Notices

OF THE

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Notices
OF THE
AMERICAN MATHEMATICAL SOCIETY

Edited by Everett Pitcher and Gordon L. Walker

CONTENTS

The Six Hundred Eighty-Ninth Meeting
Auburn University
Auburn, Alabama
November 19–20, 1971

The six hundred eighty-ninth meeting of the American Mathematical Society will be held at Auburn University in Auburn, Alabama, on Friday and Saturday, November 19–20, 1971.

By invitation of the Committee to Select Hour Speakers for the Southeastern Sectional Meetings, there will be three one-hour addresses, all of which will be presented in Room 307 of the Commons Building. Professor J. C. Cantrell of the University of Georgia will give an address on Friday at 1:00 p.m. entitled "Locally flat embeddings of manifolds." An address entitled "Quasi-analyticity and semi-groups" will be given by Professor John Neuberger of Emory University at 4:15 p.m. on Friday, and Professor Charles W. McArthur of Florida State University will give an address on Saturday at 9:00 a.m. entitled "Developments in the theory of Schauder bases."

The registration desk will be located in the entrance hall of the Commons Building, Physical Science Center, where all sessions will be held. Registration hours will be 10:00 a.m. to 5:00 p.m. on Friday, November 19, and 9:00 a.m. to 12:00 noon on Saturday, November 20.

Auburn is located on Interstate 85 approximately one-third of the way from Montgomery, Alabama, to Atlanta, Georgia. Auburn is also accessible via U.S. 280, U.S. 29, and U.S. 80. The nearest commercial airline terminals are Atlanta, Georgia (two hours driving time); Columbus, Georgia, and Montgomery, Alabama (one hour driving time, each). It is possible that several pickups can be arranged at these airports for persons flying in and for whom car rental is impractical. Persons requiring such service should write to Professor L. P. Burton, Head, Department of Mathematics, by November 15.

Meals, except for snacks, must be taken at commercial establishments. Coffee and doughnuts will be served each morning in Room 244, Commons Building. A dutch-treat beer party is planned for Friday evening.

The two motels which are within walking distance of the meetings and which are holding blocks of rooms for reservations (deadline November 10) are:

<u>University Motor Lodge</u> (25-room block)

125 North College Street
P. O. Box 831
Phone: 887-6583

Single (one person per room)	\$ 9.36
One Double Bed (two persons per room)	\$12.48
Two Double Beds (two persons per room)	\$14.56

(six blocks from Commons Building)

<u>Heart of Auburn Motel</u> (80-room block)

333 South College Street
P. O. Box 632
Phone: 887-3462

Single (one person per room)	\$ 9.88
One Double Bed (two persons per room)	\$12.48
Two Double Beds (two persons per room)	\$15.60

(three blocks from Commons Building)

Additional accommodations are as follows:

<u>Holiday Inn</u>

Birmingham Highway 280
P. O. Box 551
Phone: 887-7065

Single (one person per room)	\$ 9.45
One Double Bed (two persons per room)	\$11.55
Two Double Beds (two persons per room)	\$16.80

(five miles north of campus on Alabama 147, cocktail lounge)

Holiday Inn

Junction I-85 and U.S. 280
P. O. Box 391
Opelika, Alabama 36801
Phone: 745-6331

One Double Bed $11.55
(one person per room)

Two Double Beds $16.80
(two persons per room)

(five miles east of campus, cocktail lounge)

Stoker's Motel

1208 Opelika Highway
Auburn, Alabama 36830
Phone: 887-3481

Single $ 8.40
(one person per room)

One Double Bed $10.50
(two persons per room)

Two Double Beds $12.60
(two persons per room)

(four miles east of campus)

All reservations should be made directly with the motels as early as practicable.

PRESENTORS OF TEN-MINUTE PAPERS

Following each name is the number corresponding to the speaker's position on the program.

Anson, Dennis #58
Arnold, Jimmy T. #27
Aull, Charles E. #67
Bailes, Gordon L., Jr. #93
Baker, John M. #4
Baker, John Warren #5
Bang, Chang Mo #79
Barr, Alvin F. #104
Bass, Charles D. #49
Bean, Phillip W. #110
Beard, Jacob T.B., Jr. #105
Bode, J. J. #88
Bowman, Thomas T. #59
Boyce, William M. #45
Brauer, Alfred T. #37
Braun, Ben-Ami #19
Buckley, James J. #100
Butler, Kim Ki-Hang #94
Cain, George L., Jr. #41
Carlisle, Ronald L. #62
Carmichael, Richard D. #116
Chambless, Donald A. #74
Daigle, Roy J. #48
Daverman, Robert J. #50
Edwards, William R., Jr. #55
Eslinger, Robert C. #44
Evans, Trevor #96
Fitzgibbon, William E. #98
Fitzpatrick, Ben, Jr. #64
Fulp, R. O. #31
Garrett, C. #101
Gibson, Peter M. #34
Gilmer, Robert #26
Golightly, W. L. #22
Goodman, Adolph W. #103
Grams, Anne P. #25
Grams, William F. #112
Hall, Japheth, Jr. #76
Hallam, Thomas G. #12
Hanson, Thomas H. McH. #47
Hargrove, E. E. #65
Harley, Peter W., III #61
Harris, Ralph L. #1
Hartman, Phillip A. #75
Hays, Thomas E. #60
Heath, Robert W. #69
Heidel, John W. #9
Heinzer, William J. #23
Hinkle, Carl V., Jr. #95
Hitz, Reinhart #18
Holmes, J. P. #6
Hora, Rajinder B. #86
Horne, J. G. #57
Howard, Frederic T. #107
Husch, Lawrence S. #52
Hutton, Charlene V. #115
Iglarsh, Harvey J. #111
Johnson, Lee W. #102
Kammerer, William J. #113
Kimura, Naoki #85
Kinloch, John #30
Kreimer, Herbert F. #28
Kropa, James C. #66
Langan, Thomas J. #16
Lea, James W., Jr. #78
Leech, J. E. #92
Linton, Ronald C. #91
Lovelady, David L. #13
Luedeman, John K. #29
Lutzer, David J. #73
Marrero, Osvaldo #82
Martin, Robert H., Jr. #99
Maxfield, John E. #106
McCoy, Robert A. #43
McLean, T. Bruce #38
Minor, Lee H. #40
Moreman, Douglas #68
Nathanson, Melvyn B. #109
Norris, Eugene M. #89
Nummela, Eric C. #32
Pellerin, Arthur #80
Perkins, John C #35
Plemmons, Robert J. #36
Proctor, Thomas G. #21
Reed, George Michael #39
Reneke, James A. #20
Richardson, Gary D. #42
Rine, David C. #54
Robertson, Muril L. #7
Robinson, Daniel A. #90
Rogers, Jack W., Jr. #63
Rollins, Laddie W. #11
Rosen, Harvey #51
Schwabauer, Robert J. #97
Scott, T. J. #77
Shamma, Shawky E. #14
Sharp, Thomas Joseph #84
Sheehan, John P. #114
Sigmon, Kermit N. #56
Slaughter, Frank G., Jr. #72
Slotterbeck, Oberta A. #87
Sobczyk, Andrew #83
Spitznagel, Carl R. #46
Stafford, Raymond A. #10
Textor, Robin E. #17
Todd, Aaron R. #3
Travis, Curtis C. #8
Trotter, William T., Jr. #81
Van Doren, Kenneth R. #71
Vaughan, Nick H. #24
Wallace, Alexander D. #53
Webber, Robert P. # 33
Williams, James L. #108
Young, Eutiquio C. #15
Zaidman, Samuel #2
Zenor, Phillip L. #70

The time limit for each contributed paper is ten minutes. To maintain this schedule, the time limit will be strictly enforced.

FRIDAY, 1:00 P. M.

Invited Address, Room 307, Commons Building
Locally flat embeddings of manifolds
Professor J. C. Cantrell, University of Georgia

FRIDAY, 2:30 P. M.

Session on Functional Analysis, Room 319, Commons Building

2:30-2:40
(1) A characterization of uniformly convex spaces. Preliminary report
Mr. Ralph L. Harris, Western Carolina University (689-B2)

2:45-2:55
(2) Properties of strongly almost-periodic families of linear operators
Professor Samuel Zaidman, Université de Montréal (689-B4)

3:00-3:10
(3) A property of locally convex Baire spaces. Preliminary report
Professor Stephen A. Saxon and Mr. Aaron R. Todd*, University of Florida (689-B21)

3:15-3:25
(4) A cross norm close to the greatest cross norm
Professor John M. Baker, Western Carolina University (689-B25)

3:30-3:40
(5) Uncomplemented C(X)-subalgebras of C(X)
Dr. John Warren Baker, Florida State University (689-B26)

3:45-3:55
(6) Differentiable power associative partial groupoids with identity
Mr. J. P. Holmes, University of Florida (689-B29)

FRIDAY, 2:30 P. M.

Session on Ordinary Differential Equations, Room 224, Commons Building

2:30-2:40
(7) The equation $y'(t) = F(t, y(g(t)))$
Dr. Muril L. Robertson, Auburn University (689-B3)

2:45-2:55
(8) A note on second order nonlinear oscillations
Professor Curtis C. Travis, Vanderbilt University (689-B9)
(Introduced by Professor B. F. Bryant)

3:00-3:10
(9) A counterexample in nonlinear boundary value problems
Professor John W. Heidel, University of Tennessee (689-B13)

3:15-3:25
(10) A comparison theorem for second order linear differential equations. Preliminary report
Mr. Raymond A. Stafford, University of Tennessee (689-B15)

3:30-3:40
(11) Criteria for discrete spectrum of singular selfadjoint differential operators
Mr. Laddie W. Rollins, Georgia Institute of Technology (689-B18)
(Introduced by Professor Frank W. Stallard)

3:45-3:55
(12) Convergence of solutions of perturbed nonlinear differential equations
Professor Thomas G. Hallam, Florida State University (689-B19)

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*For papers with more than one author, an asterisk follows the name of the author who plans to present the paper at the meeting.

FRIDAY, 2:30 P. M.

Session on Partial Differential Equations, Room 352, Commons Building

2:30-2:40

(13) Existence on prescribed rectangles for a hyperbolic equation in a Banach space
Professor David L. Lovelady, University of South Carolina (689-B5)

2:45-2:55

(14) Asymptotic eigenfunctions of a scattering problem
Professor Shawky E. Shamma*, University of West Florida, and
Professor Samuel N. Karp, Courant Institute, New York University
(689-B10)

3:00-3:10

(15) Uniqueness theorems for a singular ultrahyperbolic equation
Professor Eutiquio C. Young, Florida State University (689-B12)

3:15-3:25

(16) Differential inequalities for semilinear hyperbolic operators with two independent variables
Mr. Thomas J. Langan, Naval Ship Research and Development Center,
Bethesda, Maryland (689-B14)

3:30-3:40

(17) On a singular Cauchy problem for a nonlinear hyperbolic equation
Professor Robin E. Textor, University of South Carolina (689-B23)

FRIDAY, 2:30 P. M.

Session on Summability, Integral Equations, etc., Room 252, Commons Building

2:30-2:40

(18) Necessary and sufficient conditions for convergence of continued fractions
Dr. Reinhart Hitz, Old Dominion University (689-B24)

2:45-2:55

(19) On the multiplicative completion of deleted Schauder bases in L^p, $1 \leqq p < \infty$. Preliminary report
Dr. Ben-Ami Braun, University of South Florida (689-B11)

3:00-3:10

(20) Product integral solutions for hereditary systems
Professor James A. Reneke, Clemson University (689-B28)

3:15-3:25

(21) Asymptotically equivalent sets of Stieltjes integral equations. Preliminary report
Professor Thomas G. Proctor, Clemson University (689-B27)

3:30-3:40

(22) Linearization of Volterra integral equations. Preliminary report
Mr. W. L. Golightly, Clemson University (689-B22)
(Introduced by Professor James A. Reneke)

FRIDAY, 2:30 P. M.

Session on Commutative Rings and Algebras, Room 250, Commons Building

2:30-2:40

(23) A characterization of certain 2-dimensional regular affine UFD's as polynomial rings. Preliminary report
Professor William J. Heinzer, Purdue University (689-A27)

2:45-2:55

(24) A note on the classes of semiprimary ideals and Dedekind ideals in a domain. Preliminary report
Professor Nick H. Vaughan, North Texas State University (689-A46)

3:00-3:10

(25) The equality of $(A \cap B)^n = A^n \cap B^n$ for ideals. Preliminary report
Professor Robert Gilmer and Mrs. Anne P. Grams*, Florida State
University (689-A4)

3:15-3:25

(26) On factorization into prime ideals. Preliminary report
Professor Robert Gilmer, Florida State University (689-A5)

3:30-3:40

(27) Power series rings over Prüfer domains
Dr. Jimmy T. Arnold, Virginia Polytechnic Institute and State
University (689-A9)

FRIDAY, 2:30 P. M.

Session on Commutative and/or Associative Rings, Room 254, Commons Building

2:30-2:40
(28) Separability and the Galois theory of commutative rings. Preliminary report
Professor Herbert F. Kreimer, Florida State University (689-A31)

2:45-2:55
(29) On the embedding of topological rings into rings of quotients
Dr. John K. Luedeman, Clemson University (689-A39)

3:00-3:10
(30) A one-sided characterization of Dedekind semiprime rings. Preliminary report
Dr. John Kinloch, East Tennessee State University (689-A24)

3:15-3:25
(31) Splitting locally compact abelian groups. Preliminary report
Professor R. O. Fulp, North Carolina State University (689-G4)

3:30-3:40
(32) Homological algebra of topological modules. Preliminary report
Professor Eric C. Nummela, University of Florida (689-A45)

FRIDAY, 2:30 P. M.

Session on Linear and Multilinear Algebra, Room 364, Commons Building

2:30-2:40
(33) The matrix semigroup determined by the spectral norm. Preliminary report
Mr. Robert P. Webber, University of Tennessee (689-A32)

2:45-2:55
(34) A lower bound for the permanent of a (0,1)-matrix
Professor Peter M. Gibson, University of Alabama in Huntsville (689-A17)

3:00-3:10
(35) Symmetric involutions over fields of characteristic 2. Preliminary report
Dr. John C. Perkins*, United States Army, Ft. Belvoir, Virginia, and Professor John D. Fulton, Clemson University (689-A21)

3:15-3:25
(36) Monotonicity and the generalized inverse
Professor Abraham Berman, Université de Montréal, and Professor Robert J. Plemmons*, University of Tennessee (689-A3)

3:30-3:40
(37) On classes of stochastic matrices whose absolute smallest characteristic is real
Professor Alfred T. Brauer, Wake Forest University (689-A41)

FRIDAY, 2:30 P. M.

Session on General Topology. I, Room 350, Commons Building

2:30-2:40
(38) Confluent images of tree-like spaces
Mr. T. Bruce McLean, University of Kentucky (689-G2)
(Introduced by Professor Joseph B. Fugate)

2:45-2:55
(39) On screenability and metrizability of Moore spaces
Dr. George Michael Reed, Ohio University (689-G3)

3:00-3:10
(40) The relationship of pseudo-expansiveness, expansiveness, and recurrence in transformation groups
Professor Lee H. Minor, Western Carolina University (689-G24)
(Introduced by Professor John M. Baker)

3:15-3:25
(41) Metrizable mapping compactifications
Professor George L. Cain, Jr., Georgia Institute of Technology (689-G14)

3:30-3:40
(42) Regular compactifications of convergence spaces
Professor Darrell C. Kent, Washington State University, and Mr. Gary D. Richardson*, East Carolina University (689-G13)

3:45-3:55
(43) Homeomorphism groups of Hilbert cube manifolds
Professor Robert A. McCoy, Virginia Polytechnic Institute and State University (689-G12)

FRIDAY, 2:30 P. M.

Session on Topological Groups and Generalizations, Room 217, Commons Building

2:30-2:40

(44) An implicit function theorem with applications to locally Banach semigroups with identity. Preliminary report
Dr. Robert C. Eslinger, King College (689-G16)

2:45-2:55

(45) Baxter permutations and functional composition
Dr. William M. Boyce, Bell Telephone Laboratories, Murray Hill, New Jersey (689-G28)

3:00-3:10

(46) The lattice of congruences on a band of groups
Mr. Carl R. Spitznagel, University of Kentucky (689-A7)

3:15-3:25

(47) Actions of a locally compact group with compact boundary
Professor Thomas H. McH. Hanson, University of Florida (689-A18)

FRIDAY, 2:30 P. M.

Session on Topological Embeddings, Room 356, Commons Building

2:30-2:40

(48) Complements of minimal surfaces. Preliminary report
Mr. Roy J. Daigle, University of Georgia (689-G25)

2:45-2:55

(49) Some embeddings of a disk in E^3 which support a squeezing map
Mr. Charles D. Bass, Pembroke State University (689-G15)

3:00-3:10

(50) On the absence of tame disks in certain wild cells. Preliminary report
Dr. Robert J. Daverman, University of Tennessee (689-G22)

3:15-3:25

(51) Shrinking wild cellular subsets of 2-spheres in S^3. Preliminary report
Dr. Harvey Rosen, University of Alabama (689-G27)

3:30-3:40

(52) Homeomorphisms of 3-manifolds which fail to be regular on 1-dimensional polyhedron
Professor Paul F. Duvall, Jr., Oklahoma State University, and Professor Lawrence S. Husch*, University of Tennessee (689-G7)

FRIDAY, 4:15 P. M.

Invited Address, Room 307, Commons Building
Quasi-analyticity and semigroups
Professor John Neuberger, Emory University

SATURDAY, 9:00 A. M.

Invited Address, Room 307, Commons Building
Developments in the theory of Schauder bases
Professor Charles W. McArthur, Florida State University

SATURDAY, 10:15 A. M.

Session on Automata and Semigroups, Room 319, Commons Building

10:15-10:25

(53) Skewly associative groupoids
Professor Alexander D. Wallace, University of Florida (689-G1)

10:30-10:40

(54) An elementary theory of computation
Professor David C. Rine, West Virginia University (689-A8)

10:45-10:55

(55) The left discrimination sequence of an automaton. Preliminary report
Mr. William R. Edwards, Jr.* and Professor Zamir Bavel, University of Kansas (689-C1)
(Introduced by Professor John W. Case)

11:00-11:10
(56) Idempotent topological groupoids with nonempty center
Professor Kermit N. Sigmon, University of Florida (689-H1)

11:15-11:25
(57) Homomorphic images of $(P^{-})^{n}$
Professor J. G. Horne, University of Georgia (689-H2)

11:30-11:40
(58) The structure of totally ordered sets. Preliminary report
Dr. Dennis Anson, Western Kentucky University (689-H3)

11:45-11:55
(59) The automorphism groups of compact solenoidal and cylindrical semigroups
Dr. Thomas T. Bowman, University of Florida (689-H4)

12:00-12:10
(60) ML*-semigroups. Preliminary report
Mr. Thomas E. Hays, University of Tennessee (689-H5)

SATURDAY, 10:15 A.M.

Session on General Topology. II, Room 356, Commons Building

10:15-10:25
(61) On countably paracompact spaces and closed maps
Professor Peter W. Harley III, University of South Carolina (689-G18)

10:30-10:40
(62) Monotone maps and ϵ-maps between graphs. Preliminary report
Mr. Ronald L. Carlisle, Kennesaw Junior College (689-G19)
(Introduced by Dr. Jack W. Rogers, Jr.)

10:45-10:55
(63) Inverse limits on graphs and monotone mappings
Dr. Jack W. Rogers, Jr., Emory University (689-G20)

11:00-11:10
(64) Concerning connectedness im kleinen. Preliminary report
Professor Ben Fitzpatrick, Jr., Auburn University (689-G23)

11:15-11:25
(65) Properties of local Darboux functions. Preliminary report
Mr. E. E. Hargrove, University of Alabama (689-G26)
(Introduced by Professor B. D. Garrett)

11:30-11:40
(66) Cancellative topological semigroups on a manifold. Preliminary report
Dr. James C. Kropa, Judson College (689-G21)

SATURDAY, 10:15 A.M.

Session on General Topology. III, Room 213, Commons Building

10:15-10:25
(67) Quasi-developable and weak σ-spaces. Preliminary report
Professor Charles E. Aull, Virginia Polytechnic Institute and State University (689-G5)

10:30-10:40
(68) A generalization to metric spaces of the notion of weak convergence
Mr. Douglas Moreman, Auburn University (689-G6)
(Introduced by Professor Ben Fitzpatrick, Jr.)

10:45-10:55
(69) A characterization of monotone normality. Preliminary report
Professor Robert W. Heath* and Professor David J. Lutzer, University of Pittsburgh (689-G8)

11:00-11:10
(70) Certain subsets of products of metacompact spaces are realcompact
Professor Phillip L. Zenor, Auburn University (689-G9)

11:15-11:25
(71) Inverse limits and closed mappings. Preliminary report
Mr. Kenneth R. Van Doren, Auburn University (689-G10)

11:30-11:40
(72) The closed continuous image of a metrizable space is M_1
Professor Frank G. Slaughter, Jr., University of Pittsburgh (689-G11)

11:45-11:55
(73) Another note on weak θ-refinability. Preliminary report
Dr. Harold R. Bennett, Texas Tech University, and Dr. David J. Lutzer*, University of Pittsburgh (689-G17)

SATURDAY, 10:15 A.M.

Session on Order, Lattices, etc., Room 301, Commons Building

10:15-10:25
(74) The ℓ-group of almost-finite, continuous functions
Professor Donald A. Chambless, University of Florida (689-A12)

10:30-10:40
(75) Integrally closed and complete ordered quasigroups and loops
Dr. Phillip A. Hartman, University of North Carolina at Asheville (689-A25)

10:45-10:55
(76) Order relations determined by coverings of sets. Preliminary report
Dr. Japheth Hall, Jr., Stillman College (689-A38)

11:00-11:10
(77) Monotonic permutations of chains. Preliminary report
Mr. T. J. Scott, University of Georgia (689-A35)
(Introduced by Professor Stephen H. Mc Cleary)

11:15-11:25
(78) An embedding theorem for compact lattices
Dr. James W. Lea, Jr., Middle Tennessee State University (689-A6)

SATURDAY, 10:15 A.M.

Session on Combinatorics, Room 217, Commons Building

10:15-10:25
(79) Isomorphism types of infinite symmetric graphs
Dr. Chang Mo Bang, Emory University (689-A11)

10:30-10:40
(80) Cubic graphs on twelve vertices and the line graph of a finite affine plane. Preliminary report
Mr. Arthur Pellerin* and Professor Renu Laskar, Clemson University (689-A28)

10:45-10:55
(81) A note on triangulated graphs. Preliminary report
Dr. William T. Trotter, Jr., The Citadel (689-A26)

11:00-11:10
(82) Modular Hadamard matrices and related designs. II. Preliminary report
Professor Osvaldo Marrero, Francis Marion College (689-A1)

11:15-11:25
(83) Arrangements of lines which are of class two
Professor Andrew Sobczyk, Clemson University (689-A40)

SATURDAY, 10:15 A.M.

Session on Group Theory and Generalizations. I, Room 350, Commons Building

10:15-10:25
(84) On D-groups and Y-subgroups
Professor Thomas Joseph Sharp, West Georgia College (689-A47)
(Introduced by Professor Jo W. Ford)

10:30-10:40
(85) Left translations of the free product of semigroups
Professor Rajinder B. Hora and Professor Naoki Kimura*, University of Arkansas (689-A44)

10:45-10:55
(86) Commutative semigroups with 0 such that the square of every element is 0
Professor Rajinder B. Hora* and Professor Naoki Kimura, University of Arkansas (689-A43)

11:00-11:10
(87) Wreath products and saturated formations. Preliminary report
Dr. Oberta A. Slotterbeck, University of Florida (689-A42)

11:15-11:25
(88) t-functors. Preliminary report
Mr. J. J. Bode, University of South Carolina (689-A37)
(Introduced by Professor Paul L. Sperry)

11:30-11:40
(89) More on externally induced operations. Preliminary report
Dr. Alexander R. Bednarek, University of Florida, and Dr. Eugene M. Norris*, West Virginia University (689-A15)

11:45-11:55
(90) An "extra" law for characterizing Moufang loops
Professor Orin Chein, Temple University, and Professor Daniel A. Robinson*, Georgia Institute of Technology (689-A16)

SATURDAY, 10:15 A.M.

Session on Group Theory and Generalizations. II, Room 364, Commons Building

10:15-10:25
(91) Abelian groups in which every neat subgroup is a direct summand
Mr. Ronald C. Linton, University of South Alabama (689-A23)

10:30-10:40
(92) Congruences under $\mathscr{D}$. Preliminary report
Dr. J. E. Leech, University of Tennessee (689-A22)
(Introduced by Professor Harvey Carruth)

10:45-10:55
(93) Right inverse semigroups
Mr. Gordon L. Bailes, Jr., Clemson University (689-A20)

11:00-11:10
(94) Combinatorial properties of partial transformation semigroups
Professor Kim Ki-Hang Butler* and Professor Charles D. Bass, Pembroke State University (689-A10)

11:15-11:25
(95) The injective hull of $\mathscr{P}$-torsion-free semigroups. Preliminary report
Mr. Carl V. Hinkle, Jr., Clemson University (689-A29)

11:30-11:40
(96) When are functionally free algebras free? A remark on a problem of B. M. Schein
Dr. Trevor Evans, Emory University (689-A30)

11:45-11:55
(97) Covering conditions and variety lattices
Dr. Robert J. Schwabauer, Virginia Commonwealth University (689-A33)

SATURDAY, 10:15 A.M.

Session on Complex Variables, Real Variables, Measure Theory, and Operator Theory, Room 254 Commons Building

10:15-10:25
(98) Time dependent nonlinear Cauchy problems in Banach spaces. Preliminary report
Mr. William E. Fitzgibbon, Vanderbilt University (689-B7)

10:30-10:40
(99) Generators of nonlinear semigroups in uniformly convex Banach spaces
Professor Robert H. Martin, Jr., North Carolina State University (689-B17)

10:45-10:55
(100) Integrating group-valued functions
Professor James J. Buckley, University of South Carolina (689-B8)

11:00-11:10
(101) Almost continuous functions. Preliminary report
Mrs. C. Garrett, Stillman College (689-B30)
(Introduced by Dr. Japheth Hall, Jr.)

11:15-11:25
(102) A procedure for estimating $E_n(f)$
Professor Lee W. Johnson* and Professor R. D. Riess, Virginia Polytechnic Institute and State University (689-B31)

11:30-11:40
(103) Coefficients for the area theorem. Preliminary report
Professor Adolph W. Goodman, University of South Florida (689-B1)

11:45-11:55
(104) The radius of univalence of certain classes of analytic functions. Preliminary report
Dr. Alvin F. Barr, Livingston University (689-B16)

SATURDAY, 10:15 A. M.

Session on Number Theory and General Mathematical Systems, Room 252, Commons Building

10:15-10:25
(105) A characterization of all matrix fields contained in $(Q)_n$. Preliminary report
Dr. Jacob T. B. Beard, Jr., University of Texas at Arlington (689-A19)

10:30-10:40
(106) On a restricted partition function and its tables. Preliminary report
Professor Lal M. Chawla, Professor John E. Maxfield*, Kansas State University, and Professor Marijo O. LeVan, Eastern Kentucky University (689-A13)

10:45-10:55
(107) A property of a class of nonlinear difference equations
Professor Frederic T. Howard, Wake Forest University (689-A14)

11:00-11:10
(108) Quotients determined by a correspondence. Preliminary report
Mr. James L. Williams, University of Arkansas (689-A34)
(Introduced by Professor Naoki Kimura)

11:15-11:25
(109) Sums of finite sets of integers
Professor Melvyn B. Nathanson, Southern Illinois University (689-A36)

SATURDAY, 10:15 A. M.

Session on Applied Mathematics, Functional Analysis and Statistics, Room 305, Commons Building

10:15-10:25
(110) Concerning property $R[k, f_k(n)]$. Preliminary report
Mr. Phillip W. Bean, Auburn University (689-D1)

10:30-10:40
(111) Regularity for the stopped Wiener process
Professor Harvey J. Iglarsh, Georgia Institute of Technology (689-F2)

10:45-10:55
(112) Convergence rates for U-statistics and related statistics. Preliminary report
Mr. William F. Grams* and Professor R. J. Serfling, Florida State University (689-F1)

11:00-11:10
(113) Local convergence of smooth cubic spline interpolates
Professor William J. Kammerer*, Georgia Institute of Technology, and Mr. G. W. Reddien, Jr., Vanderbilt University (689-C2)

11:15-11:25
(114) On the dynamic response of an infinite Bernoulli-Euler beam
Mr. John P. Sheehan* and Dr. Lokenath Debnath, East Carolina University (689-C3)

11:30-11:40
(115) On 2-trivial Banach spaces. Preliminary report
Miss Charlene V. Hutton, Louisiana State University (689-B20)

11:45-11:55
(116) Generalized Cauchy and Poisson integrals and distributional boundary values
Dr. Richard D. Carmichael, Wake Forest University (689-B6)

O. G. Harrold
Associate Secretary

Tallahassee, Florida

The Six Hundred Ninetieth Meeting
University of Wisconsin—Milwaukee
Milwaukee, Wisconsin
November 27, 1971

The six hundred ninetieth meeting of the American Mathematical Society will be held at the University of Wisconsin-Milwaukee, Milwaukee, Wisconsin, on Saturday, November 27, 1971. The sessions of the meeting will be held in the Science Complex, which houses the Department of Mathematics, and in the adjoining Physics and Engineering Building. These buildings are located near the corner of North Cramer Street and East Kenwood Boulevard in northeast Milwaukee.

By invitation of the Committee to Select Hour Speakers for Western Sectional Meetings, there will be two one-hour addresses. Professor Raghavan Narasimhan of the University of Chicago will address the Society at 11:00 a.m. His subject will be "Deformations of principal bundles." Professor Mary Ellen Rudin of the University of Wisconsin, Madison, will speak at 1:30 p.m. on the topic "Set theory and general topology." Both hour talks will be held in Room E180 of the Science Complex.

By invitation of the same committee there will be three special sessions of selected twenty-minute papers. Professor Morris Marden of the University of Wisconsin-Milwaukee has arranged one such session on the subject of Function Theory; the speakers will be Stephen B. Agard, Patrick R. Ahern, Herbert J. Alexander, Albert Baernstein II, Peter L. Duren, Stephen D. Fisher, Michael B. Freeman, Simon Hellerstein, Albert Marden, Joseph B. Miles, Ricardo Nirenberg, Walter J. Schneider, and Daniel F. Shea. Another special session has been arranged by Professor Frank A. Raymond of the University of Michigan on the subject of Transformation Groups; the speakers will be Gary C. Hamrick, Hsu-Tung Ku, Ronnie Lee, Reinhard E. Schultz, Tatsuo Suwa, and Philip D. Wagreich. The third special session has been arranged by Professor Thomas G. McLaughlin of the University of Illinois on the subject of Recursion Theory; the speakers will be Louise Hay, Carl G. Jockusch, Jr., Alistair H. Lachlan, Manuel Lerman, Michael Machtey, Thomas G. McLaughlin, Yiannis N. Moschovakis, James C. Owings, Jr., Marian B. Pour-El, Gerald E. Sacks, Robert I. Soare, and Hisao Tanaka. There will also be four sessions of contributed ten-minute papers.

SYMPOSIUM

On Friday afternoon, November 26, 1971, the day before the meeting itself, the University of Wisconsin-Milwaukee will sponsor a symposium on Function Theory in the 1970's as part of dedication ceremonies of the Science Complex. The dedication exercises will take place at 1:45 p.m. in Room E190 of the Science Complex and will be followed by three special lectures. At 2:00 p.m. Professor Robert C. Gunning of Princeton University will speak on "Compact Riemann surfaces: Some new problems in an old field." At 3:00 p.m. Professor Frederick W. Gehring of the University of Michigan will give a talk on "Function theory in higher dimensional Euclidean spaces." At 4:00 p.m. Professor Walter Rudin of the University of Wisconsin, Madison, will speak on "A new look at some old theorems." In addition, the special session on Function Theory mentioned earlier will be an extension of this symposium.

REGISTRATION

The registration desk will be located in the ground floor lobby of the Science Complex. The desk will be open from 1:00 p.m. to 5:00 p.m. on Friday, November 26, and from 8:00 a.m. to 4:00 p.m. on Saturday, November 27.

ACCOMMODATIONS

Dormitory accommodations in the Carl Sandburg Dormitory (3400 North Maryland Avenue) have been reserved by the Department of Mathematics for at least one hundred participants at this meeting. A single room, in room suites with bath, is $8. Indoor parking is available underneath the dormitory at a modest fee.

In addition, the following two hotels are recommended. Both are located near downtown Milwaukee about three miles from the campus; both offer free indoor parking. The rates listed are special ones for those attending the meeting.

THE PFISTER HOTEL AND TOWER
424 East Wisconsin Avenue
(at Jefferson Street)

Singles	$12.50
Doubles	17.50

THE MILWAUKEE INN
916 East State Street

Singles	$13.00
Doubles	18.00

Reservations for the dormitory or for either of these hotels should be made through the department at least two weeks in advance. A reservation form may be found on the last page of the October *Notices*.

The YMCA and YWCA are located at 915 West Wisconsin Avenue and 626 North Jackson Street, respectively, approximately three miles from the campus. Reservations should be made directly.

FOOD SERVICE

A dining room located in the Carl Sandburg dormitory will be serving meals at a cafeteria-style snack bar. The hours of operation will be posted at the registration desk. Some well-known Milwaukee restaurants are Dutch's Sukiyaki House, the John Ernst Cafe, Frenchy's Restaurant, Mader's German Restaurant, and Karl Ratzsch's Restaurant. Reservations for dinner on Friday evening can be made at the registration desk.

TRAVEL AND LOCAL INFORMATION

Milwaukee is served by Air Wisconsin, Eastern Air Lines, North Central Air Lines, Northwest Orient Air Lines, Ozark Air Lines, and United Air Lines. Amtrak offers train service from Minneapolis, Chicago, and St. Louis.

Limousine service is available from the airport to the downtown area with stops at the Pfister Hotel and Milwaukee Inn. The charge for this service is about $1.40. If enough prior commitments are received, limousine service may be arranged directly to the Sandburg Dormitory. Registrants interested in the direct service from the airport to the dormitory should so indicate on the reservation form included on the last page of the October *Notices*. Direct service will be arranged only if it is economically feasible.

Those coming by car should go as far east as possible on the East-West Freeway (I-794) and then continue east on East Clybourn Avenue to North Lincoln Memorial Drive which runs along Lake Michigan. They should take this Drive north to its end and then go west on East Kenwood Boulevard. The University is on the north side of East Kenwood Boulevard about ten blocks from Lake Michigan. The entrance to the Science Complex parking area is on the west side of North Maryland Avenue about half a block north of East Kenwood Boulevard. A twenty-five cent piece is needed for entrance.

The campus may be reached from the downtown area by bus, namely, Busline No. 30 ("Prospect-Maryland") from Wisconsin Avenue or Busline No. 15 ("Oakland-Delaware") from the corner of East Wisconsin Avenue and North Water Street. Exact fare of forty cents is necessary. East Kenwood Boulevard is the proper stop.

ENTERTAINMENT

There will be a complimentary beer and cheese party between 5:00 p.m. and 7:00 p.m. on Friday, November 26, and a tea between 4:00 p.m. and 6:00 p.m. on Saturday, November 27. Both events will be held in the Department of Mathematics Lounge, Room E495B of the Science Complex.

LIBRARY

The Mathematics Library is located in Room E370 of the Science Complex. It will be open on Friday, November 26, from 8:00 a.m. to 10:00 p.m., and on Saturday, November 27, from 7:00 a.m. to 6:00 p.m.

The time limit for each contributed paper in the general sessions is ten minutes and in the special sessions is twenty minutes. To maintain this schedule, the time limits will be strictly enforced.

SATURDAY, 8:00 A. M.

Special Session on Function Theory, Room E190, Science Complex

8:00-8:20
(1) Polynomial approximation near a point where the Cauchy-Riemann equations hold
Professor Michael B. Freeman, University of Kentucky (690-B7)

8:25-8:45
(2) A lemma concerning the mapping radius function and some of its applications. Preliminary report
Professor Karl Barth, Syracuse University, and Professor Walter J. Schneider*, Carleton University (690-B24)

8:50-9:10
(3) The moduli of extremal functions
Professor Stephen D. Fisher, Northwestern University (690-B3)

9:15-9:35
(4) Convolution inequalities and some extremal problems in function theory
Professor David Drasin, Purdue University, and Professor Daniel F. Shea*, University of Wisconsin (690-B12)

9:40-10:00
(5) The maximal Nevanlinna deficiency of a finite value in the class of entire functions of finite order λ
Professor Simon Hellerstein, University of Wisconsin (690-B14)

10:05-10:25
(6) Proof of Edrei's spread conjecture
Professor Albert Baernstein II, Syracuse University (690-B5)

10:30-10:50
(7) On Bers' boundary of Teichmüller space
Professor Albert Marden, University of Minnesota (690-B20)

SATURDAY, 8:25 A. M.

Special Session on Recursion Theory, Room 137, Physics and Engineering Building

8:25-8:45
(8) Thinning the branches of a semicomputable tree
Professor Thomas G. McLaughlin, University of Illinois (690-E12)
(Introduced by Professor Paul T. Bateman)

8:50-9:10
(9) Π_1^1 sets of sets and hyperdegrees
Professor Hisao Tanaka, Hosei University, Tokyo, Japan, and University of Illinois (690-E6)

9:15-9:35
(10) Recursion theory versus analog generability
Professor Marian Boykan Pour-El, University of Minnesota (690-E2)

9:40-10:00
(11) The recursive primitive recursive degrees are not a lattice. Preliminary report
Professor Michael A. Machtey, Indiana University (690-E4)

10:05-10:25
(12) Diagonalization and the recursion theorem
Professor James C. Owings, Jr., University of Maryland (690-E9)

10:30-10:50
(13) k-sections of type n objects. Preliminary report
Professor Gerald E. Sacks, Massachusetts Institute of Technology (690-E7)

*For papers with more than one author, an asterisk follows the name of the author who plans to present the paper at the meeting.

SATURDAY, 9:15 A. M.

Special Session on Transformation Groups, Room 135, Physics and Engineering Building

9:15-9:35
(14) Semicharacteristic classes. Preliminary report
Professor Ronnie Lee, Yale University (690-G2)

9:40-10:00
(15) Complex bordism of topologically cyclic groups and isometries
Professor Gary C. Hamrick*, University of Texas, and Professor Erich Ossa, Rheinische Friedrich-Wilhelms-Universitat, Bonn, Germany (690-G4)

10:05-10:25
(16) On spaces of equivariant self-maps
Professor Reinhard E. Schultz, Purdue University (690-G1)

SATURDAY, 9:00 A. M.

Session on Real Analysis, Room 133, Physics and Engineering Building

9:00-9:10
(17) Remark on strongly bounded vector measures. Preliminary report
Professor Joseph Diestel, Kent State University (690-B1)

9:15-9:25
(18) Regularity result for weak solutions of abstract differential equations
Professor Samuel Zaidman, Université de Montréal (690-B9)

9:30-9:40
(19) Strict essential minima
Mr. Richard J. O'Malley, Purdue University (690-B8)

9:45-9:55
(20) Lipschitz behavior of a class of integral transforms. Preliminary report
Dr. Kusum K. Soni and Dr. Raj Pal Soni*, University of Tennessee (690-B23)

10:00-10:10
(21) Summability of Jacobi series
Professor Richard A. Askey, University of Wisconsin (690-B21)

10:15-10:25
(22) Generalization of an inequality of Levinson and Fan
Mr. Daniel Segalman and Mr. Scott Lawrence*, University of Wisconsin (690-B22)
(Introduced by Professor Richard A. Askey)

SATURDAY, 9:00 A. M.

Session on Complex Analysis, Room E180, Science Complex

9:00-9:10
(23) Some subordination results for classes of univalent functions. Preliminary report
Mr. Robert Byers, University of Cincinnati (690-B10)
(Introduced by Professor Edward P. Merkes)

9:15-9:25
(24) Two-slit mappings and the Marx conjecture. Preliminary report
Professor Peter L. Duren, University of Michigan, and Professor Renate McLaughlin*, University of Michigan—Flint (690-B25)

9:30-9:40
(25) On bounded oscillation and asymptotic values of conformal strip mappings
Professor Arthur E. Obrock, Case Western Reserve University (690-B11)

9:45-9:55
(26) Uniformly equicontinuous families of conformal and quasiconformal mappings. Preliminary report
Dr. Raimo Näkki and Mr. Bruce P. Palka*, University of Michigan (690-B16)

10:00-10:10
(27) Radial limit sets on the torus
Professor Laurence D. Hoffman, Claremont Men's College (690-B15)

10:15-10:25
(28) Tangential and unrestricted nontangential limits in $N^*(U^n)$. Preliminary report
Mr. Carl Stephen Davis, University of Wisconsin (690-B19)

SATURDAY, 11:00 A. M.

Invited Address, Room E180, Science Complex
Deformations of principal bundles
Professor Raghavan Narasimhan, University of Chicago

SATURDAY, 1:30 P. M.

Invited Address, Room E180, Science Complex
Set theory and general topology
Professor Mary Ellen Rudin, University of Wisconsin

SATURDAY, 2:45 P. M.

Special Session on Function Theory, Room E190, Science Complex

2:45-3:05
(29) Quasiconformal mappings and the moduli of p-dimensional surface families
Professor Stephen B. Agard, University of Minnesota (690-B27)

3:10-3:30
(30) Inequalities for H^p functions on polydisks
Professor Peter L. Duren* and Professor Allen L. Shields, University of Michigan (690-B6)

3:35-3:55
(31) Zero sets in $H^p(U^n)$
Professor Joseph B. Miles, University of Illinois (690-B4)

4:00-4:20
(32) On polynomial hulls
Professor Herbert J. Alexander, University of Michigan (690-B18)

4:25-4:45
(33) Inner functions in the polydisc
Professor Patrick R. Ahern, University of Wisconsin (690-B28)
(Introduced by Professor Paul T. Bateman)

4:50-5:10
(34) A holomorphic extension theorem for real submanifolds of $\mathbb{C}^n$
Professor Ricardo Nirenberg, State University of New York at Albany (690-B29)

SATURDAY, 2:45 P. M.

Special Session on Recursion Theory, Room 137, Physics and Engineering Building

2:45-3:05
(35) The finite injury priority argument in α-recursion theory
Professor Manuel Lerman, Yale University (690-E1)

3:10-3:30
(36) The difficulty of splitting r.e. degrees. Preliminary report
Professor Alistair H. Lachlan, Simon Fraser University (690-E8)

3:35-3:55
(37) Classical descriptive set theory as a refinement of modern hierarchy theory. Preliminary report
Professor Yiannis N. Moschovakis, University of California, Los Angeles (690-E10)
(Introduced by Professor T. G. McLaughlin)

4:00-4:20
(38) The Friedberg-Muchnik theorem re-examined
Professor Robert I. Soare, University of Illinois at Chicago Circle (690-E3)

4:25-4:45
(39) Discrete ω-sequences of index sets. Preliminary report
Professor Louise Hay, University of Illinois at Chicago Circle (690-E11)

4:50-5:10
(40) Degrees in which the recursive sets are uniformly recursive
Professor Carl G. Jockusch, Jr., University of Illinois (690-E5)

SATURDAY, 2:45 P. M.

Special Session on Transformation Groups, Room 135, Physics and Engineering Building

2:45-3:05
(41) Differentiable actions of S^1 and S^3
Professor Hsu-Tung Ku, University of Massachusetts (690-G5)

3:10-3:30
(42) "Algebraic" 5-manifolds with fixed point free circle action
Professor Peter P. Orlik, University of Wisconsin and University of Oslo, Norway, and Professor Philip D. Wagreich*, University of Pennsylvania (690-G3)

3:35-3:55
(43) Compact quotient spaces of C^2 by affine transformation groups
Dr. Tatsuo Suwa, University of Michigan, and University of Tokyo, Japan (690-G6)

SATURDAY, 2:45 P. M.

Session on Algebra and Theory of Numbers, Room 133, Physics and Engineering Building

2:45-2:55
(44) On g-invariant measures
Professor Milton N. Parnes, State University of New York at Buffalo (690-A6)
(Introduced by Professor Richard E. Vesley)

3:00-3:10
(45) On partitions of the type $x_1^k + \ldots + x_\lambda^k = n$, $k \geqq 2$. Preliminary report
Professor Lal M. Chawla* and Professor John E. Maxfield, Kansas State University (690-A3)

3:15-3:25
(46) Proof of a basic property of positive definite binary n-adic forms
Mr. K. Demys, Santa Barbara, California (690-A1)

3:30-3:40
(47) Polycyclic groups and rings. Preliminary report
Dr. William F. Lipman, St. Anne's School, Brooklyn, New York (690-A2)

3:45-3:55
(48) On extensions of orders in abelian groups
Dr. Donald P. Minassian, Butler University (690-A5)

4:00-4:10
(49) Countable automorphism groups of groupoids
Dr. Matthew I. Gould, Vanderbilt University (690-A4)

SATURDAY, 2:45 P. M.

Session on Complex Analysis, Room E180, Science Complex

2:45-2:55
(50) A characterization of functions of bounded index
Dr. Gerd H. Fricke, Kent State University (690-B26)

3:00-3:10
(51) On radial growth and the distribution of values of a certain class of Hadamard gap series
Dr. William D. Serbyn, University of Minnesota (690-B13)

3:15-3:25
(52) Properties of domains bounded by k-circles. Preliminary report
Mr. Donald King Blevins, University of Michigan (690-B17)

3:30-3:40
(53) Toeplitz operators on certain function spaces. Preliminary report
Professor George U. Brauer, University of Minnesota (690-B2)

3:45-3:55
(54) Conjecture on John von Neumann's proof. Preliminary report
Professor Maria Z. v. Krzywoblocki, Michigan State University (690-C1)

Paul T. Bateman
Associate Secretary

Urbana, Illinois

Some Mathematical Questions in Biology
Philadelphia, Pennsylvania
December 26-27, 1971

The sixth annual symposium on Some Mathematical Questions in Biology will be held on December 26-27, 1971, in the Viennese Room of the Bellevue Stratford Hotel, Philadelphia, Pennsylvania. This symposium is cosponsored by the American Mathematical Society and the Society for Industrial and Applied Mathematics, and is being held in cooperation with Section A (Mathematics) of the American Association for the Advancement of Science. It is anticipated that the symposium will be supported by the Institute for Defense Analyses and the National Science Foundation. Registration and hotel arrangements will be announced in SCIENCE.

This is the sixth in a series of annual symposia whose purpose is to stimulate direct communication between biologists and biophysicists with some mathematical background, and mathematicians. The topics will range from studies on macromolecules and cells to those on organisms, and on evolution. The techniques discussed will include physiological and biochemical experimentation, computer simulation and numerical analysis, probability, nonlinear stability theory, and differential topology, as applied to biology.

The program, consisting of six lectures, was arranged by the AMS-SIAM Joint Committee on Mathematics in the Life Sciences. The members of the committee are Hans J. Bremermann, Hirsh Cohen, Jack D. Cowan (chairman), Murray Gerstenhaber, Alston S. Householder, Richard C. Lewontin, and Robert MacArthur.

PROGRAM

December 26, 2:00 p. m.

Chairman: Jack D. Cowan, Department of Theoretical Biology, University of Chicago

Determination of codon frequencies and sequence structure of polynucleotides
Hans J. Bremermann, University of California, Berkeley

Some biological and mathematical aspects of cellular dynamics
Joseph J. Higgins, Johnson Research Foundation, University of Pennsylvania

Evolution as a stability problem
Ilya Prigogine, Université Libre de Bruxelles, Belgium

December 27, 9:00 a. m.

Chairman: Murray Gerstenhaber, University of Pennsylvania

The horseshoe crab eye: A little nervous system whose dynamics are solvable
Bruce Knight, The Rockefeller University

Stochastic limitation on sensory performance
William M. Siebert, Massachusetts Institute of Technology

A global dynamical scheme for vertebrate embryology
René Thom, Institut des Hautes Études Scientifiques, Paris

Jack D. Cowan
Chairman

Chicago, Illinois

PRELIMINARY ANNOUNCEMENTS OF MEETINGS

The Seventy-Eighth Annual Meeting
Sahara Hotel
Las Vegas, Nevada
January 17-20, 1972

The seventy-eighth annual meeting of the American Mathematical Society will be held at the Sahara Hotel in Las Vegas, Nevada, January 17–20, 1972. The meeting is being held in conjunction with the annual meeting of the Mathematical Association of America (January 19–21). The Conference Board of the Mathematical Sciences will present a panel discussion on Performance Contracting and Mathematics in the Schools on Wednesday, January 19, at 4:00 p.m. The AMS Committee on Employment and Educational Policy will hold a panel discussion on Tuesday evening, January 18, at 8:30 p.m.

On the recommendation of the Council of the Society, the invited addresses will be scheduled without regard to the scheduling of sessions for ten-minute contributed papers. See page 485 of the April 1971 issue of these *Notices* for a more detailed explanation of this new policy. By invitation of the Committee to Select Hour Speakers for the Summer and Annual Meetings, there will be eight invited addresses. They are scheduled as follows:

Foliations of compact manifolds
Blaine Lawson, University of California, Berkeley
Monday, January 17, 9:00 a.m.

Equivariant real algebraic differential topology
Richard S. Palais, Brandeis University
Tuesday, January 18, 1:30 p.m.

The closing lemma revisited
Charles C. Pugh, University of California, Berkeley
Thursday, January 20, 2:45 p.m.

On automorphism groups of induced CR structure
Hugo Rossi, Brandeis University
Wednesday, January 19, 4:00 p.m.

Complexity of statement, computation, and proof
Jacob T. Schwartz, Courant Institute of Mathematical Sciences, New York University
Thursday, January 20, 1:30 p.m.

Powers of singular cardinals
Robert M. Solovay, University of California, Berkeley
Monday, January 17, 10:30 a.m.

Some applications of an axiomatic theory of sheaves
Myles Tierney, Rutgers University
Tuesday, January 18, 9:00 a.m.

The structure of attractors
Robert F. Williams, Northwestern University
Tuesday, January 18, 10:30 a.m.

All the hour lectures will be given in Rooms 2 and 3 of the Sahara Hotel Convention Center.

The Josiah Willard Gibbs Lecture will be given by Professor Freeman J. Dyson of the Institute for Advanced Study at 8:30 p.m. on Monday, January 17, in Rooms 2 and 3 of the Sahara Hotel Convention Center. The title of Professor Dyson's lecture is "Missed opportunities."

There will be no limit on the number of ten-minute papers presented at the meeting. The deadline for abstracts to be received in the Providence office was November 4, 1971. There is no provision for late papers. The Providence office will not be able to accept changes in abstracts. Authors are requested to notify the Providence office of papers to be withdrawn.

Two special sessions of selected twenty-minute papers are scheduled. Professor Louis Auslander of the City University of New York is arranging a session on Harmonic Analysis on Solvable and Nilpotent Lie Groups; this session will begin at 1:30 p.m. on Monday, January 17, in Room 5 of the Sahara Hotel Convention Center. The speakers will include Jonathan P. Brezin, Roe W. Goodman, Anthony W. Knapp, Adam Koranyi, Leonard F. Richardson, and S. Vagi. Professor Morris W. Hirsch of the University of California, Berkeley, is organizing a session on Foliations and Stable Manifolds; this session is scheduled for 2:45 p.m. on Tuesday, January 18, also in Room 5 of the Sahara Hotel Convention Center. The speakers at this session will be Neil H. Fenichel, Blaine Lawson, Joel Pasternak, Joseph F. Plante, Michael Shub, and William Thurston.

Professor Paul R. Halmos of Indiana University is arranging an informal session; it will begin at 2:45 p.m. on Tuesday, January 18, in Room 7 of the Sahara Hotel Convention Center. The topic is Noncommutative Approximation Theory. In contrast with classical approximation theory, which is concerned with functions, noncommutative approximation theory is concerned with operators, principally on Hilbert space. Speakers at this informal session will include William B. Arveson, Gerhard K. Kalisch, Allen L. Shields, and Joseph G. Stampfli.

COUNCIL AND BUSINESS MEETING

The Council of the Society will meet at 2:00 p.m. on Sunday, January 16, in Rooms 11 and 12 of the Sahara Hotel. These rooms are located in the North Hall section of the hotel on the second floor. The Business Meeting of the Society will be held on Wednesday, January 19, at 2:30 p.m. in Rooms 2 and 3 of the Sahara Hotel Convention Center.

In accordance with Article X, Section 1, of the bylaws, notice is given that the following resolution is on the agenda of the Business Meeting:

Resolved that

1) the American Mathematical Society will work actively for equal opportunities for women in the following areas:
   a) employment at all levels: this will include the search for and recruitment of qualified women.
   b) advancement and tenure in academic positions.
   c) admissions to graduate schools.
   d) graduate and postdoctoral fellowships and assistantships.
   e) membership on advisory boards and panels

and

2) the Society will include more women on
   a) Society programs and panels, including invited speakers and section chairmen.
   b) Society committees and governing boards.

REGISTRATION

The registration desk for this meeting will be in the lobby of the Sahara Hotel Convention Center, located on the second floor of the hotel. The desk will be open from 2:00 p.m. to 8:00 p.m. on Sunday, January 16; from 8:00 a.m. to 5:00 p.m. on Monday, January 17; from 8:30 a.m. to 4:30 p.m. on Tuesday through Thursday, January 18–20; and from 8:30 a.m. to 2:30 p.m. on Friday, January 21.

The registration fees for the meetings are as follows:

Member	$ 5.00
Student and unemployed	$ 1.00
Nonmember	$10.00

There will be no extra charge for members of the families of registered participants.

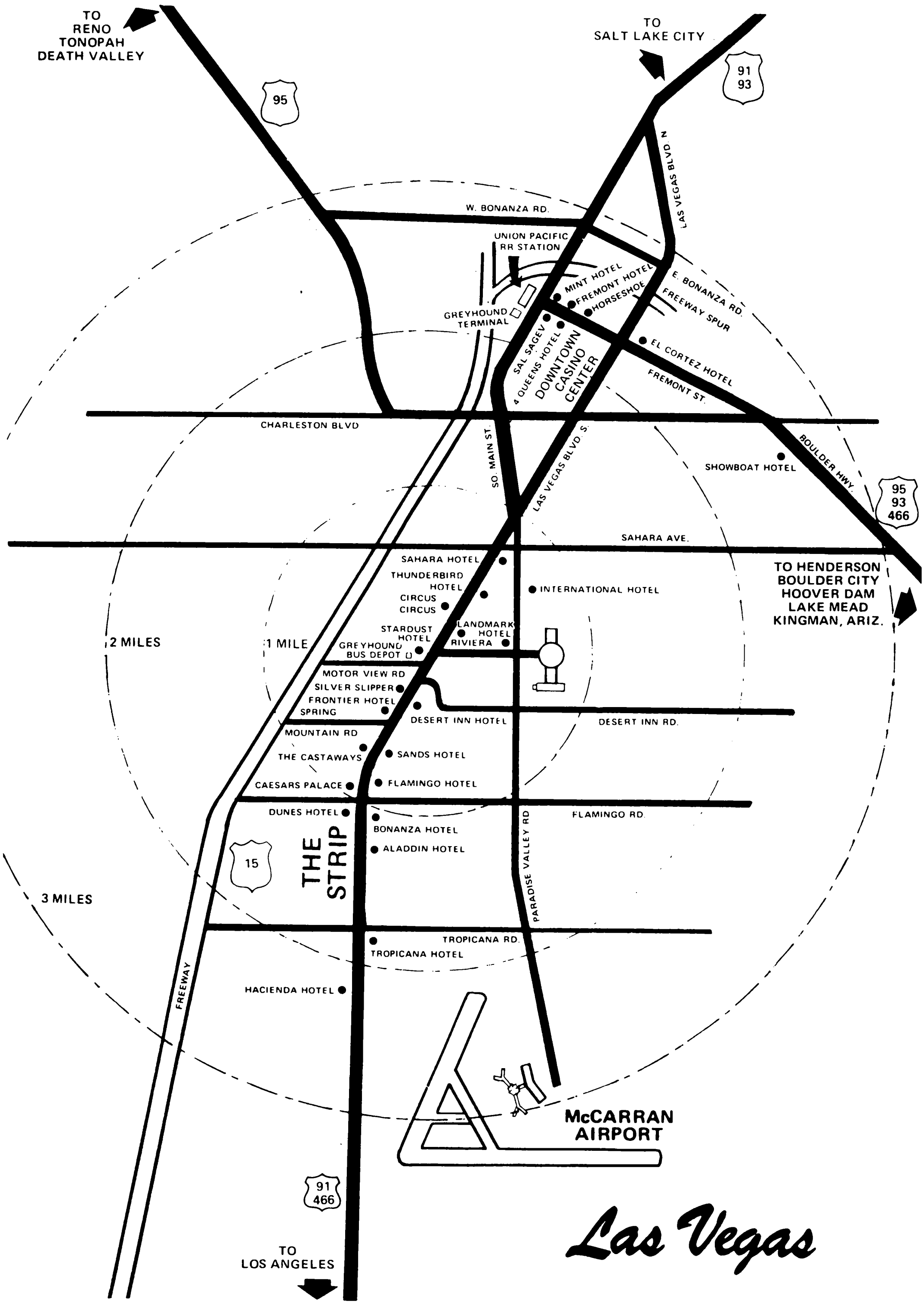
The unemployed status refers to any members currently unemployed, but actively seeking employment. It is not intended to include members who have voluntarily resigned or retired from their latest position.

EMPLOYMENT REGISTER

The Mathematical Sciences Employment Register will be maintained from 9:00 a.m. to 4:00 p.m. on Tuesday, January 18, and from 9:00 a.m. to 5:40 p.m. on Wednesday and Thursday, January 19–20, in the International Room of the Stardust Hotel. The International Room is located on the second floor of the hotel above the casino.

EXHIBITS

The book and educational media exhibits will be displayed in the Exhibit Hall, Room 1, of the Sahara Hotel Convention Center, from Monday through Thursday, January 17–20. The exhibits will be displayed from 12:00 noon to 5:00



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DUNES HOTEL
FLAMINGO RD.
BONANZA HOTEL
ALADDIN HOTEL
15
THE STRIP
PARADISE VALLEY RD
3 MILES
TROPICANA RD.
TROPICANA HOTEL
HACIENDA HOTEL
FREEWAY
McCARRAN AIRPORT
91 466
TO LOS ANGELES
Las Vegas

p.m. on Monday; from 9:00 a.m. to 5:00 p.m. on Tuesday and Wednesday; and from 9:00 a.m. to 12:00 noon on Thursday. All participants are encouraged to plan a visit to the exhibits sometime during the meeting.

The Monroe Calculator Company will sponsor a series of workshops on Wednesday, January 19, in Room 9 of the Sahara Hotel Convention Center. The workshops are scheduled for 9:00 a.m., 10:30 a.m., 1:30 p.m. and 4:00 p.m. Since attendance is limited to fifteen persons at each session, those interested in attending will have to sign in at the Monroe Calculator booth in the exhibit hall. Participants will have an opportunity to work "hands-on" with the Monroe 1600 series scientific electronic programmable calculators.

BOOK SALE

Books published by the Society will be sold for cash prices somewhat below the usual prices when these same books are sold by mail.

ACCOMMODATIONS

Accommodations for the meeting will be handled by the Las Vegas Convention Bureau Housing Office. A form for requesting accommodations will be found on the last page of these *Notices*. Persons desiring accommodations should complete this reservation form or a reasonable facsimile and send it to the Mathematics Meetings Housing Bureau, Las Vegas Convention Authority, P. O. Box 14006, Las Vegas, Nevada 89114. Reservations will be made in accordance with preferences indicated on the reservation form, insofar as this is possible, and all reservations will be confirmed. Deposit requirements vary from hotel to hotel, and participants will be informed of any such requirement at time of confirmation. REQUESTS FOR RESERVATIONS SHOULD ARRIVE IN LAS VEGAS NO LATER THAN DECEMBER 27, 1971. Saturday arrivals (as well as arrivals prior to 2:00 p.m. on Sunday) should be avoided if at all possible since Las Vegas is a busy weekend resort area.

CAESARS PALACE	
Singles	$16.00-$26.00
Twin	18.00- 28.00
Double	18.00- 28.00
Suites (1 bedroom)	65.00- 75.00
CASTAWAYS	
Singles	$14.00
Twin	16.00
Double	16.00
DUNES	
Singles	$15.00-$25.00
Twin	18.00- 28.00
Double	18.00- 28.00
Suites	54.00-150.00
FLAMINGO	
Singles	$20.00-$24.00
Twin	22.00- 30.00
Double	22.00- 30.00
Suites	46.00- 85.00
FREMONT	
Singles	$13.00
Twin	16.00
Double	16.00
FRONTIER	
Singles	$12.00-$22.00
Twin	14.00- 26.00
Double	14.00- 26.00
Suites (1 bedroom)	35.00- 95.00
HACIENDA	
Twin	$19.00-$23.00
Double	19.00- 23.00
INTERNATIONAL	
Singles	$18.00-$26.00
Twin	18.00- 26.00
Double	18.00- 26.00
Suites	48.00-145.00
LANDMARK	
Singles	$14.00-$20.00
Twin	17.00- 23.00
Double	17.00- 23.00
Suites	28.00- 90.00
SAHARA	
Singles	$16.00-$26.00
Twin	19.00- 26.00
Double	19.00- 26.00
Suites	45.00- 75.00
SANDS	
Singles	$16.00-$22.00
Twin	18.00- 25.00
Double	18.00- 25.00
STARDUST	
Singles	$12.00-$15.00
Regular Twin or Double	12.00- 17.00
"South Chateau" Twin or Double	21.00
Suites	35.00- 75.00

TROPICANA

Singles	\$14.00-\$16.00
Twin	14.00- 16.00
Double	14.00- 16.00
Suites	45.00- 75.00

THUNDERBIRD

Singles	\$14.00-\$16.00
Twin	14.00- 16.00
Double	14.00- 16.00
Suites	45.00- 75.00

NATIONAL SCIENCE FOUNDATION INFORMATION CENTER

NSF staff members will be available to provide counsel and information on all NSF programs of interest to mathematicians from 9:00 a.m. to 5:00 p.m. on January 18, 19, and 20, in Rooms C and D, Sahara Hotel. These rooms are adjacent to the registration area on the second floor of the hotel.

ENTERTAINMENT

Many scenic and recreational spots are easily accessible from Las Vegas. Red Rock Recreation Area, Valley of Fire, Lake Mead National Recreation Area, and Hoover Dam are each within an hour's drive. Local airlines offer scenic flights over and to the Grand Canyon. Death Valley is less than three hour's drive by car.

There are many excellent golf courses and tennis courts in the Las Vegas area. Entertainment in one form or another is available twenty-four hours every day and most of it is legal. A list of restaurants will be available at the Local Information Desk. This list will include details of the many lavish buffets and brunches for which Las Vegas is famous.

TRAVEL

In winter, Las Vegas is on Pacific Standard Time. There is regular airline service to the McCarran International Airport, about five miles from the Sahara Hotel. Limousine service, stopping at all of the hotels, is available from the airport. The fare is \$1.50. It should be noted, however, that because the Sahara Hotel is the last hotel at which the limousine stops, that it is not unusual for the limousine ride to take in excess of thirty minutes. Participants should join a group and take a taxi to the hotel if at all possible.

Driving time from either Phoenix or Los Angeles is about five hours, from Salt Lake City about nine hours. There is no passenger train service into Las Vegas. Greyhound and Trailways have scheduled bus service into Las Vegas; registrants can obtain the latest schedules and prices by calling the local offices of either company.

WEATHER

During January one may expect an average daily high temperature of about 54° and a low of around 32°. The record high is 76° and the record low is 8°. Usually it is sunny and clear; rain is unlikely.

MAIL AND MESSAGE CENTER

All mail and telegrams for persons attending the meetings should be addressed in care of Mathematics Meetings, Sahara Hotel, Las Vegas, Nevada 89114. Mail and telegrams so addressed may be picked up at the Mail and Information Desk located at the registration area in the lobby of the Sahara Hotel Convention Center.

A Message Center will be located in the same area to receive incoming calls for all members in attendance. The center will be open from January 17 through January 21 between 8:30 a.m. and 4:30 p.m. Messages will be recorded, and the name of any member for whom a message has been received will be posted until the message has been picked up at the Message Center. Members are advised to leave the following number with anyone who might want to reach them at the meeting: (702) 735-2111. Persons calling should specify that they want to leave a message at the Mathematics Meeting Message Center.

LOCAL ARRANGEMENTS COMMITTEE

Paul Aizley, H. L. Alder (ex officio), E. Maurice Beesley (chairman), Michael A. Golberg, Robert Clark Hooper, Kenneth A. Ross (ex officio), Lewis J. Simonoff, Gordon L. Walker (ex officio), Edward F. Wishart.

Kenneth A. Ross
Associate Secretary

Eugene, Oregon

AMERICAN MATHEMATICAL SOCIETY		MATHEMATICAL ASSOCIATION OF AMERICA and OTHER ORGANIZATIONS	
SUNDAY, January 16			
2:00 p.m.	COUNCIL MEETING		
MONDAY, January 17			
*9:00 a.m.	Sessions of Ten Minute Papers		
9:00 a.m.	Invited Address: Blaine Lawson Foliations of compact manifolds		
10:30 a.m.	Invited Address: Robert M. Solovay Powers of singular cardinals		
*1:30 p.m.	Sessions of Ten Minute Papers		
1:30 p.m.	Special Session of Twenty Minute Papers: Harmonic analysis on solvable and nilpotent Lie groups		
8:30 p.m.	Gibbs Lecture: Freeman J. Dyson Missed opportunities		
TUESDAY, January 18			
*9:00 a.m.	Sessions of Ten Minute Papers	9:00 a.m.	MAA -- BOARD OF GOVERNORS
9:00 a.m.	Invited Address: Myles Tierney Some applications of an axiomatic theory of sheaves		
10:30 a.m.	Invited Address: Robert F. Williams The structure of attractors		
*1:30 p.m.	Sessions of Ten Minute Papers		
1:30 p.m.	Invited Address: Richard S. Palais Equivariant real algebraic differential topology		
2:45 p.m.	Special Session of Twenty Minute Papers: Foliations and stable manifolds		
2:45 p.m.	Informal Session: Noncommutative approximation theory		
8:30 p.m.	Panel Discussion: Committee on Employment and Educational Policy		
WEDNESDAY, January 19			
			MAA - The Open University in England
		9:00 a.m.	The Open University and its mathematics curriculum
		10:00 a.m.	Film Showing: Open University mathematics productions
		11:00 a.m.	Panel Discussion: The implications of the Open University for mathematics education in the United States
1:15 p.m.	Cole Prize Session		
2:30 p.m.	BUSINESS MEETING		
*4:00 p.m.	Sessions of Ten Minute Papers		
4:00 p.m.	Invited Address: Hugo Rossi On automorphism groups of induced CR structure	4:00 p.m.	Conference Board of the Mathematical Sciences -- Panel Discussion: Performance contracting and mathematics in the schools
		8:00 p.m.	Mathematics Action Group -- Panel Discussion: Elitism in mathematical education

AMERICAN MATHEMATICAL SOCIETY		MATHEMATICAL ASSOCIATION OF AMERICA and OTHER ORGANIZATIONS	
THURSDAY, January 20			
			MAA -- Modernizing the mathematics curriculum in two- and four-year colleges
		9:00 a.m.	Mathematics for the liberal arts student O. W. Laing
		9:40 a.m.	Mathematics for the "captured" student S. K. Stein
		10:30 a.m.	BUSINESS MEETING
		11:20 a.m.	Retiring Presidential Address: G. S. Young The mathematical sciences in the next decade
*1:30 p.m.	Sessions of Ten Minute Papers		
1:30 p.m.	Invited Address: Jacob T. Schwartz Complexity of statement, computation, and proof		
		2:00 p.m.	CBMS - COUNCIL
2:45 p.m.	Invited Address: Charles C. Pugh The closing lemma revisited		
		4:00 p.m.	Rocky Mountain Mathematics Consortium - Board Meeting
		8:00 p.m.	CBMS - COUNCIL
FRIDAY, January 21			
			MAA -- Players, probabilities, and profits
		9:00 a.m.	The law and large numbers Richard Epstein
		10:00 a.m.	Games mathematicians play S. W. Golomb
		11:00 a.m.	How to gamble successively L. E. Dubins
		1:30 p.m.	Galois theory revisited D. K. Harrison
		2:30 p.m.	Some applications of analysis to number theory H. M. Stark
		3:30 p.m.	Generators, relations, and congruence subgroups Robert Steinberg

* Time given is approximate.

Six Hundred Ninety-Second Meeting
Biltmore Hotel
New York, New York
March 27 – 30, 1972

The six hundred ninety-second meeting of the American Mathematical Society will be held at the Biltmore Hotel, Madison Avenue at 43rd Street, New York, New York, from Monday, March 27, through Thursday, March 30, 1972.

By invitation of the Committee to Select Hour Speakers for Eastern Sectional Meetings, Professor Alex Rosenberg of Cornell University will present an hour address entitled "Commutative ring theory and the structure of Witte rings." Professor Gian-Carlo Rota of the Massachusetts Institute of Technology will also speak; the title of Professor Rota's lecture and information on two additional speakers will be announced in the January issue of these *Notices*.

There will be sessions for contributed ten-minute papers in the morning and afternoon on both Monday and Tuesday. Abstracts should be submitted to the American Mathematical Society, P.O. Box 6248, Providence, Rhode Island 02904, so as to arrive prior to the deadline of January 27, 1972.

Detailed information about travel and accommodations will appear in the January *Notices*; the final program of the meeting will appear in the February *Notices*.

SYMPOSIUM ON STOCHASTIC DIFFERENTIAL EQUATIONS

With the expected support of the Office of Naval Research, a symposium on Stochastic Differential Equations is scheduled to be held on Wednesday and Thursday, March 29 and 30. This topic was selected by the AMS-SIAM Committee on Applied Mathematics whose members are Donald G.M. Anderson, Joaquin B. Diaz, Hirsh G. Cohen, Stanislaw M. Ulam, Richard S. Varga (chairman), and Calvin H. Wilcox. The cochairmen of the Organizing Committee are J.B. Keller and H.P. McKean both of the Courant Institute of Mathematical Sciences, New York University. Further details about this symposium will be given in the January and February issue of these *Notices*.

Walter H. Gottschalk
Associate Secretary

Middletown, Connecticut

Six Hundred Ninety-Third Meeting
St. Louis University
St. Louis, Missouri
March 29-April 1, 1972

The six hundred ninety-third meeting of the American Mathematical Society will be held at St. Louis University, St. Louis, Missouri, from March 27 to April 1, 1972. St. Louis University is located at the corner of Grand Boulevard and Lindell Boulevard about two miles west of downtown St. Louis.

By invitation of the Committee to Select Hour Speakers for Western Sectional Meetings, there will be five one-hour addresses. The speakers will be Professors Gilbert Baumslag of Rice University, Allen Devinatz of Northwestern University, Lawrence Markus of the University of Minnesota, Wolfgang Schmidt of the University of Colorado, and (tentatively) Atle Selberg of the Institute for Advanced Study.

There will be sessions for contributed papers on both Friday and Saturday, March 31 and April 1. Abstracts should be submitted to the American Mathematical Society, P.O. Box 6248, Providence, Rhode Island, so as to arrive prior to the deadline of January 27, 1972. Those having time preferences for the presentation of their papers should so indicate on their abstracts. There will be a session for late papers if one is needed, but late papers will not be listed in the printed program of the meeting.

There will be four special sessions of selected twenty-minute papers. Professor William M. Boothby of Washington University is arranging one such session on Some Aspects of Differential Geometry. Another special session is being arranged by Professor Saunders Mac Lane of the University of Chicago on the subject of Category Theory. Professor Lee A. Rubel of the University of Illinois is arranging a special session on the subject of Vector Spaces of Analytic Functions. The fourth special session is being arranged by Professor Thomas E. Storer of the University of Michigan on the subject of Combinatorial Theory. Most of the papers to be presented at these four sessions will be by invitation. However, anyone contributing an abstract for the meeting who feels that his paper would be particularly appropriate for one of these special sessions should indicate this emphatically on his abstract and submit it three weeks earlier than the above deadline, namely by January 6, 1972, in order to allow time for the additional handling necessary.

Detailed information about travel and accommodations will appear in the January *Notices*. The final program of the meeting will appear in the February *Notices*.

SYMPOSIUM ON ANALYTIC NUMBER THEORY

The Society is seeking the support of the National Science Foundation for a symposium on Analytic Number Theory, to be held March 27-30. This topic was chosen by the Committee to Select Hour Speakers for Western Sectional Meetings, which consists of Paul T. Bateman (chairman), Roger C. Lyndon, and Lawrence Markus. The Organizing Committee of the symposium, responsible for the planning of the program and the choice of speakers, consists of Harold G. Diamond (chairman), Patrick X. Gallagher, Hugh L. Montgomery, Wolfgang M. Schmidt, and Harold M. Stark. Further details about this Symposium will be given in the January and February issues of the *Notices*.

Paul T. Bateman
Associate Secretary

Urbana, Illinois

LETTERS TO THE EDITOR

Editor, the *Notices*

After the decision of the Council announced in the June issue of the *Notices* that advertisers in the *Notices* be required to abide by the requirements of Title VII of the 1964 Civil Rights Act (at least in spirit), why is it that an advertisement for the University of the Witwatersrand in Johannesburg, South Africa, appeared in the August issue?

M. Solveig Espelie
Howard University

EDITOR'S NOTE

The secretary asked for the advice of the Council on whether to publish the advertisement in question (these *Notices*, page 846 of the current year). In response, the Council passed a resolution in April 1971 that the *Notices* and the rate card for advertisers should contain a statement such as the one on page 600 of the June 1971 *Notices*. The proof sheet of the advertisement from the University of Witwatersrand was returned to Mr. F. Roberts, scientific counsellor of the South African embassy, for him to verify the correctness of the text, and it carried the following statement: "All employers in the United States are required to abide by the requirements of Title VII of the Civil Rights Act of 1964, enunciating a national policy of equal employment opportunity in private employment, without discrimination because of race, color, religion, sex, or national origin. We assume that all institutions advertising positions abide by the spirit of the law, whether or not they are legally bound to do so." Mr. Roberts acknowledged the correctness of the proof, but initially did not sign the statement in the place provided on the grounds that it pertained only to the internal U.S.A. He was informed that the last sentence was written to apply to advertisers outside the United States, and that the editors must know whether the University of Witwatersrand subscribed to the statement before authorizing the publication of the advertisement. Then Mr. A.J. deV. Herholdt, registrar of the university, signed the statement. In retrospect, the editors wish that the statement had a more affirmative tone, and plan to rewrite it.

EMPLOYMENT OF NEW PH. D'S

By R. D. Anderson

On behalf of the AMS Committee on Employment and Educational Policy and with the assistance of Lincoln K. Durst of the AMS, the author tabulated on August 27, 1971, from the data then available, the employment status for 1971-1972 of the Ph.D.'s who received their degrees during the period July 1, 1970-June 30, 1971. The basic data was from departmental lists of all new Ph.D.'s received by the Society in the summer of 1971, modified by later information submitted for the Starting Salary Survey by almost 50% of the Ph.D.'s themselves. See the October 1971 *Notices* for the lists of new Ph.D.'s, the Starting Salary Survey Report, and a flow diagram representing a slight extrapolation of the data in the table on the next page.

Several comments about the classifying procedures and the data should be made.

1. The information on degrees in pure mathematics is substantially complete, probably more than 96%. The information for other areas is less complete, e.g. many computer science departments did not respond to the request for information, and such data may be only 60% to 70% complete. Moreover, there is no clear-cut basis for classifying some new Ph.D.'s in computer-oriented subjects as belonging to, or as not belonging to, the mathematical sciences. The mathematics education figure is substantially incomplete, e.g. in early 1970 government figures showed over one hundred doctorates per year in mathematics education (many of these may be only marginally in mathematics). The "Others" classification, as classified by the departments submitting the lists, includes some biometry, some biostatistics, and some computer-related subjects.

2. Canadian Ph.D.'s are included as is customary in AMS data. Numerically, however, the Canadian Ph.D.'s balance out: seventy-nine new Canadian Ph.D.'s are included, whereas eighty-three new Ph.D.'s had addresses in Canada, seventy-nine of whom obtained academic positions there. Of the remaining one hundred and five new Ph.D.'s with foreign addresses, seventy-five had academic positions for

COMMITTEE ON EMPLOYMENT AND EDUCATIONAL POLICY

The AMS Committee on Employment and Educational Policy (William L. Duren, Jr., chairman, Richard D. Anderson, John W. Jewett, and Gail S. Young) has been authorized (1) to oversee the gathering of employment data by the Society, (2) to monitor the job market and inform the membership of its findings, (3) to consider possible alternative educational programs in mathematics, and (4) to report and make recommendations to the Council. The committee, and some of its members acting as individuals, have reported to the mathematical community through articles in the *Notices* (April 1971, pp. 486-490; August 1971, pp. 718-722; November 1971, pp. 1021-1026). The report of the Fifteenth Annual Salary Survey appeared in the *Notices* (October 1971, pp. 865-871), together with a summary of the panel discussion held by the committee at the summer meeting in University Park, Pennsylvania.

The committee plans a second panel discussion at the annual meeting in January 1972 in Las Vegas, Nevada. In preparation for this session, the committee extends primarily to younger mathematicians an invitation to express their thoughts and suggestions regarding the problems of education and employment now confronting the mathematical profession. Please address comments to Dr. Lincoln K. Durst, American Mathematical Society, P.O. Box 6248, Providence, Rhode Island 02904.

1971-1972.

3. The "University" classification is basically for Ph.D. granting institutions (and includes research institutes). The "College" classification includes both bachelor's and master's level institutions.

4. Of the eighty-one listed as "Unknown," twenty-seven are from three departments which systematically did not provide data on 1971-1972 employment of their Ph.D.'s.

5. A substantial number of those taking industrial, business, or government positions apparently already had such positions while doing graduate work; e.g. nine statisticians listing jobs in government were granted their Ph.D.'s by universities in the Washington, D.C., area, and at least three of the twelve pure mathematicians listing industrial or business positions apparently had held such jobs previously.

6. Those getting degrees in January-June 1971 were originally tabulated separately from those getting degrees in July-December 1970, since the latter group (509 in all) were to be in their second postdoctoral year in 1971-1972. The patterns of the two tabulations, however, seemed rather similar; thus, the combined figures only are presented below. Perhaps the biggest difference was in the category "Not yet employed" for 1971-1972 which showed 9% of the total for the 1971 graduates as compared to 7% overall.

7. The "University" figure includes postdoctorals, research instructors, and other replacements of nontenured faculty. The total university faculty changed very little from last year according to the Salary Survey.

8. More than 600 of the new Ph.D.'s submitted starting salary data in July and August with about 35 of these showing positions for 1971-1972 not reported by their departments earlier in the summer. Perhaps another 35 (representing the other 700 of the new Ph.D. total) also had obtained positions by late August. This

1971-1972 JOB STATUS OF NEW PH. D. 'S IN MATHEMATICS

Type of employer	Area of Dissertation							
	Pure Mathematics	Applied Mathematics	Computer Science	Operations Research	Statistics	Mathematics Education	Others	Totals
University	206	48	47	14	72	2	8	397
College	344	26	10	5	36	16	1	438
Two-year college	13		1					14
Business and industry	12	11	24	11	16		3	77
Government and research laboratory	7	12	6	2	25		3	55
Foreign	102	17	18	8	31	2	10	188
Military service	2	2	2		1			7
Not seeking employment	2				1			3
Not yet employed	71	11	1	2	9		2	96
Unknown	49	7	6	4	12		3	81
Totals	808	134	115	46	203	20	30	1356

probably effectively offsets the somewhat worse employment pattern that presumably would have been shown by the August 1971 graduates (for which no dissertation titles or other data had been collected, nor could appropriately have been collected, early in the summer). It is known anecdotally that a few new Ph.D.'s did get last-minute positions in late August or early September.

9. The number of 1970-1971 nontenured faculty who were not retained by their colleges or universities and who were "Not yet employed" for 1971-1972 is not revealed by information available to the AMS at the end of August. The flow diagram of Ph.D. employment in physics for 1970, reported in Physics Today in May 1971, suggests that professional unemployment is much more likely for nonretained Ph.D.'s than for new Ph.D.'s. The AMS Committee on Employment and Educational Policy hopes to be able to get useful data on this problem for Ph.D.'s in mathematics.

10. Among the new Ph.D.'s classified as "Not yet employed" for 1971-1972, three types of individuals stood out: (a) women, most of whom were seeking jobs, presumably, near where their husbands had jobs; (b) a scattering of new Ph.D.'s from some of the most prestigious institutions; and (c) those who names identified them as Asians and who apparently came to the U.S. as students.

Comparisons with Tabulations of 1968-1969 Ph.D.'s

As reported by the author in an article in the June-July 1970 issue of the American Mathematical Monthly, a somewhat similar tabulation of positions was made for 1,164 new Ph.D.'s who received their degrees during the period of July 1, 1968-June 30, 1969. The classifications were a bit different however: the mathematical area was assigned by name of department (or occasionally by dissertation title); the "Mathematics" classification was basically pure mathematics but included some applied mathematics; and the employment information requested was for the immediate postdoctoral year. The following pattern changes seem worthy of note:

1. About 12% of the total in 1968-1969 took positions in business, industry, and government as compared to 10% in 1970-1971. The actual numbers of pure and (classical) applied mathematics Ph.D.'s who took such positions fell from seventy-one for 1968-1969 to fifty-two for 1970-1971. Presumably the economic slowdown is to blame, but, unfortunately, we now have a smaller base from which to build.

2. Only about 10% of the 1968-1969 graduates went to foreign addresses as compared to 14% of last year's graduates. (CBMS and NSF figures indicate that foreign students constitute about 20% of the total graduate enrollment in the mathematical sciences.)

3. The relative frequencies of employment roles by universities and by colleges for pure and (classical) applied mathematics were reversed-356 university and 291 college for 1968-1969, and 254 university and 370 college for 1970-1971.

4. The total of pure, (classical) applied mathematics, and mathematics education Ph.D.'s in 1968-1969 was 932 as compared to 962 in 1970-1971. Thus, the most recent growth in Ph.D. production has been in the computer and statistical applied areas.

5. For 1968-1969, only two Ph.D.'s (both in mathematics education) took positions in two-year colleges. For 1970-1971, at least fourteen Ph.D.'s did so. (Perhaps a few were missed in the tabulation because some colleges were not adequately identified.) In any event, it seems clear that there is no basis for believing that the two-year colleges will hire more than a small number of mathematics Ph.D.'s a year. Indeed, CBMS survey data collected a year ago indicate that then existing two-year colleges employed 137 additional doctorates for their mathematics teaching staff in 1970-1971, and hoped to employ 80 more in 1971-1972, but only 32 more in 1972-1973 if successful in their 1971-1972 hiring. The figure 137 for 1970-1971 presumably includes some doctorates moving from one two-year college to another, some nontenured doctorates from four-year colleges and universities, and some doctorates from disciplines not (fully) included

in the AMS figures such as mathematics education or physics. The reduction in prospective demand unfortunately indicates that we may already be approaching a level of saturation for faculty with the doctorate in the existing two-year colleges.

6. Further data on trends in employment of doctorates are given in the January and October, 1971, issues of the CBMS Newsletter.

Future Employment Prospects

The following assessments of future job prospects for Ph.D. mathematicians are those of the author and do not necessarily reflect those of the other members of the Committee on Employment and Educational Policy.

Academic Employment. For background information, readers are referred to the thoughtful and well-documented articles by Allan M. Cartter (*Science*, Volume 172, pp.132-140, 9 April 1971) and by Dael Wolfle and Charles V. Kidd (*Science*, Volume 173, pp. 784-793, 27 August 1971) which discuss the overall scientific Ph.D. supply and demand over the next fifteen or twenty years.

Academic employment for future Ph.D.'s in mathematics depends primarily on the future growth of the national mathematics faculty, since replacements due to death and retirement will average only about 200 per year, or 1% of the faculty, over the next twelve or fifteen years. The future growth of the faculty depends on (the recognition of) possible existing deficiencies in present faculties, on the growth in the student population, and on the economy and public attitude as they affect funds to be made available by society for higher education. We shall concentrate on the first two of these factors since they are more susceptible to analysis and are thus more predictable. But a few words about the economy are needed. There is no doubt that the current national economic slowdown has been a major contributing factor in the tight job market this year. There is also no doubt that a major improvement in the economy would produce significantly more jobs in academia over the next year or so; however, the evidence to follow shows that the economy is not the basic cause of the impending future job crisis.

There is little reason to believe that, over the next ten or twenty years, money will be made available for mathematics faculty growth in excess of the growth justified by existing faculty deficiencies and future student growth. It is much more likely that economic and public attitudes will serve as limiting factors. It is widely understood that the private sector of the four-year college and university community is in serious financial trouble. Thus, only very limited faculty growth can be expected from this (approximate) third of the community. Indeed, raw data from the AMS Salary Survey indicates that mathematics faculty in the private institutions declined almost 2% in the past year while the faculty in the public institutions increased almost 2%.

Deficiencies in the present faculty, as they affect job prospects, are of three possible types: (1) deficiency in numbers of faculty necessary to handle the present student body; (2) deficiency in quality of faculty (usually measured in terms of percentage of faculty having the Ph.D.); (3) deficiency in numbers or quality of faculty in special mathematical areas such as computer science. There seems to be little deficiency of any type in the third of the community consisting of Ph.D. granting public universities, except possibly for a lack of computer scientists in some places; but there probably are recognized deficiences of all types in the public colleges (bachelor's and master's level) comprising the other third of the community. In the 1960's, however, the total full-time national mathematics faculty increased by a factor of about 2.6 (roughly from 6,000 to over 15,000) while the full-time equivalent student population was increasing only by a factor of about 2.1. Reductions in teaching loads and growth in new mathematical areas probably account for the excess. At the same time, the percentage of the mathematics faculty having Ph.D.'s rose from under 48% to about 63% in 1970 (CBMS figures); probably it is at least 66% this year. The effective saturation level will probably be reached within a couple of years. The faculty size and Ph.D. percentage figures from the past several years of the AMS Salary Surveys strongly support this view. In the bachelor's level schools reporting (not a random sample), the percentage of faculty with Ph.D.'s has grown from 28% in 1967–1968 to almost 49% in 1971–1972, and in master's level schools from 43% in 1967–1968 to 60% in 1971–1972, both percentages going up about six points in the past year alone.

The aspect of the growth and improvement of the national mathematics faculty in the 1960's that has been most striking is that it has been achieved with an average annual production of about 700 Ph.D.'s (303 in 1960 and 688 in 1965), whereas we are now turning out about 1,300 a year, not counting the Canadians. Some of the growth has been with master's level faculty, and apparently much of the growth in computer areas has been with faculty trained in other disciplines.

We now consider the critical question of the future growth of the national student body. Most of the figures cited below are extracted from Cartter's article referred to above. These figures were briefly discussed in Gail Young's article in the August 1971 *Notices* but are worth repeating.

18-21 age group population (in thousands)

1965	12,154
1970	14,218
1975	16,206
1980	17,033
1985	15,445
1990	14,493 (estimated)

A rise of 20% is due from 1970 to 1980, and a corresponding drop is due from 1980 to 1990. Furthermore, at the present time about 48% of all eighteen-year olds enter (or will enter) degree-credit college studies, and another 10% enter nondegree but formal postsecondary programs.

Starting from these figures and from known college enrollments, Cartter has projected college enrollments to 1990. He assumes that by 1982, 63% of the eighteen-year olds will enter degree-credit college programs and that this figure is at or near the ultimate saturation level. (The 63% figure for 1982 seems optimistic to me since in 1970 only 78% of the eighteen-year olds were finishing, or were going to finish, high school.) He concludes that a rise in full-time equivalent college students of about 50% from 1970 to 1980 is indicated. (The rise from 1960 to 1970 was about 110%.) The tacit assumption is apparently made that the average duration of time students spend in college work will remain about constant (e.g. increases in graduate enrollment should be offset by a higher percentage of marginal students who drop out after a year or so). The numerical increase in full-time equivalent students from 1960 to 1970 was 3,390,000. Cartter's projected increase from 1970 to 1980 is 3,234,000, more or less evenly distributed over the ten years. He projects a decrease from 1980 to 1990 of 863,000.

The 9,000 total growth of the national mathematics faculty in the 1960's would appear to be an upper bound on the faculty growth in the 1970's, as the former occurred in an era of growth psychology and of relative prosperity. Perhaps a faculty growth of 7,000 would be more realistic, and even that figure may be too high. Maybe 20% of the new faculty will not be Ph.D.'s (there is much precedence for this, particularly in those disciplines with a very heavy freshman teaching load), and some of the new faculty will have Ph.D.'s from other disciplines (as has occurred in the past). A disproportionate share of the growth is expected to be in computer science and related areas.

As the faculty growth in the 1960's was achieved with an average annual production of 700 Ph.D's, and as we are now producing 1,300 Ph.D.'s annually, it is all too clear that many of the Ph.D.'s in the next decade must find employment outside academia. Indeed, the situation may be much worse since, of the various projections of future Ph.D. production as reported by Wolfle and Kidd, the most conservative (Cartter) anticipates 2,000 Ph.D.'s in the mathematical sciences per year by 1980. Certainly Ph.D. production in various computer subjects will increase markedly.

In the 1960's, over 80% of our Ph.D. production in the mathematical sciences found long-term employment in academia;

in the other physical sciences, the figure was 40%. We will be fortunate in the 1970's to have 40% to 50% of our Ph.D.'s find employment in academia, and in the 1980's there may be almost no such employment for new Ph.D.'s. As indicated in the table at the beginning of this report, it is the pure and (classical) applied mathematicians who are facing the most immediate and pronounced job crisis. Indeed, those new Ph.D.'s in any traditional discipline whose training is directed primarily toward research and teaching in academia are among those who can expect the most difficulty in finding suitable employment over the next twenty years. Furthermore, much of the academic employment will probably occur in the less prestigious schools without Ph.D. programs and without research libraries. The tight academic job market will affect both new Ph.D.'s and those nontenured faculty members who are not retained by their departments.

Nonacademic Employment. Future employment of Ph.D. mathematicians outside academia appears to fall into three categories: (1) traditional nonacademic Ph.D. employment (principally in research and development activities); (2) employment in positions of the type now held by master's or bachelor's level mathematicians; (3) employment as highly educated and intelligent people but without much explicit use of high level mathematical training.

As shown by the table at the beginning of this report, and in various other analyses of positions held by Ph.D.'s outside academia, there has been only a relatively small amount of employment of Ph.D. mathematicians in government, business, and industry (particularly of those in pure mathematics). Studies of recent nonacademic employment problems of Ph.D.'s in science emphasize research and development (R and D) activities and the past and current shortages of public and private funds for R and D. Estimates of future growth of R and D funds are in the few percent per year category. It is highly unlikely, therefore, that a growth in such jobs for Ph.D. mathematicians would significantly alleviate the conditions that will be caused by the massive imbalance in the academic supply and demand; indeed, the low numerical level of present employment of computer science Ph.D.'s makes the future nonacademic demand for Ph.D.'s even in this area quite uncertain.

It has been an assumption by Cartter and others that Ph.D.'s in science and mathematics will be able to get the jobs traditionally held by master's level people, thereby resulting in underemployment rather than unemployment per se. It is not certain whether this will in fact occur for Ph.D.'s in pure areas of science and mathematics, and if it does, it is not clear what effect it will have on salaries and working conditions. In the transition era over the next several years, such nonacademic employment may well be the best and only available.

In projections of future production and employment of Ph.D.'s in science, engineering, and mathematics, NSF and others have anticipated that society will somehow employ Ph.D.'s in nontraditional jobs, and that many Ph.D.'s will be taking positions not directly using their advanced graduate training. For the past two or three generations, society has been adapting itself to use more and more college-trained people without specific regard to the specialties of their education. Perhaps our present situation, relative to Ph.D. training, is comparable to the situation of some years ago with respect to bachelors training. As Wolfle and Kidd point out, half of all Ph.D.'s produced in the United States got their degrees in the past decade; the total is expected to double again by 1980. In any event, all major scientific disciplines and almost all other academic disciplines are going to produce far, far more Ph.D.'s than past patterns of employment can accommodate. In the long run, this fact will force society to adjust its employment practices and priorities in order to enable it to make proper use of its highly educated citizenry.

MEMORANDA TO MEMBERS

MATHEMATICAL SCIENCES EMPLOYMENT REGISTER

The International Room of the Stardust Hotel in Las Vegas, Nevada, will be the location of the Mathematical Sciences Employment Register during the annual meeting. The Employment Register will be open for three days, January 18 through January 20, 1972. Hours of operation will be from 9:00 a.m. to 4:00 p.m. on Tuesday and from 9:00 a.m. to 5:40 p.m. on Wednesday and Thursday. If necessary, evening interviews will be scheduled on Wednesday and Thursday.

Registration for the Employment Register is separate and apart from meeting registration; it is, therefore, most important that both applicants and employers sign in at the Employment Register desk as early as they can on Tuesday morning, January 18.

The new system of operation introduced in January 1971 will again be in effect for the open Register in Las Vegas, with a few administrative changes. The most significant change is that a registration fee of $10 for each employer in the open Register has been established. Also, applicants MUST BE REGISTERED for the general Mathematics Meeting before registering for the open Register; there is no open Register fee for applicants participating in interview schedules. Registration fees, location of the registration area, and hours of operation for the registration of participants for the Mathematics Meeting are listed in the preliminary announcement of the meeting included in this issue of these *Notices*.

Applicants and employers should secure an instruction sheet to acquaint themselves with the rules and operating regulations. These instruction sheets will be available on request in the International Room registration area at 9:00 a.m. on Tuesday. There will be no interviews scheduled for the first day. Please keep in mind that the registration for the open Register is separate and apart from both meeting registration and from the published listings, and it is imperative that both applicants and employers who wish to participate in the open Register sign in at the Employment Register desk as early as they can on Tuesday morning. <u>Appointments will be scheduled only for those people who have actually signed in at the Register and obtained a code number.</u> Requests for appointments can be submitted on Tuesday and Wednesday only and these interviews will be scheduled on Wednesday and Thursday respectively.

The January published lists will be mailed on approximately December 22. Applicants and employers who wish to be listed in the published list should write to the Mathematical Sciences Employment Register, P.O. Box 6248, Providence, Rhode Island 02904, for applicant qualification forms or position description forms. These forms must be completed and returned to the Employment Register not later than December 1, 1971, in order to be included in the January lists. There is no charge for listing in the published lists except when the late listing charge of $5 is applicable. Provision will be made for anonymity of applicants upon payment of $5 to defray the cost involved in handling such a listing.

A subscription to the lists, which includes three issues (January, May, and August) of both the applicants list and the positions list, is available for $30 a year; the individual issues of both lists may be purchased in January, May, or August for $15. A subscription to the applicants list alone or single copies of that list is not available. Copies of the positions list only may be purchased for $5. A subscription to the list of positions, which also includes three issues (January, May, August), is available for $12 a year. Employers who wish to display literature pertinent to available positions may do so. The charge for this service is $15. Checks should be made payable to the American Mathematical Society and sent to the address given above.

It should also be carefully noted that lists are mailed book rate (which means average delivery time from Providence to most locations is approximately 14 to 21 days) unless the purchaser either indicates a willingness in advance to pay the first class or airmail charges or includes the fee for this service when prepayment is made. The applicable postage charges, determined by the location of the purchaser, will be furnished on request to those persons who would like to take advantage of this service. A limited number of the published lists will also be available at the meeting on a first-come-first-served basis.

The Employment Register is sponsored by the American Mathematical Society, the Mathematical Association of America, and the Society for Industrial and Applied Mathematics for the purpose of establishing communication between mathematical scientists available for employment and employers with positions to fill.

NEWS ITEMS AND ANNOUNCEMENTS

NEW DOCTORAL PROGRAM AT UNIVERSITY OF IOWA

The Graduate College of the University of Iowa announces an interdisciplinary program leading to the Doctor of Philosophy in Applied Mathematical Science. The program seeks to assist the student to achieve basic command of abstract mathematics, at least one science (physical, behavioral, biological, medical), and the methods of applied mathematics. Additionally, the program seeks to develop the "attitude" of an applied mathematical scientist by emphasizing the totality of the discipline. The individual plan of study will be specifically programmed, taking into consideration the aforementioned subjects together with the student's background, interests, and goals. Students applying for admission should have an excellent background in science and mathematics. Further information and applications may be obtained from the Chairman, Program in Applied Mathematical Science, Graduate College, University of Iowa, Iowa City, Iowa 52240.

FULBRIGHT SCHOLARS

The Board of Foreign Scholarships, the presidentially appointed body supervising the Fulbright scholarship program, would like to hear from American alumni of the Fulbright program in connection with the 25th Anniversary of the initiation of the exchanges. The response could be as brief as a simple listing of name, current position, and year of grant and country. The Board would also appreciate any brief comment about what the scholarships have meant to individual scholars personally and professionally. Responses can be directed to Board of Foreign Scholarships, CU/BFS, Room 4825, Department of State, Washington, D.C. 20520.

LATIN AMERICAN TEACHING FELLOWSHIPS

The Latin American Teaching Fellowships program is now accepting applications for teaching positions in Latin American universities from natural and physical scientists and engineers with advanced degrees or who are Ph.D. candidates. Placement possibilities now exist for the 1972-1973 academic year. These opportunities are part of a service program designed to assist Latin American universities to develop more advanced technical and scientific programs. Salaries are thus geared to a moderate subsistence level rather than being competitive with North American salaries. Inquiries should be addressed to Latin American Teaching Fellowships, Fletcher School of Law and Diplomacy, Tufts University, Medford, Massachusetts 02155.

SPECIAL MEETINGS INFORMATION CENTER

The purpose of this center is to maintain a file on prospective symposia, colloquia, institutes, seminars, special years, meetings of other associations, and to notify the organizers if conflicts in subject matter, dates or geographical area become apparent. A first announcement will be published in the *Notices* if it contains a call for papers, place, date, and subject, where applicable; a second announcement must contain reasonably complete details of the meeting in order for it to be published. Information on the pre-preliminary planning will be stored in the files, and will be available to anyone desiring information on prospective conferences. All communications on special meetings should be sent to the Special Meetings Information Center of the American Mathematical Society.

CONFERENCE ON THE APPLICATION OF OPERATIONAL RESEARCH TO TRANSPORT PROBLEMS

The NATO Advisory Panel on Operational Research is sponsoring a conference on the Application of Operational Research to Transport Problems in Sandefjord, Norway, on August 14-18, 1972. The conference will focus on the problems of transport from several perspectives. It will consider solutions to transport problems derived through operational analysis, the assessments made of proposed solutions with such techniques, and the problems still requiring operational analysis efforts. Applicants from the United States should submit abstracts no later than December 15, 1971, to the Scientific Director, Dr. Murray A. Geisler, The RAND Corporation, 1700 Main Street, Santa Monica, California 90406. Applicants from the European countries should submit abstracts by December 15, 1971, to Mr. Andreas Mortensen, Norwegian Defense Research Establishment, P.O. Box 25, Kjeller, Norway. Candidates for the panel sessions must also submit abstracts of their proposed talks, along with biographical and bibliographical material, on the same dates and to the same addresses as are indicated above. Application forms and more detailed information may be obtained from Dr. Geisler and Mr. Mortensen.

RELIABILITY CONFERENCE

As part of its program for 1972, the NATO Advisory Panel on Operational Research is sponsoring a conference on Reliability Testing and Reliability Evaluation. The conference will be held in or near The Hague, September 4-8, 1972. The theme of the conference is very broad. It covers such aspects as confidence bounds for system reliability, military demonstration techniques and requirements, cost effectiveness and allocation of resources in testing, reliability of computer software and software for checking the reliability of complex equipment, overstress testing and accelerated life testing, exploitation of knowledge of relevant physical processes in modeling stochastic phenomena. Those wishing to contribute papers should send abstracts to Dr. Nancy R. Mann, Rocketdyne, North American Rockwell Corporation, 6633 Canoga Avenue, Canoga Park, California 91304, no later than February 1, 1972. Authors whose papers are selected for presentation will be notified during March 1972. Complete texts of talks must be submitted no later than June 1, 1972, for publication in the proceedings. Persons wishing to attend the conference but not present a paper should so inform Dr. Mann as soon as possible. Attendance is limited, and it may not be possible to accommodate all those who wish to participate.

SYMPOSIUM ON MATHEMATICAL METHODS OF ECONOMICS

A symposium on Mathematical Methods of Economics will be held in

Warsaw, Poland, from February through July 1972. The symposium is being organized by the Institute of Mathematics of the Polish Academy of Sciences. Although the program is still tentative, essential discussions will be carried on in three seminars: mathematical models of economic growth and related problems; dynamic programming with special emphasis on Markov chains with reward; and convex finite and infinite-dimensional analysis. A number of mathematicians and economists are being invited by the Institute of Mathematics, as well as a group of young mathematicians who have recently received the Ph.D. For this latter group of participants, there will be an introductory course in the topics to be discussed at the seminars. Funds are available only for the support of the invited speakers. Others wishing to participate in the activities of the symposium, and whose expenses will be covered from other sources, are requested to apply to the organizing committee, with suggestions for topics of discussion. Letters should include the applicant's address, academic rank, and institution, and should be addressed to Symposium on Mathematical Methods of Economics, Institute of Mathematics, Polish Academy of Sciences, Śniadeckich 8, P.O. Box 137, Warsaw 1, Poland.

THIRD INTERNATIONAL SYMPOSIUM ON MULTIVARIATE ANALYSIS

The Third International Symposium on Multivariate Analysis will be held at Wright State University, Dayton, Ohio, on June 19-24, 1972. This symposium will be sponsored by the Aerospace Research Laboratories. Several distinguished workers in the field are expected to present invited papers on a broad spectrum of topics in the area. In addition, there will be some contributed paper sessions. It is expected that the proceedings of the sessions of invited papers will be published. Those interested in presenting contributed papers should submit abstracts of their papers (not to exceed 200 words) as soon as possible, but not later than March 15, 1972. The chairman of the local arrangements committee is Professor Carl Maneri of Wright State University. Further details regarding the symposium may be obtained by writing to the symposium chairman, Dr. P.R. Krishnaiah, ARL/LB, Building 450, Aerospace Research Laboratories, Wright-Patterson Air Force Base, Ohio 45433.

CONFERENCE ON COMPUTERS IN THE UNDERGRADUATE CURRICULA

Under a grant from the National Science Foundation, the Southern Regional Education Board will sponsor a conference on Computers in the Undergraduate Curricula at Georgia Institute of Technology on June 12-14, 1972. The conference will be open to faculty of two-year and four-year colleges, and the program will consist of panel discussions, demonstrations, and sessions of invited addresses and contributed papers. The program will be devoted to broad application of the computer in helping students learn, with information being exchanged on actual experiences and plans in the use of computers in undergraduate instruction. The committee for the conference consists of John W. Hamblen, Southern Regional Education Board; Thomas Kurtz, Dartmouth College; Gerald L. Engel, Hampden-Sydney College; Glenn R. Ingram, Washington State University; Gerald P. Weeg, University of Iowa; and Fred W. Weingarten, Claremont Colleges. Further information on the conference and a copy of the call for papers may be obtained from Dr. John W. Hamblen, Project Director, Computer Sciences, Southern Regional Education Board, 130 Sixth Street, N. W., Atlanta, Georgia 30313. The deadline for submission of papers is January 15, 1972.

INTERNATIONAL CONGRESS ON RHEOLOGY

The Sixth International Congress on Rheology will be held in Lyon, France, on September 4-8, 1972. The program for the congress is still in the planning stages, but present plans call for sessions for contributed papers including sessions on biorheology and rheo-optics. Further information, as well as official notifications, may be obtained by writing to Dr. C. Smadja, VI^{e} Congres International de Rhéologie, Boite Postale n^{o} 1, 69 Lyon-Mouche, France.

ANNUAL MEETING OF ASSOCIATION FOR SYMBOLIC LOGIC

The annual meeting of the Association for Symbolic Logic will be held on December 27-28, 1971, at the Statler Hilton Hotel, Seventh Avenue and 33rd Street, New York City, in conjunction with the annual meeting of the Eastern Division of the American Philosophical Association. In addition to an invited one-hour address and sessions for contributed papers, the program is as follows:

MONDAY, DECEMBER 27
9:15 a.m.-12.:15 p.m. Symposium on the Recent History of Logic, Washington Room (in commemoration of the 35th anniversary of the founding of the Association, and in honor of Alonzo Church and Haskell B. Curry). Chairman: Dana Scott, Princeton University. Speakers: Leon Henkin, University of California, Berkeley; G. Kreisel, Stanford University; J. Barkley Rosser, University of Wisconsin, Madison.

2:00 p.m.-4:00 p.m. Symposium on Entailment, Georgian Room. Chairman: Alan Ross Anderson, University of Pittsburgh. Speakers: Dana Scott, Princeton University; H.P. Grice, University of California, Berkeley; R.K. Meyer, Indiana University.

TUESDAY, DECEMBER 28
9:00 a.m.-11:00 a.m. Symposium on English as a Formal Language, Washington Room (in memory of Richard Montague). Chairman: David Kaplan, University of California, Los Angeles. Speakers: Barbara Hall Partee, University of California, Los Angeles; Terence D. Parsons, University of Illinois, Chicago Circle.

SYMPOSIUM ON NONLINEAR RESEARCH IN THE SCIENCES AND HUMANITIES

A symposium on Nonlinear Research in the Sciences and Humanities will be held on December 28-29, 1971, in Philadelphia, Pennsylvania, during the annual meeting of the American Association for the Advancement of Science. The purpose of the symposium is to bring together specialists from diverse fields in the sciences and humanities who are engaged in examining a new and potentially powerful approach to the solution of problems unresolvable by standard formulations. Nonlinearity is a formulation of self-reflectivity and of self-organizing activity on any level (e.g. mathematical, physical, psycho-social), and like the relatively recent science of cybernetics, it is both interdisciplinary and a discipline in its own right. The chairmen of the sessions include Michael Kosok, Fairleigh Dickinson University; Richard Levins, University of Chicago; Edward Feit, University of Massachusetts; and Benjamin Nelson, New School for Social Research.

NEW AMS PUBLICATIONS

SIAM-AMS PROCEEDINGS

Volume III

MATHEMATICAL ASPECTS OF ELECTRICAL NEWTORK ANALYSIS
Edited by Herbert S. Wilf and Frank Harary

214 pages; List Price \$10.60; Member Price \$7.95

This volume constitutes the proceedings of the SIAM-AMS symposium on Mathematical Aspects of Electrical Network Analysis held in New York City in April 1969. Most of the papers apply various mathematical methods to electrical network problems, but a few of them accomplish the reverse "application," and help to solve purely mathematical problems using electrical network theory technique. The authors include R.K. Brayton, R. W. Brockett, D. A. Calahan, Shu-Park Chan, R. J. Duffin, R.J. Leake, R. Liu, Ronald Rohrer, J. Paul Roth, R. Saeks, I.W. Sandberg, R.A. Skoog, Pravin Varaiya, Dan H. Wolaver, Dante C. Youla, and J.W.T. Youngs.

Volume IV

COMPUTERS IN ALGEBRA AND NUMBER THEORY
Edited by Garrett Birkhoff and Marshall Hall, Jr.

208 pages; List Price \$12.70; Member Price \$9.53

This volume constitutes the proceedings of the SIAM-AMS symposium on Computers in Algebra and Number Theory held in New York City in March 1970. Part 1 consists of papers devoted to applications of algebraic ideas to computing with especial attention to problems of optimizing computer algorithms. The five papers in Part 2 consider number theory and combinatorial theory. The final section, Part 3, deals with the application of computers to finite groups. The authors include L.D. Baumert et al., B.J. Birch, Garrett Birkhoff, John J. Cannon, J.H. Conway, Marshall Hall, Jr., M.D. Hestenes, D.G. Higman, John McKay, J. Neubüser, Charles C. Sims, H.P.F. Swinnerton-Dyer, J.H. van Lint, Shmuel Winograd, and Hans Zassenhaus.

MEMOIRS OF THE AMERICAN MATHEMATICAL SOCIETY

Number 115

STRUCTURES IN TOPOLOGY
By Douglas Harris

102 pages; List Price \$2.00; Member Price \$1.50

The object of this Memoir is to introduce into the study of Hausdorff topological spaces methods patterned after the theory of uniform spaces, allowing concentration on the space itself when studying its extensions. To this end, the theory of structures is introduced, developed, and applied to the examination of various classes of topological spaces. It is shown that many topological constructions can be conceived as completions with respect to an appropriate structure, and that this allows the establishment of general results regarding extensions of mappings. The stress is on applications that do not involve compactness or complete regularity, in order to emphasize the power and generality of the method and its applicability to questions remote from uniform space theory. Particular classes of spaces considered include Hausdorff, compact, realcompact, Hausdorff-closed, semiregular, minimal Hausdorff, regular, RC-regular, regular-closed, locally compact, and locally H-closed spaces.

PROCEEDINGS OF THE STEKLOV INSTITUTE

Number 105

THEORY AND APPLICATIONS OF DIFFERENTIABLE FUNCTIONS OF SEVERAL VARIABLES, Part III
Edited by S.M. Nikol'skiĭ

300 pages; List Price $21.00; Member Price $15.75

This collection contains original articles by members of the section on function theory of the Mathematics Institue in the Academy of Sciences of the USSR. The authors include O.V. Besov, V.I. Burenkov, A.D. Džabrailov, A.S. Foht, G.G. Kazarjan, I.A. Kiprijanov, P.I. Lizorkin, Ju. S. Nikol'skiĭ, S.M. Nikol'skiĭ, T.S. Pigolkina, T.A. Timan, and E.A. Volkov. The papers deal with theorems on imbedding of classes of differentiable functions of several variables, compactness of sets in function spaces, and estimates of singular integrals, and also with applications to the theory of boundary-value problems in mathematical physics. The collection is intended for researchers in the theory of functions and differentiable equations and for graduate students in mathematics.

TRANSLATIONS—SERIES II

Volume 98

FIVE PAPERS ON LOGIC AND FOUNDATIONS

292 pages; List Price $14.60; Member Price $10.95

This volume of the AMS Translations-Series 2 contains the following papers: "On constructive mathematics" by A.A. Markov; "Mean value theorems in constructive analysis" by G.S. Ceĭtin; "On singular coverings and properties of constructive functions connected with them" by I.D. Zaslavskiĭ and G.S. Ceĭtin; "Certain properties of E.L. Post's apparatus of canonical calculi" by S. Ju. Maslov; and "Graph schemes with memory" by I.D. Zaslavskiĭ.

NEWS ITEMS AND ANNOUNCEMENTS

DIRECTORY OF HISTORIANS OF MATHEMATICS

A world directory of historians of mathematics is being prepared by the Commission on History of Mathematics of the International Union for the History and Philosophy of Science. Scholars who are teaching or doing research in the history of mathematics should communicate with the chairman of the Commission, Professor Kenneth O. May, Department of Mathematics, University of Toronto, Toronto 181, Canada. Please send name and address (as you wish it for mail), a statement of your special fields of interest, and the languages you read. The Commission expects to begin publication of an international journal of the history of mathematics in 1973. Meanwhile, a newsletter will be distributed.

INSTITUTE FOR ADVANCED STUDY MEMBERSHIPS

The School of Mathematics will grant a limited number of memberships, some with financial support, for research in mathematics at the Institute during the academic year 1972-1973. Candidates must have given evidence of ability in research comparable at least with that expected for the Ph.D. degree. Application blanks may be obtained from the Secretary of the School of Mathematics, Institute for Advanced Study, Princeton, New Jersey 08540, and should be returned (whether or not funds are expected from some other source) by January 15, 1972, or as soon thereafter as possible.

DOCTORATES CONFERRED IN 1970-1971

Supplementary List

The following are among those who received doctorates in the mathematical sciences and related subjects from universities in the United States and Canada during 1970-1971. This is a supplement to the list printed in the October 1971 issue of these *Notices*.

CALIFORNIA

UNIVERSITY OF CALIFORNIA, BERKELEY
(3 in Statistics)

Department of Biostatistics

Fellner, William Henry
Tissue regeneration and its relation to carcinogenesis

Moore, Dan
Evaluation of five discrimination procedures for binary variables

Selvin, Steve
A biostatistical investigation of twins and twin-study methodology

TEXAS

UNIVERSITY OF TEXAS AT AUSTIN
(1 in Applied Mathematics)

Webb, Jerry Claude
Finite-difference methods for a harmonic mixed boundary value problem

SOUTHERN METHODIST UNIVERSITY
(13 in Statistics)

Department of Statistics

Barham, Richard Hugh
Parameter reclassification in nonlinear least squares

Brock, Dwight Brandon
Statistical inference for Markov renewal processes

Brodnax, Charles Troy
Decomposition of time series into deterministic and indeterministic components

Craig, James Alton
An empirical Bayes approach to a variables sampling plan problem

Cranley, Roy
Statistical methods for evaluating multiple integrals

Davenport, James Melton
Components in a class of approximate F-tests

Harrist, Ronald Baxter
On the resolution of hypotheses employing the asymptotic independence of successive likelihood ratio statistics

Johnson, Charles Ray
A duality property for Bayes rules with applications

Mason, Robert Lee
Tests when errors are correlated in a randomized block design

Sansing, Raymond Clayton
The density of the t statistics for non-normal distributions

Webber, William Fred
On the statistical analyses of random surfaces

Wheeler, Donald Jefferson
An alternative for an F-test on variances

Young, John Coleman
Some inference problems associated with the complex multivariate normal distribution

CANADA

UNIVERSITÉ LAVAL (1 in Pure Mathematics)

Roy, Ghislain
Sur le nombre de formes différentielles associées à une variété V_m plongée dans un espace Riemannien V_n

VISITING MATHEMATICIANS

Supplementary List

The following is a continuation of the lists of visiting mathematicians printed in the August and October 1971 issues of these *Notices*.

American and Canadian Mathematicians Visiting Abroad

Name and Home Country	Host Institution	Field of Special Interest	Period of Visit
Bers, Lipman (U.S.A.)	Institut Mittag-Leffler, Djursholm, Sweden	Analysis	9/71 - 12/71
Kinderlehrer, David (U.S.A.)	Italy	Minimal Surfaces	9/71 - 6/72
Levine, Howard (U.S.A.)	Switzerland	Partial Differential Equations	9/71 - 6/72
Segal, Lee A. (U.S.A.)	Weizmann Institute of Science, Israel	Applied Mathematics	8/71 - 8/72
Waugh, W.A. O'N. (Canada)	University of Sheffield, England	Stochastic Processes	1/72 - 6/72

Foreign Mathematicians Visiting in the United States

Ikeda, Matatoshi (Turkey)	San Diego State College	Algebra	9/71 - 6/72

ERRATA

The host university for both of the following American mathematicians was given incorrectly in the August issue of these *Notices*. The correct listings are as follows.

Goldberg, Samuel I. (U.S.A.)	Technion—Israel Institute of Technology	Differential Geometry	2/72 - 8/72
Rothman, Neal J. (U.S.A.)	Technion—Israel Institute of Technology	Harmonic Analysis	9/71 - 8/72

PERSONAL ITEMS

WILLIAM B. DAY of Uniroyal has been appointed to an assistant professorship at Auburn University.

LEE W. ERLEBACH of the University of British Columbia has been appointed to a visiting assistant professorship at the University of Arizona.

STEPHAN D. FRANKLIN of the University of Chicago has been appointed to an assistant professorship at the University of California, Irvine.

ABOLGHASSEM GHAFFARI of NASA, Goddard Space Flight Center, has been appointed to a professorship at the University of Teheran, Iran.

ARTHUR GRAD of the Illinois Institute of Technology has been appointed to the presidency of the Polytechnic Institute of Brooklyn.

WILFRED M. GREENLEE of Northwestern University has been appointed to an associate professorship at the University of Arizona.

LARRY C. GROVE of Syracuse University has been appointed to an associate professorship at the University of Arizona.

ALEXANDER J. HAHN of the University of Notre Dame has been appointed to a National Science Foundation fellowship at the University of Zurich.

STEPHEN J. HARIS of the Institute for Advanced Study has been appointed to an assistant professorship at the University of Maryland.

JOHN C.HEMPERLY of Yale University has been appointed to an assistant professorship at the University of Maryland.

THEODORE LAETSCH of Illinois State University and the Aerospace Research Laboratory, Wright-Patterson AFB, Ohio, has been appointed to an associate professorship at the University of Arizona.

MALCOLM G. LANE of Duke University has been appointed to an assistant professorship at West Virginia University.

JOHN D. LOGAN of the University of Dayton, Research Institute, has been appointed to an assistant professorship at the University of Arizona.

ANTHONY G. MUCCI of the University of California, Irvine, has been appointed to an assistant professorship at the University of Maryland.

RICHARD PAVELLE of Carleton University has been appointed to a visiting assistant professorship at the University of Arizona.

ROBERT C. REILLY of the Massachusetts Institute of Technology has been appointed to an assistant professorship at the University of California, Irvine.

PETER G. SAWTELLE of the University of California, Riverside, has been appointed to an assistant professorship at the University of Missouri, Rolla.

JAMES R. SIDBURY of Auburn University has been appointed to an associate professorship at Winthrop College.

HOWARD H. STRATTON of SUNY at Albany has been appointed to a visiting associate professorship at the University of Arizona.

JAN R. STROOKER of the University of Utrecht will spend the last three months of 1971 as a visiting professor at the University of Buenos Aires.

CHARLES W. SWARTZ of New Mexico State University has been appointed to a visiting associate professorship at the University of Arizona.

ALVIN I. THALER of the University of Maryland has been appointed assistant program director for algebra in the Mathematical Sciences Section of the National Science Foundation, Washington, D.C.

SELDEN Y. TRIMBLE of Washington University has been appointed to an assistant professorship at the University of Missouri, Rolla.

EDOARDO VESENTINI of the Scuola Normale Superiore, Pisa, Italy, has been appointed to a professorship at the University of Maryland.

PROMOTIONS

To Benjamin Cheney Professor of Mathematics. Dartmouth College: ERNST SNAPPER.

To Professor. California State College, Long Beach: CHARLES W. AUSTIN, ROBERT E. MOSHER; University of Maryland: WILLIAM W. ADAMS, ELLEN CORREL.

To Associate Professor. University of Arizona: CARL L. DeVITO, BRUCE WOOD; University of Maryland: KENNETH R. BERG, ROBERT L. ELLIS.

INSTRUCTORSHIPS

University of Arizona: ROBERT A. MASSAGLI; University of California, Irvine: HARVEY C. GREENWALD, LEONARD P. SASSO.

DEATHS

Dr. WALLACE J. ECKERT of Leonia, New Jersey, died on August 24, 1971, at the age of 69. He was a member of the Society for 24 years.

Professor NORMAN E. STEENROD of Princeton University died on October 14, 1971, at the age of 61. He was a member of the Society for 35 years.

NEWS ITEMS AND ANNOUNCEMENTS

GEORGE WILLIAM HILL AND EMMY NOETHER RESEARCH INSTRUCTORSHIPS

These instructorships are offered by the Department of Mathematics of the State University of New York at Buffalo to young mathematicians with doctorates to enable them to carry forward a research program. Two awards are granted yearly with appointments for two years. The stipend for twelve months beginning September 1972 is $15,600, including generous staff benefits. Teaching load will total two one-semester courses during the twelve-month period. Upon expiration of the two-year appointment, priority consideration for a three-year appointment as assistant professor will be given and will be based upon success and potential in both research and teaching. All requirements for the Ph.D. must have been completed by September 1, 1972. A summary of post-high school educational background, as well as a sketch of past and projected research activity, should accompany the application. Letters of recommendation from three mathematicians should be submitted. All material is to be directed to Professor N.D. Kazarinoff, Chairman, Department of Mathematics, SUNY at Buffalo, 4246 Ridge Lea Road, Amherst, New York 14226, so as to arrive by January 22, 1972. Appointments will be announced by February 15.

NSF GRADUATE FELLOWSHIPS

The National Science Foundation has reopened competition for 600 Graduate Fellowships it will offer in the spring of 1972. These fellowships, available to citizens or nationals of the United States, are awarded for full-time study leading to the master's or doctor's degree in science, including the social sciences, mathematics, or engineering. As part of a restructuring of its Graduate Fellowship program, stipends have been increased to $3,600 for a twelve-month tenure, or $300 per month, regardless of year of study. No dependency allowances will be paid. The awards to be made in March 1972 will be for a period of three years, dependent on the student's satisfactory progress and the availability of NSF funds. Awards will be made only to students who have completed one year or less of graduate study. The application deadline is November 29, 1971. For copies of the announcement and application materials, please write to the Fellowship Office, National Research Council, 2101 Constitution Avenue, N.W., Washington, D.C. 20418.

ABSTRACTS OF CONTRIBUTED PAPERS

Preprints are available from the author in cases where the abstract number is starred.

The November Meeting in Auburn, Alabama
November 19—20, 1971

Algebra & Theory of Numbers

689-A1. OSVALDO MARRERO, Francis Marion College, Florence, South Carolina 29501. Modular Hadamard matrices and related designs. II. Preliminary report.

This paper continues the investigation initiated in a paper by the author and A. T. Butson ("Modular Hadamard matrices and related designs," submitted for publication). (See also Abstract 672-531, these *Notices* 17(1970), 236.) Modular Hadamard matrices having n odd and $h \equiv -1 \pmod n$ are studied for a few values of the parameters n and h. Also, some results are obtained about the related combinatorial designs; these results are concerned mainly with the column sums of the incidence matrices of the designs. It is shown that in some cases the two related type of designs are actually equivalent. (Received June 21, 1971.)

689-A2. WITHDRAWN.

*689-A3. ABRAHAM BERMAN, Centre de Recherches Mathématiques, Université de Montréal, Case Postale 6128, Montréal 101, Québec, Canada and ROBERT J. PLEMMONS, University of Tennessee, Knoxville, Tennessee 37916. Monotonicity and the generalized inverse.

Theorem 1. Let $A \in R^{m\times n}$ and let A^+ denote the generalized inverse of A. Then the following are equivalent: (a) $A^+ \geqq 0$. (b) $Ax \in AA^+R^m_+$, $x \in R(A^T) \Rightarrow x \geqq 0$. (c) $Ax \in R^m_+ + N(A^T)$, $x \in R(A^T) \Rightarrow x \geqq 0$. Definition. A is row-monotone if $Ax \geqq 0$, $x \in R(A^T) \Rightarrow x \geqq 0$. Theorem 2. Let $A \in R^{m\times n}$. Then the following are equivalent: (a) A is row-monotone. (b) $AX \geqq 0 \Rightarrow AA^+X \geqq 0$. (c) The system $Y \geqq 0$, $YA = A^+A$, is consistent. Theorem 3. Let $0 \leqq A \in R^{m\times n}$. Then the following are equivalent: (a) $A^+ \geqq 0$. (b) $A = DA^T$ where $D = \operatorname{diag}\{d_i\}, d > 0$. (c) A and A^T are row-monotone. The results are extended to complex matrices, and their corollaries include applications to Numerical Analysis. In the special case of full column rank, results of Collatz, Mangasarian and Ben-Israel, are obtained. (Received June 30, 1971.)

*689-A4. ROBERT GILMER and ANNE P. GRAMS, Florida State University, Tallahassee, Florida 32306. The equality $(A\cap B)^n = A^n \cap B^n$ for ideals. Preliminary report.

Let R be a commutative ring and n a positive integer. R is said to have property (n) if for any x, y $\in$ R, $(x,y)^n = (x^n, y^n)$; property (n)' if for any $x \in R$ and any ideal A of R such that $x^n \in A^n$, it follows that $x \in A$; and property (n)'' if for any ideals A and B of R, $(A\cap B)^n = A^n \cap B^n$. (Properties (n) and (n)' were investigated by J. Ohm [Monatsh. Math. 71(1967), 32-39] and R. Gilmer [Canad. J. Math. 20(1968), 970-983].) Theorem 1. Property (n)'' $\rightarrow$ property (n)' $\rightarrow$ property (n). Theorem 2. Let D be an integrally closed domain with identity, and let n be a positive integer greater than one. The following conditions are equivalent in D:

(1) D is a Prüfer domain. (2) Property (n)'' holds in D. (3) Property (n)'' for finitely generated ideals holds in D. A similar theorem is stated for an integrally closed ring with few zero divisors. Domains are constructed which have property (n)'' for each positive integer n but which are not integrally closed and hence not Prüfer. Finally, an example is given of a domain which has property (n)' for each positive integer but property (n)'' for no integer greater than one. (Received July 9, 1971.)

*689-A5. ROBERT GILMER, Florida State University, Tallahassee, Florida 32306. On factorization into prime ideals. Preliminary report.

Let A be a finitely generated regular ideal of the commutative ring R. If A is a finite product of prime ideals of R, then such a representation is unique if R contains no identity element, and is unique to within factors of R otherwise. The prime ideals in a product representation of A need not be finitely generated; in fact, if k and n are positive integers, then there exists an integral domain D_n with identity containing distinct prime ideals $P_1, P_2, \ldots, P_n$ such that the power product $P_1^{e_1}P_2^{e_2}\ldots P_n^{e_n}$ is finitely generated if and only if $k \leqq (e_1 + \ldots + e_n)$. (Received July 21, 1971.)

*689-A6. JAMES W. LEA, JR., Middle Tennessee State University, Murfreesboro, Tennessee 37130. An embedding theorem for compact lattices.

Let L be a compact topological lattice such that the breadth of L is less than or equal to the width n of the set of all meet irreducible elements of L. Then L can be embedded in a product of n compact chains by a join preserving homeomorphism. (Received August 5, 1971.)

689-A7. CARL R. SPITZNAGEL, University of Kentucky, Lexington, Kentucky 40506. The lattice of congruences on a band of groups.

If S is any regular semigroup, the lattice congruence θ on $\Lambda(S)$ is defined by $(\rho, \tau) \in \theta$ if and only if ρ and τ induce the same partition of the idempotents of S. A band of groups S is called θ-modular, provided that the conditions $\alpha \geqq \beta$, $(\alpha, \beta) \in \theta$, $\alpha \wedge \gamma = \beta \wedge \gamma$, $\alpha \vee \gamma = \beta \vee \gamma$, for elements $\alpha, \beta, \gamma \in \Lambda(S)$, imply that $\alpha = \beta$. Theorem 1. Let S be a band of groups, and let φ be the function defined on $\Lambda(S)$ by $\varphi(\rho) = (\rho \vee \mathcal{H}, \rho \wedge \mathcal{H})$. Then φ maps $\Lambda(S)$ isomorphically onto a sublattice of the product of the lattice of band congruences on S with the lattice of idempotent-separating congruences on S if and only if S is θ-modular. Theorem 2. Let S be a normal band of groups. Then S is θ-modular if and only if whenever e and f are idempotents with $f < e$, then $f \cdot H_e = \{f\}$. (Received August 31, 1971.)

689-A8. DAVID C. RINE, Statistics and Computer Science Department, West Virginia University, Morgantown, West Virginia 26506. An elementary theory of computation.

This paper grew out of a communication with D. Scott(1971) on the introducing of a mathematical theory of computation. The reason for setting down a mathematical theory of computation is to put forth general semantics for high-level computer languages such as FORTRAN, APL, BASIC, ALGOL 60, and PL/1.

"Procedure" is an undefined term, and the mathematical semantics of a procedure is the function from elements of the "data type" of input variables to elements of the "data type" of the output. The purpose of this paper is to provide an interesting abstract definition of "data type." The more sophisticated mathematical manipulations of "data type" have been explored categorically by Rine(1969-1970) and by Scott (1971). (Received September 8, 1971.)

*689-A9. JIMMY T. ARNOLD, Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061. Power series rings over Prüfer domains.

R denotes a commutative ring with identity. If there exists a chain $P_0 \subset P_1 \subset \ldots \subset P_n$ of $n+1$ prime ideals of R, where $P_n \neq R$, but no such chain of $n+2$ prime ideals, then we say that R has dimension n and we write dim R = n. An ideal A of R is called an ideal of strong finite type (or, an SFT-ideal) provided there exists a finitely generated ideal $B \subseteq A$ and a positive integer k such that $a^k \in B$ for each $a \in A$. We say that R is an SFT-ring if each ideal of R is an SFT-ideal. The author has previously shown that if R is not an SFT-ring, then $\dim R[[X]] = \infty$ (Abstract 682-13-6, these *Notices* 18(1971)). Theorem. For an n-dimensional Prüfer domain D, the following are equivalent: (1) D is an SFT-ring. (2) For each prime ideal P of D, there exists a finitely generated ideal A such that $P^2 \subseteq A \subseteq P$. (3) $\dim D[[X]] = n+1$. (4) $\dim D[[X]] < \infty$. (Received September 9, 1971.)

*689-A10. KIM KI-HANG BUTLER and CHARLES D. BASS, Pembroke State University, Pembroke, North Carolina 28372. Combinatorial properties of partial transformation semigroups.

The number of idempotents in the (full) transformation semigroup F(X), X finite, were investigated by Tainiter (J. Combinatorial Theory 5(1968), 370-373) in terms of their rank. In this paper the investigation is extended to the semigroup W(X) of partial transformations on X. Let $|X| = n$. Theorem 1. If D_r is the $\mathcal{D}$-class of rank r in W(X) then $|D_r| = \binom{n}{r} r! \sum_{m=r}^{n} \binom{n}{m} S(m,r)$ where S(m,r) is the Stirling number of the second kind. Corollary 2. $|W(X)| = (n+1)^n - 1 = \sum_{r=1}^{n} \binom{n}{r} r! \sum_{m=r}^{n} \binom{n}{m} S(m,r)$. Theorem 3. If I(n) is the number of idempotents in W(X) then $I(n) = \sum_{r=1}^{n} \binom{n}{r} \sum_{m=r}^{n} \binom{n-r}{m-r} r^{m-r}$. (Received September 10, 1971.)

*689-A11. CHANG MO BANG, Emory University, Atlanta, Georgia 30322. Isomorphism types of infinite symmetric graphs.

Using Comer and LeTourneau's 1-unary algebras ["Isomorphism types of infinite algebras," Proc. Amer. Math. Soc. 21(1969), 635-639], we construct 2^m pairwise nonisomorphic symmetric graphs of infinite order m to answer B. Jonsson's question ["Topics in universal algebra," Lecture notes, Vanderbilt Univ., 1971, p. 31]. R. Pierce communicated to us another way of construction using Monk and Sparks' Theorem 1 (Abstract 71T-A74, these *Notices* 18(1971), 551). (Received September 13, 1971.)

*689-A12. DONALD A. CHAMBLESS, University of Florida, Gainesville, Florida 32601. The ℓ-group of almost-finite, continuous functions.

In this paper the structure of the f-algebra D(X) of all continuous, almost-finite functions on a Stone topological space X is investigated. The maximal ring ideals of D(X) are shown to be precisely the minimal prime convex ℓ-subgroups, and appropriate modifications of methods due to Gillman and Jerison ("Rings of continuous functions," Van Nostrand, Princeton, N. J., 1960) show that the corresponding quotient fields are real-closed η_1-fields. X is identified as a space of prime subgroups of D(X) with the hull-kernel topology, and necessary and sufficient conditions are given for D(X) to have a basis, be hyperarchimedean, and be representable by real-valued functions, respectively. (Received September 13, 1971.)

*689-A13. LAL M. CHAWLA, JOHN E. MAXFIELD, Kansas State University, Manhattan, Kansas 66502 and MARIJO O. LeVAN, Eastern Kentucky University, Richmond, Kentucky 40475. On a restricted partition function and its tables. Preliminary report.

Let Q(n) be the number of partitions of n, each of which has a summand which divides every summand of the partition. Of course, such a divisor summand may occur repeatedly in a given partition of the requisite type. Let p(n) denote the number of unrestricted partitions of n, with $p(0) = 1$, and $p_m(n)$ the number of partitions, the summands in each of which are divisible by m. Then $p_1(n) = p(n)$. Let $p_m(n) = 0$ if $n < 0$. In the present paper, we prove the following properties of Q(n) and then prepare a table of the partition function $R(n) = Q(n) - p(n-1)$ up to $n = 300$. (i) $Q(n) = \Sigma_{d|n} p(d-1)$. (ii) $Q(n) = p(n-1) + R(n)$, where $Q(n) = \Sigma_{m=1}^{n} p_m(n-m)$ and $R(n) = \Sigma_{m=2}^{n} p_m(n-m)$. (iii) $\lim_{n\to\infty}(Q(n)/p(n-1)) = 1$. (iv) If $f(x,m) = (x^m/(1-x^m)(1-x^{2m})(1-x^{3m})\ldots)$ then $f(x,m) = \Sigma_{n=1}^{\infty} p_m(n-m)x^n$. (v) If $F(x) = \Sigma_{m=1}^{\infty} f(x,m)$, then $F(x) = \Sigma_{n=1}^{\infty} Q(n)x^n$. (vi) If $G(x) = \Sigma_{m=2}^{\infty} f(x,m)$, then $G(x) = \Sigma_{n=1}^{\infty} R(n)x^n$. (Received September 14, 1971.)

689-A14. FREDRIC T. HOWARD, Wake Forest University, Winston-Salem, North Carolina 27109. A property of a class of nonlinear difference equations.

Let g(n) be a rational function of n whose denominator is divisible by exactly 2^t for each n, and let $a_1, a_2, \ldots$ be any sequence of rational numbers such that for $n > 1$, $a_n = g(n)(a_1 a_{n-1} + a_2 a_{n-2} + \ldots + a_{n-1}a_1)$. If we define $\theta(n)$ to be the exponent of the highest power of 2 dividing the denominator of a_n, then we can prove by induction that $\theta(n) = n\theta(1) + 1 + (n-1)t - m$ where m is the number of nonzero terms in the base 2 expansion of n. Furthermore if $n = 2^{e_1} + 2^{e_2} + \ldots + 2^{e_m}$, $e_1 > e_2 > \ldots > e_m \geqq 0$, and $e_i - e_{i+1} > 1$ for q terms e_i, then $2^{\theta(n)} a_n \equiv (-1)^q r \pmod 4$ if either one of the following two sets of congruences holds for all n: $2^t g(n) \equiv r \equiv 2^{\theta(1)} a_1 \pmod 4$; $2^t g(n) \equiv (-1)^n r \equiv (-1)^{n+1} 2^{\theta(1)} a_1$. If $2^t g(n) \equiv (-1)^n r \equiv (-1)^n 2^{\theta(1)} a_1 \pmod 4$ for all n, then $2^{\theta(n)} a_n \equiv (-1)^{q+n} r \pmod 4$. Examples of sequences for which these theorems hold are $a_n = B_{2n}/(2n)!$ where B_{2n} is the 2nth Bernoulli number and $a_n = \binom{2n-2}{n-1}/n$. (Received September 15, 1971.)

*689-A15. ALEXANDER R. BEDNAREK, Department of Mathematics and Center for Information Research, University of Florida, Gainesville, Florida 32601 and EUGENE M. NORRIS, West Virginia University, Morgantown, West Virginia 26506. More on externally induced operations. Preliminary report.

A relation R from T to Z is difunctional if $R \circ R^{-1} \circ R$ is contained in R, for subsets $A \subseteq T$, $B \subseteq Z$, let $AR = \{z \in Z : \exists\, a \in A \ni (a,z) \in R\}$ and $RB = \{t \in T : \exists\, b \in B \ni (t,b) \in R\}$. We write tR to $\{t\}R$. We call (T,Z,m) a recursion (the term is due to A. D. Wallace) if m is a function, usually continuous, from $T \times Z$ into Z, for which we write $A \cdot B = m(A \times B)$ for all $A \subseteq T$, $B \subseteq Z$. The recursion is said to be compact (respectively, discrete) if both T and Z are compact Hausdorff (both discrete) spaces. Theorem. If (T,Z) is a compact or discrete recursion and R is a closed difunctional relation from T to Z satisfying (1) $Z = TR$, (2) $t \cdot (t'R) = t' \cdot (tR)$ for all t, t' in T, (3) $t \cdot (t' \cdot (t''R)) = t' \cdot (t \cdot (t''R))$ for all t, t', t'' in T, (4) for t, t' in T there is some t'' in T for which $t \cdot (t'R)$ is contained in $t''R$, then $R^{-1} \circ R$ is an equivalence relation on Z and $Z/(R^{-1} \circ R)$ is an abelian semigroup with multiplication $*$ satisfying $\bar{z} * \bar{z}' = \overline{tz'}$ for all t in T, where in general $\bar{z}$ is the equivalence class, modulo $R^{-1} \circ R$, containing z. Corollary (Aczél-Wallace). If (T,Z) is a compact or discrete recursion satisfying (1) $t(t'x) = t'(tx)$ for all t, t' in T, all x in Z and (2) $Z = Tz$ for some z in Z, then Z is an abelian monoid with multiplication $*$ satisfying $tz * z' = tz'$ for t in T and all z, z' in Z. (Received August 2, 1971.)

*689-A16. ORIN CHEIN, Temple University, Philadelphia, Pennsylvania 19122 and DANIEL A. ROBINSON, Georgia Institute of Technology, Atlanta, Georgia 30332. An "extra" law for characterizing Moufang loops.

Let $(G, \cdot)$ be a loop and let λ, δ, α be mappings of G into G such that $x\lambda = x \cdot x\delta = x(x\alpha \cdot x)$ for all $x \in G$. A theorem of H. Pflugfelder ["A special class of Moufang loops," Proc. Amer. Math. Soc. 26 583-586] is employed to prove that the following three conditions are equivalent: (a) $(xy \cdot z)x\alpha = x(y(z \cdot x\alpha))$ for all $x, y, z \in G$. (b) $(G, \cdot)$ is Moufang and $x\delta$ is in the nucleus of $(G, \cdot)$ for all $x \in G$. (c) $(xy)(z \cdot x\lambda) = (x \cdot yz)x\lambda$ for all $x, y, z \in G$. In particular, it follows that $(G, \cdot)$ is extra in that $(xy \cdot z)x = x(y \cdot zx)$ for all $x, y, z \in G$ (cf. Fenyves, Publ. Math. Debrecen 15(1968), 235-238) if and only if $(G, \cdot)$ satisfies the M_3 in that $(xy)(z \cdot x^3) = (x \cdot yz)x^3$ for all $x, y, z \in G$ (cf. Pflugfelder, op. cit.). (Received September 16, 1971.)

*689-A17. PETER M. GIBSON, University of Alabama, Huntsville, Alabama 35807. A lower bound for the permanent of a (0,1)-matrix.

Let A be an n-square fully indecomposable (0,1)-matrix. H. Minc [Proc. Amer. Math. Soc. 22 (1969), 117-123] has shown that $\operatorname{per} A \geq \sigma(A) - 2n + 2$, where $\sigma(A)$ is the sum of all entries of A. In this paper the following improvement of Minc's inequality is established by induction on k. Theorem. If each row sum of A is at least k then $\operatorname{per} A \geq \sigma(A) - 2n + 2 + \sum_{m=1}^{k-1} (m! - 1)$. (Received September 20, 1971.)

*689-A18. THOMAS H. McH. HANSON, University of Florida, Gainesville, Florida 32601. Actions of a locally compact group with compact boundary.

In this paper we assume that S is a locally compact group with compact boundary (i.e. S is a locally compact semigroup containing a nonempty, nowhere dense, compact subset B such that $G = S - B$ is a group, and B is in the closure of G) acting on a topological space X, and set $X' = \{x \in X : sx = tx, s, t \in S, \text{ iff } s = t\}$. We have the following results: Theorem. For each $x \in X$, Sx is a closed subset of X; if $x \in X'$, Sx is homeomorphic to S. Theorem. G acts on X', and if X' is locally compact, the orbit space X'/G is a locally compact Hausdorff space. Theorem. Suppose (a) there is a cross-section to the orbits of G in X', (b) $X = SX'$, (c) if x and y are in the cross-section such that Bx meets By, then $Gx = Gy$, and (d) if U is open in S and V is open in X', then UV is open in X. Then, X is homeomorphic to $S \times X'/G$, and the action of S on X is equivalent to the natural action of S on $S \times X'/G$, i.e. $s(t, Gx) = (st, Gx)$. (Received September 20, 1971.)

689-A19. JACOB T. B. BEARD, JR., University of Texas, Arlington, Texas 76010. A characterization of all matrix fields contained in $(Q)_n$. Preliminary report.

Let $(Q)_n$ denote the algebra of all $n \times n$ matrices over the field Q of rational numbers under normal matrix addition and multiplication. If a subring M of $(Q)_n$ is a field, then M is called a matrix field, and we say that M is a subfield of the ring $(Q)_n$. As in the case of $(GF(q))_n$ [Abstract 71T-A238, these *Notices* 18(1971)], and $(Z)_n$ [Abstract 71T-A252, these *Notices* 18(1971)], the author characterizes all subfields of $(Q)_n$. In addition, the following result is obtained. Theorem. Let F be an algebraic number field, with $[F:Q] = m$. Then F is representable as a maximal subfield of $(Q)_n$ whenever $n \geqq m$. (Received September 20, 1971.)

*689-A20. GORDON L. BAILES, JR., Clemson University, Clemson, South Carolina 29631. Right inverse semigroups.

A right inverse semigroup is a regular semigroup S in which $|aV(a)| = 1$ for all $a \in S$ where $V(a)$ is the set of inverses of a. Such semigroups are orthodox and are characterized by each $\mathcal{R}$-class containing a unique idempotent. If e and f are idempotents in a right inverse semigroup S then $e\mathcal{L}f$ if and only if $V(e) = V(f)$. Any homomorphic image of S is right inverse and the minimal right inverse congruence on a regular semigroup is determined. If E is the set of idempotents of S and $E = \bigcup_{\alpha \in Y} E_\alpha$ is the maximal semilattice decomposition of E, let S_α, for $\alpha \in Y$, be the maximal right inverse subsemigroup of S with E_α as its set of idempotents ; then S_α is left simple, and the S_α, $\alpha \in Y$, are pairwise disjoint. The following are equivalent: $\mathcal{R} = \mathcal{H}$; $\mathcal{L} = \mathcal{D}$; $\mathcal{R}$ is a congruence ; $\mathcal{L}$ is a congruence ; $S = \bigcup_{\alpha \in Y} S_\alpha$; S is a semilattice of the S_α, $\alpha \in Y$; each S_α is a $\mathcal{D}$-class ; S is a union of groups ; and S is a band of groups. If any of these hold then $\mathcal{D} = \mathcal{L} = \mathcal{J}$. An explicit construction of right inverse semigroups which are unions of groups is given along with necessary and sufficient conditions for two such semigroups to be isomorphic. Finally, completely simple and completely 0-simple right inverse semigroups are characterized. (Received September 22, 1971.)

*689-A21. JOHN C. PERKINS, Box 376, Ft. Belvoir, Virginia 22060 and JOHN D. FULTON, Clemson University, Clemson, South Carolina 29631. Symmetric involutions over fields of characteristic 2. Preliminary report.

The $n \times n$ symmetric involutions of signature s over a field K of characteristic 2 are characterized as follows: X is an $n \times n$ symmetric involutory matrix of signature s, $0 \leqq s \leqq [n/2]$, over K, if and only if X can be decomposed as $I + P^T NP$, where P is $s \times n$ of rank s, $PP^T = 0$, and N is $s \times s$ nonsingular and symmetric. These symmetric involutions are partitioned through a chain of theorems and are enumerated in the case that K is finite. (Received September 23, 1971.)

689-A22. J. E. LEECH, University of Tennessee, Knoxville, Tennessee 37916. Congruences under $\mathscr{K}$. Preliminary report.

The normal subgroup-congruence correspondence for groups is extended to semigroup congruences under $\mathscr{K}$ as follows: Let S be a semigroup with identity and let $\mathscr{D}\langle S\rangle$ be its division category. If $\mathscr{J}_S$ is the maximal congruence lying under $\mathscr{K}$, then $\mathscr{J}_S$ induces a functor $\Gamma_S : \mathscr{D}\langle S\rangle \to \underline{\underline{Gr}}$, the category of groups such that for all $x \in S = \mathcal{O}b(\mathscr{D}\langle S\rangle)$, $\Gamma_S(x)$ is the normal subgroup of $H_S(x)$, the right Schützenberger group of x, such that $x \cdot \Gamma_S(x) = \mathscr{J}(x)$. We have the following Theorem. There exists an order preserving correspondence between the subfunctors of Γ_S and the congruences under $\mathscr{K}$ defined by the map $F \to \bigcup_{x \in S}(x \cdot F(x)) \times (x \cdot F(x))$. This is one of the results of a 100^+ pp. monograph being prepared by the author with this title. (Received September 13, 1971.)

*689-A23. RONALD C. LINTON, University of South Alabama, Mobile, Alabama 36608. Abelian groups in which every neat subgroup is a direct summand.

Fuchs, Kertész and Szele ["Abelian groups in which every serving subgroup is a direct summand," Publ. Math. Debrecen 3(1953), 95-105] have classified those groups in which every pure subgroup is a direct summand. A more natural problem (at least from a purely module-theoretic viewpoint) is that of describing those abelian groups satisfying (N): Every neat subgroup is a direct summand. Theorem 1. G satisfies (N) if and only if G has one of the following forms: (1) G is a torsion group in which, for each prime p, G_p is either divisible or of the form $H + K$, where H is a direct sum of cyclic groups of order p^n and K is the direct sum of cyclic groups of order p^{n+1}. (2) $G = D + E$ where D is divisible and E is the direct sum of a finite number of mutually isomorphic rank one torsion-free groups. An abelian group G is quasi-injective if and only if each homomorphism of any subgroup A into G can be extended to an endomorphism of G. An application of Theorem 1 yields Theorem 2. G is quasi-injective if and only if G has one of the following forms: (1) G is divisible. (2) G is torsion and each G_p is the direct sum of mutually isomorphic cocyclic ($\cong Z(p^k)$, $k = 1, 2, \ldots$, or ∞) groups. (Received September 16, 1971.)

*689-A24. JOHN KINLOCH, East Tennessee State University, Johnson City, Tennessee 37601. A one-sided characterization of Dedekind semiprime rings. Preliminary report.

A ring R with 1 is a Dedekind semiprime ring if R is a hereditary, noetherian, semiprime ring which is a maximal order in its quotient ring. These rings were originally studied by J. C. Robson (J. Algebra 9(1968), 247-265). In the following we characterize the two-sided nature of these rings by means of one-sided conditions only. A ring R with 1 satisfies (LTF) if each finitely generated torsion-free left module is a submodule of a free module. Theorem 1. A ring R is a hereditary, noetherian, semiprime ring iff R satisfies: (1) R is a left Ore ring, (2) R is left hereditary, (3) R satisfies (LTF), and (4) R satisfies the DCC on finitely generated integral left ideals containing a fixed integral left ideal. Theorem 2. A ring R is a Dedekind semiprime ring iff R satisfies (1)-(4) of Theorem 1 and in addition (5) R is a maximal left order in its quotient ring. (Received September 24, 1971.)

*689-A25. PHILLIP A. HARTMAN, University of North Carolina, Asheville, North Carolina 28801. Integrally closed and complete ordered quasigroups and loops.

The results on embedding a partially ordered group in its Dedekind extension are generalized by showing that, with the appropriate definition of integral closure, any partially ordered quasigroup (loop) G can be embedded in a complete partially ordered quasigroup (loop) if and only if G is integrally closed. If G is directed as well, then its Dedekind extension is a complete lattice-ordered quasigroup (loop). Furthermore, any complete fully ordered quasigroup (loop) has, with one exception, the real numbers with their usual ordering as its underlying ordered set. The quasigroup (loop) operation however need not be ordinary addition as it is in the associative case. On the other hand, a complete strongly power associative fully ordered loop is either the integers or the real numbers with ordinary addition. (Received September 24, 1971.)

*689-A26. WILLIAM T. TROTTER, JR, The Citadel, Charleston, South Carolina 29409. A note on triangulated graphs. Preliminary report.

A graph G is said to be triangulated if every cycle of four or more distinct vertices has a chord. The vertex independence number is the least upper bound on the cardinality of subsets of vertices no two of which are adjacent. A graph G is perfect if for every induced subgraph $H \subseteq G$, the vertex independence number is equal to the number of complete subgraphs required to cover the vertices of H and the chromatic number of H is equal to the cardinality of the largest complete subgraph of H. The principal results of this paper are that infinite triangulated graphs are perfect when the vertex independence number is finite, and that triangulated graphs which have infinite vertex independence number also have an infinite set of vertices no two of which are adjacent. Some additional characterizations of triangulated graphs and connections with topology are also discussed. (Received September 27, 1971.)

689-A27. WILLIAM J. HEINZER, Purdue University, Lafayette, Indiana 47907. A characterization of certain 2-dimensional regular affine UFD's as polynomial rings. Preliminary report.

Let k be a field, let X and Y be indeterminates over k, and let $f \in k[X,Y]$ be a polynomial generator for $k[X,Y]$--i.e. there exists a polynomial g such that $k[X,Y] = k[f,g]$. Theorem. If A is a regular affine unique factorization domain such that $k[X,Y] \subset A < k[X,Y,1/f]$, then A is a polynomial ring over k.

(Received September 27, 1971.)

*689-A28. ARTHUR PELLERIN and RENU LASKAR, Clemson University, Clemson, South Carolina 29631. Cubic graphs on twelve vertices and the line graph of a finite affine plane. Preliminary report.

Rao and Rao ["A characterization of the line graph of a BIBD with $\lambda = 1$," Sankhya: Series A, 1969] showed that certain geometric properties characterize the line graph of a BIB design with parameters b, v, r, k, 1, provided $r - 2k + 1 < 0$. If $r = k + 1$, and $k > 2$, a characterization of the line graph of a finite affine plane was also given independently by Laskar [Abstract 669-11, these *Notices* 16(1969), 1046]. If $r - 2k + 1 = 0$, it can be easily checked that the only possible value for k is 2, from which it follows that $r = k + 1$ resulting in the case of the line graph of the finite affine plane. It is shown that if $r = k + 1$, one of the properties can be weakened and that for $k = 2$, there are exactly seven nonisomorphic graphs with those properties which are not the line graph of a finite affine plane. It follows that these seven nonisomorphic exceptional graphs and the line graph of a finite affine plane of order 2 are the only cubic graphs on 12 vertices with no quadrilaterals. (Received September 30, 1971.)

689-A29. CARL V. HINKLE, JR., Department of Mathematical Sciences, Clemson University, Clemson, South Carolina 29631. The injective hull of $\mathcal{P}$-torsion-free semigroups. Preliminary report.

Let S be any semigroup with 0. A sub S-set N_S of a right S-set M_S is said to be intersection large in M_S if $N_S \cap E_S \neq 0$ for each non-0 sub S-set E_S of M_S. Let $\mathcal{P}$ be the collection of all intersection large right ideals of S_S. The singular congruence $\psi = \psi_S(M)$ on M_S is defined by $\psi = \{(m,n): ms = ns$ for all $s \in E$ for some $E \in \mathcal{P}\}$. M_S is said to be $\mathcal{P}$-torsion-free if $\psi_S(M)$ is the identity. An analogue $Q = Q(S)$ of the Johnson ring of quotients is developed for semigroups with 0. In this paper we show that if S_S is $\mathcal{P}$-torsion-free then Q_S is the injective hull of S. Furthermore, Q is injective as a Q-set. As an example, the injective hull of a regular Rees matrix semigroup $M^0(G,I,J,P)$ where no two rows of P are left proportional is the semigroup of column monomial $I \times I$ matrices over the group G with 0. (Received September 30, 1971.)

*689-A30. TREVOR EVANS, Emory University, Atlanta, Georgia 30345. When are functionally free algebras free? A remark on a problem of B. M. Schein.

Schein asks (Semigroup Forum 1(1970), Research Problem), whether any band (idempotent semigroup) satisfying exactly the identities common to all bands is actually a free band. This has been answered in the negative by several authors (see Hall, Semigroup Forum 2(1971), 83-84, and the editor's comment following

Hall's paper). In this note we show that the only semigroup varieties satisfying Schein's condition that functionally free algebras in V are free in V are the varieties of left-zero semigroups, right-zero semigroups, constant semigroups and abelian groups of prime exponent. The problem is also solved for other varieties of algebras including rings and lattices. Another question suggests itself--does every variety satisfying Schein's condition have to be equationally complete and hence have every algebra in it free? In this connection, an example is given of a variety V containing algebras F, A such that F and $F \times A$ are free but A is not.
(Received October 1, 1971.)

689-A31. HERBERT F. KREIMER, Florida State University, Tallahassee, Florida 23206. Separability and the Galois theory of commutative rings. Preliminary report.

Let B be a commutative ring with 1, let G be a finite group of automorphisms of B, and let A be the subring of G-invariant elements of B. If A' is a separable A-subalgebra of B, then A' is a finitely generated, projective A-module; for each prime ideal p in A, the rank of A'_p over A_p does not exceed the order of G; and there is a finite group H of automorphisms of B such that A' is the subring of H-invariant elements of B. If, in addition, A' is G-stable, then every automorphism of A' over A is the restriction of an automorphism of B, $\mathrm{Hom}_A(A',A')$ is generated as a left A'-module by those automorphisms of A' which are restrictions of elements of G, and the Morita theorems may be used to deduce that the contraction map is an isomorphism of the lattice of G-stable ideals of A' onto the lattice of ideals of A. (Received October 1, 1971.)

689-A32. ROBERT P. WEBBER, University of Tennessee, Knoxville, Tennessee 37916. The matrix semigroup determined by the spectral norm. Preliminary report.

It is well known that the spectral norm of a matrix A is the largest of its singular values; i.e., the nonnegative square root of the largest eigenvalue of $A^H A$. Let S be the semigroup of $n \times n$ matrices whose spectral norm is less than or equal 1. Let $A = U\Lambda V^H$ be the singular value decomposition of A.
Theorem 1. Green's $\mathcal{L}$-relation on S is characterized as follows: Let A_1 and A_2 be in S. $A_1 \mathcal{L} A_2$ if and only if there exist singular value decompositions $A_1 = U_1\Lambda_1 V_1^H$ and $A_2 = U_2\Lambda_2 V_2^H$ such that $\Lambda_1 = \Lambda_2$ and $V_1 = V_2$. The other Green's relations are similarly characterized. Theorem 2. A regular element of S has a unique semi-inverse in S, and this semi-inverse is the Moore-Penrose inverse of the matrix.
Theorem 3. The maximal subgroups of S are the groups of unitary matrices of rank 1 through n. (Received October 1, 1971.)

*689-A33. ROBERT J. SCHWABAUER, Department of Mathematical Sciences, Virginia Commonwealth University, Richmond, Virginia 23220. Covering conditions and variety lattices.

Let K be the equational class of commutative semigroups defined by $(1,1,1,1,1,1) = (0,0,0,0,0,0,6)$. Let J be the variety of commutative semigroups defined by the above equation and $(1,4) = (2,3) = (3,2)$. Then the interval $[J,K]$ in the lattice of commutative semigroup varieties satisfies neither the lower nor the upper covering conditions. Thus the following holds: Theorem 1. The lattice of equational classes of commutative semigroups is neither semimodular nor dually semimodular. Obvious modifications give: Corollary. The lattice of equational classes of semigroups and any lattice of all equational classes of a given similarity type τ, where $2 \in \tau$ are neither semimodular nor dually semimodular. The interval mentioned above was suggested by an example in the Ph.D. thesis of Evelyn M. Nelson ("The lattice of equational classes of commutative semigroups," McMaster University, Hamilton, Ontario, 1970). (Received October 4, 1971.)

689-A34. JAMES L. WILLIAMS, University of Arkansas, Fayetteville, Arkansas 72701. Quotients determined by a correspondence. Preliminary report.

Let $\rho \subset X \times Y$ be a correspondence on X, Y and let Cor(X, Y) denote all correspondences on sets X, Y. For $x \in X$, $x\rho = \{y \in Y \mid (x,y) \in \rho\}$ where $\rho \in \mathrm{Cor}(X,Y)$. The quotient of X with respect to ρ is $X|\rho = \{A \in 2^X \mid A \neq \emptyset ; a,b \in A \text{ iff } b\rho = a\rho \neq \emptyset\}$. If $\alpha \in \mathrm{Cor}(X,Y)$ there are $\eta \in \mathrm{Cor}(X,X/\alpha)$ and $\alpha' \in \mathrm{Cor}(X/\alpha, Y)$ such that $\alpha = \alpha'\eta$ with η an onto map. This theorem may be characterized in terms of the analogous theorem for functions on the power sets of X and Y. Similar results may be obtained with a different definition of quotient, which does not require elements to be disjoint. (Received October 4, 1971.)

*689-A35. T. J. SCOTT, University of Georgia, Athens, Georgia 30601. Monotonic permutations of chains. Preliminary report.

The group $A(\Omega)$ of all order-preserving permutations of a chain Ω is a lattice-ordered group (when ordered pointwise). If G is a transitive lattice-ordered subgroup of $A(\Omega)$ and $G_\alpha = \{g \in G \mid \alpha g = \alpha\}$, the orbits of G_α are convex, and thus inherit a natural total order. If Δ is a G_α orbit, $\Delta' = \{\alpha g \mid \alpha \in \Delta g\}$ is called the paired orbit of Δ. A permutation of Ω is called monotonic iff it is either an order-preserving permutation or an order-reversing permutation. The collection of all monotonic permutations of Ω forms a group. A permutation group (K, Ω) is called monotonic (ℓ-monotonic) iff K contains at least one order-reversing permutation and $G = G(K) = K \cap A(\Omega)$ is transitive (a transitive lattice-ordered subgroup of $A(\Omega)$). Theorem. If K is monotonic, $[K:G] = 2$, and an orbit ($\neq \{\alpha\}$) of K_α is the union of a positive G_α orbit and a negative G_α orbit. A monotonic group (K, Ω) has the pairing property iff each K_α orbit is the union of a G_α orbit and its paired G_α orbit. There exist ℓ-monotonic groups which do not have the pairing property. Theorem. If K is ℓ-monotonic, K can be "nicely" embedded in the generalized monotonic wreath product of its o-primitive monotonic components; and each component, as well as the wreath product, has the pairing property. (Received October 4, 1971.)

*689-A36. MELVYN B. NATHANSON, Southern Illinois University, Carbondale, Illinois 62901. Sums of finite sets of integers.

Let $\mathcal{A}$ be a finite set of integers. The h-fold sum of $\mathcal{A}$, denoted $h\mathcal{A}$, is the set of all sums of h elements of $\mathcal{A}$, repetitions being allowed. The object of this paper is to describe exactly the structure of all sufficiently high sums of any finite set of integers. All latin letters stand for integers, and all script letters for finite sets of integers. Denote by $(a_1, a_2, \ldots, a_k)$ the greatest common divisor of $a_1, a_2, \ldots, a_k$. Let $[p,q]$ be the set of integers n such that $p \leqq n \leqq q$. Let $z - \mathcal{B} = \{z - d \mid d \in \mathcal{B}\}$. Theorem. Let $\mathcal{A} = \{a_0, a_1, \ldots, a_k\}$ be a finite set of integers with $a_0 = 0 < a_1 < \ldots < a_k = a$ and $(a_1, a_2, \ldots, a_k) = 1$. Then there exist nonnegative integers C and D and sets $\mathcal{C} \subset [0, C-2]$ and $\mathcal{D} \subset [0, D-2]$ such that $h\mathcal{A} = \mathcal{C} \cup [C, ha-D] \cup ha-\mathcal{D}$ for all $h \geqq a^2k$. Moreover, $C = 0$ if and only if $a_1 = 1$, and $D = 0$ if and only if $a_{k-1} = a_k - 1$. Clearly, an arbitrary finite set of integers differs from the normalized sets considered in the theorem only by a translation and contraction. (Received October 4, 1971.)

689-A37. J. J. BODE, University of South Carolina, Columbia, South Carolina 29208. t-functors. Preliminary report.

Any extension of Z (integers) by a group H induces a preradical via the Hom-Ext sequence. Dually these extensions give rise to a class of preradicals through the Tor-Tensor sequence. Such preradicals will be called t-functors. If H is a p-group the t-functor induced by an extension of Z by H is characterized by the Ulm sequence of the image of one. The characterization for p-groups yields a complete description of t-functors induced by extensions of Z by arbitrary groups. (Received October 4, 1971.)

*689-A38. JAPHETH HALL, JR., Stillman College, Tuscaloosa, Alabama 35401. Order relations determined by coverings of sets. Preliminary report.

This paper introduces a notion for dealing with problems of maximization which is more general than the notion of a poset: (V,R,E) is an order system if V is a set, R is a reflexive and transitive relation on V, and E is an equivalence relation on V such that if $x \in V$, $y \in V$, xRy and yRx, then xEy. Each reflexive relation R on a set V determines a covering $G(R) = \{V_x : x \in V\}$ of V, where $V_x = \{y \in V : xRy\}$ if $x \in V$. Let V be a set and G be a covering of V with the property that if $x \in V$, then there is a maximal subcollection G_x of G such that $x \in X$ if $X \in G_x$. G determines an order system (V, R_G, E_G) as follows: "xR_Gy" means that $G_x \subseteq G_y$ and "xE_Gy" means that $G_x = G_y$ if $x \in V$ and $y \in V$. Definitions of "totally ordered chain", "upper bound" and "maximal elements" are analogues to the usual ones for posets. Proposition. If (V,R,E) is an order system, then $R_{G(R)} = R$ and $E_{G(R)} \subseteq E$. Theorem. The following are equivalent: (i) (V, R_G, E_G) has a maximal element; (ii) there is an element b of V such that $\cap G_x = \cap G_b$ whenever $x \in (\cap G_b)$; (iii) some element of V fails to belong to any totally ordered chain having no upper bound. (Received October 4, 1971.)

*689-A39. JOHN K. LUEDEMAN, Clemson University, Clemson, South Carolina 29631. On the embedding of topological rings into rings of quotients.

Let A be a ring and H be a multiplicative subsemigroup of A composed of non-zero-divisors. An overring with identity, Q, of A is a ring of left quotients of A relative to H if each element of H is invertible in Q and each element of Q has the form $h^{-1}a$ for some $h \in H$ and $a \in A$. It is well known that A has a ring of left quotients relative to H iff $ha \cap Ah \neq \emptyset$ for all $a \in A$ and $h \in H$. When this is the case we call (A,H) a pair. The multiplicative subgroup of Q generated by H and the multiplicative inverses of members of H is denoted by $Q^*(H)$. Finally, H is quotient closed if $\{y \in A \mid Hy \cap H \neq \emptyset \text{ or } yH \cap H \neq \emptyset\} \subseteq H$. We give necessary and sufficient conditions on a pair (A,H) in which A is a topological ring and H is open and quotient closed to extend the topology on A to a ring topology on Q in such a way that $Q^*(H)$ is an open topological multiplicative subgroup of Q. These conditions are related to those in our earlier paper ("On the embedding of topological domains into quotient fields," Manuscripta Math. 3(1970), 213-220) and arise naturally when considering the embedding of products of topological domains. (Received October 4, 1971.)

*689-A40. ANDREW SOBCZYK, Clemson University, Clemson, South Carolina 29631. Arrangements of lines which are of class two.

For any set or arrangement $\mathcal{A}$ of lines in R^d, $d \geqq 2$ (cf. Grünbaum), the relation that two lines of $\mathcal{A}$ intersect determines an associated graph A of which the vertices are in correspondence with the lines of $\mathcal{A}$. The arrangement is said to be of class 1 in case the graph is complete. A nonplanar class 1 arrangement necessarily is a subset of a pencil; i.e. the lines of $\mathcal{A}$ are all concurrent. The arrangement $\mathcal{A}$ is of class 2 in case A^2 is a complete graph, i.e. if each pair of lines of $\mathcal{A}$ either intersect or have a common intersector in $\mathcal{A}$. An arrangement $\mathcal{U}$ (of infinitely many lines) is representative if it contains exactly one line parallel to each one-dimensional subspace of R^d. Theorem. Any continuous, representative arrangement $\mathcal{U}$ in R^3 is of class 2. Finite class 2 arrangements $\mathcal{A}$ are classified, with a view toward determining whether there exists a universal arrangement $\mathcal{U}$ for such arrangements $\mathcal{A}$, and also whether there is a $\mathcal{U}$ which has the property that for each positive $n \geqq 5$ there is a subarrangement of n lines which is of a prescribed higher class. (Received October 4, 1971.)

689-A41. ALFRED T. BRAUER, Wake Forest University, Winston-Salem, North Carolina 27109. On classes of stochastic matrices whose absolute smallest characteristic is real.

There exist classes of stochastic matrices whose absolute greatest nontrivial root is positive and other classes where all nontrivial roots are imaginary. It is shown in this paper that there are such matrices where the absolute smallest root is real. (Received October 4, 1971.)

*689-A42. OBERTA A. SLOTTERBECK, University of Florida, Gainesville, Florida 32601. Wreath products and saturated formations. Preliminary report.

For G a group and p a prime, let $O_{p'p}(G)$ denote the maximum normal subgroup of G that is an extension of a p'-group by a p-group. D. Parker's results on finite wreath products in locally defined formations [Proc. Amer. Math. Soc. 24(1970), 404-408] are extended to characterize $O_{p'p}(G)$ for G either a restricted or unrestricted standard wreath product of arbitrary groups. This yields, for example, characterizations of wreath products in some saturated periodic locally solvable formations in the sense of Gardiner, Hartley and Tomkinson [J. Algebra 17(1971), 177-211]. (Received October 4, 1971.)

*689-A43. RAJINDER B. HORA and NAOKI KIMURA, University of Arkansas, Fayetteville, Arkansas 72701. Commutative semigroups with 0 such that the square of every element is 0.

Let us call a semigroup with 0 Boolean, if it is commutative and $x^2 = 0$ for all element x. An example of a Boolean semigroup, which plays a crucial role, is constructed as follows: Let X be a set and let $P = P(X)$ be the set of all nonempty finite subsets of X. Define partial multiplication only for disjoint pairs as the union.

Then the set $P_0 = P \cup \{0\}$, where $0 \notin P$, is Boolean by extending the multiplication so that the other products are all 0. Coideals of P(X) is defined as dual hereditary families. Then the Reese quotient of P(X) by a coideal is again Boolean. Any Boolean S generated by a subset X is a homomorphic image of $P_0(X)$. The complete characterization of the congruence will be performed with the notion of the coideal as a main media. (Received October 4, 1971.)

*689-A44. RAJINDER B. HORA and NAOKI KIMURA, University of Arkansas, Fayetteville, Arkansas 72701. Left translations of the free product of semigroups.

Let X be a subset of a semigroup S such that the right ideal of S generated by X is S. A criterion that a function $f: X \to S$ can be extended to a left translation of S will be presented. Then as an interesting application of this, when a semigroup is the free product of a family of semigroups will be considered. This latter case includes free semigroups as a special case. (Received October 4, 1971.)

*689-A45. ERIC C. NUMMELA, University of Florida, Gainesville, Florida 32601. Homological algebra of topological modules. Preliminary report.

Consider the category of Hausdorff topological modules and continuous homomorphisms over some Hausdorff topological ring R with identity. The module F is free over the space X if (1) there is a continuous function $i_X: X \to F$, and (2) given any module M and continuous function $f: X \to M$, there is a unique homomorphism $\bar{f}: F \to M$ with $\bar{f} \cdot i_X = f$. Given X, a module F(X) free over X always exists, unique up to isomorphism. If X is a module, then i_X is an embedding, and the homomorphism $\bar{f} = q_X: F(X) \to X$ induced by $f = 1_X$ is a quotient mapping. Free modules are projective relative to the class $\underline{S}$ of short exact sequences $A \to B \to C$ (which must consist of an embedding onto a closed submodule followed by the canonical quotient morphism) for which the latter homomorphism has a continuous right inverse function. Note that the sequence $\mathrm{Ker} \to F(X) \to X$ belongs to $\underline{S}$ for every module X. The class $\underline{S}$ is suitable for constructing a relative extension bifunctor, denoted by Ext^n, of equivalence classes of n-fold $\underline{S}$-exact sequences. Moreover, Ext* can be computed via projective resolutions of the first variable. The topological projective dimension of a module is also considered. (Received September 10, 1971.)

*689-A46. NICK H. VAUGHAN, North Texas State University, Denton, Texas 76203. A note on the classes of semiprimary ideals and Dedekind ideals in a domain. Preliminary report.

Let D denote an integral domain with $1 \neq 0$ and quotient field K. An overring of D is a ring J such that $D \subset J \subset K$. If π is a general ring property, then an ideal A of D is called a π-ideal provided there exists an overring J of D such that J is a π-domain and $A = AJ \cap D$. The set of Dedekind ideals is denoted by $\mathscr{D}$ and the set of semiprimary ideals is denoted by $\mathscr{S}$. Theorem 1. If the prime ideals of D are almost Dedekind ideals, then $\mathscr{D} \subset \mathscr{S}$ if and only if the prime ideals of D are chained. Theorem 2. $\mathscr{D} = \mathscr{S}$ in D if and only if D is a rank one discrete valuation ring. (Received October 4, 1971.)

689-A47. THOMAS JOSEPH SHARP, West Georgia College, Carrollton, Georgia 30117. On D-groups and Y-subgroups.

In a paper by J. M. Irwin and J. D. O'Neill (Canad. J. Math. 22(1970), 525-544) the following question is raised: If the torsion subgroup of a product of p-groups equals $A \oplus B$, does the product equal $A' \oplus B'$, where $A' \supset A$ and $B' \supset B$? We work in a more general case and define a D-group as any abelian group with the property indicated above. A Y-subgroup is defined as a subgroup H with the property that whenever $G = A \oplus B$, then $H = (H \cap A) \oplus (H \cap B)$, and it is shown that H is a Y-subgroup if and only if H is projection-invariant. Results on D-groups and Y-subgroups are obtained, including: Lemma. If G is a D-group, then $T(A') = A$ and $T(B') = B$. Theorem 1. If $\{G_\beta\}$ is a collection of D-groups, where each T_β is fully invariant in T, then $G = \Sigma_\beta G_\beta$ is a D-group. Theorem 2. If H is a Y-subgroup of the D-group G such that T(H) is a direct summand of T, then H is a D-group. Several other results are presented. (Received October 4, 1971.)

Analysis

689-B1. ADOLPH W. GOODMAN, University of South Florida, Tampa, Florida 33620. Coefficients for the area theorem. Preliminary report.

Set $f(z) = z + \sum_{n=2}^{\infty} a_n z_n$. Let $F(z) = (f(z^p))^{1/p}$ where p is a positive integer, and define g_n by $G(z) \equiv F^{-1}(z^{-1}) \equiv z + \sum_{n=1}^{\infty} g_{np-1} z^{np-1}$. The area theorem states that if f(z) is univalent in $|z| < 1$, then $\sum_{n=1}^{\infty} (np-1)|g_{np-1}|^2 \leqq 1$. The importance of this inequality in the theory of univalent functions is well known, but the inequality is not easy to use because for large values of n the dependence of g_{np-1} on $(a_2, a_3, \ldots, a_{n+1})$ seems to be rather complicated. In this paper we obtain an explicit formula for g_{np-1} in terms of $a_2, a_3, \ldots, a_{n+1}$, valid for every integer $n \geqq 1$ and every integer $p \geqq 1$. (Received May 17, 1971.)

689-B2. RALPH L. HARRIS, Western Carolina University, Cullowhee, North Carolina 28723. A characterization of uniformly convex spaces. Preliminary report.

Theorem. Suppose that S is a normed linear space. Then S is uniformly convex if and only if S is strictly convex and has property H [see Coleman, Abstract 71T-B191, these *Notices* 18(1971), 817]. Corollary. There exists a reflexive Banach space which does not have property H. (Received July 15, 1971.)

*689-B3. MURIL L. ROBERTSON, Auburn University, Auburn, Alabama 36830. The equation $y'(t) = F(t, y(g(t)))$.

Suppose $I_1, I_2, \ldots$ is a sequence of intervals so that $p \in I_1 \subseteq I_2 \subseteq \ldots$ and I^* is the union of these intervals. Also, suppose F is a continuous function from $I^* \times B$ into B, a Banach space, so that there is a function M so that $\|F(x,y) - F(x,z)\| \leqq M(x) \cdot \|y - z\|$, for all $x \in I^*$, and $y, z \in B$. Also, assume that M is Lebesgue integrable on each I_n. Let g be a continuous function from I^* into I^* so that $g(I_n) \subseteq I_n$, for each positive integer n. Then, a certain multiple integral condition is shown to be sufficient for the unique solution of the functional differential equation $y'(t) = F(t, y(g(t)))$, $y(p) = q$, for all $t \in I^*$. (Received July 30, 1971.)

689-B4. SAMUEL ZAIDMAN, Université de Montréal, Montréal, Québec, Canada. Properties of strongly almost-periodic families of linear operators.

Consider a Banach space X and let $G(t)$, $-\infty < t < \infty \to \mathcal{L}(X,X)$ be a function such that $G(t)x$ is X-continuous and almost-periodic for any $x \in X$. Then: Theorem 1. If $M \subset X$ is a relatively compact set, the family of X-valued functions $\{G(t)x,\ x \in M\}$ is relatively compact in the strong uniform convergence on the real axis. Theorem 2. If $f(t)$ is an X-almost-periodic function then $G(t)f(t)$ is again an X-almost-periodic function. Both results were given earlier by the author in the case where $G(t)$ is a one-parameter group of linear operators. They are obtained now in a somewhat different way. Theorem 2 is a trivial consequence of the main approximation theorem in almost-periodic functions theory, but the proof here is using only basic properties and definitions of this theory. (Received September 14, 1971.)

*689-B5. DAVID L. LOVELADY, University of South Carolina, Columbia, South Carolina 29208. Existence on prescribed rectangles for a hyperbolic equation in a Banach space.

Let Y be a Banach space, and let F be a continuously Fréchet differentiable function from $Y \times Y \times Y$ to Y with $F(0,0,0) = 0$. Let b be a positive number. Theorem. There is a positive number c such that if each of (1) and (2) is true then (3) is true. (1) Each of φ and ψ is a continuously differentiable function from $[0,b]$ to Y, p is in Y, $\varphi(0) = \psi(0) = p$, $|\varphi(s) + \psi(t) - p| \leqq c$ whenever (s,t) is in $[0,b]\times[0,b]$, $|\varphi'(s)| \leqq c$ whenever s is in $[0,b]$, and $|\psi'(t)| \leqq c$ whenever t is in $[0,b]$. (2) g is a continuous function from $[0,b]\times[0,b]$ to Y such that, if (s,t) is in $[0,b]\times[0,b]$, then $|\int_0^s \int_0^t g(x,y)dydx| \leqq c$, $|\int_0^s g(x,t)dx| \leqq c$, and $|\int_0^t g(s,y)dy| \leqq c$. (3) There is exactly one continuously differentiable function u from $[0,b]\times[0,b]$ to Y such that $u(s,0) = \varphi(s)$ and $u(0,t) = \psi(t)$ whenever (s,t) is in $[0,b]\times[0,b]$ and such that $(\partial^2/\partial s\partial t)u(s,t) = g(s,t) + F(u(s,t), (\partial/\partial s)u(s,t), (\partial/\partial t)u(s,t))$ whenever (s,t) is in $(0,b)\times(0,b)$. (Received September 17, 1971.)

689-B6. RICHARD D. CARMICHAEL, Wake Forest University, Winston-Salem, North Carolina 27109. Generalized Cauchy and Poisson integrals and distributional boundary values.

Let C be an open connected cone in $\mathbf{R}^n$ and $T^C = \mathbf{R}^n + iC \subset \mathbf{C}^n$ be the corresponding tubular radial domain. Put $K(z-t) = \int_{C^*} \exp(2\pi i\langle z-t, q\rangle)dq$, $z \in T^{O(C)}$, where $C^* = \{q : \langle q,y\rangle \geqq 0,\ y \in C\}$ is the dual cone of C and $O(C)$ is the convex envelope of C. Put $Q(z;t) = K(z-t)\,\overline{K(z-t)}/K(2iy)$, $z \in T^{O(C)}$. Both $K(z-t)$ and $Q(z;t)$ are elements of D_{L^q}, $1/q + 1/p = 1$, $1 < p \leqq 2$. Let $U \in D'_{L^p}$, $1 < p \leqq 2$; and put $C(U;z) = \langle U, K(z-t)\rangle$ and $P(U;z) = \langle U, Q(z;t)\rangle$, $z \in T^{O(C)}$, which are the generalized Cauchy and Poisson integrals, respectively, of U. We show that $C(U;z)$ is an analytic function in $T^{O(C)}$; and $U \in D'_{L^p}$, $1 < p \leqq 2$, can be represented as a limit of the generalized Cauchy and Poisson integrals on the distinguished boundary of T^C. We introduce a set of functions analytic in T^C, which we denote as G_C^b, and show that the elements in G_C^b are representable by the generalized Cauchy integral of their boundary value, which is an element of D'_{L^p}, $1 < p \leqq 2$. It is seen that the space G_C^b is a generalization of the Hardy H^p spaces in tube domains. Generalizations of these and other results to disconnected tubular cones are obtained. (Received September 17, 1971.)

689-B7. WILLIAM E. FITZGIBBON, Vanderbilt University, Nashville, Tennessee 37203. Time dependent nonlinear Cauchy problems in Banach spaces. Preliminary report.

An operator A is said to have property (M) provided that whenever $\{x_n\} \subseteq D(A)$, $\lim_{n\to\infty} x_n = x_0$, and $\sup \|Ax_n\| < \infty$, it follows that $x_0 \in D(A)$ and w-lim $Ax_n = Ax_0$. Theorem. Let $\{A(t) : t \in [0, T]\}$ be a family of accretive operators such that the following are true: (1) $D(A(t))$ is time independent; (2) $R(I + \lambda(A(t))) \supseteq D(A(t))$ for all $\lambda > 0$; and (3) $\|(I + \lambda A(t))^{-1}x - (I + \lambda A(\tau))^{-1}x\| \leq \lambda|t - \tau| L(\|x\| + \|A(\tau)x\|)$ where $t, \tau \in [0, T]$, $\lambda > 0$, and $L : [0, \infty) \to [0, \infty)$ is an increasing function. Then the Cauchy problem $du(t)/dt + A(t)u(t) = 0$, $u(0) = x$, has a strong solution for a,e, $t \in [0, T]$ and $x \in D(A(0))$. Condition (M) holds if we require that X is reflexive and A(t) is demiclosed or if we require that D(A(0)) is closed and that A(t) is continuous from the strong to the weak topology. (Received September 20, 1971.)

*689-B8. JAMES J. BUCKLEY, University of South Carolina, Columbia, South Carolina 29208. Integrating group-valued functions.

$(X, \mathcal{A}, m)$ is a measure space, Y is a group, Y' = all the homomorphisms f of Y into the reals R under addition (f is continuous if Y has a topology, f is order-preserving if Y is partially ordered), I(h) = Lebesgue integral of $h : X \to R$ over X with respect to m with extension $I'(g) = \{y \in Y | f(y) = I(f(g)),$ all $f \in Y'\}$ for $g : X \to Y$. I'(g) is a coset in Y; if Y = R, then I' = I; if Y = Banach space over R, then I' = Pettis integral; I' is a linear function of the integrand; I' is additive on $\mathcal{A}$; if T is a homomorphism of Y into group G, then $T(I'(g)) \subset I'(T(g))$. Convergence and order properties of I' are also given. Example 1. Y = all the linear maps of R^n onto R^n under composition with the relative product topology. If $a = I(\ln|\det g(x)|)$ for $g : X \to Y$, then $I'(g) = \exp(a/n)\{n \times n\ A | \det A = \pm 1\}$. Example 2. Y = C[a, b] with pointwise convergence. If $g(x) = h(x, t) \in Y$ for each $x \in X$, then $I'(g) = I(h(x, t)dm(x))$. Example 3. Y = all the affine transformations of R into R under composition with the relative product topology. If $g(x) = a(x)t + b(x) \in Y$ for each $x \in X$, then $I'(g) = at\{\pm t + b | b \in R\}$, where $a = \exp(I(\ln|a(x)|))$. (Received September 20, 1971.)

*689-B9. CURTIS C. TRAVIS, Vanderbilt University, Nashville, Tennessee 37203. A note on second order nonlinear oscillations.

Positivity assumptions are removed from a well-known sufficient condition for the oscillation of all continuable solutions of the nonlinear differential equation (1) $\ddot{Y} + a(t)f(Y) = 0$, where $f(x)x > 0$ for $x \neq 0$ and $f'(x) \geqq 0$ for all x. Theorem. If $\int_1^\infty du/f(u) < \infty$, $\int_{-1}^\infty du/f(u) < \infty$, and $\int^\infty sa(s)ds = \infty$ then all continuable solutions of (1) are oscillatory. (Received September 22, 1971.)

689-B10. SHAWKY E. SHAMMA, University of West Florida, Pensacola, Florida 32504 and SAMUEL N. KARP, Courant Institute, New York University, New York, New York 10012. Asymptotic eigenfunctions of a scattering problem.

Previously [see Abstract 672-654, these *Notices* 17(1970), 271, and SIAM J. Appl. Math. 20(1971)] we discussed a generalization of separability in boundary value problems arising in potential theory. In this paper we extend the results to the scattering problems. The method is based on an integral equation whose kernel is the two-dimensional free space Green's function $(i/4) H_0^{(1)} (k|\underline{r} - \underline{r}'|)$, where $|\underline{r} - \underline{r}'|$ is the distance between two points on the scatterer. The asymptotic behavior of the eigenvalues and eigenfunctions of a larger class of kernels is obtained. The class includes the second iterated kernel of the above Green's function. It includes also the kernel $N_0(k|\underline{r} - \underline{r}'|)$ which arises in the study of several physical situations. The results are confirmed in a special case. (Received September 21, 1971.)

*689-B11. BEN-AMI BRAUN, University of South Florida, Tampa, Florida 33620. On the multiplicative completion of deleted Schauder bases in L^p, $1 \leqq p < \infty$. Preliminary report.

Boas and Pollard, "The multiplicative completion of sets of functions," Bull. Amer. Math. Soc. 54(1948), 518-522, proved that given any basis $\{f_n\}_{n=1}^{\infty}$ for $L^2(E)$ one can delete the first k basis elements and then find a bounded measurable function M such that $\{Mf_n\}_{n=k+1}^{\infty}$ is total in $L^2(E)$. We improve this result by weakening the hypothesis to accept bases of $L^p(E)$, $1 < p < \infty$ and strengthening the conclusion to read serially total. We also show that certain infinite deletions are possible. (Received September 23, 1971.)

*689-B12. EUTIQUIO C. YOUNG, Florida State University, Tallahassee, Florida 32306. Uniqueness theorems for a singular ultrahyperbolic equation.

Let X be a parallelopiped and Y be a bounded domain in the spaces $t, x_1, \ldots, x_m$ and $y_1, \ldots, y_n$, respectively. Let $Lu \equiv u_{tt} + (k/t)u_t + \Delta u - \sum_1^n D_i(a_{ij}D_ju) + cu = 0$ where k is a real parameter $-\infty < k < \infty$, Δ denotes the Laplace operator in $x_1, \ldots, x_m$, and $D_i = \partial/\partial y_i$. Consider the Dirichlet problem (*) $Lu = 0$ in Q, $u = 0$ on ∂Q, where $Q = X \times Y$. If $k > 0$, then every solution $u \in C^2(Q) \cap C^1(\bar{Q})$ of (*) vanishes identically in Q. For $k \leqq 0$, a necessary and sufficient condition is obtained for $u \equiv 0$ to be the only solution of (*). (Received September 23, 1971.)

*689-B13. JOHN W. HEIDEL, University of Tennessee, Knoxville, Tennessee 37916. A counterexample in nonlinear boundary value problems.

The nonlinear boundary value problem (1) $x'' = f(t, x, x')$, (2) $\alpha x(t_1) + \beta x'(t_1) = r_1$, $\gamma x(t_2) + \delta x'(t_2) = r_2$ is considered. It is assumed that f is continuous on $(a, b) \times R^2$ and that $a, b, \alpha, \beta, \gamma, \delta, r_1, r_2$ are real constants. If initial value problems of (1) are unique and exist on (a, b) and if $\beta\delta = 0$ then it is known that global uniqueness for (1), (2) on (a, b) implies global existence for (1), (2) on (a, b) [Lasota and Opial, Colloq. Math. 18(1967), 1-5]. These authors have also shown by example that this result is false if $\beta\delta \neq 0$ and $\alpha\delta - \beta\gamma = 0$. The purpose of this paper is to show by example that the result is also false for $\beta\delta \neq 0$ and $\alpha\delta - \beta\gamma \neq 0$. (Received September 24, 1971.)

*689-B14. THOMAS J. LANGAN, Naval Ship Research and Development Center, Hydrodynamics Branch 1552, Bethesda, Maryland 20034. Differential inequalities for semilinear hyperbolic operators with two independent variables.

Differential inequalities for semilinear hyperbolic systems are developed by means of Dini derivatives defined along the characteristics. These inequalities are consequently applicable to solutions in the wider sense of Friedrichs, that is solutions to equivalent integral equations along the characteristics. In addition, comparison theorems are obtained from the inequalities; moreover, if the semilinear hyperbolic operators are quasi-equicontinuous, there exists a mini-max solution to the system. (Received September 27, 1971.)

689-B15. RAYMOND A. STAFFORD, University of Tennessee, Knoxville, Tennessee 37916. A comparison theorem for second order linear differential equations. Preliminary report.

Assume that $q_1, q_2 \in C[a,\infty)$ for some $a > 0$ and that $q_2(t) \geqq 0$ for all $t \geqq a$. Theorem. Assume that $\mu \in C'[a,\infty)$ and that $\mu(t) > 0$ and $\mu'(t) \geqq 0$ for all $t \geqq a$. Suppose there exists $t_0 \geqq a$ such that $\int_{t_0}^t \mu^2(s)q_1(s)ds \geqq \int_{t_0}^t \mu^2(s)q_2(s)ds$ for all $t > t_0$. If the equation $\mu(t)u' = \mu'(t)u - u^2 - \mu^2(t)q_1(t)$ has a solution u on $[t_0,\infty)$ such that $u(t_0) \leqq \mu'(t_0)$, then the equation $\mu(t)v' = \mu'(t)v - v^2 - \mu^2(t)q_2(t)$ has a solution which exists for large t. Corollary 1. If there exists $t_0 \geqq a$ such that either (i) there exists $\gamma > 1/2$ such that $\int_{t_0}^t s^{2\gamma}q_2(s)ds \leqq (4(2\gamma-1))^{-1}(t^{2\gamma-1} - t_0^{2\gamma-1})$ for all $t > t_0$ or (ii) $\int_{t_0}^t sq_2(s)ds \leqq \frac{1}{4}\ln(t/t_0)$ for all $t > t_0$, then the equation $y'' + q_2(t)y = 0$ is nonoscillatory. Corollary 2. If there exists $t_0 \geqq a$ and $\epsilon > 0$ such that either (i) there exists $\gamma > 1/2$ such that $\int_{t_0}^t s^{2\gamma}q_1(s)ds \geqq ((1+\epsilon)/4(2\gamma-1))(t^{2\gamma-1} - t_0^{2\gamma-1})$ for all $t > t_0$ or (ii) $\int_{t_0}^t sq_1(s)ds \geqq ((1+\epsilon)/4)\ln(t/t_0)$ for all $t > t_0$, then the equation $y'' + q_1(t)y = 0$ is oscillatory. Example. It follows easily from the corollaries that the equation $y'' + (k/t^2)[1 - \sin(\ln t)]y = 0$ is nonoscillatory for $k \leqq \frac{1}{4}$ and is oscillatory for $k > \frac{1}{4}$. (Received September 27, 1971.)

689-B16. ALVIN F. BARR, Livingston University, Livingston, Alabama 35470. The radius of univalence of certain classes of analytic functions. Preliminary report.

Let $P(z) = \Pi_{k=1}^n (z - z_k)$ be a polynomial of degree $n > 0$, $|z_k| \geqq 1$, and $f(z) \in S^*$, the class of normalized starlike univalent functions. Theorem 1. Let $f(z) = z + a_2z^2 + \ldots$ be starlike of order α in $|z| < 1$. Then $F(z) = f(z)(P(z))^{1/n}$ is starlike and univalent in $|z| < r_0$, where r_0 is the positive root of $h(r) = 1 + (2\alpha - 3)r - 2\alpha r^2$. The result is sharp. Definition. Let H denote the class of functions of the form $f(z)(P(z))^{1/n}$, where $f(z) \in S^*$. Theorem 2. There is no number $M \geqq 1$ with $|z_k| \geqq M$ such that the class H maps $|z| < 1$ onto a starlike domain. Theorem 3. If $F(z) = f(z)(P(z))^{1/n}$ and f(z) is starlike of order α, then F(z) is starlike in $|z| < 1$ if $R \geqq (1+\alpha)/\alpha$. The result is sharp. (Received September 27, 1971.)

*689-B17. ROBERT H. MARTIN, JR., North Carolina State University, Raleigh, North Carolina 27607. Generators of nonlinear semigroups in uniformly convex Banach spaces.

Let E be a Banach space such that both E and its dual space E* are uniformly convex. If B is a subset of $E \times E$, then B is said to be dissipative if $\|x_1 - x_2 - h(y_1 - y_1)\| \geqq \|x_1 - x_2\|$ for all (x_1,y_1), $(x_2,y_2) \in B$ and $h > 0$. B is said to be maximal dissipative on $C \subset E$ if any dissipative extension of B coincides with B on $C \times E$. Now let C be a closed subset of E and let ω be a real number. Let B be a subset of $E \times E$ such that $B - \omega I = \{(x, y - \omega x) : (x,y) \in B\}$ is maximal dissipative on C and for each $x \in D(B) \cap C$ (where $D(B) = \{x \in E : (x,y) \in B$ for some $y \in E\}$) let $B^\circ x$ denote the element of minimum norm of the closed, convex set $Bx = \{y : (x,y) \in B\}$. B is said to have property $(G)^\omega$ on C if for each $\epsilon > 0$ and $x \in D(B) \cap C$, there is a $\delta > 0$ and $x_\delta \in D(B) \cap C$ such that $\|x + \delta B^\circ x - x_\delta\| \leqq \delta\epsilon$ and $\|B^\circ x_\delta\| \leqq e^{\omega(\delta+\epsilon)}\|B^\circ x\|$. Theorem. Suppose that A is a function from $D(A) \subset C$ into E such that $\overline{D(A)} = C$. Then these are equivalent: (1) A is the (strong) generator of a continuous nonlinear semigroup of type ω on E. (2) There is a subset B of $E \times E$ such that $B - \omega I$ is maximal dissipative on C, B has property $(G)^\omega$, $D(B) \cap C = D(A)$, and $B^\circ x = Ax$ for all $x \in D(A)$. (Received September 27, 1971.)

689-B18. LADDIE W. ROLLINS, Georgia Institute of Technology, Atlanta, Georgia 30332. Criteria for discrete spectrum of singular selfadjoint differential operators.

Consider the formally selfadjoint formal differential operator defined by $\tau u(x) = 1/m(x)\sum_{k=0}^{m}(-1)^k D^k p_k(x) D^k u(x)$, $a \leq x < b$, where b may be finite or infinite. The author is concerned with finding conditions on the coefficients, $p_k(x)$, which ensure a discrete spectrum for selfadjoint operators produced by the operation τ. First, the $p_k(x)$ are restricted to make the minimal operator semibounded and hence, have a Friedrichs extension. Then, if at least one $p_k(x)$, $k = 1, 2, \ldots, m$, satisfies certain conditions, the Friedrichs extension will have a compact inverse and, hence, will have a discrete spectrum. (Received September 27, 1971.)

*689-B19. THOMAS G. HALLAM, Florida State University, Tallahassee, Florida 32306. Convergence of solutions of perturbed nonlinear differential equations.

The nonlinear variation of parameters formula is used to investigate the convergence of the solutions of a nonlinear perturbed system of differential equations. The classifications of convergent systems used here are those given by C. Avramescu, Ann. Mat. Pura Appl. 81(1969), 147-168. A basic assumption is that the nonlinear unperturbed system is equi-uniformly convergent in variation. In general, a class of perturbation terms is found which preserve the convergence properties of the unperturbed system of differential equations. (Received September 29, 1971.)

689-B20. CHARLENE V. HUTTON, Louisiana State University, Baton Rouge, Louisiana 70803. On 2-trivial Banach spaces. Preliminary report.

If X is an n-dimensional subspace of a Banach space, E, let $P_E(X) = \inf\{\|P\| : P$ is a projection of E onto $X\}$ and let $\alpha_E(X) = (P_E(X)d(X, \ell^2(n))/\sqrt{n}$. Let $\alpha_n(E) = \inf\{\alpha_E(X) : X$ is an n-dimensional subspace of $E\}$ and $\alpha(E) = \lim\inf_{n\to\infty}\alpha_n(E)$. For any infinite-dimensional E, $0 \leq \alpha(E) \leq 1$. An operator $T : E \to F$ is said to be absolutely p-summing if there is an $M > 0$ such that for every $(x_i)_{i=1}^n \subset E$, $n \geq 1$, $(\sum_{i=1}^n \|Tx_i\|^p)^{1/p} \leq M \sup\{(\sum_{i=1}^n |\langle x_i, f\rangle|^p)^{1/p} : f \in E', \|f\| \leq 1\}$. A pair of Banach spaces E and F is said to be p-trivial if every operator from E to F is absolutely p-summing. Theorem 1. If $\alpha(E) = 0$ then $\langle E, F\rangle$ is not 2-trivial for any Banach space F. If E is sufficiently Euclidean (S.E.) [in sense of Retherford-Stegall] then $\alpha(E) = 0$, in particular $\alpha(\ell^p) = 0$, $1 < p < \infty$. Theorem 2. If X is a complimented subspace of E and $\alpha(X) = 0$ then $\alpha(E) = 0$. In particular, if E is a $\mathcal{L}_p$-space, $1 < p < \infty$, then $\alpha(E) = 0$. Theorem 3. Let $E_0 = E$ and $E_n = (E_{n-1})'$. Then $\alpha(E_0) = \alpha(E_n)$ for every n. In particular $\alpha(C_0) = \alpha(\ell^\infty) = \sqrt{2/\pi}$. (Received September 30, 1971.)

689-B21. STEPHEN A. SAXON and AARON R. TODD, University of Florida, Gainesville, Florida 32601. A property of locally convex Baire spaces. Preliminary report.

A locally convex space (LCS) E is a Baire LCS if it is not the union of an increasing sequence of rare sets. If the sets are required to be absolutely convex, then E is called a Baire-like space. An LCS is an unordered Baire-like space (UBLS) if it is not a countable union of absolutely convex rare sets. Theorem. The following are equivalent: (i) E is an UBLS; (ii) E is not a countable union of translates of absolutely convex rare sets; (iii) if E is a countable union of subspaces $\{F_n\}$, then some F_n is dense and barrelled. One of the

Robertson-Robertson closed-graph theorems uses only the familiar property (iii) of a Baire LCS. While in general products of Baire spaces need not be Baire, we have the Theorem. The unordered Baire-like property is productive. Theorem. If F is a subspace of countable codimension in an UBLS E, then F is an UBLS. Theorem. The closure of the image of an UBLS under an almost open continuous linear map is an UBLS. Corollary. The unordered Baire-like property is divisible. Corollary. The closure of a subspace which is an UBLS is an UBLS. (It is known that the latter three theorems hold for Baire-like spaces, and that every metrizable barrelled space is Baire-like; however, not every normed Baire-like space is an UBLS.) (Received October 1, 1971.)

689-B22. W. L. GOLIGHTLY, Clemson University, Clemson, South Carolina 29631. Linearization of Volterra integral equations. Preliminary report.

Suppose $S = [0, \infty]$, X is the set of $n \times n$ real matrices, f is a function from S to X, k is a function from $S \times S$ to X and g is a function from $S \times X$ to X. Results from the linearization of Stieltjes integral equations are used to study the solutions of (1) $h(t) = f(t) + \int_0^t k(t,s)h(s)ds + \int_0^t k(t,s)g(s,h(s))ds$ under appropriate smoothness conditions. Let H be the space of continuous functions from S to X, M be the set of functions from H to H, V_1 be the function from $S \times S$ to M defined by $V_1(x,y)f_1(t) = \int_y^x k(t,s)f_1(s)ds$ for each x, y, t in S and f_1 in H and G be the function from H to H defined by $(G\circ f_1)(s) = g(s, f_1(s))$ for each s in S and f_1 in H. Then a solution of (1) is given by $h(t) = [U(t)](t)$ where $U(t) = f + (R)\int_t^0 V_1 U + (R)\int_t^0 V_1 GU$. If W_1 is the function from $S \times S$ into M defined by $W_1(x,y)f_1 = {}_x\Pi^y[1 + V_1]f_1$ for each x, y in S and f_1 in H, then $U(t) = W_1(t,0)f + (R)\int_t^0 dW_1[t, \;]GU$. (Received October 4, 1971.)

*689-B23. ROBIN E. TEXTOR, University of South Carolina, Columbia, South Carolina 29208. On a singular Cauchy problem for a nonlinear hyperbolic equation.

In this paper the Cauchy problem for $r^2u^{2\beta}u_x^{2\gamma}u_{xx} - u_{yy} + f(x,y,u,u_x,u_x) = 0$, with initial conditions $u(x,0) = 0$, $u_y(x,0) = \varphi(x)$ prescribed on some finite, closed interval on the x-axis, is considered. An existence theorem for this problem was investigated by S. Singer (to appear in Ann. Mat. Pura Appl.) under certain restrictions. It is shown here that these restrictions are not sufficient and modified conditions are derived. By using Singer's result and a theorem of Lick (Ann. Mat. Pura Appl. 72(1966), 267-274), a uniqueness result is obtained for this problem. Then the existence and uniqueness theorems are used to demonstrate the existence of a unique solution under the conditions $f_p(x,y,u,p,q) = o(y^{\beta+\gamma-1})$ as $y \to 0$ and $\beta\gamma < 1$. (Received October 4, 1971.)

*689-B24. REINHART HITZ, Old Dominion University, Norfolk, Virginia 23508. Necessary and sufficient conditions for convergence of continued fractions.

As an application of the author's theorems relative to polynomials in the continued fraction (1) $1/b_1 + 1/b_2 + 1/b_3 + \dots$ (Abstract 687-40-1, these *Notices* 18(1971), 776) the following are proved: Theorem A. Suppose the partial denominators of the continued fraction (1) have the properties that Σb_{2p-1} is bounded and the series $\Sigma|P_{2p-1,1}b_{2p}|$ is convergent. Then the sequence $\{B_{2n}\}_{n=0}^{\infty}$ of denominators of the approximants is absolutely convergent; $\{B_{2n-1}\}_{n=1}^{\infty}$ is bounded, and the continued fraction diverges. Theorem B. Suppose the partial denominators of (1) have the properties: (i) $\{b_{2p-1}\}$ is bounded and $|\Sigma_{p=1}^{\infty} b_{2p-1}| = \infty$; (ii) each of $\Sigma|b_{2p}|$ and

$\Sigma|P_{2p-1,1}b_{2p}|$ converges; (iii) $b_1 \neq 0$, the terms of $\{b_{2p}\}$ are points of $\mathrm{Re}(u\bar{z}) \geqq 0$ and the terms of $\{b_{2p-1}\}$ are points of $\mathrm{Re}(-uz) \leqq 0$, where u is the common point of the circle $|u| = 1$, $\mathrm{Re}(u) \geqq 0$, and the perpendicular at the origin to $\mathrm{Re}(u\bar{z}) = 0$. Then $\{B_{2p}\}$ is convergent, $|\lim_{n\to\infty} B_{2p-1}| = \infty$, and the continued fraction converges; its value v satisfies the inequality $|v - (u/2 + \bar{b}_1 \mathrm{Re}(\bar{u}b_2))/(|b_1|^2 \mathrm{Re}(\bar{u}b_2) + \mathrm{Re}(ub_1))| \leqq (1/2)/(|b_1|^2 \mathrm{Re}(\bar{u}b_2) + \mathrm{Re}(ub_1))$.
(Received October 4, 1971.)

*689-B25. JOHN M. BAKER, Western Carolina University, Cullowhee, North Carolina 28723. A cross norm close to the greatest cross norm.

Let X and Y be nontrivial normed linear spaces over the field of complex numbers, E the space $\mathcal{L}(X,Y)$ of continuous linear transformations from X into Y equipped with the usual norm, and W a Y-total linear subspace of the dual Y' of Y. If λ is the least cross norm, we may consider the tensor product $X \otimes W$ canonically embedded in $(X' \otimes_\lambda Y)'$ and so obtain a cross norm λ^0 on $X \otimes W$. If $X \otimes W$, on the other hand, is canonically embedded in E', then we obtain another cross norm ψ on $X \otimes W$. A third cross norm γ on $X \otimes W$ is that of the projective topology. We have that $\lambda^0 \leqq \psi \leqq \gamma$. Theorem. (i) On $X \otimes W$, ψ is equivalent to γ whenever Y is W-pseudo-reflexive, and $\psi = \gamma$ whenever Y is W-reflexive. (ii) Taking $W = Y'$, $\lambda^0 = \gamma$ on $X \otimes Y'$ whenever X'' or Y' has the approximation property. (iii) If the canonical injection under the projective topology of $X \otimes W$ into $X \otimes Y'$ is an isometry, then $\lambda^0 = \gamma$ on $X \otimes W$ whenever X'' or Y' has the approximation property. (Received October 4, 1971.)

*689-B26. JOHN WARREN BAKER, Florida State University, Tallahassee, Florida 32306. Uncomplemented $C(X)$-subalgebras of $C(X)$.

Let $C(X)$ denote the Banach algebra of bounded, continuous scalar-valued functions on a topological space X with the supremum norm. Theorem 1. If X contains an open, 0-dimensional, compact metric subspace K with its ωth topological derivative $K^{(\omega)}$ nonempty, then there exists an uncomplemented subalgebra A of $C(X)$ with a multiplicative isometric isomorphism u from $C(X)$ onto A. One consequence of this theorem is the following "uncomplemented" analogue of A. Pełczyński's "complemented $C(S)$-subspace" theorem in his paper, "On $C(S)$-subspaces of separable Banach spaces," [Studia Math. 31(1968)]. Theorem 2. Let S be a compact metric space with $S^{(\omega)}$ nonempty. If a Banach space X contains a subspace Y isomorphic to $C(S)$, then there is a subspace Z of Y such that Z is isomorphic to $C(S)$ and Z is not complemented in X. Moreover, if S is 0-dimensional, then "isomorphic" can be replaced with "isometrically isomorphic" in the preceding statement. Theorem 1 is used to obtain an affirmative solution to a conjecture of A. Pełczyński concerning averaging operators. (Received October 4, 1971.)

*689-B27. THOMAS G. PROCTOR, Clemson University, Clemson, South Carolina 29631. Asymptotically equivalent sets of Stieltjes integral equations. Preliminary report.

Let $S = (0,\infty)$, $(G, |\ |)$ be a complete normed abelian group and denote all functions from G to G by H. A function $V: S \times S \to H$ is order additive providing $V(x,y) + V(y,z) = V(x,z)$ whenever y is between x and z. Let V_1, V_2 be order additive functions such that V_1 has its values in the group homomorphisms on G and there are order additive functions α_1, α_2 from $S \times S$ to the nonnegative numbers such that $|V_i(x,y)p - V_i(x,y)q| \leqq$

$\alpha_1(x,y)|p-q|$, $|V_2(x,y)\ 0| \leqq \alpha_2(x,y)$ for each (x,y) in $S \times S$, (p,q) in $G \times G$ and $i = 1,2$. The set of integral equations $h(t) = p + (R)\int_t^0 (V_1 + V_2)h$, p in G and the set of integral equations $k(t) = p^* + (R)\int_t^0 V_1 k$, p^* in G are called asymptotically equivalent if given any p in G there is a p^* in G so that the solutions of the corresponding equations satisfy $k(t) - h(t) \to 0$ as $t \to \infty$ and the analogous converse statement holds. Conditions are specified which guarantee asymptotic equivalence. (Received October 4, 1971.)

*689-B28. JAMES A. RENEKE, Clemson University, Clemson, South Carolina 29631. Product integral solutions for hereditary systems.

$\{X, |\cdot|\}$ is a Banach space, S is a subinterval of $[0,\infty)$ containing 0, S' is a subinterval of $(-\infty,\infty)$ containing S, and G is a linear space of functions from S' into X. Where appropriate, 0 will denote the real number, the zero of X, or the zero of G. N is a function from S into the pseudonorms on G such that (i) if (w,f) is in $G \times G$ then $N_w(f) = 0$ iff $f(u) = 0$ for all $u \leqq w$, (ii) if $u \leqq w$ then $N_u(f) \leqq N_w(f)$ for each f in G, and (iii) $\{G, N\}$ is complete. H is the set of all operators on G and K is a function from $S' \times S'$ into H such that (i) if f is in G, $[u,v]$ is a subinterval of S', t is in S', and $t \leqq u$, then $[K(u,v)f](t) = 0$, (ii) if $\{u,v,w\}$ is a monotone sequence in S' then $K(u,v) + K(v,w) = K(u,w)$, and (iii) there is a function k from S into the class of nondecreasing functions from S to the numbers such that, for each w in S, $N_w(K(u,v)0) \leqq k_w(v) - k_w(u)$ and $N_w(K(u,v)f - K(u,v)g) \leqq (L)\int_u^v N_x(f-g)dk_w(x)$, for each (f,g) in $G \times G$ and subinterval $[u,v]$ of $[0,w]$. The equation $h(u) = f(u) + [K(0,u)h](u)$ is solved for h in terms of product integrals. (Received October 4, 1971.)

*689-B29. J. P. HOLMES, University of Florida, Gainesville, Florida 32601. Differentiable power associative partial groupoids with identity.

Suppose H is a Banach space, D is an open set of H containing 0, and V is a function from $D \times D$ to H satisfying $V(x,0) = V(0,x) = x$ for each x in D. If x is in D and n is an integer greater than 1 let x^n be the product of n x's associated as follows whenever the product exists. $x^n = V(x, V(x, \ldots, V(x,x)\ldots))$. Let $x^0 = 0$ and $x^1 = x$. Suppose that V is power associative. That is for each x in D and for each pair (m,n) of nonnegative integers for which x^{n+m} exists we have $V(x^m, x^n) = x^{n+m}$. Theorem. If H, D, and V are as above and V is continuously differentiable in the sense of Fréchet on $D \times D$ then there is a positive number d so that for each x in H with $\|x\| < d$ there is a unique continuous function T_x from $[0,1]$ to H satisfying $T_x(0) = 0$, $T_x(1) = x$, and $V(T_x(s), T_x(d)) = T_x(s+t)$ whenever each of s, t and $s+t$ is in $[0,1]$. This is similar to a result of G. Birkhoff assuming V is associative in the paper "Analytical groups", Trans. Amer. Math. Soc. 43(1938), 61-101. There are partial groupoids satisfying the hypotheses of our theorem which are not associative. (Received October 4, 1971.)

689-B30. C. GARRETT, Stillman College, Tuscaloosa, Alabama 35401. Almost continuous functions. Preliminary report.

The subset D of the plane is said to have property C if and only if D is an open set and, for each point x in the X-projection of D, the subset of D with X-projection x is a vertical open interval. The statement that D is a C-set means that D is an open subset of the plane such that some open subset D' of D has property C and X-projection the same as the X-projection of D. Suppose f is a real function with domain the real line and range a bounded subset of the real line. The function f is said to be almost continuous whenever each open subset of the plane containing the graph of f contains the graph of a continuous function with the same domain as f. Theorem. In order that the function f be almost continuous, it is necessary and sufficient that each open set containing the graph of f be a C-set. (Received October 4, 1971.)

*689-B31. LEE W. JOHNSON and R. D. RIESS, Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061. A procedure for estimating $E_n(f)$.

Let F be the set of real-valued functions on $[-1,1]$ which are infinitely differentiable and satisfy $|f^{(i)}(0)| \leqq M$ for all i. A procedure is given for estimating the degree of approximation, $E_n(f)$, of a function f in F by polynomials of degree n or less. For a large subset of F, $E_n(f)$ is shown to be equal, up to terms of the form $O(1/n)/2^n(n+1)!$, to the error in interpolating f at the roots of the Chebyshev polynomial of the first kind of degree n + 1. (Received October 4, 1971.)

Applied Mathematics

*689-C1. WILLIAM R. EDWARDS, JR. and ZAMIR BAVEL, Department of Computer Science, University of Kansas, Lawrence, Kansas 66044. The left discrimination sequence of an automaton. Preliminary report.

Left discrimination of a semigroup is defined and shown to be a sufficient condition that a semigroup be isomorphic to the input semigroup of its semigroup automaton, a necessary condition if the semigroup is finite. The left discrimination sequence of an automaton is defined as a sequence of semigroups beginning with the input semigroup of the semigroup automaton of its predecessor. It is related directly to a particular monotonically decreasing sequence of subautomata of the original automaton. This sequence is shown to be preserved by homomorphisms and is extended and used in an algorithm for determining the homomorphisms on one finite automaton to another. (Received September 2, 1971.)

*689-C2. WILLIAM J. KAMMERER, Georgia Institute of Technology, Atlanta, Georgia 30332 and G. W. REDDIEN, JR., Vanderbilt University, Nashville, Tennessee 37203. Local convergence of smooth cubic spline interpolates.

In this paper we develop local error bounds for smooth cubic spline interpolates of a function f which depends only on the local smoothness of f. A typical result is the following: let $a \leqq a' < \alpha < \beta < b' \leqq b$ and let f be a bounded function defined on $[a,b]$ with $f \in C^k[a',b']$ and $0 \leqq k \leqq 4$. If for $\Delta : a = x_0 < x_1 < \ldots < x_N = b$, a uniform partition of $[a,b]$, s is the unique cubic spline satisfying $s(x_i) = f(x_i)$, $0 \leqq i \leqq N$, and $Ds(x_i) = 0$ for

$i = 0, N$, then there exists a constant K independent of Δ for which $\|D^j(f-s)\|_{L_\infty[\alpha,\beta]} \leqq K|\Delta|^{k-j}$, $0 \leqq j \leqq k$, where $|\Delta| = (b-a)/N$. The proof uses a global result of Swartz and Varga [LA-DC-11694, Los Alamos Scientific Laboratory (1970)]. (Received September 20, 1971.)

689-C3. JOHN P. SHEEHAN and LOKENATH DEBNATH, East Carolina University, Greenville, North Carolina 27834. On the dynamic response of an infinite Bernoulli-Euler beam.

This paper presents a theory of the transient Bernoulli-Euler beam problem on an elastic foundation which takes into account the effects of axial load and linear damping. An analytical solution of the steady state and the transient components has been obtained due to physically realistic load distributions. With a view to extend its practical applicability, the characteristic features of the solution are explored. Several limiting situations are investigated as special cases. It is shown that the steady state vibration can be achieved as the limit of the solution of the transient problem. (Received September 29, 1971.)

Geometry

689-D1. PHILLIP W. BEAN, Auburn University, Auburn, Alabama 36830. Concerning property $R[k, f_k(n)]$. Preliminary report.

For basic notation and definitions see a paper by J. R. Calder (J. London Math. Soc. (2) 3(1971), 422-428). Suppose T is an interval convexity on S; k is an integer and $k \geqq 2$; and f_k is a function from $I^+ - \{1\}$ into I^+ such that $f_k(2) = k+1$. If n is a positive integer and $n \geqq 2$, the statement that C(T) has property $R[k, f_k(n)]$ means that if M is a point set containing at least $f_k(n)$ points, then there exist n mutually exclusive subsets, $H_1, H_2, \ldots, H_n$, of M such that $M = \bigcup_{i=1}^n H_i$ and $\bigcap_{i=1}^n \mathrm{Co}(H_i)$ exists. Property $R[k, f_k(2)]$ is denoted by R(k). If C(T) has property R(k) for some positive integer $k \geqq 2$, the smallest positive integer, k_0, such that C(T) has property $R(k_0)$ is called the Radon number of C(T). Tverberg (J. London Math. Soc. 41(1966), 123-128) has shown that if F_0 is the collection of all convex sets in R^{k-1}, then F_0 has property $R[k, f_k(n)]$ where $f_k(n) = (n-1)k+1$ for $n, k \geqq 2$. Theorem. Suppose T is an interval convexity on S such that if M is in P(S), then $\mathrm{Co}(M) = \bigcup_{m \text{ in } M} T(m,m)$. If C(T) has property R(k), then C(T) has property $R[k, f_k(n)]$ where $f_k(n) = (n-1)k+1$ for $n, k \geqq 2$. Theorem. Suppose $\leqq$ is a partial order on S such that $C(\leqq)$ has Radon number 3. If M is a point set containing exactly $4n-6$ points, $n > 2$, then there exist n mutually exclusive subsets, $H_1, H_2, \ldots, H_n$, of M such that $M = \bigcup_{i=1}^n H_i$ and $\bigcap_{i=1}^n \mathrm{Co}(H_i)$ exists. Theorem. Suppose $\leqq$ is a partial order on S and if $n, k \geqq 2$, $f_k(n) = (2n-3)k+1$. If $C(\leqq)$ has property R(k), then $C(\leqq)$ has property $R[k, f_k(n)]$. (Received September 30, 1971.)

Statistics and Probability

*689-F1. WILLIAM F. GRAMS and R. J. SERFLING, Department of Statistics, Florida State University, Tallahassee, Florida 32306. Convergence rates for U-statistics and related statistics. Preliminary report.

Rates of convergence are provided in the central limit theorem and the strong law of large numbers for U-statistics. The results are obtained by establishing suitable bounds upon the moments of the difference between a U-statistic and its projection. Analogous conclusions for the associated von Mises statistical functions are indicated. Statistics considered for exemplification are the sample variance and the Wilcoxon two-sample statistic. (Received September 20, 1971.)

689-F2. HARVEY J. IGLARSH, Georgia Institute of Technology, Atlanta, Georgia 30332. Regularity for the stopped Wiener process.

Let $\{W(t) : t \geqq 0\}$ be a one dimensional Wiener process and let T denote the first exit time from the open set $V \subset \mathbb{R}$. For a bounded measurable $f : V \to \mathbb{R}$ define $q_t f(x) = E_x[f(W(t))\chi_{\{T>t\}}]$ and $r_t^n f(x) = E_x[f(W(t))\chi_{\{W((k/n)t) \in V, k=1,\dots,n\}}]$. Let V be either $(0,\infty)$ or $(-\beta,\beta)$, for $\beta > 0$, and set $t > 0$. Theorem. $r_t^n f \to q_t f$ uniformly as $n \to \infty$ on any subset of V bounded away from ∂V and $(d/dx)(r_t^n f) \to (d/dx)(q_t f)$ pointwise on V. Remark. Let V denote an open ball in an abstract Wiener space. Then $q_t f$ and $r_t^n f$ can be defined as above in the infinite dimensional setting. For a certain class of abstract Wiener spaces we can prove a result analogous to the above theorem where the derivatives will be Frechet derivatives in H-directions. [See L. Gross, J. Functional Analysis 1(1967), 123-181, for the definition of this type of derivative.] (Received September 27, 1971.)

Topology

689-G1. ALEXANDER D. WALLACE, University of Florida, Gainesville, Florida 32601. Skewly associative groupoids.

These are defined by the equation $(ab)c = b(ca)$, which implies the equation $(pq)(rs) = (qp)(sr)$, related to mediality. If M is one such then M^2 (exponentiation is unambiguous) is associative, and $M^4 \subset \text{cnt } M$. If tws M denotes all those $x \in M$ such that $x(ab) = x(ba)$ for all $a, b \in M$ (and dually) if aso M is the associator of M (termed nucleus by Bruck) and if E is the set of idempotents then $EM = ME \subset \text{tws } M \cap \text{cnt } M \cap \text{aso } M$, for example. Any compact groupoid has a minimal closed ideal (though not necessarily a minimal ideal) but in this case such exists and is the minimal ideal of M^2. The equivalence defined by the equation $x^2 = y^2$ is a congruence and the quotient is an abelian semigroup. The equivalence defined by $tx = ty$ (for all $t \in M$) is also a congruence. A variety of results concerning the ideal structure of M may be proved, exempli gratia, if M is connected and if J is an ideal of M then one and only one component of J is an ideal. (Received January 18, 1971.)

*689-G2. T. BRUCE McLEAN, University of Kentucky, Lexington, Kentucky 40506. Confluent images of tree-like spaces.

A space X is a curve if it is a one dimensional compact connected metric space. A curve X is a tree if it is homeomorphic to a simply connected one dimensional polyhedron and is tree-like if for each $e > 0$, there exists a tree Y and a map $f: X \to Y$, such that if $y \in Y$, $\operatorname{diam}[f^{-1}(y)] < e$. A map $f: X \to Y$ of a curve onto a curve is confluent provided if A is a subcontinuum of Y and B is a component of $f^{-1}[A]$, then $f[B] = A$. Theorem. If $f: X \to Y$ is a confluent map of a tree-like curve onto a curve, then Y is tree-like. (This answers a question raised by A. Lelek in the affirmative.) Corollary. If $f: X \to Y$ is an open map of a tree-like curve onto a curve then Y is tree-like. (Received May 24, 1971.)

*689-G3. GEORGE MICHAEL REED, Ohio University, Athens, Ohio 45701. On screenability and metrizability of Moore spaces.

A space is said to be star-screenable (strongly star-screenable) if and only if for each open covering G of S there exists a σ-pairwise disjoint (σ-discrete) open covering H of S which refines $\{st(x, G) | x \in S\}$. The following main results are obtained: (1) A space S is screenable (strongly screenable) if and only if each open covering of S has a point countable refinement which covers S and S is star-screenable (strongly star-screenable). (2) A Moore space S in which there does not exist an uncountable collection of pairwise disjoint open sets is separable if and only if it is star-screenable. (3) There exists a nonnormal, nonseparable Moore space which is strongly star-screenable. (4) Each normal, star-screenable Moore space is strongly star-screenable. (5) It is shown that normality can be replaced by strong star-screenability in certain metrization theorems concerning subspaces of Moore spaces. (Received June 23, 1971.)

*689-G4. R. O. FULP, North Carolina State University, Raleigh, North Carolina 27607. Splitting locally compact abelian groups. Preliminary report.

Let $\mathcal{L}$ denote the category of locally compact abelian groups with continuous homomorphisms as morphisms. An extension $A \overset{\alpha}{\rightarrowtail} B \overset{\beta}{\twoheadrightarrow} C$ is, by definition, a short exact sequence of morphisms of $\mathcal{L}$ such that $\alpha: A \to \alpha(A)$ is a topological isomorphism onto a closed subgroup of B and the induced map $\beta^*: B/\operatorname{Ker}\beta \to C$ is a topological isomorphism onto C. We show that a group G in $\mathcal{L}$ splits out of every X in $\mathcal{L}$ for which $G \subseteq X$ and X/G is torsion iff G is divisible (independent of the topology of G). Also G splits out of each X in $\mathcal{L}$ for which $G = X/Y$ for some torsion-free Y iff $G = \mathbb{R}^m \oplus (\oplus_\sigma Z)$ for some cardinal σ. We prove that G splits out of each X in $\mathcal{L}$ for which $G = X/Y$ for some torsion Y iff $G = \mathbb{R}^m \oplus \Pi_\lambda J_\lambda \oplus \Pi_\mu \Sigma_a \oplus (\oplus_\sigma Z)$ where J_λ is some p-adic group and Σ_a is the solenoidal group. Finally we characterize those G in $\mathcal{L}$ such that G splits out of each X in $\mathcal{L}$ for which $G \subseteq X$ and X/G is torsion free. (Received July 23, 1971.)

689-G5. CHARLES E. AULL, Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061. Quasi-developable and weak σ-spaces. Preliminary report.

Definition. A topological space X is a weak σ-space if there is a σ-disjoint network for the space such that each disjoint family is discrete w.r.t. some open subspace of X containing each member of the family. It is proved that every quasi-developable space is a weak σ-space and a weak σ-space with a point countable base is quasi-developable. Definition. A family of sets is a k family if it is pairwise disjoint for $k = 0$, relatively locally finite for $k = 1$ (locally finite in the union of the family), point finite for $k = 2$, point countable for $k = 3$. A space is CN - k if for every discrete family $\{D_a\}$, there is an open k family $\{G_a\}$ such that $D_a \subset G_a$ and $D_a \cap G_b = \emptyset$ for $a \neq b$. Theorem. A quasi-developable hereditarily CN - k space has a σ - k base for k = 0, 1, 2, 3. A quasi-developable space has a σ - k base if and only if for every subspace every open cover has a σ - k open refinement for k = 0, 1, 2, 3. (Received July 30, 1971.)

*689-G6. DOUGLAS MOREMAN, Auburn University, Auburn, Alabama 36830. A generalization to metric spaces of the notion of weak convergence.

A generalization to the context of a metric space S is developed containing the following definitions and theorems: Definitions. A point set M is said to be (1) spherically convex if for each two points A and B of M the common part of all spherical disks containing A and B is a subset of M, (2) spherically convexly perfectly compact if for each monotonic collection H of bounded and spherically convex subsets of M some point belongs to every point set that is the closure of a set in H, (3) spherically convexly closed if for each point P not in M there exists a spherically convex point set that contains all but finitely many points of M and whose closure does not contain P. Theorems. (1) If S is spherically convexly perfectly compact, the point set M is spherically convexly closed and P is a point not in M then there exists a point Q of M such that no point of M is nearer to P than is Q. (2) If M is an infinite point set and K is the set of all centers of M (W. P. Coleman, Abstract 71T-B191, these *Notices* 18(1971), 817) then K is spherically convex. (3) If S is spherically convexly perfectly compact then every infinite and bounded point set has a center. (4) No infinite point set has two centers if and only if no infinite and spherically convexly closed point set has two centers. (Received August 6, 1971.)

*689-G7. PAUL F. DUVALL, JR., Oklahoma State University, Stillwater, Oklahoma 74074 and LAWRENCE S. HUSCH, University of Tennessee, Knoxville, Tennessee 37916. Homeomorphisms of 3-manifolds which fail to be regular on 1-dimensional polyhedron.

A homeomorphism h of a metric space X is regular (positively regular) at x if for each $\epsilon > 0$ there exists $\delta > 0$ such that $d(x,y) < \delta$ implies $d(h^i(x), h^i(y)) < \epsilon$ for all i ($i > 0$). Let h be a homeomorphism of an open 3-manifold M onto itself such that h is positively regular everywhere and is regular everywhere except on the fixed point set P which is a compact 1-dimensional polyhedron topologically embedded in M. Lemma. $f(x) = \lim_{i\to+\infty} h^i(x)$ defines a homotopy equivalence of M onto P. A locally flat surface $N \subseteq M$ is a cross-section for h if (i) N separates M so that P lies in the bounded component of M - N; (ii) $N \cap h(N) = \emptyset$; (iii) for each $p \in P$, $f^{-1}(p) \cap N$ is connected. It is easy to find N which satisfies (i) and (ii) but it is unknown to the author whether

(iii) can be satisfied. Theorem 1. If there exists a cross-section N for h, then P is tamely embedded in M. Theorem 2. There exists a homeomorphism h of the 3-sphere S^3 and an arc $A \subseteq S^3$ such that h is regular on $S^3 - A$ and A is wildly embedded in S^3. The authors have an example to show that Theorem 1 cannot be extended to higher dimensional M's. (Received August 5, 1971.)

*689-G8. ROBERT W. HEATH and DAVID J. LUTZER, University of Pittsburgh, Pittsburgh, Pennsylvania 15213. A characterization of monotone normality. Preliminary report.

In [Abstract 679-G2, these *Notices* 17(1970), 1034] P. Zenor introduced the concepts of monotone normality and complete monotone normality. In this note the authors prove that for a T_1-space X, the following are equivalent: (a) X is completely monotonically normal; (b) X is monotonically normal; (c) there is a function G which associates with each pair (p, C), where $C \subset X$ is closed and $p \in X \setminus C$, and open set G(p, C) such that (i) $p \in G(p, C) \subset X \setminus C$; (ii) if $D \subset C$ are closed and if $p \notin C$ then $G(p, C) \subset G(p, D)$; (iii) if $p \neq q$ are points of X then $G(p, \{q\}) \cap G(q, \{p\}) = \emptyset$. When combined with Zenor's results [op. cit.] (which assume complete monotone normality) the above characterization yields: Let X be a T_1-space. The following are equivalent: (a) X is stratifiable; (b) $X \times M$ is monotonically normal for every metric space M; (c) $X \times I$ is monotonically normal, where I is the usual unit interval; (d) the product of countably many copies of X is monotonically normal. One can also conclude that there is no nondegenerate T_1-space Z such that the product of $\aleph_1$ copies of Z is monotonically normal. (Received August 17, 1971.)

*689-G9. PHILLIP L. ZENOR, Auburn University, Auburn, Alabama 36830. Certain subsets of products of metacompact spaces are realcompact.

The normal space X can be embedded as a closed subset in the product of a collection of copies of the real line if and only if (i) the cardinality of each discrete subset of X is nonmeasurable and (ii) X can be embedded as a closed subset in the product of a collection of metacompact T_3-spaces. (Received August 20, 1971.)

689-G10. KENNETH R. VAN DOREN, Auburn University, Auburn, Alabama 36830. Inverse limits and closed mappings. Preliminary report.

This paper concerns the question: under what conditions on the bonding maps for an inverse sequence of closed, continuous images of metric spaces is the inverse limit space a closed, continuous image of a metric space? It is shown that it is not sufficient that the bonding maps be closed. A necessary and sufficient condition for a certain "natural" mapping of a metric space onto the inverse limit space to be closed is established: the inverse limit space must be a Frechet space and a certain decomposition of the metric space must be upper semicontinuous. A condition on the bonding maps which is sufficient for the inverse limit space to be a Frechet space is established, and an example is given to show that the inverse limit space is not necessarily a Frechet space if this condition is violated. A necessary and sufficient condition for the decomposition to be upper semicontinuous is also established. (Received September 9, 1971.)

*689-G11. FRANK G. SLAUGHTER, JR., University of Pittsburgh, Pittsburgh, Pennsylvania 15213. The closed continuous image of a metrizable space is M_1.

J. Ceder ("Some generalizations of metric spaces," Pacific J. Math. 11(1961), 105-126) introduced the notions of M_1 space (a regular space with σ-closure preserving base) and stratifiable space as natural generalizations of Nagata and Smirnov's conditions for the metrizability of a regular space. Even though a topological space Y which is the image of a metrizable space under a closed, continuous mapping need not be metrizable, we show as the major result of this paper that Y will have a σ-closure preserving base. It follows that one can not obtain an example of a stratifiable space which is not M_1 by constructing a quotient space from an upper-semicontinuous decomposition of a metric space. In the course of establishing our major result, we also obtain conditions under which the image of certain collections of sets under a closed, continuous mapping will be closure preserving. (Received September 9, 1971.)

*689-G12. ROBERT A. McCOY, Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061. Homeomorphism groups of Hilbert cube manifolds.

Whittaker showed in [Ann. of Math. (2) 78(1963), 74-91] that if M and N are either both (finite-dimensional) manifolds without boundary or both compact (finite-dimensional) manifolds, then M is homeomorphic to N if and only if H(M) is isomorphic to H(N), where $H(\underline{X})$ denotes the group of homeomorphisms from the space $\underline{X}$ onto itself under the group operation of composition. The techniques used by Whittaker can be modified to show that if M and N are Hilbert cube manifolds (i.e., regular spaces such that each point has an open neighborhood which is homeomorphic to some open subset of Hilbert cube), then M is homeomorphic to N if and only if H(M) is isomorphic to H(N). (Received September 23, 1971.)

689-G13. DARRELL C. KENT, Washington State University, Pullman, Washington 99163 and GARY D. RICHARDSON, East Carolina University, Greenville, North Carolina 27834. Regular compactifications of convergence spaces.

J. F. Ramaley and O. Wyler ["Cauchy spaces. II", Math. Ann. 187(1970), 187-199] have shown that the compact regular Hausdorff convergence spaces is a reflective subcategory of all convergence spaces. We obtain the following two results (see A. M. Carstens and D. C. Kent, "A note on products of convergence spaces," Math. Ann. 182(1969), 40-44, for definitions): Theorem 1. A regular convergence space has a regular compactification (embedding sense) iff (1) it is almost pretopological and (2) its pretopological modification is a completely regular topological space. Theorem 2. A regular convergence space has a regular Stone-Čech compactification (embedding sense) iff (1) and (2) above are satisfied. (Received September 27, 1971.)

*689-G14. GEORGE L. CAIN, JR., Georgia Institute of Technology, Atlanta, Georgia 30332. Metrizable mapping compactifications.

Suppose $f: X \to f(X) = Y$ is a continuous function from one completely regular T_1 space onto another. There is associated with each possible compactification $\widetilde{X}$ of the domain space X a compactification of the mapping f; this uniquely defined mapping compactification of f is called the compactification determined by $\tilde{X}$. The major result of this paper is that if $\tilde{X}$ is metrizable, then the domain of the mapping compactification determined by it is also metrizable if and only if the range Y is. (Received September 27, 1971.)

*689-G15. CHARLES D. BASS, Pembroke State University, Pembroke, North Carolina 28372. Some embeddings of a disk in E^3 which support a squeezing map.

The problem of squeezing a disk to an arc in Euclidean 3-space is investigated. Let Δ_2 denote the unit disk in the plane; Δ_1 the set $\{(x,0) \in \Delta_2 | -1 \leq x \leq 1\}$; π the projection map of Δ_2 onto Δ_1 defined by $\pi((x,y)) = (x,0)$. An embedding g of Δ_2 into E^3 supports a squeezing map if there exists a map f of E^3 onto itself such that (i) $f|E^3 - g(\Delta_2)$ is a homeomorphism onto $E^3 - fg(\Delta_2)$, (ii) $fg(\Delta_2)$ is an arc, and (iii) $fg\pi^{-1}$ is a homeomorphism of Δ_1 onto $fg(\Delta_2)$. Some embeddings which support a squeezing map are specified as follows. Theorem. Let g be a homeomorphism of Δ_2 onto a disk D which lies in the boundary of a 3-cell C in E^3. Suppose there exists an F_σ set F in Bd C such that (i) Ext $C \cup F$ is 1-ULC and (ii) for each point p of Δ_1, $g\pi^{-1}(p)$ contains at most one point of F. Then g supports a squeezing map. Corollary. Every homeomorphism of Δ_2 onto a subdisk of Bing's 2-sphere supports a squeezing map. Similar results, announced in Abstract 71T-G51, these *Notices* 18(1971), follow as corollaries. (Received September 27, 1971.)

689-G16. ROBERT C. ESLINGER, King College, Bristol, Tennessee 37620. An implicit function theorem with applications to locally Banach semigroups with identity. Preliminary report.

Let B be a Banach space. A mapping h defined on a subset of $B \times B$ satisfies property (P) at (x_0, y_0) provided there is a one-one mapping A from a subset of B into B and positive numbers r, M, and c so that (i) if $\|x\| \leq r$, $\|A^{-1}(x)\| \leq M\|x\|$, (ii) $c < 1/M$, and (iii) if $\|x - x_0\|$, $\|y - y_0\|$, $\|z - y_0\| < r$, then $\|h(x,y) - h(x,z) - A(y-z)\| \leq c\|y - z\|$. Theorem. Suppose each of D and G is an open subset of B, h is a continuous mapping from $D \times G$ into B which satisfies property (P) at (x_0, y_0), and $h(x_0, y_0) = 0$. There is an open subset D' of D containing x_0 and a unique continuous mapping u from D' into G so that $u(x_0) = y_0$ and $h(x, u(x)) = 0$ for each x in D'. Let S be a topological semigroup with identity e. Suppose G is an open subset of S containing e and g is a homeomorphism from G onto an open subset of B. Define mappings h and k on the appropriate subsets of $g[G] \times g[G]$ by $h(x,y) = g[g^{-1}(x)g^{-1}(y)]$ and $k(x,y) = h(y,x)$. Theorem. Suppose each of h and k satisfies property (P) at $(g(e), g(e))$. There is an open subset G' of G containing e each member of which has an inverse and the inverse operation on G' is continuous. (Received September 29, 1971.)

*689-G17. HAROLD R. BENNETT, Texas Tech University, Lubbock, Texas 79409 and DAVID J. LUTZER, University of Pittsburgh, Pittsburgh, Pennsylvania 15213. Another note on weak θ-refinability. Preliminary report.

In [Abstract 71T-G111, these *Notices* 18(1971), 677] the authors introduced the notion of weak θ-refinability and announced some of the basic properties of spaces having that property. More recently we have been able to prove that the class of quasi-developable spaces (in the sense of Bennett) coincides with the class of spaces having a θ-base (in the sense of Worrell and Wicke) by combining a previously announced result (Bennett and Lutzer, ibid.) with the proposition that any quasi-developable space is (hereditarily) weakly θ-refinable. The authors wish to thank D. K. Burke for some illuminating remarks on this matter. (Received September 29, 1971.)

*689-G18. PETER W. HARLEY III, University of South Carolina, Columbia, South Carolina 29208. On countably paracompact spaces and closed maps.

The following results are proved. Theorem 1. Let X be T_1 and countably paracompact, Y a q-space, and $f: X \rightarrow Y$ a continuous closed surjection. Then $\partial f^{-1}(y)$ is countably compact for each $y \in Y$. Corollary 1. Y is countably paracompact. Theorem 2. A countably compact topological space Y is metrizable if and only if there exists a countably paracompact, symmetrizable, Hausdorff space X and a continuous closed surjection $f: X \rightarrow Y$. E. Michael ("A note on closed maps and compact sets," Israel J. Math.2(1964), 173-176) proved Theorem 1 with countable paracompactness replaced by normality. Corollary 1 generalizes a theorem of P. Zenor ("On countable paracompactness and normality," Prace Mat. 13(1969), 23-32). Theorem 2 generalizes a result of A. V. Arhangelskii and Z. Frolik ("Mappings and spaces," Russian Math. Surveys 21(1966), 115-162). (Received September 30, 1971.)

689-G19. RONALD L. CARLISLE, Kennesaw Junior College, Marietta, Georgia 30060. Monotone maps and ϵ-maps between graphs. Preliminary report.

A graph is defined to be a continuum homeomorphic to the space of a one-dimensional simplicial complex. As the paper's main result, we show that if each of G_1 and G_2 is a graph then there is an ϵ-map from G_1 onto G_2, for all $\epsilon > 0$, if and only if there is a monotone map from G_2 onto G_1. To that end, we prove that if each of G_1 and G_2 is a graph then there is a positive number δ such that the following statements are equivalent: (1) There is a δ-map of G_1 onto G_2. (2) If $\epsilon > 0$, there is a piecewise-linear ϵ-map of G_1 onto G_2. (3) There is a monotone map from G_2 onto G_1. (4) There is a piecewise-linear monotone map of G_2 onto G_1 such that at most finitely many points of G_1 have nondegenerate preimages. Examples are given which demonstrate that this relationship between monotone maps and ϵ-maps is not true of certain other classes of spaces. (Received September 30, 1971.)

689-G20. JACK W. ROGERS, JR., Emory University, Atlanta, Georgia 30322. Inverse limits on graphs and monotone mappings.

In 1935, Knaster gave an example of an irreducible continuum (i.e. compact connected metric space) K which can be mapped onto an arc so that each point-preimage is an arc. The continuum K is chainable (or arc-like). In this paper it is shown that every one-dimensional continuum M is a continuous image, with arcs as point-preimages, of some one-dimensional continuum M'. Moreover, if M is G-like, for some collection G of graphs, then M' can be chosen to be G-like. A corollary is that every chainable continuum is a continuous image, with arcs as point-inverses, of a chainable (and hence, by a theorem of Bing, planar) continuum. These investigations give rise to the study of certain special types of inverse limit sequences on graphs. (Received October 1, 1971.)

689-G21. JAMES C. KROPA, Judson College, Marion, Alabama 36756. Cancellative topological semigroups on a manifold. Preliminary report.

Let S be a manifold with or without boundary which is a topological semigroup with identity e. Mostert and Shields in [Trans. Amer. Math. Soc. 91(1959), 380-389] essentially ask: if S does not have a boundary and e is the only idempotent, is S a group? Theorem 1. If S is cancellative and e is not a boundary point, then S is a group and so does not have a boundary. Theorem 2. If S is cancellative and e is a boundary point, then the set of elements with inverses is contained in the boundary. (Received October 1, 1971.)

689-G22. ROBERT J. DAVERMAN, University of Tennessee, Knoxville, Tennessee 37916. On the absence of tame disks in certain wild cells. Preliminary report.

Theorem. For $3 \leqq k < n$ there exists a k-cell B in Euclidean n-space E^n such that no 2-cell in B is cellular relative to E^n. Corollary. Each polyhedron P of dimension at least 2 topologically embedded in B is wildly embedded in E^n. Theorem. For $n \geqq 4$ there exists an n-cell B in E^n such that no 2-cell in Bd B is cellular relative to E^n. (Received October 4, 1971.)

689-G23. BEN FITZPATRICK, JR., Auburn University, Auburn, Alabama 36830. Concerning connectedness im kleinen. Preliminary report.

A. Lelek calls a compact metric continuum X Souslinean provided each collection of mutually exclusive nondegenerate subcontinua of X is countable. Every Souslinean continuum is connected im kleinen at each point of a dense inner limiting subset. If M is a metrizable compact space then there is a compact metric continuum X which contains a topological copy N of M such that N is the set of all points of X at which X is connected im kleinen. (Received October 4, 1971.)

*689-G24. LEE H. MINOR, Western Carolina University, Cullowhee, North Carolina 28723. The relationship of pseudo-expansiveness, expansiveness, and recurrence in transformation groups.

Let (X, d) be a metric space. A map $f: X \to X$ is pseudo-expansive on (X, d) if there exists $c > 0$ such that $f^n \neq f^m$ implies $d(f^n(x), f^m(x)) > c$ for some $x \in X$ (Gerald Jungck, "Periodicity via equicontinuity," Amer. Math. Monthly 75(1968), 265-267). Each expansive homeomorphism is pseudo-expansive, and the author has shown that a surjective nonperiodic map is pseudo-expansive if and only if it is not recurrent. Examples are given to show that similar statements are not true in the setting of transformation groups. (Received October 4, 1971.)

*689-G25. ROY J. DAIGLE, University of Georgia, Athens, Georgia 30601. Complements of minimal surfaces. Preliminary report.

Two minimal surfaces for a knot are different if their complements in S^3 are not homeomorphic. Theorem. Given positive integers n and m there is a knot K(n, m) which has at least 2^m different minimal surfaces and which has Alexander Polynomial $n(n+1) - (1 + 2n(n+1))t + n(n+1)t^2$. The case $n = 1$, $m = 1$ was first established by W. R. Alford in [Ann. of Math. (2) 91(1970), 419-424]; the cases n = 1, m arbitrary were established by W. R. Alford and C. B. Schaufele in ["Topology of manifolds," Markham, Chicago, 1970, pp. 87-96]. (Received October 4, 1971.)

689-G26. E. E. HARGROVE, University of Alabama, University, Alabama 35486. Properties of local Darboux functions. Preliminary report.

Bruckner and Ceder [Jber. Deutsch. Math.-Verein. 67(1965), 93-117] have defined a Darboux mapping. If f is a mapping and G is an open cover of its domain, then f is locally Darboux with respect to G provided that for each element U of G, the restriction of f to U is Darboux. The mapping f is a local Darboux map on its domain X provided there exists an open cover G of X such that f is locally Darboux with respect to G. Theorem. If f is a function with domain X then f is Darboux if and only if f is locally Darboux with respect to each open cover of X. Theorem. If X is a hereditarily locally connected space, then each local Darboux function on X is Darboux. Conditions under which a function is continuous whenever its inverse mapping is local Darboux are also obtained. (Received October 4, 1971.)

689-G27. HARVEY ROSEN, University of Alabama, University, Alabama 35486. Shrinking wild cellular subsets of 2-spheres in S^3. Preliminary report.

Let S be a 2-sphere in S^3 whose set of wild points is an arc A that is cellular in S^3. It is examined whether or not for each $\epsilon > 0$ there is a $\delta > 0$ such that every unknotted simple closed curve in $N(A, \delta) - S$ can be shrunk to a point in $N(A, \epsilon) - S$, where $N(A, \delta)$ denotes the δ-neighborhood of A. Let f be a continuous function of S^3 onto itself such that f is 1-1 on $S^3 - A$ but f(A) is a point. Theorem. The 2-sphere f(S) is tame if and only if every point of A is a piercing point of S. (Received October 4, 1971.)

*689-G28. WILLIAM M. BOYCE, Bell Telephone Laboratories, Murray Hill, New Jersey 07974. Baxter permutations and functional composition.

Glen Baxter (Proc. Amer. Math. Soc. 15(1964), 851-855) and Baxter and Joichi (Math. Scand. 13(1963), 140-150) have studied the properties of permutations induced by pairs of continuous functions f, g on the unit interval I which commute under functional composition: fg(x) = gf(x) for all x in I. In this paper permutations satisfying their necessary conditions, called Baxter permuations, are characterized as follows: Theorem. A permutation P of the first N natural numbers is a Baxter permutation if and only if there are continuous functions f, g on I such that gf has a finite number of fixed points and the image under f of the ith crossing point of gf is the P(i)th crossing point of fg. Note that the degree of commutativity between f and g is irrelevant. (Received October 4, 1971.)

Miscellaneous Fields

*689-H1. KERMIT N. SIGMON, University of Florida, Gainesville, Florida 32601. Idempotent topological groupoids with nonempty center.

Using Alexander-Cech cohomology, it is first shown that if X is a compact idempotent topological groupoid whose center intersects each component of X, then $H^p(X;G)$ is uniquely 2-divisible for each $p \geq 1$ and each coefficient group G. It is then shown that if X is a compact connected idempotent topological groupoid with zero, then $H^p(X;G) = 0$ for each $p \geq 1$ and each G. Analogs of these results for singular cohomology, singular homology, and the homotopy groups are also given. It is also shown that an idempotent topological groupoid with nonempty center on a compact connected polyhedron must be contractible. (Received August 27, 1971.)

*689-H2. J. G. HORNE, University of Georgia, Athens, Georgia 30601. Homomorphic images of $(P^-)^n$.

Let P^- denote the multiplicative semigroup on $[0,\infty)$ and let $(P^-)^n$ denote the cartesian product of n copies of P^- and set $A = (P^-)^n$. A detailed knowledge of the possible locally compact, continuous homomorphic images of A is sought. One easily conjectures that they have the form $((P^-)^k \times (S^1)^\ell)/\{0\}^k \times (S^1)^\ell$. We do not yet take sides concerning this conjecture but offer several intermediate results, mostly having the hypothesis that a given homomorphism is one-to-one on the maximal group G of A. By way of example: Suppose S is a locally compact semigroup and $f: A \to S$ is a continuous surjection which is one-to-one on G. If, moreover, $f(H)$ is locally compact for every half-planar subsemigroup of A then f is one-to-one everywhere and hence is an isomorphism. (Received September 1, 1971.)

689-H3. DENNIS ANSON, Western Kentucky University, Bowling Green, Kentucky 42101. The structure of totally ordered sets. Preliminary report.

Let X be a totally ordered set bearing the order topology induced by the given total ordering $\leq$ and let T be a topological semigroup whose internal operation is $*$. Finally, let $T \times X \to X$ be a continuous order-preserving mapping whose values are denoted by juxtaposition. If this mapping satisfies the "action equation" [for all $t, s \in T$ and all $x \in X$, $t(sx) = (t*s)x$], then T order acts on X and the pair (T,X) is a totally ordered act. Theorem 1. Given any totally ordered space X of the type above, there exists a "natural" multiplication on $\leq$ and a "natural" transition map $\leq \times X \to X$ for which $(\leq, X)$ is a totally ordered act. Theorem 2. Given X, let $MR(X)$ be the set of all monotone [= order-preserving] retractions of X into itself. If X is connected, then $MR(X)$ is a composition semigroup which order acts on X by evaluation. Theorem 3. If X also has a largest and a smallest element, then the totally ordered acts $(MR(X),X)$ and $(\leq, X)$ are isomorphic. (Received September 7, 1971.)

*689-H4. THOMAS T. BOWMAN, University of Florida, Gainesville, Florida 32601. The automorphism groups of compact solenoidal and cylindrical semigroups.

Let H be the additive semigroup of nonnegative reals, $H^* = H \cup \infty$, and H_r^* be the Rees quotient $H^*/[r, \infty]$. Let S be a compact solenoidal semigroup, and $f: H \to S$ a dense one-parameter semigroup. If $e = e^2$ is in the minimal ideal M(S) then the map $p \to ef(p)$ can be extended to a dense one-parameter group of R into M(S). This map can be factored through the character group of the discrete reals $\hat{R_d}$. Taking the dual gives an embedding of $M(S)\hat{}$ in R_d which will be denoted by B. Let the automorphism group Aut(S) be endowed with the compact-open topology. Theorem. (i) If $S \cong H^*$, then Aut(S) is isomorphic to the multiplicative group of positive reals. (ii) If $S/M(S) \cong H_r^*$, then $\mathrm{Aut}(S) = \{1\}$. (iii) If $S/M(S) \cong H^*$ and S is not isomorphic to H^*, then Aut(S) is discrete and is isomorphic to the subgroup $\{\lambda \in P : \lambda B = B\}$ of the positive reals P under multiplication. Theorem. If S is a compact cylindrical semigroup, then the subgroup $I^*(H(1)_0)$ of Aut(S) consisting of all inner automorphisms induced by elements in the connected component $H(1)_0$ of the group of units of the identity of S is the maximal compact connected subgroup of Aut(S). (Received September 27, 1971.)

689-H5. THOMAS E. HAYS, University of Tennessee, Knoxville, Tennessee 37916. ML*-semigroups. Preliminary report.

Definition. A compact semigroup S will be called a ML-semigroup if the $\mathcal{H}$-relation is a congruence on S and $S/\mathcal{H}$ is an abelian generalized tree with idempotent endpoints and $E(S/\mathcal{H})$ is a Lawson semilattice. We give in this paper the complicated definition of an extended collection, nearly identical with that of Mislove's generalized collection, and show, using techniques similar to those of Mislove and a result of Bowman, that a ML-semigroup can be constructed from this extended collection. Conversely, we show that any ML*-semigroup can be constructed in this manner from an extended collection, where the semigroups appearing in the collection are cylindrical subsemigroups of the ML-semigroup. This generalizes the work of Mislove with semigroups over trees which in turn generalized Hofmann and Mostert's work with the hormos. (Received September 27, 1971.)

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Algebra & Theory of Numbers

*690-A1. K. DEMYS, 844 San Ysidro Lane, Santa Barbara, California 93108. Proof of a basic property of positive definite binary n-adic forms.

The set of initial assumptions (hereafter called A) being that x, y, z and n are positive integers with $n > 1$ and that $x^n + y^n - z^n = 0$, the following statements, hereafter abbreviated S, and their justifications (in parentheses) prove then that $n = 2$. $\underline{S}_1 : x + y > z$, with $x \neq y \neq z$ (A). $\underline{S}_2$: x, y, z, form a triangle ($\underline{S}_1$). $\underline{S}_3 : (x^2 + y^2 - 2xy \cos\theta)^{n/2} = z^n$, where θ is the angle between x and y (A + $\underline{S}_2$). $\underline{S}_4$: if $\cos\theta < 0$ is not an integer, then, from the expansion of the foregoing trinomial, $x^n + y^n + \Delta = z^n$, where $\Delta > 0$; this is impossible by hypothesis, and hence noninteger $\cos\theta < 0$ is impossible (A). $\underline{S}_5$: if $\cos\theta > 0$ is a noninteger, then by the same expansion $x^n + y^n + \Delta_1 - \Delta_2 = z^n$ where $\Delta_1 \neq \Delta_2$ because $x \neq y$ ($\underline{S}_1$); moreover Δ_1, $\Delta_2 \neq 0$ since $x, y \neq 0$ (A); but then the foregoing equation is impossible by hypothesis and hence a noninteger $\cos\theta > 0$ is impossible (A). $\underline{S}_6$: $\cos\theta$ is an integer or zero ($\underline{S}_4 + \underline{S}_5$). $\underline{S}_7$: if $\cos\theta$ is a nonzero integer it must be ± 1 (definition of cosine), and then $(x \pm y)^n = z^n$ or $x \pm y = z$ ($\underline{S}_3$), which, however, is impossible (A + $\underline{S}_1$). $\underline{S}_8$: $\cos\theta = 0$ is the only remaining possibility ($\underline{S}_6 + \underline{S}_7$) whence $(x^2 + y^2)^{n/2} = z^n$ ($\underline{S}_3$), which in fact has a unique solution that satisfies A, namely $n = 2$, as was to be shown. (Received July 29, 1971.)

690-A2. WILLIAM F. LIPMAN, St. Anne's School, Brooklyn, New York 11201. Polycyclic groups and rings. Preliminary report.

Let R be any finite nilpotent ring, and let R^+ be the additive group of R. It is shown that many results for finite nilpotent groups have analogs: ideals of prime order annihilate R; R satisfies a "normalizer condition" for subrings; a Burnside's Basis Theorem holds for minimal generating sets of R; and, if R^+ is a p-group, then (a) the number of subrings of R of any fixed order is congruent to 1 (mod p), and (b) if R has unique proper subrings of order p and p^2, then R^+ is cyclic. Using such results, the rings with finitely generated additive groups for which there exists an index preserving lattice isomorphism from the subrings of the ring onto the subgroups of a cyclic group are classified--with minor exceptions, they are the rings with cyclic additive groups. Therefore, a polycyclic ring is defined as a ring for which there exists a finite subinvariant series with factor rings having cyclic additive groups, and nearly all the general results of K. A. Hirsch (Proc. London Math. Soc. 44(1938), 53-60, 336-344; ibid. 49(1946), 184-194) on polycyclic groups, i.e. solvable groups with A. C. C., are shown to have ring analogs, though the proofs are not obtained by direct extension from the group case. (Received August 17, 1971.)

*690-A3. LAL M. CHAWLA and JOHN E. MAXFIELD, Kansas State University, Manhattan, Kansas 66502. On partitions of the type $x_1^k + \ldots + x_\lambda^k = n$, $k \geq 2$. Preliminary report.

Let $P_k(n)$ be the number of partitions of a positive integer n of the type $x_1^k + \ldots + x_\lambda^k = n$, $1 \leqq \lambda \leqq n$, $k \geqq 2, \ldots$ (1). For a given positive integer $m \geqq 1$, let $P_k(n, m^k)$ be the number of partitions of the type $x_1^k + \ldots + x_\lambda^k = n$, $x_1^k \cdot \ldots \cdot x_\lambda^k = m^k, \ldots$ (2). We say m is 'admissible', if $P_k(n, m^k) \geqq 1$. Let $C_{P_k}(n)$ be the number of all admissible values of m, which satisfy (2), for all the partitions $P_k(n)$ and let C_{P_k} denote the set of all these values of m. It is evident that $P_k(n) = \Sigma_{m \in C_{P_k}} P_k(n, m^k)$ and that the partitions $P_k(n)$, considered as a set, are classified into $C_{P_k}(n)$ classes $P_k(n, m^k)$. In this paper, we determine $C_{P_k}(n)$, C_{P_k} and $P_k(n, m^k)$ in terms and by means of a related pair of arithmetic functions $f_k(n)$ and $q_k(n)$ studied in [Marijo O. LeVan, "On a generalization of Chawla's two arithmetic functions," J. Natur. Sci. and Math. 9(1969), 57-66]. In [L. M. Chawla and J. E. Maxfield, "On the classification and evaluation of some partition functions and their tables," to appear] we studied the same problem for the partition function $p(n)$ and for restricted partition functions $t(n)$ and $r(n)$ by means of the related pair of arithmetic functions $f(n)$ and $q(n)$ [L. M. Chawla, "On a pair of arithmetic functions", J. Natur. Sci. and Math. 8(1968), 263-269], of which $f_k(n)$ and $q_k(n)$ are generalizations as given in [LeVan, above]. (Received September 14, 1971.)

*690-A4. MATTHEW I. GOULD, Vanderbilt University, Nashville, Tennessee 37203. Countable automorphism groups of groupoids.

Theorem. For every finite or denumerable group G there is a left-cancellative groupoid G^* of the same cardinality, such that G is isomorphic to the automorphism group of G^*. Remarks. (1) There exist in the literature several ways of constructing a groupoid whose automorphism group is isomorphic to a given group, but in all of these constructions the groupoid has no cancellation property and is larger than the group in the finite case. (2) The theorem would be false if G^* were additionally required to be commutative or associative or right-cancellative. (Received September 27, 1971.)

*690-A5. DONALD P. MINASSIAN, Butler University, Indianapolis, Indiana 46208. On extensions of orders in abelian groups.

Let N be a subgroup of the torsion-free abelian group G. Theorem 1. A partial order for N is contained in one, two or uncountably many full orders for G. Theorem 2. A full order for nonzero N is contained in one or uncountably many full orders for G. (Received September 27, 1971.)

690-A6. MILTON N. PARNES, State University of New York at Buffalo, Amherst, New York 14226. On g-invariant measures.

Let A be a set of positive integers and Γ be the well-known function, given by $\Gamma(A) = \Sigma_1^\infty \alpha(n) 2^{-n}$, where $\alpha(n)$ is the characteristic function of A, mapping 2^N onto $[0,1]$. If C is a class of subsets of the positive integers, let $\Gamma(C)$ be the set of image points. We show that a large number of classes associated with the g-invariant measures, as defined by R. Bumby and E. Ellentuck [Fund. Math. 65(1969)], are of measure zero. Among these are the class of sets whose measure is constant for the entire class of translation invariant measures on 2^N. An immediate implication is that the class $\mathcal{D}_\mu$, introduced by R. C. Buck [Amer. J. Math. 68(1946)] is of measure zero. (Received October 4, 1971.)

Analysis

*690-B1. JOSEPH DIESTEL, Kent State University, Kent, Ohio 44240. Remark on strongly bounded vector measures. Preliminary report.

Let (Ω, Σ) be a measurable space, X be a Banach space. A finitely additive map $\mu : \Sigma \to X$ is called bounded iff $\|\mu(A)\| \leqq M$ for some $M > 0$ and all $A \in \Sigma$; μ is called strongly bounded (a notion introduced by C. Rickart in the early 40's) iff given $E_n \in \Sigma$ disjoint, then $\|\mu(E_n)\| \to 0$. As is well known, strongly bounded implies bounded, simple examples show the converse is, in general, false. However, we have the surprising Theorem. If X is a separable Banach space then bounded finitely additive μ's are strongly bounded. The proof is based upon some recent results of J. K. Brooks and is surprisingly simple. Also, we show that if X is a Banach space with unconditional (Schauder) basis then each bounded finitely additive function $\mu : \Sigma \to X$ has a decomposition into purely finitely additive and countably additive parts which can be explicitly defined; previous theorems of this sort for vector measures were existential in nature. Other decomposition theorems are also given. (Received July 15, 1971.)

*690-B2. GEORGE U. BRAUER, University of Minnesota, Minneapolis, Minnesota 55455. Toeplitz operators on certain function spaces. Preliminary report.

For $p > 1$, M_p denotes the space of functions $f(z) = \sum_{n=0}^{\infty} \hat{f}(n) z^n$ analytic in the unit disc D such that $\|f\|_{M_p} = \lim\sup (1-r)^{1/p'} [\int_0^{2\pi} |f(z)|^p d\theta/2\pi]^{1/p} < \infty$ ($p' = p/(p-1)$, $z = r \exp i\theta$). For each function $\varphi(\theta) = \sum \hat{\varphi}(n) \exp in\theta$ a Toeplitz operator $T_\varphi f(z)$ is defined as follows: the Fourier series of the function $\varphi(\theta) f(r \exp i\theta)$ is denoted by $\sum_{n=-\infty}^{\infty} \hat{g}(n, r) \exp in\theta$; a point ρ_0 in the Stone-Cech compactification βI of the interval $I = [0, 1)$ and not in I is fixed; the continuous extension to βI of $\hat{g}(n, r)$ as a function of r is denoted by $g^\beta(n, \rho)$; finally $T_\varphi f(z)$ is defined as $\sum_{n>0} g^\beta(n, \rho_0) z^n$. Theorem. If $\varphi(\theta)$ has an absolutely convergent Fourier series then T_φ, considered as an operator on M_p, is invertible if and only if φ does not vanish on $[0, 2\pi]$. (Received August 5, 1971.)

690-B3. STEPHEN D. FISHER, Northwestern University, Evanston, Illinois 60201. The moduli of extremal functions.

Let D be an open, connected set in the complex plane which supports nonconstant bounded analytic functions and let p be an arbitrary but fixed point of D. Schwarz's lemma for D and p concerns the solution to the extremal problem of maximizing $|f'(p)|$ over all f which are analytic and bounded by 1 on D; the solution is unique up to multiplication by unimodular constants. It is shown here that multiplication by the solution φ is an isometry of $H^\infty(D)$ and consequently φ has unit modulus on the Silov boundary of the Banach algebra $H^\infty(D)$. In several cases it is, in fact, possible to prove that $\varphi \circ F$ is an inner function on the unit disc, where F is the uniformizer of D. (Received September 7, 1971.)

*690-B4. JOSEPH B. MILES, University of Illinois, Urbana, Illinois 61801. Zero sets in $H^p(U^n)$.

Let U^n be the n-dimensional polydisk. Theorem. If $n \geqq 2$ and $0 < p < \infty$, there exists $f \in H^p(U^n)$ such that if $g \in H^q(U^n)$ for some $q > p$ and the zero set of g contains the zero set of f, then $g \equiv 0$. This extends a previous result of Rudin, "Zeros and factorizations of holomorphic functions," Bull. Amer. Math. Soc. 72(1966), 1064-1067. The proof that f has the required properties is based on an application of Jensen's theorem to certain of the slice functions of f. (Received September 7, 1971.)

*690-B5. ALBERT BAERNSTEIN II, Syracuse University, Syracuse, New York 13210. Proof of Edrei's spread conjecture.

Let f(z) be a meromorphic function of finite lower order μ. Following Edrei (J. Analyse Math. 19 (1967), 56) we define the spread $\sigma(\tau) = \sigma(\tau, f)$ of the value τ as follows: Fix a sequence $\{r_m\}$ of Pólya peaks of order μ for $T(r) = T(r,f)$ and a positive increasing unbounded function $\Lambda(r)$ such that $\Lambda(r) = o(T(r))$ as $r \to \infty$. Then $\sigma(\tau) = \liminf_{m\to\infty} \text{meas } \{\theta : |f(r_m e^{i\theta}) - \tau| < e^{-\Lambda(r_m)}\}$ for $\tau \neq \infty$, and $\sigma(\infty) = \liminf_{m\to\infty} \text{meas } \{\theta : |f(r_m e^{i\theta})| > e^{\Lambda(r_m)}\}$. Edrei conjectured the spread relation: $\sigma(\tau) \geqq \min \{2\pi, (4/\mu)\sin^{-1}((\frac{1}{2}\delta(\tau))^{1/2})\}$, where $\delta(\tau) = \delta(\tau, f)$ denotes the Nevanlinna deficiency of τ. Earlier (J. Analyse Math. 14(1965), 83) Edrei had proved an approximate version of the spread relation which enabled him to prove that, for functions with $0 < \mu \leqq \frac{1}{2}$, either f(z) has one deficient value or $\sum \delta(\tau) < 1 - \cos \pi\mu$, where the sum is taken over all deficient values. The author is now able to prove the exact form of the spread relation. This enables Edrei to prove that, for functions with $\frac{1}{2} < \mu \leqq 1$, $\sum \delta(\tau) \leqq 2 - \sin \pi\mu$. Our proof of the spread relation is based on the following result, which seems to us to be of independent interest. Theorem. Let $m^*(re^{i\theta}) = \sup \{(1/2\pi)\int_E \log|f(re^{i\omega})|d\omega : \text{meas } E = 2\theta\}$. Then $T^*(z) = m^*(z) + N(|Z|, f)$ is subharmonic in the upper half plane. (Received September 9, 1971.)

690-B6. PETER L. DUREN and ALLEN L. SHIELDS, University of Michigan, Ann Arbor, Michigan 48104. Inequalities for H^p functions on polydisks.

Extending a theorem of Carleson [Ann. of Math. (2) 76(1962), 547-559], Duren [Bull. Amer. Math. Soc. 75(1969), 143-146; see also "Theory of H^p spaces," Academic Press, New York, 1970, Chapter 9] described the measures μ on the open unit disk such that $\int |f(z)|^q d\mu(z) \leqq C\|f\|_p^q$ for all $f \in H^p$, $0 < p \leqq q < \infty$. This theorem is now generalized to the polydisk. The condition is that $\mu(S) \leqq A(h_1 \ldots h_n)^{q/p}$, where $S = S_1 \times \ldots \times S_n$ and S_k is the set of points $re^{i\theta}$ such that $1 - h_k \leqq r < 1$ and $|\theta - \theta_k| \leqq h_k$. The proof uses a maximal function and the Marcinkiewicz interpolation theorem. Applications are a generalization to the polydisk of a theorem of Hardy and Littlewood on the convergence of integrals of the form $\int_0^1 (1-r)^{\lambda\alpha - 1}\{M_q(r,f)\}^\lambda dr$ for $f \in H^p$, a generalized Fejér-Riesz inequality, a result on interpolation in the polydisk, and a solution to a problem of Rudin ["Function theory in polydiscs," Benjamin, New York, 1969, p. 53] concerning restrictions of H^p functions to certain subsets of the polydisk. (Received September 13, 1971.)

*690-B7. MICHAEL B. FREEMAN, University of Kentucky, Lexington, Kentucky 40506. Polynomial approximation near a point where the Cauchy-Riemann equations hold.

Let $f = Q + o(|z|^2)$ be a smooth complex valued function near the origin of $\mathbb{C}^1$, where $Q(z) = \alpha z^2 + \beta z\bar{z} + \gamma \bar{z}^2$ is the second-order part of its Taylor expansion. Theorem. If f is even and $|\beta| < |\gamma|$ then there is a compact neighborhood of 0 on which the uniform algebra generated by f and the identity function comprises all continuous functions. Passing to higher dimensions, let U be a neighborhood of 0 in $\mathbb{C}^n$ and $f: U \to \mathbb{C}^m$ be smooth. Differential constraints and possibly other conditions are sought which enable a description of the uniform algebra generated by $z_1, \ldots, z_n, f_1, \ldots, f_m$. When first-order effects predominate, this algebra is relatively well understood. A second-order analysis involves the set where the system of Cauchy-Riemann equations tangential to the graph of f has less than maximal rank. In the case above when $n = 1$ this set has an isolated point at 0, while in higher dimensions it is usually much more complicated. (Received September 13, 1971.)

690-B8. RICHARD J. O'MALLEY, Department of Mathematical Sciences, Purdue University, Lafayette, Indiana 47904. Strict essential minima.

A simple proof is given of the fact that the set of strict essential minima of a real function of n variables is of measure zero. The proof uses only that a continuous function on a compact set has a maximum and the elementary fact, which seems to be new, that each set of positive measure contains a compact set which has positive upper density at each of its points. (Received September 13, 1971.)

690-B9. SAMUEL ZAIDMAN, Université de Montréal, Montréal, Québec, Canada. Regularity result for weak solutions of abstract differential equations.

Consider a Hilbert space H, and a linear closed operator A with dense domain $D(A)$. Assume that $(i\lambda - A)^{-1} \in L(H;H)$ for all real λ with $|\lambda| \geqq N > 0$ and that estimate $\|(i\lambda - A)^{-1}\| \leqq c|\lambda|^{-1}$ is verified for these λ. Let $f(t)$ be given in $L^2_{loc}(-\infty, \infty; H)$, and let $u(t)$ in the same space be a weak solution of $(d/dt - A)u = f$. Then $u(t)$ is H-continuous, $u'(t) \in L^2_{loc}(-\infty, \infty; H)$, $u(t) \in D(A)$ almost-everywhere, and $(d/dt - A)u = f$ almost-everywhere. The result was (essentially) proved by the author in 1964 (Ricerche Mat.) for the second-order equation $u'' - Bu = f$ with $\geqq 0$ selfadjoint operator B. The proof here is essentially the same. There is also a subsequent announcement by V. Barbu (Rend. Accad. Naz. Lincei) from where the present result can be probably deduced as a corollary (personal communication), but the author's proof is independent and quite straightforward. (Received September 14, 1971.)

690-B10. ROBERT BYERS, University of Cincinnati, Cincinnati, Ohio 45221. Some subordination results for classes of univalent functions. Preliminary report.

Let S be the class of all functions $f(z) = z + a_2 z^2 + \ldots$ $(a_2 \geqq 0)$ univalent in the unit disk E, S^* the the subclass of S containing the starlike functions, K the subclass of S containing the convex functions. Keogh examined necessary and sufficient conditions on complex numbers λ and μ such that $z/4 \propto \lambda z + \mu a_2 z^2 \propto f(z)$

for all $f \in S^*$ [MacIntyre Memorial Volume, "A strengthened form of the $\frac{1}{4}$ theorem for univalent functions"]. The first part of this paper is devoted to examining the same problem for the class S. Also presented are an alternate proof of Keogh's result and a generalization to a class of nonunivalent functions. In the second part of the paper we examine necessary and sufficient conditions on real numbers λ and μ such that $z/4 \propto \lambda z/(1-\mu a_2 z) \propto f(z)$ for all $f \in S$. The same result holds for the subclass S^*. When S is restricted to K, the problem $z/2 \propto \lambda z/(1-\mu a_2 z) \propto f(z)$ is examined. (Received September 17, 1971.)

690-B11. ARTHUR E. OBROCK, Case Western Reserve University, Cleveland, Ohio 44106. On bounded oscillation and asymptotic values of conformal strip mappings.

Let $w = f(z)$ denote the conformal mappings of $S = \{0 < y < \theta(x)\}$ onto $H = \{0 < v < 1\}$ which leaves $-\infty, 0, \infty$ fixed and let $\omega(x) = \sup_y \operatorname{Re} f(x+iy) - \inf_y \operatorname{Re} f(x+iy)$. If θ is positive then $\omega(x) = O(1)$ if and only if $[M(t)/L(t)] = O(1)$ where $M(t)$, $L(t)$ denote the larger, smaller of the lengths of the sides of the two largest disjoint squares which lie in S and abut along the vertical line $\{\operatorname{Re} z = t\}$. If θ is infinitely differentiable then, whenever the double series $\sum y^{2m+1} a_{2m+1,n}(x)$ coverges uniformly on compact subsets of the closure of S, its limit is $v = \operatorname{Im} f$, the harmonic measure of $\{y = \theta(x)\}$ with respect to S. (Here $m, n \geq 0$, $a_{1,0} = 1/\theta(x)$, $a_{2m+1,n} = (-1)^m D^{2m} a_{1,n}$, $D = d/dx$, and $a_{1,n} = -\sum_k \theta^{2k} a_{2k+1,\, n-k}$, for $1 \leq k \leq n$.) If θ has $2N+3$ continuous derivatives then we derive a sufficient condition under which $f(z) = f_N(z) + \lambda_N + o(1)$, where $f_N = u_N + v_N$, $v_N = \sum y^{2m+1} a_{2m+1,n}$, $(0 \leq m, n \leq N)$, $du_n = [\partial(v_{N+1} - a_{1,NH})/\partial y]dx + [\partial v_N/\partial x]dy$. There are cases where this formula is correct, but where the formulas of Warschawski and Gol'dberg-Stročik are incomplete. (Received September 13, 1971.)

*690-B12. DAVID DRASIN, Purdue University, Lafayette, Indiana 47907 and DANIEL F. SHEA, University of Wisconsin, Madison, Wisconsin 53706. Convolution inequalities and some extremal problems in function theory.

Let K be a nonnegative kernel on $(-\infty, \infty)$, with total mass one, and let φ satisfy $\varphi(x) \leq \{1 + o(1)\} \int_{-\infty}^{\infty} \varphi(t) K(x-t)\, dt$. Under some (essentially necessary) additional conditions it is shown that φ varies regularly (in the sense of Karamata) on a set G of density one, $G = \bigcup [a_n, b_n]$ where $a_n \to \infty$, $b_n - a_n \to \infty$ $(n \to \infty)$. For special choices of φ and K this yields information on the entire functions extremal for certain classical inequalities, e.g. the "$\cos \pi\rho$ inequality" of Littlewood-Valiron-Wiman. (Received September 20, 1971.)

690-B13. WILLIAM D. SERBYN, University of Minnesota, Minneapolis, Minnesota 55455. On radial growth and the distribution of values of a certain class of Hadamard gap series.

Let $f(z) = \sum_{j=1}^{\infty} c_j z^{n_j}$ be a gap series, convergent in D: $|z| < 1$, satisfying the conditions $\sum_{j=2}^{\infty} |c_{j-1}/c_j| < \infty$ and $\sum_{k=1}^{\infty} \sum_{j=k+2}^{\infty} |c_j/c_k| r_{k+1}^{n_j} < \infty$ where $r_k = \exp[-1/n_k]$. (These conditions imply that $n_{k+1}/n_k \to \infty$ as $k \to \infty$ so that f(z) has Hadamard gaps.) Suppose also that $\mu(r)$ is a positive nondecreasing function on $0 \leq r < 1$ for which $\sum_{j=1}^{\infty} \mu(r_{j+1})/|c_j| < \infty$. Theorem A. Excluding a set of measure zero, there is for each θ an r_θ such that $|f(re^{i\theta})| > \mu(r)$ whenever $r > r_\theta$. Corollary. $\lim_{r \to 1^-} f(re^{i\theta}) = \infty$ a.e.

Examples are given to compare the maximum modulus of $f(z)$ with estimates obtained from Theorem A. This work extends a result of W. Rudin (Proc. Amer. Math. Soc. 6(1955), 202-204). Theorem B. Almost all boundary points of D are Julia points for $f(z)$. The proof is based on a result of K. Meier (Math. Ann. 142 (1961), 328-344) and sharpens, for this class of functions, a recent result of J. M. Anderson (Quart. J. Math. Oxford Ser. (2) 21(1970), 247-256). (Received September 20, 1971.)

690-B14. SIMON HELLERSTEIN, University of Wisconsin, Madison, Wisconsin 53706. The maximal Nevanlinna deficiency of a finite value in the class of entire functions of finite order λ.

The still unsolved problem of determining the precise upper bound for the quantity in the title has been considered in recent years by a number of investigators. A summary of the progress on this problem is given. The solution of the problem for the class of entire functions with negative zeros by S. Hellerstein and J. Williamson (J. Analyse Math. 22(1969), 233-267) has led to some successful extensions of classical results to functions of finite order exceeding 1, in joint work with D. F. Shea (Duke Math. J. 38(1970), 489-499). These results are reviewed. Related problems are described. Some new results concerning entire functions with real zeros are also presented. (Received September 27, 1971.)

*690-B15. LAURENCE D. HOFFMANN, Claremont Men's College, Claremont, California 91711. Radial limit sets on the torus.

Let U^N denote the unit polydisc and T^N the unit torus in the space of N complex variables. A subset S of T^N is called an (RL)-set (radial limit set) if to each positive continuous function F on T^N there corresponds a function f in $H^{\infty}(U^N)$ whose radial limit f^* equals F in modulus a.e. on T^N and everywhere on S. If $N > 1$, the question of characterizing (RL)-sets is open, but two positive results are obtained. In particular, it is shown that T^N contains an (RL)-set which is homeomorphic to a cartesian product $K \times T^{N-1}$, where K is a Cantor set. Also, certain countable unions of "parallel" copies of T^{N-1} are shown to be (RL)-sets in T^N. In one variable, every subset of T is an (RL)-set; in fact, there is always a zero-free function f in $H^{\infty}(U)$ with the desired properties. It is shown, however, that when $N > 1$ there exists a circle S in T^N and a positive continuous function F on T^N to which corresponds no zero-free f in $H^{\infty}(U^N)$ with $|f^*| = F$ a.e. on T^N and everywhere on S. (Received September 27, 1971.)

690-B16. RAIMO NÄKKI and BRUCE P. PALKA, University of Michigan, Ann Arbor, Michigan 48104. Uniformly equicontinuous families of conformal and quasiconformal mappings. Preliminary report.

$\overline{R}^n$ will denote the one-point compactification of R^n, for $n \geqq 2$. Let D be a domain in $\overline{R}^n$ and F a family of K-quasiconformal mappings f of D into $\overline{R}^n$ with $f(D) = D_f$. This paper attempts to determine necessary and sufficient conditions for F to be uniformly equicontinuous in D. If $D_f = D'$ for all $f \in F$, a complete solution is given, provided that either D or D' has a "smooth" boundary. A solution for the case of variable domains is given, provided that D has a "smooth" boundary and that, for some fixed continuum $A \subset D$, the sets $f(A)$ have chordal diameter bounded away from 0. An application to the case $n = 2$, $K = 1$ yields the following result for the class $\mathscr{S}$ of normalized conformal mappings of $D = B^2$. Theorem. $F \subset \mathscr{S}$

is uniformly equicontinuous (in the Euclidean metric) on B^2 if and only if (1) $|f(z)| \leqq M$ for some $M > 0$ and for all $f \in F$, $z \in B^2$; and (2) given $r > 0$ there is a $\delta < \infty$ such that for any $f \in F$ and for any connected sets E, $E' \subset D_f$ with diam $E \geqq r$ and diam $E' \geqq r$, the extremal length of the family of curves joining E and E' in D_f is not greater than δ. (Received September 29, 1971.)

690-B17. DONALD KING BLEVINS, University of Michigan, Ann Arbor, Michigan 48104. Properties of domains bounded by k-circles. Preliminary report.

A Jordan curve Γ in the extended plane is called a k-circle if for each ordered quadruple of points (Z_1, Z_2, Z_3, Z_4) on Γ we have the inequality $|Z_1 - Z_2||Z_3 - Z_4| + |Z_2 - Z_3||Z_1 - Z_4| \leqq k|Z_1 - Z_3||Z_2 - Z_4|$. A Jordan curve is a quasiconformal circle if it is a k-circle for some k, $1 \leqq k < \infty$. Since a 1-circle is a circle, theorems concerning domains bounded by a k-circle can be obtained which interpolate between known results for a disc and for an arbitrary Jordan domain. For example, the following result interpolates between Schwarz's lemma and the Koebe $\frac{1}{4}$ theorem. Theorem. If f is a conformal map of the unit disc onto a domain D bounded by a k-circle and d is the distance from $f(0)$ to ∂D, then $f'(0) \leqq 4\theta d/\pi$, where $\theta = \pi - \arcsin(1/k)$. This inequality is sharp. We also consider coefficient estimates for such maps, harmonic measure and capacity estimates for such domains, and the H^p class of functions whose range is contained in such a domain. (Received September 30, 1971.)

690-B18. HERBERT J. ALEXANDER, University of Michigan, Ann Arbor, Michigan 48104. On polynomial hulls.

Using results of A. Browder on polynomially convex sets we deduce some properties of polynomially convex hulls of sets in $\mathbb{C}^n$. (Received September 30, 1971.)

690-B19. CARL STEPHEN DAVIS, University of Wisconsin, Madison, Wisconsin 53706. Tangential and unrestricted nontangential limits in $N^*(U^n)$. Preliminary report.

Let $U = \{|z| < 1\}$, U^n the n-polydisc, T^n the n-torus. A convex, nonnegative, nondecreasing function, F, is called strongly convex if $F(t)/t \to \infty$ as $t \to \infty$. f analytic in U^n is in $N^*(U^n)$ if there exists a strongly convex F such that $F(\log(1+|f|))$ has an n-harmonic majorant. $A(U^n)$ consists of analytic functions continuous on the closure of U^n. Suppose, for $1 \leqq i \leqq n$, $r_k^i \to 1$ and $x_k^i \to 0$ as $k \to \infty$. For f in $N^*(U^n)$, let $f_k(z_1, \ldots, z_n) = f(r_k^1 \exp(ix_k^1)z_1, \ldots, r_k^n \exp(ix_k^n)z_n)$. Theorem. For f in $N^*(U^n)$, there exists a subsequence $\{m(k)\}$ such that $f_{m(k)}(\exp(it_1), \ldots, \exp(it_n)) \to f(\exp(it_1), \ldots, \exp(it_n))$, for almost all $(\exp(it_1), \ldots, \exp(it_n))$ in T^n, where $f(\exp(it_1), \ldots, \exp(it_n))$ denotes the limit of $f(r\exp(it_1), \ldots, r\exp(it_n))$ as $r \to 1$ (a.e.). The proof makes use of a quasinorm making $N^*(U^n)$ a complete topological algebra in which $A(U^n)$ is dense. The quasinorm is given by evaluating the least n-harmonic majorant of $\log(1+|f|)$ at the origin. (Received September 30, 1971.)

690-B20. ALBERT MARDEN, University of Minnesota, Minneapolis, Minnesota 55455. On Bers' boundary of Teichmüller space.

Here a B-group is a finitely generated Kleinian group G whose regular set $\Omega(G)$ has a simply connected, invariant component $\Omega_0(G)$. Special cases are Fuchsian groups of the first kind, deformations of Fuchsian groups under quasiconformal maps of the plane (quasi-Fuchsian groups), and degenerate groups, namely groups for which $\Omega_0(G) = \Omega(G)$. Bers and Maskit have raised the question whether every B-group G which is not a Fuchsian or quasi-Fuchsian group lies on the Bers' boundary of some Teichmüller space (or is conjugate to such a group). That is, whether G is the limit of a sequence of quasi-Fuchsian groups G_n with $f_n : \Omega_0(G) \to \Omega_0(G_n)$ a conformal map and $T \mapsto f_n T f_n^{-1}$, $T \in G$, an isomorphism $G \to G_n$. In order to study this it is useful to interpret G as acting on the open unit ball B in $\mathbf{R}^3$ as well as on the Riemann sphere ∂B. Then G has a Poincaré fundamental polyhedron in B. A partial answer is provided as follows: Theorem. If G has a finite sided fundamental polyhedron, then G is a boundary group. (Received October 1, 1971.)

*690-B21. RICHARD A. ASKEY, University of Wisconsin, Madison, Wisconsin 53706. Summability of Jacobi series.

The positivity of some Cesaro mean is proven for Jacobi series $\sum_{n=0}^{\infty} a_n p_n^{(\alpha,\beta)}(x)$ for $\alpha, \beta \geqq -\frac{1}{2}$. For some (α, β) the best possible result that the $(C, \alpha+\beta+2)$ means are positive is proven. If this conjecture is true for $\alpha \geqq \beta = -\frac{1}{2}$ then $\int_0^x (x-t)^{\alpha+3/2} t^{\alpha+1} J_\alpha(t)\,dt \geqq 0$. Another formulation of this conjecture for Bessel functions is: $x^{-c}(x^2+1)^{-c}$ is a completely monotonic function for $c \geqq 1$, $c = \alpha + 3/2$. The complete monotonicity of $x^{-c-1}(x^2+1)^{-c}$ is proven for $c \geqq 0$. (Received October 1, 1971.)

690-B22. DANIEL SEGALMAN and SCOTT LAWRENCE, University of Wisconsin, Madison, Wisconsin 53706. Generalization of an inequality of Levinson and Fan.

The following inequality of Fan first appeared in Beckenbach and Bellman's "Inequalities": If $0 < x_i \leqq \frac{1}{2}$, $i = 1, \ldots, n$, then $\Pi_{i=1}^n x_i / (\Sigma_{i=1}^n x_i)^n \leqq \Pi_{i=1}^n (1-x_i)/(\Sigma_{i=1}^n (1-x_i))^n$, with equality if and only if all the x_i are equal. Fan's restriction on the x_i has been replaced by the condition that the x_i can be paired in such a way that the sum of the members of each pair is at most 1. Equality then holds if and only if either all the x_i are equal, or the sum of the members of each of the above-mentioned pairs is 1. Levinson gave a generalization of Fan's inequality in 1964, and a similar weakening of conditions also carries through in Levinson's theorem. (Received October 1, 1971.)

690-B23. KUSUM K. SONI and RAJ PAL SONI, University of Tennessee, Knoxville, Tennessee 37916. Lipschitz behavior of a class of integral transforms. Preliminary report.

Let $\varphi(x) = \int_{-\infty}^{\infty} e^{ixt} dF(t)$ where F is a distribution function. Define Λ^* and λ^* to be the classes of continuous functions φ such that $\varphi(x+h) + \varphi(x-h) - 2\varphi(x) = O(h)$ or $o(h)$ uniformly in x, as $h \to 0$. It is known [R. P. Boas, Jr., Ann. Math. Statist. 38(1967), 32-36] that (i) $\varphi \in \Lambda^*$ or λ^* if and only if $F(x) - F(\pm\infty) = O(1/|x|)$ or $o(1/|x|)$ as $|x| \to \infty$. (ii) If $0 < \gamma < 1$, $\varphi \in \mathrm{Lip}\,\gamma$ if and only if $F(x) - F(\pm\infty) = O(|x|^{-\gamma})$ as $|x| \to \infty$. In this paper the authors are concerned with the relation between the asymptotic behavior of f(t) as

$t \to \infty$ and the local behavior of Φ where $\Phi(x) = \int_0^\infty k(xt)\,df(t)$ and the kernel $k(t)$ is bounded and satisfies a uniform Lipschitz condition in $0 \leqq t < \infty$. In particular the results apply to the Sine, the Cosine and the Hankel transforms. (Received October 1, 1971.)

690-B24. KARL F. BARTH, Syracuse University, Syracuse, New York 13210 and WALTER J. SCHNEIDER, Carleton University, Ottawa 1, Ontario, Canada. A lemma concerning the mapping radius function and some of its applications. Preliminary report.

Lemma (mapping radius criteria along level lines). Let J be a Jordan domain and let α be a subarc of ∂J. Let $\omega(z)$ be the harmonic measure of α with respect to J. In addition, let $\omega(z_1) = \omega(z_2) = \lambda$. Under these conditions the mapping radius at z_1 is greater than the mapping radius at z_2 if and only if $(\partial\omega/\partial n)_{z=z_1} > (\partial\omega/\partial n)_{z=z_2}$ where $(\partial\omega/\partial n)$ is the partial derivative of ω taken in the direction of the normal to the curve $\omega = \lambda$. The following Theorem is indicative of the kinds of applications one can make of this lemma. Theorem. Let $y = f(x)$ $(\in C^1[0,1])$ be positive in the interior of $[0,1]$ and zero at its end points. In addition let $f(x)$ have only one relative maximum. Under these conditions the mapping radius function for the domain D $(= \{z = x+iy : 0 < x < 1, -f(x) < y < f(x)\})$ achieves its maximum at only one point. (Received October 4, 1971.)

690-B25. PETER L. DUREN, University of Michigan, Ann Arbor, Michigan 48104 and RENATE McLAUGHLIN, University of Michigan, Flint, Michigan 48503. Two-slit mappings and the Marx conjecture. Preliminary report.

Let S^* denote the set of functions that are starlike, univalent, and normalized in the unit disk. Let $k(z) = z(1-z)^{-2}$ denote the Koebe function. In 1932, A. Marx conjectured that for each $f \in S^*$, the derivative $f'(z)$ is subordinate to $k'(z)$. Theorem. If f has the form $f(z) = z(1-ze^{is})^{-1}(1-ze^{it})^{-1}$, then $f'(z)$ is subordinate to $k'(z)$. The proof uses methods due to M. Marden [Chapter X of M. Marden, "Geometry of polynomials," Math. Surveys, No. 3, Amer. Math. Soc., Providence, R. I., 1966]. (Received October 4, 1971.)

*690-B26. GERD H. FRICKE, Kent State University, Kent, Ohio 44240. A characterization of functions of bounded index.

The following necessary and sufficient condition for an entire function to be of bounded index is given: Theorem. An entire function f is of bounded index if and only if for each $r > 0$ there exist an integer $N = N(r) > 0$ and a constant $M = M(r) > 0$ such that for each $z \in \mathbb{C}$ there exists an integer $\ell = \ell(z)$ with $0 \leqq \ell \leqq N$ and $\max_{|\xi - z| = r}\{|f^{(\ell)}(\xi)|\} \leqq M|f^{(\ell)}(z)|$. The theorem is essential in answering two long-sought problems. Namely, the class of functions of bounded index is closed under translation and under multiplication. (Received October 4, 1971.)

*690-B27. STEPHEN B. AGARD, University of Minnesota, Minneapolis, Minnesota 55455. Quasiconformal mappings and the moduli of p-dimensional surface families.

Let T be a homeomorphism of a domain D in R^n, and let Σ be a family of p-dimensional surfaces in D. For $p > 1$, is the modular inequality Mod $\Sigma \leqq K$ Mod $T(\Sigma)$ necessary and/or sufficient for T to be quasiconformal? Results on the necessity are surveyed, topics including: Should the modulus for surface families be defined using Hausdorff measure, Lebesgue area, or the classical area integral? What is the relation between the constant K and the maximal dilatation of T? The area formula can be applied to quasiconformal surfaces. If T is quasiconformal, then (1) if Σ is composed of quasiconformal surfaces, then almost every (in the modular sense) surface in $T(\Sigma)$ is quasiconformal, and the modular inequality holds provided Lebesgue area is used in the definition; (2) almost every (in the measure theoretic sense) p-dimensional cross section of D has the property that T maps sets of zero Lebesgue p-measure onto sets of zero Hausdorff p-measure. (Received October 4, 1971.)

690-B28. PATRICK R. AHERN, University of Wisconsin, Madison, Wisconsin 53706. Inner functions in the polydisc.

The structure of inner functions in one variable is well known. For inner functions defined on the polydisc in several variables much less is known. There are many questions about factorization and boundary behavior that can be answered in the one variable case using the structure theory for which there is no known answer in the several variable case. Some positive results and some examples concerning these questions will be discussed. (Received October 4, 1971.)

690-B29. RICARDO NIRENBERG, State University of New York, Albany, New York 12203. A holomorphic extension theorem for real submanifolds of $\mathbb{C}^n$.

Let M^k be a smooth real k-dimensional submanifold of $\mathbb{C}^n$. A CR function on M^k is a C^∞ solution of the partial differential equations induced on M^k by the Cauchy-Riemann equations in $\mathbb{C}^n$. Conditions are given on M^k to ensure the extendibility of any CR function on M^k to a CR function on a bigger submanifold, or to a holomorphic function on an open set of $\mathbb{C}^n$. This generalizes for lower dimensional submanifolds the results of Hans Lewy for hypersurfaces. (Received October 4, 1971.)

Applied Mathematics

*690-C1. MARIA Z. v. KRZYWOBLOCKI, Division of Engineering Research, Michigan State University, East Lansing, Michigan 48823. Conjecture on John von Neumann's proof. Preliminary report.

In 1932 John von Neumann (at 29) announced his famous proof on the quantum theory which became a pivotal argument for the ideology of quantum physics. von Neumann proved to virtually everyone's satisfaction that no parameters previously "hidden" from quantum physics could later be discovered and permit the precise measurements that violate the uncertainty law, thereby forever exiling cause-effect from the scene of physics. From the known systems of equations, expressed in terms of "hidden" variables, one could include here integro-differential equation of Boltzmann and Navier-Stokes. The main mathematical tool used by von Neumann is the classical Hamiltonian mechanics. The used coordinates are the conjugate coordinates (p, q) and the treated particles have constant mass. Mathematical operations are performed in (p, q) phase-space. The used equation is the Schroedinger wave equation. Recently, the author proposed the wave mechanics theory of turbulence based upon the wave (quantum) mechanics, John von Neumann's theorem and Schroedinger's wave equation. There is a habit in the mathematical theory of fluid dynamics that all the operations have to be performed in domains and systems of infinitely many degrees of freedom. With reference to that the writer proposes a conjecture on a generalization of the von Neumann proof from the phase-space to a function space, from the classical mechanical system to a field. (Received July 30, 1971.)

Logic and Foundations

*690-E1. MANUEL LERMAN, Yale University, New Haven, Connecticut 06520. The finite injury priority argument in α-recursion theory.

The finite injury priority argument is a combinatorial principle used in many constructions in ordinary recursion theory. We show how to prove a generalization of this principle, which can then be used in constructions to prove many similar theorems for recursion theory on admissible ordinals. The proof given is recursion theoretic in nature, in contrast with the previous model theoretic proof of Sacks and Simpson. (Received July 19, 1971.)

690-E2. MARIAN BOYKAN POUR-EL, University of Minnesota, Minneapolis, Minnesota 55455. Recursion theory versus analog generability.

Recursive analysis provides a definition of "computable function of a real variable". The author has given a definition of a "function generable by a general-purpose analog computer (G.P.A.C.)" (the author's definition is formulated in terms of a simultaneous set of nonlinear differential equations and covers functions generable by existing G.P.A.C.). This talk is concerned with the relationship between these two concepts. Recent results as well as open problems are considered. (Received July 14, 1971.)

*690-E3. ROBERT I. SOARE, University of Illinois at Chicago Circle, Chicago, Illinois 60680. The Friedberg-Muchnik theorem re-examined.

In the well-known solution to Post's problem, Friedberg and Muchnik each constructed a pair of incomparable r.e. degrees $\underset{\sim}{a}$ and $\underset{\sim}{b}$. Subsequently, Sacks constructed r.e. degrees $\underset{\sim}{c}$ and $\underset{\sim}{d}$ such that $\underset{\sim}{c} \cup \underset{\sim}{d} = \underset{\sim}{0}'$ and $\underset{\sim}{c}' = \underset{\sim}{d}' = \underset{\sim}{0}'$. Lachlan showed that such degrees $\underset{\sim}{c}, \underset{\sim}{d}$ could have no greatest lower bound in the upper semilattice of r.e. degrees. We show that the original Friedberg-Muchnik degrees $\underset{\sim}{a}, \underset{\sim}{b}$ automatically satisfy Sacks' conditions and hence witness that the upper semilattice of r.e. degrees is not a lattice. We consider a class of "finite-injury" priority constructions formulated by Sacks. We give sufficient conditions on the construction so that Sacks' "priority set" is automatically complete. We show that any such Sacks' construction can be combined with a technique of Yates to produce a set with the desired property and recursive in a given nonrecursive r.e. set. Although not difficult, these observations apply to many constructions in the literature including Ladner's nonmitotic r.e. set, and McLaughlin's r.e. set A whose complement is regressive but not retraceable. (Received September 24, 1971.)

690-E4. MICHAEL A. MACHTEY, Indiana University, Bloomington, Indiana 47401. The recursive primitive recursive degrees are not a lattice. Preliminary report.

The primitive recursive (p.r.-) degrees were introduced by Kleene (Colloq. Math. 6(1958), 67-78) and were studied by Axt (Trans. Amer. Math. Soc. 92(1959), 85-105) who raised the question of whether the recursive p.r.-degrees are a lattice. Using computational complexity theory, this question is answered in the negative. A "natural" complexity measure in the sense of Blum (J. Assoc. Comput. Mach. 14(1967), 322-336) is introduced on the recursive functions and a p.r.-degree is said to be honest if it contains a running time function. It follows that a p.r.-degree $\underset{\sim}{a}$ is honest if and only if the union of the degrees $\leq \underset{\sim}{a}$ is a complexity class. Since the measu used has the parallel computation property, the intersection of the complexity classes determined by functions f and g is the complexity class determined by min(f, g). By the McCreight-Meyer Union Theorem (Proc. 1969 ACM Sympos. Theory of Computing, pp. 79-88) if $\underset{\sim}{a}_0, \underset{\sim}{a}_1, \ldots$ is an r.e. increasing sequence of honest p.r.-degrees then the union of all the p.r.-degrees $\underset{\sim}{b}$ such that $\underset{\sim}{b} \leq \underset{\sim}{a}_i$ for some i is a complexity class. Theorem. For any r.e. increasing sequence $\underset{\sim}{a}_0, \underset{\sim}{a}_1, \ldots$ of honest p.r.-degrees there are honest p.r.-degrees $\underset{\sim}{b}$ and $\underset{\sim}{c}$ such that $\underset{\sim}{b} \mid \underset{\sim}{c}$, $\underset{\sim}{a}_i < \underset{\sim}{b}$ and $\underset{\sim}{a}_i < \underset{\sim}{c}$ for all i, and if $\underset{\sim}{d} \leq \underset{\sim}{b}$ and $\underset{\sim}{d} \leq \underset{\sim}{c}$ then $\underset{\sim}{d} \leq \underset{\sim}{a}_j$ for some j. (Received September 2, 1971.)

*690-E5. CARL G. JOCKUSCH, JR., University of Illinois, Urbana, Illinois 61801. Degrees in which the recursive sets are uniformly recursive.

For any degree $\underset{\sim}{a}$ the following assertions (i) and (ii) are equivalent: (i) the recursive sets are uniformly of degree $\leqq \underset{\sim}{a}$; (ii) $\underset{\sim}{a}' \geqq \underset{\sim}{0}''$. For r.e. degrees $\underset{\sim}{a}$, (i) and (ii) are each equivalent to the assertion (iii): the recursive sets are contained in a family of sets uniformly of degree $\leqq \underset{\sim}{a}$. However, there exist degrees $\underset{\sim}{a}$ satisfying (iii) with $\underset{\sim}{a}' = \underset{\sim}{0}'$. In general, (iii) is equivalent to the disjunction (ii) $\vee$ (iv), where (iv) asserts that there is a complete extension of first order arithmetic of degree $\leqq \underset{\sim}{a}$. (The analogues of (i) and (iii) obtained by changing "sets" to "functions" are each equivalent to (ii).) If $\underset{\sim}{a}, \underset{\sim}{b}$ are r.e. degrees with $\underset{\sim}{a} \leqq \underset{\sim}{b}$ and

$\underset{\sim}{a} \leq \underset{\sim}{0}'$, then the assertions (v) and (vi) are equivalent: (v) the r.e. sets recursive in $\underset{\sim}{a}$ are uniformly recursive in $\underset{\sim}{b}$; (vi) $\underset{\sim}{a}'' = \underset{\sim}{b}''$. The preceding result answers (in the case $\underset{\sim}{a} = \underset{\sim}{b}$) a question raised by Yates ["Degrees of index sets. II," Trans. Amer. Math. Soc. 135(1969), 249-266]. Finally the assertions (vii) and (viii) are equivalent: (vii) $\underset{\sim}{a} \cup \underset{\sim}{0}' \geqq \underset{\sim}{0}''$; (viii) there is a function g of degree $\leqq \underset{\sim}{a}$ such that the recursive functions are precisely those partial recursive functions which have an index in the range of g. (Received September 7, 1971.)

690-E6. HISAO TANAKA, Hosei University, Tokyo, Japan and University of Illinois, Urbana, Illinois 61801. <u>Π_1^1 sets of sets and hyperdegrees</u>.

Let $\mathcal{A}$ be a set of sets of natural numbers. <u>Definition</u>. $\mathcal{A}$ has property (S) iff for every hyperdegree $\underset{\sim}{d}$ there exists a member A of $\mathcal{A}$ whose hyperdegree is $\underset{\sim}{d}$. <u>Theorem</u> 1. If $\mathcal{A}$ is a Π_1^1 set of positive measure then $\mathcal{A}$ has property (S). (G. E. Sacks independently obtained this theorem.) <u>Theorem</u> 2. If $\mathcal{A}$ is a nonmeager Π_1^1 set then $\mathcal{A}$ has property (S). The former is proved by a method as in author's paper ["A basis result for Π_1^1 sets of positive measure," Comment. Math. Univ. St. Paul. 16(1968), 115-127], whereas the latter by a forcing method as in P. G. Hinman's paper ["Some applications of forcing to hierarchy problems in arithmetic," Z. Math. Logik Grundlagen Math. 15(1969), 341-352]. (Received September 9, 1971.)

*690-E7. GERALD E. SACKS, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139. <u>k-sections of type n objects</u>. Preliminary report.

U, V, W, ... are objects of finite type. $U \leqq V$ means U is recursive in V in the sense of S. C. Kleene (Trans. Amer. Math. Soc. 91(1959), 1-52). $S_k U$ (the k-section of U) is the set of all V of type k such that $V \leqq U$. ${}^nE$ (an object of type n) is the equality predicate restricted to objects of type less than n. Let ${}^kW$ be a wellordering of all objects of type less than k. <u>Theorem</u> 1. Suppose $k < n$, U is of type n, ${}^nE \leqq U$ and ${}^kW \leqq U$; then there exists a V of type k + 1 such that $S_k U = S_k V$. <u>Corollary</u> 2. If U is of type n and ${}^nE \leqq U$, then there exists a V of type 2 such that $S_1 U = S_1 V$. <u>Theorem</u> 3. For each i, j > 0, there exists a V of type 2 whose 1-section consists of just the Δ_j^i reals. (Received September 13, 1971.)

690-E8. ALISTAIR H. LACHLAN, Simon Fraser University, Burnaby 2, British Columbia, Canada. <u>The difficulty of splitting r.e. degrees</u>. Preliminary report.

Recursively enumerable degrees $\underset{\sim}{c}$ and $\underset{\sim}{a}$ are constructed such that $\underset{\sim}{c} < \underset{\sim}{a}$ and for no r.e. degrees $\underset{\sim}{b}_0, \underset{\sim}{b}_1$ is it true that $\underset{\sim}{b}_0 \cup \underset{\sim}{b}_1 = \underset{\sim}{a}$, $\underset{\sim}{c} < \underset{\sim}{b}_0$, $\underset{\sim}{c} < \underset{\sim}{b}_1$, and $\underset{\sim}{b}_0 | \underset{\sim}{b}_1$. (Received September 13, 1971.)

690-E9. JAMES C. OWINGS, JR., University of Maryland, College Park, Maryland 20742. <u>Diagonalization and the recursion theorem</u>.

In 1938 Kleene proved his famous recursion theorem: If f is a recursive function, then for some integer c, φ_c and $\varphi_{f(c)}$ are the same partial recursive functions. The proof he gave was very clever but it was equally mysterious. We have discovered that, when f is well defined on indices of partial recursive functions (i.e., $\varphi_c \cong \varphi_d \rightarrow \varphi_{f(c)} \cong \varphi_{f(d)}$), the recursion theorem may be viewed as a simple but natural turnabout of a Cantorian diagonal argument. The same sort of statement applies to the general case, although the situation there is more complicated. We state and prove a general theorem which encompasses all fixed-point theorems closely related to the recursion theorem, such as those for Church's λ-calculus and first-order number theory. Our work will appear shortly in Notre Dame J. Formal Logic. (Received September 13, 1971.)

*690-E10. YIANNIS N. MOSCHOVAKIS, University of California, Los Angeles, California 90024. Classical descriptive set theory as a refinement of modern hierarchy theory. Preliminary report.

The classical Suslin theorem asserts that every $\underset{\sim}{\Delta}_1^1$ set A of reals is Borel; the modern Kleene theorem gives the additional information that there is a recursive function f such that if α is a $\underset{\sim}{\Delta}_1^1$-code of A, then $f(\alpha)$ is a Borel-code of A. Because of this (and many other similar examples) it is often asserted that modern hierarchy theory is a refinement of classical descriptive set theory. In this paper we notice that the classical proof of Suslin's theorem (and many other similar examples) is constructive and can be formalized in an appropriate intuitionistic system; it then follows from (a general version of) Kleene's realizability theory that recursive uniformities such as the f above must exist. From this point of view the classical work is a refinement of the modern theory since it gives both the uniformities and the constructive proofs that they work. There are some practical applications of these ideas in providing guidelines for choosing "best codings" for various sets in hierarchy theory. (Received September 14, 1971.)

*690-E11. LOUISE HAY, University of Illinois at Chicago Circle, Chicago, Illinois 60680. Discrete ω-sequences of index sets. Preliminary report.

Let $\{W_x\}$ be a standard enumeration of all r.e. sets. If A is a class of r.e. sets, $\theta A = \{x | W_x \in A\}$ is the index set of A. Definition. $\{\theta A_n\}$, $n \geqq 0$, is a discrete ω-sequence of index sets if (i) $\theta A_n <_1 \theta A_{n+1}$ for all n, and (ii) $\theta A_n \leqq \theta B \leqq \theta A_{n+1}$ implies $\theta A_n \cong \theta B$ or $\theta A_{n+1} \cong \theta B$, for all B. Theorem 1. The index sets of finite classes of finite sets form a discrete ω-sequence. Theorem 2. If $\underset{\sim}{0}' \nleqq_T S$ and $P_S = \{W_x | W_x \subset S\}$, then θP_S is at the bottom of a discrete ω-sequence. Theorem 3. Every Turing degree $\geqq 0'$ contains c discrete ω-sequences. (Received September 27, 1971.)

*690-E12. THOMAS G. McLAUGHLIN, University of Illinois, Urbana, Illinois 61801. Thinning the branches of a semicomputable tree.

Let T_1, T_2 be r.e. trees such that (1) T_2 has at least one infinite branch α, (2) every infinite branch β of T_1 satisfies $\beta \subseteq \alpha$ for some infinite branch α of T_2, (3) if α is an infinite branch of T_2 then $\beta \subseteq \alpha$ holds for some infinite branch β of T_1, and (4) there exists an r.e. set W such that if β, α are infinite branches of T_1, T_2 respectively and $\beta \subseteq \alpha$, then $\alpha - \beta = \alpha \cap W$. Under these conditions, we write $T_1 <_+ T_2$ and say that T_1 is a uniformly co-enumerable T_2-skeleton. A variety of applications to the theory of the co-simple regressive isols flows from the following basic Theorem. If f is a function recursive in $\underset{\sim}{0}^{1\sim}$ and T_2 is an r.e. tree with at least one infinite branch, then there exists an r.e. tree T_1 such that (a) $T_1 <_+ T_2$ and (b) every infinite branch of T_1 dominates f. (Received September 29, 1971.)

Topology

690-G1. REINHARD E. SCHULTZ, Purdue University, West Lafayette, Indiana 47907. On spaces of equivariant self-maps.

Let (X, φ) be a G-space, let $E(X)$ be the space of self-maps of X, and let $E(X, \varphi)$ be the subspace of equivariant maps. Under suitable conditions there is a spectral sequence converging to $\pi_*(E(X, \varphi))$ related to the Federer spectral sequence for $\pi_*(E(X/G))$. If $X = S^{\alpha n-1}$ with the free linear action of S^1 or S^3 (where $\alpha = 2$ or 4), then the image of the homotopy groups of U_n or Sp_n in $\pi_*(E(X, \varphi))$ may be studied by spectral sequence methods. Theorem 1. For infinitely many values of k, if n is sufficiently large than $\pi_k(E(X, \varphi)) \to \pi_k(G_{\alpha n})$ is not onto. Theorem 2. There are infinitely many exotic k-spheres which are not fixed point sets of semifree circle actions on homotopy $(k+2n)$-spheres for $2n \geqq k+2$. (Received August 13, 1971.)

690-G2. RONNIE LEE, Yale University, New Haven, Connecticut 06532. Semicharacteristic classes. Preliminary report.

Let $O(48)$ be the binary octahedral group and $O(48; r)$ be the group extension with subgroup Z_r and factor group $O(48)$ such that the 3-Sylow subgroup is cyclic and such that $O(48)$ acts on Z_r as follows. Elements in the commutator subgroup $T(24)$ act trivially, while the remaining elements of $O(48)$ carry each element of Z_r into its inverse. Using semicharacteristic classes we have the following: Theorem 1. For r odd and $r \geqq 3$, the group $O(48; r)$ cannot operate freely on any Z_2-homology sphere whose dimension is congruent to 3 modulo 8. Similar results also hold for the group $Q(8n, k, \ell)$ with presentation $(x, y, z : x^2 = (xy)^2 = y^{2n}, z^{k\ell} = 1, xzx^{-1} = z^r, yzy^{-1} = z^{-1})$ where $r \equiv -1 \pmod{k}$, $r \equiv 1 \pmod{\ell}$. Theorem 2. If n is odd and $n > k > \ell \geqq 1$, then the group $Q(8n, k, \ell)$ cannot operate freely on any sphere whose dimension is congruent to 3 modulo 8. (Received September 14, 1971.)

690-G3. PETER P. ORLIK, University of Wisconsin, Madison, Wisconsin 53706 and University of Oslo, Blindern, Oslo 3, Norway and PHILIP D. WAGREICH, University of Pennsylvania, Philadelphia, Pennsylvania 19104. "Algebraic" 5-manifolds with fixed point free circle action.

Suppose V is an algebraic subvariety of C^{n+1} invariant under a C^*-action σ defined by $\sigma(t, (Z_0, \ldots, Z_n)) = (t^{q_0} Z_0, \ldots, t^{q_n} Z_n)$, where $q > 1$ for all i and $(q_0, \ldots, q_n) = 1$. Let $K = V \cap S^{2n+1}$ be the intersection of V with the unit sphere about $\underline{0}$. Then K has an induced S^1 action. If V has an isolated singularity at $\underline{0}$ then K is a manifold. Theorem 1. If V has an isolated singularity, K is a 5-sphere, $X = K/S^1$ is a manifold and the action on K is not free, then X is algebraically isomorphic to CP^2. Theorem 2. Suppose V has an isolated singularity and $K/S^1 = CP^2$. Then the S^1-manifolds K as above are classified up to equivariant homeomorphism by a set of invariants $\{b; (\alpha_1, \beta_1), \ldots, (\alpha_r, \beta_r)\}$, $0 < \beta_i < \alpha_i$, $(\alpha_i, \beta_i) = 1$ where $b \in Z$ is a generalized Chern class, α_i = order of the isotropy group of an irreducible component of the set of exceptional orbits and β_i determines the representation of Z_{α_i} in the slice. The above theorems have generalizations which apply to certain varieties V which have one-isolated singularities. (Received September 16, 1971.)

*690-G4. GARY C. HAMRICK, University of Texas, Austin, Texas 78712 and ERICH OSSA, Rheinische Friedrich-Wilhelms-Universitat, Bonn, Germany. Complex bordism of topologically cyclic groups and isometries.

Let G be a compact topologically cyclic Lie group, and let $U_*(G,j)$ be the graded bordism group of manifolds with G-action commuting with a stable complex structure of the tangent bundle such that each isotropy group has codimension at least j. $U_*(G,j)$ is a module over the bordism ring U_* of stably complex manifolds. Theorem. $U_*(G,j)$ is a free U_*-module on generators in dimensions congruent to j modulo 2. P. S. Landweber has obtained this result for finite cyclic groups ("Equivariant bordism and cyclic groups," Proc. Amer. Math. Soc., to appear). The freeness of $U_*(G,0)$ leads to a similar result for $U_*(I)$, the bordism module of isometries which commute with stable complex structures. Theorem. $U_*(I)$ is a free U_*-module on uncountably many even dimension generators. (Received October 4, 1971.)

*690-G5. HSU-TUNG KU, University of Massachusetts, Amherst, Massachusetts 01002. Differentiable actions of S^1 and S^3.

Let (S^1, Σ^{2n+1}) (resp. (S^3, Σ^{4n+3})) be a free differentiable action. There is a homotopy equivalence $f: M \to CP^n$ (resp. $f: N \to QP^n$), which is transverse regular on the submanifold CP^{n-k} (resp. QP^{n-k}) with $n-k > 2$ (resp. $n-k > 1$) and let $M' = f^{-1}(CP^{n-k})$ (resp. $f^{-1}(QP^{n-k})$). Suppose that dim $M' = 0 \pmod 4$. The characteristic invariants $I_{2k}(S^1, \Sigma^{2n+1})$ (resp. $I_{4k}(S^3, \Sigma^{4n+3})$) of the free differentiable action (S^1, Σ^{2n+1}) (resp. (S^3, Σ^{4n+3})) is defined by $I_{2k}(S^1, \Sigma^{2n+1}) = \tau(M') - \tau(CP^{n-k})$ (resp. $I_{4k}(S^3, \Sigma^{4n+3}) = \tau(M') - \tau(QP^{n-k})$), where $\tau(X)$ denotes the index of the smooth manifold X. Theorem 1. Let S^1 act freely and differentiably on a homotopy $(4n+3)$-sphere Σ^{4n+3} $(n \geqq 2)$ such that the orbit space Σ^{4n+3}/S^1 admits an S^1 action with a component of the fixed point set of codimension 2. Then $I_2(S^1, \Sigma^{4n+3}) = 0$. Theorem 2. There exist infinitely many topologically distinct homotopy quaternionic projective n-spaces $(n = 3, n \neq 4)$ which do not admit differentiable S^1-actions with a component of the fixed point set of codimension 4. (Received October 4, 1971.)

690-G6. TATSUO SUWA, University of Michigan, Ann Arbor, Michigan 48104 and University of Tokyo, Tokyo, Japan. Compact quotient spaces of C^2 by affine transformation groups.

In this talk, I am going to classify the compact quotient spaces of $\mathbb{C}^2$ by properly discontinuous and fixed point free affine transformation groups G. The fundamental lemma for this is the following: Lemma. G is solvable. Then according to the first Betti number b_1 of $\mathbb{C}^2/G$, these spaces are classified; if $b_1 = 4$, $\mathbb{C}^2/G$ is a complex torus, $b_1 = 3$, an elliptic curve bundle over an elliptic curve, $b_1 = 2$, hyperelliptic surfaces and $b_1 = 1$, elliptic surfaces with multiple fibres. The higher dimensional case will be also mentioned.
(Received October 4, 1971.)

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Algebra & Theory of Numbers

*71T-A246. GEORGE D. POOLE, Texas Tech University, Lubbock, Texas 79409. Berge's theorem for infinite tournaments.

In addition to establishing some results for countably infinite tournaments which indicate similarities and dissimilarities between the finite and infinite case, the following analogue to a theorem of Berge is proved: Theorem. Suppose $G(V,e)$ is a countably infinite tournament. (a) If G contains $1 \leqq n < \infty$ hubs (vertices which connect to any other vertex), then G contains a center of radius less than three. (b) A center of radius two may be unique. (c) If $1 \leqq n \leqq \infty$, G may contain exactly n centers of radius two. (d) If G contains $1 \leqq n < \infty$ hubs, then the diameter of G is less than n (except when $n = 1$ in which case $d(G) = 1$). (Received July 19, 1971.)

71T-A247. W. RUSSELL BELDING, University of Notre Dame, Notre Dame, Indiana 46556. Invertibility in incidence rings of finite pre-orders. Preliminary report.

X is a finite set of cardinality n; $(X,*)$ is a pre-order (transitive, reflexive relation) and R is a commutative ring with identity. $I = I(X,*,R)$ is the incidence ring of $(X,*)$ over R (G. C. Rota, "Foundations of combinatorial theory. I," Z. Warscheinkeitstheorie und Verw. Gebiete 2(1964), 340-368) with zeta function z. If $f \in I$, define the $n \times n$ matrix over R, $m(f)_{ij} = f(i,j)$. Lemma. $m : I \to m(I)$ is a ring and R-module isomorphism. Lemma. f is invertible in I iff $\det(m(f))$ is a unit. Lemma. z is invertible in I iff $\det(m(z)) \neq 0$ iff $\det(m(z)) = 1$ iff $(X,*)$ is a partial order. (Received July 21, 1971.)

71T-A248. KHEE-MENG KOH, University of Manitoba, Winnipeg 19, Manitoba, Canada. On sublattices of a lattice. II.

Theorem 5. Let L be a distributive lattice. Then every sublattice is the intersection of all completely isolated sublattices containing it. Let $P(L)$, $Q(L)$ be the poset of all prime ideals and prime dual ideals of L respectively. Set $\overline{P}(L) = P(L) \cup \{L\}$, $\overline{Q}(L) = Q(L) \cup \{L\}$. Theorem 6. Let L be a distributive lattice. Then A is a convex completely isolated sublattice of L iff there exist a unique $P \in \overline{P}(L)$ and a unique $Q \in \overline{Q}(L)$ such that $A = P \cap Q$. Corollary 1. Every convex sublattice of a distributive lattice L can be represented as the intersection of all convex completely isolated sublattices containing it. If $0,1 \in L$, then the representation is

unique iff L is a finite Boolean lattice. Let X(L) be the family of all convex completely isolated sublattices of L. Corollary 2. Let L be a distributive lattice such that 0 is Λ-reducible if 0 exists. Then L is a generalized Boolean lattice iff the family $X(L) - \bar{P}(L) \cup \bar{Q}(L)$ is unordered. Theorem 7. For each positive integer n and, for each $i = 1,\ldots,n$, let L_i be a distributive lattice. Then $X(\Pi_{i=1}^{n} L_i) \subsetneq \Pi_{i=1}^{n} X(L_i)$.
(Received August 3, 1971.)

71T-A249. RAVINDER KUMAR, Ramjas College, Delhi University, Delhi, India. Discrete quasi valuation rings. Preliminary report.

R denotes commutative ring with unity and having few zero divisors (see Davis, Trans. Amer. Math. Soc. 110(1964)). An overring S of R ($\neq$ total quotient ring of R) is said to be a fractionary ideal of R if $S \subset (1/d)R$ for some nonzero divisor d of R. R is said to have property (F) if each overring of R is a fractionary ideal. Firstly, we develop a few results on quasi valuation rings on the lines of valuation rings. These results are then used to prove: (i) If each regular ideal of R is principal then R has property (F) iff R is a discrete quasi valuation ring. (ii) A D-ring has property (F) iff R is a discrete quasi valuation ring. (iii) Suppose each regular ideal of R is finitely generated; then R has property (F) iff each overring of R is a finitely generated R-module. (iv) If R satisfies a.c.c. for regular prime ideals, R is a discrete quasi valuation ring iff each regular ideal of R is a prime power (in the case of integral domains a.c.c. is not needed).
(Received August 6, 1971.) (Author introduced by Professor S. K. Jain.)

71T-A250. KIM KI-HANG BUTLER, Pembroke State University, Pembroke, North Carolina 28372. The number of partially ordered sets. Preliminary report.

An unsolved problem in combinatorial analysis asks for G*(n), the number of different partial orderings which may be defined on a finite set containing n elements. In the present paper we give a partial solution to this problem by interpreting a partial order relation as reduced idempotent Boolean matrix. The main results of this paper are as follows: An $n \times n$ Boolean matrix A is said to be reduced iff row rank of A = n = column rank of A. (i) Enumerating the partial order relations which may be defined on a finite set containing n elements is equivalent to enumerating E(n), the set of idempotent reduced $n \times n$ Boolean matrices. (ii) The set E(n) and a special subset E(n,m) are defined. (iii) The number of interest is $|E(n)| = \Sigma_{r=0}^{n(n-1)/2} |E(n,r)|$. (iv) Formulas for $|E(n,0)| = 1$, $|E(n,1)| = n(n-1)$, and $|E(n,n(n-1)/2)| = n!$ are given.
(Received August 17, 1971.)

*71T-A251. WILLIAM A. LAMPE, University of Manitoba, Winnipeg 19, Manitoba, Canada. On the independence of certain related structures of a universal algebra. III. The subalgebra lattice and congruence lattice are independent.

Suppose $\mathfrak{L}_0$ and $\mathfrak{L}_1$ are any two algebraic lattices where $\mathfrak{L}_1$ has two or more elements. A universal algebra is constructed whose subalgebra lattice is isomorphic to $\mathfrak{L}_0$ and whose congruence lattice is isomorphic to $\mathfrak{L}_1$. (Received August 23, 1971.)

*71T-A252. JACOB T. B. BEARD, JR., University of Texas, Arlington, Texas 76010. On matrix fields contained in complete matrix rings.

Let R be an arbitrary ring with identity, and let $(R)_n$ denote the complete matrix ring of all $n \times n$ matrices over R under normal matrix addition and multiplication. If a subring M of $(R)_n$ is a field, then M is called a matrix field, and we say that M is a subfield of $(R)_n$. Theorem. Let D be an integral domain having characteristic zero, and let R be the subring generated by the identity of D. Then $(R)_n$ has no subfields. This result is a prelude to others recently obtained by the author concerning the characterization of all subfields of $(R)_n$ for various choices of R, and whenever appropriate, the determination of the number of distinct subfields of $(R)_n$. (Received August 30, 1971.)

*71T-A253. THOMAS S. SHORES, University of Nebraska, Lincoln, Nebraska 68508. Artinian modules over commutative rings.

All rings of this note are commutative with unit. The ring R is locally Noetherian if the localization of R at I, R_I, is Noetherian for each maximal ideal I. R satisfies the Matlis Theorem if Artinian R-modules can be characterized as essential extensions of their finitely generated socles. The ideal A of R is subdirectly irreducible if R/A is a subdirectly irreducible ring. Also A is bounded if A contains some power of its prime radical. Theorem. The following are equivalent: (1) R satisfies the Matlis Theorem. (2) R is locally Noetherian. (3) If I is a maximal ideal of R, then the injective hull of R/I is a Loewy module whose second factor in the Loewy series is finitely generated. (4) (a) Every subdirectly irreducible ideal is a bounded primary ideal; and (b) if I is a maximal ideal of R, then I/I^2 is a finitely generated R-module. (Received September 3, 1971.)

*71T-A254. WITHDRAWN.

71T-A255. BRIAN DAY, University of Chicago, Chicago, Illinois 60637. Monoidal localisation. Preliminary report.

If a class Σ of morphisms in a (symmetric) monoidal category $\underline{C}$ satisfies the condition $s \in \Sigma \Rightarrow 1 \otimes s \in \Sigma$ then the localisation $P : \underline{C} \to \underline{C}(\Sigma^{-1})$ becomes a monoidal functor solving the appropriate universal problem. If P has a right adjoint and $\underline{C}$ is closed then $\underline{C}(\Sigma^{-1})$ is a monoidal closed category with a normal embedding in $\underline{C}$. This follows from results "On closed categories of functors" in Reports of the Midwest Category Seminar IV, Lecture Notes in Math., no. 137, Springer-Verlag, Berlin and New York, 1970, pp. 1-38. Application is made to results of Applegate and Tierney in "Iterated cotriples" (loc. cit., pp. 56-99). For example, if $M : \underline{A} \to \underline{C}$ is a monoidal models functor which preserves tensor products then the limit of the Tierney tower over $\underline{C}$ is monoidal closed if $\int^A FA \cdot (MA \otimes MB) \cong (\int^A FA \cdot MA) \otimes MB$ in $\underline{C}$ for all functors F in $[\underline{A}^{op}, Ens]$. In particular, if $\underline{A}$ is monoidal closed then so is its tower completion with respect to the Yoneda embedding $\underline{A} \to [\underline{A}, Ens]^{op}$. (Received September 10, 1971.)

71T-A256. RAYMOND BALBES, University of Missouri, St. Louis, Missouri 63121. Free pseudocomplemented semilattices. Preliminary report.

The free pseudocomplemented semilattice on n ($< \infty$) free generators is characterized as follows. For each $S \subseteq \{1,\ldots,n\}$, $0 \oplus 2^S$ denotes the algebra of subsets of $\{1,\ldots,n\}$ with $0 < T$ for all $T \in 2^S$. Theorem. Let L be the direct product of the algebras $0 \oplus 2^S$ as S ranges over the subsets of $\{1,\ldots,n\}$ and P_n the subalgebra of all elements $x \in L$ for which there is a set $T \subseteq \{1,\ldots,n\}$ such that for each $S \subseteq \{1,\ldots,n\}$: (1) $x(S) \in \{0, S \sim T\}$ and (2) $T \nsubseteq S$ implies $x(S) = 0$. Then P_n is the free pseudocomplemented semilattice on n free generators. Corollary. The number of elements in P_n is $1 + \sum_{k=0}^{n} \binom{n}{k} (2^{2^{n-k}}-1)$.

(Received September 13, 1971.)

*71T-A257. GARY CHARTRAND and S. F. KAPOOR, Western Michigan University, Kalamazoo, Michigan 49001. The square of every 2-connected graph is 1-hamiltonian.

If G is a 2-connected graph of order at least 4, then G^2 and $G^2 - v$ are hamiltonian for every vertex v of G. (Received September 13, 1971.)

71T-A258. E. M. WRIGHT, University of Aberdeen, Aberdeen, United Kingdom. Decreasing probability of connectedness of a graph as number of edges increases. Preliminary report.

An (n,q) graph is one with n nodes and q edges, in which any two different nodes are or are not joined by a single edge. F_{nq} (resp. T_{nq}) is the number of (n,q) graphs with labelled (resp. unlabelled) nodes; f_{nq} (resp. t_{nq}) is the number of these graphs which are connected; $\alpha(n,q) = f_{nq}/F_{nq}$ (resp. $\beta(n,q) = t_{nq}/T_{nq}$) is the probability that an (n,q) graph is connected. It is natural to expect that, for given n, the probabilities $\alpha(n,q)$ and $\beta(n,q)$ are (nonstrictly) increasing with q. For $\alpha(n,q)$, this is true and the proof is trivial. For $\beta(n,q)$, it is false; the simplest counterexample is that $\beta(6,9) = 20/21$, $\beta(6,10) = 14/15$. We write $N = n(n-1)/2$ so that $0 \leqq q \leqq N$. For any positive integer s we prove that $\beta(n,q) > \beta(n,q+1)$ for $N-n-s \leqq q \leqq N-n$ and $n > n_0(s)$. Indeed we prove that $1 - \beta(n,q+1) > Cn^{1/2}\{1-\beta(n,q\}$, for this range of q. Hence $\beta(n,q)$ moves sharply away from 1, just before becoming 1 for $N - n + 2 \leqq q \leqq N$. It seems possible that we can improve our result to show that $\beta(n,q) > \beta(n,q+1)$ for every q in the interval $N - An \leqq q \leqq N - n$ for any fixed A and $n > n_0(A)$, and perhaps if An is replaced by $(\frac{1}{2} - \epsilon) n \log n$, but these require much more elaborate arguments. (Received September 15, 1971.)

*71T-A259. JOHN A. BEACHY, Northern Illinois University, DeKalb, Illinois 60115. Weakly σ-injective modules.

Let ${}_R\underline{M}$ be the category of unital left modules over a ring R with 1, let σ be a torsion preradical (kernel functor) of ${}_R\underline{M}$, and let M, Q $\in$ ${}_R\underline{M}$ with injective hulls E(M), E(Q). Call Q weakly σ-injective if $f: N' \to Q$ extends to N for all $N' \subseteq N \in {}_R\underline{M}$ such that ker(f) is σ-dense in N. Recall that Q is M-injective if $f: M' \to Q$ extends to M for all $M' \subseteq M$. Theorem. The following are equivalent: (a) Q is weakly σ-injective. (b) $f: A \to Q$ extends to R for all left ideals A such that ker(f) is σ-dense in R. (c) Q is M-injective for all M such that $\sigma(M) = M$. (d) If $f: N' \to Q$ and $N = N' + \sigma(N)$, then f extends to N. (e) $0 \to Q \to N$ splits if $Q + \sigma(N) = N$. (f) $Q \supseteq \sigma(E(Q))$. Corollary. For all $M \in {}_R\underline{M}$, $M + \sigma(E(M))$ yields a "weakly σ-injective hull". Theorem. R satisfies a.c.c. for σ-dense left ideals iff every direct sum of weakly σ-injective. Theorem. Q is M-injective iff Q is weakly ρ-injective for the smallest torsion preradical ρ such that $\rho(M) = M$. Corollary. Every direct sum of M-injective modules is M-injective iff the set of left ideals each of which contains the left annihilator of a finite subset of M satisfies a.c.c. (Received September 17, 1971.)

*71T-A260. DAVID M. FOSTER, Michigan State University, East Lansing, Michigan 48823. Radicals and bimodules in varieties of algebras. Preliminary report.

Suppose V(I) is a variety of algebras. If $U \in V(I)$ and $x \in U$, let $\langle x \rangle$ denote the principal ideal of U generated by x. If M is an I-bimodule for U, then $[0|M] = \{x \in U \mid my = ym = 0 \text{ for all } m \in M,\ y \in \langle x \rangle\}$ whence $[0|M] \lhd U$. Following Andrunakievic and Rjabuhun (Soviet Math. Dokl. 5(1964)), we define general classes of I-bimodules, and from Hentzel (Proc. Amer. Math. Soc. 19(1968)) we prove Theorem 1. Suppose to each $U \in V(I)$ there is assigned a class $\Sigma(U)$ of I-bimodules such that Σ, the class of all $\Sigma(U)$, is a general class of I-bimodules. Then the class S of all $U \in V(I)$ such that $\Sigma(U) = \emptyset$ is a radical class (in the sense of Amitsur), and if S(T) denotes the S-radical of T, $T \in V(I)$, then $S(T) = \bigcap\{[0|M] \mid M \in \Sigma(T)\}$. The converse is true in some cases. Indeed, if I satisfies conditions (H), (L), and (U) (see Jacobson's "Structure and representations of Jordan algebras," Chapter 1, §6), then Theorem 2. If P is any radical property for V(I), then there exists, for each $U \in V(I)$, a class $\Sigma(U)$ of I-bimodules such that Σ, the class of all $\Sigma(U)$, is a general class of I-bimodules, and such that $P(U) = \bigcap\{[0|M] \mid M \in \Sigma(U)\}$. (Received September 20, 1971.)

*71T-A261. D. K. HALEY, Universität Mannheim, 68 Mannheim, West Germany. Equationally compact Artinian rings.

For the definition of equational compactness (e.c.) see Mycielski, "Some compactifications of general algebras," Colloq. Math. 13(1964), 1-9. R denotes an associative ring. Proposition 1. If R has a 1 and S is a matrix ring over R, then S is e.c. iff R is e.c. and S is finite dimensional over R. Proposition 2. If R is e.c. and D is the divisible part of R^+, the additive group of R, then $R \cdot D = D \cdot R = \{0\}$ and R/D is an e.c. ring. Proposition 3. If R is e.c. and R^+ is bounded torsion, then there is an e.c. ring S with 1 such that R is an ideal of S of finite index. As consequences: Theorem 1. If R is e.c. and Noetherian with 1 then the (Jacobson-) radical topology is a complete and compact topology on R. Theorem 2. If R is Artinian the following are equivalent: (i) R is e.c. (ii) R^+ is the direct sum of a finite group and a finite sum P of Prüfer groups, and $R \cdot P = P \cdot R = \{0\}$. (iii) R is a (algebraic) retract of a compact topological ring. Corollary 1. A compact topological Artinian ring is finite. Corollary 2. If R is Artinian with 1 and (algebraically) a subring of a compact topological ring, then R is finite. (Received September 20, 1971.) (Author introduced by Professor Günter H. Wenzel.)

71T-A262. SYDNEY BULMAN-FLEMING, Universität Mannheim, 68 Mannheim, West Germany. On λ-limits in semilattices. Preliminary report.

In his paper "Eine Charakterisierung gleichungskompakter universeller Algebren" (Manuskripte der Fakultät für Mathematik und Informatik der Universität Mannheim, 68 Mannheim, West Germany) G. H. Wenzel defines a λ-limit (for any ordinal λ) on a universal algebra $\mathfrak{A} = \langle A ; F \rangle$ as a homomorphism $L: \mathfrak{A}^{\omega_\lambda} \to \mathfrak{A}$ which satisfies (i) $L((a)) = a$ for every $a \in A$; and (ii) if x and y are elements of A^{ω_λ} which satisfy $x(\gamma) = y(\gamma)$ for all γ larger than some fixed $\delta < \omega_\lambda$ then $L(x) = L(y)$. The cited paper closes with the question "Does the existence of a λ-limit on a universal algebra of cardinality $\aleph_\lambda$ imply that algebra's equational compactness?" The following example provides a negative answer: let γ denote the semilattice $\langle S ; \wedge \rangle$ where S = the set of nonnegative real numbers, and $\wedge$ is the usual binary "minimum" operation. Then $L = \lim\inf : S^{\omega_1} \to S$ is a 1-limit on γ, although γ is not equationally compact. (Received September 20, 1971.) (Author introduced by Professor Günter H. Wenzel.)

*71T-A263. GÜNTER H. WENZEL, Universität Mannheim, 68 Mannheim, West Germany. Eine Charakterisierung gleichungskompakter universeller Algebren. Preliminary report.

Let ω_λ be the initial ordinal of the cardinal number $\aleph_\lambda$. A "λ-limit on the universal algebra $\mathfrak{A}$" is a homomorphism $\mathrm{Lim}_\lambda : \mathfrak{A}^{\omega_\lambda} \to \mathfrak{A}$ with the following two properties: (1) $\mathrm{Lim}_\lambda(a_\gamma)_{\gamma < \omega_\lambda} = a$ if $a_\gamma = a$ for all $\gamma < \omega_\lambda$, (2) $\mathrm{Lim}_\lambda(a\gamma)_{\gamma < \omega_\lambda} = \mathrm{Lim}_\lambda(b_\gamma)_{\gamma < \omega_\lambda}$ if there exists $\xi_0 < \omega_\lambda$ such that $a_\gamma = b_\gamma$ for all $\gamma \geqq \xi_0$. Theorem. The universal algebra $\mathfrak{A}$ of cardinality ω_α is equationally compact if and only if there are λ-limits on $\mathfrak{A}$ for all $\lambda \leqq \alpha$. Another equivalent condition is, e.g., the existence of λ-limits on $\mathfrak{A}$ for all λ. As a special case of this theorem one obtains the known characterization of algebraically compact Abelian groups due to J. Łos (see "Generalized limits in algebraically compact groups", Bull. Acad. Polon. Sci. Ser. Sci. Math. Astronom. Phys. 7(1959), 19-21). (Received September 20, 1971.)

71T-A264. GEORGE T. GEORGANTAS, State University College of New York, Buffalo, New York 14222. An isomorphism between $H^2(S/R,U)$ and R/Z. Preliminary report.

Let R be a commutative ring with 1 and of prime characteristic, and let S be a purely inseparable exponent one ring extension of R. Under certain natural conditions, the author has given an additive description of the Chase-Rosenberg group, $\mathcal{Q}(S,R)$, by showing that $R/Z \cong \mathcal{Q}(S,R)$ [see Abstract 70T-A192 these *Notices* 17(1970), 942]. Thus, by [S. Yuan, "Central separable algebras with purely inseparable splitting rings of exponent one", Trans. Amer. Math. Soc. 153(1971), 427-450], $R/Z \cong \mathcal{Q}(S,R) \cong H^2(S/R,U) \cong \mathcal{B}(\partial, S)$, which generalizes the classical field situation to rings. In the present paper, the seven-term exact sequence of Chase and Rosenberg ["Amitsur cohomology and the Brauer group", Mem. Amer. Math. Soc. No. 52(1965), p. 76] is computed for a special bicomplex giving yet another proof that $H^2(S/R, U) \cong R/Z$. In the process of carrying out the computation, a proof that $H^1(S/R, U) \cong L(C/A)$, the logarithmic derivative group, is also obtained. (Received September 21, 1971.)

*71T-A265. HERBERT S. GASKILL, Simon Fraser University, Burnaby 2, British Columbia, Canada. On transferability of semilattices.

Let $\mathfrak{S}^*$ be a semilattice and $I(\mathfrak{S}^*)$ its semilattice of ideals. Let $\mathfrak{S}$ be a finite semilattice. Definition 1. A finite semilattice is transferable provided that if φ is an embedding of $\mathfrak{S}$ in $I(\mathfrak{S}^*)$ then there is an embedding ψ of $\mathfrak{S}$ in $\mathfrak{S}^*$ which satisfies $a\psi \in b\varphi$ if and only if $a \leqq b$. Definition 2. A finite semilattice $\mathfrak{S}$ is strictly transferable in case for some linear order, $\prec$, of the join irreducibles of $\mathfrak{S}$, $\{k_0, \ldots, k_{n-1}\}$, every inequality valid in $\mathfrak{S}$ is a logical consequence of inequalities valid in $\mathfrak{S}$ of the forms (1) $k_p \leqq \sum_{i \in J} k_i$ where $k_p \prec k_i$ for each $i \in J$ or (2) $k_i \leqq k_j$. Theorem. A finite semilattice $\mathfrak{S}$ is transferable if and only if it is strictly transferable. The proof is by construction of a maximally free $\mathfrak{S}^*$ in which $\mathfrak{S}$ is embeddable in $I(\mathfrak{S}^*)$. (Received September 21, 1971.)

*71T-A266. RONALD M. SOLOMON, Yale University, New Haven, Connecticut 06520. Finite groups with Sylow 2-subgroups of type $\mathfrak{A}_{12}$.

Let $\mathfrak{A}_n$ denote the alternating group on n letters; $Sp_6(2)$, the group of all 6×6 symplectic matrices over GF(2); and $\Omega_7(3)$, the commutator subgroup of the group of all 7×7 orthogonal matrices over GF(3). Theorem. Let G be a finite simple group with Sylow 2-subgroups of type $\mathfrak{A}_{12}$. Then either G is isomorphic to $\mathfrak{A}_{12}$, $\mathfrak{A}_{13}$, or $Sp_6(2)$ or G has the involution fusion pattern of $\Omega_7(3)$. Corollary A. If G is a finite group with the involution fusion pattern of $Sp_6(2)$, then $G/O(G) \cong Sp_6(2)$. Corollary B. If G is a finite group with the involution fusion pattern of $\mathfrak{A}_{12}$, then $G/O(G) \cong \mathfrak{A}_{12}$ or $\mathfrak{A}_{13}$. A theorem of Alperin-Goldschmidt is used to prove that if G is a finite group with Sylow 2-subgroup, S, of type $\mathfrak{A}_{12}$, then the fusion of involutions in G is controlled by $N_G(J(S))$ and $C_G(Z(S))$. A signalizer functor argument then reduces the theorem to the identification of groups with known centralizers of involutions. (Received September 21, 1971.)

71T-A267. W. D. BURGESS, University of Ottawa, Ottawa, Ontario, Canada. Rings of quotients of duo rings. Preliminary report.

Let R be a duo ring (every one-sided ideal is two-sided) and let $Q_{c\ell}$ Q_r and Q_ℓ be the classical, right complete and left complete rings of quotients of R, respectively. (1) $Q_{c\ell}$ always exists (if R has a regular element), its maximal one-sided ideals are two-sided and its idempotents are central. (2) If R is Noetherian, $Q_{c\ell}$ is duo. (3) If R has ascending chain condition on right and left annihilators then $Q_{c\ell}$, Q_r and Q_ℓ coincide. (4) If R is semiprime, Q_r and Q_ℓ coincide and are strongly regular. (Received September 24, 1971.)

71T-A268. DAVID L. CARLSON, Southern Colorado State College, Pueblo, Colorado 81001. Good sequences of integers. Preliminary report.

A sequence (a_n) of integers is α-good (α real) if the sequence $(a_n\alpha)$ of real numbers is uniformly distributed mod 1; a sequence of integers is good if it is α-good for all irrational α. Theorem 1. Let $P(x) = \beta_0 + \beta_1 x + \ldots + \beta_k x^k$ be a polynomial of degree $k \geqq 1$ with real coefficients. (i) Suppose that β_k is irrational and β_j/β_k is rational for $j = 1, \ldots, k$. Then the sequence $([P(n)])$ of integer parts is α-good if and only if $1, \beta_k, \beta_k\alpha$ are linearly independent over the rational field. (ii) Otherwise, the sequence $([P(n)])$ is good. Theorem 2. Let integers r and b be given which satisfy $r \geqq 2$ and $0 \leqq b \leqq r-1$, and suppose that y is normal to base r. Write $y = [y] + \sum_{j=1}^{\infty} y_j^{(r)} r^{-j}$, where $y_j^{(r)} \in \{0, 1, \ldots, r-1\}$ for each j, and put $A_b^{(r)}(y) = \{j \mid y_j^{(r)} = b\}$. Suppose that Q(x) is a polynomial of positive degree with real coefficients, such that Q(n) is an integer whenever n is an integer. Then if $A_b^{(r)}(y) = \{a_1 < a_2 < a_3 < \ldots\}$, the sequence $(Q(a_n))$ is good. (The case $r = 2$ was settled by J. Lesca and M. Mendès France in "Ensembles normaux," Acta Arith. 17(1970), 273-282.) Proofs are based upon Weyl's criterion for uniform distribution mod 1. (Received September 17, 1971.)

*71T-A269. HENRY W. GOULD, West Virginia University, Morgantown, West Virginia 26506. A remarkable combinatorial formula of Heselden.

Using a series transformation of the author (J. Combinatorial Theory 1(1966), 233-247) it is proved that $\sum_k \binom{a+bk}{k}\binom{c-bk}{n-k} x/(x-k) = \sum_k \binom{a+bx}{k}\binom{c-bx}{n-k} x/(x-k)$, which was first found by George Heselden. We also obtain the Abel type analog $\sum_k \binom{n}{k}(a+bk)^k (c-bk)^{n-k} x/(x-k) = \sum_k \binom{n}{k}(a+bx)^k (c-bx)^{n-k} x/(x-k)$. In both cases k ranges over $0 \leqq k \leqq n$. The general equation $\sum_k Q_k(a+bk) Q_{n-k}(c-bk) x/(x-k) = \sum_k Q_k(a+bx) Q_{n-k}(c-bx) x/(x-k)$ is shown to lead to polynomials related to the well-known Bell polynomials. It is remarkable that k may be replaced by x as shown and leave the sum unchanged. The identities are true for arbitrary real real a, b, c, x, provided only that $x \neq 0, 1, \ldots, n$. (Received September 27, 1971.)

*71T-A270. ETHAN COVEN, Wesleyan University, Middletown, Connecticut 06457 and GUSTAV A. HEDLUND, Yale University, New Haven, Connecticut 06520. Sequences with minimal block growth.

Let x be a bisequence over $\{0,1\}$. For $n \geqq 1$, let $P(x,n)$ be the number of different n-blocks which appear in x. If x is Sturmian, then $P(x,n) \leqq n+1$ $(n \geqq 1)$ and any Sturmian bisequence x which is not periodic has the property that $P(x,n) = n+1$ $(n \geqq 1)$. Any bisequence x over $\{0,1\}$ for which $P(x,n) \leqq n+1$ $(n \geqq 1)$ and which is not Sturmian, satisfies $P(x,n) = n+1$ $(n \geqq 1)$, is asymptotically periodic in both senses and the asymptotic periods are relatively prime. To within duality (replacing 0 by 1 and 1 by 0) and shifting the index, this latter class is completely determined by specification of the asymptotic periods. (Received September 27, 1971.)

*71T-A271. VACLAV CHVÁTAL, McGill University, Montréal, Québec, Canada and PAUL ERDÖS, Hungarian Academy of Sciences, Budapest, Hungary. A note on hamiltonian circuits.

Theorem. Let G be a graph with at least three vertices. If, for some s, G is s-connected and contains no independent set of more than s (resp. $s+1$, resp. $s-1$) vertices then G has a hamiltonian circuit (resp. hamiltonian path, resp. hamiltonian path between any pair of vertices). The complete bipartite graphs show that this theorem is sharp. (Received September 27, 1971.)

*71T-A272. VACLAV CHVÁTAL, McGill University, Montréal, Québec, Canada. Flip-flops in hypohamiltonian graphs.

A graph is hypohamiltonian if it has no hamiltonian circuit but every point-deleted subgraph $G - u$ has at least one. We denote by $f(p)$, resp. $g(p)$, the number of nonisomorphic hypohamiltonian, resp. cubic hypohamiltonian graphs with p points. Theorem. $f(p) > 0$ whenever $p \geqq 26$, $g(p) > 0$ whenever $p \geqq 21$. Moreover, $f(21)$, $f(23)$, $f(24)$, $g(26)$, $g(34)$, $g(36)$, $g(38) > 0$ and $f(p)$, $g(2p) \to \infty$. (Received September 27, 1971.)

71T-A273. LOUIS D. NEL, Carleton University, Ottawa, Ontario, Canada. Classes of epimorphisms and monomorphisms and factorization. Preliminary report.

$\mathcal{C}$ is a complete locally small category, final objects are generators and the pullback of f and the monomorphism m_i is initial for each $i \in I$ (= any set) only if the pullback of f and $\cup_{i \in I} m_i$ is initial. Define $f \in \mathcal{L}$ to mean the pullback of f and g is initial only if the domain of g is initial; $f \in \mathcal{P}$ means f is mono and $f = h\ell$ with $\ell \in \mathcal{L}$ implies ℓ is iso. (Motivation: in categories such as all Hausdorff or all T_0-spaces with continuous maps, the $\mathcal{P}$-subobjects are precisely the topological subspaces. I thank Dr. C. M. Ringel for pointing this out.) Results. (1) $\mathcal{L}$ contains all epimorphisms and is closed under compositions. (2) $\mathcal{P}$ contains all extremal monomorphisms and is closed under compositions, intersections and unions. (3) Every $\mathcal{C}$-morphism f has a unique factorization $f = p\ell$ with $p \in \mathcal{P}$, $\ell \in \mathcal{L}$ iff the pullback of g and h can be initial only when the pullback of g and the image of h is initial. (Received September 27, 1971.)

*71T-A274. EDWARD GERRISH THURBER, Biola College, La Mirada, California 90638. The Scholz-Brauer problem on addition chains. Preliminary report.

An addition chain for a positive integer n is a set $1 = a_0 < a_1 < a_2 < \ldots < a_r = n$ of integers such that every element a_i is the sum $a_j + a_k$ of two preceding members (not necessarily distinct) of the set. Let $l(n)$ denote the minimal r for which an addition chain for n exists. Also, let $\nu(n)$ denote the number of ones in the binary representation of n, and let $c(r)$ be the smallest value of n for which $l(n) = r$. Theorem. If $\nu(n) \geqq 9$, then $l(n) \geqq [\log n/\log 2] + 4$. Theorem. The set of integers n such that $n = 2^m(23) + 7$ where $m \geqq 5$ is an infinite class of integers for which $l(2n) = l(n) = m + 8$. Theorem. $c(16) \geqq 3583$. Theorem. If $n = 2^m(19) + 1$ and $m \geqq 8$, then $l(n) = l(n+b) = m + 7$ for all integers b such that $1 \leqq b \leqq 21$. (Received September 27, 1971.)

Analysis

*71T-B246. H. M. SRIVASTAVA, University of Victoria, Victoria, British Columbia, Canada. An integral equation involving the confluent hypergeometric function of several complex variables. Preliminary report.

In the present note the author establishes an inversion theorem for a convolution transform whose kernel involves a confluent hypergeometric function of several complex variables. It is shown how the main result can be specialized to solve a number of integral equations involving special functions of interest in applied problems. An extension of the method to a general class of integral equations is also considered. (Received June 21, 1971.)

71T-B247. AVRAHAM UNGAR, Department of Applied Mathematics, University of Waterloo, Waterloo, Ontario, Canada. On an integral transform related to the wave and to the heat equations. Preliminary report.

Let $\bar{\varphi}(\omega) = W[\varphi(t)] = (i\sqrt{4\pi\omega})^{-1}\int_{-i\infty}^{i\infty} e^{t^2/4\omega}\varphi(t)dt$ be an integral transform of $\varphi(t)$, whose inverse transform is $\varphi(t) = H\bar{\varphi}(\omega) = (t/\sqrt{4\pi})\int_0^\infty \omega^{-3/2}e^{-t^2/4\omega}\bar{\varphi}(\omega)d\omega$. Theorem. $W[\varphi_{tt}] = -(d/d\omega)W[\varphi]$ and $H[\bar{\varphi}_\omega] = -(d^2/dt^2)H[\bar{\varphi}]$. Particularly, wave functions are transformed by W to heat functions and heat functions are transformed by H to wave functions (up to a (-) sign). Example. $W[\cosh(bt)] = e^{-b^2\omega}$. Theorem. $f(\psi)$ is a wave function where $\psi = \psi(x,y,z,t)$ and f is an arbitrary (differentiable) function iff ψ satisfies (*) $\nabla^2\psi = c^{-2}(\partial^2\psi/\partial t^2)$ and (**) $(\nabla\psi)^2 = c^{-2}(\partial\psi/\partial t)^2$. Example. $\psi = x - ct$ satisfies (*) and (**). With the aid of the integral transforms W and H, a plethora of functions ψ satisfying (*) and (**) could be found, as for example $\psi = (zt \pm x\sqrt{x^2+z^2-t^2})/(x^2+z^2)$ and $\psi = \lambda + i\theta$. (For definition and application of $\lambda + i\theta$, see Abstract 71T-B149, these *Notices* 18(1971), 651.) Let T be a differential operator on t. Lemma. $f(x,t)$ satisfies (***) $\Delta^n\varphi = T\varphi$ iff $R^{-1}f(R,t)$ satisfies (***), where $R^2 = x^2 + y^2 + z^2$. Examples. $f(x-t)$ and $R^{-1}f(R-t)$ are wave functions, $t^{-1/2}\exp(-x^2/4t)$ and $R^{-1}t^{-1/2}\exp(-R^2/4t)$ are heat functions. (Received July 28, 1971.)

71T-B248. WOLFGANG R. WASOW, University of Wisconsin, Madison, Wisconsin 53706. Adiabatic invariance of a simple oscillator.

J. E. Littlewood (Ann. Phys. 91(1963), 233-242) has derived asymptotic expressions, as $\epsilon \to 0+$, for the function $H(t, \epsilon) = \varphi(\epsilon t)u^2 + \varphi^{-1}(\epsilon t)(du/dt)^2$, when u is a solution of the differential equation $d^2u/dt^2 + \varphi^2(\epsilon t)u = 0$. He assumes that (i) $\varphi(\tau) > 0$, $\varphi(\pm\infty) > 0$, $\varphi^{(n)}(\pm\infty) = 0$, and $\varphi^{(n)} \in L(-\infty, \infty)$, for all $n > 0$. In the present paper, Littlewood's results are reproved, and strengthened, by using the established methods for the solution of differential equations by asymptotic series. The new result is an explicit series construction in powers of ϵ for the function $H(t, \epsilon)$. Littlewood's asymptotic expression was in terms of the unknown solution of the differential equation. The new proof of Littlewood's result that $H(\infty, \epsilon) - H(-\infty, \epsilon) = O(\epsilon^n)$ for all n is based on a calculation of the connection formula between two fundamental systems of solutions with known asymptotic expansions.
(Received September 3, 1971.)

*71T-B249. KEITH A. EKBLAW, Boise State College, Boise, Idaho 83707. On entire functions of strongly bounded index.

Let Q be the class of Euler differential operators and let $C = \{L(D) | L(D) = p_0D^k + p_1D^{k-1} + \ldots + p_k$ where $p_0, \ldots, p_k$ are polynomials and for $1 \leqq j \leqq k$ either $p_j \equiv 0$ or $\deg p_0 \geqq \deg p_j\}$. The following is shown. Theorem. Let $n \geqq 1$, $1 \leqq i, j \leqq n$, $Q_{i,j} \in Q$, $L_j \in C$ and $F_{i,j} = Q_{i,j} \cdot L_j$. If (1) (f) is a column matrix formed from entire functions $f_1, \ldots, f_n$, (2) (q) is a column matrix formed from polynomials $q_1, \ldots, q_n$, (3) $\det (Q_{i,j})_{1 \leqq i, j \leqq n}$ is a nonzero constant operator and (4) $(F_{i,j})(f) = (q)$, then $f_1, \ldots, f_n$ are entire functions of strongly bounded index. Corollary. If Q denotes the class of linear differential operators with constant coefficients then the above result is valid. (Received September 7, 1971.)

*71T-B250. CHARLES F. DUNKL and DONALD E. RAMIREZ, University of Virginia, Charlottesville, Virginia 22901. Central Sidon sets of bounded representation type. Preliminary report.

For G an infinite compact group, let $E \subset \hat{G}$ (the dual of G) be a central Sidon set of bounded representation type. Then E is a uniformly approximable central Sidon set. (As one would expect, this result is modelled after its abelian analogue due to S. Drury [C. R. Acad. Sci. Paris Sér. A 271 (1970), 162-163].) This result together with those of C. Cecchini ["Lacunary Fourier series on noncommutative groups," to appear] yields that the union of two Sidon sets in a compact Lie group is a Sidon set. (Received September 7, 1971.)

*71T-B251. JEFFREY B. RAUCH, University of Michigan, Ann Arbor, Michigan 48104 and FRANK J. MASSEY III, University of Kentucky, Lexington, Kentucky 40506. Differentiability of solutions to hyperbolic initial-boundary value problems.

Consider the hyperbolic system $Lu = (\partial_t - G)u = (\partial_t - \sum A_j(x)\partial_{x_j} - B(x))u$, defined in a cylinder $[0, T] \times \Omega$. The coefficients A_j, B and the boundary of Ω are assumed smooth, and in addition the coefficients are required to be constant outside a compact subset of $\bar{\Omega}$. We studied the mixed problem: $Lu = 0$ in $[0, T] \times \Omega$, $u(0, \cdot) = f(\cdot)$, $M(x)u(t, x) = 0$ for $x \in \partial\Omega$, where M is a smoothly varying rectangular matrix of maximal rank.

Suppose that for any $f \in \mathcal{L}_2(\Omega)$ the mixed problem has a unique strong solution $u \in C([0,T];\mathcal{L}_2(\Omega))$ (several sufficient conditions are known). Theorem. Necessary and sufficient conditions for the solution u to be in $H^s([0,T] \times \Omega) \cap C([0,T];H^s(\Omega))$ are that $u(0) \in H^s(\Omega)$ and $u(0)$ satisfies the compatibility conditions $MG^j u(0) = 0$ on $\partial\Omega$ for $0 \leqq j \leqq s-1$. If boundary conditions are incorporated in the definition of G, this is equivalent to $u(0) \in H^s(\Omega) \cap \mathcal{D}(G^s)$. Similar results are obtained when the coefficients and boundary conditions depend on t and for problems with $Lu \neq 0$, $Mu \neq 0$. (Received September 13, 1971.)

*71T-B252. HUGO TEUFEL, JR., Wichita State University, Wichita, Kansas 67208. A note on second order differential inequalities and functional differential equations.

This note presents three theorems on the nonexistence of eventually positive solutions of the inequality system $x'' + a(t)N(x) \leqq 0$, $x \geqq 0$, under the conditions on a, N, of Atkinson, Belohorec, and Osgood. The oscillation of all solutions of large classes of functional differential equations, $x'' + F(t, x(t), x(t-\tau(t))) = 0$, where $\sup_t \tau(t) < \infty$, and $0 \leqq a(t)xN(x) \leqq xF(t,x,x)$, follows as corollaries. (Received September 13, 1971.)

*71T-B253. FREDERICK W. HARTMANN, Villanova University, Villanova, Pennsylvania 19085. On Karamata's method of summation of a Cauchy product series.

The main theorem of this note extends the results of some recent publications on the summation of a Cauchy product of two series (see Swetits [Proc. Amer. Math. Soc. 23(1969), 144-146] and Ishiquro [Proc. Amer. Math. Soc. 13(1962), 695-697]). Let $K[\alpha,\beta] = (c_{n,k})$ be defined by $c_{00} = 1$; $c_{0k} = 0$, $k = 1,2,\ldots$; $[\{\alpha + (1-\alpha-\beta)z\}/\{1-\beta z\}]^n = \sum_{k=0}^{\infty} c_{n,k} z^k$, $n = 1,2,\ldots$. If $\sum_{n=0}^{\infty} a_n^* = A$, where $a_n^* = \sum_{k=0}^{\infty} c_{n,k} a_k$ then one writes $\sum a_n = A(K[\alpha,\beta])$. Theorem. Let $\alpha < 1$, $\beta < 1$ and $\alpha + \beta > 0$. Suppose $\sum a_n = A(K[\alpha,\beta])$, $\sum b_n = B(K[\alpha,\beta])$ and $\sum c_n = C(K[\alpha,\beta])$ where $c_\rho = \sum_{\rho=m+n} a_m b_n$, $\rho = 0, 1,$ then $AB = C$. (Received September 13, 1971.)

71T-B254. WITHDRAWN.

71T-B255. GLENN G. MEYERS, State University of New York, Albany, New York 12203. On Toeplitz bases in summability domains. Preliminary report.

Let T be a regular row-finite matrix. For any sequence x, let $\overset{n}{x}$ denote the nth section of x (i.e. $\overset{n}{x} = \sum_{k=1}^{n} x_k e^k$ where $e_i^k = 0$ if $k \neq i$, $e_k^k = 1$). Let $T_n x$ denote the nth T-section of x (i.e. $T_n x = \sum_{k=1}^{\infty} t_{nk} \overset{k}{x}$. Let E be an FK space. The e^k's form a basis for E if $\overset{n}{x} \to x$ for all x in E. If $T_n x \to x$ for all x in E, the e^k's form a Toeplitz basis for E. If the e^k's form a basis for E, they form a Toeplitz basis for E, but not conversely. Let A be a conservative matrix with summability field C_A. Properties of C_A related to the set $\{x^n\}_{n=1}^{\infty}$ have been studied by several authors. (See, for example, Wilansky, J. Analyse Math. 12(1964), 327-350.) We investigate conditions under which the T-sections converge to x in C_A, converge weakly to X in C_A, are weakly Cauchy in C_A, or are bounded in C_A. (Received September 16, 1971.)

*71T-B256. LAWRENCE J. DICKSON, 12727 3rd N. W., Seattle, Washington 98177. Higher order differences, zeta functions, Bernoulli numbers, and Euler polynomials.

A generalized form of the alternating zeta function is expressed as a series involving nth order differences, and convergent on $\{\mathrm{Re}(z) < n - 1\}$. This gives an elementary proof of the extendibility of the Riemann zeta function, and simple closed formulas for the Bernoulli numbers and the Euler polynomials. (Received September 17, 1971.)

*71T-B257. JOSEPH DIESTEL, Kent State University, Kent, Ohio 44242. Remarks on Schwartz spaces. Preliminary report.

Let E be a locally convex linear topological space. Recall that E is said to be a Schwartz space whenever given a Banach space X and a linear continuous operator $u: E \to X$ there exists a neighborhood V of zero in E such that u(V) is relatively compact in X. Theorem 1. Let E be a locally convex linear topological space such that E is a subspace of some power of ℓ_p and of some power of ℓ_q where $1 \leqq p \neq q < \infty$. Then E is Schwartz. Theorem 2. Let E be a locally convex linear topological space such that E is a subspace of some power of c_0 and some power of F where F is any reflexive Banach space. Then E is Schwartz. (Received September 20, 1971.)

*71T-B258. ATHANASSIOS G. KARTSATOS, University of South Florida, Tampa, Florida 33620. Maintenance of oscillation under the effect of a periodic forcing term. Preliminary report.

A necessary and sufficient condition is given for the oscillation of all solutions of the differential equation: $x^{(n)} + P(t, x, x', \ldots, x^{(n-1)}) = Q(t)$ where P, Q are continuous, $x_1 P(t, x_1, x_2, \ldots, x_n) > 0$ for every $x_1 \neq 0$, Q(t) is periodic, and all solutions of the unperturbed equation are oscillatory. This result answers a question recently raised by J. S. W. Wong. (Received September 20, 1971.) (Author introduced by Professor George Michalides.)

71T-B259. CONSTANCE M. ELSON, Ithaca College, Ithaca, New York 14850. An extension of Weyl's lemma to infinite dimensions.

A theory of distributions, analogous to Schwartz distribution theory, is formulated for an arbitrary separable Banach space, using abstract Wiener space techniques. A distribution T is harmonic on an open set U if for any test function f with support properly contained in U, $T(\Delta f) = 0$, where Δ is the generalized Laplacian. Theorem. A distribution harmonic on an open set U can be represented by a unique measure on any properly contained subset of U. The measure has smoothness properties analogous to infinite differentiability of functions. (Received September 21, 1971.) (Author introduced by Professor Leonard Gross.)

*71T-B260. GERASIMOS E. LADAS, GANGAROMS LADDE and J. PAPADAKIS, University of Rhode Island, Kingston, Rhode Island 02881. On oscillatory solutions generated by delays. Preliminary report.

Oscillation theorems are proved for the retarded differential equation (1) $y''(t) - p(t)y(t-\tau) = 0$ where $\tau > 0$, $p(t) > 0$ and continuous and $\int_{t-\tau}^{t}(t-s)p(s)ds \geqq 1$. (For example $\tau^2 p(t) \geqq 2$.) Theorem 1. Every bounded solution of (1) is oscillatory. Theorem 2. Let (2) $y(t) = \varphi(t)$, $t_0 - \tau \leqq t \leqq t_0$ and $\varphi(t)$ continuous. Then (1) - (2) has always oscillatory solutions. Theorem 3. Assume that $(\tau^2 + 2\tau)p(t) \geqq 2$. Let $y(t)$ be a bounded nonoscillatory solution of (1). Then $\lim_{t\to\infty} y(t) = \lim_{t\to\infty} y'(t) = 0$, monotonically and $|y(t)| = O(e^{-\tau t})$. These results are also extended to equations with many delays and functional arguments and generalize previous results by Ladas and Lakshmikantham [Abstract 682-39-1, these *Notices* 18(1971), 167]. (Received September 22, 1971.)

*71T-B261. WALTER R. WOODWARD, Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061. Fixed point theorems for a class of nonlinear operators.

Let X be a Banach space and K a subset of X. Call a map semi-nonexpansive (s.n.e.) if $\|Sx - Sy\| \leqq \frac{1}{2}\|Sx - x\| + \frac{1}{2}\|Sy - y\|$. (See these *Notices* 18(1971), 387.) The class of s.n.e. maps is independent from the class of nonexpansive maps: s.n.e. maps need not be continuous. We show that many of the fixed point theorems for nonexpansive maps remain valid for s.n.e. maps. Theorem 1. Let K be a bounded closed convex subset of X and M a weakly compact subset of K. If $S: K \to K$ is s.n.e. and if for each $x \in K$ (1) $\overline{co}(o(x)) \cap M \neq \emptyset$ and (2) $\overline{co}(o(x))$ has normal structure, then S has a unique fixed point in K. Theorem 1 remains valid if (2) is replaced by (3) S has diminishing orbital diameters (d.o.d.). Theorem 2. Let K be a closed bounded subset of a uniformly convex X and $S: K \to K$ a s.n.e. map. If $x \in K$, then one of the following must hold (1) S has d.o.d. at x of (2) $\frac{1}{2}(Sx + S^2x)$ is fixed by S. Similar theorems exist in a metric space. (Received September 24, 1971.)

71T-B262. JAMES H. OLSEN, North Dakota State University, Fargo, North Dakota 58102. An ergodic theorem for convex combinations of isometries. Preliminary report.

Let $\sigma_0, \sigma_1, \ldots, \sigma_n$ be nonsingular point transformations of the Lebesgue space $(X, \mathfrak{F}, \mu)$ such that for every i, j, σ_i and σ_j commute and for every $i \geqq 1$, σ_i has period a_i, with $(a_i, a_j) = 1$ for every $i \neq j$, $i, j \geqq 1$. Theorem. If σ_0 is antiperiodic, then given N and $\epsilon > 0$, there exists a nonsingular point transformation of $(X, \mathfrak{F}, \mu)$ that commutes with each σ_i, $1 \leqq i \leqq n$, has period N, and differs from σ_0 only on a set of measure less than ϵ. Standard approximation arguments allow us to conclude that if T_i is the isometry of $L_p(X, \mathfrak{F}, \mu)$, $1 < p < \infty$, induced by σ_i, $0 \leqq i \leqq n$, σ_0 arbitrary, any operator that is a convex combination of the T_i is such that the sequence $\{T^n f\}_{n=0}^{\infty}$ converges Cesàro to an L_p function a.e. for every f in $L_p(X, \mathfrak{F}, \mu)$. (Received September 24, 1971.)

71T-B263. JIMMIE R. RIDENHOUR and RAJ PAL SONI, University of Tennessee, Knoxville, Tennessee 37916. Tauberian theorems for the Hankel transform. Preliminary report.

Theorems of a Tauberian nature are obtained for the Hankel transform by the use of well-known results concerning the asymptotic behavior of the Laplace transform. The main result gives a theorem for the Hankel transform which corresponds to a result of Karamata for the Laplace transform [D. V. Widder, "The Laplace transform," Princeton, 1941, p. 191]. (Received September 24, 1971.)

71T-B264. WITHDRAWN.

71T-B265. WILLIAM D. L. APPLING, North Texas State University, Denton, Texas 76203. Finite additivity and distribution functions.

U, p, p_B, p_A^+ and the notions of subdivision, refinement and integral are as in previous abstracts of the author. "$E << D$" means E is a refinement of D. Suppose m is in p_A^+ and A is in p_B. For each s in $\mathbf{R}$ and $G << \{U\}$ let $G(s) = \{I : I \text{ in } G,\ A(I) < s\}$, and $v(s) = \inf\{\sup\{\Sigma_{E(s)} m(I) : E << D\} : D << \{U\}\}$. For each x in $\mathbf{R}$ let $g(x) = \inf\{v(y) : x < y\}$. Theorem 1. The following two statements are equivalent: (1) If x is in $\mathbf{R}$ and $0 < c$, then there is $s > x$ such that if $x < r < s$, then there is $D << \{U\}$ such that if $E << D$, then $|g(x) - \Sigma_{E(r)} m(I)| < c$; and (2) $\int_U A(I)m(I)$ exists. Theorem 2. If H is in p and is m-summable (see Riv. Mat. Univ. Parma 8(1967), 77-100), then there is a nondecreasing function g from $\mathbf{R}$ into $\mathbf{R}$ such that if h is a continuous function from $\mathbf{R}$ into $\mathbf{R}$ such that $\{|h(x)|/|x| : 1 < |x|\}$ is bounded, then $\int_{-\infty}^{\infty} h(x)dg(x) = s_m(h(H))(U)$. (Received September 27, 1971.)

71T-B266. WITHDRAWN

Geometry

71T-D28. JOHN DeCICCO, Illinois Institute of Technology, Chicago, Illinois 60616 and ROBERT V. ANDERSON, Université du Québec à Montréal, Montréal, Québec, Canada. On surfaces applicable to a pseudosphere.

There is a conformal map T, termed a special stereographic projection T upon a Euclidean plane π of a surface Σ which is applicable to a pseudosphere, such that its system of ∞^2 geodesics C_1 is depicted on π by the parallel pencil of ∞^1 straight lines which represents the simple family of its ∞^1 meridians C_1 and by the system of ∞^2 circles C_1, for each of which the center is on the y-axis. Every simple family of ∞^1 curves C which is both parallel and isothermal on Σ is pictured on Π either by the parallel pencil of ∞^1 straight lines $x = \text{constant}$, which represents the simple family of its ∞^1 circles of latitude, or by a proper pencil of ∞^1 straight lines L, for each of which its proper vertex V is on the y-axis, or else, by either an elliptic, or a parabolic, or a hyperbolic pencil of ∞^1 circles C such that the radical axis of every such pencil is the y-axis. Therefore, on a surface which is applicable to a pseudosphere there is an aggregate of ∞^2 simple families of ∞^1 curves C, each of which is both parallel and isothermal. (Received May 14, 1971.)

*71T-D29. MURRAY S. KLAMKIN, Ford Motor Company, Dearborn, Michigan 48121. Partial convexity and triangle inequalities.

An extended notion of convexity together with ordinary convexity is used to derive a number of inequalities for spherical triangles. A number of these spherical triangle inequalities reduce to know plane triangle inequalities by taking the limit as the radius of the sphere increases without bound. (Received July 9, 1971.)

71T-D30. JOSEPH A. ERBACHER, University of Southern California, Los Angeles, California 90007. A characterization of three-dimensional Riemannian manifolds of constant curvature. Preliminary report.

Let $\bar{M}$ be a C^∞ Riemannian manifold and let M be a C^∞ compact orientable submanifold of codimension 1 of $\bar{M}$, possibly with boundary. For sufficiently small s, let M_s denote the set of points lying on geodesic normals to M (and on a fixed side of M) at distance s from M. Denote the volume of M_s by A(s). Following Holgsager and Wu ("A characterization of two-dimensional Riemannian manifolds of constant curvature," Michigan Math. J. 17(1970), 297, 299) we call A(s) the growth function of M. Wu and Holgsager showed that the two-dimensional Riemannian manifolds of constant curvature c are characterized by $A'' + cA = 0$ for all M. We show Theorem. A Riemannian manifold has the property that the growth function A of each of its compact orientable hypersurfaces satisfies the linear differential equation $A''' + 4cA' = 0$ (where c is a fixed constant) if and only if it is a three-dimensional Riemannian manifold of constant curvature equal to c; in the case $c = 0$ we assume $A'' \neq 0$ for some compact orientable hypersurface. Remark. Let $D = d/ds$. If $\bar{M}$ has constant curvature c and dimension n, A(s) satisfies $(D^2 + c)(D^2 + 9c) \cdots (D^2 + (n-1)^2 c) A \neq 0$ for n even and $D(D^2 + 4c) \cdots (D^2 + (n-1)^2 c) A = 0$ for n odd. We conjecture the converse to be true for all n. (Received July 26, 1971.)

*71T-D31. BANG-YEN CHEN, Michigan State University, East Lansing, Michigan 48823. Normal curvature of pseudo-umbilical submanifolds.

Let M^n be an n-dimensional manifold immersed in an (n+p)-dimensional Riemannian manifold N^{n+p}. Let $h = \sum_{r=n+1}^{n+p} \sum_{i,j=1}^{n} h^r_{ij} \omega^i \omega^j e_r$ be the second fundamental form and $H = n^{-1} \sum h^r_{ii} e_r$ the mean curvature vector of M^n in N^{n+p}. If there exists a function f on M^n such that $\langle h(X,Y), H\rangle = f\langle X, Y\rangle$ for all tangent vectors X, Y on M^n, then M^n is called a pseudo-umbilical submanifold of N^{n+p}. The normal curvature K_N of M^n in N^{n+p} is defined by $K_N = \sum_{r,s,i,j} [\sum_k (h^r_{ik} h^s_{jk} - h^s_{ik} h^r_{jk})]^2$. Theorem 1. The Veronese surface is the only closed pseudo-umbilical surface in euclidean space (or sphere) with constant normal curvature $K_N \neq 0$ and mean curvature vector H nowhere zero. Theorem 2. The hyperbolic Veronese surface is the only closed pseudo-umbilical surface in hyperbolic space with constant normal curvature $K_N \neq 0$ and H nowhere zero. Theorem 3. The n-sphere and the submanifold $M_{m;n-m}$ are the only closed pseudo-umbilical submanifolds of dimension n in euclidean space with zero normal curvature, H nowhere zero and scalar curvature $R \geqq n(n-2)f$, where $M_{m;n-m}$ is the product of two spheres of dimensions m and n - m and of radii in the ratio of $\sqrt{m/n} : \sqrt{(n-m)/n}$. Details will be given in "Pseudo-umbilical submanifolds of a Riemannian manifold of constant curvature. III," to appear in J. Differential Geometry. (Received September 7, 1971.)

Logic and Foundations

71T-E98. JONATAN STAVI, Institute of Mathematics, Hebrew University, Jerusalem, Israel. On strongly and weakly defined Boolean terms. Preliminary report.

For notations see preceding abstract. Let B be a B.a. (possibly incomplete), $b : \delta \to B$. [A subalgebra A of B is said to be complete if whenever $X \subseteq A$ and $b = \bigcup X$ in B then $b \in A$, regular if whenever $X \subseteq A$ and $a = \bigcup X$ in A then $a = \bigcup X$ also in B.] The pair (b ; B) is called regular when the smallest complete subalgebra of B containing range (b) is regular. A B.t. f is said to be strongly defined in (b ; B) if all the joins and meets required to find f(b ; B) exist in B. f is said to be weakly defined in (b ; B) if f(b ; B') is an element of B, where B' is the normal completion of B. Consider the assertion (*): For each weakly defined B.t. f there is a strongly defined B.t. g such that $f \equiv g$. We assume that $\delta \geqq \omega$. Theorem 1. Not every pair (b ; B) satisfies (*), but every regular pair does. The question of the validity of (*) was raised by Gaifman, Fund. Math. 54(1964), 234. The negative part of Theorem 1 is a result of Theorem 2. There exists a B.a. $\mathfrak{J}$ with a complete subalgebra $\mathcal{S}$ and elements $p_{mn} \in \mathcal{S}$ $(m,n < \omega)$ such that in the normal completion of $\mathfrak{J}$, $\bigcap_n \bigcup_m p_{mn}$ is an element of $\mathfrak{J}$ and not of $\mathcal{S}$. (Received June 15, 1971.) (Author introduced by Professor Haim Gaifman.)

71T-E99. ERIK ELLENTUCK, Rutgers University, New Brunswick, New Jersey 08903. Decomposable isols. Preliminary report.

Theorem. For no cosimple isol x and $n > 1$ is $C(x,n)$ nth order indecomposable. This contrasts with the known result that there is an isol x such that $C(x,n)$ is nth order indecomposable for every $n > 0$. (Received July 12, 1971.)

*71T-E100. STEVEN K. THOMASON, Simon Fraser University, Burnaby 2, British Columbia, Canada. Semantics for tense logics.

(Contributed to the Tarski Symposium, Berkeley, June, 1971.) We consider three semantics for tense logics. The second-order semantics is the standard one, analogous to the Kripke semantics for modal logics; a structure is a pair (W, R) where R is a binary relation on the nonempty set W. The algebraic semantics is derived from results of Jónsson and Tarski ("Boolean algebras with operators, I," Amer. J. Math. 73(1951)). The first-order semantics is to the second-order as two-sorted first-order logic is to second-order logic; a structure is a triple (W, R, P) where P is an appropriate collection of subsets of W. A semantics is adequate for a tense logic T if each nontheorem of T is invalidated by some model of T in the semantics. It is shown that the latter two semantics are adequate for every tense logic, but the second-order semantics is not; in fact there is a finitely axiomatized tense logic having no second-order models and having $(N, <, P)$ as a first-order model, where P comprises all finite and cofinite subsets of N. An investigation of the model theory of the first-order semantics leads to two theorems to the effect that the second-order semantics is adequate for every tense logic meeting certain conditions. (Received September 20, 1971.)

71T-E101. GEORGE F. McNULTY, University of California, Berkeley, California 94720. Base-decidable theories and irredundant bases.

For notation see Abstract 70T-E47, these *Notices* 17(1970), 675. An equation is balanced if each variable occurs the same number of times on both sides. t projects U over V if $V \vdash \{t(u) = u : u \in U\}$ where t(u) results from substitution of u for each variable in t. $f_0, f_1, f_2, \ldots$ represent unary operations; Q represents an r-ary operation, $r \geqq 2$. If t is $f_0 \ldots f_{n-1} Q t_0 \ldots t_{r-1}$ let $t^{\#} = \{t, t_0, \ldots, t_{r-1}\}$. If t is $f_0^n f_1 \ldots f_{k-1} f_0^m x$ where f_0 differs from f_1 and f_{k-1} let $t^{\#} = \{t, f_1 \ldots f_{k-1} x\}$. Proposition 1. If T is finitely based and each member of T is balanced then T is base-decidable and ∇T is bounded above. Proposition 2. If T is any theory of commutative semigroups where $x_0 \ldots x_{n-1} = x_n^n \in T$, $xy^{n-2} = x^2 y^{n-3} \in T$, and $x^{n-1} = y^{n-1} \notin T$ for some $n \geqq 4$ then T is base-decidable and ∇T is bounded above. Theories satisfying Proposition 1 have only finitely many essentially different bases; those satisfying Proposition 2 have infinitely many. Finite algebras exist in every power > 2 satisfying Proposition 2. Theorem. Let G be a finite set of equations. If there is a term t involving an operation of rank $\geqq 2$ or two different unary operations so that t projects $\{u : \exists v[v = u \in G \vee u = v \in G]\} \cup t^{\#}$ over G then (i) $\theta[G]$ is base-undecidable (ii) $\nabla\theta[G]$ is unbounded above. V. L. Murskii [Soviet. Math. Dokl. 12(1971)] has announced a result related to part (i). Part (ii) extends a result of Alfred Tarski. (Received September 21, 1971.)

*71T-E102. MIRIAM A. LIPSCHUTZ-YEVICK, Rutgers University, New Brunswick, New Jersey 08903. Holograms of infinite strings. Preliminary report.

Let a_j denote the index of the jth symbol in the string. We assume that the number of distinct symbols used remains finite. We normalize the Gödel pattern as follows; $f(x,y) = 1$ for $a_j - 1 \leqq x < a_j$, $(j-1)(D/n) \leqq y < j(D/n)$, where D, the vertical extent of the G-pattern, is a constant. The hologram now becomes: $H(u,v) = \sum_{j=1}^{n} [\sin(u/2)/(u/2) \cdot \exp(-iu/2)(D/n) \cdot \sin(vD/2n)/(vD/2n) \cdot \exp(-iv/2n)] \cdot \exp(iua_j)\exp(ivj/D/n)$. Let J_1 be the set of indices which recur periodically and J_2 those which occur only a finite number of times or recur randomly then: $\lim_{n\to\infty} H(u,v) = \lim_{n\to\infty} (D/n) [\sum_{j=1, a_j \in J_1}^{n} + \sum_{j=1, a_j \in J_2}^{n}] = \sum_{a_j \in J_1} \exp(iua_j) \cdot [\sin(u/2)/(u/2) \cdot \exp(-iu/2)]$. Reconstruction of the hologram will approximate a set of vertical bars: $f(x,y) \cong 1$, $a_j - 1 \leqq x < a_j$, $0 \leqq y < \pi/D$, for all $j \in J_1$. (Received September 20, 1971.)

71T-E103. PUSHPA K. SATINDER, Lakehead University, Thunder Bay, Ontario, Canada. Completeness and cut-elimination in constructive infinitary arithmetics. Preliminary report.

The infinitary arithmetics, called S_n^{∞}, considered here are based on the subprimitive-recursive hierarchy $\{\mathcal{L}^n\}$ of Parsons ["Hierarchies of primitive recursive functions," Z. Math. Logik Grundlagen Math. 14(1968), 357-376], in the sense that the permitted terms and ω-rule governing functions for S_n^{∞} are drawn from $\mathcal{L}^n$. These theories are investigated, mainly, for questions of cut-elimination and completeness. It is proved that to eliminate cuts from a proof in S_n^{∞}-arithmetic $(n \geqq 2)$, a higher level arithmetic, precisely, S_{n+1}^{∞} is required. Reasons for this rise in the level of the arithmetic are discussed. Towards completeness, it is shown that, for $n \geqq 2$, the S_{n+1}^{∞}-arithmetic is complete for prenex form sentences with matrices of level complexity n. Both these results depend rather heavily on the primitive-recursion theorem of Kleene which is shown to hold at all levels (except the first two) of the hierarchy $\{\mathcal{L}^n\}$ of Parsons mentioned above. (Received September 20, 1971.) (Author introduced by Professor C. F. Kent.)

*71T-E104. GEORGE F. CLEMENTS, University of Colorado, Boulder, Colorado 80302. A minimization problem concerning subsets of a finite set.

Let F be the set of subsets of a finite set S, and for $H \subset F$, let H' denote the elements of F which are contained in some element of H. Given integers m_ℓ and $m_{\ell+1}$ does there exist a subset H of F consisting of exactly m_ℓ ℓ-element subsets of S and $m_{\ell+1}$ $(\ell+1)$-element subsets of S such that no two elements of H are related by setwise inclusion, and if such sets H do exist what is the smallest $|(\ell-1)(H')|$ can be, where $|(\ell-1)(H')|$ is the number of $(\ell-1)$-element subsets of S in H'? A generalization of this problem, which was posed by G. Katona, is solved in this paper with the help of the generalized Macauley theorem [J. Combinatorial Theory 7(1969), 230-238, MR 40#50]. (Received September 27, 1971.)

Statistics and Probability

*71T-F14. WALTER J. HENDRICKS, Case Western Reserve University, Cleveland, Ohio 44106. Hausdorff dimension in a process with stable components--an interesting counterexample. Preliminary report.

Let $X_{\alpha_1}(t)$ and $X_{\alpha_2}(t)$ be independent stable processes in R_1 of stable index α_1 and α_2 respectively, where $1 < \alpha_2 < \alpha_1 \leqq 2$. Let $X(t) \equiv (X_{\alpha_1}(t), X_{\alpha_2}(t))$ be a process in R_2 formed by allowing X_{α_1} to run on the horizontal axis and X_{α_2} on the vertical axis ; X(t) is called a process with stable components. The Blumenthal-Getoor indices of X(t) satisfy $\alpha_2 = \beta'' < \beta' = 1 + \alpha_2 - \alpha_2/\alpha_1 < \beta = \alpha_1$. Denote by dim E the Hausdorff dimension of E. It is shown that if $E = [0,1]$ and F is any fixed Borel set for which dim F $\leqq$ $1/\alpha_1$ then (with probability 1) we have dim X(E) = β' dim E and dim X(F) = β dim X(F). This shows that the results of Blumenthal and Getoor ("Sample functions of stochastic processes with stationary independent increments," J. Math. Mech. 10(1961), 493-515) for the bounds on dim X(E) for arbitrary processes X and fixed Borel sets E are the best possible, and that their conjecture that dim X(E) = dim X$[0,1]$ $\cdot$ dim E is incorrect. (Received September 16, 1971.)

Topology

*71T-G200. FRANK SIWIEC, St. John's University, Jamaica, New York 11432. KM-spaces and km-covering mappings.

For Hausdorff spaces we say that a space is KM if every compact subspace is metrizable and a mapping $f: X \to Y$ is km-covering if for every compact metrizable subspace L of Y there exists a compact metrizable subspace K of X such that f(K) = L. A space Y is KM iff every km-covering mapping onto Y is compact-covering. If X is KM, then every compact-covering mapping on X is km-covering. A space Y is sequential (resp. Fréchet, countably bisequential, locally compact + locally metrizable) iff every km-covering mapping onto Y is quotient (resp. pseudo-open, countably biquotient, biquotient). There exists a space which is regular +

Lindelöf + M + KM + nonmetrizable. Thus a space which is a compact-covering image of a metric space and which also has a perfect mapping onto a metric space, need not be metrizable. Though, by a corollary of a theorem of Filippov, if the compact-covering mapping is also an s-mapping, then the space is metrizable. (Cf. Abstract 71T-G168, these *Notices* 18(1971).) (Received July 23, 1971.)

*71T-G201. KAPIL D. JOSHI, Indiana University, Bloomington, Indiana 47401. On Lebesgue metrics.

A metric d on a topological space X (that is, a metric compatible with the topology on X) is said to be a Lebesgue metric if given any open cover $\mathcal{U}$ of X there exists $\delta > 0$ such that for each x in X, the δ-ball about x is contained in some member of $\mathcal{U}$. Among several characterizations of Lebesgue metrics is the following. Theorem 1. A metric d on X is a Lebesgue metric if and only if every continuous real-valued function on X is uniformly continuous w.r.t. d and the usual metric on the real line. Finally those spaces which admit the existence of a Lebesgue metric on them are characterized. Let the kernel of a space X be defined as the set of nonisolated points of X. (A point x in X is said to be isolated if $\{x\}$ is open.) Theorem 2. A metrizable space admits a Lebesgue metric if and only if its kernel is compact. (Received September 7, 1971.)

*71T-G202. P. L. SHARMA, Indian Institute of Technology, Kanpur, India. A structure theorem for p-class of uniformities.

Let (X, δ) be a separated proximity space. The set of all uniformities on X compatible with δ is called the p-class of δ. It is well known that a p-class contains precisely one totally bounded uniformity. A p-class having no other member is called trivial. It is also known that a Tychonoff space is pseudocompact iff the p-class of each compatible proximity is trivial. The order of a separated proximity δ on X is defined as the cardinal number of $\delta X - X$ where δX is the Smirnov compactification of (X, δ). Theorem. The p-class of a separated proximity of finite order is trivial. Corollary. The p-class of the coarsest compatible proximity on a locally compact Tychonoff space is trivial. (Received September 8, 1971.) (Author introduced by Professor B. L. Bhatia

*71T-G203. SIDNEY A. MORRIS, University of New South Wales, Kensington, New South Wales 2033, Australia. Markov and Graev free topological groups.

The precise relationship between Markov and Graev topological groups has not previously been made clear. We observe that the Markov free topological group is the free product (cf. S. A. Morris, Bull. Austral. Math. Soc. 4(1971), 17-29) of the Graev free topological group and the discrete group of integers. An analogue of this result is true in any variety of topological groups, which fact includes the result announced by Ward (Abstract 672-182, these *Notices* 17(1970), 135). As a consequence we have that topological spaces with isomorphic Graev free topological groups have isomorphic Markov free topological groups. (Received September 10, 1971.)

71T-G204. STEPHEN B. SEIDMAN, Courant Institute, New York University, New York, New York 10012. Some explicit constructions in $B(G,X)$.

Let G be an abelian topological group and X a based topological space. Then there exists a topological group $B(G,X)$, defined by McCord (Trans. Amer. Math. Soc. 146(1969), 273-297). In this paper, we construct explicit local bases for the spaces $B(G,X)$ if G and X are compact. As an application, if G is a finite abelian group, we define explicit local cross-sections of the universal principal G-bundle $B(G,I) \to B(G,S^1)$. (Received September 13, 1971.)

71T-G205. LI PI SU, University of Oklahoma, Norman, Oklahoma 73069. Some properties on the product of bitopological spaces. Preliminary report.

The product of a family of bitopological spaces $\{(X_\alpha, \mathcal{T}_\alpha, \mathcal{T}'_\alpha) : \alpha \in \Delta\}$ is pairwise T_0, $T_{1/2}$, T_1, semi-Hausdorff, Hausdorff, regular, or completely regular if each of $\{(X_\alpha, \mathcal{T}_\alpha, \mathcal{T}'_\alpha) : \alpha \in \Delta\}$ is pairwise T_0, $T_{1/2}$, T_1, semi-Hausdorff, regular, or completely regular respectively. Remark. Product space of pairwise normal spaces may not be pairwise normal. Let X be the long line and Y be the one-point compactification of X. Let $\mathcal{T} = \mathcal{T}'$ = the "usual" topology of long line and $\mathcal{T}_0 = \mathcal{T}'_0$ = the same topology of Y. Then $(X, \mathcal{T}, \mathcal{T}')$ and $(Y, \mathcal{T}_0, \mathcal{T}'_0)$ are pairwise normal. But, $(X \times Y, \mathcal{T} \times \mathcal{T}_0, \mathcal{T}' \times \mathcal{T}'_0)$ is not. (Received September 15, 1971.)

71T-G206. JOHN M. ATKINS, University of Pittsburgh, Pittsburgh, Pennsylvania 15213. A characterization of Σ and M^* spaces. Preliminary report.

Let $(X, \mathcal{T})$ be a topological space and let g be a function from $\mathbb{N} \times X$ into $\mathcal{T}$. Consider the following conditions on g: (A) if $y \in g(n,x)$, then $g(n,y) \subseteq g(n,x)$; (B) if $x \in g(n,x_n)$, then $\{x_n\}$ clusters; (C) if $g(n,x) \cap g(n,x_n) \neq \emptyset$, then $\{x_n\}$ clusters; (D) for $x \in X$, $g(n,y) \subseteq g(n,x)$ for at most finitely many distinct $g(n,y)$'s. Theorem 1. A T_1 space X is a Σ space if and only if g satisfies (A), (B) and (D). This theorem corrects a misprint in Abstract 71T-G151, these *Notices* 18(1971). Theorem 2. A T_1 space X is an M^* space if and only if g satisfies (A), (C) and (D). If, in the definition of M^*, one requires the closed covers to be closure preserving instead of locally finite, then (A) and (C) characterize such a space. (Received September 20, 1971.)

*71T-G207. RONALD H. ROSEN, University of Michigan, Ann Arbor, Michigan 48104. Contractible neighborhoods in manifold factors.

The author proves some results concerning suspensions including the following. Theorem 1. Let (X,B) be a collared compact metric pair. Suppose that $(X - B) \times R^k$ is an open $(n+k)$-manifold with $n + k \geqq 5$ and $k > 0$. Assume that X is contractible and let Y denote $X \times I^k$. Then the suspension of Y and $Y \times I$ are both topological $(n+k+1)$-cells. Theorem 2. Let (X,B) be a collared pair so that the suspension of B and the suspension of $X \times I$ are respectively homeomorphic to S^{n-1} and I^{n+1}. Then the suspension of X is homeomorphic to I^n. These two theorems generalize the author's recent work "Concerning suspension spheres," Proc. Amer. Math. Soc. 23(1969), 225-231. (Received September 22, 1971.)

*71T-G208. JAMES R. BOONE, Texas A & M University, College Station, Texas 77843. Nearness preserving conditions for inverse images.

The pseudo-open mappings were introduced by Arhangelskii (Soviet Math. Dokl. 4(1963), 1726-1729). Theorem A. A mapping $f: X \to Y$ is pseudo-open iff for each $p \in Y$ and for each $H \subset Y$ such that $p \in cl(H)$, there exists $q \in f^{-1}(p)$ such that $q \in cl(f^{-1}(H))$. The inverse image of a convergent sequence need not contain a sequence which converges to a point in the inverse image of the limit point, even if the mapping is a perfect, continuous surjection. Example. Consider the mapping φ from the Stone-Cech compactification of the natural numbers, βN, onto the one-point compactification, N^*, of N, defined by $\varphi(n) = n$, for each $n \in N$, and $\varphi(p) = \infty$, for each $p \in \beta N - N$. φ is perfect, and continuous. The sequence $\{n\}$ converges to ∞, in N^*, but no subsequence of $\{n\}$ $(=\varphi^{-1}(\{n\}))$ in βN converges to any point in $\beta N - N$ $(=\varphi^{-1}(\infty))$, because Novak (Fund. Math. 40(1953), 106-112) has shown that every infinite closed set in βN has cardinality 2^c. The notion of a sequential space is due to Franklin (Fund. Math. 57(1965), 107-115). Theorem B. If $f: X \to Y$ is a quotient mapping and fo each convergent sequence $\{p_i\}$ in Y, say $p_i \to p$, $f^{-1}(p) \cup (\cup\{f^{-1}(p_i) : i \in N\})$ is sequential as a subspace of X, then there exists a sequence, $\{q_n\}$, in X, such that $q_n \in f^{-1}(p_{i_n})$, for some subsequence $\{p_{i_n}\}$ and $q_n \to q$, for some $q \in f^{-1}(p)$. (Received September 24, 1971.)

*71T-G209. WITHDRAWN.

*71T-G210. MELVIN C. THORNTON, University of Nebraska, Lincoln, Nebraska 68508. Semirings of functions determine finite T_0 topologies.

An analogue of the Stone-Gelfand-Kolmogoroff theorem for compact Hausdorff spaces is proven for finite T_0 topological spaces. Let $C(X)$ be the semiring of continuous functions from finite T_0X into Z, the nonnegative integers with open sets of the form $\{0, 1, \ldots, n\}$. Products and sums in $C(X)$ are defined componentwise. Denote the set of nonzero semiring homomorphisms of $C(X)$ into Z by $H(X)$ and give it the compact-open topology. Then (1) X and $H(X)$ are homeomorphic. (2) $C(X)$ is semiring isomorphic to $C(Y)$ iff X is homeomorphic to Y. (3) The topology of X can be completely recovered from the inclusion relations amonḡ the ideals of $C(X)$ which are kernels of the elements in $H(X)$. (Received September 27, 1971.)

*71T-G211. JOHN J. WALSH, State University of New York, Binghamton, New York 13901. Fiber preserving cellular decompositions. Preliminary report.

Let (Y, P, B) be a locally trivial bundle with fiber M^r $(r \neq 4)$, a compact connected metric manifold with or without boundary; furthermore, assume B is a finite-dimensional locally compact metric space. Let $G = \{G[b] \mid b \in B\}^*$ be an upper semicontinuous (usc) decomposition of Y with each $G[b]$ a cellular decomposition of $P^{-1}(b)$ such that $P^{-1}(b)/G[b] \approx M$. It is assumed that if $g \in G[b]$ and $g \cap bd(P^{-1}(b)) \neq \emptyset$, then $g \subseteq bd(P^{-1}(b))$ and that if $r = 5$, then only degenerate elements of $G[b]$ meet $bd(P^{-1}(b))$. In this note we prove that $\tilde{P}: Y/G \to B$, defined by $\widetilde{P}(P^{-1}(b)/G[b]) = b$, is a locally trivial bundle with fiber M. The special cases where $Y = S^1 \times B$, $Y = S^2 \times B$, or $Y = S^3 \times B$ and P is the projection map have been proved by E. Dyer and M. E. Hamstrom. The work of Dyer and Hamstrom on completely regular mappings, that of Kirby and Edwards on local contractibility of spaces of homeomorphisms of manifolds, and that of Siebenmann on approximating cellular maps by homeomorphisms are all essential to the proof of the main result. (Received September 27, 1971.)

*71T-G212. IVAN L. REILLY and STUART N. YOUNG, University of Auckland, Auckland, New Zealand. Bitopological quasicomponents.

In a bitopological space $(X, \mathcal{T}_1, \mathcal{T}_2)$ a relation R is defined by $(x, y) \in R$ iff x and y cannot be separated by a separation of X. R is an equivalence relation and the equivalence class of x under R is its quasicomponent. Simple properties of quasicomponents are investigated. Typical results are: In a pairwise locally connected bitopological space each quasicomponent is a component; each quasicomponent Q in $(X, \mathcal{T}_1, \mathcal{T}_2)$ satisfies the equation $Q = (\mathcal{T}_1 \operatorname{cl} Q) \cap (\mathcal{T}_2 \operatorname{cl} Q)$. (Received September 27, 1971.)

Miscellaneous Fields

*71T-H6. ALAN DAVID SLOAN, Carnegie-Mellon University, Pittsburgh, Pennsylvania 15213. The relativistic polaron without cutoffs in two-space dimensions.

A 2-space dimensional version of Nelson's quantum field theory model of a massive polaron is considered (E. Nelson, J. Math. Phys. 5(1964), 1190). The nonrelativistic free nucleon Hamiltonian, H_{nuc}, is replaced with the (momentum space) operator given as multiplication on $L^2(R^2)$ by the relativistic kinetic energy function $E_m(p) = (|p|^2 + m^2)^{1/2}$ where m is the mass of the nucleon. The sharp momentum cutoffs, $E_\mu^{-1/2} \chi_\varkappa$, are replaced by real, radial C^∞ functions, $f_\varkappa$, for each integer $\varkappa > 0$, where μ is the meson mass. $f_\varkappa$ agrees with $E_\mu^{-1/2} \chi_\varkappa$ outside the radial interval $[\varkappa, \varkappa+1]$. Thus, the total Hamiltonian for the polaron with momentum cutoff $f_\varkappa$ is defined as a selfadjoint operator on $K = L^2(R^2) \otimes$ (symmetric Fock space over $L^2(R^2)$) and is bounded below. For each $\varkappa$, the nucleon mass may be adjusted to produce a total Hamiltonian, $H'_\varkappa \geqq 0$, with inf(spectrum $H'_\varkappa$) = 0. Theorem. There is a nonnegative, selfadjoint operator, H_∞, densely defined on K, such that the resolvents of $H'_\varkappa$ converge strongly to the resolvent of H_∞ as $\varkappa \to \infty$. It follows that the induced unitary groups and contraction semigroups also converge strongly. (Received September 20, 1971.)

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