For $n = 100$ these various terms are, to 20 decimals,

$$\begin{array}{r} 15.13877\ 11331\ 89030\ 86048 \\ -\ 2.30258\ 50929\ 94045\ 68402 \\ -\ 1.03051\ 03088\ 84078\ 40331 \\ -\ .00006\ 66666\ 66666\ 66667 \\ -\ .00000\ 31680\ 00000\ 00000 \\ -\ .00000\ 00059\ 39751\ 82222 \\ +\ .00000\ 00001\ 77894\ 60480 \\ +\ .00000\ 00000\ 01115\ 32887 \\ -\ .00000\ 00000\ 00032\ 28031 \\ -\ .00000\ 00000\ 00000\ 45844 \\ +\ .00000\ 00000\ 00000\ 01166, \end{array}$$

and their sum is

$$11.80560\ 58908\ 83465\ 49\cdots.$$

Thus

$$I_{100}(75) = 134001.44880\ 18810.$$

D. H. L.

[1] Such Bessel functions have been used recently at the statistical laboratory of the University of California in preparation of certain statistical tables to appear in *Annals of Math. Statistics*. See also: J. WISHART, "A note on the distribution of the correlation ratio," *Biometrika*, v. 24, 1932, p. 454, formula (27).

[2] The most extensive tables of $I_n(x)$ are in B.A.A.S., *Math. Tables*, v. 6, *Bessel Functions*, part I, Cambridge, 1937, Tables VI and VIII.

[3] J. W. NICHOLSON, *Phil. Mag.* s. 6, v. 20, 1910, p. 938–943.

[4] The $Q$'s may be checked by the relation $Q_m(1) = 4^m$; also $m(m+1)\int_0^1 t^{2m-1}Q_m(t^{-2})dt = 2^{2m+1}|B_{m+1}|$, where $B_k$ is the $k$th Bernoulli number, in the notation of Lucas.

[5] D. F. E. MEISSEL, *Astr. Nach.* v. 130, 1892, cols. 363–4.

[6] G. N. WATSON, *A Treatise on the Theory of Bessel Functions*, Cambridge 1922, p. 228.

# Mathematical Tables in Phil. Mag.

The important scientific periodical *London, Edinburgh & Dublin Philosophical Magazine and Journal of Science*, has been published under different titles since 1798, in 7 series of volumes (varying in number from 11 to 68). In recent years the usual print abbreviation of the title of this periodical has been *Phil. Mag.* In what follows it is proposed to list, with a few added notes, mathematical tables in *Phil. Mag.*, s. 3, v. 18 to s. 7, v. 34, no. 235, inclusive, 1841–1943. It will be observed that more than half of the Tables listed were published after the last world war started in 1914, a period of unparalleled scientific development. Many mathematical tables have been already published as the result of problems arising in the prosecution of the present war; the number is likely to be greatly increased in the next few years.

## S. 3

**1.** G. B. AIRY "On diffraction of an annular aperture," v. 18, 1841, p. 7. Table of $E[= J_0(e)] = (1/2\pi)\int_0^{2\pi}\cos(e\cos\theta)d\theta$, and of $E^2$, for $e = [0.0(0.2)10.0; 4D]$. Airy had an earlier table of $2J_1(e)/e$ of the same scope in Cambridge Phil. So., *Trans.*, v. 5, 1835, p. 291. In B.A.A.S., *Math. Tables*, v. 6, *Bessel Functions*, Cambridge, 1937, $J_0$ is given, among other values, for $e = [0.0(0.2)25.0; 8D]$. The value of $J_0(40)$, to $7D$, was computed by W. R. Hamilton, *Phil. Mag.*, s. 4, v. 14, 1857, p. 381.

2. J. W. LUBBOCK, "On the determination of the numerical values of the coefficients in any series consisting of sines and cosines of multiples of a variable angle," v. 33, 1848, p. 106–127. "In the following methods of developing any function of sines and cosines by means of particular values of the function, the artifice by which the coefficients are successively eliminated is taken from Leverrier's 'Développements sur plusieurs points de la théorie des perturbations des planètes.' " Lubbock gives various tables for evaluating numerical constants when the method is practically employed.

## S. 4

3. J. W. L. GLAISHER, "On a class of definite integrals—Part II," v. 42, 1871, p. 431–436, "Tables of the error function." Values of $\int_x^\infty e^{-u^2}du$ are given for $x$ = [3.00(0.01)4.50; 7*S*]. The range $x$ = [0.00(0.01)3.00; 8*D* to 11*D*] was covered in a table, with first three differences, in C. KRAMP, *Analyse des Réfractions Astronomiques et Terrestres*, Leipzig and Paris, 1799, p. 195–202. In A. A. MARKOV, *Table des Valeurs de l'Intégrale* $\int_x^\infty e^{-t^2}dt$, St. Petersburg, 1888, $x$ = [0.000(0.001)3.00(0.01)4.80; 11*D*], and first, second, and third differences are given.

## S. 5

4. H. M. TAYLOR, "On the relative values of the pieces in chess," v. 1, 1876, p. 229. Table of "chance of the king being in check" for chessboard of $n^2$ squares ($n$ even).

5. J. W. STRUTT, *baron* RAYLEIGH, "Investigations in optics, with special reference to the spectroscope," v. 8, 1879, p. 266–268. For varying values of $u$, $u^{-2}\sin^2 u$, to 4*D*, is given in T. I; $f = u^{-1}\left(\sin u - \sin\frac{u}{6}\right)$, as well as $(f/f_0)^2$, are given to 4*D* in T. II; and $f = u^{-1}\left(\sin u - 2\sin\frac{u}{8}\cos\frac{u}{3}\right)$, as well as $(f/f_0)^2$ are given in T. III. Also in *Sci. Papers of John William Strutt*, v. 1, Cambridge, 1899, p. 419–422.

6. G. B. AIRY and W. ELLIS, "On a systematic interruption in the order of numerical values of vulgar fractions when arranged in a series of consecutive magnitudes," v. 12, 1881, p. 175–178. Tabular selections from their "Logarithms of the values of all vulgar fractions, with numerator and denominator not exceeding 100, arranged in order of magnitude," Institution of Civil Engineers, *Minutes of Proc.*, v. 65, 1881, p. 271–298; 3043 fractions. Compare no. **7**, and RMT **87**.

7. J. J. SYLVESTER, "On the number of fractions contained in any 'Farey series' of which the limiting number is given," v. 15, 1883, p. 251–257. A Farey series (see *Phil. Mag.*, s. 1, v. 47, 1816, p. 385) is a system of all the unequal vulgar fractions arranged in order of magnitude, the numerator and denominator of which do not exceed a given number, $j$. The number of fractions in the series for the limit $j$, is identical with the sum of the totients of all the natural numbers up to $j$ inclusive—a totient of $n$, denoted by $\tau(n)$, meaning the number of numbers not exceeding $n$ and prime to it, $T(j) = \sum_{n=1}^{n=j}\tau(n)$. There is a table for $n$ = 1(1) 500, of $\tau(n)$, $T(n)$, $(3/\pi^2)n^2$. In Sylvester's *Coll. Math. Papers*, v. 4, Cambridge, 1912, p. 103–109 this table is extended to $n$ = 1000. For $n$ = 688, *for* $\tau(n)$ 536, *read* 336. This error is noted in J. W. L. GLAISHER, *Number-Divisor Tables* (B.A.A.S., *Math. Tables*, v. 8), Cambridge, 1940, where Sylvester's table is extended to $n$ = 10 000. See further D. H. LEHMER, *Guide to Tables in the Theory of Numbers* (Nat. Res. Council *Bull.* no. 105), Washington, D. C. 1941. Compare no. **6**.

8. C. E. HOLLAND, P. R. JONES, C. G. LAMB, "Table of zonal spherical harmonics with a short explanation and some illustrations of its use by John Perry." v. 32, 1891, p. 512–523. Also in Phys. So. London, *Proc.*, v. 11, 1892, p. 221–233. This contains the first table of $P_n(\cos\theta)$, $n$ = [1(1)7; 4*D*], $\theta = 0°(1°)90°$. It was reprinted in W. E. BYERLY, *An Elementary*

*Treatise on Fourier's Series and Spherical, Cylindrical, and Ellipsoidal Harmonics . . .*, Boston, 1895, p. 278–279. A. H. H. TALLQVIST (Soc. Sci. Fennicae, *Acta*, v. 33, no. 4, 1908) pointed out the numerous and non-trivial errors in this table which Byerly faithfully reproduced except in one case, $P_4(\cos 7°)$, where he correctly put $-0.0038$ for 0.0038 in the original. Of the 92 errors there are 38 in $P_6(\cos\theta)$ and 48 in $P_7(\cos\theta)$. Of the worst errors samples are as follows: $P_2(\cos 76°)$, *for* $-0.4112$, *read* $-0.4122$; $P_6(\cos 7°)$ *for* 0.8476, *read* 0.8492; $P_4(\cos 47°)$ *for* $-0.4252$, *read* $-0.4227$; $P_7(\cos 7°)$ *for* 0.7986, *read* 0.8016; $P_7(\cos 18°)$ *for* 0.0289, *read* 0.0248. A correct form of the table, including $P_8(\cos\theta)$, is given in Tallqvist's *Grunderna af Teorin för Sferiska Funktioner . . .*, Helsingfors, 1905, p. 386–387.

**9.** J. W. STRUTT, *baron* RAYLEIGH, "On the instability of a cylinder of viscous liquid under capillary force," v. 34, 1892, p. 152–153. There is a table of $x^2 + 1 - x^2 I_0^2(x)/I_1^2(x) = x^2[J_0^2(ix)/J_1^2(ix) + 1 + 1/x^2]$, for $x$ = 0.0, 0.2, 0.4, 0.6, 1.0, 2.0, 4.0, 6.0, to 3*D* or 4*D*. Also *Scientific Papers*, v. 3, 1902, p. 591–592.

**10.** H. A. ROWLAND, "Gratings in theory and practice," v. 35, 1893, p. 414. Table of "ghosts" (Bessel functions) $J_n^2$, for $n$ = [0(1)14; 3*D*] and varying values of the parameter 0.0 to 10.0.

**11.** J. PERRY and H. F. HUNT, "The development of arbitrary functions," v. 40, 1895, p. 508–509. Four tables of $xJ_1(x)$ to 4*D*, for varying values of a parameter.

**12.** H. NAGAOKA, "Diffraction phenomena in the focal plane of a telescope with circular aperture, due to a finite source of light," v. 45, 1898, p. 9. Tables of $I(r) = 1 - J_0^2(r) - J_1^2(r)$ for varying values of $r$, to 5*D*.

**13.** B. A. SMITH, "Tables of $\kappa J_0(x) - Y_0(x)$ and $\kappa J_1(x) - Y_1(x)$," v. 45, 1898, p. 122–123. In an article by J. H. Michell on the "wave-resistance of a ship." Tables for $x$ = [0.00(.01)10.3; 4*D*]. $\kappa J_0(x) - Y_0(x) = \int_0^\infty e^{-x\sinh\theta}d\theta - \int_0^{\pi/2}\sin(x\sin\theta)d\theta$. Smith's tables of $Y_0(x)$ and $Y_1(x)$, for $x$ = [0.00(.01)10.2; 4*D*], are given in *Mess. Math.*, n.s., v. 26, 1896, p. 98–101. This was the earliest table of functions of the second kind.

**14.** K. PEARSON and A. LEE, "On the criterion that a given system of deviations from the probable in the case of a correlated system of variables is such that it can be reasonably supposed to have arisen from random sampling," v. 50, 1900, p. 175. Improbability

$$P = e^{-\frac{1}{2}\chi^2}\left(1 + \frac{\chi^2}{2} + \frac{\chi^4}{2\cdot 4} + \cdots \frac{\chi^{n'-3}}{2\cdot 4\cdot 6\cdots(n'-3)}\right) \quad \text{for } n' \text{ odd;}$$

probability

$$P = (2/\pi)^{\frac{1}{2}}\int_\chi^\infty e^{-\frac{1}{2}\chi^2}d\chi + (2/\pi)^{\frac{1}{2}}e^{-\frac{1}{2}\chi^2}\left(\frac{\chi}{1} + \frac{\chi^3}{1\cdot 3} + \frac{\chi^5}{1\cdot 3\cdot 5} + \cdots \frac{\chi^{n'-3}}{1\cdot 3\cdot 5\cdots(n'-3)}\right) \quad \text{for } n' \text{ even.}$$

Table of $P$ for $n'$ = 3(1)20, $\chi^2$ = 1(1)10(5)30(10)70.

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**15.** W. B. MORTON, "On the propagation of polyphase currents along a number of parallel wires," v. 1, 1901, p. 566. $\eta = (\sin\pi/n)^{-2\cos(2q\pi/n)}[\sin(2\pi/n)]^{-2\cos(4q\pi/n)}$. Table of $\eta$ for $q$ = 1(1)6, $n$ = 2(1)12.

**16.** H. HILTON, "A note on van der Waals' equation" $\left[y = \frac{8\theta}{3x-1} - \frac{3}{x^2}\right]$, v. 1, 1901, p. 579–589. Tables I–II for $8\theta$ = 4, 5, 6, $\cdots$16, 24, 32, 40, 48, $\cdots$80, $-$ 8, for $x$ = .4, $\cdots$, 5.0; $8\theta$ = 4, $-$ 8, for $x$ = .3 $\cdots$ .1, $-$ .5, $\cdots -$ 5.00. Graphs. There are further tables and graphs in Hilton's "A further note on van der Waals' equation," v. 2, 1901, p. 108–118. See also various tables in J. P. Dalton, "On the saturation constants according to van der Waals' equation," v. 13, 1907, p. 517–522.

**17.** A. A. ROBB, "On the conduction of electricity through gases between parallel plates —part II," v. 10, 1905, p. 672–675. A table gives $P$, $Q$, $R$, $S$, $T$, in terms of $\cosh\omega$

$= [1.0(0.1)10; 3D]$;

$$P = \frac{4}{5}\cosh^{9/5}\omega \int_\omega^\infty \frac{d\omega}{\cosh^{9/5}\omega}; \qquad Q = \cosh^{9/5}\omega; \qquad R = \int_0^\omega \left(\frac{9}{5}\cosh^{9/5}\omega \int_\omega^\infty \frac{d\omega}{\cosh^{9/5}\omega}\right) d\omega,$$

$$S = \frac{9}{4}\int_0^\omega \cosh^{9/5}\omega d\omega; \qquad T = \frac{S}{Q}\left(\frac{1}{\tanh\omega} - P\right) + R.$$

**18.** H. G. SAVIDGE, "Tables of the ber and bei and ker and kei functions, with further formulae for their computation," v. 19, 1910, p. 49–58; also in Phys. So. London, *Proc.*, v. 22, 1910, p. 105–114. Tables of ber $x$, bei $x$, ker $x$, kei $x$, for $x$ = [1(1)30; 4S]; other tables, p. 54–57, with a similar range. SAVIDGE gave also tables of ker and kei and their first derivatives for $x$ = [0.1(0.1)10.0; 7D–9D], in B.A.A.S., *Report*, 1915, p. 36–38.

**19.** J. R. AIREY, "Tables of Neumann functions $G_n(x)$ and $Y_n(x)$," v. 22, 1911, p. 658–663. Tables of $G_0(x)$, $G_1(x)$, $Y_0(x)$, $Y_1(x)$, for $x$ = [0.1(0.1)16; 7D].

**20.** P. F. WARD, "The transverse vibrations of a rod of varying cross-section," v. 25, 1913, p. 85–106. Various tables corresponding to assumed forms of the rod; Bessel functions basic.

**21.** J. R. AIREY, "Bessel and Neumann functions of equal order and argument," v. 31, 1916, p. 521–527. The following results are given: first root of $J_{1000}(z) = 0$, to 2D; the values of $J_n(n)$ for $n$ = 6 (6D), $n$ = 48 (9D), $n$ = 750 (14D), $n$ = 109 (6D). Also of $J_n(109)$ for $n$ = [100(1)108; 6D]. Also of $G_n(n)$ and $G_{n-1}(n)$ for $n$ = [7(1)13; 6D]. And of $G_n(104)$ for $n$ = [100(1)107; 6D]. See B.A.A.S., *Report*, 1916, p. 92–96 where AIREY has elaborate tables of $J_n(n)$, $J_{n-1}(n)$, $G_n(n)$, $G_{n-1}(n)$, $Y_n(n)$, $Y_{n-1}(n)$ for $n$ = [1(1)50(5)100(10)200(20)-400(50)1000(100)2000(500)5000(1000)20000(5000)30000(10000) 50000, 100000, 500000, 1000000; 6D]. Compare nos. **23**, **37**.

**22.** J. R. AIREY, "The roots of Bessel and Neumann functions of high order," v. 32, 1916, p. 7–14. The first five zeros of $J_{10}(x)$, $J_{100}(x)$, $J_{1000}(x)$, $G_{100}(x)$, and $Y_{100}(x)$ are given to 6S. AIREY'S "The roots of the Neumann and Bessel functions," Phys. So. London, *Proc.*, v. 33, 1911, p. 219–24, has the following tables: (a) The first 40 zeros of $Y_0(x)$, $Y_1(x)$, $Y_2(x)$, to 5D; (b) The first 10 zeros of $N_0(x)$, $N_1(x)$, $J_0(x) + Y_0(x)$, $J_1(x) + Y_1(x)$, $J_0(x) - Y_0(x)$, to 5D.

**23.** G. N. WATSON, "Bessel functions of equal order and argument," v. 35, 1918, p. 369. Incidentally there is a table of $n\int_0^1 J_n(nx)dx$ for $n$ = [1(2)23; 7D]. Compare no. **21**.

**24.** H. A. WEBB and J. R. AIREY, "The practical importance of the confluent hypergeometric function," v. 36, 1918, p. 137–141, and plate VI. Table of $M(\alpha, \gamma, x)$ for $\gamma$ = [1(1)7; 4S], $\alpha = -$ 3.0(0.5)4.0, $x$ = 1(1)6(2)10.

**25.** J. R. AIREY, "The addition theorem of the Bessel functions of zero and unit orders," v. 36, 1918, p. 238–242. $J_0(x)$, $G_0(x)$, $Y_0(x)$, $J_1(x)$, $G_1(x)$, $Y_1(x)$ are given for $x$ = 9 and 10, to 14D; also auxiliary tables for the computation.

**26.** J. R. AIREY, "Bessel functions of small fractional order and their application to problems of elastic stability," v. 41, 1921, p. 201–205. First and second zeros of $J_n(x)$ are given for $n$ = [$\frac{3}{4}$, $\frac{2}{3}$, $\frac{1}{3}$, $\frac{1}{4}$; 2D] (also the first three zeros for $n$ = [$-\frac{3}{4}$, $-\frac{2}{3}$, $-\frac{1}{3}$, $-\frac{1}{4}$; 2D]), and $n$ = [$-\frac{1}{2}$(0.025)1; 4D], uncertain fourth decimal places, and the first zeros of $J_{+1/3}(x)$ and $J_{+1/6}(x)$, being also indicated. This last table is reprinted (without indicated uncertainties) in the work of Gray-Mathews-Macrobert, T. XIII, p. 317. Airey gives also the second zero, and $(2n)^{-\frac{1}{2}} \times$ first zero, of $J_n'(x)$ for $n$ = [$-\frac{1}{2}$(0.025)$\frac{1}{2}$; 4D]; and [$\frac{1}{4}(n + 1)$] $\times$ the square of the first zero of $J_n(x)$ for $n$ = [$-$ 1(0.025)0; 4D]. Compare no. **27**.

**27.** A. ONO, "On the first root of Bessel functions of fractional order," v. 42, 1921, p. 1021. With reference to Airey's paper (no. **26**), gives first zeros of $J_{-1/2}$, $J_{-2/7}$, $J_{-1/3}$, $J_0$, $J_{1/2}$, $J_1$; 3D.

**28.** H. E. H. WRINCH and D. M. WRINCH, "Tables of the Bessel functions $I_n(x)$," v. 45, 1923, p. 846–849. Table I: $I_n(x)$, $n$ = 0(1)6, $x$ = [5(1)15; 6S]; T. II: Log $\Gamma(x + 1)$ for

$x = [1(1)28; 7D]$; T. III: $(2\pi x)^{\frac{1}{2}}$, $e^x$, $e^{-x}$, $e^x/(2\pi x)^{\frac{1}{2}}$, $e^{-x}/(2\pi x)^{\frac{1}{2}}$ for $x = [1(1)20; 6S]$. See also no. **30.**

**29.** F. F. P. BISACRE, "The calculation of the skin effect in electrical conductors," v. 45, 1923, p. 1035–1038. There are tables (independently checked) for the following functions in $[r, \theta]$ and $[a + ib]$ forms: Table I: $J_0(\sqrt{-i}\,x)$, $J_1(\sqrt{-i}\,x)$; T. II: $H_0^{(2)}(\sqrt{-i}\,x)$, $H_1^{(2)}(\sqrt{-i}\,x)$; for $x = [0.0(0.2)6.0$; 4 or 5S]. The tables for the $J$'s in the $[r, \theta]$-form were calculated from the corresponding tables in the $(a + ib)$-form, due to W. S. ALDIS, R. So. London, *Proc.*, v. 66, 1900, p. 42–43. Some columns in Table II were taken from JAHNKE and EMDE, *Funktionentafeln*, Leipzig, 1909, p. 139–140.

**30.** H. E. H. WRINCH and D. M. WRINCH, "Tables of Bessel functions," v. 47, 1924, p. 62–65. Table I: $I_n(x)$, $n = 0(1)6$, $x = [16(1)37; 6S]$; T. II: $(2\pi x)^{\frac{1}{2}}$, $e^x$, $e^x/(2\pi x)^{\frac{1}{2}}$ for $x = [21(1)40; 6S]$; T. III–IV: $J_2(x)/I_2(x)$, and $J_3(x)/I_3(x)$, for $x = [1(1)15; 6D]$. The graphs of the functions in Tables III–IV are also given. These functions are of importance in certain problems in acoustics; see R. So. London, *Proc.*, v. 101A., 1922, p. 493–508. See also no. **28.**

**31.** S. P. OWEN, "Table of values of the integral $\int_0^x K_0(t)dt$," v. 47, 1924, p. 736. Table for $x = [0.02, 0.1, 0.5, 1(1)12$; 6S-7S].

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**32.** H. E. H. WRINCH and D. M. WRINCH, "The roots of hypergeometric functions with a numerator and four denominators," v. 1, 1926, p. 273–276. First paragraph: "During the last few years, the Generalized Hypergeometric Function has become of increasing importance in applied mathematics. This paper contains certain tables of the roots of certain generalized hypergeometric functions which have important applications in the theory of the vibrations of bars of various cross-sections. Attention was called to the fundamental significance in problems of morphology, of the periods of vibration of bars of varying density with cross-sections of various types, by [A.] DENDY and [J. W.] NICHOLSON [R. So. London, *Proc.*, v. 93A and 89B] in 1917." In this paper are given the first nine roots of the equation $F(\alpha; \alpha_1, \alpha_2, \alpha_3, \alpha_4; -4(x/4)^4) = 0$ for a varied range of values of the parameters.

**33.** H. P. MULHOLLAND and S. GOLDSTEIN, "The characteristic numbers of the Mathieu equation with purely imaginary parameter," v. 8, 1929, p. 839. Tables are found for the first eight characteristic numbers for purely imaginary $q$ in the Mathieu equation

$$\frac{d^2y}{dx^2} + (4\alpha - 16q\cos 2x)y = 0.$$

**34.** S. HIGUCHI, "On some closed algebraic curves and their application to dynamical problems," v. 9, 1930, p. 186–190. Tables for solutions of the equation $x^{2n} + y^{2n} = 1$, for the area of this curve, and for the volumes of its revolution about $x$ and $y$ axes; etc., for different values of $n$.

**35.** N. W. MCLACHLAN and A. L. MEYERS, "The polar form of the ker and kei functions, with applications to eddy current heating," v. 18, 1934, p. 621–624. There are tables of $N_0(z)$, $\phi_0(z)$ (degrees), $N_1(z)$, $\phi_1(z)$ (degrees), for $z = [0.0(0.1)10.0$; 5D to 8D].

$$N_0(z) = (\text{ker}^2 z + \text{kei}^2 z)^{\frac{1}{2}}; \qquad \phi_0(z) = \tan^{-1}\frac{\text{kei}\, z}{\text{ker}\, z};$$

$$N_1(z) = (\text{ker}'^2 z + \text{kei}'^2 z)^{\frac{1}{2}}; \qquad \phi_1(z) = \tan^{-1}\frac{\text{kei}'\, z}{\text{ker}'\, z} + \tfrac{1}{4}\pi.$$

**36.** J. R. AIREY, "Toroidal functions and the complete elliptic integrals," v. 19, 1935, p. 177–188. The tabulation of toroidal functions which satisfy Laplace's equation and are suitable for conditions over the surfaces of anchor-rings, is most conveniently and accurately effected, especially when the argument is large, through the complete elliptic integrals of the first and second kind, $K$ and $E$. $K(k^2) = K_1\cdot\ln(4/h^{\frac{1}{2}}) - K_2$; $E(k^2) = E_1\cdot\ln(4/h^{\frac{1}{2}}) + E_2$

Table I gives $K_1$ and $K_2$ with second differences, for $h$ = [0.00000(0.00001)0.0001(.0001) 0.001(.001)0.170; 10, or more, D]; T. II gives, to 10 or more D, with second differences, $E_1$ and $E_2$ for $h$ = 0.00000(0.00001)0.0001(0.0001)0.001(0.001)0.100.

**37.** J. R. Airey, "Bessel functions of nearly equal order and argument," v. 19, 1935, p. 233–235. There are tables, to 6D, of $J_\nu(\mu)$ for $\mu = 1(1)20$; $\nu = \overline{\mu - 1}(0.1)\overline{\mu + 1}$. Cf. no. **21**.

**38.** J. R. Airey, "The Bessel function derivatives $\frac{\partial J_\nu(x)}{\partial x}$ and $\frac{\partial^2 J_\nu(x)}{\partial x^2}$," v. 19, 1935, p. 236–243. Tables to 4D or 5D for $x = 1(1)20$; $\nu = \overline{x - 1}(0.1)\overline{x + 1}$.

**39.** L. Silberstein, "On complex primes," v. 19, 1935, p. 1104. The table gives the decomposition of primes less than 1000 of the form $4n + 1$ into sums of two squares, part of the author's ms. table for $P \leqq 25{,}033$.

**40.** W. G. Bickley and J. Nayler, "A short table of the functions $Ki_n(x)$, from $n = 1$ to $n = 16$," v. 20, 1935, p. 343–347. $Ki_n(x) = \int_0^\infty \frac{e^{-x\cosh u}du}{\cosh^n u}$, $Ki_0(x) = K_0(x)$, the Bessel function of the second kind, with imaginary argument, as defined by Watson. The tables are for $x$ = [0.00(0.05)0.2(0.1)2.0, 3.0; 9D]; $n$ = 1(1)16.

**41.** L. Silberstein, "The roots of $\cos z = z$," v. 20, 1935, p. 528–531. Approximations to the real root, to 5D, and six complex roots, $z = x + iy$ for $x$ negative, are found. The real root, to 8D, and 12 other complex roots for $x$ positive, were found much earlier by T. H. Miller, "On the numerical values of the roots of the equation $\cos x = x$," Edinburgh Math. So., *Proc.*, v. 9, 1891, p. 80–83. Compare *Scripta Math.*, v. 4, 1936, p. 100. See also R. Cooper and J. Todd, "The large roots of $\cos z = az+c$," v. 21, 1936, p. 249–262. Compare no. **50.**

**42.** J. R. Airey, assisted by L. J. Comrie, "The circular and hyperbolic functions, argument $x/\sqrt{2}$," v. 20, 1935, p. 721–731. There are here two tables: the first, p. 722–726, of sin $(x/\sqrt{2})$ and of cos $(x/\sqrt{2})$ for $x$ = [0.0(0.1)20.0; 12D]; the second, p. 726–731, of sinh $(x/\sqrt{2})$ and cosh $(x/\sqrt{2})$ for the same ranges. $\text{Sin}\,(x/\sqrt{2}) = \sqrt{2}[J_1(x) + J_3(x) - J_5(x) - J_7(x) + J_9(x) + \cdots]$ and $\cos\,(x/\sqrt{2}) = J_0(x) - 2[J_4(x) - J_8(x) + J_{12}(x) - J_{16}(x) + \cdots]$. Compare *Scripta Math.*, v. 4, 1936, p. 101–102.

**43.** J. R. Airey, "The circular sine and cosine functions, argument $\log_e x$," v. 20, 1935, p. 731–738. Bessel, Neumann, Struve, and other related functions of imaginary order $i\nu$, in ascending powers of the real variable $x$, include the factor $x^{i\nu}$, the tabulation of which involves $\cos(\nu \ln x)$ and $\sin(\nu \ln x)$. There are the following four tables: T. I: $\sin\ln(1+\rho)$ and $\cos\ln(1+\rho)$ for $\rho$ = [0.000(0.001)0.020; 10D]; T. II: $-\sin\ln(1-\rho)$ and $\cos\ln(1-\rho)$ for the same range; T. III: $\sin\ln x$ and $\cos\ln x$ for $x$ = 2, 5, 10, 100, 1000, $\pi$, to 13D or 14D; T. IV: $\sin\ln x$ and $\cos\ln x$, for $x$ = [0.1(0.1)20.0; 10D].

**44.** V. M. Falkner, "A method of numerical solution of differential equations," v. 21, 1936, p. 624–640. In connection with the discussion of solutions of $Y'' - YY' = 0$, and of $Y''' - YY'' = 0$, tables are given for $Y$, $Y'$, $Y''$, $Y'''$ for varying values of $X$.

**45.** R. C. Colwell and H. C. Hardy, "The frequencies and nodal systems of circular plates," v. 24, 1937, p. 1046. On this page are given, incidentally, two tables; in one, the values of the modified Bessel functions $I_0(x)$, for $x$=0, 1, 5, 10, 20, 30; in the second the first ten roots of $J_n(x) = 0$ for $n$ = [0(1)5; 3D], 34 of 60 values incorrect; cf. MTE **21**.

**46.** J. R. Airey, "The radiation integrals $\int_x^\infty \frac{dx}{x^\alpha(e^x - 1)}$," v. 25, 1938, p. 273–282. These integrals "occur in a number of physical and astronomical problems, *e.g.*, when $n = 1$, the X-ray analysis of crystal structures; when $n = 2$, the radiative equilibrium of a planetary nebula and the temperatures of stars and novae; when $n = 3$, temperature of the nuclei of planetary nebulae and the exchange of energy between monatomic gases and solid surfaces." $x$ = [0.0(0.1)10.0; 6D], $\alpha = 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, 2$. Compare v. 34, 1943, p. 602–606.

**47.** A. FLETCHER, "A table of complete elliptic integrals," v. 30, 1940, p. 516–519. "The following table was made in connection with work on Laplace coefficients in dynamical astronomy. Doubtless this is not the only case in which the modulus $k$ is preferable to $\sin^{-1}k$ or $k^2$ as argument, and the functions of the complementary modulus $k'$ are not required." $K = \pi/2M$. A table of $K$, $E$, and $M$, is given for $k$ = [0.00(0.01)1.00; 10D]. See also Fletcher's "Tables of the two chief Laplace coefficients," R.A.S., *Mo. Notices*, v. 99, 1939, p. 259–265.

**48.** W. G. BICKLEY and J. C. P. MILLER, "Numerical differentiation near the limits of a difference table," v. 33, 1942, p. 1–14 + 4 folding plates. See RMT **93**.

**49.** C. A. COULSON and W. E. DUNCANSON, "Some new values for the exponential integral," v. 33, 1942, p. 754–760. In T. I, $Ei(x)$ and $-Ei(-x)$ are given for $x$ = [15(1)50; 10S]. B.A.A.S., *Mathematical Tables*, v. 1, London, 1931, and MATH. TABLES PROJECT, New York, *Tables of Sine, Cosine, and Exponential Integrals*, 2 v. 1940, give values of these functions for $0 < x < 15$. J. P. GRAM, "Undersøgelser angaaende Mængden af Primtal under en given Grænse," Danske Vidensk. Selskabs, *Skrifter*, s. 6, *Naturvid. og Math.*, v. 2, p. 268–272, gives $Ei(x)$ for $x$ = [10(1)20; 20D], [5.0(0.2)20; 10D] with first differences. But $-Ei(-x)$ in the range $15 < x < 20$ had not been earlier computed. The only previous table of $-Ei(-x)$ for $x > 20$ is that of T. AKAHIRA, "Tables of $e^{-x}/x$ and $\int_x^\infty e^{-u}du/u$ from $x = 20$ to $x = 50$," Inst. Phys. Chem. Research, Tokyo, *Sci. Papers*, *Table* no. 3, 1929; $x$ = [20.00(0.02)50.00; 6S, 5S] with first and second differences. Coulson and Duncanson make no direct reference to the paper of Gram, from which it is clear that their last unit in $Ei(20)$ should be 6, not 7. "$Ei(x)$ is repeatedly needed in the evaluation of integrals concerned with nuclear structure, molecular structure and flow of heat."

**50.** A. P. HILLMAN and H. E. SALZER, "Roots of $\sin z = z$," v. 34, 1943, p. 575. In this article is a table of the first ten non-zero roots of $\sin z = z$, in the first quadrant, to 6D. Compare no. **41**.

*Names*



R. C. A.