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MISSOURI COOPERATIVE HIGHWAY RESEARCH PROGRAM
REPORT **67-9**

"TIME-DEPENDENT DEFLECTION OF A BOX GIRDER BRIDGE"

MISSOURI STATE HIGHWAY DEPARTMENT
UNIVERSITY OF MISSOURI, COLUMBIA
BUREAU OF PUBLIC ROADS

TIME-DEPENDENT DEFLECTION OF A BOX GIRDER BRIDGE

Prepared for

MISSOURI STATE HIGHWAY DEPARTMENT

ADRIAN PAUW

and

A. AKBAR SHERKAT

DEPARTMENT OF CIVIL ENGINEERING

UNIVERSITY OF MISSOURI

COLUMBIA, MISSOURI

in cooperation with

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BUREAU OF PUBLIC ROADS

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SYNOPSIS

In this report the time-dependent dead-load deflections of a continuous box girder bridge are compared with the values obtained by a rational analysis based on the "modified-elastic-modulus" method. Warpage and deflection due to shrinkage are estimated by applying the principles of superposition using an effective elastic modulus and an assumed shrinkage potential for the concrete. The study reveals that deflections due to shrinkage are relatively insensitive to the assumed value of the effective modulus of elasticity of the concrete.

While creep deflections are more sensitive to the value of the effective elastic modulus selected, within reasonable tolerances variations due to this assumption would not introduce an error greater than about 15 percent. As a result it is believed that the proposed method of analysis is adequate for normal design purposes. Because the computations become somewhat complex due to changes in effective section properties, both in the geometry of the member and due to cracking in the tension zone, a direct method for integrating the elastic curve was employed, using McCaulay's notation. This procedure was found to be especially appropriate in that it permitted a relatively simple computer analysis for the problem. The computed deflections were found to be in reasonable agreement with the field observations for the structure analyzed.

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CHAPTER I

INTRODUCTION

Objective and Scope

The problem of predicting time-dependent deflections of concrete flexural members due to creep and shrinkage is complicated by the rather wide range in properties encountered in concrete produced in the field. In the past these deflections have been determined by empirical coefficients for multiplying the instantaneous elastic deflections. These coefficients have been obtained from observed behaviour in practice.

In this report an attempt is made to develop a more rational analysis for estimating time-dependent behaviour and to evaluate this analysis by comparison with the observed field behavior of a continuous box girder bridge. The analysis for creep deflections is based on the "modified elastic modulus(1)*" method. In this method a reduced effective elastic modulus is assumed for the concrete, based on the ratio of stress to total elastic plus creep strain. The analysis for shrinkage is based on the principle of superposition by considering the effect on the section of a member, of a longitudinal force equal and opposite to the force which would have to be applied to the reinforcement to permit unrestrained shrinkage of the member without warping.

*Numbers in parentheses refer to entries in the bibliography.

The bridge structure analyzed in this report is a continuous reinforced concrete box girder. This bridge, A-992, is located in Jackson County Missouri, at the intersection of Interstate 435 over Interstate 70. Field observations of the total deformations at the mid and quarter points of the two main spans were available and have been previously reported[2]. The principal physical dimensions and the location of the deflection gage points are shown in Figures 1.1 and 1.2.

Assumptions

The analysis used in this report is based on the following assumptions:

1. The effect of time-dependent creep can be analyzed on the basis of a modified elastic modulus for the concrete.

2. The shrinkage potential is uniform throughout the entire section and has a nominal value of 0.0002 in./in.[3].

3. The sections are cracked in the tension zone except in a section of arbitrarily selected length adjacent to the point of contraflexure.

Section properties were computed on the basis of an equivalent transformed section, using a modular ratio of N to determine the equivalent concrete section of the reinforcement in the "cracked" tension zone. The equivalent incremental concrete area for the reinforcement in the compression zone or in the uncracked tension zones was obtained by multiplying the area of the reinforcement by

(N-1). For a modular ratio N = 5 the neutral axis of the sections was found to pass through the top interior fillets. For this case, the contribution to the moment of inertia of these fillets was negligible and was neglected to simplify the calculations. Where a change in reinforcement affected a length of section less than two feet, the effect of this change was neglected and the rigidity of the adjacent section with the smallest value was assumed. Since the effect of increased dead load due to cap beams and interior diaphragms was offset by increased rigidity at these sections, their effect on deflection was assumed to be negligible.

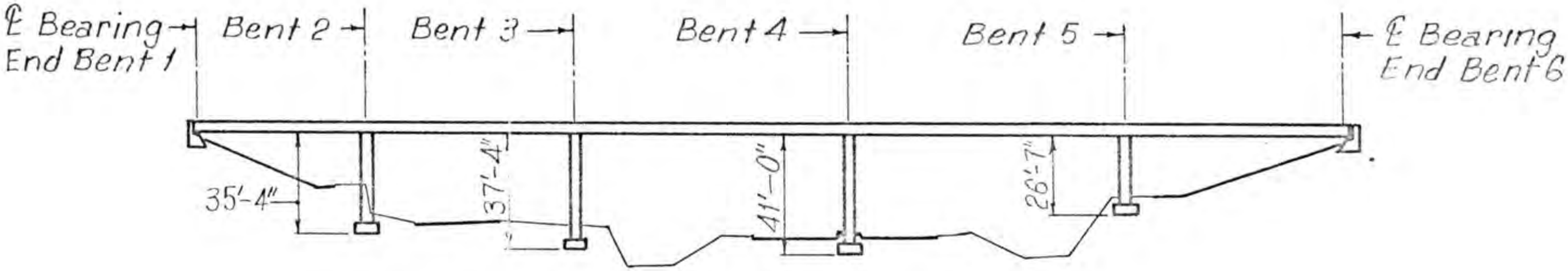
Procedure

To simplify the computational procedures and to make the method of solution more readily applicable for the analysis of similar structures, standard computer programs were developed, both for computing the section properties for various assumed modular ratios, and for the direct integration of the elastic curve of a continuous beam with stepwise variation of moment of inertia. These programs permit ready analysis of any continuous beam given the section geometry and the material properties.

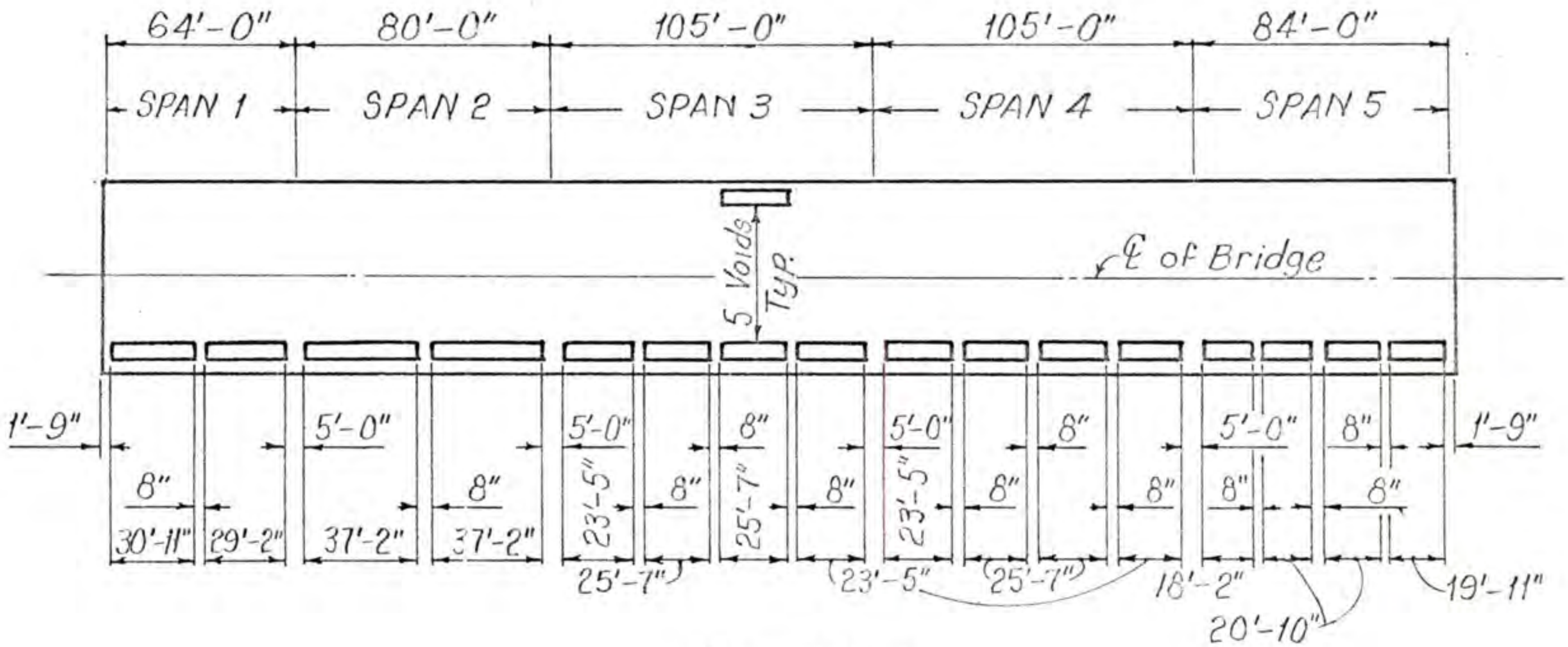
The procedure used for calculating the section properties is based on an application of the "Section Moulding[4]" technique. Using this method the section properties, including the location of the centroid and the moment of inertia, of a complicated section may readily be computed by sequential consideration of component elements of the section. This method is especially useful in locating the neutral axis of a complex "cracked" section by application of a simple iterative procedure. The computer program based on this procedure requires as input parameters defining the geometric configuration of the section, the desired modular ratio ($N = E_s/E_{ct}$*), and the shrinkage coefficient. The program output includes the section rigidity (EI), the location of neutral axis, and the shrinkage "warping" moment.

The procedure for computing the deflections, both elastic and creep and shrinkage, is based on direct integration of the elastic curve using MacAulay's notation. By this technique the deflection equation can be expressed as a continuous function of the loads or moments and section properties along the full length of the structure. This procedure is particularly suited for computer programming in a generalized form.

*Notations refer to enteries in the "Notation Used in Analytical Solution"



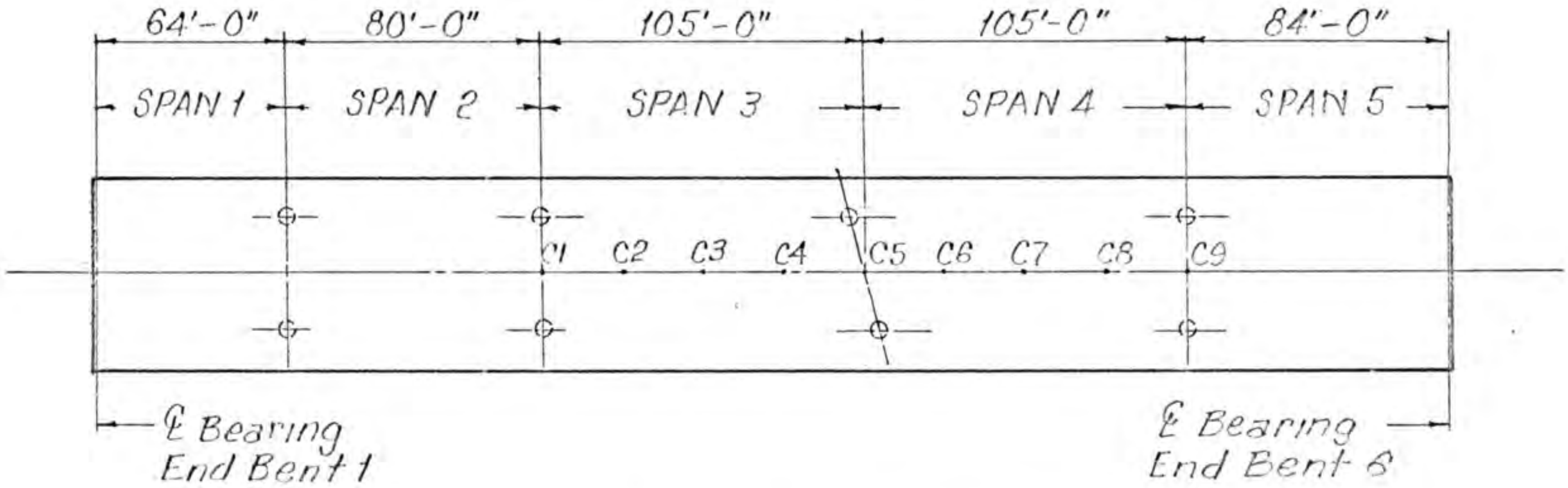
Section Along ℄ of Bridge



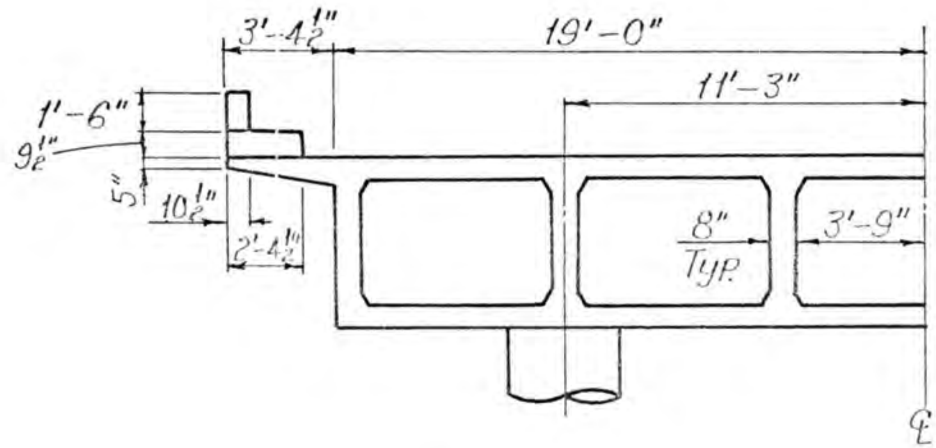
Plan of Bridge

BRIDGE A - 992

FIG. 1.1



Deflection Points



Note:
Bridge cross section is symmetrical about ℄.

Typical Cross Section

BRIDGE A - 992

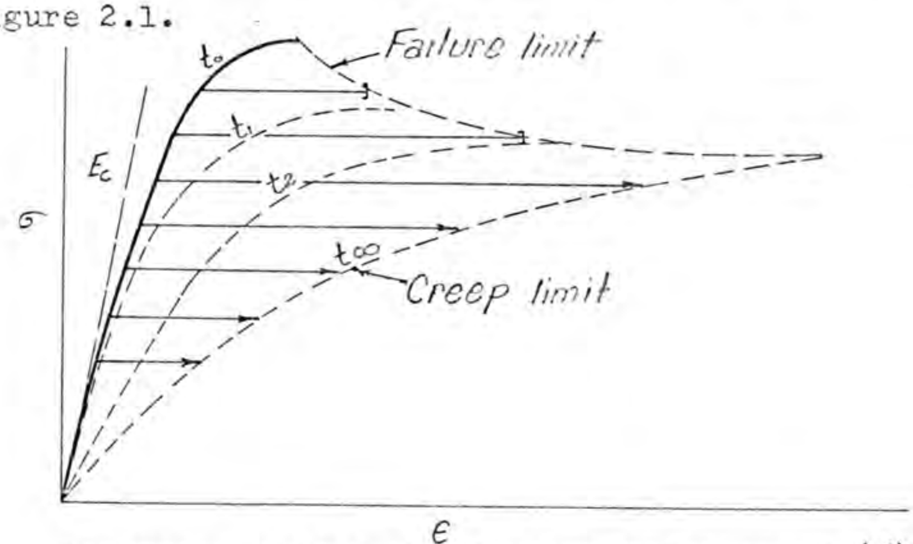
FIG. 1.2

CHAPTER II

SECTION PROPERTIES

Introduction

The deflection analysis used in this report is based on a knowledge of the effective section rigidity, EI. To determine the time-dependent deflections the "effective modulus" method was employed. This method is based on the assumption that for working stress levels, creep strains are proportional to the stress as well as being a function of the duration of the loading period, as shown schematically in Figure 2.1.



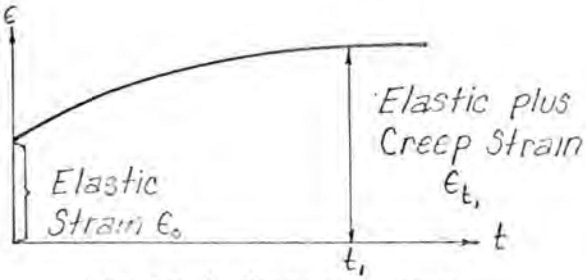
Schematic Stress-Strain-Time Relationship[5]

Fig. 2.1

Thus for a given time duration, t_1, and a sustained stress level, σ, the total elastic and creep strain would be given by the expression

$$\epsilon_{t_1} = \frac{\sigma}{E_{ct_1}} \qquad (2.1)$$

Where, E_{ct_1}, would be the effective or reduced modulus for time t_1. This modulus can be computed from the characteristics of a representative creep curve such as shown in Figure 2.2.



Typical Creep Curve

Fig. 2.2

Defining the ratio $\epsilon_{t_1}/\epsilon_0$ as a "Creep Coefficient" C_t, the effective modulus is given by

$$E_{ct} = E_c/C_t \qquad (2.2)$$

The section rigidities (EI) for the various sections in the structure were computed by the use of an equivalent concrete section or the so-called "Transformed" section method where the reinforcement area is replaced by an equivalent area of concrete equal to

$$(A_s)\,\text{equiv.} = N\,A_s \qquad (2.3)$$

where $N = E_s/E_{ct}$ (2.4)

Clearly the effective rigidity EI is not constant but decreases as E_{ct} decreases due to creep. However, this decrease is not directly proportional to E_{ct} and hence I increases as E_{ct} decreases because the elastic modulus of the reinforcement is constant. Since the designer cannot always exactly specify the desired value of N, a range of values from N = 5 to N = 25 was considered in this report in order to determine the functional relationships between the value of N selected and the rigidity of the section.

Due to changes in reinforcement and reversal of moment sign, it was necessary to analyze a total of 21 sections for each of 5 modular ratios. It was therefore desirable to develop a practical formulation for computing the moment of inertia of the section as well as the centroid of the transformed section and the centroid of the reinforcement area. This problem was further complicated for the cracked sections. In these sections the position of the neutral axis had to be determined by an iteraterive procedure before the moment of inertia could be computed.

Section Moulding

The procedure selected for computing the 105 rigidity constants is known as the "Section Moulding[4]" method. This method was especially efficient for this problem since it permits calculation of the moment of inertia by considering the effect of consecutive changes or additions to the section. Since most of the changes were due to changes in the amount of reinforcement and resulting changes in the position of the neutral axis, the method provided considerable savings in computational time.

The equations required for the section moulding method may readily be derived from the fundamental axis transfer formula:

$$I_y = I_o + Ay^2 \qquad (2.5)$$

where y is the distance between the desired axis and the parallel centroidal axes, I_o is the moment of inertia of the section about its centroidal axes and A is the area of the section.

Consider now the moment of inertia of two sections having areas A_o and A_1, let b equal the normal distance between parallel centroidal axes for A_o and A_1, and a_o and a_1 the distance from the centroidal axes of the combined area to the centroids of A_o and A_1, respectively. Furthermore, let I_o and I_1 be the moments of inertia of area A_o and A_1, respectively, about parallel centroidal axes (Fig. 2.3).

The location of the centroidal axis of the combined area is determined by taking moments, thus

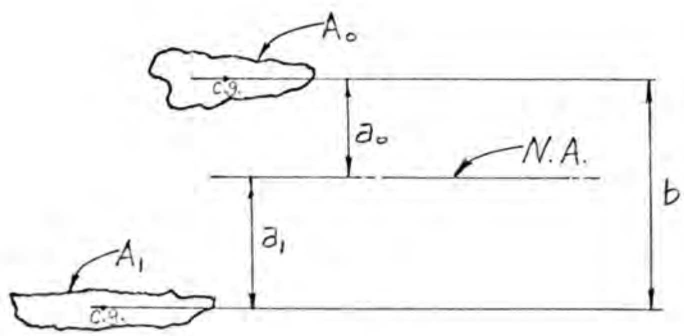


Fig. 2.3

$$a_o = \frac{A_1 b}{A_o + A_1} \quad \text{and} \quad a_1 = \frac{A_o b}{A_o + A_1} \qquad (2.6)$$

From Eq. (2.6)

$$\frac{a_o}{A_1} = \frac{a}{A_o} \quad \text{or} \quad a_o A_o = a_1 A_1 \qquad (2.7)$$

For the combined area the moment of inertia about the centroidal axis can now be evaluated from Eq. (2.5):

$$I = I_o + A_o a_o^2 + I_1 + A_1 a_1^2 \qquad (2.8)$$

Substituting the result of Eq. (2.7)

$$I = I_o + I_1 + A_o a_o (a_o + a_1)$$

$$= I_o + I_1 + A_o a_o b \qquad (2.9)$$

and substituting the value for a_o from Eq. (2.6), Eq. (2.9) takes the form:

$$I = I_o + I_1 + A_o \frac{A_1 b}{A_o + A_1} b$$

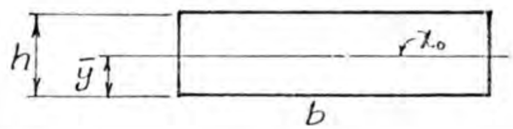
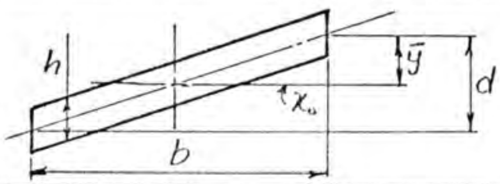
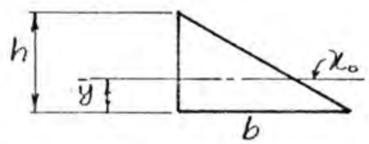
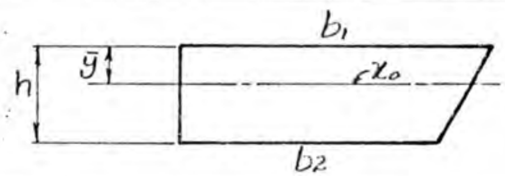
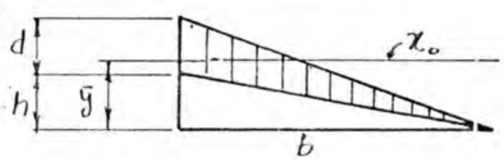
$$I = I_o + I_1 + \frac{A_o A_1}{A_o + A_1} b^2 \qquad (2.10)$$

This formulation is independent of the position of the centroidal axis of these combined areas and furthermore, the correction term is always positive for an incremental addition to the section. Clearly, Eq. (2.10) can be applied sequentially any number of times provided the centroidal axis for the combined area (determined by Eq. (2.6)) is used as the new reference axis.

Convenient formulas for the position of the centroidal axis and for the moment of inertia about the centroidal axis for the several geometrical segments required for analyzing the bridge sections in this report are summarized in Table (2.1). These formulas can readily be determined from the calculus.

TABLE 2.1

MOMENT OF INERTIA FORMULAS

Segment Configuration	Distance to Centroidal Axis	Moment of Inertia
	$\bar{y} = \frac{h}{2}$	$I_o = \frac{h^3 b}{12}$
	$\bar{y} = \frac{d}{2}$	$I_o = hb\,\frac{(h^2 + d^2)}{12}$
	$\bar{y} = \frac{h}{3}$	$I_o = \frac{h^3 b}{36}$
	$\bar{y} = \frac{b_1 h^2 - \frac{2}{3}(b_1 - b_2)\,h^2}{2b_1 h - (b_1 - b_2)h}$	$I_o = \frac{h^3}{36(b_1 + b_2)}\left(b_1^2 + 4b_1 b_2 + b_2^2\right)$
	$\bar{y} = \frac{2h + d}{3}$	$I_o = \frac{b}{36}\left(d^3 + hd(d+h)\right)$

Section Variations

Figures 2.5, 2.6, and Figures 2.7 and 2.8 show typical bridge sections for the positive and negative moment regions, and the cracked and uncracked sections, respectively. Table 2.2 shows the variation in the reinforcement in the bottom slab of the box girder and in the top of the bridge deck slab as well as the total reinforcement area in the section. The location of the reinforcement changes are shown in Figures 2.9 and 2.10. Several sections had identical reinforcement areas and distribution and these sections are identified in Table 2.3. Section rigidities, EI, were computed on the basis of an assumed elastic modulus of 29000 ksi for the reinforcement.

Variation of the effective section rigidity for the 21 different sections is shown in Figures 2.11 through 2.31, plotted as a function of N. It may be noted from these figures that changes in rigidity are relatively small as the value of N becomes greater than 20. This result is due to the fact that for the larger values of N, the reinforcement contributes a larger percentage of the rigidity and this effect is especially noticeable in "cracked" sections because an increase in N is associated with a larger effective concrete compression zone. Section input data for computing the moment of inertia as well as a print out of the computed section rigidities are given in the Appendix, together with the computer program and flow chart used for these computations.

Warping Moments

To determine the warping moments due to shrinkage the procedure proposed by Ferguson was employed[6]. This procedure employs a horizontal force C_o which would have to be applied at the centroid of the reinforcement to permit unrestricted longitudinal shrinkage of the box girder section. Since the reinforcement, however, restrains the shrinkage, a warping moment is developed equal to:

$$M_s = T_o e \qquad (2.11)$$

where, T_o = a force equal and opposite to the shrinkage force C_o

and, e = eccentricity, i.e. the distance between the centroid of the transformed section and the centroid of the reinforcement.

The force T_o was obtained on the basis of an assumed shrinkage potential of 0.0002, hence

$$T_o = 0.0002\, A_s E_s$$

$$= 5.8\, A_s \text{ kips} \qquad (2.12)$$

where, A_s is the total area of the reinforcement in the section in square inches. The warping moment therefore is

$$M_s = 5.8\, A_s e$$

where $e = \bar{y}_s - \bar{y}$ as shown in Figure 2.4.

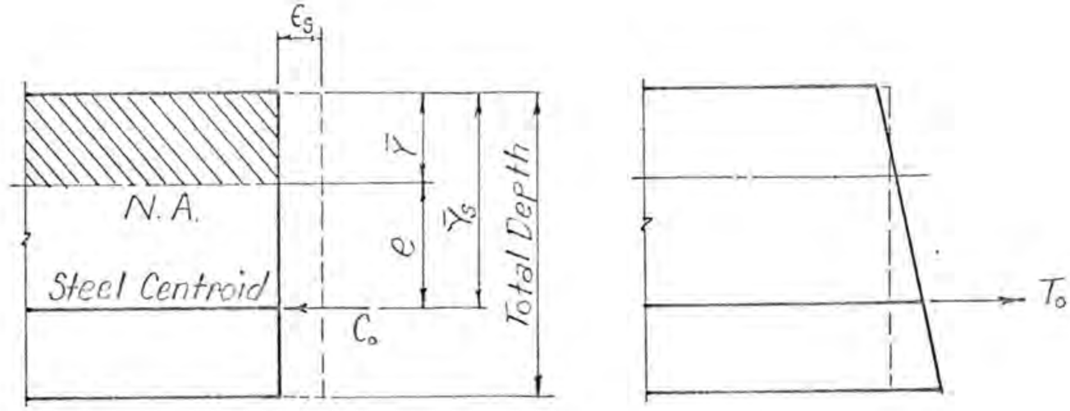
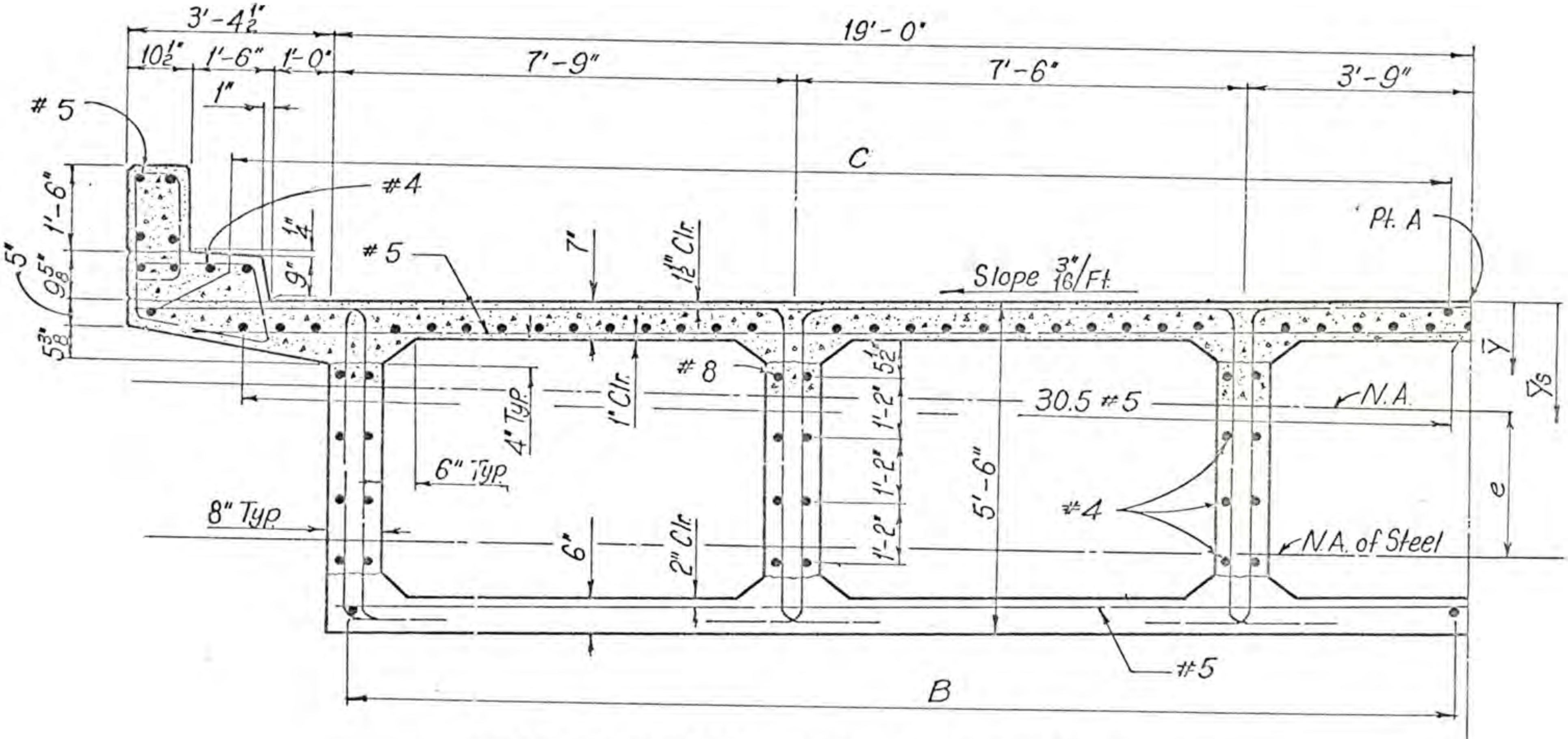


Fig. 2.4

Tables 2.4 through 2.8 show the variation in the section rigidity, eccentricity, and warping moment for each select N value.

Computer Programs

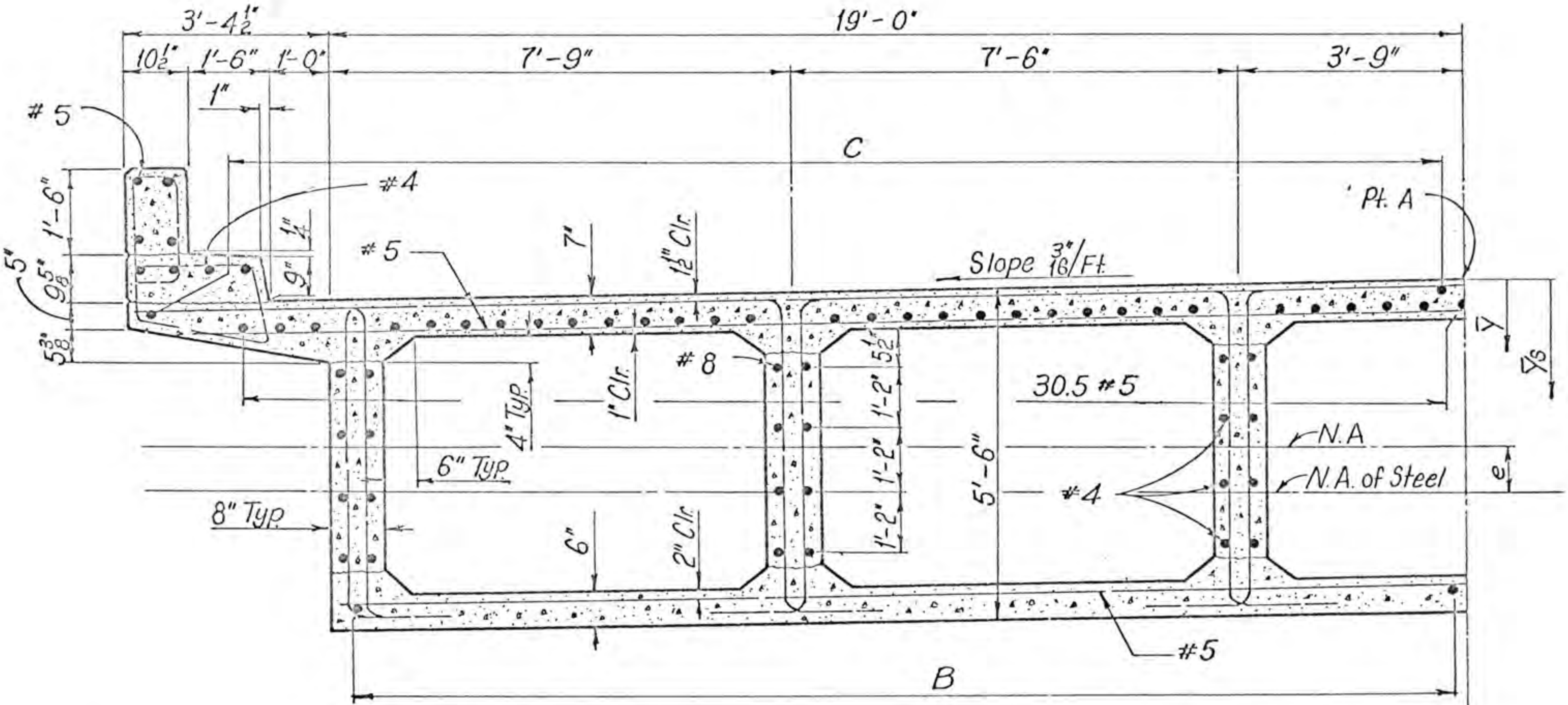
Computer programs were developed for the IBM 360/40 using Fortran IV language. The program computed not only the section rigidities, but also the centroid of the transformed section, $\bar{y}$, the centroid of the reinforcement, $\bar{y}_s$, the eccentricity of the reinforcement, e, the shrinkage force, T_o, for a shrinkage coefficient of 0.0002 and the warping moment M_s. The flow charts, data input, and computer outputs are given in the Appendix.



Note: For variable reinforcement B & C, see Table 2.2
For variable eccentricity e, see Tables 2.4 through 2.8

TYPICAL CRACKED SECTION
TENSION IN BOTTOM

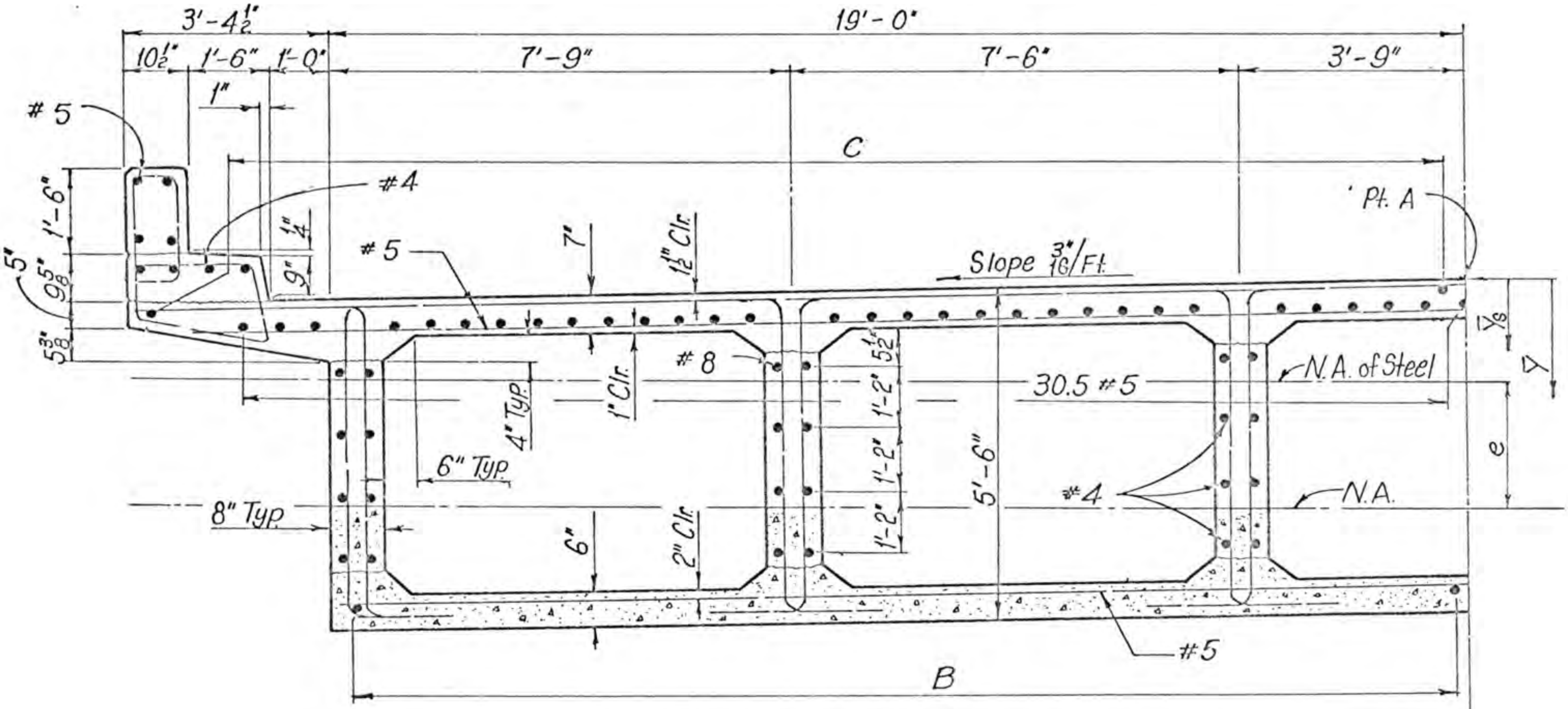
FIG. 2.5



Note: For variable reinforcement B & C, see Table 2.2
For variable eccentricity e, see Tables 2.4 through 2.8

TYPICAL UNCRACKED SECTION
TENSION IN BOTTOM

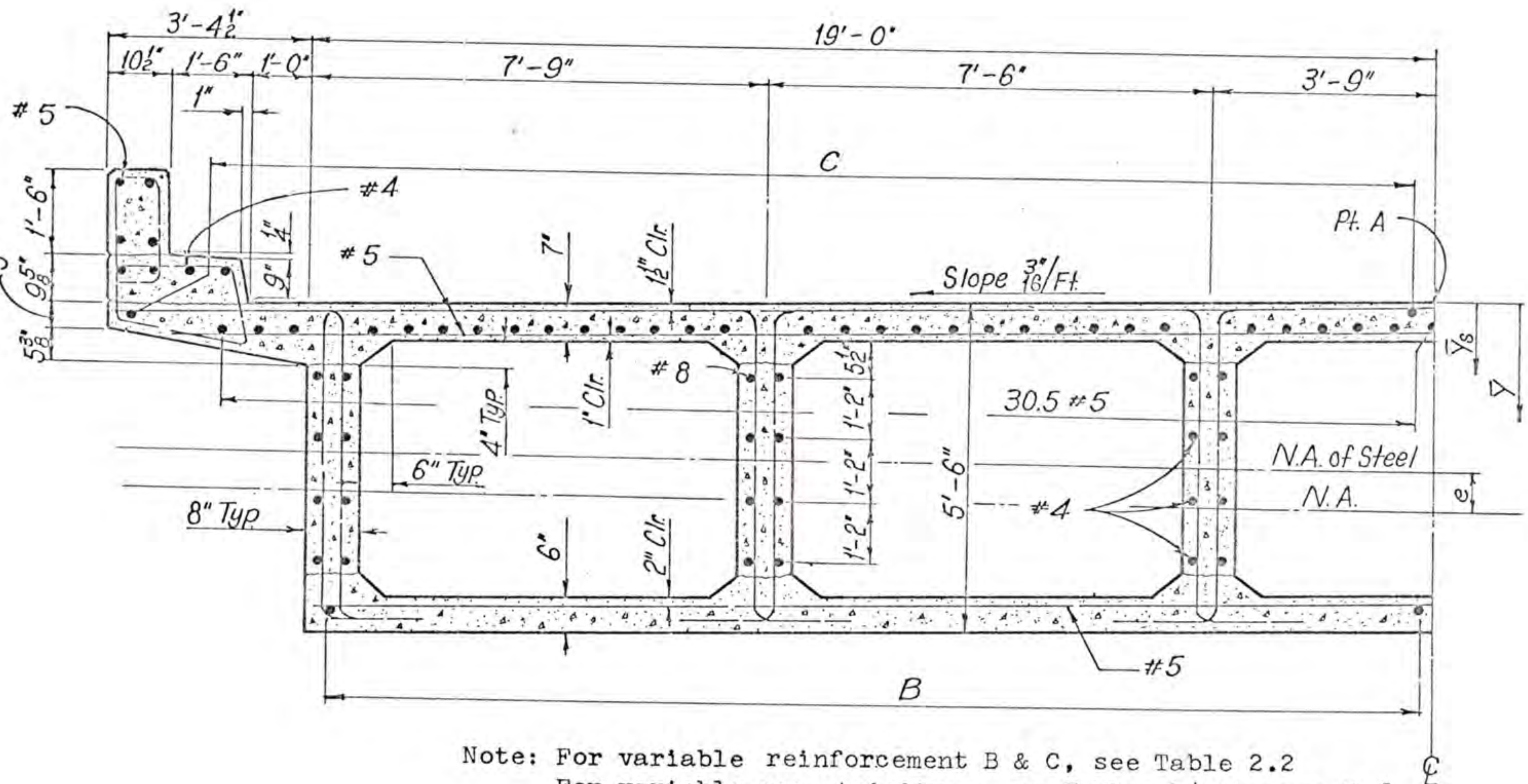
FIG. 2.6



Note: For variable reinforcement B & C, see Table 2.2
For variable eccentricity e, see Tables 2.4 through 2.8

TYPICAL CRACKED SECTION
TENSION IN TOP

FIG. 2.7



Note: For variable reinforcement B & C, see Table 2.2
For variable eccentricity e, see Tables 2.4 through 2.8

TYPICAL UNCRACKED SECTION
TENSION IN TOP

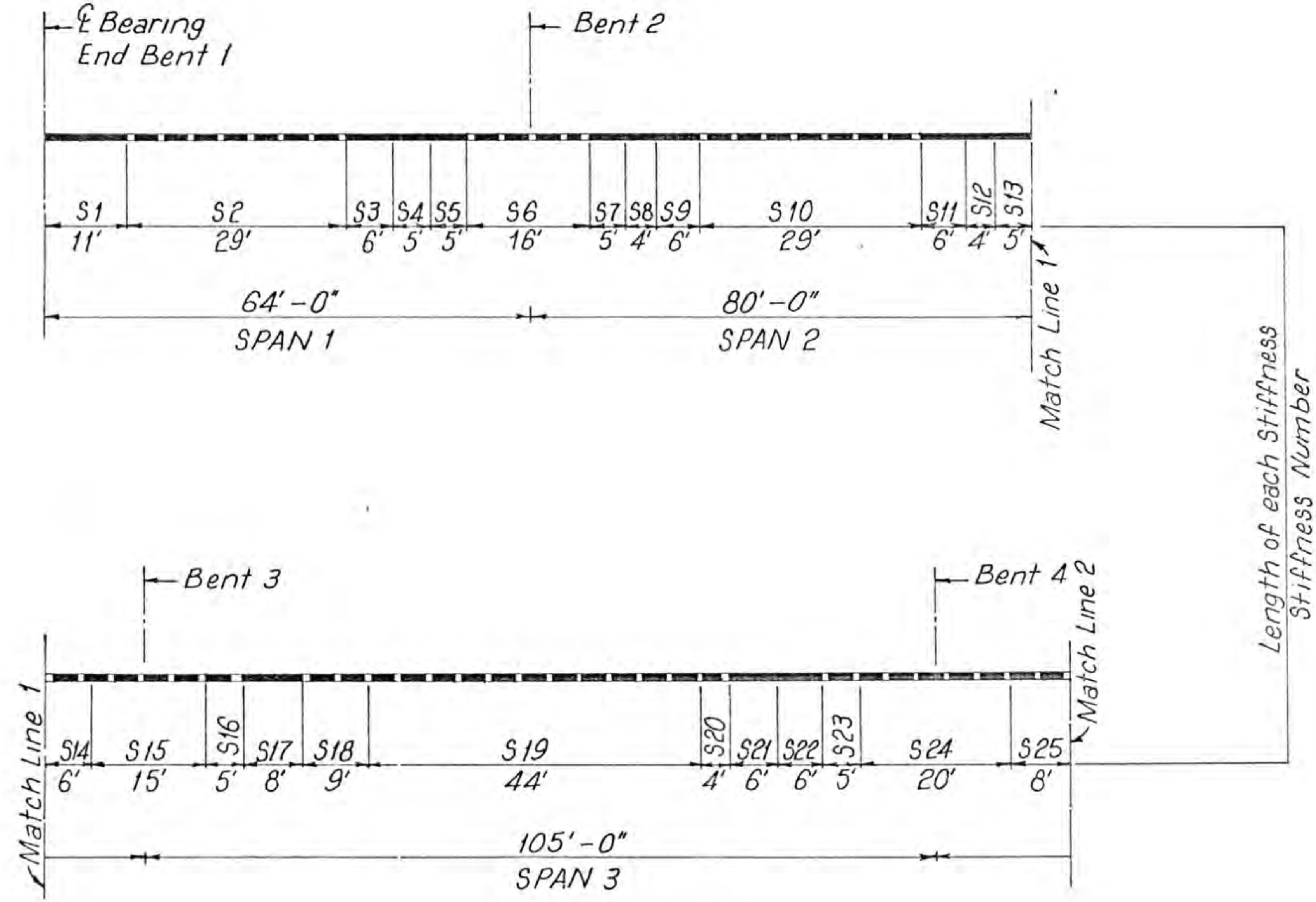
FIG. 2.8

TABLE 2.2

VARIATION IN REINFORCEMENT

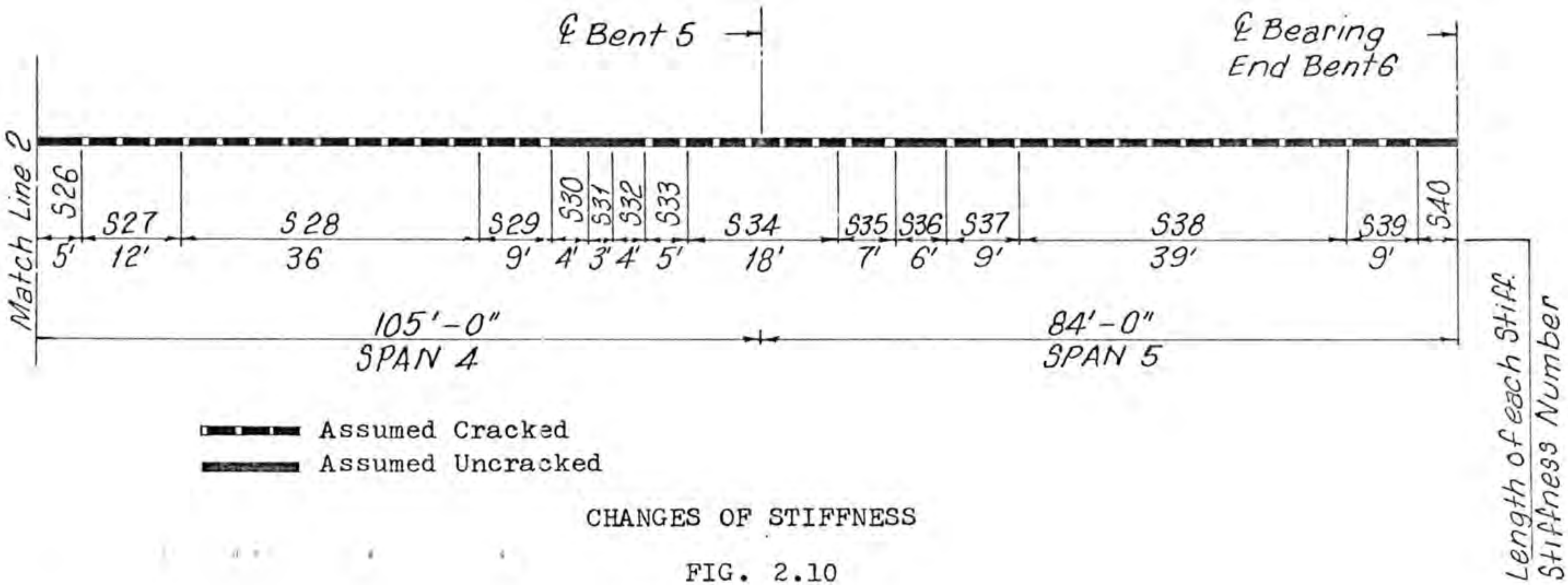
			Reinforcement in 1/2 of Box Girder Section		
S.	Cr.* or UCr.*	Location of Tension	Reinforcement in Bottom Slab - B	Reinforcement in Top of Bridge Deck - C	Total Steel Area in^2
1	UCr.	Bot.	23#10 + 11.5#6	27#8	75.435
2	Cr.	Bot.	23#10 + 22.5#6	27#8	80.275
4	UCr.	Top.	23#7 + 11.5#6	27#8	60.025
5	UCr.	Top.	23#7	27#10	67.925
6	Cr.	Top.	23#7	40.5#10	85.070
8	UCr.	Top.	23#7 + 11.5#8	27#10	77.010
9	UCr.	Bot.	34.5#8	27#8	68.420
10	Cr.	Bot.	45.5#8	27#8	77.110
12	UCr.	Top.	34.5#8	27#8 + 13.5#11	89.480
13	UCr.	Top.	23#8	27#8 + 13.5#11	80.395
14	Cr.	Top.	23#8	40.5#11	101.185
15	Cr.	Top.	23#8	53.5#11	121.465
17	UCr.	Top.	23#8 + 11.5#11	27#8 + 13.5#11	98.335
18	Cr.	Bot.	34.5#11	27#8	94.985
19	Cr.	Bot.	45.5#11	27#8	112.145
21	UCr.	Bot.	23#8 + 11.5#11	27#8	77.275
26	UCr.	Top.	23#8 + 11.5#10	27#8	95.000
27	Cr.	Bot.	34.5#10	27#8	84.980
28	Cr.	Bot.	45.5#10	27#8	98.950
30	UCr.	Bot.	23#8 + 11.5#10	27#8	73.940
40	UCr.	Bot.	23#7 + 11.5#10	27#8	69.570

*Cr. = Cracked Section
UCr. = Uncracked Section



CHANGES OF STIFFNESS

FIG. 2.9



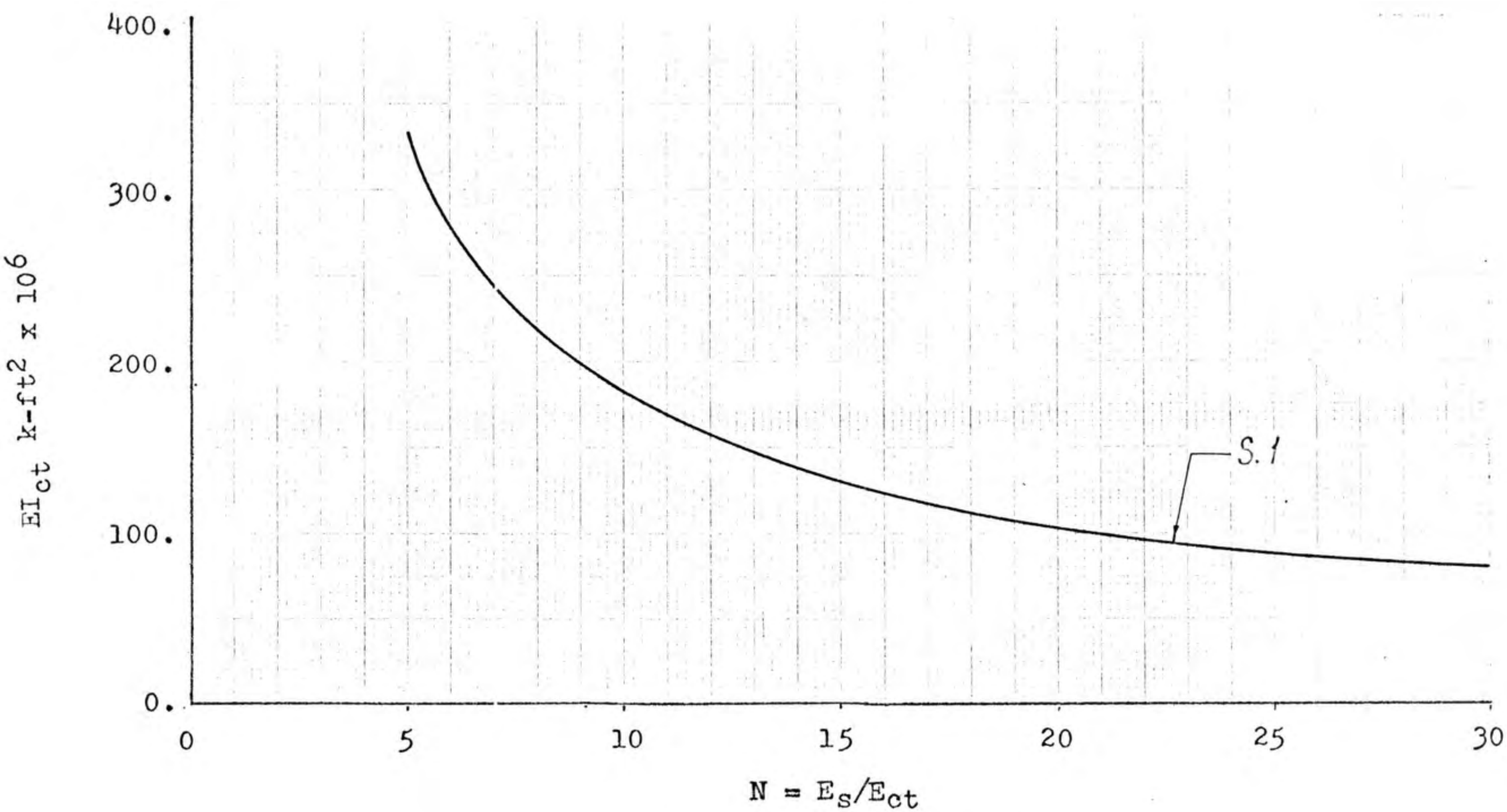
CHANGES OF STIFFNESS

FIG. 2.10

TABLE 2.3

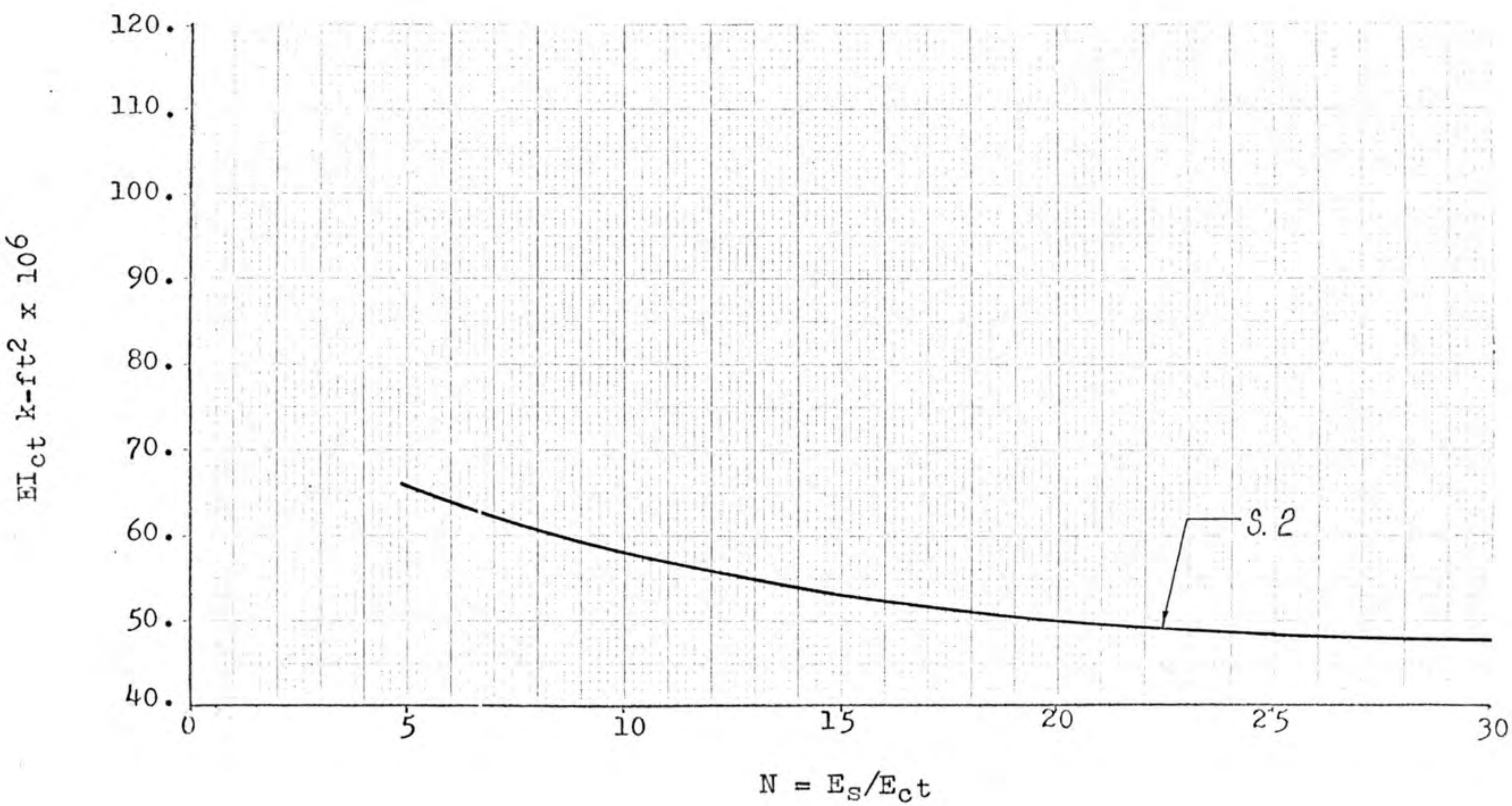
SIMILAR STIFFNESSES

S3 = S1	S23 = S14	S33 = S14
S7 = S5	S24 = S15	S34 = S15
S11 = S9	S25 = S14	S35 = S14
S16 = S14	S29 = S27	S36 = S26
S20 = S18	S31 = S26	S37 = S27
S22 = S17	S32 = S13	S39 = S27



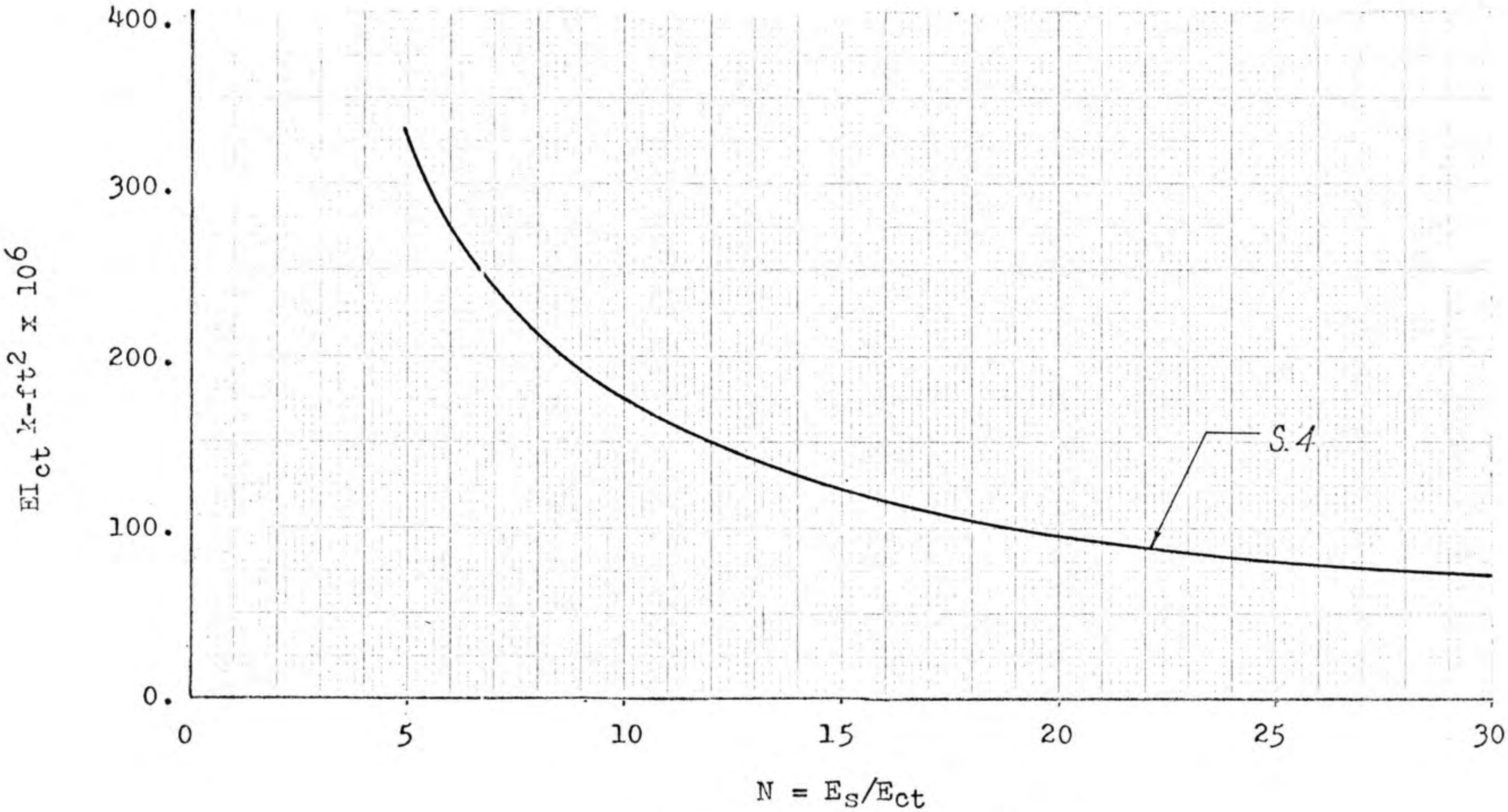
EFFECTIVE SECTION RIGIDITY
UNCRACKED SECTION

FIG. 2.11



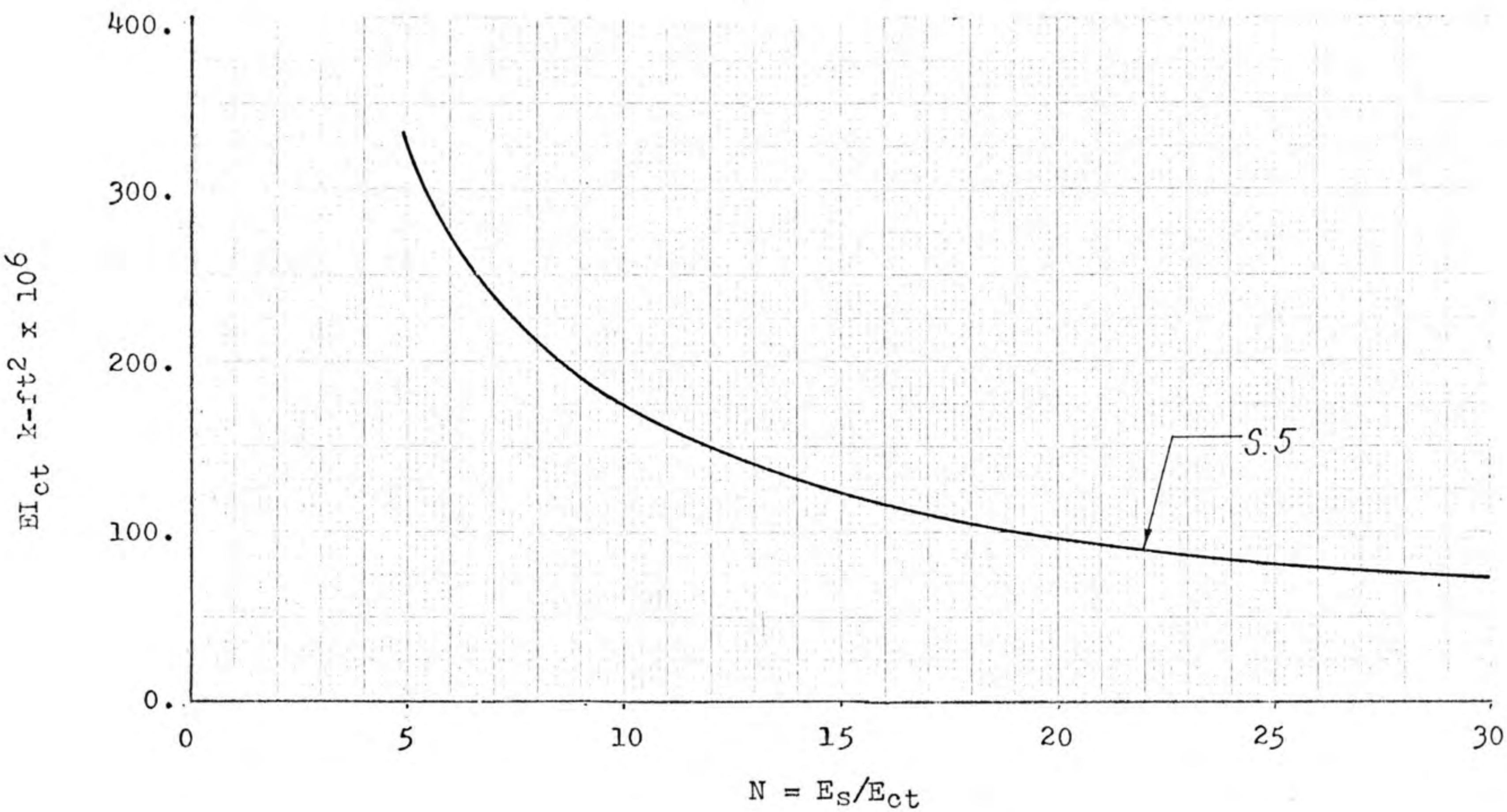
EFFECTIVE SECTION RIGIDITY
CRACKED SECTION

FIG. 2.12



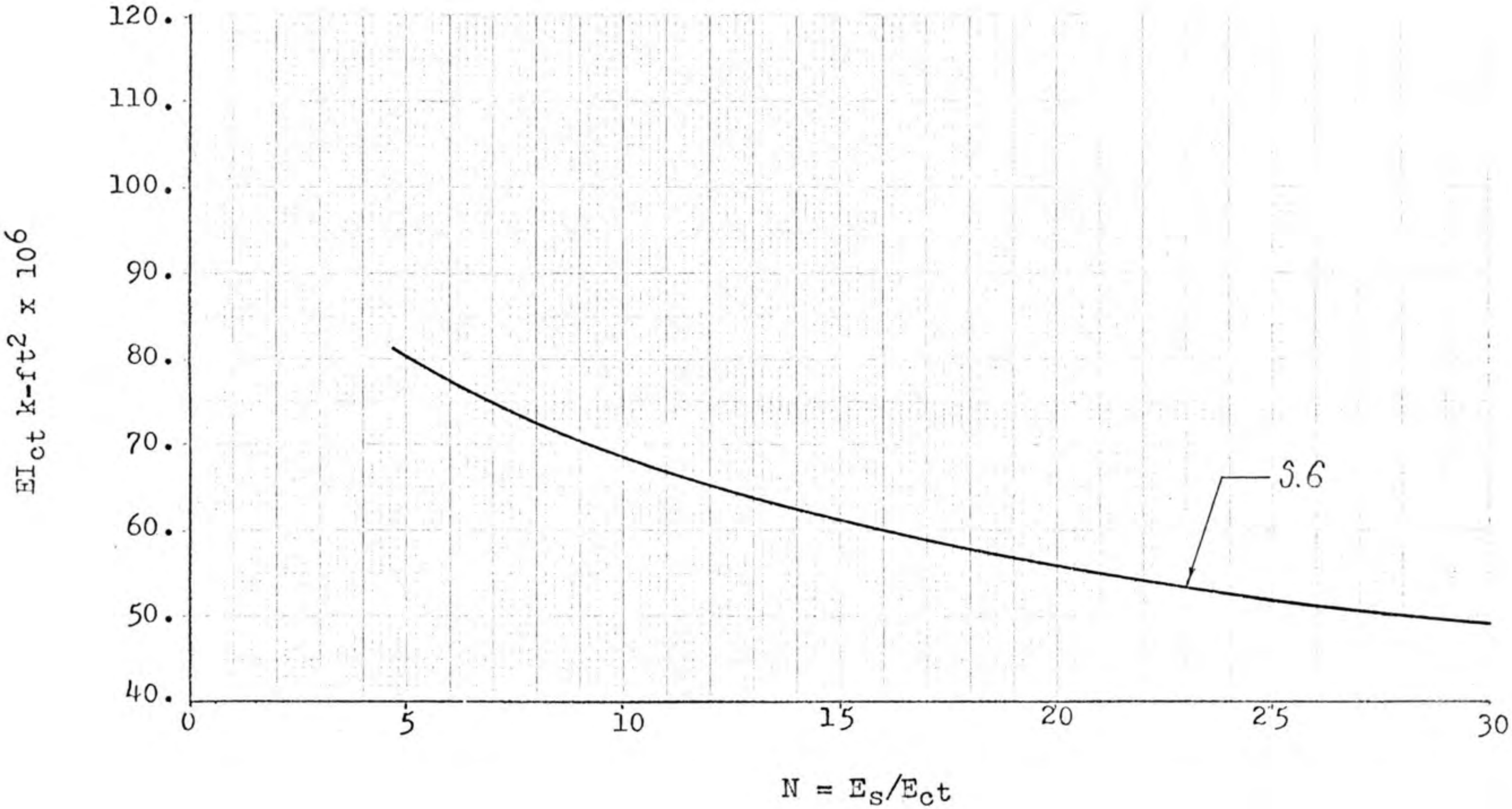
EFFECTIVE SECTION RIGIDITY
UNCRACKED SECTION

FIG. 2.13



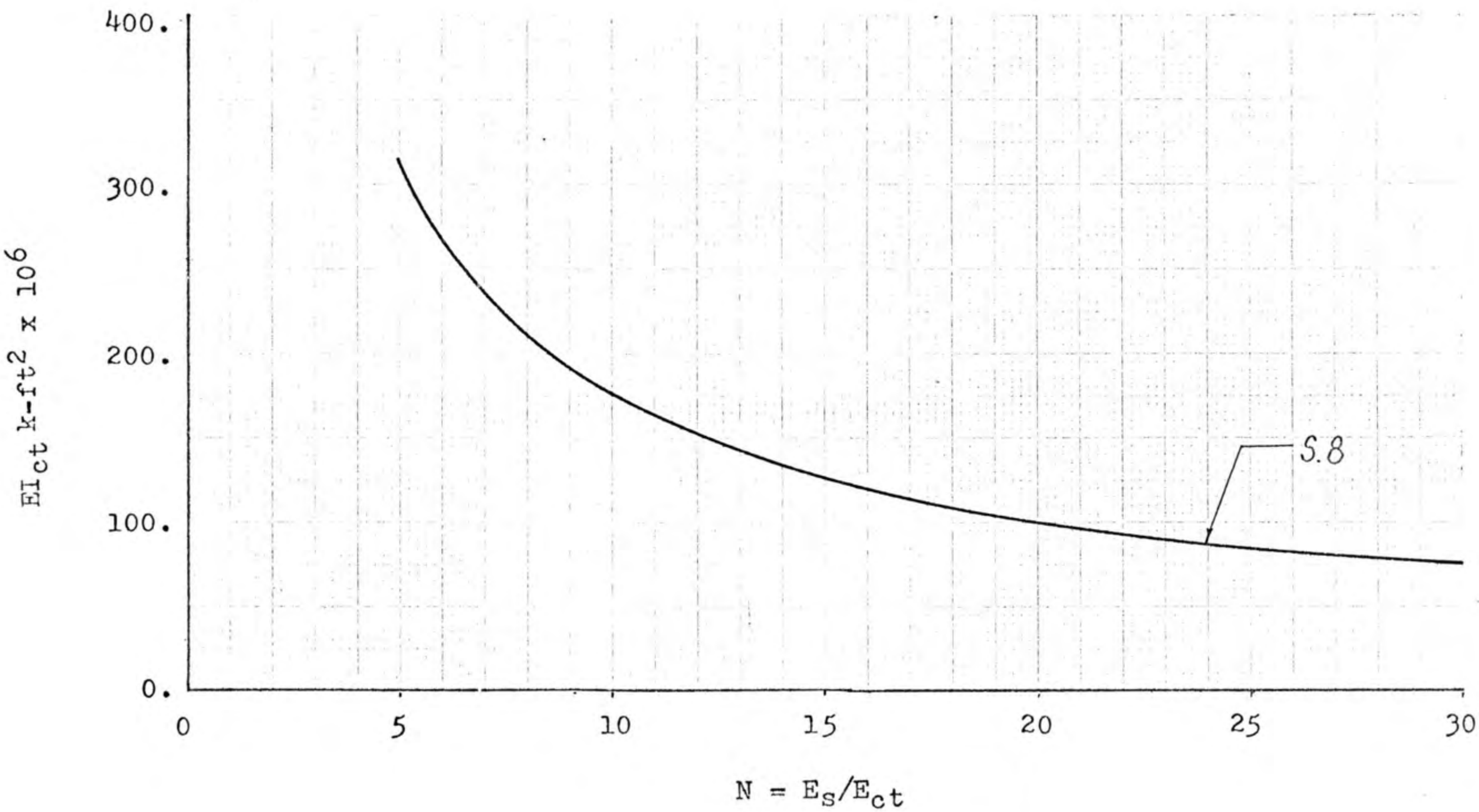
EFFECTIVE SECTION RIGIDITY
UNCRACKED SECTION

FIG. 2.14



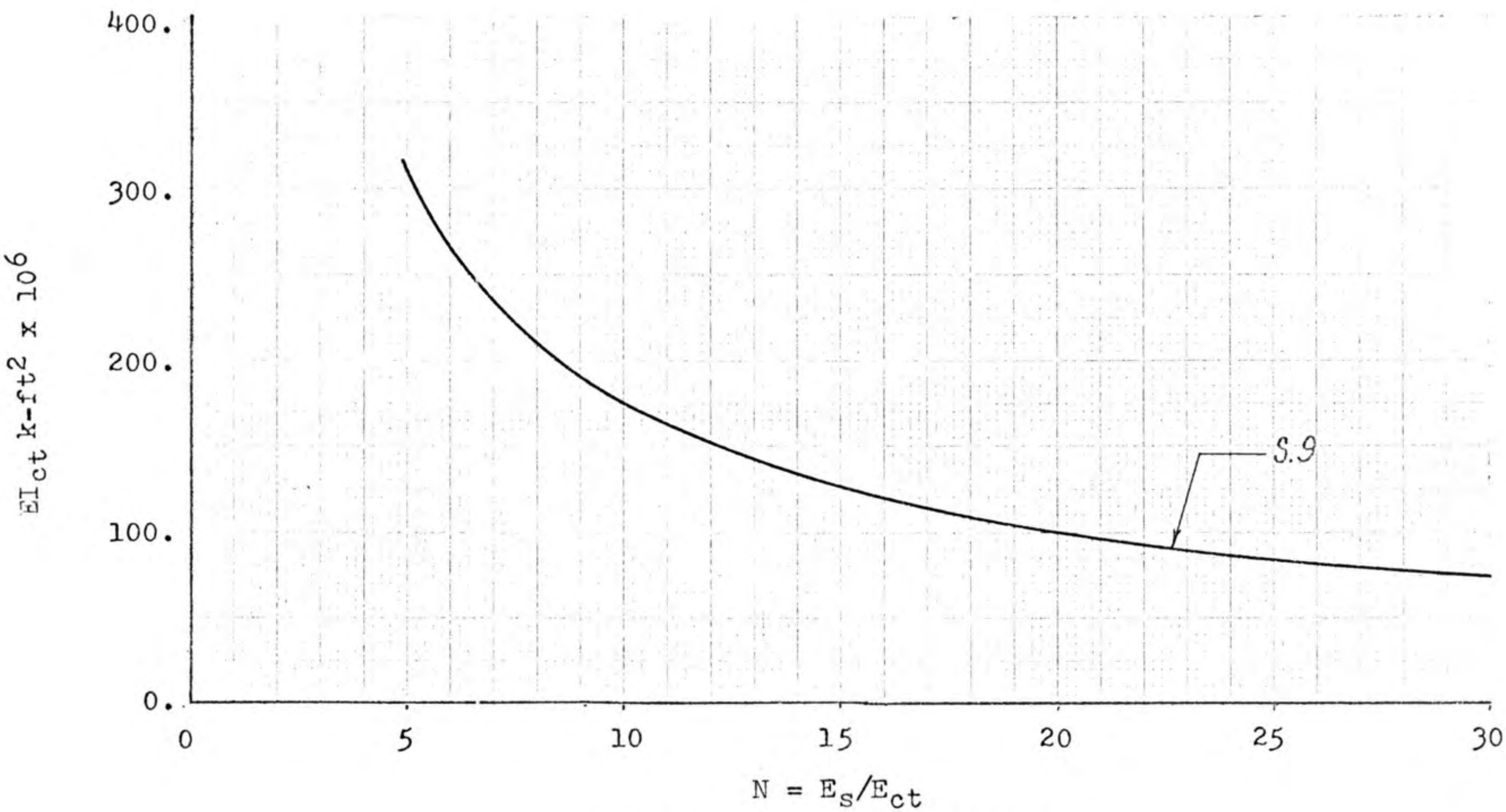
EFFECTIVE SECTION RIGIDITY
CRACKED SECTION

FIG. 2.15



EFFECTIVE SECTION RIGIDITY
UNCRACKED SECTION

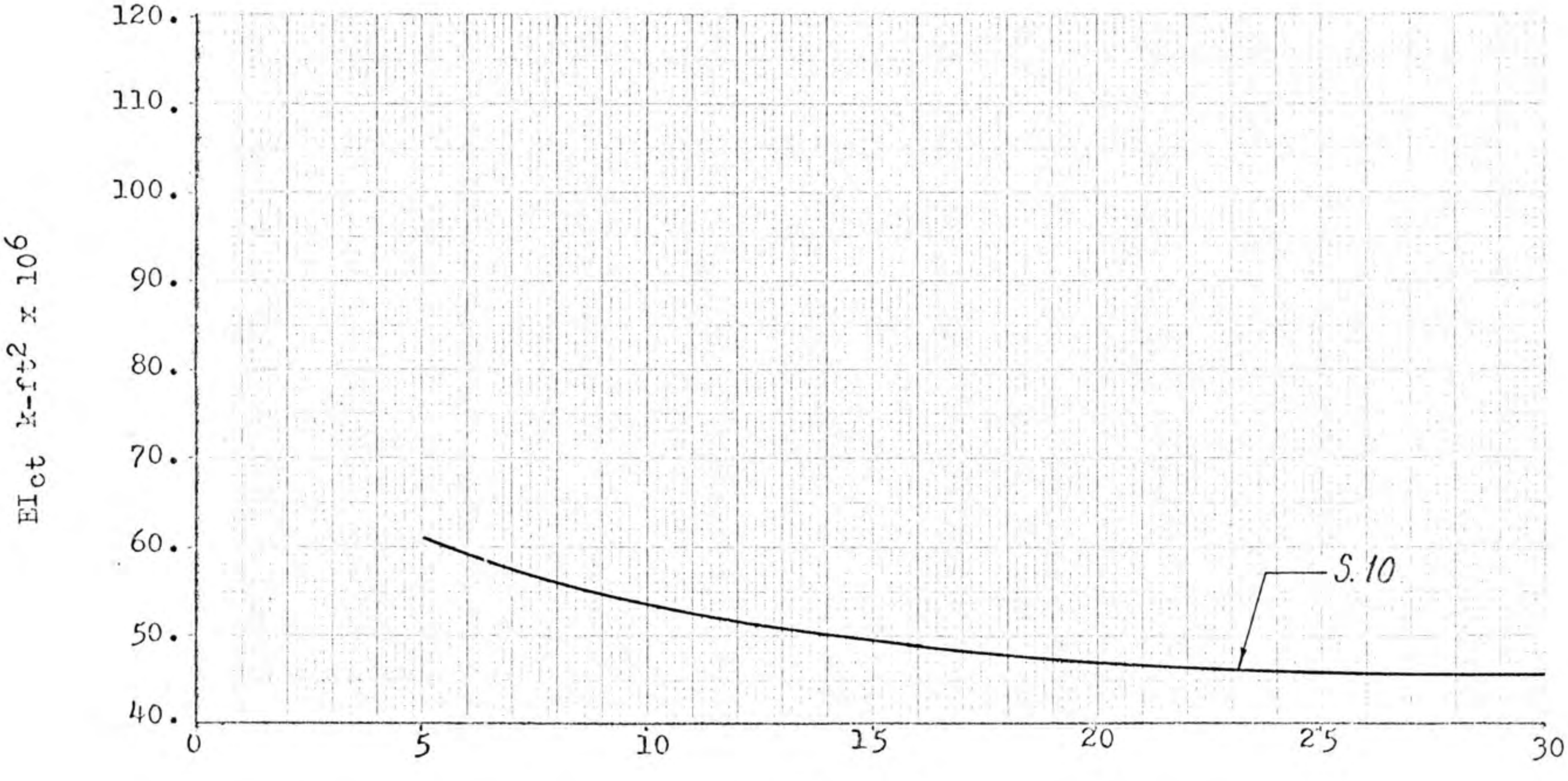
FIG. 2.16



EFFECTIVE SECTION RIGIDITY

UNCRACKED SECTION

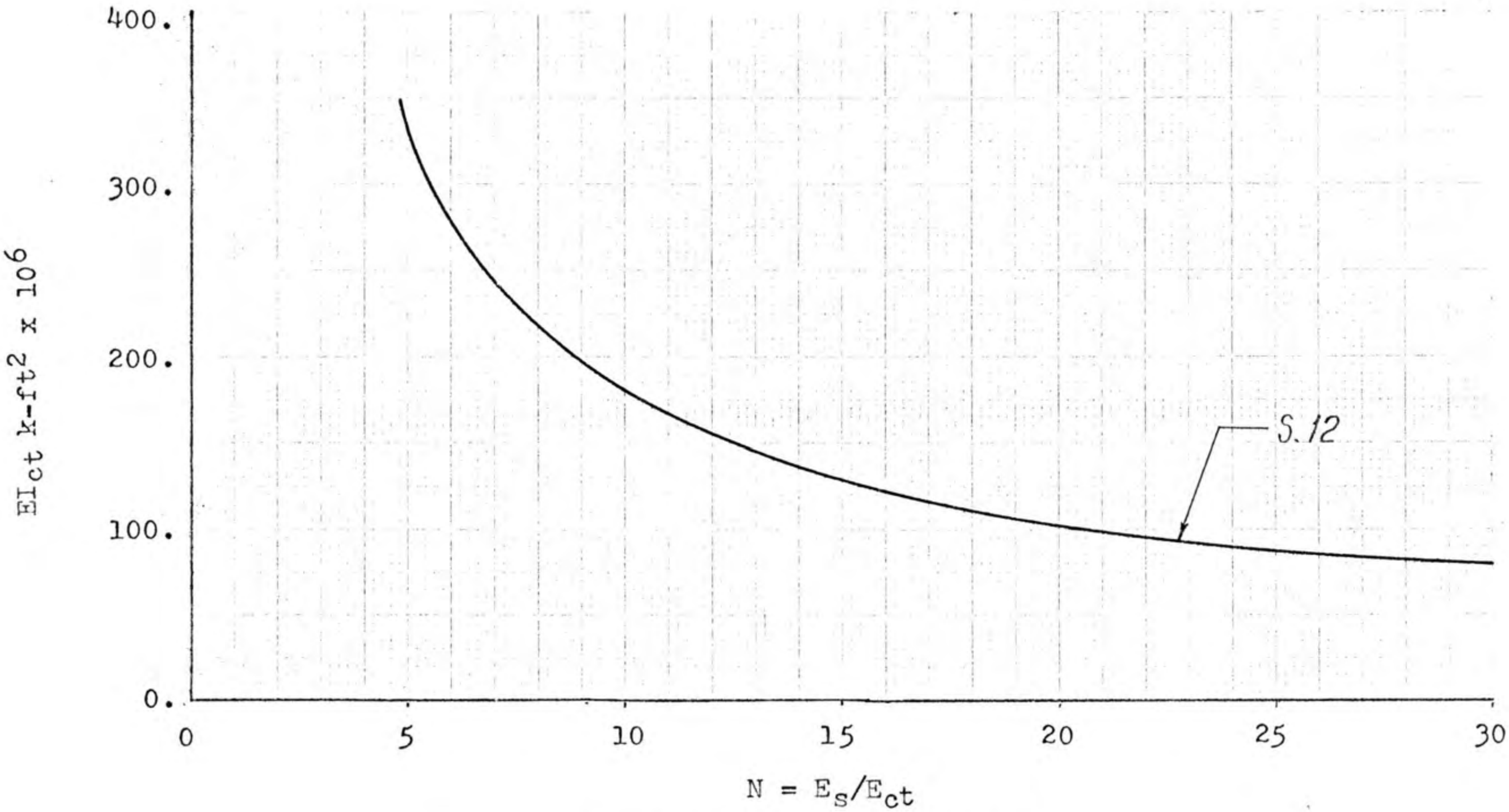
FIG. 2.17



EFFECTIVE SECTION RIGIDITY

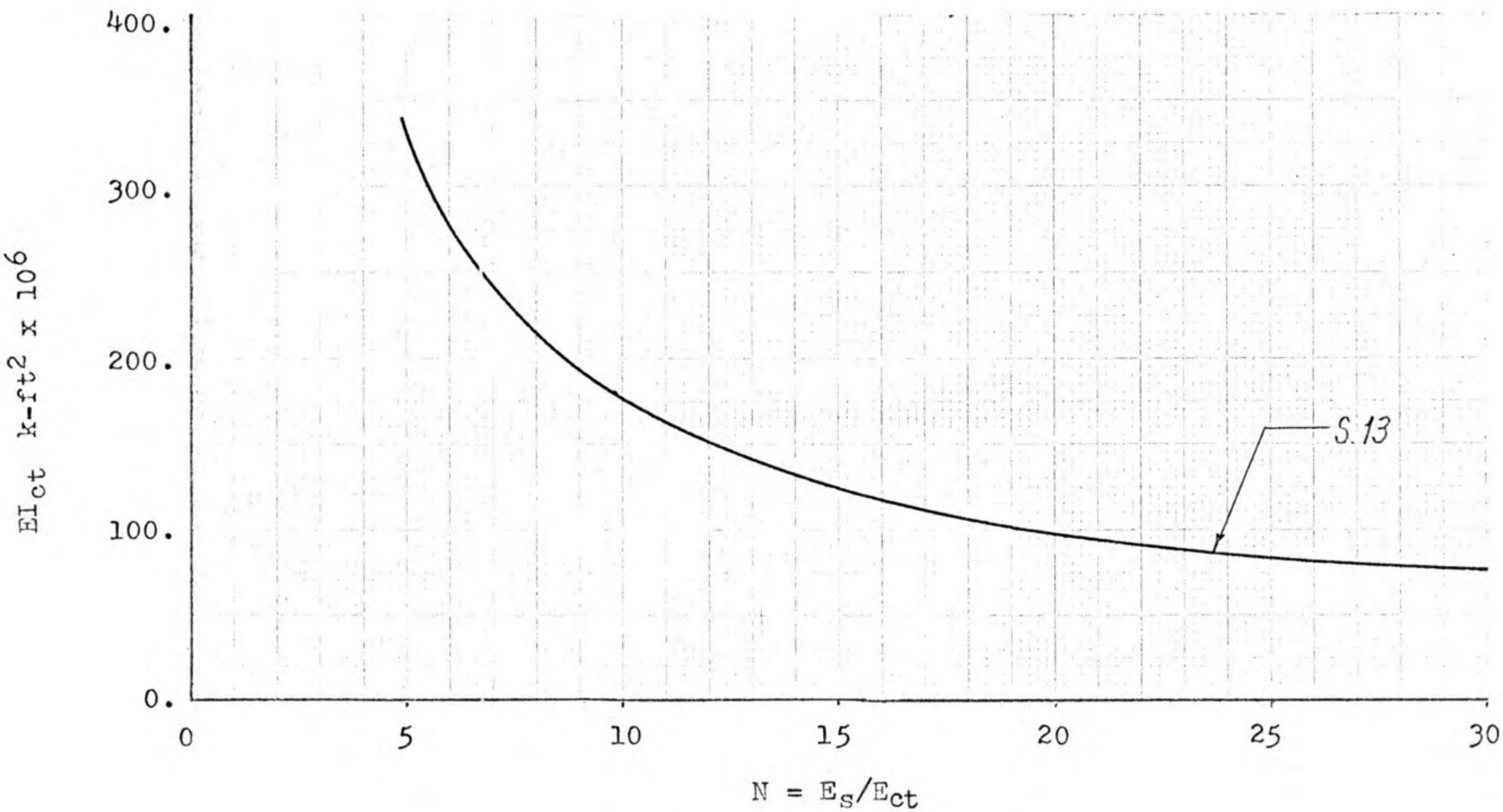
CRACKED SECTION

FIG. 2.18



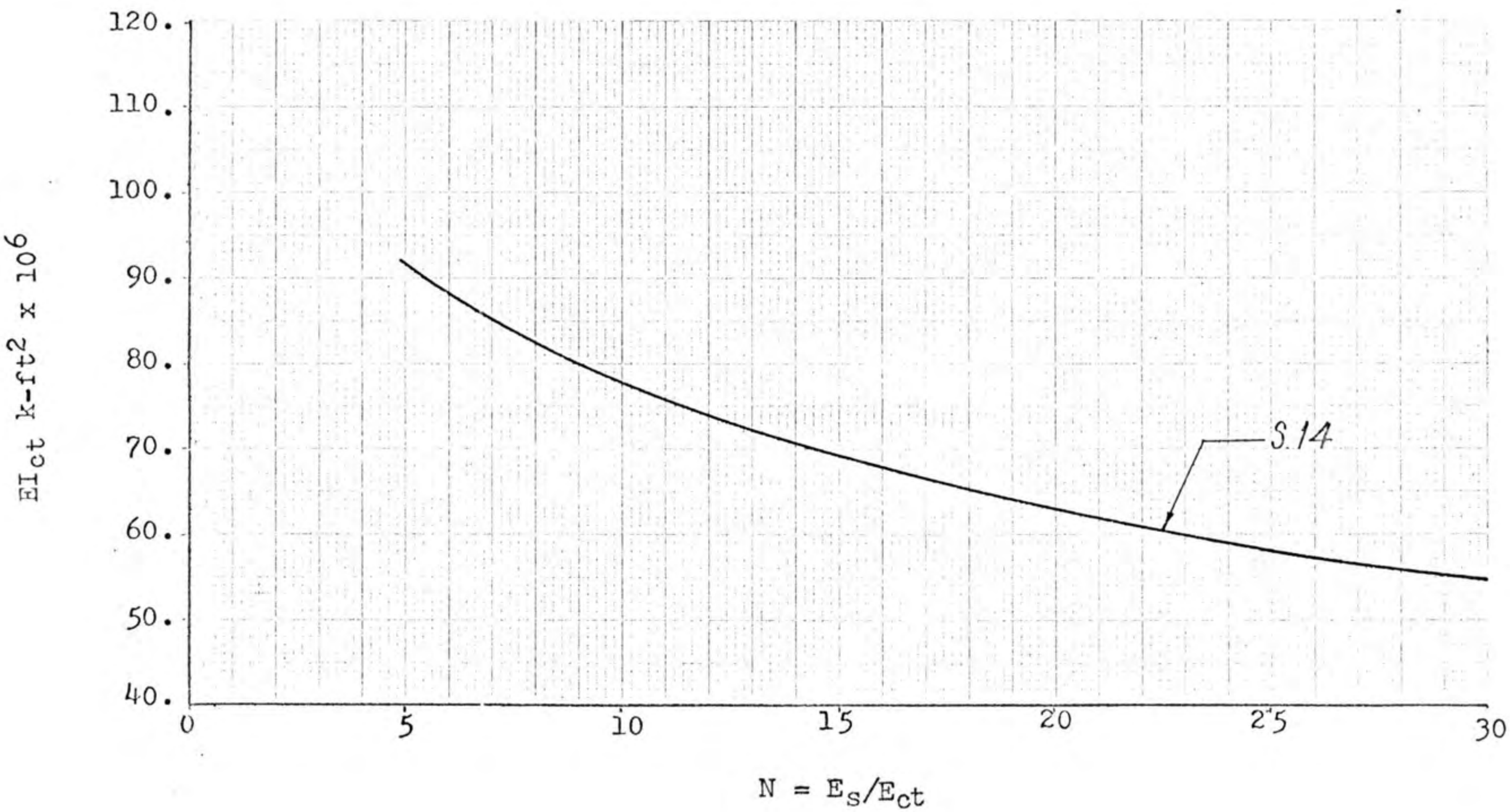
EFFECTIVE SECTION RIGIDITY
UNCRACKED SECTION

FIG. 2.19



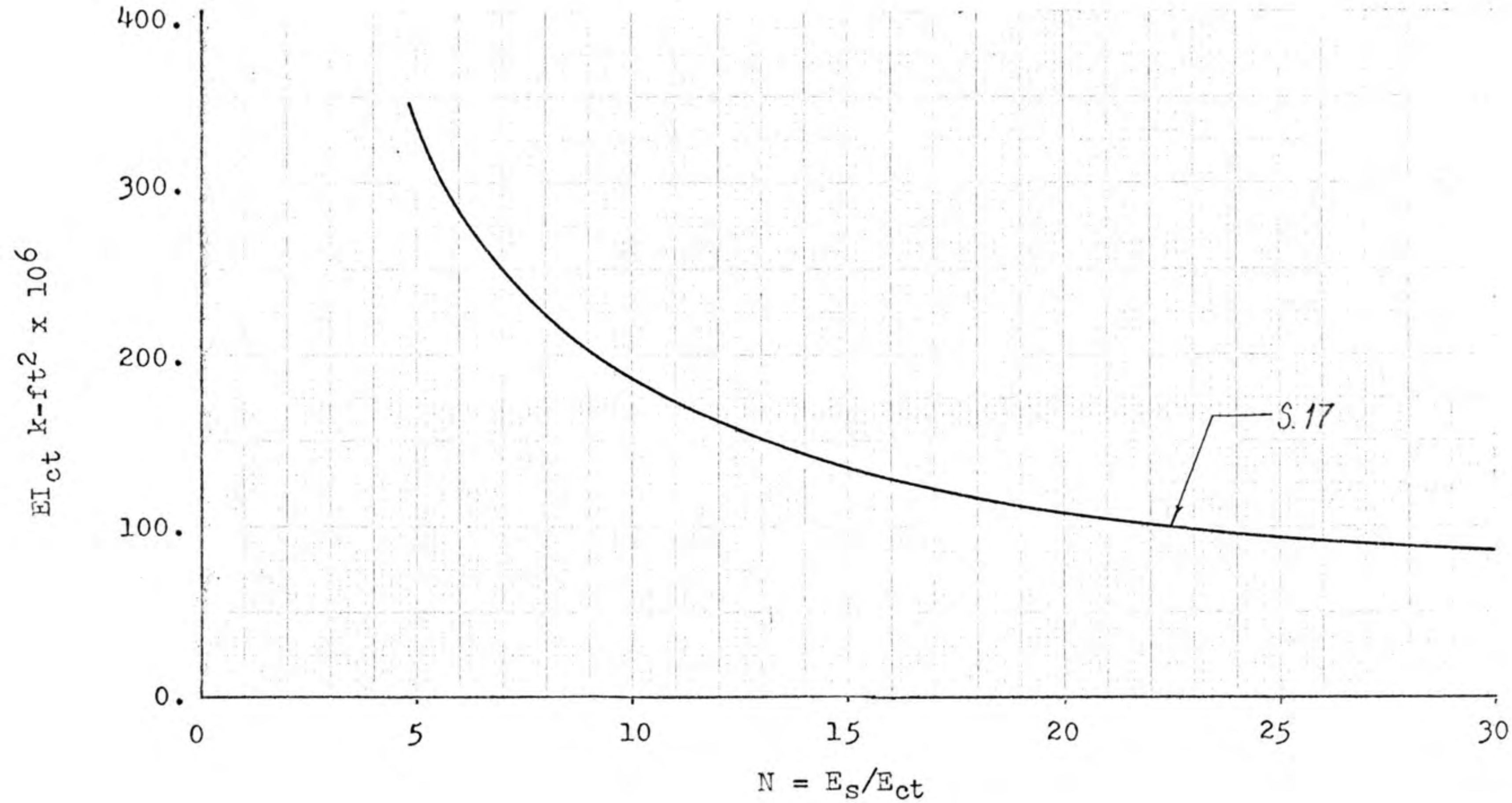
EFFECTIVE SECTION RIGIDITY
UNCRACKED SECTION

FIG. 2.20



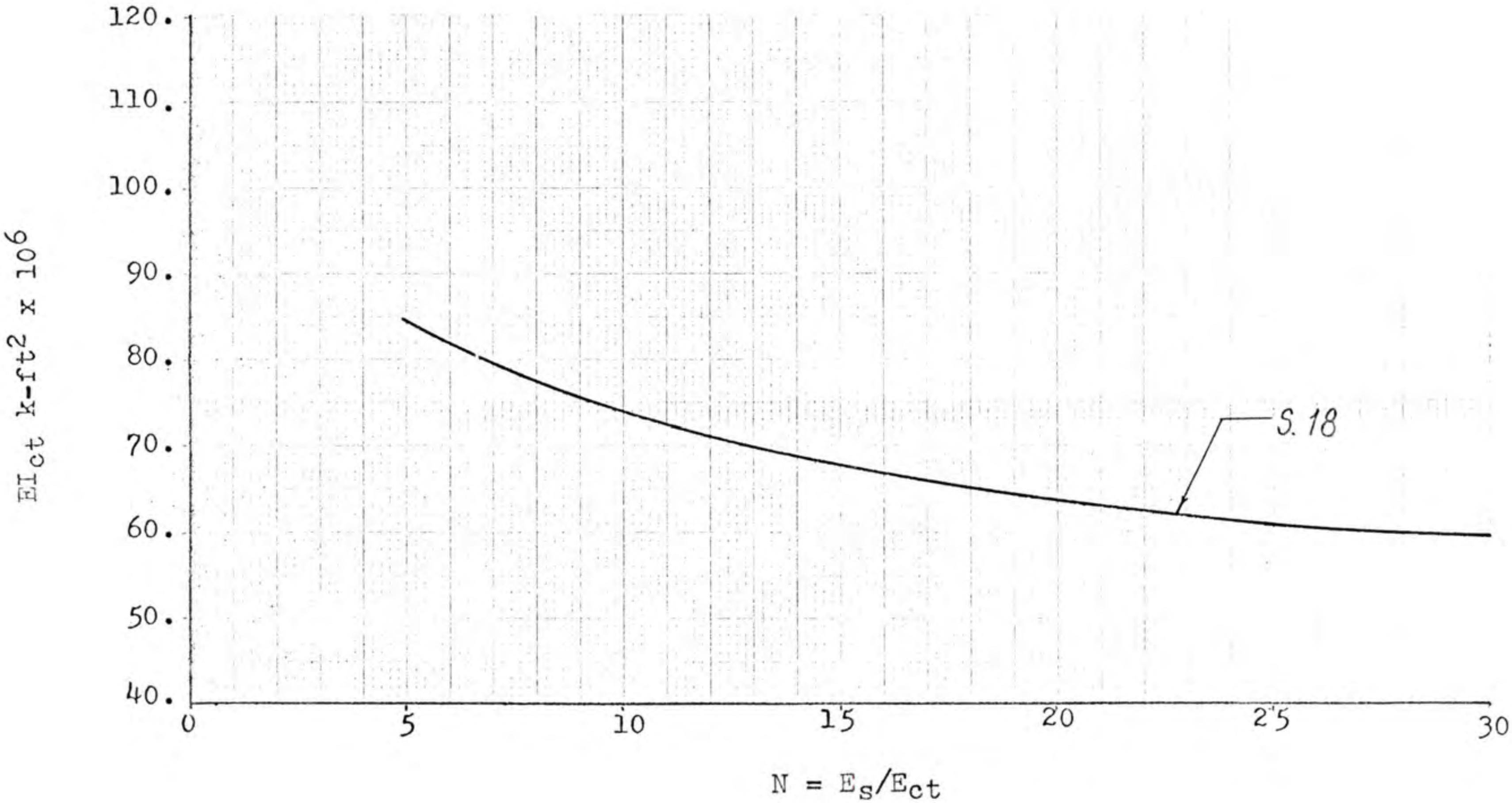
EFFECTIVE SECTION RIGIDITY
CRACKED SECTION

FIG. 2.21



EFFECTIVE SECTION RIGIDITY
UNCRACKED SECTION

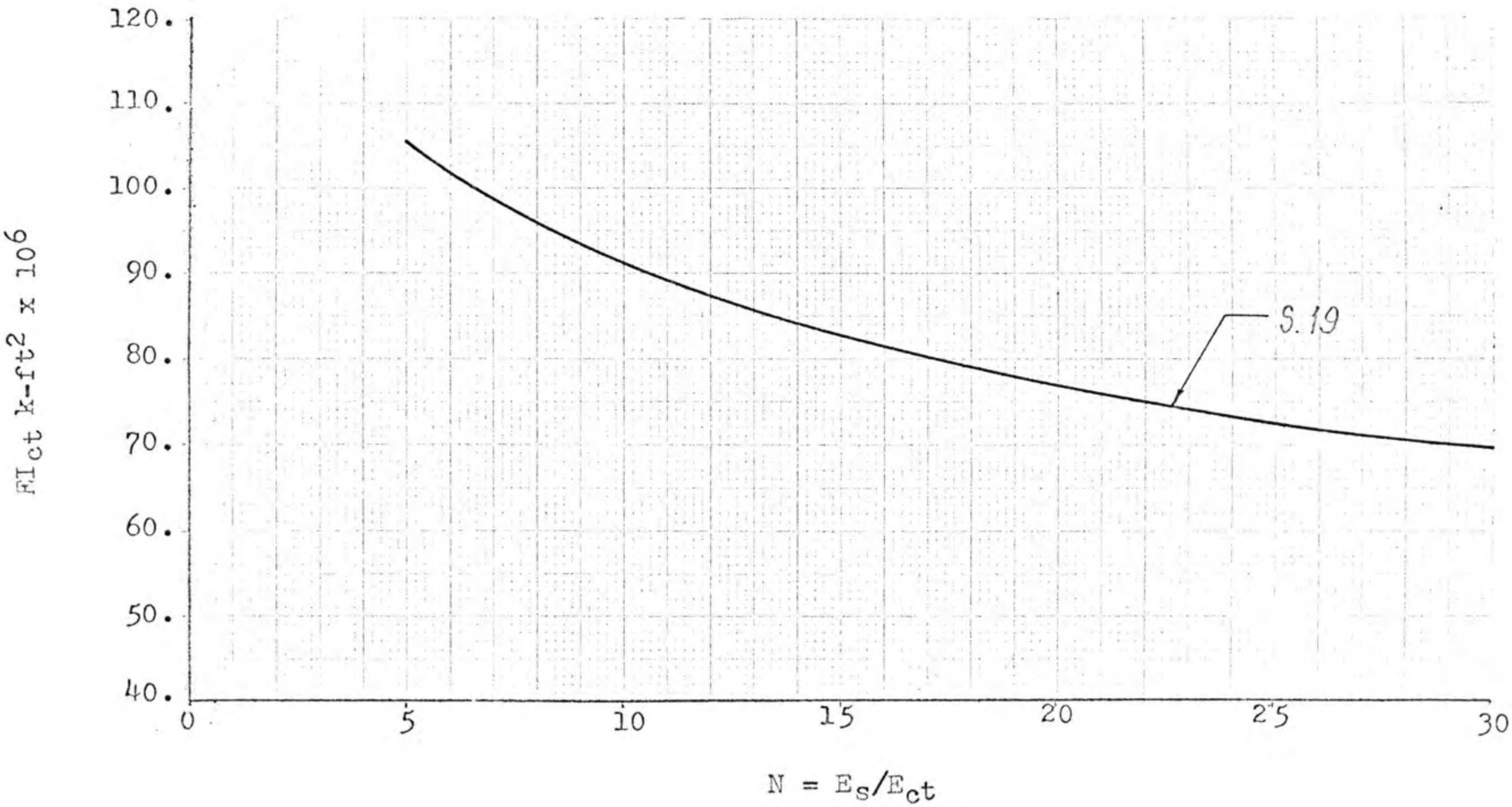
FIG. 2.22



EFFECTIVE SECTION RIGIDITY

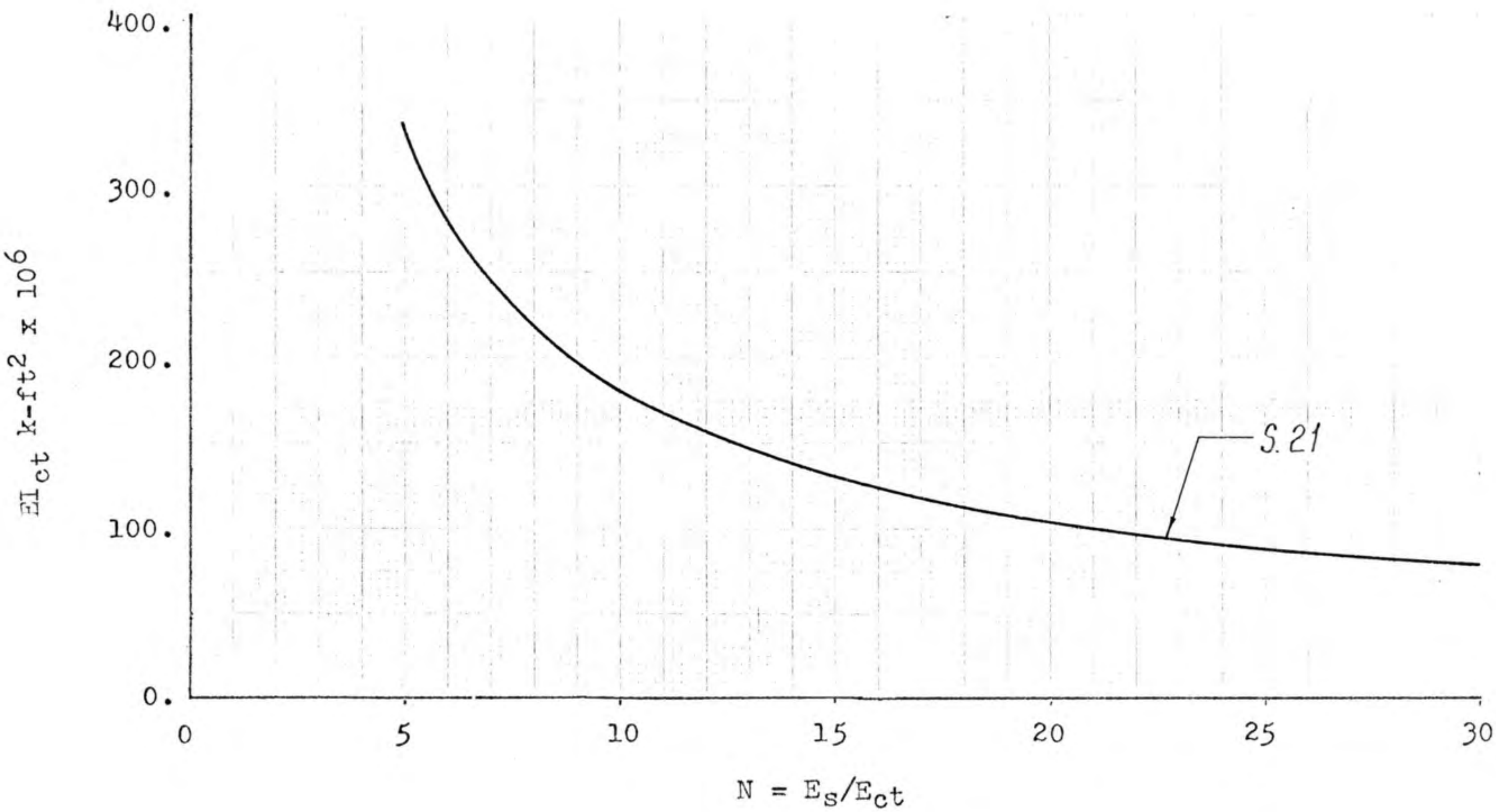
CRACKED SECTION

FIG. 2.23



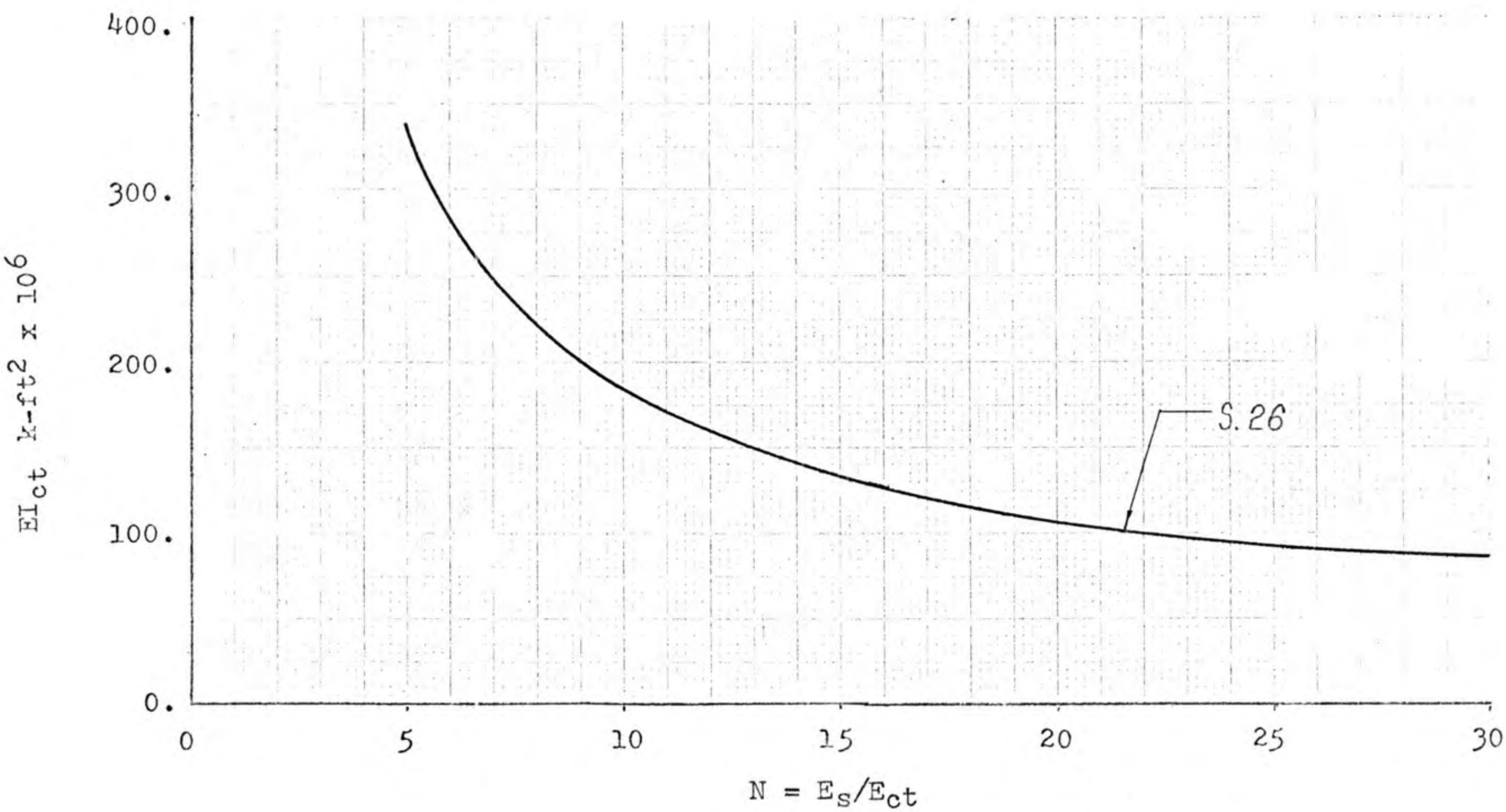
EFFECTIVE SECTION RIGIDITY
CRACKED SECTION

FIG. 2.24



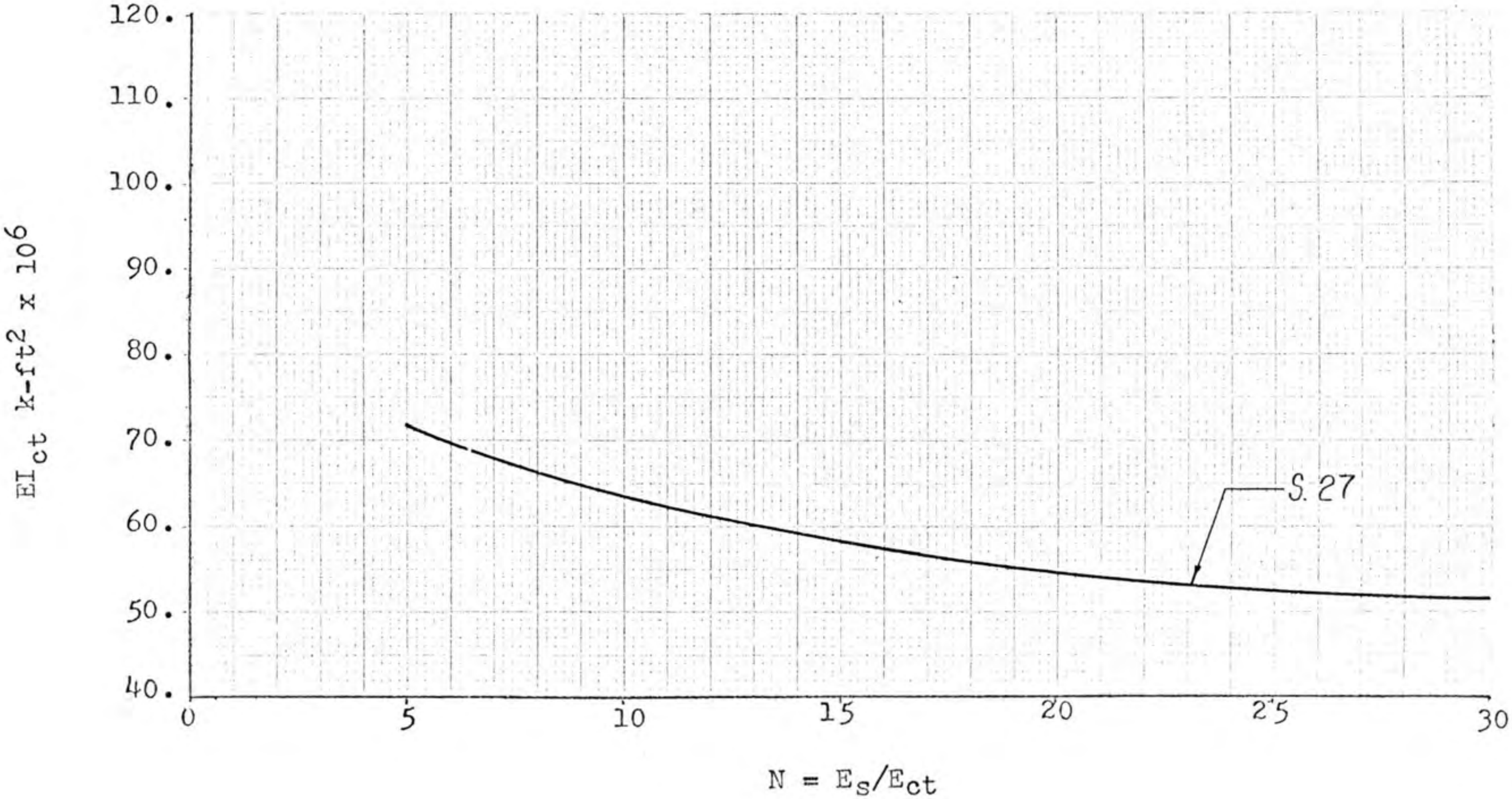
EFFECTIVE SECTION RIGIDITY
UNCRACKED SECTION

FIG. 2.25



EFFECTIVE SECTION RIGIDITY
UNCRACKED SECTION

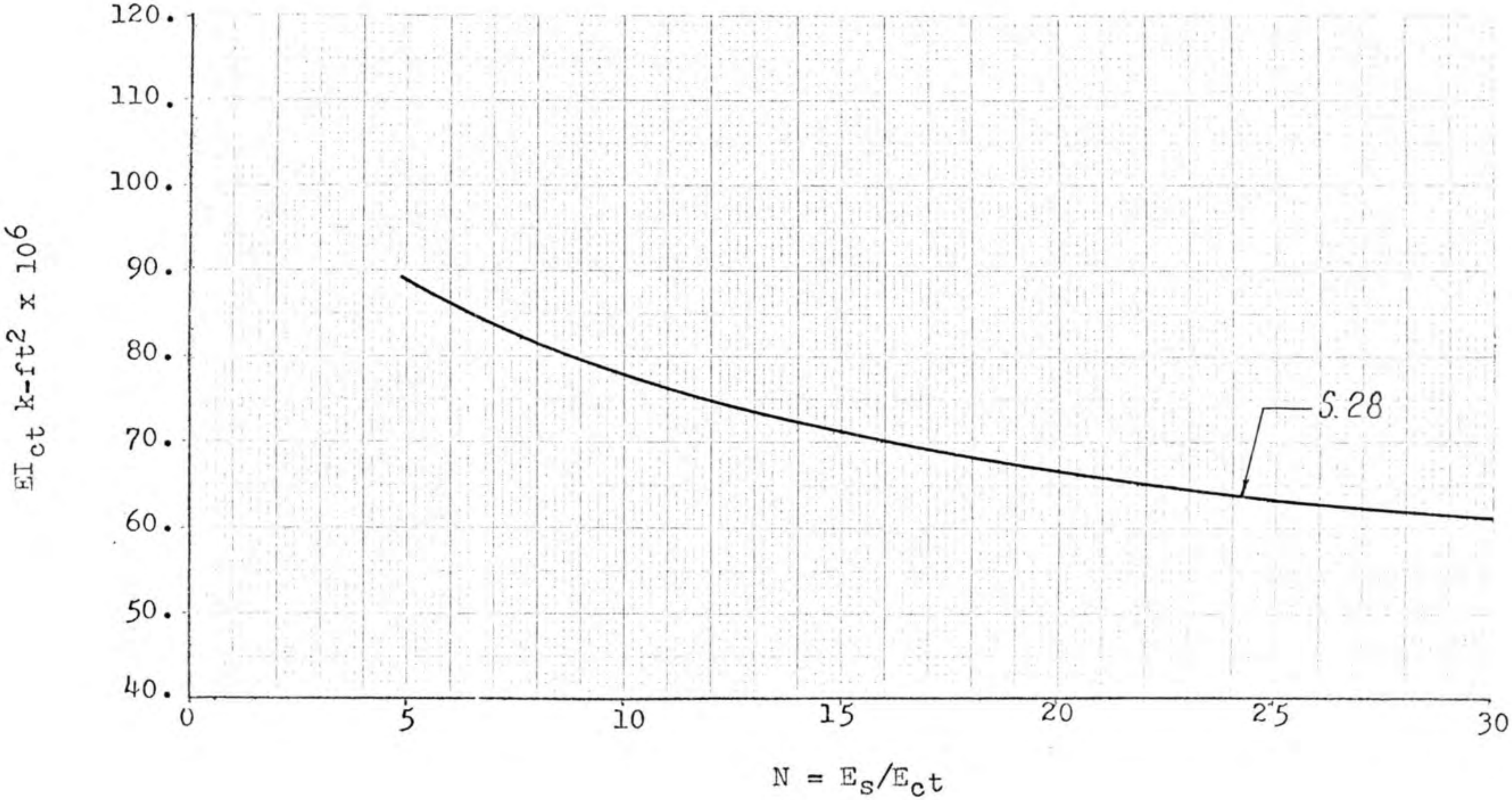
FIG. 2.26



EFFECTIVE SECTION RIGIDITY

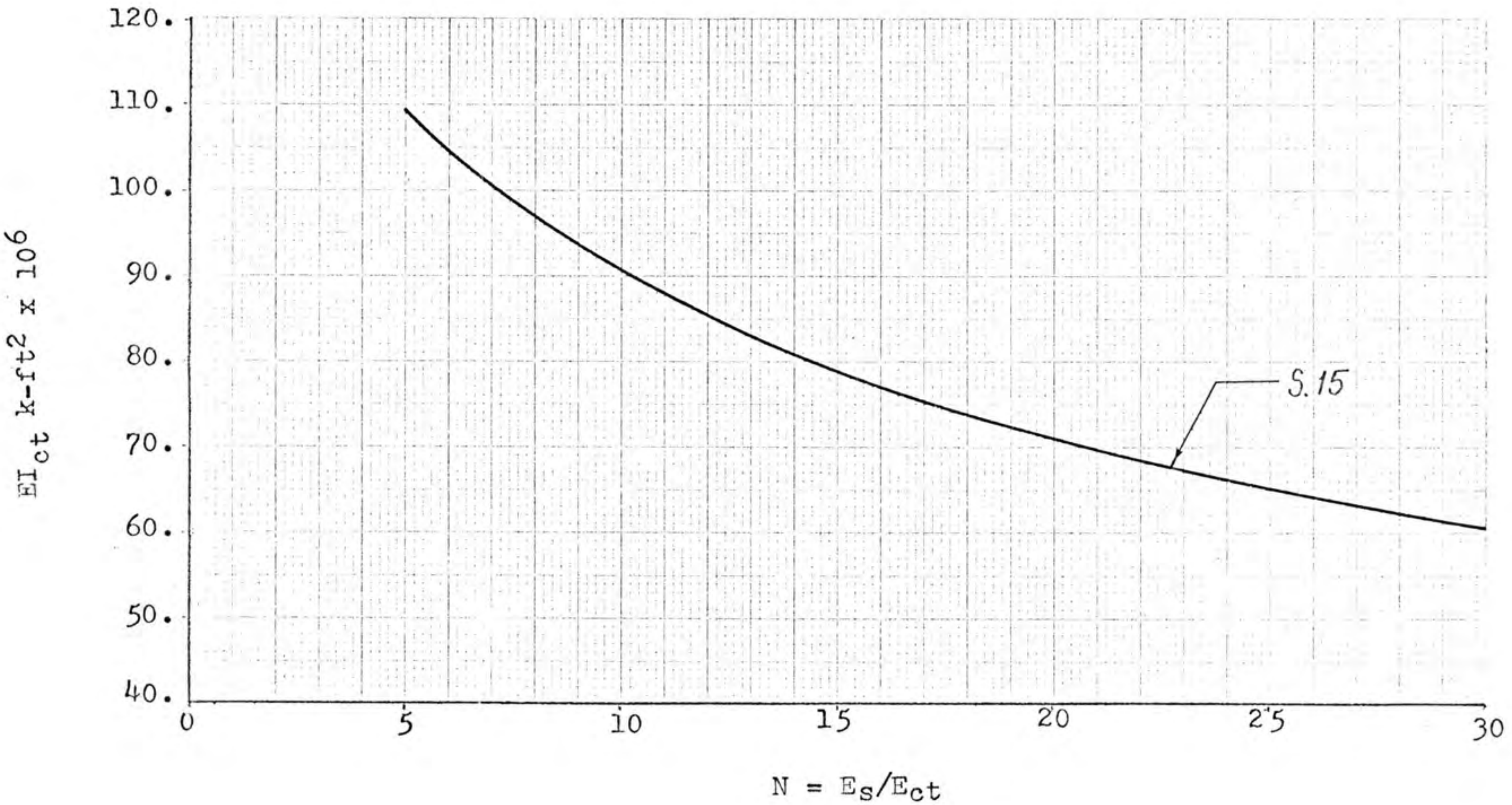
CRACKED SECTION

FIG. 2.27



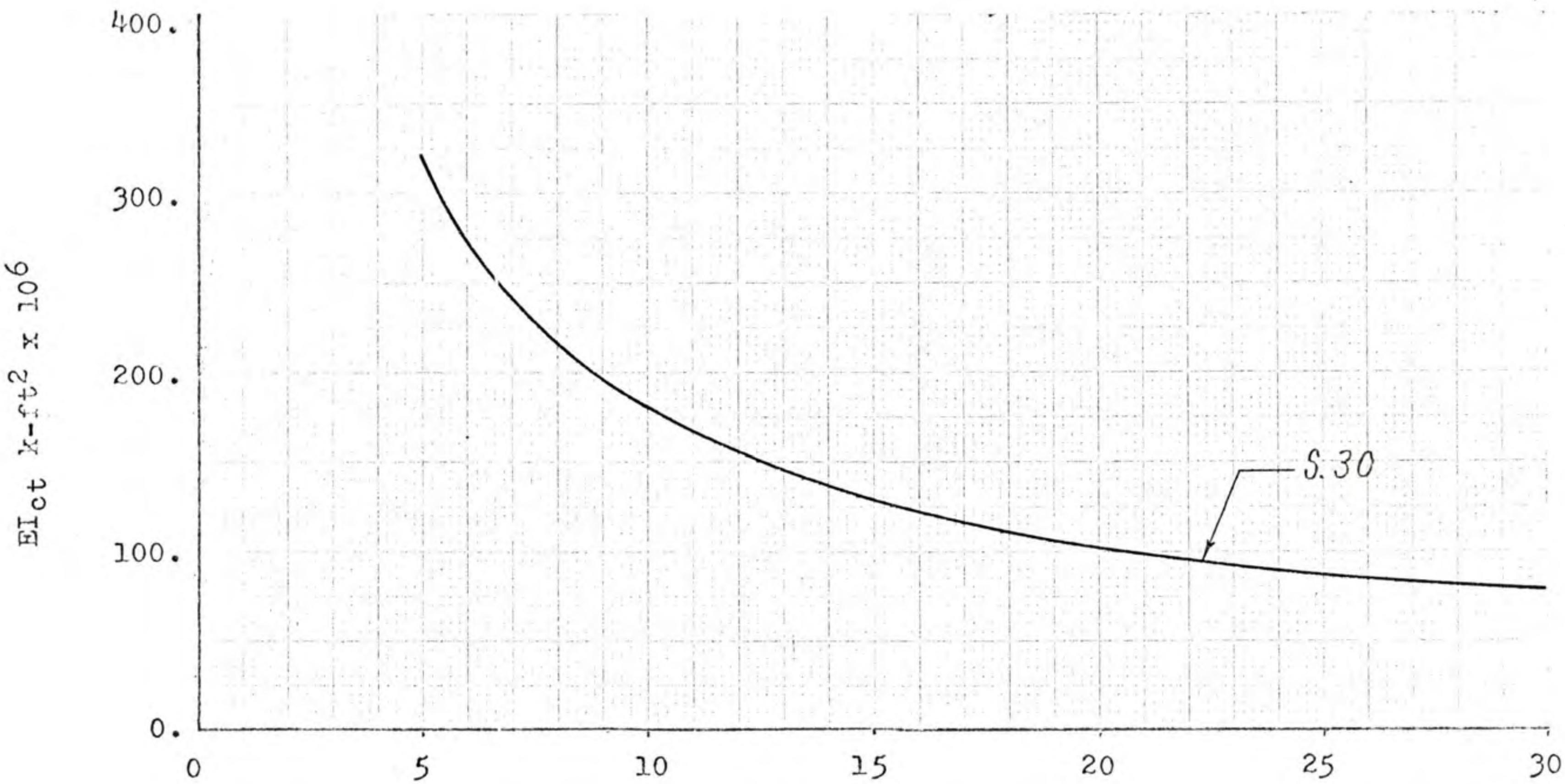
EFFECTIVE SECTION RIGIDITY
CRACKED SECTION

FIG. 2.28



EFFECTIVE SECTION RIGIDITY
CRACKED SECTION

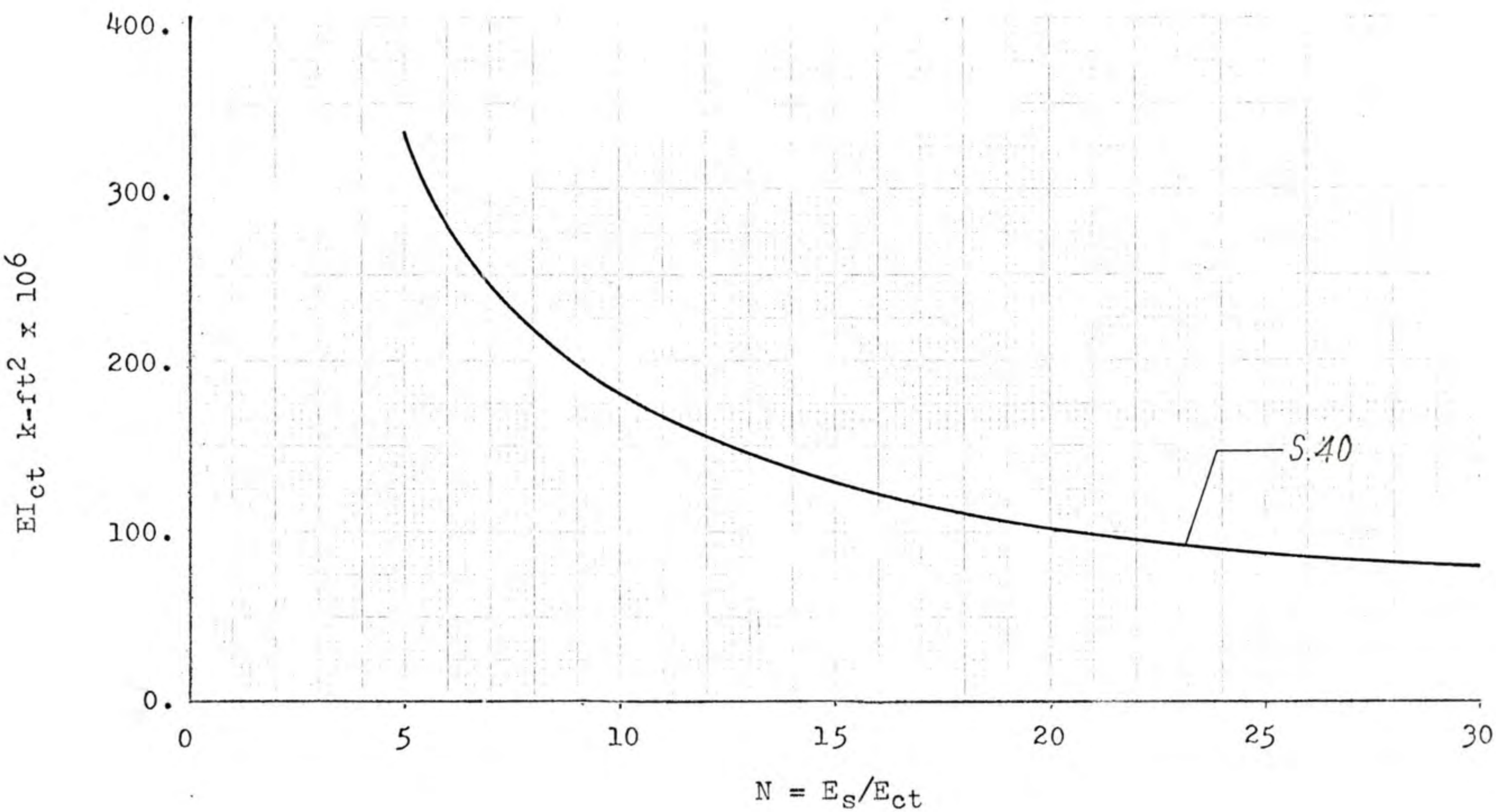
FIG. 2.29



$N = E_s/E_{ct}$

EFFECTIVE SECTION RIGIDITY
UNCRACKED SECTION

FIG. 2.30



EFFECTIVE SECTION RIGIDITY
UNCRACKED SECTION

FIG. 2.31

TABLE 2.4

SECTION PROPERTIES

N = 5

$E_c = E_s/N = 5800.$ ksi

S.	Cr.* or UCr.*	EI_{ct} k-ft^2 x 10^6	e In	T_o k	$T_o e$ k-ft
1	UCr.	332.20	5.36	876.	390.84
2	Cr.	65.23	28.25	932.	2192.50
4	UCr.	323.90	-2.07	696.	-120.00
5	UCr.	323.40	-8.64	788.	-567.16
6	Cr.	79.99	-37.45	986.	-3079.50
8	UCr.	328.30	-3.59	894.	-267.00
9	UCr.	328.30	2.38	794.	157.66
10	Cr.	60.95	27.45	894.	2046.16
12	UCr.	332.20	-3.47	1038.	-300.00
13	UCr.	327.30	-7.68	932.	-596.84
14	Cr.	91.20	-35.41	1174.	-3464.00
15	Cr.	109.10	-35.37	1408.	-4142.34
17	UCr.	337.00	-.13	1140.	-12.34
18	Cr.	84.25	31.13	1102.	2858.00
19	Cr.	105.00	33.21	1300.	3600.34
21	UCr.	333.10	6.01	896.	448.50
26	UCr.	335.20	-1.31	1102.	-120.34
27	Cr.	71.37	29.28	986.	2405.16
28	Cr.	88.98	31.69	1148.	3031.34
30	UCr.	331.40	4.75	858.	339.34
40	UCr.	329.00	2.92	808.	196.66

*Cr. = Cracked Section
UCr. = Uncracked Section

TABLE 2.5

SECTION PROPERTIES

N = 10

$E_c = E_s/N = 2900.$ ksi

S.	Cr.* or UCr.*	EI_{ct} k-ft^2 x 10^6	e In	T_o k	T_oe k-ft
1	UCr.	180.50	5.21	876.	379.66
2	Cr.	57.80	24.53	932.	1903.50
4	UCr.	172.10	-1.77	696.	102.50
5	UCr.	171.70	-7.94	788.	-521.34
6	Cr.	68.96	-31.17	986.	-2563.00
8	UCr.	176.80	-3.16	894.	-235.34
9	UCr.	176.60	2.44	794.	161.16
10	Cr.	53.93	23.85	894.	1778.16
12	UCr.	180.80	-3.02	1038.	-261.50
13	UCr.	175.80	-6.97	932.	-541.50
14	Cr.	77.76	-28.84	1174.	-2821.16
15	Cr.	90.82	-28.18	1408.	-3318.54
17	UCr.	185.70	.07	1140.	6.16
18	Cr.	74.06	26.76	1102.	2456.66
19	Cr.	91.26	28.08	1300.	3044.34
21	UCr.	181.40	5.80	896.	523.50
26	UCr.	183.90	-1.02	1102.	-94.00
27	Cr.	63.47	25.39	986.	2085.84
28	Cr.	78.02	27.12	1148.	2594.34
30	UCr.	179.60	4.64	858.	331.66
40	UCr.	177.30	2.94	808.	197.84

*Cr. = Cracked Section
UCr. = Uncracked Section

TABLE 2.6

SECTION PROPERTIES

N = 15

$E_c = E_s/N = 1933.$ ksi

S.	Cr.* or UCr.*	EI_{ct} $k\text{-}ft^2 \times 10^6$	e In	T_o k	T_oe k-ft
1	UCr.	129.30	4.89	876.	356.66
2	Cr.	53.43	21.86	932.	1696.16
4	UCr.	121.00	-1.68	696.	-97.34
5	UCr.	120.60	-7.50	788.	-492.34
6	Cr.	61.55	-26.87	986.	-2210.00
8	UCr.	125.70	-2.97	894.	-220.84
9	UCr.	125.40	2.30	794.	152.16
10	Cr.	50.49	21.31	894.	1588.50
12	UCr.	129.80	-2.81	1038.	-243.34
13	UCr.	124.70	-6.52	932.	-506.84
14	Cr.	69.04	-24.51	1174.	-2397.00
15	Cr.	79.36	-23.60	1408.	-2770.50
17	UCr.	134.80	.06	1140.	5.84
18	Cr.	68.07	23.55	1102.	2162.34
19	Cr.	83.00	24.41	1300.	2645.66
21	UCr.	130.20	5.44	896.	406.50
26	UCr.	132.90	-.95	1102.	-87.16
27	Cr.	58.32	22.53	986.	1851.00
28	Cr.	71.53	23.80	1148.	2276.34
30	UCr.	128.40	4.36	858.	311.84
40	UCr.	126.20	2.77	808.	186.50

*Cr. = Cracked Section
UCr. = Uncracked Section

TABLE 2.7

SECTION PROPERTIES

N = 20

$E_c = E_s/N = 1450.$ ksi

S.	Cr.* or UCr.*	EI_{ct} k-ft^2 x 10^6	e In	T_o k	$T_o e$ k-ft
1	UCr.	103.70	4.61	876.	336.16
2	Cr.	50.49	19.74	932.	1531.50
4	UCr.	95.43	-1.60	696.	-92.66
5	UCr.	94.96	-7.10	788.	-466.34
6	Cr.	56.16	-23.70	986.	-1949.00
8	UCr.	100.20	-2.79	894.	-208.00
9	UCr.	99.87	2.18	794.	144.16
10	Cr.	47.35	19.29	894.	1437.66
12	UCr.	104.30	-2.63	1038.	-227.34
13	UCr.	99.09	-6.13	932.	-476.34
14	Cr.	62.85	-21.37	1174.	-2090.50
15	Cr.	71.37	-20.37	1408.	-2391.34
17	UCr.	109.30	.06	1140.	5.34
18	Cr.	63.80	21.07	1102.	1934.16
19	Cr.	77.08	21.62	1300.	2354.16
21	UCr.	104.50	5.13	896.	382.84
26	UCr.	107.40	-.88	1102.	-81.16
27	Cr.	54.96	20.28	986.	1666.16
28	Cr.	66.89	21.24	1148.	2031.50
30	UCr.	102.80	4.12	858.	294.34
40	UCr.	100.60	2.63	808.	176.50

*Cr. = Cracked Section
UCr. = Uncracked Section

TABLE 2.8

SECTION PROPERTIES

N = 25

$E_c = E_s/N = 1160.$ ksi

S.	Cr.* or UCr.*	EI_{ct} k-$ft^2 \times 10^6$	e In	T_o k°	T_oe k-ft
1	UCr.	88.30	4.36	876.	318.00
2	Cr.	48.23	18.01	932.	1397.50
4	UCr.	80.09	-1.52	696.	-88.34
5	UCr.	79.57	-6.75	788.	-442.84
6	Cr.	51.99	-21.24	986.	-1746.50
8	UCr.	84.87	-2.64	894.	-196.50
9	UCr.	84.62	2.07	794.	136.84
10	Cr.	45.30	17.63	894.	1314.16
12	UCr.	89.03	-2.47	1038.	-213.50
13	UCr.	83.71	-5.78	932.	-449.16
14	Cr.	58.12	-18.99	1174.	-1857.16
15	Cr.	65.45	-17.95	1408	-2117.16
17	UCr.	94.00	.05	1140.	5.00
18	Cr.	60.51	19.08	1102.	1752.16
19	Cr.	72.56	19.43	1300.	2116.84
21	UCr.	89.14	4.84	896.	361.84
26	UCr.	92.15	-.83	1102.	-76.00
27	Cr.	52.38	18.46	986.	1516.50
28	Cr.	63.33	19.20	1148.	1836.34
30	UCr.	87.46	3.90	858.	278.66
40	UCr.	85.23	2.49	808.	177.50

*Cr. = Cracked Section
UCr. = Uncracked Section

CHAPTER III

DEFLECTION ANALYSIS

Analysis Requirements

To permit development of an appropriate computer analysis which would accommodate the numerous discontinuities in rigidity and warping moment due to section changes, as to provide for continuity, a direct method for integrating the elastic curve was selected, employing MacAulay's notation(7).

Dead Load Deflection

The basic loading diagram for the structure analyzed is shown in Figure 3.1. The changes in section rigidity are not shown but have been previously discussed in Chapter II.

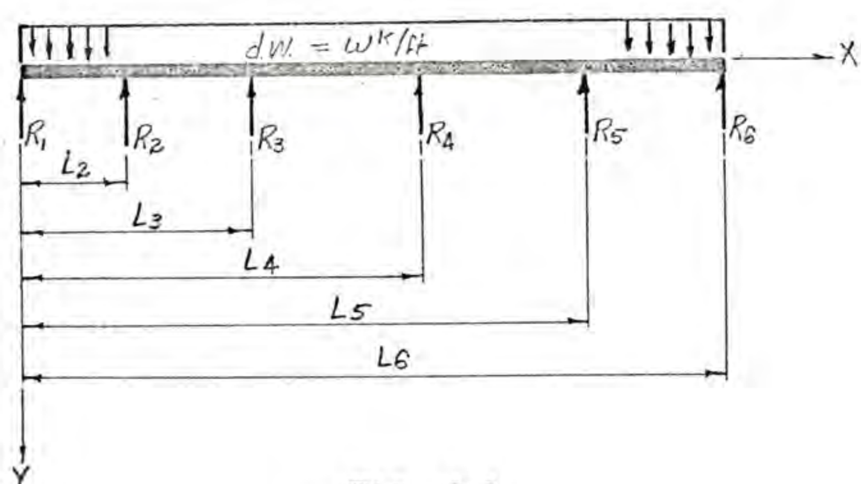


Fig. 3.1

To permit introduction of stepwise discontinuous functions, e.g. the reactions, changes in section rigidity, etc., the MacAulay notation was employed. Using this notation, a function enclosed by the symbols $\langle\ \rangle$ is defined as follows:

$$\begin{aligned} \langle x-a\rangle^n &= 0 \quad , \quad x<a \\ &= (x-a)^n \quad , \quad x \geq a \end{aligned} \qquad (3.1)$$

i.e., if the quantity within the MacAulay brackets is negative, the function assumes a value equal to zero. In particular, this notation may be used to define the unit step function, thus

$$\begin{aligned} \langle x-a\rangle^0 &= 0 \quad , \quad x<a \\ &= 1 \quad , \quad x \geq a \end{aligned} \qquad (3.2)$$

The MacAulay function can be integrated in the conventional manner to provide a continuously defined function:

$$\int\langle x-a\rangle^n dx = \frac{\langle x-a\rangle^{n+1}}{n+1} + C \qquad (3.3)$$

Integration of the elastic curve leads to integrals of the form $\int\langle x-a\rangle^n\langle x-b\rangle^0 dx$

The integrand reduces to

$$\langle x-a\rangle^n\langle x-b\rangle^0 = \langle x-a\rangle^n \qquad a>b \qquad (3.4)$$

hence two cases must be considered. For the first case $a<b$, the integral can be determined by integration by parts:

$$\int\langle x-a\rangle^n\langle x-b\rangle^0 dx = \frac{(\langle x-a\rangle^{n+1}-C_1)}{n+1}\langle x-b\rangle^0 + C$$

Since the function must be continuous at $x=b$, C_1 must take on the value $\langle b-a\rangle^{n+1}$, hence

$$\int \langle x-a\rangle^{n} \langle x-b\rangle^{0} dx = \frac{\langle x-a\rangle^{n+1} - \langle b-a\rangle^{n+1}}{n+1} \langle x-b\rangle^{0} + C\,, \quad a<b \qquad (3.5)$$

For $a > b$, $C_1 = 0$ and Eq. (3.5) reduces to

$$\int \langle x-a\rangle^{n} \langle x-b\rangle^{0} dx = \frac{\langle x-a\rangle^{n+1}}{n+1} + C\,, \qquad a>b$$

which is consistent with the integration of Eq. (3.4) applying Eq. (3.3).

In particular, when $a=0$, Eq. (3.5) reduces to

$$\int x^{n} \langle x-b\rangle^{0} dx = \frac{x^{n+1} - b^{n+1}}{n+1} \langle x-b\rangle^{0} + C \qquad (3.6)$$

The use of MacAulay's notation permits writing the shear equation for the structure shown in Fig. 3.1 as a continuous function of the dead load and the reactions R_1 to R_6.

$$\begin{aligned} V &= R_1 - wx + R_2\langle x-L_2\rangle^{0} + R_3\langle x-L_3\rangle^{0} + R_4\langle x-L_4\rangle^{0} \\ &\quad + R_5\langle x-L_5\rangle^{0} + R_6\langle x-L_6\rangle^{0} \\ &= \left(\sum_{i=1}^{6} R_i \langle x-L_i\rangle^{0}\right) - wx \end{aligned} \qquad (3.7)$$

where L_1 is defined to be equal to zero.

Integrating, the moment equation is

$$M = \left(\sum_{i=1}^{6} R_i \langle x-L_i\rangle\right) - \frac{wx^2}{2} + C \qquad (3.8)$$

Since x cannot be greater than L_6 the last term in the summation does not contribute. Furthermore, since $M = 0$ for $x = 0$, $C = 0$, hence Eq. (3.8) reduces to

$$M = \left(\sum_{i=1}^{5} R_i \langle x-L_i\rangle\right) - \frac{wx^2}{2} \qquad (3.9)$$

The relationship between the slope and deflection and the moment is given by the differential equations

$$\frac{d^2y}{dx^2} = \frac{d\varphi}{dx} = -\frac{M}{EI} \tag{3.10}$$

The forty stepwise discontinuities in section rigidity can be expressed by a series of MacAulay terms as follows:

$$\frac{1}{EI} = \frac{1}{E}\left(\frac{1}{I_1} + \left(\frac{1}{I_2} - \frac{1}{I_1}\right)\langle x - \ell_2\rangle^0 + \cdots\cdots + \left(\frac{1}{I_j} - \frac{1}{I_{j-1}}\right)\langle x - \ell_j\rangle^0\right)$$

$$= \sum_{j=1}^{40} B_j \langle x - \ell_j\rangle^0 \tag{3.11}$$

where

$$B_j = \left(\frac{1}{EI_j} - \frac{1}{EI_{j-1}}\right)$$

Substituting the results of Eqs. (3.11) and (3.9) in Eq. (3.10),

$$\frac{d^2y}{dx^2} = \frac{\omega x^2}{2}\sum_{j=1}^{40} B_j\langle x - \ell_j\rangle^0 - \sum_{i=1}^{5}\sum_{j=1}^{40} B_j R_i \langle x - L_i\rangle\langle x - \ell_j\rangle^0 \tag{3.12}$$

Integrating Eq. (3.12) yields

$$\varphi = \frac{dy}{dx} = \omega\sum_{j=1}^{40} B_j \frac{x^3 - \ell_j^3}{6}\langle x - \ell_j\rangle^0$$
$$- \sum_{i=1}^{5}\sum_{j=1}^{40} B_j R_i \frac{\langle x - L_i\rangle^2 - \langle \ell_j - L_i\rangle^2}{2}\langle x - \ell_j\rangle^0 + C_1 \tag{3.13}$$

where the constant of integration, C_1, is the slope at x = 0. Integrating Eq. (3.13) yields the equation for the elastic curve,

$$y = \omega\sum_{j=1}^{40} B_j\left\{\frac{(x^4 - \ell_j^4)}{24}\langle x - \ell_j\rangle^0 - \frac{\ell_j^3}{6}\langle x - \ell_j\rangle\right\} -$$

$$-\sum_{i=1}^{5}\sum_{j=1}^{40} B_j R_i \left\{ \frac{\langle x-L_i\rangle^3 - \langle \ell_j - L_i\rangle^3}{6}\langle x-\ell_j\rangle^0 - \frac{\langle \ell_j - L_i\rangle^2}{2}\langle x-\ell_j\rangle \right\}$$

$$+C_1 x + C_2 \qquad (3.14)$$

where the constant of integration, C_2, is the deflection at x = 0 and is equal to zero.

Eq. (3.14) contains six additional unknown constants, five reactions, R_1 to R_5, and the slope, C_1, at x = 0. These unknown constants may be determined from the boundary conditions, y = 0 when x = L_i (= 2 to 6) and M = 0 when x = L_6. (eq. 3.8)

The resulting simultaneous equations may be expressed in matrix form

$$\begin{vmatrix} A_{11} & A_{12} & 0 & 0 & 0 & 0 \\ A_{21} & A_{22} & A_{23} & 0 & 0 & 0 \\ A_{31} & A_{32} & A_{33} & A_{34} & 0 & 0 \\ A_{41} & A_{42} & A_{43} & A_{44} & A_{45} & 0 \\ A_{51} & A_{52} & A_{53} & A_{54} & A_{55} & A_{56} \\ 0 & A_{62} & A_{63} & A_{64} & A_{65} & A_{66} \end{vmatrix} \begin{vmatrix} C_1 \\ R_1 \\ R_2 \\ R_3 \\ R_4 \\ R_5 \end{vmatrix} = \begin{vmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \\ B_5 \\ B_6 \end{vmatrix}$$

Boundary Condition	
(y = 0	x = L_2)
(y = 0	x = L_3)
(y = 0	x = L_4)
(y = 0	x = L_5)
(y = 0	x = L_6)
(M = 0	x = L_6)

A standard computer program (SIMILQ)* was employed to solve these equations.

With the unknowns in Eq. (3.14) determined, the deflections at any point x on the girder could readily be determined by direct substitution. The constants were determined for the dead load w for each of the five modular ratios

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*SIMILQ is the standard subroutine for the solution of linear simultaneous equations used by the University of Missouri computer center.

(N = 5, 10, 15, 20, and 25) and the deflections calculated at the quarter points and center line of spans 3 and 4. R_6 was calculated by considering vertical equilibrium.

Deflection Due to Shrinkage

The analysis for the shrinkage deflection was made in a similar manner except that warping moments were considered rather than moments due to dead load. Since the structure is restrained at the piers (i.e. zero deflection) unknown reaction vlaues also had to be considered. The problem was simplified somewhat by the fact that changes in warping moment coincided with changes in section rigidity and therefore could be combined in a single summation.

The warping moment, M_j

$$M_j = (T_o e)_j \qquad (3.15)$$

was taken as positive when producing tension on the lower fiber of the section (i.e. the eccentricity, e, was assumed positive when the centroid of the reinforcement was below the centroid of the section). Since these moments coincided with the changes in section rigidity, the "elastic weights" could be defined by a MacAulay function of the form

$$\frac{M_s}{EI_j} = \sum_{j=1}^{40} C_j \langle x - \ell_j \rangle^0 \qquad (3.16)$$

where M_s/EI_j = elastic weight at any section

and $$C_j = \left(\frac{M_j}{EI_j} - \frac{M_{j-1}}{EI_{j-1}} \right) \qquad (3.17)$$

Hence the differential equation for the elastic curve takes the form

$$-\frac{d^2y}{dx^2} = \frac{M}{EI} = \sum_{i=1}^{5}\sum_{j=1}^{40} B_j R_i \langle x - L_i \rangle \langle x - \ell_j \rangle^0 + \sum_{j=1}^{40} C_j \langle x - \ell_j \rangle^0 \qquad (3.18)$$

Integrating, the slope equation is

$$-\varphi = -\frac{dy}{dx} = \sum_{i=1}^{5}\sum_{j=1}^{40} B_j R_i \left\{ \frac{\langle x - L_i \rangle^2 - \langle \ell_j - L_i \rangle^2}{2} \right\} \langle x - \ell_j \rangle^0 + \sum_{j=1}^{40} C_j \langle x - \ell_j \rangle + C_1 \qquad (3.19)$$

Integrating the slope equation the equation of the elastic curve is found to be:

$$-y = \sum_{i=1}^{5}\sum_{j=1}^{40} B_j R_i \left\{ \frac{\langle x - L_i \rangle^3 - \langle \ell_j - L_i \rangle^3}{6} \langle x - \ell_j \rangle^0 - \frac{\langle \ell_j - L_i \rangle^2}{2} \langle x - \ell_j \rangle \right\} + \sum_{j=1}^{40} \frac{C_j}{2} \langle x - \ell_j \rangle^2 + C_1 x + C_2 \qquad (3.20)$$

The five reactions and C_1 and C_2 again were determined from the boundary conditions y = 0 for x = L_i and M = 0 for x = L_6. These constants were determined in a manner similar to that used for the dead load deflections and substituted in Eq.

(3.20) to determine the shrinkage deflections at the desired locations. It should be noted that both the warping moment as well as the rigidity changed in each section as different values of the modular ratio were assumed; the warping moment being greater for low values of N because of the greater eccentricities associated with higher rigidities.

Results

Print outs for the deflection calculations are given in the Appendix. Computed deflection values are summarized in Table 3.1. Deflection vs modular ratio due to dead load and shrinkage for span 3 is plotted in Figure 3.2, and for span 4 in Figure 3.3.

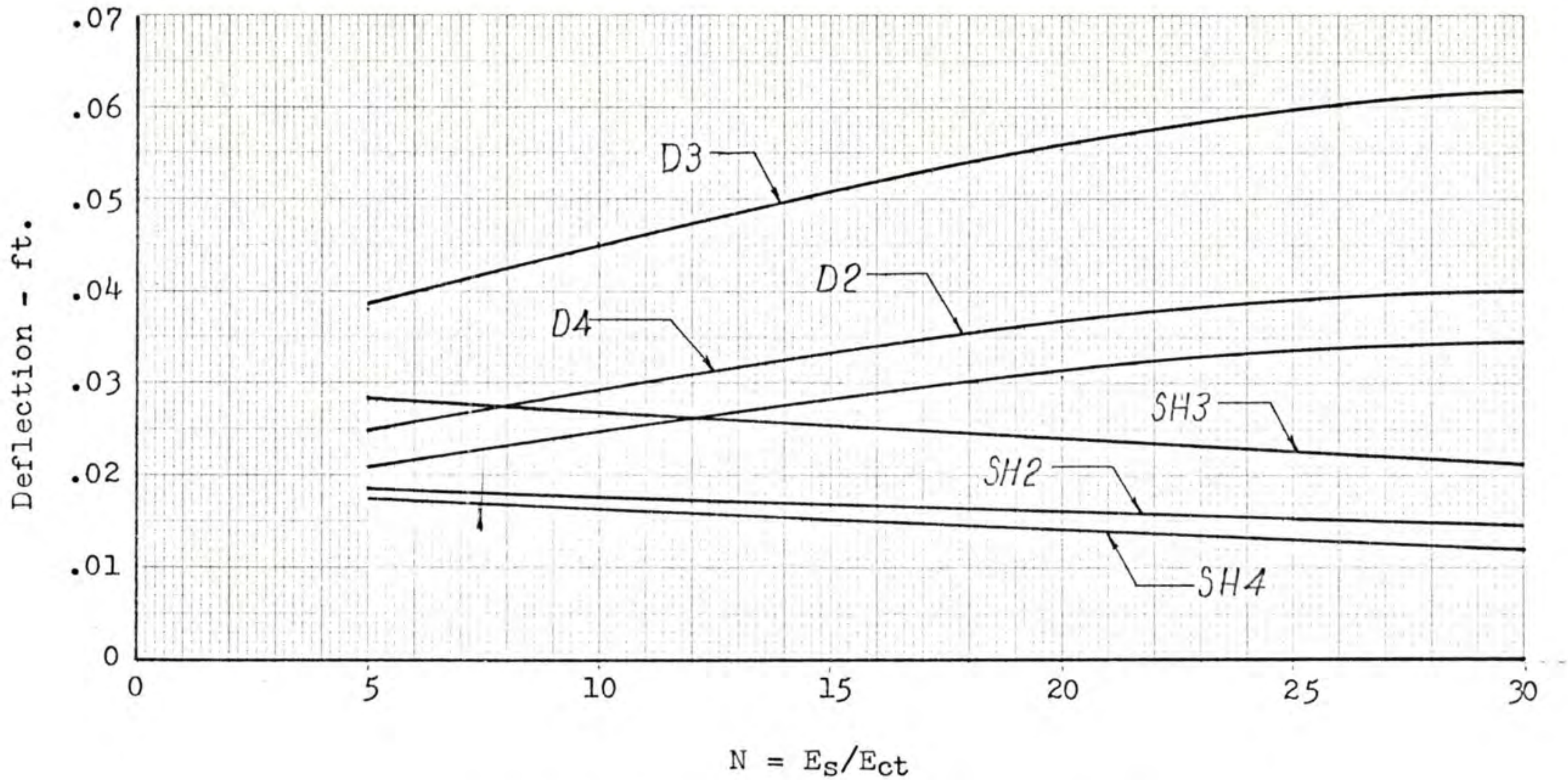
Figure 3.4 shows the deflection profile for spans 3 and 4 based on modular ratio of N = 20. This value was selected on an assumed value of the instantaneous modulus of concrete of 5×10^6 psi and a creep coefficient of 3.5. These values are realistic estimates of the quality of concrete specified for the structure.

Flow charts and detailed computer programs are given in the Appendix.

TABLE 3.1

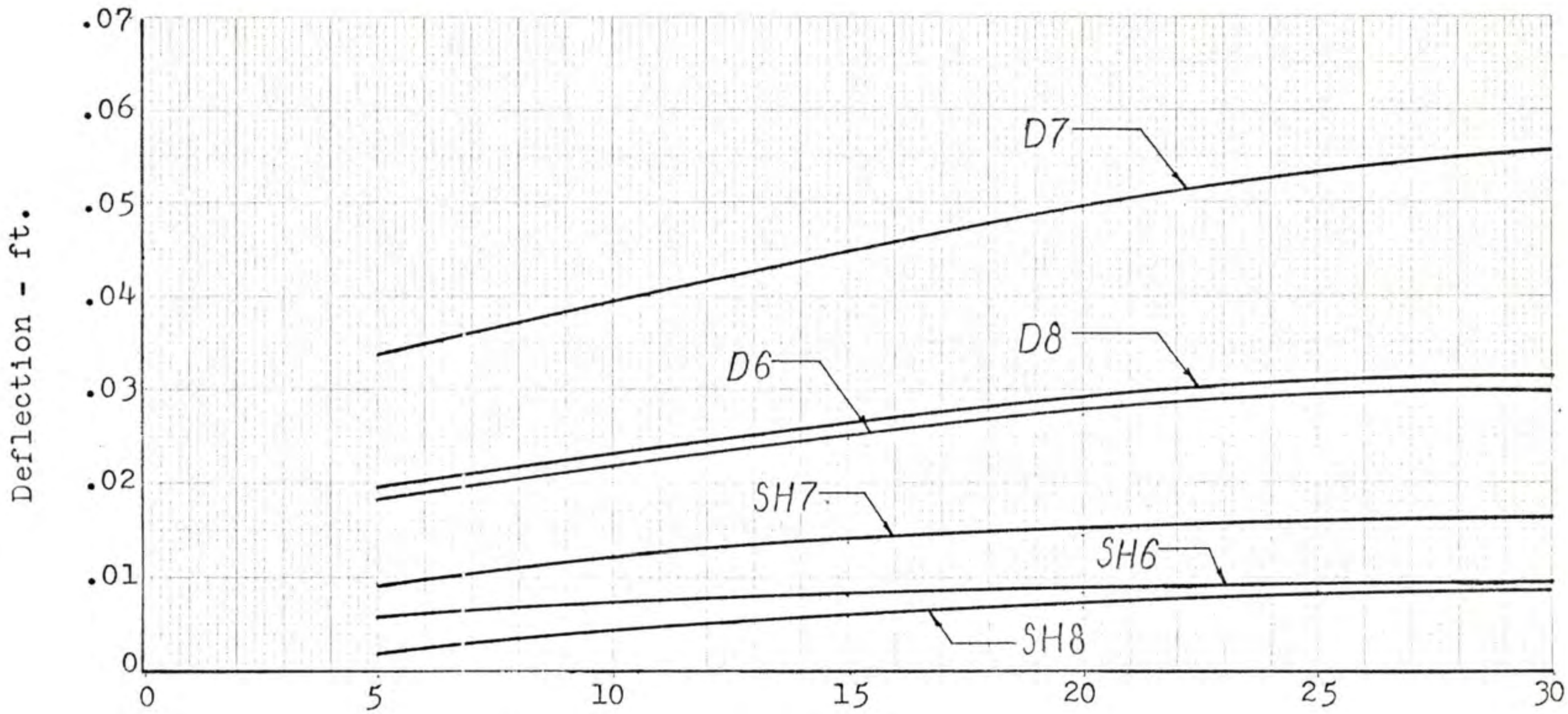
DEAD LOAD AND SHRINKAGE DEFLECTIONS

	Modular Ratio N				
	5	10	15	20	25
SPAN # 3					
Quarter Pt. D2	.0248	.0293	.0331	.0362	.0390
Quarter Pt. D3	.0383	.0451	.0507	.0554	.0597
Quarter Pt. D4	.0211	.0249	.0281	.0308	.0334
SPAN # 4					
Quarter Pt. D6	.0184	.0219	.0248	.0273	.0295
Quarter Pt. D7	.0337	.0398	.0449	.0496	.0533
Quarter Pt. D8	.0195	.0231	.0263	.0293	.0313
SPAN # 3					
Quarter Pt. SH2	.0187	.0178	.0168	.0160	.0151
Quarter Pt. SH3	.0285	.0269	.0254	.0240	.0225
Quarter Pt. SH4	.0173	.0160	.0150	.0141	.0130
SPAN # 4					
Quarter Pt. SH6	.0058	.0073	.0080	.0083	.0092
Quarter Pt. SH7	.0094	.0123	.0136	.0143	.0160
Quarter Pt. SH8	.0018	.0044	.0058	.0067	.0085



DEAD LOAD AND SHRINKAGE DEFLECTIONS
SPAN 3

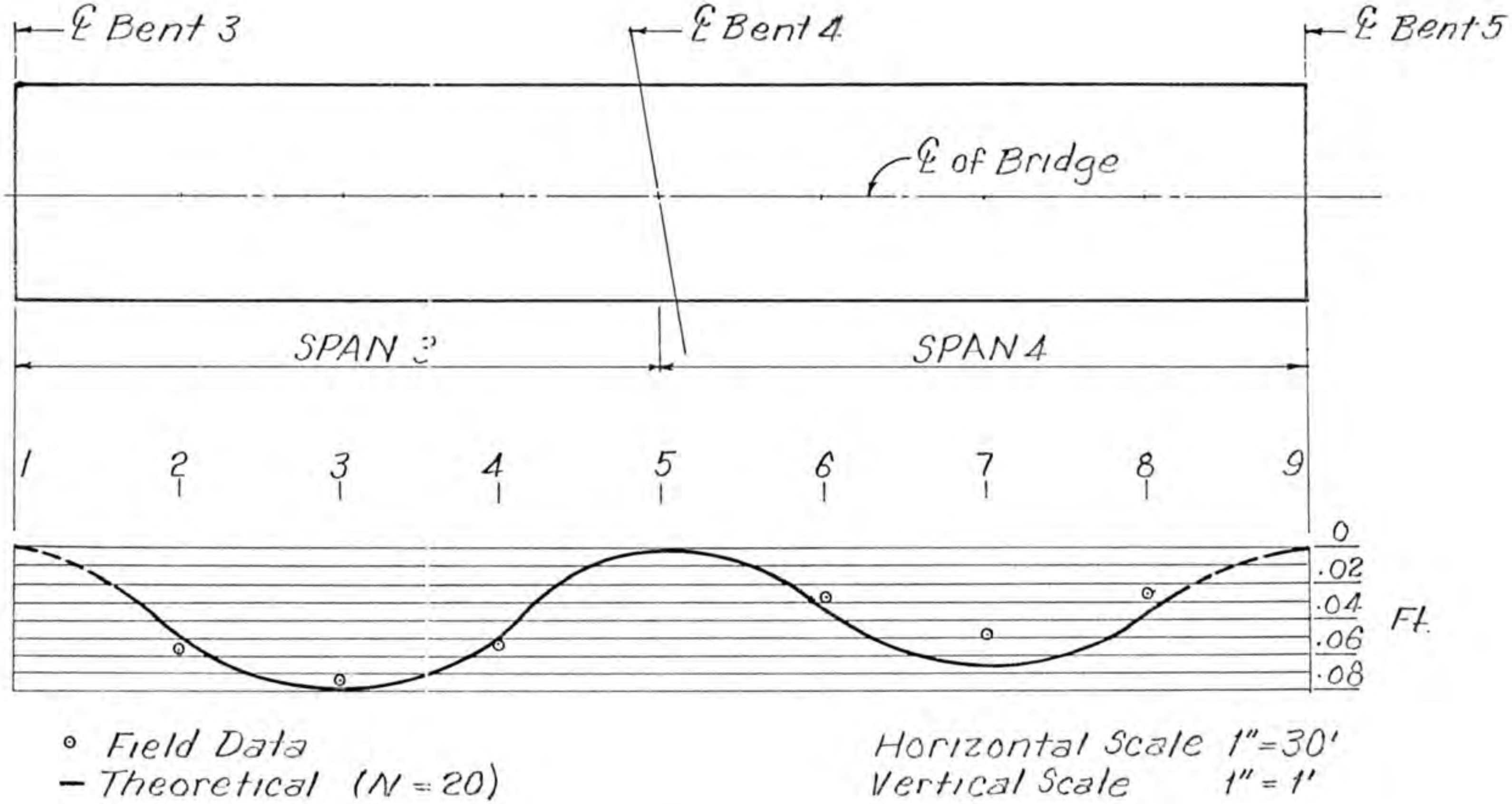
FIG. 3.2



$N = E_s/E_{ct}$

DEAD LOAD AND SHRINKAGE DEFLECTIONS
SPAN 4

FIG. 3.3



DEFLECTION - SPANS 3 & 4

FIG. 3.4

CHAPTER IV

SUMMARY AND CONCLUSIONS

The principal objective was to determine if the time-dependent deflections of a structure due to creep and shrinkage can be predicted by the designer with reasonable accuracy. The scope of this report was limited to the analysis of a box-girder bridge for which field data was available.

The analysis demonstrates that time-dependent deflection of plain reinforced concrete structures are not overly sensitive to the magnitude of the creep coefficient assumed because of the stiffening effect of the reinforcement. Similarly the shrinkage deflection is essentially independent of the creep because of a reduction in the warping moment as creep strains are developed. These phenomena are clearly demonstrated in Figures 3.2 and 3.3, showing the relationship between deflection and modular ratio.

Figure 3.4 shows that while the general form of the deflection profile predicted by the effective modulus method of analysis agrees with field behavior, the observed field deflections are somewhat smaller. This result may be attributed to several factors.

1. The analysis essentially assumed a fully cracked section in regions where normal allowable stress levels can be expected. It has been observed that in structures in actual service, the cracks in the tension zone develop gradually over a period of several months to as much as a year.

2. The method assumes a straight line stress distribution after creep takes place. Recent studies have shown that the stress distribution does not remain linear as the concrete creeps and as a result the effective modulus method over-estimates the shift in the neutral axis and the resulting rotation and deflection.

3. The analysis was based on the assumption that the bents were normal to the longitudinal axis of the bridge. Actually, bent 4, located between span 3 and 4 was slightly skewed thus reducing the effective span length. No simple method is presently available to estimate the effect of skewed supports on the deflection of a beam or girder.

The results obtained by the analysis however, indicates that this analysis provides a reasonably accurate procedure for guiding the designer. The results for the continuous structure analyzed in this report are consistent with the results on a series of simple beams studied by Jones(8).

The computer program developed in this report could be applied to analyze other structures for which field observations are available. Such an extension of this work would be useful to corroborate the findings reported in this study.

NOTATION USED IN ANALYTICAL SOLUTION

c = Subscript denoting property of concrete.

s = Subscript denoting property of steel.

t = Subscript denoting value at a given time.

i = Subscript denoting reaction number as in Fig. 3.1.

j = Subscript denoting stiffness number as in Figures 2.9 and 2.10.

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APPENDIX

COMPUTER PROGRAMS

DEFINITION OF VARIABLES

1	RN	= Ratio of Modulus of Elasticity of Steel To That of Concrete
	A	= Area of a Segment
2	AN	= Number of Typical Segments in a row
3	AP	= Cross Sectional Area of one Bar
4	AL	= Length of Each Constant Stiffness
5	AI	= BL,s Moment of Inertia
6	ACL	= Accumulated Length of Sections
	TE	= Warping Moment
7	AC	= Unit Cross Sectional Area
8	PC	= Deflection Point in Consideration
9	B1	= Segment Base
10	B2	= Shorter Base of Segment in Case of Trapezoid
11	BL1	= Accumulated Length of Spans From End
12	BL	= BL1 - Preceding Span Length
13	D	= Variable Height of Skewed Rectangle Center Line
14	ES	= Modulus of Elasticity of Steen in ksi
15	EC	= Effective Modulus of Elasticity of Concrete
16	J, JC, and JS are Dominant Characteristics	
17	K	= Location Number
18	M	= Total Number of Elements
19	N	= Number of Set of Datas
20	NBL	= Number of Spans
21	NAL	= Number of Rigidities
22	NRN	= Number of Modular Ratios
23	NPC	= Number of Points for Deflection
24	Y	= Area Moment Arm to Initial Point in Consideration
25	YBARA	= Assumed $\bar{y}$

PROGRAM EQUATIONS

To simplify the flow charting, the long equations in computer programs are given in this page.

Program Equation #1

$$XI_o = ((H(I) * B1(I) * (D(I)^2 + H(I)^2))/12.)* AN(I)$$

Program Equation #2

$$XI_o = ((H(I)^3*(B1(I)^2+4.*B1(I)*B2(I)+B2(I)^2))/36 *(B1(I)+B2(I))*AN(I)$$

Program Equation #3

$$XI_o = (B1(I)*(D(I)^3+H(I)*D(I)^2+H(I)*D(I)))/36.)*AN(I)$$

Program Equation #4

$$B1 = B(J)*(((A1(J)^3/6.) - (A2(J)^3/6.))*A3(J) - (A2(J)^2/2) *A4(J)*X(M1)$$

Program Equation #5

$$C1 = B(J)*(((4N^4 -ACL(J)^4)*A3(J)/12.) - ((ACL(J)^3) *A4(J)/3.))$$

NEUTRAL AXIS, ECCENTRICITY, MOMENT OF INERTIA, AND SHRINKAGE FORCE

```
      DIMENSION A(30),AN(30),B1(30),B2(30),J(30),JC(30),JS(30),H(30)
      DIMENSION D(30),Y(30),AP(30)
   11 FORMAT(I2)
   12 FORMAT(I2)
   13 FORMAT(I5,F5.0,F10.3)
   14 FORMAT(21X,16HFOR LOCATION NO.,I2,2X,3HAND,2X,4HN = ,F3.0/)
   15 FORMAT(3I2,7F8.3)
   16 FORMAT(2X,I2,2(4X,I2),2X,F6.3,2X,F5.3,2X,F6.3,2X,F7.3,2(2X,F6.3),2
     1X,F7.3)
   17 FORMAT(10X,I2,3X,5HTENS.,3X,5HSTEEL,3X,F8.3,3X,F8.3,12X,F8.3)
   18 FORMAT(10X,I2,3X,5HCOMP.,3X,5HSTEEL,3X,F8.3,13X,F7.3,3X,F8.3)
   19 FORMAT(10X,I2,3X,5HCOMP.,3X,5HCONC.,3X,F8.3,23X,F8.3)
   21 FORMAT(10X,I2,3X,F8.3,3X, F8.3,3X,F8.3,3X,F10.3,3X,F10.3)
   22 FORMAT(//24X,18HYBAR = ACAY/ATA = ,F6.3,1X,2HIN/)
   23 FORMAT(10X,6HYBAR =,F6.3,3X,7HAGAINST,3X,14HASSUMED YBAR =,F6.3,4X
     1,2HOK)
   24 FORMAT(/10X,6HACAS =,F7.3,1X,3HIN2,10X,7HACASY =,F8.3,1X,3HIN3)
   25 FORMAT(/24X,21HYBARS = ACASY/ACAS = ,F6.3,1X,2HIN//10X,17HED = YBA
     1RS-YBAR =,F8.3,1X,2HIN)
   26 FORMAT(10X,6HYBAR =,F6.3,3X,7HAGAINST,3X,14HASSUMED YBAR =,F6.3,4X
     1,7HNO GOOD)
   27 FORMAT(10X,I2,3X,F8.3,3X,F6.3,3X,F8.3,3X,F11.3,4X,F11.3)
   28 FORMAT(10X,18HI = 2.*(TOTAL I) =,F10.0,3HIN4/)
   29 FORMAT(10X,4HES =,F6.0,1X,3HKSI/10X,11HEC = ES/N = ,F6.0,1X,3HKSI)
   31 FORMAT(10X,19HECI = (EC*I/144.) =,E10.3,1X,5HK-FT2,/10X,6HEPSH =,F
     $5.4,1X,5HIN/IN)
   32 FORMAT(10X,24HTO = (EPSH*ES*ACAS)*2. =,F10.3,1X,1HK/10X,16HCM = TO
     $*ED/12. =,F10.3,1X,4HK-FT)
   51 FORMAT(1H1,//////////30X,11HDATA  GIVEN)
   52 FORMAT(1H1,//////////23X,24HTRANSFORMED  AREA  TABLE)
   53 FORMAT(10X,2HNO,4X,4HZONE,4X, 3HMAT,6X,4HAREA,7X,4HN*SA,3X,8H(N-1)
     1*AS,2X,8HTRANS'A')
```

```
 54 FORMAT(33X,3HIN2,8X,3HIN2,2(7X,3HIN2))
 55 FORMAT(1H1,/////////22X,27HLOCATION  OF  NEUTRAL  AXIS)
 56 FORMAT(10X,44HATA = ACCUMULATED EFFECTIVE TRANSFORMED AREA)
 57 FORMAT(10X,18HAY = TRANS'A'*Y(I))
 58 FORMAT(10X,21HACAY = ACCUMULATED AY)
 59 FORMAT(10X,23HACAS = TOTAL STEEL AREA/10X,29HACASY = ACCUMULATED A
   1CAS*Y(I)/)
 61 FORMAT(10X,2HNO,4X,8HTRANS'A',5X,3HATA,9X,1HY,10X,2HAY,10X,4HACAY)
 62 FORMAT(19X,3HIN2,7X,3HIN2,8X,2HIN,10X,3HIN3,9X,3HIN3)
 63 FORMAT(1H1,//////23X,26HMOMENT  OF  INERTIA  TABLE)
 64 FORMAT(10X,30HY1= ABSOLUTE VALUE OF (Y-YBAR),/10X,18HI1= TRANS'A'*
   1Y1**2,/10X,22HTOTAL I =TOTAL I+I0+I1/)
 65 FORMAT(10X,2HNO,3X,8HTRANS'A',5X,2HY1,8X,2HI0,12X,2HI1,9X,7HTOTAL
   1I)
 66 FORMAT(18X,3HIN2,7X,2HIN,7X,3HIN4,2(11X,3HIN4))
 67 FORMAT(10X,21HSHRINKAGE CALCULATION/10X,21H---------------------)
 68 FORMAT(10X,35HES = MODULUS OF ELASTICITY OF STEEL/10X,48HEC = EFFE
   $CTIVE MODULUS OF ELASTICITY OF CONCRETE)
 69 FORMAT(10X,28HEPSH = SHRINKAGE COEFFICIENT/10X,9HN = ES/EC/)
 71 FORMAT(1X,4HJ(I),1X,5HJC(I),1X,5HJS(I),2X,5HAN(I),2X,5HAP(I),3X,4H
   1H(I),4X,5HB1(I),4X,5HB2(I),3X,4HD(I),5X,4HY(I))
101 FORMAT(9X,63H-----------------------------------------------------
   1----------)
102 FORMAT(9X,63H-----------------------------------------------------
   1----------/)

    READ(5,11)N
499 DO 500 L=1,N
    READ(5,12)M
    READ(5,13)K,RN,YBARA
    WRITE(6,51)
    WRITE(6,14)K,RN
    WRITE(6,101)
    WRITE(6,71)
    WRITE(6,102)
    DO 85 I = 1,M
```

```
      READ(5,15)J(I),JC(I),JS(I),AN(I),AP(I),H(I),B1(I),B2(I),D(I),Y(I)
   85 WRITE(6,16)J(I),JC(I),JS(I),AN(I),AP(I),H(I),B1(I),B2(I),D(I),Y(I)
      WRITE(6,101)
      WRITE(6,52)
      WRITE(6,14)K,RN
      WRITE(6,101)
      WRITE(6,53)
      WRITE(6,54)
      WRITE(6,102)
      ACAS = 0
      ACASY = 0
      DO 90 I=1,M
      IF(J(I))10,10,20
C     TOTAL STEEL AREA IN EACH ELEMENT
   10 AS = AN(I)*AP(I)
      ACAS = ACAS+AS
      IF(JC(I))30,30,40
C     EFFECTIVE TRANSFORMED TENSION STEEL
   30 A(I) = RN*AS
      WRITE(6,17)I,AS,A(I),A(I)
      ASY = AS*Y(I)
      GO TO 95
C     EFFECTIVE TRANSFORMED COMPRESSION STEEL
   40 A(I) = (RN-1.)*AS
      WRITE(6,18)I,AS,A(I),A(I)
      ASY = AS*Y(I)
   95 ACASY = ACASY+ASY
      GO TO 90
C     CONCRETE CROSS SECTIONAL AREAS
   20 IF(JC(I))50,60,70
C     AREA OF RECTANGLAR SHAPE ELEMENT
   50 A(I) =(H(I)*B1(I))*AN(I)
      GO TO 80
C     AREA OF TRIANCLAR SHAPE ELEMENT
   60 A(I) =(H(I)*B1(I)/2.)*AN(I)
      GO TO 80
```

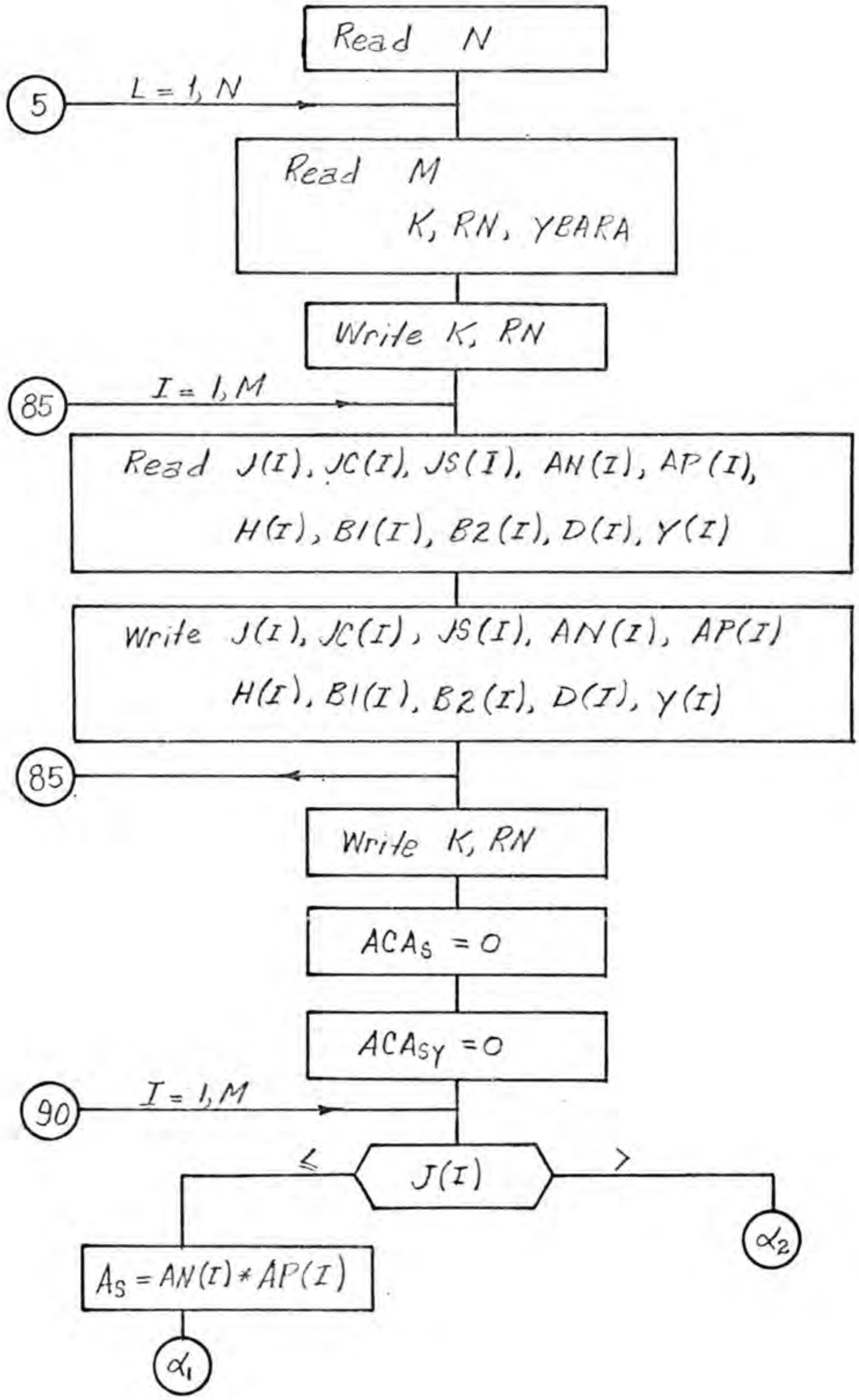
```
C       AREA OF TRAPEZOIDAL SHAPE ELEMENT
   70 A(I) =(H(I)*(B1(I)+B2(I))/2.)*AN(I)
   80 WRITE(6,19)I,A(I),A(I)
   90 CONTINUE
      WRITE(6,101)
      WRITE(6,55)
      WRITE(6,14)K,RN
      WRITE(6,56)
      WRITE(6,57)
      WRITE(6,58)
      WRITE(6,59)
      WRITE(6,101)
      WRITE(6,61)
      WRITE(6,62)
      WRITE(6,102)
      ATA =0
      ACAY =0
      DO 100 I=1,M
      AY = A(I)*Y(I)
      ACAY = ACAY+AY
      ATA = ATA+A(I)
  100 WRITE(6,21)I,A(I),ATA,Y(I),AY,ACAY
      WRITE(6,101)
      YBAR =ACAY/ATA
      WRITE(6,22)YBAR
      IF(ABS(YBAR-YBARA)-.1)110,110,120
  110 WRITE(6,23)YBAR,YBARA
      YBARS =ACASY/ACAS
      ED = YBARS-YBAR
      WRITE(6,24)ACAS,ACASY
      WRITE(6,25)YBARS,ED
      GO TO 130
  120 WRITE(6,26)YBAR,YBARA
      GO TO 499
  130 CONTINUE
```

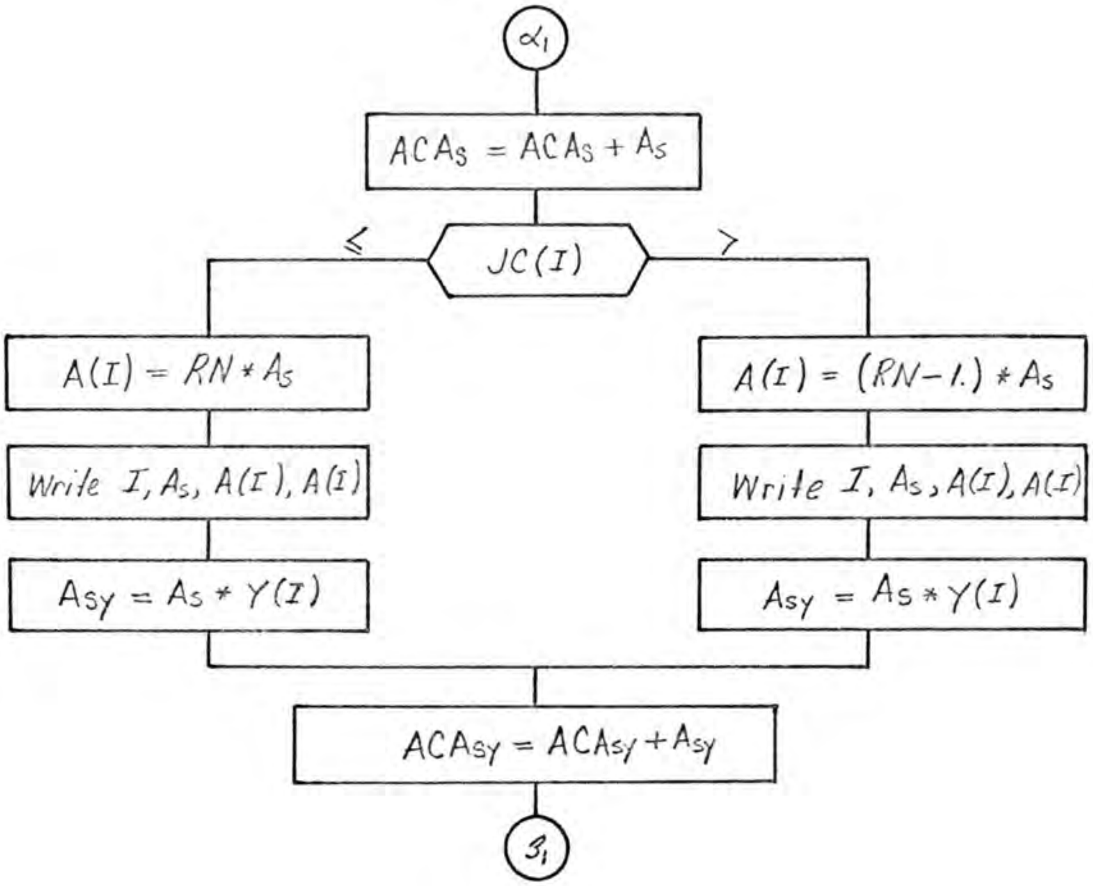
A6

```
C       MOMENT OF INERTIA CALCULATION
        WRITE(6,63)
        WRITE(6,14)K,RN
        WRITE(6,64)
        WRITE(6,101)
        WRITE(6,65)
        WRITE(6,66)
        WRITE(6,102)
        SUMI =0
        DO 200 I=1,M
        Y1=ABS(Y(I)-YBAR)
        YS = Y1**2
C       FOR TRASFERRING THE MOMENT OF INERTINA OF EACH PART TO N.A.
        XIT =A(I)*YS
C       MOMENT OF INERTIA OF EACH PART ABOUT IT'S HORIZONTAL AXIS
        IF(J(I))150,150,160
C
C       IO OF REINFORCING BARS
  150   XIO =0
        GO TO 140
  160   IF(JC(I))170,180,190
  170   IF(JS(I))210,220,220
  210   XIO=((H(I)*B1(I)*(D(I)**2+H(I)**2))/12.)*AN(I)
C       IO OF SKEWED RECTANGLE
        GO TO 140
C       IO OF RECTANGLE
  220   XIO=((H(I)**3*B1(I))/12.)*AN(I)
        GO TO 140
C       IO OF TRAPEZOID
  190   XIO=((H(I)**3*(B1(I)**2+4*B1(I)*B2(I)+B2(I)**2))/36.*(B1(I)+B2(I))
       1)*AN(I)
        GO TO 140
  180   IF(JS(I))230,240,240
C       IO OF IRREGULAR TRIANGLE
  230   XIO =(B1(I)*(D(I)**3+H(I)*(D(I)**2+H(I)*D(I)))/36.)*AN(I)
        GO TO 140
```

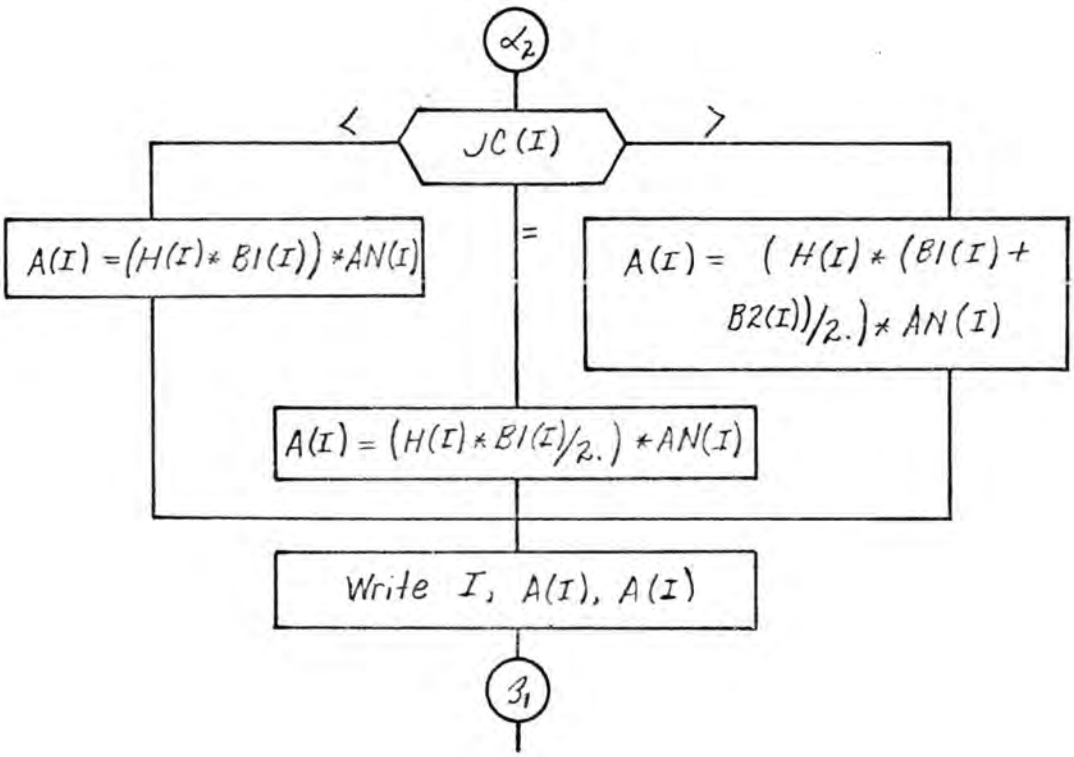
```
C       IO OF TRIANGLE
  240 XIO =((H(I)**3*B1(I))/36.)*AN(I)
C       SUMMATION OF IO AND I1 OF EACH PART
  140 TXIT =XI1+XIO
      SUMI = SUMI+TXIT
  200 WRITE(6,27)I,A(I),Y1,XIO,XIT,SUMI
      WRITE(6,101)
      SSI = 2.*SUMI
      WRITE(6,28)SSI
      WRITE(6,67)
      WRITE(6,68)
      WRITE(6,69)
C       SHRINKAGE FORCE CALCULATION
      ES = 29000.
      EC = ES/RN
      ECI = EC*SSI/144.
      EPSH = .0002
      TO =(EPSH*ES*ACAS)*2.
      CM =TO*ED/12.
      WRITE(6,29)ES,EC
      WRITE(6,31)ECI,EPSH
      WRITE(6,32)TO,CM
  500 CONTINUE
      END
```

Flow Diagram
Eccentricity, Moment of Inertia, and Shrinkage Moment
Based on Cracked Section

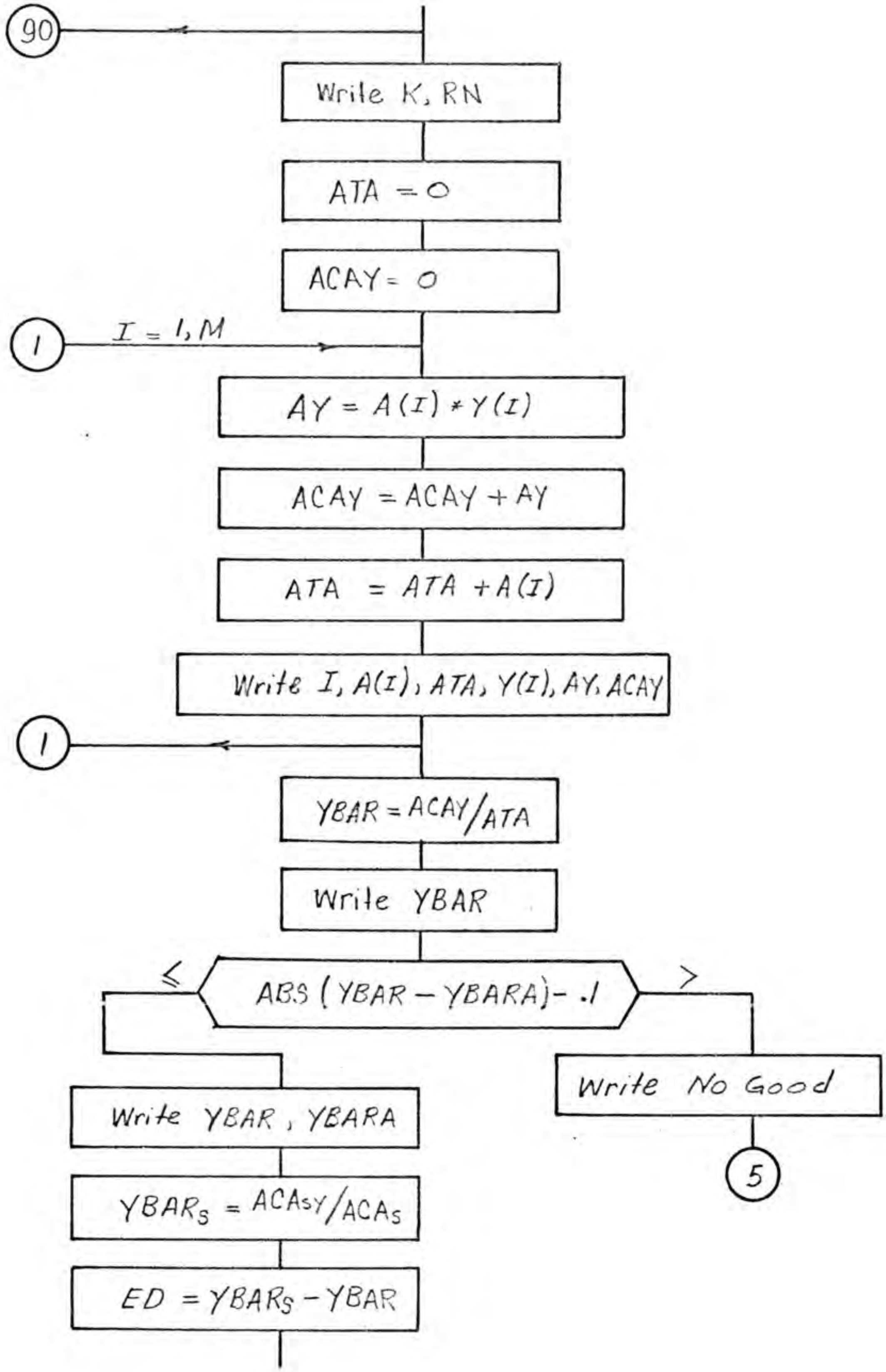


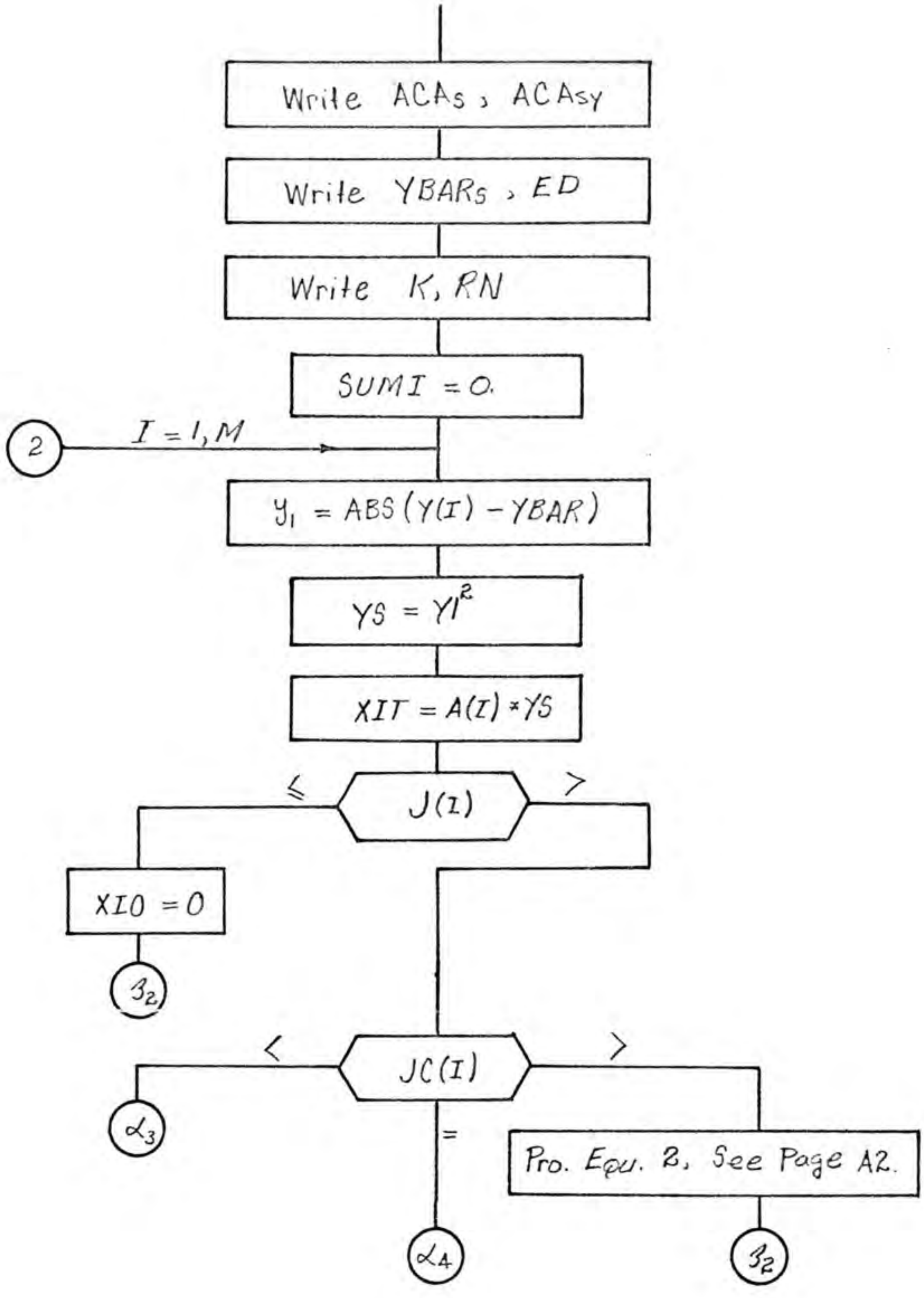


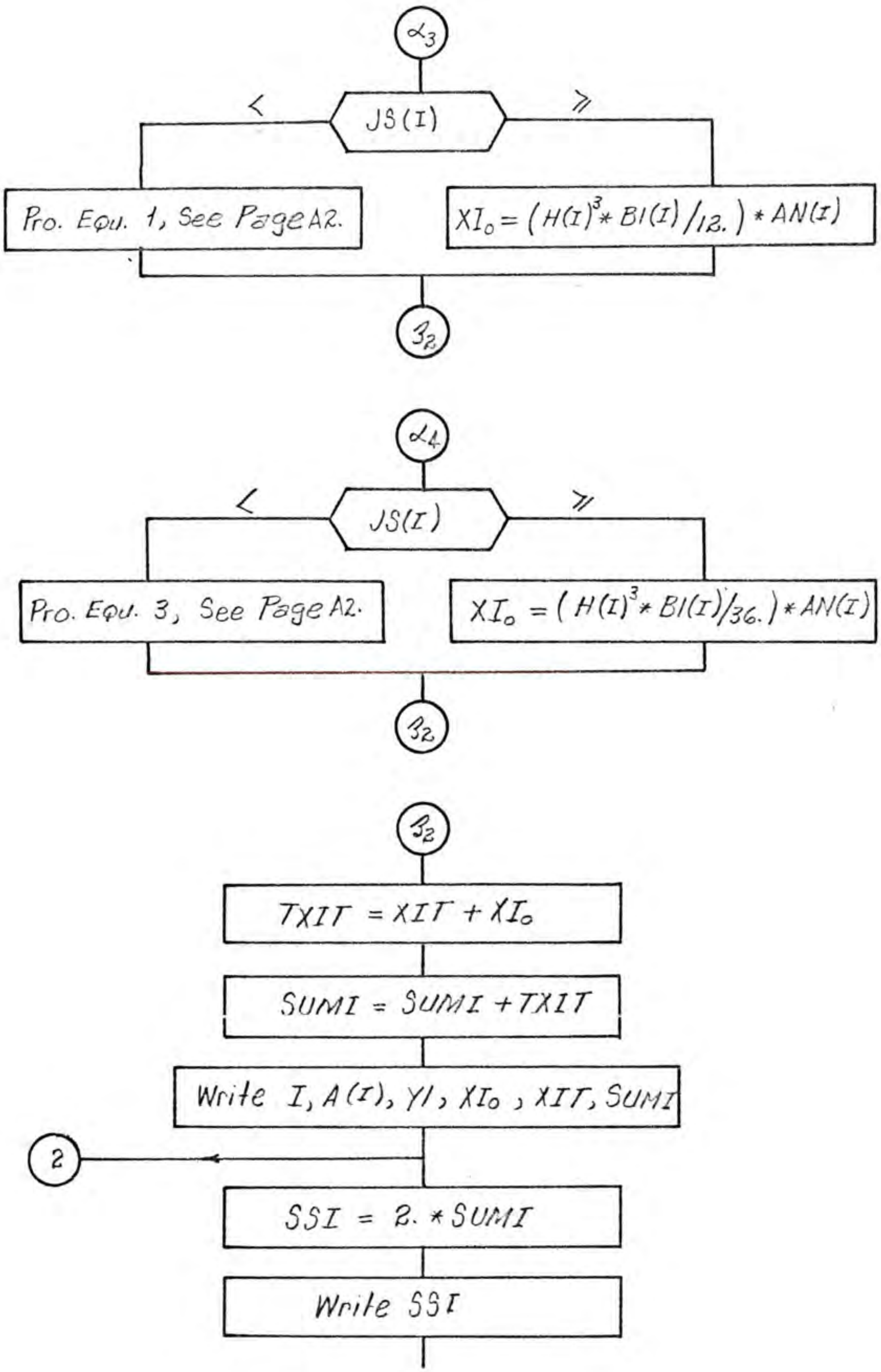
α1
ACAs = ACAs + As
≤
JC(I)
>
A(I) = RN * As
Write I, As, A(I), A(I)
Asy = As * Y(I)
A(I) = (RN-1.) * As
Write I, As, A(I), A(I)
Asy = As * Y(I)
ACAsy = ACAsy + Asy
β1



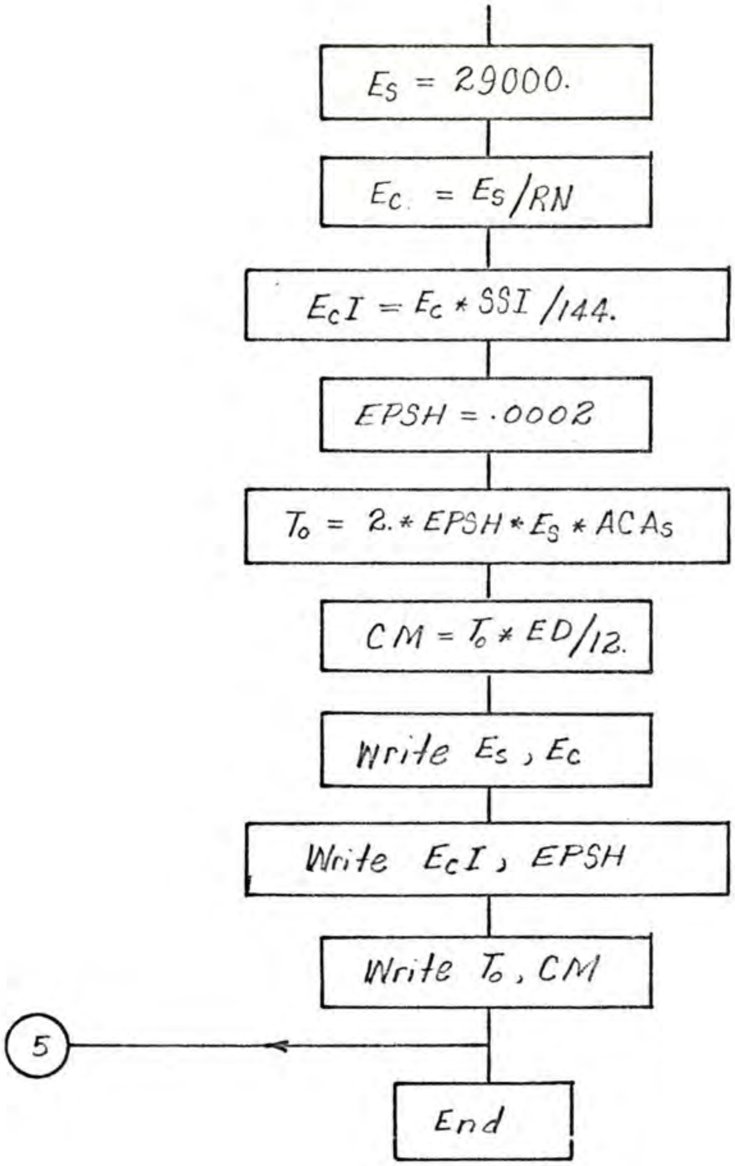
α2
<
JC(I)
>
=
A(I) = (H(I) * B1(I)) * AN(I)
A(I) = (H(I) * (B1(I) + B2(I))/2.) * AN(I)
A(I) = (H(I) * B1(I)/2.) * AN(I)
Write I, A(I), A(I)
β1


90
Write K, RN
ATA = 0
ACAY = 0
1
I = 1, M
AY = A(I) * Y(I)
ACAY = ACAY + AY
ATA = ATA + A(I)
Write I, A(I), ATA, Y(I), AY, ACAY
1
YBAR = ACAY/ATA
Write YBAR
≤
ABS (YBAR – YBARA) – .1
>
Write No Good
5
Write YBAR, YBARA
YBARS = ACASY/ACAS
ED = YBARS – YBAR


Write ACAs, ACAsy
Write YBARs, ED
Write K, RN
SUMI = 0.
2
I = 1, M
Y1 = ABS(Y(I) - YBAR)
YS = YI²
XIT = A(I) * YS
J(I)
≤
>
XIO = 0
β2
JC(I)
<
>
=
α3
Pro. Equ. 2, See Page A2.
α4
β2



α3
JS(I)
<
≥
Pro. Equ. 1, See Page A2.
XIo = (H(I)^3 * BI(I)/12.) * AN(I)
β2
α4
JS(I)
<
≥
Pro. Equ. 3, See Page A2.
XIo = (H(I)^3 * BI(I)/36.) * AN(I)
β2
β2
TXIT = XIT + XIo
SUMI = SUMI + TXIT
Write I, A(I), YI, XIo, XIT, SUMI
2
SSI = 2. * SUMI
Write SSI


Es = 29000.
Ec = Es/RN
EcI = Ec * SSI /144.
EPSH = .0002
To = 2. * EPSH * Es * ACAs
CM = To * ED/12.
Write Es , Ec
Write EcI , EPSH
Write To , CM
5
End

DATA GIVEN
FOR LOCATION NO.19 AND N = 15.

J(I)	JC(I)	JS(I)	AN(I)	AP(I)	H(I)	B1(I)	B2(I)	D(I)	Y(I)
-1	-1	0	45.500	1.560	0.0	0.0	0.0	0.0	65.109
-1	-1	0	16.000	0.200	0.0	0.0	0.0	0.0	42.609
-1	1	0	6.000	0.790	0.0	0.0	0.0	0.0	14.609
-1	1	0	30.500	0.310	0.0	0.0	0.0	0.0	6.937
-1	1	0	27.000	0.790	0.0	0.0	0.0	0.0	4.500
-1	1	0	4.000	0.200	0.0	0.0	0.0	0.0	-3.813
-1	1	0	4.000	0.310	0.0	0.0	0.0	0.0	-14.875
1	-1	-1	1.000	0.0	7.000	240.000	0.0	3.750	5.375
1	-1	0	1.000	0.0	5.000	28.500	0.0	0.0	6.250
1	-1	0	1.000	0.0	2.000	12.072	0.0	0.0	9.750
1	0	0	1.000	0.0	2.000	16.428	0.0	0.0	9.427
1	0	0	1.000	0.0	9.000	1.000	0.0	0.0	0.750
1	-1	0	1.000	0.0	9.500	27.500	0.0	0.0	-1.000
1	-1	0	1.000	0.0	18.000	10.500	0.0	0.0	-14.875
1	0	0	1.000	0.0	3.375	24.072	0.0	0.0	11.688
1	0	0	1.000	0.0	4.000	6.000	0.0	0.0	11.770
1	0	0	2.000	0.0	4.000	6.000	0.0	0.0	10.442
1	0	0	2.000	0.0	4.000	6.000	0.0	0.0	8.021
1	-1	0	3.000	0.0	10.931	8.000	0.0	0.0	14.574

TRANSFORMED AREA TABLE
FOR LOCATION NO.19 AND N = 15.

NO	ZONE	MAT	AREA IN2	N*SA IN2	(N-1)*AS IN2	TRANS'A' IN2
1	TENS.	STEEL	70.980	1064.699		1064.699
2	TENS.	STEEL	3.600	54.000		54.000
3	COMP.	STEEL	4.740		66.360	66.360
4	COMP.	STEEL	9.455		132.370	132.370
5	COMP.	STEEL	21.330		298.620	298.620
6	COMP.	STEEL	0.800		11.200	11.200
7	COMP.	STEEL	1.240		17.360	17.360
8	COMP.	CONC.	1680.000			1680.000
9	COMP.	CONC.	142.500			142.500
10	COMP.	CONC.	24.144			24.144
11	COMP.	CONC.	16.428			16.428
12	COMP.	CONC.	4.500			4.500
13	COMP.	CONC.	261.250			261.250
14	COMP.	CONC.	189.000			189.000
15	COMP.	CONC.	40.621			40.621
16	COMP.	CONC.	12.000			12.000
17	COMP.	CONC.	24.000			24.000
18	COMP.	CONC.	24.000			24.000
19	COMP.	CONC.	262.344			262.344

LOCATION OF NEUTRAL AXIS
FOR LOCATION NO.19 AND N = 15.

ATA = ACCUMULATED EFFECTIVE TRANSFORMED AREA
AY = TRANS'A'*Y(I)
ACAY = ACCUMULATED AY
ACAS = TOTAL STEEL AREA
ACASY = ACCUMULATED ACAS*Y(I)

NO	TRANS'A' IN2	ATA IN2	Y IN	AY IN3	ACAY IN3
1	1064.699	1064.699	65.109	69321.500	69321.500
2	54.000	1118.699	42.609	2300.885	71622.375
3	66.360	1185.059	14.609	969.453	72591.812
4	132.370	1317.429	6.937	918.250	73510.062
5	298.620	1616.049	4.500	1343.788	74853.812
6	11.200	1627.249	-3.813	-42.706	74811.062
7	17.360	1644.608	-14.875	-258.230	74552.812
8	1680.000	3324.608	5.375	9030.000	83582.812
9	142.500	3467.108	6.250	890.625	84473.437
10	24.144	3491.252	9.750	235.404	84708.812
11	16.428	3507.680	9.427	154.867	84863.625
12	4.500	3512.180	0.750	3.375	84867.000
13	261.250	3773.430	-1.000	-261.250	84605.750
14	189.000	3962.430	-14.875	-2811.375	81794.375
15	40.621	4003.052	11.688	474.784	82269.125
16	12.000	4015.052	11.770	141.240	82410.312
17	24.000	4039.052	10.442	250.608	82660.875
18	24.000	4063.052	8.021	192.504	82853.375
19	262.344	4325.395	14.574	3823.401	86676.750

YBAR = ACAY/ATA = 20.039 IN

YBAR =20.039 AGAINST ASSUMED YBAR =20.040 OK

ACAS =112.145 IN2 ACASY =4984.145 IN3

YBARS = ACASY/ACAS = 44.444 IN

ED = YBARS-YBAR = 24.405 IN

MOMENT OF INERTIA TABLE
FOR LOCATION NO.19 AND N = 15.

Y1= ABSOLUTE VALUE OF (Y-YBAR)
I1= TRANS'A'*Y1**2
TOTAL I =TOTAL I+I0+I1

NO	TRANS'A' IN2	Y1 IN	I0 IN4	I1 IN4	TOTAL I IN4
1	1064.699	45.070	0.0	2162725.000	2162725.000
2	54.000	22.570	0.0	27507.758	2190232.000
3	66.360	5.430	0.0	1956.640	2192188.000
4	132.370	13.102	0.0	22723.059	2214911.000
5	298.620	15.539	0.0	72105.125	2287016.000
6	11.200	23.852	0.0	6371.887	2293387.000
7	17.360	34.914	0.0	21161.637	2314548.000
8	1680.000	14.664	8826.750	361256.812	2684633.000
9	142.500	13.789	296.875	27094.574	2712024.000
10	24.144	10.289	8.048	2555.984	2714588.000
11	16.428	10.612	3.651	1850.042	2716441.000
12	4.500	19.289	20.250	1674.300	2718135.000
13	261.250	21.039	1964.818	115639.812	2835739.000
14	189.000	34.914	5103.000	230389.000	3071231.000
15	40.621	8.351	25.706	2832.931	3074089.000
16	12.000	8.269	10.667	820.523	3074920.000
17	24.000	9.597	21.333	2210.473	3077151.000
18	24.000	12.018	21.333	3466.394	3080638.000
19	262.344	5.465	2612.219	7835.316	3091085.000

I = 2.*(TOTAL I) = 6182170.IN4

SHRINKAGE CALCULATION

ES = MODULUS OF ELASTICITY OF STEEL
EC = EFFECTIVE MODULUS OF ELASTICITY OF CONCRETE
EPSH = SHRINKAGE COEFFICIENT
N = ES/EC

ES =29000. KSI
EC = ES/N = 1933. KSI
ECI = (EC*I/144.) = 0.830E 08 K-FT2
EPSH =.0002 IN/IN
TO = (EPSH*ES*ACAS)*2. = 1300.880 K
CM = TO*ED/12. = 2645.639 K-FT

DEFLECTION DUE TO DEAD LOAD

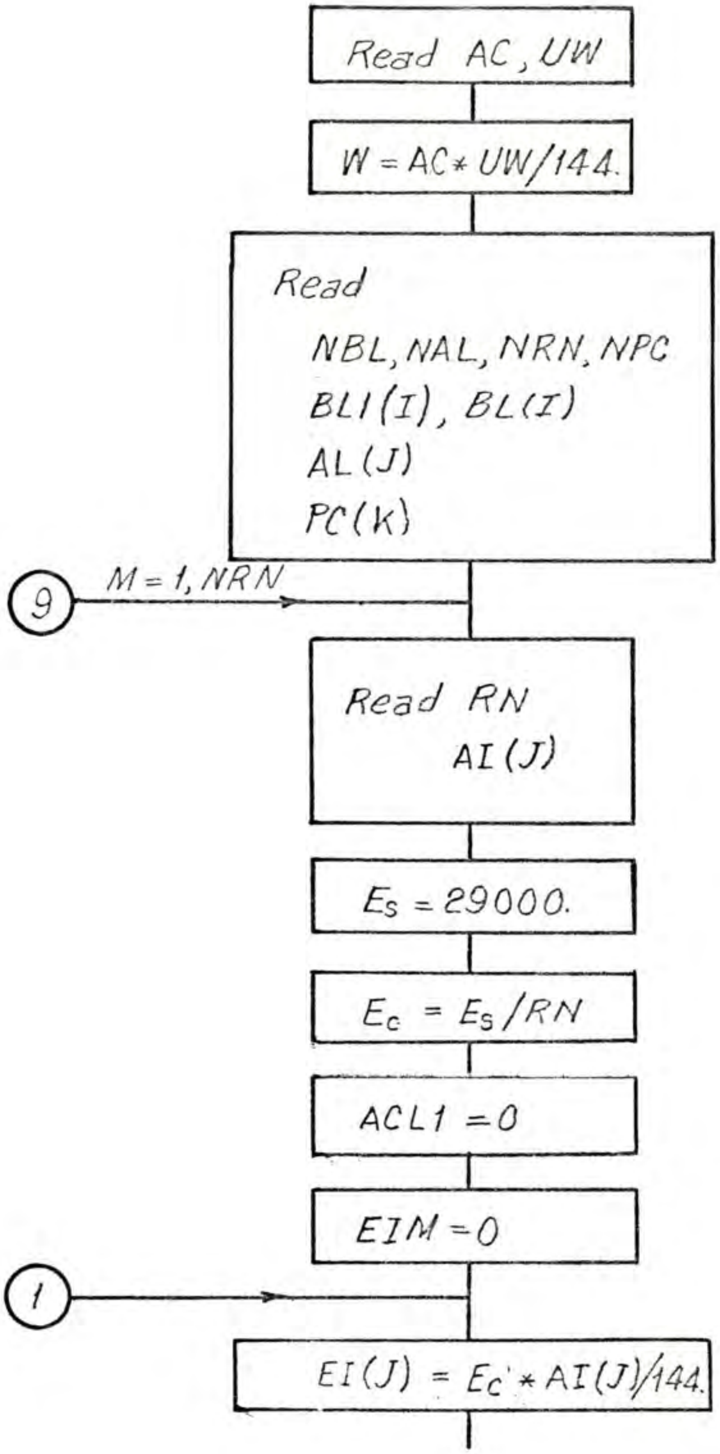
```
      DIMENSION BL(10),AL(50),AI(50),ACL(50),EI(50),B(50),PC(50)
      DIMENSION A(20,21),X(20),A1(50),A2(50),A3(50),A4(50),BL1(10)

    9 FORMAT(F10.3)
   11 FORMAT(F20.3,F10.3)
   12 FORMAT(I5)
   13 FORMAT(2F10.0)
   14 FORMAT(F8.0)
   15 FORMAT(F5.0)
   16 FORMAT(F10.0)
   17 FORMAT(10X,4HES =,F6.0,1X,3HKSI/10X,11HEC = ES/N =,F6.0,1X,3HKSI/)
   18 FORMAT(10X,32HW = AC * UNIT WEIGHT OF CONCRETE/12X,2H= ,F6.3,1X,4H
     $K/FT)
   19 FORMAT(10X,6(F6.2,1X),2X,1HX,I1,2X,F10.2)
   20 FORMAT(10X,4HC1 =,F9.3,2X,1HK/10X,4HR1 =,F9.3,2X,1HK/10X,4HR2 =,F
     $9.3,2X,1HK/10X,4HR3 =,F9.3,2X,1HK)
   21 FORMAT(10X,4HR4 =,F9.3,2X,1HK/10X,4HR5 =,F9.3,2X,1HK)
   22 FORMAT(10X,4HR6 =,F9.3,2X,1HK)
   61 FORMAT(1H1/////35X,8HFOR N = ,F3.0/27X,25HREACTIONS AND DEFLECTION
     $S/31X,16HDUE TO DEAD LOAD//)
   62 FORMAT(//37X,6HMATRIX/30X,1HA,24X,1HX,8X,1HB/)
   63 FORMAT(///10X,28HSLOPE CONSTANT AND REACTIONS/10X,28H------------
     $----------------/)

      READ(5,11)AC,UW
      W = AC*UW/144.
      READ(5,12)NBL,NAL,NRN,NPC
      READ(5,13)(BL1(I),BL(I),I=1,NBL)
      READ(5,14)(AL(J),J=1,NAL)
      READ(5,9)(PC(K),K=1,NPC)
      DO 900 M=1,NRN
      READ(5,15)RN
      READ(5,16)(AI(J),J=1,40)
      ES = 29000.
```

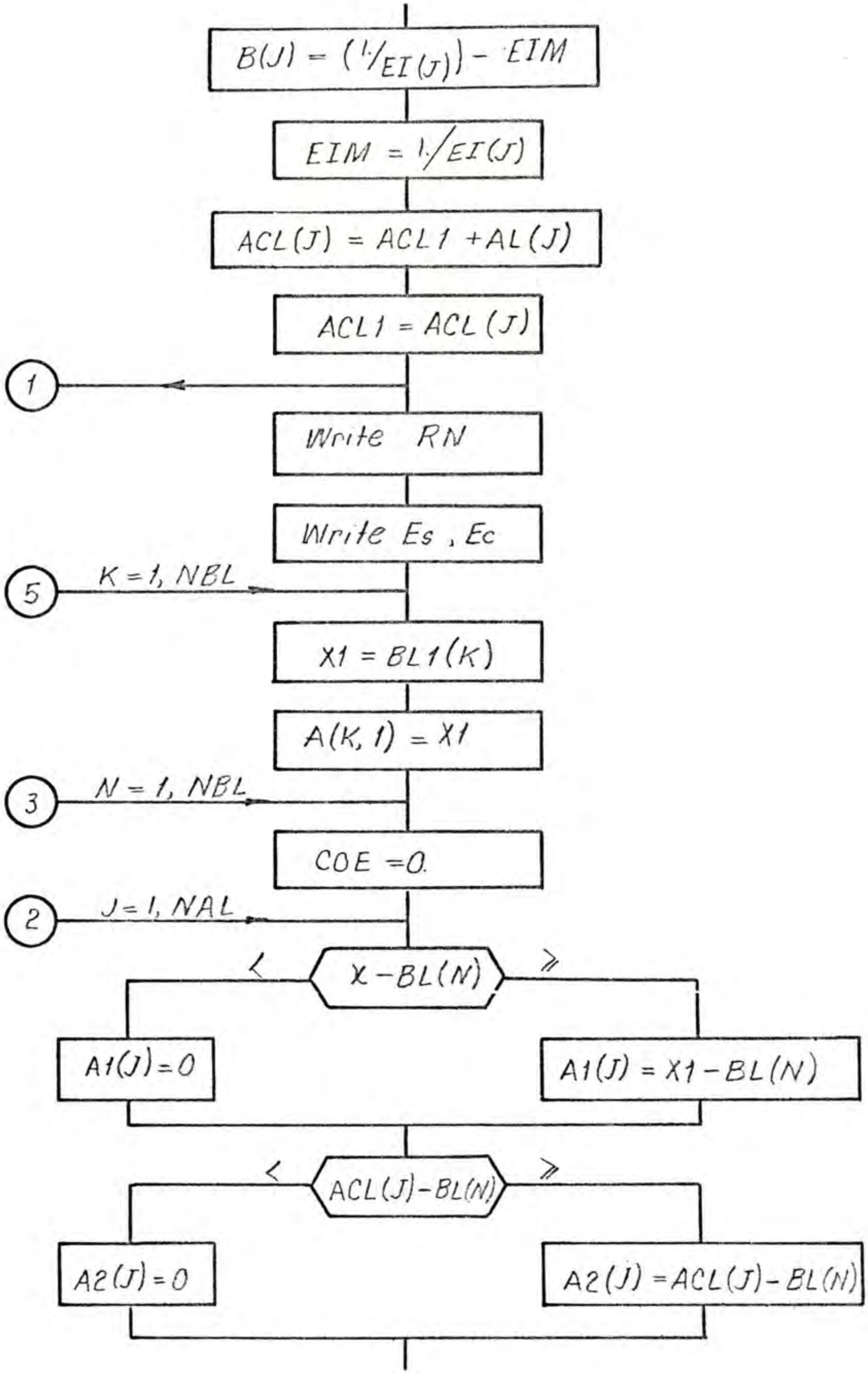
```
      EC = ES/RN
      ACLI = 0
      EIM = 0
      DO 100 J=1,NAL
      EI(J) = (EC*AI(J))/144.
      B(J) = ((1./EI(J))-EIM)
      EIM = (1./EI(J))
      ACL(J) = ACLI + AL(J)
      ACLI = ACL(J)
  100 CONTINUE
      WRITE(6,61)RN
      WRITE(6,17)ES,EC
      DO 500 K=1,NBL
      X1 = BLI(K)
      A(K,1) = X1
      DO 300 N = 1,NBL
      COE = 0.
      DO 200 J=1,NAL
      IF(X1-BL(N))30,40,40
   30 A1(J) =0
      GO TO 50
   40 A1(J) = (X1 - BL(N))
   50 IF(ACL(J)-BL(N))60,70,70
   60 A2(J) =0
      GO TO 80
   70 A2(J) = (ACL(J)-BL(N))
   80 IF(X1-ACL(J))90,110,110
   90 A3(J) =0
      GO TO 120
  110 A3(J) =1.
  120 IF(X1-ACL(J))130,140,140
  130 A4(J) =0
      GO TO 150
  140 A4(J) = (X1-ACL(J))
  150 B1=B(J)*(((A1(J)**3/6.)-(A2(J)**3/6.))*A3(J)-(A2(J)**2/2.)*A4(J))
  200 COE = COE+B1
```

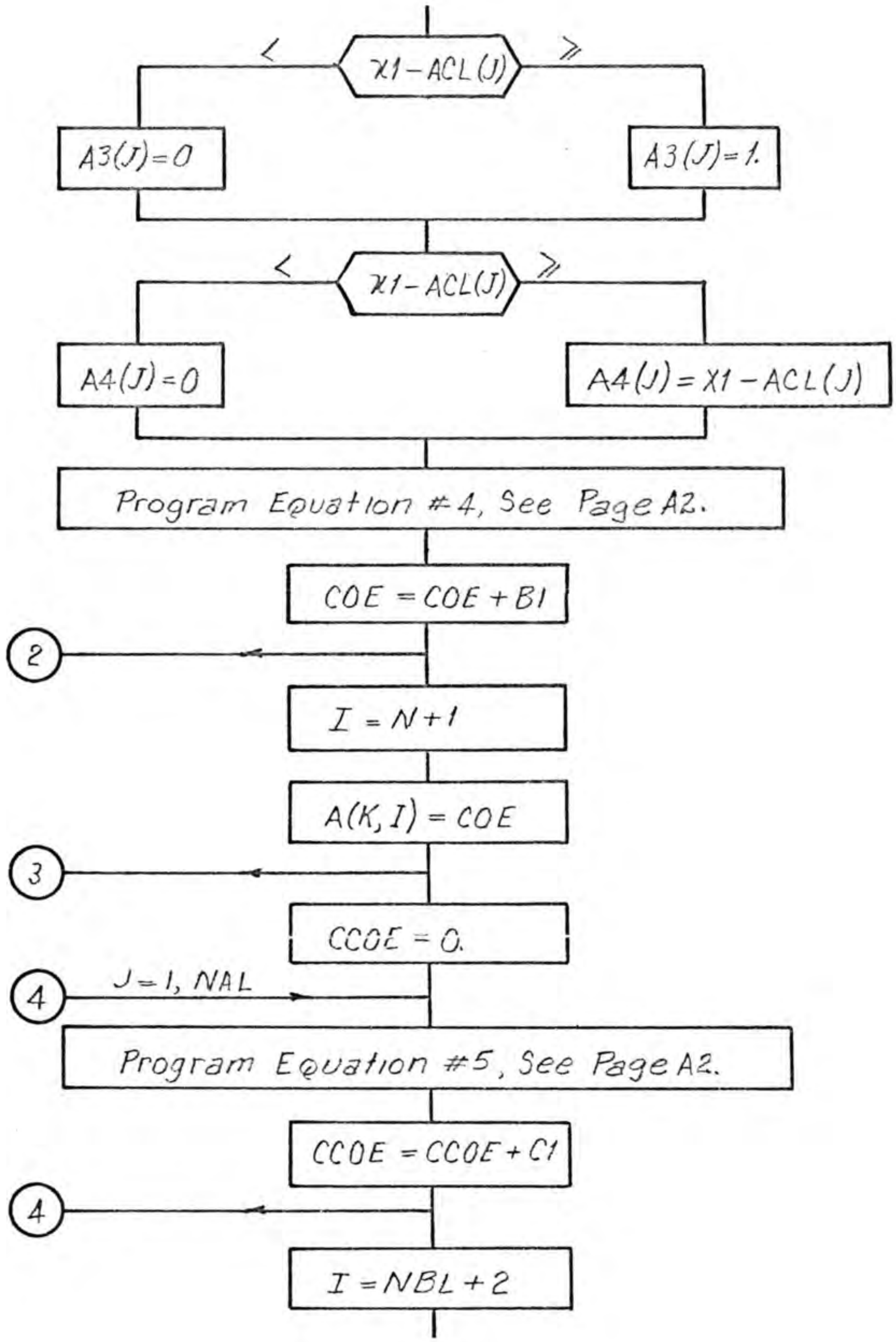
```
      I = N+1
  300 A(K,I) = COE
      CCOE = 0.
      DO 400 J=1,NAL
      C1 = B(J)*(((X1**4-ACL(J)**4)*A3(J)/12.)-((ACL(J)**3)*A4(J)/3.))
  400 CCOE = CCOE+C1
      I = NBL+2
  500 A(K,I) =(W*CCOE/2.)
      K = NBL+1
      A(K,1)=0
      A(K,2) = BL1(NBL)
      DO 700 I1=3,K
      J = I1-2
  700 A(K,I1)= BL1(NBL)-BL1(J)
      A(K,I) = (W*BL1(NBL)**2)/2.
      WRITE(6,18)W
      WRITE(6,62)
      DO 600 K=1,6
  600 WRITE(6,19)(A(K,I),I=1,6),K,A(K,7)
      N = NBL +1
      CALL SIMILQ(N,A,X,NOGO)
      IF(NOGO)160,170,160
  160 CALL EXIT
      GO TO 900
  170 WRITE(6,63)
      WRITE(6,20)(X(K),K=1,4)
      WRITE(6,21)(X(K),K=5,N)
      RNBL = W*BL1(NBL)
      DO 800 I=2,N
  800 RNBL = RNBL-X(I)
      WRITE(6,22)RNBL
      CALL DEADDE(PC,X,BL,ACL,B,W,NBL,NAL,NPC)
  900 CONTINUE
      END
```

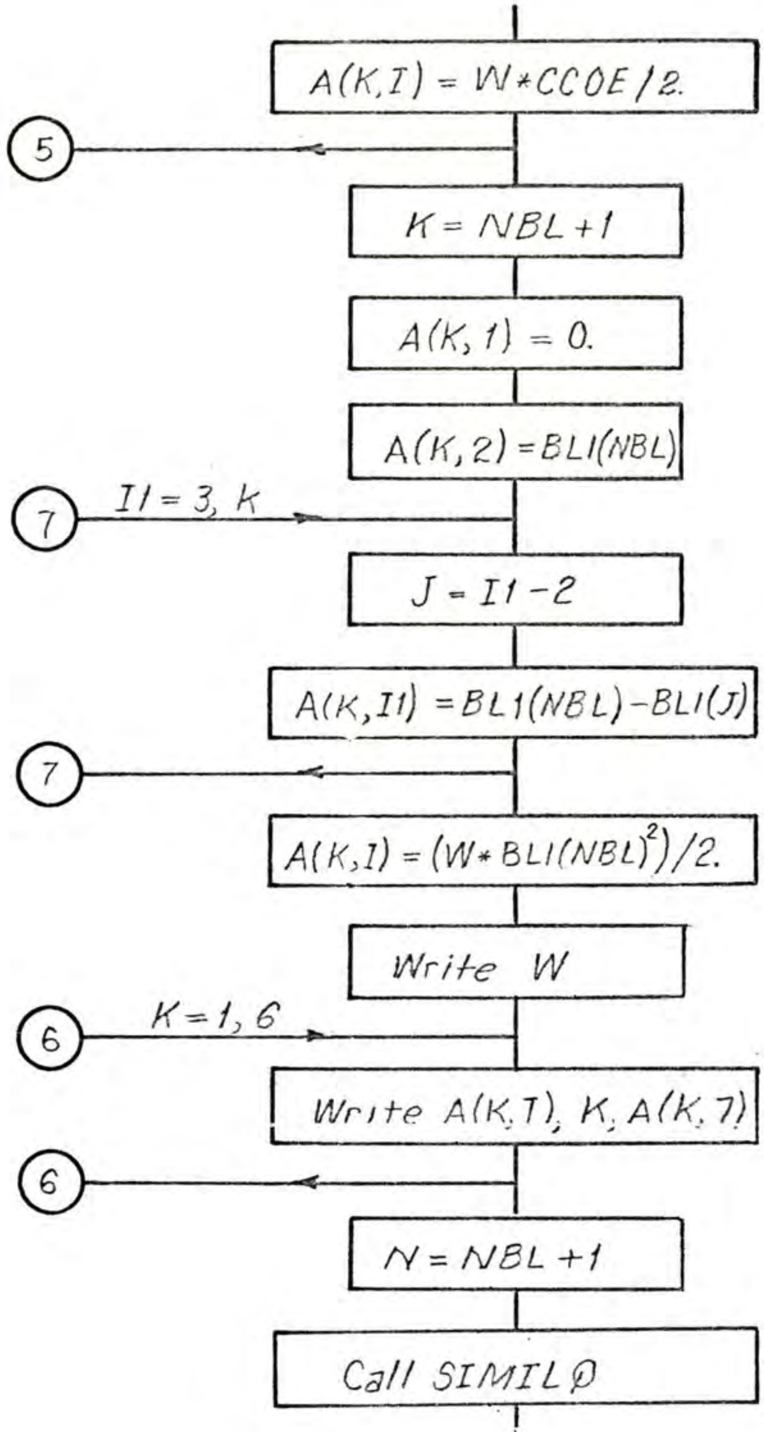
Flow Diagram

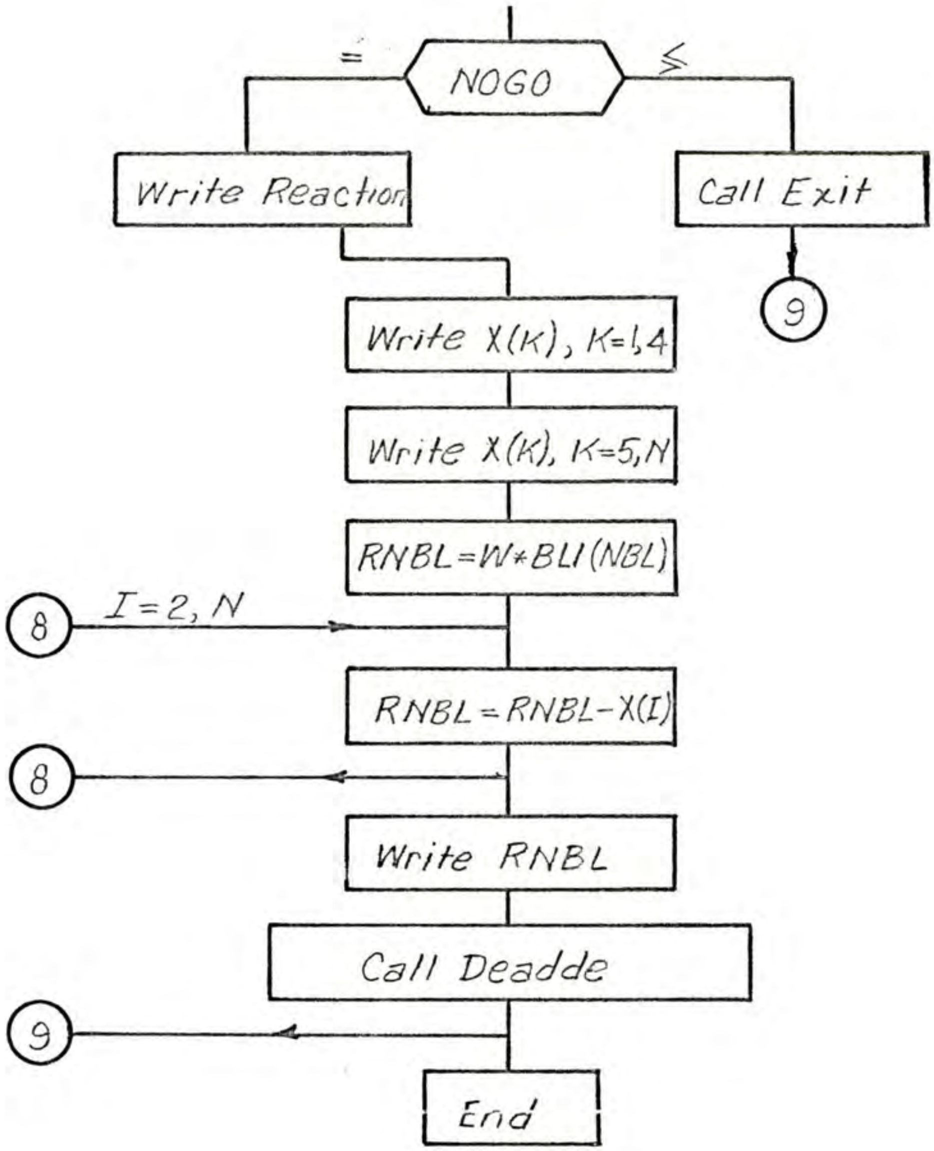
Dead Load Deflection




B(J) = (1./EI(J)) - EIM
EIM = 1./EI(J)
ACL(J) = ACL1 + AL(J)
ACL1 = ACL(J)
1
Write RN
Write Es , Ec
5
K = 1, NBL
X1 = BL1(K)
A(K, 1) = X1
3
N = 1, NBL
COE = 0.
2
J = 1, NAL
X - BL(N)
<
≥
A1(J) = 0
A1(J) = X1 - BL(N)
ACL(J) - BL(N)
<
≥
A2(J) = 0
A2(J) = ACL(J) - BL(N)


X1 - ACL(J)
<
≥
A3(J)=0
A3(J)=1.
X1 - ACL(J)
<
≥
A4(J)=0
A4(J) = X1 - ACL(J)
Program Equation #4, See Page A2.
COE = COE + B1
2
I = N+1
A(K, I) = COE
3
CCOE = 0.
4
J = 1, NAL
Program Equation #5, See Page A2.
CCOE = CCOE + C1
4
I = NBL + 2

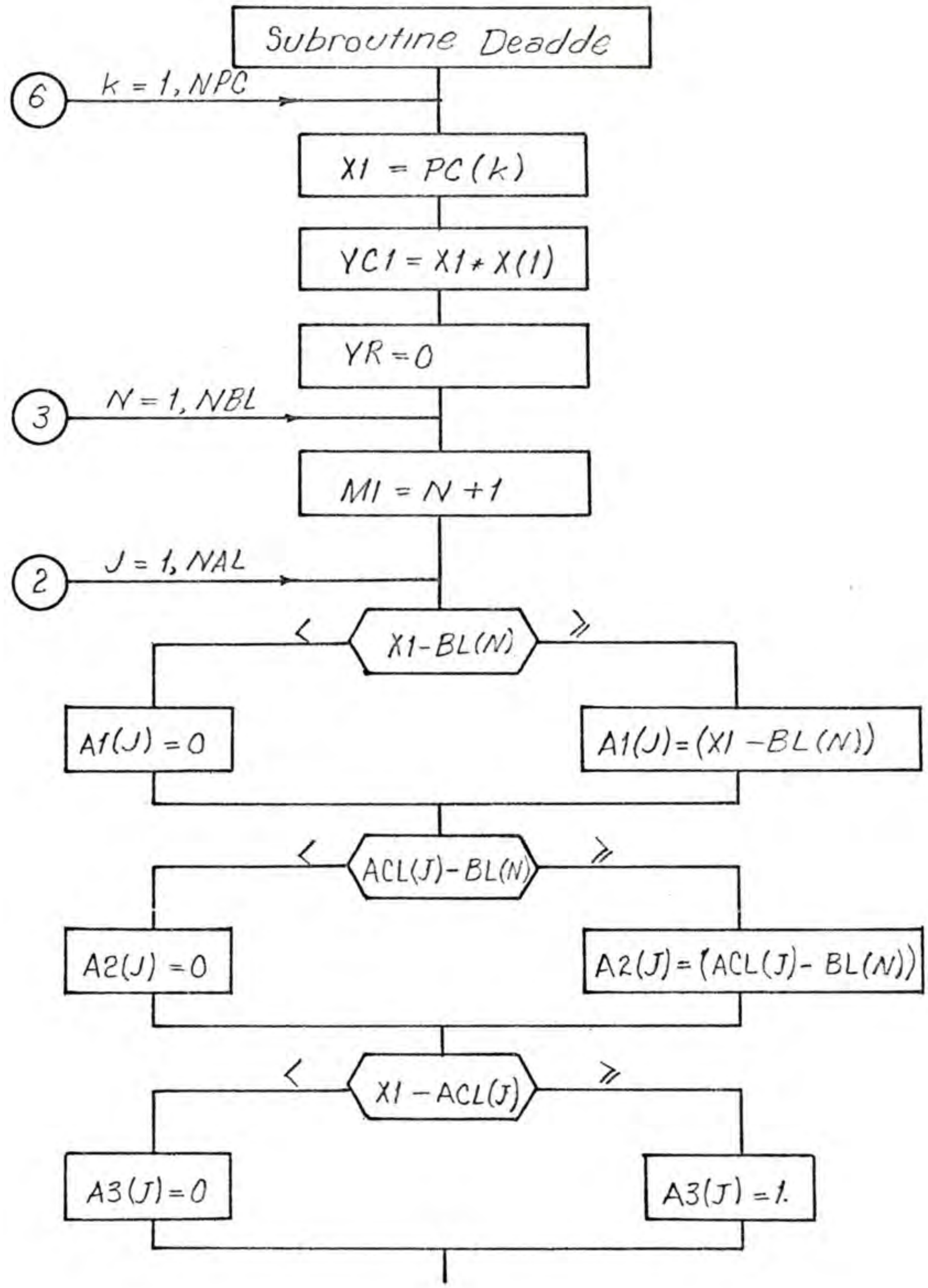

A(K,I) = W*CCOE/2.
5
K = NBL+1
A(K,1) = 0.
A(K,2) = BLI(NBL)
7
I1 = 3, K
J = I1-2
A(K,I1) = BL1(NBL)-BLI(J)
7
A(K,I) = (W*BLI(NBL)²)/2.
Write W
6
K = 1, 6
Write A(K,I), K, A(K,7)
6
N = NBL+1
Call SIMILØ

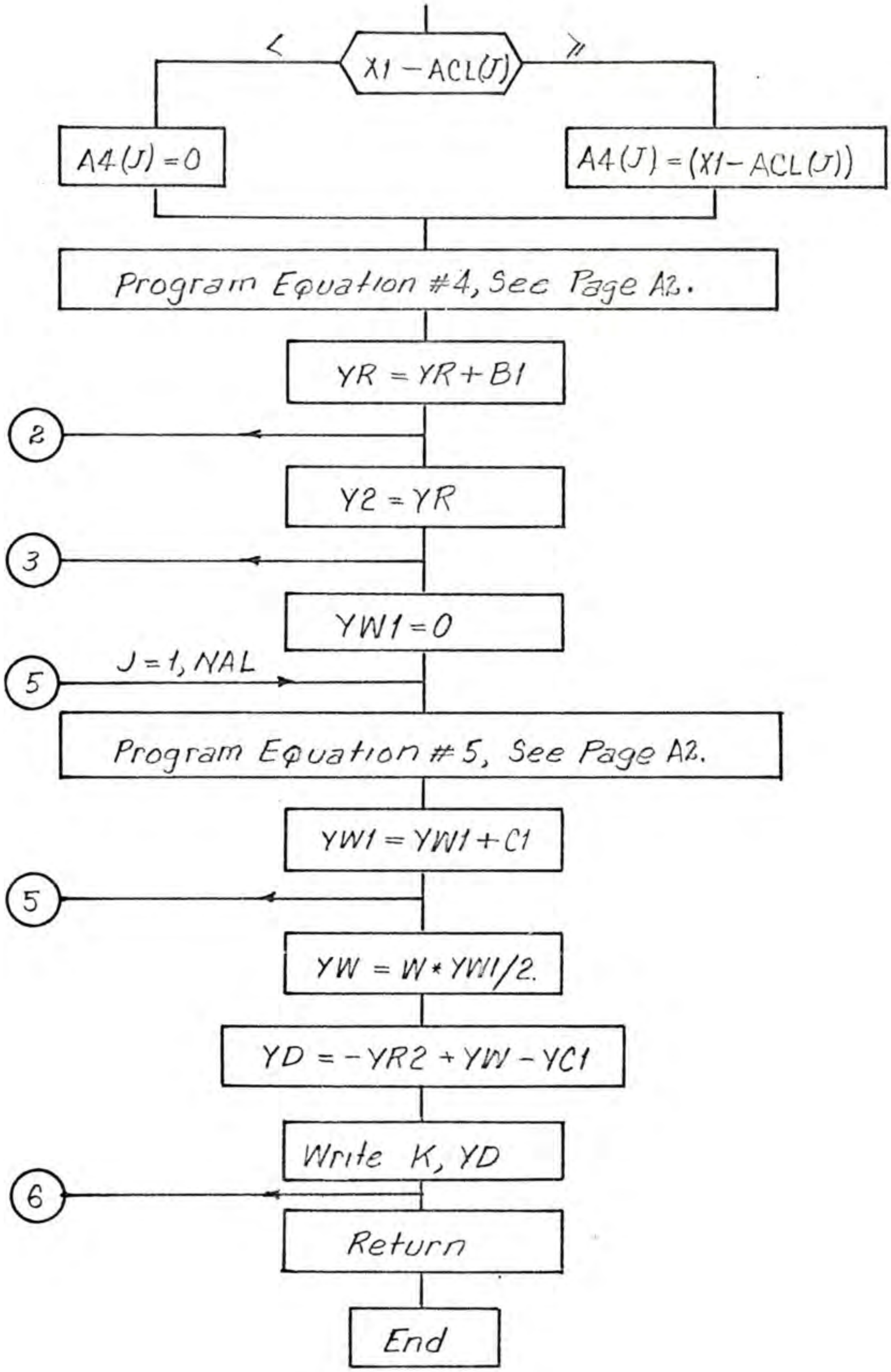

NOGO
=
≤
Write Reaction
Call Exit
9
Write X(K), K=1,4
Write X(K), K=5,N
RNBL=W*BLI(NBL)
8
I=2, N
RNBL=RNBL-X(I)
8
Write RNBL
Call Deadde
9
End

```
      SUBROUTINE DEADDE(PC,X,BL,ACL,B,W,NBL,NAL,NPC)
      DIMENSION PC(50),X(10),BL(10),ACL(50),B(50)
      DIMENSION A1(50),A2(50),A3(50),A4(50)

   31 FORMAT(10X,15HDEFLECTION AT D,I1,2H =,F10.5,1X,2HFT)
   71 FORMAT( //10X,11HDEFLECTIONS/10X,11H-----------/)

      WRITE(6,71)
      DO 600 K=1,NPC
      X1 = PC(K)
      YC1 = X(1)*X1
      YR = 0
      DO 300 N = 1,NBL
      M1 = N+1
      DO 200 J=1,NAL
      IF(X1-BL(N))30,40,40
   30 A1(J) =0
      GO TO 50
   40 A1(J) = (X1 - BL(N))
   50 IF(ACL(J)-BL(N))60,70,70
   60 A2(J) =0
      GO TO 80
   70 A2(J) = (ACL(J)-BL(N))
   80 IF(X1-ACL(J))90,110,110
   90 A3(J) =0
      GO TO 120
  110 A3(J) =1.
  120 IF(X1-ACL(J))130,140,140
  130 A4(J) =0
      GO TO 150
  140 A4(J) = (X1-ACL(J))
  150 B1=B(J)*(((A1(J)**3/6.)-(A2(J)**3/6.))*A3(J)-(A2(J)**2/2.)*A4(J))*
     $X(M1)
  200 YR = YR+B1
  300 YR2 = YR
```

```
      YW1 = 0
      DO 500 J=1,NAL
      C1 = B(J)*(((X1**4-ACL(J)**4)*A3(J)/12.)-((ACL(J)**3)*A4(J)/3.))
  500 YW1 =YW1+C1
      YW = W*YW1/2.
      YD = -YR2+YW-YC1
  600 WRITE(6,31)K,YD
      RETURN
      END
```

Flow Diagram





X1 − ACL(J)
<
≥
A4(J) = 0
A4(J) = (X1 − ACL(J))
Program Equation #4, See Page A2.
YR = YR + B1
2
Y2 = YR
3
YW1 = 0
J = 1, NAL
5
Program Equation #5, See Page A2.
YW1 = YW1 + C1
5
YW = W * YW1/2.
YD = −YR2 + YW − YC1
Write K, YD
6
Return
End

FOR N = 5.
REACTIONS AND DEFLECTIONS
DUE TO DEAD LOAD

ES =29000. KSI
EC = ES/N = 5800. KSI

W = AC * UNIT WEIGHT OF CONCRETE
= 10.663 K/FT

MATRIX

A						X	B
64.00	0.00	0.0	0.0	0.0	0.0	X1	0.07
144.00	0.01	0.00	0.0	0.0	0.0	X2	1.94
249.00	0.02	0.01	0.00	0.0	0.0	X3	15.78
354.00	0.07	0.04	0.01	0.00	0.0	X4	66.07
438.00	0.13	0.08	0.04	0.01	0.00	X5	158.14
0.0	438.00	374.00	294.00	189.00	84.00	X6	1022836.00

SLOPE CONSTANT AND REACTIONS

C1 = -0.001 K
R1 = 257.571 K
R2 = 813.776 K
R3 = 1003.641 K
R4 = 1142.990 K
R5 = 1125.857 K
R6 = 326.646 K

DEFLECTIONS

DEFLECTION AT D1 = 0.00001 FT
DEFLECTION AT D2 = 0.02479 FT
DEFLECTION AT D3 = 0.03826 FT
DEFLECTION AT D4 = 0.02111 FT
DEFLECTION AT D5 = 0.00011 FT
DEFLECTION AT D6 = 0.01838 FT
DEFLECTION AT D7 = 0.03371 FT
DEFLECTION AT D8 = 0.01950 FT
DEFLECTION AT D9 = 0.00067 FT

FOR N = 10.
REACTIONS AND DEFLECTIONS
DUE TO DEAD LOAD

ES =29000. KSI
EC = ES/N = 2900. KSI

W = AC * UNIT WEIGHT OF CONCRETE
= 10.663 K/FT

MATRIX

A						X	B
64.00	0.00	0.0	0.0	0.0	0.0	X1	0.09
144.00	0.01	0.00	0.0	0.0	0.0	X2	2.38
249.00	0.03	0.01	0.00	0.0	0.0	X3	19.12
354.00	0.08	0.05	0.02	0.00	0.0	X4	79.10
438.00	0.16	0.10	0.05	0.01	0.00	X5	188.41
0.0	438.00	374.00	294.00	189.00	84.00	X6	1022836.00

SLOPE CONSTANT AND REACTIONS

C1 = -0.001 K
R1 = 258.525 K
R2 = 813.613 K
R3 = 1002.505 K
R4 = 1144.142 K
R5 = 1122.992 K
R6 = 328.705 K

DEFLECTIONS

DEFLECTION AT D1 = 0.00002 FT
DEFLECTION AT D2 = 0.02932 FT
DEFLECTION AT D3 = 0.04511 FT
DEFLECTION AT D4 = 0.02490 FT
DEFLECTION AT D5 = 0.00029 FT
DEFLECTION AT D6 = 0.02191 FT
DEFLECTION AT D7 = 0.03976 FT
DEFLECTION AT D8 = 0.02312 FT
DEFLECTION AT D9 = 0.00063 FT

FOR N = 15.
REACTIONS AND DEFLECTIONS
DUE TO DEAD LOAD

ES =29000. KSI
EC = ES/N = 1933. KSI

W = AC * UNIT WEIGHT OF CONCRETE
= 10.663 K/FT

MATRIX

A						X	B
64.00	0.00	0.0	0.0	0.0	0.0	X1	0.10
144.00	0.01	0.00	0.0	0.0	0.0	X2	2.72
249.00	0.03	0.01	0.00	0.0	0.0	X3	21.76
354.00	0.10	0.05	0.02	0.00	0.0	X4	89.56
438.00	0.18	0.11	0.06	0.02	0.00	X5	212.84
0.0	438.00	374.00	294.00	189.00	84.00	X6	1022836.00

SLOPE CONSTANT AND REACTIONS

C1 = -0.001 K
R1 = 259.589 K
R2 = 812.969 K
R3 = 1001.825 K
R4 = 1145.113 K
R5 = 1120.511 K
R6 = 330.477 K

DEFLECTIONS

DEFLECTION AT D1 = 0.00008 FT
DEFLECTION AT D2 = 0.03307 FT
DEFLECTION AT D3 = 0.05072 FT
DEFLECTION AT D4 = 0.02808 FT
DEFLECTION AT D5 = 0.00039 FT
DEFLECTION AT D6 = 0.02484 FT
DEFLECTION AT D7 = 0.04493 FT
DEFLECTION AT D8 = 0.02630 FT
DEFLECTION AT D9 = 0.00058 FT

FOR N = 20.
REACTIONS AND DEFLECTIONS
DUE TO DEAD LOAD

ES =29000. KSI
EC = ES/N = 1450. KSI

W = AC * UNIT WEIGHT OF CONCRETE
= 10.663 K/FT

MATRIX

A						X	B
64.00	0.00	0.0	0.0	0.0	0.0	X1	0.12
144.00	0.01	0.00	0.0	0.0	0.0	X2	3.04
249.00	0.04	0.01	0.00	0.0	0.0	X3	24.11
354.00	0.11	0.06	0.02	0.00	0.0	X4	98.61
438.00	0.20	0.12	0.06	0.02	0.00	X5	233.75
0.0	438.00	374.00	294.00	189.00	84.00	X6	1022836.00

SLOPE CONSTANT AND REACTIONS

C1 = -0.001 K
R1 = 260.384 K
R2 = 812.390 K
R3 = 1001.558 K
R4 = 1145.625 K
R5 = 1118.725 K
R6 = 331.800 K

DEFLECTIONS

DEFLECTION AT D1 = 0.00004 FT
DEFLECTION AT D2 = 0.03615 FT
DEFLECTION AT D3 = 0.05543 FT
DEFLECTION AT D4 = 0.03079 FT
DEFLECTION AT D5 = 0.00040 FT
DEFLECTION AT D6 = 0.02734 FT
DEFLECTION AT D7 = 0.04956 FT
DEFLECTION AT D8 = 0.02927 FT
DEFLECTION AT D9 = 0.00056 FT

FOR N = 25.
REACTIONS AND DEFLECTIONS
DUE TO DEAD LOAD

ES =29000. KSI
EC = ES/N = 1160. KSI

W = AC * UNIT WEIGHT OF CONCRETE
= 10.663 K/FT

MATRIX

A						X	B
64.00	0.00	0.0	0.0	0.0	0.0	X1	0.13
144.00	0.01	0.00	0.0	0.0	0.0	X2	3.30
249.00	0.04	0.02	0.00	0.0	0.0	X3	26.13
354.00	0.12	0.06	0.02	0.00	0.0	X4	106.51
438.00	0.22	0.13	0.06	0.02	0.00	X5	252.06
0.0	438.00	374.00	294.00	189.00	84.00	X6	1022836.00

SLOPE CONSTANT AND REACTIONS

C1 = -0.001 K
R1 = 261.112 K
R2 = 811.764 K
R3 = 1001.475 K
R4 = 1145.927 K
R5 = 1117.330 K
R6 = 332.876 K

DEFLECTIONS

DEFLECTION AT D1 = 0.00004 FT
DEFLECTION AT D2 = 0.03897 FT
DEFLECTION AT D3 = 0.05972 FT
DEFLECTION AT D4 = 0.03339 FT
DEFLECTION AT D5 = 0.00040 FT
DEFLECTION AT D6 = 0.02953 FT
DEFLECTION AT D7 = 0.05333 FT
DEFLECTION AT D8 = 0.03128 FT
DEFLECTION AT D9 = 0.00033 FT

DEFLECTION DUE TO SHRINKAGE (WARPING) MOMENT

```
      DIMENSION BL(10),AL(50),C(50),AI(50),ACL(50),EI(50),B(50),PC(50)
      DIMENSION A(20,21),X(20),A1(50),A2(50),A3(50),A4(50),BL1(10)
      DIMENSION TE(50)
    9 FORMAT(F10.3)
   11 FORMAT(F10.0)
   12 FORMAT(I5)
   13 FORMAT(2F10.0)
   14 FORMAT(F8.0)
   15 FORMAT(F5.0)
   16 FORMAT(F10.0)
   17 FORMAT(10X,I2,F10.0,2X,E9.3,2X,E10.3,2X,F10.3,2X,E10.3)
   18 FORMAT(10X,4HES =,F6.0,1X,3HKSI/10X,11HEC = ES/N =,F6.0,1X,3HKSI/)
   19 FORMAT(10X,6(F6.2,1X),2X,1HX,I1,2X,F10.2)
   20 FORMAT(10X,4HC1 =,F9.3,2X,1HK/10X,4HR1 =,F9.3,2X,1HK/10X,4HR2 =,F
     $9.3,2X,1HK/10X,4HR3 =,F9.3,2X,1HK)
   21 FORMAT(10X,4HR4 =,F9.3,2X,1HK/10X,4HR5 =,F9.3,2X,1HK)
   22 FORMAT(10X,4HR6 =,F9.3,2X,1HK)
   61 FORMAT(1H1//29X,24HTABLE OF DISCONTINUITIES/33X,7HFOR N =,F3.0/)
   62 FORMAT(10X,31HTE = SHRINKAGE (WARPING) MOMENT/)
   63 FORMAT(10X,1HJ,7X,1HI,9X,2HEI,11X,1HB,10X,2HTE,10X,1HC)
   64 FORMAT(17X,3HIN4,6X,5HK-FT2,5X,9H1/(K-FT2),6X,4HK-FT,7X,4H1/FT)
   65 FORMAT(1H1,///35X,8HFOR N = ,F3.0,/27X,25HREACTIONS AND DEFLECTION
     $S/31X,16HDUE TO SHRINKAGE//)
   66 FORMAT(//37X,6HMATRIX/30X,1HA,24X,1HX,8X,1HB/)
   67 FORMAT( //10X,28HSLOPE CONSTANT AND REACTIONS/10X,28H------------
     $----------------/)
  101 FORMAT(9X,62H-----------------------------------------------------
     $---------)

      READ(5,12)NBL,NAL,NRN,NPC
      READ(5,13)(BL1(I),BL(I),I=1,NBL)
      READ(5,14)(AL(J),J=1,NAL)
      READ(5,9)(PC(K),K=1,NPC)
```

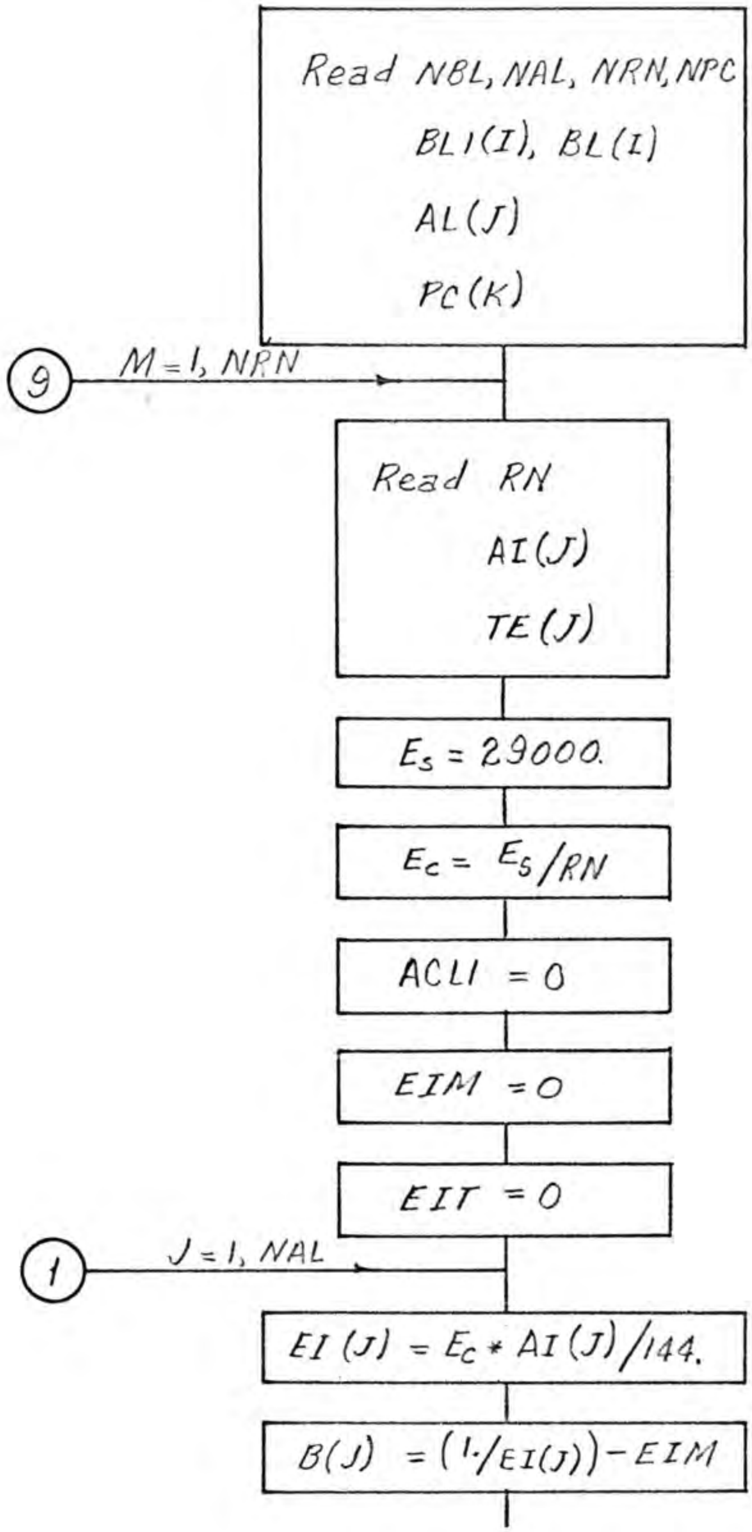
```
      DO 900 M=1,NRN
      READ(5,15)RN
      READ(5,16)(AI(J),J=1,NAL)
      READ(5,11)(TE(J),J=1,NAL)
      ES = 29000.
      EC = ES/RN
      WRITE(6,61)RN
      WRITE(6,62)
      WRITE(6,101)
      WRITE(6,63)
      WRITE(6,64)
      WRITE(6,101)
      ACL1 = 0
      EIM = 0
      EIT = 0.
      DO 100 J=1,NAL
      EI(J) = (EC*AI(J))/144.
      B(J) = ((1./EI(J))-EIM)
      EIM = (1./EI(J))
      ACL(J) = ACL1 + AL(J)
      ACL1 = ACL(J)
      C(J) = ((TE(J)/EI(J))-EIT)
      EIT = (TE(J)/EI(J))
      WRITE(6,17)J,AI(J),EI(J),B(J),TE(J),C(J)
  100 CONTINUE
      WRITE(6,101)
      WRITE(6,65)RN
      WRITE(6,18)ES,EC
      DO 500 K=1,NBL
      X1 = BL1(K)
      A(K,1) = X1
      DO 300 N = 1,NBL
      COE = 0.
      DO 200 J=1,NAL
```

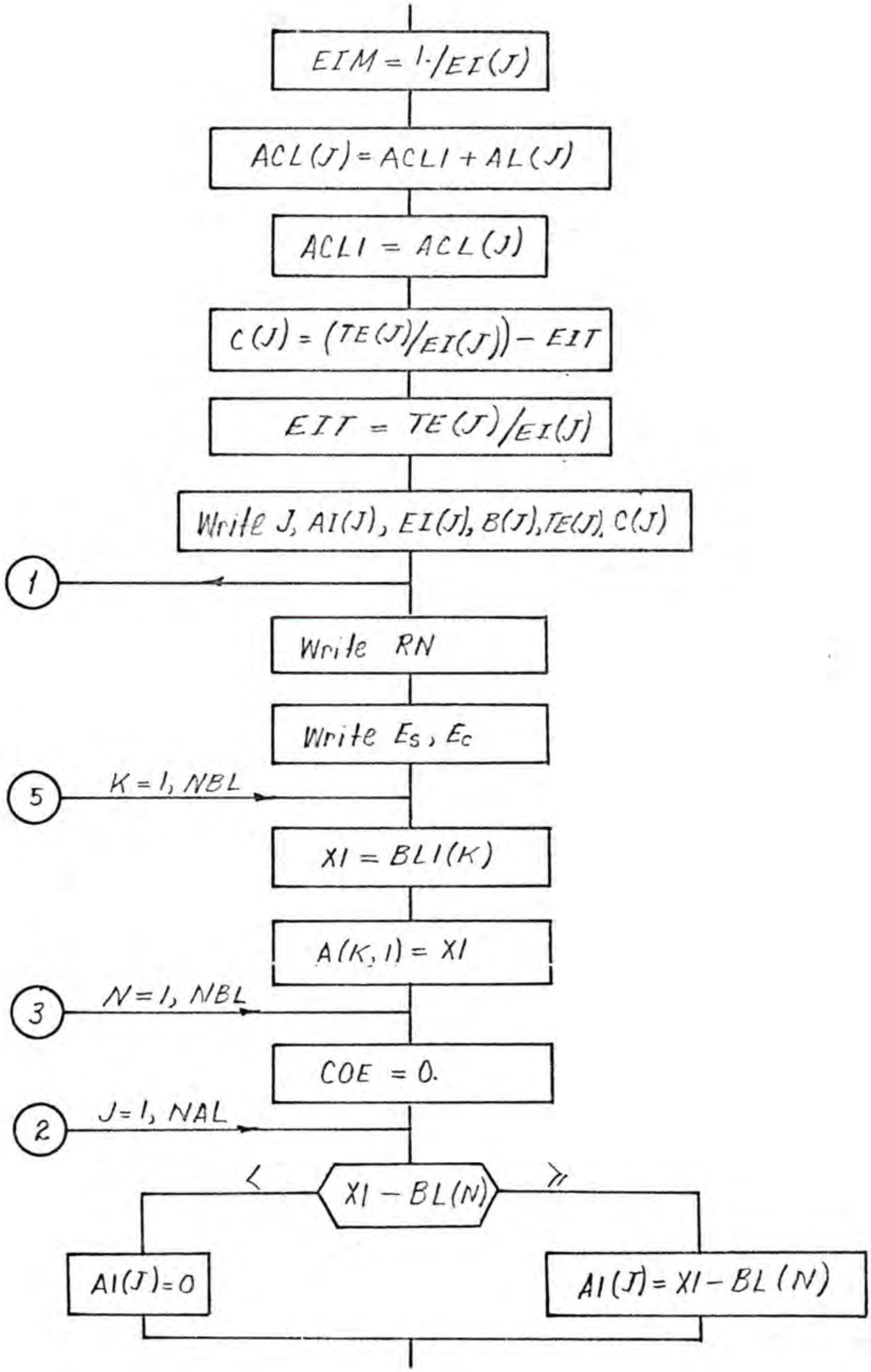
```
      IF(X1-BL(N))30,40,40
   30 A1(J) =0
      GO TO 50
   40 A1(J) = (X1 - BL(N))
   50 IF(ACL(J)-BL(N))60,70,70
   60 A2(J) =0
      GO TO 80
   70 A2(J) = (ACL(J)-BL(N))
   80 IF(X1-ACL(J))90,110,110
   90 A3(J) =0
      GO TO 120
  110 A3(J) =1.
  120 IF(X1-ACL(J))130,140,140
  130 A4(J) =0
      GO TO 150
  140 A4(J) = (X1-ACL(J))
  150 B1=B(J)*(((A1(J)**3/6.)-(A2(J)**3/6.))*A3(J)-(A2(J)**2/2.)*A4(J))
  200 COE = COE+B1
      I = N+1
  300 A(K,I) = COE
      CCOE = 0.
      DO 400 J=1,NAL
      C1 = C(J)*A4(J)**2
  400 CCOE = CCOE+C1
      I = NBL+2
  500 A(K,I) = -CCOE/2.
      K = NBL+1
      A(K,1)=0
      A(K,2) = BL1(NBL)
      DO 700 I1=3,K
      J = I1-2
  700 A(K,I1)= BL1(NBL)-BL1(J)
      CTF = 0.
      DO 750 J=1,NAL
      C2 = TF(J)*A3(J)
```

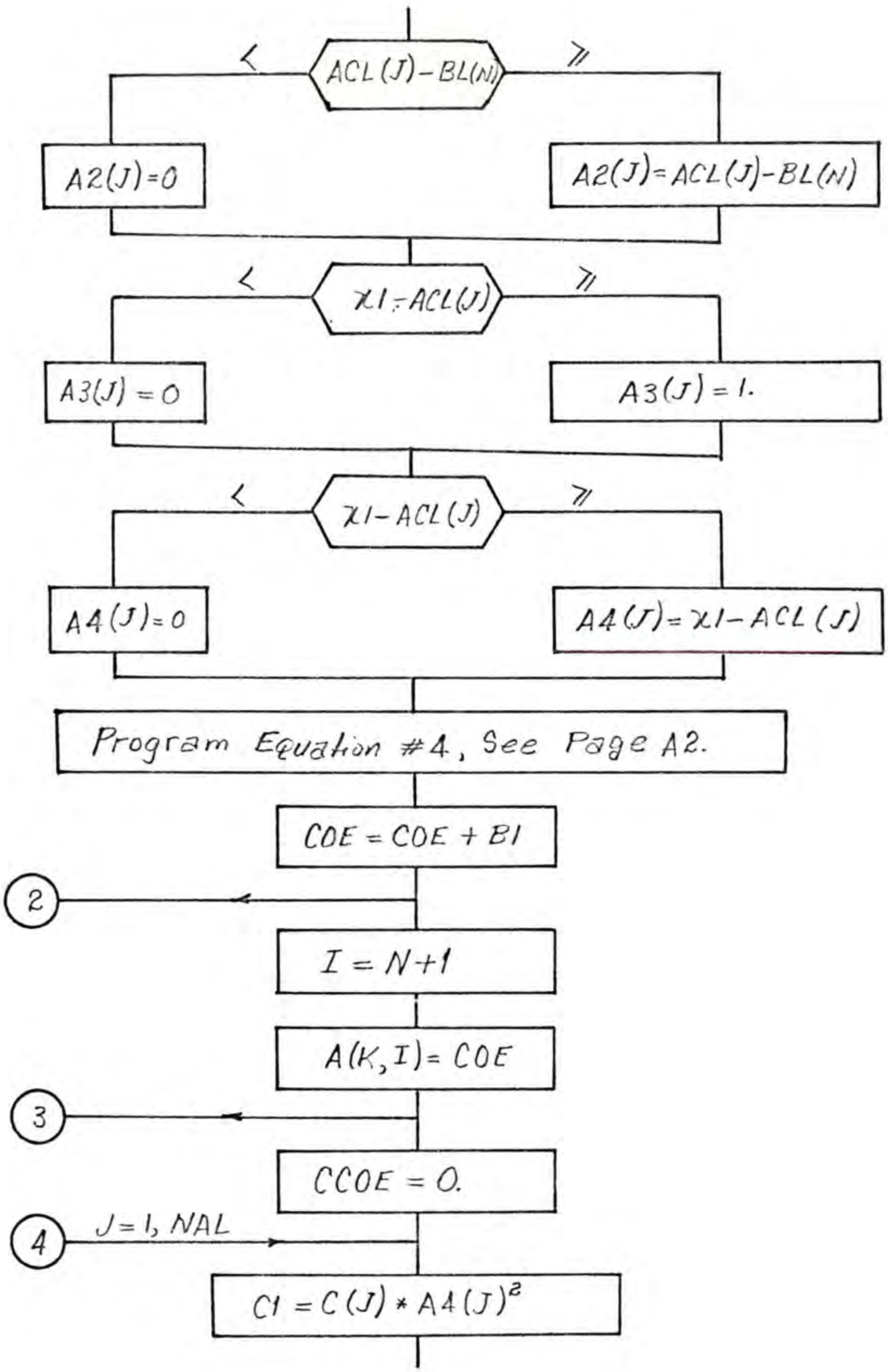
```
750 CTE = CTE+C2
    A(K,I) = -CTE
    WRITE(6,66)
    DO 600 K=1,6
600 WRITE(6,19)(A(K,I),I=1,6),K,A(K,7)
    N = NBL +1
    CALL SIMILQ(N,A,X,NOGO)
    IF(NOGO)160,170,160
160 CALL EXIT
    GO TO 900
170 WRITE(6,67)
    WRITE(6,20)(X(K),K=1,4)
    WRITE(6,21)(X(K),K=5,N)
    RNBL = 0.
    DO 800 I=2,N
800 RNBL = RNBL-X(I)
    WRITE(6,22)RNBL
    CALL SHRIDE(PC,X,BL,ACL,B,TE,NBL,NAL,NPC,C)
900 CONTINUE
    END
```

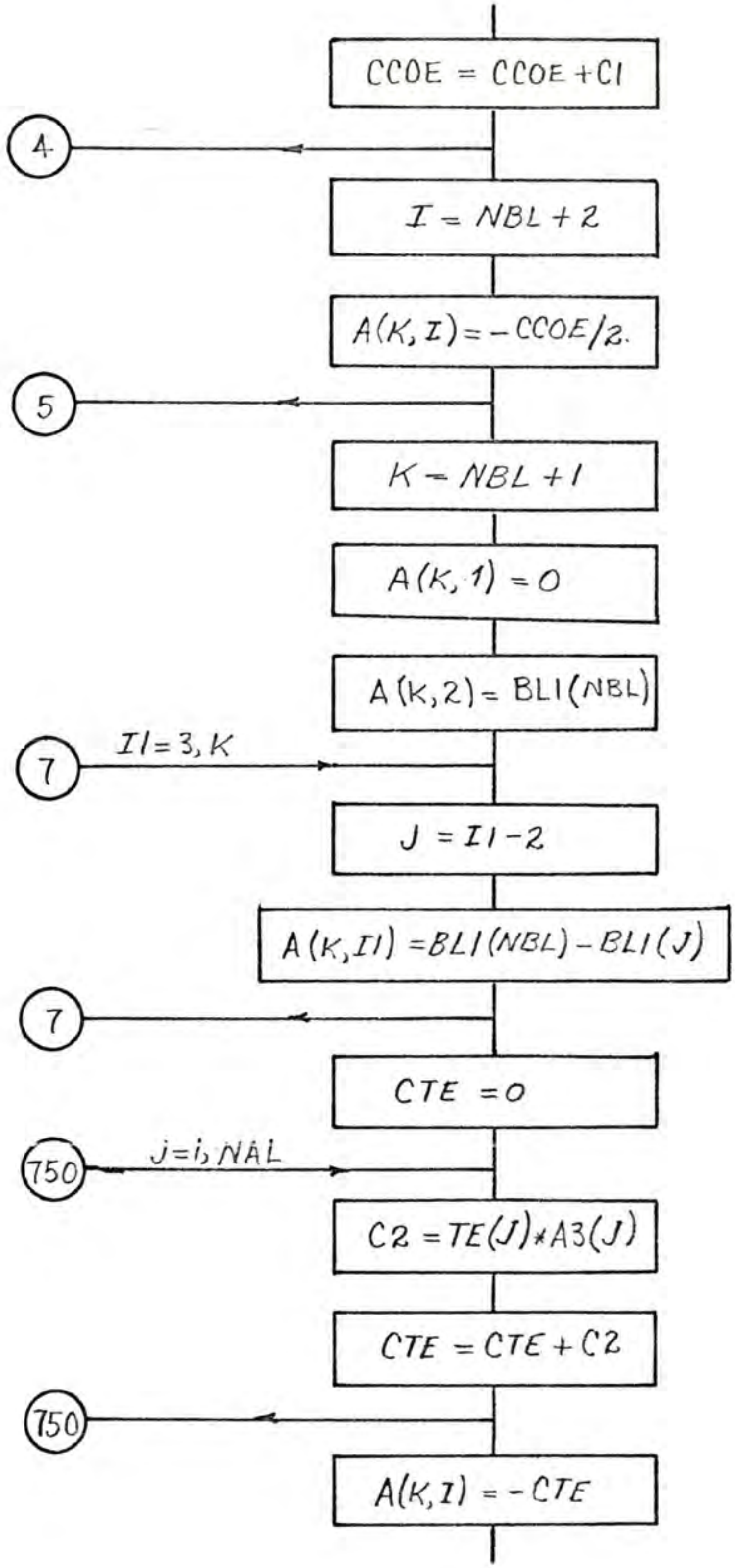
Flow Diagram

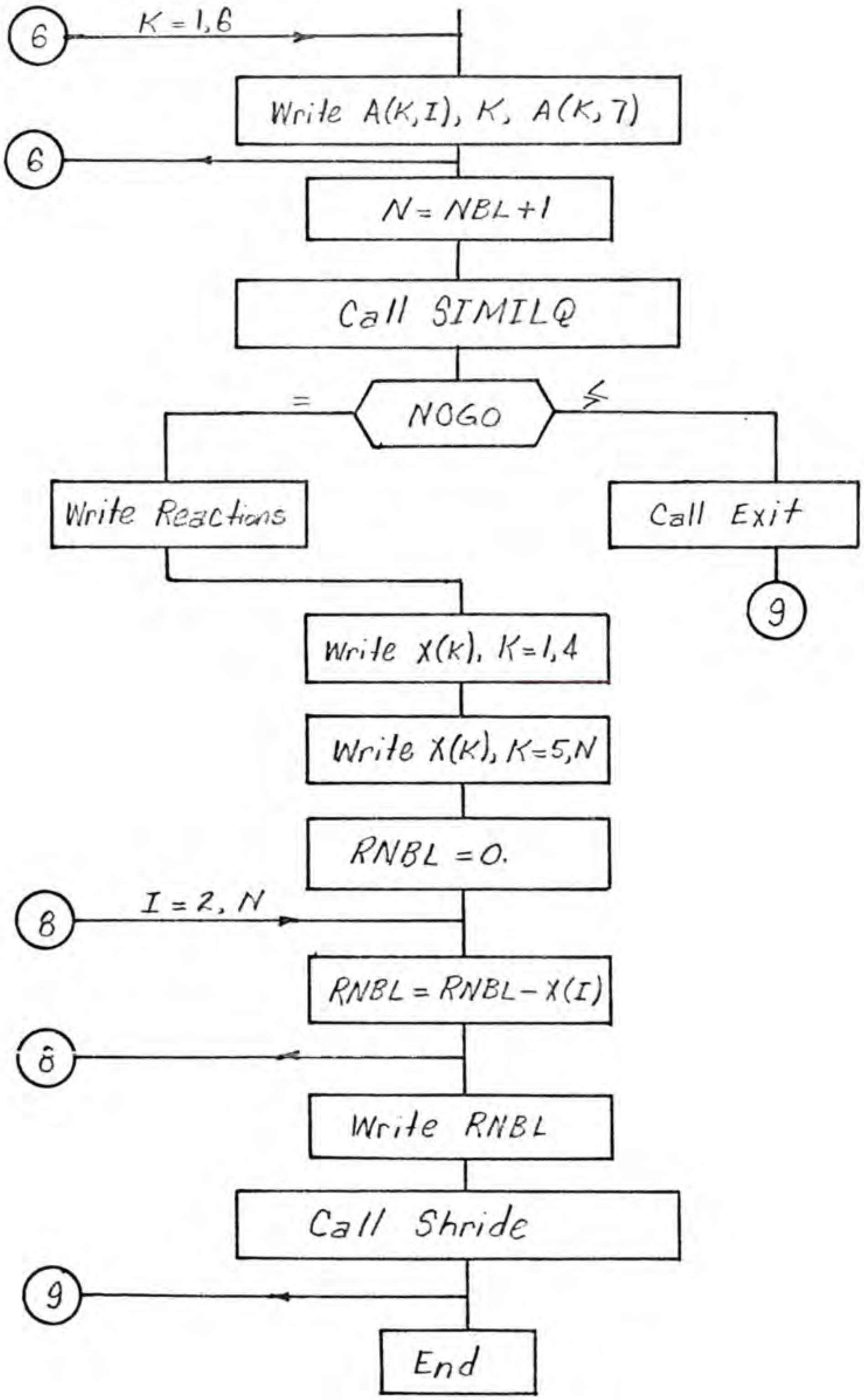
Shrinkage Deflection




EIM = 1./EI(J)
ACL(J) = ACLI + AL(J)
ACLI = ACL(J)
C(J) = (TE(J)/EI(J)) - EIT
EIT = TE(J)/EI(J)
Write J, AI(J), EI(J), B(J), TE(J), C(J)
1
Write RN
Write Es, Ec
5
K = 1, NBL
XI = BLI(K)
A(K,1) = XI
3
N = 1, NBL
COE = 0.
2
J = 1, NAL
XI - BL(N)
<
≥
AI(J) = 0
AI(J) = XI - BL(N)


ACL(J) - BL(N)
<
≥
A2(J) = 0
A2(J) = ACL(J) - BL(N)
X1 - ACL(J)
<
≥
A3(J) = 0
A3(J) = 1.
X1 - ACL(J)
<
≥
A4(J) = 0
A4(J) = X1 - ACL(J)
Program Equation #4, See Page A2.
COE = COE + B1
2
I = N+1
A(K,I) = COE
3
CCOE = 0.
4
J = 1, NAL
C1 = C(J) * A4(J)²

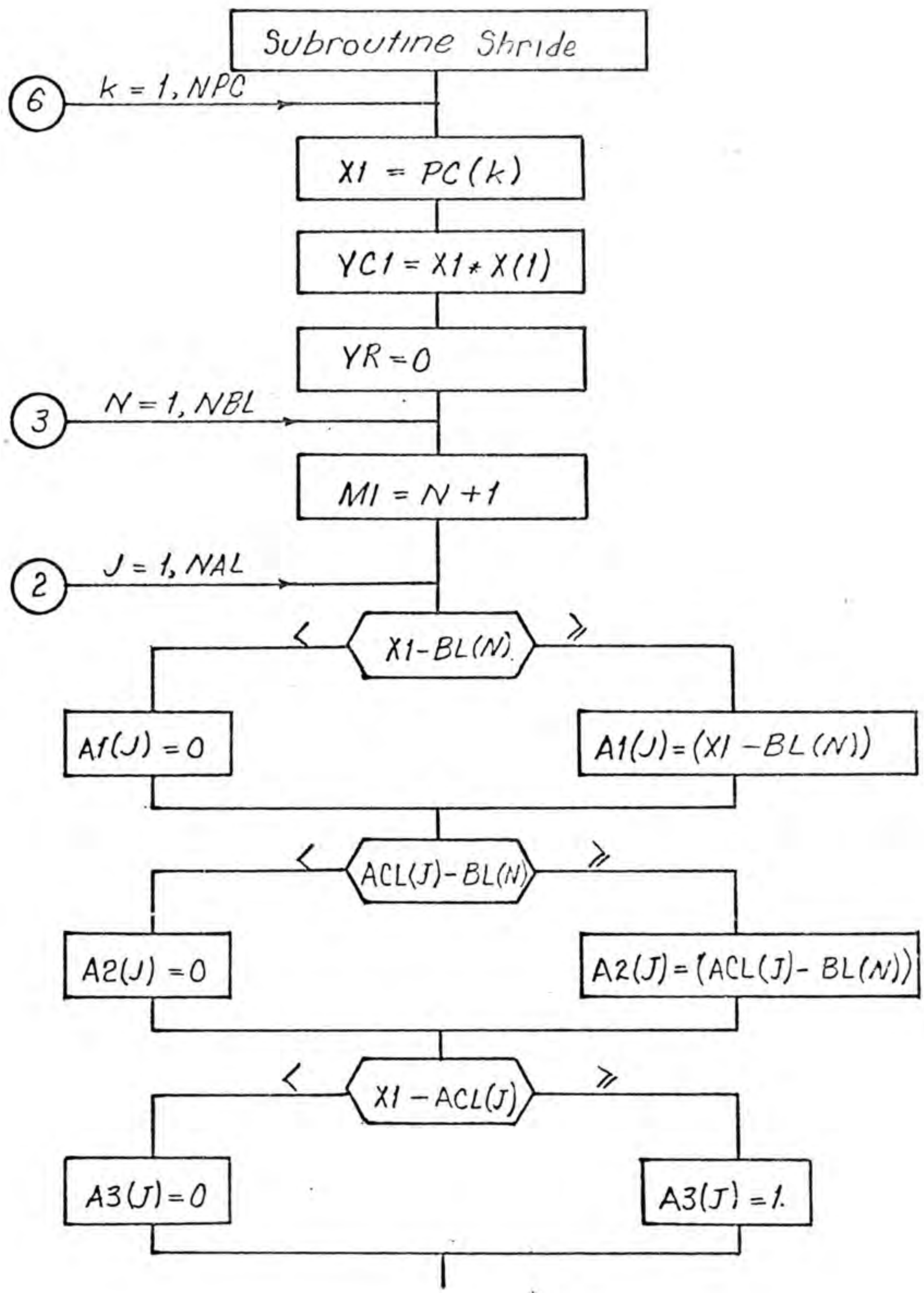

CCOE = CCOE + C1
4
I = NBL + 2
A(K, I) = - CCOE/2.
5
K = NBL + 1
A(K, 1) = 0
A(K, 2) = BL1(NBL)
I1 = 3, K
7
J = I1 - 2
A(K, I1) = BL1(NBL) - BL1(J)
7
CTE = 0
J = 1, NAL
750
C2 = TE(J) * A3(J)
CTE = CTE + C2
750
A(K, I) = - CTE

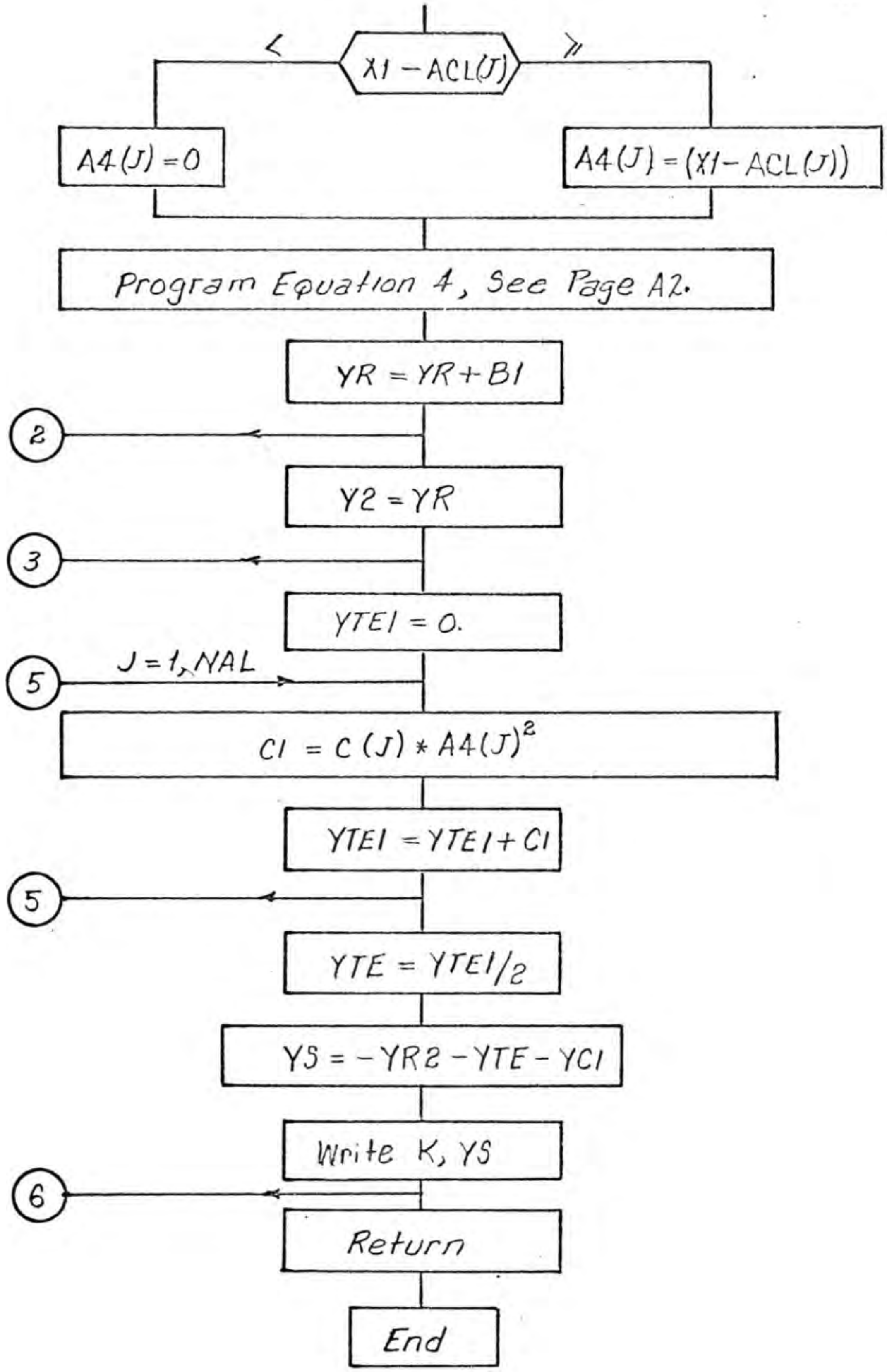

6
K = 1,6
Write A(K,I), K, A(K,7)
6
N = NBL+1
Call SIMILQ
=
NOGO
≤
Write Reactions
Call Exit
9
Write X(K), K=1,4
Write X(K), K=5,N
RNBL = 0.
8
I = 2, N
RNBL = RNBL - X(I)
8
Write RNBL
Call Shride
9
End

```
      SUBROUTINE SHRIDE(PC,X,BL,ACL,B,TE,NBL,NAL,NPC,C)
      DIMENSION PC(50),X(10),BL(10),ACL(50),B(50)
      DIMENSION A1(50),A2(50),A3(50),A4(50),TE(50),C(50)

   31 FORMAT(10X,16HLEFLECTION AT SH,I1,2H =,F10.5,1X,2HFT)
   71 FORMAT(///10X,11HDEFLECTIONS/10X,11H-----------/)

      WRITE(6,71)
      DO 600 K=1,NPC
      X1 = PC(K)
      YC1 = X(1)*X1
      YR = 0
      DO 300 N = 1,NBL
      M1 = N+1
      DO 200 J=1,NAL
      IF(X1-BL(N))30,40,40
   30 A1(J) =0
      GO TO 50
   40 A1(J) = (X1 - BL(N))
   50 IF(ACL(J)-BL(N))60,70,70
   60 A2(J) =0
      GO TO 80
   70 A2(J) = (ACL(J)-BL(N))
   80 IF(X1-ACL(J))90,110,110
   90 A3(J) =0
      GO TO 120
  110 A3(J) =1.
  120 IF(X1-ACL(J))130,140,140
  130 A4(J) =0
      GO TO 150
  140 A4(J) = (X1-ACL(J))
  150 B1=B(J)*(((A1(J)**3/6.)-(A2(J)**3/6.))*A3(J)-(A2(J)**2/2.)*A4(J))*
     $X(M1)
  200 YR = YR+B1
  300 YR2 = YR
      YTE1 = 0.
```

```
      DO 500 J=1,NAL
      C1 = C(J)*A4(J)**2
  500 YTE1 = YTE1+C1
      YTE = YTE1/2.
      YS = -YR2-YTE-YC1
  600 WRITE(6,31)K,YS
      RETURN
      END
```

Flow Diagram





<
X1 − ACL(J)
≥
A4(J) = 0
A4(J) = (X1 − ACL(J))
Program Equation 4, See Page A2.
YR = YR + B1
2
Y2 = YR
3
YTE1 = 0.
5
J = 1, NAL
C1 = C (J) * A4(J)2
YTE1 = YTE1 + C1
5
YTE = YTE1/2
YS = −YR2 − YTE − YC1
Write K, YS
6
Return
End

TABLE OF DISCONTINUITIES
FOR N = 5.

TE = SHRINKAGE (WARPING) MOMENT

J	I IN4	EI K-FT2	B 1/(K-FT2)	TE K-FT	C 1/FT
1	8248448.	0.332E 09	0.301E-08	390.833	0.118E-05
2	1619448.	0.652E 08	0.123E-07	2192.500	0.324E-04
3	8248448.	0.332E 09	-0.123E-07	390.833	-0.324E-04
4	8042046.	0.324E 09	0.773E-10	-120.000	-0.155E-05
5	8029162.	0.323E 09	0.495E-11	-567.167	-0.138E-05
6	1985976.	0.800E 08	0.941E-08	-3079.500	-0.367E-04
7	8029162.	0.323E 09	-0.941E-08	-567.167	0.367E-04
8	8151290.	0.328E 09	-0.463E-10	-267.000	0.941E-06
9	8151306.	0.328E 09	-0.711E-14	157.667	0.129E-05
10	1513170.	0.609E 08	0.134E-07	2046.167	0.331E-04
11	8151306.	0.328E 09	-0.134E-07	157.667	-0.331E-04
12	8247380.	0.332E 09	-0.355E-10	-300.000	-0.138E-05
13	8125294.	0.327E 09	0.452E-10	-596.833	-0.921E-06
14	2264304.	0.912E 08	0.791E-08	-3464.000	-0.362E-04
15	2709534.	0.109E 09	-0.180E-08	-4152.332	-0.660E-07
16	2264304.	0.912E 08	0.180E-08	-3464.000	0.660E-07
17	8367102.	0.337E 09	-0.800E-08	-12.333	0.379E-04
18	2091778.	0.843E 08	0.890E-08	2858.000	0.340E-04
19	2606966.	0.105E 09	-0.235E-08	3600.333	0.366E-06
20	2091778.	0.843E 08	0.235E-08	2858.000	-0.366E-06
21	8270902.	0.333E 09	-0.887E-08	448.500	-0.326E-04
22	8367102.	0.337E 09	-0.345E-10	-12.333	-0.138E-05
23	2264304.	0.912E 08	0.800E-08	-3464.000	-0.379E-04
24	2709534.	0.109E 09	-0.180E-08	-4152.332	-0.660E-07
25	2264304.	0.912E 08	0.180E-08	-3464.000	0.660E-07
26	8322008.	0.335E 09	-0.798E-08	-120.333	0.376E-04
27	1771836.	0.714E 08	0.110E-07	2405.167	0.341E-04
28	2209260.	0.890E 08	-0.277E-08	3031.333	0.364E-06
29	1771836.	0.714E 08	0.277E-08	2405.167	-0.364E-06
30	8226720.	0.331E 09	-0.110E-07	339.333	-0.327E-04
31	8322008.	0.335E 09	-0.346E-10	-120.333	-0.138E-05
32	8125294.	0.327E 09	0.722E-10	-596.833	-0.146E-05
33	2264304.	0.912E 08	0.791E-08	-3464.000	-0.362E-04
34	2709534.	0.109E 09	-0.180E-08	-4152.332	-0.660E-07
35	2264304.	0.912E 08	0.180E-08	-3464.000	0.660E-07
36	8322008.	0.335E 09	-0.798E-08	-120.333	0.376E-04
37	1771836.	0.714E 08	0.110E-07	2405.167	0.341E-04
38	2209260.	0.890E 08	-0.277E-08	3031.333	0.364E-06
39	1771836.	0.714E 08	0.277E-08	2405.167	-0.364E-06
40	8169402.	0.329E 09	-0.110E-07	196.667	-0.331E-04

FOR N = 5.
REACTIONS AND DEFLECTIONS
DUE TO SHRINKAGE

ES =29000. KSI
EC = ES/N = 5800. KSI

MATRIX

A						X	B
64.00	0.00	0.0	0.0	0.0	0.0	X1	-0.04
144.00	0.01	0.00	0.0	0.0	0.0	X2	-0.11
249.00	0.02	0.01	0.00	0.0	0.0	X3	-0.25
354.00	0.07	0.04	0.01	0.00	0.0	X4	-0.46
438.00	0.13	0.08	0.04	0.01	0.00	X5	-0.68
0.0	436.00	374.00	294.00	189.00	84.00	X6	8401.30

SLOPE CONSTANT AND REACTIONS

C1 = -0.001 K
R1 = -5.170 K
R2 = -1.128 K
R3 = 14.638 K
R4 = -38.686 K
R5 = 167.809 K
R6 = -137.463 K

DEFLECTIONS

DEFLECTION AT SH1 = -0.00000 FT
DEFLECTION AT SH2 = 0.01874 FT
DEFLECTION AT SH3 = 0.02850 FT
DEFLECTION AT SH4 = 0.01727 FT
DEFLECTION AT SH5 = -0.00000 FT
DEFLECTION AT SH6 = 0.00580 FT
DEFLECTION AT SH7 = 0.00939 FT
DEFLECTION AT SH8 = 0.00183 FT
DEFLECTION AT SH9 = -0.00000 FT

TABLE OF DISCONTINUITIES
FOR N =10.

TE = SHRINKAGE (WARPING) MOMENT

J	I IN4	EI K-FT2	B 1/(K-FT2)	TE K-FT	C 1/FT
1	8961912.	0.180E 09	0.554E-08	379.667	0.210E-05
2	2869876.	0.578E 08	0.118E-07	1903.500	0.308E-04
3	8961912.	0.180E 09	-0.118E-07	379.667	-0.308E-04
4	8547452.	0.172E 09	0.269E-09	-102.500	-0.270E-05
5	8526768.	0.172E 09	0.141E-10	-521.333	-0.244E-05
6	3424134.	0.690E 08	0.868E-08	-2563.000	-0.341E-04
7	8526768.	0.172E 09	-0.868E-08	-521.333	0.341E-04
8	8778542.	0.177E 09	-0.167E-09	-235.333	0.170E-05
9	8768172.	0.177E 09	0.669E-11	161.167	0.224E-05
10	2677782.	0.539E 08	0.129E-07	1778.167	0.321E-04
11	8768172.	0.177E 09	-0.129E-07	161.167	-0.321E-04
12	8979808.	0.181E 09	-0.133E-09	-261.500	-0.236E-05
13	8727696.	0.176E 09	0.160E-09	-541.500	-0.163E-05
14	3861114.	0.778E 08	0.717E-08	-2821.167	-0.332E-04
15	4509762.	0.908E 08	-0.185E-08	-3308.500	-0.147E-06
16	3861114.	0.778E 08	0.185E-08	-2821.167	0.147E-06
17	9223428.	0.186E 09	-0.748E-08	6.167	0.363E-04
18	3677240.	0.741E 08	0.812E-08	2456.667	0.331E-04
19	4532950.	0.913E 08	-0.255E-08	3044.333	0.175E-06
20	3677240.	0.741E 08	0.255E-08	2456.667	-0.175E-06
21	9006370.	0.181E 09	-0.799E-08	433.500	-0.308E-04
22	9223428.	0.186E 09	-0.130E-09	6.167	-0.236E-05
23	3861114.	0.778E 08	0.748E-08	-2821.167	-0.363E-04
24	4509762.	0.908E 08	-0.185E-08	-3308.500	-0.147E-06
25	3861114.	0.778E 08	0.185E-08	-2821.167	0.147E-06
26	9132048.	0.184E 09	-0.742E-08	-94.000	0.358E-04
27	3151616.	0.635E 08	0.103E-07	2085.833	0.334E-04
28	3874300.	0.780E 08	-0.294E-08	2594.333	0.387E-06
29	3151616.	0.635E 08	0.294E-08	2085.833	-0.387E-06
30	8918754.	0.180E 09	-0.102E-07	331.667	-0.310E-04
31	9132048.	0.184E 09	-0.130E-09	-94.000	-0.236E-05
32	8727696.	0.176E 09	0.252E-09	-541.500	-0.257E-05
33	3861114.	0.778E 08	0.717E-08	-2821.167	-0.332E-04
34	4509762.	0.908E 08	-0.185E-08	-3308.500	-0.147E-06
35	3861114.	0.778E 08	0.185E-08	-2821.167	0.147E-06
36	9132048.	0.184E 09	-0.742E-08	-94.000	0.358E-04
37	3151616.	0.635E 08	0.103E-07	2085.833	0.334E-04
38	3874300.	0.780E 08	-0.294E-08	2594.333	0.387E-06
39	3151616.	0.635E 08	0.294E-08	2085.833	-0.387E-06
40	8804390.	0.177E 09	-0.101E-07	197.833	-0.317E-04

FOR N = 10.
REACTIONS AND DEFLECTIONS
DUE TO SHRINKAGE

ES =29000. KSI
EC = ES/N = 2900. KSI

MATRIX

A						X	B
64.00	0.00	0.0	0.0	0.0	0.0	X1	-0.04
144.00	0.01	0.00	0.0	0.0	0.0	X2	-0.10
249.00	0.03	0.01	0.00	0.0	0.0	X3	-0.25
354.00	0.08	0.05	0.02	0.00	0.0	X4	-0.46
438.00	0.16	0.10	0.05	0.01	0.00	X5	-0.69
0.0	438.00	374.00	294.00	189.00	84.00	X6	5194.14

SLOPE CONSTANT AND REACTIONS

C1 = -0.001 K
R1 = -4.333 K
R2 = -0.146 K
R3 = 8.935 K
R4 = -22.537 K
R5 = 104.516 K
R6 = -86.434 K

DEFLECTIONS

DEFLECTION AT SH1 = -0.00000 FT
DEFLECTION AT SH2 = 0.01777 FT
DEFLECTION AT SH3 = 0.02686 FT
DEFLECTION AT SH4 = 0.01603 FT
DEFLECTION AT SH5 = -0.00000 FT
DEFLECTION AT SH6 = 0.00727 FT
DEFLECTION AT SH7 = 0.01226 FT
DEFLECTION AT SH8 = 0.00444 FT
DEFLECTION AT SH9 = -0.00000 FT

TABLE OF DISCONTINUITIES
FOR N =15.

TE = SHRINKAGE (WARPING) MOMENT

J	I IN4	EI K-FT2	B 1/(K-FT2)	TE K-FT	C 1/FT
1	9630294.	0.129E 09	0.773E-08	356.667	0.276E-05
2	3979984.	0.534E 08	0.110E-07	1696.167	0.290E-04
3	9630294.	0.129E 09	-0.110E-07	356.667	-0.290E-04
4	9012570.	0.121E 09	0.530E-09	-97.333	-0.356E-05
5	8980920.	0.121E 09	0.291E-10	-492.333	-0.328E-05
6	4584566.	0.616E 08	0.795E-08	-2210.000	-0.318E-04
7	8980920.	0.121E 09	-0.795E-08	-492.333	0.318E-04
8	9365068.	0.126E 09	-0.340E-09	-220.833	0.233E-05
9	9343194.	0.125E 09	0.186E-10	152.167	0.297E-05
10	3760540.	0.505E 08	0.118E-07	1588.500	0.302E-04
11	9343194.	0.125E 09	-0.118E-07	152.167	-0.302E-04
12	9671278.	0.130E 09	-0.270E-09	-243.333	-0.309E-05
13	9286716.	0.125E 09	0.319E-09	-506.833	-0.219E-05
14	5142546.	0.690E 08	0.646E-08	-2397.000	-0.307E-04
15	5911260.	0.794E 08	-0.188E-08	-2770.500	-0.191E-06
16	5142546.	0.690E 08	0.188E-08	-2397.000	0.191E-06
17	10038722.	0.135E 09	-0.706E-08	5.833	0.348E-04
18	5070062.	0.681E 08	0.727E-08	2162.333	0.317E-04
19	6182170.	0.830E 08	-0.264E-08	2645.667	0.109E-06
20	5070062.	0.681E 08	0.264E-08	2162.333	-0.109E-06
21	9695688.	0.130E 09	-0.701E-08	406.500	-0.286E-04
22	10038722.	0.135E 09	-0.263E-09	5.833	-0.308E-05
23	5142546.	0.690E 08	0.706E-08	-2397.000	-0.348E-04
24	5911260.	0.794E 08	-0.188E-08	-2770.500	-0.191E-06
25	5142546.	0.690E 08	0.188E-08	-2397.000	0.191E-06
26	9901380.	0.133E 09	-0.696E-08	-87.167	0.341E-04
27	4343876.	0.583E 08	0.962E-08	1851.000	0.324E-04
28	5327898.	0.715E 08	-0.317E-08	2276.333	0.842E-07
29	4343876.	0.583E 08	0.317E-08	1851.000	-0.842E-07
30	9566556.	0.128E 09	-0.936E-08	311.833	-0.293E-04
31	9901380.	0.133E 09	-0.263E-09	-87.167	-0.308E-05
32	9286716.	0.125E 09	0.498E-09	-506.833	-0.341E-05
33	5142546.	0.690E 08	0.646E-08	-2397.000	-0.307E-04
34	5911260.	0.794E 08	-0.188E-08	-2770.500	-0.191E-06
35	5142546.	0.690E 08	0.188E-08	-2397.000	0.191E-06
36	9901380.	0.133E 09	-0.696E-08	-87.167	0.341E-04
37	4343876.	0.583E 08	0.962E-08	1851.000	0.324E-04
38	5327898.	0.715E 08	-0.317E-08	2276.333	0.842E-07
39	4343876.	0.583E 08	0.317E-08	1851.000	-0.842E-07
40	9397126.	0.126E 09	-0.922E-08	186.500	-0.303E-04

FOR N = 15.
REACTIONS AND DEFLECTIONS
DUE TO SHRINKAGE

ES =29000. KSI
EC = ES/N = 1933. KSI

MATRIX

A						X	B
64.00	0.00	0.0	0.0	0.0	0.0	X1	-0.04
144.00	0.01	0.00	0.0	0.0	0.0	X2	-0.10
249.00	0.03	0.01	0.00	0.0	0.0	X3	-0.24
354.00	0.10	0.05	0.02	0.00	0.0	X4	-0.44
438.00	0.18	0.11	0.06	0.02	0.00	X5	-0.66
0.0	438.00	374.00	294.00	189.00	84.00	X6	3578.98

SLOPE CONSTANT AND REACTIONS

C1 = -0.001 K
R1 = -3.454 K
R2 = -0.071 K
R3 = 6.116 K
R4 = -14.663 K
R5 = 72.518 K
R6 = -60.447 K

DEFLECTIONS

DEFLECTION AT SH1 = -0.00000 FT
DEFLECTION AT SH2 = 0.01685 FT
DEFLECTION AT SH3 = 0.02536 FT
DEFLECTION AT SH4 = 0.01498 FT
DEFLECTION AT SH5 = -0.00000 FT
DEFLECTION AT SH6 = 0.00797 FT
DEFLECTION AT SH7 = 0.01361 FT
DEFLECTION AT SH8 = 0.00583 FT
DEFLECTION AT SH9 = -0.00000 FT

TABLE OF DISCONTINUITIES
FOR N =20.

TE = SHRINKAGE (WARPING) MOMENT

J	I IN4	EI K-FT2	B 1/(K-FT2)	TE K-FT	C 1/FT
1	10296480.	0.104E 09	0.965E-08	336.167	0.324E-05
2	5014120.	0.505E 08	0.102E-07	1531.500	0.271E-04
3	10296480.	0.104E 09	-0.102E-07	336.167	-0.271E-04
4	9477510.	0.954E 08	0.833E-09	-92.667	-0.421E-05
5	9430800.	0.950E 08	0.519E-10	-466.333	-0.394E-05
6	5576904.	0.562E 08	0.728E-08	-1949.000	-0.298E-04
7	9430800.	0.950E 08	-0.728E-08	-466.333	0.298E-04
8	9950748.	0.100E 09	-0.550E-09	-208.000	0.283E-05
9	9917810.	0.999E 08	0.331E-10	144.167	0.352E-05
10	4701960.	0.473E 08	0.111E-07	1437.667	0.289E-04
11	9917810.	0.999E 08	-0.111E-07	144.167	-0.289E-04
12	10361746.	0.104E 09	-0.429E-09	-227.333	-0.362E-05
13	9841134.	0.991E 08	0.507E-09	-476.333	-0.263E-05
14	6241174.	0.628E 08	0.582E-08	-2090.500	-0.285E-04
15	7087890.	0.714E 08	-0.190E-08	-2391.333	-0.241E-06
16	6241174.	0.628E 08	0.190E-08	-2090.500	0.241E-06
17	10854006.	0.109E 09	-0.676E-08	5.333	0.333E-04
18	6335658.	0.638E 08	0.653E-08	1934.167	0.303E-04
19	7654816.	0.771E 08	-0.270E-08	2344.167	0.945E-07
20	6335658.	0.638E 08	0.270E-08	1934.167	-0.945E-07
21	10382162.	0.105E 09	-0.611E-08	382.833	-0.267E-04
22	10854006.	0.109E 09	-0.416E-09	5.333	-0.361E-05
23	6241174.	0.628E 08	0.676E-08	-2090.500	-0.333E-04
24	7087890.	0.714E 08	-0.190E-08	-2391.333	-0.241E-06
25	6241174.	0.628E 08	0.190E-08	-2090.500	0.241E-06
26	10670598.	0.107E 09	-0.661E-08	-81.167	0.325E-04
27	5458454.	0.550E 08	0.889E-08	1666.167	0.311E-04
28	6643158.	0.669E 08	-0.324E-08	2031.500	0.554E-07
29	5458454.	0.550E 08	0.324E-08	1666.167	-0.554E-07
30	10212674.	0.103E 09	-0.847E-08	294.333	-0.275E-04
31	10670598.	0.107E 09	-0.417E-09	-81.167	-0.362E-05
32	9841134.	0.991E 08	0.784E-09	-476.333	-0.405E-05
33	6241174.	0.628E 08	0.582E-08	-2090.500	-0.285E-04
34	7087890.	0.714E 08	-0.190E-08	-2391.333	-0.241E-06
35	6241174.	0.628E 08	0.190E-08	-2090.500	0.241E-06
36	10670598.	0.107E 09	-0.661E-08	-81.167	0.325E-04
37	5458454.	0.550E 08	0.889E-08	1666.167	0.311E-04
38	6643158.	0.669E 08	-0.324E-08	2031.500	0.554E-07
39	5458454.	0.550E 08	0.324E-08	1666.167	-0.554E-07
40	9989248.	0.101E 09	-0.825E-08	176.500	-0.286E-04

FOR N = 20.
REACTIONS AND DEFLECTIONS
DUE TO SHRINKAGE

ES =29000. KSI
EC = ES/N = 1450. KSI

MATRIX

A						X	B
64.00	0.00	0.0	0.0	0.0	0.0	X1	-0.03
144.00	0.01	0.00	0.0	0.0	0.0	X2	-0.10
249.00	0.04	0.01	0.00	0.0	0.0	X3	-0.23
354.00	0.11	0.06	0.02	0.00	0.0	X4	-0.42
438.00	0.20	0.12	0.06	0.02	0.00	X5	-0.62
0.0	438.00	374.00	294.00	189.00	84.00	X6	2588.49

SLOPE CONSTANT AND REACTIONS

C1 = -0.001 K
R1 = -2.827 K
R2 = -0.137 K
R3 = 4.573 K
R4 = -10.113 K
R5 = 52.919 K
R6 = -44.414 K

DEFLECTIONS

DEFLECTION AT SH1 = -0.00000 FT
DEFLECTION AT SH2 = 0.01599 FT
DEFLECTION AT SH3 = 0.02401 FT
DEFLECTION AT SH4 = 0.01409 FT
DEFLECTION AT SH5 = -0.00000 FT
DEFLECTION AT SH6 = 0.00831 FT
DEFLECTION AT SH7 = 0.01430 FT
DEFLECTION AT SH8 = 0.00669 FT
DEFLECTION AT SH9 = -0.00000 FT

TABLE OF DISCONTINUITIES
FOR N =25.

TE = SHRINKAGE (WARPING) MOMENT

J	I IN4	EI K-FT2	B 1/(K-FT2)	TE K-FT	C 1/FT
1	10960970.	0.883E 08	0.113E-07	318.000	0.360E-05
2	5987336.	0.482E 08	0.941E-08	1397.500	0.254E-04
3	10960970.	0.883E 08	-0.941E-08	318.000	-0.254E-04
4	9942308.	0.801E 08	0.116E-08	-88.333	-0.470E-05
5	9877044.	0.796E 08	0.825E-10	-442.833	-0.446E-05
6	6453484.	0.520E 08	0.667E-08	-1746.500	-0.280E-04
7	9877044.	0.796E 08	-0.667E-08	-442.833	0.280E-04
8	10535734.	0.849E 08	-0.786E-09	-196.500	0.325E-05
9	10503994.	0.846E 08	0.356E-10	136.833	0.393E-05
10	5623274.	0.453E 08	0.103E-07	1314.167	0.274E-04
11	10503994.	0.846E 08	-0.103E-07	136.833	-0.274E-04
12	11051402.	0.890E 08	-0.585E-09	-213.500	-0.402E-05
13	10391892.	0.837E 08	0.713E-09	-449.167	-0.297E-05
14	7214584.	0.581E 08	0.526E-08	-1857.167	-0.266E-04
15	8124892.	0.655E 08	-0.193E-08	-2107.167	-0.239E-06
16	7214584.	0.581E 08	0.193E-08	-1857.167	0.239E-06
17	11669300.	0.940E 08	-0.657E-08	5.000	0.320E-04
18	7512022.	0.605E 08	0.589E-08	1752.167	0.289E-04
19	9007012.	0.726E 08	-0.274E-08	2106.833	0.822E-07
20	7512022.	0.605E 08	0.274E-08	1752.167	-0.822E-07
21	11066248.	0.891E 08	-0.531E-08	361.833	-0.249E-04
22	11669300.	0.940E 08	-0.580E-09	5.000	-0.401E-05
23	7214584.	0.581E 08	0.657E-08	-1857.167	-0.320E-04
24	8124892.	0.655E 08	-0.193E-08	-2107.167	-0.239E-06
25	7214584.	0.581E 08	0.193E-08	-1857.167	0.239E-06
26	11439714.	0.922E 08	-0.636E-08	-76.000	0.311E-04
27	6502898.	0.524E 08	0.824E-08	1516.500	0.298E-04
28	7861824.	0.633E 08	-0.330E-08	1836.333	0.462E-07
29	6502898.	0.524E 08	0.330E-08	1516.500	-0.462E-07
30	10857384.	0.875E 08	-0.766E-08	278.667	-0.258E-04
31	11439714.	0.922E 08	-0.582E-09	-76.000	-0.401E-05
32	10391892.	0.837E 08	0.109E-08	-449.167	-0.454E-05
33	7214584.	0.581E 08	0.526E-08	-1857.167	-0.266E-04
34	8124892.	0.655E 08	-0.193E-08	-2107.167	-0.239E-06
35	7214584.	0.581E 08	0.193E-08	-1857.167	0.239E-06
36	11439714.	0.922E 08	-0.636E-08	-76.000	0.311E-04
37	6502898.	0.524E 08	0.824E-08	1516.500	0.298E-04
38	7861824.	0.633E 08	-0.330E-08	3025.833	0.188E-04
39	6502898.	0.524E 08	0.330E-08	2405.167	-0.186E-05
40	10580846.	0.852E 08	-0.736E-08	167.500	-0.439E-04

FOR N = 25.
REACTIONS AND DEFLECTIONS
DUE TO SHRINKAGE

ES =29000. KSI
EC = ES/N = 1160. KSI

MATRIX

A						X	B
64.00	0.00	0.0	0.0	0.0	0.0	X1	-0.03
144.00	0.01	0.00	0.0	0.0	0.0	X2	-0.09
249.00	0.04	0.02	0.00	0.0	0.0	X3	-0.21
354.00	0.12	0.06	0.02	0.00	0.0	X4	-0.40
438.00	0.22	0.13	0.06	0.02	0.00	X5	-0.62
0.0	438.00	374.00	294.00	189.00	84.00	X6	-146.01

SLOPE CONSTANT AND REACTIONS

C1 = -0.000 K
R1 = -2.344 K
R2 = -0.039 K
R3 = 2.754 K
R4 = -4.409 K
R5 = 10.937 K
R6 = -6.899 K

DEFLECTIONS

DEFLECTION AT SH1 = -0.00000 FT
DEFLECTION AT SH2 = 0.01506 FT
DEFLECTION AT SH3 = 0.02250 FT
DEFLECTION AT SH4 = 0.01300 FT
DEFLECTION AT SH5 = -0.00000 FT
DEFLECTION AT SH6 = 0.00918 FT
DEFLECTION AT SH7 = 0.01599 FT
DEFLECTION AT SH8 = 0.00854 FT
DEFLECTION AT SH9 = -0.00000 FT