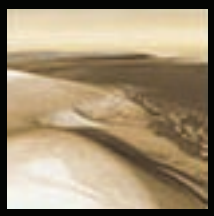
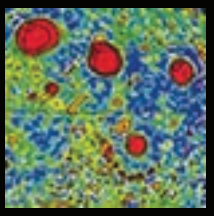
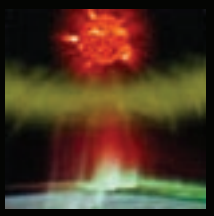
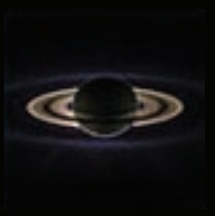
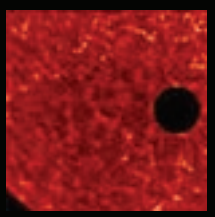
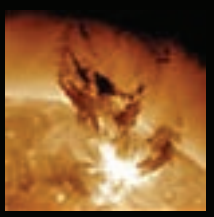
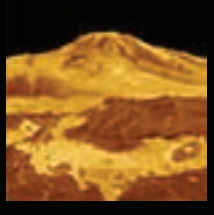
National Aeronautics and Space Administration

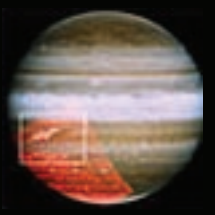











Real-World Math

Year of the Solar System

This collection of activities is based on a weekly series of space science problems distributed to thousands of teachers during the 2004-2013. They were intended for students looking for additional challenges in the math and physical science curriculum in grades 5 through 12. The problems were created to be authentic glimpses of modern science and engineering issues, often involving actual research data.

The problems were designed to be 'one-pagers' with an Answer Key as a second page. This compact form was deemed very popular by participating teachers.

This math guide was created to support the NASA/JPL Year of the Solar System (YSS) project.

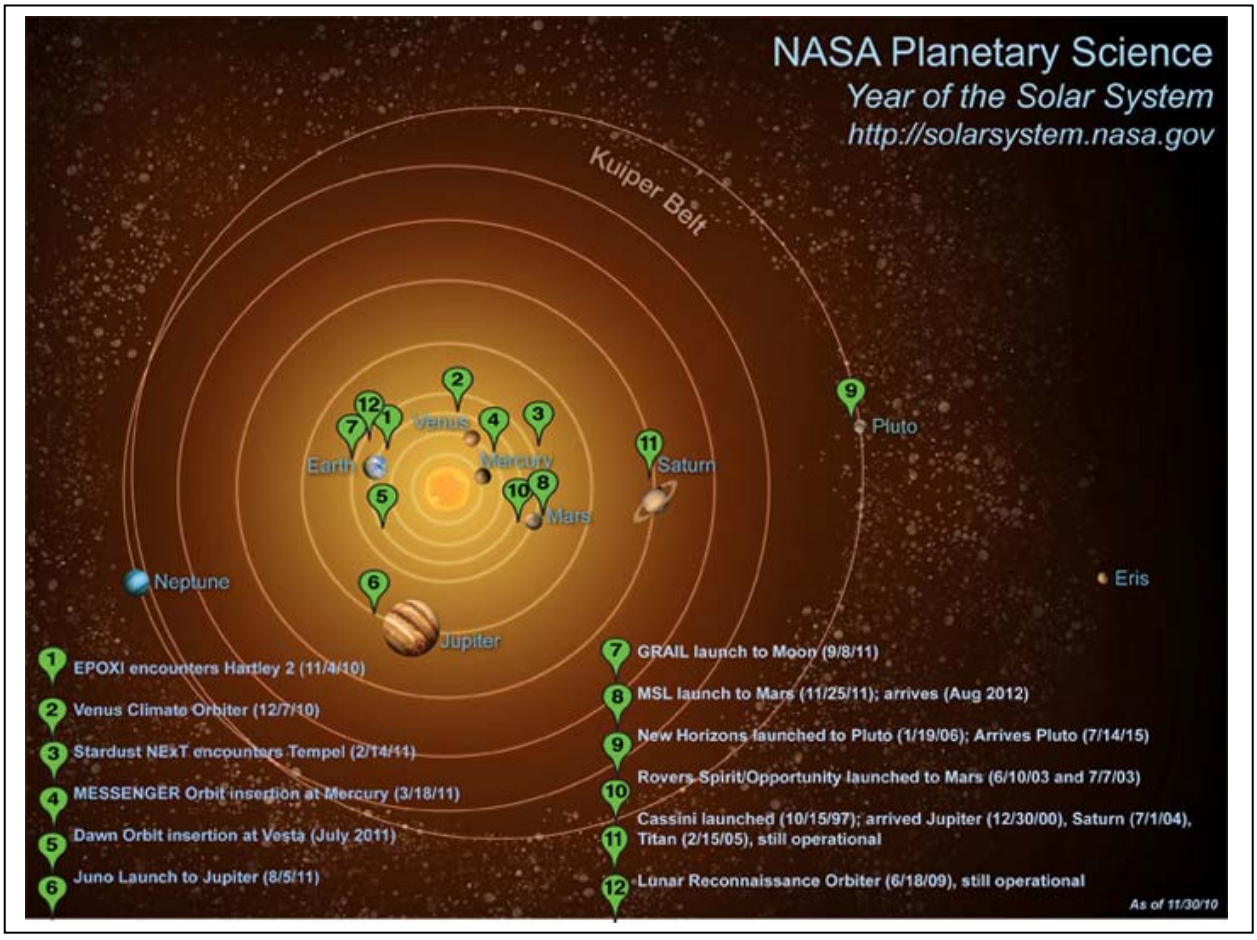
As NASA spacecraft head to, and arrive at, key locations across our solar system, YSS offers a continuing salute to the 50 year history of solar system exploration by providing an integrated picture of our new understanding of the solar system to educators and the general public!

YSS combines the amazing discoveries of past NASA planetary missions with the most recent findings of the ongoing missions, and connects them to the related planetary science topics!

This math guide offers educators and students insight into the behind-the-scenes role that mathematics plays in solar system exploration through engaging real-world problems.

For more real-world math activities about astronomy and space visit the NASA website,

http://spacemath.gsfc.nasa.gov



This booklet was created through a NASA Science Mission Directorate ROSES/EPOESS education grant NNH06ZDA001N-EPO and by a grant from the JPL/NASA Year of the Solar System program.

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Mathematics Topic Matrix

Topic	Problem Numbers																												
	1.0									2.0							3.0									4.0					
	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6
Inquiry										X																					
Technology, rulers										X									X										X		
Numbers, patterns, percentages	X			X	X						X	X					X	X		X		X				X	X				
Averages																															
Time, distance,speed	X	X										X																	X		
Areas and volumes										X	X							X			X										
Scale drawings							X												X	X	X			X					X		
Proportions	X						X										X		X	X											
Geometry		X			X																X			X							
Scientific Notation						X		X	X				X	X	X	X							X	X							X
Unit Conversions						X																									
Fractions																															
Graph or Table Analysis										X							X	X					X					X		X	
Solving for X																														X	
Evaluating Fns													X	X	X	X							X								X
Modeling													X	X	X	X		X			X			X	X						
Probability																															
Rates/Slopes																													X	X	
Logarithmic Fns																															X
Polynomials																									X						
Power Fns																									X						
Conics																															
Trigonometry																															
Integration													X	X	X	X															
Differentiation													X	X	X	X															
Limits																															

Mathematics Topic Matrix (cont'd)

Topic	Problem Numbers																														
	4.0			5.0								6.0								7.0									8.0		
	7	8	9	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	9	1	2	3
Inquiry																															
Technology, rulers					X	X	X	X				X	X	X	X					X					X							
Numbers, patterns, percentages											X										X			X							
Averages																															
Time, distance,speed														X		X	X														
Areas and volumes	X	X		X																		X								X	
Scale drawings					X	X	X	X				X	X	X	X	X				X		X			X	X						
Proportions					X	X					X	X	X	X						X					X						
Geometry													X			X				X						X					
Scientific Notation	X	X	X	X						X												X								X	
Unit Conversions	X																														
Fractions																															
Graph or Table Analysis					X												X		X				X								
Solving for X																															
Evaluating Fns			X						X	X																			X		X
Modeling																							X						X		
Probability																															
Rates/Slopes																X								X	X					X	
Logarithmic Fns																		X									X				
Polynomials																															
Power Fns																															
Conics			X																				X		X	X					
Trigonometry						X																									
Integration		X																										X			
Differentiation																															
Limits																															

Mathematics Topic Matrix (cont'd)

Topic	Problem Numbers																													
	9.0								10.0			11.0										12.0								
	1	2	3	4	5	6	7	8	1	2	3	1	2	3	4	5	6	7	8	9	10	1	2	3	4	5	6	7	8	
Inquiry					X																				X					
Technology, rulers						X					X											X	X	X		X				
Numbers, patterns, percentages									X																			X		
Averages																														
Time, distance,speed																	X	X									X			
Areas and volumes									X		X												X	X		X	X	X		
Scale drawings	X					X	X															X	X	X	X					
Proportions	X	X	X			X		X				X										X	X	X						
Geometry	X			X		X	X	X																			X			
Scientific Notation									X	X	X																X			
Unit Conversions																														
Fractions	X	X	X																											
Graph or Table Analysis						X									X						X									
Solving for X		X	X										X																	
Evaluating Fns				X														X	X	X	X						X			
Modeling						X	X						X	X		X	X											X		
Probability																														
Rates/Slopes										X			X							X	X									
Logarithmic Fns																														
Polynomials																													X	
Power Fns																														
Exponential Fns																														
Conics																														
Trigonometry																														
Integration																											X		X	
Differentiation																														
Limits																														

Mathematics Topic Matrix (cont'd)

Topic	Problem Numbers																												
	13.0											14.0				15.0													
	1	2	3	4	5	6	7	8	9	10	11	1	2	3	4	1	2	3	4	5	6	7	8	9	10	11	12	13	
Inquiry																												X	
Technology, rulers		X	X			X																							
Numbers, patterns, percentages												X			X	X	X	X	X	X	X	X				X			
Averages																		X				X					X		
Time, distance,speed																													
Areas and volumes					X	X			X	X				X															
Scale drawings		X	X																					X					
Proportions																													
Geometry		X	X		X									X														X	
Scientific Notation								X	X	X																		X	
Unit Conversions											X												X						
Fractions																													
Graph or Table Analysis															X	X	X	X					X	X					
Solving for X	X			X									X																
Evaluating Fns	X						X	X																	X			X	
Modeling					X					X			X	X															
Probability																				X	X	X				X	X		
Rates/Slopes	X				X				X		X		X	X	X														
Logarithmic Fns																													
Polynomials																													
Power Fns																													
Exponential Fns																													
Conics																													
Trigonometry																													
Integration								X																	X				
Differentiation																													
Limits																													

Mathematics Topic Matrix (cont'd)

Topic	Problem Numbers																													
	16.0												17.0						18.0											
	1	2	3	4	5	6	7	8	9	10	11	12	1	2	3	4	5	6	1	2	3	4	5	6	7	8				
Inquiry																														
Technology, rulers													X	X	X					X										
Numbers, patterns, percentages	X															X														
Averages																			X											
Time, distance,speed																			X	X	X	X			X					
Areas and volumes	X				X					X			X	X			X	X		X										
Scale drawings																					X	X			X					
Proportions			X	X		X	X	X	X																					
Geometry	X																				X	X				X				
Scientific Notation													X				X													
Unit Conversions	X									X		X		X																
Fractions																														
Graph or Table Analysis									X		X					X					X	X		X						
Solving for X		X	X	X			X	X	X																					
Evaluating Fns											X	X				X									X					
Modeling																														
Probability																														
Rates/Slopes															X				X	X	X	X	X							
Logarithmic Fns																														
Polynomials																														
Power Fns																														
Exponential Fns																														
Conics																														
Trigonometry																		X								X				
Integration																														
Differentiation																														
Limits																														

Mathematics Topic Matrix (cont'd)

Topic	Problem Numbers																																
	19.0											20.0																					
	1	2	3	4	5	6	7	8	9	10	11	1	2	3	4	5	6	7	8	9													
Inquiry																		X	X														
Technology, rulers					X																												
Numbers, patterns, percentages	X	X	X	X		X	X					X	X	X					X	X													
Averages																																	
Time, distance,speed																																	
Areas and volumes	X						X								X	X																	
Scale drawings						X	X																										
Proportions				X																													
Geometry																																	
Scientific Notation							X	X	X		X			X	X	X																	
Unit Conversions																																	
Fractions																																	
Graph or Table Analysis	X	X	X									X	X			X	X																
Solving for X																																	
Evaluating Fns								X	X		X						X																
Modeling								X	X							X		X															
Probability		X										X																					
Rates/Slopes																			X	X													
Logarithmic Fns																																	
Polynomials																																	
Power Fns																																	
Exponential Fns																																	
Conics										X	X																						
Trigonometry																																	
Integration																																	
Differentiation																																	
Limits																																	

Next Generation Science Standards

MS-ESS1 Earth's Place in the Universe

- **Performance Expectation: MS-ESS1-3**
  - Analyze and interpret data to determine scale properties of objects in the solar system.

HS – ESS1 Earth's Place in the Universe

- **Performance Expectations: HS-ESS1-1**
  - Develop a model based on evidence to illustrate the life span of the sun and the role of nuclear fusion in the sun's core to release energy in the form of radiation.
- **Performance Expectations: HS-ESS1-4**
  - Use mathematical or computational representations to predict the motion of orbiting objects in the solar system

CCMS: Common Core Mathematics Standards

Grades 6–8

CCSS.Math.Content.7.RP.A.2c Represent proportional relationships by equations

CCSS.Math.Content.6.EE.A.2c Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems

CCSS.Math.Content.6.SP.B.4 Display numerical data in plots on a number line, including dot plots, histograms, and box plots

CCSS.Math.Content.7.G.A.1 Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

CCSS.Math.Content.8.EE.A.4 Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used.

CCSS.Math.Content.8.EE.C.7b Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

CCSS.Math.Content.8.EE.C.8c Solve real-world and mathematical problems leading to two linear equations in two variables

Grades 9–12

CCSS.Math.Content.HSN-RN.A.1 Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents

CCSS.Math.Content.HSN-Q.A.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

CCSS.Math.Content.HSA-SSE.B.3c Use the properties of exponents to transform expressions for exponential functions.

CCSS.Math.Content.HSA-CED.A.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.

CCSS.Math.Content.HSF-TF.A.1 Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.

CCSS.Math.Content.HSG-GPE.A.3 (+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.

Associated NASA eClips Video Programs

NASA eClips, developed by the National Institute of Aerospace (NIA), consists of over 200 short video programs (3-10 minutes) featuring a variety of NASA missions and other topical resources. The videos are narrated by students making the content more easily accessible to grades 3 – 12 students.

Programs are available at :
http://www.nasa.gov/audience/foreducators/nasaeclips/index.html

Topic	NASA eClips Connection	Link
1.0 Scale of the Solar System	**RealWorld -- Scale Models and Rations**: This NASA video segment explains scale models, ratios, proportions and how to calculate problems with different units of measurement. Color animations clarify the use of ratios.	http://tinyurl.com/njnbl9e
2.0 Formation of the Solar System: Birth of Worlds	**Our World -- Stardust:** Visit a lab at NASA's Johnson Space Center where scientists study meteorites. Learn how aerogel, the lightest material in the world helped capture pieces of a comet and return the comet dust to Earth. See what scientists can learn about our universe from these tiny particles.	http://tinyurl.com/ozt85pb
3.0 The Planets – Investigating Our Planetary Family Tree	**Our World -- Pluto:** With more powerful telescopes, scientists are discovering smaller objects in our solar system. Find out how scientists now classify planets. See how NASA's robotic spacecraft, New Horizons, will help us learn more about the dwarf planet Pluto and similar objects in the Kuiper Belt.	http://tinyurl.com/pm7slqn
4.0 Gas Giants, Atmospheres and Weather	**Launchpad -- Global Warming:** Learn how the greenhouse effect keeps more of the sun's heat and energy within Earth's atmosphere causing temperatures on Earth to rise. This video explains the effect warmer temperatures are playing on Earth.	http://tinyurl.com/omalvln
5.0 Moons and Rings	**Our World -- Moons in Our Solar System:** Did you know astronomers have identified more than 300 moons in our solar system? How big is Ganymede? How small is Deimos? Which moons might have what it takes to support life? Follow the NASA missions to learn about these unique bodies in space.	http://tinyurl.com/ock4sp7
6.0 Leftovers from Planet Building: Asteroids	**Our World – What is the Solar System?** Find out why one amateur astronomer created an amazing graphic of the 88 largest objects in our solar system. Learn just what makes up a solar system and find out how we classify the thousands of objects in our own solar system.	http://tinyurl.com/prmjqmb

7.0 Comets: Small Bodies/Big Impacts	**Real World – Comets:** NASA engineers are finding new uses for old spacecraft as a way to study comets. Find out how a repurposed spacecraft can return to a comet for a second visit to uncover secrets about the formation of the solar system. Use angular size to see just how big this comet really is!	http://tinyurl.com/orygk6q
8.0 Volcanism in the Solar System	**Real World – Choosing the Right Lunar Excavator:** See how NASA engineers use the design process to evaluate the best choice for a new lunar excavator. Three different models are tested on location in Hawaii where the soil on Mauna Kea Volcano is similar to that on the moon.	http://tinyurl.com/qeyowbc
9.0 The Sun, Transits and Eclipses	**Launchpad – Solar Eclipses:** Join NASA to learn more about solar eclipses, especially the awe-inspiring phenomenon of total eclipses. Find out about the geometry and the distances and sizes of the sun and moon as seen from Earth that allow us to witness the sun's corona or actually be in the path of totality.	http://tinyurl.com/phsebts
10.0 Ice in the Solar System	**Launchpad – Thin Ice:** Join teacher scientists as they learn what ice can tell us about the history of winter. Find out why NASA studies ice and what thin ice sections observed through polarizing filters can help us learn. See how to use bubble patterns in ice core samples to show long-term weather patterns.	http://tinyurl.com/ppaf66y
11.0 Gravity: It's What Keeps Us Together	**Our World – Gravity in Space:** What is gravity? Find out about the balance between gravity and inertia that keeps the International Space Station in orbit. Learn why astronauts "float" in space and how the space shuttle has to slow down in order to come back to Earth.	http://tinyurl.com/pagm483
12.0 Collisions and Craters in the Solar System	**Real World – Lunar Reconnaissance Orbiter Mission:** Join NASA scientists for a look at the new Lunar Reconnaissance Orbiter, or LRO. Find out about the instruments that will make a comprehensive map of the moon and search for safe landing sites by collecting unprecedented amounts of data.	http://tinyurl.com/ns7qlfx
13.0 Water in the Solar System	**Our World – Life on Other Worlds:** Explore the possibility of finding life on other planets. See how NASA's search for water on Mars proved successful with the Phoenix Lander. Find out about extremophiles and what makes a habitable zone for life as we know it.	http://tinyurl.com/pgyhu6p
14.0 Planets grow and change over time	**Our World – Traveling to the Moon and Mars:** This video segment calculates the distance from Earth to the moon and from Earth to Mars. It also analyzes the temperature and surface of other planets and explains why Mars is targeted for human exploration.	http://tinyurl.com/qbmts4w

15.0 Planetary Shields: Magnetospheres	**Our World – The Sun, a Real Star:** Learn about the important relationship between Earth and the sun. Find out about the layers of the sun and how Earth's magnetosphere acts like a giant handkerchief to protect us from all kinds of space weather.	http://tinyurl.com/o2jooyq
16.0 Early Observations, from Telescopes to Spacecraft	**Our World – Early Hubble History:** Learn how the Hubble Space Telescope has changed the way scientists look at the universe, without the interference of Earth's atmosphere. Find out how this telescope works and a bit of history about the man for whom it is named.	http://tinyurl.com/p9xcr3c
17.0 Our Evolving Understanding of the Solar System	**Launchpad – Methane - An Indicator for Life?** What is the shape of our heliosphere and what lies beyond? How does interstellar medium affect the heliosphere? To find out, NASA launched the Interstellar Boundary Explorer, or IBEX, to map out the boundaries of our solar system.	http://tinyurl.com/qjehwok
18.0 Robotic Spacecraft: Far-Ranging Robots	**Launchpad – Curiosity Goes to Mars:** Find out why Curiosity is the best name for the largest rover ever sent to another planet. Learn about the challenges of landing on a planet with an atmosphere and the geology and chemistry questions scientists hope to answer with instruments on the Mars Science Laboratory.	http://tinyurl.com/nta9qda
19.0 The search for planets: Discovering New Worlds	**Launchpad – Kepler:** Join NASA on the Kepler Mission as this traveling telescope images the light from faraway stars to locate Earth-sized and smaller planets. Using the transit method, the Kepler telescope measures the brightness of a star and uses the data to predict habitable zones.	http://tinyurl.com/qhokfjv
20.0 Astrobiology- Are We Alone in the Universe?	**Launchpad – Astrobiology:** Are we alone in the universe? Where do we come from? Join NASA in the search for answers to these and many more questions about life in our solar system. Learn how astrobiologists use what we know about Earth to investigate Titan, Europa and other far-off worlds.	http://tinyurl.com/o3b62xa

Solar Math (2012). This is a revised and expanded version of the book Hinode Math (2008), featuring problems about the sun from many NASA science missions. The activities explore solar storms and solar structure using simple math activities. Problems range from calculating the sizes of sunspots from photographs, to investigating solar magnetism, the sunspot cycle, and solar storms using algebra and geometry. Suitable for students in grades 6-12. [99 Problems PDF: 15.7 Mby]

Lunar Math (2012). This is a revised and expanded version of Lunar Math (2008) that includes many problems for grades 3-5, as well as more challenging problems for older students. An exploration of the moon using NASA photographs and scaling activities including mathematical modeling of the lunar interior, and problems involving estimating the total mass of the moon and its atmosphere. [56 Problems PDF: 13.8 Mby]

Earth Math (2009). Students explore the simple mathematics behind global climate change through analyzing graphical data, data from NASA satellites, and by performing simple calculations of carbon usage using home electric bills and national and international energy consumption. [46 Problems PDF: 4.2 Mby]

Space Weather Math (2010). Students explore the way in which the sun interacts with Earth to produce space weather, and the ways in which astronomers study solar storms to predict when adverse conditions may pose a hazard for satellites and human operation in space. Six appendices and an extensive provide a rich 150-year context for why space whether is an important issue. [96 Problems PDF: 26.1 Mby]

Transit Math (2010). Students explore astronomical eclipses, transits and occultations to learn about their unique geometry, and how modern observations by NASA's Kepler Satellite will use transit math to discover planets orbiting distant stars. A series of Appendices reveal the imagery and history through news paper articles of the Transits of Venus observed during the 1700 and 1800s. [44 Problems PDF: 14.6 Mby]

Remote Sensing Math (2011). This book covers many topics in remote sensing, satellite imaging, image analysis and interpretation. Examples are culled from earth science and astronomy missions. Students learn about instrument resolution and sensitivity as well as how to calibrate a common digital camera, and how to design a satellite imaging system. [103 Problems PDF: 15.2 Mby]

Astrobiology Math (2011). This book introduces many topics in the emerging subject of astrobiology: The search for life beyond Earth. It covers concepts in evolution, the detection of extra-solar planets, habitability, Drake's Equation, and the properties of planets such as temperature and distance from their star. [75 Problems PDF: 33.2 Mby]

Mars Math (2012). An introduction to the planet Mars and some of the NASA missions that have studied this planet and its surface, including the Spirit and Opportunity Rovers and the much-awaited Curiosity Rover. Problems include basic scales and proportions, fractions, scientific notion, algebra and geometry. [24 Problems PDF: 7.7 Mby]

Exploring Planetary Moons (2013). This collection of activities is intended for students looking for additional challenges in the math and physical science curriculum in grades 3 through 6, but where the topics are drawn from astronomy and space science. This book introduces students to some of the most unusual places in our solar system that are not planets. Using simple proportional relationships and working with fractions, they will study the relative sizes of the larger moons in our solar system, and explore how temperatures change from place to place using the Celsius and Kelvin scales. [22 Problems PDF: 4.4 Mby]

These books are available at:

http://spacemath.gsfc.nasa.gov/books.html

Grade- 6

- Chapter 3 - **Understanding Decimals** - Students will learn about the Cassini mission and its exploration of Saturn's moons through reading a NASA press release. By viewing a NASA eClips video segment, students will learn more about these and other moons in our solar system. Then students will use decimals to compare the sizes and distances of Saturn's moons to the center of Saturn. Featured NASA Missions: Cassini

- Chapter 4 - **Number Theory and Fractions** - Students will learn about the Juno mission and its exploration of Jupiter, a giant gas planet, through reading a NASA press release. By viewing a NASA eClips video segment, students will visualize Jupiter and the other 88 largest objects in our solar system. Then students will use fractions to compare Jupiter's moons and movements.Featured NASA Missions: Juno

- Chapter 6 - **Data Collection** - During the last sunspot cycle between 1996-2008, over 21,000 flares and 13,000 clouds of plasma exploded from the suns magnetically active surface. These events create space weather. Students will learn more about space weather and how it affects Earth through reading a NASA press release and viewing a NASA eClips video segment. Then students will explore the statistics of various types of space weather storms by determining the mean, median and mode of a sample of storm events. Featured NASA Missions: SDO, ACE, STEREO

- Chapter 8 - **Measurement and Geometry** - Students learn how solar panels can be used to generate electrical power and how the size and area of the panels affects energy production. By reading a NASA press release and viewing a NASA eClips video segment, students see how solar energy is used by various NASA satellites and technology. Featured NASA Missions: Juno

Grade- 7

- Chapter 2 - **Integer Arithmetic** - Students will explore how methane molecules are produced from larger molecules, and how NASA is using signs of methane gas to search for life on other planets such as Mars. Students will read a NASA press release and view a NASA eClips video segment. Then students will use integer arithmetic to tally the number of hydrogen, oxygen and carbon atoms in a molecule and determine the number of methane atoms that can result. Featured NASA Missions: Mars Science Laboratory

- Chapter 4 - **Scale Models and Diagrams** - Students will learn more about the Lunar Reconnaissance Orbiter (LRO) through reading a NASA press release and viewing a NASA eClips video segment. Then students will explore scale modeling by measuring scaled drawings using high-resolution images of the lunar and martian surfaces. Featured NASA Missions: LRO, Opportunity Rover

- Chapter 7 - **Mean, Median and Mode** - During the last sunspot cycle between 1996-2008, over 21,000 flares and 13,000 clouds of plasma exploded from the suns magnetically active surface. Students will learn more about space weather through reading a NASA press release and viewing a NASA eClips video segment. Then students will explore the statistics of various types of space weather storms by determining the mean, median and mode of different samples of storm events. Featured NASA Missions: SDO

- Chapter 8 - **Angular Measure** - Students will learn about the Transit of Venus through reading a NASA press release and viewing a NASA eClips video that describes several ways to observe transits. Then students will study angular measurement by learning about parallax and how astronomers use this geometric effect to determine the distance to Venus during a Transit of Venus. Featured NASA Missions: Kepler, SDO

Grade -7

- Chapter 9 - **The Volume of Spheres and Cylinders** - Students will learn more about asteroids and comets through reading a NASA press release and viewing a NASA eClips video segment. Then, students will estimate and calculate volumes of comets, asteroids, and spacecraft. Featured NASA Missions: Dawn

- Chapter 10 - **Probability and Predictions** - Students will learn about the NASA Kepler mission and Earth-like planet discoveries through reading a NASA press release. They will also view a NASA eClips video describing the search for planets beyond our solar system. Then, students will study the statistics of planets outside our solar system and estimate the number of Earth-like planets in the Milky Way galaxy. Featured NASA Missions: Kepler

Grade – 8

- Chapter 1 - **Rational Number Operations** - Students will learn about the twin STEREO spacecraft and how they are being used to track solar storms through reading a NASA press release and viewing a NASA eClips video segment. Then students will examine data to learn more about the frequency and speed of solar storms traveling from the sun to Earth. Featured NASA Missions: STEREO

- Chapter 2 - **Graphs and Functions** - Students will learn about NASAs Radiation Belt Storm Probes (RBSP), Earths van Allen Radiation Belts, and space weather through reading a NASA press release and viewing a NASA eClips video segment. Then students will use simple linear functions to examine the scale of the radiation belts and the strength of Earths magnetic field. Featured NASA Missions: RBSP

- Chapter 4 - **Ratios, Proportions, Similarity** - Students will learn about the planet Mercury and the MESSENGER mission through reading a NASA press release and viewing a NASA eClips video segment. Then students will perform calculations using fractions and decimals to explore the relative sizes of planets in our solar system and orbiting other stars. Featured NASA Missions: MESSENGER

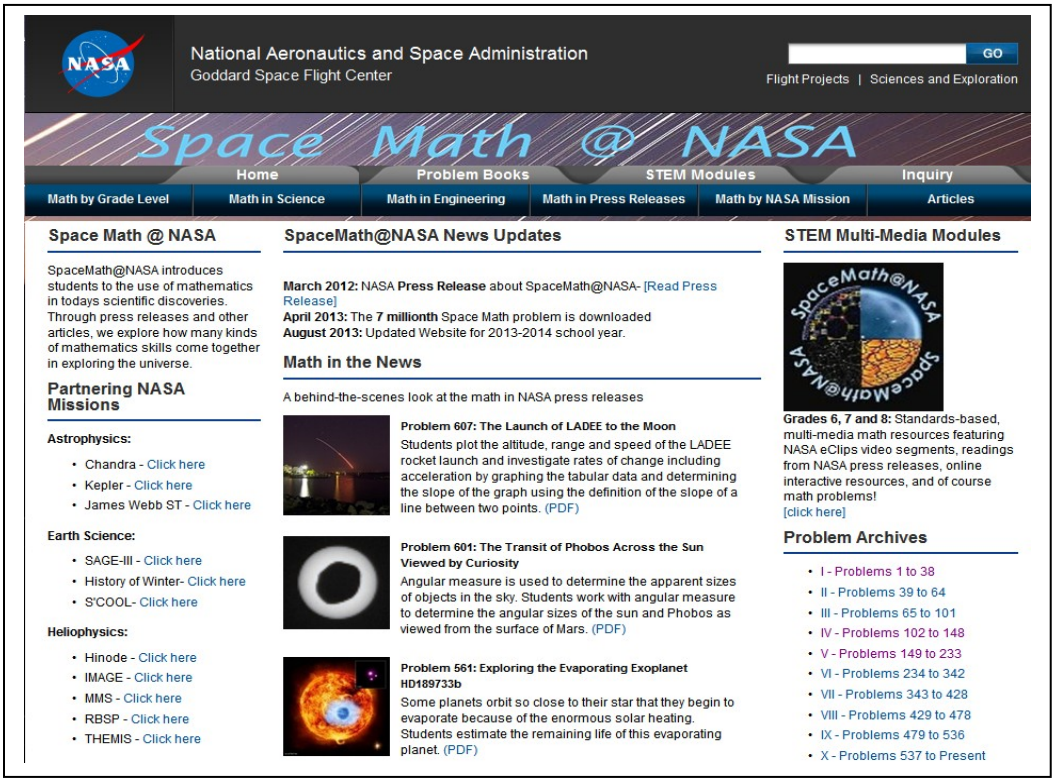
- Chapter 5 - **Geometry and Angle Properties** - Students will learn about the Mars Science Laboratory (MSL) and the Curiosity Rover through reading a NASA press release and viewing a NASA eClips video segment. Then students will use the Pythagorean Theorem to determine distance between a series of hypothetical exploration sites within Gale Crater on Mars. Featured NASA Missions: Curiosity

- Chapter 7 - **Multi-Step Equations** - Students will learn how NASA uses different types of power systems to generate electricity through reading a NASA press release and viewing a NASA eClips video segment. Then students will solve a series of problems involving multi-step equations to explore these systems in more detail. Featured NASA Missions: Cassini, Mars Science Laboratory

These resources are available at the STEM Modules resource for SpaceMath@NASA located at

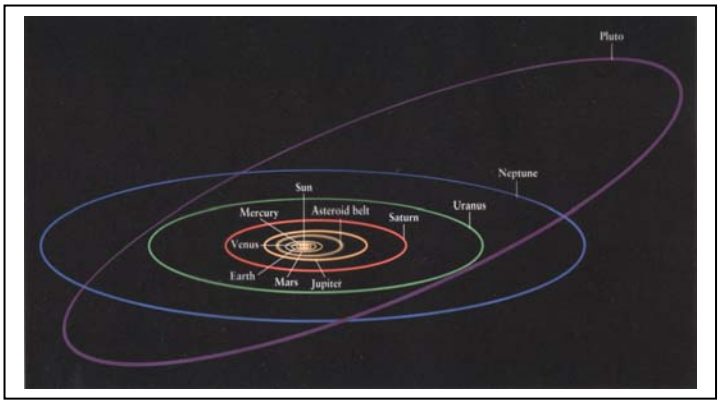
http://spacemath.gsfc.nasa.gov/media.html

http://solarsystem.nasa.gov/yss/topics.cfm



Space Math **http://spacemath.gsfc.nasa.gov**

How Big is Our Solar System?



Our solar system is so big it is almost impossible to imagine its size if you use ordinary units like feet or miles. The distance from Earth to the Sun is 93 million miles (149 million kilometers), but the distance to the farthest planet Neptune is nearly 3 billion miles (4.5 billion kilometers). Compare this to the farthest distance you can walk in one full day (70 miles) or that the International Space Station travels in 24 hours (400,000 miles).

The best way to appreciate the size of our solar system is by creating a scaled model of it that shows how far from the sun the eight planets are located. Astronomers use the distance between Earth and sun, which is 93 million miles, as a new unit of measure called the Astronomical Unit. It is defined to be exactly 1.00 for the Earth-Sun orbit distance, and we call this distance 1.00 AUs.

Problem 1 - The table below gives the distance from the Sun of the eight planets in our solar system. By setting up a simple proportion, convert the stated distances, which are given in millions of kilometers, into their equivalent AUs, and fill-in the last column of the table.

Planet	Distance to the Sun in millions of kilometers	Distance to the Sun in Astronomical Units
Mercury	57	
Venus	108	
Earth	149	
Mars	228	
Jupiter	780	
Saturn	1437	
Uranus	2871	
Neptune	4530	

Problem 2 – Suppose you wanted to build a scale model of our solar system so that the orbit of Neptune was located 10 feet from the yellow ball that represents the sun. How far from the yellow ball, in inches, would you place the orbit of Jupiter?

Answer Key

Problem 1 - The table below gives the distance from the Sun of the eight planets in our solar system. By setting up a simple proportion, convert the stated distances, which are given in millions of kilometers, into their equivalent AUs, and fill-in the last column of the table.

Answer: In the case of Mercury, the proportion you would write would be

$$\frac{149 \text{ million km}}{1 \text{ AU}} = \frac{57 \text{ million km}}{X} \quad \text{then } X = 1 \text{ AU} \times (57/149) = 0.38$$

Planet	Distance to the Sun in millions of kilometers	Distance to the Sun in Astronomical Units
Mercury	57	0.38
Venus	108	0.72
Earth	149	1.00
Mars	228	1.52
Jupiter	780	5.20
Saturn	1437	9.58
Uranus	2871	19.14
Neptune	4530	30.20

Problem 2 – Suppose you wanted to build a scale model of our solar system so that the orbit of Neptune was located 10 feet from the yellow ball that represents the sun. How far from the yellow ball, in inches, would you place the orbit of Jupiter?

Answer: The proportion would be written as:

$$\frac{30.20 \text{ AU}}{10 \text{ feet}} = \frac{5.2 \text{ AU}}{X} \quad \text{then } X = 10 \text{ feet} \times (5.2/30.2) \quad \text{so } X = \mathbf{1.72 \text{ feet}}$$

Since 1 foot = 12 inches, the unit conversion is written as

$$1.72 \text{ feet} \times \frac{12 \text{ inches}}{1 \text{ foot}} = \mathbf{20.64 \text{ inches}}.$$

Visiting the Planets at the Speed of Light!

The fastest way to get from place to place in our solar system is to travel at the speed of light, which is 300,000 km/sec (670 million miles per hour!). Unfortunately, only radio waves and other forms of electromagnetic radiation can travel exactly this fast.

When NASA sends spacecraft to visit the planets, scientists and engineers have to keep in radio contact with the spacecraft to gather scientific data. But the solar system is so vast that it takes quite a bit of time for the radio signals to travel out from Earth and back.

Problem 1 – Earth has a radius of 6378 kilometers. What is the circumference of Earth to the nearest kilometer?

Problem 2 – At the speed of light, how long would it take for a radio signal to travel once around Earth?

Problem 3 – The Moon is located 380,000 kilometers from Earth. During the Apollo-11 mission in 1969, engineers on Earth would communicate with the astronauts walking on the lunar surface. From the time they asked a question, how long did they have to wait to get a reply from the astronauts?

Problem 4 – In the table below, fill in the one-way travel time from the sun to each of the planets. Use that fact that the travel time from the Sun to Earth is 8 ½ minutes. Give your answer to the nearest tenth, in units of minutes or hours, whichever is the most convenient unit.

Planet	Distance from Sun in Astronomical Units	Light Travel Time
Mercury	0.38	
Venus	0.72	
Earth	1.00	8.5 minutes
Mars	1.52	
Jupiter	5.20	
Saturn	9.58	
Uranus	19.14	
Neptune	30.20	

Problem 1 – Earth has a radius of 6378 kilometers. What is the circumference of Earth to the nearest kilometer?

Answer: $C = 2 \pi R$ so $C = 2 \times 3.141 \times (6378 \text{ km}) =$ **40,067 km.**

Problem 2 – At the speed of light, how long would it take for a radio signal to travel once around Earth?

Answer: Time = distance/speed so
Time = 40,067/300,000 = **0.13 seconds. This is about 1/7 of a second**.

Problem 3 – The Moon is located 380,000 kilometers from Earth. During the Apollo-11 mission in 1969, engineers on Earth would communicate with the astronauts walking on the lunar surface. From the time they asked a question, how long did they have to wait to get a reply from the astronauts?

Answer: From the proportion:

$$\frac{0.13 \text{ seconds}}{40067 \text{ km}} = \frac{X}{380000 \text{ km}}$$ we have $X = (380000/40067) \times 0.13 = 1.23$ seconds.

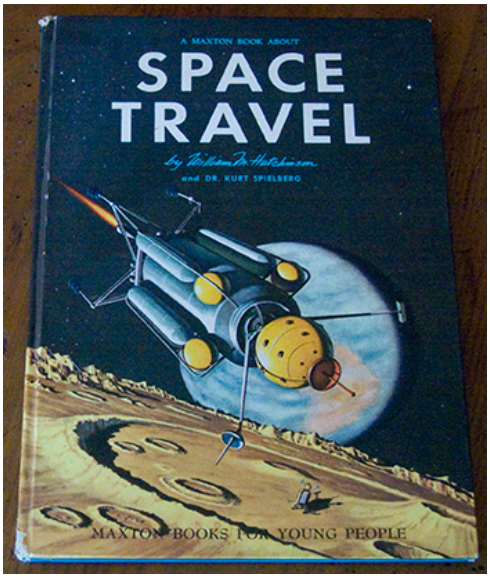
This is the one-way time for the signal to get to the moon from Earth, so the round-trip time is twice this or **2.46 seconds**.

Problem 4 – In the table below, fill in the one-way travel time from the sun to each of the planets. Use that fact that the travel time from the Sun to Earth is 8 ½ minutes. Give your answer to the nearest tenth, in units of minutes or hours, whichever is the most convenient unit.

Answer: Use simple proportions based on 8.5 minutes of time = 1.00 AU of distance.

Planet	Distance from Sun in Astronomical Units	Light Travel Time
Mercury	0.38	3.2 minutes
Venus	0.72	6.1 minutes
Earth	1.00	8.5 minutes
Mars	1.52	12.9 minutes
Jupiter	5.20	44.2 minutes
Saturn	9.58	1.4 hours
Uranus	19.14	2.7 hours
Neptune	30.20	4.3 hours

Travel Times by Spacecraft Around the Solar System



Most science fiction stories often have spaceships with powerful, or exotic, rockets that can let space travelers visit the distant planets in less than a day's journey. The sad thing is that we are not quite there in the Real World. This is because our solar system is so vast, and our rockets can't produce quite enough speed to make journeys short.

NASA has been working on this problem for over 50 years and has come up with many possible solutions. Each one is more expensive than just using ordinary fuels and engines like the ones used on most rockets!

Problem 1 – The entire International Space Station orbits Earth at a speed of 28,000 kilometers per hour (17,000 mph). At this speed, how many days would it take to travel to the sun from Earth, located at a distance of 149 million kilometers?

Problem 2 – The planet Neptune is located 4.5 billion kilometers from Earth. How many years would it take a rocket traveling at the speed of the International Space Station to make this journey?

Problem 3 – The fastest unmanned spacecraft, Helios-2, traveled at a speed of 253,000 km/hr. In the table below, use proportional math to fill in the travel times from the sun to each planet traveling at the speed of Helios-2. Give your answers to the nearest tenth in appropriate units of days or years.

Planet	Distance in millions of kilometers	Time
Mercury	57	
Venus	108	
Earth	149	
Mars	228	
Jupiter	780	
Saturn	1437	
Uranus	2871	
Neptune	4530	

Problem 1 – The entire International Space Station orbits Earth at a speed of 28,000 kilometers per hour (17,000 mph). At this speed, how many days would it take to travel to the sun from Earth, located at a distance of 149 million kilometers?

Answer: Time = Distance/speed so
Time = 149,000,000 km/ 28,000
= 5321 hours or **222 days**.

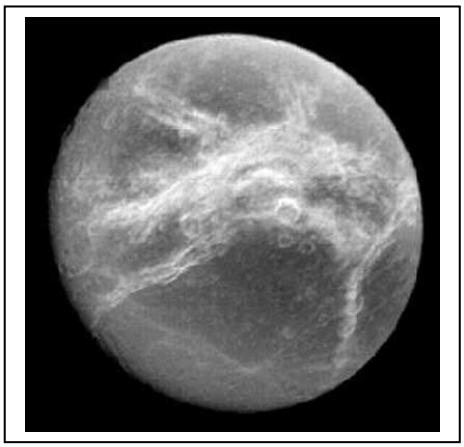
Problem 2 – The planet Neptune is located 4.5 billion kilometers from Earth. How many years would it take a rocket traveling at the speed of the International Space Station to make this journey?

Answer: Time = 4,500,000,000 km / 28,000 km/h
= 160714 hours or 6696 days or **18.3 years**.

Problem 3 – The fastest unmanned spacecraft, Helios-2, traveled at a speed of 253,000 km/hr. In the table below, use proportional math to fill in the travel times from the sun to each planet traveling at the speed of Helios-2. Give your answers to the nearest tenth in appropriate units of days or years.

Planet	Distance in millions of kilometers	Time
Mercury	57	9.4 days
Venus	108	17.8 days
Earth	149	24.5 days
Mars	228	37.5 days
Jupiter	780	128.5 days
Saturn	1437	236.7 days
Uranus	2871	1.3 years
Neptune	4530	2.0 years

A Matter of Timing



Astronomers who study planets and their satellites often have to work out how often satellites or planets 'line up' in various ways, especially when they are closest together in space.

Figure shows the satellite Dione (Courtesy: NASA/Cassini)

Problem 1 – The two satellites of Tethys and Dione follow circular orbits around Jupiter. Tethys takes about 2 days for one complete orbit while Dione takes about 3 days. If the two satellites started out closest together on July 1, 2008, how many days later will they once again be at 'opposition' with one another?

A) Find the Least Common Multiple between the orbit periods.

B) Draw two concentric circles and work the solution out graphically.

C) What is the relationship between your answer to A and B?

Problem 2 - Two planets have orbit periods of 3 years and 5 years. How long will it take them to return to the same locations that they started at?

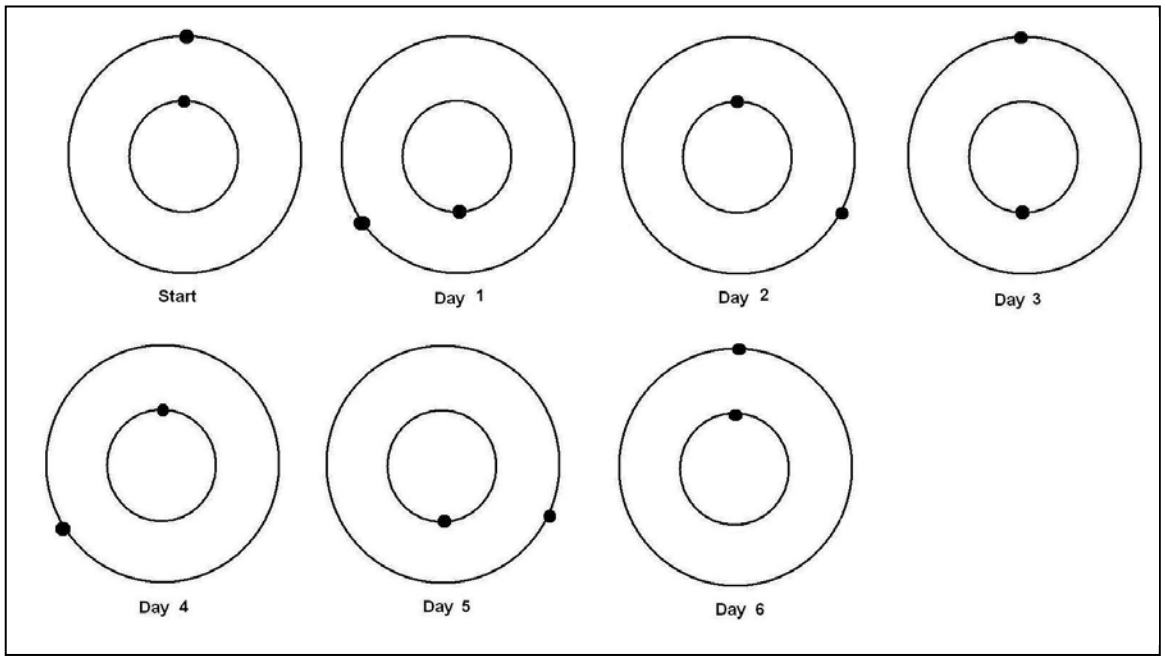
Answer Key

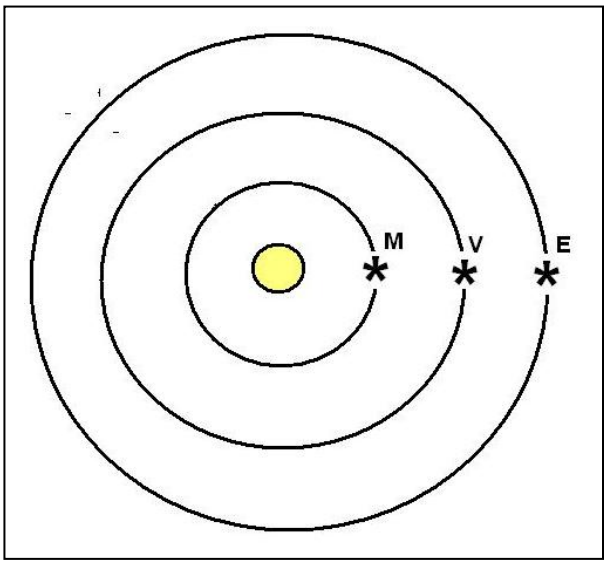
Problem 1 - A) The Least Common Multiple between 2 and 3 is 6, so it will take 6 days for the two moons to return to their original positions. B) The figure below shows the progression in elapsed days, with the moons moving counterclockwise. C) The LCF between the orbit periods tells you how long it will take for the two bodies to return to their same locations when they started.

Problem 2 - Two planets have orbit periods of 3 years and 5 years. How long will it take them to return to the same locations that they started at?
Answer; The LCM for 3 and 5 is found by forming the multiples of 3 and 5 and finding the first number they share in common.

For 3: 3, 6, 9, 12**, 15,** 18, 21, 24, 27, 30, 33, 36, ….
For 5: 5, 10, **15,** 20, 25, 30, …

The smallest common multiple is '15', so it will take the two planets 15 years to return to the positions they started with.





One of the most interesting things to see in the night sky is two or more planets coming close together in the sky. Astronomers call this a conjunction. As seen from their orbits, another kind of conjunction is sometimes called an 'alignment' which is shown in the figure to the left and involves Mercury, M, Venus, V, and Earth, E. As viewed from Earth's sky, Venus and Mercury would be very close to the sun, and may even be seen as black disks 'transiting' the disk of the sun at the same time, if this alignment were exact. How often do alignments happen?

Earth takes 365 days to travel one complete orbit, while Mercury takes 88 days and Venus takes 224 days, so the time between alignments will require each planet to make a whole number of orbits around the sun and return to the pattern you see in the figure above. Let's look at a simpler problem. Suppose Mercury takes 1/4 Earth-year and Venus takes 2/3 of an Earth-year to make their complete orbits around the sun. You can find the next line-up from one of these two methods:

Method 1: Work out the three number series like this:

Earth = 0, 1, **2**, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12,…
Mercury = 0, 1/4, 2/4, 3/4 ,4/4, 5/4, 6/4, 7/4, **8/4**, 9/4, 10/4, 11/4, 12/4,…
Venus = 0, 2/3, 4/3, **6/3**, 8/3, 10/3, 12/3, 14/3, 16/3, 18/3, 20/3, …

Notice that the first time they all coincide with the same number is at **2 years**. So Mercury has to go around the Sun 8 times, Venus 3 times and Earth 2 times for them to line up again in their orbits.

Method 2: We need to find the Least Common Multiple (LCM) of 1/4, 2/3 and 1. First render the periods in multiples of a common time unit of 1/12, then the sequences are:

Mercury = 0, 3, 6, 9, 12, 15, 18, 21, **24**,
Venus = 0, 8, 16, **24**, 32, 40, …
Earth, 0, 12, **24**, 36, 48, 60, …

The LCM is 24 which can be found from prime factorization:

Mercury: 3 = 3
Venus: 8 = 2 x 2 x 2
Earth: 12 = 2 x 2 x 3

The LCM the product of the highest powers of each prime number or 3 x 2 x 2 x 2 = 24. and so it will take 24/12 = **2 years.**

Problem 1 - Suppose a more accurate estimate of their orbit periods is that Mercury takes 7/30 Earth-years and Venus takes 26/42 Earth-years. After how many Earth-years will the alignment shown in the figure above reoccur?

Problem 1 - Suppose a more accurate estimate of their orbit periods is that Mercury takes 7/30 Earth-years and Venus takes 26/42 Earth-years. After how many Earth-years will the alignment reoccur?

Mercury = 7/30 x 365 = 85 days vs actual 88 days
Venus = 26/42 x 365 = 226 days vs actual 224 days
Earth = 1

The common denominator is 42 x 30 = 1,260 so the series periods are
Mercury = 7 x 42 = 294 so 7/30 = 294/1260
Venus = 26 x 30 = 780 so 26/42 = 780/1260
Earth = 1260 so 1 = 1260/1260

The prime factorizations of these three numbers are

294 = 2 x 2 x 3 x 7 x 7
780 = 2 x 2x 5 x 3 x 13
1260 = 2 x 2 x 3 x 3 x 5 x 7

LCM = 2 x 2 x 3 x 3 x 5 x 7 x 7 x 13 = 114,660

So the time will be 114,660 / 1260 = 91 years! In this time, Mercury will have made exactly 114,660/294 = 390 orbits and Venus will have made 114,660/780 = 147 orbits

Note to Teacher: Why did the example problem give only 2 years while this problem gave 91 years for the 'same' alignment? Because we used a more accurate approximation for the orbit periods of the three planets. Mercury actual period = 88 days but 1/4 Earth-year = 91.25 days compared to 7/30 Earth-year = 85 days. Venus actual period = 224 days but 2/3 Earth-year = 243 days and 26/42 Earth-year = 226 days.

This means that after 2 years and exactly 8 orbits (8 x 91.25 = 730 days), Mercury will be at 8/4 x 365 = 730 days while the actual 88-day orbit will be at 88 x 8 = 704 days or a timing error of 26 days. Mercury still has to travel another 26 days in its orbit to reach the alignment position. For Venus, its predicted orbit period is 2/3 x 365 = 243.3 days so its 3 orbits in the two years would equal 3 x 243.3 days = 730 days, however its actual period is 224 days so in 3 orbits it accumulates 3 x 224 = 672 days and the difference is 730-672 = 58 days so it has to travel another 58 days to reach the alignment. In other words, the actual positions of Mercury and Venus in their orbits is far from the 'straight line' we were hoping to see after exactly 2 years, using the approximate periods of 1/4 and 2/3 earth-years!

With the more accurate period estimate of 7/30 Earth-years (85 days) for Mercury and 26/42 Earth-years (226 days) for Venus, after 91 years, Mercury will have orbited exactly 91 x 365 days/88 days = 377.44 times, and Venus will have orbited 91x365/224 = 148.28 times. This means that Mercury will be 0.44 x 88d = 38.7 days ahead of its predicted alignment location, and Venus will be 0.28 x 224 = 62.7 days behind its expected alignment location. Comparing the two predictions, Prediction 1: Mercury= - 26 days, Venus= - 58 days; Prediction 2: Mercury = +26 days and Venus = - 22 days. Our prediction for Venus has significantly improved while for Mercury our error has remained about the same in absolute magnitude. In the sky, the two planets will appear closer together for Prediction 2 in 1911 years than for Prediction 1 in 2 years. If we want an even 'tighter' alignment, we have to make the fractions for the orbit periods much closer to the actual periods of 88 and 224 days.

Unit Conversions III

1 Astronomical Unit = 1.0 AU = 1.49 x 10^8 kilometers
1 Parsec = 3.26 Light years = 3 x 10^{18} centimeters = 206,265 AU
1 Watt = 10^7 ergs/sec
1 Star = 2 x 10^{33} grams

1 Yard = 36 inches	**1 meter = 39.37 inches**	**1 mile = 5,280 feet**
1 Liter = 1000 cm3	**1 inch = 2.54 centimeters**	**1 kilogram = 2.2 pounds**
1 Gallon = 3.78 Liters	**1 kilometer = 0.62 miles**	

Problem 1 – Convert 11.3 square feet into square centimeters.

Problem 2 – Convert 250 cubic inches into cubic meters.

Problem 3 – Convert 1000 watts/meter^2 into watts/foot^2

Problem 4 – Convert 5 miles into kilometers.

Problem 5 – Convert 1 year into seconds.

Problem 6 – Convert 1 km/sec into parsecs per million years.

Problem 7 - A house is being fitted for solar panels. The roof measures 50 feet x 28 feet. The solar panels cost \$1.00/cm^2 and generate 0.03 watts/cm^2. A) What is the maximum electricity generation for the roof in kilowatts? B) How much would the solar panels cost to install? C) What would be the owners cost for the electricity in dollars per watt?

Problem 8 – A box of cereal measures 5 cm x 20 cm x 40 cm and contains 10,000 Froot Loops. What is the volume of a single Froot Loop in cubic millimeters?

Problem 9 – In city driving, a British 2002 Jaguar is advertised as having a gas mileage of 13.7 liters per 100 km, and a 2002 American Mustang has a mileage of 17 mpg. Which car gets the best gas mileage?

Problem 10 – The Space Shuttle used 800,000 gallons of rocket fuel to travel 400 km into space. If one gallon of rocket fuel has the same energy as 5 gallons of gasoline, what is the equivalent gas mileage of the Space Shuttle in gallons of gasoline per mile?

Problem 11 – The length of an Earth day increases by 0.0015 seconds every century. How long will a day be in 3 billion years from now?

Problem 12 – The density of matter in the Milky Way galaxy is 7.0 x 10^{-24} grams/cm^3. How many stars are in a cube that is 10 light years on a side?

Problem 13 – At a speed of 300,000 km/sec, how far does light travel in miles in 1 year?

Problem 1 – 11.3 x (12 inches/foot)x(12 inches/foot) x (2.54 cm/1 inch)x(2.54 cm/1 inch) = **10,500 cm^2**

Problem 2 – 250 $inch^3$ x (2.54 cm/inch)^3 x (1 meter/100 cm)^3 = **0.0041 m^3**

Problem 3 – 1000 watts/$meter^2$ x (1 meter/39.37 inches)^2 x (12 inches/foot)^2 = **93.0 watts/ft^2**

Problem 4 – 5 miles x (5280 feet/mile) x (12 inches/foot)x(2.54 cm/inch)x(1 meter/100 cm) x(1 km/1000 meters) **= 8.1 km**

Problem 5 – 1 year x (365 days/year)x(24 hours/day)x(60 minutes/hr)x(60 seconds/minute) = **31,536,000 seconds.**

Problem 6 – 1 km/sec x (100000 cm/km)x(3.1 x 10^7 seconds/year) x (1 parsec/ 3.1 x 10^{18} cm) x (1,000,000 years/1 million years) = **1 parsec/million years**

Problem 7 - A) Area = 50 feet x 28 feet = 1400 ft^2. Convert to cm^2: 1400 x (12 inch/foot)^2 x (2.54 cm/1 inch)^2 = 1,300,642 cm^2. Maximum power = 1,300,642 cm^2 x 0.03 watts/cm^2 = **39.0 kilowatts**. B) 1,300,642 cm^2 x $1.00 /$cm^2$ = **$1.3 million** C) $1,300,000 / 39,000 watts = **$33.3 /watt**.

Problem 8 – Volume of box = 5 x 20 x 40 = 4000 cm^3. This contains 10,000 Froot Loops, so each one has a volume of 4,000 cm^3/10,000 loops = 0.4 cm^3/Loop. Converting this into cubic millimeters: 0.4 cm^3 x (10 mm/1 cm)^3 = **400 mm^3/Loop**.

Problem 9 – Convert both to kilometers per liter. Jaguar = 100 km/13.7 liters = 7.3 km/liter. Mustang = 17.0 x (1 km/0.62 miles) x(1 gallon/3.78 liters) = 7.25 km/liter. **They both get similar gas mileage under city conditions**.

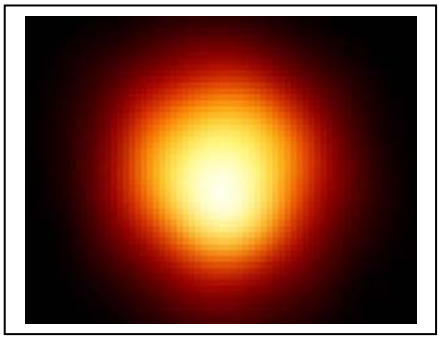
Problem 10 – 400 km x (0.62 miles/km) = 248 miles. Equivalent gallons of gasoline = 800,000 gallons rocket fuel x (5 gallons gasoline/1 gallon rocket fuel) = 4,000,000 gallons gasoline, so the 'mpg' is 248 miles/4000000 = 0.000062 miles/gallon or **16,130 gallons/mile**.

Problem 11 – 0.00015 sec/century x (1 century/100 years) x 3 billion years = 4,500 seconds or 1.25 hours. The new 'day' would be 24h – 1.25 = **22.75 hours long**.

Problem 12 –First convert to grams per cubic parsec: 7.0 x 10^{-24} grams/cm^3 x (3.1 x 10^{18} cm/parsec)^3 = 2.0 x 10^{32} grams/pc^3. Then convert to Stars/pc3: 2.0 x 10^{32} grams/pc^3 x (1 Star/2 x 10^{33} grams) = 0.1 Stars/pc^3. Then compute the volume of the cube: V = 10x10x10 = 1000 light years^3 = 1000 light years^3 x (1 parcsec/3.26 light years)3 = 28.9 Parsecs^3. Then multiply the density by the volume: 0.1 Stars/pc^3 x (28.9 Parsecs^3) = **3.0 Stars in a volume that is 10 light years on a side**.

Problem 13 – 300,000 km/sec x (3.1 x 10^7 sec/year) = 9.3 x 10^{12} km. Then 9.3 x 10^{12} km x (0.62 miles/km) = **5.7 trillion miles**.

The relative sizes of the sun and stars



Stars come in many sizes, but their true appearances are impossible to see without special telescopes. The image to the left was taken by the Hubble Space telescope and resolves the red supergiant star Betelgeuse so that its surface can be just barely seen. Follow the number clues below to compare the sizes of some other familiar stars!

Problem 1 - The sun's diameter if 10 times the diameter of Jupiter. If Jupiter is 11 times larger than Earth, how much larger than Earth is the Sun?

Problem 2 - Capella is three times larger than Regulus, and Regulus is twice as large as Sirius. How much larger is Capella than Sirius?

Problem 3 - Vega is 3/2 the size of Sirius, and Sirius is 1/12 the size of Polaris. How much larger is Polaris than Vega?

Problem 4 - Nunki is 1/10 the size of Rigel, and Rigel is 1/5 the size of Deneb. How large is Nunki compared to Deneb?

Problem 5 - Deneb is 1/8 the size of VY Canis Majoris, and VY Canis Majoris is 504 times the size of Regulus. How large is Deneb compared to Regulus?

Problem 6 - Aldebaran is 3 times the size of Capella, and Capella is twice the size of Polaris. How large is Aldebaran compared to Polaris?

Problem 7 - Antares is half the size of Mu Cephi. If Mu Cephi is 28 times as large as Rigel, and Rigel is 50 times as large as Alpha Centauri, how large is Antares compared to Alpha Centauri?

Problem 8 - The Sun is 1/4 the diameter of Regulus. How large is VY Canis Majoris compared to the Sun?

Inquiry: - Can you use the information and answers above to create a scale model drawing of the relative sizes of these stars compared to our Sun.

Answer Key

The relative sizes of some popular stars is given below, with the diameter of the sun = 1 and this corresponds to an actual physical diameter of 1.4 million kilometers.

Betelgeuse	440	Nunki	5	VY CMa	2016	Delta Bootis	11
Regulus	4	Alpha Cen	1	Rigel	50	Schedar	24
Sirius	2	Antares	700	Aldebaran	36	Capella	12
Vega	3	Mu Cephi	1400	Polaris	24	Deneb	252

Problem 1 - Sun/Jupiter = 10, Jupiter/Earth = 11 so Sun/Earth = 10 x 11 = **110 times.**

Problem 2 - Capella/ Regulus = 3.0, Regulus/Sirius = 2.0 so Capella/Sirius = 3 x 2 = **6 times.**

Problem 3 - Vega/Sirius = 3/2 Sirius/Polaris=1/12 so Vega/Polaris = 3/2 x 1/12 = **1/8 times**

Problem 4 - Nunki/Rigel = 1/10 Rigel/Deneb = 1/5 so Nunki/Deneb = 1/10 x 1/5 = **1/50.**

Problem 5 - Deneb/VY = 1/8 and VY/Regulus = 504 so Deneb/Regulus = 1/8 x 504 = **63 times**

Problem 6 - Aldebaran/Capella = 3 Capella/Polaris = 2 so Aldebaran/Polaris = 3 x 2 **= 6 times**.

Problem 7 - Antares/Mu Cep = 1/2 Mu Cep/Rigel = 28 Rigel/Alpha Can = 50, then Antares/Alpha Can = 1/2 x 28 x 50 = **700 times**.

Problem 8 - Regulus/Sun = 4 but VY CMA/Regulus = 504 so VY Canis Majoris/Sun = 504 x 4 = **2016 times the sun's size!**

Inquiry: Students will use a compass and millimeter scale. If the diameter of the Sun is 1 millimeter, the diameter of the largest star VY Canis Majoris will be 2016 millimeters or about 2 meters!

Scientific Notation I

Astronomers rely on scientific notation in order to work with 'big' things in the universe. The rules for using this notation are pretty straight-forward, and are commonly taught in most 7th-grade math classes as part of the National Education Standards for Mathematics.

The following problems involve the conversion of decimal numbers into SN form, and are taken from common astronomical applications and quantities.

1) Length of a year. 31,560,000.0 seconds

2) Speed of light: 299,792.4 kilometers/sec

3) Mass of the sun: 1,989,000,000,000,000,000,000,000,000,000,000 grams

4) Mass of Earth: 5,974,000,000,000,000,000,000,000 kilograms

5) One light-year : 9,460,500,000,000 kilometers

6) Power output of sun : 382,700,000,000,000,000,000,000,000 watts

7) Mass of an electron: 0.00000000000000000000000000000091096 kilograms

8) Energy equivalent of one electron-Volt : 0.000000000000000000016022 joules

9) Ratio of proton to electron mass: 1,836.2

10) Planck's Constant: 0.00000000000000000000000000006626068 ergs seconds

11) Radius of hydrogen atom : 0.00000000529177 centimeters

12) Radius of Earth's orbit: 14,959,789,200,000 centimeters

13) Smallest unit of physical distance: 0.0000000000000000000000000000000016 centimeters

14) Diameter of Visible Universe: 26,000,000,000,000,000,000,000,000,000 centimeters

Answer Key:

1) Length of a year. 31,560,000.0 seconds
Answer: 3.156×10^{7} seconds

2) Speed of light: 299,792.4 kilometers/sec
Answer: 2.997924×10^{5} km/sec

3) Mass of the sun: 1,989,000,000,000,000,000,000,000,000,000,000 grams
Answer: 1.989×10^{33} grams

4) Mass of Earth: 5,974,000,000,000,000,000,000,000 kilograms
Answer: 5.974×10^{24} kg

5) One light-year : 9,460,500,000,000 kilometers
Answer: 9.4605×10^{12} km

6) Power output of sun : 382,700,000,000,000,000,000,000,000 watts
Answer: 3.827×10^{26} watts

7) Mass of an electron: 0.00000000000000000000000000000091096 kilograms
Answer: 9.1096×10^{-31} kg

8) Energy equivalent of one electron-Volt : 0.000000000000000000016022 joules
Answer: 1.6022×10^{-19} Joules

9) Ratio of proton to electron mass: 1,836.2
Answer; 1.8362×10^{3}

10) Planck's Constant: 0.00000000000000000000000000006626068 ergs seconds
Answer: 6.626068×10^{-27} ergs seconds

11) Radius of hydrogen atom : 0.00000000529177 centimeters
Answer: 5.29177×10^{-9} cm

12) Radius of Earth's orbit: 14,959,789,200,000 centimeters
Answer: $1.49597892 \times 10^{13}$ cm

13) Smallest unit of physical distance: 0.0000000000000000000000000000000016 centimeters
Answer: 1.6×10^{-33} cm

14) Diameter of Visible Universe: 26,000,000,000,000,000,000,000,000,000 centimeters
Answer; 2.6×10^{28} cm

Applications of Scientific Notation 1.9

Scientific notation is an important way to represent very big, and very small, numbers. Here is a sample of astronomical problems that will test your skill in using this number representation.

Problem 1: The sun produces 3.9×10^{33} ergs per second of radiant energy. How much energy does it produce in one year (3.1×10^{7} seconds)?

Problem 2: One gram of matter converted into energy yields 3.0×10^{20} ergs of energy. How many tons of matter in the sun is annihilated every second to produce its luminosity of 3.9×10^{33} ergs per second? (One metric ton = 10^{6} grams)

Problem 3: The mass of the sun is 1.98×10^{33} grams. If a single proton has a mass of 1.6×10^{-24} grams, how many protons are in the sun?

Problem 4: The approximate volume of the visible universe (A sphere with a radius of about 14 billion light years) is 1.1×10^{31} cubic light-years. If a light-year equals 9.2×10^{17} centimeters, how many cubic centimeters does the visible universe occupy?

Problem 5: A coronal mass ejection from the sun travels 1.5×10^{13} centimeters in 17 hours. What is its speed in kilometers per second?

Problem 6: The NASA data archive at the Goddard Space Flight Center contains 25 terabytes of data from over 1000 science missions and investigations. (1 terabyte = 10^{15} bytes). How many CD-roms does this equal if the capacity of a CD-rom is about 6×10^{8} bytes? How long would it take, in years, to transfer this data by a dial-up modem operating at 56,000 bits/second? (Note: one byte = 8 bits).

Problem 7: Pluto is located at an average distance of 5.9×10^{14} centimeters from Earth. At the speed of light (2.99×10^{10} cm/sec) how long does it take a light signal (or radio message) to travel to Pluto and return?

Problem 8: The planet HD209458b, now known as Osiris, was discovered by astronomers in 1999 and is at a distance of 150 light-years (1 light-year = 9.2×10^{12} kilometers). If an interstellar probe were sent to investigate this world up close, traveling at a maximum speed of 700 km/sec (about 10 times faster than our fastest spacecraft: Helios-1), how long would it take to reach Osiris?

Problem 1: The sun produces 3.9×10^{33} ergs per second of radiant energy. How much energy does it produce in one year (3.1×10^{7} seconds)? **Answer:** $3.9 \times 10^{33} \times 3.1 \times 10^{7} = 1.2 \times 10^{41}$ ergs.

Problem 2: One gram of matter converted into energy yields 3.0×10^{20} ergs of energy. How many tons of matter in the sun is annihilated every second to produce its luminosity of 3.9×10^{33} ergs per second? (One metric ton = 10^{6} grams). **Answer:** $3.9 \times 10^{33} / 3.0 \times 10^{20} = 1.3 \times 10^{13}$ grams per second, or $1.3 \times 10^{13} / 10^{6} = 1.3 \times 10^{5}$ metric tons of mass.

Problem 3: The mass of the sun is 1.98×10^{33} grams. If a single proton has a mass of 1.6×10^{-24} grams, how many protons are in the sun? **Answer:** $1.98 \times 10^{33} / 1.6 \times 10^{-24} = 1.2 \times 10^{57}$ protons.

Problem 4: The approximate volume of the visible universe (A sphere with a radius of about 14 billion light years) is 1.1×10^{31} cubic light-years. If a light-year equals 9.2×10^{17} centimeters, how many cubic centimeters does the visible universe occupy? **Answer:** 1 cubic light year = $(9.2 \times 10^{17})^{3} = 7.8 \times 10^{53}$ cubic centimeters, so the universe contains $7.8 \times 10^{53} \times 1.1 \times 10^{31} = 8.6 \times 10^{84}$ cubic centimeters.

Problem 5: A coronal mass ejection from the sun travels 1.5×10^{13} centimeters in 17 hours. What is its speed in kilometers per second? **Answer:** $1.5 \times 10^{13} / (17 \times 3.6 \times 10^{3}) = 2.4 \times 10^{8}$ cm/sec = 2,400 km/sec.

Problem 6: The NASA data archive at the Goddard Space Flight Center contains 25 terabytes of data from over 1000 science missions and investigations. (1 terabyte = 10^{15} bytes). How many CD-roms does this equal if the capacity of a CD-rom is about 6×10^{8} bytes? How long would it take, in years, to transfer this data by a dial-up modem operating at 56,000 bits/second? (Note: one byte = 8 bits). **Answer:** $2.5 \times 10^{16} / 6 \times 10^{8} = 4.2 \times 10^{7}$ Cdroms. It would take $2.5 \times 10^{16} / 7{,}000 = 3.6 \times 10^{12}$ seconds or about 1.1×10^{5} years.

Problem 7: Pluto is located at an average distance of 5.9×10^{14} centimeters from Earth. At the speed of light (2.99×10^{10} cm/sec) how long does it take a light signal (or radio message) to travel to Pluto and return? **Answer:** $2 \times 5.98 \times 10^{14} / 2.99 \times 10^{10} = 4.0 \times 10^{4}$ seconds or 11 hours.

Problem 8: The planet HD209458b, now known as Osiris, was discovered by astronomers in 1999 and is at a distance of 150 light-years (1 light-year = 9.2×10^{12} kilometers). If an interstellar probe were sent to investigate this world up close, traveling at a maximum speed of 700 km/sec (about 10 times faster than our fastest spacecraft: Helios-1), how long would it take to reach Osiris? **Answer:** $150 \times 9.2 \times 10^{12} / 700 = 1.9 \times 10^{12}$ seconds or about 64,000 years!

When the solar system was young, planets were built as huge numbers of smaller bodies like asteroids collided with each other. Over millions of years of collisions, planets like Earth grew to their present sizes.

To see how this happened, we will model the process using a ball of clay!

Problem 1 - Take one-half pound of clay and form it into a round ball. Slice this round ball exactly in half and measure its diameter using a millimeter ruler.

Problem 2 – Take the two halves of the clay ball and re-form them into a round ball again. Now divide this ball into ten equal pieces of clay and roll each of these into round balls of about equal sizes. Slice one of these balls in half and measure its diameter.

Problem 3 – On a piece of graph paper, mark the horizontal axis with the number of small balls used from one to ten and the vertical axis with the diameter of the finished ball from 1 centimeter to 20 centimeters. Plot the diameter of one of these small balls on the graph.

Problem 4 – Take two of the small balls and roll them together into a single ball. Cut this new ball in half and measure its diameter. Mark the finished ball on the graph by its size in centimeters and the number of small balls '2'. Continue this process until you have collected all ten balls into one large ball and plot the diameter of the large ball and the number of small balls used '10'.

Problem 5 – Connect the ten points on the graph. What do you notice about the curve you plotted?

Problem 6 – If the large ball represented the final size of our Earth with a diameter of 12,000 kilometers, on this scale of the clay balls, how big would each of the ten 'planetessimals' have been that collided to form the final planet?

Problem 7 – Suppose you started with 100 equal-sized small clay balls. Would you get the same kind of plotted curve?

Problem 1 - Take one-half pound of clay and form it into a round ball. Slice this round ball exactly in half and measure its diameter using a millimeter ruler. **Answer will depend on how the students make the ball and how much clay is used.**

Problem 2 – Take the two halves of the clay ball and re-form them into a round ball again. Now divide this ball into ten equal pieces of clay and roll each of these into round balls of about equal sizes. Slice one of these balls in half and measure its diameter. **Answer will depend on how the students make the ball and how much clay is used.**

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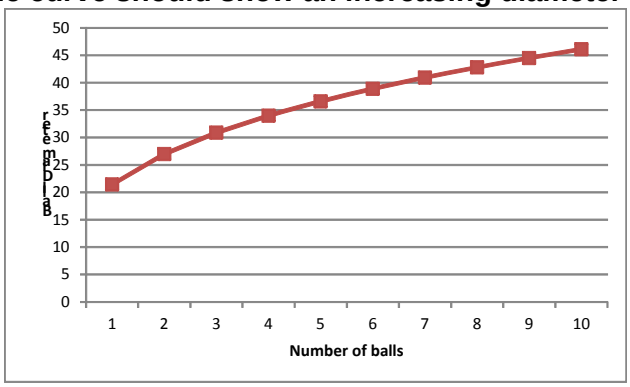
Problem 4 – Take two of the small balls and roll them together into a single ball. Cut this new ball in half and measure its diameter. Mark the finished ball on the graph by its size in centimeters and the number of small balls '2'. Continue this process until you have collected all ten balls into one large ball and plot the diameter of the large ball and the number of small balls used '10'. **Answer will depend on how the students make the ball and how much clay is used.**

Problem 5 – Connect the ten points on the graph. What do you notice about the curve you plotted? **Answer: The curve should show an increasing diameter from left to right.**



Problem 6 – If the large ball represented the final size of our Earth with a diameter of 12,000 kilometers, on this scale of the clay balls, how big would each of the ten 'planetessimals' have been that collided to form the final planet? Answer: Suppose the big balls final diameter is 45 millimeters and the small balls diameter is 20 mm. Use a simple proportion to find 12,000/45 = X/20 and so **X = 5300 km**. This is a bit larger than the diameter of our moon (3,500 km).

Problem 7 – Suppose you started with 100 equal-sized small clay balls. Would you get the same kind of plotted curve? **Answer: The curve would look the same but the scale on the horizontal axis would be changed. Each of the small balls would have a diameter of about 10 mm and represent a planetessimal with a diameter of about 12000/45 = X/10 so X = 2700 kilometers or slightly smaller than the diameter of our moon!**

Between 4.1 and 3.8 billion years ago, the surfaces of all the planets were being bombarded by asteroids and other large bodies called impactors that had formed in the solar system by this time. Astronomers call this the Late Heavy Bombardment Era, and it is the era which finalized the formation of the planets at their present sizes.

The surface of our moon shows many large round basins called mare that are all that remains of this era. Similar scars on Earth have long since vanished due to erosion, volcanism and plate tectonic activity.

Problem 1 – Using the large crater and impact basin record on the lunar surface, astronomers can estimate that Earth had about 20,000 craters over 20 km across, about 50 impact basins with diameters of 1,000 kilometers, and perhaps 5 large basins with diameters of 5,000 kilometers. If the Late Heavy Bombardment Era lasted about 300 million years, how many years elapsed between the impacts of each of the three kinds of objects during this era?

Problem 2 – A Rule-of-Thumb says that the actual diameter of an impacting body is about 1/6 the diameter of the crater it forms. What were the average sizes of the three kinds of impactors during this Era?

Problem 3 – Use the formula for the volume of a sphere to calculate A) the total volume added to Earth of the small impactors in cubic kilometers. B) the total volume added to Earth of the medium-sized impactors in cubic kilometers. C) the total volume added to Earth of the large impactors in cubic kilometers.

Problem 4 - If the radius of Earth is 6,378 km, what percentage of Earth's volume was added by each of the three kinds of impactors?

Problem 1 – Using the large crater and impact basin record on the lunar surface, astronomers can estimate that Earth had about 20,000 craters over 20 km across, about 50 impact basins with diameters of 1,000 kilometers, and perhaps 5 large basins with diameters of 5,000 kilometers. If the Late Heavy Bombardment Era lasted about 300 million years, how many years elapsed between the impacts of each of the three kinds of objects during this era?

Answer: Small: 300 million/20,000 = **15,000 years**. Medium: 300 million/50 = **6 million years**, Large: 300 million/5 = **60 million years**.

Problem 2 – A Rule-of-Thumb says that the actual diameter of an impacting body is about 1/6 the diameter of the crater it forms. What were the average sizes of the three kinds of impactors during this Era?

Answer: Small = 20 km/6 = **3 km**. Medium: 1000 km/6 = **166 km**. Large: 5000km/6 = **833 km**.

Problem 3 – Use the formula for the volume of a sphere to calculate A) the total volume added to Earth of the small impactors in cubic kilometers. B) the total volume added to Earth of the medium-sized impactors in cubic kilometers. C) the total volume added to Earth of the large impactors in cubic kilometers.

Answer: $V = 4/3\ \pi\ R^3$ and there were 20,000 of these so
Small = 20,000 x 4/3 x 3.141 x $(3\ km/2)^3$ = **282,000 km^3**
Medium: There were 50 of these so $V = 50 \times 4/3 \times 3.141 \times (166\ km/2)^3$ = **120 million km^3**
Large: There were 5 of these so $V = 5 \times 4/3 \times 3.141 \times (833\ km/2)^3$ = **1.5 billion km^3**

Problem 4 - If the radius of Earth is 6,378 km, what percentage of Earth's volume was added by each of the three kinds of impactors?

Answer: The total volume of a spherical Earth is $V = 4/3 \times 3.141 \times (6378)^3$ = 1.1 trillion km^3

So the three kinds of impactors contributed:

Small = 100% x (282000/1.1 trillion) = **0.00003 % of the final volume**.
Medium = 100% x (120 million/1.1 trillion) = **0.01 % of the final volume**.
Large = 100% x (1.5 billion/1.1 trillion) = **0.13% of the final volume**.

So the infrequent (every 60 million years) but largest impactors changed Earth's size the most rapidly during the Late Heavy Bombardment Era!

A Timeline for Planet Formation

Era	Time (years)	Description
Pre-solar Nebula Era	0.0	Collapse of cloud to form flattened disk
Asteroid Era	3 million	Formation of large asteroids up to 200 km across ends
Gas Giant Era	10 million	Rapid formation of Jupiter and Saturn ends
Solar Birth Era	50 million	Sun's nuclear reactions start to produce energy in core
Planetessimal Era	51 million	Formation of numerous small planet-sized bodies ends
T-Tauri Era	80 million	Solar winds sweep through inner solar system and strip off primordial atmospheres
Ice Giant Era	90 million	Formation of Uranus and Neptune
Rocky Planet Era	100 million	Formation of rocky planets by mergers of 50-100 smaller bodies
Late Heavy Bombardment Era	600 million	Migration of Jupiter disrupts asteroid belt sending large asteroids to impact planetary surfaces in the inner solar system.
Ocean Era	600 million	LHB transports comets rich in water to Earth to form oceans
Life Era	800 million	First traces of life found in fossils on Earth

For decades, geologists and astronomers have studied the contents of our solar system. They have compared surface features on planets and moons across the solar system, the orbits of asteroids and comets, and the chemical composition and ages for recovered meteorites. From all this effort, and with constant checking of data against mathematical models, scientists have created a timeline for the formation of our solar system.

Our solar system began as a collapsing cloud of gas and dust over 4.6 billion years ago. Over the next 600 million years, called by geologists the Hadean Era, the sun and the planets were formed, and Earth's oceans were probably created by cometary impacts. Comets are very rich in water ice.

The fossil record on Earth shows that the first bacterial life forms emerged about 600 million years after the formation of the solar system. Geologists call this the Archaen Era – The era of ancient life.

Problem 1 – If the Pre-Solar Nebula Era occurred 4.6 billion years ago, how long ago did the Rocky Planet Era end?

Problem 2 – How many years from the current time did the Late Heavy Bombardment Era end in the inner solar system?

Problem 3 – About how many years ago do the oldest fossils date from on Earth?

Problem 4 – How many years were there between the Planetessimal Era and the end of the Rocky Planet Era?

Problem 5 – If 80 objects the size of the Moon collided to form Earth during the time period in Problem 4, about how many years elapsed between these impact events?

Problem 1 – If the Pre-Solar Nebula Era occurred 4.6 billion years ago, how long ago did the Rocky Planet Era end?

Answer: On the Timeline '0.0' represents a time 4.6 billion years ago, so the Rocky Planet Era ended 100 million years after this or **4.5 billion years ago**.

Problem 2 – How many years from the current time did the Late Heavy Bombardment Era end in the inner solar system?

Answer: LHB ended 600 million years after Time '0.0' or 4.6 billion – 600 million = **4.0 billion years ago.**

Problem 3 – About how many years ago do the oldest fossils date from on Earth?

Answer: 4.6 billion – 800 million = **3.8 billion years ago**.

Problem 4 – How many years were there between the Planetessimal Era and the end of the Rocky Planet Era?

Answer: On the timeline the difference is 100 million – 51 million = **49 million years**.

Problem 5 – If 80 objects the size of the Moon collided to form Earth during the time period in Problem 4, about how many years elapsed between these impact events?

Answer: The time interval is 49 million years so the average time between impacts would have been 49 million years/80 impacts = **612,000 years**.

As they are forming, planets and raindrops grow by accreting matter (water or asteroids) at their surface.

The basic shape of a planet or a raindrop is that of a sphere. As the sphere increases in size, there is more surface area for matter to be accreted onto it, and so the growth rate increases.

In the following series of problems, we are going to follow a step-by-step logical process that will result in a simple mathematical model for predicting how rapidly a planet or a raindrop forms.

Problem 1 - The differential equation for the growth of the mass of a body by accretion is given by Equation 1 and the mass of the body is given by Equation 2

Equation 1) $\frac{dM}{dt} = 4\pi\ \rho\ V\ R(t)^2$ Equation 2) $M(t) = \frac{4}{3}\pi\ D\ R(t)^3$

where R is the radius of the body at time t, V is the speed of the infalling material, ρ is the density of the infalling material, and D is the density of the body.

Solve Equation 2 for R(t), substitute this into Equation 1 and simplify.

Problem 2 - Integrate your answer to Problem 1 to derive the formula for M(t).

Raindrop Condensation - A typical raindrop might form so that its final mass is about 100 milligrams and D = 1000 kg/m^3, under atmospheric conditions where ρ = 1 kg/m^3 and V = 1 m/sec. How long would it take such a raindrop to condense?

Planet Accretion - A typical rocky planet might form so that its final mass is about that of Earth or 5.9x10^{24} kg, and D = 3000 kg/m^3, under conditions where ρ = 0.000001 kg/m^3 and V = 1 km/sec. How long would it take such a planet to accrete using this approximate mathematical model?

Figure from 'Planetary science: Building a planet in record time' by Alan Brandon, Nature 473,460–461(26 May 2011)

Answer 1:

$$R(t)=\left(\frac{3M(t)}{4\pi D}\right)^{\frac{1}{3}} \quad \text{then} \quad \frac{dM}{dt}=4\pi\ \mathrm{V}\rho\left(\frac{3M}{4\pi D}\right)^{\frac{2}{3}} \quad \text{so} \quad \frac{dM}{dt}=4\pi V\rho\left(\frac{3}{4\pi D}\right)^{\frac{2}{3}}\ \mathrm{M}^{\frac{2}{3}}$$

Answer 2: First rearrange the terms to form the integrands:

$$\frac{dM}{M^{\frac{2}{3}}}=4\pi\mathrm{V}\rho\left(\frac{3}{4\pi D}\right)^{\frac{2}{3}}dt \qquad \text{Integrate both sides:} \quad 3M^{\frac{1}{3}}=4\pi V\rho\left(\frac{3}{4\pi D}\right)^{\frac{2}{3}}t$$

Now solve for M(t) to get the answer: $$M(t)=\left(\frac{4\pi V\rho}{3}\right)^{3}\left(\frac{3}{4\pi D}\right)^{2}t^{3}$$

Raindrop Condensation - A typical raindrop might form so that its final mass is about 100 milligrams and **D** = 1000 kg/m$^{\mathbf{3}}$, under atmospheric conditions where ρ = 1 kg/m$^{\mathbf{3}}$ and **V** = 2.0 m/sec (about 5 miles per hour). How long would it take such a raindrop to condense using this approximate mathematical model?

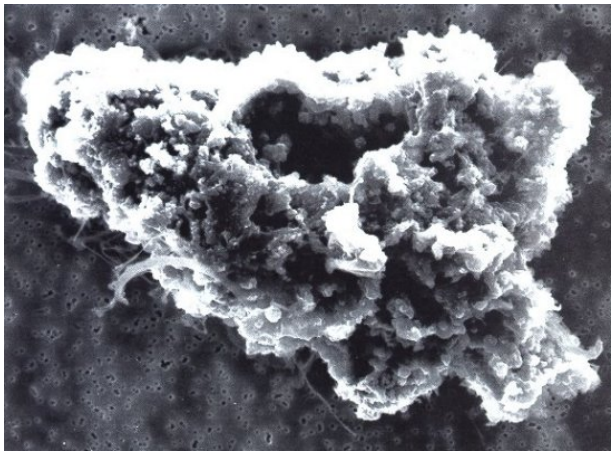
$$0.0001=\left(\frac{4\pi(2.0)(1.0)}{3}\right)^{3}\left(\frac{3}{4\pi(1000)}\right)^{2}t^{3} \qquad \text{so} \quad t^{\mathbf{3}}=2.98$$

and so it takes about **1.4 seconds**.

Planet Accretion - A typical rocky planet might form so that its final mass is about that of Earth or 5.9x10$^{\mathbf{24}}$ kg, and **D** = 3000 kg/m$^{\mathbf{3}}$, under conditions where ρ = 0.000001 kg/m$^{\mathbf{3}}$ and **V** = 1 km/sec. How long would it take such a planet to accrete using this approximate mathematical model?

$$5.9x10^{24}=\left(\frac{4\pi(1000)(0.000001)}{3}\right)^{3}\left(\frac{3}{4\pi(3000)}\right)^{2}t^{3} \qquad \text{so} \quad t^{\mathbf{3}}=1.27\text{x}10^{\mathbf{40}}$$

and so it takes about 2.3x10$^{\mathbf{13}}$ seconds or **750,000 years**.



A cosmic dust grain about 0.1mm across captured by a NASA high-altitude aircraft. Probably debris from a passing comet

Planets are built in several stages. The first of these involves small, micron-sized interstellar dust grains that collide and stick together to eventually form centimeter-sized bodies. A simple model of this process can tell us about how long it takes to 'grow' a rock-sized body starting from microscopic dust. This process occurs in dense interstellar clouds, which are known to be the birth places for stars and planets.

Problem 1 – Assume that the forming rock is spherical with a density of 3 gm/cc, a radius R, and a mass M. If the radius is a function of time, R(t), what is the equation for the mass of the rock as a function of time, M(t)?

Problem 2 – The rock grows by absorbing incoming dust grains that have an average mass of m grams and a density of N particles per cubic centimeter in the dust cloud. The particles collide with the surface of the rock at a speed of V cm/sec, what is the equation that gives the rate of growth of the rock's mass in time (dM/dt)?

Problem 3 – From your answer to Problem 1 and 2, re-write dM/dt in terms of M not R.

Problem 4 – Integrate your answer to problem 3 so determine M(t).

Problem 5 – What is the mass of the rock when it reaches a diameter of 1 centimeter if its density is 3 grams/cc?

Problem 6 – The rock begins at t=0 with a mass of 1 dust grain, $m = 8 \times 10^{-12}$ grams. The cloud density $N = 3.0 \times 10^{-5}$ dust grains/cc and the speed of the dust grains striking the rock, without destroying the rock, is V=10 cm/sec. How many years will the growth phase have to last for the rock to reach a diameter of 1 centimeter?

Problem 1 – Answer: Because mass = density x volume, we have
$M = 4/3\ \pi\ R^3\ \rho$ and so **$M(t) = 4/3\ \pi\ \rho\ R(t)^3$**

Problem 2 –**Answer:** The change in the mass, dM, occurs as a quantity of dust grains land on the surface area of the rock per unit time, dt. The amount is proportional to the surface area of the rock, since the more surface area the rock has, the more dust particles will be absorbed. Also, the rate at which dust grain mass is brought to the surface is proportional to the product of the dust grain density in the interstellar gas, times the speed of the grains landing on the surface. This leads to m x N x V where m is in grams per dust grain, N is in dust grains per cubic centimeter, and V is in centimeters/sec. The product of all three factors has the units of grams per square centimeter per second. The product of (m x N x V) with the surface area of the rock, will then have the units of grams/sec representing the rate at which the rock mass is growing. The full formula for the growth of the rock mass is then
$dM/dt = 4\ \pi\ R^2\ m\ N\ V$

Problem 3 – Answer: From Problem 1 we see that $R(t) = (3\ M(t)/\ 4\ \pi\ \rho)^{1/3}$. Then substituting into dM/dt we have $dM/dt = 4\ \pi\ m\ N\ V\ (3\ M(t)/4\ \pi\ \rho)^{2/3}$ so

$$\frac{dM(t)}{dt} = 4\pi mNV\left(\frac{3}{4\pi\rho}\right)^{\frac{2}{3}} M(t)^{\frac{2}{3}}$$

Problem 4 –Answer: Re-write the differentials and move M(t) to the side with dM to get the integrand $M(t)^{-2/3}\ dM = 4\ \pi\ m\ N\ V\ (3/4\ \pi\ \rho)^{2/3}\ dt$ Then integrate both sides to get:
$3\ M(t)^{1/3} = 4\ \pi\ m\ N\ V\ (3/4\ \pi\ \rho)^{2/3}\ t\ + c.$ Solve for M(t) to get the final equation for M(t), and remember to include the integration constant, c:

$$M(t) = \left[\frac{4}{3}\pi mNV\left(\frac{3}{4\pi\rho}\right)^{\frac{2}{3}} t + c\right]^3$$

Problem 5 –Answer: The radius will be 0.5 centimeters so, $m = 4/3\ \pi\ (0.5\ cm)^3$ x 3.0 gm/cc = **1.6 grams.**

Problem 6 – Answer: For t = 0, M(0) = m so the constant of integration is $c = m^{1/3}$ so $c = (8x10^{-12})^{1/3} = 2 \times 10^{-4}$.
Then $M(t) = (4/3\ (3.14)\ (8 \times 10^{-12}\ )(3.0 \times 10^{-5}\ )(10.0)(3/(4(3.14)\ (3.0)))^{2/3}\ t\ + 2 \times 10^{-4}\ )^3$
$M(t) = (1.9 \times 10^{-15}\ t + 2 \times 10^{-4})^3$

To reach M(t) = 1.6 grams, $t = 6.1 \times 10^{14}$ seconds or about **19 million years!**

Asteroid Gaspra about 15 km across. Image taken by the NASA Galileo spacecraft.

Planets are built in several stages. The first of these involves small, interstellar dust grains that collide and stick together to form centimeter-sized bodies. This can take millions of years. The second stage involves the formation of kilometer-sized asteroids from the centimeter-sized rocks. A simple model of this process can tell us about how long it takes to 'grow' an asteroid from rock-sized bodies.

Problem 1 – Assume that the forming asteroid is spherical with a density of 3 gm/cc, a radius R, and a mass M. If the radius is a function of time, R(t), what is the equation for the mass of the asteroid as a function of time, M(t)?

Problem 2 – The asteroid grows by absorbing incoming rocks that have an average mass of 5.0 grams and a density of N rocks per cubic centimeter in the cloud. The rocks collide with the surface of the forming asteroid at a speed of V cm/sec, what is the equation that gives the rate of growth of the asteroid's mass in time (dM/dt)?

Problem 3 – From your answer to Problem 1 and 2, re-write dM/dt in terms of M not R.

Problem 4 – Integrate your answer to problem 3 so determine M(t).

Problem 5 – What is the mass of the asteroid when it reaches a diameter of 1 kilometer if its density is 3 grams/cc?

Problem 6 – The asteroid begins at t=0 with a mass of m=5 grams. The cloud density N = 1.0×10^{-8} rocks/cc and the speed of the rocks striking the asteroid, without destroying the asteroid, is V=1 kilometer/sec. How many years will the growth phase have to last for the asteroid to reach a diameter of 1 kilometer?

Problem 1 – Answer: Because mass = density x volume, we have
$M = 4/3\ \pi\ R^3\ \rho$ and so **$M(t) = 4/3\ \pi\ \rho\ R(t)^3$**

Problem 2 –**Answer:** The change in the mass, dM, occurs as a quantity of rocks land on the surface area of the forming asteroid per unit time, dt. The amount is proportional to the surface area of the asteroid, since the more surface area the asteroid has, the more rocks will be absorbed. Also, the rate at which rock mass is brought to the surface of the asteroid is proportional to the product of the rock density in the solar nebula, times the speed of the rocks landing on the surface of the asteroid. This leads to m x N x V where m is in grams per rock, N is in rocks per cubic centimeter, and V is in centimeters/sec. The product of all three factors has the units of grams per square centimeter per second. The product of (m x N x V) with the surface area of the asteroid, will then have the units of grams/sec representing the rate at which the asteroid mass is growing. The full formula for the growth of the asteroid mass is then
$dM/dt = 4\ p\ R^2\ m\ N\ V$

Problem 3 – Answer: From Problem 1 we see that $R(t) = (3\ M(t)/\ 4\ \pi\ \rho)^{1/3}$. Then substituting into dM/dt we have $dM/dt = 4\ \pi\ m\ N\ V\ (3\ M(t)/4\ \pi\ \rho)^{2/3}$ so

$$\frac{dM(t)}{dt} = 4\pi mNV\left(\frac{3}{4\pi\rho}\right)^{\frac{2}{3}} M(t)^{\frac{2}{3}}$$

Problem 4 –Answer: Re-write the differentials and move M(t) to the side with dM to get the integrand $M(t)^{-2/3}\ dM = 4\ \pi\ m\ N\ V\ (3/4\ \pi\ \rho)^{2/3}\ dt$ Then integrate both sides to get:
$3\ M(t)^{1/3} = 4\ \pi\ m\ N\ V\ (3/4\ \pi\ \rho)^{2/3}\ t\ + c.$ Solve for M(t) to get the final equation for M(t), and remember to include the integration constant, c:

$$M(t) = \left[\frac{4}{3}\pi mNV\left(\frac{3}{4\pi\rho}\right)^{\frac{2}{3}} t + c\right]^3$$

Problem 5 – What is the mass of the asteroid when it reaches a diameter of 1 kilometer if its density is 3 grams/cc? Answer $M = 4/3\ \pi\ (50{,}000\ cm)^3$ x 3.0 gm/cc = **1.6×10^{15} grams.**

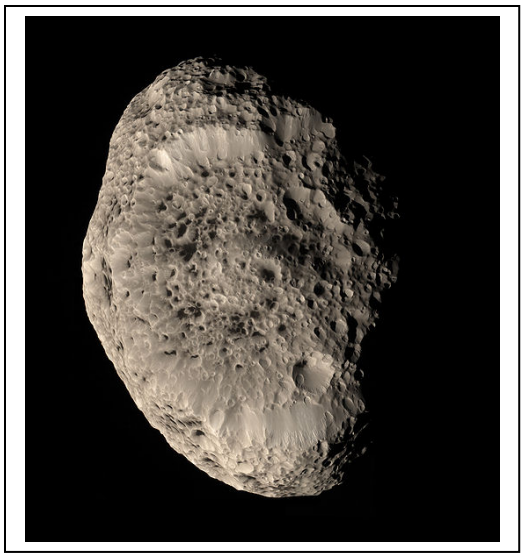
Problem 6 – The rock begins at t=0 with a mass of 1 rock, m = 5 grams. The cloud density N = 1.0×10^{-8} rocks/cc and the speed of the rocks striking the asteroid, without destroying the asteroid, is V=1 kilometer/sec. How many years will the growth phase have to last for the asteroid to reach a diameter of 1 kilometer? Answer: For t = 0, M(0) = m so the constant of integration is $c = m^{1/3}$ so c = 1.7.

Then $M(t) = (4/3\ (3.14)\ (5)(1.0 \times 10^{-8}\ )(100{,}000)(3/(4(3.14)\ (3.0)))^{2/3}\ t\ +1.7\ )^3$

$M(t) = (0.0039\ t + 1.7)^3$

So to get $M(t) = 1.6 \times 10^{15}$ grams, solve for t to get t = 29,600,000 seconds or about **342 days!**

From Asteroids to Planets



Planets are built in several stages. Dust grains grow to large rocks in a million years, then rocks accumulate to form asteroids in a few years or so. The third stage combines kilometer-wide asteroids to make rocky planets. A simple model of this process can tell us about how long it takes to 'grow' a planet by accumulating asteroid-sized bodies through collisions. Saturn's moon Hyperion (see image) is 300 km across and is an example of a 'small' planet-sized body called a planetoid.

Problem 1 – Assume that the forming planet is spherical with a density of 3 gm/cc, a radius R, and a mass M. If the radius is a function of time, R(t), what is the equation for the mass of the planet as a function of time, M(t)?

Problem 2 – The planet grows by absorbing incoming asteroids that have an average mass of 10^{15} grams and a density of N asteroids per cubic centimeter in the cloud. The asteroids collide with the surface of the forming planet at a speed of V cm/sec, what is the equation that gives the rate of growth of the planet's mass in time (dM/dt)?

Problem 3 – From your answer to Problem 1 and 2, re-write dM/dt in terms of M not R.

Problem 4 – Integrate your answer to Problem 3 so determine M(t).

Problem 5 – What is the mass of the planet when it reaches a diameter of 5000 kilometers if its density is 3 grams/cc?

Problem 6 – The planetoid begins at t=0 with a mass of m= 2×10^{15} grams. The cloud density N = 1.0×10^{-24} asteroids/cc (1 asteroid per 1000 cubic kilometers), and the speed of the asteroids striking the planet, without destroying the planet, is V=1 kilometer/sec. How many years will the growth phase have to last for the planet to reach a diameter of 5000 kilometers?

Problem 1 – Answer: Because mass = density x volume, we have
$M = 4/3\ \pi\ R^3\ \rho$ and so **$M(t) = 4/3\ \pi\ \rho\ R(t)^3$**

Problem 2 –**Answer:** The change in the mass, dM, occurs as a quantity of asteroids land on the surface area of the planet per unit time, dt. The amount is proportional to the surface area of the planet, since the more surface area the planet has, the more asteroids will be absorbed. Also, the rate at which asteroid mass is brought to the surface of the forming planet is proportional to the product of the asteroid density in the planetary nebula, times the speed of the asteroids landing on the surface of the planet. This leads to m x N x V where m is in grams per dust grain, N is in asteroids per cubic centimeter, and V is in centimeters/sec. The product of all three factors has the units of grams per square centimeter per second. The product of (m x N x V) with the surface area of the planet, will then have the units of grams/sec representing the rate at which the planet mass is growing. The full formula for the growth of the planet mass is then
$dM/dt = 4\ p\ R^2\ m\ N\ V$

Problem 3 – Answer: From Problem 1 we see that $R(t) = (3\ M(t)/\ 4\ \pi\ \rho)^{1/3}$. Then substituting into dM/dt we have $dM/dt = 4\ \pi\ m\ N\ V\ (3\ M(t)/4\ \pi\ \rho)^{2/3}$ so

$$\frac{dM(t)}{dt} = 4\pi mNV\left(\frac{3}{4\pi\rho}\right)^{\frac{2}{3}} M(t)^{\frac{2}{3}}$$

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$3\ M(t)^{1/3} = 4\ \pi\ m\ N\ V\ (3/4\ \pi\ \rho)^{2/3}\ t\ + c.$ Solve for M(t) to get the final equation for M(t), and remember to include the integration constant, c:

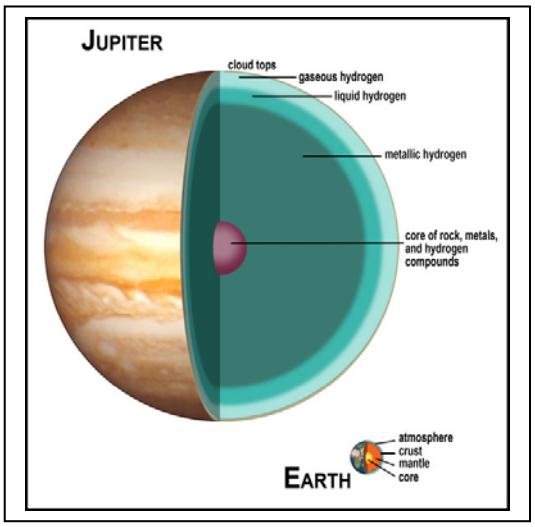
$$M(t) = \left[\frac{4}{3}\pi mNV\left(\frac{3}{4\pi\rho}\right)^{\frac{2}{3}} t + c\right]^3$$

Problem 5 – What is the mass of the planet when it reaches a diameter of 5000 kilometers if its density is 3 grams/cc? Answer $M = 4/3\ \pi\ (2.5 \times 10^8\ cm)^3 \times 3.0\ gm/cc =$ **2.0×10^{26} grams.**

Problem 6 – The planetoid begins at t=0 with a mass of 1 asteroid, $m = 2.0 \times 10^{15}$ grams. The cloud density $N = 1.0 \times 10^{-24}$ asteroids/cc (This equals 1 asteroid per 1000 cubic kilometers) and the speed of the asteroids striking the planet, without destroying the planet, is V=1 kilometer/sec. How many years will the growth phase have to last for the planet to reach a diameter of 5000 kilometers? Answer: For t = 0, M(0) = m so the constant of integration is c $= m^{1/3}$ so $c = (2.0 \times 10^{15}\ g)^{1/3} = 1.3 \times 10^5$.
Then $M(t) = (4/3\ (3.14)\ (\ 2 \times 10^{15})(1.0 \times 10^{-24}\ )(100{,}000)(3/(4(3.14)\ (3.0)))^{2/3}\ t\ +1.3 \times 10^5\ )^3$
$M(t) = (0.00015\ t + 1.3 \times 10^5\ )^3$

To get $M(t) = 2.0 \times 10^{26}$ grams will take $t = 3.9 \times 10^{12}$ seconds or about **126,000 years!**

Relative Sizes of the Planets and other Objects



The actual sizes of the major objects in our solar system range from the massive planet Jupiter, to many small moons and asteroids no more than a few kilometers across.

It is often helpful to create a scaled model of the major objects so that you can better appreciate just how large or small they are compared to our Earth.

This exercise will let you work with simple proportions and fractions to create a scaled-model solar system.

Problem 1 - Jupiter is 7/6 the diameter of Saturn, and Saturn is 5/2 the diameter of Uranus. Expressed as a simple fraction, how big is Uranus compared to Jupiter?

Problem 2 – Earth is 13/50 the diameter of Uranus. Expressed as a simple fraction, how much bigger than Earth is the planet Saturn?

Problem 3 – The largest non-planet objects in our solar system, are our own Moon (radius=1738 km), Io (1810 km), Eris (1,500 km), Europa (1480 km), Ganymede (2600 km), Callisto (2360 km), Makemake (800 km),Titan (2575 km), Triton (1350 km), Pluto (1,200 km), Haumea (950 km). Create a bar chart that orders these bodies from smallest to largest. For this sample, what is the: A) Average radius? B) Median radius?

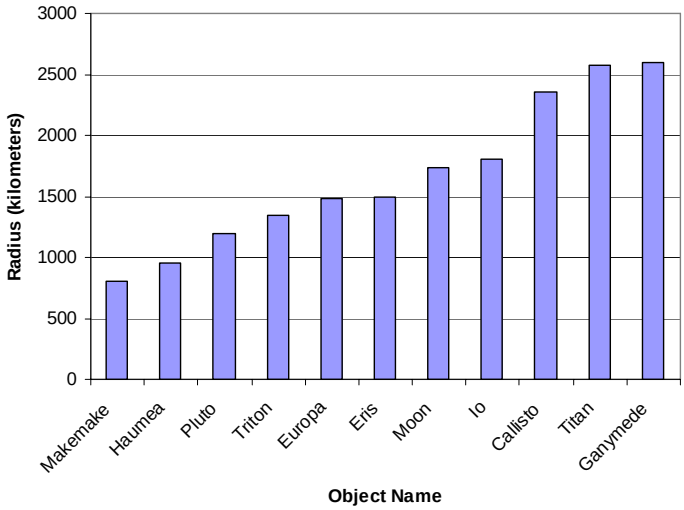
Problem 4 – Mercury is the smallest of the eight planets in our solar system. If the radius of Mercury is 2425 km, how would you create a scaled model of the non-planets if you selected a diameter for the disk of Mercury as 50 millimeters?

Problem 1 - Jupiter is 7/6 the diameter of Saturn, and Saturn is 5/2 the diameter of Uranus. Expressed as a simple fraction, how big is Uranus compared to Jupiter? Answer: 2/5 x 6/7 = **12/35**.

Problem 2 – Earth is 13/50 the diameter of Uranus. Expressed as a simple fraction, how much bigger than Earth is the planet Saturn? Answer: 5/2 x 50/13 = 250/26 = **125/13** times.

Problem 3 – The largest non-planet objects in our solar system, are our own Moon (radius=1738 km), Io (1810 km), Eris (1,500 km), Europa (1480 km), Ganymede (2600 km), Callisto (2360 km), Makemake (800 km),Titan (2575 km), Triton (1350 km), Pluto (1,200 km), Haumea (950 km). Create a bar chart that orders these bodies from smallest to largest. For this sample, what is the: A) Average radius? B) Median radius?

Answer: Average = (1738+1810+1500+1480+2600+2360+800+2575+1350+1200+950)/11 so **Average radius = 1669 km**. **Median radius = 1500 km**.

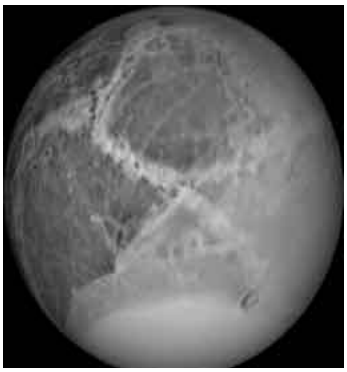
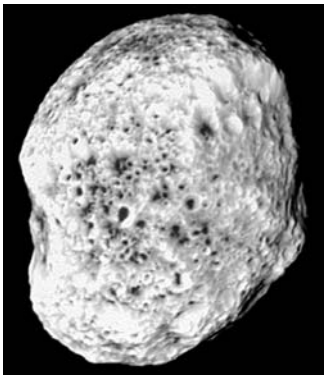


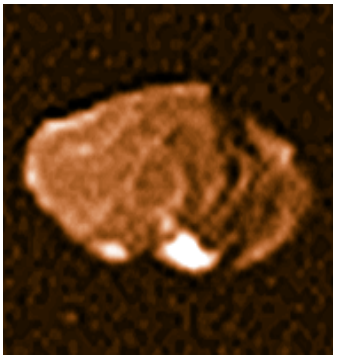
Problem 4 – Mercury is the smallest of the eight planets in our solar system. If the radius of Mercury is 2425 km, how would you create a scaled model of the non-planets if you selected a diameter for the disk of Mercury as 50 millimeters? Answer: The scaled disk diameters are shown in Column 3 in millimeters.

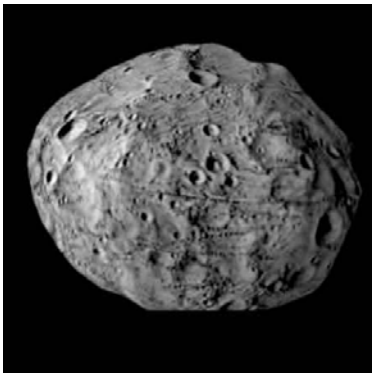
Object	Radius (km)	Diameter (mm)
Makemake	800	16
Haumea	950	20
Pluto	1200	25
Triton	1350	28
Europa	1480	30
Eris	1500	30
Moon	1738	36
Io	1810	38
Callisto	2360	48
Titan	2575	54
Ganymede	2600	54

Astronomy in the Round

Why are many astronomical bodies round? Here is an activity in which you use astronomical photographs of various solar system bodies, and determine how big a body has to be before it starts to look round. Can you figure out what it is that makes a body round?





The images show the shapes of various astronomical bodies, and their sizes: Dione (560 km), Hyperion (205 x 130 km), Tethys (530 km), Amalthea (130 x 85 km), Ida (56 x 24 km), Phobos (14 x 11 km).

Question 1) How would you define the roundness of a body?

Question 2) How would you use your definition of roundness to order these objects from less round to round?

Question 3) Can you create from your definition a number that represents the roundness of the object?

Question 4) On a plot, can you compare the number you defined in Question 3 with the average size of the body?

Question 5) Can you use your plot to estimate the minimum size that a body has to be in order for it to be noticeably round? Does it depend on whether the body is mostly made of ice, or mostly of rock?

The images show the shapes of various astronomical bodies, and their sizes: Dione (560 km), Hyperion (205 x 130 km), Tethys (530 km), Amalthea (130 x 85 km), Ida (56 x 24 km), Phobos (14 x 11 km).

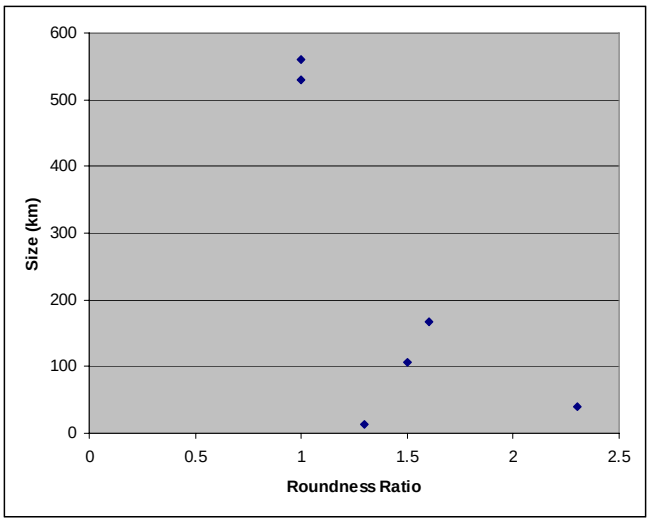
Question 1) How would you define the roundness of a body? **Answer: Students may explore such possibilities as the difference between the longest and shortest dimension of the object; the ratio of the longest to the shortest diameter; or other numerical possibilities.**

Question 2) How would you use your definition of roundness to order these objects from less round to round? **Answer. The order should look something like Ida, Amalthea, Hyperion, Phobos, Tethys and Dione.**

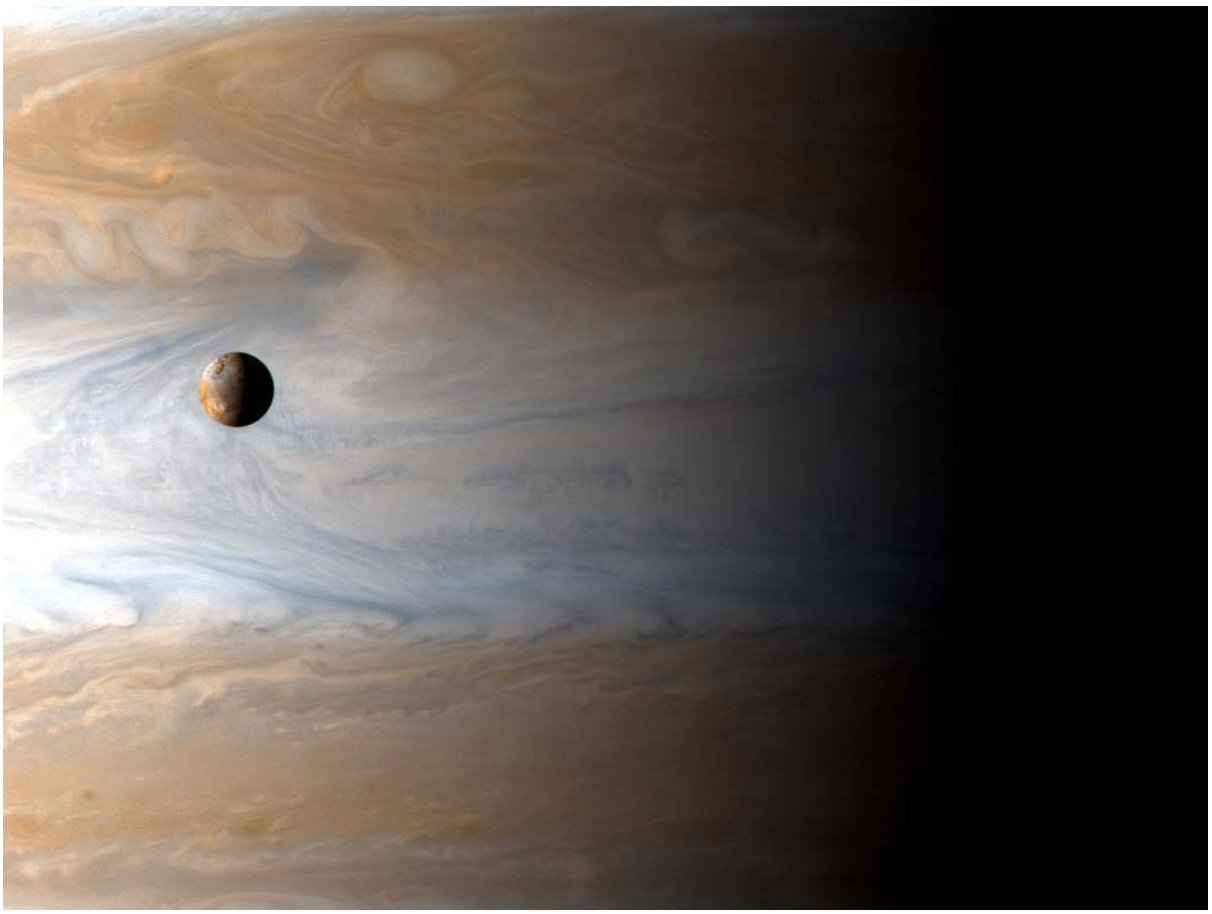
Question 3) Can you create from your definition a number that represents the roundness of the object? **Answer: If we select the ratio of the major to minor axis length, for example, we would get Dione = 1.0; Hyperion = 1.6; Tethys=1.0; Amalthea=1.5; Ida = 2.3 and Phobos = 1.3**

Question 4) On a plot, can you compare the number you defined in Question 3 with the average size of the body? **Answer: See plot below for the numerical definition selected in Question 3. We have used the average size of each irregular body defined as (L + S)/2. so Hyperion = (205+130)/2 = 167 km; Amalthea = (130+85)/2 = 107 km; Ida = (56+24)/2 = 40 km; Phobos = (14+11)/2 = 13 km.**

Question 5) Can you use your plot to estimate the minimum size that a body has to be in order for it to be noticeably round? Does it depend on whether the body is mostly made of ice, or mostly of rock? Answer: By connecting a smooth curve through the points (you can do this by eye), the data suggests that a body becomes noticeably round when it is at a size between 200-400 km. Students can obtain pictures of other bodies in the solar system and see if they can fill-in the plot better with small moons of the outer planets, asteroids (Ceres, etc) or even comet nuclei. Note, inner solar system bodies are mostly rock. Outer solar system bodies are mostly ice, so students might notice that by labeling the points as 'rocky bodies' or 'icy bodies' that they may see two different trends because ice is more pliable than rock. Students should investigate what the size (mass) has to do with roundness, and see that larger bodies have more gravity to deform their substance with.



Jupiter and Io



This NASA image of Jupiter with its satellite Io was taken by the Cassini spacecraft. (Credit: NASA/Cassini Imaging Team). The satellite is 3,600 kilometers in diameter.

The scale of an image is found by measuring with a ruler the distance between two points on the image whose separation in physical units you know. In this case, we are told the diameter of Io is 3,600 kilometers.

Step 1: Measure the diameter of Io with a metric ruler. How many millimeters in diameter?

Step 2: Use clues in the image description to determine a physical distance or length.

Step 3: Divide your answer to Step 2 by your answer to Step 1 to get the image scale in kilometers per millimeter to two significant figures.

Once you know the image scale, you can measure the size of any feature in the image in units of millimeters. Then multiply it by the image scale from Step 3 to get the actual size of the feature in kilometers to two significant figures.

Question 1: What are the dimensions, in kilometers, of this image?
Question 2: What is the width, in kilometers, of the largest feature in the atmosphere of Jupiter?
Question 3: What is the width, in kilometers, of the smallest feature in the atmosphere of Jupiter?
Question 4: What is the size of the smallest feature on Io that you can see?

This NASA image of Jupiter with its satellite Io was taken by the Cassini spacecraft. (Credit: NASA/Cassini Imaging Team). The satellite is 3,600 kilometers in diameter.

The scale of an image is found by measuring with a ruler the distance between two points on the image whose separation in physical units you know. In this case, we are told the diameter of Io is 3,600 kilometers.

Step 1: Measure the diameter of Io with a metric ruler. How many millimeters in diameter?
Answer: 10 mm

Step 2: Use clues in the image description to determine a physical distance or length.
Answer: 3,600 km

Step 3: Divide your answer to Step 2 by your answer to Step 1 to get the image scale in kilometers per millimeter to two significant figures.
Answer: 3600 km / 10 mm = 360 km/mm

Once you know the image scale, you can measure the size of any feature in the image in units of millimeters. Then multiply it by the image scale from Step 3 to get the actual size of the feature in kilometers to two significant figures.

Question 1: What are the dimensions, in kilometers, of this image?
Answer: 160 mm x 119 mm = 58,000 km x 19,000 km

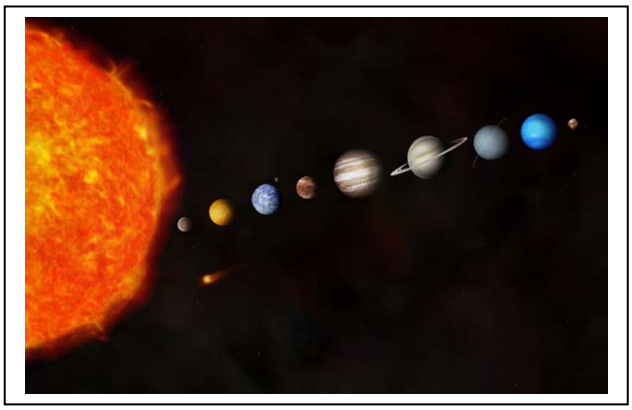
Question 2: What is the width, in kilometers, of the largest feature in the atmosphere of Jupiter?
Answer: The width of the white equatorial band is 45 mm or 16,000 km

Question 3: What is the width, in kilometers, of the smallest feature in the atmosphere of Jupiter?
Answer: The faint cloud streaks are 0.5 mm wide or 200 km across to one significant figure.

Question 4: What is the size of the smallest feature on Io that you can see?
Answer: The white spots in the southern hemisphere are 0.5 mm across or 200 km to one significant figure. This is a good time to mention that some details in an image can be artifacts from the printing process or defects in the camera itself. Students may find photocopying artifacts at 0.5 mm or less.

Note to teachers: The correct scale for Io and Jupiter will be slightly different depending on how far away the camera was when taking the picture. If the camera was very close to Io, then the scale you will infer for Io will be very different than for the more distant Jupiter because Io will take up more of the field-of-view in the image. Geometrically, for a fixed angle of separation between features on Io, this angle will subtend a SMALLER number of kilometers than the same angle on the more-distant Jupiter. However, if the distance from the camera to Jupiter/Io is very large, then as seen from the camera, both objects are at essentially the same distance and so there will be little difference between the scales used for the two bodies. Students can check this result with an inquiry assignment.

Planet Fractions and Scales



Some of the planets in our solar system are much bigger than Earth while others are smaller. By using simple fractions, you will explore how their sizes compare to each other.

Image courtesy NASA/Chandra Observatory/SAO

Problem 1 - Saturn is 10 times bigger than Venus, and Venus is 1/4 the size of Neptune. How much larger is Saturn than Neptune?

Problem 2 - Earth is twice as big as Mars, but only 1/11 the size of Jupiter. How large is Jupiter compared to Mars?

Problem 3 - Earth is the same size as Venus. How large is Jupiter compared to Saturn?

Problem 4 - Mercury is 3/4 the size of Mars. How large is Earth compared to Mercury?

Problem 5 - Uranus is the same size as Neptune. How large is Uranus compared to Earth?

Problem 6 - The satellite of Saturn, called Titan, is 1/10 the size of Uranus. How large is Titan compared to Earth?

Problem 7 - The satellite of Jupiter, called Ganymede, is 2/5 the size of Earth. How large is it compared to Jupiter?

Problem 8 - The Dwarf Planet Pluto is 1/3 the diameter of Mars. How large is the diameter of Jupiter compared to Pluto?

Problem 9 - If the diameter of Earth is 13,000 km ,what are the diameters of all the other bodies?

Note to teachers: The actual diameters of the planets, in kilometers, are as follows

Mercury	4,900 km	Jupiter	143,000 km
Venus	12,000 km	Saturn	120,000 km
Earth	13,000 km	Uranus	51,000 km
Mars	6,800 km	Neptune	50,000 km

Also: Titan = 5,100 km, Ganymede = 5,300 km Ceres = 950 km, and Pluto 2,300 km

Advanced students (Grades 4 and above) may use actual planetary size ratios as decimal numbers, but for this simplified version (Grades 2 and 3), we approximate the size ratios to the nearest simple fractions. **Students may also use the information in these problems to make a scale model of the solar system in terms of the relative planetary sizes.**

Problem 1 - Saturn is 10 times bigger than Venus, and Venus is 1/4 the size of Neptune. How much larger is Saturn than Neptune?
Answer: Neptune is 4x Venus and Saturn is 10x Venus, so Saturn is 10/4 = 5/2 times as big as Neptune.

Problem 2 - Earth is twice as big as Mars, but only 1/11 the size of Jupiter. How large is Jupiter compared to Mars?
Answer: Jupiter is 11 x Earth, and Mars is 1/2 Earth, so Jupiter is 22x Mars.

Problem 3 - Earth is the same size as Venus. How large is Jupiter compared to Saturn?
Answer: If Saturn is 10 x Venus, and Jupiter is 11 x Earth, Jupiter is 11/10 times Saturn.

Problem 4 - Mercury is 3/4 the size of Mars. How large is Earth compared to Mercury?
Answer: Mars is 1/2 x Earth, so Mercury is 3/4 x 1/2 = 3/8 x Earth

Problem 5 - Uranus is the same size as Neptune. How large is Uranus compared to Earth?
Answer: Neptune was 4x Venus, but since Venus = earth and Neptune=Uranus, we have Uranus = 4x Earth.

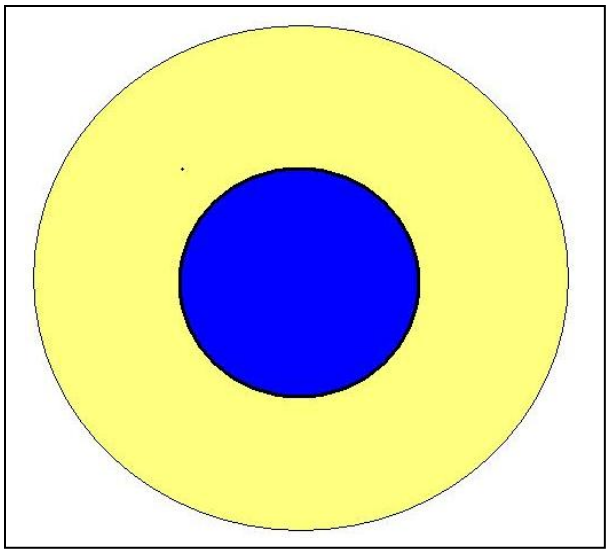
Problem 6 - The satellite of Saturn, called Titan, is 1/10 the size of Uranus. How large is Titan compared to Earth?
Answer: Titan / Uranus = 1/10, but Uranus/Earth = 4, so Titan/Earth = 3/10 x 4 = 2/5.

Problem 7 - The satellite of Jupiter, called Ganymede, is 2/5 the size of Earth. How large is it compared to Jupiter?
Answer: Earth = 1/11 Jupiter so Ganymede is 1/11 x 2/5 = 2/55 x Jupiter.

Problem 8 - The Dwarf Planet Pluto is 1/3 the size of Mars. How large is Jupiter compared to Pluto?
Answer: Jupiter = 1/11 Earth, Mars= 1/2 Earth, so Pluto= 1/3 x 1/2 = 1/6 Earth, and 1/66 Jupiter.

Problem 9 – **Answer: Students should, very nearly, reproduce the numbers in the table at the top of the page.**

How to Build a Planet from the Inside Out!



The planet Osiris orbits 7 million kilometers from the star HD209458 located 150 light years away in the constellation Pegasus. The Spitzer Space Telescope has recently detected water, methane and carbon dioxide in the atmosphere of this planet. The planet has a mass that is 69% that of Jupiter, and a volume that is 146% greater than that of Jupiter.

By knowing the mass, radius and density of a planet, astronomers can create plausible models of the composition of the planet's interior. Here's how they do it!

Among the first types of planets being detected in orbit around other stars are enormous Jupiter-sized planets, but as our technology improves, astronomers will be discovering more 'super-Earth' planets that are many times larger than Earth, but not nearly as enormous as Jupiter. To determine whether these new worlds are Earth-like, they will be intensely investigated to determine the kinds of compounds in their atmospheres, and their interior structure. Are these super-Earths merely small gas giants like Jupiter, icy worlds like Uranus and Neptune, or are they more similar to rocky planets like Venus, Earth and Mars?

Problem 1 - A hypothetical planet is modeled as a sphere. The interior has a dense rocky core, and on top of this core is a mantle consisting of a thick layer of ice. If the core volume is 4.18×10^{12} cubic kilometers and the shell volume is 2.92×10^{13} cubic kilometers, what is the radius of this planet in kilometers?

Problem 2 - If the volume of Earth is 1.1×10^{12} cubic kilometers, to the nearest whole number, A) how many Earths could fit inside the core of this hypothetical planet? B) How many Earths could fit inside the mantle of this hypothetical planet?

Problem 3 - Suppose the astronomer who discovered this super-Earth was able to determine that the mass of this new planet is 8.3 times the mass of Earth. The mass of Earth is 6.0×10^{24} kilograms. What is A) the mass of this planet in kilograms? B) The average density of the planet in kilograms/cubic meter?

Problem 4 - Due to the planet's distance from its star, the astronomer proposes that the outer layer of the planet is a thick shell of solid ice with a density of 1000 kilograms/cubic meter. What is the average density of the core of the planet?

Problem 5 - The densities of some common ingredients for planets are as follows:

Granite 3,000 kg/m^3 ; Basalt 5,000 kg/m^3 ; Iron 9,000 kg/m^3

From your answer to Problem 4, what is the likely composition of the core of this planet?

Problem 1 - The planet is a sphere whose total volume is given by V = 4/3 π R^3. The total volume is found by adding the volumes of the core and shell to get V = 4.18 x 10^{12} + 2.92 x 10^{13} = 3.34 x 10^{13} cubic kilometers. Then solving the equation for R we get R = (3.34 x 10^{13} / (1.33*3.14))$^{1/3}$ = 19,978 kilometers. Since the data are only provided to 3 place accuracy, the final answer can only have three significant figures, and with rounding this equals a radius of R = **20,000 kilometers.**

Problem 2 - If the volume of Earth is 1.1 x 10^{12} cubic kilometers, to the nearest whole number,
A) how many Earths could fit inside the core of this hypothetical planet?
Answer: V = 4.18 x 10^{12} cubic kilometers / 1.1 x 10^{12} cubic kilometers = **4 Earths.**

B) How many Earths could fit inside the mantle of this hypothetical planet?
Answer: V = 2.92 x 10^{13} cubic kilometers / 1.1 x 10^{12} cubic kilometers = **27 Earths.**

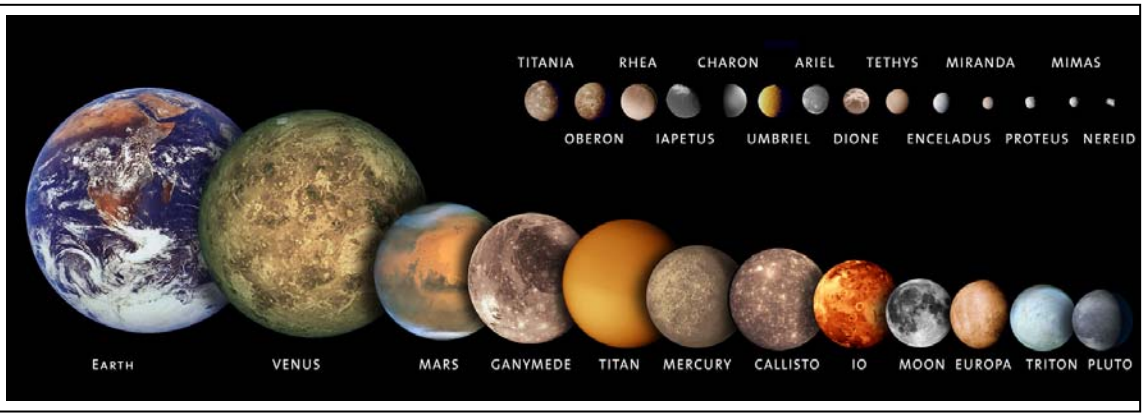
Problem 3 - What is A) the mass of this planet in kilograms? Answer: 8.3 x 6.0 x 10^{24} kilograms = **5.0 x 10^{25} kilograms.**
B) The average density of the planet in kilograms/cubic meter?
Answer: Density = total mass/ total volume
= 5.0 x 10^{25} kilograms/3.34 x 10^{13} cubic kilometers
= 1.5 x 10^{12} kilograms/cubic kilometers.
Since 1 cubic kilometer = 10^9 cubic meters,
= 1.5 x 10^{12} kilograms/cubic kilometers x (1 cubic km/10^9 cubic meters)
= **1,500 kilograms/cubic meter.**

Problem 4 - We have to subtract the total mass of the ice shell from the mass of the planet to get the mass of the core, then divide this by the volume of the core to get its density. Mass = Density x Volume, so the shell mass is 1,000 kg/ m^3 x 2.92 x 10^{13} km^3 x (10^9 m^3/km^3) = 2.9 x 10^{25} kg. Then the core mass = 5.0 x 10^{25} kilograms - 2.9 x 10^{25} kg = 2.1 x 10^{25} kg. The core volume is 4.18 x 10^{12} km^3 x (10^9 m^3/km^3) = 4.2 x 10^{21} m^3, so the density is D = 2.1 x 10^{25} kg / 4.2 x 10^{21} m^3= **5,000 kg/ m^3.**

Problem 5 - The densities of some common ingredients for planets are as follows:
Granite 3,000 kg/m^3 ; Basalt 5,000 kg/m^3 ; Iron 9,000 kg/m^3
From your answer to Problem 4, what is the likely composition of the core of this planet?
Answer: **Basalt.**

Note that, although the average density of the planet (1,500 kg/m^3) is not much more than solid ice (1,000 kg/m^3), the planet has a sizable rocky core of higher density material. Once astronomers determine the size and mass of a planet, these kinds of 'shell-core' models can give valuable insight to the composition of the interiors of planets that cannot even be directly imaged and resolved! In more sophisticated models, the interior chemistry is linked to the temperature and location of the planet around its star, and proper account is made for the changes in density inside a planet due to compression and heating. The surface temperature, which can be measured from Earth by measuring the infrared 'heat' from the planet, also tells which compounds such as water can be in liquid form.

Big Moons and Small Planets!



This diagram shows the Top-26 moons and small planets in our solar system, and drawn to the same scale.

Problem 1 – What fraction of the objects are smaller than our moon?

Problem 2 – What fraction of the objects are larger than our moon but are not planets?

Problem 3 – What fraction of the objects, including the moon, are about the same size as our moon?

Problem 4 – If Saturn's moon Titan is ½ the diameter of Earth, and Saturn's moon Dione is 1/6 the diameter of Titan, how large is the diameter of Dione compared to Earth?

Problem 5 – Oberon is 1/7 the diameter of Earth, Io is 1/3 the diameter of Earth, and Titania is 4/9 the diameter of Io. Which moon is bigger in diameter: Oberon or Titania?

Problem 1 – What fraction of the objects are smaller than our moon?
Answer: **17/26**

Problem 2 – What fraction of the objects are larger than our moon but are not planets?

Answer: Io, Callisto, Titan and Ganymede : 4/26 or **2/13**

Problem 3 – What fraction of the objects, including the moon, are about the same size as our moon?

Answer: Moon, Europa, Triton and Pluto so 4/26 = **2/13**.

Problem 4 – Saturn's moon Titan is ½ the diameter of Earth, and Saturn's moon Dione is 1/6 the diameter of Titan, how large is the diameter of Dione compared to Earth?

Answer: ½ x 1/6 = **1/12 the size of Earth**.

Problem 5 – Oberon is 1/7 the diameter of Earth, Io is 1/3 the diameter of Earth, and Titania is 4/9 the diameter of Io. Which moon is bigger in diameter: Oberon or Titania?

Answer: Oberon is 1/7 the diameter of Earth.
Titania is 4/9 the diameter of Io, and Io is 1/3 the diameter of Earth
So Titania is (4/9) x (1/3) the diameter of Earth
So Titania is 4/27 the diameter of Earth.

Comparing Oberon, which is 1/7 the diameter of Earth with Titania, which is 4/27 the diameter of Earth, which fraction is larger: 1/7 or 4/27?

Find the common denominator 7 x 27 = 189, then cross-multiply the fractions:

Oberon: 1/7 = 27/189 and Titania: 4/27 = (4x7)/189 = 28/189 so

Titania is 28/189 Earth's diameter and Oberon is 27/189 Earth's diameter, and so **Titania is slightly larger!**

Finding Mass in the Cosmos

$$F_g = \frac{G M m}{R^2}$$

$$F_c = \frac{m V^2}{R}$$

$$V = \frac{2 \pi R}{T}$$

One of the neatest things in astronomy is being able to figure out the mass of a distant object, without having to 'go there'. Astronomers do this by employing a very simple technique. It depends only on measuring the separation and period of a pair of bodies orbiting each other. In fact, Sir Issac Newton showed us how to do this over 300 years ago!

Imagine a massive body such as a star, and around it there is a small planet in orbit. We know that the force of gravity, **Fg**, of the star will be pulling the planet inwards, but there will also be a centrifugal force, **Fc**, pushing the planet outwards.

This is because the planet is traveling at a particular speed, **V**, in its orbit. When the force of gravity and the centrifugal force on the planet are exactly equal so that **Fg = Fc**, the planet will travel in a circular path around the star with the star exactly at the center of the orbit.

Problem 1) Use the three equations above to derive the mass of the primary body, M, given the period, T, and radius, R, of the companion's circular orbit.

Problem 2) Use the formula $\mathbf{M = 4 \pi^2 R^3 / (G T^2)}$ where G = 6.6726 x 10^{-11}N-m^2/kg^2 and M is the mass of the primary in kilograms, R is the orbit radius in meters and T is the orbit period in seconds, to find the masses of the primary bodies in the table below. (Note: Make sure all units are in meters and seconds first! 1 light years = 9.5 trillion kilometers)

Primary	Companion	Period	Orbit Radius	Mass of Primary
Earth	Communications satellite	24 hrs	42,300 km	
Earth	Moon	27.3 days	385,000 km	
Jupiter	Callisto	16.7 days	1.9 million km	
Pluto	Charon	6.38 days	17,530 km	
Mars	Phobos	7.6 hrs	9,400 km	
Sun	Earth	365 days	149 million km	
Sun	Neptune	163.7 yrs	4.5 million km	
Sirius A	Sirius B	50.1 yrs	20 AU	
Polaris A	Polaris B	30.5 yrs	290 million miles	
Milky Way	Sun	225 million yrs	26,000 light years	

Answer Key

Problem 1: Answer

$$\frac{G M m}{R^2} = \frac{m V^2}{R}$$

Cancil the companion mass, m, on both sides, and isolate the primary mass, M, on the left side:

$$M = \frac{R V^2}{G}$$

Now use the definition of V to eliminate it from the equation,

$$M = \frac{R}{G}\left(\frac{2 \pi R}{T}\right)^2$$

and simplify

$$M = \frac{4 \pi^2 R^3}{G T^2}$$

Problem 2:

Primary	Companion	Period	Orbit Radius	Mass of Primary
Earth	Communications satellite	24 hrs	42,300 km	**6.1×10^{24} kg**
Earth	Moon	27.3 days	385,000 km	**6.1×10^{24} kg**
Jupiter	Callisto	16.7 days	1.9 million km	**1.9×10^{27} kg**
Pluto	Charon	6.38 days	17,530 km	**1.3×10^{22} kg**
Mars	Phobos	7.6 hrs	9,400 km	**6.4×10^{23} kg**
Sun	Earth	365 days	149 million km	**1.9×10^{30} kg**
Sun	Neptune	163.7 yrs	4.5 million km	**2.1×10^{30} kg**
Sirius A	Sirius B	50.1 yrs	298 million km	**6.6×10^{30} kg**
Polaris A	Polaris B	30.5 yrs	453 million km	**6.2×10^{28} kg**
Milky Way	Sun	225 million yrs	26,000 light years	**1.7×10^{41} kg**

Note: The masses for Sirius A and Polaris A are estimates because the companion star has a mass nearly equal to the primary so that our mass formula becomes less reliable.

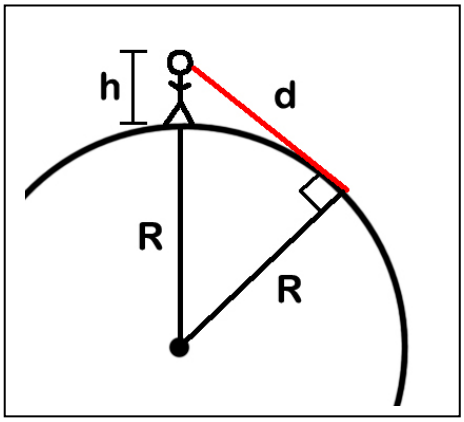
The Distance to the Martian Horizon

Imagine you and your friend standing on the surface of a perfectly flat planet. Your friend starts walking away from you, and you see her size get smaller and smaller until at a distance of 100 kilometers you can't even see her at all.

Now imagine the same experiment on a spherical planet. As many sea-farers discovered 1000 years ago, because Earth is curved, you will see the ships hull disappear from the bottom upwards, then the last thing that vanishes is the top of the main mast.

The image above comes from Johannes de Sacrobosco's Tractatus de Sphaera (*On the Sphere of the World*) written in 1230 AD. It showcases the knowledge that the appearance of ships on the horizon testified to a curved earth. A bit of simple geometry, and some help from the Pythagorean Theorem, will let you calculate the distance to the horizon on Mars as viewed from the InSight Lander!

Problem 1 – Use the Pythagorean Theorem to solve for the distance, d, in terms of h and R.



Problem 2 – R is the radius of Mars, which is 3,378 kilometers. If h is the height of an observer in meters, write a simplified equation for d when h << R. What is the distance to the martian horizon as viewed from the IDS camera located 1 meters above the ground?

Problem 3 - Mars has no ionosphere, so radio signals cannot be 'bounced' around Mars to distant locations. Instead, tall 'cell towers' have to be used. If the cell tower is 100 meters tall, how many cell towers will you need to cover the entire surface of Mars?

Problem 1 – Use the Pythagorean Theorem to solve for the distance, d, in terms of h and R.

Answer: $d^2 = (R+h)^2 - R^2$

So $d^2 = 2Rh + h^2$

And so $d = (2Rh + h^2)^{1/2}$

Problem 2 – R is the radius of Mars, which is 3,378 kilometers. If h is the height of an observer in meters, write a simplified equation for d when h << R. What is the distance to the martian horizon as viewed from the IDS camera located 1 meters above the ground?

Answer: For the formula to work, R and h must be in the same units of meters (or kilometers!). When h is much less then R, the quantity h^2 is always much, much smaller than 2Rh, so the formula simplifies to $d = (2Rh)^{1/2}$.

For Mars, h = 1 meter and R = 3378000 meters and so **d = 2599 meters or about 2.6 kilometers.**

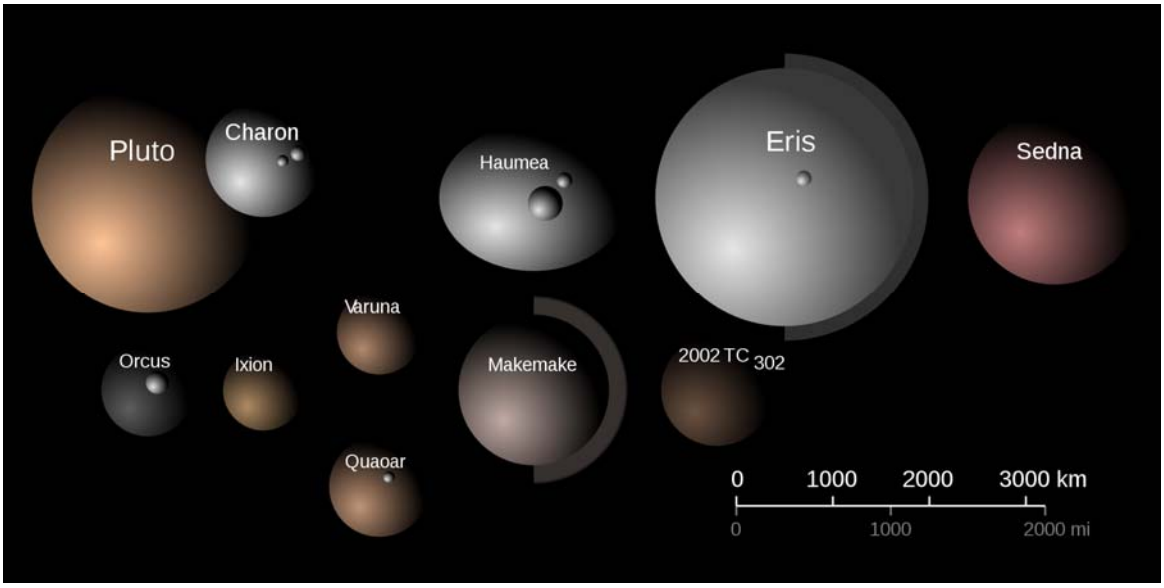
Problem 3 - Mars has no ionosphere, so radio signals cannot be 'bounced' around Mars to distant locations. Instead, tall 'cell towers' have to be used. If the cell tower is 100 meters tall, how many cell towers will you need to cover the entire surface of Mars?

Answer: h = 100 meters or 0.1 km, so the horizon distance for one cell tower has a radius of $d = (2 \times 3378 \text{ km} \times 0.1 \text{ km})^{1/2} = 26$ kilometers. The area of this reception circle is $A = \pi R^2 = 2123 \text{ km}^2$. The total surface area of Mars is $A = 4 \times \pi \times (3378)^2 = 1.43 \times 10^8 \text{ km}^2$.

Then dividing the surface area of Mars by the cell tower reception area we get $N = 1.43 \times 10^8 / 2123 =$ **67,530 cell towers**.

As of 2013, there are over 200,000 cell towers in the Unites States alone!

Dwarf Planets and Kepler's Third Law



Object	Distance (AU)	Period (years)
Mercury	0.4	0.24
Venus	0.7	0.61
Earth	1.0	1.0
Mars	1.5	1.88
Ceres	2.8	4.6
Jupiter	5.2	11.9
Saturn	9.5	29.5
Uranus	19.2	84.0
Neptune	30.1	164.8
Pluto	39.4	247.7
Ixion	39.7	
Huya	39.8	
Varuna	42.9	
Haumea	43.3	285
Quaoar	43.6	
Makemake	45.8	310
Eris	67.7	557
1996-TL66	82.9	
Sedna	486.0	

Note: Distances are given in Astronomical Units (AU) where 1 AU = earth-sun distance of 150 million km.

Astronomers have detected over 500 bodies orbiting the sun well beyond the orbit of Neptune. Among these 'Trans-Neptunian Objects (TNOs) are a growing number that rival Pluto in size. This caused astronomers to rethink how they should define the term 'planet'.

In 2006 Pluto was demoted from a planet to a dwarf planet, joining the large asteroid Ceres in that new group. Several other TNOs also joined that group, which now includes five bodies shown highlighted in the table. A number of other large objects, called Plutoids, are also listed.

Problem 1 - From the tabulated data, graph the distance as a function of period on a calculator or Excel spreadsheet. What is the best-fit: A) Polynomial function? B) Power-law function?

Problem 2 - Which of the two possibilities can be eliminated because it gives unphysical answers?

Problem 3 - Using your best-fit model, what would you predict for the periods of the TNOs in the table?

Problem 1 - From the tabulated data, graph the distance as a function of period on a calculator or Excel spreadsheet. What is the best-fit:

A) Polynomial function? **The N=3 polynomial $D(x) = -0.0005x^3 + 0.1239x^2 + 2.24x - 1.7$**

B) Power-law function? **The N=1.5 powerlaw: $D(x) = 1.0x^{1.5}$**

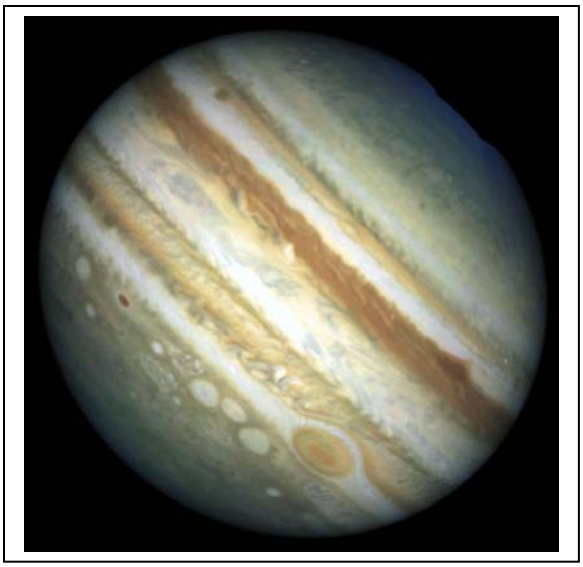
Problem 2 - Which of the two possibilities can be eliminated because it gives unphysical answers? The two predictions are shown in the table:

Object	Distance	Period	N=3	N=1.5
Mercury	0.4	0.24	-0.79	0.25
Venus	0.7	0.61	-0.08	0.59
Earth	1	1	0.66	1.00
Mars	1.5	1.88	1.93	1.84
Ceres	2.8	4.6	5.53	4.69
Jupiter	5.2	11.9	13.22	11.86
Saturn	9.5	29.5	30.33	29.28
Uranus	19.2	84	83.44	84.13
Neptune	30.1	164.8	164.34	165.14
Pluto	39.4	247.7	248.31	247.31
Ixion	39.7		251.21	250.14
Huya	39.8		252.19	251.09
Varuna	42.9		282.94	280.99
Haumea	43.3	285	286.99	284.93
Quaoar	43.6		290.05	287.89
Makemake	45.8	310	312.75	309.95
Eris	67.7	557	562.67	557.04
1996-TL66	82.9		750.62	754.80
Sedna	486		-27044.01	10714.07

Answer: The N=3 polynomial gives negative periods for Mercury, Venus and Sedna, and poor answers for Earth, Mars, Ceres and Jupiter compared to the N=3/2 power-law fit. The N=3/2 power-law fit is the result of Kepler's Third Law for planetary motion which states that the cube of the distance is proportional to the square of the period so that when all periods and distances are scaled to Earth's orbit, Period = Distance$^{3/2}$

Problem 3 - See the table above for shaded entries

The Composition of Planetary Atmospheres



All of the planets in our solar system, and some of its smaller bodies too, have an outer layer of gas we call the atmosphere. The atmosphere usually sits atop a denser, rocky crust or planetary core. Atmospheres can extend thousands of kilometers into space.

The table below gives the name of the kind of gas found in each object's atmosphere, and the total mass of the atmosphere in kilograms. The table also gives the percentage of the atmosphere composed of the gas.

Object	Mass (kilograms)	Carbon Dioxide	Nitrogen	Oxygen	Argon	Methane	Sodium	Hydrogen	Helium	Other
Sun	3.0×10^{30}							71%	26%	3%
Mercury	1000			42%			22%	22%	6%	8%
Venus	4.8×10^{20}	96%	4%							
Earth	1.4×10^{21}		78%	21%	1%					<1%
Moon	100,000				70%		1%		29%	
Mars	2.5×10^{16}	95%	2.7%		1.6%					0.7%
Jupiter	1.9×10^{27}							89.8%	10.2%	
Saturn	5.4×10^{26}							96.3%	3.2%	0.5%
Titan	9.1×10^{18}		97%			2%				1%
Uranus	8.6×10^{25}					2.3%		82.5%	15.2%	
Neptune	1.0×10^{26}					1.0%		80%	19%	
Pluto	1.3×10^{14}	8%	90%			2%				

Problem 1 – Draw a pie graph (circle graph) that shows the atmosphere constituents for Mars and Earth.

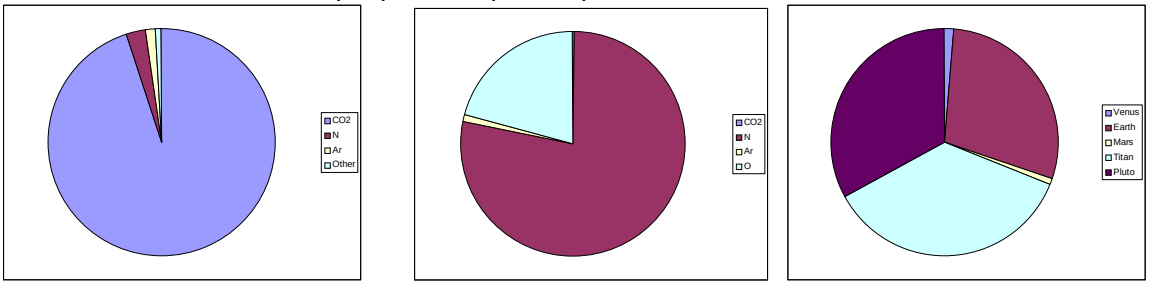
Problem 2 – Draw a pie graph that shows the percentage of Nitrogen for Venus, Earth, Mars, Titan and Pluto.

Problem 3 – Which planet has the atmosphere with the greatest percentage of Oxygen?

Problem 4 – Which planet has the atmosphere with the greatest number of kilograms of oxygen?

Problem 5 – Compare and contrast the objects with the greatest percentage of hydrogen, and the least percentage of hydrogen.

Problem 1 – Draw a pie graph (circle graph) that shows the atmosphere constituents for Mars and Earth. Answer: Mars (Ieft), Earth (middle)



Problem 2 – Draw a pie graph that shows the percentage of Nitrogen for Venus, Earth, Mars, Titan and Pluto. Answer: First add up all the percentages for Nitrogen in the column to get 271.7%. Now divide each of the percentages in the column by 271.7% to get the percentage of nitrogen in the planetary atmospheres that is taken up by each of the planets: Venus = (4/271) = 1.5%; Earth = (78/271) = 28.8%, Mars = (2.7/271)= 1.0%, Titan = (97/271)=35.8%, Pluto=(90/271)=33.2%. Plot these new percentages in a pie graph (see above right). This pie graph shoes that across our solar system, Earth, Titan and Pluto have the largest percentage of nitrogen. In each case, the source of the nitrogen is from similar physical processes involving the chemistry of the gas methane (Titan and Earth) or methane ice (Pluto).

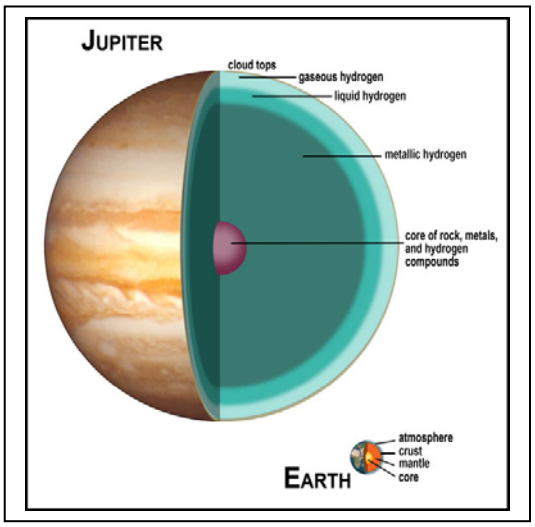
Problem 3 – Which planet has the atmosphere with the greatest percentage of Oxygen? Answer: From the table we see that **Mercury** has the greatest percentage of oxygen in its atmosphere.

Problem 4 – Which planet has the atmosphere with the greatest number of kilograms of oxygen? Answer: Only two planets have detectable oxygen: Earth and Mercury. Though mercury has the highest percentage of oxygen making up its atmosphere, the number of kilograms of oxygen is only 1000 kg x 0.42 = 420 kilograms. By comparison, **Earth** has a smaller percentage of oxygen (21%) but a vastly higher quantity: 1.4×10^{21} kg x 0.21 = **2.9×10^{20} kilograms**. (That's 290,000,000,000,000,000,000 kg)

Problem 5 – Compare and contrast the objects with the greatest percentage of hydrogen, and the least percentage of hydrogen.

Answer: The objects with the highest percentage of hydrogen are the sun, Mercury, Jupiter, Saturn, Uranus and Neptune. The objects with the least percentage are Venus, Earth, Moon, Mars, Titan, Pluto. With the exception of Mercury, which has a very thin atmosphere, the high-percentage objects are the largest bodies in the solar system. The planet Jupiter, Saturn, Uranus and Neptune are sometimes called the Gas Giants because so much of the mass of these planets consists of a gaseous atmosphere. These bodies generally lie far from the sun. The low-percentage objects are among the smallest bodies in the solar system. They are called the 'Rocky Planets' to emphasize their similarity in structure, where a rocky core and mantel are surrounded by a thin atmosphere. Most of these bodies lie close to the sun.

Measuring Atmospheric Trace Gases with Parts Per Million 4.2



Most of the planets in our solar system have two or three constituents that make up most of the atmosphere. For example, Venus and Mars have more than 98% of their atmosphere in carbon dioxide and nitrogen, while Earth has 99% of its atmosphere in nitrogen and oxygen. But trace gases with percentages below 1% are also important. For example, without the 0.3% of carbon dioxide in Earth's atmosphere, Earth would be a lifeless and frigid planet!

Scientists use 'parts per million' to represent the amounts of trace gases in a planetary atmosphere.

Examples: One year is 1 part per hundred of a century, or 1% of a century.
One year is 1 part per thousand of a millennium, or 0.1% of a millennium.
One millimeter is 1 part per million of 1 kilometer, or 0.0001% of a kilometer.

Problem 1 - The following list gives the percentages of various trace gases in the atmospheres of the indicated objects. Convert these percentages to parts-per-million (ppm) units.

Earth:	Carbon Dioxide……	0.038%.....	_____	ppm
	Neon……....	0.00182%.....	_____	ppm
	Methane…….	0.000175%.....	_____	ppm
	Water Vapor……...	5.0 %.....	_____	ppm
Mars:	Neon……....	0.00025%.....	_____	ppm
	Methane…….	0.00000105%.....	_____	ppm
Titan:	Methane…….	1.4%	_____	ppm
	Argon……....	0.0043%.....	_____	ppm
	Carbon Monoxide…...	0.0052%.....	_____	ppm
	Ethane…….	0.0013%.....	_____	ppm
Jupiter:	Methane…….	0.3%.....	_____	ppm
	Ammonia…….	0.026%.....	_____	ppm
	Ethane…….	0.00058……	_____	ppm
	Water Vapor……...	0.0004……	_____	ppm

Problem 2 - Using your ppm answers from Problem 1, by what factor does Earth have more methane than Mars as a trace gas?

Problem 3 – The amount of carbon dioxide in Earth's atmosphere is increasing by 2.5 ppm per year. If its value in 2012 was measured to be 392 ppm, by what year will it have reached 517 ppm? How old will you be then, if the growth rate continues at this rate of increase?

Problem 1 - The following list gives the percentages of various trace gases in the atmospheres of the indicated objects. Convert these percentages to parts-per-million (ppm) units.
Answer: Example: Earth CO2. 0.038% = 0.00038 then 0.00038 x 1,000,000 = 380 ppm

Earth:	Carbon Dioxide…….	0.038%.....	**380**	**ppm**	
	Neon……....	0.00182%.....	**18.2**	**ppm**	
	Methane…….	0.000175%.....	**1.75**	**ppm**	
	Water Vapor……...	5.0 %.....	**50000**	**ppm**	
Mars:	Neon……....	0.00025%.....	**2.5**	**ppm**	
	Methane…….	0.00000105%.....	**0.0105**	**ppm**	or 10.5 ppb
Titan:	Methane…….	1.4%	**14000**	**ppm**	
	Argon……....	0.0043%.....	**43**	**ppm**	
	Carbon Monoxide…..	0.0052%.....	**52**	**ppm**	
	Ethane……..	0.0013%.....	**13**	**ppm**	
Jupiter:	Methane……..	0.3%.....	**3000**	**ppm**	
	Ammonia…….	0.026%.....	**260**	**ppm**	
	Ethane…….	0.00058……	**5.8**	**ppm**	
	Water Vapor…….....	0.0004……	**4.0**	**ppm**	

Note. When the value for ppm is less than one, use parts per billion by multiplying ppm by 1000. Example, for Mars and Methane, 0.0105 ppm = 0.0105 x1000 = 10.5 ppb.

Problem 2 - Using your ppm answers, by what factor does Earth have more Methane than Mars as a trace gas?

Answer: Earth/Mars = 1.75 ppm/0.0105 ppm = **167 times more than Mars**.

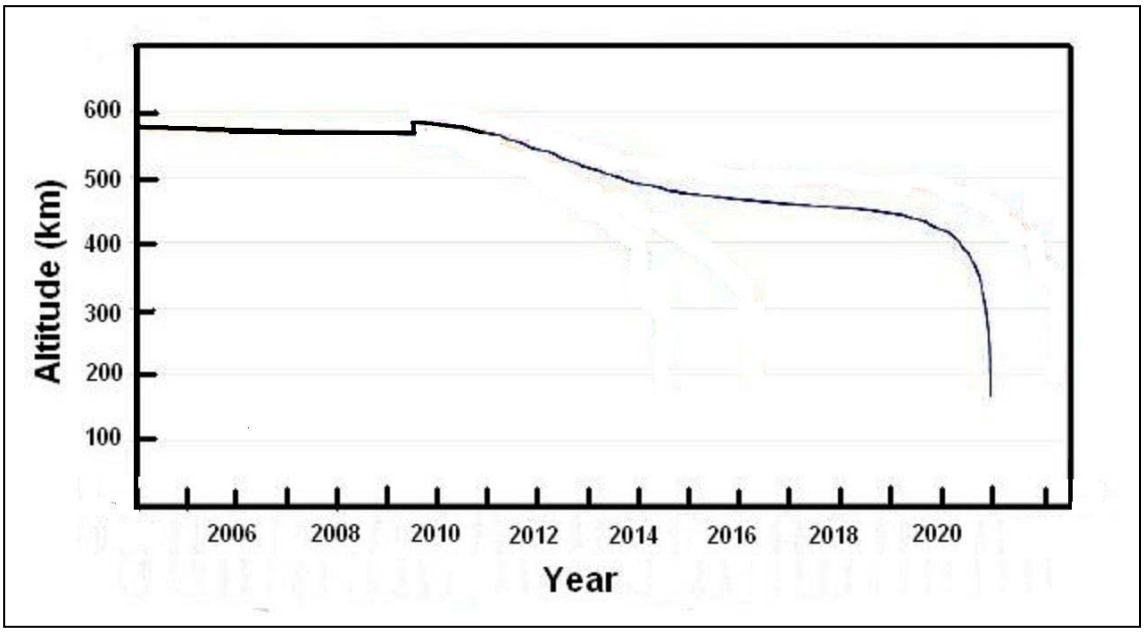
Problem 3 – The amount of carbon dioxide in Earth's atmosphere is increasing by 2.5 ppm per year. If its value in 2012 was measured to be 392 ppm, by what year will it have reached 517 ppm? How old will you be then, if the growth rate continues at this rate of increase?

Answer: To get to 517 ppm from 392 ppm it must increase by 517-392 = 125 ppm. If the increase is 2.5 ppm each year, it will take 125/2.5 = 50 years, so the year will be 2062.
If a student was 15 years old in 2012, they will be 65 years old in 2062.

Satellite Drag and the Hubble Space Telescope

The Hubble Space Telescope was never designed to operate forever. What to do with the observatory remains a challenge for NASA once its scientific mission is completed in 2012. Originally, a Space Shuttle was proposed to safely return it to Earth, where it would be given to the National Air and Space Museum in Washington DC. Unfortunately, after the last Servicing Mission, STS-125, in May, 2009, no further Shuttle visits are planned. As solar activity increases, the upper atmosphere heats up and expands, causing greater friction for low-orbiting satellites like HST, and a more rapid re-entry.

The curve below shows the predicted altitude for that last planned re-boost in 2009 of 18-km. NASA plans to use a robotic spacecraft after ca 2015 to allow a controlled re-entry for HST, but if that were not the case, it would re-enter the atmosphere sometime after 2020.



Problem 1 – The last Servicing Mission in 2009 only extend the science operations by another 5 years. How long after that time will the HST remain in orbit?

Problem 2 – Once HST reaches an altitude of 400 km, with no re-boosts, about how many weeks will remain before the satellite burns up? (Hint: Use a millimeter ruler.)

Problem 1 – The last Servicing Mission in 2009 only extend the science operations by another 5 years. How long after that time will the HST remain in orbit?

Answer: The Servicing Mission occurred in 2009. The upgrades and gyro repairs extend the satellite's operations by 5 more years, so if it re-enters after 2020 it will have about 6 years to go before uncontrolled re-entry.

Problem 2 – Once HST reaches an altitude of 400 km, with no re-boosts, about how many weeks will remain before the satellite burns up? (Hint: Use a millimeter ruler.)

Answer: Use a millimeter ruler to determine the scale of the horizontal axis in weeks per millimeter. Mark the point on the curve that corresponds to a vertical value of 400 km. Draw a line to the horizontal axis and measure its distance from 2013 in millimeters. Convert this to weeks using the scale factor you calculated. The answer should be about 50 weeks.

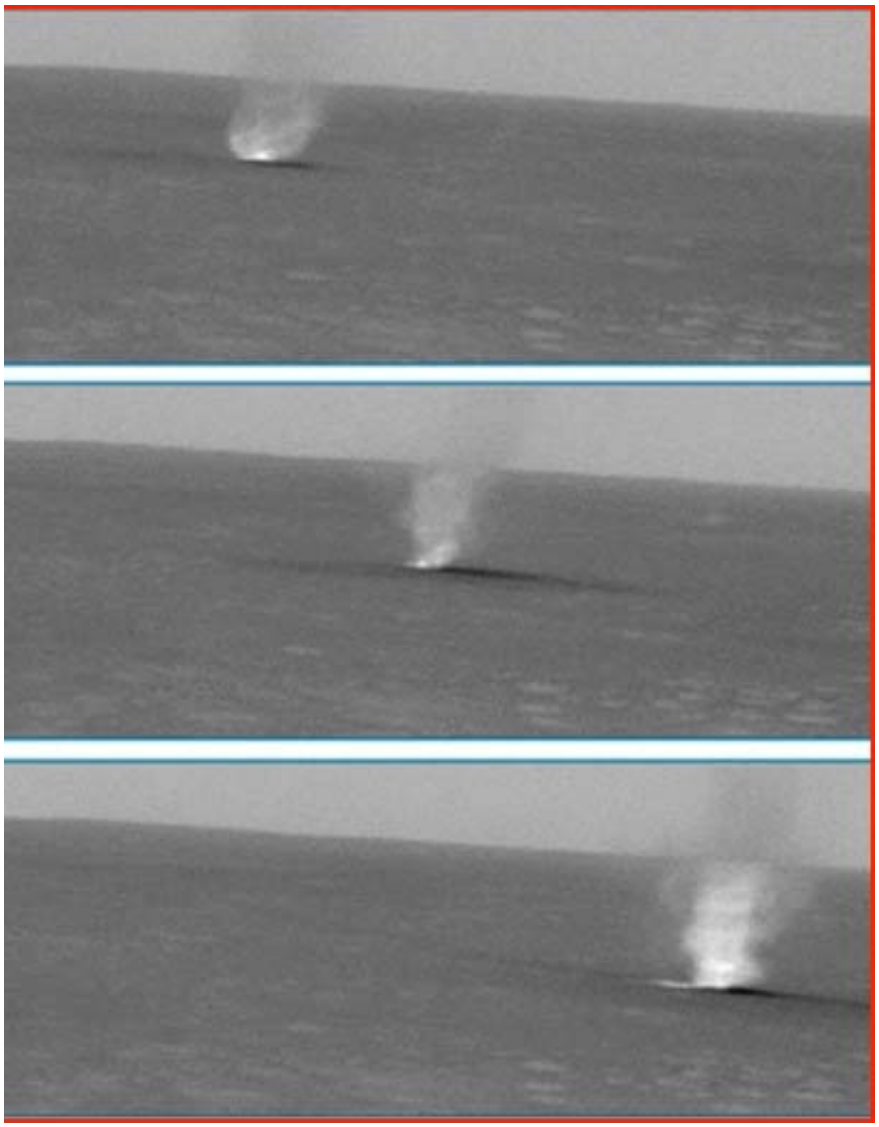
"NASA's 23-year-old Hubble Space Telescope is still going strong, and agency officials said Tuesday (Jan. 8, 2013) they plan to operate it until its instruments finally give out, potentially for another six years at least.

After its final overhaul in 2009, the Hubble telescope was expected to last until at least 2015. Now, NASA officials say they are committed to keeping the iconic space observatory going as long as possible.

"Hubble will continue to operate as long as its systems are running well," Paul Hertz, director of the Astrophysics Division in NASA's Science Mission Directorate, said here at the 221st meeting of the American Astronomical Society. Hubble, like other long-running NASA missions such as the Spitzer Space Telescope, will be reviewed every two years to ensure that the mission is continuing to provide science worth the cost of operating it, Hertz added."

(Space.com, January 9, 2013.)

The Martian Dust Devils



A dust devil spins across the surface of Gusev Crater just before noon on Mars. NASA's Spirit rover took the series of images to the left with its navigation camera on March 15, 2005.

The images were taken at:

11:48:00 (T=top)
11:49:00 (M=middle)
11:49:40 (B=bottom)

based upon local Mars time.

The dust devil was about 1.0 kilometer from the rover at the start of the sequence of images on the slopes of the "Columbia Hills."

A simple application of the rate formula

$$speed = \frac{\text{distance}}{\text{time}}$$

lets us estimate how fast the dust devil was moving.

Problem 1 - At the distance of the dust devil, the scale of the image is 7.4 meters/millimeter. How far did the dust devil travel between the top (T) and bottom (B) frames?

Problem 2 - What was the time difference, in seconds, between the images T-M, M-B and T-B?

Problem 3 - What was the distance, in meters, traveled between the images T-M and M-B?

Problem 4 - What was the average speed, in meters/sec, of the dust devil between T-B. If an astronaut can briskly walk at a speed of 120 meters/minute, can she out-run a martian dust devil?

Problem 5 - What were the speeds during the interval from T-M, and the interval M-B?

Problem 6 - Was the dust devil accelerating or decelerating between the times represented by T-B?

Answer Key

Problem 1 - At the distance of the dust devil, the scale of the image is 7.4 meters/millimeter. How far did the dust devil travel between the top and bottom frames? Answer: The location of the dust devil in frame B when placed in image T is a shift of about 65 millimeters, which at a scale of 7.4 meters/mm equals about 480 meters.

Problem 2 - What was the time difference between the images T-M, M-B and T-B? Answer: T-M = 11:49:00 - 11:48:00 - 1 minute or 60 seconds. For M-B the time interval is 40 seconds. For T-B the time interval is 100 seconds.

Problem 3 - What was the distance traveled between the images T-M and M-B? Answer: T-M = about 30 mm or 222 meters; M-B = about 35 mm or 259 meters.

Problem 4 - What was the average speed, in meters/sec, of the dust devil between T-B? If an astronaut can briskly walk at a speed of 120 meters/minute, can she out-run a martian dust devil?
Answer: Speed = distance/time so 480 meters/100 seconds = 4.8 meters/sec. This is about twice as fast as an astronaut can walk, so running would be a better option.

Problem 5 - What were the speeds during the interval from T-M, and the interval M-B? Answer: Speed(T-M) = 222 meters/60 seconds = 3.7 meters/sec. Speed(M-B) = 259 meters/40 seconds = 6.5 meters/sec.

Problem 6 - Was the dust devil accelerating or decelerating between the times represented by T-B? Answer: because the speed increased from 4.8 meters/sec to 6.5 meters/sec, the dust

Table of Global Temperature Anomalies

Year	Temperature (degrees C)	Year	Temperature (degrees C)
1900	-0.20	1960	+0.05
1910	-0.35	1970	0.00
1920	-0.25	1980	+0.20
1930	-0.28	1990	+0.30
1940	+0.08	2000	+0.45
1950	-0.05	2010	+0.63

A new study by researchers at the Goddard Institute for Space Studies determined that 2010 tied with 2005 as the warmest year on record, and was part of the warmest decade on record since the 1800s. The analysis used data from over 1000 stations around the world, satellite observations, and ocean and polar measurements to draw this conclusion.

The table above gives the average 'temperature anomaly' for each decade from 1900 to 2010. The Temperature Anomaly is a measure of how much the global temperature differed from the average global temperature between 1951 to 1980. For example, a +1.0 C temperature anomaly in 2000 means that the world was +1.0 degree Celsius warmer in 2000 than the average global temperature between 1951-1980.

Problem 1 - By how much has the average global temperature changed between 1900 and 2000?

Problem 2 - The various bumps and wiggles in the data are caused by global weather changes such as the El Nino/La Nina cycle, and year-to-year changes in other factors that are not well understood by climate experts. By how much did the global temperature anomaly change between: A) 1900 and 1920? B) 1920 to 1950? C) 1950 and 1980? D) 1980 to 2010? Describe each interval in terms of whether it was cooling or warming.

Problem 3 - From the data in the table, calculate the rate of change of the temperature anomaly per decade by dividing the temperature change by the number of decades (3) in each time period. Is the pace of global temperature change increasing, decreasing, or staying about the same since 1900?

Problem 4 - Based on the trends in the data from 1960 to 2000, what do you predict that the temperature anomaly will be in 2050? Explain what this means in terms of average global temperature in 2050.

Problem 1 - By how much has the average global temperature changed between 1900 and 2000? Answer: In 1900 it was -0.20 C and in 2000 it was +0.45, so it has changed by +0.45 - (-0.20) = **+0.65 C**.

Problem 2 - The various bumps and wiggles in the data are caused by global weather changes such as the El Nino/El Nina cycle, and year-to-year changes in other factors that are not well understood by climate experts. By how much did the global temperature change between: A) 1900 and 1920? B) 1920 to 1950? C) 1950 and 1980? D) 1980 to 2010? Describe each interval in terms of whether it was cooling or warming.Answer:
1900 to 1920: -0.25C - (-0.20C) = **-0.05 C a decrease (cooling) of 0.05 C**
1920 to 1950: - 0.05C - (-0.25C) = **+0.20 C an increase (warming) of 0.20 C**
1950 to 1980: +0.20C - (-0.05C) = **+0.25 C an increase (warming) of 0.25 C**
1980 to 2010: +0.63C - (+0.20C) = **+0.43 C an increase (warming) of 0.43 C**

Problem 3 - From the data in the table, calculate the rate of change of the Temperature Anomaly per decade by dividing the temperature change by the number of decades (3) in each time period. Is the pace of global temperature change increasing, decreasing, or staying about the same since 1900? Answer:
1900 to 1920: -0.05 C/3 decades = **-0.017 C per decade**
1920 to 1950: +0.20 C/ 3 decades = **+0.067 C per decade**
1950 to 1980: +0.25 C/3 decades = **+0.083 C per decade**
1980 to 2010: +0.43 C/3 decades = **+0.143 C per decade**.

The pace of global temperature change is **increasing in time**. It is almost doubling every 10 years.

Problem 4 - Based on the trends in the data from 1960 to 2000, what do you predict that the temperature anomaly will be in 2050? Explain what this means in terms of average global temperature in 2050.
Answer: Students may graph the data in the table, then use a ruler to draw a line on the graph between 1960 and 2000, to extrapolate to the temperature anomaly in 2050. A linear equation, T = mx + b, that models this data is b = +0.05C m = (+0.45 - 0.05)/4 decades so m = +0.10 C/decade. Then T = +0.10x + 0.05. For 2050, which is 9 decades after 1960, x=9 so T = +0.1(9) + 0.05 = **+0.95 C**. So, the world will be, on average, about **+1 C warmer** in 2050 compared to its average temperature between 1950 and 1980. This assumes a linear change in T with time.

However from Problem 3 we see that the temperature anomaly change is accelerating. The 'second order' differences are +0.033, +0.033, +0.06. If we take the average change as (0.033+0.033+0.06)/3 = +0.042 we get a more accurate 'quadratic' expression: $T = +0.042\ x^2 + 0.1x + 0.05$. For the year 2050, this quadratic prediction suggests $T = 0.042(9)^2 + 0.1(9) + 0.05$ so T = +1.32 C.

For more information about this research, see the NASA Press Release at **http://www.nasa.gov/topics/earth/features/2010-warmest-year.html**

Exponential Functions and the Atmosphere

$$P(z) = P_0 \, e^{-\frac{z}{H}} \quad \text{and} \quad H = \frac{kT}{mg}$$

A very common way to describe the atmosphere of a planet is by its 'scale height'. This quantity represents the vertical distance above the surface at which the density or pressure if the atmosphere decreases by exactly 1/e or $(2.718)^{-1}$ times (equal to 0.368).

The scale height, usually represented by the variable **H**, depends on the strength of the planet's gravity field, the temperature of the gases in the atmosphere, and the masses of the individual atoms in the atmosphere. The equation to the left shows how all of these factors are related in a simple atmosphere model for the density P. The variables are:

z: Vertical altitude in meters
T: Temperature in Kelvin degrees
m: Average mass of atoms in kilograms
g: Acceleration of gravity in meters/sec^2
k: Boltzmann's Constant 1.38x10^{-23} J/deg

Problem 1 - For Earth, g = 9.81 meters/sec^2, T = 290 K. The atmosphere consists of 22% O_2 (m= 2x2.67x10^{-26} kg) and 78% N_2 (m= 2x2.3x10^{-26} kg). What is the scale height, H?

Problem 2 - Mars has an atmosphere of nearly 100% CO_2 (m=7.3x10^{-26} kg) at a temperature of about 210 Kelvins. What is the scale height H if g= 3.7 meters/sec^2?

Problem 3 - The Moon has an atmosphere that includes about 0.1% sodium (m=6.6x10^{-26} kg). If the scale height deduced from satellite observations is 120 kilometers, what is the temperature of the atmosphere if g= 1.6 meters/sec^2?

Problem 4 - At what altitude on Earth would the density of the atmosphere P(z) be only 10% what it is at sea level, P_0?

Problem 5 - Calculate the total mass of the atmosphere in a column of air, below a height h with integral calculus. At what altitude, h, on Earth is half the atmosphere below you?

Problem 1 - Answer: First we have to calculate the average atomic mass. <m> = 0.22 (2x2.67x10^{-26} kg) + 0.78 (2x2.3x10^{-26} kg) = 4.76x10^{-26} kg. Then,

$$H = \frac{(1.38\times10^{-23})(290)}{(4.76\times10^{-26})(9.81)} =$$ **8,570 meters or about 8.6 kilometers.**

Problem 2 - Answer:

$$H = \frac{(1.38\times10^{-23})(210)}{(7.3\times10^{-26})(3.7)} =$$ **10,700 meters or about 10.7 kilometers.**

Problem 3 - Answer

$$T = \frac{(6.6\times10^{-26})(1.6)(120000)}{(1.38\times10^{-23})} =$$ **918 Kelvins.**

Problem 4 - Answer: $0.1 = e^{-(z/H)}$, Take ln of both sides, ln(0.1) = -z/H then z = 2.3 H so for H= 8.6 km, **z = 19.8 kilometers**.

Problem 5 - First calculate the total mass:

$$M = \int_0^\infty P(z)dz \qquad M = P_0\int_0^\infty e^{-\frac{z}{H}}dz \qquad M = P_0H\int_0^\infty e^{-x}dx \qquad M = P_0H$$

Then subtract the portion above you:

$$m = \int_h^\infty P(z)dz \qquad M = P_0\int_h^\infty e^{-\frac{z}{H}}dz \qquad M = P_0H\int_h^\infty e^{-x}dx \qquad M = P_0H\left[e^{-h} - 1\right]$$

To get: $dm(h) = P_0He^{-h}$

So 1/2 = $e^{-h/H}$ and h = ln(2)(8.6 km) and so the height is **6 kilometers!**

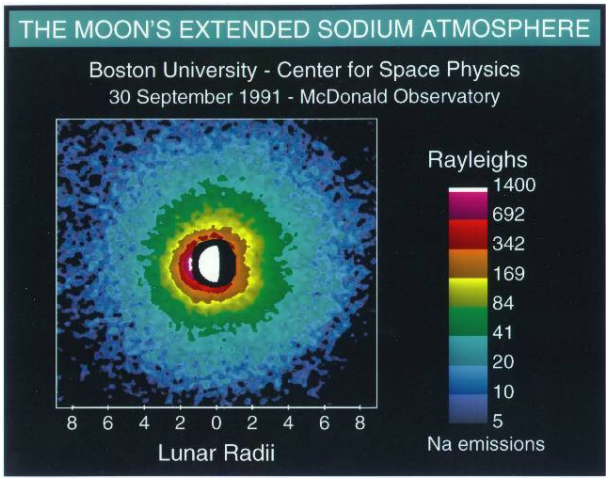
The Moon's Atmosphere!

Courtesy: T.A.Rector, I.P.Dell'Antonio (NOAO/AURA/NSF)

Experiments performed by Apollo astronauts were able to confirm that the moon does have a very thin atmosphere.

The Moon has an atmosphere, but it is very tenuous. Gases in the lunar atmosphere are easily lost to space. Because of the Moon's low gravity, light atoms such as helium receive sufficient energy from solar heating that they escape in just a few hours. Heavier atoms take longer to escape, but are ultimately ionized by the Sun's ultraviolet radiation, after which they are carried away from the Moon by solar wind.

Because of the rate at which atoms escape from the lunar atmosphere, there must be a continuous source of particles to maintain even a tenuous atmosphere. Sources for the lunar atmosphere include the capture of particles from solar wind and the material released from the impact of comets and meteorites. For some atoms, particularly helium and argon, outgassing from the Moon's interior may also be a source.



Problem 1: The Cold Cathode Ion Gauge instrument used by Apollo 12, 14 and 15 recorded a daytime atmosphere density of 160,000 atoms/cc of hydrogen, helium, neon and argon in equal proportions. What was the density of helium in particles/cc?

Problem 2: The atomic masses of hydrogen, helium, neon and argon are 1.0 AMU, 4.0 AMU, 20 AMU and 36 AMU. If one AMU = 1.6×10^{-24} grams, a) How many grams of hydrogen are in one cm3 of the moon's atmosphere? B) Helium? C) Neon? D) Argon? E) Total grams from all atoms?

Problem 3: Assume that the atmosphere fills a spherical shell with a radius of 1,738 kilometers, and a thickness of 170 kilometers. What is the volume of this spherical shell in cubic centimeters?

Problem 4. Your answer to Problem 2E is the total density of the lunar atmosphere in grams/cc. If the atmosphere occupies the shell whose volume is given in Problem 3, what is the total mass of the atmosphere in A) grams? B) kilograms? C) metric tons?

Answer Key:

Problem 1: The Cold Cathode Ion Gauge instrument used by Apollo 12, 14 and 15 recorded a daytime atmosphere density of 160,000 atoms/cc of hydrogen, helium, neon and argon in equal proportions. What was the density of helium in particles/cc?

Answer: Each element contributes 1/4 of the total particles so hydrogen = 40,000 particles/cc; helium = 40,000 particles/cc, argon=40,000 particles/cc and argon=40,000 particles/cc

Problem 2: The atomic masses of hydrogen, helium, neon and argon are 1.0 AMU, 4.0 AMU, 20 AMU and 36 AMU. If one AMU = 1.6 x 10-24 grams, a) How many grams of hydrogen are in one cm3 of the moon's atmosphere? B) Helium? C) Neon? D) Argon? E) Total grams from all atoms?

Answer: A) Hydrogen = 1.0 x (1.6 x 10^{-24} grams) x 40,000 particles = 6.4 x $10^{\mathbf{-20}}$ grams
B) Helium = 4.0 x (1.6 x 10^{-24} grams) x 40,000 particles = 2.6 x $10^{\mathbf{-19}}$ grams
C) Neon = 20.0 x (1.6 x 10^{-24} grams) x 40,000 particles = 1.3 x $10^{\mathbf{-18}}$ grams
D) Argon = 36.0 x (1.6 x 10^{-24} grams) x 40,000 particles = 2.3 x $10^{\mathbf{-18}}$ grams
E) Total = (0.064 + 0.26 + 1.3 + 2.3) x 10^{-18} grams = 3.9 x $10^{\mathbf{-18}}$ grams per cc.

Problem 3: Assume that the atmosphere fills a spherical shell with a radius of 1,738 kilometers, and a thickness of 170 kilometers. What is the volume of this spherical shell in cubic centimeters?

Answer: Compute the difference in volume between A sphere with a radius of Ri = 1,738 km and Ro = 1,738+170 = 1,908 km. V = 4/3 π $(1908)^{\mathbf{3}}$ - 4/3π $(1738)^{\mathbf{3}}$ = 2.909 x $10^{\mathbf{10}}$ $km^{\mathbf{3}}$ - 2.198 x $10^{\mathbf{10}}$ $km^{\mathbf{3}}$ = 7.1 x $10^{\mathbf{9}}$ $km^{\mathbf{3}}$

Volume = 7.1 x $10^{\mathbf{9}}$ $km^{\mathbf{3}}$ **x** (10^5 cm/km) **x** (10^5 cm/km) **x** (10^5 cm/km)
= 7.1 x 10^{24} cm^3

Note: If you use the 'calculus technique' of approximating the volume as the surface area of the shell with a radius of Ri, multiplied by the shell thickness of h = 170 km, you will get a slightly different answer of 6.5 x $10^{\mathbf{9}}$ $km^{\mathbf{3}}$ and 6.5 x 10^{24} cm^3

Problem 4. Your answer to Problem 2E is the total density of the lunar atmosphere in grams/cc. If the atmosphere occupies the shell whose volume is given in Problem 3, what is the total mass of the atmosphere in A) grams? B) kilograms?

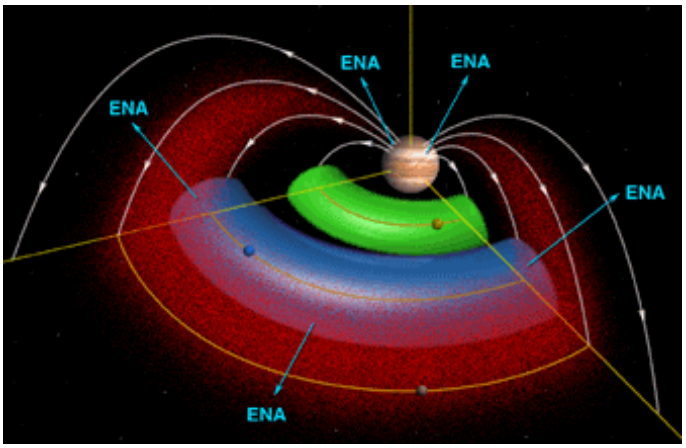
A) Mass = density x volume = (3.9 x $10^{\mathbf{-18}}$ gm/cc) x 7.1 x $10^{\mathbf{24}}$ $cm^{\mathbf{3}}$ = 2.8 x $10^{\mathbf{7}}$ grams

B) Mass = 2.8 x $10^{\mathbf{7}}$ grams x (1 kg/1000 gms) = 28,000 kilograms.

C) Mass = 28,000 kg x (1 ton / 1000 kg) = 28 tons.

Teacher note: You may want to compare this mass to some other familiar objects. Also, the Apollo 11 landing and take-off rockets ejected about 1 ton of exhaust gases. Have the students discuss the human impact (air pollution!) on the lunar atmosphere from landings and launches.

The Io Plasma Torus



The satellite of Jupiter, Io, is a volcanically active moon that ejects 1,000 kilograms of ionized gas into space every second. This gas forms a torus encircling Jupiter along the orbit of Io. We will estimate the total mass of this gas based on data from the NASA Cassini and Galileo spacecraft.

Image: Io plasma torus (Courtesy NASA/Cassini)

Problem 1 - Galileo measurements obtained in 2001 indicated that the density of neutral sodium atoms in the torus is about 35 atoms/cm^3. The spacecraft also determined that the inner boundary of the torus is at about 5 Rj, while the outer boundary is at about 8 Rj. (1 Rj = 71,300 km). A torus is defined by the radius of the ring from its center, R, and the radius of the circular cross section through the donut, r. What are the dimensions, in kilometers, of the Io torus based on the information provided by Galileo?

Problem 2 - Think of a torus as a curled up cylinder. What is the general formula for the volume of a torus with radii R and r?

Problem 3 - From the dimensions of the Io torus, what is the volume of the Io torus in cubic meters?

Problem 4 - From the density of sodium atoms in the torus, what is A) the total number of sodium atoms in the torus? B) If the mass of a sodium atom is 3.7×10^{-20} kilograms, what is the total mass of the Io torus in metric tons?

Calculus:

Problem 5 - Using the 'washer method' in integral calculus, derive the formula for the volume of a torus with a radius equal to R, and a cross-section defined by the formula $x^2 + y^2 = r^2$. The torus is formed by revolving the cross section about the Y axis.

Answer Key

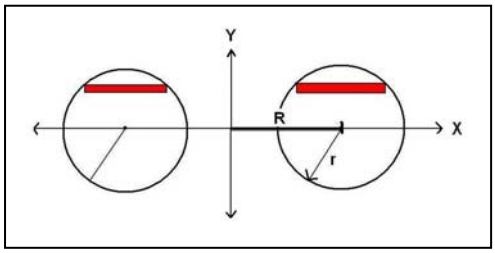
Problem 1 - The mid point between 5 Rj and 8 Rj is (8+5)/2 = 6.5 Rj so R = 6.5 Rj and r = 1.5 Rj. Then R = 6.5 x 71,300 so **R = 4.6 x 10^5 km**, and r = 1.5 x 71,300 so **r = 1.1 x 10^5 km**.

Problem 2 - The cross-section of the cylinder is πr^2, and the height of the cylinder is the circumference of the torus which equals $2 \pi R$, so the volume is just $V = (2 \pi R) \times (\pi r^2)$ or **$V = 2 \pi^2 R r^2$.**

Problem 3 - Volume = $2 \pi^2$ (4.6 x 10^5 km) (1.1 x 10^5 km)^2 so **V = 1.1 x 10^{17} km^3.**

Problem 4 - A) 35 atoms/cm^3 x (100000 cm/1 km)^3 = 3.5 x 10^{16} atoms/km^3. Then number = density x volume so N = (3.5 x 10^{16} atoms/km^3) x (1.1 x 10^{17} km^3) , so **N = 3.9 x 10^{33} atoms**. B) The total mass is M = 3.9 x 10^{33} atoms x 3.7 x 10^{-20} kilograms/atom = 1.4 x 10^{14} kilograms. 1 metric ton = 1000 kilograms, so the total mass is **M = 100 billion tons**.

Advanced Math:



$$V = 8\pi R \int_0^r \left(r^2 - y^2\right)^{1/2} dy$$

Recall that the volume of a washer is given by $V = \pi$ (R(outer)^2 - R(inner)^2) x thickness. For the torus figure above, we see that the thickness is just dy. The distance from the center of the cross section to a point on the circumference is given by $r^2 = x^2 + y^2$. The width of the washer (the red volume element in the figure) is parallel to the X-axis, so we want to express its length in terms of y, so we get $x = (r^2 - y^2)^{1/2}$. The location of the outer radius is then given by R(outer) = $R + (r^2-y^2)^{1/2}$, and the inner radius by R(inner) = $R - (r^2 - y^2)^{1/2}$. We can now express the differential volume element of the washer by $dV = \pi [(R + (r^2-y^2)^{1/2})^2 - (R - (r^2 - y^2)^{1/2})^2]$ dy. This simplifies to $dV = \pi [4R (r^2-y^2)^{1/2}]$ dy or $dV = 4 \pi R (r^2-y^2)^{1/2}$ dy. The integral can immediately be formed from this ,with the limits y = 0 to y=r. Because the limits to y only span the upper half plane, we have to double this integral to get the additional volume in the lower half-plane. The required integral is shown above.

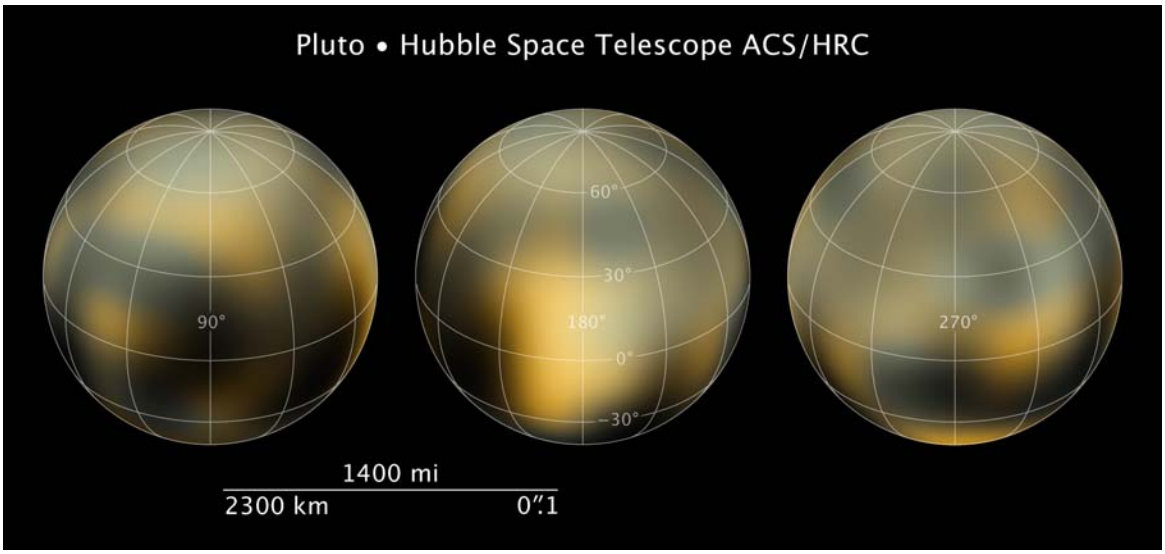
This integral can be solved by factoring out the r from within the square-root, then using the substitution U = y/r and dU = 1/r dy to get the integrand $dV = 8 \pi R r^2 (1 - U^2)^{1/2}$ dU. The integration limits now become U=0 to U=1. Since r and R are constants, this is an elementary integral with the solution $V = 1/2 U (1-U^2)^{1/2} + 1/2 \arcsin(U)$. When this is evaluated from U=0 to U=1, we get

$V = 8 \pi R r^2 [0 + 1/2 \arcsin(1)] - [0 + 1/2 \arcsin(0)]$

$V = 8 \pi R r^2 \, 1/2 (\pi/2)$

$V = 2 \pi R r^2$

The Changing Atmosphere of Pluto



Recent Hubble Space Telescope studies of Pluto have confirmed that its atmosphere is undergoing considerable change, despite its frigid temperatures. Let's see how this is possible!

Problem 1 - The equation for the orbit of Pluto can be approximated by the formula $2433600=1521x^2+1600y^2$. Determine from this equation, expressed in Standard Form, A) the semi-major axis, a; B) the semi-minor axis, b; C) the ellipticity of the orbit, e; D) the longest distance from a focus called the aphelion; E) the shortest distance from a focus, called the perihelion. (Note: All units will be in terms of Astronomical Units. 1 AU = distance from the Earth to the Sun = $1.5x10^{11}$ meters).

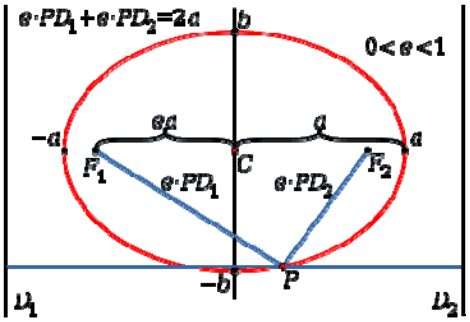
Problem 2 - The temperature of the methane atmosphere of Pluto is given by the formula

$$T(R)=\left(\frac{L(1-A)}{16\pi\sigma R^2}\right)^{\frac{1}{4}} \quad \text{Kelvin (K)}$$

where L is the luminosity of the sun ($L=4 \times 10^{26}$ watts); σ is a constant with a value of $5.67x10^{-8}$, R is the distance from the sun to Pluto in meters; and A is the albedo of Pluto. The albedo of Pluto, the ability of its surface to reflect light, is about A = 0.6. From this information, what is the predicted temperature of Pluto at A) perihelion? B) aphelion?

Problem 3 - If the thickness, H, of the atmosphere in kilometers is given by H(T) = 1.2 T with T being the average temperature in degrees K, can you describe what happens to the atmosphere of Pluto between aphelion and perihelion?

Problem 1 - Answer:



In Standard Form $2433600=1521x^2+1600y^2$ becomes $1=\frac{x^2}{1600}+\frac{y^2}{1521}$ = $\frac{x^2}{a^2}+\frac{y^2}{b^2}$

Then A) **a = 40 AU** and B) **b=39 AU**. C) The ellipticity **e = $(a^2- b^2)^{1/2}/a$ = 0.22**. D) The longest distance from a focus is just a(1 + e) = 40(1+0.22) = **49 AU**. E) The shortest distance is just a(1-e) = (1-0.22)(40) = **31 AU**. Written out in meters we have a= $6x10^{12}$ meters; b= $5.8x10^{12}$ meters; aphelion = $7.35x10^{12}$ meters and perihelion = $4.6x10^{12}$ meters.

Problem 2 - Answer: For R in terms of AU, the formula simplifies to

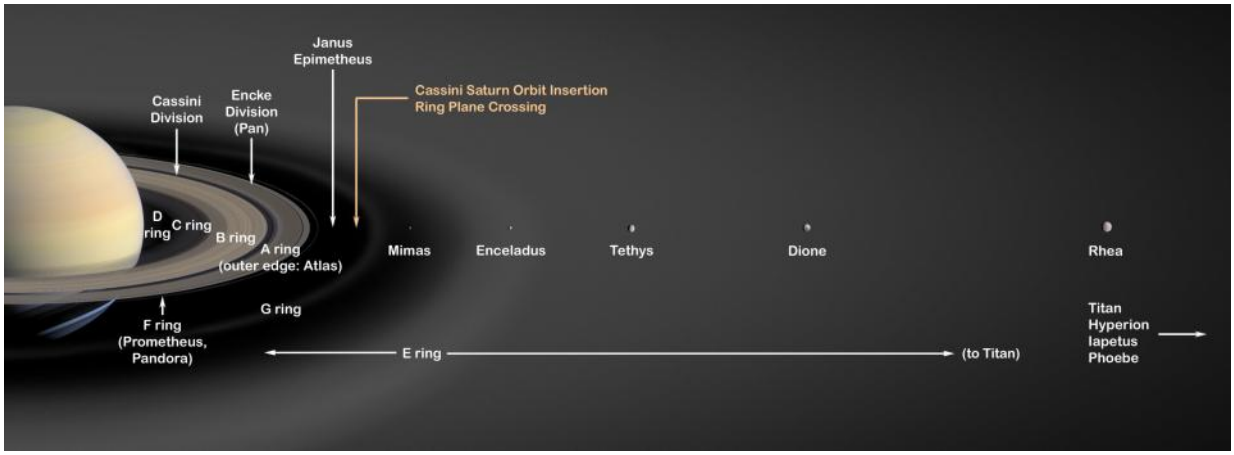
$$T(R)=\left(\frac{4x10^{26}(1-0.6)}{16(3.14)(5.67x10^{-8})(1.5x10^{11})^2R^2}\right)^{\frac{1}{4}} \quad \text{so } T(R)=\frac{223}{\sqrt{R}} \quad \text{K}$$

A) For a perihelion distance of 31 AU we have T = $223/(31)^{1/2}$ = 40 K; B) At an aphelion distance of 49 AU we have T = $223/(49)^{1/2}$ = 32 K. Note: The actual temperatures are about higher than this and average about 50K.

Problem 3 - Answer: At aphelion, the height of the atmosphere is about H=1.2x(32) = 38 kilometers, and at perihelion it is about H=1.2x(40) = 48 kilometers, so as Pluto orbits the sun its atmosphere increases and decreases in thickness.

Note: In fact, because the freezing point of methane is 91K, at aphelion most of the atmosphere freezes onto the surface of the dwarf planet, and at aphelion it returns to a mostly gaseous state. This indicates that the simple physical model used to derive H(T) was incomplete and did not account for the freezing-out of an atmospheric constituent.

The Rings of Saturn



Saturn, the most beautiful planet in our solar system, is famous for its dazzling rings. Shown in the figure above, these rings extend far into space and engulf many of Saturn's moons. The brightest rings, visible from Earth in a small telescope, include the D, C and B rings, Cassini's Division, and the A ring. Just outside the A ring is the narrow F ring, shepherded by tiny moons, Pandora and Prometheus. Beyond that are two much fainter rings named G and E. Saturn's diffuse E ring is the largest planetary ring in our solar system, extending from Mimas' orbit to Titan's orbit, about 1 million kilometers (621,370 miles).

The particles in Saturn's rings are composed primarily of water ice and range in size from microns to tens of meters. The rings show a tremendous amount of structure on all scales. Some of this structure is related to gravitational interactions with Saturn's many moons, but much of it remains unexplained. One moonlet, Pan, actually orbits inside the A ring in a 330-kilometer-wide (200-mile) gap called the Encke Gap. The main rings (A, B and C) are less than 100 meters (300 feet) thick in most places. The main rings are much younger than the age of the solar system, perhaps only a few hundred million years old. They may have formed from the breakup of one of Saturn's moons or from a comet or meteor that was torn apart by Saturn's gravity.

Problem 1 – The dense main rings extend from 7,000 km to 80,000 km above Saturn's equator (Saturn's equatorial radius is 60,300 km). If the average thickness of these rings is 1 kilometer, what is the volume of the ring system in cubic kilometers? (use $\pi = 3.14$)

Problem 2 – The total number of ring particles is estimated to be 3×10^{16} . If these ring particles are evenly distributed in the ring volume calculated in Problem 1, what is the average distance in meters between these ring particles?

Problem 3 – If the ring particles are about 1 meter in diameter and have the density of water ice, 1000 kg/m^3, about what is the diameter of the assembled body from all of these ring particles?

Problem 1 – The dense main rings extend from 7,000 km to 80,000 km above Saturn's equator (Saturn's equatorial radius is 60,300 km). If the average thickness of these rings is 1 kilometer, what is the volume of the ring system in cubic kilometers? (use $\pi = 3.14$)

Answer: The area of a ring with an inner radius of r and an outer radius of R is given by $A = \pi (R^2 - r^2)$ and its volume for a thickness of h is just $V = \pi (R^2 - r^2)h$.

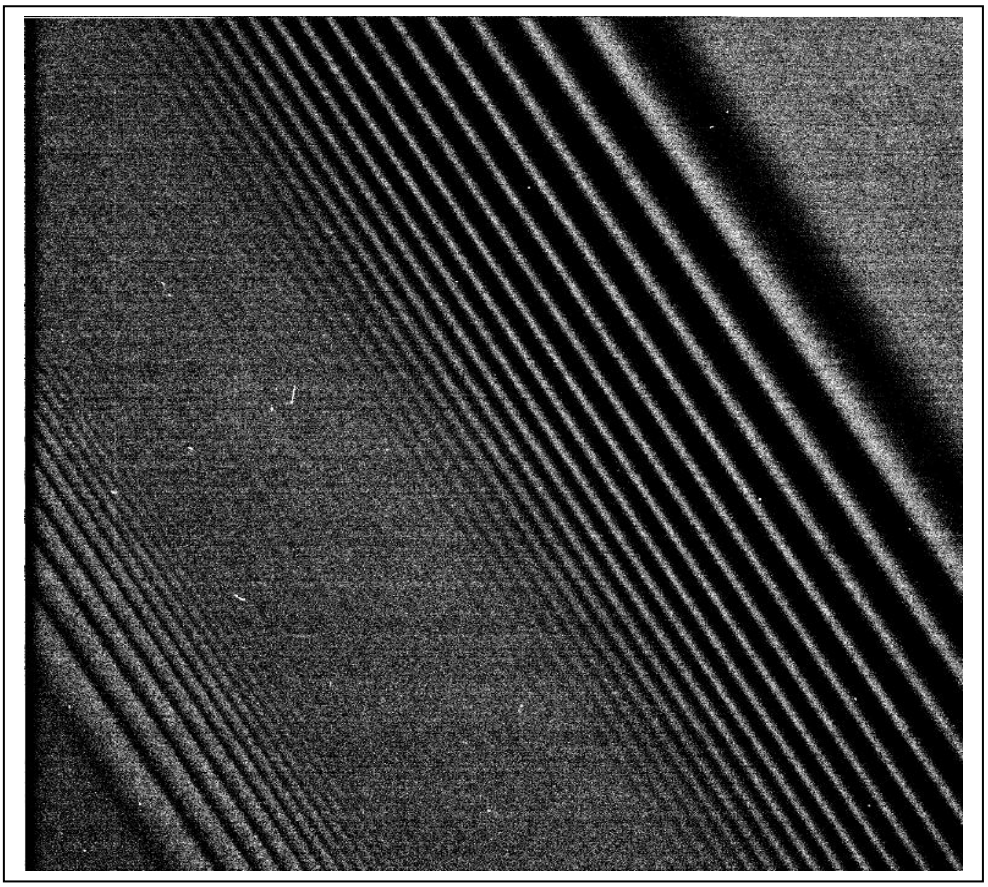
For Saturn's rings we have an inner radius r = 60300km+7000km =67300 km and an outer radius of R = 60300km+80000km = 140,300 km, and a volume of $V = 3.14 ((140300)^2 - (67300)^2) (1.0) =$ **4.75×10^{10} km^3**.

Problem 2 – The total number of ring particles is estimated to be 3×10^{16} . If these ring particles are evenly distributed in the ring volume calculated in Problem 1, what is the average distance in meters between these ring particles?

Answer: 4.75×10^{10} km^3 / 3×10^{16} = 1.6×10^{-6} km^3/particle, so each particle is found in a volume of 1.6×10^{-6} km^3. For two cubes next to each other each with a volume of $V = s^3$, their centers are separated by exactly s. The distance to the nearest ring particle is $D = (1.6 \times 10^{-6}$ km$^3)^{1/3}$ = 0.012 km, or **12 meters**!

Problem 3 – If the ring particles are about 1 meter in diameter and have the density of water ice, 1000 kg/m^3, about what is the diameter of the assembled body from all of these ring particles?

Answer: The volume of a single particle is $4/3\pi(1/2)^3 = 0.5$ meter^3. The total volume of all the 3×10^{16} particles is then $V = 1.5 \times 10^{16}$ meter^3. If this is a spherical body then $4/3\pi R^3 = 1.5 \times 10^{16}$ m^3, so R = 151185 meters or 150 kilometers in radius. The diameter is then **300 kilometers**.



This spectacular close-up image of Saturn's A ring was taken in 2004 by the Cassini spacecraft. It shows a 220-km wide snapshot of a magnified portion of the A ring, and how it dissolves into smaller ringlets. Astronomers think that these ringlets are formed by gravitational interactions with Saturn's inner moons, causing ripples and waves to form that 'bunch up' billions of ring particles into separate ringlets. Some of the bright spots you see in the dark bands may be 'shepherding moonlets' only a few kilometers in size, which keep the ring particles orbiting together.

Problem 1 – By using a millimeter ruler, determine the scale of this image in kilometers/millimeter, and estimate the width of a typical ringlet in this image.

Problem 2 – Draw a diagonal line from the upper right corner (closest to Saturn) to the lower left corner (farthest from Saturn). Number the 16 ringlets in consecutive order starting from the first complete ringlet in the upper right corner. In a table, state the width of each consecutive ringlet in millimeters and kilometers.

Problem 3 – What is the average width of the 16 ringlets you measured to the nearest kilometer?

Problem 4 – Plot the ringlet number and the ringlet width in kilometers. What can you say about the ringlet sizes in this portion of the A ring?

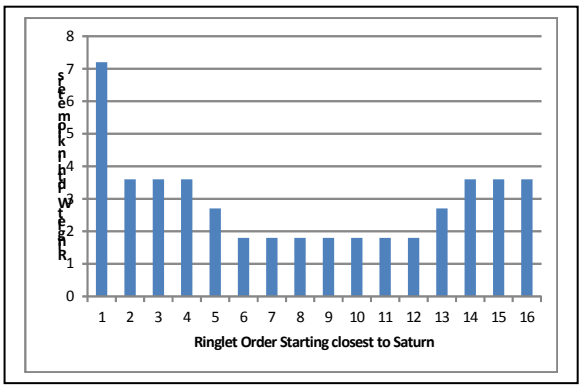
Problem 1 – By using a millimeter ruler, determine the scale of this image in kilometers/millimeter, and estimate the width of a typical ringlet in this image. Answer: When printed on standard 8 ½ x 11 paper, the width of the image is 124 millimeters, so the scale is 220 km/124mm = **1.8 km/mm**.

Problem 2 – Draw a diagonal line from the upper right corner (closest to Saturn) to the lower left corner (farthest from Saturn). Number the 16 ringlets in consecutive order starting from the first complete ringlet in the upper right corner. In a table, state the width of each consecutive ringlet in millimeters and kilometers. Answer: **See below**.

Ringlet	millimeters	kilometers
1	4	7.2
2	2	3.6
3	2	3.6
4	2	3.6
5	1.5	2.7
6	1	1.8
7	1	1.8
8	1	1.8
9	1	1.8
10	1	1.8
11	1	1.8
12	1	1.8
13	1.5	2.7
14	2	3.6
15	2	3.6
16	2	3.6

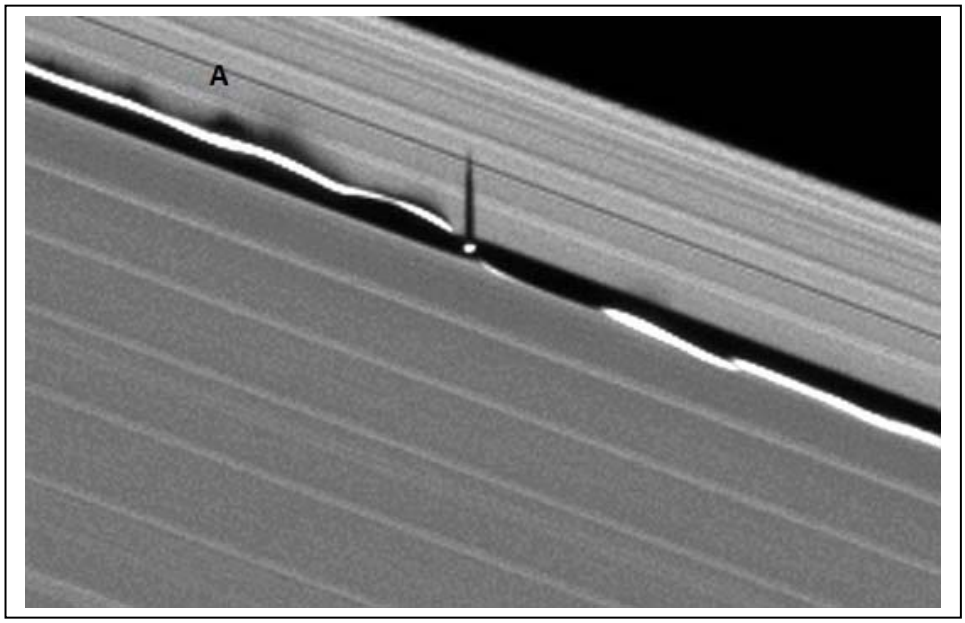
Problem 3 – What is the average width of the 16 ringlets you measured to the nearest kilometer? Answer: (6x3.6 + 7.2 +2x2.7+7x1.8)/16 = **2.9 kilometers**.

Problem 4 – Plot the ringlet number and the ringlet width in kilometers. What can you say about the ringlet sizes in this portion of the A ring?



Answer: The ringlet widths decrease as you get further from Saturn and are present with distinct dark gaps in between. As the dark gaps become narrower than about 3 km near Ringlet 12, the rings begin to increase slightly in size.

It is 'interesting' that the ringlets in the lower right corner are wider than the upper ringlets, and there are fewer of them.



Never-before-seen looming vertical structures, created by the tiny moon Daphnis, cast long shadows across Saturn's A Ring in this startling image taken by the Cassin spacecraft. The 8-kilometre-wide moon Daphnis orbits within the 42-kilometre-wide Keeler Gap in Saturn's outer A Ring, and its gravitational pull perturbs the orbits of the particles forming the gap's edges. The Keeler Gap is foreshortened and appears only about 30 km wide because the image was taken at an angle of about 45 degrees to the ring plane.

Problem 1 – If the apparent perpendicular width of the Keeler Gap is 30 km, what is the length of the shadow of Daphnis in this image?

Problem 2 – Rounded to the nearest kilometer, what is the length of Shadow A to the left from Daphnis?

Problem 3 – Create a scaled model of this ring area and its 45 degree inclination, and using right triangles, estimate the elevation angle of the sun above the ring plane.

Problem 4 – From your scaled model, what is the height of the feature that is casting Shadow A on the ring plane?

Problem 1 – If the apparent (foreshortened) perpendicular width of the Keeler Gap is 30 km, what is the corresponding foreshortened length of the shadow of Daphnis in this image?

Answer: After printing this page on standard 8 ½ x 11 paper, the perpendicular width of the Keeler Gap is about 4 millimeters, which corresponds to 30 km, so the scale is 30 km/4mm = 7.5 km/mm. The length of the moonlets shadow is 12 mm, so its projected length is 12 x 7.5 = **90 km**.

Problem 2 – Rounded to the nearest kilometer, what is the length of Shadow A to the left from Daphnis?

Answer: Students may obtain 5 mm if they measure from the inner edge of the white band, or 3 mm if they measure from the outer edge of the white band. These measurements correspond to **23 km or 38 km**.

Problem 3 – Create a scaled model of this ring area and its 45 degree inclination, and using right triangles, estimate the elevation angle of the sun above the ring plane.

Answer: For the shadow of Daphnis, the true shadow length is the hypotenuse of a 45-45-90 right triangle ABD, and its shadow length in the image is the horizontal side along segment BC of this triangle, so the hyponenuse (segment AB) length is 90 km x $(2)^{\mathbf{1/2}}$ = 127 km. The diameter of the moon is 8 km, so the sun angle for the shadow on the ring plane is a triangle whose sides are AC=8 km and BC = 127 km. The angle to the sun ABC can be measured with a protractor and is Tan(Theta) = 8km/127km so to the nearest degree, **Theta = 4 degrees**.

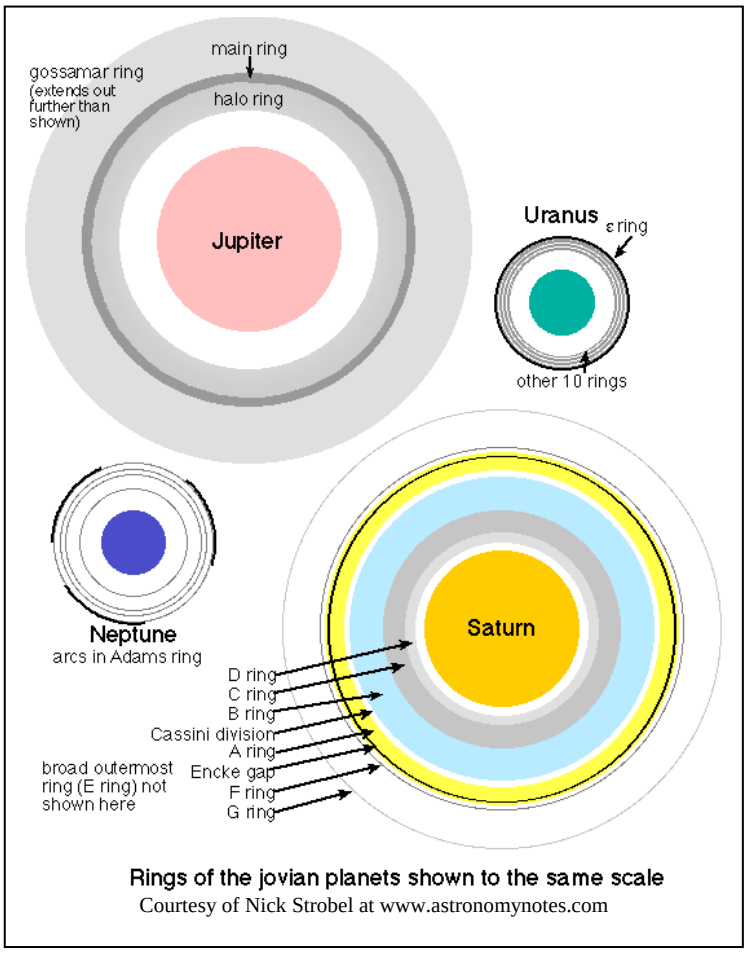
Problem 4 – From your scaled model, what is the height of the feature that is casting Shadow A on the ring plane?

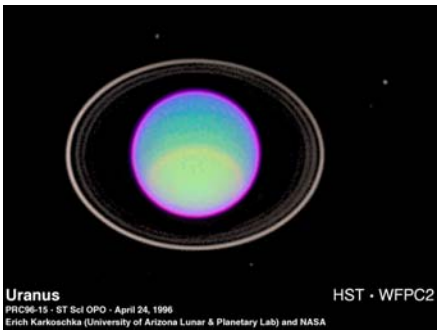
Answer: The projected height of the shadow is 23 or 38 km. The true shadow length is then
23 x $(2)^{\mathbf{1/2}}$ = 33 km or
38 x $(2)^{1/2}$ = 54 km.

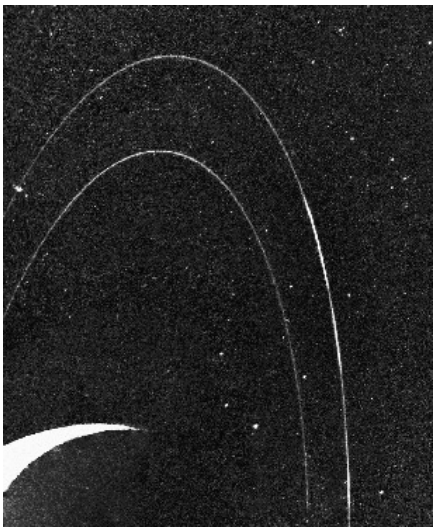
The sun angle above the ring plane is 4 degrees, so if the right triangle ABC has side BC = 33 km then the height of segment AC can be measured with a protractor as about 33xTan(4) = **2.3 km**, or using the second shadow measurement 54xtan(4) = **3.8 km**.

Note: Astronomers find it amazing that even though the A ring has a thickness of about 10 meters, the wave produced by this small moon rises over 200 times higher above the ring plane than the thickness of the rings themselves!

Comparing the Rings of the Outer Planets







The rings of Uranus (top right) and Neptune (bottom right) are similar to those of Jupiter and consist millions of small rocky and icy objects in separate orbits. The figure shows a comparison of the scales of the planets and their ring systems.

Problem 1 – Compare the extent of the ring systems for each planet in terms of their size in units of the radius of the corresponding planet. For example, 'The rings of Uranus extend from 1.7 to 2.0 times the radius of Uranus'.

Problem 2 – An icy body will be destroyed by a planet if it comes within the Tidal Limit of the planet. At this distance, the difference in gravity between the near and the far side of the body exceeds the body's ability to hold together by its own gravity, and so it is shredded into smaller pieces. For Jupiter (2.7), Saturn (2.2), Uranus (2.7) and Neptune (2.9), the Tidal Limits are located between 2.2 and 2.9 times the radius of each planet from the planet's center. Describe where the ring systems are located around each planet compared to the planets Tidal Limit. Could the rings be explained by a moon or moon's getting too close to the planet?

Problem 1 – Compare the extent of the ring systems for each planet in terms of their size in units of the radius of the corresponding planet. For example, 'The rings of Uranus extend from 1.7 to 2.0 times the radius of Uranus'.

Answer:
Jupiter from 1.4 to 2.3
Saturn from 1.1 to 3.6
Uranus from 1.7 to 2.0
Neptune from 1.7 to 2.7

Problem 2 – An icy body will be destroyed by a planet if it comes within the Tidal Limit of the planet. At this distance, the difference in gravity between the near and the far side of the body exceeds the body's ability to hold together by its own gravity, and so it is shredded into smaller pieces. For Jupiter (2.7), Saturn (2.2), Uranus (2.7) and Neptune (2.9), the Tidal Limits are located between 2.2 and 2.9 times the radius of each planet from the planet's center. Describe where the ring systems are located around each planet compared to the planets Tidal Limit. Could the rings be explained by a moon or moon's getting too close to the planet?

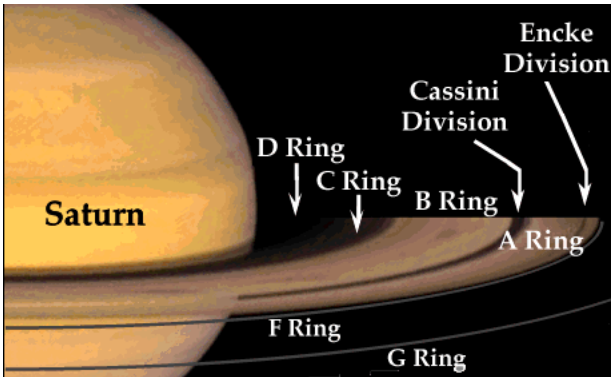
Answer:

Jupiter	1.4	2.3	(2.7)		
Saturn	1.1	(2.2)			3.6
Uranus	1.7	2.0	(2.7)		
Neptune	1.7		2.7	(2.9)	

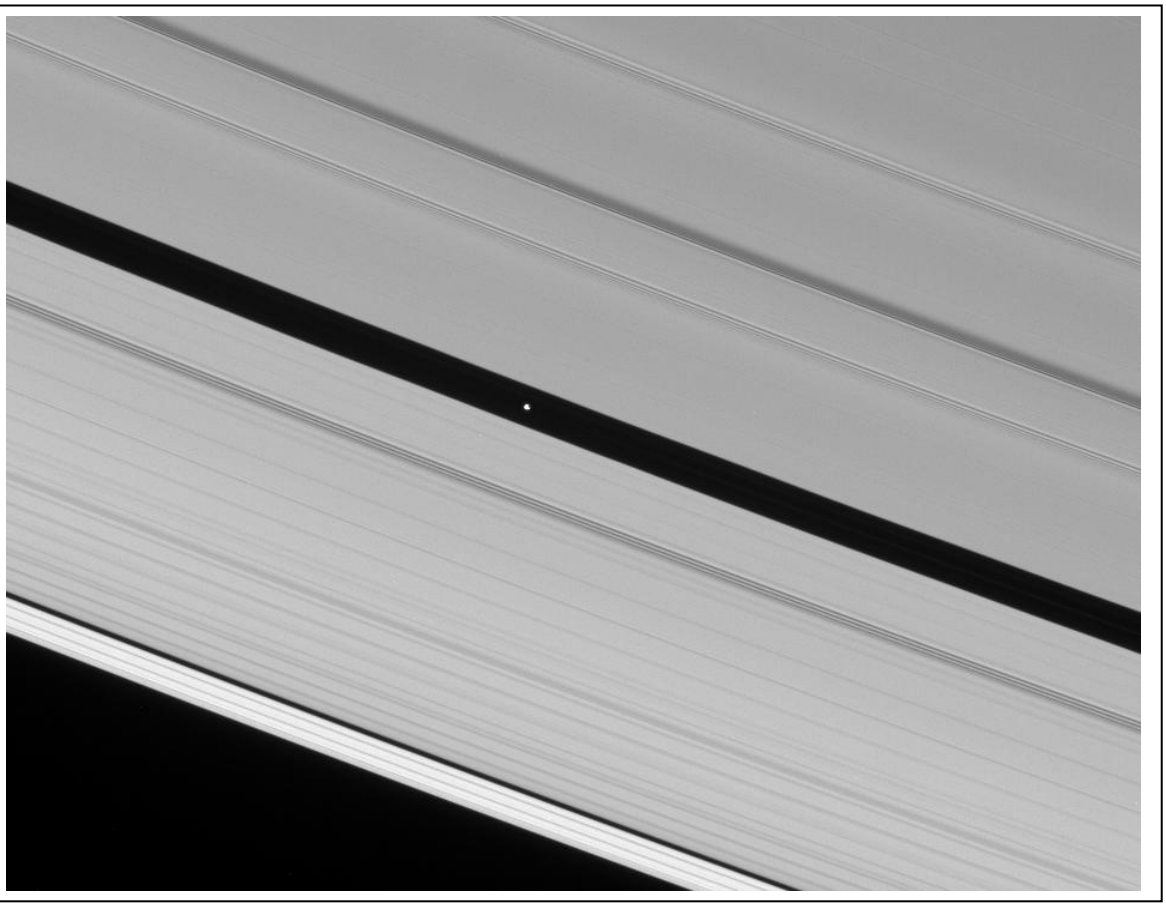
The location of the tidal radius for each planet is given in parenthesis on this scaled model. We see that for all of the planets, most of the ring material is located inside the Tidal Limit. In the case of Saturn, which seems to be the exception, there is also some ring material outside R = 2.2 Rs and includes the very sparse F and G rings. But even for Saturn, the majority of the visible rings (seen through a telescope) are inside the Tidal Limit.

The rings can be explained by moons that were tidally destroyed as they passed inside the Tidal Limit for each planet. These moons could not have been formed this close because the same tidal forces that destroy these moons would have prevented them from assembling, so the moons must have been formed outside the Tidal Limit and over millions of years their orbits carried them closer and closer to the Tidal Limit until they were finally destroyed.

Pan's Highway - Saturn's Rings



The Encke Gap is a prominent feature of Saturn's outer A-ring system that has been observed since the 1830's. The arrival of the Cassini spacecraft in July 2004 revealed the cause for this gap. A small moonlet called Pan clears out the ring debris in this region every 12 hours as it orbits Saturn!



Problem 1 - This image was taken by Cassini in 2007 and at the satellite's distance of 1 million kilometers, spans a field of view of 5,700 km x 4,400 km. With the help of a millimeter ruler, what is the scale of the image in kilometers per millimeter?

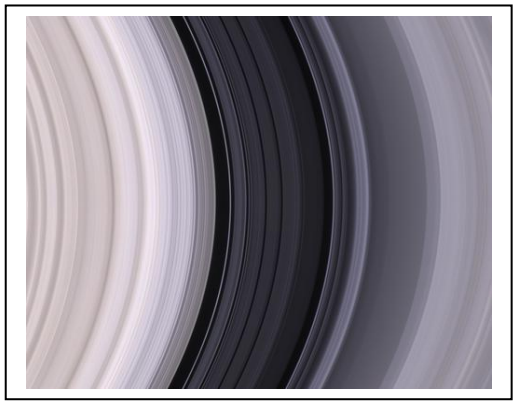
Problem 2 - Pan is that bright spot within the black zone of the Encke Gap. About how many kilometers in diameter is Pan?

Problem 3 - About how wide is the Encke Gap?

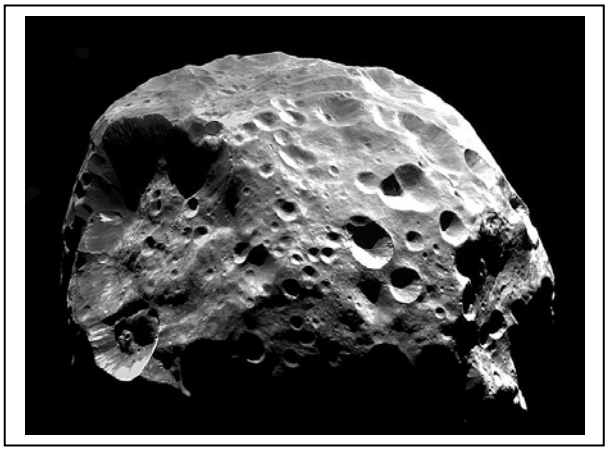
Problem 4 - About what is the smallest feature you can discern in the photo?

Problem 1 - The width of the picture is 150 millimeters, so the scale is 5,700 km/150 mm = **38 km/mm.**

Problem 2 - Pan is about 1.0 millimeters in diameter which is 38 km/mm x 1mm = **38 kilometers in diameter.**



Problem 3 - Students should measure a width of about 5.0 millimeters which is 38 km/mm x 5.0 mm = **190 kilometers**. The actual width of the Encke Gap is 325 km, but projection effects will foreshorten the gap as it appears in the photo. With the actual gap width (325 km) as the hypotenuse, and 190 km as the short side, the angle opposite the short side is the viewing angle of the camera relative to the ring plane. This angle can be found by constructing a scaled triangle and using a protractor to measure the angle, which will be about 36 degrees.

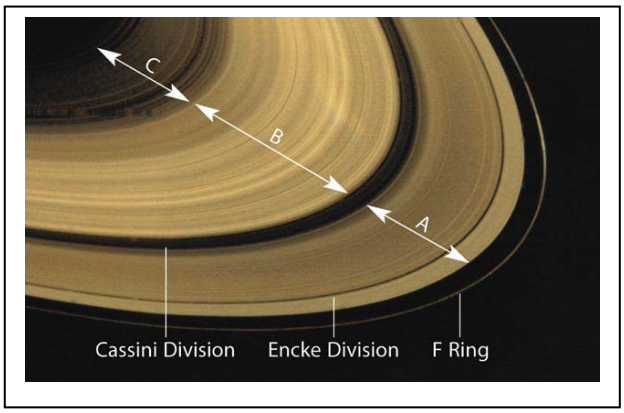


Problem 4 -

It is difficult to estimate lengths smaller than a millimeter. Students may consider using a photocopying machine to make a more convenient enlargement of the image, then measure the features more accurately. Small dark ring bands are about 0.1 mm wide, which is **about 4 km.**

NASA/Cassini mages, top to bottom:
Saturn Rings closeup showing Cassini Division and Encke Gap;
Rings closeup showing detail;
One of Saturn's outer satellites, Phoebe, is about 200 km across, and may have been a captured comet.

Orbit Speeds and Times for Saturn's Rings



The trillions of particles in Saturn's rings orbit the planet like individual satellites. Although the rings look like they are frozen in time, in fact, the rings orbit the planet at thousands of kilometers per hour! The speed of each ring particle is given by the formula:

$$V = \frac{29.4}{\sqrt{R}} \text{ km/s}$$

where R is the distance from the center of Saturn to the ring in multiples of the radius of Saturn (R = 1 corresponds to a distance of 60,300 km).

Problem 1 – The inner edge of the C Ring is located 7,000 km above the surface of Saturn, while the outer edge of the A Ring is located 140,300 km from the center of Saturn. How fast are the C Ring particles traveling around Saturn compared to the A Ring particles?

Problem 2 – The Cassini Division contains nearly no particles and is the most prominent 'gap' in the ring system easily seen from earth. It extends from 117,580 km to 122,170 km from the center of Saturn. What is the speed difference between the inner and outer edge of this gap?

Problem 3 – If the particles travel in circular orbit, what is the formula giving the orbit period for each ring particle in hours?

Problem 4 – What are the orbit times for particles near the inner and outer edge of the Cassini Division?

Problem 5 – The satellite Mimas orbits Saturn every 22.5 hours. How does this orbit period compare to the period of particles at the inner edge of the Cassini Division?

Problem 1 – The inner edge of the C Ring is located 7,000 km above the surface of Saturn, while the outer edge of the A Ring is located 140,300 km from the center of Saturn. How fast are the C Ring particles traveling around Saturn compared to the A Ring particles?

Answer: R = (60300 km + 7,000 km) / 60300 km = 1.12, so V = 23.6 km/sec
R = 140300 km/60300 km = 2.33, so V = 16.3 km/sec.

Note: The International Space Station orbits Earth at a speed of 7.7 km/s.

Problem 2 – The Cassini Division contains nearly no particles and is the most prominent 'gap' in the ring system easily seen from earth. It extends from 117,580 km to 122,170 km from the center of Saturn. What is the speed difference between the inner and outer edge of this gap?

Answer: R = 117580/60300 = 1.95 V = 17.83 km/s
R = 122170/60300 = 2.02 V = 17.52 km/s
The outer edge particles travel about 17.83 – 17.52 = 0.31 km/sec slower than the inner edge particles.

Problem 3 – If the particles travel in circular orbit, what is the formula giving the orbit period for each ring particle in hours?

Answer: Orbit circumference = 2 π r km, but r = 60300 R so C = 2 (3.141) x 60300 R , C = 379,000 R km, wnhere R is in Saturn radius units. Since the orbit speed is $V = 24.9/R^{1/2}$, then Time = C/V = 15220 $R^{3/2}$ seconds. Since 1 hour = 3600 seconds, we have **$T = 4.22\ R^{3/2}$ hours**.

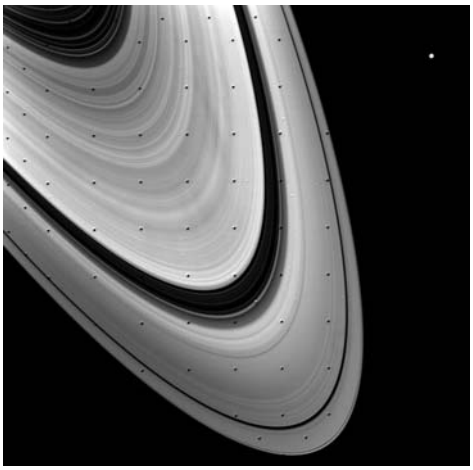
Problem 4 – What are the orbit times for particles near the inner and outer edge of the Cassini Division?

Answer: R = 1.95 so T = 11.49 hours.
R = 2.02 so T = 12.11 hours.

Problem 5 – The satellite Mimas orbits Saturn every 22.5 hours. How does this orbit period compare to the period of particles at the inner edge of the Cassini Division?

Answer: 22.5/11.49 = 1.94 which is nearly 2.0. This means that every time Mimas orbits once, the particles in the Cassini Division orbit about twice around Saturn. This is an example of an orbit resonance. Because the ring particles encounter a push from Mimas's gravitational field at the same location every two orbits, they will be ejected. This is an explanation for why the Cassini Division has so few ring particles.

How Saturn's Moon Mimas created the Cassini Division



The Cassini Division is easily seen from Earth with a small telescope, and splits the rings of Saturn into two major groups. A little detective work shows that there may be a good reason for this gap that involves Saturn's nearby moon, Mimas.

Mimas orbits Saturn once every 22 hours, and would-be particles in the Cassini Division would orbit once every 11-12 hours, so that the ratio of the orbit periods is close to 2 to 1. This creates a resonance condition where the gravity of Mimas perturbs the Cassini particles and eventually ejects them.

Imagine a pendulum swinging. If you lightly tap the pendulum when it reaches the top of its swing, and do this every other swing, eventually the small taps add up to increasing the height of the pendulum.

Problem 1 – The mass of Mimas is 4.0×10^{19} kilograms, and the distance to the center of the Cassini Division from Mimas is 67,000 kilometers. Use Newton's Law of Gravity to calculate the acceleration of a Cassini Division particle due to the gravity of Mimas if

$$\text{Acceleration} = G \frac{M}{R^2} \text{ in meters/sec}^2$$

where $G = 6.67 \times 10^{-11}$, M is the mass of Mimas in kilograms and R is the distance in meters.

Problem 2 – The encounter time with Mimas is about 2 hours every orbit for the Cassini particles. If speed = acceleration x time, what is the speed increase of the particles after each 12-hour orbit?

Problem 3 – If a particle is ejected from the Cassini Division once its speed reaches 1 km/sec, how many years will it take for this to happen?

Problem 1 – The mass of Mimas is 4.0×10^{19} kilograms, and the distance to the center of the Cassini Division is 67,000 kilometers. Use Newton's Law of Gravity to calculate the acceleration of a Cassini Division particle due to the gravity of Mimas if

$$\text{Acceleration} = G \frac{M}{R^2} \text{ in meters/sec}^2$$

where $G = 6.67 \times 10^{-11}$, M is the mass of Mimas in kilograms and R is the distance in meters.

Answer: Acceleration = 6.67×10^{-11} $(4.0 \times 10^{19}) / (6.7 \times 10^{7}$ meters$)^2$
= **5.9×10^{-7} meters/sec^2**

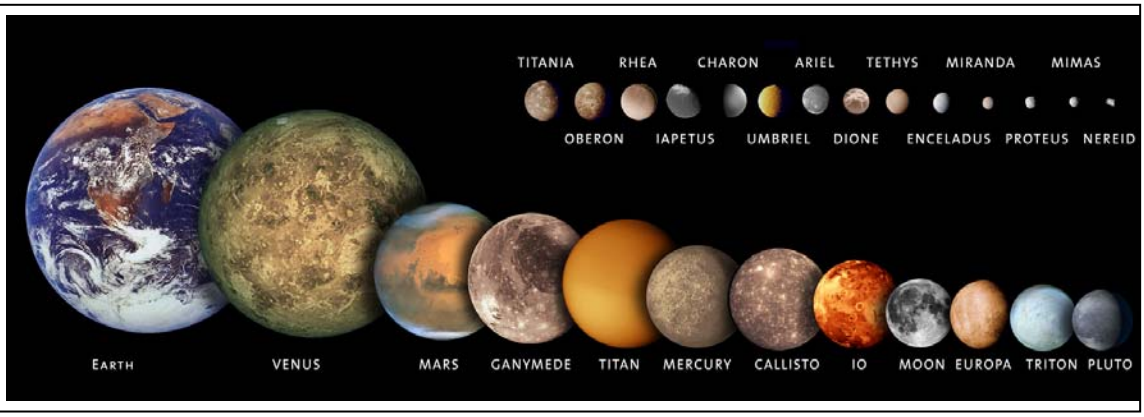
Problem 2 – The encounter time with Mimas is about 2 hours every orbit for the Cassini particles. If speed = acceleration x time, what is the speed increase of the particles after each 12-hour orbit?

Answer: 1 hour = 3600 seconds, so 2 hours = 7200 seconds and
speed = 5.9×10^{-7} m/sec^2 x 7200 sec
= 4.2×10^{-3} meters/sec per orbit.

Problem 3 – If a particle is ejected from the Cassini Division once its speed reaches 1 km/sec, how many years will it take for this to happen?

Answer: (1000 m/s)/(0.0042 m/s) = 238000 orbits. Since 1 orbit = 12 hours, we have 12 x 238,000 = 2,856,000 hours or about **326 years**.

Big Moons and Small Planets! 5.8



This diagram shows the Top-26 moons and small planets in our solar system, and drawn to the same scale.

Problem 1 – What fraction of the objects are smaller than our moon?

Problem 2 – What fraction of the objects are larger than our moon but are not planets?

Problem 3 – What fraction of the objects, including the moon, are about the same size as our moon?

Problem 4 – If Saturn's moon Titan is ½ the diameter of Earth, and Saturn's moon Dione is 1/6 the diameter of Titan, how large is the diameter of Dione compared to Earth?

Problem 5 – Oberon is 1/7 the diameter of Earth, Io is 1/3 the diameter of Earth, and Titania is 4/9 the diameter of Io. Which moon is bigger in diameter: Oberon or Titania?

Problem 1 – What fraction of the objects are smaller than our moon?
Answer: **17/26**

Problem 2 – What fraction of the objects are larger than our moon but are not planets?

Answer: Io, Callisto, Titan and Ganymede : 4/26 or **2/13**

Problem 3 – What fraction of the objects, including the moon, are about the same size as our moon?

Answer: Moon, Europa, Triton and Pluto so 4/26 = **2/13**.

Problem 4 – Saturn's moon Titan is ½ the diameter of Earth, and Saturn's moon Dione is 1/6 the diameter of Titan, how large is the diameter of Dione compared to Earth?

Answer: ½ x 1/6 = **1/12 the size of Earth**.

Problem 5 – Oberon is 1/7 the diameter of Earth, Io is 1/3 the diameter of Earth, and Titania is 4/9 the diameter of Io. Which moon is bigger in diameter: Oberon or Titania?

Answer: Oberon is 1/7 the diameter of Earth.
Titania is 4/9 the diameter of Io, and Io is 1/3 the diameter of Earth
So Titania is (4/9) x (1/3) the diameter of Earth
So Titania is 4/27 the diameter of Earth.

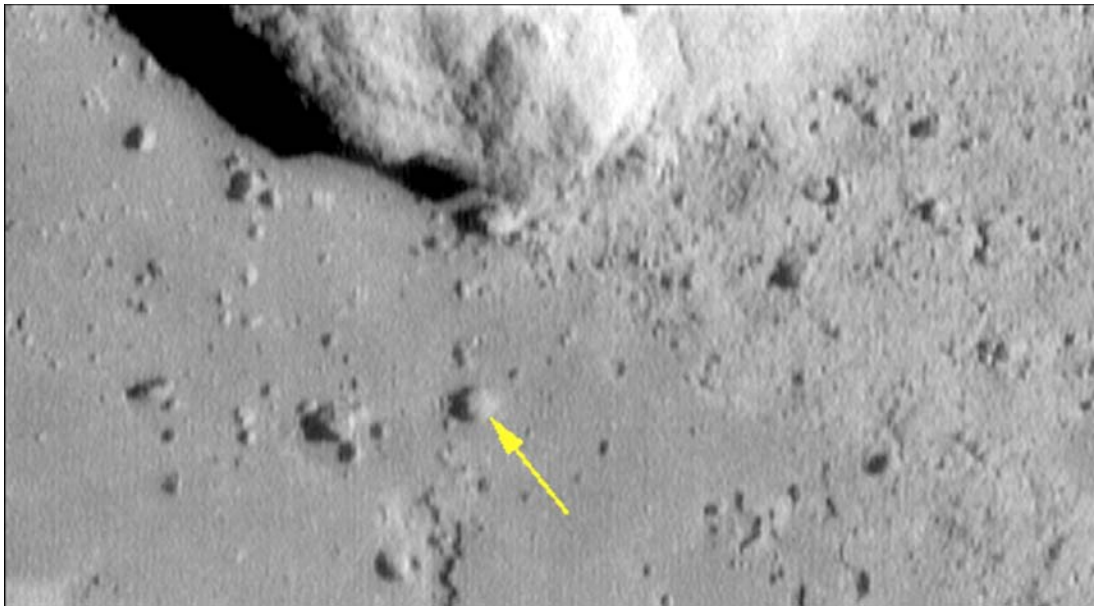
Comparing Oberon, which is 1/7 the diameter of Earth with Titania, which is 4/27 the diameter of Earth, which fraction is larger: 1/7 or 4/27?

Find the common denominator 7 x 27 = 189, then cross-multiply the fractions:

Oberon: 1/7 = 27/189 and Titania: 4/27 = (4x7)/189 = 28/189 so

Titania is 28/189 Earth's diameter and Oberon is 27/189 Earth's diameter, and so **Titania is slightly larger!**

Asteroid Eros



This NASA, NEAR image of the surface of the asteroid Eros was taken on February 12, 2001 from an altitude of 120 meters (Credit: Dr. Joseph Veverka/ NEAR Imaging Team/Cornell University). The image is 6 meters wide.

The scale of an image is found by measuring with a ruler the distance between two points on the image whose separation in physical units you know. In this case, we are told the image width is 6.0 meters.

Step 1: Measure the width of the image with a metric ruler. How many millimeters long is the image?

Step 2: Use clues in the image description to determine a physical distance or length.

Step 3: Divide your answer to Step 2 by your answer to Step 1 to get the image scale in centimeters per millimeter to two significant figures.

Once you know the image scale, you can measure the size of any feature in the image in units of millimeters. Then multiply it by the image scale from Step 3 to get the actual size of the feature in centimeters to two significant figures.

Question 1: What are the dimensions, in meters, of this image?

Question 2: What is the width, in centimeters, of the largest feature?

Question 3: What is the size of the smallest feature you can see?

Question 4: How big is the stone shown by the arrow?

Answer Key:

This NASA, NEAR image of the surface of the asteroid Eros was taken on February 12, 2001 from an altitude of 120 meters (Credit: Dr. Joseph Veverka/ NEAR Imaging Team/Cornell University)). The image is 6 meters wide.

The scale of an image is found by measuring with a ruler the distance between two points on the image whose separation in physical units you know. In this case, we are told the image width is 6 meters.

Step 1: Measure the width of the image with a metric ruler. How many millimeters long is the image?
Answer: 144 millimeters

Step 2: Use clues in the image description to determine a physical distance or length.
Answer: 6.0 meters

Step 3: Divide your answer to Step 2 by your answer to Step 1 to get the image scale in centimeters per millimeter.
Answer: 6.0 meters / 144 mm = 600 cm / 144 millimeters = 4.2 cm/mm

Once you know the image scale, you can measure the size of any feature in the image in units of millimeters. Then multiply it by the image scale from Step 3 to get the actual size of the feature in centimeters.

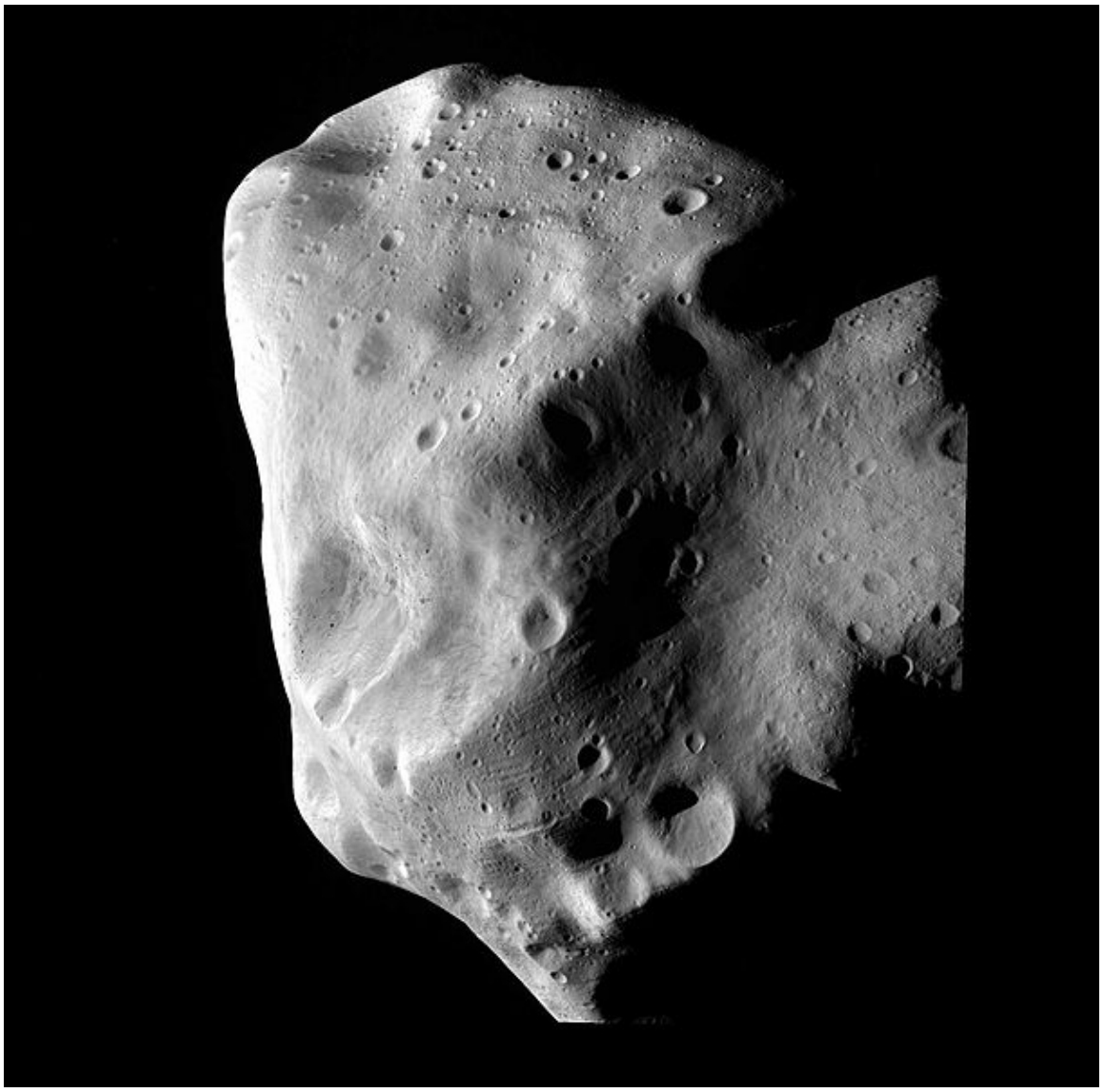
Question 1: What are the dimensions, in meters, of this image?
Answer: Height = 80 mm = 336 cm or 3.4 meters so area is 6.0 m x 3.4 m

Question 2: What is the width, in centimeters, of the largest feature?
Answer: The big rock at the top of the image is about 60 mm across or 2.5 meters.

Question 3: What is the size of the smallest feature you can see?
Answer: The small pebbles are about 0.5 millimeters across or 2.1 centimeters (about 1 inch).

Question 4: How big is the stone shown by the arrow?
Answer: 4 millimeters or 17 centimeters (about 7 inches).

A Flyby of Asteroid Lutetia by Rosetta



Problem 1 - What is the scale of this image in meters/mm if the width of the image is 130 kilometers?

Problem 2 - At a distance of 3,162 km, A) what was the angular diameter of this asteroid? B) Compared to the full moon viewed from Earth (0.5 degrees), how much larger was Lutetia?

Problem 3 - If the Rosetta spacecraft traveled at a speed of 15 km/s on a path exactly tangent to the line connecting the center of the asteroid and spacecraft at closest approach, how long after closest approach would the asteroid have an angular diameter equal to the full moon?

Image credit: ESA 2010 MPS for OSIRIS Team
MPS/UPD/LAM/IAA/RSSD/INTA/UPM/DASP/IDA. European Space Agency's Rosetta spacecraft, with NASA instruments aboard, flew past asteroid Lutetia on Saturday, July 10, 2010. Asteroid diameter about 130 km. This view is from a distance of 3,162 Km. The probe spent several hours shooting images of the irregular shaped space rock, circling more than 450 million km (280 million miles) out from the sun. The space agency says its OSIRIS camera was able to capture detail down to just a few dozen meters.

Problem 1 - What is the scale of this image in meters/mm if the width of the image is 130 kilometers?

Answer: 130 km / 152 mm = **855 meters/mm**.

Problem 2 - At a distance of 3,162 km, A) what was the angular diameter of this asteroid? B) Compared to the full moon viewed from Earth (0.5 degrees), how much larger was Lutetia?

Answer: A) $\text{Tan}(\theta) = 130/3162$ so θ = **2.3 degrees**.
B) Asteroid is 2.3/0.5 = **4.6 times** the diameter of the full moon.

Problem 3 - If the spacecraft traveled at a speed of 15 km/s on a path exactly tangent to the line connecting the center of the asteroid and spacecraft at closest approach, how long after closest approach would the asteroid have an angular diameter equal to the full moon?

Answer: To have an angular diameter of 0.5 degrees ,the spacecraft has to be 4.6 times farther away than at closest approach, or a distance of 4.6 x 3,162 = 14,545 km. From the Pythagorean theorem, the distance from the closest approach point is just $d = (14545^2 - 3162^2)^{1/2}$ = 14,197 km. To travel this distance takes T = 14197/15 = **946 seconds or 15.8 minutes.**

Dawn Spacecraft sees Asteroid Vesta Up Close!

On July 15, 2011 the NASA spacecraft Dawn completed a 2.8 billion kilometer journey taking four years, and went into orbit around the asteroid Vesta. Vesta is the second largest asteroid in the Asteroid Belt. Its diameter is 530 kilometers. After one year in orbit, Dawn departed in 2012 for an encounter with asteroid Ceres in 2015. Meanwhile, from its orbit around Vesta, it will map the surface and see features less than 1 kilometer across.

Problem 1 - Use a millimeter ruler and the diameter information for this asteroid to determine the scale of this image in kilometers per millimeter.

Problem 2 - What is the diameter of the largest and smallest features that you can see in this image?

Problem 3 - Based on the distance traveled, and the time taken by the Dawn satellite, what was the speed of this spacecraft in A) kilometers per year? B) kilometers per hour?

Problem 4 - The Space Shuttle traveled at a speed of 28,000 km/hr in its orbit around Earth. How many times faster than the Shuttle does the Dawn spacecraft travel?

Problem 1 - Use a millimeter ruler and the diameter information for this asteroid to determine the scale of this image in kilometers per millimeter.

Answer: When printed using a standard printer, the width of the asteroid is about 123 millimeters. Since the true diameter of the asteroid is 530 km, the scale is then S = 530 km / 123 mm = **4.3 kilometers per millimeter**.

Problem 2 - What is the diameter of the largest and smallest features that you can see in this image?

Answer: **Students can find a number of small features in the image that are about 1 mm across, so that is about 4.3 kilometers. Among the largest features are the 9 large craters located along the middle region of Vesta from left to right. Their diameters are about 4 to 7 millimeters or 17 to 30 kilometers across. The large depression located in the upper left quadrant of the image is about 40 mm long and 15 mm wide in projection, which is equivalent to 172 km x 65 km in size.**

Problem 3 - Based on the distance traveled, and the time taken by the Dawn satellite, what was the speed of this spacecraft in A) kilometers per year? B) kilometers per hour?

Answer: Time = 4 years, distance = 2.8 billion km, so the speed is A) S = 2.8 billion km / 4 years = **700 million km/year**. B) Converting to an hourly rate, S = 700 million km/yr x (1 year/365 days) x (1 day/24 hours) = **79,900 km/hour**.

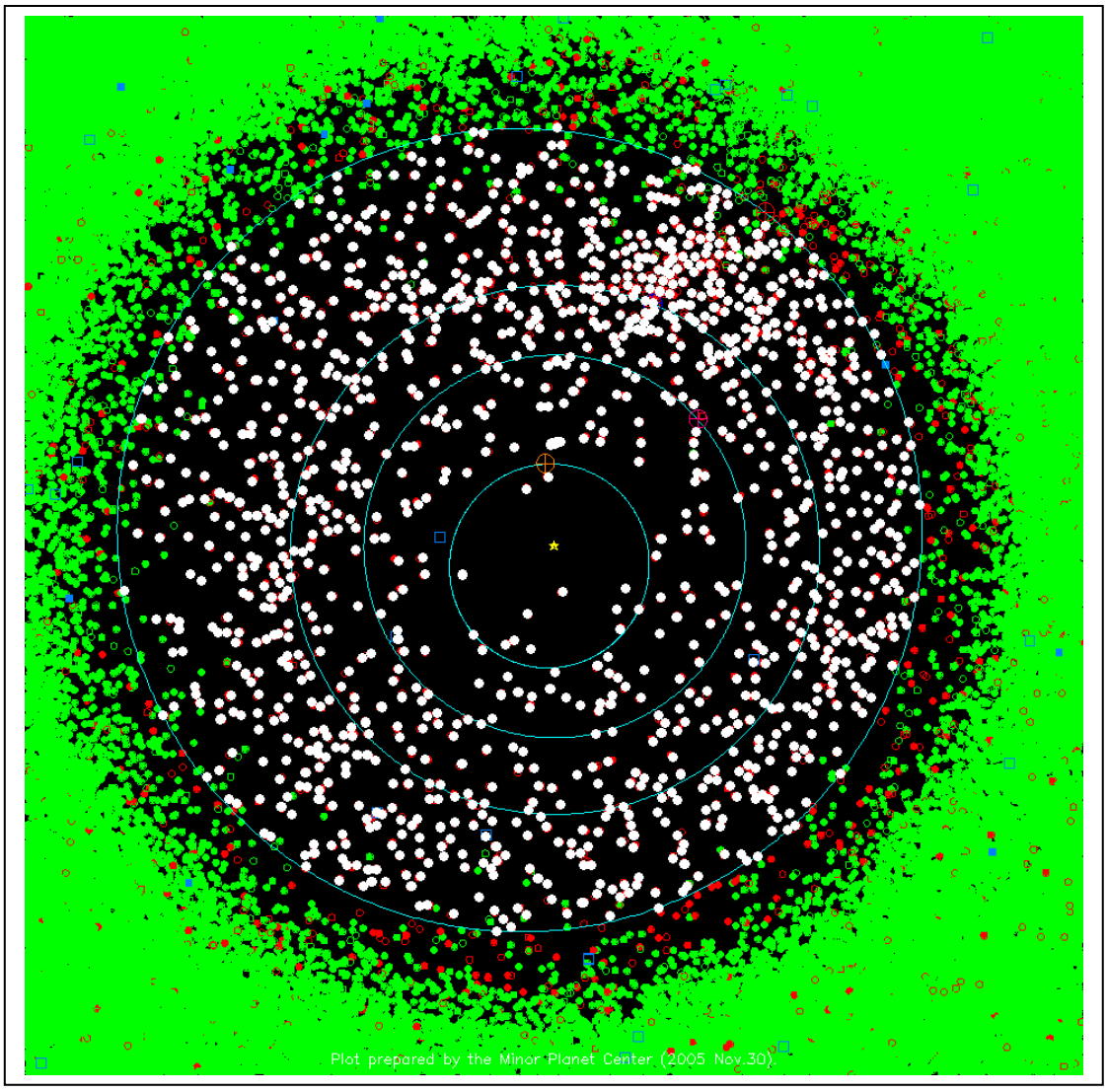
Problem 4 - The Space Shuttle traveled at a speed of 28,000 km/hr in its orbit around Earth. How many times faster than the Shuttle does the Dawn spacecraft travel?

Answer: Ratio = (79,900 km/hr) / (28,000 km/hr) = 2.9 . So Dawn is traveling at an average speed that is **2.9 times faster than the Space Shuttle!**

Note: The Space Shuttle is traveling 8 times faster that a bullet (muzzle velocity) from a high-powered M16 rifle! So Dawn is traveling 23 times faster than such a bullet!

Asteroids Between Mars and the Sun

Astronomers have catalogued and determined orbits for 30,000 minor planets in the solar system (asteroids, comets etc). Over 150,000 bodies larger than a few hundred meters across have been spotted and remain to have their orbits exactly determined. Below is a plot made on November 30, 2005 of the locations of all known objects (white dots) within the orbit of Mars whose path skirts the inner edge of the asteroid belt (green dots).



Question 1 – How many minor planets are located inside the orbit of Mercury? Venus? Earth? Mars?

Question 2 – If the radius of Earth's orbit is 150 million km, what is the scale of this figure in millions of km per millimeter?

Question 3 – About how far apart are the minor planets from each other on this particular day? Would they be a hazard for space travel?

Question 4 – How many asteroids crossed Earth's orbit on November 30, 2005?

The plot of the minor planets was obtained from the IAU, Minor Planets Center (http://cfa-www.harvard.edu/iau/lists/InnerPlot2.html). It shows the location of the known asteroids, comets and other 'minor planets' for November 30, 2005. The plot shows the orbits of Mercury, Venus, Earth and Mars. Objects that have parahelias (closest orbit location to the sun) less than 1.3 AU are shown in white circles. More details can be found at http://www.space.com/scienceastronomy/solarsystem/asteroid_toomany_011019-1.html

Question 1 – How many minor planets are located inside the orbit of Mercury? Venus? Earth? Mars? **Answer:** Students should count the plotted symbols within (or on) the first inner ring (Mercury's orbit) and get **13** symbols (don't include the sun!). For the space between Venus and Mercury, I count 119 spots which makes the total **132** minor planets inside the orbit of Venus. Between Earth and Venus there are about 280 for a total of 412 minor planets inside Earth's orbit. Between Mars and Earth, a careful student may be able to count about 833 which means there are 833+412 = **1245** minor planets inside the orbit of Mars.

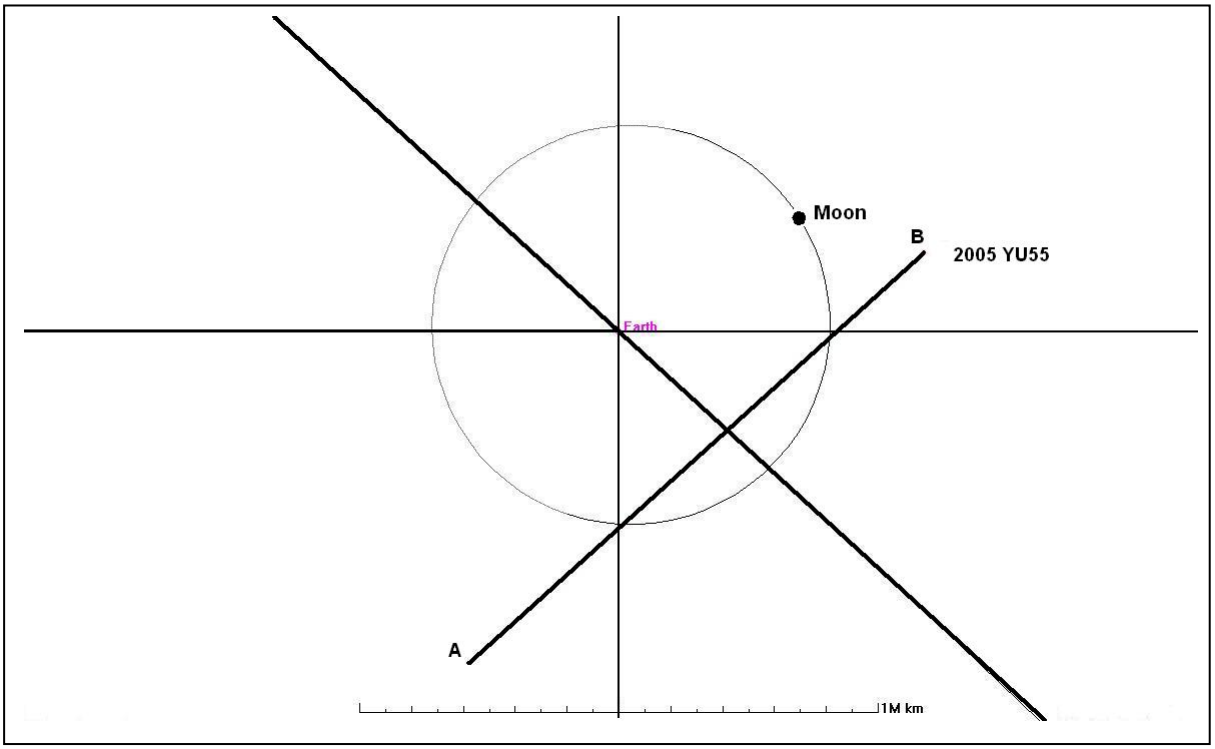
Question 2 – What is the scale of this figure in millions of km per millimeter? **Answer:** The radius of Earth's orbit is 150 million kilometers, which corresponds to 70 millimeters, so the scale is 2.1 million km per millimeter.

Question 3 – About how far apart are the minor planets from each other on this particular day? Would they be a hazard for space travel? **Answer:** Although the asteroids are only plotted as though they are located in the same 2-D plane, we can estimate from the average 'eyeball' separation between asteroids of about 2 millimeters, that they are about 4.2 million kilometers apart. A spacecraft would not collide with a typical asteroid unless it was directed to specifically target an asteroid for investigation…or impact. It is a popular myth about space travel that astronauts have to dodge asteroids when traveling to Mars or the outer solar system. Interplanetary dust grains and micro-meteoroids are, however, a much bigger hazard!!

Question 4 – How many asteroids crossed Earth's orbit on November 30, 2005? **Answer:** Just count the number of white spots that touch the line that represents the orbit of Earth. There are about 70 spots that touch the orbit line.

Could the Earth collide with them? Each dot is about 1 mm in radius, so this represents a distance of 2 million kilometers. Since Earth is only 12,000 km across and a typical asteroid is only 1 km across, collision is extremely unlikely even when the diagram seems to show otherwise. There is another way that this diagram makes the situation look worse than it is. Because the asteroid orbits can be several million miles above or below the orbit of Earth as the asteroids cross this location, there are very few close calls between Earth and any given asteroid in the current catalog. Astronomers call the ones that get close 'Near-Earth Asteroids' and there are about 700 of these known. Only **one** of these known NEAs may get close enough to Earth in the next 30 years to be a potential collision problem, but astronomers are still finding dozens more NEAs every year.

The Closest Approach of Asteroid 2005 YU55 - II



The Asteroid 2005 YU55 passed inside the orbit of our moon sometime between November 8 and November 9, 2011. The diagram shows the lunar orbit as a circle centered on Earth. The diagonal line is the orbit of Earth around the sun. The line segment AB is a portion of the orbit of the asteroid. The horizontal line at the bottom of the page is 1 million kilometers long at the scale of the figure. Point A is the location of the asteroid on November 8.438. Point B is its location one day later on November 9.438, where we have used digital days to indicate a precise hour and minute within each endpoint date in terms of Universal Time.

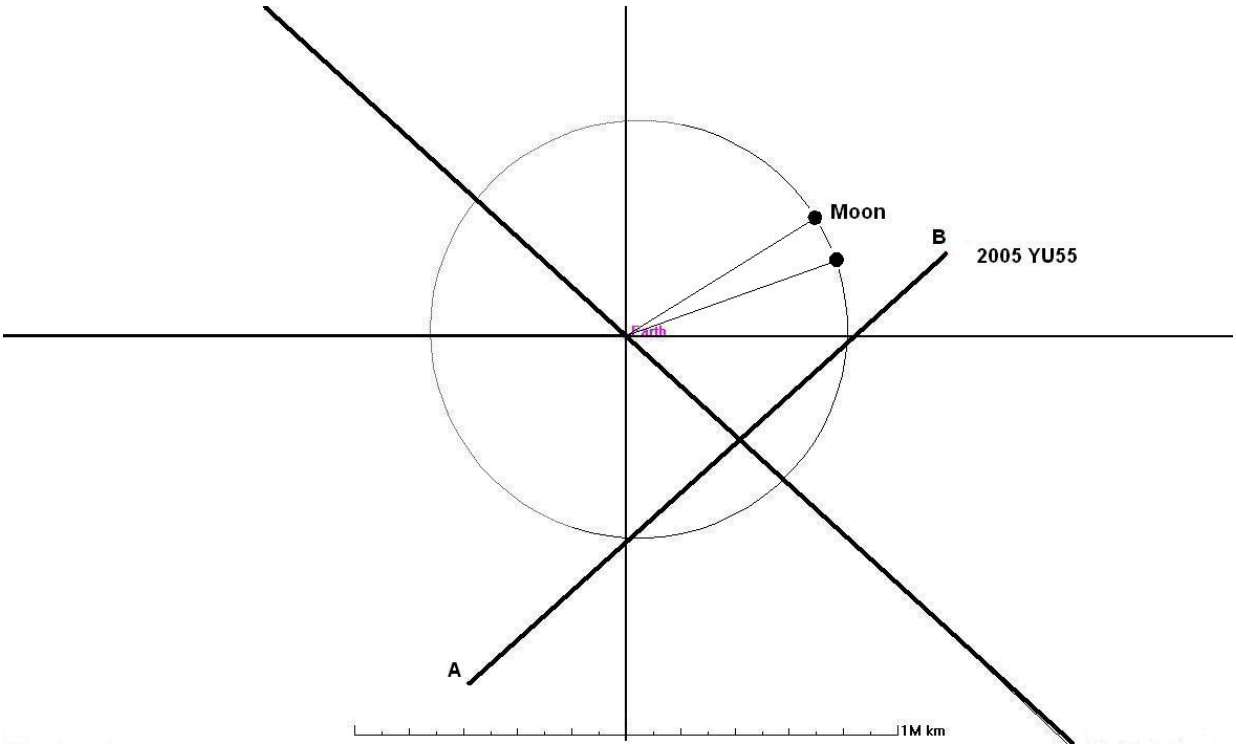
Problem 1 - The figure above shows the location of the Moon on November 9 at 10:13 Universal Time (9.438 days) in its counter-clockwise journey around earth in a circular orbit. The period of the orbit is 27.3 days. What was the date and time when the asteroid was at Point A?

The line segment AB is a portion of the orbit of the asteroid. The horizontal line at the bottom of the page is 1 million kilometers long at the scale of the figure. Point A is the location of the asteroid on November 8.438. Point B is its location one day later on November 9.438, where we have used digital days to indicate a precise hour and minute within each endpoint date in terms of Universal Time.

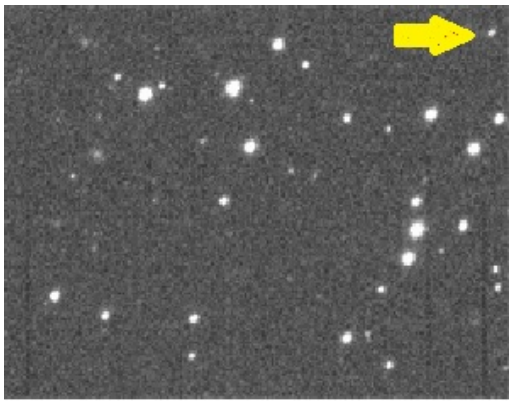
Problem 1 - The figure above shows the location of the Moon on November 9 at 10:13 Universal Time (9.438days) in its counter-clockwise journey around earth in a circular orbit. The period of the orbit is 27.3 days. What was the date and time when the asteroid was at Point A?

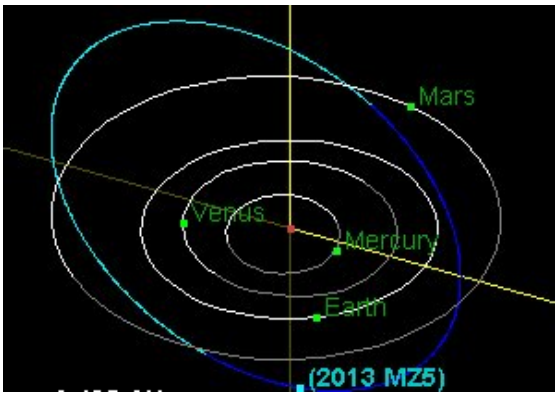
Answer: The time between Point A and Point B is exactly 1.0 days, so we need to determine the position of the moon 1 day earlier that its location in the diagram.

The moon travels 360 degrees in 27.3 days for a speed of 13.2 degrees per day, so we need to find the position of the moon 13.2 degrees clockwise of its position in the diagram. Using a protractor, the figure below shows this position.



The 10,000 Near Earth Asteroid 2013 MZ5





It was only 300 meters in diameter, but on June 18, 2013 it was discovered by the Pan-STARRS-1 survey telescope in Hawaii (top image), making it the 10,000th Near Earth Object (NEO).

NASA estimates that there are 15,000 asteroids that are 140-meters or larger with orbits close to Earth. We have now discovered 30% of them according to statistical estimates based on the rate of discovery of these NEOs. Objects of this size or larger are considered severe hazards should they impact near cities. The February 2013 'Russian' meteor was 17 meters in diameter, yet it injured over 3000 people, though it left no crater. A 140-meter asteroid impact would produce a 25-meter crater and the atmospheric shock wave would damage thousands of buildings.

The orbit is shown in the left. The figure to the left. The orbit is tilted nearly 45° to Earth's orbit plane.

The distance from Earth to 2013 MZ5 is given in the table below, every 10 days, and in units of millions of kilometers.

Date	Distance	Date	Distance	Date	Distance
May 22	115	July 1	71	August 9	82
June 1	100	July 10	69	August 19	89
June 11	88	July 20	71	August 29	97
June 21	77	July 30	76	September 8	104

Problem 1 – Graph the tabulated distance function and connect the points with a smooth curve.

Problem 2 – On what date is the asteroid closest to Earth?

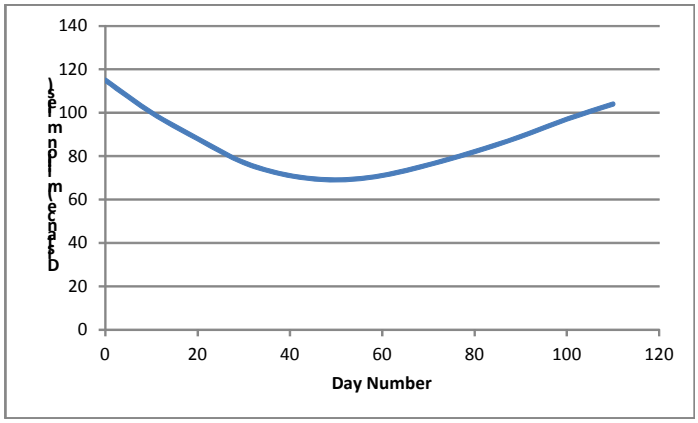
Problem 3 – The approximate formula for the approach speed of the asteroid to Earth in km/hr is given by S = 1100 T – 62000, where T is the elapsed days from May 22. How fast was it approaching Earth on June 18, the day of its discovery?

Orbit - http://ssd.jpl.nasa.gov/sbdb.cgi?sstr=2013%20MZ5;orb=1;cov=0;log=0;cad=0#orb

Data - http://ssd.jpl.nasa.gov/sbdb.cgi?sstr=2013%20MZ5

http://www.nasa.gov/centers/jpl/news/neo20130624.html
NASA - Ten Thousandth Near-Earth Object Unearthed in Space
June 24, 2013.

Problem 1 – Graph the tabulated distance function and connect the points with a smooth curve. Answer: **See graph below**.



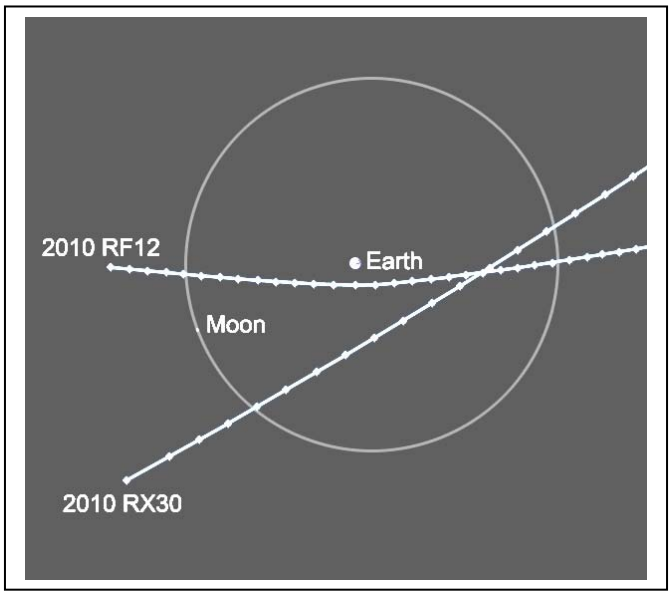
Problem 2 – On what date is the asteroid closest to Earth?

Answer: The table shows that on **July 10 (Elapsed day 50)** it reached its minimum distance of 69 million kilometers.

Problem 3 – The approximate formula for the approach speed of the asteroid to Earth in km/hr is given by S = 1100 T – 62000, where T is the elapsed days from May 22. How fast was it approaching Earth on June 18, the day of its discovery?

Answer: June 18 is elapsed day 27, so T = 27 and so S = 1100(27)-6200 = **-23,500 km/hr**. The negative sign means that the distance to Earth was decreasing.

Close Encounters of the Asteroid Kind!



The Catalina Sky Survey near Tucson, Ariz., discovered two small asteroids on the morning of Sunday, September 5, 2010 during a routine monitoring of the skies.

Asteroid 2010RX30 is about 15 meters in diameter and will pass within 248,000 kilometers of Earth. Asteroid 2010RF12, about 10 meters in diameter, will pass within 79,000 kilometers of Earth.

Both asteroids should be observable near closest approach to Earth with moderate-sized amateur telescopes.

Neither of these asteroids has a chance of hitting Earth. A 10-meter- sized near-Earth asteroid from the undiscovered population of about 50 million would be expected to pass almost daily within a lunar distance, and one might strike Earth's atmosphere about every 10 years on average. The last asteroid that was observed to enter Earth's atmosphere in this size range was the *Great Daylight Fireball of 1972* which streaked above the Grand Tetons. It was about 5 meters in diameter but skipped out of the atmosphere and never struck ground.

Small asteroids appear very faint in the sky, not only because they are small in size, but because their surfaces are very dark and reflect very little sunlight. The formula for the brightness of a typical asteroid that is spotted within a few million kilometers of Earth is given by:

$$R = 0.011\, d\, 10^{-\frac{1}{5}(m)}$$

where: R is the asteroid radius in meters, d is the distance to the asteroid from Earth in kilometers, and m is the apparent brightness of the asteroid viewed from Earth. Note, the faintest star you can see with the naked eye is about m = +6.5. The planet Venus when it is brightest in the evening sky has a magnitude of m = -2.5. The asteroid is assumed to have a reflectivity similar to lunar rock.

Problem 1 - What does the formula estimate as the brightness of these two asteroids when they are closest to Earth on September 8, 2010?

Problem 2 - Astronomers are anxious to catalog all asteroids that can potentially impact Earth and cause damage to cities. Suppose that at the typical speed of an asteroid (10 km/sec) it will take about 24 hours for it to travel 1 million kilometers (3 times lunar orbit distance). What is the astronomical brightness range for asteroids with diameters between 1 meter and 500 meters?

Problem 1 - What does the formula estimate as the brightness of these two asteroids when they are closest to Earth on September 8, 2010?
Answer:

2010RX30, R=15 meters, d = 248,000 kilometers

$15 = 0.011\ (248{,}000)10^{-.2m}$ then $0.0055 = 10^{-.2(m)}$

Log(0.0055) = -0.2m so **m = +11.3 magnitudes**

2010RF12, R=10 meters, d= 79,000 kilometers

$10 = 0.011\ (79{,}000)10^{-.2m}$ then $0.011 = 10^{-.2(m)}$

Log(0.011) = -0.2m so **m = +9.7 magnitudes**

Problem 2 - Astronomers are anxious to catalog all asteroids that can potentially impact Earth and cause damage to cities. Suppose that at the typical speed of an asteroid (10 km/sec) it will take about 24 hours for it to travel 1 million kilometers (3 times lunar orbit distance). What is the astronomical brightness range for asteroids with diameters between 1 meter and 500 meters?

Answer: First evaluate the equation for d = 1 million km and solve for m(R)

$R = 0.011\ (1.0 \times 10^{6})\ 10^{-.2m}$
$R = 1.1 \times 10^{4}\ 10^{-.2m}$

so m(R) = - 5 log10(0.000091R)

For R = 1 to 500 meters, **m = +20.2 to +6.7**

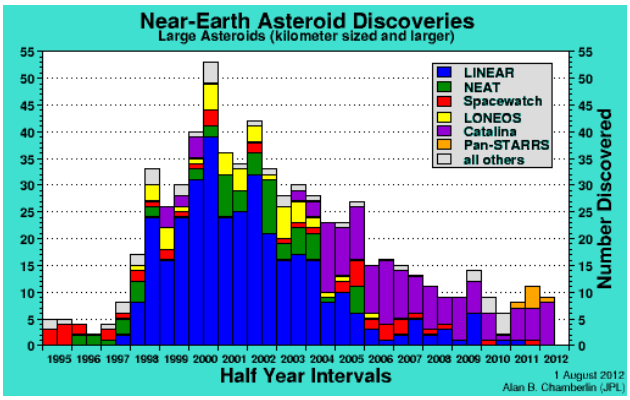
The most common asteroids have sizes between 1 meter and 50 meters, so the detection of such small, faint, and rapidly moving asteroids with ground based telescopes is a major challenge and may be a matter of luck in most cases.

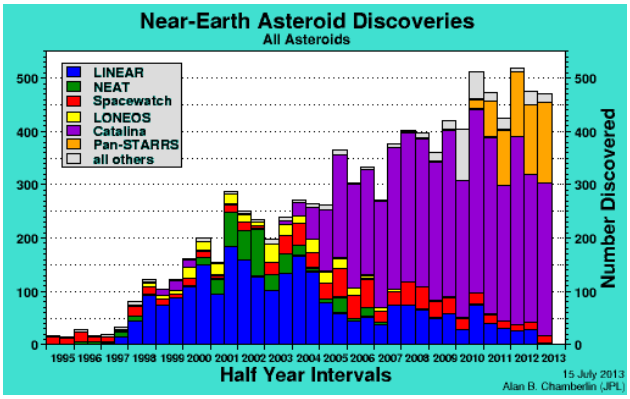
For more information, read the NASA press release at

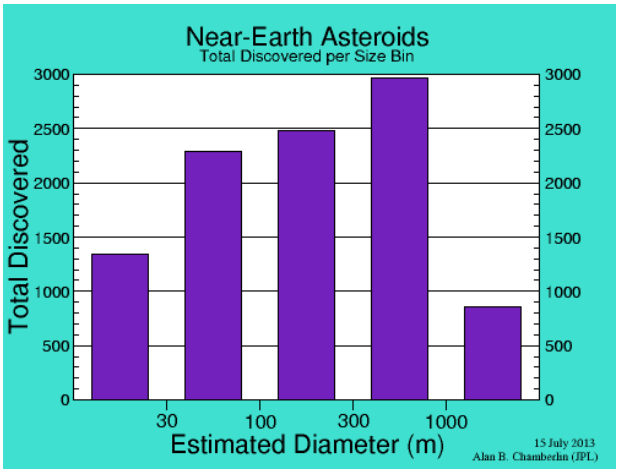
"Two Asteroids to Pass By Earth Wednesday"

http://www.nasa.gov/topics/solarsystem/features/asteroid20100907.html

How Quickly are NEOs Being Discovered?







Near Earth Objects (NEOs) are asteroids that have orbits very close to Earth's orbit in the solar system. That means that, over time, they might collide with Earth. The most devastating asteroids are larger than 1 km and can cause world-wide extinction events. Smaller bodies from 10 to 300-meters can damage cities but are too small to affect continent-sized areas.

Astronomers have, for decades detected and tracked these smaller bodies as they come near Earth, and from this determine their orbits. The graphs to the left show the progress of these searches since 1995.

Problem 1 – The top graph shows the number of NEOs detected each year. Asteroids that are a kilometer or more in size can cause extinction events. How many of these were discovered in 2012?

Problem 2 – What is the total number of asteroids discovered in 2012?

Problem 3 – What percentage of the asteroids discovered in 2012 were A) larger than 1 kilometer? B)Smaller than 1 kilometer?

Problem 4 – From the bottom figure of total discovered asteroids, what percentage are a) smaller than 1 kilometer? B) Larger than 1 kilometer?

Problem 5 – Compare the top two figures. What can you conclude about the number of 1 kilometer or larger asteroids that are yet to be discovered, compared to those smaller than 1 kilometer?

Problem 1 – The top graph shows the number of NEOs detected each year. Asteroids that are a kilometer or more in size can cause extinction events. How many of these were discovered in 2012?

Answer: The last bar in the graph shows that **9** were discovered in 2012.

Problem 2 – What is the total number of asteroids discovered in 2012?

Answer: The second graph shows that for 2012 the column indicates about **470** were discovered that year.

Problem 3 – What percentage of the asteroids discovered in 2012 were A) larger than 1 kilometer? B) Smaller than 1 kilometer?

Answer: A) P = 100% x (9/470) = **1.9%.**
B) P = 100% x (461/470) = **98.1%**

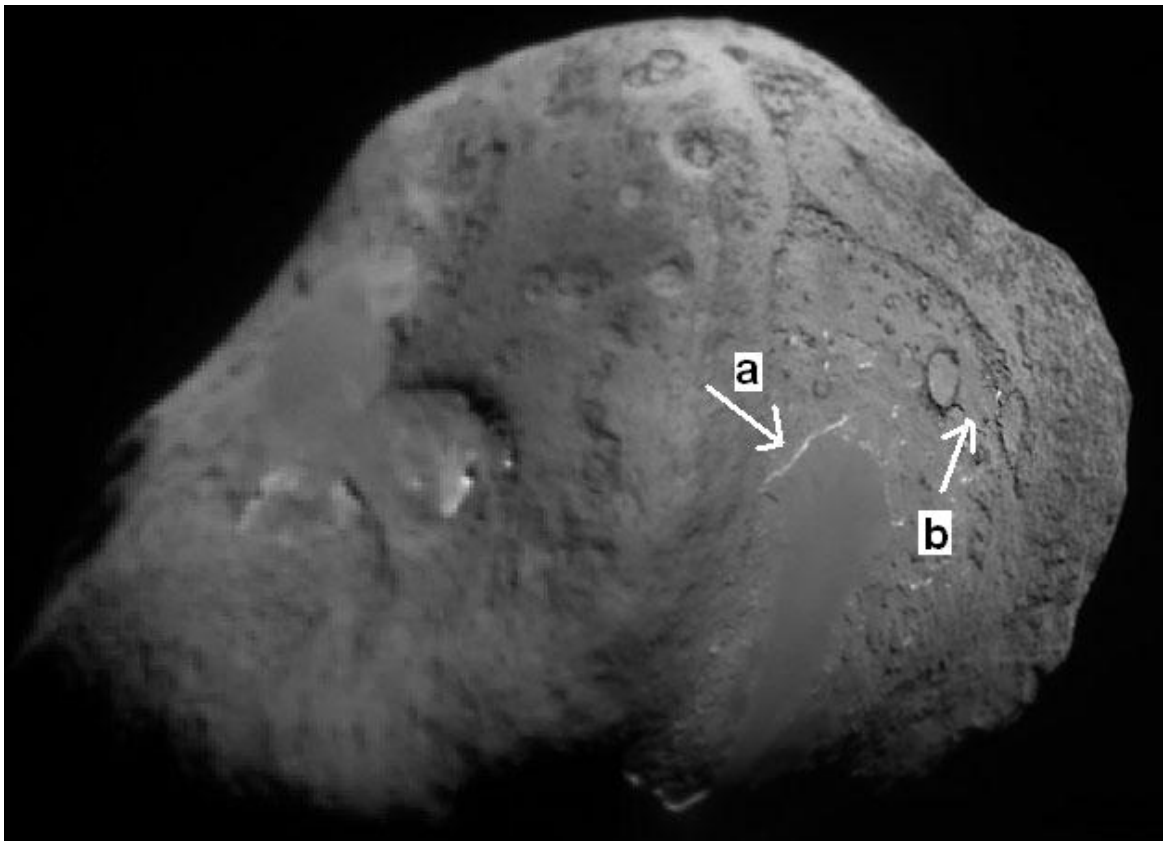
Problem 4 – From the bottom figure of total discovered asteroids, what percentage are a) smaller than 1 kilometer? B) Larger than 1 kilometer?

Answer: A) Adding up all 5 columns gives a total of 1350+2300 + 2500 + 3000 + 850 = 10,000
Then Small asteroids P = 100% x (10,000-850)/10000 = **91.5%**
B) Large asteroids P = 100% - 91.5% = **8.5%**

Problem 5 – Compare the top two figures. What can you conclude about the number of 1 kilometer or larger asteroids that are yet to be discovered, compared to those smaller than 1 kilometer?

Answer: The top graph shows that the detection rate for these large asteroids has declined steadily since 2000. This means that each year the surveys are finding fewer and fewer large asteroids that they did not previously know about. This means that the surveys have almost completely detected all of the NEO asteroids that are this large. Because these large NEOS are only 2% of the detected asteroids, the majority of NEO asteroids are much smaller than the large ones, and more numerous. The middle graph shows that the numbers we are detecting continues to grow each year with no sign of decreasing. This means that there are many more of these to be discovered than we have found already. By some estimates, we have only discovered about 5% of all that there are near Earth, which is why we have to keep searching. Once the number of new discoveries begins to follow the profile of the top figure, we will know that we have discovered the vast majority of the small asteroids that remain.

Tempel 1 - Close-up of a Comet!



On July 4, 2005, the Deep Impact spacecraft flew within 500 km of the nucleus of comet Tempel 1. This composite image of the surface of the nucleus was put together from images taken by the Impactor probe as it plummeted towards the comet before finally hitting it and excavating its own crater. The width of this picture is 8.0 kilometers.

Problem 1 - By using a millimeter ruler: A) what is the scale of this image in meters per millimeter? B) What is the approximate size of the nucleus of this comet in kilometers? C) How big are the two craters near the right-hand edge of the nucleus by the arrow? D) What is the size of some of the smallest details you can see in the picture?

Problem 2 - The white streak identified by Arrow A is a cliff face. What is the height of the cliff in meters, (the width of the white line) and the length of the cliff wall in meters?

Problem 3 - The Deep Impact Impactor probe collided with the comet at the point marked by the tip of Arrow B. If there had been any uncertainty in the accuracy of the navigation, by how many meters might the probe have missed the nucleus altogether?

Problem 1 - By using a millimeter ruler: A) what is the scale of this image in meters per millimeter? B) What is the approximate size of the nucleus of this comet in kilometers? C) How big are the two craters near the right-hand edge of the nucleus? D) What is the size of some of the smallest details you can see in the picture?

Answer: By using a millimeter ruler, what is the scale of this image in meters per millimeter? Answer: A) Width = 153 millimeters, so the scale is 8000 meters/153 mm = **52 meters/mm**. B) Width = 147 mm x 110 mm or **7.6 km x 5.7 km**. C) Although the craters are foreshortened, the maximum size gives a better indication of their 'round' diameters of about 7mm or **360 meters**. D) **Students may find features about 1 millimeter across or 50 meters.**

Note to Teacher: Depending on the quality of your printer, the linear scale of the image in millimeters may differ slightly from the 153 mm stated in the answer to Problem 1. Students may use their measured value as a replacement for the '153 mm' stated in the problem.

Problem 2 - The white streak near the center of the picture is a cliff face. What is the height of the cliff in meters, (the width of the white line) and the length of the cliff wall in meters?

Answer: The width of the irregular white feature is about 0.5 millimeters or 26 meters. The length is about 15 millimeters or 15x52 = **780 meters**.

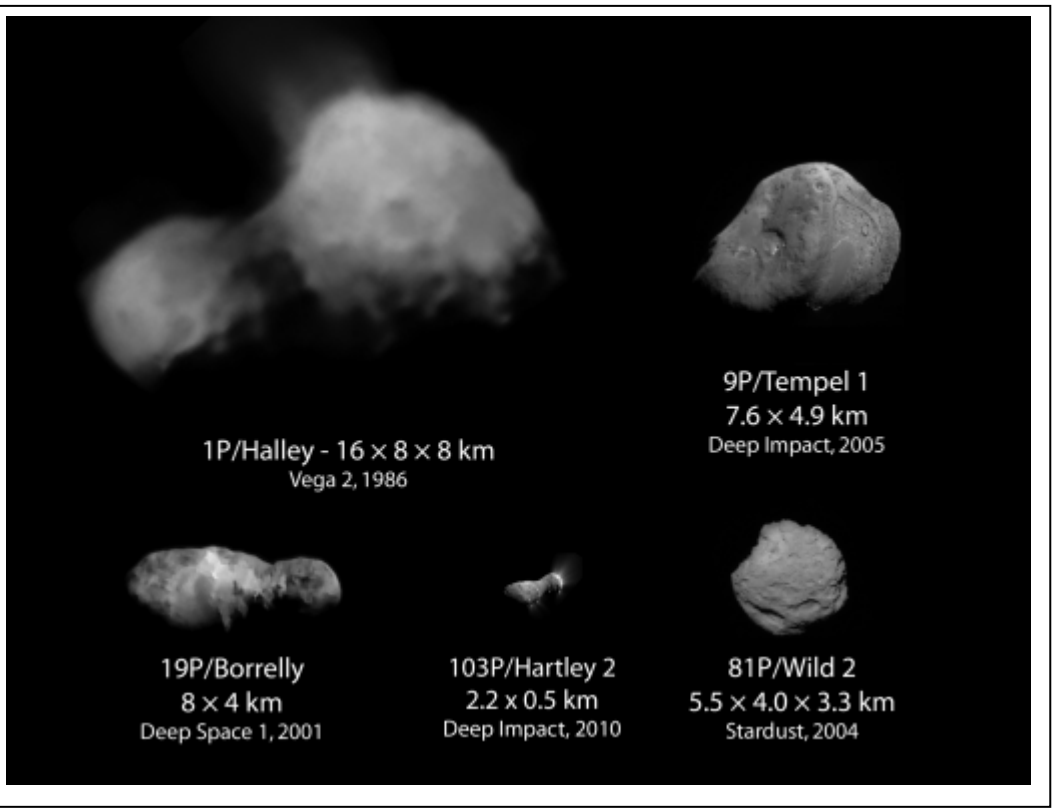
Problem 3 - The Deep Impact Impactor probe collided with the comet at the point marked by the tip of the arrow. If there had been any uncertainty in the accuracy of the navigation, by how many meters might the probe have missed the nucleus altogether?

Answer: The picture shows that the shortest distance to the edge of the nucleus is about 20 millimeters to the right, so this is a distance of about 20 x 52 = **1 kilometer**!

Note to Teacher:
Since the distance to the Earth was about 100 million kilometers, the spacecraft orbit had to be calculated to better than 1 part in 100 million over this distance in order for the probe to hit Tempel-1 as planned.

Comparing Comets Up Close with NASA Spacecraft!

7.2



Spacecraft have flown-by five comets to study the dense object which produces the dramatic head and tails of these objects as seen from Earth. The figure above shows images of the nuclear objects to the same scale.

Problem 1 – What percentage of comet nuclei are:

A) round

B) potatoe-shaped?

Problem 2 - What is the range of size, in kilometers, for the dimensions of these nuclei?

Problem 3 – If the range in size represented one side of a cube, what is the range in volumes of the nuclei in cubic kilometers?

Problem 1 – What percentage of comet nuclei are:

A) Round - answer 100% x (2/5) = **40%**

B) potatoe-shaped? - answer 100% x (3/5) = **60%**

Problem 2 - What is the range of size, in kilometers, for the dimensions of these nuclei?

Answer: The smallest dimension is for Comet Hartley 2 at 0.5 km. The largest dimension is for Halleys Comet at 16 km. **So the range is from 0.5 to 16 kilometers.**

Problem 3 – If the range in size represented one side of a cube, what is the range in volumes of the nuclei in cubic kilometers?

Answer: S = 0.5 km so volume = 0.5 x 0.5 x 0.5 = 0.125 km^3
S = 16 km, so volume = 16 x 16 x 16 = 4096 km^3.

The range of volumes is 0.125 to 4096 km^3.

Deep Impact Comet Encounter

On July 4, 2005 at 5:45 UT the 362-kilogram Impactor from NASA's Deep Impact mission, collided with the nucleus of the comet Tempel 1, causing a bright flash of light and a plume of ejected gas (see photo).

Traveling at 10.3 km/sec, the Impactor created a crater on the nucleus and ejected about 10,000 tons of material.

The average density of the comet nucleus is 400 kg/m^3 and its size can be approximated as a sphere with a radius of 3 kilometers.

Problem 1 – From the information given, what was the approximate mass of the comet nucleus in kilograms?

Problem 2 - If the Impactor's path was perpendicular to the path taken by the Comet's nucleus, conservation of momentum requires that the product of the mass of the Impactor and its speed perpendicular to the orbit must equal the product of the comet's mass and the comet's speed perpendicular to the orbit after the impact assuming no mass loss. Although the impact ejected 10,000,000 kilograms of comet material, we will ignore this effect since the comet's mass was over 45 trillion kilograms! From the information, what is the final speed of the comet nucleus perpendicular to its orbit in A) kilometers/sec? B) meters/year?

Problem 3 – How far, in kilometers, will the comet nucleus have drifted 'sideways' to its orbit after 1 million years?

Problem 4 – Suppose that the comet had been headed toward Earth, and it was predicted that in 50 years it would collide with Earth. A nuclear bomb with an explosive yield equal to 10 million tons of TNT is launched to intercept the comet nucleus and deliver a blast, whose energy is equal to that of a 7.5 x 10^8 kilogram kilogram Impactor traveling at 10.3 km/sec. Assuming that the nucleus is not pulverized, A) about how far, in kilometers, will the nucleus drift after 20 years? B) Is this enough to avoid hitting Earth (diameter = 12,000 kilometers)?

Problem 1 – From the information given, what was the approximate mass of the comet nucleus in kilograms?

Answer: The spherical volume was V = 4/3 π (3000 meters)^3 = 1.1 x 10^{11} meters^3. The density was 400 kg/m^3, so Mass = Density x Volume = 400 x 1.1 x 10^{11} = **45 trillion kilograms.**

Problem 2 -- If the Impactor's path was perpendicular to the path taken by the Comet's nucleus, conservation of momentum requires that the product of the mass of the Impactor and its speed perpendicular to the orbit must equal the product of the comet's mass and the comet's speed perpendicular to the orbit after the impact assuming no mass loss. Although the impact ejected 10,000,000 kilograms of comet material, we will ignore this effect since the comet's mass was over 45 trillion kilograms! From the information, what is the final speed of the comet nucleus perpendicular to its orbit in A) kilometers/sec? B) meters/year?

Answer: A) Vc = mi Vi/Mc = (362 kg) x (10.3 km/sec)/45 trillion kg = **8 x 10^{-11} kilometers/sec.**

B) 8 x 10^{-11} km/s x (1000 m/km) x (3600 s/hr) x (24 hr/day) x (365 d/yr) = **2.5 meters/year.**

Problem 3 – How far, in kilometers, will the comet nucleus have drifted sideways to its orbit after 1 million years?

Answer: From Problem 2, the drift is 2.5 meters/year, so after 1 million years the nucleus will have drifted about 2,500,000 meters or **2,500 kilometers.**

Problem 4 – Suppose that the comet had been headed towards Earth, and it is predicted that in 50 years it will collide with Earth. A nuclear bomb with an explosive yield equal to 10 million tons of TNT is launched to intercept the comet nucleus and deliver a blast, whose energy is equal to that of a 7.5 x 10^8 kilogram Impactor traveling at 10.3 km/sec. Assuming that the nucleus is not pulverized, A) about how far, in kilometers, will the nucleus drift after 20 years? B) Is this enough to avoid hitting Earth (diameter = 12,000 kilometers)?

Answer: Using miVi = mcVc, we get

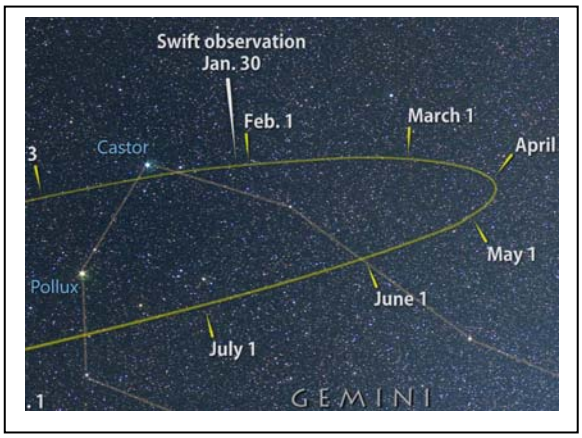
Vc = (7.5 x 10^8 kilograms) x (10.3km/sec)/45 trillion kg

= 0.00017 kilometers/sec.

A) In 20 years (20 x 3.1 x 10^7 seconds) it travels **100,000 kilometers.**

B) **Yes,** since it only needs to travel 12,000 kilometers sideways to avoid hitting Earth, the detonation did help to avoid the collision…assuming the comet wasn't fragmented into a large cloud of debris!

The Orbit of Comet ISON



Comet ISON will be carefully watched as it makes its closest approach to the sun in November, 2013. Some astronomers predict that it may break up into smaller comets because of the Sun's enormous gravity. The comet will travel to within 800,000 km of the center of the sun, or about 110,000 km from the hot solar surface! As it travels, it will also get very close to Mars and the asteroid 3362 Khufu, though no impacts are predicted!

A portion of its track across the sky is shown in the figure for January-July, 2013.

The table below gives the location of Comet ISON as it approaches the sun. The sun is located at the point (-0.5, +18.7) where all of the coordinate units are in millions of kilometers.

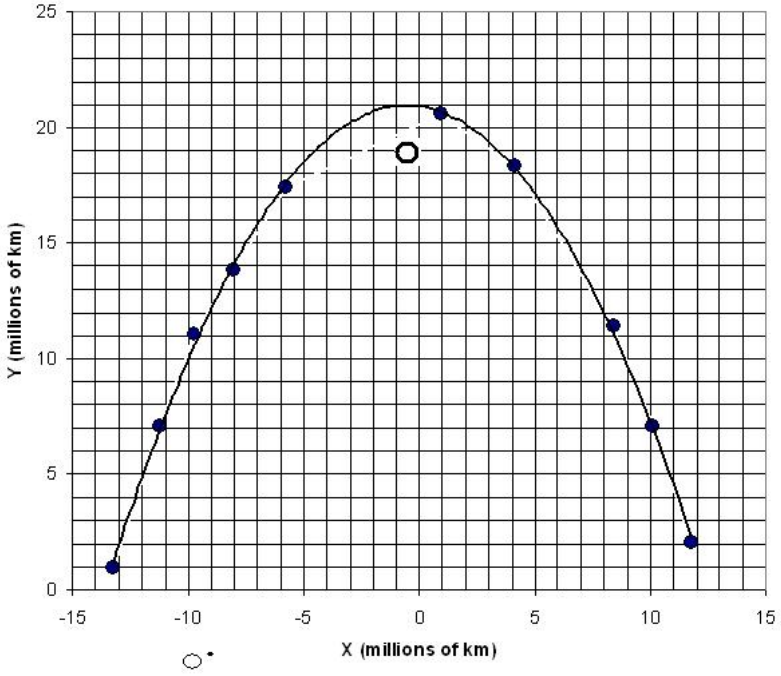
Date and Universal Time	X (million km)	Y (million km)	Distance to the sun (million km)
November 26, 18:00 UT	-13.3	+0.9	
November 27, 13:00 UT	-11.2	+7.1	
November 28, 01:00 UT	-9.7	+11.0	
November 28, 08:00 UT	-8.0	+13.8	
November 28, 14:00 UT	-5.8	+17.4	
November 28, 20:00 UT	+0.9	+20.6	
November 28, 23:00 UT	+4.1	+18.3	
November 29, 10:00 UT	+8.4	+11.4	
November 29, 19:00 UT	+10.1	+7.1	
November 30, 10:00 UT	+11.8	+2.1	

Problem 1 - Plot these points on an X-Y graph and connect the points with a smooth parabolic curve.

Problem 2 – Using either a millimeter ruler and the scale of the graph, or the Two Point Distance Formula, calculate the distance from each comet position to the sun in the table.

Problem 3 – What is your prediction for the exact time when Comet ISON is at its closest point in its orbit to the sun?

Problem 1 - Plot these points on an X-Y graph and connect the points with a smooth parabolic curve. **Note: These coordinates are valid for the orbit as known on July 29, 2013 but may change as a more precise orbit is eventually determined.**



Problem 2 – Using either a millimeter ruler and the scale of the graph, or the Two Point Distance Formula, calculate the distance from each comet position to the sun in the table.

Answer: For advanced students using the distance formula: $D = ((x2-x1)^{2} + (y2-y1)^{2})^{1/2}$. For the first point x1 at (-13.3, +0.9) and x2 at (-0.5,+18.7) so D = 21.9 million km.

Date and Universal Time	X (million km)	Y (million km)	Distance to the sun (million km)
November 26, 18:00 UT	-13.3	+0.9	**21.9**
November 27, 13:00 UT	-11.2	+7.1	**15.8**
November 28, 01:00 UT	-9.7	+11.0	**12.0**
November 28, 08:00 UT	-8.0	+13.8	**9.0**
November 28, 14:00 UT	-5.8	+17.4	**5.5**
November 28, 20:00 UT	+0.9	+20.6	**2.3**
November 28, 23:00 UT	+4.1	+18.3	**4.6**
November 29, 10:00 UT	+8.4	+11.4	**11.5**
November 29, 19:00 UT	+10.1	+7.1	**15.7**
November 30, 10:00 UT	+11.8	+2.1	**20.7**

Problem 3 – What is your prediction for the exact time when Comet ISON is at its closest point in its orbit to the sun? Answer: Students may interpolate between Points 5, 6 and 7 using any convenient method. The answers should be close to **November 28 at 14:00 UT and a distance of 725,000 km.**

Searching for Comets

Year	Total	Amateurs	Spacecraft	Observatories
2012	60	27	1	32
2011	49	28	2	19
2010	161	29	119	13
2009	227	35	188	4
2008	220	34	182	4
2007	223	35	170	18
2006	206	31	152	23
2005	221	23	169	29
2004	223	8	172	43
2003	193	7	149	37
2002	182	9	131	42
2001	149	4	107	38
2000	135	9	99	27
1999	129	18	87	24

Every year, professional astronomers and dedicated amateur astronomers use everything from simple binoculars to sophisticated computer-driven telescopes to discover new comets. The table to the left gives a count of the number of comets detected between 1999 and 2012.

Comet hunters carefully compare images of the same part of the sky over a period of days or weeks. Although stars remain fixed, comets appear as fuzzy spots of light that change their positions.

Problem 1 – During 2010, what percentage of comet discoveries were made by amateur astronomers, spacecraft and ground-based observatories?

Problem 2 – During the years 1999 to 2012, what is the average number of comets discovered by amateur astronomers and by ground-based observatories?

Problem 3 – What percentage of new comets would have been lost in 2012 had there not been any amateur astronomers searching the skies?

The tabulated data is based upon the Catalog of Comet Discoveries archive at http://www.comethunter.doc

Problem 1 – During 2010, what percentage of comet discoveries were made by amateur astronomers, spacecraft and ground-based observatories?

Answer: Amateur astronomers: 100% x (29/161) = **18%**
Spacecraft: 100% x (119/161) = **74%**
Observatories : 100% x(13/161) = **8%**

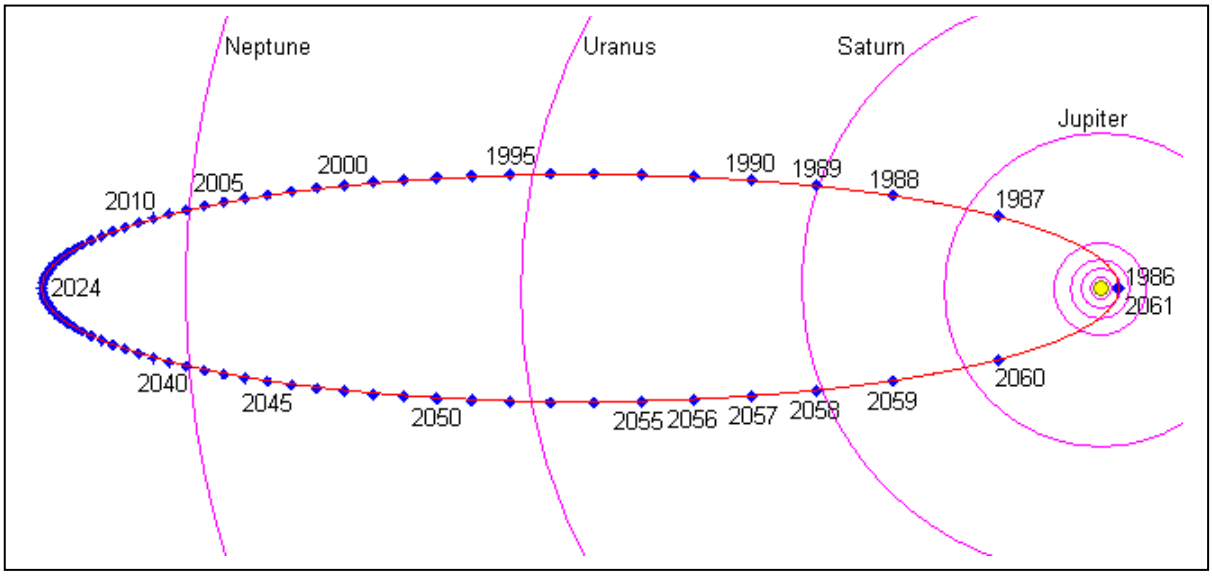
Problem 2 – During the years 1999 to 2012, what is the average number of comets discovered by amateur astronomers and by ground-based observatories?

Answer: Total for amateurs = 297 over 14 years so the average was **21 comets/year**.
For observatories: 353 over 14 years so the average is **25 comets/year**.

Problem 3 – What percentage of new comets would have been lost in 2012 had there not been any amateur astronomers searching the skies?

Answer: In 2012 the total number of comets was 60, and of these 27 were detected by amateur astronomers, so 60-27 = 33 detected by other means, then 100% x (33/60) = **55% of the comets would have remained undetected.**

Exploring Comet Orbits



Comets are giant icebergs in space, sometimes over 50 miles across, that shed water vapor as they are heated while approaching the sun. Some comets only come around once and are never seen again. Others travel on elliptical paths called orbits, that take them far beyond the orbit of Jupiter before they, once again, loop back towards the sun.

Halleys Comet, which made a pass near the sun in 1986 is one of the most famous Periodic Comets, and will return to Earth's skies in the year 2061. This figure shows the orbit of Halleys Comet to the same scale as the orbits of the planets. Each dot is the position after one Earth year has elapsed.

Problem 1 – What is the period of Halleys Comet in Years?

Problem 2 – What is the longest diameter of the elliptical orbit in kilometers if the distance between the orbits of Jupiter and Saturn is 650 million km?

Problem 3 – The distance between Saturn's orbit and the orbit if Venus is 1.3 billion km. About how fast is Halleys Comet traveling in km/year as it travels the Venus-Saturn distance?

Problem 4 – The distance between the orbits of Uranus and Neptune is 1.6 billion km. From the diagram, about how many years does it take to travel this distance, and what is the average speed of Halleys Comet during this time in km/year?

Problem 1 – What is the period of Halleys Comet in Years?

Answer: 2061 – 1986 = **75 years.**

Problem 2 – What is the longest diameter of the elliptical orbit in kilometers if the distance between the orbits of Jupiter and Saturn is 650 million km?

Answer: First determine the scale of this figure using a millimeter ruler and the actual Jupiter-Saturn distance. When printed on standard 8 ½ x 11 paper, the separation should be about 19 millimeters, so the scale is 650 million km/19mm = 34 million km/mm. The length of the ellipse is 143 mm, so the actual distance is 143 x 34 million = **4.86 billion kilometers**.

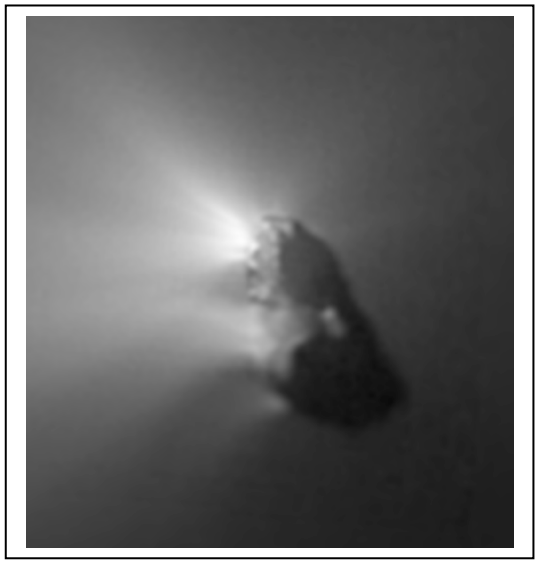
Problem 3 – The distance between Saturn's orbit and the orbit if Venus is 1.3 billion km. About how fast is Halleys Comet traveling in km/year as it travels the Venus-Saturn distance?

Answer: Counting the number of years, it takes 3 years to travel this distance, so the speed is 1.3 billion km/3 years = **433 million km/year**.

Problem 4 – The distance between the orbits of Uranus and Neptune is 1.6 billion km. From the diagram, about how many years does it take to travel this distance, and what is the average speed of Halleys Comet during this time in km/year?

Answer: Counting the year marks, it takes 12.5 years, so the speed is about 1.6 billion km/12.5 years = **128 million km/year.**

Computing the Orbit of a Comet



This historic image of the nucleus of Halley's Comet by the spacecraft Giotto in 1986 reveals the gases leaving the icy body to form the tail of the comet.

Once astronomers discover a new comet, a series of measurements of its location allows them to calculate the orbit of the comet and predict when it will be closest to the Sun and Earth.

Problem 1 – Astronomers measured two positions of Halley's Comet along its orbit. The x and y locations in its orbital plane are given in units of the Astronomical Unit, which is a unit equal to the distance from Earth to the sun (150 million km). The positions are (+10, +4) and (+14, +3). What are the two equations for the elliptical orbit based on these two points, written as quadratic equations in a and b, which are the lengths of the semi-major and semi-minor axis of the ellipse?

Problem 2 – Solve the system of two quadratic equations for the ellipse parameters a and b.

Problem 3 – What is the orbit period of Halley's Comet from Kepler's Third Law if $P^2 = a^3$ where a is in Astronomical Units and P is in years?

Problem 4 – The perihelion of the comet is defined as d = a – c where c is the distance between the focus of the ellipse and its center. How close does Halley's Comet come to the sun in this orbit in kilometers?

Problem 1 – Astronomers measured two positions of Halley's Comet along its orbit. The x and y locations in its orbital plane are given in units of the Astronomical Unit, which equals 150 million km. The positions are (+10, +4) and (+14, +3). What are the two equations for the elliptical orbit based on these two points, written as quadratic equations in a and b, which are the lengths of the semimajor and semiminor axis of the ellipse?

Answer: The standard formula for an ellipse is $x^2/a^2 + y^2/b^2 = 1$ so we can re-write this as $b^2x^2 + a^2y^2 = a^2b^2$.

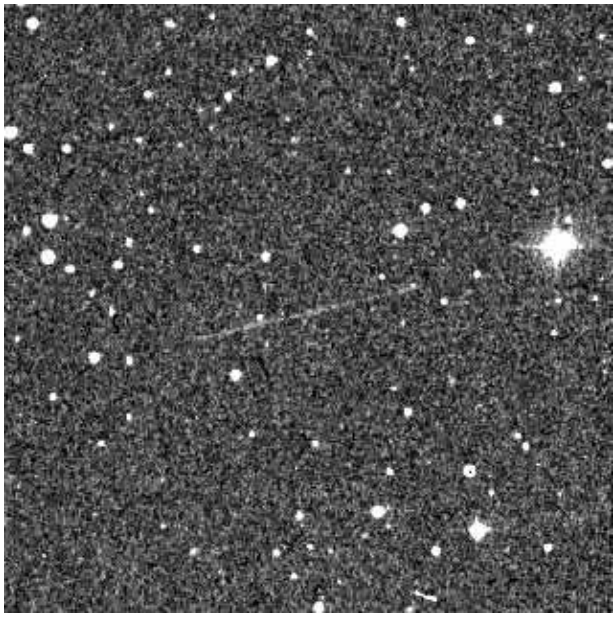
Then for Point 1 we have
$10^2b^2 + 4^2a^2 = (ab)^2$ so **$100b^2 + 16a^2 = (ab)^2$**. Similarly for Point 2 we have
$14^2b^2 + 3^2a^2 = (ab)^2$ so **$196b^2 + 9a^2 = (ab)^2$**

Problem 2 – Solve the system of two quadratic equations for the ellipse parameters a and b.
Answer:
$100b^2 + 16a^2 = (ab)^2$
$196b^2 + 9a^2 = (ab)^2$
Difference the pair to get $7a^2 = 96b^2$ so $a^2 = (96/7)b^2$.
Substitute this into the first equation to eliminate b^2 to get
$(700/96) + 16 = (7/96)a^2$ or $a^2 = 2236/7$ and so **a = 17.8 AU**.
Then substitute this value for a into the first equation to get
$5069 = 217\ b^2$ and so **b = 4.8 AU**.

Problem 3 – What is the orbit period of Halley's Comet from Kepler's Third Law is $P^2 = a^3$ where a is in Astronomical Units and P is in years? Answer: $P = a^{3/2}$ so for a = 17.8 AU we have **P = 75.1 years**.

Problem 4 – The perihelion of the comet is defined as d = a – c where c is the distance between the focus of the ellipse and its center. How close does Halley's Comet come to the Sun in this orbit in kilometers?
Answer: From the definition for c as $c = (a^2 - b^2)^{1/2}$ we have c = 17.1 AU and so the perihelion distance is just d = 17.8 – 17.1 = 0.7 AU. Since 1 AU = 150 million km, it comes to within **105 million km** of the Sun. This is near the orbit of Venus.

Spotting an Approaching Asteroid or Comet



An asteroid, or comet, viewed from Earth will be either bright or faint depending on many quantifiable factors. Of course the size of the body and its reflectivity make a big difference. So does its distance from the sun and earth at the time you see it. The brightness also depends on whether, from Earth, it is fully-illuminated like the full moon, or only partly-illuminated like the crescent moon.

Astronomers can put all of these variables together into one single equation which works pretty well to predict a body's brightness just about anywhere inside the solar system!

The streak in the photo above is the asteroid 1999AN10 (Courtesy Palomar Digital Sky Survey). Orbit data suggest that on August 7, 2027 it will pass within 37,000 kilometers of Earth. The formula for the brightness of the asteroid is given by:

$$R = 0.011\, d\ 10^{-\frac{1}{5}(m)}$$

where: R is the asteroid radius in meters, d is the distance to Earth in kilometers, and m is the apparent brightness of the asteroid viewed from Earth. Note, the faintest star you can see with the naked eye is about m = +6.5. The photograph above shows stars as faint as m = +20. The asteroid is assumed to have a reflectivity similar to lunar rock.

Problem 1 - If the distance to the asteroid at the time of closest approach in 2027 will be d = 37,000 kilometers, what is the formula R(m) for the asteroid?

Problem 2 - If the radius of the asteroid is in the domain between 200 meters and 1000 meters, what is the range of apparent brightnesses?

Problem 1 - If the distance to the asteroid at the time of closest approach in 2027 will be d = 37,000 kilometers, what is the formula R(m) for the asteroid? Answer: Substitute the given values into the equation and simplify. The formula will give the radius of the asteroid in meters as a function of its apparent brightness (called apparent magnitude by astronomers) given by m.

$R(m) = 0.011\ (37000)10^{-0.2m}$

$R(m)= 407\ 10^{-0.2m}$

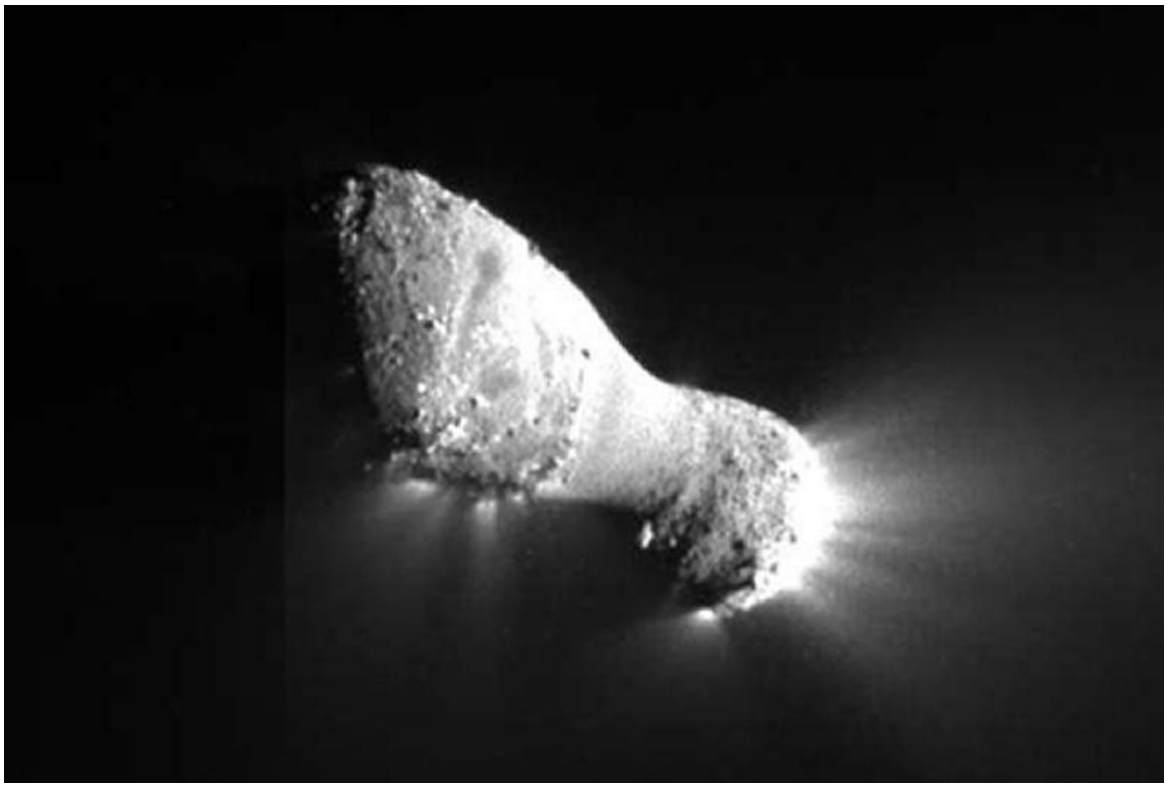
Problem 2 - If the radius of the asteroid is in the domain between 200 meters and 1000 meters, what is the range of apparent brightnesses? Answer: Solve the formula for R(m) for m(R) and evaluate for R = 200 meters and R = 1000 meters to obtain the range of the function.

$m(R) = -5 \log_{10}(R/4224)$

so m(R) for R = 200 yields m(200) = -5 (-11.3) so m(200) = +1.5
and m(1000) = -5(0.39) so m(1000) = -2.0

so Domain R: [200,1000]
and Range m: [+1.5, -2.0]

Note: The planet Venus can be as bright as m = -2.5 so this asteroid should be easily visible if it is in this size domain.



Comet Hartley 2 is seen in this spectacular image taken by the Deep Impact/EPOXI Medium-Resolution Instrument on November 4, 2010 as it flew by the nucleus at a distance of 700 kilometers. The pitted surface, free of large craters, shows a complex texture in regions where gas plumes are actively ejecting gas. The potato-shaped nucleus is 2 kilometers long and 0.4 kilometers wide at its narrowest location. (Credit: NASA/JPL-Caltech/UMD).

Problem 1 - Suppose that the shape of the comet nucleus can be approximated by the following function

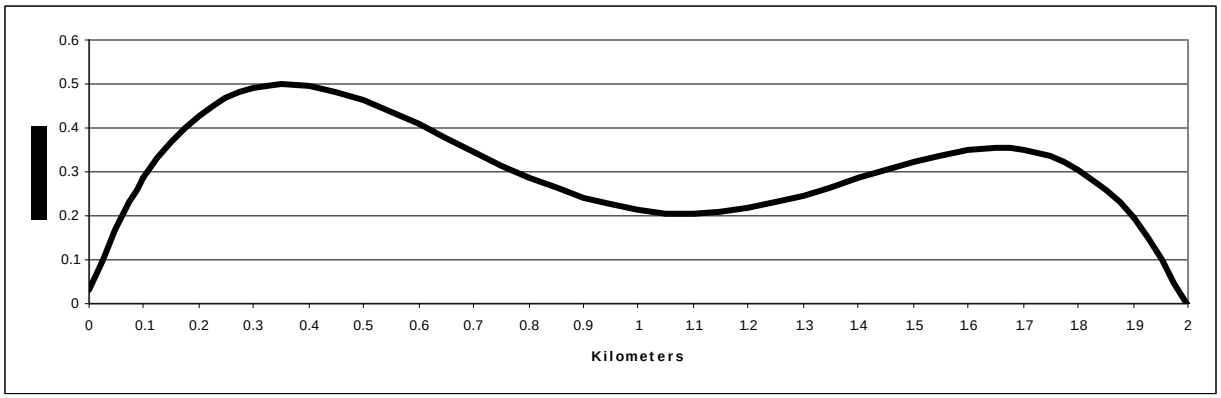
$$y(x) = -1.22x^4 + 5.04x^3 - 6.78x^2 + 3.14x + 0.03$$

rotated about the x-axis between x=0 and x=2.0, where all units are in kilometers.
A) Graph this function;
B) Perform the required volume integration by using the method of circular disks.
C) To two significant figures, what is the total volume of the nucleus in cubic meters?

Problem 2 - Assuming that the density of Comet Hartley-2 is 0.6 grams/cm^3, what is your estimate for the mass of Comet Hartley-2 in megatons? (Note: 1000 kg = 1 metric ton)

Problem 1 - Answer:

A) Graph:



B)

$$V = \int_0^2 \pi y(x)^2 dx \qquad \text{then} \qquad V = \pi \int_0^2 \left(-1.22x^4 + 5.04x^3 - 6.78x^2 + 3.14x + 0.03\right)^2 dx$$

Expand integrand and collect terms (be careful!):

$$V = \pi \int_0^2 \left(1.49x^8 - 12.30x^7 + 41.94x^6 - 76.00x^5 + 77.55x^4 - 42.28x^3 + 9.46x^2 + 0.18x + 0.0009\right) dx$$

Integrate each term:

$$V = \pi \left[0.17x^9 - 1.54x^8 + 5.99x^7 - 12.67x^6 + 15.51x^5 - 10.57x^4 + 3.15x^3 + 0.09x^2 + 0.0009x + c\right]_0^2$$

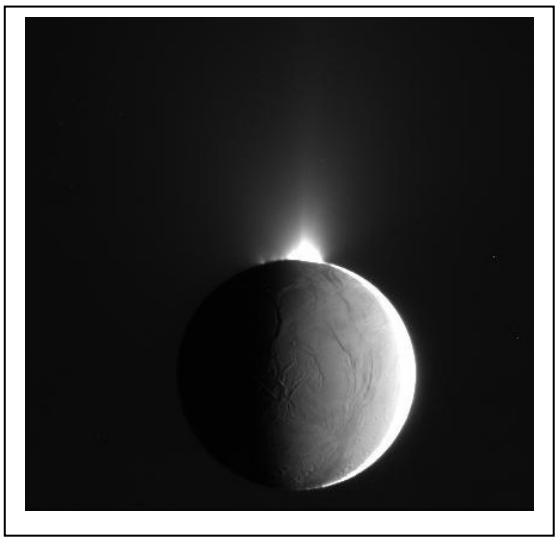
Now evaluate V(x) at the two limits to get V = V(2) - V(0): Note that the answer for V will be sensitive to the accuracy of the polynomial coefficients, here given to 4 decimal place accuracy:

V=(3.14)[0.1655(2)^9 – 1.5375(2)^8 +5.9914(2)^7 – 12.6667(2)^6 + 15.51(2)^5 – 10.57(2)^4 +3.1533(2)^3 + 0.09(2)^2 + 0.0009(2)]

V = 3.14[0.157]

So **V = 0.49 cubic kilometers**.

Problem 2 - Mass = Density x Volume; First convert the volume to cubic centimeters from cubic kilometers: V = 0.49 km^3 x (10^3 meters/1 km)^3 x (100 cm/1 meter)^3 = 4.9 x 10^{14} cm^3. Then, Mass = 0.6 gm/cm^3 x 4.9 x 10^{14} cm^3 = 2.9 x 10^{14} gm**.** Convert grams to megatons: Mass = 2.9 x 10^{14} gm x (1 kg/1000 gm) x (1 ton/1000 kg) = 2.9 x 10^8 tons or **290 megatons**.



For decades, astronomers thought that the only geologically active object in our entire solar system was Earth. During the last 50 years of spacecraft studies, we now know that there are many locations where recent and even current volcanism can be found.

This image, taken by the Cassini spacecraft, shows plumes of gas and dust ejected from cracks in the surface of Saturn's moon Enceladus. Similar plumes have been found in Jupiter's satellite Io, and Neptune's moon Triton. They are called cryovolcanos because the temperatures are so low, only 100 kelvins (173 Celsius), and instead of rocky lava they eject water, methane or other frozen gases.

A simple 'square-root' formula relates the height of a plum, h, and its ejection speed, V, to the surface gravity of the body, g :

$$V = (2gh)^{1/2}$$

where h is in meters, V is in meters/sec and g is the acceleration of gravity at the surface in meters/sec^2.

Problem 1 – Complete the following table to estimate the ejection speeds and heights of volcanic plumes on the indicated bodies.

Object	Type	G (m/s^2)	V (m/s)	H (m)
Venus	Volcano	8.8	100	
Earth	Volcano	9.8		8,000
Earth	Geyser	9.8	35	
Mars	Volcano	3.8	100	
Io	Volcano	1.8		200,000
Enceladus	Geyser	0.1	60	
Titan	Volcano	1.4	100	
Triton	Geyser	0.8		5,000

Problem 2 – If 1 meter/sec = 2.2 miles/hr, which objects have the fastest and slowest ejection speeds in mph?

Problem 3 - Calculate the average ejection speeds for volcanos and geysers. What do you notice about the kind of event and its ejection speed?

Problem 1 – Complete the following table to estimate the ejection speeds and heights of volcanic plumes on the indicated bodies.

Object	Type	G (m/s^2)	V (m/s)	H (m)
Venus	Volcano	8.8	100	**570**
Earth	Volcano	9.8	**400**	8,000
Earth	Geyser	9.8	35	**63**
Mars	Volcano	3.8	100	**1,300**
Io	Volcano	1.8	**850**	200,000
Enceladus	Geyser	0.1	60	**18,000**
Titan	Volcano	1.4	100	**3600**
Triton	Geyser	0.8	**89**	5,000

Problem 2 – If 1 meter/sec = 2.2 miles/hr, which objects have the fastest and slowest ejection speeds in mph?

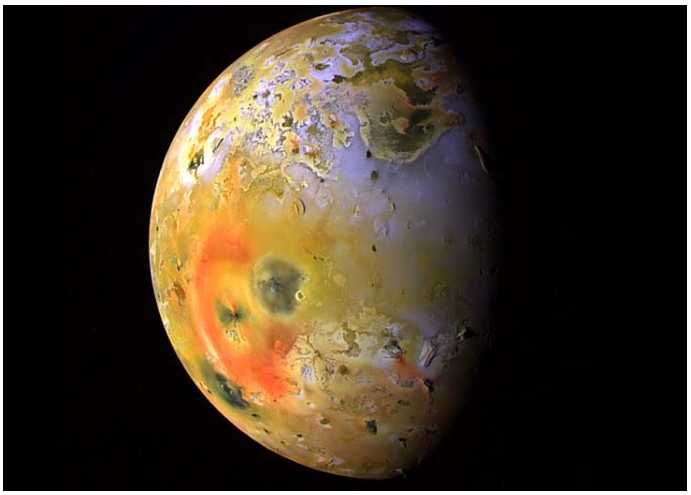
Answer: Fastest is Io at 850 m/s or 1,870 mph. Slowest is Earth geyser at 35 m/s or 77 mph.

Problem 3 - Calculate the average ejection speeds for volcanos and geysers. What do you notice about the kind of event and its ejection speed?

Answer: Volcanos: (100+400+100+850+100)/5 = 310 m/sec (682 mph).
Geysers: (35+60+89)/3 = 61 m/sec (134 mph.)

Volcanos have the highest speeds. This is because they are produced by intense pressure of magma trapped in rocky 'pipes' that reach hundreds of kilometers below the surface, while geysers are created by more modest pressures in the surface crust.

Io's Volcanoes and Resurfacing



This 1997 image taken by NASA's Galileo spacecraft shows the complex surface of Io. Sulfur dioxide frost appears in white and grey hues while yellowish and brownish hues are from other sulfurous materials. The new dark spot 400 km in diameter, surrounds a volcanic center named Pillan Patera. The spot did not exist 5 months earlier, and is the source of a 120 km high plume that has been seen erupting from this location.

Although no impact craters have been found, over 420 calderas and active vents have been mapped. About 15 are actively spewing fresh material within 175 km of each vent. This means that Io quickly resurfaces itself, covering over all of the impact craters within a million years or less.

Problem 1 – Assume Io is a sphere with a radius of 1820 km, and is covered to a depth of 1 kilometers to cover any new craters. What volume of fresh material must be produced in cubic meters?

Problem 2 – If the present surface was produced by the 420 calderas, what is the volume produced by each caldera?

Problem 3 – The typical time between large meteor impacts is about 500,000 years. How much material would have to be produced by each caldera each year to cover the surface between impacts?

Problem 4 – If any given caldera is only active for 1% of its life, what does the resurfacing rate have to be for each caldera?

Problem 5 – What is the total resurfacing rate each year in centimeters/year?

Problem 1 – Assume Io is a sphere with a radius of 1820 km, and is covered to a depth of 1 kilometers to cover any new craters. What volume of fresh material must be produced in cubic meters?

Answer: Area = 4 π R^2, and R = 1820,000 meters, so
Area = 4 x 3.14 x $(1820000)^2$ = 4.2x10^{13} m^2.
The volume of the surface shell 1km thick is that V = A x 1km = **4.2x10^{16} m^3.**

Problem 2 – If the present surface was produced by the 420 calderas, what is the volume produced by each caldera?

Answer: 4.2 x 10^{16} m^3 / 420 = **1.0x10^{14} m^3 per caldera**.

Problem 3 – The typical time between large meteor impacts is about 500,000 years. How much material would have to be produced by each caldera each year to cover the surface between impacts?

Answer: 1.0x10^{14} m^3/500000yrs = **2.0x10^8 m^3/year.**

Problem 4 – If any given caldera is only active for 1% of its life, what does the resurfacing rate have to be for each caldera?

Answer: 2.0x10^8 m^3/year would be the rate if each caldera continuously operated for 500,000 years. If they only are active for 1% of this time, then the average rate has to be 100x higher or **2.0x10^{10} m^3/yr.**

Problem 5 – What is the total resurfacing rate each year in centimeters/year?

Answer: If the 1km depth is generated over 500,000 years, then each year the depth added is 100000 centimeters/500000 yr = **0.2 centimeters/year.**

Volcanos are a Blast- Working with simple equations 8.3

Equation 1

$$V = \sqrt{2\,g\,H}$$

Equation 2

$$X = \frac{V^2}{g}$$

Equation 3

$$T = \frac{V\sqrt{2}}{g}$$

There are three equations that describe projectile motion on a planet:

Equation 1: Maximum velocity, **V**, needed to reach a height, **H**:

Equation 2: Maximum horizontal distance, **X**:

Equation 3: Time, **T**, required to reach maximum horizontal distance:

In all three equations, **g** is a constant and is the acceleration of gravity at the surface of the planet, and all units are in meters or seconds.

Problem 1 - The volcano, Krakatoa, exploded on August 26, 1883 and obliterated an entire island. The detonation was heard over 2000 kilometers away in Australia, and was the loudest sound created by Nature in recorded human history! If the plume of gas and rock reached an altitude of H=17 miles (26 kilometers) what was the speed of the gas, V, that was ejected, in A) kilometers/hour? B) miles/hour? C) What was farthest horizontal distance, X, in kilometers that the ejecta reached? D) How long, T, did it take for the ejecta to travel the maximum horizontal distance? E) About 30,000 people were killed in the explosion. Why do you think there were there so many casualties? (Note: **g** = 9.8 meters/sec^2 for Earth.)

Problem 2 - An asteroid collides with the lunar surface and ejects lunar material at a speed of V=3,200 kilometers/hr. A) How high up, H, does it travel before falling back to the surface? B) The escape speed from the lunar surface is 8,500 km/hr. From your answer to Problem 1, would a 'Krakatoa' explosion on the moon's surface have been able to launch lunar rock into orbit? (Note: **g** = 1.6 meters/sec^2 for the Moon.)

Problem 3 - Plumes of gas are ejected by geysers on the surface of the satellite of Saturn called Enceladus. If **g** = 0.1 meters/sec^2 , and H = 750 km, what is the speed of the gas, V, in the ejection in kilometers/hr?

Inquiry Problem: Program an Excel Spreadsheet to calculate the various quantities in the three equations given input data about the planet and ejecta. How does the maximum ejection velocity and height change with the value of **g** used for a variety of bodies in the solar system?

Answer Key

Problem 1 - The volcano, Krakatoa, exploded on August 26, 1883 and obliterated an entire island. The detonation was heard over 2000 kilometers away in Australia, and was the loudest sound created by Nature in recorded human history! If the plume of gas and rock reached an altitude of 17 miles (26 kilometers) what was the speed of the gas that was ejected, in

A) Use Equation 1 with H = 26,000 meters; g = 9.8 m/s^2 and get H = $(2 \times 26000 \times 9.8)^{1/2}$ = 714 m/sec, but since the input numbers are only good to two significant figures, the answer is 710 meters/sec. Then converting to km/hr we get 710 m/s x (3600 sec/hr) x(1 km/1000meters) = 2,556 km/hr but again we only report to 2 significant figures so the answer is **2,600 km/hr**.

B) 2,600 km/hr x (0.62 miles/km) = **1,600 miles/hour** to 2 significant figures

C) Use Equation 2: X = $(710 \text{ meters/sec})^2/9.8$ = 51,439 meters, which to 2 significant figures becomes 51,000 meters or **51 kilometers**.

D) Use Equation 3: T = 1.414 x (710)/9.8 = 102.4, but to 2 significant figures is **100 seconds.**

E) About 30,000 people were killed in the explosion. Why do you think there were there so many casualties? Answer: **They had less than 100 seconds to flee from the advancing ejecta cloud!** You can also ask the students to calculate the sound travel time to cross 51 kilometers (sound speed = 340 m/sec) which would take 51,000 meters/340 = 150 seconds to reach someone at 51 kilometers…so the eject would strike them BEFORE they even heard the detonation.

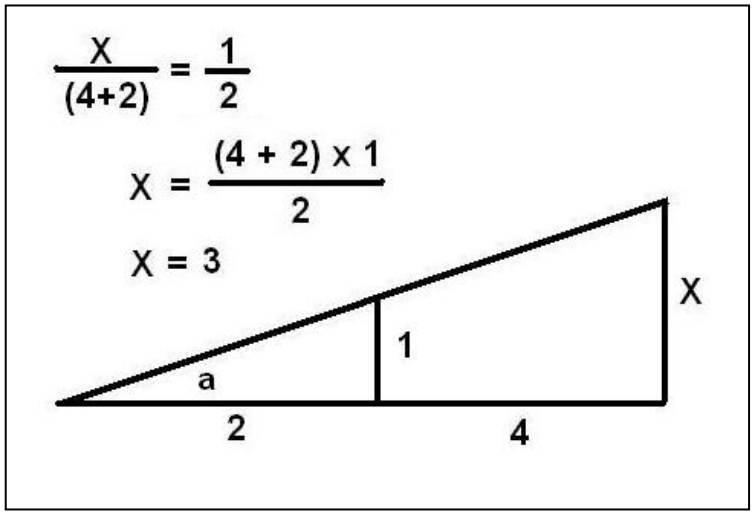
Problem 2 - Answer: 3,200 km/hr = 0.9 km/sec = **900 meters/sec**. to 2 significant figures. A) Solve Equation 1 for H… H = V2/2g so H = $(900)^2/(2 \times 1.6)$ = **250 kilometers** (2 SigFig). B) The escape speed from the lunar surface is 8,500 km/hr. From your answer to Problem 1, would a 'Krakatoa' explosion on the moon's surface have been able to launch lunar rock into orbit? (Note: g = 1.6 meters/sec^2 for the Moon.) Answer: **Yes!**

Problem 3 - Answer: From Equation V = $(2 \times 0.1 \times 750000)^{1/2}$ = 390 meters/sec = **1,400 km/hr** (2 SigFig)

Inquiry Problem: Program an Excel Spreadsheet to calculate the various quantities in the three equations given input data about the planet and ejecta. How does the maximum ejection velocity and height change with the value of **g** used for a variety of bodies in the solar system?

Answer: **There are many ways for students to program each column in a spreadsheet to calculate the variables in the equations. Students should, for instance, notice that as the surface gravity, g, increases, the maximum speed, V, changes as the square-root of g, and the values for X and T vary inversely with g.**

Angular Size and Similar Triangles

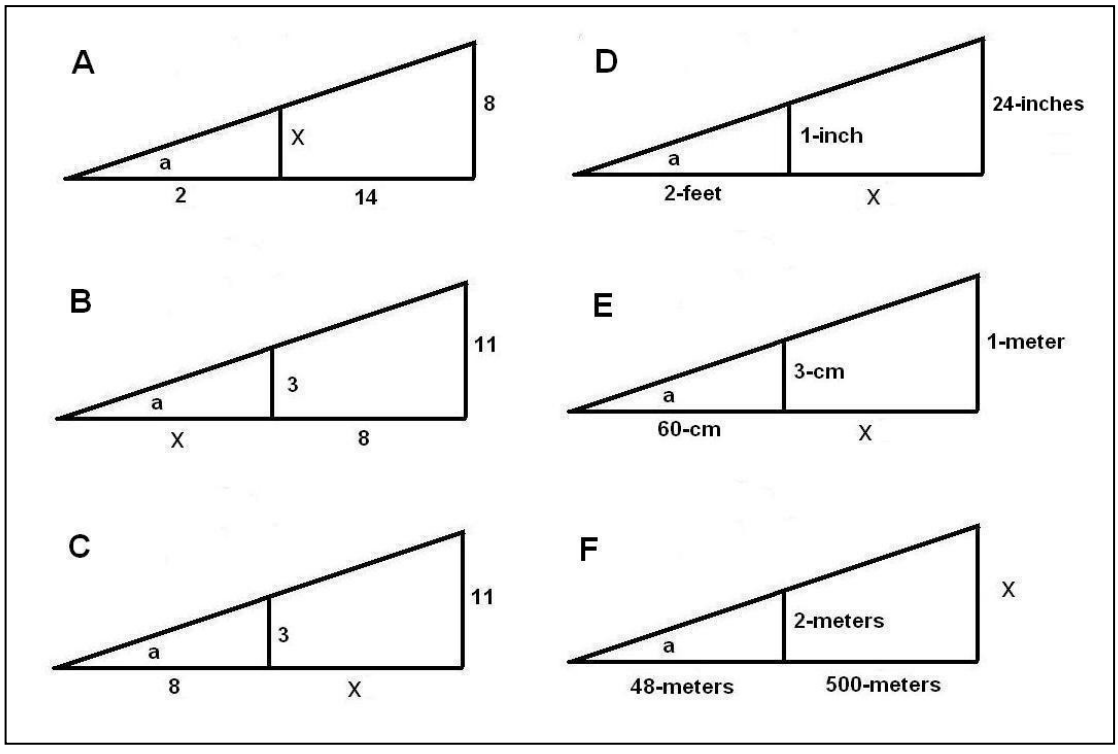


The corresponding sides of similar triangles are proportional to one another as the illustration to the left shows. Because the vertex angle of the triangles are identical in measure, two objects at different distances from the vertex will subtend the same angle, **a**. The corresponding side to 'X' is '1' and the corresponding side to '2' is the combined length of '2+4'.

Problem 1: Use the properties of similar triangles and the ratios of their sides to solve for 'X' in each of the diagrams below.

Problem 2: Which triangles must have the same measure for the indicated angle **a** ?

Problem 3: The sun is 400 times the diameter of the moon. Explain why they appear to have about the same angular size if the moon is at a distance of 384,000 kilometers, and the sun is 150 million kilometers from Earth?



Problem 1: Use the properties of similar triangles and the ratios of their sides to solve for 'X' in each of the diagrams below.

A) X / 2 = 8 / 16 so **X = 1**

B) 3 / X = 11 / (X+8) so 3 (X + 8) = 11 X ; 3X + 24 = 11 X ; 24 = 8X and so **X = 3.**

C) 3 / 8 = 11 / (x + 8) so 3 (x + 8) = 88 ; 3X + 24 = 88 ; 3X = 64 and so **X = 21 1/3**

D) 1-inch / 2-feet = 24 inches / (D + 2 feet); First convert all units to inches;
1 / 24 = 24 / (D + 24); then solve (D + 24) = 24x24 so D = 576 - 24;
D = 552 inches or 46 feet.

E) 3 cm / 60 cm = 1 meter / (X + 60 cm). 3/60 = 1 meter / (X + 0.6 m) then
3(X + 0.60) = 60 ; 3X + 1.8 = 60 ; 3X = 58.2 meters so **X = 19.4 meters.**

F) 2 meters / 48 meters = X / 548 meters ; 1/24 = X/548 ; X = 548 / 24 ; so **X = 22.8.**

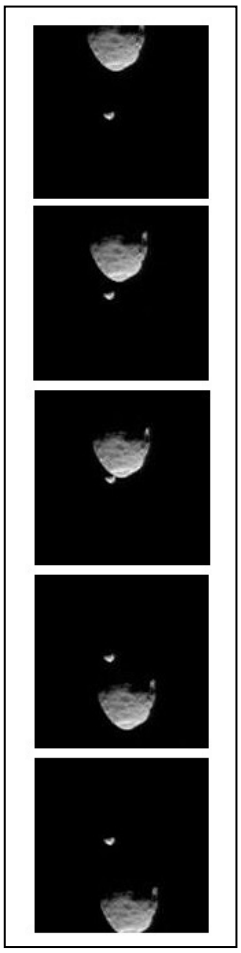
Problem 2: Which triangles must have the same measure for the indicated angle **a** ?

Answer: Because the triangle (D) has the side proportion 1-inch /24-inches = 1/24 and triangle (F) has the side proportion 2 meters / 48 meters = 1/24 these two triangles, **D and F, have the same angle measurement for angle a**

Problem 3: The Sun is 400 times the diameter of the Moon. Explain why they appear to have the same angular size if the moon is at a distance of 384,000 kilometers, and the sun is 150 million kilometers from Earth?

Answer: From one of our similar triangles, the long vertical side would represent the diameter of the sun; the short vertical side would represent the diameter of the moon; the angle **a** is the same for both the sun and moon if the distance to the sun from Earth were 400x farther than the distance of the moon from Earth. Since the lunar distance is 384,000 kilometers, the sun must be at a distance of 154 million kilometers, which is close to the number given.

The Occulting Moons of Mars



The larger of the two moons of Mars, Phobos, passes directly in front of the other, Deimos, in a new series of sky-watching images from NASA's Mars rover Curiosity. Large craters on Phobos are clearly visible in these images from the surface of Mars. No previous images from missions on the surface caught one moon eclipsing the other.

Deimos (small image), and Phobos (large image), are shown together as they actually were photographed by the Mast Camera (Mastcam) on NASA's Mars rover Curiosity on August 1, 2013.

How do we figure out how big something will look when it's far away? We draw a scale model of the object showing its diameter and its distance as the two sides of a triangle. The angle can then be measured. As the distance to the object increases, the 'angular size' of the object will decrease proportionately. A simple proportion can then be written that relates the angles to the lengths of the sides:

$$\frac{\text{Apparent Angle in degrees}}{\text{57.3 degrees}} = \frac{\text{True diameter in kilometers}}{\text{Distance in kilometers}}$$

Let's see how this works for estimating the sizes of the moons of Mars as viewed from the Curiosity rover!

Problem 1 – Earth's moon is located 370,000 km from the surface of earth, and has a diameter of 3476 km. About how many degrees across does the lunar disk appear in the sky?

Problem 2 - Deimos has a diameter of 7.5 miles (12 kilometers) and was 12,800 miles (20,500 kilometers) from the rover at the time of the image. Phobos has a diameter 14 miles (22 kilometers) and was 3,900 miles (6,240 kilometers) from the rover at the time of the image. What are the angular diameters of Phobos and Diemos as seen by the Curiosity rover?

Problem 3 – Mars is located 227 million kilometers from the sun, and the sun has a diameter of 1,400,000 kilometers. What is the angular diameter of the sun as viewed from Mars?

Problem 4 – Occasionally, Phobos and Diemos pass across the face of the sun as viewed from the surface of Mars. Will the moons create a full eclipse of the sun in the same way that Earth's moon covers the full face of the sun as viewed from Earth?

NASA Rover Gets Movie as a Mars Moon Passes Another
http://www.nasa.gov/mission_pages/msl/news/msl20130815.html
August 15, 2013

Problem 1 – Earth's moon is located 370,000 km from the surface of earth, and has a diameter of 3476 km. About how many degrees across does the lunar disk appear in the sky?

Answer: Apparent size = 57.3 x (3476/370000) = **0.5 degrees**.

Problem 2 - Deimos has a diameter of 7.5 miles (12 kilometers) and was 12,800 miles (20,500 kilometers) from the rover at the time of the image. Phobos has a diameter 14 miles (22 kilometers) and was 3,900 miles (6,240 kilometers) from the rover at the time of the image. What are the angular diameters of Phobos and Diemos as seen by the Curiosity rover?

Answer: Diemos: 57.3 x (12/20500) = **0.033 degrees**
Phobos: 57.3 x (22/6240) = **0.2 degrees**.

Note: Diemos appears to be about .033/.2 = 1/6 the diameter of Phobos in the sky.

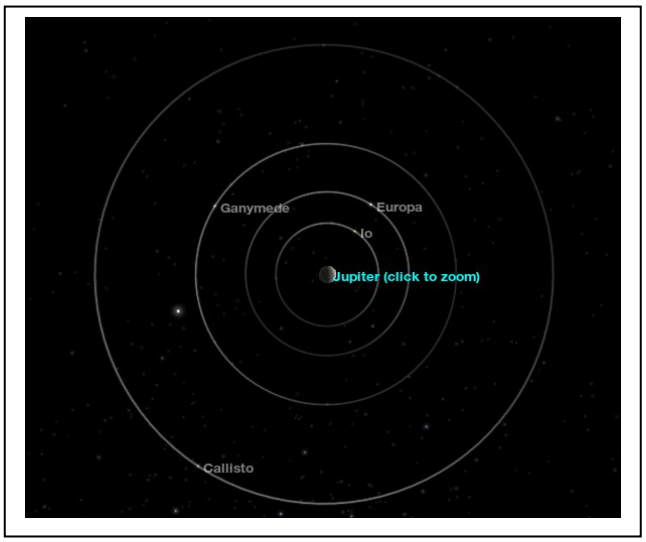
Problem 3 – Mars is located 227 million kilometers from the sun, and the sun has a diameter of 1,400,000 kilometers. What is the angular diameter of the sun as viewed from Mars?

Answer: 57.3 x (1400000/227,000,000) = 57.3 x (1.4/227) = **0.35 degrees**.

Problem 4 – Occasionally, Phobos and Diemos pass across the face of the sun as viewed from the surface of Mars. Will the moons create a full eclipse of the sun in the same way that Earth's moon covers the full face of the sun as viewed from Earth?

Answer: Diemos is only 0.033/0.35 = 1/11 the diameter of the sun in the sky as viewed from Mars, so it does not cover the full disk of the sun. As it passes across the sun it would look like a large dark spot. Phobos is 0.2/0.35 = ½ the diameter of the sun in the sky and it would not produce an eclipse like our moon does. It would look like a large black spot ½ the diameter of the sun.

Apparent Sizes of Objects from Jupiter's Moon Europa



In the future, rovers will land on the moons of Jupiter just as they have on Mars. Rover cameras will search the skies for the disks of nearby moons. One candidate for landing is Europa with its ocean of water just below its icy crust.

The figure to the left shows the orbits of the four largest moons near Europa. How large will they appear in the Europan sky compared to the Earth's moon seen in our night time skies? The apparent angular size of an object in arcminutes is found from the proportion:

$$\frac{\text{Apparent size}}{3438 \text{ arcminutes}} = \frac{\text{Diameter (km)}}{\text{Distance (km)}}$$

The table below gives the diameters of each 'Galilean Moon' together with its minimum and maximum distance from Europa. Jupiter has a diameter of 142,000 km. Europa has a diameter of 2960 km. Callisto's diameter is 4720 km and Ganymede's diameter is 5200 km. The sun hass a diameter of 1.4 million km. Our Moon has a diameter of 3476 km.

	Shortest Distance (km)	Size (arcminutes)	Longest Distance (km)	Size (arcminutes)
Europa to Callisto	1.2 million		2.6 million	
Europa to Io	255,000		1.1 million	
Europa to Ganymede	403,000		1.7 million	
Europa to Jupiter	592,000		604,000	
Europa to Sun	740 million		815 million	
Earth to Moon	356,400		406,700	

Problem 1 – From the information in the table, calculate the maximum and minimum angular size of each moon and object as viewed from Europa.

Problem 2 – Compared to the angular size of the sun as seen from Jupiter, are any of the moons viewed from Europa able to completely eclipse the solar disk?

Problem 3 – Which moons as viewed from Europa would have about the same angular diameter as Earth's moon viewed from Earth?

Problem 4 – Io is closer to Jupiter than Europa. That means that Io will be able to pass across the face of Jupiter as viewed from Europa. . In terms of the maximum and minimum sizes, about how many times smaller is the apparent disk of Io compared to the disk of Jupiter?

Problem 1 – From the information in the table, calculate the maximum and minimum angular size of each moon and object as viewed from Europa. Jupiter has a diameter of 142,000 km. Io has a diameter of 3620 km. Callisto's diameter is 4720 km and Ganymede's diameter is 5200 km. The sun has a diameter of 1.4 million km. Our Moon has a diameter of 3476 km. Answer: see below.

	Shortest Distance (km)	Size (arcminutes)	Longest Distance (km)	Size (arcminutes)
Europa to Callisto	1.2 million	**13.5**	2.6 million	**6.2**
Europa to Io	255,000	**48.8**	1.1 million	**11.3**
Europa to Ganymede	403,000	**44.4**	1.7 million	**10.5**
Europa to Jupiter	592,000	**824.7**	604,000	**808.3**
Europa to Sun	740 million	**6.5**	815 million	**5.9**
Earth to Moon	356,400	**33.5**	406,700	**29.4**

Problem 2 – Compared to the angular size of the sun as seen from Jupiter, are any of the moons viewed from Europa able to completely eclipse the solar disk?

Answer: The solar disk has an angular size between 5.9 and 6.5 arcminutes. Only Callisto at its longest distance has an angular diameter (6.2 arcminutes) close to the solar diameter and so a complete eclipse is possible.

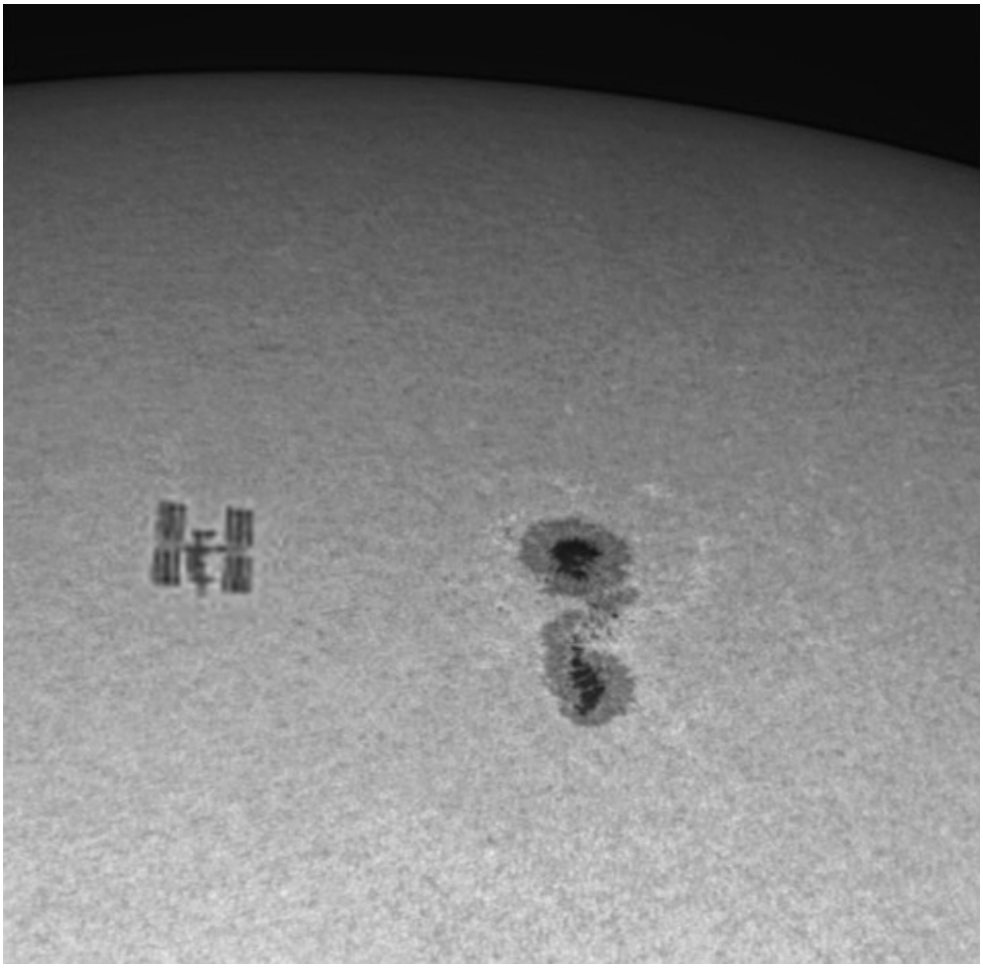
Problem 3 – Which moons as viewed from Europa would have about the same angular diameter as Earth's moon viewed from Earth?

Answer: Io and Ganymede have angular diameters between 11 and 49 arcminutes. Our moon has a range of sizes between 29.4 and 33.5 arcminutes, so at some points in the orbits of Io and Ganymede, they will appear about the same size as our moon dies in our night sky.

Problem 4 – Io is closer to Jupiter than Europa. That means that Io will be able to pass across the face of Jupiter as viewed from Europa. In terms of the maximum and minimum sizes, about how many times smaller is the apparent disk of Io compared to the disk of Jupiter?

Answer: When Jupiter appears at its largest (824 arcminutes) and Io at its smallest (11.3 arcminutes) Jupiter will be 73 times bigger than the disk of Io. When Jupiter is at its smallest (808 arcminutes) and Io is at its largest (48.8 arcminutes) Jupiter will appear to be 16.6 times larger than the disk of Io.

ISS and a Sunspot - Angular Scales



Photographer John Stetson took this photo on March 3, 2010 by carefully tracking his telescope at the right moment as the International Space Station passed across the disk of the sun.

The angular size, θ, in arcseconds of an object with a length of L meters at a distance of D meters is given by

$$\theta = 206265 \frac{L}{D}$$

Problem 1 - The ISS is 108 meters wide, and was at an altitude of 350 km when this photo was taken. If the sun is at a distance of 150 million kilometers, how large is the sunspot in A) kilometers? B) compared to the size of Earth if the diameter of Earth is 13,000 km?

Problem 2 - The sun has an angular diameter of 0.5 degrees. If the speed of the ISS in its orbit is 10 km/sec, how long did it take for the ISS to cross the face of the sun as viewed from the ground on Earth?

Problem 1 - The ISS is 108 meters wide, and was at an altitude of 350 km when this photo was taken. If the sun is at a distance of 150 million kilometers, how large is the sunspot in A) kilometers? B) compared to the size of Earth if the diameter of Earth is 13,000 km?

Answer: As viewed from the ground, the ISS subtends an angle of
Angle = 206265 x (108 meters/350,000 meters) so
Angle = 63 arcseconds.

At the distance of the sun, which is 150 million kilometers, the angular size of the ISS corresponds to a physical length of
L = 150 million kilometers x (63/206265) so
L = 46,000 kilometers.

The sunspot is comparable in width to that of the ISS and has a length about twice that of the ISS so its size is about **46,000 km x 92,000 km**.

As a comparison, Earth has a diameter of 13,000 km so the sunspot is about **3 times the diameter of Earth in width, and 6 times the diameter of Earth in length.**

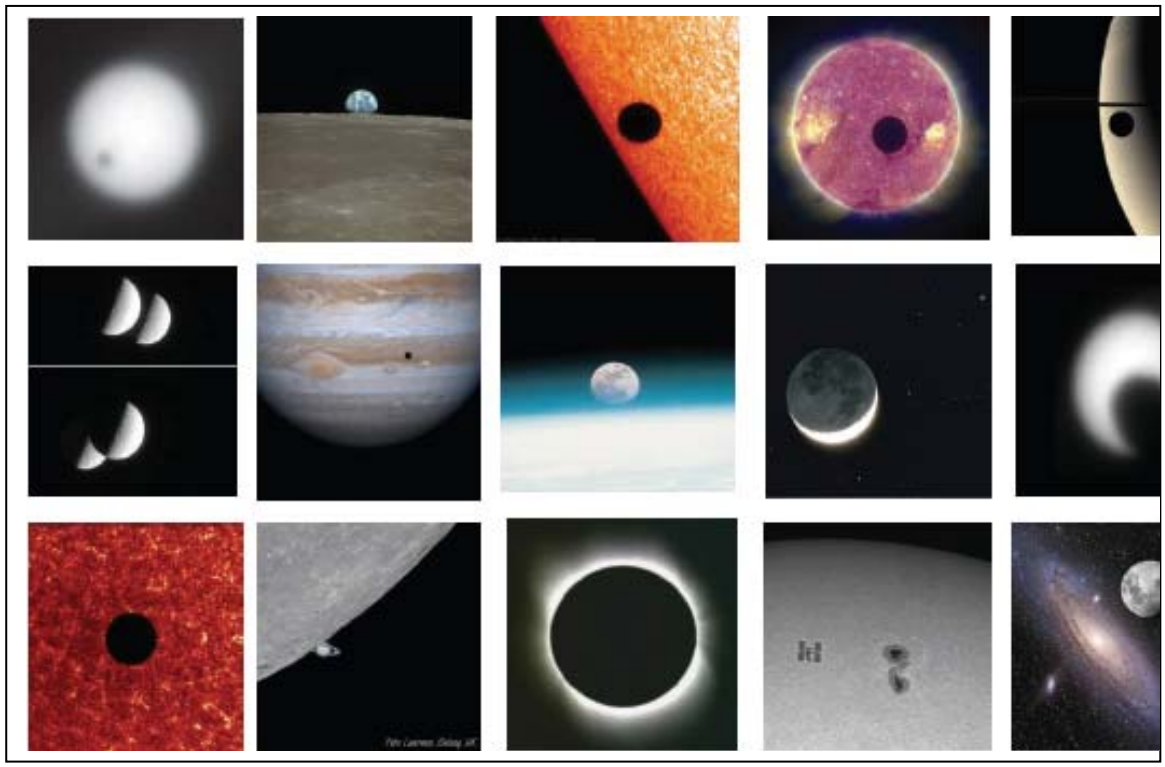
Problem 2 - The sun has an angular diameter of 0.5 degrees. If the speed of the ISS in its orbit is 10 km/sec, how long did it take for the ISS to cross the face of the sun as viewed from the ground on Earth?

Answer: From the ground, convert the speed of the ISS in km/sec to an angular speed in arcseconds/sec.

In one second, the ISS travels 10 km along its orbit. From the ground this corresponds to an angular distance of
Angle = 206265 x (10 km/350 km)
= 5900 arcseconds.

The speed is then 5900 arcseconds/sec. The diameter of the sun is 0.5 degrees which is 30 arcminutes or 1800 arcseconds. To cover this angular distance, the ISS will take
T = 1800 arcseconds / (5900 arcseconds/s) so
T = 0.3 seconds!

Eclipses, Transits and Occultations



The images above show a variety of transits, eclipses and occultations. The images are labeled from left to right as (Top Row) A, B, C, D, E; (Middle Row) F, G, H, I, J, (Bottom Row), K, L, M, N, O. Using the definitions of these three astronomical events, identify which images go along with each of the three types of events. One example is shown below.

A) Deimos and the Sun _____

B) Moon and Earth _____

C) Sun and Mercury _____

D) Sun and Moon ___Transit_

E) Rhea and Saturn _____

F) Rhea and Dione _____

G) Jupiter and Io _____

H) Earth and Moon _____

I) Moon and Star Cluster _____

J) Sun and Phobos _____

K) Sun and Venus _____

L) Moon and Saturn _____

M) Sun and Moon _____

N) Sun and Space Station _____

O) Moon and galaxy _____

A) **Transit** of Deimos across the Sun seen by Opportunity Rover
B) **Occultation** of Earth by the Moon seen by Apollo-8 astronauts
C) **Transi**t of Mercury across sun seen by TRACE?
D) **Transit** of Moon across Sun seen by STEREO satellite
E) **Transit** of satellite Rhea across Saturn seen by Cassini spacecraft
F) Rhea **occulting** Dione seen by Cassini spacecraft near Saturn
G) Io **transiting** Jupiter seen by Galileo spacecraft
H) Earth **occulting** moon seen by Space Shuttle astronauts
I) Moon **occulting** the Pleiades star cluster
J) Phobos **transiting** sun seen by Opportunity Rover on Mars.
K) Venus **transiting** the sun seen by TRACE satellite.
L) Moon **occulting** Saturn
M) **Eclipse** of the sun by the moon.
N) **Transit** of the space station across sun
O) Hypothetical **occultation** of the andromeda galaxy by the moon.

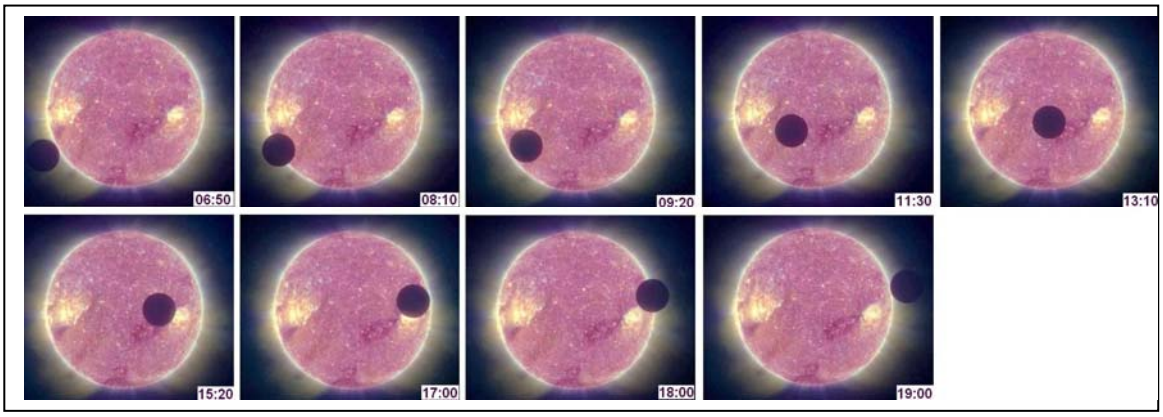
Special Image credits:

Moon occulting Saturn - Pete Lawrence (http://www.digitalskyart.com/)
pete.**lawrence**@pbl33.co.uk.

Space Station transiting sun - John Stetson

Moon and Andromeda - Adam Block (ngc1535@caelumobservatory.com
- Tim Puckett (tpuckett@mindspring.com)

Moon and Star Cluster - Jerry Lodriguss (jerry5@astropix.com)
http://www.astropix.com/

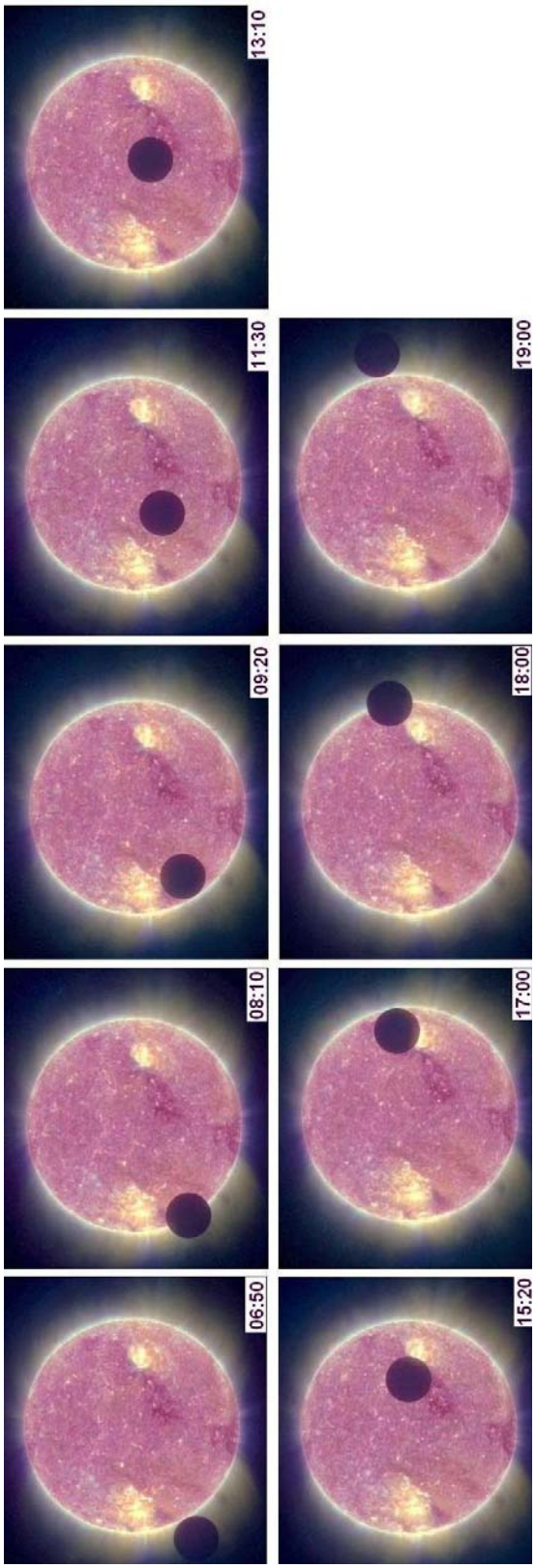


NASA's STEREO-B satellite is in an orbit around the sun at the same distance as Earth. On February 25, 2007 it took a series of pictures of the sun during the time when the moon was in transit across the sun, and when the satellite was 1.7 million km from the moon. The normal Earth-Moon distance is 380,000 km. By comparison, the distance to the sun from Earth is 149 million km.

Problem 1 - If the angular diameter of the sun was 2100 arcseconds at the time of the transit as viewed by STEREO-B, what was the diameter of the moon as viewed by STEREO-B in A) arcseconds? B) degrees?

Problem 2 - By what percentage was the sun's light dimmed during the times when the full, circular, lunar disk covered the solar surface in these images?

Problem 3 - Based on the sequence of images in the above series, with the Universal Time (hour:minutes) indicated in the lower right corner of each image, draw the light curve of this lunar transit from start to finish in terms of the percentage of sunlight visible by the STEREO-B satellite, from 93% to 100%, and the Universal Time in decimal hours since 06:00.



Problem 1 - Method 1: The diameter of the solar disk in each image is about 21 millimeters, so the scale of the images is 2100 arcseconds/21 mm = 100 arcseconds/mm. The lunar disk measures about 4mm, so the angular diameter is 4mm (100 asec/mm) = **400 arcseconds**.

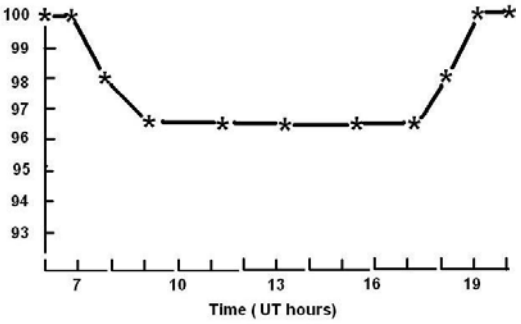
Method 2: Using the trigonometric formula
$\text{Tan}(\theta) = D/R$
$\text{Tan}(\theta) = 3400/1{,}700{,}000 = 0.002$
So $\theta = 0.11$ degrees = **410 arcseconds**.

So the diameter of the moon is about **400 arcseconds**

Problem 2 - By what percentage was the sun's light dimmed during the times when the full, circular, lunar disk covered the solar surface in these images?

Answer: By the ratio of their circular areas.
$100\% \times (400/2100)^2$ = **3.6%**

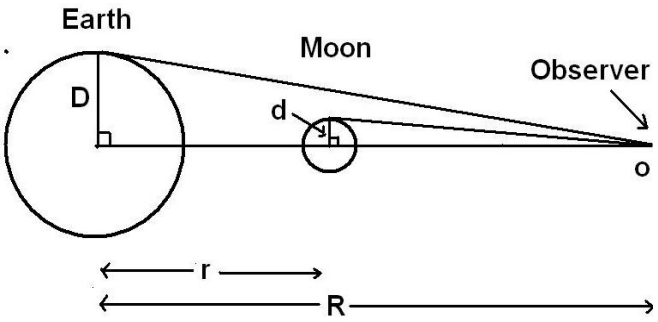
Problem 3 - Answer: See graph below:



Movies of the transit can be found at the STEREO website

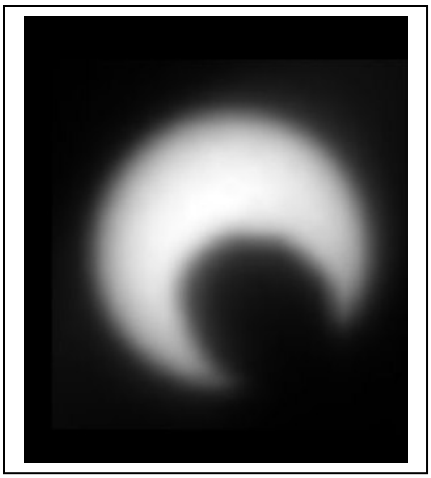
http://stereo.gsfc.nasa.gov/gallery/item.php?id=selects&iid=8

Earth and Moon Angular Sizes



In space, your perspective can change in complicated ways that sometimes go against Common Sense unless you 'do the math'. This happens very commonly when we are looking at one object pass across the face of another. Even though the Moon is 1/4 the diameter of Earth, the simple ratio of their apparent diameters will depend on how far from them YOU are when you see them.

An important 'skinny triangle' relationship for triangles states that, if the angle is less than 1 degree (< 0.017 radians), the angle measure in radians equals very nearly the sine of the angle, which is just the ratio of the opposite side to the hypotenuse: $\theta = D/R$ where D is the radius of the object in kilometers, R, is the distance to the object in kilometers, and θ is the angular radius of the object in radians. For instance, the Moon is located r = 384,000 km from Earth and it has a radius of d = 1,738 km, so its angular radius is 1,738/384,000 = 0.0045 radians. Since 1 radian = 57.3 degrees, the angular radius of the Moon is 0.0045 x 57.3 = 0.26 degrees, so its diameter is 0.52 degrees as viewed from Earth.



Problem 1 - In the figure above, assume that the diameter of the Moon is less than 1 degree when spotted by the spacecraft located at 'O'. What is the angular diameter of: A) the Moon, θ_m, in terms of d and R? B) The Earth, θ_e, in terms of D and R? and C) What is the ratio of the angular diameter of the Moon to the Earth in terms of d, D, r and R?

Problem 2 - A spacecraft is headed directly away from the Moon along the line connecting the center of Earth and the Moon. At what distance will the angular diameter of the Moon equal the angular diameter of Earth?

Problem 3 - The figure to the top left shows the martian satellite Phobos passing across the disk of the sun as viewed from the surface of Mars by the Rover Opportunity. If the ratio of the diameters is 1/2, and if r = 228 million km, d = 10 km, and D=696,000 km, about how far from Phobos was Opportunity at the time the photo was taken?

Problem 4 - The Deep Impact spacecraft observed the Moon pass across the disk of Earth as shown in the photo to the bottom left. The ratio of the disk diameters is 1/3.9, and if r = 384,000 km, d=1,786 km and D = 6,378 km, about how far from Earth, R, was the spacecraft?

Problem 5 - As the distance, R, becomes very large, in the limit, what does the angular ratio of the disk approach in the equation defined in Problem 1?

Problem 1 - What is the angular diameter of:
A) the Moon, θ_m, in terms of d and R? Answer:

$$\theta m = \frac{2\pi d}{R - r}$$

B) The Earth, θ_e, in terms of D and R? Answer:

$$\theta e = \frac{2\pi D}{R}$$

C) What is the ratio of the angular diameter of the Moon to the Earth in terms of d, D, r and R? Answer:

$$\frac{\theta m}{\theta e} = \frac{R}{R - r}\frac{d}{D}$$

Problem 2 - At what distance will the angular diameter of the Moon equal the angular diameter of Earth? Answer:

$$1 = \frac{R}{R - 384{,}000}\frac{(1{,}786)}{(6{,}378)}$$

so 3/4 R = 384,000 km, and so R = 512,000 km.

Problem 3 - If the ratio of the diameters is 1/2, and if r = 228 million km, d = 10 km, and D=696,000 km, about how far from Phobos was Opportunity at the time the photo was taken? Answer:

$$\frac{1}{2} = \frac{R}{R - 228 million}\frac{(10km)}{(696{,}000km)}$$

so 34799 R = 228 million km, and so R = 6,552 km.

Problem 4 - The ratio of the disk diameters is 1/3.9, and if r = 384,000 km, d=1,786 km and D = 6,378 km, about how far from Earth, R, was the spacecraft? Answer:

$$\frac{1}{3.9} = \frac{R}{R - 384{,}000km}\frac{(1{,}786km)}{(6{,}378km)}$$

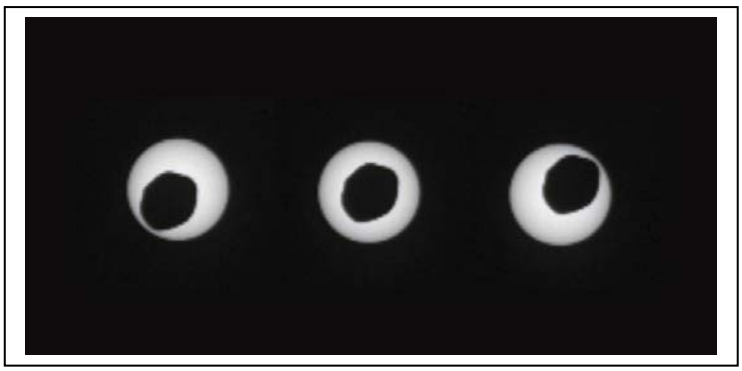
so 0.02 R = 384,000 km , and so R = 19.2 million km.
Note: the actual distance was about 30 million km for the photo shown in this problem.

Problem 5 - As the distance, R, becomes very large, in the limit, what does the angular ratio of the disk approach in the equation defined in Problem 1? Answer: As R becomes much, much larger than r (e.g the limit of r approaches infinity), then the equation approaches

$$\frac{\theta m}{\theta e} = \frac{d}{R}\frac{R}{D}$$

and since the 'R' terms cancel, we get the angular ratio approaching the physical ratio d/D of the diameters of the two bodies. In other words, although the apparent angular sizes change rapidly when you are very close to the bodies and the value of R is comparable to 'r', at very great distances, the angular ratio approaches a constant value d/D. This has many practical consequences in the search for planets around other stars as they' transit' their stars.

Transit of Phobos Across the Sun Viewed from Mars



This set of three images shows views three seconds apart as the larger of Mars' two moons, Phobos, passed directly in front of the sun as seen by NASA's Mars rover Curiosity.

Curiosity photographed this annular eclipse with the rover's Mast Camera on August 17, 2013 or 'Sol 369' by the Mars calendar.

Curiosity paused during its drive to Mount Sharp to take a set of observations that the camera team carefully calculated to record this celestial event. Because this eclipse occurred near mid-day at Curiosity's location on Mars, Phobos was nearly overhead. This timing made Phobos' silhouette larger against the sun -- as close to a total eclipse of the sun as is possible from Mars.

Angular size is given by $\Theta = 57.3 \times \frac{\text{Diameter (km)}}{\text{Distance (km)}}$ degrees

Problem 1 – At the time of the transit, Phobos which has a diameter of 11 km, was 6000 km from the surface of Mars, and Mars was 235 million km from the Sun. What are the angular diameters of the Sun and Phobos viewed from the surface of Mars if the diameter of the Sun is 1.4 million km? How large are these angles in minutes of arc?

Problem 2 – Phobos orbits at a distance of 9,400 km from the center of Mars at a speed of 2.1 km/sec. As viewed from the surface of Mars (6000 km), how fast is it traveling across the sky in arcminutes/second?

Problem 3 – To the nearest second, how long will it take for Phobos to travel completely across the disk of the sun?

Annular Eclipse of the Sun by Phobos, as Seen by Curiosity
http://www.nasa.gov/mission_pages/msl/news/msl20130828.html
Aug. 28, 2013

Problem 1 – At the time of the transit, Phobos which has a diameter of 11 km, was 6000 km from the surface of Mars, and Mars was 235 million km from the Sun. What are the angular diameters of the Sun and Phobos viewed from the surface of Mars if the diameter of the Sun is 1.4 million km? How large are these angles in minutes of arc?

Answer: Phobos: 57.3 x (11/6000) = **0.10 degrees**
or 0.1 degrees x 60 = **6 minutes of arc**

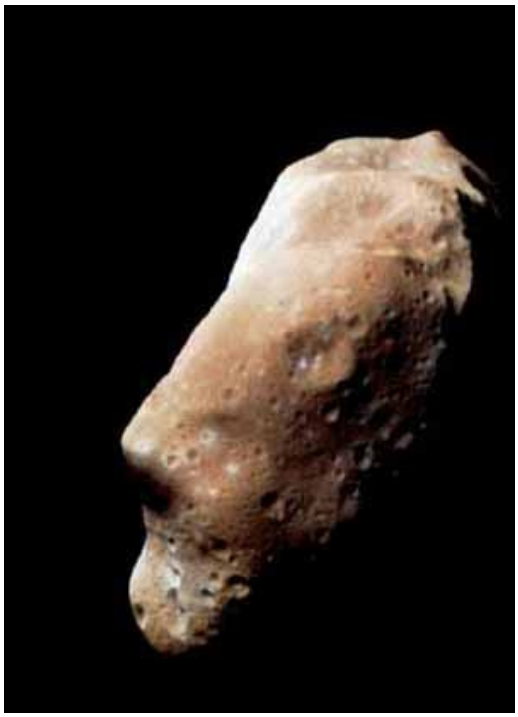
Sun: 57.3 x (1.4/235) = **0.34 degrees** or 0.34 x 60 = **20 minutes of arc**.

Problem 2 – Phobos orbits at a distance of 9,400 km from the center of Mars at a speed of 2.1 km/sec. As viewed from the surface of Mars (6000 km), how fast is it traveling across the sky in arcminutes/second?

Answer: If the object is located 6000 km from the surface and moves 2.1 km, it will appear to cover an angle of 57.3 x (2.1/6000) = 0.02 degrees. There are 60 arcminutes in 1 degree, so this angle is 1.2 arcminutes. Since this distance is traveled in 1 second, the angular speed is **1.2 arcminutes /second**.

Problem 3 – To the nearest second, how long will it take for Phobos to travel completely across the disk of the sun?

Answer: The diameter of the sun is 20 arcminutes and the diameter of Phobos is 6 arcminutes. When the center of the disk of Phobos is 3 arcminutes from the eastern edge of the sun, it is just touching the solar disk and about to start its transit. When it is 3 arcminutes from the western edge of the sun, it has just finished its transit, so the total distance it has to travel is 3 arcminutes + 20 arcminutes + 3 arcminutes or 26 arcminutes. It travels at a speed of 1.2 arcminutes per second, to it will cover 26 arcminutes in about 26/1.2 = **22 seconds**.


Asteroid Gaspara

Astronomers studying the asteroid 24-Themis detected water-ice and carbon-based organic compounds on the surface of the asteroid.

NASA detects, tracks and characterizes asteroids and comets passing close to Earth using both ground and space-based telescopes.

NASA is particularly interested in asteroids with water ice because this resource could be used to create fuel for interplanetary spacecraft.

On October 7, 2009, the presence of water ice was confirmed on the surface of this asteroid using NASA's Infrared Telescope Facility. The surface of the asteroid appears completely covered in ice. As this ice layer is sublimated (goes directly from solid to gaseous state) it may be getting replenished by a reservoir of ice under the surface. The orbit of the asteroid varies from 2.7 AU to 3.5 AU (where 1 AU is the 150 million km distance from Earth to the sun) so it is located within the asteroid belt. The asteroid is 200 km in diameter, has a mass of 1.1×10^{19} kg, and a density of 2,800 kg/m^3 so it is mostly rocky material similar in density to Earth's.

By measuring the spectrum of infrared sunlight reflected by the object, the NASA researchers found the spectrum consistent with frozen water and determined that 24 Themis is coated with a thin film of ice. The asteroid is estimated to lose about 1 meter of ice each year in a process called sublimation, so there must be a sub-surface reservoir to constantly replace the evaporating ice.

Problem 1 – Assume that the asteroid has a diameter of 200 km. How many kilograms of water ice are present in a layer 1-meter thick covering the entire surface, if the density of ice is 1,000 km/meter^3? (Hint: Volume = Surface area x thickness)

Problem 2 – Suppose that only 1% by volume of the 1-meter-thick 'dirty' surface layer is actually water-ice and that it sublimates 1 meter per year, what is the rate of water loss in kg/sec?

Problem 1 – Assume that the asteroid has a diameter of 200km. How many kilograms of water ice are present in a layer 1-meter thick covering the entire surface, if the density of ice is 1,000 km/meter^3? (Hint: Volume = Surface area x thickness)

Answer: Volume = surface area x thickness.

$SA = 4 \pi r^2$

$= 4 (3.14) (100{,}000 \text{ meters})^2$

$= 1.3 \times 10^{11} \text{ meters}^2$

Volume = $1.3 \times 10^{11} \text{ meters}^2 \times 1 \text{ meter}$

$= 1.3 \times 10^{11} \text{ meters}^3$

Mass of water = density x volume

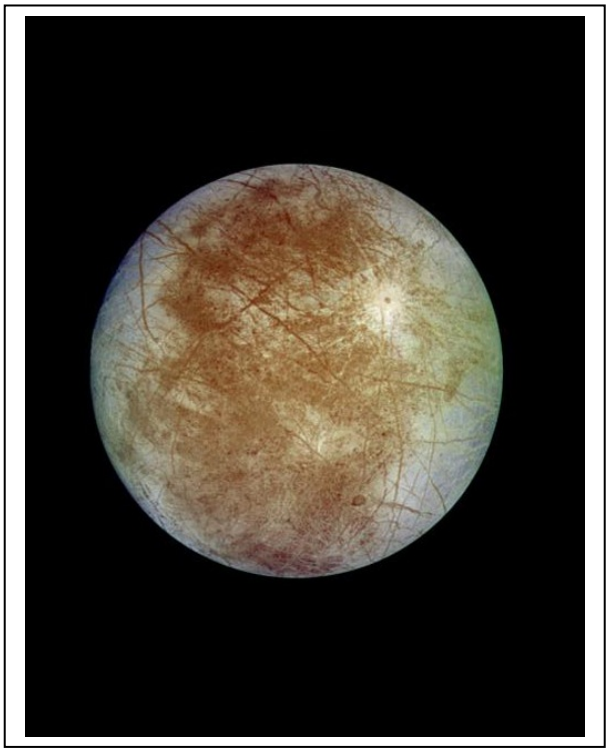
$= 1{,}000 \text{ kg/meter}^3 \times 1.3 \times 10^{11} \text{ meter}^3$

$= \mathbf{1.3 \times 10^{14} \text{ kg}}$ (or 130 billion tons)

Problem 2 – Suppose that only 1% by volume of the 'dirty' 1-meter-thick surface layer is water-ice and that it evaporates 1 meter per year, what is the rate of water loss in kg/sec?

Answer: The mass of water in the outer 1-meter layer is 1% of 1.3×10^{14} kg or 1.3×10^{12} kg. Since 1 year = 365 days x 24h/day x 60m/hr x 60 sec/min = 3.1×10^7 seconds, the mass loss is just 1.3×10^{12} kg / 3.1×10^7 sec = **42,000 kg/sec or 42 tons/sec**.

Ice or Water?



There are no known terrestrial organisms that can exist at a temperature lower than the freezing temperature of water without special adaptations. It is also believed that liquid water is a crucial ingredient to the chemistry that leads to the origin of life. To change water-ice to liquid water requires energy.

First, you need energy to raise the ice from wherever temperature it is, to 0 Celsius. This is called the Specific Heat and is 2.04 kiloJoules/kilogram C

Then you need enough energy added to the ice near 0 C to actually melt the ice by increasing the kinetic energy of the water molecules so that their hydrogen bonds weaken, and the water stops acting like a solid. This is called the Latent Heat of Fusion and is 333 kiloJoules/kilogram.

Let's see how this works!

Example 1: You have a 3 kilogram block of ice at a temperature of -20 C. The energy needed to raise it by 20 C to a new temperature of 0 C is Eh = 2.04 kiloJoules/kg C) x 3 kilograms x (20 C) = 2.04 x 3 x 20 = 122 kiloJoules.

Example 2: You have a 3 kilogram block of ice at 0 C and you want to melt it completely into liquid water. This requires Em = 333 kiloJoules/kg x 3 kilograms = 999 kiloJoules.

Example 3: The total energy needed to melt a 3 kilogram block of ice from -20 C to 0C is E = Eh + Em = 122 kiloJoules + 999 kiloJoules = 1,121 kiloJoules.

Problem 1 - On the surface of the satellite Europa (see NASA's Galileo photo above), the temperature of ice is -220 C. What total energy in kiloJoules is required to melt a 100 kilogram block of water ice on its surface? (Note: Calculate Eh and Em separately then combine them to get the total energy.)

Problem 2 - To a depth of 1 meter, the total mass of ice on the surface of Europa is 2.8×10^{16} kilograms. How many Joules would be required to melt the entire surface of Europa to this depth? (Note: Calculate Eh and Em separately then combine them to get the total energy. Then convert kiloJoules to Joules)

Problem 3 - The sun produces 4.0×10^{26} Joules every second of heat energy. How long would it take to melt Europa to a depth of 1 meter if all of the sun's energy could be used? (Note: The numbers are BIG, but don't panic!)

Problem 1 - On the surface of the satellite Europa, the temperature of ice is -220 C. What total energy is required to melt a 100 kilogram block of water ice on its surface?

Answer: You have to raise the temperature by 220 C, then
E = 2.04 x 220 x 100 + 333 x 100
= 44,880 + 33,300
= **78,180 kiloJoules.**

Problem 2 - To a depth of 1 meter, the total mass of ice on the surface of Europa is 2.8×10^{16} kilograms. How many joules would be required to melt the entire surface of Europa to this depth?

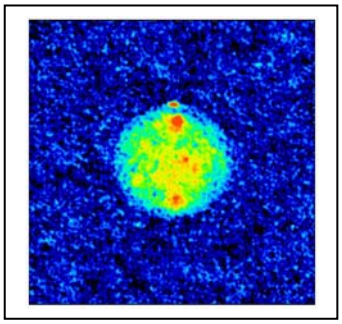
Note: The radius of Europa is 1,565 km. The surface area is $4 \times \pi \times (1{,}565{,}000\ m)^2 = 3.1 \times 10^{13}$ meters^2. A 1 meter thick shell at this radius has a volume of 3.1×10^{13} meters^2 x 1 meter = 3.1×10^{13} meters^3. The density of water ice is 917 kilograms/m^3 ,so this ice layer on Europa has a mass of $3.1 \times 10^{13} \times 917 = 2.8 \times 10^{16}$ kilograms.

Energy = $(2.04 \times 220 + 333) \times 2.8 \times 10^{16}$ kg
= 2.2×10^{19} kiloJoules
= **2.2×10^{22} Joules.**

Problem 3 - The sun produces 4.0×10^{26} Joules every second of heat energy. How long would it take to melt Europa to a depth of 1 meter if all of the sun's energy could be used?

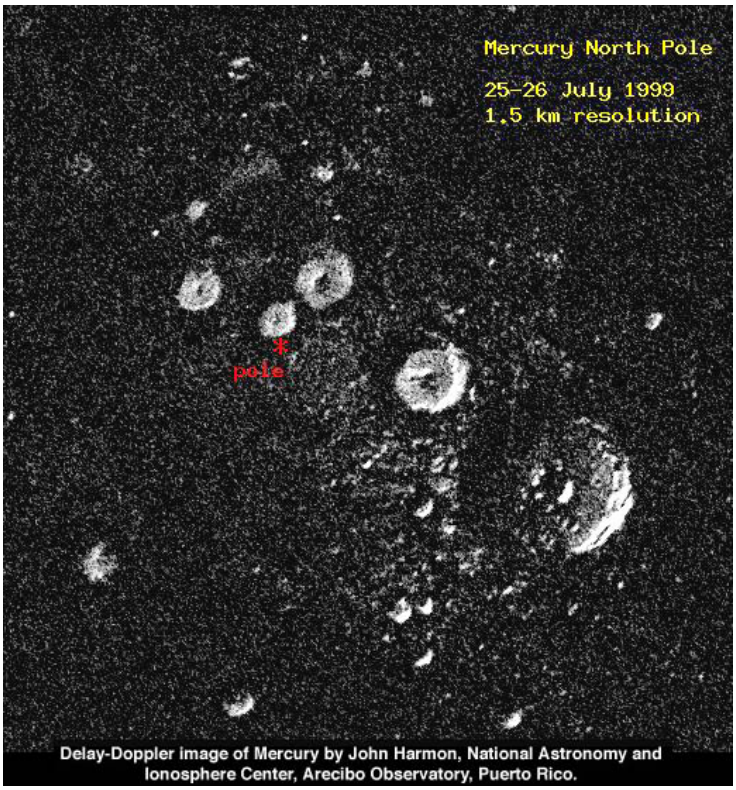
Answer: Time = Amount / Rate
= 2.2×10^{22} Joules / 4.0×10^{26} Joules
= **0.000055 seconds.**

Is there Ice on Mercury ?



The NASA MESSENGER spacecraft performed its first flyby of Mercury on January 14, 2008. In addition to mapping the entire surface of this planet, one of its goals is to shed new light on the existence of ice under the polar regions of this hot planet. Ice on Mercury? It's not as strange as it seems!

In 1991, Duane Muhleman and her colleagues from Caltech and the Jet Propulsion Laboratory, created the first radar map of Mercury. The image, shown here, contained a stunning surprise. The bright (red) dot at the top of the moon image to the left indicates strong radar reflection at Mercury's North Pole, resembling the strong radar echo seen from the ice-rich polar caps of Mars.



In 1999, astronomer John Harmon at the Arecibo Observatory in Puerto Rico, repeated the 1991 study, this time using the powerful microwave beam of the Arecibo Radio Telescope. The microwave energy reflected from mercury and was detected by the VLA radio telescope array in New Mexico, where a new image was made.

The radio-wavelength image to the left shows Mercury's North Polar Region at very high resolution. The image is 370 kilometers wide by 400 kilometers tall.

All the bright features are believed to be deposits of frozen water ice, at least several meters thick in the permanently shaded floors of the craters.

Reference: Harmon, Perillat and Slade, 2001, Icarus, vol 149, p.1-15

Problem 1 - From the information provided in the essay, what is the scale of the image in kilometers per millimeter?

Problem 2 - Measure the diameters of the craters, in kilometers, and estimate the total surface area covered by the large white patches in A) square kilometers and B) square meters.

Problem 3 - Suppose the icy deposit is mixed into the Mercurian surface to a depth of 10 meters. What is the total volume of the ice within the craters you measured in cubic meters?

Problem 4 - Suppose half of the volume is taken up by rock. What is the total remaining volume of ice?

Problem 5 - The density of ice is 917 kilograms/cubic meter. How many kilograms of ice are present?

Problem 6 - If this ice were 100% water ice, and 3.8 kilograms of water equals 1.0 gallons, how many gallons of water might be locked up in the shadowed craters of Mercury?

Answer Key:

Problem 1 - From the information provided in the essay, what is the scale of the image in kilometers per millimeter?

Answer; The image is 370 kilometers wide by 400 kilometers tall. The image is 95 millimeters wide by 104 millimeters tall. The scale is therefore about **4.0 kilometers / millimeter.**

Problem 2 - Measure the diameters of the craters, in kilometers, and estimate the total surface area covered by the large white patches in A) square kilometers and B) square meters.

Answer: Students should measure the diameters of at least the 5 large craters that form the row slanted upwards from right to left through the center of the image. Their diameters are about 90 km, 40 km, 30 km, 20 km and 25 km. The area of a circle is πR^2, so the crater areas are 6,400 km^2, 700 km^2, 314 km^2 and 490 km^2. The total area A) in square kilometers is about **7,900 km^2** or B) 7,900 x (1000 m/km) x (1000 m/km) = **7.9×10^9 $meters^2$**. Students may reasonably ask how to estimate the area of partially-filled craters such as the largest one in the image .They may use appropriate percentage estimates. For example, the largest crater is about 1/2 filled (white color in image) so its area can be represented as 6,400 x 0.5 = 3,200 km^2.

Problem 3 - Suppose the icy deposit is mixed into the Mercurian surface to a depth of 10 meters. What is the total volume of the ice within the craters you measured?

Answer: Volume = surface area x height = 7.9×10^9 $meters^2$ x 10 meters = 7.9×10^{10} $meters^2$.

Problem 4 - Suppose half of the volume is taken up by rock. What is the total remaining volume of ice?

Answer; 7.9×10^{10} $meters^2$ x 0.5 = **8.0×10^{10} $meters^2$**

Problem 5 - The density of ice is 917 kilograms/cubic meter how many kilograms of ice are present?

Answer: 8.0×10^{10} $meters^2$ x 917 $kg/meters^3$ = **7.3×10^{13} kilograms**

Problem 6 - If this ice were 100% water ice, and 3.8 kilogram of water equals 1.0 gallons, how many gallons of water might be locked up in the shadowed craters of Mercury?

Answer: 7.3×10^{13} kilograms / 3.8 kg/gallon = **1.9×10^{13} gallons or 19 trillion gallons!**

One of the best ways to explore the effects of gravity on different bodies in the solar system is to calculate what your weight wound be if you were standing on their surfaces!

Scientists use kilograms to indicate the mass of an object, and it is common for Americans to use pounds as a measure of weight. On Earth, the force that one kilogram of mass has on the bathroom scale is equal to 9.8 Newtons or a weight of 2.2 pounds.

The surface gravity of a planet or other body is what determines your weight by the simple formula W = Mg where W is the weight in Newtons, M is the mass in kilograms, and g is the acceleration of gravity at the surface in meters/sec^2. For example, on Earth, g = 9.8 m/sec, and for a person with a mass of 64 kg, the weight will be W = 64 x 9.8 = 627 Newtons. Since 9.8 Newtons equals 2.2 pounds, this person weighs 627 x (2.2/9.8) = 140 pounds.

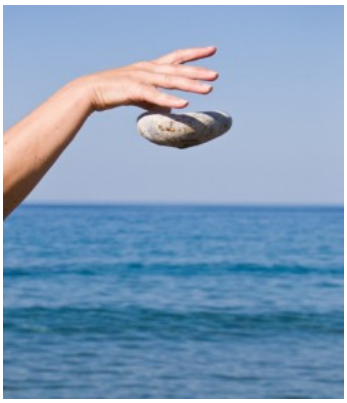
Problem 1 - Using proportional math, complete the following table to estimate the weight of a 110-pound (50 kg) person on the various bodies that have solid surfaces.

Object	Location	G (m/sec^2)	Weight (pounds)
Earth	Planet	9.8	110
Mercury	Planet	3.7	
Mars	Planet	3.7	
Io	Jupiter moon	1.8	
Moon	Earth moon	1.6	
Titan	Saturn moon	1.4	
Europa	Jupiter moon	1.3	
Pluto	Planet	0.58	
Charon	Pluto moon	0.28	
Vesta	Asteroid	0.22	
Enceladus	Saturn moon	0.11	
Miranda	Uranus moon	0.08	
Deimos	Mars moon	0.003	

Problem 1 - Using proportional math, complete the following table to estimate the weight of a 110-pound (50 kg) person on the various bodies that have solid surfaces.

Object	Location	G (m/sec^2)	Weight (pounds)
Earth	Planet	9.8	**110**
Mercury	Planet	3.7	**41.5**
Mars	Planet	3.7	**41.5**
Io	Jupiter moon	1.8	**20.2**
Moon	Earth moon	1.6	**18.0**
Titan	Saturn moon	1.4	**15.7**
Europa	Jupiter moon	1.3	**14.6**
Pluto	Planet	0.58	**6.5**
Charon	Pluto moon	0.28	**3.1**
Vesta	Asteroid	0.22	**2.5**
Enceladus	Saturn moon	0.11	**1.2**
Miranda	Uranus moon	0.08	**0.9**
Deimos	Mars moon	0.003	**0.03**

Note: For the Mars moon Deimos, which is a rocky body only 12 km (7.5 miles) in diameter, your weight would be 0.03 pounds or just ½ ounce! Astronauts that visit this moon would not 'land' on its surface but 'dock' with the moon the way that they do with visits to the International Space Station!



One of the simplest kinds of motion that were first studied carefully is that of falling. Unless supported, a body will fall to the ground under the influence of gravity. But the falling does not happen smoothly. Instead, the speed of the body increases in proportion to the elapsed time. This is called acceleration.

Near Earth's surface, the speed of a body increases 32 feet/sec (9.8 meters/sec) for every elapsed second. This is usually written as an acceleration of 32 feet/sec/sec or 32 feet/sec^2. (also 9.8 meters/sec^2) A simple formula gives you the speed of the object after an elapsed time of T seconds:

$S = 32\ T$ in feet/sec

If instead of just dropping the object, you threw it downwards at a speed of 12 feet/sec (3 meters/sec) you could write the formula as:

$S = 12 + 32T$ feet/sec

In general you could also write this by replacing the selected speed of 12 feet/sec, with a fill-in speed of S_0 to get

$S = S_0 + 32T$ feet/sec.

Problem 1 – On Earth, a ball is dropped from an airplane. If the initial speed was 0 feet/sec, how many seconds did it take to reach a speed of 130 miles per hour, which is called the Terminal Velocity? (130 mph = 192 feet/sec)

Problem 2 – On Mars, the acceleration of gravity is only 12 feet/sec^2. A rock is dropped from the edge of the huge canyon called Valles Marineris and falls 20,000 to the canyon floor. If the impact speed was measured to be 700 feet/sec (480 mph) how long did it take to impact the canyon floor?

Problem 3 – Two astronauts standing on the surface of two different objects in the solar system want to decide which object is the largest in mass. The first astronaut drops a hammer off a cliff that is exactly 5000 feet tall and measures the impact speed with a radar gun to get 100 feet/sec. It takes 8 seconds for the hammer to hit the bottom. The second astronaut drops an identical hammer off a cliff that is only 500 feet tall and also measures the impact speed at 150 feet/sec, but he accidentally gave the object a release speed of 5 feet/sec. It takes 29 seconds for the hammer to reach the ground. They can't re-do the experiments, but given this information what are the accelerations of gravity on the two bodies and which one has the highest mass?

Problem 1 – On Earth, a ball is dropped from an airplane. If the initial speed was 0 feet/sec, how many seconds did it take to reach a speed of 130 miles per hour, which is called the Terminal Velocity? (130 mph = 192 feet/sec)

Answer: 192 = 32T, so T = **6 seconds**.

Problem 2 – On Mars, the acceleration of gravity is only 12 feet/sec^2. A rock is dropped from the edge of the huge canyon called Valles Marineris and falls 20,000 to the canyon floor. If the impact speed was measured to be 700 feet/sec (480 mph) how long did it take to impact the canyon floor?

Answer: 700 = 12T, so T = **58 seconds**.

Problem 3 – Two astronauts standing on the surface of two different objects in the solar system want to decide which object is the largest in mass. The first astronaut drops a hammer off a cliff that is exactly 5000 feet tall and measures the impact speed with a radar gun to get 100 feet/sec. It takes 8 seconds for the hammer to hit the bottom. The second astronaut drops an identical hammer off a cliff that is only 500 feet tall and also measures the impact speed at 150 feet/sec, but he accidentally gave the object a release speed of 5 feet/sec. It takes 29 seconds for the hammer to reach the ground. They can't re-do the experiments, but given this information what are the accelerations of gravity on the two bodies and which one has the highest mass?

Answer: Astronaut 1. 100 = A1 x T1 T1 = 8 seconds, so A1 = 100/8 = 12 feet/sec^2.

Astronaut 2: 150 = 5 + A2 T2 T2 = 29 sec, so A2 = (150-5)/29 = 5 feet/sec^2

The acceleration measured by the second astronaut is much lower than for the first astronaut, so the first astronaut is standing on the more-massive objects. In fact, Astronaut 1 is on Mars and Astronaut 2 is on the moon!

Energy can be changed from one form to another. When you peddle a bike, your body uses up stored food energy (in calories) and converts this into kinetic energy of motion measured in joules. When you connect an electric motor to a battery, electrical energy stored in the battery is converted into rotational kinetic energy causing the motor shaft to turn.

A millstone paddle wheel uses the gravitational energy of falling water to turn the millstone wheel and perform work by grinding wheat, or even running simple machinery to cut wood in a lumber mill.

The energy in Joules of an object falling from a height near the surface of Earth can be calculated from

$$E = mgh$$

where m is the mass of the falling body in kilograms, g is the acceleration of gravity (9.8 meters/sec^2) and h is the distance of the fall in meters.

Problem 1 – Nevada Falls in Yosemite Valley California has a height of 180 meters. Every second, 500 cubic feet of water goes over the edge of the falls. If 1 cubic foot of water has a mass of 28 kilograms, how much energy does this waterfall generate every day?

Problem 2 – For a science fair project, a student wants to build a water hose powered hydroelectric plant to run a light bulb. Every second, the light bulb needs 60 Joules to operate at full brightness. If the water hose produces a steady flow of 0.2 kilograms every second, how high off the ground does the water hose have to be to turn a paddle wheel to generate the required electrical energy?

Problem 3 – A geyser on Saturn's moon Enceladus ejects water from its caldera with an energy of 1 million Joules. If g = 0.1 meters/sec^2, and the mass moved is 2000 kilograms, how high can the geyser stream travel above the surface of Enceladus?

Problem 1 – Nevada Falls in Yosemite Valley California has a height of 180 meters. Every second, 500 cubic feet of water goes over the edge of the falls. If 1 cubic foot of water has a mass of 28 kilograms, how much energy does this waterfall generate every day?

Answer: 1 day = 24 x 60 x 60 = 86,400 seconds, then the total mass is 500 x 28 x 86400 = 1.2 billion kilograms. E = 1.2 billion kg x 9.8 x 180 = **2.1 trillion joules** per day. Note: since 1 watt = 1 Joule/second, this waterfall has a wattage of 500 x 28 x 9.8 x 180 = 25 megawatts.

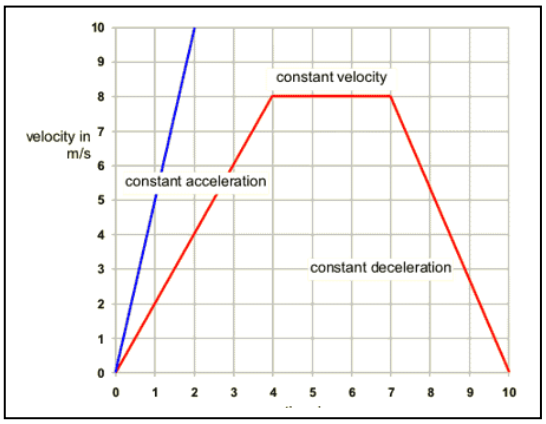
Problem 2 – For a science fair project, a student wants to build a water hose powered hydroelectric plant to run a light bulb. Every second, the light bulb needs 60 Joules to operate at full brightness. If the water hose produces a steady flow of 0.2 kilograms every second, how high off the ground does the water hose have to be to turn a paddle wheel to generate the required electrical energy?

Answer: 60 = 0.2 x 9.8 x h so h = **30.6 meters** (or 90 feet!).

Problem 3 – A geyser on Saturn's moon Enceladus ejects water from its caldera with an energy of 1 million Joules. If g = 0.1 meters/sec^2, and the mass moved is 2000 kilograms, how high can the geyser stream travel above the surface of Enceladus?

Answer: 1,000,000 = 2000 x 0.1 x h, so h = **5,000 meters or 5 kilometers**.

Reading a Speed vs Time Graph - acceleration



In the same way that speed = distance divided by time, we can also look at acceleration as the change in speed over the time that the change occurred. Both of these quantities can be thought of as rates of change or 'slopes' on a graph like the one to the left.

When the final speed is, larger than the initial speed, the slope of the line is positive (upward) and we say that the object is accelerating. When the final speed is less than the initial speed, the slope is negative (downward) and we say that the object is decelerating.

Problem 1 – A car leaves its parking spot and accelerates to 30 mph (13 m/s) in 10 seconds. It travels on a road at a constant speed of 30 mph for another 30 seconds and enters the onramp of a highway where it accelerates from 30 mph to a speed of 60 mph (26 m/s) after 6 seconds. It stays at this speed for another 2 minutes, then the car exits an off ramp, slowing to a speed of zero after 2 seconds. It then accelerates to 30 mph after 3 seconds as it merges into the local street traffic. After 1 minute at this speed the car approaches a gas station and decelerates to zero after 4 seconds. Draw a speed versus time graph in <u>metric units</u> that represents the car's journey.

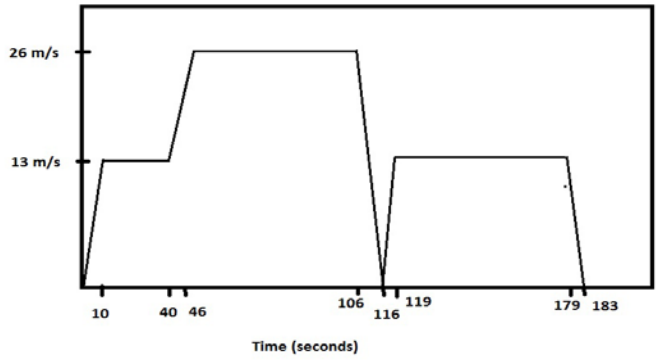
Time (Sec)	Speed (m/s)
0	0
1	3
2	4
3	7
4	10
5	12
6	15
7	16
8	20
9	23
10	25
11	27
12	31
13	34
14	36
15	39
16	43
17	46
18	49
19	53
20	56

Problem 2 – Explain how the area under a speed vs time graph gives the distance traveled, and use this to calculate the total distance traveled by the car using a combination of rectangles and triangles to calculate the total area.

Saturn V Rocket Launch Speed vs Time

Problem 3 – The table to the left shows the speed of the Saturn V rocket during a launch from the Kennedy Space Center on July 16, 1969 at 9:32:00 a.m. (EDT). What was the average acceleration of the Saturn V rocket during its first 20 seconds of constant thrust? How far did it travel during this time?

Problem 1 – A car leaves its parking spot and accelerates to 30 mph (13 m/s) in 10 seconds. It travels on a road at a constant speed of 30 mph for another 30 seconds and enters the onramp of a highway where it accelerates from 30 mph to a speed of 60 mph (26 m/s) after 6 seconds. It stays at this speed for another 1 minute, then exits an off ramp, slowing to a speed of zero after 10 seconds. It then accelerates to 30 mph after 3 seconds as it merges into the local street traffic. After 1 minute at this speed the car approaches a gas station and decelerates to zero after 4 seconds. Draw a speed versus time graph that represents the car's journey.

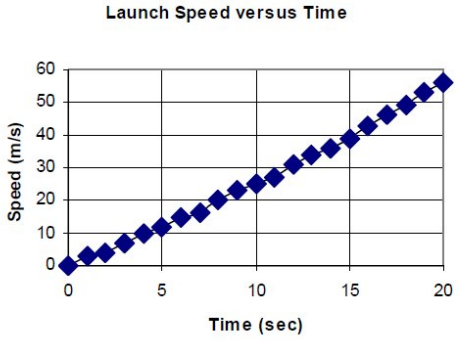


Problem 2 – Explain how the area under a speed vs time graph gives the distance traveled, and use this to calculate the total distance traveled by the car using a combination of rectangles and triangles to calculate the total area.

Answer: **Area = speed x time = meters/sec x seconds = meters. So the area under the graph has the units of distance in meters**. This figure has five triangular areas as the car is accelerating and decelerating, and three rectangular areas as the car is traveling at constant speed. The sum of the triangular areas is A = ½ time interval x speed = ½ [10 x 13 + 6 x (26-13) + 10x26 + 3x13 + 4x13) = 279 meters. The sum of the rectangular areas is A = time x speed = (30x13 + (106-46)x26 + (179-119)x13) = 2730 meters, so the total sum is 2730+279 = **3009 meters or 3 kilometers of total travel**.

Problem 3 – The table to the left shows the speed of the Saturn V rocket during a launch from the Kennedy Space Center on July 16, 1969 at 9:32:00 a.m. (EDT). What was the average acceleration of the Saturn V rocket during its first 20 seconds of constant thrust? How far did it travel during this time?

Answer: Acceleration = (56 m/s – 0 m/s)/20 sec = **28 m/s^2**. (Note this is about 2.8 times the acceleration of gravity at earth's surface so the astronauts would have felt about 2.8 times heavier). The triangular area is ½(20sec)(56 m/s) = **560 meters**.



Gravity and Escape Speed

It would be nice if we could just jump real hard and we would suddenly be in space orbiting Earth. Fortunately it is not that easy as any Olympic High Jumper will tell you!

Because of the pull of gravity, every planet, asteroid or other object in the universe has its own speed limit. If you move slower than this speed you will stay on the body. If you move faster than this speed you will escape into space. Scientists call this the **escape speed** or **escape velocity**.

It's not just a number you guess at. It depends exactly on how much mass the planet or moon has, and how far from its center you are located. That means you can predict what this speed will be as you travel to other planets. That's very handy if you are an astronaut!

For Earth, the escape speed V in kilometers/second (km/s) at a distance R from Earth's center in kilometers, is given by

$$V = \frac{894}{\sqrt{R}}$$

Problem 1 - What is the escape speed for a rocket located on Earth's surface where R = 6378 km?

Problem 2 – An Engineer proposes to launch a rocket from the top of Mt Everest (altitude 8.9 km) because its summit is farther from the center of Earth. Is this a good plan?

Problem 3 – A spacecraft is in a parking orbit around Earth at an altitude of 35,786 km. What is the escape speed from this location?

Problem 4 – To enter a circular orbit at a distance of R from the center of Earth, you only need to reach a speed that is $2^{1/2}$ smaller than the escape speed at that distance. What is the orbit speed of a satellite at an altitude of 35,786 km?

Problem 1 - What is the escape speed for a rocket located on Earth's surface where R = 6378 km?

Answer: $V = 894/(6378)^{1/2}$ = **11.19 km/s**

Problem 2 – An Engineer proposes to launch a rocket from the top of Mt Everest (altitude 8.9 km) because its summit is farther from the center of Earth. Is this a good plan?

Answer: $V = 894/(6378+8.9)^{1/2}$ = **11.18 km/s. This does not change the required escape speed by very much considering the effort to build such a launch facility at this location**.

Problem 3 – A spacecraft is in a parking orbit around Earth at an altitude of 35,786 km. What is the escape speed from this location?

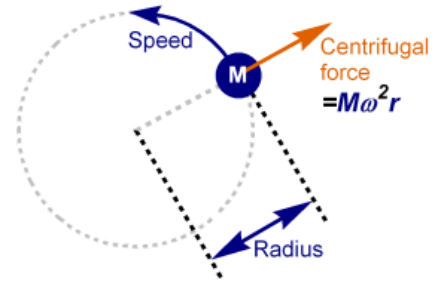
Answer: $V = 894/(6378+35,786)^{1/2}$ = **4.35 km/s**

Problem 4 – To enter a circular orbit at a distance of R from the center of earth, you only need to reach a speed that is $2^{1/2}$ smaller than the escape speed at that distance. What is the orbit speed of a satellite at an altitude of 35,786 km?

Answer: 4.35 km/s / (1.414) = **3.079 km/sec**.

Note: Satellites at an altitude of 35,800 orbit earth exactly once every day and are called geosynchronous satellites because they remain over the same geographic spot on Earth above the equator.

Circumference of the orbit = $2 \pi R$ = 2 x 3.141 x (42164 km) = 264,924 km.
Speed = 3.079 km/sec so
T = 264924/3.079 = 86,042 seconds or 23.9 hours – Earths rotation period.



Gravitron ride at an amusement park

Most science fiction stories require some form of artificial gravity to keep spaceship passengers operating in a normal earth-like environment. As it turns out, weightlessness is a very bad condition for astronauts to work in on long-term flights. It causes bones to lose about 1% of their mass every month. A 30 year old traveler to Mars will come back with the bones of a 60 year old!

The only known way to create artificial gravity it to supply a force on an astronaut that produces the same acceleration as on the surface of earth: 9.8 meters/sec^2 or 32 feet/sec^2. This can be done with bungee chords, body restraints or by spinning the spacecraft fast enough to create enough centrifugal acceleration.

Centrifugal acceleration is what you feel when your car 'takes a curve' and you are shoved sideways into the car door, or what you feel on a roller coaster as it travels a sharp curve in the tracks. Mathematically we can calculate centrifugal acceleration using the formula:

$$A = \frac{V^2}{R}$$

where V is in meters/sec, R is the radius of the turn in meters, and A is the acceleration in meters/sec^2.

Let's see how this works for some common situations!

Problem 1 - The Gravitron is a popular amusement park ride. The radius of the wall from the center is 7 meters, and at its maximum speed, it rotates at 24 rotations per minute. What is the speed of rotation in meters/sec, and what is the acceleration that you feel?

Problem 2 - On a journey to Mars, one design is to have a section of the spacecraft rotate to simulate gravity. If the radius of this section is 30 meters, how many RPMs must it rotate to simulate one Earth gravity (1 g = 9.8 meters/sec^2)?

Problem 3 – Another way to create a 1-G force is for the rocket to continuously fire during its journey so that its speed increases by 9.8 meters/sec every second. Suppose that NASA could design such a rocket system for a 60 million km trip to Mars. The idea is that it will accelerate during the first half of its trip, then make a 180-degree flip and decelerate for the remainder of the trip. If

$$\text{distance} = 1/2aT^2$$
$$\text{speed} = aT,$$

where **T** is in seconds, **a** is in meters/sec^2, and distance is in meters, how long will the trip take in hours, and what will be the maximum speed of the spacecraft at the turn-around point?

Problem 1 - The Gravitron is a popular amusement park ride. The radius of the wall from the center is 7 meters, and at its maximum speed, it rotates at 24 rotations per minute. What is the speed of rotation in meters/sec, and what is the acceleration that you feel?

Answer: For circular motion, the distance traveled is the circumference of the circle C = 2 π (7 meters) = 44 meters. At 24 rpm, it makes one revolution every 60 seconds/24 = 2.5 seconds, so the rotation speed is 44 meters/2.5 sec = 17.6 meters/sec.

The acceleration is then A = $(17.6)^2/7$ = **44.3 meters/sec^2**. Since 1 earth gravity = 9.8 meters/sec^2, the 'G-Force' you feel is 44.3/9.8 = 4.5 Gs. That means that you feel 4.5 times heavier than you would be just standing in line outside!

Problem 2 - On a journey to Mars, one design is to have a section of the spacecraft rotate to simulate gravity. If the radius of this section is 30 meters, how many RPMs must it rotate to simulate one Earth gravity (1 g = 9.8 meters/sec^2)?

Answer: The circumference is C = 2 π (30) = 188 meters. 1 RPM is equal to rotating one full circumference every minute, for a speed of 188/60 sec = 3.1 meters/second. So V = 3.1 meters/sec x RPM. Then A = $(3.1RPM)^2$ / 188 = 0.05 x RPM^2. We need 9.8 = 0.05RPM^2 so **RPM = 14**.

Problem 3 – Another way to create a 1-G force is for the rocket to continuously fire during its journey so that its speed increases by 9.8 meters/sec every second. Suppose that NASA could design such a rocket system for a 60 million km trip to Mars. The idea is that it will accelerate during the first half of its trip, then make a 180-degree flip and decelerate for the remainder of the trip. If distance = 1/2aT^2 and V = aT, how long will the trip take in hours, and what will be the maximum speed of the spacecraft at the turn-around point?

Answer: The turn-around point happens at the midway point 30 million km from Earth, so d = $3.0x10^{10}$ meters, a = 9.8 meters/sec^2, and so solving for T,

$3.0x10^{10}$ = ½ (9.8) T^2 so
T = 78,246 seconds or
for a full trip = 2x78246 seconds = **43 hours**!

Speed = 9.8 x 78246 = 767,000 meters/sec = **767 kilometers/sec**.

Distance Traveled Under Constant Acceleration

Bungee jumping has become a popular but dangerous sport. It also shows how the acceleration of gravity is connected to the total distance traveled during the fall. The distance traveled is given by the formula

$$D = \frac{1}{2} g T^2$$

Where g is the acceleration of gravity in meters/sec^2, D is the distance in meters, and T is the elapsed time in seconds. For locations near the surface of Earth, g = 9.8 meters/sec^2 (32 feet/sec^2)

Problem 1 - A confused Daredevil jumps from a plane at an altitude of 15,000 feet. How long does it take for the Daredevil to land if there is no air friction to slow him down?

Problem 2 – How fast would the Daredevil be traveling at the moment of impact if S = 32T?

Problem 3 – Once he reaches 130 mph (190 feet/sec), called the terminal velocity, his free-fall speed stops increasing. How soon after he jumps does he reach terminal velocity, and how far has he fallen from the plane?

Problem 4 - In 2012, Felix Baumgartner jumped from a high-altitude balloon at an altitude of 24 miles (127,000 feet), landing safely on the ground after 4 minutes and 19 seconds. With little atmosphere friction, he reached a maximum speed of 844 mph (1240 feet/sec). How long after he jumped did he reach this speed, and how high above the ground was he at that time?

Problem 5 – On Mars, the Valles Marineris canyon is 23,000 feet deep. If the acceleration of gravity is 12 feet/sec^2, how long would it take a rock to fall into the canyon and how fast is it traveling when it hits bottom?

Problem 1 - A confused Daredevil jumps from a plane at an altitude of 15,000 feet. How long does it take for the Daredevil to land if there is no air friction to slow him down?

Answer: $15{,}000 = \frac{1}{2}(32)T^2$, so $T^2 = 937$ and so T = **31 seconds**.

Problem 2 – How fast would the Daredevil be traveling at the moment of impact if S = 32T?

Answer: S = 32 x 31 = 992 feet/second or 676 miles/hour!

Problem 3 – Once he reaches 130 mph (190 feet/sec), called the terminal velocity, his free-fall speed stops increasing. How soon after he jumps does he reach terminal velocity, and how far has he fallen from the plane?

Answer: 190 = 32 x T so T = **6 seconds**. He has fallen $d = \frac{1}{2}(32)(6)^2$ = **576 feet**.

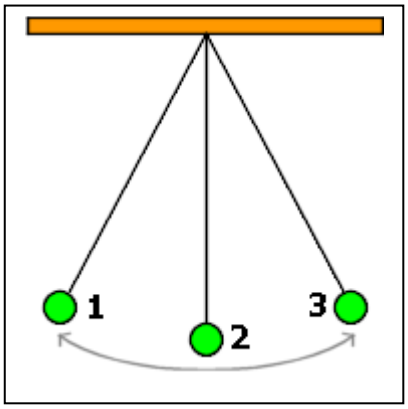
Problem 4 - In 2012, Felix Baumgartner jumped from a high-altitude balloon at an altitude of 24 miles (127,000 feet), landing safely on the ground after 4 minutes and 19 seconds. With little atmosphere friction, he reached a maximum speed of 844 mph (1240 feet/sec). How long after he jumped did he reach this speed, and how high above the ground was he at that time?

Answer: 1240 = 32 T so T = **39 seconds**.
$D = \frac{1}{2}(32)(39)^2 = 24{,}336$ feet,
so 127,000 – 24333 = **102,700 feet from the ground**.

Problem 5 – On Mars, the Valles Marineris canyon is 23,000 feet deep. If the acceleration of gravity is 12 feet/sec^2, how long would it take a rock to fall into the canyon and how fast is it traveling when it hits bottom?

Answer: $23{,}000 = \frac{1}{2}(12)T^2$ so T = **62 seconds**.
Speed = 12 x 62 = **744 feet/sec or 507 mph**.

Measuring Gravity with a Pendulum!



A pendulum is a very simple toy, but you can actually use it to measure gravity! The beat of a pendulum called its period, P, depends on the length of the pendulum, L, and the acceleration of gravity, g ,according to:

$$P = 2\pi\sqrt{\frac{L}{g}}$$

If you measure P in seconds, and know the length of the pendulum, L, in meters, you can figure out how strong the acceleration of gravity is, g, in meters/sec^2. Let's see how this works for explorers working on different planets and moons in the solar system!

Problem 1 – A mars colonist wants to make a pendulum that has a beat of 4 seconds. If the acceleration of gravity on mars is 3.8 m/sec^2, how long will the pendulum have to be in meters?

Problem 2 - A pendulum clock on the moon has a length of 2 meters, and its period is carefully measured to be 7.00 seconds. What is the acceleration of gravity on the moon?

Problem 3 – On Earth, prospectors are looking for a deposit of iron ore beneath the ground. They decide to use the acceleration of gravity to find where the iron is located because the additional iron mass should change the acceleration of gravity. They use a carefully-made pendulum with a length of 2.00000 meters and measure the period of the swing as they walk around the area where they think the deposit is located. To the nearest millionth of a second, how much will the period change if the acceleration of gravity between two spots changes from 9.80000 meters/sec^2 to 9.80010 meters/sec^2? (use π = 3.14159)

Problem 1 – A mars colonist wants to make a pendulum that has a beat of 4 seconds. If the acceleration of gravity on mars is 3.8 m/sec^2, how long will the pendulum have to be in meters?

Answer: $4 = 2\pi (L/3.8)^{1/2}$ then $L = (16/4\pi^2)*3.8$ = **1.54 meters**.

Problem 2 - A pendulum clock on the moon has a length of 2 meters, and its period is carefully measured to be 7.00 seconds. What is the acceleration of gravity on the moon?

Answer: $7.00 = 2\pi (2/g)^{1/2}$ so $g = 8\pi^2/49$ so g = **1.61 meters/sec^2**.

Problem 3 – On Earth, prospectors are looking for a deposit of iron ore beneath the ground. They decide to use the acceleration of gravity to find where the iron is located because the additional mass should change the acceleration of gravity. They use a carefully-made pendulum with a length of 2.00000 meters and measure the period of the swing as they walk around the area where they think the deposit is located. To the nearest millionth of a second, how much will the period change if the acceleration of gravity between two spots changes from 9.80000 meters/sec^2 to 9.80010 meters/sec^2? (use $\pi = 3.14159$)

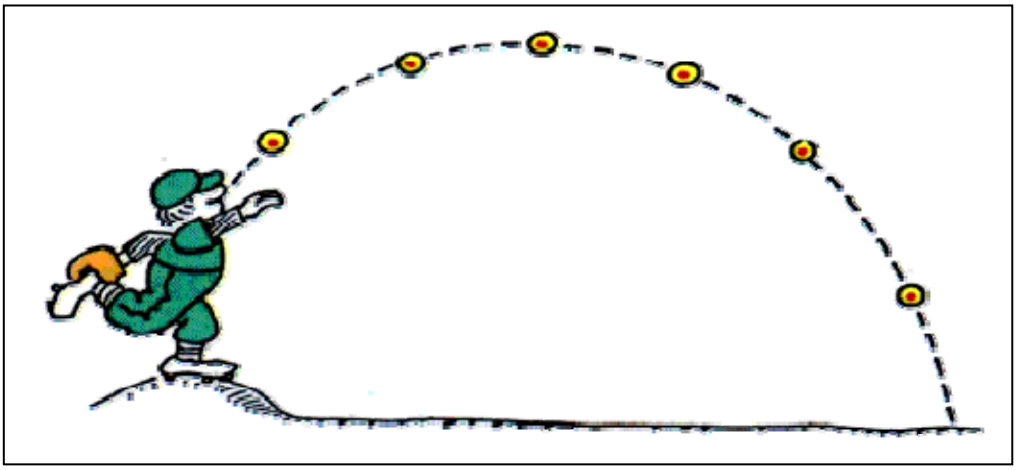
Answer: $P = 2\pi (2.00000/9.80000)^{1/2}$ = **2.838451 seconds**.
And $P2 = 2\pi (2.00000/9.80010)^{1/2}$ = **2.838437 seconds**

So the difference in period will be **0.000014 seconds or 14 microseconds.**

$$T = 2\pi\sqrt{\frac{L}{g}}\left(1 + \frac{1}{16}\theta_0^2 + \frac{11}{3072}\theta_0^4 + \cdots\right)$$

Where theta is the start angle from verticle.

The Physics of Rock Throwing



The horizontal motion of a rock (projectile) is given by the formula:

$$X = V_h T$$

Independently, the vertical motion is given by the formula

$$Y = H_0 + V_v T - \tfrac{1}{2} g T^2$$

The speed of the projectile has been described in terms of its vertical (Vv) and horizontal (Vh) speeds to that the total speed is given by the Pythagorean Theorem $S = (V_h^2 + V_v^2)^{1/2}$.

Problem 1 – A rock is tossed horizontally from the top of the Eiffel Tower at a speed of 60 mph (40 feet/sec). The Eiffel Tower stands 1,063 feet above the street. How far from the centerline of the tower does the rock land? (g = 32 feet/sec^2)

Problem 2 – On Mars (g = 12 feet/sec^2) an astronaut throws a rock up in the air so that its vertical speed is 30 feet/sec and its horizontal speed is 10 feet/sec. The rock starts at a shoulder height of 5 feet. How high does the rock travel, and how far from the astronaut does it finally reach the ground?

Problem 1 – A rock is tossed horizontally from the top of the Eiffel Tower at a speed of 60 mph (40 feet/sec). The Eiffel Tower stands 1,063 feet above the street. How far from the centerline of the tower does the rock land?

Answer: H_0 = 1063 feet, V_h = 40 feet/sec, V_v = 0.0, g = 32 feet/sec^2. The vertical equation give us the time to reach the ground (y=0) : 0 = 1063 – 16 T^2, so 1063/16 = T^2 and T = 8.1 seconds. From the horizontal motion, it travels d = 40 x 8.1 = **324 feet**.

Problem 2 – On Mars (g = 12 feet/sec^2) an astronaut throws a rock up in the air so that its vertical speed is 30 feet/sec and its horizontal speed is 10 feet/sec. The rock starts at a shoulder height of 5 feet. How high does the rock travel, and how far from the astronaut does it finally reach the ground?

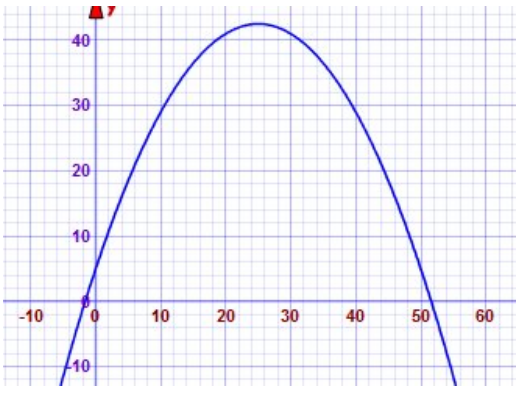
Answer: The two equations are X = 10 T and Y = 5.0 + 30T - 6T^2

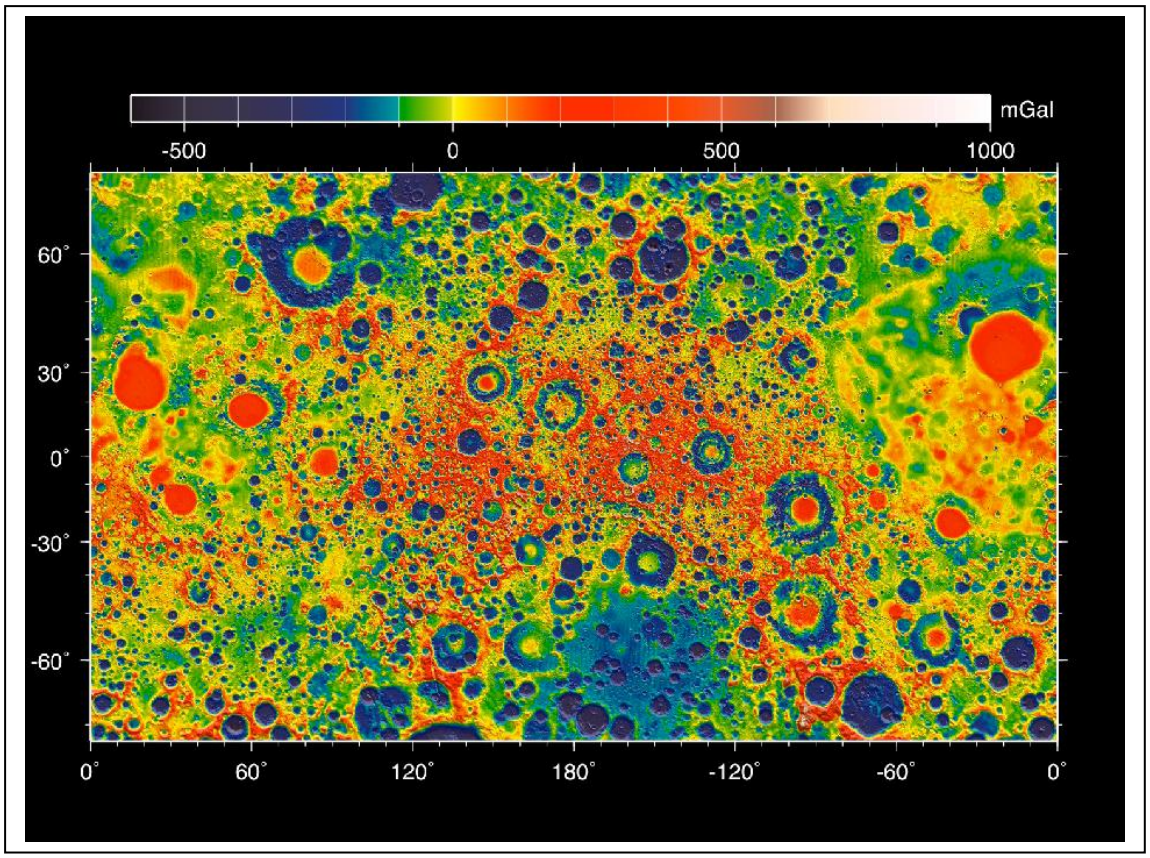
Write Y in terms of X: Y = 5.0 + 30(X/10) - 6 $(X/10)^2$ so **Y = 5.0 + 3.0X - 0.06X^2**

Solve for the roots of Y(X) = -0.06X^2 + 3.0X + 5.0 with coefficients a = -0.06, b=3.0 and c = 5.0, to get the ground intercept points using the Quadratic Formula:

X = [-3.0 +/- $(9-4(-0.06)(5.0))^{1/2}$] / (2x-0.06) so
X = (-3 + 3.19)/-0.12 = -1.6 feet , and the second root is
X = (-3 – 3.19)/-0.12 = +51.6 feet. see graph below.

The peak of the parabola is ½ way between the x-intercepts at x = (51.6-1.6)/2 = +25.0 feet
And since X = 10T, we have 25 = 10T so T = 2.5 seconds. From Y(T), the altitude of the peak is Y = 5.0 + 30(2.5) – 6$(2.5)^2$ = **42.5 feet**. From the x-intercept, it reaches a distance of **51.6 feet from the astronaut.**





During 2012, NASA's twin Grail satellites orbited the moon at altitudes of only 30 km. As they traveled, minute changes in their speeds tracked from Earth revealed changes in the gravitational field of the moon. These changes could be mapped, and revealed density changes in the lunar surface below them. In this way, scientists could look hundreds of kilometers beneath the lunar surface and explore how the surface was formed billions of years ago! On Earth, the acceleration of gravity is 9,807 cm/sec^2. The normal acceleration of gravity on the average lunar surface is 1620 cm/sec^2, but in the blue regions of the map this is as low as 1520 cm/sec^2, and in the red regions it is as high as 1920 cm/sec^2. A pendulum clock has a swinging period, T in seconds, given by the formula $T = 2\pi\sqrt{\frac{L}{g}}$ where L is the length of the pendulum in centimeters, and g is the acceleration of gravity in cm/sec^2.

Problem 1 - A lunar colony in a lunar 'blue' area has a Blue Clock with a pendulum length L = 100 cm. What is the swing period? (use π = 3.141)

Problem 2 - A lunar colony in a lunar 'red' area has an identical Red Clock. What is the swing period? (use π = 3.141)

Problem 3 - After how many swings on the Blue Clock will the clocks differ in time by 1 hour?

Problem 4 - If both clocks were synchronized to 1:00:00 am local time, what will the time on the Blue Clock and the Red Clock be when the two colony clocks are off by 1 hour relative to each other?

Problem 1 - A lunar colony in a lunar 'blue' area has a Blue Clock with a pendulum length L = 100 cm. What is the swing period?

Answer: $T = 2\ (3.141)\ (100/1520)^{1/2}$ = **1.61 seconds**.

Problem 2 - A lunar colony in a lunar 'red' area has an identical Red Clock. What is the swing period?

Answer; $T = 2\ (3.141)\ (100/1920)^{1/2}$ = **1.43 seconds**.

Problem 3 - After how many swings on the Blue Clock will the clocks differ in time by 1 hour?

Answer: Each swing on the slower Blue Clock pendulum is behind the faster Red Clock by 1.61 - 1.43 = 0.18 seconds. We want this difference to be 3600 seconds in 1 hour, which will take N = 3600/0.18 = **20,000 swings** on the Blue Clock.

Problem 4 - If both clocks were synchronized to 1:00:00 am local time, what will the time on the Blue Clock and the Red Clock be when the two colony clocks are off by 1 hour relative to each other?

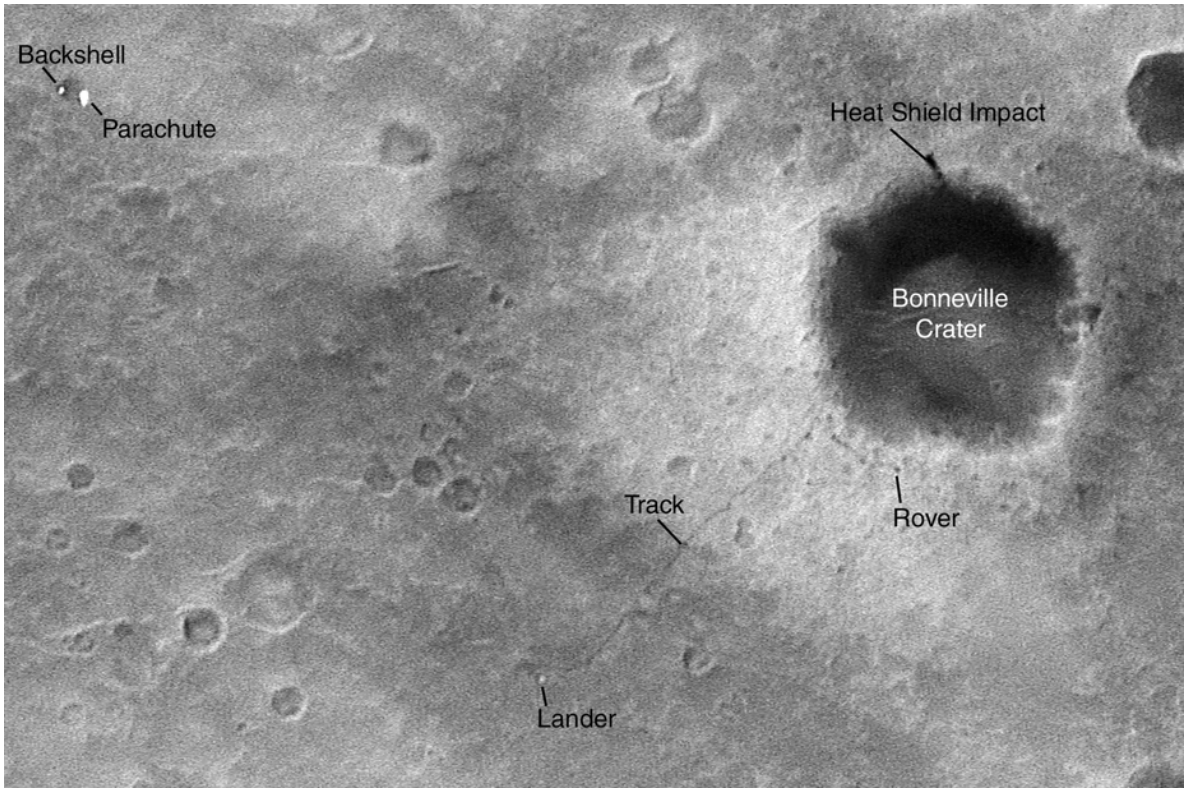
Answer: On the Blue Clock, 20,000 swings have to pass, each taking 1.61 seconds for a total time of 32,200 seconds or 8 hours, 56 minutes, 40 seconds. So the time on the Blue Clock will read **09:56:40 am local time**.

On the Red Clock, because after 20,000 swings it is exactly 1 hour behind the Blue Clock, its time will read 08:56:40 am local time. Another 'long way' to see this is that we still need 20,000 swings to add up to a 1 hour time difference, but on the Red Clock each swing is only 1.43 seconds long and so this takes 28,600 seconds or 7 hours, 56 minutes, 40 seconds. The time on the Red Clock will be **08:56:40 am local time**.

This is why colonists will NOT be using pendulum clocks on the moon!!

Note: Devices that act like pendulum clocks were once used by prospectors on Earth to search for oil and other valuable materials below ground before the advent of more accurate magnetometer-based technology. Minute changes in the pendulum period indicate changes in the density of rock below ground and these can be used to identify high-gravity, density regions (like iron ore) or low-gravity regions (like caverns). Another way to measure minute gravity changes is by the shape of a satellite orbit, or by the subtle changes in speed between two satellites on the same orbit. Lunar scientists used this orbit method with the two Grail spacecraft only 200 kilometers apart.

Mars Rover Landing Site



This NASA, Mars Orbiter image of the Mars Rover, Spirit, landing area near Bonneville Crater. The width of the image is exactly 895 meters. (Credit: NASA/JPL/MSSS). It shows the various debris left over from the landing, and the track of the Rover leaving the landing site.

The scale of an image is found by measuring with a ruler the distance between two points on the image whose separation in physical units you know. In this case, we are told the width of the image is 895 meters.

Step 1: Measure the width of the image with a metric ruler. How many millimeters wide is it?
Step 2: Use clues in the image description to determine a physical distance or length. Convert to meters.
Step 3: Divide your answer to Step 2 by your answer to Step 1 to get the image scale in meters per millimeter to two significant figures.

Once you know the image scale, you can measure the size of any feature in the image in units of millimeters. Then multiply it by the image scale from Step 3 to get the actual size of the feature in meters to two significant figures.

Problem 1: About what is the diameter of Bonneville Crater rounded to the nearest ten meters?

Problem 2: How wide, in meters, is the track of the Rover?

Problem 3: How big is the Rover?

Problem 4: How small is the smallest well-defined crater to the nearest meter in size?

Problem 5: A boulder is typically 5 meters across or larger. Are there any boulders in this picture?

Answer Key:

This NASA, Mars Orbiter image of the Mars Rover, Spirit, landing area near Bonneville Crater. The width of the crater is 200 meters. (Credit: NASA/JPL/MSSS). It shows the various debris left over from the landing, and the track of the Rover leaving the landing site.

The scale of an image is found by measuring with a ruler the distance between two points on the image whose separation in physical units you know. In this case, we are told the width of the image is 895 meters.

Step 1: Measure the width of the image with a metric ruler. How many millimeters wide is it?
Answer: 157 millimeters.

Step 2: Use clues in the image description to determine a physical distance or length. Convert to meters.
Answer: 895 meters.

Step 3: Divide your answer to Step 2 by your answer to Step 1 to get the image scale in meters per millimeter to two significant figures.
Answer: 895 m/157 mm = 5.7 meters / millimeter.

Once you know the image scale, you can measure the size of any feature in the image in units of millimeters. Then multiply it by the image scale from Step 3 to get the actual size of the feature in meters.

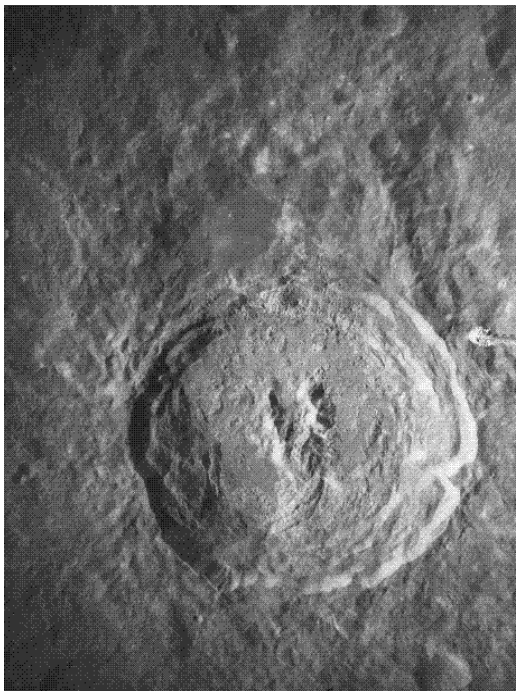
Problem 1: About what is the diameter of Bonneville Crater rounded to the nearest 10 meters?
Answer: Students answers for the diameter of the crater in millimeters may vary, but answers in the range from 30-40 mm are acceptable. Then this equals 30x5.7 = 170 meters to 40x5.7=230 meters. Students may average these two measurements to get (170+230)/2 = 200 meters.

Problem 2: How wide, in meters, is the track of the Rover?
Answer: 0.2 millimeters = 1 meter.

Problem 3: How big is the Rover?
Answer: 0.3 millimeters = 1.7 meters but since the measurement is only 1 significant figure, the answer should be 2 meters.

Problem 4: How small is the smallest well-defined crater in meters?
Answer: 2 millimeters x 5.7 = 11.4 meters, which to the nearest meter is 11 meters.

Problem 5: A boulder is typically 5 meters across or larger. Are there any boulders in this picture?
Answer: Students answers may vary and lead to interesting discussions about what features are real, and which ones are flaws in the printing of the picture. This is an important discussion because 'image artifacts' are very common in space-related photographs. 5 meters is about 1 millimeter, and there are no obvious rounded objects this large or larger in this image.



The image of Mercury's surface on the left was taken by the MESSENGER spacecraft on March 30, 2011 of the region near crater Camoes near Mercury's south pole. In an historic event, the spacecraft became the first artificial satellite of Mercury on March 17, 2011. The image on the right is a similar-sized area of our own Moon near the crater King, photographed by Apollo 16 astronauts.

The Mercury image is 100 km wide and the lunar image is 115 km wide.

Problem 1 – Using a millimeter ruler, what is the scale of each image in meters/millimeter?

Problem 2 – What is the width of the smallest crater, in meters, you can find in each image?

Problem 3 – The escape velocity for Mercury is 4.3 km/s and for the Moon it is 2.4 km/s. Why do you suppose there are more details in the surface of Mercury than on the Moon?

Problem 4 – The diameter of Mercury is 1.4 times the diameter of the Moon. From the equation for the volume of a sphere, by what factor is the volume of Mercury larger than the volume of the Moon?

Problem 5 – If mass equals density times volume, and the average density of Mercury is 5400 kg/m^3 while for the Moon it is 3400 kg/m^3, by what factor is Mercury more massive than the Moon?

Problem 1 – Using a millimeter ruler, what is the scale of each image in meters/millimeter?

Answer: Mercury image width is 80 mm, scale is 100 km/80mm = **1.3 km/mm**
Moon image width = 68 mm, scale is 115 km/68mm = **1.7 km/mm**

Problem 2 – What is the width of the smallest crater, in meters, you can find in each image?

Answer: Mercury = 0.5 mm x (1.3 km/mm) = **700 meters**.
Moon = 1.0 mm x (1.7 km/mm) = **1,700 meters**.

Problem 3 – The escape velocity for Mercury is 4.3 km/s and for the Moon it is 2.4 km/s. Why do you suppose there are more details in the surface of Mercury than on the Moon?

Answer: **On Mercury, less of the material ejected by the impact gets away, and so more of it falls back to the surface near the crater. For the Moon, the escape velocity is so low that ejected material can travel great distances, or even into orbit and beyond, so less of it falls back to the surface to make additional craters**.

Problem 4 – The diameter of Mercury is 1.4 times the diameter of the Moon. From the equation for the volume of a sphere, by what factor is the volume of Mercury larger than the volume of the Moon?

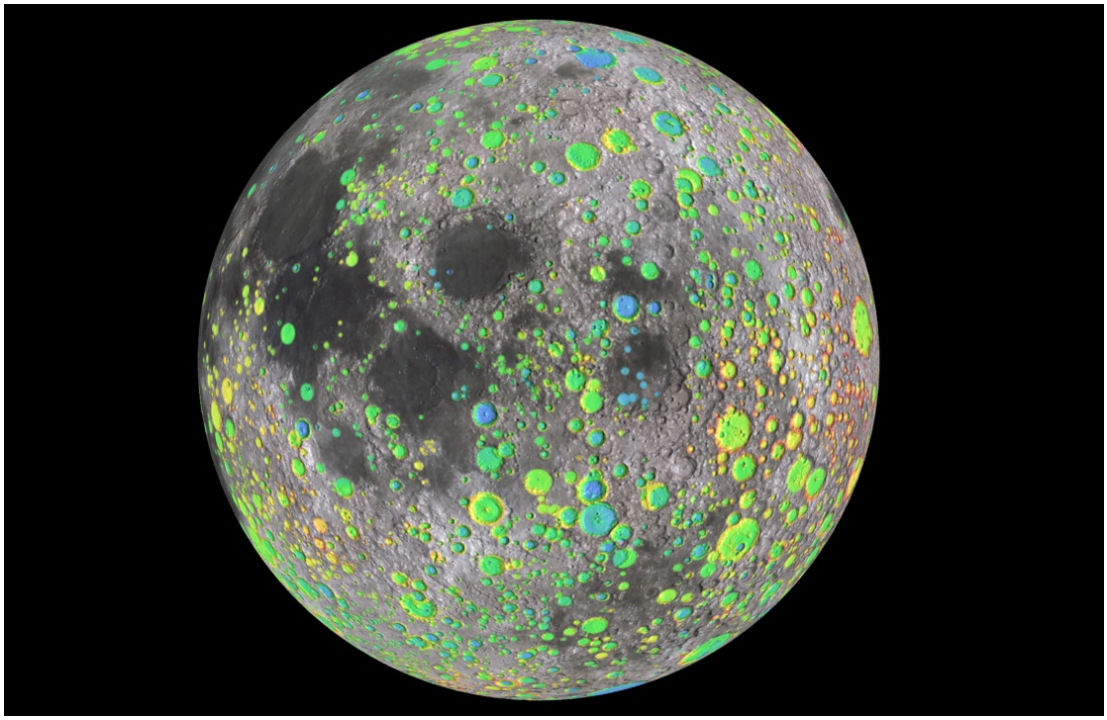
Answer: The volume of a sphere is given by $4/3\ \pi\ R^3$, so if you increase the radius of a sphere by a factor of 1.4, you will increase its volume by a factor of $1.4^3 = 2.7$ times, so the volume of Mercury is **2.7 times** larger than the volume of the Moon.

Problem 5 – If mass equals density times volume, and the average density of Mercury is 5400 kg/m^3 while for the Moon it is 3400 kg/m^3, by what factor is Mercury more massive than the Moon?

Answer: The density of Mercury is a factor of 5400/3400 = 1.6 times that of the Moon. Since the volume of Mercury is 2.7 times larger than the Moon, the mass of Mercury will be density x volume or 1.6 x 2.7 = **4.3 times than of the Moon**.

Note: Actual masses for Mercury and the Moon are 3.3×10^{23} kg and 7.3×10^{22} kg respectively, so that numerically, Mercury is 4.5 times the Moon's mass...which is close to our average estimate.

LRO Determines Lunar Cratering History



The Lunar Reconnaissance Orbiter used millions of measurements of the lunar surface to establish the history of cratering on the surface.

Problem 1 - The diameter of the moon is 3,400 kilometers. With a millimeter ruler determine the scale of the image above in kilometers/mm.

Problem 2 - How many craters can you count that are larger than 70 kilometers in diameter?

Problem 3 - If the large impacts had happened randomly over the surface of the moon, about how many would you have expected to find in the 20% of the surface covered by the maria?

Problem 4 - From your answer to Problem 3, what can you conclude about the time that the impacts occurred compared to the time when the maria formed?

Problem 1 - The diameter of the moon is 3,400 kilometers. With a millimeter ruler determine the scale of the image above in kilometers/mm.
Answer: The image diameter is 90 millimeters so the scale is 3400 km/90 mm = **38 km/mm.**

Problem 2 - How many craters can you count that are larger than 70 kilometers in diameter?
Answer: 70 km equals 2 millimeters at this image scale. There are about **56 craters larger than 2 mm on the image. Students answers may vary from 40 to 60.**

Problem 3 - If the large impacts had happened randomly over the surface of the moon, about how many would you have expected to find in the 20% of the surface covered by the maria?
Answer: You would expect to find about 0.2 x 56 = **11 craters larger than 70 km.**

Problem 4 - From your answer to Problem 3, what can you conclude about the time that the impacts occurred compared to the time when the maria formed?

Answer: **The lunar highlands were present first and were impacted by asteroids until just before the maria formed. There are few/no craters in the maria regions larger than 70 km, so the maria formed after the episode of large impactors ended.**

For more information, see the LRO press release at:

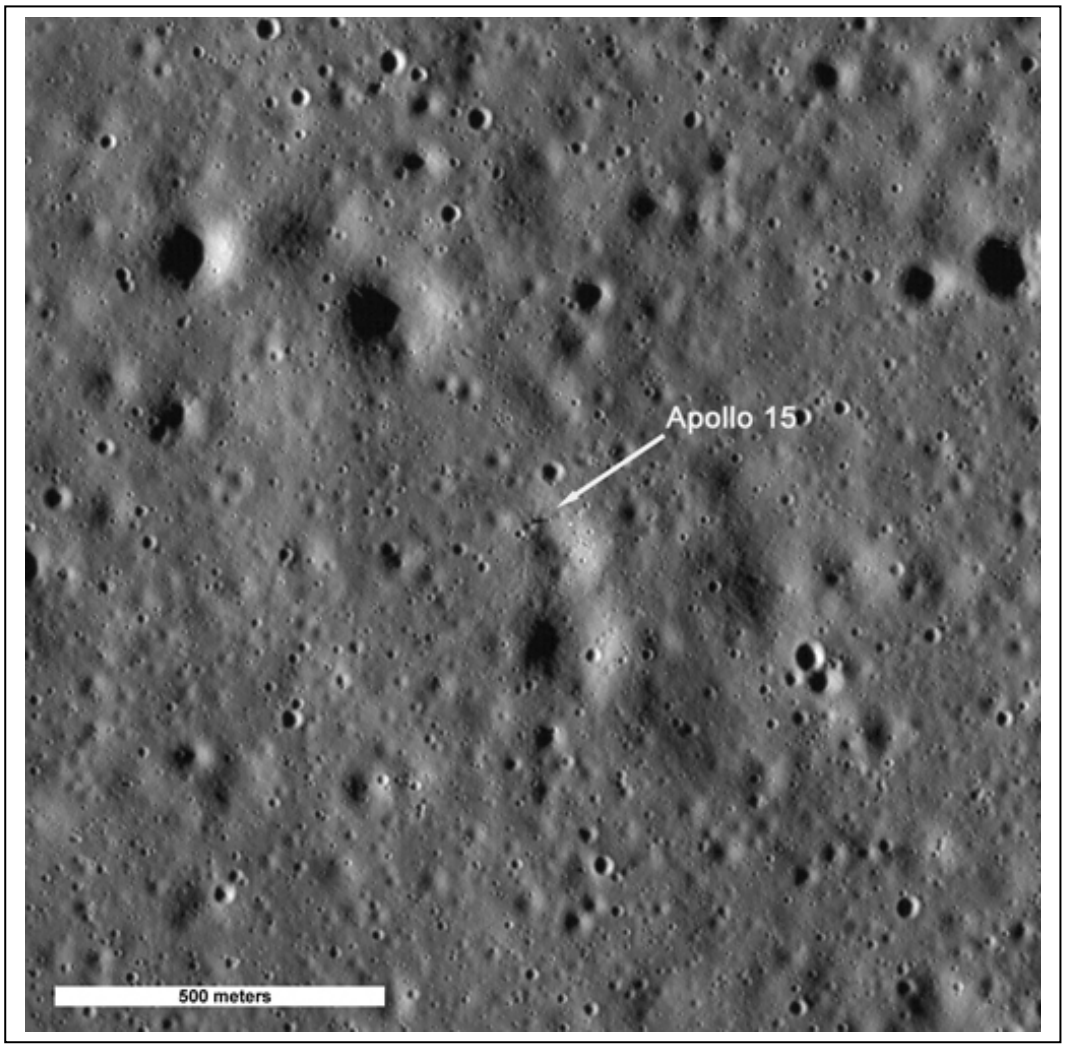
"LRO Exposes Moon's Complex, Turbulent Youth"

http://www.nasa.gov/mission_pages/LRO/news/turbulent-youth.html

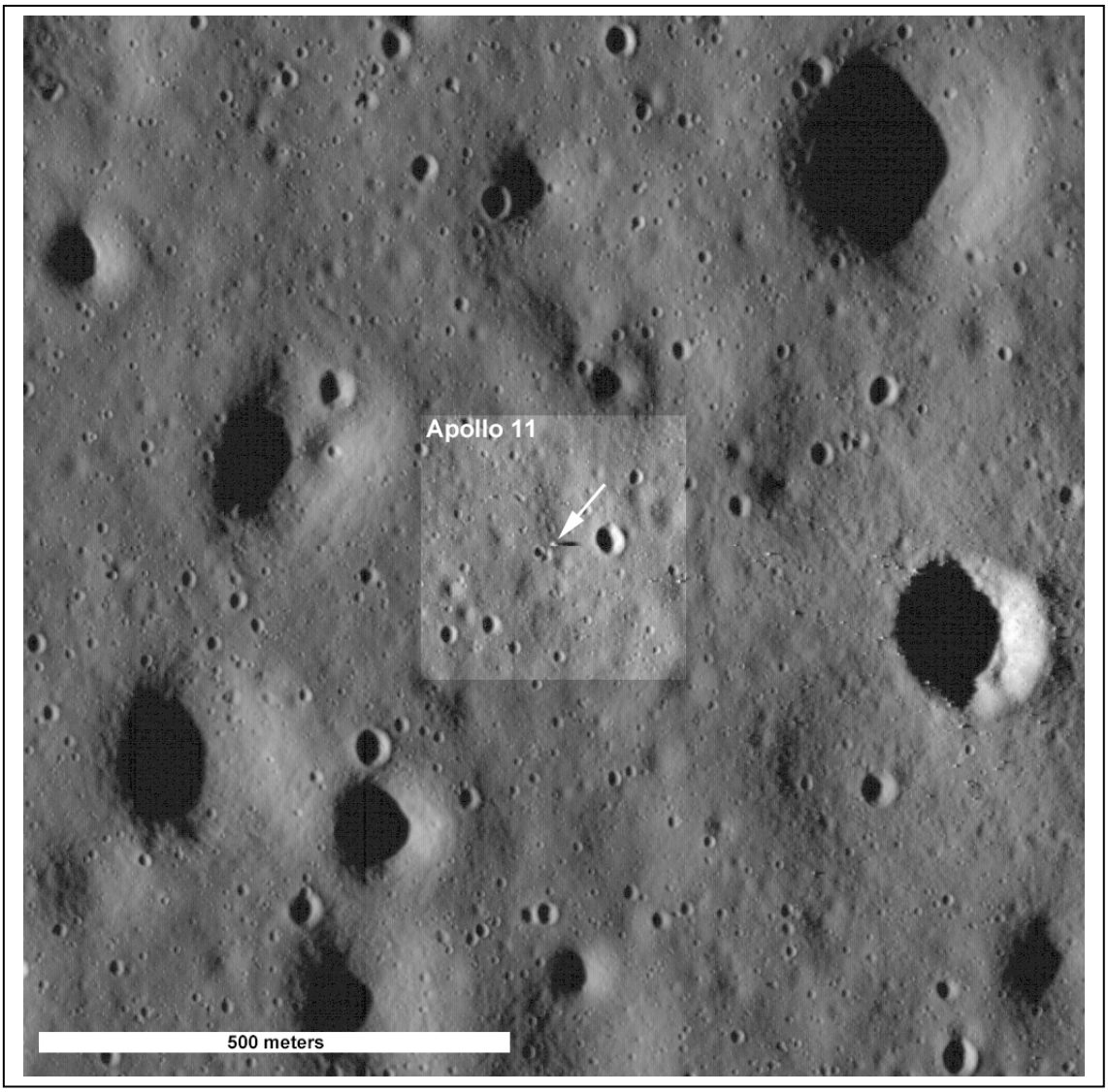
We have all sees pictures of craters on the moon. The images on the next two pages show close-up views of the cratered lunar surface near the Apollo 15 and Apollo 11 landing areas. They were taken by NASA's Lunar Reconnaissance Orbiter (LRO) from an orbit of only 25 kilometers!

Meteors do not arrive on the moon at the same rates. Very large meteors that produce the largest craters are much less common than the smaller bodies producing the smallest craters. That's because there are far more small bodies in space than large ones. Astronomers can use this fact to estimate the ages of various surfaces in the solar system by just comparing the number of large craters and small craters that they find in a given area.

Let's have a look the images below, and figure out whether Apollo-11 landed in a relatively younger or older region than Apollo 15!



This is the Apollo-15 landing area near the foot of the Apennine Mountain range. Note the bar indicating the 'scale' of the image. The arrow points to the location of the Lunar Descent Module.



This image taken by LRO is of the Apollo-11 landing area in Mare Tranquilitatus, with the arrow pointing to the Lunar Descent Module. The LDM was the launching platform for the Apollo-11 Lunar Excursion Module, which carried astronauts Neil Armstrong and Buzz Aldrin back to the orbiting Command Module for the trip back to Earth. Note the length of the '500 meter' bar, which gives an indication of the physical scale of the image. How long would it take you to walk 500 meters?

Astronomers assume that during the last 3 billion years following the so-called 'Late Heavy Bombardment Era' the average time between impacts that created craters has been constant. That means that the more time that passes, the more craters you will find, and that they are produced at a more or less steady number for each million years that passes. Also, by the Law of Superposition, younger craters lie on top of older craters.

Dating a cratered surface.

For each of the above images, perform these steps.

Step 1 - With the help of a millimeter ruler, and the '500 meter' line in the image, calculate the scale factor for the image in terms of meters per millimeter.

Step 2 - Calculate the total area of the image in square kilometers.

Step 3 - Identify and count all craters that are bigger than 20 meters in diameter.

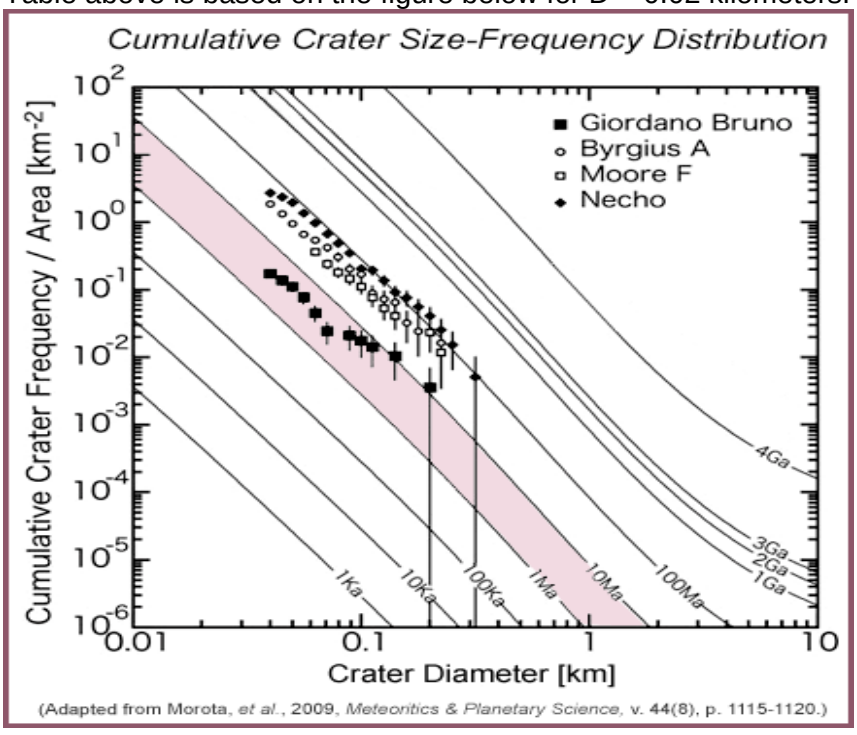
Step 4 - Divide your answer in Step 3 by the area in square kilometers in Step 2.

Step 5 - Look at the table below and estimate the average age of the surface.

Problem 1 - From your answer for each Apollo landing area in Step 5, which region of the moon is probably the youngest?

Estimated Age	Total number of craters per square kilometer
1000 years	0.0008
10000 years	0.008
100,000 years	0.08
1 million years	0.8
10 million years	8.0
100 million years	80.0
1 billion years	800.0

Table above is based on the figure below for D = 0.02 kilometers.



Another thing you might consider doing is to measure the diameters of as many craters as you can, and then plot a histogram (bar graph) of the number of craters you counted in a range of size intervals such as 5 - 10 meters, 11 to 20, 21 to 30 and so on. Because erosion (even on the moon!) tends to eliminate the smallest craters first, you can compare two regions on the moon in terms of how much erosion has occurred.

Problem 2 - Select the same-sized area on each of the Apollo images and count all the craters you can find within the size intervals you selected. How do the two landing areas compare to one another in terms of their crater frequency histograms?

Problem 3 - Which surface do you think has experienced the most re-surfacing or erosion?

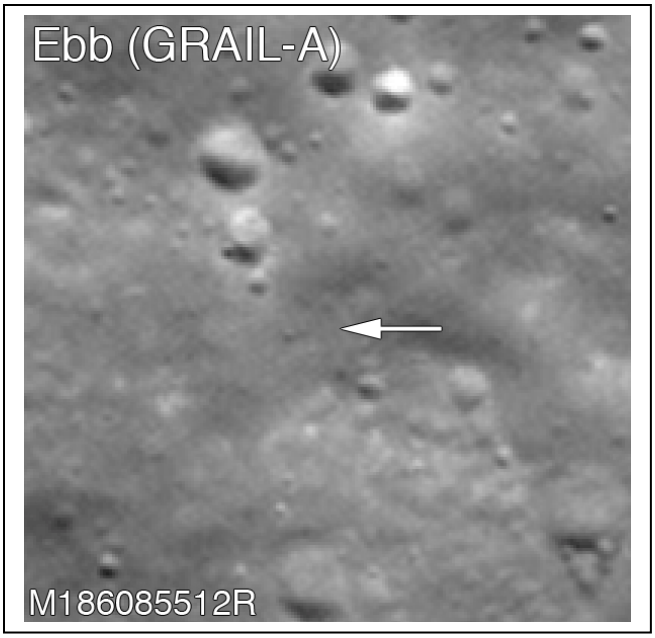
Problem 4 - Without an atmosphere, winds or running water, what do you think could have caused changes in the lunar surface after the craters were formed?

Note to Teachers: More technical information on crater dating can be found at

"How young is the Crater Giordano Bruno"

http://www.psrd.hawaii.edu/Feb10/GiordanoBrunoCrater.html

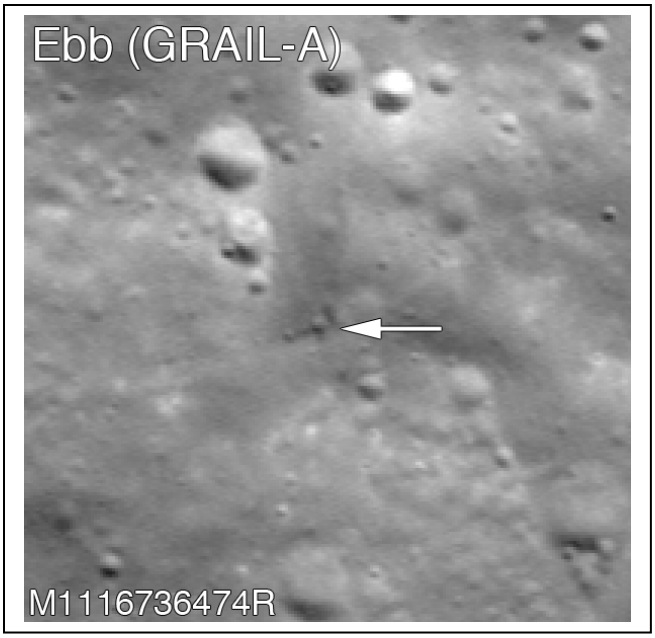
Grail Creates a New Crater on the Moon



On December 17, 2012 each of the twin Grail spacecraft ended their lunar mapping missions by crashing into the lunar surface. The two images to the left were taken by the Lunar Reconnaissance Orbiter as it flew over the impact site for the Grail-A spacecraft.

The width of each image is 213 meters.

At the moment of impact, each Grail spacecraft was traveling at a speed of 6,070 km/h and carried a mass of 200 kg.



Problem 1 – What was the diameter, in meters and feet, of the crater left behind when Ebb impacted the surface?

Problem 2 – The diameter of the Grail spacecraft was about 1-meter. How many times larger was the crater then the spacecraft?

Problem 3 – For the largest crater in the image, how large would the meteorite have to be to make the crater?

Problem 4 – Assume that the crater was a cylinder with a depth ¼ its diameter. If the density of the lunar soil is 2700 kg/m^3, how many kilograms of lunar soil were excavated by the impact?

Problem 5 – What is the ratio of the excavated mass to the spacecraft mass?

NASA's LRO Sees GRAIL's Explosive Farewell
http://www.nasa.gov/mission_pages/LRO/news/grail-results.html

Problem 1 – What was the diameter, in meters and feet, of the crater left behind when Ebb impacted the surface?

Answer: Use a millimeter ruler to measure the width of the image. You should get about 80 mm .The scale is then 213 meters/80 mm = 2.7 meters/mm. The crater is about 1mm across so this is just **2.7 meters**. Since 3 feet = 1 meter, this is about **8 feet across.**

Problem 2 – The diameter of the Grail spacecraft was about 1-meter. How many times larger was the crater then the spacecraft?

Answer: About **2.7 times larger**.

Problem 3 – For the largest crater in the image, how large would the meteorite have to be to make the crater?

Answer: The largest crater is about 9 mm across or 24 meters. Using Grail, we get 24 meters/2.7 = **8.9 meters across**.

Problem 4 – Assume that the crater was a cylinder with a depth ¼ its diameter. If the density of the lunar soil is 2700 kg/m^3, how many kilograms of lunar soil were excavated by the impact?

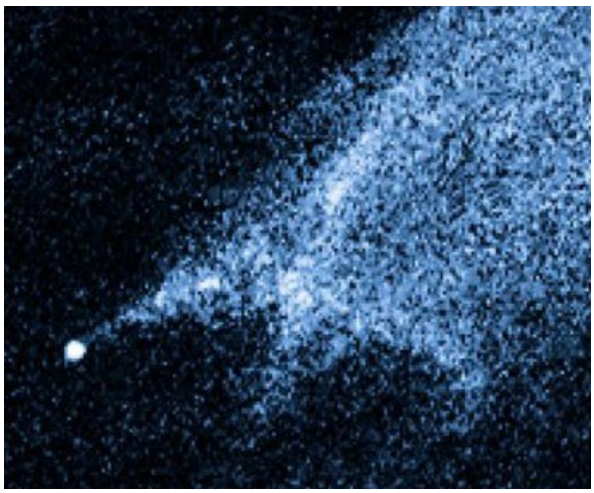
Answer: Volume = πR^2 h so V = 3.14 x $(2.7/2)^2$ x (2.7/4) = 3.8 $meters^3$.
Mass = density x volume so Mass = 2700 x 3.8 = **10,000 kilograms**.

Problem 5 – What is the ratio of the excavated mass to the spacecraft mass?

Answer: 10,000 kilograms/200 kg = **50**.

So the amount of excavated mass is 50 times the mass of the impacting spacecraft.

Hubble Spies Colliding Asteroids!



It doesn't look like much, but this picture taken by the Hubble Space Telescope on January 25, 2010 shows all that remains of two asteroids that collided! The object, called P/2010 A2, was discovered in the asteroid belt 290 million kilometers from the sun, by the Lincoln Near-Earth Asteroid Research sky survey on January 6, 2010.

Hubble shows the main nucleus of P/2010 A2, about 150 meters in diameter, lies outside its own halo of dust. This led scientists to the interpretation that it is the result of a collision.

How often do asteroids collide in the asteroid belt? The collision time can be estimated by using the formula:

$$T = \frac{1}{NAV}$$

Where N is the number of bodies per cubic kilometer, v is the speed of the bodies relative to each other in kilometers/sec and A is the cross-sectional area of the body in square-kilometers. The answer, T, will be in the average number of seconds between collisions.

Estimating A: Assume that the bodies are spherical and 100 meters in diameter .What will be A, the area of a cross-section through the body?

Estimating the asteroid speed V: At the orbit of the asteroids, they travel once around the sun in about 3 years. What is the average speed of the asteroid in kilometers/sec at a distance of 290 million kilometers?

Estimating the density of asteroids N: - This quantity is the number of asteroids in the asteroid belt, divided by the volume of the belt in cubic kilometers. A) Assume that the asteroid belt is a thin disk 1 million kilometers thick, with an inner radius of 1.6 AU and an outer radius of 2.5 AU. If 1 AU = 150 million kilometers, want is the volume? B) Based on telescopic observations, an estimate for the number of asteroids in the belt that are larger than 100 meters across is about 30 billion. From this information, and your volume estimate, what is the average density of asteroids in the asteroid belt?

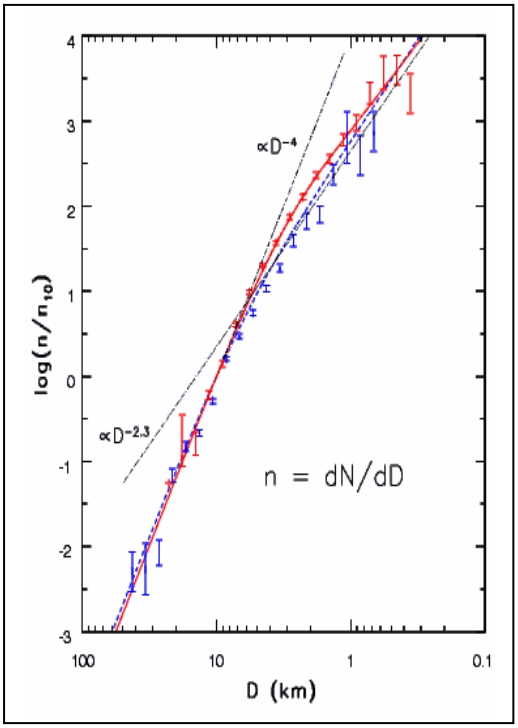
Estimating the collision time T: From the formula, A) what do your estimates for N, V and A imply for the average time between collisions in years? B) What are the uncertainties in your estimate?

Estimating A: Answer: The cross-section of a sphere is a circle, so A = πR^2 or for R = 100 meters, we have A = 3.14 $(0.05)^2$ = 8x10^{-3} km^2.

Estimating the asteroid speed V: Answer: The circumference of the orbit is C = 2 π (290 million km) = 1.8 x 10^9 kilometers, and the time in seconds is 3.0 years x (3.1 x 10^7 seconds/1 year) = 9.3 x 10^7 seconds, so the average speed V= C/T = 19 kilometers/sec. But what we want is the relative speed. If all the asteroids were going around in their orbits at a speed of 19 km/sec, their relative speeds would be zero. Example, although two cars are on the freeway traveling at 65 mph, their relative speeds are zero since they are not passing or falling behind each other.

Estimating the density of asteroids N: - Answer: R(inner) = 2.4 x 10^8 km .R(outer)= 3.4 x 10^8 km, so the volume of the disk is V = disk area x thickness = [$\pi(3.4x10^8)^2$ - $\pi(2.4 \times 10^8)^2$] x 10^6 so the volume is **1.8 x 10^{23} km^3**. Then the average asteroid density is N= 3x10^{10} asteroids/1.8 x 10^{23} km^3 = **1.7 x 10^{-13} asteroids/km^3.**

Estimating the collision time T: Answer: A) T = 1/NAV so T = 1/(1.7 x 10^{-13} x 8x10^{-3} x 19) = 3.8 x 10^{13} seconds. Since 1 year = 3.1 x 10^7 seconds, we have about **1,200,000 years between collisions**. B) Students can explore a number of places where large uncertainties might occur such as: 1) the thickness of the asteroid belt; 2) the number of asteroids; 3) their actual relative speeds. 4) The non-uniform distribution of the asteroids…not smoothly distributed all over the volume of the asteroid belt, but may be in clumps or rings that occupy a smaller actual volume. Encourage them to come up with their own estimates and see how that affects the average time between collisions. For advanced students see the problem below.



Extra Credit with Calculus: Data from the Sloan Digital Sky Survey suggests that the number of asteroids in specific size ranges follow a piecewise power-law distribution:

$$N(D) = \begin{cases} 1.8x10^9 D^{-2.3} & \text{for D <70 km} \\ 2.4x10^{12} D^{-4.0} & \text{for D>70 km} \end{cases}$$

Integrate N(D) from 100 meters to infinity to determine the number of asteroids larger than 100 meters.

Answer: $n = \int_{0.1}^{70} N(D)dD + \int_{70}^{\infty} N(D)dD$

= 2.7x10^{10} + 2.32x10^6 = 2.7x10^{10} asteroids.

Note: the figure is scaled to the number of 10-km asteroids (n_{10}) which we take to be 10,000.

On February 14, 2013 a 10,000 ton meteor about 17-meters in diameter entered Earth's atmosphere over Russia traveling at 40,000 mph (18 km/s). It detonated in the air over the town of Chelyabinsk and the explosion caused major damage to the town injuring 1,000 people. The people were hurt by flying glass when the windows of over 3000 buildings blew out over an area of about 1000 km^2. Unlike the famous Tunguska Event of 1908 which blew down 80 million trees and was nit 'discovered' for many decades afterwards, the Chelyabinsk Meteor was extensively videoed by hundreds of dashcams and cell phones as it happened.

Studies of thousands of meteor sightings by scientists can now tell us just how often asteroids of 4-meters or larger enter Earth's atmosphere. About two of these events happens each year over the entire surface area of Earth, which is 500 million km^2!

Problem 1 – The surface area of Earth is consists of 72% oceans and 28% land. Of the land area, only 3% is inhabited. How many years would you have to wait to hear about one of these large meteor events in the News?

Problem 2 – Fireballs are very bright meteors that streak across the sky. They are caused by pieces of meteors that can be 500 grams or more in mass. Astronomers estimate that 50,000 of these 'Bolides' can be seen every year over the entire surface area of Earth. From an inhabited spot on Earth, about how many Bolides should you be able to see in your lifetime if you paid attention to the sky if you could see any bolide entering over an area about 100 km^2?

Problem 3 - The kinetic energy in Joules of a large meteor is given by $KE = 1/2mV^2$ where m is its mass in kilograms and V is its speed in meters/sec. One ton of TNT explodes with an energy of 4.2×10^9 Joules. How many tons of TNT did the Chelyabinsk Meteor yield as it exploded if 1 ton = 1000 kg?

Problem 1 – The surface area of Earth is consists of 72% oceans and 28% land. Of the land area, only 3% is inhabited. How many years would you have to wait to hear about one of these large meteor events in the News?

Answer: The inhabited area of Earth is 3% of 28% of the total surface area or just 0.8% of Earth's total surface area. This is 1/125 of the full area. If you had one event per year, you would have to wait 125 years for the next one. If you had 2 events per year, you would have to wait half this time or about **62 years**. So, in a typical 70-year human lifetime, you will hear about one or two of these major impact events in the News!

Problem 2 – Fireballs are very bright meteors that streak across the sky. They are caused by pieces of meteors that can be 500 grams or more in mass. Astronomers estimate that 50,000 of these 'Bolides' can be seen every year over the entire surface area of Earth. From an inhabited spot on Earth, about how many Bolides should you be able to see in your lifetime if you paid attention to the sky if you could see any bolide entering over an area about 100 km^2?

Answer: The 50,000 bolides arrive somewhere over Earth each year. The chance that that this area is over an inhabited region of Earth is 0.8% or 1/125. So 1/125 of the bolides arrive over an inhabited area which is 50000/125 = 400 each year. For you to personally see the event, it has to happen within 100 km^2 of where you are standing. The inhabited area of Earth has an area of 1/125 x 500 million km^2 = 4 million km^2. Your 100 km^2 is only 1/40000 of this area. So you will see 400 bolides x 1/40000 = 1/100 bolides each year, or will **have to wait about 100 years to see just one**! If you watched the sky every night for your entire life, you might see one of these events!

BUT, because of our world-wide news and internet coverage, you could hear about any bolide that flashed over the inhabited area of Earth or 400 bolides each year! We only hear about a few of these because most of them are too unimpressive to get the attention of the news system. After the Chelyabinsk Meteor event on February 14, 2013, there were many announcements of fireballs or bolides over Los Angeles, San Francisco and other cities as this news topic became popular for a few weeks.

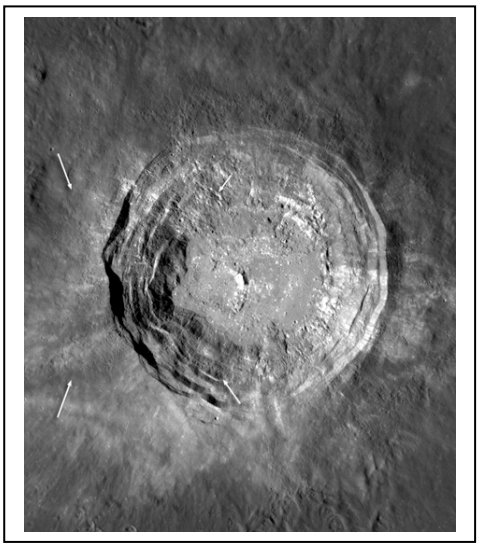
Problem 3 - The kinetic energy in Joules of a large meteor is given by $KE = 1/2mV^2$ where m is its mass in kilograms and V is its speed in meters/sec. One ton of TNT explodes with an energy of 4.2×10^9 Joules. How many tons of TNT did the Chelyabinsk Meteor yield as it exploded?

Answer: The mass was 10,000 tons or 10,000 tons x 1000 kg/1 ton = 10 million kg.
The speed was 18 km/s x 1000 m/km = 18,000 m/s, so the energy was

$KE = ½ (1.0 \times 10^7) \times (1.8 \times 10^4)^2 = 1.62 \times 10^{15}$ Joules.

This is equal to 1.62×10^{15} Joules x (1 ton/4.2×10^9Joules) = **386,000 tons of TNT** or about 10 times the energy of a small atom bomb. This is similar to the estimates found in the news reports of this event, and explains why it did so much damage!

The Volume of a Lunar Impact Crater



Lunar craters have been excavated by asteroid impacts for billions of years. This has caused major remodeling of the lunar surface as the material that once filled the crater is ejected. Some of this returns to the lunar surface hundreds of kilometers from the impact site.

An example of a lunar crater is shown in the image to the left taken of the crater Aristarchus by the NASA Lunar Reconnaissance Orbiter (LRO).

Astronomers create models of the rock that was displaced that try to match the overall shape of the crater. The shape of a crater can reveal information about the density of the rock, and even the way that it was layered below the impact area.

One such mathematical model was created for a 17-kilometer crater located at lunar coordinates 38.4 °North, and 194.9 °West. For this particular crater, its depth, D, at a distance of x from its center can be approximated by the following 4^{th}-order polynomial:

$$\mathbf{D = 0.0001x^4 - 0.0055x^3 + 0.0729x^2 - 0.2252x - 0.493}$$

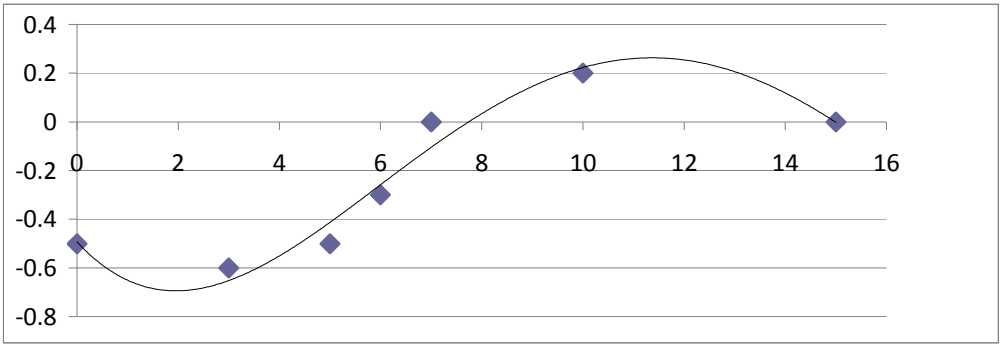
where D and x are in kilometers. The crater is symmetric around the axis x=0.

Problem 1 – Graph this function in the interval (0, +15.0) for which it provides a suitable model.

Problem 2 - To 2 significant figures, what is the approximate excavated volume of this symmetric crater using the method of inscribed and circumscribed disks?

Problem 3 – To 2 significant figures, what is the volume of this crater bounded by the function h(R) and the line y = +0.2 km, and rotated about the y-axis? (Use $\pi = 3.141$)

Problem 1 – Graph this function in the interval (0, +15.0) for which it provides a suitable model.



Problem 2 - To 2 significant figures, what is the approximate excavated volume of the symmetric crater using the method of inscribed and circumscribed disks? Answer; The volume of a disk is V = π R^2 h. For the crater, the inscribed disk has a radius of 5 kilometers and a height of 0.7 km, so its volume is Vi = 3.141 $(5)^2$ (0.7) = 55 km^3. The circumscribed disk has a radius of 10 km and a height of 0.8 km, so Vc = 3.141 $(10)^2$ (0.8) = 251 km^3. The estimated volume is then the average of these two or V = (Vc + Vi)/2 = 153 km^3. To 2 Significant Figure, the correct answer would be V = 150 km^3.

Problem 3 – To 2 significant figures, what is the volume of this crater bounded by the function h(R) and the line y = +0.2 km, and rotated about the y-axis? (Use π = 3.141) Answer: Using the method of shells, the volume differential for this problem is dV = 2π x [0.2 - h(x)] dx

The definite integral to evaluate is then $V = \int_0^{10} 2\pi x[0.2 - h(x)]dx$ Then :

$$V = 2\pi(0.2)\int_0^{10} x\,dx - 2\pi\int_0^{10} 0.0001x^5 - 0.0055x^4 + 0.0729x^3 - 0.2252x^2 - 0.493x\;dx$$

$$V = 2\pi(0.2)\frac{x^2}{2}\Big]_0^{10} - 2\pi\left(0.0001\frac{x^6}{6} - 0.0055\frac{x^5}{5} + 0.0729\frac{x^4}{4} - 0.2252\frac{x^3}{3} - 0.493\frac{x^2}{2}\right)_0^{10}$$

$$V = 2\pi(0.2)\frac{10^2}{2} - 2\pi\left(0.0001\frac{10^6}{6} - 0.0055\frac{10^5}{5} + 0.0729\frac{10^4}{4} - 0.2252\frac{10^3}{3} - 0.493\frac{10^2}{2}\right)$$

V = 2(3.141) [10 - 16.7 + 110 - 182.3 + 75.1 + 24.7] V = 6.24 [20.8]

V = 129.8 km^3 To 2 significant figures this becomes **V = 130 km^3**. So to check that Vi < V < Vo we have 55 km^3 < 130 km^3 < 251 km^3.

As a comet orbits the sun, it produces a long tail stretching millions of kilometers through space. The tail is produced by heated gases leaving the nucleus of the comet.

This image of the head of Comet Tempel-1 was taken by the Hubble Space Telescope on June 30, 2005. It shows the 'coma' formed by these escaping gases about 5 days before its closest approach to the sun (perihelion). The most interesting of these ingredients is ordinary water.

Problem 1 – The NASA spacecraft Deep Impact flew-by Temple-1 and measured the rate of loss of water from its nucleus. The simple quadratic function below gives the number of tons of water produced every minute, W, as Comet Tempel-1 orbited the sun, where T is the number of days since its closest approach to the sun, called perihelion.

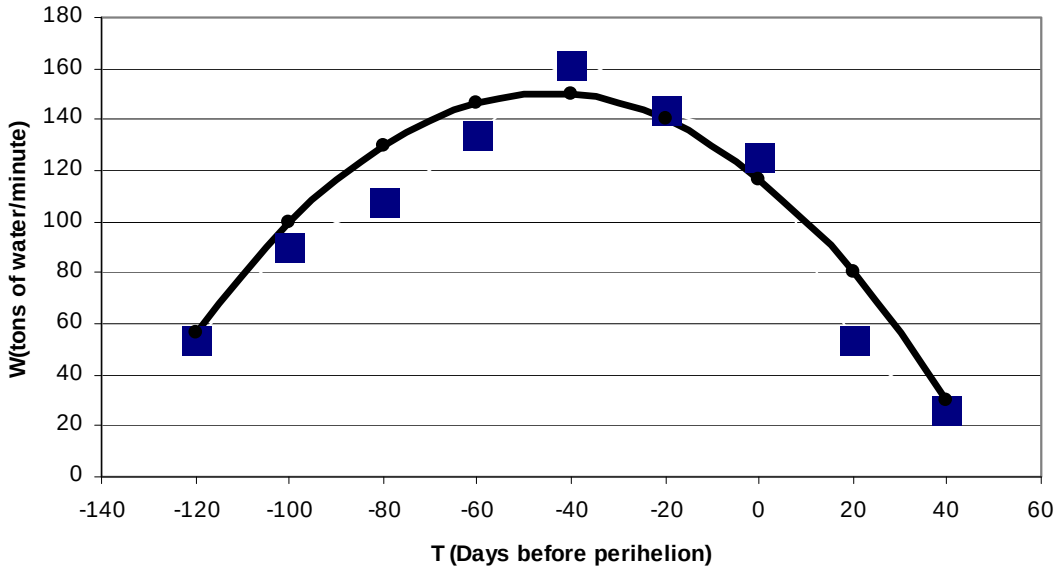
$$W(T) = \frac{(T+140)(60-T)}{60}$$

A) Graph the function W(T). B) For what days, T, will the water loss be zero? C) For what T did the comet eject its maximum amount of water each minute?

Problem 2 – To two significant figures, how many tons of water each minute were ejected by the comet 130 days before perihelion (T = -130)?

Problem 3 - To two significant figures, determine how many tons of water each minute were ejected by the comet 70 days after perihelion (T = +70). Can you explain why this may be a reasonable prediction consistent with the mathematical fit, yet an implausible 'Real World' answer?

Problem 1 – A) The graph below was created with Excel. The squares represent the actual measured data and are shown as an indicator of the quality of the quadratic model fit to the actual data.



Answer: B) The roots of the quadratic equation, where W(T)=0 are for **T=-140** days and **T=+60** days after perihelion. C) The maximum (vertex of the parabola) occurs half-way between the two intercepts at T = (-140+60)/2 or **T = -40** which indicates 40 days before perihelion.

Problem 2 – To two significant figures, how many tons of water each minute were ejected by the comet 130 days before perihelion (T = -130)?

Answer: W(-130) = (-130+140)(60+130)/60 = **32 tons/minute**

Problem 3 - To two significant figures, determine how many tons of water each minute were ejected by the comet 70 days after perihelion (T = +70). Can you explain why this may be a reasonable prediction consistent with the mathematical fit, yet an implausible 'Real World' answer?

Answer: The fitting function W(T) predicts that W(+70) = (70+140)(60-70)/60 = -35 tons per minute. Although this value smoothly follows the prediction curve, it implies that instead of ejecting water (positive answer means a positive rate of change) the comet is absorbing water (negative answer means a negative rate of change), so the prediction is not realistic.

Water on Mars!

NASA/JPL/Malin Space Science Systems

This NASA, Mars Orbiter image was taken of a crater wall in the southern hemisphere of mars from an altitude of 450 kilometers. It shows the exciting evidence of water gullies flowing downhill from the top left to the lower right.

The scale of an image is found by measuring with a ruler the distance between two points on the image whose separation in physical units you know. In this case, we are told the length of the dark bar is a distance of 300 meters.

Step 1: Measure the length of the bar with a metric ruler. How many millimeters long is the bar?

Step 2: Use clues in the image description to determine a physical distance or length. Convert this to meters.

Step 3: Divide your answer to Step 2 by your answer to Step 1 to get the image scale in meters per millimeter to two significant figures.

Once you know the image scale, you can measure the size of any feature in the image in units of millimeters. Then multiply it by the image scale from Step 3 to get the actual size of the feature in meters to two significant figures.

Question 1: What are the dimensions, in kilometers, of this image?
Question 2: How wide, in meters, are the streams half-way down their flow channels?
Question 3: What is the smallest feature you can see in the image?
Question 4: How wide is the top of the crater wall at its sharpest edge?

Answer Key:

This NASA, Mars Orbiter image was taken of a crater wall in the southern hemisphere of mars from an altitude of 450 kilometers. It shows the exciting evidence of water gullies flowing downhill from the top left to the lower right.

The scale of an image is found by measuring with a ruler the distance between two points on the image whose separation in physical units you know. In this case, we are told the length of the dark bar is a distance of 300 meters.

Step 1: Measure the length of the bar with a metric ruler. How many millimeters long is the bar?
Answer: 13 millimeters.

Step 2: Use clues in the image description to determine a physical distance or length. Convert this to meters.
Answer: 300 meters

Step 3: Divide your answer to Step 2 by your answer to Step 1 to get the image scale in meters per millimeter to two significant figures.
Answer: 300 meters / 13 mm = 23 meters / millimeter.

Once you know the image scale, you can measure the size of any feature in the image in units of millimeters. Then multiply it by the image scale from Step 3 to get the actual size of the feature in meters to two significant figures.

Question 1: What are the dimensions, in kilometers, of this image?
Answer: 134 mm x 120 mm = 3.1 km x 2.8 km

Question 2: How wide, in meters, are the streams half-way down their flow channels?
Answer: 0.5 millimeters = 12 meters.

Question 3: What is the smallest feature you can see in the image?
Answer: Sand dunes in upper left of image = 0.3 millimeters or 7 meters wide.

Question 4: How wide is the top of the crater wall at its sharpest edge?
Answer: 0.2 millimeters or 4 meters wide.

Avalanche on Mars!

This image was taken by NASA's Mars Reconnaissance Orbiter on February 19, 2008. It shows an avalanche photographed as it happened on a cliff on the edge of the dome of layered deposits centered on Mars' North Pole. From top to bottom this impressive cliff is over 700 meters (2300 feet) tall and reaches slopes over 60 degrees. The top part of the scarp, to the left of the image, is still covered with bright (white) carbon dioxide frost which is disappearing from the polar regions as spring progresses. The upper mid-toned (pinkish-brownish) section is composed of layers that are mostly ice with varying amounts of dust. The dust cloud extends 190 meters from the base of the cliff.

The scale of an image is found by measuring with a ruler the distance between two points on the image whose separation in physical units you know. In this case, we are told the cloud extends 190 meters from the base of the cliff.

Step 1: Measure the length of the dust cloud with a metric ruler. How many millimeters long is the cloud?

Step 2: Use clues in the image description to determine a physical distance or length.

Step 3: Divide your answer to Step 2 by your answer to Step 1 to get the image scale in meters per millimeter to two significant figures.

Once you know the image scale, you can measure the size of any feature in the image in units of millimeters. Then multiply it by the image scale from Step 3 to get the actual size of the feature in meters to two significant figures.

Question 1: What are the dimensions, in meters, of this image?
Question 2: What is the smallest detail you can see in the ice shelf?
Question 3: What is the average thickness of the red layers on the cliff?
Question 4: What is the total width of the reddish rock cliff?

For experts: Two sides of the right triangle measure 700 meters, and your answer to Question 4. What is the angle of the cliff at the valley floor?

The scale of an image is found by measuring with a ruler the distance between two points on the image whose separation in physical units you know. In this case, we are told the cloud extends 190 meters from the base of the cliff.

Step 1: Measure the length of the dust cloud with a metric ruler. How many millimeters long is the cloud?
Answer: 60 millimeters.

Step 2: Use clues in the image description to determine a physical distance or length.
Answer: 190 meters.

Step 3: Divide your answer to Step 2 by your answer to Step 1 to get the image scale in meters per millimeter to two significant figures.
Answer: 190 meters / 60 mm = 3.2 meters / mm

Once you know the image scale, you can measure the size of any feature in the image in units of millimeters. Then multiply it by the image scale from Step 3 to get the actual size of the feature in meters to two significant figures.

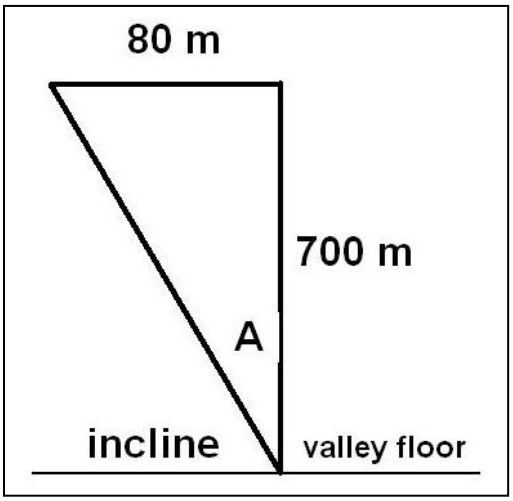
Question 1: What are the dimensions, in meters, of this image?
Answer: 140 mm x 86 mm = 448.0 meters x 275.2 meters, but to two significant figures this becomes 450 meters x 280 meters.

Question 2: What is the smallest detail you can see in the ice shelf?
Answer: 0.2 mm = 0.6 meters

Question 3: What is the average thickness of the red layers on the cliff?
Answer: 1.0 millimeter = 3.2 meters.

Question 4: What is the total width of the reddish rock cliff?
Answer: 25 millimeters = 80 meters.

For experts: Two sides of the right triangle measure 700 meters, and your answer to Question 4. What is the angle of the cliff at the valley floor?
Answer: Draw the triangle with the 700-meter side being vertical and the 80 meter side being horizontal. The tangent of the angle is (80 / 700) = 0.11 so the angle is 6.5 degrees. This is the angle from the vertical, so the incline angle from the floor of the valley is 90 - 6.5 = 84 degrees. This is a nearly vertical wall!



Kelvin Temperatures and Very Cold Things!

183 K	Vostok, Antarctica
160 K	Phobos
134 K	Superconductors
128 K	Europa summer
120 K	Moon at night
95 K	Titan
90 K	Liquid oxygen
88 K	Miranda
81 K	Enceladus summer
77 K	Liquid nitrogen
70 K	Mercury at night
63 K	Solid nitrogen
55 K	Pluto summer
54 K	Solid oxygen
50 K	Quaoar
45 K	Moon - shadowed crater
40 K	Star-forming region
33 K	Pluto wInter
20 K	Liquid hydrogen
19 K	Bose-Einstein Condensates
4 K	Liquid helium
3 K	Cosmic Background Radiation
2 K	Liquid helium
1 K	Boomerang Nebula
0 K	ABSOLUTE ZERO

To keep track of some of the coldest things in the universe, scientists use the Kelvin temperature scale which begins at 0 Kelvin, which is also called Absolute Zero. Nothing can ever be colder than Absolute Zero because at this temperature, all motion stops. The table to the left shows some typical temperatures of different systems in the universe.

You are probably already familiar with the Centigrade (C) and Fahrenheit (F) temperature scales. The two formulas below show how to switch from degrees-C to degrees-F.

$$C = \frac{5}{9}(F - 32) \qquad F = \frac{9}{5}C + 32$$

Because the Kelvin scale is related to the Centigrade scale, we can also convert from Centigrade to Kelvin (K) using the equation:

$$K = 273 + C$$

Use these three equations to convert between the three temperature scales:

Problem 1: 212 F converted to K

Problem 2: 0 K converted to F

Problem 3: 100 C converted to K

Problem 4: -150 F converted to K

Problem 5: -150 C converted to K

Problem 6: Two scientists measure the daytime temperature of the moon using two different instruments. The first instrument gives a reading of + 107 C while the second instrument gives + 221 F. A) What are the equivalent temperatures on the Kelvin scale; B) What is the average daytime temperature on the Kelvin scale?

Answer Key

$$C = \frac{5}{9}(F - 32) \qquad F = \frac{9}{5}C + 32 \qquad K = 273 + C$$

Problem 1: 212 F converted to K :
First convert to C: C = 5/9 (212 - 32) = +100 C. Then convert from C to K:
K = 273 + 100 = **373 Kelvin**

Problem 2: 0 K converted to F: First convert to Centigrade:
0 = 273 + C so C = -273 degrees. Then convert from C to F:
F = 9/5 (-273) + 32 = **-459 Fahrenheit**.

Problem 3: 100 C converted to K : K = 273 - 100 = **373 Kelvin**.

Problem 4: -150 F converted to K : Convert to Centigrade
C = 5/9 (-150 - 32) = -101 C. Then convert from Centigrade to Kelvin: K = 273 - 101 = **172 Kelvin**.

Problem 5: -150 C converted to K : K = 273 + (-150) = **123 Kelvin**

Problem 6: Two scientists measure the daytime temperature of the moon using two different instruments. The first instrument gives a reading of + 107 C while the second instrument gives + 221 F.

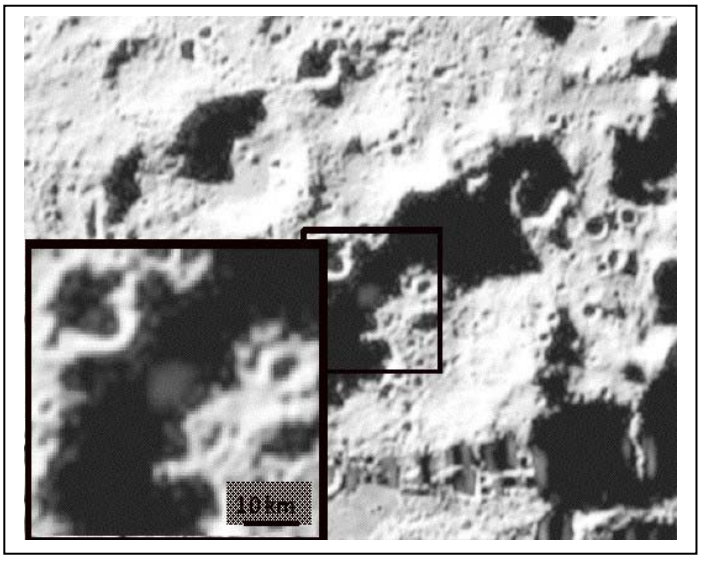
A) What are the equivalent temperatures on the Kelvin scale?;
107 C becomes K = 273 + 107 = **380 Kelvins.**
221 F becomes C = 5/9 (221 - 32) = 105 C, and so K = 273 + 105 = **378 Kelvins**.

B) What is the average daytime temperature on the Kelvin scale?
Answer: (380 + 378)/2 = **379 Kelvins**.

C) Explain why the Kelvin scale is useful for calculating averages of different temperatures. Answer: **Because the degrees are in the same units in the same measuring scale so that the numbers can be averaged.**

Note: Students may recognize that in order to average +107 C and +221 F they could just as easily have converted both temperatures to the Centigrade scale or the Fahrenheit scale and then averaged those temperatures. You may challenge them to do this, and then compare the averaged values in the Centigrade, Fahrenheit and Kelvin scales. They should note that the final answer will be the same as 379 Kelvins converted to F and C scales using the above formulas.

LCROSS 'Sees' Water on the Moon! 13.5



On October 9, 2009 the LCROSS spacecraft and its companion Centaur upper stage, impacted the lunar surface within the shadowed crater Cabeus located near the moon's South Pole. The Centaur impact speed was 9,000 km/hr with a mass of 2.2 tons.

The impact created a crater about 20 meters across and about 3 meters deep. Some of the excavated material formed a plume of debris visible to the LCROSS satellite as it flew by. Instruments on LCROSS detected about 25 gallons of water.

Problem 1 - The volume of the crater can be approximated as a cylinder with a diameter of 20 meters and a height of 3 meters. From the formula $V = \pi R^2 h$, what was the volume of the lunar surface excavated by the LCROSS-Centaur impact in cubic meters?

Problem 2 - If the density of the lunar soil (regolith) is about 3000 kilograms/meter^3, how many tons of regolith were excavated by the impact?

Problem 3 - During an impact, most of the excavated material remains as a ring-shaped ejecta blanket around the crater. For the LCROSS crater, the ejecta appeared to be scattered over an area about 70 meters in diameter and perhaps 0.2 meter thick around the crater. How many tons of regolith from the crater remained near the crater?

Problem 4 - If the detected water came from the regolith ejected in the plume, and not scattered in the ejecta blanket ,what was the concentration of water in the plume in units of tons of regolith per liter of water?

Problem 1 - The volume of the crater can be approximated as a cylinder with a diameter of 20 meters and a height of 3 meters. From the formula $V = \pi R^2 h$, what was the volume of the lunar surface excavated by the LCROSS-Centaur impact in cubic meters?

Answer: $V = (3.14) \times (10 \text{ meters})^2 \times 3 =$ **942 cubic meters**.

Problem 2 - If the density of the lunar soil (regolith) is about 3000 kilograms/meter^3, how many tons of regolith were excavated by the impact?

Answer: $3000 \text{ kg/m}^3 \times (942 \text{ meters}^3) = 2{,}800{,}000$ kilograms. Since 1000 kg = 1 ton, there were **2,800 tons of regolith excavated**.

Problem 3 - During an impact, most of the excavated material remains as a ring-shaped ejecta blanket around the crater. For the LCROSS crater, the ejecta appeared to be scattered over an area about 70 meters in diameter and perhaps 0.2 meter thick around the crater. How many tons of regolith from the crater remained near the crater?

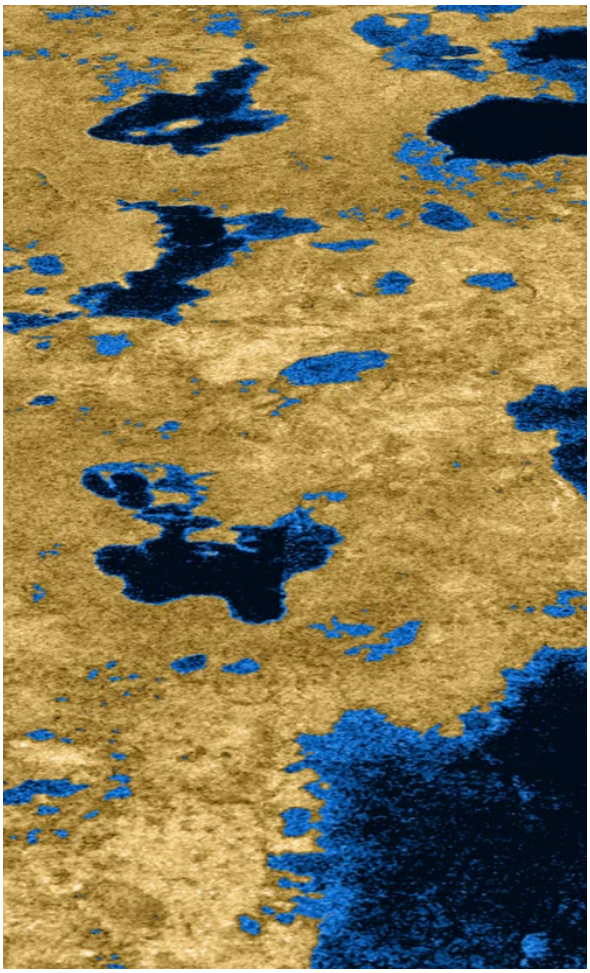
Answer: The area of the ejecta blanket is given by $A = \pi(35 \text{ meters})^2 - \pi(10 \text{ meters})^2 = 3{,}846 - 314 = 3500 \text{ meters}^2$. The volume is $A \times h = (3500 \text{ meters}^2) \times 0.2 \text{ meters} = 700 \text{ meters}^3$. Then the mass is just $M = (700 \text{ meters}^3) \times (3{,}000 \text{ kilograms/meter}^3) =$ 2,100,000 kilograms or **2,100 tons in the ejecta blanket**.

Problem 4 - If the detected water came from the regolith ejected in the plume, and not scattered in the ejecta blanket ,what was the concentration of water in the plume in units of tons of regolith per liter of water?

Answer: The amount of ejected regolith was 2,800 tons - 2,100 tons or 700 tons. The detected water amounted to 25 gallons or 25 gallons x (3.78 liters/ 1 gallon) = 95 liters. So the concentration was about C = 700 tons/95 liters = **7 tons/liter**.

Note to teacher: The estimated concentration, C, in Problem 4 is based on an approximated geometry for the crater (cylinder), an average thickness for the ejecta blanket (about 0.2 meters) and whether all of the remaining material (700 tons) was actually involved in the plume measured by LCROSS. Students may select, by scaled drawing, other geometries for the crater, and thickness for the ejecta blanket to obtain other estimates for the concentration, C. The scientific analysis of the LCROSS data may eventually lead to better estimates for C.

Lakes of Methane on Titan



A key goal in the search for life elsewhere in the universe is to detect liquid water, which is generally agreed to be the most essential ingredient for living systems that we know about.

The image to the left is a false-color synthetic radar map of a northern region of Titan taken during a flyby of the cloudy moon by the robotic Cassini spacecraft in July, 2006. On this map, which spans about 150 kilometers across, dark regions reflect relatively little of the broadcast radar signal. Images like this show Titan to be only the second body in the solar system to possess liquids on the surface. In this case, the liquid is not water but methane!

Future observations from Cassini during Titan flybys will further test the methane lake hypothesis, as comparative wind affects on the regions are studied.

Problem 1 – From the information provided, what is the scale of this image in kilometers per millimeter?

Problem 2 – What is the approximate total surface area of the lakes in this radar image?

Problem 3 - Assume that the lakes have an average depth of about 20 meters. How many cubic kilometers of methane are implied by the radar image?

Problem 4 – The volume of Lake Tahoe on Earth is about 150 km^3. How many Lake Tahoes-worth of methane are covered by the Cassini radar image?

Problem 1 – From the information provided, what is the scale of this image in kilometers per millimeter?

Answer: 150 km / 77 millimeters = **1.9 km/mm**.

Problem 2 – What is the approximate total surface area of the lakes in this radar image?

Answer: Combining the areas over the rectangular field of view gives about 1/4 of the area covered. The field of view measures 77 mm x 130mm or 150 km x 247 km or an area of 37,000 km^2. The dark areas therefore cover about 1/4 x 37,000 km^2 or **9,300 km^2.**

Problem 3 - Assume that the lakes have an average depth of about 20 meters. How many cubic kilometers of methane are implied by the radar image?

Answer: Volume = area x height = 9,300 km^2 x (0.02 km) = **190 km^3**.

Problem 4 – The volume of Lake Tahoe on Earth is about 150 km^3. How many Lake Tahoes-worth of methane are covered by the Cassini radar image?

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Answer: 190 km^3 / 150 km^3 = **1.3 Lake Tahoes**.

How Hot is that Planet?

Planets have been spotted orbiting hundreds of nearby stars, but this makes for a variety of temperatures depending on how far the planet is from its star and the stars luminosity.

The temperature of the planet will be about

$$T=273\left(\frac{(1-A)L}{D^2}\right)^{1/4}$$

where A is the reflectivity (albedo) of the planet, L is the luminosity of its star in multiples of the sun's power, and D is the distance between the planet and the star in Astronomical Units (AU), where 1 AU is the distance from Earth to the sun (150 million km). The resulting temperature will be in units of Kelvins. (i.e. 0^{o} Celsius = +273 K, and Absolute Zero is defined as 0 K).

Problem 1 - Earth is located 1.0 AU from the sun, for which L = 1.0. What is the surface temperature of Earth if its albedo is 0.4?

Problem 2 - At what distance would Earth have the same temperature as in Problem 1 if the luminosity of our sun were increased 1000 times and all other quantities remained the same?

Problem 3 - The recently discovered planet CoRoT-7b (see artist's impression above, from ESA press release), orbits the star CoRoT-7 which is a sun-like star located about 490 light years from Earth in the direction of the constellation Monoceros. If the luminosity of the star is 71% of the sun's luminosity (L = 0.71) and the planet is located 2.6 million kilometers from its star (D= 0.017 AU) what are the predicted surface temperatures of the day-side of CoRoT-7b for the range of albedos shown in the table below?

Surface Material	Example	Albedo (A)	Surface Temperature (K)
Basalt	Moon	0.06	1892
Iron Oxide	Mars	0.16	
Water+Land	Earth	0.40	
Gas	Jupiter	0.70	

Problem 1 - Earth is located 1.0 AU from the sun, for which L = 1.0. What is the surface temperature of Earth if its albedo is 0.4? **Answer: $T = 273\ (0.6)^{1/4} = 240$ K**

Note: The equilibrium temperature of Earth is much lower than the freezing point of water. Were it not for the trace gases of carbon dioxide and to a lesser extent water vapor and methane providing 'greenhouse heating' our planet would be unlivable even with an atmosphere!

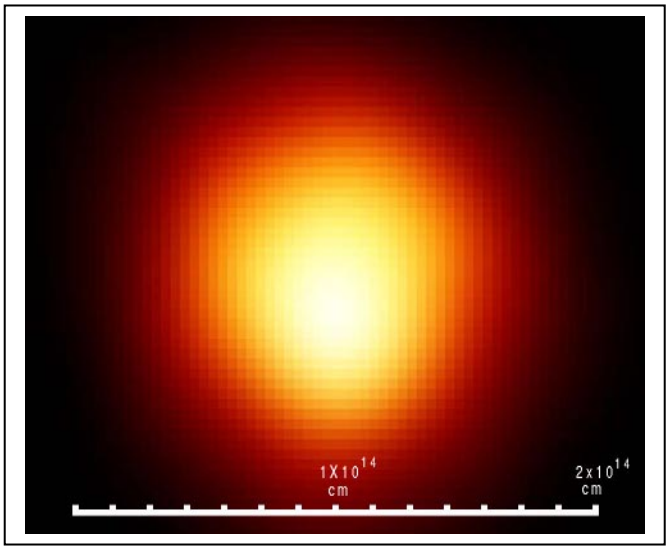
Problem 2 - At what distance would Earth have the same temperature as in Problem 1 if the luminosity of our sun were increased 1000 times and all other quantities remained the same? Answer: From the formula, T = 240 and L = 1000 so
$240 = 273(0.6 \times 1000/D^2)^{1/4}$ and so **D = 5.6 AU**. This is about near the orbit of Jupiter.

Problem 3 - The recently discovered planet CoRoT-7b orbits the star CoRoT-7 which is a sun-like star located about 490 light years from Earth in the direction of the constellation Monoceros. If the luminosity of the star is 71% of the sun's luminosity (L = 0.71) and the planet is located 2.6 million kilometers from its star (D= 0.017 AU) what are the predicted surface temperatures of the day-side of CoRoT-7b for the range of albedos shown in the table below?

Surface Material	Example	Albedo (A)	Surface Temperature (K)
Basalt	Moon	0.06	1892
Iron Oxide	Mars	0.16	1840
Water+Land	Earth	0.40	1699
Gas	Jupiter	0.70	1422

Example: For an albedo similar to that of our Moon:
$T = 273 * ((1-0.06)*0.71/(0.017)^2)^{.25}$
= **1,892 Kelvin**

Note: To demonstrate the concept of Significant Figures, the values for L, D and A are given to 2 significant figures, so the answers should be rounded to 1900, 1800, 1700 and 1400 respectively.



The amount of power that a star produces in light is related to the temperature of its surface and the area of the star. The hotter a surface is, the more light it produces. The bigger a star is, the more surface it has. When these relationships are combined, two stars at the same temperature can be vastly different in brightness because of their sizes.

Image: Betelgeuse (Hubble Space Telescope.) It is 950 times bigger than the sun!

The basic formula that relates stellar light output (called luminosity) with the surface area of a star, and the temperature of the star, is L = A x F where the star is assumed to be spherical with a surface area of $A = 4 \pi R^2$, and the radiation emitted by a unit area of its surface (called the flux) is given by $F = \sigma T^4$. The constant, σ, is the Stefan-Boltzman radiation constant and it has a value of $\sigma = 5.67 \times 10^{-5}$ ergs/ (cm^2 sec deg^4). The luminosity, L, will be expressed in power units of ergs/sec if the radius, R, is expressed in centimeters, and the temperature, T, is expressed in Kelvins. The formula then becomes,

$$L = 4 \pi R^2 \sigma T^4$$

Problem 1 - The Sun has a temperature of 5700 Kelvins and a radius of 6.96×10^5 kilometers, what is its luminosity in A) ergs/sec? B) Watts? (Note: 1 watt = 10^7 ergs/sec).

Problem 2 - The red supergiant Antares in the constellation Scorpius, has a temperature of 3,500 K and a radius of 700 times the radius of the sun. What is its luminosity in A) ergs/sec? B) multiples of the solar luminosity?

Problem 3 - The nearby star, Sirius, has a temperature of 9,200 K and a radius of 1.76 times our Sun, while its white dwarf companion has a temperature of 27,400 K and a radius of 4,900 kilometers. What are the luminosities of Sirius-A and Sirius-B compared to our Sun?

Calculus:

Problem 4 - Compute the total derivative of L(R,T). If a star's radius increases by 10% and its temperature increases by 5%, by how much will the luminosity of the star change if its original state is similar to that of the star Antares? From your answer, can you explain how a star's temperature could change without altering the luminosity of the star. Give an example of this relationship using the star Antares!

Answer Key

Problem 1 - We use L = 4 (3.141) R^2 (5.67 x 10^{-5}) T^4 to get L (ergs/sec) = 0.00071 R(cm)^2 T(K)^4 then,
A) L(ergs/sec) = 0.00071 x (696,000 km x 10^5 cm/km)^2 (5700)^4 = **3.6 x 10^{33} ergs/sec**
B) L(watts) = 3.6 x 10^{33} (ergs/sec)/ 10^7 (ergs/watt) = **3.6 x 10^{25} watts**.

Problem 2 - A) The radius of Antares is 700 x 696,000 km = 4.9 x 10^8 km.
L(ergs/sec) = 0.00071 x (4.9 x 10^8 km x 10^5 cm/km)^2 (3500)^4 = **2.5 x 10^{38} ergs/sec**
B) L(Antares) = (2.5 x 10^{38} ergs/sec) / (3.6 x 10^{33} ergs/sec) = **71,000 L(sun).**

Problem 3 - Sirius-A radius = 1.76 x 696,000 km = 1.2 x 10^6 km
L(Sirius-A) = 0.00071 x (1.2 x 10^6 km x 10^5 cm/km)^2 (9200)^4 = **7.3 x 10^{34} ergs/sec**
L = (7.3 x 10^{34} ergs/sec) / (3.6 x 10^{33} ergs/sec) = **20.3 L(sun).**
L(Sirius-B) = 0.00071 x (4900 km x 10^5 cm/km)^2 (27,400)^4 = **9.5 x 10^{31} ergs/sec**
L(Sirius-B) = 9.5 x 10^{31} ergs/sec / 3.6 x 10^{33} ergs/sec = **0.026 L(sun).**

Advanced Math:

Problem 4 *(*Note: In the discussion below, the symbol d represents a partial derivative)

$$dL(R,T) = \frac{dL(R,T)}{dR}\,dR + \frac{dL(R,T)}{dT}\,dT$$

$$dL = [\,4\,\pi\,(2)\,R\,\sigma\,T^4\,]\,dR + [\,4\,\pi\,(4)\,R2\,\sigma\,T^3\,]\,dT$$

$$dL = 8\,\pi\,R\,\sigma\,T^4\,dR + 16\,\pi\,R^2\,\sigma\,T^3\,dT$$

To get percentage changes, divide both sides by $L = 4\,\pi\,R^2\,\sigma\,T^4$

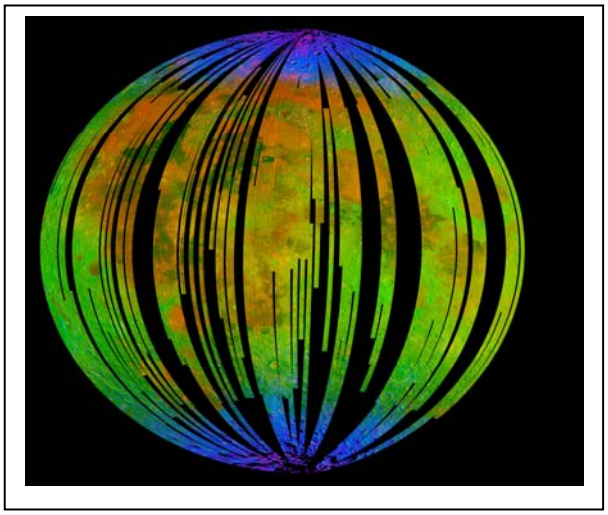
$$\frac{dL}{L} = \frac{8\,\pi\,R\,\sigma\,T^4}{4\,\pi\,R^2\,\sigma\,T^4}\,dR + \frac{16\,\pi\,R^2\,\sigma\,T^3}{4\,\pi\,R^2\,\sigma\,T^4}\,dT$$

Then dL/L = 2 dR/R + 4 dT/T so for the values given, dL/L = 2 (0.10) + 4 (0.05) = **0.40**
The star's luminosity will increase by 40%.

Since dL/L = 2 dR/R + 4 dT/T, we can obtain no change in L if 2 dR/R + 4 dT/T = 0. This means that 2 dR/R = - 4 dT/T and so, - 0.5 dR/R = dT/T. **The luminosity of a star will remain constant if, as the temperature decreases, its radius increases.**

Example. For Antares, its original luminosity is 71,000 L(sun) or 2.5 x 10^{38} ergs/sec. If I increase its radius by 10% from 4.9 x 10^8 km to 5.4 x 10^8 km, its luminosity will remain the same if its temperature is decreased by dT/T = 0.5 x 0.10 = 0.05 which will be 3500 x 0.95 = 3,325 K so L(ergs/sec) = 0.00071 x (5.4 x 10^8 km x 10^5 cm/km)^2 (3325)^4 = **2.5 x 10^{38} ergs/sec**

Water on the Moon !



The debate has gone back and forth over the last 10 years as new data are found, but measurements by Deep Impact/EPOXI, Cassini and most recently the Lunar Reconnaissance Orbiter and Chandrayaan-1 are now considered conclusive. Beneath the shadows of polar craters, billions of gallons of water may be available for harvesting by future astronauts.

The image to the left created by the Moon Minerology Mapper (M3) instrument onboard Chandrayaan-1, shows deposits and sources of hydroxyl molecules. The data has been colored blue and superimposed on a lunar photo.

Complimentary data from the Deep Impact/EPOXI and Cassini missions of the rest of the lunar surface also detected hydroxyl molecules covering about 25% of the surveyed lunar surface. The hydroxyl molecule (symbol OH) consists of one atom of oxygen and one of hydrogen, and because water is basically a hydroxyl molecule with a second hydrogen atom added, detecting hydroxyl on the moon is an indication that water molecules are also present.

How much water might be present? The M3 instrument can only detect hydroxyl molecules if they are in the top 1-millimeter of the lunar surface. The measurements also suggest that about 1 metric ton of lunar surface has to be processed to extract 1 liter (0.26 gallons) of water.

Problem 1 – The radius of the moon is 1,731 kilometers. How many cubic meters of surface volume is present in a layer that is 1 millimeter thick?

Problem 2 – The density of the lunar surface (called the regolith) is about 3000 kilograms/meter^3. How many metric tons of regolith are found in the surface volume calculated in Problem 1?

Problem 3 – The concentration of water is 1 liter per metric ton. How many liters of water could be recovered from the 1 millimeter thick surface layer if 25% of the lunar surface contains water?

Problem 4 – How many gallons could be recovered if the entire surface layer were mined? (1 Gallon = 3.78 liters).

Problem 1 – The radius of the moon is 1,731 kilometers. How many cubic meters of surface volume is present in a layer that is 1 millimeter thick?

Answer: The surface area of a sphere is given by $S = 4\pi r^2$ and so the volume of a layer with a thickness of L is $V = 4\pi r^2 L$ provided that L is much smaller than r.
$V = 4 \times (3.141) \times (1731000)^2 \times 0.001 =$ **3.76×10^{10} m^3**

Problem 2 – The density of the lunar surface (called the regolith) is about 3000 kilograms/meter^3. How many metric tons of regolith are found in the surface volume calculated in Problem 1? Answer: 3.76×10^{10} $m^3 \times (3000$ kg/$m^3) \times (1$ ton/1000 kg) = **1.13×10^{11} metric tons.**

Problem 3 – The concentration of water is 1 liter per metric ton. How many liters of water could be recovered from the 1 millimeter thick surface layer if 25% of the surface contains water? Answer: 1.13×10^{11} tons x (1 liter water/1 ton regolith) x 1/4 = **2.8×10^{10} liters of water.**

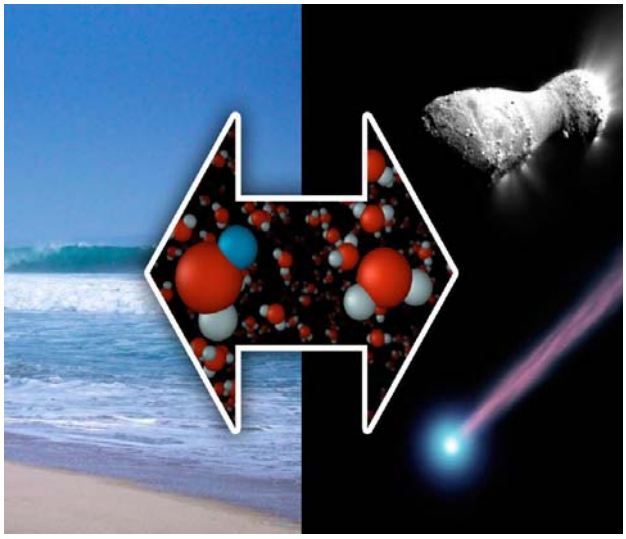
Problem 4 – How many gallons could be recovered if the entire surface layer were mined? (1 Gallon = 3.78 liters). Answer: 2.8×10^{10} liters x (1 gallon / 3.78 liters) = 7.5×10^9 gallons of water or about **8 billion gallons of water.**

Note: This is similar to the roughly '7 billion gallon' estimate made by the M3 scientists as described in the NASA Press Release for this discovery in September 2009.

For more information, visit:

Moon Minerology Mapper News - http://moonmineralogymapper.jpl.nasa.gov/

The front picture of the moon is from NASA's Moon Mineralogy Mapper on the Indian Space Research Organization's Chandrayaan-1 mission. It is a three-color composite of reflected near-infrared radiation from the sun, and illustrates the extent to which different materials are mapped across the side of the moon that faces Earth. Small amounts of water and hydroxyl (blue) were detected on the surface of the moon at various locations. This image illustrates their distribution at high latitudes toward the poles. **Blue** shows the signature of water and hydroxyl molecules as seen by a highly diagnostic absorption of infrared light with a wavelength of three micrometers. **Green** shows the brightness of the surface as measured by reflected infrared radiation from the sun with a wavelength of 2.4 micrometers. **Red** shows an iron-bearing mineral called pyroxene, detected by absorption of 2.0-micrometer infrared light.



The abundance of heavy-water in Earth's oceans is about 0.015%. The abundance of heavy-water in Hartley-2 is about 0.016%, so comets like Hartley-2 could have impacted Earth and deposited over time Earth's ocean water. (Image courtesy NASA/JPL-Caltech)

New measurements from the Herschel Space Observatory show that comet Hartley 2, which comes from the distant Kuiper Belt, contains water with the same chemical signature as Earth's oceans. This remote region of the solar system, some 30 to 50 times as far away as the distance between Earth and the sun, is home to icy, rocky bodies including Pluto, other dwarf planets and innumerable comets.

Herschel detected the signature of vaporized water in this coma and, to the surprise of the scientists, Hartley 2 possessed half as much "heavy water" as other comets analyzed to date. In heavy water, one of the two normal hydrogen atoms has been replaced by the heavy hydrogen isotope known as deuterium. The amount of deuterium is similar to the abundance of this isotope in Earth's ocean water.

The deposition of Earth's oceans probably occurred between 4.2 and 3.8 billion years ago. Suppose that the comet nuclei consisted of three major types, each spherical in shape and made of pure water-ice: Type 1 consisting of 2km in diameter bodies arriving once every 6 months, Type-2 consisting of 20 km diameter bodies arriving once every 600 years and Type-3 consisting of 200 km diameter bodies arriving every one million years.

Problem 1 - What are the volumes of the three types of comet nuclei in km^3?

Problem 2 - The volume of Earth's liquid water oceans is 1.33×10^9 cubic kilometers. If solid ice has 6 times the volume of liquid water, what is the volume of cometary ice that must be delivered to Earth's surface every year to create Earth's oceans between 4.2 and 3.8 billion years ago?

Problem 3 - What is the annual ice deposition rate for each of the three types of cometary bodies?

Problem 4 - How many years would it take to form the oceans at the rate that the three types of cometary bodies are delivering ice to Earth's surface?

Space Observatory Provides Clues to Creation of Earth's Oceans

http://www.nasa.gov/mission_pages/herschel/news/herschel20111005.html

Problem 1 - What are the volumes of the three types of comet nuclei in km^3?

Answer: $V = 4/3\ \pi\ R^3$ so

Type 1 Volume = **$4.2\ km^3$**

Type 2 Volume = **$4{,}200\ km^3$**

Type 3 Volume = **$4.2 \times 10^6\ km^3$**

Problem 2 - The volume of Earth's liquid water oceans is 1.33×10^9 cubic kilometers. If solid ice has 6 times the volume of liquid water, what is the volume of cometary ice that must be delivered to Earth's surface every year to create Earth's oceans between 4.2 and 3.8 billion years ago?

Answer: $1.33 \times 10^9\ km^3$ of water requires
$6 \times (1.33 \times 10^9\ km^3) = 8.0 \times 10^9\ km^3$ of ice.

The average delivery rate would be about
$R = 8.0 \times 10^9\ km^3$ of ice / 400 million years
= **$20\ km^3$ of ice per year**.

Problem 3 - What is the annual ice deposition rate for each of the three types of cometary bodies?

Type 1: $R1 = 4.2\ km^3/0.5$ yrs = **$8.4\ km^3/yr$**

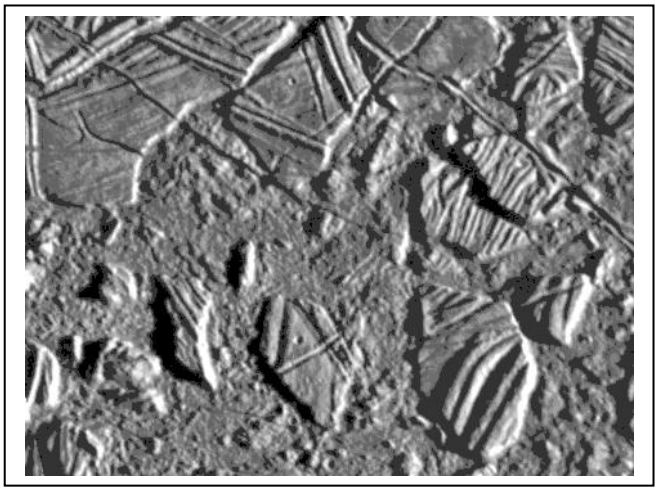
Type 2: $R2 = 4200\ km^3/600$ yrs = **$7.0\ km^3/yr$**

Type 3: $R3 = 4.2 \times 10^6\ km^3 / 1000000$ yrs = **$4.2\ km^3/yr$**

Problem 4 - How many years would it take to form the oceans at the rate that the three types of cometary bodies are delivering ice to Earth's surface?

Answer: The total deposition rate is R1+R2+R3 = $20\ km^3/yr$, so it would take
$T = 8.0 \times 10^9\ km^3$ of ice / $(20\ km^3/yr)$ = **400 million years**.

Water on Planetary Surfaces



Space is very cold! Without a source of energy, like a nearby star, water will exist at a temperature at nearly -270 C below zero and frozen solid. To create a permanent body of liquid water in which pre-biotic chemistry can occur, a steady source of energy must flow into the ice to keep it melted and in liquid form. Common sources of energy on Earth are volcanic activity, oceanic vents and fumaroles, and sunlight.

The picture above was taken by NASA's Galileo spacecraft of the surface of Jupiter's moon Europa. Its icy crust is believed to hide a liquid-water ocean beneath. The energy for keeping the water in a liquid state is probably generated by the gravity of Jupiter, which distorts Europa's shape through tidal action. The tidal energy may be enough to keep the oceans liquid for billions of years.

A common measure of energy flow or usage is the Watt. One Watt equals one Joule of energy emitted or consumed in one second.

Problem 1: How much energy, in Joules, does a 100 watt incandescent bulb consume if left on for 1 hour?

Problem 2: A house consumes about 3,000 kilowatts in one hour. How many Joules is this?

Problem 3: A homeowner has a solar panel system that produces 3,600,000 Joules every hour. How many watts of electrical appliances can be run by this system?

Water ice at 0 C requires 330,000 Joules of energy to become liquid for each kilogram of ice. Suppose the ice absorbed all the energy that fell on it. Ice doesn't really work that way, but let's suppose that it does just to make a simple mathematical model!

Problem 4: A student wants to melt a 10 kilogram block of ice with a 2,000-watt hair dryer. How many seconds will it take to melt the ice block completely? How many minutes?

Problem 1: How much energy, in Joules, does a 100 watt incandescent bulb consume if left on for 1 hour?

Answer: 100 watts is the same as 100 Joules/sec, so if 1 hour = 3600 seconds, the energy consumed is 100 Joules/sec x 3600 seconds = **360,000 Joules.**

Problem 2: A house consumes about 3,000 kilowatts in one hour. How many Joules is this?

Answer: 3,000 kilowatts x (1,000 watts/1 kilowatt) = 3,000,000 watts. Since this equals 3,000,000 Joules/sec, in 1 hour (3600 seconds) the consumption is 3,000,000 watts x 3,600 seconds = **10,800,000,000 Joules or 10.8 billion Joules.**

Problem 3: A homeowner has a solar panel system that produces 3,600,000 Joules every hour. How many watts of electrical appliances can be run by this system?

Answer: 3,600,000 Joules in 1 hour is an average rate of 3,600,000 Joules/3,600 seconds = 1,000 Joules/sec or **1,000 Watts**. This is the maximum rate at which the appliances can consume before exceeding the capacity of the solar system.

Problem 4: A student wants to melt a 10 kilogram block of ice with a 2,000-watt hair dryer. How many seconds will it take to melt the ice block completely? How many minutes?

Answer: From the information provided, it takes 330,000 Joules to melt 1 kilogram of ice. So since the mass of the ice block is 10 kilograms, it will take 330,000 x 10 = **3,300,000 Joules**

If the ice stores the energy falling on it from the hair dryer, all we need to do is to calculate how long a 2,000-watt hair dryer needs to be run in order to equal 3,300,000 Joules. This will be Time = 3,300,000 Joules/2000 watts = **1650 seconds or about 27.5 minutes.**

Proving that the unit will automatically be in seconds is a good exercise in unit conversions and using the associative law and reciprocals:

1 Joule / (1 watt) = 1 Joule / (1 Joule/sec)
= 1 Joule x (1 sec/1 joule) : Multiply be the reciprocal
= (1 Joule x 1 sec)/ 1 joule : Re-write
= 1 sec x (1 joule / 1 joule) : Re-group common terms
= 1 sec. :after cancling the 'joules' unit.

A Timeline for Planet Formation

Era	Time (years)	Description
Pre-solar Nebula Era	0.0	Collapse of cloud to form flattened disk
Asteroid Era	3 million	Formation of large asteroids up to 200 km across ends
Gas Giant Era	10 million	Rapid formation of Jupiter and Saturn ends
Solar Birth Era	50 million	Sun's nuclear reactions start to produce energy in core
Planetessimal Era	51 million	Formation of numerous small planet-sized bodies ends
T-Tauri Era	80 million	Solar winds sweep through inner solar system and strip off primordial atmospheres
Ice Giant Era	90 million	Formation of Uranus and Neptune
Rocky Planet Era	100 million	Formation of rocky planets by mergers of 50-100 smaller bodies
Late Heavy Bombardment Era	600 million	Migration of Jupiter disrupts asteroid belt sending large asteroids to impact planetary surfaces in the inner solar system.
Ocean Era	600 million	LHB transports comets rich in water to Earth to form oceans
Life Era	800 million	First traces of life found in fossils on Earth

For decades, geologists and astronomers have studied the contents of our solar system. They have compared surface features on planets and moons across the solar system, the orbits of asteroids and comets, and the chemical composition and ages for recovered meteorites. From all this effort, and with constant checking of data against mathematical models, scientists have created a timeline for the formation of our solar system.

Our solar system began as a collapsing cloud of gas and dust over 4.6 billion years ago. Over the next 600 million years, called by geologists the Hadean Era, the sun and the planets were formed, and Earth's oceans were probably created by cometary impacts. Comets are very rich in water ice.

The fossil record on Earth shows that the first bacterial life forms emerged about 600 million years after the formation of the solar system. Geologists call this the Archaen Era – The era of ancient life.

Problem 1 – If the Pre-Solar Nebula Era occurred 4.6 billion years ago, how long ago did the Rocky Planet Era end?

Problem 2 – How many years from the current time did the Late Heavy Bombardment Era end in the inner solar system?

Problem 3 – About how many years ago do the oldest fossils date from on Earth?

Problem 4 – How many years were there between the Planetessimal Era and the end of the Rocky Planet Era?

Problem 5 – If 80 objects the size of the Moon collided to form Earth during the time period in Problem 4, about how many years elapsed between these impact events?

Problem 1 – If the Pre-Solar Nebula Era occurred 4.6 billion years ago, how long ago did the Rocky Planet Era end?

Answer: On the Timeline '0.0' represents a time 4.6 billion years ago, so the Rocky Planet Era ended 100 million years after this or **4.5 billion years ago**.

Problem 2 – How many years from the current time did the Late Heavy Bombardment Era end in the inner solar system?

Answer: LHB ended 600 million years after Time '0.0' or 4.6 billion – 600 million = **4.0 billion years ago.**

Problem 3 – About how many years ago do the oldest fossils date from on Earth?

Answer: 4.6 billion – 800 million = **3.8 billion years ago**.

Problem 4 – How many years were there between the Planetessimal Era and the end of the Rocky Planet Era?

Answer: On the timeline the difference is 100 million – 51 million = **49 million years**.

Problem 5 – If 80 objects the size of the Moon collided to form Earth during the time period in Problem 4, about how many years elapsed between these impact events?

Answer: The time interval is 49 million years so the average time between impacts would have been 49 million years/80 impacts = **612,000 years**.

Table of Global Temperature Anomalies

Year	Temperature (degrees C)	Year	Temperature (degrees C)
1900	-0.20	1960	+0.05
1910	-0.35	1970	0.00
1920	-0.25	1980	+0.20
1930	-0.28	1990	+0.30
1940	+0.08	2000	+0.45
1950	-0.05	2010	+0.63

A new study by researchers at the Goddard Institute for Space Studies determined that 2010 tied with 2005 as the warmest year on record, and was part of the warmest decade on record since the 1800s. The analysis used data from over 1000 stations around the world, satellite observations, and ocean and polar measurements to draw this conclusion.

The table above gives the average 'temperature anomaly' for each decade from 1900 to 2010. The Temperature Anomaly is a measure of how much the global temperature differed from the average global temperature between 1951 to 1980. For example, a +1.0 C temperature anomaly in 2000 means that the world was +1.0 degree Centigrade warmer in 2000 than the average global temperature between 1951-1980.

Problem 1 - By how much has the average global temperature changed between 1900 and 2000?

Problem 2 - The various bumps and wiggles in the data are caused by global weather changes such as the El Nino/La Nina cycle, and year-to-year changes in other factors that are not well understood by climate experts. By how much did the global temperature anomaly change between: A) 1900 and 1920? B) 1920 to 1950? C) 1950 and 1980? D) 1980 to 2010? Describe each interval in terms of whether it was cooling or warming.

Problem 3 - From the data in the table, calculate the rate of change of the temperature anomaly per decade by dividing the temperature change by the number of decades (3) in each time period. Is the pace of global temperature change increasing, decreasing, or staying about the same since 1900?

Problem 4 - Based on the trends in the data from 1960 to 2000, what do you predict that the temperature anomaly will be in 2050? Explain what this means in terms of average global temperature in 2050.

Problem 1 - By how much has the average global temperature changed between 1900 and 2000? Answer: In 1900 it was -0.20 C and in 2000 it was +0.45, so it has changed by +0.45 - (-0.20) = **+0.65 C**.

Problem 2 - The various bumps and wiggles in the data are caused by global weather changes such as the El Nino/El Nina cycle, and year-to-year changes in other factors that are not well understood by climate experts. By how much did the global temperature change between: A) 1900 and 1920? B) 1920 to 1950? C) 1950 and 1980? D) 1980 to 2010? Describe each interval in terms of whether it was cooling or warming.Answer:
1900 to 1920: -0.25C - (-0.20C) = **-0.05 C a decrease (cooling) of 0.05 C**
1920 to 1950: - 0.05C - (-0.25C) = **+0.20 C an increase (warming) of 0.20 C**
1950 to 1980: +0.20C - (-0.05C) = **+0.25 C an increase (warming) of 0.25 C**
1980 to 2010: +0.63C - (+0.20C) = **+0.43 C an increase (warming) of 0.43 C**

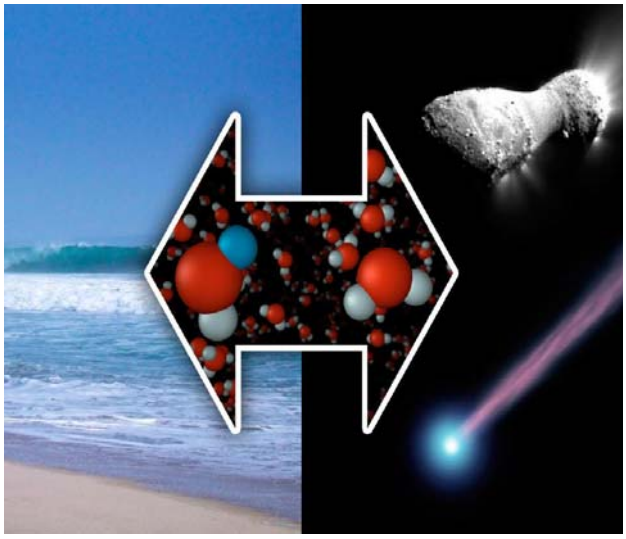
Problem 3 - From the data in the table, calculate the rate of change of the Temperature Anomaly per decade by dividing the temperature change by the number of decades (3) in each time period. Is the pace of global temperature change increasing, decreasing, or staying about the same since 1900? Answer:
1900 to 1920: -0.05 C/3 decades = **-0.017 C per decade**
1920 to 1950: +0.20 C/ 3 decades = **+0.067 C per decade**
1950 to 1980: +0.25 C/3 decades = **+0.083 C per decade**
1980 to 2010: +0.43 C/3 decades = **+0.143 C per decade**.

The pace of global temperature change is **increasing in time**. It is almost doubling every 10 years.

Problem 4 - Based on the trends in the data from 1960 to 2000, what do you predict that the temperature anomaly will be in 2050? Explain what this means in terms of average global temperature in 2050.
Answer: Students may graph the data in the table, then use a ruler to draw a line on the graph between 1960 and 2000, to extrapolate to the temperature anomaly in 2050. A linear equation, T = mx + b, that models this data is b = +0.05C m = (+0.45 - 0.05)/4 decades so m = +0.10 C/decade. Then T = +0.10x + 0.05. For 2050, which is 9 decades after 1960, x=9 so T = +0.1(9) + 0.05 = **+0.95 C**. So, the world will be, on average, about **+1 C warmer** in 2050 compared to its average temperature between 1950 and 1980. This assumes a linear change in T with time.

However from Problem 3 we see that the temperature anomaly change is accelerating. The 'second order' differences are +0.033, +0.033, +0.06. If we take the average change as (0.033+0.033+0.06)/3 = +0.042 we get a more accurate 'quadratic' expression: $T = +0.042\ x^2 + 0.1x + 0.05$. For the year 2050, this quadratic prediction suggests $T = 0.042(9)^2 + 0.1(9) + 0.05$ so T = +1.32 C.

For more information about this research, see the NASA Press Release at **http://www.nasa.gov/topics/earth/features/2010-warmest-year.html**



The abundance of heavy-water in Earth's oceans is about 0.015%. The abundance of heavy-water in Hartley-2 is about 0.016%, so comets like Hartley-2 could have impacted Earth and deposited over time Earth's ocean water. (Image courtesy NASA/JPL-Caltech)

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Type 2 Volume = **$4{,}200\ km^3$**

Type 3 Volume = **$4.2 \times 10^6\ km^3$**

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Answer: $1.33 \times 10^9\ km^3$ of water requires
$6 \times (1.33 \times 10^9\ km^3) = 8.0 \times 10^9\ km^3$ of ice.

The average delivery rate would be about
$R = 8.0 \times 10^9\ km^3$ of ice / 400 million years
= **$20\ km^3$ of ice per year**.

Problem 3 - What is the annual ice deposition rate for each of the three types of cometary bodies?

Type 1: $R1 = 4.2\ km^3/0.5$ yrs = **$8.4\ km^3/yr$**

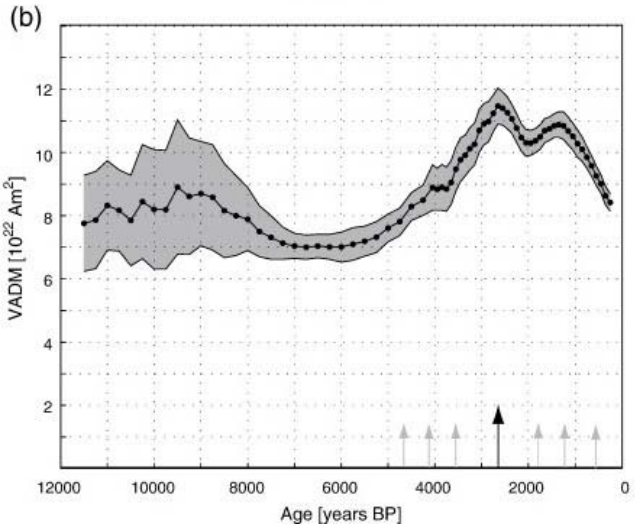
Type 2: $R2 = 4200\ km^3/600$ yrs = **$7.0\ km^3/yr$**

Type 3: $R3 = 4.2 \times 10^6\ km^3 / 1000000$ yrs = **$4.2\ km^3/yr$**

Problem 4 - How many years would it take to form the oceans at the rate that the three types of cometary bodies are delivering ice to Earth's surface?

Answer: The total deposition rate is R1+R2+R3 = $20\ km^3/yr$, so it would take
$T = 8.0 \times 10^9\ km^3$ of ice / $(20\ km^3/yr)$ = **400 million years**.

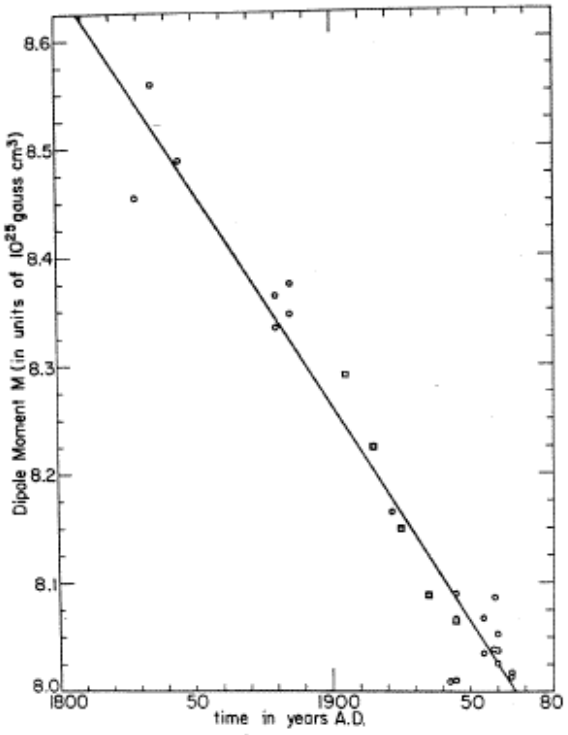
Earth's Declining Magnetism



Not only do the magnetic poles of Earth drift over time, but the entire strength of Earth's magnetic field increases and decreases. The strength of this field is commonly measured in terms of a quantity called VADM with the units of Ampere meter^2 (Am^2). For example, a 1 Ampere current circulating in a closed circle with an area of 1 meter^2 has a VADM of 1 Am^2.

The top figure shows the variations in Earth's VADM since end of the last Ice Age about 12,000 years ago. The gray area represents the range of measurement uncertainty. The current era is to the far-right of the plot.

The lower figure shows the most recent changes since 1800 using a slightly different unit scale.



Problem 1 - During the last 12,000 years, what has been the range in the VADM dipole strength as indicated by the black line?

Problem 2 - In about how many years from the present would you predict that the VADM will reach the lower end of its range in the last 12,000 years?

Problem 3 - Based on the slope of the line in the lower figure, what is the current rate-of-change of the magnetic field in terms of percent per century?

Problem 4 - If the decline continues at this pace, by what year will the strength of Earth's main dipole field be near-zero?

Problem 5 - Comparing the trends displayed by the upper plot with the lower graph, do you think the current rate-of-change exceptional?

Data from "Variations in the geomagnetic dipole moment during the Holocene and the past 50 kyr" by Mads Faurschou Knudsen, Peter Riisager, Fabio Donadini, Ian Snowball, Raimund Muscheler, Kimmo Korhonen, and Lauri J. Pesonen in the journal 'Earth and Planetary Science Letters' ,Vol. 272, pp 319-329.

Teacher note: VADM is an acronym for "Virtual Axial Dipole Moment"

Problem 1 - During the last 12,000 years, what has been the range in the VADM dipole strength as indicated by the black line? Answer: Estimating from the lowest and highest values reached by the black line we get a range from 7.0 to 11.5 x 10^{22} Am^2.

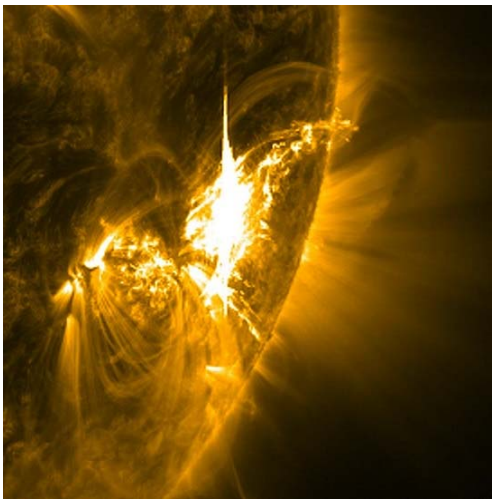
Problem 2 - In about how many years from the present would you predict that the VADM will reach the lower end of its range in the last 12,000 years? Answer: The 'current era' are the years to the far-right in the top graph. The trend shows a slope of 10.5 to 8.5 from about 1000 years ago to 250 years ago. The slope is then (8.5 - 10.5)/(250 - 1000) = 0.003/year. The lower limit of the range is at 7.0 which is 1.5 below the last plotted point that occurred 250 years ago, so -250 + 1.5/0.003 = 500 years from now. Students may also solve this problem graphically with a ruler by extending the line for 'VADM=7.0' to where it meets up with the trend line from the last 1000 years of data.

Problem 3 - Based on the slope of the line in the lower figure, what is the current rate-of-change of the magnetic field in terms of percent per century? Answer: The decrease was from 8.6 to 8.0 over 170 years. This is a percentage change of 0.6/8.6 x 100% = 6.9% over 1.7 centuries and so the rate of decrease has been 6.9%/1.7C = 4% per century.

Problem 4 - If the decline continues at this pace, by what year will the strength of Earth's main dipole field be near-zero? Answer: To go from 8.0 to 0.0 at a rate of 4%/100 years will take 100% / 4% = 25 centuries or 2500 years. Adding this to the current year, 2009 gives us the year 4509 AD. Students answer will vary depending on the actual current year.

Problem 5 - Comparing the trends displayed by the upper plot with the lower graph, do you think the current rate-of-change exceptional? Answer: This question asks whether there have been other times in the last 12,000 years when the SLOPE of the data has been at least as steep as the current slope (i.e. rate of change). Some of the line slopes around 9000 years ago seem, at least for a limited time, to be as rapid as the current era. The period between 2000 and 3000 years ago also shows a similar rapid decline. The current era does seem unique in terms of the duration of this decline which has lasted for 1,500 years.

Solar Flares



This image was taken by NASA's Solar Dynamics Observatory on July 6, 2012 and shows a brilliant X-ray solar flare erupting from the sun.

Solar flares are not all the same. Some produce less energy than others, and so astronomers classify them by their X-ray energy using four different letters: B, C, M and X. C-class flares produce 10 times more X-ray energy than B-class flares. M-class flares produce 10 times more energy than C-class flares, and X-class flares produce 10 times more energy than M-class flares. One B-class flare can produce more energy than 240,000 million tons of TNT!

The table below lists all of the M and X-class flares detected between January 1, 2013 and August 15, 2013 at a time when solar activity was near its maximum. This period of time spans the first 227 days of 2013. Also during this time, there were about 690 C-class flares and 440 B-class flares. All of these flares were seen on the side of the sun facing Earth, which represents ½ of the total surface area of the sun.

Day	Flare	Day	Flare	Day	Flare
1-5	M	5-2	M	5-20	M
1-11	M, M	5-3	M, M	5-22	M
1-13	M, M	5-5	M	5-31	M
2-17	M	5-10	M	6-5	M
3-5	M	5-12	M, M	6-7	M
3-15	M	5-13	X, M, X	6-21	M
3-21	M	5-14	X	6-23	M
4-5	M	5-15	X	7-3	M
4-11	M	5-16	M	8-12	M
4-22	M	5-17	M		

Problem 1 - What were the total number of M and X-class flares during this period of time?

Problem 2 – What were the total number of B, C, M and X-class flares detected during this period?

Problem 3 – What percentage of all flares were B, C, M and X?

Problem 4 – What was the average number of B and C-class flares seen each day?

Problem 5 – An astronaut wants to do a spacewalk on a particular day during this period. What are the odds that she will see an M or X-class flare?

The flare data was obtained from
http://www.swpc.noaa.gov/ftpmenu/warehouse/2013/2013_events.html

Problem 1 - What were the total number of M and X-class flares during this period of time?

Answer: By counting Ms in the table, there were **31 M-class and 4 X-class flares**.

Problem 2 – What were the total number of B, C, M and X-class flares detected during this period?

Answer: 690 + 440 + 31 + 4 = **1165 flares**.

Problem 3 – What percentage of all flares were B, C, M and X?

Answer: B = 100% x (440/1165) = 38%
C = 100% x (690/1165) = 59%
M = 100% x (31/1165) = 3%
X = 100% x (4/1165) = 0.3 %

Problem 4 – What was the average number of B and C-class flares seen each day?

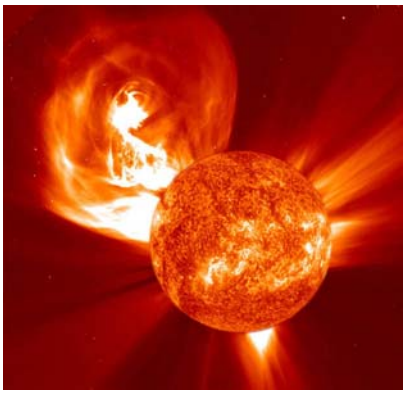
Answer: B: 440 flares/227 days = about 2 flares
C: 690 flares/227 days = about 3 flares

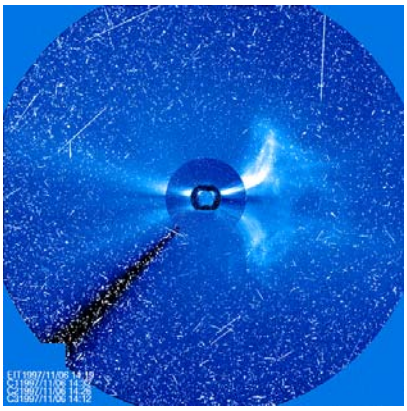
Problem 5 – An astronaut wants to do a spacewalk on a particular day during this period. What is the probability that she will see each an M or X-class flare?

Answer: There are 227 days in the sample and 35 M or X-class flares were seen, so the probability is 35/227 = **0.15** which is also stated as 15%. The low probability means that it is not likely that on a random day the astronaut will see anything. In order to have a 50/50 chance, she would have to observe for at least 4 days (4x0.15 = 0.60 which is greater than 0.50 or 50%).

Coronal Mass Ejections and Radiation Storms

15.2





Every once in a while, the sun ejects huge clouds of heated gas, called plasma, which can contains billions of tons of matter and travel at speeds of millions of miles per hour. Occasionally these are directed at earth, and when they arrive they cause brilliant aurora. They can also cause problems for electrical systems on the ground and satellite systems in space.

The top image is a composite that shows the surface of the sun and one of these 'coronal mass ejections' being released. This one is directed away from earth and is harmless to us. When we spot a CME directed towards earth, the cloud seems to form a temporary 'halo' around the edge of the sun. These Halo CMEs are ejected from the sun, and can arrive at earth about 2 to 4 days later.

Soon after a Halo CME is ejected, satellites may detect a rain storm of radiation particles that were ejected from the sun at the same time. These travel so fast that they arrive at earth in only an hour or so. Also called Solar Proton Events (SPEs), these radiation storms are very harmful to astronauts in space and to sensitive satellite electronics. Predicting when SPEs will occur is an important goal of Space Weather Research.

The table below gives the dates for the CMEs detected during the 227 days from January 1, 2013 and August 15, 2013 during the peak of our sun's current storm cycle. Yellow shading indicates that a SPE occurred on the same date.

Date	Type	Date	Type	Date	Type
1-23	Halo, non-Halo	3-15	Halo	5-22	Non-Halo
1-31	Halo	4-11	Halo	6-20	Halo
2-1	Halo	4-20	Non-Halo	7-16	Halo
2-5	Non-Halo	4-21	Non-Halo	7-26	Non-Halo
2-9	Halo	5-17	Halo	8-6	Halo, non-Halo
2-20	Halo	5-19	Halo		

Problem 1 – What is the average number of days between all of the CMEs in this sample?

Problem 2 – What percentage of CMEs are of the Halo-type?

Problem 3 – What percentage of Halo CMEs seem to produce Solar Proton Events?

Problem 4 – If you observed a CME, what is the probability that it may produce a harmful solar proton event?

Problem 1 – What is the average number of days between all of the CMEs in this sample?

Answer: There are a total of 20 CMEs in 227 days so the average interval is about 227/20 = **11 days**.

Problem 2 – What percentage of CMEs are of the Halo-type?

Answer: Of the 20 CMEs, 12 were Halo-type so the percentage is 100%x(12/20) = **60%**

Problem 3 – What percentage of Halo CMEs seem to produce Solar Proton Events?

Answer: Of the 12 Halo-type events, 4 produced SPEs so 100% (4/12) = **33%**

Problem 4 – If you observed a CME, what is the probability that it may produce a harmful solar proton event?

Answer: There were 20 CMEs total of which 4 produced SPEs, so 100% x (4/20) = **20%**.

Note: Of the 4 SPEs, three occurred with Halo CMEs so this means that ¾ or 75% of all SPEs coincide with Halo-type CMEs, however it is also true that only 33% of Halo-type CMEs produce SPEs. Not all Halo events produce solar proton events, so using halo events to predict whether an SPE will occur will lead to a large number of false-positives by about 8 false to 4 positives (8+4 = 12 halo events).

Magnetic Storms, Aurora and the Kp Index

15.3

When a solar coronal mass ejection collides with earth's magnetic field, it can produce intense aurora that can be seen from the ground. Geophysicists who study magnetic disturbances have created a 9-point scale that indicates how intense the storm is. Kp=9 is the most intense, and aurora can be seen near Earths equator for many of these. Kp=7 and 8 are strong storms that can still cause aurora and upset electrical power systems. Kp=5 and 6 are mild storms that may or may not produce intense aurora.

The table below gives the dates when magnetic storms were detected which exceeded Kp=4. The multiple entries each day indicate consecutive 3-hour measurements of the storm intensity. The data are given for the 227 days from January 1, 2013 to August 15, 2013. Highlighted boxes in yellow indicate days when Halo CMEs were detected

Date	Kp measurements	Date	Kp Measurements
3-1	5	6-2	5
3-17	6, 5, 5, 6, 6, 5	6-7	5, 6, 5
3-29	5	6-29	6, 7, 5, 6
3-30	5, 5	7-10	5, 5
4-26	5	7-11	5
5-18	5, 5	7-15	5, 5
5-24	5	8-4	5
5-25	5, 5, 5	8-5	5
6-1	6, 6, 6, 5, 6		

Problem 1 – What percentage of the days during this time period had magnetic storm events?

Problem 2 – What percentage of the magnetic storm days were more intense than Kp=5?

Problem 3 – What is the probability that of the 12 Halo CME events that occurred during this 227-day period, that these events produce a magnetic storm more intense than Kp=5?

Problem 4 – The Kp index is measured every 3 hours. From the table, what is the average duration of a storm that exceeds Kp=5 during its entire duration?

Problem 1 – What percentage of the days during this time period had magnetic storm events?

Answer: 17 days out of 227 so 100% x (17/227) = **7%**

Problem 2 – What percentage of the magnetic storm days were more intense than Kp=5?

Answer: 4 out of 17 or 100% x (4/17) = **23%.**

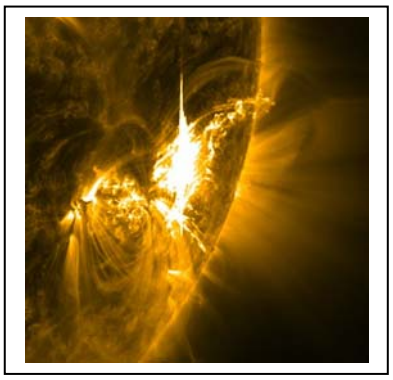
Problem 3 – What is the probability that of the 12 Halo CME events that occurred during this 227-day period, that these events produce a magnetic storm more intense than Kp=5?

Answer: Only 4 of the 12, so P = 100% x (4/12) = **33%.**

Problem 4 – The Kp index is measured every 3 hours. From the table, what is the average duration of a storm that exceeds Kp=5 during its entire duration?

Answer: There were 4 storms that exceeded Kp=5. These occurred on the dates: 3-17, 6-1, 6-7 and 6-29. The total hours for each storm were 6x3 = 18 hours (for 3-17), 5x3 = 15 hours (for 6-1); 3x3 = 9 (for 6-7) and 4x3 = 12 (for 6-29). The average for these hours is (18+15+9+12)/4 = **14 hours.**

So, once a strong magnetic storm begins, it takes about one-half a day for it to reach its maximum intensity and then fade away. If this happens in the summertime during the day, you will not see an aurora borealis because of the daylight brightness.



The following essay describes solar storms. The words that belong in the missing blanks can be found by solving the ten number sentences.

Look in the Word Bank and match the calculated number to the correct answer.

Fill-in the blank with the word you found to complete the essay and answer a question about solar storms!

Our sun is a very predictable star. Each day it rises and sets as the world turns upon its axis, and warms the Earth making life possible. But the sun is also a stormy star. It produces 1)_____ and incredible explosions of 2)_____ almost every day. Sometimes, its entire surface is speckled by 3)_____ that come and go every 11 years. In 2013, the sun was at the peak of its maximum stormy activity. This means that many more flares and explosions of gas were happening compared to other times in the 11-year cycle. Solar flares are bursts of intense 4)_____ light that can cause problems for radio communication on Earth. They also heat up the 5)_____ and cause it to expand into space. About 1000 of these flares were detected during the first 8 months of 2013.

Occasionally the sun ejects billion-ton clouds of 6)_____ called 7)_____ or CMEs. Traveling at over a million miles an hour, they can reach Earth in only a few 8)_____. When they arrive, they cause problems for satellites and our electric power grid, but they also cause beautiful 9)_____ in gthe northern and southern skies. Most CMEs are not directed towards earth and are completely 10)_____.

So, even though the sun looks the same every day, it really is a very stormy star that can sometimes create unpleasant surprises for us here on Earth!

Word Bank

-5	asteroids	+48	heat	-17	X-rays
-44	aurora	+1	ocean	+3	harmless
-15	ultraviolet	+5	days	+44	rainbows
-27	energy	0	atmosphere	-5	months
-40	rocks	-48	gas	+24	sunspots
-6	harmful	+3	plasma	-3	coronal mass ejections
-2	flares	+20	comets	-7	prominences

Solve these problems to get the Word Bank number key.

1) 1+(1-3)-(5-8)+(-6+2) =
2) 8(3-2)-2(3-8)+5(-6-3) =
3) (1-3)(-5+2)(8-6)(3-1) =
4) 3(2-6)+(-8+4)-(4-3) =
5) 5(-3+2)-2(6-2)-(+7-20) =
6) -2(+2(-3(+2(-3+1)))) =
7) -8/2 +(3+2)/(8-3) =
8) -7+23-6-(-10)+(-3)(4+1) =
9) 12/(-1/4) + (-36)/(-9) =
10) (-4)2 + 21/3 + 4 =

Answer Key

Solve these problems to get the Word Bank number key.

1)	1+(1-3)-(5-8)+(-6+2)	=	-2	6)	-2(+2(-3(+2(-3+1))))	= -48
2)	8(3-2)-2(3-8)+5(-6-3)	=	-27	7)	-8/2 +(3+2)/(8-3)	= -3
3)	(1-3)(-5+2)(8-6)(3-1)	=	+24	8)	-7+23-6-(-10)+(-3)(4+1)	= +5
4)	3(2-6)+(-8+4)-(4-3)	=	-17	9)	12/(-1/4) + (-36)/(-9)	= -44
5)	5(-3+2)-2(6-2)-(+7-20)	=	0	10)	(-4)2 + 21/3 + 4	= +3

The words are

Line 1 = -2 = flares
Line 2 = -27 = energy
Line 3 = +24 = sunspots
Line 4 = -17 = X-ray
Line 5 = 0 = atmosphere
Line 6 = -48 = gas
Line 7 = -3 = coronal mass ejections
Line 8 = +5 = days
Line 9 = -44 = aurora
Line 10 = +3 = harmless

Our sun is a very predictable star. Each day it rises and sets as the world turns upon its axis, and warms the Earth making life possible. But the sun is also a stormy star. It produces 1)____**flares**_____ and incredible explosions of 2)___**energy**_____ almost every day. Sometimes, its entire surface is speckled by 3)__**sunspots**____ that come and go every 11 years. In 2013, the sun was at the peak of its maximum stormy activity. This means that many more flares and explosions of gas were happening compared to other times in the 11-year cycle. Solar flares are bursts of intense 4)___**X-ray**____ light that can cause problems for radio communication on Earth. They also heat up the 5)___**atmosphere**_____ and cause it to expand into space. About 1000 of these flares were detected during the first 8 months of 2013.

Occasionally the sun ejects billion-ton clouds of 6)__**gas**____ called 7)___**coronal mass ejections**____ or CMEs. Traveling at over a million miles an hour, they can reach Earth in only a few 8)____**days**_____. When they arrive, they cause problems for satellites and our electric power grid, but they also cause beautiful 9)__**aurora**____ in the northern and southern skies. Most CMEs are not directed towards earth and are completely 10)___**harmless**_____.

So, even though the sun looks the same every day, it really is a very stormy star that can sometimes create unpleasant surprises for us here on Earth!

Solar Storms: Odds, Fractions and Percentages

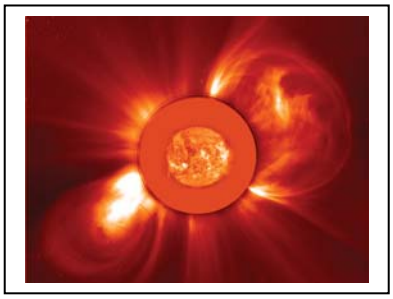
One of the most basic activities that scientists perform with their data is to look for correlations between different kinds of events or measurements in order to see if a pattern exists that could suggest that some new 'law' of nature might be operating. Many different observations of the Sun and Earth provide information on some basic phenomena that are frequently observed. The question is whether these phenomena are related to each other in some way. Can we use the sighting of one phenomenon as a prediction of whether another kind of phenomenon will happen?

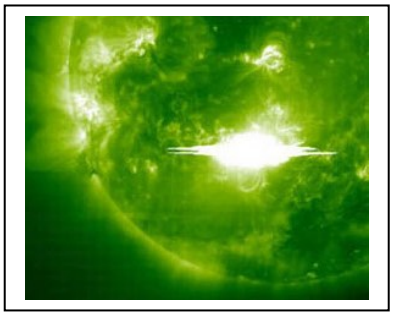
During most of the previous sunspot cycle (January-1996 to June-2006), astronomers detected 11,031 coronal mass ejections, (CME: Top image) of these 1186 were 'halo' events. Half of these were directed towards Earth.

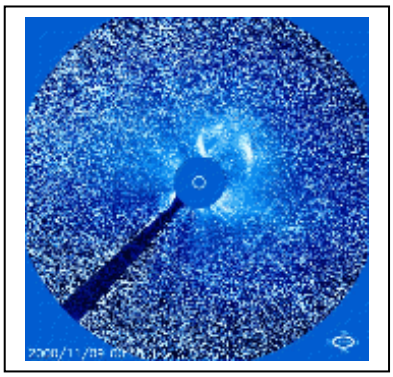
During the same period of time, 95 solar proton events (streaks in the bottom image were caused by a single event) were recorded by the GOES satellite network orbiting Earth. Of these SPEs, 61 coincided with Halo CME events.

Solar flares (middle image) were also recorded by the GOES satellites. During this time period, 21,886 flares were detected, of which 122 were X-class flares. Of the X-class flares, 96 coincided with Halo CMEs, and 22 X-class flares also coincided with 22 combined SPE+Halo CME events. There were 6 X-flares associated with SPEs but not associated with Halo CMEs. A total of 28 SPEs were not associated with either Halo CMEs or with X-class solar flares.

From this statistical information, construct a Venn Diagram to interrelate the numbers in the above findings based on resent NASA satellite observations, then answer the questions below.







1 - What are the odds that a CME is directed towards Earth?

2 - What fraction of the time does the sun produce X-class flares?

3 - How many X-class flares are not involved with CMEs or SPEs?

4 - If a satellite spotted both a halo coronal mass ejection and an X-class solar flare, what is the probability that a solar proton event will occur?

5 - What percentage of the time are SPEs involved with Halo CMEs, X-class flares or both?

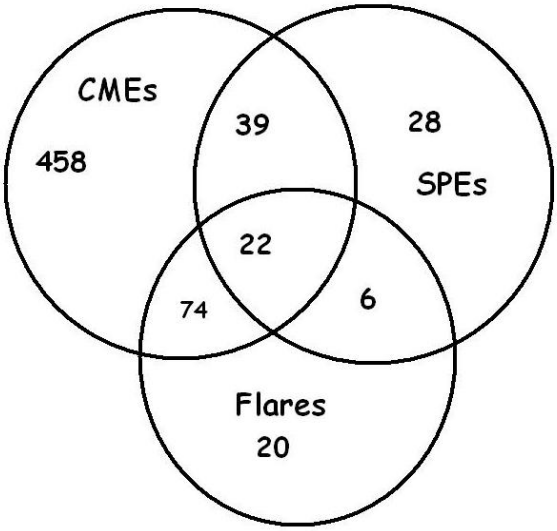
6 - If a satellite just spots a Halo CME, what are the odds that an X-class flare or an SPE or both will be observed?

7 - Is it more likely to detect an SPE if a halo CME is observed, or if an X-class flare is observed?

8 - If you see either a Halo CME or an X-class flare, but not both, what are the odds you will also see an SPE?

9 - If you observed 100 CMEs, X-class flares and SPEs, how many times might you expect to see all three phenomena?

Answer Key:



Venn Diagram Construction.

1. There are 593 Halo CMEs directed to Earth so 593 = 74 with flares + 39 with SPEs + 22 both SPEs and Flares + 458 with no SPEs or Flares..

2. There are 95 SPEs. 95 = 39 with CMEs + 6 with flares + 22 with both flares and CMEs + 28 with no flares or CMEs

3. There are 122 X-class flares. 122 = 74 With CMEs only + 6 with SPEs only + 22 both CMEs and SPEs + 20 with no CMEs or SPEs.

1 - What are the odds that a CME is directed towards Earth? 593/11031 = 0.054 **odds = 1 in 19**

2 - What fraction of the time does the sun produce X-class flares? 122/21886 = **0.006**

3 - How many X-class flares are not involved with CMEs or SPEs? 122 - 74 - 22 - 6 = **20.**

4 - If a satellite spotted BOTH a halo coronal mass ejection and an X-class solar flare, what is the probability that a solar proton event will occur? 22/(74+22) = **0.23**

5 - What percentage of the time are SPEs involved with Halo CMEs, X-class flares or both?
100% x (39+22+6 / 95) = **70.1 %**

6 - If a satellite just spots a Halo CME, what are the odds that an X-class flare or an SPE or both will be observed?
39+22+74 / 593 = 0.227 so the odds are 1/0.227 or about **1 in 4.**

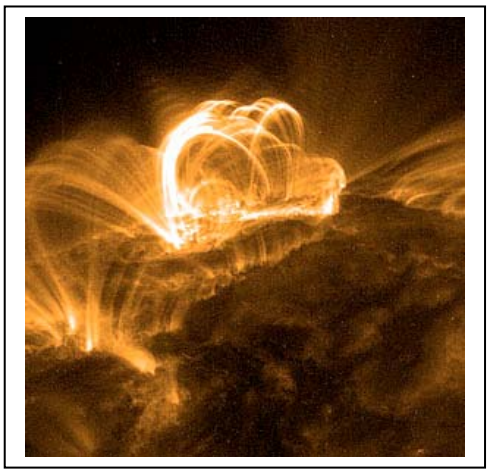
7 - Is it more likely to detect an SPE if a halo CME is observed, or if an X-class flare is observed?
(6+22)/95 = 0.295 or 1 out of 3 times for X-flares
(39+22)/95 = 0.642 or 2 out of 3 for Halo CMEs
It is more likely to detect an SPE if a Halo CME occurs by 2 to 1.

8 - If you see either a Halo CME or an X-class flare, but not both, what are the odds you will also see an SPE?
39+6 / 95 = 0.50 so the odds are 1/0.50 or **2 to 1 .**

9 - If you observed 100 CMEs, X-class flares and SPEs, how many times might you expect to see all three phenomena?

100 x 22/(95+122+593) = **3 times**

Solar Storms: Sequences and Probabilities



The Sun is an active star, which produces solar flares (F) and explosions of gas (C). Astronomers keep watch for these events because they can harm satellites and astronauts in space. Predicting when the next storm will happen is not easy to do. The problems below are solved by writing out all of the possibilities, then calculating the probability of the particular outcome!

Solar flare photo courtesy TRACE/NASA

1 – During a week of observing the sun, astronomers detected 1 solar flare (F). What was the probability (as a fraction) that it happened on Wednesday?

2 – During the same week, two gas clouds were ejected (C), but not on the same days. What is the probability (as a fraction) that a gas cloud was ejected on Wednesday?

3 – Suppose that the flares and the gas clouds had nothing to do with each other, and that they occurred randomly. What is the probability (as a fraction) that both a flare and a gas cloud were spotted on Wednesday? (Astronomers would say that these phenomena are uncorrelated because the occurrence of one does not mean that the other is likely to happen too).

Answer Key

1 – Answer: There are only 7 possibilities:

F X X X X X X X X X F X X X X X X X X X F
X F X X X X X X X X X F X X
X X **F** X X X X X X X X X F X
So the probability for any one day is **1/7.**

2 – Here we have to distribute 2 storms among 7 days. For advanced students, there are 7! / (2! 5!) = 7 x 6 / 2 = 21 possibilities which the students will work out by hand:

C C X X X X X X C **C** X X X X X X **C** C X X X X X X C C X X
C X **C** X X X X X C X C X X X X X **C** X C X X X X X C X C X
C X X C X X X X C X X C X X X X **C** X X C X X X X C X X C
C X X X C X X X C X X X C X X X **C** X X X C X X X X C C X
C X X X X C X X C X X X X C X X X X C X C
C X X X X X C X X X X C C

There are 6 possibilities (in red) for a cloud appearing on Wednesday (Day 3), so the probability is **6/21.**

3 – We have already tabulated the possibilities for each flare and gas cloud to appear separately on a given day. Because these events are independent of each other, the probability that on a given day you will spot a flare and a gas cloud is just 1/7 x 6/21 or 6/147. This is because for every possibility for a flare from the answer to Problem 1, there is one possibility for the gas clouds.

There are a total of 7 x 21 = 147 outcomes for both events taken together. Because there are a total of 1 x 6 outcomes where there is a flare and a cloud on a particular day, the fraction becomes (1 x 6)/147 = **6/147.**

How Common are X-Class Solar Flares? 15.7

	CY	J	F	M	A	M	J	J	A	S	O	N	D
1996	1							1					
1997	2											3	
1998	3				2	2			5			5	
1999	4								2		1	1	
2000	5		1	3			4	3		1		5	
2001	6			1	8		1		1	1	4	2	3
2002	7				1	1		5	4		1		
2003	8			2		3	4				7	4	
2004	9							6	2		1	2	
2005	10	6						1		10			
2006	11												2

X-Class solar flares are among the most powerful, explosive events on the solar surface. They can cause short-wave radio interference, satellite malfunctions and can even cause the premature re-entry of satellites into the atmosphere.

The table above lists the number of X-class flares detected on the sun during the last sunspot cycle which lasted from 1996 to about 2008. The second column also gives the year from the start of the 11-year sunspot cycle in 1996. The counts are listed by year (rows) and by month (columns). Study this table and answer the following questions to learn more about how common these flares are.

Problem 1 – For the years and months considered, is the distribution of months with flares a uniform distribution? Explain.

Problem 2 - The sunspot cycle can be grouped into pre-maximum (1997,1998, 1999), maximum (2000,2001,2002) and post-maximum (2003,2004,2005). For each group, calculate A) The percentage of months with no flares; and B) The average number of weeks between flares.

Problem 3 – For each group, what is the median number of flares that occurs in the months that have flares?

Problem 4 – Taken as a whole, what is the average number of flares per month during the entire 11-year sunspot cycle?

Problem 5 – We are currently in Year-3 of the current sunspot cycle, which began in 2007. About how many X-class flares would you predict for this year using the tabulated flares from the previous sunspot cycle as a guide, and what is the average number of weeks between these flares for this year?

Problem 1 – For the years and months considered, is the distribution of months with flares a uniform distribution? Explain. Answer; If you shaded in all the months with flares you would see that most occur between 1999-2002 so the distribution is not random, and is not uniform.

Problem 2 - The sunspot cycle can be grouped into pre-maximum (1997,1998, 1999), maximum (2000,2001,2002) and post-maximum (2003,2004,2005). For each group, calculate A) the percentage of months with no flares, and B) The average number of weeks between flares. Answer: A) Pre-Maximum, N = 28 months so P = 100%x28/36 = 78% . Maximum; N = 17 months so P = 100% x 17/36 = 47%; post-maximum N= 24 months so P = 100% x 24/36 = 67%. B) pre-maximum N = 22 flares so T = 36 months/22 flares = 1.6 months. Maximum: N = 50 flares so T = 36 mo/50 = 0.7 months; post-maximum: N = 48 flares so T = 36 mo/48 = 0.8 months.

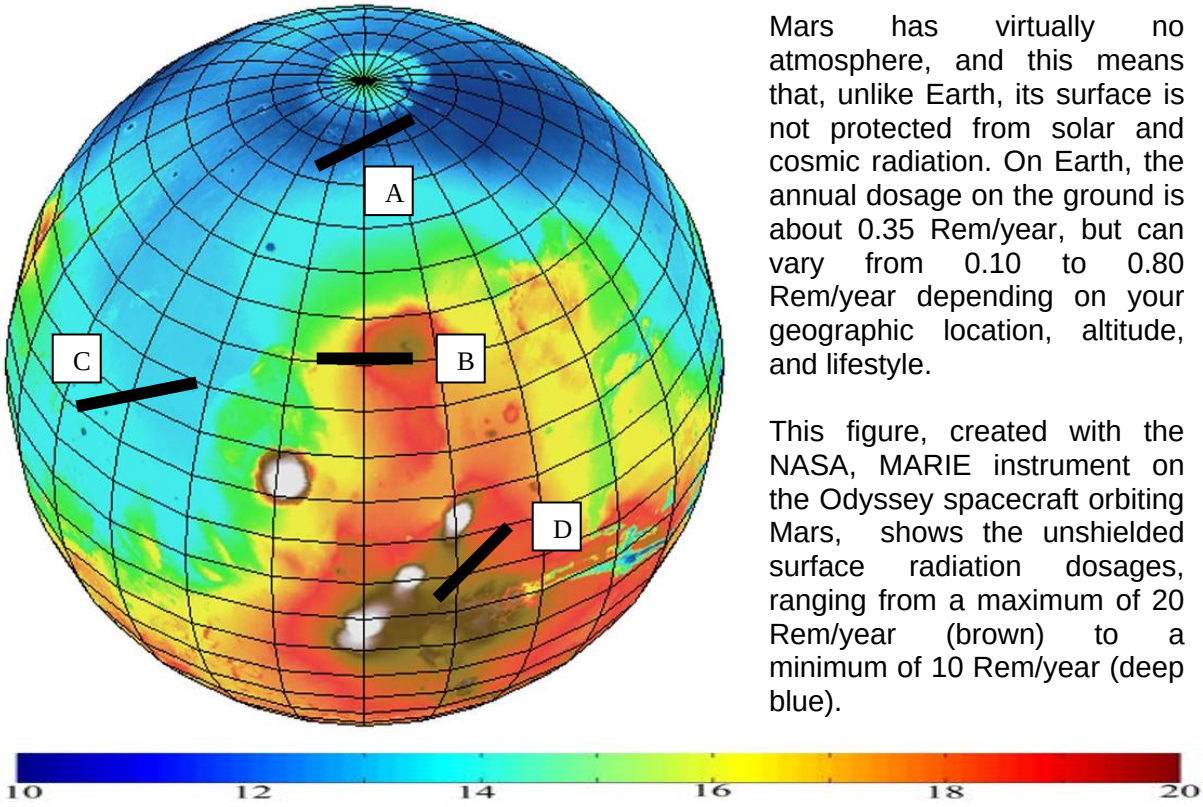
Problem 3 – For each group, what is the median number of flares that occurs in the months that have flares? Answer: Pre-maximum: 1,1,2,2,2,3,5,5 median = 3. Maximum: 1,1,1,1,1,1,1,1,2,3,3,3,4,4,4,5,8 median = 4; post-maximum 1,1,2,2,2,3,4,6,6,7 median = 6

Problem 4 – Taken as a whole, what is the average number of flares per month during the entire 11-year sunspot cycle? Answer: There were 122 flares detected during the 132 months of the sunspot cycle, so the average is 122 flares/132 months = 0.9 , which can be rounded to **1 flare/month**.

	CY	J	F	M	A	M	J	J	A	S	O	N	D	
1996	1							1						1
1997	2											3		3
1998	3				2	2			5			5		14
1999	4								2		1	1		4
2000	5		1	3			4	3		1		5		17
2001	6			1	8		1		1	1	4	2	3	21
2002	7				1	1		5	4		1			12
2003	8			2		3	4				7	4		20
2004	9							6	2		1	2		11
2005	10	6						1		10				17
2006	11												2	2
		6	1	6	11	6	9	16	14	12	14	22	5	

Problem 5 – We are in Year-3 of the current sunspot cycle, which began in 2007. About how many X-class flares would you predict for this year using the tabulated flares from the previous sunspot cycle as a guide, and what is the average number of weeks between these flares for this year? Answer: From the table we find for Year 3 that there were 14, X-class flares. Since there are 12 months in a year, this means that the average time between flares is about 14/12 = **1.2 months**.

Having a Hot Time on Mars!



Mars has virtually no atmosphere, and this means that, unlike Earth, its surface is not protected from solar and cosmic radiation. On Earth, the annual dosage on the ground is about 0.35 Rem/year, but can vary from 0.10 to 0.80 Rem/year depending on your geographic location, altitude, and lifestyle.

This figure, created with the NASA, MARIE instrument on the Odyssey spacecraft orbiting Mars, shows the unshielded surface radiation dosages, ranging from a maximum of 20 Rem/year (brown) to a minimum of 10 Rem/year (deep blue).

Astronauts landing on Mars will want to minimize their total radiation exposure during the 540 days they will stay on the surface. The Apollo astronauts used spacesuits that provided 0.15 gm/cm^2 of shielding. The Lunar Excursion Module provided 0.2 gm/cm^2 of shielding, and the orbiting Command Module provided 2.4 gm/cm^2. The reduction in radiation exposure for each of these was about 1/4, 1/10 and 1/50 respectively. Assume that the Mars astronauts used improved spacesuit technology providing a reduction of 1/8, and that the Mars Excursion Vehicle provided a 1/20 radiation reduction.

The line segments on the Mars radiation map represent some imaginary, 1,000 km exploration tracks that ambitious astronauts might attempt with fast-moving rovers, and not a lot of food! Imagine a schedule where they would travel 100 kilometers each day. Suppose they spend 20 hours a day within a shielded rover, and they study their surroundings in spacesuits for 4 hours each day.

1) Convert 10 Rem/year into milliRem/day.

2) What is the astronauts radiation dosage per day in a region (brown) where the ambient background produces 20 Rem/year?

3) For each of the tracks on the map, plot a dosage history timeline for the 10 days of each journey. From the scaling relationship defined for one day in Problem 3, calculate the approximate total dosage to an astronaut in milliRems (mRems), given the exposure times and shielding information provided.

4) Which track has the highest total dosage in milliRems? The least total dosage? What is the annual dosage that is equivalent to these 20-day trips? How do these compare with the 350 milliRems they would receive if they remained on Earth?

Having a Hot Time on Mars!

1) Convert 10 Rem/year into milliRem/hour.

Answer: (10 Rem/yr) x (1 year/ 365 days) x (1 day/24 hr) = 1.1 milliRem/hour

2) What is the astronauts radiation dosage per day in a region (brown) where the background is 20 Rem/year?

Answer: From Problem 1, 20 Rem/year = 2.2 milliRem/hour.
20 hours x (1/20) x 1.1 milliRem/hr + 4 hours x (1/8) x 1.1 milliRem/hr = 1.1 + 0.55 = 1.65 milliRem/day

3) For each of the tracks on the map, plot a dosage history timeline for the 10 days of each journey. From the scaling relationship defined for one day in Problem 3, calculate the total dosage in milliRems to an astronaut, given the exposure times and shielding information provided. The scaling relationship is that for each 20 Rems/year, the daily astronaut dosage is 0.66 milliRem/day (e.g. 0.66/20). The factor of 2 in the answers accounts for the round-trip.

Track A dosage:

2x(12 Rems/yr x 10 days x (1.65 / 20)) = 2x(9.9) = 19.8 mRem.

Track B dosage:

2x(16 Rems/yr x 3.3days + 18 Rems/yr x 3.3days + 20 Rems/yr x 3.3days)(1.65/20) = 2x(14.8) = 29.6 mRem

Track C dosage:

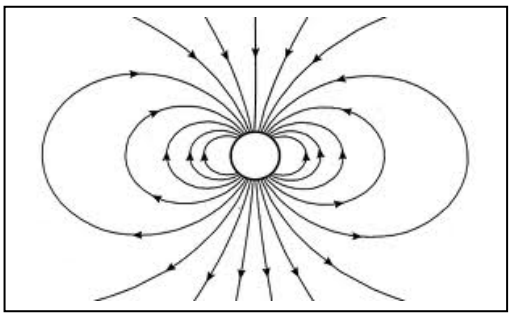
2x(12 Rems/yr x 5 days x (1.65/20) + 14 Rems/yr x 5 days x (1.65/20)) = 2x(5.0 + 5.8) = 21.6 mRem

Track D dosage:

2x(18 Rems/yr x 5 days x (1.65/20) + 20 Rems/yr x 5 days x (1.65/20)) = 2x(7.5 + 8.25) = 31.5 mRem

4) For this 20-day excursion, Track D has the highest dosage and Track A has the lowest. The equivalent annual dosage for the lowest-dosage track is 19.8 milliRem x 365 days/10 days = 722 milliRem, which is about twice the annual dosage they would receive if they remained on Earth. For the highest-dosage trip, the annualized dosage is 1,149 milliRems which is about 3 times the dosage on Earth.

Mathematical Model of Magnetic Field Lines



Magnets have a north and a south pole. If you make this magnet small enough so that it looks like a point, all you will see are the looping lines of force mapped out by iron fillings or by using a compass.

Physicists call these patterns of lines, magnetic lines of force, and they can describe them mathematically!

Problem 1 - Create a standard Cartesian 'X-Y' graph with all four quadrants shown. Select a domain [-5.0, + 5.0] and a range [-2.0, +2.0] and include tic marks every 0.1 along each axis.

Problem 2 - Plot the following points in the order given and connect them with a smooth curve.

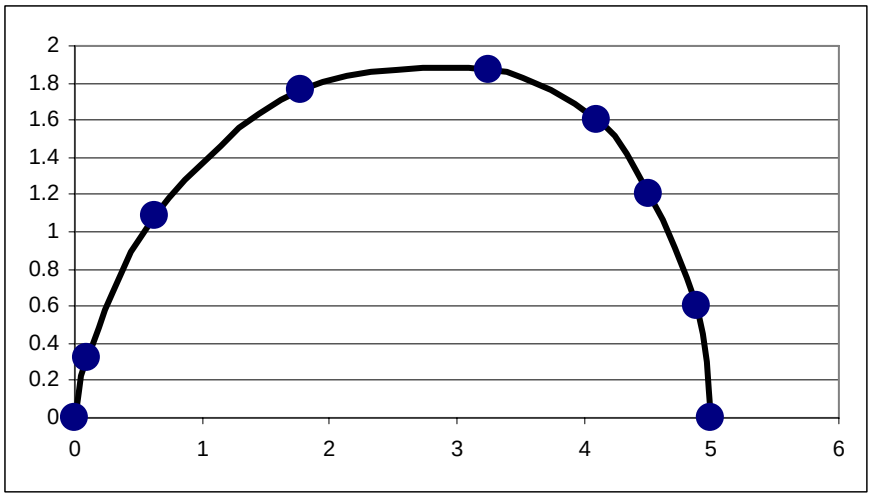
X	Y
+0.0	+0.0
+0.1	+0.3
+0.6	+1.1
+1.8	+1.8
+3.2	+1.9
+4.1	+1.6
+4.5	+1.2
+4.9	+0.6
+5.0	+0.0

Problem 2 - Reflect the curve you drew into Quadrant 4, then reflect the curve in Quadrant 1 and 4 into Quadrants 2 and 3 to complete a single magnetic line of force for a magnet located at the origin!

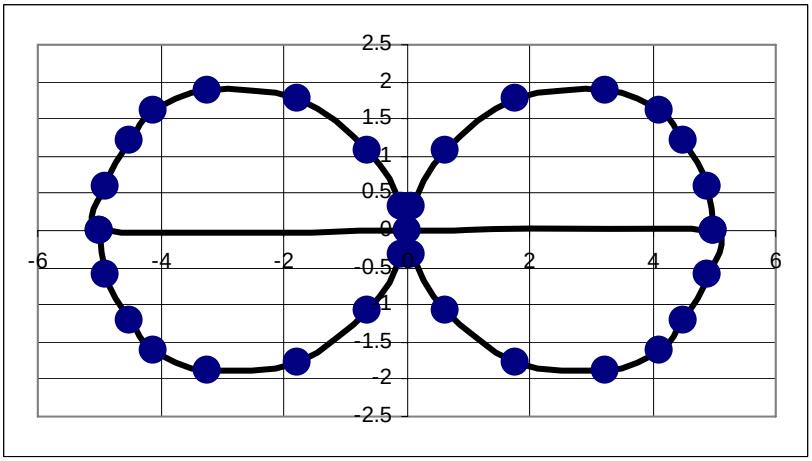
Problem 3 - Add two additional lines of force to your picture by re-scaling the figure you drew so that the X-Y coordinates are now A) 1/4 as large and B) 1.5 times larger.

Problem 1 - Create a standard Cartesian 'X-Y' graph with all four quadrants shown. Select a domain [-5.0, + 5.0] and a range [-2.0, +2.0].

Problem 2 - Plot the following points in the order given and connect them with a smooth curve.

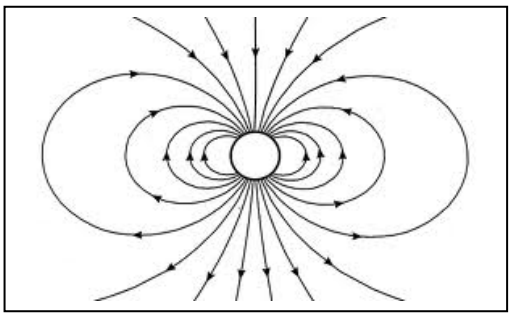


Problem 2 - Reflect the curve you drew into Quadrant 4, and then reflect the curve in Quadrant 1 and 4 into Quadrants 2 and 3 to complete a single magnetic line of force for a magnet located at the origin!



Problem 3 - Add two additional lines of force to your picture by re-scaling (dilating or contracting) the figure you drew so that the X-Y coordinates are now A) 1/4 as large (contraction) and B) 1.5 times larger (dilation).

Mathematical Model of Magnetic Field Lines



Mathematically, every point in space near a magnet can be represented by a vector, B. Because the field exists in 3-dimensional space, it has three 'components'. The equations for the coordinates of B in 2-dimensions looks like this:

$$B_r = -\frac{2M \sin\theta}{r^3} \qquad B_\theta = \frac{M \cos\theta}{r^3}$$

It is convenient to graph a magnetic field on a 2-dimensional piece of paper to show its shape. The lines that are drawn are called 'magnetic field lines', and if you placed a compass at a particular point on the field line, the direction of the line points to 'north' or 'south'.

The slope of the magnetic field at any point (R,θ) is defined by $\frac{B_\theta}{B_r}$.

From calculus, in a polar coordinate system, the slope of a line is defined by $\frac{rd\theta}{dr}$.

Problem 1 – What is the differential equation that relates $\frac{B_\theta}{B_r}$ to $\frac{rd\theta}{dr}$?

Problem 2 – Integrate your answer to Problem 1 to find the polar coordinate equation of a magnetic field line.

Problem 1 -

$$\frac{rd\theta}{dr} = \frac{M\cos\theta}{-2M\sin\theta} \quad \text{so} \quad \frac{rd\theta}{dr} = -\frac{\cos\theta}{2\sin\theta}$$

Problem 2 -

Rearrange the terms into two integrands: $$\frac{dr}{r} = -\frac{2\sin\theta}{\cos\theta}d\theta$$

The integrals become $$\int\frac{dr}{r} = -2\int\frac{\sin\theta}{\cos\theta}d\theta$$

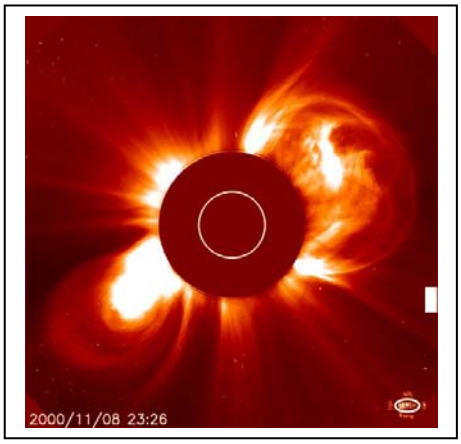
These are both logarithmic integrals that yield the solution:

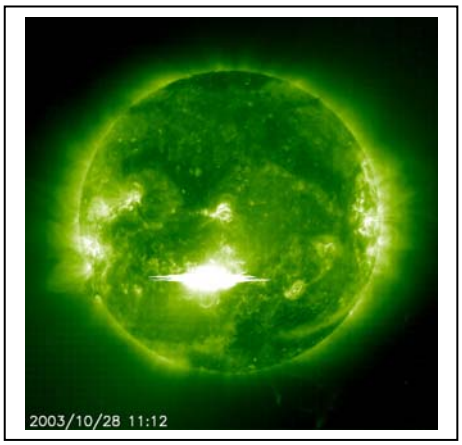
$$ln(r) + C = 2\ ln(\cos\theta) + C$$

R_0 is the distance from the center of the magnetic field to the point where the field line crosses the equatorial plane of the magnet at θ=0. Each field line is specified by a unique crossing point distance. In other words, the constants of integration are specified by the condition that r = r_0 for θ = 0, which then gives us the final form of the equation:

$$r = r_0\cos^2\theta$$

Do Fast CMEs produce intense SPEs?





The sun produces two basic kinds of storms; coronal mass ejections (SOHO satellite: top left) and solar flares (SOHO satellite: bottom left). These are spectacular events in which billions of tons of matter are launched into space (CMEs) and vast amounts of electromagnetic energy are emitted (Flares). A third type of 'space weather storm' can also occur.

Solar Proton Events (SPEs) are invisible, but intense, showers of high-energy particles near Earth that can invade satellite electronics and cause serious problems, even malfunctions and failures. Some of the most powerful solar flares can emit these particles, which streak to Earth within an hour of the flare event. Other SPE events, however, do not seem to arrive at Earth until several days latter.

Here is a complete list of Solar Proton Events between 1976-2005: http://umbra.nascom.nasa.gov/SEP/

Here is a complete list of coronal mass ejections 1996 - 2006: http://cdaw.gsfc.nasa.gov/CME_list/

Between January 1, 1996 and June 30, 2006 there were 11,031 CMEs reported by the SOHO satellite. Of these, 1186 were halo events. Only half of the halo events are actually directed towards Earth. The other half are produced on the far side of the sun and directed away from Earth. During this same period of time, 90 SPE events were recorded by GOES satellite sensors orbiting Earth. On the next page, is a list of all the SPE events and Halo CMEs that corresponded to the SPE events. There were 65 SPEs that coincided with Halo CMEs. Also included is the calculated speed of the CME event.

From the information above, and the accompanying table, draw a Venn Diagram to represent the data, then answer the questions below.

Question 1: A) What percentage of CMEs detected by the SOHO satellite were identified as Halo Events?
B) What are the odds of seeing a halo Event?
C) How many of these Halo events are directed towards Earth?

Question 2: A) What fraction of SPEs were identified as coinciding with Halo Events?
B) What are the odds that an SPE occurred with a Halo CME?
C) What fraction of all halo events directed towards earth coincided with SPEs?

Question 3: A) What percentage of SPEs coinciding with Halo CMEs are more intense than 900 PFUs?
B) What are the odds that, if you detect a 'Halo- SPE', it will be more intense than 900 PFUs?

Question 4: A) What percentage of Halo-SPEs have speeds greater than 1000 km/sec?
B) What are the odds that a Halo-SPE in this sample has a speed of > 1000 km/sec?

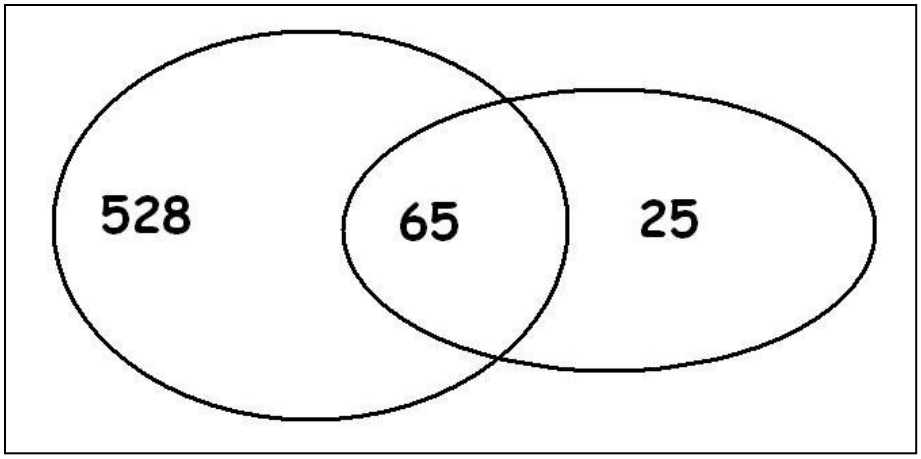
Question 5: From what you have calculated as your answers above, what might you conclude about Solar Proton Events and CMEs? How would you use this information as a satellite owner and operator?

Data Tables showing dates and properties of Halo CMEs and Solar Proton Events.

Date	CME Speed (km/s)	SPE (pfu)
November 4, 1997	785	72
November 6, 1997	1556	490
April 20, 1998	1863	1700
May 2, 1998	938	150
May 6, 1998	1099	210
May 3, 1999	1584	14
June 1, 1999	1772	48
June 4, 1999	2230	64
February 18, 2000	890	13
April 4, 2000	1188	55
June 6, 2000	1119	84
June 10, 2000	1108	46
July 14, 2000	1674	24000
July 22, 2000	1230	17
September 12, 2000	1550	320
October 16, 2000	1336	15
October 25, 2000	770	15
November 8, 2000	1738	14800
November 24, 2000	1289	940
January 28, 2001	916	49
March 29, 2001	942	35
April 2, 2001	2505	1100
April 10, 2001	2411	355
April 15, 2001	1199	951
April 18, 2001	2465	321
April 26, 2001	1006	57
August 9, 2001	479	17
September 15, 2001	478	11
September 24, 2001	2402	12900
October 1, 2001	1405	2360
October 19, 2001	901	11
October 22, 2001	618	24
November 4, 2001	1810	31700
November 17, 2001	1379	34
November 22, 2001	1437	18900
December 26, 2001	1446	779

Date	CME Speed (km/s)	SPE (pfu)
January 8, 2002	1794	91
January 14, 2002	1492	15
February 20, 2002	952	13
March 15, 2002	957	13
March 18, 2002	989	19
March 22, 2002	1750	16
April 17, 2002	1240	24
April 21, 2002	2393	2520
May 22, 2002	1557	820
July 15, 2002	1151	234
August 14, 2002	1309	24
August 22, 2002	998	36
August 24, 2002	1913	317
September 5, 2002	1748	208
November 9, 2002	1838	404
May 28, 2003	1366	121
May 31, 2003	1835	27
June 17, 2003	1813	24
October 26, 2003	1537	466
November 4, 2003	2657	353
November 21, 2003	494	13
April 11, 2004	1645	35
July 25, 2004	1333	2086
September 12, 2004	1328	273
November 7, 2004	1759	495
January 15, 2005	2861	5040
July 13, 2005	1423	134
July 27, 2005	1787	41
August 22, 2005	2378	330

Note: Solar Proton Event strengths are measured in the number of particles that pass through a square centimeter every second, and is given in units called Particle Flux Units or PFUs.



Question 1: A) What percentage of CMEs detected by the SOHO satellite were identified as Halo Events?
1186/11031 = **11%**

B) What are the odds of seeing a halo Event?
1 / 0.11 = **1 chance in 9**

C) How many of these Halo events are directed towards Earth?
From the text, only half are directed to Earth so 1186/2 = **593 Halos**.

Question 2: A) What fraction of SPEs were identified as coinciding with Halo Events?
65 table entries / 90 SPEs = **72%**

B) What are the odds that an SPE occurred with a Halo CME?
1 / 0.72 = 1 chance in 1.38 or about **2 chances in 3**

C) What fraction of all halo events directed towards Earth coincided with SPEs?
65 in Table / (528+65) Halos = **11%**

Question 3: A) What percentage of SPEs coinciding with Halo CMEs are more intense than 900 PFUs?
From the table, there are 12 SPEs out of 65 in this list or 12/65 = **18 %**

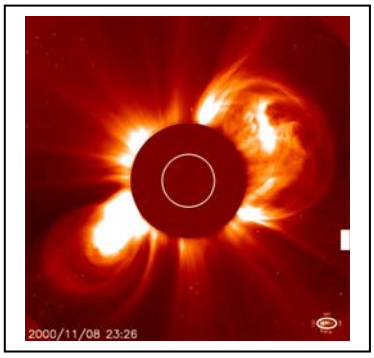
B) What are the odds that, if you detect a 'Halo- SPE', it will be more intense than 900 PFUs?
1 / 0.18 = **1 chance in 5**.

Question 4: A) What percentage of Halo-SPEs have speeds greater than 1000 km/sec?
There are 50 out of 65 or 50/65 = **77%**

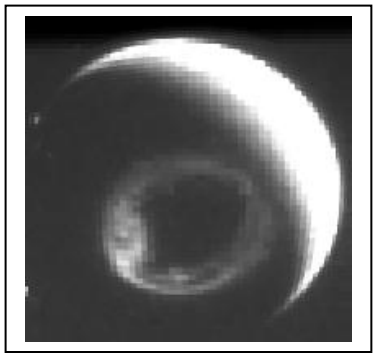
B) What are the odds that a Halo-SPE in this sample has a speed of > 1000 km/sec?
1 / 0.77 = 1 chance in 1.3 or **2 chances in 3**.

Question 5: From what you have calculated as your answers above, what might you conclude about Solar Proton Events and CMEs? How would you use this information as a satellite owner and operator?

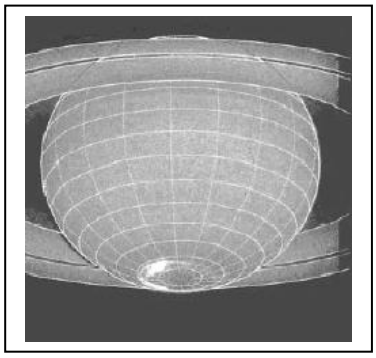
A reasonable student response is that Halo CMEs occur only 11% of the time, and of the ones directed towards Earth only 1 out of 9 coincide with SPEs. However, in terms of SPEs, virtually all of the SPEs coincide with Halo events (2 out of 3) and SPEs are especially common when the CME speed is above 1000 km/sec. As a satellite owner, I would be particularly concerned if scientists told me there was a halo CME headed towards Earth AND that it had a speed of over 1000 km/sec. Because the odds are now 2 chances out of 3 that an SPE might occur that could seriously affect my satellite. I would try to put my satellite in a safe condition to protect it from showers of high-energy particles that might damage it.



Sun - CME



Earth - Aurora



Saturn - Aurora

On November 8, 2000 the sun ejected a blast of plasma called a coronal mass ejection or CME. On November 12, the CME collided with Earth and produced a brilliant aurora detected from space by the IMAGE satellite. On December 8, the Hubble Space Telescope detected an aurora on Saturn. During the period from November to December, 2000, Earth, Jupiter and Saturn were almost lined-up with each other. Assuming that the three planets were located on a straight line drawn from the sun to Saturn, with distances from the sun of 150 million, 778 million and 1.43 billion kilometers respectively, answer the questions below:

Problem 1 – How many days did the disturbance take to reach Earth and Saturn?

Problem 2 – What was the average speed of the CME in its journey between the Sun and Earth in millions of km per hour?

Problem 3 – What was the average speed of the CME in its journey between Earth and Saturn in millions of km per hour?

Problem 4 – Did the CME accelerate or decelerate as it traveled from the Sun to Saturn?

Problem 5 – How long would the disturbance have taken to reach Jupiter as it passed Earth's orbit?

Problem 6 – On what date would you have expected to see aurora on Jupiter?

On November 8, 2000 the sun ejected a blast of plasma called a coronal mass ejection or CME. On November 12, the CME collided with Earth and produced a brilliant aurora detected from space by the IMAGE satellite. On December 8, the Hubble Space telescope detected an aurora on Saturn. During the period from November to December, 2000, Earth, Jupiter and Saturn were almost lined-up with each other. Assuming that the three planets were located on a straight line drawn from the sun to Saturn, with distances from the sun of 150 million, 778 million and 1.43 billion kilometers respectively, answer the questions below:

1 – How many days did the disturbance take to reach Earth and Saturn?
Answer: Earth = 4 days; Saturn = 30 days.

2 – What was the average speed of the CME in its journey between the Sun and Earth in millions of km per hour? **Answer:** Sun to Earth = 150 million km. Time = 4 days x 24 hrs = 96 hrs so Speed = 150 million km/96hr = 1.5 million km/hr.

3 – What was the average speed of the CME in its journey between Earth and Saturn in millions of km per hour? **Answer:** Distance = 1,430 – 150 = 1,280 million km. Time = 30 days x 24h = 720 hrs so Speed = 1,280 million km/720hrs = 1.8 million km/hr.

4 – Did the CME accelerate or decelerate as it traveled from the Sun to Saturn? **Answer:** The CME accelerated from 1.5 million km/hr to 1.8 million km/hr.

5 – How long would the disturbance have taken to reach Jupiter as it passed Earth's orbit?
Answer: Jupiter is located 778 million km from the Sun or (778 – 150 =) 628 million km from Earth. Because the CME is accelerating, it is important that students realize that it is more accurate to use the average speed of the CME between Earth and Saturn which is (1.8 + 1.5)/2 = 1.7 million km/hr. The travel time to Jupiter is then 628/1.7 = 369 hours.

6 – On what date would you have expected to see aurora on Jupiter? **Answer:** Add 369 hours (~ 15 days) to the date of arrival at Earth to get November 23. According to radio observations of Jupiter, the actual date of the aurora was November 20. Note: If we had used the Sun-Earth average speed of 1.5 million km/hr to get a travel time of 628/1.5 = 418 hours, the arrival date would have been November 29, which is 9 days later than the actual storm. This points out that the CME was accelerating after passing Earth, and its speed was between 1.5 and 1.8 million km/hr.

For more details about this interesting research, read the article by Renee Prange et al. "An Interplanetary Shock Traced by Planetary Auroral Storms from the Sun to Saturn" published in the journal Nature on November 4, 2004, vol. 432, p. 78. Also visit the Physics Web online article "Saturn gets a shock" at http://www.physicsweb.org/articles/news/8/11/2/1

Giving Particles a Boost in the Van Allen Belts

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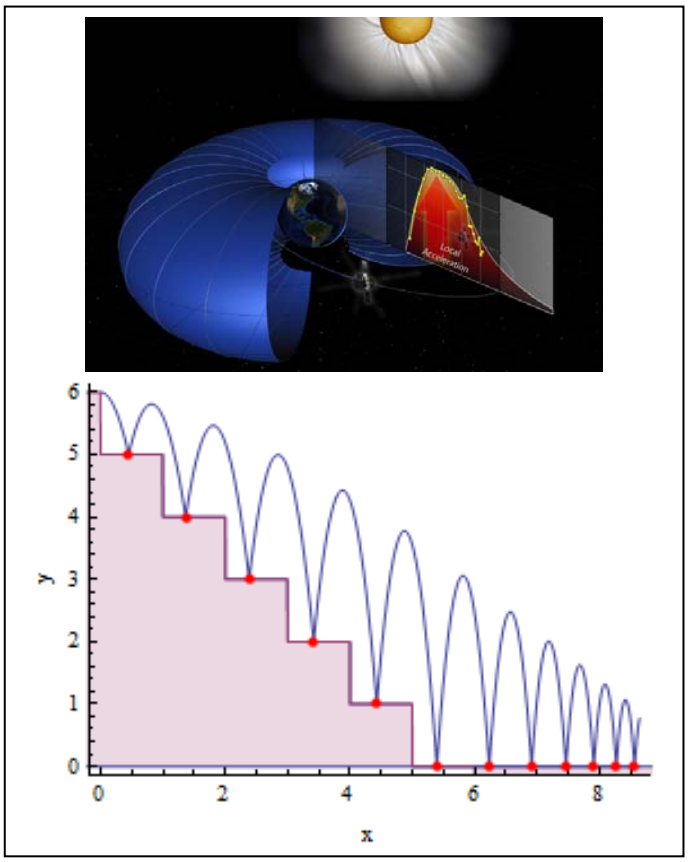


Scientists have discovered a massive particle accelerator in the heart of one of the Van Allen radiation belts. Scientists knew that something in space accelerated particles in the radiation belts to more than 99 percent the speed of light but they didn't know what that something was. New results from NASA's Van Allen Probes now show that the particles inside the belts are sped up by local kicks of energy, buffeting the particles to ever faster speeds, much like a perfectly timed push on a moving swing.

To see how this happens, imagine a ball bouncing down a long staircase as shown in the diagram to the left. Each step it falls, adds a small amount of energy to the ball so it bounces a bit higher each time. This is because gravity pulls on the ball and increases its kinetic energy after each step. As the kinetic energy increases, the ball's speed and height increases.

The formula for Kinetic Energy is K.E. = ½ m V^2 where m is the mass of the particle in kilograms, v is its speed in meters/sec and K.E. is measured in units of Joules.

Problem 1 – A small ball has a mass of 0.1 kilograms and a kinetic energy of 5 Joules, what is its speed in meters/sec?

Problem 2 – A 0.1 kilogram ball bounces down a long staircase that has 100 steps. If it gains 0.3 Joules after each step, how much kinetic energy will it have at the bottom of the staircase, and how fast will it be moving?

Problem 3 – An electron in the Van Allen belts has a mass of 9.1×10^{-31} kg. It starts out with a speed of 10,000 km/sec and reaches a speed of 150,000 km/sec after 12 hours. About how much kinetic energy does it gain every hour as it travels around the Van Allen Belts?

NASA's Van Allen Probes Discover Particle Accelerator in the Heart of Earth's Radiation Belts
July 25, 2013
http://www.nasa.gov/content/goddard/van-allen-probes-find-source-of-fast-particles/index.html

Problem 1 – A small ball has a mass of 0.1 kilograms and a kinetic energy of 5 Joules, what is its speed in meters/sec?

Answer: $5.0 = 0.5 \times 0.1 \times V^2$, so $V^2 = 100$ and V = **10 meters/sec**.

Problem 2 – A 0.1 kilogram ball bounces down a long staircase that has 100 steps. If it gains 0.3 Joules after each step, how much Kinetic Energy will it have at the bottom of the staircase, and how fast will it be moving?

Answer: $0.3 \times 100 = 30$ Joules, then $30 = \frac{1}{2}(0.1) V^2$, and V = **24 meters/sec**.

Note: The potential energy of the ball at the top of its bounce is given my E = m g h, where g = 9.8 m/sec^2 and h is the height in meters. So for this ball, m = 0.1 kg, E = 30 Joules and so its maximum bounce height is h = 30 meters.

Problem 3 – An electron in the Van Allen belts has a mass of 9.1×10^{-31} kg. It starts out with a speed of 10,000 km/sec and reaches a speed of 150,000 km/sec after 12 hours. About how much kinetic energy does it gain every hour as it travels around the Van Allen Belts?

Answer: The initial kinetic energy of the electron is
$E = \frac{1}{2}(9.1 \times 10^{-31})(10{,}000{,}000)^2 = 4.6 \times 10^{-17}$ Joules.

The final kinetic energy is $E = \frac{1}{2}(9.1 \times 10^{-31})(150{,}000{,}000)^2 = 1.0 \times 10^{-14}$ Joules,

so the electron gained $1.0 \times 10^{-14} - 4.6 \times 10^{-17} = 1.0 \times 10^{-14}$ Joules of energy.

If this was done equally over 12 hours, then the energy gained per hour was $1.0 \times 10^{-14} / 12 =$ **8.5×10^{-16} Joules each hour**.

How Telescopes Work

You have probably seen a telescope before, and wondered how it works!

Telescopes are important in astronomy because they do two things extremely well. Their large lenses and mirrors can collect much more light than the human eye, which make it possible to see very faint things. This is called Light Gathering Ability. They also make distant things look much bigger than what the human eye can see so it is easier to study details. This is called magnification.

The human eye at night is a circle about 7 millimeters in diameter, called the pupil, which lets light pass through its lens and onto the retina. A telescope can have a main mirror or lens that can be many meters in diameter.

How do you figure out how much Light Gathering Ability a telescope has compared to the human eye? Just calculate the area of the two circles and form their ratio!

Problem 1 – The human eye can have a pupil diameter of as much as 7 millimeters. Using the formula for the area of a circle, and a value of $\pi = 3.145$, what is the area of the human pupil in square millimeters?

Problem 2 - The Hubble Space Telescope mirror has a diameter of 2.4 meters, which equals 2400 millimeters. What is the area of the Hubble mirror in square millimeters?

Problem 3 – What is the ratio of the area of the Hubble mirror to the human pupil? This is called the Light Gathering Ability of the Hubble Space Telescope!

Problem 4 - The faintest stars in the sky that the human eye can see are called magnitude +6.0 stars. To see magnitude +11 stars, you need a telescope that can see 100 times fainter than the human eye. What is the diameter of the mirror or lens that will let you see these faint stars?

Problem 1 – The human eye can have a pupil diameter of as much as 7 millimeters. Using the formula for the area of a circle, and a value of π = 3.145, what is the area of the human pupil in square millimeters?

Answer: $A = 3.14 (7/2)^2 =$ **0.78 mm^2**

Problem 2 - The Hubble Space Telescope mirror has a diameter of 2.4 meters, which equals 2400 millimeters. What is the area of the Hubble mirror in square millimeters?

Answer: $A = 3.14 (2400/2)^2 =$ **4,521,600 mm^2**

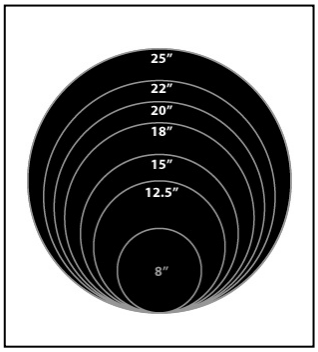
Problem 3 – What is the ratio of the area of the Hubble mirror to the human pupil? This is called the Light Gathering Ability of the Hubble Space Telescope!

Answer: 4,521,600 / 0.78 = **5,796,923 times the human eye**

Problem 4 - The faintest stars in the sky that the human eye can see are called magnitude +6.0 stars. To see magnitude +11 stars, you need a telescope that can see 100 times fainter than the human eye. What is the diameter of the mirror or lens that will let you see these faint stars?

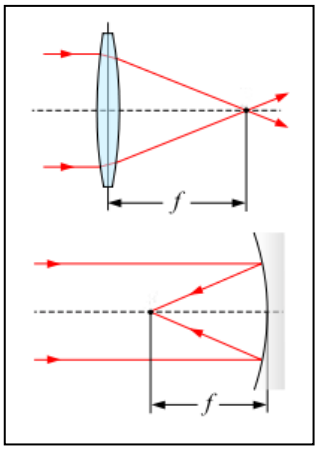
Answer: $100 = \pi R^2/0.78$ so $R^2 = 24.8$ and so R = 5.0 and **D = 10.0 millimeters**.

Focal Lengths, Apertures and F/ Numbers



Any optical system, such as a telescope, camera or microscope, can be described by a just few basic numbers.

Aperture is the main lens or mirror that gathers the light to a focus. Aperture diameter, D, is commonly measured in inches, millimeters, centimeters or even meters. The larger the aperture, the more light the system gathers and the finer details it can see. The top figure shows various aperture diameters for telescopes that can be bought.



Focal length is the distance between the center of the aperture and the point in space where distant light rays come to a focus. In the figure, both a lens and a properly-curved mirror can have focal points. The symbol, f, represents the focal length.

F/ number is a measure of the speed and clarity of the optical system. It is the ratio of the focal distance to the aperture size. Fast systems have small F/numbers such as F/1, F/2 or F/3. Slow systems have large F/ numbers such as F/8, F/15 or even F/20. In photography these are also called F-stops.

$$F/ = \frac{f}{D}$$

Problem 1 – An astronomer wants to design a telescope that takes up the least amount of space in a research satellite. The aperture has to be 254 millimeters in order to gather the most light possible and provide the clearest images. The light path between the mirror center and the focus can be folded 3 times between mirrors separated by 500 millimeters. What is the focal length of this system and the F/ number? Is this a fast or slow system?

Problem 2 – An amateur astronomer wants to buy a telescope and has a choice between three different systems that cost about the same:

System 1 : F/2.0		f = 100 mm
System 2 :	D= 10-inches,	f = 1270 mm
System 3 : F/15.0	D = 50 mm	

Fill-in the missing quantities and describe the pros and cons of each system.

Problem 1 – An astronomer wants to design a telescope that takes up the least amount of space in a research satellite. The aperture has to be 254 millimeters in order to gather the most light possible and provide the clearest images. The light path between the mirror center and the focus can be folded 3 times between mirrors separated by 500 millimeters. What is the focal length of this system and the F/ number? Is this a fast or slow system?

Answer: The focal length is 3 x 1500 millimeters = 1500 millimeters, and F/ = 1500/254 = 5.9. It is a slow system because F/ > 3.0.

Problem 2 – An amateur astronomer wants to buy a telescope and has a choice between three different systems that cost about the same:

System 1 :	F/2.0	D=200 mm	f = 100 mm
System 2 :	F/5.0	D= 254 mm	f = 1270 mm
System 3 :	F/15.0	D = 50 mm	f = 750 mm

Fill-in the missing quantities and describe the pros and cons of each system.

Answer: System 1: D = 100 mm x 2.0 = 200 mm. System 2: D = 10 inches x 25.4 mm/inch = 254 millimeters so F = 1270/254 = 5.0; System 3: f = 50 mm x 15.0 = 750 mm.

System 3 is the slowest optical system in the group, and has the smallest aperture, which means that it gathers the least amount of light and so images will appear fainter and show less detail.

System 1 and 2 are very similar in aperture so they gather about the same amount of light, however, System 1 is nearly 3 times faster and so will provide the clearest images. System 1 is also shorter that System 2 (100 mm vs 1270 mm) so it would be easier and lighter to operate.

How do Telescopes Magnify?

Early depiction of a 'Dutch telescope' from the "Emblemata of zinne-werck" (Middelburg, 1624) of the poet and statesman Johan de Brune (1588-1658). The print was engraved by Adriaen van de Venne, who, together with his brother Jan Pieters van de Venne, printed books not far from the original optical workshop of Hans Lipperhey.

Telescopes can magnify the sizes of distant objects so that the eye can see them more clearly. This is very handy for astronomers who want to study distant planets, stars and galaxies to figure out what they are!

A simple telescope, called a refractor, has two lenses. The large one collects the light from a distant objects and amplifies it so that the image is much brighter than what the eye normally sees. This is called the Objective Lens, or for reflecting telescopes, the Objective Mirror. A second lens is placed at the focus of the Objective and provides the magnification you need to study the objects.

Both the Objective and the eye lens (called the Eyepiece) have their own focus points. The distance between the lens and this focus point is called the focal length. The magnification of the telescope is just the ratio of the Objective focal length to the eyepiece focal length!

$$M = \frac{\text{Objective focal length}}{\text{Eyepiece focal length}}$$

Note, the units for the focal lengths both have to be the same units…inches…millimeters….etc.

Problem 1 – Galileo's first telescope consisted of two lenses attached to the inside of a tube. The Objective had a focal length of 980 millimeters and the eye lens had a focal length of 50 milimeters. What was the magnification of this telescope?

Problem 2 – In 1686, astronomer Christian Huygens built an 8-inch refractor with a 52 meter focal length. If he used the same magnifying eyepiece that Galileo had used, what would be the magnification of this 'long tube refractor'?

Problem 3 – An amateur builds a 20-inch reflector that has a focal length of 157 inches. He already owns three very expensive eyepieces with focal lengths of 4mm, 20mm and 35 mm. What magnification will he get from each of these eyepieces? (1 inch = 25.4 mm)

Problem 1 – Galileo's first telescope consisted of two lenses attached to the inside of a tube. The Objective had a focal length of 980 millimeters and the eye lens had a focal length of 50 milimeters. What was the magnification of this telescope?

Answer. M = 980/50 = **19.6 times**.

Problem 2 – In 1686, astronomer Christian Huygens built an 8-inch refractor with a 52 meter focal length. If he used the same magnifying eyepiece that Galileo had used, what would be the magnification of this 'long tube refractor'?

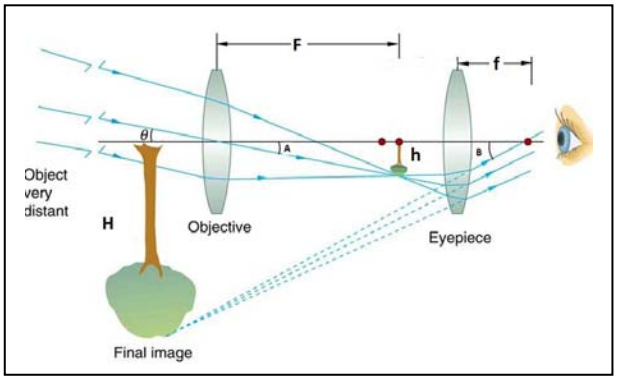
Answer: 52 meters = 52000 millimeters, then for a 50mm eyepiece, the magnification is M = 52000/50 = **1040 times**.

Problem 3 – An amateur builds a 20-inch reflector that has a focal length of 157 inches. He already owns three very expensive eyepieces with focal lengths of 4mm, 20mm and 35 mm. What magnification will he get from each of these eyepieces?

Answer: 157 inches x 25.4 mm/inch = 3988 mm, then

M = 3988/4 = 997x,
M = 3988/20 = 199 x
M = 3988/35 = 114 x

Calculating the Magnification of a Telescope



A telescope consists of an objective mirror or lens and an eyepiece. The role of the eyepiece is to change the angle, A, of the rays from the objective as they enter the eye. As the figure shows, when B > A, it appears as though the image of the tree is bigger than its actual image at the focus of the telescope objective. A simple proportion relates the image sizes to the focal lengths of the lenses:

$$\frac{H}{h} = \frac{F}{f}$$

For example, if the telescope objective has a focal length of 2000 millimeters and the eyepiece has a focal length of 4 millimeters, H/h = 2000/4 = 500, so the image h has been magnified by 500 times. The quantity F/f is the magnification.

Problem 1 – The table below gives the optical data for some large telescopes. Use this data to calculate the magnification for each indicated lens. Also fill in all other missing information. Focal lengths and aperture dimensions are given in millimeters.

Telescope	Type	Aperture	F/	Focal Length	Eyepiece F.L.	Magnification
8-inch Orion	Reflector	203		1198	10	
Obsession-20	Reflector	508	5.0		8	
1-meter	Reflector	1000	17.0			850
David Dunlop	Reflector	1880	17.3		100	
Hubble	Reflector	2400		57600		2880
Mt Palomar	Reflector		3.3	16830	28	
Yale	Refractor	1020	19.0		4	
Subaru	Reflector	8200		15000		7500
Keck	Reflector		1.75	17500	1	

Problem 2 – Suppose that the eyepiece was eliminated and the human eye was used as the eyepiece instead. If the focal length of the human eye is 25 cm, what is the magnification for the Obsession-20 telescope operating in this way? (Note: this is called Prime Focus observing).

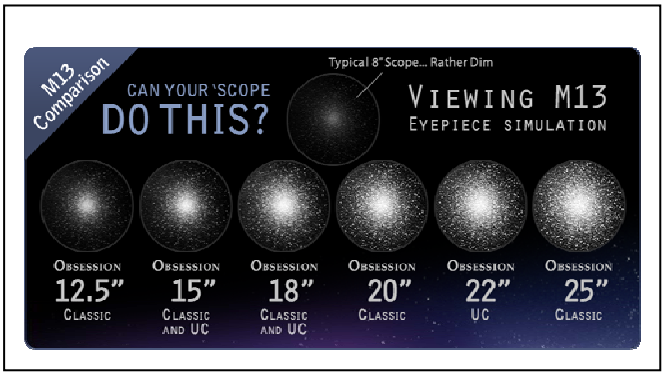
Problem 1 – The table below gives the optical data for some large telescopes. Use this data to calculate the magnification for each indicated lens. Also fill in all other missing information.

Telescope	Type	Aperture	F/	Focal Length	Eyepiece F.L.	Magnification
8-inch Orion	Reflector	203	**5.9**	1198	10	**120**
Obsession-20	Reflector	508	5.0	**2540**	8	**317**
1-meter	Reflector	1000	17.0	**17000**	**20**	850
David Dunlop	Reflector	1880	17.3	32,524	**100**	**325**
Hubble	Reflector	2400	**24**	57600	**20**	2880
Mt Palomar	Reflector	**5100**	3.3	16830	28	**601**
Yale	Refractor	1020	19.0	**19400**	4	**4850**
Subaru	Reflector	8200	**1.83**	15000	**2**	7500
Keck	Reflector	**10000**	1.75	17500	1	**17500**

Problem 2 – Suppose that the eyepiece was eliminated and the human eye was used as the eyepiece instead. If the focal length of the human eye is 25 cm, what is the magnification for the Obsession-20 telescope operating in this way? (Note: this is called Prime Focus observing).

Answer: The focal length of the Obsession-20 mirror is 2540 mm, and the eye's focal length is 250 mm, so the magnification is only about **10 times**. This means that if you look at the moon in this way, it will appear 10 times bigger 'in the sky' than without the telescope.

Note: A rule-of-thumb is that you do not use a higher magnification than 2 times the aperture size in millimeters. At higher magnifications the image remains blurry and you do not see additional details. In the table in Problem 1, the only eyepiece that violates this rule is the one selected for the Yale Telescope, and so an eyepiece with a longer focal length and lower magnification is the best to use.



This figure shows a comparison of the same faint star cluster seen with telescopes of increasing aperture size and LGA. Notice the big change between an 8-inch and am 18-inch!

The human eye is a small lens that lets-in only a small amount of light. This is useful when you are looking at a bright daytime scene, but when you are studying faint stars this becomes a problem.

A telescope has a much larger aperture than the eye and allows more light to be brought to a focus to study. This means that even stars too faint to be detected by the eye can easily be 'brightened' by the telescope so that they are easy to detect and study.

Light Gathering Ability is the property of an optical system that tells you how much brighter things will appear than what the human eye can see. It is the ratio of the area of the objective to the area of the human eye lens.

Problem 1 – A pair of binoculars has a lens with a diameter of 50 mm. If the human eye lens has a diameter of 7mm, how much more light do the binoculars gather than the human eye?

Problem 2 – Star brightness is measured on the magnitude scale where each magnitude represents an increase in intensity by a factor of 2.514. What is the brightness difference between a star with m = +1.0 and m = +6.0?

Problem 3 – The human eye can see stars as faint as m = +6.0. What size mirror will be needed so that stars as faint as +16.0 can be seen?

Problem 1 – A pair of binoculars has a lens with a diameter of 50 mm. If the human eye lens has a diameter of 7mm, how much more light do the binoculars gather than the human eye?

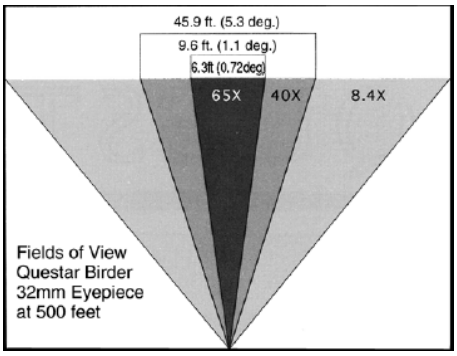
Answer: LGA = $(50/7)^2$ = 51 times more light.

Problem 2 – Star brightness is measured on the magnitude scale where each magnitude represents an increase in intensity by a factor of 2.514. What is the brightness difference between a star with m = +1.0 and m = +6.0?

Answer: The magnitude difference is m = 5.0, so the brightness difference is a factor of $(2.512)^5$ = 100 times.

Problem 3 – The human eye can see stars as faint as m = +6.0. What size mirror will be needed so that stars as faint as +16.0 can be seen?

Answer: The magnitude difference is +16.0 - +1.0 = +10.0, which is a brightness factor of 100x100 = 10,000. You need a telescope that provides a LGA of 10000 so
$10000 = (D/7mm)^2$ and so D = 700 millimeters (27-inches in diameter).



Although magnifying an object makes it appear larger, the view also gets smaller as the diagram to the left shows. Every combination of telescope and eyepiece produces its own Field of View (FOV), usually stated in angular terms. For example, a combination that gives an angular FOV of 10° in diameter will easily let you see the entire full moon, which is only 0.5° in diameter. But if you use a lens with 40 times more magnification, the FOV is now only 1/4° so you only see ½ of the full disk of the moon in the eyepiece.

By itself, an eyepiece allows incoming light to be brought to a focus for the human eye or camera. The incoming light rays can come from many different directions within a cone whose vertex is the focus point for the lens. The angle of the cone's vertex defines the FOV for the eyepiece. The table below shows the FOVs for various eyepieces that are used with telescopes:

Vendor	Model	Focal Length	Apparent FOV (°)	Actual FOV (°)	Magnification	Price
Orion	Optilux 2"	40	60	1.18	51	$140
Televue	Panoptic 2"	35	68	1.17	58	$370
Orion	FMC Plössl 2"	50	45	1.11	41	$120
Orion	DeepView 2"	42	52	1.07	48	$70
Edmund Optics	RKE Erfle 2"	32	68	1.07	64	$225
Meade	SWA 2"	32	67	1.06	64	$240
Orion	Optilux 2"	32	60	0.94	64	$140
Teleview	Panoptic 2"	27	68	0.90	75	$330
Teleview	Plössl 1.25"	40	43	0.85	51	$110
Celestron	Ultima 1.25"	35	49	0.84	58	$108

The apparent FOV for each eyepiece ranges from 45° to 68° and is a result of how the eyepiece is designed. When used in this example with an 8-inch telescope with a focal length of 2032 millimeters, the magnifications range from 41x to 75x. The resulting telescope FOV is then just FOV = Eyepiece FOV/magnification. For the Optilix 2" eyepiece, the FOV is then 60°/51 = 1.18°.

Problem 1 – An astronomer wants to design a system so that the full moon fills the entire FOV of the telescope. He uses an eyepiece with a FOV of 60°. What magnification will give him the desired FOV?

Problem 2 – An amateur astronomer upgrades to a larger telescope and keeps his old eyepieces, which have FOVs of 50°. His old telescope provided a 2.0° FOV for his most expensive eyepiece. Because the focal length of the new telescope is twice that of his older telescope, all magnifications on the new telescope will be twice as high. What will the FOV be for his most expensive eyepiece on the new telescope?

Problem 1 – An astronomer wants to design a system so that the full moon fills the entire FOV of the telescope. He uses an eyepiece with a FOV of 60°. What magnification will give him the desired FOV?

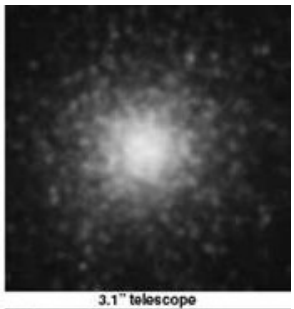
Answer: 0.5° = 60° / M so the magnification is M = **120x**.

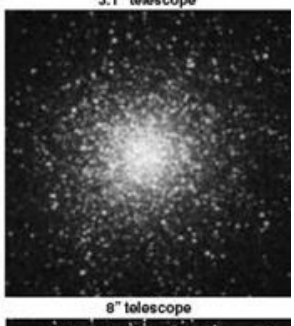
Problem 2 – An amateur astronomer upgrades to a larger telescope and keeps his old eyepieces, which have FOVs of 50°. His old telescope provided a 2.0o FOV for his most expensive eyepiece. Because the focal length of the new telescope is twice that of his older telescope, all magnifications on the new telescope will be twice as high. What will the FOV be for his most expensive eyepiece on the new telescope?

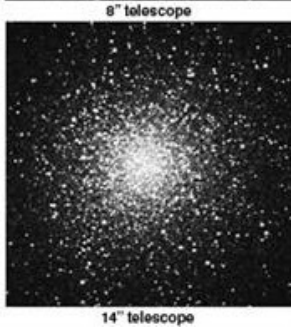
Answer: Because FOV = eyepiece FOV/magnification, if the new telescope provides magnifications that are twice the older system, then the FOV will be half as large for this eyepiece or 1.0°.

Telescope Resolution – How much detail can you see?

16.7







The images to the left show what the star cluster Messier-13 would look like to three different telescopes with apertures of 3.1, 8.0 and 14.0 inches. Notice that as the aperture increases, the fuzzy smudges seen by the smallest telescope become increasingly more clear to see as the aperture increases. This is an example of Optical Resolution, which is sometimes called the Resolving Power of a telescope.

To make the clearest photographs of stars, planets, or even people, it helps to use the largest lens or aperture to make crisp, clear images. Astronomers also want the highest resolutions possible so that they can study the smallest details on a planet surface, or in a distant galaxy.

Telescope resolution at optical wavelengths can be calculated using the simple formula:

$$R = \frac{134}{D}$$

where D is the diameter of the objective in millimeters, and R is the resolution in seconds of arc. (There are 3600 seconds of arc in 1 angular degree).

For example, a pair of binoculars with D = 50 mm, provides a resolution limit of R = 2.8 arcseconds. A small 8-inch telescope for which D = 200 mm, provides R = 0.67 arcseconds.

Problem 1 – An astronomer wants to design a system that will let him study craters on the moon that are about 0.1 arcseconds in diameter as seen from Earth. What is the minimum-sized aperture he needs to conduct his study?

Problem 2 – The Hubble Space Telescope has a diameter of 2.4 meters. What is its maximum resolution?

Problem 3 – Two telescopes are combined in an instrument called an interferometer, which creates a single telescope with a diameter of 640 meters. What is the maximum resolution of this system?

Problem 1 – An astronomer wants to design a system that will let him study craters on the moon that are about 0.1 arcseconds in diameter as seen from Earth. What is the minimum-sized aperture he needs to conduct his study?

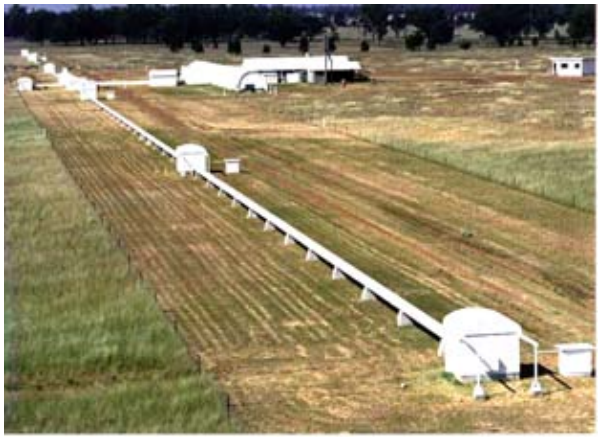
Answer: 0.1 = 134/D so D = **1340 millimeters (53 inches)**.

Problem 2 – The Hubble Space Telescope has a diameter of 2.4 meters. What is its maximum resolution?

Answer: R = 134/2400 = **0.06 arcseconds** or 60 milliarcseconds

Problem 3 – Two telescopes are combined in an instrument called an interferometer, which creates a telescope with a diameter of 640 meters. What is the maximum resolution of this system?

Answer: R = 134/640000 = **0.0002 arcseconds or 0.2 milliarcseconds**.



A photo of the Sydney University Stellar Interferometer (SUSI) is a long-baseline optical interferometer located approximately 20km west of the town of Narrabri in northern New South Wales, Australia. The equivalent diameter of the optical aperture for this instrument is 640 meters.

Buying a Telescope!

OK…So those wonderful pictures of planets, star clusters and galaxies have got your curiosity on fire. You want to have your own telescope so that you can see the universe for yourself! All you have to do is spend a few minutes on the Internet and you will see a bewildering number of choices for telescopes you can buy. Some are pretty inexpensive and cost less than $70.00, but others can cost $500.00 or more. How do you decide which one is right for you?

Remember, the bigger the objective lens or mirror, the fainter you can see stars in the sky. The longer the focal length, the higher will be the magnification. The only limit to either of these is that you should not use magnifications higher than 50x the diameter of the objective, or 2 times its diameter in millimeters. Higher magnifications only make images look worse!

Type	Objective (cm)	Focal Length (millimeters)	Maximum Magnification	Cost
Reflector	7.6	300		$64.95
Refractor	6.0	700		$54.95
Refractor	8.9	910		$300.00
Reflector	11.4	900		$129.95
Reflector	15.2	610		$319.95
Refractor	10.0	900		$749.95
Reflector	20.3	1000		$699.95
Refractor	15.2	1219		$1,199.00
Reflector	50.8	2032		$4,400.00

Problem 1 – You have a set of eyepieces with focal lengths of 2mm, 4mm and 28mm. If

$$\text{magnification} = \frac{\text{telescope focal length}}{\text{eyepiece focal length}}$$

would you be able to use all of these eyepieces with the telescopes in the table above?

Problem 2 - In terms of cost per objective area, which type of telescopes seem to be the best value: reflectors or refractors?

Problem 3 – About how much would you expect to pay for a 50.8-cm refractor?

Type	Objective (cm)	Focal Length (millimeters)	Maximum Magnification	Cost	Cost per area
Reflector	7.6	300	152x	$64.95	1.4
Refractor	6.0	700	120x	$54.95	1.9
Refractor	8.9	910	178x	$300.00	4.7
Reflector	11.4	900	228x	$129.95	1.3
Reflector	15.2	610	304x	$319.95	1.8
Refractor	10.0	900	200x	$749.95	9.6
Reflector	20.3	1000	406x	$699.95	2.2
Refractor	15.2	1219	304x	$1,199.00	6.6
Reflector	50.8	2032	1016x	$4,400.00	2.2

Problem 1 – You have a set of eyepieces with focal lengths of 2mm, 4mm and 28mm. If

$$\text{magnification} = \frac{\text{telescope focal length}}{\text{eyepiece focal length}}$$

would you be able to use all of these eyepieces with the telescopes in the table above?

Answer: The limit would be set by the 2mm eyepiece. For the telescope focal lengths in the table, this eyepiece could not be used with the telescopes shaded in yellow in the table, Telescopes 2, 3, 4, 6, 7 and 8. The 4mm would not be used on telescopes 2, 3, 4, and 6.

Problem 2 - In terms of cost per objective area, which type of telescopes seem to be the best value: reflectors or refractors?

Answer: You can get a reflector with a larger area than you can a refractor. Refractors are more expensive per unit area. From the table, reflectors cost 1.3 to 2.2 dollars per square centimeter, while refractors cost 1.9 to 9.6 dollars per square centimeter.

Problem 3 – About how much would you expect to pay for a 50.8-cm refractor?

Answer: A 15.2 cm refractor costs about 6.6 dollars per square centimeter, so a 50.8-cm refractor would cost $\pi\,(50.8/2)^2 \times 6.6 =$ **$13,374.**

Designing a Telescope System

Telescope diameter = D
Telescope focal length = F
Eyepiece focal length = f
Eyepiece field of view = FOVe
Magnification = M
Resolution = R

D, F, f are in millimeters
FOVe is in degrees
R is in arcseconds

$$M = \frac{F}{f}$$

$$\text{F/number} = \frac{F}{D}$$

$$R = \frac{134}{D}$$

Astronomers don't just go out and buy a telescope and then use it. Whether it is for use on a satellite orbiting Saturn, or in an observatory, telescopes are designed 'from the ground up' by starting from a set of goals that the research needs to accomplish. The telescope is mathematically designed to meet these research goals.

The table to the left gives the basic quantities and formulae for designing a simple telescope system. Let's see how two different research goals can lead to very different telescope systems!

System 1 – Veronica has been an amateur astronomer for 20 years and especially enjoys photographing faint galaxies and nebulae. She has owned three telescopes and plans to sell them to fund her next system. She can afford a telescope with an aperture no larger than 20-inches (500 mm), and needs it to be a fast optical system with an F-number less than 3.0. She has a set of expensive eyepieces that she will keep. Her favorite one is a 20mm Plossl with a FOV of 68°, and for best results she wants this eyepiece to have a magnification of no more than 50x. What is the best combination of aperture size and focal length for the telescope that will satisfy all of her needs?

System 2 – Leonard has a program of observing Saturn to keep track of its equatorial belt system. He needs a telescope with F/number > 10 and a resolution between 1/3 and 1/2 arcseconds. He has three eyepieces with focal lengths of f = 5mm, 10mm and 20mm that have provided him with high, medium and low magnification on his previous telescope, which got damaged in a house fire. He wants the 5mm eyepiece to provide no more than a magnification of 700x. What is the best system that meets his needs?

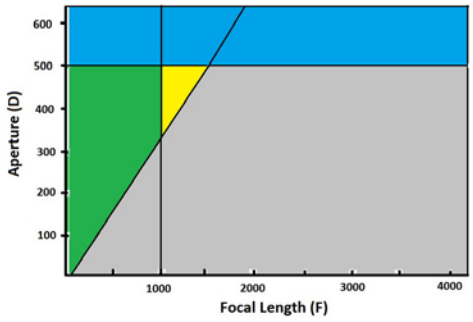
To design these systems, create a graph with the aperture diameter (vertical) and the focal length (horizontal).

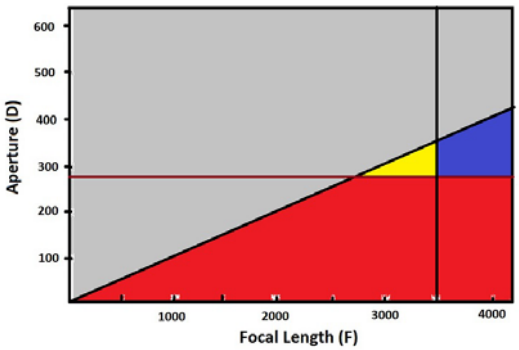
System 1 – Veronica has been an amateur astronomer for 20 years and especially enjoys photographing faint galaxies and nebulae. She has owned three telescopes and plans to sell them to fund her next system. She can afford a telescope with an aperture no larger than 20-inches (500 mm), and needs it to be a fast optical system with an F-number less than 3.0. She has a set of expensive eyepieces that she will keep. Her favorite one is a 20mm Plossel with a FOV of 68°, and for best results she wants this eyepiece to have a magnification of no more than 50x. What is the best combination of aperture size and focal length for the telescope that will satisfy all of her needs?

Answer: D < 20 inches (500 mm); F/number < 3.0; f = 20 mm, FOVe=68°, M> 50x

On the D vs F graph, draw a line representing F/D = 3.0 or F = 3.0D and shade the excluded region below this line (grey), which represents F/n > 3.0. Now draw a horizontal line for D = 500mm and shade (blue) the region above this line which represents D > 500mm. Magnification = F/f so we have F/f > 50 and F > 50f. For f = 20mm, the constraint is F > 1000 mm. Draw a vertical line at F = 1000mm and shade (green) all points to the left as the excluded region. The permitted regions is the one shown in yellow. The final plot (below left) should look like the one below.

An optimal system is near the middle of the yellow permitted region for which D = 450 mm and F = 1250mm. We then have F/number = 2.8, a magnification of 62, a telescope FOV of 68/62 = 1 degree, and a resolution of 134/450 = 0.3 arcseconds.



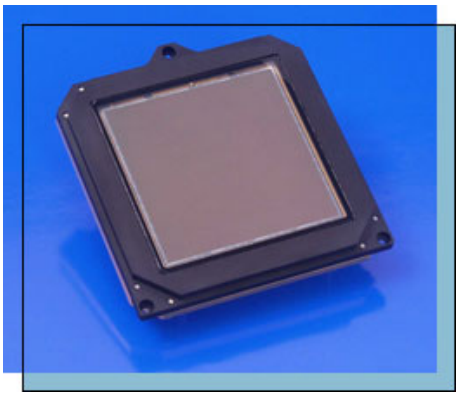


System 2 – Leonard has a program of observing Saturn to keep track of its equatorial belt system. He needs a telescope with F/number > 10 and a resolution between 1/3 and 1/2 arcseconds. He has three eyepieces with focal lengths of f = 5mm, 10mm and 20mm that have provided him with high, medium and low magnification on his previous telescope, which got damaged in a house fire. He wants the 5mm eyepiece to provide no more than a magnification of 700x. What is the best system that meets his needs?

Answer: F/number > 10; R = 1/3 to ½ arcseconds; M < 700x.
Draw a line representing F/number = 10 and shade (grey) the excluded area above this line which indicates F/number <10. For R = ½ arcseconds, draw a horizontal line at D = 270mm and shade the region (red) below this line that represents R > ½. The magnification M = F/5mm so for M=700 we have F < 3500mm. Draw a vertical line at F = 3500 and shade (blue) the excluded area to the right. The allowed region is in yellow in the diagram on the upper right.

The best system lies in the center of the triangle with D = 300mm and F = 3250 mm. This gives F/10.8; Resolution = 134/300 = 0.4 arcseconds; Lens magnifications of 650, 325 and 162.

Digital Camera Math!



This is a 9200x9200 image sensor for a digital camera. The grey area contains the individual 'pixel' elements. Each square pixel is about 8 micrometers (8 microns) wide, and is sensitive to light. The pixels accumulate electrons as light falls on them, and computers read out each pixel and create the picture.

Digital cameras are everywhere! They are in your cell phones, computers, iPads and countless other applications that you may not even be aware of.

In astronomy, digital cameras were first developed in the 1970s to replace and extend photographic film techniques for detecting faint objects. Digital cameras are not only easy to operate and require no chemicals to make the images, but the data is already in digital form so that computers can quickly process the images.

Commercially, digital cameras are referred to by the total number of pixels they contain. A '1 megapixel camera' can have a square-shaped sensor with 1024x1024 pixels. This says nothing about the sensitivity of the camera, only how big an image it can create from the camera lenses. Although the largest commercial digital camera has 80 megapixels in a 10328x7760 format, the largest astronomical camera developed for the Large Synoptic Survey Telescope uses 3200 megapixels (3.2 gigapixels)!

Problem 1 – An amateur astronomer purchases a 6.1 megapixel digital camera. The sensor measures 20 mm x 20 mm. What is the format of the CCD sensor, and about how wide are each of the pixels in microns?

Problem 2 – Suppose that with the telescope optical system, the entire full moon will fit inside the square CCD sensor. If the angular diameter of the moon is 1800 arcseconds, about what is the resolution of each pixel in the camera?

Problem 3 – The LSST digital camera is 3.2 gigapixels in a 10328x7760 format. If the long side of the field covers an angular range of 3.5 degrees, what is the angular resolution of this CCD camera in arcseconds/pixel?

Problem 1 – An amateur astronomer purchases a 6.1 megapixel digital camera. The sensor measures 20 mm x 20 mm. What is the format of the CCD sensor, and about how wide are each of the pixels in microns?

Answer: This is a square array, so s2 = 6100000 pixels and so s = 2469 pixels. The format is **2469 x 2469 pixels**. Since the width of a side is 20 mm, each pixel is about 20 mm/2469 = 0.0000081 meters or **8.1 microns** on a side.

Problem 2 – Suppose that with the telescope optical system, the entire full moon will fit inside the square CCD sensor. If the angular diameter of the moon is 1800 arcseconds, about what is the resolution of each pixel in the camera?

Answer: 1800 arcseconds/2469 pixels = **0.7 arcseconds/pixel**.

Problem 3 – The LSST digital camera is 3.2 gigapixels in a 10328x7760 format. If the long side of the field covers an angular range of 3.5 degrees, what is the angular resolution of this CCD camera in arcseconds/pixel?

Answer: 1 degree = 3600 arcseconds, so 3.5 degrees = 12600 arcseconds. Then 12600 arcseconds/10328 pixels = **1.2 arcseconds/pixel.**

Life-size model of the Webb Space Telescope.

In 2018, the new Webb Space Telescope will be launched. This telescope, designed to detect distant sources of infrared 'heat' radiation, will be a powerful new instrument for discovering distant dwarf planets far beyond the orbit of Neptune and Pluto.

Scientists are already predicting just how sensitive this new infrared telescope will be, and the kinds of distant bodies it should be able to detect in each of its many infrared channels. This problem shows how this forecasting is done.

Problem 1 - The angular diameter of an object is given by the formula:

$$\theta(R) = 0.0014 \frac{L}{R} \text{ arcseconds}$$

Create a single graph that shows the angular diameter, θ(R), for an object the size of dwarf planet Pluto (L=2,300 km) spanning a distance range, R, from 30 AU to 100 AU, where 1 AU (Astronomical Unit) is the distance from Earth to the sun (150 million km). How big will Pluto appear to the telescope at a distance of 90 AU (about 3 times its distance of Pluto from the sun)?

Problem 2 - The temperature of a body that absorbs 40% of the solar energy falling on its surface is given by

$$T(R) = \frac{250}{\sqrt{R}} \text{ Kelvins}$$

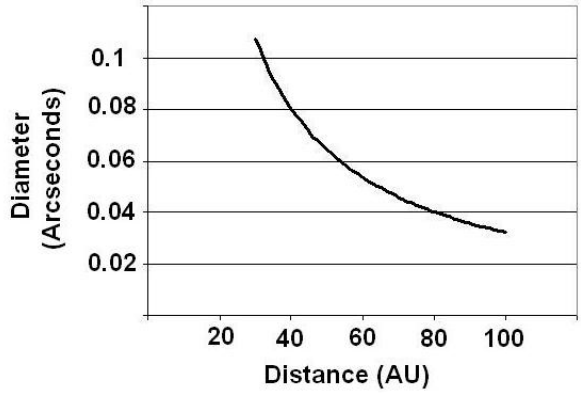
where R is the distance from the sun in AU. Create a graph that shows T(R) vs R for objects located in the distance range from 30 to 100 AU. What will be the predicted temperature of a Pluto-like object at 90 AU?

Problem 3 - A body in other outer solar system with an angular size θ(R) emits most of its light energy in the infrared and has a temperature given by T(R) in Kelvins. Its brightness in units of Janskys, F, at a wavelength of 20 microns will be given by:

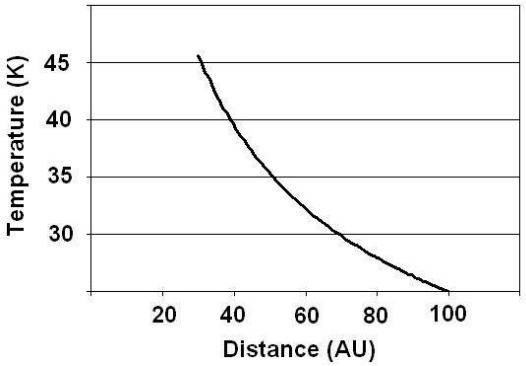
$$F(T) = \frac{120000}{(e^x - 1)} \theta(R)^2 \text{ Janskys} \qquad \text{where} \quad x = \frac{720}{T(R)}$$

From the formula for θ(R) and T(R), create a curve F(R) for a Pluto-like object. If the Webb Space Telescope cannot detect objects fainter than 4 nanoJanskys, what will be the most distant location for a Pluto-like body that this telescope can detect? (Hint: Plot the curve with a linear scale in R and a log10 scale in F.)

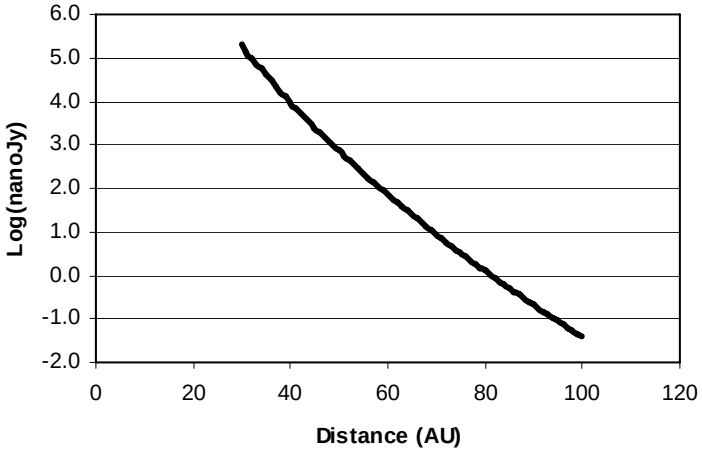
Problem 1 - Answer: At 90 AU, the disk of a Pluto-sized body will be 0.035 arcseconds in diameter.



Problem 2 Answer: At 90 AU, the predicted temperature will be about 28 K.

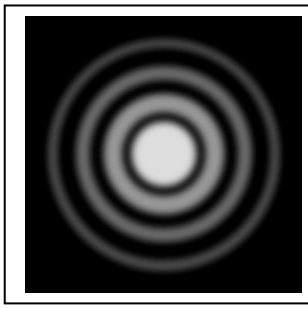


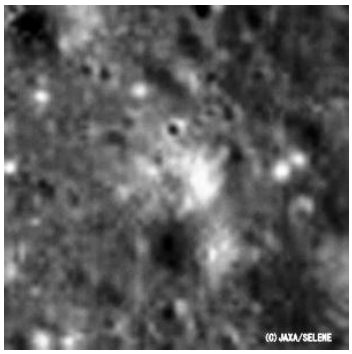
Problem 3 - Answer: At 4 nanoJanskies, Log(4 nanoJy) = 0.60 which occurs at a distance of about 40 AU. More accurate estimates using more realistic emission properties for Pluto suggest 90 AUs as an actual limit.

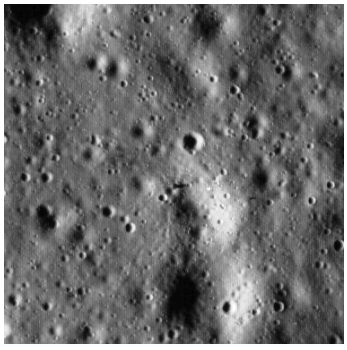


The Most Important Equation in Astronomy!

$$R = 1.22\frac{L}{D}$$







There are many equations that astronomers use to describe the physical world, but none is more important and fundamental to the research that we conduct than the one to the left! You cannot design a telescope, or a satellite sensor, without paying attention to the relationship that it describes.

In optics, the best focused spot of light that a perfect lens with a circular aperture can make, limited by the diffraction of light. The diffraction pattern has a bright region in the center called the Airy Disk. The diameter of the Airy Disk is related to the wavelength of the illuminating light, L, and the size of the circular aperture (mirror, lens), given by D. When L and D are expressed in the same units (e.g. centimeters, meters), R will be in units of angular measure called radians (1 radian = 57.3 degrees).

You cannot see details with your eye, with a camera, or with a telescope, that are smaller than the Airy Disk size for your particular optical system. The formula also says that larger telescopes (making D bigger) allow you to see much finer details. For example, compare the top image of the Apollo-15 landing area taken by the Japanese Kaguya Satellite (10 meters/pixel at 100 km orbit elevation: aperture = about 15cm) with the lower image taken by the LRO satellite (1.0 meters/pixel at a 50km orbit elevation: aperture = 0.8 meter). The Apollo-15 Lunar Module (LM) can be seen by its 'horizontal shadow' near the center of the image.

Problem 1 - The Senator Byrd Radio Telescope in Green Bank West Virginia with a dish diameter of D=100 meters is designed to detect radio waves with a wavelength of L= 21-centimeters. What is the angular resolution, R, for this telescope in A) degrees? B) Arc minutes?

Problem 2 - The largest, ground-based optical telescope is the D = 10.4-meter Gran Telescopio Canaris. If this telescope operates at optical wavelengths (L = 0.00006 centimeters wavelength), what is the maximum resolution of this telescope in A) microradians? B) milliarcseconds?

Problem 3 - An astronomer wants to design an infrared telescope with a resolution of 1 arcsecond at a wavelength of 20 micrometers. What would be the diameter of the mirror?

Answer Key

Problem 1 - The Senator Byrd Radio Telescope in Green Bank West Virginia with a dish diameter of D=100 meters is designed to detect radio waves with a wavelength of L= 21-centimeters. What is the angular resolution, R, for this telescope in A) degrees? B) Arc minutes?

Answer: First convert all numbers to centimeters, then use the formula to calculate the resolution in radian units: L = 21 centimeters, D = 100 meters = 10,000 centimeters, then R = 1.22 x 21 cm /10000 cm so R = 0.0026 radians. There are 57.3 degrees to 1 radian, so A) 0.0026 radians x (57.3 degrees/ 1 radian) = **0.14 degrees.** And B) There are 60 arc minutes to 1 degrees, so 0.14 degrees x (60 minutes/1 degrees) = **8.4 arcminutes.**

Problem 2 - The largest, ground-based optical telescope is the D = 10.4-meter Gran Telescopio Canaris. If this telescope operates at optical wavelengths (L = 0.00006 centimeters wavelength), what is the maximum resolution of this telescope in A) microradians? B) milliarcseconds?
Answer: R = 1.22 x (0.00006 cm/10400 cm) = 0.000000069 radians. A) Since 1 microradian = 0.000001 radians, the resolution of this telescope is **0.069 microradians.** B) Since 1 radian = 57.3 degrees, and 1 degree = 3600 arcseconds, the resolution is 0.000000069 radians x (57.3 degrees/radian) x (3600 arcseconds/1 degree) = 0.014 arcseconds. One thousand milliarcsecond = 1 arcseconds, so the resolution is 0.014 arcsecond x (1000 milliarcsecond / arcsecond) = **14 milliarcseconds.**

Problem 3 - An astronomer wants to design an infrared telescope with a resolution of 1 arcsecond at a wavelength of 20 micrometers. What would be the diameter of the mirror?

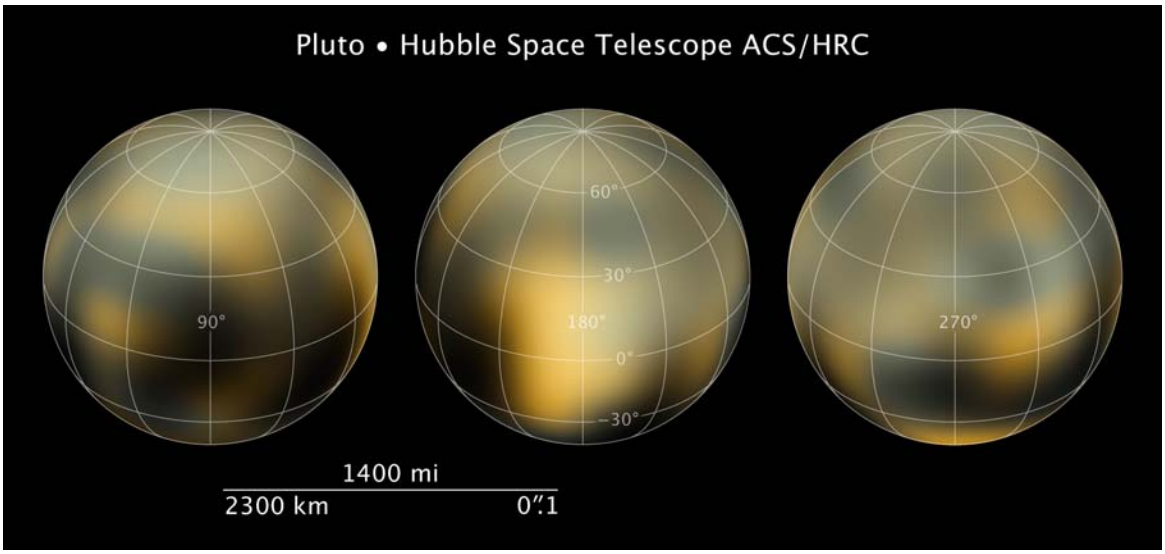
Answer: From R = 1.22 L/D we have R = 1 arcsecond and L = 20 micrometers and need to calculate D, so with algebra we re-write the equation as D = 1.22 L/R.
Convert R to radians:

R = 1 arcsecond x (1 degree/3600 arcsecond) x (1 radian / 57.3 degrees) = 0.0000048 radians.

L = 20 micrometers x (1 meter/1,000,000 micrometers) = 0.00002 meters.

Then D = 1.22 (0.00003 meters)/(0.0000048 radians) = **5.1 meters.**

Seeing a Dwarf Planet Clearly: Pluto



Recent Hubble Space Telescope studies of Pluto have confirmed that its atmosphere is undergoing considerable change, despite its frigid temperatures. The images, created at the very limits of Hubble's ability to see small details (sometimes called a telescope's resolving power), show enigmatic light and dark regions that are probably organic compounds (dark areas) and methane or water-ice deposits (light areas). Since these photos are all that we are likely to get until NASA's New Horizons spacecraft arrives in 2015, let's see what we can learn from the image!

Problem 1 - Using a millimeter ruler, what is the scale of the Hubble image in kilometers/millimeter?

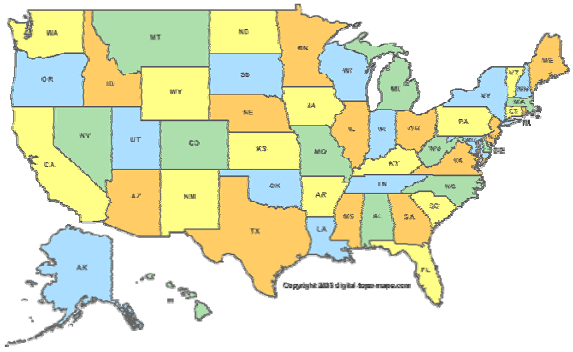
Problem 2 - What is the largest feature you can see on any of the three images, in kilometers, and how large is this compared to a familiar earth feature or landmark such as a state in the United States?

Problem 3 - The satellite of Pluto, called Charon, has been used to determine the total mass of Pluto. The mass determined was about 1.3×10^{22} kilograms. From clues in the image, calculate the volume of Pluto and determine the average density of Pluto. How does it compare to solid-rock (3000 kg/m^3), water-ice (917 kg/m^3)?

Inquiry: Can you create a model of Pluto that matches its average density and predicts what percentage of rock and ice may be present?

Problem 1 - Using a millimeter ruler, what is the scale of the Hubble image in kilometers/millimeter? Answer: The Legend bar indicates 2,300 km and is 43 millimeters long so the scale is 2300/43 = **53 km/mm**.

Problem 2 - What is the largest feature you can see on any of the three images, in kilometers, and how large is this compared to a familiar earth feature or landmark such as a state in the United States?
Answer; Student's selection will vary, but on the first image to the lower right a feature measures about 8 mm in diameter which is 8 mm x (53 km/1mm) = **424 kilometers wide**. This is about the same size as the **state of Utah**!

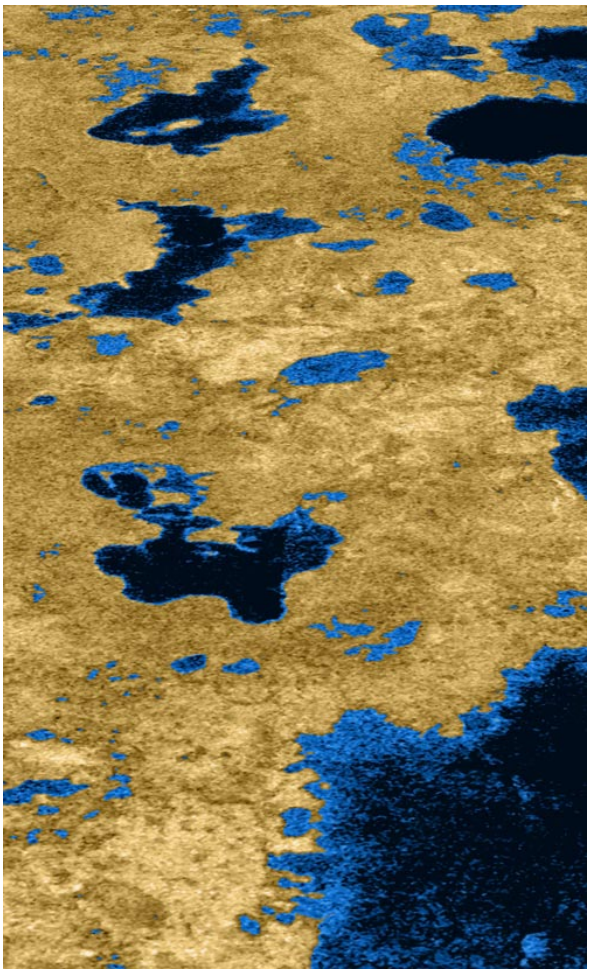


Problem 3 - The satellite of Pluto, called Charon, has been used to determine the total mass of Pluto. The mass determined was about 1.3 x 10^{22} kilograms. From clues in the image, calculate the volume of Pluto and determine the average density of Pluto. How does it compare to solid-rock (3000 kg/m^3), water-ice (917 kg/m^3)? Answer: From the image, Pluto is a sphere with a diameter of 2,300 km, so its volume will be V = 4/3 π $(1{,}250{,}000)^3$ = 8.2 x 10^{18} $meters^3$. Then its density is just D = M/V = (1.3 x 10^{22} kilograms)/ (8.2 x 10^{18} $meters^3$) so **D = 1,600 kg/m^3**. This would be about the density of **a mixture of rock and water-ice**.

Inquiry: Can you create a model of Pluto that matches its average density and predicts what percentage of rock and ice may be present?

Answer: We want to match the density of Pluto (1,600 km/m^3) by using ice (917 kg/m^3) and rock (2300 kg/m^3). Suppose we made Pluto out of half-rock and half-ice **by mass**. The volume this would occupy would be V = (0.5* 1.3 x 10^{22} kilograms/917 kg/m^3) = 7.1x10^{18} $meters^3$ for the ice, and V = (0.5* 1.3 x 10^{22} kilograms/3000 kg/m^3) = 2.2 x10^{18} $meters^3$ for the rock, for a total volume of 9.3 x 10^{18} $meters^3$ for both. This is a bit larger then the actual volume of Pluto (8.2 x 10^{18} $meters^3$) so we have to increase the mass occupied by ice, and lower the 50% by mass occupied by the rock component. The result, from student trials and errors should yield after a few iterations **about 40% ice and 60% rock.** This can be done very quickly using an Excel spreadsheet. For advanced students, it can also be solved exactly using a bit of algebra.

Lakes of Methane on Titan



A key goal in the search for life elsewhere in the universe is to detect liquid water, which is generally agreed to be the most essential ingredient for living systems that we know about.

The image to the left is a false-color synthetic radar map of a northern region of Titan taken during a flyby of the cloudy moon by the robotic Cassini spacecraft in July, 2006. On this map, which spans about 150 kilometers across, dark regions reflect relatively little of the broadcast radar signal. Images like this show Titan to be only the second body in the solar system to possess liquids on the surface. In this case, the liquid is not water but methane!

Future observations from Cassini during Titan flybys will further test the methane lake hypothesis, as comparative wind affects on the regions are studied.

Problem 1 – From the information provided, what is the scale of this image in kilometers per millimeter?

Problem 2 – What is the approximate total surface area of the lakes in this radar image?

Problem 3 - Assume that the lakes have an average depth of about 20 meters. How many cubic kilometers of methane are implied by the radar image?

Problem 4 – The volume of Lake Tahoe on Earth is about 150 km^3. How many Lake Tahoes-worth of methane are covered by the Cassini radar image?

Problem 1 – From the information provided, what is the scale of this image in kilometers per millimeter?

Answer: 150 km / 77 millimeters = **1.9 km/mm**.

Problem 2 – What is the approximate total surface area of the lakes in this radar image?

Answer: Combining the areas over the rectangular field of view gives about 1/4 of the area covered. The field of view measures 77 mm x 130mm or 150 km x 247 km or an area of 37,000 km^2. The dark areas therefore cover about 1/4 x 37,000 km^2 or **9,300 km^2.**

Problem 3 - Assume that the lakes have an average depth of about 20 meters. How many cubic kilometers of methane are implied by the radar image?

Answer: Volume = area x height = 9,300 km^2 x (0.02 km) = **190 km^3**.

Problem 4 – The volume of Lake Tahoe on Earth is about 150 km^3. How many Lake Tahoes-worth of methane are covered by the Cassini radar image?

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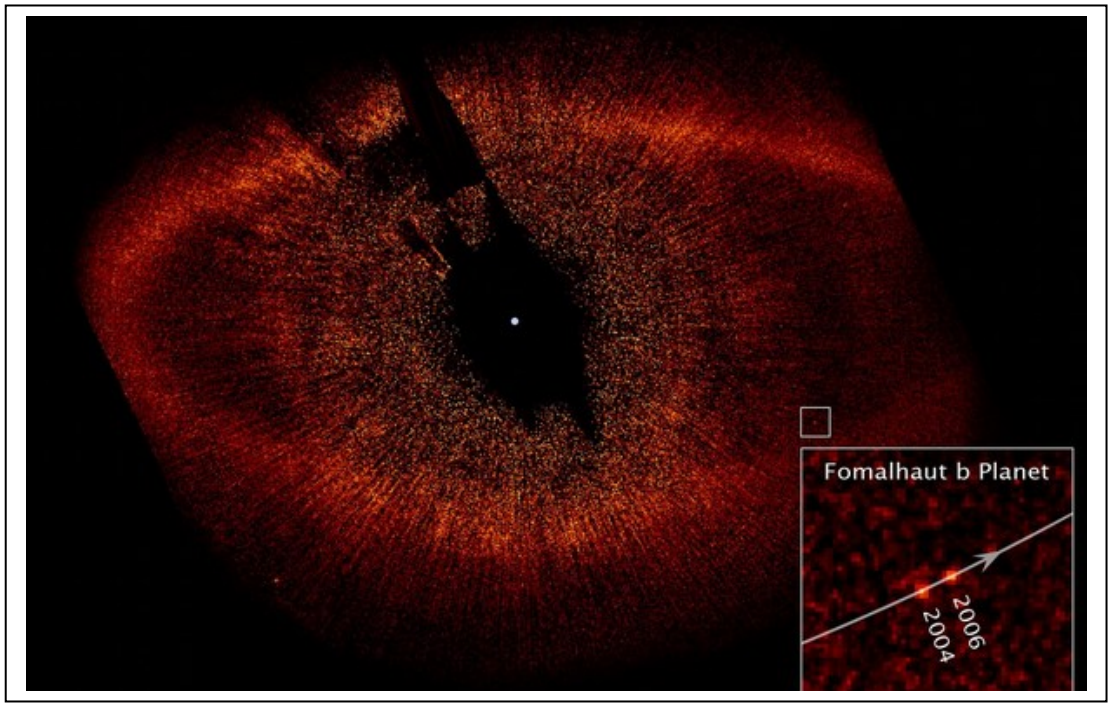
Answer: 190 km^3 / 150 km^3 = **1.3 Lake Tahoes**.

Hubble Sees a Distant Planet

The bright star Fomalhaut, in the constellation Piscis Austrinus (The Southern Fish) is only 25 light years away. It is 2000° K hotter than the Sun, and nearly 17 times as luminous, but it is also much younger: Only about 200 million years old. Astronomers have known for several decades that it has a ring of dust (asteroidal material) in orbit 133 AU from the star and about 25 AU wide. Because it is so close, it has been a favorite hunting ground in the search for planets beyond our solar system. In 2008 such a planet was at last discovered using the Hubble Space Telescope. It was the first direct photograph of a planet beyond our own solar system.

In the photo below, the dusty ring can be clearly seen, but photographs taken in 2004 and 2006 revealed the movement of one special 'dot' that is now known to be the star's first detected planet. The small square on the image is magnified in the larger inset square in the lower right to show the location of the planet in more detail.



Problem 1 – The scale of the image is 2.7 AU/millimeter. If 1.0 AU = 150 million kilometers, how far was the planet from the star in 2006?

Problem 2 – How many kilometers had the planet moved between 2004 and 2006?

Problem 3 – What was the average speed of the planet between 2004 and 2006 if 1 year = 8760 hours?

Problem 4 – Assuming the orbit is circular, with the radius found from Problem 1, about how many years would it take the planet to make a full orbit around its star?

Answer Key

Problem 1 – The scale of the image is 2.7 AU/millimeter. If 1.0 AU = 150 million kilometers, how far was the planet from the star in 2006?

Answer: The distance from the center of the ring (location of star in picture) to the center of the box containing the planet is 42 millimeters, then 42 x 2.7 AU/mm = 113 AU. Since 1 AU = 150 million km, the distance is 113 x 150 million **= 17 billion kilometers.**

Problem 2 – How many kilometers had the planet moved between 2004 and 2006?

Answer: On the main image, the box has a width of 4 millimeters which equals 4 x 2.7 = 11 AU. The inset box showing the planet has a width of 36 mm which equals 11 AU so the scale of the small box is 11AU/36 mm = 0.3 AU/mm. The planet has shifted in position about 4 mm, so this corresponds to 4 x 0.3 = **1.2 AU or 180 million km**.

Problem 3 – What was the average speed of the planet between 2004 and 2006 if 1 year = 8760 hours?

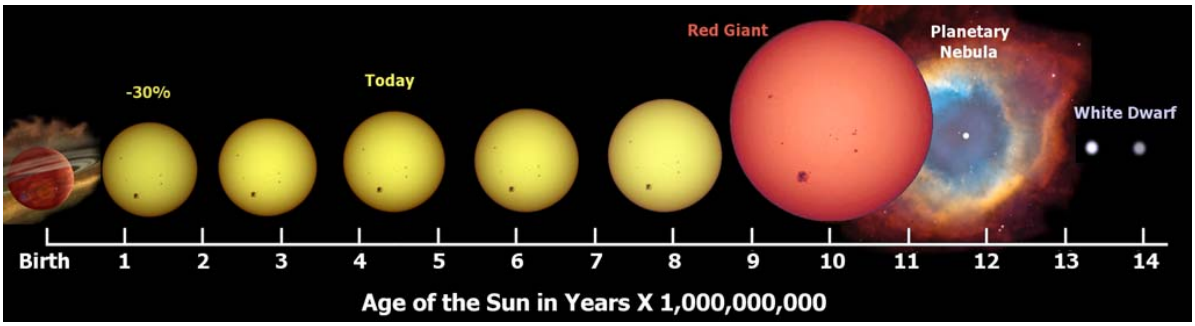
Answer: The average speed is 180 million km/17520 hours = **10,273 km/hr.**

Problem 4 – Assuming the orbit is circular, with the radius found from Problem 1, about how many years would it take the planet to make a full orbit around its star?

Answer: The radius of the circle is 113 AU so the circumference is $2 \pi R = 2 (3.141) (113 \text{ AU}) = 710$ AU. The distance traveled by the planet in 2 years is, from Problem 2, about 1.2 AU, so in 2 years it traveled 1.2/710 = 0.0017 of its full orbit. That means a full orbit will take 2.0 years/0.0017 = **1,176 years.**

Note - Because we are only seeing the 'projected' motion of the planet along the sky, the actual speed could be faster than the estimate in Problem 3, which would make the estimate of the orbit period a bit smaller than what students calculate in Problem 4.

A careful study of this system by its discoverer, Dr. Paul Kalas (UC Berkeley) suggests an orbit distance of 119 AU, and an orbit period of 872 years.



Our sun was formed 4.6 billion years ago. Since then it has been steadily increasing its brightness. This normal change is understood by astronomers who have created detailed mathematical models of the sun's complex interior. They have considered the nuclear physics that causes its heating and energy, gravitational forces that compress its dense core, and how the balance between these processes change in time. The diagram above shows the major stages in our sun's evolution from birth to end-of-life after 14 billion years. A simple formula describes how the power of our sun changes over time:

$$L = \frac{L_0}{1 + \frac{2}{5}(1 - x)}$$

where $x = t/t_0$ t_0 = 4.6 billion yrs, L_0 = 1.0 for the luminosity of the sun today.

Problem 1 – Graph the function L(x) for the age of the sun between 0 and 6 billion years

Problem 2 – By what percentage will L increase when it is 2 billion years older than it is today?

Problem 3 – A simple formula for the temperature, in kelvins, of Earth is given by:

$$T = 284[(1 - A)L]^{\frac{1}{4}}$$

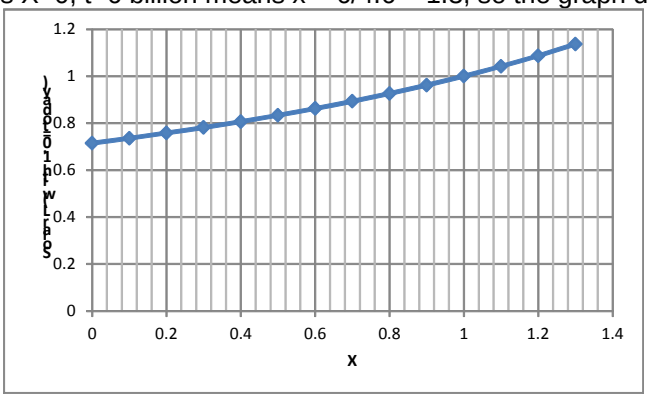
where L is the solar luminosity (today L=1.0), and A is the surface albedo, which is a number between 0 and 1, where asphalt is A=0 and A=1.0 is a perfect mirror A) What is the estimated current temperature of Earth if its average albedo is 0.4? B) What will be the estimated temperature of Earth when the sun is 5% brighter than today assuming that the albedo remains the same?

Problem 4 - Combine the two formulae above to define a new formula that gives Earth's temperature in kelvins only as a function of time, t, and albedo, A.

Problem 5 – If the albedo of Earth increases to 0.6, what will be the age of the sun when Earth's average temperature reaches 150° F (339 kelvins)? (Note: it is currently 60° F)

$$L = \frac{L_0}{1+\frac{2}{5}(1-x)}$$

Problem 1 – Graph the function L(x) for the age of the sun between 0 and 6 billion years.
Answer: t = 0 means X=0, t=6 billion means x = 6/4.6 = 1.3, so the graph domain is [0,1.3]



Problem 2 - By what percentage will L increase when it is 2 billion years older than it is today?

Answer: X = (4.6 + 2.0)/4.6 = 1.43, then L = 1 / (1+0.4(1-1.43)) so L = 1.20 this is **20% brighter** than today.

Problem 3 – A) What is the estimated current temperature of Earth if its average albedo is 0.4? B) What will be the estimated temperature of Earth when the sun is 5% brighter than today assuming that the albedo remains the same?

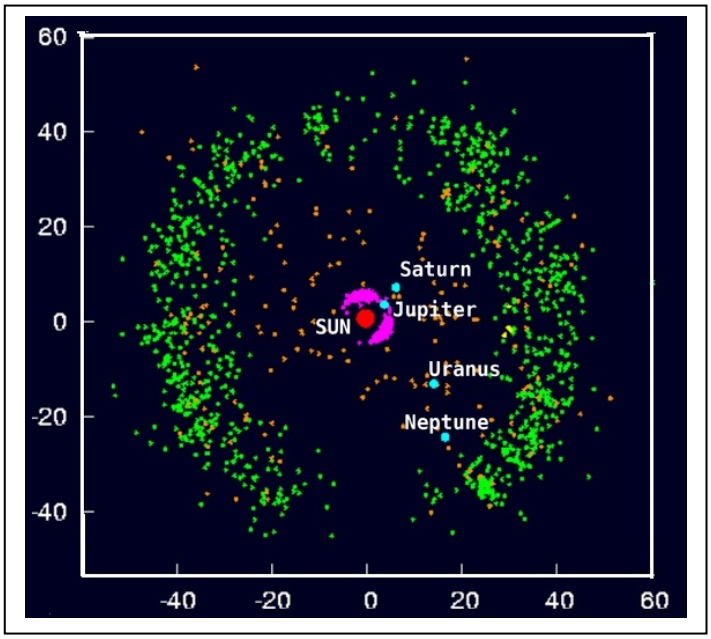
Answer: A) L = 1.0 today so $T = 284((1-0.4)x1.0)^{1/4}$ = **250 kelvins** (or -23° Celsius)
B) L = 1.05 so $T = 284(0.6x1.05)^{1/4}$ = **253 kelvins** (or -20° Celsius)

Problem 4 - Combine the formulae for L(x) and T to define a new formula, T(x,A) that gives Earth's temperature only as a function of time, x, and albedo, A, and assumes that L_0=1.0 today.

$$T(x,A) = 425\left(\frac{1-A}{7-2x}\right)^{\frac{1}{4}}$$

Problem 5 – If the albedo of Earth increases to 0.6, what will be the age of the sun when Earth's average temperature reaches 100° F (310 kelvins)? (Note: it is currently 60° F)

Answer: $310 = 425\ (0.4/(7-2x))^{1/4}$
$0.4\ (425/310)^4 = 7-2x$
$1.4 = 7 - 2x$
$2x = 5.6$ so $x = 2.8$
and the age of the sun will be t = 2.8 x 4.6 billion = **12.9 billion years**.
This occurs 12.9 – 4.6 = 8.3 billion years in the future.



Since the belt was discovered in 1992, the number of known Kuiper belt objects (KBOs) has increased to over a thousand, and more than 100,000 KBOs over 100 km (62 mi) in diameter are believed to exist.

Pluto is the largest known member of the Kuiper belt. Originally considered a planet, Pluto's status as part of the Kuiper belt caused it to be reclassified as a "dwarf planet" in 2006

This figure shows the locations of the known KBOs with the X and Y positions given in terms of Astronomical Units (AUs), where 1 AU equals the distance from Earth to the Sun (93 million miles or 149 million km).

Problem 1 – The Kuiper Belt stretches from 30 to 60 AU and has a torus shape. What is the volume of the Kuiper Belt in cubic kilometers if the volume of a torus is given by

$$V = 2\pi^2 r^2 R,$$

where R is the Kuiper Belts average distance from the sun and r is its radius?

Problem 2- It is estimated that over 100,000 objects larger than 100 km reside in the Kuiper Belt, of which 1,200 have been discovered by 2013. What is the density of the estimated 100,000 objects in objects/km^3 if they are uniformly distributed throughout the toroidal volume of the Kuiper Belt?

Problem 3 – Based upon the average density calculated in Problem 2, about what is the average distance between the Kuiper Belt Objects compared to the distance between Earth and Sun?

Problem 1 – The Kuiper Belt stretches from 30 to 60 AU and has a torus shape. What is the volume of the Kuiper Belt in cubic kilometers if the volume of a torus is given by $V = 2\pi^2 r^2 R$, where R is the Kuiper Belts average distance from the sun and r is its radius?

Answer: R = (60+30)/2 = 45 AU or $45 \times 149 \times 10^6$ km = 6.7×10^9 km
r = (60-30)/2 = 15 AU or $15 \times 149 \times 10^6$ km = 2.2×10^9 km

$V = 2\ (3.141)2\ (2.2 \times 10^9)^2\ (6.7 \times 10^9) =$ **6.4×10^{29} km^3**

Problem 2- It is estimated that over 100,000 objects larger than 100 km reside in the Kuiper Belt, of which 1,200 have been discovered by 2013. What is the density of the estimated 100,000 objects in objects/km3 if they are uniformly distributed throughout the toroidal volume of the Kuiper Belt?

Answer: $N = 10^5 / 6.4 \times 10^{29}$ km^3 = **1.6×10^{-25} objects/km^3**.

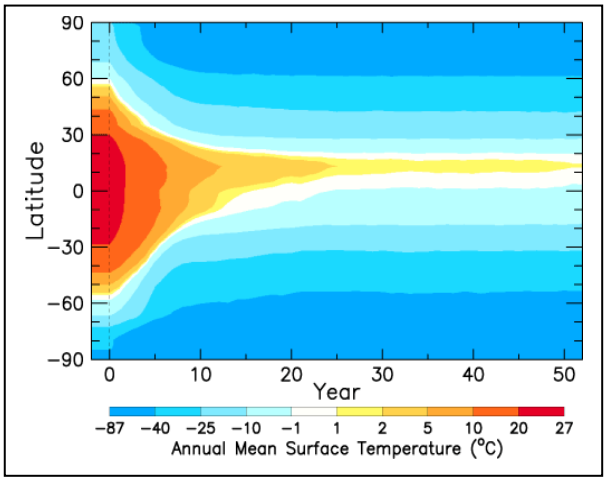
Problem 3 – Based upon the average density calculated in Problem 2, about what is the average distance between the Kuiper Belt Objects compared to the distance between Earth and Sun?

Answer: We just need to calculate the cube root of the density to get the reciprocal of this distance:

$D = 1 / (1.6 \times 10^{-25})^{1/3}$ = **183 million kilometers**.

Since the Earth-Sun distance is 149 million kilometers, the average distance between KBOs is about 1.2 AU or **1.2 times the Earth-Sun distance**.

The Temperature of Earth without Carbon Dioxide



A computer model developed by NASA scientists at the Goddard Institute for Space Science shows that without carbon dioxide, the terrestrial greenhouse would collapse and plunge Earth into an icebound state. Today, the average temperature is +15°C. Within 50 years the average temperature would drop to -21°C without the warming provided by atmospheric carbon dioxide. The delicate link between the planet's temperature and carbon dioxide has also been proved by geologic records of CO_2 levels during ice ages and interglacial periods. The temperature difference between an ice age period and an interglacial period is only 5°C. During previous ice ages, CO_2 levels were near 180 parts per million (ppm). During the warm interglacial periods the levels were near 280 ppm. Today we are living in an interglacial period that started 12,000 years ago and may last another 40,000 years. Scientists continue to worry that, as CO_2 levels approach 400 ppm, we are in uncharted territory with no historical precedent as far back as 1 million years.

Although there is no known process that would instantly remove all CO_2 from the atmosphere, this computer model is important for another reason. It helps us predict how warm a planet would be if it had no greenhouse gases, even though it is close to its star.

Problem 1 - The surface area of the Earth above a latitude of θ degrees is given by

$$A = 4\pi R^2(1-\sin\theta)$$

From the computer model, after how many years will exactly half of the surface of Earth be covered by ice caps where T < 0°C?

Problem 2 – The albedo of Earth is a number between 0 and 1 that indicates how much sunlight it reflects back into space. The higher the albedo, the more light is reflected back into space and the less heating occurs. An albedo of A = 1.0 is a perfect mirror so that all sunlight is reflected and none is absorbed to heat the planet. An albedo of A=0 is similar to asphalt and reflects no light back into space and absorbs all the light energy to heat the planet. Ice has an albedo of A=0.7 and ocean water has A=0.2. After how many years will its albedo increase to 0.6 according to the computer models?

http://www.giss.nasa.gov/research/news/20101014/
How Carbon Dioxide Controls Earth's Temperature
October 14, 2010

Problem 1 - The surface area of the Earth above a latitude of θ degrees is given by

$$A = 4\pi R^2(1-\sin\theta)$$

From the computer model, after how many years will exactly half of the surface of Earth be covered by ice caps where T < 0°C?

Answer: From the formula, we need $A/4\pi R^2 = ½$
so $½=1-\sin\theta$ and so $\theta = 30^o$ latitude

From the model, the zone where T = -1°C to +1° C reaches a latitude of 30° occurs after a time of **5 years.**

Problem 2 – The albedo of Earth is a number between 0 and 1 that indicates how much sunlight it reflects back into space. The higher the albedo, the more light is reflected back into space and the less heating occurs. An albedo of A = 1.0 is a perfect mirror so that all sunlight is reflected and none is absorbed to heat the planet. An albedo of A=0 is similar to asphalt and reflects no light back into space and absorbs all the light energy to heat the planet. Ice has an albedo of A=0.7 and ocean water has A=0.2. After how many years will its albedo increase to 0.6 according to the computer models?

Answer: The average albedo is found by averaging the albedo of the area covered by the ice caps (A=0.7) with the albedo of the area covered by the ocean (a=0.2).

The area covered by ice caps = $4\pi R^2(1-\sin\theta)$
The remaining area covered by water = $4\pi R^2 - 4\pi R^2(1-\sin\theta) = 4\pi R^2(\sin\theta)$

$$0.6\ (4\pi R^2) = 0.7\ (4\pi R^2)(1-\sin\theta) + 0.2(4\pi R^2)[\sin\theta]$$

Simplifying: $0.6 = 0.7(1-\sin\theta) + 0.2\sin\theta$
$0.6 = 0.7 - 0.7\sin\theta + 0.2\sin\theta$
$-0.1 = -0.5\sin\theta$
$\sin\theta = 0.2$
So $\theta = 12^o$

So, when the zone -1°C < T < +1°C reaches a latitude of +12°, the albedo of Earth will have increased from its current value of 0.4 to a much higher reflectivity of 0.6. According to the model graph, this happens after 10 years.

Note: Although CO_2 loss is an important cooling process, the rapid increase in albedo is a significant cause for cooling and an important 'feed back' in the process. As the planet cools, more ice appears and the albedo increases, causing more cooling and more ice to appear…

Curiosity Heads for Mount Sharp

In July 2013, Curiosity began its long journey to the base of Mt Sharp, seen in the distance in this image. Because it is operated robotically, it only travels a few meters every hour and communicates with its Earth technicians after each step. Because radio waves take 20 minutes or longer to reach earth, 40 minutes elapse before a transmitted command is received and the results of the action can be verified.

The table below gives the progress made by Curiosity during several days of operation on Mars, called Sols.

Sol	Drive	Duration (minutes)	Odometer (meters)	Azimuth (degrees)	Pitch (degrees)
345	68	83	1490	235	-1
347	69	67	1550	236	-2
349	70	72	1621	190	+1
351	71	94	1706	249	+1
354	72	65	1763	304	+1

Problem 1 – What is the average time that Curiosity drove each day?

Problem 2 – What is the average distance traveled each day?

Problem 3 – 'Azimuth' is the direction you are pointing from North so that due North is 0°, East is 90°, South is 180° and West is 270°. What is the average azimuth angle that Curiosity traveled along during the tabulated period?

Problem 4 – 'Pitch' is the tilt angle of the land, with straight up being +90°, horizontal being 0° and straight down being -90°. What is the average pitch of Curiosity's travels during this period and what can you tell about the ground over which it traveled?

Problem 5 – The direction to Mt Sharp is at an azimuth of 225°. What does Curiosity's average azimuth have to be during the next 5 days so that it is back on course to Mt Sharp?

Problem 6 – Mt Sharp is located 7.5 km from Curiosity. About how many more Sols will be required for Curiosity to get there?

Problem 1 – What is the average time that Curiosity drove each day?

Answer: T = (83+67+72+94+65)/5 = 381 minutes/5 = **76 minutes/day**

Problem 2 – What is the average distance traveled each day?

Answer: D = (60 + 71 + 85 + 57)/4 = 273 meters/4 = **68 meters/day**.

Problem 3 – 'Azimuth' is the direction you are pointing from North so that due North is 0°, East is 90°, South is 180° and West is 270°. What is the average azimuth angle that Curiosity traveled along during the tabulated period?

Answer: A = (235+236+190+249+304)/5 = 1214/5 = **243°**

Problem 4 – 'Pitch' is the tilt angle of the land, with straight up being +90°, horizontal being 0° and straight down being -90°. What is the average pitch of Curiosity's travels during this period and what can you tell about the ground over which it traveled?

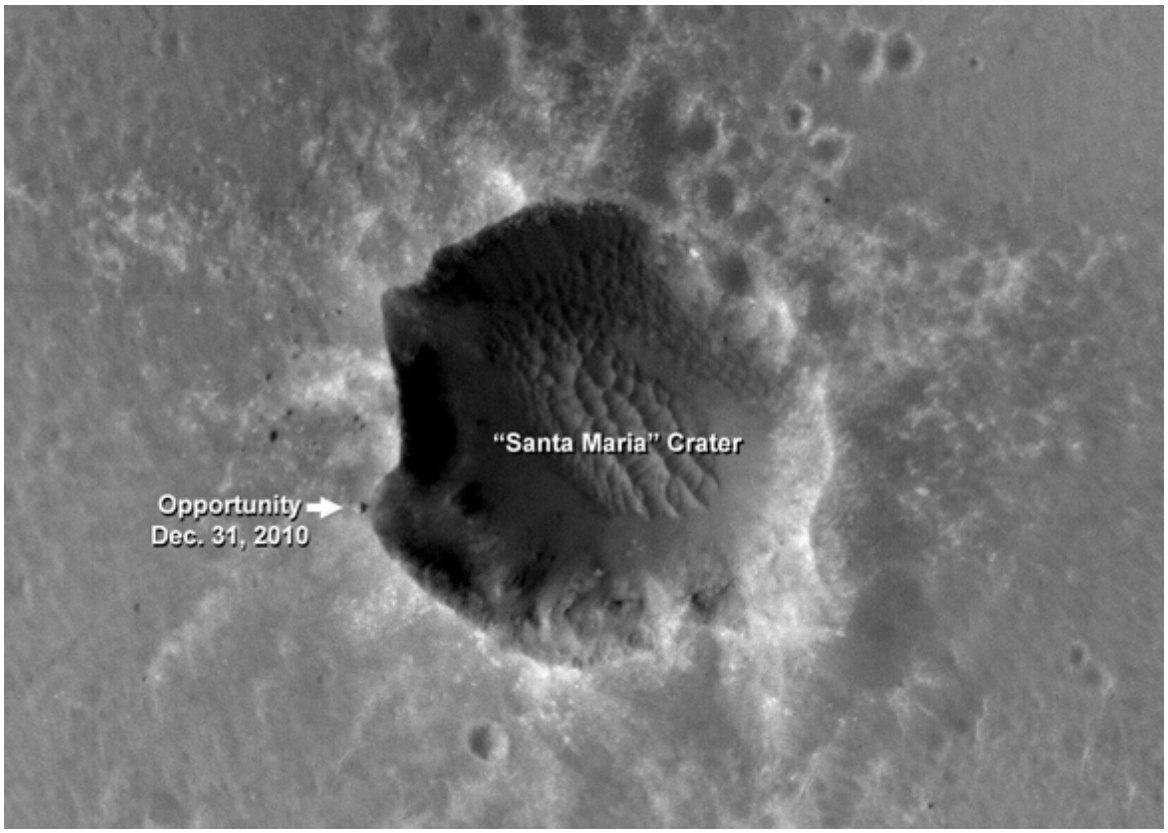
Answer: P = (-1 + -2 + +1 + +1 + +1)/5 = 0° So the terraine was very level and horizontal.

Problem 5 – The direction to Mt Sharp is at an azimuth of 225°. What does Curiosity's average azimuth have to be during the next 5 days so that it is back on course to Mt Sharp?

Answer: 225 = [5 (243) + 5(X)] / 10
2250 – 1214 = 5X so **X = 207°**

Problem 6 – Mt Sharp is located 7.5 km from Curiosity. About how many more Sols will be required for Curiosity to get there?

Answer: The average distance traveled was 68 meters/day so to travel 7500 meters will take T = 7500/68 = 110 Sols if Curiosity does not stop along the way.



The High Resolution Imaging Science Experiment (HiRISE) camera on NASA's Mars Reconnaissance Orbiter acquired this image of the Opportunity rover on the southwest rim of "Santa Maria" crater on New Year's Eve 2010. Opportunity arrived at the western edge of Santa Maria crater in mid-December and will spend about two months investigating rocks there. That investigation will take Opportunity into the beginning of its eighth year on Mars. Opportunity is imaging the crater interior to better understand the geometry of rock layers and the meteor impact process on Mars. Santa Maria is a relatively young, 90 meter-diameter impact crater, but old enough to have collected sand dunes in its interior.

Problem 1 - Using a millimeter ruler, what is the scale of this image in meters per millimeter to one significant figure?

Problem 2 - To one significant figure, about what is the circumference of the rim of this crater in meters?

Problem 3 - The rover can travel about 100 meters in one day. To one significant figure, how long will it take the rover to travel once around this crater?

Problem 4 - A comfortable walking speed is about 100 meters per minute. To one significant figure, how long would it take a human to stroll around the edge of this crater?

Problem 1 - Using a millimeter ruler, what is the scale of this image in meters per millimeter to one significant figure?

Answer: The shape of the crater is irregular, but taking the average of several diameter measurements with a ruler gives a diameter of about 55 millimeters. Since this corresponds to 90 meters according to the text, the scale of this image is about 90 meters/55 mm = 1.8 meters/millimeter, which to one significant figure becomes **2 meters/millimeter.**

Problem 2 - To one significant figure, about what is the circumference of the rim of this crater in meters?

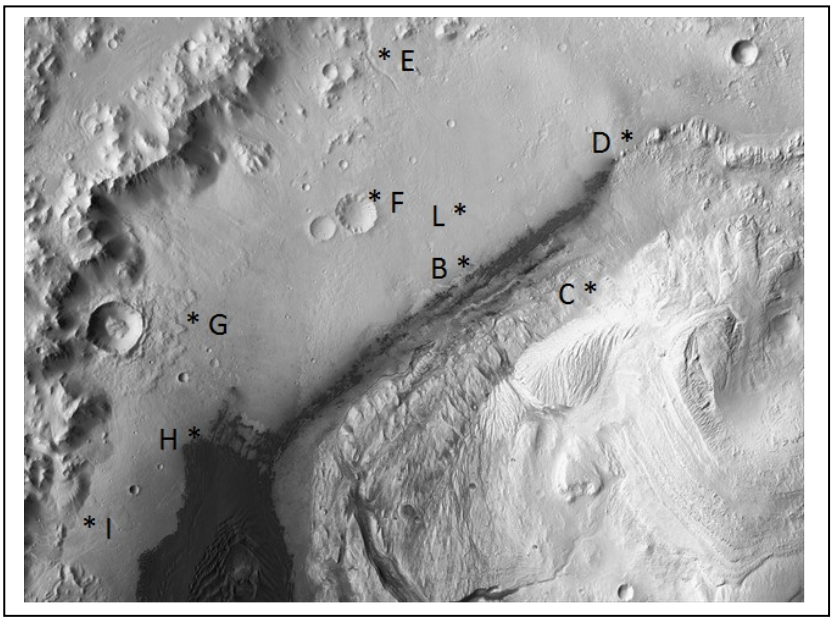
Answer: Students may use a piece of string and obtain an answer of about 200 millimeters. Using the scale of the image in Problem 1, the distance in meters is about 200 mm x 2 meters/mm = **400 meters**.

Problem 3 - The rover can travel about 100 meters in one day. To one significant figure, how long will it take the rover to travel once around this crater?

Answer: 400 meters x (1 day/100 meters) = **4 days**.

Problem 4 - A comfortable walking speed is about 100 meters per minute. To one significant figure, how long would it take a human to stroll around the edge of this crater?

Answer: 400 meters x (1 minute/100 meters) = **4 minutes**.



The table below gives the coordinates for the locations to be visited by the Curiosity Rover shown in the figure above. The X and Y coordinates are given in kilometers. Although Curiosity is free to travel between most points on the map, Point C is at a much higher elevation than the other points located in the crater floor, and a steep and impassible cliff wall exists between points B and C and runs diagonally to the lower left.

Label	Name	(X,Y)	Label	Name	(X,Y)
L	Landing Area	(45,40)	F	Crater Wall	(38,43)
B	Layered Wall	(50,35)	G	Mudslide	(17,30)
C	Alluvial Fan	(60,32)	H	Dark Sands	(17,19)
D	Summit Access	(65,50)	I	Mystery Valley	(5,10)
E	River Bed	(37,58)			

Problem 1 – Curiosity can travel at a top speed of 300 meters/hr. As soon as it lands, Curiosity will be instructed to travel to the highest priority location first, just in case the mission prematurely fails. To the nearest kilometer, what is the distance traveled, and to the nearest hour, how long will it take to travel between Point L and Point B?

Problem 2 – To the nearest kilometer, what is the distance from Point D to Point I, and to the nearest hour, how long will it take Curiosity to travel this far?

Problem 3 – One possible path Curiosity might take that connects all of the points is represented by the sequence L-B-D-C-D-E-F-B-G-H-I. To the nearest kilometer, what is the total distance traveled, and to the nearest tenth, how many days will this journey take?

Problem 1 – Curiosity can travel at a top speed of 300 meters/hr. As soon as it lands, Curiosity will be instructed to travel to the highest priority location first, just in case the mission prematurely fails. To the nearest kilometer, what is the distance traveled, and to the nearest hour, how long will it take to travel between Point L and Point B?

Answer: L (45,40) and B (50,35). Using the Pythagorean Theorem and distance formula for Cartesian points $D = ((50-45)^2 + (35-40)^2)^{1/2}$ = **7 km**. Traveling at 300 m/hr, this will take 7000m/300m = **23 hours**.

Problem 2 – To the nearest kilometer, what is the distance from Point D to Point I, and to the nearest hour, how long will it take Curiosity to travel this far?

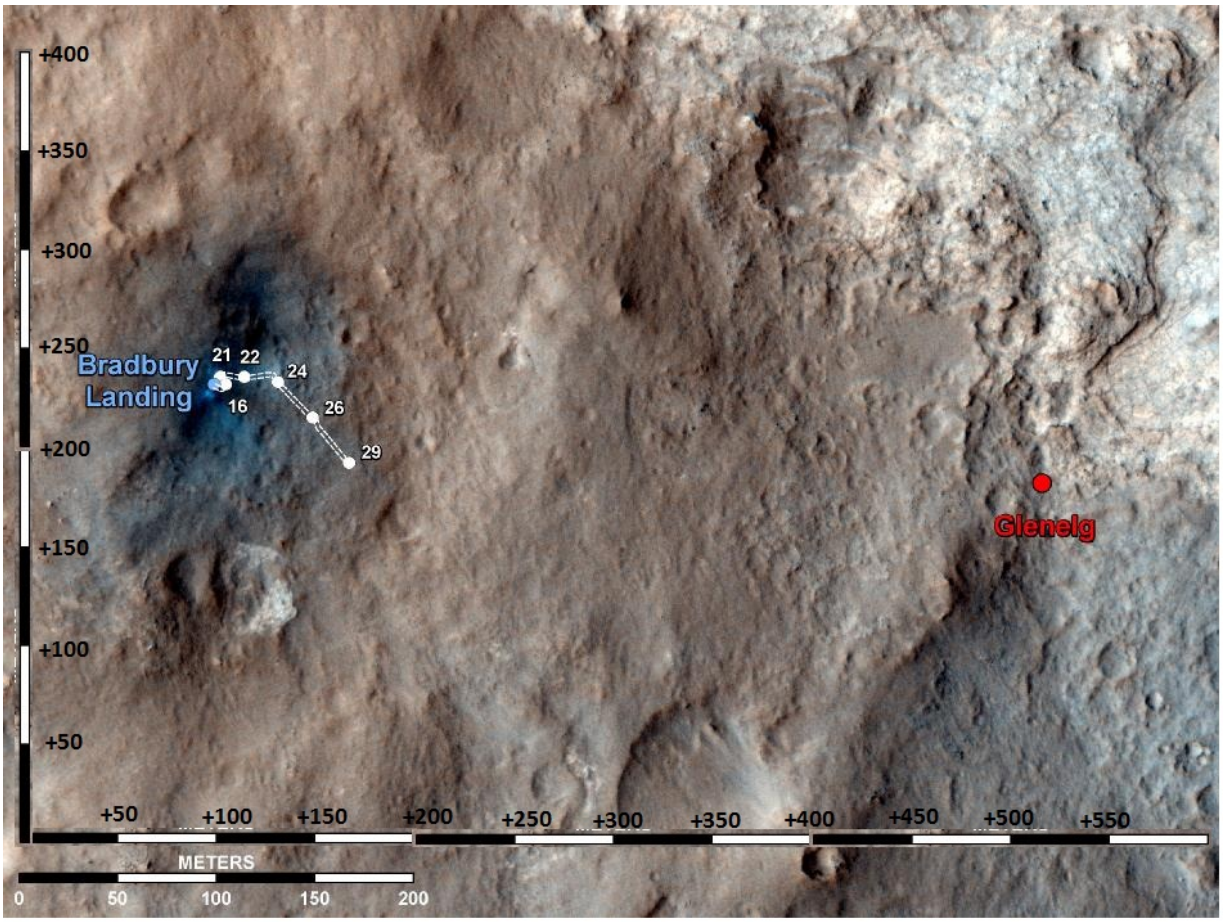
Answer: Point D (65,50), Point I (5,10). $D = ((5-65)^2 + (10-50)^2)^{1/2}$ = **72 kilometers**. Traveling at 300 m/hr, this takes 72000/300 = **240 hours** (or 10 days).

Problem 3 – One possible path Curiosity might take that connects all of the points is represented by the sequence L-B-D-C-D-E-F-B-G-H-I. To the nearest kilometer, what is the total distance traveled, and to the nearest tenth, how many days will this journey take?

$D(LB) = ((50-45)^2 + (35-40)^2)^{1/2} = 7$
$D(BD) = ((65-50)^2 + (50-35)^2)^{1/2} = 21$
$D(DC) = ((60-65)^2 + (32-50)^2)^{1/2} = 19$
$D(CD) = ((65-60)^2 + (50-32)^2)^{1/2} = 19$
$D(DE) = ((37-65)^2 + (58-50)^2)^{1/2} = 29$
$D(EF) = ((38-37)^2 + (43-58)^2)^{1/2} = 15$
$D(FB) = ((50-38)^2 + (35-43)^2)^{1/2} = 14$
$D(BG) = ((17-50)^2 + (30-35)^2)^{1/2} = 33$
$D(GH) = ((17-17)^2 + (19-30)^2)^{1/2} = 11$
$D(HI) = ((5-17)^2 + (10-19)^2)^{1/2} = 15$

Total distance traveled = **183 km**. Time = 183,000/300 = 610 hours = **25.4 days**

The Curiosity Rover on the Move!



The Curiosity Rover on Mars landed at Bradbury Station on Day 0 (Called Sol 0) and is headed for an important geological site called Glenelg. This map shows the location of the Rover until Sol 29. Also shown on the map is a coordinate grid marked in intervals of 50-meters. Bradbury Station is located at approximately (+100, +230). The table below gives the location of Curiosity for the period from Sol 29 to Sol 56. Students should use the distance formula to determine interval lengths: $d^2 = (x2-x1)^2 + (y2-y1)^2$ but they may also use millimeter rulers and the image scale to determine the distances between the points.

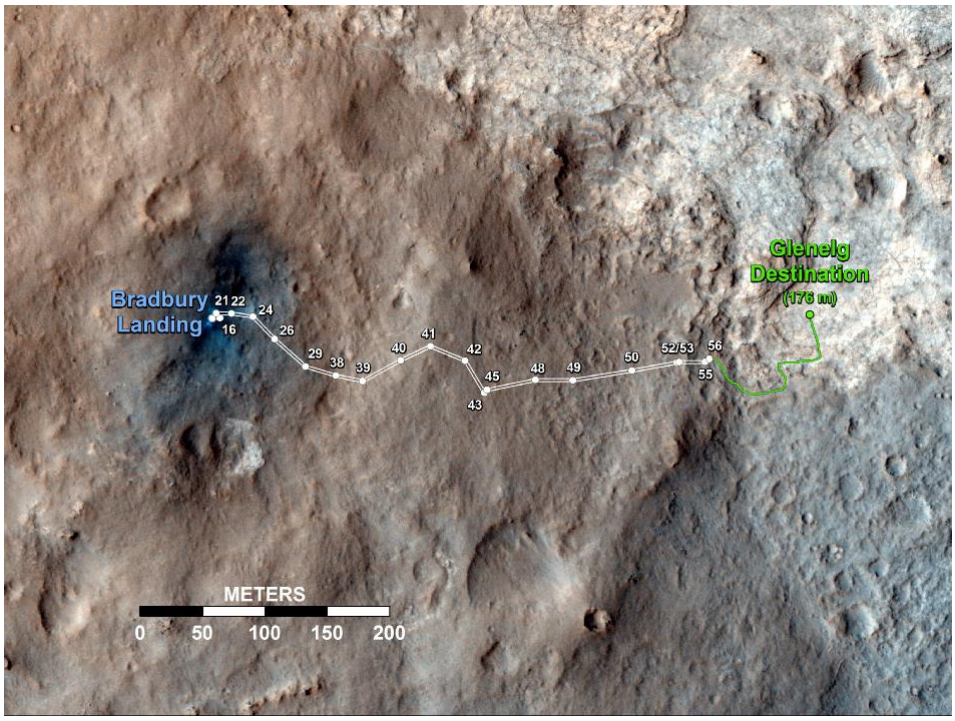
Day	X	Y	Day	X	Y
39	+210	+180	48	+360	+175
41	+270	+210	49	+390	+180
42	+300	+200	52	+470	+200
45	+315	+165	56	+500	+205

Problem 1 – Graph the additional points and connect them with line segments to show Curiosity's path across the martian landscape.

Problem 2 – During which segment was Curiosity traveling the fastest?

Problem 3 – During which segment was Curiosity traveling the slowest?

Problem 4 – What has been the average speed of Curiosity between Sol 39 and Sol 56?



Problem 1 – Graph the additional points and connect them with line segments to show Curiosity's path across the martian landscape. Answer: **See actual course above**.

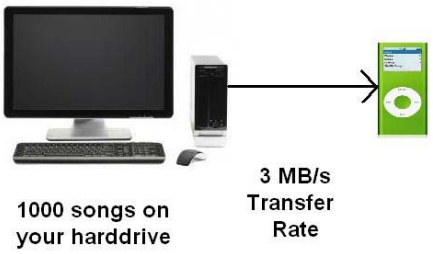
Day	X	Y	Segment Time (days)	Segment Distance (m)	Segment Speed (m/d)
39	+210	+180			
41	+270	+210	2	67	34
42	+300	+200	1	32	32
45	+315	+165	3	38	13
48	+360	+175	3	46	15
49	+390	+180	1	30	30
52	+470	+200	3	82	27
56	+500	+205	4	30	8

Example: Day 45 - Day 42 = 3 days. $D^2 = (315-300)^2 + (165-200)^2 = 1450$ so d = 38 meters and speed = 38meters/3days = 13 meters/day.

Problem 2 – During which segment was Curiosity traveling the fastest? **Between Sol 41 and Sol 42 at a speed of 34 meters per day.**

Problem 3 – During which segment was Curiosity traveling the slowest? **Between Sol 52 and Sol 56.**

Problem 4 – What has been the average speed of Curiosity between Sol 39 and Sol 56?
Total segment distance traveled = (67+32+38+46+30+82+30)=325 meters in 17 days
So **average speed = 19 meters/day.**



Transferring digital data from place to place takes time. Like water flowing into a lake, the faster it flows the more rapidly the lake fills up and overflows. With computer data, we have a similar problem. You have probably had to do this yourself many times. Each time you copy your playlist from your PC to your portable music player, you will have to wait a certain length of time. The transfer rate is fixed, so the more songs you want to transfer the longer you have to wait. Here's how this works!

Problem 1 - Suppose you want to transfer 1000 songs from your PC collection to your music Player. Each 4-minute song takes up 4 megabytes on the PC, and the cable link from your computer to your Player can handle a transfer rate of 3 million bytes/second. How many minutes does it take to transfer all your songs to the Player?

Imagine a lake fed by one large slow-moving river that brings water to it, and a second small, fast-moving river that takes water from the lake. If the rates at which the water enters and leaves the lake are not in step, the lake's water level will overflow. The InSight lander has a similar problem. It is gathering data at one rate, but transmitting it to Earth at another rate. We don't want to lose any of the data, so the data has to be stored in a memory device called a buffer.

InSight has two instruments that generate constant streams of digital data. The SEIS seismometer produces 48 megabytes/hr and the HP3 produces 2 megabytes/hr. This data is stored in a 500 megabyte buffer. Every 2 hours, the data in the buffer is transmitted to Earth at a rate of 4 megabytes/sec.

Problem 2 - How long will it take to fill up the buffer with data?

Problem 3 - How long will be required to transmit the buffer data to Earth during each 2-hour transmission cycle?

Problem 4 – The receiver on Earth can be scheduled to contact the Lander as often as once every 2 hours. How large a buffer would you need so that you could gather as much data as 4 megabytes/sec over 2 hours? How long does it take the instruments to gather this much data?

Problem 1 - You want to transfer 1000 songs from your PC collection to your music Player. Each 4-minute song takes up 4 megabytes, and the cable link from your computer to your Player can handle a transfer rate of 3 million bytes/second. How many minutes does it take to transfer all your songs?

Answer: 4000 megabytes x (1 second/ 3 megabytes) = 1333 seconds or about **22 minutes**.

The InSight lander has two instruments that generate constant streams of digital data. The SEIS seismometer produces 48 megabytes/hr and the HP3 produces 2 megabytes/hr which is stored in a 500 megabyte digital memory called a buffer. Every 2 hours, the data in the buffer is transmitted to Earth at a rate of 4 megabytes/sec.

Problem 2 - How long will it take to fill up the buffer with data?

Answer: The data enters the buffer at 50 megabytes/hr and the buffer contains 500 megabytes, so it can store data for 500 MBytes/(50Mbytes/hr) = **10 hours**.

Problem 3 - How long will be required to transmit the buffer data to Earth during each 2-hour transmission cycle?

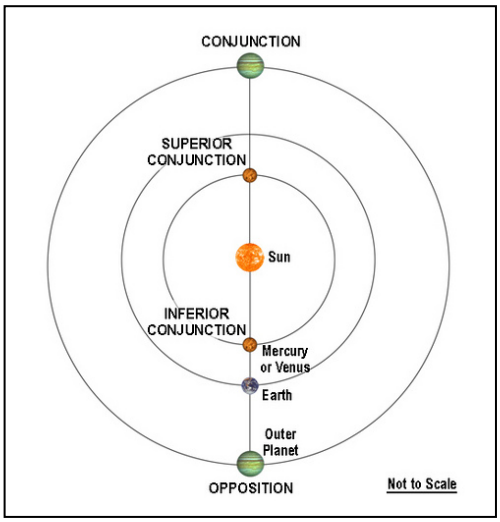
Answer: In 2 hours at a data rate of 50 megabytes/hr you have 100 megabytes stored in the buffer. At a transmission rate of 4 megabytes/sec it takes 100 Mbytes/(4 Mbytes/sec) = **25 seconds** to transmit the100 megabytes from the buffer to Earth.

Problem 4 – The receiver on Earth can be scheduled to contact the Lander as often as once every 2 hours. How large a buffer would you need so that you could gather as much data as 4 megabytes/sec over 2 hours? How long does it take the instruments to gather this much data?

Answer: 2 hours equals 2 x 3600 = 7200 seconds. At a transmission rate of 4 Mbytes/sec, this equals 7200 sec x 4 Mbytes/sec = 28,800 Megabytes or **28.8 Gigabytes.**

The instruments gather 50 Megabytes/hour, so it would take them 28,800 Megabytes/(50 MB/hr) = 576 hours or 24 days to gather this much data.

What this says is that if you had a buffer this large (28.8 gigabytes) it could store 24 days of data from InSight and only take 2 hours to transmit to Earth. The danger of waiting so long to transmit data (every 24 days) is that something could happen to the lander and you would lose all this data! That's why scientists try to download their data as often as possible.



The InSight Lander needs to be in constant communication with Earth every day the data it gathers can be sent back to Earth. As viewed from the surface of Mars, Earth never gets very far from the sun. Over the course of about 780 days, Earth travels from its farthest westward position in the sky (morning star) of 46^{o} W, to its farthest eastward position (evening star) of 41^{o} E and back in about 780 days.

When the Earth-Sun angle is near zero as viewed from Mars, Earth can either be between Mars and the sun (called Inferior Conjunction) or Earth can be on the opposite side of the sun as viewed from Mars (called Superior Conjunction).

When Mars and Earth are in inferior conjunction, Earth can receive signals from the Lander, but the Lander will have to broadcast its data almost directly at the sun, which is a hazard for the transmitter. When Earth and Mars are in superior conjunction, Neither InSight nor the Earth radar can transmit or receive data with Earth behind the sun.

Problem 1 – The following table gives the Earth-Sun angle viewed from Mars during the time InSight is operating on the martian surface. During this time, inferior conjunction occurred on May 22, 2016 and July 26, 2018, with superior conjunction on July 27, 2017. When will the next inferior conjunction occur after July 26, 2018?

Month	Angle	Month	Angle	Month	Angle	Month	Angle
9/16	+46	3/17	+24	9/17	-10	3/18	-40
10/16	+45	4/17	+18	10/17	-17	4/18	-41
11/16	+42	5/17	+13	11/17	-23	5/18	-37
12/16	+38	6/17	+7	12/17	-28	6/18	-27
1/17	+34	7/17	+1	1/18	-33	7/18	-7
2/17	+29	8/17	-5	2/18	-38		

Angular data obtained from *Eyes on the Solar System* (February 28, 2013).

Problem 2 – InSight will end its operations after one full martian year (687 days). If it lands on September 20, 2016, during which months of operation will Earth be within 10 degrees of the sun at conjunction, and unable to communicate with Earth?

Problem 3 - Graph the data in the table. During which months will the Earth-Mars angle A) be changing the most rapidly? B) the slowest?

Problem 1 – When will the next superior conjunction occur after July 26, 2018?

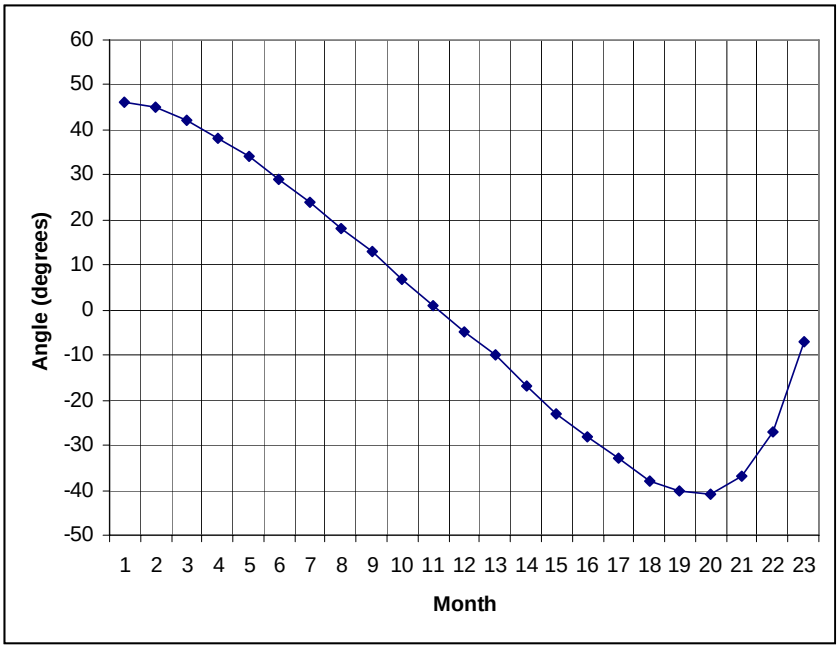
Answer: Each pair of superior and inferior conjunctions happen in cycles. The time for one cycle can be found by the time between the dates of the two listed inferior conjunctions on 5/22/2016 and 7/26/2018. Students can find the number of days between these dates manually (hard), or they can use an online calculator (fun!) like http://www.timeanddate.com/date/duration.html to get 796 days. To find the next superior conjunction, add 796 days to July 27, 2017 (eg. http://www.timeanddate.com/date/dateadd.html) to get **May 4, 2019**.

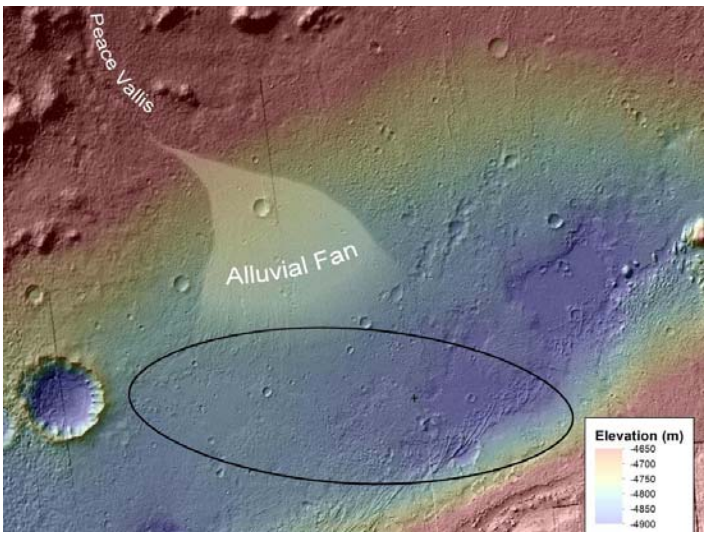
Problem 2 – InSight will end its operations after one full martian year (687 days). If it lands on September 20, 2016, during which months of operation will Earth be within 10 degrees of the sun at conjunction, and unable to communicate with Earth?

Answer: Conjunction occurs on July 27, 2017. From the table, Earth is within 10 degrees of the sun during the months of **June 2017 to August 2017** so it will be difficult to directly transmit or receive data during this time.

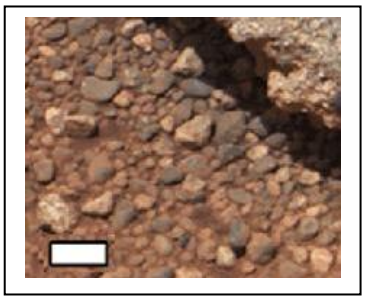
Problem 3 - Graph the data in the table. During which months will the Earth-Mars angle A) be changing the most rapidly? B) the slowest?

Answer: Students may use Microsoft Excel. X-axis may be the month number.
A) June-July, 2018.
B) During September-November, 2016 and February-April, 2018. Slow changes correspond to a nearly flat 'slope' while fast changes correspond to the steepest slope.





The Curiosity Rover discovered rounded pebbles near its original landing site marked with the 'X' in the figure. The figure also shows the elevation changes in this area. Here is what the pebbles looked like ! The white bar is 1 cm long.



Geologists studying the pebbles and the landscape believe that the water flow that moved and rounded the pebbles was at least ankle deep and perhaps waist deep. As on Earth, the pebbles were carried by fast moving water and over time became rounded by the constant scraping and bouncing. How fast was the water moving?

Calculating the stream gradient:

Problem 1 – The landing ellipse is 18 km wide. To the nearest kilometer, how far is the Curiosity 'X' from the apex of the alluvial fan near Peace Valllia?

Problem 2 – What is the change in elevation, h, between the alluvial fan vertex and the 'X'.?

Problem 3 – The stream gradient is defined as the elevation difference divided by the distance traveled. What is the stream gradient, SG, in units of meters/meters, for the water which left Peace Vallis and flowed down the alluvial fan?

Calculating the stream flow speed:

Problem 4 – On Mars, the stream flow can be approximated by $V = (2gh)^{1/2} \sin(\theta)$ where $\tan(\theta) = SG$, $g = 3.8$ meters/sec^2 is the acceleration of gravity on Mars, and h is the difference in elevation of the top and bottom of the stream. About how fast was the water flowing past the Curiosity landing area to create the pebbles?

Calculating the stream gradient:

Problem 1 – The landing ellipse is 18 km wide. To the nearest kilometer, how far is the Curiosity 'X' from the apex of the alluvial fan near Peace Valllia? Answer: **About 15 km**.

Problem 2 – What is the change in elevation, h, between the alluvial fan vertex and the 'X'.? Answer: -4650 m – (-4900 m) so h = **250 meters.**

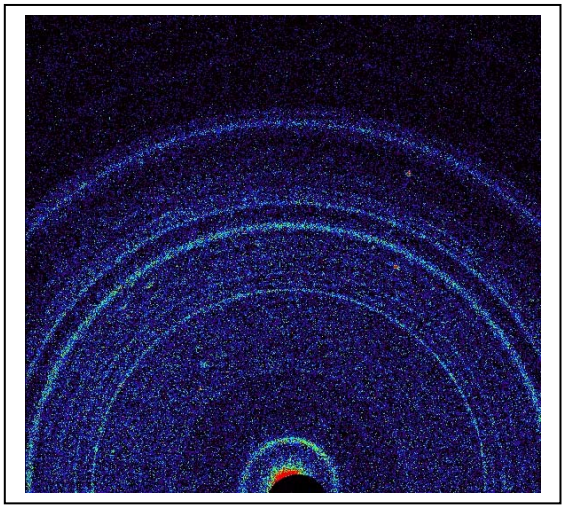
Problem 3 – The stream gradient is defined as the elevation difference divided by the distance traveled. What is the stream gradient, SG, in units of meters/meters, for the water which left Peace Vallis and flowed down the alluvial fan? 250 meters / 15 km = **17 meters/kilometer or SG=0.017 meters/meter.**

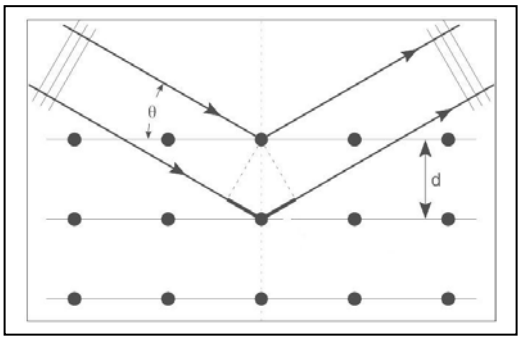
Calculating the stream flow speed:

Problem 4 – On Mars, the stream flow can be approximated by $V = (2gh)^{1/2} \sin(\theta)$ where $\tan(\theta) = SG$, and $g = 3.8$ meters/sec^2 is the acceleration of gravity on Mars. About how fast was the water flowing past the Curiosity landing area to create the pebbles?

Answer: h = 650 meters, SG=0.017, g = 3.8 meters/sec^2 . then q = 1.0 degrees.
$V = (2 \times 3.8 \times 650)^{1/2} \sin(1.0)$
V = 1.2 meters/sec.

This is about as fast as a human walking very slowly (about 0.3 miles/hr or 4.3 km/hr).





The Curiosity Rover recently use a technique called X-ray Diffraction Crystallography to determine the identity of compounds found in a rock sample on the surface of Mars. The image to the left shows what this data looks like. The exact radii of these rings, and the locations of spots along these rings, serve as a fingerprint of the shape of the mineral compound in space. We all know how human fingerprints work, and even 'DNA' fingerprinting is commonly mentioned in TV programs like NCIS or CSI. But how does this technique work?

The figure to the left shows a beam of light striking the surface of a crystal with 15 atoms arranged into three parallel planes. The light strikes the atoms and is 'defracted' into a new direction defined by the angle θ.

If two beams of light are out-of-phase by 90 degrees, when they are added together, the crests of one wave interfere with the troughs of the other wave and you end up with no light. If they are in-phase, they will add together, you get the light intensified and you also get a ring of light!

The diagram shows the added distance that the lower ray gains by being diffracted through the angle θ.

Problem 1 – From the information in the diagram, what is the extra distance, s, traveled by the x-ray light in a crystal lattice where the planes are separated by a distance d?

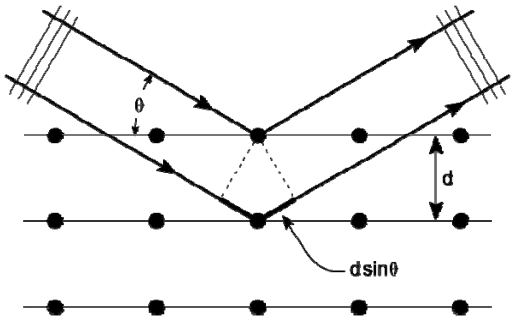
Problem 2 - If the light struck the crystal exactly face-on ($\theta=90^{o}$) how much extra distance would the second beam travel compared to the first beam that was reflected only from the top surface?

Problem 3 – If the wavelength of the x-ray light is L, what is the relationship between L and s so that the wave crests exactly match up?

Problem 4 – Suppose that in the Curiosity data, a diffraction ring is detected at an angle of incidence $\theta=2^{o}$. The Curiosity instrument uses X-rays with an energy of 6.929 keV, which have a wavelength of 1.79×10^{-10} meters. What is the separation, d, of the crystal planes in the mineral sample?

Problem 1 – From the information in the diagram, what is the extra distance, L, traveled by the x-ray light in a crystal lattice where the planes are separated by a distance d?

Answer: From the diagram below, **$s = 2d \sin\theta$**



Problem 2 - If the light struck the crystal exactly face-on ($\theta=90^{O}$) how much extra distance would the second beam travel compared to the first beam that was reflected from the top surface?

Answer: **s = 2d**

Problem 3 – If the wavelength of the x-ray light is L, what is the relationship between L and s so that the wave crests exactly match up?

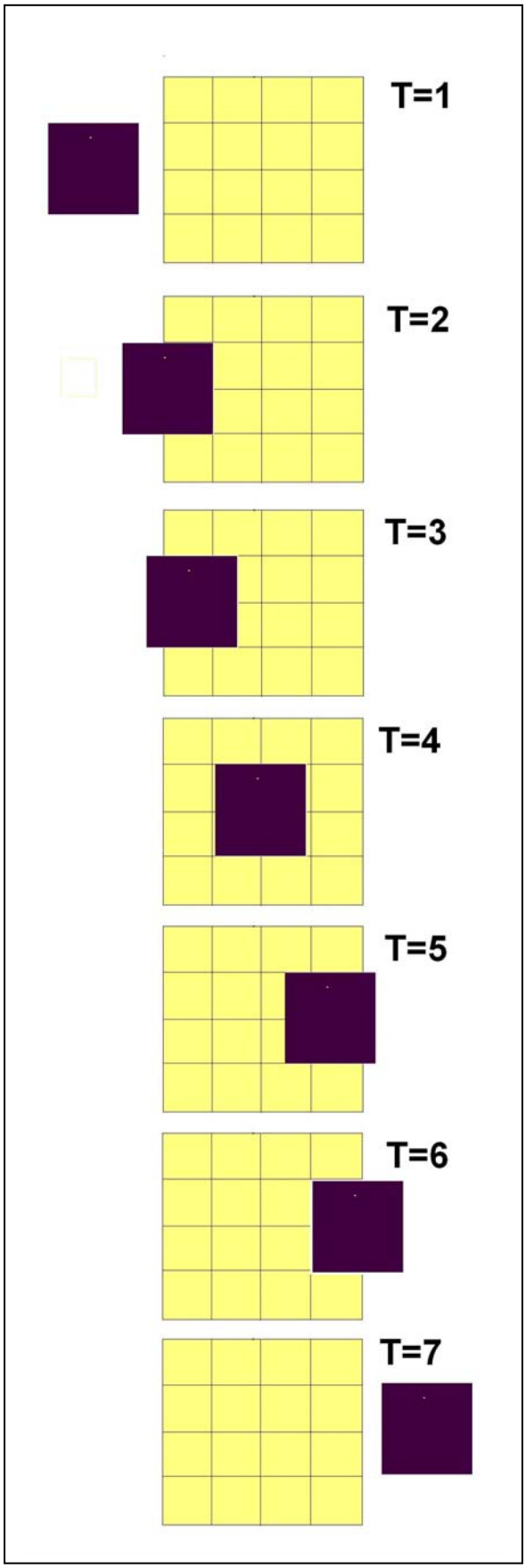
Answer: The wavelength is L and for the waves to exactly match up, the two waves can either be shifted by s=0, or by **one full wavelength** s = L. So the first non-zero condition is that L = 2 d sinθ and so

$$d = \frac{L}{2\sin\theta}$$

Problem 4 – Suppose that in the Curiosity data, a diffraction ring is detected at an angle of incidence $\theta=2^{O}$. The Curiosity instrument uses x-rays with an energy of 6.929 keV, which have a wavelength of 1.79×10^{-10} meters. What is the separation, d, of the crystal planes in the mineral sample?

Answer: $d = 1.79\times10^{-10}$ meters/$(2\sin2^{O})$ so d = **2.56×10^{-9} meters**.

Transits and Brightness Change

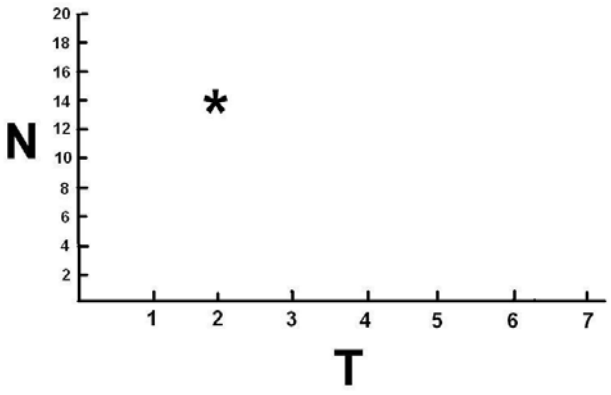


NASA's Kepler spacecraft recently announced the discovery of five new planets orbiting distant stars. The satellite measures the dimming of the light from these stars as planets pass across the face of the star as viewed from Earth. To see how this works, lets look at a simple model.

In the Bizarro Universe, stars and planets are cubical, hot spherical. Bizarro astronomers search for distant planets around other stars by watching planets pass across the face of the stars and cause the light to dim.

Problem 1 - The sequence of figures shows the transit of one such planet, Osiris (black). Complete the 'light curve' for this star by counting the number of exposed 'star squares' not shaded by the planet. At each time, T, create a graph of the number of star brightness squares. The panel for T=2 has been completed and plotted on the graph below.

Problem 2 - If you knew that the width of the star was 1 million kilometers, how could you use the data in the figure to estimate the width of the planet?



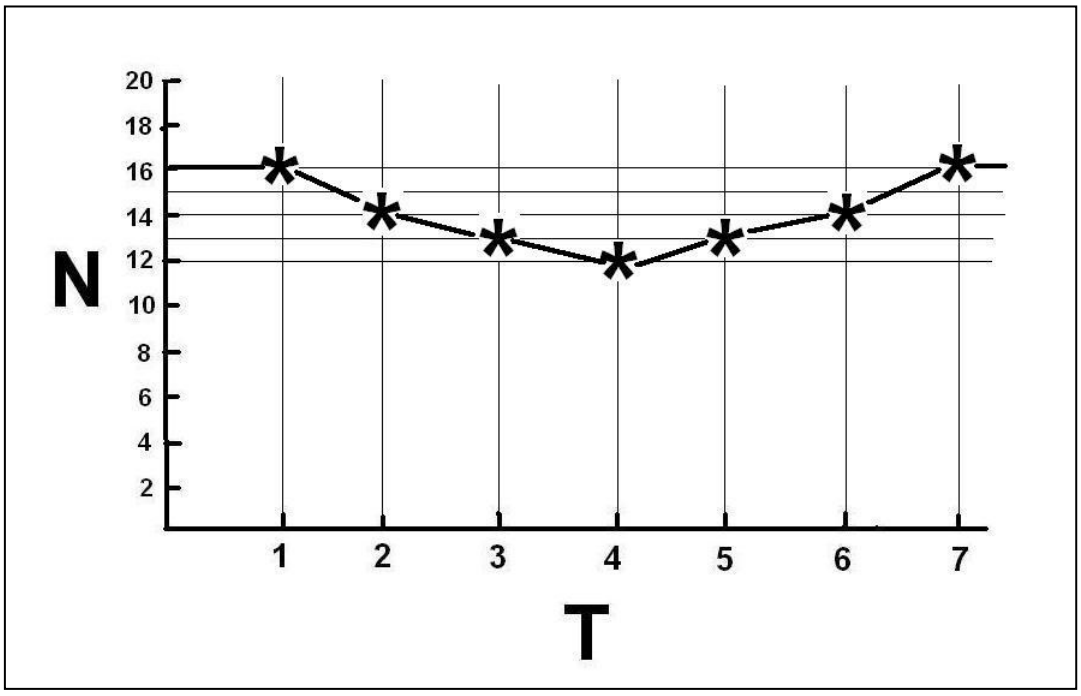
Problem 1 - The sequence of figures shows the transit of one such planet, Osiris (black). Complete the 'light curve' for this star by counting the number of exposed 'star squares' not shaded by the planet. At each time, T, create a graph of the number of star brightness squares. The panel for T=2 has been completed and plotted on the graph below.

Answer: **Count the number of yellow squares in the star and plot these for each value of T in the graph as shown below. Note, for T= 3 and 5, the black square of the planet occupies 2 full squares and 2 half squares for a total of 2 + 1/2 + 1/2 = 3 squares covered, so there are 16 - 3 = 13 squares remaining that are yellow.**

Problem 2 - If you knew that the width of the star was 1 million kilometers, how could you use the data in the figure to estimate the width of the planet?

Answer: The light curve shows that the planet caused the light from the star to decrease from 16 units to 12 units because the planet blocked 16-12 = 4 units of the stars surface area. That means that the planet squares occupy 4/16 of the stars area as seen by the astronomers. The area of the star is just the area of a square, so the area of the square planet is 4/16 of the stars area or Ap = 4/16 x Astar. Since the star as a width of Wstar = 1 million kilometers, the planet will have a width of $Wp = Wstar\sqrt{\frac{4}{16}}$ or **500,000 kilometers**.

The amount of star light dimming is proportional to the ratio of the area of the planet and the star facing the observer. The Kepler satellite can detect changes by as little as 0.0001 in the light from a star, so the smallest planets it can detect have diameters about 1/100 the size of the stars that they orbit. For a star with a diameter of the sun, 1.4 million kilometers, the smallest planet detectable by the Transit Method has a diameter about equal to 14,000 kilometers or about the size of Earth.



Period (days)	F	G	K
0-10	11	138	20
11-20	7	53	16
21-30	4	25	6
31-40	2	13	0
41-50	1	7	0
51-60	1	1	0
61-70	0	1	0
71-80	0	1	0
> 81	0	3	2
Total:	26	242	44

On June 16, 2010 the Kepler mission scientists released their first list of stars that showed evidence for planets passing across the faces of their stars. Out of the 156,097 target stars that were available for study, 52,496 were studied during the first 33 days of the mission. Their brightness was recorded every 30 minutes during this time, resulting in over 83 million high-precision measurements.

700 stars had patterns of fading and brightening expected for planet transits. Of these, data were released to the public for 306 of the stars out of a sample of about 88,000 target stars.

The surveyed stars for this study were distributed by spectral class according to F = 8000, G = 55,000 and K = 25,000. For the 306 stars, 43 were K-type, 240 were G-type, and 23 were K-type. Among the 312 transits detected from this sample of 306 stars, the table above gives the number of transits detected for each stellar type along with the period of the transit.

Problem 1 - Comparing the F, G and K stars, how did the frequency of the stars with transits compare with the expected frequency of these stars in the general population?

Problem 2 - The distance of the planet from its star can be estimated in terms of the orbital distance of Earth from our sun as $D^3 = T^2$ where D = 1.0 is the distance of Earth from the sun, and T is in multiples of 1 Earth Year. A) What is the distance of Mercury from our sun if its orbit period is 88 days? B) What is the range of orbit distances for the transiting planets in multiples of the orbit of Mercury if the orbit times range from 5 days to 80 days?

Problem 3 - As a planet passes across the star's disk, the star's brightness dims by a factor of 0.001 in brightness. If the radius of the star is 500,000 km, and both the planet and star are approximated as circles, what is the radius of the planet A) in kilometers? B) In multiples of Earth's diameter (13,000 km)?

Problem 1 - Comparing the F, G and K stars, how did the frequency of the stars with transits compare with the expected frequency of these stars in the general population?
Answer: Of the 88,000 stars F = 8000/88000 = 9%; G = 55000/88000 = 63% and K=25000/88000 = 28%.

For the 306 stars: F = 43/306 = 14%; G = 240/306 = 78% and K = 23/306 = 8%....so there were significantly fewer transits detected for K-type stars (8%) compared to the general population (28%).

Note: Sampling error accounts for s = $(306)^{1/2}$ = +/-17 stars or a +/- 6% uncertainty which is not enough to account for this difference in the K-type stars.

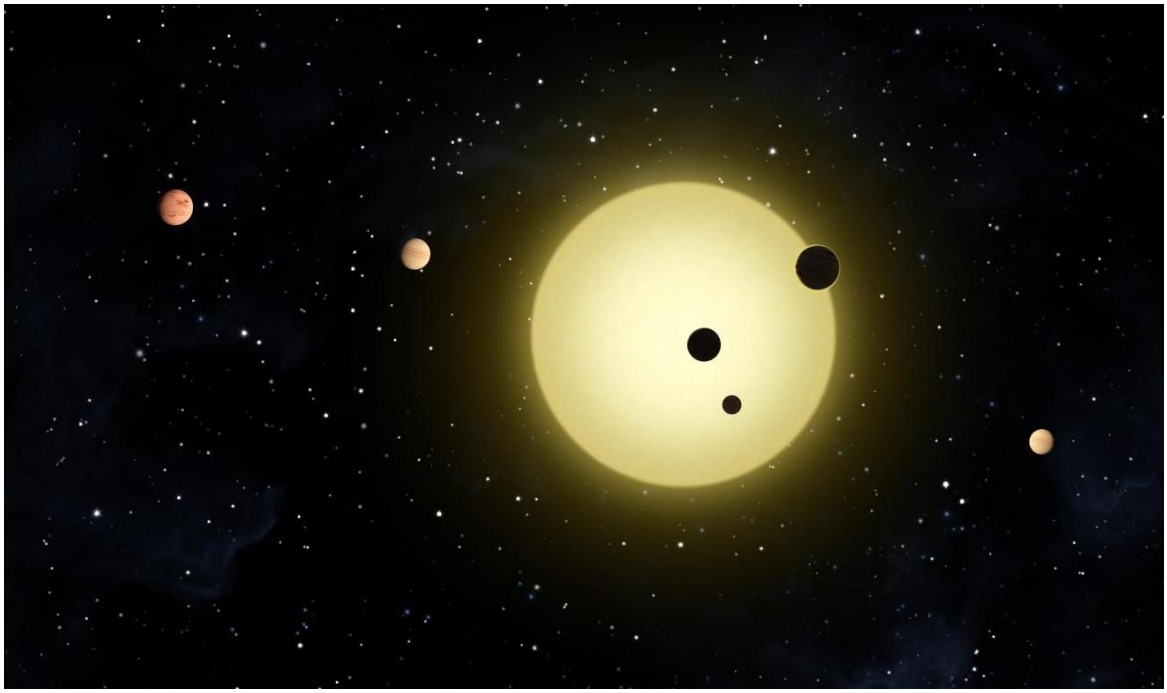
Problem 2 - The distance of the planet from its star can be estimated in terms of the orbital distance of Earth from our sun as $D^3 = T^2$ where D = 1.0 is the distance of Earth from the sun, and T is in multiples of 1 Earth Year. A) What is the distance of Mercury from our sun if its orbit period is 88 days? B) What is the range of orbit distances for the transiting planets in multiples of the orbit of Mercury if the orbit times range from 5 days to 80 days?

Answer: A) T = 88 days/365 days = 0.24 Earth Years, than $D^3 = 0.24^2$, $D^3 = 0.058$, so D = $(0.058)^{1/3}$ and so D = **0.39 times Earth's orbit distance**.
B) The time range is 0.014 to 0.22 Earth Years, and so D is in the range from 0.058 to 0.36 Earth distances. Since Mercury has D = 0.39, in terms of the orbit distance of Mercury, the transiting planets span a range from 0.058/0.39 = **0.15** to 0.36/0.39 = **0.92 Mercury orbits**.

Problem 3 - As a planet passes across the star's disk, the star's brightness dims by a factor of 0.001 in brightness. If the radius of the star is 500,000 km, and both the planet and star are approximated as circles, what is the radius of the planet A) in kilometers? B) In multiples of Earth's diameter (13,000 km)?

Answer: The amount of dimming is equal to the ratio of the areas of the planet's disk to the star's disk, so 0.001 = $\pi R^2/\pi(500{,}000)^2$ so **R = 15,800 kilometers**, which equals a diameter of 31,600 kilometers. Since Earth's diameter = 13,000 km, the transiting planet is about **2.4 times the diameter of Earth**.

Earth-sized Planets by the Score! 19.3



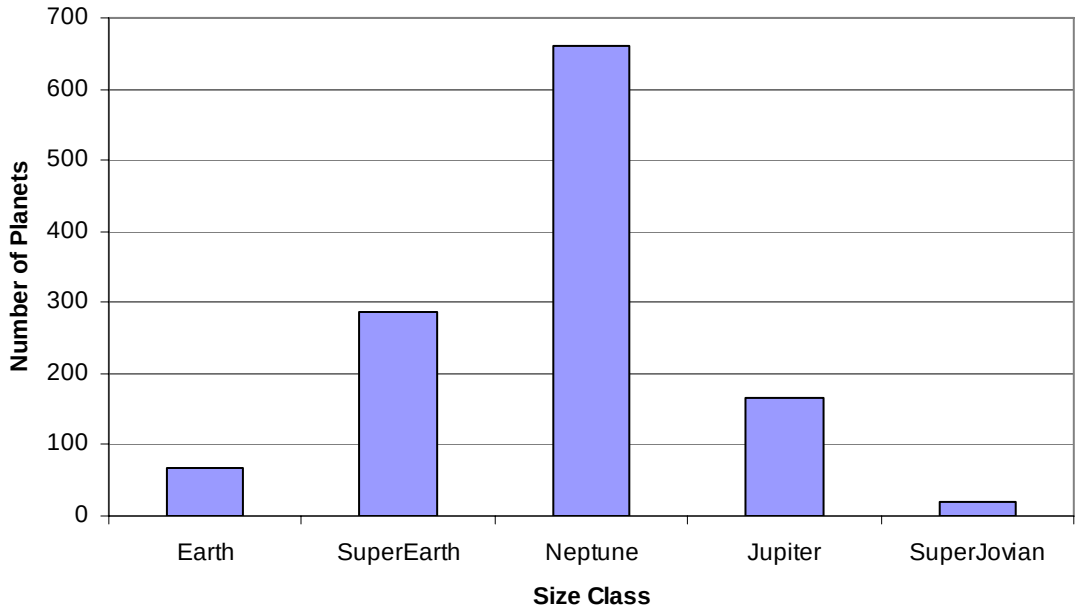
NASA's Kepler Space Observatory recently announced the results of its continuing survey of 156,453 stars in the search for planet transits. Their survey, in progress for just under one year, has now turned up 1,235 transits from among this sample of stars of which 33 were eliminated because they were too big to be true planets. About 30 percent of the remaining candidates belong to multiple-planet systems in which several planets orbit the same star. Among the other important findings are the numbers of planet candidates among the various planet types summarized as follows: Earth-sized = 68; superEarths = 288; Neptune-sized = 662; Jupiter-sized=165; superJovians=19.

Problem 1 - Create a histogram that shows the number of candidate planets among the 5 different size classes.

Problem 2 - What percentage of all the planets detected by Kepler were found to be Earth-sized?

Problem 3 - Extrapolating from the Kepler findings, which was based on a search of 156,453 stars, about how many Earth-sized planets would you expect to find if the Milky Way contains about 40 billion stars similar to the ones surveyed by NASA's Kepler Space Observatory?

Problem 1 - Create a histogram that shows the number of candidate planets among the 5 different size classes. Answer:



Problem 2 - What percentage of all the planets detected by Kepler were found to be Earth-sized? Answer: P = 100% x (68/1202) = **5.7 %**

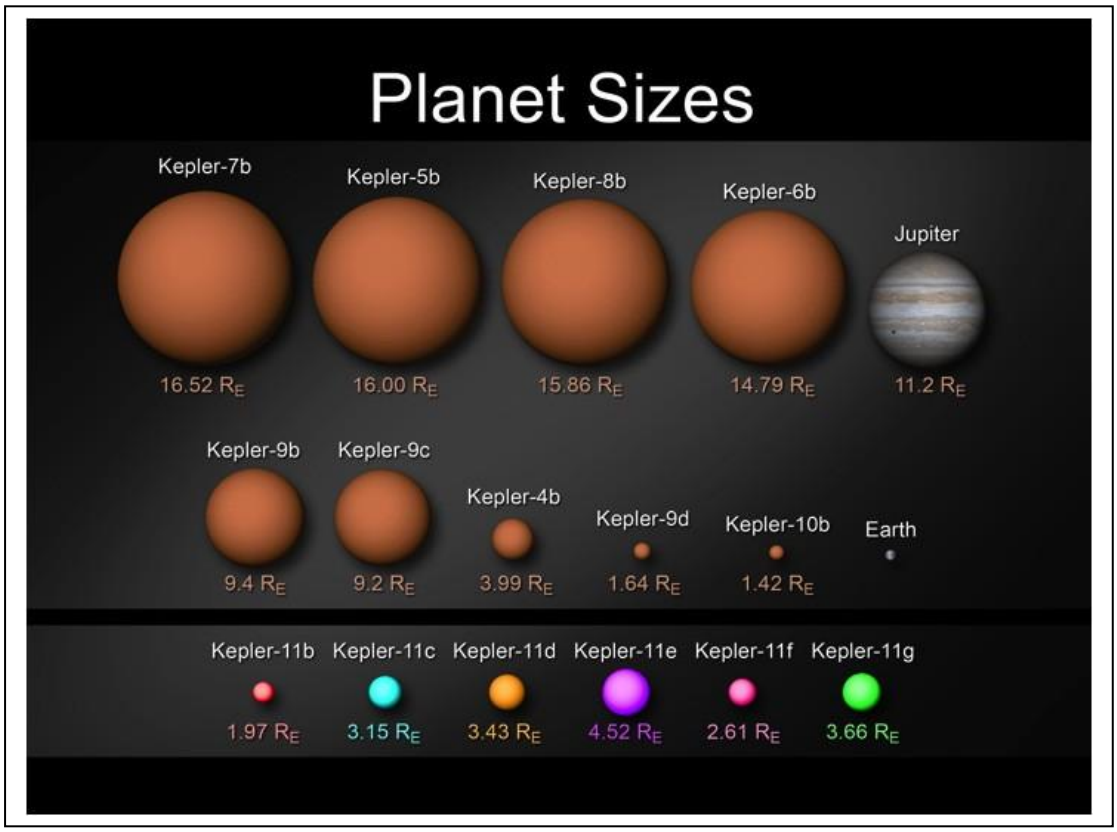
Problem 3 - Extrapolating from the Kepler findings, which was based on a search of 156,453 stars, about how many Earth-sized planets would you expect to find if the Milky Way contains about 40 billion stars similar to the ones surveyed by Kepler?

Answer: There are 40 billion candidate stars in the Milky Way, so by using simple proportions and re-scaling the survey to the larger sample size

$$\frac{68}{157,453} = \frac{X}{40 billion}$$

we get about (40 billion/156,453) x 68 planets or about **17 million Earth-sized planets**.

Note: *The Kepler survey has not been conducted long enough to detect planets much beyond the orbit of Venus in our own solar system, so in time many more earth-sized candidates farther away from their stars will be reported in the years to come. This means that there may well be considerably more than 17 million Earth-sized planets orbiting stars in the Milky Way similar to our own sun.*



This diagram shows disks representing the planets discovered in orbit around 8 different stars all drawn to the same scale. Earth and Jupiter are also shown so that you can see how big they are in comparison. Solve the problems below using fraction arithmetic to find out how big these new planets are compared to Earth and Jupiter.

Problem 1 – Kepler-5b is 8 times the diameter of Kepler-11b. Kepler-11b is twice the diameter of Earth. How big is the planet Kepler-5b compared to Earth?

Problem 2 – The planet Kepler-9c is 9./11 the diameter of Jupiter, and Kepler-11e is 1/2 the diameter of Kepler-9c. How big is the planet Kepler-11e compared to Jupiter?

Problem 3 – The planet Kepler-10b is 1/10 the diameter of Kepler-6b, and Kepler-9b is 9/15 the diameter of Kepler-6b. If Kepler-11g is 4/9 the diameter of Kepler-9b, how big is Kepler-11g compared to Kepler-10b?

Problem 1 – Kepler-5b is 8 times the diameter of Kepler-11b .Kepler-11b is twice the diameter of Earth. How big is the planet Kepler-5b compared to Earth?

Answer: It helps to set up these kinds of problems as though they were unit conversion problems, and then cancel the planet names to get the desired ratio:

$$\frac{1 \times \text{Kepler5b}}{8 \times \text{Kepler11b}} \times \frac{1 \times \text{Kepler11b}}{2 \times \text{Earth}} = \frac{1\ \text{Kepler5b}}{8 \times \text{Earth}}$$ so Kepler 5b is **16 x Earth**

Note how the 'units' for Kepler-11b have canceled out.

Problem 2 – The planet Kepler-9c is 9/11 the diameter of Jupiter, and Kepler-11e is 1/2 the diameter of Kepler-9c. How big is the planet Kepler-11e compared to Jupiter?

$$\frac{11 \times \text{Kepler9c}}{9 \times \text{Jupiter}} \times \frac{2 \times \text{Kepler11e}}{1 \times \text{Kepler9c}} = \frac{22\text{x Kepler9c}}{9 \times \text{Jupiter}}$$ so Kepler 11e = **9/22 Jupiter**

From the figure we see Kepler11e = 4.52 Re and Jupiter = 11.2 Re so Kepler11e = 4.5/11 = 9/22 Jupiter.

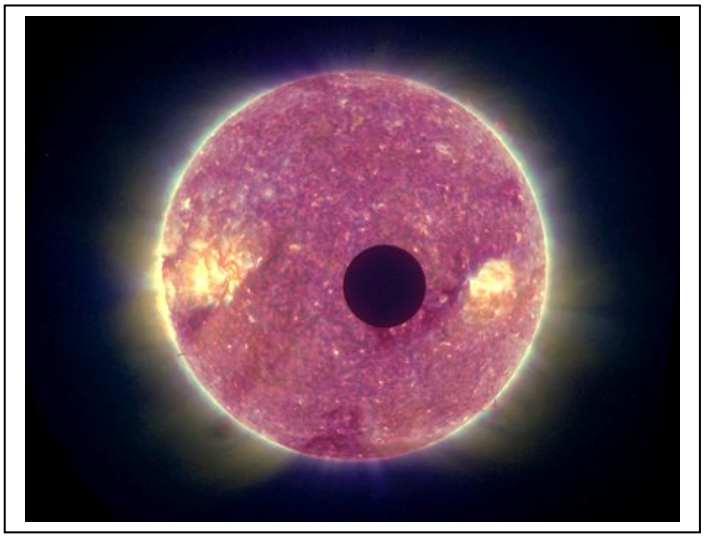
Problem 3 – The planet Kepler-10b is 1/10 the diameter of Kepler-6b, and Kepler-9b is 9/15 the diameter of Kepler-6b .If Kepler-11g is 4/9 the diameter of Kepler-9b, how big is Kepler-11g compared to Kepler-10b?

$$\frac{10\text{xKepler10b}}{1\text{xKepler6b}} \times \frac{9\ \text{Kepler6b}}{15\ \text{Kepler9b}} \times \frac{4\ \text{Kepler9b}}{9\ \text{Kepler11g}} = \frac{40\ \text{Kepler10b}}{15\ \text{Kepler11g}}$$

So 15 x the diameter of Kepler 11g is equal to 40x the diameter of Kepler 10b
And so, Kepler 11g is 40/15 times the diameter of Kepler 10b

From the figure we see that Kepler 11g is 3.66 Re and Kepler 10b is 1.4 Re which is in about the same ratio as our fractions. 3.66/1.4 = 2.6 and 40/15 = 2.7.

Kepler – The Hunt for Earth-like Planets



On March 11, 2009, NASA launched the Kepler satellite. Its 3-year mission is to search 100,000 stars in the constellation Cygnus and detect earth-sized planets. How can the satellite do this?

The image to the left shows what happens when a planet passes across the face of a distant star as viewed from Earth. In this case, this was the planet Mercury on February 25, 2007.

The picture was taken by the STEREO satellite. Notice that Mercury's black disk has reduced the area of the sun. This means that, on Earth, the light from the sun dimmed slightly during the Transit of Mercury. Because Mercury was closer to Earth than the Sun, Mercury's disk appears very large. If we replace Mercury with the Moon, the lunar disk would exactly cover the disk of the Sun and we would have a total solar eclipse.

Now imagine that the Sun was so far away that you couldn't see its disk at all. The light from the Sun would STILL be dimmed slightly. The Kepler satellite will carefully measure the brightness of more than 100,000 stars to detect the slight changes caused by 'transiting exoplanets'.

Problem 1 – With a compass, draw a circle 160-millimeters in radius to represent the sun. If the radius of the sun is 696,000 kilometers, what is the scale of your sun disk in kilometers/millimeter?

Problem 2 – At the scale of your drawing, what would be the radius of Earth (R = 6,378 km) and Jupiter (R = 71,500 km)?

Problem 3 – What is the area of the Sun disk in square millimeters?

Problem 4 – What is the area of Earth and Jupiter in square millimeters?

Problem 5 – By what percent would the area of the Sun be reduced if: A) Earth's disk were placed in front of the Sun disk? B) Jupiter's disk were placed in front of the Sun disk?

Problem 6 – For the transit of a large planet like Jupiter, draw a graph of the percentage brightness of the star (vertical axis) as it changes with time (horizontal axis) during the transit event. Assume that the entire transit takes about 1 day from start to finish.

Answer Key

Problem 1 – With a compass, draw a circle 160-millimeters in radius to represent the sun. If the radius of the Sun is 696,000 kilometers, what is the scale of your Sun disk in kilometers/millimeter? **Answer: 4,350 km/mm**

Problem 2 – At the scale of your drawing, what would be the radius of Earth (R = 6,378 km) and Jupiter (R = 71,500 km)? **Answer: 1.5 mm and 16.4 mm respectively.**

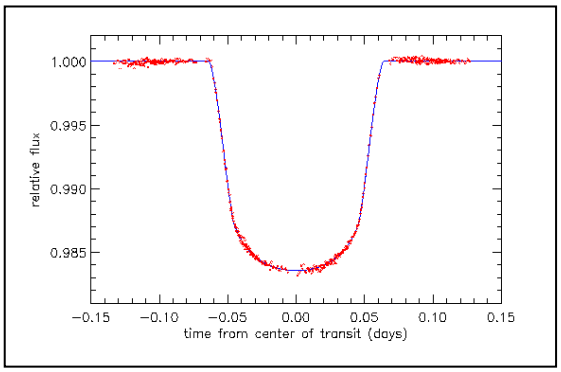
Problem 3 – What is the area of the sun disk in square millimeters? Answer; π x $(160)^2$ **= 80,400 mm^2.**

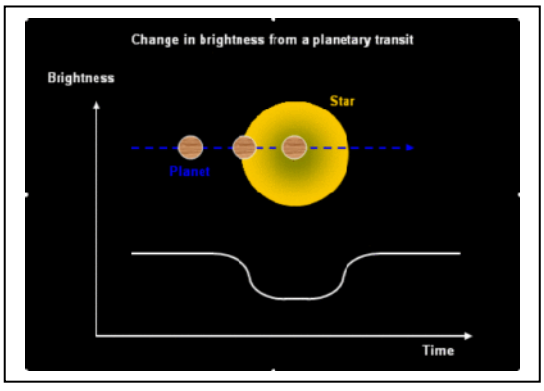
Problem 4 – What is the area of Earth and Jupiter in square millimeters? Answer: Earth = π x $(1.5)^2$ = **7.1 mm^2**. Jupiter = π x $(16.4)^2$ = **844.5 mm^2**.

Problem 5 – By what percent would the area of the sun be reduced if: A) Earth's disk were placed in front of the Sun disk? B) Jupiter's disk were placed in front of the Sun disk? Answer A) 100% x (7.1 mm^2 / 80400 mm^2) = 100% x 0.000088 **= 0.0088 %**
B) Jupiter: 100% x (844.5 mm^2 / 80400 mm^2) = 100% x 0.011 = **1.1%.**

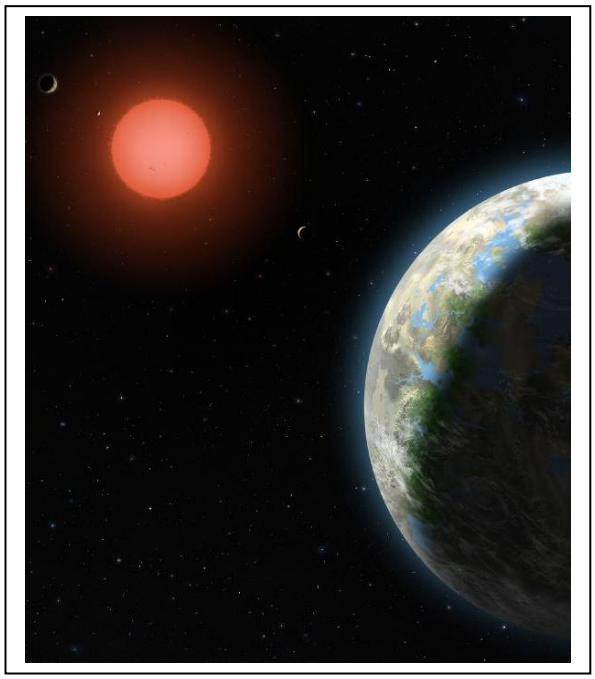
Problem 6 – For the transit of a large planet like Jupiter, draw a graph of the percentage brightness of the star (vertical axis) as it changes with time (horizontal axis) during the transit event. Assume that the entire transit takes about 1 day from start to finish.

Answer: Students should note from their answer to Problem 5 that when the planet disk is fully on the star disk, the star's brightness will dim from 100% to 100% - 1.1% = 98.9%. Students should also note that as the transit starts, the stars brightness will dim as more of the planet's disk begins to cover the star's disk. Similarly, as the planet's disk reaches the edge of the star's disk, the area covered by the planet decreases and so the star will gradually brighten to its former 100% level. The figures below give an idea of the kinds of graphs that should be produced. The left figure is from the Hubble Space Telescope study of the star HD209458 and its transiting Jupiter-sized planet.





The Earth-like Planet Gliese 581g



Professors Steven Vogt at UC Santa Cruz, and Paul Butler of the Carnegie Institution have just announced the discovery of a new planet orbiting the nearby red dwarf star Gliese 518. The star is located 20 light years from Earth in the constellation Libra. The planet joins five others in this crowded planetary system, and has a mass about three to four times Earth, making it in all likelihood a rocky planet, rather than a gas giant. The planet is tidally locked to its star which means that during its 37 day orbit, it always shows the same face to the star so that one hemisphere is always in daylight while the other is in permanent nighttime.

One of the most important aspects to new planets is whether they are in a distance zone where water can remain a liquid on the planets surface. The Habitable Zone (HZ) location around a star depends on the amount of light energy that the star produces. For the Sun, the HZ extends from about the orbit of Venus to the orbit of Mars. For stars that emit less energy, the HZ will be much closer to the star. Once an astronomer knows what kind of star a planet orbits, they can calculate over what distances the HZ will exist.

Problem 1 - What is the pattern that astronomers use to name the discovered planets outside our solar system?

Problem 2 - One Astronomical Unit (AU) is the distance between Earth and the Sun (150 million kilometers). Draw a model of the Gliese 581 planetary system with a scale of 0.01 AU per centimeter, and show each planet with a small circle drawn to a scale of 5,000 km/millimeter, based on the data in the table below:

Planet	Discovery Year	Distance (AU)	Period (days)	Diameter (km)
Gliese 581 b	2005	0.04	5.4	50,000
Gliese 581 c	2007	0.07	13.0	20,000
Gliese 581 d	2007	0.22	66.8	25,000
Gliese 581 e	2009	0.03	3.1	15,000
Gliese 581 f	2010	0.76	433	25,000
Gliese 581 g	2010	0.15	36.6	20,000

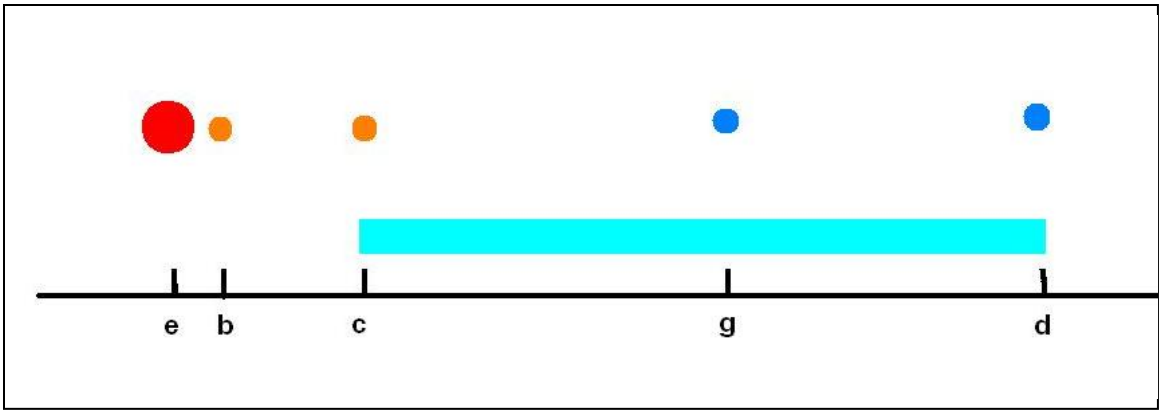
Problem 3 - The Habitable Zone for our solar system extends from 0.8 to 2.0 AU, while for Gliese 581 it extends from about 0.06 to 0.23 because the star shines with nearly 1/100 the amount of light energy as our sun. In the scale model diagram, shade-in the range of distances where the HZ exists for the Gliese 581 planetary system. Why do you think astronomers are excited about Gliese 581g?

Problem 1 - What is the pattern that astronomers use to name the discovered planets outside our solar system? Answer: **According to the order of discovery date.** Note: Gliese 581 A is the designation given to the star itself.

Problem 2 - Draw a model of the Gliese 581 planetary system with a scale of 0.01 AU per centimeter, and show each planet with a small circle drawn to a scale of 5,000 km/millimeter, based on the data in the table. Answer: **The table below gives the dimensions on the scaled diagram. See figure below for an approximate appearance. On the scale of the figure below, Gliese 581f would be located about 32 centimeters to the right of Gliese 581d.**

Planet	Discovery Year	Distance (cm)	Period (days)	Diameter (mm)
Gliese 581 b	2005	4	5.4	10
Gliese 581 c	2007	7	13.0	4
Gliese 581 d	2007	22	66.8	5
Gliese 581 e	2009	3	3.1	3
Gliese 581 f	2010	76	433	5
Gliese 581 g	2010	15	36.6	4

Problem 3 - The Habitable Zone for our solar system extends from 0.8 to 2.0 AU, while for Gliese 581 it extends from about 0.06 to 0.23 because the star shines with nearly 1/100 the amount of light energy as our sun. In the scale model, shade-in the range of distances where the HZ exists. Why do you think astronomers are excited about Gliese 581g? Answer: **See the bar spanning the given distances. Note that Gliese c, g and d are located in the HZ of Gliese 581. Because Gliese 581 g is located near the center of this zone and is very likely to be warm enough for there to be liquid water, which is an essential ingredient for life. Gliese 581c may be too hot and Gliese 581 d may be too cold.**



The Cometary Planet: HD 209458b

Artist view of planet G. Bacon (STScI/NASA)

Every 4 days, this planet orbits a sun-like star located 153 light years from Earth. Astronomers using NASA's Hubble Space Telescope have confirmed that this gas giant planet is orbiting so close to its star its heated atmosphere is escaping into space.

Observations taken with Hubble's Cosmic Origins Spectrograph (COS) suggest powerful stellar winds are sweeping the cast-off atmospheric material behind the scorched planet and shaping it into a comet-like tail. COS detected the heavy elements carbon and silicon in the planet's super-hot, $2{,}000^{o}$ F atmosphere.

Problem 1 - Based upon a study of the spectral lines of hydrogen, carbon and silicon, the estimated rate of atmosphere loss may be as high as 4×10^{11} grams/sec. How fast is it losing mass in: A) metric tons per day? B) metric tons per year?

Problem 2 - The mass of the planet is about 60% of Jupiter, and its radius is about 1.3 times that of Jupiter. If the mass of Jupiter is 1.9×10^{27} kg, and its radius is 7.13×10^{7} meters, what is the density of A) Jupiter? B) HD209458b?

Problem 3 - Suppose that, like Jupiter, the planet has a rocky core with a mass of 18 times Earth. If Earth's mass is 5.9×10^{24} kg, what is the mass of the atmosphere of HD209458b?

Problem 4 - About how long would it take for HD209458b to completely lose its atmosphere at the measured mass-loss rate?

Problem 1 - Based upon a study of the spectral lines of hydrogen, carbon and silicon, the estimated rate of atmosphere loss may be as high as 4×10^{11} grams/sec. How fast is it losing mass in: A) metric tons per day? B) metric tons per year?

Answer: A) 4×10^{11} grams/sec x (10^{-6} kg/gm) x (86,400 sec/day) x (1 ton / 1000 kg)
= **3.5×10^{10} tons/day**
B) 3.5×10^{10} tons/day x 365 days/year = **1.3×10^{13} tons/year**

Problem 2 - The mass of the planet is about 60% of Jupiter, and its radius is about 1.3 times that of Jupiter. If the mass of Jupiter is 1.9×10^{27} kg, and its radius is 7.13×10^{7} meters, what is the density of A) Jupiter? B) HD209458b?

Answer: A) $V = 4/3\ \pi\ R^3$ so V(Jupiter) = 1.5×10^{24} meters^3. Density = mass/volume so Density (Jupiter) = 1.9×10^{27} kg/1.5×10^{24} meters^3 = **1266 kg/meter^3**.
B) Mass = 0.6 M(Jupiter) and volume = $(1.3)^3$ V(Jupiter) so density = $0.6/(1.3)^3$ x 1266 kg/meter^3 =0.27x1266 = **342 kg/meter^3**.

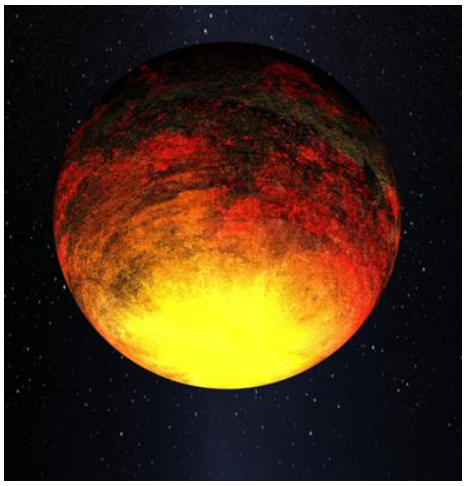
Problem 3 - Suppose that, like Jupiter, the planet has a rocky core with a mass of 18 times Earth. If Earth's mass is 5.9×10^{24} kg, what is the mass of the atmosphere of HD209458b? Answer: M(HD209458b) = 0.6x Jupiter = 1.1×10^{27} kg so M(atmosphere) = 1.1×10^{27} kg – 18x(5.9×10^{24} kg) = **9.9×10^{26} kg**.

Problem 4 - About how long would it take for HD209458b to completely lose its atmosphere at the measured mass-loss rate?

Answer: Time = Mass/rate, and the rate is 1.3×10^{13} tons/year. Since 1 metric ton = 1,000 kg, the rate is 1.3×10^{16} kg/year so that
= 9.9×10^{26} kg / (1.3×10^{16} kg/year)
= **7.6×10^{10} years**

The paper is Linsky et al., "Observations of Mass Loss from the Transiting Exoplanet HD 209458b," *Astrophysical Journal* Vol. 717, No. 2 (10 July 2010), p. 1291. They estimate nearly a trillion years, so the planet is in no danger of disappearing!

Planet Kepler-10b: A matter of gravity



NASA Artist rendition of the sizzling-hot, Earth-like world: Kepler 10b.

The Kepler Space Observatory recently detected an Earth-sized planet orbiting the star Kepler-10. The more than 8 billion year old star, located in the constellation Draco, is 560 light years from Earth. The planet orbits its star at a distance of 2.5 million km with a period of 20 hours, so that its surface temperature exceeds 2,500 F.

Careful studies of the transit of this planet across the face of its star indicates a diameter 1.4 times that of Earth, and an estimated average density of 8.8 grams/cc, which is about that of solid iron, and 3-times the density of Earth's silicate-rich surface rocks.

Problem 1 -Assume that Kepler-10b is a spherical planet, and that the radius of Earth is 6,378 kilometers. What is the total mass of this planet if its density is 8800 kg/meter^3?

Problem 2 - The acceleration of gravity on a planet's surface is given by the Newton's formula

$$a = 6.67x10^{-11}\frac{M}{R^2} \quad \text{meters/sec}^2$$

Where R is distance from the surface of the planet to the planet's center in meters, and M is the mass of the planet in kilograms. What is the acceleration of gravity at the surface of Kepler-10b?

Problem 3 - The acceleration of gravity at Earth's surface is 9.8 meters/sec^2. If this acceleration causes a 68 kg human to have a weight of 150 pounds, how much will the same 68 kg human weigh on the surface of Kepler-10b if the weight in pounds is directly proportional to surface acceleration?

Problem 1 -Assume that Kepler-10b is a spherical planet, and that the radius of Earth is 6,378 kilometers. What is the total mass of this planet if its density is 8800 kg/meter^3?

Answer: The planet is 1.4 times the radius of Earth, so its radius is 1.4 x 6,378 km = 8,929 kilometers. Since we need to use units in terms of meters because we are given the density in cubic meters, the radius of the planet becomes 8,929,000 meters.

Volume = $\frac{4}{3}\pi R^3$

so V = 1.33 x (3.141) x (8,929,000 meters)^3

V = 2.98 x 10^{21} meter^3

Mass = Density x Volume

= 8,800 x 2.98 x 10^{21}

= **2.6×10^{25} kilograms**

Problem 2 - The acceleration of gravity on a planet's surface is given by the Newton's formula

$$a = 6.67x10^{-11}\frac{M}{R^2} \text{ meters/sec}^2$$

Where R is distance from the surface of the planet to the planet's center in meters, and M is the mass of the planet in kilograms. What is the acceleration of gravity at the surface of Kepler-10b?

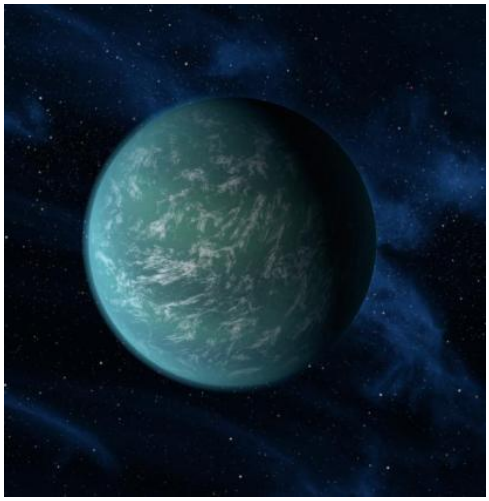
Answer: a = 6.67 x 10^{-11} (2.6 x 10^{25})/(8.929x10^6)^2

= **21.8 meters/sec^2**

Problem 3 - The acceleration of gravity at Earth's surface is 9.8 meters/sec^2. If this acceleration causes a 68 kg human to have a weight of 150 pounds, how much will the same 68 kg human weigh on the surface of Kepler-10b if the weight in pounds is directly proportional to surface acceleration?

Answer: The acceleration is 21.8/9.8 = 2.2 times Earth's gravity, and since weight is proportional to gravitational acceleration we have the proportion:

$\frac{21.8}{9.8} = \frac{X}{150lb}$ and so the human would weigh 150 x 2.2 = **330 pounds!**



An artists illustration of Kepler-22b (Credit NASA/Ames/JPL-Caltech)

NASA's Kepler mission has confirmed its first planet in the "habitable zone," the region where liquid water could exist on a planet's surface.

The newly confirmed planet, Kepler-22b, is the smallest yet found to orbit in the middle of the habitable zone of a star similar to our sun.

The planet is about 2.4 times the radius of Earth. Scientists don't yet know if Kepler-22b has a rocky, gaseous or liquid composition, but its discovery is a step closer to finding Earth-like planets.

Problem 1 - Suppose Kepler-22b is a spherical, rocky planet like Earth with an average density similar to Earth (about 5,500 kg/meter^3). If the radius of Kepler-22b is 15,000 km, what is the mass of Kepler-22b in A) kilograms? B) multiples of Earth's mass (5.97x1024 kg)?

Problem 2 - The acceleration of gravity on a planetary surface is given by the formula

$$a = \frac{GM}{R^2}$$

where M is in kilograms, R is in meters and G is the Newtonian Constant of Gravity with a value of 6.67 x 10^{-11} m^3 kg^{-1} sec^{-2} What is the surface acceleration of Kepler-22b A) In meters/sec^2? B) In multiples of Earth's surface gravity 9.8 meters/sec^2?

Problem 3 - The relationship between surface acceleration and your weight is a direct proportion. The surface acceleration of Earth is 9.8 meters/sec^2. If you weigh 150 pounds on the surface of Earth, how much will you weigh on the surface of Kepler-22b given the acceleration you calculated in Problem 2?

Problem 4 - The dimensions of a typical baseball park are determined by the farthest distance that an average batter can bat a home-run. This in turn depends on the acceleration of gravity, which is the force that pulls the ball back to the ground to shorten its travel distance. For a standard baseball field, the distance to the back-field fence from Home Plate may not be less than 325 feet, and the baseball diamond must be exactly 90 feet on a side.

A) If the maximum travel distance of the baseball scales linearly with the acceleration of gravity, what is the minimum distance to the back-field fence from Home Plate along one of the two foul lines?

Problem 1 - Suppose Kepler-22b is a spherical, rocky planet like Earth with an average density similar to Earth (about 5,500 kg/meter^3). If the radius of Kepler-22b is 15,000 km, what is the mass of Kepler-22b in A) kilograms? B) multiples of Earth's mass (5.97x10^{24} kg)?

Answer: A) First find the volume of the spherical planet in cubic meters, then multiply by the density of the planet to get the total mass.

R = 15,000 km x (1000 m/1 km) = 1.5 x 10^7 meters.

$V = 4/3\ \pi\ R^3$

$= 1.33 \times 3.14 \times (1.5 \times 10^7 \text{ meters})^3 = 1.41 \times 10^{22} \text{ meters}^3$

Then M = density x volume

= 5,500 kg/m^3 x (1.41x10^{22})

= 7.75 x10^{25} kg

B) M = 7.75x10^{25} kg / 5.97x10^{24} kg = **12.9 Earths**.

Problem 2 - A) In meters/sec^2? B) In multiples of Earth's surface gravity 9.8 meters/sec^2?

Answer: A) $a = 6.67 \times 10^{-11} (7.75 \times 10^{25}) / (1.5 \times 10^7)^2$ = **23.0 meters/sec^2**

B) 23.0 / 9.8 = **2.3 times earth's surface gravity**

Note: From the formula for M and a, we see that the acceleration varies directly with the radius change, which is a factor of 2.4 times Earth, so a = 2.4xa(earth)

Problem 3 – The relationship between surface acceleration and your weight is a direct proportion. The surface acceleration of Earth is 9.8 meters/sec^2. If you weigh 150 pounds on the surface of Earth, how much will you weigh on the surface of Kepler-22b given the acceleration you calculated in Problem 2?

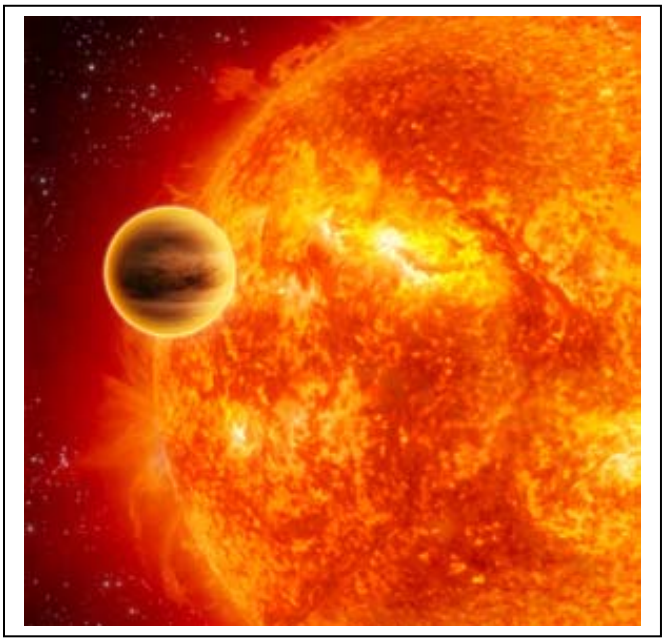
Answer: By a simple proportion: X/150 = 2.3/1.0 so x = 2.3x150 = **345 pounds**.

Problem 4 - For a standard baseball field, the distance to the back-field fence from Home Plate may not be less than 325 feet, and the baseball diamond must be exactly 90 feet on a side.

A) If the maximum travel distance of the baseball scales linearly with the acceleration of gravity, what is the distance to the back-field fence from Home Plate along one of the two foul lines? Answer: 325 / 2.3 = **141 feet**.

B) What are the dimensions of the baseball diamond? Answer: 90/2.3 = **39 feet** on a side.

Exoplanet Orbits and the Properties of Ellipses 19.10



By 2011, over 1700 planets have been discovered orbiting nearby stars since 1995. Called 'exoplanets' to distinguish them from the familiar 8 planets in our own solar system, they are planets similar to Jupiter in size, but orbiting their stars in mostly elliptical paths. In many cases, the planets come so close to their star that conditions for life to exist would be impossible.

Astronomers are continuing to search for smaller planets to find those that are more like our own Earth.

(Artist rendition: courtesy NASA)

Use the basic properties and formulae for ellipses to analyze the following approximate exoplanet orbits by first converting the indicated equations into standard form. Then determine for each planet the:

A) a= semi-major axis

B) b=semi-minor axis;

C) ellipticity $e = \frac{\sqrt{\left(a^2 - b^2\right)}}{a}$

D) 'perihelion' closest distance to star, defined as P=a(1-e);

E) 'aphelion' farthest distance from star, defined as A = a(1+e).

Problem 1: Planet: 61 Virginis-d Period=4 days $1 = 4x^2 + 5y^2$

Problem 2: Planet: HD100777-b Period=383 days $98 = 92x^2 + 106y^2$

Problem 3: Planet: HD 106252-b Period=1500 days $35 = 5x^2 + 7y^2$

Problem 4: Planet: 47 UMa-c Period= 2190 days $132 = 11x^2 + 12y^2$

Problem 1: 61 Virginis -d Period=4 days $1 = 4x^2 + 5y^2$

$$1 = \frac{x^2}{0.25} + \frac{y^2}{0.20}$$

a=0.5 b=0.45 $e = \frac{\sqrt{(a^2 - b^2)}}{a}$ = **0.43** P = (0.5)(1-0.43) = **0.28**, A=(0.5)(1+0.43) = **0.71**

Problem 2: Planet: HD100777-b Period=383 days $98 = 92x^2 + 106y^2$

$$1 = \frac{x^2}{1.06} + \frac{y^2}{0.92}$$

a=1.03 b=0.96 $e = \frac{\sqrt{(a^2 - b^2)}}{a}$ = **0.36** P = (1.03)(1-0.36) = **0.66,** A=(1.03)(1+0.36) = **1.40**

Problem 3: Planet: HD 106252-b Period=1500 days $35 = 5x^2 + 7y^2$

$$1 = \frac{x^2}{7.0} + \frac{y^2}{5.0}$$

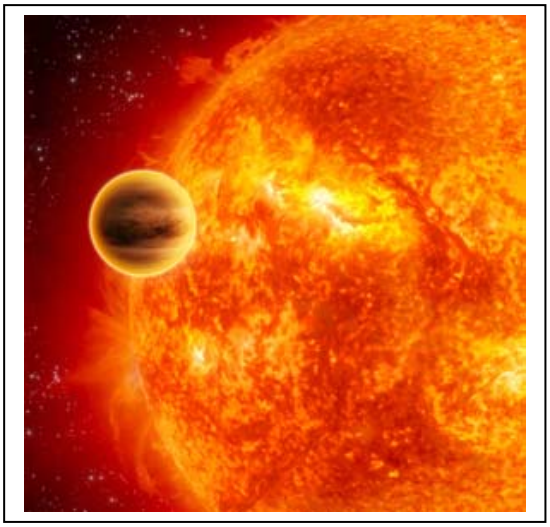
a=2.6 b=2.2 $e = \frac{\sqrt{(a^2 - b^2)}}{a}$ = **0.53** P = (2.6)(1-0.53) = **1.22**, A=(2.6)(1+0.53) = **4.0**

Problem 4: Planet: 47 UMa-c Period= 2190 days $132 = 11x^2 + 12y^2$

$$1 = \frac{x^2}{12.0} + \frac{y^2}{11.0}$$

a=3.5 b=3.3 $e = \frac{\sqrt{(a^2 - b^2)}}{a}$ = **0.33** P = (3.5)(1-0.33) = **2.35,** A=(3.5)(1+0.33) = **4.65**

Estimating the Temperatures of an Exoplanet 19.11



Because many exoplanets orbit their stars in elliptical paths, they experience large swings in temperature. Generally, organisms can not survive if water is frozen (0 C = 273 K) or near its boiling point (100 C or 373 K). Due to orbital conditions, this very narrow 'zone of life' may not be possible for many of the worlds detected so far.

Problem 1 - Complete the table below by calculating

A) The semi-minor axis distance $B = A(1-e^2)$

B) The perihelion distance $Dp = A(1-e)$

C) The aphelion distance, $Da = B(1+e)$

Problem 2 - Write the equation for the orbit of 61 Vir-d in Standard Form.

Problem 3 - The temperature of a planet similar to Jupiter can be approximated by the formula below, where T is the temperature in Kelvin degrees, and R is the distance to its star in Astronomical Units (AU), where 1 AU is the distance from Earth to the sun (150 million km). Complete the table entries for the estimated temperature of each planet at the farthest 'aphelion' distance Ta, and the closest 'perihelion;' distance Tp.

$$T(R) = \frac{250}{\sqrt{R}}$$

Problem 4 - Which planets would offer the most hospitable, or the most hazardous, conditions for life to exist, and what would be the conditions be like during a complete 'year' for each world?

Planet	A (AU)	B (AU)	e	Period (days)	Dp (AU)	Da (AU)	Ta (K)	Tp (K)
47 UMa-c	3.39		0.22	2190				
61 Vir-d	0.47		0.35	123				
HD106252-b	2.61		0.54	1500				
HD100777-b	1.03		0.36	383				
HAT-P13c	1.2		0.70	428				

Problem 1 See table below:

Planet	A (AU)	B (AU)	e	Period (days)	Dp (AU)	Da (AU)	Ta (K)	Tp (K)
47 UMa-c	3.39	**3.3**	0.22	2190	**2.6**	**4.1**		
61 Vir-d	0.47	**0.4**	0.35	123	**0.3**	**0.6**		
HD106252-b	2.61	**2.2**	0.54	1500	**1.2**	**4.0**		
HD100777-b	1.03	**1.0**	0.36	383	**0.7**	**1.4**		
HAT-P13c	1.2	**0.9**	0.70	428	**0.4**	**2.0**		

Problem 2 Write the equation for the orbit of 61 Vir-d in Standard Form.
Answer: A = 0.47 and B = 0.4

So $1 = \frac{x^2}{0.47} + \frac{y^2}{0.40}$ and also $188 = 40x^2 + 47\,y^2$

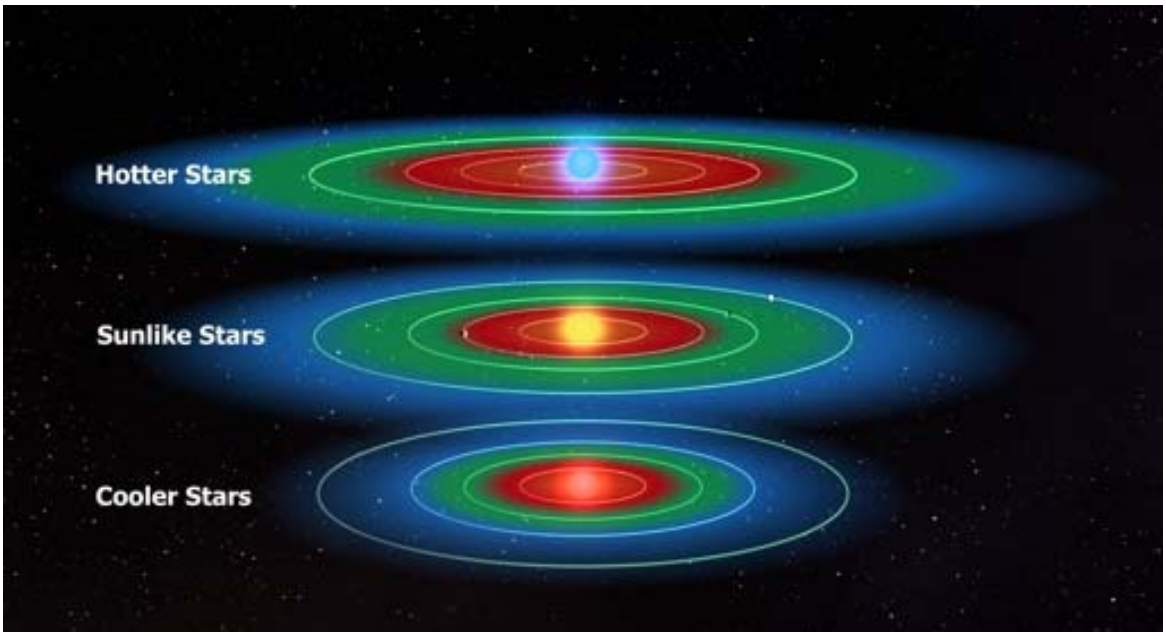
Problem 3 - See table below:

Planet	A (AU)	B (AU)	e	Period (days)	Dp (AU)	Da (AU)	Ta (K)	Tp (K)
47 UMa-c	3.39	**3.3**	0.22	2190	**2.6**	**4.1**	**154**	**123**
61 Vir-d	0.47	**0.4**	0.35	123	**0.3**	**0.6**	**452**	**314**
HD106252-b	2.61	**2.2**	0.54	1500	**1.2**	**4.0**	**228**	**125**
HD100777-b	1.03	**1.0**	0.36	383	**0.7**	**1.4**	**308**	**211**
HAT-P13c	1.2	**0.9**	0.70	428	**0.4**	**2.0**	**417**	**175**

Problem 4 - Which planets would offer the most hospitable, or most hazardous, conditions for life to exist, and what would be the conditions be like during a complete 'year' for each world?
Answer: For the habitable 'water' range between 273K and 373K, none of these planets satisfy this minimum and maximum condition. They are either too hot at perihelion 'summer' or too cold at 'winter' aphelion.

Only HD100777-b during perihelion is in this temperature range during 'summer', at a temperature of 308 K (35 C). During 'winter' at aphelion, it is at -62 C which is below the freezing point of water, and similar to the most extreme temps in Antarctica.

Note: These temperature calculations are only approximate and may be considerably different with greenhouse heating by the planetary atmosphere included.



Once you have discovered a planet, you need to figure out whether liquid water might be present. In our solar system, Mercury and Venus are so close to the sun that water cannot remain in liquid form. It vaporizes! For planets beyond Mars, the sun is so far away that water will turn to ice. Only in what astronomers call the Habitable Zone (shown in green in the figure above) will a planet have a chance for being at the right temperature for liquid water to exist in large quantities (oceans) on its surface!

The Table on the following page lists the 54 planets that were discovered by NASA's Kepler Observatory in 2010. These planets come in many sizes as you can see by their radii. The planet radii are given in terms of the Earth, where '1.0' means a planet has a radius of exactly 1 Earth radius (1.0 Re) or 6,378 kilometers. The distance to each planet's star is given in multiples of our Earth-Sun distance, called an Astronomical Unit, so that '1.0 AU' means exactly 150 million kilometers.

Problem 1 - For a planet discovered in its Habitable Zone, and to the nearest whole number, what percentage of planets are less than 4 times the radius of Earth?

Problem 2 - About what is the average temperature of the planets for which R <4.0 Re?

Problem 3 - About what is the average temperature of the planets for which R >4.0 Re?

Problem 4 - Create two histograms of the number of planets in each distance zone between 0.1 and 1.0 AU using bins that are 0.1 AU wide. Histogram-1: for the planets with R > 4.0 Re. Histogram-2 for planets with R < 4.0 Re. Can you tell whether the smaller planets favor different parts of the Habitable Zone than the larger planets?

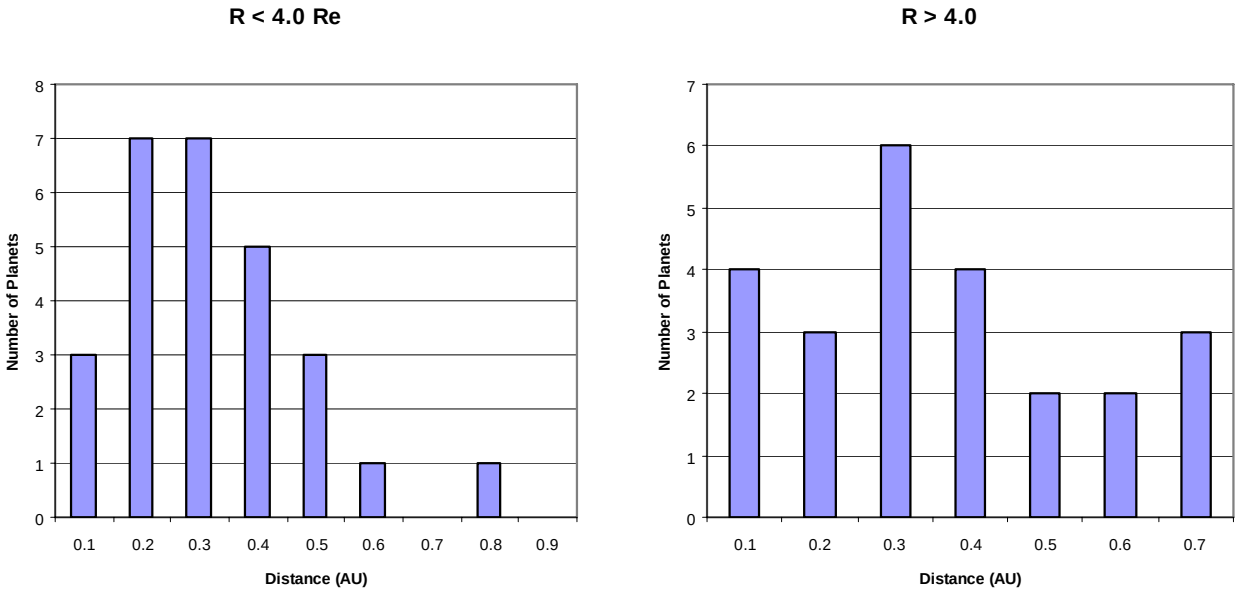
Problem 5 - If you were searching for Earth-like planets in our Milky Way galaxy, which contains 40 billion stars like the ones studies in the Kepler survey, how many do you think you might find in our Milky Way that are at about the same distance as Earth from its star, about the same size as Earth, and about the same temperature (270 - 290 K) if 157,453 stars were searched for the Kepler survey?

Problem 1 - For a planet discovered in its Habitable Zone, and to the nearest whole number, what percentage of planets are less than 4 times the radius of Earth? Answer: There are 28 planets for which R < 4.0 re, so P = 100% x (28/54) = **52%**

Problem 2 - About what is the average temperature of the planets for which R <4.0 Re? Answer; Students will identify the 28 planets in the table that have R < 4.0, and then average the planet's temperatures in Column 6. **Answer: 317 K**.

Problem 3 - About what is the average temperature of the planets for which R >4.0 Re? Students will identify the 26 planets in the table that have R > 4.0, and then average the planet's temperatures in Column 6. **Answer: 306 K**.

Problem 4 - Create two histograms of the number of planets in each distance zone between 0.1 and 1.0 AU using bins that are 0.1 AU wide. Histogram-1: for the planets with R > 4.0 Re. Histogram-2 for planets with R < 4.0 Re. Can you tell whether the smaller planets favor different parts of the Habitable Zone than the larger planets? Answer; They tend to be found slightly closer to their stars, which is why in Problem 2 their average temperatures were slightly hotter than the larger planets.



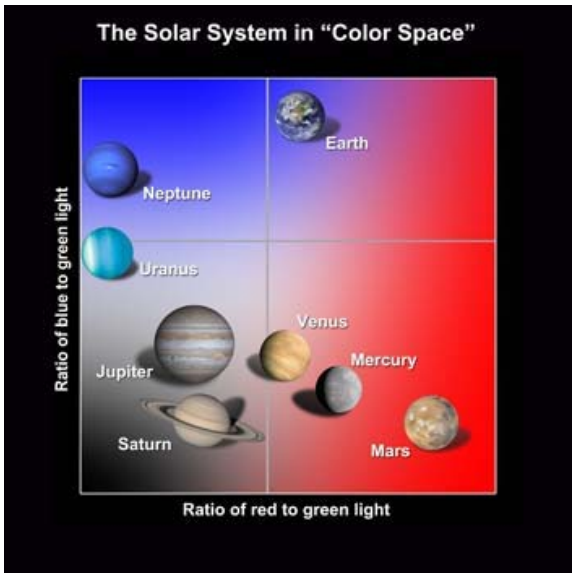
Problem 5 - If you were searching for Earth-like planets in our Milky Way galaxy, which contains 40 billion stars like the ones studies in the Kepler survey, how many do you think you might find in our Milky Way that are at about the same distance as Earth from its star, about the same size as Earth, and about the same temperature (270 - 290 K) if 157,453 stars were searched for the Kepler survey?

Answer: Students may come up with a number of different strategies and estimates. For example, they might create Venn Diagrams for the data in the table that meet the criteria given in the problem. Then, from the number of planets in the intersection, find their proportion in the full sample of 54 planets, then multiply this by the ratio of 40 billion to 157,453. Estimates near 1 million are in the right range.

Table of Habitable Zone Candidates

	Planet Name (KOI)	Orbit Period (days)	Distance To Star (AU)	Planet Radius (Re)	Planet Temp. (K)	Star Temp. (K)
1	683.01	278	0.84	4.1	239	5,624
2	1582.01	186	0.63	4.4	240	5,384
3	1026.01	94	0.33	1.8	242	3,802
4	1503.01	150	0.54	2.7	242	5,356
5	1099.01	162	0.57	3.7	244	5,665
6	854.01	56	0.22	1.9	248	3,743
7	433.02	328	0.94	13.4	249	5,237
8	1486.01	255	0.80	8.4	256	5,688
9	701.03	122	0.45	1.7	262	4,869
10	351.01	332	0.97	8.5	266	6,103
11	902.01	84	0.32	5.7	270	4,312
12	211.01	372	1.05	9.6	273	6,072
13	1423.01	124	0.47	4.3	274	5,288
14	1429.01	206	0.69	4.2	276	5,595
15	1361.01	60	0.24	2.2	279	4,050
16	87.01	290	0.88	2.4	282	5,606
17	139.01	225	0.74	5.7	288	5,921
18	268.01	110	0.41	1.8	295	4,808
19	1472.01	85	0.37	3.6	295	5,455
20	536.01	162	0.59	3.0	296	5,614
21	806.01	143	0.53	9.0	296	5,206
22	1375.01	321	0.96	17.9	300	6,169
23	812.03	46	0.21	2.1	301	4,097
24	865.01	119	0.47	5.9	306	5,560
25	351.02	210	0.71	6.0	309	6,103
26	51.01	10	0.06	4.8	314	3,240
27	1596.02	105	0.42	3.4	316	4,656
28	416.02	88	0.38	2.8	317	5,083
29	622.01	155	0.57	9.3	327	5171
30	555.02	86	0.38	2.3	331	5,218
31	1574.01	115	0.47	5.8	331	5,537
32	326.01	9	0.05	0.9	332	3,240
33	70.03	78	0.35	2.0	333	5,342
34	1261.01	133	0.52	6.3	335	5,760
35	1527.01	193	0.67	4.8	337	5,470
36	1328.01	81	0.36	4.8	338	5,425
37	564.02	128	0.51	5.0	340	5,686
38	1478.01	76	0.35	3.7	341	5,441
39	1355.01	52	0.27	2.8	342	5,529
40	372.01	126	0.50	8.4	344	5,638
41	711.03	125	0.49	2.6	345	5,488
42	448.02	44	0.21	3.8	346	4,264
43	415.01	167	0.61	7.7	352	5,823
44	947.01	29	0.15	2.7	353	3,829
45	174.01	56	0.27	2.5	355	4,654
46	401.02	160	0.59	6.6	357	5,264
47	1564.01	53	0.28	3.1	360	5,709
48	157.05	118	0.48	3.2	361	5,675
49	365.01	82	0.37	2.3	363	5,389
50	374.01	173	0.63	3.3	365	5,829
51	952.03	23	0.12	2.4	365	3,911
52	817.01	24	0.13	2.1	370	3,905
53	847.01	81	0.37	5.1	372	5,469
54	1159.01	65	0.30	5.3	372	4,886

Discovering Earth-like Worlds by their Color 20.2



Earth is invitingly blue. Mars is angry red. Venus is brilliant white. NASA astronomer Lucy McFadden, and UCLA research assistant in geochemistry Carolyn Crow, have now discovered that a planet's "true colors" can reveal important details.

Mars is red because its soil contains rusty red stuff called iron oxide. Our planet, the "blue marble" has an atmosphere that scatters blue light rays more strongly than red ones. This suggests that astronomers could use color information to identify Earth-like worlds. Their colors will tell us which ones to study in more detail.

As NASA's Deep Impact spacecraft cruised through space, its High Resolution Instrument (HRI) measured the intensity of Earth's light. HRI is a 30-cm telescope that feeds light through seven different color filters. Each filter samples the incoming light at a different portion of the visible-light spectrum, from ultraviolet and blue to near-infrared. A table showing the reflectivity of each body is shown below. The numbers indicate the percentage of light reflected by the planet at 350, 550 and 850 nanometers (nm). For example, compared to the light that it reflects at 550 nm, Venus reflects 116% more light at 850 nm.

Object	350 nm	550 nm	850nm	Object	350nm	550nm	850nm
Mercury	47	100	142	Jupiter	60	100	64
Venus	58	100	109	Saturn	45	100	78
Earth	152	100	110	Titan	34	100	88
Moon	67	100	169	Uranus	98	100	15
Mars	34	100	203	Neptune	125	100	13

Note: Table based upon data published by the astronomers in the Astrophysical Journal (March 10, 2011).

Problem 1 – One way to plot this data so that the planets can be easily separated and identified is to plot the ratio of the reflectivities for each planet where X = R(850)/R(550) and X = R(350)/R(550). For example, for the Moon, where R(350) = 61% , R(550) = 100%, and R(850) = 155% we have X = 155/100 = 1.55, and Y = 61/100 = 0.61. Using this method, calculate X and Y for each object and then plot the (X,Y) points on a graph.

Problem 2 – The planetary data in the table can be written as an ordered triplet. For example, for Mercury the reflectivities in the table above would be written as (56, 100, 177). Using the definition for X and Y in Problem 1, which of the planets below would you classify as Earth-like, Jupiter-like, or Moon-like, if the planetary reflectivities are: Planet A (61, 82, 156), Planet B (45, 35, 56), Planet C (90, 120, 67).

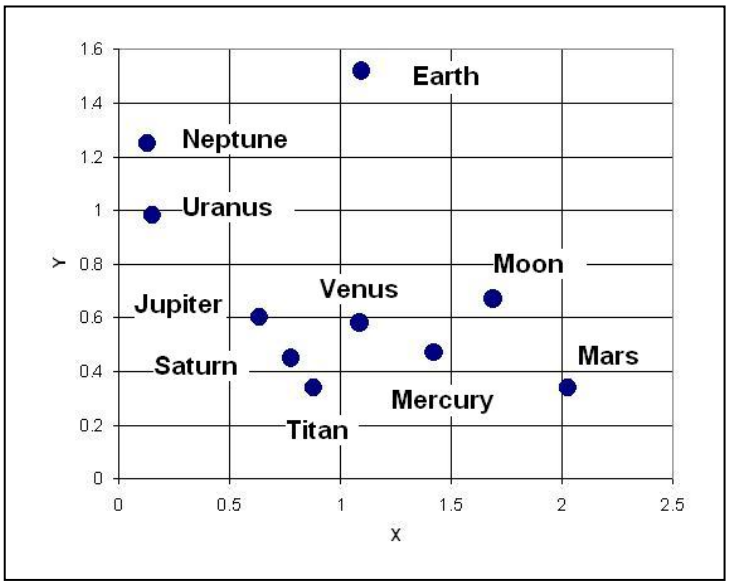
Problem 3 – Can you create a different plot for the planets that makes their differences stand out even more?

Problem 1 – One way to plot this data so that the planets can be easily separated and identified is to plot the ratio of the reflectivities for each planet where X = R(850)/R(550) and X = R(350)/R(550). For example, for the Moon, where R(350) = 61% , R(550) = 100%, and R(850) = 155% we have X = 155/100 = 1.55, and Y = 61/100 = 0.61. Using this method, calculate X and Y for each object and then plot the (X,Y) points on a graph. See graph below.

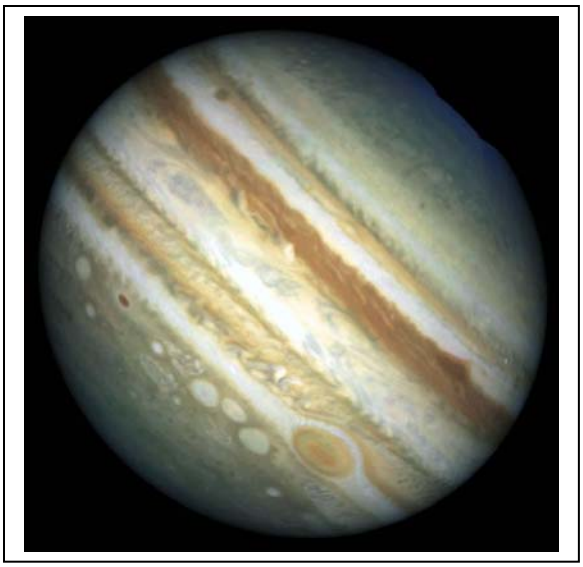
Object	350 nm	550 nm	850nm	X	Y
Mercury	47	100	142	**1.42**	**0.47**
Venus	58	100	109	**1.09**	**0.58**
Earth	152	100	110	**1.10**	**1.52**
Moon	67	100	169	**1.69**	**0.67**
Mars	34	100	203	**2.03**	**0.34**
Jupiter	60	100	64	**0.64**	**0.60**
Saturn	45	100	78	**0.78**	**0.45**
Titan	34	100	88	**0.88**	**0.34**
Uranus	98	100	15	**0.15**	**0.98**
Neptune	125	100	13	**0.13**	**1.25**

Problem 2 – The planetary data in the table can be written as an ordered triplet. For example, for Mercury the reflectivities in the table above would be written as (56, 100, 177). Using the definition for X and Y in Problem 1, which of the planets below would you classify as Earth-like, Jupiter-like, or Moon-like, if the planetary reflectivities are: Planet A (61, 82, 156), Planet B (45, 35, 56), Planet C (90, 120, 67). Answer: **Planet A has X and Y values similar to the moon; Planet B is more like Earth; and Planet C is more like Jupiter.**

Problem 3 – Can you create a different plot for the planets that makes their differences stand out even more? Answer: **There are more than 100 different ways in which students may decide to create new definitions for X and Y such as X = R(350) – R(850); Y = R(850)/R(350) and so on. Some will not visually let you see a big difference between the planet 'colors' while other may. Astronomers try many different combinations, usually with some idea of the underlying physics and how to enhance what they are looking for. There is no right or wrong answer, only ones that make the analysis easier or harder!**



The Composition of Planetary Atmospheres



All of the planets in our solar system, and some of its smaller bodies too, have an outer layer of gas we call the atmosphere. The atmosphere usually sits atop a denser, rocky crust or planetary core. Atmospheres can extend thousands of kilometers into space.

The table below gives the name of the kind of gas found in each object's atmosphere, and the total mass of the atmosphere in kilograms. The table also gives the percentage of the atmosphere composed of the gas.

Object	Mass (kilograms)	Carbon Dioxide	Nitrogen	Oxygen	Argon	Methane	Sodium	Hydrogen	Helium	Other
Sun	3.0×10^{30}							71%	26%	3%
Mercury	1000			42%			22%	22%	6%	8%
Venus	4.8×10^{20}	96%	4%							
Earth	1.4×10^{21}		78%	21%	1%					<1%
Moon	100,000				70%		1%		29%	
Mars	2.5×10^{16}	95%	2.7%		1.6%					0.7%
Jupiter	1.9×10^{27}							89.8%	10.2%	
Saturn	5.4×10^{26}							96.3%	3.2%	0.5%
Titan	9.1×10^{18}		97%			2%				1%
Uranus	8.6×10^{25}					2.3%		82.5%	15.2%	
Neptune	1.0×10^{26}					1.0%		80%	19%	
Pluto	1.3×10^{14}	8%	90%			2%				

Problem 1 – Draw a pie graph (circle graph) that shows the atmosphere constituents for Mars and Earth.

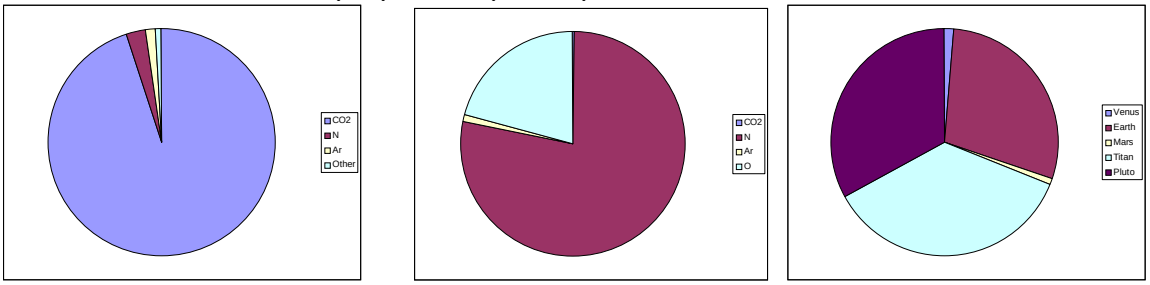
Problem 2 – Draw a pie graph that shows the percentage of Nitrogen for Venus, Earth, Mars, Titan and Pluto.

Problem 3 – Which planet has the atmosphere with the greatest percentage of Oxygen?

Problem 4 – Which planet has the atmosphere with the greatest number of kilograms of oxygen?

Problem 5 – Compare and contrast the objects with the greatest percentage of hydrogen, and the least percentage of hydrogen.

Problem 1 – Draw a pie graph (circle graph) that shows the atmosphere constituents for Mars and Earth. Answer: Mars (Ieft), Earth (middle)



Problem 2 – Draw a pie graph that shows the percentage of Nitrogen for Venus, Earth, Mars, Titan and Pluto. Answer: First add up all the percentages for Nitrogen in the column to get 271.7%. Now divide each of the percentages in the column by 271.7% to get the percentage of nitrogen in the planetary atmospheres that is taken up by each of the planets: Venus = (4/271) = 1.5%; Earth = (78/271) = 28.8%, Mars = (2.7/271)= 1.0%, Titan = (97/271)=35.8%, Pluto=(90/271)=33.2%. Plot these new percentages in a pie graph (see above right). This pie graph shoes that across our solar system, Earth, Titan and Pluto have the largest percentage of nitrogen. In each case, the source of the nitrogen is from similar physical processes involving the chemistry of the gas methane (Titan and Earth) or methane ice (Pluto).

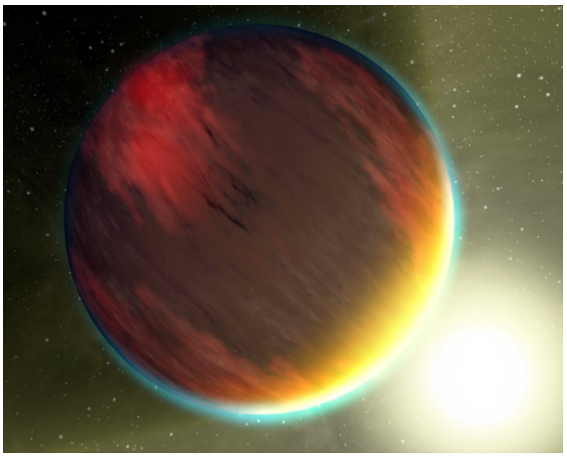
Problem 3 – Which planet has the atmosphere with the greatest percentage of Oxygen? Answer: From the table we see that **Mercury** has the greatest percentage of oxygen in its atmosphere.

Problem 4 – Which planet has the atmosphere with the greatest number of kilograms of oxygen? Answer: Only two planets have detectable oxygen: Earth and Mercury. Though mercury has the highest percentage of oxygen making up its atmosphere, the number of kilograms of oxygen is only 1000 kg x 0.42 = 420 kilograms. By comparison, **Earth** has a smaller percentage of oxygen (21%) but a vastly higher quantity: 1.4×10^{21} kg x 0.21 = **2.9×10^{20} kilograms**. (That's 290,000,000,000,000,000,000 kg)

Problem 5 – Compare and contrast the objects with the greatest percentage of hydrogen, and the least percentage of hydrogen.

Answer: The objects with the highest percentage of hydrogen are the sun, Mercury, Jupiter, Saturn, Uranus and Neptune. The objects with the least percentage are Venus, Earth, Moon, Mars, Titan, Pluto. With the exception of Mercury, which has a very thin atmosphere, the high-percentage objects are the largest bodies in the solar system. The planet Jupiter, Saturn, Uranus and Neptune are sometimes called the Gas Giants because so much of the mass of these planets consists of a gaseous atmosphere. These bodies generally lie far from the sun. The low-percentage objects are among the smallest bodies in the solar system. They are called the 'Rocky Planets' to emphasize their similarity in structure, where a rocky core and mantel are surrounded by a thin atmosphere. Most of these bodies lie close to the sun.

Organic Molecules Detected on Distant Planet! 20.4



The basic chemistry for life has been detected in a second hot gas planet, HD 209458b, depicted in this artist's concept. Two of NASA's Great Observatories – the Hubble Space Telescope and Spitzer Space Telescope, yielded spectral observations that revealed molecules of carbon dioxide, methane and water vapor in the planet's atmosphere. HD 209458b, bigger than Jupiter, occupies a tight, 3.5-day orbit around a sun-like star about 150 light years away in the constellation Pegasus. (NASA Press release October 20, 2009)

Some Interesting Facts: The distance of the planet from the star HD209458 is 7 million kilometers, and its orbit period (year) is only 3.5 days long. At this distance, the temperature of the outer atmosphere is about 1,000 C (1,800 F). At these temperatures, water, methane and carbon dioxide are all in gaseous form. It is also known to be losing hydrogen gas at a ferocious rate, which makes the planet resemble a comet! The planet itself has a mass that is 69% that of Jupiter, and a volume that is 146% greater than that of Jupiter. The unofficial name for this planet is Osiris.

Problem 1 - The mass of Jupiter is 1.9×10^{30} grams. The radius of Jupiter is 71,500 kilometers. A) What is the volume of Jupiter in cubic centimeters, assuming it is a perfect sphere? B) What is the density of Jupiter in grams per cubic centimeter (cc), based on its mass and your calculated volume?

Problem 2 - From the information provided; A) What is the volume of Osiris in cubic centimeters, if it is in the shape of a perfect sphere? B) What is the mass of Osiris in grams? C) What is the density of Osiris in grams/cc, and how does this compare to the density of Jupiter?

Problem 3 - The densities of some common ingredients for planets are as follows:

Rock ……	3.0 grams/cc
Iron ……	9.0 grams/cc
Water ……	5.0 grams/cc
Ice ……	1.0 gram/cc
Mixture of hydrogen + helium ……	0.7 grams/cc

Based on the average density of Osiris, from what substances do you think the planet is mostly composed?

Problem 1 - The mass of Jupiter is 1.9 x 10^{30} grams. The radius of Jupiter is 71,500 kilometers.
A) What is the volume of Jupiter in cubic centimeters, assuming it is a perfect sphere?
Answer: The radius of Jupiter, in centimeters, is
R = 71,500 km x (100,000 cm/1 km)
= 7.15 x 10^{9} cm.

For a sphere, V = 4/3 π R^{3} so the volume of Jupiter is
V = 1.33 x (3.141) x (7.15 x 10^{9})^{3}
V = 1.53 x 10^{30} cm ^{3}

B) What is the density of Jupiter in grams per cubic centimeter (cc), based on its mass and your calculated volume?
Answer: Density = Mass/Volume so the density of Jupiter is D = (1.9 x 10^{30} grams)/(1.53 x 10^{30} cm ^{3}) = **1.2 gm/cc**

Problem 2 - From the information provided;
A) What is the volume of Osiris in cubic centimeters, if it is in the shape of a perfect sphere?
Answer: The information says that the volume is 146% greater than Jupiter so it will be V =
V=1.46 x (1.53 x 10^{30} cm ^{3})
= **2.23 x 10^{30} cm ^{3}**

B) What is the mass of Osiris in grams?
Answer: the information says that it is 69% of Jupiter so
M = 0.69 x (1.9 x 10^{30} grams)
= **1.3 x 10^{30} grams**
C) What is the density of Osiris in grams/cc, and how does this compare to the density of Jupiter?
Answer: D = Mass/volume
= 1.3 x 10^{30} grams / 2.23 x 10^{30} cm ^{3}
= **0.58 grams/cc**

Problem 3 - The densities of some common ingredients for planets are as follows:

Rock	3 grams/cc	Ice	1 gram/cc
Iron	9 grams/cc	Mixture of hydrogen + helium	0.7 grams/cc
Water	5 grams/cc		

Based on the average density of Osiris, from what substances do you think the planet is mostly composed?

Answer: Because the density of Osiris is only about 0.6 grams/cc, the closest match would be **a mixture of hydrogen and helium.** This means that, rather than a solid planet like earth, which is a mixture of higher-density materials such as iron, rock and water, Osiris has much in common with Jupiter which is classified by astronomers as a Gas Giant!

Artist's rendition of GJ 1214b with a hypothetical moon.
(Courtesy: CfA/David Aguilar)

In December 2009, astronomers announced the discovery of the transiting super-Earth planet GJ 1214b located 42 light years from the sun, and orbits a red-dwarf star. Careful studies of this planet, which orbits a mere 2 million km from its star and takes 1.58 days to complete 'one year'. Its mass is known to be 6.5 times our Earth and a radius of about 2.7 times Earth's. Its day-side surface temperature is estimated to be 370^{o} F, and it is locked so that only one side permanently faces its star.

When a planet passes in front of its star, light from the star passes through any atmosphere the planet might contain and travels onwards to reach Earth observers. Although the disk of the planet will temporarily decrease the brightness of its star by a few percent, the addition of an atmosphere causes an additional brightness decrease. The amount depends on the thickness of the atmosphere, the presence of dust and clouds, and the chemical composition. By studying the light dimming at many different wavelengths, astronomers can distinguish between different atmospheric constituents by using specific spectral 'fingerprints'. They can also estimate the thickness of the atmosphere in relation to the diameter of the planet.

Problem 1 – Assuming the planet is a sphere, from the available information, to two significant figures, what is the average density of the planet in kg/meter^3? (Earth mass = 6.0 x 10^{24} kg; Diameter = 6378 km).

Problem 2 – The average density of Earth is 5,500 kg/m^3. Suppose that GJ 1214b has a rocky core with Earth's density and a radius of R, and a thin atmosphere with a density of D. Let R = 1.0 at the top of the atmosphere, and R=0 at the center of the planet, and assume the core is a sphere, and that the atmosphere is a spherical shell with inner radius R and outer radius R=1.0. The formula relating the atmosphere density, D, to the core radius, R, is given by:

$$1900 = (5500 - D)R^3 + D$$

A) Re-write this equation by solving for D.

B) Graph the function D(R) over the domain R:[0,1].

C) If the average density of the atmosphere is comparable to that of Venus's atmosphere for which D= 100 kg/m^3, what fraction of the radius of the planet is occupied by the Earth-like core, and what fraction is occupied by the atmosphere?

Problem 1 – Answer: Volume = 4/3 (3.141) $(2.7 \times 6378000)^3 = 2.1 \times 10^{22}$ meter^3.

Density = Mass/Volume

= $(6.5 \times 6.0 \times 10^{24}$ kg) / $(2.1 \times 10^{22}$ meter^3)

= **1,900 kg/meter^3**

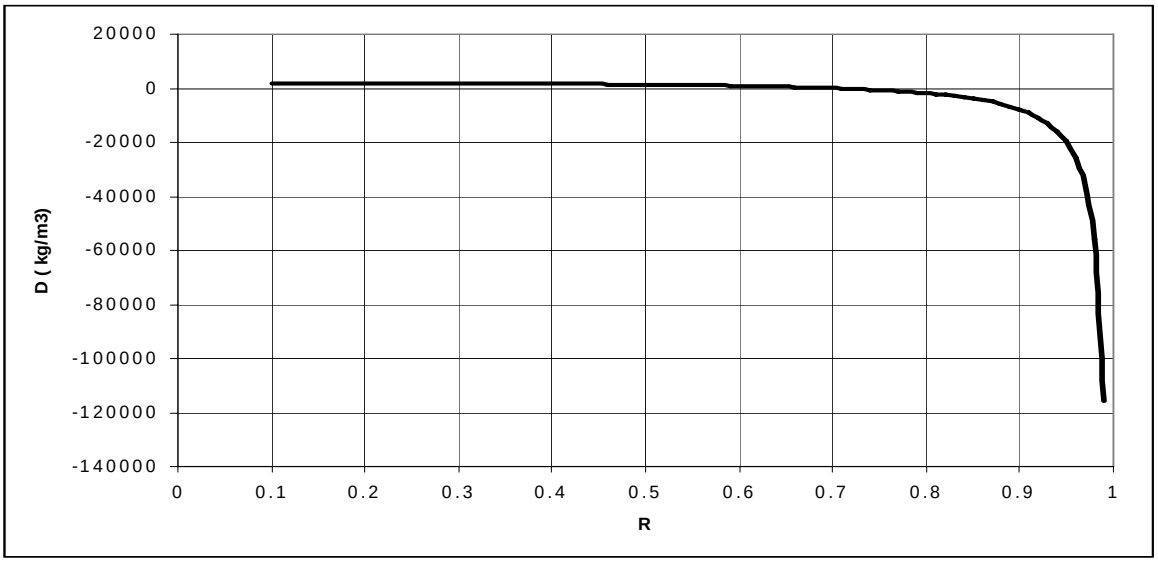
Problem 2 – A) Answer: $1900 = (5500 - D)R^3 + D$

$$1900 - D = 5500R^3 - DR^3$$

$$D\left(R^3 - 1\right) = 5500R^3 - 1900$$

$$D = \frac{5500R^3 - 1900}{R^3 - 1}$$

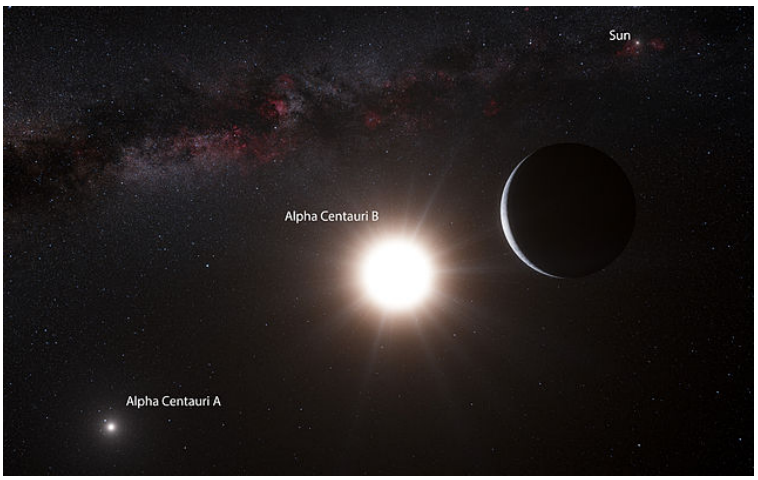
B) See below:



Note that for very thin atmospheres where D: [0, 100] the function predicts that the core has a radius of about R=0.7 or 70% of the radius of the planet. Since the planet's radius is 2.7 times Earth's radius, the core is about 0.7x2.7 = 1.9 x Earth's radius. Values for D < 0 are unphysical even though the function predicts numerical values. This is a good opportunity to discuss the limits of mathematical modeling for physical phenomena.

C) If the average density of the atmosphere is comparable to that of Venus's atmosphere for which D= 100 kg/m^3, what fraction of the radius of the planet is occupied by the Earth-like core, and what fraction is occupied by the atmosphere?

Answer: **For D=100, R = 0.71, so the core occupies the inner 71 % of the planet, and the surrounding atmospheric shell occupies the outer 29% of the planet's radius.**



Artist rendition courtesy ESO/L. Calçada/N. Risinger (skysurvey.org)

Alpha Centauri is a binary star system located 4.37 light years from our sun. The two stars, A and B, are both sun-like stars, but they are older than our sun by about 1.5 billion years.

Astronomers have used the European Space Agency's 3.6 meter telescope at La Silla in Chile to detect the tell-tail motion of Alpha Centauri B caused by an earth-sized planet in close orbit around this star.

The planet, called Alpha Centauri Bb, orbits at a distance of only six million kilometers from its parent star – closer than Mercury is to the sun. The planet is bathed in unbearable heat, and has a surface temperature of 1,200 Celsius (2,200 F or 1,500 Kelvin). This is hot enough that its surface must be mostly molten lava. Its tight orbit means a year passes in only 3.2 Earth days.

The astronomers made hundreds of measurements of the speed of the Alpha Cen B star to search for a periodic change it its speed through space. They found a change (amplitude) of about 50 cm/sec that increased and decreased with a precise period, which would only be expected from an orbiting object.

This discovery is still being confirmed through independent observations by other astronomers.

We can check the numbers in the information box ourselves. Here are a few of the measurements made of the star's speed:

Time (hours)	Speed (cm/sec)	Time (hours)	Speed (cm/sec)
6	170	48	50
10	150	56	70
21	110	71	130
33	60	83	170

Problem 1 – Graph the speed data. Draw a smooth curve through the data (which need not go through all the points) and estimate the period (in days) of the speed curve to get the orbit period of the proposed planet.

Problem 2 – Kepler's Third Law can be used to relate the period of the planet's orbit (T in years) to its distance from its star (D in Astronomical Units) using the formula

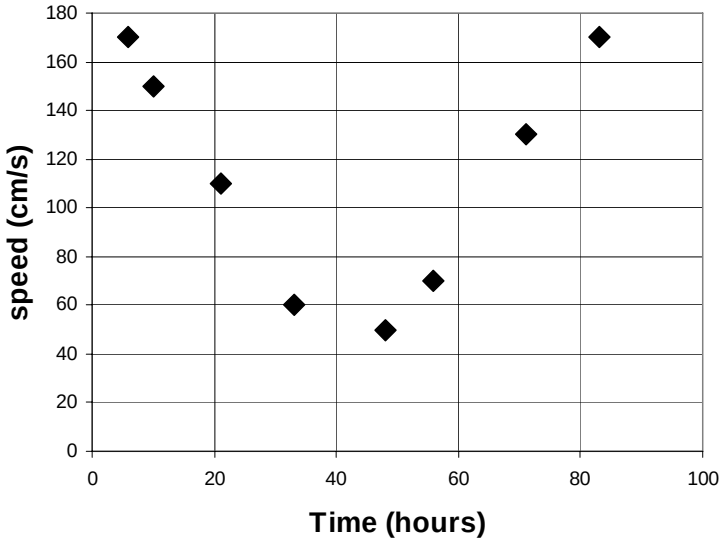
$$T^2 = D^3$$

where 1 Astronomical Unit equals the distance from Earth to our sun (150 million km). Using your estimated planet period, what is the orbit distance of the new planet from Centauri B in A) Astronomical Units? B) kilometers?

Problem 3 – What is the temperature T (in kelvins) of the new planet if its average temperature at a distance of D Astronomical Units is given by the formula:

$$T = \frac{310}{\sqrt{D}}$$

Problem 1 – Graph the speed data. Draw a smooth curve through the data and estimate the period of the speed curve to get the orbit period of the proposed planet. Answer should be about 77 hours or **3.2 days.**



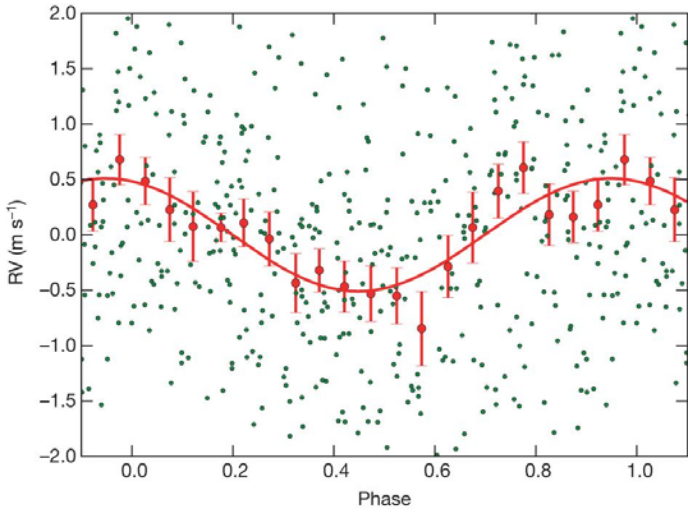
Problem 2 – Kepler's Third Law can be used to relate the period of the planet's orbit (T in years) to its distance from its star (D in Astronomical Units) using the formula

$$T^2 = D^3$$

where 1 Astronomical Unit equals the distance from the earth to our sun (150 million km). Using your estimated planet period, what is the orbit distance from Centauiri B in A) Astronomical Units? B) kilometers? Answer: T = 3.2 days or in terms of earth-years T = 3.2/365 = 0.00877 years. Then $D^3 = (0.00877)^2$, $D^3 = 7.69 \times 10^{-5}$ $D = (7.69 \times 10^{-5})^{1/3}$ **D = 0.043 Astronomical Units**. B) In kilometers, this is 0.043 AU x (150 million km/1 AU) = **6.4 million kilometers**.

Problem 3 – What is the temperature T (in kelvins) of the planet if its average temperature at a distance of D Astronomical Units. Answer: $T = 310 / (0.043)^{1/2}$ so T = **1,500 kelvins**.

The actual graph of the data published by the astronomers is shown below:



In 1961, astronomer Frank Drake devised an ingenious equation that has helped generations of scientists estimate how many intelligent civilizations may exist in the Milky Way. The 'Drake Equation' looks like this:

N = S x P x E x C x I x A x L

Where:

S = Number of stars in the Milky Way

P = Fraction of stars with planets

E = Number of planets per star in the right temperature zone

C = Fraction of planets in E actually able to support life

I = Fraction of planets in C where intelligent life evolves

A = Fraction of planets in I that communicate with radio wave technology

L = Fraction of a stars lifetime when communicating civilization exists

On Earth, bacteria have existed for nearly 4 billion years. Insects for 500 million years. Modern humans for 20,000 years. Which lifeform is the most likely to be found on a distant planet?

Known values:
S = 500 billion
P = 0.1

For our solar system:
E = 2 (Earth and Mars)
C = 0.5 (Earth)
I = 1.0 (Earth)
A = 1.0 (Earth)
L = 0.00000002 (100 yrs/4.5 billion yrs)

The great challenge is to determine from direct or indirect measurements what each of these factors might be. Fortunately, there are at least a few of these factors that we have pretty good ideas about, especially for our own planet. The estimate based on the solar system as a model is the sum of the products in the sample table N = 500 billion x 0.1 x 2 x 0.5 x 1.0 x 1.0 x 0.00000002 = 1000 civilizations existing right now.

Question 1: Based on your internet research, what do you think are the possible ranges for the factors P, E, and C? Remember to cite your sources and use only primary sources by astronomers or other scientists, not opinions by non-scientists.

Question 2: Which factors are the most uncertain and why?

Question 3: What kinds of astronomical observations might help decide what the values for E and C?

Question 4: Using the evolution of life on Earth as a guide, what could you conclude about I? What is the most likely form of life in the Milky Way?

Question 5: How would you try to estimate A using a radio telescope?

Question 6: What would the situation have to be for the value of L to be 0.000002 or 0.000000002?

Question 7: Using the Drake Equation, and a Milky Way in which 1 million intelligent civilizations exist, work backwards to create a 'typical' scenario for each factor so that a) N = 1 million. B) N = 1. Make sure you can defend your choices of the different factors.

Question 1: Based on your internet research, what do you think are the possible ranges for the factors P, E, and C?

Answer: Students may cite the following approximate ranges, or plausible variations after providing the appropriate bibliographic reference: P probably between 1% and 10% based on local planet surveys; E between 1 and 3 based on planet surveys and our own solar system; C between 0.1 and 0.5;

Question 2: Which factors are the most uncertain and why?

Answer: I , A and L. These depend on the details of evolution on non-Earth planets and it is basically anyone's guess what these numbers might be. Students may consult various internet resources or essays on the Drake Equation to get an idea of what ranges are the most talked about.

Question 3: What kinds of astronomical observations might help decide what the values for E and C?
Answer: Our best tool is to conduct surveys of nearby stars and attempt to detect earth-sized planets, not the much larger Jupiter-sized bodies that astronomers currently study.

Question 4: Using the evolution of life on Earth as a guide, what could you conclude about I? What is the most likely form of life in the Milky Way?

Answer: Bacteria were the first life forms on Earth and have survived for nearly 4 billion years. Statistically, they should be the most common life forms in the universe. Also, alien life forms are much more likely to be simple rather than complex. This is favored by the distribution of life on Earth, in which there are very few complex organisms compared to simple ones, especially if you rank them by the total mass of the species and use this to set probabilities. For example, for every billion pounds of bacteria, there are perhaps only 1 thousand pounds of species larger and more complex than insects.

Question 5: How would you try to estimate A using a radio telescope?

Answer, by measuring the radio output of thousands of stars every year and looking for signs of intelligent 'modulation' like a morse-code signal. Stars don't normally emit radio signals that vary in a precise way in time. The SETI program is continuing to conduct these kinds of surveys.

Question 6: What would the situation have to be for the value of L to be 0.000002 or 0.000000002?
Answer: A star like the sun lives for about 10 billion years. L would then be 10 billion x 0.000002 = 20,000 years or as little as 10 billion x 0.000000002 = 20 years. In the first case, a civilization has learned to survive its technological 'Childhood' and prosper. In the second case, the civilization may have perished after learning how to use radio technology, or it may still be a thriving civilization that no longer uses radio communication.

Question 7: Using the Drake Equation, and a Milky Way in which 1 million intelligent civilizations exist, work backwards to create a 'typical' scenario for each factor so that a) N = 1 million. B) N = 1.

Answer: This will depend on the values that students assign to the various factors. Make sure that each student can defend their choice. This may also be opened to class discussion as a wrap-up.

Extracting Oxygen from Moon Rocks

About 85% of the mass of a rocket is taken up by oxygen for the fuel, and for astronaut life support. Thanks to the Apollo Program, we know that as much as 45% of the mass of lunar soil compounds consists of oxygen. The first job for lunar colonists will be to 'crack' lunar rock compounds to mine oxygen.

NASA has promised $250,000 for the first team capable of pulling breathable oxygen from mock moon dirt; the latest award in the space agency's Centennial Challenges program.

Lunar soil is rich in oxides of silicon, calcium and iron. In fact, 43% of the mass of lunar soil is oxygen. One of the most common lunar minerals is *ilmenite*, a mixture of iron, titanium, and oxygen. To separate *ilmenite* into its primary constituents, we add hydrogen and heat the mixture. This hydrogen reduction reaction is given by the 'molar' equation:

$$FeTiO_3 + H_2 \text{ --- > } Fe + TiO_2 + H_2O$$

A Bit Of Chemistry - This equation is read from left to right as follows: One mole of *ilmenite* is combined with one mole of molecular hydrogen gas to produce one mole of free iron, one mole of titanium dioxide, and one mole of water. Note that the three atoms of oxygen on the left side (O_3) is 'balanced' by the three atoms of oxygen found on the right side (two in TiO_2 and one in H_2O). **One 'mole' equals 6.02 x 10^{23} molecules**.

The 'molar mass' of a molecule is the mass that the molecule has if there are 1 mole of them present. The masses of each atom that comprise the molecules are added up to get the molar mass of the molecule. Here's how you do this:

For H_2O, there are two atoms of hydrogen and one atom of oxygen. The atomic mass of hydrogen is 1.0 AMU and oxygen is 16.0 AMU, so the molar mass of H_2O is 2 (1.0) + 16.0 = 18.0 AMU. **One mole of water molecules will equal 18 grams of water by mass.**

Problem 1 -The atomic masses of the atoms in the *ilmenite* reduction equation are Fe = 55.8 and Ti = 47.9. A) What is the molar mass of ilmenite? B) What is the molar mass of molecular hydrogen gas? C) What is the molar mass of free iron? D) What is the molar mass of titanium dioxide? E) Is mass conserved in this reaction?

Problem 2 - If 1 kg of ilmenite was 'cracked' how many grams of water would be produced?

Inquiry Question - If 1 kg of ilmenite was 'cracked' how many grams of molecular oxygen would be produced if the water molecules were split by electrolysis into

$$2\ H_2O \text{ -- > } 2H_2 + O_2?$$

Problem 1 -

A) What is the molar mass of ilmenite? 1(55.8) + 1 (47.9) + 3(16.0) = **151.7 grams/mole**
B) What is the molar mass of molecular hydrogen gas? 2(1.0) = **2.0 grams/mole**
C) What is the molar mass of free iron? 1(55.8) = **55.8 grams/mole**
D) What is the molar mass of titanium dioxide? 1(47.9) + 2(16.0) = **79.9 grams/mole**
E) Is mass conserved in this reaction? **Yes. There is one mole for each item on each side, so we just add the molar masses for each constituent. The left side has 151.7 + 2.0 = 153.7 grams and the right side has 55.8 + 79.9 + 18.0 = 153.7 grams so the mass balances on each side.**

Problem 2 -

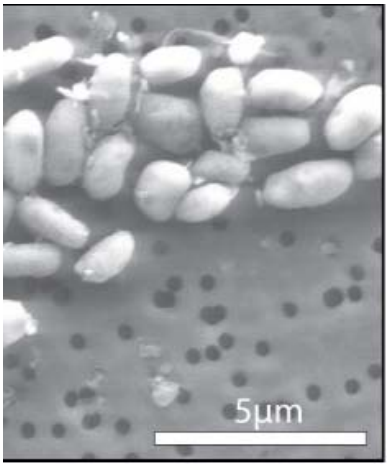
Step 1 - The reaction equation is balanced in terms of one mole of ilmenite (1.0 x $FeTiO_3$) yielding one mole of water (1.0 x H_2O) . The molar mass of ilmenite is 151.7 grams which is the same as 0.1517 kilograms, so we just need to figure out how many moles is needed to make one kilogram.

Step 2 - This will be 1000 grams/151.7 grams = 6.6 moles. Because our new reaction is that we start with 6.6 x $FeTiO_3$ that means that for the reaction to remain balanced, we need to produce 6.6 x H_2O, or in other words, 6.6 moles of water.

Step 3 - Because the molar mass of water is 18.0 grams/mole, the total mass of water produced will be 6.6 x 18.0 = **119 grams of water.**

Inquiry Question - The reaction is: $2\ H_2O$ -- > $2H_2 + O_2$

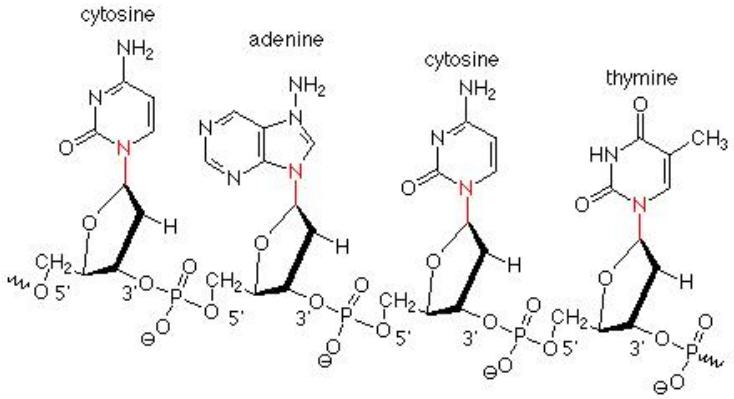
This means that for every 2 moles of water, we will get one mole of O_2. The ratio is 2 to 1. From the answer to Problem 2 ,we began with 6.59 moles of water not 2.0 moles. That means we will produce 6.6/2 = 3.3 moles of water. Since 1 molecule of oxygen has a molar mass of 2(16) = 32 grams/mole, the total mass of molecular oxygen will be 3.3 moles x 32 grams/mole = **106 grams. So, 1 kilogram of ilmenite will eventually yield 106 grams of breathable oxygen.**



Microphotograph of the new bacterium GFAJ-1 that subsists on the toxic element arsenic.

NASA researchers exploring extremophile bacteria in Mono Lake, California claimed to have discovered a new strain of bacterium GFAJ-1 in the Gammaproteobacteria group, which not only feeds on the poisonous element arsenic, but incorporates this element in its DNA as a replacement for normal phosphorus. All other known life forms on Earth use 'standard' DNA chemistry based upon the common elements carbon, oxygen, nitrogen and phosphorus.

In the search for life on other worlds, knowing that 'life' can exist that is fundamentally different than Earth life now broadens the possible places to search for the chemistry of life in the universe.



This diagram shows the elements that make up a small section of normal DNA containing the four bases represented from top to bottom by the sequence 'CACT'. They are held together by a 'phosphate backbone' consisting of a phosphor atom, P, bonded to four oxygen atoms, O. Each phosphor group (called a phosphodiester) links together two sugar molecules (dioxyribose), which in turn bond to each of the bases by a nitrogen atom, N.

Problem 1 - The atomic mass of phosphor P= 31 AMU, arsenic As= 75 AMU, hydrogen H=1 AMU and Oxygen O= 16 AMU. A) What is the total atomic mass of one phosphodiester molecule represented by the formula PO_4 ? B) For the new bacterium, what is the total atomic mass of one arsenate molecule represented by the formula AsO_4?

Problem 2 - The DNA for the smallest known bacterium, mycoplasma genetalium, has about 582,970 base pairs. Suppose that the 1,166,000 phosphodiester molecules contribute about 30% of the total mass of this organism's DNA. If arsenic were substituted for phosphorus to form a twin arsenic-based organism, by how much would the DNA of the new organism increase?

Problem 1 - The atomic mass of phosphor P= 31 AMU, arsenic As= 75 AMU, hydrogen H=1 AMU and Oxygen O= 16 AMU. A) What is the total atomic mass of one phosphodiester molecule represented by the formula PO_4? B) For the new bacterium, what is the total atomic mass of one arsenate molecule represented by the formula AsO_4 ?

Answer: A) PO_4 = 1 Phosphorus + 4 Oxygen
= 1 x 31 AMU + 4 x 16 AMU
= **95 AMU**

B) AsO_4 = 1 Arsenic + 4 Oxygen
= 1 x 75 AMU + 4 x 16 AMU
= **139 AMU**

Problem 2 - The DNA for the smallest known bacterium, mycoplasma genetalium, has about 582,970 base pairs. Suppose that the 1,166,000 phosphodiester molecules contribute about 30% of the total mass of this organism's DNA. If arsenic were substituted for phosphorus to form a twin arsenic-based organism, by how much would the DNA of the new organism increase?

Answer: The arsenic-substituted ester has a mass of 139 AMU compared to the phosphorus-based ester with 95 AMU, so the new molecule AsO_4 is 100% x (95/139) = 68% more massive than PO_4.

Since in the normal DNA the PO_4 contributes 30% of the total DNA mass, the non-PO_4 molecules contribute 70% of the normal mass.

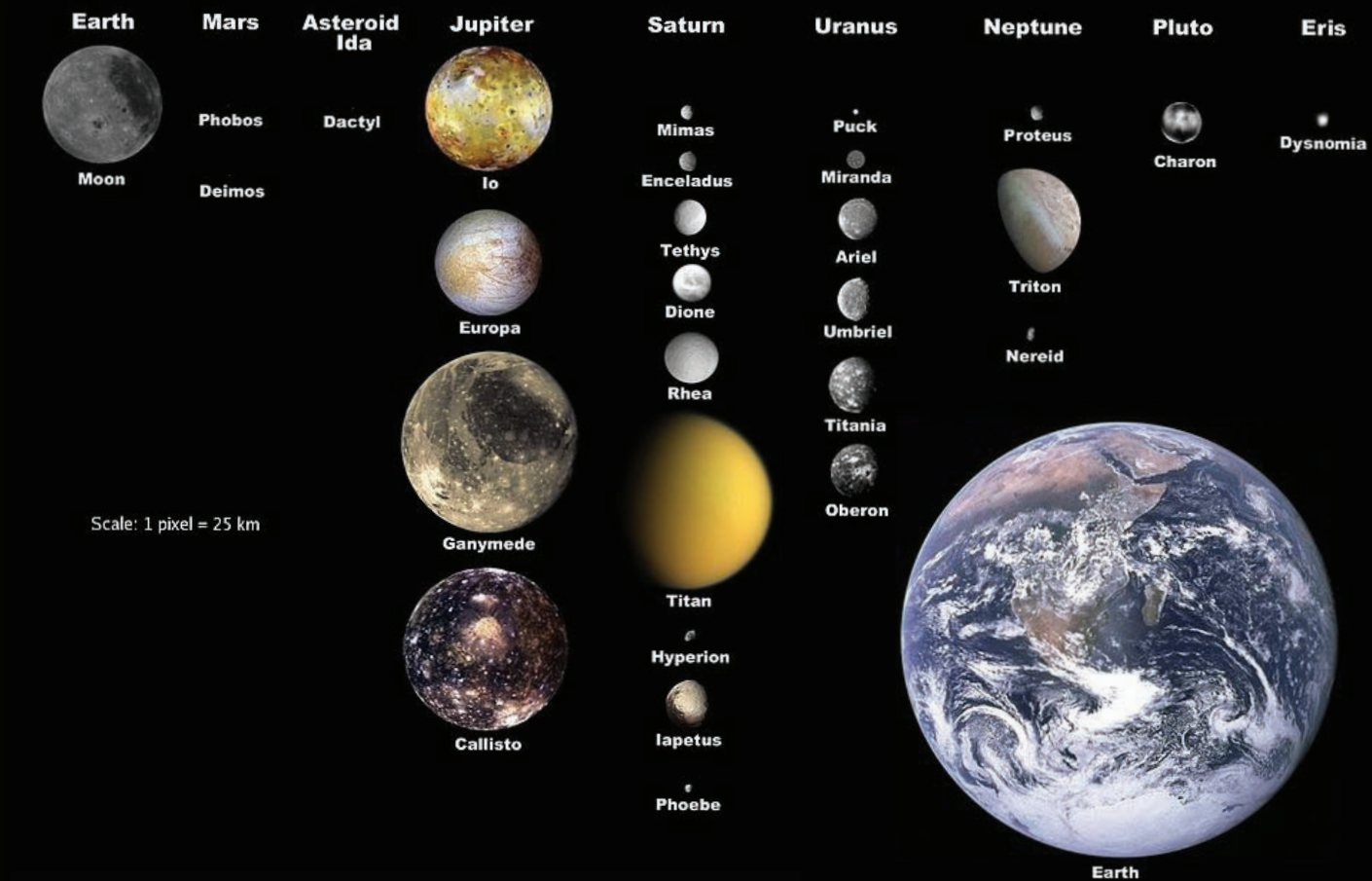
This is added to the new arsenic-based molecule mass for AsO_4 of 30% x1.68 = 50% to get a new mass that is 70% + 50% = **120% heavier** than the original, 'normal' DNA based on PO_4.

So we would predict that the DNA of the twin arsenic-based organism is only 20% more massive than the DNA of the original phosphate-based organism.

Note: Students may have a better sense of the calculation if they start with a concrete amount of 100 grams of normal DNA. Then 70 grams are in the non-PO_4 molecules and 30 grams is in the PO_4 molecules. Because AsO_4 is 68% more massive than PO_4, its contribution would be 30 grams x 1.68 = 50 grams. Then adding this to the 70 grams you get 120 grams with is 20 grams more massive than normal DNA for a gain of 120%.

New research published in 2012 now disputes the claim that the organism is truly an arsenic-based life form.

Selected Moons of the Solar System, with Earth for Scale



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