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The Empirical Comparison of Risk Models in Estimating Value at Risk and Expected Shortfall

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ABSTRACT

Risk measurement is the core content of risk management which is one of the three pillars of modern finance research. Value at risk model is the main model to depict risks of financial time series. This paper analyzes empirically the non-parametric and parametric methods to forecast Value at risk and Expected shortfall by the Dow Jones Industrial Index under the confidence level of 99%, 95% respectively, and make back-test on different models' result by Bernoulli coverage test and independence coverage test to make comparison of their applicability. The empirical analysis result is as follow: EVT is best for rare events, EWMA and GARCH are preferred for observed volatility clustering and historical simulation is most suitable for simplicity.

***Keywords**: Value at Risk, Expected Shortfall, Back-test, Violation Ratio, Risk Model.*

1. INTRODUCTION

Risk measurement is one of the most important aspects of financial investment. The common motivation of financial investment is profit maximization, however higher return usually corresponds with taking higher financial risk, the possibility of losing investment. In this paper, risk measures are evaluated and applied to a portfolio. Three of the most commonly used risk measures or mathematical methods for computing risk in finance are volatility, value at risk (VaR) and expected shortfall (ES). This project focuses on VaR and expected shortfall, since volatility has limits on valuing risk and does not account for the direction of an investment's movement.

2. METHODOLOGY

2.1. Risk Measures

2.1.1. VaR

Value at risk is a measure about the worst-case scenario, the amount of maximum loss at a certain confidence level [1]. In a mathematical sense, value at risk is a quantile on the distribution of profit and loss (Q), and can be satisfied as follows:

$$\Pr[Q \leq -\mathrm{VaR}(p)] = p \quad (1)$$

Where p is the confidence percentage level.

2.1.2. Expected Shortfall

The expected shortfall, also known as conditional VaR, is a measure that produces better incentives on expected loss when loss actually happens [2].

$$\mathrm{ES} = -\mathrm{E}[Q \mid Q \leq -\mathrm{VaR}(p)] \quad (2)$$

2.2. Risk Models

In order to forecast market risk measures, VaR and ES, the profit and loss distribution must first be estimated from historical observations [3]. There are two main techniques for forecasting risk measures, nonparametric methods and parametric methods.

2.2.1. Non-parametric Method - historical Simulation

Historical simulation is the general nonparametric method for forecasting risk; there are no assumptions of statistical models, no requirements of parameter estimations and it uses an actual empirical distribution to measure risk. Essentially, historical simulation assumes that an observed past return is expected to be the next period return and each historical observation has equal weight. Historical simulation is more suitable than alternative methods when there are structural breaks, because it is less sensitive to outliers and does not have estimation errors. However, a shortcoming of this approach is the large sample size requirement; the minimum sample size recommended for a 1%

confidence level is 1000.

2.2.2. *Parametric Method*

For parametric methods, the general form for calculating VaR of a portfolio is as follows:

$$\mathrm{VaR}(p) = -\sigma_{\mathrm{port}}\gamma(p)\vartheta \quad (3)$$

$$\sigma_{\mathrm{port}}{}^{2} = w'\Sigma w \quad (4)$$

where $\gamma(p)$ is inverse given distribution at the significance level of p, ϑ is the portfolio value, w is the portfolio weight vector for k assets and Σ is the portfolio's covariance matrix. Parameter methods used in this project can be divided into two main types, volatility time-independent and time-dependent [4].

2.2.2.1. *Volatility Time-Independent Models*

The time-independent methods assume the distribution of returns to different terms, which are normal distribution, student's t-distribution and skewed-t distribution in this article. Because it is assumed that stock prices follow a random walk, it is intuitive to assume that returns follow a normal distribution with zero meannm [5]. However, the return distribution of financial assets in the real market have fatter tails than normality and non-zero mean value, so student's t-distribution and skewed-t distribution describe the return distribution more precisely.

2.2.2.2. *EWMA*

Exponentially weighted moving average model is a volatility forecast model that builds on the simple moving average model by allocating greater weight to more recent observations, as follows:

$$\hat{\sigma}_t^2 = (1-\lambda)y_{t-1}^2 + \lambda\hat{\sigma}_{t-1}^2 \quad (5)$$

Where $\lambda\epsilon(0,1)$ is the decay factor, and is set to 0. 94 in this project according to previous analysis. EWMA is favorable for forecasting risk with the property of volatility clusters, where observed volatility will be maintained in the short-term.

2.2.2.3. *GARCH*

The GARCH model is given by:

$$\sigma_t^2 = \omega + \alpha Y_{t-1}^2 + \beta\sigma_{t-1}^2 \quad (6)$$

Similarly to EWMA, in GARCH, older returns have less impact on current volatility than more recent returns. However, the two coefficients (α, β) of GARCH are time dependent and estimated by the maximum likelihood method [6]. Optimal values of two parameters (p, q) in GARCH need to be determined. In this project, we set p and q from 1 to 3 resulting in 9 combinations of values for p and q. By using information criteria, AIC and BIC, we can determine the optimal number of lagged terms in the model [7].

2.2.2.4. *Extreme Value Theory*

Extreme value theory describes the science of estimating the tails of a distribution, and can smooth the tails of the probability distribution of portfolio changes computed by using historical simulation. When using historical simulation to estimate VaR, the higher VaR confidence level required, then the higher the standard error will be; and EVT can be used to decrease the standard error.

Suppose that F(v) is the cumulative distribution function for the variable of profit and loss over a certain time period. The probability that v lies between u and u+y with v>u is as follows:

$$F_u(y) = \frac{F(u+y) - F(u)}{1 - F(u)} \quad (7)$$

$F_u(y)$ is the right-tail of the cumulative probability distribution, and converges to a Generalized Pareto Distribution (GPD) when the threshold u increases.

$$G_{\xi,\beta}(y) = 1 - [1 + \xi\frac{y}{\beta}]^{-1/\xi} \quad (8)$$

GPD has two parameters, which can be estimated using maximum likelihood methods. The first parameter, ξ is the shape parameter that determines the tail heaviness of the distribution. For most financial investment data, the shape parameter is between 0. 1 and 0. 4. The other is the scale parameter, β. These two risk measures can be determined as shown below:

$$\mathrm{VaR} = U + \frac{y}{\beta}\{\left[\frac{n}{n_u}(1-q)\right]^{-\xi} - 1\} \quad (9)$$

$$\mathrm{ES} = \frac{\mathrm{VaR} + \beta - \xi u}{1 - \xi} \quad (10)$$

where n is the total number of observations, and n_u is the number of observations where v>u.

2.2.2.5. *Cornish-Fisher Expansion*

Cornish-Fisher Expansion, also known as modified VaR, is an alternative method to calculate VaR when the portfolio return distribution is not normally distributed. CFE takes higher moments, skewness and kurtosis, into consideration. The formula for modified VaR is:

$$\mathrm{mVaR} = \mu(X) + \sigma(X)Z_{cf} \quad (11)$$

$$Z_{cf} = q_p + \frac{(q_p^2 - 1)S(X)}{6} + \frac{(q_p^3 - 3q_p)K(X)}{24} - \frac{(2q_p^3 - 5q_p)S^2(X)}{36} \quad (12)$$

Where Z_{cf} is the CFE critical value under the significance level, p.

2.2.2.6. *Monte Carlo Simulation*

MCS is an independent method from non-parametric and parametric estimations, which simulates returns over past returns under specific distribution and uses these values to compute simulated future returns and corresponding risk measures [8].

2.3. Backtesting

Backtesting is a useful procedure for testing and determining the best-fit models. Specifically, its main purpose is to compare forecasts to historical realized returns. Its first step is to determine the length of an estimation window (W_E), and set the remaining horizon to be the testing window (W_T). Using the return from the estimation window, risk of the next period is forecasted. Then, by moving the estimation window forward, a series of forecasts, which has the same length as the testing window, can be derived from the data in the rolled estimation window. Furthermore, if the real losses exceed the estimated VaRs for specific days, a VaR violation is said to have occurred. The violation ratio, which is a main indicator of feasibility of models in backtesting, can be calculated:

$$\eta_t = \begin{cases} 1 & \text{if } y_t \leq VaR_t \\ 0 & \text{if } y_t > VaR_t \end{cases} \tag{13}$$

$$v_1 = \sum \eta_t \tag{14}$$

$$v_0 = W_T - v_1 \tag{15}$$

$$T = W_T + W_E \tag{16}$$

$$VA = \frac{\text{observed number of violations}}{\text{Expected number of violations}} = \frac{v_1}{p \times W_T} \tag{17}$$

There are different estimation window length requirements for each model. Even within the same model, different W_T will lead to varying results; so choosing an appropriate estimation window length is crucial.

Risk under-forecast occurs when the violation ratio is greater than 1, otherwise risk over-forecast occurs. If the $VA \in [0.8, 1.2]$, then the forecast is suitable. When $VA < 0.5$ or $VA > 1.5$, the model is inaccurate. If the violation ratios are the same for different methods, then the standard deviation of VaR estimates should be considered for comparison, the method with lower standard deviation is better [9, 10].

3. DATA

For the first part of the test, we chose Dow Jones Industrial Index (DJIA) as it resembles a stock market index. To mimic the index, we use the component stocks of DJIA as the basic component of the portfolio. Since DJIA is a price-weighted index, we simply sum all of the prices of the component stocks to obtain the price of the mimicking portfolio, from which we compute the return of the portfolio. All component stocks' prices are acquired by using the "hist_stock_data" function in Matlab, except for that of Visa, because of a lack of data during the sample period. Therefore, 1258 days of portfolio sample returns, from September 16, 2005 to September 16, 2010, are obtained from the 29 stocks' prices within the sample period. The portfolio returns are used as the whole sample data.

Next, to examine the role that risk measures play in a portfolio with exposure to interest rate risk, interest rate related assets are included in the portfolio. The corresponding assets used are returns of a bond index, namely S&P/BGCantor U. S. Treasury Bond Index, during the sample periods. Missing prices in the sample are computed as the average of the prices of the previous and latter days. Weights of assets in the portfolio are determined by the capability of the tangency portfolio in minimum variance portfolio theory.

4. EMPIRICAL RESULTS

4.1. Results of Stock Portfolio

Figure 1 and Figure 2 below display the features of trend and distribution of the mimicking portfolio return.

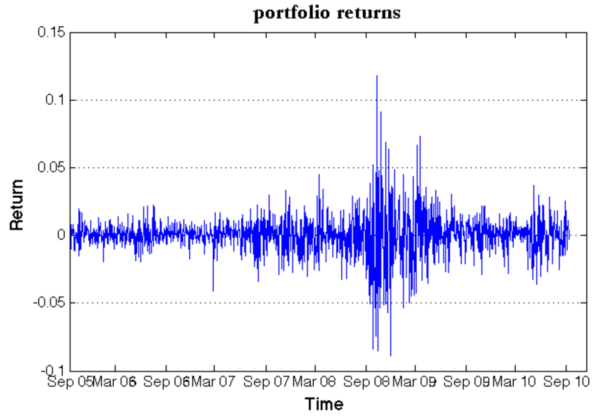

Figure 1 Portfolio Returns With Respect To Time

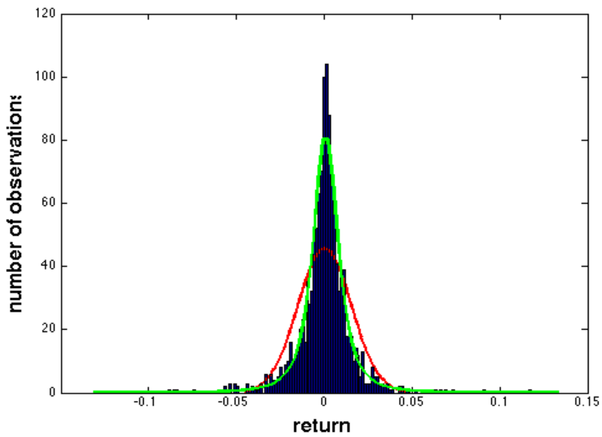

Figure 2 Histogram of Portfolio Returns and Fitting Distribution

From Figure 1, it is clear that the mean of portfolio return is around zero, which makes the assumption in normal and student's t models more reliable. But the volatilities are changing with time, as the volatilities in 2008 are the highest due to the financial crisis. Moreover, Figure 1 also shows the phenomenon of volatility clustering in this sample data, since relatively high or low volatilities endure for a period of time, probably leading to a violation of estimated risk measures within that time.

Figure 2 presents the histogram of portfolio returns, and fitting distribution under the assumption of normal (depicted as red line) and student's t distribution (depicted as green line). Once again, we could observe the zero mean property. However, the real distribution of returns is more leptokurtic than what normal distribution describes, and also fatter tails. These two features can be more precisely described by student's t distribution, which has positive excess kurtosis. Hence it is reasonable to assume returns follow student's t distribution.

Table 1 presents the results of VaR and Expected Shortfall estimates, under confidence level of 95 and 99 percent, within the whole sample period, by using different risk models [11]. The values of 95% VaR derived are approximately 0. 02, while volatility time-dependent models give relatively small VaR, since they consider conditional volatilites of returns rather than unconditional, which make more precise estimates.

Although student's t-distribution gives a smaller 95% VaR estimate compared to normal distribution, it has a distinctly larger 99% VaR value, owing to the fat tails of the t-distribution. The same observation can be made on the non-central t-distribution. However, Cornish Fisher expansion has the largest 99% VaR, giving a large penalty to extreme losses, since it revises the critical values for a given confidence level by considering higher moments of a distribution.

Obviously, ES has larger values than the corresponding VaR. The fat tail in the t-distribution has more effect on valuing risk, which results in the largest expected shortfall. Also, extreme value theory gives larger expected shortfall than the whole return distribution; hence it is more precise when estimating situations that rarely occur.

Table 2 shows the results of backtesting, and includes the mean estimation value of VaR and ES during the testing windows as well as the standard deviation. After backtesting, the difference between the results of the models appears to be narrower, especially under the 95% confidence level, owing to backtest taking the trend of returns into account and then making the estimates more precise. Volatility time-dependent models' estimations have larger standard deviation, which can be explained as time-varying volatilities of returns make VaR and ES forecasts time varying. Hence estimations from volatility time-dependent models are unstable, but consistent with recent market changes. However, this feature fades out when the confidence level rises to 99%. Same as the analysis above, fat-tails of the t-distribution has a larger effect when the confidence level is high, and which is not obvious at 95%.

Compared to EWMA, the GARCH model gives a looser view about risk, resulting in smaller values of VaR and ES, but larger volatilities of both estimators. Current volatility being dependent on lagged volatilities, but also on lagged returns in the GARCH model may explain this phenomenon.

Likewise to previous results using whole sample data, Cornish-Fisher expansion has a strict view on risk when the confidence level is high, and extreme value theory has a different view when using the estimator of expected shortfall. Estimations of Monte Carlo simulation stay low due to the nonpositive value of VaR during the beginning of time period, resulting from initial tiny variation of returns derived by MCS, which can be observed in Figure 3. More surprisingly, historical simulation seems to be a good choice for measuring risks, because of its simplicity.

In addition, from Figure 3, we could observe the trend of VaR estimation. It is obvious to find out volatility time-dependent models, i. e. , EWMA and GARCH have the more volatile VaR estimation than the other models. Results of those time-independent models are more like parallel to each other, except for Monte Carlo method, which has the similar trend but much more volatility within small time period.

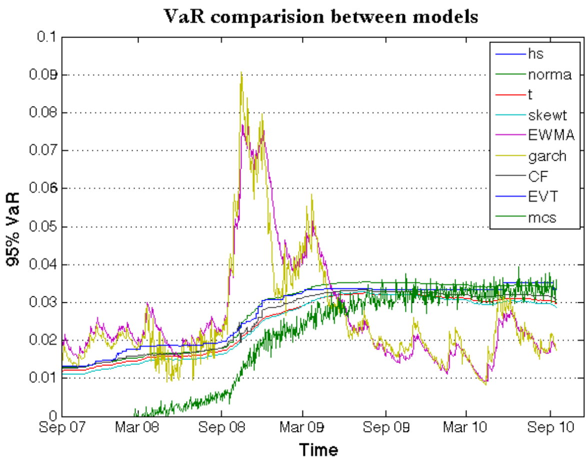

Figure 3 Backtesting 95% VaR Estimation of All Models

Table 1. VaR and ES for Whole Sample Period

Models	VaR		ES	
Confidence Level	0. 95	0. 99	0. 95	0. 99
Historical Simulation	0. 02316	0. 04981	0. 03772	0. 06201
Normal Distribution	0. 02494	0. 03528	0. 03128	0. 04041
t-dist	0. 02135	0. 04968	0. 04364	0. 09698
Skew t	0. 02056	0. 04887	0. 04283	0. 09614
EWMA	0. 01668	0. 02360	0. 02092	0. 02703
GARCH	0. 01024	0. 01449	0. 01285	0. 01660
CF Expansion	0. 02197	0. 06562	0. 04118	0. 00005
EVT	0. 02302	0. 04281	0. 05093	0. 07127
Monte Carlo	0. 02572	0. 04006	0. 03303	0. 04301

Table 2. Backtesting Results of VaR and ES

Confidence level = 95%	VaR		ES	
Models	mean	std-d	mean	std-d
Historical Simulation	0. 02729	0. 00787	0. 04081	0. 01340
Normal Distribution	0. 02731	0. 00886	0. 03425	0. 01111
t-dist	0. 02474	0. 00767	0. 04474	0. 01589
Skew-t	0. 02422	0. 00797	0. 04421	0. 01614
EWMA	0. 02659	0. 01484	0. 03335	0. 01861
GARCH	0. 02570	0. 01487	0. 03222	0. 01865
CF Expansion	0. 02581	0. 00776	0. 04020	0. 01689
EVT	0. 02729	0. 00787	0. 05458	0. 01887
Monte Carlo	0. 01924	0. 01384	0. 02578	0. 01638
Confidence level = 99%	VaR		ES	
Models	mean	std-d	mean	std-d
Historical Simulation	0. 04603	0. 01409	0. 06126	0. 02138
Normal Distribution	0. 03862	0. 01253	0. 04425	0. 01435
t-dist	0. 05181	0. 01815	0. 08931	0. 03695
Skew-t	0. 05128	0. 01841	0. 08877	0. 03714
EWMA	0. 03761	0. 02099	0. 04309	0. 02404
GARCH	0. 03634	0. 02103	0. 04164	0. 02410
CF expansion	0. 05602	0. 02039	0. 00339	0. 00195
EVT	0. 04603	0. 01409	0. 06359	0. 01999
Monte Carlo	0. 03091	0. 01843	0. 03545	0. 01990

Table 3. Violation Ratio, Bernoulli and Independence Coverage Test Results

Models	VA	BER	BER-P	IND	IND-P
Historical Sim.	1. 60950	12. 61165	0. 00038	0. 26747	0. 60504
Normal Dist.	1. 55673	10. 64958	0. 00110	0. 04029	0. 84092
t-dist	1. 87335	24. 48282	0. 00000	0. 02096	0. 88490
Skew-t	2. 03166	33. 12612	0. 00000	0. 69622	0. 40406
EWMA	1. 37203	4. 97234	0. 02576	2. 87631	0. 08989
GARCH	1. 47757	7. 98368	0. 00472	1. 55432	0. 21250
CF Expansion	1. 74142	18. 13196	0. 00002	0. 12322	0. 72557
EVT	1. 60950	12. 61165	0. 00038	0. 26747	0. 60504
Monte Carlo Sim	4. 16887	232. 18296	0. 00000	34. 23811	0. 00000

Table 4. VaR and ES for Whole Sample Period with Interest Rate Exposure

Whole Sample	VaR		ES	
Confidence level	95%	99%	95%	99%
Historical Simulation	0. 01825	0. 03916	0. 02946	0. 04897
Normal Distribution	0. 01962	0. 02775	0. 02461	0. 03180
t-dist	0. 01679	0. 03901	0. 03424	0. 07595
skew t	0. 01615	0. 03836	0. 03359	0. 07528
EWMA	0. 01278	0. 01807	0. 01602	0. 02071
GARCH	0. 01271	0. 01797	0. 01594	0. 02059
CF expansion	0. 01712	0. 05184	0. 03282	0. 00003
EVT	0. 01811	0. 03282	0. 04354	0. 05625
Monte Carlo	0. 01859	0. 02510	0. 02277	0. 02708

Table 5. Backtesting Results of VaR and ES with Interest Rate Exposure

Confidence level = 95%	VaR (95%)		ES	
	mean	std-d	mean	std-d
Historical Simulation	0. 02130	0. 00621	0. 03201	0. 01065
Normal Distribution	0. 02149	0. 00699	0. 02695	0. 00876
t-dist	0. 01945	0. 00602	0. 03514	0. 01253
skew t	0. 01902	0. 00627	0. 03471	0. 01274
EWMA	0. 02082	0. 01180	0. 02611	0. 01480
GARCH	0. 01814	0. 01215	0. 02275	0. 01523
CF expansion	0. 02016	0. 00609	0. 03194	0. 01346
EVT	0. 02130	0. 00621	0. 04375	0. 02068
Monte Carlo	0. 01423	0. 01105	0. 01921	0. 01333
Confidence level = 99%	**VaR (99%)**		**ES**	
	mean	std-d	mean	std-d
Historical Simulation	0. 03583	0. 01115	0. 04816	0. 01707
Normal Distribution	0. 03039	0. 00988	0. 03482	0. 01132
t-dist	0. 04069	0. 01431	0. 07010	0. 02920
skew t	0. 04026	0. 01453	0. 06966	0. 02937
EWMA	0. 02945	0. 01669	0. 03374	0. 01912
GARCH	0. 02566	0. 01718	0. 02939	0. 01968
CF expansion	0. 04409	0. 01634	0. 00273	0. 00159
EVT	0. 03583	0. 01115	0. 04704	0. 01458
Monte Carlo	0. 02308	0. 01519	0. 02665	0. 01649

Table 6. Violation Ratio, Bernoulli and Independence Coverage Test Results (with bonds)

Models	VA	BER	BER-P	IND	IND-P
Historical Sim.	1. 60950	12. 61165	0. 00038	0. 26747	0. 60504
Normal Dist.	1. 63588	13. 64645	0. 00022	0. 78702	0. 37500
t-dist	1. 89974	25. 84786	0. 00000	0. 22684	0. 63387
skew t	2. 00528	31. 61127	0. 00000	0. 29074	0. 58974
EWMA	1. 31926	3. 71166	0. 05403	2. 44711	0. 11774
GARCH	1. 82058	21. 84637	0. 00000	4. 59910	0. 03199
CF expansion	1. 74142	18. 13196	0. 00002	0. 12322	0. 72557
EVT	1. 60950	12. 61165	0. 00038	0. 26747	0. 60504
Monte Carlo Sim	4. 27441	245. 19094	0. 00000	33. 94315	0. 00000

In Table 3, the violation ratio, the results of Bernoulli coverage test and independence coverage test are presented. Except for the extremely large value for MCS, the violation ratios in all of the models are between 1 and 2, indicating that the observed number of violations exceeds the estimated number of violations, i. e. , all of the risk models underestimate the market risk [12]. However, volatility time-varying models have comparatively better results, with the lowest violation ratio belonging to EWMA. Therefore, inaccuracy is attributed to instability of volatility of returns, which can be obviously observed during the financial crisis in 2008.

According to the Bernoulli test, the predictability of all models is questionable, since all models reject the null hypothesis that real number of violations equals estimated number of violations, under the significance level of 5%, while EWMA cannot reject the null when the significance level declines to 1%.

With respect to independence coverage test, which examines the independence of violation observations and volatility cluster, all models, other than EWMA and MCS, claim no clustering in violations. Nevertheless, volatility clustering exists according to the return distribution. Hence EWMA precisely observe this fact when measuring risk, although the level of significance is not high enough.

4.2. Results of Portfolio with Interest Rate Exposure

Comparing the Table 4 with Table 1, by adding interest rate related assets to the portfolio, it becomes obvious that the estimated risk measures of portfolios with the bond index are generally lower than those of portfolios without the bond index. It can simply be explained that bonds have comparatively lower risk than stock assets. Therefore, inter-asset diversification that includes fixed income securities in the portfolio is able to significantly reduce the exposure to market risk, thus making portfolio resilient to any shocks in the market.

Comparing the results between Table 2 and Table 5, it is obvious that the features of the metrics previously derived from the models are preserved when adding the bond index into the portfolio. Since the weights of component assets we chose to construct the portfolio stay unchanged during the testing period, any changes in the estimated values of VaR and ES should not be observed. The effect of changes of weights of bond assets on the estimation values for various models needs to be further evaluated.

In Figure 4, the trend features of different models' VaR estimation when adding the bond index into the portfolio are unchanged. EWMA and GARCH still have the more volatile VaR estimation than the other models. Results of those time-independent models are more like parallel to each other, except for Monte Carlo method, which has the similar trend but much more volatility within small time period.

Table 6 displays the significance of backtests when the portfolio includes bonds. Even with bonds, the violation ratio is not reduced, resulting in underestimation of market risks of the models. However, the EWMA model has a more precise forecast, since we cannot reject the null in the Bernoulli test under a significance level of 5%. With regard to the GARCH model, it fails to reject the null in the independence coverage test under 5% significance level, indicating that it is more likely to observe volatility clustering with the bonds in the portfolio.

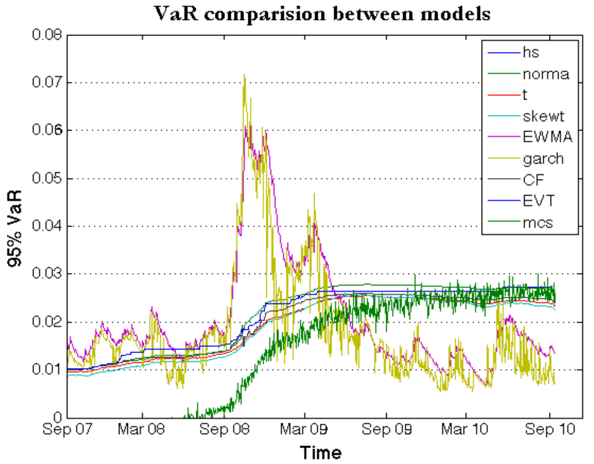

Figure 4 Backtesting 95% VaR Estimation of All Models (with bonds)

5. CONCLUSION

Theoretically, ES further considers the average loss degree in extreme cases on the basis of VaR, which can measure the extreme loss risk of a portfolio more completely. From the empirical analysis, VaR and ES are effective risk measures to value risk of investment portfolio. However, the analysis result shows that all of the risk models underestimate the market risk by estimating VaR, and the all ES of the risk models are larger than VaR. Even though inaccuracy of VaR is attributed to instability of volatility during the financial crisis, ES is more accurate than VaR to value market risk in this case. Hence, it may be beneficial to repeat this analysis on data excluding such extreme events.

Besides that, there are various risk models to measure VaR. Given their differences, some may be more appropriate for different situations. In this analysis, parametric methods and nan-parametric methods have been used to forecast market risk measures, VaR and ES. Comparing the backtest results of different models, it was observed that EVT is best for rare events, EWMA and GARCH are preferred for observed volatility clustering, historical is most suitable for simplicity and estimations provided by Monte Carlo simulation are low because of inaccurate price simulation. Furthermore, diversifying by the addition of bonds to the portfolio generally reduces risk and does not affect backtesting.

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