MODELING CREDIT RISK SPREAD AND INTEREST RATE VOLATILITY IN THE EURODOLLAR MARKET

by

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Abstract

In this thesis, I conduct an investigation into two principal issues in the Eurodollar market. The first issue examines the stochastic behaviour of the credit risk spread in the yield of the three-month Eurodollar deposits placed in a designated London bank. The second examines the volatility of the yield on the same. In both issues examined, the period covered extends from June 1, 1973 through August 19, 1996, and the sampled data analyzed are at the daily frequency.

The purpose of the first essay, "The Credit-Risk Spread in the Eurodollar Market: An Empirical Analysis" is twofold. The first is to investigate the empirical determinants of credit risk spread in the Eurodollar market. The second is to assess the adequacy of using the information in the U.S. Treasury yield curve in modeling and predicting the observed credit risk in the market. In the analysis, I use the Engle, Lilien, and Robins (1987) GARCH-in-Mean modeling method. The results indicate that the yield curve does contain information for future credit risk. In addition to the information in the yield curve, I find that other financial time series also contain significant information for future credit risk. In order to evaluate the performance of the various models examined, I use the out-of-sample forecast encompassing test, the mean absolute prediction error, and the root mean square prediction error. All the performance indicators rank the GARCH- in- Mean model, which uses all financial market information, as the ideal for modeling and predicting credit risk.

The principal purpose of the second essay, "Modeling the Volatility of Interest Rates in the Eurodollar Market," is to investigate the predictive ability of the interest-rate models within and across the following family of models: the continuous time family, the (G)ARCH family, and the factor-ARCH family. Within the factor-ARCH family, attention is focused on the models that use directly observable financial market information rather than the latent variable or unobservable factor models. To evaluate the additional benefit that accrues in using directly observable financial market factors rather than models that use just the previous level of interest rate, the combination of the previous predicted volatility and the squared innovations, three evaluation criteria are employed. These are the out-of-sample mean square prediction error, the out-of-sample forecast encompassing method, and the N-fold cross-validation mean

square prediction error. The cross-validation method indicates that the factor-ARCH model, using directly observable financial market information, best predicts the future volatility. The factor-ARCH model is also the only model whose out-of-sample forecast error cannot be explained by the other models out-of-sample forecast. On this basis, the factor-ARCH model is ranked superior to other interest rate models.

Dedication

To my family

Acknowledgment

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Chapter 1

An Overview of the Euromarket

1.1 The Background

The Euromarket is a market for securities denominated in a currency other than that of the country where the security is issued. This market consists of: the Eurocurrency market, the Eurobond market, and the Euroequity market. The Eurocurrency market deals exclusively with short-term lending and borrowing of funds denominated in a different currency. Of the Eurocurrency market, the Eurodollar market is by far the largest, at about seventy percent of the Eurocurrency market.[1] Following–in terms of the volume and the value of transaction–are the Eurodeutschemark, the Euroswiss franc, the Eurosterling, the Euroyen, and the Eurocanadian dollar market, among others. The Eurobond and the Euroequity markets deals with long-term bonds and equity issues respectively.

The markets each operate from an offshore location such as the Cayman Islands, the Bahamas, Panama, Singapore, Hong Kong and the Channel Islands. They also operate from European financial centres such as London, Paris, Frankfurt, and Luxembourg, and from North

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[1] In the remainder of this study, attention is focused on the Eurodollar market as it constitutes the largest part of the Eurocurrency market. Nonetheless, this is not to say that the other markets are not important. They are equally important, and whatever conclusion is arrived at for the Eurodollar market is also equally applicable to any of the other Eurocurrency markets as well.

American locations such as New York, Chicago, San Francisco, and Toronto. As can be observed from these locations, the market spans the globe, and is traded around the clock. Furthermore, the market operates externally in tandem with the corresponding domestic financial market. In addition, they may also operate onshore alongside their domestic counterparts.

Although the Euromarket is not specific to any particular country, it nonetheless has a significant impact on the economic and financial lifeblood of many nations. It provides alternative avenues for corporations, banks, governments and other organizations in need of a cheaper source of funds than is available domestically. Likewise, it affords portfolio and fund managers the opportunity of investing in this market in order to take advantage of the higher yields offered in this market.

The market had its beginnings in the 1950s as a result of fears by the USSR that its U.S.-dollar-denominated assets in the United States might be frozen by the U.S. government. They therefore transferred their assets to the Russian banks operating in London and Paris. The second factor leading to the development of the market was the restriction imposed by the British government on the British banks not to finance overseas trade with the pound sterling. The British banks promptly switched to the U.S. dollar as an alternative to the pound sterling. The market was further bolstered by the series of restrictive banking regulations in the U.S.-in particular, the interest rate ceiling under regulation Q that became binding towards the late 1960s and the early 1970s.[2]

Even though some of the restrictive trade and financial regulations that lead to the development of the market have been removed (for example, the interest-rate ceiling under regulation Q in the U.S., and the restriction on overseas trade financing in the United Kingdom), the market still continues to prosper. The market has grown tremendously in recent periods; transactions now amount to over a trillion dollars a year. This rapid growth and development have been attributed to a number of factors, among which are: advances in transport and telecommunication technology, growth in international trade, the global expansion of multi-

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[2]The historical development of this market is too extensive to be properly covered in this study. For a more detailed historical account of the development of the market see, for example, Sarver (1990), Dufey and Giddy (1994).

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national corporations, the desire of governments—especially of emerging capital markets and developing countries–to finance trade deficits and development projects using short-term to medium-term credits, and, most importantly, the deregulation of financial markets in several countries, particularly in the 1980s.

The last of these factors is especially important. The wave of simultaneous deregulation in the financial markets in several countries further encourages a freer movement of capital across international boundaries. As a result, it expands the investment opportunity set faced by all fund managers. It also eliminates, to a certain extent, some of the impediments to conducting international arbitrage between the domestic and external financial assets markets; i.e., that fund managers are less constrained to investing in only domestic securities. Despite the opportunities, however, the market also introduces an element of risk to which the comparative domestic debt instruments may be less prone. In the next section, therefore, I discuss some of the markets' basic characteristics as well as their risk implications for investors.

1.2   The Euromarket Features and their Risk Implications

In comparison with domestic banking operations, the Eurocurrency market is more competitive. The Eurodollar (or Eurocurrency) market offers a higher rate on deposits, and the lending rates are also much lower. These differences in the rates between the domestic market and the Eurodollar market can be attributed to a number of features intrinsic to this market. These features include the following: first, most of the banks in the Eurodollar market operate in an offshore location, and more often, they operate outside the regulatory framework set by the financial authorities of that country.[3] As such, these institutions are less-regulated than the domestic banks. For example, during the 1960s and '70s, under regulation Q in the U.S., they were not constrained by the interest-rate ceiling imposed on domestic deposits. This feature enables the Eurobanks to compete more favorably with domestic banks, which must comply

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[3]This is equally true of money placed in the International Banking Facilities (IBF) in the U.S., even though the Eurobanking activity is on-shore.

with the interest-rate ceiling when it becomes binding.[4] In addition, they need not comply with the reserve requirements on deposits as established by the central banks, resulting in the opportunity cost of funds being lower for banks operating in the Eurodollar market than it is for the domestic banks. Moreover, unlike the domestic banks, Eurobanks are not compelled to insure customer deposits, which implies an additional lower cost of operation. Finally, they operate mainly as a wholesale bank, in the sense that the size of the deposits taken by these banks is large compared to those of the domestic banks. As such, it confers economies of scale that are not available to the domestic banks. Because the Eurobanks are less regulated, they have no lender of last resort as do the domestic banks, and as the funds placed in the Eurodollar market are not insured, the deposits placed therein are, therefore, more at risk than those placed in domestic banks or U.S. Treasury securities.[5]

The banks operating in the Euromarket are of two main types. The first type operates as a subsidiary, affiliate, representative, or correspondent of a major bank. These subsidiaries or affiliates are incorporated in the offshore location for the purpose of conducting Eurobanking business. The second type involves those operating simply as foreign branches of domestic banks. Even though these banks operating in the offshore location may have "parents,"or may have a very strong tie with the domestic banks, the funds deposited in the offshore location are nonetheless neither implicitly nor explicitly guaranteed by the parent or associated bank in the domestic market. In fact, under international banking law, each foreign subsidiary, affiliate, or correspondent bank is regarded as a separate legal entity: a corporate person that can sue and be sued. In law, the parent bank is regarded as a mere shareholder, and its liability is limited

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[4]Today, in the economies of countries such as Singapore, Malaysia, Thailand and many of the emerging capital markets in South East Asia, Africa or Latin America, the monetary authority still exercises tight control on the domestic money market. It sometimes sets the maximum rate that lenders may charge on loans denominated in domestic currencies. This pre-setting of rates is similar in most respects to that of regulation Q.

[5]Depositors are only guaranteed payment up to the maximum of $100,000 in the event of the failure of a bank insured by the Federal Deposit Insurance Corporation (FDIC). For depositors with larger amounts, the maximum may represent a very small fraction of the total amount deposited. As a result, funds placed in Eurodollar deposits may not be riskier than funds placed in domestic banks. However, given that the same fund invested in a U.S. government treasury security is backed by the full faith and guarantee of the federal government, the funds placed in the Eurodollar banks are obviously more risky.

under the memorandum and articles of association to the extent of the fully paid up share capital in the subsidiary or affiliate bank. Given this state of the law, the depositor's claim on the bankrupt bank operating in the Eurodollar market ranks *pari pasu* merely as an unsecured creditor. Furthermore, should there exist insufficient funds to compensate all the creditors, the creditors have no recourse under the law to make claims on the parent bank for the amount owed. With this state of affairs, the investor stands not only to lose the interest payments, but also the principal amount deposited.

For most practical considerations and business expediency, however, rulings in British and American courts have found the parent bank of an offshore bank liable for the liabilities incurred by the subsidiaries and branch banks operating in the offshore location.[6] Also, the parent banks may, on occasion, redeem the liability of its subsidiary or foreign branches— even though it is not legally obliged to do so—especially where it thinks its reputation might be at risk. While it is possible for a depositor to recover from the parent bank the full or partial amount owed by the offshore bank, it would only be after lengthy and costly litigation. From this stance, one can now see that, in addition to the interest-rate risk, an investor may also be exposed to the default risk. The reality of default risk in this market, therefore, is important to this study.

To support the view presented above, there are some examples of internationally active banks that have failed or reported to be in distress over the years. For example, due to considerable foreign exchange losses in May-June 1974 the Franklin National Bank (Sindona) of New York, the Bankhaus Herstatt of Cologne, Lloyd's Bank-Lugano, Bank of Belgium, and Westdeutschelandes Bank all failed; and it subsequently lead to an increased perception of bank failure in the 1974-75 period.[7] Other banks also failed due, principally, to large losses

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[6]The international banking law governing the liability of banks in the event of bankruptcy of an offshore bank is too extensive to be covered here. Notwithstanding, Dufey and Giddy (1984) offer a useful exposition of the Eurocurrency deposit risk, and can be consulted by the interested reader. Also, Goodfriend (1981) explains how the sovereign risk, the jurisdictional risk, and the financial viability of the Eurobanks may impact the relative risk of the Eurodollar deposits vis-à-vis deposits held in the United States.

[7]See the plots of the absolute credit risk spread in Figure 2.1 and the relative credit risk spread in Figure 2.2. Both plots indicate that during the 1973-75 periods, the level of risk perception was relatively high compared to

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sustained by their foreign subsidiaries. Examples include Banco Ambrossiano Holdings SA (Luxembourg) in 1982, Schröeder, Munchey, Huego & Co. (Hamburg) operating in Luxembourg in 1982, Banco Ultramar (Venezuela) operating in Panama in 1983, and Barings Bank operating in Singapore in 1995. However, in the case of the Bank of Credit and Commerce International (BCCI) in 1991, its failure was due to outright fraud on the part of bank officers in the offshore locations.

The bust in the real estate market in the early 1980s also caused a number of banks to be in distress. In this category are the Penn Square Bank in 1982, the Continental Illinois Bank in 1984, and most of the largest Japanese banks in the 1990s.[8] Also in recent distress (1995-1997) is the Credit Lyonaise Bank of France. The global debt crisis of 1982-1983 further increased the perception of risk in the Eurodollar market and hence the wider spread that is subsequently observed.[9]

As can be observed from the foregoing analysis and examples, it is clear that banks operating in international markets do so on a very narrow spread (the difference between the rate at which they lend and borrow) in order to stay competitive. They are also more susceptible to adverse movement in the foreign exchange rate, or interest rates, if they are not properly hedged against these types of risks. In what follows, I attempt to explain and predict the credit risk spread in the Eurodollar market using information emanating from the following markets: the Eurodollar market itself, the foreign exchange market, the Federal Funds markets, the stock market and the U.S. Treasury bond market. This information is considered because assets markets are inter-related, and hence, in modeling the returns, the volatility, or any other type of risk in any of the markets, one must always take into account the events and

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other periods examined in the study.

[6]See the *The Financial Post* of September 12, 1997 on how the Japanese banks were affected by the collapse of the real estate market in the late 1980s. The paper reported among other things that Tokyo-Mitsubishi Ltd., the world's largest bank, had to write off $12.8billion (1.12 trillion yen) as bad debts from doubtful debt provisions or non-performing loans dating back to the 1980s.

[9]This debt crisis made depositors more aware of the extent of the risk imposed on all financial institutions due to the portfolio arrangement of the banks operating in the Eurodollar and the Eurobond markets. Because of the extent of interbank transactions, the collapse of one major bank within the system can have repercussions on others that are not directly connected with it; therefore, the global financial system is vulnerable.

developments in the other markets as well.

1.3 The Research Issues

The focus of this study is on two issues in the Eurodollar market: the credit risk spread and the volatility of the short-term interest rate. The credit risk spread, which is a measure of the credit or default risk, derives from the fact that deposits placed in the Eurodollar market are at greater risk than the alternative of placing them in domestic time deposits, commercial paper, or even default-free U.S. Treasury securities.

The first essay investigates the factors influencing the credit risk spread in the Eurodollar market. It investigates how the credit risk spread can best be modeled and predicted so as to be able to take advantage of the opportunities afforded by the credit-risk derivative instruments. These derivative instruments include the credit-risk swap, the Treasury-Eurodollar (TED) spread, the Eurodollar differentials (DIFFs), among others. These instruments have been developed and traded on exchanges such as the Chicago Mercantile Exchange (CME), and are used largely to mitigate the credit-risk exposure to which a portfolio of securities may be exposed.

In the second essay, I investigate the volatility of interest rates in the Eurodollar market. The objectives of the essay are as follows: to identify the factors governing the behaviour of volatility of interest rates in the Eurodollar market, and to develop a statistical model that best fits and predicts the volatility of interest rates in this market. The volatility model is required because, in addition to the credit risk faced by investors operating in the Eurodollar market is the interest-rate risk. Derivative instruments such as Eurodollar futures contracts, options on Eurodollar futures and forward contracts, swaps and swaptions, among others, exist to mitigate the interest-rate risk. However, in order to appropriately value these derivative contracts, one needs the "correct"estimate and model of the volatility of interest rate as an input in the valuation process. If the appropriate volatility estimate and model is not used, errors in the pricing of the securities may occur, and as a consequence, financial losses. The volatility

model is also instrumental to the appropriate calibration of the risk to which a portfolio of fixed income securities may be exposed: that is, in evaluating the value-at-risk (VaR) of the portfolio(see, for example, Jorion 1997, Phelan 1995, J. P. Morgan Bank 1995, among others).

It is important to understand the set of factors influencing the credit and interest-rate risks in the Eurodollar market for improving a portfolio's performance. This is the underlying theme of my thesis. At this juncture, I should mention that the this study adopts a purely statistical method to evaluate each of the models examined. I do acknowledge that the ideal method would have been to compare and contrast the models on the basis of the marginal gains and benefits accruing to each model relative to a benchmark model. Nonetheless, I have adopted the statistical method because of the following reason: any valuation of default-risky debt instrument and its derivatives, or the calibration of the VaR on such default-risky instruments, using the variables identified in this study would involve more than three factors, and at the moment, this is not computationally feasible.

The remainder of this thesis is organized as follows: Part I presents the first essay: "The Credit Risk Spread in the Eurodollar Market: An Empirical Analysis"and Part II presents the second essay: "Modeling the Interest-Rate Volatility in the Eurodollar Market."

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Part I

ESSAY #I

The Credit Risk Spread in the Eurodollar Market: An Empirical Analysis

Abstract

This essay analyzes the daily sampled data on credit risk spread in the Eurodollar market between June 1, 1973 and August 19, 1996. Its purpose is twofold. The first is to investigate the empirical determinants of credit risk spread in the Eurodollar market. The second is to assess the adequacy of U.S. Treasury yield curve information for modeling and predicting the observed credit risk in the Eurodollar market. In the study, I use the Engle, Lilien, and Robins (1987) GARCH-in-Mean modeling method. The results show that the yield curve does contain information for future credit risk. In addition to the information contained in the U.S. Treasury yield curve, I find that other financial time series also contain significant information for future credit risk. The out-of-sample forecast encompassing tests, the mean absolute prediction error, and the mean square prediction error, evaluation criteria all rank the GARCH-in-Mean model–which uses all financial market information–best for predicting credit risk.

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Chapter 2

The Credit Risk Spread

2.1 Introduction

When two counterparties enter into a contractual relationship, the risk that one of the parties will default in their contractual obligations is an ever-present possibility. This risk, sometimes referred to as the default risk or credit risk[1] (Fabozzi and Modigliani 1995: 5) is a pervasive problem in interbank lending: in domestic banking for borrowing and lending federal funds, and in the Eurocurrency market for interbank deposits. The situation is similar when banks and other financial institutions, such as mortgage corporations, insurance companies, investment and mutual funds, enter into a contractual relationship with their non-bank customers.

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[1]Technically speaking, there is a subtle difference between credit risk and default risk. Credit risk is associated with changes in credit quality (the ability to pay) of the counterparty, and it may not necessarily precipitate a default. However, for a default to occur, there will have to have been a change in the credit quality. It is this likelihood of default that the default risk captures. Thus, while default risk implies credit risk, the converse is not necessarily true. Despite this, most analyses ignore the subtle differences between the two concepts. This study also follows suit; i.e., that the terms "credit risk"and "default risk"are used interchangeably.

Since the true premium for credit risk is neither directly observable nor measurable, I follow the existing literature in using the yield spread between the yield on the Eurodollar instrument and the yield on a comparable risk-free U.S. Treasury security as a proxy for the credit risk. For examples, see Fabozzi and Modigliani (1995: 481), and Duffee (1996a). Consequently, this measure of credit risk or default risk is also at times referred to as credit risk spread.

Due to the existence and pervasiveness of this type of risk, the contracting parties usually demand compensation for the risk they must bear if the counterparty defaults in their obligation. The amount of compensation demanded varies over time as the perception of risk changes.

In this essay, I investigate the factors influencing the amount of compensation demanded for credit risk in the Eurodollar market, as they are particularly relevant for the following reasons.[2] First, the value of default-risky securities ultimately depends on each of the factors affecting default risk; thus, having identified a particular risk factor, one can then determine how the price of the security will be affected by changes in each of the factors. Second, identifying the factors and the effect on security prices aid in measuring and managing the default risk to which a financial institutions portfolio may be exposed. Third, the factors identified in this study provide a potential set of variables useful for predicting the TED spread.[3] In addition, a statistical model for predicting the future credit risk is developed. The model is then compared with the more commonly used models.

There have been several studies on credit (default) risk at both the theoretical and empirical levels. The studies at the theoretical level (Merton 1974; Sarig and Warga 1989) are general and are equally applicable to all forms of debt instruments that are subject to change in the credit quality of contracting parties, or an outright default by one of the contracting parties. However, on the empirical front, most of the studies have concentrated on domestic debt instrument, such as commercial paper and corporate bond issues (Fama 1984a, 1984b, 1986; Van Horne 1979; Ma, Ramesh and Peterson 1989; Clinebell, Kahl, and Stevens 1996; Duffee 1996a, 1996b; Jónsson and Fridson 1996); on municipal bonds (McInish 1980); U.S. corporate bond and Eurobond issues (Finnerty and Nunn 1985a, 1985b) and on sovereign credit risk (Feder and Ross 1982; Cantor and Parker 1996) among others. Empirical studies on credit risk in the

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[2]The credit risk spread investigated in this paper is the minimum amount of compensation required on deposits or loans to counterparties because it is the minimum mark-up on the London Interbank Offer Rates (LIBOR)—the rates that the top tier banks lend to each other. Every other bank not in that tier pays more depending on its specific credit rating or country of domicile. For a discussion of tiering in the Eurodollar market see Stigmum (1990: 890).

[3]The TED spread is the difference between the Treasury bills futures and the Eurodollar futures with the same period to maturity.

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Eurodollar market are almost non-existent.[4] This essay attempts to fill some of the vacuum in this area of the literature.

The studies mentioned above can be broadly classified into three classes. The first class includes those using solely the information in the U.S. Treasury yield curve (Duffee 1996a, 1996b; Fama 1984a, 1984b, 1986). The second includes those using solely the historical information in the time series of the observed credit risk spread (Clinebell, Kahl, and Stevens 1996). The third includes those using the other information, such as the specific characteristics of the issue and the debt issuer (Ma, Ramesh and Peterson 1989), age of issued bond (Jónsson and Fridson 1996), among others. Studies using only the yield curve information have been the most prevalent in the literature, and are examined along with the other two approaches in the next section. In the meantime, I intend to establish the link between this study and the existing literature.

As indicated above, there have been several studies on credit risk, particularly, on domestic default-risky debt instruments such as corporate bond issues and commercial papers. As in most financial and economic time series, the variable frequently offered to explain, predict, and price the different forms of risk in these debt instruments is the term structure of the interest rate in the U.S. Treasury securities market.[5] The argument commonly advanced in support of this view is that the U.S. Treasury yield curve observed on any given date contains informa-

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[4]Unlike in the domestic bond market, where actual defaults on bonds issued have been reported, there have been no reported cases of bank defaults on deposits placed by other banks in the Eurodollar market. Nonetheless, default is still a possibility recognized by all participants in the Eurodollar market when placing deposits in other banks.

[5]The yield curve, sometimes referred to as the term structure of interest rate, on any particular date contains information useful for explaining and predicting observable macroeconomic factors. For instance, it has been observed that the yield curve contains information for predicting the future movement of the following series: the short-term interest rate (Fama 1984a; Campbell and Shiller 1991), the growth rate of the economy (Estrella and Hardouvellis 1991; Harvey 1991, 1993; Haubrich and Dumbrosky 1996) or recession (Estrella and Mishkin 1996; Dueker 1997), future changes in inflation rates (Fama 1975; Mishkin 1990), the term premium in default free Treasury securities (Alles 1995; Taylor 1992; Margaritis 1994; Fama 1984a, 1984b, 1986) and the default risk premium of high-yield corporate debts (Helwedge and Kleiman 1997; Duffee 1996a, 1996b). In recent times, however, the predictive ability of the information in the yield curve has been the the subject of active debate in the literature.

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tion for all assets, real and financial, i.e., that the yield curve has predictive power for all asset prices and returns.[6] This assertion is even more tenable for interest-rate-dependent securities such as bonds, certificates of deposit, mortgages, forward rate agreements, futures, options, swaps and other forms of derivative securities, especially as these interest-rate-dependent securities are priced—using the arbitrage condition—off the default-free U.S. Treasury securities of comparable maturity. Despite this, the pertinent question arising from these studies using only the yield curve information is that can the yield curve information alone be used to explain and predict the behaviour of all financial and economic-time series?

Apart from the studies using only the yield curve information are those using solely the pure-time series of credit risk spread to explain and predict the credit risk. This strand of the literature ignores the Granger-causal effect of the other financial and economic variables. It is, however, rationalized by the argument that there are some patterns left in the time series of the data, and that the patterns can be exploited to predict future observations. For these studies, the same question arising in the studies using the yield curve information is asked: can the time series, and thus the history, of credit risk spread alone be used to explain and forecast credit risk spread? Or is there more pertinent information that has been neglected by these studies?

There are potential problems associated with using just one series. The problem with using only the term structure explanation, or just the pure-time series of credit risk spread—as in the extant literature—is that the effects of other relevant financial and economic variables may have been ignored. As a result, a correct attribution may not have been made for the effect of each of these explanatory variables on credit risk, and errors may therefore arise in assessing and predicting future levels of credit risk spread. Furthermore, as the prices of default-risky

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[6]The current price of any asset is the discounted value of all future stream-of-cash flows. The current and future levels of interest rates, therefore, is of concern for securities because of the discounting factor used in discounting the future stream-of-benefits/costs. Also, the research by Litterman and Scheinkman (1991), and Knez, Litterman and Scheinkman (1994) indicates that there are three unobservable common factors in the Treasury yield curve (the level, the slope, and the curvature) that explain over 96 percent of the returns, and thus prices, of debt instruments in the money market. It is, therefore, tempting to restrict attention to just the yield curve information when predicting the movement of asset prices, its returns, or for that matter the state of the financial market.

securities depend on the risk assessment, errors in assessing the size of credit risk may also translate into costly pricing errors on the default-risky securities. Similarly, and from a statistical perspective, an invalid inference may be drawn from the statistical models that ignores the other relevant information when modeling the observed credit risk premium.

Thus, the principal purpose of this essay is to investigate the empirical determinants of credit risk spread in the Eurodollar market. In particular, I focus on assessing the adequacy of the information content in the U.S. Treasury yield curve for modeling and predicting the level of credit risk spread observed in the Eurodollar market. For the purpose of further clarification, I am interested in the following: testing whether the U.S. Treasury yield curve contains information for modeling and predicting the credit risk spread in the Eurodollar market; identifying the specific elements of the information set in the yield curve that may be useful for modeling credit risk spread; and in testing whether or not the information in the U.S. Treasury yield curve provides a better out-of-sample forecast of credit risk spread than other models using only the pure-time series of credit risk spread, other financial market information, or a combination of all the information.

In order to asses the adequacy of the information in the U.S. Treasury yield curve, I augment the yield-curve information with the past-time series of credit risk spread, and with other financial and economic time series. I then test to see if the variables augmenting the yield curve information have any additional explanatory power for the observed credit risk spread. I controlled for the effect of these other variables because of the possible bias,[7] prediction errors, and inferential problems that may arise when they are ignored.

This essay adopts the (G)ARCH-in-Mean (GARCH-M) modeling methodology of Engle, Lilien, and Robins (1987). The authors used this model in the context of modeling the term premium (or excess return) in the U.S. Treasury securities market. Other studies modeling the term premium in the Treasury securities market have also used this modeling method, including Margaritis (1994) for New Zealand and Taylor (1992) for the United Kingdom. This

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[7]Statistically, bias implies that the parameter estimates or weights attached to each regressor may be over- or under-estimated depending on the nature of the correlation between the included and the excluded variables. One could, therefore, be making a wrong judgment as to the importance or effect of the included variables.

essay differs from the previous studies in that the method is applied to a different set of data–the credit risk spread in the Eurodollar market. In addition, I augment the information in the Treasury yield curve with other financial and economic time series.

The plan of this essay is as follows. Section 2.2 presents a brief survey of the literature on credit risk modeling in the money market. Section 2.3 examines the empirical model underlying the analysis. Section 2.4 discusses the estimation technique. Section 2.5 discusses the data analyzed. Sections 2.6 and 2.7 present the empirical results and the summary, respectively.

2.2 The Default Risk Literature: An Overview

In this section, I present a brief survey of the literature on the econometric analysis of default risk in the money market.[8] The survey covers studies on default risk premium on a broad spectrum of short-term debt instruments in both the domestic and international markets. It should also be noted at this juncture that most of the studies reviewed here only mentioned or listed the variables used as explanatory variables, without making claim as to their expected impact, or attempting to provide a justification for why they are necessary for modeling or predicting the variable of interest, i.e., the default risk spread. As a result, most of the discussion that follows in this section concentrates mainly on the type of data used, the method of analysis, the results, and the possible implications of the results for default risk modeling. In contrast to the studies reviewed here, however, in Section 2.3, I attempt to provide some rationale as to why the variables in my model may be necessary for modeling and predicting credit risk spread. The section is organized into three parts. In Section 2.2.1, I discuss the studies using solely the information in the Treasury yield curve. In Section 2.2.2, I then discuss those studies using solely the past observations of the default risk itself; and in the final part, Section 2.2.3, I discuss those studies that use specific characteristic of the particular issue and the issuer along with other information.

2.2.1 Term Structure Explanations

Here, I present studies explaining the default risk using solely the information in the term structure of the Treasury securities. The section is further divided into two parts; the first dealing with studies using specific information in the term structure, and the second with studies using the various factors extracted from the term structure. The theoretical construct underlying the empirical analysis in this section is based on the argument that the current term structure of interest rates sufficiently reflects the current state and general outlook of the

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[8]An alternative approach that is not pursued in this study is the option-pricing theoretic method pioneered by Merton (1974). For an extension and an application of this method see, for example, Duffee (1996b) and Duffie and Huang (1995).

economy. Furthermore, it is also maintained that the term structure effectively summarizes the recent developments in the financial market, and perhaps, its future behaviour as well.[9] Since the term structure contains this crucial information, it is therefore used frequently as a basis for modeling and predicting the default probability, and hence the default risk, in the default-risky assets. In addition, it is also used extensively in determining the value of assets.

2.2.1.1 Specific Term Structure Information

The studies reviewed in this part consider specific information in the treasury yield curve as the only predictor of default risk. For example, Fama (1984a, 1984b, 1986), using the forward rate premium as a proxy for the state of the U.S. economy, investigated the term premium and the default risk in the U.S. money market instruments. In the 1984 studies, he examined the relationship between the term premium in one- to six-Month U.S. Treasury securities and the implied forward risk premium[10] using the least squares regression method. The period covered in the analysis extends from February 1959 through July 1982. For the sample examined, he reported that this information in the Treasury yield curve had a significant predictive power for the term premium in the U.S. Treasury securities.

Using a similar analytical method in the 1986 study, Fama extends the analysis to include default-risky debt instruments in the U.S. money market. Specifically, in addition to the term

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[9]See, for example, the general equilibrium asset-pricing model developed by Cox, Ingersoll, and Ross (1985), and Jacobs and Jones (1985) for a theoretical exposition of the relationship between the productive capacity of the economy, consumer tastes and preferences, and the financial market equilibrium condition.

Also, Dialynas and Edington (1992) provide some theoretical and graphical analysis of the relationship between the corporate bond yield spread, the U.S. term structure, gross domestic product and industrial production. Other studies such as Estrella and Hardouvellis (1991), Harvey (1991, 1993), Estrella and Mishkin (1996), Haubrich and Dumbrosky (1996), and Dueker (1997) all formally test for the predictive power of the yield curve for forecasting the growth rate of the U.S. economy. They all reported that the slope of the yield curve has predictive power for the growth rate of the economy. Therefore, there is a close association between the yield curve observed and the future performance of the economy.

[10]The term premium is the difference between the yield of two securities with the same attributes except for the term to maturity. On the other hand, the forward premium is calculated as the difference between the forward rate implicit in the current spot rates and the short-term spot rate.

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premium in U.S. Treasury securities, he also analyzed the default risk premium on commercial papers issued by private corporations, bankers acceptances, and certificates of deposit. The result of the analysis indicates that the forward premium in the U.S. Treasury yield curve is statistically significant, and it is positively related to the risk premiums. Thus, when the forward rate rises, the term premium in the treasury instrument and the default risk premium in the default-risky securities also increase as well. Furthermore, the results also suggest that during the period examined, January 1967 through October 1984, the forward rate premium alone accounted for as much as seventy-seven per cent of the variation in the observed default risk premium. This result implies that the observed premium closely trends the U.S. business cycle which is captured by the forward premium: the forward premium, the term premium, and the default risk premium were all high in the recession periods of 1973-75 and 1979-83, and relatively low in the boom periods of 1975-78 and 1983-84. The results of this study therefore suggest that this element of the treasury yield curve contains information for predicting the time-varying term premium in the U.S. Treasury securities market as well as for predicting the default risk premium in the default-risky debt issues in the domestic market; i.e., that the variations in the default risk premium and the term premium on debt instruments is well explained by the forward rate derived from the U.S. Treasury yield curve. The modeling method used by Fama (1984a, 1984b, 1986) was also adopted by Alles (1995) while investigating the risk premium in the Australian money market. The result reported for the Australian data is also similar to that of Fama.

Similarly, Duffee (1996a, 1996b) investigated the default risk premium in corporate bond issues in the U.S. market using specific elements of the information contained in the U.S. Treasury yield curve. Specifically, he examined the relationship between the U.S. Treasury yield curve (the slope of the yield curve and the changes in the level of the interest rate) and the yield spreads of investment-grade corporate bonds[11] over U.S. Treasury securities of comparable maturity (the default risk premium). In the analysis, Duffee considered only the long-term and the medium-term investment-grade bonds in the Lehman Brothers Bond Index; and

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[11]Investment grade bonds are corporate bonds with a credit rating of Baa or better, from the Moody's investors service or the Standard and Poors' service.

the period examined extends from January 1973 through May 1995. The result indicates–for all maturity ranges and risk classes–a negative relationship between the changes in the level of the interest rate and the default risk. Also, except for the long-term A and Baa, and the medium-term A and Baa-rated bonds, the relationship was statistically insignificant. As for the link between the default risk and the slope of the U.S. Treasury yield curve, a negative and a statistically insignificant relationship was reported; the only exception being the relationship between the default premium on the long-term Baa rated bonds.

The result of Duffee (1996a, 1996b) studies shows that the changes in the level of the interest rate, or the slope of the yield curve, lack significant predictive power for the default risk premium at all maturity ranges and all risk classes. Consequently, only a weak evidence exists between either the slope of the Treasury yield curve or the changes in the level of the interest rate and the default risk of corporate bonds. These results contrast sharply with those of Fama (1984a, 1984b, 1986) who, among others, found that there exists a significant and positive relationship between the default risk on default-risky money market instruments and the U.S. Treasury yield curve information: Duffee's results, in essence, indicate that the Treasury yield curve contains only a very limited amount of information for modeling default risk; and besides, the relationships are negative. As such, other information may be necessary to augment the information in the U.S. Treasury yield curve in order to avoid the possible bias problem that may arise. The result may be suffering from the problem of bias in parameter estimates because of omitted factors.[12]

In summary, these studies show that there is no one unique element of yield curve information that could be used to model risk premium. Also, there is no consensus on the direction of impact, the size of impact, or even whether or not the information in the yield curve is sta-

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[12]Duffee mentioned that there were certain bonds issued with option-like features that were included in the Lehman Brothers index of the high-yield corporate bonds. These features have further implications for econometric modeling of the default risk. The exercise, or otherwise, of these option rights may, for example, depend directly on the current state as well as the future prospects of the economy. The measure of the state of the economy, for example, gross domestic product, is highly correlated with the change in the interest rate level, or the slope of the yield curve. So, given the correlation between the variables, a model that include term structure information only and leaves out the gross domestic variable may, therefore, be biased, and also have the wrong signs.

tistically significant. Moreover, the studies using only the term structure variables are limited in that the effect of the term-structure variables might be an under- or over-estimate of the true effect. This is the case, because these empirical models failed to control for the direct influence of other financial market and economic variables affecting the default risk premium. Given the preceding, the use as well as the significance of the yield curve information for modeling and predicting the default risk therefore remain an empirical issue.

2.2.1.2 The Multi-Factor Models

In this part, I review some of the studies that assume that money market instruments are influenced by a complex array of factors. These factors, observable and unobservable, are assumed to influence the various types of risk that a debt instrument might be subject to. Through the influence of these risks, the various factors also affect the value of the debt instruments. The studies described in this part usually follows a two step procedure to identify the various factors. In the first step, they fit a model of the bond prices using a set of factors.[13] They then compare the theoretical prices computed to the observed set of prices to determine the pricing errors. In the second step, the squared pricing error is then analyzed using for example factor analysis to determine the unobserved factors affecting the risk components in the debt instruments. Alternatively, if a large number of observable factors is used in computing the theoretical prices and the pricing errors, then principal components analysis is used to extract the principal components affecting the various types of risk to which the debt instrument might be exposed. It should be explicitly noted that this modeling approach assumes that all types of risk (default risk, and term premium, among others) are all affected by the same fundamental factors in the economy. Below, I examine specific studies using this modeling method.

In order to investigate the factors influencing the risk and return structure in the Canadian corporate bond market between January 1986 and May 1992, Kahn and Gulrajani (1993)

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[13]In this case, the factors used in computing the prices are not directly observable. They are in essence latent variables.

followed the two-step process described above. In the first step, they fitted a model of the Canadian bond prices using nine term structure factors and two yield spreads. The nine term structure factors are the pure discount Government of Canada bond prices with one, two, three, four, five, seven, ten, twenty, and thirty years to maturity. The nine vertexes of the term structure were used to capture the effect of the general trend of the financial market on bond prices and on its risk evaluation. The implicit assumption underlying the use of the vertexes is that each of the vertexes incorporates a different type of information. On the other hand, the yield spreads are the Canada–U.S. Treasury yield spreads for three-year and ten-year spot rates. The spreads were used to account for the high degree of correlation between the U.S. and Canadian bond markets, and to account for non-market forces affecting bond prices and their default risk.[14] The pricing error of the fitted model has a mean value of zero. The errors were also uncorrelated with each other, with coupon payments, with time to maturity, with yield spreads, or with term structure factors. In the second step, to explain the variances of the pricing errors, and hence the composite risks in bonds, they also employed the same set of factors used in modeling bond prices. A further analysis of the explanatory variables produced a variance-covariance term for the factors, which is composed of four blocks: the non-diagonal covariance matrix of the term structure factors, the non-diagonal matrix of the yield spread factors, and the two blocks of covariance between the term structure terms and the yield spread. In order to reduce the dimension of the problem, they used the principal component analysis to extract the principal components of the term-structure factors. They found that there were three principal components that adequately describe the term structure factors. The first factor is the non-parallel shift in the Government of Canada Treasury yield curve. This factor alone accounted for 89.2 percent of the variations in the term-structure factors. The second factor in the term structure is twist (the slope), which accounts for an additional 7.8 percent; and the third factor is the butterfly (or the curvature of the yield curve), that accounts for 2.2 percent. These three factors account for 99.2 percent of the variations in the term structure factors. This

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[14]There is a high degree of integration between the Canadian and U.S. financial markets. Thus, any event affecting the U.S. filters into the Canadian financial market. The effect of the U.S. bond market on the Canadian bond market may, therefore, be captured by these yield spreads. However, the yield spread is more likely to be due to the appreciation or depreciation of the dollar in the foreign exchange market.

result, therefore, suggests that these three characteristics of the term structure of interest rates fully summarize all the information in the variables used in modeling the price and risk of corporate bonds in Canada.

Kahn (1995) conducted a similar analysis for the U.S. corporate bond market. The period covered in their study extends from January 1980 through October 1986. However, unlike the analysis of the Canadian market, the yield spread, which was used as a factor in this instant, is the spread between the corporate bond and U.S. Treasury security with the same maturity period. The spread was used to capture the non-term structure factors such as the credit quality of bond issuers of a particular risk classification, for example, the triple-A-rated corporations. In addition, coupon payments on bonds with option-like features were adjusted to reflect the intrinsic properties of each issue. As in the Canadian bond market, in the second stage, Kahn (1995) used the principal component analysis to extract the principal components affecting the various types of risk, and thus the prices, of corporate bonds in the U.S. market. The result indicates that there were two principal components in the non-callable U.S. Treasury securities of various maturities that were taken into consideration. The first principal component–the non-parallel shift–accounts for 95.4 percent of the variations in the U.S. Treasury yield curve; while the second component– the twist (slope)–accounts for 4.1 percent. So, these two principal components jointly account for 99.5 percent of the variations in the U.S. Treasury yield curve. In all, the two principal components account for 87 percent of the variations in the risk observed in the U.S. corporate bond market. Also reported is the full factor model, with the ten factors. This model could not explain more than 88 percent of the variations in the total risk.

Murphy, Won and Gulrajani (1995) followed the method of Kahn and Gulrajani (1993) and Kahn (1995) in their analysis of the international bond market. In their study, they used the investment-grade corporate bond market in each of the G-7 countries. While they mentioned the role that the exchange rate plays in pricing bonds and evaluating risks in this international setting, they failed to include it in their empirical analysis. In other words, they used only the information in the national treasury yield curve of each country. For each country, they reported that three principal components of the treasury yield curve influence risk and prices

in each of the countries considered.

In conclusion, it is important to mention that the method and the result of the studies mentioned in this section are consistent with those of Litterman and Scheinkman (1991) and Knez, Litterman and Scheinkman (1994). As with the Litterman *et al.* (1991) and Knez *et al.* (1994) studies, they also demonstrate the fundamental importance of the treasury yield curve information for modeling the price of bonds and the various types of risks to which it might be exposed. However, in this framework, the specific factors in the treasury yield curve remain a mystery. As a result, this method of analysis will not be pursued in this study.[15]

2.2.2 Time Series Analysis

This section considers studies using the pure-time series of the default risk to model and predict the default risk itself. The justification often offered for this type of analysis is that there are patterns in the past default risk data that can be extrapolated into the future. The extrapolations then provide a basis for predicting the future level of default risk.

For instance, Clinebell, Kahl, and Stevens (1996) examined the time series of the default risk premium on high-yield long-term corporate bond issues. The time series of return on corporate bond issues, and the long-term U.S. Treasury bond issues were obtained from Ibbotson and Associates' *Stocks, Bonds, Bills and Inflation: 1991 Year Book*. The period they examined extends from January 1926 to December 1990. They maintained that the default risk premium can be modeled and predicted by using just its own previous values; hence, they estimated an autoregressive model of order one—an AR(1) model. The parameter estimate on the once-lagged default risk premium is negative, and statistically significant; and in addition, the parameter's absolute value is also less than unity. This, therefore, suggests that the default risk premium on corporate bond issues behaves as a mean-reverting process; i.e., that default risk converges to its mean value after following a cyclic pattern. Because of the cyclic nature of the convergence, it can further be inferred that investors over- or under-react each time they fail

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[15]These studies have been included to demonstrate other ways that various researchers have approach the problem, and also to highlight the importance of the treasury yield curve.

to predict the default risk correctly. With the AR(1) model they examined, they could only account for 7 percent of the variation in the observed-risk premium. Given the low explanatory power of their model, it is likely that other relevant explanatory variables (for example, the information in the U.S. Treasury yield curve, the business cycle indicators, the volatility of the default risk premium) could be used to improve the models fit as well as its predictive ability. In addition, the impact of the previous default risk may have been over- or under- estimated and may lead to errors in forecasting. Consequently, if the predicted estimates are used in valuing a default-risky debt instruments, the price is also likely to be in error.

2.2.3 Specific-Issue Features and Other Information

In this section, I examine studies using the basic characteristics of each bond issue and the issuer. I also present a sample of studies using other information such as the state of the economy as represented by the gross domestic product or its growth rate, and the age of the bond issue, among others. The specific characteristics of the issuer, such as its credit rating, indicate the ability of the borrower to pay the principal, coupon or both, as promised. If the credit rating is lower, then the default risk measure on the bond issued is also going to widen. The same effect can be observed of firms borrowing during a period of economic downturn. The widening yield spread is expected because the depressed state of the economy—more often than not—adversely affects earnings and hence the profitability of firms. When company profits are adversely affected over a prolonged period of time, the borrower's ability to repay loans as promised may also suffer. Some of the studies along these lines are presented below.

Finnerty and Nunn (1985a, 1985b) investigated the yield spread on the corporate bond issued in the U.S. market and the three-month U.S. Treasury securities, and also the spread on the corporate bonds issued by offshore divisions of U.S. multinational corporations in the Eurobond market and the three-month U.S. Treasury securities. Specifically, they inquired into the following issues: first, they wanted to test whether the yield spreads on the Eurobond issues are statistically different from those observed in the U.S. domestic corporate bond market; and second, they sought to uncover the factors influencing the observed yield spreads in the

two markets. Regarding the first issue, they argued that because both bond issues are equally risky and are identical in all respects, the observed yield spreads should be the same if the financial market is integrated (non-segmented). Their empirical analysis indicates that the yield spreads on the domestic bonds is consistently and significantly higher than those observed in the Eurobond market. This result thus suggests that the Eurobond market may constitute a cheaper source of funds for fund managers, while the domestic corporate bond market provides a more profitable investment opportunity for the same risk. This apparent differences in the yield spread would conceivably not exist if the capital markets were integrated.

With regard to the factors influencing the yield spreads in both markets, Finnerty and Nunn (1985a, 1985b) used the following variables to model and predict the observed yield spreads. The first set of variables involves the intrinsic characteristics of the bond issuer and of the issue itself. The characteristics, among others, include the size of the bond issued, the coupon on each issue, and the credit rating of the bond issuer. The economic variable used to augment the preceding information is the growth rate of the gross domestic product (GDP). The GDP growth rate is used to capture the effect of the business cycle on default risk. The size of the issue and the coupon rates, are supposed to account for the effect of the marketability, and hence the liquidity of the debt instruments. They reported that the coupon rate on each bond, the size of each issue, and the GDP growth rate are all pertinent to the spreads in both markets; and in addition, the credit rating of the bond issuer matters for only the Aa- and the A-rated bond issuers.[16] The effects of each of the variables on the default risk in the respective markets also differ significantly from each other.

In order to determine the appropriate functional specification for the default risk premium, Lamy and Thompson (1988) examined the default risk premium on a cross-section of investment grade bonds in the U.S. corporate bond primary issue market. The industrial bonds investigated are those rated Baa or higher by Moody's investors service, and were selected from

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[16]The data set used in the analysis consist of 500 newly issued U.S. dollar Eurobonds over the period 1972-1982 (World bank data). Of the 500 new issues, 173 were successfully matched with the domestic issues contained in Moody's report. These issues were matched on the basis of date of issue, credit rating of issuer, call provision, the underwriter, the issuer and other pertinent information in the data.

issues made between January 1970 and June 1983. They modeled the risk premium as a linear function of the following: the interest rate level, interest rate volatility, characteristics of the bond issuer, and the specific characteristics of the issue itself.[17] They reported a negative and insignificant relationship between the interest rate level and the default risk premium. They also reported a significant relationship between the default risk premium and bond characteristics, and between the default risk and the interest rate volatility. In addition, they reported that the relative risk measure specification provided a better fit to the data than the absolute risk measure.[18]

Similarly, in order to investigate whether the bankruptcy of a major bond issuer (the LTV corporation) on July 18, 1986 had any significant and permanent effect on the default risk premium observed in the market, Ma, Rao and Peterson (1989) also examined the high-yield corporate bond market. The risk class of bond examined include those with Moody's ratings of Baa to Bbb, and the period covered by the study extends from January 1980 to May 1987. The explanatory variables for the default risk premium are: the characteristics of the specific issue, such as the size of the issue, the convertibility and callability features; the purpose for which the bond was issued, i.e., for leverage buy-out or business expansion; the characteristics of the issuer represented by the credit rating; and the business condition measured by the yield spread between Moody's 30-year triple-A bond and the 30-year U.S. Treasury bond series. From their analysis of the data, Ma, Rao and Peterson found that the default of a major high-yield corporate bond issuer increased the perception of risk and hence the premium on new issues. However, the effect is transitory, lasting only about six months.

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[17]The interest rate level was represented by the twenty-year constant-maturity U.S. Treasury bond index on the date the industrial bond was issued. The volatility was represented by the absolute deviation of the twenty-year rate on the date the corporate bond was issued and the rate on the ten previous days. The characteristics of the bond issuer were represented by the firm's credit rating from Moody's investors service. The characteristics of the bond taken into consideration include the amount issued, sinking-fund provision and callability features.

[18]The absolute risk measure of the default risk premium considers the absolute value of the difference between the yield on the corporate bond and the yield on U.S. Treasury securities of comparable maturity. On the other hand, the relative risk measure expresses the absolute risk measure as a fraction of the level of the yield on U.S. Treasury securities of comparable maturity.

In order to test if a bank's assessment of default probability of sovereign borrowers in the Eurocredit market is reflected in the price of a sovereign loan, Feder and Ross (1982) examined the credit risk spread on a U.S.-dollar-denominated loan to 34 sovereign countries. The period of their analysis covers June-July 1979. To test the above assertion, the data on default risk probability, as perceived by bankers, were based on the weighted average of the response of ninety banks to the *Institutional Investors* June-July 1979 survey.[19] Additional explanatory variables used in the analysis include, the time to maturity on the loan, and the grace period on the loan. The results indicate that lenders expect losses on loans if there is a rescheduling of the loan or an outright default, and therefore their default probability assessment is reflected in the price of a medium-term sovereign loan. While this study gives some insight into whether the risk of default is reflected in issue prices, it is, however, silent on how the bankers arrived at their default probability assessment of each country. Other similar studies, such as Cantor and Parker (1996), indicate the economic and political factors that are taken into account when assessing sovereign credit risk.

In the context of comparing three alternative models of default risk in high-yield corporate bond issues, Helwedge and Kleinman (1997) used, as explanatory variables, the expected default rate on bond issues calculated by the rating agencies, the age of the bond issue, and the gross domestic product. The base model uses only the expected default rate. The alternatives to the base model are: the model using only the age of the issued bond, and the model augmenting the expected default rate with macroeconomic information—the gross domestic product.[20] They found a significant relationship between the explanatory variables in each of

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[19]The banks were asked to score a number of countries on their perceived default probability on a scale of one to ten. Default in this instant included the failure to make promised payments or to reschedule loans granted.

[20]The age factor is represented by a three-period lag of the total amount of the bond issued. The aging factor theory suggests that high-risk bond issuers are less likely to default in the first two years of bond issue, and are most likely to default in the third year or thereafter; the reason being that in the first two years they are more liquid and therefore can meet all outstanding obligations. Moreover, high-yield bond issuers are less likely to issue bonds when they are most weak, or when the economy is in the doldrums. They are more likely to issue when the economy is in a state of prosperity. Given the state of the business cycle, the economy is more likely to be weaker in about three years or thereafter. As a result, default is more likely to occur then. Empirical support for this can be found in Jónsson and Fridson (1996), who investigated the default rate on high-risk bonds and the age factor.

the models. On the basis of the adjusted R-square, they found that the aging model performed the best. It accounts for as much as 81 percent of the variations in the observed default risk premium. Following is the model that augments the expected default rate with the macroeconomic information (75 percent); and last is the model using only the expected default rate computed by the bond rating agency (47 percent).

In brief, this section shows that there are other factors besides the term structure of interest rate, and the time series of default risk, that could help explain the observed default risk premium in the money market. As in much of the literature, the models surveyed in this section have been silent on the role played by the monetary policy of the Federal Reserve Bank, the stock market, or the foreign exchange market in modeling default risk. These factors are by far some of the most important factors affecting the health of businesses, and thus their ability to meet financial commitments. These and other issues are taken up in the subsequent part of this study.

2.2.4 Summary, Conclusion and the Direction of Research

Above, I have presented a brief survey of the literature on credit risk modeling in the money market. However, it is by no means exhaustive. As can be observed from this survey, there are many approaches to modeling default risk; and similarly, there are many different factors that have been used to explain default risk in the money market. Also, there appears to be no one unique way or generally accepted method of modeling default risk in this literature. Despite this, what appears to be the dominant paradigm for modeling credit risk in the existing literature is to use only the information in the U.S. Treasury yield curve. For the reasons mentioned in Section 2.1, this approach to modeling credit risk in any type of security in the

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As mentioned, the GDP factor reflects the effect of the state of the business cycle on the observed default risk. In a depression, investors prefers to hold the much-safer treasury securities; as such, in a depression, investors would have to be offered a high premium to induce them to hold a corporate bond. In a period of economic boom, there is less prospect of a default, and so the observed default risk is smaller. Bond ratings have a similar effect. A bond issuer with a low credit rating has to offer a higher default premium to induce investors to hold its bond. The premium offered on the high-credit-rated bonds is lower.

money market may not be appropriate because it neglects potential information that may be contributed by other factors. Also, all the models surveyed above have been silent on the role that the stock market, the real estate market, or the foreign exchange market volatility could play in measuring default risk in the money market. The same is equally true of the role that monetary and fiscal policies could play. To this end, I investigate whether the events in these other asset markets have implications for the default risk premium in the money market, in particular, the Eurodollar market.

Furthermore, most of the existing studies on default risk have concentrated on domestic securities such as commercial papers, domestic certificates of deposit, bankers acceptances, municipal and corporate bond issues. Moreover, empirical studies on default risks in the Euromarket have largely centered on sovereign risk. Empirical studies on the credit risk in Eurodollar deposits is almost non-existent. This study attempts to contribute to this area of the literature by extending the empirical analysis into this market, and by investigating the Granger-causal relationships from other assets markets into the Eurodollar market. This study is particularly relevant, as the world financial market is becoming more fully integrated, and more Eurodollar debt instruments are being issued. Since the default by the issuers of these instrument is an ever-present possibility, it is important to understand what factors govern the dynamic behaviour of this risk in the Eurodollar market. As mentioned at the beginning of this section, in order to appropriately price a debt instrument in this market one needs the credit risk evaluation of the issuer as input. Thus, an incorrect assessment will also eventually lead to an incorrect price being placed on a debt instrument. In essence, it is important to understand the dynamics of credit risk in order to minimize the pricing errors on securities.

To conclude this section, I reiterate that this essay seeks to explain the wide fluctuations in the daily observations of credit risk in the Eurodollar market over the period extending from June 1, 1973, through August 19, 1996. The approach taken in this study is to combine the information in the current U.S. Treasury yield curve with that of past observations of the credit risk spread. In addition, this information is augmented with the historical information contained in the U.S. Treasury yield curve, stock market returns, foreign exchange rates, Federal Funds Rates, and the current predictions of the volatility of credit risk. In the next section, I

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present the empirical model as well as a brief exposition of its underlying rationale.

2.3 The Model

The previous section provides a brief survey of the default risk literature. In this section, I present the GARCH-in-Mean specification used in modeling the dynamic behaviour of credit risk in the Eurodollar market. Furthermore, the section motivates why each of the explanatory variables in the GARCH-in-Mean specification may be relevant for modeling the credit risk spread observed in this market. The section is organized into four parts as follows: Section 2.3.1 presents the GARCH-in-Mean model and the explanatory variables entering into the analysis; Section 2.3.2 further examines the relationship between the default probability and the elements in the U.S. Treasury yield curve information set; Section 2.3.3 presents the case for other factors such as the credit risk spread history, the contagious effect of volatility from one asset market to the others, the exchange rate and the monetary policy. The final part, Section 2.3.4 presents a brief summary of the section. The section is very brief concerning the empirical specification, but is more detailed as to why and how each of the elements in the information set affects the default risk spread in the Eurodollar market.

The theoretical model underlying the analysis of this section and the rest of the essay, is the principle of arbitrage pricing in the financial market. However, because this has been extensively covered in the existing literature, it is not separately examined in this study. Following next is the empirical specification for the daily observations of credit risk spread in the Eurodollar market between the sample period June 1, 1973 to August 19, 1996.

2.3.1 The GARCH-in-Mean Model

In order to test the various hypotheses of interest, I estimate the dynamic functional form examined below, using the daily sampled data for the period June 1, 1973, to August 19, 1996. The specification draws on the method of analysis used in Engle, Lilien, and Robins (1987); and it is written compactly as follows:[21]

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[21] Engle, Lilien, and Robins (1987) used the GARCH-in-Mean model to study the term premium, or the excess returns, in default-free U.S. Treasury securities. The variable that they explained is the term premium, while the explanatory variables were the yield spread between the three- and six-month treasury bill rates and the condi-

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$$CR_t = \alpha_0 + \sum_{i=1}^{10} \sum_{j=1}^{m} \alpha_{i,j} X_{i,t-j} + \gamma X_{11,t} + \epsilon_t \qquad \epsilon_t \sim N(0, \sigma_t^2) \tag{2.1}$$

$$\sigma_t^2 = \beta_0 + \beta_1 \epsilon_{t-1}^2 + \beta_2 \sigma_{t-1}^2 \tag{2.2}$$

The dependent variables in the equations above are defined as follows:

CR_t : is the relative credit risk spread observed in period t on 3-Month Eurodollar deposits. It is measured as the ratio of the credit risk spread on the Eurodollar deposits (the difference between the continuously compounded annualized equivalent yield on the 3-Month Eurodollar deposits and the continuously compounded annualized equivalent yield on 3-Month U.S. Treasury bills) to the level of the continuously compounded annualized equivalent yield on the 3-Month Treasury bills. This measure therefore expresses the credit risk spread as a proportion of the observed yield on the 3-Month Treasury security. More so, it allows for the possibility that the credit risk spread may directly be varying with the level of the observed yield on 3-Month Treasury securities. see, for example, Lamy and Thompson (1988).

σ_t^2 : is the time-varying conditional variance of the non-systematic component, the residual term (ϵ_t) in equation (2.1), of the relative credit risk spread.

The independent variables are are defined as follows:

$X_{1,t}$: is the relative credit risk spread observed in the previous period, CR_{t-1}.

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tional variance of the term premium. This method of analysis has also been used in studying the term premium in the treasury securities of other countries such as the United Kingdom (Taylor 1992), and New Zealand (Margaritis 1994).

$X_{2,t}$: is the level of the continuously compounded annualized equivalent yield on the 3-Month U.S. Treasury bill at time t.

$X_{3,t}$: is the change in the level of the continuously compounded annualized equivalent yield on the 3-Month U.S. Treasury bill at time t.

$X_{4,t}$: is the square of the change in the level of the continuously compounded annualized equivalent yield on the 3-Month U.S. Treasury bill at time t.

$X_{5,t}$: is the slope of the transformed U.S. Treasury yield curve measured at the short-term end; that is, the yield spread between 12- and 3-Month Treasury bills at time t.

$X_{6,t}$: is the slope of the transformed U.S. Treasury yield curve measured at the long-term end; that is, the yield spread between 60- and 12-Month Treasury bills at time t.

$X_{7,t}$: is square of the differences in the slope of the Treasury yield curve at the short- and the long-term end of the market at time t.

$X_{8,t}$: is the square of changes in the logged level of NYSE composite common stock price index at time t.

$X_{9,t}$: is the square of changes in the logged level of trade- weighted foreign exchange rate index of U.S. dollar vis-à-vis the G-10 countries at time t.

$X_{10,t}$: is the change in the the level of the continuously compounded annualized equivalent yield on the 7-day Federal Funds at time t.

$X_{11,t}$: is σ_t, the conditional variance of relative credit risk spread in period t.

ϵ_{t-1}^2 : is the square of the once-lagged prediction error or innovations.

σ_{t-1}^2 : is the once lagged predicted conditional variance.

Equation (2.1) above describes the dynamic behaviour of the relative credit risk spread. It is composed of two parts: the systematic component and the non-systematic component. The systematic component describes the conditional mean of the relative credit risk spread in period t given the information set Ω_t.[22] This equation states that the relative credit risk spread predicted for period t is a weighted average of the factors in the information set. The weight

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[22]The information set at time t is defined as: $\Omega_t = \{X_{1,t}, X_{2,t}, \cdots, X_{12,t}, \epsilon_{t-1}^2, \sigma_{t-1}^2\}$, and its elements are as defined above.

placed on each element of the information set is the parameter estimated for the respective variable. The second component, ϵ_t, is non-systematic. As such, it is unpredictable with respect to the elements in Ω_t. Furthermore, it has a conditional mean of zero, and a time-varying conditional variance represented by σ_t^2; and, as is indicated in equation (2.1), ϵ_t is also assumed to be normally distributed with a mean of zero, and a time-varying variance, σ_t^2.

Similarly, equation (2.2) describes the behaviour of the time-varying conditional variance, and hence, how it can be predicted. This equation further states that the conditional variance predicted for period t is also a weighted average of the squares of the past prediction error (ϵ_{t-1}^2), and the past predicted variance (σ_{t-1}^2). The weight given the respective variables is represented by the parameters, β_1 and β_2; and it is optimally determined by using, for example, the maximum likelihood method.

The model above addresses the issues raised in the previous sections. It contains the pure-time series of credit risk spread model as a special case. Similarly, models using only the information contained in the treasury yield curve can also be obtained as a special case. Furthermore, unlike the other models using only the current information in the yield curve, the model presented above explicitly allows for the historical information in each of the series including the treasury yield curve variables.

As can be observed from the specification above, the elements of the information in the U.S. Treasury yield curve are represented by the variables $X_{2,t}$ to $X_{7,t}$.[23] The squares of the change in the respective variables represent the rate of change of each variable, and it thus serves as a measure of the variability of that particular variable.[24] By including these volatility

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[23] I am aware of the literature using the characteristics of specific bonds issues, the attributes of the issuer of the debt instrument (that is, credit rating), or the purpose for which the money is being raised, to determine default risk (see, for example, Ma, Rao, and Peterson 1989; Lammy and Thompson 1988; Finnerty and Nunn 1985a, 1985b, among others) and those using solely the time series of default risk data (Clinebell, Kahl, and Steven 1996). The focus of this essay is to identify and assess the elements of the observable information in the U.S. Treasury yield curve that is useful for modeling the behaviour of the credit risk spread in the Eurodollar market. Moreover, the credit risk spread examined here is the basic assessment for banks in the top credit rating; and as banks are tiered, institutions with a lower credit rating pay more.

[24] The square of the changes in each of the variables is used to proxy the uncertainty or volatility in the respective

measures, it enables us to formally incorporate into the analysis the effect of the uncertainty existing in a particular asset market; it also enables us to ascertain the effect of the uncertainty in other assets market on the credit risk spread. In addition, each of the explanatory variables used in the above model is lagged m-periods; the only exception is the relative credit risk spread's volatility estimate. I have used the lagged explanatory variables for the following reasons. First, agents can only use the information available at time t to make forecasts for future periods, and some of the elements in the information set are only available with lags. Second, using the lag values implies that I am using predetermined values. As such, the likely problem of endogeneity that could arise in the regression is therefore avoided.

Furthermore, I introduce other financial market information that may affect the magnitude of credit risk spread in the Eurodollar market. The rationale for using each of these variables as well as their expected impact on the size of credit risk spread is discussed in the next two subsections.

2.3.2 The Default Probability and the U.S. Treasury Yield Curve

A major component of the credit risk premium is the probability of default of one of the contracting parties. If this probability is high, then the credit risk spread that is observed in the market will also be high. On the other hand, if the probability is low, then the observed credit risk spread will also be low. Thus, there exists a positive monotonic relationship between the default probability and the observed credit risk spread. Studies such as those by Duffee (1996a, 1996b) and Fama (1984a, 1984b, 1986), among others, using only the term structure informa-

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asset market. Underlying the use of the square of the variables is the implicit assumption that security prices and interest rate series in the financial market ($X_{i,t}$) follow a random walk process. That is, that

$$X_{i,t} = X_{i,t-1} + \eta_{i,t}; \qquad \eta_{i,t} \sim N(0, \sigma_{i,t}^2); \qquad E(\Delta X_{i,t}) = 0; \qquad \text{and} \qquad E(\Delta X_{i,t})^2 = \sigma_{i,t}^2$$

where $E(.)$ is the expectations operator.

Thus the expected change of the ith variable has a mean value of zero and the variance, $\sigma_{i,t}^2$. Empirical studies support interest rate series as behaving as random walk (Murphy 1990; Marsh and Rosenfeld 1983). The same is equally true for the stock prices (Cootner 1964; Malkiel 1996) and the foreign exchange rate (Alder and Lehmann 1983; Meese and Rogoff 1983, 1988).

tion implicitly assume that the default probability, and hence the default risk, is influenced solely by the information in the yield curve.

The rest of the section is organized into three parts: The first part, Section 2.3.2.1, presents the expected impact of the treasury yield curve on the credit risk spread; the second part, Section 2.3.2.3, identifies the specific elements of the yield curve information; and the third part, Section 2.3.2.2, identifies the mechanism through which U.S. Treasury securities information affects the credit risk spread in the Eurodollar market.

2.3.2.1 The Expected Impact of Yield Curve Variable

As in the previous studies, I assume that this probability is influenced by several variables. These variables include the current level of the short-term interest rate (the 3-month U.S. Treasury bill rate), the change in the short-term interest rate, and the rate of change in the short-term interest rate as measured by the squares of the first difference. In addition are the current expectations of the future short-term interest rate at time t, and the current expectation of the variability (volatility) of the future short-term interest rate. The reasons why these variables may be relevant for modeling the credit risk spread are explained below.[25]

First, a substantial proportion of the portfolio of a bank or other financial institutions is in the form of loans of varying maturities to governments, other banks and financial institutions, and commercial and industrial organizations.[26] Furthermore, in order to fund these loans,

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[25]There is quite a substantial amount of literature on the association between the U.S. interest rate, the Eurocurrency rate, the Eurobond rates, and the rates on Treasury bonds issued in other countries. For example, Tse and Booth (1996) tested for and found evidence of a common volatility and volatility spillover between the U.S. and the Eurodollar market; likewise, Kaen and Hachey (1983), Swanson (1988a, 1988b), Tse and Booth (1995), Fung and Isberg (1992), and Chan and Lee (1996) present evidence of Granger-causality between the U.S. Treasury yield and the Eurodollar deposit rates; Pigott (1993/1994) and Fujihara and Mougoué (1996) also present evidence of interdependence among domestic interest rates of the G-7 countries. The approach taken here is that the level of the interest rate in the U.S. market, which is the reference rate for all dealings in the Eurodollar deposits, affects a banks' fortune, and thus its ability to meet financial obligations.

[26]These entities may be located in the domestic market, the foreign market, or operate in both markets (for example the multinational corporations).

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these banks also accept deposits, usually on a short-term basis, from the same class of clients as well. As a consequence, the interest earnings on the financial asset side of the balance sheet, the interest cost of the financial liability side of the balance sheet, and hence the profitability of the net positions of these institutions depend to a large extent on the term structure of interest rates.[27] If, for example, the current interest rate level is high, or the interest rates rise, it may be argued that the potential earnings of banks and of the institutions that borrow from them may be lower. In addition, the cost of funding the loans is higher; and in the final analysis, the overall profitability of these institutions may be adversely affected.[28]

The reasons for the possible reduction in profitability, and hence the reduction in the ability to meet future commitments are as follows: First, a high or higher interest rate level could cause the institutions to suffer substantial capital losses on their pre-existing loan commitments, especially, when these loan contracts are fixed-rate commitments with an extended period to maturity. Second, a high or higher interest rate level may cause problems with repayment of the loan principal, accrued interest, or both. This is particularly so when the pre-existing loan contracts are of the variable interest type. When interest rates move against the borrower's original expectations (that is, their expectations of future interest rate levels and the state of the economy when entering into a loan covenant), they may have an incentive to default (see Simons 1989).

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[27] In general, it is expected that interest rates will have a significant impact on business profitability. However, given the existence of many types of financial contracts such as options, futures and forwards, swaps, caps, floors, collars and other forms of derivative contracts traded in the financial market, firms should be able to hedge these risk. In consequence, the interest rate level or its changes may not have a significant effect on profits. The studies by Flannery (1981, 1983) on the effect of the level of interest rate on bank profitability supports the view that banks have effectively hedged themselves against interest rate movement. This is especially true of the large U.S. bank holding companies. Consequently, their profitability is not necessarily affected by changes in interest rate level.

[28] Contrary to the report of Flannery (1981, 1983) studies, Flannery and James (1984), Booth and Officer (1985), Scott and Peterson (1986), Sweeney and Warga (1986), Yourougoua (1990), and Allen and Jaghani (1996) all found a significant negative relationship between interest rate level and the bank stock returns. In their analysis of individual banks, they found that some of the banks, including the largest money center banks, are not fully protected by the hedging policies instituted. This shows that though the effect of interest rate movement on profitability can be reduced, it is an empirical matter as to whether it can be eliminated altogether.

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Finally, a high or rising interest rate level may further accentuate the asymmetry of information, the adverse selection, and the moral hazard problem that banks face (see, for example, Mishkin 1997; and Stiglitz and Weiss 1981). As Stiglitz and Weiss (1981) argued, when interest rates are high, marginally profitable investment projects tend to be suspended by prudent project managers. On the other hand, risk-loving managers may still go ahead and execute the project, if they can find a financier. Because of the asymmetry of information as to the type of manager, a bank may ultimately run the risk of lending to the more risk-loving project managers when interest rates are high. Eventually, if the projects fail, the bank is left with a large number of non-performing loans that may have to be written-off it books, and thus affecting banks future profits and its equity capital.

As can be observed from the preceding analysis, the interest earnings, the rising interest cost of funding loans, and the higher amount of bad debt provisions that would have to be written-off against profits, all have a significant effect on the financial viability of a bank. As a result, the interest rate level, its changes, or its variability is expected to have a direct or indirect bearing on credit risk spread. Even though banks may be able to hedge some of the interest rate risk on the net position of its portfolio, or some of the credit risk of their customers, it is, however, not possible to sign *a priori* what the impact of interest rate change would be on profits and consequently on its own credit risk to others. A lot depends on the effectiveness of the hedging policies instituted by the bank. It therefore remains an empirical issue as to whether or not the interest rate level, its changes or variability have any predictive power for the credit risk observed in the market.

As for current expectations of the future short-term interest rate, they enter the equation because of the future level of interest rate impact on the profitability of future operations. The arguments supporting these assertions are the same as those used in the discussions on interest rate level in the preceding paragraphs.[29]

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[29]For example, previous studies by Booth and Officer (1985) show that contemporaneous (unanticipated) changes and predicted changes in the level of interest rates both have a significant and negative effect on a bank's stock performance. The previous paragraph concentrates on current earnings and profitability while the current paragraph concentrates on the future profitability of operations. I should also remark here that the future level

The other term structure factor influencing the probability of default is the future variability of interest rates. The more variable that interest rates are expected to be in the future, the more variable is the value of the financial institutions portfolio of fixed-income securities. Hence, their ability to meet their financial commitments may be adversely affected by the magnitude of the volatility of future interest rates. This effect is expected to have a positive impact on the probability of default because the profitability of future operations, and hence the future value of institutions, becomes more uncertain. In essence, when the level of the interest rate becomes more unpredictable, the higher the default probability expected by the contracting parties. The result of this is a higher level of credit risk spread. Again, as mentioned, this is an empirical issue since banks do hedge against interest rate variability as well; and depending on the effectiveness of the hedge, it may be difficult to say categorically what the impact on the credit risk spread would be.

2.3.2.2 The Elements in the Yield Curve Information Set

As I pointed out earlier in Section 2.1, the information in the yield curve has been used extensively to model and predict many financial and economic time series. In this study, I further investigate to see if the information in the U.S. Treasury yield curve is also useful for modeling and predicting the credit risk spread observed in the Eurodollar market. I also inquire into whether the information is adequate for predicting the credit risk spread; that is, does the information in the yield curve need to be complemented by other financial and macroeconomic time series to produce a more accurate estimate and forecast of credit risk spread?

In the pursuit of these objectives, I extract from the U.S. Treasury yield curve the following information set.[30] First, is the level of the yield curve, which is anchored to the shortest-term

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of interest rates may rise or fall depending on the expectations of the future growth rate of the economy, future inflation rate, or both. With regard to the inflation rate expectations, agents demand compensation for the loss in value of their money. I control separately for this effect by using the ease or tightness of credit (the federal funds rate) in the money market as a proxy for inflation expectations at the daily frequency. I discuss this further in the next section.

[30] The analysis here is similar in spirit to the Litterman and Scheinkman (1991), and the Knez, Litterman, and

maturity–the 3-month treasury bill rate. Anchoring the yield curve to the shortest-maturity instrument is appropriate because, under the expectations theory of the yield curve, the yield on longer-maturing instruments can be expressed as a weighted average of the current and expected future level of the yield on the shortest-maturity instrument.[31] This feature is used for the current short-term interest rate level influencing the default probability. In addition, the change in the continuously compounded annualized equivalent yield between successive periods, and the squares of this change are derived from the level of the yield on 3-month treasury bills. The former captures the effect of changes in yield, while the latter its variability or volatility.

The second feature derived from the U.S. Treasury yield curve is its slope. The slope serves as an indicator of the current expectations of future short-term interest rate.[32] These slopes are measured at two points on the yield curve: at the short-term end of the yield curve is the slope relating the 3- and 12-month treasury bills; and at the long-term end of the yield curve is the slope relating the 12- and 60-month treasury bills and notes. The third feature derived from the yield curve is the rate of change between the slopes. This represents the rate at which the slope of the yield curve is changing at the two points, and it serves as an indicator of current expectations of the future variability[33] of interest rates. This feature is measured as the square

Scheinkman (1994) studies. I use directly identifiable and interpretable components of the yield curve while the Litterman and Scheinkman (1991), and the Knez *et al.* (1994) studies use the principal components method to determine the major orthogonal elements of the yield curve. These principal components cannot be directly associated with any observable information in the yield curve. My work also differs from theirs in that they did not extend their analysis to testing the factors affecting the credit risk structure in the Eurodollar market.

[31] In fact, this is how the expectations theory of the term structure of interest rate is defined. See, for instance, Shiller (1990) or Campbell and Shiller (1987). Campbell and Shiller (1987) expressed further that the weights can be made dependent of the discounting factor, so that cash flows that are received far into the future are given less weight than those that are received much sooner.

[32] Alternatively, the slope of the yield curve can act as an indicator of current expectations of the future inflation rate. As mentioned earlier, I am controlling for the inflation factor and the monetary policy separately using the federal funds rate as a proxy. This enables us to separate the effect of the future interest rate due to real factors than from inflation factors.

[33] The variability of the interest rate is important for the credit risk measurement. I assume that variability consist of two parts. The first is forward looking, this is the part measured by the squares of the change in the 3-month

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of the gradient of the yield curve.

2.3.2.3 Treasury Yield curve and the Eurobanks

In general, the arguments presented thus far are especially relevant for the U.S.-based banks. However, since the Eurobanks are in a similar line of business—financial intermediation—as the U.S.-based banks, then the arguments presented in the preceding section are equally applicable.[34] Other channels through which the term structure of interest rate may have an effect on the Eurobanks are: first, the Eurobanks do devote part of their portfolio to U.S. government securities, and also make loans to other banks in the domestic market, governments of other countries and private corporations. Therefore, any changes in the interest rate directly affects the market value of the securities held in their portfolio. Though the effect of interest rate changes can be hedged, much depends on the effectiveness of interest rate hedging contracts entered into. Thus, the interest rate may have an effect on the overall performance of the bank's portfolio and hence, profitability.

The second, although indirect, method through which the yield curve changes affect the Eurobanks is that Eurobanks lend on a short-term basis to regional and money center banks, other financial institutions, commercial and industrial organizations in the U.S. and in other countries. These institutions may, in turn, also hold U.S. Treasury securities, and lend to other banks, other governments and private corporations. Given these arrangements, any unexpected movement in interest rates may also have the effect of reducing the value of securities

rate, and the squares of the difference in slopes. The second part reflects on how volatile the observed credit risk spread itself has been in the past. This part is reflected through the volatility estimate in the conditional mean, $X_{12,t}$. This is included in the model in order to separate the effect of the forward-looking measures of variability, and the previous market experience of volatility.

[34]The quoted yields on the Eurodollar instruments are dependent on the yield of a U.S. Treasury instrument with comparable maturity. So, the entire term structure of the interest rate on any given day mirrors the U.S. Treasury yield curve; except that the term structure of interest rate in the Eurodollar market lies everywhere above the U.S. Treasury yield curve because of the credit risk premium. If, for example, the U.S. yield curve shifts, or the slope changes, that is also likely to be reflected in the yield curve on the Eurodollar deposits. As such, the interest earnings on assets, the interest expense on liabilities, and thus the net revenue from operations are also affected.

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held by these institutions in their portfolios. As a result of the loss suffered by these institutions, they may not be in a position to service their debts to others as promised.

Similarly, private corporations that borrow from banks may also experience difficulty in making repayments due to an increase in the cost of re-financing outstanding debts, loss of revenue arising from a reduction in consumer spending (possibly due to the wealth effect of interest rate increase), or both. All of these directly affect the profitability of corporations and banks alike and hence their ability to meet financial commitments. As is explained here, loans to these U.S.-based banks and private corporations thus serve as another conduit through which the Eurobanks are exposed to changes in the Treasury yield curve. The Eurobanks may be able to immunize their portfolios against interest rate change on the securities they directly hold (for example, government securities); however, they may not be able to do so on risk exposures arising through a third party. This is especially the case when borrowers are adversely affected by interest rate changes.

In conclusion, it may be expected that a high interest rate level, unexpected changes in the interest rate, an expected high level interest rate, and high volatility of these rates will adversely affect the default probability, and hence the credit risk spread. However, it is difficult to say a priori what the magnitude of these effects will be. This is because banks do enter into financial contracts that can be used to both eliminate or profit, from the credit risks associated with their customers and the interest rate risk on assets and liabilities held in their portfolios. Whether or not these variables have any effect greatly depends on the effectiveness of the contracts in eliminating the potential risk due to the changes in the U.S. Treasury yield curve. Nonetheless, these variables should not be ignored in any empirical model of the credit risk spread, whether in the domestic money market or in the Euromarket. If these variables are significant, the observed data will reveal this fact.

2.3.3 The Control Variables

I will now discuss the variables outside the information contained in the current yield curve. How each of these variables affects the observed credit risk is also explained. I control for the

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effect of these variables in order to properly identify and attribute the true contribution of each of the elements in the yield curve information set, i.e., to avoid bias and an invalid conclusion from the results.

2.3.3.1 Credit Risk History

Historical information can, and does, provide a clue to the future. The past observations of credit risk spread can, therefore, be used to model and predict the future credit risk spread. The conventional wisdom behind this is that if the credit risk spread has been high in the past, then the future level of credit risk spread is also more likely to be high. Likewise, if it has been low in the past, then it is also likely to be low in the future. As such, to allow for the persistence in the level of the observed credit risk, I use the lagged values of credit risk spread as an additional explanatory variable.[35]

Similarly, the variability of the credit risk spread observed in the past may also be of interest to the contracting parties. Again, if history can be used as a guide to the future, a high variability of credit risk spread in the recent past tends to continue into the future, as does low variability. Thus from the historical data, the previous level and the previous variability can be determined and used to augment the information in the yield curve to make a forecast of future credit risk. This information has a high value, especially when there is a strong persistence in the level and variance of the series. To measure the previous variability of credit risk, I use the predicted conditional variance from the GARCH(1,1) model. Thus the model considered for this exercise is of the GARCH-in-Mean class introduced by Engle, Lilien and Robins (1987).

2.3.3.2 The Volatility Spill-Over Effect From Other Assets Markets

Asset markets do not exist in isolation, and the Eurodollar market is no exception. When agents plan their portfolio holdings, all assets (domestic or international, real or financial)

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[35]Clinebell, Kahl, and Steven (1996) consider only this variable in their analysis of the default risk in the U.S. corporate bond market.

are usually considered as perfect or near perfect substitutes. As a result, all assets markets are intrinsically linked to each other because agents frequently compare the relative prices (return) and the relative risk of the securities in the market. Because of the interdependence that exists among these securities, any stochastic shock affecting one of the assets markets eventually filters into the others. Therefore, the effect of the stochastic shock affecting one financial asset market is not localized to that particular market.[36] To incorporate the spill-over effect, or the reverberating effect, of the shocks from other assets markets into the analysis in this essay, I consider the variability of the value of the NYSE composite stock price index. This variability is measured as the squares of the first differences in the logged value of the NYSE composite stock index. It may be expected that, as the other assets markets become more volatile, the composite index will also pick up some of these effects as agents try to reallocate their assets portfolios. Furthermore, it may be expected that when the stock market becomes more volatile, the returns on financial assets become more uncertain, and a greater number of defaults may, therefore, be expected.[37] As a result, a positive correlation may be expected to prevail between

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[36]There is also ample literature on the extent of financial market integration: within the domestic financial market, within the international financial market, and across securities in these markets. For example, Rahman and Mustapha (1997) found evidence of bi-directional causality between stock market returns and bond market returns in the U.S. market. Also Christie (1982), Schwert (1989), Ferson (1989) and Zhou (1996) all found evidence of interdependence between the volatility of the U.S. Treasury securities market and the stock market. In addition, there is considerable evidence which suggests that the international stock markets are interdependent (see Koutmos 1996; and Ammer and Mei 1996). These studies show that there is a strong connection between the stock markets across countries. They also show that the stock market in each country is intricately linked to the domestic bond market. As such, the U.S. bond market is linked to the Eurocurrency, Eurobond, and the international bond market. Therefore, due to the interdependence in these markets, the information arising therefrom cannot be ignored in modeling credit risk in the Eurodollar market.

[37]The Merton (1974) model of default risk is based on the value of the firm. The market value of the firm is the principal determinant of the value of the stock traded on the NYSE. In the Merton (1974) model, defaults on loans used to finance operations are triggered when the firm's value reaches a particular threshold. The greater the volatility in the market value of the firm the higher the probability of this threshold being reached.

Similarly, in the stock market literature (see, for example, Christies 1982) a variable frequently used to explain equity premium, stock volatility, or compensation for risk is the financial leverage (the ratio of debt outstanding to the market value of the firm). If this ratio is high, for example as result of a low-market value of the firm, the compensation for risk is also expected to be high because investors expect the firm to be more likely to file for

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stock market volatility and the credit risk premium observed in the Eurodollar market.

2.3.3.3 The Federal Funds Rate

The Federal Funds Rate is one of the most watched financial market indicators by market analysts and fund managers the world over. The rate is closely watched because it generally serves as an indicator of the type of monetary policy being pursued by the Federal Reserve Board. An indication of whether the Federal Reserve intends to ease or tighten the credit condition in the money market has implications for the future performance of the economy, the future behaviour of inflation rates, or both. Given that the state of the economy can be influenced by this variable, the profitability of both the financial and the non-financial sectors can be adversely affected by a policy that results in slowing down the economy. If any of the institutions mentioned earlier are—in one form or another—indebted to the Eurobanks, then the Eurobanks' profitability can also be adversely affected. Even though the Eurobanks may not directly participate in the federal funds market, nonetheless their profitability may be affected through the chain effect of lending to domestic banks and commercial and industrial organizations. Although there is no direct connection between the federal funds market and the Eurobanks, the question of whether or not the Federal Funds Rate has any predictive power for the credit risk spread remains an empirical issue to be investigated.

2.3.3.4 Foreign Exchange Rate and Macroeconomic Activity

The foreign exchange rate may also affect the credit risk premium from at least two perspectives. The first, and indirect, is through macroeconomic activities and their effects on business profitability. Through this channel, the foreign exchange rate can still affect a bank's ability to meet its financial obligations despite that it holds a zero net balance of foreign-currency-denominated assets and liabilities. Even a bank with its main operational base in the domestic market is not insulated from this indirect effect. As explained earlier, Eurobanks do lend to U.S. regional banks, U.S. money center banks, and commercial and industrial organizations

bankruptcy, or chapter XI, protection in the event of a downturn in the economy.

based in the U.S. and other countries. If the exchange rate affects the activities of these banks and the activities of the commercial and industrial organizations in the respective domestic markets, then this may indirectly expose the Eurobanks to the exchange rate risk affecting the entities to which it lends funds. This effect may be large or small depending on the extent to which the exchange rate affect the financial position of the borrowers.[38]

The second channel through which the foreign exchange rate appreciation or deprecia-tion and its volatility may affect the credit risk spread in the Eurodollar market is, however, more direct. This effect operates through two fronts: the first is through the foreign-currency-denominated assets and liabilities held in the Eurobanks' portfolios; and the alternative is through market-making in off-balance sheet derivative contracts that are denominated in for-eign currencies. Any adverse movement in the exchange rate can seriously impair a bank's profitability, and hence a bank's ability to meet its financial commitments to its clients, even though some of the foreign exchange rate risk exposure can be hedged using, for example, forward and futures contracts, swaps, swaptions, and options, among others. But like the in-terest rate risk-hedging considered earlier, the effectiveness of the hedge remains an empirical issue.[39]

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[38]As an example, consider for the moment, a Eurobank or domestic bank lending money to a commercial and industrial organization whose principal market is outside its place of operation, or a firm engaged in tourism developments. Clearly, the fluctuations in the foreign exchange rate, or an appreciation in the exchange rate over an extended period of time, will surely put in jeopardy the loan made by the banks. Even more disturbing is the fact that this type of indirect risk exposure cannot be completely hedged by the banks.

[39]The positions in the off-balance-sheet derivative contracts are usually used to hedge the balance sheet items, or other derivative contracts outstanding. The hedge is used to lock-in a given rate of return should the exchange rate move in particular direction. However, as the pay-off on these derivative contracts is highly non-linear, and that the number of contracts taken have to be continuously adjusted to reflect the changing features of the underlying securities, the effectiveness of these hedges is critical and remains an empirical issue. For this reason, it should be accounted for while investigating what factors influence the profitability of banks and their capacity to meet their financial obligations. The empirical study by Choi and Elyasiani (1996) reports a negative relationship between the foreign exchange rate movement and the rate of return on the 59 largest banks in the U.S.. This relationship holds almost across-the-board for all the banks examined. Likewise, the study by Chamberlain, Howe, and Popper (1996) also reports a similar finding in their analysis of U.S. and Japanese banking institutions. These two studies serve to illustrate that banks may not at all times be successful at eliminating exchange rate risks when they hedge

2.3.4 Summary

In the preceding discussion, I have presented the GARCH-in-Mean model as it applies to credit risk modeling in the Eurodollar market. The reasons as to why and how each of the factors affects the credit risk have also been presented. A common theme in the factors considered above is that they each influence the performance of a financial institution, or a commercial and industrial organization that borrows from the financial institutions. In consequence, each of these institution's profitability, and hence, its ability to meet its financial obligations to others may also be affected.

It is important to mention that most of the factors mentioned above apply strictly to the domestic banks. However, as these banks and commercial entities are linked to the Eurobanks through the various financial contracts between them, these contracts serve as the conduit though which most of the variables specific to the U.S. are suspected to influence the credit risk spread in the Eurodollar market. Even though it is not the principal objective of this study to test if there is a Granger-causality from the domestic banking to the Eurodollar market, the situation here, however, enables us to indirectly conduct such a test of market integration between the Eurodollar market and the U.S. domestic money market.

To empirically test the significance of each factor, I next consider the parameter estimation method in Section 2.4. In Section 2.5, I discuss the data used in the estimation.

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against exchange rate volatility.

2.4 The Maximum Likelihood Estimation Criterion

The estimation method used in this study is the maximum likelihood, assuming normality. The procedure involves maximizing the following joint conditional probability distribution, or the likelihood function of ϵ_t up to time T, with respect to the parameter space, Γ:

$$\max_{\Gamma} L_T(\Gamma) \equiv \prod_{t=1}^{T} f(\epsilon_t|\Omega_t; \Gamma) = \prod_{t=1}^{T} \frac{1}{\sqrt{2\pi\sigma_t^2}} \exp\left(-\frac{1}{2}\frac{\epsilon_t}{\sigma_t}\right)^2 \tag{2.3}$$

$$\epsilon_t = CR_t - \alpha_0 - \sum_{i=1}^{10}\sum_{j=1}^{m} \alpha_{i,j} X_{i,t-j} + \gamma X_{11,t} \tag{2.4}$$

$$\sigma_t^2 = \beta_0 + \beta_1 \epsilon_{t-1}^2 + \beta_2 \sigma_{t-1}^2 \tag{2.5}$$

where:

$\Omega_t = \{\{X_{1,t-j}, \ldots, X_{11,t-j}\}: \; .j = 1.2.\ldots.m\}$, it represents the conditioning information set available in period t.

$\Gamma = \{\alpha_0, \alpha_{i,j}, \beta_0, \beta_1, \beta_2, \gamma; \sigma_0; \quad i = 1.2.\ldots.10. \quad j = 1.2.\ldots.m\}$, it represents the set of parameters to be estimated from the likelihood function.

As is conventional, I maximize the log likelihood function,[40] i.e., that I maximize the following objective function with respect to the parameters space, Γ.

$$\max_{\Gamma} \ln L_T(\Gamma) = -\left(\frac{T-m}{2}\right)\ln 2\pi - \frac{1}{2}\sum_{t=m}^{T}\ln\sigma_t^2 - \frac{1}{2}\sum_{t=m}^{T}\left(\frac{\epsilon_t}{\sigma_t}\right)^2 \tag{2.6}$$

In addition, the order of the lag length m is decided using the Schwartz information criteria.[41]

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[40] For the details of how to set up the likelihood function, see, for example, Kennedy (1992), Davidson and MacKinnon (1993), and Jazwinski (1970). The parameter estimate, $\hat{\Gamma}$, that maximizes the log likelihood function is estimated numerically using the Marquadt-Levenberg algorithm. For a more detailed description of the algorithm see Press, Teukolsky, Vetterling and Flannery (1992: 678) or SAS/ETS manual.

[41] There are other model selection criteria such as the Akaike information criterion that can also be used to determine the appropriate lag length. But as the Schwartz information criterion often selects the most parsimonious models, I will restrict my attention to it.

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2.5 The Data

The data series used in this study are the daily observations recorded at the close of each trading day. The data on the interest rate series are as follows: the first is the London Interbank Offer Rate (LIBOR rate) on U.S.-dollar-denominated 3-month term deposits, placed in a designated London bank; the second is the yield on U.S. Treasury securities with 3-, 12-, and 60-months to maturity; and the third is the Federal Funds Rate on 7-day federal funds. These rates are actual market quotes, on the respective securities, at the close of each business day. Following convention, the quoted rates were transformed into their continuously compounded annualized equivalent yield basis. This conversion is necessary so that the different rates are in a directly comparable form. The data on the LIBOR rates were obtained from Data Resource Inc. (DRI), while the yield on the U.S. Treasury securities and the Federal Funds Rate were obtained from the Federal Reserve Board, *Federal Statistical Releases*, Selected Interest Rate (series H15). The full sample period on all the interest rate series extends from June 1, 1973, through August 19, 1996.

The other financial time series employed comprise the following: the New York Stock Exchange (NYSE) common stock composite price index reported at the close of each business day. This index is obtained from the NYSE historical stock database. The other series is the trade-weighted foreign exchange rate index of the U.S. dollar vis-à-vis the G-10 countries. The foreign exchange rate index is as reported at the end of each business day by the Federal Reserve Board, *Federal Statistical Releases*, Foreign Exchange Rate (series H10). For both series, the full sample period also extends from June 1, 1973, through August 19, 1996.

In the subsequent analysis, all interest rate series have been transformed into their continuously compounded annualized equivalent yield basis.[42] I then compute the yield spread

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[42] The transformation to the continuously compounded annualized yield basis proceeds as follows. First, for rates on securities with less than 365-days to maturity the following formula was used in the conversion to continuously compounded annualized equivalent yield (r^c).

$$r^c = -\frac{36500}{n} \ln \left(\frac{F}{P} \right)$$

where $p = 100$, and $F = 100 \left(1 + r^q \left(\frac{n}{360} \right) \right)$ for the Eurodollar deposits which is based on bankers quoted add-on

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between the continuously compounded annualized equivalent yield on the 3-month LIBOR rate and the continuously compounded annualized equivalent yield on the 3-month U.S. Treasury bill rate. This difference is non-negative. Since the difference between these rates is due mainly to the credit quality of participants, it is termed "credit risk premium."Alternatively, it is referred to as the "credit risk spread on 3-month Eurodollar deposits."It is this spread, the credit risk spread, that this study analyzes.

The NYSE composite stock price index, and the trade-weighted foreign exchange rate series were also transformed, using a logarithmic transformation. They were then differenced once. The square of the differenced series is then used as a proxy for the variability of the stock market and the variability of the foreign exchange rate market.

The NYSE composite stock price index[43] has been employed instead of the Dow-Jones Industrial Average (DJIA), or the Standard and Poors 500 (S &P 500) index. The NYSE index is employed because it represents a broader market index, and is, therefore, more representative of the investment portfolio in U.S. business enterprises than are the DJIA and the S &P 500.[44] Analogously, the trade-weighted foreign exchange rate index–a multilateral exchange

yield basis (r^q).

For the U.S. Treasury securities, because they are based on a discount yield basis (r^d):

$$P = 100 \left(1 - r^d \left(\frac{n}{360}\right)\right) \text{ and } F = 100$$

Second, for securities with more than 365-days to maturity the following equation was solved numerically, on each date, using Newton-Raphson algorithm

For Treasury securities with two coupon payments per year:

$$f(r^c) \equiv \sum_{t=1}^{2T} \frac{c}{2} \exp(-r^c \frac{t}{2}) + F \exp(-r^c T) - P = 0 \qquad P = 100 \qquad F = 100$$

For Eurodollar deposits with one coupon payment per year:

$$f(r^c) \equiv \sum_{t=1}^{T} c \exp(-r^c t) + F \exp(-r^c T) - P = 0 \qquad P = 100 \qquad F = 100$$

[43]The composite stock price index adjusts for changes in the composition of the firms used in constructing the index, stock splits and other features of the firms that affect the firm's value.

[44]The NYSE composite price index comprises all common stocks listed on the NYSE. Each stock in the index

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rate–is used in the analysis instead of one of the bilateral exchange rates such as the U.S. dollar-pound sterling rate, the U.S. dollar-Deutschemark rate, the U.S. dollar-Yen rate among others. The trade-weighted foreign exchange rate index is preferred because fund managers of banks, insurance companies, pension funds, and mutual funds among others, often maintain investment positions in several countries that do not use the U.S. dollar as their official currency. Hence, before taking a particular position in these economies, the U.S. funds must first be converted to the respective foreign currencies. Since U.S. fund managers do not exclusively prefer one specific country to the another, it is therefore more appropriate to use a weighted average of the most-traded currencies.

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reflects its market capitalization; that is, the market value of outstanding stocks, calculated as a multiple of the number of each firm's stock outstanding and the market price of each stock. The S & P 500 index accounts for only eighty percent of the market capitalization of all the stocks listed on the NYSE (Hull 1989: 43). Similarly, the DJIA comprise only 30 "blue chip" stocks in the U.S., and it accounts for only twenty percent of the market value of the NYSE stock market capitalization (Dubofsky 1992: 241).

2.6 The Data Analysis and Empirical Results

Before presenting the empirical results, I first discuss the intrinsic features of the data in Section 2.6.1. In Section 2.6.2, I discuss the findings and their implications for modeling credit risk spread in the money market, and the Eurodollar market in particular. In Section 2.6.3, I evaluate the predictive ability of the model, and also compare its predictive power with other competing models.

2.6.1 The preliminary Data Analysis

In this Section, I present the results of the exploratory data analysis, i.e., the summary statistics for the full sample period, and the plots of the variables of interest. Table 2.2 presents the summary statistics for each of the variables taken into consideration. This table contains the result for the daily-sampled data, and it covers the full sample period, June 1, 1973 through August 19, 1996. From this table, it can be observed that over the sample period, the average of the absolute credit risk spread observed (CRD. RSK.) is positive, and it is approximately 120 basis points.[45] Also, the standard deviation around the mean at this sampling frequency is 92.4 basis points; and the range (Maximum - Minimum) is 670 basis points. From these statistics, we can infer the following: first, the mean indicates that the level of the absolute credit risk spread in the Eurodollar market is high; and second, the measures of variability suggest that the absolute credit risk spread observed during the sample period is also highly variable.

The analysis of the relative credit risk spread (CRD. REL.) shows a similar result over the sample period. The absolute credit risk spread, expressed as a proportion of the continuously compounded annualized yield observed on a 3-month U.S. Treasury bill, has a mean value of 15 percent. The standard deviation about this mean value is 9.8 percent. Also, this proportion range is from as low as 3.0 percent to a high of 95.7 percent. These statistics for the relative credit risk spread closely mimic those of the absolute credit risk spread. The time series plots of the absolute credit risk spread and the relative credit risk spread, which are contained in

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[45] A basis point is a hundredth of a percentage point, $\frac{1}{100}$.

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Figures 1 and 2 respectively, both attest to this. As can be seen from the two plots, they are virtually identical in how they clearly map out the behaviour of the series over time.

With regard to the empirical distribution of the above series, the skewness and the kurtosis measures suggest that they are not normally distributed. The empirical distribution is positively skewed and fat-tailed; and furthermore, the kurtosis displays a higher peakedness than is characteristic of a normal distribution. This positive skewness suggests that a greater proportion of the values observed for credit risk spread lies above its modal value of 31.04 basis points. From the foregoing analysis, we can therefore see that a high value of credit risk spread occurs more frequently. These preliminary results have two implications: first, since the incidence and the value of credit risk is high, it is therefore worthy of further investigation; second, any model that attempts to explain and predict credit risk spread must also take into consideration the fact that data are not normally distributed.

Another notable feature of the credit risk spread in the summary statistics presented in Table 2.2, is that the measure of dispersions is high relative to its mean value. This tends to suggest that the credit risk spread is not significantly different from zero. However, as the empirical distribution is not normal, this inference may not be valid. In order to have a better view of the data, I subdivided it into smaller subsample periods. The results for the sub-periods indicate that the mean values in relation to the standard variance are significantly different from zero. The average amount of credit risk spread in the Eurodollar contracts varies over time. Figures 1 and 2 present a graphical view of this degree of variability as well as the magnitude of the credit risk spread in each period. The graphs show that the level of credit risk was exceedingly high in the early 1970s and in the period between 1979 and 1984. It also remains highly variable during the period.

Table 2.2 also includes the summary statistics of the other series used in the analysis. The summary statistics were also computed for smaller subsamples. The mean, the variance, and the range of the remaining series generally follow the same pattern as that for the credit risk spread. These patterns can be readily observed in the time-series plots contained in Figures 3 to 7 for U.S. Treasury bills and bonds, in Figures 8 to 10 for the foreign exchange rate market,

in Figures 11 to 13 for the NYSE common stock price index, and in Figures 15 and 16 for the federal funds market rate and the Eurodollar market rate respectively. Because of the close relationships between the above time series from different markets and the absolute (or the relative) credit risk spread, it is expected that these variables will provide a good explanation and forecast of the observed credit risk spread. I will now discuss the empirical results of the statistical models used in the investigation.

2.6.2 The Empirical Results: In-Sample

This section considers the following issues. First, it discusses the specification search method, the diagnostic tests on the residuals, and the test for the structural stability of the models. Second, it discusses the impact of each element in the U.S. Treasury yield curve information set, and the impact of the other economic factors on the observed credit risk spread; and finally, it discusses the implication of the empirical findings for credit risk modeling in the Eurodollar market.

2.6.2.1 The Specification Search Method

Since the empirical results hinge on the specification, I will present the specification search method before discussing the results of the analysis. The empirical specification search method followed in this study is the dynamic linear regression model of Hendry, Pagan, and Sargan (1984) and Hendry (1995). In short, the general-to-specific modeling methodology.[46] In this regard, I start with a generous lag-length of order $m=90$ days for each of the explanatory variables in equation (2.1).[47] The only exception to this rule is the predicted volatility, variable $X_{12,t}$, in equation (2.1).

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[46]There are other empirical specification search methods such as the specific-to-general method that could be used. However, such specification search methods are fraught with problems. For example, it is difficult to control the power of the tests under the specific-to-general framework.

[47]This falls within the 7 to 100 trading days widely used for estimating the moving averages of desired financial time series in the financial market. See, for example, Jorion (1997: 168).

In addition to the preceding, I introduce four dummy variables into equation (2.1), the first three of which were used because studies such as Roberds, Runkle, and Whiteman (1996) found that the changes in the Federal Reserve operating procedures have an effect on the stochastic behaviour of financial time series.[48] Furthermore, I allow each dummy variable to interact with each of the explanatory variables at the various lags. Including the dummy variables as an independent variable allows me to determine if the intercept term significantly differs from that of the base period (1973-1979) in each of the other regime periods. The interaction dummies also allow me to determine if the slope parameters changed in each of the regime periods. The fourth dummy variable in the regression is used to isolate the effect of the extraordinary event in the stock market on Monday October 19, 1987.

I then test down this initial model to derive a more parsimonious specification. The steps taken are described next; but by way of explanation, I have specified this number of lags because the effects of changes in each of these variables may be distributed over time, i.e., that a change in the level of a particular variable will not only be effected when the change occurs, but the effect may also linger for some time into the future. The lags introduced into the model

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[48]During the sample period examined, there were changes in the Federal Reserve Bank operating procedures. Roberds, Runkle, and Whiteman (1996) in their study of the predictive power of the yield spread for short-term interest rate movement in the U.S. Treasury securities market indicate that there were four separate regimes, and each of these policy regime epochs have consequences for the observed U.S. Treasury yield curve. The four epochs used in their study are: the period of Federal Funds Rate targeting (period up to October 3, 1979); the period of non-borrowed reserves targeting (October 6, 1979, to October 6, 1982); the period of borrowed reserves targeting with lagged reserves accounting (October 7, 1982 to February 1, 1984); and the period of borrowed reserves targeting with contemporaneous reserves accounting (the period after February 2, 1984). It has also been found that the policy regime changes have implications for the dynamics of several economic time series.

Accordingly, the dummy variables used in this study are defined as follows: D7982 takes the value of one, if the date falls within October 6, 1979 and October 6, 1982 zero otherwise; D8284 takes the value of one, if the date falls within October 7, 1982 and February 1, 1984 zero otherwise; and D8496 takes the value of one, if the date falls within February 2, 1984 and August 19, 1996 zero otherwise. The base period therefore correspond to the period between June 1, 1973 through October 6, 1979. Each of these dummy variables corresponds to a particular monetary policy regime followed by the Federal Reserve. The dummy for the stock market crash of 1987 is represented by D87. It takes the value of one if date is equal to October 19, 1987 zero otherwise.

thus allow me to capture the persistence of the effect of changes in each variable.[49] Though the number of maximum lags selected here may somewhat be arbitrary, the rationale underlying my choice is that events in at most the last three months in the financial market enter into the financial agents' information set. As a result, the developments in the financial markets within the last three months may have influenced their decisions.

In order to have an efficient estimate of the parameters, and to avoid the possible multi-collinearity problem that could arise from using this number of lags, the estimation strategy followed involves the following steps. In the first step, the weight associated with each lag is approximated by the following modified gamma distributed lag function (see, for example, Judge, Griffiths, Hill, Lutkepohl, and Lee 1985: 401), so that for a particular variable $X_{i,t}$, the m-distributed lag reduces to:

$$Z_{i,t} = \sum_{j=1}^{m} \omega_j X_{i,t-j}; \qquad \omega_j = \alpha_i j^{s-1} \epsilon^{-(j-1)\theta_i} \tag{2.7}$$

$$= \alpha_i \sum_{j=1}^{m} j^{s-1} \epsilon^{-(j-1)\theta_i} X_{i,t-j} = \alpha_i X_{i,t}^*; \qquad X_{i,t}^* = \sum j^{s-1} \epsilon^{-(j-1)\theta_i} X_{i,t-j} \tag{2.8}$$

where $s \geq 1$. $i = 1, 2, \ldots, 10$.

In the second step, given a particular value for the decay rate parameter (θ_i), the lag length (m) and setting $s = 1$, then reformulate equation (2.1) as:[50]

$$CR_t = \alpha_0 + \sum_{k=1}^{4} \alpha_{0,k} D_k + \sum_{i=1}^{10} Z_{i,t} + \sum_{i=1}^{10} \sum_{k=1}^{3} D_k Z_{i,t} + \alpha_{11} X_{11,t} + \epsilon_t \tag{2.9}$$

$$CR_t = \alpha_0 + \sum_{k=1}^{4} \alpha_{0,k} D_k + \sum_{i=1}^{10} \alpha_i X_{i,t}^* + \sum_{i=1}^{10} \sum_{k=1}^{3} \alpha_{i,k} D_k X_{i,t}^* + \alpha_{11} X_{11,t} + \epsilon_t \tag{2.10}$$

In the third step, equations (2.10) along with equation (2.2) are then estimated using the estimation procedure described in Section 2.4. The fourth step repeat steps one to three with a new lower-lag order m, for the same or a new decay rate parameter (θ_i). I then compute the

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[49]There is no economic theory that suggests how many lags that can be considered in a model. Also, there is no theory one can draw on to determine how fast the lagged information decay is. These are the two main questions I grapple with in this part of the study.

[50]The dummy variable corresponding to period k is indicated by D_k. It takes the value of one when the period fall with period k, and zero otherwise.

log-likelihood value, the Schwartz information criterion, and the Akaike information criterion for the given decay rate and the given lag order. In the final step, I select the lag-order and decay rate parameter combination that gives the highest log likelihood, or the combinations that give the least Schwartz information criterion and the least Akaike information criterion.

The lag order considered in this study is $m = \{90.60.30.15.7\}$ and the decay rate parameter $\theta_i = \{\frac{1}{2}. \frac{1}{20}. \frac{1}{50}. \frac{1}{90}. \frac{1}{150}. \frac{1}{200}\}$. With these combinations, I have searched over thirty different specifications of the model. The summary result of these regressions is shown in Table 2.3. Using the procedure described above, I find that the log likelihood value, the Schwartz information criterion and the Akaike information criterion all suggest that a lag order of 30 days and a decay rate parameter of $\frac{1}{20}$ is the most consistent with the data on the relative credit risk spread.[51]

2.6.2.2 The Misspecification Tests

While conducting the specification search, a number of misspecification tests were also conducted on the models. In particular, the residual terms from the regression were tested for randomness, that is, whether or not the residual terms are independent of each other over time. The Lagrange multiplier test suggests that the residual terms were serially correlated. As a result, the models were respecified to include the lagged innovation terms. The process taken to determine the lag order of the innovation terms involves specifying a high lag order (a five-day lag) and sequentially testing down using the likelihood ratio test method. This test method indicates that a lag order of three days is appropriate for modeling the data.

In the final analysis, the model reported in Table 2.4 does take into consideration the fact that the residual terms were serially correlated and also heteroscedastic. I also tested the residual terms for normality. However, as in most analyses of daily sampled data, the Bera and Jarque (1982) test for normality on the residuals suggest that the residuals are not normally distributed. Nonetheless, given the large sample size used in the study, the inferences drawn

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[51] A similar regression for the absolute credit risk spread indicates a much larger lag order of 90-days and the decay rate parameter of $\frac{1}{100}$.

on the various test statistics are only valid on an asymptotic ground.[52]

2.6.2.3 The Analysis of In-sample Results

The result of the final model selected from the various specifications discussed in the preceding section is presented in Tables 2.4 and 2.5. In Table 2.4, panel A, the first column contain the explanatory variables. The second column contain the parameters of the conditional mean equation estimated for the base period (1973-1979). The third to the fifth columns contain the differences in the parameter estimates from the base period in respective monetary policy regime periods. Panel B of Table 2.4 contain the parameters in the heteroscedasticity model— the result of the GARCH(1,1) model used in modeling the volatility of the credit risk spread; and Panel C contains the summary statistics of the GARCH-in-Mean regression model. As can be observed from Table 2.4, the predicted volatility, the innovation terms, ϵ_{t-i}'s , and the heteroscedasticity parameters are variables common to all regimes. The t-statistics of the respective parameters is shown beneath each parameter in parenthesis. The results in Table 2.5 is the long-term net effect of each of the variables in Table 2.4.

In general, these tables convey the following information. First, Table 2.4 suggests that the term structure factors are statistically significant, and are therefore relevant for explaining and predicting credit risk spread observed in the Eurodollar market. The only exception are the variability in the level of the yield curve and the slope of the yield curve at the short-term end of the market. Second, Table 2.4 conveys the information that the intercept is statistically different from zero. Although there were substantial reduction in the intercept level in the subsequent monetary regime periods, these reduction are not significantly different from zero. In the next three subsections, I examine the effect of the term structure variables, the credit risk history, and the effect of each of the other financial market information on credit risk spread.

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[52] I am in the process of computing the Bollerslev and Wooldridge (1992) robust standard errors for the parameter estimates.

2.6.2.3.1 The Impact of U.S. Treasury Yield Curve Information Since it is one of the objectives of this study to investigate the adequacy of observable information in the U.S. Treasury yield curve, I therefore begin my analysis from this perspective. Here, I evaluate the effect of the elements of the U.S. Treasury yield curve information over the respective regime periods; and the analysis is centered on Tables 2.4 and 2.5. As the results in Table 2.4 indicates, the level of the yield curve measured by the yield on 3-Month Treasury bills, the changes in level of the yield curve, the slope of the yield curve at the long-term end, and the variability of the slopes are all statistically significant at the five percent level in explaining and predicting the credit risk spread observed in the Eurodollar market. On the other hand, despite having a positive effect on credit risk spread, the slope of the yield curve at the short-term end and the variability in the level of the yield curve are not statistically significant at the five percent level.

Table 2.4 also reveals that the effect of each element of the treasury yield curve information on credit risk spread differs significantly according to the Federal Reserve Bank operating procedure. For instance, during the base period the change in the level of the U.S. Treasury yield curve have the effect of reducing the credit risk spread by 3.9716 basis points. But in the regime periods following October 6, 1979 this effect increased from that of the base period by 4.9897 basis points in 1979-1982 period, by 5.2717 basis points in 1982-1984 period, and by 4.6947 basis points in 1984-1996 period. A similar effect is observed for the impact o f the slope of the yield curve at the long-term end of the market.

Table 2.5 shows the net effect of each of the yield curve variables, from TRSP-L to SQ-DFTRSP. The table also indicate among other things that the net effect of each yield curve factor also depends on the operating procedure of the Federal Reserve Board. For example, the effect of the level of the yield curve (0.0558 basis points), the slope of the yield curve at the long-term end of the market (0.1622 basis points), the variability of the slope of the yield curve (0.2141 basis points), and the variability of the changes in the level of the yield curve (1.0660 basis points), exerts a positive impact on credit risk spread on average. This effect however vary over the sample period, changing from a net increase in one regime period to a net decrease in the other. As a result, it is difficult to generalize whether or not the effect of each of the yield curve factor in future will be positive or negative on credit risk spread.

What is however clear from these results is that the portfolio of banks and other financial institutions, and hence their performance, is sensitive to changes in each element of the U.S. Treasury yield curve information. The result also shows that, despite the widespread use of financial contracts that can be used to immunize a portfolio against changes in yield curve factors, some of the hedges instituted may not have been effective. Consequently, without adequate insulation of the portfolio to changes in interest rate levels, a bank's fortune may be adversely affected.

In summary, in this section I have explored the impact of the observable information in the U.S. Treasury yield curve on the credit risk spread observed in the Eurodollar market. The analysis shows that the yield curve contains relevant and significant information for modeling and predicting the credit risk spread. The direction and size of the effect of the information varies with the subsample period examined; and as a result, one cannot make a sweeping generalization about the effect of the term structure factors. In the next section, I discuss the effect of the other factors influencing the relative credit risk spread in the Eurodollar market.

2.6.2.3.2 The Impact of Credit Risk History

As was discussed in Section 2.3.3, other information, apart from that contained in the U.S. Treasury yield curve, is necessary for modeling and predicting the amount of credit risk spread observed in the Eurodollar market. The results reported in Table 2.4 panel A show, for example, that the 30-day history of the observed relative credit risk spread–the lagged values of the relative credit risk spread–is statistically significant at the five percent level for determining the future level of credit risk spread. The changes in the effect of this variable in the regimes subsequent to October 6, 1979, were not significantly different from zero. In this table, the effect of the past relative credit risk spread is negative and in absolute term less than unity. This suggest that the credit risk prediction oscillates about its average value cyclically; implying that agents may be over- or under- reacting to their inability to predict the credit risk spread accurately. This part of the table further shows that the innovation terms and the predicted volatility of credit risk spread were statistically significant; and each have a positive effect on the credit risk spread observed in the market.

To show the importance of the historical information in the credit risk spread, Table 2.5

indicate that on average, a basis point increase in the past predicted volatility results in 1.0270 basis point increase in credit risk in the long-term. Likewise, for the previous innovations the effect is a net increase of 0.9267 basis points in the relative credit risk spread. And, in all regime period subsequent to October 6, 1979 the effect of both series is also positive on the relative credit risk spread. As we can see from these results, the past volatility of the credit risk spread and the historical levels of credit risk have large consequences for its current and future levels. These variables therefore somewhat account for the fact that economic agents often reflect on the past when making decisions about the future.

Further analysis of the GARCH(1,1) model for the volatility of credit risk spread–in panel B of Table 2.4–reveals that the previous predicted volatility as well as the squares of the past forecast errors are both statistically significant. This results suggest that the volatility has a long memory, and as such, if volatility has been high some time in the past, it will still have a positive effect on the level of credit risk observed in the current period. This result is consistent with the findings of Engle et al. (1987) who found that there exists a significant positive relationship between the term premium (excess return) in the treasury securities rate term structure using the ARCH-in-Mean model.

2.6.2.3.3 The Impact of Other Financial Market Information

The effect of the uncertainty in the other assets market–represented by the uncertainty in the stock market–that filters into the Eurodollar market is positive and also statistically significant at the five percent level for explaining and predicting the credit risk spread. In Table 2.4 I separate the effect of the more turbulent periods in the financial markets (D87) from the periods of relative normal market activity (SQ-DFSTK). As the result in this table show, the period of instability in the stock market have a positive impact on the credit risk spread, an increase of 12.2960 basis points. On the other hand, in period of normal market activity the variability of the stock market holds little or no information for the credit risk in the Eurodollar mark at the five percent level. In Table 2.5 we can see that over the long run, the instability in the financial market– the stock market in particular–have the effect of increasing the relative credit risk spread by 11.6938 basis points.

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Now, to consider the effect of the credit ease or otherwise in the money market, I find that the Federal Funds Rate, which serves as the proxy for this factor, is also statistically significant over the sample period examined at the five percent level. This variable has a positive effect on the credit risk spread in the base period (0.4132 basis points), but as Table 2.4 indicates, the effects are significantly lower in all periods subsequent to October 6, 1979. Table 2.5 shows that a one-hundred basis points increase in the Federal Funds Rate will result, on the average, in a net increase of about 0.0907 basis points in credit risk spread. The Federal Funds Rate had the highest net effect during the subsample period 1973-1979 (0.3861 basis points) and the effect steadily declined in the subsequent subsample periods to a net decrease of 0.1047 basis point in 1984-1996 period. This result seems plausible because, as noted earlier, this variable is significant is explaining the credit risk spread in the mid-1970s when the inflation rate was high, and in an attempt to keep inflation in check the Federal Funds Rate was also very high. For these reasons, a bank's exposure to the risk of illiquidity was also very high.[53] However, in the 1980s and the 1990s, the problem of inflation has not been as severe as that experienced in the mid-1970s and, as a result, the Federal Funds Rate has been consistently lower. As such, the risk of illiquidity is, at present, not a very serious threat to a bank's operations and performance as was the case in the mid-1970s, thus the smaller effect of this variable in influencing the amount of credit risk observed in the market.

Lastly, the result presented in Tables 2.4 and 2.5 indicates that foreign exchange rate appreciation or depreciation, and foreign exchange rate variability are also significant factors for explaining the observed credit risk spread in the Eurodollar market. The effect on the relative credit risk spread observed in the market is negative in the base period for currency depreciation or appreciation. However, in all periods subsequent to 1979, the effects increased significantly over that of the base period. As Table 2.5 indicates, the net effect is positive on the credit risk spread during 1979-1996 sub-periods. On the average, the effect of currency appreciation or depreciation is 0.1930 basis points. That the effect of exchange rate appreciation or depreciation, and of exchange rate variability, is positive lends credence to the assertion

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[53]For a further discussion of how the Federal Funds Reserves regulation may constitute a source of risk premia, especially to the U.S. banks see, for example, Barret, Brian, Slovin, and Shuhka (1988).

that despite the hedging avenues open to financial institutions, a substantial number of banks are not fully hedged. Therefore, to account for the risk occasioned by changes in the foreign exchange rate, the credit risk must be loaded with this risk factor.[54]

2.6.2.3.4 Results Implications for Credit Risk Modeling

Given the results in Tables 2.4 and 2.5 , majority of the variables included in the regression are significant for explaining the observed credit risk, and in addition, the results are largely consistent with what would be expected a priori. This therefore suggests that the included variables are relevant, and as such are properly determined. Furthermore, the results presented here have the implication that any regression model that ignores any of the information will be misspecified, and may consequently result in costly forecasting errors in determining the importance of the included variable for modeling the credit risk spread.

Also, the coefficient of variation and the adjusted coefficient of variation computed from the regression 94.32 percent and 94.25 percent respectively, indicate that the model presented in this essay fits the observed data quite well. These results, therefore, suggest that the relevant explanatory variables have been used in the model presented here for modeling and predicting credit risk in the Eurodollar market. The significance of the other financial market information, and the information in the credit risk history, in addition to the information in the U.S. Treasury yield curve, further suggests that any empirical or theoretical model that attempts to fit the credit risk spread in the Eurodollar market, must incorporate this non-yield curve information into its analysis instead of considering only one type of information.

In what follows, I present the result of the regression of the restricted versions of the model discussed above using a subset of the information set. These restricted models are compared with the model discussed here. The restricted models considered include the following: those

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[54]One may always expect increased volatility of the stock market, the foreign exchange market or any other type of financial asset market to increase the credit risk observed in the Eurodollar market, it may, however, not always be the case–as reported in this study. The possible reason is that the volatilities themselves create opportunities for banks and other financial institutions to profit from these variations if they have a superior knowledge of the market. Thus, if these banks have enough knowledge of the market it may be able to reduce the attendant risk and even profit from such fluctuations. They may, however, occasionally go astray.

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using only the U.S. Treasury yield curve information (YC); those using the stock market information, the foreign exchange market information, and the Federal Funds Rate (FMT); and those models using only the time series of the credit risk spread (TS). The estimated models are of two varieties; the pure GARCH(1,1) model and the GARCH (1,1)-in-Mean model.[55]

2.6.3 The Out-of-Sample Predictive Ability of Models

The ultimate test of any statistical model is the extent to which it answers the following questions: How well can each model predict out-of-sample? and, how well does its out-of-sample forecasting ability compare with other competing models? The answers to these questions are crucial and necessary because it is quite possible for a particular model, especially the most general model, to over-fit the data in-sample, while out-of-sample it performs badly.

To answer the questions above, I subdivided the sample data into two parts: the first, spanning June 1, 1973, through December 31, 1994, serves as the in-sample data; and the second, extending from January 1, 1995 through August 19, 1996, serves as the out-of-sample data. I then used a rolling regression method to forecast one-step ahead the credit risk spread, using each of the models mentioned earlier. In the rolling forecast, I use the in-sample data to estimate the parameters of each model, then produce the forecast for the first period in the out-of-sample data. Next, I update the data set to include the January 1, 1995 observations, and then re-estimate each models' parameters. The forecast for January 2, 1995 is then produced. This data *cum* parameter updating scheme and forecast producing is then repeated in the subsequent periods until all the data in the out-of-sample data set are depleted.

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[55]The restricted model uses the same specification as the general model, whose result is presented in Table 2.4. The only exception is that appropriate parameter restrictions are imposed on the other variables. For instance, in the pure-time series model, all the parameters of the explanatory variables are restricted to zero while the lagged-dependent variable, the dummy variables, the interaction dummies, and the predicted volatility from the GARCH(1,1) model are unrestricted in the most general form of the pure-time series model (TS1). The TS1 model thus represents a GARCH-in-Mean specification for the pure-time series model. The restricted version of this model is TS2 which is a GARCH(1,1) model. The predicted volatility parameter in the conditional mean equation, TS1, is restricted to zero.

From these forecasts, and for the respective models, I then determine the following out-of-sample forecast performance statistics: the mean square prediction error, the root mean square prediction error, and the mean absolute percent error.[56] The results of the out-of-sample one-step ahead forecast and the standardized prediction errors are presented in Tables 2.6 and 2.7. Also, Figures OSF1 to OSF4 presents the plots of the out-of-sample forecast of each model and the corresponding actual observations of credit risk. In what follows, I examine the summary statistics of the out-of-sample forecast; I then examine the mean prediction error statistic; and finally, I examine the result of the out-of-sample forecast encompassing tests.

2.6.3.1 The Summary Statistics of the Out-of-Sample Forecast

The first column of Table 2.6 contains the models used in producing the out-of-sample forecast, columns two to five contain the summary statistics of the one-step ahead, out-of-sample forecast of the relative credit risk spread; and columns six to eight contain the summary statistics of the standardized-forecast errors. The row occupied by QSP3 represents the statistics of the actual out-of-sample data. From this table, we can draw the following conclusions: first, the mean forecast of each of the models examined is not significantly different from the mean of the actual observations in the out-of-sample data. This suggests that any of the models may be suitable for modeling and predicting the relative credit risk spread. Second, when the mean forecasts are compared with the mean of the actual observations, the mean forecast of the models using the yield curve information (YC1 and YC2) are lower than the mean of the actual out-of-sample data. This suggests that on average the models using only the yield curve information may be under-predicting the relative credit risk spread.

Third, the other models using, (a) only the time series of the relative credit risk spread (TS1 and TS2); (b) the other financial market information (FMT1 and FMT2), and (c) all the information (ALL1 and ALL2) have their mean out-of-sample forecast above the mean of the actual data in the out-of-sample period. These results thus suggest that the models may be

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[56] See Section 3.5.2 of part II for a more detailed account of the rolling-forecast method and the model evaluation criteria used in this study.

over-predicting the relative credit risk spread in the out-of-sample period. Fourth, from the table, we can also see that the model which uses other financial market information contains the mean forecast that is closest to the mean of the actual data. Its standard deviation from its mean is also the lowest. The model with the farthest mean forecast is the pure-time series model (TS1). This suggests that on average TS1 model over-predict the actual relative credit risk spread. In the next subsections, I examine the result of the formal model evaluation criteria.

2.6.3.2 The Mean Absolute and Mean Square Prediction Errors

The results reported in Table 2.7 contain a more-formal method of ranking the models; i.e., on the basis of their mean absolute prediction error (MAPE), their mean square prediction error (MSPE), and their root mean square prediction error (RMSPE). From this table—and based on the three evaluation criteria used—we can see that the models that consider all available information (ALL1 and ALL2) performed best. These two models have the lowest mean absolute and mean square prediction errors. It is a bit surprising that ALL1 and ALL2 models performed better than the other model using only the yield curve information, or the models using only the past-time series of credit risk spread. The ALL1 and ALL2 models are more general, and more often the the most general model are not expected to perform well out-of-sample.

Next in rank to the models using all available information are the models that considers only the information in the U.S. Treasury yield curve (YC1 and YC2). Furthermore, Table 2.7 reveals that the models with the worst out-of-sample forecast performance are those that use only the past relative credit risk spread to predict future realizations (TS1 and TS2), and the models that considers only other financial market data in its information set (FMT1 and FMT2).

The results indicated above can be verified by inspecting the plots in Figures OSF1 to OSF4. The plot in Figure OSF1 shows that the out-of-sample forecast of the model using all available information closely trend the observed data better than the other models.

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In summary, one can now see from Table 2.7 that of the eight different specifications considered, the models that rank first and second are those which consider jointly the developments in the other financial markets. In the third and fourth positions are the models that augment the other financial market information with the pure-time series and yield curve information. The pure-time time series model and the model using the treasury yield curve information are both inferior. The result of this analysis supports the view that, apart from the bias that may result from ignoring other financial market information, it may also lead to a less accurate forecast of future credit risk spread.

2.6.3.3 The Out-of-Sample Forecast Encompassing Tests

As discussed earlier, the mean absolute and the mean square prediction error evaluation criteria both rank the general model (ALL1 and ALL2), and the model using the other financial market information (FMT1 and FMT2) ahead of the pure-time series model and the model using only the information in the treasury yield curve. In this section, the question that I intend to investigate is: are the ALL1, ALL2, FMT1 and FMT2 models superior to the others in terms of their out-of-sample forecast encompassing ability? I investigate this issue by conducting an out-of-sample forecast encompassing test. This test enables me to determine whether or not the out-of-sample forecast of the model ranked best encompasses the out-of-sample forecast of those that are ranked lower.

The out-of-sample forecast encompassing test involves testing whether or not the out-of-sample forecast of a particular model i can explain the out-of-sample forecast error of another model j, while the out-of-sample forecast of model j cannot in turn explain the out-of-sample forecast error of model i. If this is true, then model i is indeed superior to model j. If it happens that both models out-of-sample forecast can explain each other's out-of-sample forecast error, then no one model is superior to the other. The same conclusion is also applicable when both models out-of-sample forecast fail to explain each other's out-of-sample forecast errors. In the event that no one model is superior to the other, then a reasonable alternative may be to

combine the various models' forecast using, for example, the artificial neural networks.[57]

The result of the forecast encompassing test is contained in Table 2.8. As can be observed from the table, it is broken down into four blocks: The first block contains the p-values of the test of GARCH against GARCH models;[58] the second block contains the test of the GARCH against GARCH-in-Means model; the third block contains the test of the GARCH-in-Means against GARCH models; and the fourth block contains the test of the GARCH-in-Means against GARCH-in-Means models. The out-of-sample forecast error for each model i–the dependent variable–is in the first column. Columns two to nine contain the out-of-sample forecast from model j. Each element of the column serves as the explanatory variable of each of the elements of column one. For the out-of-sample forecast of model i to encompass that of model j at the 5 percent significance level, for example, then the p-value of $\beta_{j,i}^1$ has to be less than 5 percent, while the p-value of $\beta_{i,j}^1$ greater than 5 percent. The converse is true in the event that model j out-of-sample forecast encompasses that of model i. In these instances, the model whose out-of-sample forecast encompasses the other is ranked as being superior. On the other hand, if the p-values on $\beta_{j,i}^1$ and $\beta_{i,j}^1$ are both less than or greater than the 5 percent significance level, then no one model can be ranked as superior to the other.

Applying the above rules to Table 2.8, we can see that the model ranked as superior to the others within the different blocks is the most general model that uses all available information–ALL1 and ALL2. The table indicates that ALL1 and ALL2 can explain the out-of-sample forecast errors of all the other models using different information set, while at the same time, these other models cannot explain the forecast errors of ALL1 and ALL2 models. The table further shows that none of the models using only the treasury yield curve information, only the past time series of credit risk spread itself, or using other financial market information apart from

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[57]For a more detailed discussion of the out-of-sample forecast encompassing test and its empirical implementation see Section 3.5 of part II.

[58]The GARCH models are those models whose conditional mean is independent of the predicted volatility. The GARCH-in-Mean model assumes, in addition, that the conditional mean is influenced by the predicted volatility. Thus, what basically distinguishes models such as TS1 and TS2 from one another is that the predicted volatility is an extra variable in the conditional mean of TS2 while it is not in TS1. The same applies to the other models as well.

the treasury yield curve information is superior to each other. As can be observed from the table the latter set of models out-of-sample forecast each explains the others out-of-sample error.

In the final analysis, the results in Table 2.8 shows that the model that incorporates all financial market information with the treasury yield curve, and the past time series of credit risk spread, provides a better model and prediction than any other model. Thus using just a subset of the available information may lead to both bias in the parameter estimates of models and also to an inaccurate forecasts. The consequence of these effects may be financial loses that are avoidable if relevant information were used in forecasting and decision making.

2.7 Summary, Conclusions and Future Research

This essay analyzes the daily sampled data on credit risk in the Eurodollar market between June 1, 1973 and August 19, 1996. Its main objectives are as follows: to determine if the U.S. Treasury yield curve contains adequate information for modeling and predicting the credit risk spread observed in the Eurodollar market; to identify other factors that may be influencing the behaviour of the credit risk spread in the Eurodollar market; and to develop a suitable statistical model for explaining and predicting the credit risk spread. I employed the GARCH-in-Mean modeling methodology pioneered by Engle, Lilien, and Robins (1987), and obtained the following results. First, I found that the yield curve does contain information for future credit risk spread. However, such information is statistically "insufficient" for explaining and predicting the observed credit risk spread. Second, I found that besides the information in the U.S. Treasury yield curve, other factors are also relevant. These factors include the historical information on the level of credit risk spread, the variability of the level of credit risk spread, the variability of the NYSE composite stock price index, and the variability of the foreign exchange rate market. Third, the parameters of the GARCH-in-Mean model were not stable over time; they are significantly affected by the operating policies of the Federal Reserve Bank.

In addition, I also evaluated the performance of the GARCH-in-Mean model out-of-sample using four evaluation criteria. These are: the out-of-sample forecast encompassing tests, the mean squared prediction error, the root mean square prediction error, and the mean absolute prediction error. All these evaluation criteria rank the model used–the GARCH-in-Mean model specification that uses all types of financial market information–as superior to those using just the pure-time series of the relative credit risk spread, or just the information in the U.S. Treasury yield curve. In sum, the results identified significant factors that can be used to augment the yield curve information; and they also suggest that these non-yield curve factors are of vital importance in modeling and predicting credit risk spread.

It should be also noted that, despite the strong results obtained in this study, it is nonetheless devoid of certain deficiencies. Among them are: first, with regard to the time series used in this study, it fails to distinguish between the nominal and real variables. In principle, the

distinction between nominal and real variables may be important to the results. However, for lack of data on inflation rate series at the daily sampling frequency examined in this study, I therefore do not pursue this distinction. Moreover, to construct a proxy variable for the daily inflation rate expectations may unduly cloud the results of the analysis; I therefore used nominal variables throughout.

Second, the study assumes that the time of the structural breaks–period of changes in the Federal Reserve operating procedures–in the model are known for certain. This may not necessarily be so, as the effect of changes in the Federal Reserve operating procedures on financial agents' behaviour may have started before the changes are actually effected, or after the changes have been implemented. Whichever is the case depends on the credibility that the financial agents have with the Federal Reserve Board. As such, the dates used to segment the data into regime periods are only approximates.

Given the limitations above, I intend to extend the analysis of this study to models that allow time-varying parameters in order to accommodate the unknown change point of the Federal Reserve operating procedures. Also, in order to adequately control for the effect of the inflation rate expectations at the daily frequencies, I will expand the information set to include commodities futures prices such as petroleum or gold. Finally, I will consider constructing an artificial neural-network model for forecasting the credit risk spread in the Eurodollar market using either the variable identified in this study, or using the out-of-sample forecast of the various models considered in the study.

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Table 2.1: The List and Definition of Variables Used in Modeling Credit Risk Spread:

| CRD. RSK | : | is the absolute credit risk spread, measured as the difference between the continuously compounded annualized equivalent yield of the 3-Month Eurodollar deposits and the 3-Month U.S. Treasury bills. |
| CRD. REL. | : | is the relative credit risk spread, measured as the ratio of the absolute credit risk spread to the level of the continuously compounded annualized equivalent yield on the 3-Month U.S. Treasury bills, and it is multiplied by a hundred basis points $\left(\frac{CRD.RSK}{TR3M}\right) * 100$. |

Treasury Yield Curve Information:

TR3M	:	is the level of the continuously compounded annualized equivalent yield on the 3-Month U.S. Treasury bills.
DFTR3M	:	is the first difference of the level of the continuously compounded annualized equivalent yield on the 3-Month Treasury bills.
SQ-DFTR3M	:	is the square of the first difference of the level of the continuously compounded annualized equivalent yield on 3-Month Treasury bills.
TRSP-L	:	is the U.S. Treasury yield curve slope at the long end of the bond market, it is measured as the difference between the continuously compounded annualized equivalent yield on the 60-Month and 12-Month securities.
TRSP-S	:	is the U.S. Treasury yield curve slope at the short end of the bond market, it is measured as the difference between the continuously compounded annualized equivalent yield on the 12-Month and 3-Month securities.
SQ-DFTRSP	:	is the square of the difference the Treasury yield curve slope at the short- and the long-term end of the bond market.

List and Definition of Variables

Other Financial Market Information:

STK	:	is the log of the level of the New York Stock Exchange (NYSE) composite common stock price index.
DFSTK	:	is the first difference of the logged level of the NYSE composite common stock price index multiplied by a hundred $(STK_t - STK_{t-1}) * 100$.
SQ-DFSTK	:	is the squares of the first difference of logged level of NYSE composite common stock index.
FFR	:	is the level of the continuously compounded annualized equivalent yield on 7-Day Federal Fund.
DF-FFR	:	is the first difference of the level of the continuously compounded annualized equivalent yield on 7-Day Federal Funds.
SQ-DFFFR	:	is the square of the first difference of the level of the continuously compounded annualized equivalent yield on 7-Day Federal Funds.
XCH	:	is the logged level of the trade weighted exchange rate of the U.S. dollar vis-a-vis the G-10 countries.
DF-XCH	:	is the first difference of the logged level of the trade weighted exchange rate of the U.S. dollar vis-a-vis the G-10 countries. A measure of the appreciation or depreciation of the U.S. dollar vis-à-vis a basket of G-7 countries currencies.
SQ-DFXCH	:	is the squares of the first difference of the logged level of the trade weighted exchange rate of the U.S. dollar vis-a-vis the G-10 countries.
PRED. VOL.	:	is the predicted variability of credit risk spread from the GARCH(1,1) model.

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List and Definition of Variables

Dummy Variables and Innovation Terms:

D7982	:	Dummy variable, if (Oct. 6, 1979 < date <= Oct. 6, 1982) then equal 1, Else equal 0
D8284	:	Dummy variable, if (Oct. 7, 1982 < date <= Feb. 1, 1984) then equal 1, Else equal 0
D8496	:	Dummy variable, if (Feb. 2, 1984 < date <= AUG. 19, 1996) then equal 1, Else equal 0
D87	:	Dummy variable, if (date = October 19, 1987) then equal 1, Else equal 0
$\hat{\epsilon}_{t-i}$	:	is the innovations i-period ago. Effectively, it is the estimated residual for the conditional mean equation in the period $t - i$ ago; $i = 1, 2, 3$.
ARCH0	:	the intercept of the conditional variance equation.
ARCH1	:	the coefficient of the once lagged squared residual.
GARCH1	:	the coefficient of the once lagged conditional variance.

Table 2.2: The Data Summary Statistics

For the Full Sample Period: JUNE 1, 1973 TO AUG. 1996

VAR.	NOBS.	MEAN	STD. DEV.	MIN.	MAX.	SKEW.	KURT.
DAILY SAMPLED DATA							
CRD. RSK.	5637	1.196	0.924	0.096	6.795	1.870	4.513
CRD. REL.	5637	15.592	9.820	3.009	95.647	2.767	11.607
TR3M	5637	7.371	2.901	2.654	17.761	1.010	1.081
DFTB3M	5483	-7.05E-4	0.137	-1.340	1.413	0.137	16.226
SQ-DFTB3M	5483	0.019	0.081	0.000	1.996	11.701	193.780
TRSP-L	5379	1.087	1.289	-4.576	4.961	-0.978	0.871
TRSP-S	5379	-0.156	0.693	-4.585	2.606	-1.781	5.8 29
SQ-DFTRS	5379	2.176	1.788	0.000	8.256	0.680	-0.411
STK	5768	4.699	0.639	3.493	5.896	0.114	-1.357
DFSTK	5664	0.033	0.904	-21.286	8.622	- 2.446	59.285
SQ-DFSTK	5663	0.819	6.381	0.000	453.090	63.546	4463.4
FFR	5896	8.086	3.489	2.616	22.663	1.068	1.36 8
DF-FFR	5894	-5.452E-4	0.480	-7.997	7.896	0.162	40.153
SQ-DFFFR	5894	0.230	1.495	0	63.956	28.481	1100.1
XCH	5760	4.606	0.148	4.362	5.104	1.087	0.632
DF-XCH	5622	-1.666E-3	0.523	-3.547	4254	0.074	3.536
SQ-DFXCH	5622	0.274	0.644	1.41E-8	18.092	8.746	154.02

Table 2.3: Determining the Optimal Lag Length and Decay Rate

Parameter For the Independent Variables of

Credit Risk Spread in Eurodollar Market

Decay Rate — m-Days Lag l	$\frac{1}{2}$	$\frac{1}{20}$	$\frac{1}{50}$	$\frac{1}{90}$	$\frac{1}{150}$	$\frac{1}{200}$

LOG-LIKELIHOOD

	$\frac{1}{2}$	$\frac{1}{20}$	$\frac{1}{50}$	$\frac{1}{90}$	$\frac{1}{150}$	$\frac{1}{200}$
7	-10383	-10170	-10245	-10183	-10184	-10185
15	-10807	-10163	-10493	-10502	-10507	-10508
30	-10819	-10153**	-10188	-10200	-10205	- 10206
60	-10390	-10156	-10187	-10206	-10218	-10223
90	-10383	-10156	-10173	-10190	-10201	-10207

SCHWARTZ INFORMATION CRITERION

	$\frac{1}{2}$	$\frac{1}{20}$	$\frac{1}{50}$	$\frac{1}{90}$	$\frac{1}{150}$	$\frac{1}{200}$
7	21235	20816	20966	20841	20844	20845
15	22075	20802	21455	21473	21482	21484
30	21267	20782**	20852	20876	20886	20886
60	21248	20788	20849	20888	20912	20921
90	21234	20787	20823	20856	20877	20890

AKAIKE INFORMATION CRITERION

	$\frac{1}{2}$	$\frac{1}{20}$	$\frac{1}{50}$	$\frac{1}{90}$	$\frac{1}{150}$	$\frac{1}{200}$
7	20876	20452	20602	20477	20480	20481
15	20908	20438	21096	21114	21123	21125
30	21721	20418**	20488	20512	20522	20522
60	20889	20424	20485	20525	20548	20557
90	20875	20423	20459	20492	20514	20526

The sample Period: June 1, 1973 –Dec. 31, 1994; and, ** indicates the cell corresponding to the optimal lag length and decay rate.

Table 2.4: The Maximum Likelihood Estimate of GARCH-M Model

For Credit Risk Spread in Eurodollar Market During June 1, 1973 to Dec. 31, 1994

PANEL A:

	BASE PERIOD ↓	DIFFERENCE FROM BASE PERIOD ↓		
EXPL. VAR. ↓	1/6/73-5/10/79	6/10/79-6/10/82	7/10/82-1/2/84	2/2/84 - 31/12/94
INTERCEPT	20.1477	-17.4709**	-35.7633**	-16.8560*
	(3.797)	(-0.561)	(-1.177)	(-1.856)
CRD. REL.	-0.0701	-0.0463**	0.0436**	0.0186**
	(-5.462)	(-1.583)	(1.281)	(1.217)
TRSP-L	-0.7845	2.5043	0.2113**	1.2048
	(-2.854)	(3.822)	(0.253)	(2.603)
TRSP-S	0.5291**	-1.7737	0.0197**	-1.0393
	(1.623)	(-2.623)	(0.018)	(-2.076)
TR3M	0.1770	-0.2438	-0.0934**	-0.1388
	(3.793)	(-2.247)	(-0.492)	(-2.128)
DFTR3M	-3.9716	4.9897	5.2717	4.6947
	(-6.943)	(7.133)	(6.032)	(6.816)
SQ-DFTR3M	3.3770**	-3.6458**	-2.7505**	-2.6005**
	(1.366)	(-1.433)	(-0.950)	(-0.800)
SQ-DFTRSP	0.4692	-0.4748	0.0133**	-0.5188
	(3.002)	(-2.337)	(0.048)	(-2.749)
D87	12.2960			
	(2.016)			
SQ-DFSTK	-0.0639**	0.0374**	-0.0044**	0.0847**
	(-1.078)	(0.357)	(-0.044)	(1.412)
DF-XCH	-0.5147	1.0568	1.1869	0.6329
	(-3.609)	(4.072)	(5.256)	(4.280)

NOTE: ** indicates the parameters that are not statistically significant at the five per cent level; and * indicates those that are not significant at the ten per cent level. Below each parameter estimate, in parenthesis, is the t-statistics.

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The Maximum Likelihood Estimate of GARCH-M Model

For Credit Risk Spread in Eurodollar Market (continuation of Table 2.4)

	BASE PERIOD ↓	DIFFERENCE FROM BASE PERIOD ↓		
EXPL. VAR. ↓	1/6/73—5/10/79	6/10/79—6/10/82	7/10/82—1/2/84	2/2/84—31/12/96

Continuation of PANEL A:

SQ-DFXCH	0.1825**	-0.0926**	-0.5216*	-0.1787**
	(1.601)	(-0.375)	(-1.887)	(-1.505)
DF-FFR	0.4132	-0.3612	-0.3776**	-0.5233
	(3.739)	(-2.210)	(-1.634)	(-3.885)
PRED. VOL.	1.0939			
	(4.426)			
$\hat{\epsilon}_{t-1}$	0.7648			
	(42.407)			
$\hat{\epsilon}_{t-2}$	0.1478			
	(7.756)			
$\hat{\epsilon}_{t-3}$	0.0744			
	(4.577)			

PANEL B: Heteroscedasticity Parameters

ARCH0	0.0266			
	(6.162)			
ARCH1	0.0932			
	(16.808)			
GARCH1	0.9049			
	(170.310)			

PANEL C: GARCH-M Regression Summary Statistics

SSE	NOBS.	LOG-LIK	Rsq	Adj Rsq
29835.34	4906	-10142.1	0.9432	0.9425

NOTE: ** indicates the parameters that are not statistically significant at the five per cent level; and • indicates those that are not significant at the ten per cent level. Below each parameter estimate, in parenthesis, is the t-statistics.

Table 2.5: The Long Term Net Effect of Each Variable
On the Relative Credit Risk Spread in the Eurodollar Market

Period (i) — EXPL. VAR. (j) ↓	1973–1996 AVERAGE	SAMPLE PERIOD ↓			
		1973–1979	1979–1982	1982–1984	1984–1996
INTERCEPT	2.2859	18.8279	2.3977	-15.2125	3.1305
TRSP-L	0.1622	-0.7331	1.5405	-0.5584	0.3997
TRSP-S	-0.1428	0.4944	-1.1148	0.5346	-0.4852
TR3M	0.0558	0.1654	-0.0598	0.0814	0.0363
DFTR3M	-0.2113	-3.7114	0.9119	1.2665	0.6877
SQ-DFTR3M	1.0660	3.1558	-0.2408	0.6103	0.7385
SQ-DFTRSP	0.2141	0.4385	-0.0050	0.4700	-0.0472
D87	2.9235	–	–	–	11.6938
SQ-DFSTK	-0.0027	0.0597	-0.0237	-0.0665	0.0198
DF-XCH	0.1930	-0.4810	0.4856	0.6548	0.1124
SQ-DFXCH	-0.0189	0.1705	0.0805	-0.3303	0.0036
DF-FFR	0.0907	0.3861	0.0466	0.0347	-0.1047
PRED. VOL.	1.0270	1.0222	0.9798	1.0657	1.0403
INNOV. ($\hat{\epsilon}_{t-i}$)	0.9267	0.9223	0.8841	0.9615	0.9387

The long run effect were calculated as

$$\frac{\alpha_{i,1973-79} + \alpha_{i,D_k}}{1 - (\beta_{1973-79} + \beta_{D_k})}$$

where $\beta_{1979-73}$ is the coefficient of the lagged dependent variable ($CRD.REL.$) during 1979-1979 period; β_{D_k} is the slope of the cross-product of the dummy variable in period k and the lagged dependent variable. A similar definition applies to the coefficient of the variable j, and j is represented by the entries in the first column of the table.

For each row, the figures in column two–AVERAGE–is computed as a simple average of columns three to six.

Table 2.6: The Out-of-Sample Statistics for the Relative Credit Risk Spread

PRD. Model	Relative Credit Risk Forecasts				Standardized Forecast Errors		
	Mean	Std Dev	Skew.	Exc. Kurt.	Std Dev	Skew.	Exc. Kurt.
CRD. REL.	8.1997	1.7857	1.0562	2.2521	–	–	–

The GARCH Models

ALL1	8.2933	1.7211	1.2778	3.1299	0.6581	-0.2148	1.0498
TS1	9.3983	2.6965	0.8029	-0.6562	2.2180	-0.1177	0.0152
YC1	8.0427	1.8579	1.1088	2.6895	1.1291	0.4509	1.6559
FMT1	8.3569	1.7450	1.4567	3.2381	1.9158	0.0959	0.1105

The GARCH-in-Mean Models

ALL2	8.2949	1.7239	1.2775	3.1483	0.6590	-0.2349	1.1009
TS2	9.4596	2.9260	0.4704	-1.2073	2.3637	0.0168	-0.7549
YC2	8.0255	1.8119	1.2797	2.9467	1.2552	0.3799	1.3064
FMT2	8.6567	1.4590	0.6501	2.3238	1.7817	0.2998	0.2874

The out-of-Sample Statistics for the Relative credit Risk Spread and Forecast Errors using Sample Period January 1, 1995 to August 19,1996.

ALL represents the model that uses all the varoius information to predict credit risk spread one-step ahead–ALL1 is is the GARCH version and ALL2 is the GARCH-M version of the model. Similarly, TS represents the model that considers only the time series of the relative credit risk–TS1 is the GARCH version and TS2 is the GARCH-M version of the model. YC represents model that uses only the information in the yield curve–YC1 is the GARCH version and YC2 is the GARCH-M version of the model. FMT is the model that uses other financial market information outside the U.S. Treasury bond market. FMT1 is the GARCH version and FMT2 is the GARCH-M version of the model.

Table 2.7: The One-step Ahead Out-of-Sample Forecast Performance
Using the Out-of-Sample Period January 1, 1995, to August 19, 1996.

CRITERIA ↓	INFORMATION USED ↓							
	ALL		YIELD CURVE		TIME SERIES		OTHER FIN. MKT	
	ALL1	ALL2	YC1	YC2	TS1	TS2	FMT1	FMT2
MSPE	1.1205	1.1166	1.1189	1.1202	1.1302	1.1121	1.0959	1.0958
RMSPE	1.0585	1.0567	1.0578	1.0584	1.0631	1.0546	1.0468	1.0468
MAPE	0.8373	0.8335	0.8371	0.8334	0.8459	0.8390	0.8233	0.8234

Mean Square Prediction Error (MSPE), Root Mean Square Prediction Error (RMSPE) and Mean Absolute

Prediction Error (MAPE).

ALL represents the model that uses all the varoius information to predict credit risk spread one-step ahead–ALL1

is is the GARCH version and ALL2 is the GARCH-M version of the model. Similarly, TS represents the model that

considers only the time series of the relative credit risk–TS1 is the GARCH version and TS2 is the GARCH-M

version of the model. YC represents model that uses only the information in the yield curve–YC1 is the GARCH

version and YC2 is the GARCH-M version of the model. FMT is the model that uses other financial market

information outside the U.S. Treasury bond market. FMT1 is the GARCH version and FMT2 is the GARCH-M

version of the model.

Table 2.8: The Out-of-Sample Forecast Encompassing Test Statistics

The figures in table represents the p-values on β in:

$$\nu_{i,t} \equiv \hat{C}R_t - \hat{C}R_{i,t} = \beta_{i,j}^0 + \beta_{i,j}^1 \hat{C}R_{j,t} + \eta_{i,t} \qquad \eta_{i,t} \sim \mathcal{N}(0, h_t) \qquad h_t = \kappa(\hat{C}R_{i,t})$$

Model j $(\hat{C}R_{j,t}^2)$ — Model i $(\nu_{i,t})$ ↓	GARCH Models ↓				GARCH-in-Mean Models ↓			
	ALL1	YC1	TS1	FMT1	ALL2	YC2	TS2	FMT2
ALL1	–	0.189	0.145	0.992	0.000	0.409	0.795	0.302
YC1	0.000	–	0.000	0.168	0.001	0.000	0.001	0.118
TS1	0.000	0.420	–	0.540	0.000	0.019	0.001	0.006
FMT1	0.000	0.910	0.202	–	0.000	0.001	0.000	0.284
ALL2	0.000	0.224	0.127	0.974	–	0.417	0.738	0.278
YC2	0.000	0.000	0.003	0.125	0.074	–	0.000	0.027
TS2	0.000	0.000	0.000	0.082	0.011	0.000	–	0.002
FMT2	0.000	0.000	0.002	0.000	0.000	0.157	0.016	–

The out-of-sample forecast of the relative credit risk spread, $\hat{C}R_{i,t}$ and $\hat{C}R_{j,t}$, used in the analysis are those produced by the one-period ahead rolling forecast of each model. The out-of-sample period extends from January 1, 1995 through August 19, 1996.

ALL represents the model that uses all the varoius information to predict credit risk spread one-step ahead–ALL1 is is the GARCH version and ALL2 is the GARCH-M version of the model. Similarly, TS represents the model that considers only the time series of the relative credit risk–TS1 is the GARCH version and TS2 is the GARCH-M version of the model. YC represents model that uses only the information in the yield curve–YC1 is the GARCH version and YC2 is the GARCH-M version of the model. FMT is the model that uses other financial market information outside the U.S. Treasury bond market. FMT1 is the GARCH version and FMT2 is the GARCH-M version of the model.

The Absolute Credit Risk Spread
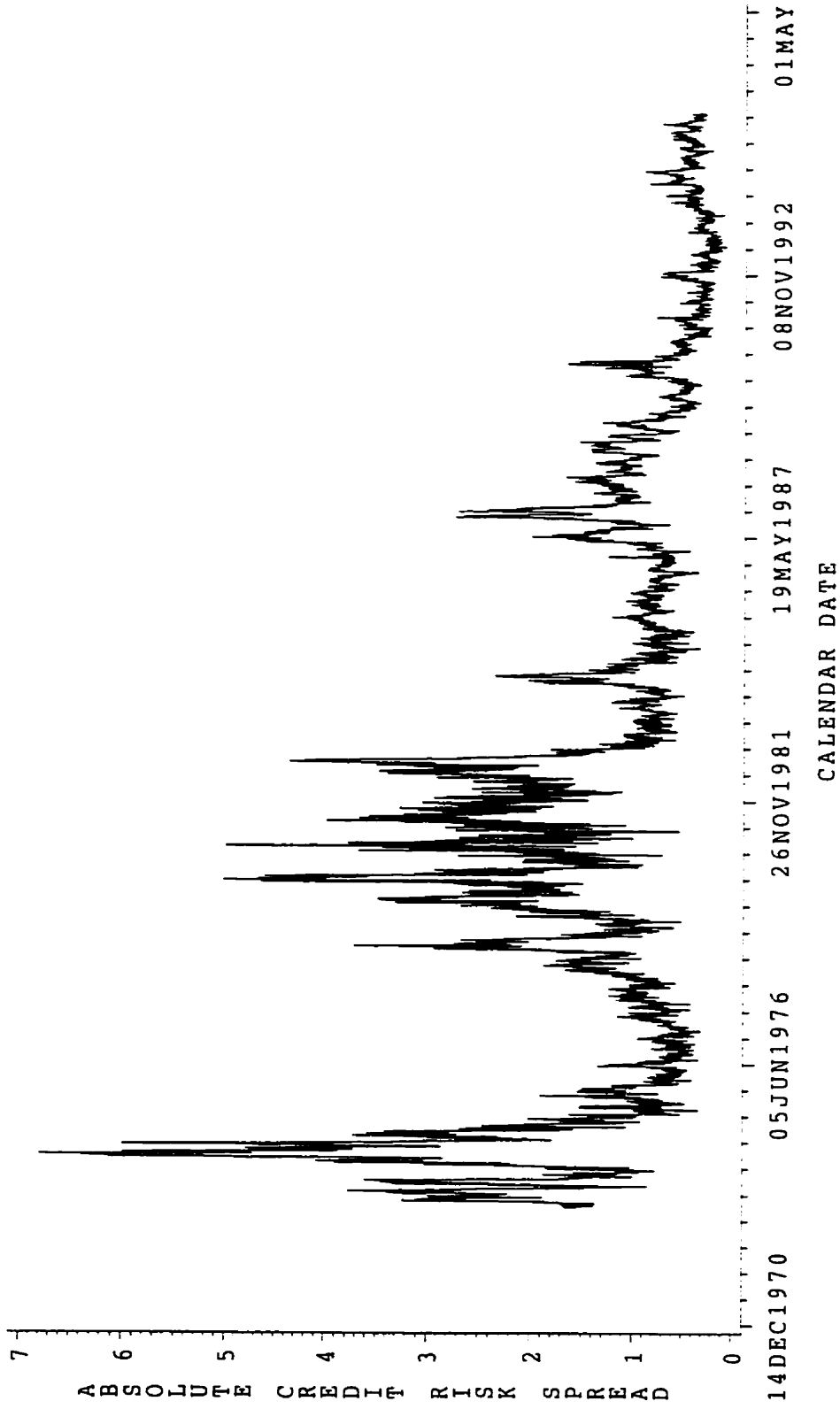
June 1, 1973 to August 19, 1996

FIGURE 1:   On the 3-Month Eurodollar Deposits

The Relative Credit Risk Spread
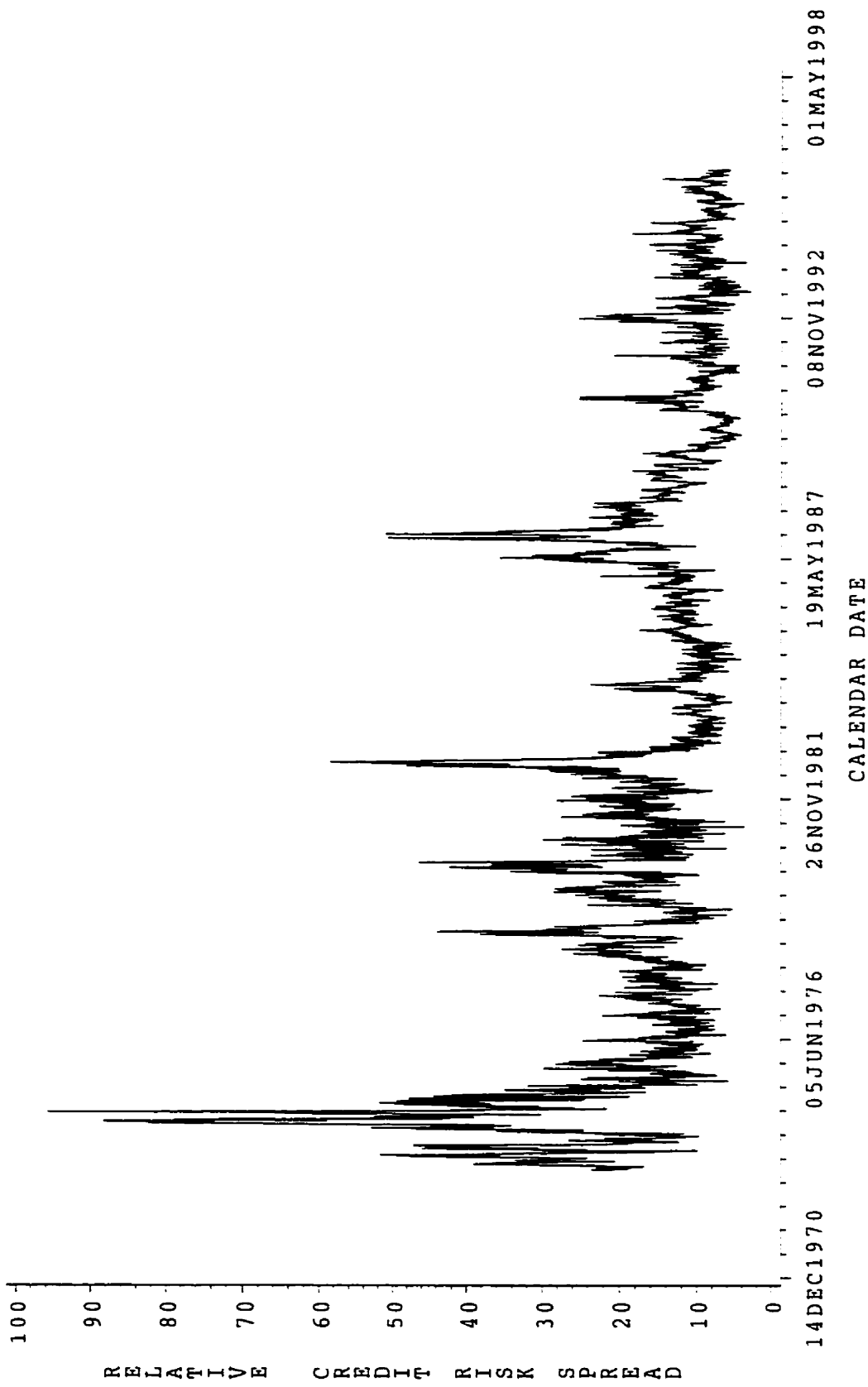During June 1, 1973 to August 19, 1996

FIGURE 2: On the 3-Month Eurodollar Deposits

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The Yield on 3-Month US Treasury Bills
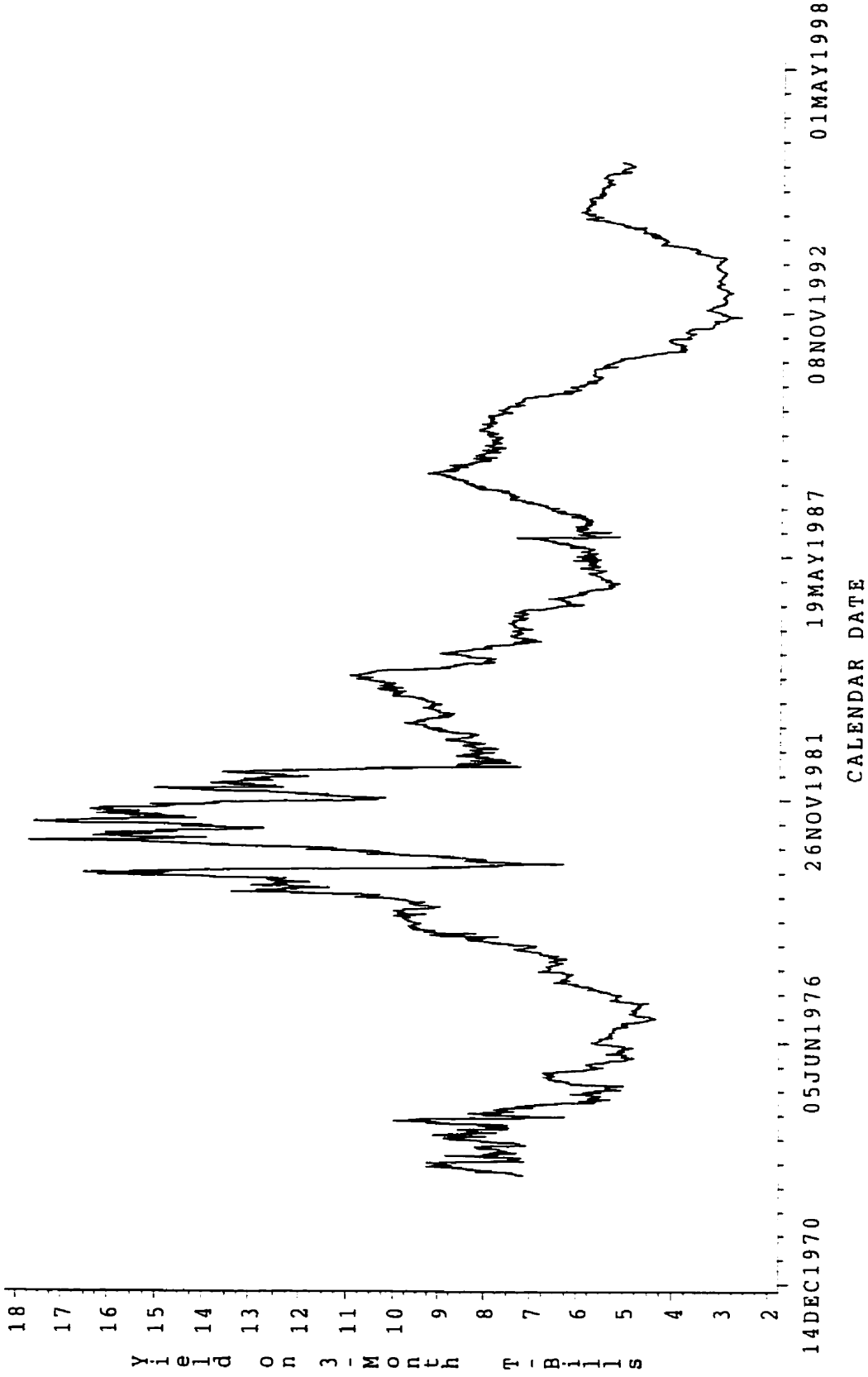During June 1, 1973 to August 19, 1996

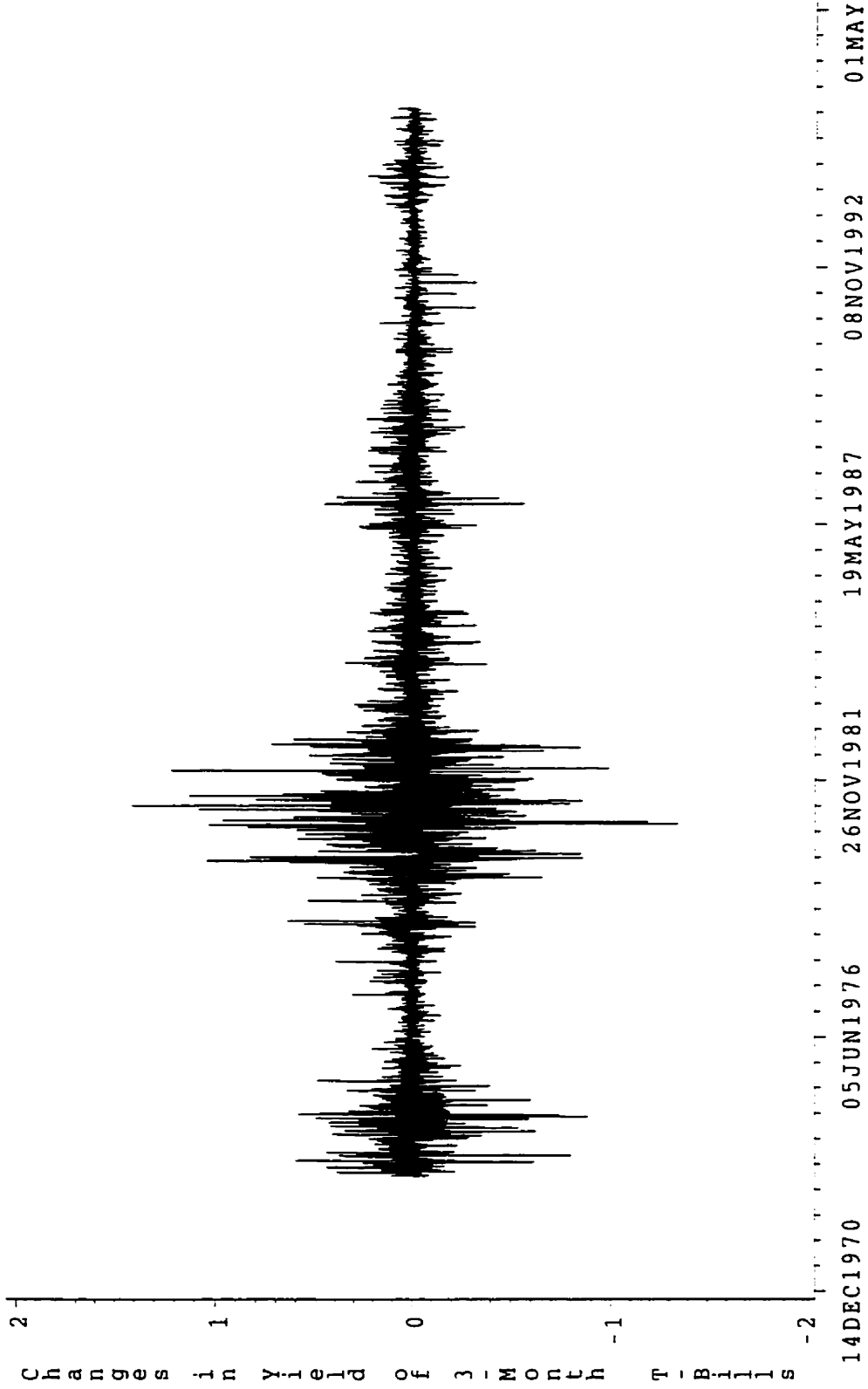FIGURE 3: The Level of Yield on 3-Month US T-Bills

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The Change in the Level of Yield on 3-Month US Treasury Bills

During June 1, 1973 to August 19, 1996

CALENDAR DATE

The Change in Level of Yield on 3-Month US T-Bills

FIGURE 4:

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The Yield Curve Slope at Short–Term End

During June 1, 1973 to August 19, 1996

FIGURE 5: The Yield Curve Slope at the Short-Term End. (Diff. 12- and 3-Month Yields)

The Yield Curve Slope at Long—Term End

During June 1, 1973 to August 19, 1996
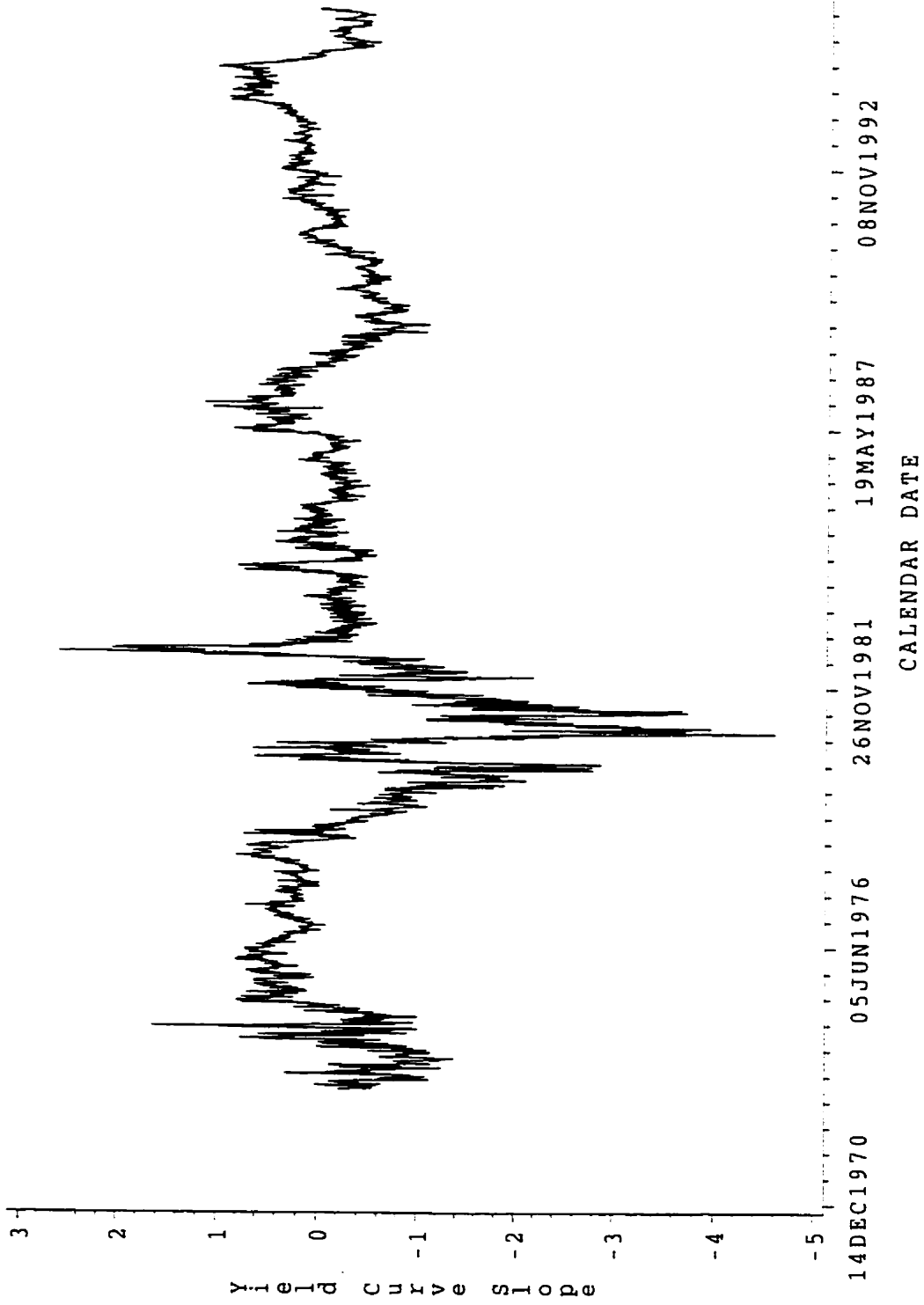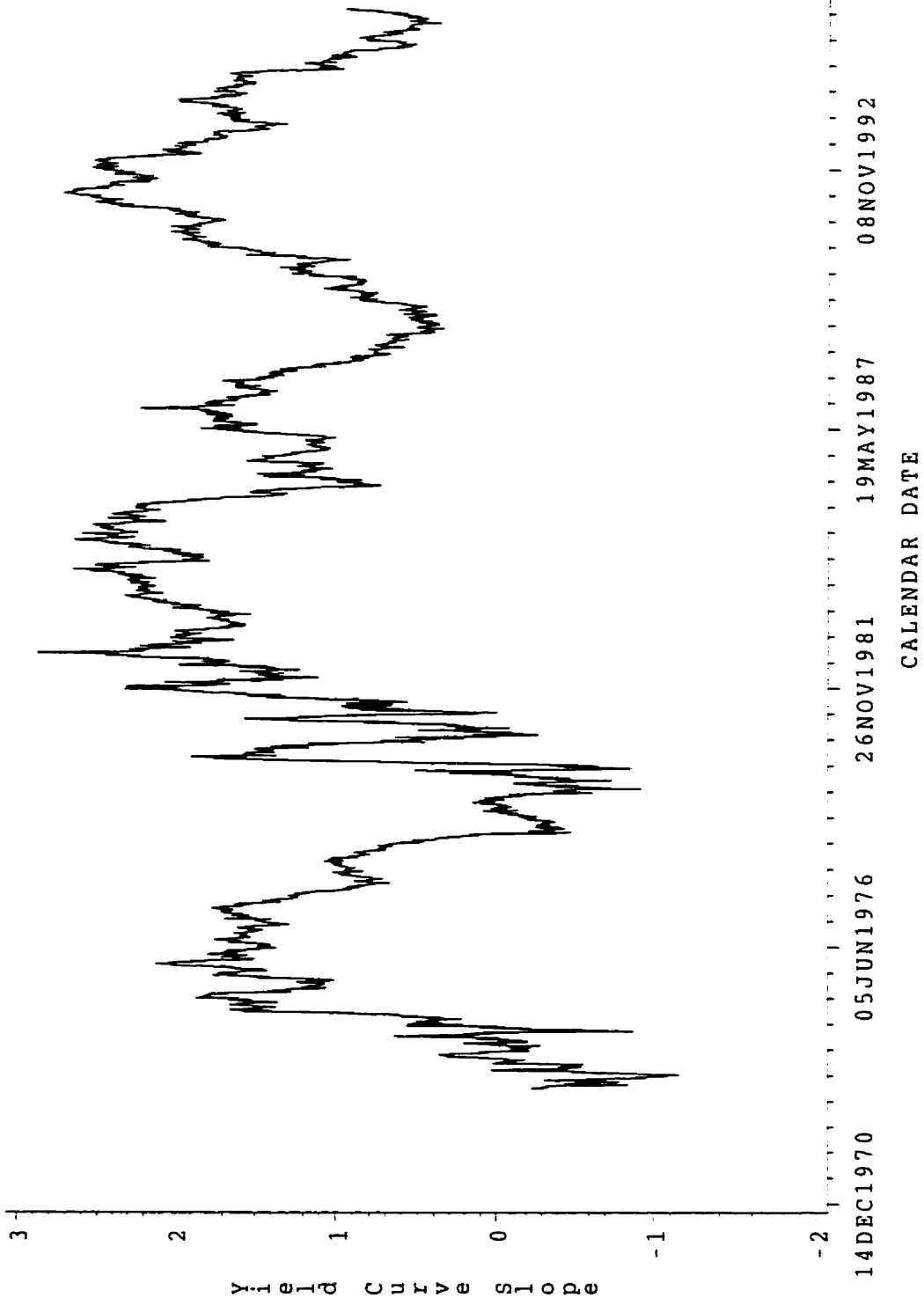

FIGURE 6: The Yield Curve Slope at the Long-Term End. (Diff. 60- and 12-Month Yields)

The Yield Curve Slope Volatility

During June 1, 1973 to August 19, 1996
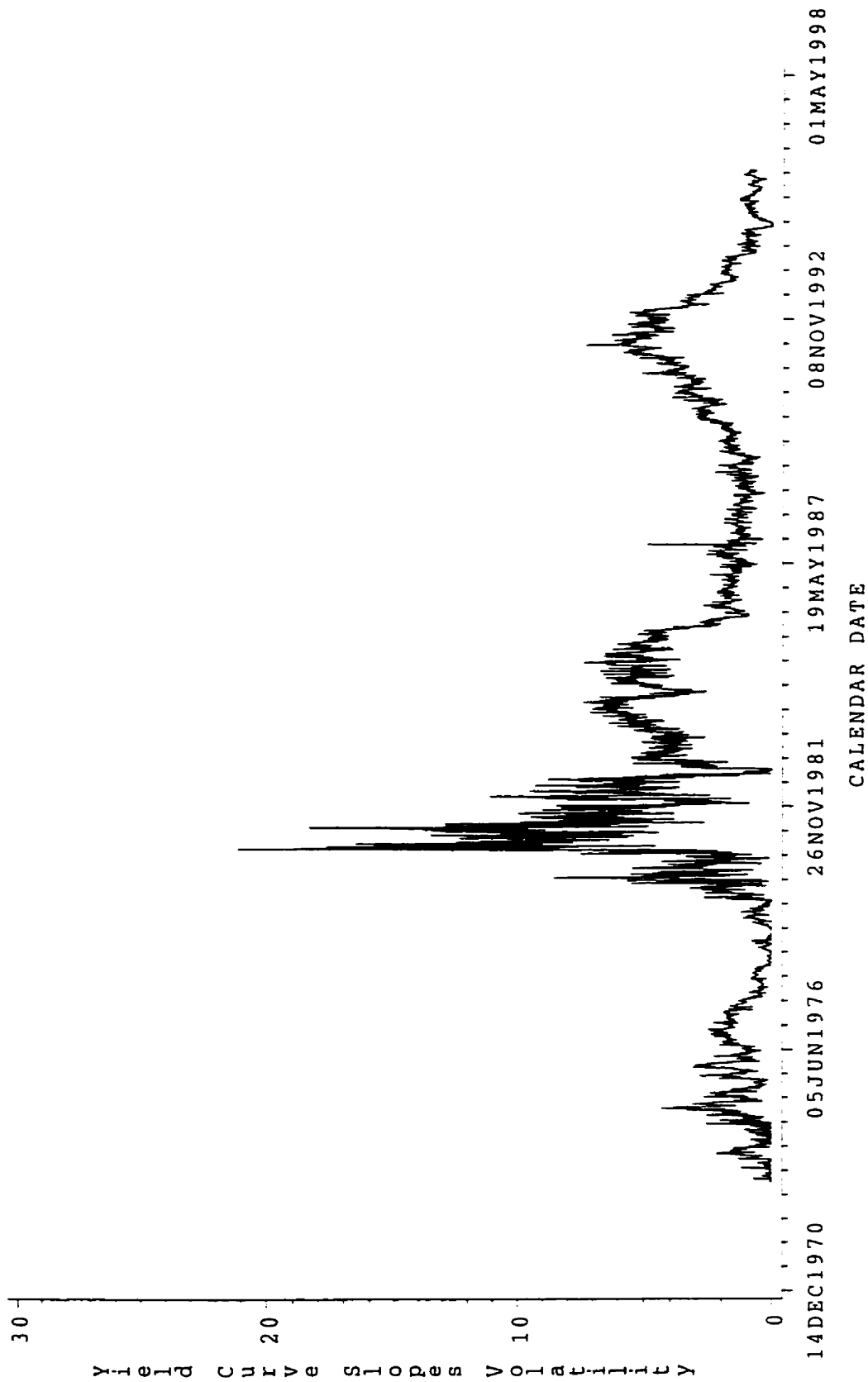
FIGURE 7: The Yield Curve Slope Volatility in the US Treasury Securities Market

The Level of Foreign Exchange Market Index

During June 1, 1973 to August 19, 1996
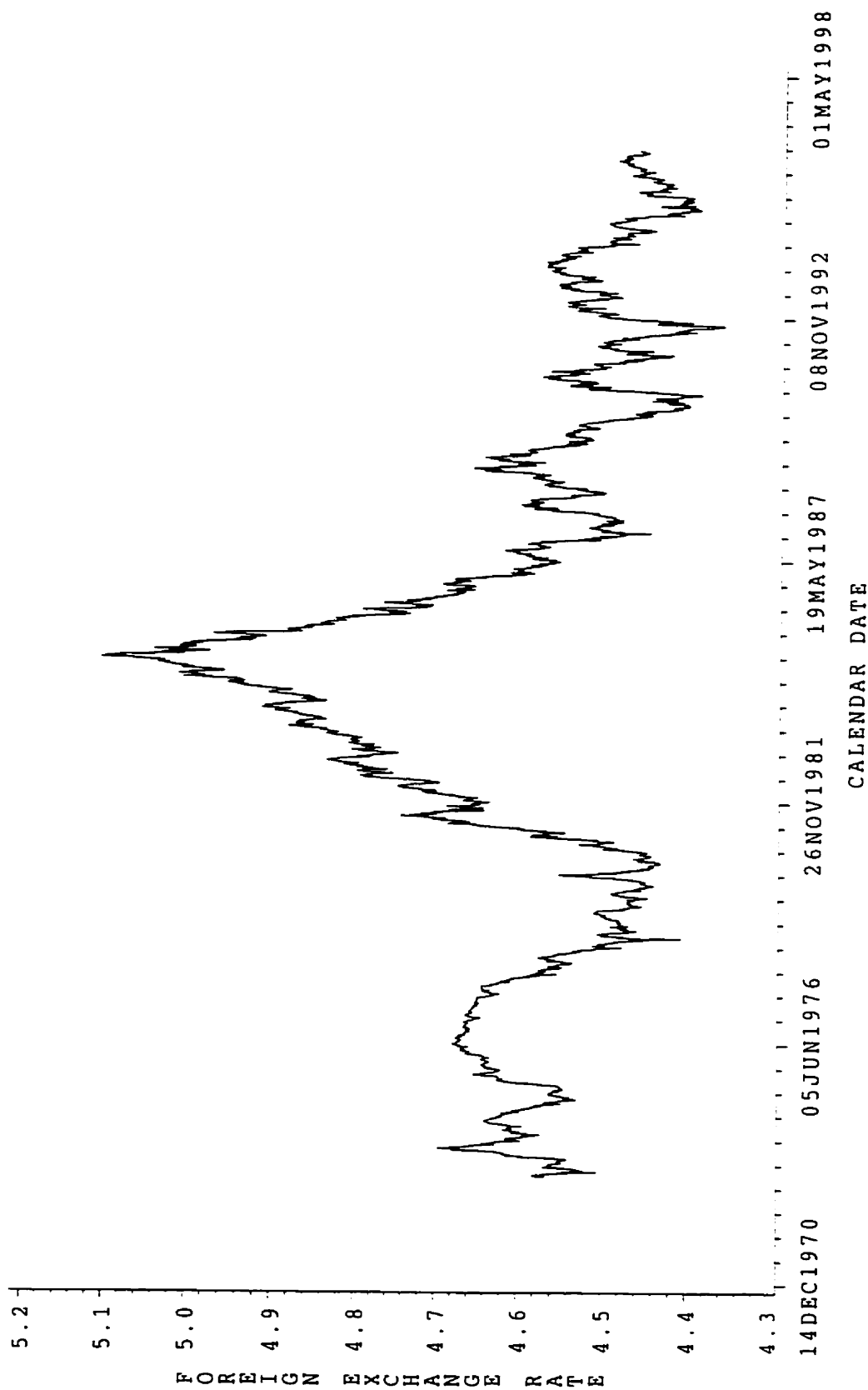
FIGURE 8:   The FOREIGN EXCHANGE MARKET BEHAVIOR

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The Change in the Level of Foreign Exchange Market Index

During June 1, 1973 to August 19, 1996

CALENDAR DATE
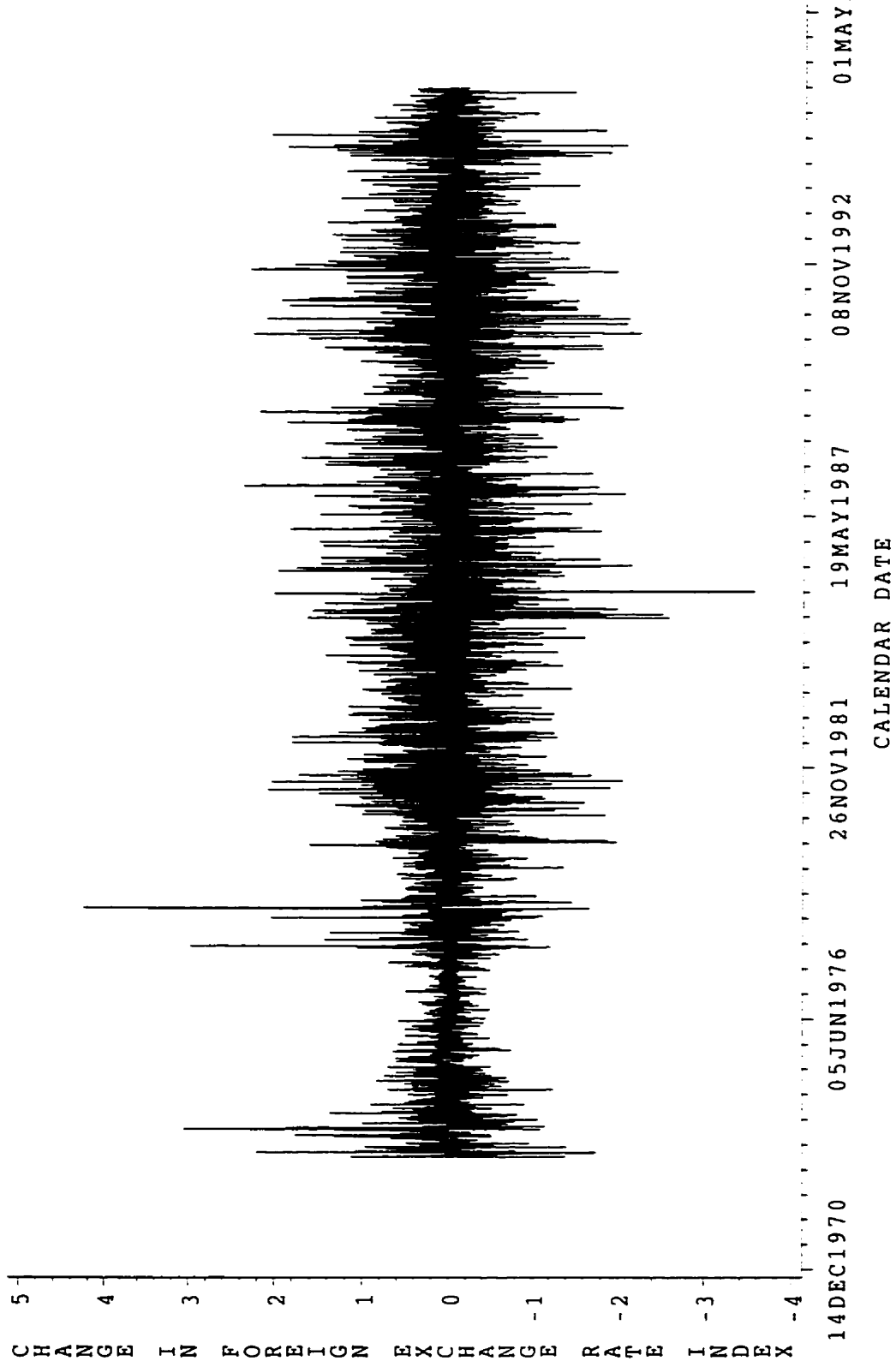
FIGURE 9: The FOREIGN EXCHANGE MARKET BEHAVIOR

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The Foreign Exchange Rate Market Volatility

During June 1, 1973 to August 19, 1996

104
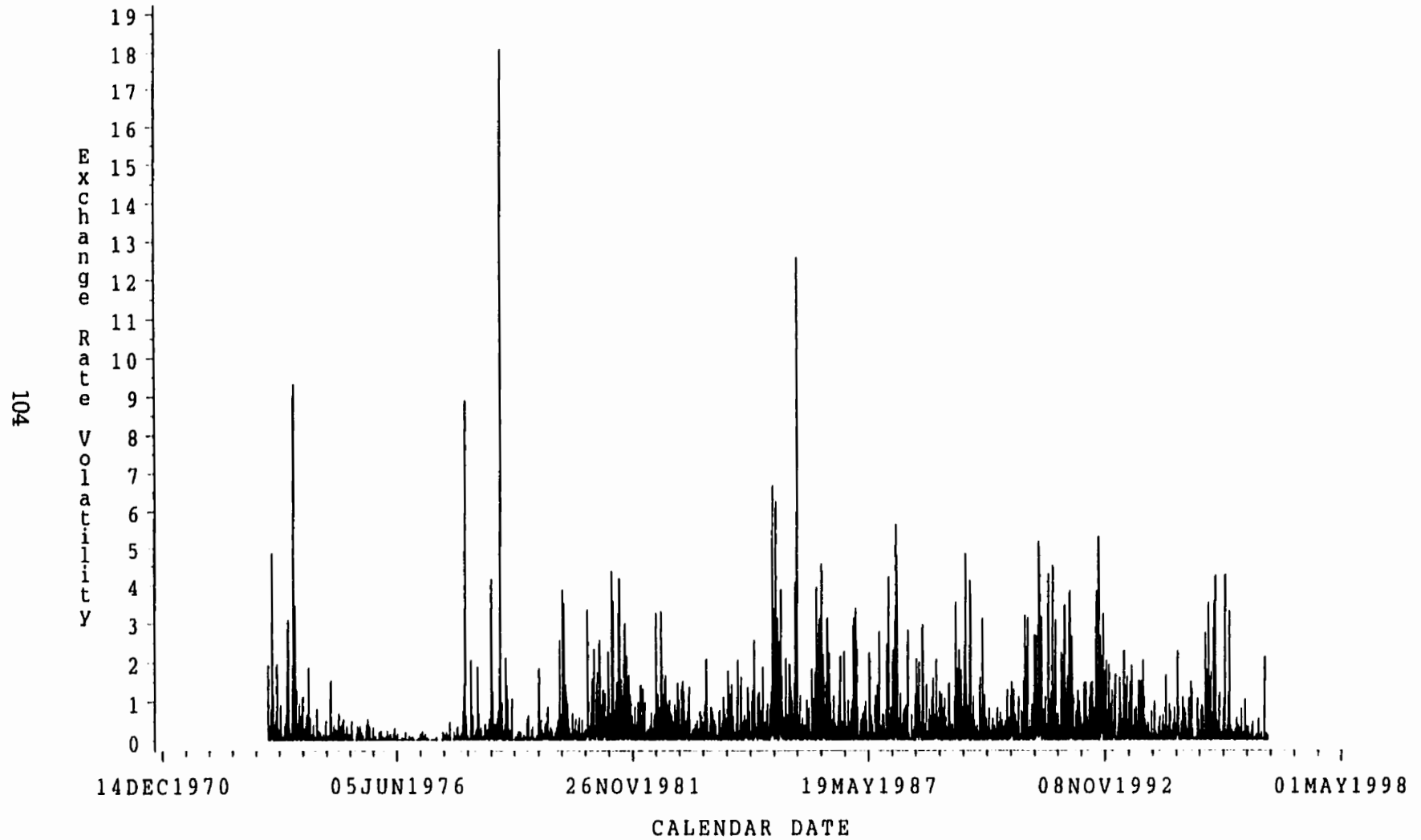
FIGURE 10:   Multi-lateral Exchange rate of US$ to G-10 Countries

The Level NYSE Common Stock Price Index

During June 1, 1973   to August 19, 1996

FIGURE 11:   The STOCK MARKET BEHAVIOR

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The Change in the Level of NYSE Common Stock Price Index

During June 1, 1973 to August 19, 1996

CHANGE IN NYSE STOCK PRICE INDEX

CALENDAR DATE

14DEC1970   05JUN1976   26NOV1981   19MAY1987   08NOV1992   01MAY1998
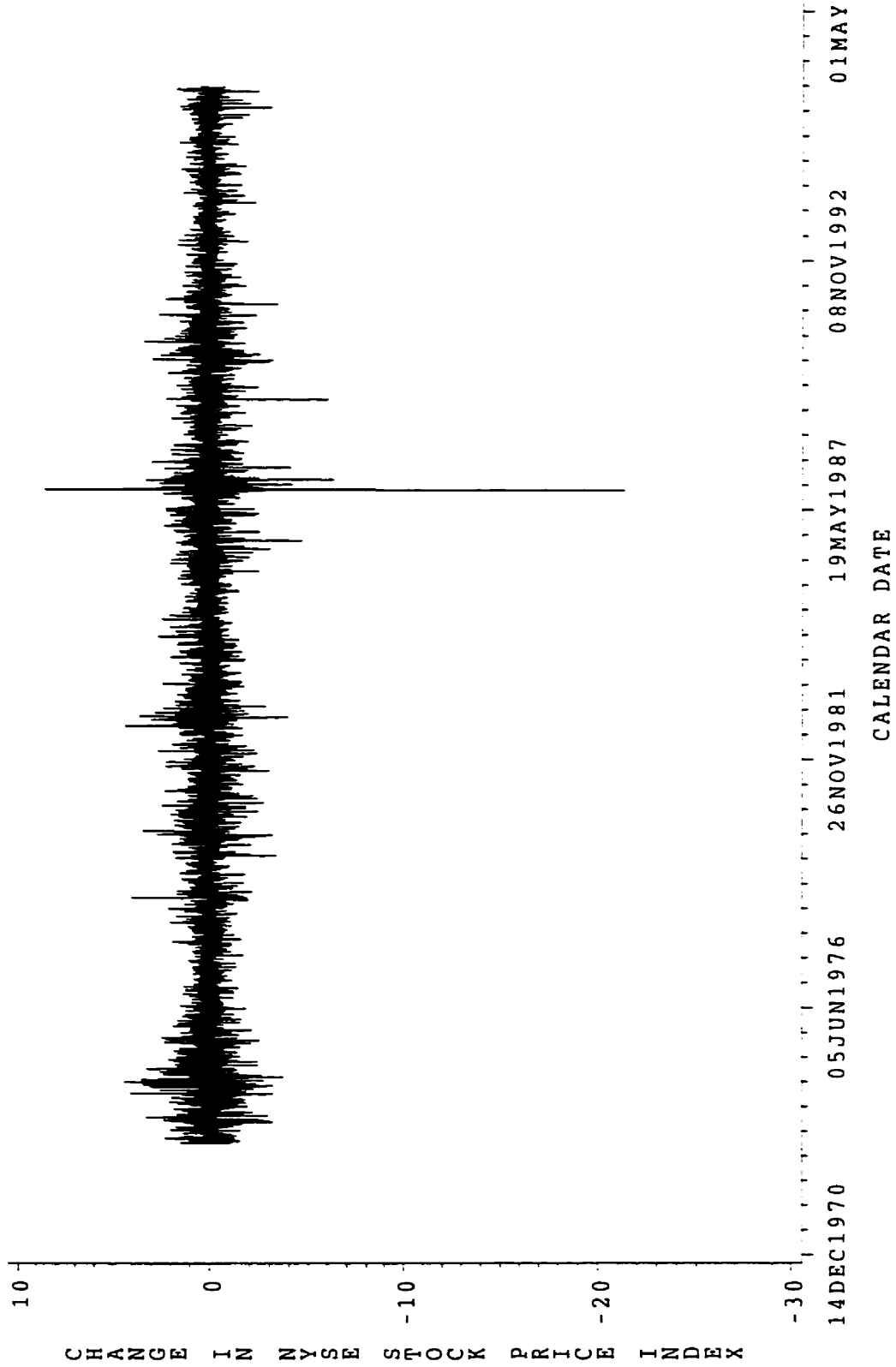
FIGURE 12:   The STOCK MARKET BEHAVIOR

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The Stock Market Volatility

During June 1, 1973 to August 19, 1996

The NYSE Composite Common Stock Index

CALENDAR DATE

FIGURE 13:

107
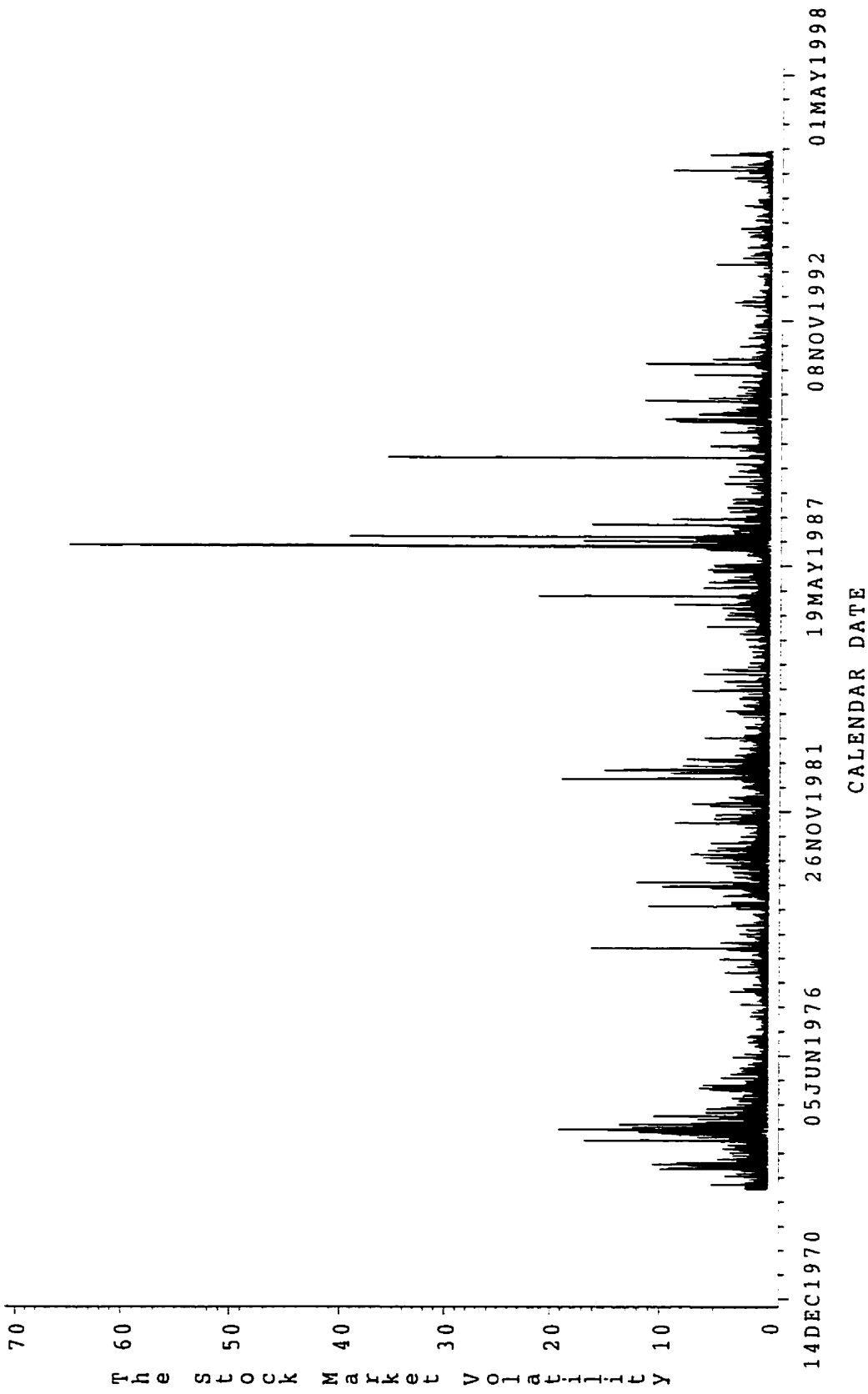
The 7-Day Federal Funds Rate Level
During June 1, 1973 to August 19, 1996
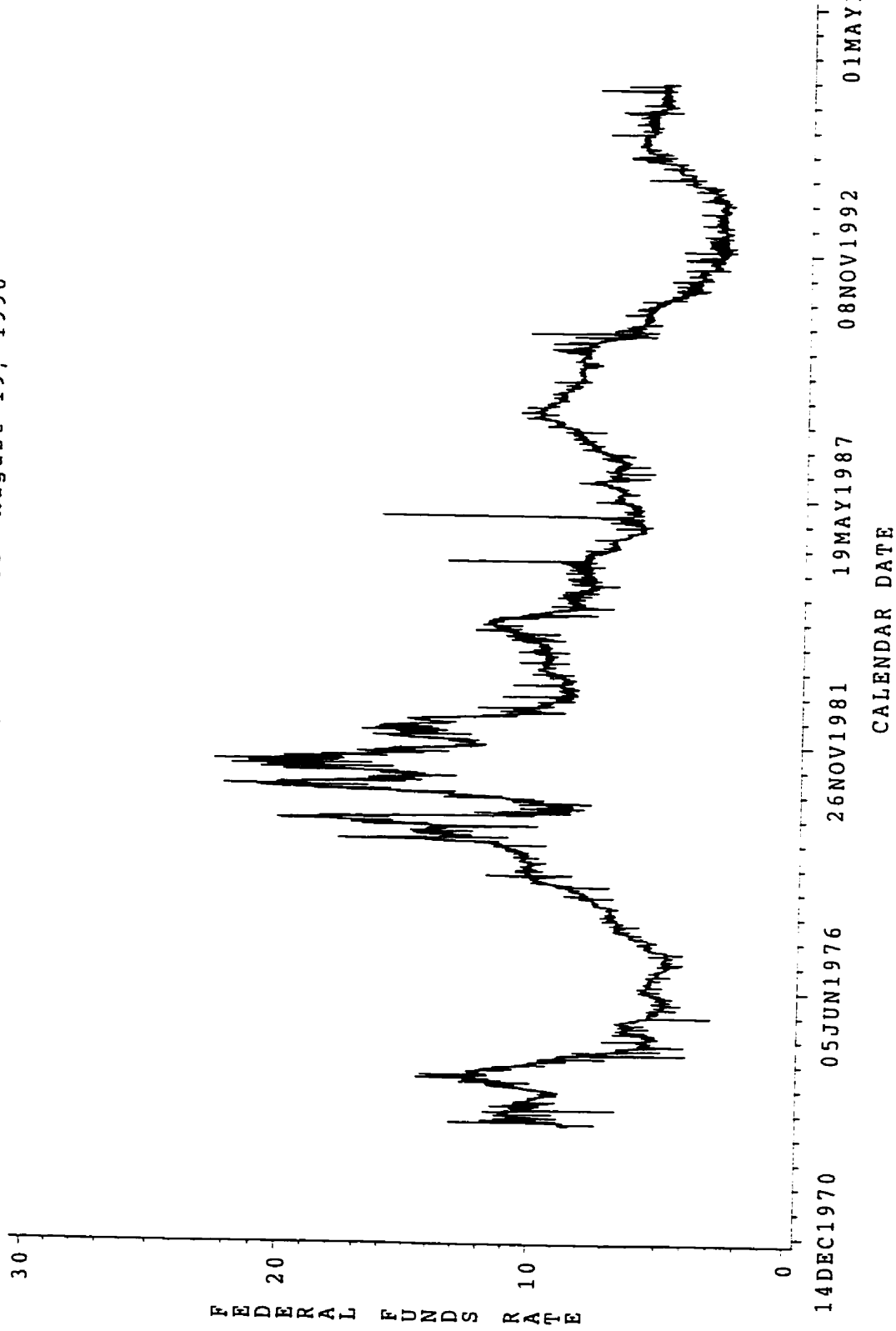
FIGURE 14: The FEDERAL FUNDS MARKET BEHAVIOUR

The Changes in the 7-Day Federal Funds Rate
During June 1, 1973 to August 19, 1996
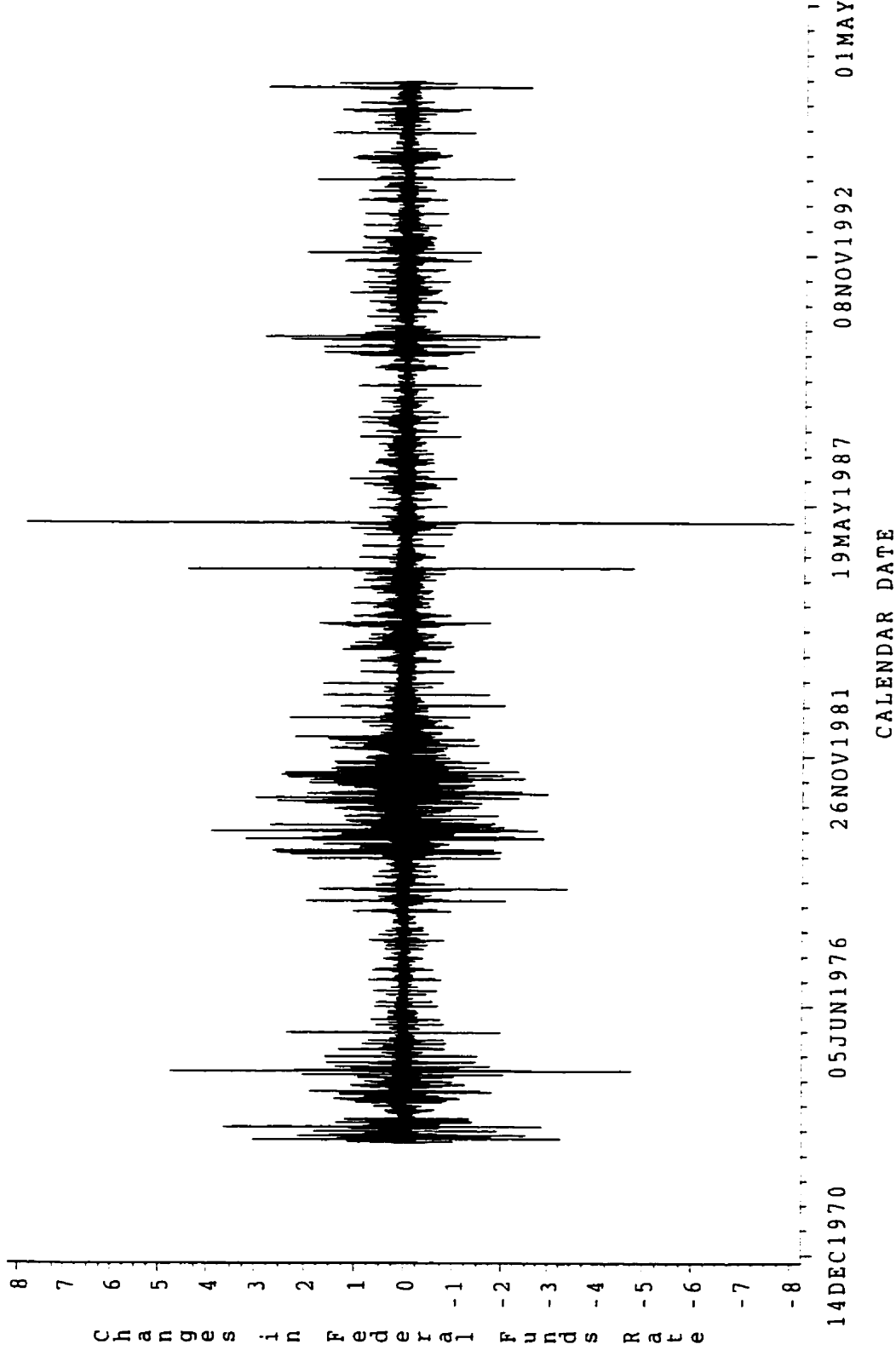
FIGURE 15: The Yield on 7-Day Federal Funds Market

The 3–Month Eurodollar Deposit Rate Volatility

During   June 1, 1973   to   August 19, 1996
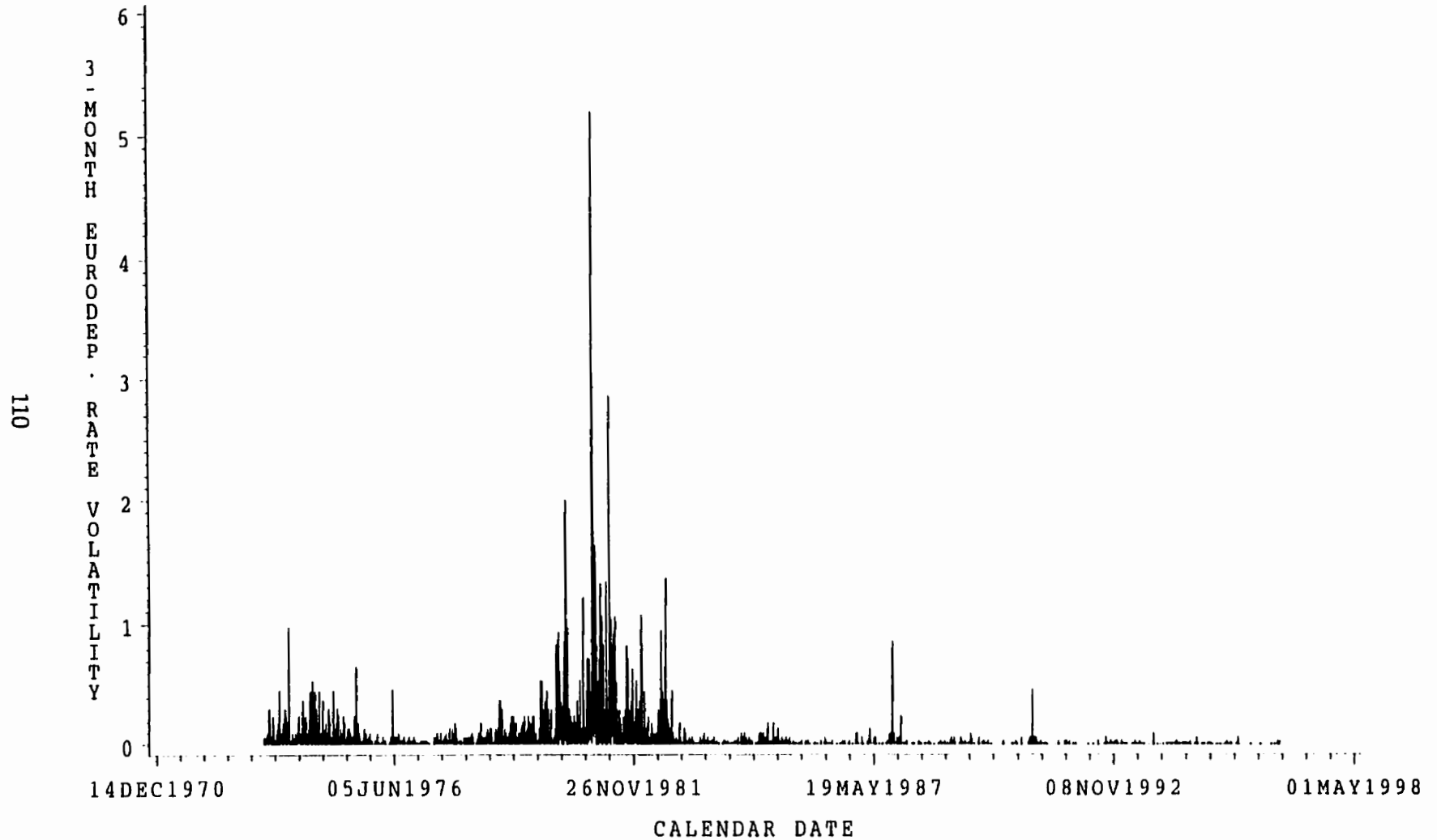
FIGURE   16:   The EURODOLLAR VOLATILITY BEHAVIOUR

The CHANGE IN 3-MONTH EURODOLLAR DEPOSIT RATE

During June 1, 1973 to August 19, 1996
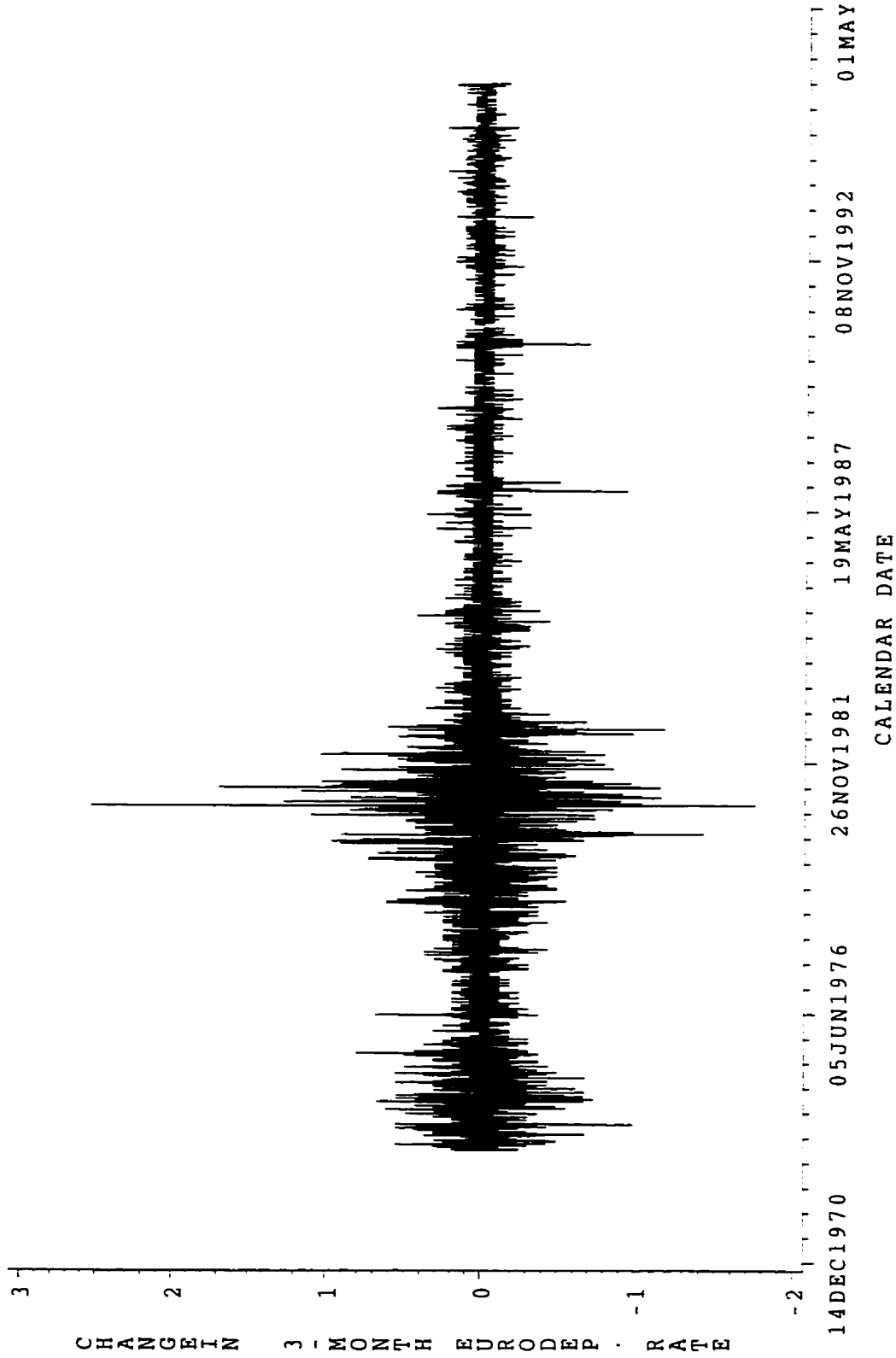
FIGURE 17: The CHANGE IN EURODOLLAR RATES

The LEVEL 3−Month Eurodollar Deposit Rate
During June 1, 1973 to August 19, 1996
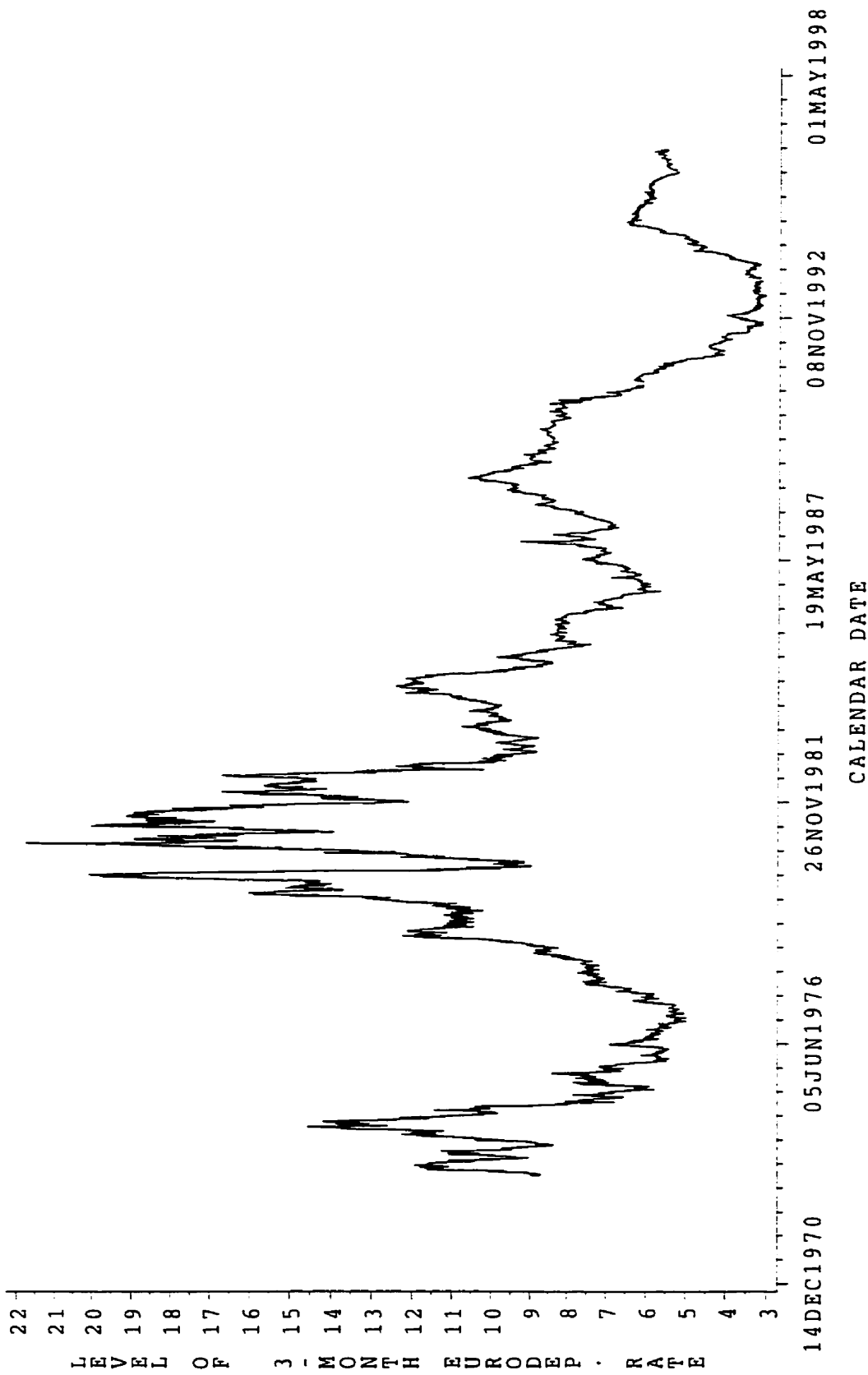
FIGURE 18: The LEVEL OF EURODOLLAR RATES

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The Out−of−Sample Forecast of Credit Risk Spread

January 1, 1995 to August 19, 1996

key: ······ Credit Risk Spread (%)   —— Predicted Credit Risk (%)
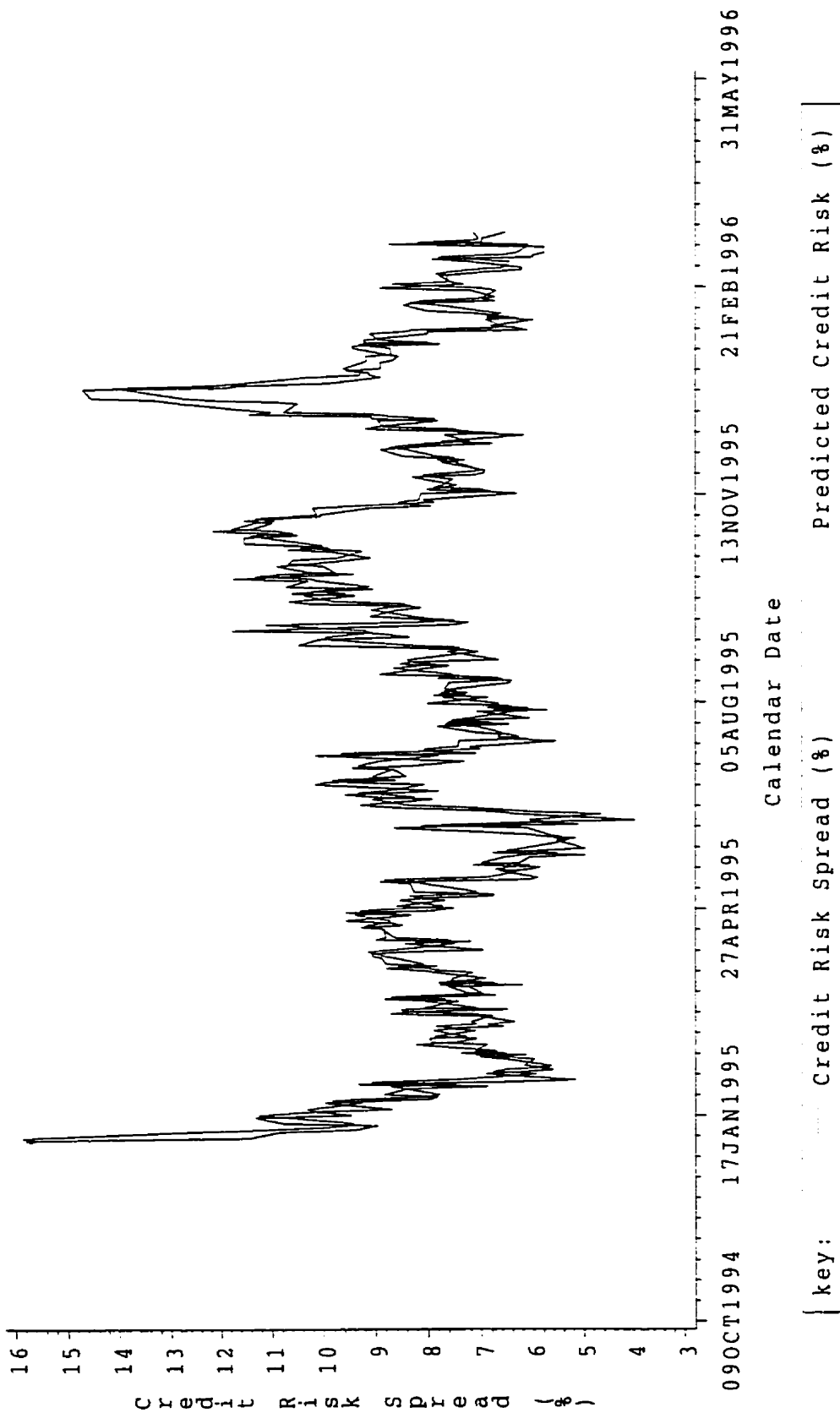
FIGURE OSF1:   One-step ahead forecast of credit risk spread in Eurodollar market
               Using All Financial Market information

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The Out-of-Sample Forecast of Credit Risk Spread

January 1, 1995 to August 19, 1996

key: ⋯⋯ Credit Risk Spread (%)　　　　Predicted Credit Risk (%)
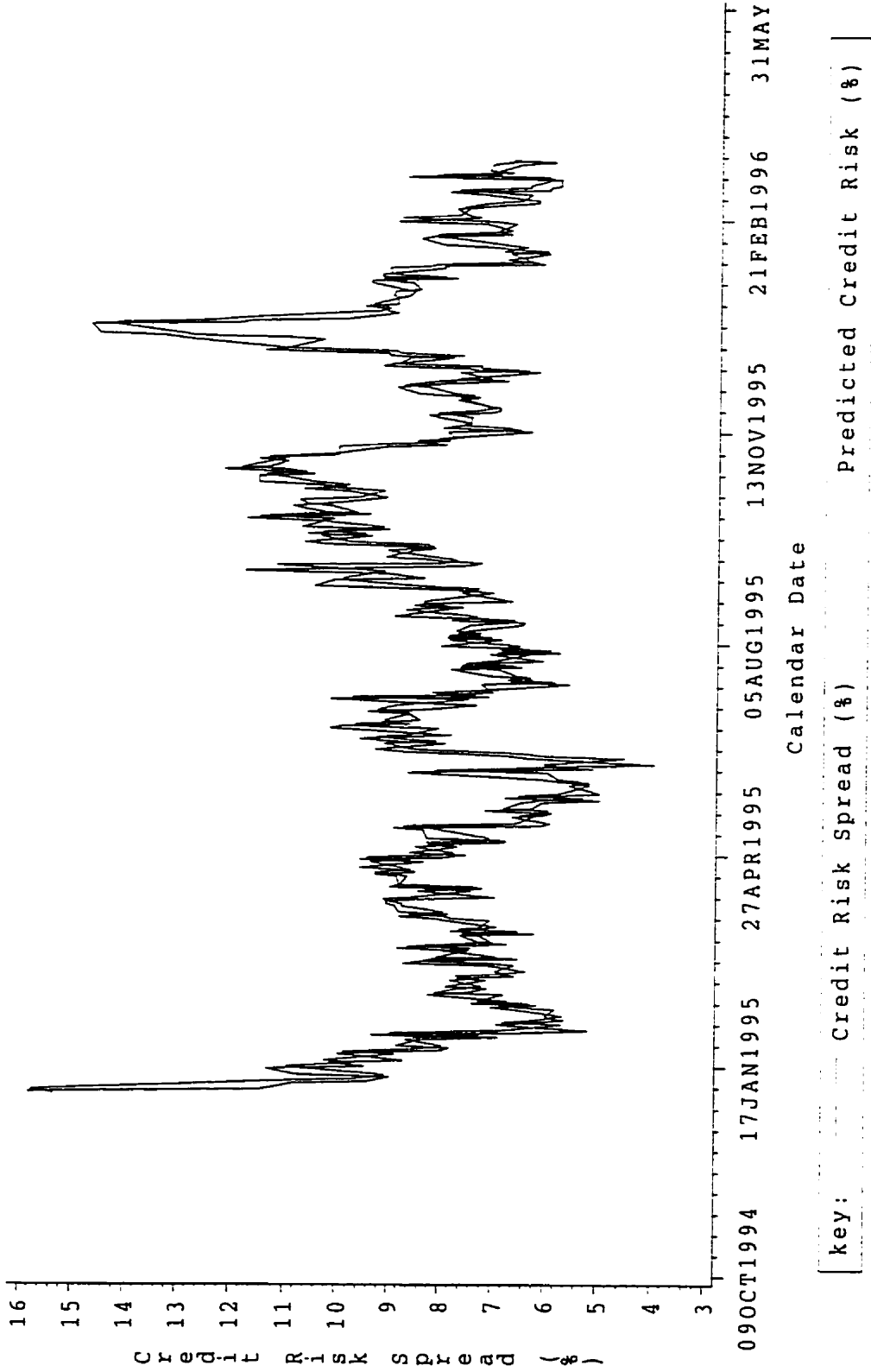
FIGURE OSF2: One-step ahead forecast of credit risk spread in Eurodollar market Using Yield Curve information

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The Out—of—Sample Forecast of Credit Risk Spread

January 1, 1995 to August 19, 1996
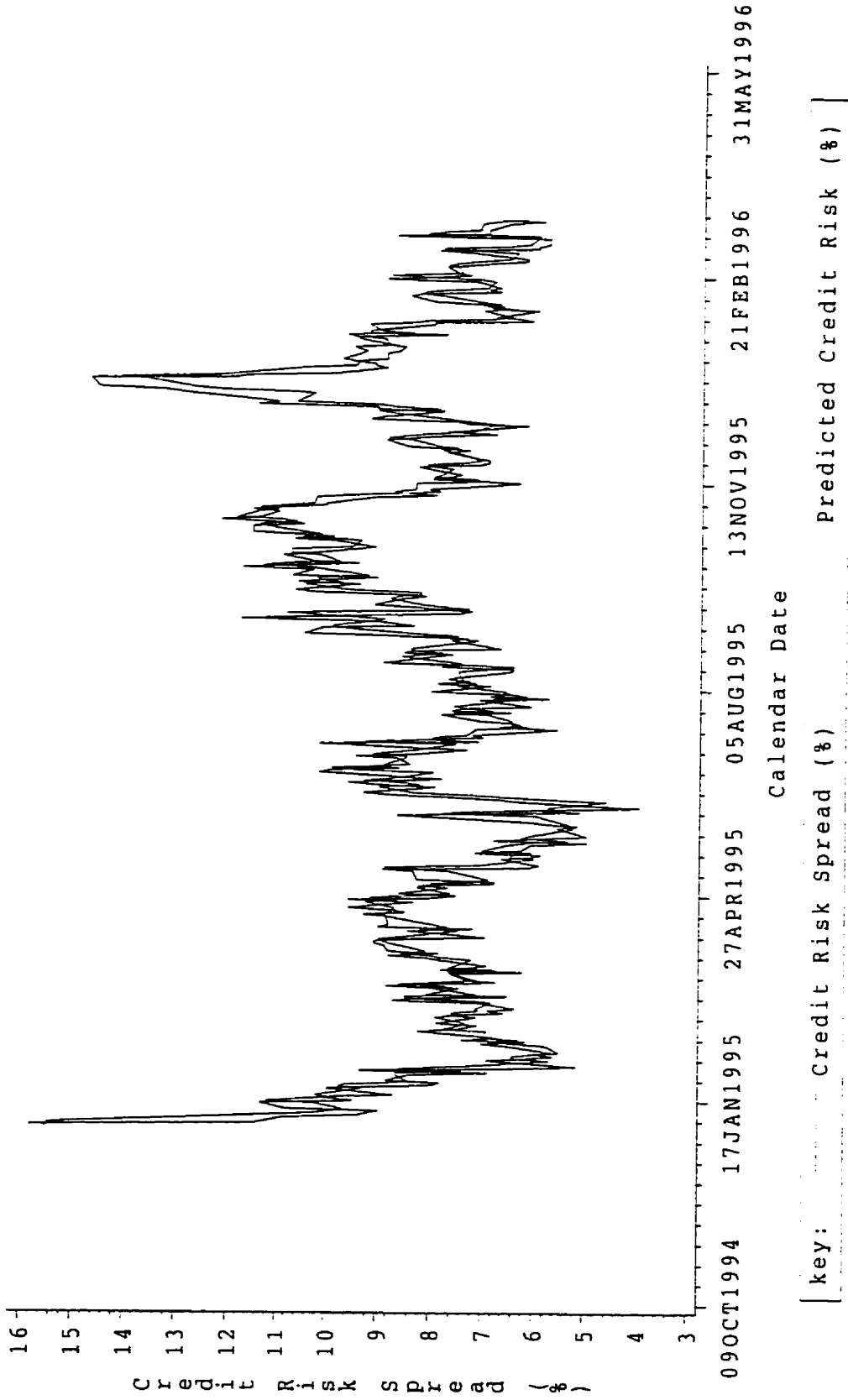
FIGURE OSF3: One-step ahead forecast of credit risk spread in Eurodollar market Using Credit Risk Time Series information

key: ········ Credit Risk Spread (%)   Predicted Credit Risk (%)

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The Out—of—Sample Forecast of Credit Risk Spread
January 1, 1995 to August 19, 1996

key: ------ Credit Risk Spread (%)   Predicted Credit Risk (%)
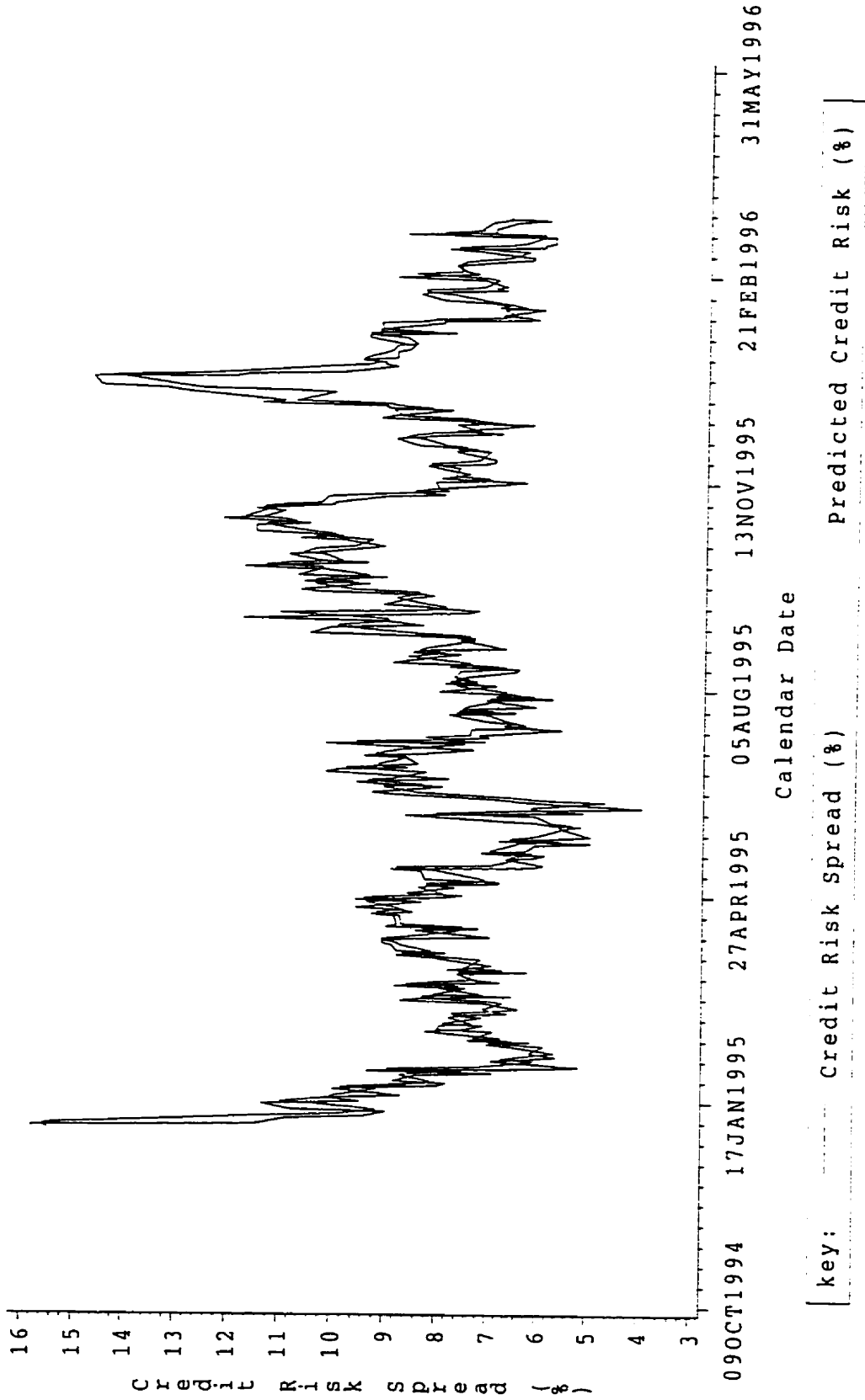
FIGURE OSF4: One-step ahead forecast of credit risk spread in Eurodollar market Using Other Financial Market Information

Part II

ESSAY #II

Modeling the Volatility of Interest rate in the Eurodollar Market

Abstract

This essay empirically examines the volatility of the short-term interest rate in the Eurodollar market. The period examined extends from January 1, 1973 through August 19, 1996. The principal purpose of the essay is to investigate the predictive ability of the models within the continuous-time family, the (G)ARCH family, and the factor-ARCH family. Within the factor-ARCH family, attention is focused on models that use directly observable financial market information rather than the latent-variable or unobservable-factor models. In order to investigate the additional benefit that accrues to using observable financial market information over the models that use just the previous level of interest rate, or the combination of the previous predicted volatility and innovations, three evaluation criteria were employed. These criteria are: the out-of-sample mean square prediction error, the out-of-sample forecast encompassing test, and the N-fold cross-validation mean square prediction error. The N-fold cross-validation test method, suggests that the factor-ARCH model that uses directly observable financial market information best predicts the future volatility. That is, that the factor model has, on average, the least out-of-sample forecast error among the class of models examined. This result thus indicates that the volatility forecast produced by the factor-ARCH model may provide a more accurate estimate of future volatility for use in the pricing of financial assets than the estimate provided by the continuous time based models and the (G)ARCH family of models. In addition, the factor-ARCH model is the only model whose out-of-sample forecast error cannot be explained by the other models' out-of-sample forecast. On this basis, the factor-ARCH model is ranked superior to other interest rate models.

Chapter 3

The Interest Rate Volatility

3.1 Introduction

The stochastic process followed by the interest rate plays a critical role in financial analysis, in particular, in the determination of an asset or a portfolio's value-at-risk (VaR), the valuation of interest-rate-dependent securities, and the general management of a fixed-income portfolio.[1] Because of the key roles played by the interest rate process, considerable effort has been devoted in the literature to developing a model that best describes its stochastic behaviour. As a

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[1]The value-at-risk (VaR) of an asset or a portfolio refers to the amount that can be lost given a normal business operation within a specific period of time and with a given confidence level. The stochastic process governing the underlying state variable is required for the purpose of simulating its possible paths when calibrating the amount that can be lost under a normal business operations. For a more detailed analysis of VaR or stress testing see, for example, Jorion (1997), Phelan (1995) and J. P. Morgan (1995).

Examples of interest-rate-dependent securities include options, swaps, swaptions, forward and futures, bonds, commercial papers, certificates of deposit and other types of fixed-income securities. The value of all these securities depend on the moments—the second moment in particular—of the interest rate. See, for example, Abken and Nandi (1996), and Lo and Wang (1995). Besides, having the knowledge of whether the interest rate will rise, fall, or become more volatile in the future is crucial for determining whether to take a long or a short position in financial contracts that can be used in off-setting the expected losses from changes in interest rates levels. For instance, Fong and Vasicek (1991) illustrate how volatility affects risk and returns of fixed-income securities, and how to manage it using the knowledge of interest rate movements.

consequence, there now exists a multitude of these models describing the dynamic behaviour of interest rates, particularly the rates on the Treasury securities. However, given the number of models available for describing the evolution of interest rates, there is an apparent problem in determining the most appropriate model to use in the context of calibrating an asset or portfolio's VaR, or in determining how much a particular asset might be worth at a point in time. In addition, the majority of the models that have been proposed use only the information from one asset market–the bond market. Thus these models all implicitly assume that the bond market is independent of the other financial assets markets, or that it is not sensitive to macroeconomic factors. Because of these limitations, I investigate–using statistical methods– an alternative model of interest rate process in the Eurodollar market. The model developed here is also evaluated and compared, in terms of its out-of-sample predictive ability, with the most commonly used interest rate processes.

The purposes of this study are, first: to formulate a volatility model that uses a set of financial market information for predicting the volatility of the interest rate in the Eurodollar market; second: to evaluate the forecast efficiency of the model developed here in relation to other volatility models that are frequently employed in modeling interest rate volatility, and third: to determine from these alternative models, the model that best predicts interest rate volatility in the Eurodollar market. As mentioned, an investigation into the foregoing issues is relevant because the volatility dynamics, and the estimates thereof, are both fundamental to the pricing of financial assets. They are also fundamental to assessing the value of the asset or portfolio that can be lost on a normal trading day; and in choosing among alternative strategies in managing a portfolio of fixed-income securities. As such, having an understanding of the volatility dynamics that best fit–out-of-sample–the interest rate data in the Eurodollar market is desirable, especially regarding appropriately pricing assets, and correctly assessing the asset's VaR.

An extensive literature exists on the volatility of interest rates, stock market returns, and on other financial market data. However, most of these studies treats each of the market as if they are independent of one another; and as such, they use only the information emanating from the particular asset market under consideration. For instance, Chan, Karolyi, Longstaff, and

Sanders (1992a, 1992b), Cheng (1996), Fisher and Zechner (1984), Leung, Sanders, and Unal (1992), Brenner, Harjes, and Kronner (1996), among others, used only the previous level of the interest rate to explain and predict interest rate volatility in the bond market. Similarly, Pagan and Schwert (1990), Engle and Ng (1993), and Donaldson and Kamstra (1996), among others, used only stock market information to predict the volatility of stock returns. Also, Baillie and Bollerslev (1990), and Bollerslev and Domowitz (1993) used only the information from the foreign exchange market to predict foreign exchange rate volatility. These studies thus implicitly assume that one financial asset market is segmented from the others; and as such, the information emanating from other assets markets may not be necessary to improve the forecast of future returns, and volatility in a specific market. This approach to modeling volatility is, however, inconsistent with the empirical evidence on the interdependence of financial assets markets. In this study, therefore, I take a different perspective by augmenting the information from the Eurocurrency market with the information from an array of other financial markets.

In addition, most of the existing studies evaluate the predictive power of different volatility models using the information from just one assets market. That is, that all models compared use only the information from that specific market alone. There exists almost no study that systematically evaluates the relative forecast efficiency of each of these models with models using information from two or more assets markets. This, thus, represents another gap in the literature, especially of the interest rate volatility in the Eurodollar market, which this study intends to bridge. Accordingly, I evaluate the interest rate volatility models based on the information from one particular market and models based on information from several markets. In the study, the models examined include those from the continuous-time-based family, the (Generalized) Autoregressive conditional heteroscedasticity [(G)ARCH] family, and the structural time series (Factor-ARCH) based family.

The principal model evaluation and selection criterion used to determine the most likely model generating the interest rate data observed in the Eurodollar market is the cross-validation method. This method of model selection has been used, and found to work well in other fields, such as meteorology (Hjorth and Holmqvist 1981), and forecast-combining with artificial neural networks in the stock market (Donaldson and Kamstra 1996). However,

it has not been applied to discriminating among models of interest rate processes. The cross-validation method has the advantage in that it can be used to discriminate between non-nested models, and that it requires a less-restrictive set of assumptions (other than the usual regularity conditions no further assumption is required).[2] On the downside for this evaluation method, is that it is computationally expensive. The other model evaluation criteria considered are the out-of-sample forecast encompassing test method, whose exponents are Chong and Hendry (1985), and the out-of-sample mean square prediction error.

The remainder of this essay is organized as follows. Section 3.2 presents a summary of the literature on the volatility models, particularly as they are applied to modeling the volatility of financial time series. It presents a brief discourse on the continuous-time-based interest rate models as well as the ARCH volatility models. Section 3.3 presents the factor-ARCH model developed in this study. Section 3.4 presents the data used in the study. Section 3.5 presents the estimation theory and the evaluation methods. Section 3.6 presents the empirical results, while Section 3.7 presents the summary and conclusion.

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[2]See, for example, Stoica, Eykhoff, Jannssen, and Söderström (1986), and Hjorth (1994, chap. 3) for some of the other nice optimality properties of the cross-validation method.

3.2 The Previous Research

In this section, I present a brief review of the existing literature on financial assets volatility modeling, particularly, as it applies to fixed-income securities. The section is organized into three parts. The first part, Section 3.2.1, discusses the continuous-time family which deals with models formulated in the continuous time. The second part, Section 3.2.2, examines the (G)ARCH family which deals with models that are considered as discrete time approximations to the models formulated in the continuous-time framework. The third part, Section 3.2.3, present a brief summary of the survey, the deficiencies and the limitations of the various modeling methods examined; and it concludes with an indication of the direction of this study.

3.2.1 The Continuous-Time Family

Most of the theoretical valuation models in finance are often formulated in a continuous space and a continuous time framework. In addition, the data generating process—sometimes referred to as the stochastic process—governing the evolution of the state variables on which the price of the respective securities depends is also assumed to operate in a continuous space and a continuous time framework. See, for example, the models by Brennan and Schwartz (1979), Cox, Ingersoll, and Ross (1985), Jacobs and Jones (1985), Heath Jarrow, and Morton (1992), among others. However, in practice, the price of the securities moves in a discrete space such as one thirty-second for the Treasury notes and bonds. Similarly, the data on prices, or on state variables, if directly measurable and observable, can only be sampled at discrete time periods. Because of these problems, most empirical implementations of the continuous-time models often resort to using the discrete time analogue of the models contemplated in a continuous-time space. Moreover, it is often assumed that the finer the time step in the discrete-time space, the closer the approximation is to the continuous time.

The discrete-time analogue to the continuous-time based[3] interest rate process is defined

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[3]The continuous time process of interest rate (r_t) is defined by the following stochastic differential equation:

$$dr_t = \kappa(\alpha - r_t)dt + \sigma r_t^{\frac{\theta}{2}} dz(t); \qquad dz(t) = \sqrt{dt}\eta_t; \qquad \eta_t \sim \mathcal{N}(0, 1)$$

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below by the following stochastic difference equation. See, for example, the studies by Chan, Karolyi, Longstaff and Sanders (1992a, 1992b), Cheng (1996), Fisher and Zechner (1984), Marsh and Rosenfeld (1983), Tse (1995) among others.

$$\Delta r_t = \alpha_0 + \alpha_1 r_{t-1} + \epsilon_t \tag{3.1}$$

$$\epsilon_t = \sqrt{h_t}\eta_t \qquad \eta_t \sim \mathcal{N}(0.1) \tag{3.2}$$

$$h_t = \sigma^2 r_{t-1}^{\beta} \tag{3.3}$$

where:

Δr_t : is the change in the level of interest rate between successive time periods; that is, between time t and $t-1$, $(r_t - r_{t-1})$.

r_{t-1} : is the level of interest rate in the previous period, time $t-1$.

ϵ_t : is the random error in period t.

Equation (3.1) above decomposes the change in the level of the interest rate into two components: The systematic drift per unit of time $(\alpha_0 + \alpha_1 r_{t-1})$, and the zero mean random component (ϵ_t). It can be observed from this equation that the drift component evolves overtime; and moreover, it varies with the interest rate level observed in the previous period.[4] On the other hand, the random component is described by equation (3.2): it has an expected value of zero; it is orthogonal to the once-lagged level of interest rate; it is serially independent over

where the mean level towards which interest rate reverts is denoted by α; the rate at which interest rate reverts to its mean level is denoted by κ; the random error which follows a Wiener process is denoted by $dz(t)$, it has a mean value of zero and a dt variance per instant; and the sensitivity (elasticity) of interest rate volatility to the interest rate level is represented by β.

The first part of the dr_t equation describes the instantaneous conditional mean while the second part describes the instantaneous random component. The instantaneous conditional variance $(\sigma^2 r_{t-1}^{\beta})$ is time-varying, and it depends only on the level of interest rate.

[4]In the empirical analysis presented later, other financial market information is included as a factor driving the instantaneous drift term. But, each set of information from the other markets tests to be statistically insignificant at the five per cent level. As a result, and in consonance with the existing literature, I use equation (3.1) throughout the analysis for the drift term. This treatment should enable me to directly compare the various volatility models as this is all that differentiates one model from the other.

time; and it has the variance, h_t, per unit of time. The variance, h_t, also varies overtime, and its behaviour is as indicated by equation (3.3). From these equations, it can be observed that the behaviour of the interest rate volatility is also assumed to be governed by the level of the interest rate in the previous period. The above relationships thus form the core of the discrete-time approximation to the continuous-time models which are widely used in the empirical finance literature.

The interest rate process defined by the equations above corresponds to the constant elasticity of variance model developed by Cox and Ross (1976) in the context of modeling stock price dynamics. The equations also nest other interest rate processes that are frequently used in the pricing of fixed-income securities, options, swaps, futures and forwards, and other forms of interest-rate-dependent securities. For example, if specific restrictions are imposed on the parameters of equations (3.1) and (3.3), then one can derive the following interest rate models:[5]

Restriction(s)	:	Result in:
$\beta = 0$,	:	the Vasicek (1977) mean-reverting model.
$\beta = \frac{1}{2}$,	:	the Cox, Ingersoll, and Ross (1985) square root model.
$\beta = 1$,	:	the Brennan and Schwartz (1979) proportional volatility model.
$\beta = 0$ and $\alpha_1 = 0$,	:	the Merton (1973) random walk with drift model.
$\beta = 1$ and $\alpha_1 = 0$,	:	the geometric Brownian motion model.
$\beta = 1. \alpha_0 = 0$ and $\alpha_1 = 0$,	:	the Dothan (1978) pure random walk model.

From the foregoing analysis, we can observe that the model defined by equations (3.1) to (3.3) is general, and that it nests other models of interest rates frequently used in the calibration

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[5] A further possible extension to equations (3.1) to (3.3) above, which will not be pursued in this study, is to allow each of the parameters to be time-varying according to a deterministic or stochastic pattern.

The Brennan and Schwartz (1979) model listed below is a two-factor model. One of the factors is the short-term (instantaneous) interest rate, and the other is the long-term interest rate. The other financial market factors have been used to proxy the long-term interest rate but they are not statistically significant. Hence, using equation (3.1) to represent the drift in this instant is appropriate.

of the VaR or in the pricing of options, swaps, futures and forward contracts, bonds and other contingent claims assets.

As noted earlier, using a particular interest rate model to price securities when in fact the underlying process is governed by another can result in mispricing these securities. Furthermore, if an incorrectly specified process is used in calibrating the VaR of securities or portfolios, it can also result in a wrong assessment of the value of the security or portfolio that can be lost. Since the stochastic process generating interest rate is of such importance in the pricing of securities and in calibrating the security's VaR, it is therefore of interest to know which of the models best conforms with the observed data in the Eurodollar market.

In the subsequent parts of this essay, I consider the unrestricted model (equations (3.1) to (3.3)), and the models implied by the following restrictions, $\beta = 0. \beta = \frac{1}{2}$. and $\beta = 1$. I have focused on these models because they are the most commonly assumed processes thought to be generating the observed interest rates. This is especially so, when interest-rate-dependent securities are being priced, or the VaR of the security is being assessed.

Despite the widespread applicability of these models, it should still be noted that they have some deficiencies and limitations. The principal limitation of the models in this particular family is that they each ignore the impact of other financial market information. Also, they fail to acknowledge the effect on interest rate volatility, the effect of other financial market information reflecting the state of the economy. As a result, each of the models in this family thus implicitly assumes that the debt instruments market is segmented from the other assets market, and that it may not be directly affected by macroeconomic factors such as changes in fiscal and monetary policies, and the growth rate of the economy, among others. However, such a conclusion will be inconsistent with the empirical findings of studies such as Borio and McCauley (1996), and Bollerslev, Engle, and Wooldridge (1988), who reported that the financial assets markets are interdependent. Also, it is inconsistent with the result of Booth and Booth (1997), Fama and French (1989), and Schwert (1990) who provide evidence that the stock and bond returns are both sensitive to macroeconomic factors. Since the stochastic shocks affecting one particular asset market have effect, or Granger-causes the returns, the volatility,

or both, in other asset markets, ignoring this other information may have consequences for predicting the future volatility as well as the inferences drawn about the models parameter estimates.

Other studies recognize the deficiencies and limitations above, and have attempted to correct the problem. For example, Taylor (1994), Anderson and Lund (1995) among others, have suggested using the stochastic volatility model.[6] Nonetheless, the stochastic volatility models still neglect other economic or other financial market information that may be relevant to forecasting volatility. In fact, the problem of estimating the parameters of the stochastic volatility model is made even more complex and cumbersome than the previous deterministic volatility model.

3.2.2 The (G)ARCH Family

The second family of models frequently employed to model volatility is the autoregressive conditional heteroscedasticity model of order p, the ARCH(p) model. The model was first developed by Engle (1982) in the context of modeling inflation uncertainty or variability; and since then, it has been widely adopted in the empirical finance literature for modeling assets prices and returns volatility as well. Although the models in this class are formulated in discrete time, the theoretical results in Nelson (1990) have shown that the models are, in fact, the discrete-time analogue, or approximate, of the diffusion processes commonly applied in pricing derivative assets, or in calibrating assets' VaR.

The ARCH(p) model was later generalized and extended to the Generalized ARCH [GARCH(p,q)] and the Exponential GARCH [EGARCH] i models by Bollerslev (1986) and Nelson (1989, 1990, 1991) respectively. From these core models (the ARCH(p), the GARCH(p,q) and the EGARCH models), various other functional forms have been suggested in the literature for modeling financial assets' prices or returns variability. But, despite the

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[6]The stochastic volatility model is similar in structure to the set up above, in equations (3.1) to (3.3). The only difference is that it assumes that the σ^2 in equation (3.3) is time-varying; that is, that σ^2 becomes σ_t^2. In particular, σ_t^2 is allowed to evolve according to its own stochastic difference equation; thus, its behaviour is not deterministic.

multiplicity of functional specification, they all retain the common feature that only the information in the previous predicted volatility and the previous innovations is used in the models.[7] Following, I discuss the specification for the conditional-mean equation and the conditioning-information set, then the most commonly adopted of the ARCH models.

3.2.2.1 The Conditional Mean Model

As is the case with the continuous-time based models, the discrete-time conditional distribution of changes in the interest rate level between successive periods can be defined in terms of two parameters: the conditional mean per unit of time, and the conditional variance per unit of time. The conditional mean may be allowed to remain the same as in equation (3.1), or it may assume some other functional form.[8] Besides, equation (3.2) remains unchanged.

However, unlike in equation (3.3) where the conditional variance of the random error of equation (3.1) depended on the level of the interest rate in the previous period, the functional form of the conditional variance, h_t, now depends on the conditioning-information set, $X = \{\{\epsilon_{t-i}\}_{i=1}^n, \{\hat{\epsilon}_{t-i}^2\}_{i=1}^k, \{\hat{h}_{t-j}\}_{j=1}^m\}$. In X, the n-history of the prediction error or the innovations is represented by $\{\epsilon_{t-i}\}_{i=1}^n$,[9] the k-history of the squared prediction error, $\hat{\epsilon}_{t-i}^2$, is represented by $\{\hat{\epsilon}_{t-i}^2\}_{i=1}^k$; and the m-history of the predicted variance is also represented by $\{\hat{h}_{t-i}\}_{i=1}^m$. Other terms allowable in the information set may include the asymmetric behavior of investors when security prices falls as opposed to when it rises.

The particular parametric form taken by h_t, the analogue of equation (3.3), in discrete time is discussed next. As mentioned earlier, there are several alternative models that can be considered in this family. But in what follows, only the ARCH(p) model developed by Engle

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[7]The literature on the ARCH models is rather too extensive to be fully covered in this study. As a result, my attention is to focus on the most commonly used of the models. For an extended discussion of the ARCH models, see, for example, Bollerslev, Chou, and Kroner (1992), Engle (1993), Bollerslev, Engle, and Nelson (1994), Bera and Higgins (1993), Pagan (1996) among others.

[8]For the purposes of maintaining consistency and easy comparability of the volatility models, in the subsequent analysis, the conditional mean equation is allowed to remain unchanged. The variables augmenting the lagged interest were statistically insignificant at the five per cent level.

[9]The n-history is the sequence of a variable up to n-periods ago. The same is true of the $k-$ and $m-$histories.

(1982), the GARCH(p,q) model developed by Bollerslev (1986) and the EGARCH model developed by Nelson (1991) will be considered. These models are specifically considered because they are the most widely adopted models in this family. The functional specification and the restrictions on each of the conditional variance models are examined in turn below.

3.2.2.2 The ARCH(p) Model

The ARCH(p) model proposed by Engle (1982) is represented below as:

$$h_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i \epsilon_{t-i}^2 \qquad (3.4)$$

where $\alpha_0 > 0$, $\alpha_i \geq 0$, and ϵ_{t-i}^2 represents the squared prediction errors or innovations i-periods ago. The ARCH(p) model above states that the conditional variance in period t is a weighted average of the past squared prediction errors, or the squared innovations, from equation (3.1). The weight assigned to the squared innovation i-period ago is given by α_i. The model captures some of the persistence frequently observed in the financial market volatility: that is, that periods of high volatility tend to follow each other in quick succession, and also, that periods of tranquillity in the market tend to follow each other as well. The main disadvantage of this model is that, in order to effectively capture the volatility observed in the financial market, particularly with the high frequency data, the order of p that is required is often very large. Consequently, using a high order of p may result in an inefficient estimation of the parameters of the model.

3.2.2.3 The GARCH(p,q) Model

The GARCH model proposed by Bollerslev (1986) enables a parsimonious representation of the ARCH(p) model. This model assumes that the conditioning-information set required by agents for predicting volatility consists of the past-squared forecast errors $\{\{\epsilon_{t-i}^2\}_{i=1}^p\}$ from the conditional mean equation, equation (3.1), and the past-predicted conditional variance, $\{\{h_{t-i}\}_{i=1}^q\}$. The GARCH (p,q) model can be written as:

$$h_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i \epsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j h_{t-j} \qquad (3.5)$$

where $p \geq q$. $\alpha_0 > 0$. $\alpha_i \geq 0 \; \forall i$. $\beta_j \geq 0 \; \forall j$, and to ensure the stationarity of the uncondi-
tional variance, the restriction that $\sum_{i=1}^{p} \alpha_i + \sum_{j=1}^{p} \beta_j < 1$ must also be imposed. However, if
this restriction fails to be satisfied, then the integrated GARCH(p,q) –the IGARCH(p,q)– model
can be considered as an alternative.

The GARCH(p,q) model stated above, expresses the conditional variance for period t as a
weighted average of the past squared innovations and the past predicted conditional variance.
The weight given to the squared innovations i-period ago is denoted by α_i; and that for the
predicted volatility in the same period is denoted by β_i. These weights must be optimally de-
termined using, for example, the maximum-likelihood estimation technique. This model, like
the ARCH(p) model it is designed to improve, also has some defects. For instance, restrictions
must be imposed on the parameters to ensure that the predicted volatility is non-negative.
Besides, it fails to incorporate some of the real empirical features of the financial market. For
example, the behaviour of financial agents following a fall in the rate of returns, or a rise in the
rate of returns, of an asset is not incorporated into the model.

3.2.2.4 The EGARCH Model

Since the financial market frequently behaves differently when the market is bullish than when
it is bearish, it is therefore necessary to reflect this fact in any empirical model that purports
to model financial market volatility. In order to capture the differential effect of a positive
and a negative change in returns to assets, Nelson (1991) proposed the EGARCH(p,q) model
of volatility. The asymmetry in asset returns is incorporated into the conditional variance
equation through the functions of the innovations augmenting the past predicted variance.

An example of a model in this class is:[10]

$$\ln h_t \;=\; \alpha_0 + \sum_{i=1}^{q} \beta_i g(z_{t-i}) + \sum_{j=1}^{p} \gamma_j \ln h_{t-j} \tag{3.6}$$

$$g(z_{t-i}) \;=\; \theta_i |z_{t-i}| + \phi_i \max(0. z_{t-i}) + \lambda_i \cosh(z_{t-i}) \quad \text{where} \quad z_{t-i} = \frac{z_{t-i}}{\sqrt{h_{t-i}}} \tag{3.7}$$

In this model, it is no longer necessary to impose the non-negativity constraints on the parameters of the model as the predicted volatility is always guaranteed to be positive. The above specification expresses the predicted volatility as a linear function of the previous predicted volatility and a function of the previous standardized prediction errors or standardized innovations.

It can be observed from the specifications above, that the models considered here—or any other model within the (G)ARCH family—use an information set which is restrictive. The information set is restrictive in the sense that only the functions of the past-prediction errors, and the past-predicted conditional variance, are taken into consideration. It thus neglects the time series of the volatility of other asset types or of other economic variables. The questions that arise from these specifications are: can a better fit to the data be obtained by using additional information from other assets markets? And, can a better prediction of the future volatility of interest rates be obtained by using the additional information?.

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[10]Other variants of this model exist. For example, in Nelson (1991) the $g(z_{t-i})$ function have the following form:

$$g(z_{t-i}) = \theta z_{t-i} + \gamma[| z_{t-i} | - E(| z_{t-i} |)]$$

In this study, I use the hyperbolic cosine function, $\cosh(.)$, in equation (3.7) on the standardized errors z_{t-i} in order to always bind it away from zero. That is, that $\cosh(z_{t-i}) > 0$. The hyperbolic cosine function is defined as follows:

$$\cosh(z_{t-i}) = \frac{1}{2}(e^{z_{t-i}} + e^{-z_{t-i}})$$

3.2.3 The Summary and Direction of the Study

The models surveyed in the two families above share a common deficiency: each model focuses on a narrow set of information for predicting the conditional variance. As such, the interdependence between or among the various types of financial assets is completely ignored. The models, as they are, implicitly suggest that the agents assumes that the debt instruments market is independent of other assets markets. In addition, they also implicitly suggest that the yields, and hence the prices of the debt instruments, are less sensitive to changes in the monetary and fiscal policies, or the changes in the state of the economy. These implicit assumptions may not be justified, as they are inconsistent with the empirical observations in the financial market. Because these models leave out vital information that may be relevant to modeling the interest rate dynamics, there can arise adverse consequences when it is used in predicting the future interest rate level or its volatility. For instance, it may result in an incorrect assessment of the effect of the included variable on future volatility. As such, it may lead to an error of judgment in choosing a strategy to adopt in managing a fixed-income portfolio. Furthermore, since interest rate processes are used in the calibration of the asset's VaR, or its value, this may also be in error because relevant information fails to be accounted for in the model describing the behaviour of interest rates.

Some suggestions have been made by researchers to fix part of the problems inherent in this family of models. For example, Harvey and Shephard (1994), Harvey, Ruiz and Shephard (1994), Anderson and Lund (1995), and Taylor (1994) have suggested the stochastic volatility model. This model relaxes the assumptions about the parameters in the above family of models by allowing the parameters to follow a time-varying stochastic model instead of a deterministic pattern. In addition, Diebold and Nerlove (1989), Engle, Ng and Rothschild (1990), and Ng, Engle and Rothschild (1992) have also suggested using the factor-based models. In the factor model they suggested, the factors are neither directly measurable nor observable, and must therefore be determined form other constructs such as using the principal component analysis or the factor analysis. Although both the stochastic volatility model and the unobservable-factor model relax some of the assumptions about the functional form of the

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model, to allow for a more flexible model; they are, however, more computationally expensive to implement than are the previous models. Moreover, the models still do not address the issue of financial market interdependence, and whether the information from the other assets markets can help in improving the volatility forecasts in a given market. It is also not clear what factors are used when unobservable factors are used in forecasting future rates or their volatility.

To address the above issues, I examine the factor-ARCH (or the structural-time series) model next. The factor-ARCH family of models that I examine augments the information considered in the models above with other financial time series information. The approach taken here directly recognizes the fact that the financial market is interdependent, and that the information generated in one market may be useful in predicting the returns and the volatility of assets in other markets.

3.3 The Model

I now develop the statistical model used in modeling interest rate volatility in the Eurodollar market. The model developed here, unlike those surveyed earlier, draws on the fact that the financial markets are interdependent. As such, the information available from other assets markets is combined in a reasonable fashion to explain and predict the prices (hence the returns) and the volatilities of those prices. The question that I ultimately want to address is whether the model developed in this section provides a better representation and a better out-of-sample forecast of the volatility of interest rates in the Eurodollar market. The factor-ARCH and the exponential factor-ARCH models are presented below.

3.3.1 The Factor-ARCH Model

Following the method of analysis used in studies such as Zhou (1996), Tse and Booth (1996), Schwert (1989), Ferson (1989), Christie (1982) among others, I use other financial time series to augment the information in the previous level of the yield on Eurodollar deposits.[11] Conse-

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[11]The studies by Zhou (1996), Schwert (1989), and Ferson (1989) examine the relationship between the volatilities of the 3-month U.S. Treasury security yield and the U.S. stock market return. They used the Granger-causality test method to establish the direction of causality; and they all found evidence that the level of the yield on the U.S. Treasury yield has significant impact on the first two conditional moments of stock prices and returns. However, in the analysis, they failed to control for the effect of other financial market information such as the foreign exchange rate volatility, the volatilities of the securities traded in the Euromarket, and the volatilities of the Federal Funds market rate among others.

Similarly, in their analysis of the variability of the 3-month U.S. Treasury bill futures and the 3-month Eurodollar deposit futures market, Tse and Booth (1996) used the TED spread, the difference between the Eurodollar futures and the corresponding maturity treasury futures markets, to augment the GARCH(1,1) model. They reported that the lagged TED spread is statistically significant in explaining the volatility observed in both futures markets. Thus their result support the view that there is a common factor driving the volatilities of both futures markets. However, as in other studies, their analysis also fails to acknowledge the possible direct and indirect effect of other financial market information such as the stock market volatility, the foreign exchange rate volatility among others on these two futures market that are closely tied to their respective cash (spot) market.

quently, the empirical model can be written mathematically as:

$$h_t = \beta_0 + \sum_{i=1}^{7} \beta_i . X_{i,t} \tag{3.8}$$

where:

h_t : is the volatility of the continuously compounded annualized equivalent yield on the 3-Month Eurodollar deposits.

$X_{1,t}$: is the level of the continuously compounded annualized equivalent yield on the 3-Month Eurodollar deposits in the previous period, r_{t-1}.

$X_{2,t}$: is the square of the standardized prediction error in the previous period, z_{t-1}^2; and z_{t-1} is as defined in equation (3.7).

$X_{3,t}$: is the predicted variance in the previous period, h_{t-1}.

$X_{4,t}$: is the square of the first difference of the continuously compounded annualized equivalent yield on 7-day Federal Funds in the previous period.

$X_{5,t}$: is the square of the first difference of the logged New York Stock Exchange common stock composite price index in the previous period.

$X_{6,t}$: is the square of the first difference of the logged trade weighted foreign exchange rate index of the US dollar vis-à-vis the G-10 countries in the previous period.

$X_{7,t}$: is the square of the spread between the continuously compounded annualized equivalent yield on the 3-Month Eurodollar deposits and the 3-Month US Treasury bills in the previous period.

The specifications above represent the unrestricted model conjectured for the factor-ARCH family. Unlike the factor models considered by Diebold and Nerlove (1989), Engle, Ng, and Rothschild (1990), Ng, Engle and Rothschild (1992) among others, these factors are related to directly measurable and observable factors.

Equation (3.8) above states that the volatility of interest rates in the Eurodollar market is a linear combination of the previous level of the interest rate in the Eurodollar market, the function of the previous prediction error, the previous predicted volatility in the Eurodollar market, the volatility of the stock market and the federal funds market, and the variability of

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the spread between the Treasury-Eurodollar deposit rates. All series are as observed in the immediate past. The optimal weight given to each independent variable is represented by the parameter associated with the respective variables, the β_i's. The parameter β_0 represents the intercept which captures the part of the volatility that is independent of any of the explanatory variables. The method of determining the optimal weights is considered later in Section 3.5.

The explanatory variables in the specification above are lagged once because most information is available only with time lags. Furthermore, I have used the square of the financial variables in the equation above to represent the uncertainty, or the volatility, associated with each of the financial market.[12] The volatility of asset prices, and hence of the returns, in the other financial markets is expected to have an impact on that of the Eurodollar deposit market because fund managers frequently move funds between the different assets in their portfolio in response to the changing market conditions. These managers react to the changing dynamics of each assets market in order to hold a mix of assets in a portfolio that is consistent with their desired risk-return objectives. This thus forms the main transmission mechanism by which events and developments in other financial assets markets are expected to spill-over into the Eurodollar market.

In addition, the past predicted volatility of interest rates in the Eurodollar market, the past innovations, and the past levels of the yield on U.S. Treasury bills are included in the model because they each have been found useful in predicting volatility in the other interest rate models examined earlier. For instance, the interest rate level constitutes a significant predictor of volatility in the continuous-time based models; and in the ARCH-type models, the past-predicted volatility as well as functions of the past-prediction error, or innovations, constitute

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[12]The implicit assumption made here is that stock prices, the interest rates, and the foreign exchange rates behave as a random walk series. Thus the change in the respective level of the prices or rates between successive periods is equal to the random component with the mean value of zero and a given variance. As such, squaring the first difference of the respective market variable is equivalent to squaring the random component term that leads to the variance of the mean value. Empirical evidence supporting the random walk behaviour of the foreign exchange rates includes that presented in Alder and Lehman (1983), and Meese and Rogoff (1983, 1988); as for the behaviour of stock prices see, for example, Cootner (1964) and Malkiel (1996); and for the behaviour of interest rates in the bond markets see, for example, Murphy (1990).

a significant predictor of volatility.

The volatility model above has the following advantages: first, one can establish if there is a Granger-causality in volatility from other assets markets into the Eurodollar market; second, one can also establish whether changes in U.S. monetary policies (the Federal Funds market rate) have any direct bearing on interest rate volatility in the Eurodollar market; and finally, one can more appropriately attribute the direct effect of each explanatory variable on the volatility of interest rates in the Eurodollar market. Despite the advantages above, the specification in (3.8) has the inherent problem that a negative-predicted volatility cannot be ruled out. As a result, an alternative specification is, therefore, also considered. This issue is discussed next.

3.3.2 The Exponential Factor-ARCH Model

The model considered here has the same explanatory variables as equation (3.8) except that the dependent variable, the lagged dependent variable, and the function of the squared prediction errors assume their natural logarithmic transform. The model can therefore be expressed as:

$$
\begin{aligned}
\ln h_t \ = \ & \beta_0' + \beta_1' X_{1,t} + \beta_2' \ln X_{2,t} + \beta_3 \ln X_{3,t} + \beta_4 X_{4,t} \\
& + \beta_5' X_{5,t} + \beta_6' X_{6,t} + \beta_7' X_{7,t}
\end{aligned}
\tag{3.9}
$$

3.4 The Data

All the time series used in this study are daily sampled data. The data on the interest rates series are as follows: the London Interbank Offer Rate (LIBOR rate) on U.S. dollar denominated 3-month term deposits, placed in a designated London bank; the yield on 3-month U.S. Treasury bills; and the 7-day U.S. Federal Funds market rate. These rates are actual market quotes on the respective securities at the close of each business day. As is conventional, the quoted rates were transformed into their continuously compounded annualized equivalent yield basis.[13] This conversion is necessary so that the different rates are directly comparable. The data on the LIBOR rates were obtained from Data Resource Inc. (DRI), while the yield on the 3-month treasury bills and the Federal Funds Rate were obtained from the Federal Reserve Board, *Federal Statistical Releases*, Selected Interest Rate (series H15).

The other financial time series employed are as follows: the New York Stock Exchange (NYSE) common stock composite price index reported at the close of each business day in the NYSE historical stock data base; and the trade-weighted foreign exchange rate index of the U.S. dollar vis-à-vis the G-10 countries. The foreign exchange rate index is also as reported at the end of each business day by the Federal Reserve Board, *Federal Statistical Releases*, Foreign Exchange Rate (series H10). The NYSE common stock price index and the trade-weighted foreign exchange rate index were also transformed using logarithmic transformation.

The NYSE composite stock price index has been employed instead of the Dow-Jones Industrial Average (DJIA) and the Standard and Poors 500 (S & P 500) index. This is because the NYSE index represents a broader market index, and hence, is more representative of the

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[13]The transformation to a continuously compounded annualized equivalent yield basis (r^c) is based on the following conversion formula.

$$r^c = -\frac{36500}{n} \ln \frac{F}{P}$$

where $P = 100$. $F = 100(1 + r^q(\frac{n}{360}))$ for the future value of the Eurodollar deposits which is based on bankers quoted add-on yield basis (r^q). For the 3-Month Treasury bill and the Federal Funds rate, because they are are based on a discount yield basis (r^d):

$$P = 100(1 - r^d(\frac{n}{360}))$$

and F =100.

performance of the U.S. business investment portfolio than is the DJIA and the S & P 500 indexes.[14] Similarly, the trade-weighted foreign exchange rate index is used in the analysis instead of one of the bilateral exchange rates such as the U.S. dollar-pound sterling rate, the U.S. dollar-Deutschemark rate, the U.S. dollar-Yen rate among others. The trade-weighted foreign exchange rate index is preferred because fund managers of banks, insurance companies and pension funds, mutual funds among others often establish investment positions in several countries that do not use the U.S. dollar as their official currency. Hence, before a particular investment position can be taken in these economies, the U.S. funds must first be converted to the respective foreign currencies. Since not one country is exclusively preferred by U.S. fund managers, it is therefore more appropriate to use a weighted average of the most traded currencies.

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[14]The NYSE common stock composite price index comprise of all common stocks listed on the New York Stock Exchange. Each stock reflects its market capitalization, that is, the market value of outstanding stocks calculated as a multiple of the number of each firms' stock outstanding and the market price of each stock. The S & P 500 index accounts for only eighty per cent of the market capitalization of all the stocks listed in the New York Stock Exchange (Hull (1989: 43)). Similarly, the DJIA comprise of only thirty "blue chip" stocks in the U.S., and it accounts for only twenty per cent of the market value of NYSE stock market capitalization (Dubofsky (1992: 241)).

3.5 Estimation Theory and The Evaluation Criteria

The estimation procedure used in this study is the maximum likelihood technique. The performance evaluation criteria, and hence the model selection criteria, considered are; the cross-validation method, the out-of-sample forecast encompassing test method, and the out-of-sample root mean square prediction error. I briefly describe each of these techniques below.

3.5.1 The Maximum Likelihood Estimation Criterion

The estimation method used in this study is the maximum likelihood procedure, assuming normality of the residual terms. The steps involved in setting up the likelihood or the log-likelihood function that is to be maximized is as follows. See, for example, Kennedy (1992), Russell and MacKinnon (1993), and Jazwinski (1970).

STEP 1:   Given that the density of ϵ_t in equation (3.1) is assumed normal, derive the conditional density of ϵ_t. This conditional density, $f(\epsilon_t \mid \Omega_t \colon \Gamma)$, is also normal.

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STEP 2: Then, define the joint conditional probability distribution function (or the likelihood function) for ϵ_t up to time T as:

$$L_T(\Gamma) \equiv \prod_{t=1}^{T} f(\epsilon_t|\Omega_t; \Gamma) = \prod_{t=1}^{T} \frac{1}{\sqrt{2\pi h_t}} \exp\left(-\frac{1}{2}\frac{\epsilon_t^2}{h_t}\right) \qquad (3.10)$$

$\epsilon_t \quad := \quad \Delta r_t - \alpha_0 - \alpha_1 r_{t-1}$ represents the residual of changes in the conditional mean of the continuously compounded annualized equivalent yield on the 3-Month Eurodollar deposits, equation (3.1)

$h_t \quad := \quad g(\Omega_t, \Gamma')$ represents the conditional variance of ϵ_t at time t. The specific functional form assumed by $g(\Omega_t, \Gamma')$ depends on the type of the volatility model being investigated: one of equations (3.3) to (3.9) as the case may be. Γ' is the set of parameters characterizing the respective volatility model.

$\Omega_t \quad := \quad \{\{X_{i,t}\}_{i=1}^{T}, \{\epsilon_{t-i}\}_{i=1}^{p}\}$ represents the conditioning information set available in period t.

$\Gamma \quad := \quad \{\alpha_0, \alpha_1, \Gamma'\}$ represents the set of parameters to be estimated from the likelihood function.

STEP 3: Now, maximize the likelihood function defined in step 2 with respect to the parameter set, Γ. Following convention the log likelihood function is maximized; and in this regard, the following objective function is maximized with respect to the parameters of interest, Γ.

$$\max_{\Gamma'} \ln L_T(\Gamma') = -\left(\frac{T-1}{2}\right)\ln 2\pi - \frac{1}{2}\sum_{t=1}^{T}\ln(g(\Omega_t, \Gamma')) - \frac{1}{2}\sum_{t=1}^{T}\left(\frac{\epsilon_t}{\sqrt{g(\Omega_t, \Gamma')}}\right)^2$$
$$(3.11)$$

where $g(\Omega_t, \Gamma')$ is as defined in step 2.

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Step 3 above concludes the parameter estimation[15] phase of the analysis. The next phase is to produce an out-of-sample forecast for each volatility model on the basis of the information at hand: the model parameter estimates obtained in step 3, and the conditioning information set, Ω_t. I next discuss the forecasting phase as well as the model evaluation procedures.

3.5.2 The Performance Evaluation Criteria

Since the primary objective of this study is to compare and contrast the predictive ability of the various models frequently used in modeling volatility against the factor-ARCH model developed here, I will, therefore, define the metric for evaluating each of these models. The evaluation criteria considered are: the out-of-sample root mean square prediction error, which is the most commonly adopted method; the out-of-sample forecast encompassing ability of each model against the others; and the mean square prediction error from the N-fold cross-validation method. Each of these evaluation methods is examined briefly below.

3.5.2.1 The Cross-Validation Test Criterion

The cross-validation method involves the following steps:

STEP 1: Estimate the parameters of each model separately, leaving out $\frac{T}{N}$ of the total observations as "out-of-sample" data; i.e., that the estimation data set, or the "in-sample" data, consist of only $T - \frac{T}{N}$ sampled data. T is the total number of observations in the data set, and N is the desired number of cross-validations.

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[15]The parameter estimate that maximizes the log-likelihood function, equation (3.11), is estimated numerically using the Marquadt-Levenberg algorithm. For a more detailed description of the algorithm see Press, Teukolsky, Vetterling, and Flannery (1992:678) or SAS/ETS manual.

STEP 2: Use the parameters estimated from the "in-sample"data to make predictions for the $\frac{T}{N}$ observations left out in step one, the "out-of-sample"data. Then, compute the following: the forecast errors for the conditional mean ($\hat{\epsilon}_{m,t+i}$); the predicted conditional variance ($\hat{h}_{t+i}$) and its forecast error ($\hat{\epsilon}_{h,t+i}$); and last, the predicted log-likelihood ($L(\hat{\Gamma})_j$). These measures are computed as follows.

$$\Delta \hat{r}_{t+i} = \hat{a}_0 + \hat{a}_1 r_{t-1+i}: \qquad \hat{h}_{t+i} = g(\Omega_{t+i-1} \cdot \hat{\Gamma}') \tag{3.12}$$

$$\hat{\epsilon}_{m,t+i} = \Delta r_{t+i} - \Delta \hat{r}_{t+i} \qquad i = 1,2,\ldots,\frac{T}{N} \tag{3.13}$$

$$\hat{\epsilon}_{h,t+i} = \hat{\epsilon}^2_{m,t+i} - \hat{h}_{t+i} \qquad i = 1,2,\ldots,\frac{T}{N} \tag{3.14}$$

$$L(\hat{\Gamma})_j = -\frac{T}{2N} \ln 2\pi - \frac{1}{2}\sum_{i=1}^{\frac{T}{N}} \ln \hat{h}_{t+i} - \frac{1}{2}\sum_{i=1}^{\frac{T}{N}} \left(\frac{\hat{\epsilon}_{m,t+i}}{\sqrt{\hat{h}_{t+i}}}\right)^2 \tag{3.15}$$

As is conventional in related studies on forecasting volatility, the $\hat{\epsilon}^2_{m,t+i}$ in equation (3.14) is used as a proxy for h_{t+i}.[16] For examples, see the studies by Lopez (1995) and Diebold and Lopez (1995) in the context of evaluating the out-of-sample forecast of volatility models; Pagan and Schwert (1990), and Franses and Van Dijk (1996) in the context of forecasting stock market volatility, and Lee (1992) in the context of testing for heteroscedasticity.

STEP 3: Repeat steps one and two until all observations in the data set have been used.

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[16]Of course, there are other series that have been used in the literature to proxy for the unobserved volatility. For instance, Akgiray (1989) uses the weighted average of daily squared residuals during the month to estimate monthly volatility; and others, including Parkinson (1980) uses the extreme values—the differences between the high-low—of securities observed at the daily, weekly or monthly frequencies to gauge market volatility.

STEP 4: Now, using the forecast errors and the predicted log-likelihood computed in the step three, compute the mean square prediction error for the conditional mean and the conditional variance, and the predicted log-likelihood for each model. These metrics are computed as:

$$\mathbf{MSE}_k = \frac{1}{T} \sum_{i=1}^{T} \epsilon_{k.t+i}^2 \qquad k = m.h \tag{3.16}$$

$$\mathbf{LLF} = \sum_{j=1}^{N} L(\hat{\Gamma})_j \tag{3.17}$$

where m indicates the conditional mean, and h the conditional variance, of changes in LIBOR rates.

The model judged to be the best from the various specifications considered is the one with the least cross-validated mean square prediction error, or the maximum-predicted log-likelihood, or both.[17]

This method of evaluating models has been adopted because of its ability to discriminate between non-nested models, and because it requires a less-restrictive assumption on the model being evaluated–it requires only that the regularity condition be met (see, for example, Stoica, Eykhoff, Janssen, and Söderström 1986). Also, the technique has been applied, and found to work well, in other areas, such as meteorology (Hjorth and Holmqvist 1981), forecast-combining with artificial neural networks in the stock market (Donaldson and Kamstra 1996), among others, but it has not been applied to discriminating between or among the interest rate models. In addition, the method also allows all observations to be used in evaluating a model rather than just a small subset of the data.

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[17]Since the maximum-likelihood procedure was used in estimating the parameters of the model, the cross-validated likelihood value is the most natural measure. In addition, the predicted-likelihood value implicitly takes into consideration both the conditional mean and the conditional variance prediction errors at the same time.

3.5.2.2 The Mean Square Prediction Error Criterion

The second evaluation criteria considered is the out-of-sample mean square prediction error. In producing the mean square prediction error for each volatility model, the sample data is partitioned into two non-overlapping samples. The first subsample extends from June 1, 1973 through December 31, 1990; and the second subsample extends from January 1, 1991 through August 19, 1996. The first serves as the in-sample data, while the second serves as the out-of-sample data. Following this, I then use the rolling-regression method to forecast one-step ahead the volatility for the out-of-sample period.[18]

This regression method entails the following sequences. In order to produce the first out-of-sample forecast for each model, with this method, I run a regression for each volatility model on the in-sample data from June 1, 1973 through December 31, 1990. Then, the parameters estimated for each volatility model are used along with the other information to produce a one-step ahead out-of-sample forecast for the respective models. Next, I update the parameter estimated for each model by using the information up to and including January 1, 1991, and re-estimate each of the volatility models. The updated parameters of each model, along with the most recent information set is then used to produce the forecast for the second period in the out-of-sample data–the period corresponding to January 2, 1991. Again, the information set is then updated to include observations up to and including January 2, 1991; the parameters of each volatility model are then updated once more by re-estimating each of the models with the new sample data. Then, the one-step-ahead out-of-sample forecast for the period corresponding to January 3, 1991 is then produced. This information *cum* parameter-updating scheme, and the production of the one-period ahead out-of-sample forecast is repeated until the last period in the out-of-sample data.

Accordingly, from the rolling regression, the out-of-sample mean square prediction error (MSPE) and the root mean square error (RMSPE) for the interest rate volatility are then com-

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[18]For other studies that use this method see, for example, Gunter and Aksu (1989), and Donaldson and Kamstra (1997).

puted as:

$$MSPE = \frac{1}{N}\sum_{i=1}^{N}(\hat{\epsilon}_{t+i}^2 - \bar{h}_{t+i})^2 \tag{3.18}$$

$$RMSPE = 100 * \sqrt{\frac{1}{N}\sum_{i=1}^{N}(\hat{\epsilon}_{t+i}^2 - \bar{h}_{t+i})^2} \tag{3.19}$$

The RMSPE expresses the mean square prediction errors in basis points. These loss functions consider over- and under-prediction of volatility as equally bad. Consequently, the model with the least RMSPE is preferred according to this criterion.[19]

3.5.2.3 The Out-of-Sample Forecast-Encompassing Test Criterion

The out-of-sample forecast encompassing method follows the same procedure as the MSPE criterion both in the way the sampled data is partitioned, and the way the out-of-sample forecasts are obtained. Notwithstanding, the out-of-sample forecast encompassing criterion evaluates the performance of a model differently than does the MSPE. The forecast encompassing tests evaluate the performance of a model on the basis of its forecast encompassing ability; that is, the ability of a particular model to reproduce, or improve the forecast of an alternative model, while the alternative model cannot, in turn, be used to improve on the forecast of the first model.[20] In what follows, I discuss first the intuition underlying this evaluation method; and second, its empirical implementation.

Basically, the forecast encompassing test can be viewed as a test on the weighted combi-

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[19]Other loss functions, such as the mean absolute error (MAE), the mean square percent error (MS%E), and the mean absolute percent error (MAPE), can equally be defined from the prediction errors obtained in the out-of-sample forecast. However, the MSPE defined above is used for the following reasons. First, the MSPE imposes a higher penalty on larger-forecast error than does the MAE. Second, the financial losses suffered by economic agents are directly related to the size of the forecast errors rather than the relative size of the errors which the MS%E observes.

[20]This performance evaluation criteria has been advocated by researchers such as Chong and Hendry (1986), Hendry (1995), Gourieroux and Montford (1994) among others. Also, it has been used in earlier studies of the financial market. See, for example, the studies by Donaldson and Kamstra (1996, 1997), and Harrald and Kamstra (1997) who employed it in the context of modeling stock return volatility.

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nation of the out-of-sample forecast from two models, models i and j, at time t. This linear combination of the out-of-sample forecast from the two models is represented as:

$$\sigma_t^2 = \alpha_0 + \alpha_1 \hat{\sigma}_{i,t}^2 + \alpha_2 \hat{\sigma}_{j,t}^2 \tag{3.20}$$

where:

σ_t^2 := is the variance of the 1-step ahead period we are interested in forecasting. In the case examined here, this variable is not observed; in its place I use $\hat{\epsilon}_t^2$ which has expectations equal to σ_t^2.

$\hat{\sigma}_{i,t}^2$:= is the 1-step ahead forecast of σ_t^2 produced by model i in period t.

$\hat{\sigma}_{j,t}^2$:= is the 1-step ahead forecast of σ_t^2 produced by model j in period t.

α_0 := is the intercept term.

α_1 := is the weight attached to the forecast of model i.

α_2 := is the weight attached to the forecast of model j.

Now, given the specification above and the null hypothesis that model i is the true model for predicting σ_t^2, then the parameter estimates for α_1 from a least squares regression should not be significantly different from unity, and the estimate for α_0 and α_2 should jointly not be significantly different from zero. Similarly, if under the null hypothesis, model j is the true model for predicting σ_t^2, then α_2 should not be significantly different from unity, while α_0 and α_1 should jointly not be significantly different from zero.

As multicollinearity can arise from carrying out a least squares regression of equation (3.20), the model is reformulated under the null that model i is the true model as:

$$v_{i,t} \equiv \hat{\epsilon}_t^2 - \hat{\sigma}_{i,t}^2 = \beta_{i,j}^0 + \beta_{i,j}^1 \hat{\sigma}_{j,t}^2 + \eta_{i,t} \tag{3.21}$$

where $v_{i,t} \equiv \hat{\epsilon}_t^2 - \hat{\sigma}_{i,t}^2$ represents the forecast error from model i in period t.[21]

Now, to test the null hypothesis that either models out-of-sample forecast encompasses the others forecast, the tests on $\beta_{i,j}^1$ and $\beta_{j,i}^1$ must both be performed concurrently. In this

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[21]Similarly, when the null hypothesis that the true model is model j, then the model can be reformulated as:

$$v_{j,t} \equiv \hat{\epsilon}_t^2 - \hat{\sigma}_{j,t}^2 = \beta_{j,i}^0 + \beta_{j,i}^1 \hat{\sigma}_{i,t}^2 + \eta_{j,t}$$

case, if the t-test on $\beta^1_{i,j}$ is statistically insignificant–say at the five per cent level–this indicates that the out-of-sample forecast error produced by model i is orthogonal to the out-of-sample forecast produced by model j. Consequently, model j's forecast cannot help to improve on the forecast produced by model i. Furthermore, if the t-test on $\beta^1_{j,i}$ is statistically significant, then the forecast produced by model i can help to improve on the forecast produced by model j. This suggests that model i's forecast can help in improving the forecast produced by model j, while model j's forecast cannot in turn be used in improving the forecast of model i. Given this scenario, model i is said to forecast encompass model j, and, as a result, model i is ranked as being superior to model j. Conversely, if the reverse is true, then model j is ranked higher than model i.

Above, I have discussed two possible outcomes of the test: that model i's forecast encompasses that of model j, and that model j's forecast encompasses that of model i. Of course, there are other possible outcomes. It is quite possible that $\beta^1_{i,j}$ and $\beta^1_{j,i}$ are both statistically significant. In that case, this implies that the forecasts from both models help in improving each others out-of-sample forecast. Likewise, it is also possible that both parameters are insignificant. In this case, this implies that the out-of-sample forecast of neither model can help to improve on the forecast of the other. In the two possible outcomes considered here, none of the models can be ranked as superior to the other. In this instance, the particular model selected for forecasting by an investigator then depends on other extenuating circumstances such as the use to which the forecast will be put, or the dominant paradigm for modeling. A more common strategy adopted in the recent literature is to combine the models in some fashion so that the information in each model is used to augment those in other models. For examples, see Donaldson and Kamstra (1996, 1997), Hallman and Kamstra (1989), and Gunter and Aksu (1989).

3.5.3 Summary

In the preceding sections, I have discussed the three evaluation criteria used in this study; the cross-validation method, the mean square prediction error, and the out-of-sample forecast

encompassing capability of the models. Currently, there exist no consensus in the existing literature as to which is the most preferred criterion to use in discriminating between or among models. As a result, all the criteria are used in this study.

3.6 The Empirical Results

In this section, I present the results of the application of the model evaluation and selection procedure discussed in the preceding section. The section is arranged into three parts: the first examines the results of the out-of-sample mean square prediction error; the second examines the result of the forecast encompassing test; and the third examines the result of the N-fold cross-validation method.

3.6.1 The Mean Square Prediction Error

Table 3.2 presents the results of the out-of-sample mean square prediction error (MSPE), and the root mean square prediction error (RMSPE) computed for each model in the respective family of models examined. The results in this table are based on the computation method described in Section 3.5.2; and the out-of-sample data employed for this analysis are those from the sample period extending from January 1, 1991 to August 19, 1996. The table is arranged into three parts, the continuous-time family, the GARCH family, and the Factor-ARCH family.

The first part of the table presents the results for the models within the continuous-time family: the Cox, Ingersoll, and Ross (1985) square root model (CIR), the Cox and Ross (1976) constant elasticity of variance model (CEV), the Brennan and Schwartz (1979) proportional volatility model (PRP) and the Vasicek (1977) constant volatility model (VAS). As can be observed from this table, the CEV model performed the best out-of-sample. It has a root mean square prediction error of one basis point.[22] That the CEV model performed the best out-of-sample may be a bit surprising as it is the most unrestricted of the models examined within the continuous-time family. Usually, unrestricted models do badly out-of-sample. Following closely, is the model of Brennan and Schwartz (1979) that suggests that the volatility of interest rates varies in direct proportion to the previous interest rate level. This model also has a root mean square prediction error of 1.41 basis points. The CIR square root model and the VAS model then follow in respective order. The former has a root mean square prediction error of

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[22]A basis point is equivalent to a hundredth of one full percentage point, i.e., $\frac{1}{100}$.

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2.83 basis points while the latter has 4.36 basis points. One reason that accounts for the difference in the root mean square prediction error of the Vasicek (1977) model from the others, is that the model assume that volatility is time-invariant.

The second part of Table 3.2 presents the results for the models in the (G)ARCH family. The models examined in this class include the following: the autoregressive conditional heteroscedasticity model of order p, ARCH (p), where p=4 and p=2; the generalized ARCH (p,q) model, where p=2, q=1 and p=1, q=1; and the exponential GARCH (EGARCH) model of the form stated in equation (3.6), the unrestricted form is represented by EGA1 and its restricted version EGA2. In general, this part of the table suggests that all the models examined within this family have a similar mean square prediction error. On average, the root mean square prediction error for the models examined within this family is about 2.24 basis points. Of the models examined within this family, the EGARCH model has the least root mean square prediction error at 2.00 basis points. As such, it can be considered as the best model by the measure of the mean square prediction error and the root mean square prediction error.

The last part of the table presents the results for the factor-ARCH family. The average of the root mean square prediction error for the models from this family is also about 2.24 basis points. The table also suggests that the exponential factor-ARCH models–E-FAC1 and E-FAC2– have the least mean square prediction error for forecasting future interest rate volatility in the Eurodollar market.

When the models in each family are compared with the models from another family, the table shows that the Cox and Ross (1976) constant elasticity of variance model and the Brennan and Schwartz (1979) proportional volatility model, both from the continuous-time family, dominate the other models. They both have a lower mean square prediction error–and lower root mean square prediction error–than the other models. Following closely is the exponential factor-ARCH model, which fared better than the models in the (G)ARCH family, the Cox, Ingersoll, and Ross (1985) square root model and the constant volatility model proposed by Vasicek (1977).

3.6.2 The Out-of-Sample Forecast-Encompassing Capabilities

Tables 3.3 presents the result of the application of the out-of-sample forecast encompassing test.[23] As indicated at the top of the table, the results presented are based on a least squares regressions of the out-of-sample forecast error of model i, $v_{i,t}$, on the out-of-sample forecast of volatility produced by model j, $\hat{\sigma}^2_{j,t}$ after correcting for the possible heteroscedasticity in the residuals of this regression. The out-of-sample forecast error of model i, the dependent variable, is shown in the first column. The regressor, the out-of-sample forecast of volatility produced by model j is shown along columns two to eight. The numbers in cells i and j represents the p-value on the $\beta^1_{i,j}$ parameter.

Given the p-value in each cell, in order to establish whether or not model i's forecast encompasses the out-of-sample forecast of model j, one must examine the p-values in cell i and j (the p-value on $\beta^1_{i,j}$) and cell j and i (the p-value on $\beta^1_{j,i}$) concurrently. For model i's out-of-sample forecast to encompass model j's forecast, it has to be that model i's out-of-sample forecast explains model j's out-of-sample forecast error, while model j's out-of-sample forecast cannot in turn explains model i's out-of-sample forecast error. In which case, the p-value on $\beta^1_{j,i}$ is less than five percent, while the p-value on $\beta^1_{i,j}$ is greater than five per cent.

Looking at the rows and columns spanned by the CEV model, for example, it can be observed from Table 3.3 that this model encompasses the out-of-sample forecast produced by the Brennan and Schwartz (1979) model (PRP) and the restricted version of the exponential factor-ARCH model (EFAC2) at the five per cent significance level. Notwithstanding, the CEV model is itself encompassed by two other models: the ARCH(2) and the GARCH(1,1) models. The restricted version of the EGARCH model (EGA2) and the CEV model out-of-sample forecast have significant information for explaining each others out-of-sample forecast error. In addi-

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[23]The results contained in this table are similar to those on the regression of $\frac{v_{i,t}}{\hat{\sigma}^2_{i,t}} \equiv \left(\frac{\hat{\sigma}^2_{i,t}}{\hat{\sigma}^2_{i,t}} - 1 \right)$, as the dependent variable, on $\frac{1}{\hat{\sigma}^2_{i,t}}$, $\frac{\hat{\sigma}^2_{j,t}}{\hat{\sigma}^2_{i,t}}$ as the explanatory variables. See Table 3.4.

The table reported here is derived from a larger set of table that contains all models used in each family of models. Each of the model here represents the restricted versions of the more general variant, and each has tested to be insignificantly different from the unrestricted form. In addition, the out-of-sample forecast encompassing capability of the unrestricted versions is similar to the one presented here.

tion, the out-of-sample forecast produced by the CEV model and the restricted version of the factor-ARCH model (FAC2) are not statistically significant in explaining each others forecast error. With this analysis of the CEV model vis-à-vis the other models, the results in Table 3.3 suggest that this model is superior to two other models in the sense that it encompasses their out of sample forecast. However, the model is also inferior to two other models in the sense that its out-of-sample forecast is encompassed by these other models. Besides, the CEV model is found to be neither superior nor inferior to two other models because they each explain the others out-of-sample forecast or both fails to.

A similar type of analysis for the other models was also conducted for the other models as well; and, a further analysis of Table 3.3 suggests the following about the other models examined. First, the Brennan and Schwartz (1979) model (PRP) out-of-sample forecast is encompassed by four other models; the CEV, the ARCH(2), the GARCH(1,1), and the FAC2 models. This model however fails to encompass any other model. As a result, the Brennan and Schwartz (PRP) model is ranked lowest in terms of its forecast encompassing ability; that is, that other models out-of-sample forecast can explain the forecast error of this model, while its out-of-sample forecast cannot explain their forecast error.

Second, two other models, the ARCH(2) and the GARCH(1,1) models, each models out-of-sample forecast encompasses the CEV, the PRP and the exponential factor-ARCH (EFAC2) models. Nonetheless, the out-of-sample forecast error from the ARCH(2) and the GARCH(1,1) models is in turn explained by at least one other model's out-of-sample forecast of volatility: the ARCH(2) forecast error is explained by the out-of-sample forecast from EGARCH and the GARCH(1,1) models; and the forecast error of the GARCH (1,1) model is also explained by the out-of-sample forecast from ARCH(2) model. These results therefore, suggest that the ARCH (2) and the GARCH (1,1) models may not be absolutely superior to each other.

Third, the factor-ARCH model encompasses two other models; the Brennan and Schwartz (1979) model, and the EGARCH models. Unlike the ARCH (2), and the GARCH(1,1) models, the forecast error of the factor-ARCH model cannot be explained by the out-of-sample forecast of any other other model. Consequently, it is not inferior to any other model. On this basis,

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the factor-ARCH model can be said to be the best of the models examined for modeling and forecasting interest rate volatility.

To sum up, among all the models examined, only the factor-ARCH (FAC2) model out-of-sample forecast is never encompassed by any other model. It is, therefore, ranked on this basis as the best model for modeling and predicting the credit risk spread. The next best set of models are the GARCH(1,1) and the ARCH(2) models. But all three of these models fails to encompass a number of other models; and as such, there is no one model from these three top models that is absolutely superior to the others.

3.6.3 The Cross-Validation Results

The results of the evaluation methods presented in Sections 3.6.1 and 3.6.2 relate to the out-of-sample period extending from January 1, 1991 through August 19, 1996. As a consequence, it can be argued that the results of the out-of-sample performance analysis reported may be period-specific. This further raises the question: if a particular model does well on a given criteria within a specific period, is it also likely to perform similarly when the market conditions are different in a different period? To address this issue of the robustness of the model performance, the N-fold cross-validation was carried out following the steps described in Section 3.5.2.

In Table 3.5, Panels A, B, and C, contain the results of the N-fold cross-validated mean square prediction error; and in Table 3.6, Panels A, B, and C, contain the results of the cross-validated log-likelihood values. The first column of each panel lists the rule used in setting the number of cross-validations; while the second to the last column contain the out-of-sample forecast performance of the models listed under each panel. As the first columns of each panel indicate, the 10-, 50-, and 100-fold cross-validations were carried out instead of the traditional leave-one-out cross-validations.[24] The number in each cell represents the mean square predic-

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[24]These N-fold cross-validations were used for the following reasons. Given the number of sample data (5897), it is computationally expensive to do a leave-one-out cross-validation. Moreover, as pointed out in Shao (1993) a leave-one-out cross-validation suffers from the deficiency that it is asymptotically inconsistent in that it does not

tion error for the respective N-fold cross-validation and the corresponding model.

In general, it can be observed from Table 3.5 that there are substantial differences in the performance of each of the volatility models examined. The discussion in the rest of is section is, therefore, focused on the cross-validated mean square prediction error from each model in each family.[25] Panel A reports the results of the out-of-sample performance of the models within the continuous-time family. Within this family, the model with the least out-of-sample prediction error for volatility is the Brennan and Schwartz (1979) model. This model assumes that the volatility of interest varies directly with the previous level of interest rate. Thus, whenever the interest rate is high the predicted volatility is also expected to be high. The converse is also true. Following in respective order are, the Cox, Ingersoll, and Ross (1985) square root model, the Vasicek (1977) model, and last is the most general constant elasticity of variance (CEV) model. Also notable in Panel A is the substantial difference in the cross-validated mean square prediction error of the conditional variance from the Brennan and Schwartz (1979) model and the other volatility models in this class.

Panel B reports the result of the cross-validation on models within the (G)ARCH family. The cross-validated mean square prediction errors of the models in this family are similar to each other—with the EGARCH model somewhat preferred—and also is better than the continuous-time models. Panel C reports the cross-validation results for the models in the factor-ARCH family, and they favor the restricted version of the factor-ARCH model (FAC2).

Taking the results contained in Table 3.5 into perspective, the models that performed

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select with probability one, the model with the best predictive ability, as the sample size (T) increases asymptotically, i.e., as $T \to \infty$. The study further shows, through a Monte Carlo experiment, that the problem can be rectified by using a leave-N-out cross-validations instead. In this case $N > 1$.

[25]The mean square prediction error for the conditional mean equation is not reported separately. This is because the cross-validated mean square prediction error for all interest rate models and N-fold cross-validation are almost the same. This suggests that all the models have a similar performance for estimating the conditional mean of changes in the level of interest rate. This result may not at all be surprising since all models use the same functional specification for the conditional mean equation. As noted earlier, the same functional specifications were used to model the conditional mean of changes in the level of interest rate. Because the other financial market information test to be insignificantly different from zero when regressed on the changes in the level of interest rates.

worst in terms of their cross-validated out-of-sample forecast accuracy are those from the continuous-time family. The models from the (G)ARCH family performed better; and the factor-ARCH family performed best. As the tables show, the restricted version of the factor-ARCH model provides the least cross-validated mean square prediction error. As we can see from these rankings, the factor-ARCH (FAC2) model may therefore be the most suitable for predicting volatility, for calibrating the VaR of an asset or a portfolio, and for valuing securities. If this is the case, the current practice of using the one-factor or two-factor models in valuing contingent claim assets may have to be modified and extended to multi-factor models; and in this instant, the factors would have to be an observable series from the other financial markets.[26]

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[26]The cross-validated log-likelihood computed from the cross-validation method is given in Table 3.6. As we can observed from the table, it gives a different ranking to the models within and across families. The constant elasticity of variance model is ranked best in the continuous time family; the ARCH(4) model is ranked best within the (G)ARCH family; and the FAC2 model is also ranked as the best model within the factor-ARCH family. All the above models have the highest cross-validated log-likelihood within each family. But, when the models are compared against each other, the factor-ARCH model is ranked least, the ARCH(4) model is ranked higher, and the constant elasticity of variance model is ranked highest.

3.7 Summary, Conclusions and Future Research

This essay empirically examines the volatility of the short-term interest rate in the Eurodollar market. The period examined extends from January 1, 1973 through August 19, 1996. The principal purpose of the essay is to investigate the predictive ability of the models within the continuous-time family, the (G)ARCH family and the factor-ARCH family. Within the factor-ARCH family, attention is focused on models that use directly observable financial market information rather than the latent variables or the unobservable factor models. In order to investigate the additional benefit that accrues in using observable financial market information over the models that use just the previous interest rate level, or the combination of the previous predicted volatility and innovations, three evaluation criteria were employed. These are, the out-of-sample mean square prediction error, the out-of-sample forecast encompassing test criterion, and the N-fold cross-validation mean square prediction error.

The N-fold cross-validation method suggests that the factor-ARCH model that uses directly observable financial market information best predicts the future volatility; i.e., that the factor model has, on average, the least out-of-sample forecast error among the class of models examined. This result suggests that the volatility forecast produced by the factor-ARCH model may provide a more accurate estimate of future volatility for use in the pricing of financial assets than the estimate provided by the continuous-time based models and the (G)ARCH family of models. The result also suggests that the factor-ARCH model best describes the dynamics of interest rate volatility; and so would be valuable in calibrating the assets or portfolio's VaR. In addition, the results of the out-of-sample forecast encompassing tests also lend some support to the factor-ARCH model: first, it is the only model whose out-of-sample forecast errors cannot be explained by the out-of-sample forecast of volatility from other models; and second, its out-of-sample forecast encompasses the forecasts of two other models, the Brennan and Schwartz (1979) model from the continuous-time family, and the EGARCH model from the (G)ARCH family.

Although, the N-fold cross-validation and the forecast encompassing test results do lend some support to the factor-ARCH model, the result of the out-of-sample mean square pre-

diction error for the sample period January 1, 1991 to August 19, 1996 did not give such an unequivocal support to this model. In fact, the out-of-sample mean square prediction error shows that the factor-ARCH model is dominated by both the constant elasticity of variance model and the Brennan and Schwartz (1979) model. Also, the cross-validated log-likelihood ranks both the constant elasticity of variance and the ARCH(4) model ahead of the factor-ARCH model. As we can see from these results, it is apparent that there is no one model that is uniformly superior to the others under all the different evaluation criteria applied.

Due to the ambiguity in determining the one best model from among the best models, an alternative strategy that may be viable is to combine the forecast produced by these three top models. The optimally combined forecast may then be used to provide an estimate of future interest rate volatility. These combined forecasts may produce a superior forecast of volatility that can be used in pricing interest-rate-dependent financial assets, or in calibrating an asset's VaR. This combined forecasting framework is the subject of continuing work.

I should also mention the two major limitations of this study. The first is that the models have been evaluated using pure statistical evaluation criteria rather than the economic benefits or costs that might arise from using each of this models. In future studies, the economic gain or loss evaluation criterion will be employed to assess the different models evaluated here. The second is that the out-of-sample forecast analysis and evaluations have been restricted to just one-day-ahead periods. As portfolio managers and security traders may also require forecasts for periods farther into the future, in subsequent studies, the analysis in this essay will be extended to periods such as 15-, 30-, 60-, 90-, or 180-day ahead period.

Two conclusions that stand out from the above analysis are: first, that the models in the continuous-time family rank at the bottom in terms of their forecast encompassing capability when compared with other models from the (G)ARCH and the factor-ARCH families; and second, that none of the other models' out-of-sample forecast can explain the forecast errors of the factor-ARCH model. As a result, the factor-ARCH model using directly observable factors deserves further investigation.

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Table 3.1: The Volatility Models: List and Definition of Variables

FAMILY	MODEL:	FUNCTIONAL FORM:
Continuous Time	Vasicek (VAS)	$\Delta r_t = \alpha_0 + \alpha_1 r_{t-1} + \epsilon_t$ $\epsilon_t \sim \mathcal{N}(0.h_t) \qquad h_t = \sigma^2$
	Cox, Ingersoll, and Ross (CIR)	$\Delta r_t = \alpha_0 + \alpha_1 r_{t-1} + \epsilon_t$ $\epsilon_t \sim \mathcal{N}(0.h_t) \qquad h_t = \sigma^2 r_{t-1}^{\frac{1}{2}}$
	Brennan and Schwartz (PRP)	$\Delta r_t = \alpha_0 + \alpha_1 r_{t-1} + \epsilon_t$ $\epsilon_t \sim \mathcal{N}(0.h_t) \quad h_t = \sigma^2 r_{t-1}$
	Constant Elasticity of Variance (CEV)	$\Delta r_t = \alpha_0 + \alpha_1 r_{t-1} + \epsilon_t$ $\epsilon_t \sim \mathcal{N}(0.h_t) \quad h_t = \sigma^2 r_{t-1}^{\beta}$
(G)ARCH	ARCH(p) [ARp]	$\Delta r_t = \alpha_0 + \alpha_1 r_{t-1} + \epsilon_t$ $\epsilon_t \sim \mathcal{N}(0.h_t) \quad h_t = \delta_0 + \sum_{i=1}^{p} \delta_i \epsilon_{t-i}^2$
	GARCH(p,q) [GApq]	$\Delta r_t = \alpha_0 + \alpha_1 r_{t-1} + \epsilon_t \qquad \epsilon_t \sim \mathcal{N}(0.h_t)$ $h_t = \delta_0 + \sum_{i=1}^{p} \delta_i \epsilon_{t-i}^2 + \sum_{i=1}^{q} \beta_i h_{t-i}$
	EGARCH [EGA1]	$\Delta r_t = \alpha_0 + \alpha_1 r_{t-1} + \epsilon_t$ $\epsilon_t \sim \mathcal{N}(0.h_t) \qquad z_t = \frac{\epsilon_t}{\sqrt{h_t}}$ $\ln h_t = \alpha_0 + \alpha_1 \ln h_{t-1} + \alpha_2 \ln h_{t-2} + \alpha_3 |z_{t-1}|$ $+\alpha_4 max(0.z_{t-1}) + \alpha_5 cosh(z_{t-1})$
	EGARCH [EGA2]	$\Delta r_t = \alpha_0 + \alpha_1 r_{t-1} + \epsilon_t$ $\epsilon_t \sim \mathcal{N}(0.h_t) \qquad z_t = \frac{\epsilon_t}{\sqrt{h_t}}$ $\ln h_t = \alpha_0 + \alpha_1 \ln h_{t-1} + \alpha_2 \ln h_{t-2} + \alpha_3 |z_{t-1}|$

The continuation of Table 3.1

The Volatility Models: List and Definition of Variables

FAMILY	MODEL:	FUNCTIONAL FORM:
Factor-ARCH	FAC1	$\Delta r_t = \alpha_0 + \alpha_1 r_{t-1} + \epsilon_t \quad \epsilon_t \sim N(0, h_t)$ $h_t = \beta_0' + \beta_1' X_{1,t} + \beta_2' X_{2,t} + \beta_3 X_{3,t} + \beta_4 X_{4,t}$ $\qquad + \beta_5' X_{5,t} + \beta'_6 X_{6,t} + \beta_7' X_{7,t}$
	FAC2	$\Delta r_t = \alpha_0 + \alpha_1 r_{t-1} + \epsilon_t \quad \epsilon_t \sim N(0, h_t)$ $h_t = \beta_0' + \beta_1' X_{1,t} + \beta_2' X_{2,t} + \beta'_6 X_{6,t} + \beta_7' X_{7,t}$
	E-FAC1	$\Delta r_t = \alpha_0 + \alpha_1 r_{t-1} + \epsilon_t \quad \epsilon_t \sim N(0, h_t)$ $\ln h_t = \beta_0' + \beta_1' X_{1,t} + \beta_2' \ln X_{2,t} + \beta_3 \ln X_{3,t} + \beta_4 X_{4,t}$ $\qquad + \beta_5' X_{5,t} + \beta'_6 X_{6,t} + \beta_7' X_{7,t}$
	E- FAC2	$\Delta r_t = \alpha_0 + \alpha_1 r_{t-1} + \epsilon_t \quad \epsilon_t \sim N(0, h_t)$ $\ln h_t = \beta_0' + \beta_1' X_{1,t} + \beta_2' \ln X_{2,t} + \beta'_6 X_{6,t} + \beta_7' X_{7,t}$

Table 3.2: The Out-of-Sample Forecast Mean Square Prediction Error For the Volatility Models Between January 1, 1991 to August 19, 1996.

The Continuous-Time Family

	CIR	CEV	PRP	VAS
MSPE$\times 10^{-4}$	8.00	1.00	2.00	1.90
RMSPE	2.83	1.00	1.41	4.36

The Garch Family

	ARCH (p)		GARCH (p,q)		Exponential GARCH	
	ARCH (4)	ARCH(2)	GARCH (2,1)	GARCH (1,1)	EGA1	EGA2
MSPE$\times 10^{-4}$	5.00	6.00	5.00	5.00	4.00	4.00
RMSPE	2.24	2.45	2.24	2.24	2.00	2.00

The factor-ARCH Family

	Factor-ARCH		Exponential Factor-ARCH	
	FAC1	FAC2	EFAC1	EFAC2
MSPE$\times 10^{-4}$	7.00	6.00	3.00	3.00
RMSPE	2.65	2.45	1.82	1.82

The one-period ahead out-of-sample forecast of volatility $\hat{\sigma}_{t,t}^2$ used in the analysis are those produced by the rolling regression of each model. MSPE represents the mean square prediction error, and the RMPSE is the root mean square prediction error expressed in basis points.

Table 3.3: Volatility Models: Out-of-Sample Forecast Encompassing Test Statistics (1)

The figures in table represents the p-values on β in:

$$v_{i,t} \equiv \hat{\epsilon}_{i,t}^2 - \hat{\sigma}_{i,t}^2 = \beta_{i,j}^0 + \beta_{i,j}^1 \sigma_{j,t}^2 + \eta_{i,t} \qquad \eta_{i,t} \sim \mathcal{N}(0, h_t) \qquad h_t = \kappa(\sigma_{i,t}^2)^2$$

Model j $(\hat{\sigma}_{j,t}^2)$ → Model i $(v_{i,t})$ ↓	CONT. TIME CLASS		ARCH CLASS			FACTOR CLASS	
	CEV	PRP	AR2	EGA2	GA11	FAC2	EFAC2
CEV	–	0.4708	0.0001	0.0001	0.0001	0.2073	0.8804
PRP	0.0001	–	0.0001	0.0001	0.0001	0.0001	0.2641
AR2	0.2733	0.8007	–	0.0001	0.0001	0.8429	0.6881
EGA2	0.0001	0.0015	0.0001	–	0.4428	0.0010	0.6549
GA11	0.9492	0.6000	0.0001	0.2251	–	0.4498	0.6059
FAC2	0.3185	0.1399	0.2054	0.9508	0.6129	–	0.6629
EFAC2	0.0007	0.1345	0.0358	0.2426	0.0201	0.8206	–

Note 1: The one-period ahead out-of-sample forecast of volatility, $\hat{\sigma}_{i,t}^2$ and $\hat{\sigma}_{j,t}^2$, used in the analysis are those produced by the rolling regression of each model. The out-of-sample period extends from January 1, 1991 to August 19, 1996.

Note 2: The numbers in the cells i and j represents the p-value on the $\beta_{i,j}^1$ parameter. These p-values indicate whether or not the out-of-sample forecast of volatility from model j have any significant predictive power for the out-of-sample forecast error of model i, at a particular significance level. For instance, if the p-value on $\beta_{i,j}^1$ is greater than say 5%, this suggests that the out-of-sample forecast from model j cannot help to explain the out-of-sample forecast error from model i. So, the out-of-sample forecast of volatility from model j cannot be used to improve upon the forecast from model i. However, if the p-value on $\beta_{i,j}^1$ is lower than say 5%, then the out-of-sample forecast of volatility from model j is significant in explaining the out-of-sample forecast error from model i. Consequently, the out-of-sample forecast of volatility from model j can be used in improving the forecast of volatility from model i.

Table 3.4: Volatility Models: Out-of-Sample Forecast Encompassing Test Statistics (2)

The figures in the table below represents the p-values on $\mathcal{J}$ in:

$$\frac{\hat{\varepsilon}_{i,t}^2 - \hat{\sigma}_{i,t}^2}{\hat{\sigma}_{i,t}^2} = \mathcal{J}_{i,j}^0 \frac{1}{\sigma_{i,t}^2} + \mathcal{J}_{i,j}^1 \frac{\sigma_{j,t}^2}{\sigma_{i,t}^2} + \eta_{i,t} \qquad \eta_{i,t} \sim \mathcal{N}(0, h_t) \qquad h_t = \kappa(\sigma_{i,t}^2)^2$$

MOD j → MOD i ↓	CONT. TIME CLASS		ARCH CLASS			FACTOR CLASS	
	CEV	PRP	AR2	EGA2	GA11	FAC2	EFAC2
CEV	–	0.6849	0.0001	0.2629	0.0001	0.0001	0.5034
PRP	0.0001	–	0.0001	0.5533	0.0001	0.0415	0.0429
AR2	0.3148	0.8166	–	0.6287	0.0001	0.7590	0.7539
EGA2	0.3404	0.4006	0.3383	–	0.3866	0.8633	0.0485
GA11	0.1788	0.1259	0.0001	0.0663	–	0.2880	0.6497
FAC2	0.2281	0.1891	0.6381	0.5290	0.2676	–	0.9444
EFAC2	0.8105	0.1104	0.0149	0.0547	0.0014	0.0152	–

Note 1:   The one-period ahead out-of-sample forecast of volatility, $\hat{\sigma}_{i,t}^2$ and $\hat{\sigma}_{j,t}^2$, used in the analysis are those produced by the rolling regression of each model. The out-of-sample period extends from January 1, 1991 to August 19, 1996.

Note 2:   The numbers in the cells i and j represents the p-value on the $\mathcal{J}_{i,j}^1$ parameter. These p-values indicate whether or not the out-of-sample forecast of volatility from model j have any significant predictive power for the out-of-sample forecast error of model i, at a particular significance level. For instance, if the p-value on $\mathcal{J}_{i,j}^1$ is greater than say 5%, this suggests that the out-of-sample forecast from model j cannot help to explain the out-of-sample forecast error from model i. So, the out-of-sample forecast of volatility from model j cannot be used to improve upon the forecast from model i. However, if the p-value on $\beta_{i,j}^1$ is lower than say 5%, then the out-of-sample forecast of volatility from model j is significant in explaining the out-of-sample forecast error from model i. Consequently, the out-of-sample forecast of volatility from model j can be used in improving the forecast of volatility from model i.

Table 3.5: Volatility Models: Cross-Validated Mean Square Prediction Error
Sample Period June 1973 to Aug. 1996

Panel A:

Conditional Variance Models in the Continuous Time Family:

MODELS — N-Fold ↓	VAS	CIR	PRP	CEV
N=10	0.0236	0.0232	0.0210	0.0241
N=50	0.0232	0.0226	0.0206	0.0239
N=100	0.0230	0.0224	0.0203	0.0238

Panel B:

Conditional Variance Models in the (G)ARCH Family:

MODELS — N-Fold ↓	ARCH (p)		GARCH (p,q)		Exponential GARCH	
	p=4	p=2	p=2, q=1	p=1, q=1	EGA1	EGA2
N=10	0.0204	0.0206	0.0206	0.0204	0.0234	0.0242
N=50	0.0201	0.0204	0.0201	0.0201	0.0172	0.0172
N=100	0.0200	0.0206	0.0202	0.0201	0.0172	0.0172

Panel C:

Conditional Variance Models in the Factor-ARCH Family:

MODELS — N-Fold ↓	Factor-ARCH		Exponential Factor-ARCH	
	FAC1	FAC2	E-FAC1	E-FAC2
N=10	0.0170	0.0162	0.0336	0.0327
N=50	0.0167	0.0158	0.0374	0.0355
N=100	0.0165	0.0157	0.0387	0.0367

Table 3.6: Volatility Models: Cross-Validated Log-Likelihood
Sample Period: June 1973 to Aug. 1996

Panel A:

Conditional Variance Models in the Continuous Time Family:

MODELS — N-Fold ↓	VAS	CIR	PRP	CEV
N=10	286.77	1233.08	2709.69	89733.2
N=50	1148.66	1715.49	2784.02	41121.8
N=100	1286.13	1799.84	2819.53	39037.6

Panel B:

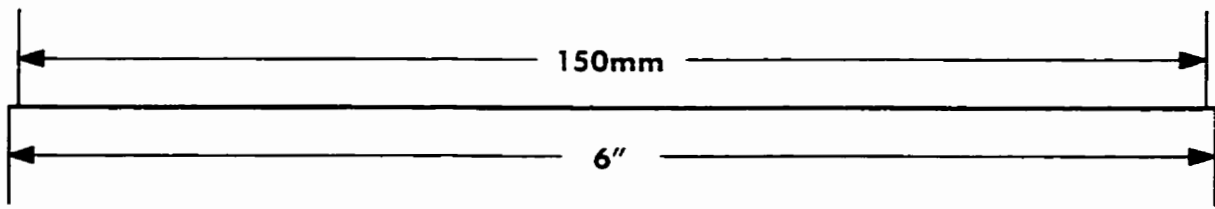Conditional Variance Models in the (G)ARCH Family:

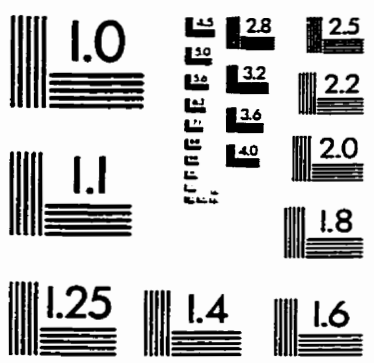| MODELS — N-Fold ↓	ARCH (p)		GARCH (p,q)		Exponential GARCH	
	p=4	p=2	p=2, q=1	p=1, q=1	EGA1	EGA2
N=10	3675.00	3552.04	-102261	-137287	-78603.5	-110777.0
N=50	3707.59	3600.19	-35775.0	-65661.90	-72080.6	-90182.6
N=100	3730.57	3609.76	-32847.6	-105697.0	-70680.5	-88253.8

Panel C:

Conditional Variance Models in the Factor-ARCH family:

MODELS — N-Fold ↓	Factor-ARCH		Expo. Factor-ARCH	
	FAC1	FAC2	E-FAC1	E-FAC2
N=10	-1735.95	2056.23	-4828.90	-5168.31
N=50	2341.57	2653.83	-5092.03	-5447.84
N=100	-2643.69	3041.16	-5105.78	-5464.62

IMAGE EVALUATION
TEST TARGET (QA-3)

1.0

1.1

1.25 1.4 1.6

1.8

2.0

2.2

2.5

2.8

3.2

3.6

4.0

|←———————————— 150mm ————————————→|

|←———————————— 6″ ————————————→|
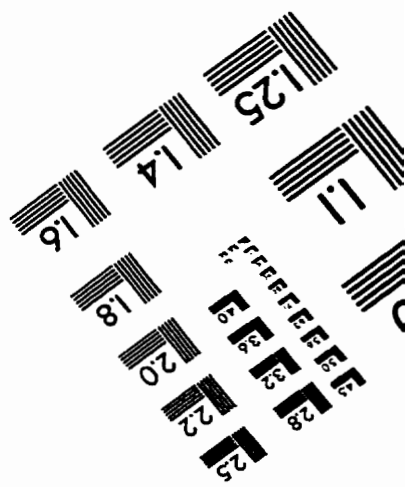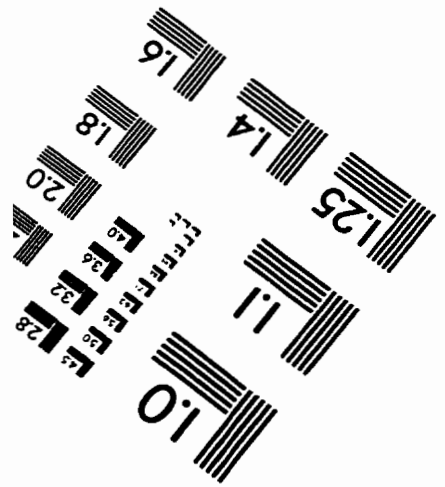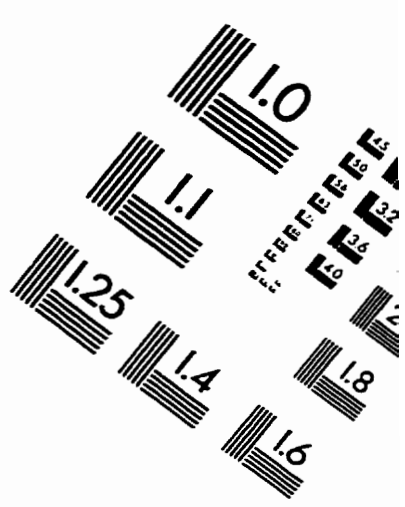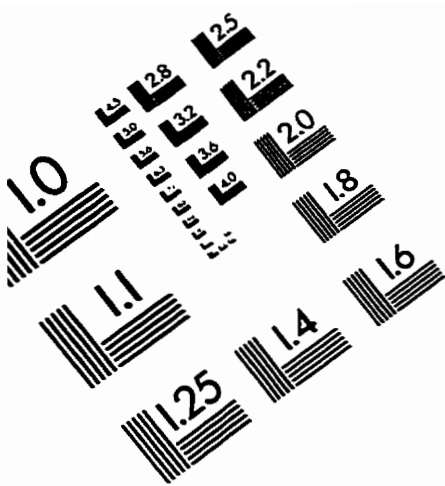
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