

Techniques of Water-Resources Investigations of the United States Geological Survey

Chapter B3

TYPE CURVES FOR SELECTED PROBLEMS OF FLOW TO WELLS IN CONFINED AQUIFERS

By J. E. Reed

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CONTENTS

	Page		Page
Abstract	1	Summaries of type-curve solutions, etc.—Continued	
Introduction	1	Solution 6: Constant discharge from a partially penetrating well in a leaky aquifer	29
Summaries of type-curve solutions for confined ground-water flow toward a well in an infinite aquifer ..	5	Solution 7: Constant drawdown in a well in a leaky aquifer	34
Solution 1: Constant discharge from a fully penetrating well in a nonleaky aquifer (Theis equation)	5	Solution 8: Constant discharge from a fully penetrating well of finite diameter in a nonleaky aquifer	37
Solution 2: Constant discharge from a partially penetrating well in a nonleaky aquifer	8	Solution 9: Slug test for a finite diameter well in a nonleaky aquifer	45
Solution 3: Constant drawdown in a well in a nonleaky aquifer	13	Solution 10: Constant discharge from a fully penetrating well in an aquifer that is anisotropic in the horizontal plane	46
Solution 4: Constant discharge from a fully penetrating well in a leaky aquifer	18	Solution 11: Variable discharge from a fully penetrating well in a leaky aquifer	49
Solution 5: Constant discharge from a well in a leaky aquifer with storage of water in the confining beds	25	References	52

ILLUSTRATIONS

		Page
PLATE	1. Type-curve solutions for confined ground-water flow toward a well in an infinite aquifer	In pocket
FIGURE	0.1. Graph showing the relation of $1/u$, $W(u)$ type curve and t,s data plot	2
	0.2. Graph showing the application of the principle of superposition to aquifer tests	3
	1.1. Cross section through a discharging well in a nonleaky aquifer	6
	1.2. Graph showing type curve of dimensionless drawdown ($W(u)$) versus dimensionless time ($1/u$) for constant discharge from an artesian well (Theis curve)	Plate 1
	2.1. Cross section through a discharging well that is screened in a part of a nonleaky aquifer	9
	2.2. Graph showing four selected type curves of dimensionless drawdown ($W(u)+f$) versus dimensionless time ($1/u$) for constant discharge from a partially penetrating artesian well	Plate 1
	2.3. Graph of the drawdown correction factor f_c versus ar/b	10
	2.4. Example of output from program for partial penetration in a nonleaky artesian aquifer	17
	3.1. Cross section through a well with constant drawdown in a nonleaky aquifer	18
	3.2. Graph showing type curve of dimensionless discharge ($G(\alpha)$) versus dimensionless time (α) for constant drawdown in an artesian well	Plate 1
	3.3. Graph showing type curves of dimensionless drawdown ($A(\tau,\rho)$) versus dimensionless time (τ/ρ^2) for constant drawdown in a well in a nonleaky aquifer	Plate 1
	4.1. Cross section through a discharging well in a leaky aquifer	21
	4.2. Graph showing type curve of $L(u,v)$ versus $1/u$	Plate 1
	4.3. Graph showing type curve of the Bessel function $K_0(x)$ versus x	Plate 1
	4.4. Example of output from program for computing drawdown due to constant discharge from a well in a leaky artesian aquifer	24
	5.1. Cross sections through discharging wells in leaky aquifers with storage of water in the confining beds, illustrating three different cases of boundary conditions	27
	5.2. Graph showing dimensionless drawdown ($H(u, \beta)$) versus dimensionless time ($1/u$) for a well fully penetrating a leaky artesian aquifer with storage of water in leaky confining beds ..	Plate 1

	Page
FIGURE 5.3. Example of output from program for computing drawdown due to constant discharge from a well in a leaky aquifer with storage of water in the confining beds	30
6.1. Cross section through a discharging well that is screened in part of a leaky aquifer	32
6.2. Eight selected type curves of dimensionless drawdown ($W(u, \beta) + f$) versus dimensionless time ($1/u$) for constant discharge from a partially penetrating well in a leaky artesian aquifer	Plate 1
6.3. Example of output from program for partial penetration in a leaky artesian aquifer	33
7.1. Cross section through a well with constant drawdown in a leaky aquifer	35
7.2. Graph showing type curve of dimensionless discharge ($G(\alpha, r_w/B)$) versus dimensionless time (α) for constant drawdown in a well in a leaky aquifer	Plate 1
7.3. Graph showing ten selected type curves of dimensionless drawdown versus dimensionless time for constant drawdown in a well in a leaky aquifer	Plate 1
7.4. Example of output from program for constant drawdown in a well in a leaky artesian aquifer	38
8.1. Cross section through a discharging well of finite diameter	39
8.2. Graph showing five selected type curves of $F(u, \alpha)$, and the Theis solution, versus $1/u$	40
8.3. Graph showing eight selected type curves of $F(u, \alpha, \rho)$ for $\alpha = 10^{-4}$, and the Theis solution, versus $1/u$	Plate 1
8.4. Example of output from program for drawdown inside a well of finite diameter due to constant discharge	44
8.5. Example of output from program for drawdown outside a well of finite diameter due to constant discharge	44
9.1. Cross section through a well in which a slug of water is suddenly injected	46
9.2. Graph showing ten selected type curves of $F(\beta, \alpha)$ versus β	Plate 1
9.3. Example of output from program to compute change in water level due to sudden injection of a slug of water into a well	48
10.1. Plan view showing coordinate axes	49
11.1. Cross section through a well with variable discharge	51
11.2. Graph showing eleven selected type curves for linearly varying discharge, $Q(t) = Q_0(1 + ct)$	Plate 1
11.3. Example of output from program to compute the convolution integral for a leaky aquifer	53

TABLES

	Page
TABLE 1.1. Values of Theis equation $W(u)$ for values of $1/u$	7
2.1. Listing of program for partial penetration in a nonleaky artesian aquifer	57
3.1. Values of $G(\alpha)$	19
3.2. Values of $A(\tau, \rho)$	20
4.1. Selected values of $W(u, r/B)$	22
4.2. Selected values of $K_0(x)$	23
4.3. Listing of program for radial flow in a leaky artesian aquifer	66
5.1. Values of $H(u, \beta)$ for selected values of u and β	28
5.2. Listing of program for radial flow in a leaky artesian aquifer with storage of water in the confining beds	72
6.1. Listing of program for partial penetration in a leaky artesian aquifer	75
7.1. Values of $G(\alpha, r_w/B)$	36
7.2. Listing of program for constant drawdown in a well in an infinite leaky aquifer	83
8.1. Values of the function $F(u, \alpha, \rho)$	41
8.2. Listing of programs for constant discharge from a fully penetrating well of finite diameter	91
9.1. Values of H/H_0	47
9.2. Listing of program to compute change in water level due to sudden injection of a slug of water into a well	99
11.1. Listing of program to compute the convolution integral for a leaky aquifer	102

SYMBOLS AND DIMENSIONS

[Numbers in parentheses indicate the solutions to which the definition applies. If no number appears, the symbol has only one definition in this report]

<i>Symbol</i>	<i>Dimension</i>	<i>Description</i>
a	Dimensionless	$\sqrt{K_z/K_r}$.
b	L	Aquifer thickness.
b'	L	Thickness of confining bed (4, 6, 7, 11); specifically the upper confining bed (5).
b''	L	Thickness of lower confining bed.
d	L	Depth from top of aquifer to top of pumped well screen.
d'	L	Depth from top of aquifer to top of observation-well screen.
H	L	Change in water level in well.
H_0	L	Initial head increase in well.
h	L	Change in water level in aquifer.
K	LT^{-1}	Hydraulic conductivity of aquifer.
K_r	LT^{-1}	Hydraulic conductivity of the aquifer in the radial direction.
K_z	LT^{-1}	Hydraulic conductivity of the aquifer in the vertical direction.
K'	LT^{-1}	Hydraulic conductivity of confining bed (4, 6, 7); specifically the upper confining bed (5).
K''	LT^{-1}	Hydraulic conductivity of lower confining bed.
l	L	Depth from top of aquifer to bottom of pumped well screen.
l'	L	Depth from top of aquifer to bottom of observation-well screen.
Q	L^3T^{-1}	Discharge rate.
$Q(t)$	L^3T^{-1}	Discharge rate.
r	L	Radial distance from center of pumping, flowing, or injecting well.
r_c	L	Radius of well casing or open hole in the interval where the water level changes.
r_u	L	Effective radius of well screen or open hole for pumping, flowing, or injecting well.
S	Dimensionless	Storage coefficient.
S_s	L^{-1}	Specific storage of aquifer.
S'_s	L^{-1}	Specific storage of confining beds.
S'	Dimensionless	Storage coefficient of upper-confining bed.
S''	Dimensionless	Storage coefficient of lower confining bed.
s	L	Drawdown in head (change in water level).
s_1	L	Drawdown in upper confining bed.
s_2	L	Drawdown in lower confining bed.
s_{ic}	L	Constant drawdown in discharging well.
T	L^2T^{-1}	Transmissivity.
T_{xx}, T_{yy}, T_{zz}	L^2T^{-1}	Components of the transmissivity tensor in any orthogonal x-, y-axis system.
$T_{\epsilon\epsilon}, T_{\eta\eta}$	L^2T^{-1}	Transmissivities along two principal axes, ϵ and η , such that $T_{\epsilon\eta}=0$.
t	T	Time.
t'	Dimensionless	Variable of integration.
u	Dimensionless	$r^2S/4Tt$ (2, 6); variable of integration (3, 7, 9).
v	Dimensionless	Variable of integration.
x	Dimensionless	Dummy variable (2, 5); variable of integration (3).
x, y	L	Distances from the pumped well for an arbitrary rectangular coordinate system (10).
y	Dimensionless	Variable of integration (1, 2, 4, 5, 6).
z	L	Depth from top of aquifer, also, specifically, the depth to bottom of a piezometer (2, 6); depth below top of upper confining bed (5).
z	Dimensionless	Dummy variable (10).
α	Dimensionless	T/Sr_{ic}^2 .
β	Dimensionless	Variable of integration.
Θ	Dimensionless	Angle between x axis and ϵ axis.
ϵ, η	L	Distances from pumped well in a coordinate system colinear with principal axes of transmissivity tensor.
ρ	Dimensionless	r/r_u .
τ	Dimensionless	T/Sr_{ic}^2 .

TYPE CURVES FOR SELECTED PROBLEMS OF FLOW TO WELLS IN CONFINED AQUIFERS

By J. E. Reed

Abstract

This report presents type curves and related material for 11 conditions of flow to wells in confined aquifers. These solutions, compiled from hydrologic literature, span an interval of time from Theis (1935) to Papadopoulos, Bredehoeft, and Cooper (1973). Solutions are presented for constant discharge, constant drawdown, and variable discharge for pumping wells that fully penetrate leaky and nonleaky aquifers. Solutions for wells that partially penetrate leaky and nonleaky aquifers are included. Also, solutions are included for the effect of finite well radius and the sudden injection of a volume of water for nonleaky aquifers. Each problem includes the partial differential equation, boundary and initial conditions, and solutions. Programs in FORTRAN for calculating additional function values are included for most of the solutions.

Introduction

The purpose of this report is to assemble, under one cover and in a standard format, the more commonly used type-curve solutions for confined ground-water flow toward a well in an infinite aquifer. Some of these solutions are only published in several different journals; some of these journals are not readily obtainable. Other solutions which are included in several references (for example, Ferris and others, 1962; Walton, 1962; Hantush, 1964a; Lohman, 1972) are included here for completeness.

The need for a compendium of type curves for aquifer-test analysis was recognized by Robert W. Stallman, who initiated the work on it. However, ill health and the press of other duties prevented him from personally carrying out his concept, but he never ceased to advocate the need for the compendium. Although it is reduced in scope from his original concept, this

report should be recognized to be a result of Stallman's foresight and endeavors in the field of ground-water hydrology.

The type-curve method was devised by C. V. Theis (Wenzel, 1942, p. 88) to determine the two unknown parameters, S and T , in the equations

$$s = (Q/4\pi T)W(u)$$

and

$$u = r^2 S / (4Tt),$$

where s is the drawdown in water level in response to the pumping rate Q in an aquifer with transmissivity T and storage coefficient S . The distance r from the pumping well, and the elapsed time t since pumping began, combine with S and T to define a dimensionless variable u and corresponding dimensionless response $W(u)$. Briefly, the method consists of plotting a function curve or type curve, such as $(1/u, W(u))$ on logarithmic-scale graph paper, and plotting the time-drawdown ($t-s$) data on a second sheet having the same scales. This is equivalent to expressing the preceding equations as

$$\log s = \log Q/4\pi T + \log W(u)$$

and

$$\log 1/u = \log t + \log 4T/r^2 S.$$

If the two sheets are superimposed and matched, keeping coordinate axes parallel, as shown in figure 0.1, the respective coordinate

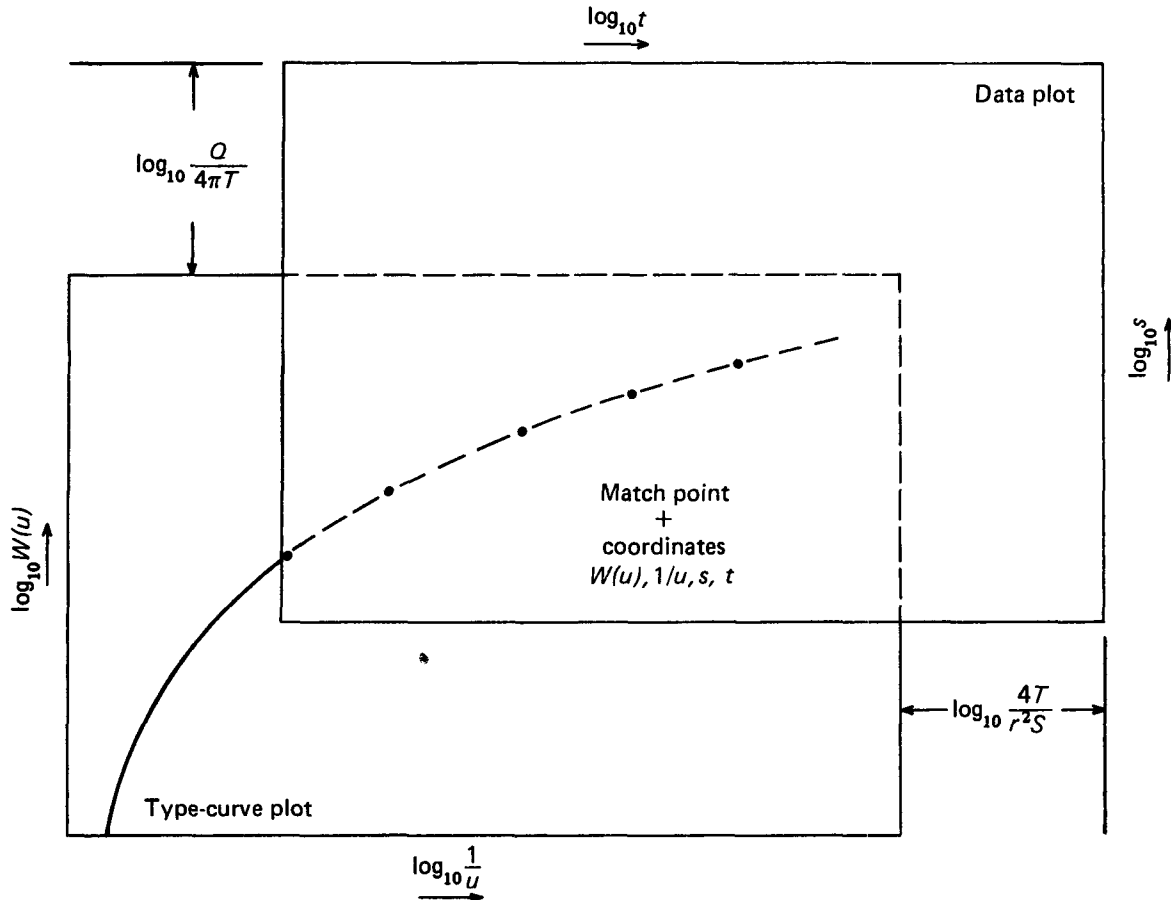


FIGURE 0.1.—Relation of $1/u, W(u)$ type curve and t, s data plot. Modified from Stallman (1971, p. 5, fig. 1).

axes will be related by constant factors: $s/W(u) = C_1$ and $t/(1/u) = C_2$. The values of these two constants are

$$C_1 = Q/(4\pi T)$$

and

$$C_2 = r^2 S/(4T).$$

Thus, a common match point for the two curves may be chosen, and the four coordinate points— $W(u)$, $1/u$, s , and t —recorded for the common match point. T can be obtained from the equation $T = QW(u)/(4\pi s)$, and then S can be solved from the equation $S = 4Tut/r^2$, where $W(u)$, $1/u$, s , and t are the match-point values.

It is apparent that the type curves, and data, can be plotted in several ways. That is, the function curve, using $W(u)$ as an example, could be plotted as $(u, W(u))$ with corresponding

data plots of $(1/t, s)$ or $(r^2/t, s)$; or could be plotted as $(1/u, W(u))$ with corresponding data plots of (t, s) or $(t/r^2, s)$. The type-curve method is covered more fully by Ferris, Knowles, Brown, and Stallman (1962, p. 94).

The type curves presented in this report are shown on two different plots. One plot has both logarithmic scales with 1.85 inches per log-cycle, such as K and E 467522.¹ The other plot is arithmetic-logarithmic scale with the logarithmic scale 2 inches per log-cycle and the arithmetic scale with divisions at multiples of 0.1, 0.5, and 1.0 inches, such as K and E 466213.

Other methods exist for analysis of aquifer-test data. Among them are methods based on plots of data on semi-log paper, developed by

¹The use of brand names in this report is for identification purposes only and does not imply endorsement by the U S Geological Survey

Jacob (Ferris and others, 1962, p. 98) and by Hantush (1956, p. 703). These methods are useful, but they are beyond the scope of this report.

Aquifer tests deal with only one component of the natural flow system. The isolation of the effects of one stress upon the system is based upon the technique of superposition. This technique requires that the natural flow system can be approximated as a linear system, one in which total flow is the addition of the individual flow components resulting from distinct stresses.

The use of the principle of superposition is implied in most aquifer-test analyses. The term "superposition," as here applied, is derived from the theory of linear differential equations. If the partial-differential equation is linear (in the dependent variable and its derivatives), two or more solutions, each for a given set of boundary and initial conditions, can be summed algebraically to obtain a solution for the combined conditions. For instance, consider a situation (fig. 0.2) where a well has been pumping for some time at a constant rate Q_0 , and the drawdown trend for that pumping rate has been established. Assume that the pumping rate increases by some amount ΔQ at

some time t_1 . Then the drawdown for that step increase in rate will be the change in drawdown from that occurring due to the pumpage Q_0 .

Programs, written in FORTRAN, for calculating additional function values are included for most of the solutions. Some of the type-curve solutions would require an unreasonably long tabulation to include all the possible combinations of parameters. An alternative to a tabulation is the computer program that can calculate type-curve values for the parameters desired by the user. The programs could be easily modified to calculate aquifer response to more than one well, such as well fields or image-well systems (Ferris and others, 1962, p. 144). The programs have been tested and are probably reasonably free from error. However, because of the large number of possible parameter combinations, it was possible to test only a sample of possible parameter values. Therefore, errors might occur in future use of these programs.

"An aquifer test is a controlled field experiment made to determine the hydraulic properties of water-bearing and associated rocks" (Stallman, 1971). The areal variability of hydraulic properties in an aquifer limits aquifer tests to integrating these properties within the

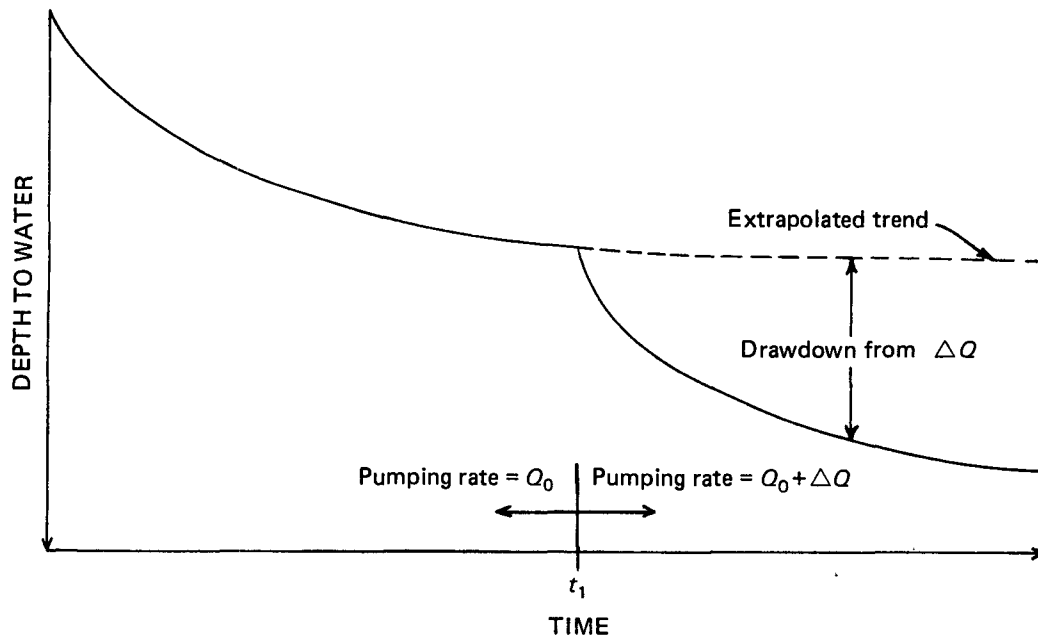


FIGURE 0.2.—The application of the principle of superposition to aquifer tests.

cone of depression produced during the test. Aquifer-test solutions are based on idealized representations of the aquifer, its boundaries, and the nature of the stress on the aquifer. The type-curve solutions presented in this report all have certain assumptions in common. The common assumptions are that the aquifer is horizontal and infinite in areal extent, that water is confined by less permeable beds above and below the aquifer, that the formation parameters are uniform in space and constant in time, that flow is laminar, and that water is released from storage instantaneously with a decline in head. Also implicit is the assumption that hydraulic potential or head is the only cause of flow in the system and that thermal, chemical, density, or other forces are not affecting flow. In addition to these common assumptions are special assumptions that characterize each solution summary. An important first step in aquifer-test analysis is deciding which simplified representations most closely match the usually complex field conditions.

Generally the best start in the analysis of aquifer-test data is with the most general set of type curves that apply to the situation, keeping in mind limitations of the method and effects that cause departures from the theoretical results. For example, the most general set of type curves for constant discharge presented in this report is for leaky aquifers with storage of water in the confining beds, *solution 5*. This includes, as a limiting case, the curve for a nonleaky aquifer. The most severe limitation on this set of curves is that they apply only at early times, as specified in *solution 5*.

Some of the effects that cause departure from the theoretical curves are partial penetration, finite well radius, and variable discharge for the pumped well. The effects of partial penetration must be considered when $r/b < 1.5$, and because vertical-horizontal anisotropy is probably a common condition, these effects should be considered for $r/b < 10$. The effect of finite well radius should be considered for early times, as specified in *solution 8*. The effects of variable discharge depend upon the manner of the variation. A change in discharge is more important if the change is monotonic, either continually increasing or decreasing. This fact is shown by the type curves for *solution 11*,

where a monotonic change of 10 percent caused a significant departure from the Theis curve. If the discharge variation consists of random "noise" about a constant discharge, a 10-percent variation is not significant. The most general set of type curves for tests on flowing wells is *solution 7*, for leaky aquifers, which includes nonleaky aquifers as a limiting case. The only set of curves for slug tests is given in *solution 9*.

A recurring problem in type-curve solution for unknown hydrologic parameters is that of nonuniqueness. That is, function curves for different parameter values sometimes have similar shapes. An example of this is given by Stallman (1971, p. 19 and fig. 6). He indicated that the selection of the conceptual model is very important in interpreting the test results. Equally important is adequate testing of the conceptual model. Corroboration of the conceptual model is indicated by similar results for hydrologic parameters from data collected at varying distances from the pumped well, depths within the aquifer, and at different observation times. However, proof of suitability of the conceptual model ultimately rests on field investigations and not on curve matching.

As an example of similar curve shapes for different situations, consider the case of constant discharge in a nonleaky aquifer with exponentially varying thickness. The thickness, b , is equal to $b_0 \exp[-2(X - X_0)/a]$, where b_0 and X_0 are the thickness and X -coordinate, respectively, at the site of the discharging well and a is a parameter. The drawdown for this situation is given by Hantush (1962, p. 1529):

$$s = (Q/4\pi K b_0) \exp(r/a \cos \Theta) W(u, r/a),$$

where

$$W(u, \beta) = \int_u^\infty (\exp(-y - \beta^2/4y)/y) dy,$$

$$u = r^2 S_s / 4Kt,$$

Q is the discharge, r is the distance from the discharging well, Θ is the angle, with apex at the discharging well, between the observation

well and the positive X -axis, K is the hydraulic conductivity of the aquifer, and S_s is the specific storage coefficient of the aquifer. This solution is similar to the equation describing drawdown in a leaky artesian aquifer (Hantush, 1956, p. 702), which is

$$s = (Q/4\pi T) W(u, r/B),$$

with $T = Kb$, $B = \sqrt{Tb'/K'}$, and b' and K' are the thickness and hydraulic conductivity, respectively, of the leaky confining bed. The other symbols are used as above.

These two functions have the same shape when plotted on logarithmic paper, and drawdown resulting from one function could be matched to a type curve of the other function. Suppose, as an example, that the "observed data" are described by the function for the aquifer with exponentially changing thickness. Suppose, also, that the hydrologist is unaware of the variation in thickness and that the family of type curves for leaky aquifers without storage in the confining beds, *solution 4*, has been chosen for analysis of the "observed data." Matching the data plots to the type curves and solving for unknown parameters by the methods suggested in *solution 4* gives for the ratio of K_a , the apparent hydraulic conductivity, to K , the true hydraulic conductivity, $K_a/K = \exp((r/a) \cos \Theta)$. The ratio would be close to one only in the vicinity of the discharging well. The diffusivity, K/S_s , would be determined correctly, but the apparent specific storage coefficient would have the same percentage error as the apparent hydraulic conductivity. Most important of all, the erroneous conclusion would be that the aquifer is leaky, with leakage parameter $B = \sqrt{Kb'b'/K'} = a$. This somewhat contrived example illustrates a principle in the interpretation of aquifer-test data. Conclusions about the hydrologic constraints on the response of the aquifer to pumping should not be based on the shape of the data curves. Inferences may be made from these curves, but they must be verified by other hydrologic and geologic data. Therefore, proof of the suitability of the conceptual model must come from field investigations.

Many of the old reports of the U.S. Geological Survey contain references to the terms "coeffi-

cient of transmissibility" and "field coefficient of permeability." These terms, which were expressed in inconsistent units of gallons and feet, have been replaced by transmissivity and hydraulic conductivity (Lohman and others, 1972, p. 4 and p. 13). Transmissivity and hydraulic conductivity are not solely properties of the porous medium; they are also determined by the kinematic viscosity of the liquid, which is a function of temperature. Field determinations of transmissivity or hydraulic conductivity are made at prevailing field temperatures, and no corrections for temperature are made.

Summaries of Type-Curve Solutions for Confined Ground-Water Flow Toward a Well in an Infinite Aquifer

Solution 1: Constant discharge from a fully penetrating well in a nonleaky aquifer (Theis equation)

Assumptions:

1. Well discharges at a constant rate, Q .
2. Well is of infinitesimal diameter and fully penetrates the aquifer.
3. Aquifer is not leaky.
4. Discharge from the well is derived exclusively from storage in the aquifer.

Differential equation:

$$\partial^2 s / \partial r^2 + (1/r) (\partial s / \partial r) = (S/T) (\partial s / \partial t)$$

Boundary and initial conditions:

$$s(r, 0) = 0, r \geq 0 \quad (1)$$

$$s(\infty, t) = 0, t \geq 0 \quad (2)$$

$$Q = \begin{cases} 0, t < 0 \\ \text{constant} > 0, t \geq 0 \end{cases} \quad (3)$$

$$\lim_{r \rightarrow 0} r \frac{\partial s}{\partial r} = -\frac{Q}{2\pi T}, t \geq 0 \quad (4)$$

Equation 1 states that initially drawdown is zero everywhere in the aquifer. Equation 2

states that the drawdown approaches zero as the distance from the well approaches infinity. Equation 3 states that the discharge from the well is constant throughout the pumping period. Equation 4 states that near the pumping well the flow toward the well is equal to its discharge.

Solution (Theis, 1935):

$$s = \frac{Q}{4\pi T} \int_u^\infty \frac{e^{-y}}{y} dy$$

$$u = \frac{r^2 S}{4Tt},$$

where

$$\int_u^\infty \frac{e^{-y}}{y} dy = W(u) = -0.577216 - \log_e u + u - \frac{u^2}{2! \cdot 2} + \frac{u^3}{3! \cdot 3} - \frac{u^4}{4! \cdot 4} + \dots$$

Comments:

Assumptions made are applicable to artesian aquifers (fig. 1.1). However, the solution may be applied to unconfined aquifers if drawdown is small compared with the saturated thickness

of the aquifer and if water in the sediments through which the water table has fallen is discharged instantaneously with the fall of the water table. According to assumption 2, this solution does not consider the effect of the change in storage within the pumping well. Assumption 2 is acceptable if

$$t > 2.5 \times 10^2 r_c^2 / T$$

(Papadopoulos and Cooper, 1967, p. 242), where r_c is the radius of the well casing in the interval over which the water-level declines, and other symbols are as defined previously. Figure 1.2 on plate 1 is a logarithmic graph of $W(u) = 4\pi s T / Q$ plotted on the vertical coordinates versus $1/u = 4Tt / (r^2 S)$ plotted on the horizontal coordinates. The test data should be plotted with s on the vertical coordinates and corresponding values of t or t/r^2 on the horizontal coordinates.

Values of $W(u)$ for u between 0 and 170 may be computed by using subroutine EXPI of the IBM System/360 Scientific Subroutine Package. Table 1.1 gives values of $W(u)$ for selected values of $1/u$ between 1×10^{-1} and 9×10^{14} , as calculated by this subroutine.

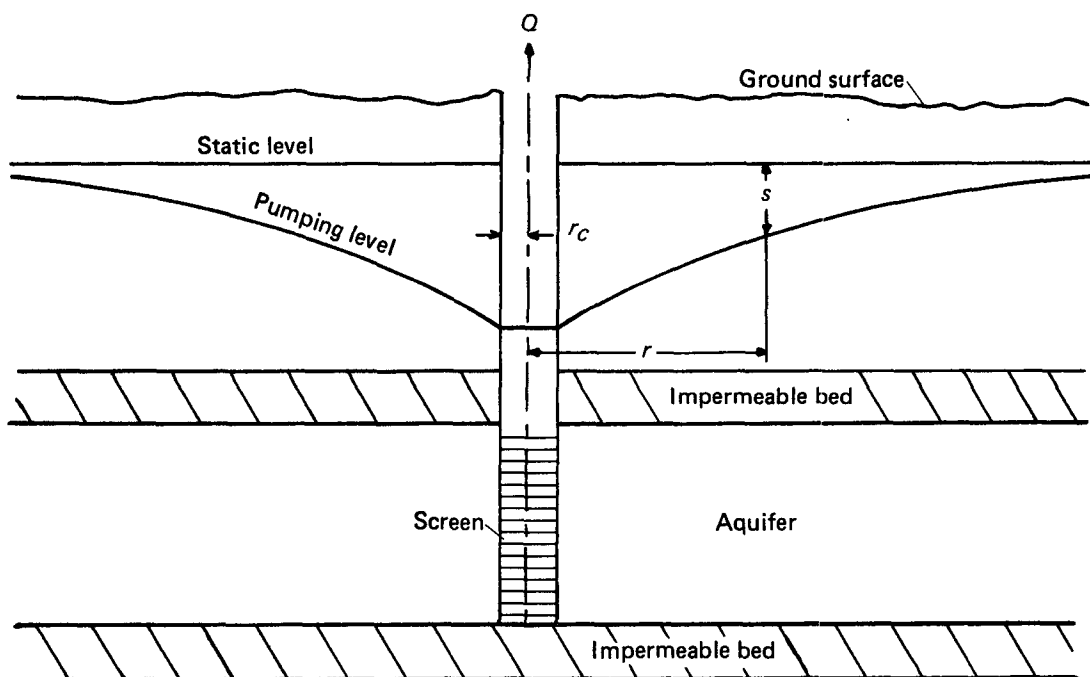


FIGURE 1.1.—Cross section through a discharging well in a nonleaky aquifer.

TABLE 1.1.—Values of Theis equation $W(u)$ for values of $1/u$

$1/u$	$1/u \times 10^{-1}$	1	10	10^2	10^3	10^4	10^5	10^6
1.0	0.00000	0.21938	1.82292	4.03793	6.33154	8.63322	10.93572	13.23830
1.2	0.0003	29255	1.98932	4.21859	6.51369	8.81553	11.11804	13.42062
1.5	0.0017	39841	2.19641	4.44007	6.73667	9.03866	11.34118	13.64376
2.0	0.0115	55977	2.46790	4.72610	7.02419	9.32632	11.62886	13.93144
2.5	0.0378	70238	2.68126	4.94824	7.24723	9.54945	11.85201	14.15459
3.0	0.0857	82889	2.85704	5.12990	7.42949	9.73177	12.03433	14.33691
3.5	0.1566	94208	3.00650	5.28357	7.58359	9.88592	12.18847	14.49106
4.0	0.2491	1.04428	3.13651	5.41675	7.71708	10.01944	12.32201	14.62459
5.0	0.4890	1.22265	3.35471	5.63939	7.94018	10.24258	12.54515	14.84773
6.0	0.7833	1.37451	3.53372	5.82138	8.12247	10.42490	12.72747	15.03006
7.0	1.1131	1.50661	3.68551	5.97529	8.27659	10.57905	12.88162	15.18421
8.0	1.4641	1.62342	3.81727	6.10865	8.41011	10.71258	13.01515	15.31774
9.0	1.8266	1.72811	3.93367	6.22629	8.52787	10.83036	13.13294	15.43551

$1/u$	$1/u \times 10^7$	10^8	10^9	10^{10}	10^{11}	10^{12}	10^{13}	10^{14}
1.0	15.54087	17.84344	20.14604	22.44862	24.75121	27.05379	29.35638	31.65897
1.2	15.72320	18.02577	20.32835	22.63094	24.93353	27.23611	29.53870	31.84128
1.5	15.94634	18.24892	20.55150	22.85408	25.15668	27.45926	29.76184	32.06442
2.0	16.23401	18.53659	20.83919	23.14177	25.44435	27.74693	30.04953	32.35211
2.5	16.45715	18.75974	21.06233	23.36491	25.66750	27.97008	30.27267	32.57526
3.0	16.63948	18.94206	21.24464	23.54723	25.84982	28.15240	30.45499	32.75757
3.5	16.79362	19.09621	21.39880	23.70139	26.00397	28.30655	30.60915	32.91173
4.0	16.92715	19.22975	21.53233	23.83492	26.13750	28.44008	30.74268	33.04526
5.0	17.15030	19.45288	21.75548	24.05806	26.36064	28.66322	30.96582	33.26840
6.0	17.33263	19.63521	21.93779	24.24039	26.54297	28.84555	31.14813	33.45071
7.0	17.48677	19.78937	22.09195	24.39453	26.69711	28.99969	31.30229	33.60487
8.0	17.62030	19.92290	22.22548	24.52806	26.83064	29.13324	31.43582	33.73840
9.0	17.73808	20.04068	22.34326	24.64584	26.94843	29.25102	31.55360	33.85619

¹Value shown as 0.00000 is nonzero but less than 0.000005.

Solution 2: Constant discharge from a partially penetrating well in a nonleaky aquifer

Assumptions:

1. Well discharges at a constant rate, Q .
2. Well is of infinitesimal diameter and is screened in only part of the aquifer.
3. Aquifer has radial-vertical anisotropy.
4. Aquifer is not leaky.
5. Discharge from the well is derived exclusively from storage in the aquifer.

Differential equation:

$$\frac{\partial^2 s}{\partial r^2} + \frac{1}{r} \frac{\partial s}{\partial r} + a^2 \frac{\partial^2 s}{\partial z^2} = \frac{S}{T} \frac{\partial s}{\partial t}$$

$$a^2 = K_z/K_r$$

This is the differential equation for nonsteady radial and vertical flow in a homogeneous confined aquifer with radial-vertical anisotropy.

Boundary and initial conditions:

$$s(r, z, 0) = 0, \quad r \geq 0, \quad 0 \leq z \leq b \quad (1)$$

$$s(\infty, z, t) = 0, \quad t \geq 0 \quad (2)$$

$$\partial s(r, 0, t) / \partial z = 0, \quad r \geq 0, \quad t \geq 0 \quad (3)$$

$$\partial s(r, b, t) / \partial z = 0, \quad r \geq 0, \quad t \geq 0 \quad (4)$$

$$\lim_{r \rightarrow 0} r \frac{\partial s}{\partial r} = \begin{cases} 0; & 0 < z < d \\ -Q/(2\pi K_r(l-d)); & d < z < l \\ 0; & l < z < b \end{cases} \quad (5)$$

Equation 1 states that initially the drawdown is zero everywhere in the aquifer. Equation 2 states that the drawdown approaches zero as the distance from the pumped well approaches infinity. Equations 3 and 4 state that there is no vertical flow at the upper and lower boundaries of the aquifer. This means that vertical head gradients in the aquifer are caused by the geometric placement of the pumping well screen, and not by leakage. Equation 5 states that near the pumping well the flow is radial, that the flow toward the well is equal to its discharge, that the discharge is distributed uniformly over the well screen, and that no radial flow occurs above and below the screen.

Solution:

I. For the drawdown in a piezometer, a solution by Hantush (1961a, p. 85, and 1964a, p. 353) is given by

$$s = \frac{Q}{4\pi T} \left[W(u) + f\left(u, \frac{ar}{b}, \frac{l}{b}, \frac{d}{b}, \frac{z}{b}\right) \right], \quad (6)$$

where

$$W(u) = \int_u^\infty \frac{e^{-y}}{y} dy$$

and

$$f\left(u, \frac{ar}{b}, \frac{l}{b}, \frac{d}{b}, \frac{z}{b}\right) = \frac{2b}{\pi(l-d)} \sum_{n=1}^{\infty} \frac{1}{n} \left(\sin \frac{n\pi l}{b} - \sin \frac{n\pi d}{b} \right) \cos \frac{n\pi z}{b} W\left(u, \frac{n\pi ar}{b}\right) \quad (7)$$

$$W(u, x) = \int_u^\infty (\exp(-y - x^2/4y)/y) dy$$

$$u = \frac{r^2 S}{4Tt}$$

$$a = \sqrt{K_z/K_r}$$

An alternate form of this solution for $a=1$ is given by Hantush (1961a, p. 85):

$$s = \frac{Qb}{8\pi T(l-d)} \left[M\left(u, \frac{l+z}{r}\right) + M\left(u, \frac{l-z}{r}\right) + f'\left(u, \frac{b}{r}, \frac{l}{r}, \frac{z}{r}\right) - M\left(u, \frac{d+z}{r}\right) - M\left(u, \frac{d-z}{r}\right) - f'\left(u, \frac{b}{r}, \frac{d}{r}, \frac{z}{r}\right) \right], \quad (8)$$

in which

$$f'\left(u, \frac{b}{r}, \frac{x}{r}, \frac{z}{r}\right) = \sum_1^{\infty} \left[M\left(u, \frac{2nb+x+z}{r}\right) - M\left(u, \frac{2nb-x-z}{r}\right) + M\left(u, \frac{2nb+x-z}{r}\right) - M\left(u, \frac{2nb-x+z}{r}\right) \right] \quad (9)$$

and

$$M(u, \beta) = \int_u^\infty \frac{e^{-y}}{y} \operatorname{erf}(\beta \sqrt{y}) dy$$

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-y^2} dy.$$

II. For the drawdown in an observation well (Hantush, 1961a, p. 90, and 1964a, p. 353),

$$s = \frac{Q}{4\pi T} \left[W(u) + \bar{f} \left(u, \frac{ar}{b}, \frac{l}{b}, \frac{d}{b}, \frac{l'}{b}, \frac{d'}{b} \right) \right], \quad (10)$$

where $W(u)$ is as defined previously and

$$\begin{aligned} \bar{f} \left(u, \frac{ar}{b}, \frac{l}{b}, \frac{d}{b}, \frac{l'}{b}, \frac{d'}{b} \right) &= \frac{2b^2}{\pi^2(l-d)(l'-d')} \\ &\cdot \sum_{n=1}^{\infty} \frac{1}{n^2} \left(\sin \frac{n\pi l}{b} - \sin \frac{n\pi d}{b} \right) \\ &\cdot \left(\sin \frac{n\pi l'}{b} - \sin \frac{n\pi d'}{b} \right) W \left(u, \frac{n\pi ar}{b} \right), \quad (11) \end{aligned}$$

where $W(u, x)$ and u are as defined previously.

Comments:

Assumptions apply to conditions shown in figure 2.1. The effects of partial penetration need to be considered for $ar/b < 1.5$. There must be a type curve for each value of ar/b , d/b , l/b , and either z/b for piezometer, or l'/b and d'/b for observation wells. Because the number of possible type curves is large, only samples of curves for selected values of the parameters are shown in figure 2.2 on plate 1.

For large values of time, that is, for $t > b^2 S / (2a^2 T)$ or $t > bS / (2K_z)$, the effects of partial penetration are constant in time, and

$$W \left(u, \frac{n\pi ar}{b} \right)$$

can be approximated by

$$2K_0 \left(\frac{n\pi ar}{b} \right)$$

(Hantush, 1961a, p. 92). $K_0(x)$ is the modified Bessel function of the second kind of order zero.

Equation 6 then becomes

$$s = \frac{Q}{4\pi T} W(u) + \delta s = \frac{Q}{4\pi T} [W(u) + f_s],$$

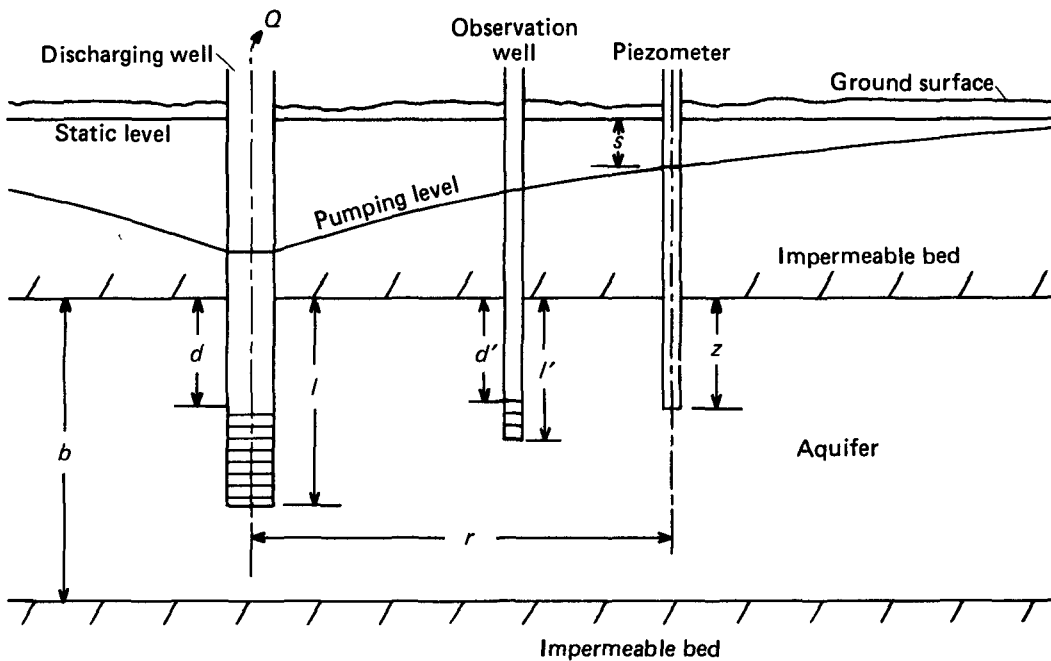


FIGURE 2.1.—Cross section through a discharging well that is screened in a part of a nonleaky aquifer.

where
$$\delta s = \frac{Q}{4\pi T} f_s,$$

and f_s is given in equation 7

with $W\left(u, \frac{n\pi ar}{b}\right)$ replaced by $2K_0\left(\frac{n\pi ar}{b}\right)$.

Figure 2.3 shows plots of f_s as tabulated by Weeks (1969, p. 202-207). In using these curves, it should be noted that f_s for a given r , b , and z_1, l_1, d_1 is equal to f_s for the same r , b , and $z_2=b-z_1, l_2=b-d_1$, and $d_2=b-l_1$. Figure 2.3 can be used to find f_s by interpolation and

then constructing type curves of $W(u)+f_s$ in the manner described by Weeks (1964, p. D195).

For small values of time

$$t < \frac{(2b-l-z)^2 S}{20T}$$

(Hantush, 1961b, p. 172), equation 8 can be approximated by

$$s = \frac{Qb}{8\pi T(l-d)} \left[M\left(u, \frac{l+z}{r}\right) - M\left(u, \frac{d+z}{r}\right) + M\left(u, \frac{l-z}{r}\right) - M\left(u, \frac{d-z}{r}\right) \right].$$

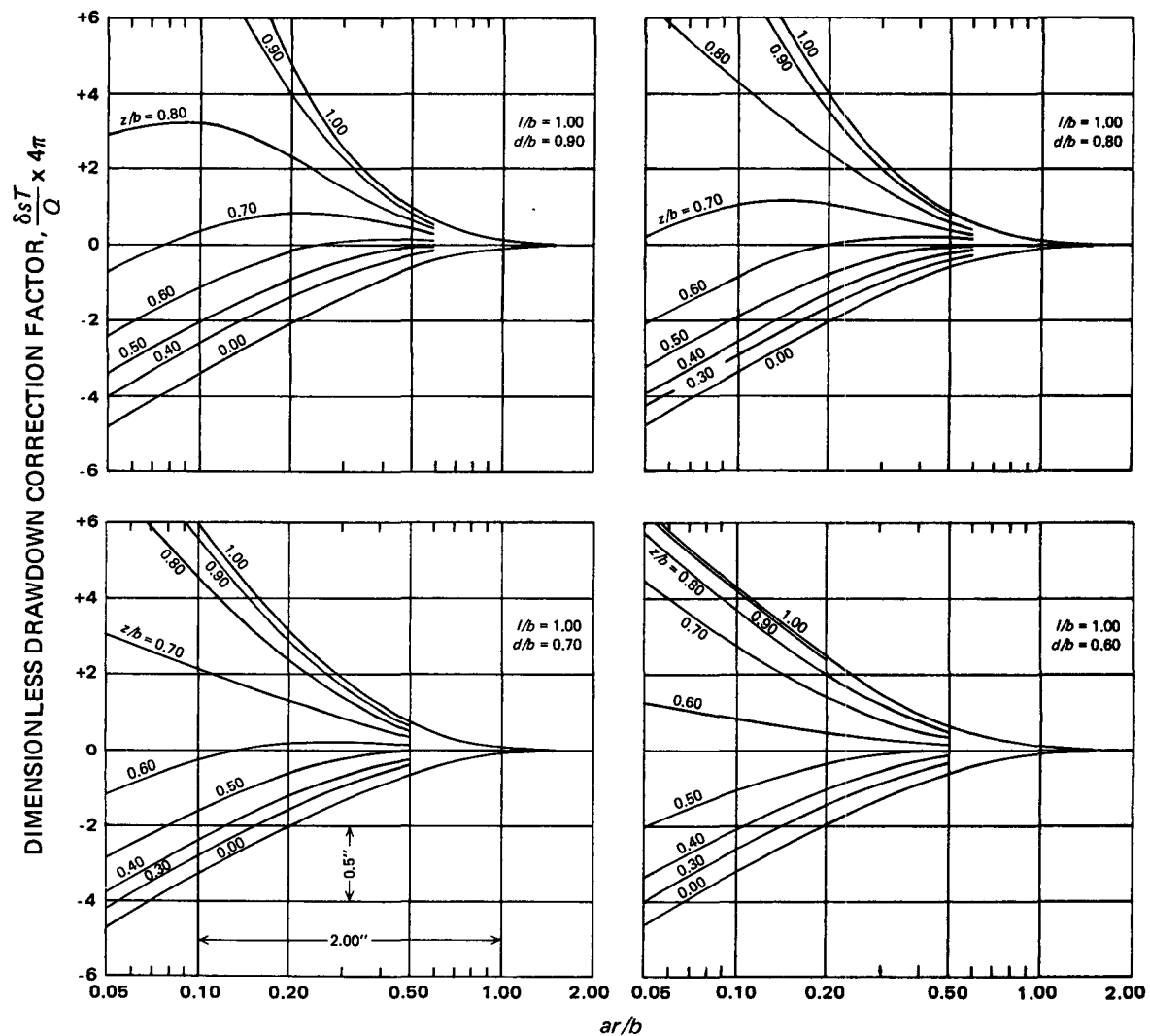


FIGURE 2.3.—The drawdown correction factor f_s versus ar/b , from tables of Weeks (1969).

An extensive table of $M(u, \beta)$ has been prepared by Hantush (1961c).

Although r/b for a given observation well probably would be known, however, the conductivity ratio a^2 would not be. Thus, it would not be known which ar/b curve should be matched. In other words, not only T and S , but also the conductivity ratio a^2 must be determined. A criterion for determining the match between data curves and type curves is that the values of ar/b for different observation wells should all indicate the same " a ". Plotting the drawdown data for several observation wells on a single t/r^2 plot and matching to sets of type

curves, a different set for each " a ", is a useful approach.

Figure 2.2 was prepared from data calculated by the FORTRAN program listed in table 2.1. This program computes " s " from either equation 6 or 10, depending on the input data. The input data consist of cards containing the parameters coded in specific formats. Readers unfamiliar with FORTRAN format items should consult a FORTRAN language manual. The first card contains: the aquifer thickness (b), coded in columns 1-5, in format F5.1; the depth to bottom of pumped well screen (l), coded in columns 6-10, in format F5.1; the

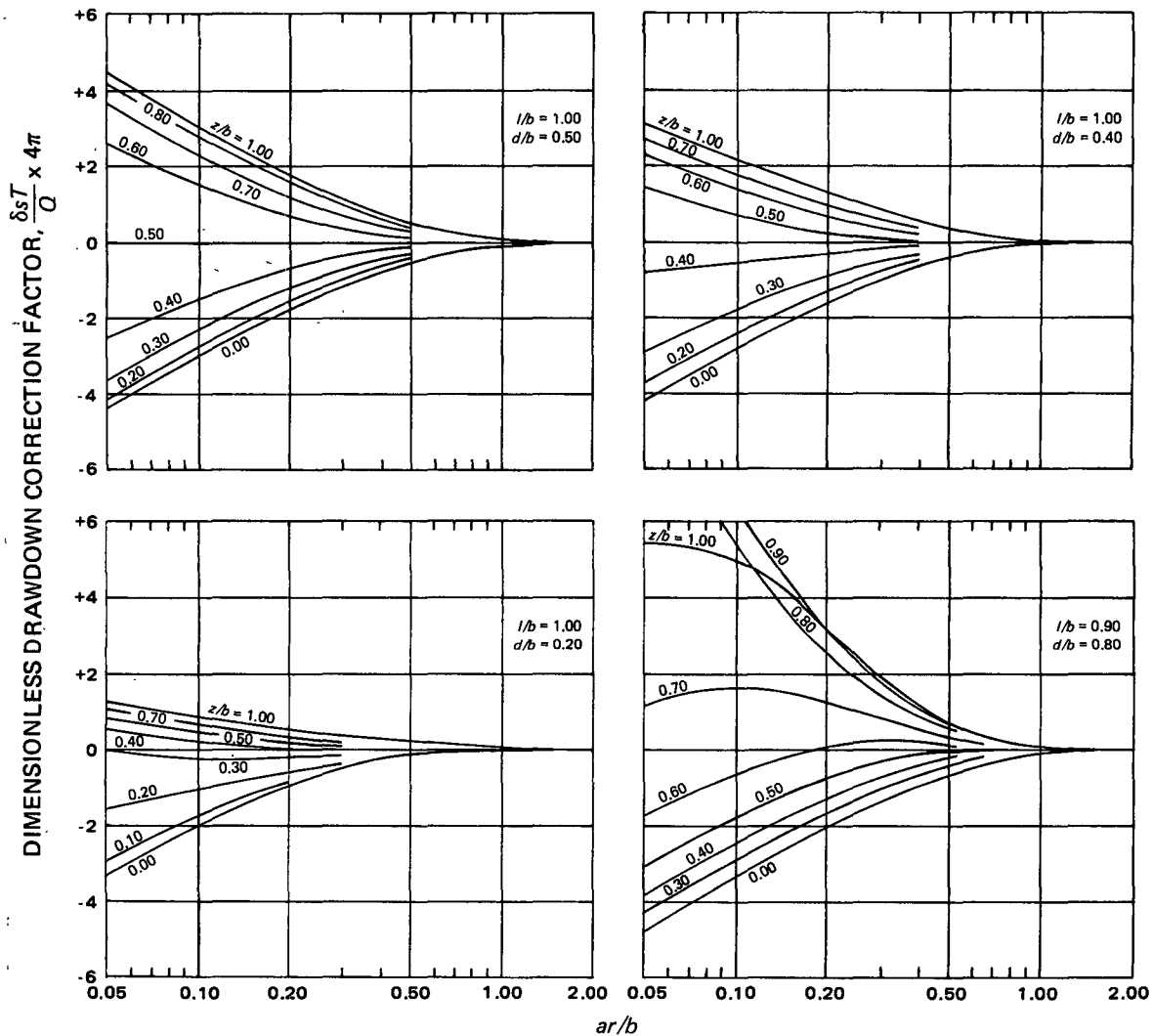


FIGURE 2.3.—Continued.

depth to top of pumped well screen (d), coded in columns 11–15, in format F5.1; the number of observation wells and (or) piezometers, coded in columns 16–20, in format I5; the smallest value of $1/u$ for which computation is desired, coded in columns 21–30, in format E10.4; the largest value of $1/u$ for which computation is desired, coded in columns 31–40, in format E10.4. The ratio of the largest $1/u$ value to the smallest $1/u$ value should be less than 10^{12} . Following this card is a group of cards containing one card for each observation well or piezometer. These cards are coded for an observation well as: distance from pumped well mul-

tiplied by the square root of the ratio of the vertical to horizontal conductivity ($r\sqrt{K_z/K_r}$), in columns 1–5, in format F5.1; depth to bottom of observation well screen (l'), coded in columns 6–10, in format F5.1; depth to top of observation well screen (d'), coded in columns 11–15, in format F5.1. A card would be coded for a piezometer as follows: distance from pumped well multiplied by the square root of the ratio of the vertical to horizontal conductivity ($r\sqrt{K_z/K_r}$), in columns 1–5, in format F5.1; and total depth of piezometer (z), in columns 11–15, in format F5.1. The output from this program is tables of computed function values,

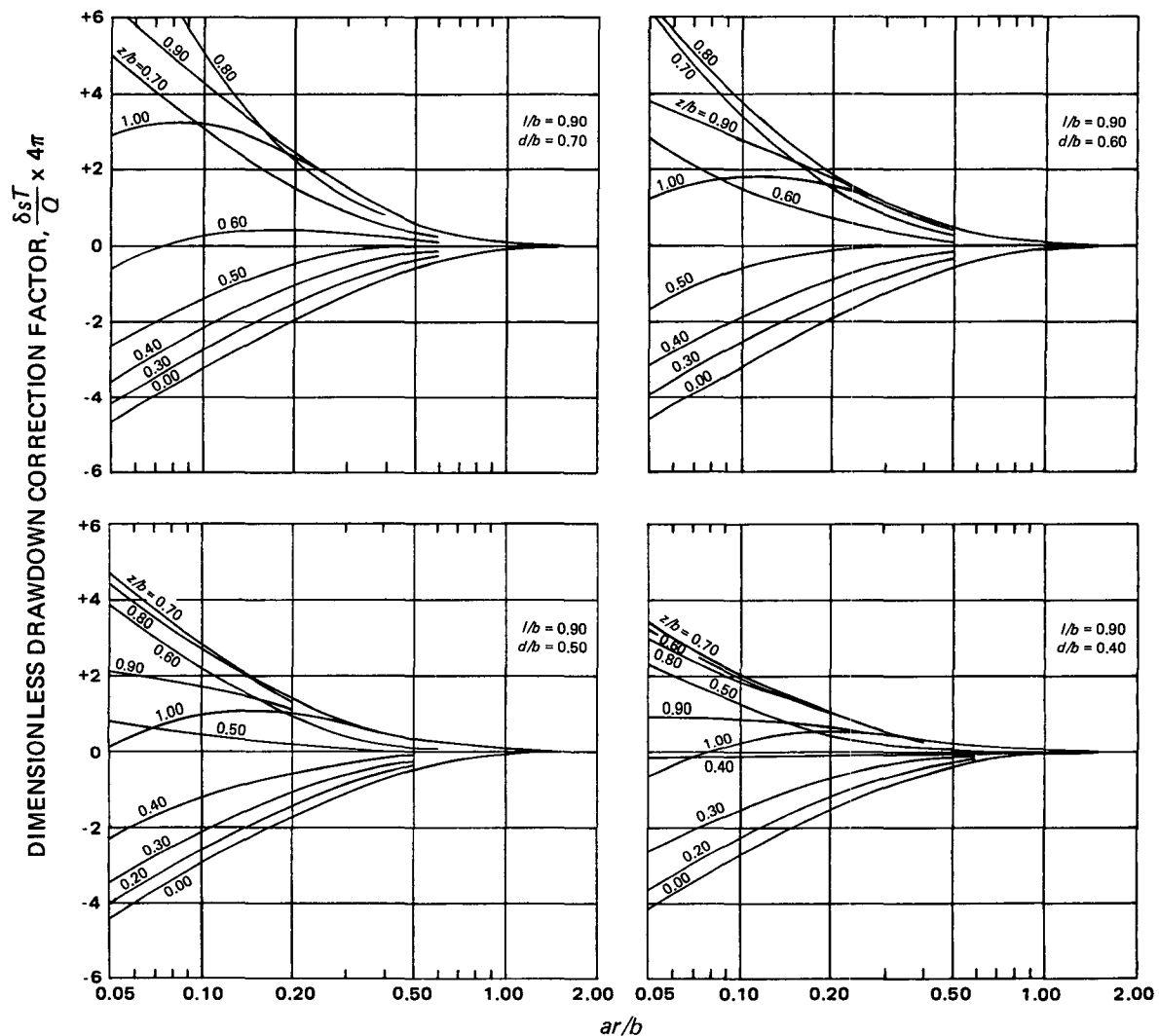


FIGURE 2.3.—Continued.

an example of which is shown in figure 2.4. Subroutines DQL12, BESK, and EXPI are from the IBM Scientific Subroutine Package and a discussion of them is in the IBM SSP manual.

Solution 3: Constant drawdown in a well in a nonleaky aquifer

Assumptions:

1. Water level in well is changed instantaneously by s_w at $t = 0$.
2. Well is of finite diameter and fully penetrates the aquifer.

3. Aquifer is not leaky.

4. Discharge from the well is derived exclusively from storage in the aquifer.

Differential equation:

$$\frac{\partial^2 s}{\partial r^2} + \frac{1}{r} \frac{\partial s}{\partial r} = \frac{S}{T} \frac{\partial s}{\partial t}$$

This is the differential equation describing nonsteady radial flow in a homogeneous isotropic confined aquifer.

Boundary and initial conditions:

$$s(r,0) = 0, r \geq r_w \tag{1}$$

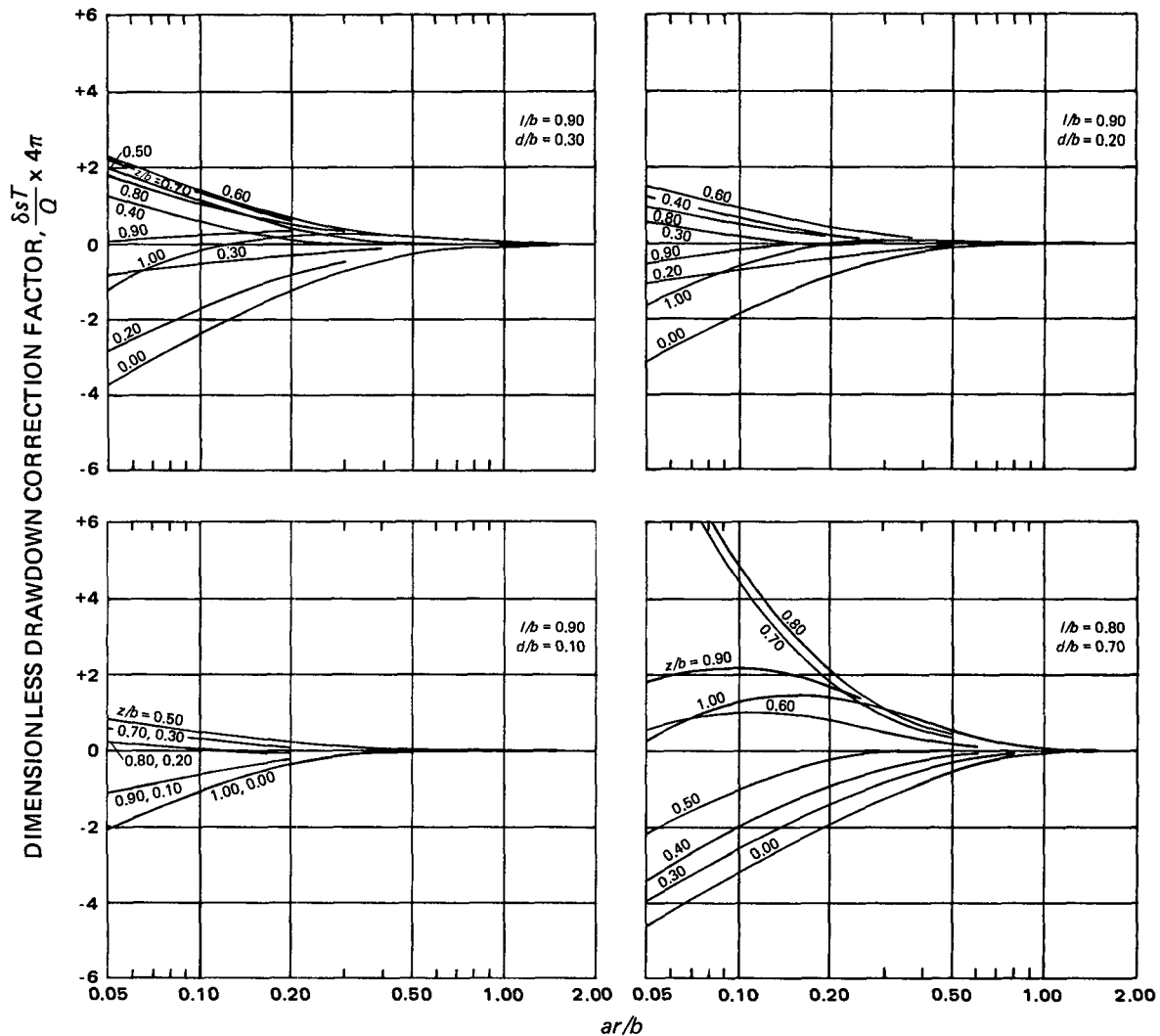


FIGURE 2.3.—Continued.

$$s(r_w, t) = \begin{cases} 0, & t < 0 \\ s_w = \text{constant}, & t \geq 0 \end{cases} \quad (2)$$

$$s(\infty, t) = 0, \quad t \geq 0 \quad (3)$$

Equation 1 states that initially the drawdown is zero everywhere in the aquifer. Equation 2 states that, as the well is approached, drawdown in the aquifer approaches the constant drawdown in the well, implying no entrance loss to the well. Equation 3 states that the drawdown approaches zero as the distance from the well approaches infinity.

Solutions:

I. For the well discharge (Jacob and Lohman, 1952, p. 560):

$$Q = 2\pi T s_w G(\alpha),$$

where

$$G(\alpha) = \frac{4\alpha}{\pi} \int_0^\infty x e^{-\alpha x^2} \left\{ \frac{\pi}{2} + \tan^{-1} \left[\frac{Y_0(x)}{J_0(x)} \right] \right\} dx$$

and

$$\alpha = \frac{Tt}{Sr_w^2}.$$

II. For the drawdown in water level (Hantush, 1964a, p. 343):

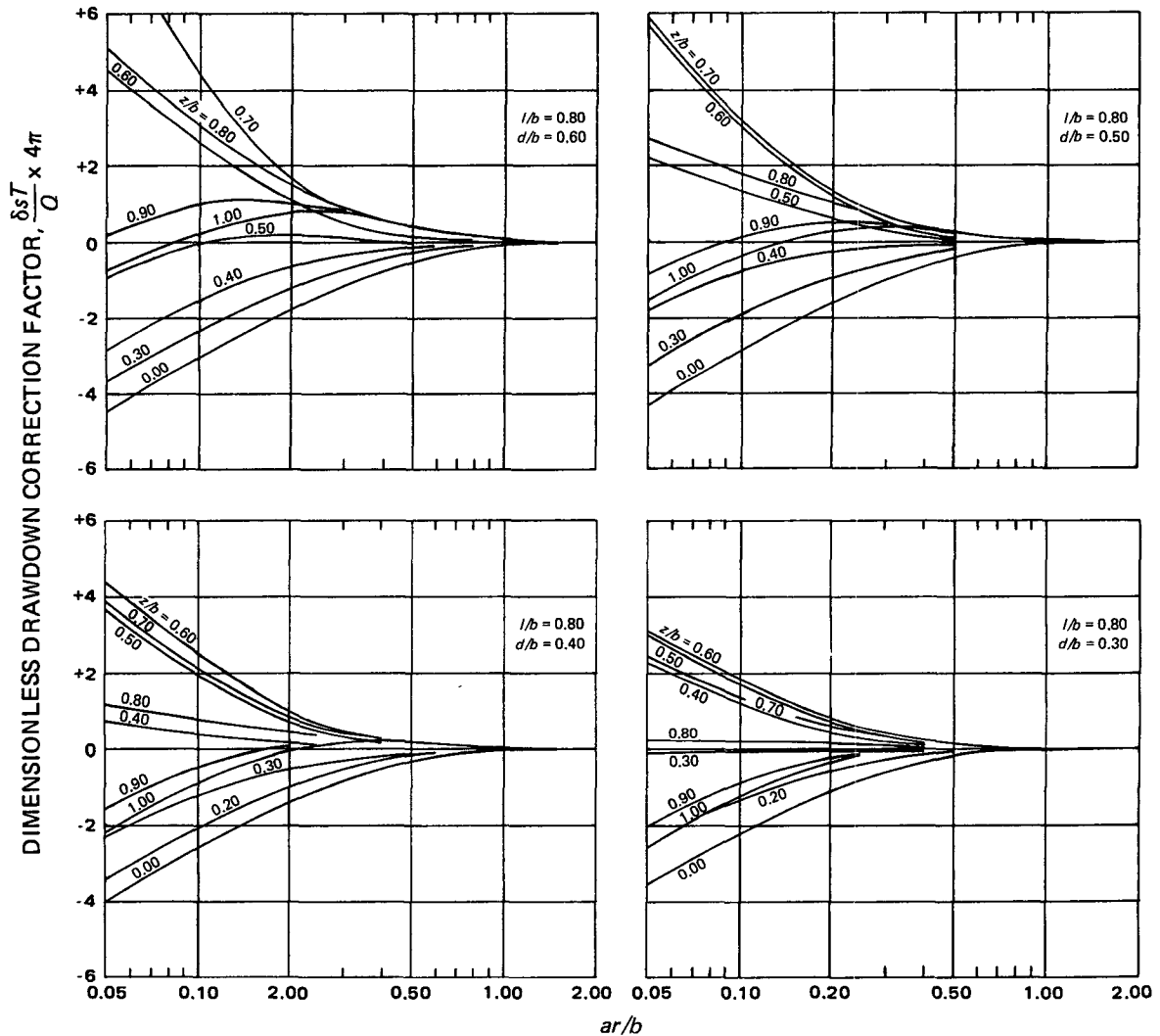


FIGURE 2.3.—Continued.

$$s = s_w A(\tau, \rho),$$

where $A(\tau, \rho) = 1$

$$-\frac{2}{\pi} \int_0^\infty \frac{J_0(u) Y_0(\rho u) - Y_0(u) J_0(\rho u)}{J_0^2(u) + Y_0^2(u)} \exp(-\tau u^2) \frac{du}{u},$$

and $\tau = \alpha = \frac{Tt}{Sr_w^2},$

$$\rho = \frac{r}{r_w}.$$

Comments:

Boundary condition 2 requires a constant drawdown in the discharging well, a condition

most commonly fulfilled by a flowing well, although figure 3.1 shows the water level to be below land surface.

Figure 3.2 on plate 1 is a plot from Lohman (1972, p. 24) of dimensionless discharge ($G(\alpha)$) versus dimensionless time (α). Additional values in the range α greater than 1×10^{12} were calculated from $G(\alpha) \approx 2/\log(2.2458\alpha)$ (Hantush, 1964a, p. 312). Function values for $G(\alpha)$ are given in table 3.1. The data curve consists of measured well discharge versus time. After the data and type curves are matched, transmissivity can be calculated from $T = Q/2\pi s_w G(\alpha)$, and the storage coefficient can be

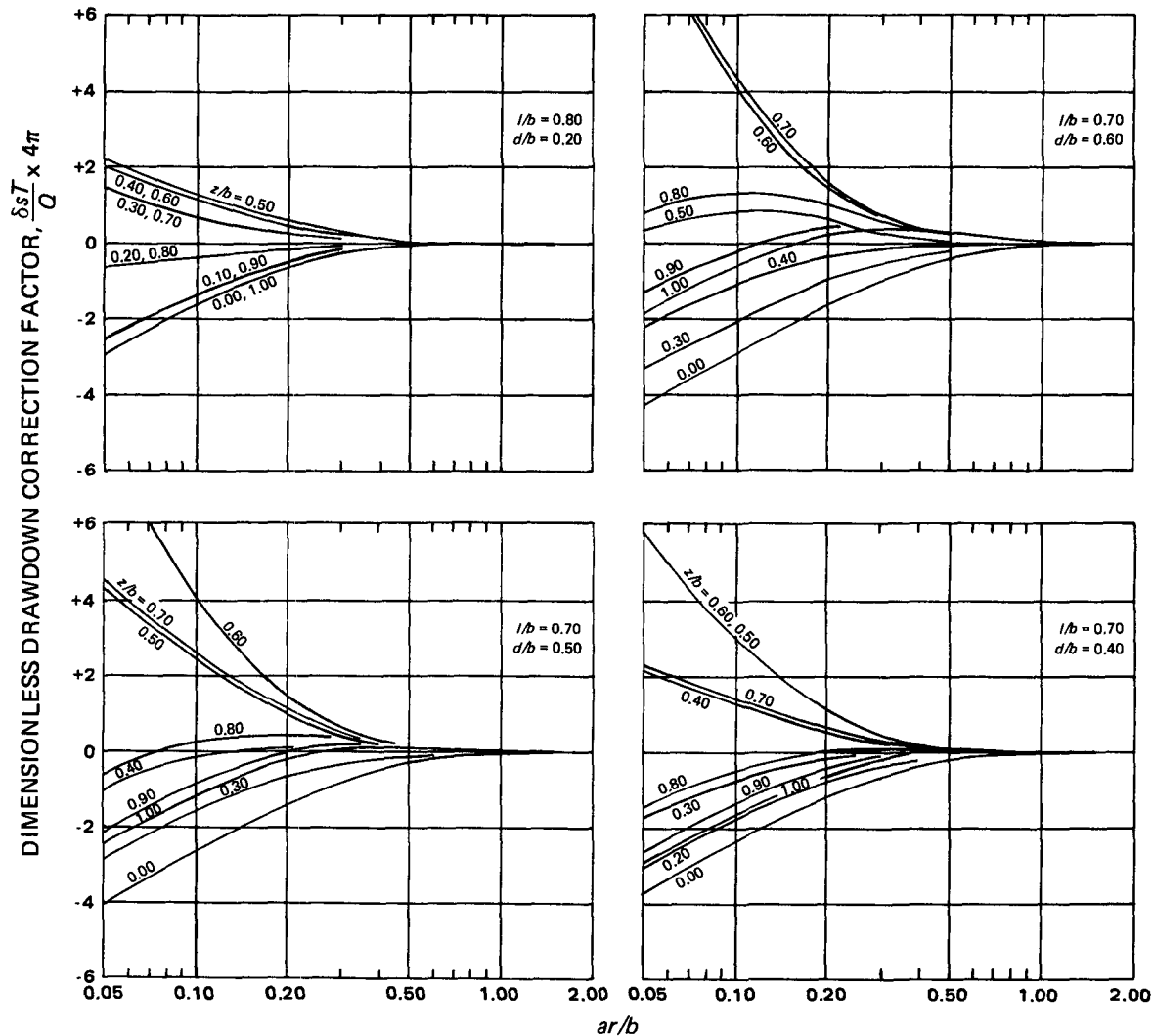


FIGURE 2.3.—Continued.

calculated from $S = Tt/\alpha r_w^2$, where $(\alpha, G(\alpha))$ and (t, Q) are matching points on the type curve and data curve, respectively.

Similarly, data curves of drawdown versus time may be matched to figure 3.3 on plate 1; this is a plot of dimensionless drawdown $(A(\tau, \rho) = s/s_w)$ versus dimensionless time $(\tau/\rho^2 = Tt/Sr^2)$. After the data and type curves are matched, the hydraulic diffusivity of the aquifer can be calculated from the equality $T/S = (\tau/\rho^2)(r^2/t)$. Usually s_w is known, and some of the uncertainty of curve matching can be eliminated by plotting s/s_w versus t because only horizontal translation is then required. If

r_w is also known, the particular curve to be matched can be determined from the relation $\rho = r/r_w$. Generally, however, the effective radius, r_w , differs from the actual radius and is not known. The effective radius can often be estimated from a knowledge of the construction of the well and the water-bearing material, or it can be determined from step-drawdown tests (Rorabaugh, 1953). Figure 3.3 was plotted from table 3.2. For $\tau \leq 1 \times 10^3$, the data are from Hantush (1964a, p. 310). For $\tau > 1 \times 10^3$, values of drawdown in a leaky aquifer, as $r_w/B \rightarrow 0$, were used. (See solution 7.) Where 0.000 occurs in table 3.2, $A(\tau, \rho)$ is less than 0.0005.

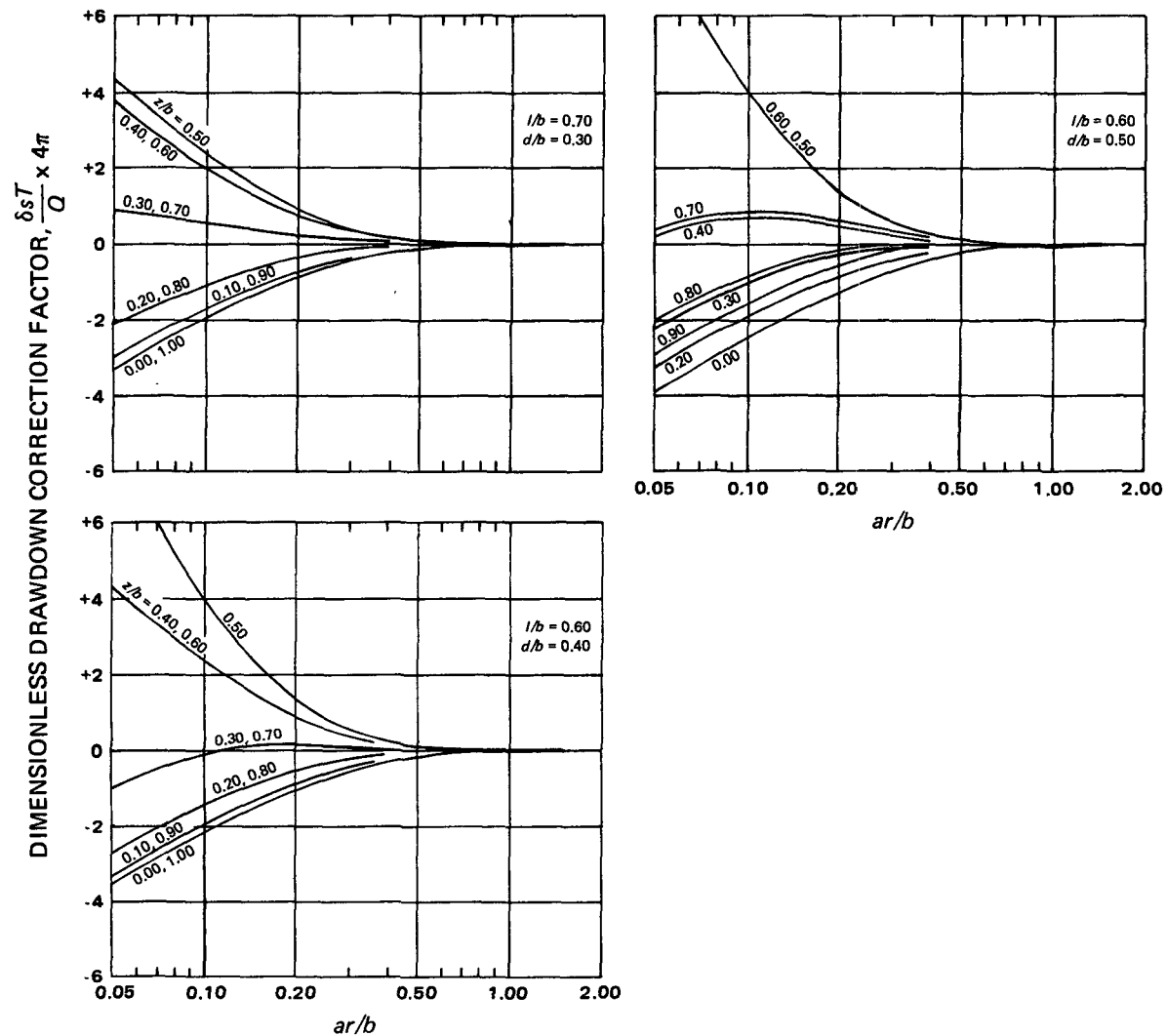


FIGURE 2.3.—Continued.

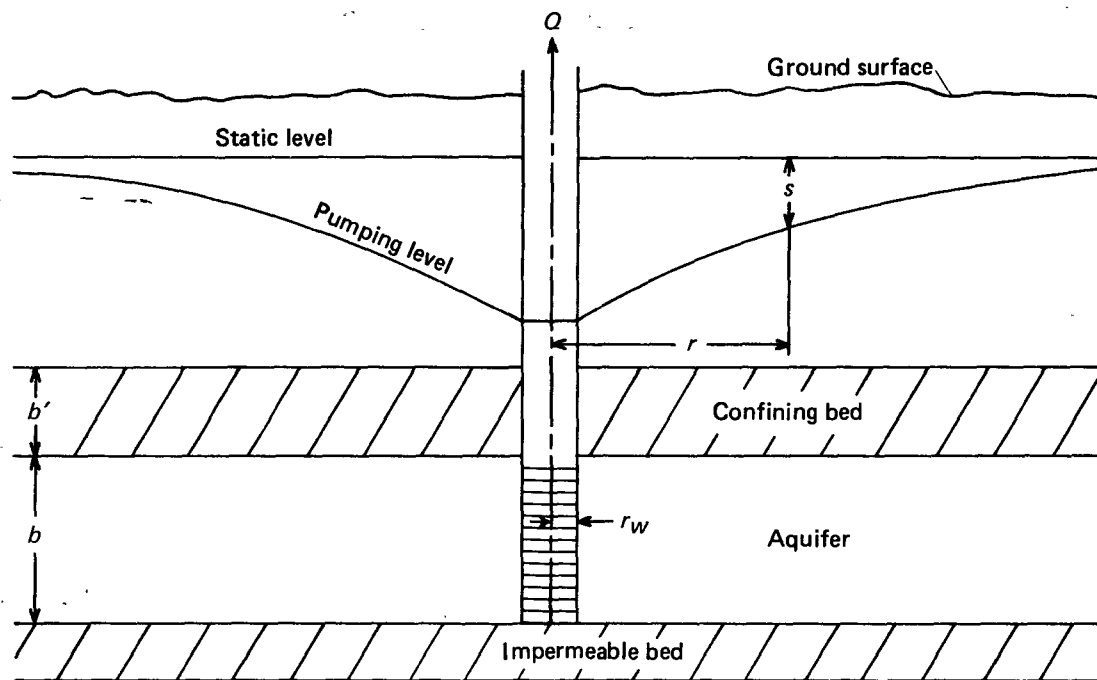


FIGURE 2.4.—Example of output from program for partial penetration in a nonleaky artesian aquifer.

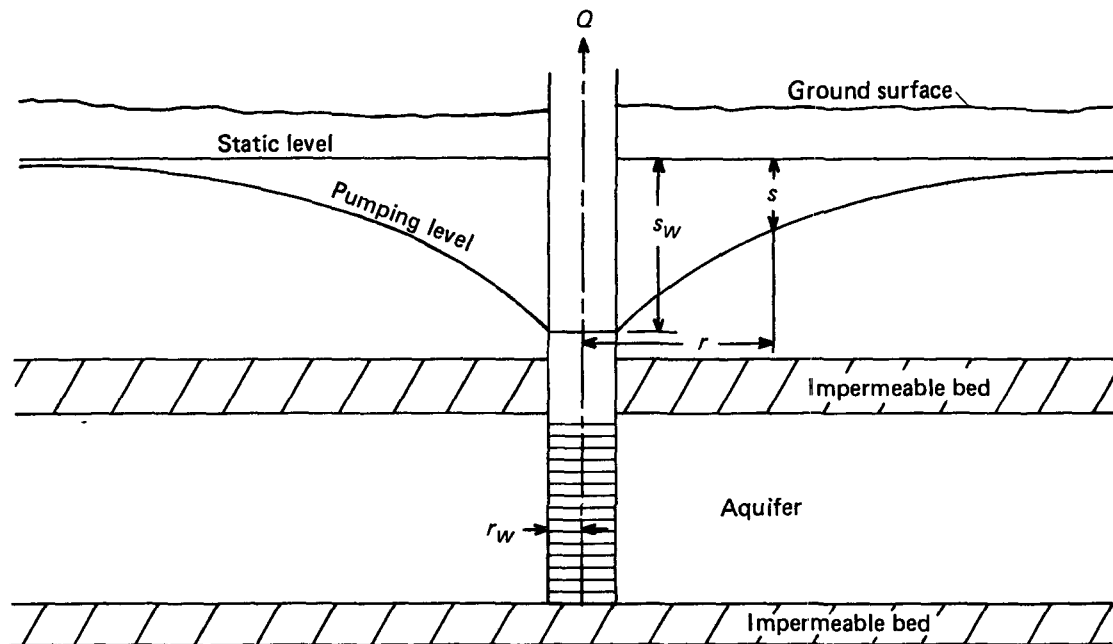


FIGURE 3.1.—Cross section through a well with constant drawdown in a nonleaky aquifer.

Solution 4: Constant discharge from a fully penetrating well in a leaky aquifer

Assumptions:

1. Well discharges at a constant rate, Q .
2. Well is of infinitesimal diameter and fully penetrates the aquifer.
3. Aquifer is overlain, or underlain, everywhere by a confining bed having uniform hydraulic conductivity (K') and thickness (b').
4. Confining bed is overlain, or underlain, by an infinite constant-head plane source.
5. Hydraulic gradient across confining bed changes instantaneously with a change in head in the aquifer (no release of water from storage in the confining bed).
6. Flow in the aquifer is two-dimensional and radial in the horizontal plane and flow in the confining bed is vertical. This assumption is approximated closely where the hydraulic conductivity of the aquifer is sufficiently greater than that of the confining bed.

Differential equation:

$$\frac{\partial^2 s}{\partial r^2} + \frac{1}{r} \frac{\partial s}{\partial r} - \frac{sK'}{Tb'} = \frac{S}{T} \frac{\partial s}{\partial t}$$

This is the differential equation describing nonsteady radial flow in a homogeneous isotropic aquifer with leakage proportional to drawdown.

Boundary and initial conditions:

(1)

$$s(\infty, t) = 0, t \geq 0 \quad (2)$$

$$Q = \begin{cases} 0, & t < 0 \\ \text{constant} > 0, & t \geq 0 \end{cases} \quad (3)$$

$$\lim_{r \rightarrow 0} r \frac{\partial s}{\partial r} = -\frac{Q}{2\pi T} \quad (4)$$

Equation 1 states that the initial drawdown is zero. Equation 2 states that drawdown is small at a large distance from the pumping well. Equation 3 states that the discharge from the well is constant and begins at $t=0$. Equation 4 states that near the pumping well the flow toward the well is equal to its discharge.

TABLE 3.1.—Values of $G(\alpha)$

[Modified from Lohman (1972, p 24)]

α	$\alpha \times 10^{-4}$	10^{-3}	10^{-2}	10^{-1}	1	10	10^2	10^3	10^4	10^5
1	.569	18.34	6.13	2.249	0.985	0.534	0.346	0.251	0.1964	0.1608
2	.404	13.11	4.47	1.716	.803	.461	.311	.232	.1841	.1524
3	.331	10.79	3.74	1.477	.719	.427	.294	.222	.1777	.1479
4	.287	9.41	3.30	1.333	.667	.405	.283	.215	.1733	.1449
5	.257	8.47	3.00	1.234	.630	.389	.274	.210	.1701	.1426
6	.235	7.77	2.78	1.160	.602	.377	.268	.206	.1675	.1408
7	.218	7.23	2.60	1.103	.580	.367	.263	.203	.1654	.1393
8	.204	6.79	2.46	1.057	.562	.359	.258	.200	.1636	.1380
9	.193	6.43	2.35	1.018	.547	.352	.254	.198	.1621	.1369
α	$\alpha \times 10^6$	10^7	10^8	10^9	10^{10}	10^{11}	10^{12}	10^{13}	10^{14}	10^{15}
1	0.1360	0.1177	0.1037	0.0927	0.0838	0.0764	0.0704	0.0651	0.0605	0.0566
2	.1299	.1131	.1002	.0899	.0814	.0744	.0686	.0636	.0593	.0555
3	.1266	.1106	.0982	.0883	.0801	.0733	.0677	.0628	.0586	.0549
4	.1244	.1089	.0968	.0872	.0792	.0726	.0671	.0622	.0581	
5	.1227	.1076	.0958	.0864	.0785	.0720	.0666	.0618	.0577	
6	.1213	.1066	.0950	.0857	.0779	.0716	.0662	.0615	.0574	
7	.1202	.1057	.0943	.0851	.0774	.0712	.0658	.0612	.0572	
8	.1192	.1049	.0937	.0846	.0770	.0709	.0655	.0609	.0569	
9	.1184	.1043	.0932	.0842	.0767	.0706	.0653	.0607	.0567	

Solution (Hantush and Jacob, 1955, p. 98):

$$s = \frac{Q}{4\pi T} \int_u^\infty \frac{e^{-z} - \frac{z}{4B^2}}{z} dz \quad (5)$$

where $u = r^2 S / 4Tt$

$$B = \sqrt{\frac{Tb'}{K'}} \quad (6)$$

Comments:

As pointed out by Hantush and Jacob (1954, p. 917), leakage is three-dimensional, but if the difference in hydraulic conductivities of the aquifer and confining bed are sufficiently great, the flow may be assumed to be vertical in the confining bed and radial in the aquifer. This relationship has been quantified by Hantush (1967, p. 587) in the condition $b/B < 0.1$. In terms of relative conductivities, this would be $K/K' > 100 b/b'$. Assumption 5, that there is no change in storage of water in the confining bed, was investigated by Neuman and Witherspoon (1969b, p. 821). They concluded that this assumption would not affect the solution if

$$\beta < 0.01, \text{ where } \beta = \frac{r}{4b} \sqrt{\frac{K'S_s'}{KS_s}}$$

Assumption 4, that there is no drawdown in water level in the source bed lying above the confining bed, was also examined by Neuman and Witherspoon (1969a, p. 810). They indicated that drawdown in the source bed would have negligible effect on drawdown in the pumped aquifer for short times, that is, when

$\frac{Tt}{r^2 S} < 1.6 \frac{\beta^2}{(r/B)^4}$. Also, they indicated (1969a, p. 811) that neglect of drawdown in the source bed is justified if $T_s > 100T$, where T_s represents the transmissivity of the source bed. Figure 4.1, a cross section through the discharging well, shows geometric relationships. Figure 4.2 on plate 1 shows plots of dimensionless drawdown compared to dimensionless time, using the notation of Cooper (1963) from Lohman (1972, pl. 3). Cooper expressed equations 5 and 6 as

$$L(u, v) = \int_u^\infty \frac{e^{-v - \frac{u^2}{v}}}{y} dy, \quad (7)$$

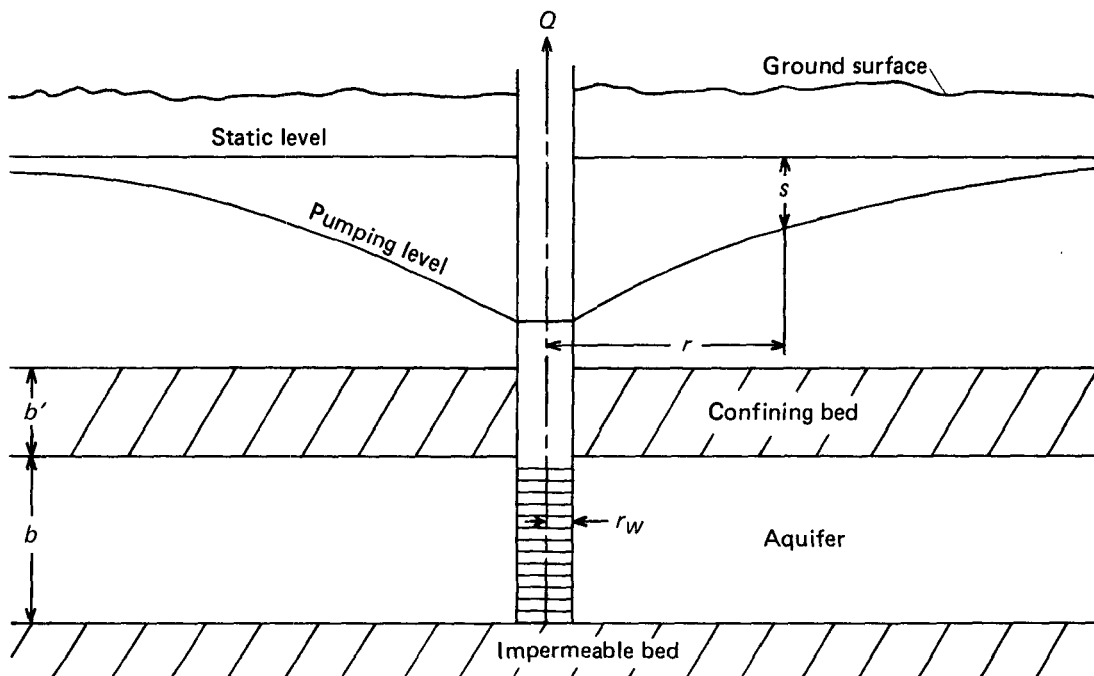


FIGURE 4.1.—Cross section through a discharging well in a leaky aquifer.

The drawdown in the steady-state region is given by the equation (Jacob, 1946, eq. 15)

$$s = \frac{Q}{2\pi T} K_0(x),$$

where $K_0(x)$ is the zero-order modified Bessel function of the second kind and

$$x = r \sqrt{\frac{K'}{Tb'}}.$$

Data for steady-state conditions can be analyzed using figure 4.3 on plate 1. The drawdowns are plotted versus r and matched to figure 4.3. After choosing a convenient match point with coordinates (s,r) and $(K_0(x),x)$ the parameters are computed from the equations

$$T = \frac{Q}{2\pi s} K_0(x) \text{ and } \frac{K'}{b'} = \frac{xT}{r^2}.$$

Values of $K_0(x)$ from Hantush (1956) are given in table 4.2.

A FORTRAN program for generating type-curve function values of equation 7 is listed in table 4.3. Using the notation $L(u,v)$ of Cooper (1963), the function is evaluated as follows. For $u \geq 1$,

$$L(u,v) = \int_u^\infty (1/y) \exp(-y-v^2/y) dy = \int_u^\infty f(y) dy.$$

This integral is transformed into the form

$$\int_0^\infty e^{-x} \left[\exp\left(-u - \frac{v^2}{x+u}\right) \frac{1}{x+u} \right] dx$$

evaluated by a Gaussian-Laguerre quadrature formula. For $v^2 < u < 1$,

$$L(u,v) = \int_1^\infty f(y) dy + \int_u^1 f(y) dy.$$

The first integral is evaluated by a Gaussian-Laguerre quadrature formula, as previously described. The second integral is evaluated using a series expansion, as

$$\int_u^1 f(y) dy = s(1-s(u)),$$

where

$$s = \log u \left[\sum_{n=0}^\infty \frac{(v^2)^n}{(n!)^2} \right] + \sum_{m=1}^\infty \left[\frac{(-1)^m}{m} \left[u^m - \left(\frac{v^2}{u} \right)^m \right] \left[\sum_{n=0}^\infty \frac{(v^2)^n}{(m+n)!n!} \right] \right].$$

For $u < 1$ and $u \leq v^2$,

$$L(u,v) = 2K_0(2v) - \int_{\frac{v^2}{u}}^\infty f(y) dy$$

(Cooper, 1963, p. C50),

where K_0 is the zero-order modified Bessel function of the second kind. The integral in the above expression is evaluated by the Gaussian-Laguerre procedure, as described previously.

Input data for this program consist of three cards with the numeric data coded by specific FORTRAN formats. Readers unfamiliar with FORTRAN format items should consult a FORTRAN language manual. The first card contains: the smallest value of $1/u$ for which computation is desired, coded in columns 1-10 in format E10.5; the largest value of $1/u$ for which computation is desired, coded in columns 11-20 in format E10.5. The table will include a range of $1/u$ values spanning these two coded values if the span is less than or equal to 12 log cycles. The next two cards contain 12 values of r/B , all coded in format E10.5, in columns 1-10, 11-20, 21-30, 31-40, 41-50, 51-60, 61-70, and 71-80 of the first card and columns 1-10, 11-20, 21-30, and 31-40 of the second card. Zero (or blank) coding is permissible in this field, but computation will terminate with the first zero (or blank) value encountered. An example of the output from this program is shown in figure 4.4.

TABLE 4.2.—Selected values of $K_0(x)$

[From Hantush (1956, p. 704)]

N	$x = NX10^{-2}$	$x = NX10^{-1}$	$x = N$
1	4.7212	2.4271	0.4210
1.5	4.3159	2.0300	.2138
2	4.0285	1.7527	.1139
3	3.6235	1.3725	.0347
4	3.3365	1.1145	.0112
5	3.1142	.9244	.0037
6	2.9329	.7775	-----
7	2.7798	.6605	-----
8	2.6475	.5653	-----
9	2.5310	.4867	-----

w (U,R/B)

		R/B								
1/U	1	0.10E-05	0.30E-05	0.10E-04	0.30E-04	0.10E-03	0.30E-03	0.10E-02	0.30E-02	0.10E-01
0.100E	01	0.2194	0.2194	0.2194	0.2194	0.2194	0.2194	0.2194	0.2194	0.2194
0.150E	01	0.3984	0.3984	0.3984	0.3984	0.3984	0.3984	0.3984	0.3984	0.3984
0.200E	01	0.5598	0.5598	0.5598	0.5598	0.5598	0.5598	0.5598	0.5598	0.5598
0.300E	01	0.8289	0.8289	0.8289	0.8289	0.8289	0.8289	0.8289	0.8289	0.8289
0.500E	01	1.2226	1.2226	1.2226	1.2226	1.2226	1.2226	1.2226	1.2226	1.2226
0.700E	01	1.5066	1.5066	1.5066	1.5066	1.5066	1.5066	1.5066	1.5066	1.5066
0.100E	02	1.8229	1.8229	1.8229	1.8229	1.8229	1.8229	1.8229	1.8229	1.8229
0.150E	02	2.1964	2.1964	2.1964	2.1964	2.1964	2.1964	2.1964	2.1964	2.1964
0.200E	02	2.4679	2.4679	2.4679	2.4679	2.4679	2.4679	2.4679	2.4679	2.4679
0.300E	02	2.8570	2.8570	2.8570	2.8570	2.8570	2.8570	2.8570	2.8570	2.8570
0.500E	02	3.3547	3.3547	3.3547	3.3547	3.3547	3.3547	3.3547	3.3547	3.3547
0.700E	02	3.6855	3.6855	3.6855	3.6855	3.6855	3.6855	3.6855	3.6855	3.6855
0.100E	03	4.0379	4.0379	4.0379	4.0379	4.0379	4.0379	4.0379	4.0379	4.0379
0.150E	03	4.4401	4.4401	4.4401	4.4401	4.4401	4.4401	4.4401	4.4401	4.4401
0.200E	03	4.7261	4.7261	4.7261	4.7261	4.7261	4.7261	4.7261	4.7261	4.7261
0.300E	03	5.1299	5.1299	5.1299	5.1299	5.1299	5.1299	5.1299	5.1299	5.1299
0.500E	03	5.6394	5.6394	5.6394	5.6394	5.6394	5.6394	5.6394	5.6394	5.6394
0.700E	03	5.9753	5.9753	5.9753	5.9753	5.9753	5.9753	5.9753	5.9753	5.9753
0.100E	04	6.3315	6.3315	6.3315	6.3315	6.3315	6.3315	6.3315	6.3315	6.3315
0.150E	04	6.7367	6.7367	6.7367	6.7367	6.7367	6.7367	6.7367	6.7367	6.7367
0.200E	04	7.0242	7.0242	7.0242	7.0242	7.0242	7.0242	7.0242	7.0242	7.0242
0.300E	04	7.4295	7.4295	7.4295	7.4295	7.4295	7.4295	7.4295	7.4295	7.4295
0.500E	04	7.9402	7.9402	7.9402	7.9402	7.9402	7.9402	7.9402	7.9402	7.9402
0.700E	04	8.2766	8.2766	8.2766	8.2766	8.2766	8.2766	8.2766	8.2766	8.2766
0.100E	05	8.6332	8.6332	8.6332	8.6332	8.6332	8.6332	8.6332	8.6332	8.6332

FIGURE 4.4.—Example of output from program for computing drawdown due to constant discharge from a well in a leaky artesian aquifer.

Solution 5: Constant discharge from a well in a leaky aquifer with storage of water in the confining beds

Assumptions:

1. Well discharges at a constant rate, Q .
2. Well is of infinitesimal diameter and fully penetrates the aquifer.
3. Aquifer is overlain and underlain everywhere by confining beds having hydraulic conductivities K' and K'' , thicknesses b' and b'' , and storage coefficients S' and S'' , respectively, which are constant in space and time.
4. Flow in the aquifer is two dimensional and radial in the horizontal plane and flow in confining beds is vertical. This assumption is approximated closely where the hydraulic conductivity of the aquifer is sufficiently greater than that of the confining beds.
5. Conditions at the far surfaces of the confining beds are (fig. 5.1):

Case 1. Constant-head plane sources above and below.

Case 2. Impermeable beds above and below.

Case 3. Constant-head plane source above and impermeable bed below.

Differential equations:

For the upper confining bed

$$\frac{\partial^2 s_1}{\partial z^2} = \frac{S'}{K'b'} \frac{\partial s_1}{\partial t} \quad (1)$$

For the aquifer

$$\frac{\partial^2 s}{\partial r^2} + \frac{1}{r} \frac{\partial s}{\partial r} + \frac{K'}{T} \frac{\partial}{\partial z} s_1(r, b', t) - \frac{K''}{T} \frac{\partial}{\partial z} s_2(r, b' + b, t) = \frac{S}{T} \frac{\partial s}{\partial t} \quad (2)$$

For the lower confining bed

$$\frac{\partial^2 s_2}{\partial z^2} = \frac{S''}{K''b''} \frac{\partial s_2}{\partial t} \quad (3)$$

Equations 1 and 3 are, respectively, the differential equations for nonsteady vertical flow in the upper and lower semipervious beds. Equation 2 is the differential equation for nonsteady two-dimensional radial flow in an aquifer with leakage at its upper and lower boundaries.

Boundary and initial conditions:

Case 1: For the upper confining bed

$$s_1(r, z, 0) = 0 \quad (4)$$

$$s_1(r, 0, t) = 0 \quad (5)$$

$$s_1(r, b', t) = s(r, t) \quad (6)$$

For the aquifer

$$s(r, 0) = 0 \quad (7)$$

$$s(\infty, t) = 0 \quad (8)$$

$$\lim_{r \rightarrow 0} r \frac{\partial s(r, t)}{\partial r} = - \frac{Q}{2\pi T} \quad (9)$$

For the lower confining bed

$$s_2(r, z, 0) = 0 \quad (10)$$

$$s_2(r, b' + b + b'', t) = 0 \quad (11)$$

$$s_2(r, b' + b, t) = s(r, t) \quad (12)$$

Case 2: Same as case 1, with conditions 5 and 11 being replaced, respectively, by

$$\frac{\partial s_1(r, 0, t)}{\partial z} = 0 \quad (13)$$

$$\frac{\partial s_2(r, b' + b + b'')}{\partial z} = 0 \quad (14)$$

Case 3: Same as case 1, with condition 11 being replaced by condition 14.

Equations 4, 7, and 10 state that initially the drawdown is zero in the aquifer and within each confining bed. Equation 5 states that a plane of zero drawdown occurs at the top of the upper confining bed. Equations 6 and 12 state that, at the upper and lower boundaries of the aquifer, drawdown in the aquifer is equal to drawdown in the confining beds. Equation 8 states that drawdown is small at a large distance from the pumping well. Equation 9 states that, near the pumping well, the flow is equal to the discharge rate. Equation 11 states that a plane of zero drawdown is at the base of the lower confining bed. Equation 13 states that

there is no flow across the top of the upper confining bed. Equation 14 states that no flow occurs across the base of the lower confining bed.

Solutions (Hantush, 1960, p. 3716):

I. For small values of time (t less than both $b'S'/10K'$ and $b''S''/10K''$):

$$s = \frac{Q}{4\pi T} H(u, \beta), \quad (15)$$

where
$$u = \frac{r^2 S}{4Tt}$$

and
$$\beta = \frac{r}{4} \left(\sqrt{\frac{K'S'}{b'TS}} + \sqrt{\frac{K''S''}{b''TS}} \right)$$

$$H(u, \beta) = \int_u^\infty \frac{e^{-y}}{y} \operatorname{erfc} \frac{\beta\sqrt{u}}{\sqrt{y(y-u)}} dy$$

$$\operatorname{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty e^{-y^2} dy.$$

II. For large values of time:

A. Case 1, t greater than both $5b'S'/K'$ and $5b''S''/K''$

$$s = \frac{Q}{4\pi T} W(u\delta_1, \alpha), \quad (16)$$

where u is as defined previously

and
$$\delta_1 = 1 + (S' + S'')/3S,$$

$$\alpha = r \sqrt{\frac{K'/b'}{T} + \frac{K''/b''}{T}}$$

$$W(u, \alpha) = \int_u^\infty \frac{\exp(-y - x^2/4y)}{y} dy.$$

B. Case 2, t greater than both $10b'S'/K'$ and $10b''S''/K''$

$$s = \frac{Q}{4\pi T} W(u\delta_2), \quad (17)$$

where
$$\delta_2 = 1 + (S' + S'')/S$$

$$W(u) = \int_u^\infty \frac{e^{-y}}{y} dy.$$

C. Case 3, t greater than both $5b'S'/K'$ and $10b''S''/K''$

$$s = \frac{Q}{4\pi T} W(u\delta_3, r \sqrt{\frac{K'/b'}{T}}), \quad (18)$$

where

$$\delta_3 = 1 + (S'' + S'/3)/S$$

and $W(u, \alpha)$ is as defined in case 1.

Comments:

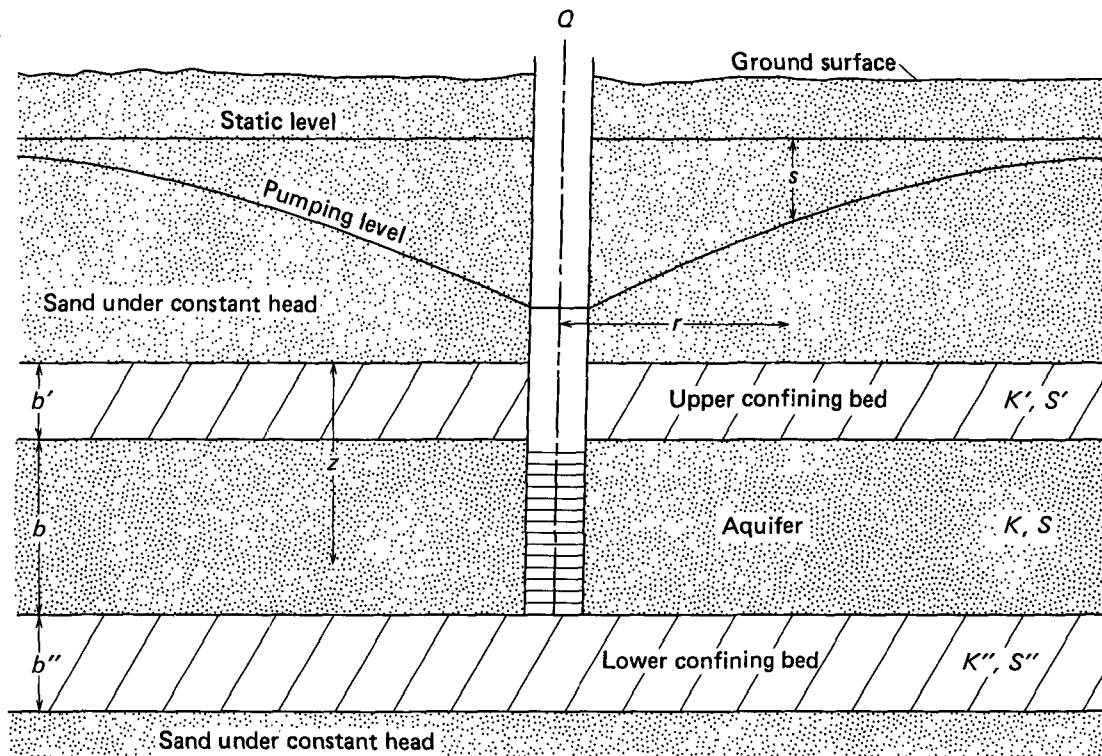
A cross section through the discharging well is shown in figure 5.1. The flow system is actually three-dimensional in such a geometric configuration. However, as stated by Hantush (1960, p. 3713), if the hydraulic conductivity in the aquifer is sufficiently greater than the hydraulic conductivity of the confining beds, flow will be approximately radial in the aquifer and approximately vertical in the confining beds. A complete solution to this flow problem has not been published. Neuman and Witherspoon (1971, p. 250, eq. II-161) developed a complete solution for case 1 but did not tabulate it. Hantush's solutions, which have been tabulated, are solutions that are applicable for small and large values of time but not for intermediate times.

The "early" data (data collected for small values of t) can be analyzed using equation 15. Figure 5.2 on plate 1 shows plots of $H(u, \beta)$ from Lohman (1972, pl. 4). Hantush (1961d) has an extensive tabulation of $H(u, \beta)$, a part of which is given in table 5.1. The corresponding data curves would consist of observed drawdown versus t/r^2 . Superposing the data curves on the type curves and matching the two, with graph axes parallel, so that the data curves lie on or between members of the type-curve family and choosing a convenient match point ($H(u, \beta)$, $1/u$), T and S are computed by

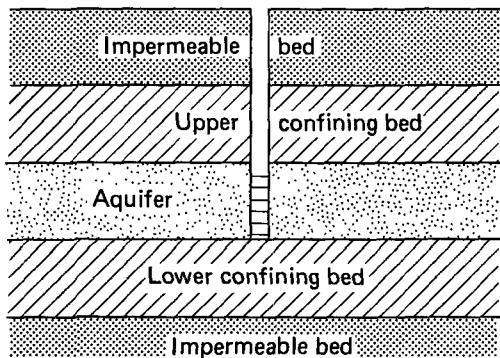
$$T = \frac{Q}{4\pi s} H(u, \beta),$$

$$S = 4T \frac{t}{r^2} \bigg/ \frac{1}{u}.$$

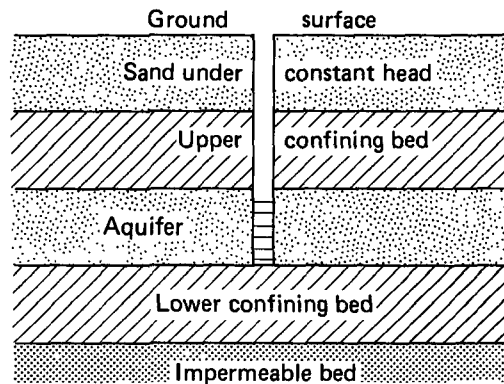
If simplifying conditions are applicable, it is possible to compute the product $K'S'$ from the β value. If $K''S''=0$, $K'S'=16\beta^2 b'TS/r^2$, and if $K''S''=K'S'$,



CASE 1



CASE 2



CASE 3

FIGURE 5.1.—Cross sections through discharging wells in leaky aquifers with storage of water in the confining beds, illustrating three different cases of boundary conditions.

$$K'S' = \frac{16\beta^2}{r^2} TS \frac{b'b''}{b'+b''+2\sqrt{b'b''}}$$

The curves in figure 5.2 are very similar from $\beta=0$ to about $\beta=0.5$. Therefore, the β val-

ues in this range are indeterminate. There is also uncertainty in curve matching for all β values because of the fact that it is a family of curves whose shapes change gradually with β . This uncertainty will be increased if the data covers a small range of t values. The problem

TABLE 5.1.—Values of $H(u, \beta)$ for selected values of u and β [From Hantush (1961d) Numbers in parentheses are powers of 10 by which the other numbers are multiplied, for example $963(-4) = 0.0963$]

u	β							
	0.03	0.1	0.3	1	3	10	30	100
1×10^{-9}	12.3088	11.1051	10.0066	8.8030	7.7051	6.5033	5.4101	4.2221
2	11.9622	10.7585	9.6602	8.4566	7.3590	6.1579	5.0666	3.8839
3	11.7593	10.5558	9.4575	8.2540	7.1565	5.9561	4.8661	3.6874
5	11.5038	10.3003	9.2021	7.9987	6.9016	5.7020	4.6142	3.4413
7	11.3354	10.1321	9.0339	7.8306	6.7337	5.5348	4.4487	3.2804
1×10^{-8}	11.1569	9.9538	8.8556	7.6525	6.5558	5.3578	4.2737	3.1110
2	10.8100	9.6071	8.5091	7.3063	6.2104	5.0145	3.9352	2.7858
3	10.6070	9.4044	8.3065	7.1039	6.0085	4.8141	3.7383	2.5985
5	10.3511	9.1489	8.0512	6.8490	5.7544	4.5623	3.4919	2.3662
7	10.1825	8.9806	7.8830	6.6811	5.5872	4.3969	3.3307	2.2159
1×10^{-7}	10.0037	8.8021	7.7048	6.5032	5.4101	4.2221	3.1609	2.0591
2	9.6560	8.4554	7.3585	6.1578	5.0666	3.8839	2.8348	1.7633
3	9.4524	8.2525	7.1560	5.9559	4.8661	3.6874	2.6469	1.5966
5	9.1955	7.9968	6.9009	5.7018	4.6141	3.4413	2.4137	1.3944
7	9.0261	7.8283	6.7329	5.5346	4.4486	3.2804	2.2627	1.2666
1×10^{-6}	8.8463	7.6497	6.5549	5.3575	4.2736	3.1110	2.1051	1.1361
2	8.4960	7.3024	6.2091	5.0141	3.9350	2.7857	1.8074	.8995
3	8.2904	7.0991	6.0069	4.8136	3.7382	2.5984	1.6395	.7725
5	8.0304	6.8427	5.7523	4.5617	3.4917	2.3661	1.4354	.6256
7	7.8584	6.6737	5.5847	4.3962	3.3304	2.2158	1.3061	.5375
1×10^{-5}	7.6754	6.4944	5.4071	4.2212	3.1606	2.0590	1.1741	.4519
2	7.3170	6.1453	5.0624	3.8827	2.8344	1.7632	.9339	.3091
3	7.1051	5.9406	4.8610	3.6858	2.6464	1.5965	.8046	.2402
5	6.8353	5.6821	4.6075	3.4394	2.4131	1.3943	.6546	.1685
7	6.6553	5.5113	4.4408	3.2781	2.2619	1.2664	.5643	.1300
1×10^{-4}	6.4623	5.3297	4.2643	3.1082	2.1042	1.1359	.4763	963(-4)
2	6.0787	4.9747	3.9220	2.7819	1.8062	.8992	.3287	494(-4)
3	5.8479	4.7655	3.7222	2.5937	1.6380	.7721	.2570	315(-4)
5	5.5488	4.4996	3.4711	2.3601	1.4335	.6252	.1818	166(-4)
7	5.3458	4.3228	3.3062	2.2087	1.3039	.5370	.1412	103(-4)
1×10^{-3}	5.1247	4.1337	3.1317	2.0506	1.1715	.4513	.1055	390(-5)
2	4.6753	3.7598	2.7938	1.7516	.9305	.3084	.551(-4)	169(-5)
3	4.3993	3.5363	2.5969	1.5825	.8006	.2394	.355(-4)	713(-6)
5	4.0369	3.2483	2.3499	1.3767	.6498	.1677	.190(-4)	205(-6)
7	3.7893	3.0542	2.1877	1.2460	.5589	.1292	.120(-4)	821(-7)
1×10^{-2}	3.5195	2.8443	2.0164	1.1122	.4702	.955(-4)	.695(-5)	274(-7)
2	2.9759	2.4227	1.6853	.8677	.3214	.487(-4)	.205(-5)	226(-8)
3	2.6487	2.1680	1.4932	.7353	.2491	.308(-4)	.888(-6)	
5	2.2312	1.8401	1.2535	.5812	.1733	.160(-4)	.261(-6)	
7	1.9558	1.6213	1.0979	.4880	.1325	.982(-5)	.106(-6)	
1×10^{-1}	1.6667	1.3893	.9358	.3970	.966(-4)	.552(-5)	.365(-7)	
2	1.1278	.9497	.6352	.2452	.468(-4)	.149(-5)	.307(-8)	
3	.8389	.7103	.4740	.1729	.281(-4)	.592(-6)		
5	.5207	.4436	.2956	.1006	.130(-4)	.151(-6)		
7	.3485	.2980	.1985	.646(-4)	.714(-5)	.534(-7)		
1×1	.2050	.1758	.1172	.365(-4)	.337(-5)	.151(-7)		
2	458(-4)	395(-4)	264(-4)	.760(-5)	.487(-6)			
3	122(-4)	106(-4)	707(-5)	.196(-5)	.102(-6)			
5	108(-5)	934(-6)	624(-6)	.167(-6)	.672(-8)			
7	109(-6)	941(-7)	629(-7)	.165(-7)				
1×10	391(-8)	339(-8)	227(-8)					
2								
3								
5								
7								

can be avoided, if data from more than one observation well are available, by preparing a composite data plot of s versus t/r^2 . This data plot would be matched by adding the constraint that the r values for the different data curves representing each well fall on proportional β curves.

The "late" data (for large values of t) can be analyzed using equations 16, 17, and 18; these equations are forms of summaries 1, $W(u)$, and 4, $L(u, v)$. However, for cases 1 and 3, the late data fall on the flat part of the $L(u, v)$ curves and a time-drawdown plot match would be indeterminate. Thus, only a distance-drawdown

match could be used. Drawdown predictions, however, could be made using the $L(u, v)$ curves.

Assumption 5, that no drawdown occurs in the source beds, has been examined by Neuman and Witherspoon (1969a, p. 810, 811) for the situation in which two aquifers are separated by a less permeable bed. This is equivalent to case 3 with $K''=0$ and $S''=0$. They concluded that (1) $H(u, \beta)$, in the asymptotic solution for early times, would not be affected appreciably because the properties of the source bed have a negligible effect on the solution for $Tt/r^2S \leq 1.6\beta^2/(r/B)^4$, which is equivalent to $t \leq S'b'/10K'$, where $B = \sqrt{Tb'/K'}$; and (2) if $T_s > 100T$, where T_s represents the transmissivity of the source bed, it is probably justified to neglect drawdown in the unpumped aquifer.

Table 5.2 is a listing of a FORTRAN program for computing values of $H(u, \beta)$ for $u \geq 10^{-60}$ using a procedure devised and programmed by S. S. Papadopoulos. Input data for this program consists of three cards. The first card contains the beginning value of $1/u$, coded in columns 1-10, in format E10.5, and the ending (largest) value of $1/u$, coded in columns 11-20, in format E10.5. The next two cards contain 12 values of β , coded in columns 1-10, 11-20, . . . , and 71-80 on the first card and columns 1-10, 11-20, . . . , 31-40 on the second card, all in format E10.5. The function is evaluated as follows (S. S. Papadopoulos, written commun., 1975):

$$H(u, \beta) = \int_u^\infty (e^{-u/y}) \operatorname{erfc}(\beta\sqrt{u}/\sqrt{y(y-u)}) dy \\ = \int_u^\infty f dy,$$

where f represents the integrand. For $\beta=0$, $H(u, \beta) = W(u)$, where $W(u)$ is the well function of Theis. Because $\operatorname{erfc}(x) \leq 1$ for $x \geq 0$, it follows that $H(u, \beta) \leq W(u)$, and for $u > 10$, $W(u) \approx 0$ and therefore for $u > 10$, $H(u, \beta) \approx 0$. The tables of $H(u, \beta)$ indicate that $H(u, \beta) \approx 0$ for $\beta > 1$ and $\beta^2 u > 300$. For an arbitrarily small value of u , the integral can be considered as the sum of three integrals

$$\int_u^\infty f dy = \int_u^{u_1} f dy + \int_{u_1}^{u_2} f dy + \int_{u_2}^\infty f dy,$$

$$\text{where } u_2 = (u/2)(1 + \sqrt{1+10^{20}\beta^2/u}),$$

$$\text{and } u_1 = (u/2)(1 + \sqrt{1+0.025\beta^2/u}).$$

The significance of u_2 and u_1 is that

$$\operatorname{erfc}(\beta\sqrt{u}/\sqrt{y(y-u)}) \approx 1 \text{ for } u > u_2$$

and

$$\operatorname{erfc}(\beta\sqrt{u}/\sqrt{y(y-u)}) \approx 0 \text{ for } u < u_1.$$

Therefore,

$$\int_u^{u_1} f dy \approx 0,$$

and

$$\int_{u_2}^\infty f dy \approx W(u_2),$$

where $W(u_2)$ is the well function of Theis. The function can be evaluated as

$$H(u, \beta) \approx W(u) \text{ for } u > u_2$$

$$H(u, \beta) \approx \int_u^{u_2} f dy + W(u_2) \text{ for } u_1 < u < u_2$$

$$\text{and } H(u, \beta) \approx \int_{u_1}^{u_2} f dy + W(u_2) \text{ for } u < u_1.$$

If $u_2 > 10$, then

$$\int_{u_1}^{u_2} f dy = \int_{u_1}^{10} f dy, W(u_2) \approx 0.$$

An example of output from this program is shown in figure 5.3.

Solution 6: Constant discharge from a partially penetrating well in a leaky aquifer

Assumptions:

1. Well discharges at a constant rate, Q .
2. Well is of infinitesimal diameter and is screened in only part of the aquifer.
3. Aquifer has radial-vertical anisotropy.

H(U, BETA)

		BETA				
1/U		0.30E-01	0.10E 00	0.30F 00	0.10E 01	0.30F 01
0.100E	02	1.6667	1.3894	0.9358	0.3970	0.0965
0.150E	02	1.9953	1.6531	1.1203	0.5010	0.1374
0.200E	02	2.2308	1.8401	1.2536	0.5812	0.1733
0.300E	02	2.5626	2.1010	1.4435	0.7023	0.2320
0.500E	02	2.9759	2.4228	1.6853	0.8677	0.3214
0.700E	02	3.2428	2.6296	1.8457	0.9836	0.3897
0.100E	03	3.5196	2.8443	2.0164	1.1122	0.4702
0.150E	03	3.8256	3.0826	2.2112	1.2647	0.5717
0.200E	03	4.0369	3.2483	2.3499	1.3767	0.6498
0.300E	03	4.3259	3.4775	2.5459	1.5394	0.7683
0.500E	03	4.6754	3.7598	2.7938	1.7516	0.9305
0.700E	03	4.8969	3.9425	2.9576	1.8953	1.0447
0.100E	04	5.1247	4.1338	3.1317	2.0507	1.1715
0.150E	04	5.3756	4.3486	3.3301	2.2306	1.3225
0.200E	04	5.5488	4.4996	3.4712	2.3602	1.4335
0.300E	04	5.7871	4.7109	3.6704	2.5452	1.5951
0.500E	04	6.0787	4.9747	3.9220	2.7819	1.8062
0.700E	04	6.2565	5.1474	4.0880	2.9396	1.9494
0.100E	05	6.4623	5.3297	4.2643	3.1082	2.1042
0.150E	05	6.6816	5.5361	4.4650	3.3014	2.2837
0.200E	05	6.8353	5.6821	4.6076	3.4394	2.4131
0.300E	05	7.0498	5.8874	4.8087	3.6349	2.5979
0.500E	05	7.3170	6.1454	5.0624	3.8827	2.8344
0.700E	05	7.4915	6.3149	5.2297	4.0467	2.9921
0.100E	06	7.6754	6.4944	5.4072	4.2212	3.1606
0.150E	06	7.8834	6.6983	5.6090	4.4202	3.3538
0.200E	06	8.0304	6.8427	5.7523	4.5617	3.4917
0.300E	06	8.2369	7.0462	5.9544	4.7616	3.6872
0.500E	06	8.4960	7.3024	6.2091	5.0141	3.9351
0.700E	06	8.6662	7.4710	6.3770	5.1807	4.0991
0.100E	07	8.8463	7.6497	6.5549	5.3576	4.2730
0.150E	07	9.0507	7.8528	6.7573	5.5589	4.4726
0.200E	07	9.1955	7.9968	6.9010	5.7018	4.6141
0.300E	07	9.3995	8.1998	7.1034	5.9035	4.8141
0.500E	07	9.6560	8.4554	7.3586	6.1578	5.0656
0.700E	07	9.8249	8.6237	7.5267	6.3255	5.2332
0.100E	08	10.0038	8.8022	7.7049	6.5033	5.4101
0.150E	08	10.2070	9.0050	7.9075	6.7055	5.6114
0.200E	08	10.3512	9.1489	8.0512	6.8490	5.7544
0.300E	08	10.5543	9.3517	8.2539	7.0513	5.9561
0.500E	08	10.8101	9.6072	8.5092	7.3063	6.2104
0.700E	08	10.9785	9.7754	8.6773	7.4744	6.3781
0.100E	09	11.1570	9.9538	8.8556	7.6525	6.5559
0.150E	09	11.3599	10.1566	9.0583	7.8550	6.7581
0.200E	09	11.5039	10.3004	9.2021	7.9988	6.9016
0.300E	09	11.7067	10.5032	9.4048	8.2014	7.1040
0.500E	09	11.9622	10.7585	9.6602	8.4566	7.3590
0.700E	09	12.1305	10.9269	9.8284	8.6248	7.5270
0.100E	10	12.3089	11.1052	10.0067	8.8031	7.7052

FIGURE 5.3.—Example of output from program for computing drawdown due to constant discharge from a well in a leaky aquifer with storage of water in the confining beds.

4. Aquifer is overlain, or underlain, everywhere by a confining bed having uniform hydraulic conductivity (K') and thickness (b').
5. Confining bed is overlain, or underlain, by an infinite constant-head plane source.
6. Hydraulic gradient across confining bed changes instantaneously with a change in head in the aquifer (no release of water from storage in the confining bed).
7. Flow is vertical in the confining bed.
8. The leakage from the confining bed is assumed to be generated within the aquifer so that in the aquifer no vertical flow results from leakage alone.

Differential equation:

$$\partial^2 s / \partial r^2 + 1/r \partial s / \partial r + \alpha^2 \partial^2 s / \partial z^2 - s K' / T b' = S/T \partial s / \partial t$$

$$\alpha^2 = K_z / K_r$$

This is the differential equation describing nonsteady radial and vertical flow in a homogeneous aquifer with radial-vertical anisotropy and leakage proportional to drawdown.

Boundary and initial conditions:

$$s(r, z, 0) = 0, \quad r \geq 0, \quad 0 \leq z \leq b \quad (1)$$

$$s(\infty, z, t) = 0, \quad 0 \leq z \leq b, \quad t \geq 0 \quad (2)$$

$$\partial s(r, 0, t) / \partial z = 0, \quad r \geq 0, \quad t \geq 0 \quad (3)$$

$$\partial s(r, b, t) / \partial z = 0, \quad r \geq 0, \quad t \geq 0 \quad (4)$$

$$\lim_{r \rightarrow 0} r \frac{\partial s}{\partial r} = \begin{cases} 0, & \text{for } 0 < z < d \\ -Q / (2\pi K_r (l-d)), & \text{for } d < z < l \\ 0, & \text{for } l < z < b \end{cases} \quad (5)$$

Equation 1 states that, initially, drawdown is zero. Equation 2 states that drawdown is small at a large distance from the pumping well. Equations 3 and 4 state that there is no vertical flow at the upper and lower boundaries of the aquifer. This means that vertical head gradients in the aquifer are caused by the geometric placement of the pumping well screen and not by leakage. Equation 5 states that near the pumping well the discharge is

distributed uniformly over the well screen and that no radial flow occurs above and below the screen.

Solution:

I. For the drawdown in a piezometer, a solution by Hantush (1964a, p. 350) is given by

$$s = Q/4\pi T \{W(u, \beta) + f(u, ar/b, \beta, d/b, l/b, z/b)\},$$

where

$$W(u, \beta) = \int_u^\infty \frac{e^{-y - \frac{\beta^2}{4y}}}{y} dy$$

$$u = \frac{r^2 S}{4Tt}$$

$$\beta = \sqrt{\frac{r^2 K'}{T b'}}$$

$$a = \sqrt{K_z / K_r}$$

$f(u, ar/b, \beta, d/b, l/b, z/b)$

$$= 2b/\pi(l-d) \sum_{n=1}^{\infty} l/n (\sin n\pi l/b - \sin n\pi d/b) \cdot \cos(n\pi z/b) W\left(u, \sqrt{\beta^2 + (n\pi ar/b)^2}\right).$$

II. For the drawdown in an observation well

$$s = Q/4\pi T \{W(u, \beta) + \bar{f}(u, ar/b, \beta, d/b, l/b, d'/b, l'/b)\},$$

where

$$\bar{f}(u, ar/b, \beta, d/b, l/b, d'/b, l'/b)$$

$$= 2b^2/\pi^2(l-d)(l'-d')$$

$$\cdot \sum_{n=1}^{\infty} 1/n^2 (\sin n\pi l/b - \sin n\pi d/b)$$

$$\cdot (\sin n\pi l'/b - \sin n\pi d'/b) W\left(u, \sqrt{\beta^2 + (n\pi ar/b)^2}\right)$$

Comments:

The geometry is shown in figure 6.1. The differential equation and boundary conditions are based on the assumption that vertical flow in the aquifer is caused by partial penetration of the pumping well and not by leakage. Hantush (1967, p. 587) concluded that this assumption is correct if $b\sqrt{K'/Tb'} < 0.1$. The solutions are based on a uniform distribution of flow over the screen of the pumped well. Depending on friction losses within the well, a more realistic assumption might be constant drawdown over

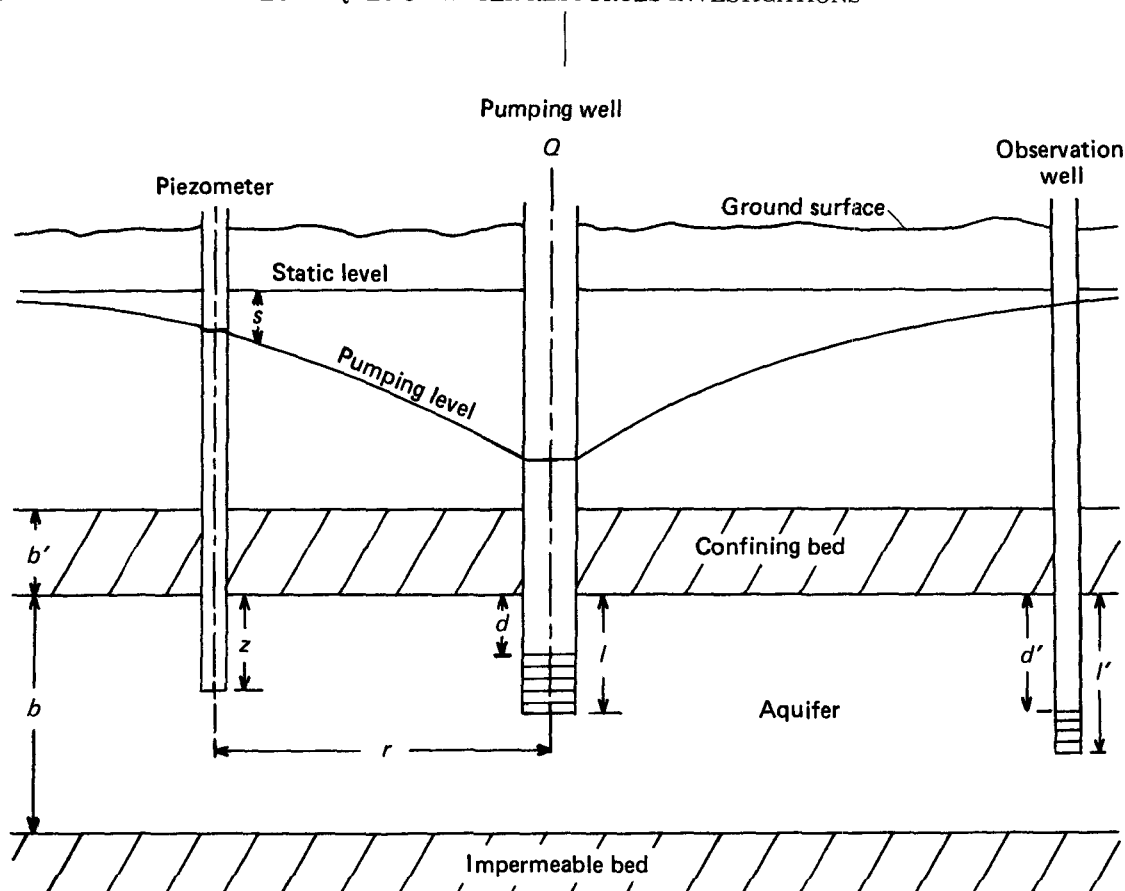


FIGURE 6.1.—Cross section through a discharging well that is screened in part of a leaky aquifer.

the screen of the pumped well; this assumption would imply nonuniform distribution of flow. Hantush (1964a, p. 351) postulates that the actual drawdown at the face of the pumping well will have a value between these two extremes. The solutions should be applied with caution at locations very near the pumped well. The effects of partial penetration are insignificant for $r > 1.5 b/a$ (Hantush, 1964a, p. 350), and the solution is the same for the solution 4.

Because of the large number of variables involved, presentation of a complete set of type curves is impractical. An example, consisting of curves for selected values of the parameters, is shown in figure 6.2 on plate 1. This figure is based on function values generated by a FORTRAN program.

The computer program formulated to compute drawdowns due to pumping a partially penetrating well in a leaky aquifer is listed in table 6.1. Input data to this program consists of cards coded in specific FORTRAN formats. Readers unfamiliar with FORTRAN format

items should consult a FORTRAN language manual. The first card contains: aquifer thickness (b), coded in format F5.1 in columns 1–5; depth, below top of aquifer, to bottom of pumping well screen (l), coded in format F5.1 in columns 6–10; depth, below top of aquifer, to top of pumping well screen (d), coded in format F5.1 in columns 11–15; number of observation wells and piezometers, coded in format I5 in columns 16–20; smallest value of $1/u$ for which computation is desired, coded in format E10.4 in columns 21–30; largest value of $1/u$ for which computation is desired, coded in format E10.4 in columns 31–40. The next two cards contain 12 values of r/B , all coded in format E10.5, in columns 1–10, 11–20, 21–30, 31–40, 41–50, 51–60, 61–70, and 71–80 of the first card and columns 1–10, 11–20, 21–30, and 31–40 of the second card. Computation will terminate with the first zero (or blank) value coded. Next is a series of cards, one card per observation well or piezometer, containing: radial distance from the pumped well multiplied

by the square root of the ratio of vertical to horizontal conductivity ($r\sqrt{K_z/K_r}$), coded in format F5.1 in columns 1-5; depth, below top of aquifer, to bottom of observation well screen (code blank for piezometer), coded in format F5.1, in columns 6-10; depth, below top of aquifer, to top of observation well screen (total depth for a piezometer), coded in format F5.1,

in columns 11-15. Output from this program is a table of function values. An example of the output is shown in figure 6.3.

Because most aquifers are anisotropic in the $r-z$ plane, it is generally impractical to use this solution to analyze for the parameters. However, it can be used to predict drawdown if the parameters are determined independently.

W(U,R/BR)+F(U,R/B,R/BR,L/B,D/B,Z/B), Z/B= 0.50, SQRT(KZ/KR)*R/B= 0.10, L/R= 0.70, D/B= 0.30

I R/BR									
1/U	0.10E-05	0.10E-04	0.10E-03	0.10E-02	0.10E-01	0.10E 00	0.10E 01	0.10E 02	
0.100E 01	0.5478	0.5478	0.5478	0.5478	0.5478	0.5468	0.4631	0.0001	
0.150E 01	0.9901	0.9901	0.9901	0.9901	0.9900	0.9878	0.7872	0.0001	
0.200E 01	1.3804	1.3804	1.3804	1.3804	1.3803	1.3764	1.0398	0.0001	
0.300E 01	2.0043	2.0043	2.0043	2.0043	2.0042	1.9964	1.3767	0.0001	
0.500E 01	2.8381	2.8381	2.8381	2.8381	2.8379	2.8221	1.6931	0.0001	
0.700E 01	3.3737	3.3737	3.3737	3.3737	3.3735	3.3499	1.8159	0.0001	
0.100E 02	3.9049	3.9049	3.9049	3.9049	3.9046	3.8700	1.8826	0.0001	
0.150E 02	4.4488	4.4488	4.4488	4.4488	4.4483	4.3975	1.9094	0.0001	
0.200E 02	4.7951	4.7951	4.7951	4.7951	4.7944	4.7291	1.9143	0.0001	
0.300E 02	5.2379	5.2379	5.2379	5.2379	5.2369	5.1455	1.9155	0.0001	
0.500E 02	5.7539	5.7539	5.7539	5.7539	5.7525	5.6135	1.9155	0.0001	
0.700E 02	6.0864	6.0864	6.0864	6.0864	6.0844	5.9001	1.9155	0.0001	
0.100E 03	6.4390	6.4390	6.4390	6.4389	6.4363	6.1859	1.9155	0.0001	
0.150E 03	6.8411	6.8411	6.8411	6.8411	6.8372	6.4816	1.9155	0.0001	
0.200E 03	7.1271	7.1271	7.1271	7.1271	7.1220	6.6669	1.9155	0.0001	
0.300E 03	7.5309	7.5309	7.5309	7.5309	7.5233	6.8854	1.9155	0.0001	
0.500E 03	8.0404	8.0404	8.0404	8.0403	8.0278	7.0788	1.9155	0.0001	
0.700E 03	8.3763	8.3763	8.3763	8.3762	8.3588	7.1556	1.9155	0.0001	
0.100E 04	8.7326	8.7326	8.7326	8.7323	8.7076	7.2002	1.9155	0.0001	
0.150E 04	9.1377	9.1377	9.1377	9.1373	9.1005	7.2199	1.9155	0.0001	
0.200E 04	9.4252	9.4252	9.4252	9.4247	9.3758	7.2239	1.9155	0.0001	
0.300E 04	9.8305	9.8305	9.8305	9.8298	9.7568	7.2250	1.9155	0.0001	
0.500E 04	10.3412	10.3412	10.3412	10.3400	10.2199	7.2251	1.9155	0.0001	
0.700E 04	10.6776	10.6776	10.6776	10.6759	10.5099	7.2251	1.9155	0.0001	
0.100E 05	11.0343	11.0343	11.0343	11.0318	10.7990	7.2251	1.9155	0.0001	

W(U,R/BR)+F(U,R/B,R/BR,L/B,D/B,L'/B,D'/B), L'/B= 0.51, D'/B= 0.49, SQRT(KZ/KR)*R/B= 0.10, L/B= 0.70, D/H= 0.30

I R/BR									
1/U	0.10E-05	0.10E-04	0.10E-03	0.10E-02	0.10E-01	0.10E 00	0.10E 01	0.10E 02	
0.100E 01	0.5477	0.5477	0.5477	0.5477	0.5477	0.5468	0.4631	0.0001	
0.150E 01	0.9899	0.9899	0.9899	0.9899	0.9899	0.9876	0.7871	0.0001	
0.200E 01	1.3801	1.3801	1.3801	1.3801	1.3801	1.3761	1.0396	0.0001	
0.300E 01	2.0038	2.0038	2.0038	2.0038	2.0037	1.9959	1.3764	0.0001	
0.500E 01	2.8372	2.8372	2.8372	2.8372	2.8371	2.8213	1.6927	0.0001	
0.700E 01	3.3727	3.3727	3.3727	3.3727	3.3725	3.3488	1.8153	0.0001	
0.100E 02	3.9037	3.9037	3.9037	3.9037	3.9034	3.8688	1.8821	0.0001	
0.150E 02	4.4475	4.4475	4.4475	4.4475	4.4470	4.3962	1.9089	0.0001	
0.200E 02	4.7937	4.7937	4.7937	4.7937	4.7930	4.7277	1.9138	0.0001	
0.300E 02	5.2365	5.2365	5.2365	5.2365	5.2356	5.1441	1.9150	0.0001	
0.500E 02	5.7525	5.7525	5.7525	5.7525	5.7511	5.6122	1.9150	0.0001	
0.700E 02	6.0850	6.0850	6.0850	6.0849	6.0830	5.8987	1.9150	0.0001	
0.100E 03	6.4376	6.4376	6.4376	6.4375	6.4349	6.1845	1.9150	0.0001	
0.150E 03	6.8397	6.8397	6.8397	6.8397	6.8358	6.4802	1.9150	0.0001	
0.200E 03	7.1257	7.1257	7.1257	7.1257	7.1206	6.6655	1.9150	0.0001	
0.300E 03	7.5295	7.5295	7.5295	7.5295	7.5219	6.8840	1.9150	0.0001	
0.500E 03	8.0390	8.0390	8.0390	8.0389	8.0264	7.0775	1.9150	0.0001	
0.700E 03	8.3749	8.3749	8.3749	8.3748	8.3574	7.1542	1.9150	0.0001	
0.100E 04	8.7312	8.7312	8.7312	8.7309	8.7062	7.1988	1.9150	0.0001	
0.150E 04	9.1363	9.1363	9.1363	9.1359	9.0991	7.2185	1.9150	0.0001	
0.200E 04	9.4238	9.4238	9.4238	9.4233	9.3743	7.2225	1.9150	0.0001	
0.300E 04	9.8291	9.8291	9.8291	9.8284	9.7554	7.2236	1.9150	0.0001	
0.500E 04	10.3398	10.3398	10.3398	10.3386	10.2185	7.2237	1.9150	0.0001	
0.700E 04	10.6762	10.6762	10.6762	10.6745	10.5085	7.2237	1.9150	0.0001	
0.100E 05	11.0329	11.0329	11.0328	11.0304	10.7976	7.2237	1.9150	0.0001	

FIGURE 6.3.—Example of output from program for partial penetration in a leaky artesian aquifer.

Solution 7: Constant drawdown in a well in a leaky aquifer

Assumptions:

1. Water level in well is changed instantaneously by s_w at $t=0$.
2. Well is of finite diameter and fully penetrates the aquifer.
3. Aquifer is overlain, or underlain, everywhere by a confining bed having uniform hydraulic conductivity (K') and thickness (b').
4. Confining bed is overlain, or underlain, by an infinite constant-head plane source.
5. Hydraulic gradient across confining bed changes instantaneously with a change in head in the aquifer (no release of water from storage in the confining bed).
6. Flow in the aquifer is two dimensional and radial in the horizontal plane and flow in the confining bed is vertical. This assumption is approximated closely where the hydraulic conductivity of the aquifer is sufficiently greater than that of the confining bed.

Differential equation:

$$\partial^2 s / \partial r^2 + (1/r) \partial s / \partial r - s K' / T b' = (S/T) \partial s / \partial t$$

This differential equation describes nonsteady radial flow in a homogeneous isotropic confined aquifer with leakage proportional to drawdown.

Boundary and initial conditions:

$$s(r,0) = 0, r \geq 0 \quad (1)$$

$$s(r_w, t) = s_w, t \geq 0 \quad (2)$$

$$s(\infty, t) = 0, t \geq 0 \quad (3)$$

Equation 1 states that, initially, drawdown is zero. Equation 2 states that at the wall or screen of the discharging well, drawdown in the aquifer is equal to the constant drawdown in the well, which assumes that there is no entrance loss to the discharging well. Equation 3 states that the drawdown approaches zero as distance from the discharging well approaches infinity.

Solutions (Hantush, 1959):

I. For the discharge rate of the well,

$$Q = 2\pi T s_w G(\alpha, r_w/B),$$

where

$$G(\alpha, r_w/B) = (r_w/B) K_1(r_w/B) / K_0(r_w/B) + (4/\pi^2) \exp[-\alpha(r_w/B)^2] \int_0^\infty \left\{ u \exp(-\alpha u^2) [J_0^2(u) + Y_0^2(u)] \right\} \cdot du / [u^2 + (r_w/B)^2],$$

$$\text{and} \quad \alpha = Tt/Sr_w^2,$$

$$B = \sqrt{Tb'/K'}.$$

K_0 and K_1 are zero-order and first-order, respectively, modified Bessel functions of the second kind. J_0 and Y_0 are the zero-order Bessel functions of the first and second kind, respectively.

II. For the drawdown in water level

$$s = s_w (K_0(r/B) / K_0(r_w/B))$$

$$+ (2/\pi) \exp(-\alpha r_w^2/B^2) \int_0^\infty \frac{\exp(-\alpha u^2)}{u^2 + (r_w/B)^2}$$

$$\cdot \frac{J_0(ur/r_w) Y_0(u) - Y_0(ur/r_w) J_0(u)}{J_0^2(u) + Y_0^2(u)} u du \quad (4)$$

with α , B , K_0 , J_0 , and Y_0 as defined previously.

Comments:

A cross section through the discharging well is shown in figure 7.1. The boundary conditions most commonly apply to a flowing artesian well, as is shown in this illustration.

Figure 7.2 on plate 1 is a plot of dimensionless discharge ($G(\alpha, r_w/B)$) versus dimensionless time (α) from data of Hantush (1959, table 1) and Dudley (1970, table 2). Selected values of $G(\alpha, r_w/B)$ are given in table 7.1. The corresponding data curve should be a plot of observed discharge versus time. The data curve is matched to figure 7.2 and from match points ($\alpha, G(\alpha, r_w/B)$) and (t, Q), T and S are computed from the equations

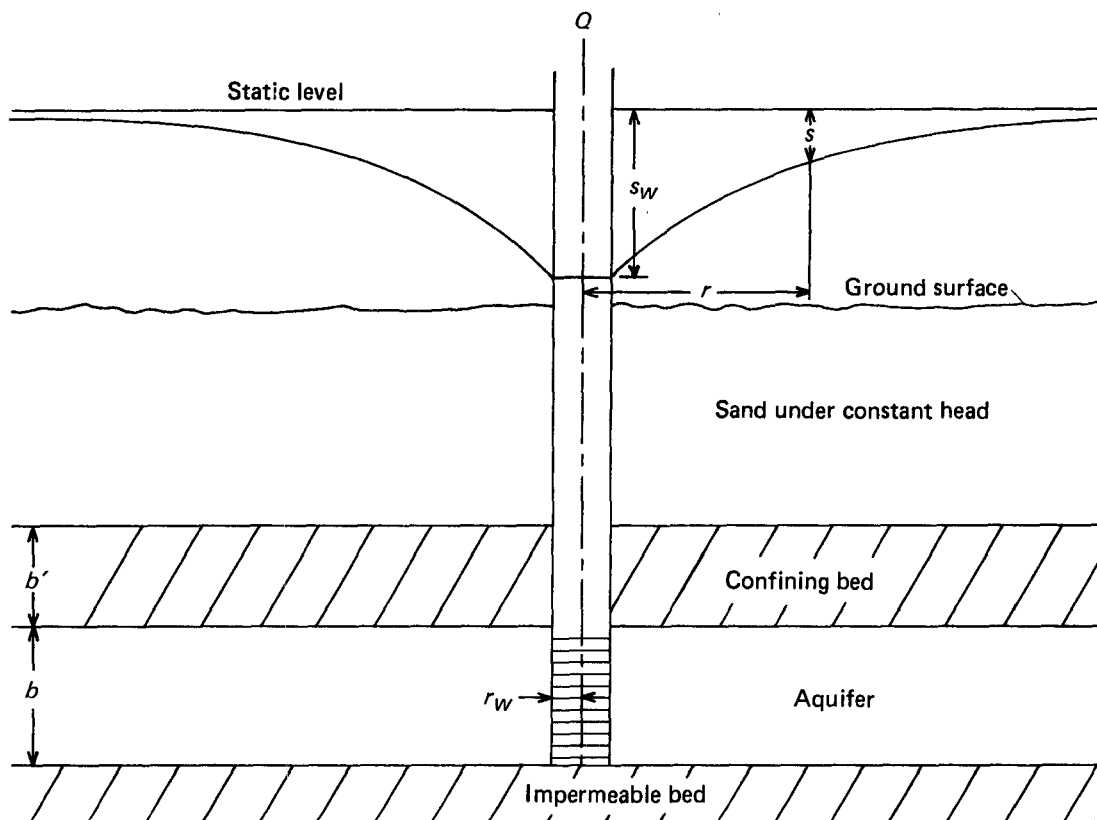


FIGURE 7.1.—Cross section through a well with constant drawdown in a leaky aquifer.

$$T = Q/(2\pi s_w G(\alpha, r_w/B))$$

and
$$S = Tt/(\alpha r_w^2).$$

Figure 7.3 on plate 1 contains plots of dimensionless drawdown (s/s_w) versus dimensionless time ($\alpha r_w^2/r^2$). The corresponding data plot would be observed drawdown versus observation time. Matching the data and type curves by superposition and choosing convenient match points ($s/s_w, \alpha r_w^2/r^2$) and (s, t), the ratio of transmissivity to storage coefficient can be computed from the relation

$$T/S = (\alpha r_w^2/r^2)(r^2/t).$$

Figure 7.3 was plotted from function values generated by a FORTRAN program. This program is listed in table 7.2. The input data for this program consist of three cards coded in specific formats. Readers unfamiliar with

FORTRAN format items should consult a FORTRAN language manual. The first card contains: the smallest value of alpha for which computation is desired, coded in format E10.5 in columns 1-10; the largest value of alpha for which computation is desired, coded in format E10.5 in columns 11-20. The output table will include a range in alpha spanning these two values up to a limiting range of nine log cycles. The second card contains 13 values of r_w/B . These coded values are the significant figures only and should be greater or equal to 1 and less than 10. The power of 10 by which each of these coded values is multiplied is calculated by the program. Zero (or blank) coding is permissible, but the first zero (or blank) value will terminate the list. The 13 values, all coded in format F5.0, are coded in columns 1-5, 6-10, 11-15, 16-20, 21-25, 26-30, 31-35, 36-40, 41-45, 46-50, 51-55, 56-60, and 61-65. The third card contains the radius of the control well and distances to the observation wells.

TABLE 7.1.—Values of $G(\alpha, r_w/B)$ [Values for $r_w/B \leq 1 \times 10^{-2}$ and $\alpha \geq 1 \times 10^2$ are from Hantush (1959, table 1), others are from Dudley (1970, table 2)]

α	r_w/B								
	0	6×10^{-3}	1×10^{-2}	2×10^{-2}	6×10^{-2}	1×10^{-1}	2×10^{-1}	6×10^{-1}	1×10^0
1×10^{-1}	2.24	2.24	2.24	2.25	2.25	2.25	2.26	2.31	2.43
2	1.71	1.71	1.71	1.71	1.72	1.72	1.73	1.81	1.96
5	1.23	1.23	1.23	1.23	1.23	1.24	1.25	1.38	1.61
1×10^0	.983	.983	.983	.984	.986	.990	1.01	1.18	1.49
2	.800	.800	.800	.801	.804	.809	.834	1.07	1.44
5	.628	.628	.628	.629	.633	.642	.682	1.01	1.43
1×10^1	.534	.534	.534	.535	.541	.554	.611		
2	.461	.461	.461	.462	.472	.491	.569		
5	.389	.389	.389	.390	.407	.438	.548		
1×10^2	.346	.346	.346	.349	.374	.417	.545		
2	.311	.311	.312	.316	.353	.408			
5	.274	.275	.276	.284	.341	.406			
1×10^3	.251	.252	.255	.266	.339				
2	.232	.234	.239	.255					
5	.210	.215	.222	.249					
1×10^4	.196	.204	.216	.248					
2	.185	.197	.213						
5	.170	.192	.212						
1×10^5	.161	.191							
2	.152								
5	.143								
1×10^6	.136								
2	.130								
5	.123	.191	.212	.248	.339	.406	.545	1.01	1.43

α	r_w/B								
	0	1×10^{-3}	2×10^{-3}	6×10^{-3}	1×10^{-2}	2×10^{-2}	6×10^{-2}	1×10^{-1}	2×10^{-1}
1×10^4	0.196	0.196	0.196	0.196	0.196	0.196	0.196	0.196	0.197
2	.185	.185	.185	.185	.185	.185	.185	.185	.185
5	.170	.170	.170	.170	.170	.170	.170	.170	.173
1×10^5	.161	.161	.161	.161	.161	.161	.162	.162	.167
2	.152	.152	.152	.152	.152	.152	.153	.155	.163
5	.143	.143	.143	.143	.143	.143	.144	.148	.161
1×10^6	.136	.136	.136	.136	.136	.137	.139	.144	.159
2	.130	.130	.130	.130	.130	.131	.135	.143	.159
5	.123	.123	.123	.123	.123	.124	.133	.142	.158
1×10^7	.118	.118	.118	.118	.118	.120			
2	.114	.114	.114	.114	.114	.116			
5	.108	.108	.108	.108	.110				
1×10^8	.104	.104	.104	.105	.108				
2	.100	.100	.101	.103	.107				
5	.0958	.0958	.0966	.102					
1×10^9	.0927	.0930	.0943						
2	.0899	.0906	.0927						
5	.0864	.0880	.0916						
1×10^{10}	.0838	.0867	.0914						
2	.0814	.0862							
5	.0785	.0860							
1×10^{11}	.0764	.0860	.0914	.102	.107	.116	.133	.142	.158
2									
5									

The control well radius (r_w) is coded first, in columns 1–8 in format F8.2. The distances (r) to the observation wells (maximum of nine) are coded next, in monotonic increasing order (smallest r first, largest r last), in columns 9–16, 17–24, 25–32, 33–40, 41–48, 49–56, 57–64, 65–72, and 73–80, all in format F8.2. If two or more observation wells have the same distance, this common distance should be coded only once, the function values will apply to all wells at the same distance from the control

well. If the number of observation wells is less than nine, the remaining columns on the card should be left blank.

The integral in equation 4 is approximated by

$$\int_0^{\infty} f(u, \alpha, r_w/B) du \doteq \sum_{i=1}^{8000} f(-\Delta u/2 + i\Delta u, \alpha, r_w/B) \Delta u .$$

This expression is a composite quadrature with equally spaced abscissas. The abscissas are chosen at the midpoints of the intervals instead of the ends because the integrand is singular at $u=0$. The value of Δu used is related to α and is $\Delta u \leq 10^{-3}/\sqrt{\alpha}$. The r_w/B values then selected by the program satisfy $r_w/B \geq 10 \Delta u$. These two constraints, though empirical, are related to the behavior of the integrand; the first constraint is related to the term e^{-au} as u becomes large, and the second to $u/(u^2+(r_w/B)^2)$ as u becomes small.

The Bessel functions $K_0(r/B)$, $K_0(r_w/B)$ are evaluated by the IBM subroutine BESK. A description of this subroutine may be found in the IBM Scientific Subroutine Package.

The Bessel functions of the second kind in the integrand, $Y_0(u)$ and $Y_0(ur/r_w)$, are evaluated using IBM subroutine BESY, which is discussed in IBM SSP manual. The Bessel functions $J_0(u)$ and $J_0(ur/r_w)$ are evaluated for arguments less than four by a polynomial approximation consisting of the first 10 terms of the series expansion

$$J_0(x) = \sum_{n=0}^{\infty} (-1)^n (x^2/2)^n / (n!)^2.$$

For arguments greater than or equal to four, the asymptotic expansion is used

$$J_0(x) = P \cos(x - \pi/4) + Q \sin(x - \pi/4).$$

P and Q are calculated by the algorithm used in IBM subroutine BESY.

The output from this program consists of tables of function values, an example of which is shown in figure 7.4.

Solution 8: Constant discharge from a fully penetrating well of finite diameter in a nonleaky aquifer

Assumptions:

1. Well discharges at a constant rate, Q .
2. Well is of finite diameter and fully penetrates the aquifer.
3. Aquifer is not leaky.
4. Discharge from the well is derived from a depletion of storage in the aquifer and inside the well bore.

Differential equation:

$$\partial^2 s / \partial r^2 + (1/r) \partial s / \partial r = (S/T) \partial s / \partial t, \quad r \geq r_w$$

This differential equation describes nonsteady radial flow in a homogeneous isotropic aquifer in the region outside the pumped well.

Boundary and initial conditions:

$$s(r_w, t) = s_w(t), \quad t > 0 \quad (1)$$

$$s(\infty, t) = 0, \quad t > 0 \quad (2)$$

$$s(r, 0) = 0, \quad r \geq r_w \quad (3)$$

$$s_w(0) = 0 \quad (4)$$

$$(2\pi r_w T) \partial s(r_w, t) / \partial r - (\pi r_w^2) \partial s_w(t) / \partial t = -Q, \quad t > 0 \quad (5)$$

Equation 1 states that the drawdown at the well bore is equal to the drawdown inside the well, assuming that there is no entrance loss at the well face. Equation 2 states that drawdown is small at a large distance from the pumping well. Equations 3 and 4 state that, initially, drawdown in the aquifer and inside the well is zero. Equation 5 states that the discharge of the well is equal to the sum of the flow into the well and the rate of decrease in storage inside the well.

Solution (Papadopoulos and Cooper, 1967; Papadopoulos, 1967):

$$s = (Q/4\pi T) F(u, \alpha, \rho),$$

where

$$F(u, \alpha, \rho) = (8\alpha/\pi) \int_0^{\infty}$$

$$\frac{[(1 - \exp(-\beta^2 \rho^2 / 4u)) [J_0(\beta \rho) A(\beta) - Y_0(\beta \rho) B(\beta)]]}{\{[A(\beta)]^2 + [B(\beta)]^2\} \beta^2} d\beta$$

and

$$B(\beta) = \beta J_0(\beta) - 2\alpha J_1(\beta),$$

$$A(\beta) = \beta Y_0(\beta) - 2\alpha Y_1(\beta),$$

$$u = r^2 S / 4Tt,$$

$$\alpha = r_w^2 S / r_c^2,$$

and

$$\rho = r / r_w.$$

J_0 and Y_0 , J_1 and Y_1 , are zero-order and first-order Bessel functions of the first and second kind, respectively.

Z(ALPHA,R/RW,RW/B), R/RW= 100.

		RW/B												
ALPHA	I	0.10E-03	0.15E-03	0.20E-03	0.30E-03	0.50E-03	0.70E-03	0.10E-02	0.15E-02	0.20E-02	0.30E-02	0.50E-02	0.70E-02	0.10E-01
0.100E	05	0.114	0.114	0.114	0.114	0.113	0.113	0.113	0.112	0.112	0.109	0.102	0.091	0.074
0.150E	05	0.142	0.142	0.142	0.141	0.141	0.141	0.141	0.140	0.138	0.134	0.122	0.107	0.082
0.200E	05	0.161	0.161	0.161	0.161	0.161	0.161	0.160	0.159	0.157	0.151	0.135	0.115	0.086
0.300E	05	0.188	0.188	0.188	0.188	0.188	0.188	0.187	0.184	0.181	0.173	0.150	0.123	0.088
0.500E	05	0.221	0.221	0.221	0.221	0.220	0.220	0.218	0.214	0.209	0.196	0.162	0.128	0.089
0.700E	05	0.242	0.242	0.242	0.241	0.241	0.240	0.237	0.232	0.225	0.208	0.167	0.130	0.089
0.100E	06	0.263	0.262	0.262	0.262	0.261	0.260	0.257	0.250	0.240	0.218	0.169	0.130	0.089
0.150E	06	0.285	0.285	0.285	0.284	0.283	0.281	0.277	0.267	0.254	0.225	0.170	0.130	0.089
0.200E	06	0.300	0.300	0.300	0.299	0.298	0.295	0.289	0.277	0.262	0.228	0.171	0.130	0.089
0.300E	06	0.321	0.321	0.320	0.319	0.317	0.313	0.305	0.289	0.269	0.231	0.171	0.130	0.089
0.500E	06	0.345	0.345	0.344	0.343	0.339	0.333	0.322	0.299	0.275	0.232	0.171	0.130	0.089
0.700E	06	0.360	0.360	0.359	0.357	0.352	0.344	0.330	0.303	0.276	0.232	0.171	0.130	0.089
0.100E	07	0.375	0.375	0.374	0.371	0.364	0.355	0.337	0.305	0.277	0.232	0.171	0.130	0.089
0.150E	07	0.391	0.391	0.389	0.386	0.376	0.364	0.342	0.306	0.277	0.232	0.171	0.130	0.089
0.200E	07	0.402	0.401	0.400	0.396	0.384	0.368	0.344	0.307	0.277	0.232	0.171	0.130	0.089
0.300E	07	0.417	0.416	0.414	0.408	0.392	0.373	0.345	0.307	0.277	0.232	0.171	0.130	0.089
0.500E	07	0.435	0.432	0.429	0.421	0.399	0.376	0.346	0.307	0.277	0.232	0.171	0.130	0.089
0.700E	07	0.445	0.442	0.438	0.427	0.401	0.376	0.346	0.307	0.277	0.232	0.171	0.130	0.089
0.100E	08	0.456	0.452	0.446	0.435	0.403	0.377	0.346	0.307	0.277	0.232	0.171	0.130	0.089
0.150E	08	0.467	0.461	0.454	0.437	0.403	0.377	0.346	0.307	0.277	0.232	0.171	0.130	0.089
0.200E	08	0.474	0.467	0.458	0.439	0.404	0.377	0.346	0.307	0.277	0.232	0.171	0.130	0.089
0.300E	08	0.483	0.473	0.462	0.440	0.404	0.377	0.346	0.307	0.277	0.232	0.171	0.130	0.089
0.500E	08	0.492	0.479	0.465	0.440	0.404	0.377	0.346	0.307	0.277	0.232	0.171	0.130	0.089
0.700E	08	0.497	0.482	0.466	0.440	0.404	0.377	0.346	0.307	0.277	0.232	0.171	0.130	0.089
0.100E	09	0.501	0.483	0.467	0.440	0.404	0.377	0.346	0.307	0.277	0.232	0.171	0.130	0.089

FIGURE 7.4.—Example of output from program for constant drawdown in a well in a leaky artesian aquifer.

The drawdown inside the pumped well is obtained at $r = r_w$ and can be expressed as (Papadopoulos and Cooper, 1967, p. 242):

$$s_w = (Q/4\pi T) F(u_w, \alpha),$$

where $F(u_w, \alpha) = F(u, \alpha, 1),$

and $u_w = r_w^2 S/4tT.$

Comments: A cross section through the discharging well is shown in figure 8.1. The geometry, except for the region of the well bore, is the same as for solution 1 (Theis solution). It is apparent from figure 8.2 and 8.3 (on plate 1) that $F(u, \alpha, \rho)$ approaches $W(u),$ the Theis solution, as time becomes large.

Papadopoulos (1967, p. 161) stated that for $t > 2.5 \times 10^3 r_c^2/T,$ or $\alpha \rho^2/u > 10^4,$ the function $F(u, \alpha, \rho) = W(u).$ Papadopoulos and Cooper (1967, p. 242) stated that for $t > 2.5 \times 10^2 r_c^2/T,$ or $\alpha/u_w > 10^3,$ the function $F(u_w, \alpha)$ can be closely approximated by $F(u_w, \alpha) = W(u_w).$ An examination of the type curves and function values indicates that $F(u_w, \alpha) \approx W(u_w)$ (less than 5-percent error) for $\alpha/u_w > 10^2,$ and hence t should only be greater than $25 r_c^2/T$ for drawdown in the pumped well.

Figures 8.2 and 8.3 were prepared from function values given in Papadopoulos and Cooper (1967) and Papadopoulos (1967), which are reproduced in table 8.1. For drawdown observations in the pumped well, the method of analysis is to plot drawdown versus time and

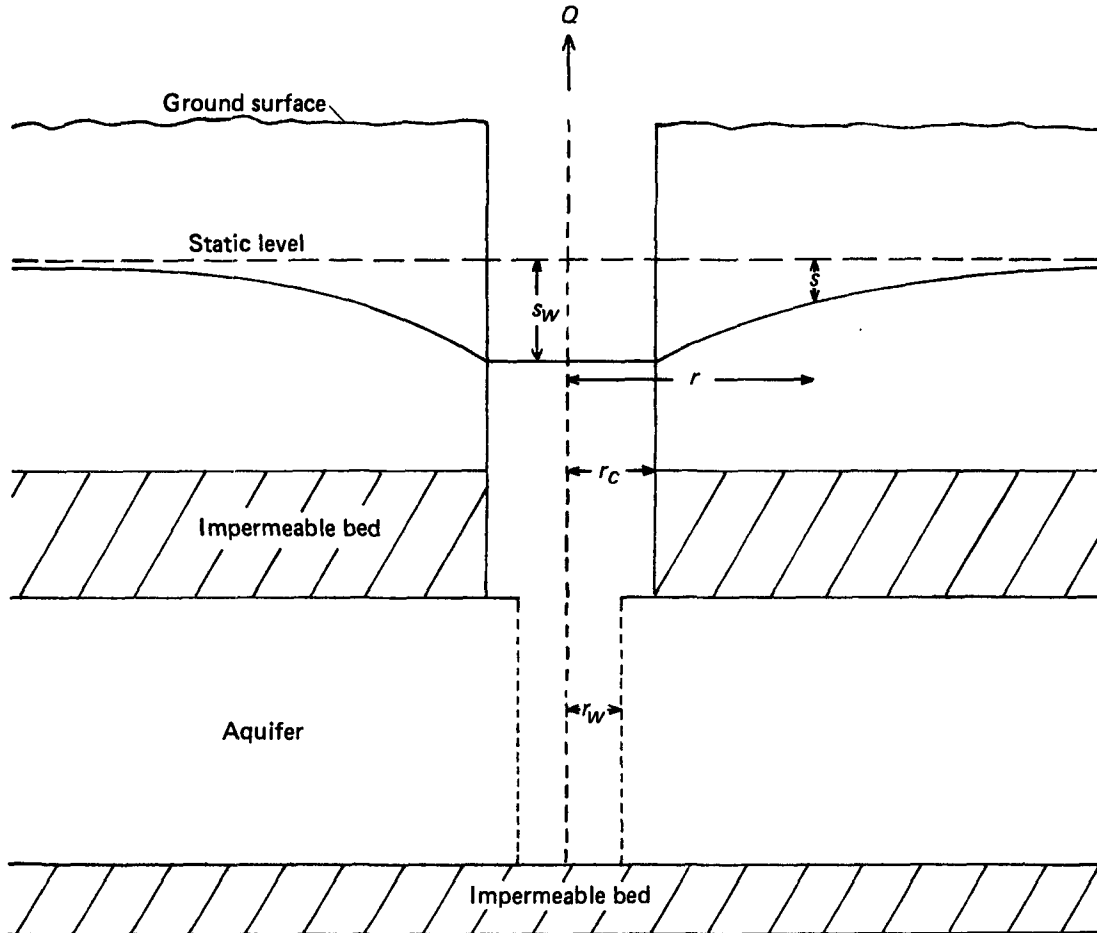


FIGURE 8.1.—Cross section through a discharging well of finite diameter.

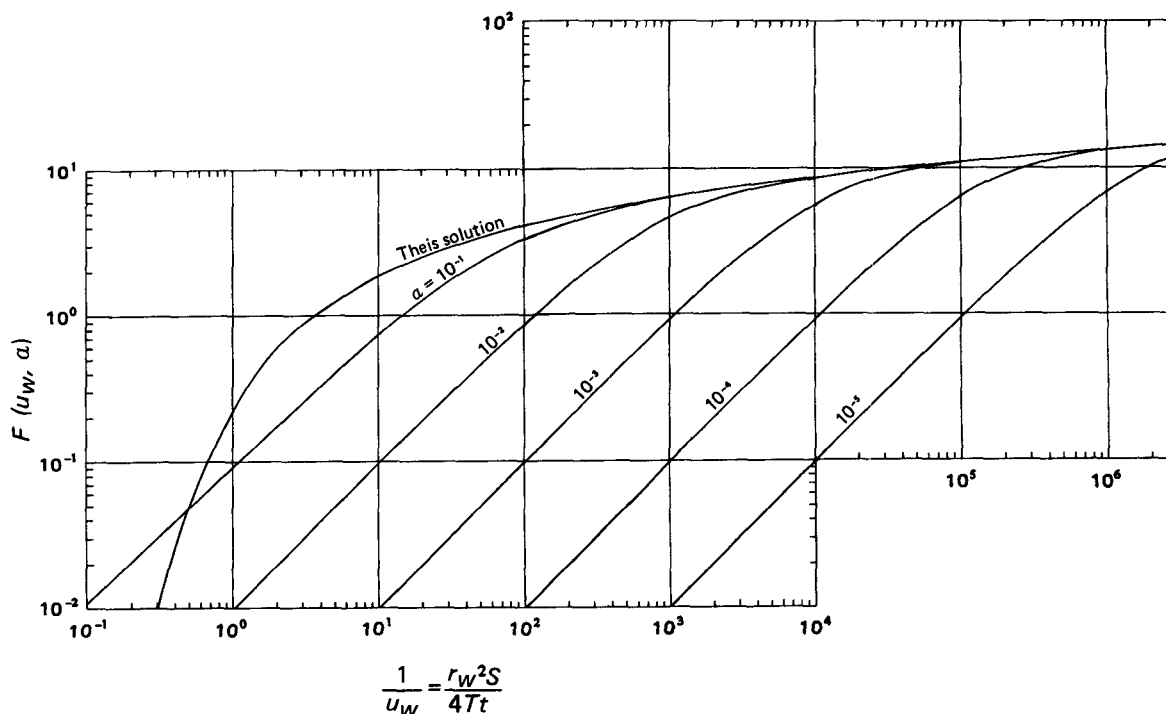


FIGURE 8.2.—Five selected type curves of $F(u_w, \alpha)$, and the Theis solution, versus $1/u_w$.

then superimpose the plot on figure 8.2. After match points of (s, t) and $(F(u_w, \alpha), 1/u_w)$ are chosen, the transmissivity can be computed from the relation $T = (Q/4\pi s) F(u_w, \alpha)$. Then, the storage coefficient can be determined from $S = (4 T t / r_w^2) / (1/u_w)$.

For observations not in the pumped well, two procedures are available for analyzing the data. To analyze the data from a single observation well, a family of type curves of $F(u, \alpha, \rho)$ versus $1/u$ for different values of α can be plotted for the ρ value appropriate for the observation well, using values in table 8.1. This procedure produces a family of type curves similar to that shown for $\rho = 1$ in figure 8.2. If ρ for the observation well is between ρ values in table 8.1, function values can be interpolated. Using this approach, the data for the observation well are plotted as drawdown versus time and matched to the best-fitting member of the plotted type curves. Transmissivity and storage coefficient can be calculated from $T = (Q/4\pi s) F(u, \alpha, \rho)$ and $S = (4 T t / r^2) / (1/u)$.

Drawdowns at more than one observation point may be combined by preparing a composite plot of the drawdowns at each observation

well versus t/r^2 . This composite plot would be analyzed by matching it to a family of type curves of $F(u, \alpha, \rho)$ versus $1/u$ for constant α . An example of such a type-curve family for $\alpha = 10^{-4}$ is shown in figure 8.3. This method requires multiple sheets of type curves, one sheet for each value of α . When the data curves are matched to the type-curve family, care should be taken to insure that the data for each well fall on the type curve having the appropriate ρ value. This will be possible for all the data for only one value of α . Transmissivity and storage coefficient are calculated from $T = (Q/4\pi s) F(u, \alpha, \rho)$ and $S = 4 T (t/r^2) / (1/u)$.

In both of these methods of plotting and comparing data, an alternate computation of storage coefficient is $S = r_c^2 \alpha / r_w^2$. However, as pointed out by Papadopoulos and Cooper (1967, p. 244), the shapes of type curves differ only slightly when α changes by an order of magnitude, therefore the determination of S is sensitive to choosing the "correct" curve. Papadopoulos and Cooper (1967, p. 244) suggest that if S can be estimated within an order of magnitude, the value of α to be used for matching the data can be decided.

TABLE 8.1.—Values of the function $F(u, \alpha, \rho)$

[Values for $\rho = 1$ from Papadopoulos and Cooper, 1967. Other values from Papadopoulos, 1967]

u	ρ							
	1	2	5	10	20	50	100	200
For $\alpha = 10^{-1}$								
2×10^0	4.88×10^{-2}	1.96×10^{-2}	1.75×10^{-2}	2.41×10^{-2}	3.48×10^{-2}	4.24×10^{-2}	4.48×10^{-2}	4.50×10^{-2}
1	9.19	7.01	9.55	1.41×10^{-1}	1.85×10^{-1}	2.09×10^{-1}	2.14×10^{-1}	2.15×10^{-1}
5×10^{-1}	1.77×10^{-1}	1.95×10^{-1}	3.21×10^{-1}	4.44	5.20	5.49	5.55	5.59
2	4.06	5.78	9.42	1.13×10^0	1.19×10^0	1.22×10^0		
1	7.34	1.11×10^0	1.60×10^0	1.76	1.80			
5×10^{-2}	1.26×10^0	1.84	2.33	2.43	2.46			
2	2.30	2.97	3.28	3.34	3.35			
1	3.28	3.81	4.00	4.03				
5×10^{-3}	4.26	4.60	4.70	4.72				
2	5.42	5.58	5.63	5.64				
1	6.21	6.30	6.33					
5×10^{-4}	6.96	7.01						
2	7.87	7.93						
1	8.57	8.63						
5×10^{-5}	9.32							
2	10.24							
For $\alpha = 10^{-2}$								
2×10^0	4.99×10^{-3}	2.13×10^{-3}	2.11×10^{-3}	3.52×10^{-3}	7.47×10^{-3}	2.03×10^{-2}	3.44×10^{-2}	4.35×10^{-2}
1	9.91	7.99	1.32×10^{-2}	2.69×10^{-2}	6.12×10^{-2}	1.42×10^{-1}	1.91×10^{-1}	2.11×10^{-1}
5×10^{-1}	1.97×10^{-2}	2.40×10^{-2}	5.40	1.21×10^{-1}	2.63×10^{-1}	4.65	5.31	5.51
2	4.89	8.34	2.33×10^{-1}	5.12	9.15	1.16×10^0	1.20×10^0	1.22×10^0
1	9.67	1.93×10^{-1}	5.67	1.12×10^0	1.58×10^0	1.78	1.81	
5×10^{-2}	1.90×10^{-1}	4.16	1.18×10^0	1.95	2.32	2.44	2.46	
2	4.53	1.03×10^0	2.42	3.11	3.29	3.34	3.35	
1	8.52	1.87	3.48	3.90	4.00	4.03		
5×10^{-3}	1.54×10^0	3.05	4.43	4.65	4.71	4.72		
2	3.04	4.78	5.52	5.61	5.63	5.64		
1	4.55	5.90	6.27	6.31	6.33			
5×10^{-4}	6.03	6.81	6.99	7.01				
2	7.56	7.85	7.92	7.94				
1	8.44	8.59	8.63					
5×10^{-5}	9.23	9.30						
2	10.20	10.23						
1	10.87	10.93						
5×10^{-6}	11.62	11.63						
2	12.54							
1	13.24							

TABLE 8.1.—Values of the function $F(u, \alpha, \rho)$ —Continued

u	ρ							
	1	2	5	10	20	50	100	200
For $\alpha = 10^{-3}$								
2×10^0	5.00×10^{-4}	2.15×10^{-4}	2.15×10^{-4}	3.70×10^{-4}	8.35×10^{-3}	3.05×10^{-3}	8.38×10^{-3}	1.50×10^{-2}
1	9.99	8.11	1.37×10^{-3}	2.95×10^{-3}	7.58×10^{-3}	2.81×10^{-2}	7.56×10^{-2}	1.47×10^{-1}
5×10^{-1}	2.00×10^{-3}	2.45×10^{-3}	5.77	1.42×10^{-2}	3.90×10^{-2}	1.54×10^{-1}	3.23×10^{-1}	4.78
2	4.99	8.71	2.67×10^{-2}	7.24	2.03×10^{-1}	6.59	1.02×10^0	1.17×10^0
1	9.97	2.07×10^{-2}	7.16	2.01×10^{-1}	5.41	1.38×10^0	1.70	1.79
5×10^{-2}	1.99×10^{-2}	4.66	1.74×10^{-1}	4.87	1.19×10^0	2.27	2.40	2.45
2	4.95	1.29×10^{-1}	5.05	1.31×10^0	2.52	3.22	3.32	3.35
1	9.83	2.70	1.04×10^0	2.38	3.59	3.96	4.02	
5×10^{-3}	1.95×10^{-1}	5.47	1.96	3.68	4.50	4.69	4.72	
2	4.73	1.31×10^0	3.81	5.23	5.55	5.63	5.64	
1	9.07	2.39	5.34	6.13	6.28	6.32		
5×10^{-4}	1.69×10^0	3.98	6.57	6.92	7.00	7.02		
2	3.52	6.44	7.77	7.90	7.93			
1	5.53	7.95	8.55	8.61	8.63			
5×10^{-5}	7.63	9.02	9.28	9.31				
2	9.68	10.12	10.22	10.24				
1	10.68	10.88	10.93					
5×10^{-6}	11.50	11.59	11.62					
2	12.49	12.53	12.54					
1	13.21	13.23	13.24					
5×10^{-7}	13.92	13.93						
2	14.84							
1	15.54							
For $\alpha = 10^{-4}$								
2×10^0	5.00×10^{-5}	2.17×10^{-5}	2.18×10^{-5}	3.73×10^{-5}	8.46×10^{-5}	3.16×10^{-4}	9.56×10^{-4}	3.83×10^{-3}
1	1.00×10^{-4}	8.15	1.38×10^{-4}	2.98×10^{-4}	7.77×10^{-4}	3.23×10^{-3}	1.01×10^{-2}	3.42×10^{-2}
5×10^{-1}	2.00	2.47×10^{-4}	5.81	1.45×10^{-3}	4.10×10^{-3}	1.80×10^{-2}	5.62	1.75×10^{-1}
2	5.00	8.76	2.71×10^{-3}	7.54	2.27×10^{-2}	1.03×10^{-1}	3.04×10^{-1}	7.10
1	1.00×10^{-3}	2.09×10^{-3}	7.34	2.16×10^{-2}	6.69	2.97	7.92	1.43×10^0
5×10^{-2}	2.00	4.72	1.82×10^{-2}	5.55	1.74×10^{-1}	7.30	1.62×10^0	2.24
2	5.00	1.32×10^{-2}	5.56	1.74×10^{-1}	5.36	1.87×10^{-0}	2.95	3.28
1	9.98	2.81	1.23×10^{-1}	3.86	1.14×10^0	3.08	3.84	4.02
5×10^{-3}	1.99×10^{-2}	5.88	2.64	8.13	2.17	4.25	4.63	4.71
2	4.97	1.53×10^{-1}	6.89	1.97×10^0	4.14	5.47	5.60	5.63
1	9.90	3.10	1.36×10^0	3.44	5.61	6.24	6.31	6.33
5×10^{-4}	1.97×10^{-1}	6.18	2.53	5.26	6.71	6.98	7.01	
2	4.81	1.48×10^0	4.95	7.33	7.82	7.92	7.94	
1	9.34	2.72	7.03	8.37	8.57	8.62		
5×10^{-5}	1.77×10^0	4.65	8.65	9.20	9.29	9.32		
2	3.83	7.87	10.02	10.19	10.23	10.24		
1	6.25	9.92	10.83	10.91	10.93			
5×10^{-6}	8.99	11.23	11.57	11.62	11.63			

2	11.74	12.40	12.52	12.54
1	12.91	13.17	13.23	13.24
5 × 10 ⁻⁷	13.78	13.90	13.93	
2	14.79	14.83		
1	15.51	15.53		
5 × 10 ⁻⁸	16.22	16.23		
2	17.14			
1	17.84			

For $\alpha = 10^{-5}$

2 × 10 ⁰	5.00 × 10 ⁻⁶	2.27 × 10 ⁻⁶	2.48 × 10 ⁻⁶	4.19 × 10 ⁻⁶	9.00 × 10 ⁻⁶	3.21 × 10 ⁻⁵	9.77 × 10 ⁻⁵	3.15 × 10 ⁻⁴
1	1.00 × 10 ⁻⁵	8.36	1.44 × 10 ⁻⁵	3.07 × 10 ⁻⁵	7.89 × 10 ⁻⁵	3.27 × 10 ⁻⁴	1.04 × 10 ⁻³	3.44 × 10 ⁻³
5 × 10 ⁻¹	2.00	2.51 × 10 ⁻⁵	5.94	1.47 × 10 ⁻⁴	4.14 × 10 ⁻⁴	1.84 × 10 ⁻³	6.02	2.00 × 10 ⁻²
2	5.00	8.87	2.74 × 10 ⁻⁴	7.61	2.31 × 10 ⁻³	1.08 × 10 ⁻²	3.61 × 10 ⁻²	1.19 × 10 ⁻¹
1	1.00 × 10 ⁻¹	2.11 × 10 ⁻⁴	7.42	2.18 × 10 ⁻³	6.85	3.30	1.10 × 10 ⁻¹	3.50
5 × 10 ⁻²	2.00	4.77	1.84 × 10 ⁻³	5.65	1.82 × 10 ⁻²	8.90	2.92	8.57
2	5.00	1.34 × 10 ⁻³	5.64	1.80 × 10 ⁻²	5.92	2.89 × 10 ⁻¹	8.91	2.12 × 10 ⁰
1	1.00 × 10 ⁻²	2.84	1.26 × 10 ⁻²	4.09	1.36 × 10 ⁻¹	6.49	1.80 × 10 ⁰	3.34
5 × 10 ⁻³	2.00	5.96	2.74	9.03	3.01	1.35 × 10 ⁰	3.14	4.40
2	5.00	1.56 × 10 ⁻²	7.43	2.47 × 10 ⁻¹	8.06	3.03	5.01	5.52
1	9.99	3.20	1.55 × 10 ⁻¹	5.15	1.60 × 10 ⁰	4.75	6.06	6.27
5 × 10 ⁻⁴	2.00 × 10 ⁻²	6.54	3.20	1.04 × 10 ⁰	2.96	6.31	6.90	6.99
2	4.98	1.66 × 10 ⁻¹	8.08	2.45	5.58	7.71	7.89	7.93
1	9.93	3.34	1.58 × 10 ⁰	4.28	7.54	8.52	8.61	8.63
5 × 10 ⁻⁵	1.98 × 10 ⁻¹	6.62	2.93	6.63	8.90	9.21	9.31	
2	4.86	1.59 × 10 ⁰	5.86	9.36	10.10	10.22	10.24	
1	9.49	2.95	8.53	10.60	10.86	10.92		
5 × 10 ⁻⁶	1.82 × 10 ⁰	5.15	10.67	11.48	11.59	11.62		
2	4.03	9.08	12.28	12.49	12.53	12.54		
1	6.78	11.76	13.12	13.21	13.23	13.24		
5 × 10 ⁻⁷	10.13	13.41	13.88	13.92	13.93			
2	13.71	14.68	14.83	14.85				
1	15.13	15.46	15.54					
5 × 10 ⁻⁸	16.05	16.20						
2	17.08	17.14						
1	17.81	17.84						
5 × 10 ⁻⁹	18.51							
2	19.40							
1	20.15							

The early parts (short time) of the curves in figure 8.2 are straight lines. According to Papadopoulos and Cooper (1967, p. 244), these represent conditions under which all the water pumped is derived from storage within the well. The straight lines approached by the curves satisfy the equations

$$F(u_w, \alpha) = \alpha/u_w$$

and

$$s_w = Qt/\pi r_c^2 = \frac{\text{volume of water discharged}}{\text{area of well}}$$

Therefore, as pointed out by Papadopoulos and Cooper (1967, p. 244), data that fall on this straight part of the type curves do not indicate information about the aquifer characteristics.

Table 8.2 is a listing of two FORTRAN programs by S. S. Papadopoulos that evaluate

$F(u_w, \alpha)$ and $F(u, \alpha, \rho)$. The input data to both programs consists of cards coded in specified format (readers unfamiliar with FORTRAN language format should refer to a FORTRAN language manual). Input to the programs is one or more groups of data, each group of data consisting of two cards. The first card contains one value of alpha in columns 1-10, coded in format E10.5. The program to evaluate $F(u, \alpha, \rho)$ also requires a value of rho on this card in columns 11-20. This value of rho, which must be greater than one, is also coded in format E10.5. The second card contains 16 values of u coded in columns 1-5, 6-10, . . . , 75-80 in format 16F5.0. The $F(u_w, \alpha)$ or $F(u, \alpha, \rho)$ values will be printed in the order that the u values are coded. If less than 16 values of u are desired, the remaining columns on the card may be left blank. Outputs from these two programs are shown in figures 8.4 and 8.5.

F(UW, ALPHA) FOR ALPHA= 1.00000E-04

UW	INTEGRAL	INTEGRAL ERROR	F(UW, ALPHA)	X (PEAK)	Y (PEAK)
2.00000E 00	1.54210E 03	-6.98844E-02	4.99991E-05	5.96561E-03	5.55886E 05
1.00000E 00	3.08412E 03	-1.39817E-01	9.99956E-05	5.96561E-03	1.11177E 06
5.00000E-01	6.16789E 03	-2.74775E-01	1.99980E-04	5.96561E-03	2.22353E 06
2.00000E-01	1.54184E 04	-6.97533E-01	4.99907E-04	5.96561E-03	5.55875E 06
1.00000E-01	3.08331E 04	-1.39715E 00	9.99695E-04	5.96560E-03	1.11173E 07
5.00000E-02	6.16529E 04	-2.71364E 00	1.99896E-03	5.96559E-03	2.22335E 07
2.00000E-02	1.54061E 05	-6.97112E 00	4.99507E-03	5.96559E-03	5.55764E 07
1.00000E-02	3.07919E 05	-1.39383E 01	9.98359E-03	5.96554E-03	1.11128E 08
5.00000E-03	6.15138E 05	-2.78767E 01	1.99445E-02	5.96549E-03	2.22157E 08
2.00000E-03	1.53334E 06	-6.82757E 01	4.97152E-02	5.96527E-03	5.54652E 08
1.00000E-03	3.05367E 06	-1.38658E 02	9.90083E-02	5.96493E-03	1.10684E 09
5.00000E-04	6.06085E 06	-2.76458E 02	1.96509E-01	5.96425E-03	2.20389E 09
2.00000E-04	1.48475E 07	-6.79220E 02	4.81397E-01	5.96223E-03	5.43712E 09
1.00000E-04	2.88072E 07	-1.30780E 03	9.34008E-01	5.95886E-03	1.06380E 10
5.00000E-05	5.45352E 07	-2.50960E 03	1.76818E 00	5.95237E-03	2.03734E 10
2.00000E-05	1.18065E 08	-5.40026E 03	3.82800E 00	5.93415E-03	4.49196E 10

FIGURE 8.4.—Example of output from program for drawdown inside a well of finite diameter due to constant discharge.

F(U, ALPHA, RHO) FOR ALPHA= 1.00000E-05, RHO= 2.00000E 00

U	INTEGRAL	INTEGRAL ERROR	F(U, ALPHA, RHO)
9.99999900E-04	6.29273600E 02	5.45096700E-01	3.20486300E-02
5.00000000E-04	1.28359500E 03	1.11649700E 00	6.53728800E-02
1.99999900E-04	3.26376700E 03	2.47402200E 00	1.66222200E-01
1.00000000E-04	6.55423000E 03	3.31468400E 00	3.33803700E-01
5.00000000E-05	1.30015800E 04	3.53750700E 00	6.62164900E-01
2.00000000E-05	3.11692500E 04	3.54940500E 00	1.58743500E 00
9.99999900E-06	5.79505700E 04	3.54602200E 00	2.95139600E 00
4.99999900E-06	1.01023500E 05	3.53222000E 00	5.14508300E 00
1.99999900E-06	1.78237100E 05	3.62180400E 00	9.07753300E 00
1.00000000E-06	2.30897600E 05	3.66347000E 00	1.17595100E 01
4.99999900E-07	2.63222100E 05	3.68847000E 00	1.34057800E 01
1.99999900E-07	2.88201800E 05	3.52180300E 00	1.46779900E 01

FIGURE 8.5.—Example of output from program for drawdown outside a well of finite diameter due to constant discharge.

Solution 9: Slug test for a finite-diameter well in a nonleaky aquifer

Assumptions:

1. A volume of water, V , is injected into, or is discharged from, the well instantaneously at $t=0$.
2. Well is of finite diameter and fully penetrates the aquifer.
3. Aquifer is not leaky, and flow is in radial direction only.

Differential equation:

$$\partial^2 h / \partial r^2 + (1/r) \partial h / \partial r = (S/T) \partial h / \partial t, \quad r > r_w$$

This differential equation describes nonsteady radial flow in a homogeneous isotropic aquifer beyond the radius of the injected well.

Boundary and initial conditions:

$$h(r_w, t) = H(t), \quad t > 0 \quad (1)$$

$$h(\infty, t) = 0, \quad t > 0 \quad (2)$$

$$2\pi r_w T \frac{\partial h(r_w, t)}{\partial r} = \pi r_w^2 \frac{\partial H(t)}{\partial t}, \quad t > 0 \quad (3)$$

$$h(r, 0) = 0, \quad r > r_w \quad (4)$$

$$H(0) = H_0 = V / \pi r_w^2 \quad (5)$$

Equation 1 states that the head change in the aquifer at the face of the well is equal to that inside the well; one assumes that there is no exit loss at the well face. Equation 2 states that the head change approaches zero as distance from the discharging well approaches infinity, a condition which will be approximated if boundaries of the aquifer are sufficiently distant from the discharging well. Equation 3 states that near the well the radial flow is equal to the rate of change in volume of water inside the well. Equations 4 and 5 state that initially the head change is zero in the aquifer, and the head increase or decrease inside the well is equal to H_0 .

Solution (Cooper and others, 1967):

$$h = (2H_0/\pi) \int_0^\infty (\exp(-\beta u^2/\alpha) \{ J_0(ur/r_w) \cdot [uY_0(u) - 2\alpha Y_1(u)] - Y_0(ur/r_w) \cdot [uJ_0(u) - 2\alpha J_1(u)] \} / \Delta(u)) du, \quad (6)$$

$$\text{where} \quad \alpha = r_w^2 S / r_c^2, \\ \beta = Tt / r_c^2,$$

$$\text{and} \quad \Delta(u) = [uJ_0(u) - 2\alpha J_1(u)]^2 \\ + [uY_0(u) - 2\alpha Y_1(u)]^2.$$

J_0 and Y_0 , J_1 and Y_1 , are zero-order and first-order Bessel functions of the first and second kind, respectively.

The head, H , inside the well, obtained by substituting $r=r_w$ in equation (6) is

$$H/H_0 = F(\beta, \alpha),$$

where

$$F(\beta, \alpha) = (8\alpha/\pi^2) \int_0^\infty (\exp(-\beta u^2/\alpha) / u \Delta(u)) du$$

and where α , β , $\Delta(u)$ are as defined previously.

Comments: Figure 9.1 is a cross section showing geometric configuration along the well bore. The volume of water injected into or discharged from the well is $\pi r_w^2 H_0$. The water-level data in the injected well, expressed as a fraction of H_0 , is plotted versus time on semi-logarithmic graph paper. This plot is superimposed on figure 9.2, keeping the baselines the same and sliding horizontally until a match or interpolated fit is made. A match point for β , t , and α is picked from the two graphs. Transmissivity is calculated from $T = \beta r_c^2 / t$ and storage coefficient from $S = \alpha r_c^2 / r_w^2$. As pointed out by Cooper, Bredehoeft, and Papadopoulos (1967, p. 267), the determination of S by this method has questionable reliability because of the similar shape of the curves, whereas the determination of T is not as sensitive to choosing the correct curve. Figure 9.2 on plate 1 is plotted from data in table 9.1, which contains original material from two sources (Cooper and others, 1967; and Papadopoulos and others, 1973).

Table 9.2 is a listing of a FORTRAN program by S. S. Papadopoulos that evaluates $F(\beta, \alpha)$. Input to the program consists of cards coded in a specific format (readers unfamiliar with FORTRAN formats should refer to a FORTRAN language manual). Input consists of two or more cards, each containing a single value of

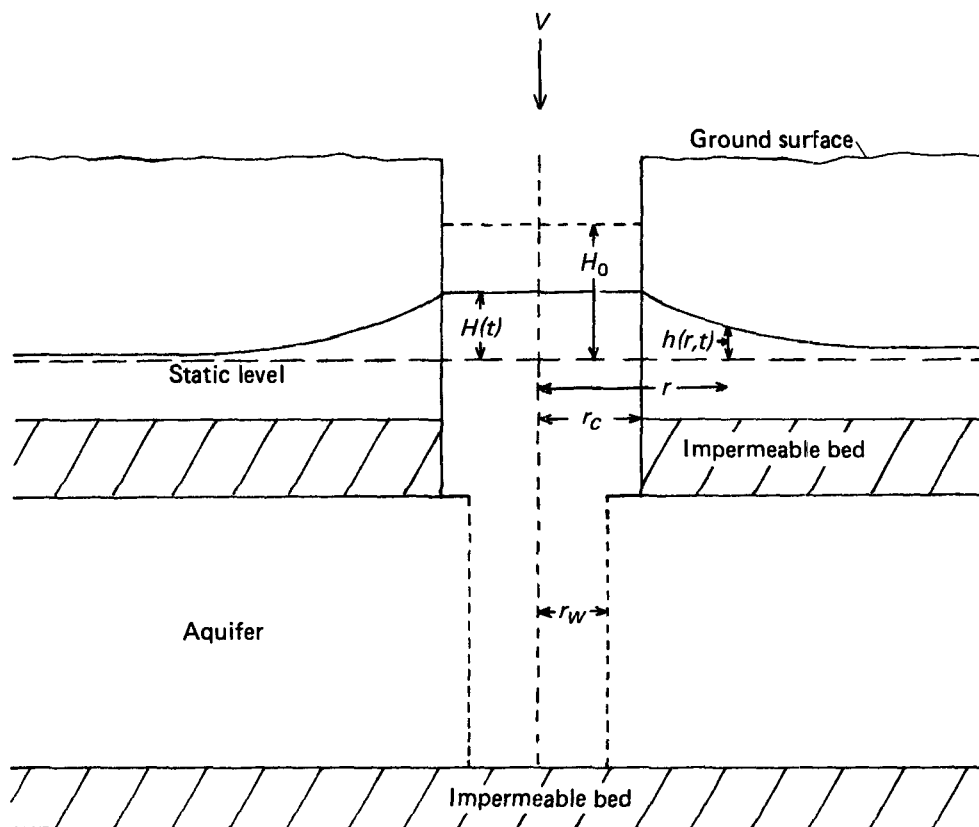


FIGURE 9.1.—Cross section through a well in which a slug of water is suddenly injected.

α coded in format F16.5. The first $\alpha \leq 0$ will signal program termination. Output from the program is shown in figure 9.3.

Solution 10: Constant discharge from a fully penetrating well in an aquifer that is anisotropic in the horizontal plane

Assumptions:

1. Well discharges at a constant rate, Q .
2. Well is of infinitesimal diameter and fully penetrates the aquifer.
3. Aquifer is anisotropic in the horizontal plane.
4. Aquifer is not leaky.
5. The transmissivity of the aquifer, T , is a two-dimensional symmetric tensor.

Differential equation:

$$T_{xx} \partial^2 s / \partial x^2 + 2T_{xy} \partial^2 s / \partial x \partial y + T_{yy} \partial^2 s / \partial y^2 + Q \delta(x) \delta(y) = S \partial s / \partial t.$$

This differential equation describes nonsteady flow in a homogeneous anisotropic aquifer with a constantly discharging well at $x=y=0$. The Dirac delta function is represented as $\delta(z)$ and has the following properties: $\delta(z)=0$ if $z \neq 0$ and $\int_{-\infty}^{\infty} \delta(z) dz = 1$.

Boundary and initial conditions:

$$s(x, y, 0) = 0 \quad (1)$$

$$s(\pm\infty, y, t) = 0 \quad (2)$$

$$s(x, \pm\infty, t) = 0 \quad (3)$$

TABLE 9.1.—Values of H/H_0

From Cooper, Bredehoeft, and Papadopoulos, 1967						
Tt/r^2	α	10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-5}
10^{-3}	1.00	0.9771	0.9920	0.9969	0.9985	0.9992
	2.15	.9658	.9876	.9949	.9974	.9985
	4.64	.9490	.9807	.9914	.9954	.9970
10^{-2}	1.00	.9238	.9693	.9853	.9915	.9942
	2.15	.8860	.9505	.9744	.9841	.9883
	4.64	.8293	.9187	.9545	.9701	.9781
10^{-1}	1.00	.7460	.8655	.9183	.9434	.9572
	2.15	.6289	.7782	.8538	.8935	.9167
	4.64	.4782	.6436	.7436	.8031	.8410
10^0	1.00	.3117	.4598	.5729	.6520	.7080
	2.15	.1665	.2597	.3543	.4364	.5038
	4.64	.07415	.1086	.1554	.2082	.2620
	7.00	.04625	.06204	.08519	.1161	.1521
	1.00	.03065	.03780	.04821	.06355	.08378
10^1	1.40	.02092	.02414	.02844	.03492	.04426
	2.15	.01297	.01414	.01545	.01723	.01999
	3.00	.009070	.009615	.01016	.01083	.01169
	4.64	.005711	.004919	.006111	.006319	.006554
	7.00	.003722	.003809	.003884	.003962	.004046
10^2	1.00	.002577	.002618	.002653	.002688	.002725
	2.15	.001179	.001187	.001194	.001201	.001208

From Papadopoulos, Bredehoeft, and Cooper, 1973						
Tt/r^2	α	10^{-6}	10^{-7}	10^{-8}	10^{-9}	10^{-10}
10^{-3}	1	0.9994	0.9996	0.9996	0.9997	0.9997
	2	.9989	.9992	.9993	.9994	.9995
	4	.9980	.9985	.9987	.9989	.9991
	6	.9972	.9978	.9982	.9984	.9986
10^{-2}	8	.9964	.9971	.9976	.9980	.9982
	1	.9956	.9965	.9971	.9975	.9978
	2	.9919	.9934	.9944	.9952	.9958
	4	.9848	.9875	.9894	.9908	.9919
10^{-1}	6	.9782	.9819	.9846	.9866	.9881
	8	.9718	.9765	.9799	.9824	.9844
	1	.9655	.9712	.9753	.9784	.9807
	2	.9361	.9459	.9532	.9587	.9631
10^0	4	.8828	.8995	.9122	.9220	.9298
	6	.8345	.8569	.8741	.8875	.8984
	8	.7901	.8173	.8383	.8550	.8686
	1	.7489	.7801	.8045	.8240	.8401
10^1	2	.5800	.6235	.6591	.6889	.7139
	3	.4554	.5033	.5442	.5792	.6096
	4	.3613	.4093	.4517	.4891	.5222
	5	.2893	.3351	.3768	.4146	.4487
	6	.2337	.2759	.3157	.3525	.3865
	7	.1903	.2285	.2655	.3007	.3337
	8	.1562	.1903	.2243	.2573	.2888
	9	.1292	.1594	.1902	.2208	.2505
	1	.1078	.1343	.1620	.1900	.2178
10^2	2	.02720	.03343	.04129	.05071	.06149
	3	.01286	.01448	.01667	.01956	.02320
	4	.008337	.008898	.009637	.01062	.01190
	5	.006209	.006470	.006789	.007192	.007709
	6	.004961	.005111	.005283	.005487	.005735
	8	.003547	.003617	.003691	.003773	.003863
10^3	1	.002763	.002803	.002845	.002890	.002938
	2	.001313	.001322	.001330	.001339	.001348

F(BETA, ALPHA) FOR ALPHA= 1.00D-01

BETA	H/H0
1.00D-03	0.9769
2.00D-03	0.9670
4.00D-03	0.9528
6.00D-03	0.9417
8.00D-03	0.9322
1.00D-02	0.9238
2.00D-02	0.8904
4.00D-02	0.8421
6.00D-02	0.8048
8.00D-02	0.7734
1.00D-01	0.7459
2.00D-01	0.6418
4.00D-01	0.5095
6.00D-01	0.4227
8.00D-01	0.3598
1.00D 00	0.3117
2.00D 00	0.1786
3.00D 00	0.1196
4.00D 00	0.0876
5.00D 00	0.0681
6.00D 00	0.0553
7.00D 00	0.0463
8.00D 00	0.0396
9.00D 00	0.0346
1.00D 01	0.0306
2.00D 01	0.0141
3.00D 01	0.0091
4.00D 01	0.0067
5.00D 01	0.0053
6.00D 01	0.0044
7.00D 01	0.0037
8.00D 01	0.0032
9.00D 01	0.0029
1.00D 02	0.0026
2.00D 02	0.0013
4.00D 02	0.0006
6.00D 02	0.0004
8.00D 02	0.0003
1.00D 03	0.0003

FIGURE 9.3.—Example of output from program to compute change in water level due to sudden injection of a slug of water into a well.

Equation 1 states that, initially, drawdown is zero. Equations 2 and 3 state that the drawdown approaches zero as distance from the discharging well approaches infinity, a condition which will be approximated if boundaries of the aquifer are sufficiently distant from the discharging well.

Solution (Papadopoulos, 1965, p. 23):

$$s = (Q/4\pi\sqrt{T_{xx}T_{yy}-T_{xy}^2}) W(u_{xy}), \quad (4)$$

where

$$W(u) = \int_u^\infty (e^{-v}/v) dv$$

and

$$u_{xy} = (S/4t)(T_{xx}y^2 + T_{yy}x^2 - 2T_{xy}xy)/(T_{xx}T_{yy} - T_{xy}^2). \quad (5)$$

If the coordinate axes x and y are the same as the principal axes ϵ and η (fig. 10.1) of the transmissivity tensor, the preceding equation for drawdown becomes

$$s = (Q/4\pi\sqrt{T_{\epsilon\epsilon}T_{\eta\eta}}) W(u_{\epsilon\eta}),$$

where

$$u_{\epsilon\eta} = (S/4t)(T_{\epsilon\epsilon}n^2 + T_{\eta\eta}\epsilon^2)/T_{\epsilon\epsilon}T_{\eta\eta}.$$

Comments: The method of type-curve solution as outlined by Papadopoulos (1965, p. 26) requires observation of drawdown in at least three observation wells. First, choose a convenient rectangular coordinate system with the pumped well at the origin. Then, plot the observed drawdown versus t on logarithmic paper. Match these plots to the $W(u)$ type curve given in solution 1. Choose a match point of (t, s) and $(1/u_{xy}, W(u_{xy}))$ for each well and compute $T_{xx}T_{yy} - T_{xy}^2 = (QW(u_{xy})/4\pi s)^2$ for each well. Match points for all observation wells should yield approximately the same value of $(T_{xx}T_{yy} - T_{xy}^2)$. Usually they will not and judgment must be used to obtain an "average" value. Substituting this value and the three values of (x, y) in equation 5 gives three equations in three unknowns ST_{xx} , ST_{yy} , and ST_{xy} . These equations are of the form

$$\begin{aligned} y^2(ST_{xx}) + x^2(ST_{yy}) - 2xy(ST_{xy}) \\ = 4tu_{xy}(T_{xx}T_{yy} - T_{xy}^2). \end{aligned}$$

Solve these three equations to determine T_{xx} , T_{xy} , and T_{yy} in terms of S , and S may be determined from

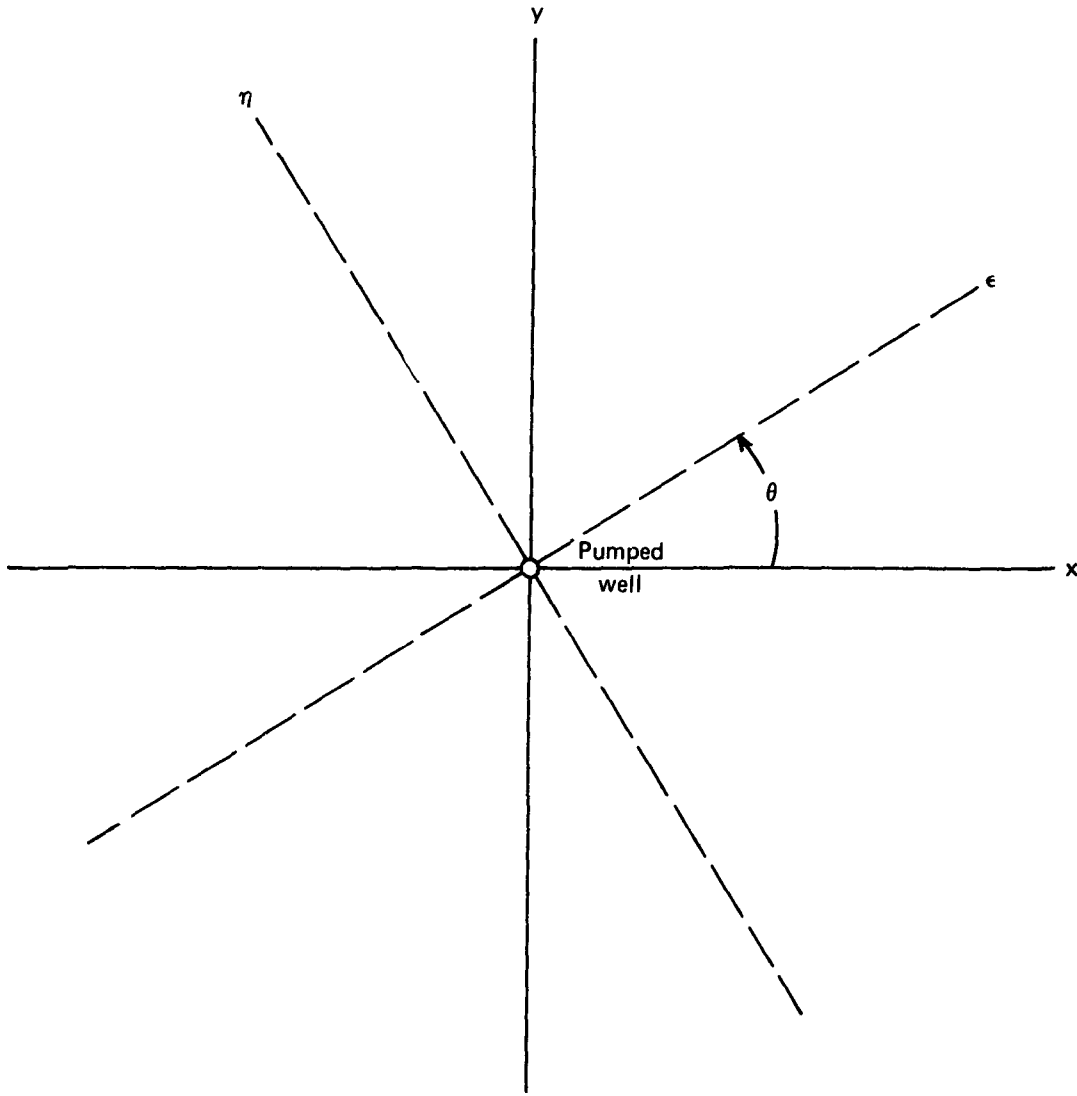


FIGURE 10.1.—Plan view showing coordinate axes.

$$S = \sqrt{(ST_{xx}ST_{yy} - (ST_{xy})^2)/(T_{xx}T_{yy} - T_{xy}^2)}$$

Then, compute T_{xx} , T_{yy} , and T_{xy} from ST_{xx} , ST_{yy} , and ST_{xy} . $T_{\epsilon\epsilon}$, $T_{\eta\eta}$, and Θ (the angle between the x and the ϵ axis) may be calculated from the relations (Papadopoulos, 1965, p. 28)

$$T_{\epsilon\epsilon} = 1/2(T_{xx} + T_{yy} + ((T_{xx} - T_{yy})^2 + 4T_{xy}^2)^{1/2})$$

$$T_{\eta\eta} = 1/2(T_{xx} + T_{yy} - ((T_{xx} - T_{yy})^2 + 4T_{xy}^2)^{1/2})$$

$$\Theta = \arctan ((T_{\epsilon\epsilon} - T_{xx})/T_{xy})$$

Solution 11: Variable discharge from a fully penetrating well in a leaky aquifer

Assumptions:

1. Well discharge changes as a specified function of time.
2. Well is of infinitesimal diameter and fully penetrates the aquifer.
3. Aquifer is overlain, or underlain, everywhere by a confining bed having uniform hydraulic conductivity (K') and thickness (b').

4. Confining bed is overlain, or underlain, by an infinite constant-head plane source.
5. Hydraulic gradient across confining bed changes instantaneously with a change in head in the aquifer (no release of water from storage in the confining bed).
6. Flow in the aquifer is two-dimensional and radial in the horizontal plane and flow in the confining bed is vertical. This assumption will be approximated closely where the hydraulic conductivity of the aquifer is sufficiently greater than that of the confining bed.

Differential equation:

$$\frac{\partial^2 s}{\partial r^2} + \frac{1}{r} \frac{\partial s}{\partial r} - \frac{sK'}{Tb'} = \frac{S}{T} \frac{\partial s}{\partial t}$$

This is the differential equation describing nonsteady radial flow in a homogeneous isotropic aquifer with leakage proportional to drawdown.

Boundary and initial conditions:

$$s(r,0)=0 \quad (1)$$

$$s(\infty,t)=0 \quad (2)$$

$$\lim_{r \rightarrow 0} r \frac{\partial s}{\partial r} = -\frac{Q(t)}{2\pi T}, t \geq 0 \quad (3)$$

Equation 1 states that, initially, drawdown is zero. Equation 2 states that drawdown is zero at large distances from the pumped well. Equation 3 states that near the pumped well the radial flow is equal to the discharge of the pumped well, which is a function of time.

Solution:

Solutions for certain discharge functions have been published by Abu-Zied and Scott (1963), and Werner (1946) for a nonleaky aquifer, and by Hantush (1964a) for both leaky and nonleaky aquifers. For arbitrary discharge functions for leaky aquifers, a solution using the convolution integral has been presented by Moench (1971, eq. 3):

$$s = (1/4\pi T) \int_0^t (Q(t')/(t-t')) \cdot \exp[-A/(t-t') - (t-t')K'/Sb'] dt', \quad (4)$$

where $Q(t)$ is the discharge function of time and $A = r^2 S/4T$. A numerical integration scheme is generally necessary to evaluate the above equation.

For type curves, a more useful form of equation 4 is

$$s = (Q_r/4\pi T) \int_0^t [Q(t')/Q_r(t-t')] \cdot \exp[-A/(t-t') - (t-t')K'/Sb'] dt', \quad (5)$$

or

$$s = (Q_r/4\pi T) SO(t), \quad (6)$$

where $SO(t)$, read "system output function," represents the integral expression in equation 5, and Q_r is an arbitrary discharge that eliminates dimension from the integral expression. For example, Q_r could be the initial, final, or average discharge, according to the needs of the user.

Comments: Figure 11.1 is a cross section through the discharging well. This situation is the same as for solution 4, except for the varying discharge of the well. The effect of finite well radius (r_w) was investigated by Hantush (1964b, p. 4224), who concluded that for $t > 25r_w^2 S/T$ and $r_w/\sqrt{Tb'/K'} < 0.1$ the drawdown could be represented closely by the convolution integral.

Figure 11.2 on plate 1 shows a selected set of type curves for linear change in discharge in a nonleaky aquifer. The solution for this type of discharge function has been presented by Werner (1946, p. 706). The discharge function for figure 12.2 is $Q(t) = Q_0(1+ct)$, and the resulting drawdown is

$$s = (Q_0/4\pi T) W(u) \{1 + ct [u + 1 - e^{-u}/W(u)]\},$$

where $W(u)$ is the well function of Theis. Substituting A/u for t in the above expression gives

$$s = (Q_0/4\pi T) W(u) \cdot (1 + cA \{1 + (1/u) [1 - e^{-u}/W(u)]\}),$$

or

$$s = (Q_0/4\pi T) SO(t),$$

where $SO(t)$ represents

$$W(u) (1 + cA \{1 + (1/u) [1 - e^{-u}/W(u)]\}).$$

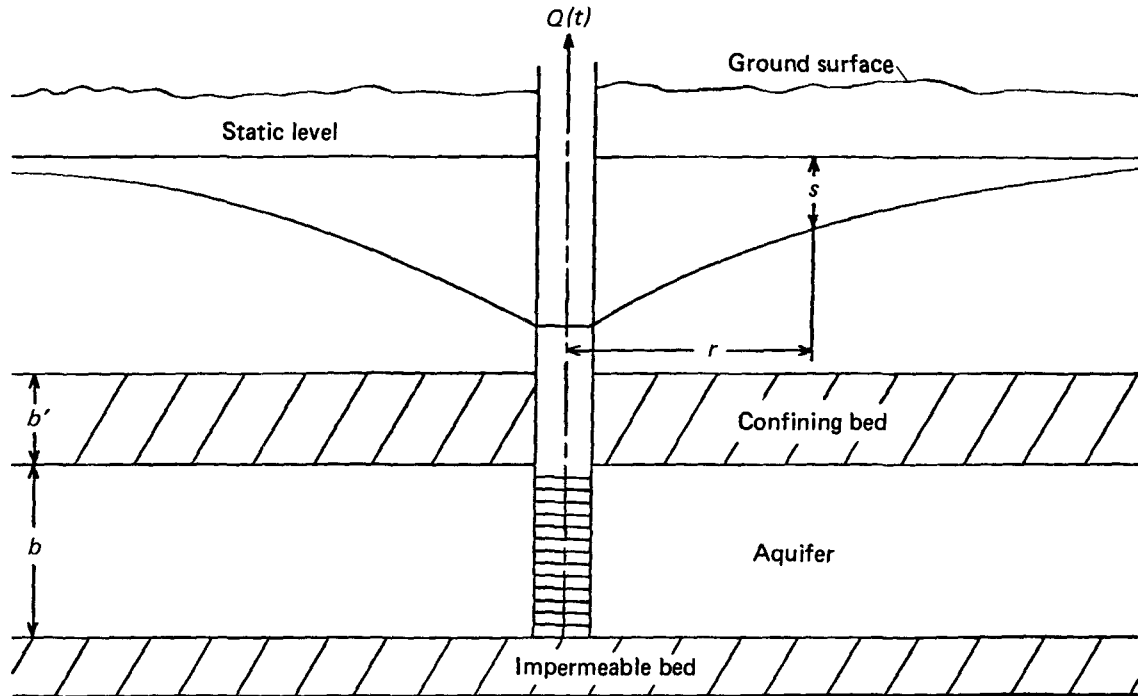


FIGURE 11.1.—Cross section through a well with variable discharge.

This substitution permits the plotting of a family of type curves, each curve specified by a value of cA .

Table 11.1 is the listing of a FORTRAN program designed to evaluate the above convolution integral for five different discharge functions. Three of these discharge functions are those devised by Hantush (1964a, p. 343, 344), who presented solutions for drawdown resulting from these functions. These three discharge functions are:

$$(a) \quad Q(t) = Q_s [1 + \delta \exp(-t/t^*)],$$

$$(b) \quad Q(t) = Q_s [1 + \delta/(1 + t/t^*)],$$

and (c) $Q(t) = Q_s [1 + \delta/\sqrt{1 + t/t^*}],$

where Q_s is the ultimate steady discharge and δ and t^* are parameters defining a particular function. The first discharge function, for an exponentially decreasing discharge (case "a" of Hantush, 1964a) is virtually the same as the discharge function of Abu-Zied and Scott (1963). Besides the three functions of Hantush, the program also includes discharge as a fifth-

degree polynomial of time, $Q(t) = \sum_{i=0}^5 a_i t^i$, where the a_i are the coefficients of the polynomial, and as a piecewise linear function of time with eight segments,

$$Q(t) = a_j + b_j(t - t_{j-1})$$

for

$$t_{j-1} < t \leq t_j, j = 1, 2, \dots, 8,$$

where a_j and b_j are parameters defining the j^{th} line segment. The program uses a different, but equivalent to equation 4, expression for the convolution integral

$$s = (1/4\pi T) \int_0^t (Q(t-t')/t') \cdot \exp(-A/t' - t'K'/Sb') dt'.$$

The program uses a sum to approximate the convolution integral. It chooses a starting value of t' that satisfies $r^2S/4Tt' + K't'/Sb' = 100$. If such a value of t' does not exist, that is, $(r^2S/4T)(K'/Sb') > 2500$, then a value of zero is assigned for the integral value. The ending point of the interval is picked as 10 times the

starting point. The integral over this interval is approximated by a trapezoidal sum using 500 subdivisions of the interval. A new interval is then constructed using the previous end point as a new starting point and a new ending point equal to 10 times the new starting point. This new interval is again evaluated by a trapezoidal sum of 500 segments. This summation procedure over intervals that are successively an order of magnitude larger continues until either $t' = t$ or $(r^2 S / 4 T t') + (K' t / S b') > 101$. Input to this program consists of cards coded in specific formats. Readers unfamiliar with FORTRAN formats should refer to a FORTRAN language manual. Input consists of one or more groups of data, each group consisting of the following. First, one card containing the beginning time of the period of analysis in columns 1–10, coded in format E10.3; the ending time coded in columns 1311–20, in format E10.3; and a discharge index (a number from 1 through 5) coded in column 25, in format I1; and a reference discharge, QR , coded in columns 31–40, in format E10.3. The discharge index, IQ , selects a discharge function, $Q(t)$, in the following manner. If $IQ = 1$, the discharge function is exponentially decreasing,

$$Q(t) = Q_s [1 + \delta \exp(-t/t^*)].$$

This is case (a) of Hantush (1964a, p. 343). If $IQ = 2$, the discharge function is hyperbolically decreasing,

$$Q(t) = Q_s [1 + \delta / (1 + t/t^*)].$$

This is case (b) of Hantush (1964a, p. 344). If $IQ = 3$, the discharge function is the same as case (c) of Hantush (1964a, p. 344),

$$Q(t) = Q_s [1 + \delta / \sqrt{1 + t/t^*}].$$

If $IQ = 4$, the discharge function is a fifth-degree polynomial of time,

$$Q(t) = \sum_{i=0}^5 a_i t^i.$$

If $IQ = 5$, the discharge function is a piecewise-linear function of time with eight or less segments,

$$Q(t) = a_j + b_j (t - t_{j-1})$$

for $t_{j-1} < t \leq t_j, j = 1, 2, \dots, 8.$

The reference discharge, QR , is used to determine the form of the output from the program: If QR is coded as zero (or blank), the output shows t, s (as defined by eq. 4), and $Q(t)$. If a value greater than zero is coded for QR , the output shows $1/u, SO(t)$ (as defined by eq. 6), and $Q(t)/QR$.

Second, there are one or more cards containing parameters of the discharge function. If $IQ = 1, 2$, or 3 , then it consists of one card containing: QST , the ultimate steady discharge, coded in columns 1–10, in format E10.3; $DELTA$, a rate parameter, coded in columns 11–20, in format E10.3; $TSTAR$, a time parameter, coded in columns 21–30, in format E10.3. If $IQ = 4$, it is one card containing the six polynomial coefficients. They are coded in the order a_0, a_1, \dots, a_5 , in columns 1–10; 11–20, \dots , 51–60 all in format E10.3. If $IQ = 5$, then the program requires four cards, each card containing $t_j, a_j, b_j, t_{j+1}, a_{j+1}, b_{j+1}$; the four cards representing $j = 1, 3, 5, 7$. The last part of each set of data consists of two or more cards containing coded values for: distance from pumped well, in columns 1–10; storage coefficient, in columns 11–20; transmissivity, in columns 21–30; and ratio of hydraulic conductivity to thickness for the confining bed, in columns 31–40, all in format E10.3. A blank card is used to signal the end of each set of data. Output from this program is shown in figure 11.3.

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R**2*S/(4*TRANS)= 1.000E-04, K'/(S*B')= 2.500E 03, QR= 1.257E 05

1/U	1/U*10** 0		1/U*10** 1		1/U*10** 2		1/U*10** 3	
	SO(T)	Q(T)/QR	SO(T)	Q(T)/QR	SO(T)	Q(T)/QR	SO(T)	Q(T)/QR
1.0	0.185	1.000E 00	0.819	1.000E 00	0.842	1.000E 00	0.842	1.000E 00
1.5	0.317	1.000E 00	0.837	1.000E 00	0.842	1.000E 00	0.842	1.000E 00
2.0	0.421	1.000E 00	0.841	1.000E 00	0.842	1.000E 00	0.842	1.000E 00
3.0	0.566	1.000E 00	0.842	1.000E 00	0.842	1.000E 00	0.842	1.000E 00
5.0	0.715	1.000E 00	0.842	1.000E 00	0.842	1.000E 00	0.842	1.000E 00
7.0	0.780	1.000E 00	0.842	1.000E 00	0.842	1.000E 00	0.842	1.000E 00

R**2*S/(4*TRANS)= 1.000E-04, K'/(S*B')= 2.500E 01, QR= 1.257E 05

1/U	1/U*10** 0		1/U*10** 1		1/U*10** 2		1/U*10** 3	
	SO(T)	Q(T)/QR	SO(T)	Q(T)/QR	SO(T)	Q(T)/QR	SO(T)	Q(T)/QR
1.0	0.219	1.000E 00	1.805	1.000E 00	3.815	1.000E 00	4.829	1.000E 00
1.5	0.397	1.000E 00	2.167	1.000E 00	4.111	1.000E 00	4.849	1.000E 00
2.0	0.558	1.000E 00	2.427	1.000E 00	4.296	1.000E 00	4.853	1.000E 00
3.0	0.826	1.000E 00	2.793	1.000E 00	4.515	1.000E 00	4.854	1.000E 00
5.0	1.216	1.000E 00	3.244	1.000E 00	4.708	1.000E 00	4.854	1.000E 00
7.0	1.495	1.000E 00	3.530	1.000E 00	4.785	1.000E 00	4.854	1.000E 00

R**2*S/(4*TRANS)= 1.000E-04, K'/(S*B')= 2.500E-01, QR= 1.257E 05

1/U	1/U*10** 0		1/U*10** 1		1/U*10** 2		1/U*10** 3	
	SO(T)	Q(T)/QR	SO(T)	Q(T)/QR	SO(T)	Q(T)/QR	SO(T)	Q(T)/QR
1.0	0.219	1.000E 00	1.823	1.000E 00	4.036	1.000E 00	6.307	1.000E 00
1.5	0.398	1.000E 00	2.196	1.000E 00	4.437	1.000E 00	6.700	1.000E 00
2.0	0.560	1.000E 00	2.468	1.000E 00	4.721	1.000E 00	6.975	1.000E 00
3.0	0.829	1.000E 00	2.857	1.000E 00	5.123	1.000E 00	7.356	1.000E 00
5.0	1.223	1.000E 00	3.354	1.000E 00	5.627	1.000E 00	7.820	1.000E 00
7.0	1.507	1.000E 00	3.684	1.000E 00	5.958	1.000E 00	8.110	1.000E 00

R**2*S/(4*TRANS)= 1.000E-04, K'/(S*B')= 2.500E-03, QR= 1.257E 05

1/U	1/U*10** 0		1/U*10** 1		1/U*10** 2		1/U*10** 3	
	SO(T)	Q(T)/QR	SO(T)	Q(T)/QR	SO(T)	Q(T)/QR	SO(T)	Q(T)/QR
1.0	0.219	1.000E 00	1.823	1.000E 00	4.038	1.000E 00	6.332	1.000E 00
1.5	0.398	1.000E 00	2.197	1.000E 00	4.440	1.000E 00	6.737	1.000E 00
2.0	0.560	1.000E 00	2.468	1.000E 00	4.726	1.000E 00	7.024	1.000E 00
3.0	0.829	1.000E 00	2.857	1.000E 00	5.130	1.000E 00	7.429	1.000E 00
5.0	1.223	1.000E 00	3.355	1.000E 00	5.639	1.000E 00	7.939	1.000E 00
7.0	1.507	1.000E 00	3.686	1.000E 00	5.975	1.000E 00	8.275	1.000E 00

FIGURE 11.3.—Example of output from program to compute the convolution integral for a leaky aquifer.

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SUPPLEMENTAL DATA

TABLE 2.1.—Listing of program for partial penetration in a nonleaky artesian aquifer

```

C *****PPN 1
C PPN 2
C PURPOSE PPN 3
C TO COMPUTE TYPE CURVE FUNCTION VALUES FOR PARTIAL PENETRATION PPN 4
C IN A NONLEAKY AQUIFER USING EQUATIONS 1 AND 9A OF HANTUSH,M,S., PPN 5
C 1961,DRAWDOWN AROUND A PARTIALLY PENETRATING WELL; HYDRAULIC PPN 6
C DIV, JUUR., AM. SOC. CIVIL ENGINEERS PROC., P. 83-98, PPN 7
C INPUT DATA PPN 8
C 1 CARD = FORMAT (3F5,1,I5,2E10,4) PPN 9
C B = AQUIFER THICKNESS PPN 10
C L = DEPTH, BELOW TOP OF AQUIFER, TO BOTTOM OF PUMPING PPN 11
C WELL SCREEN PPN 12
C D = DEPTH, BELOW TOP OF AQUIFER, TO TOP OF PUMPING WELL PPN 13
C SCREEN PPN 14
C NUM = NUMBER OF OBSERVATION WELLS OR PIEZOMETERS TIMES PPN 15
C NUMBER OF VALUES OF KZ/KR, PPN 16
C SMALL = SMALLEST VALUE OF 1/U FOR WHICH COMPUTATION IS PPN 17
C DESIRED PPN 18
C LARGE = LARGEST VALUE OF 1/U FOR WHICH COMPUTATION IS PPN 19
C DESIRED PPN 20
C NUM CARDS (ONE FOR EACH OBS. WELL OR PIEZOMETER AND FOR EACH 1PPN 21
C VALUE OF R*SQRT(KZ/KR), = FORMAT (3F5,1) PPN 22
C R = RADIAL DISTANCE FROM PUMPED WELL TIMES SQRT(KZ/KR), PPN 23
C LPRIME = DEPTH, BELOW TOP OF AQUIFER, TO BOTTOM OF OBS, PPN 24
C WELL SCREEN (ZERO FOR PIEZOMETER) PPN 25
C DPRIME = DEPTH, BELOW TOP OF AQUIFER, TO TOP OF OBS. WELL PPN 26
C SCREEN (TOTAL DEPTH FOR PIEZOMETER) PPN 27
C SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED PPN 28
C DQL12,SERIES,BESK,FCT,L,F,EXPI PPN 29
C PPN 30
C *****PPN 31
C REAL*8 U PPN 32
C REAL*4 L,LB,LPB,LPRIME,LARGE PPN 33
C DIMENSION ARRAY(13,12), IARG(12), ARG(13), A(12), C(12) PPN 34
C DATA ARG/1.,1.2,1.5,2.,2.5,3.,3.5,4.,5.,6.,7.,8.,9./ PPN 35
C DATA A/12*1 N*1/,C/12*10**1/ PPN 36
C IRD=5 PPN 37
C IPT=6 PPN 38
C READ (IRD,6) B,L,D,NUM,SMALL,LARGE PPN 39
C LB=L/B PPN 40
C DB=D/B PPN 41
C IBEGIN=ALOG10(SMALL) PPN 42
C IEND=ALOG10(LARGE)+1. PPN 43
C JLIMIT=IEND-IBEGIN PPN 44
C IF (JLIMIT.GT.12) JLIMIT=12 PPN 45
C DO 5 K=1,NUM PPN 46
C READ (IRD,6) R,LPRIME,DPRIME PPN 47
C RB=R/B PPN 48
C LPB=LPRIME/B PPN 49
C DPB=DPRIME/B PPN 50
C DO 1 I=1,13 PPN 51
C ARG(I)=ARG(I) PPN 52
C DO 1 J=1,JLIMIT PPN 53
C IARG(J)=IBEGIN+J-1 PPN 54
C Y=ARG(I)*10.** (IBEGIN+J-1) PPN 55
C U=1./Y PPN 56
C X=U PPN 57
C CALL EXPI(X,WU,DUMMY) PPN 58
C 1 ARRAY(I,J)=WU+F(U,RB,LB,DB,LPB,DPB) PPN 59
C IF (LPB=0.) 2,2,3 PPN 60

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TABLE 2.1.—Listing of program for partial penetration in a nonleaky artesian aquifer—Continued

```

2 WRITE (IPT,7) DPB,RB,LB,DB                      PPN 61
GO TO 4                                           PPN 62
3 WRITE (IPT,8) LPB,DPB,RB,LB,DB                 PPN 63
4 WRITE (IPT,9) (A(I),C(I),IARG(I),I=1,JLIMIT)   PPN 64
DO 5 I=1,13                                       PPN 65
WRITE (IPT,10) ARG(I),(ARRAY(I,J),J=1,JLIMIT)   PPN 66
5 CONTINUE                                        PPN 67
STOP                                             PPN 68
C                                               PPN 69
C                                               PPN 70
C                                               PPN 71
6 FORMAT (3F5,1,I5,2E10,4)                       PPN 72
7 FORMAT ('1',1W(U)+F(U,R/B,L/B,D/B,Z/B), Z/B=1,F5,2,1, SQRT(KZ/KR)*PPN 73
1R/B=1,F5,2,1, L/B=1,F5,2,1, D/B=1,F5,2,1, U=1/N')
8 FORMAT ('1',1W(U)+F(U,R/B,L/B,D/B,L1/B,D1/B), L1/B=1,F5,2,1, D1/PPN 74
11/B=1,F5,2,1, SQRT(KZ/KR)*R/B=1,F5,2,1, L/B=1,F5,2,1, D/B=1,F5,2,1,PPN 75
2, U=1/N')
9 FORMAT (10I,2X,1N1,1X,12(2A4,I2))              PPN 77
10 FORMAT ((1,1,F4,1,12(F9,4,1X)))                PPN 78
END                                               PPN 79
REAL FUNCTION F*4(U,RB,LB,DB,LPB,DPB)           F 1
*****                                           F 2
C                                               F 3
C                                               F 4
FUNCTION F                                       F 5
C                                               F 6
PURPOSE                                         F 7
TO COMPUTE DEPARTURES FROM THEIS CURVE CAUSED BY PARTIAL
C PENETRATION OF PUMPED WELL.                   F 8
C USAGE                                         F 9
F(U,RB,LB,DB,LPB,DPB)                          F 10
C DESCRIPTION OF PARAMETERS                    F 11
ALL REAL, U DOUBLE PRECISION                    F 12
C U = R**2*8/4*T*TIME (RADIAL DISTANCE SQUARED * STORAGE
C COEFFICIENT / 4*TRANSMISSIVITY * TIME        F 14
RB = R/B ( RADIAL DISTANCE / AQUIFER THICKNESS ) F 15
C LB = L/B ( FRACTION OF AQUIFER PENETRATED BY PUMPED WELL) F 16
C DB = D/B ( FRACTION OF AQUIFER ABOVE PUMPED WELL SCREEN) F 17
C LPB = L1/B (FRACTION OF AQUIFER PENETRATED BY OBS, WELL, ZERO
C FOR PIEZOMETER)                               F 19
C DPB = D1/B (FRACTION OF AQUIFER ABOVE OBS, WELL SCREEN, TOTAL
C DEPTH FOR PIEZOMETER)                         F 21
C SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED F 22
DQL12,SERIES,BESK,FCT,L                        F 23
C METHOD                                        F 24
SUMS THE SERIES THROUGH N*PI*R/B EQ 20         F 25
C                                               F 26
*****                                           F 27
REAL*8 U,V                                       F 28
REAL*4 L,N,LB,LPB                               F 29
SUM=0.                                           F 30
N=0.                                             F 31
PIRB=3,141593*RB                                F 32
PILB=3,141593*LB                                F 33
PIDB=3,141593*DB                                F 34
IF (LPB=0.) 1,1,4                                F 35
C CHECKS FOR WELL OR PIEZOMETER                 F 36
1 PIZB=3,141593*DPB                              F 37
2 N=N+1.                                         F 38
V=N*PIRB/2.                                       F 39
IF (V.GT,10.) GO TO 3                            F 40
C TRUNCATES SERIES WHEN V>10                   F 41
X=L(U,V)/N                                       F 42

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TABLE 2.1.—Listing of program for partial penetration in a nonleaky artesian aquifer—Continued

```

      CALL DQL12(FCT,F)                                     L 46
      L=F+SERIES(U,V)                                       L 47
C     INTEGRAL 1 TO INF, BY G=L QUAD., INTEGRAL U TO 1 BY SERIES EXP,   L 48
      GO TO 5                                               L 49
      4 TWOV=2,*V                                           L 50
      CALL BESK(TWOV,0,BK,IER)                               L 51
      CALL DQL12(FCT,F)                                     L 52
      L=2,*BK=F                                             L 53
C     2K0(2V)=INTEGRAL V**2/U TO INF,                          L 54
      5 RETURN                                             L 55
      END                                                 L 56=

      REAL FUNCTION SERIES*8(U,V)                           SER 1
C     *****SERIES*****                                SER 2
C     C                                                     SER 3
C     C     FUNCTION SERIES                               SER 4
C     C     PURPOSE                                     SER 5
C     C     TO EVALUATE S(1)-S(U), WHERE S IS A SERIES EXPANSION OF SER 6
C     C     INTEGRAL(EXP(-Y*V**2/Y)DY/Y) GIVEN BY: S= SUM, M=0 TO INFINITY, SER 7
C     C     (F(M)*SUM, N=0 TO INF, ((V**(2*N)/((N!)*(M+N)!)) WHERE F(M)= SER 8
C     C     LOG(U) IF M=0 AND = ((-1)**M/M)*(U**M-(V**2/U)**M) IF M>0. SER 9
C     C     DESCRIPTION OF PARAMETERS                    SER 10
C     C     BOTH DOUBLE PRECISION                       SER 11
C     C     U = R**2*S/4*T*TIME (RADIAL DISTANCE SQUARED * STORAGE SER 12
C     C     COEFFICIENT / 4*TRANSMISSIVITY * TIME     SER 13
C     C     V = R/2*SQRT(K'/(T*B'))=ONE-HALF RADIAL DISTANCE*SQUARE ROOT SER 14
C     C     (HYD. COND. OF CONFINING BED/TRANSMISSIVITY*THICKNESS SER 15
C     C     OF CONFINING BED)                           SER 16
C     C     SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED SER 17
C     C     NONE                                         SER 18
C     C     METHOD                                         SER 19
C     C     SUMMATION IS TERMINATED FOR THE INNER SERIES WHEN A TERM SER 20
C     C     BECOMES LESS THAN 5,E-7/N AND FOR OUTER SERIES WHEN A TERM SER 21
C     C     BECOMES LESS THAN 5,E-7                     SER 22
C     C     *****SERIES*****                            SER 23
C     C     *****SERIES*****                            SER 24
      REAL*8 DLOG,DABS,S(2),VUM,UU                         SER 25
      REAL*8 TEST,U,UM,EM,EN,SUM1,SUM,SIGN,V,VSQ,VSQU,RMUL,TERM,TERM1 SER 26
      TEST=5,D=07                                          SER 27
      VSQ=V*V                                              SER 28
      UU=U                                                 SER 29
      DO 6 I=1,2                                           SER 30
C     EVALUATES SERIES FOR LOWER LIMIT = U AND UPPER LIMIT = 1   SER 31
      IF (I,EQ,2) U=1.                                     SER 32
      UM=1.                                                SER 33
      EM=1.                                                SER 34
      SUM1=0.                                              SER 35
      SIGN=1.                                              SER 36
      VUM=1.                                               SER 37
      VSQU=VSQ/U                                           SER 38
      1 EM=EM+1.                                           SER 39
      IF (EM=,1) 2,3,3                                     SER 40
C     CHECKS FOR M=0                                         SER 41
      2 RMUL=DLOG(U)                                       SER 42
      TERM1=1.                                             SER 43
      GO TO 4                                              SER 44
      3 UM=UM*U                                             SER 45
      IF (VUM,LT,1,D=30) VUM=0.                           SER 46
      VUM=VUM*VSQU                                         SER 47
      RMUL=(UM-VUM)/EM                                     SER 48
      TERM1=TERM1/EM                                       SER 49

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TABLE 2.1.—Listing of program for partial penetration in a nonleaky artesian aquifer—Continued

```

4 SIGN=-SIGN                                SER 51
  SUM=TERM!                                  SER 52
  TERM=TERM!                                  SER 53
  EN=0,                                       SER 54
5 EN=EN+1,                                    SER 55
  TERM=TERM*VSQ/(EN*(EN+EM))                SER 56
  SUM=SUM+TERM                               SER 57
  IF (TEST,LE,DABS(RMUL*EN*TERM)) GO TO 5   SER 58
C TRUNCATES INNER SERIES IF OUTER TERM*INNER TERM < 5,E=7 SER 59
  SUM1=SUM1+SIGN*RMUL*SUM                   SER 60
  IF (EM,LT,,1) GO TO 1                     SER 61
  IF (TEST,LE,DABS(RMUL*SUM)) GO TO 1      SER 62
C TRUNCATES OUTER SERIES IF OUTER TERM*INNER SUM < 5,E=7 SER 63
6 S(I)=SUM!                                  SER 64
  U=UU                                       SER 65
  SERIES=S(2)=S(1)                          SER 66
  RETURN                                     SER 67
  END                                        SER 68=
  REAL FUNCTION FCT*(X)                      FCT 1
C *****FCT 2
C FCT 3
C FUNCTION FCT                              FCT 4
C FCT 5
C PURPOSE                                   FCT 6
C TO COMPUTE FCT(X)=EXP(-Z+V**2/(X+Z))/(X+Z) FCT 7
C DESCRIPTION OF PARAMETERS                FCT 8
C X = THE DOUBLE PRECISION VALUE OF X FOR WHICH FCT IS COMPUTED FCT 9
C SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED FCT 10
C NONE                                     FCT 11
C METHOD                                    FCT 12
C FORTRAN EVALUATION OF FUNCTION           FCT 13
C FCT 14
C *****FCT 15
C REAL*8 X,V,Z,P,DEXP                      FCT 16
C COMMON /C1/ V,Z                          FCT 17
C IF (X) 1,2,2                              FCT 18
1 FCT=0,                                    FCT 19
  GO TO 4                                    FCT 20
2 P=Z+V**2/(X+Z)                            FCT 21
  IF (P=5.01) 3,3,1                        FCT 22
3 FCT=DEXP(-P)/(X+Z)                        FCT 23
4 RETURN                                    FCT 24
  END                                        FCT 25=
  SUBROUTINE DQL12(FCT,Y)                   DL12 380
C DL12 10
C .....DL12 20
C SUBROUTINE DQL12                          DL12 30
C DL12 40
C DL12 50
C PURPOSE                                   DL12 60
C TO COMPUTE INTEGRAL(EXP(*X)*FCT(X), SUMMED OVER X DL12 70
C FROM 0 TO INFINITY),                    DL12 80
C DL12 90
C USAGE                                    DL12 100
C CALL DQL12 (FCT,Y)                       DL12 110
C PARAMETER FCT REQUIRES AN EXTERNAL STATEMENT DL12 120
C DL12 130
C DESCRIPTION OF PARAMETERS                DL12 140
C FCT = THE NAME OF AN EXTERNAL DOUBLE PRECISION FUNCTION DL12 150
C SUBPROGRAM USED,                        DL12 160
C Y = THE RESULTING DOUBLE PRECISION INTEGRAL VALUE, DL12 170

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TABLE 2.1.—Listing of program for partial penetration in a nonleaky artesian aquifer—Continued

C		DL12 180
C	REMARKS	DL12 190
C	NONE	DL12 200
C		DL12 210
C	SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED	DL12 220
C	THE EXTERNAL DOUBLE PRECISION FUNCTION SUBPROGRAM FCT(X)	DL12 230
C	MUST BE FURNISHED BY THE USER,	DL12 240
C		DL12 250
C	METHOD	DL12 260
C	EVALUATION IS DONE BY MEANS OF 12-POINT GAUSSIAN-LAGUERRE	DL12 270
C	QUADRATURE FORMULA, WHICH INTEGRATES EXACTLY,	DL12 280
C	WHENEVER FCT(X) IS A POLYNOMIAL UP TO DEGREE 23,	DL12 290
C	FOR REFERENCE, SEE	DL12 300
C	SHAO/CHEN/FRANK, TABLES OF ZEROS AND GAUSSIAN WEIGHTS OF	DL12 310
C	CERTAIN ASSOCIATED LAGUERRE POLYNOMIALS AND THE RELATED	DL12 320
C	GENERALIZED HERMITE POLYNOMIALS, IBM TECHNICAL REPORT	DL12 330
C	TR00,1100 (MARCH 1964), PP,24-25.	DL12 340
C		DL12 350
C	DL12 360
C		DL12 370
C		DL12 390
C		DL12 400
C	DOUBLE PRECISION X,Y,FCT	DL12 410
C		DL12 420
C	X=,3709912104446692 D2	DL12 430
C	Y=,814807746742624 D=15*FCT(X)	DL12 440
C	X=,2848796725098400 D2	DL12 450
C	Y=Y+,3061601635035021 D=11*FCT(X)	DL12 460
C	X=,2215104037939701 D2	DL12 470
C	Y=Y+,1342391030515004 D=8*FCT(X)	DL12 480
C	X=,1711685518746226 D2	DL12 490
C	Y=Y+,1668493876540910 D=6*FCT(X)	DL12 500
C	X=,1300605499330635 D2	DL12 510
C	Y=Y+,836505585681980 D=5*FCT(X)	DL12 520
C	X=,962131684245687 D1	DL12 530
C	Y=Y+,2032315926629994 D=3*FCT(X)	DL12 540
C	X=,6844525453115177 D1	DL12 550
C	Y=Y+,2663973541865316 D=2*FCT(X)	DL12 560
C	X=,4599227639418348 D1	DL12 570
C	Y=Y+,2010238115463410 D=1*FCT(X)	DL12 580
C	X=,2833751337743507 D1	DL12 590
C	Y=Y+,904492222116809 D=1*FCT(X)	DL12 600
C	X=,1512610269776419 D1	DL12 610
C	Y=Y+,2440820113198776 D0*FCT(X)	DL12 620
C	X=,6117574845151307 D0	DL12 630
C	Y=Y+,3777592758731380 D0*FCT(X)	DL12 640
C	X=,1157221173580207 D0	DL12 650
C	Y=Y+,2647313710554432 D0*FCT(X)	DL12 660
C	RETURN	DL12 670
C	END	DL12 68=
C	SUBROUTINE BESK(X,N,BK,IER)	BESK 410
C		BESK 10
C	BESK 20
C		BESK 30
C	SUBROUTINE BESK	BESK 40
C		BESK 50
C	COMPUTE THE K BESSEL FUNCTION FOR A GIVEN ARGUMENT AND ORDER	BESK 60
C		BESK 70
C	USAGE	BESK 80
C	CALL BESK(X,N,BK,IER)	BESK 90

TABLE 2.1.—Listing of program for partial penetration in a nonleaky artesian aquifer—Continued

C		BESK 100
C	DESCRIPTION OF PARAMETERS	BESK 110
C	X =THE ARGUMENT OF THE K BESSEL FUNCTION DESIRED	BESK 120
C	N =THE ORDER OF THE K BESSEL FUNCTION,DESIRED	BESK 130
C	BK =THE RESULTANT K BESSEL FUNCTION	BESK 140
C	IER=RESULTANT ERROR CODE WHERE	BESK 150
C	IER=0 NO ERROR	BESK 160
C	IER=1 N IS NEGATIVE	BESK 170
C	IER=2 X IS ZERO OR NEGATIVE	BESK 180
C	IER=3 X ,GT, 170, MACHINE RANGE EXCEEDED	BESK 190
C	IER=4 BK ,GT, 10**70	BESK 200
C		BESK 210
C	REMARKS	BESK 220
C	N MUST BE GREATER THAN OR EQUAL TO ZERO	BESK 230
C		BESK 240
C	SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED	BESK 250
C	NONE	BESK 260
C		BESK 270
C	METHOD	BESK 280
C	COMPUTES ZERO ORDER AND FIRST ORDER BESSEL FUNCTIONS USING	BESK 290
C	SERIES APPROXIMATIONS AND THEN COMPUTES N TH ORDER FUNCTION	BESK 300
C	USING RECURRENCE RELATION,	BESK 310
C	RECURRENCE RELATION AND POLYNOMIAL APPROXIMATION TECHNIQUE	BESK 320
C	AS DESCRIBED BY A,J,M,HITCHCOCK, 'POLYNOMIAL APPROXIMATIONS	BESK 330
C	TO BESSEL FUNCTIONS OF ORDER ZERO AND ONE AND TO RELATED	BESK 340
C	FUNCTIONS', M,T,A,C., V,11,1957,PP,86-88, AND G.N. WATSON,	BESK 350
C	'A TREATISE ON THE THEORY OF BESSEL FUNCTIONS', CAMBRIDGE	BESK 360
C	UNIVERSITY PRESS, 1958, P, 62	BESK 370
C		BESK 380
C	BESK 390
C		BESK 400
C	DIMENSION T(12)	BESK 420
C	BK=,0	BESK 430
C	IF(N)10,11,11	BESK 440
C	10 IER=1	BESK 450
C	RETURN	BESK 460
C	11 IF(X)12,12,20	BESK 470
C	12 IER=2	BESK 480
C	RETURN	BESK 490
C	20 IF(X=170,0)22,22,21	BESK 500
C	21 IER=3	BESK 510
C	RETURN	BESK 520
C	22 IER=0	BESK 530
C	IF(X=1,)36,36,25	BESK 540
C	25 A=EXP(-X)	BESK 550
C	B=1./X	BESK 560
C	C=SQRT(B)	BESK 570
C	T(1)=B	BESK 580
C	DO 26 L=2,12	BESK 590
C	26 T(L)=T(L-1)*B	BESK 600
C	IF(N=1)27,29,27	BESK 610
C		BESK 620
C	COMPUTE KO USING POLYNOMIAL APPROXIMATION	BESK 630
C		BESK 640
C	27 GO=A*(1,2533141=-,1566642*T(1)+,08811128*T(2)-,09139095*T(3)	BESK 650
C	2+,.1344596*T(4)-,2299850*T(5)+,3792410*T(6)-,5247277*T(7)	BESK 660
C	3+,.5575368*T(8)-,4262633*T(9)+,2184518*T(10)-,06680977*T(11)	BESK 670
C	4+,.009189383*T(12))*C	BESK 680
C	IF(N)20,28,29	BESK 690
C	28 BK=GO	BESK 700
C	RETURN	BESK 710

TABLE 2.1.—Listing of program for partial penetration in a nonleaky artesian aquifer—Continued

C		BESK 720
C	COMPUTE K1 USING POLYNOMIAL APPROXIMATION	BESK 730
C		BESK 740
	29 G1=A*(1,2533141+,4699927*T(1)-,1468583*T(2)+,1280427*T(3)	BESK 750
	2=,1736432*T(4)+,2847618*T(5)-,4594342*T(6)+,6283381*T(7)	BESK 760
	3=,6632295*T(8)+,5050239*T(9)-,2581304*T(10)+,07880001*T(11)	BESK 770
	4=,01082418*T(12))*C	BESK 780
	IF(N=1)20,30,31	BESK 790
	30 BK=G1	BESK 800
	RETURN	BESK 810
C		BESK 820
C	FROM K0,K1 COMPUTE KN USING RECURRENCE RELATION	BESK 830
C		BESK 840
	31 DO 35 J=2,N	BESK 850
	GJ=2,*(FLOAT(J)-1,)*G1/X+G0	BESK 860
	IF(GJ=1,0E70)33,33,32	BESK 870
	32 IER=4	BESK 880
	GO TO 34	BESK 890
	33 G0=G1	BESK 900
	35 G1=GJ	BESK 910
	34 BK=GJ	BESK 920
	RETURN	BESK 930
	36 B=X/2,	BESK 940
	A=,5772157+ALOG(B)	BESK 950
	C=B*B	BESK 960
	IF(N=1)37,43,37	BESK 970
C		BESK 980
C	COMPUTE K0 USING SERIES EXPANSION	BESK 990
C		BESK1000
	37 G0=-A	BESK1010
	X2J=1,	BESK1020
	FACT=1,	BESK1030
	HJ=.0	BESK1040
	DO 40 J=1,6	BESK1050
	RJ=1,/FLOAT(J)	BESK1060
	IF(X2J,LT,1,E=40) X2J=0,	BESK1061
C	PREVIOUS STATEMENT ADDED TO IBM SUBROUTINE TO CORRECT UNDERFLOW	BESK1062
C	PROBLEM ON WATFOR COMPILER	BESK1063
	X2J=X2J*C	BESK1070
	FACT=FACT*RJ*RJ	BESK1080
	HJ=HJ+RJ	BESK1090
	40 G0=G0+X2J*FACT*(HJ=A)	BESK1100
	IF(N)43,42,43	BESK1110
	42 BK=G0	BESK1120
	RETURN	BESK1130
C		BESK1140
C	COMPUTE K1 USING SERIES EXPANSION	BESK1150
C		BESK1160
	43 X2J=B	BESK1170
	FACT=1,	BESK1180
	HJ=1,	BESK1190
	G1=1,/X+X2J*(,5+A-HJ)	BESK1200
	DO 50 J=2,8	BESK1210
	X2J=X2J*C	BESK1220
	RJ=1,/FLOAT(J)	BESK1230
	FACT=FACT*RJ*RJ	BESK1240
	HJ=HJ+RJ	BESK1250
	50 G1=G1+X2J*FACT*(,5+(A-HJ)*FLOAT(J))	BESK1260
	IF(N=1)31,52,31	BESK1270
	52 BK=G1	BESK1280
	RETURN	BESK1290
	END	BESK1300

TABLE 2.1.—Listing of program for partial penetration in a nonleaky artesian aquifer—Continued

```

SUBROUTINE EXPI(X,RES,AUX)
.....
SUBROUTINE EXPI
PURPOSE
  COMPUTES THE EXPONENTIAL INTEGRAL =EI(=X)
USAGE
  CALL EXPI(X,RES)
DESCRIPTION OF PARAMETERS
  X      = ARGUMENT OF EXPONENTIAL INTEGRAL
  RES    = RESULT VALUE
  AUX    = RESULTANT AUXILIARY VALUE
REMARKS
  X GT 170 (X LT =174) MAY CAUSE UNDERFLOW (OVERFLOW)
  WITH THE EXPONENTIAL FUNCTION
  FOR X = 0 THE RESULT VALUE IS SET TO =1,E75
SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
  NONE
METHOD
  DEFINITION
  RES=INTEGRAL(EXP(=T)/T, SUMMED OVER T FROM X TO INFINITY).
  EVALUATION
  THREE DIFFERENT RATIONAL APPROXIMATIONS ARE USED IN THE
  RANGES 1 LE X, X LE =9 AND =9 LT X LE =3 RESPECTIVELY,
  A POLYNOMIAL APPROXIMATION IS USED IN =3 LT X LT 1.
.....
IF(X=1,)2,1,1
1 Y=1,/X
  AUX=1,=Y*((((Y+3,377358E0)*Y+2,052156E0)*Y+2,709479E=1)/(((Y*
11,072553E0+5,716943E0)*Y+6,945239E0)*Y+2,593888E0)*Y+2,709496E=1)
  RES=AUX*Y*EXP(=X)
  RETURN
2 IF(X+3,)6,6,3
3 AUX=((( ((( (( (7,122452E=7*X=1,766345E=6)*X+2,928433E=5)*X=2,335379E=4
1)*X+1,664156E=3)*X=1,041576E=2)*X+5,555682E=2)*X=2,500001E=1)*X
2+9,999999E=1
  RES=-1,E75
  IF(X)4,5,4
4 RES=X*AUX=ALOG(ABS(X))=5,772157E=1
5 RETURN
6 IF(X+9,)8,8,7
7 AUX=1,=((( (5,176245E=2*X+3,061037E0)*X+3,243665E1)*X+2,244234E2)*X
1+2,486697E2)/((( (X+3,995161E0)*X+3,893944E1)*X+2,263818E1)*X
2+1,807837E2)
  GOTD 9
8 Y=9,/X
  AUX=1,=Y*((((Y+7,659824E=1)*Y=7,271015E=1)*Y=1,080693E0)/(((Y
1*2,518750E0+1,122927E1)*Y+5,921405E0)*Y=8,666702E0)*Y=9,724216E0)
9 RES=AUX*EXP(=X)/X
  RETURN
END

```

EXPI 350
EXPI 10
EXPI 20
EXPI 30
EXPI 40
EXPI 50
EXPI 60
EXPI 70
EXPI 80
EXPI 90
EXPI 100
EXPI 110
EXPI 120
EXPI 130
EXPI 140
EXPI 150
EXPI 160
EXPI 170
EXPI 180
EXPI 190
EXPI 200
EXPI 210
EXPI 220
EXPI 230
EXPI 240
EXPI 250
EXPI 260
EXPI 270
EXPI 280
EXPI 290
EXPI 300
EXPI 310
EXPI 320
EXPI 330
EXPI 340
EXPI 360
EXPI 370
EXPI 380
EXPI 390
EXPI 400
EXPI 410
EXPI 420
EXPI 430
EXPI 440
EXPI 450
EXPI 460
EXPI 470
EXPI 480
EXPI 490
EXPI 500
EXPI 510
EXPI 520
EXPI 530
EXPI 540
EXPI 550
EXPI 560
EXPI 570
EXPI 580
EXPI 590
EXPI 60=

TABLE 4.3—Listing of program for radial flow in a leaky artesian aquifer

```

C *****WUB 1
C WUB 2
C PURPOSE WUB 3
C TO COMPUTE A TABLE OF VALUES OF THE LEAKY AQUIFER WELL WUB 4
C FUNCTION =  $w(U, R/B)$  = HANTUSH, M.S., AND JACOB, C.E., 1955, WUB 5
C NON-STEADY RADIAL FLOW IN AN INFINITE LEAKY AQUIFER; AM, WUB 6
C GEOPHYS. UNION TRANS., V. 36, NO. 1, P. 95-100. WUB 7
C INPUT DATA WUB 8
C 1 CARD = FORMAT(2E10,5) WUB 9
C USMALL = SMALLEST VALUE OF 1/U FOR WHICH COMPUTATION IS WUB 10
C DESIRED, WUB 11
C ULARGE = LARGEST VALUE OF 1/U FOR WHICH COMPUTATION IS WUB 12
C DESIRED, WUB 13
C 2 CARDS = FORMAT(8E10,5) WUB 14
C BDAT = 12 VALUES OF R/B FOR TABLE, WUB 15
C SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED WUB 16
C L,SERIES,FCT,BE3K,DQL12 WUB 17
C WUB 18
C *****WUB 19
C REAL*4 L WUB 20
C REAL*8 U,V WUB 21
C DIMENSION ARRAY(73,12), Y(73), BDAT(12), YNUM(6) WUB 22
C DATA YNUM/1.,1.5,2.,3.,5.,7./ WUB 23
C IRD=5 WUB 24
C IPT=6 WUB 25
C READ (IRD,6) USMALL,ULARGE WUB 26
C READ (IRD,6) BDAT WUB 27
C IBEGIN=ALOG10(USMALL) WUB 28
C IEND=ALOG10(ULARGE)+.99999 WUB 29
C ILIMIT=(IEND-IBEGIN)*6+1 WUB 30
C IF (ILIMIT.GT.73) ILIMIT=73 WUB 31
C DO 1 I=1,12 WUB 32
C IF (BDAT(I).EQ.0.) GO TO 2 WUB 33
1 CONTINUE WUB 34
C NB=12 WUB 35
C GO TO 3 WUB 36
2 NB=I-1 WUB 37
3 II=0 WUB 38
C DO 4 I=1,ILIMIT WUB 39
C II=II+1 WUB 40
C IF (II.GT.6) II=1 WUB 41
C IEXP=IBEGIN+(I-1)/6 WUB 42
C Y(I)=YNUM(II)*10.**IEXP WUB 43
C U=1./Y(I) WUB 44
C DO 4 J=1,NB WUB 45
C V=BDAT(J)/2. WUB 46
4 ARRAY(I,J)=L(U,V) WUB 47
C WRITE (IPT,7) (BDAT(I),I=1,NB) WUB 48
C DO 5 I=1,ILIMIT WUB 49
5 WRITE (IPT,8) Y(I),(ARRAY(I,J),J=1,NB) WUB 50
C STOP WUB 51
C WUB 52
C WUB 53
C 6 FORMAT (8E10,5) WUB 54
C 7 FORMAT ('1', 'w(U,R/B)'/10',10X,'1 R/B'/ ' ',6X,'1/U ',12E10,2) WUB 55
C 8 FORMAT (' ',E10,3,12F10,4) WUB 56
C END WUB 57
C REAL FUNCTION L*4(U,V) L 1
C ***** L 2
C L 3
C FUNCTION L L 4
C L 5

```

TABLE 4.3—Listing of program for radial flow in a leaky artesian aquifer—Continued

```

C      PURPOSE                                     L      6
C      TO COMPUTE THE INTEGRAL( EXP(-Y=V**2/Y)/Y) SUMMED OVER Y FROM L      7
C      U TO INFINITY(WELL FUNCTION FOR LEAKY AQUIFERS). L      8
C      DESCRIPTION OF PARAMETERS L      9
C      BOTH DOUBLE PRECISION L     10
C      U = R**2*S/4*T*TIME (RADIAL DISTANCE SQUARED * STORAGE L     11
C      COEFFICIENT / 4*TRANSMISSIVITY * TIME L     12
C      V = R/2*SQRT(K'/(T*B'))=ONE-HALF RADIAL DISTANCE*SQUARE ROOT L     13
C      (HYD. COND. OF CONFINING BED/TRANSMISSIVITY*THICKNESS L     14
C      OF CONFINING BED) L     15
C      SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED L     16
C      DQL12,SERIES,BESK,FCT L     17
C      METHOD L     18
C      IN THE FOLLOWING F=EXP(-Y=V**2/Y)/Y L     19
C      (1) U>=1, USES A GAUSSIAN-LAGUERRE QUADRATURE FORMULA TO L     20
C      EVALUATE INTEGRAL(F) FROM U TO INF. L     21
C      (2) V**2<U<1, USES THE G=L QUADRATURE TO EVALUATE INTEGRAL(F) L     22
C      FROM ONE TO INF AND A SERIES EXPANSION TO EVALUATE INTEGRAL(F) L     23
C      FROM U TO ONE. L     24
C      (3) U<1, U<=V**2, USES THE REPRESENTATION INTEGRAL(F) FROM U L     25
C      TO INF. = 2*K0(2*V)=INTEGRAL(F) FROM V**2/U TO INF. L     26
C      EVALUATES THE ZERO ORDER MODIFIED BESSEL FUNCTION OF SECOND L     27
C      KIND WITH IBM SUBROUTINE, EVALUATES INTEGRAL BY G=L QUAD. L     28
C      L     29
C      ***** L     30
C      EXTERNAL FCT L     31
C      REAL*8 U,V,Z,F,VV,SERIES L     32
C      COMMON /C1/ VV,Z L     33
C      VV=V L     34
C      IF (U=1.) 1,2,2 L     35
C      CHECKS IF U<1 L     36
C      1 Z=V*V/U L     37
C      IF (Z=1.) 3,4,4 L     38
C      CHECKS IF V**2/U < 1 L     39
C      2 Z=U L     40
C      CALL DQL12(FCT,F) L     41
C      L=F L     42
C      INTEGRAL U TO INF, EVALUATED BY GAUSS-LAGUERRE QUADRATURE L     43
C      GO TO 5 L     44
C      3 Z=1. L     45
C      CALL DQL12(FCT,F) L     46
C      L=F+SERIES(U,V) L     47
C      INTEGRAL 1 TO INF, BY G=L QUAD., INTEGRAL U TO 1 BY SERIES EXP. L     48
C      GO TO 5 L     49
C      4 TWOV=2.*V L     50
C      CALL BESK(TWOV,0,BK,IER) L     51
C      CALL DQL12(FCT,F) L     52
C      L=2.*BK*F L     53
C      ZK0(2V)=INTEGRAL V**2/U TO INF, L     54
C      5 RETURN L     55
C      END L     56
C      REAL FUNCTION SERIES*8(U,V) SER     1
C      ***** SER     2
C      FUNCTION SERIES SER     3
C      SER     4
C      SER     5
C      PURPOSE SER     6
C      TO EVALUATE S(1)=S(U), WHERE S IS A SERIES EXPANSION OF SER     7
C      INTEGRAL(EXP(-Y=V**2/Y)DY/Y) GIVEN BY: S= SUM, M=0 TO INFINITY, SER     8
C      (F(M)*SUM, N=0 TO INF., (V**2*(2*N))/((N!)*(M+N)!)) WHERE F(M)= SER     9
C      LOG(U) IF M=0 AND = ((-1)**M/M)*(U**M=(V**2/U)**M) IF M>0, SER    10
C      DESCRIPTION OF PARAMETERS SER    11
C      BOTH DOUBLE PRECISION SER    12
C      U = R**2*S/4*T*TIME (RADIAL DISTANCE SQUARED * STORAGE SER    13
C      COEFFICIENT / 4*TRANSMISSIVITY * TIME SER    14
C      V = R/2*SQRT(K'/(T*B'))=ONE-HALF RADIAL DISTANCE*SQUARE ROOT SER    15
C      (HYD. COND. OF CONFINING BED/TRANSMISSIVITY*THICKNESS SER    16
C      OF CONFINING BED) SER    17

```

TABLE 4.3—Listing of program for radial flow in a leaky artesian aquifer—Continued

```

C   SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED          SER 18
C   NONE                                                  SER 19
C   METHOD                                                 SER 20
C   SUMMATION IS TERMINATED FOR THE INNER SERIES WHEN A TERM SER 21
C   BECOMES LESS THAN  $5.E-7/N$  AND FOR OUTER SERIES WHEN A TERM SER 22
C   BECOMES LESS THAN  $5.E-7$                                SER 23
C   *****                                              SER 24
C   REAL*8 DLOG,DABS,S(2),VUM,UU                          SER 25
C   REAL*8 TEST,U,UM,EM,EN,SUM1,SUM,SIGN,V,VSQ,V$QU,RMUL,TERM,TERM1 SER 26
C   TEST=5,D=07                                           SER 27
C   VSQ=V*V                                               SER 28
C   UU=U                                                  SER 29
C   DO 6 I=1,2                                           SER 30
C   EVALUATES SERIES FOR LOWER LIMIT = U AND UPPER LIMIT = 1 SER 31
C   IF (I,EQ,2) U=1.                                     SER 32
C   UM=1,                                                 SER 33
C   EM=-1,                                               SER 34
C   SUM1=0,                                              SER 35
C   SIGN=-1,                                             SER 36
C   VUM=1,                                              SER 37
C   VSQU=VSQ/U                                          SER 38
C   1 EM=EM+1,                                           SER 39
C   IF (EM=.1) 2,3,3                                     SER 40
C   CHECKS FOR M=0                                       SER 41
C   2 RMUL=DLOG(U)                                       SER 42
C   TERM1=1,                                             SER 43
C   GO TO 4                                              SER 44
C   3 UM=UM*U                                            SER 45
C   IF (VUM.LT.,1,D=30) VUM=0,                          SER 46
C   VUM=VUM*VSQU                                         SER 47
C   RMUL=(UM*VUM)/EM                                    SER 48
C   TERM1=TERM1/EM                                       SER 49
C   4 SIGN=-SIGN                                         SER 50
C   SUM=TERM1                                           SER 51
C   TERM=TERM1                                           SER 52
C   EN=0,                                                SER 53
C   5 EN=EN+1,                                           SER 54
C   TERM=TERM*VSQ/(EN*(EN+EM))                          SER 55
C   SUM=SUM+TERM                                         SER 56
C   IF (TEST,LE,DABS(RMUL*EN*TERM)) GO TO 5             SER 57
C   TRUNCATES INNER SERIES IF OUTER TERM*N*INNER TERM <  $5.E-7$  SER 58
C   SUM1=SUM1+SIGN*RMUL*SUM                              SER 59
C   IF (EM.LT.,.1) GO TO 1                               SER 60
C   IF (TEST,LE,DABS(RMUL*SUM)) GO TO 1                 SER 61
C   TRUNCATES OUTER SERIES IF OUTER TERM*INNER SUM <  $5.E-7$  SER 62
C   6 S(I)=SUM1                                          SER 63
C   U=UU                                                 SER 64
C   SERIES=S(2)-S(1)                                     SER 65
C   RETURN                                              SER 66
C   END                                                 SER 67
C   REAL FUNCTION FCT*(X)                                FCT 1
C   *****                                              FCT 2
C   FUNCTION FCT                                         FCT 3
C   FCT 4
C   FCT 5
C   PURPOSE                                             FCT 6
C   TO COMPUTE  $FCT(X)=EXP(-Z-V**2/(X+Z))/(X+Z)$           FCT 7

```

TABLE 4.3—Listing of program for radial flow in a leaky artesian aquifer—Continued

C	DESCRIPTION OF PARAMETERS	FCT	8
C	X = THE DOUBLE PRECISION VALUE OF X FOR WHICH FCT IS COMPUTED	FCT	9
C	SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED	FCT	10
C	NONE	FCT	11
C	METHOD	FCT	12
C	FORTRAN EVALUATION OF FUNCTION	FCT	13
C	*****	FCT	14
C	*****	FCT	15
C	REAL*8 X,V,Z,P,DEXP	FCT	16
C	COMMON /C1/ V,Z	FCT	17
C	IF (X) 1,2,2	FCT	18
C	1 FCT=0.	FCT	19
C	GO TO 4	FCT	20
C	2 P=Z+V**2/(X+Z)	FCT	21
C	IF (P=5.01) 3,3,1	FCT	22
C	3 FCT=DEXP(=P)/(X+Z)	FCT	23
C	4 RETURN	FCT	24
C	END	FCT	25
C	SUBROUTINE DQL12(FCT,Y)	DL12	380
C	DL12	10
C	DL12	20
C	DL12	30
C	SUBROUTINE DQL12	DL12	40
C	PURPOSE	DL12	50
C	TO COMPUTE INTEGRAL(EXP(-X)*FCT(X), SUMMED OVER X	DL12	60
C	FROM 0 TO INFINITY).	DL12	70
C		DL12	80
C		DL12	90
C	USAGE	DL12	100
C	CALL DQL12 (FCT,Y)	DL12	110
C	PARAMETER FCT REQUIRES AN EXTERNAL STATEMENT	DL12	120
C		DL12	130
C	DESCRIPTION OF PARAMETERS	DL12	140
C	FCT = THE NAME OF AN EXTERNAL DOUBLE PRECISION FUNCTION	DL12	150
C	SUBPROGRAM USED.	DL12	160
C	Y = THE RESULTING DOUBLE PRECISION INTEGRAL VALUE.	DL12	170
C		DL12	180
C	REMARKS	DL12	190
C	NONE	DL12	200
C		DL12	210
C	SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED	DL12	220
C	THE EXTERNAL DOUBLE PRECISION FUNCTION SUBPROGRAM FCT(X)	DL12	230
C	MUST BE FURNISHED BY THE USER.	DL12	240
C		DL12	250
C	METHOD	DL12	260
C	EVALUATION IS DONE BY MEANS OF 12-POINT GAUSSIAN-LAGUERRE	DL12	270
C	QUADRATURE FORMULA, WHICH INTEGRATES EXACTLY,	DL12	280
C	WHENEVER FCT(X) IS A POLYNOMIAL UP TO DEGREE 23.	DL12	290
C	FUR REFERENCE, SEE	DL12	300
C	SHAO/CHEN/FRANK, TABLES OF ZEROS AND GAUSSIAN WEIGHTS OF	DL12	310
C	CERTAIN ASSOCIATED LAGUERRE POLYNOMIALS AND THE RELATED	DL12	320
C	GENERALIZED HERMITE POLYNOMIALS, IBM TECHNICAL REPORT	DL12	330
C	TROO,1100 (MARCH 1964), PP,24-25.	DL12	340
C		DL12	350
C	DL12	360
C	DL12	370
C		DL12	390
C		DL12	400
C	DOUBLE PRECISION X,Y,FCT	DL12	410
C		DL12	420
C	X=.3709912104446692 D2	DL12	430
C	Y=.814807746742624 D=15*FCT(X)	DL12	440

TABLE 4.3—Listing of program for radial flow in a leaky artesian aquifer—Continued

```

X=,2848796725098400 D2
Y=Y+,3061601635035021 D=11*FCT(X)
X=,2215109037939701 D2
Y=Y+,1342391030515004 D=8*FCT(X)
X=,1711685518746226 D2
Y=Y+,1668493876540910 D=6*FCT(X)
X=,1300605499330635 D2
Y=Y+,836505585681980 D=5*FCT(X)
X=,962131684245687 D1
Y=Y+,2032315926629994 D=3*FCT(X)
X=,6844525453115177 D1
Y=Y+,2663973541865316 D=2*FCT(X)
X=,4599227639418348 D1
Y=Y+,2010238115463410 D=1*FCT(X)
X=,2833751337743507 D1
Y=Y+,904492222116809 D=1*FCT(X)
X=,1512610269776419 D1
Y=Y+,2440820113198776 D0*FCT(X)
X=,6117574845151307 D0
Y=Y+,3777592758731380 D0*FCT(X)
X=,1157221173580207 D0
Y=Y+,2647313710554432 D0*FCT(X)
RETURN
END
SUBROUTINE BESK(X,N,BK,IER)
.....
SUBROUTINE BESK
  COMPUTE THE K BESSEL FUNCTION FOR A GIVEN ARGUMENT AND ORDER
  USAGE
  CALL BESK(X,N,BK,IER)
  DESCRIPTION OF PARAMETERS
  X =THE ARGUMENT OF THE K BESSEL FUNCTION DESIRED
  N =THE ORDER OF THE K BESSEL FUNCTION DESIRED
  BK =THE RESULTANT K BESSEL FUNCTION
  IER=RESULTANT ERROR CODE WHERE
  IER=0 NO ERROR
  IER=1 N IS NEGATIVE
  IER=2 X IS ZERO OR NEGATIVE
  IER=3 X ,GT. 170, MACHINE RANGE EXCEEDED
  IER=4 BK ,GT. 10**70
  REMARKS
  N MUST BE GREATER THAN OR EQUAL TO ZERO
  SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
  NONE
  METHOD
  COMPUTES ZERO ORDER AND FIRST ORDER BESSEL FUNCTIONS USING
  SERIES APPROXIMATIONS AND THEN COMPUTES N TH URDEK FUNCTION
  USING RECURRENCE RELATION,
  RECURRENCE RELATION AND POLYNOMIAL APPROXIMATION TECHNIQUE
  AS DESCRIBED BY A,J,M,HITCHCOCK,'POLYNOMIAL APPROXIMATIONS
  TO BESSEL FUNCTIONS OF ORDER ZERO AND ONE AND TO RELATED
  FUNCTIONS', M,T,A,C., V.11,1957,PP,86-88, AND G.N. WATSON,
  'A TREATISE ON THE THEORY OF BESSEL FUNCTIONS', CAMBRIDGE
  UNIVERSITY PRESS, 1958, P. 62

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DL12 450
DL12 460
DL12 470
DL12 480
DL12 490
DL12 500
DL12 510
DL12 520
DL12 530
DL12 540
DL12 550
DL12 560
DL12 570
DL12 580
DL12 590
DL12 600
DL12 610
DL12 620
DL12 630
DL12 640
DL12 650
DL12 660
DL12 670
DL12 68=
BESK 410
BESK 10
BESK 20
BESK 30
BESK 40
BESK 50
BESK 60
BESK 70
BESK 80
BESK 90
BESK 100
BESK 110
BESK 120
BESK 130
BESK 140
BESK 150
BESK 160
BESK 170
BESK 180
BESK 190
BESK 200
BESK 210
BESK 220
BESK 230
BESK 240
BESK 250
BESK 260
BESK 270
BESK 280
BESK 290
BESK 300
BESK 310
BESK 320
BESK 330
BESK 340
BESK 350
BESK 360
BESK 370

```

TABLE 4.3—Listing of program for radial flow in a leaky artesian aquifer—Continued

```

C
C ..... BESK 380
C ..... BESK 390
C ..... BESK 400
  DIMENSION T(12) BESK 420
  BK=,0 BESK 430
  IF(N)10,11,11 BESK 440
10 IER=1 BESK 450
  RETURN BESK 460
11 IF(X)12,12,20 BESK 470
12 IER=2 BESK 480
  RETURN BESK 490
20 IF(X=170,0)22,22,21 BESK 500
21 IER=3 BESK 510
  RETURN BESK 520
22 IER=0 BESK 530
  IF(X=1,)36,36,25 BESK 540
25 A=EXP(-X) BESK 550
  B=1./X BESK 560
  C=SQRT(B) BESK 570
  T(1)=B BESK 580
  DO 26 L=2,12 BESK 590
26 T(L)=T(L-1)*B BESK 600
  IF(N=1)27,29,27 BESK 610
C
C COMPUTE K0 USING POLYNOMIAL APPROXIMATION BESK 620
C BESK 630
C BESK 640
27 G0=A*(1,2533141=,1566642*T(1)+,08811128*T(2)=,09139095*T(3) BESK 650
  2+,1344596*T(4)=,2299850*T(5)+,3792410*T(6)=,5247277*T(7) BESK 660
  3+,5575368*T(8)=,4262633*T(9)+,2184518*T(10)=,06680977*T(11) BESK 670
  4+,009189383*T(12))*C BESK 680
  IF(N)20,28,29 BESK 690
28 BK=G0 BESK 700
  RETURN BESK 710
C
C COMPUTE K1 USING POLYNOMIAL APPROXIMATION BESK 720
C BESK 730
C BESK 740
29 G1=A*(1,2533141+,4699927*T(1)=,1468583*T(2)+,1280427*T(3) BESK 750
  2=,1736432*T(4)+,2847618*T(5)=,4594342*T(6)+,6283381*T(7) BESK 760
  3=,6632295*T(8)+,5050239*T(9)=,2581304*T(10)+,07880001*T(11) BESK 770
  4=,01082418*T(12))*C BESK 780
  IF(N=1)20,30,31 BESK 790
30 BK=G1 BESK 800
  RETURN BESK 810
C
C FROM K0,K1 COMPUTE KN USING RECURRENCE RELATION BESK 820
C BESK 830
C BESK 840
31 DO 35 J=2,N BESK 850
  GJ=2.*(FLOAT(J)-1.)*G1/X+G0 BESK 860
  IF(GJ=1,0E70)33,33,32 BESK 870
32 IER=4 BESK 880
  GO TO 34 BESK 890
33 G0=G1 BESK 900
35 G1=GJ BESK 910
34 BK=GJ BESK 920
  RETURN BESK 930
36 B=X/2, BESK 940
  A=,5772157+ALOG(B) BESK 950
  C=B*B BESK 960
  IF(N=1)37,43,37 BESK 970
C
C COMPUTE KU USING SERIES EXPANSION BESK 980
C BESK 990

```

TABLE 4.3—Listing of program for radial flow in a leaky artesian aquifer—Continued

```

C
37 GO=A
   X2J=1,
   FACT=1,
   HJ=0
   DO 40 J=1,6
   RJ=1./FLOAT(J)
   IF(X2J,LT,1,E=40) X2J=0,
C PREVIOUS STATEMENT ADDED TO IBM SUBROUTINE TO CORRECT UNDERFLOW
C PROBLEM ON WATFOR COMPILER
   X2J=X2J*C
   FACT=FACT*RJ*RJ
   HJ=HJ+RJ
40 GO=GO+X2J*FACT*(HJ=A)
   IF(N)43,42,43
42 BK=GO
   RETURN
C
C COMPUTE K1 USING SERIES EXPANSION
C
43 X2J=B
   FACT=1,
   HJ=1,
   G1=1./X+X2J*(.5+A-HJ)
   DO 50 J=2,8
   X2J=X2J*C
   RJ=1./FLOAT(J)
   FACT=FACT*RJ*RJ
   HJ=HJ+RJ
50 G1=G1+X2J*FACT*(.5+(A-HJ)*FLOAT(J))
   IF(N=1)31,52,31
52 BK=G1
   RETURN
   END

```

BESK1000
 BESK1010
 BESK1020
 BESK1030
 BESK1040
 BESK1050
 BESK1060
 BESK1061
 BESK1062
 BESK1063
 BESK1070
 BESK1080
 BESK1090
 BESK1100
 BESK1110
 BESK1120
 BESK1130
 BESK1140
 BESK1150
 BESK1160
 BESK1170
 BESK1180
 BESK1190
 BESK1200
 BESK1210
 BESK1220
 BESK1230
 BESK1240
 BESK1250
 BESK1260
 BESK1270
 BESK1280
 BESK1290
 BESK130=

TABLE 5.2—Listing of program for radial flow in a leaky artesian aquifer with storage of water in the confining beds

```

C *****LST 1
C LST 2
C PURPOSE LST 3
C TO COMPUTE TYPE CURVE FUNCTION VALUES FOR M(U,BETA) == LST 4
C MANTUSH,M.S.,1960, MODIFICATION OF THE THEORY OF LEAKY LST 5
C AQUIFERS: JOUR, GEOPHYS, RES., V. 65, NO. 11, P. 3713-3725. LST 6
C THE COMPUTATIONAL ALGORITHM WAS DEVISED AND PROGRAMMED BY LST 7
C S.S.PAPADOPULUS. LST 8
C INPUT DATA LST 9
C 1 CARD = FORMAT(2E10,5) LST 10
C USMALL = SMALLEST(BEGINNING) VALUE OF 1/U. LST 11
C ULARGE = LARGEST(ENDING) VALUE OF 1/U. LST 12
C 2 CARDS = FORMAT(8E10,5) LST 13
C BDAT = 12 VALUES OF BETA (ZERO) OR BLANK VALUES ARE LST 14
C PERMISSIBLE IF LESS THAN 12 DESIRED, *WILL TERMINATE LST 15
C AT FIRST ZERO (OR BLANK VALUE). LST 16
C SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED LST 17
C H,DGG32,MUR,* = MUST BE INCLUDED IN DECK, LST 18
C DSQRT,DEXP,DERFC,DLOG = MUST BE IN COMPUTER LIBRARY. LST 19
C LST 20
C *****LST 21
C REAL*8 U,BETA,H LST 22
C DIMENSION ARRAY(73,12), Y(73), BDAT(12), YNUM(6) LST 23
C DATA YNUM/1.,1.5,2.,3.,5.,7./ LST 24
C IRD=5 LST 25
C IPT=6 LST 26
C READ (IRD,6) USMALL,ULARGE LST 27
C HEAD (IRD,6) BDAT LST 28
C IBEGIN=ALOG10(USMALL) LST 29
C IEND=ALOG10(ULARGE)+.99999 LST 30
C ILIMIT=(IEND-IBEGIN)*6+1 LST 31
C IF (ILIMIT,GT,73) ILIMIT=73 LST 32
C DO 1 I=1,12 LST 33
C IF (BDAT(I),EQ,0.) GO TO 2 LST 34
1 CONTINUE LST 35
NB=12 LST 36

```

TABLE 5.2—Listing of program for radial flow in a leaky artesian aquifer with storage of water in the confining beds—
Continued

```

GO TO 3
2 NB=1
  II=0
3 DO 4 I=1,ILIMIT
  IEXP=IBEGIN+(I-1)/6
  II=II+1
  IF (II,GT,6) II=1
  Y(I)=YNUM(II)*10,**IEXP
  U=1./Y(I)
  DO 4 J=1,NB
  BETA=BDAT(J)
4 ARRAY(I,J)=H(U,BETA)
  WRITE (IPT,7) (BDAT(I),I=1,NB)
  DO 5 I=1,ILIMIT
5 WRITE (IPT,8) Y(I),(ARRAY(I,J),J=1,NB)
  STOP

C
6 FORMAT (BE10,5)
7 FURMAT ('I',H(U,BETA)'/0',10X,'I BETA'/' ',6X,'/U ',12E10,2)
8 FORMAT (' ',E10,3,12F10,4)
END
DOUBLE PRECISION FUNCTION H(U,B)
*****
C
C C
C C C
FUNCTION H
C C C
PURPOSE
C C C
TO COMPUTE THE INTEGRAL OF
C C C
EXP(-Y)*ERFC(B*SQRT(U)/SQRT(Y*(Y+U)))/Y SUMMED OVER Y
C C C
FROM U TO INFINITY (FUNCTION H(U,BETA) OF HANTUSH).
C C C
DESCRIPTION OF PARAMETERS
C C C
BOTH DOUBLE PRECISION
C C C
U = R**2*S/(4*T*TIME), (RADIAL DISTANCE SQUARED * STORAGE
C C C
COEFFICIENT / (4 * TRANSMISSIVITY * TIME), U MUST BE > 1.0=60,
C C C
B = (R/4)*(SQRT(K'*S'/(B'*T*S)+K''S''/(B''*T*S))),
C C C
K',S',B' = HYD. COND., STORAGE COEFF., THICKNESS OF
C C C
UPPER CONFINING BED,
C C C
K'',S'',B'' = HYD. COND., STORAGE COEFF., THICKNESS OF
C C C
LOWER CONFINING BED,
C C C
METHOD
C C C
I. FOR U < 1.0=60, NO COMPUTATION IS MADE,
C C C
II. FOR B=0, H(U,0)=w(U) (THEIS WELL FUNCTION),
C C C
III. H(U,B)=0 IF
C C C
  1. U > 10,
C C C
  2. B > 1 AND B**2*U > 300,
C C C
IV. ERFC(ARG)=0 FOR ARG > 40 AND H(U,B) = H(UB,B)
C C C
FOR U < Y < UB WHERE UB IS THE U CORRESPONDING TO ARG = 40
C C C
SINCE H(UB,B) < w(UB) THEN FOR UB > 10, H(U,B) = 0,
C C C
ERFC(ARG) = 1 FOR ARG < 2.E=10 AND H(UUB,B) = w(UUB)
C C C
WHERE UUB IS THE U CORRESPONDING TO ARG = 2.E=10,
C C C
IF UUB > 10, H(U,B) = INTEGRAL FROM UB TO 10,
C C C
IF UUB < 10, H(U,B) = INTEGRAL FROM UB TO UUB + w(UUB)
C C C
*****
C
IMPLICIT REAL*8(A=H,D=Z)
COMMON UUU,BBB
EXTERNAL HUB
UUU=U
BBB=B
IF (U,GT,1.0=60) GO TO 1
WRITE (6,7)
STOP
1 IF (B,EQ,0,0) GO TO 5
IF (U,GT,10,0) GO TO 6
BU=B*B*U
IF (B,GT,1.0,AND,BU,GE,300,0) GO TO 6
H1=0,0
UP=10,0
UB=0,5*U*(1,0+DSQRT(1,0+0,025*B*B/U))
IF (UB,GT,UP) GO TO 6
UUB=0,5*U*(1,0+DSQRT(1,0+1,020*B*B/U))
IF (UUB,GT,UP) GO TO 2
H1=w(UUB)
UP=UUB
2 H2=0,0
XL=UB
3 XU=10,*XL
IF (XU,GE,UP) XU=UP
CALL DQG32(XL,XU,HUB,AREA)
H2=H2+AREA
XL=XU
IF (XL,EQ,UP) GO TO 4
GO TO 3
4 H=H1+H2
RETURN
5 H=w(U)
RETURN

```


TABLE 5.2—Listing of program for radial flow in a leaky artesian aquifer with storage of water in the confining beds—
Continued

```

6 H=0,0
RETURN
C
7 FORMAT ('0', 'U TOO SMALL FOR COMPUTATION')
END
C
SUBROUTINE DQG32(XL,XU,FCT,Y)
C
C .....
C
C SUBROUTINE DQG32
C
C PURPOSE
C TO COMPUTE INTEGRAL(FCT(X), SUMMED OVER X FROM XL TO XU)
C
C USAGE
C CALL DQG32 (XL,XU,FCT,Y)
C PARAMETER FCT REQUIRES AN EXTERNAL STATEMENT
C
C DESCRIPTION OF PARAMETERS
C XL = DOUBLE PRECISION LOWER BOUND OF THE INTERVAL.
C XU = DOUBLE PRECISION UPPER BOUND OF THE INTERVAL.
C FCT = THE NAME OF AN EXTERNAL DOUBLE PRECISION FUNCTION
C SUBPROGRAM USED.
C Y = THE RESULTING DOUBLE PRECISION INTEGRAL VALUE.
C
C REMARKS
C NONE
C
C SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C THE EXTERNAL DOUBLE PRECISION FUNCTION SUBPROGRAM FCT(X)
C MUST BE FURNISHED BY THE USER,
C
C METHOD
C EVALUATION IS DONE BY MEANS OF 32=POINT GAUSS QUADRATURE
C FORMULA, WHICH INTEGRATES POLYNOMIALS UP TO DEGREE 63
C EXACTLY. FOR REFERENCE, SEE
C V.I.KRYLOV, APPROXIMATE CALCULATION OF INTEGRALS,
C MACMILLAN, NEW YORK/LONDON, 1962, PP.100=111 AND 337=340.
C
C .....
C
C DOUBLE PRECISION XL,XU,Y,A,B,C,FCT
C A=.500*(XU+XL)
C B=XU-XL
C=.498631930924740800*B
C Y=.35093050047350480=2*(FCT(A+C)+FCT(A=C))
C=.492805755772634200*B
C Y=Y+.8137197365452840=2*(FCT(A+C)+FCT(A=C))
C=.482381127793753200*B
C Y=Y+.12696032654631030=1*(FCT(A+C)+FCT(A=C))
C=.467453037968869800*B
C Y=Y+.17136931456510720=1*(FCT(A+C)+FCT(A=C))
C=.448160577883026100*B
C Y=Y+.21417949011113340=1*(FCT(A+C)+FCT(A=C))
C=.424683806866285000*B
C Y=Y+.25499029631188090=1*(FCT(A+C)+FCT(A=C))
C=.397241897983971200*B
C Y=Y+.29342046739267770=1*(FCT(A+C)+FCT(A=C))
C=.366091059370144800*B
C Y=Y+.32911111388180920=1*(FCT(A+C)+FCT(A=C))
C=.331522133465107600*B
C Y=Y+.36172897054424250=1*(FCT(A+C)+FCT(A=C))
C=.293857878620381200*B
C Y=Y+.39096947893535150=1*(FCT(A+C)+FCT(A=C))
C=.253449954466114700*B
C Y=Y+.41655962113473380=1*(FCT(A+C)+FCT(A=C))
C=.210675638065317700*B
C Y=Y+.43826046502201910=1*(FCT(A+C)+FCT(A=C))
C=.165934301141063800*B
C Y=Y+.45586939347881940=1*(FCT(A+C)+FCT(A=C))
C=.119643681126068500*B
C Y=Y+.46922199540402280=1*(FCT(A+C)+FCT(A=C))
C=.7223598079139820=1*B
C Y=Y+.47819360039637430=1*(FCT(A+C)+FCT(A=C))
C=.24153832643869160=1*B
C Y=B*(Y+.48270044257363900=1*(FCT(A+C)+FCT(A=C)))
RETURN
END
C
DOUBLE PRECISION FUNCTION HUB(X)
C *****
C
C FUNCTION HUB
C PURPOSE
C TO COMPUTE VALUES OF THE INTEGRAND OF H(U,B)
C DESCRIPTION OF PARAMETER
C X = DOUBLE PRECISION, POINT AT WHICH INTEGRAND IS EVALUATED.

```

TABLE 5.2—Listing of program for radial flow in a leaky artesian aquifer with storage of water in the confining beds—
Continued

```

C      METHOD                                          HUB  9
C      FURTKAN EVALUATION OF FUNCTION,              HUB 10
C                                                    HUB 11
C      *****HUB 12
C      IMPLICIT REAL*8(A=H,O=Z)                     HUB 13
C      COMMON UUU,BBB                                HUB 14
C      ARG=DSQRT((BBB*BBB*UUU)/(X*X-X*UUU))          HUB 15
C      HUB=DEXP(-X)*DERFC(ARG)/X                     HUB 16
C      RETURN                                         HUB 17
C      END                                            HUB 18=
C
C      DOUBLE PRECISION FUNCTION W(U)                KU  1
C      *****KU  2
C      FUNCTION W                                     KU  3
C      PURPOSE                                       KU  4
C      TO EVALUATE THE WELL FUNCTION OF THIS,      KU  5
C      DESCRIPTION OF PARAMETER                     KU  6
C      U = DOUBLE PRECISION, ARGUMENT FOR WELL FUNCTION, KU  7
C                                                    KU  8
C      *****KU  9
C      IMPLICIT REAL*8 (A=H,O=Z)                     KU 10
C      IF (U,LE,0.0) GO TO 2                          KU 11
C      IF (U,GT,100.) GO TO 3                          KU 12
C      IF (U,GE,1.0) GO TO 1                          KU 13
C      W=,57721566+U*(,99999193+U*(,24991055+U*(,05519968+U*(,00976004
KU 14
KU 15
KU 16
KU 17
KU 18
KU 19
KU 20
KU 21
KU 22
KU 23
KU 24
KU 25
KU 26
KU 27
KU 28
KU 29
KU 30=
C      1 ENUM=DEXP(=U)*(,2677737343+U*(8,6347608925+U*(18,0590169730+U*(8,5
KU 18
KU 19
KU 20
KU 21
KU 22
KU 23
KU 24
KU 25
KU 26
KU 27
KU 28
KU 29
KU 30=
C      1733287401+U)))
C      DEN=U*(3,9584969228+U*(21,0996530827+U*(25,6329561486+U*(9,5733223
KU 20
KU 21
KU 22
KU 23
KU 24
KU 25
KU 26
KU 27
KU 28
KU 29
KU 30=
C      1454+U)))
C      W=ENUM/DEN
C      GO TO 4
C      2 WRITE (6,5) U
C      STOP
C      3 W=0.0
C      4 RETURN
C
C      5 FUMAT ('0',5X,'W(U) NOT DEFINED FOR U=',1PD15,8)
C      END

```

TABLE 6.1.—Listing of program for partial penetration in a leaky artesian aquifer

```

C      *****PPL  1
C      PURPOSE                                         PPL  2
C      TO COMPUTE TYPE CURVE FUNCTION VALUES FOR PARTIAL PENETRATION PPL  3
C      IN A LEAKY AQUIFER USING EQ. 73 OF HANTUSH,M.S., 1964, PPL  4
C      HYDRAULICS OF WELLS IN CHOW, VEN TE, ADVANCES IN HYDRUSCIENCE, PPL  5
C      VOL. 1: ACADEMIC PRESS, NEW YORK, P. 281-442. PPL  6
C      INPUT DATA                                     PPL  7
C      1 CARD = FORMAT (3F5,1,15,2E10,4)             PPL  8
C      B = AQUIFER THICKNESS                           PPL  9
C      E = DEPTH, BELOW TOP OF AQUIFER, TO BOTTOM OF PUMPING PPL 10
C      WELL SCREEN                                     PPL 11
C      D = DEPTH, BELOW TOP OF AQUIFER, TO TOP OF PUMPING WELL PPL 12
C      SCREEN                                           PPL 13
C      NUM = NUMBER OF OBSERVATION WELLS OR PIEZOMETERS TIMES PPL 14
C      NUMBER OF VALUES OF KZ/KR,                    PPL 15
C      SMALL = SMALLEST VALUE OF 1/U FOR WHICH COMPUTATION IS PPL 16
C      DESIRED                                          PPL 17
C      LARGE = LARGEST VALUE OF 1/U FOR WHICH COMPUTATION IS PPL 18
C      DESIRED                                          PPL 19
C      2 CARDS = FORMAT(8E10,5)                        PPL 20
C      BDATA = 12 VALUES OF R/BR, NON ZERO VALUES SHOULD BE PPL 21
C      FIRST, WILL TERMINATE AT FIRST ZERO (OR BLANK) VALUE. PPL 22
C      NUM CARDS (ONE FOR EACH OBS. WELL OR PIEZOMETER AND FOR EACH PPL 23
C      VALUE OF R*SQRT(KZ/KR) = FORMAT (3F5,1)        PPL 24
C      R = RADIAL DISTANCE FROM PUMPED WELL TIMES SQRT(KZ/KR), PPL 25
C      LPRIME = DEPTH, BELOW TOP OF AQUIFER, TO BOTTOM OF OBS. PPL 26
C      WELL SCREEN (ZERO FOR PIEZOMETER)              PPL 27
C      DPRIME = DEPTH, BELOW TOP OF AQUIFER, TO TOP OF OBS. WELL PPL 28
C

```

TABLE 6.1.—Listing of program for partial penetration in a leaky artesian aquifer—Continued

C	SCREEN (TOTAL DEPTH FOR PIEZOMETER)	PPL 30
C	SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED	PPL 31
C	DQL12,SERIES,BESK,FCT,L,FL	PPL 32
C	*****	PPL 33
C	*****	PPL 34
	REAL*8 U,V	PPL 35
	REAL*4 L,LB,LPB,LPRIME,LARGE	PPL 36
	DIMENSION ARRAY(55,12), ARG(6), BDAT(12), Y(55)	PPL 37
	DATA ARG/1,,1,5,2,,3,,5,,7,/,	PPL 38
	DATA ARRAY/660*0,/,Y/55*0,/,	PPL 39
	IRD=5	PPL 40
	IPT=6	PPL 41
	READ (IRD,9) B,E,D,NUM,SMALL,LARGE	PPL 42
	READ (IRD,14) BDAT	PPL 43
	DO 1 I=1,12	PPL 44
	IF (BDAT(I).EQ,0,) GO TO 2	PPL 45
1	CONTINUE	PPL 46
	NB=12	PPL 47
	GO TO 3	PPL 48
2	NB=I-1	PPL 49
3	LB=E/B	PPL 50
	DB=D/B	PPL 51
	IBEGIN=ALOG10(SMALL)	PPL 52
	IEND=ALOG10(LARGE)+.1	PPL 53
	JLIMIT=IEND-IBEGIN	PPL 54
	IF (JLIMIT.GT,9) JLIMIT=9	PPL 55
	ILIMIT=6+JLIMIT+1	PPL 56
	DO 8 K=1,NUM	PPL 57
	READ (IRD,9) R,LPRIME,DPRIME	PPL 58
	RB=R/B	PPL 59
	LPB=LPRIME/B	PPL 60
	DPB=DPRIME/B	PPL 61
	DO 4 I=1,ILIMIT	PPL 62
	INDEX=(I-1)/6	PPL 63
	IEXP=IBEGIN+INDEX	PPL 64
	II=I-INDEX*6	PPL 65
	Y(I)=ARG(II)*10,**IEXP	PPL 66
	U=1./Y(I)	PPL 67
	DO 4 J=1,NB	PPL 68
	BETA=BDAT(J)	PPL 69
	V=BETA/2.	PPL 70
4	ARRAY(I,J)=L(U,V)+FL(U,RB,BETA,LB,DB,LPB,DPB)	PPL 71
	IF (LPB=0.) 5,5,6	PPL 72
5	WRITE (IPT,10) DPB,RB,LB,DB	PPL 73
	GO TO 7	PPL 74
6	WRITE (IPT,11) LPB,DPB,RB,LB,DB	PPL 75
7	WRITE (IPT,12) (BDAT(I),I=1,NB)	PPL 76
	DO 8 I=1,ILIMIT	PPL 77
	WRITE (IPT,13) Y(I),(ARRAY(I,J),J=1,NB)	PPL 78
8	CONTINUE	PPL 79
	STOP	PPL 80
C		PPL 81
C		PPL 82
	9 FORMAT (3F5,1,I5,2E10,4)	PPL 83
10	FORMAT ('11','W(U,R/BR)+F(U,R/B,R/BR,L/B,D/B,Z/B), Z/B='F5,2,1, SQRT(KZ/KK)*R/B='F5,2,1, L/B='F5,2,1, D/B='F5,2)	PPL 84
	1RT(KZ/KK)*R/B='F5,2,1, L/B='F5,2,1, D/B='F5,2)	PPL 85
11	FORMAT ('11','W(U,R/BR)+F(U,R/B,R/BR,L/B,D/B,L'/B,D'/B), L'/B='F5,2,1, D'/B='F5,2,1, SQRT(KZ/KK)*R/B='F5,2,1, L/B='F5,2,1, D/B='F5,2)	PPL 86
	1F5,2,1, D'/B='F5,2,1, SQRT(KZ/KK)*R/B='F5,2,1, L/B='F5,2,1, D/B='F5,2)	PPL 87
	2B='F5,2)	PPL 88
12	FORMAT ('01,9X,'1 R/BR'/ '1,5X,'1/U '1,12E10,2)	PPL 89
13	FORMAT (' '1,E10,3,12F10,4)	PPL 90
14	FORMAT (8E10,5)	PPL 91
	END	PPL 92=

TABLE 6.1.—Listing of program for partial penetration in a leaky artesian aquifer—Continued

```

C REAL FUNCTION FL*(U, RB, BETA, LB, DB, LPB, DPB) FL 1
C ***** FL 2
C FUNCTION FL FL 3
C FL 4
C FL 5
C PURPOSE FL 6
C TO COMPUTE DEPARTURES FROM HANTUSH-JACOB LEAKY AQUIFER CURVE FL 7
C CAUSED BY PARTIAL PENETRATION OF PUMPED WELL. FL 8
C USAGE FL 9
C FL(U, RB, BETA, LB, DB, LPB, DPB) FL 10
C DESCRIPTION OF PARAMETERS FL 11
C ALL REAL, U DOUBLE PRECISION FL 12
C U = R**2*S/4*T*TIME (RADIAL DISTANCE SQUARED * STORAGE FL 13
C COEFFICIENT / 4*TRANSMISSIVITY * TIME FL 14
C RB = R/B ( RADIAL DISTANCE / AQUIFER THICKNESS ) FL 15
C BETA = R*SQRT(K'/B'T) = (RADIAL DISTANCE * SQUARE ROOT FL 16
C (HYD. COND. OF CONFINING BED/THICKNESS OF CONFINING FL 17
C BED * TRANSMISSIVITY OF AQUIFER)) FL 18
C LB = L/B ( FRACTION OF AQUIFER PENETRATED BY PUMPED WELL) FL 19
C DB = D/B ( FRACTION OF AQUIFER ABOVE PUMPED WELL SCREEN) FL 20
C LPB = L'/B (FRACTION OF AQUIFER PENETRATED BY OBS, WELL, ZERO FL 21
C FOR PIEZOMETER) FL 22
C DPB = D'/B (FRACTION OF AQUIFER ABOVE OBS, WELL SCREEN, TOTAL FL 23
C DEPTH FOR PIEZOMETER) FL 24
C SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED FL 25
C DQL12,SERIES,BESK,FCT,L FL 26
C METHOD FL 27
C SUMS THE SERIES THROUGH N*PI*R/B EQ 20 FL 28
C FL 29
C ***** FL 30
C REAL*8 U,V,DSQRT FL 31
C REAL*4 L,N,LB,LPB FL 32
C SUM=0, FL 33
C N=0, FL 34
C BETSQ=BETA*BETA FL 35
C PIRBSQ=9.869604*RB*RB FL 36
C PILB=3.141593*LB FL 37
C PIDB=3.141593*DB FL 38
C IF (LPB=0.) 1,1,4 FL 39
C CHECKS FOR WELL OR PIEZOMETER FL 40
C 1 PIZB=3.141593*DPB FL 41
C 2 N=N+1, FL 42
C V=SQRT(BETSQ+N*N*PIRBSQ)/2, FL 43
C IF (V,GT,10.) GO TO 3 FL 44
C TRUNCATES SERIES WHEN V>10 FL 45
C X=L(U,V)/N FL 46
C SUM=SUM+(SIN(N*PILB)-SIN(N*PIDB))*COS(N*PIZB)*X FL 47
C GO TO 2 FL 48
C 3 FL=.6366198*SUM/(LB=DB) FL 49
C GO TO 7 FL 50
C 4 PILPB=3.141593*LPB FL 51
C PIDPB=3.141593*DPB FL 52
C 5 N=N+1 FL 53
C V=SQRT(BETSQ+N*N*PIRBSQ)/2, FL 54
C IF (V,GT,10.) GO TO 6 FL 55
C TRUNCATES SERIES WHEN V>10 FL 56
C X=L(U,V)/N FL 57
C SUM=SUM+(SIN(N*PILB)-SIN(N*PIDB))*(SIN(N*PILPB)-SIN(N*PIDPB))*X/N FL 58
C GO TO 5 FL 59
C 6 FL=.2026424*SUM/((LB=DB)*(LPB=DPB)) FL 60
C 7 RETURN FL 61
C END FL 62=

```


TABLE 6.1.—Listing of program for partial penetration in a leaky artesian aquifer—Continued

```

REAL FUNCTION FCT*8(X)
*****
FUNCTION FCT
PURPOSE
  TO COMPUTE  $FCT(X) = \exp(-Z - V^2/(X+Z)) / (X+Z)$ 
DESCRIPTION OF PARAMETERS
  X = THE DOUBLE PRECISION VALUE OF X FOR WHICH FCT IS COMPUTED
SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
  NONE
METHOD
  FORTRAN EVALUATION OF FUNCTION
*****
REAL*8 X,V,Z,P,DEXP
COMMON /C1/ V,Z
IF (X) 1,2,2
1 FCT=0,
  GO TO 4
2 P=Z+V**2/(X+Z)
  IF (P-S,D1) 3,3,1
3 FCT=DEXP(-P)/(X+Z)
4 RETURN
  END
SUBROUTINE DQL12(FCT,Y)
.....
SUBROUTINE DQL12
PURPOSE
  TO COMPUTE INTEGRAL( $\exp(-X) * FCT(X)$ , SUMMED OVER X
  FROM 0 TO INFINITY),
USAGE
  CALL DQL12 (FCT,Y)
  PARAMETER FCT REQUIRES AN EXTERNAL STATEMENT
DESCRIPTION OF PARAMETERS
  FCT = THE NAME OF AN EXTERNAL DOUBLE PRECISION FUNCTION
  SUBPROGRAM USED,
  Y = THE RESULTING DOUBLE PRECISION INTEGRAL VALUE,
REMARKS
  NONE
SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
  THE EXTERNAL DOUBLE PRECISION FUNCTION SUBPROGRAM FCT(X)
  MUST BE FURNISHED BY THE USER,
METHOD
  EVALUATION IS DONE BY MEANS OF 12-POINT GAUSSIAN-LAGUERRE
  QUADRATURE FORMULA, WHICH INTEGRATES EXACTLY,
  WHENEVER FCT(X) IS A POLYNOMIAL UP TO DEGREE 23.
  FOR REFERENCE, SEE
  SHAO/CHEN/FRANK, TABLES OF ZEROS AND GAUSSIAN WEIGHTS OF
  CERTAIN ASSOCIATED LAGUERRE POLYNOMIALS AND THE RELATED
  GENERALIZED HERMITE POLYNOMIALS, IBM TECHNICAL REPORT
  TR00.1100 (MARCH 1964), PP.24-25.
.....

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TABLE 6.1.—Listing of program for partial penetration in a leaky artesian aquifer—Continued

C		DL12 370
C		DL12 390
C		DL12 400
	DOUBLE PRECISION X,Y,FCT	DL12 410
C		DL12 420
	X=,3709912104446692 D2	DL12 430
	Y=,814807746742624 D=15*FCT(X)	DL12 440
	X=,2848796725098400 D2	DL12 450
	Y=Y+,3061601635035021 D=11*FCT(X)	DL12 460
	X=,2215109037939701 D2	DL12 470
	Y=Y+,1342391030515004 D=8*FCT(X)	DL12 480
	X=,1711685518746226 D2	DL12 490
	Y=Y+,1668493876540910 D=6*FCT(X)	DL12 500
	X=,1300605499330635 D2	DL12 510
	Y=Y+,836505585681980 D=5*FCT(X)	DL12 520
	X=,962131684245687 D1	DL12 530
	Y=Y+,2032315926629994 D=3*FCT(X)	DL12 540
	X=,6844525453115177 D1	DL12 550
	Y=Y+,2663973541865316 D=2*FCT(X)	DL12 560
	X=,4599227639418348 D1	DL12 570
	Y=Y+,2010238115463410 D=1*FCT(X)	DL12 580
	X=,2833751337743507 D1	DL12 590
	Y=Y+,904492222116809 D=1*FCT(X)	DL12 600
	X=,1512610269776419 D1	DL12 610
	Y=Y+,2440820113198776 D0*FCT(X)	DL12 620
	X=,6117574845151307 D0	DL12 630
	Y=Y+,3777592758731380 D0*FCT(X)	DL12 640
	X=,1157221173580207 D0	DL12 650
	Y=Y+,2647313710554432 D0*FCT(X)	DL12 660
	RETURN	DL12 670
	END	DL12 68=
	SUBROUTINE BESK(X,N,BK,IER)	BESK 410
C		BESK 10
C		BESK 20
C		BESK 30
C	SUBROUTINE BESK	BESK 40
C		BESK 50
C	COMPUTE THE K BESSEL FUNCTION FOR A GIVEN ARGUMENT AND ORDER	BESK 60
C		BESK 70
C	USAGE	BESK 80
C	CALL BESK(X,N,BK,IER)	BESK 90
C		BESK 100
C	DESCRIPTION OF PARAMETERS	BESK 110
C	X =THE ARGUMENT OF THE K BESSEL FUNCTION DESIRED	BESK 120
C	N =THE ORDER OF THE K BESSEL FUNCTION DESIRED	BESK 130
C	BK =THE RESULTANT K BESSEL FUNCTION	BESK 140
C	IER=RESULTANT ERROR CODE WHERE	BESK 150
C	IER=0 NO ERROR	BESK 160
C	IER=1 N IS NEGATIVE	BESK 170
C	IER=2 X IS ZERO OR NEGATIVE	BESK 180
C	IER=3 X .GT. 170, MACHINE RANGE EXCEEDED	BESK 190
C	IER=4 BK .GT. 10**70	BESK 200
C		BESK 210
C	REMARKS	BESK 220
C	N MUST BE GREATER THAN OR EQUAL TO ZERO	BESK 230
C		BESK 240
C	SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED	BESK 250
C	NONE	BESK 260
C		BESK 270
C	METHOD	BESK 280
C	COMPUTES ZERO ORDER AND FIRST ORDER BESSEL FUNCTIONS USING	BESK 290
C	SERIES APPROXIMATIONS AND THEN COMPUTES N TH ORDER FUNCTION	BESK 300
C	USING RECURRENCE RELATION.	BESK 310

TABLE 6.1.—Listing of program for partial penetration in a leaky artesian aquifer—Continued

C	RECURRENCE RELATION AND POLYNOMIAL APPROXIMATION TECHNIQUE	BESK 320
C	AS DESCRIBED BY A,J,M,HITCHCOCK,'POLYNOMIAL APPROXIMATIONS	BESK 330
C	TO BESSEL FUNCTIONS OF ORDER ZERO AND ONE AND TO RELATED	BESK 340
C	FUNCTIONS', M,T,A,C., V,11,1957,PP,86-88, AND G.N. WATSON,	BESK 350
C	'A TREATISE ON THE THEORY OF BESSEL FUNCTIONS', CAMBRIDGE	BESK 360
C	UNIVERSITY PRESS, 1958, P. 62	BESK 370
C		BESK 380
C	BESK 390
C		BESK 400
	DIMENSION T(12)	BESK 420
	BK=0	BESK 430
	IF(N)10,11,11	BESK 440
10	IER=1	BESK 450
	RETURN	BESK 460
11	IF(X)12,12,20	BESK 470
12	IER=2	BESK 480
	RETURN	BESK 490
20	IF(X=170,0)22,22,21	BESK 500
21	IER=3	BESK 510
	RETURN	BESK 520
22	IER=0	BESK 530
	IF(X=1,.)36,36,25	BESK 540
25	A=EXP(-X)	BESK 550
	B=1./X	BESK 560
	C=SQRT(B)	BESK 570
	T(1)=B	BESK 580
	DO 26 L=2,12	BESK 590
26	T(L)=T(L-1)*B	BESK 600
	IF(N=1)27,29,27	BESK 610
C		BESK 620
C	COMPUTE KO USING POLYNOMIAL APPROXIMATION	BESK 630
C		BESK 640
27	G0=A*(1,2533141=-,1566642*T(1)+,08811128*T(2)-,09139095*T(3)	BESK 650
	2+,1344596*T(4)-,2299850*T(5)+,3792410*T(6)+,5247277*T(7)	BESK 660
	3+,5575368*T(8)-,4262633*T(9)+,2184518*T(10)+,06680977*T(11)	BESK 670
	4+,009189383*T(12))*C	BESK 680
	IF(N)20,28,29	BESK 690
28	BK=G0	BESK 700
	RETURN	BESK 710
C		BESK 720
C	COMPUTE K1 USING POLYNOMIAL APPROXIMATION	BESK 730
C		BESK 740
29	G1=A*(1,2533141+,4699927*T(1)-,1468583*T(2)+,1280427*T(3)	BESK 750
	2-,1736432*T(4)+,2847618*T(5)-,4594342*T(6)+,6283381*T(7)	BESK 760
	3-,6632295*T(8)+,5050239*T(9)-,2581304*T(10)+,07880001*T(11)	BESK 770
	4-,01082418*T(12))*C	BESK 780
	IF(N=1)20,30,31	BESK 790
30	BK=G1	BESK 800
	RETURN	BESK 810
C		BESK 820
C	FROM KO,K1 COMPUTE KN USING RECURRENCE RELATION	BESK 830
C		BESK 840
31	DO 35 J=2,N	BESK 850
	GJ=2,*(FLUAT(J)=1,)*G1/X+G0	BESK 860
	IF(GJ=1,0E70)33,33,32	BESK 870
32	IER=4	BESK 880
	GO TO 34	BESK 890
33	G0=G1	BESK 900
35	G1=GJ	BESK 910
34	BK=GJ	BESK 920
	RETURN	BESK 930
36	B=X/2,	BESK 940

TABLE 6.1.—Listing of program for partial penetration in a leaky artesian aquifer—Continued

```

A=,5772157+ALOG(B)                                BESK 950
C=B*B                                               BESK 960
IF(N=1)37,43,37                                    BESK 970
C                                                    BESK 980
C COMPUTE K0 USING SERIES EXPANSION                BESK 990
C                                                    BESK 1000
37 GO=A                                             BESK 1010
  X2J=1,                                           BESK 1020
  FACT=1,                                         BESK 1030
  HJ=,0                                           BESK 1040
  DO 40 J=1,6                                     BESK 1050
  RJ=1,/FLOAT(J)                                 BESK 1060
  IF(X2J,LT,1,E=40) X2J=0,                       BESK 1061
C PREVIOUS STATEMENT ADDED TO IBM SUBROUTINE TO CORRECT UNDERFLOW BESK 1062
C PROBLEM ON WATFOR COMPILER                      BESK 1063
  X2J=X2J*C                                       BESK 1070
  FACT=FACT*RJ*RJ                                 BESK 1080
  HJ=HJ+RJ                                       BESK 1090
40 GO=GO+X2J*FACT*(HJ=A)                         BESK 1100
  IF(N)43,42,43                                   BESK 1110
42 BK=GO                                          BESK 1120
  RETURN                                          BESK 1130
C                                                    BESK 1140
C COMPUTE K1 USING SERIES EXPANSION                BESK 1150
C                                                    BESK 1160
43 X2J=B                                           BESK 1170
  FACT=1,                                         BESK 1180
  HJ=1,                                           BESK 1190
  G1=1,/X+X2J*(,5+A=HJ)                         BESK 1200
  DO 50 J=2,8                                     BESK 1210
  X2J=X2J*C                                       BESK 1220
  RJ=1,/FLOAT(J)                                 BESK 1230
  FACT=FACT*RJ*RJ                                 BESK 1240
  HJ=HJ+RJ                                       BESK 1250
50 G1=G1+X2J*FACT*(,5+(A=HJ)*FLOAT(J))          BESK 1260
  IF(N=1)31,52,31                                 BESK 1270
52 BK=G1                                          BESK 1280
  RETURN                                          BESK 1290
  END                                             BESK 1300

```

TABLE 7.2.—Listing of program for constant drawdown in a well in an infinite leaky aquifer

```

C ***** Z 1
C ***** Z 2
C PURPOSE Z 3
C TO COMPUTE A TABLE OF FUNCTION VALUES FOR DRAWDOWN IN A Z 4
C LEAKY ARTESIAN AQUIFER IN RESPONSE TO A STEP CHANGE IN Z 5
C WATER LEVEL IN THE CONTROL WELL, FUNCTION VALUES ARE Z 6
C EXPRESSED AS A FRACTION OF DRAWDOWN IN CONTROL WELL (S/SW), Z 7
C REFERENCE = HANTUSH, M.S., 1959, NONSTEADY FLOW TO FLOWING Z 8
C WELLS IN LEAKY AQUIFERS: JOUR. GEOPHYS. RESEARCH, V. 64, Z 9
C NO. 8, P. 1043-1052, Z 10
C INPUT DATA Z 11
C 1 CARD = FORMAT(2E10,5) Z 12
C TSMALL = SMALLEST VALUE OF ALPHA FOR WHICH COMPUTATION Z 13
C IS DESIRED, Z 14
C TLARGE = LARGEST VALUE OF ALPHA FOR WHICH COMPUTATION Z 15
C IS DESIRED, Z 16
C 1 CARD = FORMAT(13F5,0) Z 17
C BOAT = 13 VALUES OF RW/B, NON ZERO VALUES SHOULD BE GE 1 Z 18
C AND LT 10, FIRST ZERO (OR BLANK) WILL TERMINATE THE Z 19
C LIST, AT LEAST ONE NON ZERO VALUE MUST BE CODED, INPUT Z 20
C VALUES ARE MULTIPLIED BY POWER OF TEN DETERMINED BY Z 21
C PROGRAM FROM ALPHA, Z 22

```

TABLE 7.2.—Listing of program for constant drawdown in a well in an infinite leaky aquifer—Continued

```

C      1 CARD = FORMAT(10F8,2)                                Z 23
C      RW = RADIUS OF CONTROL WELL,                          Z 24
C      RDAT = 9 VALUES OF RADIAL DISTANCE OF OBSERVATION POINTS Z 25
C      FROM CONTROL WELL, SHOULD BE CODED WITH SMALLEST NUMBER Z 26
C      FIRST, THEN BY INCREASING DISTANCE, THE FIRST ZERO    Z 27
C      (OR BLANK) VALUE WILL TERMINATE COMPUTATION,           Z 28
C
C      METHOD                                                    Z 29
C      EVALUATES EQ. 13 OF HANTUSH, EVALUATION OF BESSEL FUNCTIONS Z 30
C      BY SUBROUTINES BESK AND BESY AND FUNCTION JO. EVALUATES Z 31
C      INTEGRAL BY SUM, I=1 TO 8000, F((DELTA U)*(I-.5))*(DELTA U), Z 32
C      CHOOSES INITIAL DELTA U = .001/SQRT(SMALLEST ALPHA) AND USES Z 33
C      THIS VALUE FOR ALL RW/B GE 10*(DELTA U), FOR SMALLER RW/B, Z 34
C      DIVIDES DELTA U BY 10 AND MULTIPLIES SMALLEST ALPHA BY 100, Z 35
C
C      REMARKS                                                  Z 36
C      SMALLEST RW/B GE .01/SQRT(SMALLEST ALPHA)              Z 37
C      SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED            Z 38
C      BESK,BESY,JO                                            Z 39
C
C      *****                                                Z 40
C      REAL*8 SUM1,SUM2                                         Z 41
C      REAL*4 KOBP,KOB,JO,JOPU,JOU,Y(8000),J(8000),F(8000),FT(8000), Z 42
C      1 FB(8000),RDAT(9),TDAT(6),BDAT(13),ARRAY(25,9,13),B(13),T(25) Z 44
C      DATA FT/8000*0.,/,FB/8000*0.,/                          Z 45
C      DATA RDAT/9*1.,/                                         Z 46
C      DATA ARRAY/2925*0.,/,TDAT/1.,1.5,2.,3.,5.,7./          Z 47
C      IRD=5                                                      Z 48
C      IPT=6                                                       Z 49
C      READ (IRD,24) TSMALL,TLARGE                                Z 50
C      READ (IRD,23) BDAT                                         Z 51
C      READ (IRD,22) RW,RDAT                                       Z 52
C      IBEGIN=ALOG10(TSMALL)                                        Z 53
C      IEND=ALOG10(TLARGE)+.99999                                  Z 54
C      IF ((IBEGIN/2*.2).LT,IBEGIN) IBEGIN=IBEGIN-.1            Z 55
C      ISPAN=IEND-IBEGIN                                           Z 56
C      MLIMIT=(ISPAN+1)/2                                          Z 57
C
C      COMPUTES INITIAL DELTA U (DU) = .001/SQRT(SMALLEST ALPHA) Z 58
C      DU=.001/SQRT(TDAT(1)*10,**IBEGIN)                          Z 59
C
C      EXPONENT (JBEGIN) OF SMALLEST RW/B IS COMPUTED FROM EXPONENT Z 60
C      (IBEGIN) OF SMALLEST ALPHA.                                Z 61
C      JBEGIN=-IBEGIN/2-2                                          Z 62
C      DO 1 I=1,13                                                 Z 63
C      IF (BDAT(I).EQ,0.) GO TO 2                                  Z 64
C      1 CONTINUE                                                  Z 65
C      NB=13                                                       Z 66
C      GO TO 3                                                      Z 67
C      2 NB=I-1                                                     Z 68
C      3 CONTINUE                                                  Z 69
C      DO 4 I=1,9                                                  Z 70
C      IF (RDAT(I).EQ,0.) GO TO 5                                  Z 71
C      4 RDAT(I)=RDAT(I)/RW                                        Z 72
C      NR=9                                                         Z 73
C      GO TO 6                                                      Z 74
C      5 NR=I-1                                                     Z 75
C      6 DO 21 M=1,MLIMIT                                          Z 76
C      NUM=8000                                                    Z 77
C      START=DU/2.                                                Z 78
C      U=START                                                     Z 79
C      DO 7 I=1,NUM                                               Z 80
C      U=U+DU                                                      Z 81
C      CALL BESY(U,0,Y(I),IDUMY)                                   Z 82
C      7 J(I)=JO(U)                                                Z 83
C      DO 19 IR=1,NR                                               Z 84

```

TABLE 7.2.—Listing of program for constant drawdown in a well in an infinite leaky aquifer—Continued

```

RHO=RDAT(IR)                                Z 85
U=START                                       Z 86
DO 8 I=1,NUM                                  Z 87
U=U+DU                                        Z 88
CALL BESY(RHO*U,0,YOPU,IDUMY)                Z 89
JOPU=JO(RHO*U)                                Z 90
JOU=J(I)                                       Z 91
YOU=Y(I)                                       Z 92
8 F(I)=(JOPU*YOU+YOPU*JOU)/(JOU*JOU+YOU*YOU)  Z 93
DO 19 IT=1,25                                  Z 94
INDEX=(IT-1)/6                                  Z 95
IEXP=IBEGIN+INDEX                              Z 96
II=IT-INDEX*6                                  Z 97
TAU=TDAT(II)*10,**IEXP                        Z 98
T(IT)=TAU                                       Z 99
U=START                                       Z 100
NUMT=NUM                                        Z 101
DO 9 I=1,NUMT                                    Z 102
U=U+DU                                        Z 103
FTEST=F(I)                                       Z 104
IF (ABS(FTEST),LT,1,E=30) GO TO 10            Z 105
XTEST=TAU*U*U                                    Z 106
IF (XTEST+69,) 10,10,9                          Z 107
9 FT(I)=FTEST*EXP(XTEST)                        Z 108
GO TO 11                                         Z 109
10 NUMT=I-1                                       Z 110
FT(I)=0,                                         Z 111
11 DO 19 IB=1,13                                  Z 112
JINDEX=(IB-1)/NB                                  Z 113
JEXP=JBEGIN+JINDEX                              Z 114
JJ=IB-JINDEX*NB                                  Z 115
BETA=BDAT(JJ)*10,**JEXP                          Z 116
B(IB)=BETA                                       Z 117
U=START                                       Z 118
BSQ=BETA*BETA                                    Z 119
NUMB=NUMT                                       Z 120
DO 12 I=1,NUMB                                    Z 121
U=U+DU                                        Z 122
FTEST=FT(I)                                       Z 123
IF (ABS(FTEST),LT,1,E=30) GO TO 13            Z 124
12 FB(I)=FTEST/(U+BSQ/U)                          Z 125
GO TO 14                                         Z 126
13 NUMB=I-1                                       Z 127
FB(I)=0,                                         Z 128
14 SUM1=0,                                         Z 129
SUM2=0,                                         Z 130
DO 15 I=1,NUMB,2                                  Z 131
SUM1=SUM1+FB(I)                                  Z 132
15 SUM2=SUM2+FB(I+1)                              Z 133
XINT=(SUM1+SUM2)*DU                              Z 134
CALL BESK(RHO*BETA,0,KOBP,IDUMY)                Z 135
CALL BESK(BETA,0,KOB,IDUMY)                    Z 136
RATIO=0,                                         Z 137
IF (KOBP,GT,0,) RATIO=KOBP/KOB                  Z 138
XTEST=TAU*BSQ                                    Z 139
IF (XTEST+30,) 16,17,17                          Z 140
16 XPT=0,                                         Z 141
GO TO 18                                         Z 142
17 XPT=EXP(XTEST)                                Z 143
18 Z=RATIO+.6366198*XPT*XINT                      Z 144
IF ((Z,LT,0,)AND,(Z,GT,=5,E=5)) Z=0,E0        Z 145
19 ARRAY(IT,IR,IB)=Z                             Z 146

```

TABLE 7.2.—Listing of program for constant drawdown in a well in an infinite leaky aquifer—Continued

```

DO 20 K=1,NR                                Z 147
WRITE (IPT,25) RDATA(K),B                    Z 148
WRITE (IPT,26) (T(I),(ARRAY(I,K,L),L=1,13),I=1,25) Z 149
C 20 CONTINUE                                Z 150
C EXPONENT OF SMALLEST RW/B DECREASED BY ONE EACH TIME THROUGH LOOP Z 151
JBEGIN=JBEGIN-1                              Z 152
C EXPONENT OF SMALLEST ALPHA INCREASED BY TWO EACH TIME THROUGH LOOP Z 153
IBEGIN=IBEGIN+2                              Z 154
C DELTA U (DU) IS DIVIDED BY 10 EACH TIME THROUGH THE LOOP          Z 155
21 DU=.1*DU                                  Z 156
STOP                                          Z 157
C                                             Z 158
22 FORMAT (10F8,2)                            Z 159
23 FORMAT (13F5,0)                            Z 160
24 FORMAT (2E10,5)                            Z 161
25 FORMAT ('11','Z(ALPHA,R/RW,RW/B), R/RW=1,F6,0/101,9X,'1 RW/B1/'('1', Z 162
13X,'ALPHA 11',13E9,2))                      Z 163
26 FORMAT ('1',E10,3,13F9,3)                 Z 164
END                                           Z 165
REAL FUNCTION JO*4(X)                          JO 1
*****                                       JO 2
C                                             JO 3
C FUNCTION JO                                JO 4
C                                             JO 5
C PURPOSE                                    JO 6
C TO COMPUTE THE ZERO ORDER J BESSEL FUNCTION FOR A GIVEN          JO 7
C ARGUMENT.                                  JO 8
C USAGE                                       JO 9
C JO(X)                                       JO 10
C DESCRIPTION OF PARAMETER                    JO 11
C X = REAL*4, ARGUMENT OF JO BESSEL FUNCTION DESIRED.              JO 12
C SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED JO 13
C NONE,                                       JO 14
C METHOD                                       JO 15
C POLYNOMIAL APPROXIMATION FOR X<4 AND ASYMPTOTIC SERIES FOR      JO 16
C X GE 4, THE POLYNOMIAL APPROXIMATION IS THE FIRST 10 TERMS OF   JO 17
C THE POWER SERIES FOR JO(X) (MILLER, K.S., 1957,                  JO 18
C ENGINEERING MATHEMATICS; RINEHART AND CO., INC., NEW YORK,     JO 19
C P. 120). THE ASYMPTOTIC EXPANSION OF JO(X) IS GIVEN ON P. 82   JO 20
C OF BOWMAN, FRANK, 1958, INTRODUCTION TO BESSEL FUNCTIONS;      JO 21
C DOVER PUBLICATIONS INC., NEW YORK, THE TERMS P ('A*P0') AND     JO 22
C Q ('B*Q0') OF THE ASYMPTOTIC EXPANSION ARE COMPUTED BY AN      JO 23
C ALGORITHM FROM IBM SUBROUTINE BESY.        JO 24
C                                             JO 25
C *****                                       JO 26
C IF (X=4.) 1,3,3                                       JO 27
C COMPUTE JO BY FIRST 10 TERMS OF POWER SERIES JO 28
1 A=X*X/4,                                       JO 29
  B=1,                                           JO 30
  DO 2 I=1,10                                    JO 31
    C=11.-I                                       JO 32
2 B=1.+B*(A/(C*C))                               JO 33
  JO=B                                           JO 34
  GO TO 4                                         JO 35
C COMPUTE JO BY ASYMPTOTIC SERIES              JO 36
3 T1=4./X                                       JO 37
  T2=T1*T1                                       JO 38
  P0=(((.-.0000037043*T2+.0000173565)*T2-.0000487613)*T2+.00017343)* JO 39
1 T2=.001753062)*T2+.3989423                    JO 40
  Q0=(((.0000032312*T2-.0000142078)*T2+.0000342468)*T2-.0000869791) JO 41
1 *T2+.0004564324)*T2=.01246694                JO 42
  A=2.0/SQRT(X)                                  JO 43

```

TABLE 7.2.—Listing of program for constant drawdown in a well in an infinite leaky aquifer—Continued

```

B=A*T1                                JO 44
C=X=.7853982                          JO 45
JO=A*P0*COS(C)=B*Q0*SIN(C)           JO 46
4 RETURN                               JO 47
END                                    JO 48=
SUBROUTINE BESY(X,N,BY,IER)           BESY 410
                                        BESY 10
.....BESY 20
SUBROUTINE BESY                       BESY 30
                                        BESY 40
PURPOSE                                BESY 50
  COMPUTE THE Y BESSEL FUNCTION FOR A GIVEN ARGUMENT AND ORDERBESY 60
USAGE                                  BESY 70
  CALL BESY(X,N,BY,IER)               BESY 80
                                        BESY 90
DESCRIPTION OF PARAMETERS              BESY 100
  X =THE ARGUMENT OF THE Y BESSEL FUNCTION DESIRED          BESY 110
  N =THE ORDER OF THE Y BESSEL FUNCTION DESIRED            BESY 120
  BY =THE RESULTANT Y BESSEL FUNCTION                       BESY 130
  IER=RESULTANT ERROR CODE WHERE                            BESY 140
    IER=0 NO ERROR                                         BESY 150
    IER=1 N IS NEGATIVE                                    BESY 160
    IER=2 X IS NEGATIVE OR ZERO                            BESY 170
    IER=3 BY HAS EXCEEDED MAGNITUDE OF 10**70             BESY 180
REMARKS                                BESY 190
  VERY SMALL VALUES OF X MAY CAUSE THE RANGE OF THE LIBRARY BESY 200
  FUNCTION ALOG TO BE EXCEEDED                               BESY 210
  X MUST BE GREATER THAN ZERO                               BESY 220
  N MUST BE GREATER THAN OR EQUAL TO ZERO                  BESY 230
SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED              BESY 240
  NONE                                                       BESY 250
METHOD                                                    BESY 260
  RECURRENCE RELATION AND POLYNOMIAL APPROXIMATION TECHNIQUE BESY 270
  AS DESCRIBED BY A.J.M.HITCHCOCK,'POLYNOMIAL APPROXIMATIONS BESY 280
  TO BESSEL FUNCTIONS OF ORDER ZERO AND ONE AND TO RELATED BESY 290
  FUNCTIONS', M.T.A.C., V,11,1957,PP.86-88, AND G.N. WATSON, BESY 300
  'A TREATISE ON THE THEORY OF BESSEL FUNCTIONS', CAMBRIDGE BESY 310
  UNIVERSITY PRESS, 1958, P. 62                             BESY 320
.....BESY 330
CHECK FOR ERRORS IN N AND X              BESY 340
                                        BESY 350
IF(N)180,10,10                                         BESY 360
10 IER=0                                               BESY 370
IF(X)190,190,20                                        BESY 380
                                        BESY 390
BRANCH IF X LESS THAN OR EQUAL 4        BESY 400
                                        BESY 410
20 IF(X=4,0)40,40,30                                  BESY 420
                                        BESY 430
  COMPUTE Y0 AND Y1 FOR X GREATER THAN 4 BESY 440
                                        BESY 450
30 T1=4,0/X                                           BESY 460
T2=T1*T1                                               BESY 470
P0=(((=,0000037043*T2+,0000173565)*T2=,0000487613)*T2 BESY 480
                                        BESY 490
                                        BESY 500
                                        BESY 510
                                        BESY 520
                                        BESY 530
                                        BESY 540
                                        BESY 550
                                        BESY 560
                                        BESY 570

```

TABLE 7.2.—Listing of program for constant drawdown in a well in an infinite leaky aquifer—Continued

```

1  +,00017343)*T2=,001753062)*T2+,3989423          BESY 580
Q0=(((((,0000032312*T2=,0000142078)*T2+,0000342468)*T2          BESY 590
1  =,0000869791)*T2+,0004564324)*T2=,01246694          BESY 600
P1=(((((,0000042414*T2=,0000200920)*T2+,0000580759)*T2          BESY 610
1  =,000223203)*T2+,002921826)*T2+,3989423          BESY 620
Q1=(((((=,0000036594*T2+,00001622)*T2=,0000398708)*T2          BESY 630
1  +,0001064741)*T2=,0006390400)*T2+,03740084          BESY 640
A=2,0/SQRT(X)          BESY 650
B=A*T1          BESY 660
C=X=,7853982          BESY 670
Y0=A*P0*SIN(C)+B*Q0*COS(C)          BESY 680
Y1=-A*P1*COS(C)+B*Q1*SIN(C)          BESY 690
GD TO 90          BESY 700
C          BESY 710
C          BESY 720
C          BESY 730
      COMPUTE Y0 AND Y1 FOR X LESS THAN OR EQUAL TO 4          BESY 740
40 XX=X/2,          BESY 750
   X2=XX*XX          BESY 760
   T=ALOG(XX)+,5772157          BESY 770
   SUM=0,          BESY 780
   TERM=T          BESY 790
   DO 70 L=1,15          BESY 800
   IF(L=1)50,60,50          BESY 810
50 SUM=SUM+1./FLOAT(L=1)          BESY 820
60 FL=L          BESY 830
   TS=T-SUM          BESY 840
   IF(ABS(TERM),LE,1,E=40) TERM=0,          BESY 841
   TERM=(TERM*(=X2)/FL**2)*(1,=1,/(FL*TS))          BESY 850
70 Y0=Y0+TERM          BESY 860
   TERM = XX*(T=,5)          BESY 870
   SUM=0,          BESY 880
   Y1=TERM          BESY 890
   DO 80 L=2,16          BESY 900
   SUM=SUM+1./FLOAT(L=1)          BESY 910
   FL=L          BESY 920
   FL1=FL-1,          BESY 930
   TS=T-SUM          BESY 940
   IF(ABS(TERM),LE,1,E=40) TERM=0,          BESY 941
   TERM=(TERM*(=X2)/(FL1*FL))*((TS=,5/FL)/(TS+,5/FL1))          BESY 950
80 Y1=Y1+TERM          BESY 960
   PI2=,6366198          BESY 970
   Y0=PI2*Y0          BESY 980
   Y1=-PI2/X+PI2*Y1          BESY 990
C          BESY 1000
C          BESY 1010
C          BESY 1020
90 IF(N=1)100,100,130          BESY 1030
C          BESY 1040
C          BESY 1050
      RETURN EITHER Y0 OR Y1 AS REQUIRED          BESY 1060
C          BESY 1070
100 IF(N)110,120,110          BESY 1080
110 BY=Y1          BESY 1090
      GO TO 170          BESY 1100
120 BY=Y0          BESY 1110
      GO TO 170          BESY 1120
C          BESY 1130
C          BESY 1140
C          BESY 1150
      PERFORM RECURRENCE OPERATIONS TO FIND YN(X)          BESY 1160
130 YA=Y0          BESY 1170
      YB=Y1
      K=1

```


TABLE 7.2.—Listing of program for constant drawdown in a well in an infinite leaky aquifer—Continued

	RETURN	BESK 460
11	IF(X)12,12,20	BESK 470
12	IER=2	BESK 480
	RETURN	BESK 490
20	IF(X=170,0)22,22,21	BESK 500
21	IER=3	BESK 510
	RETURN	BESK 520
22	IER=0	BESK 530
	IF(X=1,)36,36,25	BESK 540
25	A=EXP(-X)	BESK 550
	B=1./X	BESK 560
	C=SQRT(B)	BESK 570
	T(1)=B	BESK 580
	DO 26 L=2,12	BESK 590
26	T(L)=T(L-1)*B	BESK 600
	IF(N=1)27,29,27	BESK 610
		BESK 620
C		BESK 630
C	COMPUTE K0 USING POLYNOMIAL APPROXIMATION	BESK 640
C		BESK 650
27	G0=A*(1,2533141-,1566642*T(1)+,08811128*T(2)+,09139095*T(3)	BESK 660
	2+,1344596*T(4)+,2299850*T(5)+,3792410*T(6)+,5247277*T(7)	BESK 670
	3+,5575368*T(8)+,4262633*T(9)+,2184518*T(10)+,06680977*T(11)	BESK 680
	4+,009189383*T(12))*C	BESK 690
	IF(N)20,28,29	BESK 700
28	BK=G0	BESK 710
	RETURN	BESK 720
C		BESK 730
C	COMPUTE K1 USING POLYNOMIAL APPROXIMATION	BESK 740
C		BESK 750
29	G1=A*(1,2533141+,4699927*T(1)+,1468583*T(2)+,1280427*T(3)	BESK 760
	2+,1736432*T(4)+,2847618*T(5)+,4594342*T(6)+,6283381*T(7)	BESK 770
	3+,6632295*T(8)+,5050239*T(9)+,2581304*T(10)+,07880001*T(11)	BESK 780
	4+,-01082418*T(12))*C	BESK 790
	IF(N=1)20,30,31	BESK 800
30	BK=G1	BESK 810
	RETURN	BESK 820
C		BESK 830
C	FROM K0,K1 COMPUTE KN USING RECURRENCE RELATION	BESK 840
C		BESK 850
31	DO 35 J=2,N	BESK 860
	GJ=2.*(FLOAT(J)=1,)*G1/X+G0	BESK 870
	IF(GJ=1,0E70)33,33,32	BESK 880
32	IER=4	BESK 890
	GO TO 34	BESK 900
33	G0=GJ	BESK 910
35	G1=GJ	BESK 920
34	BK=GJ	BESK 930
	RETURN	BESK 940
36	B=X/2,	BESK 950
	A=,5772157+ALOG(B)	BESK 960
	C=B*B	BESK 970
	IF(N=1)37,43,37	BESK 980
C		BESK 990
C	COMPUTE K0 USING SERIES EXPANSION	BESK 1000
C		BESK 1010
37	G0=-A	BESK 1020
	X2J=1,	BESK 1030
	FACT=1,	BESK 1040
	HJ=,0	BESK 1050
	DO 40 J=1,6	BESK 1060
	RJ=1./FLOAT(J)	BESK 1061
	IF(X2J,LT,1,E=40) X2J=0,	BESK 1062
C	PREVIOUS STATEMENT ADDED TO IBM SUBROUTINE TO CORRECT UNDERFLOW	

TABLE 7.2.—Listing of program for constant drawdown in a well in an infinite leaky aquifer—Continued

```

C      PROBLEM ON WATFOR COMPILER                                BESK1063
      X2J=X2J*C                                                  BESK1070
      FACT=FACT*RJ*RJ                                           BESK1080
      HJ=HJ+RJ                                                  BESK1090
40     GO=GO+X2J*FACT*(HJ=A)                                     BESK1100
      IF(N)43,42,43                                             BESK1110
42     BK=GO                                                    BESK1120
      RETURN                                                    BESK1130
C
C      COMPUTE K1 USING SERIES EXPANSION                         BESK1140
C
C      43 X2J=B                                                  BESK1170
      FACT=1,                                                    BESK1180
      HJ=1,                                                       BESK1190
      G1=1./X+X2J*(.5+A=HJ)                                     BESK1200
      DO 50 J=2,8                                               BESK1210
      X2J=X2J*C                                                  BESK1220
      RJ=1./FLOAT(J)                                           BESK1230
      FACT=FACT*RJ*RJ                                           BESK1240
      HJ=HJ+RJ                                                  BESK1250
50     G1=G1+X2J*FACT*(.5+(A=HJ)*FLOAT(J))                   BESK1260
      IF(N=1)31,52,31                                           BESK1270
52     BK=G1                                                    BESK1280
      RETURN                                                    BESK1290
      END                                                        BESK130*

```

TABLE 8.2.—Listing of programs for constant discharge from a fully penetrating well of finite diameter

```

C*****FAR 1
C      FAR 2
C      PURPOSE                                                 FAR 3
C      COMPUTES FUNCTION VALUES OF F(U,ALPHA,RHO) FOR RHO > 1 = FAR 4
C      PAPAOPULOS,I,S. AND COOPER,H,H.,JR., 1967, DRAWDOWN IN FAR 5
C      A WELL OF LARGE DIAMETER; WATER RESOURCES RESEARCH, V. 3, FAR 6
C      NO. 1, P. 241-244.                                       FAR 7
C      PROGRAM BY S,S,PAPAOPULOS,                               FAR 8
C      INPUT DATA = ONE OR MORE GROUPS, EACH GROUP CODED AS FOLLOWS FAR 9
C      1 CARD = FORMAT(2E10,5)                                   FAR 10
C      ALPHA = RW**2*S/RC**2 = RADIUS OF WELL (SCREEN          FAR 11
C      OR OPEN BORE IN AQUIFER) SQUARED * STORAGE              FAR 12
C      COEFFICIENT / RADIUS OF CASING (OVER INTERVAL OF        FAR 13
C      WATER LEVEL CHANGE) SQUARED,                            FAR 14
C      RHO = R/RW = DISTANCE FROM PUMPED WELL / RADIUS OF      FAR 15
C      WELL (SCREEN OR OPEN BORE IN AQUIFER), MUST BE          FAR 16
C      GREATER THAN ONE,                                       FAR 17
C      1 CARD = FORMAT(16E5,0)                                  FAR 18
C      U = 16 VALUES OF U = R**2*S/(4*T*TIME) = DISTANCE FROM FAR 19
C      PUMPED WELL SQUARED * STORAGE COEFFICIENT /            FAR 20
C      4 * TRANSMISSIVITY * TIME. IF LESS THAN 16 DESIRED,    FAR 21
C      BLANK OR ZERO VALUES MAY BE CODED FOR THE REST,       FAR 22
C      SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED            FAR 23
C      PEAK,SIMP,APEKE,EXBSL1,JY0,JY1,ROOTS = MUST BE IN DECK, FAR 24
C      FAR 25
C*****FAR 26
      DIMENSION V(40,40),U(16)                                   FAR 27
      COMMON XPK,YPK                                             FAR 28
      COMMON/PBLK/A,B,RHO                                       FAR 29
      EXTERNAL EXBSL1                                           FAR 30
      1 READ (5,16,END=15) ALPHA,RHO                            FAR 31
      IF (ALPHA) 15,15,2                                        FAR 32

```

TABLE 8.2.—Listing of programs for constant discharge from a fully penetrating well of finite diameter—Continued

2	WRITE (6,17) ALPHA,RHO	FAR	33
	WRITE (6,18)	FAR	34
3	READ (5,19) U	FAR	35
	DO 14 II=1,16	FAR	36
	IF (U(II)) 1,1,4	FAR	37
4	A=ALPHA+ALPHA	FAR	38
	B=0,25/U(II)	FAR	39
	CALL APEKE(EXBSL1)	FAR	40
	CALL PEAK(EXBSL1)	FAR	41
	IF (XPK=1,0E-8) 5,6,6	FAR	42
5	WRITE (6,20) XPK,U	FAR	43
	GO TO 3	FAR	44
6	IF (XPK=3,0) 8,7,7	FAR	45
7	WRITE (6,21) XPK,U	FAR	46
	GO TO 3	FAR	47
8	EPS=0,000001	FAR	48
	HBAR=0,007*XPK	FAR	49
	CALL SIMPS(0,0,XPK,EPS,HBAR,SUM,DEL,EXBSL1)	FAR	50
	XM1=((3,14159265*7,0)/(8,0*(RHO=1,)))+1,E=6)*RHO/2.	FAR	51
	DX1=XM1*(1,0E=6)*RHO	FAR	52
	DXN=(2,0*3,14159265*RHO)/(5,*(RHO=1,))	FAR	53
	DL=3,14159265*RHO/(RHO=1,)	FAR	54
	CALL ROOTS(XM1,DX1,RT1,EXBSL1)	FAR	55
	HBAR=0,007*(RT1=XPK)	FAR	56
	CALL SIMPS(XPK,RT1,EPS,HBAR,TRM1,ERR1,EXBSL1)	FAR	57
	SUM=SUM+TRM1	FAR	58
	DEL=DEL+ERR1	FAR	59
	X1=RT1	FAR	60
	I=1	FAR	61
9	XM=X1+DL	FAR	62
	CALL ROOTS(XM,DXN,X2,EXBSL1)	FAR	63
	HBAR=0,007*(X2=X1)	FAR	64
	CALL SIMPS(X1,X2,EPS,HBAR,TRM,ERR,EXBSL1)	FAR	65
	V(1,I)=ABS(TRM)	FAR	66
	DEL=DEL+ERR	FAR	67
	I=I+1	FAR	68
	IF (I=40) 10,10,11	FAR	69
10	X1=X2	FAR	70
	GO TO 9	FAR	71
11	EST=0,0	FAR	72
	DO 12 K=2,40	FAR	73
	M=41=K	FAR	74
	DO 12 J=1,M	FAR	75
12	V(K,J)=V(K-1,J+1)-V(K-1,J)	FAR	76
	DO 13 N=1,40	FAR	77
	L=N=1	FAR	78
	DELV=(=0,5)**L*V(N,1)	FAR	79
13	EST=EST+(0,5)*DELV	FAR	80
	SUM=SUM+EST	FAR	81
	PUAR=4,0*A*RHO*SUM/3,14159265	FAR	82
	WRITE (6,22) U(II),SUM,DEL,PUAR	FAR	83
14	CONTINUE	FAR	84
	GO TO 1	FAR	85
15	STUP	FAR	86
		FAR	87
		FAR	88
16	FORMAT (2E10,5)	FAR	88
17	FORMAT ('1',1F(U,ALPHA,RHO) FOR ALPHA=1,1PE13,5,1, RHO=1,1E13,5)	FAR	89
18	FORMAT (1H0,12X,1H0,16X,8HINTEGRAL,9X,14HINTEGRAL ERROR,6X,14HF(U,	FAR	90
	1ALPHA,RHO)/1H)	FAR	91
19	FORMAT (16E5,0)	FAR	92

TABLE 8.2.—Listing of programs for constant discharge from a fully penetrating well of finite diameter—Continued

1	ROOT=XM	ROO	23
	GO TO 10	ROO	24
2	IF (YM*YL) 7,3,4	ROU	25
3	ROOT=XL	ROO	26
	GO TO 10	ROU	27
4	IF (YM*YR) 8,5,6	ROO	28
5	ROOT=XR	ROO	29
	GO TO 10	ROU	30
6	WRITE (6,11) XL,XR	ROO	31
	STOP	ROO	32
7	XR=XM	ROO	33
	YR=YM	ROO	34
	GO TO 9	ROO	35
8	XL=XM	ROU	36
	YL=YM	ROO	37
9	XM=(XL+XR)/2,0	ROO	38
	ROOT=XM	ROO	39
10	RETURN	ROO	40
C		ROO	41
11	FORMAT (1H ,10X,27HNO ROOT IN INTERVAL XM=DX =,1PE20,8,5X,11HAND XROO		42
	1M+DX =,1PE20,8/)	ROO	43
	END	ROO	44
	SUBROUTINE APEKE(EXBSL)	APE	1
C	*****	APE	2
C		APE	3
C	PURPOSE	APE	4
C	GETS FIRST APPROXIMATION TO PEAK POSITION	APE	5
C		APE	6
C	*****	APE	7
	COMMON XPK,YPK	APE	8
	XPK=0,0	APE	9
	YPK=0,0	APE	10
	DO 2 I=1,17	APE	11
	X=10,0*(I-9)	APE	12
	Y=EXBSL(X)	APE	13
	IF (Y=YPK) 3,3,1	APE	14
1	XPK=X	APE	15
	YPK=Y	APE	16
2	CONTINUE	APE	17
3	RETURN	APE	18
	END	APE	19
	SUBROUTINE PEAK(EXBSL)	PEA	1
C	*****	PEA	2
C		PEA	3
C	PURPOSE	PEA	4
C	ATTEMPTS TO FIND POSITION OF MAXIMUM FOR INTEGRAND	PEA	5
C		PEA	6
C	*****	PEA	7
	COMMON XPK,YPK	PEA	8
	YPK=EXBSL(XPK)	PEA	9
	DO 13 L=1,200	PEA	10
	DX=0,01*XPK	PEA	11
	XL=XPK-DX	PEA	12
	YL=EXBSL(XL)	PEA	13
	XR=XPK+DX	PEA	14
	YR=EXBSL(XR)	PEA	15
	DEN=YR+YL-YPK-YPK	PEA	16
	IF (DEN) 1,9,1	PEA	17
1	X=XPK+0,5*(YR-YL)*DX/DEN	PEA	18
2	IF (X) 3,4,4	PEA	19

TABLE 8.2.—Listing of programs for constant discharge from a fully penetrating well of finite diameter—Continued

3	X=0,0	PEA	20
4	Y=EXBSL(X)	PEA	21
	IF (YH=Y) 6,6,5	PEA	22
5	Y=YR	PEA	23
	X=XR	PEA	24
6	IF (YL=Y) 8,8,7	PEA	25
7	Y=YL	PEA	26
	X=XL	PEA	27
8	IF (Y=YPK) 14,14,12	PEA	28
9	IF (YR=YPK) 11,10,10	PEA	29
10	X=XPK+DX+DX	PEA	30
	GO TO 2	PEA	31
11	X=XPK-DX-DX	PEA	32
	GO TO 2	PEA	33
12	YPK=Y	PEA	34
	XPK=X	PEA	35
13	CONTINUE	PEA	36
14	RETURN	PEA	37
	END	PEA	38
	SUBROUTINE SIMPS(Q,R,EPS,HBAR,AREA,DEL,F)	SIM	1
C	*****	SIM	2
C	PURPOSE	SIM	3
C	TO DETERMINE THE INTEGRAL OF A FUNCTION, F, FROM Q TO R,	SIM	4
C	USING SIMPSON'S RULE,	SIM	5
C	DESCRIPTION OF PARAMETERS	SIM	6
C	ALL REAL	SIM	7
C	Q = LOWER LIMIT OF INTEGRAL	SIM	8
C	R = UPPER LIMIT OF INTEGRAL	SIM	9
C	EPS = DESIRED ACCURACY	SIM	10
C	HBAR = MINIMUM DIVISION OF THE INTERVAL	SIM	11
C	AREA = COMPUTED VALUE OF INTEGRAL BETWEEN Q AND R	SIM	12
C	DEL = COMPUTED ESTIMATE OF ERROR	SIM	13
C	F = THE INTEGRAND (FUNCTION REFERENCE)	SIM	14
C	METHOD	SIM	15
C	USES SIMPSON'S RULE TO COMPUTE A SUM APPROXIMATING THE INTEGRAL	SIM	16
C	USES INITIAL H=(R-Q)/2, COMPUTES A SEQUENCE OF SUMS BY HALVING	SIM	17
C	H EACH TIME, COMPUTES ESTIMATE OF ERROR (DEL) AS (PREVIOUS	SIM	18
C	SUM - CURRENT SUM)/15, COMPUTATION STOPS WHEN 1) H<HBAR,	SIM	19
C	2) ABS(DEL)<ABS(EPS*CURRENT SUM), IF HBAR IS LE 0,	SIM	20
C	THEN HBAR=.007*(R-Q),	SIM	21
C		SIM	22
C		SIM	23
C	*****	SIM	24
	H=R-Q	SIM	25
	IF (H) 1,1,2	SIM	26
1	AREA=0,0	SIM	27
	DEL=0,0	SIM	28
	GO TO 10	SIM	29
C	R MUST BE GREATER THAN Q	SIM	30
2	SP=1,0E35	SIM	31
	S3=0,0	SIM	32
	S1=F(Q)+F(R)	SIM	33
	IF (HBAR) 3,3,4	SIM	34
3	HBAR=0,007*H	SIM	35
4	S2=0,0	SIM	36
	X=Q+0,5*H	SIM	37
5	S2=S2+4,0*F(X)	SIM	38
	X=X+H	SIM	39
	IF (X=R) 5,5,6	SIM	40
6	SC=(S1+S2+S3)*H*0,16666667	SIM	41

TABLE 8.2.—Listing of programs for constant discharge from a fully penetrating well of finite diameter—Continued

```

      DEL=0,066666667*(SP=SC)                      SIM 42
      IF (ABS(DEL)=ABS(EP9*SC)) 7,8,8              SIM 43
      7 AREA=SC=DEL                                SIM 44
      GO TO 10                                     SIM 45
      8 S3=S3+0,5*S2                               SIM 46
      H=0,5*H                                       SIM 47
      IF (H=HBAR) 7,9,9                            SIM 48
      9 SP=SC                                       SIM 49
      GO TO 4                                       SIM 50
      10 RETURN                                    SIM 51
      END                                           SIM 52
      SUBROUTINE JY0(X,J0,Y0)                       JY0 1
C*****JY0 2
C                                           JY0 3
C      PURPOSE                                     JY0 4
C      COMPUTES BESSEL FUNCTIONS OF THE FIRST AND SECOND KIND, JY0 5
C      ZERO ORDER, FOR POSITIVE ARGUMENTS,       JY0 6
C      SEE NBS AMS 55, P. 369-370.              JY0 7
C      DESCRIPTION OF PARAMETERS = ALL REAL      JY0 8
C      X= ARGUMENT, MUST BE >0                 JY0 9
C      J0 = RETURNED FUNCTION VALUE, J0(X)      JY0 10
C      Y0 = RETURNED FUNCTION VALUE, Y0(X)      JY0 11
C                                           JY0 12
C*****JY0 13
      REAL J0                                       JY0 14
      IF (X=3,0) 1,2,3                             JY0 15
      1 IF (X) 4,4,2                               JY0 16
      2 Z=(0,33333333*X)**2                         JY0 17
      J0=1,0-Z*(2,2499997-Z*(1,2656208-Z*(0,3163866-Z*(0,0444479-Z*(0,00JY0 18
      139444=0,00021*Z))))                         JY0 19
      Y0=0,63661977*ALOG(0,5*X)+J0+0,36746691+Z*(0,60559366-Z*(0,7435038JY0 20
      14=Z*(0,25300117-Z*(0,04261214-Z*(0,00427916=0,00024846*Z)))) JY0 21
      RETURN                                       JY0 22
      3 Z=3,0/X                                    JY0 23
      F=0,79788456-Z*(0,77E=6+Z*(0,0059274+Z*(0,00009512-Z*(0,00137237-ZJY0 24
      1*(0,00072805=0,00014476*Z))))              JY0 25
      P=0,78539816+Z*(0,04166397+Z*(0,00003954-Z*(0,00262573-Z*(0,000541JY0 26
      125+Z*(0,00029333=0,00013558*Z))))        JY0 27
      Q=SQRT(1,0/X)                               JY0 28
      J0=Q*F*COS(X=P)                             JY0 29
      Y0=Q*F*SIN(X=P)                             JY0 30
      4 RETURN                                    JY0 31
      END                                           JY0 32
      SUBROUTINE JY1(X,J1,Y1)                       JY1 1
C*****JY1 2
C                                           JY1 3
C      PURPOSE                                     JY1 4
C      COMPUTES BESSEL FUNCTIONS OF THE FIRST AND SECOND KIND, JY1 5
C      FIRST ORDER, FOR POSITIVE ARGUMENTS,     JY1 6
C      SEE NBS AMS 55, P. 370.                 JY1 7
C      DESCRIPTION OF PARAMETERS = ALL REAL      JY1 8
C      X= ARGUMENT, MUST BE >0                 JY1 9
C      J1 = RETURNED FUNCTION VALUE, J1(X)      JY1 10
C      Y1 = RETURNED FUNCTION VALUE, Y1(X)      JY1 11
C                                           JY1 12
C*****JY1 13
      REAL J1                                       JY1 14
      IF (X=3,0) 1,2,3                             JY1 15
      1 IF (X) 4,4,2                               JY1 16
      2 Z=(0,33333333*X)**2                         JY1 17

```

TABLE 8.2.—Listing of programs for constant discharge from a fully penetrating well of finite diameter—Continued

```

J1=X*(0.5-Z*(0.56249985-Z*(0.21093573-Z*(0.03954289-Z*(0.00443319-JY1 18
1Z*(0.00031761=0.00001109*Z)))))) JY1 19
Y1=0.63661977*ALOG(0.5*X)*J1+(=0.6366198+Z*(0.2212091+Z*(2.1682709JY1 20
1=Z*(1.3164827-Z*(0.3123951-Z*(0.0400976=0.0027873*Z)))))/X JY1 21
RETURN JY1 22
3 Z=3.0/X JY1 23
F=0.79788456+Z*(0.156E+5+Z*(0.01659667+Z*(0.00017105-Z*(0.00249511JY1 24
1=Z*(0.00113653=0.00020033*Z)))))) JY1 25
P=0.78539816-Z*(0.12499612+Z*(0.0000565-Z*(0.00637879-Z*(0.0007434JY1 26
18+Z*(0.00079824=0.00029166*Z)))))) JY1 27
Q=SQRT(1.0/X) JY1 28
J1=Q*F*8IN(X=P) JY1 29
Y1=Q*F*COS(X=P) JY1 30
4 RETURN JY1 31
END JY1 32=
C*****FUA 1
C FUA 2
C PURPOSE FUA 3
C COMPUTES FUNCTION VALUES OF F(UW,ALPHA) = FUA 4
C PAPADOPULOS,I,S. AND COOPER,H,H.,JR., 1967, DRAWDOWN IN FUA 5
C A WELL OF LARGE DIAMETER; WATER RESOURCES RESEARCH, V, 3, FUA 6
C NO. 1, P. 241-244. FUA 7
C PROGRAM BY S,S,PAPADOPULOS. FUA 8
C INPUT DATA = ONE OR MORE GROUPS, EACH GROUP CODED AS FOLLOWS FUA 9
C 1 CARD = FORMAT (E10,5) FUA 10
C S = (ALPHA) = RW**2*S/RC**2 = RADIUS OF WELL (SCREEN FUA 11
C OR OPEN BORE IN AQUIFER) SQUARED * STORAGE FUA 12
C COEFFICIENT / RADIUS OF CASING (OVER INTERVAL OF FUA 13
C WATER LEVEL CHANGE) SQUARED. FUA 14
C 1 CARD = FORMAT(16E5,0) FUA 15
C U = 16 VALUES OF UW = RW**2*S/(4*T*TIME) = RADIUS OF FUA 16
C PUMPED WELL SQUARED * STORAGE COEFFICIENT / FUA 17
C 4 * TRANSMISSIVITY * TIME, IF LESS THAN 16 DESIRED, FUA 18
C BLANK OR ZERO VALUES MAY BE CODED FOR THE REST. FUA 19
C SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED FUA 20
C PEAK,SIMP,APEKE,EXBSL2,JY0,JY1 = MUST BE INCLUDED IN DECK, FUA 21
C FUA 22
C*****FUA 23
COMMON XPK,YPK FUA 24
COMMON/PBLK/A,B FUA 25
EXTERNAL EXBSL2 FUA 26
DIMENSION U(16) FUA 27
EPS=0.0001 FUA 28
1 READ (5,13,END=12) S FUA 29
IF (S) 1,1,2 FUA 30
2 READ (5,14) U FUA 31
WRITE (6,15) S FUA 32
DO 11 I=1,16 FUA 33
UW=U(I) FUA 34
IF (UW) 1,1,3 FUA 35
3 B=0.25/UW FUA 36
A=S+B FUA 37
CALL APEKE(EXBSL2) FUA 38
CALL PEAK(EXBSL2) FUA 39
IF (XPK=1.0E+8) 4,5,5 FUA 40
4 WRITE (6,16) UW,S,XPK,YPK FUA 41
GO TO 11 FUA 42
5 IF (XPK=1.0E8) 7,7,6 FUA 43
6 WRITE (6,17) UW,S,XPK,YPK FUA 44
GO TO 11 FUA 45
7 HBAR=0.007*XPK FUA 46

```


TABLE 9.2.—Listing of program to compute change in water level due to sudden injection of a slug of water into a well

```

C***** FBA 1
C FBA 2
C PURPOSE FBA 3
C COMPUTES FUNCTION VALUES OF F(BETA,ALPHA) = THE SLUG TEST FBA 4
C FUNCTION = COOPER,H,H.,JR., BREDEHOEFT,J,D., AND PAPADOPULOS, FBA 5
C I,S., 1967, RESPONSE OF A FINITE-DIAMETER WELL TO AN FBA 6
C INSTANTANEOUS CHARGE OF WATER; WATER RESOURCES RESEARCH, FBA 7
C V, 3, NO. 1, P. 263-269, FBA 8
C PROGRAM BY S,S.PAPADOPULOS. FBA 9
C INPUT DATA FBA 10
C 1 OR MORE CARDS = FORMAT(F16,5) FBA 11
C A = (ALPHA) = RW**2*S/HC**2 = RADIUS OF WELL (SCREEN OR FBA 12
C OPEN BORE IN AQUIFER) SQUARED * STORAGE COEFFICIENT FBA 13
C / RADIUS OF CASING (OVER INTERVAL OF WATER LEVEL FBA 14
C CHANGE) SQUARED. FBA 15
C SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED FBA 16
C PRX,DJY0,DJY1,DSIMPS = MUST BE INCLUDED IN DECK FBA 17
C METHOD FBA 18
C THIS PROGRAM CALCULATES THE SLUG TEST FUNCTION, F(BETA,ALPHA), FBA 19
C FOR VALUES OF BETA RANGING FROM 0.001 TO 1000.0 BY INCREMENTING FBA 20
C BETA ACCORDING TO DATA ARRAY BB(I), AVERAGE COMPUTATION TIME FBA 21
C IS ABOUT 30 SECONDS PER VALUE OF ALPHA ON IBM 360/155. FBA 22
C FBA 23
C***** FBA 24
C DOUBLE PRECISION A,B,PI,ZZ,EPS,Y,X1,X2,TERM,FAB,DATAN,DEL,HBAR FBA 25
C DIMENSION ZZ(40), BB(39) FBA 26
C COMMON A,B,PI FBA 27
C EXTERNAL PRX FBA 28
C DATA ZZ/0.0+0,1.0=10,1.0=9,1.0=8,1.0=7,1.0=6,1.0=5,1.0=4, FBA 29
C 1 1.0=3,1.0=2,1.0=1,2.0=1,3.0=1,4.0=1,5.0=1,6.0=1,7.0=1,8.0=1, FBA 30
C 2 9.0=1,1.0=0,2.0=0,3.0=0,4.0=0,5.0=0,6.0=0,7.0=0,8.0=0, FBA 31
C 3 9.0=0,1.0=1,2.0=1,3.0=1,4.0=1,5.0=1,6.0=1,7.0=1,8.0=1, FBA 32
C 4 9.0=1,1.0=2,1.25D+2,1.5D+2/ FBA 33
C DATA BB/.001,.002,.004,.006,.008,.01,.02,.04,.06,.08,.1,.2,.4,.6, FBA 34
C 18,1,2,3,4,5,6,7,8,9,10,20,30,40,50,60,70,80,90,1 FBA 35
C 200,200,400,600,800,1000./ FBA 36
C PI=4,*DATAN(1.0D+00) FBA 37
C EPS=0.00001 FBA 38
C 1 READ (5,6) A FBA 39
C IF (A,LE,0.0) GO TO 5 FBA 40
C WRITE (6,7) A FBA 41
C WRITE (6,8) FBA 42
C DO 4 I=1,39 FBA 43
C B=BB(I) FBA 44
C Y=0.0 FBA 45
C DO 2 L=1,39 FBA 46
C X1=ZZ(L) FBA 47
C X2=ZZ(L+1) FBA 48
C HBAR=0. FBA 49
C CALL DSIMPS(X1,X2,EPS,HBAR,TERM,DEL,PRX) FBA 50
C Y=Y+TERM FBA 51
C IF (L,GT,20,AND,TERM,LT,EPS) GO TO 3 FBA 52
C 2 CONTINUE FBA 53
C 3 FAB=4,*A*Y/(PI*PI) FBA 54
C 4 WRITE (6,9) B,FAB FBA 55
C GO TO 1 FBA 56
C 5 STOP FBA 57
C FBA 58
C FBA 59
C 6 FORMAT (F16,5) FBA 60
C 7 FORMAT ('1',41X,'F(BETA,ALPHA) FOR ALPHA=',1PD9,2/) FBA 61
C 8 FORMAT ('01',53X,'BETA',13X,'H/H0'/) FBA 62
C 9 FORMAT ('1',51X,1PD8,2,10X,0PF6,4) FBA 63
C END FBA 64

```

TABLE 9.2.—Listing of program to compute change in water level due to sudden injection of a slug of water into a well—
Continued

```

      DOUBLE PRECISION FUNCTION PRX(X)                                PRX  1
C*****                                                                    PRX  2
C                                                                    PRX  3
C      PURPOSE                                                        PRX  4
C      COMPUTE VALUES OF THE INTEGRAND FOR F(BETA,ALPHA)           PRX  5
C      DESCRIPTION OF PARAMETER                                       PRX  6
C      X = DOUBLE PRECISION = ARGUMENT OF INTEGRAND                 PRX  7
C                                                                    PRX  8
C*****                                                                    PRX  9
      DOUBLE PRECISION A,B,PI,XX,X,C,F1,F2,J0,Y0,J1,Y1             PRX 10
      DOUBLE PRECISION DLOG,DSQRT,DEXP                             PRX 11
      COMMON A,B,PI                                                PRX 12
      XX=DSQRT(A*X/B)                                              PRX 13
      IF (X) 6,1,2                                                PRX 14
      1 PRX=(PI*PI)/(16,*A*B)                                       PRX 15
      GO TO 6                                                      PRX 16
      2 IF (X,LT,150,) GO TO 3                                       PRX 17
      PRX=0,0                                                       PRX 18
      GO TO 6                                                      PRX 19
      3 IF (XX,GT,0,0001) GO TO 4                                       PRX 20
      C=DEXP(5,772156649D=01)/2,                                       PRX 21
      F1=PI*X*(1,=A)                                               PRX 22
      F2=X*DLOG(C*C*A*X/B)+4,*B                                       PRX 23
      PRX=(B*PI*PI*DEXP(-X))/(A*(F1*F1+F2*F2))                   PRX 24
      GO TO 6                                                      PRX 25
      4 IF (XX,LT,50,) GO TO 5                                       PRX 26
      PRX=(PI*DEXP(-X))/(2,*XX*(X+4,*A*B))                         PRX 27
      GO TO 6                                                      PRX 28
      5 CALL DJY0(XX,J0,Y0)                                          PRX 29
      CALL DJY1(XX,J1,Y1)                                          PRX 30
      F1=(XX*J0=2,*A*J1)                                           PRX 31
      F2=(XX*Y0=2,*A*Y1)                                           PRX 32
      PRX=DEXP(-X)/(X*(F1*F1+F2*F2))                                PRX 33
      6 RETURN                                                       PRX 34
      END                                                            PRX 35=

      SUBROUTINE DJY0(X,J0,Y0)                                       DJO  1
C*****                                                                    DJO  2
C                                                                    DJO  3
C      PURPOSE                                                        DJO  4
C      COMPUTES BESSEL FUNCTIONS OF THE FIRST AND SECOND KIND,     DJO  5
C      ZERO ORDER, FOR POSITIVE ARGUMENTS,                          DJO  6
C      DESCRIPTION OF PARAMETERS = ALL DOUBLE PRECISION             DJO  7
C      X = ARGUMENT, MUST BE >0                                     DJO  8
C      J0 = RETURNED FUNCTION VALUE, J0(X)                          DJO  9
C      Y0 = RETURNED FUNCTION VALUE, Y0(X)                          DJO 10
C                                                                    DJO 11
C*****                                                                    DJO 12
      DOUBLE PRECISION Z,J0,Y0,F,P,Q,U,W,X,DLOG,DCOS,DSIN,DSQRT   DJO 13
      IF (X=3,0) 1,2,3                                             DJO 14
      1 IF (X) 4,4,2                                               DJO 15
      2 Z=(X/3,0)**2                                               DJO 16
      J0=1,0=Z*(2,2499997=Z*(1,2656208=Z*(0,3163866=Z*(0,0444479=Z*(0,00DJO 17
      139444=0,00021*Z))))))                                       DJO 18
      W=(0,500)*X                                                  DJO 19
      Y0=0,63661977*DLOG(W)*J0+0,36746691+Z*(0,60559366=Z*(0,74350384=Z*DJO 20
      1(0,25300117=Z*(0,04261214=Z*(0,00427916=0,00024846*Z)))))) DJO 21
      RETURN                                                       DJO 22
      3 Z=3,0/X                                                    DJO 23
      F=0,79788456=Z*(0,77D=6+Z*(0,0055274+Z*(0,00009512=Z*(0,00137237=ZDJO 24
      1*(0,00072805=0,00014476*Z))))))                             DJO 25
      P=0,78539816+Z*(0,04166397+Z*(0,00003954=Z*(0,00262573=Z*(0,000541DJO 26
      125+Z*(0,00029333=0,00013558*Z))))))                         DJO 27

```


TABLE 9.2.—Listing of program to compute change in water level due to sudden injection of a slug of water into a well—
Continued

C*****		DSI	24
	DOUBLE PRECISION H,HBAR,AREA,DEL,S1,S2,S3,SC,SP,X,A,B,EPS,F,DABS	DSI	25
C	AREA OF F FROM A TO B,EPS IS DESIRED ACCURACY, HBAR THE MINIMUM	DSI	26
C	ALLOWABLE INTERVAL, DEL THE ESTIMATE OF THE ERROR	DSI	27
	H=B-A	DSI	28
	IF (H) 1,1,2	DSI	29
1	AREA=0,0	DSI	30
	DEL=0,0	DSI	31
	GO TO 10	DSI	32
2	SP=1,0035	DSI	33
	S3=0,0	DSI	34
	S1=F(A)+F(B)	DSI	35
	IF (HBAR) 3,3,4	DSI	36
3	HBAR=0,007*M	DSI	37
4	S2=0,0	DSI	38
	X=A+0,5*M	DSI	39
5	S2=S2+4,0*F(X)	DSI	40
	X=X+M	DSI	41
	IF (X=B) 5,5,6	DSI	42
6	SC=(S1+S2+S3)*H*0,16666666667	DSI	43
	DEL=0,06666666667*(SP=SC)	DSI	44
	IF (DABS(DEL)=DABS(EPS*SC)) 7,8,8	DSI	45
7	AREA=SC=DEL	DSI	46
	GO TO 10	DSI	47
8	S3=S3+0,5*S2	DSI	48
	H=0,5*M	DSI	49
	IF (H=HBAR) 7,9,9	DSI	50
9	SP=SC	DSI	51
	GO TO 4	DSI	52
10	RETURN	DSI	53
	END	DSI	54

TABLE 11.1.—Listing of program to compute the convolution integral for a leaky aquifer

C*****		HRT	1
C		HRT	2
C	PURPOSE	HRT	3
C	COMPUTES CHANGES IN WATER LEVEL, H(R,T), IN RESPONSE TO	HRT	4
C	VARYING DISCHARGE USING THE CONVOLUTION INTEGRAL FOR	HRT	5
C	LEAKY AQUIFERS = EQ. 3 OF MOENCH, ALLEN, 1971, GROUND-WATER	HRT	6
C	FLUCTUATIONS IN RESPONSE TO ARBITRARY PUMPAGE; GROUND WATER,	HRT	7
C	V, 9, NO. 2, P. 4-8.	HRT	8
C	INPUT DATA = ONE OR MORE GROUPS, EACH GROUP CODED AS FOLLOWS	HRT	9
C	1 CARD = FORMAT(2E10,5,4X,I1,5X,E10,5)	HRT	10
C	TBEGIN = SMALLEST VALUE OF TIME FOR OUTPUT.	HRT	11
C	TEND = LARGEST VALUE OF TIME FOR OUTPUT.	HRT	12
C	IQ = INDICATES FORM OF DISCHARGE FUNCTION, Q(T).	HRT	13
C	IQ=1,2,3 REFER TO DISCHARGE FUNCTIONS IN	HRT	14
C	HANTUSH, M, S., 1964, HYDRAULICS OF WELLS IN CHOW,	HRT	15
C	VEN TE, ED., ADVANCES IN HYDROSCIENCE, VOL. 11	HRT	16
C	ACADEMIC PRESS INC., NEW YORK, P. 281-442.	HRT	17
C	IQ=1, Q(T) IS AN EXPONENTIAL FUNCTION, CASE A,	HRT	18
C	P. 343 OF HANTUSH.	HRT	19
C	IQ=2, Q(T) IS A HYPERBOLIC FUNCTION, CASE B,	HRT	20
C	P. 344 OF HANTUSH.	HRT	21
C	IQ=3, Q(T) IS AN INVERSE SQUARE ROOT FUNCTION,	HRT	22
C	CASE C, P. 344 OF HANTUSH.	HRT	23

TABLE 11.1.—Listing of program to compute the convolution integral for a leaky aquifer—Continued

```

C          IQ=4, Q(T) IS A FIFTH-DEGREE POLYNOMIAL,          HRT 24
C          IQ=5, Q(T) IS A PIECEWISE LINEAR FUNCTION OF      HRT 25
C          TIME (EIGHT SEGMENTS),                            HRT 26
C          QR = REFERENCE DISCHARGE, ZERO OR BLANK FOR PROJECTION, HRT 27
C          1 OR 4 CARDS, DEPENDING ON IQ,                    HRT 28
C          IF IQ=1,2,3 = 1 CARD = FORMAT(3E10,3)            HRT 29
C          QST = EVENTUAL CONSTANT DISCHARGE,                HRT 30
C          DELTA = RATE PARAMETER,                            HRT 31
C          TSTAR = TIME PARAMETER,                            HRT 32
C          IF IQ=4 = 1 CARD = FORMAT(6E10,3)                HRT 33
C          AQ(6) = 6 VALUES = THE POLYNOMIAL COEFFICIENTS HRT 34
C          WITH A0 FIRST AND A5 LAST,                         HRT 35
C          IF IQ=5 = 4 CARDS = FORMAT(6E10,3)                HRT 36
C          TI(I),AI(I),BI(I),TI(I+1),AI(I+1),BI(I+1),I=1,3,5,7 HRT 37
C          PARAMETERS OF THE PIECEWISE LINEAR FUNCTION        HRT 38
C          (8 SEGMENTS), CODED 2 SEGMENTS PER CARD, FIRST    HRT 39
C          AND SECOND SEGMENTS ON FIRST CARD, THEN SEQUENTIALLY HRT 40
C          ON SUCCEEDING CARDS, EACH SEGMENT HAS THREE        HRT 41
C          PARAMETERS WHICH ARE IN CODING ORDER                HRT 42
C          TI = ENDING TIME OF THE SEGMENT,                   HRT 43
C          AI = DISCHARGE AT BEGINNING OF SEGMENT,            HRT 44
C          BI = RATE OF CHANGE IN DISCHARGE DURING SEG,       HRT 45
C          THE DISCHARGE FUNCTION IN EACH SEGMENT HAS THE     HRT 46
C          FORM  $Q(T) = AI(I) + BI(I) * (T - TI(I-1))$ , IF LESS THAN 8 HRT 47
C          SEGMENTS ARE NEEDED, BLANKS CAN BE CODED FOR       HRT 48
C          SUCCEEDING SEGMENTS,                               HRT 49
C          2 OR MORE CARDS = FORMAT(4E10,3)                  HRT 50
C          R = RADIAL DISTANCE FROM PUMPED WELL, BLANK OR ZERO HRT 51
C          SIGNALS PROGRAM AS END TO GROUP OF DATA,          HRT 52
C          S = STORAGE COEFFICIENT                            HRT 53
C          T = TRANSMISSIVITY                                  HRT 54
C          PM = (P1/M1) = HYD. COND. OF CONFINING BED DIVIDED HRT 55
C          BY THICKNESS OF CONFINING BED,                     HRT 56
C          SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED        HRT 57
C          CONVOL,Q = MUST BE INCLUDED IN DECK,               HRT 58
C                                                            HRT 59
C*****                                                      HRT 60
C          DIMENSION      D(12),IEX(12),X(6),H(12,6),QS(12,6),CP(12),CT(12) HRT 61
C          DIMENSION H1(12),H2(12),Q1(12),Q2(12)              HRT 62
C          DIMENSION H3(12),H4(12),Q3(12),Q4(12)              HRT 63
C          COMMON AQ(6),TI(9),AI(9),BI(9),QST,DELTA,TSTAR    HRT 64
C          DATA CP/12*' T*/',CT/12*'1/U*/',D/12*'10**'/      HRT 65
C          DATA H1/12*' S(1',H2/12*'R,T)'/,Q1/12*' 1',Q2/12*'Q(T)'/ HRT 66
C          DATA H3/12*' S1',H4/12*'O(T)'/,Q3/12*' Q(T)',Q4/12*'')/QR'/ HRT 67
C          DATA X/1.,1.5,2.,3.,5.,7./                        HRT 68
C          TI(1)=0.                                           HRT 69
C          N=500                                               HRT 70
C          1 READ (5,18,END=17) TBEGIN,TEND,IQ,QR              HRT 71
C          IF (IQ,LT,4) READ (5,19) QST,DELTA,TSTAR           HRT 72
C          IF (IQ,EQ,4) READ (5,19) AQ                         HRT 73
C          IF (IQ,EQ,5) READ (5,19) (TI(I),AI(I),BI(I),I=2,9) HRT 74
C          WRITE (6,24)                                         HRT 75
C          2 READ (5,19) R,S,T,PM                               HRT 76
C          IF (R,EQ,0.) GO TO 1                                 HRT 77
C          A=R*R*S/(4.*T)                                       HRT 78
C          B=PM/S                                                HRT 79
C          Y=A*LOG10(TBEGIN)                                     HRT 80

```

TABLE 11.1.—Listing of program to compute the convolution integral for a leaky aquifer—Continued

IF (Y) 3,5,4	HRT	81
3 Y=Y-.001	HRT	82
GO TO 5	HRT	83
4 Y=Y+.001	HRT	84
5 IBEGIN=Y	HRT	85
Y=ALOG10(TEND)	HRT	86
IF (Y) 6,8,7	HRT	87
6 Y=Y-.001	HRT	88
GO TO 8	HRT	89
7 Y=Y+.001	HRT	90
8 IEND=Y	HRT	91
M=IEND-IBEGIN+1	HRT	92
IF (M,GT,12) M=12	HRT	93
DO 10 I=1,M	HRT	94
IEX(I)=IBEGIN+I-1	HRT	95
Y=10.** (IBEGIN+I-1)	HRT	96
DO 10 J=1,6	HRT	97
TIME=X(J)*Y	HRT	98
IF (QR,GT,0.) TIME=A*TIME	HRT	99
CALL CONVOL(TIME,A,B,N,IQ,SUM)	HRT	100
IF (QR,GT,0.) GO TO 9	HRT	101
H(I,J)=SUM/(12.5664*T)	HRT	102
QS(I,J)=Q(TIME,IQ)	HRT	103
GO TO 10	HRT	104
9 H(I,J)=SUM/QR	HRT	105
QS(I,J)=Q(TIME,IQ)/QR	HRT	106
10 CONTINUE	HRT	107
K=M	HRT	108
IF (M,GT,6) K=6	HRT	109
IF (QR,GT,0.) GO TO 11	HRT	110
WRITE (6,20) A,B,(CP(I),D(I),IEX(I),I=1,K)	HRT	111
WRITE (6,21) (H1(I),H2(I),Q1(I),Q2(I),I=1,K)	HRT	112
GO TO 12	HRT	113
11 WRITE (6,25) A,B,QR,(CT(I),D(I),IEX(I),I=1,K)	HRT	114
WRITE (6,21) (H3(I),H4(I),Q3(I),Q4(I),I=1,K)	HRT	115
12 DO 13 J=1,6	HRT	116
WRITE (6,22) X(J),(H(I,J),QS(I,J),I=1,K)	HRT	117
13 CONTINUE	HRT	118
IF (M,LE,6) GO TO 2	HRT	119
K1=K+1	HRT	120
IF (QR,GT,0.) GO TO 14	HRT	121
WRITE (6,23) (CP(I),D(I),IEX(I),I=K1,M)	HRT	122
WRITE (6,21) (H1(I),H2(I),Q1(I),Q2(I),I=K1,M)	HRT	123
GO TO 15	HRT	124
14 WRITE (6,26) (CT(I),D(I),IEX(I),I=K1,M)	HRT	125
WRITE (6,21) (H3(I),H4(I),Q3(I),Q4(I),I=K1,M)	HRT	126
15 DO 16 J=1,6	HRT	127
WRITE (6,22) X(J),(H(I,J),QS(I,J),I=K1,M)	HRT	128
16 CONTINUE	HRT	129
GO TO 2	HRT	130
17 STOP	HRT	131
C	HRT	132
18 FORMAT (2E10.5,4X,I1,5X,E10.5)	HRT	133
19 FORMAT (6E10.3)	HRT	134
20 FORMAT ('01','R**2*S/(4*TRANS)=',1PE10.3,' ',X11/(S*B11)=',E10.3/10'	HRT	135
1,2X,'T',5X,6(2A4,I2,9X))	HRT	136
21 FORMAT (' ',4X,6(2A4,2X,2A4,1X))	HRT	137

TABLE 11.1.—Listing of program to compute the convolution integral for a leaky aquifer—Continued

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22 FORMAT (' ',F4,1,6(OPF8,3,1PE11,3))          HRT 138
23 FORMAT ('0',2X,'T',5X,6(2A4,I2,9X))          HRT 139
24 FORMAT (1H1)                                HRT 140
25 FORMAT ('0',1R**2*S/(4*TRANS)=',1PE10,3,', ' K'/(S*B')=',E10,3,', HRT 141
    1QR=',E10,3/'0',1X,'1/U',4X,6(2A4,I2,9X))    HRT 142
26 FORMAT ('0',1X,'1/U',4X,6(2A4,I2,9X))        HRT 143
    END                                          HRT 144
SUBROUTINE CONVOL(TIME,A,B,N,IQ,SUM)            CON  1
C*****CONVOL*****                          CON  2
C                                             CON  3
C    PURPOSE                                  CON  4
C    COMPUTES VALUES OF THE CONVOLUTION INTEGRAL FOR LEAKY CON  5
C    AQUIFERS, THE INTEGRAL IS, FROM 0 TO T, OF CON  6
C     $Q(T-T')/T' \cdot \exp(-A/T' - B \cdot T') \cdot DT'$  CON  7
C    DESCRIPTION OF PARAMETERS                CON  8
C    A,B,SUM ARE REAL; N,IQ ARE INTEGER,      CON  9
C     $A = R^{*2} \cdot S / (4 \cdot T)$  = RADIAL DISTANCE SQUARED * STORAGE CON 10
C    COEFFICIENT / 4 * TRANSMISSIVITY,      CON 11
C     $B = P' / (S \cdot M')$  = HYD. COND. OF CONFINING BED DIVIDED BY CON 12
C    AQUIFER STORAGE COEFFICIENT * THICKNESS OF CONF. BED, CON 13
C    N = NUMBER OF INCREMENTS FOR EACH INTERVAL OF THE SUM, CON 14
C    IQ = INDICATES FORM OF DISCHARGE FUNCTION, CON 15
C    SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED CON 16
C    Q                                        CON 17
C    METHOD                                    CON 18
C    APPROXIMATES INTEGRAL BY SUMMING THE TRAPEZOIDAL RULE APPLIED CON 19
C    TO A SEQUENCE OF SEGMENTS, LOWER LIMIT OF FIRST SEGMENT IS CON 20
C    PICKED AT POINT WHERE EXPONENT > =100 . CON 21
C    IF SUCH A POINT DOES NOT EXIST (A*B > 2500) A FUNCTION VALUE CON 22
C    OF 0 IS RETURNED, UPPER LIMIT = 10 * LOWER LIMIT FOR EACH CON 23
C    SEGMENT, USES INCREMENT OF DELTA T' = (U-L)/N WHERE N IS THE CON 24
C    NUMBER OF INCREMENTS IN THE CALL, CEASES SUMMATION WHEN CON 25
C    EXPONENT < =101 .                      CON 26
C                                             CON 27
C*****CONVOL*****                          CON 28
    REAL*8 DSUM                                CON 29
    REAL*4 NEWT,NEWTP,NEWX,NEWF                CON 30
    DSUM=0,D+0                                CON 31
    IS=0                                       CON 32
C    INITIAL T' COMPUTED FROM A,B             CON 33
    AB=A*B                                     CON 34
    IF (AB,GE,2500,) GO TO 7                   CON 35
    IF (B,GT,0,) GO TO 2                      CON 36
1   OLDTP=.01*A                               CON 37
    GO TO 3                                    CON 38
2   OLDTP=(1,-SQRT(1,-AB/2500,))*50./B       CON 39
    IF (OLDTP,EQ,0,) GO TO 1                 CON 40
C    INITIAL T=T'                             CON 41
3   OLDTP=TIME-OLDTP                          CON 42
    OLDX=-A/OLDTP-B*OLDTP                    CON 43
    OLDY=Q(OLDTP,IQ)*EXP(OLDX)/OLDTP         CON 44
C    END OF SUMMATION SEGMENT IS 10 TIMES THE BEGINNING CON 45
4   ENDT=10.*OLDTP                            CON 46
    IF (ENDT,LT,TIME) GO TO 5                CON 47
    IF (OLDTP,GE,TIME) GO TO 7               CON 48
    IS=1                                       CON 49
    ENDT=TIME                                 CON 50

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TABLE 11.1.—Listing of program to compute the convolution integral for a leaky aquifer—Continued

```

C   DELTA T' IS COMPUTED FROM LENGTH AND NUMBER OF INCREMENTS           CUN  51
5  DELT=(ENDT-OLDT)/N                                                    CON  52
   DO 6 I=1,N                                                            CON  53
C   T' IS INCREMENTED BY DELTA T'                                       CON  54
   NEWT=OLDT+DELT                                                         CON  55
   NEWX=A/NEWT-B*NEWT                                                     CON  56
C   TERMINATES SUMMATION WHEN EXP(-A/T'-B*T') < 1.37E-44              CON  57
   IF (NEWX,LT,=.101,) GO TO 7                                           CON  58
   NEWTP=TIME=NEWT                                                        CON  59
   NEWF=Q(NEWT, IQ)*EXP(NEWX)/NEWT                                       CON  60
   DSUM=DSUM+(NEWF+OLDF)*DELT                                           CON  61
   OLDT=NEWT                                                              CON  62
   OLDF=NEWF                                                              CON  63
6  CONTINUE                                                              CON  64
   IF (IS,GT,0) GO TO 7                                                  CON  65
C   IF T' < T, BEGINS A NEW SEGMENT                                     CON  66
   GO TO 4                                                                CON  67
7  SUM=DSUM/2.0+0                                                        CON  68
   RETURN                                                                  CON  69
   END                                                                    CUN  70

      FUNCTION Q(TIME, IQ)                                               Q   1
C*****                                                                    Q   2
C   PURPOSE                                                                Q   3
C   COMPUTES THE DISCHARGE FUNCTION, Q(T)                                Q   4
C   DESCRIPTION OF PARAMETERS                                             Q   5
C   TIME = REAL = ELAPSED TIME SINCE BEGINNING OF DISCHARGE,           Q   6
C   IQ = INTEGER = INDICATES FORM OF DISCHARGE FUNCTION,               Q   7
C   IQ=1,2,3, CASES A,B,C, RESPECTIVELY, OF HANTUSH, M.S.,             Q   8
C   1964, HYDRAULICS OF WELLS IN CHOW, VEN TE, ED.,                    Q   9
C   ADVANCES IN HYDROSCIENCE, VOL. 1: ACADEMIC PRESS,                   Q  10
C   NEW YORK, P. 343,344,                                                Q  11
C   IQ=4, DISCHARGE IS A FIFTH DEGREE POLYNOMIAL OF TIME,              Q  12
C   IQ=5, DISCHARGE IS A PIECEWISE LINEAR FUNCTION OF UP TO            Q  13
C   8 SEGMENTS,                                                           Q  14
C   METHOD                                                                  Q  15
C   FORTRAN EVALUATION OF FUNCTIONS,                                     Q  16
C*****                                                                    Q  17
COMMON AQ(6),TI(9),AI(9),BI(9),QST,DELTA,TSTAR                           Q  18
GO TO (1,2,3,4,5), IQ                                                  Q  19
1  Q=QST*(1.+DELTA*EXP(-TIME/TSTAR))                                     Q  20
   RETURN                                                                Q  21
2  Q=QST*(1.+DELTA/(1.+TIME/TSTAR))                                     Q  22
   RETURN                                                                Q  23
3  Q=QST*(1.+DELTA/SQRT(1.+TIME/TSTAR))                                 Q  24
   RETURN                                                                Q  25
4  Q=AI(1)+TIME*(AQ(2)+TIME*(AQ(3)+TIME*(AQ(4)+TIME*(AQ(5)+TIME*AQ(6)
1))))                                                                    Q  26
   RETURN                                                                Q  27
5  DO 6 I=2,9                                                            Q  28
   IF (TIME.LE.TI(I)) GO TO 7                                           Q  29
6  CONTINUE                                                              Q  30
   I=9                                                                    Q  31
7  Q=AI(I)+BI(I)*(TIME-TI(I-1))                                         Q  32
   RETURN                                                                Q  33
   END                                                                    Q  34

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