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## NUCLEAR CONSTANTS

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NUCLEAR CONSTANTS

## Issue No. 7

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## DELAYED NEUTRONS AND THE PHYSICS OF FISSION

## B. P. Maksyutenko

## New method for determining relative yields of delayed neutrons

In reactor kinetics calculations it is necessary to know the absolute and relative yields of delayed neutrons resulting from the fission of various substances by neutrons of different energies. The yield of delayed neutrons from the $i^{\text {th }}$ precursor $Y_{i}$ can be found if one known the yield of a given mass in fission, $P\left(A_{i}\right)$, the probability $P\left(Z_{i}, Z_{p i}\right)$ of a given charge $Z_{i}$ occurring in the given mass $A_{i}$ in the case of the most probable charge $Z_{p i}$, and the probability of radiation of delayed neutrons by this precursor $P_{n i}$ :

$$
\begin{equation*}
Y_{i}=P\left(A_{i}\right) P_{n i} \Sigma P\left(Z_{i}, Z_{p i}\right) \tag{1}
\end{equation*}
$$

where $\Sigma$ denotes that a cumulative yield is being considered. Since $P_{n i}$ is independent of energy, $P\left(A_{i}\right)$ can be determined experimentally, and we would only have to find $P\left(Z_{i}, Z_{p i}\right)$ in order to calculate $Y_{i}$ for any energy of fission-producing neutrons. However, the intensity of existing fast neutron sources is not high enough to make this feasible. At the present time the charge distribution has been established only for the case of ${ }^{235} \mathrm{U}$ fission by thermal neutrons and 15 MeV neutrons. Thus we are left with the purely experimental method of determining the delayed neutron yields. This involves analysing the whole of the delayed neutron decay curve and determining the ratio of the yields from different precursors (for example, $Y_{i} / Y_{1}$, where $Y_{1}$ is the yield of a group with half-life $\sim 55 \mathrm{sec}$ ), and then finding the total absolute yield (from all precursors together, in another experiment). These data may be also used to calculate the absolute yield of delayed neutrons from any precursor.

With the aid of radiochemical and mass spectrometric methods it has so far been possible to identify 37 fragment-precursors, i.e. to establish their $Z_{i}$ and $A_{i}$ values and, in addition, their $P_{n i}$ values and their half-lives. Unfortunately expansion of the decay curve by the best available method the least squares method - serves to segregate only six contributors at most. Hence we get only a very rough solution, because each of the six separate groups represents the relative yield of delayed neutrons from several precursors. It would be desirable to segregate a larger number of these in the expansion. M. Z. Tarasko has found a way of doing this $[1,2]$ and has demonstrated its possibilities by obtaining a numerical solution for a system of equations with a thirtieth-order Gilbert matrix. Now the question is to what extent a given specific statistical selection (i.e. a given experimental result) is adequate for extracting a larger number of exponents. This will be illustrated below by comparing the results obtained by expansion of the delayed neutron decay curve resulting from thermal fission of ${ }^{235} U$ with radiochemical data. But now let us consider the possibilities offered by this expansion.

In Table 1 the delayed neutron precursors are arranged in descending order of half-life. Some of them have widely differing half-lives whilst others agree within the limits of experimental error. The latter have been grouped together. In the expansion of the decay curve, as we can see from Table l, the net yields of three bromine isotopes (with masses $87,88,89$ ) can be determined. Thence we can find the probability of a given charge occurring in fission:

$$
\begin{equation*}
P\left(Z_{i}, Z_{p i}\right)=\frac{Y_{i}}{P_{n i} P\left(A_{i}\right)} \tag{2}
\end{equation*}
$$

where the right-hand side is determined entirely from experimental data. Since from Eq. (2) we know that

$$
\begin{equation*}
P\left(Z, Z_{p}\right)=\frac{1}{\sigma \sqrt{2} \pi} \exp \left\{-\frac{\left(z-Z_{p}\right)^{2}}{2 \sigma \frac{2}{Z}}\right\},\left(\Delta Z_{p}=Z-Z_{p}\right) \tag{3}
\end{equation*}
$$

and since from Eq. (3) it is apparent that

$$
\begin{equation*}
Z_{p}=k A+\ell \tag{4}
\end{equation*}
$$

we can now proceed to the $P(Z, A)$ dependence $[4]$ " The scheme for this transformation of co-ordinates, i.e. the transition from $A=$ const., assumed by wahl [2], to $Z=$ const., is illustrated in Fig. 1.

Such a transformation results in a quadratic parabola on the semilogarithmic scale:

$$
\begin{equation*}
\ln P(Z, A)=a A^{2}+b A+q \tag{5}
\end{equation*}
$$

where the coefficients $a, b$ and $q$ are a combination of the coefficients k , $\ell$ and $\sigma$. Assuming $\mathrm{A}=87,88$, etc. and solving the system of equations in expression (5), we obtain the values of $k, \ell$ and $口$ as well as $Z_{p}$ and $\Delta Z_{p}$ for the bromine isotopes.

In addition, we can find the distribution of the probabilities of radiation along the $A$ axis for these fragments (sirice $Z=$ const.):

$$
P\left(A_{i}\right)_{Z=\text { const. }}=Y_{i} / P_{n i}
$$

Thus, as shown schematically in Fig. 2, we find two distributions,

$$
P\left(A_{i}\right)_{Z}=\text { const. and } P\left(Z_{i}\right)_{A}=\text { const. }
$$

in two mutually perpendicular planes. If the primary mass distribution is used in this construction, we obtain the fragment energy surface without allowance for the gamma radiation energy.

Now let us turn again to the determination of $Z_{p}$ for fragments and to delayed neutron yields from the other precursors. The values of $P\left(Z, A_{i}\right)$ can be found for two isotopes of iodine (with masses 137 and 138) using expression (2) in the same way as for the bromine isotopes. Let us assume that the distribution width is the same in both cases (i.e. for bromine and iodine isotopes); it can easily be shown that the relationship between the values of $P(Z, A)$ for the three isotopes of bromine and iodine (the latter are denoted by istrokes) should then be the following:

$$
\begin{equation*}
\frac{P_{1} P_{3}}{P_{2}^{2}}=\frac{P_{1}^{\prime} P_{3}^{\prime}}{P_{2}^{2}} \tag{6}
\end{equation*}
$$

where the indices $1,2,3$ denote the values of $P(Z A)$ for bromine- 87,88 and 89 and iodine-137, 138 and 139. All the quantities in expression (6) are known except $P_{2}^{\prime}$ - the value of $P(Z, A)$ for iodine-139. By determining this
and then using the corresponding values of $P_{I I}$ and $P(A)$ for iodine-139, we find the delayed neutron yield from this precursor. In this way we isolate the contribution of the iodine-139 precursor from the delayed neutron yield of the group of precursors with a mean half-life of $\sim_{2} \mathrm{sec}$.

Furthermore, as can be seen from Table l, apart from the isotopes there are also isobars, for example bromine-87, selenium-87 and arsenic-87; bromine-88 and selenium-88, etc. Since

$$
\begin{equation*}
\frac{Y_{i}}{Y_{1}}=\frac{P_{n i}}{P_{n_{1}}} \cdot \frac{P\left(z-1, Z_{p}\right)}{P\left(Z, Z_{p}\right)} \tag{7}
\end{equation*}
$$

(the subscript 1 refers,for example, to bromine-87 and the subscript $i$ to selenium-87) and

$$
\begin{equation*}
\ln \frac{P\left(Z-1, Z_{p}\right)}{P\left(Z, Z_{p}\right)}=\frac{1}{\sigma 2}\left(Z-Z_{p}-0.5\right) \tag{8}
\end{equation*}
$$

it is possible, using the value of $Z_{p}$ determined for a fragment with mass 87 , to calculate from equation (8) the ratio of the probabilities of occurrence of a given charge and then to determine from equation (7) the ratio of the delayed neutron yield from selenium- 87 to that from bromine-87. A similar calculation can be done for ${ }^{87}$ As.

After isolating the delayed neutron contribution of the precursors ${ }^{139}$ I and ${ }^{88}$ Se in the group with a half-life of $\sim_{2}$ sec, we know that the remainder must be the contribution from the precursor ${ }^{85}$ As plus a slight addition from ${ }^{92} \mathrm{Kr}$. In the same way it is possible to separate the contribution of ${ }^{87}$ Se from ${ }^{93} \mathrm{Rb}$ and split up the group with a half-life of 5.9 sec.

We can sum up the procedure as follows. First we use the new method of expanding the decay curves, to classify the precursors according to the only criterion available - the precursor half-life. The second stage involves constructing the charge distribution for the bromine isotopes and then using it to calculate the delayed neutron yields from the pure precursors contained in the groups, which are a mixture of the yields from precursors with half-lives coinciding within the limits of experimental error.

From the above analysis it is clear that we can not only determine the delayed neutron yields from a larger number of precursors than is possible with the normal least squares method but that we can also find the charge distribution parameters for any energy of fission-inducing neutrons. Apart from the above formulae the following relationship derived from Eq. (3) may be used to determine $\Delta Z_{p}$ in the case of individual precursors:

$$
\begin{equation*}
2 \sigma^{2} \ln \frac{P\left(z_{i}, Z_{p i}\right)}{P\left(Z_{1}, Z_{p_{1}}\right)}=\Delta z_{p_{i}}^{2}-\Delta z_{p_{1}}^{2} \tag{9}
\end{equation*}
$$

Here the subscript 1 relates to bromine- 87 and the subscript $i$ to any other precursor.

Thus, if expansion to a larger number of exponentials is performed, we can:

1. Find the relative yields of delayed neutrons from pure precursors instead of a mixture of them, and furthermore from that part of the precursors which makes the predominant contribution to the total delayed neutron yield in both thermal and fast fission;
2. Find the charge distribution parameters and the most probable charge for delayed neutron precursor fragments resulting from fast fission of various substances; and
3. Determine the energy surfaces of fission fragments (bromine isotopes) and investigate how these change with a variation in energy of the neutrons causing fission.

Cumulative yields of fission products for $Z=$ const. We propose to show that there is another method of determining delayed neutron yields which is based not on the charge and mass distribution but on knowing the cumulative yields of precursor fragments (fission products).

Let us assume that the distribution of cumulative fission product yields when $2=$ const. is:

$$
P\left(A, A_{p}\right)=\frac{1}{\sigma_{Z} \sqrt{2}^{2} \pi} \quad e-\frac{\left(A-A_{p}\right)^{2}}{2 \sigma_{Z}^{2}}
$$

where $P\left(A, A_{p}\right)$ is the probability of a fragment of mass $A$ occurring with the most probable value of $A_{p}$ (for given $Z$ ) and $\sigma_{Z}$ is the distribution width. Table 2 shows calculated values of $\sigma_{Z}$ and $A_{p}$ for isotopes of bromine, krypton, rubidium, iodine and caesium based on data in Ref. [6]. Fig. 3 shows the fission product mass distributions for $Z=$ const. The mean width $\boldsymbol{\sigma}_{Z}=$ $1.482 \pm 0.085$. The values closest to this are observed in the case of rubidium, iodine and caesium isotopes, their mean value being $\bar{\sigma}_{Z}=1.509 \pm 0.064$. The width of the krypton isotope distribution differs considerably from the mean.

The relationship between $Z$ and $A_{p}$ is illustrated in Fig. 4. The straight lines running through the points for the bromine and rubidium isotopes and through the second pair of points - for iodine and caesium isotopes - are parallel, the slope being:

$$
K=\frac{Z_{2}-Z_{1}}{A_{p_{2}}-A_{p_{1}}}=\frac{2}{3.89}=0.515
$$

The point for krypton isotopes falls outside this relationship. If the same rule is assumed in this case, the krypton isotopes should be assigned the value $A_{p}=90$. The broken curve in Fig. 3 shows the distribution of krypton isotope fission products on the assumption that its width is 1.509 and the peak is at $A_{p}=90$.

Table 3 shows values of $A-A$ for isotopes of bromine, krypton, rubidium, iodine and caesium as well as experimentally determined and calculated fission product yield probabilities normalized to the yield of rubidium isotopes for $\sigma_{Z}=1.509$. In Fig. 5 the small circles indicate experimentally determined values of $P\left(A_{1} A_{p}\right)$.

As can be seen from Table 2, the Gaussian distribution parameters were calculated for three mass numbers in each group of isotopes. The yields of rubidium isotopes, for example, are known for six mass numbers and differ by three orders of magnitude. However, as can be seen from Table 3 and Fig. 5, there is good agreement with the calculated values.

The results obtained can now be used to determine the delayed neutron yields. Since

$$
P\left(A, A_{p}\right)=Y_{i} / P_{n i}
$$

the delayed neutron yields of three bromine isotopes (with masses 87, 88, 89) can be used to find the distribution paraneters for $P\left(A, A_{p}\right)$ and then the delayed neutron yields from 90 Br and ${ }^{91} \mathrm{Br}$. Since the distribution width is identical and the delayed neutron yields of ${ }^{137}$ I and ${ }^{138}$ I are known, the yield of ${ }^{139}$ I and its delayed neutron yield can be found. In short, the whole of the procedure described in the previous section can be carried out using the law established for the mass distribution with $Z=$ const. instead of the charge distribution. The accuracy of determination of the delayed neutron yields will be better, moreover, since there is no need in these calculations to make use of mass yield measurements.

Practical applications. As we have mentioned, the mathematical possibilities of the new method have been demonstrated by obtaining a numerical solution of a system of equations with a 30 th order Gilbert matrix [1.]. Now we must investigate to what extent our experimental results (from the point of view of statistics and resolution) enable us to solve the problem in hand, which is to isolate a larger number of precursors than can be segregated by the least squares method.

The total count we normally obtain under the decay curve when investigating the yields (depending on the energy of the fission-producing neutrons and on the element) is from half a million to several million pulses (in the series of measurements from 5 to 20). This statistical accuracy would seem adequate for determining a larger number of parameters than the six obtained by the least squares method. There are other factors affecting the accuracy of the results but dealing with them all would occupy a whole book. We shall concentrate on one principal factor. In this case the problem should be formulated as follows: what must be the total count under the composite decay curve and what must be the channel widths in the measurements for it to be possible to determine the initial intensities of two or more exponents with given half-lives when the curve is expanded into its components. This condition may be written comparatively easily for a composite curve consisting of two exponents, although the equation obtained is quite difficult to solve but, when there are more than two, even the formulation is unknown. In that case the only feasible method which we found for checking the correctness of the expansion of the decay curve was a series of mathematical experiments, in which the complete decay curve is expanded for different half-life combinations (the basis for their selection can be proved sound). A delayed neutron decay curve resulting from thermal fission of ${ }^{235}$
was used for this purpose and the results of the expansion are compared with radiochemical and mass spectrometric data in the section below entitled "Analysis of results".

Another equally difficult problem is calculating the errors in the ratio of the group yields, but we shall not dwell on this here for the reasons mentioned above. We have selected the simplest method, whereby both the results themselves and the errors are determined as the mean of the results of expanding seven decay curves, each of which is the sum of seven to nine exponents obtained in individual measurements.

A further mathematical problem is that of analysing precursor build-up and decay. Heretofore the decay curve has been represented as a sum of exponents:

$$
\begin{equation*}
N=\sum_{i} a_{i} \exp \left(-\lambda_{i}{ }^{t}\right) \tag{10}
\end{equation*}
$$

where $N$ is the counting rate and $a_{i}$ and $\lambda_{i}$ are the initial intensity and decay constant of the $i^{\text {th }}$ group of delayed neutrons. This is valid as long as we are considering beta decay of precursor fragments formed instantaneously during fission (direct yield). In the case of ${ }^{87} \mathrm{Br}$ and ${ }^{88} \mathrm{Br}$ the yield of their precursors from beta decay of ${ }^{87}$ Se and ${ }^{88}$ Se is appreciable and allowance should be made for the cumulative yield of bromine isotopes via the chain


This gives Eq. (10) the form

$$
\begin{equation*}
N=\Sigma_{i}{ }_{i} e^{-\lambda} i^{t}+a_{s e}\left(e^{-\lambda} B r^{t}-e^{-\lambda} S e^{t}\right) \tag{11}
\end{equation*}
$$

i.e. contributors with negative yield appear. Fig. 6 shows the decay of bromine after disconnection of the source of fission-producing neutrons as well as the build-up and decay of bromine resulting from decay of selenium. Although the latter contribution is small, the beginning of the decay curve is distorted. A detailed analysis of this problem indicated that when the decay curve is expanded from the sixth second after removal of the source there are no significant distortions in the results.

Precursor half-lives. Many factors have pointed to an incorrect determination of the half-lives of several isotopes. Analysis of various ways of expanding the decay curves has corroborated this suspicion, and we were obliged to resort to a systematic recalculation of half-lives.

Ref. [5] indicated a linear dependence of the logarithm of the half-life $T$ on the mass number $A$ of a precursor fragment for bromine and iodine isotopes, i.e.

$$
\begin{equation*}
\ln T=r A+b \tag{12}
\end{equation*}
$$

(see Fig. 7). The physical reason for this relation is the linear dependence of the beta decay energy $Q_{B}$ on $A$, so that $E q$. (12) may also be written

$$
\begin{equation*}
\ln T=r^{\prime} Q_{B}+b^{\prime} \tag{13}
\end{equation*}
$$

Fig. 8 shows the family of curves for even and odd isotopes of bromine, krypton, rubidium, iodine and caesium. Their behaviour is regular with the exception of the rubidium isotopes. The nuclei of the rubidium isotopes have no anomalous features distinguishing them from the neighbouring elementisotopes of bromine and krypton and such an anomaly in the half-life dependence on A is not justified. Using the most reliable half-life values for ${ }^{89} \mathrm{Rb},{ }^{90} \mathrm{Rb}$ and ${ }^{91} \mathrm{Rb}$ we found from Eq. (12) the half-lives of the rubidium isotopes with mass numbers 92,93 and 94 and corrected the value of the half-life for ${ }^{139}$ I. This was the first correction which we had to make to the precursor table (see Table ll).

Probability of delayed neutron radiation. Of course distortions in the determination of half-lives were also bound to cause distortions in the experimentally determined values of $P_{\text {II }}$ for the isotopes in question. Analysing the different ways of expanding the decay curves confirms this conclusion, which was in fact the reason for carrying out the investigations.

Amiel [6] showed that the proportion of beta decays resulting in delayed neutron radiation $P_{I I}$ is related to the energy $\Delta E$ by the expression

$$
\begin{equation*}
P_{n}=K(\Delta E)^{m} \tag{14}
\end{equation*}
$$

where

$$
\Delta E=Q_{B}-B_{n}
$$

Here $Q_{B}$ is the beta decay energy of a precursor fragment and $B_{n}$ is the neutron binding energy in the emitter nucleus. The best value of $m$ found by Amiel on the basis of all available experimentally determined values for $P_{\text {II }}$ is 1.51 .

Fig. 9 shows the dependence of $P_{n}$ on $\Delta E$ in double logarithmic scale. As with the half-lives, the rubidium isotopes display anomalous behaviour. The dependence of $\Delta E$ on $A$ for $Z=$ const. was investigated and found to be linear, i.e.

$$
\begin{equation*}
\Delta \mathrm{E}=\mathrm{sA}+\mathrm{p} \tag{15}
\end{equation*}
$$

Table 4 shows the results of calculating the coefficient $S$ for even and odd isotopes of the above elements (in cases where this is possible) and for all isotopes irrespective of parity. From Figs 10 and 11 and Table 4 it can be seen that the light fragments - bromine, krypton and rubidium isotopes - have practivally the same value of the coefficient $S$, whilst in the case of the heavy fragments the values differ for even and odd A. This table also presents values of $A_{0}$ - the mass corresponding to $\Delta E=0$, i.e. $P_{I I}=0$. Thus for all the above elements we have established the lower limit for the mass of a fragment at which the isotope of a given element can no longer be a delayed neutron precursor $\left(P_{I I}=0\right.$ for all $\left.A<A_{0}\right)$. The ultimate aim of the investigation was to correct the values of $P_{\text {II }}$ for rubidium isotopes and to calculate it for those isotopes for which experimental data were lacking. We assumed that the most reliable values of $P_{\text {II }}$ were those determined for bromine isotopes, so we used them to find $m$ for the light fragments and to calculate $P_{I I}$ for rubidium and krypton isotopes. Table 5 shows the calculated values of $m$ for both light and heavy fragments, An anomalous value is observed for odd iodine isotopes. The value of $P_{\text {II }}$ for ${ }^{137}$ I is one of the most reliable. On the other hand, the $m$ obtained from experimentally determined $P_{\text {II }}$ for odd caesium isotopes is the same as that obtained from two calculated values of $P_{\text {II }}$ for odd xenon isotopes ( $m=1.676$ and 1.688). Using $m=1.69$, we get $P_{\text {II }}=17.2 \%$ for ${ }^{139}$ I. Expansion of the decay curve shows that this is approximately twice the true value. The reason for this anomaly is hard to explain at present.

After correcting the values of the half-lives and $P_{\text {II }}$, the precursor table looks somewhat different: there is now a group with a half-life of $14 \mathrm{sec}\left({ }^{92} \mathrm{Rb}\right)$ whilst the group with a half-life of $2.67 \mathrm{sec}\left({ }^{94} \mathrm{Rb}\right)$ disappears. There is also a change in the delayed neutron yields from various precursors. These were found by multiplying the calculated values of $P\left(A_{1} A_{p}\right)$ by the corrected values for $P_{I I}$. This is the second correction we made to the precursor table.

Methods of measurement and results. Now let us turn to the method of measurement used. We shall not describe it in detail, since it is quite well known, but shall dwell only on a few important cletails. A sample of powdered, $90 \%$ enriched ${ }^{235}$ U enclosed in a cylindrical aluminium container 35 mm in dianeter and weighing 0.7 g was irradiated f'or 5 min and then transferred to the counter unit located 2.5 m from the target. At that instant the neutron source was disconnected. The neutrons were obtained from the $T(p, n)^{3}$ He reaction in a Van de Graaff generator and were slowed down by a polyethylene block. The decay curves were recorded by a 5l2channel analyser with a channel width of one second.

The decay curves were expanded from the 6 th second after conclusion of irradiation of the sample. The channel width was set at one second from the 6 th to the 60 th channel and at 5 seconds thereafter to the end of the decay curve.

The procedure for expanding the decay curves is described in Ref. [7]. Initially expansion is done by the least squares method and Table 6 shows the results of such expansion to four exponentials. 'To obtain a more precise comparison, the same half-lives were used as in Ref. [ 8$]$ for the case of thermal fission of ${ }^{235}$ U. In Ref. [8] the expansion was performed to six exponentials, but in fact 12 parameters were determined (relative yields and half-lives). Since the problem was non-linear in this case, it is not surprising that the accuracy of the results in Ref. [8] is inferior, despite the very good statistical accuracy. Our problem is linear (only the relative yields are determined) and the number of parameters is only onemithird of that in Ref. [8].

After preliminary expansion by the least squares method, giving the background and total count in the period from termination of irradiation to commencement of recording and from termination of recording to infinity, the area under the curve is normalized and the curve is expanded by the new method $[1,7]$. Table 7 shows the results of expanding the measured decay curves. Also given are the ratios of the yields of the delayed neutron groups obtained by calculation, incorporating the corrections mentioned above. Table 8 presents the results of expanding by the least squares method [8] into nine groups (this can be done artificially). Table 9 shows the relative delayed neutron yields obtained by radiochemical and mass spectrometric methods [6]. Table 10 shows relative delayed neutron yields
calculated from the relative fission product yields, the charge distribution and the $P_{I I}$ systematics (we shall call these "systematics data").

Comparison of the results shows that expansion of the decay curves by the new method gives considerably better agreement with the radiochemical investigations than the least squares method (9 exponentials).

It should be noted that the new method has further possibilities and that the results can be even better. For this we need more accurate determination of the half-lives by radiochemical and mass spectrometric methods and more accurate definition of $P_{I I}$.

This concludes the first stage in the calculation of the relative delayed neutron yields which involves using the only criterion we have available at present - the half-life - to segregate contributors with half-lives from 55 to 1.7 sec (the shortest-lived group with $T=1.7 \mathrm{sec}$ of course includes the residue of all shorter-lived groups and is thus essentially a "barrier" group).

The second stage involves using physical laws to separate precursors having half-lives that coincide within the limits of experimental error, since the mathematical possibilities are now exhausted. The relation

$$
P\left(A, A_{p}\right)=Y / P_{n}
$$

enables the mass distribution parameters to be calculated for bromine isotopes (masses 87, 88, 89). These parameters allow us to determine the delayed neutron yields from other bromine isotopes. Further, knowing that the distribution width is the same for isotopes of different elements, and knowing also the delayed neutron yields and values of $P_{\text {II }}$ for two isotopes of iodine (137 and 138) and rubidium (92 and 93), we can find the delayed neutron yields for any iodine or rubidium isotope. These elements make by far the largest contribution to the total delayed neutron yield. Moreover, although the measurement and treatment of the decay curve are commenced when the contribution of the short-lived groups is practically zero, it is still possible to determine their yield.

Thus, although we started off with the more modest and limited aim of trying to identify the contributions of the pure precursors of bromine isotopes, the possibilities of the mathematical method and the physical laws are such as to enable us to segregate the bulk of the precursors.

An investigation of the delayed neutron yields from fast fission of various nuclei is also a basis for studying fission fragment mass and charge distribution behaviour.

Analysis of results. We have deliberately left this section to the end so as not to obscure the main arguments.

Resolution. By this term we mean here the accuracy of determination of the relative yields of contributors to the composite decay curve. This is governed by the channel grouping selected (the choice of channel widths in the analyser), the statistical accuracy of the measurements and the method of calculation. The following table denonstrates that under sufficiently rigorous conditions - a difference in half-life of $6-7 \%$ - the yields differ by factors of $1.5-2$ (the yields of the 15.5 and 14.4 sec and the 6.3 and 5.9 sec groups). This means that the yields are still not equalized: in the limiting case where the half-lives are the same the yields must have an equal probability. This shows that the choice of width grouping is correct, the statistics are completely satisfactory and the method is sufficiently accurate. The weak contributors, as experience shows, cannot be isolated even with a large difference in half-lives (for example ${ }^{134} \mathrm{Sb}$, the yield of which is $7 \%$ of the yield of bromine-87).

| $\mathrm{T}(\mathrm{sec})$ | $\mathrm{Y}_{\mathrm{i}} / \mathrm{Y}_{1}$ |
| :--- | :--- |
| 55.65 | 1 |
| 24.4 | 4.10 |
| 15.5 | 1.59 |
| 14.4 | 1.08 |
| 6.3 | 0.58 |
| 5.9 | 0.84 |
| 4.45 | 4.8 |
| 2.1 | 6.57 |
| 1.7 | 1.24 |

Comparison of calculations by the new method and the method of least squares. Even the artificial version of the calculation by the least squares method (Table 8) gives correct. yield ratios (i.e. in agreement with radiochemical and mass spectrometric data - Tables 9 and 10 , groups 1 and 2) only for ${ }^{137}$ I and ${ }^{87} \mathrm{Br}$; the yield of ${ }^{88} \mathrm{Br}$ is distorted by an additional contribution ( $\sim 30 \%$ ) from ${ }^{92} \mathrm{Rb}$; the yield of ${ }^{138}$ I is 2.5 times too low ( $300 \%$ error of determination) and the following groups represent the yield from a mixture of four or five precursors.

The new method of calculation makes it possible to identify the yields of a larger number of contributors in the same half-life range, i.e. it has better resolution and, moreover, it gives much better agreement with the radiochemical data, as can be seen by comparing Tables 7, 9 and l0. This is the great advantage of the new method. However, its possibilities have yet to be exhausted. More accurate results (even better agreement with radiocheraical data) will be obtainable if certain half-life values are determined more accurately.

Calculation of delayeu neutron yields from short-lived isotopes and identification of contributors from mixed gruups. Below we calculate the delayed neutron yields from ${ }^{90} \mathrm{Br}(\mathrm{T}=1.6 \mathrm{sec}),{ }^{139} \mathrm{I}(\mathrm{T}=3.2 \mathrm{sec})$ and $140_{I}(T=0.8 \mathrm{sec})$. Since the decay curve is analysed only from the sixth second after the source of fission-inducing neutrons has been switched off, the contribution of groups with short half-lives is only a few per cent and includes components from other contributors with similar half-lives. Therefore the accuracy with which the yield of a group with $T 1.6-1.7$ sec can be determined by direct expansion of the decay curve is low whilst the contribution of the group with $T \sim 0.8 \mathrm{sec}$ cannot be determined at all. On the other hand, the contribution of ${ }^{139}$ I cannot be separated from the contribution of ${ }^{93} \mathrm{Rb}$ and ${ }^{137_{\mathrm{Te}}}$ (see Table 7) because their half-lives are identical. Using the cumulative yields from isotopes of bromine with masses 87,88 and 89 and of iodine with masses 137 and 138, (obtained by expansion of the decay curve) together with the values for $P_{\text {II }}$, we were able to determine the delayed neutron yields of the above isotopes from expressions $5 a$ and 6.

It would be possible to show the errors associated with calculating the yields of isotopes which are not obtained directly by expansion of the decay curves, or which are isolated from mixed groups. However, a better illustration can be obtained by comparing the calculation results with existing experimental data from radiochemical investigations. The results are given in the table below.

The good agreement between the calculated and radiochemical (experimental) data is a fine illustration of the possibilities of the new method. It might be considerably better still in the case of the iodine isotopes if there were not some uncertainty in the value of $P_{\text {II }}$ "

| A | $Y_{i} / Y_{1}$ |  | $\mathrm{P}_{\mathrm{II}}$ | $\mathrm{P}\left(\mathrm{A}, \mathrm{A}_{\mathrm{p}}\right)$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | exp. | calc. |  | exp. | calc. |
| Bromine isotopes |  |  |  |  |  |
| 87 | 1 | - | 2.5 | 0.400 |  |
| 88 | $1.78{ }^{+}$ | - | 4 | 0.445 |  |
| 89 | $2.82{ }^{+}$ | - | 7 | 0.400 |  |
| 90 | $2.80+3.76{ }^{++}$ | 3.45 | 12 |  | 0.288 |
| Iodine isotopes |  |  |  |  |  |
| 137 | $3.91{ }^{+}$ | - | 4.8 | 0.815 |  |
| 138 | $1.2^{+}+1.78^{++}$ | - | 2.5 | 0.490 |  |
| 139 | $1.17^{++}$ | 0.94 | 6 |  | 0.156 |
| 140 | $0.56+0.51{ }^{++}$ | 0.32 | 12 |  | 0.026 |

$+\quad$ From expansion of our decay curve.
++ Data from radiochemical investigations.

Charge distribution. We shall not return to this problem because it has been considered in previous papers $[1,9]$. In particular, it is shown in Ref. [9] how the method of successive approximations can be used to isolate the direct yield from the cumulative yield for bromine isotopes. The accuracy with which the charge can be determined is $0.1-0.2$ charge units essentially just as good as the direct method.

## Explanations for Tables 7-10

1. $a_{i}$ is the absolute delayed neutron yield from the $i^{\text {th }}$ precursor in $10^{4}$ fissions;
2. "Total" is the number of delayed neutrons from a given precursor recorded from the end of irradiation to infinity:

$$
\begin{equation*}
N_{n i}=\int_{0}^{\infty} a_{i} e^{-\lambda i t} d t=a_{i}^{\tau} i \tag{16}
\end{equation*}
$$

This quantity characterizes the relative statistical accuracy of the yield determination for each precursor. $\Sigma N_{n i}$ is the total delayed neutron yield from all groups from the end of irradiation to infinity. Since the calculation is performed in ralative units, the tables include values for $a_{i} T_{i}$. It is a familiar fact that even with a different set of expansion parameters - half-lives one can obtain practically the same accuracy of fitting to the experimental data. This is clear from the figures presented.

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Table 1

Delayed neutron precursors and values of $P_{I I}$ and $Z_{p}$

| No. | $T_{I / 2} \mathrm{sec}$ | Precursor | $P_{\text {II }} q_{2}$ | $Z_{p}$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 2 | 3 | 4 | 5 |


| I. | $55,65 \pm 0,20$ | $B r^{81}$ | $2,5 \pm 0,4$ | 34,52 |
| :---: | :---: | :---: | :---: | :---: |
| 2. | $24,4 \pm 0,4$ | $I^{137}$ | $4,8 \pm 1,3$ | 53,26 $\pm 0,12$ |
|  | $24,3 \pm 0,2$ | Cs ${ }^{\text {N4, }}$ | 0,073 $\pm 0,01 \mathrm{I}$ | $54,97 \pm 0,04$ |
| 3. | $15,85 \pm 0,10$ | $\mathrm{Br}^{88}$ | $4,0 \pm 1,4$ | 34,91 - |
| 4. | II,3 $\pm 0,3$ | $S^{8134}$ | $0,03 \pm 0,02$ | $5 \mathrm{I}, 77 \pm 0,1 \mathrm{I}$ |
| 5. | 6.3 | $I^{138}$ | $2,5+0,6$ | $53,45 \pm 0.10$ |
| 6. | $5,39 \pm 0,04$ | $\mathrm{RB}^{93}$ | I,65上0,30 | $37,39 \pm 0,10$ |
|  | $5,8 \pm 0,5$ | $S e^{81}$ | $0,4 \pm 0,1$ | 34.52 |
| 7. | $4,48 \pm 0,02$ | R ${ }^{92}$ | 0,012 $\pm 0, \mathrm{CO4}$ | $36,8 I \pm 0,04$ |
|  | $4,45 \pm 0,30$ | $\mathrm{Br}^{89}$ | $7 \pm 2$ | $35.42 \pm 0,12$ |
| 8. | $3,5 \pm 0,5$ | Te ${ }^{137}$ | ? | $53,26 \pm 0,12$ |
| 9. | $2,67 \pm 0,04$ | R8 ${ }^{94}$ | $I I, I \pm I, I$ | $37,84 \pm 0,15$ |
| 10. | $2,2 \pm 0,3$ | Se ${ }^{88}$ | $6,4 \pm 2.5$ | $34.91-$ |
|  | $2,028 \pm 0,012$ | $A^{65}$ | $22,0 \pm 5$. | 33,68 |
|  | $2,0 \pm 0,5$ | $I^{139}$ | $6,0 \pm$ I, 7 | $53,82 \pm 0,12$ |
|  | $1,86 \pm 0,01$ | $K r^{92}$ | $0,04 \pm 0,0007$ | $36,81 \pm 0,04$ |
| IJ. | $I, 73 \pm 0,01$ | $X e^{141}$ | $0,054 \pm 0,009$ | $54.97 \pm 0,04$ |
|  | $I, 696 \pm 0.02 I$ | $58^{135}$ | $8+2$ | $52,40 \pm 0,15$ |
|  | $I, 69 \pm 0,13$ | $C S^{143}$ | I, $13 \pm 0,25$ | $55,92 \pm 0,10$ |
|  | $1,68 \pm 0,02$ | $\mathrm{Cs}^{142}$ | 0,27 $\pm 0,07$ | $55,36 \pm 0,04$ |
| 12. | $1,6 \pm 0,6$ | $\mathrm{Br}^{90}$ | II, $5 \pm 0,4$ | $35,84 \pm 0,10$ |
| I3. | I, $4 \pm 0,4$ | As ${ }^{87}$ | - ? | 34,52 |
|  | $I, 29 \pm 0,0 I$ | $K r^{93}$ | 2,6+0,5 | $37,39 \pm 0,10$ |
|  | $I, 24 \pm 0,02$ | $X e^{142}$ | 0,4540,08 | $55,30 \pm 0,04$ |
| I4. | $I, 05 \pm 0, I 4$ | $\mathrm{Cs}{ }^{144}$ | I, $\mathrm{IO} \pm 0,25$ | $56,40 \pm 0,25$ |
|  | $>\mathrm{I}$ | $X e^{143}$ | ? | $55,92 \pm 0,09$ |
|  | $>I$ | $X e^{144}$ | ? | $56,40 \pm 0,25$ |
|  | $>\mathrm{I}$ | $X e^{145}$ | ? | 56.88 |
| 15. | $\sim 0.8$ | $I^{140}$ | $12+8$ | $54,34 \pm 0,03$ |
| I6. | $>0, \mathrm{I} 5$ | $K r^{95}$ | ? | $38,40 \pm 0,19$ |
| I7. | - $\sim 0.4$. | $\mathrm{Br}^{96}$ | ? | $36,32 \pm 0,09$ |
|  | $0,36 \pm 0,02$ | KR ${ }^{35}$ | 7, 1040,03 | $38,40 \pm 0,19$ |
|  | $\sim 0_{0}, 3$ | $I^{141}$ | ? | $54,97 \pm 0,04$ |

Table 1 (continued)

| 1 | 2 | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: |
| I8. | $0,23 \pm 0,02$ | RS ${ }^{96}$ | $12,7 \pm I, 5$ | 38,20 $\pm 0,24$ |
|  | $\sim 0.2$ | $\mathrm{Kr}^{34}$ | ? | $37,84 \pm 0,15$ |
| I9. | $0,135 \pm 0,010$ | R697 | $>20$ | 33,14 |

$+\quad$ The grouping of precursors with half-lives below 1.7 sec is arbitrary, as can be seen from the table.

Table 2
Distribution width and values of $A$ for cumulative yields of fission prodacts

|  | Bromine <br> isotopes |  | Kryptonisotopes |  | topes |  | Iodine isotopes |  | Caesium isotopes |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A | $P\left(A, A_{2}\right)$ | A | $P\left(A, A_{P}\right)$ | A | A $A_{p}$ ) | A | $P\left(A, A_{p}\right)$ | A | $P\left(A, A_{p}\right)$ |
| 87 | 2,28 | 92 | 1,68 | 92 | 5,18 | 137 | 4,II | I4I | 4,60 |
| 88 | 2,78 | 93 | 0.53 | 93 | 4,0 | 138 | 2,63 | 142 | 3, 1 |
| 89 | 2,42 | 94 | 0,08 | 94 | I, 9 | I39 | I,IO | 143 | I,43 |
| $G$ | I,721 |  | I, 265 |  | I,434 |  | I,469 |  | I,623 |
| $A_{D}$ | 88,09 |  | 91,64 |  | 91,98 |  | I36,57 |  | I40,46 |

Table 3
Distribution of mass yields of fission products for $Z=$ const.

| El ement | A | $A-A_{p}$ | $\begin{aligned} & P\left(A, A_{p}\right) \\ & \text { exper. } \end{aligned}$ | $P\left(A, A_{p}\right)$ norm. exper. | $\begin{gathered} P\left(A, A_{p}\right) \\ \text { calc! } \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $B r$ | 87 | - I,09 | 2,28 | 4,24 | 4,0 |
|  | 88 | - 0,09 | 2,78 | 5, I' | 5,17 |
|  | 89 | 0,91 | 2,42 | 4,50 | 4,30 |
|  | 90 | I,9I | I, 40 | 2,60 | 2,33 |
|  | 91 | 2,91 | 0,40 | 0,74.4 | 0,82 |
| 28 | 92 | 0,02 | 5,18 | 5,18 | 5,18 |
|  | 93 | 1,02 | 4,0 | 4,0 | 4,12 |
|  | 94 | 2,02 | I, 9 | I, 9 | 2,II |
|  | 95 | 3,02 | 0,66 | 0,66 | 0,70 |
|  | 96 | 4,02 | 0,17 | 0,17 | 0,149 |
|  | 97 | 5,02 | 0,02 | 0,02 | 0,0205 |
| I | 137 | 0,43 | 4,II | 4,98 | 4,97 |
|  | 138 | I, 43 | 2,68 | 3,24 | 3,30 |
|  | 139 | 2,43 | I, 10 | I, 33 | 1,4I |
|  | I40 | 3.43 | $\underline{0} 21$ | O2, 21 | O2389 |
| Cs | I4I | 0,54 | 4,60 | 4,86 | 4,86 |
|  | 142 | I,54 | 3,1 | 3,27 | 3,07 |
|  | 143 | 2,54 | I, 43 | I,5I | I, 25 |
|  | 144 | 3.54 | 0,4I | 0.43 | 0.33 |
| Kr | 90 | 0 | - | - | 5,17 |
|  | 91 | I | - | - | 4,14 |
|  | 92 | 2 | 1,68 | 2,14 | 2,14 |
|  | 93 | 3 | 0,43 | 0,678 | 0,728 |
|  | 94 | 4 | 0,08 | 0,102 | 0,153 |

Table 4
Values of coefficients $S$ and $A_{o}$ in the relation $\Delta E=S A+p$

| Element | Value of $S$ |  |  | Value of $A_{0}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\begin{gathered} \text { Even } \\ \text { A } \end{gathered}$ | $\begin{gathered} \text { Odd } \\ \text { A } \end{gathered}$ | $\begin{gathered} \text { All } \\ \text { A } \end{gathered}$ | Even A | $\begin{gathered} \text { Odd } \\ \text { A } \end{gathered}$ | $\underset{\text { A }}{\text { All }}$ |
| As | 2,100 | - | 2,100 | 83,56 | - | 83,25 |
| Se | , | - | 0,610 | , | - | 85.57 |
| Br | 0,093 | 0,848 | 0,930 | 86,05 | 85,60 | 85,84 |
| Kr | 0,990 | 0,890 | 0,932 | 91,75 | 90,32 | 91,47 |
| RB | 0,923 | 0,908 | 0,921 | 91. 52 | 91,39 | 91,48 |
| Y | - | 0,970 | 0.970 | - | 96,87 | , |
| Sn | - | - | 2,510 | - | - | 132,97 |
| SB | I. 50 | - | I, 50 | 133,09 | - | 132,92 |
| Te | - | - | 0,400 | - | - | 134,88 |
| I | 0,320 | 0,640 | 0,676 | 120,38 | 134,75 | 134,96 |
| Xe | 0,265 | 0,318 | 0,295 | 140,00 | I40,4I | I40,51 |
| $\mathrm{CS}_{5}$ | 0.425 | 0,480 | 0,465 | I39,55 | I40,08 | I39,93 |

Table 5
Values of $\mathrm{P}_{\text {II }}$ and $m$ for delayed neutron precursors

| El ement | A | Assumed <br> value of $\mathrm{P}_{I I}$ | m | Experimental value of ${ }^{P}$ II | Calculated value |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  | $\begin{gathered} \text { Our } \\ \text { own } \\ \text { values } \end{gathered}$ | Ref. [6] |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| Br | 87 | $2,5 \pm 0,5$ |  | $2,5 \pm 0,5$ |  |  |
|  | 88 | $4.0 \pm \mathrm{I}$ |  | $4,0 \pm 1$ |  |  |
|  | 89 | $7 \pm 2$ | 1,29 | $7 \pm 2$ |  |  |
|  | 90 | $12 \pm 3$ |  | $12 \pm 3$ |  |  |
|  | 91 | - |  | - | 13.9 | $13,8 \pm 1.6$ |
|  | 92 | - |  | - | 18,6 | 20,2 $\pm 2,3$ |
| Kr | 92 | - |  | 0,040 $\pm 0,007$ | 0,32 | 0,5 |
|  | 93 | - | I. 29 | 3,3土0,5 | 4,28 | $3,5 \pm 2,7$ |
|  | 94 | - |  | - | 5,44 | $4.7 \pm 0,5$ |
| RB | 95 | - |  | - | 8,77 | $8,3 \pm 1,0$ |
|  | 92 | - |  | 0,012 $\pm 0,004$ | 0,70 | 0,5 |

Table 5 (continued)

| 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| RB | 93 | - |  | $\mathrm{I}, 8 \pm 0,5$ | 3,2I | $2,5 \pm 1,9$ |
|  | 94 | - | I,29 | $7,5 \pm 1,9$ | 5,60 | $4,8 \pm 0,5$ |
|  | 95 | - |  | $7,1 \pm 0,9$ | 8,8 | $8,3 \pm 1,0$ |
|  | 96 | - |  | $12,7 \pm 1,5$ | I2, I | $\mathrm{I} 2, \mathrm{I} \pm \mathrm{I}, 4$ |
|  | 97 | - |  | 20 | I5,8 | I6,0 $\pm$ I, 9 |
| $Y$ | 97 | - |  | - | 0,14 | 0,5 |
|  | 98 | - | I, 29 | $0,8 \pm 0,4$ | I, 20 | 0,8 |
|  | 99 | - |  | - | 4,9 | $4,2 \pm 0,5$ |
| I | I38 | $2,5 \pm 0,6$ |  | $2,5 \pm 0,6$ | - | $3,8 \pm 2,9$ |
|  | 140 | $12 \pm 8$ | 2,56 | $12 \pm 8$ | - | 9,8 $\pm \mathrm{I}, \mathrm{I}$ |
| Cs | 142 | $0,27 \pm 0,1$ | 2,35 | $0,27 \pm 0,1$ | - |  |
|  | 144 | I, $10 \pm 0,3$ | I,IO | $\mathrm{I}, 10 \pm 0,3$ | - |  |
| I | 137 | $4,8 \pm 1,3$ |  | $4,8 \pm \mathrm{I}, 3$ | - | $2,2 \pm 1,7$ |
|  | 139 | $6 \pm 2$ | 0,292 ? | $6 \pm 2$ | - | $6,7 \pm 0,8$ |
| Cs | I4I | 0,073 $\pm 0,011$ | I,68 | 0,073 $\pm 0,011$ | - | 0,5 |
|  | 143 | $1,13 \pm 0,25$ |  | I, $13 \pm 0,25$ | - | $3,0 \pm 2,3$ |
|  | 145 | - |  | - | 6,9 | $4.7 \pm 0,5$ |
| Xe | 143 | I, $5 \pm$ I, I | I,69 | - |  | $1,5 \pm 1,1$ |
|  | 145 | $2,2 \pm 1.7$ |  | - |  | $2,2 \pm 1,7$ |
| I | $137{ }^{+}$ |  | I,69 |  | 2,94 | $2,2 \pm 1,7$ |
|  | I39 |  |  |  | I7,2 | $6,7 \pm 0,8$ |

+ We assumed that: $P_{I I}\left({ }^{137} \mathrm{I}\right)=4.8 ; \mathrm{m}=1.69$.


## Table 6

Relative delayed neutron yields in thermal fission of ${ }^{235} U$ obtained through expansion of the decay curve by the least squares method

| $\begin{aligned} & \mathrm{T} \mathrm{I} / 2 \\ & \mathrm{sec} \end{aligned}$ |  |  |
| :---: | :---: | :---: |
|  | Own data | Data from Ref. [8] ${ }^{+}$ |
| 55,72 | I | I |
| 22,72 | 6,I59 $\pm 0,054^{++}$ | $6,65 \pm 0,65$ |
| 6,22 | $5,757 \pm 0,054$ | $5,95 \pm 0,89$ |
| 2,30 | II,23 $\pm 0.19$ | I2,0 $\pm 1,3$ |

$+\quad$ Expansion into six groups by least squares method.

+ The error in the half-life is not taken into account; if it were, the error in the yield ratio for any group would be from 4 to $5 \%$.

Table 7
Relative delayed neutron yields obtained through expansion of the ${ }^{235} \mathrm{U}$ decay curves by the new method

| No. group | $\begin{array}{r} \mathrm{T}^{+} \\ \mathrm{sec} \end{array}$ | $\begin{gathered} \mathrm{T} \\ \mathrm{sec} \end{gathered}$ | Precursor | $Y_{i} / Y_{i}{ }^{++}$ |  | $Y_{i} / Y_{i}{ }^{\text {X }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| I. | 55,65 | 55,65 | $\mathrm{B}^{¢ 7}$ | I |  | I |
| 2. | 24,4 | 24,9 | $\mathrm{CS}^{141}$ | 0,06 | 3,73 | $3,91 \pm 0,14$ |
|  |  | 24,4 | $\mathrm{I}^{137}$ | 3,67 |  |  |
| 3. | I6,3 | 16,3 | B 88 |  | 2,06 | 1,78 $+0,10$ |
| 4. | I4,0 | I4,0 | $\mathrm{R}^{92}$ | 0,67 |  |  |
|  |  | II, 3 | $58^{134}$ | 0,05 | 0,72 | $\mathrm{I}, \mathrm{II} \pm 0,09$ |
| 5. | 6,3 | 6,3 | I $\mathrm{Se}^{87} 8$ | I,27 | I,35 | I, $22 \pm 0,10$ |
|  |  | 5,9 | Se ${ }^{\text {d }}$ | 0,08 |  |  |
| 6. | 4,45 | 4,45 | Br9 | - | 3,02 | 2,82 $\pm 0,18$ |
| 7. | 3,3 | 3,5 | $\mathrm{Te}^{137}$ | 0,II |  |  |
|  |  | 3,3 | R ${ }^{93}$ | 2,45 | 3,87 | $4,32 \pm 0,28$ |
|  |  | 3,2 | I 139 | I, $31{ }^{\text {8** }}$ |  |  |
| 8. | 2,I | 2,5 | $\mathrm{CF}_{5}^{42}$ | 0,15 |  |  |
|  |  | 2,2 | $\mathrm{Se}{ }^{88}$ | 0,87 |  |  |
|  |  | 2,I5 | ${ }^{885}$ | I,64 | 2,98 | 3,27 $\pm 0,33$ |
|  |  | 2 | $Y^{98}$ | 0,3I |  |  |
|  |  | I, 86 | $\mathrm{K}^{92}$ | 0,OI |  |  |
| 9. | I,7 | I. 73 | $\mathrm{Xe}^{\text {I4I }}$ | 0,01 |  | - |
|  |  | I,696 | $\mathrm{SH}_{6}^{135}$ | 0,60 | 0,86 | $2,14 \pm 0,40$ |
|  |  | I,69 | $\mathrm{Cs}^{1 / 3}$ | 0,25 |  |  |
|  |  | I, 6 | 890 |  | 2,82 |  |

$+\quad$ The half-life on which the expansion of the decay curve was based.

+ Calculated value.
$x$ From expansion of the decay curve.
E When $P_{\text {II }}=5 \%$.

Table 8
Results of expanding to nine exponentials by the least squares method ${ }^{+}$

|  | Group No.* |  |
| :--- | :---: | :---: |
| $I$ | 54,5 | $Y_{i} / Y_{i}$ |
| 2 | 24,4 | $I$ |
| 3 | $I 6,3$ | $3,63 \pm 0,90$ |
| 4 | 6,3 | $2,9 \pm I, 2$ |
| 5 | 4,4 | $0,5 \pm \pm, 6$ |
| 6 | $.2,0$ | $6,2 \pm \pm 2,0$ |
| 7 | $(I, 6+2,4)$ | $2,2 \pm \pm 2,7$ |

+ Data from Ref. [8].
* We show only seven of the nine groups since the other two have shorter half-lives and cannot be compared with our results.

Table 9
Relative delayed neutron yields in thermal fission of ${ }^{235} \mathrm{U}$ (radiochemical data)

| Group No. | T sec | Precursor | $a_{i}$ | $a_{i} / a_{i}$ | Total |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 2 | 3 | 4 | 5 | 6 |
| I | 54,5 | $\mathrm{Br}^{87}$ | $5,9 \pm 0,4$ | I | 55,65 |
| 2 | 24,4 | $I^{187}$ | $2 \mathrm{I}, 7 \pm 3,6$ |  |  |
|  | 24,9 | $\mathrm{CS}^{\mathrm{ml}}$ | 0,345 $\pm 0,05$ | $3,82 \pm 0,69$ | 93,I4 |
| 3 | I6, 3 | $\mathrm{Br}^{88}$ | $I 2, I \pm 3,3$ | $2,15 \pm 0,60$ | 34,08 |
| 4 | II, 3 | S $8^{138}$ | $0,280 \pm 0,004$ | 0,048 0,001 | 0,54 |
| 5 | 6,3 | $I^{138}$ | $10,3 \pm 1,6$ |  |  |
|  | 5,86 | $R 8^{93}$ | - 6,60. | - - |  |
|  | 5.9 | Se ${ }^{81}$ | 0,56 $\pm 0,14$ | $3,05 \pm 0,51$ | I8,48 |
| 6 | 4.5 | $\mathrm{Br}^{59}$ | $18,8 \pm 5,7$ |  |  |
|  | 4,48 | $R B^{92}$ | $0,06 \pm 0,08$ | $3,3 \pm 1,0$ | 14,68 |
| 7 | 2,67 | R $B^{94}$ | I7,2 $\pm 2.4$ | $2,98 \pm 0,46$ | 7,96 |
| 8 | 2,028 | $A 5^{85}$ | $9,7 \pm 0,6$ |  |  |

Table 9 (continued)

| 1 | 2 | 3 | 4 | 5 | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 9 | 2,0 | $I^{139}$ | $12,7 \pm 3,6$ |  |  |
|  | 2,2 | $\mathrm{Se}^{88}$ | $5,1 \pm 2,0$ |  |  |
|  | I,86 | $\mathrm{Kr}^{92}$ | 0,07 | $4,83 \pm 0,74$ | 9,76 |
|  | 1,696 | $9^{58}{ }^{135}$ | $3,5 \pm 0,3$ |  |  |
|  | I,69 | $\mathrm{Cs}^{1 / 43}$ | I, 6 |  |  |
|  | 1,68 | $\mathrm{Cs}^{1 / 2}$ | 0,84 | $I, 0 \pm 0, I$ | I,70 |
|  | 1,73 | \%e ${ }^{\text {I4I }}$ | 0,06 |  |  |
| 10 | I, 6 | Bra | $16,2 \pm 5,2$ | $2,8 \pm \pm 0,9$ | 4,48 |
| II | I, 4 | $A^{\text {s }}{ }^{87}$ | ? | 2,8 | 4,48 |
|  | I,29 | $\mathrm{Kr}^{93}$ | 1,38 |  |  |
|  | I,24 | $X{ }^{142}$ | 0,14 | $0,26 \pm$ ? | 0,34+? |

Table 10
Relative delayed neutron yields in thermal fission of ${ }^{235}$ (systematics data)

| No. group | $T$ sec | Precursor | $\mathrm{a}_{i}$ | $a_{i} / a_{i}$ | Total |
| :---: | :---: | :---: | :---: | :---: | :---: |
| I | 54,5 | $\mathrm{Br}^{87}$ | 5,7 | I | 55,65 |
| 2 | 24.4 | $I^{137}$ | 19,7 |  |  |
|  | 24,9 | $\mathrm{Cs}^{\text {²+1 }}$ | 0,33 | 3,59 | 87,60 |
| 3 | I6,3 | $\mathrm{Br}{ }^{88}$ | $\mathrm{I} 2, \mathrm{I} \pm 3,3$ | 2,23 | 35,33 |
| 4 | II,3 | $58^{134}$ | 0,18 | 0,032 | 0,36 |
| 5 | 6,3 | $I^{138}$ | 6,73 |  |  |
|  | 5,89 | R $8^{93}$ | 7,2 |  |  |
|  | 5,9 | $\mathrm{Se}^{87}$ | 0,44 | 2,59 | I5,65 |
| 6 | 4,5 | $\mathrm{Br}^{88}$ | I6,94 |  |  |
|  | 4,48 | R882 | 0,06 | 3,05 | 13,55 |
| 7 | 2,67 | R894 | I4,2 | 2,54 | 6,79 |
| 8 | 2,028 | As ${ }^{\text {cs }}$ | 8,0 |  |  |
|  | 2.0 | $\mathrm{I}^{639}$ | 6,60 |  |  |
|  | 2.2 | $5 e^{88}$ | 4,I6 |  |  |
|  | I,86 | $\mathrm{Kr}^{\mathbf{2 x}}$ | 0,07 | 3,43 | 6,94 |

Table 10 (continued)


Table ll
Half-lives calculated from systematics and obtained by identification

| Element | Tcalc., sec | Tident., sec |
| :---: | :---: | :---: |
| R8 ${ }^{22}$ | I4, $0 \pm 1,2$ | 4,48 |
| R883 | $3,3 \pm 0,5$ | 5,9 |
| $\mathrm{RG}^{3}$ | 0.8 | 2.67 |
| $\mathrm{I}^{\text {I39 }}$ | 3,2 | 2,5 |



Fig. 1 Scheme for constructing the charge distribution curve for $Z=$ const.


Fig. 2 Radiation probability distribution.


Fig. 3 Fission product mass distribution for $2=$ const.


Fig. $4 \quad Z$ versus $A_{p}$.


Fig. 5 Fission product mass distribution.


Fig. 6 Build-up and decay in the $\mathrm{Se} \stackrel{\beta}{\rightarrow} \mathrm{Br}$ chain.
Arbitrary scale
$-32-$


Fig. 7 Dependence of $T$ on A.


Fig. 8 Dependence of $T$ on $Q_{B}$.


Fig. 2 Dependence of $P_{n}$ on $\Delta E$.


Fig. 10 Dependence of $\Delta E$ on $A$.
MeV


Fig. 11 Dependence of $\triangle E$ on $A$.

# MEASUREMENT OF ${ }^{235} \mathrm{U}$ AND ${ }^{239} \mathrm{Pu}$ FISSION CROSS-SECTIONS WITH A NEUTRON SLOWING-DOWN TIME SPECTROMECER 

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## Introduction

The fission cross-sections of ${ }^{235} \mathrm{U}$ and ${ }^{239} \mathrm{Pu}$ were neasured on a neutron spectrometer using the neutron slowing-down time in lead $[1,2]$. The energy range of the spectrometer enables the relative energy dependence of the crossmsections to be determined from $\sim 50 \mathrm{keV}$ down to thermal. The crossmsection curves were normalized to the thermal fission cross-section obtained from additional measurements in a graphite prism moved up against the main lead moderator prism*/.

## Method of measuring cross-sections

In the main measuring channel of the lead moderator (Fig. l) we studied the count in a fission chamber cortaining a layer of the material of interest as a function of slowing-down time, $I_{f}(t)$. The neutron density $I_{B}(t)$ was measured with a deteotor having an efficiency proportional to $\sim 1 / v\left(\mathrm{BF}_{3}\right.$-counter) . Then, as shown in kef. [2], we have

$$
\begin{equation*}
\frac{I_{f}(t)}{I_{B}(t)}=K_{f}<\sigma_{f}(E) \cdot \sqrt{E}>=K_{f} \cdot \sigma_{f}(\bar{E}) \cdot \sqrt{E} \cdot(1+\delta) \tag{1}
\end{equation*}
$$

where $v(t)$ is the mean neutron velocity at an instant of slowing down, $\sigma_{f}(E)$ is the fission cross-section for nuclei of the substance under study, $<\sigma_{f}(E) \sqrt{B}>$ represents averaging over the neutron $\operatorname{spectrum} N(E, t)$ in the moderator at time $t$, $\delta$ is a snall correction allowing for the substitution $<\sigma_{f}(E) \sqrt{E}>\rightarrow_{f}(\bar{E}) \cdot \sqrt{E}$ and related to the wiath of the neutiron energy spectrum $N(E, t)$ and the energy dependence of the cross-section under investigation, $\sigma_{f}(E)$, and $K_{f}$ is the normalizing factor.

The mean neutron energy and the slowing-down time $t$ ( $\mu \mathrm{sec}$ ) are connecued by the relation $[2]$

$$
\begin{equation*}
\overline{\mathrm{E}}=183 /(\mathrm{t}+0.3)^{2}[\mathrm{keV}] \tag{2}
\end{equation*}
$$

* This method of normalizing cross-sections in a graphite prism was
proposed by A.A. Bergman.

If the correction $\delta$ can be neglected, the expression for determining the neutron energy dependence of the crossmsection takes the form

$$
\begin{equation*}
\sigma_{f}(\bar{E})=\frac{I_{f}(t)}{I_{B}(t)} \cdot \frac{1}{K_{f} \sqrt{\bar{E}}} \tag{3}
\end{equation*}
$$

The energy dependence of the crossmsections can be normalized $[2]$ to resolved resonances having known parameters $[3]$ or to the thermal cross-section. However, with weak spectrometer resolution it is not always possible to isolate resonances with reliably determined parameters in the crossmsection being measured. Furthermore, if the crossmsections have low resonances with energies of the order of a few tens of eV, the statistical measuring accuracy in the thermal region will be unsatisfactory in consequence of the large drop in neutron density in the moderator [2]:

$$
\begin{equation*}
I_{B}(t)=\text { const. } t^{-0.35} \cdot e^{-t / T} \tag{4}
\end{equation*}
$$

where $T$ is the mean neutron lifetime ( $\sim 890 \mu \mathrm{sec}$ ) in the lead prism, and in consequence of the long times relative to the neutron burst ( $\sim 2000 \mu \mathrm{sec}$ ) to which the thermal region corresponds in this case ( $\sim 1 / v$ law). A sufficiently intense thermal neutron spectrum can be obtained in a graphite prism placed close to the main lead moderator prism (Fig. l). When the times relative to the neutron burst exceed $1500 \mu \mathrm{sec}$, virtually total thermalization occurs in a graphite prism with a characteristic dimension of $\sim 1 \mathrm{~m}$. This permits the thermal fission crossmections of ${ }^{235} \mathrm{U}$ and ${ }^{239} \mathrm{Pu}$ to be reliably normalized.

## Method of normalizing to the thermal crossmsection <br> in a graphite prism

A graphite prism with dimensions of $120 \times 60 \times 60 \mathrm{~cm}$ was used for normalization to the thermal cross-section. A boron detector was employed to investigate the dependence of neutron density $I_{B}^{t h}(t)$ in the graphite prism on the time relative to the neutron burst. The results of measurements of $I_{B}^{\text {th }}(t)$ in Fig. 2 show that the curve of $\ln I_{B}^{t h}(t)$ has a linear trend which sets in at $t>1000 \mu s e c$ after formation of an equilibrium spectrum. In this time range the integral neutron density in the graphite prism is ten times higher than in the measuring channel of the lead prism.

The graphite prism was also used to investigate the time dependence of the fission chamber count, $I_{f}^{t h}(t)$. By analdgy with Eq. (l) we have

$$
\begin{equation*}
\frac{I_{f}^{t h}(t)}{I_{B}^{t h}(t)}=K_{f}<\sigma_{f}(E) \cdot \sqrt{E}>^{t h} \tag{5}
\end{equation*}
$$

from which we obtain an expression for calculating the normalizing factor from measurements in the graphite prism:

$$
\begin{equation*}
K_{f}=\frac{I_{f}^{t h}(t)}{I_{B}^{t h}(t)} \cdot \frac{1}{\sigma_{f}^{\operatorname{th}} \sqrt{E} t h} \tag{6}
\end{equation*}
$$

where $\bar{\sigma}_{f}^{t h}$ is the thermal crossmsection averaged over the Maxwellian spectrum in the graphite prism with mean energy $\vec{E}^{\text {th }}$.

## Results of measurements and their treatment

The energy dependence $\sigma_{f}(E)$ for ${ }^{235}$ was investigated with a fission chamber containing $15 \mathrm{mg} \mathrm{PuO}_{2}$, and the dependence $\sigma_{f}(E)$ for ${ }^{239} \mathrm{Pu}$ with a fission chamber containing 1.6 mg and $1.2 .4 \mathrm{mg} \mathrm{PuO}_{2}$. The chambers were filled with a mixture of argon ( 200 mmHg ) and $\mathrm{CO}_{2}(10 \mathrm{mmHg})$.

The amplifier circuit of the ionization chamber includes an amplifier with a "Siren" type discriminator. The time dependence of the detector counts was studied with the 256-channel analyser of the Measuring and Recording Centre at the Lebedev Physics Institute of the USSR Academy of Sciences [4].

The results of the measurements contain a correction for the deviation of the boron cross-section $\sigma_{10_{B}}(n, \alpha)$ from the $\sim 1 / v$ law. The correction was calculated with allowance for an expression describing the energy dependence $\sigma_{10_{B}}(E)$ derived from Ref. [5],

$$
\begin{equation*}
\sigma_{10_{B}}(\text { barm })=\frac{610.3}{\sqrt{E(\mathrm{eV})}}-0.286 \tag{7}
\end{equation*}
$$

and is $\sim 10 \%$ when $E=40 \mathrm{keV}$ and $\sim 2 \%$ when $E=2 \mathrm{keV}$.
The correction for weak spectrometer resolution, $\delta$, is calculated on the basis of theoretical estimates of the resolution [2]. To determine $\delta$ we expand $<\sigma_{f}(E) . \sqrt{E}>$ in terms of $E-E$, restricting ourselves to the first two terms:

$$
\begin{equation*}
\left\langle\sigma_{f} \cdot V_{E}\right\rangle=\left(\left.\sigma_{f} V_{E}\right|_{\bar{E}}+\left.\frac{\bar{E}^{2}}{2} \cdot \frac{d^{2}\left(\sigma_{f} \cdot V_{E}\right)}{d E^{2}}\right|_{\bar{E}} \cdot \frac{\Delta \bar{E}^{2}}{\bar{E}^{2}}\right. \tag{8}
\end{equation*}
$$

Obviously, the correction $\delta$ is a relative quantity in the second term of the expansion:

$$
\begin{equation*}
\delta=\left.\frac{\bar{E}^{2}}{2 \sigma_{f}}\left(\frac{d^{2} \sigma_{f}}{d E^{2}}+\frac{1}{E} \cdot \frac{d \sigma_{f}}{d E}-\frac{1}{4} \cdot \frac{\sigma_{f}}{E^{2}}\right)\right|_{E} \cdot \frac{\Delta \bar{E}^{2}}{\bar{E}^{2}} \tag{9}
\end{equation*}
$$

If the crossmection obeys the $\sigma_{f} \sim l / v$ law, then $\delta=0$; if $\sigma_{f}=$ const., then

$$
\begin{equation*}
\delta=-\frac{1}{8} \cdot \frac{\Delta \overline{\mathrm{E}}^{2}}{\overline{\mathrm{E}}^{2}} \tag{9a}
\end{equation*}
$$

The correction to the cross-sections estimated in accordance with Eq. (9) is not more than $2 \%$ at $E \simeq 50 \mathrm{keV}$, when the resolution of the spectrometer is at its worst (~ $100 \%$ ).

## Discussion of results

The results of the measurements of the energy dependence of the fission cross-sections of ${ }^{235} \mathrm{U}$ and ${ }^{239} \mathrm{Pu}$ nuclei are shown in Fig. 3. The fission cross-section ratio $\sigma_{f}{ }^{239} \mathrm{Pu} / \sigma_{f}{ }^{235} \mathrm{U}$ is shown in Fig. 4.

The crossmections were normalized to the thermal spectrum in the graphite prism. The thermal cross-section values used for calculating the normalizing factor are given in Table l. The normalizing factors calculated for the fission cross-section of ${ }^{239} \mathrm{Pu}$ on the basis of the resonance at $E_{0}=0.296 \mathrm{eV}$ and the resolved group of resonances at $E_{0}=7.85-32.3 \mathrm{eV}$ with well-known parameters [3] agree within the error limits with the value calculated from the thermal cross-section (Table 2).

The standard deviation of the measurements is due mainly to the normalizing error, the statistical error ( $\sim 2 \%$ ) and the error in extrapolating the time/energy relation (2) into the range of small ( $\mathrm{E}<1 \mathrm{eV}$ ) and large ( $E>10 \mathrm{keV}$ ) energies [2]. The error in normalizing to the thermal crossmsection ( $\sim 3 \%$ ) is largely attributable to statistics and counting instability in the graphite prism.

It should be noted that the effect of diffusion cooling, estimated from formulae given in Ref. [6], reduces the temperature of the spectrum in the graphite prism by $12 \%$. But, as follows from the graphs in Ref. $[7]$, the value of $\bar{\sigma}_{f}^{t h} \sqrt{E}{ }^{\text {th }}$ used for normalizing the fission crossm sections does not vary more than $0.2 \%$ as a result of this.

The results of our measurements of $\sigma_{f}(E)$ for ${ }^{235} U$ in the energy range above 5 keV are on average 9-12\% lower than the data recommended by Hart [8] and the disagreement with the results of Knoll [9] at 30 keV is about $10 \%$.

In the $0.07-10.0 \mathrm{keV}$ energy range our results are in good agreement within the limits of measuring error with averaged selected data [10-14] which we assigned to the mean value of the averaged range of energies.

The results of our measurements of $\sigma_{f}(E)$ for ${ }^{239} \mathrm{Pu}$ in the energy range $E<6 \mathrm{keV}$ agree with the averaged data reported from Harwell [5] and the recommended data of Hart [3] and Fursov et al. [15].

For energies $E>6 \mathrm{keV}$ there is a systematic deviation averaging $12-15 \%$ from the data given in Refs $[5,8,15]$. The disagreement with the averaged selected data $[14,16,17]$ is probably also partially due to the inadequate resolution of our method of measurement compared with those used by the authors of the selected data.

The values obtained for the cross-section ratio ${ }^{239} \sigma_{f} /{ }^{235} \sigma_{f}$ are in good agreement with averaged values of Gilboy and Knoll [18]. Agreement with the recommended values of Hart [8] is very good. The recommended data of Davey [19] appear somewhat high compared with our experimental data.

Table 1
Thermal cross-section values used for normalizing the graphite prism measurements

|  | ${ }^{235}$ | ${ }^{239} \mathrm{Pu}$ |
| :---: | :---: | :---: |
| $\mathrm{E}=0.0253 \mathrm{eV}$ | $580.2 \pm 1.8 \mathrm{barm}[20]$ | $741.6 \pm 3.1$ barm [20] |
| f | 0.977 [7] | $1.052[7]$ |
| $\bar{\sigma}_{f}^{t h}$ | 566.8 barm | 740.6 barn |

Table 2
Normalizing factors calculated for measurements on ${ }^{39}$ Pu using a fission chamber

| Method of calculation | $\mathrm{K}_{\mathrm{f}}$ |
| :--- | :---: |
| From the thermal cross-section in the <br> graphite prism | $0.487 \pm 0.015$ |
| From the group of resonances at <br> $\mathrm{E}_{\mathrm{o}}=7.85-32.3 \mathrm{eV}$ |  |
| From the resonance at $0.500 \pm 0.057$ <br> $\mathrm{E}_{\mathrm{O}}=0.296 \mathrm{eV}$  | $0.514 \pm 0.051$ |

Table 3
Numerical values of ${ }^{23} 35$ fission crossmection

| $\mathrm{E}, \mathrm{eV}$ | ${ }_{\mathrm{f}}$, barn | E, eV | $\sigma_{\mathrm{f}}$, barm |
| :---: | :---: | :---: | :---: |
| 43500 | $1,84 \pm 0,14$ | 105 | $22, I I \pm 0,86$ |
| 28100 | $2,07 \pm 0,13$ |  |  |
| 19700 | $2,33 \pm 0,13$ |  |  |
| 13900 | $2,55 \pm 0.13$ |  |  |
| II200 | $2,80 \pm 0.14$ |  |  |
| 8840 | $3,0 \mathrm{I} \pm 0, \mathrm{I} 4$ |  |  |
| 7180 | $3,28 \pm 0,15$ |  |  |
| 5940 | $3,55 \pm 0,15$ |  |  |
| 4700 | $4.00 \pm 0.16$ |  |  |
| 3600 | $4,61 \pm 0,18$ |  |  |
| 2570 | 5,35 $\ddagger$ 0,2I |  |  |
| 1970 | $6,24 \pm 0,24$ |  |  |
| 1660 | $6,88 \pm 0,27$ |  |  |
| I400 | $7,56 \pm 0,28$ |  |  |
| II70 | $8,13 \pm 0,33$ |  |  |
| 950 | $8,66 \pm 0,35$ |  |  |
| 800 | $9,88 \pm 0,39$ |  |  |
| 700 | $10.58 \pm 0.43$ |  |  |
| 580 | II, $75 \pm 0.47$ |  |  |
| 460 | I2,26 $\pm 0.49$ |  |  |
| 370 | $13.37 \pm 0,56$ |  |  |
| 310 | $15,50 \pm 0,62$ |  |  |
| 265 | $18,22 \pm 0,73$ |  |  |
| 220 | 20,05 $\pm 0,80$ |  |  |
| 185 | $19,57 \pm 0.78$ |  |  |
| I65 | $20,05 \pm 0,80$ |  |  |
| 145 | $20,53 \pm 0,82$ |  |  |
| 125 | $21,32 \pm 0,85$ |  |  |

Table 4
Numerical values of ${ }^{239}$ Pu fission crossm section

| E, eV | $\sigma_{f}$, barn | E, eV | $\sigma_{f}$, barn |
| :---: | :---: | :---: | :---: |
| 43500 | $I, 4 I \pm 0, I I$ | IOS | $26,0 \pm 1,0$ |
| 28100 | $1,44 \pm 0,10$ |  |  |
| 19700 | I, $54 \pm \mathrm{C,09}$ |  |  |
| 13900 | I, $65 \pm 0,09$ |  |  |
| II200 | I,72 $\pm 0,08$ |  |  |
| 8840 | $I, 85 \pm 0,08$ |  |  |
| 7180 | $1,97 \pm 0,08$ |  |  |
| 5940 | $2,14 \pm 0,09$ |  |  |
| 4700 | $2,33 \pm 0,09$ |  |  |
| 3600 | $2,53 \pm 0,10$ |  |  |
| 2570 | $2,80 \pm 0, I I$ |  |  |
| 1970 | $3,31 \pm 0,13$ |  |  |
| 1660 | $3,94 \pm 0,16$ |  |  |
| 1400 | $4,54 \pm 0,18$ |  |  |
| II70 | $5,42 \pm 0,22$ |  |  |
| 950 | $5,75 \pm 0,23$ |  |  |
| 800 | $5,86 \pm 0,24$ |  |  |
| 700 | $6,20 \pm 0.25$ |  |  |
| 580 | $7,88 \pm 0,32$ |  |  |
| 460 | $9,43 \pm 0,38$ |  |  |
| 370 | 9,6I $\pm 0,39$ |  |  |
| 310 | II, $87 \pm 0,48$ |  |  |
| 265 | $15,02 \pm 0,60$ |  |  |
| 220 | I6,55 $\pm 0,66$ |  |  |
| I85 | $16.62 \pm 0,66$ |  |  |
| I65 | I6,57 $\pm 0,66$ |  |  |
| 145 | $17,42 \pm 0,70$ |  |  |
| I25 | $19,68 \pm 0.79$ |  |  |

## Table 5

Numerical values of fission crosemsection ratio

$$
\sigma_{f}(E)^{239} \mathrm{Pu} / \sigma_{f}(E)^{235} \mathrm{U}
$$

| E, eV | $\sigma_{f}^{239} / \sigma_{f}^{235}$ | E, eV | $\sigma_{f}^{239} / \sigma_{f}^{235}$ | E, eV | $\sigma_{f}^{239} / \sigma_{f}^{235}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 43500 | 0,763 $\pm 0,086$ | 3100 | 0,537 $\pm 0,031$ | 460 | 0,767 $\pm 0,043$ |
| 28100 | $0,691 \pm 0,63$ | 2220 | 0,519+0,30 | 370 | 0,687 $+0,040$ |
| 19700 | $0,657 \pm 0,054$ | 1790 | 0,530 0,030 | 310 | 0,76I $\pm 0,043$ |
| 13900 | $0,643 \pm 0,046$ | 1500 | J,604 $\pm 0,034$ | 215 | 0,823 $\pm 0,047$ |
| II200 | C,6II $\pm 0,040$ | 1250 | 0,636 $\pm 0,036$ | 220 | 0,823 $\pm 0,047$ |
| 8840 | $0,615 \pm 0,040$ | 1070 | 0,666土0,038 | 185 | 0,846 40,148 |
| 7180 | $0,599 \pm 0,037$ | 880 | 0,640 0,036 | 165 | 0,820 0,047 |
| 5940 | $0,603 \pm 0,035$ | 760 | 0,570 0,033 | 145 | 0,847 $+0,048$ |
| 4700 | $0,580 \pm 0,033$ | 640 | 0,625 $\pm 0,036$ | I25 | 0,916 $\pm 0,052$ |
| 3600 | $0,548 \pm 0,031$ | 520 | $0,775 \pm 0,043$ | 105 | I, $175 \pm 0,067$ |

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Fig. 1 Scheme of main lead moderator prism and of the graphite prism for normalizing crossmsection curves to the thermal value.

Key:
1 = prism of lead moderator
$2=$ position of zirconium-tritium target
3 = channel in which fast neutron burst is recorded
4 = main measuxing channel
$5=$ graphite prism
6 = measuring channel of graphite prism


Fig. 2 Dependence of neutron density in the graphite prism on time relative to the neutron burst, $I_{B}^{\text {th }}(t)$.

Key:
$I=$ graphite prism located at the side of the lead moderator closest to the target
$2 .=$ graphite prism at the side furthest from the target


Fig. 3
Energy dependence of fission crossmections of ${ }^{235} \mathrm{U}$ and
239 ${ }^{239}$ Pu nuclei.


-     - данвне нас тояжед padoti [our data] --[19] -..- [8]

Fig. 4 Energy dependence of the fission cross-section ratio, $\underset{f}{239} / \sigma_{f}^{235}$.

# MEASUREMENT OF THE PARAMETER a(E) FOR ${ }^{239}$ Pu WITH A 

 NEUTRON SLOWING-DOWN TIME SPECTROMETERA.A. Bergman, Yu.Ya. Stavissky, V.R. Chelnokov, A.E. Samsonov, V.A. Tolstikov, A.N. Medvedev

## Introduction

Systematic measurements of the parameter $\alpha(E)=\sigma_{c}(E) / \sigma_{f}(E)-$ the ratio of the neutron radiative capture crossmsection to the fission crossm section - for plutonium-239, which largely determines the conversion ratio of fast reactors, are being performed in many laboratories. Estimates of $\alpha(E)$ from experimental values of the total cross-sections and the fission crossmsections indicate that the accuracy with which $a(E)$ can be obtained is no better than $30 \%$ [1]. Therefore, direct measurements are very important.

The difficulty with direct measurements of $\alpha(E)$ lies in separating the radiative capture and fission events, since both are accompanied by the emission of prompt gamma rays. Where time-of-flight measurements are concerned, the experimental values contain a considerable uncertainty due to scattering on nuclei of the specimen and to the difficulty of measuring the background. The method with which we are concerned here, using a neutron slowing-down time spectrometer with a lead prism [2, 3], permits measurements of $\alpha(E)$ in the energy range below $\sim 50 \mathrm{keV}$. The essence of the method is that sample and detector are placed in an isotropic neutron field which is not affected by neutron scattering on nuclei of the sample or the detector material. The low gamma background of the spectrometer makes it possible to record gamma rays with a gas proportional counter, in which the recording efficiency is proportional to the radiation energy [4, 5]. The efficiency of recording a capture or fission event is accordingly proportional to the total energy of the gamma cascade and is independent of possible variations in the gamma-ray spectrum.

The relative energy dependence of $\alpha(E)$ was normalized with a wellthermalized neutron spectrum produced in a graphite prism placed close to the main lead moderator prism. The necessary thermal spectrum constants are already known with a high degree of accuracy [6, 7].

1. Measuring procedure, detectors and counting equipment

A sample of ${ }^{239} \mathrm{Pu}$ and a gamma detector were placed in the measuring channel of a lead moderator (Fig. l) and the dependence of the detector count on the slowing-down time, $I_{\gamma}(t)$, was measured:

$$
\begin{align*}
I_{\gamma}(t) & =\text { const } \cdot\left[\varepsilon_{c} \cdot \sigma_{c}(E)+\varepsilon_{f} \cdot \sigma_{f}(E)\right] \cdot \varphi(t) \\
& =K_{\gamma}\left[\sigma_{c}(E)+B \cdot \sigma_{f}(E)\right] \cdot \varphi(t) \tag{1}
\end{align*}
$$

where $B=\varepsilon_{f} / \varepsilon_{c}$ is the ratio of the efficiencies of recording a fission event and a capture event from instantaneous gamma rays, $\varphi(t)$ is the neutron flux, and $K_{Y}\left(\bar{n}_{x}, M_{x}, \varepsilon_{c}^{x}\right)$ is the normalizing factor, which depends on the effective thickness of the sample $\bar{n}_{x}$, the count of the monitor $M_{x}$ and the efficiency of recording a capture event for nuclei of the sample under investigation.

Measurements with the gamma detector were alternated with counts in a fission chamber, $I_{f}(t)$, containing layers of ${ }^{239} \mathrm{Pu}$ :

$$
\begin{equation*}
I_{f}(t)=K_{f} \cdot \sigma_{f}(E) \cdot \varphi(t) \tag{2}
\end{equation*}
$$

where $K_{f}\left(\bar{n}_{K}, M_{K}, \varepsilon_{K}\right)$ is the normalizing factor, which depends on the effective thickness of the layer in the chamber, $\bar{n}_{K}$, the count of the monitor, $M_{K}$, and the efficiency of the chamber, $\varepsilon_{K}$.

Dividing expression (1) by (2), we obtain the basic expression for determining $\alpha(E)$ :

$$
\begin{equation*}
\alpha(E)=\frac{K_{f}}{K_{\gamma}} \cdot \frac{I_{\gamma}(t)}{I_{f}(t)}-B \tag{3}
\end{equation*}
$$

The mean neutron energy $E(\mathrm{keV})$ and the slowing-down time $t$ ( $\mu \mathrm{sec}$ ) are linked by the empirical relationship [3]

$$
\begin{equation*}
E=183 /(t+0.3)^{2} \tag{4}
\end{equation*}
$$

For recording prompt gamma rays from fission and radiative capture, a gas proportional counter is used $[4,5]$ in which the efficiency of recording gamma rays is approximately proportional to their energy. In this case the efficiency of recording capture and fission events is affected very little by variations in the gammamay spectrum and is determined solely by the total gamma-ray energy per interaction:

$$
\begin{align*}
& \varepsilon_{\mathrm{c}}=\text { const } \cdot \mathrm{B}_{\mathrm{n}}  \tag{5}\\
& \varepsilon_{\mathrm{f}}=\text { const } \cdot \mathrm{E}_{\mathrm{f}}
\end{align*}
$$

where $B_{n}$ is the neutron binding energy in the nucleus and $E_{f}$ is the total gamma-ray cascade energy per fission event.

From expression (5) it follows that the normalizing constants $K_{f} / K_{\gamma}$ and $B$, which are characteristics of the detectors, are constant over the investigated range of energies.

The amplifier circuit of the detectors incorporates a UIS-2 broad band anti-saturation amplifier. The time dependence of the detector count was analysed by the Measuring and Recording Centre of the Lebedev Physics Institute [8]. Since the time resolution of the spectrometer is $\sim 15 \%$, the analyser has groups of channels with pulse widths ranging from $0.25 \mu \mathrm{sec}$ at the beginning to $64 \mu \mathrm{sec}$ at the end of the cycle.

The low resolution of the spectrometer rules out reliable determination of the normalizing constants from resonances with well-known values of $\alpha\left(E_{0}\right)$, so the usual practice is to normalize on the basis of the thermal values. As the graphite prism (Fig. 1) is moved up to the lead "cube" and fed with the neutron pulse from the lead, alternate measurements are made inside it of $\Delta I_{f}^{t h}$ with the fission chamber and of $\Delta I^{\text {th }}$ of the sample in question with the gamma counter as well as of $\Delta I_{\gamma, t^{\gamma}}^{\gamma}$ of a non-fissionable standard sample:

$$
\begin{align*}
& \Delta I_{f}^{t h}=K_{f} \cdot \sigma_{f}^{t h} \cdot \Delta \varphi^{t h}  \tag{6}\\
& \Delta I_{\gamma}^{t h}=K_{\gamma} \cdot\left(\sigma_{c}^{t h}+B \sigma_{f}^{t h}\right) \cdot \Delta \varphi^{t h}  \tag{7}\\
& \Delta I_{\gamma s t}^{t h}=K_{\gamma}^{s t} \cdot \sigma_{c s t}^{t h} \cdot \Delta \varphi^{t h} \tag{8}
\end{align*}
$$

where $\sigma_{f}^{t h}, \sigma_{c}^{\text {th }}$ and $\sigma_{c s t}^{t h}$ are the thermal fission and radiative capture crosssections averaged over the neutron spectrum in the graphite prism and $\Delta \varphi^{\text {th }}$ is the thermal neutron flux.

Relations (7) and (8) are dependent because the condition in expression (5) is fulfilled for the proportional gamma counter, so that for samples with uniform geometry we have
$\frac{K_{Y}}{K_{Y}^{s t}}=\frac{\bar{n}_{x} \cdot M_{x} \cdot \varepsilon_{c}^{x}}{\bar{n}_{s t} \cdot M_{s t} \cdot \varepsilon_{c}}=\frac{\bar{n}_{x} \cdot M_{x} \cdot B_{n}^{x}}{\bar{n}_{s t} \cdot M_{s t} \cdot B_{n}^{s t}}$
After simple transformations we obtain from relations (6), (7) and (8) the normalizing constants:

$$
\begin{align*}
& B=\frac{\sigma_{c s t}^{t h}}{C_{1} \cdot \sigma_{f}^{t h}} \cdot \frac{\Delta I_{\gamma}^{t h}}{\Delta I_{\gamma s t}^{t h}}-\alpha^{t h}  \tag{10}\\
& \frac{K_{f}}{K_{\gamma}}=\left(\alpha^{t h}+B\right) \cdot \frac{\Delta I_{f}^{t h}}{\Delta I_{\gamma}^{t h}} \tag{11}
\end{align*}
$$

where $\alpha^{\text {th }}=\sigma_{c}^{t h} / \sigma_{f}^{t h}$ is the ratio of the thermal radiative capture and fission cross-sections averaged over the thermal spectrum.

## 2. Measurements in the lead cube

The measurements in the lead moderator of the spectrometer were performed with a neutron burst frequency of 312.5 and 625 Hz and a burst duration of $\sim 0.5 \mu \mathrm{sec}$.

Measurements of the effect due to the sample were alternated with measurements of the gamma background (background sample in gamma counter) and measurements of the natural radioactive background from the specimen as well as measurements with the fission chamber. During the measurements the mean square spread of the counts in the channels was monitored, an especially important procedure in the case of small slowing-down times.

In measurements with the gamma detector the signal to background ratio is $\sim 25 \%$ in the energy range $0.1-2.0 \mathrm{keV}$ and $\sim 15 \%$ in the energy range $2.0-8.0 \mathrm{keV}$. Above $E \simeq 8.0 \mathrm{keV}$ the signal to background ratio deteriorates owing to the increase in background close to the neutron burst; this means that it is essential for the position of the burst to remain stable relative to the beginning of the cycle.

## (a) Allowing for activation

The activation of the specimen, $A$, was determined from the ratio of the count rate during measurement with a sample in the gamma counter $I_{\gamma}(t)$ to the count rate during measurement with the fission chamber $I_{f}(t)$ at the end of a cycle lasting $3200 \mu \mathrm{sec}$, when it can be assumed with sufficient certainty that the cross-section under investigation follows the $\sim 1 / v$ law:

$$
\begin{equation*}
\frac{I_{\gamma}(t)-A}{I_{f}(t)}=\frac{\Delta I_{\gamma}^{t h}}{\Delta I_{f}^{t h}} \tag{12}
\end{equation*}
$$

The correction for activation of the specimen decreases sharply with decrease in $t$ and does not exceed $1 \%$ in the $0.1-50 \mathrm{keV}$ energy range.

## (b) Allowing for recycle neutrons

The correction for the contribution of recycle neutrons in a $3200 \mu \mathrm{sec}$ cycle is negligible. The contribution of recycle neutrons in a $1600 \mu \mathrm{sec}$ cycle is determined from the time dependence $I_{\gamma}(t)$ in measurements with a $3200 \mu \mathrm{sec}$ cycle, in the time range from 1600 to $3200 \mu \mathrm{sec}$.

The correction for the contribution of recycle neutrons in a series of measurements with a $1600 \mu \mathrm{sec}$ cycle is $\sim 0.5 \%$ in the $0.1-50 \mathrm{keV}$ energy range.
(c) Correction for counting losses

The correction for counting losses is determined by the finite resolving time ( $\tau \simeq 4 \mu \mathrm{sec}$ ) of the counting system. This correction is important with small $t$ because of the higher background close to the neutron burst. It was introduced in the results of the measurements with the sample and in the background measurements, in accordance with the expressions

$$
\begin{align*}
n(t) & =\frac{m(t)}{1-\eta}  \tag{13}\\
\eta & =\frac{1}{f \Delta_{K}} \cdot \int_{-\tau}^{t} m(t) d t \tag{14}
\end{align*}
$$

where $m(t)$ is the number of pulses counted in the channel situated in the range from $t$ to $t+\Delta_{K}$ per unit time, $n(t)$ is the number of pulses entering that channel per unit time, and $f$ is the neutron burst repetition frequency.

The correction for counting losses is $\sim 4 \%$ at $E=50 \mathrm{keV}$ and drops sharply to $\sim 0.5 \%$ at $\mathrm{E} \simeq 5 \mathrm{keV}$.

## (d) Correction for spectrometer resolution

The correction to $\alpha(E)$ for poor spectrometer resolution ( $\Delta E / E \simeq 35 \%$ at $E \simeq 1 \mathrm{keV}$ and $\Delta E / E \simeq 70 \%$ at $E \simeq 40 \mathrm{keV}$ ) was calculated from the theoretical evaluations of the neutron spectrum in Ref. [3].

The correction for resolution in the $0.1-50 \mathrm{keV}$ energy range does not exceed ~5\%.

## 3. Measurements in the graphite prism

The measurements in the graphite prism which is moved towards the main prism of the lead moderator were performed with a neutron burst frequency of 312.5 Hz and a width of $\sim 2.0 \mu \mathrm{sec}$.
(a) Thermalization of the neutron spectrum

A well-thermalized spectrum is necessary for normalizing the constants obtained from the graphite prism measurements to thermal values.

Measurements with a boron detector ( $\mathrm{BF}_{3}$-counter) in the graphite prism (Fig. 2) show that the thermal neutron density decreases with a constant relaxation of $\alpha_{0}=710 \mathrm{sec}^{-1}$. Using this value and data from Refs $[9,10]$ estimates were made of the thermalization time in the graphite prism and these indicated that a thermal spectrum with near-Maxwellian distribution is established at times of $t>1000 \mu \mathrm{sec}$.

To check the establishment of thermal equilibrium, measurements were made of the ratio of the count from the ${ }^{239} \mathrm{Pu}$ fission chamber to that of the boron detector, and of the ratio of the count from the gama detector with the cadmium sample to that of the boron detector (Fig. 3), as functions of time relative to the neutron burst. As can be seen from Fig. 3, an equilibrium thermal spectrum can be guaranteed at $t>1500 \mu \mathrm{sec}$.
(b) Allowing for activation

For the measurements irl the graphite prism with the gamma counter, allowance must be made for activation of the sample, which is quite significant in the case of ${ }^{239}$ Pu. This is done on the assumption that the length of the cycle ( $3200 \mu \mathrm{sec}$ ) is small compared with the lifetimes of the radioactivities produced. We take the difference between the counts of the gamma detector over two equal time intervals of $\sim 800 \mu \mathrm{sec}$ at the end of a cycle, $\Delta I_{\gamma}^{\text {th }}$. A similar procedure is followed for the fission chamber count, in that the quantities $\Delta I_{\gamma}^{\text {th }}$ and $\Delta \mathrm{I}_{\mathrm{f}}^{\mathrm{th}}$ in the ratio $\left(\Delta I_{\gamma}^{t h} / \Delta I_{f}^{t h}\right)$ are reduced to a single thermal flux.
(c) Allowing for neutron flux depression

The flux depression in the prism during the cycle due to neutron absorption in the specimen was estimated from the experimental ratio of the count from the boron detector with the cadmium sample inserted in the channel of the prism to the count wi.thout the cadmium sample (Fig. 4). The results of the measurements show that the effect of depression in the time range up to $32.00 \mu \mathrm{sec}$ is not more than $5 \%$. Allowing for the difference in absorption in the samples under investigation and the cadmium sample, we find that this effect is less than $0.5 \%$ for the samples under investigation.

Absorption during passage of neutrons through the sample was allowed for on the assumption that each neutron traversing the sample covers an identical path equal to the mean distance through the sample $\overline{\boldsymbol{l}}\left(\overline{\mathrm{n}}=\sqrt{\boldsymbol{l}}, \mathrm{at} / \mathrm{cm}^{2}\right)$. As show in Ref. [11], when $\bar{n} \sigma \sim 5$, the difference in the paths traversed may be neglected without giving rise to errors greater than $3-5 \%$. In our samples $\bar{n} \sigma \sim 0.2$, so that this difference can be neglected.

The relative reduction in the count due to attenuation of the neutron flux is

$$
\begin{equation*}
\mu=\frac{\int_{e}(v) \cdot v\left(1-e^{-\overline{\mathrm{n}} \sigma}\right) d v}{\overline{\mathrm{n}} \sigma^{\operatorname{th}} \cdot \int \mathrm{e}(v) \cdot v_{0} d v}=1-\frac{\overline{\mathrm{n}} \sigma^{\text {th }}}{V}+\frac{\left(\overline{\mathrm{n}}^{\mathrm{th}}\right)^{2}}{3} \tag{15}
\end{equation*}
$$

where $Q(v)$ and $v$ are neutron density and velocity respectively, $\sigma(v)={ }_{\sigma}^{\text {th }} v_{0} h$ is the neutron absorption cross-section and $\sigma^{\text {th }}$ is the absorption crosssection at an energy equal to kT ( $\nu_{0}=2200 \mathrm{~m} / \mathrm{sec}$ ).

The numerator in expression (15) describes the neutron absorption in the sample with allowance for attenuation of the neutron flux, whilst the denominator represents absorption without attenuation of the flux. Integration was performed after expanding the exponential term. Succeeding terms of the expansion give divergent integrals but the validity of expression (15) was verified by numerical integration for different thicknesses of the samples investigated.

A correction for absorption was introduced in the results of the measurements with samples and also in the background measurement (background container of the gamma counter). The correction for absorption in the samples used in the measurements in the graphite prism did not exceed $13 \%$ 。

## 4. Results and discussion

Measurements with the proportional gamma counter were performed for $\mathrm{PuO}_{2}$ samples of different effective thicknesses ( $\overline{\mathrm{n}}=4.4 \times 10^{21} ; 4.2 \times 10^{20}$ and $1.7 \times 10^{20} \mathrm{at} / \mathrm{cm}^{2}$ ) with a ${ }^{240} \mathrm{Pu}$ content of $\sim 1.8 \%$. For normalizing purposes measurements were also carried out in the graphite prism with samples of ${ }^{107} \mathrm{Ag}$ and ${ }^{109} \mathrm{Ag}\left(\overline{\mathrm{n}}=4.3 \times 10^{21} \mathrm{at} / \mathrm{cm}^{2}\right)$ and ${ }^{197} \mathrm{Au}(\overline{\mathrm{n}}$ $\left.=1.8 \times 10^{21} \mathrm{at} / \mathrm{cm}^{2}\right)$. The fission effect was measured with ionization fission chambers filled with a mixture of $\mathrm{A}(200 \mathrm{mmHg})$ and $\mathrm{CO}_{2}(10 \mathrm{mmHg})$. Because of the high intensity of the spectrometer only a comparatively
small quantity of fissile material need be used in the chamber and it is fairly easy to separate the fission fragment count from the alpha particle background. For our measurements we used fission chambers containing 1.6 and $12 \mathrm{mg}^{239} \mathrm{PuO}_{2}$.

The thermal crossmections used for determining the normalizing constants are given in Table 1. The absolute error due to normalization is shown in Table 2 as a function of $a(E)$. From the measurements in the graphite chamber we obtained the value $B=0.802+0.056$.

The results of our measurements of the energy dependence of $\alpha(E)$ for ${ }^{239} \mathrm{Pu}$ are shown in Fig. 5 and compared with the data of other authors. The numerical values of $\alpha(E)$ are shown in Table 3. Our own data in the $0.2-7 \mathrm{keV}$ energy range are in agreement with averaged data obtained by the time-of-flight method in linear accelerators $[16-18]$ and in the neutron spectrometer of the cyclotron at the Institute of Experimental and Theoretical Physics [15].

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## Table l

Thermal cross-sections used for normalization

## Table 2

Absolute error in $a(E)$ due to normalization

| $\alpha(E)$ | $\pm \Delta 4$ |
| :---: | :---: |
| 0,40 | 0,06 |
| 0,60 | 0,07 |
| 0,80 | 0,08 |
| $I, 00$ | 0,10 |
| 1,20 | $0, I I$ |

Note: The normalization error derives largely from the measuring statistics in the graphite prism ( $1-2 \%$ ), the error in the absolute weight of the samples ( $\sim 4 \%$ ) and the uncertainty in the dependence of the gamma counter efficiency on the gamma-ray energy ( $\sim 4 \%$ ).

## Table 3

$$
\text { Numerical values of } \alpha(E) \text { for }{ }^{239} \mathrm{Pu}
$$

| E, keV | $\alpha(\mathrm{E})$ |
| :---: | :---: |
| 6.0 | 0,96 $\pm 0,12$ |
| 5,0 | $1,00 \pm 0,12$ |
| 4,0 | 1,05 $\pm 0,15^{*}$ ) |
| 3,2 | 1,38 $\pm 0,12$ |
| 2.6 | $1,15 \pm 0,12$ |
| 2,17 | $1,36 \pm 0,12$ |
| 5.77 | $1,40 \pm 0,12$ |
| 1,47 | I,I9 $\pm 0,15^{*}$ ) |
| I,23 | $1, I R \pm 0, I I$ |
| 1.05 | $\underline{1,15} \pm 0,10$ |
| 0,92 | $1,16 \pm 0,10$ |
| 0,80 | $1,13 \pm 0,10$ |
| 0,70 | $1,20 \pm 0, I I$ |
| 0,60 | $I, 20 \pm 0, I T$ |
| 0,53 | $1,15 \pm 0,10$ |
| 0,47 | $0,95 \pm 0,13^{*}$ |
| 0,40 | $1,01 \pm 0,09$ |
| 0,35 | $1,16 \pm 0,10$ |
| 0,30 | $1,10 \pm 0,10$ |
| 0,27 | $0,94 \pm 0,10$ |
| 0,23 | $0,90 \pm 0,10$ |
| 0,20 | $0,81 \pm 0,09$ |

Note: The table shows the statistical error in the measurements of $a(E)$ except in cases marked with an asterisk, where the total error is given.


Fig. 1 Measuring set-up
1 = Lead "cube"
2 = Position of zirconium-sodium target
3 = Channel in which the fast neutron burst is recorded
4 = Main measuring channel
$5=$ Graphite prism, $60 \times 60 \times 120 \mathrm{~cm}^{3}$
$6=$ Channel of graphite prism


Fig. 2 Dependence of neutron density in the graphite prism on time relative to the neutron burst $I_{B}^{t h}(t)$ :
$l=$ Graphite prism placed at the side of the lead "cube" closest to the target;
2 = Graphite prism placed at opposite side.


Fig. 3 Ratios of the count rate of the ${ }^{239}$ Pu fission chamber $I_{f}^{t h}(t)$ and the count rate of the gamma detector with cadmium sample $I_{\gamma}^{C d}(t)$ to the count rate of the boron detector $I_{B}^{t h}(t)$ in the graphite prism.


Fig. 4 Ratio of the count rate of the boron detector with cadmium sample in the channel of the graphite prism $I_{B}^{C d}(t)$ to the count rate without cadmium $I_{B}^{\text {th }}(t)$ as a function of time relative to neutron burst.


Fig. 5 Energy dependence of $\alpha(E)$ for ${ }^{239}$ Pu.

- Own data
- Sukhoruchkin, 1970 Ryabov, 1970
- Scintillation tank
- Fission chamber $220 \mathrm{nsec} / \mathrm{m}$
- Fission chamber $15 \mathrm{nsec} / \mathrm{m}$
\& Hopkins, Diven, 1962
- Spivak, 1956
* De Saussure et al. 1966
- Andreev, 1958

Gwin et al, 1970

- metal foils
- fission chamber
- Sowerby et al. 1970
$\Delta$ Czirr, Lindsey, 1970
+ Farrell et al., 1970
- Muradyan, 1971
- Kazansky, 1971

ENERGY AND MASS DISTRIBUTIONS OF FISSION FRAGMENTS PRODUCED IN SPONTANEOUS ${ }^{244}$ Cm FISSION<br>I.D. Alkhazov, O.I. Kostochkin, S.S. Kovalenko L.Z. Malkin, K.A. Petrzhak, V.I. Shpakov

The authors present the results of bilateral measurements of the kinetic energies of fragments from spontaneous fission of ${ }^{244} \mathrm{Cm}$ which were carried out using surface-barrier silicon detectors.

A preparation of isotopically pure ${ }^{244} \mathrm{Cm}$ ( $<0.001$ wt\% ${ }^{242} \mathrm{Cm}$ ) approximately 3 mm in diameter, coated onto an aluminium base, was placed between two semiconductor detectors. The thickness of the preparation together with its carrier was less than $10 \mu \mathrm{~g} / \mathrm{cm}^{2}$. The thickness of the base was $80 \mu \mathrm{~g} / \mathrm{cm}^{2}$. The distance between the detectors was set at 1 cm . The diameter of the sensitive area of the detectors was 7 mm . The working surfaces of the detectors were diaphragmed to avoid losses in the pulse amplitude measurements when fragments strike the edges of the sensitive area of the detectors. The geometrical efficiency of recording fission fragments was $7 \%$. The detectors used in the measurements were selected from commercially available models on the basis of energy resolution, reverse current and the permissible bias voltage. The resolution of the detectors used, measured with ${ }^{239} \mathrm{Pu}$ alpha particles was about l- $2 \%$, the reverse current was no more than tenths of a microampere and the bias voltage was not less than 25 V .

The energy calibration of the apparatus was carried out with the aid of unilateral fragment spectra from thermal fission of ${ }^{235}$, using the method proposed by schmitt [1].

The time resolution of the apparatus ( $4 \times 10^{-7} \mathrm{sec}$ ) permitted an alpha particle loading of the detector of the order of $3-4 \times 10^{4} \mathrm{sec}^{-1}$. On that basis approximately three spontaneous fissions per minute were recorded. The probability of alpha particle pulses covering fragment pulses was about $3 \%$.

To avoid drift of the detector characteristics due to radiation damage caused by the high alpha activity of the ${ }^{244} \mathrm{Cm}$ preparation, not more than $10^{4}$ spontaneous fission events were recorded by one pair of detectors. In all, about 65000 spontaneous fissions of ${ }^{244} \mathrm{Cm}$ were recorded.

The peak to valley ratio was $14: 1$ in the ${ }^{235} U$ calibration spectra and 4:1 in the ${ }^{244} \mathrm{Cm}$ energy spectra. The peak to valley ratio in the 244 Cm mass distribution was $15: 1$. The comparable value obtained by Bennett and Stein [2] for ${ }^{252}$ Cf was $12: 1$.

The accuracy with which the energies and accordingly the masses of the fission fragments can be measured depends on the stability of operation of the apparatus during measurement, the accuracy of the energy calibration, the superposition of alpha particle pulses on fragment pulses and the statistical measuring errors. During measurement the continuous stability of both the amplification circuits and of the pulse height measurement was checked every 30 minutes with the aid of a precision pulse height generator and a reference ${ }^{239} \mathrm{Pu}$ alpha particle source and, in addition, the positions of the peaks in the unilateral fragment spectra were checked every three hours. Peak displacement did not exceed $1 \%$ 。

The calibration errors are compounded from errors in Schmitt's calibration formulae [1] and errors in the determination of the positions of the peaks in the unilateral spectra of ${ }^{233} \mathrm{U}$ and ${ }^{244} \mathrm{Cm}$ fragments. The errors in the coefficients do not in any case exceed the errors given by Schmitt [3] for the kinetic energies of fragments from thermal fission of ${ }^{235} \mathrm{U}$, i.e. $0.6 \%$. The errors in fixing the peaks are determined by the statistics accumulated in each series of measurements with one pair of detectors. As already indicated, the number of ${ }^{244} \mathrm{Cm}$ fission events was of the order of $10^{4}$, the same as the number of fissions in the calibration spectrum. An idea of the accuracy of the energy calibration may be gained from the following consideration. Schmitt $[1]$ proposes determining the calibration constants from the formulae

$$
a=\frac{A_{1}}{P_{L}-P_{H}} \quad a^{\prime}=\frac{A_{2}}{P_{L}-P_{H}} \quad b:=A_{3}-a P_{L} \quad b{ }^{\prime}=A_{4}-a^{\prime} P_{L}
$$

where $P_{H}$ and $P_{L}$ represent the position of the peaks of heavy and light groups of fragments in the unilateral energy spectrum, and $A_{1}, A_{2}, A_{3}$ and $A_{4}$ are constants relating to the particular fissionable nucleus. The constants for ${ }^{244} \mathrm{Cm}$ were determined from the position of the ${ }^{235} \mathrm{U}$ and ${ }^{244} \mathrm{Cm}$ peaks in each series of measurements. The constants for the different series did not differ from each other by more than $0.2 \%$. Their mean values are as follows:

$$
\begin{aligned}
& A_{1}=23.952 \\
& A_{2}=0.0355 \\
& A_{3}=91.437 \\
& A_{4}=0.1398
\end{aligned}
$$

The correction to the mean values of the kinetic energies to allow for overlapping of the pulses from alpha particles and fragments is the product of the overlapping probability and the height of the pulse from the alpha particle, i.e. not more than 200 keV . The appropriate correction was made to the data given below. An over-all estimate of the above measuring errors showed that the standard deviations in the determination of the mean kinetic energies and the masses are no more than ~ $1 \%$.

The results of the measurements were reduced to a form corresponding to fragments before the emission of prompt fission neutrons by a method similar to that proposed by Schmitt [3]. To determine the number of neutrons emitted after fission, the total number of neutrons in each fission event was measured with a liquid scintillation counter and the distribution of neutrons between fragments as a function of mass was calculated from Terrell's "universal" curve [4].

The data obtained are shown in Tables 1-3. Table 4 shows the mean total kinetic energy $\left\langle\mathrm{E}_{\mathrm{K}}\right\rangle$, the mean energies $<\mathrm{E}_{\mathrm{L}}>$ and $<\mathrm{E}_{\mathrm{H}}>$ and the mean masses $<M_{L}>$ and $<M_{H}>$ of the light and heavy peaks, the standard deviations of the mass distribution $\sigma_{M}$ and the total kinetic energy distribution $\sigma_{F_{K}}$. The table also includes similar data from the literature. It will ${ }^{K}$ be seen that our own data differ from those of other authors. These discrepancies can be explained as follows. In Ref. [8], where the measurements were performed with semiconductor detectors, no correction was made for the mass-dependent pulse-height defect of pulses from fission fragments, so that the fragment energies are too low. Moreover, in Ref. [8] and also Ref. [7], the energy calibration was based on old and too low values of the kinetic fragment energies for thermal fission of ${ }^{235} \mathrm{U}$ obtained by Fraser and Milton [9]. If Schmitt's data $[3]$ had been used in Ref. [7], the results would have agreed well with ours. In Ref. [6], the total kinetic energy is clearly too high owing to the poor time resolution of the equipment.

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## Table 1

> Distribution of the number of ${ }^{244} \mathrm{Cm}$ spontaneous fissions in relation to the total kinetic energy and mass of the light fragment before neutron escape

## Mass

| MeV |  |
| :---: | :---: |
|  |  |
|  |  |


| 86 | 88 | \% 0 | 92 | 94 | $\%$ | 48 | 100 | 10 | 104 | 106 | 108 | 110 | 112 | 114 | 116 | 118 | 120 | 122 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |  | 1 | 2 | 4 | 7 | 9 | 7 | 4 | 2 | 1 |  |
|  |  |  |  |  |  |  |  |  | 1 | 4 | 9 | 14 | 18 | 14 | 6 | 3 | 2 | I |
|  |  |  |  |  |  |  |  |  | 3 | $B$ | 21 | 5 | 29 | 23 | II | 4 | 2 |  |
|  |  |  |  |  |  |  | 1 | 1 | 9 | 16 | 36 | 50 | 47 | 35 | 16 | 4 | 2 |  |
|  |  |  |  |  |  | 1 | 2 | 4 | 21 | 36 | 64 | 77 | 76 | 53 | 24 | 7 | 3 | I |
|  |  |  |  |  |  | 2 | 4 | 10 | 58 | 83 | 108 | 114 | 123 | 76 | 35 | 10 | 4 | 1 |
|  |  |  |  |  | 2 | 4 | 8 | 23 | 72 | 131 | 160 | 166 | 156 | 103 | 47 | 14 | 4 | 2 |
|  |  |  |  |  | 3 | 5 | 19 | 52 | 125 | 194 | 235 | 218 | 197 | 129 | 60 | 19 | 6 | 2 |
|  |  |  |  |  | 4 | 9 | 41 | 104 | 191 | 263 | 310 | 287 | 234 | 152 | 69 | 24 | 10 | 5 |
|  |  |  | 1 | 3 | 6 | 24 | 77 | 155 | 261 | 351 | 385 | 559 | 272 | 172 | 84 | 51 | 12 | 7 |
|  |  |  | 2 | 4 | 9 | 37 | 122 | 224 | 341 | 450 | 454 | 401 | 303 | 190 | 9 | 35 | 16 | IJ |
|  |  | 1 | 2 | 6 | 17 | 64 | 116 | 306 | 419 | 511 | 507 | 438 | 330 | 206 | 105 | 39 | 17 | 11 |
|  |  | 2 | 4 | 11 | 31 | 101 | 230 | 360 | 476 | 556 | 544 | 457 | 340 | 214 | 107 | 39 | 18 | 12 |
|  | 1 | 2 | 7 | 16 | 56 | 145 | 283 | 416 | 533 | 570 | 539 | 439 | 335 | 207 | 102 | 38 | 20 | 12 |
|  | 1 | 2 | 10 | 31 | 04 | 1/9 | 338 | 463 | 538 | 5\%. | . 222 | 436 | 30\% | 189 | 90 | 4 | 22 | $1 ;$ |
|  | 3 | 3 | 17 | 50 | 99 | 214 | 561 | 488 | 575 | 572 | 495 | 396 | 270 | 164 | 82 | 3 | 20 | 18 |
|  | $s$ | 10 | 29 | 67 | 121 | 236 | 386 | 450 | 545 | 532 | 459 | 354 | 235 | 141 | 74 | 33 | 23 | 20 |
|  | 10 | 21 | 4 | 83 | 140 | 24. | 131 | +bl | 505 | 491 | 415 | 297 | 198 | 122 | 68 | 52 | 24 | 19 |
| 5 | 15 | 30 | 57 | 98 | 162 | 20.1 | $3 \cdot \mathrm{St}$ | 451 | 455 | 43 | 358 | 248 | 171 | 106 | 62 | 30 | 24 | 20 |
| 7 | 20 | 58 | 69 | 112 | 181 | -15 | 327 | Sch | 367 | 362 | 296 | 210 | 141 | 9 | 57 | 29 | 22 | 18 |
| to | 23 | 44 | \% | 120 | 187 | 257 | 21 | 311 | 227 | 234 | 238 | 168 | 136 | 78 | 49 | 26 | 21 | 19 |
| 14 | 29 | 54 | B. | 130 | 182 | 231 | 250 | 24 | 268 | 232 | 184 | 150 | 92 | 61 | $4]$ | 24 | 22 | 22 |
| 19 | 4 | S4 | 86 | 127 | 165 | 203 | 222 | 228 | 217 | 180 | 142 | 104 | 64 | 49 | 34 | 22 | 22 | 22 |
| 24 | 59 | 6 | 87 | 119 | 150 | 173 | 188 | 171 | 164 | 138 | III | 83 | 61 | 42 | 30 | 20 | 20 | 19 |
| 27 | $4 \hat{2}$ | 59 | 78 | 108 | 133 | 145 | 156 | 158 | 123 | 105 | 87 | 67 | 48 | 38 | 28 | 18 | 20 | 20 |
| 28 | $4{ }^{2}$ | 54 | 74 | 99 | 117 | 12. | 122 | 102 | 93 | 80 | 58 | 55 | 45 | 54 | 25 | 17 | 19 | 19 |
| 28 | 42 | 55 | 64 | 88 | $\%$ | 102 | 95 | 75 | 67 | 52 | 52 | 43 | 8 | 30 | 23 | 15 | 15 | 14 |
| 23 | 43 | 52 | 61 | 69 | \% | 8 ? | 72 | 59 | 53 | 48 | 40 | 34 | 50 | 27 | 21 | 13 | E2 | 10 |
| 27 | 47 | 47 | 49 | 51 | 59 | 62 | 5') | 52 | 40 | 32 | 31 | 27 | 25 | 23 | 19 | LI | 10 | 11 |
| 26 | 54 | 39 | 31 | 59 | 47 | 48 | 42 | 6 | 31 | 30 | 26 | 23 | 22 | 21 | 18 | 18 | 12 | 13 |
| 24 | 31 | 8 | 35 | 35 | 38 | 36 | 32 | 28 | 26 | 24 | 23 | 22 | 20 | 19 | 16 | 11 | 12 | 12 |
| 22 | 28 | 28 | 3 | 27 | 27 | 36 | 25 | 22 | 20 | 19 | 21 | 21 | 20 | 18 | 15 | 10 | 10 | 9 |
| 20 | 25 | 23 | 22 | 21 | 20 | 20 | 20 | 17 | 15 | 16 | 19 | 18 | 17 | 16 | 15 | 9 | 10 | 10 |
| 16 | 18 | 19 | 20 | 18 | 16 | 17 | 18 | 15 | 12 | 11 | L5 | 15 | 15 | 12 | 11 | 8 | 3 | II |

## Table ?

Mean total kinetic energy $\left\langle E_{k}\right\rangle$ of ${ }^{244} C m$ spontaneous fission fragments and its standard deviation in relation to light fragment mass

| m, /aew/ | 86 | 88 | 90 | 92 | 94 | 96 | 98 | I00 | 102 | 104 | 106 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 166,9 | 168,9 | 170,8 | I73,5 | 176,0 | 178,3 | 181.3 | 184,4 | I86.8 | 198,8 | 190,4 |
|  | 8.4 | 9,0 | 9,1 | 9.3 | 9,4 | 9.8 | 9,9 | 10,2 | 10, c | 10,2 | 10,4 |
| $w_{1} / \mathrm{acu} /$ | 108 | 110 | II2 | II4 | 116 | II8 | 120 | 122 |  |  |  |
| <E.>//I3日/ | 191,2 | 192,3 | 193,2 | 192,6 | 190,2 | 187, 2 | İ8,2 | [19,3 |  |  |  |
| $\sigma_{E_{\alpha} / \text { / }}^{\text {/ }}$ \% $/$ | 10, 8 | 10,6 | II,8 | 12,6 | 13,8 | 14,5 | 14,3 | 13.6 |  |  |  |

Table 3

| $E_{k}$ | 154 | 156 | 158 | 160 | 162. | $\pm 64$ | 166 | 168 | 170 | 172 | 174 | ! 76 | 178 | 180 | 182 | 184 | 186 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\left.\begin{array}{c} \mathrm{MeV} \\ \left\langle M_{\mathrm{M}}\right\rangle \end{array}\right\rangle$ | 99,9 | 97,4 | 99.3 | 99,3 | 99,5 | 99,3 | 99,4 | I 100 | 100,? | 01,0 | 101. 4 | 102,0 | 102.5 | 103,0 | 103, 5 | 104,0 | 104,5 |
| $\underset{\sigma_{m}}{\sigma_{u}}$ | II, ${ }^{\text {, }}$ | It, I | 10.7 | 10,4 | IO, 1 | 9,6 | 9,I | 8.8 | 8.6 | 8,3 | 7.9 | 7,5 | 7.1 | 6.7 | 6,2 | 6,2 | 6,0 |
| $\mathrm{E}_{\mathrm{MeV}}$ | I88 | 190 | 192 | 194 | 196 | 198 | 200 | 202 | 204 | 206 | 208 | 210 | 212 | 214 | 216 | 218 | 220 |
| $\left\langle M_{i}\right\rangle$ | 105,0 | 105,5 | 105,9 | 106,3 | 106,8 | 107,2 | 10\%,6 | 108,0 | 108,3 | 08,7 | 109,2 | 109,5 | 109,8 | 110,3 | 110,8 | If, 3 | II, 5 |
| $\underset{\sim}{\sigma_{M}}$ | 5.7 | 5.4 | 5,3 | 5,1 | 3.0 | 4,9 | 4.7 | 4,6 | 4.4 | 4,3 | 4.2 | 4.0 | 3,9 | 3.7 | 3,6 | 3.7 | 3,6 |

Table 4
Energy and mass distribution characteristics of fragments from spontaneous fission of ${ }^{244} \mathrm{Cm}$

|  | Smith et <br> al. [5] | $\begin{aligned} & \text { Malkin }{ }^{\text {et }} \\ & \text { al. }[6] \end{aligned}$ | Bolshov et $\text { al. }[7]$ | $\begin{gathered} \text { Vei-Ven } \\ \text { Chelnokov }[3] \end{gathered}$ | Our own data |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Mean total kinetic energy ( MeV ) < $\mathrm{E}_{\mathrm{K}}>$ | 185.5 |  | $182.3 \pm 2.3$ | $180.2 \pm 3$ | $188.6 \pm 1.6$ |
| Standard deviation of total kinetic energy distribution (MeV) $\sigma_{\mathrm{F}_{\mathrm{K}}}$ |  |  | 11.9 |  | 11.5 |
| Standard deviation of mass distribution (amu) $\sigma_{M}$ |  |  |  | $\begin{gathered} \text { Half width } \\ 19 \end{gathered}$ | 5.9 |
|  | M o st | $\mathrm{r} \circ \mathrm{babl}$ | values |  | Mean values |
| Energy of light fragment peak ( MeV ) $\mathrm{E}_{\mathrm{L}}$ | 105.5 | $117 \pm 4$ |  | 103.4 ${ }^{+1} .5$ | $107.5 \pm{ }^{+1} .2$ |
| Energy of heavy fragment peak ( MeV ) $\mathrm{E}_{\mathrm{H}}$ | 80 | $86 \pm 3$ |  | $76.8 \pm 1.5$ | $81.1 \pm 1.0$ |
| Mass of light fragment peak (amu) $\mathrm{M}_{\mathrm{L}}$ |  |  |  | 104 $\pm 0.5$ | $104.6 \pm 1.0$ |
| Mass of heavy fragment peak (amu) $\mathrm{M}_{\mathrm{H}}$ |  |  |  | $140 \pm 0.6$ | $139 \pm 1.4$ |

# ABSOLUTE FISSION FRAGMENT YIELDS FROM ${ }^{241}$ Pu FISSION INDUCED BY SLOW NEXTRONS 

N.V. Skovorodkin, A.V. Sorokina, S.S. Bugorkov, K.A. Petrzhak, A.S. Krivokhatsky

For the development of reactor engineering we need to know the fission characteristics of ${ }^{241} \mathrm{Pu}$, including its fission product yields. At the commencement of this investigation data were available on the stable isotope yields of the heavy frasment peak $[1,2]$ and the yields of various radioactive isotopes $\left({ }^{135} \mathrm{I},{ }^{137} \mathrm{Cs},{ }^{140} \mathrm{Ba}\right)$ [3] and, when our investigation was finished, two further publications appeared $[4,5]$ which for all practical purposes do not duplicate our data. Using the radiochemical method we determined directly the absolute cumulative yields of ${ }^{89} \mathrm{Sr},{ }^{90} \mathrm{Sr},{ }^{95} \mathrm{Zn},{ }^{99}{ }_{\mathrm{Mo}},{ }^{11 l_{\mathrm{Ag}}}$,
 the relative (to ${ }^{144} \mathrm{Ce}$ ) cumulative yields of 16 radioactive isotopes of rare earth elements and ${ }^{91} \mathrm{Y}$ resulting from slow fission of ${ }^{241} \mathrm{Pu}$. The targets were irradiated in a water-moderated and cooled reactor with a neutron spectrum having a cadmium ratiofor gold of $\sim 3$.

The plutonium for the targets was cleaned by ion exchange. The targets were dissolved in the presence of isotopic carriers and special conditions were created to promote exchange of antimony, molybdenum and zirconium with the carriers. The separation and cleaning of all the isotopes was done by ion exchange chromatography with a minimum of deposition. The chemical yield of Mo and Ce was determined by colorimetry, the yield of Sr by the complexometric method and the yield of the other isotopes by gravimetric analysis. The rare earth elements and yttrium were also separated by ion exchange chromatography using alpha-hydroxy isobutyrate of ammonium [6]. The carrier employed was lanthanum so that we were able to separate all the elements virtually without carrier.

The absolute number of disintegrations was measured in a $4 \mathbb{B}$ flow type propane counter. The active substance was coated on gold-plated organic films with a surface density of $\sim 10 \mathrm{mg} / \mathrm{cm}^{2}$. The maximum surface density of the active substance was $50 \mathrm{mg} / \mathrm{cm}^{2}$. The absolute activity of the rare earth isotopes was measured in the $4 \pi B$ counter through an $A 1$ filter with a thickness of $6 \mathrm{mg} / \mathrm{cm}^{2}$. The correction for beta particle absorption was determined experimentally for each isotope. The decay
curves were analysed by computer by the least squares method. The standard deviation of the total number of atoms of all the various isotopes was $1.5-2 \%$ except for ${ }^{141} \mathrm{Ce}$ and ${ }^{144} \mathrm{Ce}$ where it was 2.7 and $3.3 \%$ respectively.

The number of fissions in the plutonium target for radiochemical analysis was determined from the number of tracks produced in a mica detector by the fission of a known amount of plutonium coated on the mica detector, which was irradiated in the reactor together with the target in a special assembly. To allow for the contribution of ${ }^{239} \mathrm{Pu}$ fissions, we used ${ }^{239} \mathrm{Pu}$ and ${ }^{241} \mathrm{Pu}$ fission cross-sections determined for the reactor spectrum in Ref. [7]. The fission fragment recording efficiency of the mica was determined with a calibrated ${ }^{252}$ Cf source as $96.7 \pm 0.7 \%$. The contribution from ${ }^{239}$ Pu fissions was not more than 5-8\%. The standard deviation of the values obtained for the number of fissions in the plutonium target was $2 \mathbf{2} .5 \%$. The yields obtained are giver in Tables 1 and 2. In the calculations allowance was made for the cumulative yield of precursors [6].

Tablel
Absolute cumulative yields


## Table 2

Relative and absolute cumulative yields

| Isotope | Half-life used | Yield relative $\text { to }{ }^{144} \mathrm{Ce}^{* /}$ | Absolute cumulative yield in \% |
| :---: | :---: | :---: | :---: |
|  | $\begin{array}{ll} 3,85 & \mathrm{~h} \\ 32,5 I & \mathrm{~d} \\ 33,40 & \mathrm{~h} \\ I 3,59 & \mathrm{~d} \\ 5,98 & \mathrm{~h} \\ I I, 06 & \mathrm{~d} \\ 2,64 & \mathrm{yr} \\ I, 8 & \mathrm{~h} \\ 53,09 & \mathrm{~h} \\ 28,40 & \mathrm{~h} \\ 47, I & \mathrm{~h} \\ 9,4 & \mathrm{~h} \\ I 5,2 I & \mathrm{~d} \\ I 5, I 5 & \mathrm{~h} \\ I 8,0 & \mathrm{~h} \\ 7,20 & \mathrm{~d} \\ 58,8 & \mathrm{~d} \end{array}$ | $\begin{array}{ll} I, I 0 & \pm 0,02 \\ 1, I 7 & \pm 0,02 \\ 0,952 & \pm 0,020 \\ I, 06 & \pm 0,01 \\ 0,739 & \pm 0,022 \\ 0,574 & \pm 0,008 \\ 0,577 & \pm 0,023 \\ 0,361 & \pm 0,010 \\ 0,373 & \pm 0,014 \\ 0,207 & \pm 0,010 \\ 0,128 & \pm 0,003 \\ 0,0401 & \pm 0,0010 \\ 0,0417 & \pm 0,0007 \\ 0,0319 & \pm 0,0008 \\ 0,0113 & \pm 0,0002 \\ 0,00200 & \pm 0,00004 \\ 0,409 & \pm 0,007 \end{array}$ | $\begin{array}{ll} 4,50 & \pm 0, I 7 \\ 4,78 & \pm 0, I 8 \\ 3,88 & \pm 0, I 6 \\ 4,3 I & \pm 0, I 5 \\ 3,01 & \pm 0, I 4 \\ 2,34 & \pm 0,09 \\ 2,35 & \pm 0,13 \\ I, 47 & \pm 0,05 \\ I, 52 & \pm 0,08 \\ 0,846 & \pm 0,050 \\ 0,522 & \pm 0,022 \\ 0, I 63 & \pm 0,007 \\ 0, I 70 & \pm 0,006 \\ 0,130 & \pm 0,006 \\ 0,0462 & \pm 0,00 I 8 \\ 0,00815 & \pm 0,00032 \\ I, 67 & \pm 0,06 \end{array}$ |
| */ The slight differences in the relative yields compared with Ref. [6] are explained by the fact that our table includes the results of one further experiment. The yields for $156_{\mathrm{Sm}},{ }^{156}$ Eu and ${ }^{15} 3_{\mathrm{Sm}}$ have been calculated on the basis of a more accurate half-life for ${ }^{15} 6_{\text {Sm }} 9.4 \pm 0.1 \mathrm{~h}[8]$. |  |  |  |

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CALCULATED CROSS-SECTIONS FOR ELASTIC AND INELASTIC SCATTERING OF $0.3-1.5 \mathrm{MeV}$ NEUTRONS BY ATOMIC NUCLEI

I.K. Averyanov, A.E. Savelev, B.M. Dzyuba

## Introduction

In a previous paper we considered an optical model of elastic and inelastic scattering of $2-6 \mathrm{MeV}$ neutrons.*)

The object of the calculations described here is to investigate the optical potential parameters for lower incident neutron energies ( $E_{n}=0.3-1.5 \mathrm{MeV}$ ). The optical potential parameters were determined from experimental data on differential elastic scattering cross-sections for neutrons with energies of $0.3,0.5,0.8$ and 1.5 MeV ; the scattering nuclei had $A=23-238$. The Hausen-Feshbach method was used for estimating the differential crossmections for elastic scattering of neutrons by a compound nucleus.

## Results of calculations

As before, we used the standard optical potential:

$$
\begin{aligned}
V(r)= & V_{1}\left\{1+\exp \left(\frac{r-R}{a}\right)\right\}^{-1}+i V_{2} \exp \left\{-\left(\frac{r-R}{b}\right)^{2}\right\} \\
& +V_{3}\left(\frac{\hbar}{\mu c}\right)^{c} \frac{1}{r} \frac{d}{d r}\left\{1+\exp \left(\frac{r-R}{a}\right)\right\}^{-1}(\vec{\sigma} \vec{l}), R=r_{0} A^{1 / 3}
\end{aligned}
$$

The geometric parameters of the optical potential, $\Gamma_{0}, a, b$ and the parameter of the spinmorbital interaction $V_{3}$ were fixed as constants for all nuclei and for all incident neutron energies. The parameters $V_{1}$ and $V_{2}$ were varied to obtain a satisfactory description of the experimental data on the differential crossmsections for elastic scattering of neutrons.

The results of the calculations are compared with the corresponding experimental data in Figs lm of Appendix I. In the figures points indicate the experimental data, the broken lines indicate the theoretical differential crossmsections for potential elastic scattering and the solid lines are the sum of the theoretical differential crossmections for potential elastic scattering and the differential cross sections for elastic scattering by a
*/ AVERYANOV, I.K., SAVFLEN, A.E., DZYUBA, B.M., Bjul. inf. centr. jad. Dannym, issue No. 6 (1969) 236.
compound nucleus. The constant potential parameters and the parameters $V_{1}$ and $V_{2}$ obtained by the trial and error method for all the nuclei under investigation are shown in Table l.

It is clear that for nuclei starting with Zn and increasing in weight, the parameter $V_{2}$ is constant for all nuclei and varies only with neutron energy (at $E_{n}=1.5 \mathrm{MeV} V_{2}$ is constant for all nuclei). The parameter $V_{1}$ varies at all energies, decreasing as the atomic weight of the target nucleus increases. This variation is due to the dependence of $V_{1}$ on the neutron excess in the nucleus. For certain nuclei lighter than Zn , a considerable spread of $V_{1}$ and $V_{2}$ values is observed at neutron energies below 1 MeV . This is connected with specific features of these nuclei which are apparent in the resonance structure of the cross-sections.

In Figures l-3 of Appendix II the theoretical differential cross sections for inelastic scattering of neutrons (curves) are compared with the corresponding experimental data (points). The theoretical curves were obtained by the Hausenmeshbach method for calculating the elastic and inelastic scattering crossmsections for scattering of neutrons by a compound nucleus. The optical potential parameters used in deriving the theoretical curves in Appendix II are presented in Table 2.

All the experimental data on the differential crossmsections for elastic and inelastic scattering of neutrons and also the target nuclei characteristics were taken from Refs [1-26].

A comparison of the parameters $V_{1}$ and $V_{2}$ in Tables 1 and 2 shows that as a rule they are in good agreement.

The satisfactory agreement of the theoretical differential crosssections with the experimental data allows the hope that with the aid of the parameters in Table l it may be possible to predict with a certain degree of accuracy the differential crossmsections for elastic and inelastic scattering of neutrons by nuclei for which experimental data are still lacking. This is possible for nuclei which are characterized by a smooth dependence of the crossmections on neutron energy ( $Z n$ and heavier nuclei). In the case of lighter nuclei it is possible to predict the crossmsections averaged over the resonances.

Table 1

$$
a=0.65 \mathrm{f} ; \quad \mathrm{b}=0.98 \mathrm{f} ; \quad \mathrm{r}_{0}=1.25 \mathrm{f} ; \quad \mathrm{V}_{3}=6 \mathrm{MeV}
$$

|  | $\mathrm{E}_{\mathrm{n}}=0.3 \mathrm{MeV}$ |  | $\mathrm{E}_{\mathrm{n}}=0.5 \mathrm{MeV}$ |  | $\mathrm{E}_{\mathrm{n}}=0.8 \mathrm{MeV}$ |  | $\mathrm{E}_{\mathrm{n}}=1.5 \mathrm{MeV}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | MeV |  | MeV |  | MeV |  | MeV |  |
|  | $V_{1}$ | $T_{2}$ | $V 1$ | $V_{2}$ | $V$ | $V_{2}$ | $V_{1}$ | $V_{2}$ |
| Na | 65,0 | 2,0 |  |  |  |  | 67.0 | 4.6 |
| Mg | 54,5 | 2.0 |  |  |  |  |  |  |
| $A E$ | 56, 0 | 2,0 |  |  |  |  | 59,0 | 4,6 |
| Si | 55,0 | 2,0 | 57.0 | 2,5 | 56,0 | 3,5 | 59,0 | 4,6 |
| $p$ |  |  |  |  |  |  | 55,0 | 4,6 |
| $\ldots$ | 55,0 | 38.0 |  |  |  |  | 52,6 | 4,6 |
| Ca |  |  |  |  |  |  | 54,0 | 4,6 |
| $\mathrm{T}_{i}$ |  |  | 45,0 | 5.0 |  |  |  |  |
| Cr | 45,0 | 5.0 | 45.0 | 5,0 | 47,0 | 5,0 |  |  |
| Fe | 44.0 | 5.0 | 45,0 | 5.0 | 50,0 | 15,0 |  |  |
| Co |  |  |  |  | 56,0 | 3,5 | 49,0 | 4,6 |
|  | 56,0 | 2,0 |  |  |  |  |  |  |
| Cu |  |  |  |  |  |  | 49,0 | 4.6 |
| \% | 53,3 | 2.0 | 54,8 | 2,5 | 56, 8 | 3,5 | 48,0 | 4.6 |
| $\frac{1}{z}$ |  |  |  |  |  |  | 48,0 | 4,6 |
| Zr | 54,8 | 2.0 |  |  | 54,2 | 3,5 | 48,0 | 4.6 |
| N6 |  |  |  |  |  |  | 48,0 | 4,6 |
| Cd | 49,0 | 2,0 | 50,3 | 2,5 | 49,4 | 3,5 |  |  |
| ${ }_{\text {Te }}$ | 47,4 | 2,0 | 48.5 | 2,5 | 47, 5 | 3,5 |  |  |
| Te | 47.0 | 2,0 |  |  |  |  |  |  |
| Au |  |  | 45,3 | 2.5 | 44,3 | 3.5 | 46,0 | 4.6 |
| reg | 47,5 | 2,0 | 43,0 | 2,5 | 46,0 | 3,5 |  |  |
| $\mathrm{Pb}_{\mathrm{Bi}}$ | 45,5 | 2.0 | 45,0 | 2,5 | 45,0 | 3,5 | 45,0 | 4,6 |
| Th | 46,5 | 2.0 | 45.0 | 2.5 | 45.0 | 3,5 |  |  |
| U | 4I, 8 | 2.0 | 4I, 0 | 2,5 | 40,0 | 3,5 | 41,0 | 4,6 |

Table 2

$$
a=0.65 \mathrm{f} ; \quad \mathrm{b}=0.98 \mathrm{f} ; \quad r_{0}=1.25 \mathrm{f} ; \quad V_{3}=6 \mathrm{MeV}
$$

| Nucleus | MeV |  |  |
| :---: | :---: | :---: | :---: |
|  | $\mathrm{E}_{\mathrm{n}}$ | $\mathrm{V}_{1}$ | $\mathrm{V}_{2}$ |
| $N a^{23}$ | $0,98$ | $66,0$ | $4,0$ |
|  | $I, 50$ | $67,0$ | $4,6$ |
| $V^{51}$ | - 51 | 49,8 | 4,7 |
| Mo ${ }^{95}$ | -,78 | 59.0 | 3,5 |
| $\mathrm{Ag}^{107}$ | I. 10 | 50,0 | 4, I |
| Ta 181 | 0.71 | 42,5 | 3,2 |
| $W^{184}$ | O. 50 | 42, 9 | 2,5 |
| $T h^{232}$ | 0,56 | 4I, 7 | 2,7 |
| $]^{238}$ | 0,55 | 40,8 | 2,7 |

Key to Figures (Appendix I and II)

$$
\begin{array}{ll}
\text { Moв } & =\mathrm{MeV} \\
\text { мб/стер } & =\mathrm{mb} / \mathrm{sr} \\
\text { Bи.м. } & =\text { Centre of mass angle (cma) } \\
\varepsilon_{\text {ур }} & =\text { Level }
\end{array}
$$

Appendix 1


Fig. 1


Fig. 2


Fig. 3

cma


Fig. 5



Fig. 7



Fig. 2

## Appendix II



Fig. 1


Fig. 2


Fig. 3

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OPTTMUM PARAMETERS OF THE NEUTRON OPTICAL POTENTIAL OF THE ${ }^{238} 8_{U}$ NUCLEUS<br>G.V. Anikin, A.G. Dovbenko, L.Ya. Kazakova, V.E. Kolesov, V.I. Popov, G.N. Smirenkin, A.S. Tishin

## Introduction

In the 20 years which have passed since the first papers on the optical model appeared $[1,2]$, it has become common practice to calculate the crosssections for the interaction of fast neutrons with different nuclei by a method based on solution of the Shroedinger equation with complex potential. Essentially the optical model is the only sufficiently universal means of calculation capable of reproducing the main features of the experimental data on fast neutron scattering which characterize the entrance channel of a reaction.

With the statistical theory first developed by Hauser and Feshbach [3] it is also possible to calculate the cross-sections of the inelastic processes accompanying the formation of a compound nucleus. Thus with suitable development the optical model could become the basis for a theoretical description and evaluation of experimental data on the neutron cross-sections of a wide range of elements which are very important for reactor construction.

A detailed description of the various nuclear reactions requires knowledge of the "sticking" probabilities $T_{\ell, j}$ for different values of the orbital and total moments, imparted by a neutron to a compound nucleus. In the optical calculation the probabilities $T_{\ell, j}$ are obtained quite naturally. However, in Ref. [4] it is shown that their values depend very much on the shape and values of the parameters of the optical potential employed. Unfortunately, there is as yet no agreement as to what the analytical form or the potential parameters in the optical model should be. It is true that definite progress has been made in the interpretation of the optical potential. Some authors have tried to obtain it from the familiar characteristics of the nucleonnucleon interaction (see for example Ref. [5]). But in most cases we find a purely practical approach to the problem, based on a desire to find a good, and as far as possible economic, means of approximating experimental results of a particular kind; in this respect our study is no exception. Our main problem was to find an optical potential for ${ }^{238} \mathrm{U}_{\mathrm{U}}$ which would provide a more or less reliable basis for calculating neutron cross-sections, in particular for channel analysis of the fission process and also for interpreting
experimental data on small-angle neutron scattering [6]. ${ }^{238}$ U was chosen as one of the heavy nuclei which have been subjected to the most thorough experimental investigation. Besides, the use of an optical model and its physical substantiation are in practice closely related problems.

An important physical criterion of the optical potential is its universality in the sense that for a given nucleus the same potential parameters must at least give the best fit of the theoretical values to the experimental ones for different types of interaction cross-sections. In the ideal case the potential selected for the best description of the entrance channel characteristics, for example the total cross-sections, should automatically provide good agreement between other theoretical cross-sections and the experimental values.

In our work, potentials derived from the condition that they must give the best description of the total crossmections and the angular distributions of elastically scattered neutrons, are further evaluated in terms of their description of inelastic scattering.

Recently developed non-local interaction theories [7] indicate that the parameters of the local potential should have a specific energy dependence. The dependence observed in practice applied only to the real part of the potential. For the bulk of the imaginary part it is usual to obtain an increase with rising neutron energy instead of the predicted decrease [8].

The ${ }^{238} U$ nucleus is non-spherical. For such nuclei there exist crosssection calculations based on the use of a generalized optical model [9]. Experience shows that calculations involving a non-spherical potential are very time-consuming even when modern computers are used [10].

Bearing in mind above all the convenience of practical evaluations and calculations, we have concentrated our attention here on finding the average parameters of a potential in simple form, in the range of energies relevant to fast reactor physics. In selecting the parameters we considered experimental data on the scattering of $0.075-15.2 \mathrm{MeV}$ neutrons.

The energy-independent parameters for the ${ }^{238}{ }_{U}$ nucleus published earlier by Auerbach and Moore [11] relate to a narrower energy range ( $0.57-1.25 \mathrm{MeV}$ ). It is undesirable to use these parameters beyond the limits of the stated range [4], since they give a low value of the cross-section for the formation of a compound nucleus.

## Procedure for fitting the parameters

The shape of the optical potential was selected as
$-U_{0}=V_{0} \cdot f(r)+i\left(W_{1} \cdot f(r)+W_{2} \cdot g(r)\right)+V_{S O} \cdot h(r) \overrightarrow{\varepsilon \sigma} \cdot$
The form factor of the real part of the potential is written in the usual form proposed by Woods and Saxon:

$$
f(r)=\left[1+\exp \left(\frac{r-R_{1}}{a}\right)\right]^{-1}
$$

It is also used for the "volume" component of the absorption term, which includes moreover a second component allowing for the contribution of "surface" absorption. The radial dependence of the latter is in the form of a derivative of the real part:

$$
g(r)=4 \exp \left(\frac{r-R_{2}}{b}\right) /\left[1+\exp \left(\frac{r-R_{2}}{b}\right)\right]^{2}
$$

However, this expression includes additional parameters - the radius $R_{2}$, which determines the localization of the absorption peak, and the parameter $b$ defining the width of this peak. The Thomas form is used in the spin-orbital term for $h(r)$ :

$$
h(r)=\left(\frac{\hbar}{m_{\pi} c}\right)^{2} \frac{1}{r}\left|\frac{d f(r)}{d r}\right|
$$

In these formulae $R_{1}=r_{1} \sqrt[3]{A}, R_{2}=r_{2} \sqrt[3]{A}$, $A$ is the atomic weight of the nucleus, $m_{\pi}$ is the mass of a $\pi$-meson, $c$ is the velocity of light and $\bar{n}$ is Planck's constant.

Calculation of the cross-sections and the fitting of the parameters was done with a programme similar to that published in the paper by Popov et al. [12]. The potential parameters were selected solely on the basis of experimental data characterizing the reaction entrance channel, i.e. the total cross-section and the angular distributions of elastically scattered neutrons.

The calculated angular distributions included a correction for elastic scattering by a compound nucleus. This correction, which is quite considerable at low energies, was calculated semi-empirically as the difference between the calculated crbss-section for the formation of a compound nucleus and the experimental crossmection for inelastic interactions divided by $4 \pi$, with the assumption of isotropic scattering.

The automatic procedure for fitting the parameters may be summed up as follows:

1. The initial potential was used to calculate the total and differential cross-sections for 14 neutron energies in the chosen energy range. Then we estimated the sum of the squares of the relative deviations of the theoretical curves from the experimentally determined cross-sections, i.e.

$$
\begin{align*}
& H^{2}=H_{l}^{2}+H_{2}^{2}=\frac{1}{14}\left[\frac { 1 } { N } \sum _ { n = 1 } ^ { 1 4 N } \sum _ { i = 1 } ^ { N } \left(\frac{\sigma^{\exp }\left(\vartheta_{i}, E_{n}\right)-\sigma^{\text {theor. }}\left(\vartheta_{i}, E_{n}\right)}{\left.\sigma^{\exp \cdot\left(\vartheta_{i}, E_{n}\right)}\right)^{2}}\right.\right. \\
& \left.+B \cdot \sum_{n=1}^{14}\left(\frac{\sigma_{t}^{\exp } \cdot\left(E_{n}\right)-\sigma_{t}^{\text {theor. }}\left(E_{n}\right)}{\sigma_{t}^{\exp } \cdot\left(E_{n}\right)}\right)^{2}\right] \tag{1}
\end{align*}
$$

The factor $B$ allows for the best statistical accuracy of the experimental total cross-sections compared with the angular distributions;
2. A test method was used to determine the direction of variation of one of the parameters in which $H$ decreases. In the selected direction the parameter was varied successively 2-3 times by a standard step, the value of which was selected separately for * each parameter. If the relative minimun in $H$ was not attained, the step was doubled but the change of a given parameter with increased step was continued only in the following parameter variation cycle whereupon the same procedure was repeated for another parameter;
3. If the step exceeded the standard value in reaching the relative minimum for any of the parameters, it was halved until the position of the minimum could be found with the accuracy of the standard step. For the parameters $V_{o}, V_{s o}, W_{1}$ and $W_{2}$ the standard step was 0.1 MeV and for the parameters $a, r_{1}$ and $r_{2}$ it was 0.01 f . Attainment of the absolute minimum was noted by the absence of a reduction in $H$ with variation of any of the parameters in either of the two directions.

It should be noted that at the end of the $H^{2}$ minimizing process it is more economicai to use the other procedure included in the programme, whereby the transition to the following parameter js made only after the relative minimum is attained. This reduces the number of tests.

The programme provides for limiting the variation of a parameter if the rate of decrease in the sum $H^{2}$ does not exceed a given level. It is clearly advantageous to use this procedure in two cases - at the commencement of the operation, when it is desirable to segregate the group of parameters which have the strongest effect on the minimization process (we shall call these "strong" parameters), and in the case where one of the parameters has a tendency to drift into the physically unreal region without any significant improvement in the approximation of the experimental data. This situation often occurs with "weak" parameters, for example the spin-orbit parameter.

The use in our programme of the quantity $H^{2}$ instead of the more usual $\chi^{2}$, where the errors in the experimental values come under the summation sign in the denominator, derives, on the one hand, from the absence of errors in a certain number of the angular distributions employed and, on the other hand, from a desire to assign slightly more weight to the differential cross-sections at large scattering angles. The latter contain much more information about the detailed structure of the optical potential than the first diffraction peak, although it makes a major contribution to the integral elastic scattering cross-section.

## Results

The form of the optical potential used here enables three of its variants to be investigated, i.e. potentials with purely surface absorption ( $W_{1}=0$ ), purely volume absorption ( $W_{2}=0$ ) and combined absorption ( $W_{1} \neq 0 ; W_{2} \neq 0$ ). The investigation was performed as follows. For the surface and volume variants we found those sets of parameters which gave the best approximation of the total cross-sections alone $\left(B=10^{6}\right)$ and the angular distributions alone $(B=0)$ as well as sets which took both these groups of experimental data into account ( $B=9$ ) . The groups of parameters obtained are given in Table l, the top line of which shows the values of $B$ (see formula (l)) that determine the nature of the experimental data to which the corresponding group of parameters relates. The factor $B=9$ means that the accuracy with which we know the total crossmsections is at least three times better than the accuracy of the angular distributions.

It should be noted that the attempt to describe the total crosssections alone did not yield a set of parameters with combined absorption. The automatic search gave either surface or volume absorption.

The last two lines of the table show figures for $H_{1}$ and $H_{2}$, i.e. the standard deviations of the theoretical curves from the experimental values (for the total cross-sections this is the averaged curve).

Figs l-3 show cross-sections calculated with the parameters from Table 1 and compare them with experimental data taken largely from Refs [13] and [14]. Fig. 4 reproduces inelastic neutron scattering cross-sections with excitation of different levels of the ${ }^{238} U$ nucleus, also in comparison with experimental data. As in Fig. 3, the results are given for a group of parameters selected with the coefficient $B=9$ for the three types of absorption. The cross-sections $\sigma\left(n, n^{\prime}\right)$ were calculated using the programme of Kolesov and Dovbenko [15], compiled with allowance for the HauserFeshbach statistical theory but with no allowance for level width fluctuations.

## Discussion of results

1. The main conclusion to be drawn is that for $a^{238} \mathrm{U}$ nucleus it is possible to find groups of parameters which, on average, satisfactorily reproduce the experimental data on neutron scattering over the whole range of energies relevant to reactor physics.
2. If the object is to describe all the experimental data on scattering, the best results are supplied by the potential with combined absorption. Generally speaking, the difference in the results obtained with the three types of parameters is quite small and each individual type of cross-section can be reproduced by any of them.
3. From Table $l$ it can be seen that the groups with $B=0$ and with $B=10^{6}$ differ appreciably, especially in regard to the parameter a. A possible reason for this is the non-spherical nature of the ${ }^{238}{ }_{U}$ nucleus, which was not allowed for in our calculation. It may be that for reproducing the angular distributions it is essential to have a sharper nuclear boundary, whereas the total crossmsection of prolate and chaotically grouped nuclei as a whole can be effectively described by a potential with a more blurred edge.
4. Around $15-18 \mathrm{MeV}$ in the case of the spin-orbit coupling, as can be seen from Table l, there is an individual isolated minimum in the value of $H^{2}$, if the search is performed with $B=0$ or $B=9$, i.e. allowing for the angular distributions. The reason for this is as follows: in the range around $15-18 \mathrm{MeV}$ the parameter $V_{\text {so }}$ fluctuates when we try to follow the energy dependence of the parameters of any of the three potentials without defining any one of them. However, with such values of the spin-orbit parameter it is impossible to get a good description of the total crosssections with an energy-independent optical potential.

Of course, in the search for a compromise it is the "weak" parameters which are farthest from their true values. Besides, the real value of the parameter $V_{\text {so }}$ can be established only with the aid of neutron polarization data.
5. Note that in the $0.6-2 \mathrm{MeV}$ energy range the theoretical curves badly reproduce the experimental angular distributions. In Ref. [ll] there is an analogous situation with energies of 1.17 and 1.25 MeV , although the parameters in that case were selected for a narrower energy range. If we assume variation of the parameters with energy, an ideal description of the experimental angular distributions is possible for virtually all the neutron energies under consideration here. However, some of the parameters reveal an energy dependence which is difficult to explain physically. Thus, for example, the parameter a in the potential with volume absorption at energies $0.6-1.25 \mathrm{MeV}$ decreases to $0.2-0.3 \mathrm{f}$. With surface absorption in the same energy range the parameters $W_{2}$ and $V_{\text {so }}$ increase 3 or 4 times. Without excluding the possibility of exrors in the experimental data, we may take it that the irregularity in the variation of the parameters probably arises from the simplified form of the interaction potential.

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Neutron optical potential groups

| Absorption | Surface |  |  | Volume |  |  |  | Combined |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| No. of <br> group I | II | III | IV | I | II | III | IV | II | III |
| $810^{6}$ | 9 | 9 | 0 | $10^{6}$ | 9. | 9 | 0 | 9 | 9 |
| Ve, MeV 4I, 5 | 43,2 | 4I,9 | 43,3 | 44,3 | 43 | 39, I | 42,2 | 40,9 | 38,9 |
| $\mathbf{r i f}_{1} \mathrm{f}$ I,3I | I,28 | I, 3 | I,29 | I,25 | I,26 | 1,35 | I, 3I | 1,31 | I,35 |
| a, f 0,36 | 0,29 | 0,25 | 0,27 | 0,6 | 0,56 | 0,36 | 0,3 | 0,46 | 0,3I |
| Wh Mev - | - | - | - | 3,6 | 4,5 | 4,2 | 5 | 2,I | I,5 |
| $\mathrm{W}_{2} \cdot \mathrm{MeV} 4$ | 4,7 | 5,6 | 10 | - | - | - | - | 5 | 5 |
| $z_{2}: f \quad \mathrm{I}, \mathrm{I7}$ | I, I7 | I, 05 | I, I | - | - | - | - | I,06 | I,26 |
| B,f 0,95 | I, 0 | 0,95 | 0,72 | - | - | - | - | 0,72 | 0,64 |
| Vso MeV 4,9 | 3,3 | 17,8 | 18,5 | 0 | 3,I | 14,9 | 17 | 0 | 16,4 |
| $\mathrm{H}^{2} \quad 0,137$ | 0,077 | 0,054 | 0,037 | 0,I | 0,057 | 0,058 | 0,04 | 0,06I | 0,054 |
| $H_{2}^{2} \times 10^{-3} 0,17$ | I,2 | 3,6 | 7 | 0,79 | I, 2 | 2,2 | 9,7 | 0,64 | I, 4 |



Fig. 1 Total interaction cross-sections for ${ }^{238}{ }_{U}$ calculated with volume (a), surface (b) and combined (c) absorption. The theoretical curves correspond to the parameter group numbers given in Table l:



Fig. 2 Crossmections for the formation of a compound nucleus. The notation is the same as in Fig. l. The experimental points represent the total inelastic interaction crosssection.


Fig. 3 Angular distributions of elastically scattered neutrons. The theoretical curves are given for the No. II group of parameters with combined (—_), volume ( - - ) and surface (- - - ) absorption.
key


Fig. 4 Excitation functions of various ${ }^{238} \mathrm{U}$ levels. The notation for the theoretical curves is the same as in Fig. 3.

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DIFFERENTIAL CROSSOSECTIONS FOR INELASTIC SCATTERING<br>OF NEJTRONS BY Cr, Mn, Fe, Co, Ni, Cu, Y, Zr, Nb, W AND Bi NUCLEI<br>G.N. Lovchikova, O.A. Salnikov, G.V. Kotelnikova<br>A.M. Trufanov, N.I. Fetisov

## Introduction

This paper supplies differential cross-sections for inelastic scattering of neutrons in the form of graphs and tables for an initial neutron energy of 14.36 MeV .

The differential cross-sections were measured at $30^{\circ}$ intervals over the neutron scattering angle range $31^{\circ}-151^{\circ}$ with an uncertainty in the angle of $\Delta \vartheta= \pm 8^{\circ}$. The energy spread due to this uncertainty and to the thickness of the target was 0.14 MeV , so that the initial neutron energy was $14.36 \pm 0.14 \mathrm{MeV}$.

The measurements were performed by the time-of-flight method in cylindrical geometry, the resolving time of the spectrometer was $5-8 \mathrm{nsec}$, the flight path was 2 m and the neutron recording threshold $\sim 100 \mathrm{keV}$. The spectrometer is described in more detail in Ref. [1].

The differential crossmsections incorporate corrections for multiple scattering and neutron flux attenuation in the sample; these were calculated by the Monte Carlo method.

In analysing the results it is of interest to consider the angular distribution of scattered neutrons as a function not only of the scattering angle $\vartheta$ but also their energy, i.e. $\sigma\left(E_{O}, E, \vartheta\right)$. For this reason the experimentally obtained spectra of inelastically scattered neutrons at different scattering angles were split up into several energy intervals - from 0 to 3.0 MeV , from 3.0 to 4.2 MeV , from 4.2 to 5.4 MeV , from 5.4 to 6.4 MeV and from 6.4 to 14.4 MeV . Differential crossmections were calculated for each energy interval. The method of measuring the spectra and the method of processing the results are described in Refs $[1]$ and $[2]$.

The data on the angular distributions of inelastic neutron scattering are given in the form of a series $[3]: \sigma(\mu)=\frac{1}{4 \pi} \sum_{i=0}^{\sum_{i}} a_{l} P_{\ell}(\mu)$, where $a_{\ell}$ is the expansion coefficient, $P_{\ell}$ is the Legendre polynomial of order $\ell$ and $\mu$ is $\cos \vartheta$.

Three terms of the expansion were sufficient for representing the existing angular distributions. The curves drawn through the experimental points resulting from analytical expansion of the angular distributions in a Legendre polynomial series are shown in Figs l-ll.

Table 1 shows the Legendre polynomial expansion coefficients.
The differential crossmections obtained by the authors are shown in Table 2. The errors indicated on the graphs and in the tables are the mean square statistical errors which do not include the error of the cross-section for scattering by carbon.

The uncertainty in the normalization of the spectra to scattering by carbon is $10 \%$. Figs 12-14 compare the differential cross-sections for cobalt, zirconium and bismuth measured by other authors [4] and the differential cross-sections obtained by the present authors for approximately the same energy ranges (denoted by black dots).

In conclusion the authors wish to thank M.D. Bityutskaya and E.S. Chermichenko for their help with the calculations and with the formulation of the data.

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Table 1
Coefficients for expanding the differential cross-sections in Legendre polynomial series

| El ement | Chromium |  |  |  | Manganese |  | Iron |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Energy range, MeV |  |  |  |  | 9 |  |  |  | ${ }^{+}$ |
| $3,0+4,2$ | 0,I88 | 0,0502 | 0,0132 | 0,203 | 0,0470 | 0,026 | 0.191 | 0,0442 | 0,0066 |
| $4.2+5.4$ | 0,1IO | 0,0374 | 0,0071 | 0,113 | 0,0241 | 0,019 | 0,117 | 0, 0280 | 0,0044 |
| $5.4+6.4$ | 0,059 | 0,0264 | 0,0043 | 0,068 | 0,0132 | 0,013 | 0.0177 | 0,0178 | 0,0013 |
| $6,4+I 4,4$ | 0,199 | 0,0877 | 0.0205 | 0,I67 | 0,0223 | 0,045 | 0,730 | 0,0.438 | -0.0295 |
| $0+I 4,4$ | I, 4I9 | 0,2460 | 0,0012 | I,608 | 0.1395 | 0,146 | I,6I4 | 0,2376 | 0.0071 |


| Element | Cobalt |  |  | Nickel |  |  | Copper |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Energy range, MeV | $\mathrm{a}_{0}$ | $\mathrm{a}_{1}$ | $\mathrm{a}_{2}$ |  | $a_{0} a_{1}$ |  | $\mathrm{a}_{0}$ | a | $\mathrm{a}_{2}$ |
| $3,0+4,2$ | 0,215 | 0.0399 | 0,02II | 0,128 | 0.0355 | 0,0138 |  |  |  |
| $4,2+5,4$ | 0,116 | 0,0238 | 0,0146 | 0,072 | 0,0214 | 0,0106 | 0,2143 0,1237 | 0,0538 0,0365 | 0,0509 0,0049 |
| 5,4 $\div 6,4$ | 0,069 | 0,0154 | 0.0110 | 0,045 | 0,0139 | 0,0081 | 0,0705 | 0,0269 | 0,0009 |
| 6,4 $+14,4$ | 0,183 | 0,052I | 0,0570 | 0, 134 | 0,0445 | 0.0352 | 0,2269 | 0,II65 | 0,0028 |
| $0 \div I 4,4$ | I, 772 | 0,1563 | 0,1391 | I, 056 | 0,I82? | 0,1409 | I,9352 | 0,336 5 | -0,0139 |

Table 1 (continued)

| Element | Yttrium |  |  | Zirconium |  |  | Niobium |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Energy range, MeV | ${ }^{0}$ | $\mathrm{a}_{1}$ | $\mathrm{a}_{2}$ | $\mathrm{a}_{0}$ | $a_{1}$ | $\mathrm{a}_{2}$ | $a_{0}$ | $\mathrm{a}_{1}$ | $\mathrm{a}_{2}$ |
| 3,0 $+4,2$ | 0,229 | C,0573 | 0,0244 | 0,209 | 0,0314 | 0,025I | 0,270 | 0,0489 | -0,0005 |
| $4,2+5,4$ | 0,120 | 0,0376 | 0,0093 | 0,IOI | 0,0213 | 0,0133 | 0,130 | 0,0339 | 0,0030 |
| $5,4+6,4$ | 0,071 | 0,0265 | 0,0046 | 0,059 | 0,015I | 0,0082 | 0,071 | 0,0233 | C,0033 |
| $6,4+14,4$ | 0.163 | 0,0827 | 0,0079 | 0,I55 | 0,05I8 | 0,0079 | 0,155 | 0,0664 | 0,0126 |
| O 414,4 | 2,577 | 0,3395 | 0,2785 | 2,298 | 0,2056 | 0,1296 | 2,738 | 0,2214 | -0,1170 |


| Element | Tungsten |  |  | Bismuth |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Energy range, MeV | $a_{0}$ | $\mathrm{a}_{1}$ | $\mathrm{a}_{2}$ | $\mathrm{a}_{0}$ | $\mathrm{a}_{1}$ | $a_{2}$ |
| $3,0+4,2$ | 0,317 | 0,I2II | 0,0141 | 0,4114 | 0.0837 | 0,0573 |
| $4,2+5,4$ | 0,152 | 0,0732 | 0,0189 | 0,1705 | 0,05i2 | 0,0424 |
| $5,4+6,4$ | 0,036 | 0,0529 | 0,0163 | 0,0842 | 0,0319 | C,0304 |
| $6,4+\mathrm{I4,4}$ | 0,208 | 0,1580 | 0,0665 | 0,1524 | 0,0632 | 0,0656 |
| $\mathrm{C}+\mathrm{I} 4,4$ | 3,474 | 0,6463 | $-0,0557$ | 4,0039 | 0,3823 | 0,4103 |

Table？
Differential cross－sections $(n, n)+(n, 2 n) m b / s r$

Chromium

| $\theta^{\wedge E}$ ！ | 0－2，98 | 2，98－4，22 | 4，22－5，37！ | 5，37－6，43 | 6，43－14，32 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $31^{\circ}$ | $67,5 \pm 1,4$ | 18，5 $\pm 0,9$ | II， $\mathrm{I} \pm 0,9$ | $7, I \pm 0,8$ | 19，7 $\pm 2,3$ |
| $61^{0}$ | $74,8 \pm I, 4$ | ［7，4土0，9 | IO，9 ${ }_{\underline{2}} 0,8$ | 7，2£0，7 | $22,0 \pm 1,7$ |
| $91^{\circ}$ | 68，I $\pm \mathrm{I}, 3$ | I4， $4 \pm 0,8$ | 8，4土0，7 | $5,3 \pm 0,6$ | I4， $9 \pm 1,4$ |
| $12.1{ }^{\circ}$ | $6 \mathrm{I}, 2 \pm \mathrm{I}, 2$ | 12，2土0，7 | $6,7 \pm 0,6$ | 4， $\mathrm{I}+0,5$ | IC， $4 \pm 1,2$ |
| $15 \mathrm{~J}^{\circ}$ | 63， $5 \pm 1,3$ | 12，4土0，7 | 6，7さ0，6 | $4, \mathrm{I} \pm 0,5$ | II， $7 \pm 1,2$ |

Manganese

| $\triangle E E!$ | $0-2,98$ | $!2,98-4,22$ | $4,22-5,37$ | $5,37-6,43$ | $6,43-I 4,32$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $3 I^{\circ}$ | $84,2 \pm I, 6$ | $20,4 \pm I, 0$ | $I I, 3 \pm 0,9$ | $6,7 \pm 0,8$ | $I 6, I \pm I, 7$ |
| $6 I^{\circ}$ | $9 I, I \pm I, 6$ | $I 8,0 \pm 0,8$ | $I 0, I \pm 0,7$ | $6, I \pm 0,6$ | $I 4,7 \pm I, 2$ |
| $9 I^{\circ}$ | $80,9 \pm I, 4$ | $I 4,6 \pm 0,7$ | $7,9 \pm 0,6$ | $4,7 \pm 0,5$ | $I 0,9 \pm I, 0$ |
| $I 2 I^{\circ}$ | $79,9 \pm I, 4$ | $I 4,4 \pm 0,7$ | $7,9 \pm 0,6$ | $4,8 \pm 0,5$ | $I 2,2 \pm I, 0$ |
| $I 5 I^{\circ}$ | $85,9 \pm I, 5$ | $I 4,0 \pm 0,7$ | $8,3 \pm 0,6$ | $5,2 \pm 0,5$ | $I 4, I \pm I, 0$ |

Iron

| $\theta^{\Delta E}$ ！ | 0－2，98 | ！2，98－4，22 | 4，22－5，37 | 5，37－6，43 | 6，43－14，32 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $31^{\circ}$ | $81,7 \pm 1,8$ | $19.5 \pm 1.2$ | 12，7士1，2 | 8， $6 \pm 1,1$ | 26，8＋2， 8 |
| $61^{\circ}$ | $84,7 \pm 1,7$ | 15，6＋0，9 | 9，0 $\pm 0,8$ | $5,7 \pm 0,7$ | 16，3＋1，6 |
| $9 \mathrm{I}^{\circ}$ | $82,4 \pm I, 6$ | I6， $0 \pm 0,9$ | $10, \mathrm{I} \pm 0,9$ | 6，9＋0，8 | $2 I, 4 \pm I, 7$ |
| $\underline{1210}$ | $68,4 \pm \mathrm{I}, 4$ | I3， $\mathrm{I} \pm 0,8$ | $8,2 \pm 0,7$ | $5,5 \pm 0,7$ | I8，4 $\pm 1,5$ |
| I5I ${ }^{\circ}$ | $76, I \pm I, 6$ | I2， $5 \pm 0,8$ | $7,5 \pm 0,7$ | $4,8 \pm 0,6$ | $12,9 \pm \mathrm{I}, 2$ |

Cobalt

| $\theta$ | 0－2，98 | 2，98－4， | 4，22－5，37 | 5，37－6，43 | 6，43－14，32 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $31^{0}$ | 93，4 $\pm \mathrm{I}, 6$ | 20，7士0，9 | II， $5 \pm 0,8$ | $7,1 \pm 0,7$ | 19，3＋2，0 |
| $61^{0}$ | 102，0土1， 6 | 18，9£0，8 | ID，İO，7 | 6，2 $\pm 0,6$ | 17，2iI， 4 |
| $91^{\circ}$ | 91， $6 \pm 1.5$ | $16,0 \pm 0,7$ | 8，4土0，6 | 4，9 $\pm 0,5$ | 12，3iI． 2 |
| I21 ${ }^{\circ}$ | 89，4 $\pm 1,4$ | 15， $5 \pm 0,7$ | $8,3 \pm 0,6$ | 4，9＋0，5 | II，9i $\mathrm{I}_{\mathrm{i}} \mathrm{O}$ |
| $151{ }^{\circ}$ | 97，6ıI， 6 | $15,5 \pm 0,7$ | $8,3 \pm 0,6$ | 5，0＋0，5 | 13，6 $\pm$ I，J． |

Nickel

| $8 E E:$ | $0-2,98$ | $2,98-4,22$ | $A, 22-5,37$ | $5,37-6,43$ | $6,43-I 4,32$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $3 I^{0}$ | $60,3 \pm I, I$ | $I 3,8 \pm 0,7$ | $8, I \pm 0,6$ | $5, I \pm 0,6$ | $I 5,2 \pm I, 9$ |
| $6 I^{0}$ | $58, I \pm I, 0$ | $I 0,7 \pm 0,6$ | $6,0 \pm 0,5$ | $3,8 \pm 0,5$ | $I I, 8 \pm I, 2$ |
| $9 I^{0}$ | $48,8 \pm 0,9$ | $9,8 \pm 0,5$ | $5,6 \pm 0,5$ | $3,6 \pm 0,4$ | $I 0, I \pm I, 0$ |
| $I 2 I^{0}$ | $5 I, 0 \pm 0,9$ | $8,9 \pm 0,5$ | $4,7 \pm 0,4$ | $2,9 \pm 0,4$ | $7,7 \pm 0,9$ |
| $I 5 I^{\circ}$ | $52,8 \pm I, 0$ | $8,3 \pm 0,5$ | $4,8 \pm 0,4$ | $3, I \pm 0,4$ | $9,8 \pm 0,9$ |

Copper

| $\theta^{\triangle E}$ | 0－2．98 | 2，98－4，22 | 4，22－5，37 | 5，37－5，43 | 6，43－14，32 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $3 \mathrm{I}^{\circ}$ | 97，2 $\pm 1.7$ | $2 \mathrm{I}, 0 \pm 1, \mathrm{I}$ | 12，3 $\pm 1,0$ | 7，9さI，0 | $24,7 \pm 2,9$ |
| $6 \mathrm{I}^{0}$ | I． $23,5 \pm 2, \mathrm{I}$ | 19，5 $\pm$ I， 0 | II， $6 \pm 0,9$ | 7，6さ0，9 | 23，6＋2，0 |
| $91^{\circ}$ | 103， $6 \pm 1,8$ | 16，6＋0，8 | 9，5 $\pm 0,7$ | 6，İO， 6 | 17，9＋1，4 |
| I210 | 91， $4 \pm 1,5$ | 14，5 50,7 | 8，3土0，6 | 5， $\mathrm{I} \pm 0,6$ | 12，7士1，2 |
| $151^{0}$ | 98， $8 \pm 1,7$ | I4， $0 \pm 0,7$ | 7，6さ0，6 | 4，4土0，6 | 10，4土I， 3 |

Yttrium

| $\stackrel{\wedge}{\triangle}$ | 0－2，98 | 2，98－4，22 | 4，22－5，37 | ！5，37－6，43 | 6，43－I4，32 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $3 \mathrm{I}^{\circ}$ | 163，4＋3，5 | 24，2土1，7 | I3，6さI， 6 | $8,6 \pm 1,5$ | 25，7 $\pm 4,7$ |
| $61^{\circ}$ | 177， $7 \pm 3,7$ | 18， $4 \pm 1,4$ | $9,2 \pm 1.3$ | 5，1̇I， 3 | 10，0 $0 \times 3$ I |
| $91^{\circ}$ | 147，2£3，2 | I8， $8 \pm 1,3$ | 10， $3 \pm 1$ ， 2 | 6，2土I，I | I5，0 2 ， 7 |
| I2I ${ }^{\circ}$ | $140,9 \pm 3,2$ | I4， $8 \pm 1,3$ | $7.9 \pm 1$ ， 2 | 4，6さI，I | 10，4 42,7 |
| 1510 | I69，4土4，0 | 15，7さ1，4 | 7，3土1，3 | 3， $8 \pm 1,2$ | 6，5＋2，9 |

Zirconium

| ${ }_{\theta} \stackrel{\Delta}{*}^{\text {E }}$ | 0－2，98 | 2，98－4，22 | 4，22－5，37 | 5，37－6，43 | 6，43－14，32 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $31^{\circ}$ | ［43， $4 \pm 2,6$ | 20， $5 \pm 1$ ，？ | IO， $4 \pm$ I，I | 6，3 $\pm$ I， 0 | ［7，9 $\pm 3,2$ |
| $61^{\circ}$ | 150，9さ2，6 | I6， $8 \pm 1,0$ | 8，4£0，9 | $4,9 \pm 0,8$ | 12，7さ2，0 |
| $91^{\circ}$ | 138， $\mathrm{C}+2,5$ | I $6, \mathrm{I} \pm \mathrm{I}, 0$ | 7，9 $\pm 0,9$ | $4,6 \pm 0,8$ | 13， $6 \pm 2,0$ |
| I21 ${ }^{\circ}$ | $130,8 \pm 2,3$ | 15， $2 \pm 0,9$ | 7，0 $\pm 0,8$ | $3,8 \pm 0,7$ | 9，5土I， 8 |
| I5I ${ }^{\circ}$ | 142，5さ2，4 | 15，6さ0，9 | 7，3土0，8 | $4, \mathrm{I} \pm 0,7$ | 9，3土I， 6 |

Niobium

| $\theta$ | 0－2，98 | 2，98－4， | 4，22－5， | 5，37－6，4 | 6，43－14 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $31^{\circ}$ | I54，3 $\pm 2.7$ | 24，5 $\pm 1,3$ | 12，8さI， 2 | 7，5 $\pm 1.1$ | 17， $8 \pm 3,1$ |
| $61^{0}$ | $187,3 \pm 3,0$ | 23，6 $\pm 1,2$ | II， $4 \pm 1,0$ | 6，4 $\pm 0.9$ | I4，422，I |
| $9 \mathrm{I}^{0}$ | I7C， $8 \pm 2,8$ | 22，0＋1，I | 10， $5 \pm 0,9$ | $5.7 \pm 0,8$ | II， $8 \pm 1,7$ |
| $121^{\circ}$ | 159， $1 \pm 2,7$ |  | 8，6＋0，8 | $4,6 \pm 0,8$ | 9，6 $\pm 1,7$ |
| I51 ${ }^{\circ}$ | 162，8さ2，7 | 18，5士I，0 | $8,3 \pm 0,8$ | $4,3 \pm 0,7$ | 8，3＋1，6 |

Tungsten

| $\theta$ AE | 0－2，98 | 12，93－4，22 | ！4，22－5，37 ！ | 5，37－6，43 | 6，43－14，32 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $31^{\circ}$ | 216，5 $\pm 4,3$ | 34，7＋2，3 | I8，7＋2，I | II， $5 \pm 2,0$ | 32，2＋5，9 |
| $6 \mathrm{I}^{0}$ | 233，4土4，2 | 28，7ıI．9 | 14，3＋1，8 | $8,2 \pm \mathrm{I}, 7$ | 19，6＋4， 1 |
| $9 \mathrm{I}^{\circ}$ | 224，844， 1 | $25,7 \pm 1.8$ | II， $9 \pm 1$ ， 6 | 6，7土1，5 | 16， $2+3,5$ |
| $121{ }^{\circ}$ | $199,3 \pm 3,8$ | $19.5 \pm 1.5$ | 8， $3 \pm 1,4$ | 4，2 $\pm 1,3$ | 8，0ㅍ3，2 |
| $155^{\circ}$ | $194,5 \pm 3,8$ | 17．7 $\pm 1.5$ | 7，7さ1，3 | 4，İI， 3 | 9，5 $\pm 3,0$ |

Bismuth
$\left.\begin{array}{|rrrrrrr}\hline 0 & 0-2,98 & 2,98-4,22 & 4,22-5,37 & 5,37-6,43 & 6,43-I 4,32 \\ \hline 3 I^{\circ} & 272, I \pm 4,4 & 43,4 \pm 2,0 & 2 I, I \pm I, 7 & I 2, I \pm I, 5 & 29,7 \pm 4,4 \\ 6 I^{0} & 254,7 \pm 4,0 & 32,7 \pm I, 6 & I 2,7 \pm I, 3 & 5,4 \pm I, 2 & 4,8 \pm 2,9 \\ 9 I^{\circ} & 247,2+3,9 & 30,7 \pm I, 5 & I 2,6 \pm I, 2 & 6,3 \pm I, I & I 3,2 \pm 2,5 \\ I 2 I^{0} & 245, I \pm 3,8 & 30,5 \pm I, 4 & I I, 9 \pm I, I & 5,6 \pm I, 0 & I O, I \pm 2,3 \\ I 5 I^{0} & 253,5+4,0 & 28,9 \pm I, 4 & I I, 6 \pm I, I & 5,6 \pm I, 0 & 9,9 \pm 2,2\end{array}\right]$

Key to graphs（pp 113－136）

$$
\mathrm{M} \ni_{\mathrm{B}}=\mathrm{MeV}
$$

Лаб．＝Lab．

мбарн／$\quad=\mathrm{mbarn} / \mathrm{sr}$ стерадиан

смещение нуля
$=$ Suppressed zero






Fe


Co



Cu
$\left(n, n^{\prime}\right)$


$$
\begin{gathered}
C u \\
\left(n, n^{2}\right)
\end{gathered}
$$



Y












ENERGY SPECTRA OF INELASTICALLY SCATYEERED NEUTRONS FOR Cr, Mn, $\mathrm{Fe}, \mathrm{Co}, \mathrm{Ni}, \mathrm{Cu}, \mathrm{Y}$, $\mathrm{Zr}, \mathrm{Nb}, \mathrm{W}$ AND Bi<br>O.A. Salnikov, G.N. Lovchikova, G.V. Kotelnikova, A.M. Trufanov, N.I. Fetisov

This paper supplies the energy spectra of inelastically scattered neutrons with initial energy of 14.4 MeV for scattering angles of $31^{\circ}, 61^{\circ}$, $91^{\circ}, 121^{\circ}$ and $151^{\circ}$ with angular resolution of $\pm 8^{\circ}$.

The spectra were measured by the timeof-flight method in cylindrical geometry. The resolving time of the spectrometer was $6-8 \mathrm{nsec}$, the analyser channel width 2.6 nsec , the flight path 2 m and the neutron recording threshold ~ 100 keV .

The measuring procedure is described in Refs [1] and [2]. When the spectra were processed, corrections were made for counting losses due to the dead time of the spectrometer, the effect of the position of the collimator and the scatterer on the count of the monitor, for background due to cosmic rays and sample activation, and for multiple neutron scattering in the sample and attenuation of the direct flux. This correction was calculated by the Monte Carlo method using a programme developed in the Institute of Physics and Power Engineering. All the spectra were normalized to the same monitor count and were reduced to the same geometrical conditions for each element separately.

Since for all nuclei the primary neutron energy exceeds the neutron binding energy in the target nucleus, the measured spectra are in fact composite spectra of neutrons from the $\left(n, n^{\prime}\right)+(n, 2 n)+(n, p n)$ reaction. The data for the energy spectra are given in the form of graphs and tables. The errors of the different points in the energy spectra are mean square errors which take into account the statistical errors in the measurement of the spectra, errors in measuring the efficiency and the uncertainty of the flight path.

The spectra provided can be used for reactor shielding and core calculations and for other practical purposes as well as for theoretical studies of nuclear properties.

In conclusion the authors wish to thank M.D. Bityutskaya and E.S. Chernichenko for performing numerous calculations and formulating the results.

## Tables l-ll

```
    Key: Xpom = Chromium
            Maprанец = Manganese
            Железо = Iron
            Кобальт = Cobalt
            Никель = Nickel
            Медь = Copper
            Иттрий = Yttrium
            Цирконий = Zirconium
            Ниобий = Niobium
            Вольфрам = Tungsten
            Buсмут = Bismuth
            HoB = MeV
Усредн苗ый спектр = Average spectrum
```

TABLE I
$X P O H \quad$ TaOnйа.

| $\begin{gathered} m \cdot v \\ 8,9: 98 \end{gathered}$ | $31^{\circ}$ | $62^{\circ}$ | $91^{\circ}$ | $121^{\circ}$ | $151{ }^{\circ}$ | усрепннвния cnerry |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 2 | 3 | 4 | 5 | 6. | 2 |
| 0,102 | 4,26. | ?,01 | 12,97 | 10.00 | 6,66 | 8,53 |
| $0,1.5$ | 6,32 | II.12 | 17.77 | 14,56 | 13,2I | 12,65 |
| 0,182 | 8.82 | 10,22 | 24,44 | 20,07 | 19,17 | 18,14 |
| 0.221 | IL.93 | 23,57 | 30,82 | 25,73 | 24,31 | 23,29 |
| 0,241 | 13.50 | 25,95 | 33,56 | 28,11 | 26,82 | 25,59 |
| 0,265 | 15,67 | 30.51 | 38,13 | 31,04 | 30,60 | 29,19 |
| 0,292 | I7.94 | 35.19 | 42,68 | 34,25 | 34.4I | 32, 0 |
| 0.223 | 21,2i | 41.73 | 48,56 | 39,92 | 38.31 | 38,07 |
| 0.360 | 2A,40 | 49,75 | 53,12 | 43,40 | 45,22 | 43.18 |
| 0,403 | $20,90-43,65$ | 58,77 $\pm 16,30$ | $58,65 \pm 14,90$ | 49,42 $\pm 16.13$ | 52,51 $\pm 16,48$ | $49,65 \pm 10,81$ |
| 0,455 | $34,41 \pm 15,76$ | $70,14 \pm 12,10$ | 64,71+- II, 10 | $55,51 \pm 11.08$ | SI, $13 \pm 12,21$ | 67,18 $\pm 5,62$ |
| 0,5-8 | $41.14 \pm 10.23$ | 79,05 $\pm 9,53$ | 71,56 $\pm 9,28$ | 62,22 $\pm 0.17$ | 7I,61 $\pm 9.78$ | $65,12 \pm 4,33$ |
| 0,50\% | $50,51 \pm 8,72$ | $88,30 \pm 8,89$ | 77,74 $\pm 8,40$ | $68.89 \pm 0.16$ | 84,14 $\pm 8,87$ | 73,91 $\pm 3,85$ |
| 0.699 | 32,6I $\pm 8,00$ | 98,7i $\pm$ B. 52 | $84,79 \pm 7,92$ | 76,55 $\pm 7,56$ | 98,48 $\pm 8,67$ | $84,23 \pm 3,65$ |
| 0,809 | 79,10 7 7,87 | $117,86 \pm 8,62$ | -7,04 $\pm 7.72$ | 85,23 $\pm 7,39$ | 168,99 $\pm 8,66$ | $95,10 \pm 3,61$ |
| 0,901 | $95,30 \pm 8,08$ | 114,21 $\pm$, 57 | 97,26 $\pm 7.70$ | 92,27 $\pm 7,39$ | III.74 $\pm 8,50$ | 102,15 $\pm 3,61$ |
| I, I62 | 102,23 $\pm 8,17$ | I10,46 $\pm 8.30$ | 97,88 $\pm 7,54$ | 91,91 $\pm 7,14$ | $106,00 \pm 7,97$ | 101,70 $\pm 3,50$ |
| I, 3C8 | IOI, 61. $\pm 11,51$ | 105,07 $\pm 10,63$ | 06,31 $\pm 10,17$ | $88.71 \pm 9.86$ | 99,77 $\pm 10,38$ | 98,29 $\pm 4,70$ |
| I, 365 | 100, $19 \pm$ II, 09 | 102,30 $\pm 10.18$ | $95,20 \pm 9,73$ | 67,02 $\pm 9,50$ | $97,40 \pm 10,10$ | 96,5¢ $\pm 4,53$ |
| I, 426 | 3, $30 \pm 10,70$ | 100, $\mathrm{C2} \pm 9,84$ | 93,33 | $84,92 \pm 9,17$ | 94, $56 \pm 3,43$ | $94,20 \pm 4,36$ |
| 1.491 1.650 | $05,6 \pm$ <br> $94,05 \pm 10,43$ <br> 8.00 | 07, $9.9 \pm 9.58$ | 90, $01 \pm 8.05$ | $62,03 \pm 8,73$ | 90, $86 \pm 9,17$ | $91,55 \pm 4,20$ |
| I,650 I, 635 | $94,05 \pm 10,00$ $92,83 \pm 9,80$ | $94,43 \pm 9.14$ $9,1,65 \pm 0,85$ | $463.55 \pm 8.70$ $85,98 \pm 8.42$ | $79,77 \pm 8,24$ $77,17 \pm 7,95$ | $87,63 \pm 8,68$ | $89,00 \pm 4,01$ |
| 1,635 | $92,83 \div 9,80$ $90,47 \pm 9,43$ | $9,165 \pm 0,85$ $89,02 \pm 8,59$ | 85,98 $\pm$ 8,42 | $77,17 \pm 7,95$ $74,53 \pm 7,61$ | $84,53 \pm 8,28$ | 86,45士 3, 88 |
| I,801 | 87,25 $\pm 9,17$ | $85,86 \pm 8,32$ | 79,94 $\pm 7.81$ | 74, $71,43 \pm 7,61$ | $01.32 \pm 7.53$ $77.37 \pm 7.73$ | $83,71 \pm 3.74$ $60,60 \pm 3,61$ |
| 1.893 | $85,17 \pm 8.65$ | $82.51 \pm 8.02$ | $76,55 \pm 7,55$ | $69,37 \pm 7,04$ | 74,05 7,29 | 77,32 $\pm 3,47$ |
| 1.953 | 82,20 $\pm 8.60$ | 79,35 $\pm 7.73$ | 73,17 $\ddagger$ 7,28 | $65,25 \pm 6,75$ | $70.25 \pm 7.03$ | 71,06 $\pm 3,35$ |
| 2,301 | $78,54 \pm$ $75,51 \pm$ 7.85 7.85 | $75,34 \pm 7,39$ | $69,38 \pm 669$ | $61,77 \pm 6,38$ | $65.07 \pm$ ¢, 5S | 70,40+ 3.18 |
| 2,2,8 | $75,1 \pm 7.86$ $71.72 \pm 7.18$ | $72,16 \pm 7,06$ $68,48 \pm 6,74$ | 65,33 $\pm 6.57$ | $58,14 \pm 6,11$ | $61.81 \pm 6,25$ | 66,55 3 3,04 |
| 2,481 | 67,93 $\pm 7.11$ | 64,55 = 6,40 | $57.28 \pm 5.36$ | $54,48 \pm 5,76$ $50,62 \pm 5,41$ | $57,53 \pm 5,87$ $53,29 \pm 5,56$ | $\begin{aligned} & 62,73 \pm 2.88 \\ & 68.73 \pm 2.72 \end{aligned}$ |
| 2,635 | $64.13 \pm 6.71$ | 60,57 $\pm 6,08$ | $53.13 \pm 5.52$ | $46.75 \pm 5.06$ | 48,81 | $68,73-2,12$ $64,58-2,56$ |
| 2,801 | 60,14 $\pm 6.42$ | 56,57 $\pm$, 77 | $49.02 \pm 5.21$ | $43,02 \pm 4.77$ | $44,53 \pm 4,81$ | $50,65 \pm 2,43$ |
| 2,912 | 66,32 $\ddagger 6,02$ | 52,85 $\pm 5,46$ | $45,19 \pm 4,8.4$ | $39,45 \pm 4,44$ | $40,53 \pm 4,52$ | $40,87 \pm 2,27$ |
| 3,182 | $52,51 \pm 5,76$ $49,45 \pm 5,43$ | $49,24 \geq 5,15$ | $41,53 \pm 4,61$ | 35,90 $\pm 4,18$ | $36,78 \pm 4,21$ | $43.21 \pm 2.15$ |
| 3,603 3,647 | $49,46 \pm 5,43$ | 45,52 $\pm 4,89$ | $37.84 \pm 4,34$ | $32,44 \pm 3,90$ | $32,97 \pm 3,57$ | $39,45 \pm 2,02$ |
| 3,617 3,919 | $44,10 \pm 5,09$ $39,82 \pm 4.75$ | $41,65 \pm 4,00$ | $34.14 \pm 4.04$ | $29,68 \pm 3,60$ | $29.25 \pm 3,64$ | $35,60 \pm 1,83$ |
| 3,919 4,222 | $39,82 \pm$ $35,54 \pm$ 4,77 | $37,80 \pm 4,32$ | $30.57 \pm 3.77$ | $25.48 \pm 3.34$ | $25.65 \pm 3.29$ | $31,85 \pm 1,76$ |
| 4,222 4,562 | $35,54 \pm 4,47$ $31.44 \pm 4,09$ | $34,00 \pm 4,01$ $30,46 \pm 3,74$ | $27,10 \pm 3,51$ $23,76 \pm 3,21$ | $22,25 \pm 3,09$ | $22,30 \pm 3,03$ | $20,24 \pm 1,64$ |
| 4,945 | 27,50 3,81 | $30,46 \pm 3,74$ $27,13 \pm 3,50$ | $23,76 \pm 3,21$ $20,71 \pm 2,99$ | $19,23 \pm 2,61$ $16,47 \pm 2,57$ | $19,22 \pm 2,00$ | $21,82 \pm 1.50$ |
| 6,379 | 23,74 $\pm 3,49$ | $23,15 \pm 3,23$ | $20,71 \pm$ $17,82 \pm 2,76$ | $16,47 \pm 2,57$ $13,93 \pm 2,37$ | $16,48 \pm 2,55$ | 21,60 $\pm 1,40$ |
| 5,072 | $20,35 \pm 3,23$ | 20,78 $\pm 2,97$ | $17,82 \pm 2,76$ $15,22 \pm 2,51$ | $13,93 \pm 2,37$ $11,68 \pm 2,16$ | $14,02 \pm 2,29$ $11,85 \pm 2,10$ | $18,67 \pm 1.28$ |
| 8,437 | $117.19 \pm 2.95$ | 17,86 $\mp 2,73$ | $15,22 \pm 2,61$ $12,83 \pm 2,30$ | $11,68 \pm 2,16$ $0,65 \pm 1,33$ | $\begin{array}{r}11,35 \pm 2,16 \\ 9,91 \pm \\ \hline 1,93\end{array}$ | $1 \mathrm{U}, 90 \pm \mathrm{I}, 18$ |
| 7,007 | 14,29 $\pm 2,75$ | 15,16 $\pm 2,49$ | $10,02 \pm$ 10,11 | 7,65 7 , 1,33 | $9,31 \pm 1.93$ $8,20 \pm 1,76$ | $13,49 \pm 1,00$ |
| 7,842 | II,61 $\pm 2.47$ | 12,55 $\pm 2,25$ | $1,65 \pm 1,87$ | 6,25 $\pm 1,62$ | $8,20 \pm 1,76$ $6,69 \pm 1,69$ | II,22 $\pm 0.99$ |
| 8,725 | 9,27 $\pm 2,25$ | $10,21 \pm 2,00$ | 6,96 $\pm 1,64$ | $4,92 \pm 1,16$ | $6,69 \pm 1.69$ $5,40 \pm 1,45$ | 9,15 $\pm 0,89$ |
| 9,767 | $7,30 \pm 2,11$ | $8,00 \pm 1,75$ | 5,37 $\pm 1,44$ | 3,69 $\pm 1,26$ | $5,40 \pm 1,45$ $4,41 \pm 1,25$ | $7,35 \pm 0,85$ |
| 11,008 | 5,46 $\pm 2,12$ | $6,58 \pm 1,49$ | $1,34 \pm 1,44$ 4,23 | 2,97 $\pm 1,11$ | $4,41 \pm 1,25$ $3,43 \pm 1,09$ | $5,87 \pm 0,71$ |
| 32,503 | $4,22 \pm 2,16$ | $4,69 \pm 1,30$ | $4,34 \pm 1,23$ $3,38 \pm 1,06$ | 2,08 $\pm 0,93$ | $\begin{array}{ll}3,43 \pm & 1,09 \\ 2,62 \pm & 0,90\end{array}$ | $4,55 \pm 0,65$ |
| 14,328 | 3,13 $\pm 2,58$ | $3.56 \pm 1.20$ | $0,36 \pm 1,06$ 2,46 | $2,08 \pm 0,93$ $1,53 \pm 0,83$ | $2,62 \pm 0,90$ $1,94 \pm 0,77$ | $3,44 \pm 0,60$ $2,52 \pm 0,67$ |

TABLE 2
MAP「AHEil
Tаблйда 2

| $\begin{gathered} \operatorname{mex} \\ 4 . \\ \hline \end{gathered}$ | $3{ }^{0}$ | $61^{0}$ | $91^{\circ}$ | $121^{0}$ | $151^{\circ}$ | усредненния с口ектр |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 2 | 3 | 4 | 5 | $\theta$ | 7 |
| C．ice | 0,10 | 10.49 | 7，66 | 10，68 | 14， 3 | 8，51 |
| C．136 | 0，10 | 13，：6 | 12，32 | 14，65 | 19，94 | 12，15 |
| 0.182 | 0,10 | 20.63 | 19，42 | 2I， 43 | 27，68 | 17，85 |
| 0，221 | 2.74 | 28，18 | 26，16 | 28，31 | 34，74 | 24，03 |
| 0,241 | － 8 | 31，73 | ${ }_{28,66}$ | 31,33 | 38.86 | 86，92 |
| 0.265 | 7,97 | 37.47 | 33，73 | 36，48 | 44，67 | 32.07 |
| 0，292 | I2，${ }^{2}$ | 42，37 | 33，45 | 41，79 | 49，55 | 36，90 |
| c， 03 | 19， 3 | 49.59 | 43，82 | 46，99 | 56，11 | 4：3， 13 |
| 0，360 | －26，54 | 57.02 | 51，51 | 53，67 | 66，10 | 52，97 |
| $0,4 \mathrm{Cl}^{3}$ | $46,47 \pm 14,09$ | $65,05 \pm 13,82$ | 57，65 $\pm 11,82$ | $62,75 \pm 12,51$ | 75，66 $\pm 17.10$ | 60，36 $\pm 6,26$ |
| 0.450 | $39,43 \pm 10,34$ | $76,33 \pm 10,65$ | 64，90 $\pm 9,34$ | ＇22，64 $\pm 10,57$ | 07，02 $\pm 12,55$ | 68， $28 \pm 4,80$ |
| 0，519 | $44,46 \pm 8,32$ | £3，94 | 72，46 $\pm$ 8，03 | 79，00 $\pm 0,76$ | 89，19 $\pm 10.19$ | 16，71 $\pm 3,97$ |
| 0.694 | 56，71 $\pm 7.52$ | 87，00 $\pm 8,37$ | 79，14 $\pm \quad 7.51$ | 87，31 $\ddagger$ 7，90 | 105，70 $\pm 9,15$ | 85，12 $\ddagger 3,62$ |
| c，609 | 157，53 $\pm 7.08$ | 102．55 $\pm 7,59$ | 83，\％$\ddagger$ | 89，26 $\ddagger$ 7，31 | 103，19 $\pm 8.27$ | $83,27 \pm 3,35$ |
| 0，5：03 | 77， 60 \％ 6,92 | 104，41 $\pm 7.73$ | 88，92 $\pm$ 6，88 | 80，16 $\pm 6$ 6，83 | $100,93 \pm 7$ 7，66 | 91，99 $\pm 3,22$ |
| 0.60 S | 33，35 $\pm 705$ | $10.03 \pm 7.05$ | $92,18 \pm 6.86$ | 87，34 $\ddagger$－ 56 | 101，74 $=7.45$ | 94，73 $\ddagger$ 3，17 |
| I，İS | 95，99 | 101，25 $\ddagger$ 7，27 | 92，10 $\ddagger$ 6，72 | 86，44 $\pm$ 6，35 | $100,74 \pm 7,21$ | 95，30 $\pm 3,12$ |
| I， $3 \times 1$ | 97，69 $\pm 9,72$ | 57，56 $\pm 8,92$ | 90，20 $\pm 8,46$ | $85,08 \pm 3,25$ | 96，79 $\pm$ 9，00 | 93，48 $\pm$ 3，97 |
| I， 3 G5 | 97， $28 \pm$ ¢ $9,6 \mathrm{I}$ | 25，69 $\pm$ 8，57 | $89.49 \pm$ B，11 | 84，888 $\quad 8.09$ | $94,40 \div 8,57$ | 92，65 $\pm 3,85$ |
| 1，426 | 97，19 $\ddagger$ ， 20.0 | 94，42 $\pm 8,25$ | $87,72 \pm 8,03$ | 83，53 | 91，52 ${ }^{\text {－}}$－ 0,27 | 90，48 |
| 1，495 | $96,17 \pm 3010$ | 22，02 $\ddagger 8.09$ | 35，72 $\pm 7,63$ | 81，90 $\pm 7.55$ | 86，95 $\ddagger 7,31$ | 89，55 $\ddagger$ |
| I． 560 | 95，18 518.82 | 89，94 $\pm 7,32$ | 83，67 $\ddagger 7,37$ | 80，27 $\ddagger \quad 7,29$ | 83，34 $\pm 7,37$ | 89，48 $\ddagger$ 3，47 |
| I．635 | 03，62 $\ddagger 8.64$ | $87,01 \pm 7.58$ | $81,48 \pm 7,27$ | 78，27 | $79.92 \pm \quad 7.12$ | 84，24 $\begin{aligned} & \text { ¢ } \\ & \text { 3，} \\ & \text { 8，} \\ & \text { 87 }\end{aligned}$ |
| 1，715 | 91， $77 \pm 8,37$ | $85.41 \pm 7.38$ | 79，02 $\ddagger$ ¢，94 | 76，76 $=6,76$ | 76，54 $\ddagger$ 6，87 | 81，78 $\ddagger$ ¢， 28 |
| 1，501 | 29，36 | 82，${ }^{5}$ ¢ 7.15 | $75,50 \pm 6,63$ | 73，87 $\pm$ 6，54 | 72，5I $\pm 6.46$ | 79，30 $\ddagger$ 3，15 |
| ［1，893 | $\begin{array}{lll}86,66 \\ 83,38 \\ \text { i } & 8,00 \\ 0,64\end{array}$ | 79，29 | 72，78 5 ¢，56 | $70,63 \pm 6,36$ | $68,32 \pm 6,24$ | $75,54 \pm 3,03$ |
| I， 939 2,105 | $\begin{array}{ll}33,38 \pm \\ \cdots 8,46 \\ \pm & 7,64 \\ 7\end{array}$ | 73，53 $\pm 6,53$ |  | $67,35 \pm 6,02$ | 69， $00 \pm$ 5， 89 | 191，04 $\pm 2,90$ |
| 2，105 | $78,46 \pm 7,30$ | 70，53 $\pm 6.25$ | 63，89 ${ }^{+}$5，76 | $62,23 \pm 5.67$ | 58，75 $\ddagger 5.44$ | 66，77 $-.2,73$ |
| 2，218 | $76,00 \pm 7.10$ | $67,33 \pm 5,98$ $03,04 \pm 50$ | $60.57 \pm 5.64$ | 59，u9 $\pm 5.45$ | $56.54 \pm 5.24$ | 63，70 $\pm 2,64$ |
| 2,346 8,481 | $71,15 \pm 6,65$ $66,6 E \pm 6136$ | $63,04 \pm 5,70$ $58,01 \pm 5,33$ | 55，9y $\pm 5.23$ | 54，64 $\pm 5,09$ | E0，94 $\pm 4,31$ | 59，15 $\pm 2,48$ |
| 2,481 2,635 | E6，66 $\pm 6,36$ $62,23 \pm 6,01$ | $58,01 \pm 5,33$ $54,80 \pm 5,06$ | $51.413 \pm 4,82$ | $50,3.4 \pm 4.75$ | 46，95 $\pm 1,53$ | $54,85 \pm 2,33$ |
| 2，801 | 52，23 $57,33 \pm 5,00$ | 51,30 <br> 50,34 <br> 4,76 | $17,35 \pm 4,63$ $42,96 \pm 4,25$ | $16,37 \pm 4,44$ $42,44 \pm 4,14$ | $43,25 \pm 4,2 ?$ $39,65 \pm 4,01$ | $50,80 \pm 2,20$ |
| 2，902 | $52,10 \pm 5,24$ | $45,32 \pm 4,44$ | 30， $51 \pm 3,86$ | $31,04 \pm 3,79$ | $35,57 \pm 3,63$ | $46,05 \pm 2,05$ $41,94 \pm 1,89$ |
| 3，102 | $47,60 \pm 4,91$ | $41,90 \pm 4,11$ | $34,60 \pm 3,66$ | $34,13 \pm 3,51$ | $32,55 \pm 3,40$ | 30，20 $\pm 1.77$ |
| 3，403 | $42,68 \pm 4,48$ | 37，51 $\pm 3,81$ | $30,34 \pm 3,31$ | 29，93 $\pm 3,19$ | 29，04 $\pm 3,14$ | $33,90 \pm 1,62$ |
| 3，647 | $37,08 \pm 4,18$ | $33,12 \pm 3,53$ | $26,75 \pm 2,90$ | 26，49 $\pm 2,92$ | $25,93 \pm 2,85$ | $33,09 \pm 1,62$ 36,09 |
| 3，919 | 33，26 $\pm 3,83$ | $29,54 \pm 3,22$ | 23，34 $\pm 2.78$ | 23，13 $\pm 2,65$ | $23,13 \pm 2,62$ | 36，09 $26,19 \pm 1,36$ |
| 4，222 4.562 | $29,44 \pm 3,13$ $25,52 \pm 3,26$ | $26,23 \pm 3,00$ $22,95 \pm 2,70$ | 20， $12 \pm 2.60$ | $20,48 \pm 2,44$ | $20,99 \pm 2,47$ | $23.57 \pm 1.27$ |
| 4,562 4,946 | $\begin{array}{lll}25,52 \pm & 3,26 \\ 21,51 \pm \\ 2,94\end{array}$ | $22,95 \pm 2,70$ $19,25 \pm 2,42$ | $17.84 \pm 2,32$ $1504 \pm 20.12$ | $17,79 \pm 2,24$ $15,66 \pm 2,00$ | 10，62 $\pm 2.25$ | $20.04 \pm 1,16$ |
| 5，379 | $28.50 \pm 2,65$ | 16，86 $\pm 2.127$ | $15,04 \pm 2,12$ $12,99 \pm 1,97$ | $15,66 \pm 2.00$ $13.14 \pm 1.85$ | $16,95 \pm 2,01$ $14,14 \pm 1.89$ | $17.36 \pm 1.04$ |
| 6．8\％\％ | 15，18 $\pm 2,39$ | $13.88 \pm 2.03$ | $10,55 \pm 1.67$ | $13,14 \pm 1.85$ $10,42 \pm 1.63$ | $14,14 \pm 1,89$ $11,10 \pm$ | 15，15 $\pm 0.66$ |
| 6.437 | 12，05＋2， 20 | $11,43 \pm 1.78$ | 8，55 | 10,80 $9,00 \pm 1.47$ | $\begin{array}{rl}1,16 & 1,1,5 \\ 9, L 6: 17\end{array}$ | $12,44 \pm 0,85$ $16,31 \pm 0,76$ |
| 7.037 | 10，34 $\pm 1,96$ | $9.69 \pm 1.66$ | $7,12 \pm 1,38$ | 7，63 | $8,48 \pm 1,36$ | $12,31 \pm 0.86$ $8,65 \pm 0,69$ |
| 7.042 3.725 | $8,65 \pm$ <br> $6,39 \pm$ <br> 1,67 <br> 1,61 | $7,48 \pm 1,52$ $5,84 \pm 1,26$ | $5,92 \pm 1,22$ | 6，15士 1，19 | 7，14 $\pm 1,22$ | $\begin{array}{lll}8.65 \pm & 0.69 \\ 7.15 \pm & 0.54\end{array}$ |
| 9，767 |  | 1，27 $\ddagger 1,02$ | 4，49 $\ddagger 1,05$ $3,35 \pm 0,86$ | $4,85 \pm 1.04$ $3,61 \pm 0,84$ | 5，61 $\pm 1,14$ $4,35 \pm 0,04$ | $5.4 .1 \pm 0.54$ |
| I\％．cce | 3，32 $\pm 1.25$ | 2， $0 . \pm 0,7 \%$ | $2,26 \pm 0,62$ | 2，56 $\pm$（，0．04 | $4,35 \pm 0,188$ $2,99 \pm 0,66$ | $1,07 . \pm 0.15$ $2,82 \pm 0.37$ |
| 12.500 | $1,81 \pm 0,98$ 1009 |  | $1,12 \pm 0,07$ | $1.04 \pm 0.15$ | $1,94 \pm 0,16$ | $1,82 \pm 0,37$ $1,1,6 \pm 0,27$ |
| 14，3：0 | $1,09 \pm 0,78$ | $0.75 \pm 0,32$ | $0.50 \pm 0,21$ | 0．84 $\pm 0.29$ | $1,23 \pm 0,33$ | $1,16 \pm 0.27$ $0.88 \pm 0.19$ |

TABLE 3
XE. 30
"аллие: 3


TABLE 4
KODA』トT
Rosnays 4


2ABLE 5
HLKESB
Tagrumas

| E，哏 | $31^{3}$ | $61^{\circ}$ | $91^{\circ}$ | $121{ }^{\circ}$ | $152^{0}$ |  cnertp |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2 | 2 | 3 | 4 | 5 | 6 | 7 |
| C． 102 | 6.42 | $2{ }^{2} 15$ | 1E．a＇7 | 17， 17 | 14.40 | 15，30 |
| 0，：36 | 10.18 | 31，${ }^{2} 4$ | 20，7\％ | 23，59 | ac，it | 21，93 |
| 1，joi | 2．i，吅 | 4．4，0i | 30.33 | 33.49 | $3 \mathrm{C}, 36$ | $3 \mathrm{C}, 62$ |
| C．2is | 20．45 | 4， 30 | 38，65 | 41.66 | 40.20 | 32.14 |
| C，241 | 22，i4 | 52， 30 | 42，70 | 4j， 53 | 44． 24 | 42，87 |
| C．21s | 禹，我 | $67,6: 4$ | 4\％，57 | 50， 82 | ［ 6.17 | 40．35 |
| C，20： | えい，${ }^{\text {¢ }}$ | 73．85 |  | 5S． 74 | 56.03 | 54.02 |
| C，3\％， 3 | 35.24 | U2，7is | 56，ic | 6i，${ }^{\text {c }}$ 8 | 65.17 | 60，81 |
| 0，360 | ia， 80 | 93，4y | 65，90 | 71，50 | 75， 5 | 70，17 |
| 0,403 | 43，4：$\pm$ E1，38 | I60，35 1 19，29 | 74，54 $\pm 21.57$ | $60,14 \pm 28.56$ | C6，43 $\pm 19,98$ | 8， $58 \pm 9,03$ |
| C． 4 SE | $63,4 i=3 i, 6 j$ | Si4，4］ $\pm 14,59$ | $84,87 \pm 14,05$ | 91， 8 \％ $\pm 13,74$ | $98.45 \pm 14,64$ | $92,00 \pm 6,43$ |
| C． 518 | $25,57 \pm 12,01$ | I15，25 $\pm 11,97$ | 66，73 $\pm 15.96$ | $00,89 \pm 11,27$ | III．57 $\pm 11,90$ | $10 \mathrm{i}, \mathrm{k3} \pm 5,20$ |
| C，594 | ㅇN， $95 \pm i c .98$ | ii5， 56 \％J 50,52 | 101，76 $\pm$ 9，72 | IC4．40 $\ddagger 9.87$ | I19，i3 $\pm 10,69$ | $100,1 u \pm 4,62$ |
| 0，609 | i16，i4 $\pm$ 20，4i | IIE， $49 \pm 0,86$ | $1159.63 \pm 9.63$ | IIT，4i $\pm 9.31$ | 227，52 $\ddagger$ IC，20 | $115,84 \pm 4,37$ |
| r，0．3 | $15 \mathrm{~S}, 800 \pm 20,41$ | 123．25 $\pm$ 9，49 | $\underline{1,5,93} \pm 6,55$ | i17．55 $\pm$－${ }^{\text {c4 }}$ | ［35，35 $\pm 10.05$ | 123，18 $\pm 4,26$ |
| C， $\mathrm{FS}_{4}$ | 146． $3 \mathrm{E} \pm$ IG， 4 S | 124， $\mathrm{CA} \pm 9,25$ | 105．60 $\pm 0.13$ | 115，72 $\pm 13,64$ | $132,4< \pm 9,63$ | $123,20 \pm 4,14$ |
| I， 262 $i, 303$ | $255,43 \pm 10.63$ <br> $29,60 \pm 22,58$ | $\begin{aligned} & 118,63 \\ & 113,05 \pm .75\end{aligned}$ |  | 107，23 $\pm 7,97$ | $118.39 \pm$ 8，63 | 115，84 $\ddagger 3,85$ |
| 2，303 | 229，60 i $6,42,58$ 2， 2， 2 | $113.05 \pm 10,86$ $110,75 \pm 10.50$ | $\begin{array}{rrr}96,53 \pm & 10,10 \\ 9 i, 66 \pm & 9,35\end{array}$ | $100.57 \pm 113.14$ | IGT， $07.10,57$ | $103,77 \pm 4,87$ |
| 1,565 1,426 |  | $110,73 \pm 16,50$ $107,79 \pm 10,68$ | $\begin{array}{ll}91,66 \pm & 9,35 \\ 80,34 \pm & 8,96\end{array}$ | $99.24 \pm 9.90$ $05.54 \pm 13.53$ | ICS，C4 $\pm 10.20$ | 106，03 $\pm 4,63$ |
| 1，441 | Ii8，40 415,30 | 104．40 $\pm 9.76$ | $\begin{array}{ll}80,341 & 8,96 \\ 8,024 & 8,86\end{array}$ | $\begin{array}{ll}95,54 \pm & 3,53 \\ 92,64 \pm & 3,04\end{array}$ | 98,91  <br> $4,6 \pm \pm$ 9,75 | $\begin{aligned} 102,87 & \pm, 50 \\ 99,46 & \pm 4,33\end{aligned}$ |
| 1，550 | 115， $27 \pm 10,80$ | 101．55 $\pm 9.44$ | 64，97土 8，35 | 42，89 $\pm$ 1，7i | $90.44 \pm 0.74$ | 96，45 $\pm 4,13$ |
| I， 635 | 111，56 $\pm$ ic． 65 | $W_{0} 72 \pm 0.12$ | 82，72t $\quad 7.99$ | $87.12 \pm 8.34$ | 86，5i $\ddagger 8.42$ | 93，3他 4，00 |
| 1.755 | 160，924 IU． 13 | 65，59 $\pm 0,64$ |  | 84，38 $\pm$ 8， 65 | 82，74 $\ddagger 0.02$ | 90，23 $\pm 3,87$ |
| 1,861 1,693 | $163,50 \pm 9,15$ $99.76 \pm 9,60$ | 81，93 $58,72,56$ | $77.90 \pm 7.5 i$ | $81,10 \pm \quad 7,55$ | $76.51 \pm 7.72$ | 96．69 53.59 |
| 1,033 $i, 930$ | $\begin{array}{lll}79,76 \pm & 9,60 \\ 95,05 \pm & 0.23\end{array}$ | $\begin{array}{lll}\text { E8，72 } & \pm & 6,24 \\ 84,37 & \pm & 7,96\end{array}$ | $\begin{array}{ll}74,924 & 7,14 \\ 72.20 \pm & 7.27\end{array}$ | 7\％．56 $\ddagger 7.44$ | $74,18 \pm \quad 7,33$ | $82,93 \div 3,63$ |
| 2，10i | Si，67 $\ddagger$ 8，R8 | 60，30 $\ddagger$ \％， 68 | 6， 0 9， 6.74 | $\begin{array}{ll}73.83 \pm & 7.19 \\ 69,78 \pm & 6.70\end{array}$ | $70,15 \pm$ <br> $65,94 \pm$ <br> 6.79 | $\begin{array}{lll} 79,31 & 3.28 \\ 75,39 & 3,30 \end{array}$ |
| 2，218 | $8^{4}, 19 \pm 0,57$ | 70．116 $\pm 7.31$ | C6． $29 \pm 6.11$ | $65,77 \pm$ ¢， 48 | $61.27 \pm 6.34$ | $71, \mathrm{Ct} \pm 3,16$ |
| 2,346 2,484 | $42,77 \pm 14.14$ $74.14 \pm 7,76$ | $71,11 \pm 6.37$ | 61． $16 \pm 6.34$ | $61.33 \pm C .15$ | $45,76 \pm 5,55$ | $66,76 \pm 3,02$ |
| 2,484 2,635 | $76.14 \pm 7.76$ $73.15 \pm 7.41$ | $66,30 \pm 6.03$ $61,39 \pm 6.20$ | $57.93+5.93$ | $56,54 \pm 5,80$ | 12， $24 \pm 5,50$ | $62,31 \pm 2,85$ |
| 2，86i | $73,15 \pm 7.41$ $60,12 \pm 6,53$ |  | $53.93 \pm 5.51$ | 52， $25 \pm 4.40$ | $47.83 \pm 5.18$ | 57，71 $\pm 2.69$ |
| 2，50， 2 | $63.37 \pm 12,63$ |  | $49,00 \pm 5,38$ $45,32 \pm 5,02$ | $47,57 \pm 5,06$ | $43,45 \pm 1,46$ | ［3］， $13 \pm 2$ ¢ 54 |
| 3，182 | $50.74 \pm 6.30$ | 10，55 $\pm 3,22$ | $45,92 \pm 5,02$ $43,12 \pm 4.60$ | $43,22 \pm 4,73$ $34,04 \pm 4,47$ | $35,45 \pm 4,31$ $35,77 \pm 4,<2$ | 14，64 $\pm 2,39$ |
| 3.163 | $43.73 \pm 5,17$ | $41,66 \pm 1,87$ | 4，．12 $30.24 \pm 4,40$ | $39,04 \pm 4,47$ $34,79 \pm 4,11$ |  | $44,14 \pm 2,24$ $411,16 \pm 2,11$ |
| 3.6197 | Nat． 41755.57 | $37,26 \pm 4,57$ | 34，15 $\pm 4,15$ |  | 3， $26.6 \pm 3,92$ cu， $40 \pm 3,64$ | $4(1,16 \pm 2,11$ $35,91 \pm 1,95$ |
| 3，613 | 43．3v $\pm 5.41$ | 3i， $47 \pm 1,25$ | 30，6c $\pm 3,7 \%$ | $26,80 \pm 3,52$ | 25， $6.2 \pm 3.37$ | $35,97 \pm 1,96$ $31,14 r_{1} \pm 1,82$ |
| 6，2\％2 | $34,11 \pm 1,84$ $33,69 \pm 4,46$ |  |  | 23，22 $\pm 3,21$ | $22.67 \pm .3,16$ |  |
| 4，56： 4 4， | 33，69 $=1,4!$ ， | $24,91 \pm 3.66$ | 23， $5 \times \pm 3.35$ | 19， $18 \pm 2,93$ | 20，（\％） $\pm 2,84$ | ：4．13 $\pm$ ！，65 |
| 4,943 4,309 | $23,3.4 .14$ $25,10 \pm 3,8: 8$ | $\because, 60 \pm 0,36$ | ：0，49 $\pm 2,97$ | 17，04 $\pm 2,71$ | $17.65 \pm 2,06$ | 21， $21 \pm 1,44$ |
| 5， 5 \％2 | 21，83 $-2,5:$ | $13,61=3.11$ iC．01 $\pm$ c．：4t | 17，54 士 2，81 | $14,15 \pm 2,19$ | 15， $2 \mathrm{y} \pm 2,46$ | 16，20 $\pm 1.33$ |
| G，4：17 | 18，55 $\pm$ ，，cil ${ }^{\text {c }}$ |  |  | 12，60 $=2.27$ | 13． $26 \pm 2,05$ | 15， $69 \pm$ i， 23 |
| 7，4\％ 7 | $15, \therefore 7 \leq 3.14$ | $11,56 \pm 2,16$ | $10,32 \pm 2,98$ $10,64 \pm 2,16$ | $10,: 1 \pm 2,12$ | $11,40 \pm 2,0.5$ | 1：3， $13 \pm 1,13$ |
| 7，842 | 12， $6 \pm \pm 2, r 6$ | 1， $6,4 \pm 2,23$ |  | $4.80 \pm 1,42$ $6,76 \pm 1.72$ | 9．72 51.94 | $11,19 \pm 1,64$ |
| 8， 2125 | IU．1． 1.2 ， 6 | － $7,95 \pm 1,50$ | 7，16 $\pm 1.69$ | $6,76 \pm 1,72$ $5,32 \pm 1,46$ | 6， $13 \pm 1.70$ | $5.12 \pm 0,95$ |
| 9，767 | 4，37 52.46 | 6．7．$\pm 1.72$ | $5.70 \pm 1.50$ | $2,32 \pm 1,06$ $1,15 \pm 1,30$ | $6,67 \pm 1,53$ | \％，14 $\pm$ c 1.85 |
| 11．cos | C， $6.5 \pm 2,35$ | 5，35 $\pm 1.60$ | 1，37 $\pm 1.35$ | $1,15 \pm 1,39$ $2,15 \pm 1.17$ | $6,62 \pm 1,37$ $4,34 \pm 1,17$ | $0,11 \pm 0,78$ |
| 12.5015 14.328 | 5，16 $\pm 2,54$ $3,91 \pm 2,6 i$ | $4,21 \pm 1,35$ | $3.11 \pm 1,63$ | $2,67 \pm 1,00$ | $3.44 \pm$ U．97 | $4,73 \pm 0,73$ $3,63 \pm 0,70$ |
|  | 3，01 $=2,6$ | 3，19 $\pm 1.23$ | $2.35 \pm 1,10$ | 1，70 $\pm 0,88$ | $2 . r .9 \pm 0.83$ | c， $2 \pm 0.65$ |

TABLE 6
МЕд五
Таблица 6

|  | $31^{\circ}$ | $\begin{array}{r} M_{c} \lambda \\ \hline \end{array}$ | $63^{\circ}$ | $93^{\circ}$ | $123{ }^{\circ}$ | $153{ }^{\circ}$ | усреддённии ctrentp |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0，102 | 38，88 | 0，100 | 86，14 | 5I， 65 | 38，32 | 54，65 | 53，93 |
| 0，136 | 51，30 | 0，132 | II6，52 | 69，02 | 52，05 | 72，45 | 72，27 |
| 0，182 | 71，36 | 0,178 | 157．${ }^{2} 2$ | 93，i土 | 70，56 | 98，30 | 98，25 |
| 0，221 | 87，12 | 0，215 | 191，80 | 113，15 | 86.78 | II9，63 | II9，69 |
| 0，241 | 95， 14 | 0，235 | 209，69 | I23，63 | 94，25 | 131，19 | 190， 78 |
| 0，265 | 105，26 | 0，257 | 23．1． 54 | 136，03 | 104，05 | 146，40 | 144，65 |
| 0，292 | 113，24 | 0，284 | 256，95 | I50，64 | II5，73 | 159，73 | －59，26 |
| 0，323 | 121， 13 | 0，314 | 273，51 | 160，51 | 128．57 | 178， 70 | I74，2I |
| －0，360 | 130,35 | 0，350 | 283，33 | 147，58 | I42，65 | 196．41 | 180．67 |
| 0，453 | 140，19 | 0，392 | 288，64 | 208，27 | I63，06 | 215，95 | 203，22 |
| 0，455 | $149,87 \pm 20,30$ | 0，443 | 287，7I 亡 26,62 | $223,62 \pm 21,72$ | 186， 63 上 19，31 | $235,42 \pm 21,22$ | 216，25 $\pm 9,83$ |
| 0,513 | 155，79 $\pm 16,80$ | 0，503 | 277，75 722,83 | $234,59 \pm 19,92$ | 205，78 $\pm 17,13$ | 239，27 $\pm 18,66$ | 222，66 $\pm 8,58$ |
| C， 504 | $169,31 \pm 15,34$ | 0，578 | 259，03 $\pm 20,58$ | 240，3I $\pm$ I9，10 | $214,45 \pm 16,06$ | 257，37 $\pm 17,27$ | 226，10 $\pm 7.95$ |
| 0.689 | $162,04 \pm 14,71$ | 0.670 | 266， $60 \pm 19,46$ | 242， $59 \pm 18.64$ | 216，02 $\ddagger$［5，35 | $251,67 \pm 16,33$ | $267,85 \pm 7,50$ |
| C，365 | 188，79 $\ddagger$ I 4,31 | 0，705 | 26J， $45 \pm 18,56$ | 23：3，35 $\pm 17,72$ | 200， $65 \pm 14,53$ | 220，10 $\pm \pm 5,23$ | 222，05 $=7,23$ |
| 0， 0.5 | 187，31 主 73.74 | 0． 93. | $246,51 \pm$ I7．42 | 213，94 $=16,27$ | 195，27 $\pm 13,62$ | 204，04 | 20， $2 \times 6,76$ |
| I， 180 | $186,54 \pm 13,12$ | 1． 100 | $221.45 \pm 15,75$ | 190，54 $\pm 14,67$ | 176，39 $\pm 12,43$ | 184， $50=12,80$ | 190，54 $\pm 6,18$ |
| 1，383 | 174， $52 \pm 16,65$ | 1， 2 ＇it | $203, \mathrm{C}_{4}$ ix 13.54 | 176，07 $亠 16,23$ | 163，39 $\pm 14,62$ | 271，05 $\pm 5,08$ | I\％， 00 － 7,28 |
| 1． 565 | I72，37 $\pm 16,25$ | 1，327 | IS3， $16 \pm 17.89$ | I7I，49 $\pm 15.72$ | $168,53 \pm 14,18$ | I60，I9 $\pm 14,56$ | 1ヶ3，35 $\pm 7,05$ |
| 1，426 | I68，54 | 1，286 | $191.35 \pm 17,15$ | 166，05 $\pm 15,00$ | $153,00 \pm 13,58$ | I60，68 $\pm 14.11$ | 157， $02 \pm 6,78$ |
| $1.49]$ <br> 1.560 | $565,04+15,27$ $761,02+14,80$ | 1．449 | $184,20 \pm 16,50$ | $160,35 \pm 14,47$ | I47，2I $\pm 13,01$ | 154，74 $\pm 13,47$ | 162，32 $\mathrm{I}_{6}^{6,53}$ |
| 1.560 1,605 | I61，9？$\ddagger 14,80$ | 1．517 | $177.72 \pm 55,81$ | 155，29 $\pm 13,52$ | I4I， $47 \pm 12.43$ | $148,89 \pm \pm 2,54$ | 150， $80 \pm 6,63$ |
| 1，603 | $107,25 \pm 14,47$ $153,08 \pm 14,02$ | 1.569 I， 667 | $171,79+15,30$ $165 ; 38 \pm 14,86$ | J50，34 $\pm 13,56$ | I36，07 $\pm 11,95$ | 142，74 $\pm 12,35$ | I5L， $64 \pm 6,08$ |
| 1，801 | $120,108 \pm 14,02$ $147,76 \pm 13,66$ | 1,667 1，751 | $165,38 \pm 14,86$ $153,16 \pm 14,32$ | $145,09 \pm 13,04$ $139,43 \pm 12,64$ | $130,78 \pm I 1,43$ $125,19 \pm 10,99$ |  | $146,22 \pm 5,36$ $\pm 0,73 \pm 566$ |
| I，893 | $142,99 \pm \pm 3,35$ | 1，961 | $150,53 \pm 13,75$ | 132，68 $\pm 12,10$ | II8，7I $\pm 10,49$ | I22， $67 \pm 10,89$ | 133，63＝5，45 |
| 1，993 | 士37，75 $\pm 12,50$ | 1，933 | I43，3i $\pm 13,22$ | $126,80 \pm 15,66$ | I12，66 $\pm 10,11$ | II5，84 $\pm 10,36$ | 127，27 $\pm 5,24$ |
| 2，101 | I3I，64 $\pm$ I2，44 | 2， 043 | $135,90 \pm 12,56$ | 120， $11 \pm 11,12$ | 106，45 $\pm 9,59$ | 109，01 $\pm 9,88$ | $120,63 \pm 5,01$ |

TABJE 6 （continued）
a a 上 D

| MA | $31^{0}$ |  | $63^{\circ}$ | $93^{\circ}$ | $123^{0}$ | $1.53{ }^{\circ}$ | Усреднёнинй Спектр |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2，218 | $125,66 \pm 12,04$ | 2，15\％ | 123，68 $\pm 12,14$ | 113．10 $\pm 10.64$ | $1010.02 \pm$ ソ，I6 | 102，US $\pm 9,34$ | 114，09 $\pm 4,80$ |
| 2，346 | 118，96 $\pm$ II， 45 | 2，281 | $121.04 \pm 11.59$ | $166,65 \pm 10,60$ | 93．53 $\pm 2.65$ | S4，8） $\mathrm{I}_{\text {¢ }} 8.8$ ？ | $10 \% .60 \pm 4,50$ |
| 2，484 | $112,07{ }^{ \pm} 10,98$ | 2，416 | 113，24 $\pm 11,05$ | 99，23 $\pm 9.57$ | $87.69 \pm 8.20$ |  | $100,05 \pm 4.35$ |
| 2，035 | 104，7I $\ 10,45$ | 2.503 | IO4， $99 \pm 10,41$ | $92.05 \pm 0.00$ | 31， $23 \pm r, 75$ | $06.79 \pm 5,81$ | 92，75 7 ， 09 |
| 2，801 | 97，29 $\pm 0,90$ | 2，724 | 97，II $\pm 9,90$ | $84,95 \pm 3.45$ | 74，60 $\pm 5,30$ | 74，C2 $+7,28$ | $85,61 \pm 3.65$ |
| 2，582 | 90，35 $\pm$ 9，40 | 2，900 | 89， $83 \pm 3,37$ | $77.72 \pm$ \％，64 | 68， $25 \pm 0,70$ |  | $73,65 \pm 3,65$ |
| 3.182 | $83,60 \pm 9,02$ | 3，095 | $82,13 \pm 8,85$ | $70,73 \pm 7,35$ | $62,32 \pm 6,36$ | $61,50 \pm 6,38$ | 7I， $57 \pm 3,43$ |
| 3，403 | $76,22 \pm 8,45$ | 3，310 | $74,76 \pm 8,30$ |  | ¢ $0,37 \pm 5,89$ | $54,72 \pm 5,95$ | 65，24 $\pm 3,20$ |
| 3，647 | $88.52 \pm 7.95$ | 3，543 | $67.04 \pm 7,83$ | $57.65 \pm 6,26$ | $5 \mathrm{SCO} \mathrm{0} \pm 5.00$ | $45,91 \pm 5,50$ | 58，76 $\pm 3,00$ |
| 3，919 | $61,75 \pm 7,49$ | 3，812 | $60,70 \pm 7,30$ | $51,24 \pm 5.86$ | $40,00 \pm 5,09$ | $40, C 3 \pm 5,10$ | $52,37 \pm 2,79$ |
| 4，222 | $54,85 \pm 7,00$ | 4，103 | $53,93 \pm 6,80$ | $45,09 \pm 5,39$ | $33,55 \pm 4,70$ | $37,35 \pm 4,68$ | $46,16 \pm 2,59$ |
| 4，5e2 | $48,28 \pm 6,50$ | 4，439 | $47,79 \pm 6,35$ | $32,4 i \pm 4,33$ | $34,51 \pm 4,33$ | $32,2 \pm \pm 4,30$ | $40,44 \pm 2,40$ |
| 4，945 | $42,19 \pm 6,14$ | 4，812 | $41,76 \pm 5,06$ | 34，14 $\pm 4,50$ | $29,72 \pm 3,98$ | $27,30 \pm 3,50$ | $35,03 \pm 2,22$ |
| 5，379 | 36，53 5 5，70 | 5，235 | $36,33 \pm 5,40$ | $20.48 \pm 4.11$ | $25,21 \pm 3,57$ | $22,78 \pm 3,50$ | $30,69 \pm 2,04$ |
| 5，872 | $31,57 \pm 5,33$ | 5，715 | $31,68 \pm 5,62$ | $25,27 \pm 3,76$ | $21,20 \pm 3,30$ | $18,85 \pm 3.20$ | $25,53 \pm 1,89$ |
| 6.437 | $27,20 \pm 5,04$ | ¢， 266 | 27，23 $\pm 4.61$ | $21.42 \pm 3.40$ | $17,60 \pm 20.90$ | I5，33 $\pm 2,98$ | 2I， $\mathrm{ma}_{7} \pm \mathrm{I}, \mathrm{m}$ |
| 7，087 | $23,04 \pm 4,72$ | 6，900 | 23，17 $\pm 4,20$ | 17，99 $\pm 3,01$ | 14，42 $\pm 2,70$ | 12，28 $\pm 2,70$ | 18，18 $\pm 1.59$ |
| 7，842 | I9，25 $\pm 4,39$ | 7，636 | $19,35 \pm 3,74$ | $14,85 \pm 2,72$ | 11，48 $\pm 2.40$ | 9，02 $\pm 2,45$ | 14，91 $\pm 1,44$ |
| 8，725 | 16，9\％ $\pm 4,13$ | 8.407 | $16,20 \pm 3,34$ | $12,27 \pm 2,38$ | 9，09 $\pm 2,17$ | 7，53 52,22 | 12，20 |
| 9，767 | $12,07 \pm 5,32$ | 9．513 | $23,06 \pm 2,55$ | 2，37 $\pm 2,03$ | $6,9 i \pm i, 83$ | 5，11 $\pm 1,94$ | 9，57 |
| 11， 018 | 10，35 $\pm 3,76$ | IC，7\％ 5 | $10,47 \pm 2,49$ | $7,85 \pm 1.74$ | 5，11 $\pm 1,57$ | 3，94 $\pm 1,64$ | 7，25 |
| I2，503 | $8,02 \pm 3,76$ | I2，I84 | $3,04 \pm 2,00$ | $5,82 \pm 1.43$ | 3，58 $\pm 1,24$ | く，88 $\pm 1,50$ | 5，67 $\pm 0,98$ |
| 14，328 | $6,00 \pm 3,44$ | 13，966 | $5,64 \pm$ I， 72 | $4,35 \pm 1,19$ | 2，34 $\pm \mathrm{I}, \mathrm{CO}$ | 2，03 $\pm$ I，I3 | $4,12 \pm 0,36$ |

Таблида 7


TABLE 7 (continued)
ท 1 т P И й

| $\begin{aligned} & \text { MEMOS } \\ & \hline \end{aligned}$ | $31^{\circ}$ | $61^{\circ}$ | $n^{\circ}$ | $121{ }^{\circ}$ | $1.01{ }^{\circ}$ | Усреднённыиі cпemp |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| I | 2 | 3 | 4 | 5 | 6 | 7 |
| 2,218 | 41,33 $\pm 5,01$ | $35,45 \pm 4,28$ | $31,82 \pm 3,05$ | $26,69 \pm 3,90$ | $32.59 \pm 4.47$ | $33,08 \pm 1,44$ |
| 2,346 | 38,33 $\pm 4,71$ | 32,27 4,02 | 2,44 4.30 | 26.0. $5 \pm 3.65$ | 29.6\% $\pm 4.14$ | 31,14 $\pm 1,81$ |
| 2,484 | 35,29 $\pm 4,46$ | 29,26 $\pm 3,76$ | $27.15 \pm 3.48$ | $23.50 \pm 3.40$ | $25,74 \pm 3.87$ | $83.33 \pm 1.70$ |
| 2,635 | $3 \mathrm{3}, 28 \pm 4.17$ | $25.42 \pm 3.53$ | 24,92 $\pm 3,28$ | $21.10 \pm 3,1 \%$ | 23,44 $\pm 3,59$ | $25.73 \pm 1.59$ |
| 2,80I | 29,35 $\pm 3,89$ | 33,71 $\pm 3,31$ | 22, $76 \pm 3.6$ | $10.87 \pm 2,05$ | $21,25 \pm 3.33$ | 23,19 $\pm 1,48$ |
| 2,982 | $25,66 \pm 3,68$ | 2I,21 $\pm 3,11$ | 20.75 | 16,8ri $\pm 2,77$ | $13,85 \pm 3,11$ | $2 \mathrm{C}, 67 \pm$ I,40 |
| 3,182 | 24,19 $\pm 3,47$ | $18,92 \pm 2,93$ | $18,84 \pm 2,73$ | $15,09 \pm 2,59$ | $16,60 \pm 2,91$ | 18,73 $\pm 1,21$ |
| 3,403 | 21,78 $\pm 3,25$ | 16,69 $\pm 2,76$ | $16.52 \pm 2.55$ | $13,41 \pm 2,44$ | 14,42 $\pm 2,72$ | 16,65 $\pm 1,23$ |
| 3,647 | $19,41 \pm 3.10$ | $14,58 \pm 2.61$ | I5,06 $\pm 2,40$ | 11,78 $\pm 2,29$ | 12,38 $\pm 2,54$ | $14,64 \pm 1,16$ |
| 3,919 | 17,20 $\pm 2,91$ | $12,58 \pm 2.45$ | 13,27 $\pm 2,26$ | $10,30 \pm 2,10$ | $10,50 \pm 2,38$ | 12,77 $\pm \mathrm{I}, 09$ |
| 4,222 | $15,08 \pm 2,71$ | $10,73 \pm 2,30$ | 11,60 $\pm 2,12$ | $8,93 \pm 2,01$ | 8,78 $\pm 2,20$ | $11,02 \pm 1,02$ |
| 4,562 | $13,14 \pm 2,54$ | 0,04 $\pm 2,15$ | $10,64 \pm 1,96$ | $7,65 \pm 1,83$ | $7,24 \pm 2,05$ | $9,42 \pm 0,93$ |
| 4,945 | 11,41 $\pm 2,39$ | 7,5i $\pm 2,01$ | $8,57 \pm 1,84$ | $6,50 \pm \mathrm{I}, 76$ | $5,89 \pm 1,91$ | $7.98 \pm 0.89$ |
| 5,379 | $9,84 \pm 2,22$ $8,46 \pm 2,00$ | $6,17 \pm 1,88$ | 7,21 $\ddagger 1,70$ | $5,45 \pm 1,1,4$ | $4.71 \pm 1.78$ | $6,67 \pm 0,83$ |
| 5,872 6,437 | $8,46 \pm 2,08$ $7,19 \pm 1.96$ | $5,01 \pm 1,75$ | $6,02 \pm 1,06$ | $4.52 \pm 1.53$ | $3.72 \pm \pm .66$ | $5.54 \pm 0,77$ |
| 6,437 7,087 | $7,19 \pm 1.96$ $6,04 \pm 1.82$ | $3,09 \pm 1,63$ $3,09 \pm 1.01$ | $4,95 \pm 1,43$ $3,99 \pm 1,33$ | $3,70 \pm 1,43$ $2,96 \pm 1,33$ | $2,37 \pm 1,55$ $2,15 \pm$ | $4,54 \pm 0,72$ $3,05 \pm 0,67$ |
| 7.842 | $4,95 \pm 1,68$ | $2,09 \pm 1,37$ | 3,13 $\pm 1,19$ | 2,29 $2,1,21$ | $1,00 \pm 1.41$ $1,50 \pm 8$ | $3,65 \pm 0,67$ $2,34 \pm 0,00$ |
| 8,725 | $4,01 \pm 1,65$ | $1,64 \pm 1,21$ | 2,42 $\pm 1,06$ | 1,73 $\pm 1,63$ | $1,09 \pm 1.15$ | $2,18 \pm 0,54$ |
| 9,767 | $3.24 \pm 1.45$ | $1.14 \pm+1.5$ | $1,79 \pm 0.92$ | 1,Iô $\pm$ U,95 | $0.68 \pm 1.01$ | $1,00 \pm 0,49$ |
| 11,008 | $2,65 \pm 1,46$ | $0.75 \pm 0.91$ | I,32 $\pm 0,58$ | $0.35 \pm 0.82$ | $0,43 \pm 4.85$ | $1,20 \pm 0,44$ |
| 12,503 | $1.99 \pm 1.53$ | $0,48 \pm 0,83$ | $1,06 \pm 0,68$ | $0,64 \pm 0,70$ | 0,27 $\pm 0,72$ | $0,88 \pm 0,42$ |
| 14,328 | I,52 $\pm 1,48$ | $0.26 \pm 0.77$ | $0.74 \pm 0.61$ | $0,43 \pm 0,62$ | $0,12 \pm 0,62$ | $0,61 \pm 0,39$ |

TABLE 8
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Табгица 8

| $M \mathrm{~V} \gamma_{\mathrm{E}, \mathrm{MOS}}^{8}$ | $31^{\circ}$ | $61^{\circ}$ | $91^{\circ}$ | $12.1{ }^{\circ}$ | 1510 | усреднённвй लnemp |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0，102 | $18,28{ }^{2}$ | 25，90 ${ }^{3}$ | 26，19 ${ }^{4}$ | $24.05{ }^{5}$ | $29.97{ }^{6}$ | $24,68{ }^{7}$ |
| 0，136 | 25， 28 | 37.54 | 37，05 | 33，93 | 4I， 69 | 35，30 |
| 0，182 | 35，85 | 50，26 | 50，99 | 44，80 | 55，51 | 47,48 |
| 0，22I | 4i，0？ | 61，56 | 62，57 | 54，8I | 67，94 | 56，18 |
| 0，24I | 48，09 | 68，25 | 69，25 | 60．91 | 75，40 | 64，38 |
| 0，265 | 54，27 | 76，50 | 76，40 | 66，08 | 83，28 | 71，20 |
| 0，292 | 60，48 | 85，36 | 87，${ }^{1}$ | 74，32 | 93，25 | 80．10 |
| 0，323 | 66，66 | 94，38 | 90，41 | 82，65 | 10i， 84 | 88，39 |
| 0，360 | 74，6I | 105，62 | 107，43 | 92，56 | II4，II | 39.87 |
| 0，403 | 83，5； $\pm 20,55$ | II9，36 $\pm 18,72$ | II8，9＇\％ $\pm 20,68$ | 102，66 $\pm$ I5，26 | 126， $56 \pm 19.41$ | IIO，30 $\pm 8,51$ |
| 0，455 | 97，03 $\pm 5.5,29$ | I38，02 $\pm$ I4，76 | I29，17 $\pm$ I5，73 | II5，14 $\pm$ I3，60 | 137，32 | 123，34 $\pm 6,64$ |
| 0，518 | $109,49 \pm 12,60$ | I44，21 $\pm \pm 2,65$ | 133，86 $\pm$ I2，7I | 122，02 $\pm$ I2，00 | I36，95 $\pm 12,27$ | 129，30 $\pm 5,55$ |
| 0，594 | 115，30 $\pm$ II， 23 | I47，95 | I36，25 $\pm$ II， 33 | $124,70 \pm 10,63$ | 135，08 $\pm 10,98$ | 132，06 $\pm 4.99$ |
| 0.639 | $220,47 \underset{ \pm}{ \pm} 10,42$ | I53，66 ${ }^{ \pm}$II， 22 | $135,36 \pm 10,54$ | 125，77＋$+9,90$ | 133，26 $+10,10$ | 103，70 $\pm 4,67$ |
| C．803 | $120,00 \pm 3.63$ | I54，44 $\pm 10,88$ | 131，T＇心 $\pm 9.75$ | 122，97 $\pm 9,22$ | I30， $08 \pm 9,51$ | $132.74 \pm 4.4 \mathrm{I}$ |
| 0，951 | 129，97 $\pm$ ¢ 9 ， 72 | 148，28 $\pm 10,33$ | 126，4I $\pm 9,25$ | I19，27 $\pm 8,73$ | 126，52 $\pm 9.06$ | $130.09 \pm 4.21$ |
| 1，162 | 130，71 $\pm 9.50$ | I3ï，91 $\pm 9,55$ | 120，56 $\pm 8.71$ | 114，75 $\pm 8,28$ | $121.77 \pm 8.61$ | 124．94 $\pm 4,00$ |
| 1，308 | I27，55 $\ddagger \mathrm{II}, \mathrm{SB}$ | 128，15 $\pm 10,95$ | II4．4？$\pm 10.58$ | 109．86 $\pm 10.32$ | IIT，I2 $\ddagger 10.50$ | $119,43 \pm 2,86$ |
| 1，365 | I25，51 $\pm$ II，52 | 124，19 $\pm 10.44$ | 111，52 510.13 | I07，20 $\pm 9,90$ | IIt，73 $\ddagger$ IO，08 | IIE，54 $\pm 4,66$ |
| I，425 | 122，52 $\ddagger$ II， 18 | 19，40 $\pm 10,00$ | 107，77 $\underset{ \pm}{ \pm} 9.85$ | $103,90 \pm 9.57$ | III．83 $\pm 9,74$ | $113,09 \pm 4,51$ |
| I，49I | IIE， $43 \pm 10, \mathrm{re}$ | 114．19 | 103，28 $\pm 9.31$ | $160,05 \pm 9.06$ | 107， $00 \pm 9,37$ | IC8，79 $\pm 4,35$ |
| I， 560 | IIS， $27 \pm 10,27$ | $103,76 \pm 8,18$ | $98,67 \pm$＋ 0.83 | $95.07 \pm 8.64$ | 106，0¢ +0.91 | 1C4．40 $\pm 4,15$ |
| I，CEE | $10.73 \pm 9.90$ | $102.98 \pm 5.70$ | $93,92 \pm 0.55$ | 91．81 $\pm 8.25$ | 90，40 $\pm 8,55$ | 99，58 $\pm 3,93$ |
| 2，75 | 102，00 $\ddagger 9,53$ | $06,87 \pm 5,32$ | $93,85 \pm 8.05$ | $87.17 \pm$ \％ 54 | $94.48 \pm 8.13$ | 94，40 $+3,75$ |
| T，801 | 09，27 40,45 | $91.35 \pm 7,83$ | $83,38 \pm \begin{aligned} & \pm \\ & 7 \\ & 7\end{aligned}$ | $81,96 \pm 7,33$ | 88，77 $\ddagger 7,61$ | $83,77 \pm 3,51$ |
| I． 895 | 93,13 $06,05 \pm 8000$ | 6， $13 \pm 700$ |  | $75,46 \pm 7$ 7，011 | $82,85 \pm 7,21$ | $82,78 \pm .3,36$ |
| I． 960 2．101 | $86,05 \%$ <br> 80,42 | $77,08 \pm 6,98$ $9,14 \pm 6,45$ | $71,97 \pm 6.82$ |  | $76.68 \pm 6.83$ | $76,75 \pm$ 2， 5 \％ |
| R：101 | $80.42 \div 3.62$ | $0.44 \div 0.45$ | $65,55 \pm 6.29$ | $65,29 \pm 6.36$ | 70，55－6，30 | $70,45 \pm 2,95$ |

TABLE 8 (continued)

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TABLE 9
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Le5sxua 9


TABLE 9 (continued)
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| $m_{e} 0$ | $31^{2}$ | $61^{\circ}$ | $92^{\circ}$ | $121^{\circ}$ | 1515 | усиедавпии c:erap |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| I | 2 | 3 | 4 | \% | 6 | \% |
| 2,218 | 99,09 $\pm 9,40$ | IU3,46 $\pm$ y, 01 | $34,7 \% \pm 8,20$ | $84.13 \pm 1 / 07$ | $00,06 \pm$ r,00 | -8. |
| 2,316 | 50,92 $\pm 0,66$ | $94,50 \pm 8,39$ | $45,30 \pm r, 70$ | rir.49 $\pm$ \%,ib | 79,30 $\pm 7,32$ | 85, 5i $\pm 3.51$ |
| 2,484 | $83,51 \pm 8,23$ | $86,10 \pm 5,80$ | 19,24 $\pm 7.11$ | $70,26 \pm 0,65$ | $71,87 \pm 0.74$ | 70, $20 \pm 3,20$ |
| 2,635 | $76,15 \pm 7,67$ | 77,8i $\pm$ r, 24 | 71,52 $\pm 6.08$ | $63,13 \pm E, 16$ | $62,59 \pm 6,14$ | 70,45 $\pm 3,42$ |
| 2,001 | $68,15 \pm 7,60$ | $69,65 \pm 6,61$ | 64,26 $\pm$ c,07 | 50,13 $\pm 6,05$ | U6,44 $\pm$ - 60.0 | 62,81 $\pm 2,78$ |
| 2,582 | $61,65 \pm 0,59$ | CI, $54 \pm 6,69$ | $5 \%, 10 \pm 5,56$ | $49.19 \pm 5.15$ | $45,70 \pm 5.15$ | 35,5 5 2,56 |
| 3.182 | 54,75 $\pm 6,10$ | $54,22 \pm 5.61$ | 50,61 $\pm$ - 15 | $4,34 \pm 4,73$ | $43,36 \pm 4.71$ | $45,24 \pm 2,06$ |
| 3.403 | $48,75 \pm 5.62$ | $47,24 \pm 5,15$ | $43,83 \pm 4.66$ | $37.59 \pm 4.53$ | $3 \mathrm{y}, 23 \pm 2,29$ | $42,94 \pm 2,26$ |
| 3,647 | $42.71 \pm 5,20$ | $40,73 \pm 4,70$ | $38,29 \pm 4,27$ | $32,05 \pm 3,95$ | $31.75 \pm 3,00$ | ST, 60 $\pm$ I, 93 |
| 3,919 | $37,12 \pm 4,07$ | $35,00 \pm 4,32$ | $36,47 \pm 5,80$ | $27.04 \pm 3,39$ | $26,68 \pm 3,54$ | $31,64 \pm$ 1, 02 |
| 4,22\% | 31, $77 \pm 4,40$ | $29,41 \pm 3,92$ | 27,37 $\pm$ - 5,02 | 22, $27 \pm 3,29$ | $21,90 \pm 3,61$ | $26.66 \pm$ I,66 |
| 4,562 | $27,35 \pm 4,13$ | $24,63 \pm 3,61$ | $2 \mathrm{k}, 77 \pm 3,13$ | $18,05 \pm 3,01$ | IB, $6 \in \pm 2,21$ | $22,20 \pm=.52$ |
| 4,945 | 23,36 $\pm 3,91$ | 20,50 $\pm 3,31$ | $10,82 \pm 2,00$ | 15,26 $\pm 2,73$ | $14,59 \pm 2,04$ | $16,51 \pm 2.40$ |
| 5,379 | 13,49 $\pm 3,49$ | $10,69 \pm 3,00$ | 16, $22 \pm 2,62$ | I2,31 $\pm 2,49$ | 11,60 $\pm 2,4,4$ | 15,12 $\pm \mathrm{I}, 27$ |
| 5,8\%2 | I6,İ $\pm$ 2,24 | $13.80 \pm 2,83$ | 12,20 $\pm 2,30$ | 9,85 $\pm 2,29$ | 2,19 $\pm 2,20$ | $12,23 \pm 5.27$ |
| 6,437 | İ, $\mathbf{1 0} \pm 3,63$ | 11,00 $\pm$ - 5i, | $9.67 \pm 2.17$ | $7.78 \pm 2.08$ | 7.13 $\pm$ L, UT | 9,05 5 I.U7 |
| 7.60 | 1u, טi $\pm 2,73$ | 0,74 $\pm 2,34$ | $7,05 \pm 1.08$ | $0,65 \pm 1,8 y$ | 0,44 $\pm 1.04$ | $7,07 \pm L, 9{ }_{6}$ |
|  | 0, $60 \pm 2,05$ | $6,15 \pm 2,16$ | 5,\% | $4.00 \pm 2.82$ | $4,06 \pm 1,00$ | $5,83=4,85$ |
| 8,723 | 6,30 $\pm 2,107$ | $6,13 \pm 1.07$ | \%, $20 \pm$-. 000 | 3,44 $\pm 1,52$ | <,90 $\pm$ 1, S | $4,43 \pm 0,00$ |
| $9.60 \%$ | $4,92 \pm 2,12$ | 3,94 $\pm$ - Cl | 3,07 $\pm 1,34$ | $2,61 \pm 1,01$ | <, $2 \pm \pm$ U | Э, $2 \mathrm{~L} \pm 0,70$ |
| Il, Nos | $3,50 \pm 2.09$ | 2,0. $\pm$ - 0 , | $2,12 \pm 1,10$ | $6.73=\therefore 10$ | $\therefore 4^{\prime} \pm 1,07$ | $2,03=0,62$ |
| 12,503 | $2,41 \pm 2,17$ | I, $\mathrm{CJ} \pm \mathrm{I}, \mathrm{i} 4$ | 1.40 $\pm 6.00$ | 1,21 $\pm 0,89$ | $0,91 \pm 0.85$ | I, 50.0 .57 |
| 14,528 | 1,51 $\pm 2.01$ | I, it $\pm$, © | U, $0.0 \pm$ C,ri | $0.73 \pm 1.75$ | -,52 $\pm 0,67$ | $0,90 \pm 0,51$ |


Таблица IO


TABLE 10 (continued)
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TABLE 11
В ИOM YT
Таблица II


TABLE 11 (continued)
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山ролыдйсие таӧниц II

| $\begin{array}{r} \operatorname{Mex} \theta \\ E \quad M כ B \\ \hline \end{array}$ | $31^{0}$ | $61^{\circ}$ | $91^{\circ}$ | $121^{\circ}$ | $151{ }^{\circ}$ | Усренининй спектр |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| I | 2 | 3 | 4 | 5 | 6 | 7 |
| 2,218 | II7,8I $\pm 9.73$ | 100,44 $\pm 8,24$ | $98,39 \pm 7,99$ | 99,47 $\pm 8.00$ | 97,31 $\pm 8.01$ | 10\%,68 $\pm 3,78$ |
| 2,346 | 107,53 $\pm$ 9,02 | 91,03 $\pm 7.64$ | 80,50 | 89,33 $\pm 7,30$ | $86.92 \pm 7.29$ | $92,66 \pm 3.47$ |
| 2,484 | 97,22 $\pm 8,30$ | $81,77 \pm 7,06$ | '88,75 $\pm$ E.72 | 79,34 $\pm 0.61$ | $76.73 \pm 6,56$ | 82,76 $\pm 3,16$ |
| 2,635 | $87,05 \pm 7,60$ | $72,53 \pm 6,37$ | 69,21 $\pm 6,01$ | $69,75 \pm 5,99$ | $67.00 \pm 3.91$ | $73,12 \pm 2,86$ |
| 2,80] | 77,2I $\pm 6,96$ | $63,54 \pm 5,73$ | $60,22 \pm 5,52$ | $60,54 \pm 5,35$ | $57,88 \pm 5,30$ | $63,83 \pm 2,59$ |
| 2,932 | 68,25 $\pm 6,30$ | $55,22 \pm 5,26$ | $51.99 \pm 4,9 i$ | $52,18 \pm 4,77$ | $49,65 \pm 4,70$ | 55,46 $\pm 2,33$ |
| 3, I82 | 59,96 $\pm 6,75$ | $47,43 \pm 4,68$ | 44,49 $\pm 4,33$ | $44,55 \pm 4,29$ | $42,19 \pm 4,21$ | $47,73 \pm 2,09$ |
| 3,403 | $52,20 \pm 5,25$ | 40,08 $\pm 4,19$ | $37,54 \pm 4,01$ | $37,43 \pm 3,79$ | 35,34 $\pm 3,76$ | $40,52 \pm \mathrm{I}, 89$ |
| 3,647 | 44,93 $\pm 4,75$ | $33,26 \pm 3,82$ | $31,18 \pm 3,55$ | 30,31 $\pm 3,32$ | 29,12 $-3,3 \pm$ | $35,86 \pm 1,69$ |
| 3,919 | $38.29 \pm 4,30$ | 27,11 $\pm 3,39$ | $25,58 \pm 3,10$ | $25,00 \pm 2,97$ | $23.76 \pm 2,94$ | 27,95 $\pm 1,51$ |
| 4,222 | $32,42 \pm 3,93$ | $21,74 \pm 3,01$ | $20,71 \pm 2,90$ | $20,04 \pm 2,64$ | 19,19 $\pm 2,67$ | $22,02 \pm 1,37$ |
| 4,562 | $27,21 \pm 3,56$ | $17,00 \pm 2,79$ | 16,56 $\pm 2,59$ | $15,81 \pm 2,33$ | 15,31 $\pm 2,36$ | $18.38 \pm 5.23$ |
| 4,945 | $22,73 \pm 3,24$ | 13,02 $\pm 2,51$ | I2,12 $\pm 2,28$ | $12,32 \pm 2,11$ | 12,00 $\pm 2,15$ | I4,65 $\pm$ 1,II |
| 5,379 | $18,80 \pm 2,99$ | 9,65 $\pm 2,27$ | 10,30 $\pm 2,17$ | 9,46 $\pm 1,91$ | 9,40 $\pm$ I,96 | 11,52 $\pm 1 . \mathrm{C} 2$ |
| 5,872 | 15,51 $\pm 2,75$ | 6,94 $\pm 2,16$ | $8,04 \pm 1.99$ | $7.19 \pm 1.72$ | $7,21 \pm 1,77$ | $8,58 \pm 0,54$ |
| 6,437 | $12,63 \pm 2,50$ | $4,78 \pm 1,97$ | $6,16 \pm 1,75$ | 5,37 $\pm 1.60$ | $5,45 \pm 1,6 I$ | $6,80 \pm 0,95$ |
| 7.087 | IU, 14 $\pm 2,34$ | 3,14 $\pm 1,77$ | $4,70 \pm 1,68$ | $3,97 \pm 1,47$ | $4,04 \pm 1,50$ | $5,20 \pm 0,79$ |
| 7,842 | $7,93 \pm 2,12$ | 1,94 $\pm 1,68$ | $3,63 \pm 1.52$ | $2,88 \pm 1,32$ | $2,91 \pm 1,34$ | $3,84 \pm 0,72$ |
| 8,725 | $6.13 \pm 1.93$ | $1,09 \pm 1,50$ | $2,64 \pm \mathrm{I}, 28$ | 2,07 $\pm 1,18$ | $2,07 \pm$ I,19 | $2,80 \pm 0,64$ |
| -9,767 | $4,78 \pm I, 81$ |  | $1,90 \pm I, 17$ | $1.33 \pm 1.06$ | I,50 $\pm 1,05$ | $1.93 \pm 0,58$ |
| I1, U0\% | 3,71 $\pm 1,82$ |  | I, $58 \pm 1,01$ | I, $02 \pm 0.89$ | I,OI $\pm 0,8 r$ | 1,46 $\pm 0,02$ |
| I2, 303 | $2,69 \pm 1,85$ |  | I,I9 $\pm$ U, 34 | $0.81 \pm 0.75$ | $0,61 \pm 0,72$ | I, CO $\pm 0,50$ |
| 14,3:28 | $1,90 \pm 1,79$ |  | $0,86 \pm 0,77$ | $0.68 \pm 0.67$ | $0,39 \pm 0,63$ | $0,77 \pm 0,47$ |

Figs 1-55
Pис. $=$ Fig.
Key: (Oтн. = Relative units единицы $=\mathrm{MeV}$
Мәв


Pric. I.


Puo. 2.


Puc. 3.


Puc. 4.


Pre. 5.


Prc. 6.


Pис. 7.


Рис. 8.


Prac. 9.


Puc. IO.


Рис. II.


Pषс. I2.


Pre. 13.


Pric. I4.


Pис. 15.


Pre. I6.

Рис. 17.


Puc. 18.

Рис. 19.


Рис. 20.



Рис. 22.


Pac. 23.


Pис. 24.


Рис. 25.


Puc, 26,


Pre. 27.

$5 \%$


Pис. 29.


Рис. 30.


Puc. 31 .

Рис. 32.

- 179 -


Prc. 33.


Pre. 34.


Pra. 35.


Pис. 36.


Рис. 37.


Рис. 38.


Pис. 39.


Pre. 40.


FInc. 4 I.


Puc. 42.


Pme. 43.


Рис. 44.


Pac. 45.


Pac. 46.


Pzc. 47.


Pис. 48.


Puc. 49.


Рис. 50.


Pис. 5I.


Pис. 52.


Pric. 53.


Рис. 53.


Puc. 54.


Fig. 55

## REFERENCES

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ISOMER RATIOS AND GAMMA SPECTRA IN THE RAIIIATIVE CAPTURE OF THERMAL NEJTRONS

A.G. Dovbenko, A.V. Ignatyruk, V.A. Tolstikov

## Introduction

Present investigations of the mechanisn of neutron radiative capture by nuclei involve, essentially, the interpretation of three groups of experimental data: total radiative capture cross-sections, cross-sections for the formation of the isomeric state in radiative capture, and spectra of the cascade ganma rays produced. Wide use is made of statistical models of nuclear reactions to describe these data theoretically. If the total radiative capture crossmsections are investigated with such a model, it is possible to obtain data on the level density of the compound nucleus formed with excitation energy close to or above the neutron binding energy [1] , and the cross-section for the formation of isomeric states provides a basis for studyirg the relationship between level density and angular monentum [2]. A statistical description of the gamma spectra produced in radiative capture provides a means of studying the level density for excitation energies considerabiy below the neutron binding energy [3]. It energes that the shape of the spectmum is sensitive even to the discrete primary low-excited nuclear levels $[4]$.

Investigation of the gamm spectra of radiative capture provides the most complete information on the radiative capture mechanism and, in the case of many nuclei, the difference in the shape of the gama spectrum from the results of calculating spectra with the statistical model has served as a basis for developing new models of direct [5] and semi-direct (or collective) [6] capture mechanisms. It is clear that for these nuclei the total radiative capture crossmsections and the oross-sections for the formation of the isomeric states should al so be described by a direct or collective capture model. However, the results of numerous investigations have so far been very inconsistent. The radiative capture cross-sections, the absolute values of the radiation widths and the isomeric ratios at neutron energies of not more than a few MeV are reasonably well described by the statistical model over practically the whole mass range of stable nuclei $[7,8]$ investigated, whereas the gamma spectra of many nuclei are harder than predicted by the statistical model [9].

To get a consistent explanation of the above experimental data we must determine to what extent the discrepancies in the gamma spectra can be accounted for by the parameters of the statistical model and establish the contribution of the direct process to the radiative capture crosssection for neutrons in the intermediate energy range.

In this study a statistical model is used to calculate the gamma spectra associated with thermal neutron capture, and also to calculate the ratios of the level populations in the ground and long-lived isomeric states formed as a result of these gamma transitions. Unlike earlier authors $[3,4]$, in calculating the spectra we took account of the dependence of level density on angular momentum. We also investigated the effect on the shape of the spectrum and on the isomeric ratios of different theories concerning the energy dependence of the excited state density and the dependence of the gamma transition probability on energy and on the angular momentum transmitted.

## 1. Basic relations

For describing multistage decay of an excited nucleus via gamma-ray emission it is convenient to use the method developed in Ref. [3]. Let us consider this process, taking into account the effect of angular momentum on the probability of radiative transitions. We shall denote with $y_{k}(E, J, t)$ the population after emission of the $k^{t h}$ gamma quantum of a state with energy $E$ and spin $J$ at instant $t$. The variation of population with time is described in Bohr's classical model of compound nucleus decay $[10]$ by the kinetic equation

$$
\begin{gather*}
\frac{\partial y_{k}(E, J, t)}{\partial t}=\sum_{J^{\prime}} \int_{E}^{\varepsilon_{0}} y_{k-1}\left(E_{,}^{\prime} J^{\prime}, t\right) \Gamma_{\gamma}\left(E^{\prime} J^{\prime} \longrightarrow E, J\right) d E^{\prime}-  \tag{1}\\
-y_{k}(E, J, t) \Gamma_{\gamma}(E, J)
\end{gather*}
$$

where $\Gamma_{\gamma}(E, J)$ is the total radiative decay width of the state ( $E, J$ ) and $\Gamma_{\gamma}\left(E^{\prime}, J^{\prime} \rightarrow E, J\right)$ is the width of radiative transition from the state ( $E^{\prime}, J^{\prime}$ ) to the state $(E, J), E_{0}$ being the maximum excitation energy.

The initial condition for equation (1) is written

$$
\begin{equation*}
y_{k}(E, J, O)=\delta_{k 0} y_{0}(E, J), \tag{2}
\end{equation*}
$$

where $y_{0}(E, J)$ is the population of the state $(E, J)$ at the initial moment of time. When a thermal neutron is captured the nucleus has an excitation energy $E_{0}=B_{n}$ and momentum $J_{0}=|I \pm l / 2|$, where $B_{n}$ is the neutron binding energy and I is the target nucleus spin. It is then convenient to write

$$
\begin{equation*}
y_{0}(E, J)=\delta_{J J_{0}} \delta\left(E-E_{0}\right) \tag{2a}
\end{equation*}
$$

Integrating Eq. (1) with respect to time, we obtain

$$
\begin{align*}
y_{k}(E, I, \infty)-y_{x}(E, J, 0) & =\sum_{y^{2}} \int_{E}^{E_{0}} d E^{\prime} \Gamma_{p}\left(E_{1}^{\prime} J^{\prime} \rightarrow E, J\right) \int_{0}^{\infty} d t_{k-1}\left(E_{0}^{\prime} J_{,}^{\prime} t\right)-  \tag{3}\\
& -\Gamma_{p}(E, y) \int_{0}^{\infty} d t_{y_{k}}\left(E_{\lambda} y_{2} t\right)
\end{align*}
$$

$$
\begin{align*}
& \text { We then determine } \\
& \qquad W_{k}(E, J)=\Gamma_{\gamma}(E, J) \int_{0}^{\infty} d t y_{k}(E, J, t), \\
& S\left(E, J^{\prime} \rightarrow E, J\right)=\frac{\sqrt{\gamma}\left(E^{\prime} J^{\prime} \rightarrow E^{\prime} J\right)}{\sqrt{\gamma}\left(E^{\prime}, J^{\prime}\right)} \tag{4}
\end{align*}
$$

Here $W_{k}(E, J)$ is the relative probability that in the gamma decay process the excited nucleus was in the state ( $E, J$ ) after emission of the $k$ gamma quantum and $S\left(E^{\prime}, J^{\prime} \rightarrow E, J\right)$ gives the relative probability of radiative transition to the state $(E, J)$.

Since any excited state will decay in an infinite period of time, we have $y(E, J, \infty)=0$ and equation (3) can be re-written in the form

$$
\begin{equation*}
W_{k}(E, y)=\sum_{J^{\prime}} \int_{E}^{E_{0}} d E^{\prime} S\left(E^{\prime} y^{\prime}-E, \mathcal{F}\right) W_{k-1}\left(E, J^{\prime}\right)_{i-} y_{k}(E, y, O) \tag{5}
\end{equation*}
$$

If there is an isomeric state in the system its population may be defined as

$$
\begin{equation*}
W\left(E_{m}, I_{m}\right)=\sum_{k=0}^{\infty} W_{k}^{\prime}\left(E_{m}, Y_{m}\right) \tag{6}
\end{equation*}
$$

The isomeric ratio describes the relative probability of formation of a nucleus in the isomeric state and is usually determined experimentally as

$$
\eta=\frac{\sigma_{m}}{\sigma_{m}+\sigma_{g}}
$$

where $\sigma_{m}$ and $\sigma_{g}$ are the crossesections for formation of the isomeric and ground states in the given reaction. On the basis of the above relations the isomeric ratio is determined as follows:

$$
\begin{equation*}
\eta=\frac{W\left(E_{m}, J_{m}\right)}{W\left(E_{g}, Y_{g}\right)+W\left(E_{m}, J_{m}\right)}=W\left(E_{m}, J_{m}\right), \tag{7}
\end{equation*}
$$

where the initial condition (2a) is explicitly taken into account.

$$
\text { Since } \Gamma_{\gamma}\left(E, J \rightarrow E-\varepsilon \gamma, J^{\prime}\right) \text { determines the probability of emission of a }
$$ gamma quantum with energy $\varepsilon \boldsymbol{\gamma}$ during the transition from the state ( $E, J$ ) to the state $\left(E-\varepsilon \gamma, J^{\prime}\right)$, the number of gamma quanta emitted at instant $t$ from the state ( $E, J$ ) may be written as

$$
\begin{equation*}
n_{r}^{(E, y)}\left(\varepsilon_{p}, t\right)=\sum_{y^{\prime}} r_{\gamma}\left(E, y-E-\varepsilon_{\gamma}, y^{\prime} \sum_{k=0}^{\infty} y_{x}(E, y, t)\right. \tag{8}
\end{equation*}
$$

The number of gamma quanta with energy $\varepsilon_{\boldsymbol{\gamma}}$ emitted during decay of the state ( $E, J$ ) may thus be written

$$
\begin{align*}
& n_{\gamma}(E, J)\left(E_{\gamma}\right)=\sum_{j^{\prime}} \gamma_{\gamma}\left(E, y \rightarrow E-E_{\gamma,}, y^{\prime}\right) \int_{k=0}^{\infty} d t \sum_{k=0}^{\infty} y \kappa(E, y, t)=  \tag{9}\\
& =\sum_{Y^{\prime}} S\left(E, \gamma \rightarrow E-E_{j}, J^{\prime}\right) \sum_{x=0}^{\infty} W_{i}(E, y)
\end{align*}
$$

For the total number of gamma rays with energy $\varepsilon \gamma$ we obtain

$$
\begin{equation*}
n_{\gamma}\left(\varepsilon_{p}\right)=\sum_{y, y^{\prime} \varepsilon_{c}}^{\varepsilon_{0}} d E S\left(E, Y \rightarrow E-\varepsilon_{p}, y^{\prime}\right) W(E, Y) \tag{10}
\end{equation*}
$$

The gamma-ray spectrum will now be defined as

$$
\begin{equation*}
I\left(\varepsilon_{p}\right)=\frac{\varepsilon_{p} n_{p}\left(\varepsilon_{p}\right)}{\int_{0}^{\varepsilon_{0}} n_{p}\left(\varepsilon_{p}\right) d \varepsilon_{r}} \tag{11}
\end{equation*}
$$

On the basis of the initial condition selected in expression (2a) above, we have

$$
\begin{equation*}
E_{0}=\int_{0}^{E_{0}} I\left(\varepsilon_{\gamma}\right) d \varepsilon_{\gamma} \tag{12}
\end{equation*}
$$

The total number of gamma quanta emitted is given by the formula

$$
\begin{equation*}
n_{\gamma}=\int_{0}^{\varepsilon_{0}} n_{\gamma}\left(\varepsilon_{\gamma}\right) d \varepsilon_{\gamma} \tag{13}
\end{equation*}
$$

The above relations make use of the underlying assumption of Bohr's compourd nucleus model that the mode of decay of the excited state is independent of its mode of formation. In this case the correlations of the gamma quanta must be determined solely by conservation of angular momentum and there should be no other correlations. The presence of such correlations would mean that the whole approach was incorrect.

In the statistical model the widths are expressed in terms of the density of the final ruclear states and to describe this it is normal to use the Fermi gas model [10]. This method was used to calculate the gammamay spectra (neglecting angular momentum) in Refs $[3]$ and $[4]$ and to calculate the isomeric ratios in Refs [11-13]. Below we shall skow that existing data on radiation widths and level densities are not accurate enough.

For analysing isomeric ratios it is common to use the highly simplified description of Huizenga and Vandenbosch [2] in which only a small number of cascades is considered $(\sim 3-4)$ and the isomeric ratio is determined simply by the spin dependence of the level density. The simplicity of this approach is of course very attractive but the results of Refs [11-13] show that the accuracy is not good enough for a reliable quantitative analysis of experimental data.

## 2. Description of mean radiation widths

Let us now consider existing information on radiation width characteristics. Simple relationships for the widths of gamma transitions of different multipolarity from state i to state $j$ were obtained by Weisskopf on the basis of single-particle estimates [10]. Comparison of these estimates with existing data for light nuclei has shown that on average they are valid, although there are fairly large discrepancies in individual transitions [14]. If the states of the excited nucleus have sufficient density, we are interested mainly in the value of the radiation width averaged over a large number of possible transitions. For mean widths of different multipolarity we have

$$
\begin{align*}
& \Gamma_{r i j}(E 1)=\frac{6.8 \cdot 10^{-2}}{\partial_{0}} \cdot A^{2 / 3} \varepsilon_{\gamma}^{3} \rho_{i}^{-1} \\
& \Gamma_{\gamma i j}(E 2)=\frac{4.9 \cdot 10^{-8}}{\partial_{0}} A^{4 / 3} \varepsilon_{\gamma}^{5} \rho_{i}^{-1}  \tag{14}\\
& \Gamma_{\gamma i j}(M 1)=\frac{2.1 \cdot 10^{-2}}{\partial_{0}} \varepsilon_{\gamma}^{3} \cdot \rho_{i}^{-1}
\end{align*}
$$

where $A$ is the mass number, $\varepsilon \gamma$ is the gammaray energy, $p_{i}$ is the density of the initial states with given spin and parity and $D_{0}$ characterizes the mean distance between degenerate single-particle levels (sub-shells). The numerical values of the constants were obtained by wilkinson $[14]$ and correspond to the measurement of $\Gamma_{\gamma}$ in $e V, \varepsilon_{\gamma}$ and $D_{o}$ in $M e V$, and $e_{i}$ in $\mathrm{MeV}^{-1}$. Initially, Weisskopf estimated $D_{o}$ at $\sim 0.5 \mathrm{MeV}$ but the value now used is $D_{0} \sim 15 \mathrm{MeV}$. It is often included in a constant whose value is found by fitting to experimental data [8,9].

Recently, another approach has been evolved for calculating $\Gamma_{\gamma}$ based on the principle of a detailed balance linking the direct ( $n, \gamma$ ) and the inverse ( $\gamma, n$ ) reactions. Here it is assumed that the behaviour of the El transitions in the nucleus formed as a result of the ( $n, \gamma$ ) reaction is governed by the same physical processes as the giant dipole resonance in photonuclear reactions. Approximating the photoabsorption cross-section by means of Lorentz's formula on the assumption that its validity extends to energies of $6-8 \mathrm{MeV}$ - which are of interest from the point of view of the gamma radiation emitted during neutron capture - Axel $[15]$ obtained an expression for the width $\Gamma_{\text {yo }}$ of El transitions to the ground state:

$$
\begin{equation*}
\Gamma_{\text {roi }}=1 \cdot 13 \cdot 10^{6} A \frac{\varepsilon_{r}^{4} \cdot \Gamma_{g}^{2}}{\left(\varepsilon_{\gamma}^{2}-\varepsilon_{q}^{2}\right)^{2}+\varepsilon_{\gamma}^{2} \Gamma_{g}^{2}} \cdot \rho_{i}^{-1} \tag{15}
\end{equation*}
$$

where $\varepsilon g$ is the energy of the giant resonance and $\Gamma g$ is its width. To obtain a relation for evaluating a large number of nuclei, Axel [15] recommended $\Gamma g=5 \mathrm{MeV}$ and $\varepsilon_{g}=80 \cdot \mathrm{~A}^{-1 / 3}$. Then for gamma quanta with energies of $6-8 \mathrm{MeV}$ expression (15) takes the simple form

$$
\begin{equation*}
\Gamma_{p o i}=6.1 \cdot 10^{-15} \varepsilon_{\mu}^{5} A^{8 / 3} \rho_{i}^{-1} \tag{16}
\end{equation*}
$$

where $\Gamma_{\gamma \circ i}$ and $D_{i}=\frac{l}{Q_{i}}$ are in the same units and $\varepsilon \gamma$ is in $M e V$. Note once
again that expressions (15) and (16) were obtained only for transitions to the ground state. Therefore their use for calculating the radiation widths of transitions to excited states rests on the assumption that the shape of the photoabsorption energy dependence is determined solely by the gamma quanta energy and is independent of whether the nucleus is in the ground or the excited state.

Note also that even the experimental data on the width of the transition to the ground state relate only to the $15-20 \mathrm{MeV}$ region in the photoabsorption cross-section where a giant dipole resonance exists, and that there are practically no experimental data for the $6-10 \mathrm{MeV}$ region, so that extrapolation of expression (15) to lower gamma transition energies also requires verifican tion.

One of the ways of experimentally verifying the dependence of $\Gamma_{\gamma}$ on the energy of the gamma quanta emitted is to analyse the gamma spectra resulting from radiative capture of neutrons with energies in a range $\Delta E$ which contains a sufficiently large group of resonances ("average capture"). Now, thanks to the development of a new technique - Ge-Li gamma detectors - it has become possible to obtain neutron capture gamma spectra with very good resolution, by dividing the high energy part of the spectrum into separate lines.

In an experiment of this type the intensities of the gamma lines in the high energy part of the spectrum are proportional to the radiation widths $\Gamma \gamma, i, j$ in the sense defined above, and one can obtain the dependence of the radiation width on gamma-ray energy. The results of these investigations are given in Fig. 1 [16], which shows the radiation widths $\Gamma \gamma, i, j$ of the high energy part of the capture gamma spectrum for the isotopes ${ }^{156}$ Gd and ${ }^{158}{ }_{\text {Gd. }}$. The results of the experiment are compared with the energy dependence of $\Gamma_{\gamma i j}$ according to Weisskopf $\left(\sim \varepsilon_{\gamma}^{3}\right)[10]$, Axel $[15]$ and with the dependence $\sim \varepsilon_{\gamma}^{5}$. It can be seen that the values obtained for $\Gamma_{\gamma i j}$ in the $6.5-8.5 \mathrm{MeV}$ energy range do not agree with the energy dependence calculated according to Weisskopf. The experimental dependence is more sharply defined than $\varepsilon_{\gamma}^{3}$ and can be described approximately by both $\varepsilon_{\gamma}^{5}$ and expression (15). It is impossible, however, to draw any more definite conclusions, since the experimental data were obtained for a limited energy range and their accuracy is insufficient. Analysis of the spectra for other nuclei such as ${ }^{182} \mathrm{Ta},{ }^{184} \mathrm{~W},{ }^{190} \mathrm{Os}$ and ${ }^{196} \mathrm{Pt}$ also shows that the dependence of「үij on $\varepsilon_{\gamma}$ is in any case more sharply defined that $\varepsilon_{Y}^{3}[16]$.

Considerably less study has been devoted to the mean widths of the Mi and E 2 transitions. Recently obtained experimental data $[16]$ have shown that the mean widths of M1 transitions for gamma-ray energy about equal to the neutron binding energy are an order of macnitude less than the widths of the El transitions. Such a relationship is in quantitative agreement with an estimate of the widths based on expression (14). It is not ciear, however, to what extent the result obtained depends on the energy range considered. is it contains a giant min resonance, the mean radiative widths of the magetic transitions in this energy range may be comparable with the mean widths of the electric dipole transitions. Bollinger [16] attempted to analyse the effect of the giant M1 resonance by studying capture gamma-ray spectra fron the reactions ${ }^{117} \operatorname{Sn}(n, \gamma)^{118} \operatorname{Sn}$ and ${ }^{119} \operatorname{Sn}(n, \gamma)^{120}$ Sn. The energy depandence of the widths which he obtained is shown in Fig. 2. Owing to the paucity of experimental data available it is impossible to establish derinitaly whether there is a giant resonance in the intermediabe neutrons of Ml transitions, but it appears thet the transition intensity in the $6 \mathbf{- 9} \mathrm{MeV}$ energy range fluctiatee rather strongly.

From expression (14) ono would expect electric quadrupole transitione to be considerably weaker than dipole transitions, so that their contribution to the mean radiation width would be negligible. But here too it is not ceriain whether this relationship holds for low-energy transitions. The collective characteristics of the lower excited states may be such that riadrupole transitions are amplified, and this must be allowed for in calculating the mean radiation widths of nuclei in the low excitation energy range.

For calculating thermal capture gamma-ray specira $[3,4]$ and the corresponding total radiation widths [8], it is usually assumed that the nucleus decays only by way of electric dipole transitions. In calculating the isomeric ratios $[2,11,12]$ a small contribution of other transitions is admitted only at very low excitation energies. This brief examination of the data on mean radiation widths indicates that the assumption that EJ transitions dominate throughout the excitation energy range may not be correct; at any rate it is not possible at the moment to assess the accuracy of this approximation.

If the number of excited states involved in the decay process is large enough, the spectral density of the gamma quanta will be proportional to the number of states:

$$
\begin{equation*}
\Gamma_{\gamma}\left(\varepsilon_{\gamma}\right)=\Gamma_{\gamma i j}\left(\varepsilon_{\gamma}\right) \rho(u, I, \pi) \tag{17}
\end{equation*}
$$

Here $\Gamma_{i j}\left(\varepsilon_{\gamma}\right)$ coincides with the widthsdefined above - see expression (14) and $Q(u, I, \pi)$ is the density of final states with excitation energy $u=E-\varepsilon_{\gamma}$, the corresponding momentum $I$ and parity $\pi$.

Since the relationships given above for the population of states and gammaray spectra contain the relative decay probability

$$
\begin{equation*}
S\left(E_{0}^{\prime} Y^{\prime} \longrightarrow E . Y\right)=\frac{\Gamma_{\Gamma}\left(E^{\prime} Y^{\prime} \rightarrow E, Y\right)}{\Gamma_{r}(E, Y)}, \tag{18}
\end{equation*}
$$

the results of the spectrum calculations do not depend on the constants used in expression (14), and are influenced only by the energy dependence of $Q(u, J)$ and $\Gamma_{\gamma i j}\left(\varepsilon_{\gamma}\right)$. Levels of different parity are distributed on average with equal probability $[17]$ so that even in deriving the basic relationships we did not classify the transitions according to parity. Below we shall consider only dipole transitions and ignore the quadrupole contribution. The energy dependence in expression (18) should then be determined as a composite energy dependence of El and M1 transitions with allowance for their relative intensity.

## 3. Density of excited nuclear states

A description of the mean radiation width relying on the statistical approach (17) requires us to know the density of the excited nuclear states over the whole range of excitation energies and angular momenta investigated. Since there are no direct experimental data for this wide range, it is necessary when calculating to employ various model-based estimates of level density which are fitted to existing experimental data only at certain points.

It is most common in such calculations to use simple analytical relationships for level density obtained with the Fermi gas model [17]:

$$
\begin{align*}
& \rho(u, y)=\frac{2 y+1}{2 \sqrt{2 \pi} \sigma^{3}} \exp \left\{-\frac{(y+1 / 2)^{2}}{2 \sigma^{2}}\right\} \rho(u),  \tag{19}\\
& \rho(u)=\frac{\sqrt{\pi}}{12 a^{1 / 4} u^{3 / 4}} \exp \{2 \sqrt{a u}\} \tag{20}
\end{align*}
$$

The basic parameters of this model are the level density parameters, a, and the spin dependence parameter

$$
\begin{equation*}
\sigma^{2}=\mp \sqrt{\frac{u}{a}}=\frac{\sigma}{\pi^{2}} \bar{m}^{2} \sqrt{a u} \tag{21}
\end{equation*}
$$

Since this parameter depends on the excitation energy $u$, it is more convenient to use the moment of inertia $F=\frac{6}{\pi^{2}} a<m^{2}>$ as a parameter, or even $<m^{2}>-$ the mean square of the projection of the single-particle moment of states close to the Fermi energy.

For $u$ in these expressions we use the effective excitation energy, defined as

$$
u=E-\delta^{r}, \text { where } \delta^{\sigma}= \begin{cases}0 & \text { for odd-odd nuclei }  \tag{22}\\ \Delta_{N} \text { or } \Delta_{z} & \text { for odd nuclei } \\ \Delta_{N}+\Delta_{z} & \text { for even-even nuclei }\end{cases}
$$

This way of determining the effective excitation energy corresponds to the phenomenological calculation of residual interactions in nuclei with different parity of the nucleon number. The pairing parameters $\Delta$ are determined here on the basis of the even-odd nuclear mass differences [18, 19].

For a large number of nuclei the level density parameter, $a$, was found by analysing experimental data on neutron resonance density [8, 19, 20]. Selection of the parameter ${\left\langle m^{2}>\right.}^{2}$ involves some uncertainty which leads to inconsistency in the parameter a derived by different authors. Thus the authors of Refs $[8,19]$ used a value of $\left\langle\mathrm{m}^{2}\right\rangle=0.146 \mathrm{~A}^{2 / 3}$ whereas in later investigations the value employed is $\left\langle m^{2}\right\rangle=0.24^{2 / 3}[20]$. The latter value corresponds better to the theoretical determination of $\left.<\mathrm{m}^{2}\right\rangle[21]$.

The chief merit of the Fermi gas model is the simplicity of the results it yields - see expressions (19), (20) and (21). However, existing theory on the structure of the excited nucleus raises doubts as to the suitability of such a simple model for describing level density in a wide excitation energy range. A lot of experimental data are in direct contradition to Fermi gas model results [22]. To describe a large set of nuclear characteristics it is more logical to use a superfluid nuclear model based on the fundamental relations of the theory of superconductivity $[22]$. The expression for level density obtained with this approach is more complex. The following relation was used in this investigation:

$$
\begin{equation*}
\rho(u)=\left[(2 \pi)^{3} f_{z E} f_{N N}\left(f_{\beta \beta}^{(z)}+f_{\beta \beta}^{(N)}\right)\right]^{-\frac{1}{2}} \exp \left\{S_{z}+S_{N}\right\} \tag{23}
\end{equation*}
$$

$S_{Z}$ is nuclear entropy, determined for protons as

$$
\begin{equation*}
S_{z}=2 g_{z} \beta \Delta z \sum_{n=1}^{\infty}(-)^{n+1} K_{2}\left(n \beta \Delta_{z}\right) \tag{24}
\end{equation*}
$$

where $K_{V}(x)$ is the Macdonald function. The pre-exponential functions are:

$$
\begin{aligned}
& f_{z z}=g_{z} \int_{0}^{\infty} d x\left\{\frac{1}{2^{\prime} \operatorname{ch}^{2}\left(\frac{\beta \sqrt{x^{2}}+\Delta z}{2}\right)}+\frac{\Delta_{z}^{2} t h^{e} \frac{\beta \sqrt{x^{2}+\Delta_{z}^{2}}}{2}}{\sqrt{x^{2}+\Delta_{z}^{2}}}\right\}, \\
& f_{\beta \beta}=g_{z} \int_{0}^{\infty} d x\left\{\frac{x^{2}-\Delta_{z}^{2}}{2 c^{2} \frac{\beta \sqrt{x^{2}+\Delta_{z}^{2}}}{2}}+\frac{\Delta_{z}^{2}\left(x^{2}-\Delta_{z}^{2}\right)}{\left.\sqrt{x^{2}+\Delta_{z}^{2}} t h \frac{v^{6} \sqrt{x^{2}+\Delta_{z}^{2}}}{2}\right\}}\right.
\end{aligned}
$$

Relations (24) and (25) take a similar form for neutrons, it being necessary only to replace the parameters $g_{Z}$ and $\Delta_{Z}$ by $g_{N}$ and $\Delta_{N}$. The parameter $\Delta$ defines the effect of pair correlations and is obtained by solving the equation

$$
\begin{array}{ll}
\ln \frac{\Delta_{z}}{\Delta_{0 z}}=2 \sum_{n=1}^{\infty}(-)^{n+1} n \beta \Delta_{z} k_{z}\left(n \beta \Delta_{z}\right) & \text { when } \beta>\beta_{k p}^{(z)}  \tag{26}\\
\Delta_{z}=0 & \text { when } \beta \leqslant \beta_{k p}^{(z)}
\end{array}
$$

where $\beta_{\mathrm{kp}}^{(Z)}=1.76 / \Delta_{\mathrm{oZ}} \mathrm{Z}^{*} \quad$ A similar equation is written for $\Delta_{N}$.
The above expressions are functions of the thermodynamic temperature $t=B^{-1}$. For a given excitation energy $u$, the temperature $t$ is the solution of the equation

$$
\begin{equation*}
u=\frac{t}{2}\left(s_{N}+s_{Z}\right)+1 / 4 g_{Z}\left(\Delta_{O Z}^{2}-\Delta_{Z}^{2}\right)+1 / 4 g_{N}\left(\Delta_{O N}^{2}-\Delta_{N}^{2}\right) \tag{27}
\end{equation*}
$$

The dependence of the thermodynamic functions on temperature for this model is shown in Fig. 3.

Thus in the superfluid nuclear model the level density energy dependence is determined by four parameters: the density of single-particle proton states $g_{Z}$ and neutron states $g_{N}$ and the pair correlation parameters $\Delta_{o} Z$ and $\Delta_{\text {OV. }}$. These parameters are equivalent to the parameters a and $\delta$ in the Fermi gas model. If it is assumed that $\Delta_{o Z}=\Delta_{o N}=\Delta_{D}$ and $g_{Z}=g_{N}=g / 2$, the level density in Eq. (23) will be a function of only two parameters. This reduction in the number of parameters considerably simplifies their determination in analyses of experimental data; however, theoretical determination of the parameters $\Delta_{o Z}$ and $\Delta_{o N}$ shows that they almost always differ [23]. It should be noted that in the superfluid nuclear model $\Delta_{o}$ does not vanish
for an odd number of protons or neutrons, and the observed even-odd differences in the excited state densities are associated with the difference of $1 / 4 g \Delta_{0}^{2}$ for even and odd nuclei [22].

The dependence of level density on angular momentum in the superfluid model is similar to relation (19) in the Fermi gas model except that the spin dependence parameter takes a different form:

$$
\begin{equation*}
\sigma^{2}=t\left(\vec{F}_{z}+\bar{F}_{N}\right) \tag{28}
\end{equation*}
$$

The moment of inertia is determined as

$$
\begin{array}{ll}
\mathcal{F}_{z}=2 m_{z}^{2} \sum_{n=1}^{\infty}(-)^{n+1} \beta n \Delta K_{1}(\beta n \Delta) & \text { when } \beta>\beta_{k_{p}}^{(z)}  \tag{29}\\
\mathcal{F}_{z}=m_{z}^{2} g_{z} & \text { when } \beta \leqslant \beta_{F p}
\end{array}
$$

The expression for $\mathrm{F}_{\mathrm{N}}$ is similar.
In the general case $\bar{m}_{Z}^{2} \neq \bar{m}_{N}^{2}$, but this difference has very little effect on the calculation of the level density spin dependence. To reduce the number of parameters in the calculation, it was assumed that $\bar{m}_{Z}^{2}=\bar{m}_{N}^{2}=0.25 A^{2 / 3}$, where the numerical value of the coefficient is obtained from calculations on the single-particle spectrum of the shell model [21].

For near magic nuclei the level density energy dependence is more complex and cannot be described by any of the above models for constant values of the level density parameter, a (or g) [22]. For such nuclei it is accordingly best to make direct use of level density calculations on a given spectrum of single-particle states of the shell model. The results of these calculations provide a fairly good description of the experimental data on the excited state densities of magic nuclei in the barium-cerium range ( $A \sim 140$ ) and in the lead range $[24]$.

Calculations of the low energy part of the spectrum of cascade gamma radiation are considerably affected by the discrete character of the spectrum of the primary excited nuclear states [4]. If an experimental scheme of the primary nuclear levels exists, it is convenient, when calculating the radiation widths to separate out the transitions to explicit low-lying states or, what amounts to the same thing, to represent the level densities in the low energy region in the form

$$
\begin{equation*}
\rho(u, J)=\sum_{i} \delta\left(u-E_{i}\right) \delta_{j J_{i}}, \tag{30}
\end{equation*}
$$

where $E_{i}$ and $J_{i}$ are the energy and spin of the $i^{\text {th }}$ excited level.
This method of determining the density also enables allowance to be made for the probability of radiative transition to the ground state and for the effects of an energy gap in the spectrum of excited states of even-even nuclei.

## 4. Results of calculations

For calculating gamma-ray spectra and the relative population of the isomeric levels it is necessary to determine the population of excited states resulting from transitions with a given number of gamma quanta - see Eq. (5). This was found by successive calculation of the probabilities of transitions in each cascade, but for these calculations it is particularly important to choose the optimum number of cascades. This problem can be avoided by summing Eq. (5) in terms of the possible cascades and reducing the solution of the resulting integral equation to a system of differential equations $[3,4]$. However, this approach is convenient only for a fairly simple energy dependence of the radiation width $\left(\sim \varepsilon_{\gamma}^{3}\right)$; and since the behaviour of the radiation width can be more complex, we preferred the method of successive cascade analysis for calculating the spectra. Comparison of the results of those calculations with the results of solution of the differential equations in Refs $[3,4]$ showed that the optimum number of cascades is ~ 50 and that the calculations are not sensitive to any further increase in the number of cascades.

Troubetskoy and Strutinsky $[3,4]$ have shown that the low energy part of the gamma-ray spectrum cannot be obtained with a continuous level distribution function (level density) close to the ground state and that it is necessary to allow explicitly for the discrete character of the spectrum of the primary excited nuclear states - see Eq. (30). In our calculations in the low excitation energy range we therefore used discrete levels, the energy and angular momentum of which were selected in accordance with Refs [25] and [26]. This choice of levels also removes to some extent the indeterminacy associated with the use of the level density formulae expressions (19)-(27) - in the low excitation energy range. If excited states arose (not the observed isomeric states) which could not decay via dipole transitions, we then took into account transitions of higher multipolarity. The consideration of the latter was analogous to the assumptions conceming gamma transitions of the final cascade used by Huizenga and Vandenbosch [2] for investigating the isomeric ratios. These transitions have practically no effect on the gammarray spectra calculations and usually
have very little effect on the calculation of the population of the isomeric states. They are mainly required for correct normalization of the isomeric ratio - see Eq. (7).

Let us now see how our calculations are affected by differences in the energy characteristics of the excited state density and in the energy dependence of the radiation width.

Fig. 4a shows the experimental gamma-ray spectrum for thermal neutron capture by a Hf nucleus [ 26$]$ and the results of calculating the spectrum of the ${ }^{177} \mathrm{Hf}(\mathrm{n}, \mathrm{Y})^{178}$ Hf reaction for thermal capture to a state with momentum $J_{0}=3$. The spectra resulting from the formation of a compound nucleus with momentum $J_{0}=4$ differ only marginally from those given in the figure. Fig. $4 b$ shows the excited state density used in the calculations; the broken line shows the boundary below which the discrete level scheme was used. The Fermi gas model parameters were taken from Ref. [20] and correspond to experimental data on neutron resonance density. The parameters of the superfluid nuclear model were selected so that the density of the nuclear states in this model at an excitation energy equal to the neutron binding energy was the same as the neutron resonance density. This was achieved by selecting the quantity $g=g_{Z}=g_{N}$ with parameters $\Delta_{o N}$ and $\Delta_{o Z}$ taken from Ref. [23]. The results of the calculations are also greatly affected by the choice of different relations for the radiation widths - see expressions (14) and (15) and by the difference in the level density characteristics. The Fermi gas model with the usual radiation width dependence ( $\sim \varepsilon^{2 \mathcal{L}+l}$ ) gives spectra which are too soft. This conclusion is not in contradiction with the calculations in Refs [3] and [4], since they employ a level density which is too low in the high excitation energy range. Use of the Lorentz radiation width relation - Eq. (15) - improves the agreement with experimental data and it can be seen that with suitable choice of level density it is possible to achieve quite a good description of the experimental conditions; the level density curve should then fall between the curves given in Fig. 4b. When allowance is made for discrete levels in the range $\varepsilon \gamma \sim 1.2 \mathrm{MeV}$, lines are obtained in positions corresponding to the experimental ones, but their intensity is $3-4$ times higher. The Lorentz relation also gives intense lines in the high gamma-ray energy range which are not present in the experimental data. Since our calculations have an average statistical character they cannot be claimed to describe the separate lines, the intensity of which depends considerably on the structure of the wave functions of the initial and final states of the corresponding transitions. The integral spectral characteristics obtained in the calculations, the mean number of gamma quanta per capture and the isomeric ratio for the $8^{-}$state, are given in Table 1.

Table 1


The mean number of quanta is calculated from a spectrum with $\varepsilon \gamma \geqslant 0.1 \mathrm{MeV}$, i.e. the contribution of very soft gamma rays is not allowed for. A corrected value for gamma peak intensity is used for calculating the mean number of gamma quanta with $\varepsilon_{\gamma} \sim 1.2 \mathrm{MeV}$. This correction has a value $\Delta n_{\gamma}=0.4-0.6$ in the different variants of the calculations. The calculations performed give the correct order of magnitude of the isomeric ratio.

The isomeric ratios provide a means of studying the behaviour of the level density spin dependence, but such a study is very much limited by the fact that we can observe the population of only two states. The information would be more complete if the population of a larger number of states were investigated for a given nucleus. A method for the experimental derivation of this information was considered in Ref. [27]. The mean population of low-lying states with different spin was derived from the gamma peak intensity; the relative populations obtained for radiative capture of a thermal neutron by a ${ }^{164}$ Dy nucleus are shown in Fig. 5a [27]. Fig. 5b shows the total gamma-ray spectrum for this reaction [26]. With relations (5)-(7) it is possible to calculate the population of these states and Fig. 5 compares the results of the calculation with experimental data. Only the results of the Fermi gas model calculations are shown, because for this nucleus the spectra calculated with the level density - Eq. (23) - are considerably at variance with experiment. As with the reaction ${ }^{177} \mathrm{Hf}(\mathrm{n}, \mathrm{Y})^{178} \mathrm{Hf}$, considered above, the use of the Lorentz relation for the radiation width gives too high values for the intensity of the gamma lines in the high energy part of the spectrum. Fig. 5b shows how variation of the level density parameter, $a$, affects the results.

Fig. 6 shows the relative populations of low-lying discrete levels in the reaction ${ }^{176}{ }_{L u}(n, \gamma)^{177}$ Lu for a thermal neutron $[27]$. As in the case of Dy, the gamma-ray spectrum of ${ }^{177}$ Lu is best described by the Fermi gas model, so the results of calculating the relative population of the states are given only for this model. The distributions indicate how the choice of momentum of the initial state affects the calculation of the relative population, and it may be concluded that in the case of thermal neutron capture the state with momentum $J_{0}=13 / 2$ is to be preferred.

## 5. Discussion of results

The calculations have shown that allowance for the effect of angular momentum on the radiative transition probability produces a spectrum of cascade gamma quanta softer than the spectrum obtained in calculations not allowing for this effect [3, 4]. The spectra of intermediate and heavy nuclei (far from the magic numbers) calculated with a radiation width energy dependence of $\sim \varepsilon_{\gamma}^{3}$ and with level density according to the Fermi gas model, are usually much softer than the experimental spectra. Using the Lorentz curve to describe the radiation width gives harder spectra which describe the main part of the spectrum quite satisfactorily. However, in the case of discrete low-excited states this dependence usually gives unduly high values for the intensity of high-energy gamma transitions.

For even-even nuclei the agreement of the theoretical spectra with the experimental is better if the level density energy dependence obtained with the superfluid nuclear model is used. This model is much more consistent than the Fermi gas model in describing a wide range of experimental data [22]. However, before it can be used widely for calculations of this kind, it is necessary to have a more clearly defined selection of model parameters for a wide range of nuclei.

With allowance made for the discrete structure of the low-excited states, the calculations describe sufficiently well the mean population of levels with different spin. The spin dependence parameter employed, $\left\langle\mathrm{m}^{2}>=0.25 \mathrm{~A} 2 / 3\right.$ corresponds to the solid-state value of the moment of inertia [21]. The integral characteristics of the radiative capture spectra for thermal neutrons - the mean number of gamma quanta and the isomeric ratio - are less sensitive to the choice of model parameters than the shape of the spectrum, and therefore a good description of these does not always serve to verify the adequacy of the model employed.

It should be noted in connection with the statistical description of gammaray spectra considered in this paper that correct choice of the radiation width and level density parameters does not produce the hard spectra observed for nuclei with nearmagic proton or neutron numbers. The best example of this is provided by the spectra for thermal neutron capture by nuclei in the $\mathrm{Au}-\mathrm{Pb}$ range [26]. The impossibility of describing them statistically has already been discussed $[9]$ and the above method should not be used for describing the isomeric ratios in the case of these nuclei.


Fig. 1 Gamma energy versus mean width of electric dipole transitions for ${ }^{156} G$ and ${ }^{158}$ Gd nuclei [16].


Fig. 2 Gamma energy versus mean width of magnetic dipole transitions for ${ }^{118}$ Sn and ${ }^{120}$ Sn nuclei [16].


Fig. 3 Temperature dependence of the thermodynamic functions of the superfluid nuclear model. The broken line shows the behaviour of the Fermi gas model level density (Eq. (19)).


Fig. 4 (a) Gamma cascade spectra for thermal neutron capture. The experimental spectrum was obtained for a natural mixture of Hf isotopes [26]; the calculated results are for the reaction ${ }^{177_{H f}(n, y) \text {. The solid line shows the }}$ results of calculations with level density according to the superfluid nuclear model - Eq. (23); the broken line shows the results obtained using the Fermi gas model Eq. (20).
$1=$ Energy versus radiation width - Eq. (14);
2 = Curve of Eq. (15).
(b) The excited state density used in the calculations.



Fig. 5 (a) Relative populations of low-lying states of a ${ }^{165}$ Dy nucleus ${ }^{\top}$ are experimental data, . are calculations in the Fermi gas model with radiation width according to Eq. (14), $x$ are the same with radiative width according to Eq. (15).
(b) Gamma-ray spectra. The notation is the same as in Fig. 4a.



Fig. 6 Relative populations of low-lying states of ${ }^{177} \mathrm{Lu}$. Notation is the same as in Fig. 5a.

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# CROSS-SECTIONS FOR THE RADIATIVE CAPTURE OF NEUTRONS <br> BY SILVER, ${ }^{197} \mathrm{Au},{ }^{232} \mathrm{Th}$ AND ${ }^{238} \mathrm{C}_{\mathrm{U}}$ NUCLEI <br> Yu.Ya. Stavissky, V.A. Tolstikov, V.B. Chelnokov, A.E. Samsonov, A.A. Bergman 

## Introduction

The cross-sections for the radiative capture of neutrons by silver, ${ }^{197}{ }_{A u}$, ${ }^{232}$ Th and $238_{U}$ nuclei are of interest for nuclear theory and reactor design.

This paper presents the results of measurements of the average crosssections for the radiative capture of neutrons by $\mathrm{Ag},{ }^{197} \mathrm{Au},{ }^{232} \mathrm{Th}$ and ${ }^{238} \mathrm{U}$ nuclei in the energy range below 50 keV , performed with a neutron spectrometer on the basis of the slowing down time in lead $[1,2]$.

The energy dependence of the crossmsections was normalized with respect to the resolved low-order resonances and also to the thermal crossmections from measurements in a graphite prism placed close to the lead moderator [3].

The data obtained here are compared with the results of other authors.

## Measuring procedure

In the channel of the lead moderator the count of prompt gamma rays from radiative capture in each sample was measured against the slowingmown time $J_{\gamma}(t)$ and the neutron density $J_{B}(t)$, using a detector with an efficiency proportional to $\sim 1 / v\left(\mathrm{BF}_{3}\right.$ - counter). Then, as shown in Ref. [7] , we have

$$
\begin{equation*}
\sigma_{c}(E)=\frac{J_{\gamma}(t)}{J_{B}(t)} \cdot \frac{(t+0.3)}{K_{x}} \tag{1}
\end{equation*}
$$

where $\sigma_{c}(E)$ is the cross-section for radiative capture by nuclei of the sample, and $K_{x}\left(\pi, M, \varepsilon_{c}\right)$ is the normalizing factor depending on the effective thickness of the sample $\bar{n}$, the monitor count $M$ and the efficiency of recording a capture event $\varepsilon_{c}$.

The mean neutron energy $E(k e V)$ and the slowing-down time $t$ ( $\mu \mathrm{sec}$ ) are related by the empirical expression $[2]$

$$
\begin{equation*}
E=\frac{183}{(t+0.3)^{2}} \tag{2}
\end{equation*}
$$

## Detector

A gas proportional counter was used for detecting the prompt capture gamma rays. By surrounding its walls with lead (with total thickness $d>R e$, where Re is the path of a secondary electron formed by a gamma quantum) and filling it to a high pressure ( $10 \mathrm{~atm}+4 \% \mathrm{CO}_{2}$ ) it is possible to obtain an approximately linear dependence of gamma-ray recording efficiency $\varepsilon \gamma$ on gamma energy Ey:

$$
\begin{equation*}
\varepsilon_{\gamma} \simeq \text { const. } E_{\gamma} \tag{3}
\end{equation*}
$$

In this case the recording efficiency for a radiative capture event $\varepsilon_{c}$ is determined solely by the total gamma-ray cascade energy, i.e. the neutron binding energy in the nucleus $B_{n}$ (since the neutron kinetic energy may be neglected):

$$
\begin{equation*}
\varepsilon_{c} \simeq \text { const. } B_{n} \tag{3a}
\end{equation*}
$$

and is unaffected by variations in the gamma-ray spectrum from resonance to resonance [5].

The accuracy of expression (3a) can be estimated from measurements on the proportional gamma counter with different samples. Since the normalizing factor $K_{x}$, calculated from resonances with known parameters or from the thermal cross-section $[3,7]$ and referred to the sample thickness $\vec{n}$ and the neutron flux, depends only on the capture event recording efficiency $\varepsilon_{c}$, it is possible to determine the function $\varepsilon_{c}\left(B_{n}\right)$.

The values of the normalizing factors obtained by measurements with the gamma counter for different samples are shown in Fig. l. Processing the results by the least squares method gives a value $K_{x} / \bar{n} \times B_{n}=0.945 \pm 0.037$, whence it follows that expression (3a) is fulfilled for the gamma detector used in the measurements with an accuracy of $\pm 4 \%$.

## Measurements and processing of results

The measurements were performed at a neutron burst frequency $f=625$ and 312.5 Hz . The neutron burst time was set at the lowest value ( $\sim 0.5 \mathrm{msec}$ ) in those measurements in which it was important to obtain the energy dependence of the cross-sections in the keV energy range (with small slowing-down times).

The amplification circuit of the gamma counter includes an antisaturation UIS-2 amplifier with a "Siren" amplifier discriminator. The time dependence of the detector count was investigated on the 256 channel analyser of the Measuring and Recording Centre at the Lebedev Physics Institute [6]. .

During the measurements of capture in the sample and the gamma background of the spectrometer (without sample), the activation of the detector was maintained at saturation (counter irradiated for $\sim 10$ min before the start of each series) so that it could be discounted in the processing of the results. The capture measurements were alternated with measurements of the natural gamma background of the samples (thorium and uranium) on the deactivated counter. The contribution of the natural background, which was constant at equal intervals over the whole time range, was allowed for by simple subtraction. The activation of the sample was determined from the ratios of the number of counts recorded with a sample in the ganma counter and the number recorded with the boron detector (with no activation) in the analyser channels corresponding to the energy range in which the investigated cross-section approximately follows the $1 / \mathrm{v}$ law:

$$
\begin{equation*}
\frac{J_{\gamma}^{\prime}-a}{J_{B}^{\prime}}=\frac{J_{\gamma}^{\prime \prime}-a}{J_{B}^{\prime \prime}}=\text { const } \tag{4}
\end{equation*}
$$

whence it is not difficult to obtain an expression that will give the correction for activation:

$$
\begin{equation*}
a=J_{\gamma}^{\prime}-J_{B}^{\prime} \cdot \frac{J_{\gamma}^{\prime}-J_{\gamma}^{\prime \prime}}{J_{B}^{\prime}-J_{B}^{\prime \prime}} \tag{5}
\end{equation*}
$$

The estimation of the other corrections and possible measuring errors, performed in accordance with Ref. [7], results in at slight error (less than $1-1.5 \%$ ) in the cross-sections investigated.

The energy dependence of the crossmsections was normalized to resolved resonances with known parameters $[7]$ and to thermal capture crossmsections obtained from measurements in the graphite prism placed close to the main prism of the lead moderator [3]. In addition, measurements with a gamma detector for which expressions (3) and (3a) are satisfied make it possible to normalize with respect to measurements on samples with wellestablished radiative capture cross-sections. Then, if the investigated sample and the standard sample have the same geometry, we find

$$
\begin{equation*}
K_{x}=K_{s t} \frac{\bar{n}_{x} \cdot M_{x} \cdot \varepsilon_{c}^{x}}{\bar{n}_{s t} \cdot M_{s t} \cdot \varepsilon_{c}^{s t}}=K_{s t} \frac{\bar{n}_{x} \cdot M_{x} \cdot B_{n}^{x}}{\bar{n}_{s t} \cdot M_{s t} \cdot B_{n}^{s t}} \tag{6}
\end{equation*}
$$

where $K_{x}, \bar{n}_{x}, M_{x}, \varepsilon_{c}^{x}$ and $B_{n}^{x}$ are respectively the normalizing factor, the sample thickness, the monitor count, the recording efficiency for capture events and the nuclear binding energy of the sample; and $\mathrm{K}_{\mathrm{st}}, \overline{\mathrm{n}}_{\mathrm{st}}, \mathrm{M}_{\text {st }}$, $\varepsilon_{c}^{s t}$ and $B_{n}^{s t}$ are the same quantities for the standard sample. The values of the normalizing factors obtained in measurements with the samples investigated are listed in Table l. The errors quoted are mainly statistical but are also due in part to uncertainties in the resonance parameters employed [4].

## Results and discussion

Silver. The energy dependence of the crossmsection for radiative capture by silver nuclei was measured with samples of the natural isotopic mixture in two effective thicknesses ( $4.3 \times 10^{21}$ and $2.2 \times 10^{21} \mathrm{at} / \mathrm{cm}^{2}$ ). The results are given in Fig. 2.

The data obtained here are in agreement with the results of previous measurements with a slowing-down time spectrometer $[7]$ (normalized to the resonance at $E_{0}=5.19 \mathrm{eV}$ ). Agreement with the other data presented in Fig. 2 is good within the limits of measuring error.

Gold-127. The energy dependence of the cross-section for radiative capture by ${ }^{197}$ Au nuclei was measured with samples of two effective thicknesses $\left(1.8 \times 10^{21}\right.$ and $\left.6.0 \times 10^{20} \mathrm{at} / \mathrm{cm}^{2}\right)$. The results of the measurements are shown in Fig. 3.

The data obtained here are also in agreement with earlier measurements made with a slowing-down time spectrometer $[45]$ (normalized to the resonance at $\left.E_{0}=4.9 \mathrm{eV}\right)$. Agreement with the data of other authors obtained mainly by the time-of-flight method is good.

Thorium-232. $\sigma_{c}(E)$ for ${ }^{232}$ Th nuclei was measured with samples of $\mathrm{ThO}_{2}$ in three effective thicknesses ( $9.6 \times 10^{21}, 4.0 \times 10^{21}$ and $1.0 \times 10^{21} \mathrm{at} / \mathrm{cm}^{2}$ ). The results of the measurements are given in Fig. 4 and compared with data of other authors.

The data from Refs [8] and [14-20] are given in their original form whereas the data from Refs [10-13] have been renormalized as described below. Note that the numerical data of Refs $[8]$ and $[17-20]$ were not available to us, the corresponding data being derived from the graphs in those publications.

Macklin and co-workers [13] measured the crossmsection for ${ }^{232} \mathrm{Th}$ by the activation method. In calculating the crossmsection from the experimental data the authors used $K=0.9$ as the gamma quantum yield per decay event for gamma quanta with $\mathrm{E}_{\gamma}=311 \mathrm{keV}$. However, according to the decay scheme of ${ }^{233} \mathrm{~Pa}[22]$, only $\mathrm{K}_{1}=0.36$ beta decays would give quanta with an energy of $3 l l \mathrm{keV} ; \mathrm{K}_{2}=0.59$ beta decays will give quanta with $\mathrm{E}_{\mathrm{Y}}=31 \mathrm{keV}$ in a cascade with quanta of lower energy. Obviously, then, only $\mathrm{K}_{2} / 2$ quanta will reach the crystal unaccompanied by a cascade quantum and will be recorded at the $3 l l \mathrm{keV}$ photopeak. The other half will be recorded in the aggregate peak of the cascade. However, because the soft quanta will be absorbed very efficiently in the sample and the crystal container, there will be an additional fraction of $3 l \mathrm{lkeV}$ gamma rays recorded in the $3 l \mathrm{keV}$ photopeak. Thus the true fraction $K$ of gamma quanta per beta decay lies between the values $K_{1}+K_{2} / 2$ and $K_{1}+K_{2}$. For making estimates it is natural to take $K=\left(K_{1}+3 / 4 K_{2}\right) \pm K_{2} / 4$; the corrected cross-section then becomes $560 \pm 150 \mathrm{mb}$.

The data of Ref. [10] were renormalized using an $\sigma_{c}{ }^{127}$ I for fast neutrons based on the curve from the atlas [23]. Also, the latest recommended value of $\sigma_{c}^{T h} 127_{I}=6.2 \mathrm{~b}$ was used instead of $5.66[23]$.

In Ref. $[12]$ the $\sigma_{c}{ }^{232}{ }_{\text {Th }}$ curve was measured in relation to the crosssection of the ${ }^{10} B(n, \alpha)$ reaction. The results have been remevaluated by us and adjusted to give $\sigma_{c}(24 \mathrm{keV})=615 \mathrm{mb}[16]$.

The results in Ref. [ll] have been renormalized with allowance for the new fission crossmsections from Ref. [21].

Our data in the energy range above $\sim 5 \mathrm{keV}$ agree within the limits of measuring error with the data of most other authors. In this range and also in the range below $\sim 5 \mathrm{keV}$ there is, however, significant disagreement with the data of Refs [17] and [20]. It is difficult to pinpoint the causes of this disagreement without having detailed information on these particular studies. However, it should be noted that when comparing the data of different authors it is essential to allow for differences in resolution and sample thickness (blocking effect).

Uranium-238. The energy dependence of the cross-section for radiative capture by ${ }^{238} \mathrm{U}$ nuclei was measured with $\mathrm{U}_{3} \mathrm{O}_{8}$ samples; of three effective thicknesses $\left(7.1 \times 10^{21}, 3.9 \times 10^{21}\right.$ and $1.3 \times 10^{21} \mathrm{at} / \mathrm{cm}^{2}$ ). The results of the measurements are shown in Fig. 5.

Preliminary data on the cross-sections for radiative capture of neutrons in 238 were published earlier [24]. Here these data are given in definitive form after more precise definition of a number of experimental constants associated with the normalization of the crossmsection energy dependence.

In Fig. 5 our data are compared with the results of other authors, which are presented in their original form apart from Ref. $[12]$, where, at the suggestion of one of the authors, the data were renormalized to $\sigma_{c}$ $(23.5 \mathrm{keV})=439 \mathrm{mb}$.

In comparing our data with the data of Ref. $[20]$ one should remember that the latter were obtained with a ${ }^{238} \mathrm{U}$ specimen thinner than ours $\left(\bar{n}=1.3 \times 10^{21} \mathrm{at} / \mathrm{cm}^{2}\right)$, so that the blocking effect should be comparatively slight.

In conclusion the authors wish to thank Mr. I.Ya. Barit for giving them the opportunity to use the spectrometer and the facilities of the measuring centre at the Lebedev Physics Institute; also Mr. Yu.A. Dmitrenko, Mr. V.M. Polyakov and the staff of the Radiogroup for their help with the measurements, and I.V. Syutkina and E.N. Zhukova for their assistance in processing the results.

Table 1
Normalizing factors obtained in measurements with the samples under investigation

| Sample | Method | $\mathrm{K}_{\mathrm{x}}$ |
| :---: | :---: | :---: |
| Silver | ${ }^{109} \mathrm{Ag}$ resonance $\mathrm{E}_{0}=5.19 \mathrm{eV}$ |  |
|  | Thermal cross-section b |  |
| Gold-197 | ${ }^{197} \mathrm{Au}$ resonance $\mathrm{E}_{\mathrm{O}}=4.906 \mathrm{eV}$ |  |
|  | Thermal cross-section b |  |
| Thorium-232 | ${ }^{232}$ Th resonances |  |
|  | With respect to gold-197 |  |
| Uranium-238 | ${ }^{238}$ U resonance eV |  |
|  | With respect to gold-197 |  |

## Table 2

Numerical cross-sections for radiative capture of neutrons by Ag nuclei ( $\sigma_{c}(E)$ )

|  | ${ }_{c}{ }_{c}$, barm |
| :---: | :---: |
| 43600 | $0,78 \pm 0,09$. |
| 34600 | $0,84 \pm 0,09$ |
| 28200 | 0,94 $\pm 0,09$ |
| 23200 | $0,99 \pm 0,08$ |
| 19800 | $1,06 \pm 0,08$ |
| 16800 | $1,10 \pm 0,08$ |
| 13500 | $1,21 \pm 0,08$ |
| II000 | $1,28 \pm 0,08$ |
| 8850 | $1,35 \pm 0,08$ |
| 7200 | 1,42 $\pm 0,09$ |
| 5950 | $\mathrm{I}, 47 \pm 0,09$ |
| 5000 | $1,61 \pm 0,10$ |
| 450 | $1,63 \pm 0,10$ |
| 3600 | I,78 $\pm 0.11$ |
| 2900 | $\mathrm{I}, 88 \pm 0, \mathrm{II}$ |
| 2450 | $2,05 \pm 0,12$ |
| 1950 | $2,25 \pm 0,13$ |
| I550 | $2,49 \pm 0,15$ |
| 1300 | 2,90 $\pm 0,17$ |
| 1100 | $3,09 \pm 0,18$ |

Table 3
Numerical cross-sections for radiative capture of neutrons by ${ }^{197}$ Au nuclei ( $\left.\sigma_{c}(E)\right)$

| $\mathrm{E}, \mathrm{eV}$ | $\sigma_{0}$, barm |
| :---: | :---: |
| 43600 | $0,45 \pm 0,05$ |
| 34600 | $0,52 \pm 0,05$ |
| 28200 | $0,61 \pm 0,05$ |
| 23200 | $0,73 \pm 0,05$ |
| 19800 | $0,82 \pm 0,05$ |
| I6800 | $0,92 \pm 0,06$ |
| 15100 | $0,96 \pm 0,06$ |
| I3IOO | I,03 $\pm 0,06$ |
| I1250 | $\mathrm{I}_{1} \mathrm{I} 2 \pm 0,07$ |
| 9800 | $1.24 \pm 0,07$ |
| 7900 | $1,45 \pm 0,09$ |
| 6400 | $\mathrm{I}, 68 \pm 0,10$ |
| 5800 | $\mathrm{I}_{\mathrm{n}} 92 \pm 0,12$ |
| 4900 | $2, \mathrm{II} \pm 0, \mathrm{I} 3$ |
| 4 COO | $2,35 \pm 0,14$ |
| 3600 | 2,56 $\pm 0,16$ |
| 3200 | $2,77 \pm 0,17$ |
| 2750 | 3,12 $\pm 0,19$ |
| 2400 | 3,42 $\pm 0,2 \mathrm{I}$ |
| 2200 | $3,64 \pm 0,22$ |
| 1920 | $3,98 \pm 0,24$ |
| 1780 | $4,34 \pm 0,26$ |
| 1550 | $4,93 \pm 0,30$ |
| I350 | 5,51 $\pm 0,34$ |
| II30 | $6,02 \pm 0,36$ |

Table 4
Numerical cross-sections for radiative capture of neutrons by ${ }^{232} \mathrm{Th}$ nuclei $\left(\sigma_{c}(E)\right)$

| $F$, eVr | $\sigma_{\mathrm{C}}$, barn |
| :---: | :---: |
| 34600 | $0,50 \pm 0,06$ |
| 23200 | $0,63 \pm 0,05$ |
| $I 7300$ | $0,73 \pm 0,05$ |
| $I 3100$ | $0,85 \pm 0,06$ |
| 9350 | $0,96 \pm 0,06$ |
| 7600 | $I, 00 \pm 0,06$ |
| 6200 | $I, 05 \pm 0,07$ |
| 5000 | $I, I 4 \pm 0,08$ |
| 4100 | $I, 23 \pm 0,09$ |
| 3300 | $I, 34 \pm 0, I 0$ |
| 2700 | $I, 66 \pm 0, I I$ |
| 2300 | $I, 82 \pm 0, I 3$ |
| $I 750$ | $I, 90 \pm 0, I 5$ |
| 1350 | $I 330$ |

## Table 5

Numerical crossmsections for radiative capture of neutrons by ${ }^{238} \mathrm{U}$ nuclei ( $\left.\sigma_{c}(E)\right)$
$\mathrm{E}, \mathrm{eV} \quad \sigma_{\mathrm{c}}$, barm

30500
25100
21000
17300
15400
13800
II700
10370
9200
7470
$6 I 60$
5170
4480
3820
3410
2990
2640
2350
2100
1720
1430
I200
IIIO
$0,4 I \pm 0,05$
$0,45 \pm 0,04$
$0,50 \pm 0,04$
$0,58 \pm 0,04$
$0,59 \pm 0,04$
$0,64 \pm 0,04$
$0,65 \pm 0,04$
$0,69 \pm 0,05$
$0,69 \pm 0,05$
$0,74 \pm 0,06$
$0,82 \pm 0,06$
$0,87 \pm 0,07$
$0,90 \pm 0,07$
$0,98 \pm 0,07$
I, $05 \pm 0,08$
I,II $\pm 0,08$
$I, I 4 \pm 0,08$
I, $20 \pm 0,09$
$1,23 \pm 0,09$
$I, 30 \pm 0,09$
I, $44 \pm 0, I I$
$\mathrm{I}, 64 \pm 0,13$
$1,78 \pm 0,14$


Fig. 1 Reduced normalizing factor versus $B_{n}$ for different samples (measurements with a proportional gamma counter).


Fig. 2 Energy dependence of the cross-section for radiative capture of neutrons by silver nuclej.

$$
\begin{gathered}
\begin{array}{c}
\vec{n}=4,3.10^{2 I} I_{a t / \mathrm{cm}_{2}^{2}}^{2} \\
\left.O \quad \bar{n}=2,2 . I 0^{2 I_{a t / c m^{2}}}\right\} \text { data of this investigation } \\
O-[I 6] ; \quad \square-[43] ; O-[44] ; \\
\Delta-[26]
\end{array}
\end{gathered}
$$



Fig. 3 Energy dependence of the crossmection for radiative capture of neutrons by gold-197 nuclei.

$$
\begin{aligned}
& \left.\begin{array}{l}
0=I, 8 \cdot 10^{2 I} \mathrm{I}_{\mathrm{aT}} / \mathrm{cm}_{2}^{2} \\
0=6,0 \cdot 10^{20} 0_{\mathrm{aT} / \mathrm{cm}^{2}}
\end{array}\right\} \text { data of this investigation }
\end{aligned}
$$



Fig. 4 Energy dependence of the cross-section for radiative capture of neutrons by thorium-232 nuclei.

$$
\left.\begin{array}{rl} 
& =9,6 . I O^{2 I} \mathrm{ar} / \mathrm{cm}^{2} \\
0 & =4,0 . I O^{2 I} \\
\mathrm{ar} / \mathrm{cm}^{2} \\
O & =I, 0 . I O^{2 I} \mathrm{ar} / \mathrm{cm}^{2}
\end{array}\right\} \text { this investigation }
$$


$x-[19]$. $5-[20]$.


Fig. 5. Energy dependence of the cross-section for radiative capture of neutrons by uranium-238 nuclei.

$$
\left.\begin{array}{rl}
0 & =7, I \cdot I 0^{2 I} \mathrm{ar} / \mathrm{cm}_{2}^{2} \\
0 & =3,9 . I 0^{2 I} \mathrm{aT} / \mathrm{cm}_{2}^{2} \\
0 & =I, 3 . I 0^{2 I} \mathrm{aT} / \mathrm{cm}^{2}
\end{array}\right\} \text { this investigation }
$$

$$
\begin{aligned}
-[13], \square-[12], & \Delta-[25], \\
\square-[27], 0-[28], & -[29], \cdots-[30] .
\end{aligned}, 0-[26],
$$

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RADIATIVE CAPTURE OF NEUTRONS BY ${ }^{238}$ U IN THE BR-5 REACTOR CORE SPECTRUM (URANIUM CARBIDE VARIANT)<br>V.I. Ivanov, V.A. Tolstikov

## Introduction

The cross-sections for radiative capture of neutrons in ${ }^{238} \mathrm{U}$ are very important for fast reactor calculations because radiative capture in ${ }^{238} \mathrm{~J}$ is one of the factors that determine the breeding of nuclear fuel in fast power reactors.

A number of authors have performed critical analyses of experimental data on $\sigma_{n, \gamma}$ for ${ }^{238} U$ over a wide range of neutron energies. Taking these analyses as a basis and using theoretical calculations in the low neutron energy range, it has been possible to obtain recommended curves of average radiative capture cross-sections for ${ }^{238} \mathrm{U}$ over the eV to 15 MeV energy range. The recommended data have been used to establish a system of multigroup constants for radiative capture in ${ }^{238} \mathrm{U}$ which may be used for reactor calculations.

In order to check and correct the systems of multigroup constants, measurements of $\sigma_{n, y}$ for ${ }^{238} \mathrm{U}_{\mathrm{U}}$ have been made in known (or at any rate comparatively well known) broad neutron spectra.

In this investigation measurements were performed in the core spectrum of the BR-5 reactor, fuelled with $90 \%$ enriched ${ }^{2} 35 \mathrm{U}$ in carbide form.

## Experimental set-up

For the experiment one of the fuel assemblies in the first complete row of the reactor core ( 4.6 cm from its vertical axis) was replaced by an experimental assembly in the form of a hexahedral cassette which, instead of fuel elements, supported a stainless steel can containing spectrometric samples of ${ }^{238} \mathrm{U}$. The samples covered the region from the lower boundary of the core to the steel reflector formed by the tops of the assemblies.

Fig. l shows schematically the position of the samples inside the reactor as well as some of its structural features.

Irradiation was carried out for about somewhat cver a day ( $1.008 \times 10^{5} \mathrm{sec}$ ) at nominal reactor power. The flux at the centre of the reactor was taken to be $(5.61 \pm 0.65) \times 10^{14} \mathrm{n} / \mathrm{cm}^{2} \mathrm{sec}^{*}$.

* See Appendix.

Calculations allowing for neutron flux distribution showed that sample No. 1 located 4.7 cm from the core centre received an integral flux of $5.48 \pm 0.63 \times 10^{15} \mathrm{n} / \mathrm{cm}^{2}$, and sample No. $2(\mathrm{r}=5.6 \mathrm{~cm})$ a flux of $5.41 \pm 0.62 \times 10^{15} \mathrm{n} / \mathrm{cm}^{2}$. After irradiation the experimental assembly was cut up by remote handing in the hot cell and the uranium samples were removed for measurement.

## Measuring procedure

The alpha activity of the unirradiated and the irradiated samples was investigated with a semiconductor alpha-spectrometer incorporating a surfacebarrier silicon detector with a working surface of about $2 \mathrm{~cm}^{2}$ and resolution of about 40 keV .

## Results of the measurements

After a few small experimental corrections, the radiative capture cross sections may be calculated from the formula

$$
\begin{equation*}
\sigma_{n, \gamma}=\frac{1}{(\varphi \cdot t)} \cdot \frac{T_{1 / 2}^{239} \mathrm{Pu}}{\mathrm{~T}_{1 / 2}^{238} \mathrm{U}} \cdot\left(\frac{\mathrm{~A}^{239} \mathrm{Pu}}{{ }_{A}^{238}{ }_{\mathrm{U}}}\right) \tag{1}
\end{equation*}
$$

where ${ }_{238}{ }_{1 / 2}^{239} \mathrm{Pu}$ is the half-life of ${ }^{239} \mathrm{Pu}$ equal to $2.44 \times 10^{4} \mathrm{yr}[1]$, and $\mathrm{T}_{1 / 2}^{238} \mathrm{U}$ is the half-life of ${ }^{238} \mathrm{U}$ equal to $4.51 \times 10^{5} \mathrm{yr}[1]$. These are mean values based on data in Ref. [1]; gt is the integrated neutron flux, which for sample No. 1 was $5.48 \pm 0.63 \times 10^{-19} \mathrm{n} / \mathrm{cm}^{2}$ and for sample No. 2 was $5.41 \pm 0.62 \times 10^{13} \mathrm{n} / \mathrm{cm}^{2}$; and $\mathrm{A}^{239} \mathrm{Pu} / \mathrm{A}^{238} \mathrm{U}$ is the experimentally determined ratio of the alpha activities for ${ }^{239} \mathrm{Pu}$ and ${ }^{238}{ }_{U}$, equal to $1.641 \pm 0.029$ for sample No. 1 and $1.649 \pm 0.031$ for sample No. 2 .

Substituting these constants in formula (1) we obtain $\sigma_{n, \gamma}^{238}=162 \pm 19 \mathrm{mb}$ for sample No. $1(r=4.7 \mathrm{~cm})$
${ }_{\mathrm{O}, \mathrm{Y}}^{238}=165 \pm 20 \mathrm{mb}$ for sample No. $2(\mathrm{r}=5.6 \mathrm{~cm})$

The error in the crossmsection is determined by (a) the uncertainty in the half-lives of ${ }^{239} \mathrm{Pu}$ and ${ }^{238}$, which does not exceed $\pm 0.5 \%$, (b) the error in the determination of the integrated flux, equal to $11.6 \%$ (see Appendix) and (c) the error in measurements of $A^{239} \mathrm{Pu} / \mathrm{A}^{238} \mathrm{U}$ - the ratio of the alpha activities of plutonium and uranium - which does not exceed $\pm 1.5 \%$.

The total error in the determination of $\sigma_{n, \gamma}$ for ${ }^{238}$ U is $\pm 12 \%$. Fig. 2 shows the distribution of the numbers of neutron captures $\left(\sigma_{n, \gamma} \varphi\right)_{z} /\left(\sigma_{n, \gamma}\right)_{0}$ over the core of the $B R-5$ reactor.

## Comparison of the experimental results with theory

Ref. [2] provides theoretical values of neutron spectra for various distances from the reactor centre.

Averaging the recommended group constants for $\sigma_{n, \gamma}$ of ${ }^{238} U[3]$ over the neutron spectrum, we obtain $136.3,138.4,136$ and 136.9 mb for $\mathrm{r}=0.9$, $3.2,4.6$ and 6.1 crn from the reactor centre.

As indicated above, the measurements gave values of 161 and 164 mb for distances of $r=4.7$ and 5.6 cm respectively. Thus the results of calculation and experiment differ by $15 \%$, although they agree within the error limits.

## APPENDIX

## Neutron flux in reactor centre

As a result of the appearance of new experimental data we have revised the multigroup constants for the fission cross-sections of ${ }^{235} \mathrm{U}$ and ${ }^{239} \mathrm{Pu}$ which were used previously $[2]$ as reference values in the experimental determination of the neutron flux in the reactor centre.

The experiment to determine the neutron flux in the reactor centre was based on an average fission cross-section for ${ }^{239} \mathrm{Pu}$ of 1.79 b . This value was obtained by averaging the 26 group constants for the ${ }^{239}$ Pu fission crosssection $[4]$ over the theoretical neutron spectrum in the reactor centre $[2]$.

The revision of the constants resultod in a value for the average ${ }^{239} \mathrm{Pu}$ fission crossmection in the reactor centre of 1.67 b . Allowing for this, and using the experimental data of Ref. [2], we found $\varphi_{\text {max }}=5.61 \pm 0.65$ $x 10^{14} \mathrm{n} / \mathrm{cm}^{2} \mathrm{sec}$ for the flux in the core centre.

The authors wish to thank all the staff of the BF-5 reactor for their help in irradiating the experimental assembly, Mr. A.I. Gentosh for preparing the ${ }^{238}{ }_{U}$ samples and N.A. Bulanova and E.N.. Zhukova for their help in preparing the paper.

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Fig. 1 Schematic arrangement of ${ }^{238} U$ samples in the $B R-5$ reactor and some design features of the reactor
$1=$ Top cover - biological shield; $2=$ Steel reflector formed by tops of fuel assemblies; $3=$ Part of fixed nickel reflector; $4=$ Fixed nickel reflector; $5=$ Space inside fuel assembly; $6=$ Steel end reflector; 7 =Core; $8=$ Experimental assembly; $9={ }^{238} \mathrm{U}$ samples; $10=$ Movable compensating cylinder; $11=$ Shield compensator; $12=$ Part of bottom reflector with the shafts of the fuel assemblies; $133=$ Central bypass channel; $14=$ Bottom reflector; $15=0 K-50$ channel. (irradiation channel) at $\mathbf{r}=430 \mathrm{~mm}$ from centre of reactor; $16=$ Central pipe with sodium coolant; $17=$ Space for dropping control elements; $18=$ Water tank of biological shield.


Fig. 2 Capture distribution in ${ }^{238} \mathrm{U}$ along the vertical at a distance $r=4.6 \mathrm{~cm}$ from axis of $\mathrm{BR}-5$ reactor (relative trend)

## MEAN RADIATION WIDTHS OF NEUTRON RESONANCES

S.M. Zakharova

This contribution is a supplement to Ref. [ $\mathrm{Z}-70]$ and contains (see Table l) experimental data on the radiation widths of neutron resonances published in the period from January to September 1970. The averaging was performed on the same assumptions as were made in Ref. $[\mathrm{Z}-70]$, i.e. if the discrepancy between different published results does not go beyond the limits of the indicated experimental errors, then

$$
\begin{equation*}
\overline{\Gamma_{\gamma}}=\frac{\sum_{i=1}^{n} \Gamma_{\gamma i} \cdot \frac{1}{\left(\Delta \Gamma_{r_{i}}\right)^{2}}}{\sum_{i=1}^{n} \frac{1}{\left(\Delta \Gamma_{r_{i}}\right)^{2}}} \quad \text { and } \quad \overline{\Delta \Gamma_{\gamma}}=\frac{1}{\sqrt{\sum_{i=1}^{n} \frac{1}{\left(\Delta \Gamma_{\xi_{i}}\right)^{2}}}} \tag{1}
\end{equation*}
$$

whereas in all other cases

$$
\begin{equation*}
\overline{\Gamma_{r}}=\frac{\sum_{i=1}^{n} \Gamma_{r_{i}}}{n} \quad \text { and } \quad \overline{\Delta \Gamma_{r}}=\sqrt{\frac{\sum_{i=1}^{p}\left(\bar{\Gamma}_{r}-\Gamma_{r_{i}}\right)^{2}}{n-i}} \tag{2}
\end{equation*}
$$

where $n$ is the number of references if we have the mean radiation width of a given resonance, or the number of resonances if we have the mean radiation width of a given isotope. T'able l contains only those resonances for which new data have appeared. The mean radiation widths $\Gamma_{\gamma}{ }_{\gamma}$ os of the remaining resonances used for obtaining the mean radiation widths of the isotopes (see table column headed Remarks) may be found in Ref. [. Z-70]. The notation used in Table 1 is the same as in Ref. [2-70]. The mean radiation widths obtained with formula (2) are indicated with an asterisk.

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Note: For remaining references contained in Table l consult Ref. [2-70].

Table 1
Key:

| изотоп | $=$ Isotope |
| :--- | :--- |
| MB | $=\mathrm{mV}$ |
| эв | $=\mathrm{eV}$ |
| рез | $=$ res |
| Работя | $=$ Ref. |
| Примечания | $=$ Remarks |

При усреднении
учтены резонансы = Averaging with resonances
КЭВ
$=\mathrm{keV}$
Mэв $\quad=\mathrm{MeV}$
метод площади = Area method
метод 中ормы = Shape method
щ-66 = ShCh-66
принято, что = Assuming
OT $=$ of
Продолжение $=$ (continued)
без учита = omitting
При усреднении =This resonance omitted from averaging этот резонанс не учт劳H

Table 1




Table 1 continued


Table 1 continued


Table 1 continued



Table 1 continued


GAMMA RAYS FROM RADIOACTIVE CAPTURE OF FAST NEUTRONS IN Fe , Ni AND Cu<br>A.T. Bakov, O.A. Shcherbakov

The radiative capture of neutrons is one of the main processes involved in the interaction of neutrons with nuclei in the reactor spectrum region. For estimating the energy release in core structural materials, calculating the composition and size of the shielding and so on, it is often necessary to have a detailed knowledge of the capture gamma ray spectrum and its dependence on neutron energy. At present there exist "group spectra" of gamma rays from the radiative capture of thermal neutrons $[1]$, which are usually used for such calculations. But, as was pointed out long ago and has been stated again more recently in a number of publications [2, 3], the gamma-ray spectra associated with radiative capture depend on the energy of the absorbed neutrons.

In this study we measured the energy spectra of gamma rays from radiative capture of fast neutrons by $\mathrm{Fe}, \mathrm{Ni}$ and Cu nuclei. For each element samples of the natural isotopic mixture were used. The neutrons were produced by the $T(p, n)^{3}$ He reaction in a Van de Graaff accelerator. The measurements were performed in annular geometry at kinetic incident neutron energies in the ranges $150 \mathrm{~m} 370,360-560$ and $580-840 \mathrm{keV}$.

The gammamray detector was a single-crystal scintillation spectrometer with stilbene crystal ( $\phi=50, \mathrm{~h}=45 \mathrm{~mm}$ ). The measured pulse distributions were processed by the smoothing differentiation method. After averaging the results of several measurements and subtracting the background, the gammaray energy spectra were corrected with the aid of a 90 th order correction matrix (energy step 100 keV ) and corrections were made for self-absorption of gamma rays in the sample.

Table 1 shows the gammarray yields per 100 captures in $500-\mathrm{keV}$ wide energy intervals, calculated by the formula

$$
Y_{\Delta E}=100 \frac{\int_{E_{i}}^{D_{i+l}} \varphi\left(E_{\gamma}\right) \mathrm{dE}_{\gamma}}{\int_{0} \varphi\left(\mathrm{E}_{\gamma}\right) \mathrm{E}_{\gamma} d E_{\gamma}}\left(\overline{\mathrm{B}}_{n}+\bar{E}_{n}\right)
$$

where $\Delta E=E_{i+l}-E_{i}, \varphi\left(E_{\gamma}\right)$ is the capture gamma-ray energy spectrum, $\overline{\mathrm{B}}_{\mathrm{n}}$ is the mean neutron binding energy and $\overline{\mathrm{E}}_{\mathrm{n}}$ is the mean neutron kinetic energy.

In calculating $\bar{B}_{n}$ we assumed that the contribution of the individual isotopes to the mean binding energy would be proportional to their fraction in the natural isotopic composition of the element. The last two columns of Table 1 show the number of gamma quanta per 100 captures and the energy spread of captured neutrons due to the thickness of the target and the geometry of the experiment,

For the "smooth" kind of spectra shown in Table 1 an important factor is the error associated with differentiation of the pulse distributions. The error is below $15 \%$ except for those spectra marked with an asterisk, where it is 25-30\%.

For comparison Table 2 presents data on capture gamma-ray yields from Ref. [3]. Comparison of the gamma-ray spectra griven in Tables 1 and 2 for the various neutron energies shows satisfactory agreement when allowance is made for measuring errors.

The authors wish to thank Yu. A. Kazansky for his constant interest in the work and the valuable adrice he rendered during; the discussion of the results.

Table 1 and Table 2

Key:

| Элемент | $=$ Element |
| :--- | :--- |
| Мэв | $=\mathrm{MeV}$ |
| кэв | $=\mathrm{keV}$ |


| Рле- | 1,0 1,5 <br> 1,5 2,0 | 2.0 <br> 12.5 | $\left\lvert\, \begin{aligned} & 2,5 \\ & 3,0\end{aligned}\right.$ | 3,0 3,5 | 3,5 4,0 | [ ${ }_{\text {E }}$ | $\begin{aligned} & 5,0 \\ & 5,5 \\ & \hline \end{aligned}$ | 5,5 6,0 | 6,0 <br> 6,5 | 6,5 7,0 | 7,0 7,5 | 7,5 <br> 8,0 | $\begin{array}{\|l} 8,0 \\ 8,5 \end{array}$ | $\left[\begin{array}{l} 8,5 \\ 9,0 \end{array}\right.$ | 19,0 9,5 <br> 9,5 10,0 |  | $E_{n}$ $\mathrm{K} 9 \mathrm{~B}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Cu | 13 13,5 | 14,6 | 15,7 | 16,0 | 14,8 | 15,4 12,8 | 12,5 | 10,4 | 10,9 | 10,7 | 9,6 | 8,0 | I, 4 | 0 | 00 | 187 | 150+370 |
|  | 13,0 13,6 | 14,5 | 15,8 | I6,I | 15,0 | 15,2 13,8 | 12,5 | 9,9 | 9,9 | 10.5 | 9,3 | 7,7 | 4,2 | I, 0 | 00 | 189 | 360+560 |
|  | 17,4 16,2 | 16,7 | 18,8 | 17,0 | I5,0 | I5, 1 14,6 | 12,5 | II,0 | 9.4 | 10,5 | 8,9 | 7,7 | 4.5 | 1,3 | 00 | 213 | 580+840 |
| $F e$ | I7, I I6, 7 | [3,8 |  |  | II, 0 | 10,2 [2, 1 | 9,7 | 8.7 | II, 7 | 4.0 |  |  |  |  |  | 183 | 150+370 |
|  | 20,0 16,8 | 14,6 | 13,2 | II. 5 | I2,I | [2,0 9, 8 | If.0 | 9.1 | 7.9 | 5,7 | 9,2 | 15,4 | 9.4 | 2,6 | $0,7 \quad 0,8$ | 187 | $360+560$ |
|  | 22,213,6 | 15,0 | 12,3 | 9,4 | 9,2 | 12,79,4 | II, 9 | 7.7 | 10,6 | 3,0 | 13,7 | 9,2 | 12,2 | 3.9 | I, $2 \quad 0,8$ | I89 | $580+840$ |
| $N$ | I9,8 15, I | 13.5 | 10,4 | 10, I | 9,1 | 7,6 7,6 | 9,3 | 10,0 | 8.8 | 6,7 | 9,6 | 8,2 | 13,5 | 14,2 | 5,2 0,3 | 182 | 150+370 |
|  | 20,4 18,7 | [4,3 | 12,4 | 10,6 | II, 0 | $8,3 \quad 8,7$ | 9, I | 10, 2 | 8,I | 7.6 | 7.4 | 8,6 | II, I | 13.0 | 7,2 1,3 | I92 | $360+560$ |
|  | 28.7 | 19,2 |  | 18,6 |  | 15,8 | I6,8 |  | 23,4 |  | I5,4 |  | 24.7 |  | 13,0 | 178 | 580+840 |

## TABLE 2



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ANGULAR DISTRIBUTIONS OF PHOTONEUTRONS IN THE INTERACTION OF 23-MeV ELECTRONS WITH COPPER, TUNGSTEN AND LEAD TARGETS
V.P. Kovalev, V.P. Kharin, V.V. Gordeev

Using the LUE-25 linear electron accelerator and targets of copper, tungsten and lead, measurements were made of the angular distributions of photoneutrons in relation to target thickness and diameter.

The experiments were performed with a straight $23-\mathrm{MeV}$ electron beam directed at the centre of the target. The targets were placed 20 cm from the output window of the accelerator. The threshold reaction ${ }^{31} P(n, p){ }^{31}$ Si was used for detecting neutrons. The experimental set-up was similar to that used in Ref. [1].

The angular distributions of photoneutrons for lead targets are shown in Fig. 1. The thickness of the lead targets varied from 11 to 53 mm and the diameter from 30 to 80 mm . Statistical measuring errors were less than $3 \%$. The forward shift of the angular distributions is due to inelastic loss of neutron energy with increasing target thickness [2]. Similar results were obtained for the copper and tungsten targets. The table shows the results for copper and tungsten targets of various dimensions. The data were normalized at $\varphi=90^{\circ}$.

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Fig. 1 Angular distributions of photoneutrons for a lead target as a function of diameter ( $\phi-\mathrm{mm}$ ) and thickness ( $\mathrm{h}-\mathrm{mm}$ )

Table 1

Title: Angular distributions of photoneutrons in relation to diameter and thickness of copper and tungsten targets. Primary electron energy 23 MeV .

| Кеу: | $=$ Target material |
| :--- | :--- |
| Матепиал мишени | $=$ Target diameter $\phi(\mathrm{mm})$ |
| Диаметр мишени | $=$ Thickness $\mathrm{h}(\mathrm{mm})$ |
| Толщина | $=$ Angle |
| Угол | $=$ Tungsten |
| Вольфрам | $=$ Copper |
| Медь |  |


| Yorepuos <br>  | (ヵ) | \% ${ }^{20 \%}$ | $30^{\circ}$ | $50^{\circ}$ | $70^{\circ}$ | $90^{\circ}$ | 110 ${ }^{\circ}$ | $130^{\circ}$ | 150' |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & 1 \\ & 0 \\ & 8 \\ & 8 \\ & 8 \\ & 0 \\ & 0 \end{aligned}$ | 40 | 33 | 1,23 | 1.32 | 11.28 | 1.00 | 129 | 1.30 | 1.25 |
|  | - | 6.64 | 1.20 | 11.20 | 17 | 11.00 | \% 19 | 120 | 1.12 |
|  | - | 10 | 1.07 | 11,19 | 1,44 | 1.00 | ¢13 | 1,17 | 1.06 |
|  | $\rightarrow$ | 135 | 099 | 101 | 0.9 | 1,00 | 1.00 | 1,00 | 0,97 |
|  | -m | 24 | 073 | 0.79 | 0.94 | 1.00 | 1.00 | 0.90 | 0.92 |
| $\begin{aligned} & 0 \\ & 0 \\ & 2 \end{aligned}$ | 40 | 15 | 1.16 | 11.08 | 1104 | 1.00 | 1.01 | 1.06 | 1,17 |
|  | $\rightarrow$ | 30 | 0.90 | 0.91 | 0.98 | 1.00 | 1097 | 0.95 | 100 |
|  | - | 45 | 072 | 0.87 | 0.95 | 1.00 | 0.97 | 0.303 | 0.91 |
|  |  | 50 | 0.65 | 0,8: | 0.92 | 1,00 | 10.90 | 0.35 | 0.98 |
|  | $\pi$ | 75 | 0.59 | 0.73 | 0.93 | 11.00 | 1,05 | 0.9 .5 | 0.89 |
|  | $\cdots$ | 100 | 0.55 | 0.71 | 0.82 | 1,00 | 8.04 | 1,02 | 1.0 : |
|  | 50 | -- | 0.47 | 0.70 | 0.84 | 1.00 | 1.01 | 1,12 | 1.16 |
|  | 80 | $\cdots$ | 0.52 | 073 | 0.92 | 1,00 | 11.05 | 1,23 | 1.21 |
|  | 100 | $\rightarrow-$ | 051 | 062 | 0.87 | 1.00 | 1,00 | 1,26' | 1.50 |
|  | 135 | -4- | 0.69 | 070 | 0.82 | 1,00 | 1.08 | 1.66 | 1,80 |

PHOTONEJTRON YIELDS AS A FUNCTION OF THE DIAMETER AND THICKNESS OF COPPER, IUNGSTEN AND LEAD TARGETS
V.P. Kovalev, V.P. Kharin, V.V. Gordeev

The results of this study are an extension of investigations aimed at developing a pulsed photoneutron source with isotropic distribution on the basis of a linear electron accelerator [l].

To this end photoneutron yields were investigated experimentally as a function of the diameter and thickness of copper, lead and tungsten targets at an electron energy of 23 MeV .

The total photoneutron yield was measured from neutron stopping in a paraffin sphere 30 cm in diameter. A thin indium foil at the centre of the sphere served as slow neutron detector. The experiments were performed with a straight electron beam directed at the centre of the target. The targets were placed 20 cm away from the output window of the accelerator. The neutron detector was situated at an angle of $90^{\circ}$ to the direction of the incident electron beam.

The experimental results are given in the figures in the form of relative yields of photoneutrons from copper, tungsten and lead as a function of the thickness (Fig. 1) and the diameter (Fig. 2) of the targets.

The statistical measuring error was less than $2 \%$.

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Fig. 1 Neutron yields versus thickness of copper, tungsten and lead targets:
$X X X=C u$ target, diameter 40 mm
$000=W$ target, diameter 30 mm
पOD= Pb target, diameter 40 mm


Fig. 2 Neutron yields versus diameter of copper and lead. targets:
$\mathrm{XXX}=\mathrm{Cu}$ target 100 mm thick
$\square \square \square=\mathrm{Pb}$ target 48 mm thick

ABSOLUTE YIELDS OF THE REACTIONS ${ }^{12} \mathrm{C}(\gamma, \mathrm{n})$ AND ${ }^{16} \mathrm{O}(\gamma, \mathrm{n})$

V.P. Kovalev, S.P. Kapchigashev

The activation method was used to measure the yields of ( $\gamma, \mathrm{n}$ ) reactions from carbon and oxygen at maximum bremsstrahlung beam energies of 22 and 24 MeV . The bremsstrahlung source was a tungsten target 6 mm thick. The distance from the target to the samples was 25 cm . The current supplied to the target was $3-5 \mu \mathrm{~A}$. The samples used were thin Plexiglas $\left(\mathrm{C}_{5} \mathrm{H}_{8} \mathrm{O}_{2}\right)$ and distilled water. The irradiation time was 10 min . The induced activity of the samples was measured with a scintillation gamma spectrometer, in order to separate the annihilation gamma quanta with energy 0.51 MeV .

The data obtained were normalized to the known yield of the ${ }^{63} \mathrm{Cu}(\gamma, n){ }^{62} \mathrm{Cu}$ reaction [1].

The yield of the ( $\gamma, n$ ) reaction for the element under investigation, $\sigma_{x}(E \gamma m)$, was determined from the formula
where $n_{x}$ and $n_{C u}$ are the nuclear densities of the sample and of copper, respectively; $N_{x}(0)$ and $N_{C u}(0)$ are the counting rates at the moment of termination of irradiation; $t_{0}$ is the irradiation time, and $\varepsilon_{B}^{x}$ and $\varepsilon_{B}^{C u}$ are the respective output efficiencies of gamma quanta with energy 0.511 MeV resulting from $B+$ decay.

The results of the measurements together with the data of other authors are presented in Table 1.

From the table it can be seen that the yields of the ( $\gamma, n$ ) reactions from carbon at $E=22 \mathrm{MeV}$ and $\mathrm{E}=24 \mathrm{MeV}$ are in good agreement with the data of Price, Haslam and coworkers, and are 4.5 times lower than the values obtained by Montalbetti and co-workers. For the yield of the ( $\gamma, n$ ) reaction from oxygen our data are in satisfactory agreement with the data of other authors apart from Price and Thorson. Note the paucity of data on the yield of photoneutrons from ${ }^{12} \mathrm{C}$ and ${ }^{16} \mathrm{O}$ for bremsstrahlung with a maximum energy of 24 MeV .

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## Table 1

Title: Absolute yields of the reactions ${ }^{12} O(\gamma, n)^{11} C$ and ${ }^{16} O(\gamma, n)^{15} O$

| Key: |  |  |
| :---: | :---: | :---: |
|  | нейтроны | in neutrons |
| в единицах моль-рентген $=$ in mole-roentgen |  |  |
| Элементы = Elements |  |  |
| $\left\{\begin{array}{l} \text { Результаты данной }=\text { Present work } \\ \text { работы } \end{array}\right.$ |  |  |
| Прайс = Price (1950) |  |  |
| Хаслам и др./1951/= Haslam et al. (1951) |  |  |
| Монталбетти и др. = Montalbetti et al. (1953) |  |  |
| Натанс и Халперн = Nathans and Halperm (1954) |  |  |
| Барбер и др. = Barber et al. (1955) |  |  |
| Кук $\quad=$ Cook (1957) |  |  |
| Topcoh $\quad=$ Thorson (1957) |  |  |
| Хаслам и др. = Haslam et al. (1961) |  |  |
| Литература = Literature |  |  |
|  | См. ссылкуу | = See Ref. |

## TABLE 1

## 

не与трони
в единицях ноль-рөнтген

| Элешенты | $c^{2}$ |  |  | $0^{16}$ |  |  | Литературв |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 22 | 23 | 24 | 22 | 23 | 24 |  |
| Результаты <br> денно月 <br> работы | $0.6 .10^{4}$ | - | $4.5 .10^{4}$ | $1.72 .10^{4}$ | - | I. $10^{5}$ | См. ссилжу |
| преис (1950) | 0,67.10 ${ }^{4}$ | - | - | $0,67.10^{4}$ | - | - | I |
|  | $0,75.10^{4}$ | - | 4.35.10 ${ }^{4}$ | $1.68 .10^{4}$ | - | 4,53.10 ${ }^{4}$ | -1' |
| Монталбетти <br> - pp .(1953) | $2,7.10^{4}$ | - | - | $3,2.10^{4}$ | - | - | - |
|  | ) | - | - | 3,2.10 ${ }^{4}$ | - | - | -' |
| Барбер др.(I955) | - | - | - | $2,3.10^{4}$ | - | - | -n- |
| Kys (1957) | - | - | - | I.5.10 ${ }^{4}$ | - | - | - |
| Tорсон (1957) | ) - | - | - | $0,76.10^{4}$ | - | - | -" |
| $\begin{aligned} & \text { Xбслаи } \\ & \text { גр. } 196 \mathrm{I}) \end{aligned}$ | 1.15.10 ${ }^{4}$ | 2,7 |  | $2,37.10^{4}$ | - | - | -"- |

REACTIONS ITVOLVING LIGIIT NUCLEI AND CAUSED BY CHARGED
PARTICLES ARISING FROP 14.6 MeV inEUTRON INTERACTIONS

B.L. Lebedev, F. Nasyrov

Calculations and measurements were performed to establish the yields and cross-sections of the ${ }^{11} \mathrm{~B}(\mathrm{p}, \mathrm{n})^{11} \mathrm{C},{ }^{10_{B}} \mathrm{~B}(\mathrm{p}, \alpha)^{7} \mathrm{Be},{ }^{16} \mathrm{O}(\mathrm{p}, \alpha)^{13_{\mathrm{N}}},{ }^{18} \mathrm{O}(\mathrm{p}, \mathrm{n})^{18} \mathrm{~F}$, ${ }^{13} \mathrm{C}(\mathrm{p}, \mathrm{n})^{13}{ }_{\mathrm{N}}, \quad{ }^{12} \mathrm{C}(\mathrm{d}, \mathrm{n})^{13} \mathrm{~N}$ and ${ }^{7} \mathrm{Li}(\mathrm{p}, \mathrm{n})^{7} \mathrm{Be}$ reactions in $\mathrm{H}_{3} \mathrm{BO}_{3}, \mathrm{H}_{3}{ }^{1 \mathrm{O}_{\mathrm{BO}}}$, $\left(\mathrm{C}_{2} \mathrm{H}_{4}\right)_{n},\left(\mathrm{C}_{2} \mathrm{D}_{4}\right)_{n}, \mathrm{LiOH}$ and $(\mathrm{COOH})_{2} \cdot 2 \mathrm{H}_{2} \mathrm{O}$ induced by recoil protons and deuterons produced in the elastic scattering of 14.6 NeV neutrons. Experiments were performed to determine the yields of the ${ }^{1 \epsilon_{1}} O(t, n)^{18}{ }_{F}$ reaction in $\mathrm{H}_{3} \mathrm{BO}_{3}, \mathrm{H}_{3}{ }^{1 \mathrm{O}_{\mathrm{BO}}}$ and LiOH induced by tritons resulting from the interaction of neutrons with boron and lithium nuclei.

The irradiation of various materials with neutrons induces various nuclear reactions, including reactions that give rise to charged particles such as protons, deuterons, tritons, alpha particles, etc. The charged particles in turn cause secondary reactions in various elements. These secondary reactions are thus a source of varying degrees of radioactivity in reactor structural materials and neutron shield materials. For example ${ }^{13} \mathrm{~N}$ and ${ }^{18} \mathrm{~F}$ are formed in reactor cooling water through the reactions ${ }^{16} O(p, n)^{13} \mathrm{~N}$ and ${ }^{18} O(p, n)^{18} \mathrm{~F}[1-3]$, and $7^{7 e}$ is formed in boron shielding through the ${ }^{10} B(p, \alpha)^{7}$ Be reaction. Moreover, reactions induced by secondary charged particles are used to obtain radioactive isotopes in reactors [4-8]. Naude and Peisach [9] measured the crossmsections of a number of reactions in boron, carbon, nitrogen and oxygen, caused by recoil protons and deuterons arising from the interactions of 14.5 MeV neutrons.

Here we have calculated and measured the yields and the crossmsections of reactions induced by secondary charged particles in certain reactor and biological shield materials, following irradiation with $14.6-\mathrm{MeV}$ neutrons; the materials in question were polyethylene - $\left(\mathrm{C}_{2} \mathrm{H}_{4}\right)_{n}$, deutero-polyethylene $\left(C_{2} D_{4}\right)_{n}$, boric acid in the natural isotopic mixture $-\mathrm{H}_{3} \mathrm{BO}_{3}$, boric acid $85 \%$ enriched in ${ }^{10} \mathrm{~B}_{-\mathrm{H}_{3}}{ }^{1 \mathrm{O}_{\mathrm{BO}}}{ }_{3}$, lithium hydroxide -LiOH and oxalic acid ( COOH$)_{2} \cdot 2 \mathrm{H}_{2} \mathrm{O}$.

## Calculation of the yields and cross-sections <br> of secondary reactions

The number of reactions in one gram of a substance due to secondary charged particles arising from the interaction of monoenergetic neutrons with that substance is given by

$$
\begin{equation*}
Q=n_{i} n \sigma_{n^{N}}^{N} \cdot \int_{0}^{E_{\max }} \varphi(E) d E \quad E \quad \int_{0} \frac{\sigma\left(E^{\prime}\right)}{\left|\frac{d E^{\prime}}{d \xi}\right|} d E^{\prime} \tag{1}
\end{equation*}
$$

where $n$ is the number of nuclei of the element in which fast charged particles are formed by interaction with neutrons, $n_{i}$ is the number of nuclei of the element in which reactions are induced by secondary charged particles, $\sigma_{n}$ is the cross-section for the formation of charged particles, $N$ is the neutron flux per unit area, $\varphi(E)$ is the secondary charged particle spectrum, $\sigma(E)$ is the cross-section for the reaction induced by the charged particles, and $\frac{d E}{d \xi}$ is the energy lost by a charged particle along its track, in $\mathrm{MeV} / \mathrm{g} / \mathrm{cm}^{2}$. Changing the order of integration in Eq. (1), we find

$$
Q=n_{i} n \sigma_{n} N \cdot \int_{0}^{E_{\max }} \frac{\sigma(E) P(E)}{\left|\frac{d E}{d \xi}\right|} d E,
$$

where

$$
P(E)=\int_{E}^{E_{\max }} \varphi(\mathbb{E}) \mathrm{dE}
$$

is the probability that a charged particle will be formed with energy greater than E, i.e. the integral secondary charged particle spectrum. From Eq. (2) it follows that the mean cross-section for secondary charged particles is

$$
\begin{equation*}
\bar{\sigma}=\frac{Q}{n_{i} n \sigma_{n} N \cdot \int_{0}^{\frac{E_{\max }}{} \frac{P(E)}{\left|\frac{d E}{d \xi}\right|} d E}} \tag{4}
\end{equation*}
$$

Here we have calculated the yields and cross-sections of a number of secondary reactions induced by recoil protons and deuterons resulting from the irradiation of the substances under study with $14.6-\mathrm{MeV}$ neutrons. The differential and integral recoil proton spectra were calculated on the assumption of isotropic scattering of neutrons by protons, and the recoil deuteron spectra were calculated from the differential cross-sections for neutron scattering by deuterons supplied in Ref. [10].

The integral recoil proton and deuteron spectra are shown in Fig. l. The cross-sections for neutron scattering by protons and deuterons were taken to be $0.67 \mathrm{~b}[11]$ and $0.61 \mathrm{~b}[10]$ respectively. The slowing-down capacities of the investigated substances for recoil protons and deuterons were calculated from the proton and deuteron ranges in different elements [12, 13]. The yields and the cross-sections of secondary reactions induced by protons and deuterons, calculated according to formulae (1) and (4), are shown in Table l. Only recoil protons and deuterons produced by elastic neutron scattering were taken into account, since the number produced by other mechanisms is negligible in the substances with which we are concerned here. The last column of the table shows the sources of the excitation functions for charged particle reactions used in the calculations.

## Measurement of the yields and cross-sections of secondary reactions

The measurements were performed by the activation method. We studied
 ${ }^{13} C(p, n)^{13} N_{N}$ and ${ }^{12} C(d, n)^{13}{ }_{N}$ induced by recoil protons and deuterons in materials irradiated with neutrons of energy $14.6 \pm 0.2 \mathrm{MeV}$. In addition, we were able to determine the yjelds of the ${ }^{16} \mathrm{O}(\mathrm{t}, \mathrm{n})^{l / 8} \mathrm{~F}$ reaction in $\mathrm{H}_{3} \mathrm{BO}_{3}$, $\mathrm{H}_{3}{ }^{10} \mathrm{BO}_{3}$ and LiOH induced by tritons from neutron interactions with ${ }^{10} 0_{\mathrm{B}}$, ${ }^{1 l_{B}}$ and ${ }^{7}$ Li nuclei.

Neutrons were obtained in the $T(d, n)^{4}$ He reaction with a low voltage generator which accelerates deuterons to an energy of 120 keV . The samples were in the form of tablets 25 mm in diameter and $5-10 \mathrm{~mm}$ thick. For irradiation they were set at $0^{\circ}$ to the deuteron beam at a distance of 15 mm from the target.

After irradiation the activity induced in the specimens was recorded by a singlemrystal scintillation spectrometer with a NaI(Tl) crystal $80 \times 80 \mathrm{~mm}$ in diameter and an AI-256 multichannel analyser. The yield of the reactions producing ${ }^{13_{N}},{ }^{11} C$ and ${ }^{18} F$ was determined from the counting rate of pulses in the photopeak from 0.511 MeV gamma cuanta accompanying the decay of these nuclei, and the yield of the ${ }^{10_{B}}(p, \alpha){ }^{7} \mathrm{Be}$ and ${ }^{7} \mathrm{Li}(p, n){ }^{7} \mathrm{Be}$ reactions was determined from the counting rate of pulses from 0.480 MeV gamma quanta accompanying the decay of ${ }^{\prime} 7_{\mathrm{Be}}$.

The absolute yields of the secondary reactions were determined by comparison with the yield of a reaction having a well-known cross-section,
viz. the ${ }^{65} \mathrm{Cu}(\mathrm{n}, 2 \mathrm{n})^{64} \mathrm{Cu}$ reaction: the cross-section for this was taken as 970 mb [11]. The data on gamma quanta yield per disintegration, gamma quanta energy and half-lives were taken from Ref. [14].

Table $I$ shows the measured secondary reaction yields compared with the theoretical values. It can be seen that there is good agreement. An exception is the formation of 18 F in $\mathrm{H}_{3} \mathrm{BO}_{3}, \mathrm{H}_{3} \mathrm{IO}_{3}$ and LiOH, where the experimental yieids considerably exceed the theoretical. We believe this is due to the contribution of the ${ }^{16} O(t, n)^{18}$ F reaction with tritons produced in neutron interactions with ${ }^{11} B,{ }^{10} B$ and $7_{\text {Li }}$. The table also includes the experimental values of the average cross-sections for recoil protons and deuterons, determined from the experimental yields by means of Eq. (4). Some of these cross-sections are compared with the results of Naude and Peisach [9] and it can be seen that their values are higher than ours. This is due partially to the fact that we averaged over the range from 0 to $\mathbb{E}_{\max }$ whereas Naude and Peisach averaged from the threshold energy $E_{\text {thr }}$ to $E_{\max }$. The disagreement is also explained to some extent by the fact that they used different standard reactions, and the difference in neutron fluxes to the samples determined with these reactions was as much as $40 \%$. In view of this it can be considered that our results are in agreement with the results of Ref. [9].

From our results it will be seen that in certain materials containing boron and lithium which are not activated directly by neutrons it is possible for short-lived and indeed long-lived activity to arise through the formation of $7_{\mathrm{Be}}$ in reactions induced by secondary charged particles, and this must be taken into account in the design of biological shields.

Table 1
Yields and cross-sections of secondary reactions



Fig. 1 Integral spectra of recoil protons (1) and deuterons (2) produced in elastic scattering of 14.6 MeV neutrons

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# KINEMATIC ANALYSIS AND TABLES OF NEJTRON ENERGY AND ENERGY SPREAD VALUES FOR ( $\mathrm{p}, \mathrm{n}$ ) REACTIONS 

G.A. Borisov, R.D. Vasilev, V.F. Shevchenko

The ( $p, n$ ) reactions produced with electrostatic accelerators are widely used as sources of monoenergetic neutrons. To make practical use of such sources, it is necessary to know how neutron energy varies with the proton escape angle and proton energy and also to know the factors causing neutron energy spread. These questions are the subject of Refs [1-9], in which neutron energies and energy spreads were determined with kinematic equations. The results of the calculations are presented in the form of tables and graphs.

To assist with the calibration of different types of neutron detectors for monoenergetic neutrons and the measurement of cross-sections, the authors of this article have compiled energy tables for neutrons produced in the ${ }^{3} \mathrm{~T}(\mathrm{p}, \mathrm{n}){ }^{3} \mathrm{He},{ }^{7} \mathrm{Li}(\mathrm{p}, \mathrm{n}){ }^{7} \mathrm{Be},{ }^{45} \mathrm{Sc}(\mathrm{p}, \mathrm{n})^{45} \mathrm{Ti},{ }^{51} \mathrm{~V}(\mathrm{p}, \mathrm{n}){ }^{51} \mathrm{Cr}$ and ${ }^{65} \mathrm{Cu}(\mathrm{p}, \mathrm{n}){ }^{65} \mathrm{Zn}$ reactions. In addition, a quantitative assessment was made of the factors responsible for the neutron energy spread, principal among these being uncertainties in the proton energy values and the target thickness, as well as the Doppler effect due to thermal motion of the target atoms. The authors also estimated the effect on neutron energy of the uncertainty in the masses of the particles involved in the reaction and in the reaction energy $Q$. The results obtained differ from those published earlier in that they supply more detailed neutron energy data for the above reactions in the proton energy range from the corresponding thresholds to approximately 3 MeV together with additional information on how the Doppler effect influences the neutron energy spread at various target temperatures, as well as data from a comparison of neutron energies obtained using a relativistic equation, two non-relativistic approximations and one classical approximation. The calculations were performed using more accurately defined values for particle mass and $Q[10,11]$. In this abridged report only those results will be presented which are not reproduced in the literature or which differ from published data.

The neutron energies presented in Tables 2 and 3 were calculated with the relativistic equation (34) described in Ref. [5]. Apart from the data given in the tables, neutron energy values were calculated on the basis of two nonrelativistic and one classical approximation in accordance with formulae (38) and (55) also from Ref. [5]. Comparison of the results shows that the relativistic equation should be used for calculating the energies of neutrons
from the ${ }^{3} \mathrm{~T}(p, n)^{3} \mathrm{He}$ and ${ }^{7} \mathrm{Li}(p, n)^{7}$ Be reactions, because if the calculation is done with the non-relativistic or classical approximation there is a difference in neutron energy values of the order of $0.1 \%$ which can reach several per cent in the nearnthreshold proton energy range. This difference is negligible (about 100 eV ) for the ${ }^{45} \mathrm{Sc}(\mathrm{p}, \mathrm{n})^{45} \mathrm{Ti},{ }^{51} \mathrm{~V}(\mathrm{p}, \mathrm{n}){ }^{51} \mathrm{Cr}$ and ${ }^{65} \mathrm{Cu}(\mathrm{p}, \mathrm{n}){ }^{65} \mathrm{Zn}$ reactions.

A quantitative assessment of the factors responsible for the neutron energy spread was carried out by the method described in Ref. [5]. The results of evaluations for neutron escape angles of $0^{\circ}, 60^{\circ}$ and $120^{\circ}$ in the laboratory system of comordinates are presented in Table 4. The spread in neutron energies, $\Delta E_{1}$ and $\Delta E_{2}$, is due to the energy spread in the beam or target and to the Doppler effect, respectively. $\Delta E_{1}$ was determined from the relation $\Delta E_{1}=f(E+\Delta E)-f(E-\Delta E)$, where $E$ is proton energy and $\Delta E$ is the proton energy increment, taken to be $0.5 \mathrm{keV} . \Delta \mathrm{E}_{2}$ was calculated from the formula $\Delta E_{2}=2\left(E_{3 \text { max }}-E_{3}\right)[5]$, where $E_{3 \text { max }}$ is the maximum neutron energy allowing for the Doppler effect and $E_{3}$ is neutron energy without allowance for the Doppler effect. The values of $\Delta \mathrm{E}_{2}^{\prime}$ and $\Delta \mathrm{E}_{2}^{\prime \prime}$ given in Table 4 relate to target temperatures of $\sim 20$ and $\sim 200^{\circ} \mathrm{C}$ respectively. From Table 4 it is clear that the energy spread of protons in the beam or target has the most significant effect on the energy distribution of the neutrons.

The effect of the uncertainty in the values of 2 , conditionally taken to be 1 keV and leading to the uncertainty in the neatron energy values $\Delta E_{3}$, was estimated from the formulae supplied in Ref. [6.]. The resulting data are listed in Table 4 and can be used for correcting the neutron energy values in Tables 1,2 and 3 in the event of more accurate values of $Q$ becoming available. The uncertainty in neutron energy resulting from the uncertainty in the masses is about $0.01 \%$.

Table 1

Title: Neutron energy $T_{n}$ (keV) for angle $\vartheta$ (degrees) and proton energy $T_{p}$ (keV) in the laboratory system of comordinates

Key:
Maв
$=\mathrm{MeV}$
к3 B
$=\mathrm{keV}$
Table 2
Title: As above.
Key: As above.

Table 3

Title: As above.
Key: As above.

Table 4

No caption.
 системе координат

| $\theta$ | TP II47, ${ }^{4}$ | II50 | II55 | II60 | II65 | 1170 | II75 | II80 | IIS5 | 1190 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 288,19 | 291,45 | 297,65 | 303,8I | 309,94 | 316, 0 | 322,09 | 328, II | 334,II | 340,08 |
| 10 | 279,50 | 282,74 | 288,90 | 295,02 | 301, II | 307.15 | 313,17 | 319,15 | 325,10 | 331,03 |
| 20 | 254, 47 | 257,66 | 263,70 | 269,7I | 275,67 | 201.60 | 287.49 | 293,34 | 299,16 | 304,95 |
| 30 | 216,13 | 219,23 | 225,10 | 230,92 | 236,70 | 242,43 | 248,12 | 253,77 | 259,39 | 264,97 |
| 40 | I69,II | 172, $\mathrm{C9}$ | 177,74 | 183,33 | 188,87 | 194,34 | I99,77 | 205,16 | 210,50 | 215,80 |
| 50 | II9,06 | I2I,93 | 127,34 | 132,66 | 137,90 | I43.08 | I48,19 | 153,25 | I58,26 | I63,23 |
| 60 | 72,04 | 74,79 | 79,93 | 84,95 | 89,86 | 94.67 | 99,4I | 104,08 | 108,68 | II3,23 |
| 70 | 33,71 | 36,35 | 4I, I8 | 45,80 | 50,26 | 54,60 | 58,84 | 63,00 | 67,09 | 7I, II |
| 80 | 8,69 | II, I6 | I5,4I | 19,32 | 23,04 | 20,64 | 30,15 | 33,59 | 36,97 | 40,3I |
| 90 |  | I, 30 | 3,80 | 6,29 | 8,78 | II, 28 | 13,77 | I6,27 | 18,76 | 2I,25 |
| IOO |  | 0.15 | 0,94 | 2,05 | 3,35 | 4,77 | 6,29 | 7,88 | 9,52 | II, 20 |
| 110 |  | 0,05 | 0,35 | 0,86 | I,54 | 2,33 | 3,22 | 4,20 | 5,25 | 6,35 |
| 120 |  | 0,02 | 0,18 | 0,47 | 0,36 | I, 34 | I,9I | 2,54 | 3,24 | 3,99 |
| 130 |  | O, OI | 0,II | 0,30 | 0,56 | 0,89 | I,28 | I. 73 | 2,22 | 2,77 |
| 140 |  | 0,01 | 0,03 | 0,22 | 0.4 I . | 0,65 | 0,95 | I,29 | I. 67 | 2,09 |
| 150 |  | 0,OI | 0,06 | 0,17 | 0,33 | 0,52 | 0,76 | I, 04 | I. 36 | I,70 |
| 160 |  | 0.01 | 0,05 | 0,15 | 0,28 | 0,45 | 0,66 | 0,90 | I, I8 | I,48 |
| I70 |  | 0,OI | 0,05 | 0,13 | 0,26 | 0,4I | 0,6I | 0,83 | I, 08 | I, 36 |
| I80 |  | 0,OI | 0,05 | 0,13 | 0,25 | 0,40 | 0,59 | 0,81 | I, 05 | I,33 |

 $Q=-0,76384$ лэв.

TABLE 2
 системе гоорданат

|  | 1920 | 1922 | 1925 | 11930 | 1935 | 194 C | 1945 | 11050 | 1355 | 1960 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | I21,30 | I24,4I | 129.00 | 136. 50 | 143,34 | I5I,03 | 158,10 | 165,00 | 172,93 | 176, 1 I |
| 10 | II7,64 | 120,75 | 125,33 | 132,81 | 140,13 | 147,30 | 154,34 | 161,28 | 168, IT | 174.50 |
| 20 | 107,II | IIO,2i | 114,76 | 122,19 | 129,44 | 136,54 | 143,50 | 150,35 | 157.20 | 153,76 |
| 30 | 90,97 | 94,05 | 98,56 | 105,90 | 113,04 | 120, CI | 126,83 | I33,54 | If0,it | 145,65 |
| 40 | 7I, 18 | 74,23 | 78,68 | 85,88 | 92,34 | 99,62 | IC0, 25 | IL2,74 | Iis, 12 | 125,41 |
| 50 | 50,12 | 53,13 | 57,50 | 64,49 | 71,20 | 77.70 | 84,04 | 90,23 | 80,30 | 102.29 |
| 60 | 30,32 | 33,29 | 37,52 | 44,19 | 50,53 | 56,63 | 62,55 | 68,34 | 74, 01 | 79,58 |
| 70 | 14,19 | 17,08 | 2I,05 | 27,18 | 32,93 | 38,46 | 43,81 | 49, 05 | 54,18 | 59.24 |
| 80 | 3,66 | 6,31 | 9,72 | 14,89 | I9,77 | 24,49 | 29,10 | 33,64 | 38,12 | 42,55 |
| 90 |  | 1,50 | 3,75 | 7.49 | II, 23 | 14, 98 | 18,72 | 22,45 | 26,20 | 20,95 |
| 100 |  | 0,36 | 1,45 | 3, 7 ? | 6,38 | 9,16 | 12,04 | 15,00 | I8,0i | 21, $\times$ |
| IIO |  | 0,13 | 0,67 | 2,06 | 3,83 | 5,83 | 8,00 | 10,29 | 12,67 | 15.14 |
| 120 |  | 0,07 | 0,37 | 1. 27 | 2,50 | 3,90 | 5,60 | 7,39 | 9,28 | II. ${ }^{\text {n }}$ |
| 130 |  | 0,04 | 0,24 | 0.07 | I,77 | 2,89 | 4,17 | 5,59 | 7,13 | 8.77 |
| 140 |  | 0,03 | 0,18 | 0,65 | I, 36 | 2,25 | 3,30 | 4,47 | 5.76 | 7.15 |
| 120 |  | 0,02 | 0,14 | 0,53 | I, I2 | I, 87 | 2,76 | 3,78 | 4,00 | 6,12 |
| 168 |  | 0,02 | 0,12 | 0,40 | 0,97 | I,64 | 2,44 | 3,30 | 4,37 | 5,48 |
| 170 |  | 0,02 | O,II | 0.40 | 0,90 | I,52 | 2,27 | 3,13 | 4,03 | 5,12 |
| 180 |  | 0,02 | 0.11 | 0. $\mathrm{H}^{\text {a }}$ | 0,88 | I, 42 | 2,22 | 3,05 | 3,93 | 5,00 |
|  |  |  |  |  |  |  |  |  |  |  |

## TABLE 3

Энергия неитрсюон $T_{n}$ (кэз) , пля угла $\theta$ (град) и энериии потонов
$T_{p}$ (кэв) в лабораториой системе
тоординат

|  | $\mathrm{T}_{\mathrm{p}}$ | $0^{\circ}$ | $110^{\circ}$ | $20^{\circ}$ | $30^{\circ}$ | $140^{\circ}$ | $50^{\circ}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{n}_{\substack{4 \\ \hline}}$ | 2906 | 5.18 | 5.01 | 4,52 | 3,78 | 2,86 | I,88 |
|  | 2950 | 60,39 | 60,06 | 59,08 | 57,52 | 55,46 | 53,02 |
|  | 3000 | II5,50 | II5,06 | II3,76 | III,67 | 108,88 | 105,54 |
|  | 3050 | I68,98 | I68,45 | 166,90 | I64,39 | 161,03 | 156,99 |
|  | 3100 | 221,66 | 22I,06 | 219,28 | 2I6,4I | 212,57 | 207,93 |
|  | 3150 | 273,85 | 273,18 | 271,20 | 268,00 | 263,72 | 258,54 |
|  | 3200 | 325,69 | 324,96 | 322,80 | 319,31 | 314,62 | 308,93 |
|  | 3250 | 377.29 | 376,50 | 374,17 | 370,39 | 365,32 | 359,17 |
|  | 3300 | 428,70 | 427,86 | 425,36 | 421,31 | 4I5,88 | 409,28 |
|  | 3400 | 531,08 | 530,14 | 527.33 | 522,78 | 5I6,66 | 509,22 |
| $\stackrel{N}{\vdots}$ | I566 | 3,94 | 3,87 | 3,65 | 3,32 | 2,90 | 2,43 |
|  | 1600 | 43,86 | 43,68 | 43,16 | 42,32 | 4I,2I | 39,88 |
|  | 1650 | 97, I8 | 96,92 | 96,16 | 94,94 | 93,30 | 91,32 |
|  | 1700 | 149,24 | 148,93 | 147,99 | I46,47 | I44, 42 | 141,95 |
|  | 1750 | 200,76 | 200, 39 | 199,29 | 197,51 | 195,13 | 192,22 |
|  | 1800 | 251,95 | 25I,53 | 250,29 | 248,28 | 245,58 | 242,29 |
|  | 1850 | 302,93 | 302,46 | 301,09 | 298,86 | 295,87 | 292,21 |
|  | 1900 | 35.3.75 | 353,25 | 351,75 | 349,31 | 346,04 | 342,04 |
|  | 1950 | 404,46 | 403,91 | 402,29 | 390,66 | 396, Iz | 301,79 |
|  | 2000 | 455, 63 | 454,49 | 452,75 | 449,93 | 446,13 | 441,49 |
| $\underset{i}{N}$ | 2166 | 3,08 | 2,96 | 2,78 | 2,50 | 2,15 | I,77 |
|  | 2170 | 8,6I | 8,53 | 8,28 | 7.90 | 7,40 | 6,82 |
|  | 2180 | 20,6I | 20,49 | 20,15 | 19,60 | I8,88 | 18,02 |
|  | 2190 | 31.90 | 31,76 | 31,35 | 30,68 | 29,80 | 28,75 |
| そ | 2200 | 42,91 | 42,75 | 42,28 | 4I,52 | 40,51 | 39,29 |
|  | 2220 | 64,47 | 64,28 | 63.71 | 62,79 | 61,56 | 60,08 |
|  | 2240 | 85,69 | 85,47 | 84,82 | 83,76 | 82,35 | 80,64 |
|  | 2260 | 106,70 | I06,45 | 105,73 | 104,55 | 102,98 | 101,07 |
|  | 2280 | 127,56 | 127,29 | I26,50 | 125,22 | 123,50 | I2I, 4I |
|  | 2300 | I48, 32 | I48,03 | 147.17 | 145,79 | 143,93 | I4I, 67 |






| $0^{\circ}$ |  |  |  |  |  |  | $60^{\circ}$ |  |  |  |  | $120^{\circ}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathrm{E}_{\mathrm{p}}$ | $\mathrm{E}_{n}$ | $\triangle E_{i}$ | $\triangle \mathrm{E}_{2}^{\prime}$ | $\Delta E_{2}^{\prime \prime}$ | $\Delta E_{3}$ | E | $\triangle E_{1}$ | $\triangle E_{2}^{\prime}$ | $\mathrm{E}_{2}^{\prime \prime}$ | $\Delta E_{5}$ | E | - | $\Delta E_{2}^{\prime}$ | ${ }^{1} E_{8}$ | $\triangle E_{5}$ |
| - | I88I | 37,87 | 7,09 | I, 87 | 2,40 | 6,45 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 |
| $\stackrel{ }{\sim}$ | 1900 | 84,47 | I, 87 | 0,55 | 0,70 | 2,10 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | ©,00 | C,00 | 0.00 | 0,60 | 0,00 |
| 5 | 1820 | I2I,24 | I,56 | 0,43 | 0,56 | I. 74 | 30,32 | I, 52 | 0,54 | 0,70 | 2,74 | 0,00 | -,00 | 0, 00 | 0,00 | 0,01 |
| 4 | 2000 | 230,52 | I,25 | 0,3I | 0,40 | I,39 | 125,95 | I,02 | 0,30 | 0,46 | 1,17 | 29, 42 | 0.49 | C,2I | 0,27 | 0,58 |
| $\stackrel{\square}{4}$ | 2500 | 785,65 | I,06 | 0,19 | 0,24 | I, I3 | 586,38 | 0,90 | 0,33 | 0,42 | I, 00 | 321,6I | 0,62 | 0,34 | 0,44 | 0,74 |
|  | IO20 | 775,74 | 7,20 | 2,34 | 2,99 | 8,30 | 0,00 | 0,00 | 0, 00 | 0,00 | 0,00 | C,CO | 0.00 | C,00 | 0,00 | 0,00 |
| ${ }_{-}^{\text {z }}$ | II60 | 303,7.7 | I, 22 | 0,29 | 0,36 | I, 47 | 84,95 | 0,89 | 0,50 | 0,65 | I,39 | 0,47 | 0,07 | C,03 | 0, C6 | 0,io |
| 5 | 1200 | 35I, 90 | I,I8 | C,26 | 0,33 | I,39 | I22,20 | 0,89 | 0,44 | - 0,50 | I, 23 | 5, $\mathrm{S}_{4}$ | 0.77 | O, II | 0,I4 | 0,27 |
| 家 | 2000 | I202,50 | I, 02 | 0, I3 | 0,17 | I,II | 727,6i | 0.72 | 0,43 | 0,55 | 0,95 | 245.65 | 0, 34 | 0,33 | 0,42 | C,55 |
| 5 | 2500 | I7II, 32 | I,Gi | 0,II | 0,14 | I,CS | 10:0,02 | 0,72 | 0,47 | 0,60 | 0,02 | 418,49 | $\mathrm{C}_{2} 34$ | 0,42 | 0,53 | 0,57 |
|  | 2908 | 8,67 | I, 60 | 0,25 | 0,32 | I, 02 | 4, 10 | 1,35 | C, 22 | 0,28 | I, 37 | 0,71 | 0,56 | 0,10 | 0,12 | 0,58 |
| 5 | 2910 | II, 72 | I,46 | 0,23 | 0,29 | I. 49 | 6,68 | I, 23 | 0,20 | 0,26 | I,26 | I, 96 | 0,70 | O.II | 0, 14 | 0,69 |
|  | 3000 | 1I5,49 | I, 08 | $0 \times 5$ | 0.19 | I, IO | 105.79 | 工. $\mathrm{CO}_{2}$ | 0,16 | 0,2: | [iC4 | 79.50 | 0,80 | C, $\mathrm{I}_{2}$ | $C_{i} 2 \mathrm{I}$ | 0,92 |
|  | 5008 | 5,83 | I,36 | 0, i4 | 0,10 | I. 46 | 4,46 | I, 16 | 0,23 | 0,27 | I, 24 | I, BI | C.75 | 0,09 | O.II | 0,72 |
|  | 1600 | 43,85 | I, 10 | 0,10 | 0,13 | I, II | 38,38 | I, 03 | 0,II | 0,I4 | 1,05 | 29,37 | 0,89 | O,II | 0, I4 | 0,92 |
|  | 17C0 | 149,26 | I, 04 | 0,09 | 0.1 I | I, 05 | 139, I3 | I,00 | 0.10 | 0,13 | $\underline{T}$ | $120 \times 5$ | I, I 2 | 0.13 | 0.15 | Q, 95 |
|  | 2567 | 4,56 | I, $\mathrm{L}^{7}$ | 0,16 | 0,2I | I,52 | 2.73 | I,25 | 0,14 | 0.18 | 1, 00 | 0,69 | 0,69 | 0,03 | O,II | 0,67 |
|  | 2163 | 5,97 | 1,38 | 0,15 | 0,20 | I,39 | 3,89 | I,I9 | 0,24 | 0, I7 | $I_{5} 20$ | i, 59 | 0,75 | 0,09 | 0, i2 | 0,77 |
|  | 2180 | 3I, 90 | I,II | 0.12 | 0,15 | I. 13 | 27,57 | I, 04 | 0,12 | 0.15 | I, 06 | 20,57 | 0,90 | 0,II | 0,14 | 0,91 |

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RECOIL PROTON SPECTRA IN A HYDROGEN-FILLED
PROPORTIONAL COUNTER
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#### Abstract

Measurement of monoenergetic neutron fluxes by the recoil proton method involves determining the quantity $N_{i n t}$, the number of interactions of incident neutrons with a known number of hydrogen nuclei. The recoil protons arising from elastic scattering of neutrons by hydrogen nuclei have a rectangular energy spectrum in the range $0-E_{n}$, where $E_{n}$ is the incident neutron energy. Since the detector recording the recoil proton energy has finite geometry and the electronic equipment has a certain energy threshold, it is not possible to measure the recoil proton energy spectrum without distortion. For this reason $N_{i n t}$ can be determined only by comparing experimental results with a theoretical recoil proton spectrum. The accuracy of the value of $N_{\text {int }}$ obtained in this way depends upon how well the experimental conditions are allowed for in the calculations.


Figure 1 shows a typical experimental set up for measuring a monoenergetic neutron flux. A proportional counter filled with hydrogenous gas is irradiated by a plane-parallel neutron beam. It is assumed that the ionization produced by a recoil proton in the gas of the counter is proportional to the energy lost. The differential spectrum of the recoil proton energy losses (hereafter called recoil proton spectrum) is determined experimentally in the sensitive volume of the counter. The shape of this spectrum is govermed both by the recoil protons produced in the sensitive volume of the counter with length H and diameter 2 R and by the protons escaping from the insensitive volume at the front with a length equal to the maximum range of a recoil proton, i.e. of a proton with energy $E_{p}=E_{n}$. Some recoil protons will lose part of their energy in the wall of the counter or in the gas outside the sensitive volume.

Thus, the problem is to calculate the differential recoil proton spectrum $Q(x)$ formed in the counter and the quantity $K_{H}$, which will enable us to determine the spectrum of recoil protons produced in the sensitive volume of the counter, for which the number of hydrogen nuclei is known. Here and below $x=E_{p} / E_{n}$, i.e. the recoil proton energy in incident neutron energy units, whilst the index 1 denotes normalization to 1 .

Clearly,

$$
\begin{equation*}
K_{H}=1+\frac{N_{1}}{N_{V}} \tag{1}
\end{equation*}
$$

where $\mathbb{N}_{1}$ is the number of tracks of recoil protons escaping from region 1 (Fig. l) in the sensitive volume and $N_{v}$ is the number of tracks of recoil protons formed in the sensitive volume (regions 2, 3 and 4).

We can then write

$$
\begin{equation*}
N_{\text {int }}=\frac{N_{1001}(x)}{\varepsilon_{v}(x)} \tag{2}
\end{equation*}
$$

where cou $(x)$ is the number of recoil protons recorded with energy $>x$;

$$
\begin{align*}
& \varepsilon_{\mathrm{V}}(x)=K_{H} \cdot \varepsilon_{1}(x)  \tag{3}\\
& \varepsilon_{1}(x)=\int_{x}^{1} \rho_{1}(x) d x \tag{4}
\end{align*}
$$

Several publications describe the results of calculations of $Q_{V}(x)=K_{H} \cdot Q_{1}(x)$ performed by the analytical method $[1-3]$ and the Monte Carlo method $[4]$. All these calculations were done on the following assumptions:
(1) Neutron trajectories parallel to counter axis;
(2) Interaction density constant over counter volume.

The range-energy relationship is given either in the form of a table of average ranges $[2,4]$ or by the approximate formula $R=c x E^{3 / 2}$, where c is a constant $[1,3]$. The calculations performed by the analytical method are notabe for the limitations imposed on the maximum range of recoil protons; the most rigorous limitations are imposed in Refs $[1,2]$, which have $R\left(E_{n}\right)<2 R$, whilst Ref. [3] has $R\left(E_{n}\right)<6 R$. These calculations will be compared below.

In order to increase the accuracy of neutron flux measurements, it is necessary to calculate recoil proton spectra with more careful allowance for the conditions under which they are recorded. This makes it possible to assess the quality of data from earlier calculations as well as their range of applicability - and hence the reliability of crossmections obtained with them.

The recoil proton spectra were calculated here on the following assumptions:
(1) Neutron trajectories parallel to counter axis;
(2) Density of interactions between neutrons and hydrogen nuclei in the sensitive volume of the counter inversely proportional to $\left(R_{u}+z\right)^{2}$, where $R_{u}$ is the distance from the neutron source to the point $Z=0$ with $Z$ varying in the range $0-H+q$ (Fig. I);
(3) Incident neutron energy equally distributed over the range from $E_{n}-\Delta E$ to $E_{n}+\Delta E ;$
(4) The energy recorded by the counter is a random value distributed according to the normal law with parameters EQ and $\sigma(E Q)$, where EQ is the energy lost by a recoil proton in the sensitive volume, calculated on the basis of the geometrical conditions of track formation and the rangemenergy relationship:
$\sigma\left(E_{Q}\right)=\frac{\eta\left(E_{p}\right)}{2.36}$. Ep. It was assumed also that the dependence of the counter resolution on recoil proton energy $\eta\left(E_{p}\right)$ is described by the formula

$$
\begin{equation*}
\eta\left(E_{p}\right)=\frac{A}{\sqrt{E_{p}}} \tag{5}
\end{equation*}
$$

where the constant $A$ is determined by experiment.
The calculation was done by the Monte Carlo method. The programme was compiled so that by appropriate selection of the values of $R_{u}, \Delta_{E}$ and $A$ it would be possible to exclude the effect of any one of these factors on the results of the calculation. Likewise, it is possible to analyse separately the effect of any of assumptions Nos 2-4 on the shape of the calculated spectrum. In particular, by putting $R_{u}=\infty, \Delta E=0$ and $A=0$, we obtain a recoil proton spectrum calculated on the same assumptions as in Refs $[1-4]$.

Fig. 2 shows the recoil proton spectra, $Q_{v}(x)$, calculated by the methods described in Refs $[1-3]$ and with our programme using the Monte Carlo method (this of course agrees with the calculation performed with the programme described in Ref. [4]). On the whole the spectra are in quite good agreement except for the earlier work [1]. If we take as our standard the
spectrum calculated by the Monte Carlo method, the spectrum calculated according to Ref. [2] is lower in the centre and higher at the ends than the Monte Carlo spectrum. With the spectrum calculated by the method described in Ref. [3] we find the reverse situation.

A more accurate impression of the differences between these calculations is gained by comparing $\varepsilon_{v}(x)$ for the different spectra. These quantities and their values in relation to $\varepsilon_{v}(0.6)$ for the Monte Carlo spectrum are given in the table: also given are the corresponding values of $K_{H}$ and $\varepsilon_{1}(0.6)$.

Thus we can see from the foregoing discussion that the differences in $\varepsilon_{v}(x)$ calculated by the various methods depend both on the shape of the spectrum (the trend of $\varepsilon_{1}(x)$ ) and on $K_{H}$. The closer the calculation conditions are to the limitations imposed on a particular analytical method, the more the shape of the "analytical" spectrum obtained by that method will differ from the Monte Carlo spectrum. The difference in the values of $\mathrm{K}_{\mathrm{H}}$ largely depends on the form of the rangemenergy relationship used in the calculation. Since an experimental range-energy relationship was used in Ref. $[2]$ and an approximate empirical formula $R(E)=c \times E_{p}^{n}$ where $n=3 / 2$ in Ref. [3], we nay expect that the $K_{H}$ calculated in Ref. [3] will be higher than the true values, since $n \sim 1$ for low energy protons entering the sensitive volunie of the counter from the frontal volume. These conclusions are confirmed by the results show in the table. The calculation conditions are close to the limit for Ref. [2]; accordingly the essential difference in $\varepsilon_{V}(x)$ is attributable to the difference in $\varepsilon_{l}(x)$, and the values of $K_{H}$ are approximately the same. For the counter involved (see caption to Fig. 2) the calculation of the spectrum at $E_{n}=900 \mathrm{keV}$ will be the limiting case for the method according to Ref. [3], and the following results are obtained: $\varepsilon_{1}(0.6)=0.1077, K_{H}=1.140$. Calculation by the Monte Carlo method gives $\varepsilon_{1}(0.6)=0.0874, K_{H}=1.069$. In this case the difference in $\varepsilon_{V}(x)$ is due to the significant difference in the values of $\varepsilon_{1}(x)$ and $K_{H}$.

Thus, the analytical methods of calculating recoil proton spectra described in Refs [2, 3] give approximately equal results for conditions not close to the limitations imposed. But, since the limitations imposed in Ref. [3] are less rigorous and the results are obtained in the form of an analytical formula and not as tables, as in Ref. [2], they should be preferred ir. cases where the experimental conditions approximate the assumptions made in the calculation.

As remarked above, our programme enables spectra to be calculated with more detailed allowance for experimental conditions. Fig. 3 shows the results of calculating a recoil proton spectrum, $Q_{1}(x)$, with allowance for the point neutron source, the spread of incident neutron energy around the mean value and the energy resolution of the proportional counter. For comparison Fig. 3 also shows a spectrum calculated with the same assumptions as in Ref. [4]. Calculations with separate allowance for each of the above factors show that the differences in the spectra for $x<\frac{E_{n}-\Delta E}{E_{n}}$ or $x<\frac{E_{n}-3 \sigma\left(E_{n}\right)}{E_{n}}$ are caused by variations in the density of the interactions in the sensitive volume of the counter. Thus, if the experiment and the corresponding calculation are performed in conditions where $R_{u}$ does not satisfy the condition $R_{u} \gg H$, the results of the calculation will differ greatly from the case where $R_{u}=\infty$. For $R_{u}=\infty$ we have $\varepsilon_{1}(0.6)=0.265$; $K_{H}=1.040$, and for $R_{u}=6 \mathrm{~cm}, \varepsilon_{1}(0.6)=0.284, K_{H}=1.297$. The corresponding difference in the values of $N_{\text {int }}$ is $25 \%$. If $R_{u} \gg H$, the spectra will coincide at the recoil proton energies indicated above and the $N_{i n t}$ calculated from them will agree to within the limits of error.

It is well known that for the kind of target normally used the neutron energy spread is comparable with the mean energy at neutron energies below 100 keV ; moreover, the resolution of the proportionel counter at such recoil proton energies has a strong effect on the spectrum shape. Inde. these experimental conditions the polychromaticity of the neutron source and the counter resolution will affect the shape of the recoil proton spectrum over the whole recoil proton energy range. Therefore, these factors must be allowed for in the calculation of the spectra if distortion is to be avoided. But it should be remmbered that in this energy range there will be a greater error in the calculation than at energies above 100 keV , since the rangem energy relationship and the dependence of the resolution on proton energy are less accurately known.

| Method of <br> calculation | $\varepsilon_{\mathrm{v}}(0.6)$ | Rel. value <br> $\varepsilon_{\mathrm{v}}(0.6)$ | Standardization <br> coeff. $\mathrm{K}_{\mathrm{H}}$ | $\varepsilon_{1}(0.6)$ |
| :---: | :---: | :---: | :---: | :---: |
| $[1]$ | 0.322 | 1.16 | 1.045 | 0.308 |
| $[2]$ | 0.263 | 0.95 | 1.045 | 0.252 |
| $[3]$ | 0.278 | 1.01 | 1.049 | 0.265 |
| Monte Carlo | 0.276 | 1 | 1.040 | 0.266 |

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Fig. 1 Scheme of cylindrical recoil proton counter


Fig. 2 Recoil proton spectra calculated by various methods (counter filled with hydrogen)
$E_{n}=500 \mathrm{keV}, H=21 \mathrm{~cm}, R=1.55 \mathrm{~cm}$,
$\mathrm{p}=1 \mathrm{~atm} \cdot \mathrm{abs} \cdot, \mathrm{R}(500 \mathrm{keV})=3.05 \mathrm{~cm}$, $1-[3], 2-$ Monte Carlo method, $3-[2], 4-[1]$.


Fig. 3. Differential recoil proton spectra calculated by the Monte Carlo method (counter filled with hydrogen)

$$
\begin{aligned}
E_{n} & =500 \mathrm{keV}, H=21 \mathrm{~cm}, R=1.55 \mathrm{~cm}, \\
p & =1 \text { atm.abs., } R(500 \mathrm{keV})=3.05 \mathrm{~cm}, \\
1 & -R_{u}=\infty, \Delta E=0, A=O\left(\eta\left(\mathbb{E}_{p}\right)=0\right), \\
2 & -R_{u}=6 \mathrm{~cm}, \Delta \mathbb{E}=25 \mathrm{keV}, \eta\left(\mathbb{E}_{p}=764 \mathrm{keV}\right)=4.5 \%
\end{aligned}
$$

## MAXIMUM RESOLVING POWER OF IONIZING RADIATION DETECTORS

I.V. Gordeev, Yu.S. Gerasimov, V.A. Koshelev

In order to assess the maximum resolving power of various types of ionizing radiation detectors, it is necessary to know the Fano factor $[1,2]$. The Fano factor may be written
$F=\frac{\left\langle(I-\bar{I})^{2}\right\rangle}{I}$,
where quantum-mechanical and statistical averaging is performed (I is ionization in each collision event).

To obtain a correct result, it is necessary to know the crossmsections of all processes occurring when a particle passes through a substance (ionization, excitation, charge exchange, elastic scattering) as well as the energy transmitted in each çollision event (the energy transmitted and the square of the energy transmitted averaged over the cross-section of a given process).

We calculated the Fano factor for the scattering of hydrogen protons and atoms by hydrogen atoms. For these processes the calculation is simple and reliable and is of practical value for investigations of neutrons passing through organic substances and biological tissues and also for work involving hydrogen-filled chambers. The results show that the Fano factor for the passage of heavy particles through a substance may considerably exceed the Fano factor for the passage of electrons through the same substance . The higher the ratio of the mass of the incident particle to the mass of an atom of the substance, the higher the Fano factor. This "unusual" result is largely due to the contribution of elastic scattering to the Fano factor. This contribution is negligible for electron scattering but is considerable when heavy particles are involved.

At comparatively low energies an appreciable contribution is also made by the charge exchange process. Consequently the resolving power of an instrument may be considerably lower in the case of ionization caused by heavy particles than it is in the case of ionization caused by electrons.

The contribution due to elastic scattering and ionization was calculated theoretically in the Boron approximation, which is valid at proton energies above 25 keV . This was done because there is still an almost complete lack of data on the cross-sections for elastic scattering by hydrogen and on the energy distribution of secondary electrons formed as a result of ionization.

Mainly experimental data were used for the remaining processes, supplemented where necessary by theoretical data.

The results of the calculations are given in the form of tables and graphs. Stopping powers and ion-formation energies $\mathcal{E}_{\text {were }}$ also calculated.

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Mean energy of ion-formation during passage of electrons and protons through helium

|  | I | I,055 | I,2I7 | I, 27 | I,35 | I,4I42 | I,49 | I, 72 | I,922 | 2,008 | 2,46 | 2,84 | 4,48 | 6,35 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $E_{e}(\mathrm{eV})$ | 13,5 | I5 | 20 | 2I, 8 | 24,5 | 27,2 | 30 | 40 | 50 | 54,5 | BI,7 | 108 | 272 | 544,6 |
| $\mathrm{E}_{\mathrm{p}}(\mathrm{keV})$ | 25 | 27,5 | 36,7 | 40 | 43,9 | 50 | 55, I | 73,4 | 91, 8 | 100 | 150 | 200 | 500 | 1000 |
|  | - | - | - | - |  | 62,4 | 49,8 | 29, I | 32,4 | 34,6 | 37,3 | 40,1 | 42,I | 42,2 |
|  | 27,9 | 26.7 | 28,2 | 28,5 | 28,9 | 29,3 | 29,6 | 30,2 | 3I, 4 | 32,3 | 37,8 | 39,2 | 4I, 5 | 4I,7 |

$\mathrm{V} / \mathrm{v}_{0}$ is the ratio of the incident particle velocity to the velocity of a Bohr electron, $v_{o}=2.1877 \times 10^{8} \mathrm{~cm} / \mathrm{sec}$.
$\mathrm{E}_{\mathrm{e}}$ is the incident electron energy in eV ; $\mathrm{E}_{\mathrm{p}}$ is the incident proton energy in keV .
$\omega_{\mathrm{e}}$ and $\omega_{\mathrm{p}}$ are the differential energies of ion-formation for incident electrons and protons respectively, expressed in eV per ion pair.

Differential Fano factor, $\mathrm{F}_{\text {dif }}$, for passage of electrons and protons through helium

|  | 0,448 | 0,635 | 0,778 | 0,896 | I | I, 22 | I, 27 | I,36 | I, 414 | I,49 | I, 92 | 2,008 | 3,I7 | 4,48 | 5,31 | 6,3 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{B}_{\mathbf{e}}(\mathrm{eV})$ | - | - | - | - | - | - | - | 25 | 27,32 | 30 | 50 | 54,46 136, 1272,3 381,2 544,6 |  |  |  |  |
| $\mathrm{B}_{\mathrm{p}}(\mathrm{keV})$ | 5 | IO | I5 | 20 | 25 | 36,72 | 40 | 45,9 | 50 | 55,08 | 91,8 | 100 | 250 | 500 | 700 | 1000 |
| (diff.) | 0,44 | 0,52 | 0,57 | 0,53 | 0,5 | 0,45 | 0,415 | 0,38 | 0,27 | 0,26 | 0,3I | 0,35 | 0,62 | 0,71 | 0,75 | 0,8 |
| (diff.) | - | - | - | - | - | - | - | I | 0,82 | 0,53 | 0,25 | 0,3I | 0,32 | 0,35 | 0,38 | 0,4 |
|  | - | - | - | - | - | - | - | 0,38 | 0,33 | 0,49 | I, 21 | I,I3 | I, 94 | 2,03 | I,97 | 2,0 |
| $\mathrm{V} / \mathrm{v}_{\mathrm{o}}, \mathrm{E}_{\mathrm{e}}$ and $\mathrm{E}_{\mathrm{p}}$ are the same as in Table l. <br> $F_{e}$ and $F_{p}$ are the differentiai Fano factors for incident electrons and protons respectively (dimensionless quantities) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

# AGTIVATION DETECTORS FOR NEUTRON DETECTION <br> (Review) <br> R.D. Vasilev, E.A. Grigorev, V.P. Yaryna 

In the All-Union Scientific Research Institute for Physicom-Technical and Radiotechnical Measurements a programme has started to select, process and standardize nuclear physics constants for measurements of neutron field characteristics by the activation method. This survey, which marks the first step towards fulfilling this task, is devoted to selecting the most up-to-date data on isotopes widely used for neutron-activation measurements, and is designed to supplement and improve on earlier surveys $[7,16,27$, 29, 49, 50].

## 1. Resonance detectors

Table $l$ contains a list of isotopes suitable for use as resonance detectors for neutrons on the basis of the ( $n, y$ ) reaction. Columns 2-4 of the table show the chemical symbol of the isotope, its percentage concentration in the natural mixture and the activation reaction. Data on the use of these reactions for recording neutrons are contained in Refs $[3,7$, $16,17,19,29,60]$; in particular, for ${ }^{164}$ Dy and $7^{C l}$, see Ref. [27], for $P$ see Ref. [55], for Lu see Ref. [31], for Rh see Ref. [23], for ${ }^{50} \mathrm{Cr}$ see Ref. [26] and for ${ }^{63} \mathrm{Cu}$ and $F$ see Ref. [25]. $T_{\frac{1}{2}}(\operatorname{col} .5)$ is the half-life of the product nucleus resulting from the activation reaction. The half-life data are taken mainly from $\operatorname{Refs}[8,15]$; otherwise the source is indicated beside the half-life in the table. $\lambda\left[\mathrm{sec}^{-1}\right]$ is the decay constant, calculated from the half-life (in seconds) with the formula

$$
\begin{equation*}
\lambda=\frac{0.693}{T_{\frac{1}{2}}} \tag{1}
\end{equation*}
$$

$\mathrm{E}_{0}[\mathrm{eV}](\operatorname{col} .6)$ is the energy of the fundamental resonance of the reaction. For ${ }^{164}$ Dy and ${ }^{31} \mathrm{P}$ the first resonance of the activation reaction lies above 100 keV , so they may be used as detectors with a cross-section subject to the $1 / v$ law. The data are taken mainly from $\operatorname{Refs}[1,2,16]$.
$\sigma_{\text {act }}$ is the cross-section for the ( $n, \gamma$ ) reaction induced by thermal neutrons. The data are taken mainly from Ref. [1].
$J_{\mathrm{R}}[$ barns $]$ (col. 8) is the resonance integral of the activation reaction determined by the formula

$$
\begin{equation*}
J_{\mathrm{R}}=\int_{0.55}^{\infty} \sigma(E) \frac{\mathrm{dE}}{\mathrm{E}} \tag{2}
\end{equation*}
$$

The data were sel eoted from Refs $[16,18,22,23,27,29,30]$; the resonance integral of the ${ }^{197} \mathrm{Au}(\mathrm{n}, \gamma)$ reaction (1550 b) was taken as the standard.
$E_{B} \mathrm{MeV}$ (\%) (col. 9) is the beta particle energy from decay of the radioactive product; the figure in brackets shows the extermal yield of particles with that energy as a percentage of the number of disintegrations.
$\mathrm{E}_{\gamma} \mathrm{MeV}(\%)(c o l .10)$ is the gamma energy and the external yield of gamma rays with that energy as a percentage of the number of disintegrations. The decay data were selected from $\operatorname{Refs}[3,5,7,8,10,12,14,15]$.

Information on associated reactions occurring simultaneously with the basic reaction in detectors consisting of natural isotopic mixtures is given in column ll. This column also indicates certain decay characteristics as well as the thermal neutron reaction crossmsection or the resonance integral.

## 2. Resonance parameters

Table 2 contains information on the parameters of the fundamental and neighbouring resonances for the most wiclely used detectors. Columns 2 and 3 show the chemical symbol of the isotopendetector and $E_{R}(e V)$ - the energies of the fundamental (see column 6, Table l) and neighbouring resonances of the $(n, \gamma)$ reaction. Column 4 shows the spin of the ground state of the target nucleus (I) and column 5 shows the total momentum of the compound nucleus (J). Sufficient data are supplied for calculating the statistical factor $g$ :

$$
\begin{equation*}
g=\frac{2 \mathrm{~J}+1}{2(2 \mathrm{I}+1)} \tag{3}
\end{equation*}
$$

$\Gamma$, eV (col. 6) is the total resonance width, $\Gamma_{\gamma}, \mathrm{eV}(\operatorname{col} .7)$ the width for gammamray escape, $\Gamma_{n}$, eV (col. 8) the width for neatron escape, and $\sigma_{0}$, barn (col. 9) the experimentally measured total cross-section at the resonance.

The table was compiled mainly on the basis of data from Ref. [1]. In the case of ${ }^{197}$ Au a careful analysis of the contributions made by the various resonances to the activation process has been jerformed by Brisbois [38]. The diversity of the data on $\Gamma_{\gamma}$ for ${ }^{23}$ Na has been
analysed in Ref. [35]. The first reliable data on $\Gamma_{n}$ for ${ }^{37} \mathrm{Cl}$ are given in Ref. [40]. The data on $\Gamma_{\gamma}$ for ${ }^{37_{C l}}$ are very approximate [28], although it had already been concluded in Ref. [3] that the 25.5 keV resonance made the primary contribution to activation. The bibliography includes references to investigations of the resonances of $\mathrm{Rh}[41] ;{ }^{63} \mathrm{Cu}$ and ${ }^{65} \mathrm{Cu}[34]$; $\mathrm{Mg}, \mathrm{Fe}$ and $\mathrm{S}[36]$; and $\mathrm{Dy}[37]$.

## 3. Threshold detectors

Table 3 contains a list of isotopes used as fast neutron detectors on the basis of threshold reactions of the ( $n, f$ ), ( $n, n^{\prime}$ ), ( $\left.n, p\right),(n, \alpha)$ and ( $n, 2 n$ )types.

Columns 2-4 show the chemical symbol of the isotope, its percentage concentration in the natural mixture and the activation reaction.

Data on the use of these reactions for recording neutrons are contained in Refs $[45,49,50,52,54,57]$ for ${ }^{115} \mathrm{In},{ }^{32} \mathrm{~S},{ }^{58} \mathrm{Ni},{ }^{24} \mathrm{Mg},{ }^{27} \mathrm{Al},{ }^{56}{ }_{\mathrm{Fe}}$, ${ }^{31} \mathrm{P}$ and ${ }^{64} \mathrm{Zn}$, in Refs $[49,51]$ for ${ }^{237} \mathrm{~Np},{ }^{232} \mathrm{Th}$ and ${ }^{238}{ }_{\mathrm{U}}$, in Refs $[26,53,70]$ for ${ }^{54} \mathrm{Fe}$, in Ref. [55] for ${ }^{103} \mathrm{Rh}$, in $\operatorname{Refs}[52,57,63]$ for Ti , in Refs $[56,62]$ for ${ }^{35} \mathrm{Cl}$, in Ref. [45] for ${ }^{28}$ Si, in Ref. [ 57$]$ for ${ }^{63} \mathrm{Cu}$, in Ref. $[50]$ for ${ }^{65} \mathrm{Cu}$ and in Ref. [45] for $F$ and I.
$\mathrm{T}_{\frac{1}{2}}$ (col. 5) is the half-life of the product nucleus resulting from the activation reaction. The data are taken mainly from Refs [8-15].
$\lambda\left[\mathrm{sec}^{-1}\right]$ is the decay constant calculated from the half-life in seconds using formula (1).
$\mathrm{E}_{\text {eff }} \mathrm{MeV}$ (col. 6) and $\sigma_{\text {eff }}$, mbarm (col. 7) are the effective reaction threshold and the effective crossmsection at the threshold, which are related by the expression

$$
\begin{equation*}
\int_{0}^{\infty} \sigma(E) \varphi(E) d E=\sigma_{e f f} \int_{E_{e f f}}^{\infty}(E) d E \tag{4}
\end{equation*}
$$

where $\sigma(E)$ is the activation cross-section and $\varphi(E)$ is the differential neutron spectrum.

The shape of the neutron spectrum and the criteria for choosing $\mathrm{E}_{\text {eff }}$ and $\sigma_{\text {eff }}$ are described in the original sources and will not be repeated here; all the recommended values of $E_{\text {eff }}$ and $\sigma_{\text {eff }}$ satisfy the ${ }^{235} \mathrm{U}_{\mathrm{U}}$ fission spectrum with an error not exceeding $10 \%$. The source from which the values of $E_{\text {eff }}$ and $\sigma_{\text {eff }}$ were derived is indicated in the table. $\bar{\sigma}$ mbarn (col. 8) is the mean cross-section in the fission spectrum, determined by the relation

$$
\begin{equation*}
\int_{0}^{\infty} \sigma(E) \varphi(E) d E=\bar{\sigma} \int_{0}^{\infty} \varphi(E) d E \tag{5}
\end{equation*}
$$

$\mathrm{E}_{B^{\prime}} \mathrm{MeV}(\%)$ (col. 9) is the beta particle energy from decay of the radioactive product; the external yield of particles with that energy is show in brackets as a percentage of the number of disintegrations.
$\mathrm{E}_{\gamma}, \mathrm{MeV}$ (\%) (col. 10) is the energy and external yield of gamma rays as a percentage of the number of disintegrations. The decay data were derived from $\operatorname{Refs}[3,5,7,8,10,12,14,15]$.

Information on the associated reactions occurring simultaneously with the basic reaction in detectors consisting of the natural isotopic mixtures is civen in column 11.

Column 12 gives the reference fron which the energy dependence of the activation reaction cross-section is taken.

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## Table 1

Resonance detectors

## Key:



## Table 2

Resonance parameters
Key:

| Изотоп | $=$ Isotope |
| :--- | :--- |
| ЭВ | $=\mathrm{eV}$ |
| барн | $=$ barn |

## Table 3

Threshold detectors

Key:

| Изотоп | = Isotope |
| :---: | :---: |
| $\left[\begin{array}{l} \text { Содержание } \\ \text { изотоп. } \end{array}\right.$ | $=$ Concentration \% |
| Реакция | $=$ Reaction |
| сек | $=\mathrm{sec}$ |
| Э в | $=\mathrm{eV}$ |


| Мэв | $=\mathrm{MeV}$ |
| :--- | :--- |
| эфф | $=$ eff |
| мбарн | $=$ mbarn |
| $\left[\begin{array}{ll}\text { Сопутствуюцие } \\ \text { реакции } & =\text { Accompanying reactions } \\ \text { Литература } & =\text { Reference } \\ \text { см。 } & =\text { see }\end{array}\right.$. |  |

```
Translator's note: the key to Russian abbreviations in columns 5 and 11
                            of Tables 1 and 3 is as follows:
y or час =h
    IH or }\pi\textrm{H}=\textrm{d
    \sigma = b
    мбарн =mb
    akT = act
    лет =yr
    содерж = content
    сек = sec
    M or мин =min
```


## Резонанснне детепторы

|  | $\begin{aligned} & E_{0} \\ & \partial b^{2} \end{aligned}$ |  | $J_{R}$ | Es aflob | altab |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | 8 | - - 9 |  | $\vdots$ - - - - - |
|  | $1 / 1$ | $800 \pm 100$ | 377 | $\begin{aligned} & 0.29(2) \\ & \mathrm{I.19(13)} \\ & 1.26(85) \end{aligned}$ | 0.095(I4) |  |
|  | $1 / 1$ | 0.19 $\pm 0.01$ | 0.092 | 1.707(100) | - | ${ }^{31} P(n, p)^{3}\left\{i ; 2,64\right.$ rac; $E_{\beta} 1,48(100)$ б 30 suiapt. |
|  | . 005 | $2100+150$ | 978 | $0.176(I)$ <br> $0.384(3)$ | $0.208(I)$ <br> $0.113(3.2)$ | $97,40 \% \%^{175}\left\langle u(n, 5)^{176 m} L u ; 3,7 \mathrm{zac} ;\right.$ |
| ${ }^{108}{ }^{17} H_{f}{ }^{*}$ I.I88•Iti ${ }^{-6}$ |  |  | 978 | $0.497(90)$ |  | $E_{\beta} 1,1(E, 5) 1,2(93,5): E_{\delta} 0,09(6,5)$ デR 4760 . |
|  |  |  |  | I.88(I,85) | 0.556(1,9) | ${ }^{103} \operatorname{Bin}\left(n, n^{\prime}\right)^{103 N " R h ; ~} 57,5$ suus; |
| ${ }^{104} \mathrm{Pd} \mathrm{c}^{\left(\frac{1}{3}\right.} 2.613 \cdot 10^{-3}$ | 1,257 $\pm 0.002$ | $1 \mathrm{I}_{ \pm}$ | 83 | 2,44(98) | 0.05I(100) | Er $0,02(100) ; \bar{\sigma} 1,095$. |
|  | I.45I $\pm 0.02$ | $198 \pm 3$ | 3440 | $\begin{aligned} & 0.60(8) \\ & 0.87(36) \end{aligned}$ | $\begin{aligned} & \text { 2.I2(I7) } \\ & \mathrm{I} .77(2) \\ & \mathrm{I} .29(80) \end{aligned}$ | $4,23 \%{ }^{113} \mathrm{M}(n, r)^{114 \mathrm{~mm}} \mathcal{I n}_{n} ; 5 \mathrm{O}_{g 4}$; $E_{\gamma} 0,192 ; y_{\pi} 1050 \delta$. |
|  |  |  |  |  | 0.13+1,52( |  |
| Lu 97,40 Lu(nd) lu $3.69 \pm 0.04 \mathrm{4ac}$. |  |  |  | I.I(6.5) |  |  |
| $165,1 \quad 165,1665^{176} H^{\beta}{ }^{\beta} 5.217 \cdot 10^{-5}$ | $2.61 \pm 0.01$ | 18.4 $\pm 0.9$ | 476 | I.2(93.5) | 0.09(6.5) | $0,384(3) 0,497(00) ; E_{C} 0,208(7)$ |
|  |  |  |  | 0.38(I) | I.6I(I.7) |  |
| 166 | $3.92 \pm 0.01$ | $64 \pm 6$ | 860 | 1.77(47) | 2,53(33) |  |
| $62^{2} 7.156 \cdot 10^{-6}$ |  |  |  | 185(52) | 1.37(9) |  |
|  |  |  |  |  | 0.08(48) |  |




## Table 2

Дараметри резонансов

|  |  | ER | I | J | ${ }_{7}{ }^{-}$ | $\bar{r}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\therefore$ I | :2_ | 3 | 4 |  | --6 | 7 | : _ - $\boldsymbol{8}^{\text {- }}$ - - | - - $\underline{9}$ |
|  |  | I,457 |  | 5 | $(75 \pm 2) \cdot 10^{-3}$ | $(72 \pm 2) \cdot 10^{-3}$ | $(3.04 \pm 0.05) 10^{-3}$ | $38500 \pm 1000$ |
| I | ${ }_{49}{ }^{15} \mathrm{M}$ | 3.86 9.12 | 9/2 | 4 | $(8 I \pm 4) \cdot 10^{-3}$ $(82+40) 10^{-3}$ | $\begin{aligned} & (81 \pm 4) \cdot 10^{-3} \\ & (80+40) 10^{-3} \end{aligned}$ | $\begin{aligned} & (0.354 \pm 0.015) \cdot 10^{-3} \\ & (1.57+0.16) 10^{-3} \end{aligned}$ | $-3 \quad 480$ |
|  |  | 4.906 |  | 2 | $0.140 \pm 0.03$ | $0.124 \pm 0.003$ | $0.0156 \pm 0.0004$ | $37000 \pm 500$ |
| 2 | ${ }_{79}{ }^{\text {atue }}$ | 58.1 |  | I | $0.115 \pm 0.015$ | 0.112 0.015 | $=(2,3 \pm 0.14) I 0^{-3}$ | -3- |
|  |  | 60.3 | $3 / 2$ | 2 | $0.206 \pm 0.020$ | $0.130 \pm 0.020$ | $0.076 \pm 0.005$ | $6_{6} \Gamma^{2}=450 \pm 130$ |
|  |  | 78.7 |  | I | $0.157 \pm 0.015$ | $0.140 \pm 0.015$ | $0.0167 \pm 0.0008$ | $3_{\Delta} \Gamma^{2}=32 \pm 8$ |
|  |  | 5.20 |  | I | $0.153 \pm 0.003$ | $0.140 \pm 0.003$ | $0.0125 \pm 0.0001$ | $34000 \pm 1000$ |
| 3 | ${ }_{47}^{109} \mathrm{Mg}$ | 30.5 | I/2 | I | $0.132 \pm 0.013$ | $0.125 \pm 0.013$ | (8.0 $\pm 0.3) 10^{-3}$ | $3530 \pm 640$ |
|  |  | 40.2 |  | I | $0.142 \pm 0.019$ | $0.137 \pm 0.019$ | (5.0+0.4) $10^{-3}$ | $6_{0} \Gamma^{2}=34 \pm 10$ |
| 4 | ${ }_{.2}^{15} \mathrm{Sm}$ | 8.03 | 0 |  | $0.201 \pm 0.008$ | $0.071 \pm 0.010$ | $0.13 \overline{0}^{+} 0.005$ | $210000 \pm 2000$ |
|  |  | I8.84 |  |  | $0.369 \pm 0.007$ | $0.052 \pm 0.006$ | $0.317 \pm 0.005$ | $b_{0} \Gamma_{\gamma}=(7 \pm 1) I 0^{3}$ |
| 5 | ${ }_{74}^{16} \mathrm{~W}$ | 171.5 | 0 |  | $0.092 \pm 0.020$ | $0.065 \pm 0.020$ | $0.027 \pm 0.003$ | $4500 \pm 1200$ |
|  |  | 219 |  |  | $0.602 \pm 0.040$ | $0.062 \pm 0.006$ | $0.540 \pm 0.040$ | $6_{0} R_{\gamma}=450 \pm 50$ |
| 6 | ${ }_{57}^{139}$ La | 72.4 | 7/2 |  | $0.150 \pm 0.030$ | $0,120 \pm 0.030$ | $0.030 \pm 0.003$ |  |
| 7 | ${ }_{27}^{59} \mathrm{Co}$ | 132 | $7 / 2$ | 4 | $5.57 \pm 0.10$ | $0.45 \pm 0.05$ | $5.12 \pm 0.04$ | $9700=1800$ |
|  |  | 337 |  | 2 | $22.5 \pm 1.0$ | $0.5 \pm 0.1$ | $22 \pm 1$ | $3{ }_{3} \stackrel{\text { J }}{=}(3.040 .6) 10^{6}$ |
| 8 | ${ }_{25}^{55}$ dln | 1098 | $5 / 2$ | 3 |  |  | $14.6 \pm 0.7$ b | $3{ }^{2}(2.8 \pm 0.7) 10^{4}$ |
|  |  | 2377 |  | 3 |  |  | $400 \pm 10$ | - |
|  |  | 577 |  | 2 | $1,41 \pm 0.05$ | $0.55 \pm 0.07$ | $0.86 \pm 0.03$ | 1550 |
| 9 | ${ }_{29}^{23} \mathrm{Cu}$ | 2060 | 3/2 | I | $43.8 \pm 2.0$ | 0.4 | $43.5 \pm 2.0$ | 626 |
|  |  | 2660 |  | 2 | 4.7 |  | $4.5 \pm 0.5$ |  |
|  |  | 2900 |  | I |  | $0.35 \pm 0.04$ | $424 \pm$ I3 |  |
| 10 | ${ }_{1}$ Na | 54000 | $3 / 2$ | 2 | $1400 \pm 200$ |  | $g K_{n}=750 \pm 40$ |  |
|  |  | 5500 |  |  |  | $2.9 \pm 0.9$ | $1600 \pm 200$ | 473 |
| II | ${ }_{2} \mathrm{Cr}$ | 28700 | 0 |  |  |  | $510 \pm 50$ |  |
|  |  | 38700 |  |  |  |  | $1820 \pm 460$ |  |
|  |  | 8700 |  |  |  | 0.5 | $g)_{6}=40_{ \pm} 7$ |  |
| I2 | ${ }^{17} \mathrm{Cl}$ | 25500 | 3/2 | I |  | -25 | $g h_{a}=250 \pm 15$ |  |
|  |  | 47000 |  | 2 |  | $\sim 14$ | $g_{4}=236 \pm 12$ |  |

## Пороговне детекторы







```
    26 "
            2.558*10-8 [48] [47]
```




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    28 "3}\mp@subsup{}{0}{4
    * NNi I.I7.To-3 [48] [3]
```


## Chapter II. REACTOR CONSTAITTS AND PARAMETERS

NEUTRON FLUX RELAXATION LENGTHS AND RETHERMALIZATION LENGTHS<br>IN GRAPHITE AND WATER (RESULTS OF A STUDY OF NEUTRON THERMALIZATION EXPERIMENTS)<br>G.Ya. Trukhanov, Yu.A. Safin

## Introduction

For a number of years investigations have been carried out in the I.V. Kurchatov Atomic Energy Institute on neutron thermalization in a graphitewater system with a large temperature gradient over a wide range of graphite temperatures from 133 to $823^{\circ} \mathrm{K}$.

Reports on these investigations containing preliminary analyses of the experimental data have been presented at IAEA conferences [1, 2]. More recently publications have appeared $[3-7]$ giving a detailed analysis of the experimental data obtained in both the low and the high graphite temperature ranges.

In this article the data of Refs $[3-7]$ on neutron relaxation and rethermalization lengths in graphite and water are combined and generalized, and a detailed comparison is made with the results of other authors [11, 12].

A short description is given of the experimental set-up and the method used to derive the primary relaxation length and the rethermalization lengths from the experimental data.

## 2. Experimental section

The experimental equipment consisted of a graphite prism measuring $100 \times 100 \times 59.5 \mathrm{cn}^{3}$ with a water tank adjacent to one of its sides ( $100 \times 100 \mathrm{~cm}^{2}$ ) (Fig. 1). In the initial series of measurements (series A/1/) an oblong tank measuring $175 \times 175 \times 50 \mathrm{~cm}^{3}$ was used but in subsequent measurements (series $B / 2 /$ ) this was replaced by a cylinder 38 cm in diameter.

The graphite was heated electrically by elements positioned on the five faces of the graphite prism. Stainless steel pipes for circulating the coolant were installed in the same plane as the heating elements. In order to obtain a low graphite temperature ( $1.33^{\circ} \mathrm{K}$ ), liquid nitrogen was used.

The graphite prism and the water tank were separated by a heat shield consisting of two stainless steel foils 100 microns thick and one aluminium foil 200 microns thick. The other four faces of the graphite prism were insulated with a $20-\mathrm{cm}$ thick layer of ultra light-weight chamotte, which
in turn was covered with a layer of boron carbide. The prism was then enclosed in an aluminium tank.

The water moderated and cooled WNR-2 reactor was used as the external neutron source. The reactor provided a means of obtaining a wide ( $50 \times 50 \mathrm{~cm}$ ) beam of thermal neutrons. One of the faces of the graphite prism was placed close to the reactor core. The thermal neutron spectrum measured on this face (with a nearmaxwellian distribution) and the spectrum temperature depend on both the reactor power and the graphite temperature. Neutron beans were extracted from various points of the graphite with the aid of a continuous channel 17 mm in diameter passing through the centre of the prism along the axis of symmetry of the system and filled with graphite inserts of different length. The neutron beam was extracted from the water via a movable aluminium tube closed at the end, having a diameter of 14 mm . The neutron spectra in the beams extracted from different points in the graphite and the water were measured by the timemof-flight method.

The measurements were performed with a mechanical chopper having one plane-parallel slit 5 mm wide and a l60-channel time analyser. The $150 \mathrm{~mm}-$ diameter rotor of the mechanical chopper was made from glass-fibre-reinforced plastic containing boron. The neutron detector employed was an end-window proportional counter filled with enriched $\mathrm{BF}_{3}$ gas. The resolution in the time-of-flight measurements was $20 \mu \mathrm{sec} / \mathrm{m}$.

The graphite and water temperatures at which measurements were carried out are indicated in Table l. The temperature nonuniformity was $7 \%$ in the graphite and $3 \%$ in the water.

Experiments at identical graphite and water temperature were carried out for checking purposes.

The neutron spectrum was calculated from the counting rate in the channels of the time analyser, using a familiar relation [8] which takes into account the rotor transmission function ${ }^{* /}$, counter efficiency ${ }^{* *}$, the dependence of the effective flight path on neutron energy (by displacement of the effective neutron recording centre in the counter), the dependence of neutron absorption and scattering in air between the end of the tube and the detector, and the neutron background.

No corrections were made for selector resolution in the thermal range or for neutron counting losses, since they were less than the statistical measuring error.

* The effect due to penetration of the wall of the rotor slit was taken into account.
** The counter efficiency was determined experimentally.

Neutron spectra consisting of a vector neutron flux $\varphi(z, v, l)$ were measured at the following distances from the temperature jump:
(a) In graphite: $0 ; 0.5 ; 1.0 ; 2 ; 4.5 ; 29.5 ; 19.5 ; 29.5 ; 39.5$; 49.5; 59.5 cm ;
(b) In water: $0 ; 0.1 ; 0.2 ; 0.3 ; 0.5 ; 0.7 ; 1 ; 1.5 ; 2 ; 3 ; 5$ and 7.5 cm .
By way of example ${ }^{* * *}$ Fig. 2 shows the spectra of the vector flux $\varphi(z, v, l)$ in graphite and water at different distances from the thermal barrier (graphite temperature $594^{\circ} \mathrm{K}$, water temperature $302^{\circ} \mathrm{K}$ ).

The experimental spectra were used to obtain the mean values $\bar{v}, \bar{v}^{2}$ and $\overline{1 / v}$ averaged over the neutron density $v^{-1} \propto \varphi(z, v, 1)$ and the corresponding temperatures of the neutron gas (see for example Ref. [7]). In addition, by fitting the Maxwellian curve to the experimental distribution of $v . \varphi(z, v, l)$ in the region of the distribution peak, a neutron gas temperature $T=\frac{m}{4 k}\left(\frac{l}{l / v_{m}}\right)^{2}$ was obtained, where $\frac{1}{v_{m}}$ is the inverse velocity corresponding to the most probable velocity in the $v . \varphi(v, z, l)$ distribution. Figs $3-6$ show the spatial distribution of this temperature at various graphite and water temperatures. Ir addition, the integral thermal neutron density in water was measured by the activation method in the series $B$ experiments. Figs 7-9 show the results of these measurements at different graphite and water temperatures in semilogarithmic scale.

## 3. Method of obtaining neutron flux relaration <br> lengths from experimental data

The results of the experiments (spatial trend of neutron gas temperature and integral thermal neutron density) enable us to estimate the neutron flux relaxation length in water. The neutron flux relaxation lengths are by definition equal to the reciprocals of the discrete eigenvalues of problems arising in the separation of the variables in the kinetic equation. They are determined solely by the properties of the medium, irrespective of the nature of the sources. This means that they contain information on the scattering law of the medium and that, being derived from experimental data, they can be useful for checking theoretical models of the scattering kernel. On the other hand, knowledge of some of the primary neutron flux relaxation lengths and the corresponding eigenfunctions makes it possible in some cases to construct approximate solutions of the kinetic equation with a fair degree of accuracy.
***/For detailed information on the space-energy neutron distribution in this system see $\operatorname{Ref} s[6,7]$.

As shown in Ref. [3], which explains the meaning of the relaxation lengths obtained by various methods of solving the kinetic equation, the diffusion length $L_{0}$, which is the reciprocal of the zero-eth discrete eigenvalue $x_{0}$ and the first relaxation length $L_{1}=x_{1}^{-1}$ (and in certain cases also $L_{2}=x_{2}^{-l}$ ) can be obtained by analysing the asymptotic behaviour of the density or any other integral quantities (such as mean neutron velocity or neutron gas temperature). We shall demonstrate this using the example of the integral neutron density and the neutron gas temperature, confining ourselves to the plane case and the condition that there are only two discrete relaxation lengths, $L_{0}$ and $L_{1}$. The consideration is based on Ref. [3]. In our assumptions, the asymptotic part of the vector neutron flux (for $Z>1 / x^{*}$ ) where $x^{*}=\min \left[\Sigma_{t}(E)\right]$ is the boundary between the continuous and discrete parts of the spectrum of eigenvalues of the kinetic equation, $\Sigma_{t}(E)$ being the total cross-section for neutron interaction with the substance) can be represented in the form

$$
\begin{equation*}
\varphi(z, \mu, E)=\sum_{n=0 ; 1} C_{n} \cdot f_{n}(\mu, E) \cdot \exp \left(-\frac{Z}{L_{n}}\right) \tag{I}
\end{equation*}
$$

Here $f_{n}(\mu, E)(n=0 ; 1)$ are eigenfunctions corresponding to the discrete eigenvalues $x_{0}$ and $x_{1}$, and $C_{n}(n=0 ; 1)$ are expansion coefficients.

For the neutron integral density we have

$$
\begin{equation*}
n(z) \approx \sum_{n=0 ; 1} P_{n} \cdot \exp \left(-\frac{z}{L_{n}}\right) \quad\left(z>\frac{1}{x^{*}}\right) \tag{2}
\end{equation*}
$$

where $P_{n}$ are the corresponding angle and energy integrals ${ }^{*}$ in Eq. (1).
Hence it follows that, if we describe the experimental neutron density $n_{\exp }(Z)$ by the asymptotic part ( $Z>\frac{1}{x^{*}}$ ) with an expression like Eq. (2), it is in fact possible to obtain the fundamental relaxation length $L_{0}$ and the primary relaxation length $L_{1}$ provided the experimental conditions correspond to those with which formula (2) was obtained.

* The eigenfunctions $f_{0}(\mu, E)$ and $f_{1}(\mu, E)$ are solutions of the sourceless kinetic energy equation (sourceless also at infinity). This means that $f_{0}(\mu, E)$ and $f_{1}(\mu, F)$ do not contain $a \frac{1}{F}$ term. Therefore the upper limit in the energy integration of these fufictions can be taken as infinite.
* See footnote above.

Let us now find the asymptotic expression for neutron gas temperature. Using the same assumptions made when obtaining formula (2), we obtain for $z>\frac{1}{x^{*}}:$

$$
\begin{equation*}
T(z, \mu)=\frac{E^{1 / 2} \cdot \varphi(z, \mu, E) d E}{\int E^{-1 / 2} \cdot \varphi(z, \mu, E) d E}=\frac{\overline{x_{0}}(\mu)+C(\mu) \cdot \vec{E}(\mu) \cdot \exp \left(-x / L_{2}\right)}{1+C^{0}(\mu) \exp \left(-z / \mu_{e}\right)} \tag{3}
\end{equation*}
$$

$$
C(\mu)=\frac{C_{1} \cdot \int f_{1}(\mu, E) E^{-\frac{1}{2}} d E}{C_{0} \cdot \int f_{0}(\mu, E) \cdot E^{-1 / 2} d E} ; E_{n}=\frac{f_{\operatorname{t}}(\mu, E) \cdot E^{1 / 2} d E}{\int f_{n}(\mu, E) E^{-1 / 2} d E}
$$

$$
\begin{equation*}
\text { where } \quad L_{2}^{-1}=L_{1}^{-1}-L_{0}^{-1} \quad(n=0 ; 1) \tag{5}
\end{equation*}
$$

Expression (3) can be converted to

$$
\begin{equation*}
T(z, \mu) \approx T_{2}+\left(T_{1}-\cdot T_{2}\right) \frac{g}{b^{\prime}+\exp \left(\frac{Z_{2}}{L_{2}}\right)} \tag{6}
\end{equation*}
$$

where $T_{2}=E_{0}(\mu): b=C(\mu) ;\left(T_{1}-T_{2}\right) \cdot g=\left[E_{1}(\mu)-\vec{E}_{0}(\mu)\right] \cdot C_{(\mu)}$

When $Z>I_{r}$ we obtain the relation

$$
\begin{equation*}
T(z, \mu) \approx T_{2}+\left(T_{1}-T_{2}\right) g \cdot \exp \left(-\frac{z}{L_{2}}\right) \tag{7}
\end{equation*}
$$

which coincides with the familiar asymptotic expression for neutron gas temperature, $T(Z)=T_{\infty}+\left(T_{0}-T_{\infty}\right) \exp \left(\frac{-}{2} \frac{2}{2}\right)$, if $L_{r}$ is taken to represent the somealled neutron gas temperature relaxation length (see, for example, Ref. [2]).

Approximating the asymptotic behaviour of the experimental neutron gas temperature with formula (6) or (7) will give a relaxation length $\mathrm{L}_{r^{\prime}}$ which is related to the neutron flux relaxation lengths by expression (5), provided the experimental conditions correspond to those under which expressions (6) or (7) were obtained.

Thus, if the diffusion length $L_{o}$ is known, the primary neutron flux relaxation length can be obtained from the neutron gas temperature relaxation length with the formula

$$
\begin{equation*}
L_{1}=\frac{L_{0} L_{2}}{L_{0}+L_{2}} \tag{8}
\end{equation*}
$$

Note that in formula (4) the neutron gas temperature is determined as proportional to the mean square of the neutron velocity. However, the neutron gas temperature is often determined as being proportional to the square of the mean velocity $\bar{\gamma}(\mathcal{X}, \mu)\left(T_{\gamma} \sim \bar{\gamma}^{2}\right)$, or inversely proportional to the square of the mean inverse neutron velocity $(\pi / r),\left(T_{1} / r\left(\frac{T}{\gamma}\right)^{-2}\right)$. In these cases it is necessary, using formulae similar to Eqs (6) or (7), to analyse

$$
\begin{equation*}
\sqrt{T_{r}(z, \mu)} \sim \bar{\gamma}(z, \mu) \quad \text { and } \quad \frac{1}{\sqrt{T_{1 / r}(z, \mu)}} \sim\left(\frac{1}{\gamma(z, \mu)}\right) \tag{8a}
\end{equation*}
$$

respectively with the asymptotic form.
All the calculations performed above can easily be generalized to the case where there are more than two discrete eigenvalues. Obviously the procedure for obtaining the lengths $L_{0}$ and $L_{1}$ will be the same (provided $L_{1}$ is considerably greater than $L_{2}$ ) in an analysis of the region where $Z>L_{2}$. However, the problem of establishing succeeding neutron flux relaxation lengths is much more complicated because higher eigenvalues, if they exist, are usually similar to each other and difficult to separate.
4. Method of obtaining neutron rethermalization lengths from experimental data

In accordance with the conventional terminology $[1,9,11]$, rethermalization length is the name given to one of the two relaxation lengths of the method of overlapping groups in the $\mathrm{P}_{1}$ approximation. The second of these is called the diffusion length. The difference between these relaxation lengths and the true neutron flux relaxation lengths considered above, as shown in Refs [4, 5], is that they may be used to try to describe the neutron flux over the whole range of variation of the spatial variable with uniform accuracy, whereas the true relaxation lengths give the correct asymptotic behaviour of the neutron flux. We shall demonstrate this using the equations of the method of overlapping groups in the $P_{1}$ approximation for a twomregion plane infinite system, In accordance with the basic assumption of the method of overlapping groups, the neutron flux $\psi(z, v)$ at any point $Z$ for either of the regions of the two region plane system is represented as

$$
\begin{equation*}
\psi(z, r)=\sum_{e=1,2} \psi_{e}(z) \cdot \chi_{e}(r) . \tag{9}
\end{equation*}
$$

where $\chi_{1}(v)$ and $\chi_{2}(v)$ are trial functions representing a neutron spectrum in an infinite medium filled with the substance of the given region. In the $P_{1}$ approximation the weighting functions $\psi_{1}(Z)$ and $\psi_{2}(Z)$ satisfy the familiar system of equations given in Ref. [1, 9]:

$$
\begin{align*}
& D_{1} \Delta \Psi_{1}(z)-\left(\Sigma_{a_{1}}+\Sigma_{R}^{1 \rightarrow 2}\right) \Psi_{1}(z)+\Sigma_{R}^{2 \rightarrow Y_{2}}(z)=-S_{1}(z) \\
& D_{2} \Delta \Psi_{2}(z)-\left(\sum_{a R}+\Sigma_{R}^{2 \rightarrow 1}\right) \Psi_{2}(z)+\sum_{R}^{1 \rightarrow 2} \Psi_{1}(z)=-S_{2}(z) \tag{10}
\end{align*}
$$

where the notation of the physical quantities is that conventionally employed.

The fundamental difference between Eq. (9) and Eq. (1) is that instead of the eigenfunctions and the eigenlengths for the given medium Eq. (9) contains certain other functions and lengths. It is clear that, when the trial functions $\chi_{\boldsymbol{\ell}}(\mathrm{v})$ badly reproduce the eigenfunctions of the problem, formula (9) will give an unreliable asymptotic representation of the neutron flux. However, formula (9) is not designed to give the correct asymptotic form. It is arrived at describing with uniform accuracy the neutron spectrum over the whole range of variation of $Z$.

This means that the relaxation lengths of the method of overlapping groups are not universal. Their degree of universality is evidently directly related to their proximity to the true relaxation lengths. Thus the relaxation lengths of the method of overlapping groups, including the rethermalization length, are of interest for the following reasons: firstly, they are a kind of approximation to the corresponding true relaxation lengths; secondly, they can be used in conjunction with the method of overlapping groups for solving reactor problems; and, thirdly, they are a certain integral characteristic of the experiment.

The relaxation lengths of the method of overlapping groups (diffusion length and rethermalization length) are characteristic numbers in a system of conventional uniform differential equations with corstant coefficients [10]. They can be found if the parameters $D_{\boldsymbol{l}}, \Sigma_{a l}(\boldsymbol{l}=1,2)$ and the rethermalization cross-sections $\Sigma_{R}^{l \rightarrow 2}$ and $\Sigma_{R}^{2 \rightarrow 1}$ are known. Obviously these relaxation lengths will depend not only on the properties of the medium but al so on the form of the trial functions.

In the particular case where one of the rethermalization cross-sections may be neglected (in the first region $\sum_{R} \underset{R}{ } \underline{2}=0$, this means that the trial function $\chi_{1}(v)$ has a near-Maxwellian distribution with the temperature
of the first region; in the second region, $\Sigma_{R}^{2 \rightarrow 1}=0$, it means that the trial function $\chi_{2}(v)$ has a near-Maxwellian distribution with the temperature of the second region), we obtain for the first region

$$
\begin{equation*}
L_{D_{1}}=\sqrt{\frac{\Phi_{1}}{\sum_{a_{1}}}} \quad \text { and } \quad L_{2 t}^{2 \rightarrow 1}=\sqrt{\frac{D_{2}}{\sum_{R}^{2 \rightarrow 1}+\sum_{a_{2}}}} \tag{11}
\end{equation*}
$$

and for the second region

$$
\begin{equation*}
L_{D_{2}}=\sqrt{\frac{\phi_{2}}{\sum_{a_{2}}}} \quad \text { and } L_{2 t}^{1 \rightarrow 2}=\sqrt{\frac{\infty_{1}}{\sum_{R} \cdot 2^{2}+\sum_{a_{1}}}} \tag{12}
\end{equation*}
$$

where $L_{D 1}$ and $L_{D 2}$ are the diffusion lengths and $L_{r t}^{2^{\rightarrow l}}$ and $L_{r t}^{1^{2}}$ are the rethermalization lengths.

The rethermalization length can also be derived from the neutron gas temperature distribution, using expressions (6) and (7), if the following conditions are observed.

A relationship such as Eq. (6) for neutron gas temperature is obtained with the method of overlapping groups when in the system of equations in expression (10) the source terms $S_{1}$ and $S_{2}$ for one of the regions can be equated to zero and, consequently, the solution of the system within the limits of that region can be put in the form of the sum of two exponents (with coefficients), one of which contains as parameter the diffusion length $L_{D}$ and the other the rethermalization length $L_{r t}$. The neutron gas temperature relaxation length, $L_{r}$, obtained with the method of overlapping groups is associated with the diffusion length $L_{D}$ and the rethermalization length $L_{r t}$ by a relation similar to Eq. (5):

$$
\begin{equation*}
L_{2}^{-1}=L_{2 t}^{-1}-L_{D}^{-1} \tag{5a}
\end{equation*}
$$

Thus with Eq. (6) it is indeed possible to derive the rethermalization length from the experimental neutron gas temperature distribution, provided our assumptions regarding the thermal neutron source correspond to the experimental conditions. It should be borme in mind, however, that Eq. (6) must be used over the whole range of variation of the spatial variable $Z$ (note that in the case of the true relaxation lengths formula (6) is fulfilled only when $Z>1 / x^{*}$ ). A relation such as Eq. (7) for the neutron gas temperature is obtained with the method of overlapping groups in the case where,
for the system of equations in expression (10), apart from the source terms $S_{1}$ and $S_{2}$ being equated to zero, it is assumed that the diffusion coefficients and absorption cross-sections have no energy dependence, i.e. when $D_{1}=D_{2}$ and $\Sigma_{a l}=\Sigma_{a 2}$. If these assumptions correspond to the experimental conditions, formula (7) may be used to obtain the rethermalization length from the experimental neutron gas temperature. In this case the parameter minimization range should coincide with the whole range of variation of the spatial variable.
5. Results

### 5.1. Relaxation lengths in water

Using the method described in section 3, we analysed the asymptotic part of the spatial distribution of the integral themnal neutron density (Figs 7-9) and the asymptotic behaviour of the experimental neutron gas temperature in water (Figs 3-6).

The behaviour of the integral thermal neutron density was analysed as follows: on the basis of Eq. (2) the asymptotic part of the neutron density was described by a function $n(z)=P_{0} e^{-x / L o t} P_{1} e^{-\mathcal{Z}}$. The fundamental length $L_{o}$ and the corresponding weight $P_{o}$ were determined by graph (Fig. 7). Then $P_{0} \cdot \exp \left(-\notin / L_{0}\right)$ was subtracted from the initial curve (the curve of $n(Z)-P_{0} \exp \left(-Z_{/}^{\prime}\right)$ is given in the same figure under the curve of $\left.n(Z)\right)$ and the first relaxation length $L_{l}$ was evaluated in the same way. The results of the analysis are given in Table 2 .

No temperature dependence of the fundamental relaxation length $L_{o}$ in the $300-343^{\circ} \mathrm{K}$ water temperature range was detected within the limits of experimental error. The fact that the value obtained is on average somewhat higher than the values supplied by other authors (see, for example, Ref. [16]), is apparently due to the contribution of epithermal neutrons slowing down in water with a length of about $2 \mathrm{~cm}[2]$.

Table 2 also shows relaxation lengths $L_{r}$ obtained by analysing the asymptotic behaviour of the neutron gas temperature in water with the method described in section 3. The parameters $g, b$ and $1 / L_{r}$ were determined by minimizing the corresponding functionals by the method of successive approximations, using the "Tempel-3" and "Tempel-2" programmes [10] based on formulae (6) and (7). The minimization $\sqrt{*}$ was done in the range $Z=0.7-8 \mathrm{~cm}$ (here $Z$ is the distance from the thermal barrier in water).

[^0]The primary neutron flux relaxation lengths $L_{1}$ were determined from the relaxation lengths $L_{r}$ with aid of relation (5) ${ }^{*}$.

The mean value of the first relaxation length in water, $L_{1}$, based on activation measurements, is $\sim 0.44 \mathrm{~cm}$, whilst the mean lengths derived with formulae (6) and (7) for the asymptotic part of the neutron gas temperature are $\sim 0.32$ and $\sim 0.40$ cm respectively.

The relaxation lengths $L_{l}$ derived from activation measurements are less reliable because the graph subtraction process involves large errors. The disagreement with the relaxation lengths obtained with formulae (6) and (7) is quite considerable. However, it should be noted that formula (6) is preferable to formula (7) for analysing experiments in the asymptotic part, because it begins to describe the asymptotic part of the experimental curve at lower values of $Z$ than formula (7). In our case this is important because there are very few points in the asymptotic part.

The values obtained for $L_{1}$ are much lower than the maximum possible value for water of $\frac{1}{x^{*}}=0.74 \mathrm{~cm}$. On the one hand this may mean that only a fundamental relaxation length exists in water and the values of $L_{1}$ obtained are the result of an attempt to describe a continuous spectrum of eigenvalues with one discrete eigenvalue. On the other hand, the fact that they all fall into a definite range ( $0.27-0.42 \mathrm{~cm}$ ) within the error limits may indicate that a quasi-discrete relaxation length exists in the range of the continuous spectmun for water, much as quasi-discrete damping constants can exist in the non-steady-state thermalization problem (see, for example, Refs [13, 14]. Unfortunately, no final conclusion can be drawn as to the existence of $L_{1}$ in water on the basis of this experiment, because there are not enough $Z$ points in the asymptotic part.

### 5.2. Rethermalization lengths in graphite and water

The rethermalization lengths in graphite and water were obtained by analysing the spatial dependence of the neutron gas temperature using the method described in section 4 .

The parameters $g$, $b$ and $\frac{1}{L_{r}}$ were determined by minimizing the corresponding functionals by the method of successive approximations, using the "Tempel-3" programme [10] based on formula (6). Minimization was done in the range $Z=0-3 \mathrm{~cm}$ for water and $Z=0-29.5 \mathrm{~cm}$ for graphite (here $Z$ is the distance from the thermal barrier). The rethermalization lengths $L_{r t}$

[^1]for graphite and water were calculated from relaxation lengths obtained with the method of overlapping groups, using Eq. (5a). The results obtained for graphite are given in Table 3 and for water in Table: 4.

It can be seen that the neutron rethermalization length in graphite depends strongly on the graphite temperatare. This reflects the chemioel bond effects in graphite. The results agree well with Bennett's results [11], which indicates that the rethermalization length in graphite approximates the true primary relaxation length.

The rethermalization characteristics of waier were calculated with the "Emodis" programme [10]. The differential scattering cross-sections for water were calculated with the Nelkin model [1.5] using G.F. Liman's "PRADIS" programme. The results of the calculations are given in Table 5.

The neutron rethermalization lengths in water obtained here by analysing the experimental curve of neutron gas temperature and those obtained in Refs [11, 12] are at variance, coming within the range $0.26-0.57 \mathrm{~cm}$. No specj.fic dependence or the temperature of the neighbouring region was observed. The neutron rethermalization lengths in water obtained with the aid of the Nelkin model do depend on the temperature of the neighbouring region, though only very slightly.

The reason for the disagreement, in our opinion, may lie in the fact that the rethermalization lengths were obtained with constants $D(E), \Sigma_{a}(E)$, $\Sigma_{S}(E)$ and $\mu(E)$ derived from independent experiments and averaged over the trial functions of both regions. This means that the rethermalization lengths depend directly on the choice of initial constants and the form of the trial functions.

In addition, the disagreement of the rethermalization lengths in water indicates that a neutron physics characteristic of water such as the rethermalization length is not universal. This disagreement, and also the fact that all the rethermalization lengths are much lower than the limiting value for the true discrete relaxation leneths in water ( $\sim 0.74 \mathrm{~cm}$ ), may he taken as additional evidence of there being no primary relaxation length in water.

Table 1
Key:
Серия $A / 1=$ series $A / 1 /$
Серия $\bar{B} / 2=$ series $B / 2 /$
Table?
Neutron flux relaxation lengths in water
Key:

| Номер эксперимента | $=$ Number of experiment |
| :--- | :--- |
| По какой формуле вычислено | $=$ Formula employed |
| Длины релаксации, см | $=$ relaxation lengths, cm |

Table 3
Rethermalization lengths in graphite

Table 4
Rethermalization lengths in water

## Table 5

Retheralization properties of water (Nelkin model*/)

* Scattering by oxygen was taken into account in accordance with the gas model (the mass of the nucleus was assumed to be 16).

Table 1

|  |  | Серия A/I/ |  |  |  | Серия 5/2/ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $T_{c}$ | ${ }^{\circ} \mathrm{K}$ | 443 | 594 | 725 | 823 | 133 | 133 | 303 | 359 | 443 |
| $T_{H_{2}} \mathrm{O}$ | ${ }^{\circ} \mathrm{K}$ | 238 | 302 | 305 | 305 | 297 | 343 | 303 | 304 | 294 |

Неравномерность температури по объему граіпта составляла $7^{\circ}$, а воин $3^{\circ}$.
Экоперименти при одинаковой темпратуре граМита й воли бнли иговеденн с цельш нонтроля.

Длини релаксадии потока нейтронов в воде

| Номөр өкоперимента |  | I | 2 | 3 | 4 | 5 | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| По raroit dopмуде внपाсллепо | Te | 133 | 133 | 443 | 594 | 725 | 823 |
|  |  | 297 | 343 | 298 | 302 | 305 | 305 |
| (2) | $L 0$ | $3,0 \pm 0.15$ | 3,04土0,15 | $3,2 \pm 0,2$ | - | - | - |
|  | 14 | 0,45 $\pm 0,15$ | 0,43 $\pm 0,15$ | - | - | - | - |
| (6) | $L_{2}$ | $0,29 \pm 0,03$ | 0,30 $+0,03$ | 0,37 $\pm 0,04$ | 0,43 $\pm 0,04$ | 0,36 $\pm 0,04$ | 0,40 $+0,04$ |
|  | $L_{1}$ | $0,27 \pm 0,03$ | 0,27 $\pm 0,03$ | 0,33 $\pm 0,03$ | 0,37 $\pm 0,04$ | $0,32 \pm 0,04$ | $0,35 \pm 0,03$ |
| (7) | $L_{2}$ | $0,40 \pm 0,04$ | 0,45 $\pm 0,05$ | 0,50+0,05 | 0,47 $\pm 0,05$ | 0,50 $\pm 0,05$ | 0,48 $\pm 0,05$ |
|  | $L_{1}$ | $0,35 \pm 0,04$ | 0,39+0,04 | 0,42 $\pm 0,04$ | 0,40 $\pm 0,04$ | 0,42 $\pm 0,04$ | 0,4I $\pm 0,04$ |

Table 3


|  | I | 2 | 3 | 4 | 5 | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $T_{C}, \mathrm{o}_{\mathrm{K}}$ | 133 | I33 | 443 | 594 | 725 | 823 |
| $T_{\mathrm{H}_{2}} \mathrm{O}, \mathrm{O}_{\mathrm{K}}$ | 297 | 343 | 298 | 302 | 305 | 305 |
| $T_{1}, O_{K}$ | I55 | 155 | 454 | 596 | 726 | 830 |
| $L_{2}$ ，om | $18 \pm 3$ | I5，6 $\pm 3,2$ | 7，4土0，7 | 6，3＋0，6 | 5，T I O， 5 | 4，9＋0，5 |
| $L_{2 t}, \mathrm{~cm}$ $L_{2}[11] \mathrm{cm}$ | $10 \pm 1$ | $9,2 \pm I, I$ $9,6 \pm 0,5^{\text {n }}$ ） | 5，6さ0，6 | $4,9+0,5$ $4,2+0,238)$ | 4，İ0， 4 $3,5+0,2 \times m)$ | $4,0 \pm 0,4$ $3,5+0,2$ |
| Ct［ 11$]$ ， | － | 9，0 0,0 |  | 4，2土0，2 | $3, \pm \pm 0,2$ | 3，5 $\pm 0,2$ |

※）$T_{\mathrm{e}}=144^{\circ} \mathrm{K}, \quad T_{\mathrm{H}_{2} \mathrm{O}}=283^{\circ} \mathrm{K}$
（3x）$T_{C}=523^{\circ} \mathrm{K}, \quad T_{\mathcal{H}_{2} O}=299^{\circ} \mathrm{K}$
（צx）$T_{C}=690^{\circ} \mathrm{K}, \quad T_{H_{2} O}=308^{\circ} \mathrm{K}$
（ $T_{c}=828^{\circ} \mathrm{K}, \quad T_{\mathrm{H}_{2} \mathrm{O}}=315^{\circ} \mathrm{K}$


|  | I | 2 | 3 | 4 | 5 | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $T_{C}, \mathrm{o}_{\mathrm{H}}$ | 133 | 133 | 443 | 594 | 725 | 8\%3 |
| $\mathrm{TH}_{\mathrm{H}_{2}} \mathrm{O}^{0} \mathrm{O}$ | 297 | 343 | 298 | 302 | 305 | 305 |
| $T_{2},{ }_{0}$ | 307 | 356 | 303 | 310 | 3 O 0 | 315 |
| $L_{2}, \mathrm{~cm}$ | $0,30 \pm 0,03$ | 0,47 $\pm 0,05$ | 0,52 $\pm 0,05$ | $0,44 \pm 0,04$ | $0,45 \pm 0,04$ | $0,48 \pm 0,05$ |
| $L_{2 t, c m}$ | 0,26 $\pm 0,03$ | $0.4 \mathrm{I} \pm 0,04$ | 0,44土0,04 | $0,38 \pm 0,04$ | $0,39 \pm 0,04$ | $0,41 \pm 0,04$ |
| $L_{2 L}[11], \mathrm{cm}$ | - | - | 0,35 $\pm 0,04^{*}$ | $0,36 \pm 0,04 * *)$ | $0,55 \pm 0,05^{* \% \% \%}$ | - |

x) $T_{H_{2} \mathrm{O}}=292^{\circ} \mathrm{K}, \quad T_{\mathrm{C}}=410^{\circ} \mathrm{K}$
(天) $\quad T_{H_{2} \mathrm{O}}^{2}=293^{\circ} \mathrm{K}, \quad T_{G}=558^{\circ} \mathrm{K}$
*a*) $\quad T_{H_{2}} \mathrm{O}=295^{\circ} \mathrm{K}, \quad T_{C}=720^{\circ} \mathrm{K}$
Table 5
Ретериалиаадионнне овоНотва воды (иодель Нелкинан)

|  | $\mathrm{T}_{2} \cdot{ }^{\circ} \mathrm{H}$ | $\mathrm{T}_{\mathrm{H}_{2} \mathrm{O}}{ }^{\circ} \mathrm{K}$ | $L_{\text {D2 }}$, CM | $D_{1}, \mathrm{cH}$ | $\Sigma_{S 1}{ }^{\text {cu }}{ }^{-\mathrm{I}}$ | $\Sigma_{R}^{1 \rightarrow 2}-1$ | Lret, ${ }^{\text {cx }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 155 | 307 | 297 | 3,00 | 0,123 | 3,73 | 0,735 | 0,339 |
| 155 | 356 | 343 | 3,12 | 0,119 | 3,74 | 0,82I | 0,374 |
| 380 | 310 | 304 | 2,99 | 0,187 | 2,82 | 0,689 | 0,514 |
| 453 | 305 | 300 | 2,83 | 0,189 | 2,72 | 0,765 | 0,491 |
| 596 | 310 | 300 | 2,85 | 0,201 | 2,54 | 0,748 | 0,514 |
| 725 | 310 | 300 | 2,85 | 0,2II | 2,4I | 0,724 | 0,534 |
| 883 | 310 | 300 | 2,85 | 0,2I7 | 2,35 | 0,710 | 0,547 |




Fig. 1. Scheme of experimental rig
$1=$ Core of WWR-2 reactor, $2=$ Reflector, $3=$ Thermal insulation, $4=$ Reactor shield, $5=$ Graphite, $6=$ Channels for extracting beam, 7 = Heat shields, $8=$ Cadmium plus boron carbide, $9=$ Aluminium tank, $10=$ Water, $11=$ Movable shield of WWR-2 reactor, $12=$ Collimators, $13=$ Mechanical chopper, $14=$ Vacuum pipe, $15=$ Detector shield, 16 = Detector.


Fig. 2. Neutron spectra in graphite (at a distance of $29.5 \mathrm{~cm}(\bullet)$; 4.5 cm ( © ) from the thermal barrier) and in water (at a distance $0.2 \mathrm{~cm}(\boldsymbol{\Delta})$ and 3.0 cm ( $\boldsymbol{A}$ ) from the thermal barrier) at a graphite temperature of $594^{\circ} \mathrm{K}$ and a water temperature of $302^{\circ} \mathrm{K}$.


Fig. 3. Neutron temperature distribution in graphite and water at a graphite temperature of $133^{\circ} \mathrm{K}$ and water temperature of $297^{\circ} \mathrm{K}$ and $343^{\circ} \mathrm{K}$.

| $-\quad=$ neutron temperature |  |
| :--- | :--- |
| $---\quad$ temperature of the medium |  |
| 0 | $=$ neutron temperature |
| $---\quad=$ temperature of the medium |  |$\quad\left\{\begin{array}{l}\text { Water temperature } 297{ }^{\circ} \mathrm{K}\end{array}\right.$



Fig. 4. Neutron temperature distribution in graphite and water at a graphite temperature of $359^{\circ} \mathrm{K}$ and a water temperature of $304^{\circ} \mathrm{K}$.
$0 \quad=$ neutron temperature

-     - = $=$ temperature of the medium


Fig. 5. Neutron temperature distribution in graphite and water at a graphite temperature of $443^{\circ} \mathrm{K}$ and a water temperature of $298^{\circ} \mathrm{K}$ ( $\mathbf{K}=$ neutron temperature, $--=$ temperature of the medium) and at a graphite temperature of $725^{\circ} \mathrm{K}$ and a water temperature of $305^{\circ} \mathrm{K}$ ( $\bar{\delta}=$ neutron temperature, $--=$ temperature of the medium).


Fig. 6. Neutron temperature distribution in graphite and water at a graphite temperature of $594^{\circ} \mathrm{K}$ and a water temperature of $302^{\circ} \mathrm{K}$ ( $\delta=$ neutron temperature, $-\boldsymbol{T}^{-}=$temperature of the medium) and at a graphite temperature of $823^{\circ} \mathrm{K}$ and a water temperature of $305^{\circ} \mathrm{K}(\nabla=$ neutron temperature, $--=$ temperature of the medium).


Nig. 7. Intergral thermal neutron density in water at a graphite temperature of $133^{\circ} \mathrm{K}$ and a water temperature of $297^{\circ} \mathrm{K}$ ( 0 = integral thermal neutron density, = integral thermal neutron density after deduction of the term $P_{0} \ell^{-Z / L} L_{0}$ ).


Fig. 8. Integral thermal neutron density in water at a graphite temperature of $133^{\circ} \mathrm{K}$ and a water temperature of $343^{\circ} \mathrm{K}$. Notation same as Fig. 7.


Fig. 9. Integral thermal neutron flux in water
$0=$ graphite temperature $443^{\circ} \mathrm{K}$, water temperature $294^{\circ} \mathrm{K}$

- = graphite temperature $359^{\circ} \mathrm{K}$, water temperature $304^{\circ} \mathrm{K}$


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Yu.P. Elagin

A resonance integral is the name usually given to the quantity

$$
\begin{equation*}
I=\int_{E_{H}}^{\infty} G(E) \frac{d E}{E} \tag{1}
\end{equation*}
$$

i.e. the cross-section integrated over the Fermi slowing-down spectrum. The lower limit $E_{H}$ is usually the effective cadmium cut-off.

Resonance integrals of fission, radiative capture and absorption are distinguished by the form of the crossmsection under the integral in formula (l). A number of publications supply data on activation resonance integrals. In principle, the activation integral should coincide numerically with the capture integral, but often these values are identified separately in accordance with the method of measurement. Normally the mechanism of only one disintegration is analysed in activation measurement and the resultant activity may or may not be proportional to the total absorption cross-section.

Near an isolated resonance the cross-section of, say, radiative capture $\sigma_{\gamma}(E)$ is described by the Breit-Wigner formula:

$$
\begin{equation*}
\sigma_{C}(E)=\sigma_{0} \frac{F}{\Gamma} \sqrt{E_{0}}\left[1+\left(\frac{E-E_{0}}{\frac{C}{2}}\right)^{2}\right]^{-1} \tag{2}
\end{equation*}
$$

where $\sigma_{0}=4 \pi \lambda_{0}^{2} g \frac{\Gamma_{n}}{\Gamma}$ is the total maximum cross-section in the resonance, $E$ and $E_{o}$ are the incident neutron and resonance energies respectively, $\Gamma$ is the total resonance width, $\Gamma_{n}$ and $\Gamma_{\gamma}$ are the partial widths for emission of a neutron and a gamma quantum, $2 \pi \lambda_{0}$ is the neutron wavelength at the resonance energy and $g$ is the statistical weight of the level. If the conditions $E_{0} \gg k T$ and $E \gg \Gamma$ are fulfilled and if it is assumed that $\sqrt{\frac{E_{0}}{E}} \sim 1$
in the resonance region, we obtain

$$
\begin{equation*}
I_{r}=4120 g \frac{r_{11} r_{r}}{E_{o}^{2} r} \tag{3}
\end{equation*}
$$

It is usually assumed that the total capture resonance integral $I_{\gamma}$ is described sufficiently accurately by the expression

$$
\begin{equation*}
I_{\gamma^{r}}=\sum_{i} I_{i}^{(i)}+I_{i^{\prime}}^{\frac{1}{v}} \tag{4}
\end{equation*}
$$

where all the resonances $i$ are summed and

$$
\begin{equation*}
I_{\gamma}^{\frac{1}{v}}=\frac{\operatorname{Gr}(2200 \mathrm{~s} / \mathrm{cen})}{2} \tag{5}
\end{equation*}
$$

The resonance integrals of fission and absorption can be represented in a form similar to Eq. (4).

Note that the $I^{l / v}$ component of the integral is sometimes omitted from the resonance integrals supplied by investigators.

The table below lists the resonance integrals of infinite dilution for isotopes with $Z \geqslant 90$. The data are arranged as follows:
(1) The first column indicates the type of resonance integral -

$$
\begin{aligned}
\text { abs } & =\text { absorption } \\
\text { fiss } & =\text { fission } \\
\text { act } & =\text { activation } \\
\text { capt } & =\text { radiative capture }
\end{aligned}
$$

If it is known that a particular experimental value does not contain the $I^{l / v}$ term, this is indicated with an asterisk.
(2) The second column shows the value and the associated error (in barns) obtained by the author(s).
(3) The third column shows the cut-off $E_{H^{*}}$. The value of $E_{H}$ is given in tenths of electron-volts. The symbol TP corresponds to the thermal spectrum.
(4) The fourth column indicates the method used to obtain the resonance integral, and the following abbreviations are used:

ACT - activation method
TOF - time-of-flight
BUR - burn-up in reactor
GAM - total gamma radiation of fission products
ION - ionization chamber
MSA - mass spectrographic analysis of fission products
OSC - oscillator method
EST - estimated from known data
CAL - calculated from resonance parameters
REA - reactivity method: variations in reactivity
REC - recommended value
FIS - fission counter
COM - comparison (back to back)
THE - theoretical estimate
(5) The fifth column shows the normalization standard employed. In most measuring procedures the values obtained are normalized to some standard. The most generally used standards are the isotopes ${ }^{197} \mathrm{Au}$ and ${ }^{59} \mathrm{Co}$. The values adopted for the resonance integrals of these isotopes have varied to some extent with time. The brackets contain the value of the integral (in barns) used by the author(s).
(6) The sixth column gives the reference to the publication from which the data were obtained. The following abbreviations were employed:

| AE - Aktiebolaget atomenergi, report series | Sweden |
| :--- | :--- |
| AECL - Atomic Ener. of Canada Limited, C.R., rep. ser. | Canada |
| AEEN - Winfrith report series | UK |
| ANL - Argonne National Lab. report series | USA |
| ANS - Trans. American Nucl. Soc. | USA |
| AT - Atomn. Energ. | USSR |
| BAP - Bull. American Phys. Soc. | USA |
| BAPS - Bull. American Phys. Soc. | USA |
| BICJD - Bjul. inf. centr jad Dannym | USSR |
| BNL - Brookhaven National Lab. report series | USA |

CEA/R - Centre d'etude nucleaires, Saclay, rep. ser. France CYP - Canadian Journal of Physics Canada
CRC - Nat. Res. Coun. of Canada, C.R. rep. ser. Canada
CRRP - Chalk River report series Canada
DP - Du Pointde Memours Co. Savannah

- river lab. reports USA

GA - General Atomic Div., General Dynamic Corp. rep. ser. USA
IN - Reports Idaho op-office, AEC UK
JNE - Journ. Nuclear Energy Netherlands
NP - Nuclear Physics USA
NSE - Nuclear Science and Engineering Japan
NST - Nuclear Science and Technology
Nucl - Nucleonics USA
ORNL - Oak Ridge Nat. Lab. report series USA
PR - Physical Review USA
WAPD/T - Westinghouse, Atomic Power Div. rep. ser. USA
WASH - AEC, Washington reports to the NCSAG UK
PrNE - Progress in nuclear energy, London
55 Geneva. International Conference on the Peaceful Uses of Atomic Energy (Proc. Conf. Geneva, 1955) UN, New York (1956).
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68 Washington. Neutron Cross-Sections and Technology (Proc. Conf. Washington, D.C., 1968) Goldman, D.T., Ed., Gov. Printing Office, NBS Spec. Publication No. 299, Washington (1968).
(7) The seventh column indicates the laboratory where the results were obtained, and the following abbreviations are used:

AE-AB - Atomenergi, Studsvik, Stockholm Sweden
ANL - Argonne National Laboratory USA
BET - Westinghouse, Bettis Atomic Power Lab., Pittsburgh USA
BNL - Brookhaven National Laboratory USA
BNW - Battelle-Northwest, Richland, Wash. USA
CRC - Chalk River, Ontario Canada
FAR - Fontenay-aux-Roses, Seine France
GA - General Atomic, San Diego, Califormia USA
GEL - B.C.M.N. EURATOM, Geel Belgium

| HAR - AERE, Harwell | UK |
| :---: | :---: |
| Hanf - Hanford Laboratories | USA |
| JAE - Japan Atomic Energy Research Inst. | Japan |
| KAP - Knolls Atomic Power Lab., New York | USA |
| KFK - Kermforschungszentrum, Karlsruhe | FRG |
| LRL - Lawrence Radiation Lab., Livermore | USA |
| MTR - Phillips Petroleum Co. Idaho Falls | USA |
| ORL - Oak Ridge National Laboratory | USA |
| SRL - Savannah River Laboratory | USA |
| SWD - AB Atomenergi, Stockholm | Sweden |
| WES - Westinghouse Research, Pittsburgh | USA |
| WIN - AEE, Winfrith | UK |
| The eighth column gives the date (year) of publication. |  |
| etermine the resonance integral (see formula (1)), the microscopic |  |
| tion is integrated over the l/E flux. However, very few experi- |  |
| stems have exactly this spectrum. Sometimes investigators make |  |
| s to the measured values to allow for spectra deviating from the |  |
| by the authors without any assessment of |  |

Table

## Resonance integrals of isotopes with $Z \geqslant 90$

Key:

| Ak | = ACT |
| :---: | :---: |
| ВП | $=\mathrm{TOF}$ |
| Вж | $=B U R$ |
| Га | $=\mathrm{GAM}$ |
| ИК | $=$ ION |
| MC | $=\mathrm{MSA}$ |
| Oc | = OSC |
| Oц | = EST |
| РП | = CAL |
| Pe | $=\mathrm{REA}$ |
| Рк | $=\mathrm{RBC}$ |
| CH | $=\mathrm{FIS}$ |
| Cp | $=\mathrm{COM}$ |
| Te | $=$ THE |

```
Mин = min
AO = AT
```

Женева = Geneva
Сан-Диего = San Diego
Вашинг = Washing.
Париж = Paris
БИЦЯД = BICJD
БИЦЯД5 $=$ ВКDC

Table
Резонанснне интегралн изотопов с 90.

| 11 | - 2 | 3 | 4 | 5 | 6 | $?$ | 8 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Th |  |  |  |  |  |  |  |
| abs | $84 \pm 4$ | 5 | Oc | B | CEA/R-2486 | far | 64 |
| Th - 229 |  |  |  |  |  |  |  |
| fiss | 240 |  |  |  | A才 8,47 |  | 60 |
| Th -230 |  |  |  |  |  |  |  |
| capt | $996 \pm 40$ | 5 | Ar | $\mathrm{Co}(74)$ | CJP 40,194 | CRC | 62 |
| abs | $1020 \pm 30$ | 0,5 |  |  | PR 176,1421 | ANL | 68 |
| Th-232 |  |  |  |  |  |  |  |
| act | 69,8 |  | Oc | Au(1558) | PrNE 1,179 | ORL | 56 |
| act | $67 \pm 5$ | 5 | AR | Au | JNE 2,243 | ORL | 56 |
| act | $67 \pm 3$ | 5 | $\mathrm{AK}^{\prime}$ | Au, In | 2,22 |  | 57 |
| act | $61,8 \pm 12,0$ |  | Pe | Li $(32,2)$ | 3,507 |  | 57 |
| abs | 93 |  | PII |  | NSE 4,649 | AI | 58 |
| abs | $85 \pm 10$ | 5 | OL |  | NSE 6,100 | ORL | 59 |
| abs | $106 \pm 10$ |  | Oc | Au(1513) | JNE 12,32 | HAR | 60 |
| act | $85 \pm 10$ | 5 | AK | Au(1565) | JNE 11,95 | ORL | 60 |
| act | $83 \pm 6$ | 3,5 | $A K$ | Au(1510) | AEEH 163 | WIM | 62 |
| act | $84 \pm 5$ |  | AK | Au(1561) | GA -3069 | GA | 62 |
| act | 82,7さ1,8 | 5 | AK | Au(1462) | TSE 19,244 | KFK | 64 |
| abs | $87 \pm 4$ |  | Oc | $\mathrm{Au}(1540)$ | CEA -2486 | PAR | 64 |
| abs | 81, $2 \pm 3,4$ | 5 | Pe | Au (1579) | NSE 21,406 | HTR | 65 |
| abs | 82,5 $\pm 1,7$ | 5 |  | Au | TSE 22,121 | BET | 65 |
| capt | $79 \pm 4$ |  | PII |  | FP 76,196 | HAR | 66 |
| abs | 84 | 5 | PK |  | Nucl 24,108 | GA | 66 |
| abs | $86 \pm 5$ | 5 |  |  | PR 155,1330 | BNL | 67 |
| Th - 233 |  |  |  |  |  |  |  |
| capt | $500 \pm 150$ | 3 |  |  |  | ORL | 58 |
| capt | $400 \pm 100$ | 5 | On |  | ISE 6,100 | ORI | 59 |
| act | $400 \pm 100$ | 5 | AK | $\mathrm{Au}(1555)$ | JNE 11,95 | ORL | 60 |
| Pa - 23.31 |  |  |  |  |  |  |  |
| abs | 1200 | 1 |  |  | BAPS 4,414 | MTR | 59 |
| 2 ab | $1200 \pm 200$ |  |  |  | WASE 1029 | UTR | 60 |
| abs | $1560 \pm 55$ | 1 | PII |  | FSE 12,243 | HTR | 62 |
| abs | 480 | 5 | PR |  | Nucl 24,108 | GA | 66 |
| Per 233 |  |  |  |  |  |  |  |
| abs | 1072 | 3 | PK |  | 58 Іенева | ORL | 58 |
| abs | $1200 \pm 400$ | 5 | OLI |  | ISE 6,100 | ORL | 59 |
| act | $470 \pm 90$ | 5 | Ar | Co(75) | CJP 38,751 | CRC | 60 |
| ( $-\mathrm{I}, 2$ мин ) |  |  |  |  |  |  |  |



| capt | $700 \pm 100$ | 5 | OL |  | NSE 6，100 | ORL | 59 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| abs | 665 | 5 | Pr |  | Nucl 24，108 | GA | 66 |
| capt | 700 | 5，5 | PK |  | BNL 982，22 | BNL | 66 |
| 0－235 |  |  |  |  |  |  |  |
| fiss | 271 |  | $A_{k}$ | Au（1558） |  | ANL． | 55 |
| fiss | 292 | 4 |  |  | WASH－192 |  | 57 |
| fiss | $300 \pm 50$ | 5 | OL |  | NSE 6，100 | ORL | 59 |
| fiss | $276 \pm 11$ | 5 | 「a | Au（1535） | NSE 9，341 | BET | 61 |
| Piss | $263 \pm 12$ | 6 | $\Gamma \mathrm{Ca}$ | $\mathrm{Au}(1535)$ | DP－817 | SRL | 63 |
| Piss | $272 \pm 8$ | 4，5 | Ta | Au（1535） | CRRP－1183 | CRC | 64 |
| fiss | $298 \pm 14$ | 4，5 | 「a | In（2790） | CRRP－1483 | CRC | 64 |
| Piss | 27549 |  | 「a | Au | EANDC（E）331 | SWD | 64 |
| fiss | $274 \pm 10$ |  |  |  | BNL 325 Sup2 |  | 64 |
| fiss | $288 \pm 18$ | 3，9 | Ak | Au（1553） | ANS 7，78 | EAP | 64 |
| fiss | 292 | 4，1 | On |  | GA－5944 | GA | 64 |
| fiss | $279 \pm 8$ | 5 | 「a | $\mathrm{Au}(1550)$ | AE 181 | $A B$ | 65 |
| fiss | $269 \pm 16$ | 5 | AK | Co $(72,5)$ | 66 Сан－диего | KAP | 66 |
| P1ss | 280 | 5 | PK |  | Nucl 24，108 | GA | 66 |
| P1ss | $275 \pm 16$ | 5 |  | Co | TSE 29，1 | BET | 67 |
| fiss | 222，0土2，0 | 62 | Cp |  | 68 Вашинг．，475 | GEL | 68 |
| fiss | 258 | 5 | СД |  | ISE 35,350 | LRL | 69 |
| flss | $274 \pm 11$ | 5 |  |  | AЭ 28，359 |  | 70 |
| abs | $450 \pm 100$ | 5 | On |  | TSE 6，100 | ORL | 59 |
| abs | 445 | 4，1 | OL |  | GA－5944 | GA | 64 |
| abs | 420 | 5 | Pr |  | Hucl 24，108 | GA． | 65 |
| abs | 380 |  |  |  | 68 Вашинг．，I2？ | ISRL | 68 |
| act | $271 \pm 25$ |  |  |  | ANL -5800 | ANL． | 58 |
| capt | $150 \pm 50$ | 5 | On |  | INSE 6，100 | ORL | 59 |
| sapt | $144 \pm 5$ |  | HC | Au（1535） | EANDC（can）20L | CRC | 63 |
| capt | $148 \pm 7$ | 3，9 | MC | $\mathrm{Au}(1535)$ | ANS 7，78 | IAP | 64 |
| capt | $133+7$ | 5 | MC | Co $(72,5)$ | 66 Сан－Диего | KAP | 66 |
| capt | 140 | 5 | Pk |  | Hucl 24，108 | GA | 66 |
| capt | $143 \pm 7$ | 5 |  |  | 66 Парккх 2,17 | CRC | 66 |
| capt | $136+8$ | 5 |  |  | ISSE 29，4 | BET | 67 |
| capt | $13 \mathrm{~A} \pm 8$ | 1，${ }^{\text {a }}$ |  |  | 49 24，351 |  | 68 |
| －-236 |  |  |  |  |  |  |  |
| abs | 310 |  |  |  | BAP－1，487 66 | BNL | 56 |
| act | 400 |  |  |  | WASE－191 | ORI | 56 |
| act | 350 |  |  |  | AINL－5800 | AIIL | 58 |
| act | $350 \pm 40$ |  |  | Au | 58 тенева | ORL | 58 |
| act | $257 \pm 22$ | 5 | Ak | Co $(48,6)$ | JNB 7，81 | CRC | 58 |
| oapt | $400 \pm 100$ | 5 | O4 |  | ISS 6，100 | ORL | 59 |


|  | 2 | 3 | 4 | 1 - 5 | 1-_---6 | $?$ | 18 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| capt | 381 | 5 | AR | $\mathrm{Au}(1558)$ | HASH 9041 | HTR | 62 |
| capt | 320 | 5 | PE |  | Hucl 24,108 | GA | 66 |
| capt | $400 \pm 40$ | 5,5 | $\mathrm{P}_{\mathrm{K}}$ |  | BNL 982,22 | BNL | 66 |
| capt | $350 \pm 25$ | 5 |  |  | GA 9057 | GA | 68 |
| capt | $417 \pm 25$ | 5 | AK |  | NSB 32,265 | SRT | 68 |
| abs | $400 \pm 40$ |  |  |  | 68 Вашииг. I27I | SRL | 68 |
| oapt | $350 \pm 25$ | 5 |  |  | NP A141, 577 |  | 70 |
| capt | 300 | 5 | PII |  | БицЋД 5, 159 |  | 68 |
| U - 237 |  |  |  |  |  |  |  |
| abs | 290 |  |  |  | 68 Вашинг., 2271 | SRL | 68 |
| J - 238 |  |  |  |  |  |  |  |
| abs | $276 \pm 12$ | 5 |  |  | PR 99,10 | BNL | 55 |
| act | $281 \pm 20$ | 5 | MC | Au(1558) | JNB 2,243 | ORL | 56 |
| act | $279 \pm 20$ |  |  |  | PR 105,661 | ANL | 57 |
| abs | $224 \pm 40$ |  | Oc | Li(71) | A3 3,507 |  | 57 |
| abs | 269 |  | PII |  | NSE 4,649 | AI | 58 |
| act | $280 \pm 15$ | 5 | On |  | NSE 5,100 | ORI | 59 |
| abs | $285 \pm 25$ |  | 0 c | Au(1513) | JNEA 12,32 | HAR | 60 |
| act | $282 \pm 8$ | 5 | AK | $\mathrm{Au}(1535)$ | NSB 14,358 | WES | 62 |
| abs | $280 \pm 10$ | 6,2 |  | Au | DP -817 | SRL | 63 |
| act | $277 \pm 10$ | 5,5 | $\Gamma \mathrm{Ca}$ | Au(1535) | ANS $7{ }_{\\|} 27$ | SRL | 64 |
| capt | $280 \pm 12$ |  | Pr |  | BNL325 Sup 2 | BNL | 66 |
| capt | 278 | 5 | Pra |  | Nuel 24, 108 | GA | 66 |
| abs | 270 |  |  | Au(1579) | NSE 25,12 |  | 66 |
| Fp-237 |  |  |  |  |  |  |  |
| abs | $870 \pm 130$ |  | 0 c | $\mathrm{Au}(1510)$ | 58 7онәва | HAR | 58 |
| abs |  | 27 |  |  | AD 6,569 |  | 59 |
| abs | $945 \pm 430$ |  | 0 C | Au(1513) | JNRA 1?,32 | HAR | 60 |
| capt | 500 | 5,5 | ${ }^{P} \mathrm{PR}$ |  | BNL 983,22 | BNL | 60 |
| abs | 905 | 5 |  | Au(1558) | ANS 10,259 | MTE | 57 |
| abs | 850 |  |  |  | 68 Ванинг. ,I27I | SRL | 68 |
| abs | $900 \pm 30$ | 5 | Pe | Au | Is -1195 | MTIR | 68 |
| crpt | $715 \pm 5$ | 5 | PII |  | БицДД 5. 159 |  | 68 |
| 耳p - 238 |  |  |  |  |  |  |  |
| abs | $1500 \pm 500$ |  |  |  | 68 Вашинг.,I27I | SRL | 68 |
| 耳p - 239 |  |  |  |  |  |  |  |
| ebs | 415 | 5 | Pr |  | Nuel 24,108 | GA | 66 |
| Pu - 238 |  |  |  |  |  |  |  |
| ospt | $3260 \pm 280$ | 5 | 10 | Co $(36,4)$ | CJP 3:i,147 | CRC | 57 |
| fiss | $25 \pm 5$ | 6 |  |  | 58 सereba | CRC | 58 |
| Ifse | 25 | 5,5 | Pr |  | BNL 982,22 | B8L | 66 |
| orpt | 150 | 5,5 | $\mathrm{F}_{2}$ |  | BKL 982,22 | BKL | 66 |



| 1 | 2 | 3 | 4 | 5 | 1 | 6 | 17 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| abs | $2800 \pm 500$ | 1,5 | Oq |  | NSE | 6,100 | ORL | 59 |
| abs | 712 | 3 | Pr |  | Nucl | 24,108 | GA | 66 |
| abs | $1389 \pm 15$ |  |  |  | WASH | -1136,43 |  | 69 |
| capt | $1000 \pm 300$ | 1,5 | On |  | NSB | 6,100 | ORL | 59 |
| capt | 139 | 5 | Pr |  | Nucl | 24,108 | GA | 66 |
| capt | 260 | 5,5 | Pr |  | BNL | 982,22 | BNL | 66 |
| Pu - 242 |  |  |  |  |  |  |  |  |
|  | 0,6 |  |  |  | ANS | 10,228 | BNL | 67 |
| abs | $1275 \pm 30$ | 5 | HC | Co $(48,6)$ | CJP | 35,147 | CRC | 57 |
| capt | $1300 \pm$ 20 | 1,5 | OX |  | NSE | 6,100 | BNL | 59 |
| capt | $1050 \pm 150$ | 5 |  | Co | PR | 114,505 | ANL | 59 |
| abs | $1280 \pm 60$ |  | Ax | Co(75) | ORLN | -3679,13 | ORL | 64 |
| capt | 1100 | 5 | Pr |  | Nuel | 24,108 | GA | 66 |
| capt | 1150 | 5,5 | Px |  | BNL | 982,22 | BNL | 66 |
| ant | 1180 | 9,2 | AE | Co(75) | 68 Ba | тинг., I279 | SRI | 68 |
| capt | 1061 | 5 | PII |  |  | 凹Д 5, I59 |  | 68 |
| Pu - 244 ( |  |  |  |  |  |  |  |  |
| capt | $35 \pm 7$ |  |  | $\mathrm{Au}(1558)$ | MASH | 1136,51 | UTE | 69 |
| Anm - 249 |  |  |  |  |  |  |  |  |
| fiss | 8,5 | 5,5 | Te |  | BNJ. | 982,22 | BNL | 66 |
| fiss | $21 \pm 2$ | 5 |  | Au | AS | 23,316 |  | 67 |
| fiss | $21 \pm 2$ | 5 |  |  | AS | 28,359 |  | 70 |
| act | 900 |  | Ar | Au(1558) | WASH | 1053,76 | MTH | 64 |
| capt | 1600 | 5,5 | Te |  | BNL | 982,22 | BNL | 66 |
| capt | $1470 \pm 135$ | 5. | PII |  | PR | 114,505 | ANL | 59 |
| capt | $\begin{aligned} & 2100 \pm 200 \\ & (-16 \text { чac }) \end{aligned}$ | 5 |  | Au | AS | 23,316 |  | 67 |
| capt | $\begin{aligned} & 300 \pm 30 \\ & (-152 \text { ч. }) \end{aligned}$ | 5 |  | Au | A9 | 23,316 |  | 67 |
| cept | $\begin{aligned} & 850 \pm 60 \\ & (-16 \text { प8c }) \end{aligned}$ | 5 |  | Co $(74,6)$ | WASH | 1136,53 | MTR | 69 |
| capt | $\begin{aligned} & 250 \pm 40 \\ & (-152 \text { ч. } \end{aligned}$ | 5 |  | $\operatorname{Co}(74,6)$ | WASH | 1136,53 | MTR | 69 |
| capt | 1472 | 5 | PII |  | Бич | , 5, 159 |  | 68 |
| Am - 242 |  |  |  |  |  |  |  |  |
| Am - $242^{\text {min }}$ |  |  |  |  |  |  |  |  |
| abs | $7000 \pm 2000$ | 5 |  | Co $(74,6)$ | WAST | 1136,53 | MTR | 69 |
| figs | 1570 | 5 |  |  | PR | 166,1219 | IRL | 68 |
| 1188 | $1570 \pm 110$ | 5 |  |  | NSE | 32,131 | IRL | 68 |
| As - 243 |  |  |  |  |  |  |  |  |
| 1188 | 1,5 |  | Te |  | AISS | 10,228 | BSIL | 67 |



| 1 | , | 3 | 4 | -_5 |  | 6 | 1.7 | 1 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| capt | 1850 |  | Te |  | ANS | 10,228 | ENL | 67 |
| aot | 1240 | 5,5 |  | Co(75) | 68 B | Вашинг. , I279 | SRS | 68 |
| CP - 250 |  |  |  |  |  |  |  |  |
| fiss | 85 |  | Te |  | ANS | 10,228 | BNL | 67 |
| capt | 940 |  | Te |  | ANS | 10,228 | BNL | 67 |
| act | 5300 |  |  | Co(75) | 68 B | Вашинг. , I279 | SRL | 68 |
| CX - 251 |  |  |  |  |  |  |  |  |
| f1as | 445 |  | Te |  | AHS | 10,228 | ENL | 67 |
| capt | 172 |  | Te |  | ANS | 10,228 | BNL | 67 |
| aot | 980 |  |  | Co(75) | 68 B | Вамииг. ,I279 | SRL | 68 |
| CI - 252 |  |  |  |  |  |  |  |  |
| P188 | 5 |  | Te |  | ANS | 10,228 | BINL | 67 |
| capt | 1800 |  | Te |  | ANS | 10,228 | BNL | 67 |
| act | 42 | 9,2 |  | $\mathrm{Co}(75)$ | 68 B | Вашинг. , 1279 | SRL | 68 |
| act | 43,5+3 | 5 |  |  | ESE | 27,228 | ORI | 69 |
| capt | 44 |  |  |  | ORNL | L 4428 | ORL | 69 |
| 91-253 |  |  |  |  |  |  |  |  |
| 1iss | 198 |  | Te |  | ANS | 10,228 | BNL | 67 |
| oapt | 540 |  | Te |  | ANS | 10,228 | BNL | 67 |
| Of-254 |  |  |  |  |  |  |  |  |
| 1188 | 28 |  | Te |  | ANS | 10,228 | BNL | 67 |
| capt | 1650 |  | Te |  | ARS | 10,228 | BNL | 67 |
| capt | 1650 |  |  |  | ORNT | -4428 | ORL | 69 |
| $\mathrm{Bm}-253$ |  |  |  |  |  |  |  |  |
| not | 3600 | 5.5 |  | Co(75) | 68 B | Ваминг. ,I279 | SRL | 68 |

USE OF THE ITERATION METHOD FOR RAPID UNFOLDING OF A SPECTRUM OF ARBITRARY FORM

V.S. Troshin, E.A. Kramer-Ageev, K.D. Vasilev<br>E.I. Grigorev, G.B. Tarmovsky, V.P. Yaryna

Work is being carried out in the All-Union Scientific Research Institute of Physico-Technical and Radiotechnical Measurements aimed at standardizing the methods and equipment used for neutron measurements, and the Institute is now preparing, in conjunction with the Moscow Engineering-Physics Institute, to issue instructions for unfolding fast neutron reactor spectra from measurements of induced activity in threshold detectors. These instructions are centred around a rapid method for unfolding fast neutron spectra [1], using the effective threshold crossmsections.

The effective threshold crossmsection calculated for different types of spectra are given in Refs $[2-5]$. Ref. [5] recommends optimum values of the effective threshold ( $E_{\text {eff }}$ ) and the effective threshold crossmsection ( $\sigma_{\text {eff }}$ ) for a group of spectra comprising the fission neutron spectrum and fission neutron spectra after passage of neutrons through layers of polyethylene, carbon, iron, lead and nickel. The values obtained agree with EURATOM recommendations [4].

When unfolding neutron spectra not included in the above group, errors may arise due to differences between the recommended effective threshold crossmsections and the crossmsections corresponding to the spectrum being constructed. To increase the accuracy of unfolding such spectra, we propose a method of iterative refinement of the results of the unfolding which involves redetermining the values of $\sigma_{\text {eff }}$ for constant values of $\mathrm{E}_{\text {eff }}$. This method of iterative refinement simplifies the calculation compared with the method employed in Ref. [6], in which the effective reaction threshold is redetermined for each iteration.

The unfolding of a spectrum by the rapid iterative method is done in the following sequence:
(1) The neutron spectrum is unfolded by the method described in Ref. [1], using the effective threshold crossmsections supplied in Refs $[1,5]$ (zero approximation).
(2) Using the exponential form of the spectrum obtained in the zero approximation $\left[\rho^{\circ}(E)\right]$ and the well-known dependence of reaction crossmsection on energy $\left[\sigma_{i}(E)\right]$, we calculate the activation integrals $R_{i}^{0}$ for the $i^{\text {th }}$ reaction:
$R_{i}^{0}=\int_{0}^{\infty} G_{i}(E) \cdot l^{0}(E) d E \simeq \sum_{k=1}^{n} \varphi^{0}\left(E_{k}\right) \cdot \int_{E_{k}}^{E_{k+1}} G_{i}(E) \cdot e^{-\mu \mu_{k}^{n}(E-E k)} \cdot d E$.
where $f^{\circ}\left(E_{k}\right)$ is the differential flux density at the lower boundary of the $k^{\text {th }}$ interval, and the coefficient $\mu_{k}^{0}$ is calculated from the formula

$$
\begin{equation*}
\mu_{k}^{0}=\frac{1}{E_{k+1}-E_{k}} \cdot \ln \frac{\rho^{0}\left(E_{k}\right)}{\rho^{0}\left(E_{k}+1\right)} . \tag{2}
\end{equation*}
$$

For convenience we have tabulated the values of the integrals

$$
\begin{equation*}
I_{i}\left(\Lambda_{\kappa}\right)=\int_{E_{\kappa}}^{E_{k}+1} \sigma_{i}(E) \cdot e^{-\mu_{\kappa}\left(E-E_{\kappa}\right)} \cdot d E \tag{3}
\end{equation*}
$$

for $-2 \leqslant \mu_{k} \leqslant 2$ for the intervals $0.5-1.5 ; 1.5-2.5 ; \ldots$; 9.5-10.5 and $10.5-17 \mathrm{MeV}$. The dependence of $J_{i}$ on $\mu_{k}$ is given in Figs l-12, and the following numerical key has been adopted for the reactions:

$$
\begin{aligned}
& \text { 1-237 } N(n, f) ; 2-{ }^{103} R h(n, n) ; 3-15 I_{n}\left(n, r_{1}\right) ; 4-{ }^{235} U(n, f) ; \\
& 5-{ }^{32} T h(n, f) ; 6{ }^{3 \prime} p(i n, p) ; 7-{ }^{c^{4}} z_{i n}(n, p) ; 8-{ }^{32} \mathrm{~J}^{\prime}(n, p) ; \\
& \text { 9- }{ }^{59} \mathrm{Ni}(r, p) ; 10-{ }^{54} \mathrm{Pe}(n, p) ; 11-{ }^{35} C l(n, \alpha) ; 12-{ }^{27} A P(n, p) \text {; } \\
& 13^{-28} S_{i}(n, p) ; 14-{ }^{50} z x(n, p) ; 15^{-50} F e(n, p) ; 16^{-59} \mathrm{Co}(n, d) ;
\end{aligned}
$$

> For the reactions ${ }^{237} \mathrm{~Np}(n, f),{ }^{103} \mathrm{Rh}\left(n, n^{\prime}\right)$ and ${ }^{15} \mathrm{In}\left(n, n^{\prime}\right)$, the contribution of neutrons with energy less than 0.5 MeV was taken into account when calculating the integral $J(\mu)$ in the range $0.5-1.5 \mathrm{MeV}$. For the remaining reactions this contribution is negligible.
(3) The effective threshold crossmsections are derived from the calculated activation integrals in a first approximation:

$$
\begin{equation*}
\left.\vec{O}_{i \text { eff }}^{\prime}=\frac{R_{i}^{\prime}}{\infty^{\circ}\left(E_{i \text { eff }}\right)}\right) \tag{4}
\end{equation*}
$$

where $\phi^{\circ}\left(\mathrm{E}_{\mathrm{i} \text { eff }}\right)$ is the integral flux density of neutrons with energy above $E_{i}$ eff, obtained in the zero approximation.
(4) The integral neutron flux densities are determined in a first approximation, using the experimental values of the activation integrals and the effective threshold cross-sections obtained in a first approximation:

$$
\begin{equation*}
q^{1}\left(E_{i \text { eff }}\right)=\frac{R_{i} \text { exp }}{G_{i}^{x} \text { eff }} \tag{5}
\end{equation*}
$$

and the spectrum is unfolded in a first approximation [1]. The iteration process is repeated until the differential flux densities in the centres of the energy intervals obtained in two successive approximations agree to within $\sim 15 \%$ (this figure is accounted for by the errors in the method used and the calculations).

By way of example Fig. 13 shows a spectrum unfolded on the basis of two iterations which differ considerably from the fission spectrum.

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Fig. 1


Fig. 2


Fig. 3


Fige 4



Fig. 6


Fig. 7


Fig. 8


Fig. 2



Fig. 11


Fig. 12


Fig. 13 Results of unfolding a spectrum similar to the spectrum from a Po-Be source by the rapid method using two iterations.
(a) Integral spectra
$0, \triangle, \square$ are the integral neutron flux densities in the first, second and third approximations respectively. $\Phi_{0}, \Phi_{1}, \Phi_{2}$ are the zero, first and second approximations respectively.
(b) Differential spectra
$\qquad$ = initial spectrum
_ . _ . _ = unfolded spectrum - zero approximation

-     -         -             -                 - = unfolded spectrum - first approximation
_ = unfolded spectrum - second approximation


## NEUTRON SPECTRUM CALCULATIONS IN THE $P_{1}$ APPROXIMATION

## V.S. Shulepin

In Ref. [1] a system of equations was obtained for calculating the neutron spectrum in any region of a reactor in the $P_{1}$ approximation. From these results, the following algebraic equations can be written for determining the slowing-down spectrum:

$$
\left.\begin{array}{l}
\tilde{\varphi}_{1}^{j}+\sum_{0 y}^{j} \varphi_{0}^{j}=\sum_{c=1}^{j=1} \sum_{0}^{\varepsilon j} \varphi_{0}^{e}+S_{v}^{j}+S_{s}^{j},  \tag{1}\\
\frac{1}{3} L^{j} \varphi_{0}^{j}+\sum_{1 y}^{j} \widetilde{\varphi}_{1}^{j}=\sum_{c=1}^{j-1} \sum_{1}^{c \rightarrow j} \tilde{\varphi}_{1}^{\prime},
\end{array}\right\}
$$

where

$$
\begin{equation*}
\varphi_{0}^{j}=\int \varphi_{0}^{j}(\vec{z}) d v, \quad \vec{\varphi}_{i}^{j}=\int \operatorname{div} \vec{\varphi}_{i}^{j}(\vec{z}) d v \tag{la}
\end{equation*}
$$

(The integrals are taken over the volume of the region);
$\varphi_{0}^{j}(\vec{r}), \varphi_{1}^{j}(\vec{r})$ are the coefficients for expanding a flux of neutrons of the $j^{\text {th }}$ energy group $\vec{\varphi}^{j}(\vec{r}, \vec{\Omega})$ in a Legendre polynomial series relative to the angular variable $[2]$;
$\Sigma_{o y}^{j}, \Sigma_{l y}^{j}$ are the group removal cross-sections;
$\stackrel{\ell \rightarrow j}{\Sigma}{ }_{0} \quad \Sigma_{1}$ are the cross-sections for transition from group $\ell$ to group $j$;
$S_{v}^{j}, S_{s}^{j}$ are the volume and surface sources of the region respectively; and
$L_{v}^{j}$ is the leakage parameter.
The various possible ways of selecting the quantities $S_{v}^{j}, S_{S}^{j}$ and $L^{j}$ are discussed in Ref. [1].

In this paper we derive a formula with which the desired quantities $\varphi_{o}^{j}$ may be written concisely. Let us determine $\tilde{\varphi}_{1}^{j}$ from the first equation of the system in expression (1) above and $\tilde{\varphi}_{1}^{\ell}$ from similar equations, and then substitute these expressions in the second equation, so that we have:

$$
\begin{align*}
& \frac{1}{3} L^{j} \varphi_{0}^{j}+\sum_{1 y}^{j}\left[\sum_{c=1}^{j-1} \sum_{0}^{\epsilon-j} \varphi_{0}^{e}-\sum_{0 y}^{j} \varphi_{0}^{j}+S_{j}^{j}+S_{s}^{j}\right]= \\
= & \sum_{e=1}^{j-1} \sum_{1}^{e-j}\left[\sum_{k=1}^{e-1} \sum_{0}^{k \rightarrow c} \varphi_{0}^{k}-\sum_{0 y}^{c} \varphi_{0}^{e}+S_{v}^{e}+S_{s}^{e}\right] \tag{2}
\end{align*}
$$

The latter equation may be written as follows:

$$
\begin{align*}
& \frac{1}{3} \frac{L^{j}}{\sum_{i+y}^{j}} \varphi_{0}^{j}-\sum_{0 y}^{j} \varphi_{0}^{j}+\sum_{i=i}^{j-1}\left(\sum_{0}^{c+j}+A_{j}^{e}\right) \varphi_{0}^{l}+ \\
& +S_{z}^{j}+S_{s}^{j}-\frac{1}{\sum_{i y}} \sum_{e=1}^{j}\left(S_{v}^{e}+S_{s}^{e}\right)=0 \tag{3}
\end{align*}
$$

where:

Note that $\stackrel{p \rightarrow q}{\Sigma_{0}}=\sum_{i}^{p \rightarrow q}=0$ when $p \geqslant q$. It will be seen that equations (2) and (3) are completely equivalent for any $j$. The final expression for $\varphi_{0}^{j}$ takes the form:

All values of $\varphi_{o}^{j}$ can be calculated successively $(j=1,2, \ldots$.$) with$ formula (4).

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## P.E. Bulavin

In Ref. $[1]$ the neutron balance method $[2]$ was used to obtain the following expression for the effective boundary conditions for a "black" rod situated in a non-absorbing infinite medium:

$$
\begin{equation*}
\gamma=4 / 3-a \varphi(a) \tag{1}
\end{equation*}
$$

where
$r_{0}=$ the radius of the absorbing rod and $\Sigma_{s}=$ the macroscopic scattering cross-section of the medium.

However, the expression for the function $\varphi(a)$ can be greatly simplified by integrating analytically over the angles $\psi$ and $v$. In fact let us differentiate $Q(a)$ with respect to the parameter a. The following expression is then obtained for the function $W(a)=a \frac{\alpha, 0(a)}{\alpha a}$

The expression for the function $W(a)$ may now be integrated over the angle $\psi$, as a result of which we obtain

$$
\begin{equation*}
\left.W_{(a)}=\frac{2}{\pi} \int_{0}^{\pi / 2} \varphi_{\operatorname{in}}{ }^{2} v d v \int_{0}^{\infty} e^{-5}\left[\left(\frac{a}{\xi \varphi \operatorname{in} \sigma}+\frac{\xi \varphi \operatorname{in} v}{a}\right) \operatorname{arctg} \frac{\xi \varphi \operatorname{in} v}{a}-\frac{\pi}{2} \frac{\xi \sin \theta}{a}\right]\right] d \frac{1}{5} \tag{4}
\end{equation*}
$$

Integrating expression (4) over the angle $v$, we obtain

$$
\begin{equation*}
W(a)=\frac{2}{3} \int_{0}^{\infty} e^{-\xi}\left[\frac{\left(\frac{\xi}{a}+\frac{a}{\xi}\right)^{2}}{\sqrt{1+\xi^{2}} / a^{2}}-\frac{a^{2}}{\xi^{2}}-\frac{\xi}{a}-\frac{3}{2}\right] d \xi \tag{5}
\end{equation*}
$$

or, substituting ax for $\xi$, we obtain

$$
\begin{equation*}
W(a)=\frac{2}{3} a \int_{0}^{\infty} e^{-a x}\left[\frac{(x+1 / x)^{2}}{V 1+x^{2}}-1 / x^{2}-x-\frac{3}{\varepsilon}\right] d x \tag{6}
\end{equation*}
$$

Dividing Eq. (6) by a, integrating the result over a and applying the condition $\varphi(\infty)=0$, we obtain

$$
\begin{equation*}
Q(a)=\frac{2}{3} \int_{0}^{\infty} \frac{e^{-a x}}{x}\left[\frac{3}{2}+x+1 / x^{2}-\frac{(x+1 / x)^{2}}{\sqrt{1+x^{2}}}\right] d x \tag{7}
\end{equation*}
$$

or, substituting $\xi / a$ for $x$, we obtain:

$$
\begin{equation*}
P(a)=\frac{2}{3} \int_{0}^{\infty} \frac{e^{-\xi}}{\xi}\left[\frac{3}{2}+\frac{\xi}{a}+\frac{a^{2}}{\xi^{2}}-\frac{\left(\frac{\xi}{a}+\frac{a}{s}\right)^{2}}{\sqrt{1+\xi} \sqrt{a^{2}}}\right] d \xi \tag{8}
\end{equation*}
$$

When a tends to 0 , the limit $W(0)=-1$ is obtained from formula (5) whence follows the logarithmic character of the function $\varnothing(\mathrm{a})$ :

$$
\begin{equation*}
p(a)=-\ln 2 a, a<1 . \tag{9}
\end{equation*}
$$

When $a \gg 1$, by expanding the expression in square brackets in Eq. (8) into a power series of $\xi / a$ and confining ourselves to two terms of the series, we obtain

$$
\begin{equation*}
Q(a)=\frac{2}{3 a}-\frac{1}{4 a^{2}}, a>1 \tag{10}
\end{equation*}
$$

The results of calculating the functions $\boldsymbol{\phi}(a)$ and $W(a)$ from formulae (7) and (6) on the "Nairi" computer are presented in Table 1.

The accuracy of the calculations is estimated at not less than $\pm 0.0001$ for the function $\varphi(a)$ and $\pm 0.0005$ for the function $W(a)$ when $a<0.1$. When $a \geqslant 0.1$ there is an accuracy of $\pm 1$ in the last significant figure.

The calculations of the function $\varphi(a)$ differ considerably from the results given in Ref. [1] for small values of a (maximum deviation $12.6 \%$ for $a=0.025$ ).

Table 1
Values of the functions $\varphi(a)$ and $W(a)$

| $a$ | 0,025 | 0,05 | 0,075 | 0,1 | 0,2 | 0,3 | 0,4 | 0,5 | 0,6 | 0,7 | 0,8 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\phi(a)$ | 3,0258 | 2,4099 | 2,0682 | 1,8372 | 1,3279 | 1,0670 | 0,9008 | 0,7833 | 0,6949 | 0,6255 | 0,5694 |
| $W(a)$ | $-0,9099$ | $-0,8628$ | $-0,8216$ | $-0,7851$ | $-0,6797$ | $-0,6058$ | $-0,5492$ | $-0,5037$ | $-0,4660$ | $-0,4341$ | $-0,4066$ |
| $a$ | 0,9 | $I$ | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| $\Phi(a) 0,5229$ | 0,4837 | 0,2788 | 0,1965 | 0,1518 | 0,1237 | 0,1043 | 0,09023 | 0,07948 | 0,07102 | 0,06419 |  |
| $W(a)-0,3826$ | $-0,3614$ | $-0,2342$ | $-0,1737$ | $-0,138 \mathrm{I}$ | $-0,1145$ | $-0,09784$ | $-0,08538$ | $-0,7573$ | $-0,06803$ | $-0,06175$ |  |

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CHAPTER III

RADIATION SHIELDING CHARACTERISTICS AND PARAMETERS<br>RADIATION AND NUCLEAR SAFETY

SPATIAL—ENERGY DISTRIBUTION OF FAST NEUTRONS IN TWO-LAYER IRON-WATER SHIELDING

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Fairly extensive theoretical and experimental data are already available on fast neutron transport in homogeneous shielding layers of varying thickness [1]. However, this question has been far less well investigated in the case of inhomogeneous shields $[2,3,4]$.

Here the authors report on a study of the space-energy distribution of a flux of fast neutrons with energy $E>0.4 \mathrm{MeV}$ in barrier geometry in two-layer iron-water shielding. The thickness of the first layer, made of iron, was 20 cm in the first case and 35 cm in the second. The thickness of the adjacent water shield was 15 cm in both cases. The source used was a plane collimated neutron beam with a reactor spectrum [5]. The transverse dimensions of the shielding layers and the beam were $\sim 70 \mathrm{~cm}$. The neutrons were detected by means of a singlecrystal scintillation spectrometer with a stilbene crystal 3 cm in diameter and 2 cm high, with gamma-background discrimination based on de-excitation time [6]. The signal and the background were recorded with the shielding layers in place, and the intensity of the incident neutron beam was measured with the shielding removed. The pulse height distributions were recorded with a "diaphragm" 256-channel analyser and were converted into energy spectra by differentiation; the neutron recording efficiency and the light output of the stilbene crystal were allowed for by the method described in Ref. $[7]$.

The penetration of fast neutrons in the shielding compositions described above was calculated by the multigroup method of discrete ordinates in the $2 D_{7} P_{7}$ approximation $[1]$. The calculations were performed in plane unidimensional geometry for a monodirectional neutron beam, using the POZ-l programme [8]. The group constants used for iron and water were obtained
by averaging over the neutron spectra in an infinite homogeneous medium consisting of the appropriate material. These constants are given in Ref. [l]. They were verified by calculation of the experimental data for fast neutron fields in extended iron-water shields [1, 9]. Fig. l shows experimental and theoretical data on the energy distributions of a fast neutron flux with energy $E>0.4 \mathrm{MeV}$ behind the iron-weter shields described above. (Geometrical attenuation of the beam has been omitted in the presentation of the experimental data.) The figure ellso shows experimental data on the energy spectrum of a monodirectional neutron beam impinging on the shields under investigation. By averaging these data within the group intervals we obtained the data on the incident beam spectrum on which our subsequent calculations were based.

For comparison Fig. 1 also shows the calculated energy spectrum of a neutron flux behind 15 cm of water $[1]$ for the same incident beam spectrum together with similar data for layers of iron 20 and 35 cm thick [9]. In the case of the 20 cm thick iron layer experimental data from Ref. [9] are also shown and these agree well with the calculated values. Note that Ref. [9] includes a comparison of theoretical and experimental results for neutron energy spectra behind 10 and 30 cm thick iron shields. Good agreement was also obtained in these cases.

From Fig. 1 we see that there is good agreement in the 1.4 to 6.5 MeV range between experimental and theoretical data on neutron spectra behind iron-water shields (the results agree within the limits of experimental error). However, a significant deviation (by more than a factor of 2) is observed for energies less than 1.4 MeV . This is partially attributable to the fact that the constants which we used for iron [9] and water [10] for neutrons with $\mathrm{E}<1 \mathrm{MeV}$ have not been proved experimentally. Another possible reason is that, strictly speaking, when calculating inhomogeneous shields consisting of thin layers (say, less than 4-5 free path lengths), one ought to use group constants obtained by averaging not over the spectra in an infinite homogeneous medium (as was done here) but over the integral neutron spectrum in each layer [l]. For calculating these integral spectra it is necessary to allow for mutual effects of these layers. Unfortunately no algorithms have yet been devised to allow for this effect. An improvement can be obtained by using the results of an additional calculation with a large number of groups for averaging the cross-sections (this calculation may, of course, be done in a lower approximation).

Also, averaging over the neutron spectrum in an infinite homogeneous medium consisting of a homogeneous mixture of iron and water nuclei can be used as a simpler approximation in the above problem.

Analysis of the fast neutron spectra obtained behind iron-water shields shows competition between two trends: "softening" of the spectrum by the layer of iron (due to inelastic scattering by iron nuclei) and "hardening" of the spectrum by the succeeding mass of water (due to intense slowing-down of neutrons by hydrogen nuclei). These tendencies are also to be observed in monolayer shields. For example, a 20 cm iron layer increases the ratio of the neutron flux in the $0.4 \mathrm{ml.4} \mathrm{MeV}$ range to the flux with $\mathrm{E}>1.4 \mathrm{MeV}$, $\frac{F(0.4-1.4 \mathrm{MeV})}{F(E>1.4 \mathrm{MeV})}$, by approximately a factor of 10 compared with the incident beam spectrum shown in Fig. l (for which the ratio is ~1.5). At the same time this ratio decreases by almost a factor of 3 behind 15 cm of water with the same incident beam. The two shields together also reduce this ratio but to a lesser extent (by a factor of approximately 2.4). Thus, a mass of water situated behind a 20 cm thick iron layer reduces $\frac{F(0.4-1.4 \mathrm{MeV})}{F(E>1.4 \mathrm{MeV})}$ by a factor of approximately 24.

The neutron spectrum behind a two-layer iron-water shield is, so to speak, half way between the spectrum in water and the spectrum in iron (at comparable distances). Note that this has already been demonstrated for water shields less than 20 cm thick in Ref. [3], where the authors investigated the space-energy distribution of fast neutrons from monoenergetic sources, $D(d, n)$ and $T(d, n)$ in metal-water shields.

Increasing the thickness of the iron layer causes its "softening" capacity to increase, so that at a thickness of 35 cm the ratio $\frac{\mathrm{F}(0.4-1.4 \mathrm{MeV})}{\mathrm{F}(\mathrm{E}>1.4 \mathrm{MeV})}$ is more than 20 times greater than in the incident beam spectrum. However, the attentuation of the beam of relatively "soft" neutrons is also enhanced.

It is interesting to note that the fast neutron spectrum behind a twolayer shield consisting of a layer of iron ~ $25-30 \mathrm{~cm}$ thick ( 20 cm according to experimental data, 35 cm according to calculations) and a somewhat thinner mass of water ( $\sim 15 \mathrm{~cm}$ ) is very similar in shape to the incident beam spectrum. Evidently, it would be possible to choose other iron-water combinations which would also maintain the shape of the incident beam spectrum. Clearly, if the iron layer thickness is increased, the thickness of the water layer must also be increased (but of course to a lesser extent).

One further conclusion to be drawn from an analysis of the above spectra is that in two-layer iron-water shields a 15 cm thick water layer is inadequate for a specific "water" spectrum to be established. This will begin to show up only when the thickness of the iron layer is considerably less than 20 cm .

Fig. 2 shows the spatial flux distributions of fast neutrons in the two-layer iron-water shields described above for energies $E>2.5 \mathrm{MeV}$, $\mathrm{E}>1.4 \mathrm{MeV}$ and $0.4 \mathrm{MeV}<\mathrm{E}<1.4 \mathrm{MeV}$.

Of interest here is the dependence of the neutron distribution close to the boundary of the iron layer on the material of the succeeding layer. This dependence is particularly marked for neutrons with $E<1.4 \mathrm{MeV}$.

For comparison Fig. 2 also shows calculated neutron flux distributions in water. The attenuation effect of the water varies with its position. For example, for a neutron flux with $E>1.4 \mathrm{MeV}$, the attenuation factor of a 15 cm thick mass of water is $\sim 6.6$ when it is irradiated directly with a unidirectional beam, ~ 15.5 when it is located behird a 20 cm layer of iron, and ~ 19 when located behind a 35 cm iron layer. This difference is due primarily to the change in the angular distribution of the neutron flux on passage through the iron layer and, secondly, to the resulting change in the neutron energy spectrum. The difference between the corresponding attenuation values for neutrons with $\mathrm{E}>2.5 \mathrm{MeV}$ is slightly less: instead of a factor of $\sim 3$, as in the case of neutrons with $\mathrm{E}>1.4 \mathrm{MeV}$, it is only ~ 1.6. For neutrons in the range $0.4 \mathrm{MeV}<\mathrm{E}<1.4 \mathrm{MeV}$ the corresponding attenuation factors calculated as $\sim 20, \sim 230$ and $\sim 580$. This strong dependence of attenuation on the position of the water layer is attributable to the different degree of "softening" of the neutron spectrum in these cases.

For comparison, Fig. 2 also shows experimental and theoretical data on the dependence of a fast neutron flux with different energies behind layers of iron on the thickness of these layers, $t$. As can be seen, for neutrons with $\mathrm{E}<2.5 \mathrm{MeV}$ the nature of this dependence can differ appreciably from the spatial flux distribution $F_{0}(x)$ within iron layers that are backed by a layer of water.
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Key:

| $\frac{\text { нейтрон }}{\text { cm }^{2} \mathrm{PT}}$ | $=\mathrm{n} / \mathrm{cm}^{2} \mathrm{~W}$ |
| :--- | :--- |
| Мэв | $=\mathrm{MeV}$ |
| Вода | $=$ water |
| Железо | $=$ iron |

Fig. 1 The energy dependence of the fast neutron flux in layers of iron and water

Lines indicate calculated data; points indicate experimental data



Fig.
Рис. I. Знергетические раслрсделения потока бистрих нейтронов за слоями железа п ноды.

линии - расчет, точки - экслеримент:
$\cdots \times$ - слой воды 15 cx [I]




Fig. 2 Spatial flux distributions of fast neutrons of various energies in iron-water shields irradiated by a plane unidirectional beam.

Results of calculations:
$\qquad$ $=2-1$ ayer shield, $35 \mathrm{~cm} \mathrm{Fe}+15 \mathrm{~cm}_{2} \mathrm{O}$
_ _ _ _ = 2-layer shield, $20 \mathrm{~cm} \mathrm{Fe}+15 \mathrm{~cm} \mathrm{H}_{2} \mathrm{O}$
— . . _ = 15 cm layer of $\mathrm{H}_{2} \mathrm{O}$ [1]
T = Results of neutron flux measurements behind iron-water shields

For comparison, the figure also includes data on $\mathrm{F}_{0}(\mathrm{t})$ - the neutron flux behind an iron barrier as a function of the barrier thickness t [9].
I indicates experimental data

- $x$ - indicates calculated data


Рис．＇2．Пространственнве распределения потока бистрих нейтронов раздичих энергии в хелезоводних замитах при их обдучених пиоскии ноноваправденным пучкои．

Резуднтати расчетов：

$$
\begin{aligned}
& \text { Для сравненкя приведены данные по зависикости } F_{0}(t) \text { потока неитро } \\
& \text { нов за барьерои из хелеза от толмины } t \text { этого барьера [9]: } \\
& \text { 空 - эксперянент, —x一 - расчет. }
\end{aligned}
$$

## DIFFERENTIIAL ALBEDOS OF FAST NEUTRONS FOR CARBON AND BORON CARBIDE

T.A. Germogenova, V.A. Klimanov, M.G. Kobozev, V.P. Mashkovich, E.I. Panfilova, O.G. Petrov, A.P. Suvorov

For solving many problems concerned with the physics of shielding, dosimetry and radiation technology it is necessary to know the differential characteristics of the back-scattered radiation field of unidirectional neutron beams. This problem has in recent years attracted the attention of a large number of investigators. A brief review of work on neutron backscattering is presented in Ref. [1].

This paper reports calculated differential albedo characteristics of plane unidirectional neutron beams impinging on plare barriers of various thicknesses made of carbon and boron carbide. The calculations were performed for plane unidimensional geometry by a special albedo method $[2]$ in the $2 \mathrm{D}_{\mathrm{N}} \mathrm{P}_{\mathrm{N}}$ approximation of the discrete ordinates method [3]. We used the group constants from Ref. [4] together with a group with $\mathrm{E}>10.5 \mathrm{MeV}$ supplemented by data on scattering anisotropy from Refs $[5,6]$.

Tables l-4 show the calculated differential flux albedo $A\left(\Delta E_{0}, \vartheta_{0} ; \Delta E, \vartheta, \varphi\right)$ of a plane unidirectional fast neutron beam. These values characterize the probability of a neutron in the energy group $\Delta_{E_{0}}$, incident on a scattering medium at angle $\vartheta_{0}$, being reflected at a single solid angle, described by the angle $\vartheta$ and the azimuthal angle $\varphi$, and thereby falling into the group $\triangle E$. The angles $\vartheta$ and $\vartheta_{o}$ are read off from the normal to the surface of the scatterer, the azimuthal angle $\varphi$ from the projection of the beam direction onto the surface of the medium. The results are normalized to an incident flux of intensity $10^{4} \mathrm{n} / \mathrm{cm}^{2}$. The calculations were performed for nine groups of neutrons with energy $E>0.1 \mathrm{MeV}$ in the $2 \mathrm{D}_{7} \mathrm{P}_{7}$ approximation. It was found that for neutrons with $E>4 \mathrm{MeV}$ the $\mathrm{P}_{7}$-approximation of the scattering angular distribution is inadequate for describing all the irregularities of its angular dependence. In some cases the $P_{7}$-representation of the scattering angular distribution for carbon and boron assumes negative values for certain scattering angles and its use in albedo calculations accordingly leads, for some values of $\vartheta$, to physically meaningless negative albedo values. Tables l-4 show the albedo values only for neutrons with $E_{0}<4 \mathrm{MeV}$. These values relate to layers of carbon 15 and 25 cm thick and
layers of boron carbide (density $1.3 \mathrm{~g} / \mathrm{cm}^{3}$ ) 5 and 10 cm thick. When calculating the differential angular albedos for neutrons with $\mathrm{E}_{\mathrm{o}}>4 \mathrm{MeV}$ it is necessary to use a higher approximation of the scattering angular distribution. From the data supplied in the above references it can be seen that a significant azimuthal dependence of the albedos exists only for neutrons with energy $E>0.8 \mathrm{McV}$ at large $\vartheta_{0}$ and $\vartheta$.

The theoretical data were verified experimentally with plane scatterers made of carbon 20 and 40 cm thick. The horizontal channel of the IRT-2000 reactor of the Moscow Engineering-Physics Institute was used as the neutron source. The neutron beam diameter where it intersected the surface of the scatterer at normal incidence was 7 cm . The fast neutron detector used was a scintillation counter based on a $5 \times 30 \mathrm{~mm} \mathrm{ZnS}$ crystal, having virtually constant sensitivity to fast neutrons with a recording threshold of $E_{t h r} \simeq 1.0 \mathrm{MeV}[7]$. The detector was put 2.0 m from the scatterer. The scattering medium measured 1.8 m across. A special mechanical system was used to set the necessary angles of $\vartheta_{0}$, $\vartheta$ and $\varphi$. The above experimental set-up enabled measurements to be made of the flux differential albedos of fast neutrons with a threshold energy $E_{t h r} \simeq 1.0 \mathrm{MeV}$. The theoretical values of this quantity were determined by integrating the data obtained during this investigation for separate monogroups of $\Delta E_{o}$ with allowance for the energy distribution of reactor neutrons incident on the scatterer. The experimental values of the differential numerical albedo were obtained for the angles $\vartheta_{0}$ and $\vartheta=0-75^{\circ}$ and $\varphi=0^{\circ}$ and $180^{\circ}$. The maximum disagreement between the theoretical and experimental values did not exceed 25\%. The figure compares the theoretical and experimental data on the differential flux neutron albedo for a 25 cm layer of carbon.

## Key for tables 1 through 4c

$$
\begin{aligned}
\text { град } & =\text { deg } \\
\text { Mән } & =\text { MeV }
\end{aligned}
$$

Table la
Differential albedos of neutrons for a 15 cm layer of carbon, $\Delta \mathrm{E}_{\mathrm{o}}=2.5-4 \mathrm{MeV}$ Table lb

Differential albedos of neutrons for a 15 cm layer of carbon, $\Delta \mathrm{E}_{0}=1.4-2.5 \mathrm{MeV}$ Table lc

Differential albedos of neutrons for a 15 cm layer of carbon, $\Delta \mathrm{E}_{\mathrm{o}}=0.8-1.4 \mathrm{MeV}$ Table 2a

Differential albedos of neutrons for a 25 cm layer of carbon, $\Delta \mathrm{E}_{\mathrm{o}}=2.5-4 \mathrm{MeV}$
Table 2b
Differential albedos of neutrons for a 25 cm layer of carbon, $\Delta \mathrm{E}_{\mathrm{o}}=1.4-2.5 \mathrm{MeV}$ Table 2c

Differential albedos of neutrons for a 25 cm layer of carbon, $\Delta E_{0}=0.8 \mathrm{ml} .4 \mathrm{MeV}$ Table 3a

Differential albedos of neutrons for a 5 cm layer of boron carbide, $\Delta E_{0}=2.5-4 \mathrm{MeV}$

Table 3b
Differential albedos of neutrons for a 5 cm layer of boron carbide, $\Delta E_{0}=1.4-2.5 \mathrm{MeV}$

## Table 3c

Differential albedos of neutrons for a 5 cm layer of boron carbide, $\Delta E_{0}=0.8-1.4 \mathrm{MeV}$

## Table 4a

Differential albedos of neutrons for a 10 cm layer of boron carbide, $\Delta E_{0}=2.5-4 \mathrm{MeV}$

Table $4 b$
Differential albedos of neutrons for a 10 cm layer of boron carbide, $\Delta E_{0}=1.4 m 2.5 \mathrm{MeV}$

## Table 4c

Differential albedos of neutrons for a 10 cm layer of boron carbide, $\Delta \mathrm{E}_{\mathrm{O}}=0.8 \mathrm{~m} .4 \mathrm{MeV}$

Дифференциальные альбедо веитронов для слоя углероде $I 5$ ом, $\Delta E_{0}=2,5-4$ Мэв

| Q., | $\Delta E$, | $\varphi=0^{\circ}$ |  |  |  | $\varphi=90^{\circ}$ |  |  |  | $\varphi=180^{\circ}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| гряд | ${ }^{\text {H88 }}$ | $\theta=0^{\circ}$ | $30^{\circ}$ | $45^{\circ}$ | $75^{\circ}$ | $0^{0}$ | $30^{\circ}$ | $45^{\circ}$ | $75^{\circ}$ | $0^{0}$ | $30^{\circ}$ | $45^{\circ}$ | $50$ |
| 0 | 2,5-4,0 | 2322 | 2340 | 2493 | 3496 | 23.2 | 2340 | 2493 | 3496 | 2322 | 2340 | 2487 | 3497 |
|  | 1,4-2,5 | II472 | IIOI4 | 10468 | 8000 | 11472 | IIOI4 | 10468 | 8000 | II472 | IIOI4 | 10468 | 8000 |
|  | 0,8-1,4 | 2362 | 2393 | 2428 | 2485 | 2362 | 2393 | 2428 | 2485 | 2362 | 2393 | 2428 | 2485 |
|  | 0,4- 0,8 | I36I | 1356 | 1347 | I223 | I36I | 1356 | I347 | 1223 | I36I | 1356 | I347 | 1223 |
|  | 0,2-0,4 | 651 | 644 | 634 | 549 | 651 | 644 | 634 | 549 | 651 | 644 | 634 | 549 |
| 30 | 2,5-4,0 | 2199 | 2482 | 2772 | 3135 | 2199 | 2306 | 2507 | 3424 | 2199 | 2334 | 2407 | 3131 |
|  | I, $4-2,5$ | 10340 | 8715 | 7820 | 5684 | 10384 | 10015 | 9576 | 7578 | 10426 | II502 | II739 | 10840 |
|  | 0,8-I,4 | 2304 | 2350 | 2391 | 2472 | 2303 | 2334 | 2368 | 2428 | 2303 | 2318 | 2344 | 2380 |
|  | 0,4-0,8 | I3I9 | 1314 | 1306 | 1187 | I3I9 | 1315 | I506 | II88 | I3I9 | 1315 | 1307 | II89 |
|  | 0,2-0,4 | 630 | 624 | 614 | 532 | 630 | 624 | 614 | 532 | bsu | 624 | 614 | 532 |
| 45 | 2,5-4,0 | 2165 | 2555 | 2738 | 3085 | 2159 | 2310 | 2520 | 3317 | 2154 | 2218 | 2342 | 2849 |
|  | I, $4-2,5$ | 9089 | 7248 | 6402 | 4966 | 9143 | 8872 | 8547 | 7068 | 9197 | 10870 | II520 | 11950 |
|  | 0,8-I,4 | 2218 | 2269 | 2313 | 2407 | 2217 | 2247 | 2281 | 2344 | 2215 | 2285 | 2247 | 2276 |
|  | 0,4- 0,8 | 1260 | I256 | I248 | II36 | I260 | 1257 | 1249 | II38 | 1260 | 1257 | 1250 | II40 |
|  | 0,2-0,4 | 601 | 595 | 585 | 508 | 601 | 595 | 585 | 508 | 601 | 595 | 585 | 508 |
| 75 | 2,5-4,0 | 1749 | 1670 | 1783 | 4859 | 1750 | 1824 | 1917 | 2417 | 1750 | 1667 | 1647 | 2187 |
|  | 1,4-2,5 | 估38 | 3154 | 2966 | 3459 | 4180 | 4196 | 4220 | 4391 | 4221 | 5978 | 7048 | 10950 |
|  | 0,3-1,4 | 1496 | I544 | 1584 | I7II | 1495 | I5I9 | I546 | 1630 | I 495 | 1493 | 1507 | 1543 |
|  | 0,4-0,8 | 814 | 812 | 808 | 747 | 814 | 8 I 3 | 609 | 749 | 814 | 813 | 810 | 75I |
|  | 0,2-0,4 | 383 | 379 | 374 | 327 | 383 | 379 | 374 | 327 | 383 | 379 | 373 | 327 |

дифференциөльные өльбөдо пеитронов длм слои углорода $15 \mathrm{~cm}, \Delta E_{5}=1,4-2,5$ млв

| $\begin{gathered} \hat{\theta}_{0_{1}} \\ \text { spoд } \end{gathered}$ | $\underbrace{}_{M} E_{B}$ | $\theta=0^{\circ} \quad 1,30^{\circ}=45^{\circ}$ |  |  |  |  | $\underline{\varphi}=2$ | $45^{\circ}$ |  |  | 4 | $0^{\circ}$ $-45^{\circ}$ | $.35^{\circ}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 1,4-2,5 | 3090 | 3540 | 3930 | 5170 | 3090 | 3540 | 3930 | 5170 | 3090 | 3540 | 3930 | 5170 |
|  | 0,8-1,4 | 5670 | 5690 | 5710 | 5670 | 5670 | 5690 | 5710 | 5670 | 5670 | 5690 | 5710 | 5760 |
|  | 0,4-0,8 | 2553 | 2552 | 2545 | 2378 | 2553 | 2552 | 2545 | 2578 | 2553 | 2552 | 2545 | 2378 |
|  | 0,2-0,4 | 1206 | 1194 | 1176 | 1030 | 1206 | 1594 | 1176 | 1030 | 1206 | II94 | 1176 | 1030 |
| 30 | 1,4-2,5 | 3343 | 3877 | 4166 | 6218 | 3327 | 3633 | 3919 | 5125 | 3311 | 3050 | 3251 | 4535 |
|  | 0,8-I,4 | 5400 | 5297 | 5c75 | 5218 | 5404 | 5431 | 5461 | 5484 | 5408 | 5579 | 5677 | 5830 |
|  | 0,4-0,8 | 2453 | 2455 | 2451 | 2297 | 2453 | 2452 | 2447 | 2290 | 2453 | 2450 | 2443 | 2283 |
|  | 0,2-0,4 | 1155 | II44 | 1127 | 988 | II55 | II44 | 1127 | 988 | II55 | II44 | 1127 | 988 |
| 45 | 1,4-2,5 | 3416 | 3839 | 4207 | 6703 | 3404 | 3612 | 3831 | 5052 | 3391 | 2996 | 2982 | 4131 |
|  | 0,8-I,4 | 5061 | 4928 | 4904 | 4834 | 5066 | 5102 | 5143 | 5238 | 5072 | 5303 | 5442 | 5747 |
|  | 0,4-0,8 | 2318 | 2322 | 2319 | 2183 | 2318 | 2318 | 2313 | 2173 | 2318 | 2315 | 2308 | 2162 |
|  | 0,2-0,4 | 1087 | 1077 | 1062 | 932 | 1087 | 1077 | 1062 | 932 | 1087 | 1077 | 1062 | 933 |
| 75 | 1,4-2,5 | 2600 | 3312 | 3874 | 6820 | 2587 | 2760 | 2920 | 4190 | 2576 | 2416 | 2387 | 2346 |
|  | 0,8-1,4 | 3145 | 3053 | 3061 | 3166 | 3150 | 3209 | 3283 | 3642 | 3155 | 3412 | 3604 | 4349 |
|  | $0,4-0,8$ | 1456 | 1465 | 1470 | 1422 | 1456 | 1460 | 1463 | 1409 | 1456 | 1456 | 1456 | 1395 |
|  | 0,2-0,4 | 663 | 658 | 650 | 578 | 663 | 658 | 650 | 578 | 663 | 658 | 650 | 579 |



| $\theta 0$ | $\Delta E$, | $\varphi=0^{\circ}$ |  |  |  | $\varphi=90^{\circ}$ |  |  |  | $\varphi=180^{\circ}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| тред | M ${ }^{\text {a }}$ | $\theta=0^{\circ}$ | $30^{\circ}$ | $45^{\circ}$ | $75^{\circ}$ | $0^{0}$ | $30^{\circ}$ | $45^{\circ}$ | $75^{\circ}$ | $0^{0}$ | $30^{\circ}$ | $45^{\circ}$ | $75^{\circ}$ |
| 0 | 0,8-I,4 | 4100 | 4335 | 4640 | 6290 | 4100 | 4335 | 4640 | 6290 | 4100 | 4335 | 4640 | 6290 |
|  | 0,4-0,8 | 6533 | 6590 | 6645 | 6657 | 6533 | 6590 | 6645 | 6657 | 6533 | 6590 | 6645 | 6657 |
|  | 0,2-0,4 | 2716 | 2702 | 2678 | 2435 | 2716 | 2702 | 2678 | 2435 | 2716 | 2702 | 2678 | 2435 |
| 30 | 0,8-I,4 | 4083 | 4644 | 5125 | 7473 | 4075 | 4305 | 4600 | 6266 | 4064 | 3975 | 4094 | 5082 |
|  | 0,4-0,8 | 6266 | 6306 | 6347 | 6311 | 6266 | 6321 | 6380 | 6430 | 6266 | 6327 | 6393 | 6510 |
|  | 0,2-0,4 | 2585 | 2574 | 2553 | 2331 | 2585 | 2573 | 2551 | 2327 | 2585 | 2571 | 2549 | 2323 |
| 45 | 0,8-I,4 | 4030 | 4724 | 5280 | 7970 | 4016 | 4240 | 4524 | 6120 | 4003 | 3773 | 3810 | 4510 |
|  | 0,4-0,8 | 5912 | 5940 | 5973 | 5940 | 5912 | 5970 | 6030 | 6130 | 5913 | 5982 | 6053 | 6235 |
|  | 0,2-0,4 | 2417 | 2408 | 2390 | 2193 | 2417 | 2406 | 2387 | 2187 | 2417 | 2404 | 2385 | 2181 |
| 75 | 0,8-I,4 | 3170 | 3984 | 4610 | 8000 | 3150 | 3320 | 3540 | 4910 | 3135 | 2710 | 2610 | 2730 |
|  | 0,4-0,8 | 3728 | 3720 | 3746 | 3810 | 3730 | 3790 | 3870 | 4210 | 3731 | 3840 | 3935 | 4350 |
|  | 0,2-0,4 | 1431 | 1432 | 1430 | 1356 | 1431 | 1430 | 1426 | 1347 | I43I | I428 | 1422 | $\pm 340$ |

Диффөрөнциальнқе өльбөдо неИтронов для слон углерода $25 \mathrm{~cm}, \Delta E_{0}=2,5-4$ Мзв

| $\theta$ | $\Delta E$, | $p=0^{\circ}$ |  |  |  | $\Psi=90^{\circ}$ |  |  |  | $\varphi=180^{\circ}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| rpea | ${ }^{48}$ | $\theta=0^{\circ}$ | $30^{\circ}$ | 5 | $75^{\circ}$ | $0^{0}$ | - $30^{\circ}$ | $45^{\circ}$ | $75^{\circ}$ | $10^{0}$ | 3 | $45$ | $75^{\circ}$ |
| 0 | 2,5-4,0 | 2536 | 2541 | 2586 | 3590 | 2536 | 2545 | 2686 | 3500 | 2536 | 2541 | 2686 | 3590 |
|  | I, $4-2,5$ | I2510 | II950 | II310 | 8470 | I2510 | 11950 | II310 | 8470 | 12510 | I5950 | 11310 | 8470 |
|  | 0,8-I,4 | 3167 | 3173 | 3173 | 3033 | 3167 | 3173 | 3173 | 3033 | 3167 | 3173 | 3173 | 3033 |
|  | 0,4-0,8 | 2040 | 2010 | 1970 | 1696 | 2040 | 2010 | 1970 | 1696 | 2040 | 2010 | 1970 | 1695 |
|  | 0,2-0,4 | 1090 | 1066 | 1037 | 858 | 1090 | 1066 | 1037 | 858 | 1090 | 1066 | 1037 | 858 |
| 30 | 2,5-4,0 | 2388 | 2663 | 2940 | 3222 | 2388 | 2485 | 2672 | 3515 | 2388 | 2500 | 2560 | 3240 |
|  | I, 4-2,5 | I1250 | 9560 | 8500 | 6164 | IT300 | 10880 | 10376 | 8065 | II340 | 12330 | 12570 | 11340 |
|  | 0,8-I,4 | 3033 | 3056 | 3066 | 2967 | 3033 | 3040 | 3042 | 2921 | 3033 | 3023 | 3017 | 2873 |
|  | 0,4-0,8 | I936 | 1910 | 1875 | 1617 | 1936 | I9II | 1873 | 1618 | 1936 | I9II | 1876 | I6I9 |
|  | 0,2-0,4 | IO3I | 1010 | 983 | 815 | IO3I | IOII | 983 | 815 | 1031 | IOIO | 983 | 815 |
| 45 | 2,5-4,0 | 2377 | 2710 | 2880 | 31.60 | 2321 | 2463 | 2660 | 3390 | 2512 | 2570 | 2857 | 3618 |
|  | I, 4-2,5 | 9860 | 7960 | 7055 | 5374 | 9913 | 9600 | 9235 | 7484 | 9962 | II630 | 12330 | 12320 |
|  | 0,8-I,4 | 2858 | 2888 | 2904 | 2838 | ¢ 258 | 2886 | 2871 | 2774 | 2858 | 2843 | 2836 | 2706 |
|  | 0,4-0,8 | 1807 | 1784 | 1752 | 1516 | 1807 | 1785 | 1753 | 1518 | I8U\% | 1735 | 1754 | I5I9 |
|  | 0,2-0,4 | 958 | 939 | 915 | 760 | 958 | 940 | 915 | 760 | 958 | 940 | 915 | 760 |
| 75 | 2,5-4,0 | 1798 | 1716 | 18.6 | 4882 | 1799 | 1870 | 1960 | 2439 | 2370 | 2481 | 2692 | 3299 |
|  | I, $4-2,5$ | 4394 | 3393 | 3189 | 3600 | 4435 | 4440 | 4450 | 4533 | 4477 | 6230 | 7335 | II090 |
|  | 0,8-1,4 | 1757 | 1795 | 1822 | 1879 | 1756 | 1770 | 1784 | 1798 | 1755 | 2744 | I744 | I7IO |
|  | 0,4-0,8 | 1049 | 1038 | 1024 | 906 | 1049 | 1039 | 1025 | 908 | 1049 | 1040 | 1026 | 970 |
|  | $0,2-0,4$ | 542 | 532 | 520 | 437 | 542 | 533 | 520 | 437 | 542 | 533 | 520 | 437 |

двфферевцдөльние өльбедо веятровов для олоя углероде $25 \mathrm{ox}, \Delta E_{0}=I, 4-2,5$ ивв

| $\begin{array}{r} \theta_{0}, \\ \text { град } \end{array}$ | $\Delta E$, <br> Mas | $\varphi=0^{\circ}$ |  |  |  | $\varphi=90^{\circ}$ |  |  |  | $\varphi=180^{\circ}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\theta=0^{\circ}$ | $30^{\circ}$ | $45^{\circ}$ | $75^{\circ}$ | $0^{0}$ | $30^{\circ}$ | $45^{\circ}$ | $75^{\circ}$ | $0^{0}$ | $30^{\circ}$ | $45^{\circ}$ | $75^{\circ}$ |
| 0 | 1,4-2,5 | 3480 | 3935 | 4300 | 5410 | 3480 | 3935 | 4300 | 5410 | 3480 | 3935 | 4300 | 5410 |
|  | 0,8-I,4 | 6350 | 6330 | 6310 | 6060 | 6350 | 6330 | 6310 | 6060 | 6350 | 6330 | 6310 | 6060 |
|  | 0,4-0,8 | 3190 | 3170 | 3130 | 2800 | 3190 | 3170 | 3130 | 2800 | 3190 | 3170 | 3130 | 2800 |
|  | 0,2-0,4 | 1652 | 1623 | 1584 | I334 | 1652 | 1623 | 1584 | 1334 | 1652 | 1623 | 1584 | 1334 |
| 30 | 1,4-2,5 | 3700 | 4225 | 4500 | 6410 | 3682 | 3980 | 4245 | 5320 | 3420 | 3740 | 4330 | 5200 |
|  | 0,8-I,4 | 6016 | 5880 | 5830 | 5580 | 6020 | 6018 | 6010 | 5840 | 6025 | 6170 | 6230 | 6190 |
|  | 0,4-0,8 | 3040 | 3016 | 2980 | 2680 | 3038 | 3013 | 2977 | 2673 | 3038 | 3010 | 2973 | 2666 |
|  | 0,2-0,4 | 1565 | 1538 | 1502 | 1267 | 1565 | 1538 | 1502 | 1267 | 1565 | 1538 | 1502 | 1267 |
| 45 | I, 4-2,5 | 3728 | 4144 | 4496 | 6870 | 3715 | 3915 | 4116 | 5220 | 3383 | 3570 | 4362 | 5014 |
|  | 0,8-I,4 | 5600 | 5440 | 5380 | 5200 | 5606 | 5616 | 5625 | 5552 | 5611 | 5818 | 5925 | 6060 |
|  | 0,4-0,8 | 2836 | 2819 | 2789 | 2521 | 2836 | 2815 | 2783 | 2511 | 2836 | 2811 | 2778 | 2500 |
|  | 0,2-0,4 | 1452 | 1428 | 1395 | 1180 | 1452 | 1428 | 1395 | II80 | I 452 | 1428 | 1395 | 1180 |
| 75 | 1,4-2,5 | 2708 | 3418 | 3975 | 6880 | 2696 | 2835 | 3020 | 4252 | 3290 | 3260 | 3860 | 4646 |
|  | 0,8-I,4 | 3350 | 3250 | 3244 | $3 \times 85$ | 3356 | 3406 | 3470 | 3760 | 3360 | 3610 | 3790 | 4470 |
|  | 0,4-0,8 | 1670 | 1670 | 1663 | 1560 | 1670 | 1666 | 1657 | 1546 | 1670 | 1661 | 1650 | 1532 |
|  | 0,2-0,4 | 820 | 808 | 792 | 683 | 820 | 808 | 792 | 683 | 820 | 808 | 793 | 683 |



| $\underset{\text { rpoi }}{\theta_{0}}$ | $\underset{M a s}{\Delta E}$ | $\varphi=0^{\circ}$ |  |  |  | $\psi=90^{\circ}$ |  |  |  | $\varphi=180^{\circ}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\theta=0^{\circ}$ | $1-3$ | $145^{\circ}$ |  | $10^{\circ}$ | $30^{\circ}$ | $45^{\circ}$ | 1 75 | $0^{\circ}$ | $30^{\circ}$ | - | 75 |
| 0 | 0,8-I, 4 | 4370 | 4600 | 4890 | 6460 | 4370 | 4600 | 4890 | 6460 | 4370 | 4600 | 4890 | 6460 |
|  | 0.4-0.8 | 6980 | 7010 | 7030 | 6910 | 6980 | 7010 | 7030 | 6910 | 6980 | 7010 | 7030 | 6910 |
|  | 0,2-0.4 | 3080 | 3050 | 3000 | 2670 | 3080 | 3050 | 3000 | 2670 | 3080 | 3050 | 3000 | 2670 |
| 30 | 0,8-I,4 | 4330 | 4860 | 5340 | 7600 | 4319 | 4537 | 4814 | 6532 | 4300 | 4450 | 4850 | 6070 |
|  | 0,4-0,8 | 6660 | 6680 | 6690 | 6540 | 6662 | 6695 | 6727 | 6657 | 6662 | 6700 | 6740 | 6737 |
|  | 0,2-0,4 | 2910 | 2890 | 2850 | 2540 | 2914 | 2885 | 2844 | 2534 | 2914 | 2884 | 2842 | 2531 |
| 45 | 0,8-I, 4 | 4239 | 4923 | 5460 | 8070 | 4226 | 4440 | 4710 | 6225 | 4206 | 4310 | 4815 | 5650 |
|  | 0,4-0,8 | 6253 | 6260 | 6270 | 6140 | 6254 | 6290 | 6330 | 6320 | 6254 | 6300 | 6350 | 6430 |
|  | $0.2-0.4$ | 2700 | 2680 | 2645 | 2370 | 2702 | 2677 | 2642 | 2366 | 2701 | 2675 | 2639 | 2361 |
| 88 |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 75 |  | 3240 | 4050 | 4670 | 8040 | 3223 | 3390 | 3600 | 4950 | 3970 | 3993 | 4320 | 4950 |
|  | 0,4-0,8 | 3854 | 3840 | 3860 | 3880 | 3856 | 3910 | 3980 | 4270 | 3857 | 3960 | 4040 | 4420 |
|  | 0,2-0,4 | 1542 | 1538 | I529 | I 425 | 1542 | 1535 | I525 | 1416 | 1542 | 1533 | 152I | 1408 |

дифференциельняе өльбедо неитронов для олоя кароидя борв $5 \mathrm{om}, \Delta E_{0}=2,5-4$ мяв

| $\theta_{0}$ | $\begin{aligned} & \Delta E, \\ & M_{\theta B} \end{aligned}$ | $\varphi=0^{\circ}$ |  |  |  | Y= $90^{\circ}$ |  |  |  | $\varphi=180^{\circ}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| rpait |  | $\theta=0^{\circ}$ | $30^{\circ}$ | $45^{\circ}$ | $75^{\circ}$ | $0^{0}$ | $30^{\circ}$ | $45^{\circ}$ | $75^{\circ}$ | $0^{0}$ | $30^{\circ}$ | $45^{\circ}$ | $75^{\circ}$ |
| 0 | 2,5-4 | 2120 | 1800 | 1560 | 2060 | 2120 | 1800 | 1560 | 2060 | 2120 | 1800 | 1560 | 2060 |
|  | $1,4-2,5$ | 5320 | 5215 | 5090 | 4430 | 5320 | 5215 | 5090 | 4430 | 5320 | 5215 | 5090 | 4430 |
|  | 0,8-I,4 | 1013 | 1030 | 1050 | II50 | 1013 | 1030 | 1050 | 1150 | 1013 | 1030 | 1050 | I150 |
|  | 0,4-0,8 | 415 | 422 | 430 | 451 | 415 | 422 | 430 | 45I | 415 | 422 | 430 | 451 |
|  | 0,2-0,4 | 188 | 189 | 190 | 185 | 188 | 189 | I 90 | 185 | 188 | 189 | 190 | 185 |
| 30 | 2,5-4 | 1675 | L26I | 1345 | 3030 | 1690 | ISIO | 1590 | 2 L 20 | I710 | 2145 | 2140 | I7IO |
|  | I,4-2,5 | 4920 | 4310 | 3980 | 3230 | 4940 | 4850 | 4750 | 4260 | 4950 | 5440 | 5630 | 5800 |
|  | 0,8-I,4 | 992 | 990 | 1007 | 1135 | 993 | 1010 | 1031 | II43 | 974 | 1033 | 1065 | II83 |
|  | 0,4-0,8 | 412 | 419 | 427 | 448 | 4 I 2 | 419 | 428 | 449 | 412 | 420 | 428 | 449 |
|  | $0,2-0,4$ | 186 | I86 | 187 | I8I | 186 | 187 | I88 | I83 | I86 | 188 | 189 | 186 |
| 45 | 2,5-4 | 1337 | 1239 | 1535 | 3450 | 1351 | 1283 | 1277 | 2190 | 1365 | 1970 | 2150 | 1940 |
|  | 1,4-2,5 | 4460 | 3710 | 3370 | 2830 | 4480 | 4430 | 4360 | 4040 | 4510 | 5240 | 5590 | 6320 |
|  | 0,8-I,4 | 984 | 960 | 979 | II32 | 965 | 983 | 1007 | 1226 | 966 | 1014 | 1053 | 1189 |
|  | 0,4-0,8 | 406 | 413 | 42I | 442 | 406 | 413 | 422 | 443 | 406 | 414 | 422 | 444 |
|  | 0,2-0,4 | I82 | 182 | 182 | I77 | 182 | 183 | 185 | 180 | 182 | 185 | 186 | 184 |
| 75 | 2,5-4 | 1043 | 1610 | 1990 | 5090 | 1031 | 1129 | 1266 | 2230 | 1019 | 910 | II2I | 2345 |
|  | $1,4-2,5$ | 2410 | 1880 | 1780 | 2170 | 2435 | 2470 | 2520 | 2820 | 2460 | 3350 | 3910 | 5990 |
|  | $0,8-1,4$ | 724 | 738 | 770 | 979 | 725 | 744 | 769 | 897 | 725 | 770 | 812 | 989 |
|  | 0,4-0,8 | 318 | 324 | 331 | 352 | 318 | 325 | 332 | 354 | 318 | 325 | 333 | 356 |
|  | 0,2-0,4 | I4I | 140 | 140 | 137 | I4I | 142 | 143 | 143 | I4I | 144 | I46 | 148 |

Диффөрениивльные өльбөдо неитронов для слоя карбидо боря $5 \mathrm{~cm}, \Delta E_{0}=I, 4-2,5$ Мав

| $\theta$ 0, | $\Delta E$ | $\varphi=0^{0}$ |  |  |  | $f=90^{\circ}$ |  |  |  | $\varphi=180^{\circ}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| гряд | Mab | $\theta=0$ | $30^{\circ}$ | $45^{\circ}$ | $75^{\circ}$ | $0^{0}$ | $30^{\circ}$ | $45^{\circ}$ | $75^{\circ}$ | $0^{0}$ | $30^{\circ}$ | $45^{\circ}$ | $75^{\circ}$ |
| 0 | 1,4-2,5 | 2995 | 3285 | 3560 | 4610 | 2995 | 3285 | 3560 | 4610 | 2995 | 3285 | 3560 | 4610 |
|  | 0,8-1,4 | 4490 | 4490 | 4520 | 4750 | 4480 | 4490 | 4520 | 4750 | 4480 | 4490 | 4520 | 4750 |
|  | 0,4-0,8 | I210 | 1235 | 1265 | I 365 | 1210 | 1235 | 1265 | 1365 | 1210 | 1235 | 1265 | 1365 |
|  | 0,2-0,4 | 436 | 439 | 447 | 431 | 436 | 439 | 44 I | 431 | 436 | 439 | 44 I | 43 I |
| 30 | I, $4-2,5$ | 3093 | 3373 | 3547 | 4995 | 3086 | 3300 | 3516 | 4544 | 3079 | 3034 | 3234 | 4463 |
|  | $0,8=I, 4$ | 4240 | 4064 | 4032 | 4268 | 4246 | 4272 | 4313 | 4613 | 4253 | 4512 | 4662 | 5 I4I |
|  | 0,4-0,8 | IL77 | 1206 | 1238 | 1343 | 1177 | I<OI | I23I | 1330 | II76 | 1197 | 1224 | 1316 |
|  | 0,2-0,4 | 423 | 426 | 428 | 419 | 423 | 426 | 429 | 419 | 423 | 426 | 429 | 419 |
| 45 | $1,4-2,5$ | 3088 | 3269 | 3465 | 5248 | 3084 | 3241 | 3419 | 4456 | 3079 | 2980 | 3074 | 4273 |
|  | 0,8-I,4 | 3959 | 3745 | 3715 | 3996 | 3767 | 4006 | 4062 | 4431 | 3976 | 4330 | 4543 | 5225 |
|  | 0,4-0,8 | IL33 | 1164 | II 95 | 1301 | 1133 | [157 | I185 | 1282 | 1133 | 1150 | 1175 | 1263 |
|  | 0,2-0,4 | 406 | 409 | 4II | 402 | 406 | 409 | 412 | 452 | 406 | 409 | 412 | 403 |
| 75 | $1,4-2,5$ | 2308 | 2659 | 3031 | 5479 | 2305 | 2419 | 2574 | 3664 | 2302 | 2376 | 2468 | 3016 |
|  | 0,8-1,4 | 2556 | 2405 | 2424 | 2689 | 2533 | 2600 | 2689 | 3235 | 2540 | 2898 | 3171 | 4338 |
|  | $0,4-0,8$ | 805 | 831 | 857 | 952 | 895 | 823 | 844 | 924 | 805 | 814 | 8 73 | 896 |
|  | $0.2-0,4$ | 281 | 283 | 285 | 281 | 281 | 284 | 286 | 282 | 281 | 284 | 286 | 283 |



| $\theta$ | $\Delta E$ | $\varphi=0^{\circ}$ |  |  |  | $\varphi=90^{\circ}$ |  |  |  | $\varphi=180^{\circ}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| rpad | M88 | $\theta=0^{\circ}$ | $30^{\circ}$ | $45^{\circ}$ | $75^{\circ}$ | $0^{0}$ | $30^{\circ}$ | $45^{\circ}$ | $79^{\circ}$ | $0^{0}$ | $30^{\circ}$ | $45^{\circ}$ | $75^{\circ}$ |
| 0 | 0,8-1,4 | 3899 | 3890 | 3925 | 4900 | 3895 | 3890 | 3925 | 4900 | 3895 | 3890 | 3925 | 4900 |
|  | $0,4-0,8$ | 4750 | 4780 | 4815 | 4930 | 4750 | 4780 | 4815 | 4930 | 4750 | 4780 | 4815 | 4930 |
|  | $0,2-0,4$ | I272 | 1285 | 1300 | 1310 | 1272 | 1285 | 1300 | 1310 | 1272 | 1285 | 1300 | 1310 |
| 30 | 0,8-I,4 | 3648 | 3598 | 3777 | 5739 | 3653 | 3683 | 3761 | 4857 | 3659 | 3896 | 4017 | 4518 |
|  | $0,4-0,8$ | 4525 | 4417 | 4402 | 4460 | 4530 | 4567 | 4613 | 4778 | 4535 | 4732 | 4855 | 5183 |
|  | $0,2-0.4$ | 1232 | 1248 | I265 | 1279 | 1232 | I244 | 1258 | 1270 | 1232 | 1241 | 1253 | I260 |
| 45 | 0,8-I,4 | 3394 | 3481 | 3798 | 6126 | 3399 | 3467 | 3586 | 4791 | 3404 | 3702 | 3891 | 4464 |
|  | $0,4-0,8$ | 4254 | 4IU8 | 4085 | 4157 | 4260 | 4305 | 4360 | 4581 | 4260 | 4530 | 46\% | 5175 |
|  | $0,2-0,4$ | 1178 | 1195 | I2II | 1230 | II78 | 1190 | 1203 | I2I? | II78 | 1185 | I196 | 1203 |
| 75 | 0,8-I,4 | 2462 | 3055 | 3539 | 6098 | 2452 | 2586 | 2767 | 4024 | 2443 | 2405 | 2579 | $36 I I$ |
|  | 0,4-0,8 | 2742 | 2623 | 2622 | 2781 | 2748 | 2810 | 2888 | 3297 | 2754 | 3046 | 3260 | 4129 |
|  | 0,2-0,4 | 805 | 820 | 833 | 865 | 804 | 814 | 825 | 848 | 804 | 808 | 816 | 829 |

Аиффврвниияльнне альбодо неитронов для слоя корбида борв IO см, $\Delta E_{0}=2,5-4$ Мэв


Дифференцигльнне өлбедо неятронов для олоя көрбидв бора $10 \mathrm{ck}, \Delta E_{0}=1,4-2,5$ мөв

| $\theta 0$ | $\Delta E,$ | $\varphi=0^{\circ}$ |  |  |  | $\varphi=90^{\circ}$ |  |  |  | $\varphi=180^{\circ}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| гред |  | $\theta=0^{\circ} 130^{\circ}$ |  |  | $75^{\circ}$ | $0^{0}$ | $30^{\circ}$ | $45^{\circ}: 75^{\circ}$ |  |  |  | $45^{\circ} 175^{\circ}$ |  |
| 0 | I, $4-2,5$ | 3860 | 4144 | 4400 | 5184 | 3860 | 4144 | 4400 | 5184 | 3360 | 4144 | 4400 | 5184 |
|  | 0,8-I,4 | 5850 | 5822 | 5791 | 5639 | 5850 | 5822 | 5791 | 5639 | 5850 | 5822 | 5791 | 5639 |
|  | 0,4-0,8 | 2100 | 2103 | 2 IO | 2006 | 2100 | 2103 | 2104 | 2006 | 2100 | 2103 | 2104 | 2006 |
|  | 0,2-0,4 | 912 | 303 | 889 | 779 | 922 | 903 | 885 | 779 | 912 | 903 | 889 | 779 |
| 30 | $I, 4-2.5$ | 3902 | 4180 | 4330 | 5540 | 3895 | 4100 | 4294 | 5077 | 3890 | 3816 | 39:4 | 4990 |
|  | 0,8-1,4 | 5500 | 5376 | 5150 | 5080 | 5510 | 5500 | 5486 | 5432 | 5515 | 5750 | 5854 | 5970 |
|  | 0,4-0,8 | 2010 | 2020 | 2 n 22 | 1940 | 2010 | 2013 | 2015 | I930 | 2010 | 2010 | 2010 | 1913 |
|  | 0,2-0,4 | 869 | 86 I | 848 | 744 | 869 | 86 I | 848 | 744 | 869 | 86 I | 848 | 745 |
| 45 | I,4-2,5 | 3816 | 3990 | 4170 | 5740 | 3811 | 3960 | 4114 | 4934 | 3806 | 3680 | 3750 | 4744 |
|  | 0,8-I,4 | 5084 | 4825 | 4747 | 4722 | 5093 | 5100 | 5112 | 5166 | 5103 | 5440 | 5614 | 5910 |
|  | 0,4-0,8 | 1890 | [903 | 1909 | 1845 | 1890 | 1895 | 1900 | 1823 | I890 | 1888 | 1887 | 1804 |
|  | 0,2-0,4 | 814 | 806 | 794 | 700 | 814 | 806 | 794 | 700 | 8 I 4 | 806 | 795 | 700 |
| 75 | I, 4-2,5 | 2600 | 2950 | 3318 | 5680 | 2595 | 2704 | 2851 | 3855 | 2592 | 2658 | 2740 | 3200 |
|  | 0,8-I,4 | 3003 | 2865 | 2865 | 2990 | 3010 | 3065 | 3140 | 3550 | 3017 | 3370 | 3630 | 4655 |
|  | 0,4-0,8 | II68 | IIS 3 | II97 | 1202 | II68 | II76 | II83 | II75 | II68 | II66 | II69 | II47 |
|  | 0,2-0,4 | 484 | 430 | 475 | 425 | 484 | 480 | 476 | 426 | 484 | 488 | 476 | 427 |

Диффөрендиальные альбөдо неитровов для слок карбидв бора 10 см, $\triangle E_{0}=0,8-I, 4$ Мэв

| $\theta$ | $\Delta E \text {, }$ | $\varphi=0^{\circ}$ |  |  |  | $\varphi=90^{\circ}$ |  |  |  | $\varphi=180^{\circ}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| sрод |  | $\theta=0^{\circ}$ | $30^{\circ}$ | $45^{\circ}$ | 75 | 0 | 30 |  |  | $0^{0}$ |  | $45^{\circ}$ | 75 |
| 0 | 0,8-I,4 | 4680 | 4650 | 4646 | 5366 | 4680 | 4650 | 4646 | 5366 | 4680 | 4650 | 4646 | 5366 |
|  | 0,4-0,8 | 5800 | 5790 | 5770 | 5560 | 5800 | 5790 | 5770 | 5560 | 5800 | 5790 | 5770 | 5569 |
|  | 0,2-0,4 | I890 | I885 | 1870 | 1730 | 1890 | 1885 | 1870 | I730 | 1890 | 1885 | 1870 | 1730 |
| SO | 0,8-I,4 | 4360 | 4290 | 4440 | 6170 | 4367 | 4374 | 4420 | 5282 | 4373 | 4533 | 4680 | 4934 |
|  | 0,4-0,8 | 5490 | 5340 | 5270 | 5030 | 5497 | 5495 | 5489 | 5353 | 5502 | 5664 | 5736 | 5759 |
|  | 0,2-0,4 | 1806 | 1803 | 1793 | I664 | 1806 | 1800 | I787 | I655 | 1805 | 1795 | 1782 | 1645 |
| 45 | 0,8-I, 4 | 4020 | 4091 | 4388 | 6514 | 4025 | 4074 | 4167 | 5167 | 4030 | 4314 | 4478 | 4836 |
|  | 0,4-0,8 | 5 III | 4926 | 4857 | 4665 | 5117 | 5127 | 5137 | 5090 | 5124 | 5359 | 5480 | 5686 |
|  | 0,2-0,4 | I694 | 1693 | 1686 | 1574 | 1693 | 1688 | 1679 | I56I | I694 | 1683 | I6? | I54? |
| 75 | 0,8-I,4 | 2697 | 3289 | 3766 | 6240 | 2687 | 2815 | 2990 | 4160 | 2677 | 2632 | 2800 | 3750 |
|  | 0,4-0,8 | 3090 | 2954 | 2935 | 2980 | 3095 | 3143 | 3200 | 3500 | 3100 | 3380 | 3577 | 4330 |
|  | 0,2-0,4 | 1034 | 104 I | 1043 | rOI2 | 1034 | 1035 | 1034 | 995 | 1034 | 1029 | 1026 | 977 |



Fig. 1. Differential flux albedo of neutrons (with $E_{0}, E>1 \mathrm{MeV}$ ) for a 25 cm layer of carbon at angles of $\vartheta_{0}=60^{\circ}$ and $\varphi=0^{\circ}$ :

—_ indicates calculated data.

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# PARAMETERS OF THE REFLECTION FROM IRON OF A FILTERFD FAST NEUTRON BEAM 

I.V. Goryachev, A.P. Suvorov, L.A. Trykov

The use of the albedo concept for solving engineering physics problems such as the passage of neutrons through channels and slots in shields, the determination of the characteristics of scattered radiation fields in closed volumes, step-by-step calculation of radiation fields in separate zones of multi-layer shielding with the assignment of irradiation and reflection boundary conditions allowing for the effect of adjacent zones $[1,2]$ and so on requires us to determine the reflection parameters of neutrons which have already passed through a certain mass of shielding material. The fact that many materials have a pronounced cross-section resonance structure leads to filtration of the source neutrons at the cross-section minima and to the associated effect of "filtered" neutrons interacting with the material of the reflecting layer. Thus it can be assumed, for example, that the albedo of a reactor spectrum beam for materials with a cross-section resonance structure will be very different from the albedo of a neutron beam transmitted through a layer of material similar to the material of the reflecting layer and depleted in resonance neutrons.

The aim of this investigation was to verify experimentally the lower albedo for iron associated with a filtered neutron beam, to establish the reflection parameters for the conditions in question, and also to develop methods of allowing for resonance self-shielding of cross-sections in multigroup calculations of neutron transmission through shielding.

The methods used for the experimental investigations were similar to those described in Ref. [3]. The neutron source was a zero-power research reactor with a 16 cm thick stainless steel reflector. The initial neutron beam leaving the surface of the reflector was filtered with an iron prism 27 cm thick, installed in a channel of the reactor's biological shield. The neutron detector employed was a singlemorystal scintillation spectrometer with gamma discrimination based on de-excitation time [3]. The flux and dose albedos were determined from measurements of the neutron energy distributions. The specimen used for investigating the reflection characteristics was a 19.5 cm thick iron strip.

The differential flux albedos were calculated by a special method based on a kinetic equation in the $2 D_{7} P_{7}$-approximation of the discrete ordinates method [2]. The calculations were performed for seven energy groups with the ROZ-III programme, using the group constants employed for calculating the albedos in Ref. [4] multiplied by an additional blocking factor. This factor was determined as the ratio of the blocked totel cross-section, obtained by measuring the transmission of a filtered beam in narrow geometry [5], to the total crossmsection used in calculations for an unfiltered neutron beam [4]. In the latter values, resonance self-shielding of the cross-sections was taken into account by an approximate "homogenization" method [6]. However, the degree of self-shielding for an unfiltered beam is much less than for a filtered beam. Table 1 shows the blocked total crosssections for a filtered beam and the corresponding values of the additional blocking factor $f$. The differential elastic and inelastic scattering cross-sections were multiplied by this factor, i.e. it was assumed that these cross-sections are shielded to exactly the same extent.

The nature of the experimental and theoretical results obtained is illustrated in Figs 1 and 2, which show the energy distributions of reflected neutrons for different incident beam angles $\gamma$ both for a scalar flux (Fig. l) and for an angular flux (Fig. 2).

The results indicate that the albedo of the prefiltered beam neutrons is the albedo of a beam with a "smooth" spectrum, i.e. with no pronounced reasonance structure. From Fig. l it can be seen that the reflected neutron intensity for a filtered beam in the energy range $E<2.5-3 \mathrm{MeV}$ is several times lower than the intensity of neutrons reflected from an iron strip of the same thickness when its surface is irradiated with an unfiltered beam having a reactor spectrum. The normalization of the integral intensity was identical for both incident beams. The shape of the infiltered beam spectrum and that of the filtered beam spectrum (smoothed by averaging over the resolution of the spectrometer - Fig. l), differ somewhat. However, as the present authors have already established in previous investigations, slight variations in the shape of the incident beam spectrum cause only insignificant variations in the reflected neutron intensity. Therefore the difference observed between the reflected neutron intensities of the filtered and unfiltered beams is caused by the difference in the fine microstructure of their spectra and not by the difference in the smooth (averaged) shape of their spectra.

The data in Fig. l also indicate how the effect of the microstructure of the incident beam spectrum decreases as the angle at which the beam strikes the surface of the reflecting layer becomes larger. This is directly due to a reduction in the neutron cross-sections as a result of their self-shielding and to a corresponding increase in the probability of straight-line streaming and small-angle scattering.

This conclusion is also backed up by results of calculations of the integral flux albedos. To illustrate this, Table 2 shows the flux albedos of neutrons in the $0.4-1.4 \mathrm{MeV}$ range (same energy range before and after reflection).

Fig. 2 shows the experimental and calculated energy distributions of an angular flux of reflected neutrons for different angles $\psi$ with normal incidence of a filtered beam onto the iron strip. The angle $\psi$ is measured from the normal.

Table 3 compares the differential angular albedos calculated for unfiltered $[7]$ and filtered neutron beams in the $0.4-1.4 \mathrm{MeV}$ range (same range before and after reflection). Here we were considering a plane passing through the projection of the incident neutron beam direction onto the surface of the reflecting layer (i.e. with azimuthal angle $\varphi=0^{\circ}$ ).

As can be seen, filtration of the beam considerably reduces the differential angular albedos as well (by factors of up to 2-3). However, the effect is much less for angles of $\psi$ far from the normal. This is particularly true in the case of oblique beam incidence giving "sliding" reflection (such that the reflected beam direction deviates little from the direction of incidence).

Table 4 compares the theoretical and experimental values of the integral flux and dose albedos for unfiltered and filtered beams of fast neutrons with $E>100 \mathrm{keV}$ (for the experimental conditions described above) at different incident beam angles $\psi$.

Note that the data in Table 4 for the unfiltered beam relate to a strip 15 cm thick and the data for the filtered beam to a strip 19.5 cm thick. However, there should be very little difference between the fast neutron albedos of an unfiltered beam for these two thicknesses, particularly in the case of oblique beam incidence.

The large difference between the experimental and theoretical data for the filtered beam (factors of 2-2.5) - far exceeding the very slight difference observed in the case of the unfiltered beam (no more than 20-30\%) is explained by the approximate nature of the assumptions made in deriving the group constants for calculating the filtered beam albedos. In the first place, the values of the total cross-section $\langle\sigma\rangle$ (Table l) used in the calculations do not, generally speaking, characterize the spectrum of neutrons striking the reflecting layer, but represent rather the average characteristics of the neutron spectrum in the filter. Strictly speaking, the total cross-section for such a calculation should be derived from an analysis of the transmission curve at the very end of the filter. Such cross-sections would be smaller than those which were employed in the calculation and, if they were used instead, beam filtration would be shown to reduce the albedo values even more. In the second place, the validity of the assumption that the elastic and inelastic scattering cross-sections have identical self-shielding has not been verified; and, in the third place, the assumption that there is no resonance self-shielding of the angular distributions in neutron scattering is unjustified. As shown in Ref. [9] for neutrons with $\mathrm{E}>1.5 \mathrm{MeV}$, and in Ref. [10] for $1-2 \mathrm{MeV}$ neutrons, filtration of the beam has the effect of slightly increasing the anisotropy in the case of scattering into the forward half-space and of increasing the isotropism in the case of scattering at angles exceeding $90^{\circ}$, compared with the unfiltered beam. None of these effects were allowed for in the above calculation because the literature as yet offers no detailed data on the effect of resonance self-shielding of the differential scattering crosssections.

Thus filtration gives a bean depleted in resonance neutrons and thereby substantially reduces the albedo of an iron reflector 19.5 cm thick. The experimental data indicate a dose albedo lower by factors of $2.6-3$ and a flux albedo lower by factors of 2.8-3.6. This albedo reducing effect is greater with normal beam incidence and smaller with oblique incidence. The albedo versus angle of incidence curve is generally the same as for an unfiltered beam. However, in the large incident angle range the albedo is somewhat higher for the filtered beam than would follow from the pattern established for the unfiltered beam. This is due to the increase in anisotropy occurring
with scattering into the forward half-space and to the increase in isotropy occurring with scattering at angles larger than $90^{\circ}$, which are characteristic of a filtered beam as mentioned above. This is also the reason why there is greater isotropism in the angular distribution of the reflected neutrons for a filtered beam than for an unfiltered beam.

It would also be of great practical interest to investigate the albedos of filtered beams for iron strips more than 20 cm thick. From the physics of the phenomena analysed here one would expect a much closer relationship between neutron albedo and strip thickness for filtered beams than for unfiltered beams.

It should be noted that in the case of neutron transmission through realistic shields (i.e. not in narrow beam geometry) consisting of materials with a pronounced resonance structure of the cross-sections, the flux also becomes depleted in resonance neutrons; but this depletion is of course less than in narrow beam geometry and the albedo reducing effect of beam filtration is correspondingly slighter. In other words, for the calculation of real shields the data presented here represent in a certain sense an upper limit on the effect (for $\sim 20 \mathrm{~cm}$ thick iron strips). Nevertheless it is clear that when calculating neutron transmission through a channel in an iron shield for example, one should, generally speaking, use values for the albedos of the "inleakage" component which differ from the albedos of the "direct visibility" component.

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## Table 1

Blocked total cross-sections $<\infty$ and additional blocking factors $f$ for neutrons of different energies

Key:

| Номер группы | $=$ number of group |
| ---: | :--- |
| Энергия, Мэв | $=$ energy, MeV |
| барн | $=$ barn |

## Table 2

Theoretical flux albedos of neutrons in the $0.4-1.4 \mathrm{MeV}$ range

Key:

| град | = degrees |
| :---: | :---: |
| Нефильтров.) <br> пучок | $=\text { unfiltered beam }[7]$ |
| Фильтров. | = filtered beam |
| пучок |  |

Table 3
Calculated differential angular albedos of neutrons in the $0.4-1.4 \mathrm{MeV}$ range (per steradian)

## Key:

град $\quad=$ degrees
Нефильтров•) = unfiltered beam [7]
пучок
Фильтров.пучок $=$ filtered beam

Table 4
Integral flux and dose albedos of fast neutrons with $\mathrm{E}>100 \mathrm{keV}$ for an iron strip*

Key:


$$
\text { расчет } \quad=\text { calculation }
$$

* Note: the figure above the line is the flux albedo and that below the line is the dose albedo.

Table

Значе:ия бдокированного полного сечения $\langle\boldsymbol{\sigma} \boldsymbol{\text { и факторя дополнительои блокировки }}$
\& для неитронов резличннх анеррин

| Ном⿵人 <br> групли | Зnepris, M | $\begin{gathered} \langle\sigma\rangle \\ \sigma \theta р ; \end{gathered}$ | $\mathcal{F}$ |
| :---: | :---: | :---: | :---: |
| I | 4. - 6,5 | 3,68 | I, 0 |
| 2 | 2,5-4 | 2,90 | 0,86 |
| 3 | I, $4-2.5$ | I. 32 | 0,446 |
| 4 | 0,8-1.4 | 0,65 | 0,29 |
| 5 | 0,4-0,8 | 0,55 | 0,204 |
| 6 | 0,2-0.4 | 0,44 | 0,191 |
| 7 | $0,1-0,2$ | 0,50 | 0,347 |

Table 2

Ресчетныя потоковые вльбедо невтронов с звергиед 0.4-I. 4 M3


Table 3
Расчөтнне дифјөренциальные угдовье альбедо неятронов с энергиеи 0,4-I,4 Мэв (на стерадиан)

| $\underset{r p b_{\pi}}{\gamma}$ | $\begin{gathered} \Psi \\ \text { гряд } \end{gathered}$ | Нөфилттов. пучок [7] | Фильтров. аучон |
| :---: | :---: | :---: | :---: |
| 0 | 0 | 0,148 | 0,055 |
| 0 | 60 | 0,146 | 0,066 |
| 60 | 0 | 0,095 | 0,033 |
| 60 | 60 | 0,107 | 0,069 |

Table 4
Иетегральнне потоковне и дозовве альбе до бострдх не市ронов с е $>100$ кэв для делезнои плостини *)

| $\begin{gathered} \gamma, \\ \text { rpag } \end{gathered}$ | Нефильтрованныи пучок |  | Фильтрованнић пучок |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Эксперимент [8] | расчет [8] | эксперимөнт | ресчer |
| 0 | 0.77 | 1,06 | 0,2I | 0,51 |
|  | 0,82 | 0,85 | 0,27 | 0,48 |
| 30 | 0,67 | 0,96 | 0,19 | 0,50 |
|  | 0,73 | 0,75 | 0,25 | 0,47 |
| 50 | 0,58 | 0,79 | 0.183 | 0.46 |
|  | 0,63 | 0,62 | 0,23 | 0,43 |
| 70 | 0.37 | 0,54 | 0.132 | 0,36 |
|  | 0,40 | 0,40 | 0.153 | 0,34 |





Fig. 1. Energy distributions of a scalar flux of neutrons reflected from an iron strip for different incident beam angles $\gamma$ :

-     -         -             - . filtered beam
$\qquad$ unfiltered beam
- filtered beam spectrum

The histograms show the calculated intensities of reflected neutrons for filtered beams.

Fig. 2. Energy distributions of an angular flux of reflected neutrons for different angles $\psi$ with normal incidence of the filtered beam on the iron strip. $I$ indicatie experimental data; the histograms represent calculated values.

## INVESTIGATION OF THE ERROR INVOLVED IN VARIOUS APPROXIMATIONS OF THE DISCRETE ORDINATES METHOD IN REACTOR SHIELDING CALCULATIONS

## A.P. Suvorov, V.A. Utkin

The discrete ordinates method $[1-4]$ is being used more and more for calculating reactor shielding. When calculating with programmes based on this method, the work involved as well as the amount of computing time increase greatly with the number of the approximation; hence it is important to study the nature of the convergence of the solution to the accurate solution in relation to this number. This is particularly important for the solution of optimization problems, when it is necessary to calculate a large number of variants, and therefore undesirable to use a high order approximation.

Moreover, whilst the limitations of the computer memory quite often do nct allow calculations of alternative compositions with a large number of spatial nodes $[3]$ in high approximations, this is quite feasible in the case of sufficiently low-order approximations. However, no detailed study has yet been made of the errors involved in these approximations of the discrete ordinates method or of the difference betwecn the resuits obtained with them and reactor shielding calculations performed with higher approximations•

In this paper the authors compare the results of calculations in the different approximations of the discrete ordinates method for scalar and angular fast neutron fluxes in the heterogeneous iron-water shield of a water moderated and cooled reactor. The shield was a four-layer construction consisting of two layers of iron 10 cm thick alternating with two layers of water 30 cm thick (Fig. 1). The core composition was assumed to be similar to that of the reactor at the TES 3 nuclear power station [5].

The calculations were performed for nine neutron groups in the $0.1-14 \mathrm{MeV}$ range. The iron and water constants from Ref. [3] used in the calculations were verified by calculations based on experimental data on fast neutron distributions in homogeneous shields consisting of water or iron [3] and in heterogeneous shields of the same materials [6].

The calculations were performed with the ROZ-2 programme $[7]$ in the $2 D_{N} P_{L}$ approximation of the discrete ordinates method (the $2 D_{N}$-approximation of the angular flux and the $\mathrm{P}_{\mathrm{L}}$-approximation of the scattering angular distribution) for different $N$ and L. The $2 \mathrm{D}_{\mathrm{N}}$ approximation of the
angular flux [3] involves approximating the flux with values at the 2 N angular nodes, for which the binary Gaussian nodes (roots of the equation $P_{N}(2 \mu \pm 1)=0$ ) are chosen. The $P_{L}$-representation of the scattering angular distribution means that the expansion of the angular distribution into Legendre polynomials is restricted to the ( $L+1$ ) - term (including the zero term).

Since the ROZ-2 programme is intended for calculating neutron fields (and gamma quanta) in non-fertile media only, the reactor shield calculation is performed for a specific distribution of fission neutron sources in the core. This distribution was obtained by a calculation with the reactor programme in the $P_{1}$ approximation of the spherical harmonics method $[8]$, using a 26-group system of constants (9). The preserce of fertile material in the core was taken into account, as suggested in Ref. [10], by an approximate method whereby it is assumed, in compiling the group constants for uranium, that neutrons produced in the fission of nuclei by neutrons of a given group either remain partially in that group or are produced with lower energies.

The calculated spatial fast neutron flux distributions in the reactor core and the iron-water shield performed in the $2 \mathrm{D}_{12} \mathrm{P}_{12}$-approximation, together with the virtually identical results of the calculation in the $2 \mathrm{D}_{7} \mathrm{P}_{7}$-approximation, are given in Fig. 1. These calculations of the scalar neutron flux for a shield of this particular thickness may be regarded as accurate for all practical purposes. This conclusion is confirmed by comparing the results with calculations using the ROZ-j programme [11] which can be regarded as even more accurate, since in this case the $2 \mathrm{D}_{12^{-}}$ approximation was used not only for the angular neutron flux but also for the angular distribution of scattering by hydrogen nuclei $[7,3]$ (the $P_{12}$-approximation of the angular distribution was used for the other elements). The adoption of this "discrete" specification of the scattering angular distribution for hydrogen nuclei $[7,11,3]$ j.s justified in view of the poor convergence of the $P_{L}$-representation of thj.s angular distribution. For example, even the $P_{12}$-representation often leads to negative values of the angular distribution at many angles where arc cos $\mu_{s}>50 \%$ and sometimes also at smaller angles.

Good agreement is found on comparing the calculations of the scalar neutron flux with $E>2.5 \mathrm{MeV}$ in the $2 \mathrm{D}_{12}$-approximation of the angular flux
and in the $\mathrm{P}_{12}$-approximation of the scattering angular distribution for hydrogen nuclei, performed with the ROZ-2 programme, and in the $2 D_{12}$-approximation, performed with the ROZ-5 programme. For the shielding composition with which we are concerned here the maximum difference in the neutron fluxes was less than $2 \%$ in the iron layers and less than $1 \%$ in the water layers.

Jumping ahead a little, it should be noted here that, although this difference is small, it is somewhat greater than the difference in the results of the $2 D_{12} P_{12}$-calculations and the $2 D_{N} P_{L}$-calculations for $\mathrm{N}, \mathrm{L} \sim 5$. Thus the difference between the algorithms used for solving the problem (matrix factorization in the ROZ-2 programme and the iteration method in the ROZ-5 programme) and between the respective programme codes had more effect on the attenuation values than a reduction in the order of the approximation.

In view of the closeness of the results of the different approximations there is no point in comparing the neutron flux distributions in the usual semilog graphs. Instead, the numerical results of the different approximations are compared in Tables l-5. To facilitate comparison, the Tables include data on the discontinuity of the neutron flux values at the boundaries of the different layers compared with calculations using the $2 \mathrm{D}_{12} \mathrm{P}_{12}$ (or $2 \mathrm{D}_{7} \mathrm{P}_{7}$ ) approximation.

The following conclusions may be drawn from an analysis of Tables 1-5:

1. If the accuracy of the calculated scalar neutron flux ( $\sim 20 \%$ ) is considered adequate (this compares with the present accuracy of measuring neutron spectra), then for attenuations ( $\alpha$ ) of more than $10^{1}-10^{5}$ it is necessary to use the $2 \mathrm{D}_{\mathrm{N}} \mathrm{P}_{\mathrm{L}}$-approximations with $N \geqslant 3, L \geqslant 2$, i.e. approximations of a higher order than $2 \mathrm{D}_{3} \mathrm{P}_{2}$. Then with increasing $N$ and $L$ the error in the approximations decreases, and for approximations with $N, L \geqslant 5$ the error in the scalar flux calculations for attenuations of less than $10^{5}-10^{6}$ (due to the finiteness of the approximation) is virtually zero (the error due to inaccuracies in the constants and other factors is not considered here);
2. When $N=1$ and 2, the solution of the problem is strongly dependent on the value of L. This is because the representation
of the scattering angular distribution of fast neutrons by its values at only two or four angular points is - owing to the poor convergence of the $P_{L}$-approximation of the angular distribution closely dependent on $L$. As a result the data of the $2 D_{1} P_{L^{-}}$and $2 D_{2} \mathrm{P}_{\mathrm{L}}$-approximations may differ by several factors from the accurate values;
3. On the whole the error in the calculations increases systematically with flux attenuation, and in this case $2 \mathrm{D}_{\mathrm{N}} \mathrm{P}_{\mathrm{N}}-$ approximations at large distances from the core yields results which are too low rather than too high. The: accuracy of the $2 D_{N} P_{N}$ calculations is somewhat greater when $N$ increases than when $L$ increases;
4. The error in the calculations has a relatively weak dependence on neutron energy.

Fig. 2 shows the results of calculations in tre $2 \mathrm{D}_{12} \mathrm{P}_{12}$-approximation for an angular flux of neutrons escaping from the shielding composition under consideration. As can be seen, the angular distributions of these neutrons are well spread out in the forward direction. However, the form of this spread is reproduced differently by different $2 D_{N} P_{L}$-approximations.

Let us describe the degree of spread with the coefficient of nonuniformity of the angular neutron flux distribution, which is the ratio of the maximum angular flux (in the forward direction) to the mean angular flux (i.e. averaged over all angles of neutron escape). Table 6 shows the values of this coefficient for different approximations.

It should be noted that, to obtain the coefficient of non-uniformity of the angular flux, it is necessary to know the values of the flux at $\mu=1$, and these are not calculated in the $2 D_{N} P_{L}$-approximation because the only solutions of $F$ considered are those at the Gaussian nodes $\mu_{n}$ Strictly speaking, to obtain the angular flux for other angles it would be necessary to use the following interpolation formula:

$$
\begin{equation*}
F(\mu)=\sum_{n} F\left(\mu_{n}\right) \operatorname{Ln}(\mu) \tag{1}
\end{equation*}
$$

Here $L_{n}(\mu)$ are the Lagrangian interpolation factors.

$$
\begin{equation*}
\operatorname{Ln}(\mu)=\frac{\prod_{n}^{\prime} \neq n}{\prod_{n^{\prime} \neq n}\left(\mu-\mu_{n}^{\prime}\right)} \tag{2}
\end{equation*}
$$

To simplify our calculations the ordinary linear interpolation ( with respect to $\mu$ ) was used here in calculating $F(\mu=1$ ).

The coefficient of non-uniformity of the angular flux, characterizing the degree of forward spread, also to a large extent characterizes the penetrating power of the flux over large distances. Judging from Table 6, the low order approximations systematically understate the values of this coefficient, but the characteristics of the angular flux approach the accurate solution fairly monotonically as the order of the approximation is increased. However, in this case we consider the most accurate results to be those obtained in the calculation using the ROZm programme mentioned above, i.e. in the $2 \mathrm{D}_{12}$-approximation of the angular distribution of scattering by hydrogen nuclei.

Table 6 shows that the error in the approximations can increase considerably for the attenuation range beyond $10^{5}-10^{6}$. Therefore the set of approximations capable of providing a satisfactory description of these attenuations must be reduced. This will obviously involve the use of approximations with $N \geqslant 5, L \geqslant 3$, i.e. approximations of a higher order than $2 \mathrm{D}_{5} \mathrm{P}_{3}$. Special investigations will have to be carried out to establish more accurately the limits of applicability of the various approximations for describing the higher attenuations characteristic of biological shielding.

Note further $[7]$ that if there are any hydrogen-containing materials in the shield (as for example in our case), then, because of the bad convergence of the $\mathrm{P}_{\mathrm{L}}$-approximation of the "hydrogen" angular distribution mentioned above, calculations performed in the $2 \mathrm{D}_{\mathrm{N}} \mathrm{P}_{\mathrm{L}}$-approximations will not give a very good description of the angular neutron flux travelling in the direction of the radiation source, i.e. when $\mu<0$. The error of the low-order approximations is particularly large when the calculations give negative values of the angular flux for many different angles. However, when $N$ and L are large enough the integral flux travelling back towards the source can often be described with relatively good accuracy even in cases where hydrogen is present [12]. But we will not consider this problem further here.

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Table 1
Differences in fast neutron fluxes calculated in the $2 D_{N} P_{L}-$ and $2_{12} \mathrm{P}_{12}$-approximations at the core boundary, \%

Table 2
Differences in fast neutron fluxes calculated in the $2 \mathrm{D}_{\mathrm{N}} \mathrm{P}_{-}$- and


## Table 3

Differences in fast neutron fluxes calculated in the $2 D_{N} P_{L}$ - and ${ }^{2 \mathrm{D}}{ }_{12} \mathrm{P}_{12}$-approximations at a distance of 40 cm from the core, $\%$

## Table 4

Differences in fast neutron fluxes calculated in the $2 \mathrm{D}_{\mathrm{N}} \mathrm{P}_{\mathrm{L}}$ - and ${ }_{2 D}{ }_{12} P_{12}$-approximations at a distance of 50 cm from the $\mathrm{No} \mathrm{L} \mathrm{L}, \%$

## Table 5

Differences in fast neutron fluxes calculated in the $2 D_{1}-$ and ${ }^{2}{ }_{12} P_{12}$-approximations at a distance of 80 cm from the core, \%

## Table 6

Coefficient of non-uniformity of the angular distribution of a neutron flux behind an iron-water shield 80 cm thick in the $2 \mathrm{D}_{\mathrm{N}} \mathrm{P}_{\mathrm{L}}$-approximation

Key for the above tables:

$$
M \partial_{B}=\mathrm{MeV}
$$

Table 1

Резличие значенин потожа бшстрых вентронов, рассчитаввцх в $2 \mathcal{D N}_{\sim} P_{L}$ и $\mathcal{D}_{12} P_{12}$ - приблигениях, ва границе эктивно月 30вы, \%

| E, ${ }^{\text {¢ }}$ 98 |  | I | 2 | 3 | 5 | 7 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| >2,5 | I | I. 35 | 6,27 | 4,22 | 22,8 | 31.6 |
|  | 2 | 0,89 | 0,4I | -0,40 | -0,49 | 1,83 |
|  | 3 | 0,57 | -0,01 | 0,05 | -0,04 | -0,06 |
|  | 5 | 0,60 | 0,01 | 0.10 | 0,02 | 0,OI |
|  | 7 | 0,63 | 0,03 | 0,08 | 0,08 | - |
| 71.4 | 1 | -0,13 | 5,75 | 6,57 | 24.8 | 29.5 |
|  | 2 | 0,26 | -0,02 | -0,32 | -0,37 | 0,55 |
|  | 3 | 0,98 | 0,30 | 0,04 | 0,02 | -0,05 |
|  | 5 | 0.75 | 0,26 | 0,04 | $0,0^{2} 7$ | 0,02 |
|  | 7 | 0,60 | 0,2I | 0,04 | 0,03 | - |
| O,I-I.4 | I | -3,09 | 7,86 | 8.48 | 24,4 | 3I,7 |
|  | 2 | 0,44 | 0,57 | 0,30 | C. 87 | I,08 |
|  | 3 | 0,39 | 0,12 | 0.II | 0,10 | 0,07 |
|  | 5 | 0,34 | 0,I4 | 0,02 | $0,0^{2} 6$ | $0,0^{2} 8$ |
|  | 7 | 0.26 | 0,16 | 0,01 | $0,0^{2} 2$ |  |

## Table 2

Ряаличив анячения потоня бистрых неитронов, рассчитввввх в $2 \boldsymbol{D N}_{N} P_{L}-42 D_{12} P_{12}$ - приблипениях, нв расстоявди IO см от вктивноЯ зонн, \%


Table 3

Рязлвчие значений потокө бистрых неятронов, рассчитанных в $2 D_{N} P_{L}$ и $2 ฎ_{12} P_{12}$ приблихениях, на расстояния 40 cx at artивнои зони, \%

| $\mathrm{E}_{\text {, }}$ प98 | $\alpha$ |  | I | 2 | 3 | 5 | 7 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 72,5 | $6.10{ }^{2}$ | I | -93,8 | -92,0 | -55,0 | 4,85 | 8I, 6 |
|  |  | 2 | -24,2 | $-24,0$ | $-9,0$ | 17.9 | 42,0 |
|  |  | 3 | -I8,9 | $\bigcirc 0,45$ | 0,62 | 0,69 | 2,13 |
|  |  | 5 | -19,0 | -0,69 | 0,19 | 0,01 | 0,02 |
|  |  | 7 | -I9, I | -0,85 | 0,15 | $-0,0^{2} I$ | - |
| >I,4 | 6,9.10 ${ }^{2}$ | I | -94,2 | -92,4 | -57,3 | -2,46 | 55.8 |
|  |  | 2 | -26,0 | $-25.8$ | -97.7 | -L2,5 | 30,0 |
|  |  | 3 | $-20.5$ | -0,29 | 0,55 | 0,61 | I,44 |
|  |  | 5 | -20,6 | -4,50 | 0,15 | 0,024 | $0,0^{3} 5$ |
|  |  | 7 | $-20.7$ | -0,72 | 0.13 | $-0.0{ }^{2}$ | - |
| 0,I-I,4 | $8,6.10^{2}$ | I | -94.4 | -92.I | -60,0 | -48.5 | 40,1 |
|  |  | 2 | -26,8 | $-26.3$ | $-9.58$ | 5,83 | 19.5 |
|  |  | 3 | -2I, 5 | -0.16 | 0,44 | 0.48 | 0.93 |
|  |  | 5 | -21,6 | -0, 29 | 0,05 | 0,06 | 0,026 |
|  |  | 7 | $-21.7$ | -0,48 | 0,04 | 0,01 |  |

## Table 4

Рвзличие значении потока бнстрых нептронов, рассчитянных в $2 D_{N} P_{L}-42 D_{12} P_{12}$ - приолихениях, на расстояани 50 cx от зктиннод зоны. \%

| S, Мэв | $\alpha$ |  | I | 2 | 3 | 5 | 7 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $>2.5$ | 3.3.10 ${ }^{3}$ | $I$ | --97,6 | -96,7 | -60,6 | 23,8 | 184 |
|  |  | 2 | $-38.2$ | $-38,1$ | -I9, I | 16,0 | 54.2 |
|  |  | 3 | $-28.7$ | -I. 25 | D,2I | 0,27 | -0,04 |
|  |  | 5 | $-28,7$ | -I,38 | 0.20 | 0,01 | -0.04 |
|  |  | 7 | -28,7 | -I,4I | 0,19 | $-0.0{ }^{2} 2$ | - |
| >1,4 | 3,0.10 ${ }^{3}$ | $I$ | -97.1 | -95,8 | -63.1 | S,70 | 120 |
|  |  | 2 | $-36,6$ | -36,6 | $-17.5$ | IO, I | 36,0 |
|  |  | 3 | -28, 1 | -I. 23 | 0.37 | 0.19 | 1.54 |
|  |  | 5 | -28.1 | -13.0 | 0,15 | 0,01 | $-0.0{ }^{2} 9$ |
|  |  | 7 | -28.1 | -13.3 | 0.14 | $-0,00^{2}$ |  |
| $0.1-1.4$ | $1.7 .10^{3}$ | 1 | -95.2 | -92,8 | $-60.3$ | -74,2 | 81,0 |
|  |  | 2 | -29.8 | $-9.43$ | -5,56 | 6.15 | 25.7 |
|  |  | 3 | -23.6 | -0,23 | 0,29 | 0, 33 | I,07 |
|  |  | 5 | -23, | 0,33 | 0,05 | -0,02 | -0,II |
|  |  | 7 | -23.7 | 0,40 | 0,03 | -0, $0^{2} 3$ | - |

## Table 5

Различя аначевии потока бнстрых ноцтровов, рассчитанннх в $2 D_{N} P_{L}-$ и $2 D_{12} P_{22}$ - прибпихенпах, ва расстовнип 80 cm от активной зонн, $\%$


Table 6

सоэффициент неравномерности углового распредедення дотокв вейтонов за хелезо-водной защитод толщинон 80 cm в $2 D_{N} P_{L}$ - приблидява耳

| E. M38 | $N$ | $L=I$ | $L=2$ | $L=3$ | $L=5$ | $L=7$ | $20_{12}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $>4$ | 2 | 2.10 | 2,12 | 2,19 | 2.40 | 2,39 | 4,58 |
|  | 3 | 2,69 | 2,96 | 3,20 | 3,24 | 3,30 |  |
|  | 5 | 3,06 | 3,62 | 4,09 | 4,25 | 4,26 |  |
|  | 7 | 3,42 | 4. 29 | 4,4I | 4,53 | 4,54 |  |
| $>2.5$ | 2 | 2.03 | 2,03 | 2,I5 | 2,27 | 2,28 | 4,28 |
|  | 3 | 2.49 | 2,80 | 2,99 | 3,02 | 3.04 |  |
|  | 5 | 2,77 | 3,15 | 3,70 | 3.80 | 3,81 |  |
|  | 7 | 3,07 | 3,49 | 3.93 | 4,OI | 4,02 |  |
| $>\mathrm{I}, 4$ | 2 | I. 94 | I,94 | 2,09 | 2,17 | 2.09 | 3.75 |
|  | 3 | 2,33 | 2.64 | 2,81 | 2.83 | 2.84 |  |
|  | 5 | 2,56 | 2,93 | 3,39 | 3,45 | 3.46 |  |
|  | 7 | 2,79 | 3.23 | 3,57 | 3,62 | 3,62 |  |



Fig. 1. Flux distribution of fast neutrons of different energies in the iron-water shield of a water moderated and cooled reactor. (Broken line $=$ source intensity distribution; $q=$ number of fission neutrons per $\mathrm{cm}^{3}$ ).


Fig. 2. Angular distributions of neutrons of different energy escaping from iron-water shield 80 cm thick.

## BREMSSTRAHLUNG YIELDS OF ELECTRONS WITH AN END-POINT ENERGY OF 22.5 MeV AS A FUNCTION OF THE ATOMIC NUMBER CF A TARGET OF VARIABLE THICKNESS

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V.P. Kovalev, V.P. Kharin, V.V. Gordeev
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According to the theory of Bethe ard Heitler [1_7 radiative energy losses for an "infinitely thin" target are proportional to $z^{2}$ ( $Z$ being the atomic number of the target). For the "thick" total absorption targets and "medium" thickness targets used in practical applications, deviations of the Bremsstrahlung yields from the $Z^{2}$ law may be expected owing to the increase in non-radiative electron energy losses, absorption in the target and multiple emission of photons by a single electron [2]. These processes begin to appear at thicknesses as little as D.1 Xo (Xo being the radiation width). Buechner and co-workers [3] analysed experimental data on Bremsstrahlung yields for total absorption targets made of $\mathrm{Al}, \mathrm{Be}$, $\mathrm{Cu}, \mathrm{Ag}, \mathrm{W}$ and $\mathrm{Au}\left(E_{\gamma}{ }^{\max }=1.25-2.35 \mathrm{MeV}\right)$ in order to find the dependence of the yield on $Z$. They showed that for targets whose thickness slightly exceeds the electron range the Bremsstrahlung yield in the energy range investigated increases linearly with increasing $Z$. For medium thickness targets it can evidently be expected that the yield will vary as $\sim Z^{k}$, where $1<k<2$.

In order to verify this assumption we used the LUF-25 linear accelerator operating at an electron energy of 22.5 MeV to measure the forward Bremsstrahlung yields in the $0-12^{\circ}$ angle range for targets of $\mathrm{Al}, \mathrm{Ti}, \mathrm{Cu}$, Mo, Ta and W of various thicknesses. The experimental setmp and procedure are described in Ref. [5].

The experimental results are presented in Fig. 1 in the form of yields relative to aluminium. The errors in measuring the yields were 6-9\%. The target thickness ( $t$ ) is expressed as a fraction of the total electron range ( $R$ ). The curve for thin targets ( $t=0.02 R$ ) agrees with the $z^{2}$ dependence and for thicker targets the above assumption is confirmed: the Bremsstrahlung yield varies as $Z^{k}$, tending to the linear as the thickness increases.


Fig. 1

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# CALCULATION OF ATOMIC EXCITATION AND ELASTIC SCATTERING CROSS-SECTIONS NECESSARY FOR COMPUTING THE STOPPING POWER AND IONIZATION ENERGY OF A SUBSTANCE 

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The stopping power and ionization energy of a substance exposed to a flux of charged particles (and neutrons) can be determined theoretically if the cross-sections of elastic scattering, excitation, ionization and charge exchange are known and, in the case of ionization and elastic scattering, the differential cross-sections as well [1]. For many substances these cross-sections are either not known or have been determined only at isolated points. The situation is particularly bad in the low energy range, but as it happens knowledge of the cross-sections at low incident particle energies is important for calculating the effects associated with secondary electrons.

A common method suitable for calculating the excitation crosssections of any atomic substance is proposed here. The wave functions needed to calculate the matrix elements were found by two methods: the Bates-Damgaard method $[2]$ and a method taking into account the atomic potential (calculated by the Hartree-Fock method) in which the correct asymptotic form of the wave functions is conserved.

The cross-sections were calculated in parallel by three methods: the Born approximation, Ochkur's first exchange approximation and Ochkur's second exchange approximation.

In Ochkur's first approximation [3], the scattering exchange amplitude $g(q)$ is expressed by the direct scattering amplitude $f(q)$ as follows:

$$
\begin{equation*}
f(q)=\frac{q^{2}}{x_{a}^{2}} \cdot f(q) \tag{1}
\end{equation*}
$$

In Ochkur's second approximation [4]

$$
\begin{equation*}
f(q)=\frac{q^{2}}{x_{a}^{2}+2\left|\varepsilon_{b}\right|} \cdot f(q) \tag{2}
\end{equation*}
$$

in atomic units. Here $q$ is the transmitted pulse, $K_{a}$ is the initial pulse of the incident particle and $E_{b}$ is the excitation energy in atomic units for the final state (reckoned from the ionization potential).

The calculations were performed for several elements of the first and second groups of the periodic table and, for checking purposes, for atomic hydrogen (for which the precise wave functions are known). The results show that the exchange term always has a large effect on the results in the range of low incident particle energies. As might be expected, Ochkur's second approximation gives better results than the first approximation. However, it is even more important to allow for the atomic potential distribution when calculating the wave functions, and to maintain their correct asymptotic form.

These wave functions not only give better agreement with experiment in the high energy range than the Bates-Damgaard wave functions [2], but also reduce the cross-section peak in the low energy range. These results make it possible to select a common method of calculating wave functions and cross-sections which is suitable for any atoms in the periodic table. The corresponding cross-sections will be used for calculating stopping power, ionizing energy, the Fano factor $[5]$ and other atomic and nuclear constants.

The calculation was done on the BESM-6 computer.

Table 1
Cross-section for excitation of the $6 p{ }^{1} P_{1}$ level of barium by
electrons

Key:
Энергия электронод= Electron energy, eV Э в

Сечения возбужления=Excitation cross-sections
"Сщиваемые"волновые= "Joined" wave functions
функции
"Асимптотические" $=$ "Asymptotic" wave functions
волновые функции

Legend under table:

The Table shows cross-sections calculated in Ochkur's second exchange approximation. Election energies are expressed in electron volts and the cross-sections in urits of $10^{-16} \mathrm{~cm}^{2}$.

The second column shows cross-sections calculated with the "joined" wave functions and the third column those calculated with the Bates-Damgaard "asymptotic" wave functions.

Table 1

Сечевие воабухдения $6 \rho P_{1}$ уровня бария элентронаии

| Энергия | Сечения возбугдевия | $\left(10^{-16} \mathrm{~cm}^{2}\right)$ |
| :---: | :---: | :---: |
| злектро- <br> HO日, <br> 3B | "Саивәемые" волновые Функции | "Асимптотические" волновне Фувкциу |
| 2,357 | 33.53 | 50,88 |
| 2,487 | 45,35 | 69,06 |
| 2,617 | 53,16 | 81,I6 |
| 2,727 | 58,05 | 88,82 |
| 2,927 | 64.4I | 98,93 |
| 3,127 | 68,66 | IO5,8 |
| 3,327 | 7I,67 | IIO,5 |
| 3,827 | 75,06 | II6,8 |
| 4,427 | $75,81=7,581 \times 10^{-15}$ | II8,7 $=1,187 \times 10^{-14}$ |
| 5,027 | 74,84 | II7,9 |
| 6,327 | 70,69 | I12,4 |
| 7,427 | 66,75 | 106,7 |
| 8,527 | 62,99 | IOI, 2 |
| 12,33 | 52,42 | 85,01 |
| 15,43 | 46,15 | 75,15 |
| 18,53 | 41,30 | 67.43 |
| 21,63 | 37,45 | 61.26 |
| 24,73 | 34,30 | 56,21 |
| 27,83 | 3I,69 | 52,00 |
| 30,93 | 29,49 | 48,42 |
| 37,33 | 25,85 | 42,52 |
| 42,33 | 23,62 | 38,90 |
| 62,43 | 17,74 | 29,29 |
| 82,53 | 14,34 | 23,73 |
| 152,3 | 8,859 | 14,70 |
| 202,4 | 7,042 | II. 70 |
| 252,5 | $5,879 \times 10^{-16}$ | $9,776 \times 10^{-16}$ |


 $10^{-16} \mathrm{cx}^{2}$.

 Байтся П Дамгяярд.



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CALCULATION OF TRANSITION PROBABILITIES AND ATOMIC EXCITATION FUNCTIONS NECESSARY FOR COMPUTING THE STOPPING POWER OF A SUBSTANCE<br>Yu.S. Gerasimov, I.V. Gordeev

As is well known, the excitation function for excitation of atomic levels by charged particles or neutrons is expressed by the "oscillator force" [l], which is the square of the modulus of the matrix transition element multiplied by the transition energy expressed in rydberg (or by the neutron binding energy):

$$
\begin{equation*}
\left.f_{i \kappa}=\frac{E_{\kappa}-E_{i}}{g_{i} \cdot R h} \cdot\left|\left\langle\psi_{\kappa}\right| V\right| \Psi_{i}\right\rangle\left.\right|^{2} \tag{1}
\end{equation*}
$$

where $E_{k}$ and $E_{i}$ are the initial and final transition energies, $\psi_{i}$ and $\psi_{k}$ are the initial and final atomic wave functions respectively, and $V$ is the interaction potential.

For electromagnetic transitions in the dipole approximation we have

$$
\begin{equation*}
f_{i x}=\frac{1}{3 \cdot g_{i}} \cdot \frac{E_{n}-E_{i}}{R h_{2}}\left|<\psi_{k}\right| \vec{z}\left|\psi_{i}>\right|^{2} \tag{2}
\end{equation*}
$$

where $g_{i}$ is the statistical weight of the initial state and $\vec{r} i s$ the radiusvector.

The main difficulty in calculating excitation functions is to choose the correct wave functions of an atom of the target. The wave functions must have the correct asymptotic behaviour and must also be solutions of the equation for the atomic (or residual nuclear) core. The method of constructing wave functions is easier to check in the case of atomic interactions, since the atomic potential (the interaction potential for an atom and an incident particle) is either well known or can be calculated reliably.

The calculations were performed in the central-symmetrical field approximation. In this approximation an atom is represented in the form of a "core" whose potential has central symmetry and an extermal "optical" electron situated in the field of this potential [2]. The potential distribution was calculated by the Hartree-Fock method with complete allowance for exchange effects, spin-orbit coupling effects, relativistic corrections and other effects. However, the wave function obtained as the solution
of the Hartree-Fock equations $[2]$ was not used as the wave function of the "optical" electron. If the atomic energy levels are sufficiently well known, the wave functions can be obtained by a "semi-empirical" method $[3,4]$ based on familiar atomic level theory. The desired wave functions are solutions of the equation for an electron located in the Hartree-Fock atomic potential, with the difference that the energy parameter, $E$, employed is not the Hartree-Fock level energy but the experimental value of this energy. Here, it is assumed that the correction to the level energy obtained from the Hartreerock approximation is in fact the best approximation of the wave function to its "true" value $[3,4]$.

In the case of an atom this equation is written

$$
\begin{equation*}
\frac{d^{2} Y}{d \tau^{2}}+\left[\frac{2 z(z)}{z}-2 \varepsilon-\frac{e(e+1)}{\tau^{2}}\right] \cdot \psi(x)=0 \tag{3}
\end{equation*}
$$

where $\psi(r)$ is the radial part of the wave function $[2], 2 \pi(\underline{T})$ is the atomic potential, $2 \boldsymbol{\varepsilon}$ is the level energy and $\mathcal{L}$ is the orbital quantum number. Atomic units are used throughout $[2]$.

We must obtain solutions of equation (3) at the given potential, $V(x)=\frac{Z(\tau)}{2}$ and they must be solutions that exhibit the correct asymptotic behaviour. These solutions can be obtained by "asymptotic" methods [5]. In fact, Eq. (3) is similar to the Whittaker equation, but its solution, with an arbitrary function $Z(r)$, cannot be expressed as a single analytical formula apart from the case where $Z(r)=$ const. (when the solution is the Whittaker function) [5]. The asymptotic methods of solving the differential equations in the complex region give solutions in the vicinity of the singular points of the equation $[5]$ (in this case the singular points are the two points $r_{1}=0$ and $r_{2}=\infty$ ), and thereby ensure the correct asymptotic behaviour of the solutions. But, since the asymptotic solutions are quite accurate even in regions far from the singular points [5], the two solutions relating to the two singular points 0 and $\infty$ can be "joined" at an intermediate point namely the "boundary" of the atom, where the logarithmic derivatives $\frac{\varphi^{\prime}}{\Phi}$ of the two solutions are equal. The complete solution of Eq. (3) is then

$$
\begin{equation*}
\psi(z)=N \cdot\left[c \cdot \varphi_{1}(z)+\varphi_{2}(x)\right] \tag{4}
\end{equation*}
$$

where $C$ is the coupling constant, $N$ the normalizing factor and $\varphi_{1}(r)$ and $\varphi_{2}(r)$ the asymptotic wave functions.

This method was employed to calculate the wave functions and the "oscillator forces" for transitions in atoms of the first and second groups in the periodic table, using the atomic potential $V(r)=\frac{Z(r)}{r}$ calculated by the Hartree-Fock method. This potential, which is approximated by a smooth function composed of exponents, was introduced in Eq. (3).

It should be noted that the two wave functions $\varphi_{1}(r)$ and $\varphi_{2}(r)$ "joined" in Eq. (4) satisfy simultaneously both boundary conditions at the points $\varphi_{1}(r)$ and $\varphi_{2}(r)$, only their asymptotic behaviour being "incorrect" in the regions which do not correspond to them. $\varphi_{1}(r)$ behaves "incorrectly" when $r \rightarrow \infty$, and $\varphi_{2}(r)$ likewise when $r \rightarrow 0$. Therefore it was possible to check the correctness of the asymptotic behaviour of the wave function when $r \rightarrow \infty$ and the correct allowance for the atomic potential in the internal region.

For this reason the oscillator forces were calculated with three forms of the wave functions:

$$
\begin{align*}
& \psi(z)=N\left[c \cdot \varphi_{1}(z)+\varphi_{2}(\tau)\right]  \tag{4}\\
& \psi_{1}(\tau)=N_{1} \cdot \varphi_{t}(\tau)  \tag{5}\\
& \psi_{2}(\tau)=N_{2} \cdot \varphi_{2}(\tau) \tag{6}
\end{align*}
$$

where $N, N_{1}, N_{2}$ are the respective normalizing factors, $\varphi_{2}(r)$ has the correct asymptotic behaviour when $r \rightarrow \infty$ and $\varphi_{1}(r)$ behaves correctly in the internal region of the atom. The wave function in Eq. (6) actually coincides with the Bates-Dangaard wave function.

Some of the results are given in the table in which the following notation is used: $f$ is the oscillator force calculated with the aid of the "joined" wave function in Eq. (4), $f_{1}$ is the oscillator force calculated with the wave function in Eq. (5), $f_{2}$ is the oscillator force calculated with the Bates-Damgaard wave function in Eq. (6), and $f_{\text {exp }}$ is an experimental value. The calculation was done on the BESM-4 computer. For hydrogen all three methods give practically identical results and agree with the experimental values. Differences exist only in the third or fourth significant figure.

The following conclusions may be drawn. For atomic calculations, at any rate, correct asymptotic behaviour of the wave function when $r \rightarrow \infty$ is extremely important. Wave functions with incorrect asymptotic behaviour are generally unsuitable for calculating physical quantities. However, allowance for the effect of the potential in the internal region of the atom is also important and will considerably improve the agreement with experiment, particularly in the case of excitation of the low-lying levels.


Fig. 1

Table 1

Key:

| Элемент | $=$ Element |
| :--- | :--- |
| Переход | $=$ Transition |
| эксп. | $=$ exp. |


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METHOD OF ESTIMATING COOLANT FISSION FRAGMENT ACTIVITY AND THE RELEASES OF NUCLEAR POWER PLANTS WITTH RODTYPE FUEL ELEMENTS COOLED BY BOILING WATER

A.G. Guseinov, M.G. Kobozev, Yu.V. Kharizomenov

## 1. Introduction

The activity of the primary circuit (surfaces of steam pipes, separators, turbine, condenser, ejector, etc.) during reactor operation is largely determined by the activity of the isotope formed in the ${ }^{16} O(n, p)^{16} N$ reaction with a half-life of 7.4 sec and gamma emission with $E_{\gamma}=6.1-7.1 \mathrm{MeV}$.

When a reactor with zirconium-clad rod elements is shut down, the primary circuit activity will be governed mainly by fission fragment activity. In our calculations it was assumed that there were no fuel cladding failures likely to lead to emergency situations.

The basic assumptions for our investigation were:

1. Fission fragments get into the coolant from fuel elements with microscopic cracks in the cladding;
2. The proportion of failed elements is $0.1 \%$ of the total in the core;
3. The processes of precipitation, entrainment and distribution of the fragments between steam and water take place in accordance with a model used for calculating corrosion products.

To allow for differences in the behaviour of gaseous and solid fission fragments, we introduce two sets of coefficients, $(\omega, \gamma, k)^{g}$ and $(w, \gamma, k)$, for gaseous and solid fragments respectively:
$w^{i}\left(\mathrm{sec}^{-1}\right)=$ coefficient of fragment precipitation from coolant;
$\gamma^{i}\left(\sec ^{-2}\right)=$ coefficient of entrainment of fragments from the surface of any part of the circuit;
$k=$ coefficient of distribution of fragments between steam and water;
i = serial number of a fragment;
$q=$ serial number of a section of the circuit.

A precise assessment of the fission fragment build-up in the coolant circuit would necessitate considering all possible decay chains of fragments escaping from defective elements. However, the process of considering and solving all the transport and build-up equations, with allowance for the time-dependent yields of different fragments, would be very complex and would require too much computer time. Moreover, after two years' reactor operation the system will be saturated with concentrations of the principal fission fragments and these will account for practically all the circuit activity. Therefore, if we want to know the activity of the primary circuit in a reactor which has been in operation more than two years, we can use the absolute cumulative fission fragment yields supplied in Ref. [l], which remain steady for a long time after the fission of one ${ }^{235}{ }_{U}$ nucleus.

By absolute fragment yield we mean the probability of formation of a specific type of fragment (x) in a given fission event, expressed as a percentage:

$$
\begin{equation*}
y \%=\frac{\mathcal{N}(x)}{P} \cdot 100 \% \tag{1}
\end{equation*}
$$

where $N(x)$ is the number of atoms of the fragment $x, p$ is the number of fissions in the ${ }^{235}$ sample and $y(\%)$ is the absolute percentage yield of the fragment $x$.

In this approximation we obtain results which are slightly too high for fission fragments with half-lives of more than one year and also for their decay products. There are approximately 21 such fragments. Note that for practical purposes the only long-lived fragment of interest here is ${ }^{137} \mathrm{Cs}\left(\mathrm{T}_{\frac{1}{2}}=30 \mathrm{yr}, \mathrm{E}_{\gamma}=0.661 \mathrm{MeV}\right)$.

Thus in this approximation there is no need to consider all the decay chains and accumulations of each fission fragment in correlation with the other fragments. With only slight loss of accuracy it is possible to consider the processes of build-up, precipitation and entrainment of fragments independently of other fragments and their accumulated yields. Such an approach is justified by the acceptable accuracy of the results and by the simplicity and completenesss of the treatment.

In the present study the model is applied on a differential basis to the various sections of the circuit, although the values of the coefficients ( $w, \gamma, k$ ) employed are known accurately in orders of magnitude and, in view
of this, it can scarcely be expected that dividing the circuit into separate sections will give more accurate results than considering the circuit as a whole. However, the differential approach to calculating the activity seems desirable if the procedures we intend to follow subsequently involve variation of parameters, comparison of theoretical and experimental results, and refinement of the coefficients for different reactors.

The rate of fragment emission (gaseous and solid) from the fuel elements was determined with the formula $[2]$

$$
\begin{equation*}
\sigma_{i}=\left(\frac{v_{i}}{\sqrt{\lambda_{i}}}\right) \cdot \frac{3,29 \cdot 10^{13} \cdot \overline{N_{\alpha}} \cdot y_{i} \cdot n_{\alpha} \cdot m \cdot \eta \cdot 10^{-5}}{m \sqrt{\lambda_{i}}}\left[\frac{\text { nuclei }}{\mathrm{sec}}\right] \tag{2}
\end{equation*}
$$

Here: $i$ is the number of the fragment;
$v_{i}$ is the probability of fission fragment escape from a fuel element can;
$3.29 \times 10^{13}$ (fission/sec) is the number of fissions corresponding to a power of 1 kW ;
$\bar{N}_{k}(k W)$ is the mean power of a fuel channel;
$n_{k}$ is the number of channels in the reactor;
$y_{i}(\%)$ are the fission yields;
$\eta$ (\%) is the relative number of defective fuel elements;
$\lambda_{i}$ is the decay constant of the $i^{\text {th }}$ fragment; and
$m$ is the number of elements in a fuel channel.
The parameter $\frac{\gamma_{i}}{\sqrt{\lambda_{i}}}$ for fragments with half-lives $T_{\frac{1}{2}} \geqslant 1000 \mathrm{sec}$ is $10^{-5}-10^{-6}[2]$ In our calculations we took it to be $10^{-5}$ for all fragments.

## 2. Balance equations for volume and surface activities

For a better understanding of the problem, let us consider the flow diagram of a nuclear heat and power plant with a boiling water reactor, as shown in Fig. 1. The following notation is used:
$P_{q}^{b}(\mathrm{~kg})$ is the quantity of water in the $q^{\text {th }}$ section of the circuit;
$P_{q}^{n}(\mathrm{~kg})$ is the quantity of steam in the $q^{\text {th }}$ section;
$S_{q}\left(m^{2}\right)$ is the internal area of the $q^{\text {th }}$ section wet by the coolant;
$G_{1}(\mathrm{~kg} / \mathrm{sec})$ is the coolant flow rate through the reactor;
$G_{2}(\mathrm{~kg} / \mathrm{sec})$ is the steam flow rate at the outlet from the drumseparator;
$G_{T}(\mathrm{~kg} / \mathrm{sec})$ is the steam flow to the turbine;
$G_{3}(\mathrm{~kg} / \mathrm{sec})$ is the steam flow to the boilers at rated load;
$G_{3}^{\prime}(\mathrm{kg} / \mathrm{sec})$ is the additional steam flow to the boilers at peak load;
$G_{n p}(\mathrm{~kg} / \mathrm{sec})$ is the water consumption on blow-down;
$A_{q i}(\mathrm{Ci} / \mathrm{kg})$ is the mean volumetric activity of the coolant in the $\mathrm{q}^{\text {th }}$ section of the circuit due to the $i^{\text {th }}$ fragment;
$C_{q i}\left(C i / m^{2}\right)$ is the mean surface activity of the $q^{\text {th }}$ section of the circuit due to the $i^{\text {th }}$ fragment;
$t$ (sec) is the operating time of the reactor;
$S_{q}\left(m^{2}\right)$ is the internal surface of the $q{ }^{\text {th }}$ section of the circuit wet by the coolant; and
$P_{q}^{b}(\mathrm{~kg})$ and $P_{q}^{n}(\mathrm{~kg})$ are the quantity of steam and water in the $q^{\text {th }}$ section of the circuit.

The values used for calculating the fragment migration parameters are listed in Table l. As can be seen from the table, the gaseous fragments do not settle on the surfaces but are to be found either in the steam or the water. Only the solid fragments are precipitated. The distribution coefficient of solid fragments between steam and water is taken as $6.5 \times 10^{-4}$ although, as follows from Ref. [3], the quantity "k" can vary within the range $6 \times 10^{-5}-0.1$ for different isotopes.

Let us write the balance equations for the volumetric and surface activity of different sections of the circuit at peak load operation of the plant, when there is an additional flow of steam to the boilers, so that $G_{3}^{\prime} \neq 0$ (see Fig. 1 ).

We shall number the various sections of the circuit in the following sequence:
$1=$ core
$2=$ separator tubes leading up from the reactor to the drum-separator
3 = drum-separator

$$
\begin{aligned}
4 & =\text { turbine } \\
5 & =\text { condenser } \\
6 & =\text { boilers } \\
7 & =\text { condensate pump } \\
8 & =\text { deaerator } \\
9 & =\text { feed pump } \\
10 & =\text { circulation pump }
\end{aligned}
$$

Then the balance equations for the volumetric activity due to the $i^{\text {th }}$ fragment may be written as follows:

$$
\begin{align*}
& \frac{\Delta A_{1} i}{d t}=\frac{6 \cdot \lambda_{i}}{3+\cdot e^{c} \cdot p_{1}}+\frac{G_{1}}{p_{1}} A_{1, i}+\frac{\gamma S_{1} C_{1 i}}{p_{1}}-A_{1 i}\left(\lambda_{i}+\omega+\frac{G_{x}}{p_{2}}\right)  \tag{1}\\
& \frac{d A_{21}}{d t}=\frac{G_{1}}{P_{1}} A_{1 i}+\frac{r J_{2} C_{2 i}}{r_{2}}-A_{2 i}\left(\lambda i+w+\frac{G_{1}}{\beta_{2}}\right)  \tag{i}\\
& \frac{d A_{3 i}}{d t}=\frac{G_{1}}{P_{3}} A_{2 i}+\frac{\gamma S_{3} C_{3 i}}{\rho_{2}^{2}}-A_{3 i}\left(\lambda_{i}+\delta_{t} \frac{B_{1} \cdot \alpha_{1} G_{2}}{P_{3} \cdot P_{3}}+\frac{P_{3}(1-\alpha) /\left(S_{1}-C_{2}\right)}{P_{3}^{2} \cdot P_{3}}\right)(3)  \tag{3}\\
& \frac{d A_{i i}}{d t}=\frac{P_{3} \times G_{7}}{P_{3}^{\prime \prime} P_{4}} A_{3 i}+\frac{r_{i} S_{4} G_{i}}{P_{4}}-A_{i 1}\left(A_{i}+\omega+\frac{G_{k}+G_{3}}{P_{4}}\right)  \tag{4}\\
& \frac{d A_{5 i}}{d t}=\frac{G_{k}}{P_{5}} A_{4 i}+\frac{\gamma^{S_{5} C_{5 i}}}{\rho_{5}}-A_{5 i}\left(\lambda_{i}+w+\frac{(1-k) p_{5} G_{k}}{P_{5}^{6} P_{5}}+\frac{P_{5} k G_{k}}{P_{5}^{n} \cdot P_{5}}\right)  \tag{5}\\
& \frac{d A_{6 i}}{d t}=\frac{G_{1}-G_{x}}{P_{i}} A_{9 i}+\frac{r S_{6} C_{6 i}}{P_{6}}+\frac{P_{3} K\left(G_{2}-G_{G}\right)}{P_{3}^{n} \cdot P_{6}} A_{3 i}-A_{6 i}\left(\lambda_{i}+5+\frac{P_{6}(+\alpha)\left(G_{3}-G_{x}\right)}{P_{6}^{s} \cdot P_{6}}(G)\right. \\
& \left.+\frac{p_{6} \cdot \alpha \cdot 6_{3}}{p_{6} \cdot p_{6}}\right) \\
& \frac{d A_{2 i}}{d t}=\frac{(1-K) p_{5} \cdot G_{x}}{P_{5}^{G} \cdot P_{7}} A_{5 i}+\frac{\gamma S_{+} C_{7 i}}{\rho_{p}}-A_{7 i}\left(\lambda_{i}+\omega+\frac{G_{k}}{\rho_{p}}\right)  \tag{7}\\
& \frac{d A_{g i}}{d t}=\frac{(1-\kappa) P_{s}\left(G_{2}-G_{\alpha}\right)}{P_{6}^{d} \cdot P_{g}} A_{6 i}+\frac{\gamma S_{d} C_{8 i}}{P_{d}}+\frac{G_{K}}{\rho_{\sigma}} A_{A i}-A_{d i}\left(\lambda_{i}+\omega_{+}\right.  \tag{8}\\
& \left.+\frac{(1-k)\left(G_{2}+G_{20}\right)}{P_{2} b}+\frac{\left(G_{k}+G_{3}\right) \cdot k}{P_{8}^{n}}\right) \\
& \frac{d A_{g i}}{d t}=\frac{p_{g}(1-k)\left(G_{2}+G_{n g}\right)}{p_{g} b^{6} \cdot P_{g}} A_{s i}+\frac{\gamma S_{g} C_{g i}}{p_{g}}-A_{g i}\left(\lambda_{i}+w+\frac{G_{z}+G_{g g}}{p_{g}}\right) \tag{9}
\end{align*}
$$

$$
\begin{align*}
& -A_{10 i}\left(\lambda_{i}+\omega+\frac{G_{1}}{P_{10}}\right) \tag{IO}
\end{align*}
$$

The balance equations for the surface activity of the solid fragments for the various sections are similar:

$$
\begin{equation*}
\frac{d C_{q i}}{d t}=A_{q}: \frac{P_{q} \cdot w}{S_{q}}-C_{q}\left(\lambda_{i}+\gamma_{q}^{T}\right) \tag{11}
\end{equation*}
$$

where $q=1,2,3, \ldots 10, \gamma_{q}^{\top}=\gamma_{T}^{\top}$ for $q=4$, and $\gamma_{\gamma}^{\top}{ }^{\top} \gamma^{\top}$ for $q \neq 4$. Here $i$ is the serial number of the fragment and $q$ is the number of the circuit section.

As can be seen from the system of equations set out in expressions ( 1,11 ), the quantity $A_{q i}(t)$ represents the activity of $l \mathrm{~kg}$ of coolant in any $q^{\text {th }}$ section. This quantity characterizes the activity of steam or water as the case may be. For example, in the fourth section (the turbine) $A_{4 i}(t)$ characterizes the mean activity of 1 kg of steam, whilst in the eighth, ninth and tenth sections $A_{q i}(t)$ characterizes the activity of water.

In our system of equations it is also assumed that the decontamination factor of the biow-down water in the filter is $100 \%$ and that fully decontaminated water flows from the filter to the deaerator. In addition, the coefficients of precipitation of fragments from water and steam are considered to be identical.

The balance equations take no account of the part of the circuit linking the turbine with the regenerative feed heater and the condenser (see Fig. 1), because this has little effect on the results.

The system of equations is solved for the uniform initial conditions

$$
\left.\begin{array}{l}
\left.A_{i}(t)\right|_{t=0}=0 \\
\left.C_{q i}(t)\right|_{t=0}=0
\end{array}\right\}
$$

## 3. Asymptotic solution of the system of equations 5-6

Let us write this system as follows:

$$
\begin{gather*}
\frac{d A_{q}}{d t}=\sum_{q} a_{q q} A_{q}^{\prime}+f_{q}+\frac{r_{q} s_{q}}{p_{q}} C_{q}  \tag{8}\\
A q l_{t=0}=0 \\
\frac{d C_{q}}{d t}=\frac{w p_{q}}{s_{q}} A_{q}-\left(A+r_{q}\right) C_{q} \\
\left.C_{q}\right|_{t=0}=0 \\
q, q^{\prime}=12, \ldots, 10
\end{gather*}
$$

In the system of equations in expression (8) the coefficients $a_{q q}$, and the source term $f_{q}$ have the following values:

$$
\begin{align*}
& f_{1}=\frac{6 \lambda}{37 x^{\prime \prime} \cdot p} ; \quad f q_{1}=0 \text {; } \\
& a_{11}=\left(\lambda+w+\frac{G_{1}}{P_{1}}\right) ; a_{22}=\left(\lambda+w+\frac{G_{1}}{P_{2}}\right) \text {, } \\
& a_{3, ~}=\left(\lambda+w+\frac{K G_{2}}{P_{3}^{2}}+\frac{(1-k)\left(G_{1}-G_{2}\right)}{p_{3}^{2}}\right) ; a_{4, y}=\left(\lambda+w+\frac{G_{x+}+G_{3}}{p_{41}}\right) ; \\
& a_{55}=\left(\lambda+w+\frac{(1-k) \sigma_{k}}{p_{5}^{2}}+\frac{k \sigma_{k}}{p_{5}^{n}}\right) ; \quad a_{66}=\left(\lambda+w+\frac{(1-k)\left(\sigma_{y}-\sigma_{x}\right)}{\rho_{6}}+\frac{\kappa \sigma_{3}}{p_{6}^{n}}\right) ; \\
& a_{7,7}=\left(\lambda+w+\frac{G_{x}}{P^{7}}\right) ; a_{88}=\left(\lambda+w+\frac{(1-k)\left(G_{2}+G_{2 p}\right)}{p_{s} b^{2}}+\frac{k^{\prime}\left(G_{x}+G_{3}\right)}{p_{5}^{n}}\right) \text {; } \\
& a_{9.9}=\left(\lambda+w+\frac{G_{2}+G_{22}}{p_{9}}\right) ; a_{1010}=\left(\lambda+\sigma+\frac{G_{1}}{p_{10}}\right) ;  \tag{9}\\
& a_{1,10}=\frac{G_{1}}{P_{1}} ; a_{211}=\frac{G_{1}}{P_{2}} ; a_{3.2}=\frac{G_{1}}{P_{3}} ; a_{4.3}=\frac{P_{3} \cdot P_{0} \cdot G_{5}}{P_{3}^{0} \cdot P_{4}} ; \\
& a_{5.4}=\frac{G_{k}}{P_{5}} ; \quad a_{6.3}=\frac{p_{3} \times\left(G_{2}-G_{r}\right)}{P_{3}^{n} \cdot p_{6}} ; a_{6.4}=\frac{G_{0}-G_{2}}{p_{6}} ; \\
& a_{7.5}=\frac{(1-k) \rho_{5} \cdot G_{x}}{\rho_{5}{ }^{6} \cdot \rho_{7}} ; a_{2.6}=\frac{\rho_{0}(1-k)\left(\sigma_{2}-G_{x}\right)}{p_{6}{ }^{d} \cdot p_{5}} ; a_{81}=\frac{G_{2}}{\rho_{0}} ; \\
& a_{9.8}=\frac{P_{1}(1-R)\left(\sigma_{2}+\sigma_{p p}\right)}{P_{2}^{6} \cdot P_{9}} ; a_{10.3}=\frac{P_{3}(1-\alpha)\left(\sigma_{1}-G_{2}-C_{y}\right)}{P_{2}^{6} \cdot p_{10}} ; a_{10.3}=\frac{S_{2}+\sigma_{2}}{P_{10}}
\end{align*}
$$

while for the remainder $a_{q q^{\prime}}=0$.
Since we were estimating fragment activity for a long period of reactor operation, we looked for asymptotic values of the quantities $A_{q}$ and $C_{q}$ for $t \quad 0$, which should be supplied by the system of equations in expression (8) on the basis of the physical picture of the activity build-up.

In fact, it can be shown that the solution of the system for the case where the effect of the surface activity term is small (as in our case: $\gamma_{\mathrm{q}}=5 \times 10^{-8}-2 \times 10^{-6}\left(\mathrm{sec}^{-1}\right), w=3.5 \times 10^{-5}\left(\mathrm{sec}^{-1}\right)$ takes the form

$$
\begin{align*}
& \text { form } \\
& A_{q}\left(\frac{1}{l}\right)=\sum_{k=1}^{q} \frac{\alpha_{q}^{(\alpha)}}{a_{x \alpha}}\left(1-e^{-a_{k \alpha} t}\right)+\int_{0}^{\tau} \sum_{k=1}^{\varepsilon} \beta_{\varepsilon}^{(\alpha)} e^{-(t-z) a_{\alpha \alpha}} \sum_{<=1}^{\infty} \lambda^{e} F_{e}(\tau) d \tau  \tag{10}\\
& q_{q}=1,2, \ldots, 10 ; \quad \lambda=a_{k .10}<1
\end{align*}
$$

and the coefficients $\alpha_{\mathrm{q}}^{(k)}$ and $\mathrm{B}_{\mathrm{q}}^{(k)}$ are found from the following expressions:

$$
\left.\begin{array}{l}
\alpha_{1}^{(1)}=f_{1}+\frac{\delta q s_{1} c_{1}^{\infty}}{p_{2}}  \tag{11}\\
\beta_{1}^{(1)}=1
\end{array}\right\}
$$

All succeeding $\alpha_{q}^{(k)}$ and $B_{q}^{(k)}$ are found with recurrent relations of the form

$$
\begin{equation*}
\alpha_{q}^{(\alpha)}=\frac{a_{q, q-1} \cdot \alpha_{q-1}^{(\alpha)}}{a_{q, q}-a_{k \cdot x}} ; \alpha_{q}^{q}=\frac{\gamma_{q} s_{q}}{p_{q}} C_{q}^{\infty}-\sum_{k=1}^{q-1} \alpha_{q}^{(\alpha)} \tag{12}
\end{equation*}
$$

$$
\begin{equation*}
\beta_{q}^{k<q}(k)=\frac{a_{q \cdot q-1} \cdot \beta_{q-1}^{(k)}}{a_{q \cdot q \cdot}-a_{k \cdot K}} ; \beta_{q}^{q}=-\sum_{k=1}^{q-1} \beta_{q}^{(k)} \tag{13}
\end{equation*}
$$

The functions $F_{e}(t)$ then take the form

$$
\begin{equation*}
F_{e}(t)=a_{1.10}\left\{\gamma_{1}^{(c)}+\sum_{k=1}^{10} \gamma_{2}^{(c) k} e^{-e_{e, e} t}+\ldots+\frac{t^{2}}{c!} \sum_{k=1}^{10} \gamma_{e r i}^{(0) x} e^{-\varepsilon_{e c} t}\right\} \tag{14}
\end{equation*}
$$

where the coefficients $\gamma_{l}^{(e)}, \ldots . \gamma_{e+2}^{(e) k}$ depend on $\alpha_{e}^{(k)}$ and $B_{e}^{(k)}$.
Substituting Eq. (14) in Eq. (10), we obtain for the volumetric activities

$$
\begin{equation*}
A_{q}(t)=\sum_{k=1}^{q} \frac{\alpha_{q}^{(x)}}{a_{k-\alpha}}\left(1-e^{-a_{x x} t}\right)+a_{1,10} \sum_{e=0}^{\infty} \lambda^{c} \gamma_{1}^{(2)} \sum_{k=1}^{q} \frac{\beta_{q}^{(k)}}{\alpha_{k \alpha}}+ \tag{15}
\end{equation*}
$$

plus the terms containing the products $\frac{t^{m}}{m!} e^{-a} \mathrm{lck}^{t}(m=1,2, \ldots .$.$) .$ When $t \rightarrow \infty$, Eq. (15) gives

$$
\begin{equation*}
A_{q}^{\infty}=\sum_{k=1}^{q} \frac{\alpha_{k}^{(c)}}{a_{k \cdot N}}+a .1 .0 \sum_{i=0}^{\infty}\left(a_{r .10}\right)^{c} \gamma_{1}^{(\alpha)} \sum_{k=1}^{q} \frac{\beta_{q}^{(c)}}{a_{N i N}}=\text { costs } \tag{16}
\end{equation*}
$$

Rewriting the system in expression (8) in matrix form we have

$$
\begin{align*}
& \left\{\begin{array}{l}
\frac{d \bar{B}}{d t}=\bar{C} \bar{B}+\bar{F} \\
\bar{B} L_{t=0} \\
=0
\end{array}\right.  \tag{17.}\\
& \text { where } \bar{B}=\left\{\begin{array}{c}
A_{2} \\
\vdots \\
c_{10} \\
c_{2} \\
\vdots \\
c_{15}
\end{array}\right\}, \bar{F}=\left\{\begin{array}{c}
A_{1} \\
\vdots \\
\vdots \\
0
\end{array}\right\} \tag{18}
\end{align*}
$$

The elements $C_{e, m}$ of the matrix $\hat{C}$ are the coefficients

$$
\left.\begin{array}{l}
a_{e, m}(\text { see }(g)), \text { when } c, m<10  \tag{19}\\
f_{c, e+10}=\frac{\gamma_{e} s_{c}}{p_{c}} ; c \leq 10 \\
d_{e, c-10}=\frac{w s p_{c-10}}{s_{2-10}} ; e>10 \\
g_{e, c}=\left(\lambda+\gamma_{e-10}\right) ; c>10
\end{array}\right\}
$$

From Eq. (16) it follows that, in order to find the asymptotic solution of Eq. (17), we can put $\frac{d \bar{B}}{d t}=0$. Then for infinite reactor operating time we have $\bar{B}_{\infty}=-\hat{C} \bar{P}$, where $\hat{\mathrm{C}}^{-1}$ is a matrix which is the reciprocal of $\hat{C}$.

Solution of the system in expression (8) is considerably simplified for the case of gaseous fission fragments ( $k=1 ; w=0 ; \gamma q=1$ ). In this case all $\mathrm{C}_{\mathrm{q}} \equiv 0$ and the system in expression (8) ( $\mathrm{q}=1,2, \ldots, 10$ ) takes the following form:

$$
\begin{align*}
& \frac{d A_{1}}{d t}=f_{1}-\left(\lambda+\frac{\sigma_{1}}{P_{L}}\right) A_{1} \\
& \frac{d A_{2}}{d t}=\frac{G}{P_{2}} A_{L}-\left(\lambda+\frac{G_{1}}{P_{2}}\right) A_{2} \\
& \frac{d A_{3}}{d t}=\frac{G_{1}}{P_{3}} A_{2}-\left(\lambda+\frac{G_{2}}{P_{s}^{2}}\right) A_{3} . \\
& \frac{d A_{4}}{d t}=\frac{G_{5} P_{3}}{P_{4} \cdot \rho_{3}^{3}} A_{3}-\left(\lambda t \frac{G+G_{3}}{\rho_{4}}\right) A_{4}  \tag{20}\\
& \frac{d A_{5}}{d t}=\frac{G_{x}}{p_{s}} A_{4}-\left(\lambda+\frac{G_{k}}{\rho_{s}^{s}}\right) A_{5} \\
& \frac{d A_{6}}{a \cdot t}=\frac{P_{3}\left(G_{2}-G_{6}\right)}{P_{3}^{2} \cdot P_{6}} A_{3}+\frac{G_{G}-G_{6}}{P_{6}} A_{6}-\left(\lambda+\frac{G_{3}}{P_{8}}\right) A_{6} \\
& A_{q} l_{t=0}=0 \quad q=1,2, \ldots 6
\end{align*}
$$

Analysing the solution of the system in expression (20), we see that the solution for $t \geqslant 0.5$ years is practically the same as for $t=\infty$ and can be sequentially determined from the system

$$
\begin{align*}
& A_{1}=\frac{A_{1}}{\lambda+\frac{G_{1}}{P_{1}}} ; A_{2}=\frac{G_{1}}{P_{2}} A_{1} /\left(\lambda+\frac{G_{1}}{\rho_{2}}\right), \\
& A_{3}=\frac{G_{1}}{P_{3}} A_{2} /\left(\lambda+\frac{G_{2}}{P_{3}^{n}}\right) ; A_{4}=\frac{G_{4} P_{3}}{P_{4} \cdot P_{3}^{n}} A_{3} /\left(\lambda+\frac{G_{x}+G_{3}}{\rho_{4}}\right),  \tag{21}\\
& A_{5}=\frac{G_{k}}{P_{5}} A_{4} /\left(\lambda+\frac{G_{k}}{P_{5}^{n}}\right), \\
& A_{6}=\left[\frac{P_{3}\left(G_{2}-G_{5}\right)}{P_{3}^{n} \cdot P_{6}} A_{3}+\frac{G_{1}-G_{x}}{P_{6}} A_{4}\right] /\left(\lambda+\frac{G_{3}^{0}}{P_{6}^{\prime}}\right)
\end{align*}
$$

The asymptotic solution of the system of equations in expression (1, 11) gives the following values for the volumetric activity of the different sections:*/

$$
\begin{equation*}
\sum_{i} A_{q i} \approx\left(10^{-5} \div 10^{-3}\right) \mathrm{Ci} / \mathrm{kg} \tag{2la}
\end{equation*}
$$

* These values were obtained on the assumption of a reactor power of
125 MW .


## 4. Estimating discharges of radioactive fission fragments

As can be seen from the flow diagram of the nuclear heat and power plant, the radioactive fragments accumulating in the circuit are removed from the 5 th, 6 th and 8 th sections as the coolant is circulated. In our calculations it was assumed that the radioactive fragments present in the steam part of the above sections are entirely removed from the circuit (see the balance equations for the $5 \mathrm{th}, 6$ th and 8 th sections).

The gaseous and solid fragments are distributed between the steam and water in accordance with the distribution coefficient

$$
K=\left\{\begin{array}{l}
1 \text { for gaseous fragments } \\
6.5 \times 10^{-4} \text { for solid fragments }
\end{array}\right.
$$

From the balance equations for the volumetric activity it is easy to find relations for estimating the amount of radioactive discharges from sections 5, 6 and 8:

$$
\begin{align*}
& B_{5 i}(\mathrm{Ci} / \mathrm{d})=A_{5 i} \frac{P_{5} \cdot K \cdot G_{K}}{P_{5}} 0.865 \cdot 10^{5} ; B_{5}=\sum_{i} B_{5 i}=\frac{P_{5} K G_{5}}{P_{5}{ }^{0} 0.865 \cdot 10 \sum_{i} \sum_{i j}} \tag{22}
\end{align*}
$$

Here $B_{5 i}, B_{6 j_{h}}$ and $B_{8 i}$ are the discharges of radioactive fission products due to the $i^{\text {th }}$ fragment from the 5 th, 6 th and 8 th sections respectively. $B_{5}, B_{6}$ and $B_{8}$ are the total discharges ( $\mathrm{Ci} / \mathrm{d}$ ) from these sections. The total discharge of radioactive fragments into the vent pipe will be

$$
\begin{equation*}
B=\sum_{i}\left(B_{r i}+B_{6 i}+B_{i i}\right)=B_{5}+B_{6}+B_{2} \quad \mathrm{Ci} / \mathrm{d} \tag{22a}
\end{equation*}
$$

According to our estimates the total discharge of radioactive fragments into the vent pipe (for this particular heat and power plant) is ~ $27 \mathrm{Ci} / \mathrm{d} . \mathrm{MW}$.

Our discharge estimates, referred to unit capacity, are shown in Table 2. The results agree satisfactorily with datia for nuclear power stations with boiling water reactors $[4-6]$ (see Tables 3 and 4)/

Our estimates also show that a hundredfold increase in the distribution coefficient of solid fragments (K) leads to a hundredfold increase in the radioactive discharges due to solid fragments. In this case the volumetric activities for the different sections $\sum_{i} B_{q i}(\mathrm{Ci} / \mathrm{kg})$, vary on average by a factor of 3-5.

A large variation in the parameter $K$ is most significant for sections where discharges of radioactive fragments occur or where $\frac{Q_{\text {steam }} Q_{\text {water }}}{Q_{1}}$ which means that the volumetric activity increases considerably in those sections with increasing $K$.

## Conclusions

1. The assumptions made in order to obtain upper estimates of the dose situation at various parts of the circuit are well justified and values obtained with the model employed are in satisfactory agreement with experimental data $[4]$, both for the activity of the different sections of the circuit and for the radioactive discharges.

Our analysis also included a detailed analysis of the fission fragment spectrum, but the results are omitted here for the sake of brevity. 2. The fission fragment activity of the primary circuit - for a nuclear power plant of the type considered - is most significant in relation to releases of radioactivity.
3. To obtain a more universal set of parameters ( $w, \gamma, k$ ) it will be necessary to analyse the sensitivity of the results to variations of all the parameters, and to compare the theoretical data with experimental values for similar nuclear power plants.

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## Table 1

Parameters for describing fragment migration
Key:

$$
\begin{aligned}
\text { газ } & =\text { gas } \\
\text { тв } & =\text { sol. } \\
\text { сек } & =\mathrm{sec}
\end{aligned}
$$

## Table 2

$$
\text { Radioactive discharges from sections } 5,6 \text { and } 8,
$$ referred to unit capacity

Key:

```
No yчactков \(=\) No. of sections
кюри/сутки мвт \(=\mathrm{Ci} / \mathrm{d} . \mathrm{MW}\)
\begin{tabular}{|c|c|}
\hline \[
\begin{gathered}
\text { TВ. осколков } \\
\mathrm{B}_{\left[\begin{array}{c}
\text { сут. } \\
\frac{\text { кют }}{}
\end{array}\right]}
\end{gathered}
\] & \[
=\mathrm{B}_{[\mathrm{Ci} / \mathrm{d} \cdot \mathrm{MW}]}^{\mathrm{solid} \text { fragments }}
\] \\
\hline \[
\begin{gathered}
\text { газ осколков } \\
{\left[-\frac{\text { кюри }}{\text { сут.мвт }}\right]}
\end{gathered}
\] & \[
=B_{[\text {gaseous fragments }}^{[\mathrm{Ci} / \mathrm{d} \cdot \mathrm{MW}]}
\] \\
\hline \[
\begin{aligned}
& \mathrm{B}^{\mathrm{T}}+\mathrm{\Gamma} \text { гз осколк, } \\
& {\left[\frac{\text { кюри }}{\text { сут } \cdot \mathrm{MBT}}\right]}
\end{aligned}
\] & \[
\begin{aligned}
& \left.\mathrm{B}_{[\mathrm{Ci}}^{\text {solid }}+\mathrm{MW}\right]
\end{aligned}
\] \\
\hline
\end{tabular}
```

Radioactive discharges from foreign nuclear power stations*/
Key:

| АӘС/мощность | = Name and capacity | Выб росы Һкюри/сутки | $\begin{aligned} & =\text { Discharges (Ci/d.MW) } \\ & \text { MBT } 7 \end{aligned}$ |
| :---: | :---: | :---: | :---: |
| P/a газы | = radioactive gases | аэрозоли | = aerosols |
| Дрезден 1 | = Dresden-1 (700 MW) | Биг-Рок-Пойнт | T $=$ Big Rock Point (157 MW) |
| Гумбольт-Бей | = Humboldt Bay (165 | ) Елк-Ривер | $=$ Elk River ( 58 MW ) |

* The release data for these stations were cibtained after the gases had been cooled for $18-30 \mathrm{~min}$ and passed through aerosol filters.

Table 4
Activity of radioactive inert gases discharged (Ci/d.MW)

## KeV:

| АЭС | $=$ Nuclear power station |
| :--- | :--- |
| KRB | $=$ KRB |
| Гарильяно | $=$ Garigliano |
| Bonus | $=$ Bonus |

Рассматриваемая Nuclear heat and power plant (Fig. 1) АТЭЦ

Table 1
Пяраиетры для описания ииграции осколков

| $\omega^{\text {ms }}$ | $r^{\text {aeg }}$ | $*^{*}$ | $\omega^{\text {ref }}$ | $8^{76}$ | $k^{30}$ | $\gamma_{T}$ | 疗 | 2 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\operatorname{cex}^{-1}$ | cer $^{-1}$ | - | $\mathrm{cer}^{-1}$ | $\mathrm{ces}^{-1}$ | - | $\mathrm{cer}^{-\mathrm{I}}$ | cen ${ }^{-1 / 2}$ |  |
| 0 | I | I | 3,5.10-5 | $5.10^{-8}$ | $6,5.10^{-4}$ | $2.10^{-6}$ | $10^{-5}$ | 0.I |

Table?
Радиоактивные вибросц с участков $5,6,8$, приведөннве к единице мощности

| 火е участков | 5 | 6 | 8 |  |
| :---: | :---: | :---: | :---: | :---: |
|  | 0,08 | 0,004 | 0,15 | 0,23 |
| $B$ Brionemed | 20,70 | 5,75 | - | 26,45 |
|  | 20,76 | 5,754 | 0.15 | 26,68 |

 согласуртся с данмами для АЭС с кипимим јеанторами $4-6$ (сн. таблицд 3 44).

Table 3

Радиоактивные вибросы зарубедных дכС（х）

| АЗС（мощность） | Внбросн（иори／сутки цвт） |  |
| :---: | :---: | :---: |
|  | P／a 5834 | аэрозол11 |
| Дрезден I（ 700 мвт） | 3.1 | $0.430 .10^{-6}$ |
| Биг－Рок－Пойт（I57 uвt） | 19 | 0，64．10 ${ }^{-3}$ |
| Гэぃбольч－Бэด（165 мas） | 7.4 | 0，36．10 ${ }^{-4}$ |
| Enf－Pubep（ 58 мвт） | ．0，16 | 0，52．10 ${ }^{-7}$ |

[^2]
## Table 4

Активность внброса радиоактивных благородных газов（к／сут．чвт）

| A3C | Изотопы |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 85 | 87 | 88 | 133 | 135 | I35 | 138 |
| $K R B$ | $\begin{aligned} & 0, \mathrm{I} 25- \\ & 0,193 \end{aligned}$ | $0,174-$ | $0,36-$ | $\begin{aligned} & 0,275- \\ & 0,34 \end{aligned}$ | $\begin{aligned} & 0,39- \\ & 0,72 \end{aligned}$ | $0,325-$ | $\begin{aligned} & 0,94- \\ & 1,2 \end{aligned}$ |
| Гарильлпо | 0，0228 | $0,0^{2} 52$ | － | $0,0^{2} 26$ | 0,0298 | － | 0，05 |
| Bonus | $0,0^{3} 36$ | $0,0^{3} 2$ | $0,0^{3} 22$ | $0,0^{4} 6$ | $0,0^{3} 32$ | － | 0，00II |
| Рассцатриваецая АTコU | 0，077 | 0，252 | 0，237 | 0，062 | 0，220 | 0，375 | I，I |



## Fig. 1

Flow diagram of nuclear heat and power plant
Key:


## CHAPTER IV

## PROGRAMMING, INFORMATION AND STANDARDIZATION QUESTIONS

## ALGOL PROGRAMMES FOR DETERMINING NUCLEAR PARAMETERS FROM <br> AN ANALYSIS OF THE EXCITATION FUNCTION <br> (ERICSON'S STATISTICAL THEORY)

## A.I. Baryshnikov

Practical and theoretical investigations carried out since 1960 have shown convincingly that excitation functions measured with high energy resolution of the primary particles for compound -nucleus reactions at high excitation energy (12-30 MeV) have a fluctuation structure (Fig. 1). According to Ericson's statistical theory these fluctuations do not have a resonance character and cannot be explained by the individual nuclear levels. The fluctuation is the result of interference between the many partial levels existing within a certain energy interval, called the coherent energy $\Gamma$, which is the reciprocal of the lifetime of the compound state.

Even in his very early publications [1, 2] Ericson pointed to the amount of information derivable from the excitation functions and proposed methods of analysing them to determine a number of nuclear parameters.

Subsequent investigations have explored and expancled these possibilities, and the various methods of determining nuclear parameters from the excitation functions and angular distributions have been analysed in detail by the author in a review paper.

The present paper supplies ALGOL programmes which can be used for determining nuclear parameters from correlation analysis of the excitation function and for comparing the theoretical probability distribution with the experimental histogram of the distribution, $Z\left(\frac{\sigma}{\langle\sigma\rangle}\right)$.
I. ALGOL programme for correlation analysis (APKA)

With this programme it is possible to determine:
(1) The auto and cross-correlation functions for all exit channels of
$R^{i j}(C)=\frac{\left.\left\langle\sigma^{\text {the }}(E+\varepsilon)-\left\langle\sigma^{i}\right\rangle\right]\left[\sigma^{j}(E)-\left\langle\sigma^{j}\right\rangle\right]\right\rangle}{2\left\langle\sigma^{i}\right\rangle\left\langle\sigma^{j}\right\rangle}+$
$+\frac{\left.\left\langle\sigma^{i}(E)-\left\langle\sigma^{i}\right\rangle\right]\left[\sigma^{j}(E+\varepsilon)-\left\langle\sigma^{j}\right\rangle\right]\right\rangle}{\left\langle\left\langle\sigma^{i}\right\rangle\left\langle\sigma^{j}\right\rangle\right.}$;

If i $\neq j$, the cross-correlation function is determined.
If $i=j$, the auto-correlation function is determined.
(2) When $i=j$ and $\varepsilon=0$ the coefficient of the autowcorelation function is determined:

$$
\begin{equation*}
C(0)=R^{i j}(\varepsilon=0) \tag{2}
\end{equation*}
$$

(3) The coefficient of the cross-correlation function

$$
\begin{equation*}
C^{i}(0)=\frac{R^{i j}(\varepsilon=0)}{\sqrt{c^{i}(0) C^{j}(0)}}, \tag{3}
\end{equation*}
$$

the numerical value of which determines the degree of correlation between the channels " $i$ " and " $j$ ".
(4) The normalized mean square deviation, which is numerically equal to the coefficient of the automeorrelation function ( $\varepsilon=0$ ):

$$
\begin{equation*}
C(0)=\frac{\left.s^{\prime} \sigma^{2}\right\rangle-\langle\sigma\rangle^{2}}{\langle \}^{2}} \tag{4}
\end{equation*}
$$

(5) The coherent energy (by several methods):
(a) From the condition that the auto-correlation function decreases by a factor of two in relation to $R(\varepsilon=0)=C(0)$, since with $\varepsilon=\Gamma$ we have

$$
\begin{equation*}
P(G=P)=\frac{1}{2} C(O) \tag{5}
\end{equation*}
$$

(b) By comparing the Lorentz function $R(\Gamma, \varepsilon)$ with the autocorrelation function $R(\varepsilon)$ when $\varepsilon \geqslant \Gamma$, since

$$
\begin{equation*}
R(F, E)=C(O) \frac{F^{2}}{F^{2}+\varepsilon^{2}}=R(\varepsilon \leqslant \Gamma) \tag{6}
\end{equation*}
$$

(c) By analysing the fluctuations of the auto-correlation function when $\varepsilon>2 \Gamma$ :

$$
\begin{equation*}
\left\langle F^{2}(\varepsilon)\left[\frac{J-\varepsilon}{I}\right]\right\rangle=f^{2}(\varepsilon=0) \frac{\pi}{2}\left(\frac{1}{I}\right), \tag{7}
\end{equation*}
$$

where $I$ is the range of energy variation of the excitation function.

If the excitation function has a constant mean value over the whole range $I$, the auto-correlation function oscillates about the $\varepsilon$-axis. These oscillations are due to the use of a finite range of averaging of $I$ for the excitation function.

If the mean cross-section of the excitation function is not constant over the range $I$, the auto-correlation function will not oscillate about the $\varepsilon$-axis and the results of the auto-correlation analysis ( $C(0)$ and $\Gamma$ ) will not be true.

Methods of eliminating the effect of the mean cross-section not being constant are described in the review paper. ALGOL programes for these calculations have been compiled but will not be published until they have been checked numerically.

## II. ALGOL programme for calculating the probability distribution (RVEG)

The RVEG programme can be used to determine:
(1) The experimental histogram of the probability distribution $Z\left(\frac{\sigma}{<0\rangle}\right)$ for the excitation function of interest, for different variation steps of the argument $\frac{\sigma}{<\sigma\rangle}$ (from 0.4 to 0.2 ), where $\sigma$ and $\left.<0\right\rangle$ are the cross-section and the mean cross-section respectively.
(2) The effective number of reaction channels $N_{\text {eff }}$ (number of partial levels of the compound nucleus which participate in a given reaction channel) and the proportion of the direct interaction process, $-\psi_{\sigma} \quad y_{\Delta}=\frac{\sigma \infty}{-\sigma\rangle}$; this is done by comparing the theoretical probability distribution $\mathrm{P}\left(\frac{\sigma}{\langle\sigma\rangle}\right)$ with the experimental histogram $Z\left(\frac{\sigma}{\langle\sigma\rangle}\right)$ with simultaneous consideration of the dual experimental relationship between $N$ and $V_{D}$ :

$$
\begin{align*}
& \text { a) }  \tag{9}\\
& C(0)=\frac{I}{N}\left(I-H_{D}^{2}\right) \tag{8}
\end{align*}
$$

$\mathrm{J}_{\mathrm{N}-1}$ is the $\mathrm{N}-1$ order Bessel function from the imaginary argument. After putting the Bessesl function in the form of a series ( N being an integer) we have

$$
\begin{equation*}
J_{H-1}(2 i \sqrt{x})=i^{i-1} \sqrt{x^{\infty}} \sum_{j=0} \frac{y j}{(v-1+j)!j!}, \tag{9x}
\end{equation*}
$$

and the probability distribution becomes

$$
\begin{equation*}
P(y)=\left(\frac{N}{1-y_{2}}\right)^{N} y^{N-1} \cdot e^{-\frac{N\left(y+U_{D}\right)}{1-y s}} \sum_{j=0}^{\infty} \frac{\left[N^{2} \frac{y(1)}{\left.N-L_{D}\right)}\right]}{(N-1+j)!j!} \tag{9a}
\end{equation*}
$$

The normalized mean square deviation is also determined here from the equation

$$
\begin{equation*}
C(O)=\frac{\left\langle\sigma^{2}\right\rangle-\langle G\rangle^{2}}{\langle G\rangle^{2}} \tag{4}
\end{equation*}
$$

(3) The maximum possible number of effective channels in the reaction is determined from equation (8) for $y_{D}=0$ :

$$
\begin{equation*}
A^{\max }(\theta)=\frac{1}{C(\theta, \theta)} \tag{10}
\end{equation*}
$$

The results of processing the excitation functions (Fig. l) of the reaction ${ }^{52} \mathrm{Cr}\left(\mathrm{pp}_{1}\right){ }^{52} \mathrm{Cr}$ with these programmes are illustrated in Figs (2) and (3).

In the case of $N$ not being an integer $[3,4]$ we have

$$
\begin{gather*}
P(y)=\left(\frac{N}{1-y D}\right)^{N} y^{x 1} e^{\left.-\frac{m\left(y+y_{0}\right)}{1-y m} \sum_{j=0}^{\infty} \frac{\left[N^{2} \frac{y s}{d i}\right] d}{P(N-1+j) j}\right]}  \tag{10a}\\
P(m)=(m-1) \Gamma(m-1)
\end{gather*}
$$

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Fig. 1. Excitation functions of the reaction ${ }^{52} \mathrm{Cr}(\mathrm{p}, \mathrm{p}){ }^{52} \mathrm{Cr}$. The exit channel of the reaction is shown on the graphs.


Fig. 2. $x=$ auto-correlation function of the reaction $0=$ Lorentz function


Fig. 3. Experimental histogram of $Z\left(\frac{\sigma}{\sigma>}\right)$ and the theoretical probability distribution $P\left(\frac{\sigma}{\langle\sigma\rangle}\right)$ for the reaction ${ }^{52} \mathrm{Cr}\left(\mathrm{pp}_{11}\right)^{52} \mathrm{Cr}$.

## APKA Programme

Initial data

INTEGER
M - maximum number of excitation functions investigated
N - maximum number of points in the excitation function
$D$ - number of points in the auto- and cross-correlation functions
I,J - number of excitation function (number of reaction channel)

REAL
Q - energy variation step in excitation function in keV

ARRAY
$F[1: M, 1: N]$ - array of excitation functions
Operating variables used for controlling the output of:

## INTEGER

K - the number of the point in the auto- and cross-correlation function ( $\varepsilon$ is the deviation from energy $E$ )
L - the number of the point in the excitation function (value of energy E)
W - the number of the point of the auto-correlation function

REAL
A - the sum of the values of the excitation function; the ratio $R[K] / R[0]$; the difference in the values of the auto-correlation and Lorentz functions
$B$ - the sum of the squares of the excitation functions
SI - the mean value of the cross-section of the $J$ excitation function being analysed; the value of the coherent energy in keV obtained by analysing the fluctuations of the autowcorrelation function (see equation (7)).
V - the cross-correlation coefficient
Cl - the normalized mean square deviation
E - the sum of the products of the terms in the autom and crosscorrelation functions; the variable value of the coherent energy in the Lorentz function

```
    V - the sum of the products of the terms in the alutom and cross-
        correlation functions; the maximum number of points in the
        excitation function
SUM - the intermediate value
    Bl - the mean value of the square of the excitation functions
    G= Sl+G2
```

ARRAY
$T[1: M, 1: N]$ - the deviations of the excitation function from
the mean values
$S[1: M] \quad$ - the mean values of all $M$ excitiation functions
$\mathrm{LO}[\mathrm{O}: \mathrm{D}] \quad$ - the Lorentz function
$\mathrm{R}[\mathrm{O}: \mathrm{D}]$ - the values of the autom and cross-correlation functions
$\mathrm{P}[1: N] \quad$ - the excitation function being analysed
$C O[1: M] \quad$ - the values of the automecreletion functions when $K=0$
( $\varepsilon=0$ )

```
BEGIN
LNTEGER MT,N,D,N,L,X,Y;
AEAL A, B, 5I, BI,CI, E,V, SUM, a;
COMMENT (input of initial data)
COD('A/O-2', M,N,D,Q);
GEGIN
ARARY F,T[I:M, I:N], S, CO, CTYO[I:M], LO,R[OO:D], P[ITNJ;
COMMENT DO 'F'700'S'10'L0'S1'P'70'T'700'R'51
    '\angleO'51'R: 51' 'P'>0;
    COMMENT (input of excitation function array), ,
    cos('R10-z', F);
    COMMENT (calculation of mean cross-section and
normalized mean square deviation of the J excitation function
with print-out and filling of T array)
FOR J:=1 STEP I UNTEL N DO
PEQIN A:=0; B:=0; CI:=0;
FOR L:=1 STEP I UNTIL N DO
    SEGMY P[<L]: = F[J.人];
    A:=A+P[<]; B:=B+P[<]*2.
    END;
B1:=AN;
S[JI:=A/N;
CI:=AS/(SL IJfL2)-1;
    C[JO[]]:=C1;
    FQR L:=1 STEP 1 UNTIL N DO
    T[J, LJ:=F[J],L]-S[J]; St:= S[J];
    .cod(rP2-10; 1, P, S1,C1)
    LEND; J:=0;
    COMMENT (computation of the autom and crossmcorrelation functions
    and the cross-correlation coefficient with print-out of their values
    And filling, of the CO array)
    HM:z:={+1;
    FOR K:=0 STEP I UNTLL DTOO
    BEGIN E:=0; V:=0; ROR L:= I STEO I UNTTL N-K DO
```


## - $473^{-}$

```
VBEGKN \(E:=E+r[J, \angle] \times r[I, \angle+K]\);
    \(v:=v+T[J, \angle+K] * T[I, \angle] ;\)
    END;
    SUM: \(=0.5^{\prime} *(E+V) /(S[J] * S[I]) ;\)
    \(R[\alpha]:=\operatorname{sum} /(N-\alpha)\)
END;
\(v:=0 ; k:=0 ;\)
IF \(I \neq J\) THEN
\(v:=R[K] /(S Q R T(C I J O[I] * C T J O[J]))\)
ELSE CO[L]: \(=\) R[K];
code ( \({ }^{\prime} 02-10\); \(\left.J, ~ I, R, V\right)\);
COMMENT (proceed tc calculation of coherent energy from
auto-correlation function). -
IF \(I=J\) THEN GOTO \(H 1\) ELSE
COMMENT (proceed to calculation of the cross-correlation functions
between \(J\) channel and the following \(I(I>J)\) reaction channels)
IF \(\angle \angle M\) THEN COTO HH ELSE
COMMENT (/Droceed to next excitation function)
If J<M THEN GOTO \(A\);
COMMENT (calculate coherent energy, \(\Gamma\) and Lorentz function)
H1:
BEGEN INTEGEP W;
COMMENT (determine coherent energy \(\Gamma\) in keV from fluctuation of
automcorrelation function \([\text { see equation (7) }]^{7}\) and
print out)
    BEGIN REAL \(G, G 1, G 2 ; A:=5 M M:=G:=M:=A E=0\);
    \(H 2: K:=K+1 ; A:=R[K] / R[O] ;\)
    IF \(x=J\) THEN
    BEESN
    \(W:=2 * K\); G1: \(=W\);
    END;
    IF A>0.5 THEN GOTO H2;
    \(W:=61\);
```

$\mathrm{v}:=\mathrm{N} ;$ sum: $=0 ;$ st:=0;
EOR K:=W STEP 1 UNTKL D DO
sum: $=\operatorname{sen}+R[K]+2 *(E-K / V) ;$
S1: $=2 * V * S U M * Q /(3.14 / 57 *(D-W) * R(0]+2) ;$
$\operatorname{COD}\left({ }^{\prime} R 2-10\right.$ ', $S 1$ );
COMMENT (determine $\Gamma$ in keV by comparing the auto-correlation and the Lorentz functions and calculate the Lorentz function with print out)

BEGIN $A:=0 ; B:=0 ; E:=0 ; \mathrm{V}:=0 ; \operatorname{SUM}:=0 ; \kappa:=0.5 * W$;
$\operatorname{cod}^{\prime}\left({ }^{( } P_{2}-10 ;\right.$; $\left.K, G 1\right) ;$
IF $\mu=1$ THEN $E:=0$;
IF $\mu=2$ THEN $E:=0.5$;
XF K=3 THEN E: $=\%$;
IE M<10 TNEN
BEGTN SF K $\times 4$ THEN E: $=\pi-3$
END
ELSE
$E:=x-4 ;$
codl(p2-10;' E/;
H3: $E:=E+0.1$; SUM: $=0$;
FOR K: $=0$ STEP 1 UNTIL $0.5 * W$ DO
BEGIN
$A:=R[K]-R[O] * E+2 /(E+2+K+2) ;$
SUM: $=S U M+A$
END
G2: =E*Q; G1: $=E$;
SE SUM>O THEN GaTO H3;
$\operatorname{cod}$ ('A2-10), E, sum);
$E:=61$;
FOR $K:=0$ STEP 1 UNTIL $D$ DQ
LO[K]: : R[O]* EA2/(ER2 + KN2);
$\operatorname{cod}$ ('P2 -10;' O2, LO)
END:
COMMENT (determine mean coherent energy with print out)

```
6:=0.5*(s1+62); s1:=0;
cod('p2-10; 6)
END
END;
```

COMMENT (proceed to calculation of the cross-correlation functions between the $J$ channels and the following $I$ ( $I>J$ ) reaction channels)

IF TRM THEN GOTO NAS;
COMMENI (print out the values of the automerrelation function for $K=0 \quad(\varepsilon=0)$
$\operatorname{cod}\left({ }^{\prime} p_{2}-10\right.$ ' $\operatorname{co}$ );
$\operatorname{cod}($ 'PR2-10', co);
END
END;

## RVEG Programme

```
Initial data
```


## INTEGER

P-maximum number of excitation functions investigated
$W$ - maximum number of points in the excitation function
Q - maximum number of points in the probability distribution curve

## ARRAY

$F[1: P, 1: W]$ - array of excitation functions
$C[1: P] \quad-a r r a y$ of normalized mean square deviations for the excitation functions investigated / calculated /

Operating variables used for controlling the output of:

INTEGER DD - the volume of print out
N - number of effective channels
I - number of excitation function (reaction channel)
$L$ - number of cell in array $M$; number of terms in expansion of the Bessel function
$K$ - number of interval along the axis ${ }^{\circ} \mathrm{Kos}$
Q1 - range of renormalization of histogram

REAL
Al - sum of all points and mean value of excitation function; intermediate values

A2 - intermediate values
$J$ - variation step of $\sigma /<\infty ;$ value of $\psi$ in the probability distribution $P(y)$
Il - intermediate value
M - maximum number of effective channels; intermediate values
D - proportion of the process of direct interaction
$\mathrm{Zl}, \mathrm{B}, \mathrm{Bl}, \mathrm{B} 2, \mathrm{~B} 3, \mathrm{~B} 4, \mathrm{~B} 5, \mathrm{B6}, \mathrm{SUM}, \mathrm{R}$ - intermediate values
$V$ - normalized mean square deviation of the I excitation function

## ARRAY

NI [1:W]- excitation function being analysed NI[1:W]- array $\sigma /<\alpha>$ of function being analysed
$Z[1: Q]$ - experimental and normalized histogram
$\operatorname{PR}[1: Q]$ - array of distribution probability $P(\mathcal{Y})$
$\operatorname{PRP}[1: Q]$ - array of distribution probability $P(4)$
$\operatorname{PRR}[1: Q]$ - arrays of distribution probability after the cyclic permutation necessary for comparing them
$\operatorname{PE}[1: D D]$ - array of numerical output
$\Gamma[1: 10]-$ gamma function array (1.0, .95135, .91817, .89747, .88726, .88623, .89352, .90864, .93138, .96177)

KEGIN
INTEGER $P, G, T, L, K, W, Q 1, L 1, D D, Z f ;$
RERL AT, R2, J, It, N;
COMMENI (input of initial data)
$\operatorname{COD}\left({ }^{\left(P 1 O-22^{\prime}, P, W, Q\right) ; ~}\right.$
DD: = $3 \| Q+w+25 ;$
GEGEN $\operatorname{ARPRY} \mathrm{F}[1: \rho, 1: W], C[1: P], M 1, N 1[1: W], Z, P_{R}, P_{R P}$,
$\operatorname{PRR}[1: 0], \operatorname{PE}[1: \Delta D], \Gamma[1: 10] ;$

COMMENT (input of initial data)

$H: T:=I+1 ; A 1:=A 2:=0$;
EOR $\angle 1:=1$ STEP 1 UNTIL DD $D P$ PE[L1]:=0;
$\angle 1:=1$;
$\operatorname{PE}[\angle t]:=I_{i}$
COMMENT (calculation of mean square normalized deviation)

PEGIN EOR $\angle:=1$ STEP 1 UNTIL W DO
RESIN
$N \in[L]:=F[J, L]$;
$\operatorname{PE}[L+1]:=N 1(\angle) ;$
$A_{1}:=A_{1}+N S[L] ;$
$A 2:=A 2+N 1[L]+2$
END;
A1: $=A 1 / W$;
A2: $=A 2 / w$;
$C[I]:=A 2 /\left(A 1, \rho^{2}\right)-1 ;$
A2: $=C[I] ;$
$H:=w+2 ; P E[L 1]:=A 1 ; P E[L 1+1]:=A 2 ;$
LI: $=\angle 1+2$;
COMMENT (construction of experimental histogram of the probability distribution $Z\left({ }^{\sigma}{ }^{\sigma} \infty>\right)$ of the I excitation function for variation step of ${ }^{\sigma}$ Ko> $=(0.4 ; 0.3 ; 0.2)$ )
FOR L:=1 STEP $\pm$ UHITILS W DO MI[L]: $=N 1[\angle] / A 1 ;[I:=2.5$;

```
    H4:T1:= [1-0.5; EOR K:=1 SIEQ 1 UNTIL Q DO z[k]:~0;
    L:=0; J:=0.2 = I1;
    PE[LL]:=J;
    H5:K:=O; L:=L+1; IF L<W INEN
    BEGKN
    H6:K:= K+1;
    COMMENT (itransfer to new range of }\mp@subsup{}{}{\sigma}/<0>\mathrm{ )
    IF M1[L]>0.2*KNI1 [HEN GOTO HG
    ELSE
    IF M1[L]>0.2*(K-1)*I1 TNEN_
    COMMENT (transfer to new value of excitation function)
    BEGIN Z[k]:= E[k]+1; 自TO H5.
    END
    END;
    COMMENT (normalize area under ntstogram to unity and record
    corrected histogram)
    01:=0*0,2/7;
    EOR K:=1 STEP 1 UNTIL Q1 DQ
    Z[k]:= Z[k]/(0.2*I1*w);
    FQR X:=1 STEP 1 UNTIL Q DO
    PE[LL+k]:= = [k]]; L1:= L1+Q+1;
    PE[L1]:= 989898989; \angle1:= L1+1;;
    CONNENT (transfer to new variation step of % Kos )
    IF I1>1 THEN QOTO H4
END
    COMMENT (calculate theoretical probability distribution P( K/kos)
    of excitation function being analysed for variation step of
    %/<\alpha> = 0.2, print out proportion of direct interaction process,
    number of effective channels N}\mathrm{ and probability distribution at
    speciffic value of c[\]])
EOR K:=1 STEP 1 UNITL Q IQ PR [K]:=0;
REGIN REAL M,D,B,B1,B2,B3,B4, B5,B6,SUM,R,V;
N:=0;V:=C[J];M:=1/V;
```

```
PE[L1]:=V; AE[LL+L]:=M;
PE[<1+2]:=I; PE [<1 +3]:= 999999999;
COD ('P2-10', DE);
FOR L1:=1 STEP IUNTKL DD DO PE[L1]:=0;
L1:=f;
H3:N:=N+1; IT:=L;
IE L1>125 THEN
BEENN COD ('P2-10', PE);
EOR \angle1:=1 SIEP 1 UNIIL_ BD ID
PE[Ll]:=0;
L1:=1
ENZ;
    ELSE L1:=L1+1;
    Hнн: B1:= 1-V*N;
    IE B1<0 THEN
BEGIN N:=N-1+0.1;M:=1/C[I]; ZK=Z/+1;
    IF NSM THEN GOTQ HHH
END;
PE[L1]:=T; PE [LL+1]:=N;
COMMENT (transfer to control point at end of programme)
IE BI<O THEN EOTO HZ;
D:= SQRT (B1);
PE[L1+2]:=D;
B1:= = [z];
A2:=1;
CONMENI (transfer te eliminate part of programme)
IF N=1 THEN GOTO H1;
    COMMENN (transfer to end of programme, recording limiting number
    of channels N = 29, and re->recording the distribution and the
    control point)
    LE N>29 THEN QOTR HZ;
    COMrEETY (transfer to eliminate part of programme)
    IF N<18 THEN OOTO NZH;
    17:=1;
    IF Z=1 THEN B:=N-1Z ELXEE B:=ENTTER (N)-1R;
```

```
FOR K:=1 STEP1 UNTLLL RS DO
M: = M*(N-K);
A2:= 1/M; IF XX=/INEN A:N SLSE B:OFNTJER(N);
FQ囚 K:=1 STEP1 UNTIE 17OQ B/:=B/N/N-QrM/;
COMMENT (transfer to eliminate part of programme)
IF N~18 THEN GOTO HI;
HZH: IF {T, THEN B:=N-2 ELSE B:=FNTTER(N) - 2;
FOR K:=1 STEP 1 UNTELA PO
B1:= B1*(N-K);
H1:A1:=1/B1;B:=N/1-D); B2:= B& (O:E*N');
B3:= EXP(-0.2*B*D);
FOR K:={ STEPP 1 UNIIL Q RQ
3EEIN J =0.2*K-0.1;
    B4:= Jf(0.5*(N-1));
    B5:= B^J+D*B; BG: = ExP (-0.2*Bッ.7);
    COMMENT (In the absence of direct intraction D a j)
    IF B.5=0 TNEN
    BEGIN
    L:=0;
```




```
    IF PR[K]<1OP(-18) IHEN PA [K]:=0
    END;
    COMMENT (in the presence of direct interaction D F Oj
    IF BaS THEN
BEGIN
    PE [L1+3]:= L; PE [L1+4]:= 979797979
END;
IF BS>0 THEN
BEGIN L:=0; SUM:=10% (-15);
    R:= 10† (-15);
    H2:L:= L+1;
    R:=R* 85/((N+L-I)*L); SUM: = SUM+R;
    B1:=R;
```


## If $R<5 \times 10415$ THEN

BEEGN
$I I:=R ; M:=S M M ;$
COMNENT (transfer to increase the number of terms in the expansion of the Bessel function)
IF $R / S U M>0.01$ THEN GOTO H2;
COMMENT (print out number of effective channels)
$P E[\angle 1+3]:=L ; D E[\angle 1+4]:=969696969$
END;
EF $R>5$ * 10915 THEN
TEGK
$2:=\angle-1 ;$ sumt $:=$ Mi $100(-15) ; R:=I 1+10 f(-15)$;
SUM: $=$ SUM $=\log (-8)$;
$R:=R=10 f(-8) ;$
ZM: L: $=\angle+1$;
$R:=R=85 /((N+\angle-1) \times L) ;$ SUM: $=$ SUM $+R$;
COMMENT (transfer to record distribution, input of overflow)

IF SUM>10¢M THEN GOTO MZM;
COMMENT (transfer to increase number of terms in the
expansion of the Bessel function)
IF R/SUM $>0.01$ THEN GOTO $2 M$;
COMNENT ; print out number of effective channels)
$P E[L 1+3]:=\angle ; P E[L 1+4]:=559595959$
END;
IE. $B 1 \geq 5 * 10$ P 15 THEN
BEGIN
Sum: $=$ sum $+10 f(-15) * 77 * 109(-8)$;

$* B 2 * B 6 *(A 2 \times 1 O f 8) * B 3 * B 2 * B 6 \times B 3 * B 2 * B 6 \times B 2 * S U P \times 10 \nmid 12$,
EVE
ELSE


```
A2*10P8-83*B2*B6 - B3*B2*B6*B2* Sum-10+(-3);
IF PR[K]<10f(-18) THEN PR[K]:=0'
END
END;
MZM;
COMMENT (print out proportion of direct interaction process and
distribution of probabilities for N # 2)
L1:= L1+4;
EQR K:=1 STED 1 UNTLL Q QO
PE[L1+K]:=PR[K]; \angle1:= L1+Q +1;PE[L1]:= 919:91919;
COMMGNT (transfer for cyclic overflow of arrays PRR, PR, PRP and
eliminate part of programme operating only when N=2)
IF N>2 THEN GOTO HS;
COMMENT (oalculation for N-= 2)
IF N=1 IHEN EPR K:=1 STEP 1 UNIIL Q DO PRP[K]:=0;
N:=N+1; cov($2-10',N);
L1:=L1+1;
PE[L1]:=I; PE [L1+1]:=N;
B1:=1-V#N;
COMMENT (transfer to end of programme, recording zero array
of PRR for N=2)
IF B1<0 IHEN GONG HZ1;
D:= SRRT(B1);
PE [L1+2]:= 2;
A1:=1;
B:=N/(1-D); B2:= Bf(0.2*N); B3: : ExP(-0.2*B-D);
EOR K:=1 STEP & ENTITL Q DQ
BEGIN J:= 0.2**-d.1;
B4:= Jf (0,5* (N-1));
85:=B*J*D*B;B6:= EXP (-0.2*B*7);
IE B5=0 THEN
BEGIN SUM;=1; PRP[K]:=A1*10%5*B3NB2*BCNB4*B3*B2*
    B6*BY*B3*B2*B6*B3*B2*A6*B3*B2*B6*10f(-5);
```

$\angle:=0$;
IF PRP $[K]<\cot (-18)$ THEN PRP $[K]:=0$
END;
IE BS=O THEN
BESIN
$\operatorname{PE}[L 1+3]:=\angle ; P E[L I+4]:=979797979$
END;
IF B5>0 THEN
BEGEN L:=0; SUM: $=10 P(-15) ; R:=10 p(-15)$;
H7:L: $=2+1$;
$R:=R=B 5 /((N+2-1) * L) ;$ SUMT $=$ SUM $+R ; B E:=R$;
IE $R<5 \% 10 f 15$ TREN
BEGIN
It: $=R$; $M:=$ SUM;
CONMENT (transfer to increase number of terms in the expansion of the Bessel function)

IE $R /$ SUn $>0.01$ ITEEN GOTO $H Z$;
COMFENT (record number of effective channels)
$\operatorname{PE}[2 t+3]:=\angle ; \operatorname{PE}[L t+4] ;=569696969$
END;
IF R>5:10115 IHEN
BEGLN
$\angle:=L-1 ;$ SUM $:=7=10$ ( -15 ); $R:=11 \times 10 f(-15) ;$
$R:=R * 10 f(-8) ;$
SUM: $=$ SUM $\times 10(-8)$;
$2 M 1: \angle:=\angle+1$;
$R:=R=B 5 /((N+L-1)-L) ;$ SUM: $=5 U M+R$;
COMMENT (transfer to record probability distribation in view of overflow)
IE SUH $210 / 5$ THEN EOTS MZM1;
COMMENT (transfer to increase number of terms in the expansion of the Bessel function)

IF $R / S U M>0.01$ THEN $\underline{\text { OOTO }} 2 M 1$ :
COMMENT ( record number of effective channeis)

```
            PE[L1+3]:= L; PE [LL+4]:= 959595959
                END;
                IF B1>5*10415 IHEN
                BEGIN
                A2:=1;
                SUM:= SUM + 10%(-15)* A=10 (--8);
                            PRP[K]:=A1*10/5*B3*B2*B6* B3* Q2*BL*B2+SUM*10912;
                                    B3=1018*
                                    B2*B6*(A2* (Of S)* B3*B2NB6*B3*G2*BS*B2 *SUM* NOF 12;
                                    END;
                                    ELSE
                                    PRP[K]:=A1*10年*B3*B2*B6*B4*B3*1095*B6*B4*B3*
                                    *B2*B6* 10/5*B3*B2*B6*B3*B2*B6*E2*SUM:
                                    IF PRP[K]< 10 f(-18) THEN PRP[K]:=0
END
END:
M2M1:
COMMENT (print out proportion of direct interaction process and
probability distribution for N = 2)
LL:= LI+W;
FOR K:=1 STEP 1 UNIIL Q DO
PE[L1+K]:= PRP[K]; L1:=L1+Q+1;PE[L1]:= 929292029;
COMMENT (transfer to compare histogram with theoretical probability
distribution for two neighbourtng N)
IE, N=2 THEN GOTO H9;
H8: FOR K:=1 SIEP 1 UNIIS Q DQ
BEGIN PRR[K]:= PR[K];PR[K]:= PRP[K];PRP[K]: =PRR[K]
END;
H9:B1:=0; B2:=0;
FOR K:=1 STEP.1UNTIL Q DQ
BEGIN B:=PR[K]-2[K]; A1:=B1:B姩;
R:=PRP[K]-2[K]; B2:= B2+RY'2
END;
L1:= L1+1;PE[L1]:= 9.87654321;
```

COMMENT (transfer to increase $N$ in theoretical probability distribution).
TE B2<Bt THEN GOTO $H^{3}$; .
$P E[\angle 1]:=123456789 ; K:=0$;
MM: K: = 1 ; $\angle:=K+1$;
$B 1:=P R P[K]-\operatorname{PRP}[L] ;$
CUMMENI (transfer to determine point with maximum value in
-probabjlity distribution)
IE $B 1<0$ THEN GQTO MM:
$K:=\langle-1$;
$B 1: x z[k]-P_{R} P[K]$;
COMMENT (transfer to inorease $N$ in theoretical probability
distribution
IF B1>0 THEN GOTO H3;
$K:=0$;
MR1: $K:=K+1 ; L:=K+1 ;$
B1: = $2[k]-2[L]$;
IE B1<0 THEN GOTO MM1;
$K:=\langle-1$;
$B 1:=2[k]-\operatorname{PRP}[k] ;$
$P E[L 1]:=818181818 ;$
If $\mathrm{B} 1>0$ THEN GOTG Hz ;
H2:
H21:
$P E[L 1]:=191919191$
END;
$\cos \left({ }^{\prime} \rho_{2}-10\right.$ ', $\left.P E\right)$;
IF $L<P$ THEN gOTO H
END
ENA;

# COMPARISON OF TECHNIQUES AND METHODS FOR MEIASURING THE 

 PARAMETERS OF INTENSE NEUTRON FIELDS (FIRSTT STAGE)G.A. Borisov, R.D. Vasilev, N.B. Galiev, E.I. Grigorev, V.P. Yaryna<br>1. Introduction

At the present time the All-Union Research Institute of PhysicoTechnical and Radiotechnical Measurements (VNIIFPRI) is carrying out work aimed at standardizing all the techniques of measuring the basic parameters of neutron fields (integral neutron flux density, differential flux density, etc.) based on the activation method.

One task will be to establish the possible causes of disagreement in the results of neutron field parameter measurements performed in different organizations. The causes of such disagreement may be:

1. Differences in the nuclear physics constants and activation cross-sections used;
2. Errors in measuring the activity of activation detectors;
3. Inaccuracy in determining the number of nuclei of an isotope in a detector;
4. Errors in the method of interpreting the activity values of the detectors used.

VNIIFTRI is proposing that in 1971-72 all interested organizations should collaborate in a comparison of the methods and techniques needed to measure differential neutron spectra.

The aim of the programme is, firstly, to compare the characteristics of the activation detectors used in different organizations in order to ciscover the differences responsible for disagreement and find ways of eliminating them and, secondly, to establish the most simple and accurate methods of unfolding differential neutron spectra.

The information contained in the answers to the questionnaire will enable the Institute to undertake a broad programe of work on the standardizam tion of neutron fiel parameter measurements. The information will also be used for comparing methods of unfolding neutron spectra and for comparisons of activations obtained with the most widely used detectors.

## 2. Comparison programme

The proposed comparisons of detector characteristics and methods of unfolding spectra will be performed in two stages.

In the first stage the participants in the comparison will send the Institute information on the reactions and detectors they employ, including the nuclear physics characteristics of the detectors, and will indicate the detectors which are most widely used and which they feel should be compared first of all. This information should be included in the questionnaire supplied.

In the second stage the Institute will send out to the organizations which have answered the questionnaire the values of the activation integrals calculated for one or several test spectra and cross-sections used by a given participant, and they will be invited to unfold a test spectrum or spectra. The Institute will compare the results of the unfolding. All participants in the comparison will be informed of the results.

## 3. Conclusion

All the organizations taking part in the comparison will be informed of the results of the analysis of the questionnaires. Similar intercomparisons operated by CMEA or the International Bureau of Weights and Measures might become an important source of information on nuclear physics constants and cross-sections of interest to those participating in the survey. The questionnaire should also show whether it would be desirable later on to hold a conference on the standardization of methods and techniques of measuring neutron field parameters.

## QUESTIONNAIRE

## A. Activation detectors

The answers to the first 15 points in the questionnaire should be incorporated in a special table, the number of the question corresponding to the number of the table column. A specimen table is attached.

1. What reactions do you use for spectrum measurement?
2. Indicate the values of $\mathrm{E}_{\text {eff }}$ used and the reference.
3. Indicate the values of $\sigma_{\text {eff }}$ used and the reference.
4. Indicate the half-life of the product nucleus and the reference.
5. What chemical compound is used for your detector?
6. What type of material is used and what is its enrichment?
7. What are the shape, size and weight of the detector?
8. How do you determine the number of nuclei of the isotope in the detector?
9. Minimum neutron flux density with energy above $E_{\text {eff }}$.
10. Irradiation time.
11. Cooling time after irradiation.
12. Method of measuring the activity.
13. Type of radiation measured and its energy.
14. Correction coefficients and constants used in measuring activity (gamma radiation yield, self-absorption coefficient, internal conversion coefficient etc.).
15. Method of calibrating the measuring equipment.
16. Supply tables of the activation cross-sections for the reactions you use, showing the cross-section values together with the permissible error and the amount of detail you find necessary for unfolding a differential neutron spectrum.
17. Indicate those detectors which in your opinion should be included in the first comparison.
18. Would you welcome the centralized issue of tested activation detector kits?

## B. Methods of unfolding differential neutron spectra

1. Describe briefly your method or methods of unfolding a spectrum. What in your opinion is the confidence level of the unfolded spectrum? Where was this method first described?
2. Would you welcome the centralized issue of standard computer programes for unfolding differential neutron spectra?
3. What in your opinion would be of most help in standardizing the process of unfolding neutron spectra by the activation method?

Table

| Key: |  |
| :---: | :---: |
| Реакция $=$ | Reaction |
| $E_{\ni \Phi \Phi}, \mathrm{M} \mathrm{\vartheta 日}=$ | $\mathrm{E}_{\text {eff }}, \mathrm{MeV}$ |
| $\sigma_{\text {зфф }}$, мбарн $=$ | $\sigma_{\text {eff }}$, mbarn |
| $\mathrm{T}_{1 / 2}=$ | $\mathrm{T}_{\frac{1}{2}}$ |
| Химическое $=$ соединение | Chemical compound |
| Марка и = обогащение | Type and enrichment |
| Форма и $\underset{\text { меры }}{\text { раз- }}=$ | Shape and size |
| Число ядер = | Number of nuclei |
| $\begin{aligned} & \text { Мин.поток, } \\ & \text { нейтр./смсек } \end{aligned}$ | Minimum flux, $\mathrm{n} / \mathrm{cm}$. sec |
| Время облучения $=$ | Irradiation time |
| Время вьдержки $=$ | Cooling time |
| Метод измерения $=$ активности | Method of measuring activity |
| Излучение и его = энергия, мэв | Radiation and its energy, MeV |
| $\left[\begin{array}{l} \text { Поправочные } \\ \text { коэффициенты и } \\ \text { константы } \\ \text { Метод градуи- }= \\ \text { ровки } \end{array}\right.$ | Correction coefficients and constants <br> Method of calibration |
| дня = | Days |
| $\left\{\begin{array}{l} \text { О.С.Ч. естест- }= \\ \text { венная смесь } \\ \text { изотопов } \\ \text { Таблетка } \end{array}\right.$ | 0.S.Ch. natural isotopic mixture <br> Tablet, $\oint 10 \mathrm{~mm}, 800 \mathrm{mg}$ |


| $\left\{\begin{array}{l} \text { Взвешивание на } \\ \text { аналитических } \\ \text { весах } \end{array}\right.$ |  | Weighing on analytical balance |
| :---: | :---: | :---: |
| 2 часа | $=$ | 2 hours |
| 2-3 дня | = | 2-3 days |
| Гамма спектрометр | = | Gamma spectrometer |
| Вьгод $\begin{array}{r}\text {-лучей на }\end{array}$ распад 33\% | $=$ | Gammarray yield per disintegration 33\% |
| Градуировка фотоэффективности О.С.Г.И. | $=$ | Calibration of photoefficiency O.S.G.I. |


| - | ¢Ревкдия | ЕөФ¢ | 6 өфф | 78 | Химическое | Марка и обога- щения | ¢орм8 | $\begin{aligned} & \text { १иссло } \\ & \text { ядер } \end{aligned}$ |  |  | В Время <br> вы - <br> дерх- <br> ки |  |  | Поправочнве коэффициенTS K KOH- <br>  | $\begin{aligned} & \text { Метод } \\ & \text { rps дуи- } \\ & \text { ровки } \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 12 | II | I2 | 13 | 14 | 15 |
| 1 | $J^{22}\left(44^{2 n}\right)$ | II | 1000 | $\begin{aligned} & 13.1 \pm \\ & 0.5 \\ & \text { дHR } \end{aligned}$ | $\mathrm{C}_{7} \mathrm{H}_{7} \mathrm{O}$ | 0.c.ч. ectectвенняя <br>  nob | $\begin{aligned} & \text { Tвблет- } \\ & -\quad \mathrm{Hg} \mathrm{ma} \\ & \text { 10 m } \\ & 800 \mathrm{ur} \end{aligned}$ | Взвеши <br> вуние <br> Н8 8 Ha <br> литичөс <br> KhX <br> becex | $-10^{9}$ | $\underset{4 \theta c a}{2}$ | $\begin{aligned} & 2-3 \\ & \text { дня } \end{aligned}$ | Гамма-спектрометр | ${ }^{-0.65}$ | выход б-лучеи ня распяд $33 \%$ | Гродуировкa фотозффективности O.C.I'.K. |

RECOMMENDED EFFECTIVE THRESHOLD AND CROSS-SECTION VALUES

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The effective threshold method for describing threshold crossm sections is described in Refs [1, 2]. The table below contains the recommended effective thresholds $E_{\text {eff }}$, the effective cross-sections $\sigma$ and the errors in the effective crossmsections for the chosen class of spectra [3]. The methods by which the recommended data were determined as well as a complete list of the references used are contained in Ref. [3].

Table
Key:


## REFFERENCES

[1] KRAMER-AGEEV, E.A., TROSHIN, V.S., Atomn. Energ. 291 (1970).
[2] KHARIZOMENOV, Yu.V., SUVOROV, A.P., Bjul. inf. Centr. jad Dannym, Issue No. 3 (1966) 462.
[3] KRAMER-AGEEV, E.A., et al., Izmer. Tekh. (in press).

## CORRIGENDA

to article by S.M. Zakharova entitled "80 and 2l-group cross-sections for absorption of neutrons by ${ }^{237} \mathrm{~Np}$ and isotopes of gadolinium" (Bjul. inf. Centr. jad. Dannym

Issue No. 5 (1968) 189).

1. On page 193 the expression $D(u, J)=\frac{\bar{D}}{2 g}$, where $\bar{D}=\left\{\begin{array}{r}2 D \text { observed when } I=0 \\ D \text { observed when } I \neq 0\end{array}\right.$ should be replaced by $D(u, J)=\vec{D}$, where $\bar{D}=\left\{\begin{array}{c}2 D \text { observed when } I=0 \\ D \text { observed when } I \neq 0\end{array}\right.$
2. On page 193 the expression $r_{0}=1.25 f$ should be replaced by $r_{0}=1.31 \mathrm{f}$.
3. In the article it is stated that the factor $S$, correcting for the fluctuation of the mean neutron widths, had been taken into account. In fact, in the programme used for the computer calculation (Ref. [3] in the article) $S$ is assumed to be 1.

[^0]:    * For the case where the graphite temperature was $-33^{\circ} \mathrm{K}$ and water temperature $297^{\circ} \mathrm{K}$, minimization was performed in the range $Z=0.5-2.1 \mathrm{~cm}$.

[^1]:    * In the analysis of the series $B$ experiments the relaxation lengths in water were obtained on the assumption that the dimensions of the system were infinite in the transverse direction. Numerical evaluations of the effect of two-dimensionality indicate that the corrections to the relaxation lengths are only fractions of a per cent and may be neglected.

[^2]:    天 Ha рассиотренньх АЭС дазчня по енбросам подучены после 18 － 30 минут выдеркв выбрасивяенах газов и очпстки их с понопзд азрозольпих чильтрод．

