Notices

OF THE

AMERICAN MATHEMATICAL SOCIETY

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November 1972
Issue No. 141

Calendar

NOTE: This Calendar lists all of the meetings which have been approved by the Council up to the date at which this issue of the *Notices* was sent to press. The summer and annual meetings are joint meetings of the Mathematical Association of America and the American Mathematical Society. The meeting dates which fall rather far in the future are subject to change. This is particularly true of the meetings to which no numbers have yet been assigned.

Meeting Number	Date	Place	Deadline for Abstracts* and News Items**
701	January 25–29, 1973 (79th Annual Meeting)	Dallas, Texas	Nov. 14, 1972
702	April 14, 1973	Stanford, California	Feb. 26, 1973
703	April 18–21, 1973	New York, New York	Feb. 26, 1973
704	April 27–28, 1973	Evanston, Illinois	Feb. 26, 1973
705	June 16, 1973	Bellingham, Washington	May 3, 1973
706	August 20–24, 1973 (78th Summer Meeting)	Missoula, Montana	June 28, 1973
707	October 27, 1973	Cambridge, Massachusetts	
	November 16–17, 1973	Atlanta, Georgia	
	November 24, 1973	Tucson, Arizona	
	January 15–19, 1974 (80th Annual Meeting)	San Francisco, California	
	January 23–27, 1975 (81st Annual Meeting)	Washington, D. C.	
	January 22–26, 1976 (82nd Annual Meeting)	San Antonio, Texas	

*Deadline for abstracts not presented at a meeting (by title). January 1973 issue: November 7, 1972
February 1973 issue: January 11

**Deadline for news items for the February issue of these *Notices* is January 18, 1973.

OTHER EVENTS

December 27, 1972	Session of Contributed Papers in Biomathematics, AAAS Meeting, Washington, D. C.
August 21–28, 1974	International Congress of Mathematicians Vancouver, B. C., Canada

Abstracts should be submitted on special forms which are available in most departments of mathematics; forms can also be obtained by writing to the headquarters of the Society. Abstracts to be presented at the meeting *in person* must be received at the headquarters of the Society in Providence, Rhode Island, on or before the deadline for the meeting.

The *Notices* of the American Mathematical Society is published by the American Mathematical Society, 321 South Main Street, P. O. Box 6248, Providence, Rhode Island 02904 in January, February, April, June, August, October, November and December. Price per annual volume is $10.00. Price per copy $3.00. Special price for copies sold at registration desks of meetings of the Society, $1.00 per copy. Subscriptions, orders for back numbers (back issues of the last two years only are available) and inquiries should be addressed to the American Mathematical Society, P. O. Box 6248, Providence, Rhode Island 02904. Second class postage paid at Providence, Rhode Island, and additional mailing offices.

Notices
OF THE
AMERICAN MATHEMATICAL SOCIETY

Everett Pitcher and Gordon L. Walker, Editors
Wendell H. Fleming, Associate Editor

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The Six Hundred Ninety-Eighth Meeting
University of California, San Diego
La Jolla, California
November 18, 1972

The six hundred ninety-eighth meeting of the American Mathematical Society will be held at the University of California, San Diego, in LaJolla, California, on Saturday, November 18, 1972.

By invitation of the Committee to Select Hour Speakers for Far Western Sectional Meetings, there will be two invited addresses. Professor Marc A. Rieffel of the University of California, Berkeley, will lecture at 11:00 a.m.; he will speak on "Induced representations and Morita theorems for C*-algebras." Professor Ichiro Satake of the University of California, Berkeley, will give the second address at 2:00 p.m.; the title of his lecture is "On the arithmetic of tube domains." Both lectures will be given in Room 2250 of Building 2D. Sessions for contributed papers will be held in Building 2A. Late papers will be accepted for presentation at the meeting, but late papers will not be listed in the printed program of the meeting.

The registration desk will be located in the area of Room 2113 of Building 2A. Registration will begin at 8:30 a.m. on Saturday.

The following hotels and motels are located in La Jolla, California (zip code 92037). Each has a pool, and all but the Rancho La Jolla have restaurants. Reservations should be made directly with the hotel or motel; some of the rates given below are "university rates." Advance deposits are usually required.

ANDREA VILLA MOTEL
2402 Torrey Pines Road
Phone: (714) 459-3311

Single	\$10 up
Double	12 up
Twin	14 up

HOTEL DEL CHARRO
2380 Torrey Pines Road
Phone: (714) 454-6134

Single	\$11 up
Double	13 up
Twin	13 up

RANCHO LA JOLLA MOTEL
2420 Torrey Pines Road
Phone: (714) 454-4239

Single	\$ 9 up
Double	11 up

HOLIDAY INN LA JOLLA
Interstate 5 at La Jolla Village Drive
Phone: (714) 453-5500

Single	\$13 up
Double	17 up

HOTEL LA VALENCIA
1132 Prospect Street
Phone: (714) 454-0771

Single	\$12 up
Double	14 up

TORREY PINES INN
11480 No. Torrey Pines Road
Phone: (714) 453-4420

Single	\$12 up
Double	15 up

The motels nearest to campus are the Holiday Inn and Torrey Pines Inn; these motels are approximately one mile from campus. A city bus runs from La Jolla to the campus with convenient stops near the Del Charro and La Valencia hotels.

Lunch will be available on and off campus. A list of eating establishments will be available at the registration desk.

Several airlines serve the San Diego International Airport. There is no limousine service to La Jolla; taxi fare to the campus is about \$7. Car rentals are available at the airport. La Jolla is located approximately fourteen miles north of San Diego and is easily accessible via Interstate 5. To reach the campus, take the La Jolla Village Drive exit and follow the signs which will lead directly to the Muir campus. There will be an AMS reserved parking area in the North Parking Lot of Building 2A.

PROGRAM OF THE SESSIONS

The time limit for each contributed paper is ten minutes. To maintain this schedule, the time limit will be strictly enforced.

SATURDAY, 9:00 A. M.

Session on Algebra and Theory of Numbers, Room 2113, Building 2A

9:00-9:10 (1) Notes on N-semigroups. Preliminary report. Professor TAKAYUKI TAMURA, University of California, Davis (698-A8)

9:15-9:25 (2) Saturated chains in Noetherian rings. Professor STEPHEN J. McADAM, University of Texas, Austin (698-A5)

9:30-9:40 (3) A note on overrings of a domain. Preliminary report. Professor NICK H. VAUGHAN, North Texas State University (698-A6)

9:45-9:55 (4) Number fields with no proper subfields. Professor STEPHEN J. PIERCE, University of Toronto (698-A7)

10:00-10:10 (5) The equation of Ramanujan-Nagel and $[y^2]$. Dr. DAVID G. MEAD, University of California, Davis (698-A3)

10:15-10:25 (6) Bessel series expansions of the Epstein zeta function and the functional equation. Professor AUDREY A. TERRAS, University of California, San Diego (698-A10)

SATURDAY, 9:00 A. M.

First Session on Analysis, Room 2313, Building 2A

9:00-9:10 (7) Time-dependent solutions of the viscous incompressible flow past a circular cylinder by the method of series truncation. Preliminary report. Professor V. A. PATEL, California State University, Humboldt (698-C1) (Introduced by Professor Charles E. Snygg)

9:15-9:25 (8) On unsteady non-Newtonian flow in a rotating system. Miss UMA BASU, University of Calcutta, India, and Dr. LOKENATH DEBNATH*, East Carolina University (698-C2)

9:30-9:40 (9) Finding Riemann functions. Dr. DAVID H. WOOD, Naval Underwater Systems Center, New London Laboratory, Connecticut (698-B11)

9:45-9:55 (10) Existence and uniqueness of solutions to a fourth-order nonlinear eigenvalue problem. Dr. JERRY F. KUZANEK, University of Redlands (698-B2)

10:00-10:10 (11) Strong stability and perturbations of systems of Volterra integral equations. Preliminary report. Professor JIM M. CUSHING, University of Arizona (698-B10)

10:15-10:25 (12) A stability preservation result for small ∘ perturbed Volterra integral equations. Professor JOHN M. BOWNDS* and Professor JIM M. CUSHING, University of Arizona (698-B12)

SATURDAY, 9:00 A. M.

Session on Topology, Room 2402, Building 2A

9:00-9:10 (13) Pseudo-paracompact spaces. Preliminary report. Professor CHIEN WENJEN, California State University, Long Beach (698-G6)

9:15-9:25 (14) Concerning Jones's function K. Dr. CHARLES L. HAGOPIAN, California State University, Sacramento (698-G2)

9:30-9:40 (15) Monotone decompositions of continua into simple closed curves and generalized simple closed curves. Professor ELDON J. VOUGHT, California State University, Chico (698-G3)

9:45-9:55 (16) The higher homotopy groups of k-spun knots and links. Preliminary report. Mr. WILLIAM A. McCALLUM, Florida State University (698-G1)

10:00-10:10 (17) Higher relations among BP operations. Preliminary report. Professor RAPHAEL ZAHLER, Rutgers University, Douglass College (698-G4)

10:15-10:25 (18) On the finite determination of some homotopy types. Dr. RICHARD A. BODY and Professor ROY R. DOUGLAS*, University of British Columbia, and Professor CASPAR R. CURJEL, University of Washington (698-G5)

*For papers with more than one author, an asterisk follows the name of the author who plans to present the paper at the meeting.

SATURDAY, 11:00 A. M.

Invited Address, Room 2250, Building 2D

(19) Induced representations and Morita theorems for C*-algebras. Professor MARC A. RIEFFEL, University of California, Berkeley (698-B13)

SATURDAY, 2:00 P. M.

Invited Address, Room 2250, Building 2D

(20) On the arithmetic of tube domains. Professor ICHIRO SATAKE, University of California, Berkeley (698-A9)

SATURDAY, 3:30 P. M.

General Session, Room 2113, Building 2A

3:30-3:40 (21) A factorization theorem for a certain class of graphs. Dr. SUKHAMAY KUNDU, IBM, T. J. Watson Research Center, Yorktown Heights, New York (698-A4) (Introduced by Dr. Alan G. Konheim)

3:45-3:55 (22) On strong starters. Mr. KENNETH B. GROSS, University of Southern California (698-A1)

4:00-4:10 (23) Axiomatic bases for equational theories of natural numbers. Preliminary report. Mr. CHARLES FONTAINE MARTIN, University of California, Berkeley (698-E1)

4:15-4:25 (24) Finite sums of sequences within partitions of N. Professor NEIL B. HINDMAN, California State University, Los Angeles (698-B1)

4:30-4:40 (25) Bishop's constructive intermediate value theorem. Professor HENRY CHENG, New Mexico State University (698-B3)

SATURDAY, 3:30 P. M.

Second Session on Analysis, Room 2402, Building 2A

3:30-3:40 (26) Bounded polyharmonic functions and the dimension of the manifold. Mr. NORMAN MIRSKY* and Professor LEO SARIO, University of California, Los Angeles, and Professor CECILIA Y. WANG, Arizona State University and University of California, Los Angeles (698-B7)

3:45-3:55 (27) Parabolicity and existence of bounded biharmonic functions. Professor LEO SARIO, University of California, Los Angeles, and Professor CECILIA Y. WANG*, Arizona State University and University of California, Los Angeles (698-B6)

4:00-4:10 (28) N-manifolds carrying bounded but no Dirichlet finite harmonic functions. Mr. DENNIS S. HADA* and Professor LEO SARIO, University of California, Los Angeles, and Professor CECILIA Y. WANG, Arizona State University and University of California, Los Angeles (698-B5)

4:15-4:25 (29) Bounded harmonic but no Dirichlet-finite harmonic. Professor YOUNG KOAN KWON, University of Texas, Austin (698-A2)

4:30-4:40 (30) B-convexity of Siegel domains of the second kind. Dr. SU-SHING CHEN, University of Florida (698-B4)

4:45-4:55 (31) Distribution of supports of representing measures for H. Dr. UPADHYAYULA V. SATYANARAYANA, California State University, Northridge (698-B8)

5:00-5:10 (32) B(S) as an operator subalgebra. Preliminary report. Professor DONALD E. MYERS, University of Arizona (698-B9)

R. S. Pierce
Acting Associate Secretary

Honolulu, Hawaii

The Six Hundred Ninety-Ninth Meeting
University of North Carolina
Chapel Hill, North Carolina
November 24-25, 1972

The six hundred ninety-ninth meeting of the rican Mathematical Society will be held at niversity of North Carolina at Chapel Hill, el Hill, North Carolina, on Friday and Satur- November 24–25, 1972.

By invitation of the Committee to Select Speakers for Southeastern Sectional Meet- there will be two one-hour addresses, both ich will be presented in the Peabody Hall orium. Professor James K. Brooks of the rsity of Florida will give an address at p.m. on Friday entitled "Measure and inte- ion theory in Banach spaces," and Professor rt Gilmer of Florida State University will an address entitled "Some results on poly- ial rings over a commutative ring" at 9:00 on Saturday.

There will be sessions for contributed rs both Friday afternoon and Saturday morn- There will be two special sessions held in ition to the regular sessions. Professor D. Anderson of Louisiana State University is nging a session on Infinite Dimensional To- gy and Professor Christopher Hunter of rida State University is arranging a session New Areas for Applied Mathematics (i.e., s other than those relating to Physics and ineering).

The registration desk will be located in illips Hall. Registration hours will be from 0 a.m. to 5:00 p.m. on Friday, November and from 9:00 a.m. to noon on Saturday, ember 25. The sessions will be held in llips Hall and Peabody Hall.

Chapel Hill is located on U.S. 15–105 and lso accessible from Interstate 85 via N.C. It is 17 miles from the Raleigh-Durham port, which is served by Eastern, United, ta, and Piedmont airlines. Limousine ser- e is available from the airport to the Carolina The limousine fare is $3.25 and the trip es approximately forty minutes. Taxis are o available from the Tarheel Cab Company at one way for one person, or $4 each for two or re persons.

Meals are available only at commercial ablishments. The cafeteria in the Carolina will be available for lunch and dinner, and the dining room will be open for all three meals. There will be a cocktail hour at 5:00 p.m. and a beer party at 8:00 p.m. on Friday in the Newman Center which is on Pittsburgh Street across from the Carolina Inn. Tickets for these events may be purchased at the registration desk at the time of registration.

A block of rooms is being reserved in the Carolina Inn with November 10 as deadline. Requests for reservations should be made directly to the hotel. All reservations should be made directly with the hotel and motels as soon as possible.

CAROLINA INN
West Cameron Avenue
(one block from Phillips Hall)
Phone: (919) 933-2001

Single (one person per room)	$ 9.00 – $15.00
One Double Bed (two persons per room)	11.00 – 14.00
Two Beds (two persons per room)	12.00 – 19.00

HOLIDAY INN
15–105 Bypass
(three miles from campus)
Phone: (919) 929-2171

Single (one person per room)	$13.00
One Double Bed (two persons per room)	17.00
Two Beds (two persons per room)	19.00

UNIVERSITY MOTEL
Raleigh Road
P.O. Box 2118
(one mile from campus)
Phone: (919) 942-4132

Single (one person per room)	$10.50 – $14.00
One Double Bed (two persons per room)	14.00
Two Beds (two persons per room)	14.00 – 18.00

Messages may be left for delivery by phoning (919) 933-2028.

PRESENTERS OF PAPERS

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Boyte, J. M. #52
Brewer, J. W. #105
• Brooks, J. K. #1
Brown, J. B. #112
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• Invited one-hour lectures

PROGRAM OF THE SESSIONS

The time limit for each contributed paper in the general sessions is ten minutes and in the special sessions is twenty minutes. To maintain this schedule, the time limits will be strictly enforced.

FRIDAY, 1:00 P. M.

Invited Address, Room 08, Peabody Hall Auditorium

(1) Measure and integration theory in Banach spaces. Professor JAMES K. BROOKS, University of Florida (699-B33)

FRIDAY, 2:15 P. M.

Special Session on New Areas in Applied Mathematics, Room 08, Peabody Hall Auditorium

2:15–2:35 (2) Human skin as an elastic membrane. Dr. DONALD A. DANIELSON, University of Virginia (699-C6) (Introduced by Professor Christopher Hunter)

2:45–3:05 (3) A stochastic normative stock price model. Dr. WILLIAM M. BOYCE* and Dr. D. M. KREPS, Bell Telephone Laboratories, Inc., Murray Hill, New Jersey (699-C5)

3:15–3:35 (4) Some problems arising from the interaction between biology and mathematics. Dr. H. ROBERT van der VAART, North Carolina State University (699-C8)

3:45–4:05 (5) Modelling the atmospheric circulation of the planet Mars. Dr. STEVE BLUMSACK, Florida State University (699-C4) (Introduced by Professor Christopher Hunter)

4:15–4:35 (6) Sociology wants mathematics. Dr. I. RICHARD SAVAGE, Florida State University (699-C7)

*For papers with more than one author, an asterisk follows the name of the author who plans to present the paper at the meeting.

FRIDAY, 2:15 P. M.

ecial Session on Infinite Dimensional Topology, Room 233, Phillips Hall

:15-2:35 (7) Periodic action on (I-D) spaces. Professor RAYMOND Y.-T. WONG, University of California, Santa Barbara (699-G21)

:45-3:05 (8) Sums of Hilbert cube factors. Professor JAMES E. WEST, Cornell University (699-G29)

:15-3:35 (9) Hyperspaces of compact connected polyhedra. Preliminary report. Professor RICHARD M. SCHORI* and Professor DOUGLAS W. CURTIS, Louisiana State University (699-G22)

:45-4:05 (10) Spaces of subcontinua of compact connected polyhedra. Preliminary report. Professor DOUGLAS W. CURTIS* and Professor RICHARD M. SCHORI, Louisiana State University (699-G23)

:15-4:35 (11) Triangulation and classification of Hilbert cube manifolds. Preliminary report. Professor THOMAS A. CHAPMAN, University of Kentucky (699-G27)

FRIDAY, 2:30 P. M.

ession on Analysis I, Operator Theory, Room 328, Phillips Hall

:30-2:40 (12) Hermitian and adjoint abelian operators on certain Banach spaces. Preliminary report. Professor RICHARD J. FLEMING* and Professor JAMES E. JAMISON, Memphis State University (699-B32)

:45-2:55 (13) Infinite-dimensional manifolds modelled on abstract Wiener spaces. Professor SANDRIA N. KERR, Winston-Salem State University (699-B30)

:00-3:10 (14) A functional equation which characterizes polynomial operators with applications to uniqueness. Professor M. ZUHAIR NASHED, Georgia Institute of Technology (699-B29)

:15-3:25 (15) On unilateral shift and the norm closure of the range of a derivation. Dr. HONG WHA KIM, Bucknell University (699-B22)

:30-3:40 (16) Linear topologies for continuity of Sario's linear operator method. II. Professor PAUL A. NICKEL, North Carolina State University (699-B20)

:45-3:55 (17) Multi-parameter resolvents. Preliminary report. Dr. RONALD SHONKWILER, Georgia Institute of Technology (699-B13)

:00-4:10 (18) A Laplace-Stieltjes transform of Mikusinski operator functions. Dr. LARRY W. WILSON, Old Dominion University (699-B10) (Introduced by Professor David Rudd)

:15-4:25 (19) Compact, nuclear, and Hilbert-Schmidt composition operators in H^2. Professor JOEL H. SHAPIRO*, Michigan State University, and Professor PETER D. TAYLOR, Queen's University (699-B8)

:30-4:40 (20) Continuous dependence of critical points. Professor DAVID L. LOVELADY, Florida State University (699-B3)

:45-4:55 (21) Fourier kernels and slowly varying functions. Preliminary report. Professor KUSUM K. SONI* and Professor RAJ PAL SONI, University of Tennessee (699-B28)

FRIDAY, 2:30 P. M.

ssion on Analysis II, Functional Analysis, Room 215, Phillips Hall

30-2:40 (22) Normed Köthe spaces as intermediate spaces L_1 and L_∞. Professor STUART MILLS, Louisiana State University (699-B4)

5-2:55 (23) A series characterization of Hilbert space. Professor WILLIAM H. RUCKLE, Clemson University (699-B6)

0-3:10 (24) Banach spaces of $\{p_i\}$-summable sequences. Dr. RALPH GELLAR and Dr. ROBERT SILBER*, North Carolina State University (699-B7)

5-3:25 (25) Representation of distributions in O'_α as boundary values of functions in tube domains. Dr. RICHARD D. CARMICHAEL, Wake Forest University (699-B9)

0-3:40 (26) Operators of type ℓ^p. Preliminary report. Miss CHARLENE V. HUTTON, Louisiana State University (699-B12)

5-3:55 (27) On Banach spaces for which the quotient of the bidual by the space itself is separable. Preliminary report. Professor RALPH D. McWILLIAMS, Florida State University (699-B14)

0-4:10 (28) Permanence properties of absolute continuity conditions. Preliminary report. Mr. PAUL W. LEWIS, North Texas State University (699-B16)

4:15-4:25	(29)	A characterization of the topology of compact convergences on C(X). Preliminary report. Professor WILLIAM ALAN FELDMAN, University of Arkansas (699-B19)
4:30-4:40	(30)	Convex topology. Preliminary report. Mr. DOUGLAS MOREMAN, Auburn University (699-B23)
4:45-4:55	(31)	Decompositions of operators. Preliminary report. Dr. PAUL CARLTON WOODS, Auburn University, Montgomery (699-B27)
5:00-5:10	(32)	Duals of certain spaces with the Dunford-Pettis property. Preliminary report. Professor CHARLES P. STEGALL, State University of New York, Binghamton (699-B31)

FRIDAY, 2:30 P. M.

Session on Algebra I, Associative Rings and Algebras, Room 383, Phillips Hall

2:30-2:40	(33)	Prime ideals and sheaf representation of a pseudo symmetric ring. Dr. GOOYONG SHIN, North Carolina State University (699-A33) (Introduced by Professor Kwangil Koh)
2:45-2:55	(34)	Relatively weakly finitely generated modules. Preliminary report. Mr. ROBERT CUMBIE, Samford University (699-A28) (Introduced by Dr. W. D. Peeples, Jr.)
3:00-3:10	(35)	The group of inner higher derivations. Preliminary report. Dr. NICKOLAS HEEREMA and Mr. BRIAN J. WESSELINK*, Florida State University (699-A23)
3:15-3:25	(36)	Prime ideals of generalized rings of extended functions on compact Hausdorff spaces. Preliminary report. Ms. JO ELLEN PERRY, North Carolina State University (699-A21) (Introduced by Professor Kwangil Koh)
3:30-3:40	(37)	On antipodes in pointed Hopf algebras. Professor EARL J. TAFT* and Professor ROBERT LEE WILSON, Rutgers University (699-A16)
3:45-3:55	(38)	On centrally splitting. Preliminary report. Dr. ROBERT L. BERNHARDT. University of North Carolina, Greensboro (699-A12)
4:00-4:10	(39)	A theorem about finitely generated dual cones and generalized inverses. Preliminary report. Mr. I. FENNELL BURNS, Auburn University (699-A26)
4:15-4:25	(40)	Simultaneous real orthogonal diagonalization of rectangular complex matrices. Professor PETER M. GIBSON, University of Alabama, Huntsville (699-A17)
4:30-4:40	(41)	A regular determinant of binomial coefficients. Professor PHILIP C. TONNE. Emory University (699-A4)
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FRIDAY, 2:30 P. M.

Session on Algebra II, Group Theory, Generalizations and Nonassociative Algebras, Room 324, Phillips Hall

2:30-2:40	(43)	Elementary semigroups. Professor CARL A. EBERHART, University of Kentucky (699-A36)
2:45-2:55	(44)	Analogue of Pontryagin character theory for topological semigroups. Dr. THOMAS T. BOWMAN, University of Florida (699-A34)
3:00-3:10	(45)	Finite separability, automorphisms of free algebras and some decision problems. Professor TREVOR EVANS, Emory University (699-A31)
3:15-3:25	(46)	The maximal abelian subsemigroup of B_n. Dr. KIM KI-HANG BUTLER and Dr. JAMES R. KRABILL*, Pembroke State University (699-A29)
3:30-3:40	(47)	The Rees theorem for locally compact semigroups. Dr. W. SHEFFIELD OWEN, Auburn University (699-A15)
3:45-3:55	(48)	Maximal ideal complements which are subsemigroups. Preliminary report. Professor JOHN C. PROPES, University of Tennessee, Nashville (699-A11) (Introduced by Professor Anne P. Grams)
4:00-4:10	(49)	The Induced Morphism Theorem: A result in categorical relation theory. Professor TEMPLE HAROLD FAY, Hendrix College (699-A10)
4:15-4:25	(50)	The free compact lattice generated by a topological semilattice. Preliminary report. Professor JAMES W. STEPP, University of Houston (699-A9)
4:30-4:40	(51)	Differentiable subgroupoids. Preliminary report. Mr. JOHN P. HOLMES, Auburn University (699-A14)

FRIDAY, 2:30 P. M.

Session on General Topology I, Room 367, Phillips Hall

2:30-2:40 (52) δ_p and countably paracompact spaces. Preliminary report. Mr. JAMES M. BOYTE, Appalachian State University (699-G35) (Introduced by Professor Ernest P. Lane)

2:45-2:55 (53) On isomorphic groups and homeomorphic spaces. Professor JEONG S. YANG, University of South Carolina (699-G34)

3:00-3:10 (54) P-sets in $\beta N - N$. Professor ROBERT E. ATALLA, Ohio University (699-G33)

3:15-3:25 (55) Product spaces with compactness-like properties. II. Professor JERRY E. VAUGHAN, University of North Carolina (699-G32)

3:30-3:40 (56) Expanding connected topologies. Professor DONALD F. REYNOLDS*, West Virginia University, and Professor JOE A. GUTHRIE and Professor H. EDWARD STONE, University of Pittsburgh (699-G26)

3:45-3:55 (57) A d-sequential completion for quasi-metric spaces. Dr. FRANK L. SCOTT, Wake Forest University (699-G18)

4:00-4:10 (58) On defining a space by a weak base. Dr. FRANK E. SIWIEC, City University of New York, John Jay College (699-G17)

4:15-4:25 (59) Strongly confluent mappings. Mr. B. B. EPPS, JR., University of Houston (699-G16) (Introduced by Professor A. Lelek)

4:30-4:40 (60) A note on perfect images of spaces having a G_δ-diagonal. Professor FRANK G. SLAUGHTER, JR., University of Pittsburgh (699-G15)

4:45-4:55 (61) Certain subsets of θ-refinable spaces are realcompact. Professor PHILLIP L. ZENOR, Auburn University (699-G13)

5:00-5:10 (62) Embedding characterizations for expandable spaces. Professor JAMES CLARENCE SMITH*, Virginia Polytechnic Institute and State University, and Professor JOSEPH C. NICHOLS, Radford College (699-G12)

FRIDAY. 2:30 P. M.

Session on General Topology II, Topological Semigroups, Manifolds and Cell Complexes, Room 332, Phillips Hall

2:30-2:40 (63) Inducing functions difunctionally. Preliminary report. Professor EUGENE M. NORRIS*, University of South Carolina, and Professor A. R. BEDNAREK, University of Florida (699-G19)

2:45-2:55 (64) Semigroups which admit few embeddings. Preliminary report. Professor KENNETH D. MAGILL, JR., State University of New York, Buffalo (699-G9)

3:00-3:10 (65) Topologies on semigroups of functions. Preliminary report. Professor SARASWATHI SUBBIAH, Canisius College (699-G7) (Introduced by Professor Kenneth D. Magill, Jr.)

3:15-3:25 (66) Recognizing closed cells. Preliminary report. Dr. DENNIS C. HASS, Randolph-Macon Woman's College (699-G31)

3:30-3:40 (67) Stable classification of Q^∞-manifolds. Preliminary report. Mr. RICHARD E. HEISEY, Cornell University (699-G30)

3:45-3:55 (68) Homology of coverings of spun CW pairs with applications to knot theory. Mr. WENDELL LEWIS MOTTER, Florida State University (699-G20)

4:00-4:10 (69) Equivalence of two notions of shape for metric spaces. Dr. SIBE MARDESIC, University of Pittsburgh (699-G14)

4:15-4:25 (70) A short proof of Mac Lane's planarity theorem. Dr. PETER V. O'NEIL, College of William and Mary (699-G2)

SATURDAY, 9:00 A. M.

Invited Address, Room 08, Peabody Hall Auditorium

(71) Some results on polynomial rings over a commutative ring. Professor ROBERT GILMER, Florida State University (699-A8)

SATURDAY, 10:15 A. M.

Special Session on Infinite Dimensional Topology, Room 233, Phillips Hall

10:15-10:35 (72) Applications of infinite-dimensional topology. Dr. ROSS GEOGHEGAN, State University of New York, Binghamton (699-G28)

10:45–11:05	(73)	A finite-dimensional space having many of the topological properties of separable Hilbert space ℓ_2. Professor R. RICHARD SUMMERHILL, Kansas State University (699-G25)
11:15–11:35	(74)	Spaces of homeomorphisms. Professor WILLIAM K. MASON, Rutgers University (699-G24)
11:45–12:05	(75)	Function spaces on manifolds. Dr. WILLIAM E. HAVER, University of Tennessee (699-G4)

SATURDAY, 10:15 A. M.

Session on Analysis III, Differential Equations and Measure Theory, Room 328, Phillips Hall

10:15–10:25	(76)	Extensions of vector-valued Stieltjes measures. Dr. FRANKLIN P. WITTE, Virginia Polytechnic Institute and State University (699-B17)
10:30–10:40	(77)	A nonoscillation theorem for the superlinear case of second order differential equations $y'' + yF(y^2,x) = 0$. Mr. KUO-LIANG CHIOU, University of Tennessee (699-B26) (Introduced by Professor John W. Heidel)
10:45–10:55	(78)	Functional differential equations. Preliminary report. Professor MURIL L. ROBERTSON, Auburn University (699-B24)
11:00–11:10	(79)	Limit point criteria for positive definite fourth order differential operators. Professor DON B. HINTON, University of Tennessee (699-B18)
11:15–11:25	(80)	More bounds for eigenvalues. Preliminary report. Professor WILLIAM B. DAY, Auburn University (699-B11)
11:30–11:40	(81)	Oscillatory solutions for a generalized sublinear differential equation. Professor JOHN W. HEIDEL*, University of Tennessee, and Professor I. T. KIGURADZE, Tbilisi State University, U.S.S.R. (699-B5)
11:45–11:55	(82)	Closed orbit solutions for Bernoulli's equation. Dr. G. DiANTONIO, Pennsylvania State University, Capitol Campus (699-B2)

SATURDAY, 10:15 A. M.

Session on Analysis IV, Sequences and Summability, Room 330, Phillips Hall

10:15–10:25	(83)	On finding the distribution function for an orthogonal polynomial set. Dr. WILLIAM R. ALLAWAY, Lakehead University (697-B1)
10:30–10:40	(84)	A summability integral. Mr. RANDOLPH CONSTANTINE, JR., Clemson University (699-B25)
10:45–10:55	(85)	A divergent weighted orthonormal series of broken line Franklin functions. Dr. COKE S. REED, Auburn University (699-B21) (Introduced by Professor Ben Fitzpatrick, Jr.)
11:00–11:10	(86)	Error bounds for interpolating cubic splines under various end conditions. Professor THOMAS R. LUCAS, University of North Carolina, Charlotte (699-B15)
11:15–11:25	(87)	Moment sequences in Hilbert space. Professor GORDON G. JOHNSON, University of Houston (699-B1)

SATURDAY, 10:15 A. M.

Session on Applied Mathematics, Numerical Analysis and Probability, Room 332, Phillips Hall

10:15–10:25	(88)	Best Tschebyscheff approximation by interpolation polynomials. Preliminary report. Mr. CLINTON P. FUELLING, American University (699-C1)
10:30–10:40	(89)	Convergence and error estimates for Hermite and Fejér-Hermite interpolation. Preliminary report. Professor R. D. RIESS, Virginia Polytechnic Institute and State University (699-C2)
10:45–10:55	(90)	On forced oscillations in a rotating stratified liquid. Miss MANJUSRI MAJUMDAR, University of Calcutta, India, and Dr. LOKENATH DEBNATH*, East Carolina University (699-C3)
11:00–11:10	(91)	Finite range random fields and energy fields. Professor WAYNE G. SULLIVAN, Georgia Institute of Technology (699-F1)
11:15–11:25	(92)	Rates of convergence in the normal approximation for sequences of martingale differences. Preliminary report. Dr. WILLIAM F. GRAMS*, Vanderbilt University, and Dr. R. J. SERFLING, Florida State University (699-F2)

SATURDAY, 10:15 A. M.

Session on Algebra III, Number Theory; Algebraic Number Theory, Field Theory, and Polynomials; Combinatorics, and Lattices, Room 265, Phillips Hall

10:15-10:25 (93) Polynomials and linear transformations over finite fields. Dr. THERESA P. VAUGHAN, Duke University (699-A32)

10:30-10:40 (94) A Galois theory for fields K/k finitely generated. Dr. NICKOLAS HEEREMA and Mr. JAMES K. DEVENEY*, Florida State University (699-A22)

10:45-10:55 (95) Factorization of irreducible polynomials over a finite field with the substitution $x^{q^r} - x$ for x. I. Professor ANDREW F. LONG, JR., University of North Carolina, Greensboro (699-A19)

11:00-11:10 (96) Prime divisors of the van der Pol numbers. Professor FREDRIC T. HOWARD, Wake Forest University (699-A5)

11:15-11:25 (97) Extendable ℓ-permutation groups. Dr. THOMAS J. SCOTT, Georgia College (699-A7)

11:30-11:40 (98) On 4-lattice graphs. Dr. RENU LASKAR, Clemson University, and Mr. ARTHUR PELLERIN*, University of North Carolina, Charlotte (699-A18)

11:45-11:55 (99) Row-decreasing, pointed, infinite matrices of cardinals and classes of abelian groups. Professor CHANG MO BANG, Emory University (699-A30)

12:00-12:10 (100) On the construction of cyclic quasigroups. Professor CHARLES C. LINDNER, Auburn University (699-A35)

SATURDAY, 10:15 A. M.

Session on Algebra IV, Commutative Rings and Algebras, Room 324, Phillips Hall

10:15-10:25 (101) Order exact sequences of semivalue groups. Preliminary report. Dr. JOE L. MOTT and Mr. MICHEL F. SCHEXNAYDER*, Florida State University (699-A27)

10:30-10:40 (102) Prime ideals in GCD-domains. Professor PHILIP B. SHELDON, Virginia Polytechnic Institute and State University (699-A25)

10:45-10:55 (103) Unique factorization in Noetherian domains. Dr. JOHN R. HEDSTROM, University of North Carolina, Charlotte (699-A24)

11:00-11:10 (104) Three questions concerning Dedekind domains. Preliminary report. Professor ROBERT GILMER, Florida State University, and Professor ANNE P. GRAMS*, University of Tennessee, Nashville (699-A20)

11:15-11:25 (105) Krull dimension of polynomial rings. Professor JAMES W. BREWER*, Professor PHILLIP MONTGOMERY and Professor EDGAR A. RUTTER, University of Kansas, and Professor WILLIAM J. HEINZER, Purdue University (699-A13)

11:30-11:40 (106) Locally polynomial rings and invertible ideals. Professor PAUL M. EAKIN, University of Kentucky, and Professor WILLIAM J. HEINZER*, Purdue University (699-A6)

11:45-11:55 (107) A theorem of Radon and Hurwitz and orthogonal projective modules. Professor ANTHONY V. GERAMITA* and Professor NORMAN J. PULLMAN, Queen's University (699-A3)

12:00-12:10 (108) A note on dimension sequences. Preliminary report. Mr. THOMAS G. PARKER, Florida State University (699-A2)

SATURDAY, 10:15 A. M.

Session on General Topology III, Room 383, Phillips Hall

10:15-10:25 (109) Regularly closed maps. Professor RAYMOND F. DICKMAN, JR., Virginia Polytechnic Institute and State University (699-G11)

10:30-10:40 (110) Completeness properties and Baire spaces. Preliminary report. Dr. JOHANNES M. AARTS, Delft Institute of Technology, The Netherlands, and Dr. DAVID J. LUTZER*, University of Pittsburgh (699-G10)

10:45-10:55 (111) On scattered spaces. Preliminary report. Dr. V. KANNAN, Madurai University, India, and Professor M. RAJAGOPALAN*, Memphis State University (699-G8)

11:00-11:10 (112) Lusin density and other categoric densities. Professor JACK B. BROWN, Auburn University (699-G6)

11:15-11:25 (113) The compactifications to which an element of $C^*(X)$ extends. Professor RICHARD E. CHANDLER* and Dr. RALPH GELLAR, North Carolina State University (699-G5)

11:30-11:40 (114) Maximal connected Hausdorff spaces. Dr. J. PELHAM THOMAS, Western Carolina University (699-G3)

11:45-11:55 (115) Ind and ind coincide for certain completely normal spaces. Professor JAMES AUSTIN FRENCH, David Lipscomb College (699-G1)

O. G. Harrold
Associate Secretary

Tallahassee, Florida

NEWS ITEMS AND ANNOUNCEMENTS

SABBATICAL LECTURESHIPS AT THE UNIVERSITY OF MASSACHUSETTS

The Department of Mathematics and Statistics of the University of Massachusetts, Amherst, Massachusetts, expects to have available a limited number of Sabbatical Lectureships for the academic year 1973–1974. These lectureships will be open to faculty members of four-year colleges, or universities without a Ph. D. program in mathematics, who wish to spend their sabbatical leaves at the University of Massachusetts. Normally these individuals will be expected to have attained a Master's degree in mathematics, but not a Ph. D. Participants in the program will be half-time lecturers on the university faculty and will be required to teach one course per semester. They will also be expected to enroll in two courses and one seminar. Stipends up to $6, 000 for the academic year are available. In addition, tuition will be waived. Those interested should write to Professor Murray Eisenberg, Acting Head, Department of Mathematics and Statistics, University of Massachusetts, Amherst, Massachusetts 01002.

JOINT MATHEMATICS PROGRAM ESTABLISHED BY CATHOLIC, GEORGETOWN, AND GEORGE WASHINGTON UNIVERSITIES

A joint mathematics program between Catholic, Georgetown, and George Washington Universities was agreed upon recently. The program, which is to begin in the fall of 1973, has been worked out under the aegis of the Consortium of Universities of the Washington Metropolitan Area. Location of the program will rotate between the three institutions during the first three years: George Washington University the first year, Catholic University the second year, and Georgetown University the third year. The rotation will continue on this schedule. Over the first three years, some seventy-five different courses will be available to graduate students; existing courses will be maintained. Among the areas which are scheduled for expansion are applied mathematics and analysis. Professor Arnold Stokes of Georgetown University will serve as chairman of the executive committee for the program. The nine-member committee includes the chairmen of the departments of mathematics of the three universities and two other faculty members from each of the institutions.

NATIONAL SCIENCE FOUNDATION GRADUATE FELLOWSHIPS FOR 1973–1974

The NSF Graduate Fellowships will be awarded for study or work leading to master's or doctor's degrees in the mathematical, physical, medical, biological, engineering, and social sciences, and in the history and philosophy of science. Applicants must be citizens of the United States and will be judged solely on the basis of ability. The annual stipend will be $3, 600 for a twelve-month tenure with no dependency allowances. Applicants will be required to take the Graduate Record Examinations designed to test scientific aptitude and achievement. These examinations, administered by the Educational Testing Service, will be given on December 9, 1972, at designated centers throughout the United States and in certain foreign countries. Panels of eminent scientists appointed by the National Research Council will evaluate qualifications of applicants. Final selection will be made by the Foundation, with awards to be announced on March 15, 1973. The deadline for submission of applications is November 27, 1972. Further information and application materials may be obtained from the Fellowship Office, National Research Council, 2101 Constitution Avenue, Washington, D. C. 20418.

MATHEMATICAL REVIEWS SECTIONS

An increase in the MATHEMATICAL REVIEWS subscription rate, with various options, has been approved by the Council and the Board of Trustees: institutional members, $210; individual members, $60; reviewers, $30. Individuals may order a full subscription, OR they may subscribe to one or more sets of fifty-nine sections (tear sheets). These are the sections that appear on the cover of MATHEMATICAL REVIEWS. A number (Class 1, Class 2, or Class 3) has been assigned to each section. The first sectional subscription is $12, no matter what the class; all additional subscriptions are $10 for Class 1, $8 for Class 2, and $6 for Class 3. In the case of multiple subscriptions, the section with the lowest class number will be considered the first subscription. Full information on prices of these new subscriptions may be obtained from Sales and Subscriptions, American Mathematical Society, P. O. Box 6248, Providence, Rhode Island 02904.

The Seven Hundredth Meeting
Case Western Reserve University
Cleveland, Ohio
November 25, 1972

The seven hundredth meeting of the American Mathematical Society will be held at Case Western Reserve University, Cleveland, Ohio, on Saturday, November 25, 1972. The sessions of the meeting will be held in the Sears Library Building, which is in the center of the main campus of the Case Institute of Technology.

By invitation of the Committee to Select Hour Speakers for Western Sectional Meetings, there will be two one-hour addresses. Professor Calvin R. Putnam of Purdue University will address the Society at 11:00 a.m.; his subject will be "Almost normal operators, their spectra and invariant subspaces." Professor Mary-Elizabeth Hamstrom of the University of Illinois at Urbana-Champaign will speak at 1:45 p.m. on the topic "Homeomorphism and embedding spaces, PL and TOP." Both hour addresses will be presented in Sears 422.

By invitation of the same committee there will be two special sessions of selected twenty-minute papers. Professor Lamberto Cesari of the University of Michigan has arranged one such session for Saturday morning on the subject of Optimization Theory and Optimal Control; the speakers will be Lamberto Cesari, Henry G. Hermes, Edward J. McShane, and Roberto Triggiani. The other special session has been arranged by Professor Alan C. Woods of the Ohio State University on the subject of the Geometry of Numbers; the speakers will be Peter W. Aitchison, Michael N. Bleicher, Thomas W. Cusick, Bohuslav B. Diviš, Lawrence C. Eggan, Richard B. Lakein, John M. Masley, Bretislav Novak, George B. Purdy, and Mushfequr Rahman. There will also be five sessions for the presentation of contributed ten-minute papers.

SYMPOSIUM

On Friday, November 24, 1972, the day before the meeting itself, Case Western Reserve University will sponsor a symposium on the subject of Optimization Theory and Optimal Control, to be held in Sears 404. The schedule of speakers and their titles is as follows: 10:30 a.m., Professor Wendell H. Fleming, Brown University, "The dynamic programming method in optimal stochastic control theory"; 11:30 a.m., Professor Thomas S. Angell, University of Delaware, "Existence of optimal controls for hereditary systems"; 2:00 p.m., Professor Rudolf E. Kalman, University of Florida, "General control theory"; 3:15 p.m., Professor Robert M. Goor, University of Kentucky, "Existence theorems for parametric problems of optimization"; 3:45 p.m., Professor Marc Q. Jacobs, University of Missouri, "Optimum settling problems and functional controllability for time-lag systems"; 4:15 p.m., Professor V. Jurdjevic, University of Toronto, "Geometric theory of controllability and observability"; 4:45 p.m., Professor Jack Warga, Northeastern University, "Hyper relaxed adverse controls." In addition the special session on Optimization Theory and Optimal Control on Saturday morning will be an extension of this symposium.

REGISTRATION

The registration desk will be located in the main lobby of the Sears Library Building. (The sessions will be held on the floor above.) The desk will be open from 10:00 a.m. to 5:00 p.m. on Friday, November 24, and from 8:00 a.m. to 3:00 p.m. on Saturday, November 25.

ACCOMMODATIONS

Because of its proximity to the Case Western Reserve University, Howard Johnson's Motor Hotel at University Circle will be convenient to most participants at this meeting. Two other hotels in the center of Cleveland, the Sheraton Cleveland and the Statler-Hilton Hotel, are also recommended. All three hotels offer free indoor parking. The secretary of the Department of Mathematics at Case Western Reserve University will make reservations for the participants; reservations should be mailed to arrive prior to November 11, 1972. The mailing address is Department of Mathematics, Case Western Reserve University, Cleveland, Ohio 44106.

HOWARD JOHNSON'S MOTOR HOTEL
(Euclid Avenue and E. 107th Street)

Single	$14–$15
Double	18– 19

SHERATON CLEVELAND HOTEL
(Public Square: center of Cleveland)

Single	$14
Double	16
Twin	18

STATLER-HILTON HOTEL
(Euclid Avenue and E. 12th Street)

Single	$15–$19
Double	21– 24
Twin	22– 27

FOOD SERVICE

Tomlinson Hall and Thwing Hall will be open on Friday and Saturday; both serve meals cafeteria style and have snack bars. The Howard Johnson's Restaurant, across the street from the Sears Library Building, serves meals and snacks at all hours, and the campus abuts "Little Italy", which has several good Italian restaurants. In addition, Cleveland has many other good restaurants, including Hungarian, German, Greek, Chinese, and French cuisine, as well as sea-food houses; locations will be available at the registration desk.

TRAVEL AND LOCAL INFORMATION

Cleveland is served by most major airlines, but rail travel has virtually vanished. The airport is on the far western side of Cleveland, and the University is on the eastern side. Air travellers are advised to take the Rapid Transit from the airport to the University-Cedar stop if they are using the Howard Johnson's Hotel (which is about a block and a half north of the University-Cedar stop), or to the Terminal Tower stop if they are using the Sheraton Cleveland Hotel, which is located at the Terminal Tower. The Terminal Tower stop may also be used for the Statler-Hilton Hotel. Exact fare of seventy-five cents is required on the Rapid Transit.

Motorists coming to the University area will save time if the city is approached from the east since the University is located at about Euclid Avenue and E. 107th Street. All Ohio road maps will have a general map of Cleveland.

In addition to the parking facilities at the hotels, low-cost parking is available behind Severance Hall, which is located across Euclid Avenue from the Sears Library Building.

ENTERTAINMENT

There will be a free beer party at 8:00 p.m. on Friday, November 24, on the top floor of the Howard Johnson's Hotel. On Friday and Saturday evenings the Cleveland Orchestra and Chorus will perform the Verdi Requiem; participants who wish to attend a performance should write directly to the box office, Severance Center, University Circle, Cleveland, Ohio 44106. The Cleveland Museum is also in the University Circle, and will be open on the days of the Conference. For those interested in theatre, four plays will be given by professional companies; more information may be obtained from the secretary of the Department of Mathematics.

PROGRAM OF THE SESSIONS

The time limit for each contributed paper in the general sessions is ten minutes and in the special sessions is twenty minutes. To maintain this schedule, the time limits will be strictly enforced.

SATURDAY, 8:30 A. M.

Special Session on the Geometry of Numbers, Room 401, Sears Library Building

8:30-8:50 (1) A new algorithm for Egyptian fractions. Professor MICHAEL N. BLEICHER, University of Wisconsin (700-A4)

9:00-9:20 (2) Approximation properties of some complex continued fractions. Preliminary report. Professor RICHARD B. LAKEIN, State University of New York, Buffalo (700-A12)

9:30-9:50 (3) On cyclotomic fields Euclidean for the norm map. Dr. JOHN MYRON MASLEY, University of Illinois, Chicago Circle (700-A3)

10:00-10:20 (4) Two finiteness theorems in the Minkowski theory of reduction. Professor PETER W. AITCHISON, University of Manitoba (700-A8) (Introduced by Professor Alan C. Woods)

10:30-10:50 (5) A remark on the theory of lattice points. Dr. BŘETISLAV NOVAK, Charles University, Prague, Czechoslovakia, and University of Illinois (700-A11) (Introduced by Professor Alan C. Woods)

SATURDAY, 8:30 A. M.

Special Session on Optimization Theory and Optimal Control, Room 404, Sears Library Building

8:30-8:50 (6) Controllability and observability of dynamical systems in Banach space with bounded operators. Preliminary report. Mr. ROBERTO TRIGGIANI, University of Minnesota (700-C6)

9:00-9:20 (7) Controllability and the topology of reachable sets. Preliminary report. Professor HENRY G. HERMES, University of Colorado (700-B1)

9:35-9:55 (8) Existence theorems for problems of optimization with distributed and boundary controls. Preliminary report. Professor LAMBERTO CESARI, University of Michigan (700-B2)

10:15-10:35 (9) Stochastic optimal control theory. Professor EDWARD J. McSHANE, University of Virginia (700-C5)

*For papers with more than one author, an asterisk follows the name of the author who plans to present the paper at the meeting.

SATURDAY, 9:00 A. M.

Session on Analysis and Topology, Room 408, Sears Library Building

9:00-9:10 (10) An introduction to relations among inequalities. Dr. CARLOS ALBERTO INFANTOZZI, Instituto de Estudios Superiores, Montevideo, Uruguay (700-B8)

9:15-9:25 (11) An estimate for the rate of convergence of convolution products of sequences. Professor RANKO BOJANIC and Mrs. YOU-HWA LEE*, Ohio State University (700-B5)

9:30-9:40 (12) Operators satisfying a sequential growth condition. Preliminary report. Professor BHUSHAN LAL WADHWA, Cleveland State University (700-B6)

9:45-9:55 (13) On peak interpolation sets for the polydisc algebra. Mr. HOWARD LEWIS PENN, Eastern Michigan University and University of Michigan (700-B10)

10:00-10:10 (14) Some properties of real tangent cones. Preliminary report. Professor KEITH M. KENDIG, Cleveland State University (700-B7)

10:15-10:25 (15) Strongly bounded differential systems. Professor STEPHEN R. BERNFELD, University of Missouri, Columbia (700-B9)

10:30-10:40 (16) An example of a local flow on a manifold. Professor DENIS L. BLACKMORE, Newark College of Engineering (700-G2)

SATURDAY, 9:15 A. M.

Session on Combinatorics and Graph Theory, Room 409, Sears Library Building

9:15-9:25 (17) The number of trees in a wheel. Professor FRANK HARARY and Mr. ALLEN JOHN SCHWENK*, University of Michigan, Professor PETER V. O'NEIL, College of William and Mary, and Professor RONALD C. READ, University of Waterloo (700-A10)

9:30-9:40 (18) A maximal toroidal graph which is not a triangulation. Professor FRANK HARARY and Mr. ALLEN JOHN SCHWENK, University of Michigan, Professor PAUL C. KAINEN*, Case Western Reserve University, and Professor ARTHUR THOMAS WHITE II, Western Michigan University (700-A14)

9:45-9:55 (19) The Ramsey multiplicity of a graph. Professor FRANK HARARY*, University of Michigan, and Professor GEERT C. E. PRINS, Wayne State University (700-A9)

10:00-10:10 (20) Acyclic colorings of planar graphs. Professor BRANKO GRÜNBAUM, University of Washington (700-A15)

10:15-10:25 (21) Cubic irreducibly nonprojective planar graphs. Preliminary report. Professor HENRY H. GLOVER and Professor JOHN P. HUNEKE*, Ohio State University (700-A18)

10:30-10:40 (22) The n-genus of a graph. Preliminary report. Dr. VANCE FABER, University of Colorado, Denver Center (700-A17)

SATURDAY, 11:00 A. M.

Invited Address, Room 422, Sears Library Building

(23) Almost normal operators, their spectra and invariant subspaces. Professor CALVIN R. PUTNAM, Purdue University (700-B4)

SATURDAY, 1:45 P. M.

Invited Address, Room 422, Sears Library Building

(24) Homeomorphism and embedding spaces, PL and TOP. Professor MARY-ELIZABETH HAMSTROM, University of Illinois (700-G1)

SATURDAY, 3:00 P. M.

Special Session on the Geometry of Numbers, Room 401, Sears Library Building

3:00-3:20 (25) Lattice points on strictly convex curves and surfaces. Professor BOHUSLAV B. DIVIŠ, Ohio State University (700-A6)

3:30-3:50 (26) View-obstruction problems. Dr. THOMAS W. CUSICK, State University of New York, Buffalo (700-A5)

4:00-4:20 (27) The lattice triple packing of spheres. Preliminary report. Dr. GEORGE B. PURDY, University of Illinois (700-A13)

4:30-4:50 (28) Domain of action method. Professor MUSHFEQUR RAHMAN, Eastern Illinois University (700-A7)

5:00–5:20 (29) Cassini ovals. Preliminary report. Professor LAWRENCE C. EGGAN, Illinois State University (700–A19)

SATURDAY, 3:00 P. M.

Session on Numerical Analysis and Applied Mathematics, Room 404, Sears Library Building

3:00–3:10 (30) The numerical evaluation by splines of the Fourier transform and the Laplace transform. Dr. SHERWOOD D. SILLIMAN, Cleveland State University (700–B3)

3:15–3:25 (31) Some results on the T + m–transformation. Mr. ROLAND F. STREIT, University of Illinois (700–C1)

3:30–3:40 (32) An iterative solution of linear operator equations. Professor CHARLES W. GROETSCH, University of Cincinnati (700–C4)

3:45–3:55 (33) Wilson operator product expansions and the Dürr–Winter identification of gauge potentials. Professor HENDRICUS G. LOOS, Cleveland State University (700–C3)

4:00–4:10 (34) Association of wave mechanics with discontinuities in Blasius' parametric function. Preliminary report. Professor MARIA Z. v. KRZYWOBLOCKI, Michigan State University (700–C2)

SATURDAY, 3:00 P. M.

Session on Algebra and Geometry, Room 408, Sears Library Building

3:00–3:10 (35) $\mathfrak{F}$-projectors of finite solvable groups. Preliminary report. Professor JOSEPH A. TROCCOLO, Cleveland State University (700–A16)

3:15–3:25 (36) Witt's multiplicative forms over R(t). Professor JOHN S. HSIA and Professor ROBERT P. JOHNSON*, Ohio State University (700–A1)

3:30–3:40 (37) Structure spaces in vector lattices. Professor WILLIAM G. CHANG, Cleveland State University (700–A2)

3:45–3:55 (38) Nontriply colinear sets of points. Preliminary report. Professor HENRY W. LEVINSON*, Mr. G. F. POLLICE, and Professor MAHMOUD SAYRAFIEZADEH, Rutgers University (700–D1)

SATURDAY, 3:00 P. M.

Session on Statistics and Probability, Room 409, Sears Library Building

3:00–3:10 (39) Moments of h–statistics, using ordered partitions. Preliminary report. Dr. B. C. GUPTA, Universidade Federal do Rio de Janeiro, Brasil (700–F1) (Introduced by Professor Guilherme M. de La Penha)

3:15–3:25 (40) Asymptotic distribution of extremes of dependent observations with random sample size. Professor JANOS GALAMBOS, Temple University (700–F4)

3:30–3:40 (41) On continuous zero–two law. Mr. WILLIAM E. WINKLER, Ohio State University (700–F3)

3:45–3:55 (42) Weak infinitesimal generators of a class of jump–perturbed Markov processes. Dr. JOSEPH M. COOK, Argonne National Laboratory, Argonne, Illinois (700–F2)

Paul T. Bateman
Associate Secretary

Urbana, Illinois

Session of Contributed Papers in Biomathematics
Washington Hilton Hotel
Washington, D. C.
December 30, 1972

A one-day session of contributed papers in Biomathematics will be held on Saturday, December 30, 1972, in the Hemisphere Room of the Washington Hilton Hotel, Washington, D. C. This session is cosponsored by the American Mathematical Society and the Society for Industrial and Applied Mathematics, and is being held in cooperation with Section A (Mathematics) of the American Association for the Advancement of Science.

The program, consisting of a series of contributed papers, was arranged by a committee selected by the presidents of the American Mathematical Society and the Society for Industrial and Applied Mathematics.

PROGRAM

December 30, 9:00 a.m.

Chairman: S. Levin, Section of Ecology and Systematics, Division of Biological Sciences and Center for Applied Mathematics, Cornell University

The insulin threshold secretory process and the analysis of insulin measurements in vivo
Abraham Silvers, Department of Medicine, Stanford University

Diffusion in artificial kidney dialyzers
Stephen M. Ross, Department of Nuclear Engineering, University of Washington

A network problem arising from capillary flow
R. B. Kellog, Institute for Fluid Dynamics & Applied Mathematics, University of Maryland

Biomathematical model of an aneurysm of the circle of Willis
Jane Cronin Scanlon, Hill Center for the Mathematical Sciences, Rutgers University

Symmetry in embryonic development
Herbert Jehle, Department of Physics, University of Maryland

Source of the logical peculiarity of taxonomic definitions
Mary B. Williams, Department of Statistics, North Carolina State University

Trees of macromolecular sequences
David Sankoff, Centre de Recherches Mathématiques, Université de Montréal

2:00 p.m.

Chairman: Jack D. Cowan, Department of Theoretical Biology, University of Chicago

On the fundamental equations of nervous impulse transmission
S. P. Hastings, Department of Mathematics, State University of New York at Buffalo

Cooperative interactions between endogenously active cells
A. Sastre, Center for Applied Mathematics, Cornell University

Neural population and motoneuron pools
J. Feldman, Department of Theoretical Biology, University of Chicago

Cyclic group model for the coding of adaptive information in the visual system
A. A. Harkavy, State University of New York, College at New Paltz

Properties of a neural model for memory
J. A. Anderson, Rockefeller University

The nonlinear dynamics of recurrently interacting neurons or neuron populations
S. Grossberg, Department of Mathematics, Massachusetts Institute of Technology

Jack D. Cowan, Chairman
AMS-SIAM Committee on
Mathematics in the
Life Sciences

Chicago, Illinois

PRELIMINARY ANNOUNCEMENTS OF MEETINGS

Seventy-Ninth Annual Meeting
Fairmont Hotel
Dallas, Texas
January 25 – 28, 1973

The seventy-ninth annual meeting of the American Mathematical Society will be held at the Fairmont Hotel in Dallas, Texas, from Thursday, January 25, through Sunday, January 28, 1973. The meeting will be held in conjunction with the annual meeting of the Association for Symbolic Logic (January 25 and 26), and the annual meeting of the Mathematical Association of America (January 27 through January 29). The National Council of Teachers of Mathematics will meet jointly with MAA on Saturday and Sunday, January 27 and 28.

On the recommendation of the Council of the Society, a panel discussion will take place on Friday evening at 8:30 p.m. The panel discussion, chaired by Professor Raoul H. Bott of Harvard University, will consider graduate education in mathematics in the coming decade. Members of the panel will be Professor Peter L. Duren, University of Michigan; Professor Arthur Ogus, Princeton University; Professor Calvin C. Moore, University of California, Berkeley; and Professor Jacob T. Schwartz, New York University.

There will be one set of Colloquium Lectures, to be given by Professor Michael F. Atiyah of the Institute for Advanced Study. The title of his series of four lectures will be "The index of elliptic operators." The lectures will be given at 1:30 p.m. on Thursday, at 1:30 p.m. on Friday, at 4:30 p.m. on Saturday, and at 1:30 p.m. on Sunday.

By invitation of the Committee to Select Hour Speakers for Annual and Summer Meetings, there will be seven invited hour addresses. The speakers, times, and titles are as follows: Professor William K. Allard of Princeton University, 9:00 a.m. on Thursday, "Some unsolved regularity problems in the calculus of variations"; Professor Samuel Gitler of the Instituto Politécnico Nacional in Mexico City, 10:30 a.m. on Thursday, "Cohomology operations and obstructions"; Professor Barbara L. Osofsky of Rutgers University, 3:00 p.m. on Thursday, "The subscript of $\aleph_n$ and the nonvanishing of Ext^n and $\varprojlim^{(n)}$"; Professor Henry B. Mann of the University of Arizona, 9:00 a.m. on Friday, "Additive group theory: A progress report"; Professor Lester E. Dubins of the University of California, Berkeley, 10:30 a.m. on Friday, "The abstract gambler's problem"; Professor Michael E. Fisher of Cornell University, 3:00 p.m. on Friday, "The hunt for singularities in statistical mechanics"; Professor Nicholas M. Katz of Princeton University, 4:30 p.m. on Friday, topic to be announced in the January issue of these *Notices*.

The Josiah Willard Gibbs Lecture will be presented by Professor Jürgen K. Moser of the Courant Institute of Mathematical Sciences, New York University, at 8:30 p.m. on Thursday, January 25. The tentative title of Professor Moser's lecture is "The stability concept in dynamical systems."

There will be no limit on the number of contributed ten-minute papers to be presented at the meeting. Sessions will be held morning and afternoon on both Thursday and Friday, and during the afternoon only on Saturday and Sunday. Abstracts should be submitted to the American Mathematical Society, P. O. Box 6248, Providence, Rhode Island 02904, so as to arrive prior to the deadline of November 14, 1972. Because of the large number of contributed papers expected, no provision can be made for late papers.

On Thursday, January 25 there will be six special sessions of selected twenty-minute papers. Professor Joanne Elliott of Rutgers University is arranging a special session on Probability and Related Topics in Analysis; the tentative list of speakers includes Donald L. Burkholder, A. M. Garsia, Ronald K. Getoor, Richard F. Gundy, P. M. Millar, Stanley A. Sawyer, Martin L. Silverstein, Daniel W. Stroock, and William A. Veech. Professor John W. Gray of the University of Illinois and Professor Saunders Mac Lane of the University of Chicago are arranging a special session on Category Theory; the tentative list of speakers includes Horst Herrlich, Pierre J. Malraison, Jr., David C. Newell, and Paul H. Palmquist. Professor Mary Ellen Rudin of the University of Wisconsin is arranging a session on General Topology; the tentative list of speakers includes W. Wistar Comfort, Jun-iti Nagata, Jack Segal, and Franklin D. Tall. Professor Lowell I. Schoenfeld of the State University of New York at Buffalo is arranging a special session on Number Theory; the tentative list of speakers includes George E. Andrews, A. O. L. Atkin, Bruce C. Berndt, Itshak Borosh, John D. Brillhart, Leonard Carlitz, Mohindar S. Cheema, Harvey Cohn, Richard K. Guy, Richard B. Lakein, R. Sherman Lehman, Carlos J. Moreno, Morris Newman, and Karl K. Norton. Professors Michio Suzuki and John H. Walter are arranging a special session on Finite Groups; the tentative list of speakers includes Michael Aschbacher, Robert H. Gilman, George Glauberman, David M. Goldschmidt, Marshall Hall, Jr., Koichiro Harada, Donald G. Higman, William Kantor, Arunas Rudvalis, Leonard L. Scott, Jr., Gary M. Seitz, Ernest E. Shult, Louis Solomon, and David Wales. Professor John Todd of the California

Institute of Technology is arranging a special session on Numerical Mathematics; the tentative list of speakers includes Walter Gautschi, William B. Jones, Herbert B. Keller, Isaac J. Schoenberg, Martin H. Schultz, Lawrence F. Shampine, David M. Young, Jr., and Hans J. Zassenhaus.

On Friday, January 26 there will be six more special sessions of selected twenty-minute papers. Professor Mary W. Gray of American University is arranging a special session on Ring Theory; the tentative list of speakers includes E. Graham Evans, Jr., Kent R. Fuller, Robert Gordon, Aron V. Jategoankar, Dana May Latch, Lawrence S. Levy, M. Susan Montgomery, Thomas S. Shores, Wolmer V. Vasconcelos, and Roger A. Wiegand. Professor Paul R. Halmos of Indiana University is arranging a special session on Compact Perturbations of Operators; the tentative list of speakers includes Lewis A. Coburn, James A. Deddens, Peter A. Fillmore, Douglas N. Clark, Carl M. Pearcy, and Peter M. Rosenthal. Professor Erik Hemmingsen of Syracuse University is arranging a special session on Maps of Manifolds on Manifolds; the tentative list of speakers includes Philip T. Church, Christopher Lacher, Morris L. Marx, Louis F. McAuley, Daniel R. McMillan, Jr., Thomas M. Price, William L. Reddy, and D. Wilson. Professor Peter A. Loeb of the University of Illinois is arranging a special session on Nonstandard Analysis; the tentative list of speakers includes Allen R. Bernstein, Donald Brown, C. Ward Henson III, Peter A. Loeb, Wilhelmus A. J. Luxemburg, Lawrence C. Moore, Jr., Rohit J. Parikh, Abraham Robinson, K. D. Stroyan, Frank A. Wattenberg, and Volker B. Weispfenning. Professor Gian-Carlo Rota of Massachusetts Institute of Technology is arranging a special session on Combinatorial Theory; the tentative list of speakers (all under 35 years of age) includes Edward A. Bender, Kenneth P. Bogart, Richard A. Brualdi, Thomas H. Brylawski, T. Dowling, S. Fisk, M. Fredman, Ladnor D. Geissinger, Jay R. Goldman, William H. Graves, Curtis Greene, Lawrence H. Harper, Douglas G. Kelly, Robert J. McEliece, Neil Robertson, Bruce L. Rothschild, Richard P. Stanley, Neil L. White, Stanley G. Williamson, and Richard M. Wilson. Professor Mitchell H. Taibleson is arranging a special session on Harmonic Analysis over Local Fields; the tentative list of speakers includes Jia-arng Chao, Stephen S. Gelbart, John Gosselin, Charles Gulizia, Edwin Hewitt, Keith L. Phillips, Paul J. Sally, and Mitchell H. Taibleson.

COUNCIL AND BUSINESS MEETING

The Council of the Society will meet at 2:00 p.m. on Wednesday, January 24, 1973, in the Gold Room (also known as Room A) of the Fairmont Hotel.

The Business Meeting of the Society will be held on Saturday afternoon in the Regency Ballroom.

PREREGISTRATION AND REGISTRATION

The registration desk for this meeting will be in the lobby of the International Ballroom, located on the lobby level of the Fairmont Hotel. The desk will be open from 2:00 p.m. to 8:00 p.m. on Wednesday, January 24; from 8:00 a.m. to 5:00 p.m. on Thursday, January 25; from 8:30 a.m. to 4:30 p.m. on Friday through Sunday, January 26–28; and from 8:30 a.m. to 2:30 p.m. on Monday, January 29.

Participants who wish to preregister only should complete lines 1–7 of the form found in the October *Notices*, last page. Those who preregister will pay a lower registration fee than those who register at the meeting, as indicated in the schedule below. Preregistrants will be able to pick up their badges and programs when they arrive at the meeting. Complete instructions on procedure for making hotel reservations is given in the section entitled ACCOMMODATIONS.

The registration fees for the meeting are as follows:

	Preregistration (by mail received prior to December 20)	At meeting
Member	\$ 7	\$10
Student or unemployed member	1	1
Nonmember	12	15

There will be no extra charge for members of the families of registered participants.

The unemployed status refers to any member currently unemployed and actively seeking employment. It is not intended to include members who have voluntarily resigned or retired from their latest position.

Students are considered to be only those currently working toward a degree who do not receive an annual compensation totaling more than \$7,000 from employment, fellowships, and scholarships.

Checks for the preregistration fee should be mailed to arrive not later than December 20, 1972. Participants need not utilize the services of the Housing Bureau to make room reservations but it is essential to complete the preregistration section (lines 1–7) of the form found on the last page of the October *Notices* to take advantage of the lower registration fee.

EMPLOYMENT REGISTER

The Mathematical Sciences Employment Register will be maintained from 9:00 a.m. to 4:00 p.m. on Friday, January 26, and from 9:00 a.m. to 5:40 p.m. on Saturday and Sunday, January 27–28, in Room A (Gold Room) of the Fairmont Hotel. Room A is located on the banquet level of the hotel.

EXHIBITS

The book and educational media exhibits will be displayed in the International Ballroom of the Fairmont Hotel, from Thursday through Sunday, January 25–28. The exhibits will be displayed from noon to 5:00 p.m. on Thursday; from 9:00 a.m. to 5:00 p.m. on Friday and Saturday; and from 9:00 a.m. to noon on Sunday. All participants are encouraged to plan a visit to the exhibits sometime during the meeting.

SUMMARY OF ACTIVITIES

	AMERICAN MATHEMATICAL SOCIETY	OTHER ORGANIZATIONS
	WEDNESDAY, January 24	
2:00 p.m.	COUNCIL MEETING	
2:00 p.m. - 8:00 p.m.	REGISTRATION	REGISTRATION
	THURSDAY, January 25	
8:00 a.m. - 5:00 p.m.	REGISTRATION	REGISTRATION
morning		Sessions of the Association for Symbolic Logic
9:00 a.m. - 5:00 p.m.	Sessions of Contributed Papers	
9:00 a.m.	Invited Address: William K. Allard Some unsolved regularity problems in the calculus of variations	
9:00 a.m.	Special Sessions of Twenty-Minute Papers: Probability and Related Topics in Analysis Category Theory General Topology Number Theory Finite Groups Numerical Mathematics	
10:30 a.m.	Invited Address: Samuel Gitler Cohomology operations and obstructions	
12:00 noon - 5:00 p.m.	EXHIBITS	EXHIBITS
afternoon		Sessions of the ASL
1:30 p.m.	Colloquium Lectures: The index of elliptic operators, Lecture I Michael F. Atiyah	
3:00 p.m.	Invited Address: Barbara L. Osofsky The subscript of $\aleph_n$ and the nonvanishing of Ext^n and $\varprojlim^{(n)}$	
4:30 p.m.		ASL -- Retiring Presidential Address: Mathematical problems Abraham Robinson
8:30 p.m.	Gibbs Lecture: Jürgen K. Moser The stability concept in dynamical systems (tentative title)	
	FRIDAY, January 26	
8:30 a.m. - 4:30 p.m.	REGISTRATION	REGISTRATION
morning		Sessions of the ASL
9:00 a.m. - 4:00 p.m.	EMPLOYMENT REGISTER	EMPLOYMENT REGISTER
9:00 a.m. - 4:00 p.m.		Mathematical Association of America -- BOARD OF GOVERNORS
9:00 a.m. - 5:00 p.m.	Sessions of Contributed Papers	
9:00 a.m. - 5:00 p.m.	EXHIBITS	EXHIBITS
9:00 a.m.	Invited Address: Henry B. Mann Additive group theory: A progress report	
9:00 a.m.	Special Sessions of Twenty-Minute Papers: Ring Theory Compact Perturbations of Operators Maps of Manifolds on Manifolds Nonstandard Analysis Combinatorial Theory Harmonic Analysis Over Local Fields	
10:00 a.m. - 11:00 a.m.		Association for Women in Mathematics -- Panel on employment and affirmative action - BUSINESS MEETING

SUMMARY OF ACTIVITIES

	AMERICAN MATHEMATICAL SOCIETY	OTHER ORGANIZATIONS
	FRIDAY, January 26	
10:00 a.m. - 12:30 p.m.	ROCKY MOUNTAIN MATHEMATICS CONSORTIUM -- Annual Meeting	
10:30 a.m.	Invited Address: Lester E. Dubins The abstract gambler's problem	
afternoon		Sessions of the ASL
1:30 p.m.	Colloquium Lectures II Michael F. Atiyah	
2:30 p.m.		National Association of Mathematicians -- BUSINESS MEETING
3:00 p.m.	Invited Address: Michael E. Fisher The hunt for singularities in statistical mechanics	
4:30 p.m.	Invited Address: Nicholas M. Katz Title to be announced	
7:00 p.m.		MAA - Motion Pictures
8:30 p.m.	Panel Discussion: Graduate education in mathematics in the coming decade Raoul H. Bott (moderator) Peter L. Duren Calvin C. Moore Arthur Ogus Jacob T. Schwartz	
	SATURDAY, January 27	
8:30 a.m. - 4:30 p.m.	REGISTRATION	
9:00 a.m. - 5:00 p.m.	EXHIBITS	
9:00 a.m. - 5:40 p.m.	EMPLOYMENT REGISTER	
9:00 a.m.		Joint Session -- Mathematical Association of America with the National Council of Teachers of Mathematics: New directions in mathematics education
9:00 a.m. - 9:50 a.m.		Mathematics education for the elementary teacher J. L. Kelley
10:00 a.m. - 10:50 a.m.		The mathematics program for elementary teachers at Indiana University M. D. Thompson
11:00 a.m. - 11:50 a.m.		Individualized instruction - a viable alternative J. W. Riner, Jr. and B. K. Waits
afternoon	BUSINESS MEETING	
afternoon	Sessions of Contributed Papers	
afternoon		National Association of Mathematicians -- GENERAL MEETING
4:30 p.m.	Colloquium Lectures III Michael F. Atiyah	
7:00 p.m.		MAA - Motion Pictures
	SUNDAY, January 28	
8:30 a.m. - 4:30 p.m.	REGISTRATION	
9:00 a.m.		MAA-NCTM -- Joint Session
9:00 a.m. - 9:50 a.m.		Research in the teaching and learning of mathematics Jeremy Kilpatrick
9:00 a.m. - 12:00 noon	EXHIBITS	
9:00 a.m. - 5:40 p.m.	EMPLOYMENT REGISTER	
10:00 a.m. - 10:50 a.m.		MAA - BUSINESS MEETING

SUMMARY OF ACTIVITIES

	AMERICAN MATHEMATICAL SOCIETY	OTHER ORGANIZATIONS
	SUNDAY, January 28	
11:00 a.m. - 11:50 a.m.		MAA - Panel Discussion: Why, what and how to publish Harley Flanders Leonard Gillman Paul R. Halmos Robert R. Korfhage (moderator) Beatrice Shube
11:50 a.m. - 12:15 p.m.		General discussion by the panel and the audience
1:30 p.m.	Colloquium Lectures IV Michael F. Atiyah	
afternoon	Sessions of Contributed Papers	
1:45 p.m. - 3:45 p.m.		Conference Board of the Mathematical Sciences -- Panel Discussion: Mathematical modeling in the decision sciences Thomas E. Caywood Alan Goldman (moderator) Gordon Raisbeck Robert M. Thrall
7:00 p.m.		MAA - Motion Pictures
	MONDAY, January 29	
8:30 a.m. - 2:30 p.m.	REGISTRATION	
9:00 a.m.		Sessions of the MAA
9:00 a.m. - 9:50 a.m.		The history of mathematics as a pedagogical tool Philip S. Jones
10:00 a.m. - 10:50 a.m.		Topology for the undergraduate R. H. Bing
11:00 a.m. - 11:50 a.m.		Sheaves, sets and categories Saunders Mac Lane
1:30 p.m. - 2:20 p.m.		Structure theorems in the theory of abelian groups Laszlo Fuchs
2:30 p.m. - 3:20 p.m.		The use of computers for formal mathematics Jean E. Sammet
3:30 p.m. - 4:20 p.m.		New trends in the application of mathematics Robert M. Thrall

AUDIO TAPES AND BOOK SALE

Audio tapes of invited addresses and books published by the Society will be sold for cash prices somewhat below the usual prices when these same books and tapes are sold by mail.

ACCOMMODATIONS

A form for requesting accommodations will be found in the October *Notices*, last page. The use of the housing services requires preregistration for the meeting. Persons desiring accommodations should complete the form (or a reasonable facsimile) and send it to Mathematics Meetings Housing Bureau, c/o American Mathematical Society, P. O. Box 6248, Providence, Rhode Island 02904. The AMS will forward the reservation forms to the Dallas convention bureau which will handle accommodations. Reservations will be made in accordance with preferences indicated on the reservation form, insofar as this is possible, and all reservations will be confirmed. Deposit requirements vary from hotel to hotel, and participants will be informed of any such requirement at the time of confirmation. REQUESTS FOR RESERVATIONS SHOULD BE MAILED TO ARRIVE IN PROVIDENCE NO LATER THAN DECEMBER 20, 1972.

ADOLPHUS	
Singles	\$14.00-\$ 20.00
Doubles	16.00- 22.00
Twins	19.00- 25.00
Triples	8.00 per person
Quadruples	6.50 per person
Suites	60.00- 100.00 (2-room)
BAKER	
Singles	\$12.50-\$ 18.50
Doubles	15.50- 21.50
Twins	18.50- 23.50
Doubles or twins (with rollaway— 3 persons)	21.00- 27.00
Suites	39.00- 45.00 (2-room)
	60.00- 80.00 (3-room)
FAIRMONT	
Singles	\$18.00-\$ 41.00
Doubles and twins	24.00- 49.00
Suites	60.00- 90.00 (2-room)
	80.00- 100.00 (3-room)
HOLIDAY INN (DOWNTOWN)	
Singles	\$18.00 &\$ 20.00
Doubles (two double beds)	20.00 & 24.00
PLAZA	
Singles	\$10.00-\$ 14.00
Doubles	15.00- 17.00
Twins	16.00- 18.00
SHERATON-DALLAS	
Singles	\$18.00-\$ 28.00
Doubles and twins	24.00- 34.00
Suites	70.00 and up (2-room)
	95.00 and up (3-room)
STATLER HILTON	
Singles	\$18.00-\$ 26.00
Doubles	25.00- 33.00
Twins	27.00- 33.00

NATIONAL SCIENCE FOUNDATION INFORMATION CENTER

NSF staff members will be available to provide counsel and information on all NSF programs of interest to mathematicians from 9:00 a.m. to 5:00 p.m. on January 26, 27, and 28, in Room M. This room, also known as the Directors Room, is located on the banquet level of the Fairmont Hotel.

ENTERTAINMENT

There are many things to see and do in the Dallas area. Available at the Registration Desk will be brochures describing the various tours and places of interest around the city. Information concerning the interesting shopping villages will also be available.

Dallas has numerous art galleries and museums, many of which are located on the grounds of the State Fair of Texas. Within a thirty-mile radius are some eleven institutions of higher learning with many outstanding exhibits and collections, such as the Owens Art Museum at SMU.

Legitimate theaters abound in Dallas, including several dinner theaters. Like all major cities, Dallas has many restaurants of high quality offering a variety of cuisines. The most complete guide to restaurants will be found in the Visitor's Guide to Dallas, a magazine, which should be available in all the hotel rooms, and which will be available at the Registration Desk. This magazine also gives a complete listing of the art, music, and drama currently offered in the city. Restaurants are listed by specialty, and average price categories are included. There will also be a guide to dining near the Fairmont available at the Registration Desk.

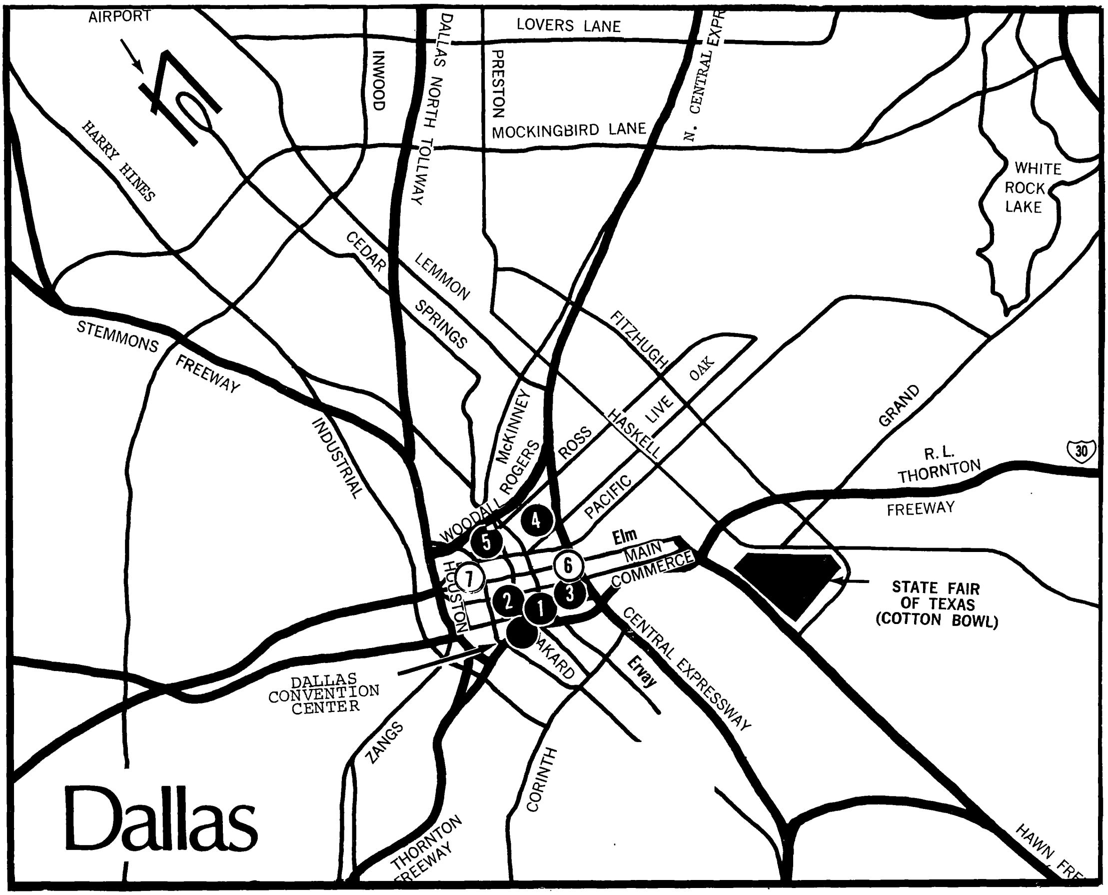
TRAVEL AND LOCAL INFORMATION

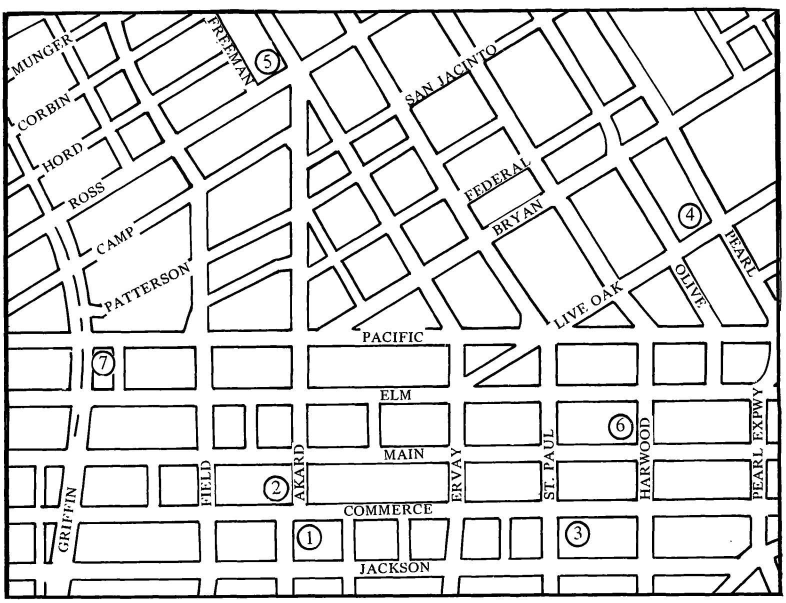
Dallas is located midway between the east and west coasts, and is easily accessible by air or ground transportation. It has more air service available than any other southwestern city. The major airlines serving Dallas include American, Braniff, Continental, Delta, Eastern, Frontier, National, Ozark, Texas International, and others.

The bus lines serving Dallas include the Continental Trailways and the Greyhound Bus Lines. The bus terminals are located in downtown Dallas.

Railroad service from Chicago to Fort Worth is provided by Amtrak. Fort Worth is located 26 miles to the west of Dallas. Again this year special Amtrak accommodations are being arranged for those AMS members who would like to travel to the Dallas AMS meeting by rail. Plans call for at least one private sleeping car for exclusive use of mathematicians aboard Amtrak's Texas Chief. The group will leave Chicago on Wednesday, January 24, 1973, for the overnight trip to Dallas. The return trip will begin Monday, January 29, 1973. Those interested should contact J. J. Uhl, Department of Mathematics, University of Illinois, Urbana, Illinois 61801. Frequent bus service is available between Fort Worth and Dallas.

Limousine service, stopping at all downtown hotels, is available from the airport. The





1. Baker
2. Adolphus
3. Statler-Hilton
4. Sheraton-Dallas
5. Fairmont
6. Plaza
7. Holiday Inn-Downtown

DETAIL OF HOTEL AREA

fare is $1.50. Taxicab service is also available at a charge of $2.50. Participants should join a group, and take a taxicab to the hotel if at all possible.

Rent-A-Car Service is available at very reasonable rates. Those persons who come to Dallas by automobile will find that most of the listed hotels offer free parking to their guests.

WEATHER

The winter months in Dallas are very mild, and the weather is normally pleasant with clear and sunshiny days. During January, the Dallas average maximum temperature is 55.2°, and the minimum is 35.7°; normal precipitation is 2.2 inches. Rarely does Dallas have ice and snow.

MAIL AND MESSAGE CENTER

All mail and telegrams for persons attending the meetings should be addressed in care of Mathematics Meetings, Fairmont Hotel, Dallas, Texas 75201. Mail and telegrams so addressed may be picked up at the Mail and Information Desk located at the registration area in the lobby of the International Ballroom located on the lobby level of the hotel.

A Message Center will be located in the same area to receive incoming calls for all members in attendance. The center will be open from January 25 through January 29 between 8:30 a.m. and 4:30 p.m. Messages will be recorded, and the name of any member for whom a message has been received will be posted until the message has been picked up at the Message Center. Members are advised to leave the following numbers with anyone who might want to reach them at the meeting: (214) 651-0207 or 651-0265.

LOCAL ARRANGEMENTS COMMITTEE

H. L. Alder (ex officio), William L. Ayres, Paul T. Bateman (ex officio), Frank Lopez, William K. McNabb, Anita Priest, Wolfgang Rindler, Everett D. Roach, Argelia V. Rodriguez, David W. Starr (chairman), Gordon L. Walker (ex officio), and Basil Wall.

Paul T. Bateman
Associate Secretary

Urbana, Illinois

MEMORANDA TO MEMBERS

MATHEMATICAL SCIENCES EMPLOYMENT REGISTER

OPEN REGISTER

The Gold Room (also known as Room A) of the Fairmont Hotel in Dallas, Texas, will be the location of the Mathematical Sciences Employment Register OPEN REGISTER during the annual meeting. The OPEN REGISTER will operate for three days, January 26 through January 28, 1973. Hours of operation will be from 9:00 a.m. to 4:00 p.m. on Friday, and from 9:00 a.m. to 5:40 p.m. on Saturday and Sunday. If necessary, evening interviews will be scheduled on Saturday and Sunday.

Registration for the Employment Register is separate and apart from meeting registration; it is, therefore, most important that both applicants and employers sign in at the Employment Register desk as early as they can on Friday morning, January 26.

The system of operation introduced in January 1971 will again be in effect in Dallas. Applicants and employers MUST BE REGISTERED for the general Mathematics Meeting before registering for the OPEN REGISTER. There is no Register fee for applicants participating in interview schedules; employers are required to pay a $10 fee. Location of the general meeting registration area, fees, and hours of operation for the registration of participants for the Mathematics Meeting are listed in the preliminary announcement of the annual meeting included in this issue of these *Notices*.

Applicants and employers should secure an instruction sheet to acquaint themselves with the rules and operating regulations. These instruction sheets will be available on request in the Gold Room registration area at 9:00 a.m. on Friday. There will be no interviews scheduled for the first day. Please keep in mind that the registration for the OPEN REGISTER is separate and apart from meeting registration, and it is imperative that both applicants and employers who wish to participate in the OPEN REGISTER sign in at the Employment Register desk as early as they can on Friday morning. Appointments will be scheduled only for those people who have actually signed in at the Register and obtained a code number. Requests for appointments can be submitted on Friday and Saturday only, and these interviews will be scheduled on Saturday and Sunday respectively.

The published lists of the Mathematical Sciences Employment Register have been replaced by a bi-monthly publication, Employment Information for Mathematicians. Information on this new list appears on page 271 of the October 1972 issue of these *Notices*.

The Mathematical Sciences Employment Register is sponsored by the American Mathematical Society, the Mathematical Association of America, and the Society for Industrial and Applied Mathematics for the purpose of establishing communication between mathematical scientists available for employment and employers with positions to fill.

QUERIES

edited by Wendell H. Fleming

As a result of a suggestion of the AMS Committee to Monitor Problems in Communication, a QUERIES column will be published in each issue of these *Notices*. The column, which is published under the associate editorship of Wendell H. Fleming, welcomes questions from members regarding mathematical matters such as details of, or references to, vaguely remembered theorems, sources of exposition of folk theorems, or the state of current knowledge concerning published conjectures. When appropriate, replies from readers may be edited into a definitive composite answer and published in a subsequent column. All answers received to QUERIES will ultimately be forwarded to the questioner. Consequently, all items submitted for consideration for possible publication in this column should include the name and complete mailing address of the person who is to receive the replies. The queries themselves, and responses to such queries, should be addressed to Professor Wendell H. Fleming, American Mathematical Society, Post Office Box 6248, Providence, Rhode Island 02904.

4. Cleve B. Moler (Department of Mathematics, University of New Mexico, Albuquerque, New Mexico 87106). Can somebody recommend a good source where I can learn about the connection of mathematics and various biological processes such as photosynthesis?

5. Allen L. Shields (Department of Mathematics, University of Michigan, Ann Arbor, Michigan 48104). Somebody at Michigan was working on the Hilbert problems and there is supposed to be a survey which says which ones have been solved and what has been done. We couldn't locate the survey. Is there a source that gives the exact reference of the current state of Hilbert problems?

6. Allen L. Shields (Department of Mathematics, University of Michigan, Ann Arbor, Michigan 48104). Blaschke products clearly predate Blaschke. Does anyone have more information on this point?

NEWS ITEMS AND ANNOUNCEMENTS

ITALIAN NATIONAL RESEARCH COUNCIL FELLOWSHIPS

The Italian National Research Council will award twenty fellowships to citizens of other countries who intend to do research in mathematics at Italian universities during the academic year 1973–1974. Each fellowship will carry a monthly stipend of 180, 000 Italian lire (approximately $300) for a maximum of twelve months. In addition, recipients will be given a reimbursement of their travel expenses. Applications, written in English, French, or Italian, should be addresses to Consiglio Nazionale delle Ricerche, Servizio Affari Scientifici e Tecnologici, Ufficio Attività di Ricerca, Sezione Borse di Studio, Piazzale delle Scienze 7, Roma, Italy. Applications should include the following information: date and place of birth, citizenship, and residence; nature of the proposed research; names of Italian mathematicians with whom the applicant would like to associate and collaborate; knowledge of Italian (if any) or other foreign languages; address. A brief curriculum vitae and two letters of recommendation should accompany the application. Applications with all the supporting documents must reach the offices of the Consiglio Nazionale delle Ricerche before January 31, 1973. Further information may be obtained by writing to Alessandro Figà-Talamanca, Istituto Matematico, Università di Genova, Via L. B. Alberti, 4, 16132-Genova, Italy.

INSTITUTE FOR ADVANCED STUDY MEMBERSHIPS

The School of Mathematics will grant a limited number of memberships, some with financial support, for research in mathematics at the Institute during the academic year 1973–1974. Candidates must have given evidence of ability in research comparable at least with that expected for the Ph. D. degree. Application blanks may be obtained from the Secretary of the School of Mathematics, Institute for Advanced Study, Princeton, New Jersey 08540, and should be returned (whether or not funds are expected from some other source) by January 15, 1973, or as soon thereafter as possible.

SPECIAL MEETINGS INFORMATION CENTER

The purpose of this center is to maintain a file on prospective symposia, colloquia, institutes, seminars, special years, meetings of other associations, and to notify the organizers if conflicts in subject matter, dates or geographical area become apparent. A first announcement will be published in the *Notices* if it contains a call for papers, place, date, and subject, where applicable; a second announcement must contain reasonably complete details of the meeting in order for it to be published. Information on the pre-preliminary planning will be stored in the files, and will be available to anyone desiring information on prospective conferences. All communications on special meetings should be sent to the Special Meetings Information Center of the American Mathematical Society.

First and third Friday of each month
SEMINAR ON STRUCTURE AND PROGRAMMING
Institute de Recherche d'Informatique et d'Automatique (IRIA)
Rocquencourt, France
Information: C. Kaiser, Institute de Recherche d'Informatique et d'Automatique, Domaine de Voluceau-78, Rocquencourt, France

Third Saturday of each month
SEMINAR IN NUMERICAL ANALYSIS
Kent State University, Kent, Ohio
Program: Three 45-minute lectures with breaks for discussion
Speakers: Richard Varga, Kent State University; Charles Hall, University of Pittsburgh; Sherwood Silliman, Cleveland State University; David Kahaner, University of Michigan; Joseph Jerome, Northwestern University; and others as yet unannounced
Information: G. Rodrique or R. Varga, Department of Mathematics, Kent State University, Kent, Ohio 44242

December 2, 1972
ONTARIO MATHEMATICS MEETING (25)
MacMaster University, Hamilton, Ontario
Information: Dr. J. Csima, Department of Mathematics, MacMaster University, Hamilton, Ontario

December 14–16, 1972
MEETING OF UNIVERSITY OF BRITISH COLUMBIA AND COMMUNITY COLLEGES OF BRITISH COLUMBIA
University of British Columbia, Vancouver, British Columbia
Program: Lectures, panel discussions, and contributed papers
Abstracts: 300 words to Professor Cayford
Information: Professor Afton H. Cayford, Department of Mathematics, University of British Columbia, Vancouver, British Columbia

December 18–22, 1972
SIXTH TEXAS SYMPOSIUM ON RELATIVISTIC ASTROPHYSICS
Americana Hotel, New York, New York
Lectures: James Arthur Lectures devoted to "Physics at the origin of time"
Information: Professor E. L. Schucking, Department of Physics, New York University, New York, New York 10003

December 18–22, 1972 (tentative)
SECOND PITTSBURGH INTERNATIONAL CONFERENCE ON GENERAL TOPOLOGY AND ITS APPLICATIONS
Pittsburgh, Pennsylvania
Sponsors: Carnegie-Mellon University, University of Pittsburgh
Speakers: A. Arhangel'skiĭ, series on recent developments in general topology in the USSR, and on cardinal invariants; several hour lectures and shorter lectures
Abstracts: As soon as possible
Information: Professor Robert W. Heath, Department of Mathematics, University of Pittsburgh, Pittsburgh, Pennsylvania 15213

January 14 - February 23, 1973
SUMMER RESEARCH INSTITUTE OF THE AUSTRALIAN MATHEMATICAL SOCIETY
University of Queensland, Brisbane, and Apollo Apartments, Gold Coast, Australia
Topics: Differential equations and functional analysis
Information: R. Vyborny, Director of the Summer Research Institute, Department of Mathematics, University of Queensland, St. Lucia, Brisbane, Australia

January 15–May 8, 1973
SEMESTER IN VALUE DISTRIBUTION THEORY AND RELATED TOPICS IN DIFFERENTIAL GEOMETRY
Tulane University, New Orleans, Louisiana
Program: Wilhelm Stoll of Notre Dame University, Michael Cowen of Princeton University, and Ivan Cnop of Brussels will be in residence for the entire semester. S. S. Chern, H. Wu, and M. Green from the University of California, Berkeley; I. Singer and J. King from the Massachusetts Institute of Technology; B. Shiffman from Yale University; P. Griffiths from Harvard University; J. Carlson from Stanford University; J. Hirschfelder from Washington University; and R. Harvey from Rice University will visit Tulane for periods of approximately two weeks each during the semester.
Information: Professor Robert O. Kujala or Professor Albert L. Vitter III, Department of Mathematics, Tulane University, New Orleans, Louisiana 70118

February 3, 1973
ONTARIO MATHEMATICS MEETING (26)
University of Toronto, Toronto, Ontario
Information: Dr. G. F. D. Duff, Department of Mathematics, University of Toronto, Toronto 5, Ontario

March 21–23, 1973
SYMPOSIUM ON PROBABILITY AND RELATED TOPICS
Carleton University, Ottawa, Ontario
Lectures: K. L. Chung, Stanford University; W. H. J. Fuchs, Cornell University; M. Kac, Rockefeller University; C. Herz, McGill University; A. Joffe, Université de Montréal; J. C. Taylor, McGill University; J. B. Walsh, University of British Columbia; W. A. O N. Waugh, University of Toronto
Abstracts: Abstracts for contributed papers due on

February 1, 1973
Information: Professor M. Csorgo, Department of Mathematics, Carleton University, Colonel By Drive, Ottawa, Ontario, K1S 5B6

March 22–23, 1973
SEVENTH ANNUAL PRINCETON CONFERENCE ON INFORMATION SCIENCES AND SYSTEMS
Princeton University, Princeton, New Jersey
Contributed papers: Thirty-minute papers (fifty-word abstracts), fifteen-minute papers (detailed summaries); deadline January 10, 1973
Information: Professor T. Pavlidis, Department of Electrical Engineering, Princeton University, Princeton, New Jersey 08540

March 24, 1973
ONTARIO MATHEMATICS MEETING (27)
L'Université d'Ottawa, Ottawa, Ontario
Information: Dr. C. M. Wong, Department of Mathematics, l'Université d'Ottawa, Ottawa, Ontario

March 26–29, 1973
CONFERENCE ON THE THEORY OF ORDINARY AND PARTIAL DIFFERENTIAL EQUATIONS
University of Dundee, Scotland
Information: I. M. Michael, University of Dundee, DD1 4HN, Dundee, Scotland

May 27 - June 2, 1973
INTERNATIONAL CONFERENCE ON INFINITE DIMENSIONAL HOLOMORPHY
University of Kentucky, Lexington, Kentucky
Speakers: Sean Dineen, Michel Herve, Christer Kiselman, Pierre Lelong, Leopoldo Nachbin, Charles Rickart, Josef Siciak, Lucian Waelbroeck
Information: Professor Thomas L. Hayden, Department of Mathematics, University of Kentucky, Lexington, Kentucky 40506

June 1973 (dates not yet announced)
CONFERENCE ON ANALYTICAL METHODS IN CELESTIAL MECHANICS
Washington, D. C.
Topics: Classical celestial mechanics, configurations of equilibrium of rotating fluid bodies, motion of rigid or deformable bodies about their centroids
Abstracts: Deadline January 5, 1973; not more than 500 words
Information and abstracts: Dr. Paolo Lanzano, Code 7840, Naval Research Laboratory, Washington, D.C. 20390

June 18–20, 1973
SIAM 1973 NATIONAL MEETING
Sheraton Mount Royal Hotel, Montreal, Canada
Program: Symposia on nonnumerical computer mathematics and numerical analysis
Abstracts: Deadline March 16, 1973
Information: SIAM, 33 South 17th Street, Philadelphia, Pennsylvania 19103

September 17–21, 1973
EIGHTH AUSTRIAN MATHEMATICAL CONGRESS (INTERNATIONAL)
Vienna, Austria
Information: Professor H. J. Stetter, Technischen Hochschule, Vienna, Austria

October 8–10, 1973
SIAM–IMS 1973 JOINT FALL MEETING
Conference Center, University of Iowa, Iowa City, Iowa
Program: Symposia on applied probability and stochastic processes
Abstracts: Deadline July 6, 1973
Information: SIAM, 33 South 17th Street, Philadelphia, Pennsylvania 19103

September 3–7, 1974
FIFTH INTERNATIONAL HEAT TRANSFER CONFERENCE
Tokyo, Japan
Abstracts: Deadline March 1, 1973; one typewritten page; English only
Information: Professor E. R. G. Eckert, Heat Transfer Laboratory, Department of Mechanical Engineering, University of Minnesota, Minneapolis, Minnesota 55455

NEWS ITEMS AND ANNOUNCEMENTS

CORPORATE AND INSTITUTIONAL ASSOCIATE MEMBERS

The Society acknowledges with gratitude the support rendered by the following corporations who held either Corporate Memberships or Institutional Associateships in the Society during this calendar year.

Corporate Members

Academic Press, Incorporated
Bell Telephone Laboratories, Incorporated
Ford Motor Company
General Motors Corporation
International Business Machines Corporation
North American Rockwell Corporation
Radio Corporation of America
TRW, Incorporated

Institutional Associates

Addison-Wesley Publishing Company
Chelsea Publishing Company
Dover Publications
Princeton University Press
Shell Development Company
Springer-Verlag, New York, Incorporated

LATIN AMERICAN TEACHING FELLOWSHIPS

The Latin American Teaching Fellowships program is now accepting applications for positions in Latin America from individuals in the social and natural sciences, engineering, business, law, and medicine who hold Ph.D.'s, professional degrees, or who are Ph.D. candidates. Placement possibilities exist for the 1973–1974 academic year. These opportunities are part of a service program designed to assist Latin American universities to develop more advanced programs. Salaries are thus geared to moderate subsistence level rather than being competitive with North American salaries. Inquiries should be addressed to Latin American Teaching Fellowships, Fletcher School of Law and Diplomacy, Tufts University, Medford, Massachusetts 02155.

HIGHER EDUCATION GUIDELINES, Executive Order 11246

The Department of Health, Education, and Welfare has just published a manual entitled Higher Education Guidelines. This manual is designed to assist college and university presidents in complying with Executive Order 11246 as amended by Executive Order 11375, which deals with nondiscrimination under Federal contracts. All universities and colleges with Federal contracts must comply with these orders and the Office of Civil Rights in HEW is responsible for enforcement in institutions of higher education. Copies of the guidelines are available from the Regional Office for Civil Rights or from the Public Information Office, Office for Civil Rights, Department of Health, Education, and Welfare, Washington, D. C. 20201.

Discussed in this publication are legal provisions of the Order, personnel policies and practices, and the development of affirmative action programs; and the attached appendixes contain information on other civil rights laws affecting institutions of higher education over which HEW has enforcement responsibility. Of immediate concern to academic personnel involved with employment is recruitment, and for their information the following statement is quoted in its entirety from Higher Education Guidelines.

"Recruitment is the process by which an institution or department within an institution develops an applicant pool from which hiring decisions are made. Recruitment may be an active process, in which the institution seeks to communicate its employment needs to candidates through advertisement, word-of-mouth notification to graduate schools or other training programs, disciplinary conventions or job registers. Recruitment may also be the passive function of including in the applicant pool those persons who on their own initiative or by unsolicited recommendation apply to the institution for a position.

"In both academic and nonacademic areas, universities must recruit women and minority persons as actively as they have recruited white males. Some universities, for example, have tended to recruit heavily at institutions graduating exclusively or predominantly nonminority males, and have failed to advertise in media which would reach the minority and female communities, or have relied upon personal contacts and friendships which have had the effect of excluding from consideration women and minority group persons.

"In the academic area, the informality of word-of-mouth recruiting and its reliance on factors outside the knowledge or control of the university makes this method particularly susceptible to abuse. In addition, since women and minorities are often not in word-of-mouth channels of recruitment, their candidacies may not be advanced with the same frequency or strength of endorsement as they merit, and as their white male colleagues receive.

"The university contractor must examine the recruitment activities and policies of each unit responsible for recruiting. Where such an examination reveals a significantly lower representation of women or minorities in the university's applicant pool than would reasonably be expected from their availability in the work force, the contractor must modify or supplement its recruiting policies by vigorous and systematic efforts to locate and encourage the candidacy of qualified women and minorities. Where policies have the effect of excluding qualified women or minorities, and where their effects cannot be mitigated by the implementation of additional policies, such policies must be eliminated.

"An expanded search network should include not only the traditional avenues through which promising candidates have been located (e.g. in the case of academic appointments, direct letters to graduate departments, or in the case of nonacademic appointments, advertising in community newspapers). In addition, to the extent that it is necessary to overcome underutilization, the university should search in areas and channels previously unexplored.

"Certain organizations such as those mentioned in Revised Order No. 4 may be prepared to refer women and minority applicants. For faculty and administrative appointments, disciplinary and professional associations, including committees and caucus groups, should be contacted and their facilities for employee location and referral used.

"Particularly in the case of academic personnel, potentially fruitful channels of recruitment include the following:

a. advertisements in appropriate professional journals and job registries;
b. unsolicited applications or inquiries;
c. women teaching at predominantly women's colleges, minorities teaching at predominantly minority colleges;
d. minorities or women professionally engaged in nonacademic positions, such as industry, government, law firms, hospitals;
e. professional women and minorities working at independent research institutions and libraries;
f. professional minorities and women who have received significant grants or professional recognition;
g. women and minorities already at the institution and elsewhere working in research or other capacities not on the academic ladder;
h. minority and women doctoral recipients, from the contractor's own institution

and from other institutions, who are not presently using their professional training;

i. women and minorities presently candidates for graduate degrees at the institution and elsewhere who show promise of outstanding achievement (some institutions have developed programs of support for completion of doctoral programs with a related possibility of future appointment);

j. minorities and women listed in relevant professional files, registries and data banks, including those which have made a particularly conscientious effort to locate women and minority persons.

"It should be noted that a contractor is required to make explicit its commitment to equal employment opportunity in all recruiting announcements or advertisements. It may do this by indicating that it is an 'equal opportunity employer.' It is a violation of the Executive Order, however, for a prospective employer to state that only members of a particular minority group or sex will be considered.

"Where search committees are used to locate candidates for appointment, they can best carry out the above measures when they are composed of persons willing and able to explore new avenues of recruitment. Effective search committees should, if possible, include among their members women and minority persons.

"Policies which exclude recruitment at predominantly minority colleges and universities restrict the pool of qualified minority faculty from which prospective appointees may be chosen. Even if the intent of such policies may be to prevent the so-called 'raiding' of minority faculty by predominantly white institutions, such policies violate the nondiscrimination provision of the Executive Order since their effect is to deny opportunity for employment on grounds relating to race. Such policies have operated to the serious disadvantage of students and teachers at minority institutions by denying them notice of research and teaching opportunities, assistantships, endowed professorships, and many other programs which might enhance their potential for advancement, whether they choose to stay at a predominantly minority institution or move to a nonminority institution.

"Minorities and women are frequently recruited only for positions thought to be for minorities and women, such as equal employment programs, ethnic studies, or women's studies. While these positions may have a particular suitability for minority persons and women, institutions must not restrict consideration of women and minorities to such areas, but should actively recruit them for any position for which they may be qualified."

NEWS ITEMS AND ANNOUNCEMENTS

COMPUTER SCIENCE CONFERENCE EMPLOYMENT REGISTER

An employment register will be available at the Computer Science Conference, sponsored by eighteen universities and industrial organizations, to be held at the Neil House in Columbus, Ohio, February 20–22, 1973. Both prospective employees and employers must file their registration on official forms. These forms may be obtained from and completed forms should be returned to Orrin E. Taulbee, Department of Computer Science, University of Pittsburgh, Pittsburgh, Pennsylvania 15213. Employers should request one form for each type of position available; only one form is needed in the case of several identical positions. Closing date for acceptance of forms is February 15, 1973; late listing will not be accepted. Complete information on this service can be obtained directly from Professor Taulbee at the preceding address.

PHILOSOPHIA MATHEMATICA ESSAY COMPETITION

The recipients of the first Philosophia Mathematica Essay Competition were announced recently; they were Victor K. Sapojnikoff and David K. Sapojnikoff, both of Princeton University. Essays for the second competition should be submitted by September 30, 1974, to Professor J. Fang, Department of Philosophy, Memphis State University, Memphis, Tennessee (until August 15, 1973), 7543 Calmbach, Federal Republic of Germany (after August 15, 1973). The specific topic for this second competition is "The topological vs. the algebraic."

ERRATUM

A part of the last sentence of the News Item entitled "Sex, Race, and Citizenship of New Doctorates," which appeared on page 308 of the October 1972 *Notices* was omitted. The complete sentence is as follows: "At Canadian universities, 48 candidates were identified as men and 4 as women; of the men, 3 were identified as Chinese and 2 as Caucasian; 4 candidates (including 1 woman) were identified as U.S. citizens."

SENIOR-LEVEL JOBS
by R. D. Anderson

The primary motivation for the present report is the widely felt concern for the problems of the nonretained assistant professor; there is much recent anecdotal evidence of university and departmental policies which limit appointments or promotions to senior-level positions, i.e. positions as full or associate professors. This report presents statistical evidence concerning recent and prospective changes in the distributions of senior faculty in the mathematics departments of U.S. universities. It does not propose solutions to the various problems inherent in the overall situation. Clearly, the mathematics community will have a hard time balancing its conflicting interests in maintaining a large number of temporary positions in research departments for beginning Ph.D.'s and in retaining a large number of deserving junior faculty in the research community.

It should be noted that the terms "senior-level position" and "tenure position" are not synonymous. Some senior-level positions are visiting positions and some nonsenior-level positions do, in fact, carry tenure. The AMS data used does not involve tenure designations per se. See the article by Joseph P. LaSalle [these *Notices* 19(1972), 69-73] for a discussion of certain tenure questions.

The sources for this study are the raw data used in the AMS annual salary survey reported in the October *Notices* for 1969, 1971, and 1972. (The data for 1970 are not available to the author.) The salary survey in the *Notices* lists total numbers of faculty reported in various categories for departments in the mathematical sciences in the U.S. and Canada. In the present report, only U.S. mathematics departments (not departments in other mathematical sciences) have been considered, and the study has been restricted to Ph.D.-producing departments only. The classifications (Group I to Group IV) are the usual AMS ones (but without Canadian departments), where I includes the mathematics departments in the top two categories in the ACE report (27 in all); II includes the other ACE rated mathematics departments (39 in all); III includes unrated departments from universities which have produced three or more mathematics Ph.D.'s in the past three years (63 in all); and IV includes the other unrated Ph.D.-producing departments (28 in all). The numbers are those of the 1972 classification.

The author felt that the phenomena of faculty distribution changes were more appropriately studied only for U.S. mathematics departments since Canadian practices, rules, and procedures may well be different, and since many of the other mathematical science departments (e.g. computer science) have been newly formed and are still growing or likely to grow. It should be noted that the ACE report was updated between the October 1969 and October 1971 *Notices*, thus changing the classifications for Groups I, II, and III. Also, the III and IV lists change as departments do or do not produce Ph.D.'s.

The AMS data are derived from forms submitted by departments in the summer preceding the October *Notices* and giving the numbers of faculty members in the various ranks for the preceding year and for the following year. All departments are requested to submit the information.

Table I gives the number of mathematics departments submitting usable returns for each of the three years.

Table I

Group	Summer 1969	Summer 1971	Summer 1972
I	19	19	14
II	18	31	24
III	41	45	46
IV	15	17	16

JOB PROSPECTS FOR SEPTEMBER 1973

The recently passed federal revenue sharing bill should make academic job prospects for September 1973 somewhat better than they were for this year. The revenue sharing bill makes available to state legislatures an average of about $35,000,000 per state per year, with such funds to continue for at least five years. Regardless of possible specific allocations of these funds to needs other than higher education, overall legislative money for budget increases should be more plentiful this coming year than last. Surely higher education should get more liberal increases of funds than has been the case for the past two years. Because of an existing need for more faculty in some departments in the mathematical sciences, we can anticipate that additional faculty positions for mathematicians will be funded (even though the overall student load in four-year colleges and universities may not increase very much).

The effect of the current revenue sharing program on annual increases for higher education, however, will probably last only one or possibly two years. The twenty-year faculty employment prospects determined essentially by student demographic factors are as discouraging as ever. (See the *Notices*, November 1971, p. 1025.)

R. D. A.

For Tables II and III, we restrict ourselves to doctorate-holding faculty only, since in Ph. D.-producing departments nondoctorate-holding faculty perform rather different functions at different pay scales and their positions may not be easily filled by doctorate-level faculty. In 1972–1973 the percentages of the total faculty in the four categories, Group I to Group IV, which did not have doctorates were 1%, 5%, 12%, and 16% respectively.

Table II gives the number of the doctorate-holding senior-level faculty as a decimal fraction of all doctorate-holding faculty reported for the academic years 1969–1970, 1971–1972, and 1972–1973 following submission of the reports.

Table II

Group	1969–1970	1971–1972	1972–1973
I	.611	.675	.692
II	.587	.596	.641
III	.537	.584	.632
IV	.557	.550	.597

Table III gives the annual increase in the number of doctorate-holding senior-level faculty as a percentage of the total doctorate-holding senior-level faculty for the preceding year.

Table III

Group	Fall 1968–Fall 1969	Fall 1970–Fall 1971	Fall 1971–Fall 1972
I	8.2%	1.5%	1.3%
II	9.9%	6.4%	3.9%
III	8.6%	7.6%	8.3%
IV	9.9%	7.6%	5.2%

There is very little clear precedence on which to base precise future expectations but some experience, and the joint evidence in Tables II and III, strongly suggest that resistance (in some cases institutional) to increases in the number and percentage of senior-level faculty occurs when the ratio of senior-level faculty to the total faculty gets too large, perhaps when it approaches .70. After noting this figure empirically, I have been told that a 70% figure is an overall bound used in a comparable context in the University of California system. In any event, many research departments will consider it necessary to maintain a substantial number of young transient faculty to keep their research programs alive and vital over a period of time. Even if the 70% bound is too low, there clearly is some bound which apparently is being rapidly approached. If the relatively slow rate of increase in the number of faculty in senior-level positions, observable from the fall of 1969 to the fall of 1972, is continued for just three more years, then we shall have considerably more than 70% of our faculty in senior-level positions by 1975. For known demographic reasons, there seems little basis for optimism that the total faculty size of the existing Ph. D.-producing departments will grow significantly over the next 15 or 20 years. The size has been almost stable for the past two years.

The import of the figures cited above is that there is a very real likelihood that the total yearly increase in the number of senior-level positions in Ph. D.-producing departments will very soon drop dangerously close to zero. This will mean that, nationally, the number of young mathematicians getting senior-level appointments in Ph. D.-producing departments may be little more than the "steady-state" number of senior-level faculty who retire, die, or otherwise leave the system. Over the next 10 or 15 years the average annual numbers expected to retire or die (for those departments now in Groups I, II, III, and IV) are only about 12, 18, 25, and 10, respectively. These figures are based on a rate of 1% per year of total faculty and are closely verified by counts available to the AMS for the past year. The figure of 1% is stable for 10 or 15 years and then rises to about 6% of current faculty by the middle 1990's. Based on data collected by the AMS this summer, it is estimated that perhaps another 10 senior-level mathematicians leave faculty positions each year (for instance to industry or a deanship) eventually to be replaced by young U. S. mathematicians.

It is sobering to realize that the total number of senior-level positions available to young mathematicians in all Ph. D.-producing mathematics departments may shortly be only about 75 per year, i. e. only enough for about one graduate per year per department now in Group I or II. From 1958 to 1968, with an average production of Ph. D.'s less than half that of the current production, the number of young mathematicians who eventually got such senior-level positions averaged perhaps 250 to 300 per year. For example, in that period about 30 Ph. D.'s from my own department, L. S. U., have senior-level positions in departments now listed as producing Ph. D.'s.

The AMS does not yet have clear long-term statistical evidence showing the amount of increase or replacement of senior-level faculty attributable to internal promotions as distinct from outside hiring. Furthermore, it seems certain that the patterns will tend to change as more very able young assistant professors not being retained in the more prestigious departments seek other positions. There will probably be more employment of such people by departments in Groups II, III, and IV seeking to strengthen their faculties. Nevertheless, we can expect the honest pressure of concern for the junior members of one's own faculty to produce a good many internal promotions rather than senior-level hiring from the outside.

Finally, we say a word about early retirement. Whereas a gradually instituted program of early retirement would have only a minor effect on the total number of academic jobs available, it would appear to have a much more marked effect on the number of senior-level positions available to younger mathematicians (since retirees leave senior-level positions). For example, phasing in to a five-year earlier average retirement over a ten-year period would increase the (ten-year) steady-state death and retirement replacement number in Ph. D.-producing departments from about 65 a year to almost 100 per year.

PERSONAL ITEMS

PETER G. CASAZZA of the University of Iowa has been appointed to an assistant professorship at the University of Alabama in Huntsville.

JACK E. CLARK of Stanford University has been appointed to an assistant professorship at the University of Massachusetts, Amherst.

JAMES A. COCHRAN of Bell Telephone Laboratories, Whippany, New Jersey, has been appointed to a professorship at Virginia Polytechnic Institute and State University.

MAURICE FRANK, JR., of the Illinois Institute of Technology has been appointed to an assistant professorship at the University of Massachusetts, Amherst.

WILLIAM H. LING of Rensselaer Polytechnic Institute has been appointed to an assistant professorship at Union College, Schenectady, New York.

PAUL NELSON, JR., of the Oak Ridge National Laboratory is on leave for the academic year 1972–1973. He will spend the year at Texas Tech University as a visiting professor.

ALFONS OOMS of Yale University has been appointed to an assistant professorship at the University of Massachusetts, Amherst.

ARUNAS RUDVALIS of Michigan State University has been appointed to an assistant professorship at the University of Massachusetts, Amherst.

ERIK SCHREINER of Western Michigan University has been appointed to a visiting associate professorship at the University of Massachusetts, Amherst.

THOMAS I. SEIDMAN of Carnegie-Mellon University has been appointed to an associate professorship at the University of Maryland, Baltimore County.

ALBERT WILANSKY of Lehigh University has been awarded a grant in the Senior Fulbright-Hays program concurrent with his leave of absence which he will spend at the University of Reading, England.

NICH O. WILLIAMS of Auburn University has been appointed to an assistant professorship at the College of Charleston.

PROMOTIONS

To Dean of Faculty and to Professor. University of Alabama in Huntsville: JAMES M. HORNER.

To Dean, School of Science and Engineering. University of Alabama in Huntsville: JAFAR HOOMANI.

To Chairman, Department of Mathematics. Howard University: JAMES A. DONALDSON.

To Chairman, Department of Mathematics and to Associate Professor. University of Alabama in Huntsville: F. LEE COOK.

To Professor. Hofstra University: ROBERT BUMCROT, AZELLE WALTCHER; Southern Illinois University, Edwardsville: CLELLIE C. OURSLER.

To Associate Professor. Southern Illinois University, Edwardsville: IRVING J. KESSLER; Southern Methodist University: MONTIE MONZINGO.

To Assistant Professor. Howard University: GUTTALU R. VISWANATH.

INSTRUCTORSHIPS

Georgia Institute of Technology: CATHERINE C. AUST; Queensborough Community College: GERALD FLYNN; Rice University: RICHARD S. ELMAN; Southeast Missouri State University: RICHARD WIRT.

DEATHS

Professor JOHANNES DE GROOT of the University of Amsterdam died on September 11, 1972, at the age of 58. He was a member of the Society for 15 years.

Professor Emeritus SOLOMON LEFSCHETZ of Princeton University died on October 5, 1972, at the age of 88. He was a member of the Society for 29 years.

Professor LUCIEN W. NEUSTADT of the University of Southern California died on October 9, 1972, at the age of 44. He was a member of the Society for 21 years.

Professor C. O. SEGERDAHL of Stockholm University died on March 1, 1972, at the age of 59. He was a member of the Society for 3 years.

VISITING MATHEMATICIANS

Supplementary List

The list of visiting mathematicians includes both foreign mathematicians visiting in the United States and Canada, and Americans visiting abroad. Note that there are two separate lists.

American and Canadian Mathematicians Visiting Abroad

Name and Home Country	Host Institution	Field of Special Interest	Period of Visit
Addison, John W., Jr. (U.S.A.)	Oxford University	Theory of Models	academic year 1972–1973
Fleishman, Bernard A. (U.S.A.)	Technical Institute of Delft, Netherlands	Nonlinear Differential Equations	1/73 - 6/73
Gale, David (U.S.A.)	Eidgenossische Technische Hochschule, Zürich	Optimal Economic Development	winter 1973
Lanford, Oscar E., III (U.S.A.)	Institut des Hautes Etudes Scientifiques, Bur-sur-Yvette	Global Analysis	academic year 1972–1973
Matkowsky, Bernard J. (U.S.A.)	Tel-Aviv University, Israel	Applied Mathematics	9/72 - 8/73
Morrey, Charles B., Jr. (U.S.A.)	University of Florence	Functional Analysis	fall quarter 1972
Ogg, Andrew P. (U.S.A.)	University of Paris	Number Theory	spring quarter 1973
Sachs, Rainer K. (U.S.A.)	Cambridge University, England	Relativity	academic year 1972–1973
Seidenberg, Abraham (U.S.A.)	Institut des Hautes Etudes Scientifiques, Bur-sur-Yvette, University of Paris, Orsay	Algebraic Varieties	fall quarter 1972
Sundaresan, K. (U.S.A.)	Institute of Mathematics, Polish Academy of Sciences	Functional Analysis	academic year 1972–1973
Weinstein, Alan D. (U.S.A.)	Institut des Hautes Etudes Scientifiques, Bur-sur-Yvette	Function Theory	winter 1973

Foreign Mathematicians Visiting in the United States

Bondesson, Magnus (Sweden)	Kent State University	Numerical Analysis	9/72 - 6/73
Dieudonné, Jean (France)	University of California, San Diego	Algebra, Analysis, Topology	12/72 - 3/73
Ganelius, Tord (Sweden)	University of California, San Diego	Analysis	9/72 - 8/73
Kenku, Monsur A. (Nigeria)	University of California, Berkeley		academic year 1972–1973
Lindenstrauss, Joram (Israel)	University of California, Berkeley	Functional Analysis	winter & spring quarters 1972–1973
Miranda, Mario (Italy)	University of California, Berkeley	Differential Geometry	academic year 1972–1973
Ruelle, David P. (France)	University of California, Berkeley	Statistical Mechanics	spring quarter 1973
Rulf, Benjamin (Israel)	Rensselaer Polytechnic Institute	Applied Mathematics	9/72 - 8/73

ABSTRACTS PRESENTED TO THE SOCIETY

Preprints are available from the author in cases where the abstract number is starred.

The papers printed below were accepted by the American Mathematical Society for presentation by title. The abstracts are grouped according to subjects chosen by the author from categories listed on the abstract form. The miscellaneous group includes all abstracts for which the authors did not indicate a category.

An individual may present only one abstract by title in any one issue of the *Notices* but joint authors are treated as a separate category. Thus, in addition to abstracts from two individual authors, one joint abstract by them may also be accepted for an issue.

Algebra & Theory of Numbers

72T-A266. PHILIP G. BUCKHIESTER, Clemson University, Clemson, South Carolina 29631. Rank r solutions to the matrix equation $XAX^T = C$, A nonalternate of odd order, C alternate, over $GF(2^y)$. Preliminary report.

Let B be a $t \times t$ matrix over $GF(2^y)$, a finite field of order 2^y. If B has nonzero diagonal, then B is said to be a nonalternate matrix. If B is symmetric and has 0 diagonal, B is said to be an alternate matrix. Let A be a symmetric, nonalternate matrix of order n, n odd, over $GF(2^y)$ and let C be an alternate matrix of order s over $GF(2^y)$. By using Albert's canonical forms for symmetric matrices over fields of characteristic two, the number of $s \times n$ matrices X of rank r over $GF(2^y)$ such that $XAX^T = C$ is determined. (Received May 24, 1972.)

*72T-A267. GEORGE A. GRÄTZER, University of Manitoba, Winnipeg, Manitoba R3T 2N2, Canada and WILLIAM A. LAMPE, University of Colorado, Boulder, Colorado 80302. Type 2 and type 3 representations of complete lattices. Preliminary report.

The methods of Abstract 71T-A211, these *Notices* 18(1971), 937, and of Abstract 72T-A240, these *Notices* 19(1972), A-683, can be brought together to prove the following result. For a complete lattice let $\mathfrak{m}(L)$ denote the smallest regular cardinal $\mathfrak{m}$ for which L is $\mathfrak{m}$-algebraic. $\mathfrak{m}(L)$ always exists; in fact, $\mathfrak{m}(L) \leqq |L|$. Theorem 1. Let L be a complete lattice. Then there exists an algebra of characteristic $\mathfrak{m}(L)$ such that L is isomorphic to the congruence lattice of $\mathfrak{A}$ and $\Theta \vee \Phi = \Theta\Phi\Theta\Phi$ for any pair of congruences Θ and Φ of $\mathfrak{A}$ (type 3 congruences). Theorem 2. Let L be a complete modular lattice. Then there exists an algebra of characteristic $\mathfrak{m}(L)$ such that L is isomorphic to the congruence lattice of $\mathfrak{A}$ and $\Theta \vee \Phi = \Theta\Phi\Theta$ for any pair of congruences Θ and Φ of $\mathfrak{A}$ (type 2 congruences). (Received August 3, 1972.)

72T-A268. K. McDOWELL, McMaster University, Hamilton, Ontario, Canada. On commutative coherent rings.

Let $\underline{C}$ be the class of commutative coherent rings R with unit with the following property: If I is any finitely generated proper ideal of R, M any finitely presented R-module and if I is contained in the set of zero divisors of M, then there exists $0 \neq m$ in M with $Im = 0$. This seems to be the proper class in which to generalize the theories of grade and R-sequences developed for Noetherian rings. All Noetherian rings are in $\underline{C}$ as are all coherent Bézout rings (e.g. von Neumann regular rings and valuation domains). Theorem. If R is a faithfully flat directed union of domains in $\underline{C}$, then R is in $\underline{C}$. This theorem yields examples of local non-Noetherian rings in $\underline{C}$ with arbitrary given weak global dimension. Grade is introduced homologically and induction is facilitated by the fact that for a coherent local ring R and nonzero divisor x in the radical, grade R = grade R/(x) + 1. If R is a local ring in $\underline{C}$, (i) the length of any maximal R-sequence equals the grade of R and (ii) p.dim M + grade M = w.gl.dim R for finitely presented M and w.gl.dim R $< \infty$. (Received September 6, 1972.) (Author introduced by Professor Bruno J. Mueller.)

*72T-A269. DAVID B. SINGMASTER, Polytechnic of the South Bank, London SE 1, England. The eigenvalues of the icosahedron and dodecahedron.

The eigenvalues of a graph are the eigenvalues of its adjacency matrix (a_{ij}), defined by $a_{ij} = 1$ if vertices i and j are adjacent, = 0 otherwise. Eigenvalues for the n-simplex, n-cube and n-octahedron are known. Using a standard computer subroutine, I have found the following eigenvalues. Icosahedron: 5, $\sqrt{5}$ (3 times), -1 (5 times), $-\sqrt{5}$ (3 times). Dodecahedron: 3, $\sqrt{5}$ (3 times), 1 (5 times), 0 (4 times), -2 (4 times), $-\sqrt{5}$ (3 times). Elementary checks on the trace, determinant and rank confirm the correctness of the results. Computer printout of the eigenvalues and eigenvectors is available on request. (Received August 9, 1972.)

72T-A270. E. M. WRIGHT, University of Aberdeen, Aberdeen, United Kingdom. Unlabelled graphs with many nodes and edges. V. Preliminary report.

An (n, q) graph has n unlabelled nodes and q undirected edges, each pair of different nodes being not joined or joined by just one edge. We write $T = T(n,q)$ for the number of different (n, q) graphs, $t = t(n,q)$ for the number of these which are connected and $\beta = \beta(n,q) = t/T$ for the probability that an (n, q) graph is connected. I have shown (Proc. Amer. Math. Soc. (to appear); Abstract 71T-A258, these *Notices* 18(1971), 1094, and Abstract 72T-A184, these *Notices* 19(1972), A-569) that, contrary to what one might expect, β does not increase steadily with q (even in a nonstrict sense). We write $N = n(n-1)/2$. I can now show that, for any fixed positive A and large enough n, we have $\beta(n,q) < \beta(n,q+1)$ for $n(A+\log n)/2 < q < q_1$ and $\beta(n,q) > \beta(n,q+1)$ for $q_1 \leqq q \leqq N-n$, while, of course, $\beta = 1$ for $N-n+2 \leqq q \leqq N$. Here q_1 can be calculated with a possible error of 1. This closes the gap near q_1 and extends the range downwards of my previous result (Abstract 72T-A184, ibid.). (Received September 1, 1972.)

*72T-A271. JOE W. FISHER and HARVEY E. WOLFF, University of Texas, Austin, Texas 78712. Decomposition theories for abelian categories.

Both the classical approach to decomposition theories and Fisher's technique of constructing decomposition theories from radical functions are extended to and exploited in the context of abelian categories. These two different approaches to decomposition theories for abelian categories intertwine in one theorem from which flows necessary and sufficient conditions for the existence of the tertiary, primary, and Bourbaki's $\underline{P}$-primary decomposition theories. (Received September 5, 1972.)

72T-A272. MICHAEL E. ADAMS, University of Bristol, Bristol BS8 1TW, England. The Frattini sublattice of a distributive lattice. Preliminary report.

For a distributive lattice L with a sublattice L_1 we define a separating set (also considered by K. M. Koh in Abstract 72T-A141, these *Notices* 19(1972), A-508) over the prime ideals of L. In terms of these sets sublattices are uniquely represented and maximal sublattices may be characterised. Let $\Phi(L)$ be the Frattini sublattice of L. Using the topological representation introduced by H. A. Priestley, "Representation of distributive lattices by means of ordered Stone spaces," Bull. London Math. Soc. 2(1970), 186-190, we show: (1) If L is a distributive lattice there exists a distributive lattice L_1 such that $L \cong \Phi(L_1)$. Let θ_B denote the congruence relation on a distributive lattice that is generated by the maximal relatively complemented ideal and filter; should either one of these fail to exist it is replaced by the identity congruence. (2) If L is a (0, 1) distributive lattice then there exists a distributive lattice L_1 such that (a) every sublattice of L_1 can be extended to a maximal sublattice, (b) $L_1/\theta_B \cong L$ and (c) $\Phi(L_1) = \emptyset$. (3) If L is a (0, 1) distributive lattice there exists a distributive lattice L_1 with a congruence θ such that (a) every sublattice of L_1 is the intersection of maximal proper sublattices and (b) $L_1/\theta \cong L$. Points (2) and (3) give negative answers to questions raised by Koh. (Received September 8, 1972.) (Author introduced by Dr. B. Rotman.)

*72T-A273. WALLACE S. MARTINDALE III, University of Massachusetts, Amherst, Massachusetts 01002. On semiprime P.I. rings.

A recent theorem due to L. Rowen (any nonzero ideal of a semiprime P.I. ring R contains a nonzero central element of R) is the main tool used to settle in the affirmative the following two conjectures of J. W. Fisher: If R is a semiprime P.I. ring, then (1) the maximal right quotient ring of R is P.I., (2) the maximal right and left quotient rings of R coincide. (Received September 11, 1972.)

72T-A274. ARUN V. JATEGAONKAR, Cornell University, Ithaca, New York 14850. The AR property and localization in Noetherian rings. Preliminary report.

Theorem 1. Let $\mathfrak{s}$ be a semiprime ideal in a right Noetherian ring R, $\mathcal{C}(\mathfrak{s}) = \{c \in R \mid [c+\mathfrak{s}]$ is regular in $R/\mathfrak{s}\}$ and ρ be the left exact radical for mod-R defined by $\mathcal{C}(\mathfrak{s})$. Then the following conditions are equivalent: (1) Given a right ideal K of R, there exists an ideal H of R and $n \in \mathbb{Z}^+$ such that R/H is a right order in a right Artinian ring, $\mathfrak{s}^n \subsetneq H \subseteq \mathfrak{s}$ and $K \cap H \subseteq \rho$-closure $(K\mathfrak{s})$. (2) If M is a cyclic essential extension of a uniform right R-submodule N of $R/\mathfrak{s}$ then $M\mathfrak{s}^n = 0$ for some $n \in \mathbb{Z}^+$ and if $\rho(M/N) = M/N$ then $M\mathfrak{s} = 0$. (3) $\mathcal{C}(\mathfrak{s})$ is a right Ore set in R and $J(R_{\mathfrak{s}})$ has the right AR-property, $R_{\mathfrak{s}}$ being the r.q. ring of R w.r.t. $\mathcal{C}(\mathfrak{s})$. If R is Noetherian, let the symbolic power $H_n(\mathfrak{s})$ be defined as in Goldie [J. Algebra 5(1967), 89-105]. Then the above three conditions are equivalent with the following condition: (4) Given a right ideal K of R, $K \cap H_n(\mathfrak{s}) \subseteq \rho$-closure $(K\mathfrak{s})$ for some $n \in \mathbb{Z}^+$. Theorem 2. Let R be a semilocal right Noetherian ring. J(R) has the right AR property iff (*) every f.g. right R-module with essential socle has finite length. If these conditions hold then $J^{\omega}(R) = (0)$. We do not know whether (*) always holds when R is (semilocal and) Noetherian. (Received September 12, 1972.)

*72T-A275. HARRY LAKSER, University of Manitoba, Winnipeg, Manitoba R3T 2N2, Canada. A note on the lattice of sublattices of a finite lattice.

K. M. Koh, "A note on the lattice of sublattices of a lattice" (preprint), proved that the lattice of sublattices of a lattice L is modular if and only if L is a chain. We prove Theorem. Let L be a finite lattice. The following four conditions are equivalent: (i) the lattice of sublattices of L is a graded lattice; (ii) the lattice of sublattices of L is a graded lattice where the height of a sublattice of L is its cardinality; (iii) the lattice of sublattices of L is lower semimodular; (iv) L has no sublattice isomorphic to the direct product of a two-element chain with a three-element chain. (Received September 12, 1972.) (Author introduced by Professor George A. Grätzer.)

*72T-A276. STEPHEN V. ULLOM, University of Illinois, Urbana, Illinois 61801. On the exponent of classgroups.

The integral group ring ZG of a finite group G is contained in a maximal order A of QG. Denote by C(B) the locally free class group of an order B in QG. Let e(G) be the exponent of the group D(ZG) which is the kernel of the epimorphism $C(ZG) \to C(A)$. Theorem. Suppose G has order p^n, p prime. Then e(G) divides p^{n-1}. If p = 2, then e(G) divides p^{n-2}. Theorem. For the symmetric group S_n, the class group $C(ZS_n) = D(ZS_n)$ has no p-torsion for primes $p > n/2$. The proofs use Jacobinski's description of D(ZG). (Received September 12, 1972.)

72T-A277. THOMAS C. CRAVEN, Cornell University, Ithaca, New York 14850. Every Boolean space is homeomorphic to the space of orderings of a field. Preliminary report.

Knebusch, Rosenberg and Ware [Bull. Amer. Math. Soc. 77(1971), 205-210] have pointed out that the set X of orderings on a field can be topologized to make a Boolean space (compact, Hausdorff and totally disconnected). This can also be found in Milnor and Husemoller, "Symmetric bilinear forms," Princeton, N. J., 1971. The former have called the sets of orderings $W(a) = \{< \text{ in } X \mid a < 0\}$ the Harrison subbasis of X. This subbasis is closed under symmetric difference and complementation. It is proved here that, given any Boolean space X, there exists a formally real field F such that X is homeomorphic to the space of orderings on F. The proof of this uses results due to Ershov [Math. Notes 6(1969), 577-582]. Also, an example is given of a Boolean space and a subbasis of clopen sets closed under symmetric difference and complementation which cannot be the Harrison subbasis of any formally real field. (Received September 18, 1972.)

*72T-A278. ROBERT GILMER and THOMAS G. PARKER, Florida State University, Tallahassee, Florida 32306 and ANNE P. GRAMS, University of Tennessee, Nashville, Tennessee 37203. Zero divisors in power series rings. Preliminary report.

Let R be a commutative ring with identity, let $\{X\} \cup \{X_\lambda\}$ be a set of indeterminates over R, and let $R^{((3))} = R[[\{X_\lambda\}]]_3$ be the full power series ring in the set $\{X_\lambda\}$ of indeterminates over R. The ring R has property (ρ) (respectively, (μ)) if each element of R[[X]] with a regular coefficient (respectively, unit coefficient) is regular in R[[X]]. Theorem 1. The following conditions are equivalent: (a) R has property (ρ). (b) If $f \in R^{((3))}$ and if the ideal A_f of R generated by the coefficients of f is regular in R, then f is regular in $R^{((3))}$. The analogue, for property (μ), of Theorem 1 is also valid. Sufficient conditions in order that R have property (ρ) are that R be zero-dimensional or that the zero ideal of R is primary; if (0) is an intersection of primary ideals of R, then R has property (μ). Theorem 2. If $f \in R^{((3))}$ and if $g \in R[[\{X_\lambda\}]] - \{0\}$ has exactly k nonzero monomials, then $B_f^k B_g = B_f^{k-1} B_{fg}$, where B_h denotes the additive subgroup of R generated by the coefficients of the element h of $R^{((3))}$. Theorem 2 is a generalization of the Dedekind-Mertens lemma; it arises in an investigation of conditions under which the analogue of McCoy's theorem for polynomial rings is valid for power series rings. (Received September 18, 1972.)

*72T-A279. TORRENCE D. PARSONS, Pennsylvania State University, University Park, Pennsylvania 16802. Path-star Ramsey numbers. Preliminary report.

Let P_m denote a path with m vertices and $K_{1,n}$ a star of degree n. This note establishes the following formulas for the generalized Ramsey numbers $r(P_m, K_{1,n})$, $m \geqq 2$ and $n \geqq 1$: (a) $m + n - 1$ if $n \equiv 1 \pmod{m-1}$; (b) $m + n - 2$ if $n \equiv 0, 2 \pmod{m-1}$, or if $n \not\equiv 1 \pmod{m-1}$ and $n \geqq (m-3)^2$; (c) $2n - 1$ if $n \leqq m \leqq 2n - 1$; (d) m if $m \geqq 2n - 1$. (Received September 25, 1972.)

*72T-A280. WALTER F. TAYLOR, University of Colorado, Boulder, Colorado 80302. Varieties without doubleton algebras. Preliminary report.

Theorem. A variety V of algebras contains no two-element algebra if and only for some M and N there exist binary terms α_j $(0 \leq j < M)$ and ternary terms β_k^σ $(0 \leq k \leq 2N;$ σ any function $\sigma : \{0, \ldots, M-1\} \to \{0,1\})$, such that the following $2^M(2N+2)$ equations hold identically in $V: x_0 = \beta_0^\sigma(x_0, x_1, x_{\sigma[0]})$, $\beta_{2i}^\sigma(x_0, x_1, \alpha_{[i]}) = \beta_{2i+1}^\sigma(x_0, x_1, \alpha_{[i]})$, $\beta_{2i+1}^\sigma(x_0, x_1, x_{\sigma[i]}) = \beta_{2i+2}^\sigma(x_0, x_1, x_{\sigma[i+1]})$, $\beta_{2N}^\sigma(x_0, x_1, x_{\sigma[2N]}) = x_1$, where $0 \leq i < N$, σ ranges over functions $\sigma : \{0, \ldots, M-1\} \to \{0,1\}$ and $[i]$ denotes the unique member of $\{0, \ldots, M-1\}$ equivalent to i modulo M. Theorem. If J is any finite set of integers whose complement is closed under multiplication, then there exists a similar "Mal'cev condition" describing the class of varieties having no algebra of power n for $n \in J$. (Received September 27, 1972.)

*72T-A281. HENRY W. GOULD, West Virginia University, Morgantown, West Virginia 26506. The design of the four binomial identities of Moriarty.

This paper is concerned with a set of twelve binomial identities which occur naturally in three sets of four identities. Group I is typified by $\sum_{k=0}^n (-1)^k \binom{k}{a} \binom{n+k}{2k+1} 2^{2k} = (-1)^{n-1} \binom{n+a}{2a+1} 2^{2a}$; Group II by $\sum_{k=0}^n \binom{2n+1}{2k+1} \binom{k}{n-a} = \binom{n+a}{2a} 2^{2a}$; Group III by $\sum_{k=0}^a \binom{2n+1}{2k+1} \binom{n-k}{a-k} = (2n+1)\binom{n+a}{2a} 2^{2a}/(2a+1)$. Each set of four identities is shown to be equivalent to a set of two identities which are called the Moriarty identities (see Abstract 72T-A80, these *Notices* 19(1972), A-426). Various proofs are given and the work shows how these diverse combinatorial identities can be seen as fourfold manifestations of the Moriarty formulas. By this means an order is shown to some of the formulas in the author's recently published book ("Combinatorial identities," rev. ed., Morgantown, W. Va., 1972) listing 555 combinatorial identities. The first identity cited above was called to the author's attention by David Zeitlin. (Received September 27, 1972.)

*72T-A282. DAVID ZEITLIN, 1650 Vincent Avenue North, Minneapolis, Minnesota 55411. A conjecture for integer sequences.

Let M be a positive integer and define the integer sequence, U_n, by the linear recurrence relation $U_{n+2} = MU_{n+1} + U_n$, with $U_0 = 0$ and $U_1 = 1$. Let P be the positive root of $x^2 - Mx - 1 = 0$. Let $F = M/(M+1)$. We conjecture now that (*) $[PU_kU_n + F] = U_kU_{n+1}$ holds as follows: (a) $n \geq k \geq 2$, (b) $n \geq 2$ for $k = 1$, and (c) for all n if $k = 0$ (trivially so), where $[x]$ is the usual greatest integer function. Using two identities for linear sequences above, we can show that (*) is equivalent (**) $[P^kU_n + F] = U_{n+k}$ for all three cases. For computation purposes, (*) is preferred. Remarks. For $M = 1$, $U_n \equiv F_n$, the Fibonacci sequence; and for $M = 1$, (**) gives as a special case the recent result (proved analytically) by Anaya and Crump, Fibonacci Quart. 10(1972) 207-211. Our generalization of (*) is omitted here because of space limitations. Our (*) and (**) can be readily programmed, using Fortran; such programs can be used to obtain counterexamples. An analytic proof of (*) may be complicated, due to the nature of P. (Received September 27, 1972.)

*72T-A283. V. R. CHANDRAN, J. 9, Judicial Quarters, Race Course, Madurai-2, India. On duo-rings. VI.

A duo-ring is one in which every one-sided ideal is two-sided. Commutative rings and skewfields are trivial examples of duo-rings. As Professor N. Jacobson has asked (personal communication) for nontrivial examples of duo-rings, we provide such ones in this paper. We also provide examples of left duo-rings which are not right duo. We also give an example of a nil duo-ring which is a question by Professor Hyman Bass (during his visit at Madurai University, June, 1971). Example. Let F be a field with two elements 0, 1. Let F[x,y,z] denote the algebra generated by three indeterminates x, y, z over F, satisfying (without constant term), the following relations: (1) $x^2 = y^2 = z^2 = 0$; (2) $xy = yz = zx$; (3) $yx = zy = xz$. We prove F[x,y,z] is a nil duo-ring. In §2, we consider the embeddability of a duo-ring in a duo-ring with identity element. Thus, we prove Theorem. Let R be a duo-ring with Jacobson radical zero. Then R can be embedded in a duo-ring with 1. The problem is open when R is a duo-ring with $J(R) \neq 0$. (Received September 28, 1972.)

*72T-A284. R. ARTHUR KNOEBEL, New Mexico State University, Las Cruces, New Mexico 88001. Inversion formulas. Preliminary report.

This common setting for diverse inversions uses matrix multiplication over some semirings. Let S be a nonempty set which has two binary, commutative and associative operations +, $\times$ s.t. $\times$ distributes over +, and constants 0, 1 for which $s = s + 0 = 0 + s = s \times 1 = 1 \times s$ and $0 = s \times 0 = 0 \times s$. For a family $\{\alpha_i\}_{i \in I}$ of functions $\alpha_i : I \to S$, let A be the I by I matrix with elements $\alpha_i(j)$; similarly B has entries $\beta_i(j)$. Assume that each column of A and B has only a finite number of nonzero entries. Theorem. The matrix product AB = 1 iff for all $f, g : I \to S$, if $g(j) = \sum_{i \in I} f(i) \times \alpha_i(j)$ for $j \in I$, then $f(i) = \sum_{j \in I} g(j) \times \beta_j(i)$ for $i \in I$. In this case, if f', $g' : I^n \to S$ and $g'(j) = \sum_i f'(i) \times \alpha_{i_1}(j_1) \times \ldots \times \alpha_{i_n}(j_n)$ for $i, j \in I^n$, then $f'(i) = \sum_j g'(j) \times \beta_{j_1}(i_1) \times \ldots \times \beta_{j_n}(i_n)$. Three applications. (1) Rota's development of Möbius functions is obtained when S is the real numbers, I is a locally finite p.o. set with minimum element, and $\alpha_i(j) = 1$ if $i \leq j$, 0 otherwise. (2) The finite Fourier transform comes about for S the complex numbers, $I = \{0, 1, \ldots, n-1\}$, and $\alpha_i(j) = (1/n) \exp(-ij2\pi\iota/n)$. The Rademacher-Walsh expansion arises when n = 2. (3) The disjunctive normal form of logic is just when $A = \{0,1\} = I$ is the two-element Boolean algebra, and A = B = 1. (Received September 28, 1972.)

Analysis

72T-B298. WILLIAM D. L. APPLING, North Texas State University, Denton, Texas 76203. Quasi-mutual singularity and digital representations.

Setting and notions are as in previous abstracts of the author. Suppose each of g, h and g - h is in p_A^+. Suppose Z is the set to which k belongs iff k is in p_A^+ and for some function B from F into $\{0, 1\}$, $k = \int Bg$. Theorem. The following seven statements are equivalent: (1) $0 = \int_U \min\{h, g-h\}$; (2) if $0 < c$, then there is a subdivision $\{P, Q\}$ of U such that $\max\{h(P), g(Q) - h(Q)\} < c$; (3) for some $K > 1$, $\int \min\{g, Kh\} = h$; (4) for all $K > 1$, $\int \min\{g, Kh\} = h$; (5) h is in Z; (6) h is in $\overline{Z}$ (i.e., Z plus its closure, where $\|w\| = \int_U |w|$ for w in p_{AB}); (7) $\int |2h - g| = g$. Corollary. If M is a C-set and linear space and A is the associated "nearest point" function, then A(g) is in Z. (Received April 26, 1972.)

72T-B299. STEFAN BERGMAN, Stanford University, Stanford, California 94305. On boundary values of solutions of a differential equation in three variables.

Integral operators $\varphi_n(X, Z, Z^*; g) = \sum_{\nu=0}^{\{\frac{1}{2}n\}} X^{n-2\nu} \psi^{(n, n-2\nu)}(Z, Z^*; g)$, $X = x$, $Z = \frac{1}{2}(z + iy)$, $Z^* = \frac{1}{2}(z - iy)$, $\varphi^{(n,n)}(Z, Z^*; g) = g(Z) + \sum_{p=1}^{\infty} q^{(n,n,p)} L^{[p]}(g)$, $\psi^{(n, n-2\nu)}(Z, Z^*; g) = \sum_{p=\nu}^{\infty} q^{(n, n-2\nu, p)} L^{[p]}(g)$, $\nu = 1, 2, \ldots$, have been introduced in Bergman, Ergebnisse der Mathematik, vol. 23, Springer-Verlag, New York, 1969, pp. 69 ff. The above operators transform an analytic function g(Z) into a (complex) solution of (1) $\Delta_3 \psi + F(y, z)\psi = 0$, where $\Delta_3 \psi = \psi_{xx} + \psi_{yy} + \psi_{zz}$, and F is an analytic function which (continued to complex values of y and z) is regular in a sufficiently large domain; $q^{\cdots}$ are functions which depend only on $F(y,z)$; $L_p(g) = \int_0^Z\int_0^{Z_1}\cdots\int_0^{Z_{p-1}} g(Z_p)\,dZ_p\ldots dZ_1$; $\{\frac{1}{2}n\} = \frac{1}{2}n$ if n is even, $\{\frac{1}{2}n\} = \frac{1}{2}(n-1)$ if n is odd. Let $g(Z)$, $Z = \xi + i\eta$, be an analytic function of the complex variable Z which is regular in $\xi^2 + \eta^2 < r^2$. Then $\varphi_n(X, Z, Z^*; g)$ is defined in the three-dimensional domain $[-\infty < x < \infty, \xi^2+\eta^2 < r^2]$. Let $\varphi = \sum_{\nu=0}^{\{\frac{1}{2}N\}} \sum_{m=0}^{\infty} \sum_p A_{\nu mp} X^{N-2\nu} Z^m Z^{*p}$ be the series development of a solution of (1). We assume that $\sum_{m=0}^{\infty} |A_{0m0}| r^{2m} = C < \infty$. Then for every $X = X_0$, when approaching in the Z, Z* plane, φ possesses nontangential boundary values almost everywhere on $|Z| = r$. The exceptional set is independent of the coefficient $F(y,z)$ of (1). (Received July 31, 1972.)

72T-B300. WITHDRAWN.

72T-B301. SAMUEL ZAIDMAN, Université de Montréal, Montréal, Québec, Canada. Weak solutions of differential equations in reflexive Banach spaces.

Let X be a reflexive Banach space, X* its dual. Let $A;\mathcal{D}(A) \subset X \to X$ be a linear closed operator with dense domain and A* its adjoint. For $-\infty \leqq a < b \leqq +\infty$ define the class $K_{A^*}(a,b)$ of functions $\varphi(t)$ in $C^1(a,b;X^*)$ with compact support in (a,b), such that $\varphi(t) \in \mathcal{D}(A^*)$ $\forall$ $t \in (a,b)$ and $(A^*\varphi)(t)$ is continuous, $a < t < b \to X^*$. Let $f(t) \in L^p_{loc}(a,b;X)$ be given. The class $W^{(a,b)}_{A,f}$ is composed of functions $u(t) \in L^p_{loc}(a,b;X)$, verifying $\int_a^b \langle \varphi'(t) + (A^*\varphi)(t), u(t)\rangle\,dt = -\int_a^b \langle \varphi(t), f(t)\rangle\,dt$ for any $\varphi(t) \in K_{A^*}(a,b)$, where $\langle\cdot,\cdot\rangle$ is the duality between X and X*. Let now $\alpha_\epsilon(t)$, $\in C_0^1(R^1)$, $= 0$ for $|t| \geqq \epsilon$, and let $(u*\alpha_\epsilon)(t)$ be the $C^1(a+\epsilon, b-\epsilon;X)$-function defined by the integral $\int_{t-\epsilon}^{t+\epsilon} u(\tau)\alpha_\epsilon(t-\tau)\,d\tau$. We have Theorem 1. The convolution $(u*\alpha_\epsilon)(t)$ belongs to $\mathcal{D}(A)$ for $a+\epsilon < t < b-\epsilon$ and $(u*\alpha_\epsilon)'(t) = A(u*\alpha_\epsilon)(t) + (f*\alpha_\epsilon)(t)$, $a+\epsilon < t < b-\epsilon$. Also, we have Theorem 2. Let $u(t)$, $f(t)$ be continuous, $a < t < b \to X$; also $u(t) \in W^{(a,b)}_{A,f}$, $u(t) \in \mathcal{D}(A)$ and Au is continuous, $a < t < b \to X$. Then $u(t) \in C^1(a,b;X)$ and $u'(t) = Au(t) + f(t)$, $a < t < b$. (Received August 28, 1972.)

72T-B302. J. L. BRENNER, 10 Phillips Road, Palo Alto, California 94303. Some simultaneous equations in matrices.

Let A_1, A_2, A_3 be square matrices of dimension $r_1 \times r_1$, $r_2 \times r_2$, $r_3 \times r_3$, respectively. Necessary and sufficient conditions for the existence of X_1, X_2, X_3 satisfying $A_1 = X_1X_2X_3$, $A_2 = X_2X_3X_1$, $A_3 = X_3X_1X_2$ are (the Flanders conditions) that the elementary divisors of A_i corresponding to nonzero proper values be the same, and that the elementary divisors corresponding to the proper value 0 deviate in degree by at most one unit. For $r > 3$, conditions for the solvability of $A_i = (\Pi_{j=i}^r X_j)(\Pi_1^{i-1} X_j)$ are more complicated; an extra condition on the degrees of the elementary divisors corresponding to 0 is involved. Other simultaneous matrix equations are analyzed. This article will appear in Duke Math. J. (Received September 5, 1972.)

*72T-B303. RICHARD C. GILBERT, California State University, Fullerton, California 92634. Simplicity of differential operators on an infinite interval.

A closed symmetric operator A in a Hilbert space is said to be simple if there is no nontrivial reducing subspace in which A is selfadjoint. Previously the author showed that the minimal closed symmetric operator T_0 in $L^2(a,b)$ determined by a formally selfadjoint linear ordinary differential operator L is simple if L is regular or quasi-regular at a or b (J. Differential Equations 11(1972), 672-681). Let T_0 be defined in $L^2(-\infty,\infty)$ by $L = d_2(a^*)^4 + d_1(a^*)^3a + d_0(a^*)^2a^2 + d_1a^*a^3 + d_2a^4$, where d_0, d_1, d_2 are real and $a = 2^{-1/2}(t + d/dt)$,

$a^* = 2^{-1/2}(t - d/dt)$. The deficiency index and simplicity of T_0 are studied for various values of d_0, d_1, d_2. In particular, if $d_2 = 0$, $|d_0| < 2|d_1|$, T_0 has deficiency index (2,2) but is not simple. The main tool is the fact that T_0 can be defined by means of an infinite matrix relative to the basis of Hermite functions. Then one can use the work of Čistjakov on the deficiency index of an infinite matrix (Mat. Sb. 85 (127) (1971), 474-503). (Received September 5, 1972.)

72T-B304. JOSEPH L. WALSH, University of Maryland, College Park, Maryland 20742. Padé approximants as limits of rational functions of best approximation, real domain.

Let the function $f(x) \equiv a_0 + a_1x + \ldots + a_nx^n + R_n$, $a_0 \neq 0$, of class $C^{(\nu+1)}[0,1]$ or of some class $C^{(\nu+1)}[0,r]$, $r > 0$, in $[0,\epsilon]$ for ϵ (> 0) sufficiently small and fixed n and ν, and let $R_{n\nu}(\epsilon, x)$ denote the function of type (n,ν) of best approximation to $f(x)$ in the (uniform) sense of Tchebycheff on the interval $\delta : 0 \leqq x \leqq \epsilon$. Suppose we have $\Delta_{n-1\ \nu-1} \neq 0$; then as ϵ approaches zero $R_{n\nu}(\epsilon, x)$ approaches the Padé function $P_{n\nu}(x)$ on any closed set where $P_{n\nu}(x)$ is analytic. (Received September 5, 1972.)

72T-B305. J. H. RIZVI, University of Western Ontario, London 72, Ontario, Canada and University of Karachi, Karachi 32, Pakistan. Absolute logarithmic summability.

Let $S_n = \sum_{r=0}^{n} u_r$, $L(x) = -(1/\mathrm{Log}(1-x))\sum_{n=0}^{\infty}(S_n/(n+1))x^{n+1}$. If $L(x) \to S$ as $x \to 1-$, then the sequence $\{S_n\}$ is L-convergent to S. If $L(x) \in Bv(0,1)$ and $L(x) \to S$ as $x \to 1-$, then $\{S_n\}$ is absolutely L-convergent or $|L|$-convergent to S. Let $\{S_n^\lambda\}$ be the sequence of the Cesaro means of order λ of the sequence $\{S_n\}$. Let $S^\lambda(x) = (1-x)\sum_{n=0}^{\infty} S_n^\lambda x^n$. If $S^\lambda(x) \in Bv(0,1)$ and $S^\lambda(x) \to S$ as $x \to 1-$, then $S_n \to S\ |A,\lambda|$. Let $\{h_n\}$ be a regular Hausdorff transform (H_χ-transform) of the sequence $\{S_n\}$, generated by the function $\chi(t)$ of bounded variation in (0,1). If $h_n \to S(L)$, then $S_n \to \ell(LH_\chi)$ and if $h_n \to S\ |L|$, then $S_n \to S\ |LH_\chi|$. The following main results are established: Theorem 1. If a is real, $(n+a)V_n = S_n$ $(n = 0, 1, 2, \ldots)$ and $S_n \to S(L)$, then $V_n \to 0\ |L|$. Theorem 2. The method $|L|$ is translative. Theorem 3. If $-1 < \lambda \leqq 1$, and $S_n \to S\ |A,\lambda|$, then $S_n \to S\ |L|$. Theorem 4. If H_χ is a regular Hausdorff method and $S_n \to S\ |L|$, then $S_n \to S\ |LH_\chi|$. (Received September 5, 1972.)

72T-B306. BARADA KINKAR RAY, Regional Engineering College, Durgapur-9, West Bengal, India. On a fixed point theorem in a metric space. Preliminary report.

In this note the following theorem has been proved. Theorem. Let X be a metric space with metrics d and ρ such that (i) $d(x,y) \leqq \rho(x,y)$; (ii) $T: X \to X$ is a mapping such that $\rho(Tx,Ty) \leqq \alpha\rho(x,Tx) + \beta\rho(y,Ty)$, $\alpha > 0$, $\beta > 0$, $\alpha + \beta < 1$ for all $x, y \in X$; (iii) T is continuous at $z \in X$ with respect to d; (iv) there exists a point $x_0 \in X$ such that $\{T^n x_0\}$ has a convergent subsequence $\{T^{n_k} x_0\}$ converging to z in the metric d. Then T has a unique fixed point. (Received September 14, 1972.) (Author introduced by Professor Sudhanshu K. Ghoshal.)

72T-B307. SUDHANSHU K. GHOSHAL and BARADA KINKAR RAY, Regional Engineering College, Durgapur-9, West Bengal, India. Fixed point theorems and a sequence of mappings. Preliminary report.

The following theorems, two of which generalise some earlier known results, have been given. Theorem 1. Let T be a mapping of a complete metric space X onto itself. If there exists a mapping M of X into itself which has a right inverse and which makes $M^{-1}TM$ satisfy the condition $\rho(M^{-1}TMx, M^{-1}TMy) \leqq \alpha\rho(x, M^{-1}TMx) + \beta\rho(y, M^{-1}TMy)$, $\alpha > 0$, $\beta > 0$, $\alpha + \beta < 1$, then T has a unique fixed point. Theorem 2. Let (i) $T_n : X \to X$ be a sequence of mappings with fixed points a_n for $n = 1, 2, \ldots$, (ii) T_n converge uniformly to

T where $T: X \to X$ is a mapping such that $d(Tx, Ty) \leqq \alpha d(x, Tx) + \beta d(y, Ty)$, $\alpha + \beta < 1$, $\alpha > 0$, $\beta > 0$, and "a" is the fixed point of T. Then a_n converges to a. Theorem 3. Let X be a complete metric space with metrics d and ρ such that $d(x,y) \leqq \rho(x,y)$ for each pair $x, y \in X$. Let X be complete with respect to d and let $T_i: X \to X$, $i = 1, 2$, be two mappings in (X, d) and $\rho(T_1x, T_2y) \leqq \alpha\rho(x, T_1x) + \beta\rho(y, T_2y)$, $\alpha > 0$, $\beta > 0$, $\alpha + \beta < 1$. Then T_1 and T_2 have a unique common fixed point in X. (Received September 14, 1972.)

*72T-B308. R. WONG, University of Manitoba, Winnipeg, Manitoba R3T 2N2, Canada. On a nonlinear Volterra integral equation. Preliminary report.

Recently Handelsman and Olmstead [SIAM J. Appl. Math. 22(1972), 373-384] have investigated the asymptotic behavior of the solutions to the nonlinear integral equation $\varphi(t) = (1/\sqrt{\pi}) \int_0^t (t-s)^{-\frac{1}{2}} \{f(s) - \varphi^n(s)\}\, ds$, $t \geqq 0$, $n \geqq 1$, where $f(t)$ is nonnegative, bounded and locally integrable on $[0, \infty)$. Furthermore, it is assumed that $f(t)$ has an asymptotic expansion of the form $f(t) \sim \sum_{m=0}^{\infty} a_m t^{-\gamma_m}$ as $t \to \infty$. They conjecture that for $a_0 \geqq 2$ and $n = 2$, $\varphi(t) = O(t^{-\frac{1}{2}} \log t)$ as $t \to \infty$. We prove in fact the following Theorem. Let $n \geqq 1$. If $a_0 = 1$, then the solution $\varphi(t)$ satisfies $\varphi(t) = O(t^{-\frac{1}{2}} \log t)$ as $t \to \infty$. If $a_0 > 1$ then $\varphi(t)$ satisfies $\varphi(t) = O(t^{-\frac{1}{2}})$ as $t \to \infty$. (Received September 15, 1972.) (Author introduced by Professor Robert Willis Quackenbush.)

*72T-B309. RAYMOND L. JOHNSON, University of Maryland, College Park, Maryland 20742. Lipschitz spaces, Littlewood-Paley spaces and convoluteurs. Preliminary report.

We study the ${}_aL_p$ spaces of Herz (J. Math. Mech. 18(1968), 283-324). It is shown that the ${}_aL_p$ norm is equivalent to the ℓ^p norm of the L^a norm taken over dyadic annuli. Applications to inclusion theorems between ${}_aL_p$ spaces and the Lorentz spaces, to the literature, and to the problem of characterization of the translation invariant operators on H^1 and L^p are given. (Received September 15, 1972.)

72T-B310. WITHDRAWN.

72T-B311. JAMES RAGO, Northwestern University, Evanston, Illinois 60201. Compact independent subsets of locally compact abelian groups.

E denotes a compact independent subset of an arbitrary nondiscrete locally compact abelian group G; $\mathcal{R}$ denotes the Raikov system generated by $E \cup -E$ on G. The Williamson conjecture is proved and a theorem of Drury is generalized; see Drury (Proc. Cambridge Philos. Soc. 64(1968), 1011-1013). Theorem. If $\mu \in M(G)$ such that $\hat{\mu}$ vanishes off the set of symmetric multiplicative linear functionals on $M(G)$, then μ is singular with respect to $\mathcal{R}$. The following theorem is used in the development: for each $P \in \mathcal{R}$ there exists a compact perfect set $K \subset G$, disjoint from P, such that group $P \cap$ group $K = \{0\}$, and (i) K is a Kronecker set if G is an I-group, (ii) K is a K_q set otherwise, for some $q \in Z^+$. Also, a theorem of Rudin is correctly proved and generalized: if $H \subset G$ is a closed nondiscrete subgroup with Haar measure h, and $x \in G$, then $h[(x + \text{group } E) \cap H] = 0$; see Rudin, "Fourier analysis on groups," Interscience, New York, 1962, Theorem 5.3.6. (Received September 18, 1972.) (Author introduced by Professor Colin C. Graham.)

72T-B312. LEONARD GALLAGHER, Catholic University of America, Washington, D. C. 20017. Independence in topological and measure structures. Preliminary report.

Let Ω denote the set of all finite sequences of 0's and 1's, and let Z denote the set of all maps $\alpha: \Omega \to \{0, 1\}$. Let C denote the Cantor discontinuum (i.e. the set of all sequences $\beta: \omega \to \{0, 1\}$), and let $[0, 1]^C$

denote the space of continuous functions $f: C \to [0,1]$ with the supremum metric. We define $F: Z \to [0,1]^C$ putting $F(\alpha)(\beta) = \sum_{r=1}^{\infty} \alpha(\beta_0, \ldots, \beta_{r-1})/2^r$ for all $\alpha \in Z$ and $\beta \in C$. Hence F is continuous and a Borel measure μ can be defined over $[0,1]^C$ by $\mu(X) = m(F^{-1}(X))$, where m is the natural probability measure on Z. Theorem 1. For μ-almost every $f \in [0,1]^C$, f is not 1-1, and $f(C)$ is a perfect set of Lebesgue measure 0. Let h be a real-valued, nondecreasing function with $h(0) = 0$ and $h(t) > 0$ for all $t > 0$, and continuous on the right for $t \geqq 0$. A set $N \subseteq \mathbb{R}^n$ is said to be "h-null" in $\mathbb{R}^n$ if for every $\epsilon > 0$ there exist bounded, open sets $G_t^k \subseteq \mathbb{R}^k$ such that $N \subseteq \bigcup_{k=1}^{\infty} P_{t=1}^n G_t^k$ and $\sum_{k=1}^{\infty} \Pi_{t=1}^n h(d(G_t^k)) < \epsilon$, where $d(G_t^k)$ is the diameter of G_t^k. Theorem 2. Let $R_k \subseteq \mathbb{R}^{n(k)}$ for $k < \omega$, where $n(k)$ are natural numbers and (1) each R_k is "h-null" for $h(t) = -1/\log t$, (2) $(x_1, \ldots, x_{r_k}) \in R_k$ implies $x_i \neq x_j$ for $1 \leqq i < j \leqq r_k$. Then for μ-almost every $f \in [0,1]^C$, $(f(C))^{r_k} \cap R_k = \emptyset$ for all $k < \omega$. A corollary of Theorem 2 extends the theorem of Jan Mycielski (Abstract 70T-A159, these *Notices* 17(1970), 809) to a case where the supposition "first category" is replaced by "h-null". (Received September 18, 1972.) (Author introduced by Professor Jan Mycielski.)

72T-B313. ERWIN O. KREYSZIG, University of Karlsruhe, Karlsruhe, Federal Republic of Germany. Ordinary differential equations for certain Bergman generating functions.

The partial differential equation $Lu := u_{zz^*} + b(z,z^*)u_{z^*} + c(z,z^*)u = 0$ (z, z^* complex, b and c analytic) is said to be of class P if there is a Bergman integral operator for the equation whose generating function g is a polynomial in the variable of integration t with coefficients depending on z and z^*, which are obtained by solving a system of nonhomogeneous linear partial differential equations of second order. It is shown that for a certain subclass of P one may obtain a single homogeneous linear ordinary differential equation in which the independent variable is a function of z, z^*, t and g is a solution of the equation. This entails representations of g in terms of hypergeometric functions, Legendre's associated functions, and — in some cases — even elementary functions. These results also contribute to the theory of operators introduced by H. Florian (Monatsh. Math. 69(1965), 18-29). (Received September 18, 1972.)

*72T-B314. KOHUR N. GOWRISANKARAN, McGill University, Montreal 110, Quebec, Canada. Measurability of lattice operations on a cone.

The following result concerning the measurability of lattice operations on a cone is proved. Theorem. Let X be a Hausdorff locally convex real topological vector space. Let C be a closed proper convex cone with vertex at the origin, generating X and such that C is a lattice in its own order. Let B be a compact metrisable base for C and let further the continuous positive linear functionals on X separate the points of C. Then, the mappings $C \times C \to C$ given by $(x,y) \to \sup(x,y)$ and $(x,y) \to \inf(x,y)$ are Borel, viz., the inverse image of any Borel set of C under each of these mappings is a Borel set of $C \times C$. (Received September 18, 1972.)

*72T-B315. ANDRE de KORVIN and LAURENCE E. KUNES, Indiana State University, Terre Haute, Indiana 47809. Some compactness properties in L_ρ spaces. I. Preliminary report.

For notations see Abstract 72T-B277, these *Notices* 19(1972), A-701. Proposition 1. The following conditions are equivalent: (1) σ^* is Hausdorff under the topology τ defined by the seminorms $p_{G,A}$ as $G \in \mathfrak{A}_{\rho'}[\mu]$ and $A \in \Sigma_0$. (2) The closure of finite sums of the form $\sum \alpha_i G(A_i)$ as $A_i \in \Sigma_0$ and $G \in \mathfrak{A}_{\rho'}[\mu]$ is all of X. (3) τ is stronger than the weak* topology of σ^*. Proposition 2. Assume σ^* is Hausdorff under the τ topology; the following conditions are equivalent: (1) σ^* is Hausdorff under the γ topology defined by the seminorms $p_{G,A}$ as G is fixed and $A \in \Sigma_0$. (2) If $\int f\,dy^*G = 0$ for all $f \in M_\rho$ and $A \in \Sigma_0$ then $\int_A f\,dy^*n = 0$ for all $f \in M_\rho$, $A \in \Sigma_0$ and for all $n \in \mathfrak{A}_{\rho'}[\mu]$. (3) γ is stronger than the weak* topology. Theorem. Let T be a continuous linear operator from L_ρ into X, let $G \in \mathfrak{A}_{\rho'}[\mu]$ be the corresponding measure; then if T is a compact operator then σ^* is τ compact. If σ^* is τ compact then the restrictions of T to functions in L_ρ vanishing off $A \in \Sigma_0$ are compact operators. (Received September 22, 1972.)

*72T-B316. RICHARD A. ALO, Carnegie-Mellon University, Pittsburgh, Pennsylvania 15213 and ANDRE de KORVIN and LAURENCE E. KUNES, Indiana State University, Terre Haute, Indiana 47809. Some compactness properties in L_ρ spaces. II. Preliminary report.

For notations see Abstract 72T-B277, these *Notices* 19(1972), A-701. Let γ' be the topology on σ^* induced by $p_{G,\Omega}$. For $y^* \in \sigma^*$ define the operator $\langle T,y^*\rangle$ on M_ρ by $\langle T,y^*\rangle(f) = \langle T(f),y^*\rangle$. $\langle T,y^*\rangle_A$ will denote the restriction of $\langle T,y^*\rangle$ to $L_\rho(A)$ where $L_\rho(A)$ denotes functions of L_ρ vanishing off A. Proposition 1. If T is a compact operator and if for all $A_n \in \Sigma_0$ with $A_n \searrow \emptyset$, $\|\langle T,y^*\rangle_{A_n}\| \to 0$ then σ^* is compact in the γ' topology and G is ρ' c.a. Proposition 2. Let $G \in \mathfrak{A}_{\rho'}[\mu]$ where $\rho(\chi_K) < \infty$ for every compact set K of Σ. The following conditions are equivalent: (1) G is ρ' c.a. (2) For every $A_n \in \Sigma_0$ with $A_n \searrow \emptyset$ there exist open Baire sets U_n such that $A_n \subset U_n$ and $\int f_n\,dG \to 0$ uniformly for $f_n \in M_\rho$, $\rho(f_n) \leq 1$, $f_n = 0$ off U_n. (Received September 22, 1972.)

*72T-B317. ANDRE de KORVIN, Indiana State University, Terre Haute, Indiana 47809 and RICHARD A. ALO and CHARLES ALEX CHENEY, Carnegie-Mellon University, Pittsburgh, Pennsylvania 15213. Abstract martingales. II. Preliminary report.

For notations see Abstract 72T-B276, these *Notices* 19(1972), A-701. Let M be a B* algebra, ρ a positive linear functional on M and D a directed set. For $\alpha \in D$, let N_α be a subspace of M. $\{x_\alpha, N_\alpha\}$ is called a martingale if $N_\alpha \subset N_\beta$ if $\alpha \leq \beta$, $x_\alpha \in N_\alpha$ for every α, $\rho(ax_\alpha) = \rho(ax_\beta)$ if $\alpha \leq \beta$ and $a \in N_\alpha$. Assume that * is an isometry on M; define $\|x\|_2 = [\rho(x^*x)]^{1/2}$; the completion of M in $\|\ \|_2$ will be denoted by $L_2(M)$. Theorem. Let $\{x_n, N_n\}$ be a martingale on $L_2(M)$ with $\|x_n\|_2 < K$; assume that N_n is closed under *. Then there exists $x_\infty \in L_2(M)$ such that x_n converges to x_∞ in the $L_2(M)$ norm. If $N_\infty = \overline{\bigcup N_n}$ then $\{\{x_n\} \cup \{x_\infty\}, \{N_n\} \cup \{N_\infty\}\}$ is a martingale on $L_2(M)$. Some properties analogous to those of classical martingales are shown. Martingales on spaces of type $L_1(M)$ will next be considered. (Received September 22, 1972.)

72T-B318. W. C. SISARCICK, Marshall University, Huntington, West Virginia 25701 and GERD H. FRICKE and SWARUPCHAND M. SHAH, University of Kentucky, Lexington, Kentucky 40506. A characterization of entire functions of exponential type and M-bounded index.

An entire function f is said to be of M-bounded index if there exists an integer N such that $\mathrm{Max}_{0 \leq j \leq N}\{\mathrm{Max}_\theta|f^{(j)}(re^{i\theta})|/j!\} \geq \mathrm{Max}_\theta|f^{(j)}(re^{i\theta})|/j!$ for all $r \geqq 0$ and all integers j. The least such integer N is called the M-index of f. Theorem 1. An entire function f is of exponential type if and only if f is of M-bounded index N for some integer $N \geqq 0$. If f is of exponential type T then $T \leq N + 1$, where N is the M-index of f. The upper bound on T is sharp. Theorem 2. Let $A = \{f(z) = e^{\alpha z+\beta}\prod_{n=1}^{\infty}(1 - z/a_n) \mid \alpha, \beta, a_n \in \mathbb{C}, 0 < |a_1| \leq |a_2| \leq \ldots \leq |a_n| \leq \ldots$, if $|a_n| = |a_{n+1}|$ then $\arg a_n \leq \arg a_{n+1}$, and $\sum_{n=1}^{\infty} 1/|a_n| < \infty\}$. Let $\rho(f,g) = |\alpha_f - \alpha_g| + |\beta_f - \beta_g| + \sum_{n=1}^{\infty}|1/a_{n_f} - 1/a_{n_g}|$ for f, g, $\in$ A. Then (A, ρ) is an incomplete metric space of 2nd category and is topologically complete. (Received September 25, 1972.)

72T-B319. SUDHANSHU KUMAR GHOSHAL, ABHA GHOSHAL and M. ABU MASOOD, Regional Engineering College, Durgapur-9, West Bengal, India. Gronwall's type vector differential inequality and applications to nonselfadjoint partial differential equations. Preliminary report.

Our work on the generalization of Gronwall's inequality for nonselfadjoint partial differential equations of hyperbolic type has been further generalized to study systems of nonselfadjoint partial differential equations. We have reduced the vector integral inequality to a vector partial differential inequality with the help of generalized Riemann functions. As in our previous work, this inequality may be used to study a uniqueness theorem, continuous dependent test, comparison theorem, etc. (Received September 26, 1972.)

72T-B320. SUDHANSHU KUMAR GHOSHAL and M. ABU MASOOD, Regional Engineering College, Durgapur-9, West Bengal, India. Generalized Gronwall's inequality and its applications to nonselfadjoint, linear and nonlinear partial differential equations. Preliminary report.

Gronwall's inequality in two dimensions has been generalized to study nonselfadjoint differential equations of hyperbolic type, both linear and nonlinear. Some special cases of our results are applicable to the case of selfadjoint hyperbolic differential equations. Our result is dependent on a Riemann function. This inequality has important applications in the theory of differential equations in connection with the proof of uniqueness of solutions, continuous dependence of the solution on the initial data, stability of the solution and its comparison criteria. (Received September 26, 1972.)

72T-B321. WILLIAM R. EMERSON, Courant Institute, New York University, New York, New York 10012. The Pointwise Ergodic Theorem for amenable groups. Preliminary report.

Renaud (Amer. J. Math. 93(1971), 52-64) incorrectly proved a generalization of the Pointwise Ergodic Theorem for an arbitrary summing sequence in an amenable topological group. We show: Theorem 1. General summing sequences $\{U_n : n \in N\}$ do not necessarily yield the Pointwise Ergodic Theorem in an amenable group. Theorem 2. Imposing the further condition (A): $|U_n^{-1} U_n| < K|U_n|$ for some $K > 0$ independent of n is sufficient to insure the Pointwise Ergodic Theorem (where $|\ |$ denotes left Haar measure). See the cited paper for definitions and notation. (Received September 27, 1972.)

*72T-B322. DOUGLAS MOREMAN, Auburn University, Auburn, Alabama 36830. "Weak Cauchy sequences" in metric spaces. Preliminary report.

This paper is a continuation of work mentioned in Abstracts 689-G6, 72T-B193, 72T-B60 and 72T-B243, these *Notices* 18(1971), 1065 and 19(1972), A-580, A-312 and A-692. In the context of a metric space S, d having an intersectional convexity C: Definition. A sequence α of points is said to close convexly provided that α is infinite and bounded and such that if β and γ are infinite subsequences of α then the distance from $C(\beta^*)$ to $C(\gamma^*)$ is zero. Theorem. If S is a normed linear space then a sequence α of points closes convexly if and only if α is a weak Cauchy sequence. Theorems are presented which relate to the question of which spaces S, d, C may be embedded in a space in which every sequence of points that closes convexly also converges convexly. (Received September 27, 1972.)

Applied Mathematics

72T-C62. NABIL A. KHABBAZ, Computer Science Department, University of Iowa, Iowa City, Iowa 52240. Homomorphic characterization of languages in the hierarchy. Preliminary report.

Let $\mathcal{L}$ be the family of labeled linear grammars, $\mathcal{CF}$ be the family of context free languages. Define $\mathcal{L}_0 = \mathcal{CF}$, $\mathcal{L}_i = \mathcal{L}/\mathcal{L}_{i-1}$ $\forall$ $i > 0$, where $\mathcal{L}/\mathcal{L}_{i-1} = \{L(\mathcal{J}) \mid \mathcal{J} = (G, A),\ G \in \mathcal{L},\ A \in \mathcal{L}_{i-1}\}$ and where $L(\mathcal{J}) = \{x \in \Sigma^* \mid \sigma \Rightarrow^*_{\pi} x, \pi \in A\}$. Σ, σ are respectively the alphabet and start symbol of G. Then $\{\mathcal{L}_j\}_{j \geq 0}$ forms a hierarchy based on the fact that long strings in $L(\mathcal{J}) \in \mathcal{L}_j$, can be put in the form $\Pi_{i=1}^{2^j} u_i v_i w_i x_i y_i$ such that $\Pi_{i=1}^{2^j} u_i v_i^k w_i x_i^k y_i \in L(\mathcal{J})$ $\forall$ $k \geq 0$ and such that some $v_i x_i \neq \epsilon$ while all $|v_i w_i x_i| \leq q$, q is a natural number and Π denotes concatenation. Similarly based on the homomorphic characterization of $K \in \mathcal{L}_0$ in terms of a Dyck language we have Definition. Let $\Sigma = \{a_1, a_2, \ldots, a_n\}$, $\Sigma' = \{a_1', a_2', \ldots, a_n'\}$ and $\Delta = \Sigma \cup \Sigma'$. Let $G = (\{\sigma\}, \Delta, \sigma, P)$ where $P = \{\sigma \rightarrow \sigma a_i \sigma a_i' \sigma, \sigma \rightarrow \epsilon \mid i = 1, \ldots, n\}$; then $D_0 = L(G)$ is called a Dyck language in $\mathcal{L}_0$. Let $\mathcal{C}_j : \Delta \rightarrow \Delta^j$ be 1-1, onto maps from Δ into 2^j distinct copies of Δ, and let $\mathcal{C}_j : \Delta^* \rightarrow (\Delta^j)^*$ denote the homomorphic extension of $\mathcal{C}_j$. Define $D_j = \{\mathcal{C}_1(x)\mathcal{C}_2(x^R)\mathcal{C}_3(x)\mathcal{C}_4(x^R)\ldots\mathcal{C}_{2^j-1}(x)\mathcal{C}_{2^j}(x^R) \mid x \in D_0\}$, then D_j is called a generalized Dyck language in $\mathcal{L}_j$ (x^R is the reverse of x). Theorem. $\forall$ $j \geq 0$, $K \in \mathcal{L}_j$ iff $K = h(D_j \cap R)$, for some generalized Dyck language $D_j \in \mathcal{L}_j$, some homomorphism h and some regular set R. (Received April 17, 1972.) (Author introduced by Professor Arthur C. Fleck.)

*72T-C63. ABRAHAM BERMAN, Centre de recherches mathématiques, Université de Montréal, Montréal 101, Québec, Canada. Generalized interval programming.

The theory of interval programming is extended to complex spaces, to include optimization problems of the form: Maximize Re $c^H x$ s.t. $b - Ax \in S$, $Ax - a \in T$, where $A \in C^{m \times n}$, $a, b \in C^m$, $c \in C^n$ and S and T are closed convex cones in C^m. A duality theorem for generalized interval programming is derived. (Received June 16, 1972.)

72T-C64. C. J. EVERETT, Los Alamos Scientific Laboratory, University of California, Los Alamos, New Mexico 87544. Free energy in concentration space.

The free energy of a system of perfect gases, undergoing a reversible reaction $\sum a_i A_i \leftrightarrow \sum b_j B_j$ governed by the law of mass action is studied mathematically as a function on the trajectory of the system in a space of concentrations. It is shown that the fundamental law of chemical thermodynamics $\Delta G^0 + RT_0 \ln K(T_0) = 0$ holds: (1) iff the Gibbs free energy G is minimal at equilibrium on the path of a system reacting at constant pressure, and also (2) iff the Helmholtz free energy A is minimal at equilibrium on the path of a system reacting at constant volume. Moreover, assuming the above law, A is also minimal in case (1) iff $\sum a_i = \sum b_j$, and G is minimal in case (2) iff $\sum a_i = \sum b_j$. The trajectories coincide for reactions of the latter type. (Received July 24, 1972.)

*72T-C65. W. JOHN WILBUR, Andrews University, Berrien Springs, Michigan 49104. Orthocomplementations and locally convex spaces. Preliminary report.

A simple characterization, using Hilbert space concepts, of those complete infinite dimensional Hausdorff locally convex topological vector spaces which have an orthocomplementation on their lattice of closed subspaces is given. From this it follows that all such orthocomplemented lattices are canonically isomorphic to a sublogic of the logic of closed subspaces of a Hilbert space. It also follows that if a complete barreled space has an orthocomplementation on its lattice of closed subspaces it is a Hilbert space. These results are valid over both the real and complex fields. (Received September 5, 1972.)

*72T-C66. OTOMAR HÁJEK, Case Western Reserve University, Cleveland, Ohio 44106. Boundedness of target set core. Preliminary report.

The core of a target set T, for a control system, consists of all points which remain in T for all times $t \geqq 0$ under some admissible control. In case the system is finite-dimensional, linear and autonomous: $\dot{x} = Ax - p$, $p(\cdot) \in P$, $x(\text{end}) \in T$, with the constraint set P nonvoid, compact and convex, and the target set of the form $T = L + K$, where $L = \{x : Mx = 0\}$ is a linear subspace and the "error term" K is compact, we have a characterization of boundedness of core(T) independent of the shape of P. Theorem. core(T) is compact if and only if either it is empty or the system $\dot{x} = Ax$, $y = Mx$ is observable. (Received September 8, 1972.)

*72T-C67. FRANK STENGER, University of Utah, Salt Lake City, Utah 84112. Trapezoidal-type quadratures over a contour. Preliminary report.

Let D be a simply connected domain with boundary C and boundary points a and b, and let φ denote a conformal map of D onto $D' \equiv \{w : |\operatorname{Im} w| < \pi/2\}$, such that $\varphi(a) = -\infty$ and $\varphi(b) = \infty$. Let f be analytic in D, such that $N_D(f) \equiv \lim_{(C' \to C,\, C' \in D)} \int_{C'} |f(z)\,dz| < \infty$, and such that $\int_{\varphi^{-1}(u+L)} |f(z)\,dz| \to 0$ as $u \to \pm\infty$, where $L = \{iy : -\pi/2 \leqq y \leqq \pi/2\}$. Then for any $q > 1$, (*) $\left|\int_a^b f(z)\,dz - \log q \sum_{m=-\infty}^{\infty} f(z_m(q))/\varphi'(z_m(q))\right| \leqq N_d(f) \exp(-\pi^2/\log q)/[1 - \exp(-\pi^2/\log q)]$, where $z_m(q) = \varphi^{-1}(m \log q)$. For example, if $D = D'$, then (*) yields the well-known trapezoidal formula (1) $\int_{-\infty}^{\infty} f(x)\,dx \cong \log q \sum_{m=-\infty}^{\infty} f(m \log q)$; if $D = \{z : |z| < 1\}$, then (*) yields the formula (2) $\int_1^1 f(x)\,dx \cong \log q \sum_{m=-\infty}^{\infty} (2q^m/(1+q^m)^2) f((q^{m-1})/(q^m+1))$; while if $D = \{z : \operatorname{Re} z > 0\}$ then (*) yields the formula (3) $\int_0^{\infty} f(x)\,dx \cong \log q \sum_{m=-\infty}^{\infty} q^m f(q^m)$. The formulas are accurate even if f has singularities at a or b. For example, if $H = \{f : f(z) = \sum_0^{\infty} a_n z^n$ and $\|f\|^2 = \sum_0^{\infty} |a_n|^2 < \infty\}$, and if $f \in H$, then taking $q = \exp(\pi/(N/2)^{1/2})$, and summing from $-N$ to N in formula (2), the error of formula (2) is $O(\exp(-\pi(N/2)^{1/2}))\|f\|$ as $N \to \infty$. (Received September 18, 1972.)

*72T-C68. WILLIAM B. GEARHART, University of Utah, Salt Lake City, Utah 84112. Use of parametric linear programming in vectorial approximation.

The problem of computing approximations subject to several criteria of approximation is considered in the setting of vectorial approximation. For the case of linear approximation with sup norms the vectorial approximations are the solutions of certain constrained minimization problems. Solutions of these constrained minimization problems can be obtained with techniques from parametric linear programming. These techniques work directly with the deviations, rather than weights, as would be used in a weighted-sum approach. Also it is shown that, using a modified LU decomposition form of the simplex method, one can increase the number of approximating functions without performing any additional computation. (Received September 15, 1972.) (Author introduced by Professor Robert E. Barnhill.)

72T-C69. SWAPNA SEN, University of Calcutta, Calcutta 9, India and LOKENATH DEBNATH, East Carolina University, Greenville, North Carolina 27834. On unsteady hydromagnetic flows in an electrically conducting rotating fluid.

An unsteady analysis is made of the hydromagnetic flow due to elliptic harmonic oscillations of an infinite plate in an incompressible, homogeneous, viscous and electrically conducting fluid which is uniformly rotating in the presence of a uniform magnetic field. The initial value problem is solved by using the Laplace transform technique. The unsteady velocity field and the induced magnetic field are calculated explicitly and the structures of the associated hydromagnetic boundary layers are determined. The simultaneous effects of rotation and the electromagnetic force on the flow phenomenon are investigated. It is shown that the corresponding steady state analysis of Hide and Roberts can readily be recovered as a particular case of the present work. Special attention is given to the physical interpretation of the mathematical results obtained. Some interesting features of the hydromagnetic flow are discussed. (Received September 22, 1972.)

72T-C70. SUKLA MUKHERJEE and KALYAN KUMAR BAGCHI, University of Calcutta, Calcutta 9, India and LOKENATH DEBNATH, East Carolina University, Greenville, North Carolina 27834. On propagation of dispersive long waves on a rotating sea due to travelling atmospheric disturbances.

An initial value investigation is made of the transient development of dispersive long waves on the surface of a homogeneous rotating shallow ocean due to an arbitrary travelling wind stress distribution acting on the free surface of the ocean. Special emphasis is given to the effects of the Coriolis force on the wave motions for various ranges of the parameter which is the ratio of the frequency of the imposed travelling disturbances and the Coriolis parameter. The integral solution for the free surface displacement function is obtained by using the generalized function technique and an asymptotic analysis of the integral is carried out for large times and distances. Some physically realizable travelling wind stress distributions which may be oscillatory or transient in nature are included in the analysis for determination of the characteristic features of the wave motions. It is predicted that the solution asymptotically approaches to the ultimate steady state as $t \to \infty$. Several limiting cases of interest are discussed. (Received September 22, 1972.)

*72T-C71. DONALD ALVIN ALTON, Department of Computer Science, University of Iowa, Iowa City, Iowa 52240. Operator embeddability in computational complexity.

We generalize the embedding theorem of McCreight and Meyer (Conf. Rec. ACM Sympos. on Theory of Computing, 1969, pp. 79-88) to arbitrary total effective operators. This also generalizes Theorem 1 of our previous abstract ("Diversity of speed-ups and embeddability," Abstract 72T-C59, these *Notices* 19(1972), A-711). Theorem. Given Φ a complexity measure, $\prec$ a countable partial order, F a total effective operator, and R a computable function, there exists a computable c such that for all i, j, and m: $\Phi_{c(i)}(x)$ is total and strictly increasing. If $\varphi_m = \varphi_{c(i)}$, then: (a) $\Phi_m(x) > R(x)$ a.e.; (b) $j \prec i$ implies $F(\Phi_{c(j)})(x) < \Phi_m(x)$ a.e.; (c) i not $\prec j$ and $i \neq j$ implies $F(\Phi_{c(j)})(x) < \Phi_m(x)$ i.o. (Received September 25, 1972.)

*72T-C72. ANDREW VOGT, Oregon State University, Corvallis, Oregon 97331. A derivation of the Poincaré group.

Let $f: R^4 \to R^4$ be a bijection such that whenever p and q lie on a common light ray, fp and fq lie on a common light ray. Then f is in the group generated by the Poincaré group and dilatations. The proof of this fact is based on Zeeman's theorem that causality implies the Lorentz group. (Received September 15, 1972.) (Author introduced by Professor David Carter.)

Logic and Foundations

72T-E101. GREGORY J. CHAITIN, Mario Bravo 249, Buenos Aires, Argentina. On the difficulty of generating all binary strings of complexity less than n. Preliminary report.

Complexity is taken in the information-theoretic sense, i.e. the complexity of something is the number of bits in the shortest program for computing it on the standard universal computer. Let $\alpha(n) =$ min max (the length of P in bits, the time it takes P to halt), where the minimum is taken over all programs P whose output is the set of all binary strings of complexity less than n. Let $\beta(n) = \max f(n)$, where the maximum is taken over all number-theoretic functions f of complexity less than n. Let $\gamma(n) = \sum$ (the length of S), where the sum is taken over all binary strings S of complexity less than n. Take $f \approx g$ to mean that there are c and c' such that for all n, $f(n) \leqq g(n+c)$ and $g(n) \leqq f(n+c')$. Theorem. $\alpha \approx \beta \approx \gamma$. (Received June 19, 1972.)

72T-E102. JONATHAN STAVI, Hebrew University of Jerusalem, Jerusalem, Israel. Cardinal collapsing with reals. Preliminary report.

Theorem 1. Let $\varkappa$ be a regular cardinal $> \aleph_0$. If $\varkappa$ is not strongly Mahlo, there is a complete, countably generated Boolean algebra $\mathcal{B}$ satisfying the $<\varkappa$-chain condition (i.e. there are no $\varkappa$ disjoint elements) such that $\|\check{\varkappa} = \aleph_1\| = 1$ in $V^{(\mathcal{B})}$. The proof is based on ideas and methods of R. B. Jensen and R. M. Solovay, "Some applications of almost disjoint sets," Proc. Internat. Colloq. (Jerusalem, 1968), edited by Y. Bar-Hillel, North-Holland, Amsterdam, 1970, §3. The theorem yields an improvement of their results, namely: Theorem 2. Let M be a transitive countable model of ZFC, and let $\varkappa$ be a regular cardinal $> \aleph_0$ in M. There is an $a \subseteq \omega$ such that $N = M[a]$ is a Cohen extension of M, and $\varkappa = \aleph_1^N$, and every $\lambda > \varkappa$ which is a cardinal in M is a cardinal in N. There are obvious applications to consistency results in set theory. Theorem 3. Let $\varkappa$ be a cardinal that is not strongly Mahlo. Then $\varkappa$ is the power of some complete, countably generated Boolean algebra iff $(\exists n \in \omega)(\varkappa = 2^n)$ or $(\exists \mu > \aleph_0)$ [μ is regular and $\varkappa = 2^{\underline{\mu}}$] (equivalently — $\varkappa = 2^\nu$ for some ν or $\varkappa = \sup\{2^\nu | \nu < \mu\}$ for some weakly inaccessible μ). The "if" direction is proved by the Boolean algebras constructed for Theorem 1. (Received August 9, 1972.) (Author introduced by Dr. Haim Gaifman.)

*72T-E103. SAHARON SHELAH, Hebrew University of Jerusalem, Jerusalem, Israel. Various results in model-theory.

Theorem 1. In every cardinal $\lambda > \aleph_0$ there is a rigid dense linear order (Ehrenfeucht and Lauchli prove it for $\lambda \geqq 2^{\aleph_0}$) (in fact there are 2^λ such nonisomorphic orders). Theorem 2. The Hanf number of omitting complete types is $\beth_{\omega_1}$. That is, for every $\alpha < \omega_1$ there is a countable language L, a complete theory T in L, and a complete type p in L such that T has a model of cardinality λ which omits p iff $\lambda \leqq \beth_\alpha$. (This solves a problem of Morley (Berkeley Sympos., 1962).) Theorem 3. Suppose T is a (first-order) theory, and for every $m < \omega$ and finite subset T' of T, there is a model M of T', $\aleph_0 > |P^M| > |Q^M|^m > m^m$ (P, Q are one-place predicates). Then if $|T| \leqq \aleph_0$, then T has a model M, $|Q^M| = \aleph_0$, $|P^M| = 2^{\aleph_0}$. Moreover if $|T| \leqq \mu \leqq \lambda$ and there are $\alpha < \mu^+$ and a tree with μ nodes and $\geqq \lambda$ branches of height α, then T has a model M, $|Q^M| = \mu$, $|P^M| = \lambda$. Theorem 4 (ZFC + M). Suppose T_1, T are countable first-order theories, $T_1 \supseteq T$, T is complete, and T has uncountably many complete types (or even is $\aleph_0$-unstable = not totally transcendental).

Then in any $\lambda > \aleph_0$ there are $\min(2^\lambda, 2^{2^{\aleph_0}})$ nonisomorphic models of T which are reducts of models of T_1. (M means there is a measurable cardinal.) (If $T_1 = T$, M is not necessary.) (Received August 21, 1972.)

72T-E104. JAN MYCIELSKI, University of Colorado, Boulder, Colorado 80302. Remarks on capacitability. Preliminary report.

Let $\mathcal{K}$ be a property of subsets of compact metric spaces (CMS), which is invariant under inverse images of continuous maps of one space onto another. $BC(\mathcal{K})$ is the sentence "If X and Y are CMS and $A \subseteq X \times Y$ satisfies $\mathcal{K}$ then for every finite Borel measure μ over X there exists a Borel set $B \subseteq X$ and a Borel-measurable function $f: B \to Y$ such that $f \subseteq A$ and μ[(projection of A into X) - B] = 0." Let C be a CMS and χ be a finite capacity over the class of closed subsets of C which is continuous on the right and alternating of order ∞ (see G. Choquet, Ann. Inst. Fourier (Grenoble) 5(1953/4), 131-295). Theorem. If $BC(\mathcal{K})$ then every subset of C satisfying $\mathcal{K}$ is χ-capacitable. The proof uses only the axiom of choice for countable collections (AC_0). It is based on Choquet's "integral representation theorem" (ibid., §49) on account of the fact that for CMS his proof needs only AC_0. Examples of properties (classes) $\mathcal{K}$ satisfying $BC(\mathcal{K})$ are: (1) Σ_2^1, if there are only $\aleph_0$ constructible reals (see D. D. Shochat, Thesis, University of California, 1972); (2) the class of all projective sets, assuming projective determinateness (see Shochat, ibid.); (3) the class of all sets if we are in the universe of Levy-Solovay (see Solovay, Ann. of Math. (2) 92(1970), 1-56) or if we assume full determinateness (Solovay, unpublished). In those examples a variant of BC also holds in which meagre replaces measure 0. (Received September 5, 1972.)

72T-E105. JOSEPH A. SGRO, University of Wisconsin, Madison, Wisconsin 53706. Completeness theorems for topological models. Preliminary report.

Let L(Q) be the language formed by adding a new quantifier symbol Q. The intended interpretation of $Qx\varphi(x)$ is that the set defined by $\varphi(x)$ is "open". Take $\mathfrak{A}$ to be a model of L, $q \subseteq \mathcal{P}(A)$, and form $(\mathfrak{A}, q)$ where q is the interpretation of Q. If q is a topology on A, then $(\mathfrak{A}, q)$ is called topological. A more explicit presentation of L(Q) and $(\mathfrak{A}, q)$ is found in Keisler (Ann. Math. Logic 1(1970), 1-93). Theorem. Let T be an L(Q) theory. Then there is a topological model $(\mathfrak{A}, q)$ of T iff T is consistent with: (A1) first order predicate calculus, (A2) $Qx\varphi(x) \leftrightarrow Qy\varphi(y)$, (A3) $(\varphi \leftrightarrow \psi) \to (Qx\varphi \leftrightarrow Qx\psi)$, (A4) $Qx(x \neq x)$, (A5) $Qx(x = x)$, (A6) $Qx\varphi \wedge Qx\psi \to Qx(\varphi \wedge \psi)$, and (A7) $\forall x Qy \varphi(x, y) \to Qy \exists x \varphi(x, y)$. Theorem. Let T be countable. Then there is a totally disconnected, separable, and metrizable topological model $(\mathfrak{A}, q)$ of T iff T is consistent with (A1)—(A7) and (A8) $\forall x Qy(x \neq y)$. Theorem. Let T be countable. Then there is a topological model $(\mathfrak{A}, q)$ of T which is a topological group iff T is consistent with (A1)—(A7), (A9) group axioms, (A10) $Qx\varphi(x) \to Qx\varphi(x^{-1})$, (A11) $Qx\varphi(x) \to Qx\varphi(x \cdot x)$, (A12) $Qx\varphi(x) \to Qx\varphi(x \cdot y)$, and (A13) $Qx\varphi(x) \to Qx\varphi(y \cdot x)$. Analogues of Los's ultraproduct theorem, Craig's interpolation theorem, and Beth's definability theorem have also been proved. (Received September 8, 1972.)

72T-E106. JOHN T. BALDWIN, Michigan State University, East Lansing, Michigan 48823, ALISTAIR H. LACHLAN, Simon Fraser University, Burnaby 2, British Columbia, Canada and RALPH N. McKENZIE, University of California, Berkeley, California 94720. Universal Horn classes categorical in power.

Theorem. Let T be a countable universal Horn theory having an infinite model. The following are equivalent: (1) T is m-categorical for all $m > 1$; (2) T is ω-categorical; (3) T is ω_1-categorical and has a finite model with more than one element; (4) T has a finite model with more than one element and the theory of the infinite models of T is complete. This improves results of Baldwin, Abstract 72T-A97, these *Notices* 19(1972), A-431 (errata: replace "no finite members" by "a finite member") and Lachlan, Abstract 72T-E66, these *Notices* 19(1972), A-598. (Received September 8, 1972.)

*72T-E107. LEONARD P. SASSO, JR., University of California, Irvine, California 92664. Deficiency sets and bounded information reducibilities. Preliminary report.

Let Δ_α be the set of deficiency sets as defined in Dekker ["A theorem on hypersimple sets," Proc. Amer. Math. Soc. 5(1954), 791-796] of an r.e. set α of natural numbers and let $\underset{\sim}{\alpha}$, $w(\alpha)$, $tt(\alpha)$, $m(\alpha)$, and $1(\alpha)$ be respectively the Turing, weak truth table, truth table, many-one, and one-one degrees of α as defined in Friedberg and Rogers ["Reducibility and completeness for sets of integers" Z. Math. Logik Grundlagen Math. 5(1959), 117-125]. It was shown by Dekker that $\Delta_\alpha \subseteq \underset{\sim}{\alpha}$ for each r.e. set α. Our main purpose is to show that for nonrecursive, r.e. sets α and β: (i) $\Delta_\alpha \subseteq m(\alpha)$ may hold, (ii) Δ_α may contain sets of incomparable w degree, and (iii) $\Delta_\alpha \cap \Delta_\beta$ may be nonempty for α and β of incomparable w degree while $\Delta_\alpha \cap \Delta_\beta$ might be empty when $1(\alpha) = 1(\beta)$. In showing (i) we introduce self-deficient sets ($\alpha \in \Delta_\alpha$) and show that every r.e. degree contains one but that not all nonrecursive deficiency sets are self-deficient. If w is replaced by tt in (ii) and (iii) then $\underset{\sim}{\alpha}$ may be arbitrary while in the w cases α may be made recursive in any given nonrecursive, r.e. set. The original constructions yield complete sets. (i) employs an e-state priority argument with followers. The remaining constructions are finite injury, diagonal sabotage arguments. The Turing degree restrictions employ modified permitting techniques. (Received September 13, 1972.)

*72T-E108. DON L. PIGOZZI, Iowa State University, Ames, Iowa 50010. On the decision problem for equational theories. II: The join of theories.

For notation and terminology see [Abstract 72T-E63, these *Notices* 19(1972), A-597]. The join of θ and Φ is the set of all consequences $\sigma = \tau$ of $\theta \cup \Phi$ such that $\mathrm{Op}(\sigma = \tau) \subseteq \mathrm{Op}(\theta) \cup \mathrm{Op}(\Phi)$. The theorem of Part I can be used to establish the following results. Theorem 1. The Turing degree of the join of any θ and Φ with $\mathrm{Op}(\theta) \cap \mathrm{Op}(\Phi) = 0$ is the join of the T-degrees of θ and Φ. An example is given showing the condition $\mathrm{Op}(\theta) \cap \mathrm{Op}(\Phi) = 0$ cannot be replaced by $\theta \cap E = \Phi \cap E$ where E is the set of all $\sigma = \tau$ such that $\mathrm{Op}(\sigma = \tau) \subseteq \mathrm{Op}(\theta) \cap \mathrm{Op}(\Phi)$. Let Γ be a fixed but arbitrary set of operation symbols containing at least one symbol of rank $\geqq 2$. Theorem 2. For any r.e. T-degree δ there exists an equation $\sigma = \tau$ with $\mathrm{Op}(\sigma = \tau) \subseteq \Gamma$ generating a theory of T-degree δ. Theorem 3. Let E be the set of all equations with operation symbols in Γ which individually generate decidable theories. Then E is a maximal Σ_3-set in the Kleene-Mostowski hierarchy. The proof is based on Boone-Rogers, Math. Scand. 19(1966), 185-192. (Received September 18, 1972.)

72T-E109. DANIEL M. ANDLER, University of California, Berkeley, California 94720. Models of uncountable theories categorical in power.

Let T be a first-order theory categorical in some $\lambda > |T|$ and thus, by Shelah's theorem, in all $\mu > |T|$ ("Categoricity of uncountable theories," Proc. Tarski Sympos. (to appear)). The terminology is drawn from his paper. Here the models in $|T|$ are studied. One develops a dimension theory for weakly minimal formulas (wmf) as Marsh did for strongly minimal formulas (Baldwin and Lachlan, J. Symbolic Logic 36(1971), 79-96). If $\mathfrak{A}$ is a model and θ a wmf, the subset of $\mathfrak{A}$ defined by θ is given by its spectrum of dimensions. A weakly minimal (wm) theory is defined as having $v = v$ as wmf partitioned by φ. If T is wm and categorical in $\lambda > |T|$, models of T with an infinite indiscernible set are characterized by a single infinite dimension. These models form a chain $\langle \mathfrak{A}_\beta ; \omega \leq \beta \rangle$ where (i) $\mathfrak{A}_{\beta+1}$ is a minimal prime extension of $\mathfrak{A}_\beta$, (ii) $\mathfrak{A}_\beta$ has dimension $|\beta|$, is maximally $|\beta|$-saturated, and nonhomogeneous if $|\beta| < |T|$. Thus there are just $|\alpha| + 1$ infinite-dimensional models in $|T| = \aleph_\alpha$ and one model in each $\mu > |T|$ (an instance of Shelah's theorem). A principle verified by known examples yields a full description of the models of T. There are $|\alpha| + \aleph_0$ of them in $|T|$, a special case of a conjecture of Shelah. Theories in a language $L_0 \cup C$, $|C| > |T|$, C a set of constants, are studied, as known examples have this form. Some generalizations are proved and open problems discussed. (Received September 21, 1972.)

72T-E110. F. GONZALEZ ASENJO and J. TAMBURINO, University of Pittsburgh, Pittsburgh, Pennsylvania 15213. Logic of antinomies.

Antinomies are formulas which are both true and false. Using essentially the same truth tables introduced in an earlier paper (see Abstract 615-6, these *Notices* 11(1964), 663) first a propositional calculus is constructed, then a predicate calculus. Each one includes antinomies and is not trivially inconsistent (not all formulas are provable); furthermore, each is complete in the sense that the theorems are exactly those formulas which are either true or antinomic. As an application of these calculi, an antinomic set theory is presented in which Russell's paradox becomes a theorem, but its antinomic character is not contagious. (Received September 25, 1972.)

72T-E111. WITHDRAWN.

72T-E112. STEVEN R. GIVANT, University of California, Berkeley, California 94720. A representation theorem for universal classes of algebras in which all members are free. Preliminary report.

By extending results of Urbanik (Fund. Math.(1963)) and applying results implicit in the author's Abstract 72T-E98, these *Notices* 19(1972), A-717, a complete description is obtained, up to p.e. (polynomial equivalence), of universal classes of algebras K in which all algebras are K-free. We restrict ourselves here to the case when K is a variety. Let $\mathfrak{A} = \langle A, +, M_\lambda \rangle_{\lambda \in F}$ be a vector space over the s.f. (skew field) $\mathfrak{F}$ with universe F (+ denotes group addition and M_λ (unary) scalar multiplication by λ). Let J be the set of finitary operations Q on A such that for some $n < \omega$ and $\lambda \in {}^nF$ with $\sum_{i<n} \lambda_i = 1$ (in $\mathfrak{F}$) we have $Q(x_0, \ldots, x_{n-1}) = \sum_{i<n} M_{\lambda_i} x_i$ for all $x \in {}^nA$. Then $\langle A, Q \rangle_{Q \in J}$ is called the affine algebra derived from $\mathfrak{A}$. Let $V(\mathfrak{F})$ be the variety of vector spaces over an s.f. $\mathfrak{F}$ and $A(\mathfrak{F})$ the class of affine algebras derived from $V(\mathfrak{F})$. Lemma. For every s.f. $\mathfrak{F}$, $A(\mathfrak{F})$ is a variety; if $2 < |F| < \omega$, $A(\mathfrak{F})$ is p.e. to a variety with one binary operation.

Theorem. Let K be a nondegenerate variety all of whose members are K-free. Then K is p.e. to one of the following varieties: (a) $V(\mathfrak{F})$ for some s.f. $\mathfrak{F}$; (b) $A(\mathfrak{F})$ for some s.f. $\mathfrak{F}$; (c) the unary algebras $\langle A, f \rangle$ where f is constant on A; (d) the unary algebras $\langle A, f \rangle$ where $fx = x$ for every $x \in A$. (Received September 26, 1972.)

72T-E113. CHARLES FONTAINE MARTIN, University of California, Berkeley, California 94720. Term order and identities with addition, multiplication, exponentiation, and positive integer constants. Preliminary report.

For notation, see Abstract 698-E1, these *Notices* 19(1972). Conjecture (C1) (op. cit.) can be reformulated to apply to the structure $\mathfrak{P}' = \langle \omega, +, \cdot, \uparrow, 1 \rangle$ with a distinguished element 1. Consider (E9) $x \cdot 1 = x$, (E10) $x^1 = x$, (E11) $1^x = 1$. Conjecture (C3) (Tarski): $\{E1, \ldots, E11\} \vdash Eq(\mathfrak{P}')$. (C3) is still open. Let T_x be the set of terms occurring in $Eq(\mathfrak{P}')$ which have a single variable, x. Let F be the set of functions $f : \omega \to \omega$

represented by terms in T_x. For f, g $\in$ F, define f $\succ$ g iff $(\exists m)(\forall n)(n > m \rightarrow f(n) > g(n))$. Skolem in Norske Vid. Selsk. Forh. (Trondheim) 29(1956) formulates a conjecture seemingly related to (C3): (C4) F is well ordered and has order type ϵ_0. Skolem (op. cit.) showed that (C4) holds for a proper subset F' of F. Ehrenfeucht (unpublished result) showed that F is well ordered by $\succ$, but the order type is still unknown. The following two results of the author partially confirm (C4) and (C3); the first of them improves Skolem's result. Let U be the set of all terms τ occurring in Eq($\mathfrak{P}'$) such that if α^β is a subterm of τ, then every occurrence of + in α must be inside a constant subterm. Theorem 1. The set of functions represented by terms of $T_x \cap U$ is well ordered by $\succ$ with order type ϵ_0. Theorem 2. For any terms τ, σ in U, $\tau = \sigma$ holds in $\mathfrak{P}'$ iff $\{E1, \ldots, E11\} \vdash \{\tau = \sigma\}$. (Received September 27, 1972.)

Statistics and Probability

*72T-F13. LANE YODER, Purdue University, West Lafayette, Indiana 47907. The Hausdorff dimensions of the graph and image of n-parameter Brownian motion in m-space.

The paper extends results of Lévy and Taylor from one parameter Brownian motion to the multiparameter case. The following theorem is proved. For n-parameter Brownian motion in m-space, the Hausdorff dimensions of the graph and image are a.s. $\min\{2n, n+m/2\}$ and $\min\{2n, m\}$ respectively. (Received September 5, 1972.)

Topology

*72T-G181. M. K. SINGAL and SUNDER LAL, Institute of Advanced Studies, Meerut University, Meerut, U.P., India. Biquasi-proximity spaces and compactification of a pairwise proximity space.

A biquasi-proximity space (X, δ_1, δ_2) is a nonempty set X endowed with two quasi-proximities δ_1 and δ_2. Like quasi-proximity spaces which are equivalent to topological spaces, biquasi-proximity spaces are equivalent to bitopological spaces. In this note we obtain characterizations of pairwise T_0, pairwise T_1, pairwise R_0, pairwise R_1, pairwise T_2, pairwise Urysohn, and pairwise regular spaces, in terms of the two quasi-proximities without explicit reference to the induced topologies. In case the two quasi-proximities are identical these characterizations reduce to the corresponding results in quasi-proximity spaces [Metzer, Kyungpook Math. J. 11(1971), 123-138]. Next using Swart's definition of compactness in bitopological space [Indag. Math. 33(1971), 135-145] we construct a compactification of a quasi-proximity space (X, δ, δ^{-1}), δ^{-1} being the conjugate quasi-proximity of δ. This compactification is similar and reduces, if δ is a proximity, to the Smirnov compactification of a proximity space. (Received August 16, 1972.) (Authors introduced by Dr. Shashi Prabha Arya.)

*72T-G182. H. E. WHITE, JR., 251 North Blackburn Road, Rt. #5, Athens, Ohio 45701. On Blumberg's theorem.

A pseudo-base for a topological space X is a family $\mathcal{P}$ of open sets such that every nonempty open subset of X contains a nonempty element of $\mathcal{P}$. Proposition 1. If the Baire space X has a σ-disjoint pseudo-base, then every real valued function defined on X is continuous on a dense subset. Corollary. If the quasi-regular Baire space X has a dense, metrizable, subspace, then every real valued function defined on X is continuous on a dense subset. Proposition 2 [CH]. Suppose the locally compact, Hausdorff space X has the property that every nonempty G_δ subset has nonempty interior. Then every real valued function defined on X is continuous on a dense subset. (Received August 30, 1972.)

*72T-G183. WILLIAM E. HAVER, University of Tennessee, Knoxville, Tennessee 37916. Closure of the space of homeomorphisms on a manifold.

Let $\overline{H(M)}$ denote the space of all mappings of the compact manifold M onto itself which can be approximated arbitrarily closely by homeomorphisms on M. $\overline{H_\partial(M)}$ denote the elements of $\overline{H(M)}$ that equal the identity on the boundary of M. We prove the following: (1) $\overline{H(M)}$ is homogeneous. (2) $\overline{H(M)}$ is weakly locally contractible. (3) Let B^2 be the 2-ball. Given $\epsilon > 0$, there is a mapping $\varphi : \overline{H_\partial(B^2)} \times I \to \overline{H_\partial(B^2)}$ such that for each $f \in \overline{H_\partial(B^2)}$, $\varphi(f, 0) = f$, diam $\varphi(f, I) < \epsilon$, $\varphi(f, t)$ is a homeomorphism for each $t \in (0, 1]$. (4) $\overline{H_\partial(B^2)}$ is an ANR. (5) If M^2 is a compact 2-manifold, $\overline{H_\partial(M^2)}$ is locally contractible. (Received September 5, 1972.)

72T-G184. KOK-KEONG TAN, Dalhousie University, Halifax, Nova Scotia, Canada and TSU-TEH HSIEH, Carleton University, Ottawa, Ontario, Canada. Metrization and contractive mappings. Preliminary report.

Let (X, τ) be a T_2-space whose topology τ is generated by a family $\mathcal{D}$ of pseudometrics on X. A map $f : X \to X$ (1) is uniformly strictly contractive (u.s.c.) w.r.t. $\mathcal{D}$ iff $\exists$ C with $0 \leq C < 1$ such that $\forall\, d \in \mathcal{D}$, $d(f(x), f(y)) \leq Cd(x, y)$, $\forall\, x, y \in X$; (2) has strong diminishing orbital diameter (s.d.o.d.) w.r.t. $\mathcal{D}$ iff $\forall\, d \in \mathcal{D}$, $\forall\, x \in X$, $d(x, f(x)) > 0 \Rightarrow \lim_{N\to\infty} d(\{f^{N+k}(x)\}_{k=0}^{\infty}) < d(\{f^K(x)\}_{K=0}^{\infty})$. Theorem 1. Suppose $f : X \to X$ is u.s.c. w.r.t. $\mathcal{D}$. If (X, τ) is second countable and $d(X) < \infty$, $\forall\, d \in \mathcal{D}$, then there is a metric ρ on X generating τ such that f is strictly contractive w.r.t. $\{\rho\}$. Theorem 2. Suppose $f : X \to X$ has s.d.o.d. (respectively is nonexpansive, contractive, isometric, or asymptotically regular) w.r.t. $\mathcal{D}$. If (X, τ) is second countable, then there is a metric ρ on X generating τ such that f has diminishing orbital diameter (respectively is nonexpansive, contractive, isometric, or asymptotically regular) w.r.t. $\{\rho\}$. (Received September 11, 1972.)

*72T-G185. STEPHEN B. SEIDMAN, George Mason University, Fairfax, Virginia 22030. Cofibrations in function spaces.

Let $p : E \to B$ be a continuous surjection, where E is compact. If X is a metric space, let X^E have the metric induced from X, and let $p_X : X^B \to X^E$ be the induced map. X^B may be regarded as the subset of X^E consisting of those maps constant on all fibers of p. If $\epsilon > 0$, we define $N(E, X, \epsilon) \subseteq X^E$ to be $\{f \mid \sup_{b\in B}\{\text{diam}\,(f(p^{-1}(b)))\} < \epsilon\}$. X is p-admissible if, given $\epsilon > 0$, there exist $\delta > 0$ and a map Φ_X: $N(E, X, \delta) \times I \to N(E, X, \epsilon)$, such that (1) $\Phi_X(f, 0) = f$ for all $f \in N(E, X, \delta)$; (2) $\Phi_X(f, 1) \in X^B$ for all $f \in N(E, X, \delta)$; (3) if $f_0(p^{-1}(b_0)) = \{x_0\}$ for some $f_0 \in N(E, X, \delta)$, $b_0 \in B$, and $x_0 \in X$, then $(\Phi_X(f_0, t))\,(p^{-1}(b_0)) = \{x_0\}$ for all $t \in I$. It follows easily that if X is p-admissible, $P_X : X^B \to X^E$ is a cofibration. Theorem 1. If X_1 and X_2 are p-admissible, $X_1 \times X_2$ is p-admissible. Theorem 2. If p is a fibration and B is weakly locally contractible, then B^n, R^n, and S^n are p-admissible for all n. If p is locally trivial, the result holds without restriction on B. (Received September 18, 1972.)

*72T-G186. EARL PERRY, West Georgia College, Carrollton, Georgia 30117. A note on hereditarily duodic continua.

In an earlier abstract (Abstract 72T-G112, these *Notices* 19(1972), A-546), the author defined the terms duod and hereditarily duodic continuum. Example. Hereditarily duodic continua A and B whose intersection contains a point which is a cut point of each of them, but which is not a cut point of $A \cup B$. Theorem 1. Suppose each of A and B is a hereditarily duodic continuum, $A \cap B$ is a continuum, each cut point of A lies in $A \cap B$, $A' = A - (A \cap B)$ is not connected, but A'^{-} is connected. Then each cut point of A'^{-} lies in $\beta_{A'}$ and $\beta_{A'}$ is a continuum. Theorem 2. Suppose each of A and B is a hereditarily duodic continuum, $A \not\subset B$, $B \not\subset A$, $A \cap B$ is a continuum, each cut point of A lies in $A \cap B$, and each cut point of B lies in $A \cap B$. Then each of $A' = A - (A \cap B)$ and $B' = B - (A \cap B)$ must have infinitely many components. (Received September 18, 1972.)

*72T-G187. JAMES G. WILLIAMS, Bowling Green State University, Bowling Green, Ohio 43403. Regarding arcwise accessibility in the plane.

Suppose S is a bounded relatively closed subset of the upper half-plane H, and that F is the set of all points on the x-axis which can be reached from the line $y = 1$ by an arc lying in S. Question. Which sets F arise in this way? It is (with mild restirctions on S) necessary and sufficient that F be an $\mathfrak{F}_{\sigma\delta}$ set. The corresponding problem where S is open is also briefly discussed. The results of the paper prove sharpness for a theorem by J. Gresser on almost arcwise accessibility, and also demonstrate some possibilities for sectioning bounded sets. (Received September 25, 1972.)

72T-G188. DALLAS EUGENE WEBSTER, State University of New York at Buffalo, Amherst, New York 14226. A fake counterexample to a conjecture of Zeeman.

Let K be the 2-complex formed by sewing two 2-cells to the wedge of two circles a and b by the words a^2b^3 and a^3b^4 respectively. Then $K \times I$ collapses and hence K is not a counterexample to Zeeman's conjecture on contractible 2-complexes, contrary to the beliefs of many. Note that a more general result has been obtained by Lickorish ($a^pb^q \cdot 2$ and a^rb^s with $|ps - qr| = 1$) using much nicer arguments. (Received September 25, 1972.)

*72T-G189. B. J. BALL, University of Georgia, Athens, Georgia 30601. Shapes of saturated subsets of compacta.

If X is a compact metric space, the decomposition space X/C_X, where C_X is the collection of all components of X, will be denoted by $\square X$, and $p_X : X \to \square X$ will denote the natural projection map. Generalizing a result of K. Borsuk, it is shown that if X and Y are compacta having the same shape, then there is a homeomorphism $\Lambda : \square X \to \square Y$ such that for every compact set $A \subset \square X$, the sets $p_X^{-1}(A)$ and $p_Y^{-1}(\Lambda(A))$ have the same shape. The requirement that A be closed in $\square X$ is essential if Borsuk's definition of (strong) shape for noncompact spaces is used, but in order that $p_X^{-1}(A)$ and $p_Y^{-1}(\Lambda(A))$ should have the same shape in the weaker sense of R. H. Fox, it is sufficient that A be a locally compact subset of $\square X$. (Received September 28, 1972.)

72T-G190. JAMES W. CARTER, 653 Orange Grove, South Pasadena, California 91030. Banach, Hilbert spaces and manifolds modeled thereon.

Every Hilbert space is separable, the only cases being (up to isomorphism) E^k and ℓ^2. Let M be a connected, T_2 (differentiable) manifold; if $\overline{\overline{M}} > c$, then M admits no Riemannian metric. The almost periodic functions with exponent 2 in the sense of Besicovitch have orthogonal dimension $\aleph_0$. Let M_S be a manifold modeled on the Banach space S. Let H be a Hilbert space over the field F (either R or C), and let O(S) be the collection of all open sets of S. Then $\overline{\overline{O(H)}} = \overline{\overline{O(M_H)}} = \overline{\overline{C(H_1, H_2)}} = \overline{\overline{L(H_1, H_2)}} = c$ if M is paracompact; further, M_H is imbeddable within H. To complete the generalization of the Whitney imbedding theorem, M_B is imbeddable in the Banach space $\Pi F \oplus \Pi B$, where (in particular) the products are finite if M is compact. (Conjecture. $\overline{\overline{O(M_B)}} = \overline{\overline{M}}\, h$, where h is the algebraic dimension of B.) (Received September 12, 1972.)

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Algebra & Theory of Numbers

698-A1. KENNETH B. GROSS, University of Southern California, Los Angeles, California 90007. On strong starters.

Let Ω be an abelian group of finite order $2n+1$. A strong starter for Ω is a set of unordered pairs of elements of Ω, $X = \{(x_i, y_i) | i = 1, 2, \ldots, n\}$, such that (i) the elements $x_1, y_1, x_2, y_2, \ldots, x_n, y_n$ are all the nonzero elements of Ω, (ii) the set of differences $\{\pm(x_i - y_i) | i = 1, 2, \ldots, n\}$ contains each of the nonzero elements of Ω, and (iii) the sums $(x_i + y_i)$, $i = 1, 2, \ldots, n$, are nonzero elements of Ω and are distinct. Given strong starters for abelian groups G and H such that $3 \nmid |H|$, a construction is given for a strong starter for the group $G \oplus H$. It is noted that in the same manner one can construct a starter with an adder for the group $G \oplus H$ given arbitrary starters with adders for the groups G and H such that $3 \nmid |H|$. A connection is developed between strong starters and the patterned starters of Stanton and Mullin ("Construction of room squares", Ann. Math. Statist. 39(1968), 1540-1548). This connection is used to show the existence of strong starters for the cyclic groups of all odd orders less than or equal to 55 and greater than 9. Some applications of strong starter theory are developed. (Received August 17, 1972.)

*698-A2. YOUNG KOAN KWON, University of Texas, Austin, Texas 78712. Bounded harmonic but no Dirichlet-finite harmonic.

The purpose of the present note is to announce that for each $n \geqq 3$ there exists a Riemannian n-manifold, which carries nonconstant bounded harmonic functions but no nonconstant Dirichlet-finite harmonic functions. (Received August 17, 1972.)

*698-A3. DAVID G. MEAD, University of California, Davis, California 95616. The equation of Ramanujan-Nagel and $[y^2]$.

In this paper we give a completely arithmetic test to determine whether or not a power product belongs to the differential ideal $[y^2]$. (The test can easily be generalized to $[y^n]$ and can be carried out on a computer.) When trying to obtain a necessary condition for a power product, the elements of whose weight sequences $\leqq 3$ ("Differential ideals", Proc. Amer. Math. Soc. 6(1955), 420-432), to belong to $[y^2]$, one is led to the question of what Mersenne numbers are triangular numbers. This is equivalent to a problem stated by S. Ramanujan in 1913 and first solved by Nagel in 1948. Several other published solutions have appeared since then, and in one of these the answer was needed to settle a question concerning error-correcting codes.

(Received September 21, 1972.)

*698-A4. SUKHAMAY KUNDU, IBM Corporation, T. J. Watson Research Center, Yorktown Heights, New York 10598. A factorization theorem for a certain class of graphs.

Let G be a graph on vertices $\{v_1, v_2, \ldots, v_n\}$ and $\langle k_i \rangle = \langle k_1, k_2, \ldots, k_n \rangle$ be a sequence of natural numbers. A factor of G is a subset of the lines of G of which exactly k_i are incident with the vertex v_i. Let $\langle G \rangle$ denote the degree sequence of G and $\langle k_i \rangle + \langle k_i' \rangle$ denote the sequence $\langle k_i + k_i' \rangle$. A class F of graphs has been identified for which the following theorem holds. Theorem 1. Let G be a graph whose complement graph G^c belongs to F. Then G contains a $\langle k_i \rangle$ factor if and only if the sequence $\langle H \rangle + \langle k_i \rangle$ is graphical for every line induced subgraph $H \subseteq G^c$. The following properties show that class F is indeed very large: (1) F contains all planar graphs, square graphs of trees, and all graphs of girth $\geqq 5$; (2) the class F is hereditary, i.e., a point induced subgraph of a graph F is also in F; (3) the line graphs and the total graphs belong to F; (4) F is closed under the operation of cartesian product (in fact, $G \times H \in F$ if and only if $G, H \in F$); and finally (5) a disconnected graph belongs to F if each connected component is in F. From (2) and (3) it follows that a graph $G \in F$ if and only if its total graph belongs to F. $K_{3,3}$ is the smallest graph not in F and Theorem 1 is false in that case.

(Received September 28, 1972.)

*698-A5. STEPHEN J. McADAM, University of Texas, Austin, Texas 78712. Saturated chains in Noetherian rings.

Let R be a commutative Noetherian domain with identity and let x be an indeterminate. If P is a prime in R and Q is a prime in R[x] such that $Q \cap R = P$ but $Q \neq PR[x]$, we will call Q an upper to P. Proposition. If (R,M) is a local domain and $n > 1$ then the following are equivalent: (i) There is a rank 1 prime of R with corank $n - 1$. (ii) There are infinitely many rank 1 primes of R with corank $n - 1$. (iii) There is an upper K to 0 in R[x] such that $K \subset MR[x]$ and $\mathrm{rank}(MR[x]/K) = n - 1$. (iv) There is an upper K to 0 and an upper Q to M such that $K \subset Q$ and $\mathrm{rank}(Q/K) = n$. We also prove that if P is a rank n prime in a Noetherian ring, then there are only finitely many primes Q satisfying $P \subset Q$, $\mathrm{rank}(Q/P) = 1$ and $\mathrm{rank}\, Q > n + 1$. We deduce several consequences of these two results. For instance if P is a prime in a Noetherian domain R and if in R[x] there is a saturated chain of primes of length n from 0 to PR[x], then in R there is a saturated chain of primes of length n from 0 to P. (Received September 29, 1972.)

*698-A6. NICK H. VAUGHAN, North Texas State University, Denton, Texas 76203. A note on overrings of a domain. Preliminary report.

Let D denote an integral domain with $1 \neq 0$ and quotient field K. An overring of D is a ring J such that $D \subset J \subset K$. Theorem 1. If $\mathfrak{F}$ is a finite family of Noetherian overrings J of D such that for each proper ideal A of D there exists $J \in \mathfrak{F}$ such that $AJ \cap D = A$, then D is a Noetherian domain. Theorem 2. If $\mathfrak{F}$ is a finite family of Noetherian integrally closed overrings J of D such that for each proper ideal A of D there exists $J \in \mathfrak{F}$ such that $AJ \cap D = A$, then D is a Noetherian integrally closed domain. (Received October 4, 1972.)

*698-A7. STEPHEN J. PIERCE, University of Toronto, Toronto, Ontario, Canada. Number fields with no proper subfields.

Let F be a totally real number field of degree $r > 1$ over the rationals Q. Assume that the normal closure K of F has a solvable Galois group G over Q. Then $[F:Q] = p^n$, p a prime and $n \geqq 2$. Moreover, G is a Frobenius group with elementary abelian kernel of order p^n. Write $F = Q(\lambda_1)$, where λ_1 is a unit of F. Let $\lambda_1, \ldots, \lambda_r$ be the conjugates of λ_1. If the Frobenius complement of G has prime order q, then the multiplicative group generated by $\lambda_1, \ldots, \lambda_{r-1}$ is free abelian if and only if $p^n - 1 = q(p-1)$. (Received October 5, 1972.)

*698-A8. TAKAYUKI TAMURA, University of California, Davis, California 95616. Notes on N-semigroups. Preliminary report.

A commutative cancellative archimedean semigroup without idempotent is called an N-semigroup. Theorem. Let G be an abelian group, R^+ the set of all positive real numbers. Let $\varphi: G \to R^+$ be a function satisfying: (1) $\varphi(\epsilon) = 1$ where ϵ is the identity element of G. (2) $\varphi(\alpha) + \varphi(\beta) - \varphi(\alpha\beta)$ is a nonnegative integer for all $\alpha, \beta \in G$. (3) For each $\alpha \in G$ there is a positive integer m such that $m\varphi(\alpha) - \varphi(\alpha^m) > 0$. Let $S = \{(x+\varphi(\alpha), \alpha) : \alpha \in G$, x nonnegative integers$\}$. Define an operation on S by $(x+\varphi(\alpha),\alpha) \cdot (y+\varphi(\beta),\beta) = (x+y+\varphi(\alpha) + \varphi(\beta), \alpha\beta)$. Then S is an N-semigroup. Every N-semigroup can be obtained in this manner. S is determined by G and φ. Note that S is a subdirect product of a positive real additive semigroup and an abelian group, but all subdirect products of a positive real additive semigroup and an abelian group need not be an N-semigroup. (Received October 5, 1972.)

698-A9. ICHIRO SATAKE, University of California, Berkeley, California 94720. On the arithmetic of tube domains.

• Let U be a real Euclidean space, M a lattice in U, and Ω a self-dual open convex cone in U. We assume Ω to be rational; that means its automorphism group $G = \mathrm{Aut}(\Omega)$ is an algebraic group defined over Q with respect to the Q-structure of U defined by M. Let Γ be an arithmetic subgroup of G leaving M fixed. We consider the tube domain $U + \Omega i$ in U_C and its quotient space $\tilde{\Gamma}\backslash U + \Omega i$, where $\tilde{\Gamma} = \Gamma \cdot M$ is a semidirect product viewed as a discontinuous affine transformation group of $U + \Omega i$. Our main purpose is to construct in a canonical manner a certain (reducible) projective variety, which may be viewed as a "monoidal transform" of the point at infinity of the quotient space. This kind of projective variety has appeared in the desingularization process of Igusa (Siegel modular case) and Hirzebruch (Hilbert modular case), and plays an important role in the general theory of desingularization of the arithmetic quotients of symmetric domains. (Received October 5, 1972.)

*698-A10. AUDREY A. TERRAS, University of California at San Diego, La Jolla, California 92037. Bessel series expansions of the Epstein zeta function and the functional equation.

For the Epstein zeta function of an n-ary positive definite quadratic form, $n - 1$ generalizations of the Selberg-Chowla formula (J. Reine Angew. Math. 227(1967), 86-110) are obtained. Further, it is shown that these $n - 1$ formulas suffice to prove the functional equation of the Epstein zeta function by mathematical induction. Finally some generalizations of Kronecker's first limit formula (C. L. Siegel, Lectures in Advanced Analytic Number Theory, Tata Institute, Bombay, 1961, p. 17) are found. (Received October 5, 1972.)

*698-B1. NEIL B. HINDMAN, California State University, Los Angeles, California 90032. Finite sums of sequences within partitions of N.

Graham and Rothschild have asked [Trans. Amer. Math. Soc. 159(1971), 291] if, whenever $N = A_1 \cup A_2$, there exists a sequence $\langle x_n \rangle_{n=1}^{\infty}$ and an i such that $\sum_{n \in F} x_n \in A_i$ whenever F is a nonempty finite subset of N. Theorem. If $N = \bigcup_{i=1}^{t} A_i$ then there exists i and a sequence $\langle x_n \rangle_{n=1}^{\infty}$ such that $\sum_{n \in F} x_n \in A_i$ whenever F is a nonempty finite subset of N. Corollary (Continuum Hypothesis). There exists an ultrafilter p on N such that, for every A in p, $\{x : A - x \in p\} \in p$ (where $A - x = \{y \in N : y + x \in A\}$). The corollary answers an (unpublished) question of F. Galvin. (Received September 21, 1972.)

698-B2. JERRY F. KUZANEK, University of Redlands, Redlands, California 92373. Existence and uniqueness of solutions to a fourth-order nonlinear eigenvalue problem.

The Euler equations for an isoperimetric eigenvalue problem consist of a singular Sturm-Liouville equation coupled to a second-order nonlinear ordinary differential equation. An existence proof for solutions to the isoperimetric problem is given, thereby proving the existence of solutions to the Euler equations. Next, it is proved that for each eigenvalue any solution to these equations provides a weak relative minimum of the functional for the isoperimetric problem. The proof that this solution is unique follows from the strict convexity of this functional with respect to all admissible functions. (Received September 21, 1972.)

698-B3. HENRY CHENG, New Mexico State University, Las Cruces, New Mexico 88001. Bishop's constructive intermediate value theorem.

Let $f : [a,b] \to R$ be a real-valued continuous function on a proper $(a < b)$ closed interval $[a,b]$ such that $\alpha \equiv \inf\{f(x) : x \in [a,b]\} < \beta \equiv \sup\{f(x) : x \in [a,b]\}$. Classically, the intermediate value theorem asserts that for each $\gamma \in [\alpha, \beta]$ there exists a real number $c \in [a,b]$ such that $f(c) = \gamma$. The proof depends essentially on the principle of the excluded middle and hence it is not constructive. Moreover, there is a counterexample to the classical theorem in the sense of Brouwer that excludes the possibility of ever obtaining a constructive proof. Therefore, we are led to state and prove Bishop's constructive substitute: For all except countably many real numbers γ in $[\alpha, \beta]$ the set $\{x \in [a,b] : f(x) = \gamma\}$ is compact. In particular we observe that a compact set is nonvoid. The proof will depend on two results, the first one being an extension of Bishop's basic result on compact spaces, as stated in his book "Foundations of constructive analysis," McGraw-Hill, New York, 1967. (Received September 22, 1972.)

698-B4. SU-SHING CHEN, University of Florida, Gainesville, Florida 32601. B-convexity of Siegel domains of the second kind.

A complex manifold M is said to be B-convex if the subset $\hat{K}_B = \{x \in M \mid |f(x)| \leqq \|f\|_K$ for all $f \in B(M)\}$ (B(M) = the algebra of bounded holomorphic functions on M) is compact provided K is a compact subset of M. A Siegel domain D of the second kind (not necessarily affine homogeneous) is shown to be complete with respect to the Carathéodory distance. Thus, D is B-convex, hence is a domain of holomorphy. Consequently, a bounded homogeneous domain in $\mathbb{C}^n$ is B-convex, hence is a domain of holomorphy. B-convexity is much stronger than holomorphy. For instance, the domain $\{(z_1, z_2) \in C^2 \mid |z_1| < |z_2| < 1\}$ is a domain of holomorphy but is not B-convex. (Received September 18, 1972.)

698-B5. DENNIS S. HADA and LEO SARIO, University of California, Los Angeles, California 90024 and CECILIA Y. WANG, Arizona State University, Tempe, Arizona 85282 and University of California, Los Angeles, California 90024. N-manifolds carrying bounded but no Dirichlet finite harmonic functions.

Central to the classification theory of Riemann surface is the strictness of the inclusion relations $O_G < O_{HP} < O_{HB} < O_{HD}$ where O_G denotes the class of parabolic surfaces, and O_{HP}, O_{HB}, O_{HD} are the classes of surfaces void of nonconstant harmonic functions which are positive, bounded, or Dirichlet finite, respectively. For Riemannian manifolds only the relations $O_G < O_{HP} < O_{HB} \subset O_{HD}$ have been known. An example is given which confirms the remaining strict inclusion $O_{HB} < O_{HD}$ for all dimensions. In the process complete characterizations of the Poincaré N-ball in O_{HB} and O_{HD} shall also be obtained. (Received October 2, 1972.)

*698-B6. LEO SARIO, University of California, Los Angeles, California 90024 and CECILIA Y. WANG, Arizona State University, Tempe, Arizona 85282 and University of California, Los Angeles, California 90024. Parabolicity and existence of bounded biharmonic functions.

The existence of bounded biharmonic functions has exhibited interesting dependence on the dimension of the base manifold. Typically, such functions exist on the punctured Euclidean N-space $E^N : 0 < |x| < \infty$ for $N = 2, 3$, but not for any $N \geqq 4$ (Sario and Wang, "Generators of the space of bounded biharmonic functions," Math. Z. 127(1972), 273-280). The present paper deals with the problem: Is there any relation between the parabolicity of a manifold and the existence of bounded biharmonic functions, and does the dimension on the manifold have any bearing on the question? Denote by H^2B the class of bounded biharmonic functions. In contrast with the case of bounded harmonic functions, which are known not to exist on any parabolic manifold, it is shown that there exist parabolic manifolds of any dimension which carry H^2B functions. More generally, the totality of Riemannian manifolds for any N is decomposed into disjoint nonempty classes $O_G^N \cap \tilde{O}_{H^2B}^N$, $\tilde{O}_G^N \cap \tilde{O}_{H^2B}^N$, $\tilde{O}_G^N \cap O_{H^2B}^N$, and $O_G^N \cap O_{H^2B}^N$, where $\tilde{O}$ stands for the complement of O. (Received October 3, 1972.)

*698-B7. NORMAN MIRSKY and LEO SARIO, University of California, Los Angeles, California 90024 and CECILIA Y. WANG, Arizona State University, Tempe, Arizona 85282 and University of California, Los Angeles, California 90024. Bounded polyharmonic functions and the dimension of the manifold.

Let H^kB be the class of bounded polyharmonic functions of order k. Denote by $O_{H^kB}^N$ the class of Riemannian N-manifolds which do not carry functions u satisfying $u \in H^kB$, $u \notin H^hB$ for $h < k$. Consider the punctured space $E_\alpha^N : 0 < |x| < \infty$, $x = (x^1, \ldots, x^N)$ with the metric $ds = r^\alpha |dx|$, α a constant. Theorem 1. There exist no H^kB functions on E_α^N for any α if $N \geqq 2k$. For $N < 2k$ there are infinitely many α for which there exist functions $u \in H^kB$, and for these α the generators of the space H^kB are surface spherical harmonics. Theorem 2. $E_\alpha^N \notin O_{H^kB}^N$ implies $E_\alpha^N \notin O_{H^hB}^N$ for all $h > k$ and all N if $\alpha = 0$. There exist E_α^N for which this is no longer true. (Received October 3, 1972.)

698-B8. UPADHYAYULA V. SATYANARAYANA, California State University, Northridge, California 91324. Distribution of supports of representing measures for H.

Let $\mathcal{B}_1$ denote the fiber above 1 in the maximal ideal space $\mathcal{B}$ of the Banach algebra H^∞ of bounded analytic functions on the unit disc D. Several results on the supports of the representing measures of homomorphisms in $\mathcal{B}_1$ are obtained. The following are typical: Theorem. If Γ is any curve in D to 1 above a Stolz ray at 1 and φ a homomorphism in $\mathcal{B}_1$ in the closure of a net below Γ which is not lower tangential, then there exists a $\psi \in \bar{\Gamma} \setminus \Gamma$ $(\subset \mathcal{B}_1)$ such that the support of ψ (of the representing measure of ψ) is a subset of the support of φ. Moreover, if μ_h denotes the (unique) measure representing h and supported on the Silov boundary of H^∞, the above ψ can be chosen in such a way that if K is any closed subset of the upper part of the support of φ with $\mu_\varphi(K) = k$ then $\mu_\psi(K) \geqq k$. Theorem. Let φ be a radial homomorphism and Γ any curve in D to 1. Then there exists a $\psi \in \overline{\Gamma}\backslash\Gamma$ such that the support of ψ is a subset of the support of φ. Also the supports of certain "minimal" radial homomorphisms are characterized completely in terms of certain measurable subsets of the unit circle C. Basically the paper is devoted to an intrinsic study of supports and several results obtained are of the above nature. (Received October 4, 1972.)

698-B9. DONALD E. MYERS, University of Arizona, Tucson, Arizona 85721. B(S) as an operator subalgebra. Preliminary report.

Let $S = \{z \mid -a < R(z) < a\}$, B(S) the bounded analytic functions on S with the sup-norm topology. $H^2(S) = \{f \mid f$ and in S, $\sup_{-a<x<a}[\int_{-\infty}^{\infty} |f(x+iy)|^2 \, dy]^{1/2} < \infty\}$. B(S) is imbedded as a commutative subalgebra of $L(H^2, H^2)$ by $\varphi \overset{\eta}{\to} T_\varphi$, $(T_\varphi f)(z) = \varphi(z) f(z)$. $H^2(S) \not\subset B(S)$ and $B(S) \not\subseteq H^2(S)$ but $\eta(B(S) \cap H^2(S))$ is a closed ideal in $\eta(B(S))$. A number of results are obtained concerning the structure and topology of B(S), an operator algebra. (Received October 4, 1972.)

*698-B10. JIM M. CUSHING, University of Arizona, Tucson, Arizona 85721. Strong stability and perturbations of systems of Volterra integral equations. Preliminary report.

Consider the system of Volterra integral equations (N) $u(t) = \varphi(t) + \int_a^t [K(t,s)u(s) + f(t,s,u(s))] \, ds$, $t \geqq a \geqq t_0$, where $\varphi(t)$, $K(t,s)$ and $f(t,s,z)$ are continuous on $t \geqq t_0$, $t, s \geqq t_0$, and $\{t, s \geqq t_0\} \times \{z \in R^n; |z| < b\}$, $b > 0$, respectively. Here $f(t,s,0) \equiv 0$. We say: (1) (N) is strongly stable on a normed space N of functions φ if to $\epsilon > 0$ there exists a $\delta > 0$ (independent of $a \geqq t_0$) such that if $|u(t^*)| \leqq \delta$ for some $t^* \geqq t_0$ then $|u(t)| \leqq \epsilon$ for all $t \geqq t_0$; (2) (N) is uniformly adjointly stable on N if for $\epsilon > 0$ there exists a $\delta > 0$ (independent of $a \geqq t_0$) such that $|\varphi|_N \leqq \delta$ implies $|u(t)| \leqq \epsilon$ for $t \geqq t_0$. The linear problem $f \equiv 0$ is denoted (L). Theorem. If (L) is uniformly adjointly stable on R^n and if $|f(t,t,z)| \leqq \gamma_1(t)|z|$ and $|f_t(t,s,z)| \leqq \gamma_2(t,s)|z|$ with $\int_{t_0}^{+\infty} (\gamma_1(s) + \int_{t_0}^{s} \gamma_2(s,u) \, du) \, ds < +\infty$, then the strong stability of (L) on any space N implies the strong stability of (N) on N. Other perturbation results for these and other generalizations of the notion of strong stability for ordinary differential equations are possible. The proofs use a "fundamental solution" and "variation of constants formula" for linear systems (L). (Received October 4, 1972.)

*698-B11. DAVID H. WOOD, Naval Underwater Systems Center, New London Laboratory, New London, Connecticut 06375. Finding Riemann functions.

Most of the few known Riemann-Green functions for the equations $U_{xy} + H(x,y)U = 0$, determined by continuous coefficient functions $H(x,y)$, are found by "guessing" a function $g(x,y)$ and assuming that the Riemann function has the form $R = f(g(x,y))$. Substituting R into the equation for U yields an ordinary differential equation for the unknown function f. The guesswork is unnecessary because the only possible "guess" is easily found. Theorem. If R is a nonlinear function of some $g(x,y)$, then R is a function of $\iint H(x,y) \, dx \, dy$ which may be assumed to be of the form $X(x)Y(y)$ without losing generality. Now the candidate for $g(x,y)$ is known and the next result can be used. Theorem. R is a function of $g(x,y)$ if and only if $g_x(x,y) g_y(x,y)/H(x,y)$ is a function of $g(x,y)$. Finally, R is obtained from an O.D.E. Theorem. If $g_x(x,y)g_y(x,y)/H(x,y) = k(g(x,y))$, then $R = f(g(x,y))$ where f is uniquely determined by the ordinary differential equation $k(t)f''(t) + f'(t) + f(t) = 0$ and a single initial condition.

The handful of examples due to Riemann, Cohn and Daggit are included in the above results, as are many more. Theorem. If $X(x)$, $Y(y)$ and $f(t)$ have continuous second derivatives with $X(0) = Y(0) = 0$ and $f(0) = 1$, then $R = f(X(x-x_0)Y(y-y_0))$ for some continuous and uniquely determined $H(x,y)$. (Received October 5, 1972.)

*698-B12. JOHN M. BOWNDS and JIM M. CUSHING, University of Arizona, Tucson, Arizona 85721. A stability preservation result for small ∘ perturbed Volterra integral equations.

Consider the linear Volterra equations: (L) $v(x) = \varphi(x) = \int_a^x K(x,t)\,v(t)\,dt$ and (P) $u(x) = \varphi(x) + \int_a^x [K(x,t)\,u(t) + f(x,t,u(t))]\,dt$, $x \geqq a$. The authors have previously defined what is meant by a solution being respectively stable, uniformly stable, and asymptotically stable on a normed space of "initial functions" (J. Applicable Anal. (to appear)). It is well known from the theory of ordinary differential equations that some sort of strong uniformity assumption (such as uniform asymptotic stability) must be made in order for small ∘ perturbed equations to preserve certain stability characteristics from a given unperturbed, linear equation. Unfortunately, uniform asymptotic stability apparently does not have an obvious natural counterpart for Volterra integral equations. However, it is possible to prove the following Theorem. Let the null solution to (L) be stable on $N = \{\varphi \in C'[a,\infty) : \|\varphi\| = \sup[|\varphi(x)| + |\varphi'(x)|] < +\infty,\ x \geqq a\}$, and assume $|f(x,x,z)| = \circ(|z|)$ uniformly in $x \geqq a$ as $z \to 0$ and $(x-a)|f_x(x,t,z)| = \circ(|z|)$ uniformly in $a \leqq t \leqq x$ as $z \to 0$. Then stability, uniform stability, and asymptotic stability on any normed space are respectively preserved from (L) to (P). The proof uses representation theorems previously established by the authors in the above reference. (Received October 5, 1972.)

698-B13. MARC A. RIEFFEL, University of California, Berkeley, California 94720. Induced representations and Morita theorems for C-algebras.

• We will show how induced representations for C*-algebras can be defined, in analogy with the familiar change of rings operations, and we will indicate how Mackey's definition of induced representations for locally compact groups is a special case of this definition for C*-algebras. These considerations will lead to generalizations to noncommutative C*-algebras of the "C*-modules" which Kaplansky defined for commutative C*-algebras. These generalizations of C*-modules can be used to define functors between the categories of Hermitian modules (that is, the Hilbert spaces of ordinary continuous nondegenerate *-representations) over two C*- algebras. The category of Hermitian modules over a C*-algebra is a linear category which also has an involution on the collection of its morphisms (consisting of taking the adjoints of operators). In addition, the spaces of morphisms can be equipt with several topologies, notably the ultraweak operator topology. In terms of these structures it is natural to define normal linear *-functors, in analogy with normal maps between von Neumann algebras. We will indicate how every normal linear *-functor can be represented by means of a functor defined by the use of a generalized C*-module, thus providing an Eilenberg-Watts type theorem for these functors. Furthermore, if a linear *-functor establishes an equivalence between the categories of Hermitian modules over two C*-algebras, then it is represented by an appropriate analogue of "invertible" bimodules, thus providing a Morita type theorem for such an equivalence. Related matters will be discussed. (Received October 5, 1972.)

Applied Mathematics

*698-C1. V. A. PATEL, California State University at Humboldt, Arcata, California 95521. Time-dependent solutions of the viscous incompressible flow past a circular cylinder by the method of series truncation. Preliminary report.

Semianalytic solutions of the Navier-Stokes equations are calculated for two-dimensional, unsteady, incompressible flow past a circular cylinder. The governing partial differential equations of motion are reduced, by using the method of series truncation, to a system consisting of parabolic partial differential equations and ordinary differential equations which are solved numerically. The results are compared with available experimental data and other solutions for Reynolds numbers $Re = 1, 2, 6, 7$ and 10. In particular, the investigation is concerned with the drag coefficient, pressure distribution, standing eddy length and stream-lines pattern. (Received September 14, 1972.)

*698-C2. UMA BASU, Centre of Advanced Study in Applied Mathematics, University of Calcutta, Calcutta 9, India and LOKENATH DEBNATH, East Carolina University, Greenville, North Carolina 27834. On unsteady non-Newtonian flow in a rotating system.

An incompressible visco-elastic fluid is bound by an infinite rigid horizontal disk at $z = 0$. Both the fluid and the disks are in a state of solid body rotation with a uniform angular velocity Ω about the z-axis. At time $t = 0+$, small amplitude nontorsional oscillations are impulsively imposed on the disk or the disk is impulsively moved with a constant acceleration so that an unsteady motion is set up in the rotating fluid. An analysis is then made of the unsteady flow generated in the non-Newtonian rotating system. The initial value problem is solved by the Laplace transform treatment combined with the theory of residues. The unsteady velocity field related to present configurations is explicitly calculated and the structure of the associated boundary layer is then determined. This analysis provides the existence of Stokes-Ekman-Elastic boundary layers of thicknesses of the order $(\nu/(2\Omega \pm \omega \mp 2k\omega))^{1/2}$. Special emphasis is given on the limiting behaviors of the solution $t \to \infty$ and the significant interaction of the elastic parameter and rotation of the fluid is examined. The drag and the lateral stress on the disk are obtained explicitly and the effects of the elastic parameter on these physical quantities are discussed. Some physical interpretation of the mathematical results are given. It is shown that in the absence of the elastic parameter the results of this paper are in accord with the corresponding results of the Newtonian rotating fluid. (Received October 4, 1972.)

Logic and Foundations

698-E1. CHARLES FONTAINE MARTIN, University of California, Berkeley, California 94720. Axiomatic bases for equational theories of natural numbers. Preliminary report.

Let $\{A_n : n \in \omega\}$ be the Ackermann sequence of binary operations on ω, the set of natural numbers. $A_0 = +$ (addition), $A_1 = \cdot$ (multiplication), and $A_2 = \uparrow$ (exponentiation). Let $Eq(\mathfrak{A})$ be the set of equations holding for the algebraic structure $\mathfrak{A}$. Define $S \vdash T$ if every equation in T may be derived from tautologies and equations in S by replacements and substitutions. Consider the equations: (E1) $x + (y + z) = (x + y) + z$, (E2) $x \cdot (y \cdot z) = (x \cdot y) \cdot z$, (E3) $x + y = y + x$, (E4) $x \cdot y = y \cdot x$, (E5) $x \cdot (y + z) = x \cdot y + x \cdot z$, (E6) $x^{y+z} = x^y \cdot x^z$, (E7) $(x \cdot y)^z = x^z \cdot y^z$, (E8) $(x^y)^z = x^{y \cdot z}$. Let $\mathfrak{N} = \langle \omega, +, \cdot, \uparrow \rangle$. Doner-Tarski (Fund. Math. 65(1969), 95-127) conjectured essentially what follows: (C1) $\{E1, \ldots, E8\} \vdash Eq(\mathfrak{N})$, and (C2) $Eq(\mathfrak{N}) \vdash Eq\langle \omega, A_n \rangle_{n \in \omega}$. The author has shown _Theorem_ 1. $\{(x^y)^z = (x^z)^y\} \vdash Eq\langle \omega, \uparrow \rangle$ and $\{E2, E4, E7, E8\} \vdash Eq\langle \omega, \bullet, \uparrow \rangle$. _Theorem_ 2. If σ and τ are terms with variables, $+$, $\bullet$, and $\uparrow$ such that $+$ never occurs in a base of exponentiation and $\sigma = \tau$ is in $Eq(\mathfrak{P})$, then $\{E1, \ldots, E8\} \vdash \{\sigma = \tau\}$. However, (C1) fails, since it is not true that $\{E1, \ldots, E8\} \vdash \{(x^u + x^u)^v \bullet (y^v + y^v)^u = (x^v + y^v)^u \bullet (y^u + y^u)^v\}$. In fact, _Theorem_ 3. There is no finite $S \subseteq Eq(\mathfrak{P})$ such that $S \vdash Eq(\mathfrak{P})$. (C2) is still open, but the author has shown a weaker result: _Theorem_ 4. $\emptyset \vdash Eq\langle \omega, A_3, \ldots, A_n, \ldots \rangle$. (Received September 5, 1972.)

Topology

698-G1. WILLIAM A. McCALLUM, Florida State University, Tallahassee, Florida 32306. _The higher homotopy groups of k-spun knots and links._ Preliminary report.

A ball configuration K_μ^n of multiplicity μ is a smooth proper embedding of the disjoint union of μ copies of the standard n-ball B_i^n $(i=1,2,\ldots,\mu)$ in B^{n+2}. One obtains the $(n+k)$-dimensional link L_μ^{n+k} of multiplicity μ by k-spinning K_μ^n as follows: $S^{n+k+2} = (S^k \times B^{n+2}) \cup (D^{k+1} \times \partial B^{n+2})$ identified along $S^k \times \partial B^{n+2} = \partial D^{k+1} \times \partial B^{n+2}$ and $S_i^{n+k} = (S^k \times B_i^n) \cup (D^{k+1} \times \partial B_i^n)$ identified along $S^k \times \partial B_i^n = \partial D^{k+1} \times \partial B_i^n$ (where D^k is the standard k-dimensional disk). _Theorem._ If $B^{n+2} - K_\mu^n$ is aspherical and if $(x_1, x_2, \ldots, x_m : r_1, r_2, \ldots, r_p)$ is a presentation of $\Pi_1(S^{n+k+2} - L_\mu^{n+k})$ with $x_1, x_2, \ldots, x_u$ the images of the generators of $\Pi_1(S^{n+1} - K_\mu^n)$ under the inclusion map, then $\Pi_1(S^{n+k+2} - L_\mu^{n+k}) = 0$ $(1 < i \leqq k)$ and $(x_{u+1}^*, x_{u+2}^*, \ldots, x_m^* : \sum_{j=u+1}^m (\partial r_i / \partial x_j) x_j^*)$ is a presentation of $\Pi_{k+1}(S^{n+k+2} - L_\mu^{n+k})$ as a left $Z\Pi_1$ module. (Received September 13, 1972.)

*698-G2. CHARLES L. HAGOPIAN, California State University, Sacramento, California 95819. _Concerning Jones's function K._

For each point x of a continuum M, F. Burton Jones defines K(x) to be the closed set consisting of x and all points y of $M - \{x\}$ such that M is not aposyndetic at x with respect to y ["Concerning non-aposyndetic continua," Amer. J. Math. 70(1948), 403-413]. _Theorem._ If M is either a plane continuum that does not have infinitely many complementary domains or an arcwise connected subcontinuum of an indecomposable plane continuum, then K(x) is connected for each point x of M. (Received September 18, 1972.)

*698-G3. ELDON J. VOUGHT, California State University, Chico, California 95926. _Monotone decompositions of continua into simple closed curves and generalized simple closed curves._

By a _generalized simple closed curve_ (g.s.c.c.) we mean a compact, Hausdorff continuum which is separated by the omission of any two of its points. If, in addition, the continuum is metric we mean a _simple closed curve_ (s.c.c.). A collection $\{G\}$ of subsets of a continuum M is _bi-saturated_ if for each $G \in \{G\}$ and $p \notin G$ there exist $G_1, G_2 \in \{G\}$ such that $G_1 \cup G_2$ separates G from p in M. _Theorem._ Let M be a compact, Hausdorff continuum that is separated by no subcontinuum. If there exists a bi-saturated collection $\{G\}$ of subsets of M such that each $G \in \{G\}$ is a non-region-containing continuum, then M has a monotone, u.s.c. decomposition whose quotient space is a g.s.c.c. If M is metric the converse is also true. _Theorem._ Let M be a compact, metric continuum that is separated by no subcontinuum. If M is hereditarily decomposable, then M has a unique monotone, u.s.c. decomposition the elements of which have void interior and for which the quotient space is an s.c.c. The primary tool used in both theorems is the aposyndetic set function T first defined by F. B. Jones (Amer. J. Math. 67(1941), 545-553). (Received October 2, 1972.)

698-G4. RAPHAEL ZAHLER, Rutgers University, Douglass College, New Brunswick, New Jersey 08903. Higher relations among BP operations. Preliminary report.

Let p be a prime greater than 3, and let BP be the p-primary Brown-Peterson spectrum. The algebra of BP-cohomology operations $BP^*(BP)$ corresponds to the Steenrod algebra of ordinary Z_p-cohomology. Then the following Adem-type relations hold in $BP^*(BP)$: w_p: $(-p+v_1r_1)r_p + (r_{p-1} - v_1r_p)r_1 = 0$; w_{p+2}: $r_2r_p + (-r_{0,1} - ((p+1)/2)r_{p+1} + \frac{1}{2}v_1r_{1,1})r_1 = 0$; w_{2p+1}: $(r_{0,1} - r_{p+1})r_p + ((1/(2p+1))\binom{2p+1}{p} r_{2p} + v_1r_{p,1} + cv_1^2r_{0,2})r_1 = 0$, where $c \in Q_p$ is a constant. There are also the following higher-order relations: $w^{(3)}_{p+1}$, $w^{(3)}_{2p}$, and $w^{(3)}_{2p+2}$ (relations among the w_i: $w^{(3)}_k$ is of grade $2k(p-1)$), and $w^{(4)}_{2p}$, $w^{(4)}_{2p+2}$ (relations among the $w^{(3)}_i$). These and similar higher relations will be used to study the stable homotopy of spheres and V(n)-spectra in forthcoming work of the author and P. E. Thomas. (Received October 2, 1972.)

698-G5. RICHARD A. BODY and ROY R. DOUGLAS, University of British Columbia, Vancouver, British Columbia, Canada and CASPAR R. CURJEL, University of Washington, Seattle, Washington 98195. On the finite determination of some homotopy types.

Let A be a candidate for the integral cohomology ring of a simply-connected, finite CW-complex (i.e., A is a graded-commutative, associative, simply-connected, finite dimensional algebra over the integers). Let T(A) be the set of homotopy types of simply-connected, finite CW-complexes, whose integral cohomology ring is isomorphic to A. Stasheff's fifth problem [James D. Stasheff, "H-space problems," Lecture Notes in Math., vol. 196, "H-spaces: Neuchatel (Suisse) August 1970", Springer-Verlag, Berlin and New York, pp. 123-124, Problem 5, and Curjel and Douglas, "On Stasheff's fifth problem," Abstract 687-55-2, these *Notices* 18(1971), 787]. For which rings A is T(A) a finite set? Partial answer. T(A) is finite, whenever A tensored with the rational field is isomorphic to a tensor product of truncated polynomial algebras, each on one generator. (See also Richard A. Body, "H* and some rather nice spaces," Thesis, University of British Columbia, September 1972.) (Received October 4, 1972.)

698-G6. CHIEN WENJEN, California State University, Long Beach, California 90801. Pseudo-paracompact spaces. Preliminary report.

A family of open sets in a topological space X is called a proximate cover if the union of the sets in the family is dense in X and is called a minimum cover if it is a cover and none of its proper subfamilies is a proximate cover. A regular space is pseudo-normal if each pair of disjoint regular closed sets (a set is called regular closed if it is the closure of its interior) is contained in two disjoint open sets respectively and is said to be pseudo-paracompact if each minimum open cover has a locally finite open refinement. A diagonal neighborhood U of a space X is locally finite if $\{U(x) \mid x \in X\}$ is a locally finite cover of X. Denote the least cardinal number of a space, i.e., the smallest cardinal number of all families of cofinals in the set of locally finite diagonal neighborhoods, by $\aleph$ (see Wenjen, Proc. Japan Acad. 43(1967), 121-124). (I) The product of a pseudo-paracompact space and a compact space is pseudo-paracompact. (II) The following are equivalent statements for a pseudo-normal space X: (a) X is pseudo-paracompact, (b) each net of open sets in X with power $\geqq \aleph$ has a cluster point, and (c) each minimal open cover of X with power $\geqq \aleph$ has a proximate subcover of power $< \aleph$ (cf. Wenjen, ibid., Theorem 1). (III) (Generalized Tamano's theorem) Let βX be the Stone-Čech compactification of a completely regular space X. If $X \times \beta X$ is pseudo-normal, then X is pseudo-paracompact. (IV) The Sorgenfrey plank, Tychonoff plank, and Moore (Niemytzki) plane are pseudo-normal and pseudo-paracompact. (Received October 4, 1972.)

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Algebra & Theory of Numbers

699-A1. WITHDRAWN.

*699-A2. THOMAS G. PARKER, Florida State University, Tallahassee, Florida 32306. A note on dimension sequences. Preliminary report.

A sequence $a_0, a_1, a_2, \ldots$ of positive integers is said to be a dimension sequence if there exists a commutative ring R with identity such that the (Krull) dimension of R is a_0 and the dimension of $R[X_1, \ldots, X_n]$ is a_n for $n \geqq 1$. Theorem 1. Let $s = \{a_i\}_{i=0}^{\infty}$ be a strictly increasing sequence of positive integers. The following two conditions are equivalent: (1) s is a dimension sequence. (2) (i) $[a_0/1] \geqq [a_1/2] \geqq [a_2/3] \geqq \ldots$, where "[ ]" denotes the greatest integer function; and (ii) if $[a_i/(i+1)] = [a_{i+1}/(i+2)]$, where $a_i = [a_i/(i+1)](i+1) + r_1$ and $a_{i+1} = [a_{i+1}/(i+2)](i+2) + r_2$, then $r_1 < i$ implies $r_2 \leqq r_1$. The proof of Theorem 1 is based on results contained in a preprint of Jimmy T. Arnold and Robert Gilmer. (Received August 25, 1972.)

*699-A3. ANTHONY V. GERAMITA and NORMAN J. PULLMAN, Queen's University, Kingston, Ontario, Canada. A theorem of Radon and Hurwitz and orthogonal projective modules.

Definition. A Radon-Hurwitz (R-H) family of orthogonal matrices, $\{A_1, \ldots, A_s\}$, is one that satisfies the two conditions (i) $A_i = -A_i^t$ and (ii) $A_iA_j = -A_jA_i$ $(i \neq j)$. If n is any integer and $n = 2^a b$, b odd, then write $a = 4c + d$, $0 \leqq d < 4$. If we let $\rho(n) = 8c + 2^d$ then Radon showed ["Linear Scharen Orthogonaler Matrizen," Abh. Math. Sem. Univ. Hamburg 1(1922), 1-14]: (i) a real R-H family of matrices of order n has fewer than $\rho(n)$ members; (ii) $\exists$ real R-H family of matrices of order n having exactly $\rho(n) - 1$ members. We prove the same theorem for integer matrices and give an application to the study of projective modules. Let R be a commutative ring. A stably-free R-module is the kernel of an epimorphism $\alpha : R^n \to R^m$ for some n, m. If, in addition, $\alpha\alpha^t = 1$ we call the kernel orthogonal. We study the generic occurrence of this type of module and can say precisely what the rank of the largest free summand of the generic module is if $\alpha : R^n \to R$. (Received August 30, 1972.)

699-A4. PHILIP C. TONNE, Emory University, Atlanta, Georgia 30322. A regular determinant of binomial coefficients.

Let n be a positive integer and suppose that each of $\{a_p\}_1^n$ and $\{c_p\}_1^n$ is an increasing sequence of nonnegative integers. Let M be the $n \times n$ matrix such that $M_{ij} = C(a_i, c_j)$, where $C(m,n)$ is the number of combinations of m objects taken n at a time. We give an explicit formula for the determinant of M as a sum of nonnegative quantities. Further, if $a_i \geqq c_i$, $i = 1, 2, \ldots, n$, we show that the determinant of M is positive. (Received September 11, 1972.)

*699-A5. FREDRIC T. HOWARD, Wake Forest University, Winston-Salem, North Carolina 27109. Prime divisors of the van der Pol numbers.

The van der Pol numbers $V_0, V_1, V_2, \ldots$ are defined by means of $x^3[6x(e^x+1)-12(e^x-1)]^{-1} = \sum_{n=0}^{\infty}(V_n/n!)x^n$. Introduced by B. van der Pol in 1957 ["Smoothing and 'unsmoothing,'" in M. Kac, "Probability and related topics in physical sciences," pp. 223-235], these numbers apparently have properties similar to those of the Bernoulli numbers. In fact, van der Pol raised the question of whether or not an analogue of the Staudt-Clausen theorem exists for V_n. If such an analogue exists, it is certainly not obvious. However, by using recurrence formulas and a relationship between V_n and the Bessel function, we can show that for $n > 0$, $2V_{2n} \equiv 1 \pmod 4$ and $3V_{2n} \equiv r + 2 \pmod 3$ if $2n \equiv r \pmod 3$, $r = 0, 1, 2$. Similar results hold for V_{2n+1} since $2V_{2n+1} = -(2n+1)V_{2n}$, $n > 1$. Results concerning prime divisors larger than 3 can be found in ["The van der Pol numbers and a related sequence of rational numbers" by F. T. Howard, Math. Nachr. 42(1969), 89-102]. (Received September 11, 1972.)

*699-A6. PAUL M. EAKIN, University of Kentucky, Lexington, Kentucky 40506 and WILLIAM J. HEINZER, Purdue University, West Lafayette, Indiana 47907. Locally polynomial rings and invertible ideals.

Theorem. Let $R \subset A$ be integral domains with A finitely generated over R. If for each prime P of R, $A \otimes R_P$ is a polynomial ring in one variable over R_P, then A is the symmetric algebra of an invertible ideal in R. Corollary. With R and A as above, if R is a unique factorization domain (UFD), then A is a polynomial ring over R. Theorem. Let A be a 2-dimensional affine UFD over an algebraically closed field k. If there exists $f \in A$ such that $f - \lambda$ is irreducible in A for each $\lambda \in k$ and such that $k(f)[A]$ is a polynomial ring in one variable over the field $k(f)$, then A is a polynomial ring over k and in fact there exists $g \in A$ so that $A = k[f, g]$. Corollary. Suppose A is an integral domain, t an indeterminate, and $A[t] = k[X, Y, Z]$ is a polynomial ring in 3 variables over a field k. If $A \cap k[X, Y]$ properly contains k, then A is a polynomial ring over k. (Received September 14, 1972.)

*699-A7. THOMAS J. SCOTT, Georgia College, Milledgeville, Georgia 31061. Extendable ℓ-permutation groups.

Monotonic permutations and ℓ-monotonic groups were introduced by this author in Abstract 689-A35, these *Notices* 18(1971), 1048. Suppose Ω is a chain which admits an order-reversing permutation (orp), and that (H, Ω) is an ℓ-subgroup of $A(\Omega)$, the group of all order-preserving permutations (opps) of Ω. Then we say that H is extendable if there is an ℓ-monotonic permutation group (K, Ω) such that $H = K \cap A(\Omega)$. Theorem 1. Suppose that (H, Ω) is the regular representation of an ordered group. Then H is extendable iff H is abelian; and then H uniquely determines its extension. Theorem 2. If (H, Ω) is full periodically 0-primitive [see S. H. McCleary, Pacific J. Math. 43(1972), 366], then H is uniquely extendable. Theorem 3. Suppose that $A(\Omega)$ is 0-2-transitive. Then the group $B(\Omega)$, which consists of all opps of Ω with bounded support, is extendable, but not uniquely extendable. The group $BA(\Omega)$, which consists of all opps with support bounded above, is not extendable. (Received September 15, 1972.)

*699-A8. ROBERT GILMER, Florida State University, Tallahassee, Florida 32306. Some results on polynomial rings over a commutative ring.

• Let R be a commutative ring with identity of finite dimension n_0, and for each positive integer k, let n_k be the dimension of the polynomial ring $R[X_1,\ldots,X_k]$ in k indeterminates over R. The sequence $\{n_i\}_{i=0}^{\infty}$ is the dimension sequence for R and the sequence $\{d_i\}_{i=1}^{\infty}$, where $d_i = n_i - n_{i-1}$ for each i, is the difference sequence for R. The question arises as to what sequences of nonnegative integers can be realized as the dimension sequence of a ring. Several restrictions on the sequences $\{n_i\}$ and $\{d_i\}$ are known. For example, $n_k + 1 \leqq n_{k+1} \leqq 2n_k + 1$ for each k, and the difference sequence is eventually constant, bounded above by $n_0 + 1$. Let S be the set of sequences $\{m_i\}_{i=0}^{\infty}$ of nonnegative integers such that the corresponding difference sequence $\{t_i\}_{i=1}^{\infty}$, where $t_i = m_i - m_{i-1}$, satisfies the following conditions: (1) $m_0 + 1 \geqq t_1 \geqq t_2 \geqq \ldots$; (2) for some k, $1 \leqq t_k = t_{k+1} = \ldots$. Theorem 1. If F is a field of infinite transcendence degree over its prime subfield and if s is in S, then there is an integrally closed quasi-local domain D with quotient field F such that s is the dimension sequence of D. Let T be the set of all sequences $\sup\{s_1,\ldots,s_n\}$, where $\{s_i\}_1^n$ is a finite subset of S and where the supremum is taken in the cardinal order. Theorem 2. Each dimension sequence is in T. Theorem 3. If F is as in Theorem 1 and if t is in T, then there is an integrally closed semi-quasi-local domain D with quotient field F such that t is the dimension sequence of D; hence T is the set of dimension sequences. The only dimension sequence $\{n_i\}_0^{\infty}$ with $n_0 = 0$ is the sequence $\{0, 1, 2,\ldots\}$, and hence the following result is definitive. Theorem 4 (Parker). If $\{a_i\}_{i=0}^{\infty}$ is a strictly increasing sequence of positive integers and if $b_i = [a_i/(i+1)]$ for each i, then $\{a_i\}_0^{\infty}$ is in T if and only if the following conditions are satisfied: (1) $a_0 = b_0 \geqq b_1 \geqq \ldots \geqq 1$; (2) for all i, if $b_i = b_{i+1}$, if $a_i = q_ib_i + r_1$ where $0 \leqq r_1 \leqq i$, and if $a_{i+1} = q_{i+1}b_{i+1} + r_2$, where $0 \leqq r_2 \leqq i+1$, then $r_1 < i$ implies that $r_2 \leqq r_1$. (Received September 18, 1972.)

*699-A9. JAMES W. STEPP, University of Houston, Houston, Texas 77004. The free compact lattice generated by a topological semilattice. Preliminary report.

Let S denote a topological semilattice, let M denote the min interval (the unit interval with multiplication given by $xy = \min x,y)$, and let Hom(S, M) be the set of continuous homomorphisms from S into M. It is known that S need not be embeddable in a product of min intervals even if Hom(S, M) separates points. It is shown that there is a largest homomorphic image $\tau(S)$ of S for which $\tau(S)$ is embeddable in a product of min intervals. Let $e: \tau(S) \to PM$ be the usual embedding and let $\beta(S) = \overline{e(\tau S)}$. Then conditions are given for $\beta(S) \cong B(S)$ (the Bohr compactification of S). Also, it is shown that if CL(S) is the free compact lattice generalized by S, then $CL(B(S)) \cong CL(S)$. Several examples are given. (Received September 21, 1972.)

*699-A10. TEMPLE HAROLD FAY, Hendrix College, Conway, Arkansas 72032. The Induced Morphism Theorem: A result in categorical relation theory.

For basic definitions see Klein, "Relations in categories," Illinois J. Math. 14(1970), 536-550. Let $\mathcal{C}$ be a locally small, finitely complete $\mathcal{E}$-$\mathcal{M}$ bicategory having coequalizers of kernel pairs. The congruence generated by a morphism f with domain X is denoted cong(f) and is the equalizer of $f\pi_1$ and $f\pi_2$ where π_1 and π_2 are the projections of $X \times X$. Let $\mathcal{E}$-epimorphisms have the property that if f is $\mathcal{E}$-epic and cong(f) $\leqq$ cong(g), then there exists a morphism h such that hf = g. Theorem. The class of $\mathcal{E}$-epimorphisms is precisely the class of regular epimorphisms. The Induced Morphism Theorem. Let the $\mathcal{E}$-epics be the regular epimorphisms and let (R,j) be a subobject of $X \times Y$ such that $\rho_1 j$ is regular epic (ρ_1 is the projection of $X \times Y$). Let E and F be the congruences determined by the regular epics $g: X \to Z$ and $h: Y \to W$ respectively satisfying $R^{-1} \circ (E \circ R) \leqq F$ or $(R^{-1} \circ E) \circ R \leqq F$. Then there exists a morphism k such that $kg\rho_1 j = h\rho_2 j$. Moreover, if $(R \circ F) \circ R^{-1} \leqq E$ or $R \circ (F \circ R^{-1}) \leqq E$, then k is an isomorphism. This result generalizes some results of Bednarek and Wallace, "A relation-theoretic result with applications in topological algebra", Math. Systems Theory 1(1963), 217-224, and of Riguet, "Quelques propriétés des relations difonctionalles", C. R. Acad. Sci. Paris 230(1950), 1999-2000. (Received September 25, 1972.)

*699-A11. JOHN C. PROPES, University of Tennessee, Nashville, Tennessee 37203. Maximal ideal complements which are subsemigroups. Preliminary report.

For a maximal ideal M of a semigroup S, $S\backslash M$ is either a degenerate subset of $S\backslash SS$ or S/M is o-simple. Theorem. Let S be an o-simple semigroup and let λ be a function, not the identity, from $\{1, 2, \ldots, n\}$ into itself, for $n > 1$. Then $S\backslash o$ is a subsemigroup of S if and only if $s_1 s_2 \ldots s_n$ contained in $S\backslash o$ implies $s_{1\lambda} s_{2\lambda} \ldots s_{n\lambda} \in S\backslash o$. (Received September 27, 1972.)

699-A12. ROBERT L. BERNHARDT, University of North Carolina, Greensboro, North Carolina 27412. On centrally splitting. Preliminary report.

Let $\mathcal{T}$ denote a TTF class, in the sense of Jans (Pacific J. Math. 15(1965), 1249-1259). Let $(\mathcal{T}, \mathfrak{F})$ and $(\mathcal{C}, \mathcal{T})$ be the torsion theories associated with $\mathcal{T}$. Call $\mathcal{T}$ centrally splitting if every module M is the direct sum of its two torsion submodules, so that $M = M_t \oplus M_c$. If $\mathcal{T}$ is a hereditary torsion class and if $\mathcal{T}_1$ is the smallest torsion class containing R_t, then $\mathcal{T}_1$ is centrally splitting if and only if R_t is generated by a central idempotent. However, when $\mathcal{T}$ is a hereditary torsion class, then R_t can be generated by a central idempotent without $\mathcal{T}$ being centrally splitting. Finally when $\mathcal{T}$ is a hereditary torsion class, then $\mathcal{T}$ is a centrally splitting TTF class if and only if $\mathcal{T}$ is closed under injective envelopes and $\mathfrak{F}$ is closed under homomorphic images. This last result was proved for semiperfect rings by Rutter (Proc. Amer. Math. Soc. 34(1972), 389-396). (Received September 28, 1972.)

*699-A13. JAMES W. BREWER, PHILLIP MONTGOMERY and EDGAR A. RUTTER, University of Kansas, Lawrence, Kansas 66044 and WILLIAM J. HEINZER, Purdue University, Lafayette, Indiana 47907. Krull dimension of polynomial rings.

Let R be a commutative ring with $1 \neq 0$ and let $X_1, \ldots, X_n$ be indeterminates over R. For Q a prime ideal of $R[X_1, \ldots, X_n]$, put $P = Q \cap R$. This paper investigates the relationship between the ranks of Q and $P[X_1, \ldots, X_n]$. The main positive result, whose proof is quite elementary, is Theorem 1. $\mathrm{rank}(Q) = \mathrm{rank}(P[X_1, \ldots, X_n]) + \mathrm{rank}(Q/P[X_1, \ldots, X_n]) + n$. This result is used to recapture several classical results concerning the Krull dimension of $R[X_1, \ldots, X_n]$. Numerous examples are also presented, among them one which shows that the strong S-rings of Kaplansky ["Commutative rings," Allyn and Bacon, 1970] are not stable under polynomial extension. (Received September 29, 1972.)

699-A14. JOHN P. HOLMES, Auburn University, Auburn, Alabama 36830. Differentiable subgroupoids. Preliminary report.

Suppose S is a topological space, D is an open set of S containing the point e, and V is a function from $D \times D$ into S satisfying $V(x, e) = V(e, x) = x$ for each x in D. If there is a neighborhood U of e such that

U contains exactly one square root of e, we will say the partial groupoid (S, D, V, e) is type A. A closed subset F of D is a subgroupoid of (S, D, V, e) if and only if $V(F \times F)$ is contained in $F \cup (S \setminus D)$. A subgroupoid F is said to be differentiable if and only if there are open sets G and B of S containing e such that if x is in $F \cap G$ then there is a unique continuous function T_x from $[0,1]$ into $F \cap B$ satisfying $T_x(0) = e$, $T_x(1) = x$, and $V(T_x(s), T_x(t)) = T_x(s+t)$ whenever each of s, t, and $s+t$ is in $[0,1]$. Theorem. Suppose, in the partial groupoid (S, D, V, e), that S is a Banach space, V is continuously differentiable on $D \times D$, and V is power associative. If f is a continuous local homomorphism defined on D into a type A partial groupoid (R, E, M, d) then $f^{-1}(\{d\})$ is a differentiable subgroupoid of (S, D, V, e). (Received October 2, 1972.)

*699-A15. W. SHEFFIELD OWEN, Auburn University, Auburn, Alabama 36830. The Rees theorem for locally compact semigroups.

It has been known for some time (Wallace, Proc. Nat. Acad. Sci. U.S.A. 42(1956), 430-432) that a compact (completely) simple semigroup S is homeomorphic to $[Se \cap E(S)] \times eSe \times [eS \cap E(S)]$ where e is any idempotent and $E(S)$ is the set of all idempotents. Moreover, eSe is a group, and $Se \cap E(S)$ and $eS \cap E(S)$ are left and right trivial semigroups, respectively. This may be generalized as follows: (1) "compact" above may be replaced by "locally compact"; (2) if S is a locally compact completely 0-simple semigroup with no zero-divisors, the set of nonzero idempotents of S is compact (assuming 0 is not isolated); (3) in case the semigroup has zero-divisors, the full analogue of the product structure obtained by Wallace is lacking. However, one has a local product structure at each (nonzero) idempotent. (Received October 2, 1972.)

*699-A16. EARL J. TAFT and ROBERT LEE WILSON, Rutgers University, New Brunswick, New Jersey 08903. On antipodes in pointed Hopf algebras.

Let H be a finite-dimensional Hopf algebra over a field K. The question of whether the antipode S of H has finite order remains open. The first author has given examples (Proc. Nat. Acad. Sci. U.S.A. 68 (1971), 2631-2633) where S has any given even order. Here we show that S has finite order if H is pointed (as coalgebra) and characteristic K is positive. In fact, $S^{2ep^m} = I$, where e is the exponent of the group of group-like elements of H and $p^m \geqq n > p^{m-1}$, where $H = H_n$ in the coradical filtration of H. If characteristic K is zero, H is pointed and S has finite order, then $S^{2e} = I$. The key to the proof is to build up a basis for the terms in the coradical filtration of a pointed coalgebra by elements x which are nearly primitive ($\Delta x = x \otimes a + b \otimes x$, a, b in G(H)) modulo previous terms in the filtration. (Received October 2, 1972.)

699-A17. PETER M. GIBSON, University of Alabama, Huntsville, Alabama 35807. Simultaneous real orthogonal diagonalization of rectangular complex matrices.

Let A^T and A^* denote the transpose and the transposed conjugate, respectively, of the complex matrix A. M. H. Pearl [Canad. J. Math. 19(1967), 344-349] proved that if A is an $m \times n$ complex matrix then there exist real orthogonal matrices P and Q such that PAQ is a diagonal matrix if and only if AA^* and A^*A are real. R. C. Thompson [Canad. Math. Bull. 12(1969), 805-808] generalized this by showing that if Γ is a set of $m \times n$ complex matrices then there exist real orthogonal matrices P and Q such that PAQ is a diagonal matrix for every $A \in \Gamma$ if and only if AB^*, B^*A, AB^T, and B^TA are symmetric for all $A, B \in \Gamma$. The requirement that AB^T and B^TA be symmetric is redundant. Theorem. If Γ is a set of $m \times n$ complex matrices, then there exist real orthogonal matrices P and Q such that PAQ is a diagonal matrix for every $A \in \Gamma$ if and only if AB^* and B^*A are symmetric for all $A, B \in \Gamma$. (Received October 2, 1972.)

*699-A18. RENU LASKAR, Clemson University, Clemson, South Carolina 29631 and ARTHUR PELLERIN, University of North Carolina, Charlotte, North Carolina 28205. On 4-lattice graphs.

A 4-lattice graph may be defined as a graph G, whose vertices can be identified with ordered quadruplets on n symbols, such that two vertices are adjacent if the corresponding quadruplets have three common coordinates. If $d(x,y)$ denotes the distance between two vertices x and y and $\Delta(x,y)$ denotes the number of vertices adjacent to both x and y, then a 4-lattice graph G has the following properties: (b_1) The number of vertices is n^4. (b_2) It is connected and regular of degree $4(n-1)$. (b_3) It is edge-regular with edge-degree $n-2$. (b_4) $\Delta(x,y) = 2$, if $d(x,y) = 2$. Theorem. If $n > 11$, then any graph G (without loops and multiple edges) having the properties $(b_1)-(b_4)$ must be a 4-lattice graph. (Received October 2, 1972.)

*699-A19. ANDREW F. LONG, JR., University of North Carolina, Greensboro, North Carolina 27412. Factorization of irreducible polynomials over a finite field with the substitution $x^{q^r} - x$ for x. I.

Let GF(q) denote the finite field of order $q = p^n$, where p is an arbitrary prime and $n \geqq 1$. Let Q(x) denote an irreducible polynomial of degree s over GF(q). Let $(r,s) = d$ and put $s = ds'$ and $r = dr'$. Let $r' = p^k \ell$, $(p,\ell) = 1$ and $k \geqq 0$, and put $D = p^k d$. Let $\rho_{s,d}(x) = \sum_{j=0}^{s-1} x^{q^{dj}}$. If $Q(x) \mid \rho_{s,d}(x)$, then $Q(x^{q^r} - x)$ is the product over GF(q) of irreducibles of degree st, $t \mid r'$. For each t dividing r' the number of irreducibles of degree st is $\sum N(vt,q)/t$, where $N(vt,q)$ is the number of elements of degree vt over GF(q) and the summation extends over all divisors v of d such that $(t, d/v) = 1$. If $Q(x) \nmid \rho_{s,d}(x)$, then $Q(x^{q^r} - s)$ is the product over GF(q) of irreducibles of degree $p^{k+1}st$, $t \mid \ell$. For each t dividing ℓ, the number of irreducibles of degree $p^{k+1}st$ is $\sum N(vt,q)/p^{k+1}t$ where the summation extends over all divisors v of D such that $(t, D/v) = 1$. (Received October 2, 1972.)

*699-A20. ROBERT GILMER, Florida State University, Tallahassee, Florida 32306 and ANNE P. GRAMS, University of Tennessee, Nashville, Tennessee 37203. Three questions concerning Dedekind domains. Preliminary report.

Let G be a countable abelian group, and let S be a subset of G. Theorem 1. If G is a torsion group or if $-S = \{-s \mid s \in S\} \subseteq S$, then there is a Dedekind domain D with class group G such that S is the set of classes that contain maximal ideals of D if and only if S generates G. Theorem 2. If S generates G, then there is a Dedekind domain D with class group G such that the set of class groups of overrings of D is $\{G/H_\lambda\}$, where $\{H_\lambda\}$ is the family of subgroups of G generated by the family of subsets of S. Theorem 3. If D is a Dedekind domain, and if the set of maximal ideals of D is countable, then each overring of D is an intersection of two quotient rings of D. (Received October 2, 1972.)

699-A21. JO ELLEN PERRY, North Carolina State University, Raleigh, North Carolina 27607. Prime ideals of generalized rings of extended functions on compact Hausdorff spaces. Preliminary report.

A generalized ring is a vector space on which a product satisfying the usual ring axioms is defined but not meaningful for every pair of elements. A generalized ring $\mathcal{C}^\infty(Q)$ of extended real valued continuous functions on a compact Hausdorff space Q which assume infinite values only on nowhere dense separating sets is discussed. Here $\mathcal{C}^\infty(Q)$ contains the ring C(Q) of real valued continuous functions on Q. Through the use of G. Ya. Ivanova's characterization of maximal ideals of $\mathcal{C}^\infty(Q)$ (Siberian Math. J. 12(1971), 501-508), it is shown that every maximal ideal of $\mathcal{C}^\infty(Q)$ is a prime ideal. Also, if S is a multiplicatively closed subset of C(Q) not containing 0, then there exists a prime ideal P of $\mathcal{C}^{\infty}(Q)$ such that $P \cap S = \emptyset$. (Received October 3, 1972.)

*699-A22. NICKOLAS HEEREMA and JAMES K. DEVENEY, Florida State University, Tallahassee, Florida 32306. A Galois theory for fields K/k finitely generated.

Let K be a field of characteristic $p \neq 0$. A subgroup G of the group $H^{\infty}(K)$ of infinite higher derivations on K is Galois if it is the group of all d in $H^{\infty}(K)$ having a given subfield h in its field of constants where K is finitely generated over h. We prove: G is Galois if and only if it is the closed group (in the higher derivation topology) generated over K by a finite, abelian, independent normal iterative set F of higher derivations or equivalently, if and only if it is a closed group generated by a normal subset possessing a dual basis. A subfield h over which K is finitely generated is Galois if and only if h is regular in K. Given K/h finitely generated we show: if K/h is separable, $K/h(K^{p^n})$ is modular for all n; $\cap\{h(K^{p^n})\}$ is the separable algebraic closure of h in K; K/h is regular if and only if $K/h(K^{p^n})$ is modular for all n. (Received October 3, 1972.)

699-A23. NICKOLAS HEEREMA and BRIAN J. WESSELINK, Florida State University, Tallahassee, Florida 32306. The group of inner higher derivations. Preliminary report.

Let R be a ring with unity having center Z. Given $W = \{w_i \mid i = 0, \ldots, t; w_i \in R, w_0 = 1\}$, the set $d_W = \{d_i \mid i = 0, \ldots, t\}$ of maps on R, where $d_n(a) = \Sigma\{w_{n-i}\, af_i \mid i = 0, \ldots, n\}$ and $f_i = \Sigma\{(-1)^{i-j} w_{k_1} w_{k_2} \cdots w_{k_{i-j}} \mid j = 0, \ldots, i-1;\ k_1 + \ldots + k_{i-j} = i;\ k_s \geqq 1,\ f_0 = 1\}$, is a rank t higher derivation which we call an inner higher derivation. (For a related definition, see Jacobson, J. Math. Pures Appl. 36(1957), 224.) If $\mathcal{H}_t(R)$ is the group of rank t higher derivations on R, and if $\mathcal{J}_t(R)$ is the group of rank t inner higher derivations on R, then $\mathcal{J}_t(R)$ is an invariant subgroup of $\mathcal{H}_t(R)$. Let $\mathcal{U}_t$ be the group of unimodular units of $R[[X]]/X^{t+1}R[[X]]$, and let $Z_t = \{\Sigma_{i=0}^{t} w_i X^i + X^{t+1}R[[X]] \in \mathcal{U}_t \mid w_i \in Z\}$. Theorem 1. The sequence $0 \to Z_t \to \mathcal{U}_t \xrightarrow{\varphi} \mathcal{J}_t \to 0$ is an exact sequence of group homomorphisms, where $\varphi(\Sigma_{i=0}^{t} w_i X^i + X^{t+1}R[[X]]) = d_W$. Theorem 2. If R is semisimple Artinian and finitely generated as a Z module, then every rank t higher derivation is inner. (Received October 3, 1972.)

*699-A24. JOHN R. HEDSTROM, University of North Carolina, Charlotte, North Carolina 28213. Unique factorization in Noetherian domains.

Let K be a field, $S = k[x_1, \ldots, x_n]$ a polynomial ring in n indeterminates over k. Theorem 1. Let $P = (\varphi_1, \ldots, \varphi_{n-d})$ be a prime ideal of S with $\varphi_j(x_1, \ldots, x_n) = \psi_j(x_1, \ldots, x_d) + x_{d+j}^{n_j}$, $1 \leqq j \leqq n-d$, n_j nonnegative integers. Then the finite integral domain $R = S/P$ is a unique factorization domain. Theorem 2. Let $P \subsetneq S$ be a prime ideal and $R = S/P$ be integrally closed with quotient field K. If T is a normalization of R with quotient field L and K/L is a purely inseparable extension then R is a unique factorization domain. Theorem 3. Let P be a homogeneous prime ideal of S of dimension $n - h$. Assume P is generated by h homogeneous elements of odd degree and $P \subsetneq (x_1, \ldots, x_n)$. Then $R = S/P$ is a unique factorization domain. Theorem 4. Let R be a local Gorenstein domain containing a prime element p. If the maximal ideal M^* of $R^* = R/(p)$ has finite R^* projective dimension then R is a unique factorization domain. Theorem 5. Let S be a local ring, P a prime ideal of S such that $R = S/P$ is a flat S module. If S is a unique factorization domain then so is R. (Received October 3, 1972.)

*699-A25. PHILIP B. SHELDON, Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061. Prime ideals in GCD-domains.

The purpose of this paper is to study the prime ideal structure of GCD-domains in relation to the properties of prime ideals in Bezout domains and UFD's. The investigation focuses on the PF-prime ideals of the GCD-domain D, which are defined to be the (nonzero) proper prime ideals satisfying one of the following equivalent conditions: (i) gcd(a, b) is in P whenever a and b are in P. (ii) D_P is a valuation ring. It is shown that UFD's are characterized among GCD-domains by the property that every PF-prime ideal is principal. Further results suggest that PF-primes play much the same role in GCD-domains in general that principal primes play in UFD's. Main Theorem. Every proper prime ideal in a GCD-domain is a union of PF-prime ideals. Theorem. Every GCD-domain is an intersection of essential valuation overrings. Corollary. A GCD-domain with only finitely many (maximal) PF-primes is a Bezout domain. Additional results are obtained which yield other sufficient conditions for a GCD-domain to be a Bezout domain. Theorem. A GCD-domain is a Bezout domain iff its partially-ordered set of prime ideals forms a tree. Corollary. A one-dimensional GCD-domain is a Bezout domain. Corollary. A GCD-domain with linearly ordered prime ideals is a valuation ring. (Received October 3, 1972.)

699-A26. I. FENNELL BURNS, Auburn University, Auburn, Alabama 36830. A theorem about finitely generated dual cones and generalized inverses. Preliminary report.

A cone, K, in R^n is proper if it is closed, convex, pointed, and full. A cone is said to be finitely generated if it is the set of all convex combinations (i.e. nonnegative linear combinations) of a finite set of nonzero vectors (called generating vectors). In case a cone K is finitely generated, a matrix P whose columns are generating vectors of K is called a generating matrix of K. If no column of P is a convex combination of the other columns then P is said to be a minimal generating matrix. If P is an $n \times m$ matrix, then any matrix X which satisfies $PXP = P$ is called a (1)-inverse of P and is denoted $P^{(1)}$. Theorem. If P $n \times m$ is a minimal generating matrix for the proper cone K and $P^{T(1)}$ is a (1)-inverse for P^T then K* is contained in the cone D generated by $P^{T(1)}$. Moreover, D = K* if and only if m = n. Therefore, K* is contained in the intersection of the set of all finitely generated cones having a generating matrix from the collection of (1)-inverses of P^T. (Received October 4, 1972.)

699-A27. JOE L. MOTT and MICHEL F. SCHEXNAYDER, Florida State University, Tallahassee, Florida 32306. Order exact sequences of semivalue groups. Preliminary report.

If B and C are integral domains contained in a field K, then we have the following exact sequence of abelian groups: (1) $0 \to U(C)/U(B) \to K^*/U(B) \to K^*/U(C) \to 0$. The group $K^*/U(B)$ partially ordered by $B^*/U(B)$ is the semivalue group of B. We discuss when (1) is order exact or lexicographically exact. If I is an ideal of a commutative ring C, if σ is the natural homomorphism of C onto C/I, and if A is a subring of C/I, call $B = \sigma^{-1}(A)$ the composite of A and C over I. Some of the ideal theoretic properties of a composite of two integral domains are reflected by the order exact sequence (1). For instance, if C is a local domain with maximal ideal I, then B is a GCD-domain if and only if C is a valuation domain and A is a GCD-domain with quotient field C/I. Gilmer's construction J [J. Reine Angew. Math. 239/240 (1970), 153-162] is the composite of a direct sum of integral domains A_1 and a semilocal Bezout domain C over the Jacobsen radical $\bigcap_{i=1}^{n} M_i$ of

C. The Krull dimension of J equals $\max\{\dim A_i + \dim C_{M_i}\}$, and J is an elementary division domain (EDD) if and only if each A_i is an EDD with quotient field $C_{M_i}/M_iC_{M_i}$. (Received October 4, 1972.)

*699-A28. ROBERT CUMBIE, Samford University, Birmingham, Alabama 35209. Relatively weakly finitely generated modules. Preliminary report.

Let R be an associative ring with unit; L, M left R-modules. M is said to be L-WFG (WFG for weakly finitely generated) if $\mathrm{Hom}_R(M, \ )$ commutes with direct sums of copies of L. In [C. R. Acad. Sci. Paris 268(1969), 930-933] Rentschler calls M a Σ-module if $\mathrm{Hom}_R(M, \ )$ commutes with direct sums. Theorem 1. If M is L-WFG for all modules L then M is a Σ-module. L is said to be a test module if any L-WFG module is a Σ-module. Theorem 2. Any cogenerator for the category of left R-modules is a test module. Theorem 3. If R is a commutative ring and R_M is a principal ideal domain for each maximal ideal M of R, then any test module is a cogenerator. Corollary. Over a Dedekind domain a test module is a cogenerator. (Received October 4, 1972.)

*699-A29. KIM KI-HANG BUTLER and JAMES R. KRABILL, Pembroke State University, Pembroke, North Carolina 28372. The maximal abelian subsemigroup of B_n.

B. M. Schein asked for the maximal abelian subsemigroup of B_n in Question 6, "Semigroups of binary relations", Miniconference on Algebraic Semigroup Theory, Szeged, Hungary, Aug. 29-Sept. 1, 1972. Let B_n be the semigroup of all $n \times n$ Boolean relation matrices over the Boolean algebra $\{0,1\}$ of order 2, and let C_n be the set of $n \times n$ circulant matrices over this algebra. That is, for $C \in C_n$, any row $(c_0, c_1, \ldots, c_{n-1})$ is followed by $(c_{n-1}, c_0, c_1, \ldots, c_{n-2})$. Theorem 1. C_n is the maximal abelian subsemigroup of B_n. N. S. Mendelsohn ("Directed graphs with the unique path property", Proc. Colloq. on Combinatorial Theory and Its Applications, Tihany, Hungary, Aug. 1969), I. J. Good (J. London Math. Soc. 21(1946), 167-169), and N. G. de Bruijn (Nederl. Akad. Wetensch Proc. 49(1946), 758-764) each discovered a specific subclass of graphs with unique path property of order n. For definition, see Mendelsohn. For these graphs, the adjacency matrix satisfies $A \in B_n$, $A^p = J_n$ for some $p > 0$. Another subclass is given in Theorem 2. For $C \in C_n$, there exists a $p > 0$ such that $C^p = J_n$ if and only if $(i_1, i_2, \ldots, i_s, n) = 1$ and for every divisor d of n there exist i_j and i_k which are incongruent modulo d. Here the i_j are the subscripts of the 1's in the first row. (Received October 4, 1972.)

*699-A30. CHANG MO BANG, Emory University, Atlanta, Georgia 30322. Row-decreasing, pointed, infinite matrices of cardinals and classes of abelian groups.

Let K be the set of all nonnegative integers, and by a cardinal we mean any symbol in $K \cup \{\infty\}$. A *-matrix α consists of both a cardinal α_0 and an infinite square matrix $[\alpha_{ij}]_{i,j=1,2,\ldots}$ such that each row $\alpha_i = (a_{i1}, a_{i2}, \ldots)$ is a decreasing ($\geqq$) sequence of cardinals. For each nonempty subset $I \subseteq K$, let M_I be the class of all *-matrices α such that each row α_i is a zero vector for all $i \in K - I$. Each M_I forms a lattice with componentwise order relation. We show several propositions of M_I. For example, if $|I| < \infty$ every sequence in M_I contains an increasing subsequence, M_I satisfies DCC but not ACC, each *-matrix covers, and is covered by, only finitely many *-matrices, etc. A nonempty subset T of M_I is called a *-ideal if $\alpha \leqq \beta \in T$ implies $\alpha \in T$ and each chain in T has its supremum in T. We obtain several properties of *-ideals of M_I. For example, if we assume $|I| < \infty$, every *-ideal of M_I is finitely generated, every *-ideal covers, and is covered by, exactly finitely many *-ideals in M_I, *-ideals in M_I satisfy DCC but not ACC, etc. As an application, we completely determine all classes of abelian groups closed under taking isomorphic groups, subgroups, and direct limits, which was asked by Fuchs and answered by Hill in another way. (Received October 5, 1972.)

*699-A31. TREVOR EVANS, Emory University, Atlanta, Georgia 30322. Finite separability, automorphisms of free algebras and some decision problems.

An algebra A has the finite separability property (with respect to f.g. subalgebras) if for any f.g. subalgebra $S \subseteq A$ and $x \notin S$, there is a finite homomorphic image $A\alpha$ of A such that $x\alpha \notin S\alpha$. If A is an f.p. algebra this property implies a positive solution to the generalised word problem for A. The first part of this paper shows that various known embedding theorems for partial latin squares imply the finite separability property for various varieties of loops and quasigroups. The second part of the paper consists of universal algebraic remarks. If an f.g. free algebra F in a variety has both the properties of residual finiteness and finite separability, then any endomorphism of F which induces automorphisms on all finite relatively free homomorphic images of F is actually an automorphism. This yields an algorithm for deciding whether a set of elements of F is a free generating set. It is also shown that if an f.g. algebra in a variety has a residually finite semigroup of endomorphisms, then the algebra is hopfian, thus simplifying earlier results of the author that residual finiteness of the algebra implies both of these properties. (Received October 5, 1972.)

699-A32. THERESA P. VAUGHAN, Duke University, Durham, North Carolina 27706. Polynomials and linear transformations over finite fields.

Let $GF(q^n)$ denote the finite field of order q^n, where $q = p^r$ for some prime p and integer $r > 0$. The algebra of linear transformations of $GF(q^n)$ over $GF(q)$ is represented by the algebra of all polynomials of the form $P(x) = \sum_{i=0}^{n-1} a_i x^{q^i}$ $(a_i \in GF(q^n))$ equipped with addition and composition of polynomials (modulo $x^{q^n} - x$) and scalar multiplication by elements of $GF(q)$. The subalgebra $\mathcal{O}$ of polynomials of the form $F(x) = \sum_{i=0}^{n-1} a_i x^{q^i}$ $(a_i \in GF(q))$ has been extensively studied by O. Ore [Trans. Amer. Math. Soc. 35(1933) and 36(1934)]. Theorem. The algebra $\mathcal{O}$ is isomorphic to the algebra of polynomials $f(T)$, where $T: x \to x^q$ and $f(x) \in GF(q)[x]$. Theorem. The operator T has a cyclic vector, and if $(n, q) = 1$, T is semisimple. Using these theorems, it is possible to derive and extend many of Ore's results. For example, an irreducible polynomial $F(x)$ in $GF(q)[x]$ is said to belong to $f(T)(x)$ if $f(T)(x)$ is the unique monic element of $\mathcal{O}$ of least degree which is divisible by $F(x)$. Theorem. If $F(x)$ belongs to $f(T)(x)$, then the number of linearly independent roots of $F(x)$ (in $GF(q^n)$) is the degree of $f(x)$. (Received October 5, 1972.)

*699-A33. GOOYONG SHIN, North Carolina State University, Raleigh, North Carolina 27607. Prime ideals and sheaf representation of a pseudo symmetric ring.

Almost symmetric rings and pseudo symmetric rings are introduced as generalizations of Lambek's symmetric ring. Symmetric rings are almost symmetric and almost symmetric rings are pseudo symmetric, but not conversely in either case. A representation of a pseudo symmetric ring with identity is obtained as the ring of global sections of a sheaf over the prime ideal space. Minimal prime ideals of a pseudo symmetric ring have the same characterization as for the commutative case. A characterization is obtained for a pseudo symmetric ring with certain right quotient ring to have compact minimal prime ideal space. Hofmann (Bull. Amer.

Math. Soc. 78(1972), 311) asks whether there is a ring, with identity outside the commutative case, outside the strongly harmonic case, and outside rings satisfying one condition of his, which is isomorphic to the ring of global sections of a sheaf with certain stalks over the maximal ideal space. One such a ring is exhibited.

(Received October 5, 1972.)

*699-A34. THOMAS T. BOWMAN, University of Florida, Gainesville, Florida 32601. Analogue of Pontryagin character theory for topological semigroups.

For S an abelian continuous-inverse semigroup, an algebraic and topological description of the character group $S^{\wedge}$ of S is given. The paper also gives necessary and sufficient conditions for S to be naturally isomorphic to $S^{\wedge\wedge}$. Combined with the work of others, this completely determines when duality holds for locally compact semigroups. (Received October 5, 1972.)

*699-A35. CHARLES C. LINDNER, Auburn University, Auburn, Alabama 36830. On the construction of cyclic quasigroups.

Let $F(x,y)$ be the free groupoid on two generators x and y. Define an infinite class of words in $F(x,y)$ by $w_0(x,y) = x$, $w_1(x,y) = y$, and $w_{i+2}(x,y) = w_i(x,y)\,w_{i+1}(x,y)$. An identity of the form $w_{3n}(x,y) = x$ is called a cyclic identity and a quasigroup satisfying a cyclic identity is called a cyclic quasigroup. The most extensively studied cyclic quasigroups have been models of the identity $y(xy) = x$. The more general notion of cyclic quasigroups was introduced by N. S. Mendelsohn. In this paper a new construction for cyclic quasigroups is given. This construction is useful in constructing large numbers of nonisomorphic quasigroups satisfying a given cyclic identity or a consequence of a cyclic identity. The construction is based on a generalization of A. Sade's singular direct product of quasigroups. (Received October 5, 1972.)

*699-A36. CARL A. EBERHART, University of Kentucky, Lexington, Kentucky 40506. Elementary semigroups.

A semigroup is elementary provided it is generated by an element a and a nonvoid subset B of inverses of a. The structure of elementary inverse semigroups has been described by Gluskin, Mat. Sb. 41 (81) (1957), 23-36, and Eberhart and Selden, Trans. Amer. Math. Soc. 168(1972), 53-66. In this paper we investigate the structure of elementary regular semigroups. Among other things we establish the existence and uniqueness of free elementary orthodox semigroups and describe the structure of elementary semigroups with identity. (Received October 5, 1972.)

Analysis

*699-B1. GORDON G. JOHNSON, University of Houston, Houston, Texas 77004. Moment sequences in Hilbert space.

Necessary and sufficient conditions are given in order that the moment sequence $\{c_n\}_{n=0}^{\infty}$ of a function of bounded variation on $[0,1]$ is square summable, and it is established that the set of all square summable moment sequences is a dense linear subspace in the space of all square summable number sequences. (Received April 24, 1972.)

*699-B2. G. DiANTONIO, Pennsylvania State University, Middletown, Pennsylvania 17057. Closed orbit solutions for Bernoulli's equation.

Necessary and sufficient conditions for the existence of closed orbits, by using the author's alternate plane method, are given. (Received April 19, 1972.)

*699-B3. DAVID L. LOVELADY, Florida State University, Tallahassee, Florida 32306. Continuous dependence of critical points.

Let Λ be a first-countable topological space and let Y be a Banach space with norm $|\ |$. Let A be a continuous function from $\Lambda \times Y$ to Y and let α be a continuous function from Λ to $(0,\infty)$. Theorem. Suppose that $|x-y-c[A(\lambda,x) - A(\lambda,y)]| \geqq [1+c\alpha(\lambda)]|x-y|$ whenever (c,λ,x,y) is in $(0,\infty) \times \Lambda \times Y \times Y$. Let U be that unique function from Λ to Y with the property that $A(\lambda,U(\lambda)) = 0$ whenever λ is in Λ. Then U is continuous. (Received September 7, 1972.)

*699-B4. STUART MILLS, Louisiana State University, Baton Rouge, Louisiana 70803. Normed Köthe spaces as intermediate spaces L_1 and L_∞.

A Banach space X is an intermediate space of the Banach spaces X_1 and X_2 if $X_1 \cap X_2 \subset X \subset X_1 + X_2$ with norm-reducing embeddings. Theorem. $L_1 \cap L_\infty$ and $L_1 + L_\infty$ are associate Orlicz spaces, and for every Orlicz space $L_{M\Pi}$, there is an equivalent Orlicz norm $\|\cdot\|_{M\Pi}$, for which it becomes an intermediate space of L_1 and L_∞. This theorem is extended to the general setting of a Köthe space. Theorem. If Λ is a universal Köthe space, then $L_1 \cap L_\infty \subset \Lambda \subset L_1 + L_\infty$. Furthermore, if Λ is normed, i.e., $\Lambda = L_\rho$, then there exists an equivalent universally rearrangement invariant norm ρ_1 such that L_{ρ_1} is an intermediate space of L_1 and L_∞. (Received September 14, 1972.)

*699-B5. JOHN W. HEIDEL, University of Tennessee, Knoxville, Tennessee 37916 and I. T. KIGURADZE, Institute of Applied Mathematics, Tbilisi State University, Tbilisi, U.S.S.R. Oscillatory solutions for a generalized sublinear differential equation.

A criterion is given for the existence of oscillatory solutions for $u'' + f(t,u) = 0$ which generalizes a result for the sublinear case of $u'' + q(t)u^\gamma = 0$ (J. W. Heidel and Don B. Hinton, "The existence of oscillatory solutions for a nonlinear differential equation," SIAM J. Math. Anal. 3(1972), 344-351; and Kuo-liang Chiou, "The existence of oscillatory solutions for the equation $\ddot{y} + qy^r = 0$, $0 < r < 1$," Proc. Amer. Math. Soc., to appear). The present theorem is the analogue of a result of Izyumova for the generalized superlinear case (D. V. Izyumova, "On the conditions for the oscillation and nonoscillation of solutions of nonlinear second-order differential equations," Differential Equations 2(1966), 814-821). (Received September 18, 1972.)

*699-B6. WILLIAM H. RUCKLE, Clemson University, Clemson, South Carolina 29631. A series characterization of Hilbert space.

We prove the following Theorem. A reflexive Banach space is isomorphic to a Hilbert space if and only if it has the following property. (a) A series $\Sigma_n x_n$ in E converges absolutely whenever $\Sigma_n T_{x_n}$ converges absolutely for each T in $L(E, \ell^2)$. (Received September 18, 1972.)

*699-B7. RALPH GELLAR and ROBERT SILBER, North Carolina State University, Raleigh, North Carolina 27607. Banach spaces of $\{p_i\}$-summable sequences.

A class of Banach spaces is defined which generalizes the classical ℓ_p spaces. Given a sequence

$\{p_i\}$ of real numbers satisfying $1 \leq p_i \leq K$ for all i, $\ell\{p_i\}$ is the class of all sequences $\{z_i\}$ of complex numbers such that $\sum|z_i|^{p_i} < \infty$. $\ell\{p_i\}$ is a vector space implicitly normed by the equation $\sum|z_i/\|\{z_i\}\||^{p_i} = 1$, which is shown to define a complete norm. In the appropriate sense, the dual of $\ell\{p_i\}$ is $\ell\{q_i\}$, where the q_i are the traditional conjugates of the p_i, and the norms are dual only up to an equivalence. The case in which the q_i are unbounded and/or infinite needs and is given special attention. For example, if the q_i are unbounded, the $\{q_i\}$-summable sequences do not form a vector space, but the set of all sequences, some positive scalar multiple of which is $\{q_i\}$-summable, do. It is shown that members of this class of spaces can have the following property: the (unweighted) shift operator in one direction always sends sequences in the space back into the space, but the shift in the other direction can go out of the space. (Received September 22, 1972.)

*699-B8. JOEL H. SHAPIRO, Michigan State University, East Lansing, Michigan 48823 and PETER D. TAYLOR, Queen's University, Kingston, Ontario, Canada. Compact, nuclear, and Hilbert-Schmidt composition operators in H^2.

Let φ be an analytic function taking the unit disc into itself, and let C_φ denote the composition operator defined on H^2 by $C_\varphi f = f \circ \varphi$. We give a sufficient condition for the compactness of C_φ which shows that there exist compact composition operators on H^2 which are not Hilbert-Schmidt; and we show that C_φ fails to be compact whenever φ has an angular derivative at some point of the unit circle. In addition we show that C_φ is a nuclear (i.e. trace class) operator on H^2 whenever φ takes the unit disc into a polygon inscribed in the unit circle, but not every nuclear composition operator on H^2 arises from such a φ. In fact there exist univalent maps φ such that C_φ is nuclear, yet $\varphi(|z| < 1)$ is a smooth Jordan domain whose boundary touches the unit circle. (Received September 25, 1972.)

699-B9. RICHARD D. CARMICHAEL, Wake Forest University, Winston-Salem, North Carolina 27109. Representation of distributions in O'_α as boundary values of functions in tube domains.

Let $\alpha = (\alpha_1, \ldots, \alpha_n)$ be an arbitrary n-tuple of real members. O_α is the space of all complex-valued C^∞ functions such that for every n-tuple N of nonnegative integers there exists a constant $K_N > 0$ such that $|D^N\varphi(t)| \leq K_N(1+|t_1|)^{\alpha_1} \ldots (1+|t_n|)^{\alpha_n}$ for all $t \in R^n$. We define an appropriate convergence in O_α and denote O'_α as the space of all continuous linear functionals on O_α. Let C be an open connected cone, and let O(C) be its convex envelope. We define Cauchy and Poisson integrals of elements in O'_α, which are functions of the complex variable $z \in T^{O(C)} = R^n + iO(C)$, and obtain their properties. We represent elements in O'_α as the boundary values of these integrals on the distinguished boundary of the tube domain $T^{O(C)}$. (Received September 27, 1972.)

*699-B10. LARRY W. WILSON, Old Dominion University, Norfolk, Virginia 23508. A Laplace-Stieltjes transform of Mikusinski operator functions.

A Stieltjes integral of continuous complex valued functions of a real variable with respect to operator valued functions is defined and developed. This integral is used to define a Laplace-Stieltjes transform for operator valued functions. The transform is shown to be in one sense a generalization of the usual Laplace-Stieltjes transform. (Received October 2, 1972.)

699-B11. WILLIAM B. DAY, Auburn University, Auburn, Alabama 36830. More bounds for eigenvalues. Preliminary report.

A method introduced by W. Leighton (J. Math. Anal. Appl. 35(1971), 381-388) for bounding eigenvalues has been extended to include problems of the form $-y'' + p(x)y = \lambda y$ when $p(x) \geqq 0$ on $[0,1]$. The boundary conditions are the general homogeneous conditions: $y(0) - ay'(0) = y(1) + by'(1)$ where $0 \leqq a, b \leqq \infty$. Upper and lower bounds for the eigenvalues of these problems are obtained, and these bounds may be made as close together as desired thereby precisely estimating λ. (Received October 2, 1972.)

699-B12. CHARLENE V. HUTTON, Louisiana State University, Baton Rouge, Louisiana 70803. Operators of type ℓ^p. Preliminary report.

Let E and F denote B-spaces. For $T \in \mathcal{L}(E,F)$ let $\alpha_k(T) = \inf\{\|T - A\|\}$ where the infimum is taken over all $A \in \mathcal{L}(E,F)$ of rank $\leqq k$, and let $\rho_p(T) = (\sum_{k=1}^{\infty} \alpha_k(T)^p)^{1/p}$. Then $\ell^p(E,F) = \{T \in \mathcal{L}(E,F): \rho_p(T) < +\infty\}$. This class of operators was first introduced by A. Pietsch. Theorem 1. If $T: \ell^r \to \ell^q$ is a diagonal operator, $T \sim (\lambda_i)_{i=1}^{\infty}$ with $\lambda_i \searrow 0$ then $T \in \ell^p(\ell^r, \ell^q)$ iff (a) $(\lambda_i)_{i=1}^{\infty} \in \ell^p$ for $1 \leqq r \leqq q \leqq \infty$; (b) $\sum_{k=1}^{\infty} (\sum_{i=k+1}^{\infty} \lambda_i^s)^{1/s} < +\infty$ for $\infty \leqq q < r \leqq 1$ and $1/r + 1/s = 1/q$, taking $s = q$ if $r = \infty$ ((b) is due to P. Johnson if $\infty < q < r < 1$). Corollary. If $T \in \ell^1(\ell^{\infty}, \ell^1)$ then T is strongly $(1+\epsilon)/2$-summable (in the sense of Grothendieck) for every $\epsilon > 0$. Theorem 2. If $T \in \ell^{2/3}(E,F)$ then T is fully nuclear (in the sense of Retherford-Stegall). Corollary. $\mathcal{L}(E,F) = \ell^{2/3}(E,F)$ implies $\min\{\dim(E), \dim(F)\} < +\infty$. This improves a result of A. Pietsch. Clearly if $T \in \ell^p(E,F)$ then $T^* \in \ell^p(F', E')$. Theorem 3. If F has the approximation property then $T \in \ell^p(E,F)$ iff $T^* \in \ell^p(F',E')$ iff $T^{**} \in \ell^p(E'',F'')$. (Received October 2, 1972.)

699-B13. RONALD SHONKWILER, Georgia Institute of Technology, Atlanta, Georgia 30332. Multi-parameter resolvents. Preliminary report.

Let A^k be a selfadjoint operator in the Hilbert space H for $k = 1, 2, \ldots, m$ and a positive selfadjoint operator in H for $k = m+1, \ldots, m+n$. Let $Q^k_{\lambda^k} = \lambda^k(I - \lambda^k A^k)^{-1}$, $\operatorname{Im} \lambda^k \neq 0$ unless $\lambda^k = 0$, be the resolvent of A^k. If the spectral functions of the operators A^k commute pairwise then so will the resolvents Q^k and in this case we define the (m,n)-parameter resolvent Q as the composition $Q(\lambda^1, \ldots, \lambda^{m+n}) = Q^1_{\lambda^1} \cdots Q^{m+n}_{\lambda^{m+n}}$. We may characterize an (m,n)-parameter resolvent intrinsically by its values on a restricted subset of its λ^k domain. Let $S(u) = S(u^1, \ldots, u^{m+n}) = Q(iu^1, \ldots, u^{m+n})$ where $u \in \Gamma = R^m \times (-\infty, 0]^n$. Theorem. S(u) is the restriction of an (m,n)-parameter resolvent iff (i) $S(-u^1, \ldots, u^{m+n}) = S^*(u)$; (ii) for every subset $\{k_1, \ldots, k_p\} \subset \{1, \ldots, m+n\}$ the joint limit $S(u)/u^{k_1} \cdots u^{k_p}$ exists in the strong operator topology as the u^k's tend to zero. In particular $S(u)/\pi u \to i^m I$ as all coordinates tend to zero where $\pi u = u^1 \cdots u^{m+n}$; and (iii) the (m,n)-parameter resolvent equation holds, for $u_1, u_2 \in \Gamma$, $\pi(u_2 - u_1) S(u_2) S(u_1) = i^m \pi u_2 \pi u_1 \Delta_{u_1}^{u_2} S$ where $\Delta_{u_1}^{u_2} S = \sum (-1)^{\sum k_j} S(u^1_{k_1}, \ldots, u^{m+n}_{k_{m+n}})$ and the summation is taken over all $k_1, \ldots, k_{m+n} = 1, 2$. (Received October 2, 1972.)

699-B14. RALPH D. McWILLIAMS, Florida State University, Tallahassee, Florida 32306. On Banach spaces for which the quotient of the bidual by the space itself is separable. Preliminary report.

It is shown first that a theorem of Johnson and Rosenthal ("On w*-basic sequences and their applications to the study of Banach spaces," to appear) for spaces X for which X'' is separable, namely that each infinite-dimensional closed subspace of X or of X' contains an infinite-dimensional reflexive subspace, holds more generally for spaces X such that X''/JX is separable, where J is the canonical embedding. Related results are then given for such spaces. For example, if X''/JX is separable, Z_1 a w*-closed subspace of X', and Y_1 a norm-closed subspace of Z_1, then Y_1 contains a reflexive subspace W_1 with Z_1/W_1 separable if and only if the polar of Y_1 in Z_1' is separable. If X''/JX is separable, and if X has a reflexive subspace L such that X/L is separable or equivalently X' has a separable w*-closed subspace M such that X'/M is reflexive, then a closed subspace Y of X contains a reflexive subspace W such that X/W is separable if and only if the polar of Y in X' is separable. (Received October 2, 1972.)

*699-B15. THOMAS R. LUCAS, University of North Carolina, Charlotte, North Carolina 28213. Error bounds for interpolating cubic splines under various end conditions.

If $f \in C^5[a,b]$ and s is the corresponding interpolating $C^2[a,b]$ cubic spline over a uniform mesh $\{x_i\}_0^n$ with end conditions $f'(a) = s'(a)$, $f'(b) = s'(b)$, it is shown that at the points x_i, $x_i + .5h$, $x_{i+1}; x_i + \lambda h$; $x_i + 5h$ respectively, $0 \leq i \leq n-1$, the error $f' - s'$; $f'' - s''$; $f''' - s'''$ is of order $O(h^4)$; $O(h^3)$; $O(h^2)$, where $\lambda = (3 \pm \sqrt{3})/6$. These point error bounds are better by a factor of h than the well-known global error bounds. In addition for sufficiently smooth f, uniform meshes and appropriate end conditions, it is shown that certain finite difference type operators acting on the second derivatives of the interpolating cubic spline approximate f'', f''' and f^{iv} at the interior knots with error of order $O(h^4)$. To achieve this, new end conditions are proposed and developed. Finally, all of the above results are shown to hold locally over locally uniform meshes for locally smooth functions. (Received October 2, 1972.)

*699-B16. PAUL W. LEWIS, North Texas State University, Denton, Texas 76203. Permanence properties of absolute continuity conditions. Preliminary report.

Theorem 1. If m is a representing measure (Batt and Berg, J. Functional Analysis 4(1969)), then m is regular iff m is countably additive. Corollary. If H is a compact Hausdorff space, F is a Banach space, and $L: C(H) \to F$ is an operator with representing measure m, then L is weakly compact iff m is regular. Definition. If m and n are representing measures, $m, n: \Sigma \to B(E, F^{**})$, then we say that n is strongly absolutely continuous with respect to m and write $n \lll m$ provided that $n(A)x \in \bigcup_{\pi(A)} \{\Sigma m(A_i)x_i\}$, where x and x_i belong to the closed unit ball of E, A is a Borel set, and $\pi(A)$ denotes all Borel partitions of A. An example is given where m is countably additive, $n \lll m$, and n is not countably additive. If $m: \Sigma \to B(E, F^{**})$ is a representing measure, then let $m_x: \Sigma \to F^{**}$ be the induced measure for each $x \in E$. Theorem 2. If m and n are representing measures, m is countably additive, and $n_x \lll m_x$ for each $x \in E$, then n is countably additive. (Received October 3, 1972.)

699-B17. FRANKLIN P. WITTE, Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061. Extensions of vector-valued Stieltjes measures.

Using a result of J. K. Brooks we prove a generalization of the classical extension theorem for Stieltjes measures. Let X be a Banach space, T the unit interval, and S the algebra of subsets of T generated by intervals of the form [a, b) and [a, 1]. Theorem. Suppose that X contains no subspace isomorphic to c_0, and that $z: T \to X$ generates the Stieltjes measure $m: S \to X$. Then m has a unique extension to a Borel measure if and only if (i) z is left-continuous, and (ii) z is of weak bounded variation. A counterexample in c_0 shows that (i) and (ii) are not sufficient in general. Using the Theorem we can prove a corrected version of a result of Cramér for Stieltjes measures in Hilbert space. (Received October 3, 1972.)

*699-B18. DON B. HINTON, University of Tennessee, Knoxville, Tennessee 37916. Limit point criteria for positive definite fourth order differential operators.

Let $L(y) = [(ry'')' - qy']' + py$ where the coefficients are real, continuous, and satisfy $r(x) > 0$, $q(x) \geqq 0$, and $p(x) \geqq k > 0$. Denote by m the number of linearly independent $\mathcal{L}_2[a,\infty)$ solutions of $L(y) = 0$. For positive functions w and σ the following conditions are considered. (C_1) The functions $(qw)^{1/2}w'/w$, $(qw)^{1/2}/x$, $(rw)^{1/4}w'/w$, $(rw)^{1/4}/x$, $(rw)^{1/4}r'/r$, and $(rw)^{1/2}w''/w$ are $O(1)$ as $x \to \infty$. (C_2) There is a number $\theta_1 \geqq 0$ such that either $(qw')' \leqq \theta_1 pw$ or $(q/p)^{1/2}|w'/w| \leqq \theta_1/2$. (C_3) Either $(\int_a^t \sigma\, dx)^2 \leqq (\sigma qw)(t)$ or $\int^\infty \sigma\, dx < \infty$ and $(\int_t^\infty \sigma\, dx)^2 \leqq (\sigma qw)(t)$. Define θ_2, θ_3 and θ_4 by: $\theta_2 = \sup_{a\leqq x}|r^{1/2}w'/q^{1/2}w|$, $\theta_3 = \sup_{a \leqq x}|r^{1/2}w''/p^{1/2}w|$, and $\theta_4 = \sup_{a\leqq x}|r^{1/2}w''/(\sigma w)^{1/2}|$. Theorem. Suppose conditions (C_1) and (C_2) hold. Then $m = 2$ if either of the following is satisfied. (i) $\theta_1/2 + 2\theta_2 + \theta_3 < 1$; (ii) (C_3) holds and $\theta_1/2 + 2\theta_2 + 4\theta_4 < 1$; (iii) $(rw')' \leqq 0$ and $\theta_1/2 + \theta_3 < 1$. (Received October 3, 1972.)

*699-B19. WILLIAM ALAN FELDMAN, University of Arkansas, Fayetteville, Arkansas 72701. A characterization of the topology of compact convergences on C(X). Preliminary report.

The function space of all continuous real-valued functions on a realcompact topological space X is denoted by C(X). It is shown that a topology τ on C(X) is a topology of uniform convergence on a collection of compact subsets of X if and only if (*) $C_\tau(X)$ is a locally m-convex algebra and a topological vector lattice. Thus, the topology of compact convergence on C(X) is characterized as the finest topology satisfying condition (*). Consequences for A-convex algebras and convergence structures are discussed. (Received October 3, 1972.)

699-B20. PAUL A. NICKEL, North Carolina State University, Raleigh, North Carolina 27607. Linear topologies for continuity of Sario's linear operator method. II.

Our purpose here is the consideration of the isomorphism arising in the linear operator method of L. Sario, such isomorphism having been shown to be topological for the quotientized compact open topology [Abstract 687-31-2, these *Notices* 18(1971), 922]. Recalling the notation there, we let $\Phi: H(W) \to H(W')$ be the mapping sending each harmonic function p on the Riemann surface W into its restriction $p|_{W'}$, a singularity function defined on the regular boundary neighborhood W'. Rodin and Sario [Kōdai Math. Sem. Rep. (1967)] have shown that $\Phi: H(W)/K \to H(W')/LC(\alpha)$ is an isomorphism, where K is the set of all constant functions of H(W) and $LC(\alpha)$ is the set of all regular singularity functions on W'. Theorem 1. The mapping Φ is topological when each of the spaces E (domain of Φ) and F (range of Φ) is equipped with the quotient of the K-strict topology β_K. Theorem 2. The same is true when each space is equipped with the weak measure topology α_K. Each of these topologies is defined by using a set M_K of positive continuous functions on $W - \{\zeta_1, \ldots, \zeta_\ell\}$ vanishing at the boundary, to modify in a natural way, the α and β topologies of Rubel and Shields [Ann. Inst. Fourier (Grenoble) (1966)]. (Received October 5, 1972.)

699-B21. COKE S. REED, Auburn University, Auburn, Alabama 36830. A divergent weighted orthonormal series of broken line Franklin functions.

Set $(a_1, a_2, \ldots) = (0, 1/2, 1/4, 1/8, 3/8, 5/8, 7/8, 1/16, \ldots)$. Set $f_0(x) = 1$ and for each positive integer N, set $f_N(x) = 0$ if $0 \leqq x \leqq a_N$ and set $f_N(x) = x - a_N$ if $a_N \leqq x \leqq 1$. There exist a differentiable function g on [0,1] and a strictly increasing continuously differentiable function M on [0,1] such that if

$\varphi_1, \varphi_2, \ldots$ is the orthonormal function sequence obtained from $f_1, f_2, \ldots$ using the Gram-Schmidt process where the inner product $((h,k)) = \int_0^1 hk\, dM$, then the number sequence $\alpha_p = \sum_{i=1}^{p}((\varphi_i, g))\varphi(1)$ is unbounded. (Received October 4, 1972.)

*699-B22. HONG WHA KIM, Bucknell University, Lewisburg, Pennsylvania 17837. On unilateral shift and the norm closure of the range of a derivation.

Let H be a separable Hilbert space over the complex field $\mathbb{C}$ and $\delta_A(X) = AX - XA$ be an inner derivation on the algebra $\mathcal{L}(H)$ of all bounded operators on H. $R(\delta_A)^-$ stands for the norm closure of the range of δ_A in $\mathcal{L}(H)$ and $\{A\}'$ for the commutant of A. U stands for the unilateral shift of multiplicity 1 and U* its adjoint. The main purpose of this paper is to prove the existence of an operator which is not contained in $R(\delta_A)^- \cap \{A\}'$ for any A in $\mathcal{L}(H)$. Theorem. $U^* \notin R(\delta_A)^- \cap \{A\}'$ and $U \notin R(\delta_A)^- \cap \{A\}'$ for any A in $\mathcal{L}(H)$. (Received October 4, 1972.)

*699-B23. DOUGLAS MOREMAN, Auburn University, Auburn, Alabama 36830. Convex topology. Preliminary report.

Generalizations to metric spaces and to more abstract settings of theorems relating the notions of weak convergence, weak topology, weak Cauchy sequence, centers, near point properties, normal structure and fixed points of transformations, uniformly convex space, and so forth. A review of work mentioned in Abstracts 689-G6, 72T-B193 and 72T-B60, these *Notices* 18(1971), 1065, and 19(1972), A-580 and A-312, and some extensions of that work. (Received October 4, 1972.)

699-B24. MURIL L. ROBERTSON, Auburn University, Auburn, Alabama 36830. Functional differential equations. Preliminary report.

Sufficient conditions are given for g and F for the unique solution of the equation $y'(t) = F(t, y(g(t)))$ using a method given by Siu (Math. Z. 90(1965), 391-392). These results are compared with those of the author (Abstract 689-B3, these *Notices* 18(1971), 1052). (Received October 5, 1972.)

699-B25. RANDOLPH CONSTANTINE, JR., Department of Mathematical Sciences, Clemson University, Clemson, South Carolina 29631. A summability integral.

If n is a positive integer, f is a function from [0,1] into the complex numbers, and μ is a complex sequence, we define $L_n(f,\mu) = \sum_{p=0}^{n}\binom{n}{p} f(p/n)\,\Delta^{n-p}\mu_p$. Theorem. If f is a complex quasi-continuous function on [0,1] and μ is the moment sequence of a function φ of bounded variation, $\mu_n = \int_0^1 I^n d\varphi$ for $n = 0, 1, \ldots$, then the number sequence $L_{\cdot}(f,\mu)$ converges. It is known that the limit is $\int_0^1 f\, d\varphi$ if f is continuous. A formula is given for the limit which involves direct sum decompositions of the spaces Q[0,1] (= the space of all quasi-continuous functions on [0,1]) and BV[0,1] and employs the mean integral. (Received October 5, 1972.)

699-B26. KUO-LIANG CHIOU, University of Tennessee, Knoxville, Tennessee 37916. A nonoscillation theorem for the superlinear case of second order differential equations $y'' + yF(y^2, x) = 0$.

The differential equation (1) $y'' + yF(y^2, x) = 0$, where $yF(y^2,x)$ is continuous for $x > 0$ and $|y| < \infty$, $F(t,x)$ is nonnegative for $x > 0$ and $t \geqq 0$, and $F(t,x)$ is nondecreasing in t for fixed x, is considered and the following theorem is established. Theorem. Let $F(t,x)$ be of class C' and assume that: (a) there exist positive functions φ and ψ such that φ'' and ψ''' are continuous on $(0,\infty)$, and (i) $\psi(\varphi/\psi)' < 0$, (ii) $(\varphi\psi'/\psi)'' \leqq$

0; (b) $z'' + p(x)z = 0$ is nonoscillatory, where $p(x)$ is positive and continuous, and $F(Cf(x), x) < p(x)$ for $x > b$ where $f(x)$ is continuous for $x > b$ and $\int_a^x (-\psi(\varphi/\psi)')^{-1}\,ds \leqq Cf(x)$ for some positive constants C, a, and b; (c) if $G(t,x)$ is defined by $G(t,x) = \int_0^t F(s,x)\,ds$, then $\varphi G(\alpha\psi, x)$ is nonincreasing in x for every $\alpha > 0$. Then (1) is nonoscillatory. This theorem is applied to the well-known equation (2) $y'' + q(x)y^{2n-1} = 0$, $n > 1$, $x > 0$, where $q(x)$ is a positive continuous function on $x > 0$ and gives the following result. <u>Corollary</u>. If $q(x)x^{n+1}(\log x)^{\beta}$ is nonincreasing eventually where $\beta > 0$, then (2) is nonoscillatory. This improves a result of Nehari (J. Differential Equations 5(1969), 452-460) and shows that a reasonable conjecture suggested by Coffman and Wong (Trans. Amer. Math. Soc. 147(1970), 357-366) is not valid. (Received October 5, 1972.)

*699-B27. PAUL CARLTON WOODS, Auburn University, Montgomery, Alabama 36109. <u>Decompositions of operators.</u> Preliminary report.

Let E be a locally convex Hausdorff space. A family $\{T_i x : i \in I\}$ of (continuous, linear) operators on E is called a <u>decomposition of operators</u> for E provided E is the closed linear span of $\{T_i x : i \in I, x \in E\}$ and for each nonzero x in E there is an i in I such that $T_i x \neq 0$. If, in addition, $T_i \circ T_j = \delta_{ij} T_i$, $\{T_i : i \in I\}$ is called a <u>Markuschevich decomposition</u> for E. This concept generalizes the notions of Markuschevich basis and Schauder decomposition. Definitions of a shrinking and a boundedly complete decomposition of operators are given. A locally convex space with a decomposition of operators with closed ranges is semireflexive if and only if the ranges are semireflexive and the decomposition is both shrinking and boundedly complete. This theorem contains the result of W. B. Johnson [Trans. Amer. Math. Soc. 149(1970), 171-177] concerning Markuschevich bases and the result for Schauder decompositions proved by T. A. Cook [Math. Ann. 182(1969), 232-235].
(Received October 5, 1972.)

699-B28. KUSUM K. SONI and RAJ PAL SONI, University of Tennessee, Knoxville, Tennessee 37916. <u>Fourier kernels and slowly varying functions.</u> Preliminary report.

Let $F(x) = \int_0^\infty k(xt)f(t)\,dt$. We obtain the asymptotic behavior of $F(x)$, $x \to 0$, when $f(t) \sim t^{-\alpha} L(t)$, $t \to \infty$. The transform kernel k satisfies rather general conditions and $L(t)$ is a slowly varying function. If k is a Fourier kernel so that the inversion holds, then under some additional conditions, the asymptotic relationship is reciprocal. Our results are related to those given by Yong for the trigonometric series [J. Math. Anal. Appl. 33(1971), 24-34; 38(1972), 1-14], Pitman for the sine and the cosine transform [J. Austral. Math. Soc. 8(1968), 423-443] and Ridenhour and Soni for the Hankel transform [Abstract 71T-B263, these *Notices* 18(1971), 1104].
(Received October 5, 1972.)

*699-B29. M. ZUHAIR NASHED, Georgia Institute of Technology, Atlanta, Georgia 30332. <u>A functional equation which characterizes polynomial operators with applications to uniqueness.</u>

A functional equation related to Taylor's theorem in normed spaces is considered, and its most general solutions are characterized. As a byproduct, some simple local and global uniqueness results for solutions of polynomial operator equations are obtained and illustrated by problems from generalized inverses.
(Received October 5, 1972.)

*699-B30. SANDRIA N. KERR, Winston-Salem State University, Winston-Salem, North Carolina 27102. Infinite-dimensional manifolds modelled on abstract Wiener spaces.

An example is constructed of an infinite-dimensional manifold modelled on an abstract Wiener space, having certain crucial properties of the Riemann-Wiener manifolds defined by Kuo (Trans. Amer. Math. Soc. 159(1971), 57-78). This example has form $f^{-1}(0)$ for a suitable regular map defined on a Banach space and is constructed by applying standard manifold techniques to classes of maps described below: A triple (i, H, B) is called a scale if B is a Banach space, H a Hilbert space, and $i: H \to B$ a continuous linear injection whose image is dense in B. A scale is an abstract Wiener space if the spaces are real and the B-norm is a measurable norm when pulled back to H. With proper identifications we may write $B^* \subset H \subset B$, where B^* is the dual of B. Given scales (i, H, B) and (i', H', B'), a bounded linear map $T: B \to B'$ is called a scale map if $TH \subset H'$ and $TB^* \subset B'^*$; T is an S-map if $TB \subset B'^*$. (Received October 5, 1972.)

699-B31. CHARLES P. STEGALL, State University of New York, Binghamton, New York 13901. Duals of certain spaces with the Dunford-Pettis property. Preliminary report.

Theorem. There exists a separable Banach space X with the Schur property (hence the Dunford-Pettis property) such that X^* does not have the Dunford-Pettis property. Proof. Let $X = \{(r_{ni}): n = 1, 2, 3, \ldots, i = 1, \ldots, n,\ r_{ni}$ real (or complex) numbers, and $\|(r_{ni})\| = \sum_{n=1}^{\infty} (\sum_{i=1}^{n} (r_{ni})^2)^{1/2} < +\infty\}$. Then X is a Banach space isomorphic to a subspace of ℓ_1 (this is essentially the Kintchine inequality); thus X has the Schur property. Define $T: X \to \ell_2$ $T((r_{ni})) = (\sum_{i=n}^{\infty} r_{ni})_{i=1}^{\infty}$; it follows from the author's "Banach spaces whose duals contain $\ell_1(\Gamma)$ with applications to the study of dual $L_1(\mu)$ spaces," to appear Trans. Amer. Math. Soc., that the operator T is onto and $T^*(\ell_2)$ is complemented in X^*. Thus X^* has a reflexive, complemented subspace and cannot have the Dunford-Pettis property. (Received October 5, 1972.)

699-B32. RICHARD J. FLEMING and JAMES E. JAMISON, Memphis State University, Memphis, Tennessee 38152. Hermitian and adjoint abelian operators on certain Banach spaces. Preliminary report.

Let X be a complex linear space endowed with a semi-inner product $[\cdot, \cdot]$. An operator A on X will be called Hermitian if $[Ax, x]$ is real for all $x \in X$; A is said to be adjoint abelian if $[Ax, y] = [x, Ay]$ for all $x, y \in X$. Recently, Schneider and Turner [Abstract 691-15-5, these *Notices* 19(1972), A-66] have characterized matrices which represent Hermitian operators (on finite dimensional spaces with absolute norm) in terms of a certain equivalence relation on the coordinates. We extend this characterization to a certain class B of Banach spaces which includes ℓ_p-sums of Hilbert spaces. Theorem 1. Let $X \in B$. Then X is a direct sum of Hilbert spaces X_i and an operator A on X with operator matrix (A_{ij}) is Hermitian if and only if the matrix (A_{ij}) is diagonal and each A_{ii} is a Hermitian operator on the Hilbert space X_i. Theorem 2. Let X be an ℓ_p-sum of Hilbert spaces. Then an operator A on X is adjoint abelian if and only if A has an operator matrix representation $A = (A_{ij})$ which satisfies: (1) $A_{ij} = 0$ if $\dim X_i \neq \dim X_j$; (2) for each K there is one and only one j such that $A_{kj} \neq 0$; (3) if $A_{kj} \neq 0$, then $A_{kj} = A_{jk}^*$; (4) there is $\lambda > 0$ such that $\lambda^{-1} A_{kj}$ is an isometry for each $A_{kj} \neq 0$. Theorem 3. The class of Hermitian operators and the class of adjoint abelian operators are the same on an isomorph X of Hilbert space if and only if X is isometric to Hilbert space. (Received October 5, 1972.)

699-B33. JAMES K. BROOKS, University of Florida, Gainesville, Florida 32601. Measure and integration theory in Banach spaces.

• A survey on recent developments in the theory of measure and integration theory in Banach spaces in the following areas will be given: weak and strong compactness in the space of vector measures, operators on function spaces, Radon-Nikodym theorems for vector measures, interchange of limit theorems, extensions and decompositions of vector measures, representations of weak and strong integrals, existence of control measures. (Received October 5, 1972.)

Applied Mathematics

699-C1. CLINTON P. FUELLING, American University, Washington, D. C. 20016. Best Tschebyscheff approximation by interpolation polynomials. Preliminary report.

Let $X = (x_0^*, x_1^*, \dots, x_{n+1}^*)$ be an alternate set for the best Tschebyscheff approximating polynomial of degree n of a continuous function f(x) on [a, b]; N be the set of indices of x_i^* in X such that $x_i^* \neq a$ and $x_i^* \neq b$; $p(x; z_0, z_1, \dots, z_n)$ be the interpolating polynomial of f(x) determined by the interpolating points $z_0, z_1, \dots, z_n$; and $(z_0^*, z_1^*, \dots, z_n^*)$ be a set of interpolation points of the best approximating polynomial. Consider the system of equations $g_i(z_0, z_1, \dots, z_n, e) = p(x_i; z_0, z_1, \dots, z_n) - f(x) + (-1)^i e$ for $i = 0, 1, \dots, n+1$ and $p'(x_i; z_0, z_1, \dots, z_n) = f'(x_i)$ for $i \in N$. Assuming that $f(x) \in C^2[a, b]$, $f''(x_i^*) \neq p''(x_i^*; z_0^*, z_1^*, \dots, z_n^*)$ for $i \in N$, and $f'(z_j^*) \neq p'(z_j^*; z_0^*, z_1^*, \dots, z_n^*)$ for $j = 0, 1, \dots, n$, then a set of interpolation points that determine the best approximating polynomial is found by the Newton functional iteration method, $G'(z_0^{(k)}, z_1^{(k)}, \dots, z_n^{(k)}, e^{(k)}) \cdot (z_0^{(k+1)} - z_0^{(k)}, z_1^{(k+1)} - z_1^{(k)}, \dots, z_n^{(k+1)} - z_n^{(k)}, e^{(k+1)} - e^{(k)})^T = -G(z_0^{(k)}, z_1^{(k)}, \dots, z_n^{(k)}, e^{(k)})$, where G is the functional with elements g_i, G' is the Jacobian of G, and k is the iteration index. (Received October 3, 1972.)

*699-C2. R. D. RIESS, Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24060. Convergence and error estimates for Hermite and Fejér-Hermite interpolation. Preliminary report.

Four types of Hermite and Fejér-Hermite interpolation are considered. Error estimates are derived for each process for functions of different orders of continuity. Convergence results are proved for Fejér-Hermite interpolation at the roots of the Chebyshev polynomial of the second kind which are analogous to those of D. L. Berman at the roots of the Chebyshev polynomial of the first kind. (Received October 4, 1972.)

*699-C3. MANJUSRI MAJUMDAR, Centre of Advanced Study in Applied Mathematics, University of Calcutta, Calcutta 9, India and LOKENATH DEBNATH, East Carolina University, Greenville, North Carolina 27834. On forced oscillations in a rotating stratified liquid.

A study is made of an unsteady axisymmetric motion set up in an inviscid, incompressible, uniforml rotating stratified liquid by a small harmonic oscillation of a body immersed in the fluid. The initial value problem is solved by the Laplace transform treatment and the principal features of the flow are determined by singularities of the Laplace inversion integral and the relative magnitudes of the angular velocity Ω of basic rotation, the frequency ω of the forcing motion and the Brunt Väisälä frequency N. It is shown that when the forcing frequency lies between 2Ω and N, the disturbance propagates in the form of travelling waves which are radically modified by both rotation and stratification. On the other hand, when the frequency of the forcing motion is less or greater than N and 2Ω, the excited unsteady flow resembles an irrotational motion. It is also found that in the absence of stratification, the present analysis reduces to that of a rotating fluid. The properties of the resulting flow associated with various frequencies involved in the problem are analyzed with physical implications. Several special cases of interest are recovered from this rotating stratified flow analysis. (Received October 4, 1972.)

699-C4. STEVE BLUMSACK, Florida State University, Tallahassee, Florida 32306. Modelling the atmospheric circulation of the planet Mars.

Mathematical theory and techniques have played an important role in the advancement of knowledge of the earth's atmospheric circulation. The planet Mars presents another opportunity for the use of mathematics, particularly when the sparseness of detailed observations is considered. The relevant parameters and data appropriate to Mars are presented followed by a discussion of some relevant physical processes that take place in the Martian atmosphere. In particular, the generation of winds, weather systems and dust storms is discussed. The role of mathematics in studying these processes is considered, especially approximation techniques for solving partial differential equations and numerical analysis. (Received October 5, 1972.)

699-C5. WILLIAM M. BOYCE and D. M. KREPS, Bell Telephone Laboratories, Inc., Murray Hill, New Jersey 07974. A stochastic normative stock price model.

A model is described for the valuation of shares of common stock in a corporation which assumes random (but correlated) future earnings, postulates investor rationality, and does not require a finite horizon. The model recognizes the central role of the earnings growth trend in the present-day security analysis. The problems encountered in computing a normative stock price under such a relatively simple model indicate that progress in the field of finance may well depend on bringing to bear much more advanced mathematics than has been used in the past. (Received October 5, 1972.)

699-C6. DONALD A. DANIELSON, Department of Applied Mathematics and Computer Science, University of Virginia, Charlottesville, Virginia 22901. Human skin as an elastic membrane.

The equations governing the deformation of human skin are derived. First, the equations of an anisotropic, elastic membrane undergoing large deformations are recorded. Next, the kinematical condition is derived which restricts the skin to slide over the surface of a rigid foundation. Last, stress-strain relations are proposed which fit the known experimental data for skin. As a special case of this theory, the equations of a simple model for the flexure of a joint are solved. In another special case of the theory, the equations of a homogeneous, isotropic, elastic membrane lying in a plane and undergoing small deformations are solved by complex variable techniques for the case of a large sheet having a circular hole and subject to biaxial tension at infinity. Finally, some problems of interest to physicians and plastic surgeons are discussed. (Received October 5, 1972.)

699-C7. I. RICHARD SAVAGE, Florida State University, Tallahassee, Florida 32306. Sociology wants mathematics.

The current sociological literature contains many examples where the verbal presentation cries out for a formal (mathematical) structure. There are other examples where sociological writings have explicitly used mathematical models which need clarification and analysis. Twelve examples are given of these wants of mathematics. These examples can serve as an introduction for sociologists and mathematicians of the potential role of mathematics in sociology. Some of the examples suggest major research efforts, such as, the further

development of social psychology by more complete formal models and other examples suggest major developmental programs, such as, a decision theoretic approach to population prediction. The examples do not yield new areas of mathematics coming from sociology. Clearly, many of the published papers could have been clarified and strengthened by modest mathematical efforts in their writing and editing. But the full benefits of mathematical analysis usually would require a substantial effort to bring the available mathematical tools to productive development of the sociological content. (Received October 5, 1972.)

699-C8. H. ROBERT van der VAART, Department of Statistics, North Carolina State University, Raleigh, North Carolina 26707. Some problems arising from the interaction between biology and mathematics.

The interaction between mathematics and biology may take many forms, some of which are very much unlike the traditional image of mathematical activities. If mathematicians are to make a lasting and fruitful contribution to the development of biology, they should be willing to work along lines that are potentially quite different from what they expected or have been trained to expect. Several examples will be mentioned of interactions between the two fields to illustrate the general points made. A by-product is that a mathematician who is of substantial help to biological research may find himself out of favor with his own department, which will usually judge him by standards of mathematical originality rather than by the standards of the science to which he has made a good contribution: Good biology may turn out to be the result of the application of uninteresting mathematics. The key is whether the mathematical model is biologically sound and useful. (Received October 5, 1972.)

Statistics and Probability

*699-F1. WAYNE G. SULLIVAN, Georgia Institute of Technology, Atlanta, Georgia 30332. Finite range random fields and energy fields.

The lattice gas model is based on the integer lattice Z^{ν} with each lattice point either empty or occupied by exactly one of a finite number of particles. A problem in probability theory is to determine which probability measures on the space of configurations of the lattice correspond to a given family of conditional probabilities at each point of the lattice. In statistical mechanics one seeks probability measures which are consistent with a given potential on the lattice. It is shown that in a certain case these two problems are equivalent. Each finite range, positive, consistent family of conditional probabilities has a finite range potential and conversely. (Received September 29, 1972.)

699-F2. WILLIAM F. GRAMS, Vanderbilt University, Nashville, Tennessee 37235 and R. J. SERFLING, Department of Statistics, Florida State University, Tallahassee, Florida 32306. Rates of convergence in the normal approximation for sequences of martingale differences. Preliminary report.

Let $\{X_i\}_{i\geq 1}$ be a sequence of martingale differences (i.e., $E[X_i]=0$ and $E[X_i|X_1,X_2,\ldots,X_{i-1}]=0$ wp 1 for all i). Let $S_n=\sum_1^n X_i$, $B_n^2=E[S_n^2]$, $n=1,2,\ldots,\sigma_j^2=E_j^![X_j^2]$, where $E_j^![\cdot]$ denotes conditional expectation given $X_1,X_2,\ldots,X_{j-1}$, and let $\Phi(t)=(2\pi)^{-1/2}\int_{-\infty}^t e^{-u^2/2}\,du$. Assume that (a) $E[\sigma_j^2]\leqq M_1$ for all j, (b) $E[|X_j|^{2+\delta}]\leqq M_2E[\sigma_j^2]$ for all j and some $0<\delta\leqq 1$, and (c) $\sum_1^n E[|\sigma_j^2-E(\sigma_j^2)|]=O(B_n^{\alpha})$, $n\to\infty$, for some $-\infty\leqq\alpha\leqq 2$. Under these conditions we prove that $\sup_t|P[S_n/B_n\leqq t]-\Phi(t)|=O(B_n^{-\rho})$, where $\rho=\min\{(2-\alpha)/3,\ \delta/(1+\delta)\}$. Assume, further, that (d) $\sum_1^n E[|E_j^!(X_j^3)|]=O(B_n^{\beta})$, $n\to\infty$, for some $-\infty\leqq\beta\leqq 2$ and (e) $E[|X_j|^{3+\gamma}]\leqq M_3E(\sigma_j^2)$ for all j and some $0<\gamma\leqq 1$. Under conditions (a) – (e) we prove that

$\sup_t |P[S_n/B_n \leq t] - \Phi(t)| = O(B_n^{-\theta})$, where $\theta = \min\{(2-\alpha)/3, (3-\beta)/4, (1+\gamma)/(2+\gamma)\}$. These results provide rates which in some cases are faster than those of Heyde and Brown (Ann. Math. Statist. 41(1970), 2161-2165) and in other cases are similar but under alternative conditions. Also, these results extend Ibragimov's (Theor. Probability Appl. 8(1963), 83-89) rate given for a special randomly indexed sum of martingale differences. (Received October 3, 1972.)

Topology

*699-G1. JAMES AUSTIN FRENCH, David Lipscomb College, Nashville, Tennessee 37203. Ind and ind coincide for certain completely normal spaces.

The following theorem and corollaries give an answer to the question raised by Nagata on p. 68, General Topology Appl. 1(1971). Definition. For every space X, X is closure totally paracompact if and only if for every basis G of X there exists a closed cover H of X such that (1) for every $h \in H$, there exists $g \in G$ such that $h \subset \bar{g}$, (2) X has the weak topology with respect to H, and (3) for every $h \in H$, there exists a collection R of closed sets such that (a) $h - \mathrm{int}(h) = \bigcup R$, (b) $h - \mathrm{int}(h)$ has the weak topology with respect to R, and (c) for every $r \in R$, there exists $g \in G$ such that $r \subset B(g)$. Theorem. For every space X, if (1) X is completely normal, (2) for every $p \in X$, if U is an open set containing p, then there is an open set V containing p such that $\overline{V} \subset U$, (3) for every closed subset M of X, for every positive integer n, if K is a collection of closed subsets of M, K covers M, M has the weak topology with respect to K, and every element of K has Ind $\leq$ n, then Ind M $\leq$ n, and (4) X is closure totally paracompact, then ind X = Ind X. Corollary. If X is a closure totally paracompact space and X is either totally normal or X is a σ-Dowker space, then ind X = Ind X (σ-Dowker space definition, Math. Reviews, Jan. 1972, p. 219). (Received August 16, 1972.)

*699-G2. PETER V. O'NEIL, College of William and Mary, Williamsburg, Virginia 23185. A short proof of Mac Lane's planarity theorem.

This paper gives a short, elementary proof of Mac Lane's planarity theorem. (Received August 23, 1972.)

*699-G3. J. PELHAM THOMAS, Western Carolina University, Cullowhee, North Carolina 28723. Maximal connected Hausdorff spaces.

A connected topological $(X, \mathcal{T})$ space is said to be maximal connected if no refinement $\mathcal{T}'$ of $\mathcal{T}$ makes $(X, \mathcal{T}')$ connected. The question of the existence of maximal connected Hausdorff spaces remains open. An MI space is one in which every dense subset is open. A nasty space is one in which no point has a local base which is linearly ordered under set inclusion. Every maximal connected space is MI and every maximal connected Hausdorff space is nasty. There exist connected, MI, nasty Hausdorff spaces. (Received September 7, 1972.)

*699-G4. WILLIAM E. HAVER, University of Tennessee, Knoxville, Tennessee 37916. Function spaces on manifolds.

Let M be a compact manifold and let $\overline{H(M)}$ denote the space, under the compact open topology, of all continuous functions of M onto itself which can be approximated arbitrarily closely by homeomorphisms of M onto itself. Theorem 1. $\overline{H(M)}$ is homogeneous. Theorem 2. Every locally contractible metric space that is the countable union of finite dimensional compacta is an ANR. Let N be a manifold which is triangulated as a finite simplicial complex and let PLH(N) denote the space of all piecewise linear homeomorphisms of N onto itself. PLH(N) is locally contractible (Cernavskii) and is the countable union of finite dimensional spaces (Geoghegan). Therefore we have the following Corollary. PLH(N) is an ANR. (Received September 14, 1972.)

*699-G5. RICHARD E. CHANDLER and RALPH GELLAR, North Carolina State University, Raleigh, North Carolina 27607. The compactifications to which an element of $C^*(X)$ extends.

All spaces are completely regular. Two compactifications αX and γX of a space X are equivalent if there is a homeomorphism $h: \alpha X \to \gamma X$ which restricted to $X \subseteq \alpha X$ is the identity mapping onto $X \subseteq \gamma X$. If h is continuous but not a homeomorphism we say $\alpha X \geqq \gamma X$. For $f \in C^*(X)$ let $K(f)$ denote a set of compactifications of X satisfying: (i) for each $\alpha X \in K(f)$ there is a map $f^{\alpha} \in C^*(\alpha X)$ extending f, and (ii) if γX is any compactification to which f extends then there is an element of $K(f)$ equivalent to γX. Theorem 1. αX is a minimal element of $K(f)$ iff f^{α} is 1-1 on $\alpha X \setminus X$. Corollary. If X is realcompact and not compact then for no $f \in C^*(X)$ is $K(f) = \{\beta X\}$. Theorem 2. If $|\beta X \setminus X| \leqq \aleph_0$ then there is an $f \in C^*(X)$ for which $K(f) = \{\beta X\}$. Examples show that if $\aleph_0 < |\beta X \setminus X| \leqq c$ then it may or may not be the case that $K(f) = \{\beta X\}$.
(Received September 18, 1972.)

*699-G6. JACK B. BROWN, Auburn University, Auburn, Alabama 36830. Lusin density and other categoric densities.

A metric space is G_{II} (F_{II}) iff no open (perfect) set is 2nd category in itself. It is shown in Hausdorff's "Set theory" that completeness $\to F_{II} \to G_{II}$, but the implications are not reversible. Definitions. M is Lusin in itself iff every nowhere dense in M subset L of M is a union of countably many sets of local cardinality less than c. An L_1 set is the union of countably many sets, each Lusin in itself. An L_2 set is one which is not an L_1 set. A metric space is L_2-dense (T_2-dense) iff no open set is an L_1 set (the union of a 1st category set and an L_1 set). Theorem. In perfect metric spaces, $F_{II} \to T_2\text{-density} \begin{smallmatrix} \nearrow L_2\text{-density} \searrow \\ \searrow G_{II} \nearrow \end{smallmatrix}$ uncountable density, but none of the implications is reversible, and G_{II} and L_2-density are not comparable. Each of these notions of density has been shown to be the necessary and sufficient condition in some space to ensure the existence of a certain kind of continuous or differentiable restriction of an arbitrary real valued function. These relationships are discussed. (Received September 22, 1972.)

699-G7. SARASWATHI SUBBIAH, Canisius College, Buffalo, New York 14208. Topologies on semigroups of functions. Preliminary report.

S(X) denotes the semigroup, under composition, of all continuous selfmaps of the topological space X. A topology on S(X) is admissible if the mapping which sends (f, x) into $f(x)$ is continuous. It is completely admissible if S(X), with this topology, is a topological semigroup. Various results are obtained concerning the family of all admissible topologies and the family of all completely admissible topologies. For example, there is an extensive class of spaces such that for any space X of the class, the following statements are equivalent: (1) The admissible topologies on S(X) form a complete sublattice of the lattice of all topologies. (2) The compact-open topology is admissible. (3) The space X is locally compact. Moreover, for any space X, the compact-open topology is admissible if and only if it is completely admissible. Set-open topologies in general are examined in some detail and it is shown, among other things, that the set-open topologies always form a complete sublattice of the lattice of all topologies and the admissible set-open topologies form a complete sublattice if and only if the compact-open topology is admissible. (Received September 25, 1972.)

699-G8. V. KANNAN, Madurai University, Madurai, India and M. RAJAGOPALAN, Memphis State University, Memphis, Tennessee 38111. On scattered spaces. Preliminary report.

In this paper we characterise all metric spaces which are countable and scattered. We use this and characterise all countable regular spaces which admit a continuous bijection onto compact, T_2 space. This gives different proofs of some theorems of Katetov and Belnov. (Received September 25, 1972.)

699-G9. KENNETH D. MAGILL, JR., State University of New York at Buffalo, Amherst, New York 14226. Semigroups which admit few embeddings. Preliminary report.

S(X) denotes the semigroup, under composition, of all continuous selfmaps of the topological space X. In the main result, two classes of spaces are given such that if X is from the first and Y is from the second, then for each isomorphism φ from S(X) into S(Y), there exists a unique idempotent v of S(Y) and a unique homeomorphism h from X onto the range of v such that $\varphi(f) = h \circ f \circ h^{-1} \circ v$ for each f in S(X). The first class of spaces is fairly extensive while quite the opposite is true of the second. The second class, however, does contain the closed interval I and the space R of real numbers. For these two spaces, it follows that for any space X from the first class, S(X) can be embedded in S(I) if and only if X is homeomorphic to either I, the two-point discrete space or the one-point space and S(X) can be embedded in S(R) if and only if X is homeomorphic to either I, R, the two-point discrete space, the one-point space, or a half-open interval. Spaces at the other end of the spectrum, that is, spaces whose semigroups admit many embeddings are also discussed. (Received September 27, 1972.)

699-G10. JOHANNES M. AARTS, Delft Institute of Technology, Delft, The Netherlands and DAVID J. LUTZER, University of Pittsburgh, Pittsburgh, Pennsylvania 15213. Completeness properties and Baire spaces. Preliminary report.

The following results will appear in a forthcoming paper which studies the completeness properties introduced by Frolik, deGroot and Oxtoby. (1) Concerning Čech-completeness: Considering only completely regular spaces, both open maps and closed irreducible maps preserve the existence of a dense, Čech-complete subspace. The Tychonoff product of $\underline{c}$ copies of the real line does not contain a dense, Čech-complete subspace, nor does the Sorgenfrey line. The box product of countably many compact T_2-spaces may fail to contain a dense Čech-complete subspace. (2) Concerning subcompactness [Indag. Math. 25(1963), 761-767]: Considering only regular spaces, if X is locally subcompact, then X is subcompact and open mappings preserve subcompactness, provided either the domain or the range is locally metrizable. The Sorgenfrey line is subcompact but not base-compact; the same is true of the space given in Theorem 9 of [Duke Math. J. 17(1950), 317-327] and there is a space which is countably subcompact but not subcompact. A space which is subcompact and has weight $\underline{m} \geq \aleph_0$ is an absolute $G(\underline{m})$ space. (3) In Moore spaces, subcompactness is M. E. Rudin's weak completeness and there is a Moore space which is not a dense subset of any Moore space which is a Baire space. (Received September 27, 1972.)

*699-G11. RAYMOND F. DICKMAN, JR., Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061. Regularly closed maps.

All spaces are Hausdorff and all maps are continuous. A map $f: X \rightarrow Y$ is regularly closed if whenever A is regular closed in X, $f(A)$ is closed in Y. A subset A is far from the remainder (f.f.r.) if whenever $\mathcal{U}$ is an open ultrafilter without cluster points, there exists $U \in \mathcal{U}$ such that $A \cap \overline{U} = \emptyset$. A map $f: X \rightarrow Y$ is said to be an absolutely closed map if there does not exist a space Z and a map $F: Z \rightarrow Y$ such that X is a proper dense subset of Z and $F|X = f$. In a recent paper G. Viglino (Trans. Amer. Math. Soc., to appear) has shown that every map can be extended to an absolutely closed map. Theorem. A map $f: X \rightarrow Y$ is absolutely closed if and only if it is a regularly closed map with f.f.r. point inverses. Corollary. Every map can be extended to a regularly closed map with f.f.r. point inverses. Remark. If a point inverse is f.f.r., it is absolutely closed relative to X. (Received September 28, 1972.)

*699-G12. JAMES CLARENCE SMITH, Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061 and JOSEPH C. NICHOLS, Radford College, Radford, Virginia 24141. Embedding characterizations for expandable spaces.

Properties of various types of expandable spaces have been shown to play an important role in the study of collectionwise normal, strongly normal and paracompact spaces. Alo and Shapiro ["Countably paracompact, normal and collectionwise normal spaces", Indag. Math.] introduced the notion of strongly P-embedded subsets and proved the following Theorem. A space X is strongly normal iff every closed subset of X is strongly P-embedded in X. In this paper we introduced a number of new types of embeddings, similar to strong P-embeddings, and use them to characterize expandable, collectionwise normal and strongly normal spaces. (Received September 28, 1972.)

*699-G13. PHILLIP L. ZENOR, Auburn University, Auburn, Alabama 36830. Certain subsets of θ-refinable spaces are realcompact.

According to Worrell and Wicke, a space S is θ-refinable if and only if for every open cover U of S there is a countable family C of open covers of S each element of which refines U such that if p is a point of S, then some member of C is finite at p ["Topological spaces," Canad. J. Math. 17(1965), 820-830]. Theorem. The normal space X is realcompact if and only if (i) each discrete subset of X is realcompact and (ii) X can be imbedded as a closed subspace in the product of a collection of θ-refinable T_3-spaces. (Received September 29, 1972.)

699-G14. SIBE MARDEŠIĆ, University of Pittsburgh, Pittsburgh, Pennsylvania 15213. Equivalence of two notions of shape for metric spaces.

The notion of shape was first introduced by K. Borsuk for compact metric spaces [Proc. Internat. Sympos. Topology and its Applications (Herceg-Novi, 1968), pp. 98-104]. It was then generalized to compact Hausdorff spaces by S. Mardešić and J. Segal [Fund. Math. 72(1971), 41-59] and to arbitrary metric spaces by R. Fox [Fund. Math. 74(1972), 47-71]. The author has recently proposed a notion of shape for arbitrary topological spaces, which was an extension of the notion for compact Hausdorff spaces. It is now proved that this notion also extends the Fox notion of shape. (Received October 2, 1972.)

*699-G15. FRANK G. SLAUGHTER, JR., University of Pittsburgh, Pittsburgh, Pennsylvania 15213. A note on perfect images of spaces having a G_δ-diagonal.

In this paper we construct an example which shows that the perfect image of a space which can be condensed onto a metric space (also called a submetrizable space) is not necessarily submetrizable. Our example will also show that even if a perfect map is applied to an hereditarily paracompact space with G_δ-diagonal, the range of the map need not have G_δ-diagonal. In the course of constructing our example, we describe a simple technique for constructing at most two-to-one closed mappings having the Michael line as domain. (Received October 2, 1972.)

*699-G16. B. B. EPPS, JR., University of Houston, Houston, Texas 77004. Strongly confluent mappings.

A mapping $f: X \to Y$ from a continuum X onto a continuum Y is strongly confluent provided that if C is a connected set in Y then each component of $f^{-1}(C)$ is mapped onto all of C by f. The following theorem generalizes a theorem of G. T. Whyburn ["Interior transformations on compact sets", Duke Math. J. (1937), 370]. Theorem 1. Let X be a locally connected continuum and let $f: X \to Y$ be an open mapping from X onto Y. Then f is strongly confluent. Let A be the class of all curves X such that every connected subset of X is path-connected. Using Theorem 1 the author obtains: Theorem 2. If X is in A and if $f: X \to Y$ is a confluent mapping from X onto Y, then Y is in A. (Received October 2, 1972.)

*699-G17. FRANK E. SIWIEC, City University of New York, John Jay College, New York, New York 10010. On defining a space by a weak base.

Arhangel'skii has defined the concepts of a weak base and a g-first countable space (= gf-axiom of countability = weak first axiom of countability). We say that a space X is g-second countable if for each $n \in N$ there exists a subset B_n of X and corresponding to this subset there also exists a collection B_n of (not necessarily all) pairs (B_n, x) with x being a point of B_n, such that a set U is open in X iff for each point x in U there exists a pair (B_n, x) for which $B_n \subset U$. A g-metrizable space is defined similarly. A regular g-second countable space is cosmic, and a g-metrizable space is a σ-space. Regular + g-second countable $\to$ g-metrizable $\to$ symmetrizable $\to$ g-first countable $\to$ sequential. A Frechet space which satisfies any of the preceding properties satisfies a corresponding stronger property, e.g., a Frechet g-metrizable space is metrizable. For each σ-space, a g-metrizable space may be defined by enlarging the topology. Arhangel'skii has claimed that a g-first countable space which is a closed image of a metric space is metrizable. We may see this by showing that a g-first countable space satisfies a weakening of a property of Michael in his "Quintuple quotient quest". (Received October 2, 1972.)

699-G18. FRANK L. SCOTT, Wake Forest University, Winston-Salem, North Carolina 27109. A d-sequential completion for quasi-metric spaces.

A quasi-pseudometric space is called d-sequentially complete if every d-Cauchy sequence converges; a sequence $\{x_n : n \in N\}$ is d-Cauchy if for every $\epsilon > 0$ there exists an integer $n(\epsilon)$ such that $d(x_n, x_m) < \epsilon$ if $n \geqq m \geqq n(\epsilon)$ [R. A. Stoltenberg, Proc. London Math. Soc. (3) 17(1967), 226-240]. Using these definitions the following is proved: Theorem. Every bounded quasi-metric space can be isometrically embedded as a dense subspace of a d-sequentially complete quasi-pseudometric space of real valued functions. (Received October 2, 1972.)

*699-G19. EUGENE M. NORRIS, University of South Carolina, Columbia, South Carolina 29208 and A. R. BEDNAREK, University of Florida, Gainesville, Florida 32601. Inducing functions difunctionally. Preliminary report.

After Riguet [C. R. Acad. Sci. Paris 230(1950), 1999-2000], a relation R from a set X to a set Y is difunctional if $R \cdot R^{-1} \cdot R \leqq R$, where, in general, we write $R \cdot S = \{(x,y) : \exists\, z.\ (x,z) \in R \text{ and } (z,y) \in S\}$. We write $XR = \{y \in Y : \exists\, x \in X.\ (x,y) \in R\}$ and $RY = XR^{-1}$. Theorem. If X and Y are both compact Hausdorff or both discrete spaces, and if R and S are closed difunctional relations from X to Y and from Y to X, respectively, which satisfy the conditions (1) $X = RY$ and $Y = SX$ and (2) $R^{-1} \cdot R \leqq S \cdot S^{-1}$, then there is a unique continuous function $h : X/R \cdot R^{-1} \to Y/S \cdot S^{-1}$ making a commutative diagram with the projections of R upon X and Y and with the canonical quotient maps. If, additionally, $S \cdot S^{-1} \leqq R^{-1} \cdot R$, then h is a homeomorphism. This result subsumes that of Bednarek and Wallace [Math. Systems Theory 1(1967), 217-224] and certain of Riguet's theorems, and casts some topological light on results of Lambek [Canad. J. Math. 10 (1958), 45-55]. (Received October 2, 1972.)

*699-G20. WENDELL LEWIS MOTTER, Florida State University, Tallahassee, Florida 32306. Homology of coverings of spun CW pairs with applications to knot theory.

Let K be a connected CW complex and L a connected subcomplex, define $\chi_p(K, L)$, the p-spin of the CW pair (K, L) as $S^p \times K \cup D^{p+1} \times L$ identified along $S^p \times L$; here (D^{p+1}, S^p) is the standard disk, sphere pair $(p \geqq 1)$. If $(\tilde{K}, \pi)$ is a regular covering of K with G the group of covering translations, then a regular covering $\widetilde{\chi_p(K, L)}$ of $\chi_p(K, L)$ is $S^p \times \tilde{K} \cup D^{p+1} \times \tilde{L}$ identified along $S^p \times \tilde{L}$ where $\tilde{L} = \pi^{-1}(L)$. Here $\gamma \in G$ acts on cells $\sigma \times \tau$ of $\widetilde{\chi_p(K, L)}$ by $\gamma(\sigma \times \tau) = \sigma \times (\gamma\tau)$. Using the Eilenberg-Zilber theorem, a tensor-product chain complex C_* is constructed by which the homology of $\widetilde{\chi_p(K, L)}$ is calculated as a module over ZG, the integral group ring of G. One proves that $H_i(\widetilde{\chi_p(K, L)}) \cong_{ZG} H_i(\tilde{K})$, $i \leqq p$; and $H_i(\widetilde{\chi_p(K, L)}) \cong_{ZG} H_{p+n}(\tilde{K}) \oplus H_n(\tilde{K}, \tilde{L})$, $i = p + n$, $n \geqq 1$. By means of the Fox free derivatives for a "geometric presentation" of $\Pi_1(K)$, one constructed using all the generators of $\Pi_1(L)$, and C_*, a presentation is derived for $H_1(\tilde{K}, \tilde{L})$. When K is homotopy equivalent to the complement of κ, a classical (3,1) knot, this gives a presentation of the $(p+1)$ homotopy group of the complement of the knot obtained by p-spinning κ. Other results are obtained for the homology of coverings of complements of p-spun higher-dimensional knots. (Received October 2, 1972.)

*699-G21. RAYMOND Y.-T. WONG, University of California, Santa Barbara, California 93106. Periodic action on (I-D) spaces.

Let E be a space homeomorphic $(\cong)$ to F^∞ or F_f^∞ for some normed linear space F. Let n be a prime. Suppose $\alpha, \beta : E \to E$ are fixed point free periodic homeomorphisms of period n. The orbit spaces E/α, E/β are E-manifold of (Eilenberg-Mac Lane) type $(Z_n, 1)$. Since n, m classify the homotopy types of type (Z_n, m) spaces, which in turn classify E-manifolds, then $E/\alpha \cong E/\beta$. Theorem. There is a homeomorphism $h : E \to E$ such that $\alpha \circ h = h \circ \beta$. In the above theorem we also say α is conjugate to β. Now let β_0 be any homeomorphism of X. We say β_0 revolves trivially at $x \in X$ provided there are arbitrarily small homotopically trivial neighborhoods $\{U\}$ of x such that $\beta_0(U) = U$. Theorem. Let $\alpha_0, \beta_0 : Q \to Q$ be homeomorphisms of the Hilbert cube Q having exactly $0 \in Q$ as fixed point. Suppose α_0, β_0 are periodic of period n, n a prime $\geqq 2$ and suppose β_0 revolves trivially at 0, then α_0 is conjugate to β_0 if and only if α_0 revolves trivially at 0. Theorem. Suppose β_0 is as above, then $Q/\beta_0 \times Q \cong Q$. (Received September 29, 1972.)

*699-G22. RICHARD M. SCHORI and DOUGLAS W. CURTIS, Louisiana State University, Baton Rouge, Louisiana 70803. Hyperspaces of compact connected polyhedra. Preliminary report.

Let 2^X be the hyperspace of nonempty closed subsets of a compact metric space X, with the Hausdorff metric. Schori and West have previously shown that $2^\Gamma \approx Q$, the Hilbert cube, for every nondegenerate finite connected graph Γ. We prove the following Theorem. $2^K \approx Q$ for every nondegenerate, compact connected polyhedron K. The most general and still open question in this direction, posed by Wojdyslawski in 1938, is whether $2^X \approx Q$ for every nondegenerate Peano continuum X. Lemma. Let X be a compactum and (X_i, f_i) an inverse sequence of subcompacta such that (i) $\lim X_i = X$, (ii) $d(f_i, id) < 2^{-i}$ for each i, and (iii) $\{f_i \circ \ldots \circ f_j\}_{j=i}^{\infty}$ is an equi-uniformly continuous family of maps for each i. Then $X \approx \operatorname{inv\,lim}(X_i, f_i)$. The theorem is proved by constructing an increasing sequence of finite connected graphs $\{K_i\}$ in K converging to K, and maps $\varphi_i : K_{i+1} \to C(K_i)$ (the subcontinua of K_i) such that the induced maps $f_i : 2^{K_{i+1}} \to 2^{K_i}$ are retractions and near-homeomorphisms, and satisfy conditions (ii) and (iii) of the lemma. Then $2^K \approx \operatorname{inv\,lim}(2^{K_i}, f_i)$, and by a theorem of Morton Brown, $\operatorname{inv\,lim}(2^{K_i}, f_i) \approx Q$. (Received October 2, 1972.)

*699-G23. DOUGLAS W. CURTIS and RICHARD M. SCHORI, Louisiana State University, Baton Rouge, Louisiana 70803. Spaces of subcontinua of compact connected polyhedra. Preliminary report.

Let C(X) be the space of nonempty subcontinua of a continuum X, with the Hausdorff metric. West has shown that if D is a dendron with a dense set of branch points, then $C(D) \approx Q$. We use this, and the lemma of the preceding abstract, in proving the following Theorem. $C(K) \times Q \approx Q$ for every compact connected polyhedron K, and $C(K) \approx Q$ iff K is nondegenerate and contains no open 1-cell. We construct a sequence of finite connected graphs $\{K_i\}$ and maps $\varphi_i : K_{i+1} \to C(K_i)$ as before, and consider the induced maps $g_i : C(K_{i+1}) \to C(K_i)$. It is shown that each $C(K_i) \times Q \approx Q$ and each map $g_i \times id_Q : C(K_{i+1}) \times Q \to C(K_i) \times Q$ is a near-homeomorphism. If K is nondegenerate and contains no open 1-cell, we consider a corresponding sequence $\{K_i^*\}$ of connected 1-dimensional ANR's in K; each K_i^* is obtained from K_i by adjoining a countably infinite number of "stickers" in such a way that the branch points are dense in K_i^*, K_i^* contains no additional simple closed curves, $K_i^* \subset K_{i+1}^*$, and $K_i^* \subset \bigcup_{i=1}^{\infty} K_i$. Then $C(K_i^*) \approx Q$, and there exist retractions $g_i^* : C(K_{i+1}^*) \to C(K_i^*)$, obtained by a slight modification and extension of the maps $\{g_i\}$, such that the $\{g_i^*\}$ are near-homeomorphisms and satisfy conditions (ii) and (iii) of the lemma. Thus $C(K) \approx \operatorname{inv\,lim}(C(K_i^*), g_i^*) \approx Q$. (Received October 2, 1972.)

699-G24. WILLIAM K. MASON, Rutgers University, New Brunswick, New Jersey 08903. Spaces of homeomorphisms.

Denote by $H(I^n)$ the space of self-homeomorphisms of the n-cell which are fixed on the boundary ($H(I^n)$ has the sup norm topology). Theorem. If $n \neq 4$, then the following are equivalent: (1) $H(I^n)$ is an absolute retract, (2) Z-sets in $H(I^n)$ are s-deficient, (3) given $\epsilon > 0$ and a Z-set K in $H(I^n)$, there is a map $f : H(I^n) \to H(I^n) - K$ which moves no point more than ϵ. A closed set K in $H(I^n)$ is a Z-set if U - K is a nonempty homotopically trivial set whenever U is an open, nonempty, homotopically trivial subset of $H(I^n)$. A set K in $H(I^n)$ is s-deficient if there is a homeomorphism of $H(I^n)$ onto $H(I^n) \times s$ (s is the countable product of real lines), taking K into $H(I^n) \times \{\text{origin}\}$. (Received October 2, 1972.)

699-G25. R. RICHARD SUMMERHILL, Kansas State University, Manhattan, Kansas 66502. A finite-dimensional space having many of the topological properties of separable Hilbert space ℓ_2.

Theorem. For any integer $k \geqq 0$, there exists a k-dimensional topologically complete separable metric space P^k with the following properties: (1) Any k-dimensional compactum X can be embedded in P^k. (2) Any σ-compact subset of P^k is strongly negligible in P^k. (3) If f and g are embeddings of a compactum into P^k, then there exists a homeomorphism $h: P^k \to P^k$ such that $hf = g$. (4) If X and Y are compact subsets of P^k, then $(P^k/X, \hat{X})$ and $(P^k/Y, \hat{Y})$ are homeomorphic if and only if X and Y have the same shape. The space P^k is the k-dimensional universal pseudo-interior in E^{2k+2} constructed by Ross Geoghegan and the author in "Pseudo-boundaries and pseudo-interiors in Euclidean spaces and topological manifolds" (see Abstract 72T-G110, these *Notices* 19(1972), 545-546). (Received October 2, 1972.)

*699-G26. DONALD F. REYNOLDS, West Virginia University, Morgantown, West Virginia 26506 and JOE A. GUTHRIE and H. EDWARD STONE, University of Pittsburgh, Pittsburgh, Pennsylvania 15213. Expanding connected topologies.

Let (X, τ) be a topological space and let $A \subseteq X$. Denote by $\tau(A)$ the topology having $\tau \cup \{A\}$ as subbase. Theorem. Let (X, τ) be connected and let $A \subseteq X$. Then $(X, \tau(A))$ is connected if and only if there exists no nontrivial $C \subseteq X$ such that all of the following hold: (i) $C \cap A$ belongs to $\tau|A$, (ii) $(X-C) \cap A$ belongs to $\tau|A$, (iii) $C \cap (X-A)$ belongs to $\tau|(X-A) \cup (X-C)$, (iv) $(X-C) \cap (X-A)$ belongs to $\tau|(X-A) \cup C$. Corollary 1. Let (X, τ) be connected and let $A \subseteq X$. If $X - A$ is connected and no component of A is τ-closed, then $(X, \tau(A))$ is connected. Corollary 2. Let (X, τ) be connected, and let $A \subseteq X$ be connected. If no union of components of $X - A$ is open in X, then $(X, \tau(A))$ is connected. Corollary 3. Let (X, τ) be connected, and let $A \subseteq X$ be connected with connected complement. If A is not closed, then $(X, \tau(A))$ is connected. Corollary 4. Let (X, τ) be connected and let A be dense in some τ-open set. Then $(X, \tau(A))$ is connected. Corollary 3 was first proved by Borges [Canad. J. Math. 19(1967), 474-487] and follows from either of Corollaries 1 or 2. (Received October 3, 1972.)

699-G27. THOMAS A. CHAPMAN, University of Kentucky, Lexington, Kentucky 40506. Triangulation and classification of Hilbert cube manifolds. Preliminary report.

A Hilbert cube manifold, or Q-manifold, is a separable metric space which has an open cover by sets which are homeomorphic to open subsets of the Hilbert cube Q. We announce here some results which extend to the noncompact case recent results of the author's concerning compact Q-manifolds and the invariance of Whitehead torsion. Theorem 1. Every Q-manifold can be triangulated, i.e. every Q-manifold is homeomorphic to $P \times Q$, for some locally compact polyhedron P. Theorem 2. If P_1, P_2 are locally compact polyhedra and $f: P_1 \to P_2$ is a proper homotopy equivalence, then f is an infinite simple homotopy equivalence (in the sense of Siebenmann) iff the map $f \times \mathrm{id}: P_1 \times Q \to P_2 \times Q$ is proper homotopic to a homeomorphism. Corollary 1. If P_1, P_2 are locally compact polyhedra and $f: P_1 \to P_2$ is a homeomorphism, then f is an infinite simple homotopy equivalence. Corollary 2. If X, Y are Q-manifolds, then X is homeomorphic to Y iff the polyhedra involved in any triangulations of X and Y, respectively, have the same infinite simple homotopy type. (Received October 4, 1972.)

699-G28. ROSS GEOGHEGAN, State University of New York, Binghamton, New York 13901. Applications of infinite-dimensional topology.

Certain spaces of maps arising in analysis and certain spaces of cycles arising in the calculus of variations ("geometric measure theory") will be discussed. The questions will be: (1) Does infinite-dimensional topology help us in the study of these spaces? (2) What do these naturally arising infinite-dimensional spaces tell us about how infinite-dimensional topology ought to develop? (Received October 4, 1972.)

699-G29. JAMES E. WEST, Cornell University, Ithaca, New York 14850. Sums of Hilbert cube factors.

A brief introduction is given to the recently developed theory of Hilbert cube factors (spaces X with the property that $X \times Y$ is homeomorphic to the Hilbert cube for some space Y) and its uses. This is followed by a presentation of the Sum Theorem for Hilbert cube factors. A union of two Hilbert cube factors is itself a Hilbert cube factor provided that their intersection is one. (Received October 4, 1972.)

699-G30. RICHARD E. HEISEY, Cornell University, Ithaca, New York 14850. Stable classification of Q^{∞}-manifolds. Preliminary report.

Let F be one of (1) the conjugate of a separable infinite-dimensional Banach space with its bounded weak* topology, (2) a separable, reflexive, infinite-dimensional Banach space with its bounded weak topology, (3) $Q^{\infty} = \varinjlim Q^n$, where Q is the Hilbert cube. Let M and N denote paracompact connected F-manifolds. Theorem 1. There is a closed split embedding $f: M \to F$ and an open embedding $g: M \times F \to F$. Theorem 2 (Stable Classification). If $f: M \to N$ is a homotopy equivalence, then there is a homeomorphism $h: M \times F \to N \times F$ such that h is homotopic to $f \times \mathrm{id}$. Theorem 3. If U is an open subset of F, then $U \times F$ is homeomorphic to U. Corollary 4. Two open subsets of F are homeomorphic if and only if they have the same homotopy type. Remark. Theorems 1 and 2 are true more generally; in particular when $F = R^{\infty} = \varinjlim R^n$ (see also Abstract 72T-G141, these *Notices* 19(1972), A-611). (Received October 4, 1972.)

699-G31. DENNIS C. HASS, Randolph-Macon Woman's College, Lynchburg, Virginia 24504. Recognizing closed cells. Preliminary report.

Let M be a compact connected metrizable generalized m-manifold, $m > 1$. Theorem. If M is a product space $I \times Y$, and if its boundary is a manifold, then each of M and the interior of Y is a manifold. (I is the unit interval.) This generalizes [C. H. Edwards, Jr., "Cartesian factorization of compact 3- and 4-manifolds," Duke Math. J. 30(1963), 355-361]. Let $m = 4$. Theorem. If $M = X \times Y$ is a product space with a sphere boundary, then each of X, Y, and M is a closed cell. This generalizes [R. H. Bing, "A set is a 3-cell if its cartesian product with an arc is a 4-cell," Proc. Amer. Math. Soc. 12(1961), 12-19]. Let $m > 1$. Theorem. If M is a product space $X \times Y$ where the dimension of X is 1 or 2, and if the boundary of M is a sphere, then each of M and X is a closed cell. Corollary. If M is a product space with sphere boundary and $m < 6$, then M is a closed cell. These theorems depend on [D. C. Hass, "The ends of product manifolds," Proc. Amer. Math. Soc., to appear]. (Received October 4, 1972.)

*699-G32. JERRY E. VAUGHAN, University of North Carolina, Chapel Hill, North Carolina 27514. Product spaces with compactness-like properties. II.

The first part of this paper was announced in these *Notices* 19(1972), A-665. The second part of the paper contains a sufficient condition for a countable product of spaces to be $[m,n]$-compact. For infinite cardinal numbers $m \leqq n$, a space is said to be $[m,n]$-compact provided that every open cover of cardinality less than or equal to n has a subcover of cardinality less than m. A space is said to satisfy property $(2)_{m,n}$ provided for every collection of subsets F with card $F \leqq n$ which satisfies the condition that if $F' \subset F$ and card $F' < m$, then $\bigcap F' \neq \emptyset$, there exists a compact set K and a collection G such that card $G \leqq n$, if $G' \subset G$ and card $G' < m$ then $\bigcap G' \in G$, and, considered as a filter base, G is finer than both F and the filter base of open sets containing K. Theorem. If each space X_i satisfies $(2)_{m,n}$ and m is regular and $\sum\{n^r : r < m\} = n$, then $\Pi\{X_i : i=1,2,\ldots\}$ is $[m,n]$-compact. Corollary (N. Noble). A countable product of Lindelöf P-spaces (i.e., every G_δ is open) is a Lindelöf space. (Received October 4, 1972.)

*699-G33. ROBERT E. ATALLA, Ohio University, Athens, Ohio 45701. P-sets in $\beta N - N$.

Let T be a nonnegative regular matrix, F the filter of sets A such that T-lim $\chi_A = 1$, and K_F the corresponding closed set in $\beta N - N$. Henriksen and Isbell (Abstract 608-116, these *Notices* 11(1964), 90-91) showed that K_F is perfect and a 'P-set', i.e., is interior to any G_δ set which contains it. Assuming the continuum hypothesis, we prove Theorem. There exists a family of 2^c pairwise disjoint perfect nowhere dense P-sets contained in K_F, each the support set of a regular Borel probability measure. This improves our previous announcement (Abstract 678-B4, these *Notices* 17(1970), 931-932). (Received October 4, 1972.)

*699-G34. JEONG S. YANG, University of South Carolina, Columbia, South Carolina 29208. On isomorphic groups and homeomorphic spaces.

For a topological space X and a topological group G, let $C(X, Z)$ be the topological group of all continuous functions from X into Z endowed with the compact-open topology and with the pointwise multiplication. In this note, we are interested in the group structure of $C(X, Z)$ in relation to the topological structure of X. It is clear that if X and Y are homeomorphic spaces, then $C(Y, Z)$ and $C(X, Z)$ are isomorphic under an isomorphism which maps every constant function on Y into the corresponding constant function on X. It is shown that the reverse situation holds under certain conditions. (Received October 4, 1972.)

*699-G35. JAMES M. BOYTE, Appalachian State University, Boone, North Carolina 28607. δ_p and countably paracompact spaces. Preliminary report.

Definition. Let X be a space and let $A \subset X$. A collection of sets C is locally finite with respect to A if and only if for each $x \in A$ there exists an open set V, $x \in V$, such that V intersects only a finite number of members of C. Definition. A space X is called δ_p if and only if given any countable open cover C of X and given any closed set F contained in any member of C, then there exists an open refinement of C that is locally finite with respect to F. Theorem. The following are equivalent for a space X. (i) The space X is countably paracompact. (ii) The space X is δ_p and countably metacompact. (iii) If I is the closed unit interval, then $X \times I$ is δ_p. (iv) If C is an open cover of X, then there exists a countable collection $\{L_i | i=1,2,3,\ldots\}$ of open refinements of C such that for each $x \in X$ there is some L_i that is locally finite with respect to X. (Received October 5, 1972.)

The November Meeting in Cleveland, Ohio
November 25, 1972

Algebra & Theory of Numbers

*700-A1. JOHN S. HSIA and ROBERT P. JOHNSON, Ohio State University, Columbus, Ohio 43210. Witt's multiplicative forms over $R(t)$.

An anisotropic quadratic form φ is called round if $\varphi \cong a\varphi$ whenever φ represents $a \neq 0$. Witt has used the concept of a round form (a generalization of Pfister's concept of a strongly multiplicative form) to give new simple proofs of results of Pfister on the structure of the Witt ring over fields (see e.g. F. Lorenz ["Quadratische Formen über Körpern," Lecture Notes in Math., no. 130, Springer-Verlag, 1970]). The authors have previously determined all round forms over a global field. Now all round forms over $R(t)$, the field of rational functions in one variable over the reals, are explicitly determined. Connections with Pfister's strongly multiplicative forms (over $R(t)$) and with the algebraic K-theory groups $k_n = K_n/2K_n$ of Milnor are shown. (Received August 21, 1972.)

*700-A2 WILLIAM G. CHANG, Cleveland State University, Cleveland, Ohio 44115. Structure spaces in vector lattices.

The representation theorem of Johnson and Kist [Canad. J. Math. 20(1962), 517-528] is used to study the structure spaces of an archimedean vector lattice. The structure spaces separated by a vector lattice are characterized and some embedding theorems are obtained via certain maps between structure spaces; e.g., Theorem. Let L' be an archimedean vector lattice; L be a subvector lattice of L'; and L, L' share an order unit ℓ. Then there exists a continuous onto function $\rho: \mathfrak{M}(L') \rightarrow \mathfrak{M}(L)$, where $\mathfrak{M}(L')$, $\mathfrak{M}(L)$ are the structure spaces of maximal ideals of L' and L, respectively. If L is order dense in L', the mapping is tight. Definition. A vector lattice has the countable interpolation property if, for countable sets A and B with $a \leqq b$ for all $a \in A$, $b \in B$, there exists x so that $a \leqq x \leqq b$ for all $a \in A$, $b \in B$. Theorem. Let X be a perfectly normal compact space. Then the set of extended valued continuous functions is an algebra if and only if X is an F-space. (Received October 5, 1972.)

*700-A3. JOHN MYRON MASLEY, University of Illinois at Chicago Circle, Chicago, Illinois 60680. On cyclotomic fields Euclidean for the norm map.

Let $\mathcal{O}_m = \mathbb{Z}[\zeta]$ where ζ is a primitive mth root of unity. Theorem 1. There are precisely 29 distinct domains $\mathcal{O}_m$ which are P.I.D.'s. They are classified by $m \neq 23, 39, 46, 52, 56, 72, 78$ for which $\varphi(m) \leqq 24$. For which of these P.I.D.'s does the norm map of the mth cyclotomic field to the rational numbers induce a Euclidean algorithm? A partial answer is given in Theorem 2. If $\varphi(m) \leqq 4$, then $\mathcal{O}_m$ is Euclidean for the norm map. (Received September 1, 1972.)

700-A4. MICHAEL N. BLEICHER, University of Wisconsin, Madison, Wisconsin 53706. A new algorithm for Egyptian fractions.

A rational number p/q is said to be written in Egyptian form if it is presented as a sum of reciprocals of distinct positive integers $n_1, n_2, \ldots, n_k$. The new algorithm here presented is based on the continued fraction expansion of the original fraction. It has the advantage of relatively short length, while keeping the n_i below the very reasonable bound of q^2. This method also ties in the best lower approximations to p/q with the subsums of the Egyptian expansion. Because it is based on the continued fraction the method is extendable to irrational numbers. (Received September 12, 1972.)

*700-A5. THOMAS W. CUSICK, State University of New York at Buffalo, Amherst, New York 14226. View-obstruction problems.

Let S^n denote the region $0 < x_i < \infty$ $(i=1,2,\ldots,n)$ of n-dimensional Euclidean space E^n. Suppose C is a closed convex body in E^n which contains the origin as an interior point. Define αC for each real number $\alpha \geqq 0$ to be the set of all $(\alpha x_1,\ldots,\alpha x_n)$, where $(x_1,\ldots,x_n)$ is a point in C. Define $C + (m_1,\ldots,m_n)$ to be the body obtained by adding the point $(m_1,\ldots,m_n)$ in E^n to every point of C. Define the point set $\Delta(C,\alpha)$ to be the set of all points of the form $C + (m_1+\frac{1}{2},\ldots,m_n+\frac{1}{2})$, where $m_1,\ldots,m_n$ are nonnegative integers. The view-obstruction problem for C is the problem of finding the constant $K(C)$ defined to be the lower bound of those α such that any half-line L given by $x_i = a_i t$ $(i=1,2,\ldots,n)$, where the a_i are positive real numbers and the parameter t runs through $[0,\infty)$, intersects $\Delta(C,\alpha)$. One simple choice for C is the n-dimensional cube with side 1. In the paper it is shown that the problem of finding $K(C)$ in this case is equivalent to a certain problem in Diophantine approximation, and this problem is solved in two and three dimensions. The case where C is chosen to be the n-dimensional ball with diameter 1 is also discussed, but here the exact value of $K(C)$ is found only in the two-dimensional case; a conjecture is given for the general case. (Received September 13, 1972.)

*700-A6. BOHUSLAV B. DIVIŠ, Ohio State University, Columbus, Ohio 43210. Lattice points on strictly convex curves and surfaces.

Let C be a strictly convex closed curve in the plane passing through N lattice points. Let ℓ be its length and A the area enclosed. Jarnik proved in 1925 that $N \leqq (3/\sqrt[3]{2\pi})\ell^{2/3} + O(\ell^{1/3})$ and this result is best possible. We prove that $N \ll A^{1/3}$ and this result up to the value of the constant implied is also best possible. If F is a strictly convex closed surface in three dimensions with surface area S and passing through N lattice points then $N \ll S^{3/4}$. (Received September 14, 1972.)

700-A7. MUSHFEQUR RAHMAN, Eastern Illinois University, Charleston, Illinois 61920. Domain of action method.

For a star-domain define a norm-distance function N and a Minkowski distance function M. A point set S is N-admissible if $N(P,Q) \geqq 1$, whenever $P,Q \in S$, $P \neq Q$. The domain of action $D(P) = D(P,M,S)$ of a point P relative to M, S is defined as the closure of the set of all points X in the plane for which $M(P,X) \leqq M(Q,X)$, $Q \in S$, $Q \neq P$. The domain of action method is developed and employed to estimate the density of an N-admissible point set for the star-domain: $2|y|(\sqrt{3}|x| - |y|) \leqq 1$, if $|y| \leqq \sqrt{3}|x|$; and $y^2 - 3x^2 \leqq 1$, if $|y| \geqq \sqrt{3}|x|$. (Received September 13, 1972.)

700-A8. PETER W. AITCHISON, University of Manitoba, Winnipeg, Manitoba R3T 2N2, Canada. Two finiteness theorems in the Minkowski theory of reduction.

Minkowski proved two important finiteness theorems concerning the reduction theory of positive definite quadratic forms. A positive definite quadratic form in n variables may be considered as an ellipsoid in n-dimensional Euclidean space, R^n, and then the two results can be investigated more generally by replacing the ellipsoid by any symmetric convex body in R^n. We show here that when $n \geqq 3$ the two finiteness theorems hold only in the case of the ellipsoid. This is equivalent to showing that Minkowski's results do not hold in a general Minkowski space, namely in a Euclidean space where the unit ball is a general symmetric convex body instead of the sphere or ellipsoid. (Received September 20, 1972.)

*700-A9. FRANK HARARY, University of Michigan, Ann Arbor, Michigan 48104 and GEERT C. E. PRINS, Wayne State University, Detroit, Michigan 48202. The Ramsey multiplicity of a graph.

A proper graph G has no isolated points. Its Ramsey number r(G) is the minimum p such that every 2-coloring of the edges of K_p contains a monochromatic G. The Ramsey multiplicity R(G) is the minimum number of monochromatic G in any 2-coloring of $K_{r(G)}$. With just two exceptions, we determine R(G) for the proper graphs with at most 4 points. For the stars $K_{1,n}$ it is shown that R = 2n when n is odd and R = 1 when n is even. We conclude with the conjecture that for a proper graph, R(G) = 1 if and only if G = K_2 or $K_{1,n}$ with n even. (Received September 11, 1972.)

*700-A10. FRANK HARARY and ALLEN JOHN SCHWENK, University of Michigan, Ann Arbor, Michigan 48104, PETER V. O'NEIL, College of William and Mary, Williamsburg, Virginia 23185 and RONALD C. READ, University of Waterloo, Waterloo, Ontario, Canada. The number of trees in a wheel.

Cayley's classic results enumerating labeled and unlabeled trees may be viewed as counting the number of nonisomorphic spanning trees in a complete graph. Analogous methods serve to derive a generating function for the number of unlabeled spanning trees in a wheel, complementing previous results on the number of labeled spanning trees. The exact numbers of unlabeled trees for $n \leqq 100$ have been calculated by computer.
(Received September 11, 1972.)

700-A11. BŘETISLAV NOVAK, Charles University, Prague, Czechoslovakia and University of Illinois, Urbana, Illinois 61801. A remark on the theory of lattice points.

Let P(x) denote the known lattice remainder term in the theory of lattice points in ellipsoids. The aim of this paper is to show that the important information about the behaviour of the function P(x) is possible to obtain by investigation of the function (ρ is complex, Re $\rho > 0$) $P_\rho(x) = (1/\Gamma(\rho)) \int_0^x P(t)(x-t)^{\rho-1}\,dt$. (Received September 18, 1972.)

700-A12. RICHARD B. LAKEIN, State University of New York at Buffalo, Amherst, New York 14226. Approximation properties of some complex continued fractions. Preliminary report.

We consider geometrically defined simple continued fractions whose partial quotients are integers of $F = Q((-m)^{1/2})$, m = 1, 2, 3, 7 or 11. Given complex x, let the nth convergent to x be p_n/q_n, and define $M_n(x) = (|q_n|\,|q_n x - p_n|)^{-1}$. Then associated with each type of CF is a constant B = inf $M_n(x)$ (taken over all x, n). For the standard real CF, B = 1, i.e., $|x - p_n/q_n| < 1/|q_n|^2$ for all x, n; and the result is best possible. This constant B is evaluated for the various CF's and related constants are considered. For example, one CF over Z(i) has B = 1, while another has B = 0. Similar questions are also considered for certain geometric generalizations of these CF's. (Received September 13, 1972.)

*700-A13. GEORGE B. PURDY, Center for Advanced Computation, University of Illinois, Urbana, Illinois 61801. The lattice triple packing of spheres. Preliminary report.

We say that a lattice Λ in n-dimensional Euclidean space E_n provides a k-fold packing for spheres of radius 1 if, when open spheres of radius 1 are centered at the points of Λ, no point of space lies in more

than k spheres. The multiple packing constant $\Delta_k^{(n)}$ is the smallest determinant of any lattice with this property. In the plane, the first three multiple packing constants $\Delta_2^{(2)}$, $\Delta_3^{(2)}$, and $\Delta_4^{(2)}$ are known, due to the work of Blundon, Few, and Heppes. In E_3, $\Delta_2^{(3)}$ is known, because of work by Few and Kanagasabapathy, but no other multiple packing constants are known. In Chapter 2 we show that $\Delta_3^{(3)} \leqq 8\sqrt{38}/27$ and give evidence that $\Delta_3^{(3)} = 8\sqrt{38}/27$. We show, in fact, that a lattice with determinant $8\sqrt{38}/27$ gives a local minimum of the determinant among lattices providing a 3-fold packing for the unit sphere in E_3. (Received September 25, 1972.)

*700-A14. FRANK HARARY and ALLEN JOHN SCHWENK, University of Michigan, Ann Arbor, Michigan 48104, PAUL C. KAINEN, Case Western Reserve University, Cleveland, Ohio 44106 and ARTHUR THOMAS WHITE II, Western Michigan University, Kalamazoo, Michigan 49001. A maximal toroidal graph which is not a triangulation.

Let S be a surface. Call a graph G S-maximal if G can be embedded in S, $G \subset S$, but $G + x \not\subset S$ for any line x not already in G. G is S-triangular if G can be embedded in S so that every region has three sides. If G is S-triangular, it is clearly S-maximal. However, the converse, while true for the sphere, is false in general; that is, the statement "G is S-maximal implies G is S-triangular" is false. This implication can fail for the trivial reason that G has too few vertices, e.g., K_5 is torus-maximal but any triangulation of the torus has ≧ vertices. We present a nontrivial example: $K_8 - C_5$, which is torus-maximal but not torus-triangular. This is the smallest such example. (Received October 2, 1972.)

*700-A15. BRANKO GRÜNBAUM, University of Washington, Seattle, Washington 98195. Acyclic colorings of planar graphs.

A k-coloring of a graph G is any partition of the vertices of G into k "colors" so that neighboring vertices have different colors. A k-coloring of a graph is called acyclic provided for each choice of two colors the subgraph spanned by vertices of those two colors is a forest (i.e., no circuit is bichromatic). We prove that every planar graph has an acyclic 9-coloring, as well as results which imply and strengthen theorems on "point-arboricity" obtained by G. Chartrand, S. Hedetniemi, H. V. Kronk, S. K. Stein, and C. E. Wall. The conjecture that every planar graph has an acyclic 5-coloring is still open, as are many related questions. (Received October 4, 1972.)

700-A16. JOSEPH A. TROCCOLO, Cleveland State University, Cleveland, Ohio 44115. $\mathfrak{F}$-projectors of finite solvable groups. Preliminary report.

Let G be a finite solvable group. $Q(G) = \{H/H_0 \mid H_0 \trianglelefteq H \leq G\}$. Definition. If $\mathfrak{F}$ is any subset of Q(G) then E is an $\mathfrak{F}$-projector of G if: (1) whenever $E \leqq F \leqq G$ and $F/F_0 \in \mathfrak{F}$ then $EF_0 = F$, and (2) E is a minimal subgroup of G with respect to (1). Theorem. If $\mathfrak{F}$ is closed with respect to the taking of epimorphic images within G then the $\mathfrak{F}$-projectors of G are a single conjugacy class of subgroups. The $\mathfrak{F}$-projectors of G may be characterized by the maximal chains of subgroups joining them to G. Definition. A complemented chief factor H/K of G is $\mathfrak{F}$-crucial if G/H is its own $\mathfrak{F}$-projector but G/K is not. A complement M to an $\mathfrak{F}$-crucial chief factor is an $\mathfrak{F}$-crucial maximal subgroup. Theorem. E is an $\mathfrak{F}$-projector of G if and only if there is a chain $G = G_0 > G_1 > \ldots > G_n$ such that G_i is an $\mathfrak{F}$-crucial maximal subgroup of G_{i-1} for $i \geqq 1$, G_n has no $\mathfrak{F}$-crucial maximal subgroups and E is an $\mathfrak{F}$-projector of G_n. (Received October 5, 1972.)

700-A17. VANCE FABER, University of Colorado, Denver, Colorado 80302. The n-genus of a graph. Preliminary report.

If G is a graph, $\gamma_n(G)$ is the smallest integer which is the genus of a compact orientable 2-manifold M upon which G has an embedding f such that $M\backslash f(G)$ is a disjoint union of open 2-cells (faces) and such that each face has at least n distinct edges of G in its boundary. ($\gamma_n(G)$ does not have to exist.) (For additional information on embedding of graphs see J. W. T. Youngs, "Minimal imbeddings and the genus of a graph," J. Math. Mech. 12(1963), 303-315.) $\hat{\gamma}_n(G)$ is defined similarly with unorientable 2-manifolds. Euler's equation is used to find lower bounds for $\gamma_n(G)$ and $\hat{\gamma}_n(G)$ when G is a complete graph, a complete bipartite graph or a cube. (Ringel, Youngs, etc. gave $\gamma_3(G)$ for complete graphs (1968) and $\gamma_3(G)$ and $\gamma_4(G)$ for complete bipartite graphs (1967) and cubes (1955).) Theorem. $\gamma_4(K_{8s+5}) = 8s^2 + 5s + 1$, where K_{8s+5} is the complete graph on $8s + 5$ vertices. (Received October 5, 1972.)

*700-A18. HENRY H. GLOVER and JOHN P. HUNEKE, Ohio State University, Columbus, Ohio 43210. Cubic irreducibly nonprojective planar graphs. Preliminary report.

A characterization of all cubic (trivalent) graphs which do not embed in the real projective plane are given in the sense that Kuratowski characterized all nonplanar graphs. Specifically, a cubic graph is nonprojective planar if and only if it contains a homeomorphic copy of an irreducibly nonprojective planar graph, that is, a nonprojective planar graph which would embed in the projective plane if it were missing any of its edges. All irreducibly nonprojective planar graphs are given, with a proof that the list is complete by analyzing disjoint θ-graphs (i.e. graphs homeomorphic to θ) within graphs of various girths. (Received October 5, 1972.)

700-A19. LAWRENCE C. EGGAN, Illinois State University, Normal, Illinois 61761. Cassini ovals. Preliminary report.

Perron and others have used Cassini ovals $O(\alpha, \beta, k) = \{z \in C; |\alpha - z|\,|\beta - z| = k\}$, where α, β are in the complex numbers C and k is a positive real constant, to obtain results in Diophantine approximations. We discuss their geometry including coverings by Cassini ovals, and further applications to Diophantine approximations. (Received October 5, 1972.)

Analysis

700-B1. HENRY G. HERMES, University of Colorado, Boulder, Colorado 80302. Controllability and the topology of reachable sets. Preliminary report.

Let $\mathscr{S}$ be a collection of smooth tangent vector fields on a manifold M and let $\mathscr{S}_x = \{X(x) \in TM_x : X \in \mathscr{S}\}$. A solution of $\mathscr{S}$ is an a.c. map $\varphi : [0, \infty) \to M$ with $d\varphi/dt \in \mathscr{S}_{\varphi(t)}$ a.e. $\mathscr{S}$ is controllable on M if every pair of points of M can be joined by a solution of $\mathscr{S}$. $\mathscr{S}$ is locally controllable at time $t_1 > 0$ along a solution φ if all points in some neighborhood of $\varphi(t_1)$ can be attained from $\varphi(0)$ by solutions of $\mathscr{S}$. $\mathscr{S}$ has the accessibility property if attainable sets from each point have nonempty interior. Computable algebraic criteria for $\mathscr{S}$ to have the accessibility property are given by Sussmann and Jurdjevic ("Controllability of non-linear systems", J. Differential Equations 12(1972), 95-116). Jurdjevic ("Certain controllability properties of analytic control systems", SIAM J. Control 10(1972), 354-360) shows that if $\mathscr{S}$ is a set of right invariant vector fields on a compact Lie group, the accessibility property implies $\mathscr{S}$ is controllable. We show that if $\mathscr{S}$ is a commuting two field on the two torus T^2, $\mathscr{S}$ is controllable. Also, there exist smooth (noncommuting) two fields on T^2 which are not controllable. Other results of this nature, together with results on local controllability and its consequences, are given. (Received September 8, 1972.)

*700-B2. LAMBERTO CESARI, University of Michigan, Ann Arbor, Michigan 48104. Existence theorems for problems of optimization with distributed and boundary controls. Preliminary report.

Let $G \subset E^{\nu}$ be bounded, open, of the Morrey type, and Γ, a part of the boundary of G, with area measure μ. Let S be a topological space, and L, J, M, K operators, not necessarily linear, from S into $(L_1(G))^r$, $(L_1(\Gamma))^{r'}$, $(L_1(G))^s$, $(L_1(\Gamma))^{s'}$. We are interested in the existence of elements $x \in S$ and measurable functions $u: G \to E^m$, $v: \Gamma \to E^{m'}$, which minimize a functional $I[x,u,v] = \int_G f_0(t, (Mx)(t), u(t))\,dt + \int_\Gamma g_0(t,(Kx)(t), v(t))\,d\mu$, subject to state equations (strong form) $(Lx)(t) = f(t, (Mx)(t), u(t))$, $t \in G$ (a.e.), $(Jx)(t) = g(t, (Kx)(t), v(t))$, $t \in \Gamma$ (μ-a.e.), and constraints $(Mx)(t) \in A(t)$, $u(t) \in U(t, (Mx)(t))$, $t \in G$ (a.e.), and $(Kx)(t) \in B(t)$, $v(t) \in V(t, (Kx)(t))$, $t \in \Gamma$ (μ-a.e.). Here $f_0(t,y,u)$, $f(t,y,u)$, $g_0(t,\mathring{y},v)$, $g(t,\mathring{y},v)$ are given functions, and $A(t)$, $B(t)$, $U(t,y)$, $V(t,\mathring{y})$ given variable sets. Weak forms of state equations are also taken into consideration. The existence of optimal solutions is proved under the sole hypotheses of closure of L, J, M, K, under growth conditions on f_0, f, g_0, g and upper semicontinuity conditions with respect to the state variables only y, $\mathring{y}$ of the relevant sets $Q(t,y) = [z^0 \geqq f_0,\ z=f,\ u \in U]$, $R(t,\mathring{y}) = [z^0 \geqq g_0,\ z=g,\ v \in V]$, namely, weak intermediate properties between property (Q) and Kuratowski's property (which in turn may be a consequence of slightly stronger growth hypotheses or Lipschitz conditions). (Received September 14, 1972.)

*700-B3. SHERWOOD D. SILLIMAN, Cleveland State University, Cleveland, Ohio 44115. The numerical evaluation by splines of the Fourier transform and the Laplace transform.

Quadrature formulae (q.f.) for the numerical evaluation of the Fourier, cosine, sine, and Laplace transforms are given in terms of the transformed function f at equally spaced points and, except for the Fourier transform, also in terms of certain derivatives of f at 0. The approximations are constructed by integrating a spline interpolant or by requiring the q.f. to be exact for a particular sequence of basis splines, the so-called B-splines. Expressions for the error are obtained by utilizing a particular monospline. (Received September 5, 1972.)

700-B4. CALVIN R. PUTNAM, Purdue University, West Lafayette, Indiana 47907. Almost normal operators, their spectra and invariant subspaces.

• This lecture will be concerned with certain operators on a Hilbert space which share some of the properties possessed by normal operators. Included among these "almost normal" operators are for instance those which are subnormal, or hyponormal, or which have resolvents satisfying certain growth conditions. It is known that various sparseness properties of the spectra (such as being nowhere dense, or of planar measure zero, or being a subset of a smooth curve) are associated with various degrees of normality of the corresponding operators. Some of the known results and problems along these lines will be reviewed. In addition, some questions concerning the existence of invariant subspaces for certain almost normal operators having spectra of two parts intersecting in a point or in a cross-section of linear measure zero will be considered. (Received September 18, 1972.)

*700-B5. RANKO BOJANIC and YOU-HWA LEE, Ohio State University, Columbus, Ohio 43210. An estimate for the rate of convergence of convolution products of sequences.

Let (a_n) and (p_n) be two sequences. The convolution product of (a_n) and (p_n) is the sequence (c_n) defined by $c_n = \sum_{k=0}^{n} p_k a_{n-k}$, $n = 0, 1, 2, \ldots$. It is known that if the series $\sum_{k=0}^{\infty} p_k$ converges absolutely and the sequence (a_n) converges, then the sequence (c_n) also converges. More generally, it is known that if (p_n) is a sequence of real numbers such that the series $\sum_{k=0}^{\infty} p_k x^k$ has positive radius of convergence R, and if (a_n) is a sequence of positive numbers such that $a_n/a_{n+1} \to \lambda$ $(n \to \infty)$, where $0 < \lambda < R$, then $c_n/a_n \to \sum_{k=0}^{\infty} p_k \lambda^k$ $(n \to \infty)$. A theorem which gives an estimate of the rate of convergence of the sequence (c_n/a_n) if some information about the rate of convergence of the sequence (a_n/a_{n+1}) is known can be stated as follows: Theorem. Suppose that the series $\sum_{k=0}^{\infty} p_k x^k$ has a positive radius of convergence R and suppose that a sequence of positive numbers (a_n) satisfies the condition $|a_n/a_{n+1} - \lambda| \leqq \delta_n$, $n = 0, 1, 2, \ldots$, where (δ_n) is a decreasing sequence such that $\delta_n \to 0$ and $\delta_{n+1}/\delta_n \to 1$ $(n \to \infty)$. If $0 < \lambda < R$, then $\limsup_{n\to\infty} \delta_n^{-1} |c_n/a_n - \sum_{k=0}^{\infty} p_k \lambda^k| \leqq \sum_{k=1}^{\infty} k p_k \lambda^{k-1} < \infty$.
(Received October 2, 1972.)

*700-B6. BHUSHAN LAL WADHWA, Cleveland State University, Cleveland, Ohio 44115. Operators satisfying a sequential growth condition. Preliminary report.

A bounded operator T on a Hilbert space H is called a sequentially G_1 operator if for each λ in the boundary of the spectrum of T there exists a sequence λ_n converging to λ such that $\|(\lambda_n - T)^{-1}\| \leqq \{\mathrm{dist}[\lambda_n, \sigma(T)]\}^{-1}$ for all n. It is shown that a sequentially G_1 algebraic operator is normal. This result has an interesting application to the theory of ρ-dilation in the sense that it generalizes and at the same time simplifies the proof of a recent result of Furuta (Acta Sci. Math. (Szeged) 33(1972), 119–122) concerning C_ρ-operators. It is also shown that all compact perturbations of a sequentially G_1 operator are in the class $\bar{R}_1$. (Received October 2, 1972.)

*700-B7. KEITH M. KENDIG, Cleveland State University, Cleveland, Ohio 44115. Some properties of real tangent cones. Preliminary report.

The notion of tangent space at a simple point of a real or complex analytic variety V has been extended to "tangent cone" $C_p(V)$ at an arbitrary point $p \in V$, simple or not; in the complex case a number of basic properties are known (e.g., $\dim_p V = \dim C_p(V)$, $C_p(V)$ is defined by the ideal of initial forms at p, etc.). For a real variety, we show these properties hold at each point off a subvariety of codimension $\geqq 2$ in V; counterexamples show this is the sharpest result in this direction. (Received October 2, 1972.)

700-B8. CARLOS ALBERTO INFANTOZZI, Instituto de Estudios Superiores, Montevideo, Uruguay An introduction to relations among inequalities.

The subject has really a greater scope. This paper concerns itself mainly with the: (a) "means inequality": (P) $G \leqq A$; (b) "Cauchy-Schwarz inequality": (CS) $|(u,v)| \leqq \|u\| \cdot \|v\|$; (c) "triangle inequality": (T) $\|a+b\| \leqq \|a\| + \|b\|$; (d) "Bernoulli inequality": (B_1) $(1+x)^m \geqq 1 + mx$ $(m \geqq 1, m \leqq 0, x > -1)$, (B_2) $(1+x)^m \leqq 1 + mx$ $(0 \leqq m \leqq 1, x > -1)$; (e) "classical inequality": (C_1) if $\alpha + \beta = 1$, then $a^\alpha b^\beta \leqq \alpha a + \beta b$ (α and β nonnegative), (C_2) $a^\alpha b^\beta \geqq \alpha a + \beta b$ (α or β nonpositive); (f) "Hölder inequality": (H_1) if $\alpha + \beta = 1$, then $\sum a^\alpha b^\beta \leqq [\sum a]^\alpha [\sum b]^\beta$ $(0 \leqq \alpha \leqq 1)$, (H_2) $\sum a^\alpha b^\beta \geqq [\sum a]^\alpha [\sum b]^\beta$ $(\alpha \geqq 1)$; (g) "I inequality": (I_1) $\sum ab \leqq [\sum a^\alpha]^{1/\alpha} \cdot [\sum b^\beta]^{1/\beta}$ ($1/\alpha + 1/\beta \geqq 1$, α and β positive), (I_2) $\sum ab \geqq [\sum a^\alpha]^{1/\alpha} [\sum b^\beta]^{1/\beta}$ ($1/\alpha + 1/\beta \leqq 1$, α or β, at least, negative); (h) "Minkowski inequality": (M_1) if $z_i = x_i + y_i$, then $[\sum z^p]^{1/p} \leqq [\sum x^p]^{1/p} + [\sum y^p]^{1/p}$ $(p \geqq 1)$, (M_2) $[\sum z^p]^{1/p} \geqq [\sum x^p]^{1/p} + [\sum y^p]^{1/p}$ $(p \leqq 1)$. (The simplified notations of Hardy-Littlewood-Pólya, "Inequalities", GITTL, Moscow, 1948; 2nd ed., Cambridge, at the University Press, 1952, are used.) It is shown that the following relations of implication are verified: (P) $\to$ (B_2) $\to$ (B_1) $\to$ (C_2) $\to$ (C_1) $\to$ (H_1) $\to$ (I_1) $\to$ (I_2) $\to$ (H_2) $\to$ (M_2) $\to$ (M_1) $\to$ (T) $\to$ (CS) $\to$ (P). Then, one has established the interesting fact that all the preceding inequalities, from a logical point of view, are equivalent. (In particular, the two relations contained in each inequality (d), (e), (f), (g) and (h) are equivalent to one another.) (Received October 2, 1972.)

*700-B9. STEPHEN R. BERNFELD, University of Missouri, Columbia, Missouri 65201. Strongly bounded differential systems.

The notion of strongly (sometimes referred to as absolutely) bounded systems was first introduced, by J. Auslander and Seibert (Ann. Inst. Fourier (Grenoble) 14(1964), 237-267) in the setting of a dynamical system by means of prolongational limit sets. We present here a consequence of strongly bounded differential systems by using Liapunov functions and exhibiting admissible classes of perturbations which preserve the boundedness properties of solutions. The results are similar in spirit to those of Vrkoc (Czechoslovak Math. J. 17(1969), 725-738) with one major difference. Whereas Vrkoc has shown that integral stability is equivalent to strong stability, we show that integral boundedness implies strong boundedness but that strong boundedness does not necessarily imply integral boundedness. (Received October 4, 1972.)

700-B10. HOWARD LEWIS PENN, Eastern Michigan University, Ypsilanti, Michigan 48197 and University of Michigan, Ann Arbor, Michigan 48104. On peak interpolation sets for the polydisc algebra.

In "Function theory in the polydisc" by Walter Rudin, Theorem 6.1.2 asserts the equivalence of five properties defined for compact subsets of the n-torus, T^n. Two of these properties are N, which states that K is a null set for every measure on T^n which annihilates $A(U^n)$, and PI, which states that for every $g \in C(K)$ there corresponds an $f \in A(U^n)$ such that $f(w) = g(w)$ for every $w \in K$ and $|f(z)| \subset \|g\|_K$ for every other $z \in \overline{U}^n$. There is a gap in the proof that $N \Rightarrow PI$. What is actually proved is that $N \Rightarrow PI_T$ where PI_T is defined the same as PI except that $|f(z)| \subset \|g\|_K$ only for every other $z \in T^n$. The gap can be filled by showing that K is a null set for every measure defined on $\overline{U}^n$ which annihilates $A(U^n)$ and applying Bishop's theorem. (Received October 5, 1972.)

Applied Mathematics

*700-C1. ROLAND F. STREIT, University of Illinois, Champaign, Illinois 61820. Some results on the T + m-transformation.

In this paper a simple test is given which enables one to determine when the transformed series generated by the T + m-transformation is uniformly better convergent than the original infinite series. A question was posed by H. L. Gray and W. D. Clark in their paper "On a class of nonlinear transformations and their applications to the evaluation of infinite series," J. Res. Nat. Bur. Standards Sect. B 73B(1969), 251-274. This question is answered and numerical examples are presented which demonstrate the theoretical results. (Received August 21, 1972.)

700-C2. MARIA Z. v. KRZYWOBLOCKI, Michigan State University, East Lansing, Michigan 48823. Association of wave mechanics with discontinuities in Blasius' parametrical function. Preliminary report.

Omitting the description of the physical aspects of the modern wave mechanics (wavy nature of matter, its oscillatory character) the writer concentrates only on the formal, mathematical aspects of the problem. The fundamental equation of the wave mechanics is the Schroedinger wave equation, a linear partial differential equation of the second order involving the Laplacian in the three-dimensional complex space for the wave function. After decomposition into two real parts it gives two equations corresponding to the Euler equation of the conservation of momentum and conservation of mass (continuity) in the classical, deterministic inviscid fluid dynamics. The first of these has terms corresponding to the kinetic energy, static pressure and the action of the external force field. Using the concept of the "diabatic" flow field, the latter field is attributed to the viscous, dissipative force field in the Navier-Stokes equation sense. By this chain of relations one constructs an association between the wave mechanics, Schroedinger equation and the Blasius solution of the laminar, "viscous", boundary layer flow along a flat plate. The Blasius parametric argument "eta" is a (not one-to-one) transformation of one argument into two independent physical coordinates (x, y). Its second order mixed partial derivatives are not equal. They depend upon the sequence of the differentiations; eta is a discontinuous function. (Received August 25, 1972.)

*700-C3 HENDRICUS G. LOOS, Cleveland State University, Cleveland, Ohio 44115. Wilson operator product expansions and the Dürr-Winter identification of gauge potentials.

Since the field operators of quantum field theory are unbounded, products of field operators have infinite matrix elements. As a result, field equations involving products of field operators are meaningless, and lead to infinities in the scattering matrix. A modern approach towards removal of the infinities is to make the field equations meaningful from the start, by using the "finite part" :AB: of the operator product AB. :AB: being defined by a Wilson expansion. It is shown that the finite part $:\bar{\psi}\gamma_\mu L^i\psi:$ (where ψ is a multicomponent Dirac spinor field of dimension $\frac{3}{2}$, γ_μ are the Dirac matrices, and the L^i form a basis of the Lie algebra for the gauge group) transforms as a linear connection in the internal space (fibre), in which ψ is a vector, if the Wilson expansion for $\bar{\psi}\psi$ defines $:\bar{\psi}\psi:$ uniquely, and if this expansion is Lorentz-, gauge-, and space inversion-invariant. This finding supports the ad hoc Dürr-Winter identification (Nuovo Cimento A (10) 70(1970), 467-501). The result is valid for the gauge groups U(1) and SU(n), $n \geqq 2$, and is obtained by using properties of the Dirac algebra and the Lie algebra for the gauge group. (Received September 29, 1972.)

*700-C4. CHARLES W. GROETSCH, University of Cincinnati, Cincinnati, Ohio 45221. An iterative solution of linear operator equations.

An iterative method studied in a previous note of the author (J. Math. Anal. Appl. 40(1972)) is applied to approximate solutions of linear operator equations in reflexive Banach spaces. The main tools are Dotson's generalization of the Eberlein ergodic theorem and the DeMoivre-Laplace theorem. The principal results are used to obtain theorems on approximate solution of linear operator equations in Hilbert space and the approximation in L^p-norm of a certain functional equation in the space L^∞. (Received October 2, 1972.)

700-C5. EDWARD J. McSHANE, University of Virginia, Charlottesville, Virginia 22903. Stochastic optimal control theory.

Deterministic control theory is a subject of many aspects. If we add in the complication of random disturbances, there are so many different ways in which these disturbances can affect the development of the system, the gathering of information on which the choice of control is based, and the cost function being minimized, that it is hardly proper to speak of a theory at all; what exists today is an aggregate of studies of problems, all involving the optimization of something or other when the system is affected by some sort of random disturbance concerning which we have some statistical information, which for mathematical purposes we are willing to regard as trustworthy. Instead of trying to present a unified theory, we shall attempt to portray the situation by discussing two problem types. The first is the linear type, in which the mathematics is at least relatively simple, and in which solutions have been obtained in such specific form as to be used in industry. The second is nonlinear, in which even the mathematical modeling of the system offers difficulties, and confusions, miscalled paradoxes, abound; and even among the variegated soundly established results few seem to offer any reasonable hope of computability. (Received October 2, 1972.)

700-C6. ROBERTO TRIGGIANI, University of Minnesota, Minneapolis, Minnesota 55455. Controllability and observability of dynamical systems in Banach space with bounded operators. Preliminary report.

The system $\mathcal{L}: \dot{x} = Ax + Bu$, X and U Banach spaces, A and B bounded operators on X and $U \rightarrow X$, respectively, is ϵ-state controllable on $[0, T]$ in case the arbitrary initial point can be steered on $[0, T]$ by admissible controls to a dense set of X. It is established by using a consequence of the Hahn-Banach theorem that this is the case iff cl sp$\{A^n Bu$, all $u \in U$, $n \geq 0\} = X$. However if B is compact and X has a basis, it is proved by using the Baire category theorem that $\mathcal{L}$ is never exactly $(\epsilon = 0)$-state controllable, even if we leave the final instant free; in particular this holds for $\mathcal{L}_m: \dot{x} = Ax + \sum_{i=1}^m b_i u_i$, that is though ϵ-state controllable iff cl sp$\{A^n b_i$, $i = 1, \ldots, m; n \geq 0\} = X$. Add now to $\mathcal{L}$ an observation equation $y = [h_1 x, \ldots, h_r x]$, $h_j \in X^*$. Then $\mathcal{L}$ is output controllable on $[0, T]$ (i.e. $y(T, x_0, u) = y_1$ for $x_0 \in X$ and $y_1 \in R^r$ arbitrary and u admissible) iff the r sequences $h_j(Bu)$, $h_j(ABu)$, $h_j(A^2 Bu), \ldots$, $j = 1, \ldots, r$, are linearly independent on U; in particular for $\mathcal{L}_m$ iff $h_j(b_1), \ldots, h_j(b_m)$, $h_j(Ab_1), \ldots, h_j(Ab_m), \ldots$ are linearly independent. Output controllability is always assured by ϵ-state controllability and linear independence of $h_1, \ldots, h_r$. ϵ-state controllability admits a dual version for observability. Also the above results generalize to the time dependent case under suitable conditions of smoothness. (Received September 11, 1972.)

Geometry

*700-D1. HENRY W. LEVINSON, G. F. POLLICE and MAHMOUD SAYRAFIEZADEH, Rutgers University, New Brunswick, New Jersey 08903. Nontriply colinear sets of points. Preliminary report.

A set of points (in E^2) is called ntc iff no three of them are colinear. Maximal ntc sets are not proper subsets of any other ntc sets. Some maximal ntc sets are boundaries of convex sets, provided these boundaries do not contain line segments. Theorem 1. Any ntc set may be extended to a maximal ntc set. Corollary. There are ntc sets which are dense in E^2. Theorem 2. The set of lines determined by pairs of points of a maximal ntc set covers (pointwise) E^2. Corollary. Each maximal ntc set in E^2 is of cardinality $\aleph_1$. The ntc property may be dualized, resulting in corresponding statements. We note that the ntc property is an incidence property and hence applicable to elements of projective planes. Thus it may provide a partial analogue to convexity in finite geometries. (Received October 3, 1972.)

Statistics and Probability

*700-F1. B. C. GUPTA, Universidade Federal do Rio de Janeiro, ZC-00 Rio de Janeiro, Brasil. Moments of h-statistics, using ordered partitions. Preliminary report.

In case the parent population is completely specified, exact sampling distribution can often be obtained. But if the parent population is not completely known, an approximate form of the sampling distribution of a statistic is obtained by using its lower moments. In order to simplify the problem Fisher (Proc. London Math Soc. 30(1928)) proposed statistics k_r as unbiased estimates of the population cumulant $\varkappa_r$. Kendall (Ann. Eugenics 11(1942), 300-305) worked to find another set of semi-invariant statistics which may be simpler than k-statistics. Dwyer (Ann. Math. Statist. 8(1937)) proposed statistics h_r as unbiased estimates of the central moment μ_r with $E(h_1) = \mu_1'$. Tracy and Gupta (Bull. Int. Statist. Inst. 38th Session, 1971) proposed generalized h-statistics which estimate products of moments unbiasedly. Gupta (Ph.D. thesis, University of Windsor, 1971) showed that h-statistics, for $r > 1$, are semi-invariant and are simpler than k-statistics. He also gave relations of generalized h-statistics with other symmetric functions such as power sums, power product sums, polykays, etc. Now in order to find the moments of h-statistics it is needed to express powers and products of generalized h-statistics as linear combinations of the same. The aim of the present paper is to provide a method, using ordered partitions, to express the powers and products of generalized h-statistics as linear combinations of the same. (Received August 11, 1972.)

*700-F2. JOSEPH M. COOK, Argonne National Laboratory, Argonne, Illinois 60439. Weak infinitesimal generators of a class of jump-perturbed Markov processes.

Markov processes for realistic models of nuclear reactors have nondifferentiable semigroups of transition functions in the usual topologies, so the initial-value problem for these processes has been neglected. However, Dynkin's w-infinitesimal generator theory can be used to prove that problem well-posed. Let a stochastically continuous Markov process with w-infinitesimal generator $A^\sim$ be given by the semigroup U acting on the Banach space $\mathscr{V}(X)$ of totally bounded measures on phase space X. Let measurable $X_n \uparrow X$ such that each $\mathscr{V}(X_n)$ reduces U. (In branching processes n is the particle number.) Define $B = B_0 + B_1$ where $(B_0\mu)(Y) = \int_Y b_0\, d\mu$ for a jump-rate $b_0(x) = -\langle B_1 \delta_x\rangle \leqq 0$ bounded on each X_n. B_0^* and B_1^* are reduced by a subspace defined in Dynkin's theory. Let U_0 be the subprocess with termination density $-b_0$. Define $S_1^{(0)} = U_0$ and $S_1^{(n)}(t) = \int_0^t U_0(t-s)B_1 S_1^{(n-1)}(s)\, ds$. Theorem. $\sum_0^\infty S_1^{(n)}$ determines a stochastically continuous Markov process which is nonterminating iff $\lim_{n\to\infty} \langle B_1 \int_0^1 S_1^{(n)}(s)\, ds\, \delta_{(\cdot)}\rangle = 0$, i.e., iff its w-infinitesimal generator is $A^\sim +' B^*$ where $+'$ is a strong sum of operators. (Received October 2, 1972.)

*700-F3. WILLIAM E. WINKLER, Ohio State University, Columbus, Ohio 43210. On continuous zero-two law.

Let $\{X_t\}$, $0 \leqq t < \infty$, be a Markov process with state space $(E, \mathscr{E})$. Let m be a σ-finite measure on $(E, \mathscr{E})$ and let the $L_\infty(E, \mathscr{E}, m)$ operator induced by the transition probability $P_t(x, A)$, $x \in E$, $A \in \mathscr{E}$, be conservative and ergodic for all $t > 0$. Let (m) abbreviate m modulo 0. For fixed $\alpha > 0$, set $h^\alpha(x) = \lim_t \|P_t(x, \ ) - P_{t+\alpha}(x, \ )\|$, where $\|\cdot\|$ is the total variation. Theorem. Either $h^\alpha(x) = 0$ (m) for almost every real α or $h^\alpha(x) = 2$ (m) for almost every real α. In particular, if $\{X_{k\alpha}\}$, $k = 1, 2, \ldots,$ is recurrent in the sense of Harris for every $\alpha > 0$, then $h^\alpha(x) = 0$ (m) for almost every real α. In proving the theorem we use the fact that if B, a subset of the reals, has positive Lebesgue measure, then $B - B = \{x - y \mid x, y \in B\}$ contains an open interval around the origin. This paper will appear in Annals of Probability. (Received October 4, 1972.)

*700-F4. JANOS GALAMBOS, Temple University, Philadelphia, Pennsylvània 19122. Asymptotic distribution of extremes of dependent observations with random sample size.

In many practical applications (waiting time, the time period up to the first failure of a machine with a large number of components, etc.) we are interested in the asymptotic distribution of the maximum of random variables $X_1, X_2, \ldots, X_N$ satisfying the properties below: (i) The X's are identically distributed with distribution function $F(x)$. (ii) The X's can be grouped into a number of subsets so that the groups are asymptotically independent for large X and that no subset contains a positive percentage of the X's as $N \to +\infty$. Let $N = N(n)$ be random variables such that N/n tends to a positive random variable as $n \to +\infty$. Let further c_n be such that, as $n \to +\infty$, $n[1-F(c_n)] \to w$. The result then is that under a mild assumption on the dependence of the X's within groups, $\lim P(Z_N < c_N) = e^{-w}$, as $n \to +\infty$, where $Z_N = \max(X_1, \ldots, X_N)$. This extends the author's recent result (Ann. Math. Statist. 43(1972), 516-521) where N (=n) is assumed to be constant. The tool is a sieve argument and a method developed by J. Mogyoródi (Magyar Tud. Akad. Mat. Fiz. Oszt. Közl. 17 (1967), 75-83) for the case of independent random variables. (Received October 5, 1972.)

Topology

700-G1. MARY-ELIZABETH HAMSTROM, University of Illinois, Urbana, Illinois 61801. Homeomorphism and embedding spaces, PL and TOP.

• This talk will survey the work that has been done and is being done on the homotopy properties of the space of self-homeomorphisms of a manifold and the space of embeddings (with restrictions on the kind of embedding) of one manifold in another. The local homotopy properties of such spaces will also be discussed. The categories involved will be PL and TOP. Hopefully, the talk will be addressed to nonspecialists as well as specialists. (Received September 18, 1972.)

700-G2. DENIS L. BLACKMORE, Newark College of Engineering, Newark, New Jersey 07102. An example of a local flow on a manifold.

Let p be a point of a smooth n-dimensional manifold. If n is even it is easy to construct a local flow about p such that p is an isolated critical point and no orbit except the stationary one at p has p as a limit point. We call such a flow a nonnull flow about p (NN-flow). Mendelson has conjectured that NN-flows do not exist on odd dimensional manifolds. We show that Mendelson's conjecture is false by constructing an NN-flow on any smooth manifold whose dimension is an odd integer exceeding one. (Received October 2, 1972.)

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