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TO THE DIRECTOR OF THE
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WASHINGTON, D. C.

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No. 73

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UNITED STATES GEOLOGICAL SURVEY

J. W. POWELL, DIRECTOR

THE

VISCOSITY OF SOLIDS

BY

CARL BARUS.

WASHINGTON

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1891

CONTENTS.

ILLUSTRATIONS.

PREFACE.

This bulletin is a second contribution to the general investigation of the physical contents of rocks, the experiments concerning which follow a general plan devised by Mr. Clarence King.

Questions bearing directly on the viscosity of rock masses make up so large a part of Mr. King's geological observations, that the duty of inquiring into the physics of this enormously complicated subject devolved seriously upon me. Above all things some form of reliable working hypothesis was to be discovered; and this is what the present bulletin endeavors to accomplish. I believe the physical hypothesis has been found and that the data afford substantial corroboration of Maxwell's theory of the viscosity of solids.

The method by which I attempt to arrive at the result in question consists in investigating the details of viscous behavior of steel. Steel has a phenomenal quality, in that it can be made to pass continuously from an extreme of brittle hardness almost to soft plasticity. Hence its value in serving as a test of any theory.

Curiously enough, viscous deformation (Nachwirkung) is strikingly pronounced at the hard and brittle end of the scale, whereas soft steel is relatively rigid. These facts are established in Chapter I, of which they form the larger part. To interpret the striking results of Chapter I it is necessary to consider them from a more general point of view. Consequently Chapter II investigates them further with regard to their general truth under variable conditions of temperature and of stress. But the full interpretation is not attempted until Chapter III, where, together with other relevant matter, they are shown to bear directly on the fundamental hypotheses of Maxwell's theory of the viscosity of solids.

Chapter IV applies a magnetic method to the study of the behavior of a strained atomic configuration, and is thus an important deduction from Chapter III.

Chapter V finally exhibits the solidity of the three states of aggregation. The results show an astounding range of variation, inasmuch as viscosity on passing from extreme fluid to extreme solid changes from at most 10^{-5}, to at least 10^{20}.

By selecting steel as the chief source of evidence I gain a second important advantage. The researches of Dr. Strouhal and myself have

thrown new light on a number of the physical qualities of steel, some of which are closely allied to the subject in hand. Thus there exists a reciprocal relation between the earlier bulletins of the Geological Survey, No. 14, No. 27 (pp. 30–60), No. 35, and No. 42 (pp. 98–131), and the present bulletin, by which clearness of conception regarding the physics of the iron-carburets is promoted.

The notation has been made uniform throughout the work, and the data reduced to the same standard. This occasioned much extra labor, since it involved a new computation of the chief results of Chapters II and III. However, seeing that the unity of the book could be much enhanced thereby, the time was not grudgingly spent.

Meantime the researches on solid viscosity have been considerably extended. I have now nearly ready for the press a sequel[1] to this volume, in which the *molecular* changes which promote viscous motion are specially discussed. The results enable me to submit a general theory of solid molecular structure, and they therefore fitly make up a second part of the continued series of researches of which this bulletin constitutes the first part.

I may add in concluding, that the researches on steel were begun by Dr. Strouhal and myself and continued to such a stage of progress as made it advisable for us to publish subsequent investigations together. But the serious delays incident to epistolary communication at best, and the great distance between us (Prague–Washington), has dictated, very much to our regret, a dissolution of literary partnership. The science of to-day is more and more exactingly a science of specialized responsibilities, and the connection between a statement and its author can not be too direct or too obvious.

CARL BARUS.

LABORATORY U. S. GEOLOGICAL SURVEY,
Washington, D. C., March 15, 1890.

[1] Cf. Annual Report of the Director for the fiscal year 1888–'89.

THE VISCOSITY OF SOLIDS.

By Carl Barus.

CHAPTER I.

By Carl Barus and Vincent Strouhal.

THE VISCOSITY OF STEEL AND ITS RELATIONS TO TEMPER.

INTRODUCTION.

1. In Bulletin No. 42, U. S. Geol. Survey, p. 131, we expressed the belief that the qualities of retaining magnetism exhibited by steel would probably stand in relation to the viscous properties of the metal. In the present chapter we make a first search for such a relation. The work is restricted to torsional viscosity because it usually exists in larger amount, and is much more accurately measurable than any of the other manifestations of viscosity. Possibly it may be brought more rationally into comparison with magnetic phenomena. We therefore develop partially (i. e. so far as is necessary for our especial purposes) a very sensitive differential method for the study of torsional viscosity.[1] Having applied it to steel, we compare the results obtained with the known behavior of permanent linear magnets, tempered under like conditions. During the course of the experiments we incidentally come upon certain ulterior results (viscosity of iron, glass).

In our magnetic comparisons we have endeavored to avoid prematurity. Many inferences of the original draught have been cropped. We do not wish to ignore the magnetic effect of the chemical changes of steel. We have merely endeavored to *circumscribe* the magnetic effect of carburation, by exploring the magnetic importance of certain mechanical properties of steel. To do this we have studied, abstractly as it were, the occurence of permanent magnetism in a medium of continuously varying viscosity, without insisting on an inherent relation between retentiveness and viscosity.

[1] Independently of ourselves, as we subsequently found, a similar method was used by Boltzmann and by O. E. Meyer (Pogg. Ann., vol. 154, 1875, p. 357).

LITERARY NOTES.

2. The necessary existence of viscosity in solids was pointed out, discovered, and its nature described by Weber;[1] but it is to Kohlrausch's[2] extensive researches that we owe the bulk of our present knowledge. A new method of research was indicated by Sir William Thomson,[3] which method was developed, independent of Thomson we believe, by Streintz[4] and by Schmidt.[5] Thomson[6] concisely defined the terms now in English use and followed by Maxwell (Heat, Chapter II).

PLAN OF RESEARCH.

3. We proposed at first to study the viscosity of steel from two distinct standpoints: one part of our work was to consider viscosity in its relations to temperature; the other part, viscosity in its relations to hardness. Indeed, we have as yet no data at all of this kind for steel. For other substances Kohlrausch[7] and after him Streintz[8] and Pisati[9] had already studied viscosity and temperature with some detail though in a direction differing somewhat from our own. Very recently, however, Schroeder[10] has experimented in a line of research quite identical with the one upon which we had determined; and though his data do not include steel, the results obtained for other metals are sufficiently pronounced to make a special investigation of steel appear superfluous, at least so far as regards its present bearing on our work. Schroeder[11] finds "Der bei 100° begonnene Verlauf der Nachwirkung wird durch Abkühlung des Drahtes auf Zimmertemperatur unterbrochen; nach

[1] Weber: Götting. Gelehrt. Anz., St. 8, 1835; Pogg. Ann., vol. 24, 1835, p. 247; ibid., vol. 54, 1841, p. 1; Comm. Soc. Götting., vol. 3, 1841, p. 45. Weber's researches express the amount of viscous deformation in terms of time by a hyperbolic formula with three constants.

[2] Kohlrausch: Pogg. Ann., vol. 119, 1863, p. 337; ibid., vol. 128, 1866, pp. 207, 399; ibid., vol. 155, 1875, p. 579; ibid., vol. 158, 1876, p. 337. The author subjected fibers of a great number of substances to torsional, tensile and flexural stress, and he discusses the amount of deformation in each case in its dependence on time and on temperature. The results are too elaborate to be excerpted here. The author finds it necessary to use exponential relations with three and even four constants to describe the results completely. In this place we advert, moreover, to the papers of Neesen (Pogg. Ann., vol. 153, 1874, p. 498; ibid., vol. 157, 1876, p. 579; Wied. Ann., vol. 7, 1879, p. 460), of Braun (Pogg. Ann., vol. 119, 1876, p. 337), of Wiedemann (Wied. Ann., vol. 6, 1879, p. 496) which supply details of critical discussion. Perhaps we may even add Pernet's well known thermometric researches (Carl's Repertorium, vol. 11, 1875, p. 257, and elsewhere).

[3] Sir William Thomson: Phil. Mag. (IV), vol. 30, 1865, p. 63. Thomson refers the loss of energy of of vibration to the existence of molecular friction.

[4] Streintz: Pogg. Ann., vol. 153, 1874, p. 387; Wien. Ber., vol. 69, part 2d, 1874, p. 355. Using a vibration method the author discusses elaborately the effect of viscosity on the logarithmic decrement.

[5] Schmidt: Wied. Ann., vol. 2, 1877, pp. 48, 241, supplements Thomson's and Streintz's results, and is able to express logarithmic decrement in terms of time by Weber's formula of three constants. We may add Cohn (Wied. Ann., vol. 6, 1879, p. 403), Hopkinson (Phil. Trans., 1877), Warburg (Wied. Ann., vol. 13, 1881, p. 141), Auerbach (Wied. Ann., vol. 14, 1881, p. 308), Kohlrausch (Pogg. Ann., vol. 128, 1886), Ewing (Phil. Trans., 1885, p. 545) and others who discuss viscosity as manifesting itself in phenomena of conductivity (Cohn), residual static charge (Hopkinson, Kohlrausch), magnetism (Auerbach, Warburg, Ewing and others).

[6] Thomson: loc. cit.

[7] Kohlrausch: Pogg. Ann., vol. 128, 1866, p. 216; ibid., vol. 158, 1876, p. 371.

[8] Streintz: Wien. Ber., vol. 69, 1874, p. 337; also Schmidt, loc. cit.

[9] Pisati: Wien. Ber., vol. 80, 1879, p. 427. Also Gaz. Chim. Ital., 1876, 1877.

[10] Schroeder: Wied. Ann., vol. 28, 1876, p. 369.

[11] Schroeder: Ibid., p. 388.

erneuter Erwärmung auf 100° wird die vorher unterbrochene Drehung fortgesetzt." We pointed out[1] that in case of drawn metals, of quenched glass (Rupert's drops) and we inferred also in case of tempered steel, energy has been stored up in virtue of the rigidity of the material; and that on exposure to temperature (annealing), the available excess of stress is made to produce the observed viscous deformation precisely in the way in which Schroeder has found it for applied torsional stress. Nevertheless, the behavior of steel, when considered in all its details, is much more complicated and is, indeed, quite unique, as will be indicated in the course of this chapter.

METHOD OF EXPERIMENT.

APPARATUS.

For the reasons given the scope of this chapter has been limited to a study of the viscosity of steel, in its relations to temper. We endeavor therefore to determine the difference of viscosity which corresponds to a given known difference of hardness. The following differential method of measurement, the principle of which can be made clear in a few words, naturally suggests itself:

Bifilar apparatus.—Given a pair of steel wires, one of which is always glass-hard, the other of known temper; let them be made the threads of a *bifilar suspension* provided with suitable means for the measurement of the angular motion of the line defined by the lower double point of attachment, relative to the upper line of attachment; finally let a twist of $+T$ be imparted to one steel wire, a twist of $-T$ to the other. If now the system be left to itself, and if the two wires be equally viscous the

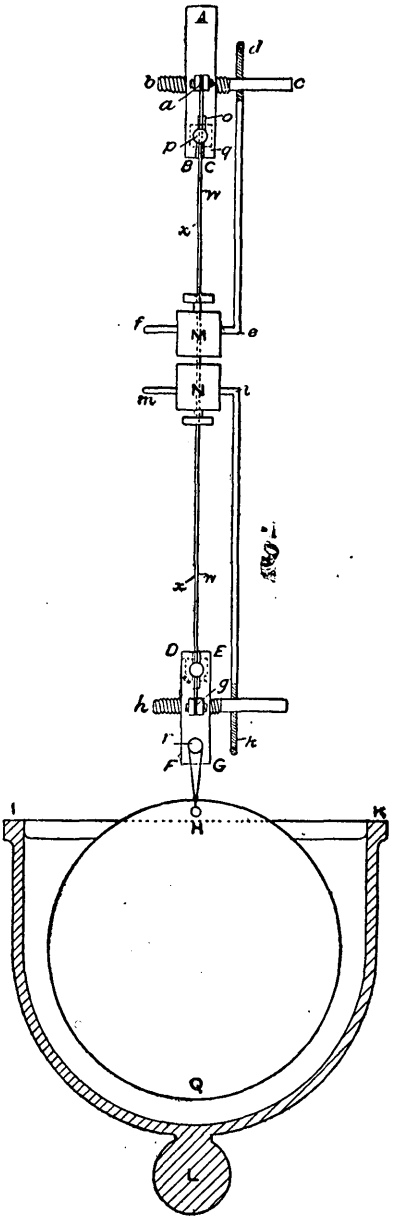

FIG. 1.—Bifilar apparatus for torsional viscosity; scale ¼.

[1] Bull. U. S. Geol. Survey, No. 14, 1885, p. 196; Am. Jour. Sci. (III), vol. 31, 1866, p. 452.

bifilar system cæteris paribus will remain at rest. If on the other hand the viscosities be unequal, the lower free part of the bifilar will move around the vertical let fall from the upper part. The sign of this angular motion will be that of the twist of the wire of greater viscosity; its amount cæteris paribus directly dependent on the difference of viscosity of the pair of wires under experiment. One of the forms of apparatus actually used is given in the annexed diagrams, Figs. 1 to 3, the former on a scale of $\frac{1}{4}$. The couple of steel wires *awwg* and *axxg* to be tested, are fastened above to a massive piece of brass *ABC* and below to a similar piece *DEGF*. *ABC* is fixed firmly to the wall at some distance from it, and all incidental motion or displacement is registered by the mirror *M*, adjustable at pleasure by aid of the screw-arms *fed* and *bc*. *DEGF* is free to move under the influence of the bifilar couples and motion is similarly registered by the mirror *N* adjustable at pleasure by aid of the screw arms *mlk* and *h*. To keep the line of symmetry of the piece *DEGF* vertical, to deaden incidental vibrations, and to give general stability to the whole bifilar adjustment, we suspended a heavy lead disk *HQ* (5 lbs.) from the screw *r*. *HQ* is partially immersed in water contained in a glass bell-jar *ILK* also (adjustably) attached to the wall.

The steel wires are inserted and then fastened in the apparatus as follows: To keep the two steel wires (about 33^{cm} long) under a permanent strain of torsion, each is softened about 2^{cm} from the ends inward. The soft parts are then bent into a hook-like loop just large enough to slide over the screws of the little brass bolts *a* and *g*. Having fastened the lower ends by the bolt *g*, the upper ends are each twisted 180° against each other and then also fastened by the bolt *a*.

The ends of this twisted system are now to be secured to the blocks of brass *ABC* and *DEGF*. One method of fastening is given ($\frac{1}{2}$ actual size) in Fig. 2, the parts of which are lettered similarly to Fig. 1. The two wires, *w* and *x*, are pressed firmly against a little rectangular piece of steel *o*, by aid of the bolt *pp'* and the nut *qq'*. The bolt is provided with a square hole near its head, through which *o* and the steel wires may easily be passed. A little pin prevents *o* from rotating around the axis of the screw, thus diminishing liability to breakage. In the apparatus figured it is necessary to slide the bolts *pp'* over the wires *w*, *x*, before the torsion is applied by fastening *a*.

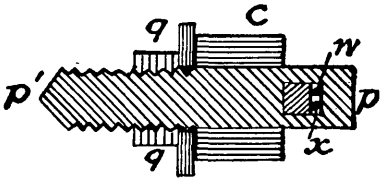

FIG. 2.—Device for suspension; scale, $\frac{1}{2}$.

In Fig. 3 the same apparatus is given in side elevation (sectional), and as the same lettering is here used as in Figs. 1, 2, this diagram is at once intelligible. It indicates in a clearer way the method of insert-

ing and fastening the twisted systems *wx*, *wx*, with reference to which wires the experiments are made.

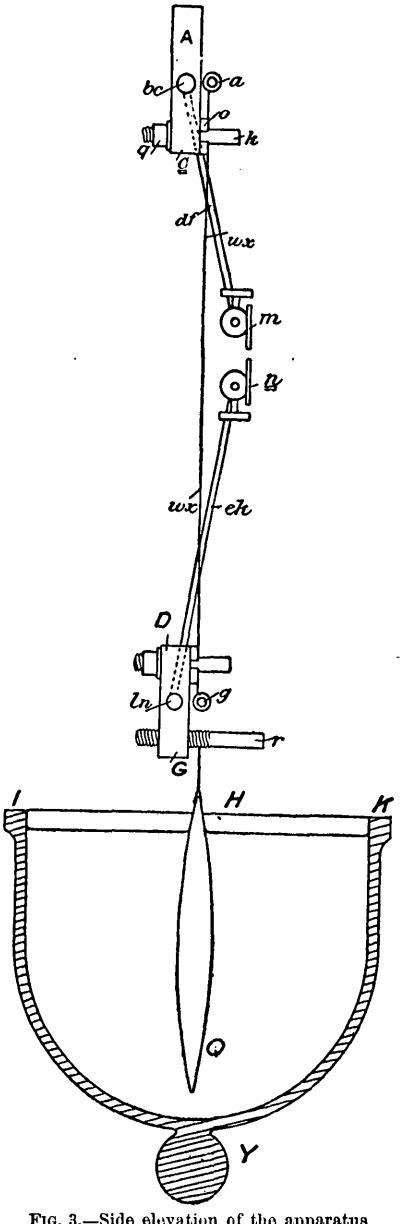

FIG. 3.—Side elevation of the apparatus shown in Fig. 1.

In a more convenient form of apparatus, perhaps, we replace the brass blocks *ABC* and *DEGF* by little hand vises, the axle of the lower one of which has been perforated and fitted with a screw subserving the purposes of *r* in the figure. By aid of a disc of lead vertical symmetry of the whole apparatus is here also maintained.

The advantage of this form of apparatus, apart from our own special purposes, is that with a single adjustment, the viscous movement can be traced with extreme accuracy for an indefinite length of time. There is no doubt that in some of the cases examined such motion will demonstrably exist even for a year after the torsion has been imparted. Kohlrausch's method is an absolute method; but his wire must be twisted and untwisted once for each observation. The method of logarithmic decrement is difficult of interpretation, and is really a complex method and limited to short intervals of time.

In experimenting with glass it was necessary to modify the apparatus so that twists less than $T=180°$ could be applied. This is easily done by perforating the screw of the upper bolt and passing the glass fiber through the hole. Torsion in any amount may then be applied to the projecting fiber.

5. *Tubular apparatus.*—Our second (tubular) apparatus combines with the advantages of the one just sketched, the ulterior desideratum of enabling the observer to follow the viscous detorsion immediately after applying stress; and therefore also the advantage of exhibiting the

effect of positive torsion immediately succeeding negative torsion and vice versa; or of studying the effect produced by an indefinite number of alternations of the sign of the twist. In the second form of apparatus the wire is introduced into a narrow tube just large enough to surround it. The tube is fixed above and fastened to the wire below. Twist is imparted to the wire and through it to the tube. The observations, in other respects, are conducted as above.

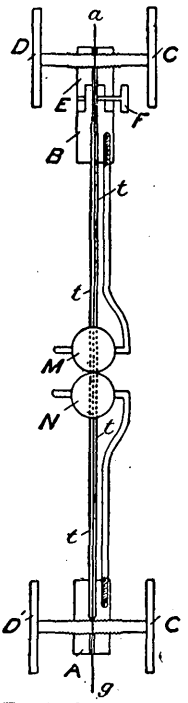

FIG. 4.—Sectional elevation of the tubular apparatus.

Figure 4 is a sectional elevation of the tubular apparatus. Here E is a massive cylinder of brass, in the lower socket of which the reduced end of the cylinder B fits snugly, so as to be free to rotate around the vertical, unless fastened by the steel pin F. E and B are axially perforated, and B carries the tube *tttt*, which in its turn carries the lower massive cylinder of brass A. The steel wire to be tested, ag, is inserted through the perforations in E, B, A, and securely fastened against rotation by the lateral screws CD and $C'D'$. N is the movable mirror, adjustably fastened to the lower bob A; M is the fixed mirror (fiducial mark) adjustably fastened to B.

The cylinder B is stationary, being secured by a clamp not shown in the figure. Twist is imparted to the system of tube *tttt* and wire ag, by withdrawing the pin F, rotating E, and then again inserting the pin.

6. *Unifilar apparatus.*—A third form of apparatus in which the wires are countertwisted unifilarly is described in Chapter II.

METHOD OF OBSERVING.

7. In both forms of apparatus, angles are registered by Gauss' method of telescope[1] and scale. We secure extreme accuracy by using two mirrors, one of which (stationary) furnishes a reliable fiducial mark for the other (movable). These mirrors are easily so adjusted that their respective scale-images appear *simultaneously* in the field of the telescope, one above the other and intersected by the same cross-hair. Indeed, it would not be difficult so to arrange this device that the stationary image could be used as a vernier on the moving image.

METHOD OF TEMPERING.

8. *Quenching.*—The wires used (Stubs's steel) were hardened galvanically as described elsewhere.[2] To Prof. H. A. Rowland, who placed his

[1] Three wood screws, the head of one of which is planed, of another hollowed out conically, make a good adjustable plane-dot-slot arrangement.

[2] Bull. U. S. Geol. Survey, No, 14. p. 29; Wied. Ann., 1880, vol. 11, p. 932. Cf. § 55.

dynamo-electric machine at our disposal for this purpose, and favored us with the benefit of his advice during almost the entire afternoon, we owe our cordial thanks.

The apparatus for quenching deserves special description, being simplified in general character and made available for divers ulterior purposes. It is given perspectively in Fig. 5, slightly tilted forward to

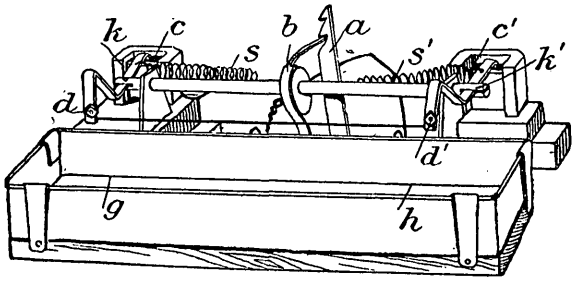

FIG. 5.—Galvanic apparatus for quenching steel.

show the parts. Here *gh* is the trough of sheet iron in which quenching is to take place. The wire *dd'*, kept slightly stretched by two horizontal beak-shaped arms *dk*, *d'k'* which are swiveled on the axle *kk'*, is suspended horizontally just above the water in *gh*. A spring at *b*, tends to snap the wire into the water, unless prevented by the notched stop *a*. *b*, therefore, tends to rotate the axle *kk'* which to secure insulation is made of hard rubber. Finally the rear ends of the said beak-shaped arms, are bent downward and amalgamated. During the suspension of the steel wire they dip into oblong adjustable troughs of mercury *c*, *c'* which communicate with a battery or dynamo. Finally, two spiral springs, *s*, *s'*, by drawing the rear ends of the arms together, (swivel at *k* and *k'*) keep the wire stretched, as has been stated, even at red heat. The level of mercury in *c* and *c'* is so adjusted that the current of the battery is broken the moment the wire touches the surface of the water.

The method of operating is easily understood. Having closed the current and heated the wire to the degree of redness desired, the pin at *a* is quickly released, which whips the wire suddenly into the water and at the same time, as already stated, automatically breaks the currents. The cold wire will be found quenched glass-hard, or indeed harder, so that specific resistance may run as high as $s=50$ microhms. As regards homogeneity, too, these wires are exceptionally excellent, as will be indicated below, twenty wires of about 150^{cm} each in length having for certain special purposes been examined per 2^{cm} of length of each.

The present apparatus offers many conveniences by which quenching is appreciably facilitated. This will be seen by comparing it with the older galvanic apparatus.[1] Alterations, moreover, are easily introduced by which quenching may be effected in water or metal of any tempera-

[1] Bull. U. S. Geol. Survey, No. 14, 1885, p. 29.

ture (simply heating the trough *gh* with burners placed underneath); and by which the rod may be quenched from any temperature of red heat corresponding to known intensity of the galvanic current which heats the wire.

9. *Annealing.*—The annealing was effected by drawing the glass-hard wires through a zone of constant temperature by clockwork.[1] Cf. § 55.

This operation is also of such importance and so difficult to accomplish in case of very long wires, that an apparatus by which it is satisfactorily done deserves description. It is given in perspective view in Fig. 6, and consists essentially of a Barus' *boiling tube T*, containing the substance boiling at the temperature at which annealing is to take place. Water 100°, aniline 185°, diphenylamine 310°, mercury 360°, sulphur 450°, are usually sufficient for annealing work. Other substances are given elsewhere.[2] These substances are easily boiled by aid of the ring burner surrounding the tube, and annealing is commenced when the thermal distribution has become stationary. The central tube of the boiling-tube *T* is prolonged downward by attaching the closed tube *t*, long enough to envelope the cold steel wire to be annealed. This wire (or wires) is suspended from a very fine copper filament, the upper end of which is fastened to the drum or disc *C* of the adjustable clock. When the clock has been set in motion the wire is slowly drawn through the zone of ebullition, by the revolving drum *C*. All parts are attached to a ring stand as shown, and easily adjustable. In the figure the wires are given at *w* prolonged, in which position they are about half way through the zone of ebullition. If *h* be the height of the zone and *R* the radius of the disc *C* revolving once per hour, then

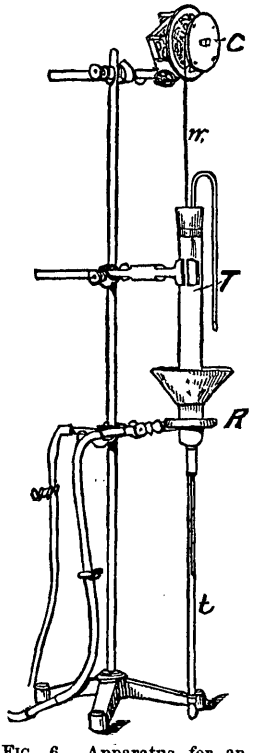

FIG. 6.—Apparatus for annealing long steel wire.

$$t=\frac{h}{2\pi R}$$

is the time of annealing in hours. Again by making $h=2\pi$, the time of exposure in hours is the reciprocal of the radius of the disc in centimeters.

This method of annealing is exceedingly convenient at high temperatures, as only a very narrow zone of constant temperature is called for. Between 500° and 1,000° such work is done by replacing the annealing tube by an annealing crucible of porcelain or fire clay of the

[1] Barus: Am. Jour. Sci. (III), 1886, vol. 32, p. 279.
[2] Barus: Bull. U. S. Geol. Survey, No. 54, 1889, p. 87.

form given in Fig. 7, the crucible being placed in a suitable Fletcher furnace, as shown in Bulletin No. 54. Cadmium, zinc bismuth, may then be used for high temperature boiling points and the rods suitably protected by atmospheres of hydrogen or carbon dioxide.

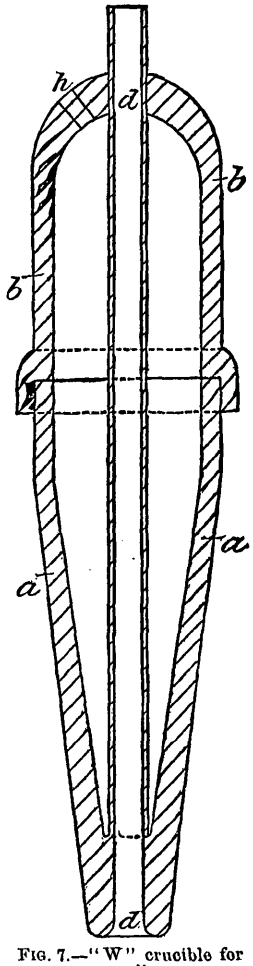

FIG. 7.—"W" crucible for annealing.

ELIMINATION OF ERRORS.

10. The steel wire to be used in these experiments are all about 0·1^{cm} in thickness. Hence to determine the condition of equilibrium or motion in the torsional apparatus described, it is necessary to consider the respective moments of the bifilar couple (M_b), of the torsional couple ($M\tau$) and of the flexural couple (M_f), by which the movable end is actuated.

The moment of the bifilar couple has the well known form[1]

$$M_b=\tfrac{1}{4}\frac{\lambda\lambda'}{l}mg \sin \alpha \quad . \quad . \quad . \quad . \quad (1)$$

where λ and λ' are respectively the distance apart of the upper ends and the lower ends of the bifilar wires, l the vertical distance and α the horizontal angle between the lines λ and λ', mg the weight of the whole suspended adjustment. To mg should be added one-half the weight of the wires.

The moment of the torsion couple is $M_\tau=\tau\alpha$, where τ is the sum of the torsional coefficients of the wires. This applies because the sum of the stored torsional moments, for $\alpha=0$, is $T+(-T)=0$. We may give M_τ an experimentally convenient form by substituting for τ the approximate value derived in elastics. We obtain

$$M\tau=\frac{2\pi}{5}\frac{E\rho^4}{l}\alpha \quad . \quad . \quad . \quad . \quad (2)$$

where ρ is the radius of the steel wires used, E the *c. g. s.* value[2] of the modulus of elasticity of steel.

The moment of the flexural couple is more involved. If p be the distance between vertical planes passing through the bifilar wires, and if f be the horizontal component of the flexural force, then the flexural couple is $M_f=fp$. We have however by geometry, $p=(\lambda\lambda' \sin \alpha)/s$,

[1] Kohlrausch Leitfaden, 1884, p. 167. Maxwell, 1881, vol. ii, p. 108.

[2] Poisson's coefficient is in the following estimate taken at $\frac{1}{4}$: If E' be the current technical modulus referred to the kilogram and square millimeter, $E=E'g\times10^6$.

where s is the sum of the horizontal projections of the wires. We have moreover given us in elastics

$$E=\frac{4}{3}f\frac{l^3}{\pi\rho^4}\frac{1}{\frac{s}{2}},$$

and therefore the couple in question becomes

$$M_f=\frac{3\pi}{8}\frac{\lambda\lambda'\rho^4}{l^3}E\sin\alpha \quad . \quad . \quad . \quad (3)$$

The equations 1, 2, 3 enable us to choose the dimensions of the wire and of the apparatus as well as to select conditions of experiment such that the torsional couple may alone be effective. The following are extremely unfavorable phases of the best available values, in question: $\lambda=\lambda'<0{\cdot}2^{cm}$; $l>25^{cm}$; $m<2500^g$; $\alpha<0{\cdot}2^\circ$;

$$\rho=0{\cdot}041^{cm};\ E=2\times10^{12}\ (ml^{-1}t^{-2}).$$

In view of the small values of α it is sufficient to compare the coefficients only. We obtain nearly

$$M_b:M_\tau:M_f=1000:280000:20.$$

Hence it appears that in the case of the chosen dimension the most unfavorable effect of the bifilar and flexural couples combined is less than 0·4 per cent of the torsional couple. Moreover the torsional couple becomes more predominant as the angle of deviation, α, increases, so that no serious inaccuracy from the discrepancies here enumerated need be apprehended.

11. When we operate with glass-hard steel, we encounter so many mechanical difficulties, that the conditions postulated in the foregoing analysis can not be immediately assumed. We can not, for instance, adjust the tensions of the two wires precisely to equality. Nor is it easy so to store the permanent torsion that the parallelism of the lines joining the upper and lower points of suspension remains intact. Under these circumstances flexural and bifilar couples may have very different values from the ones accepted, and the moments as a whole, will form a complex aggregate. Although it is improbable that the flexural and bifilar discrepancies will exceed the limits of error investigated, it is none the less desirable to determine them by direct experiment. This can be done with convenience and great accuracy.

Let the water be removed from the jar ILK, and the system be put in vibration around a vertical and in such a way that the angle α remains small. Then the circumstances of motion are given by

$$\frac{4\pi^2K}{Z^2}=\frac{1}{4}\frac{\lambda\lambda'}{l}mg+\tau+\frac{\delta M_f}{\delta\alpha},$$

where K is the moment of inertia of the bifilar body, Z the time of a complete vibration. If we replace the steel wires by threads for which

τ and $\delta M_f / \delta \alpha$ are zero, we may find the value of the bifilar coefficient in terms of the other two. We cite the following typical experiment:

Steel wires: $Z_s = 0{\cdot}025$ sec.; $l = 26^{cm}$; $\lambda < 0{\cdot}1^{cm}$;

Brass capillary wire: $Z_b = 0{\cdot}35$ sec.; $l = 30^{cm}$; $\lambda = 0{\cdot}2^{cm}$; $\lambda' = 0{\cdot}3^{cm}$.

If we reduce the results for capillary brass wire to the l and λ which obtain for steel, we deduce $Z_b = 0{\cdot}40$ sec. Hence the bifilar coefficient is less than 0·4 per cent of the combined torsional and flexural coefficients, in very unfavorable values of the chosen dimensions. This experimental result agrees with the one calculated above.

12. The following static method is better, being exhaustive and final. Let the lower ends of the pair of identical steel wires, A and B, be fastened with the bolt. Let rotation at this end be temporarily checked. Then we may store like degrees of torsion by twisting the upper ends of the wires in the following three ways:

a. Let the wire A be twisted $+360°$ and then fastened to B. Left to itself the upper end of the system will rotate $-180°$ so that the residual torsion of A is $+180°$; of B is $-180°$. The total couple actuating the bifilar body then is of the form $\pm M_b + M_\tau \pm M_f$.

b. Let the wires A and B be twisted $+180°$ and $-180°$ respectively and then fastened together. Left to itself the upper end of the system will not move. The residual torsion is therefore the same as in case *a*; but the total couple here actuating the bifilar body is $M_\tau \pm 0$. This is the case premised in the analysis which introduces this paragraph.

c. Let the wire B be twisted $-360°$ and then fastened to A. Left to itself the upper end will rotate $+180°$, so that the residual torsions on A and B are the same as in cases *a* and *b*; but the total couple which in case of viscous motion actuates the bifilar body has now the form $\mp M_b + M_\tau \mp M_f$.

This is the device: We are able to *commutate*, as it were, the combined flexual and bifilar couples relative to the torsional couple of *fixed* sign. In the results below this test is frequently applied and we will there show that for the chosen dimensions the torsion couple only need be considered.

13. In the case of the tubular apparatus, Fig. 4, the present precautions are at once superfluous. Here each wire is introduced into the tube and compared with the next wire introduced under like circumstances. The tubular apparatus is specially adapted for the investigation of viscous detorsion in its dependence on the applied torsional stress, as well as on time. But the same work may also well be done with the bifilar.

The viscous effect of the relative sections of the wires and allied observations, with a bearing on viscosity proper, can more expediently be made after the experimental data have been communicated. It is convenient to insert a few remarks on the rigidity of efficient parts of the above apparatus here. In the tubular adjustment, if d be the diameter of the wire, d_1 and D_1 the diameters of the tube, if l symbolize the effective

length, T the amount of stored twist, G the rigidity, we have, in view of the equal couples,

$$T\frac{\pi d^4 G}{32l}=T_1\frac{\pi(D^4{}_1-d^4{}_1)G_1}{32l_1} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Now in order to make sharp comparisons on the viscosity of solids, in order to investigate its dependence both on time and on stress, the apparatus is to be so adjusted that *like stresses produce like strains* in the substances compared. Hence put $T=T_1$, whence

$$\frac{d^4}{D^4{}_1-d^4{}_1}=\frac{G_1 l}{G l_1}=\text{const.}\frac{l}{l_1} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

This applies to the bifilar apparatus for $d_1=0$. The equation (2) shows at once how to study viscosity under conditions in which like stresses produce like strains upon like sectional areas of the viscous solids compared.

EXPERIMENTAL DATA.

NOTATION.

14. In Bulletin 14 we used the word "retentiveness" in the restricted German sense to designate magnetic stability, or the property in virtue of which a magnet resists such influences as temperature, percussion, time. The common English use of the term is much broader than this and refers to residual induction[1] in a general way. Fortunately the context of our paper is sufficiently clear and does not admit of serious misapprehension. Inasmuch as measurement of total induction has not yet fallen properly within the scope of our work, we purposely withhold many obvious allusions to magnetic permeability. We moreover retain the term permanent magnetization to denote the magnetic moment *per unit of mass* of a permanently saturated rod, for the sake of uniformity with our earlier notation. Its ratio to magnetic intensity is therefore the density of steel.

To avoid troublesome circumlocution we use the adjective "viscous" with the very broad meaning of "pertaining to viscosity." Such expressions as "viscous phenomena" though not elegant are convenient. Similarly we often speak without confusion of "linear magnetization" where "magnetization of a linear rod" is meant. Cf. § 55.

INTRODUCTORY EXPLANATIONS.

15. The following tables, Nos. 1 to 26, contain results for the viscosity of steel in different states of temper. To these are appended other tables of a miscellaneous kind. For convenience of comparison we insert a little index. The apparatus used in obtaining the data are desig-

[1] Mr. Ewing proposes a new and very elegant extension of the current use of "retentiveness." He defines it as "resistance to any change of magnetic state which (magnets) exhibit whenever the magnetic field in which they are placed suffers any change."—Phil. Trans., vol. 2, 1885, p. 526.

nated by the Roman numerals in the first column. Nos. I to IV are different forms of the bifilar adjustment; No. V is the tubular instrument. The index also contains the numbers of the rods or fibers joined together to make the bifilar couple and gives other information readily understood. Glass and iron fibers are lettered. It is to be remembered that in the steel measurements one of the pair of wires is always glass-hard, the other annealed as specified.

INDEX TO TABLES.

I. EXPERIMENTS PROPER.

Temper of the rod coupled with glass-hard.

Apparatus No.	Annealed at 20°.	Annealed at 100°.	Annealed at 190°.
I.	Table 1, Rods 47, 48.	Table 3, Rods 3, 4. Table 4, Rods 43, 44.	
II.	Table 2, Rods 1, 2.	Table 5, Rods 5, 6. Table 6, Rods 45, 46.	Table 9, Rods 11, 12.
III.		Table 7, Rods 7, 8.	Table 10, Rods 13, 14. Table 11, Rods 19, 20.
IV.		Table 8, Rods 9, 10.	Table 12, Rods 15, 16.

Apparatus No.	Annealed at 360°.	Annealed at 450°.	Annealed at 1000° (soft).
I.		Table 15, Rods 23, 24.	Table 17, Rods 53, 54 (drawn). Table 18, Rods 27, 28 (soft).
II.	Table 13, Rods 17, 18.	Table 16, Rods 25, 26.	Table 19, Rods 29, 30 (commercial drawn).
III.			Table 20, Rods 31, 32 (commercial annealed).
IV.	Table 14, Rods 21, 22.		Table 21, Rods 33, 34 (annealed from glass-hard).

II. MISCELLANEOUS EXPERIMENTS.

Apparatus No.	Glass fiber, *a*.	Glass fiber, *b*.	Iron drawn. *h*, *c*, *k*.
I.	Table 22, Rod 35.	Table 23, Rods 36, 42.	Table 25, Rod 49.
II.			Table 24, Rod 37.
III, IV.			Table 27, Rod 50.
V.			

Apparatus No.	Iron (soft). *g*, *d*, *m*, *n*.	Brass tube (thick). *e*.	Brass tube. (thin). *f*.
I.	Table 26, Rod 38.		
I.	Table 28, Rod 55.		
I.	Table 29, Rod 56.		
V.		Table 38, Rods 39, 40.	Table 39, Rod 41.

Apparatus.	Miscellaneous soft steel.
III.	Table 30, Rods 51, 52.
III.	Table 31, Rods 61, *p* (nickel).
III.	Table 35, Rods 57, 58.
IV.	Table 36, Rods 59, 60.
III.	Table 37, Rods 57, 58, 59, 60 (quadrifilar).
IV.	Table 32, Rods 62, *q* (drawn copper).
I.	Table 33, Rods 63, *r* (soft copper).
I.	Table 34, Rods 64, *s* (soft copper).

EXPERIMENTS PROPER.

16. *Notation.*—In Tables 1 to 26, R denotes the distance between mirror and scale, l the efficient length of the steel (bifilar) wires, λ the distance between the upper and lower points of suspension. T (in degrees) is the stored torsion, imparted to the whole length (30^{cm}) of wire, positively or negatively as specified, whereas $\varphi - \varphi'$ (in radians) is the amount of angular motion (differential detorsion) of the bifilar system due to viscosity, per centimeter of length of the system of two steel wires. Positive values of T and $\varphi - \varphi'$ refer to the same angular direction, and $\varphi - \varphi'$ is arbitrarily put zero (to fix the co-ordinates) at the time one hour after the twist T has been imparted. This is the best conveniently available method for comparing the divers data. Other methods would have compelled us to wait for the subsidence of motion, certainly several months in each case. Moreover, to obtain extreme degrees of accuracy, it is necessary to study viscosity under circumstances of constant temperature, and in a place where the viscous yielding may proceed unaccompanied by tremor or vibratory motion of the apparatus. For these nice experiments we are not now prepared.

The tables furthermore contain the date, the number of hours (h_0) elapsed since the beginning of the experiment, and the arbitrary deflection, N (radians per centimeter of bifilar length), for each of the times specified. The radius of the wires is given under ρ. Finally the sense of the couples actuating the bifilar body is indicated by τ (torsion), b (bifilar), f (flexure).

In practice N is the angle between the normals of the upper and the lower mirror (Fig. 1) divided by the efficient length l, of the wires.

17. The symbol $(\varphi - \varphi')$ has been chosen to indicate the essentially differential character of this variable, as will be fully exhibited. Here it is merely necessary to point out that φ refers to the viscous yielding of one of the counter-twisted wires and φ' to the other. In succeeding chapters we shall usually replace T by the rate of twist $\tau = T / l$. In the present chapter, since there is no means of measuring the total viscous yield during the interval of observation, T is the initial value 180° given by the method of adjustment, § 12. As the experiment continues, T is reduced in value.

It is to be remembered that one of the two wires is always glass-hard.

18. *Rods annealed* at 25° and at 100°.—The data obtained are given in Tables 1 to 8.

TABLE 1.—APPARATUS I.

$R=217^{cm}$; $l=27^{cm}$; $\lambda<0{\cdot}2^{cm}$. Rods: No. 47, glass-hard, $T=-180°$; No. 48, glass-hard, $T=+180°$. $2\rho=0{\cdot}082^{cm}$.

Date.	$N\times10^3$	h_0	$(\phi-\phi')\times10^3$	Date.	$N\times10^3$	h_0	$(\phi-\phi')\times10^3$
Aug. 21, 6h 08m	(*)	0·00	(*)	Aug. 25, 9h 05m	0·615	91·92	0·156
6 21	0·438	0·22	−0·030	7 01	0·632		
7 08	0·468	1·00	+0·000	26, 9h 10m	0·653	114·52	0·194
22, 9h 57m	0·543	15·82	0·075	4 08	0·670		
5 58	0·552	23·83	0·084	27, 9h 11m	0·695	140·26	0·237
6 54	0·555	24·77	0·087	7 36	0·714		
23, 9h 08m	0·568	39 00	0·100	28, 9h 21m	0·734	164·00	0·276
12 31	0·572	42·38	0·104	6 55	0·755		
5 37	0·577	47·48	0·109	29, 10h 12m	0·780	188·11	0·318
24, 9h 17m	0·588	67·12	0·126	6 17	0.792		
5 13	0·600						

* Adjusted $\tau-b-f$.

TABLE 2.—APPARATUS II.

$R=203^{cm}$; $l=26^{cm}$; $\lambda<0{\cdot}2^{cm}$. Rods: No. 1, glass-hard, $T=+180°$; No. 2, glass-hard, $T=-180°$. $2\rho=0{\cdot}082^{cm}$.

Date.	$N\times10^3$	h_0	$(\phi-\phi')\times10^6$	Date.	$N\times10^3$	h_0	$(\phi-\phi')\times10^3$
June 11, 4h 30m	(*)	0·00	(*)	June 12, 8h 35m	1·920	16·08	0·015
5 00	1·896	0.50	−0·009	9 25	1·922	16·92	0·017
5 30	1·905	1·00	±0·000	10 10	1·922	17·67	0·017
6 15	1·909	1·75	+0·004	1 5	1·926	20·58	0·021
7 00	1·910	2·50	0·005	2 20	1·926	21·83	0·021
7 33	1·913	3·06	0·008	5 00	1·926	24·50	0·021
				5 50	1·926	25·33	0·021
				8 30	1·926	28 00	0·021
				13, 8h 30m	1·926	40·00	0.021

* Adjusted $\tau-b-f$.

TABLE 3.—APPARATUS I.

$R=209^{cm}$; $l=27^{cm}$; $\lambda<0.2^{cm}$. Rods: No. 3, glass-hard, $T=-180°$; No. 4, annealed at 100°, 10h, $T=+180°$. $2\rho=0{\cdot}082^{cm}$.

Date.	$N\times10^3$	h_0	$(\phi-\phi')\times10^3$	Date.	$N\times10^3$	h_0	$(\phi-\phi')\times10^3$
June 24, 8h 30m	(*)	0·00	(*)	June 26, 8h 23m	0·755	47·88	0·561
9 00	0·139	0·50	−0·055	12 04	0·767	51·57	0·573
9 30	0·194	1·00	±0·000	3 17	0·778	54·78	0·584
10 56	0·277	2·43	+0·083	6 50	0·792	58·33	0·598
11 49	0·312	3·32	0·118	27, 8h 50m	0·825	72·33	0·631
12 37	0·338	4·12	0·144	3 04	0·834	78·57	0·640
1 52	0·373	5·37	0·179	28, 8h 09m	0·857	95·65	0·663
3 15	0·401	6·75	0·207	11 29	0·857	98·98	0·663
4 01	0·418	7·52	0.224	5 22	0·859	104·87	0·665
4 55	0·433	8·42	0·239	29, 8h 07m	0·864	118·63	0·670
6 5	0·453	9·58	0·259				
25, 8h 41m	0·619	24·18	0·425				
11 35	0·640	27·08	0·446				
5 25	0·682	32·92	0·488				
6 58	0·692	34·47	0·498				

* Adjusted $\tau-b-f$.

TABLE 4.—APPARATUS I.

$R=217^{cm}$; $l=27^{cm}$; $\lambda<0{\cdot}2^{cm}$. Rods: No. 43, glass-hard, $T=-180°$; No. 44, annealed at 100°, 9^h, $T=+180°$. $2\rho=0{\cdot}082$.

Date.	$N\times10^3$	h_0	$(\phi-\phi')\times10^3$	Date.	$N\times10^3$	h_0	$(\phi-\phi')\times10^3$
Aug. 8, 5^h 00^m	(*)	0·00	(*)	Aug. 14, 9^h 11^m	−0·447	140·19	1·618
5 17	1·265	0·28	−0·125	5 12	−0·509		
6 17	1·127	1·28	+0·013	15, 1^h 34^m	−0·635	167·07	1·790
7 33	1·043	2·55	+0·097	6 35	−0·664		
9, 9^h 05^m	0·736	16 08	0·404	16, 1^h 27^m	−0·755	191·18	1·909
11 56	0·686	18·93	0·454	6 55	−0·783		
1 06	0 664	20·10	0·476	17, 9^h 22^m	−0·858	213·15	2·028
7 05	0·579	26·08	0·561	6 55	−0·918		
10, 10^h 02^m	0·427	41·03	0·713	18, 9^h 15^m	−0·984	236·36	2·139
1 19	0·388	44·32	0·752	5 29	−1·015		
6 20	0·333	49·33	0·807	19, 9^h 03^m	−1·066	260·82	2·220
11, 10^h 06^m	0·184	69·15	1·001	6 35	−1·094		
1 15	0 147			20, 9^h 16^m	−1·136	285·00	2·289
7 06	0·085			6 44	−1·162		
12, 9^h 22^m	−0·042	92·43	1·220	21, 9^h 07^m	−1·196	304·12	2·336
5 30	−0·117						
13, 9^h 09^m	−0·245	116·10	1·422				
1 28	−0 285						
4 41	−0·316						

* Adjusted $\tau+b+f$.

TABLE 5.—APPARATUS II.

$R=203^{cm}$; $l=26^{cm}$; $\lambda<0{\cdot}2^{cm}$. Rods: No. 5, glass-hard, $T=-180°$; No. 6, annealed at 100°, 9^h, $T=+180°$. $2\rho=0{\cdot}082^{cm}$.

Date.	$N\times10^3$	h_0	$(\phi-\phi')\times10^3$	Date.	$N\times10^3$	h_0	$(\phi-\phi')\times10^3$
June 13, 9^h 15^m	(*)	0 00	(*)	June 14, 9^h 00^m	2·393	23·75	0·486
10 15	1·907	1·00	0·000	10 30	2·401	25·25	0·494
11 15	2·031	2·00	0·124	1 3	2·407	27·80	0·500
12 05	2·100	2·83	0·193	2 6	2·409	28·85	0·502
1 25	2·162	4·17	0·255	3 50	2·409	30·58	0·502
3 16	2·247	6·02	0·340	5 27	2·412	32·20	0·505
5 05	2·296	7·83	0·389	†6 53	2·412	33·60	0·505
6 50	2·345	9·58	0·438				
7 30	2·358	10·25	0·451				

* Adjusted, $\tau-b-f$. † Subsequent movement apparently retrograde.

TABLE 6.—APPARATUS II.

$R=202^{cm}$; $l=27^{cm}$; $\lambda<0{\cdot}2^{cm}$. Rods: No. 45, glass-hard, $T=-180°$; No. 46, annealed at 100°, 10^h, $T=+180°$. $2\rho=0{\cdot}082^{cm}$.

Date.	$N\times10^3$	h_0	$(\phi-\phi')\times10^3$	Date.	$N\times10^3$	h_0	$(\phi-\phi')\times10^3$
Aug. 8, 6^h 10^m	(*)	0·00	(*)	Aug. 14, 9^h 11^m	0·440	139·03	1·020
6 19	1·502	0·15	−0 062	5 12	0·399		
7 32	1·422	1·37	+0·018	15, 1^h 34^m	0·316	165·91	1·133
9, 9^h 55^m	1·195	15·75	0·245	6 35	0·298		
12 31	1·169	18·35	0·271	16, 1^h 27^m	0·234	190·02	1·218
7 52	1·098	25·70	0·342	6 55	0·211		
10, 10^h 02^m	1·007	39·87	0·433	17, 9^h 22^m	0·163	211·98	1·299
1 19	0·993	43·15	0·447	6 55	0·119		
6 19	0·948	48·15	0·492	18, 9^h 15^m	0·070	235·20	1·382
11, 10^h 45^m	0·853	68·19	0·615	5 29	0·046		
1 14	0·833			19, 9^h 03^m	0·007	259·65	1·440
7 05	0·789			6 35	−0·008		
12, 9^h 22^m	0·709	91·26	0·756	20, 9^h 16^m	−0·040	283·83	1.488
5 30	0·658			6 44	−0·056		
13, 9^h 09^m	0·576	114·93	0·891				
1 28	0·544						
4 41	0·526						

* Adjusted, $\tau+b+f$.

TABLE 7.—APPARATUS III.

$R=400^{cm}$; $l=27^{cm}$; $\lambda<0.2^{cm}$. Rods: No. 7, glass-hard, $T=-180°$; No. 8, annealed at 100°, 10^h, $T=+180°$. $2\rho=0.082^{cm}$.

Date.	$N\times10^3$	h_0	$(\phi-\phi')\times10^3$	Date.	$N\times10^3$	h_0	$(\phi-\phi')\times10^3$
July 1, 12^h 40^m	(*)	0·00	(*)	July 6, 8^h 40^m	1·890	120·61	1·137
1 20	3·122	0 67	−0·128	1 53	1·854		
2 45	2·904	2·08	±0·000	5 16	1·826		
3 25	2 956	2·76	+0·038	7, 8^h 53^m	1·730	144·55	1·300
4 37	2 903	3·95	0·091	1 30	1·691		
5 39	2·869	4·98	0·125	5 16	1·660		
6 59	2·832	6·32	0·162	8, 9^h 16^m	1·547	168·59	1·485
2, 9^h 22^m	2·627	20·70	0·367	12 43	1,515		
1 46	2·578	25·10	0·416	5 48	1·465		
5 11	2·554	28·52	0·440	9, 8^h 35^m	1·366	192·02	1·657
3, 8^h 20^m	2·435	43·67	0·559	12 11	1·339		
12 14	2·397	47·57	0·597	5 18	1·307		
6 30	2·331	53·83	0·663	10, 8^h 07^m	1·225	215·87	1·798
4, 9^h 47_m	2·233	73·10	0·800	12 09	1·200		
12 09	2·208			5 20	1·162		
3 08	2·183			11, 9^h 08^m	1·080	236·47	1·914
6 00	2·153						
5, 8^h 40^m	2·061	95·80	0·965				
12 00	2·033						
4 44	1·992						

* Adjusted, $\tau+b+f$.

TABLE 8.—APPARATUS IV.

$R=308^{cm}$; $l=26^{cm}$; $\lambda<0.2^{cm}$. Rods: No. 9, glass-hard, $T=-180°$; No. 10, annealed at 100°, 10^h, $T=+180°$. $2\rho=0.082^{cm}$.

Date.	$N\times10^3$	h_0	$(\phi-\phi')\times10^3$	Date.	$N\times10^3$	h_0	$(\phi-\phi')\times10^3$
July 1, 1^h 40^m	(*)	0·00	(*)	July 5, 8^h 41^m	1·876	94·81	0·718
2 46	1·185	1·10	0·000	12 00	1·899		
3 26	1·213	1·77	0·028	4 45	1·935		
4 39	1·246	2·82	0·061	6, 8^h 42^m	2·027	119·63	0·874
5 40	1·265	4·00	0·080	1 54	2·063		
7 00	1·290	5.33	0·105	5 17	2·088		
2, 9^h 23^m	1·420	19·72	0·236	7, 8^h 54^m	2·185	143·58	1·034
1 48	1·459	24·13	0·274	1 32	2·222		
5 12	1·482	28·53	0·297	5 18	2·249		
3, 8^h 22^m	1·577	42·70	0·392	8, 9^h 18^m	2·362	167·61	1·206
12 15	1·605	46·58	0·420	12 43	2·385		
6 30	1·641	52·83	0·456	5 49	2·427		
4, 9^h 49^m	1·729	72·12	0 574	9, 8^h 37^m	2·524	191·04	1·366
12 10	1·747			12 11	2·547		
3 09	1·770			5 19	2·581		
6 01	1·791			10, 8^h 07^m	2·655	210·45	1·470

* Adjusted. $\tau-b-f$.

The data of these tables are graphically given in the accompanying Fig. 8, deformation $(\varphi-\varphi')$, as ordinate, time in hours as abscissæ. Some irregularity of behavior is visible, due to the fact to be further indicated in chapters II and III, that the viscosity of glass-hard steel can be enormously reduced by twisting. Indeed a glass-hard wire twisted back and forth as far as maximum viscosity shows about the same value of that quality as a rod annealed at 100° to the maximum of permanent hardness. In their mean relations, however, the results of this figure are satisfactory and easily comparable with the new results now to be adduced.

The ordinates increase in the order of greater viscosity so that the abscissæ are glass-hard wires.

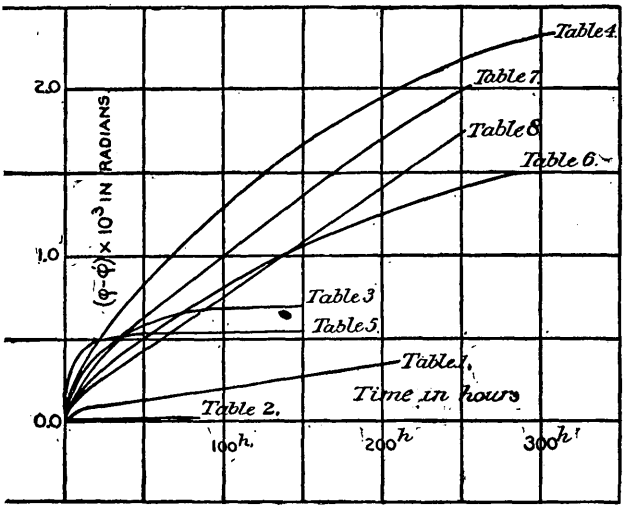

FIG. 8.—Viscous deformation of steel annealed at 20° and at 100°, compared with glass-hard steel.

19. *Rods annealed at* 190°.—The data obtained are as follows:

TABLE 9—APPARATUS II.

$R=200^{cm}$; $l=27^{cm}$; $\lambda<0{\cdot}2^{cm}$. Rods: No. 11, glass-hard, $T=-180°$; No 12, annealed at 190°, 2^h; $T=+180°$. $2\rho=0{\cdot}082^{cm}$.

Date.	$N\times 10^3$	h_0	$(\phi-\phi')\times 10^3$	Date.	$N\times 10^3$	h_0	$(\phi-\phi')\times 10^3$
July 1, $4^h\ 00^m$	(*)	0·00	(*)	July 2, $9^h\ 20^m$	2·215	17·33	0.625
$4\ 35$	2·913	0·58	−0·073	$1\ 45$	2·133	21·75	0·707
$5\ 36$	2·767	1·60	+0·073	$5\ 09$	2·080	25·15	0·760
$6\ 58$	2·655	2·97	0·185	July 3, $8^h\ 19^m$	1·891	40·32	0.949
				$12\ 12$	1·843	44·20	0·997
				$6\ 28$	1.757	50·47	1·803

* Adjusted, $\tau+b+f$.

TABLE 10—APPARATUS III.

$R=370^{cm}$; $l=26^{cm}$; $\lambda<0{\cdot}2^{cm}$. Rods: No. 13, glass-hard, $T=-180°$; No. 14, annealed at 190°, 1^h; $T=+180°$. $2\rho=0{\cdot}082^{cm}$.

Date.	$N\times10^3$	h_0	$(\phi-\phi')\times10^3$	Date.	$N\times10^3$	h_0	$(\phi-\phi')\times10^3$
June 14, 12^h 30^m	(*)	0·00	(*)	June 21, 8^h 22^m	3·732	169·91	2·692
1 10	1·025	0·67	—0·065	12 15	3·763		
2 05	1·198	1·58	+0·108	5 44	3·810		
3 51	1·402	3·35	0.312	†7 17	3·823		
4 21	1·448	3·85	0·358	22, 9^h 10^m	3·913	193·38	2·856
5 29	1·525	4·98	0·435	1 18	3·943		
6 55	1·606	6·42	0·516	2 20	3·948		
7 51	1·648	7·35	0.558	6 42	3·980		
15, 9^h 04^m	2·022	20·57	0·932	23, 8^h 30^m	4·056	214·86	2·971
10 00	2·042	21·50	0·952	12 43	4·077		
11 15	2·073	22·75	0·983	5 42	4·111		
1 10	2·121	24·67	1·031	24, 9^h 02^m	4·197	240·38	3·132
2 58	2·165	26·47	1·075	12 39	4·222		
4 17	2·195	27.78	1·105	4 58	4·248		
5 41	2·223	29·18	1·133	25, 8^h 44^m	4·335	264·13	3·266
6 18	2·233	29·80	1·143	11 38	4·351		
16, 8^h 00^m	2·467	48·26	1·460	†5 31	4·383		
9 32	2·492			26, 8^h 29^m	4·448	289·18	3·385
1 07	2·556			12 10	4·467		
3 19	2·597			3 12	4·483		
5 50	2·638			6 52	4·504		
17, 8^h 34^m	2·861	73·43	1·878	27, 8^h 52^m	4·575	311·49	3·502
10 29	2·899			3 06	4·609		
12 43	2·942			28, 8^h 11^m	4·699	335·87	3·631
3 35	2·995			11 30	4·720		
5 29	3·043			5 25	4·744		
6 45	3·066			29, 8^h 11^m	4·811	361·44	3·754
18,11^h 45^m	3·236	99·02	2·181	3 46	4·854		
4 39	3·283			5 52	4·866		
†6 09	3·293			30, 8^h 53^m	4·943	384·41	3·872
19, 9^h 07^m	3·410	120·92	2·346	12 26	4·962		
12 19	3·432			5 25	4·982		
2 25	3·445			July 1, 8^h 47^m	5·028	404·28	5·028
5 50	3·458						
20, 8^h 53^m	3·567	145·23	2·515				
10 53	3·581						
3 26	3·622						
7 44	3·652						

* Adjusted, $\tau-b-f$. † Readjusted after this observation.

TABLE 11—APPARATUS III.

$R=400^{cm}$; $l27^{cm}$; $\lambda<0{\cdot}2^{cm}$. Rods: No. 19, glass-hard, $T=-180°$; No. 20, annealed at 190°, 1^h; $T=+180°$. $2\rho=0{\cdot}082^{cm}$.

Date.	$N\times10^3$	h_0	$(\phi-\phi')\times10^3$	Date.	$N\times10^3$	h_0	$(\phi-\phi')\times10^3$
July 11, 3^h 20^m	(*)	0·00	(*)	July 15, 10^h 46^m	1·491	94·27	2·013
4 10	—0·037	0·83	—0·037	4 26	1·548		
4 20	±0·000	1·00	±0·000	16, 9^h 18^m	1·695	118·32	2·232
5 22	+0·161	2·03	+0·161	1 44	1·740		
7 00	0·329	3·67	0·329	5 55	1·780		
12, 7^h 37^m	0·913	16 28	0·913	17, 8^h 47^m	1·899	141·90	2·438
7 55	†	16·58	†	12 53	1·940		
9 02	0·466	17·70	0·960	6 02	1·992		
10 55	0·523	19·58	1·017	18, 8^h 44^m	2·096	164·25	2 617
5 27	0·696	26·12	1·190	2 26	2·151		
13, 8^h 29^m	0·952	41·15	1·446	19, 9^h 39^m	2 304	189·73	2·829
12 29	0·999	45·15	1·493	12 20	3·331		
5 08	1·061	49·80	1·555	5 12	2·371		
14, 8^h 54^m	1·230	69·49	1·765				
12 51	1·270						
4 41	1·314						

* Adjusted, $\tau+b+f$. † Adjusted, $\tau-b-f$.

TABLE 12—APPARATUS IV.

$R=300^{cm}$; $l=27^{cm}$; $\lambda<0{\cdot}2^{cm}$. Rods, No. 15, glass-hard, $T=-80°$; No. 16, annealed at 190°, 1^h; $T=+180°$. $2\rho=0{\cdot}082^{cm}$.

Date.	$N\times10^3$	h_0	$(\phi-\phi')\times10^3$	Date.	$N\times10^3$	h_0	$(\varphi-\phi')\times10^3$
July 11, $4^h\ 25^m$	(*)	0·00	(*)	July 16, $9^h\ 19^m$	1·849	117·26	1·919
$4\ 42$	−0·210	0·28	−0·175	$1\ 45$	1·885		
$4\ 55$	−0·143	0·50	−0·108	$5\ 58$	1·917		
$5\ 25$	−0·035	1·00	±0·000	17, $8^h\ 49^m$	2·022	140·84	2·093
$7\ 00$	+0·153	2·58	+0·188	$12\ 54$	2·052		
12, $7^h\ 38^m$	0·685	15·22	0·720	$6\ 03$	2·099		
$5\ 28$	0·937	25·05	0·972	18, $8^h\ 45^m$	2·198	163·35	2·256
13, $8^h\ 29^m$	1·179	40·07	1·214	$2\ 47$	2·243		
$12\ 29$	1·222	44·07	1·257	19, $9^h\ 40^m$	2·378	188·66	2·444
$5\ 09$	1·280	48.73	1·315	$12\ 21$	2·419		
14, $8^h\ 54^m$	1·434	68·40	1·508	$5\ 13$	2·429		
$12\ 52$	1·473			20, $8^h\ 32^m$	2·517	209·68	2·562
$4\ 42$	1·512			$11\ 40$	2·538		
15, $10^h\ 47^m$	1·670	93·20	1·730				
$4\ 26$	1·720						

* Adjusted, $\tau-b-f$.

The results of these tables are given in Fig. 9, in the way already indicated in Fig. 8. The agreement here is much more uniform than in the former case. Rods show pronounced increase of viscosity.

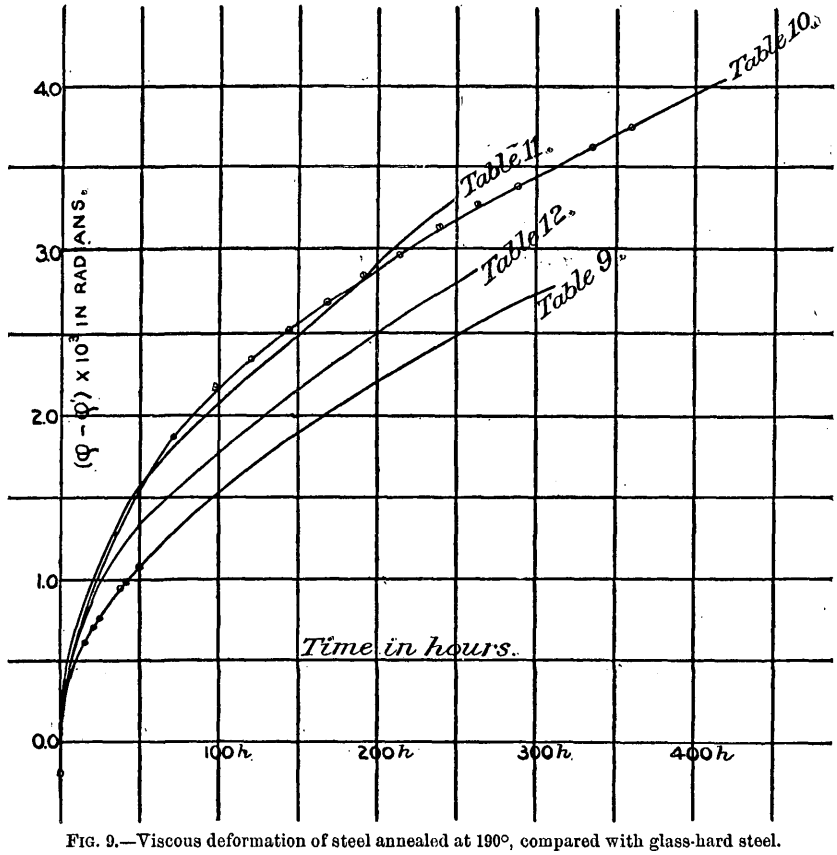

FIG. 9.—Viscous deformation of steel annealed at 190°, compared with glass-hard steel.

20. *Rods annealed at* 360°. The data obtained are as follows:

TABLE 13.—APPARATUS II.

$R = 200^{cm}$; $l = 27^{cm}$; $\lambda < 0{\cdot}2^{cm}$. Rods: No. 17, glass-hard, $T = -180°$; No. 18, annealed at 350°, 1h, $T = +180°$. $2\rho = 0{\cdot}082^{cm}$.

Date.	$N \times 10^3$	h_0	$(\phi - \phi') \times 10^3$	Date.	$N \times 10^3$	h_0	$(\phi - \phi') \times 10^3$
July 4, 10h 35m	(*)	0·00	(*)	July 10, 8h 05m	0·545	145·93	3·092
10 53	−2·767	0·30	−0·267	12 08	0·589		
12 08	−2·457	1·55	+0·043	5 19	0·643		
3 07	−2·118	4·53	0·382	11, 9h 08m	0·797	171·67	3·348
5 59	−1·929	7·40	0·570	11 34	0·821		
5, 8h 39m	−1·423	22·07	1·076	5 21	0·881		
11 59	−1·318	25·40	1·181	6 59	0·892		
5 43	−1·187	31·13	1·312	12, 7h 36m	0·993	193·93	3·533
6, 8h 40m	−0 882	46 08	1·617	5 26	1·072		
1 52	−0·773	51·28	1·726	13, 8h 27m	1·163	218·10	3·686
5 15	−0·703	54·67	1·796	12 27	1·180		
7, 8h 50m	−0·455	74·59	2·127	5 08	1·215		
1 26	−0·367			14, 8h 53m	1·299	242·20	3·823
5 15	−0·295			12 48	1·323		
8, 9h 15m	−0·056	98·66	2·513	4 40	1·348		
12 42	+0·007			15, 10h 45m	1·446	267·00	3·965
5 47	0·089			4 25	1·483		
9, 8h 34m	0·274	122·09	2·826	16, 9h 17m	1·565	286·70	4·065
12 10	0·321						
5 17	0·383						

* Adjusted $\tau - b - f$.

TABLE 14.—APPARATUS IV.

$R = 316^{cm}$; $l = 26^{cm}$; $\lambda < 0{\cdot}2^{cm}$. Rods: No. 21, glass-hard, $T = -180°$; No. 22, annealed at 360°, 1h, $T = +180°$ $2\rho = 0{\cdot}082^{cm}$.

Date.	$N \times 10^3$	h_0	$(\phi - \phi') \times 10^3$	Date.	$N \times 10^3$	h_0	$(\phi - \phi') \times 10^3$
June 14, 2h 45m	(*)	0·00	(*)	June 21, 8h 23m	4·374	167·66	3·290
3 52	1·155	1·12	0·025	12 15	4·403		
4 22	1·250	1·62	0·120	5 45	4·447		
5 30	1·415	2·75	0·285	7 16	4·456		
6 55	1·558	4·17	0·428	22, 9h 11m	4·541	191·61	3·434
7 53	1·632	5·13	0·502	1 19	4·558		
15, 9h 05m	2·198	18.33	1·068	4 13	4·573		
†10 00	2·231	19·25	1·101	6 43	4·583		
11 16	2·273	20·52	1·143	23, 8h 30m	4 666	214·22	3·569
1 11	2·340	22·43	1 210	12 43	4·699		
3 00	2·400	24·25	1·270	5 42	4·732		
4 16	2·442	25·57	1 312	24, 9h 03m	4·829	238·15	3·722
5 42	2·478	26·95	1·348	12 40	4·853		
6 19	2·490	27·57	1·360	4 59	4·873		
16, 8h 02m	2·802	46·03	1·785	25, 8h 45m	4·973	261·88	3·875
9 33	2·837			11 39	4·997		
1 09	2·928			5 30	5·044		
3 20	2·978			26, 8h 30m	5·143	286·87	4·048
5 50	3·028			†1 28	5·177		
17, 8h 35m	3·302	71·19	2·296	6 53	5·213		
10 30	3·349			27, 8h 53m	5·303	309·25	4·196
12 44	3·405			3 07	5·350		
3 36	3·465			28, 8h 13m	5·450	333·64	4·346
5 30	3·503			11 31	5·471		
6 45	3·531			5 26	5·508		
18, 8h 53m	3 741	95·12	2·667	29, 8h 12m	5·595	359·19	4·505
11 45	3·770			3 45	5·647		
4 40	3·829			5 53	5·662		
†6 10	2·850			30, 8h 54m	5·740	382·18	4·629
19, 9h 09m	4·000	118·69	2·904	12 27	5·757		
12 19	4·026			5 26	5·780		
2 27	4·037			July 1, 8h 48m	5·848	402·05	5·848
5 51	4 071						
20, 8h 54m	4·193	143 01	3·105				
10 59	4·211						
3 25	4·250						
7 45	4·286						

* Adjusted $\tau - b - f$. † Readjusted.

Figure 10 contains a graphic representation of the preceding tables on the plan already described (cf. Table 8).

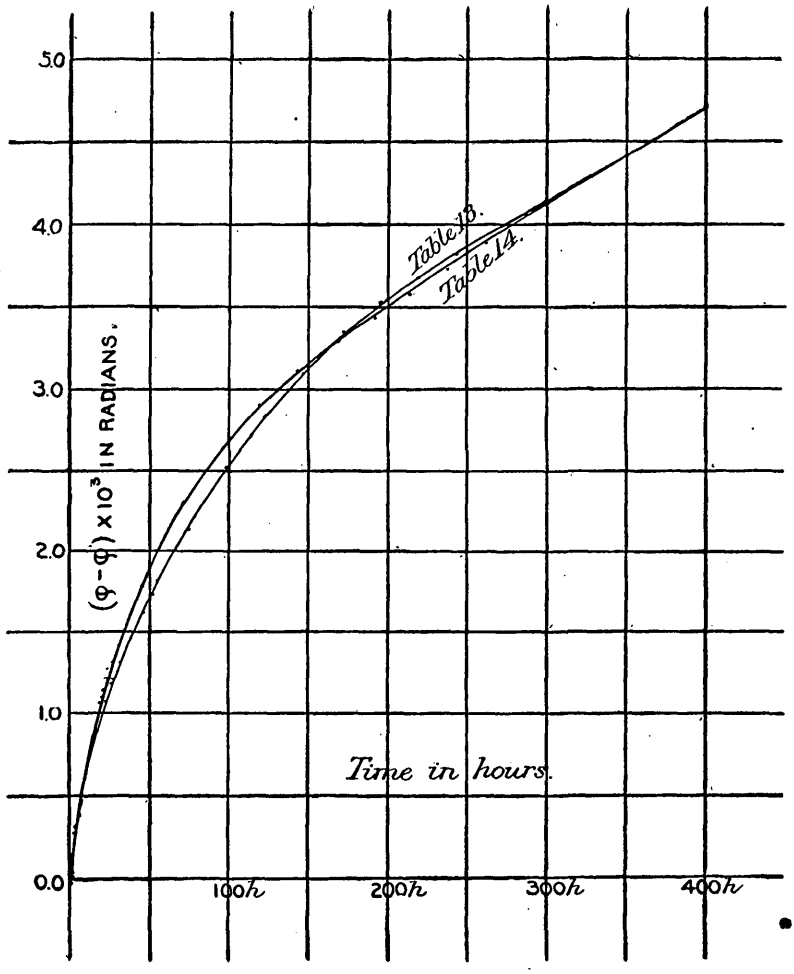

FIG. 10.—Viscous deformation of steel annealed at 360°, compared with glass-hard steel.

21. *Rods annealed at* 450°.—The data in hand are as follows:

TABLE 15.—APPARATUS I.

$R=217^{cm}$; $\lambda=26^{cm}$; $l<0{\cdot}2^{cm}$. Rods: No. 23, glass-hard, $T=-180°$; No. 24, annealed at 450°, 1^h, $T=+180°$. $2\rho=0{\cdot}082^{cm}$.

Date.	$N\times10^3$	h_0	$(\phi-\phi')\times10^3$	Date.	$N\times10^3$	h_0	$(\phi-\phi')\times10^3$
July 21, 2^h 50^m	(*)	0·00	(*)	July 29, 8^h 56^m	−0·673	189·80	3·814
3 17	3·276	0·45	−0·176	12 36	−0·714		
4 44	2·901	1·90	+0·198	4 22	−0·758		
5 21	2·805	2·52	0·294	30, 8^h 52^m	−0·937	213·85	4·085
6 13	2·698	3·38	0·401	1 05	−0·991		
22, 5^h 40^m	1·924	17·83	1·175	4 07	−1·030		
11 58	1·798	21·13	1·301	31, 9^h 28^m	−1·210	237·82	4·345
5 30	1·606	26·67	1·493	12 36	−1·249		
7 10	1·552	28·33	1·547	3 55	−1.281		
23, 7^h 49^m	1·307	40·98	1·792	Aug. 2, 9^h 38^m	−1·639	286·13	4·768
12 30	1·141	45·67	1·958	1 11	−1·672		
3 31	1·072	48·68	2·027	4 05	−1·696		
24, 9^h 13^m	0·769	69·91	2·393	3, 9^h 38^m	−1·807	310·22	4·926
12 52	0·702			1 14	−1·826		
4 10	0·647			4 18	−1·844		
26, 8^h 53^m	0·126	117·40	3·019	4, 1^h 31^m	−1·947	336·12	5·055
12 07	0·081			4 23	−1·962		
3 42	0·033			5, 1^h 36^m	−2·052	360·67	5·162
27, 8^h 49^m	−0·146	141·60	3·290	5 25	−2·073		
12 26	−0·193			6, 9^h 12^m	−2·139	382·48	5·257
4 04	−0·235			5 26	−2·175		
28, 9^h 11^m	−0·423	166·03	3·564	7, 9^h 15^m	−2·225	407·13	5·346
1 07	−0·468			6 41	−2·268		
4 17	−0·503						

* Adjusted $\tau+b+f$.

TABLE 16.—APPARATUS II.

$R=198^{cm}$; $\lambda=26^{cm}$; $l<0{\cdot}2^{cm}$. Rods: No. 25, glass-hard, $T=-180°$; No. 26, annealed at 450°, 1^h, $T=+180°$. $2\rho=0{\cdot}082^{cm}$.

Date.	$N\times10^3$	h_0	$(\phi-\phi')\times10^3$	Date.	$N\times10^3$	h_0	$(\phi-\phi')\times10^3$
June 15, 12^h 50^m	(*)	0·00	(*)	June 21, 8^h 20^m	5·364	145·55	3·177
1 07	1·968	0·28	−0·282	12 15	5·401		
1 15	2·046	0·42	−0·204	†5 40	5·466		
1 30	2·152	0·67	−0·098	7 19	5·478		
1 45	2·239	0·92	−0·011	22, 9^h 08^m	5·573	169·47	3·355
2 55	2·475	2·08	+0·225	1 16	5·597		
4 15	2·648	3·42	0·398	4 10	5·617		
5 39	2·776	4·82	0·526	6 40	5·632		
6 17	2 825	5·45	0·575	23, 8^h 27^m	5·717	192·10	3·499
16, 7^h 56^m	3·447	19·10	1·197	12 40	5·748		
9 30	3·506	20·67	1·256	5 41	5·781		
1 06	3·632	24.27	1·382	24, 9^h 00^m	5·880	216·01	3 660
3 17	3·703	26·45	1·453	12 37	5·909		
5 49	3·780	28·98	1·530	4 55	5·941		
17, 8^h 33^m	4·134	49·06	2·035	25, 8^h 42^m	6·045	239·76	3·820
10 26	4·191			11 36	6·068		
12 41	4·258			5 28	6 111		
3 31	4·337			26, 8^h 27^m	6·206	264·29	3·989
5 26	4·378			12 05	6·227		
6 44	4·410			6 05	6·283		
18, 8^h 48^m	4·656	72·98	2·471	27, 8^h 51^m	6·367	287·11	4·140
11 43	4·690			3 02	6·412		
4 38	4·759			28, 8^h 08^m	6·516	311·51	4·326
6 07	4·778			11 29	6·535		
19, 9^h 05^m	4·953	96·53	2·741	5 24	6·678		
12 10	4·982			29, 8^h 09^m	6·662	337·07	4·451
2 23	4·998			3 43	6·714		
5 50	5·031			5 50	6·728		
20, 8^h 12^m	5·170	120·71	2·963	30, 8^h 52^m	6·811	350·08	4·577
10 52	5·179			12 29	6·825		
3 24	5·232			5 24	6·845		
7 42	5·270			July 1, 8^h 39^m	6·910	379·82	4·660

* Adjusted, $\tau-b-f$. † Readjusted.

The results of these tables are given in the Chart Fig. 11 on the plan already clearly indicated. The ordinates increase in the order of greater viscosity.

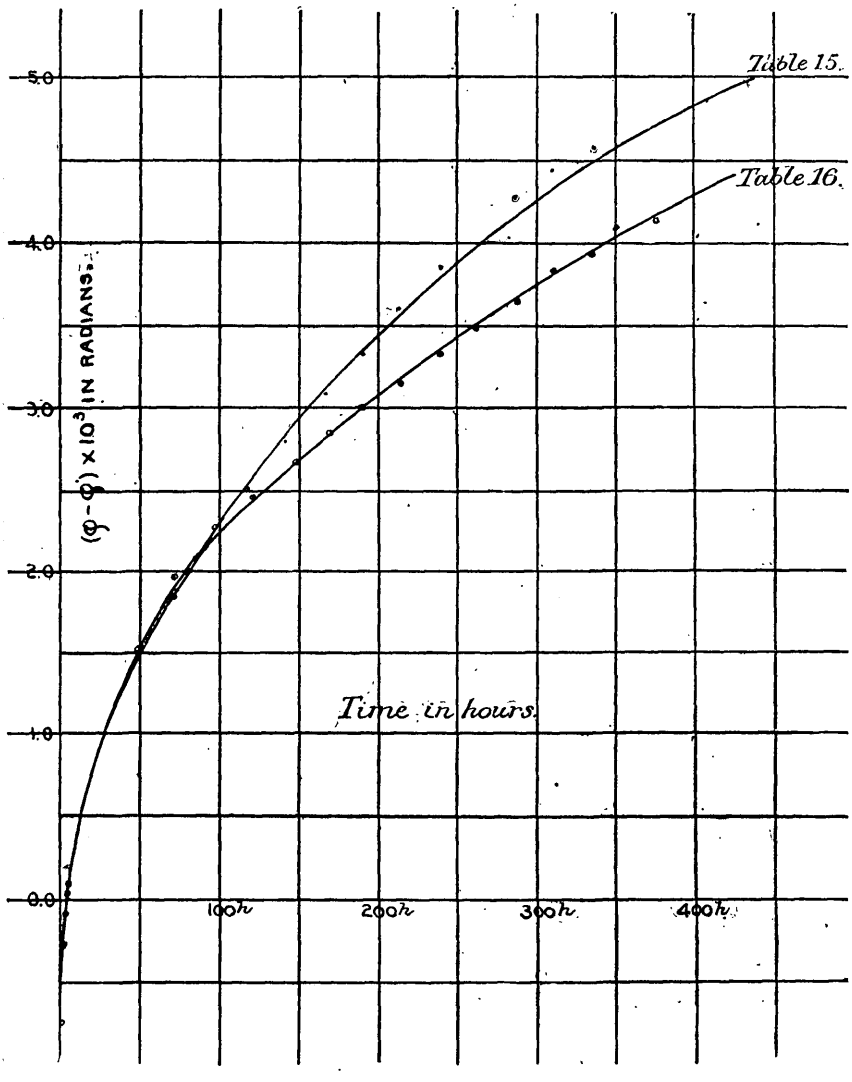

FIG. 11.—Viscous deformation of steel annealed at 450°, compared with glass-hard steel.

22. *Soft rods.*—The data in hand are as follows:

TABLE 17.—APPARATUS I.

$b=27^{cm}$; $R=202^{cm}$. Rods: No. 53, steel glass-hard; $2\rho=0{\cdot}082^{cm}$; $T=-180°$; No. 54, steel drawn; $2\rho=0{\cdot}082^{cm}$; $T=+180°$.

Date.	h	$N\times10^3$	h_0	$(\phi-\phi')\times10^3$	Remarks.
Sept. 28, 2h 00m	0·00	.....	0·00	.....	Adjusted.
2 10	0·17	0·789	0·17	−0·239	
2 30	0·50	0·661	0·50	−0·111	
3 36	1·60	0·486	1·60	+0·064	
8 00	6·00	+0·171	6·00	+0·379	
29, 10 18	20·30	−0·206	20·30	+0·756	

TABLE 18.—APPARATUS I.

$R=215^{cm}$; $l=27^{cm}$; $\lambda<0{\cdot}2^{cm}$. Rods: No. 27, glass-hard, $T=-180°$; No. 28, soft, $T=+180°$. $2\rho=0{\cdot}082$.

Date.	$N\times10^3$	h_0	$(\phi-\phi')\times10^3$
June 9, 2h 30m	(*)	0·00	(*)
3 10	1·652	0·67	−0·118
3 13	1·677	0·71	−0·093
3 21	1·730	0·85	−0·040
3 47	1·860	1·28	+0·090
4 00	1·910	1·50	0·140
4 10	1·940	1·67	0·170
4 32	2·004	2·03	0·234
5 15	2·102	2·75	0·332
5 35	2·140	3·08	†0·370
6 00	2·185	3·50	0·415
6 15	2·210	3·75	0·440
10, 8h 50m	2·938	20·09	1·227
9 35	2·964		
9 55	2·973		
10 15	2·985		
10 45	3·002		
11 20	3·023		
11 45	3·038		
12 17	3·056		
10, 1h 30m	3·100	26·64	1·448
2 15	3·128		
3 05	3·153		
4 00	3·191		
4 40	3·214		
5 25	3·238		
6 25	3·265		
†7 40	3·287		
8 05	3·296		
8 20	3·304		
11, 8h 06m	3·542	47·34	1·895
9 28	3·571		
10 21	3·588		
10 45	3·598		
11 45	3·617		
12 11	3·628		
2 30	3·680		
3 48	3·709		
4 50	3·733		
5 50	3·753		
7 00	3·777		
7 35	3·787		
12, 8h 37m	3·989	71·44	2·318
9 25	4·005		
10 10	4·017		
1 05	4·076		
2 55	4·108		
4 58	4·146		
5 50	4·162		
8 30	4·202		
13, 8h 40m	4·376	95·89	2·702
10 15	4·405		
12 06	4·436		
1 25	4·458		
3 16	4·485		
5 07	4·517		
6 50	4·545		
7 30	4·556		
June 14, 9h 04m	4·736		
10 30	4·757	120·33	3·046
1 00	4·796		
2 06	4·813		
3 50	4·836		
5 27	4·853		
6 52	4·866		
7 50	4·873		
15, 9h 00m	4·990	143·30	3·272
10 00	4·998		
11 15	5·015		
1 05	5·037		
2 55	5·060		
4 15	5·071		
5 38	5·082		
6 15	5·086		
16, 7h 58m	5·205	166·22	3·488
9 30	5·223		
1 05	5·265		
3 15	5·285		
5 48	5·310		
17, 8h 30m	5·455	191·54	3·758
10 26	5·485		
12 40	5·520		
3 30	5·550		
5 27	5·570		
7 43	5·590		
18, 8h 46m	5·705	215·31	3·970
11 43	5·720		
4 37	5·760		
6 08	5·775		
19, 9h 04m	5·880	238·52	4·125
12 09	5·890		
5 50	5·915		
20, 8h 50m	6·010	263·18	4·272
10 50	6·023		
3 23	6·055		
7 40	6·080		
21, 8h 20m	6·140	287·88	4·408
12 15	6·160		
5 38	6·200		
7 19	6·210		
22, 9h 07m	6·270	311·80	4·525
1 16	6·290		
4 09	6·305		
6 40	6·315		
23, 8h 28m	6·375	334·44	4·633
12 40	6·405		
5 40	6·430		

* Adjusted, 7—b—f. † Readjusted.

TABLE 19.—APPARATUS II.

$R=202^{cm}$; $l=28^{cm}$; $\lambda<0{\cdot}2^{cm}$. Rods: No. 29, glass-hard, $T=-180°$; No, 30, commercial, drawn, $T=+180°$. $2\rho=0{\cdot}082^{cm}$.

Date.	$N\times10^3$	h_0	$(\phi-\phi')\times10^3$	Date.	$N\times10^3$	h_0	$(\phi-\phi')\times10^3$
July 21, 10h 45m	(*)	0.00	(*)	July 29, 8h 56m	−0·709	193·90	3·012
11 50	2·263	1·08	0·000	12 39	−0·749		
12 18	2·206	1·55	0·057	4 23	−0·788		
2 03	2·045	3·30	0·218	30, 8h 54m	−0·957	217·96	3·269
3 52	1·926	5·12	0·337	1 06	−1·010		
5 22	1·847	6 62	0·416	4 07	−1·050		
22, 8h 40m	1·416	21·92	0·847	31, 9h 42m	−1·230	242·01	3·526
11 58	1·341	25·22	0·922	12 37	−1·263		
5 31	1·217	30·77	1·046	3 57	−1·297		
7 44	1·187	32·98	1·076	Aug. 2, 9h 39m	−1·644	290·23	3·934
23, 7h 50m	0·984	45·08	1·279	1 12	−1·677		
12 32	0·896	49·78	1·367	4 06	−1·693		
3 33	0·851	52·80	1·412	3, 9h 40m	−1·780	314·33	4·049
24, 9h 16m	0·624	74.03	1·737	1 15	−1·784		
12 53	0·499			4 19	−1·793		
4 12	0·456			4, 1h 32m	−1·872	340·22	4·139
26, 8h 55m	−0·025	121·53	2·292	4 24	−1·880		
12 08	−0·011			5, 1h 37m	−1·965	364·77	4·237
3 47	−0·051			5 25	−1·982		
27, 8h 51m	−0·211	145·71	2·516	6, 9h 13m	−2·044	386·59	4·324
12 27	−0·255			5 27	−2·078		
4 05	−0·293			7, 9h 17m	−2·133	411·24	4·412
28, 9h 20m	−0.487	170·18	2·781	6 42	−2·165		
1 09	−0·517						
4 18	−0·550						

* Adjusted, $\tau+b+f$.

TABLE 20.—APPARATUS III.

$R=370^{cm}$; $l=27^{cm}$; $\lambda<0{\cdot}2^{cm}$. Rods: No. 31, glass-hard, $T=-180°$; No. 32, commercial, softened, $T=+180°$. $2\rho=0{\cdot}082^{cm}$.

Date.	$N\times10^3$	h_0	$(\phi-\phi')\times10^3$	Date.	$N\times10^3$	h_0	$(\phi-\phi')\times10^3$
July 22, 9h 00m	(*)	0·00	(*)	July 27, 8h 54m	−2·305	123·51	3·055
9 08	0·949	0·13	−0·249	12 30	−2·356		
11 59	0·331	2·98	+0·368	4 08	−2·404		
5 32	0·099	8·53	0·600	28, 9h 26m	−2·612	147·98	3·355
7 11	−0·193	10·18	0·893	1 11	−2·658		
23, 7h 52m	−0·643	22·87	1·343	4 20	−2·695		
12 35	−0·794	27·58	1·494	29, 8h 59m	−2·875	171·72	3·624
3 25	−0·880	30·42	1·580	12 42	−2·923		
24, 9h 21m	−1·246	48·35	1·946	4 28	−2·974		
12 57	−1·320	51·95	2·020	30, 8h 57m	−3·160	195·80	3·913
4 16	−1·384	55·27	2·084	1 10	−3·217		
26, 8h 57m	−1·993	99·31	2·749	4 17	−3·261		
12 10	−2·050						
3 49	−2·105						

* Adjusted $\tau+b+f$.

TABLE 21.—APPARATUS IV.

$R=314^{cm}$; $l=27^{cm}$; $\lambda<0{\cdot}2^{cm}$. Rods: No. 33, glass-hard, $T=-180°$; No. 34, annealed at 1000°, $T=+180°$. $2\rho=0{\cdot}082^{cm}$.

Date.	$N\times10^3$	h_0	$(\phi-\phi')\times10^3$	Date.	$N\times10^3$	h_0	$(\phi-\phi')\times10^3$
July 21, 4h 15m	(*)	0·00	(*)	July 29, 9h 02m	−2·324	189·18	3·467
4 43	1·249	0·47	−0·149	12 45	−2·367		
5 23	1·089	1·13	+0·010	4 31	−2·409		
5 45	1·022	1·50	0·077	30, 9h 00m	−2·583	212·61	3·730
6 12	0·957	1·95	0·142	1 13	−2·633		
22, 8h 42m	0·162	16·45	0·937	4 22	−2·673		
12 00	0·042	19·75	1·057	31, 9h 52m	−2·850	236·61	3·932
5 33	−0·250	25·30	1·249	12 41	−2·882		
7 12	−0·185	26·95	1·284	4 01	−2·912		
23, 7h 53m	−0·448	39·63	1·547	Aug. 2, 10h 02m	−3·264	284·95	4·389
12 36	−0·559	44·35	1·658	1 24	−3·291		
3 36	−0·628	47·35	1·727	4 10	−3·312		
24, 9h 23m	−0·924	68·64	2·082	3, 9h 47m	−3·423	308·93	4·534
12 59	−0·985			1 22	−3·436		
4 19	−1·038			4 24	−3·442		
26, 9h 05m	−1·540	116·14	2·685	4, 1h 38m	−3·548	334·78	4·654
12 14	−1·581			4 26	−3·560		
3 52	−1·633			5, 1h 40m	−3·680	359·31	4·768
27, 9h 03m	−1·807	140·35	2·951	5 27	−3·675		
12 34	−1·851			6, 9h 14m	−3·733	381.10	4·855
4 10	−1·895			5 28	−3·777		
28, 9h 29m	−2·061	164·79	3·222	7, 9h 22m	−3·829	405·80	4·946
1 16	−2·125			6 43	−3·863		
4 23	−2·159						

* Adjusted, $\tau+b+f$.

The results of these tables are given in Fig. 12 on the plan already indicated. The observed differences are probably more largely due to the glass-hard wires than to the soft wires of the twisted system. To compare soft wires these must be specially countertwisted so as to secure greater accuracy of measurement.

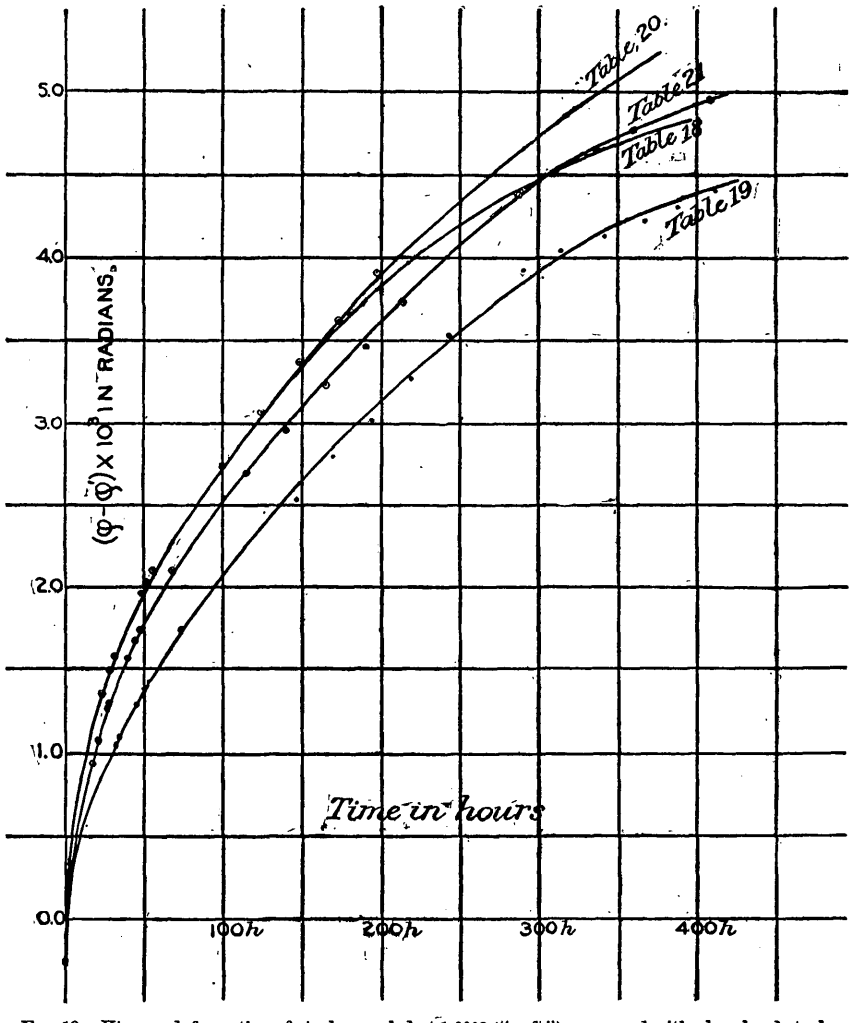

FIG. 12.—Viscous deformation of steel annealed at 1,000° ("soft") compared with glass-hard steel.

REMARKS ON THE TABLES.

23. In experimenting with couples of two glass-hard wires, or with couples in which one of the wires is annealed at 100°, the degree of hardness of the harder wire is an exceedingly important factor. Hence these curves (Tables 3, 4, 5, 6, 7, 8) appear under a variety of forms. Only Tables 4, 6, 7, 8 are used in the digest constructed below. The results in Tables 15, 19, 21 change their curvature after August 1.

Since the same error is found in all the wires then examined it is probably a result of temperature. When minutely traced (cf. Table 18, for instance), the curves show a sinuous outline which is quite marked and probably also due to temperature or to tremors. If the curves for stated temperatures and times of annealing fail to coincide, it is due to differences in the tempers of the glass-hard wires, to similar differences in the tempers of the annealed wires, to preexisting strains accidentally imparted to one wire or the other, to lack of perfect uniformity of temper throughout the efficient lengths, to unavoidable inequalities of the sectional areas of the pairs of wires. The last-named error probably also affects the curvature. Viscous rotation of the arms which carry the mirrors, or yielding of the cement, is nil and is not to be apprehended, since the fixed mirror is a fiducial mark for the movable mirror. In the above work we do not indicate the amount of permanent torsion left in each wire after the experiment is finished. Though an important desideratum, it does not fall within the scope of this chapter but will be given in Chapter III in connection with other work there discussed. The effect of temperature on twisted systems of pairs of identical steel wires we investigate in Chapter II, and then discuss the amount of detorsion due to annealing at stated temperatures and times. The soft ends introduce no serious error because soft steel is more viscous than hard steel. The amount of twist registered is that between the upper and lower points of attachment. The included wires must be of uniform hardness.

The fact that an effect of the bifilar and flexural components is wholly absent in these results is proved by Table 11, in which the curve for $\tau+b+f$ is followed without break of continuity by the curve $\tau-b-f$, and the total curve is practically identical with the locus for Table 10. It is proved, moreover, by Tables 1, 2, in which couples of glass-hard wires show no greater difference than is at once attributable to unavoidable differences of hardness; and by Table 30 below, in which $\tau+b+f$ and $\tau-b-f$ produce practically identical zero-effects on two soft wires. It is generally proved by the distribution of the 50 rods examined in a diagram of viscosity conformably with the respective tempers of the rods. If a couple of one hard and one soft rod possessed a smaller total of viscosity than a couple of two hard rods, then the bifilar and flexural couples might produce an effect in the former case and not in the latter. The absence of all effect in the case of two soft rods as well as in the case of two hard rods shows that the discrepancy in question is nil. The important bearing of this result will be indicated below.

MISCELLANEOUS EXPERIMENTS.

24. *Glass fibers.*—In Tables 22, 23 we cite the results obtained when one of the steel rods is replaced by a fiber of glass. The mean thickness or the diameter $2\rho_g$ of glass was intended to be that of the steel rod $2\rho_s$; but it is smaller in Table 22 and considerably larger in Table 23. It is

impossible to store a greater total torsion than $T_1-(-T_2)=90°$ in the given system, without breaking the glass fiber. The equality of T_1 and T_2 is assumed merely for convenience in designation (cf. pp. 12, 40). In Table 23, moreover, the glass-hard rod used during the first half of the experiment is replaced by an annealed rod in the second half, leaving the glass fiber unaltered. The suspension here is practically unifilar, since it is nearly impossible mechanically to clutch the glass fiber without breaking it.

TABLE 22.—APPARATUS I.

$R=215^{cm}$; $l=27^{cm}$; $\lambda<0{\cdot}2^{cm}$. Rods: No. 35, glass-hard steel, $T_1=+45°$; No. *a*. glass fiber, $T_2=-45°$; $\rho_s=0{\cdot}082^{cm}$; $\rho_g=0{\cdot}070^{cm}$.

Date.	$N\times10^3$	h_0	$(\phi-\phi')\times10^3$	Date.	$N\times10^3$	h_0	$(\phi-\phi')\times10^3$
June 29, 3h 20m	(*)	0·00	(*)	July 3, 8h 17m	0·974	88·95	1·075
3 42	−0·129	0.37	−0·028	12 12	0·991	92·87	1·092
4 20	−0·101	1·00	±0·000	6 27	1·026	99·12	1·127
5 30	−0·052	2·17	+0·049	4, 9h 45m	1·098	114·42	1·199
5 51	−0·031	2·52	0·070	12 09	1·109	116·82	1·210
30, 8h 50m	+0·351	17·50	0·452	3 06	1·116	119·77	1·217
12 23	0.413	21·05	0·514	5 59	1·145	122·65	1·246
1 57	0.440	22·62	0·541	5, 9h 39m	1·213	138·32	1·314
5 22	0·486	26·03	0·587	12 00	1·228	140·67	1·329
July 1, 8h 37m	0·652	41·28	0·753	4 43	1·257	145·38	1·358
2 12	0·706	46·87	0·807	6, 8h 40m	1·318	161·33	1·419
4 36	0·732	49·27	0·833	1 52	1·339	166·53	1·440
5 37	0·741	50·28	0·842	5 15	1·355	169·92	1·456
6 57	0·753	51·62	0·854	7, 8h 49m	1·420	185·48	1·521
2, 9h 19m	0·854	65·98	0·955	1 27	1·443	190·12	1·444
1 45	0·879	70·42	0·980				
5 09	0·899	73·82	1·000				

* Adjusted.

TABLE 23.—APPARATUS I.

$R=223^{cm}$; $l=23^{cm}$. Rods: No. 36, glass-hard steel, $T_1=+45°$; No. *b*, glass fiber, $T_2=-45°$; $2\rho_s=0{\cdot}082^{cm}$; $0{\cdot}085^{cm}<2\rho_g<0{\cdot}120^{cm}$.

Date.	$N\times10^3$	h_0	$(\phi-\phi')\times10^3$	Date.	$N\times10^3$	h_0	$(\phi-\phi')\times10^3$
July 7, 4h 15m	(*)	0·00	(*)	July 12, 7h 35m	3·100	111·33	−0·235
4 26	2·995	0·18	−0·130	Glass-hard rod replaced by soft steel rod, viz: No. 42, annealed, 450°, 1h, $T_1=+45°$, $2\rho=0{\cdot}082^{cm}$.			
4 38	2·934	0·38	−0·069				
5 15	2·865	1·00	±0·000	July 12, 9h 39m	(*)	0·00	(*)
8, 9h 14m	2·823	16·98	+0·042	9 49	1·358	0·17	−0·023
12 41	2 841	20·43	0·024	10 54	1·331	1·25	+0·004
4 07	2·858	23·87	0·007	5 25	1·303	7·77	0·032
5 47	2·863	25·53	0·002	13, 8h 26m	1·278	22·78	0·057
9, 8h 33m	2·912	40·30	−0·047	12 27	1·270	26·80	0·065
12 10	2·928	43·92	−0·063	5 06	1·270	31·45	0·065
5 16	2·940	49·02	−0 075	14, 8h 50m	1·258	47·18	0·077
10, 8h 04m	2 983	63·82	−0·118	12 48	1·258	51·15	0·077
12 08	2 996	67·88	−0·131	4 40	1·255	55·02	0·080
5 18	3·010	73·05	−0·145	15, 10h 45m	1·245	73·10	0·090
11, 9h 07m	3·053	88·87	−0·188	4 25	1·245	78·77	0·090
11 23	3·058	91·13	−0·193	16, 9h 16m	1·237	95·62	0·098
5 21	3·077	97·10	−0·212	1 42	1·237	100·05	0·098
6 59	3·081	98·73	−0·216	5 54	1·236	104·25	0·099

* Adjusted.

The results obtained with glass are given in Fig. 13. The thinner glass fiber is uniformly less viscous than steel. The thicker fiber lies between hard steel and soft steel in viscosity. But the occurrence of the maximum is noteworthy. Glass is here initially more and ultimately less viscous than hard steel.

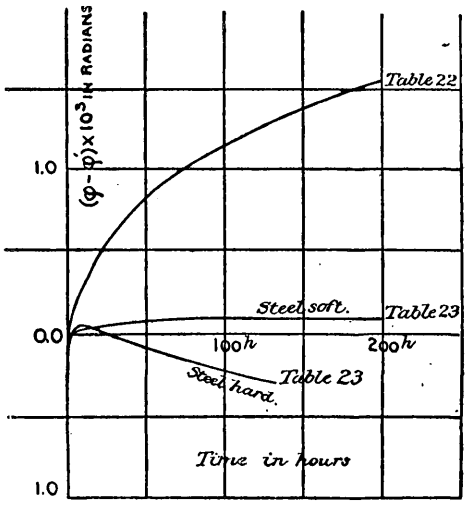

FIG. 13.—Viscous deformation of steel as compared with glass.

25. *Iron, soft and drawn.*—Tables 24, 25, 26, 27, contain a part of our results in which one steel wire is replaced by a wire of wrought iron, annealed or drawn as specified. That the amounts of torsion are equal in angle (T) is again assumed for convenience of designation only. ρ_s is the radius of steel, ρ_i of the iron wire. The applied couple twists soft iron beyond the limits of elasticity and the amount of instantaneous detorsion is here probably as large as $\frac{1}{2}(T_1+T_2)$. In case of steel instantaneous detorsion is nearly zero.

TABLE 24.—APPARATUS I.

$R=223^{cm}$; $l=26^{cm}$; $\lambda<0{\cdot}2^{cm}$. Rods: No. 37, steel, annealed at 450°, 1h, $T=+180°$; No. *c*, wrought iron, drawn, $T=-180°$. $2\rho_s=0{\cdot}082^{cm}$; $2\rho_i=0{\cdot}083^{cm}$.

Date.	$N\times10^3$	h_0	$(\phi-\phi')\times10^3$	Date.	$N\times10^3$	h_0	$(\phi-\phi')\times10^3$
July 17, 4h 20m	1·955	0·20	—0·387	July 18, 8 h27m	2·177	16·32	—0·165
4 23	2·032	0·25	—0·310	2 24	2·122	22·27	—0·220
4 27	2·113	0·32	—0·229	19, 9h 37m	2·032	41·48	—0·310
4 51	2·285	0·72	—0·057	12 19	2 025	44·18	—0 317
5 20	2·342	1·20	±0·000	5 10	2·014	49·03	—0·328
6 01	2·362	1·88	+0·020	20, 8h 29m	1·984	64·35	—0·358
				3 45	1·977	71·62	—0·364

* Adjusted $\tau-b-f$.

TABLE 25.—APPARATUS I.

R=217^{cm}; l=27^{cm}; λ<0·2^{cm}. Rods: No. 49, glass-hard, T=−180°; 2ρ=0·082^{cm}; No. h, wrought iron, drawn, T=+180°; 2ρ=0·080^{cm}.

Date.	h	$N\times10^3$	h_0	$(\phi-\phi')\times10^3$	Remarks.
Aug. 31, 5^h 45^m	0·00	.....	0·00	.....	Adjusted, $\tau+b+f$.
5 55	0·17	0·967	0·17	−0·267	
6 30	0·75	0·785	0·75	−0·085	
Sept. 1, 8^h 58^m	15·22	+0·017	15·22	+0·683	
1 00	19·25	−0·072	19·25	0·772	
6 12	24·45	−0·166	24·45	0·866	
2, 9^h 05^m	39·33	−0·376	39·33	1·076	
5 34	47·82	−0·470	47·82	1·170	
3, 10^h 02^m	64·28	−0·626	69·23	.....	
7 55	74·17	−0·713	.....	1·370	
4, 9^h 02^m	87·28	−0 805	92·60	.....	
7 40	97·92	−0 878	.....	1·542	
5, 9^h 50^m	112·08	−0·970	116·42	.....	
6 30	120·75	−1·032	.....	1·701	
6, 9^h 05^m	135·33	−1·124	135·33	1·824	Untwisted.

TABLE 26.—APPARATUS II.

R=200^{cm}; l=27^{cm}; λ<0·2^{cm}. Rods: No. 38, steel, annealed at 450°, 1^h, T=+180°; No. d, wrought iron, soft, T=−180°. $2\rho_s$=0·082^{cm}; $2\rho_i$=0·112^{cm}.

Date.	$N\times10^3$	h_0	$(\phi-\phi')\times10^3$
July 16, 9^h 40^m	(*)	0·00	(*)
9 48	0·714	0·13	−0·026
11 14	0·745	1·57	+0·005
1 48	0·758	4·13	0·018
5 47	0·750	8·23	0·010
17, 8^h 47^m	0·731	23·12	−0·009
Thick iron wire replaced by thin wire. Rod No. 9, wrought iron, soft; T=−180°; 2ρ=0·083^{cm}.			
17, 5^h 26^m	0·755	‡ 0·00	−0·019
5 30	0·745	0·07	−0·009
5 36	0 736	0·17	±0·000
6 00	0·718	0·57	+0·018
July 18, 8^h 33^m†	0.146	15·12	(0·153)
2 35	0·151	21·15	(0·148)
19, 9^h 38^m	0·128	(40·20)	(0·171)
12 20	0·119	(42·90)	(0·180)
5 10	0·110	(47·73)	(0·189)
20, 8^h 28^m	0 082	(63 03)	(0·217)
3 44	0 064	(70·32)	(0·235)

* Accident; new adjustment; data otherwise good, but distinguished by parentheses.

† Adjusted $\tau-l-f$.

‡ Adjusted $\tau+l+f$, but the time not accurately noted.

TABLE 27.—APPARATUS II.

R=202^{cm}; l=26^{cm}; λ<0.2^{cm}. Rods: No. 50, annealed at 350°, 1^h, T=+180°; No. k, wrought iron, drawn, T=−180°. $2\rho_i$=0·084^{cm}; $2\rho_s$=0·082^{cm}.

Date.	$N\times10^3$	h_0	$(\phi-\phi')\times10^3$
Aug. 23, 2^h 00^m	(*)	0·00	(*)
2 13	2·052	0·22	−0·452
2 20	1·914	0·33	−0·314
3 11	1·599	1·18	+0·001
5 37	1·417	3·62	0·183
7 11	1·392	5·18	0·208
24, 9^h 17^m	1·385	19·28	0·215
5 13	1·411	27·22	0·189
25, 9^h 05^m	1·435	43·08	0·165
7 00	1·448	53·02	0·152
26, 9^h 10^m	1·460	70·65	0·135
4 08	1·469		
Aug. 27, 9^h 10^m	1·483	96·38	0·114
7 36	1·490		
28, 9^h 21^m	1·492	120·13	0·107
6 55	1·494		
29, 10^h 11^m	1·500	144·23	0·100
6 17	1·501		
30, 9^h 11^m	1·505	163·18	0·095
31, 1^h 19^m	1·515	193·66	0·085
6 00	1·515		
Sept. 1, 9^h 19^m	1·517	211·32	0·083
2, 9^h 05^m	1·524	237·29	0·074
1 30	1·528		

* Adjusted $\tau+l+f$.

Figure 14 contains the results of Tables 24, 25, 26 and 27. The signs of ordinate and twist agree for the wire of greater viscosity. The phenomena are here of a complicated kind, showing that viscous motion in iron and steel is not similar. The sharp maximum of Table 24 is specially noteworthy; as is also the less pronounced maximum of Table 27—both of which hold for drawn iron. A maximum is also apparent in the thick, soft wire of Table 26; whereas the thin, soft wire of this table and the soft wire iron of Table 25 show no retrogression.

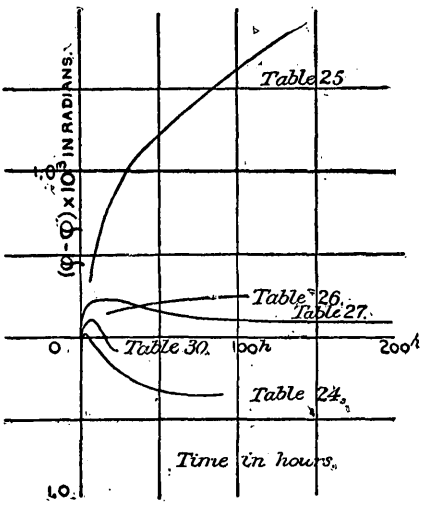

FIG. 14.—Viscous deformation of steel compared with iron, soft and drawn.

Other allied data are as follows:

TABLE 28.—APPARATUS I.

R=202cm; l=27cm; Rods: No. 55, steel, annealed at 450°, 2ρ=0·082cm; T positive; No. m wrought iron, soft, 2ρ=0·080cm; T negative.

Date.	h	$N \times 10^3$	h_0	$(\phi - \phi') \times 10^3$	Date.	h	$N \times 10^3$	h_0	$(\phi - \phi') \times 10^3$
Sept. 6, 9h 30h	0·00	.....	0·00	.....	Sept. 9, 10h 05m	72·58	0·535	76·79	0·307
9 42	0·20	0·200	0·20	—0·050	6 30	81·00	0·580		
9 52	0·37	0·223	0·37	—0·027	10, 3h 32m	102·03	0·664	104·26	0·421
1 57	4·45	0·307	4·45	+0·057	8 00	106·50	0·678		
7 12	9·70	0·333	9·70	+0·083	11, 9h 03m	119·55	0·721	124·99	0·478
7, 9h 11h	23.68	0·375	23·68	0·125	7 55	130·42	0·735		
6 11	32·68	0·397	32·68	0·147	12, 9h 43m	144·22	0·751	147·95	0·506
8, 10h 42h	49·20	0·444	53·70	0·211	5 10	151·67	0·761		
7 42	58·20	0·478							

TABLE 29.—APPARATUS II.

$R=200^{cm}$; $l=27^{cm}$. Rods: No. 56, steel, annealed at 450°; $2\rho=0{\cdot}082^{cm}$; No. *n*, iron, soft; $2\rho=0{\cdot}083^{cm}$; $T_s=+90°$; $T_i=-90°$.

Date.	h	$N\times10^3$	h_0	$(\phi-\phi')\times10^3$	Date.	h	$N\times10^3$	h_0	$(\phi-\phi')\times10^3$
Sept. 6, 10h 00m	0·00	.....	0·00	.....	13, 10h 20m	168·33	3·067	173·75	1·012
10 12	0·20	2·072	0·20	—0·008	9 10	179·17	3·116		
1 57	3·95	2·118	3·95	+0·038	14, 9h 00m	191·00	3·153	196·12	1·089
7 12	9·20	2·128	9·20	+0·048	7 14	201·23	3·184		
7, 9h 11m	23·22	2·163	23·22	0·083	15, 9h 00m	215·00	3·225	220·20	1·155
6 11	32·22	2·205	32·22	0.125	7 24	225·40	3·244		
8, 10h 42m	48·70	2·307	53·20	0·259	16, 9h 34m	239·57	3·280	244·35	1·213
7 42	57·70	2·371			7 07	249·12	3·307		
9, 10h 05m	72·08	2·477	76·29	0·425	17, 9h 20m	263·33	3·347	268·16	1·283
6 30	80·50	2·539			7 00	273·00	3·379		
10, 3h 32m	101·53	2·685	103·77	0·620	18, 9h 10m	287·17	3·415	293·15	1·353
8 00	106·00	2·715			9 08	299·13	3·451		
11, 9h 03m	119·05	2·807	124·48	0.766	19, 11h 55m	313·92	3·491	313·92	1·411
7 55	129·92	2·885			20, 9 06	335·10	3·541	335·10	1·461
12, 9h 43m	143·72	2·938	147·45	0·879	21, 9 22	359·37	3·559	359·37	1·479
5 10	151·17	2·980			28, 2 33	532·55	3·767	532·55	1·687

Permanent set 180° in iron.

TABLE 30.—APPARATUS III.

$R=370^{cm}$; $l=26^{cm}$; $\lambda<0{\cdot}2^{cm}$. Rods: No. 51, soft, $T=-180°$; No. 52, annealed at 450°; $T=+180°$. $2\rho=0{\cdot}082^{cm}$.

Date.	$N\times10^3$	h_0	$(\phi-\phi')\times10^3$	Date.	$N\times10^3$	h_0	$(\phi-\phi')\times10^3$
Aug. 20, 12h 35m	(*)	0·00	(*)	Aug. 25, 9h 05m	0·581	92·80	0·027
12 50	0·291	0·25	—0·004	7 01	0·568		
6 44	0·306	6·15	+0·011	26, 9h 10m	0·568	115·40	0·014
21, 9h 07m	0·314	20·53	0·019	4 08	0·556		
5 00	0·311	28·42	0·016	27, 9h 10m	0·548	144·13	—0·009
21, 5h 15m	(†)	0·00	(†)	7 36	0·530		
5 25	0·546	0·16	—0·002	28, 9 20m	0·530	164·87	—0·023
7 08	0·551	1·88	+0·003	6 55	0·520		
22, 9h 57m	0·556	16·70	0·008	29, 10h 1m2	0.515	188·99	—0·037
6 54	0·561	25·65	0·013	6 17	0·507		
23, 9h 08m	0·561	39·88	0·013	30, 9h 11m	0·505	207·93	—0·043
5 37	0·561	48·36	0·013				
24, 9h 18m	0·581	68·00	0·033				
5 13	0·581						

* Adjusted $\tau-b-f$. † Readjusted $\tau+b+f$.

Table 30 shows very clearly, the relatively insignificant difference which exists between steel annealed at 450° and soft steel, so far as viscosity is concerned. This difference is less than that between steel

and iron, as well as different in character. The results of Tables 28, 29, 30 containing further indication of the behavior of soft metal are given in Fig. 15.

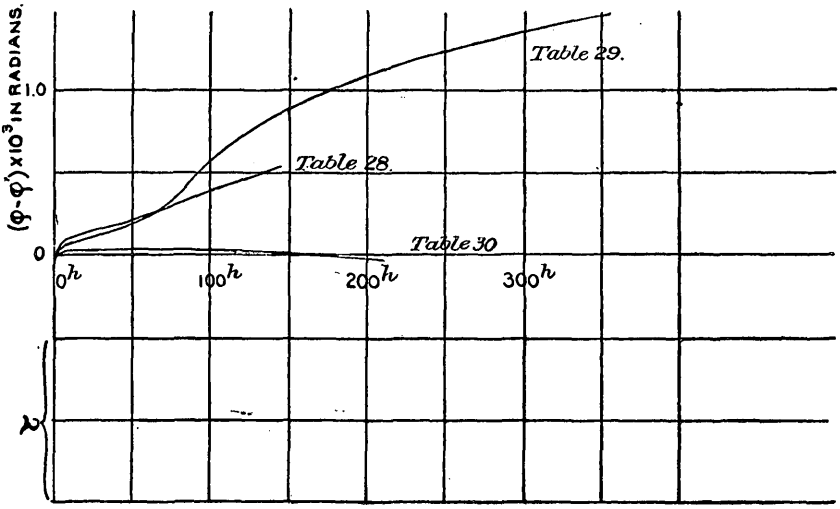

FIG. 15.—Viscous deformation of steel compared with iron.

For the purpose of further fixing the ideas I add some data with other metals. The results are to be constructed graphically in Figs. 16, 17. These relative results are difficult to interpret because of the varying amounts of permanent set incident to the experiments. To meet these errors the method must be varied as indicated in Chapters II and III.

Nickel and copper.—The data in hand are as follows:

TABLE 31.—APPARATUS III.

R=370cm; l=27cm. Rods: No. 61, steel, annealed soft; 2ρ=0·044cm; No. p nickel, annealed soft, 2ρ=0·048cm; T_s=+180°; T_n=−180°.

Date.	h	$N\times10^3$	h_0	$(\phi-\phi')\times10^3$	Remarks.
Sept. 15, 11h 30m	0·00	.....	0·00	.....	Adjusted.
11 45	0·25	0·346	0·25	−0·004	
7 22	7·87	0·371	7·87	+0·021	
16, 9h 34m	22·07	0·386	22·07	0·036	
7 15	31·75	0·396	31·75	0·046	
17, 9h 20m	45·83	0·396	50·72	0·049	
7 07	55·62	0·401			
18, 9h 09m	69·65	0·396	75·65	0·023	
9 09	81·65	0·351			
19, 11h 55m	96·42	0·346	96·42	−0·004	
20, 9 05	117·68	0·352	117·68	+0·002	
28, 11 30	312·00	0·371	312·00	+0·021	

TABLE 32.—APPARATUS IV.

$R=314^{cm}$; $l=27^{cm}$. Rods: No. 62, steel annealed at 450°; $2\rho=0{\cdot}082^{cm}$; No. q copper drawn; $2\rho=0{\cdot}083^{cm}$; $T_s=+90°$; $T_c=-90°$.

Date.	h	$N\times10^3$	h_0	$(\phi-\phi')\times10^3$	Date.	h	$N\times10^3$	h_0	$(\phi-\phi')\times10^3$
Sept. 10, 3h 06m	0·00	.....	0·00	.....	Sept. 15, 9h 00m	113·90	1·082	119·11	0·388
3 30	0·40	0·670	0·40	−0·030	7 25	124·32	1·094		
8 00	4·90	0·850	4·90	+0·150	16, 9h 35m	138·48	1·106	143·27	0·409
11, 9h 00m	17·90	0·940	17·90	0·240	7 10	148·07	1·112		
7 55	28·82	0·963	28·82	0·263	17, 9h 19m	162·22	1·118	167·12	0·418
12, 9h 45m	42·65	0·995	47·12	0·305	7 07	172·02	1·118		
6 41	51·58	1·016			18, 9h 10m	186·07	1·118	19·207	0·412
13, 10h 19m	67·22	1·022	72·66	0·328	9 10	198 07	1·106		
9 12	78·10	1·034			19, 11h 55m	236·82	1·112	236·82	0·412
14, 9h 00m	89·90	1·046	95·03	0·352	20, 9 05	245·98	1·117	245·98	0·417
7 15	100·15	1·058							

Permanent set in copper 180°.

TABLE 33.—APPARATUS I.

$R=202^{cm}$; $l=27^{cm}$. Rods: No. 63, steel, soft drawn, $2\rho=0{\cdot}082^{cm}$; No. r copper, softened; $2\rho=0{\cdot}080^{c}$; T_s positive. T_c negative.

Date.	h	$N\times10^3$	h_0	$(\phi-\phi')\times10^3$	Date.	h	$N\times10^3$	h_0	$(\phi-\phi')\times10^3$
Sept. 3, 1h 55m	0·00	.....	0·00	.....	Sept. 14, 9h 00m	19·08	0·427	19·08	0·033
2 06	0·18	0·468	0·18	−0·003	1 23	23·47	0·426	23·47	0·034
9 10	7·25	0·447	7·25	+0·013					

Permanent set in copper nearly 360°.

TABLE 34.—APPARATUS I.

$R=202^{cm}$; $l=27^{cm}$. Rods: No. 64, steel, soft; $2\rho=0{\cdot}082^{cm}$; $T=+45°$; No. s copper, softened, $2\rho=0{\cdot}155^{cmm}$ $T=-45°$.

Date.	h	$N\times10^3$	h_0	$(\phi-\phi')\times10^3$	Date.	h	$N\times10^3$	h_0	$(\phi-\phi')\times10^3$
Sept. 14, 1h 40m	0·00	.....	0·00	.....	Sept. 17, 9h 20m	67·67	1·179	72·55	0·186
1 47	0·12	0·920	0·12	−0·080	7 05	77·42	1·193		
1 58	0·30	0·942	0·30	−0·058	18, 9h 10m	91·50	1·208	97·47	0·214
7 13	5·55	1·044	5·55	+0·044	9 06	103·43	1·221		
15, 9h 00m	19·33	1·112	19·33	0·112	19, 11h 55m	117·75	1·235	117·75	0·235
7 22	29·70	1·140	29·70	0·140	20, 9 05	139·42	1·250	139·42	0·250
16, 9h 33m	43·88	1·157	48·65	0·163	21, 9 20	163·67	1·260	163·67	0·260
7 06	53·43	1·167							

Permanent set of copper 270°.

The viscosity of nickel as compared with soft steel (Fig. 17) is worthy of note. The small deformations of copper are the result of the unavoidably small stress intensities which actuate the viscous motion.

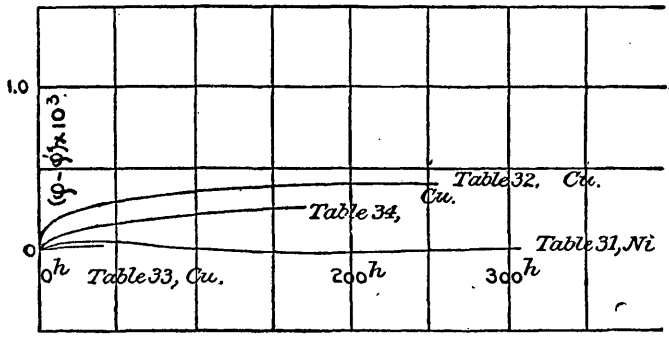

FIG. 16.—Viscous deformation of copper and nickel compared with steel annealed at 450°.

27. *Steels annealed at* 450° *and* 1000°.—The data in hand are as follows:

TABLE 35.—APPARATUS III.

$R=370^{cm}$; $l=27^{cm}$. Rods: No. 57, steel annealed at 450°; $T=+180°$; No. 58, steel, commercial, softened; $T=-180°$; $2\rho=0{\cdot}082^{cm}$.

Date.	h	$N\times10^3$	h_0	$(\phi-\phi')\times10^3$	Date.	h	$N\times10^3$	h_0	$(\phi-\phi')\times10^3$
Sept. 4, 9h 20m	0·00	.....	0·00	.....	Sept. 8, 10h 42m	97·37	0·485	101·87	0·190
9 33	0·22	0·294	0·22	−0·006	7 42	106·37	0·495		
7 40	10·33	0·371	10·33	+0·071	9, 10h 02m	120·70	0·505	124·93	0·210
5, 9h 47m	24·45	0·411	24·45	0·111	6 30	129·17	0·515		
6 30	33·17	0·425	33.17	0·125	10, 3h 30m	150·17	0·525	150·17	0·225
6, 9h 05m	47·75	0·450	52·79	0·153					
7 10	57·83	0·455							
7, 9h 10m	71·83	0·465	76·34	0·167					
6 11	80·85	0·470							

TABLE 36.—APPARATUS IV.

$R=314^{cm}$; $l=27^{cm}$. Rods: No. 59, steel annealed at 450°, $2\rho=0{\cdot}082$, T positive; No. 60, steel softened from hardness, $2\rho=0{\cdot}082$, T negative.

Date.	h	$N\times10^3$	h_0	$(\phi-\phi')\times10^3$	Date.	h	$N\times10^3$	h_0	$(\phi-\phi')\times10^3$
Sept. 4, 9h 40m	0·00	.....	0·00	.....	Sept. 7, 9, 10m	71·50	0·078	76·00	0·022
9 55	3·25	0·105	0·25	−0·002	6 11	80·50	0·084		
7 40	10·00	0·096	10·00	+0·007	8, 10h 42m	97 03	0·090	101·53	0·013
5, 9h 47m	24·12	0·084	24·12	0·019	7 42	106·03	0·089		
6 30	32·83	0·084	32·83	0·019	9, 10h 02m	120·37	0·102	124·60	0·001
6, 9 05m	47·42	0·078	52·46	0·022	6 30	128·83	0·102		
7 10	57·50	0·084							

The results for soft steels are constructed in Fig. 17, and corroborate Table 30 above, with reference to the small differences between steel annealed at 450° and steel annealed at 1000°.

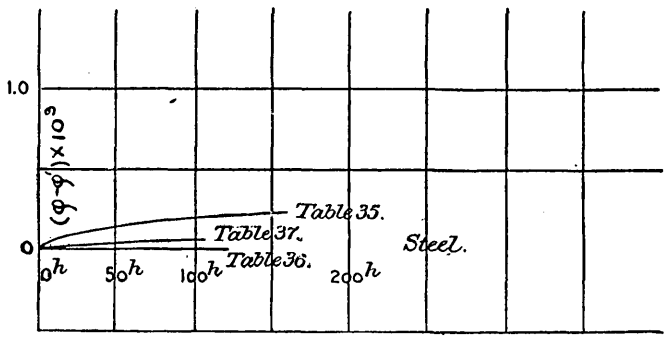

FIG. 17.—Viscous deformation of steel annealed at 450°, compared with soft steel.

28. *Quadrifilar arrangement.*—Table 37 contains results for a quadrifilar as explained below. The small differences of viscosity in question are scarcely sufficient to give character to the motion when the stress given is applied to such thin wires. Further experiments were not made.

TABLE 37.—APPARATUS III.

Quadrifilar suspension; $l=27^{cm}$; $R=370^{cm}$. Rods: No. 58, soft; $T_1=(+180°)+90°$; No. 57, annealed at 450°; $T_2=(-180°)+90°$; No. 60, soft; $T_3=(+0°)-90°$; No. 59, annealed at 450°; $T_4=(-0°)-90°$, $\rho=0{\cdot}082^{cm}$.

Date.	h	$N\times10^3$	h_0	$(\phi-\phi')\times\frac{}{10^3}$	Date.	h	$N\times10^3$	h_0	$(\phi-\phi')\times\frac{}{10^3}$
Sept. 10, 3h 45m	0·00	.....	0·00	.....	Sept. 13, 10h 20m	66·58	0·856	72·02	0·055
4 03	0·30	0·906	0·30	0·000	9 12	77·45	0·846		
8 00	4·25	0·906	4·25	0·000	14, 9h 00m	89·25	0·846	94·36	0·060
11, 9h 00m	17·25	0·898	17·25	0·008	7 14	99·48	0·846		
7 55	28·17	0·871	28·17	0·035	15, 9h 00m	113·25	0·866	113·25	0·040
12, 9h 45m	42·00	0·866	46·46	0·038					
6 41	50·93	0·871							

29. *Tubular apparatus.*—Table 38 and 39 contain our results with the tubular apparatus. ρ_1 is the inner radius, ρ_2 the outer radius of the tube. Q and q denote the right sections of brass tube and steel wire respectively. Of course sections of metallic surface are meant. The inclosed wire is twisted alternately in opposite directions as is indicated by the sign of T. The result is similarly indicated by the sign of $(\varphi-\varphi')$. Subscripts s and b refer to steel and brass respectively. Table 38 contains results for a thick tube, Table 39 for a thin tube. Hard or soft rods are inserted, as indicated.

TABLE 38.—APPARATUS (tubular) II.

$R=260^{cm}$; $l=22^{cm}$. Rods: No. e, brass, $2\rho_1=0{\cdot}188^{cm}$, $2\rho_2=0{\cdot}320^{cm}$. No. 89, steel, glasshard, $2\rho=0{\cdot}127^{cm}$; $Q=0{\cdot}053^{cm^2}$; $q=0{\cdot}0127^{cm^2}$; $Q/q<4{\cdot}5$.

Date.	$N\times10^3$	h_0	$(\phi-\phi')\times10^3$	Date.	$N+10^3$	h_0	$(\phi-\phi')\times10^3$
July 7, 9h 02m	0·308	*0·00	+0·002	July 14, 8h 57m	0·248	*0·00	±0·000
12 15	0·390	3·22	−0·080	10 27	0·248	1·50	±0·000
5 18	0·420	8·27	−0·110	12 45	0·248	3·80	±0·000
8, 9h 19m	0·460	24·28	−0·150	4 45	0·250	7·80	+0·002
12 48	0·465	27·77	−0·155	15, 10h 46m	0·250	25·82	+0·002
5 50	0·476	32·80	−0·166	15, 10h 46m	.....	*0·00	.....
9, 8h 30m	0·500	47·47	−0·190	12 35	0·310	1·82	−0·005
9, 8h 50m	0·456	†0·00	−0·056	4 27	0·306	5·68	−0·009
12 12	0·337	3·37	+0·063	16, 9h 20m	0·299	22·57	−0·016
5 20	0·283	8·50	+0·117	Original hard steel rod, No. 39, again inserted.			
10, 8h 09m	0·222	23·32	+0·178				
12 09	0·208	27·32	+0·192	16, 9h 25m	0·948	*0·00	+0·032
5 18	0·191	32·47	+0·209	11 26	1·009	2·02	−0·029
11, 9h 20m	0·160	48·50	+0·240	1 45	1·028	4·33	−0·048
11 35	0·151	50·75	+0·249	5 58	1·052	8·55	−0·072
4 15	0·142	55·42	+0·258	17, 8h 50m	1·093	23·42	−0·113
11, 4h 15m	0·540	*0·00	+0·060	12 55	1·102	27·50	−0·122
4 45	0·575	0·50	+0·025	6 04	1·125	32·65	−0·145
5 23	0·597	1·13	+0·003	18, 8h 27m	1·151	47·03	−0·171
7 00	0·630	2·75	−0·030	2 28	1·160	53·05	−0·180
12, 7h 39m	0·718	15·40	−0·118	19, 9h 41m	1·185	72·27	−0·205
5 29	0·743	25·23	−0·143	12 20	1·186	74·92	−0·206
13, 8h 30m	0·779	40·25	−0·179	5 15	1·186	79·83	−0·206
Glass-hard rod replaced by a soft steel rod, viz: No. 40, annealed, 450°, 1h, $2\rho=0{\cdot}127^{cm}$.				20, 8h 28m	1·207	95·06	−0·227
				20, 10h 50m	0·458	†0·00	−0·062
13, 10h 04m	0·876	*0·00	+0·004	11 18	0·416	0·47	−0·020
12 30	0·886	2·43	−0·004	11 50	0·396	1·00	±0·000
5 05	9·890	7·02	−0·010	1 53	0·354	3·05	+0·042
14, 8h 55m	0·894	22·85	−0·014	3 44	0·330	4·90	+0·066
				6 35	0·303	7·75	+0·093
				21, 10h 02m	0·247	23·20	+0·149
				12 18	0·239	25·47	+0·157

* Total twist $T_s-(-T_b)=180°$, twist of steel T_s, positive.
† Total twist $-T_s+(-T_b)=180°$, twist of steel, positive.

TABLE 39.—APPARATUS (tubular) V.

$R=260^{cm}$; $l=22^{cm}$. Rods: No. f, brass tube, $\begin{cases}2\rho_1=0{\cdot}19^{cm}.\\2\rho_2=0{\cdot}24^{cm}.\end{cases}$ No. 41, steel, annealed at 450°, 1h, $2\rho=0{\cdot}082^{cm}$. $Q=0{\cdot}0155^{cm2}$; $q=0{\cdot}0054^{cm2}$; $Q/q>2{\cdot}5$.

Date.	$N\times10^3$	h_0	$(\phi-\phi')\times10^3$	Date.	$N\times10^3$	h_0	$(\phi-\phi')\times10^3$
July 22, 3h 38m	.....	†0·00	.....	July 28, 9h 32m	1·476	120·14	0·392
3 48	3·602	0·17	+0·032	1 18	1·498		
5 39	3·541	2·02	−0·029	4 26	1·502		
7 13	3·518	3·58	−0·053	29, 9h 04m	1·511	143·86	0·417
23, 7h 54m	3·465	‡16·27	−0·105	12 48	1·518		
12 02	3·435	20·40	−0·135	4 33	1·521		
23, 12h 57m	1·026	*0·00	−0·074	30, 9h 01m	1·651	*167·90	0·533
3 37	1·147	2·67	+0·047	1 15	1·654		
24, 9h 28m	1·289	20·52	0·189	4 16	1·660		
1 03	1·300	24·10	0·200	31, 9h 54m	1·671	191·94	0·572
4 23	1·307	27·43	0·207	12 44	1·672		
26, 9h 08m	1·398	68·18	0·298	4 03	1·672		
12 17	1·409	71.33	0·309	Aug. 2, 9h 04m	1·689	239·95	0·592
3 58	1·417	75·02	0·317	1 27	1·692		
27, 9h 08m	1·433	95·71	0·337	4 11	1·695		
12 38	1·437			5, 1h 42m	1·727	338·63	0·627
4 13	1·441			5 27	1·727		

* Adjusted anew. † Total twist $T_s-(T_b)=180°$, twist of steel positive.
‡ Total twist $-T_s+(+T_b)=180°$, twist of steel negative.

The results of Tables 38, 39 are given graphically in Fig. 18, on a scale ten times as large as in the earlier figures. The figure shows the gradual reduction of viscous motion in the consecutive alternate twists.

We have not yet made the tubular apparatus as sensitive in its indications as is the bifilar. To obtain the full benefit of this adjustment long tubes and wires should be used. The results are nevertheless sufficiently sharp and pronounced and are cited here for their important bearing in the inferences to be drawn for steel. Fine glass tube is to be preferred to brass.

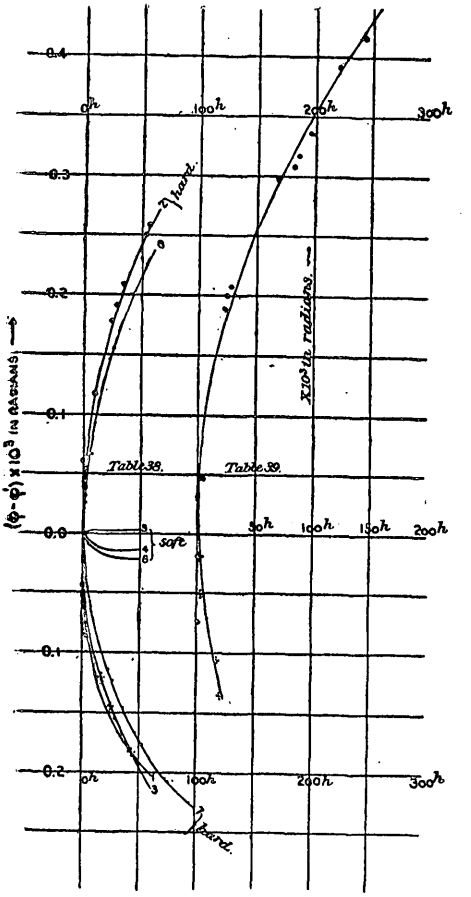

FIG. 18.—Results obtained with the tubular apparatus.

DISCUSSION.

30. *Interpretation of* $(\varphi-\varphi')$. Before proceeding further it is necessary to give the quantity $(\varphi-\varphi')$ a more specific interpretation than appears from a persual of the above paragraphs. Since the wires are 30cm long

and but .08cm in diameter, the considerations are much simplified by conceiving the twist helix of each wire to be developed, or rolled out on a plane surface. Fig. 19 indicates the result in a diagrammatic way. Here *ab* and *bd* are the developed position of two corresponding fibers (helices) of the countertwisted wires *A* and *A'*, *df* and *ag* being the lengths, *l* and *bf* and *bg* the cylindrical semicircumferences of the two wires, respectively. For in the usual cases above $bf = T = bg = 180°$. Hence *df* is usually more than 100 times as long as *bf*, and the same ratio holds for *ag* and *gb*. Hence also the angle *dbf* expresses the rate of twist; and for want of data indicating the precise relation between the rigidities of steel of different tempers, the angles *dbf* and *abg* must throughout the present volume be regarded equal.

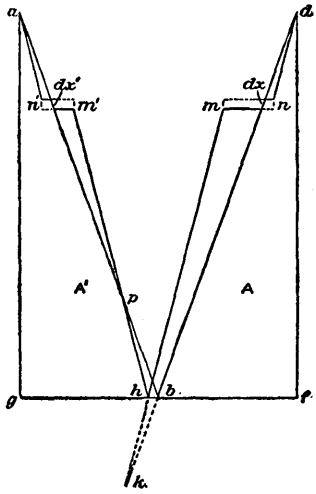

FIG. 19.—Diagram.

Suppose now that at any section of the wires, and at a distance x and x' respectively above the lower end, viscous sliding takes place, on both the plane surfaces of an elementary cylinder, of the length dx and dx' respectively. Now it is clear from the equality of couples, that the sliding on the two plane faces of the cylinder dx will be equal, and opposite in direction. The same is true of the sliding on the two ends of dx'. It is also clear that throughout this sliding the elastic equilibrium of the system of two wires is continually being disturbed. Therefore the viscous motion must be accompanied by purely elastic motion of the whole system, such that throughout every stage of viscous deformation the rate of twist is kept constant. From these conditions it follows that after a very short time *dt*, the originally straight fibers *db* and *ba* will make up a broken line *dnmh* and *hm'n'a*; that furthermore *dn* and *hm* must be parallel, *hm'* and *n'a* be also parallel, and that the angles *mhf* and *m'hg* are equal. Clearly *hb* corresponds to the motion observed at the movable mirror of the bifilar suspension (Fig. 1), and may therefore easily be interpreted.

Prolong the lines *db* and *mh* till they meet at *k*. The triangles *pbh* and *hbk* are equal in all their parts. Hence $pb = bk = y$.

Let 2φ be the amount of viscous angular motion of any section of the wire *A*, relative to another section at a unit distance from it, under the given conditions of adjustment. Let $2\varphi'$ have the same signification for *A'*. Then since the same couple acts throughout the length of either wire, $mn = 2\varphi dx$ and $m'n' = 2\varphi' dx'$, no matter what be the value of x or x'.

Let $d\psi$ be the angular motion at the mirror, so that $d\psi=hb$. Then for an inspection of the figure it follows that

$$\frac{2\varphi dx}{d\psi}=\frac{l}{y}+1 \qquad \text{and} \qquad \frac{2\varphi' dx'}{d\psi}=\frac{l'}{y'}-1.$$

whence by elimination, since

$$l=l',\ y=y',\ dx=dx',\ d\psi=(\varphi-\varphi')dx', \quad \ldots\ (1)$$

and by integration

$$\varphi-\varphi'=\psi/l \quad \ldots\ (2)$$

31. The quantity $\varphi-\varphi'$ defined by equation (2) is clearly identical with the $(\varphi-\varphi')$ of the above tables. The signification of the symbol is therefore evident. In case of any stated twist stored between the two bifilar wires, the two end sections of the unit of length of one wire slide viscously 2φ radians relative to each other, whereas the end sections of the other wire slide $2\varphi'$ radians relative to each other.

VISCOSITY AND TEMPER.

32. *Graphic digest.*—The results of Tables 1 to 21 may be discussed in two ways: We may either accept some definite and applicable law like that of Weber or that of Kohlrausch, and calculate the mean constants for each set of results, or we may construct them graphically and then calculate the coordinates of the mean curve for each set. The latter is decidedly the better way, because it is less arbitrary and more convenient. In Table 40 the mean results in question are thus summarized. The first two columns indicate the tables from which in each case the selections are made, and the tempers of the rods selected. The remaining columns contain the differences of viscous detorsion, $(\varphi-\varphi')$, in radians per centimeter of length of the bifilar, one of the wires of which is invariably glass-hard, the other annealed as stated. $(\varphi-\varphi')$ is arbitrarily fixed at zero, for one hour after a twist of $T+180°$ and $T-180°$ has been imparted to the soft and hard wire, respectively.

Instead of making all references to glass-hard wires, it would have been more in keeping with the purposes of the work to make such references to soft wire; but at the outset of the work, the high viscosity of annealed steel, as compared with hard steel, was not known, nor even conjectured.

In the tables and figures the data are positive in the direction of increasing viscosity.

TABLE 40.—*A digest of mean results, Tables 1 to 26.*

Table No.	Annealed at	Angular motion ($\phi-\phi'$) in radians.							
		50h	100h	150h	200h	250h	300h	350h	400h
1.....	20°	0·13	0·19	0·26	0·34	.....	.....	.....	.....
2.....	20°	0·02	0·03	0·03	0·04	.....	.....	.....	.....
Mean .	.....	0·08	0·11	0·15	0·19	.....	.....	.....	.....
6.....	100°, 10h	0·50	0·81	1·06	1·26	1.42	.....	.....	.....
8	...do	0·44	0·75	1·08	1·41	1·24	.....	.....	.....
7.....	...do	0·62	1·00	1·85	1·70	1·99	.....	.....	.....
4.....	...do	0·81	1·28	1·66	1·95	2·18	.....	.....	.....
Mean .	...do	0·59	0·96	1·41	1·58	1·71	.....	.....	.....
9.....	190°, 1h	1·07	1·52	1·88	2·20	2·50	.....	.....	.....
12.....	...do	1·34	1·79	2·16	2·51	2·81	.....	.....	.....
10.....	...do	1·55	2·18	2·55	2·89	3·17	.....	.....	.....
11.....	...do	1·56	2·06	2·50	2·91	3·32	.....	.....	.....
Mean .	...do	1·38	1·89	2·27	2·63	2.95	.....	.....	.....
13.....	360°, 1h	1·70	2·54	3·12	3·56	3·86	4·15	4·42	4·70
14.....	...do	1·90	2·68	3·15	3·51	3·82	4·14	4·42	4·70
Mean .	...do	1·80	2·61	3·13	3·53	3·84	4·14	4·42	4·70
16.....	450°, 1h	2·03	2·74	3·20	3·57	3·91	4·23	4·52	4·80
15.....	...do	1·98	2·78	3·42	3·92	4·37	4·75	5·08	5·34
Mean .	...do	2·01	2·76	3·31	3·75	4·14	4·49	4·80	5·07
19.....	1000c	1·36	2·06	2·64	3·12	3·55	3·93	4·20	.....
21.....	...do	1·78	2·50	3·10	3·62	4·07	4·45	4·72	.....
18.....	...do	1·94	2·74	3·32	3·81	4·20	4·47	4·69	.....
20.....	...do	1·95	2·75	3·36	3·90	4·35	4·74	5·05	.....
Mean .	...do	1·76	2·51	3·10	3·61	4·04	4·40	4·67	.....

All radii identically $2\rho = 0{\cdot}082^{cm}$. Like signs of ($\phi-\phi'$) and of N refer to the same angular direction.

The results of Table 40 are graphically constructed in Fig. 20, time in hours as abscissa, difference of angular detorsion ($\varphi-\varphi'$), in radians, as ordinate.

33. *Immediate results.*—Table 40 and Fig. 20 lead to this curiously remarkable result: If we abstract for the moment from the states of temper extreme hard and extreme soft, *the viscosity of steel decreases in proportion as the hardness of the metal increases.* We call to mind here that the torsion imparted was not sufficiently large to produce marked permanent set (cf. Chapter III). If we express its intensity by $(0{\cdot}4E)\frac{\pi\rho^4}{2l}T$, and introduce the constants of the apparatus

$$T=\pi,\ l=30^{cm},\ 2\rho=0{\cdot}082^{cm},$$

we find that the moment of the applied torsion couple did not exceed 0·5 kg. on centimeter of arm. (Cf. remark on stress value, p. 48.) If, following Sir William Thomson,[1] we agree that "the molecular friction

[1] Thomson: loc. cit., or Thomson and Tait, Natural Philos., 1883, vol. 2, p. 303. Our present conception of the viscosity of liquids, as well as the hypotheticated proportionality of frictional resistance and velocity were introduced by Newton (Principia, Liber 2, sect. 9, "resistentia quæ oritur ex defectu lubricitatis").

in elastic solids may properly be called viscosity of solids," then our deduction may be stated, *the molecular friction in case of steel is greater in proportion as the metal is softer.*[1] Examples of such relations in divers substances are not unknown. Hard steel as regards viscosity and hardness may be typified by sealing-wax; soft steel by tallow.[2] Nevertheless the continuous variations of these properties exhibited by steel is as unique as it is striking. Indeed we felt diffidence in reporting this result and have taken pains to substantiate it.

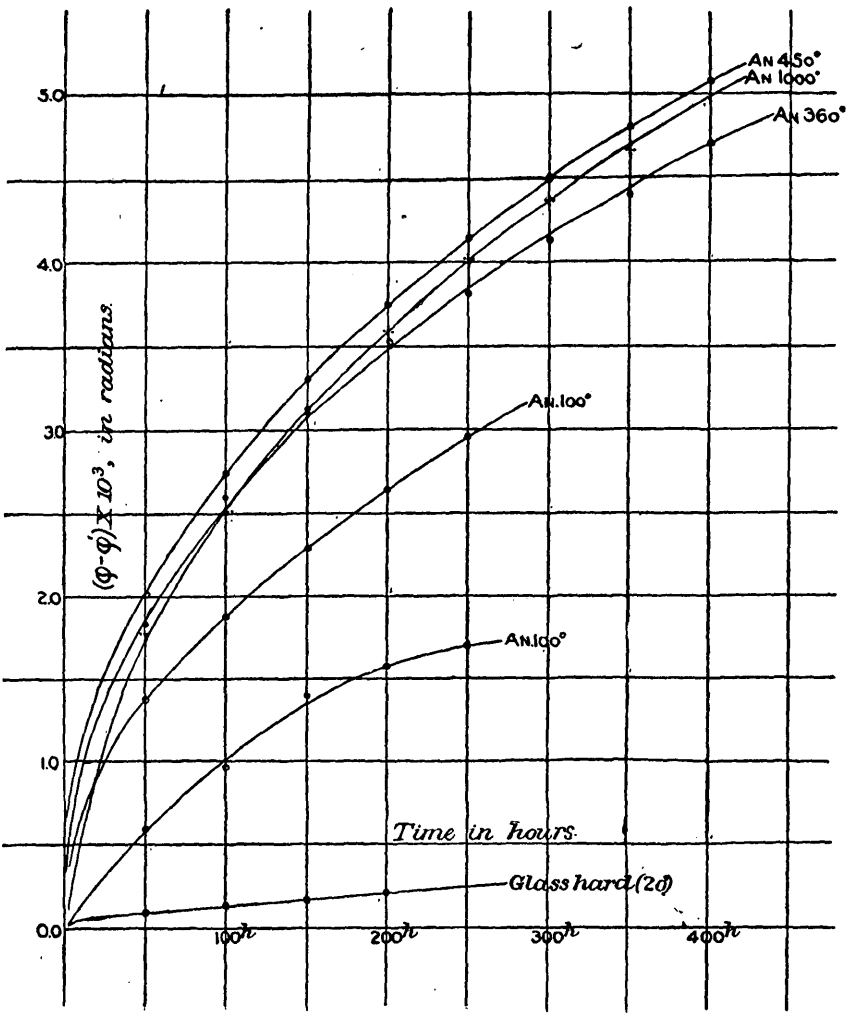

FIG. 20.—Viscous deformation of steel, as related to temper.

[1] Degrees of thermo-electric hardness are here specially in place. Cf. Bull. U. S. Geol. Survey, No. 14, p. 65.

[2] These examples (tallow, sealing wax) are given by Maxwell: Heat, Appleton, New York, 1883, p. 296.

34. The motion of the bifilar body of the above apparatus can be interpreted from two standpoints: Either it is due to the torsional couple and the result of viscous yielding of the harder wire relatively to the softer; or it may be due to the bifilar and flexural couple and is then the result of viscous yielding of the softer wire relatively to the harder (page 29). But the bifilar and flexural couples have been proved to be zero and to produce a zero effect. Hence the inference above italicized is alone admissible. Again, in the bifilar apparatus where steel is twisted against glass (Table 23) the soft steel is demonstrably more viscous than glass. Hard steel, as shown by its behavior with the same fiber, is less viscous than soft steel. We do not wish to say that it is less viscous than glass because the sectional area of the latter fiber is larger. Again, in Table 38, which contains the data obtained with the tubular apparatus, soft steel yields viscously at about the same rate as the brass tube of more than *four* times the sectional area of the steel wire. Hard steel under the same circumstances yields at a very much greater rate than brass.

35. Our results for degrees of hardness higher than "Annealed at 100°, 10^h" are to be regarded incomplete because of the magnetic importance of those degrees. As steel passes in hardness from "Annealed at 450°" to "Annealed at 1000°" (soft), it probably marches through maximum viscosity. This result is pretty clearly indicated by Table 40 and Fig. 20. Here also the results are to be regarded incomplete because of the magnetic importance of the (soft) degrees of hardness in question. Our methods of annealing between 500° and 1000° are not as yet satisfactory. It will be shown below that all remarks here made refer to intensities of stress less than the value (0·5 kg., cm.) given at the outset of this section. (Cf. Chapter II.)

36. *Viscosity and electrics of steel.*—If we compare the results of Table 40 or of Fig. 20 with the known thermo-electric behavior[1] of steel wires, we detect a very striking similarity in contour and position of corresponding members of the viscous and thermo-electric families of curves. Both phenomena practically subside in the first phase of annealing; the effect of temperature becomes rapidly less as higher degrees are approached. The same relations hold between viscosity and electrical resistance of steel.

37. *Viscosity and hardness.*—The relations between hardness and viscosity here encountered may perhaps be conceived somewhat as follows: Suppose stress to be so distributed in a solid that its application at any interface is nowhere sufficient to produce rupture. Then that property of a solid in virtue of which it resists very small forces (zero-forces) acting through very great intervals of time (∞-times) may be termed the viscosity of the solid. That property in virtue of which it resists the action of very large forces (∞-forces relatively) acting through zero-time may be termed the hardness of the solid. Since the application of

[1] U. S. Geol. Survey, Bull. 14, pp. 54, 55.

forces in such a way as accurately to meet either of these cases is rare, we have in most practical instances mixtures of viscous resistances and hardness to encounter. We may reasonably conceive that in the case of viscous motion the molecules slide into each other or even partially through each other by interchange of atoms, so that the molecular configuration is being continually reconstructed; that in the other case (hardness) the molecules are urged over and across each other and that therefore the intensity of cohesion is in this case more or less thoroughly impaired. In most cases of scratching, the action is indeed accompanied by physical discontinuity of the parts tangentially strained. The interpenetration of the molecules of a viscous substance is necessarily favored by temperature. Hence we infer the experimental result that the viscous influence of temperature is marked. If Clausius's theory of electrolysis be correct, then a certain instability or imperfect uniformity in the molecular structure of solids follows at once from the fact that many solids, notably glass,[1] may be electrolyzed even at moderately high temperatures (300°). The important bearing of all these remarks on Maxwell's theory of viscosity of solids will be given in Chapter III.

38. *Residual phenomenon.*—The stored torsional stress imparts a strain to the solid. Viscous detorsion therefore is accompained by a residual phenomenon. The observed deformation will continue until the applied tendency to change of form is reduced in value to the evoked and increasing tendency against change of form. If the applied stress be removed, the reciprocating stress becomes apparent and produces viscous effects of its own kind, as Kohlrausch has shown. The result has many magnetic and electrical analogies, among which the phenomenon of residual static charge is most obvious. In the tubular apparatus (Table 38), the residual deformation of the preceding twist may be superimposed on the deformation immediately in progress. Whether the two residual phenomena here annul each other so that the primary detorsion only is exhibited remains to be seen.

In liquids there is no such reaction unless it be the reciprocating force of galvanic polarization. In polarization, however, the mechanism is of an obviously chemical kind. In solids it is believed to be not chemical.

SECTIONAL AREAS OF BIFILAR WIRES.

39. The essential peculiarity of the bifilar apparatus is this, that the two wires are twisted by identical couples.[2] The absolute value of these couples during the experiment remains constant to 2 or 3 per cent. If the sectional areas of the wires be identical our apparatus leads to re-

[1] Warburg: Wied. Ann., vol. 21, 1884, p. 622. Literary notes are there given. Warburg is able to replace $\frac{2}{3}$ of the sodium of glass by sodium of the anode.

[2] Many years ago we compared the longitudinal resilience of hard and soft steel by fastening one end of thin wires in a vise and bending them with a weight applied at the other. We found but insignificant differences. Hence the stored torsions of two steel wires, hard and soft, produced by equal couples are cæteris paribus of equal angular value.

sults which come very close to Newton's definition of viscosity. Newton suggested that the internal friction of liquids is cæteris paribus directly proportional to the difference of velocity between nearly contiguous surfaces. In the bifilar apparatus the torsional viscosities of two substances are equal if for identical strains and equal sectional areas torsional change of form occurs at like rates per unit of length. This is the condition of rest or zero-motion of the suspended body. Such inferences as are here given must, however, be drawn with extreme caution. There are certain cardinal differences between solid and fluid viscosity, which will be indicated in Chapter III and which must first be decided upon as prerequisites to conclusions like the present. It is doubtful whether Thomson and Tait's definition of solid viscosity can be at once accepted.

We do not at present wish to do more than advert to an important deduction here; it is obvious that if the sections of the wires be so chosen that the motion of the bifilar body is zero, the viscosity of the wires must be inversely related to those sections. This principle apparently enables us to arrange solids in a scale of viscosity. We may formulate it approximately thus:

Consider an elementary ring of either wire, whose height is dz and whose right sectional area is $2\pi r dr$. Let df be the amount of tangential force uniformly distributed over this area. At the time t let the velocity of the upper surface relatively to the lower be cr, where c is a time-function and cæteris paribus a characteristic of the viscosity of the wire. Then if μ_t be the coefficient of viscosity at the time t we have

$$df = \mu_t \frac{cr 2\pi r dr}{dz} = 2\pi \mu_t \frac{cr^2}{dz} dr$$

If we multiply by r and then integrate between zero and ρ (thickness $= 2\rho$), the numerical result is the part of the impressed torsional couple which corresponds to the length dz. A similar integral holds for the other wire, to distinguish which it is merely necessary to accentuate f, μ_t, c, r, ρ. The sum of the two integrals is zero. If, moreover, we put $c = c'$ in view of the state of rest of the bifilar body, we find that the viscosities (μ_t, μ'_t) of the two substances (wires) are to each other inversely as the squares of the respective sectional areas by which the motion of the bifilar suspension is annulled.

40. Unfortunately the problem is much more complex than it here appears. The dependence of torsional deformation on time in case of a single wire is obviously related to the character of the molecule. When two different substances in wire form are twisted bifilarly against each other, the effect will rarely be such that the torsional yieldings continually equalize each other, no matter what relations of section may be chosen. The motion-curve of the bifilar-suspension will show maxima or even points of circumflexion, such as have been actually encountered in Tables 23, 24, 25, etc., and the definition of relative viscosities of the two wires will become correspondingly involved. Apart from these

complications which are inherent in solid structure, a modification of the apparatus such as is given in Chapter II, or the present tubular apparatus, may easily subserve the purpose of classification by initial tangents.

41. Again, μ_t for solids is not merely a function of time but very essentially a function of stress. Above we show (page 12) how by simply adjusting the lengths of the bifilar wires, viscosities may be compared at a given temperature, as time-functions with identical values of the parameters stress, strain (sectional area). We do not believe that viscous detorsion in this full relation has ever been rigidly investigated. The results would lead to families of curves.

Given a quadrifilar arrangement of four viscously identical wires. Let the twist T_{12} be stored between any two of them and then let the twist $T_{12,34}$ be additionally stored between these two as one, and the third and fourth wire as one. Such a device enables us in the above way to study viscosity in its simple dependence on strain, for all values of stress.

VISCOSITY AND STRAIN.

42. *Steel and glass.*—When we commenced the present research a comparison of the viscosity of glass and steel appeared desirable. In Tables 22, 23, such comparisons have been attempted, though we regret that the limitations of our available time have restricted our pursuit of them. In Table 22, the sectional area of glass fiber is less than that of steel wire; nevertheless the viscous yielding of the fiber is so much more rapid than that of the wire, that we may reasonably infer degrees of viscosity of the same order in the two substances. In Table 23, the sectional area of glass exceeds that of steel. Hence these data prove that the torsional viscosity of annealed steel is greater than that of glass. The viscosity of hard steel during the first ten hours of detorsion is certainly very much greater than that of glass. During the remainder of the time it is decidedly less. The curve passes through a maximum for which point the rates of viscous detorsion of glass and of glass-hard steel exactly coincide. Since the sectional area of glass is greater, we safely infer that the viscosity of glass is not uniformly greater than that of glasshard steel. It is well to call to mind, however, that the sum of the torsions is here only $T_s+T_g=90°$. Moreover, since for equal couples and dimensions $E_gT_g=E_sT_s$, where E is Young's modulus[1] of resilience for glass and for steel respectively and T the corresponding torsion; and since $E_g/E_s=1/3$ approximately, it appears that $T_g=3T_s$ nearly. Hence for equal stresses, strains are thoroughly unequal, and in the same measure are these comparisons not thoroughly just.

43. *Steel and iron.*—A similarly important desideratum is the comparison of the viscosities of steel and of iron. In Tables 24, 25, 26,

[1] To obtain an estimate it is sufficient to accept the same ratio of modulus of torsion to modulus of longitudinal resilience for each case. Poisson's ratios for glass and steel are about as 26 to 30.

etc., such comparisons are made, though further results are not superfluous. It appears distinctly that during the first five or ten hours of detorsion the viscosity of iron is in a strikingly pronounced manner less than that of steel. As detorsion continues the viscosity of soft iron continually remains below that of steel, whereas the viscosity of drawn iron grows temporarily greater than steel but finally reaches the same value. In this case the motion-curve passes through an exceedingly sharp maximum, at which the viscous yielding of iron and of steel occurs at like rates. Having exhibited greater viscosity in drawn iron it finally, by circumflexion, merges into a horizontal asymptote, which probably indicates subsidence of motion in each wire. These unexpected results show that the viscosity of steel is not uniformly greater than that of iron. Obviously glass-hard steel for the given stress is very much less viscous than iron.

The sum of the twists is here nominally $T_s-(-T_i)=360°$. Particularly in the case of soft iron its efficient value is very decidedly less, however. The applied torsion carries the iron wire much beyond the limits of elasticity; and so much of the stress vanishes instanter that the "after-action" subsides within relatively small limits. The viscosity of (soft) iron is either less or greater than that of hard steel according as the applied torsional stress surpasses or falls below a certain critical value. This result again shows the importance of stress-measurement, as a means of coordinating the lingering of purely viscous deformation and the instantaneous (?) deformation.

In the above results the curvature of the (absolute) motion curve for a *single* wire increases uniformly from hard steel to soft steel; increases, moreover, from soft steel to iron. Curvature is greater in the glass curve than in hard steel curves. So far as we have observed (steel, iron, glass) viscous deformation occurs more nearly at uniform rates (linearly) in proportion as hardness is greater. Cf. Chapter III, where similar results are given under "accommodation." Curvatures for early time are meant, the later parts of curves being asymptotes.

44. *Effect of quenching.*—The remarks just made on the viscosities of glass and iron suggest this plausible inference: if glass and steel be alike subjected to a stated process of quenching, and if after the operation has been performed glass be found to have retained a very high intensity of strain (Rupert's drop), then, a fortiori, steel, the substance of greater viscosity, must have retained a similarly high intensity of strain. This reasoning, however, is incomplete and must be approached with caution. We pointed out[1] that the low degrees of thermal conductivity possessed by glass were favorable to the retention of strain. We also remarked[2] that the occurrence of Gore's phenomenon of sudden expansion at red heat distinguished iron and steel from all other metals, and that in spite of relatively good thermal conductivity, iron and steel

[1] Am. Jour. Sci. (III), 1886, vol. 31, p. 451.
[2] Bull. U. S. Geol. Survey, No. 14, p. 99.

possessed virtually all the favorable conditions of glass for retaining strain. Whether differences of viscosity are sufficient to account for the unlikeness of behavior during quenching we are unable to say. It will be necessary to compare iron and steel at higher than ordinary temperature and also under greater values of applied stress than was easily feasible in the above experiments. In other words, iron and soft steel are nearly equally viscous for small values of stress. After stress exceeds a certain intensity, the permanent set suddenly imparted to iron is enormously great as compared with steel. Similar relations are true for soft steel when compared with hard steel. It is this class of "sudden" phenomena which come into play during quenching. The primary effect of quenching steel is chemical hardness. Strains are retained in the steel so modified, just as they are in glass; whereas in soft steel or soft iron the result would be permanent set. In Chapters II and III, a more complete explanation than is here possible will be given, so that further consideration may be waived for the present.

The occurrence of sudden and gradual deformation in a single substance suggests that ordinary *static* friction is probably a viscous phenomenon.

45. We add in passing that the observed inefficiency of temperatures less than 200° in producing marked viscous deformation in a Rupert's drop proves that mere interference of thermal expansion with the conditions of strain cannot be the primary cause of its variations; that strain variations essentially depend on diminished viscosity.

46. *Steel and cast-iron.*—We also add in passing that the importance of strain as associated with glasshardness is emphasized by the mass-constants of the cast-irons. The densities of these metals[1] range between the maximum of ca. 7·6 for white cast-iron and the minimum of ca. 6·9 for gray cast-iron. Hence density increases in marked degree in proportion as total carbon is more and more nearly combined. Quite the reverse of this is true for steel where density decidedly decreases as total carbon is more and more nearly combined. This discrepancy we interpreted as a strain of dilitation and carefully compared it with the analogous behavior of glass in our earlier papers.

47. *Stress intensity estimated.*—We add finally the following data from the elastics of glass and of steel. The æolotropic expansion produced by quenching glass or steel we showed elsewhere[2] to amount to 0.005. The volume resilience of glass and of steel, according to Professor Everett's measurements,[3] is 4×10^{11} and 2×10^{12} respectively. Hence if per square centimeter p be the stress-value for the given expansion, we find approximately $p=10\times10^{9}$ dynes for steel and 2×10^{9} dynes for glass. Now per square centimeter the tenacity of steel is 8×10^{9} dynes; the tenacity of glass $0{\cdot}6\times10^{9}$ dynes. The approximate ratio of stress to

[1] Cf. Bloxham's Chemistry, Lea, Philad., 1873, p. 342. Bull. U. S. Geol. Survey, No. 14, pp. 76, 77.

[2] Am. Jour. Sci. (III), 1866, vol. 32, pp. 190, 191. By æolotropic expansion we refer to the part of the total expansion which produces explosiveness and the polariscopic phenomenon.

[3] Everett: Phil. Trans., 1866, p. 369. The above numbers are rounded from Everett's values.

tenacity is therefore estimated at 1·3 for steel and 3·3 for glass. This shows that in both cases stress and tenacity are of the same order; that stress is in excess; that but for the peculiarly favorable, symmetrically arched structure of the quenched globule, rupture would ensue in glass certainly; probably also in steel. This accords with the explosive tendency of a Prince Rupert's drop and with the less pronounced liability of steel to crack on quenching.[1] If therefore quenched glass and quenched steel are under mean stress intensities of several thousand atmospheres, then in discussing the corresponding viscous properties they must be brought into relation with these high values of peculiar stress.

MAGNETIC RELATIONS.

48. *Viscosity and magnetic intensity.*—If again we abstract from the extreme states of hardness, we find that both the viscosity and the moment of linear magnetization per unit of mass, of a permanently saturated steel rod, increase in marked degree from hard to soft. This is a singularly striking result, inasmuch as the conditions of magnetic stability (following Hopkinson we shall call them coercive force), conditions which at first sight we would be inclined to associate with viscosity, decrease as viscosity increases. Hence permanently saturated linear magnetic intensity and viscosity on the one hand, magnetic stability or coercive force and hardness on the other, seem to belong together.

49. *Magnetic and viscous maxima in steel.*—The minimum of permanent linear intensity of saturated steel rods has no viscous equivalent; but we have not yet studied the viscosity of extremes of hard steel minutely, nor have we as yet sufficiently sharp data for the magnetization of very long rods (length /diameter >100) in its relation to temper. In the extreme soft region, on the other hand, the occurrence of a unique maximum of magnetization seems to be coincident with the occurrence of maximum viscosity. The magnetic maximum so far as our results go is apparently much more clearly pronounced than the viscous maximum. We remark in general that as the ratio of length to diameter increases, the minimum of permanent magnetization shows a tendency to move from soft to hard. Furthermore, it is proved that the permanent magnetization of soft rods is greater in proportion as they are more and more nearly linear.[2] Hence as our march approaches the linear limit we observe an unmistakable tendency toward more detailed similarity between the variations of the magnetic and the viscous properties of steel.

Mr. Hopkinson[3] in his memoir on the magnetization of iron introduces a definition of coercive force, defining it as "that reversed magnetic force which just suffices to reduce the induction to nothing after the

[1] Mr. J. M. Batchelder has just communicated to us his interesting results (Jour. Franklin Inst. (3), 1844, vol. 8, p. 133), in which of twelve massive pieces of quenched steel, eight subsequently cracked and one actually exploded.

[2] Bull. 14, U. S. Geol. Survey, pp. 140 to 143.

[3] Hopkinson: Phil. Trans., 1885, vol. 2, p. 463.

material has been subjected temporarily to very great magnetizing force." There is a slight objection to this definition, inasmuch as it introduces a somewhat vaguely complex state of zero-magnetization.[1] Wiedemann[2] in comparing magnetization to and fro with viscous torsional accommodation has been most explicit on these points. But the feature in question is hard to improve and the definition is valuable. Mr. Hopkinson's extensive experiments show that coercive force is invariably increased by hardness. Together with others we have proved that hard steel is alike well adapted to withstand the influence of percussion.

50. *Steel and iron.*—These general relations between viscosity and maximum permanent linear intensity observed for steel are sustained by iron. According to data contained in a paper of the lamented Dr. L. M. Cheesman[3] the permanent magnetization of drawn iron exceeds that of soft iron. The magnetic intensity of permanently saturated drawn iron is comparable with that of soft steel and quite equal to that of hard steel. More recently and even more elaborately Ewing has discussed the subject. He finds "in the absence of mechanical or other disturbance, soft iron is far more retentive than either hard iron or steel," an exceedingly remarkable result.[4] Our experiments show that the intensity of applied stress is an important factor in determining the viscous behavior of iron; that for moderate stress values it *ultimately* approaches very closely or even exceeds the maximum viscosity of steel; that it is much greater than the viscosity of hard steel.

REMARKS.

51. Among the results of this paper is the light thrown on the crucial importance of the physical changes which steel undergoes during annealing at high temperatures, i. e., when subjected to the action of temperatures between 500° and 1000°. Prof. W. F. Barrett,[5] who made valuable researches on this subject, pointed out the more important coincidences (elongation, crepitation and ticking, recalescence, magnetic and thermo-electric anomalies). In our our earlier work we unfortunately overlooked these papers. Tait, however, had before this connected his peculiar iron line (thermo-electrics) with Gore's phenomena. The occurrences in question may now be summarized as follows: Gore's[6] sudden

[1] The unmagnetic state, though dependent on the details of the process of tempering, is a distinct magnetic state. Similarly we may regard the saturated magnetic state, though allowance must here be made for the occurrence of cyclic magnetization, a phenomenon suggested by Fromme (Wied. Ann., 1878, vol. 4, p. 89) and (Wied. Ann., 1881, vol. 14, p. 308), concisely, interpreted by Warburg (Wied. Ann., 1881, vol. 13, p. 141) and which Ewing (Phil. Trans., 1885, vol. 2, p. 545) has recently discussed generically under the name of "hysteresis."

[2] Wiedemann: Wied. Ann., 1879, vi, p. 512.

[3] Cheesman: Am. Jour. Sci. (III), 1882, vol. 24, p. 183. Lengths 100 diameters and less.

[4] Ewing: op. cit., p. 541.

[5] W. F. Barrett: Journal London Inst., July, 1873; Rept. Brit. Assoc., 1875, vol. 45, pp. 259–60; Phil. Mag. (5), December, 1873; ibid, January, 1874; Proc. Royal Dub. Soc., 1886; Am. Jour. Sci. (III), 1887, vol. 33, p. 308. For recent work on recalescence see Hopkinson: Proc. Roy. Soc., 1889, vol. 45, p. 455.

[6] Gore: Proc. Roy. Soc., 1869, vol. 17, p. 260.

volume expansion; Tait's[1] sinuously broken thermo-electric diagram-line; Barrett's (loc. cit.) recalescence, crepitation, and elongation phenomena; Barrett's behavior of manganese steel; Gore-Baur's[2] sudden disappearance of magnetic quality; the passage of carbon from uncombined to combined; Jeans's[3] critical cementation temperature. Further, Chernoff-Barus's[4] sudden appearance of hardness in quenched steel, Fromme's[5] maximum density, Chwolson's[6] minimum resistance, our own unique maximum of magnetization[7] and probable maximum of viscosity, Howe's[8] sudden decrease of rigidity, are all referred to this interval, approximately to the same temperature. To determine the interdependence of these phenomena it is obviously first necessary to devise methods of operating on steel adapted to the interval in question. In other words, our introductory problem is the annealing of steel without oxidation and without carburation, at measured (high) temperatures, during stated times. It is in this direction that our investigation will ultimately proceed. In the present work, however, the thermal relations of viscosity still command paramount attention; and their importance gains much when it is called to mind that before a satisfactory physical theory of viscosity has been formulated, the light which data like the above throw on ulterior electrical phenomena will not be searching.

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[1] Tait: Trans. Roy. Soc., Edinburgh, 1872–'73, vol. 27, p. 125.

[2] Gore: Phil. Mag., IV., 1869, vol. 33, p. 59; Baur. Wied. Ann., 1880, vol. 11, p. 408; Ibid., 1870, vol. 40, p. 170.

[3] Jeans: "Steel, its history, etc." London, Spon, 1880.

[4] Chernoff: Vortrag. geb. in der Russ. techn. Ges., April und May, 1878. Barus: Wied. Ann., 1879, vol. 7, p. 405.

[5] Fromme: Wied. Ann., 1879, vol. 8, p. 354. We were unable to find Fromme's maximum: but we will search again. All these results are crucially dependent on the method of annealing, as stated in the text.

[6] Chwolson: Carl's Repertorium, 1878, vol. 14, p. 26.

[7] Barus and Strouhal: Bull. U. S. Geol. Survey, No. 14, pp. 145, 198.

[8] H. M. Howe: Scientific American Supplement, April, 1889, p. 11055.

CHAPTER II.

THE VISCOSITY OF STEEL AND ITS RELATIONS TO TEMPERATURE.

INTRODUCTION.

52. This chapter is to be restricted to a discussion of the relation between torsional viscosity and temperature, as observed with steel in different states of hardness. Some mention of the effect of stress on the amount of viscous motions in solids is, however, unavoidable; and the experiments lead naturally to the investigation of a more general method, by which the instantaneous deformation and the gradual deformation produced by stress may be coordinated. The data already show that imperceptible gradations lead from the purely viscous deformation which follows strains within the elastic limits, to the sudden, permanent set which follows strains beyond the elastic limits. As a whole the data bear directly on the truth of Maxwell's theory of viscosity. (Cf. Chapter III.)

METHOD OF MEASUREMENT.

53. *Apparatus.*—Instead of countertwisting two wires suspended bifilarly, as was done in Chapter I, it is for many purposes preferable to suspend and twist them unifilarly. In this case they may be kept at different temperatures throughout different parts of their lengths, and the adjustment therefore has special advantages for the class of experiments indicated in the present chapter.

In the actual work it is expedient to compare two identical wires, one of which is kept hot, the other cold. These wires are shown in the annexed diagram (Pl. I) at *ab* and *cd*. The ends, having been bent loop-shaped, are screwed near the middle of the apparatus to a cross-piece of iron carrying the adjustable mirror M. This is the index referred to in the preceding paragraph, and angular deviations are read off by Gauss's method. To deaden lateral vibrations the mirror is also provided with a cross-vane, *mm*, dipping appropriately into a fixed dish of water. In later experiments this dish was replaced by an annular vessel surrounding the wire. The upper end of the system of wires is screwed to a round steel rod, *af*, which fits snugly in the perforation of the massive fixed piece of brass *gh*. A steel pin, *p*, passing through both brass and steel rod, secures the latter to the former. The lower end of the wires is similarly fastened to the steel rod *dh* and the fixed

piece of brass *kl*. To keep the wires tense they are stretched by a weight, RQ; and a part of the lower fixed brass *kl* has therefore been cut away in such a manner that the lower pin *q* may slide along a plane. The system of wires is therefore free to expand.

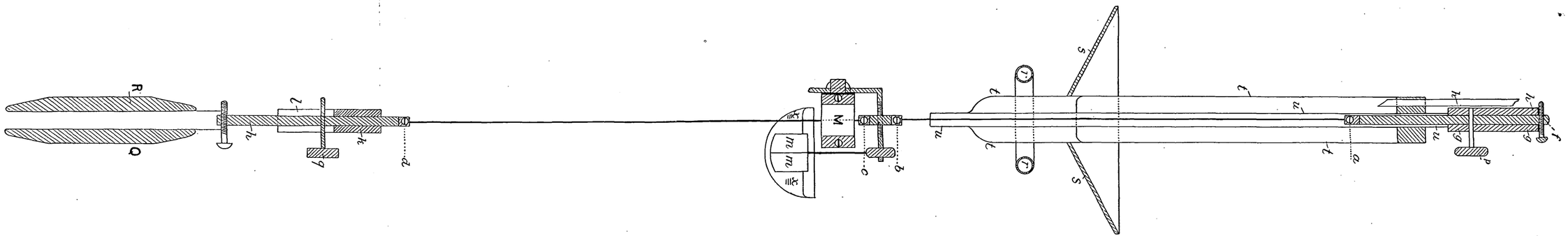
To heat the upper wire, a given part of it was surrounded by a special form of vapor bath, without being necessarily in contact with it. This has been described elsewhere.[1] Steam (100°), aniline vapor (190°), and mercury vapor (360°) were consecutively introduced, and errors due to radiation were carefully avoided by appropriate screens.

The vapor bath mentioned is seen in the figure at *ttt*, being a boiling tube containing the liquid whose boiling point is to be utilized and surrounded at its lower end by the ring burner *rr*. The asbestus screen *ss* protects the upper end of the tube from direct radiation. It is desirable to bring as much of the wire within the central tube *uu* as possible. If necessary the upper end of this tube may be loosely closed with asbestus to obviate currents of air. By carefully regulating the ring burner there is no difficulty in maintaining ebullition for hours.

By removing and then re-inserting the pin *q*, the wires may be twisted in multiples of 90° for each length, or in multiples of rate of twist (τ) of 3° for each wire. For the given dimensions this is the effect of a couple of 250 g. on one centimeter, for each 3° of rate of twist. In later forms of this apparatus a divided circle was attached to the lower piece *dh*, thus enabling the observer to vary the value of τ and to measure the amount of permanent set at the end of the experiment.

54. A few critical remarks on the efficiency of the apparatus are in place. In most of the experiments made the torsional deformation is so great that special fiducial marks to register the possible motion of the fixed pieces *gh* and *kl* are superfluous. Indeed, for angles as large as those observed, Gauss's method of angular measurement is no longer conveniently applicable, because of the number of corrections which become essential, and in future measurements it will be expedient to use other methods. The large deflections, however, made it possible to use the Gauss method even when the apparatus is thrown into unavoidable vibration; for instance, when the upper wire is surrounded by boiling mercury. In the present work it did not seem necessary to take special precautions for jacketing the lower wire. Its temperature is that of the surrounding air, very nearly, and it is not heated by radiation. In the case of steam and of aniline vapor, the length of the hot part of the upper wire is sharply measurable, but in the present work this could not be so satisfactorily done for mercury vapor. Regarding the mode of heating, there are two methods available: the wire may either be heated and the twist then applied, or the wire may be twisted to the desired amount before heating. The former of these methods eliminates the time error, but it is difficult to obtain an accurate reading for the zero-point, i. e., the scale reading for the twisted system before

[1] Bull. U. S. Geol. Survey No. 54, p. 86 et esq., where the form of *boiling tube* used is fully given.



UNIFILAR APPARATUS.

heat is applied. Hence the other method was adopted, in which the invariability of the zero referred to can be satisfactorily tested before each experiment. The error is the time consumed in heating the wire, and in the second part of the observation, the time consumed in cooling the wire after heating. Special means for cooling were not applied. When the motion is great, it is desirable to have the index-mirror adjustment light, though the device used was not perhaps objectionably heavy.

The immediate effect of heating the upper wire is an expansion of the system. Hence if the part of the lower fixed brass cylinder *kl*, along which the pin *q* is free to slide, be not plane and true, there will be a tendency to rotate the system. If, however, the twist be alternately applied positively and negatively, this expansion error is eliminated by commutation. This safeguard was invariably applied. When the twist stored amounts to 180° for each wire, its sign may often be reversed and readings may be made without necessarily readjusting the mirror. For smaller twists the mirror must be readjusted. Larger twists were not applied because of liability to permanent set of the wire so twisted.

55. The wires used are Stubs's best steel, hardened and tempered electrically, as described elsewhere.[1]

A few words relative to the technical terms used to designate the temper of steel may here find place. Prof. Roberts-Austen[2] has endeavored to give definiteness to the notation by proposing that the operation by which steel is made as soft as possible (heating to redness with very slow cooling) be termed "annealing"; whereas operations by which any other degree of hardness between extreme hard and extreme soft is imparted to steel, be termed "tempering". From a physical point of view I see little importance in such a distinction, except perhaps that under annealing *exceptionally* slow cooling is implied—i. e., to anneal steel, it is essential to cool the hot metal much more slowly, than is usually necessary in tempering. Otherwise an extreme of soft state will not be obtained. In general, however, since in any case of tempering it is necessary to act on a rod of a higher degree of hardness than the one to be produced, it makes little difference whether the rod be described as having been tempered at a temperature x, during a time y, or annealed at a temperature x, during a time y. Sufficiently slow cooling after the heating is evidently and always understood. In short the whole process is continuous in character, and a rod tempered at red heat is virtually annealed. Hence in the present work I have not thought it needful to discriminate between annealing and tempering, and I use the words interchangeably.

56. *Theory of the apparatus.*—In a general way the theory of this apparatus is similar to that given in Chapter I, § 31. There are special points however to which it is necessary to call attention specifically. In

[1] Cf. above, Chapter I, § 8 et seq. Also Bull. U. S. Geol. Survey, No. 14, 1885, p. 29.

[2] Nature, vol. 41, 1889, pp. 11, 42.

the bifilar suspension the twist helices of the two wires were right handed and left handed respectively. In the present instance the sign of the twist is the same throughout its length. Given a continuous straight steel wire of length L, to which a convenient rate of twist, τ, has been imparted. Consider two right sections whose distance apart is the unit of length, and let 2φ be the amount of viscous angular motion of the first relative to the second, during the given small time t, for the stated rate of twist τ. To fix the ideas, let the wire be adjusted vertically, and provided with an index to register angular motion at a distance l' above the lower end. At every section the viscous motion is such, that if the contiguous parts immediately below the section slide in a given direction, the parts immediately above it slide, in equal amount, in the opposite direction. Again, of the two equal and opposite viscous motions which take place on any section, only the part nearest the index will influence it. Finally throughout the whole viscous detorsion the twist is kept constant throughout the wire in virtue of its elasticity.

In Fig. 21 I have developed the conditions which here obtain, on a plan similar to that described in § 31. The length of the two wires A and A', is shown at $fd=l'$ and $da=l$, respectively, so that $af=l+l'=L$. Again $gf=T\rho$, where T is the twist for the length L and ρ the common radius of the wires. Hence gf is small as compared with af. The developed position of the twisted fiber is shown at ag, so that tan gaf is the twist $T=L\tau$. Virtually af and ag are equal in length and the position of the index is at d or the corresponding point of junction b.

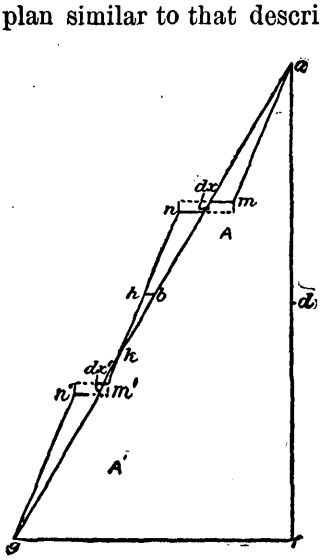


Fig. 21.

Consider the viscous sliding which takes place on the two end faces of the elementary cylinders dx and dx', distant x and x' respectively from the lower end of the system. Then in a very short time, the originally straight fiber abg is changed to a broken line $amnh$, $hm'n'g$. Here $mn=2\varphi dx$, $m'n'=2\varphi' dx'$, and $bh=d\psi$. If k be the point of intersection of the broken line $amnm'n'g$ and the straight line ag, let $bk=y$. Then an inspection of the diagram shows

$$\frac{2\varphi dx}{d\psi}=\frac{l+y}{y} \text{ and } \frac{2\varphi' dx'}{d\psi}=\frac{l'-y}{y} \quad \ldots\ldots (1)$$

If for y its geometrical value in terms of L, l, x, x', etc., be inserted, and if for brevity $2\varphi dx=a$ and $2\varphi' dx'=a'$, the following relation results:

$$\frac{a-a'}{d\psi}=\frac{2(L+x-x')-L(a+a')/a}{L+x-x'-l(a+a')/a} \quad \ldots\ldots (2)$$

Unfortunately this is too complicated for use except where $L=2l=2l'$, or where the two wires of the system are of equal length with an index at the point of junction. In this case $\varphi dx-\varphi' dx'=d\psi$ . . . . . (3)

57. This premised, the conditions of experiment may at once be introduced. Let the parts of the wire below the index, the parts whose position is 0 to l' be kept at a given constant temperature, and be of the same temper throughout. Also let those parts of the system above the index be of the same, or any other uniform temper; but suppose them heated to different constant temperatures. Thus let the viscous detorsion between $x=0$ and $x=l'$ be $2\varphi'$; between $x=l'$ and $x=\beta$ be $2\varphi_3$; between $x=\beta$ and $x=\alpha$ be 2φ; between $x=\alpha$ and $x=L$ be $2\varphi_1$. The differences φ_1, φ, φ_3 are evoked by differences of temperature of the parts of the wire to which these data refer, whereas φ' may differ from all of these by an increment of temper as well as of temperature. Then the influence of the viscous detorsion in each of the parts in question on the index whose position is $x=l'$ will be in the aggregate

$$\psi=\varphi_3(\beta-l')+\varphi(\alpha-\beta)+\varphi_1(L-\alpha)-\varphi' l' \quad . \quad . \quad . \quad . \quad (4)$$

Equation (4) results at once from a suitable integration of (3).

In the present chapter $\varphi_3=\varphi_1=\varphi'$, since the tempers and temperatures of these parts are alike. Hence

$$\psi=(\varphi-\varphi')(\alpha-\beta) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

It will usually be convenient to put $\alpha-\beta=a$.

58. If a series of detorsions φ be observed at $\theta°$, and another series, Φ, at $\Theta°$; if $\varphi=\varphi_0 F(\theta)$ and $\psi=n\,/\,2R$ (Gauss's method of angular measurement), then $\varphi_0(F(\Theta)-F(\theta))=(N-n)\,/\,RL$, where R is the distance between mirror and scale in *cm* and where N and n are the scale parts (*cm*) corresponding to $\Theta°$ and $\theta°$, respectively. Hence the distance of the individual curves for $\Theta°$ and $\theta°$ apart varies directly as φ_0. This result, though simple enough, has special bearing on the text below.

59. *Digression.*—In the interest of certain correction to be applied to some of my early results, I will inquire what correction is to be applied when the elastic restitution which accompanies viscous motion, is disregarded. In this case the differential equation corresponding to (3) is

$$d\psi=\frac{l'}{x}\varphi dx \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3')$$

and equation (4) is replaced by

$$\psi=l'\left(\varphi_1 ln\frac{L}{\alpha}+\varphi ln\frac{\alpha}{\beta}+\varphi_3 ln\frac{\beta}{l'}\right)-l\varphi' ln\frac{L}{l'} \quad . \quad . \quad . \quad . \quad (4')$$

Equation (5) finally by

$$\psi=l(\varphi-\varphi')ln\frac{\alpha}{\beta} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5')$$

Thus to reduce such data as are given in tables 42 to 51, if calculated by (5′) to data calculated by (5), the factor would be 0·66; for tables 52 to 54, the factor would be 0·78, etc., or in general $ln\alpha\,/\beta$.

EXPERIMENTAL RESULTS.

60. *Residual twist.*—The following experiments have an introductory and suggestive character. Two glass-hard wires with their ends bent loop-shaped are fastened firmly together at one end of each. The other ends are twisted τ° against each other and also fastened. This *twisted system* of glass-hard wires, or system in which a rate of twist of 3° or 6° has been stored, are then annealed at divers temperatures during stated times as shown in the tables. The amount of twist lost during annealing is measured by determining the angle between the planes of the upper and lower loops of each wire before and after annealing. A gallows arrangement by which a needle is suspended over a divided circle by the steel wire itself is used for measurement.

The tables contain the number, length (L) and diameter (2ρ) of the glass-hard wires used for each couple; also the temperature and the time (h) in hours, during which the couples are annealed. They contain, furthermore, the amount of viscous detorsion (Δ_1, Δ_2) or twist lost during annealing, for each of the times specified, and each of the wires; finally, the amount of twist stored in each wire of the system. This amount is equal to the twist nominally applied, minus the angle between the plane of the loops. Allowance is made for the amount of permanent set produced by mere manipulatory twisting of the cold wires. When this exceeds 10°, the Δ is discarded. The error of angular measurement is probably not greater than a few degrees.

In the second part of the table the wires S and T are soft.

TABLE 41.—RESIDUAL TWIST. RODS GLASS-HARD.

No.	Annealed at	h	Δ_1	Δ_2	Twist.
A and B L=31cm 2ρ=0·082cm	100°	0·00 1·08 2·08 3·58	0·0 57° 64 70	— — — —	180°—7°
C and D L=31cm 2ρ=0·082cm	100°	0·00 0·17 0·67 2·00	— — — —	—0·0 —36° —64 —70	180°—10°
E and F L=31cm 2ρ=0·82cm	190°	0·00 0·10 0·80	— — —	—0·00 —123° —153	180°—25°
G and H L=31cm 2ρ=0·082cm	190°	0·00 0·10 1·00	— — —	—0.0 —104° —130	180°—2°
I and J L=31cm 2ρ=0·082cm	190°	0·00 0·08 1·20	0 39° 57	0 52° 69	90°—10°
K and L L=31cm 2ρ=0·082cm	360°	0·00 0·30	0 166°	0 148°	180°—20°
M and N L=31cm 2ρ=0·082cm	360°	0·00 0·30	0 180°	0 166°	180°—10°
O and P L=31cm 2ρ=0·082cm	360°	0·00 0·30	0 77°	0 71°	90°—8°
Q and R L=31cm 2ρ=0·043cm	360°	0·00 0·30	0 156°	0 165°	180°—8°
S and T L=31cm 2ρ=0·043cm	360°	0·00 0·30	0 48°	0 54°	180°—2°

61. This table contains results which may be expressed as follows: the viscous detorsion produced by the action of any temperature on a twisted system of glass-hard steel wires, increases at a rate gradually diminishing through infinite time, diminishing slowly in case of low temperatures ($<100°$), rapidly at first and then again slowly at high temperatures ($>200°$); so that the residual twist corresponding to any given temperature is reached asymptotically. Moreover, the strain carried by the glass-hard twisted system is almost completely annulled when the temperature at which annealing takes place exceeds 350°. These results are strikingly similar to the *thermoelectric effect*[1] produced by annealing glass-hard steel, that the present purely mechanical result may safely be used to interpret the electrical result: in both experiments we observe strains disappearing under like conditions.

The table contains another important result: by comparing the second part of the table with the first, it appears that the effect of temperature in decreasing the viscosity of steel is greater in proportion as steel is harder. For where the twist is stored between two glass-hard wires it is found to have almost completely vanished after annealing; but where the same twist is stored between two soft wires less than one-third of it has vanished after annealing. The curious inference that in so far as its viscous properties are concerned, steel is much more susceptible to temperature when it is hard than when it is soft, will be carefully discussed in the following pages by aid of the apparatus already described.

In Fig. 22 I have shown graphically the results reached by annealing twisted systems of glass-hard wires at 360°. The form of the wire before annealing is given at *C*, the shape being that of an elongated C. This same wire is then twisted to the form of an elongated *S* and then annealed in this twisted condition. After annealing, the cold wire will be found to be permanently S-shaped, all temporarily stored twist having vanished during the exposure. This shows that hard steel during the period of occurrence of the annealing in question, is almost a viscous fluid.

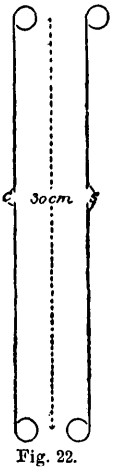


Fig. 22.

TORSIONAL VISCOSITY AND TEMPERATURE.

62. *Notation.*—The data of the following thirteen tables give a clear description of the viscosity of steel for temperatures between 0° and 400°, and for all degrees of hard ness. These data are readily intelligible. θ denotes the temperature of the hot part, θ' the temperature of the cold part of the wires in the apparatus (Pl. I). The distance between mirror and scale was 360^{cm} throughout. L, l, β, a, have the signification already given, being the total length of system of two wires, the distance of mirror

[1] Bull. U. S. Geol. Survey No. 14, 1885, pp. 55, 95.

above their lower end, the length of their cold and of their hot parts, respectively, in centimeters. Hence $a=\alpha-\beta$. The tempers of the rods (diameter 2ρ) are expressed by the temperature at which the originally glass-hard rod was annealed. Soft rods are annealed at red heat. The tabular arrangement contains the date and the time (h_0) in hours and fractions of an hour, of each of the angular detorsions ($\varphi-\varphi'$), in radians. 2φ has been defined, §§ 31, 56. It denotes the angular viscous motion between two right sections of the hot wire, whose distance apart is the unit of length, when the temperature of the included wire is $\theta°$ and the original rate of twist is τ for the given diameter 2ρ. $2\varphi'$ has the same meaning for the cold wire. Identical signs of τ and $(\varphi-\varphi')$ refer to angular motions in the same sense, and since τ and $(\varphi-\varphi')$ agree for the cold rod this is invariably of greater viscosity.

63. *Viscosity at* 100°.—The data in hand are given in Tables 42 to 47.

TABLE 42.

$\theta=100°$; $\theta'=20°$. $L=60^{cm}$. $l=30^{cm}$. $a=26^{cm}$. $\beta=34^{cm}$. Rods: No. 1, soft; No. 2, soft } $2\rho=0{\cdot}082^{cm}$.

Date.	h_0	$(\phi-\phi')\times 10^3$		Date.	h_0	$(\phi-\phi')\times 10^3$	
Feb. 18, 0h 53m	0·00	0·00	*	Feb. 19, 2h 44m	1·45	−1·30	Steam off.
0 56	0·05	−1·04	Steam on.	3 00	1·72	−1·16	
1 00	0·12	−1·10		25	2·13	− ·69	
15	0·37	−1·22		4 55	3·63	− ·63	
45	0·87	−1·43		24, 10h 42m	0·00	+0·00	‡
2 45	1·87	−1·61		10 45	0·05	·75	Steam on.
3 50	2·95	−1·81		59	0·29	1·06	
4 30	3·63	−1·97		11 12	0·50	1·16	
4 33	3·67	−1·85	Steam off.	27	0·75	1·20	
44	3·85	−1·56		59	1·30	1·26	
54	4·02	−1·46		12 23	1·66	1·28	
19, 9 30	20·67	−1·24		1 15	2·50	1·30	
19, 1h 17m	0·00	−0·00	†	1 22	2·65	1·08	Steam off.
1 25	0·13	−1·14	Steam on.	30	2·80	·89	
56	0·65	−1·29		50	3·13	·76	
2 14	0·95	−1·35		2 12	3·50	·67	
40	1·38	−1·40		3 30	4·80	·63	

* τ (cold rod) − 6°, imparted after heating.
† τ (cold rod) − 6°, imparted before heating.
‡ τ (cold rod) + 6°, imparted before heating.

TABLE 43.

$\theta=100°$; $\theta'=20°$. $a=26^{cm}$. $l=30^{cm}$. $L=60^{cm}$. $\beta=34^{cm}$. Rods: No. 3, ann'ld, 100° 9h; No. 4, ann'ld, 100° 9h. $2\rho=0{\cdot}082^{cm}$.

Date.	h_0	$(\phi-\phi')\times 10^3$		Date.	h_0	$(\phi-\phi')\times 10^3$	
Feb. 24, 4h 36m	0.00	− 0·00	*	Feb. 26, 10h 02m	0·00	0·00	‡
4 43	0.12	− 3·05	Steam on.	10 10	0·13	− 7·97	Steam on.
45	0.15	− 3·75		15	0·21	− 8·84	
55	0.32	− 8·81		20	0·30	− 9·80	
5 10	0.57	−10·61		25	0·39	−10·42	
5 15	0.65	− 8·21	Steam off.	30	0·46	−10·95	
25, 9 40	17.07	− 6·95		44	0·70	−12·15	
25, 10h 00m	0.00	0·00	†	55	0·89	−12·86	
10 06	0.10	5·91	Steam on.	11 04	1·03	−13·31	
10	0.18	9·40		16	1·22	−13·89	
20	0.32	12·09		43	1·70	−14·89	
25	0.42	13·06		57	1·92	−15·38	
30	0.50	13·41		12 55	2·87	−16·80	
34	0.56	13·80		1 09	3·10	−16·08	Steam off.
45	0.75	13·21	Steam off.	30	3·41	−15·64	
11 00	1.00	12·71		55	3·88	− 15·56	
28	1.47	12·47		3 12	5·16	−15·16	
1 40	3.67	12·27					
26, 9 37	23.62	12·03					

* τ (cold rod) $-6°$ imparted before heating.
† τ (cold rod) $+6°$ imparted before heating, immediately after *.
‡ τ (cold rod) $-6°$ imparted before heating, immediately after †.

TABLE 44.

$\theta=100°$; $\theta'=20°$; $a=26^{cm}$; $l=30^{cm}$; $L=60^{cm}$; $\beta=34^{cm}$. Rods: No. 5, annealed, 190°, 2h; No. 6, annealed, 190°, 2h; $2\rho=0{\cdot}082^{cm}$.

Date.	h_0	$(\phi-\phi')\times 10^3$		Date.	h_0	$(\phi-\phi')\times 10^3$	
Feb. 26, 3h 57m	.....	0·00	*	Feb. 28, 3h 55m	1·83	−6·86	Steam off.
4 15	0·00	0·00		4 05	2·00	−6·48	
4 20	0·09	−3·22	Steam on.	54	2·82	−6·12	
25	0·17	−3·86		Mar. 5, 10h 20m	0·00	0·00	*
34	0·32	−4·50		10 26	0·10	−1·59	Steam on.
40	0·42	−4·79		30	0·16	−1·95	
51	0·60	−5·10		35	0·25	−2·05	
5 10	0·92	−5·50		44	0·40	−2·16	
5 17	1·03	−5·02	Steam off.	56	0·60	−2·30	
28, 9 52	41·62	−3·84		11 01	0·68	−1·93	Steam off.
28, 9h 58m	.....	0·00	†	06	0·76	−1·59	
10 20	0·00	0·00		13	0·88	−1·27	
10 27	0·12	2·41	Steam on.	23	0·05	−1·04	
34	0·23	5·34		31	1·18	− ·93	
41	0·35	6·02		5, 11h 33m	0·00	0·00	†
51	0·52	6·55		11 35	0·06	2·04	Steam on.
11 02	0·70	6·96		54	0·36	5·12	
33	1·22	7·59		12 03	0·50	5·53	
54	1·57	7·87		11	0·63	5·74	
12 17	1·95	8·13		12 15	0·70	5·52	Steam off.
50	2·50	8·42		24	0·85	5·01	
1 04	2·73	7·52	Steam off.	35	1·03	4·74	
15	2·92	7·21		45	1·20	4·58	
26	3·10	7·07		5, 1h 46m	0·00	−0·00	‡
42	3·37	6·97		1 52	0·10	−2·84	Steam on.
12, 1h 49m	.....	0·00	‡	2 03	0·28	−4·97	
2 05	0·00	0·00		08	0·37	−5·27	
2 15	0·17	−4·04	Steam on.	15	0·48	−5·51	
20	0·25	−4·86		24	0·63	−5·75	
30	0·42	−5·74		2 31	0·75	−5·32	Steam off.
35	0·50	−6·03		41	0·92	−4·75	
51	0·77	−6·57		51	1·08	−4·51	
3 13	1·12	−6·96		3 11	1·41	−4·33	
20	1·25	−7·20					
45	1·67	−7·53					

* τ (cold rod) $-6°$, imparted before heating.
† τ (cold rod) $+6°$, imparted before heating, immediately after *.
‡ τ (cold rod) $-6°$, imparted before heating, immediately after †.

TABLE 45.

$T=100°$; $T'=20°$; $a=26^{cm}$; $b=30^{cm}$; $L=60^{cm}$; $k=34^{cm}$. Rods: No. 7, annealed, 360°; No. 8, annealed, 360°; $2\rho=0{\cdot}083^{cm}$.

Date.	h_0	$(\phi-\phi')\times 10^3$		Date.	h_0	$(\phi-\phi')\times 10^3$	
Mar. 1, 10h 30m	0·00	0·00	(*)	Mar. 1, 2h 09m	0·48	−2·95	
				35	0·92	−3·22	
10 35	0·08	1·44		3 04	1·33	−3·36	Steam on.
47	0·28	1·79		24	1·73	−3·43	
59	0·48	1·91		43	2·05	−3·49	
11 12	0·70	1·99	Steam on.				
22	0·87	2·03		4 55	2·25	−2·97	
45	1·25	2·11		5 03	2·40	−2·77	Steam off.
12 00	1·50	2·15		25	2·75	−2·58	
12 15	1·75	1·52		1, 4h 40m	0·00	0·00	(‡)
57	2·45	1·20	Steam off.	4 47	0·10	1·55	
1 17	2·78	1·17		55	0·25	2·31	Steam on.
37	3·12	1·16		5 05	0·42	2·63	
1, 1h 40m	0·00	0·00	(†)	15	0·58	2·75	
47	0·12	−1·42		5 22	0·70	2·45	
51	0·18	−2·27	Steam on.	2, 9h 42m	17·03	1·65	Steam off.
58	0·30	−2·70					

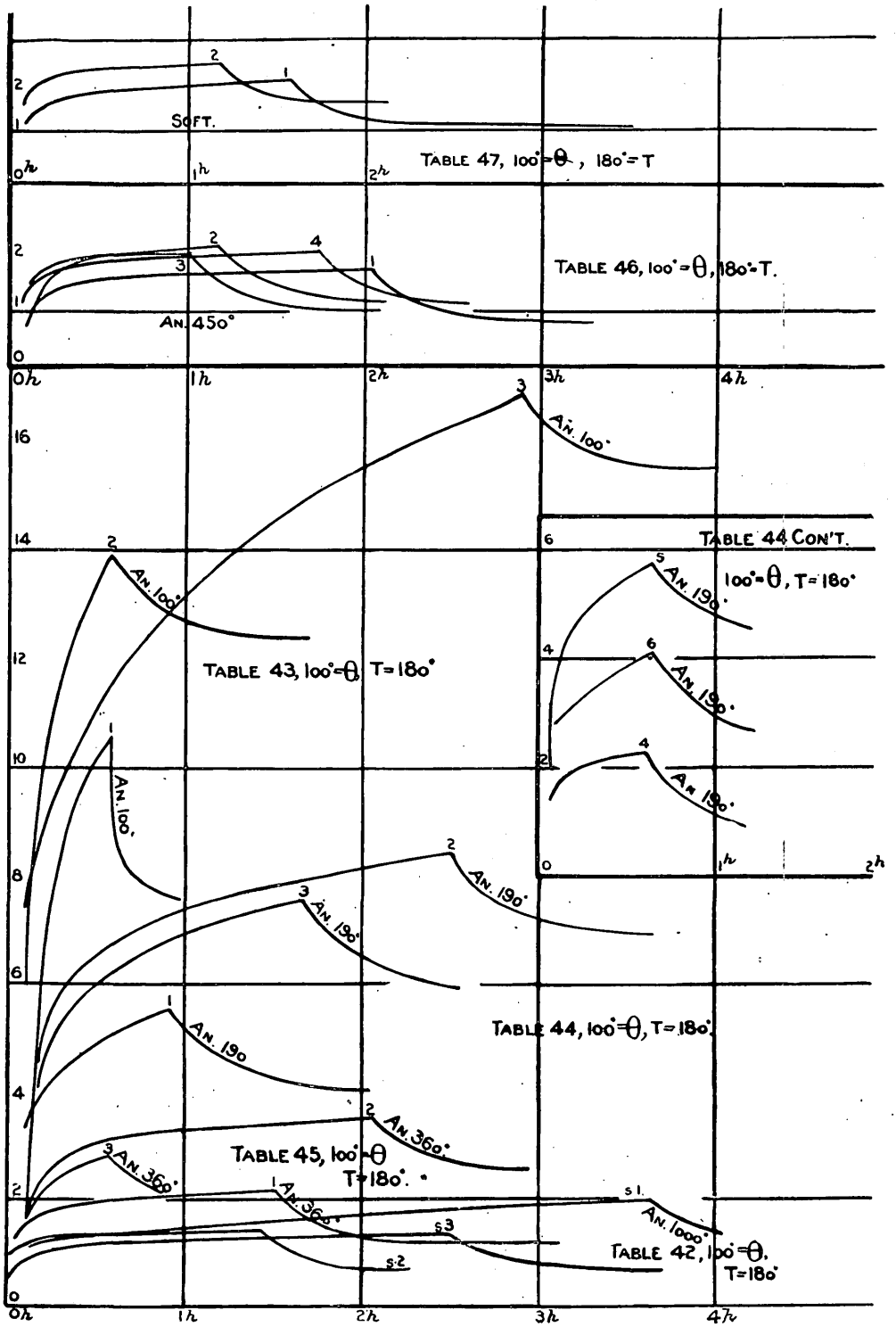
* τ (cold rod) +6°, imparted before heating.
† τ (cold rod) −6°, imparted before heating, immediately after *.
‡ τ (cold rod) + 6°, imparted before heating, immediately after †.

TABLE 46.

$\theta=100°$; $\theta'=20°$; $L=60^{cm}$; $l=30^{cm}$; $a=26^{cm}$; $\beta=34^{cm}$. Rods: No. 9, annealed, 480°; No. 10, annealed 480°; $2\rho=0{\cdot}083^{cm}$.

Date.	h_0	$(\phi-\phi')\times 10^3$		Date.	h_0	$(\phi-\phi')\times 10^3$	
Mar. 2, 10h 53m	0·00	0·00	(*)	Mar. 3, 10h 05m	0·05	·94	
11 00	0·07	·90		10	0·13	1·63	
10	0·17	1·42		15	0·21	1·85	
21	0·28	1·53		25	0·38	1·94	Steam on.
31	0·38	1·60		31	0·48	1·97	
40	0·47	1·64	Steam on.	43	0·68	2·00	
58	1·05	1·68		11 02	1·00	2·07	
12 15	1·22	1·71		11 07	1·08	1·84	
55	2·02	1·44		15	1·21	1·50	
				25	1·38	1·26	Steam off.
1 00	2·07	1·55		37	1·55	1·13	
07	2·14	1·26		55	1·88	1·05	
1 24	2·31	·96	Steam off.	12 02	2·00	1·03	
34	2·41	·89		3, 12h 12m	0·00	0·00	(§)
43	2·50	·85					
2 00	3·07	·81		12 17	0·08	−1·23	
2, 2h 05m	0·00	0·00	(†)	22	0·16	−1·61	
				28	0·26	−1·45	
2 14	0·15	−1·26		33	0·35	−1·84	
19	0·23	−1·66		44	0·53	−1·92	Steam on.
24	0·31	−1·86		55	0·72	−1·97	
32	0·45	−1·99	Steam on.	1 08	0·90	−2·02	
45	0·66	−2·08		19	1·10	−2·05	
3 00	0·92	−2·15		30	1·30	−2·07	
08	1·05	−2·18		56	1·74	−2·10	
15	1·16	−2·21					
3 23	1·30	−1·85		2 00	1·80	−1·92	
32	1·45	−1·54		06	1·90	−1·65	
38	1·55	−1·41	Steam on.	12	2·00	−1·46	Steam off.
47	1·70	−1·31		22	2·16	−1·30	
58	1·88	−1·25		33	2·35	−1·20	
4 11	2·10	−1·21		49	2·61	−1·15	
3, 10h 02m	0·00	0·00	(‡)				

* τ (cold rod) + 6°, imparted before heating.
† τ (cold rod) −6°, imparted before heating, immediately after *.
‡ τ (cold rod) + 6°, imparted before heating.
§ τ (cold rod) −6°, imparted before heating, immediately after ‡.



VISCOUS DEFORMATION AT 100° FOR DIFFERENT STATES OF TEMPER.

TABLE 47.

$\theta = 100°$; $\theta' = 20°$; $a = 26^{cm}$; $l = 30^{cm}$; $L = 60^{cm}$; $\beta = 34^{cm}$. Rods: No. 11, soft; No. 12, soft; $2\rho = 0{\cdot}082^{cm}$.

Date.	h_0	$(\phi-\phi')\times 10^3$		Date.	h_0	$(\phi-\phi')\times 10^3$	
Mar. 4, 10h 08m	0·00	0·00	(*)	Mar. 4, 12h 59m	2·85	1·08	Steam off.
10 13	0·08	1·05		1 40	3·53	1·05	
18	0·16	1·45		4, 1h 57m	0·00	0·00	(†)
24	0·28	1·56		2 04	0·11	−1·69	
33	0·41	1·61		09	0·20	−1·93	
47	0·65	1·68	Steam on.	19	0·36	−2·11	Steam on.
57	0·83	1·73		28	0·51	−2·13	
11 12	1·06	1·79		40	0·71	−2·17	
20	1·20	1·83		3 06	1·15	−2·24	
40	1·53	1·91		3 10	1·21	−2·14	
11 45	1·62	1·76		16	1·31	−1·85	
11 55	1·78	1·34		24	1·45	−1·68	Steam off.
12 05	1·95	1·26	Steam off.	33	1·60	−1·58	
15	2·11	1·19		44	1·78	−1·52	
29	2·35	1·13					

* τ (cold rod) + 6°, imparted before heating.
† τ (cold rod) − 6°, imparted before heating, immediately after *.

In Pl. II I have represented the results of these Tables 42 to 47 graphically, $(\varphi-\varphi')10^3$ as a function of time. The plate therefore shows the amount of viscous deformation, at 100°, for each of the states of temper in question. The numerals in brackets indicate the number of twists applied, alternately in opposite directions. It is significant that in proportion as harder wires are operated upon, the (2) twist is in advance of the first, the third being usually intermediate as regards viscous deformation. Even in case of 190°, the oscillatory effect of counter-twisting is marked. It is obscure in soft rods. All this has an important bearing on the discussion in Chapter III, where the phenomenon of "accommodation" is discussed. It may be noted that the two parts of Table 44 are not immediately continuous; hence the difference of behavior of parts (1), (2), (3), and parts (4)′, (5)′, (6)′ of the table.

64. *Viscosity at* 190°.—The data in hand are as follows:

TABLE 48.

$\theta=190°$; $\theta'=20°$; $a=26^{cm}$; $l=30^{cm}$; $L=60^{cm}$; $\beta=34^{cm}$. Rods: No. 5, annealed, 190°; No. 6, annealed, 190°; $2\rho=0{\cdot}082^{cm}$.

Date.	h_0	$(\phi-\phi')\times 10^3$		Date.	h_0	$(\phi-\phi')\times 10^3$	
Mar. 7, 9h 55m	0·00	0·00	(*)	Mar. 8, 9h 50m	0·25	9·81	Vapor on.
10 00	0·08	— 4·97	Vapor on.	53	0·30	10·70	
04	0·15	—14·80		56	0·35	11·09	
10	0·25	—12·39		10 00	0·42	12·26	
19	0·40	—15·81		08	0·55	13·52	
7, 12h 27m	0·00	0·00	(†)	15	0·67	14·48	
12 33	0·10	14·61	Vapor on.	27	0·87	15·84	
36	0·15	22·98		40	1·08	17·11	
40	0·22	26·83		47	1·20	17·89	
42	0·25	28·36		11 00	1·42	17·11	Vapor off.
12 55	0·47	27·01	Vapor off.	20	1·75	16·73	
1 40	1·22	26·23		55	2·33	16·63	
7, 2h 35m	0·00	0·00	(‡)	8, 12h 00m	0·00	0 00	(‖)
2 45	0·17	—11·42	Vapor on.	08	0·13	— 7·23	Vapor on.
47	0·20	—12·26		10	0·17	— 8·34	
53	0·30	—13·48		12	0·20	— 9·12	
3 10	0·58	—15·40		15	0·25	— 9·99	
19	0·73	—15·71		18	0·30	—10·71	
43	1·13	—14·28		21	0·35	—11·28	
3 53	1·30	—16·99	Vapor off.	25	0·42	—11·95	
4 07	1·53	—16·68		30	0·50	—12·68	
45	2·17	—16·58		35	0·58	—13·34	
8, 9 00	18·42	—16·58		40	0·67	—13·85	
8, 9h 35m	0·00	0·00	(§)	45	0·75	—14·29	
9 43	0·13	4·14	Vapor on.	50	0·83	—14·69	
45	0·17	6·54		1 00	1·00	—15·36	
48	0·22	8·85		1 12	1·20	—14·80	Vapor off.
				30	1·50	—14·49	
				45	1·75	—14·45	

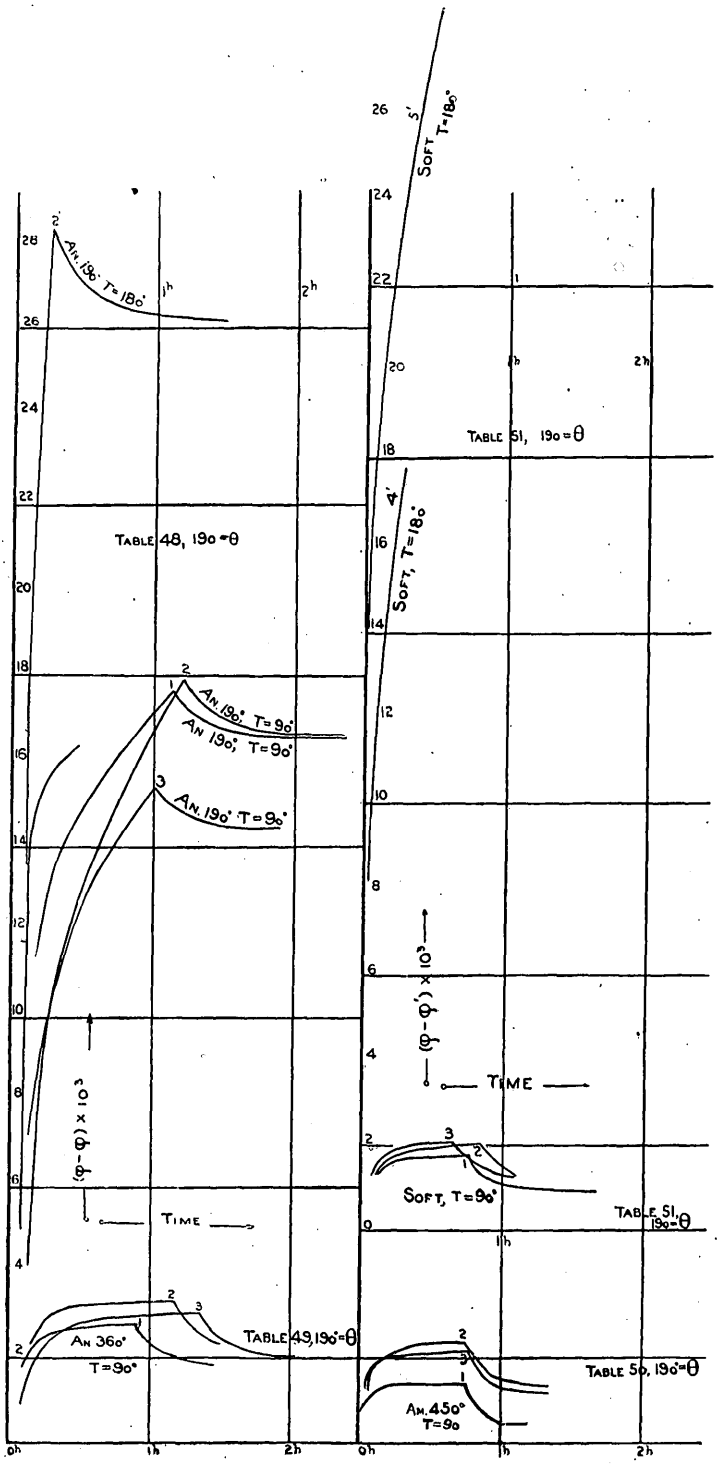
* τ (cold rod) — 6°, imparted before heating.
† τ (cold rod) + 6°, imparted before heating, immediately after *.
‡ τ (cold rod) — 3°, imparted before heating, immediately after †.
§ τ (cold rod) + 3°, imparted before heating, immediately after ‡.
‖ τ (cold rod) — 3°, imparted before heating, after §.

TABLE 49.

$\theta=190°$; $\theta'=20°$; $a=26^{cm}$; $l=30^{cm}$; $L=60^{cm}$; $\beta=34^{cm}$. Rods: No. 7, annealed, 360°; No. 8, annealed, 360°; $2\rho=0{\cdot}082^{cm}$.

Date.	h_0	$(\phi-\phi')\times 10^3$		Date.	h_0	$(\phi-\phi')\times 10^3$	
Mar. 8, 2h 11m	0·00	0·00	(*)	Mar. 8, 5h 01m	1·33	—2.57	Vapor off.
2 20	0·15	2·09	Vapor on.	10	1·48	—2·34	
25	0·23	2·47		9, 9h 27m	0·00	0·00	(‡)
36	0·42	2·63		9 32	0·08	·84	Vapor on.
3 05	0·90	2·77		40	0·21	2·01	
3 12	1·02	2·26	Vapor off.	10 35	1·13	3·01	
35	1·40	1·86		48	1·35	3·04	
8, 3h 41m	0·00	0·00	(†)	11 02	1·58	2·29	Vapor off.
3 50	0·15	—2·25	Vapor on.	07	1·66	2·19	
57	0·27	—2·99		17	1·83	2·09	
4 08	0·45	—3·21		26	1·98	2·05	
26	0·75	—3·25					
52	1·18	—3·32					

* τ (cold rod) + 3°, imparted before heating.
† τ (cold rod) — 3°, imparted before heating, immediately after *.
‡ τ (cold rod) + 3°, imparted before heating, immediately after †.



VISCOUS DEFORMATION AT 190° FOR DIFFERENT STATES OF TEMPER AND STRAIN.

stress $\tau=3°$ ((1), (2), (3), in Pl. II); whereas, its behavior toward stress $\tau=6°$ ((4)′, (5)′), is just like a viscous fluid. This is one of the most noteworthy results of these tables. It will be specially referred to in Chapter III.

It is also to be observed that viscous oscillation does not occur in the case of soft rods (Table 51, parts (1), (2), (3)).

65. *Viscosity at* 360°.—The data in hand are as follows:

TABLE 52.

$\theta=360°$; $\theta'=20°$; $a=9^{cm}$; $l=30^{cm}$; $L=60^{cm}$; $\beta=34^{cm}$. Rods: No. 11, soft; No. 12, soft; $2\rho=0{\cdot}082^{cm}$.

Date.	h_0	$(\phi-\phi')\times 10^3$		Date.	h_0	$(\phi-\phi')\times 10^3$	
Mar 11, 11h 25m	0·00	0·0	*	Mar. 11, 2h 21m	0·30	46·8	Vapor on.
				25	0·37	48·0	
11 34	0·15	— 7.8	Vapor on.	35	0·53	49·1	
36	0·18	—10·1					
39	0·23	—11·6		2 54	0 85	46·6	Vapor off.
44	0·32	—12 9		3 01	0·97	46·4	
47	0·37	—14·0		28	1·42	46·2	
53	0·47	—14·9		11, 4h 25m	0·00	0·0	‡
11 59	0·57	—13·1	Vapor off.	4 32	0·10	—17·4	Vapor on.
12 05	0·67	—12·4		34	0·15	—24·9	
10	0·75	—12·0		36	0·18	—28·1	
18	0·88	—11·8		39	0·23	—31·5	
27	1·03	—11·5		41	0·27	—32·7	
1 13	1·80	—11·1		45	0·33	—34·4	
11, 2h 03m	0·00	0·0	†	51	0·43	—35·6	
				57	0·53	—36·9	
2 06	0·05	7·7	Vapor on.	5 10	0·75	—38·8	
09	0·10	20·0					
11	0·13	27·7		5 17	0·87	—36·1	Vapor off.
12	0·15	35·0		26	1·02	—35·0	
15	0·20	41·7		12, 9h 25m	17·00	—33·8	
18	0·25	44·5					

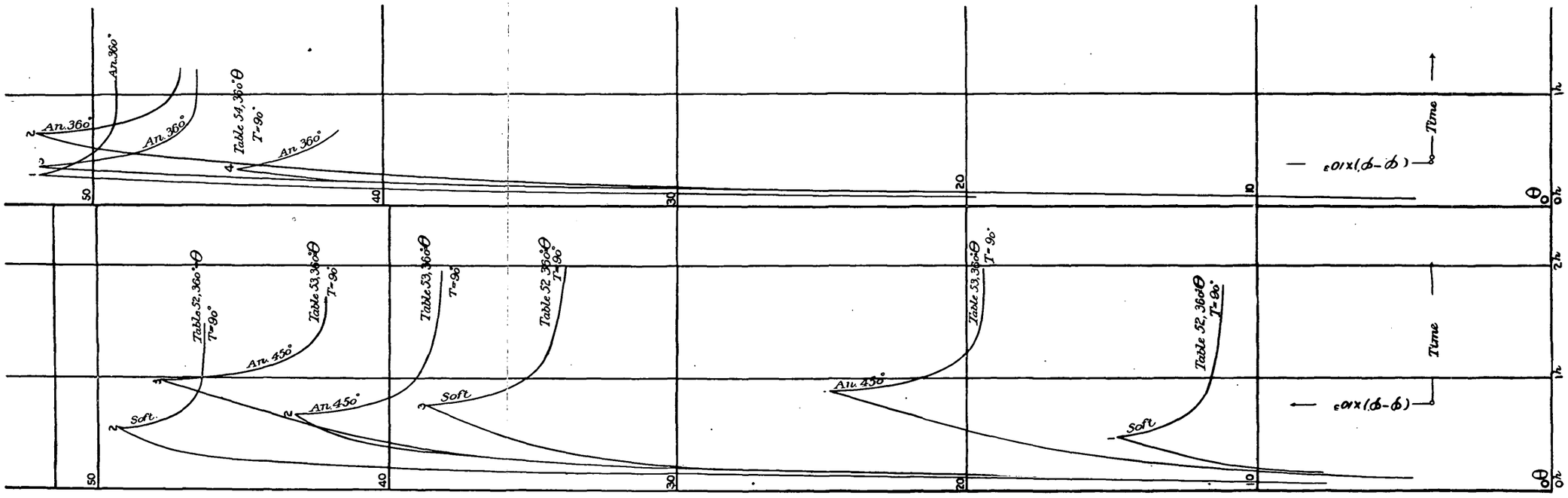
* τ (cold rod) $-3°$, imparted before heating.
† τ (cold rod) $+3°$, imparted before heating, immediately after *.
‡ τ (cold rod) $-3°$, imparted before heating, immediately after †.

TABLE 53.

$\theta=360°$; $\theta'=20°$; $a=9^{cm}$; $l=30^{cm}$; $L=60^{cm}$; $\beta=34^{cm}$. Rods: No. 9, annealed, 450°; No. 10 annealed, 450°. $2\rho=0{\cdot}082^{cm}$.

Date.	h_0	$(\phi-\phi')\times 10^3$		Date.	h_0	$(\phi-\phi')\times 10^3$	
Mar. 12, 10h 04m	0·00	0·0	*	Mar. 12, 1h 10m	0·92	39·4	Vapor off.
				2 00	1·75	38·2	
10 10	0·10	— 4·7	Vapor on.	12, 2h 15m	3·00	0·0	‡
31	0·45	—19·2					
49	0·75	—23·1		2 19	0·06	— 9·9	Vapor on.
57	0·83	—24·8		20	0·08	—26·3	
				25	0·16	—30·3	
11 15	1·18	—20·1	Vapor off.	27	0·20	—33·0	
27	1·38	—19·6		28	0·22	—39·0	
59	1·92	—19·3		36	0·35	—43·3	
12, 12h 15m	0·00	0·0	†	47	0·63	—47·6	
				3 13	0·96	—44·8	
12 25	0·17	28·6	Vapor on.	3 20	1·08	—42·9	Vapor off.
27	0·20	32·2		32	1·28	—42·5	
30	0·25	35·3		48	1·55		
36	0·35	39·0					
45	0·50	42·1					
55	0·67	43·3					

* τ (cold rod) $-3°$, imparted before heating.
† τ (cold rod) $+3°$, imparted before heating, immediately after *.
‡ τ (cold rod) $-3°$, imparted before heating, immediately after †.



VISCOUS DEFORMATION AT 360° FOR DIFFERENT STATES OF TEMPER.

TABLE 54.

$\theta=360°$; $\theta'=20°$; $a=9^{cm}$, $l=30^{cm}$; $L=60^{cm}$; $\beta=34^{cm}$. Rods: No. 7, annealed, 360°; No. 8, annealed, 360°.

Date.	h_0	$(\phi-\phi')\times 10^3$		Date.	h_0	$(\phi-\phi')\times 10^3$	
Mar. 14, 10h 00m	0 00	0·0	*	Mar. 14, 3h 19m	0·00	0·0	‡
10 03	0·07	−19·6		3 22	0·05	4·6	
05	0·10	−26·7		23	0·07	8·9	
06	0·12	−32·0		24	0·08	14·0	
07	0·13	−36·8	Vapor on.	25	0·10	19·3	
09	0·17	−41·3		27	0·12	26·4	
10	0·18	−44 8		28	0·15	30·1	Vapor on.
13	0·23	−49·5		29	0·17	34·7	
15	0·27	−51·9		32	0·22	41·1	
10 19	0·33	−51·1		33	0·23	43·9	
31	0·53	−49·5	Vapor off.	35	0·27	46·0	
40	0·68	−49·3		37	0·30	49·1	
11 04	1·08	−49·1		40	0·35	51·5	
14, 1h 32m	0·00	0·0	†	3 55	0·60	47·2	
1 38	0·10	16·9		4 12	0·88	46·4	Vapor off.
40	0·13	25·2		26	1·12	46·4	
43	0·18	32·2		14, 4h 45m	0·00	0·0	§
46	0·23	37·6	Vapor on.	4 49	0·07	− 6·4	
51	0·32	42·7		50	0·08	− 9 8	
59	0·45	47·0		52	0 12	−24·4	
2 02	0·50	48·9		54	0·15	−31·5	Vapor on.
11	0·65	51·9		56	0·18	−36·8	
2 26	0·90	47·6	Vapor off.	57	0·20	−40·2	
50	1·27	46·8		5 05	0·33	−45·0	
				5 24	0·65	−41·7	Vapor off.

* τ (cold rod) $-3°$, imparted before heating.
† τ (cold rod) $-3°$, imparted before heating, after *,
‡ τ (cold rod) $+3°$, imparted before heating, immediately after †.
§ τ (cold rod) $-3°$, imparted before heating, immediately after ‡.

In the twist (2) and (3) (Pl. IV.), the latest torsion favorable to motion stored up in the wires during the period of twist (1), asserts itself. Hence the actual stress for the former case is very decidedly greater than in twist (1). The chart again shows, that viscosity is dependent on the stress-intensity producing viscous motion, in very marked degree.

In Pl. IV the results of Tables 52, 53, 54 are graphically given, on a plan symmetrical with the above. A larger time-scale would perhaps have been preferable.

This plate preserves the character of the preceding charts. Steel annealed at 360° here shows maximum rate of viscous deformation. Indeed, the rapidity of motion for this case is extreme, so that the rod must be considered a viscous fluid. The four alternations of twist [(1), (2), (3), (4)], as before, give indications of gradually increasing viscosity; and the march toward the limit is again oscillatory.

The chart is also instructive when the behavior of steel annealed at 450° and of soft steel is considered. During twist (1) the former shows less viscosity than the latter; and both are enormously less viscous than steel annealed at 360°. But in twist (2) and twist (3) this character changes in very large degree. The reasons have already been adverted to, in considering Fig. 22 B, where stress as well as temper varies.

66. *Remarks on the tables.*—A few remarks on the tables are here in place. Large angles ψ, for the measurement of which Gauss's method is inconvenient, were calculated directly from the tangents. The error is everywhere within 1 per cent. In Table 44 and more particularly in special experiments, the observations on the constancy of the scale reading when the temperatures of both upper and lower wires are identical, prove that errors due to differences of viscosity in the cold wires are negligible. No motion is perceptible until heat has been applied. The difficulty encountered in fixing the zero of time has already been mentioned. The curves are therefore correct in their vertical dimensions, but may have been shifted laterally as much as is indicated in each table by the interval between the parts marked "vapor on" and "vapor off." It has also been stated that the results for mercury are too large because of the difficulty in accurately defining the length of the hot part *a*. Finally two sets of experiments were made with soft wire (Rods, Nos. 1, 2, 11, 12) because the viscous properties of soft wire are not sharply definable. The curves lie sometimes above, sometimes below the mean zone for steel annealed at 450°. This vagueness of the soft state is largely due to strains incidentally impressed. When a positive twist immediately follows a negative twist, or vice versa, the values of $(\varphi - \varphi')$, cæteris paribus, are larger than for the case of an untwisted wire. It is therefore to be noted that in the arrangement of apparatus employed the permanent set of any prolonged torsion is added to the following torsion, particularly in the case of high θ.

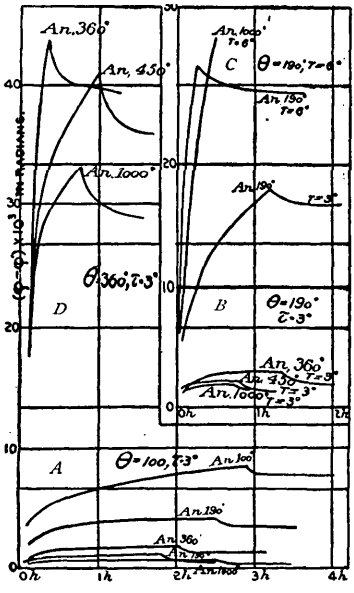


FIG. 23.—Viscous deformation increasing with time, for different states of temper and of temperature. Mean values.

DEDUCTIONS.

67. *Viscosity and temperature.*—The behavior of a given steel wire varies with the character and with the amount of twist it has received. These variations are not insignificant. Cf. Figs. 22 A, 22 B, 22 C. Hence it is expedient to construct a diagram of the mean viscous motion for each degree of hardness and then to discuss the secondary variations with reference to this diagram. In Figs. 23 A, 23 B, 23 C, 23 D $(\varphi - \varphi')$, the angular torsion in radians is exhibited as a function of time in hours, when wires in all degrees of hardness are exposed to degrees of temperature (θ) of 100°, 190° and 360° respectively. The series is made complete when the rate of twist (τ) is 3°, the values of $(\varphi - \varphi')$ corresponding to $\theta = 100°$ (Tables 42 to 47) where $\tau = 6°$ having

been divided by 2 to effect this reduction. This operation is probably not always rigorous[1]; but it is sufficiently correct for the present purposes. In the case of $\theta=190°$ experiments were made both for $\tau=3°$ (Fig. 23 B), and for $\tau=6°$ (Fig. 23 C). Each curve consists of two distinct parts, an anterior ascending branch showing the motion at the high temperature θ, and a descending branch showing the (elastic) effect of cooling from θ to room temperature (θ'). Figures 23 A to 23 D and Tables 41, and 42 to 54 clearly show that the viscous detorsion ($\varphi-\varphi'$) exhibited by steel is very much more influenced by temperature when the steel is hard than when it is soft. In other words, viscosity decreases with temperature (cæteris paribus) at much greater rates in hard steel than in soft steel. Again, at the same temperature θ, the differences of viscosity are very large when the temper of steel lies between glass-hard and annealed at 350°. Intermediate differences within this interval are larger in proportion as mean hardness is greater. When temper lies between annealed at 350° and soft, differences of viscosity at the same θ are relatively small. In general therefore the variations of viscosity due to temper are marked occurrences during the first phase[2] of annealing, and nearly vanish during the second phase, a result which corroborates the close analogy between the viscous effects of temper, the thermoelectric effect and the electric resistance effect of temper already pointed out.

68. For the same degree of hardness viscosity increases at an accelerated rate with temperature θ. In Fig. 23, where $\theta=100°$, the large viscous interval between Annealed at 100° and Annealed at 360° is in striking contrast with the smaller viscous interval Annealed at 360° to Annealed at 1000°, notwithstanding the fact that to avoid erroneous comparisons the latter interval has rather been chosen too large than too small. Indeed, the values for Annealed at 1000° in Table 47 would place the locus for soft wire even above the curve Annealed at 450° in Fig. 23, suggesting the occurrence of a maximum viscosity for $\theta=100°$. Passing from Fig. 23 to Fig. 23 B ($\theta=190°$), the interval between Annealed at 190° and Annealed at 360° is phenomenally increased. The interval Annealed at 360° to Annealed at 1000° is also increased, but only slightly. Again, passing from Fig. 23 C to 23 D ($\theta=360°$), the interval of the second phase of annealing (Annealed at 360° to Annealed at 1000°) is largely increased.[3] The marked tendency of a steel wire annealed from hardness at $\theta°$ to suffer viscous deformation when exposed to $\theta°$, is the chief result of the present paper. This tendency decreases with great rapidity in proportion as the temperature of exposure falls below θ.

[1] Cf. Weidmann: Wied. Ann., 1886, vol. 22, pp. 220, 222. He believes proportionality between stress and deformation to obtain.

[2] Am. Jour. Sci. (III), 1886, vol. 31, p. 443.

[3] Of course the degree of hardest temper to be exposed to θ for viscous comparisons is Annealed at θ. Harder wires would be annealed at this temperature and the concomitant effects due to changes of temper erroneously confounded with simple viscous motion.

As temperature increases, glass seems to lose rigidity much less rapidly than hard steel, but probably not less rapidly than soft steel. The magnetic instability of glass-hard steel is probably due to its extreme susceptibility of temperature, since every change of temper is the cause of loss of magnetic moment. This close relation between hardness and magnetism is good evidence in favor of the essentially strained character of hard steel. The full bearing of all these points is made the subject of Chapter III.

69. *Behavior of other metals.*—Kohlrausch,[1] Streintz,[2] Schmidt,[3] and Pisati[4] have discussed the effect of temperature on viscosity. More minute investigations on the relation between viscosity and temperature have recently been made with silver, platinum, iron, and german silver, and for temperatures within 100°, by Schroeder.[5] It is in place to advert in passing to certain important accordances between Schroeder's results and the present results. In the data above, one or more alternations of the sign of twist are applied to the steel wires (Tables 42 to 54, particularly Table 44), and the amount of deformation decreases (cæteris paribus) with the number of torsions applied gradually toward a limit. This fact has been studied by Wiedemann[6] and at even an earlier date though in a somewhat less relevant way, by Streintz[7] under the term "accommodation." Schroeder has apparently enlarged the importance of these observations by showing that repeated alternations of temperature from low to high values have the same effect.[8] It remains to be seen, however, whether this is not an immediate result of the fact that Schroeder's hard-drawn wires are annealed by exposure to 100°. Indeed the results of Chapter III make this altogether probable. A second result of Schroeder's,[9] viz: that the amount of "after-action," as well as the amount of change of after-action due to stated increments of temperature is greatest in silver, of intermediate value in iron, and smallest in german silver, has an important bearing on the present experiments. The present results taken together with the earlier paper of Barus and Strouhal[10] show conclusively that the viscosity of steel and the variation of viscosity due to temperature increase in like order, and in ways which throughout the course of the phenomena are thoroughly analogous. A final result of Schroeder's bearing on the present paper, viz: that the viscous detorsion occurring at 100° is arrested by suddenly lowering the temperature

[1] Kohlrausch: Pogg. Ann., 1866, vol. 128, p. 216; 1876, vol. 158, p. 371.

[2] Streintz: Wien. Berichte, 1874, vol. 69, p. 337.

[3] Schmidt: Wied. Ann., 1877, vol. 2, p. 264.

[4] Pisati: Wien. Berichte, 1879, vol. 80, p. 427.

[5] Schroeder: Wied. Ann., 1886, vol. 28, p. 369.

[6] Wiedemann: Wied. Ann., 1879, vol. 6, p. 512.

[7] Streintz: Pogg. Ann., 1874, vol. 153, p. 406.

[8] "Ebenso wie das log. Decrement bei Torsionschwingungen zeigt auch die Nachwirkung unter dem Einflusse wiederholter Temperaturänderungen eine Accommodation."

[9] "Sowohl die Nachwirkung wie die Änderung derselben mit der Temperatur ist am grössten beim Silberdraht, geringer beim Eisen, am kleinsten beim Neusilberdraht."

[10] Am. Jour. Sci. (III), 1887, vol. 33, pp. 25, 26.

as far as 20° is again fully corroborated by the behavior of steel; but the character of the viscous motion while the temperature either rises or falls does not so fully appear, because a large part of the retrograde movement observed during cooling is here to be ascribed to concomitant changes of the modulus of elasticity produced by temperature. In other words, relatively large variations of elasticity are superposed upon and obscure the nature of the viscous detorsion during the interval of transition from high to low temperature, or from low to high temperature.

70. It is because of the mixed character of the retrograde movement that I refrain from using the data for the construction of the temperature relations of the rigidity of steel. If a device for cooling with sufficient rapidity to annul the viscous movement be applied, then the present method may be used at once for investigating the effect of temperature on rigidity.

71. *Viscosity and pressure.*—In view of the great importance of viscous deformations in questions of dynamic geology, I will insert a few remarks relative to the effect of pressure on viscosity, such as follow at once from the above experiments.

At a depth of ten miles below the surface, the temperature of the stratum of earth is certainly as high as 450° C.[1] At this temperature glass is practically a viscous fluid as appears at once from experiments by Dr. Strouhal and myself, on the strain effect of quenching exhibited by the Prince Rupert drop.[2] We found all strain to have vanished when the temperature of annealing reached this limit. Special experiments on the torsion viscosity of glass made with the above apparatus shows a similarly marked effect at 370°, so that glass which is less viscous than soft steel at mean atmospheric temperatures maintains this state of inferiority when temperature is raised. It is not necessary to tabulate the experiments with glass here. They are difficult of reduction, because glass-fiber can not easily be drawn to have the same uniform section throughout its length. Again, at these temperatures glass easily devitrifies, a phenomenon which decreases its viscosity enormously. The omission is easily justified since Warburg and others have proved that even at 300° glass behaves like an electrolyte (cf. Chapter I).

It follows that if the earth were of glass, and if the viscous effect of pressure were nil, the behavior of strata at even ten miles below the surface would be that of a viscous fluid. Indeed the strata there and to some extent above this level, would yield to the application of even small intensities of shear.

Again, if the earth were made of steel in the state of maximum viscosity, the rigidity ten miles below the surface would be nil, so far at least as the action of *continuous* shearing stress is concerned.[3]

[1] Cf. Thomson and Tait Nat. Philosophy, I, part 2, 1883, pp. 476 to 477.

[2] Bull. U. S. Geol. Survey, No. 42, 1888, p. 109.

[3] Considerations of this kind have been made specifically and carried out in detail by Mr. Clarence King.

It follows finally that, if the earth, throughout depths of this insignificant value, behaves like an elastic solid, the effect of pressure on viscosity must be of importance.

72. *Sudden and gradual deformation.*—In the experiments for $\theta=100°$ the rate of twist $\tau=6°$ was uniformly applied. The results were then approximately reduced to $\tau=3°$ by merely halving the values of $(\varphi-\varphi')$. Pl. III shows that the same strain, $\tau=6°$, applied for $\theta=190°$, is too large for convenient measurement by Gauss's method. On applying $\tau=3°$ directly, the completed series of results is obtained. Comparing the loci of Pl. III and Fig. 23 C, it appears clearly that $(\varphi-\varphi')$ increases very much more rapidly than τ. Moreover, if the rate of increase of τ be 2, the rate of increase of $(\varphi-\varphi')$ is certainly as much as 3 in case of steel annealed at 190°, and even more than 10 in case of soft steel (annealed at 1000°). It follows therefore very probably that the viscous relations of soft steel to hard steel vary enormously and even change sign as the stress producing viscous motion passes from low to high values. This is an important deduction. For rates of twist less than $\tau=3°$ for the diameter $2\rho=0{\cdot}082^{cm}$, steel is less viscous, and as regards viscosity much more susceptible to the influence of temperature, in proportion as it is harder. For rates of twist greater than $\tau=6°$ steel is less viscous and more susceptible to the influence of temperature in proportion as it is softer.[1] The complete coördination of these facts, in other words the full expression of the viscosity of steel as a function of hardness for all degrees of temperature θ and all values of stress τ, will be the key for the explanation of the mechanical behavior of steel and its important bearing on magnetic, electrical and other properties of the metal. To elucidate these remarks I will complete the description of the apparatus with which the present results were obtained, leaving theoretical inferences for discussion in Chapter III.

73. In the apparatus, Pl. I, suppose the lower (cold) wire to be in connection with clockwork, in such a way that it may be twisted uniformly, at any given velocity, variable at pleasure. Suppose the method of adjusting the index to be such as is suitable for the measurement of angles of any magnitude. If the clock be set in motion from $\tau=0$, the strain will increase at some determinate arbitrary rate. For a given value of θ, therefore, a family of curves may be obtained in which the angular motion of the index ψ_1 for a given wire must be expressible as a function of the time during which twisting has taken place and the rate at which the strain is increased. In other words, if c be the rate of rotation of the lower end of the lower wire, the experimental results may be symbolized by

$$\psi_1 = f_1(t, c_1)$$
$$\psi_2 = f_2(t, c_2)$$
$$\psi_n = f_n(t, c_n),$$

[1] Cf. Carus Wilson: Phil. Mag., 1890, vol. 29, p. 200.

where t is the symbol of time. It follows that the strain per centimeter of the total length L, at any time t, will be ct/L diminished by the amount of viscous motion.[1] Hence it is experimentally possible and fully feasible to pass from the family of curves $\psi_1, \psi_2, \ldots \psi_n$ to a similar family,

$$(\varphi - \varphi') = F(t, \tau),$$

by an appropriate method of graphic solution. $(\varphi - \varphi')$ thus expressed as a function of time and strain, for all degrees of hardness and all degrees of temperature θ, is the complete solution of the problem in hand.

I take pleasure in acknowledging that the greater number of experiments in this paper are due to Mrs. Anna H. Barus.

[1] If c' be the rate of the index for perfectly elastic wires, then $\psi - c't$, and $2\,c't - \psi$ correspond respectively to the viscous motion and the strain intensity at the time t. The curves ψ will in general be circumflex, passing from an initial tangent c' to an asymptote which is the rate of rotation of the lower end of the wires.

CHAPTER III.

MAXWELL'S THEORY OF THE VISCOSITY OF SOLIDS AND ITS PHYSICAL VERIFICATION.

INTRODUCTION.

74. The viscosity of solids has been theoretically discussed in the memoirs of O. E. Meyer, Boltzmann, Neesen, Warburg, Maxwell, and Butcher. Views of a distinctly theoretical kind have also been given by Weber and Kohlrausch, and more recently by Nissen. In almost all cases, excepting alone Butcher's[1] work, which formulates the theory of Maxwell,[2] the problem has been approached from distinct points of view.

Despite the diversity of methods of discussion and the elaboration of evidence, the results do not in any case so fully represent the phenomenon as to lead to general acquiescence in one elementary physical hypothesis. Boltzmann's theory is perhaps the most powerful and is elegantly worked out; but it is purely mathematical in character. Maxwell's theory has the broadest physical basis, although left by its author in shape merely of a terse verbal sketch.

Now it seems to me, if indeed I may venture any assertion, that Maxwell's theory is a version of Williamson's[3] theory of etherification, and of Clausius's[4] theory of electrolysis. The transition made is from unstable groupings of atoms to unstable groupings of molecules. But preserving minutely all the essentials of Maxwell's argument, the experiments of this paper justify the assumption that viscosity is a phenomenon evoked by certain changes of molecular structure, the inherent nature of which is ultimately chemical. I say chemical because if molecular break up occur, cardinal questions at once arise as to the manner of removal of the debris; and the phenomenon thus depends not only on the past history, but on the immediate future history of the typical mean configuration. The analogy of the three theories is very close, so that they admit of generic classification. They are examples of the invasion of statistical method into liquid and solid molecular kinetics.

The behavior of steel when regarded as a viscous solid and in the light of known facts,[5] is convincingly in favor of the view to be advocated;

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[1] Butcher: Proc. Lond. Math. Soc., 1878, vol. 3.

[2] Maxwell: "Constitution of Bodies," Ency. Brit., 9th ed., 1876, vol. 6, p. 310.

[3] Williamson: Ann. d. Chem. u. Pharm., 1851, vol. 77, p. 37.

[4] Clausius: Pogg. Ann., 1857, vol. 100, p. 353; ibid., 1857, vol. 101, p. 338.

[5] I refer in particular to the work of Dr. Strouhal and myself. These papers, systematically discussed and enlarged, are embodied with much new matter in the bulletins of the U. S. Geological Survey, viz: Bull. No. 14, 1885, pp. 1–226; Bull. No. 27, 1886, pp. 30, 61; Bull. No. 35, 1886, pp. 11–60; Bull. No. 42, 1887, pp. 98–131. Other references are given in the text.

and it was the direct bearing of some of the results on Clausius's theory of electrolysis, that led me to suspect a chemical explanation,[1] before I became aware of the existence of Maxwell's article. To show how clearly Maxwell's theory interprets the complex and almost anomalous phenomena of viscosity exhibited by steel, is the chief endeavor of the present paper; but I shall also add other matter.

HISTORICAL SKETCH OF THE THEORIES OF SOLID VISCOSITY.

75. It is desirable to pass in brief review the divers hypotheses on the nature of solid viscosity to which I have referred.

O. E. Meyer's[2] *theory* is the earliest and most direct. It discusses the action of elastic forces in a medium of imperfect elasticity, and develops formulæ to express the diminution of stress in virtue of the occurrence of internal friction.[3] The results to which Meyer's formula eventually leads are incomplete and were not fully verified by subsequent experiment. The theory is therefore sharply antagonized by Boltzmann,[4] by Streintz,[5] and by Kohlrausch.[6] In later paper Meyer[7] partially assents to these adverse views, acknowledging that the theory does not reproduce the phenomenon actually observed. It also fails, as Kohlrausch (loc. cit.) pointed out, in predicting an insufficiently slow time of occurrence. After giving reasons for dissenting from Boltzmann's and from Neesen's hypotheses, Meyer proceeds to partially develop an older idea of Weber's.[8] This physicist referred viscosity in solids, to partial molecular rotation, a view adopted by Kohlrausch,[9] by whom it has been more clearly interpreted. The rotations underlying Weber's phenomenon are considered identical with the rotations of molecule postulated by Clausius[10] in discussing shear. Following Meyer and others, "elastische Nachwirkung" is a possible occurrence in liquids.

76. *Boltzmann's*[11] *theory*, amplifying deductions of Lamy and of Clebsch, is based on the assumption that the elastic forces are dependent not only on the present but on the preceding deformations of the body. The effect of earlier states of stress on the existing stress diminishes with the intervening time but is independent of intervening states of

[1] Am. Jour. Sci. (III), 1887, vol. 33, p. 28. It is much to be regretted that Maxwell's theory was published out of the line of a physicist's usual routine reading.

[2] Meyer: Pogg. Ann., 1874, vol. 151, p. 108.

[3] Following the usage of the term by Navier, Cauchy, Poisson, St. Venant, Stokes, Stefan. Cf. Meyer, loc. cit.

[4] Boltzmann: Pogg. Ann., Ergänzb., 1876, vol. 7, p. 624.

[5] Streintz: Pogg. Ann., 1875, vol. 155, p. 588; ibid., 1874, vol. 153, p. 405.

[6] Kohlrausch: Pogg. Ann., 1877, vol. 160, p. 225.

[7] Meyer: Wied. Ann., 1878, vol. 4, p. 249.

[8] Weber: Pogg. Ann., 1835, vol. 34, p. 247; ibid., 1841, vol. 54, p. 1.

[9] Kohlrausch: Pogg. Ann., 1866, vol. 128, p. 413; cf. also ibid., 1863, vol. 119, p. 337.

[10] "Wenn ein solcher Körper fremden Kräften unterworfen wird, die von verschiedenen Seiten ungleich auf ihn wirken, also z. B. nach einer Dimension gedehnt wird, während er nach anderen Dimensionen frei bleibt oder gar zusammengedrückt wird, dann die Moleküle neben ihrer Verschiebung sich auch etwas drehen können, indem sie in Bezug auf ihre Kraftrichtung den ungleichen Spannungen etwas folgen. . . . " Pogg. Ann., 1849, vol. 76, p. 66.

[11] Boltzmann: Pogg. Ann., Ergänzb., 1876, vol. 7, p. 624.

stress. Different viscous deformations are superposable. Boltzmann's theory, therefore, presupposes the phenomenon[1] and brings the laws of viscosity tersely into formulæ. If ω is an interval of time reckoned back from $t-\omega$, when the strain $\theta_{t-\omega}$ existed, then Boltzmann's law may be clearly exhibited in its application to the problem of vibration of a viscous solid. Given a wire of the solid of length l and radius R. Let the upper end be fixed, and the lower end be attached to a heavy bob, whose moment of inertia for the given conditions is K. Then the equation of motion is (slow oscillation presupposed)

$$D-K\frac{d^2\theta_t}{dt^2}=\frac{\pi R^4}{2l}\left\{\mu\theta_t\int_0^\infty\psi(\omega)\dot{\theta}_{t-\omega}d\omega\right\},$$

where D is the moment of the applied couple, μ Lamy's constant, and ψ some function of ω. Replying to Meyer's critique that a theory of this kind is at variance with the present state of knowledge in atomistics, Boltzmann[2] disclaims all present purpose to connect his theory with definite physical hypotheses. He points out, however, that the assumed dependence of the existing strain on foregoing deformations is easily justified when the simultaneous changes of molecular configuration are taken into account; for it is not necessary to suppose that the elastic forces *as such* have any dependence on the preexisting stress. The changes of configuration in question are closely similar to Maxwell's, so that in this respect Boltzmann's theory may be looked upon as one form of mathematical development of Maxwell's physical hypothesis. I may add that Boltzmann tested his theory with data of Kohlrausch, Neesen, and Streintz. A special series of experiments subsequently undertaken by Kohlrausch[3] give additional strength to Boltzmann's deductions. The theory does not, however, predict permanent set.

77. *Neesen's theory.*—A theory similar to Boltzmann's, but atomistic in character, is due to Neesen.[4] It also assumes the occurrence of solid viscosity. Neesen distinguishes the forces producing and retarding motion and the final purely elastic forces which obtain when viscous motion has subsided. Neesen practically postulates a change in the constants of elasticity. Warburg[5] objects to Neesen's deductions because they contain no reference to the form of the molecule. Meyer (loc. cit.) fails to find in it definite causal relations to the observed viscous motion.

Braun's[6] research, though largely experimental in character, deserves mention here, because of special light which it throws on the superposition of different viscous deformations. Excepting in glass,[7] he finds

[1] Kohlrausch: Pogg. Ann., 1877, vol. 160, p. 227.
[2] Boltzmann: Wied. Ann., 1878, vol. 5, p. 430.
[3] Kohlrausch: Pogg. Ann., 1877, vol. 160, p. 225.
[4] Neesen: Pogg. Ann., vol. 1876, 157, p. 579.
[5] Warburg: Wied. Ann., 1878, vol. 4, p. 233.
[6] Braun: Pogg. Ann., 1876, vol. 159, p. 337; cf. Kohlrausch: Pogg. Ann., 1877, vol. 160, p. 227.
[7] I do not believe that glass is an exception. The occurrence of devitrification, for instance, decreases the viscosity of glass enormously, as I found. Other points of view will be adduced in another paper.

that these molecular motions do not take place independently of each other. He concludes that elastic and viscous deformations owe their occurrence to forces of different origin, and he refers viscous motion to the partial rotations postulated by Weber and Kohlrausch.

Warburg's theory.—Warburg,[1] following out the suggestions contained in Braun's results, formulates a new theory, in which viscosity is the result of partial rotations of molecules of a form other than spherical.

78. *Nissen's*[2] *theory* is unique. He considers the ether in the space surrounding the body, in its relations to the ether within the intermolecular spaces of the body; and he bases his theory on the conditions under which the external ether enters the said intermolecular spaces when the body is deformed by stress. He thus obtains both a time and a thermal effect. In many respects this curious theory seems to anticipate Osborne Reynolds[3] in recognizing the importance of the "dilitancy" of a granular medium.

79. *Maxwell's*[4] *theory* would require more extended comment; but the

[1] Warburg: Wied. Ann., 1878 vol. 4, p. 232.

[2] Nissen: Inaug. Dissert., Bonn. 1880. (Not accessible to me.)

[3] Reynolds: Phil. Mag., V, 1885, vol. 20, p. 469.

[4] Maxwell: Encyclop. Brit., 9th ed., 1876, vol. 6, p. 311.

The theory is here subjoined in full:

The molecules of all bodies are in motion. In gases and liquids the motion is such that there is nothing to prevent any molecule from passing from any part of the mass to any other part; but in solids we must suppose that some, at least, of the molecules merely oscillate about a certain mean position, so that, if we consider a certain group of molecules, its configuration is never very different from a certain stable configuration about which it oscillates.

This will be the case even when the solid is in a state of strain, provided the amplitude of the oscillations does not exceed a certain limit, but if it exceeds this limit the group does not tend to return to its former configuration, but begins to oscillate about a new configuration of stability, the strain in which is either zero, or at least less than in the original configuration.

The condition of this breaking up of a configuration must depend partly on the amplitude of the oscillations, and partly on the amount of strain in the original configuration; and we may suppose that different groups of molecules even in a homogeneous solid are not in similar circumstances in this respect.

Thus we may suppose that in a certain number of groups the ordinary agitation of the molecules is liable to accumulate so much that every now and then the configuration of one of the groups breaks up, and this whether it is in a state of strain or not. We may even in this case assume that in every second a certain proportion of these groups break up, and assume configurations corresponding to a strain uniform in all directions.

If all the groups were of this kind, the medium would be a viscous fluid.

But we may suppose that there are other groups the configuration of which is so stable that they will not break up under the ordinary agitation of the molecules unless the average strain exceed a certain limit, and this limit may be different for different systems of these groups.

Now if such groups of greater stability are disseminated through the substance in such abundance as to build up a solid framework, the substance will be a solid, which will not be permanently deformed except by a stress greater than a certain given stress.

But if the solid also contains groups of smaller stability and also groups of the first kind which break up of themselves, then when a strain is applied the resistence to it will gradually diminish as the groups of the first kind break up, and this will go on till the stress is reduced to that due to the more permanent groups. If the body is now left to itself, it will not at once return to its original form but will do so only when the groups of the first kind have broken up so often as to get back to their original state of strain.

This view of the constitution of a solid, as consisting of groups of molecules some of which are in different circumstances from others, also helps to explain the state of the solid after a permanent deformation has been given to it. In this case some of the less stable groups have broken up and assumed new configurations, but it is quite possible that others, more stable, may still retain their original configuration, so that the form of the body is determined by the equilibrium between these two set of groups; but if, on account of rise of temperature, increase of moisture, violent vibration, or any other cause, the breaking up of the less stable groups is facilitated, the more stable groups may again assert their sway, and tend to restore the body to the shape it had before its deformation.

terms in which his views are expressed are so precise, that it is impossible to abbreviate them. cf. §99. Aside from the remarks of the next paragraph, the ideas underlying Maxwell's theory have been given by many others, indeed by almost all the chief writers on solid viscosity; but Maxwell carries them through consistently to a complete theory.

I have stated that Maxwell's theory is the analogue of Clausius's theory of electrolysis. Where the latter uses "Theil molecüle" and electromotive force to effect chemical decomposition, Maxwell has unstable configurations and stress available to produce permanent deformation. In Clausius's case the number of decomposable molecules (i. e. unstable configurations as regards the action of electromotive force), in any given case of actual electrolysis, is practically infinite. This corresponds to Maxwell's viscous fluid, hard or soft. In a viscous solid, molecular configurations are present in all degrees of stability, with a sufficient preponderance of stable configuration to constitute a solid framework. The relative number of unstable configurations varies with the viscosity of the material. If, therefore, I conceive the case of an electrolyte exhausting itself with respect to electrical conductivity, by the chemical decomposition induced by current, until conduction cease, I have the analogue of a solid which is reaching the limit of viscous deformation. cf. §107.

80. From this analogy it follows that a solid (?) electrolyte is necessarily viscous; whereas a viscous solid is only an electrolyte when the molecules break up into parts oppositely charged. Again, a viscous solid (?) is probably more viscous when undergoing electrolytic decomposition than when no current passes through it.[1] Experiments to the same effect can, however, be made with greater facility, if the solid operated on is such that *special* instability of molecular configuration is superinduced by heat, instead of electrical action. Such a solid is hard steel, in which, in addition to the ordinary thermal instability, what may be called a *carburation instability* of molecular configuration asserts itself, even at mean atmospheric temperatures, and in the homogeneous metal. Inasmuch therefore as the gist of Maxwell's theory is instability of configuration, it follows that the evidence which can be derived with reference to it, from hard steel, must be unique in character: for despite the extreme hardness and elasticity of tempered steel, instability of molecular configuration *demonstrably* exists,[2] and is distributed uniformly throughout the metal; moreover the number of unstable groups can be made to vary over an enormous range, at pleasure.

I must distinctly state, however, at the outset, that Maxwell limits his considerations to configurations of molecules. The responsibility of fusing Clausius's and Maxwell's theories rests with me. The step is

[1] I have since been at some pains to verify this surmise, working with glass at 360°. But the experiments thus far are not decisive, because the amount of current passing through glass is not only very small, but soon ceases entirely even in the case of thin-walled tubes (Warburg, Wied. Ann., vol. 21, p. 622). On the other hand, if the current passes the electrolyte in strict accordance with Ohms law, it will produce no *special* decomposition, and will not therefore affect the viscosity of the solid electrolyte.

[2] Barus and Strouhal: Am. Jour. Sci. (III) vol. 32, 1886, p. 276.

dictated by the behavior of steel, in which the integrity of the molecule is certainly invaded without producing essential differences in the character or history of the viscous phenomena (§ 98). I may note that the occurrence of chemical change makes the hypothesis verifiable.

EXPERIMENTAL RESULTS.

APPARATUS.

81. The remarks made in the preceding sections clearly show, that notwithstanding the abundance of experimental data in hand, further researches tending to throw light on the ultimate nature of viscosity are urgently called for. To obtain such I made a new set of experiments on the plan indicated in Chapter II, modifying the apparatus there described in the following respects. The lower wire of the system is provided with a torsion circle of large radius, by aid of which the amount of permanent set of the wires can be accurately registered. In this way also any amount of twist can be permanently stored, at the operator's pleasure. Again the basin X in Pl. I was replaced by a ring-shaped basin surrounding the wires so that the wire could be twisted without removing the vane. In other respects the present experiments are like the above, except that the individual viscous detorsions were not watched for so long a period as occurs in Chapter II.

82. *Theory of the apparatus.*—This has been fully given in Chapter II, §§ 56 to 59.

83. *Measurements with steel.*—The following tables exhibit the new results for steel. About twenty rods were examined. Table 55, after enumerating the rod ("No."), and stating the temperature at which it was annealed (abbreviated "An.") from glasshardness, gives the amount of twist, τ (radians), temporarily imparted per unit of length, and $2(\varphi+\varphi')$, the mean amount of viscous detorsion, in radians per unit of length, observed immediately after the end of the experiment. $2(\varphi+\varphi')$ is therefore the mean viscous effect of τ impressed on the system of two vertical wires. Hence $\tau+2(\varphi+\varphi')=2\pi/L$. Furthermore, θ' is the temperature of the lower wire, θ that of the upper wire, and $(\varphi-\varphi')/\tau$ (radians) is the amount of viscous detorsion, as observed at the index between the wires, at the time specified, per unit of τ. Regarding this differential quantity, which is the chief datum of the tables, it is merely necessary to call to mind that 2φ is the viscous detorsion, per unit of length of the upper wire, for the rate of twist τ; and $2\varphi'$ has the same signification relatively to the lower (normal) wire. The reference to the unit of τ is a convenience permissible when τ, as in the present work, has nearly the same value throughout.[1]

The normal wire, No. 7, with which all the other steel wires are compared, is annealed from hardness at 450°, and has been twisted back

[1] In how far such reductions are generally permissible, cf. Weidmann: Wied. Ann., 1886, vol. 19, pp. 220, 222.

and forth till viscosity is practically unchanged by further twisting within the same limits. It is therefore in a state of extreme viscosity, and at the same time less liable to permanent set than a soft steel wire. Its dimensions are $l'=30^{cm}$, radius $=\rho=0{\cdot}0405^{cm}$, so that $l'=l$. The wire of unknown viscosity is examined at 20° (nearly), and immediately after at 100°. Two experiments are made at each temperature with τ alternately positive and negative. When τ and $(\varphi-\varphi')/\tau$ have the same signs (the usual case), the lower wire, No. 1, is of greater viscosity. In case of $\theta=100°$, only a part of the upper wire, length $a=\alpha-\beta$, could be heated; the remainder being kept at the lower temperature θ'. Time is conveniently given in minutes, reckoned after twisting.

84. *Viscous behavior of the rods.*—This is given in the following table:

TABLE 55.—VISCOUS DETORSIONS OF HARD STEEL.

$l=30^{cm}$. $\rho=0{\cdot}0405^{cm}$. $a=28{\cdot}5^{cm}$. $\beta=31{\cdot}5^{cm}$.

No. An.	τ $2(\phi+\phi')$	θ θ'	Time.	$\frac{\phi-\phi'}{\tau}\times10^3$	No. An.	τ $2(\phi+\phi')$	θ θ'	Time.	$\frac{\phi-\phi'}{\tau}\times10^3$
2	−·1045	20	3	+0·00	2	−·1027	100	2	− 0·00
450°		20	17	·03	450°		20	6	− ·92
			50	·05				18	− 1·97
	−·0003		60	·06				29	− 2·49
								45	− 3·08
						−·0020		58	− 3·47
	+·1027	20	1	−0·00		+·1027	100	2	+ 0·00
		20	8	− ·06			20	20	1·71
			28	− ·12				33	2·56
	+·0020		52	− ·12		+·0020		47	3·21
3	−·1033	20	2	+0·00	3	−·1027	100	2	− 0·00
450°		20	6	·17	450°		20	11	− ·88
			19	·34				20	− 1·21
			28	·43				34	− 1·51
	−·0014		44	·49		−·0020		48	− 1·97
	+·1030		2	−0·00		+·1027	100	2	+ 0·00
			8	− ·03			20	10	3·11
			20	− ·03				18	4.65
	+·0017		30	− ·03		+·0020		24	5·50
4	−·1033	22	2	−0·00	4	−·1007	100	3	− 0·00
360°		22	17	− ·19	360°		22	15	− 1·74
			42	− ·24				28	− 2·48
	−·0014		54	− ·24		−·0041		46	− 3·05
	+·1007	22	1	+0·00		+·0998	100	2	+ 0·00
		22	33	1·47			22	12	5·32
			44	1·61				22	7·62
	+·0041		64	1·77				37	9·86
						+·0049		49	11.07
5	−·1027	23	2	−0·00	5	−·1019	100	3	− 0·00
360°		23	15	− ·19	360°		23	22	− 2·24
			21	− ·25		−·0029		32	− 2·94
	−·0020		34	− ·28					
	+·1021	23	2	+0·00		+·1015	100	2	+ 0·00
		23	12	·25			23	7	2·12
			19	·29				20	4·04
	+·0026		27	·31		+·0032		27	4·74
6	−·1035	19	2	−0·00	6	−·1023	100	1	− 0·00
190°		19	5	− ·12	190°		19	4	− 3·69
			17	− ·28				12	− 8·02
			35	− ·18				20	−10·39
								29	−12·95
	−·0012					−·0024		36	−13·35
	+·1035	19	2	+0·00		+·1003	100	1	+ 0·00
		19	19	·00			19	5	8·18
			31	·06				10	13·29
								15	16·98
								20	19·53
								26	22·14
								32	24·10
	+·0012					+·0044		38	25·87

TABLE 55.—Continued.

$l=30^{cm}$. $\rho=0{\cdot}0405^{cm}$. $a=28{\cdot}5^{cm}$. $\beta=31{\cdot}5^{cm}$.

No. An.	$\frac{\tau}{2(\phi+\phi')}$	$\frac{\theta}{\theta'}$	Time.	$\frac{\phi-\phi'}{\tau}\times10^3$	No. An.	$\frac{\tau}{2(\phi+\phi')}$	$\frac{\theta}{\theta'}$	Time.	$\frac{\phi-\phi'}{\tau}\times10^3$
7	−·1027	20	2	− 0·00	7	−·1007	100	1	− 0·00
190°		20	8	− ·64	190°		20	3	− 3·88
	−·0020		18	− 1·02				8	−11·10
								19	−19 47
								33	−25·02
						−·0041		40	−26·95
	+·1009	20	1	+ 0·00		+·1003	100	2	+ 0 00
		20	9	1·17			20	9	10·81
			20	1·77				16	16·03
			30	2·08				26	20·67
	+·0038		50	2·58		+·0044		40	24·75
8	−·1035	20	2	− 0·00	8	−·1007	100	3	− 0 00
190°		20	12	− ·24	190°		19	11	− 9 02
			19	− ·37				20	−14·51
	−·0012		27	− ·37				32	−18·67
						−·0041		55	−23·88
	+·1030	20	2	+ 0·00		+·0995	100	2	+ 0·00
		20	9	·43			19	11	17·05
			17	·71				22	26·59
			33	·99				28	29·30
	+.0017		44	1·15		+·0052		41	34·59
9	−·1033	19	2	− 0 00	9	——	100	2	− 0·00
100°		19	8	− ·55	100°	——	19	5	−17·50
	−·0014		27	− 1·48				8	−26·93
	+·1023	19	3	+ 0·00		Accident.			
		19	10	1·03					
			31	2·33					
	+·0023		42	2·74					
10	−·1041	20	1	− 0·00	10	−·0974	100	1	− 0·00
100°		20	12	− ·92	100°		20	3	−20·38
			25	− 1·34				7	−32·33
	−·0006		38	− 1·65				16	−56·23
								19	−62·31
						+·0073		21	−65·63
	+·1023	20	2	+ 0·00		+·0968	100	2	+ 0·00
		20	21	1·74			20	5	15·08
	+·0023		45	2·48				8	24·41
								12	33·30
								18	42·96
								22	48·14
								25	51·79
								32	59·43
								35	61·23
						+·0078		42	67·76
11	−·1033	21	1	− 0·00	11	+.1007	100	2	− 0·00
100°		21	8	− ·46	100°		20	5	− 8·56
			32	− ·80				10	−16·18
	−·0014		48	− ·89				16	−22 06
								22	−26·49
						−·0041		26	−29·03
	+·1027	21	2	+ 0·00		+·0966	100	3	+ 0·00
		21	12	1·18			20	5	5·79
			26	1·83				16	30·88
			37	2·17				23	40 79
	+·0020		50	2·46				31	49·84
								37	55·57
						+·0081		45	62·39

TABLE 55—Continued.

$l=30^{cm}$. $\rho=0{\cdot}0405^{cm}$. $a=28{\cdot}5^{cm}$. $\beta=31{\cdot}5^{cm}$.

No. An.	τ $2(\phi+\phi')$	θ θ'	Time.	$\frac{\phi-\phi'}{\tau}\times10^3$
12 25°	+·1007	22 22	2 .5	+ 0·00 2·28
			11	4·24
			29	6 88
	+·0041		36	7·57
	−·0993	22 22	2 13	− 0·00 − 5·11
			20	− 6·32
	−·0053		25	− 7·08
13 25°	−·1007	20 20	3 12	− 0·00 − 2·43
			22	− 3·63
	−·0041		36	− 4·74
	+·1001	20 20	2 6	+ 0·00 2·00
			14	3·91
			26	5·46
			31	5·94
	+·0047		41	6·61
	−·1019	20 20	4 9	− 0·00 − ·75
			17	− 1·56
			23	− 2·00
			36	− 2·75
			45	− 3·13
	−·0029		59	− 3·75
14 25°	−·1003	20 20	2 10	− 0·00 − 3 99
			21	− 5·95
			31	− 7 03
	−·0044		42	− 7·23
	+·0989	20 20	2 7	+ 0 00 2·89
			14	4·75
			18	5 49
	+·0057		23	6·18
	−·1021	20 20	5 15	− 0·00 − 1·37
			37	− 2·84
			57	− 3·57
			63	− 3·86
	−·0026		82	− 4·40
	+·1019		1	+ 0·00
	+·0029		111	7·74
17 100°	+·1041	22 22	2 12	+ 0·00 1·04
			22	1·52
	+·0006		34	1·89
	−·1033	22 22	1 15	− 0·00 − 1·66
			28	− 2·21
	−·0014		40	− 2·59
	+·1038	22 22	2 9	+ 0·00 ·79
			19	1·32
	+·0009		27	1·56

85. *Mean values deduced.*—The following Table 56 is interpolated from the preceding, and contains mean values of $(\varphi-\varphi')/\tau$, as derived from the two twists, τ, alternately positive and negative. The justification of this mode of obtaining data for a chart is given below. Besides these data Table 56 contains the number and temper, and the electrical constant[1] (specific resistance, s_0, microhms, *cc*, 0° *C.*), as well as the differences s_0-s_0' and $s_{100}-s_{20}'$, in which the subscripts are the temperatures at which s is taken, and s' is the constant of the normal rod, No. 1. Hence these electrical differences correspond to $(\varphi-\varphi')/\tau$, when $\theta=0°$ *C.* and when $\theta=100°$ *C.*, respectively. For No. 1, $s_0'=18{\cdot}6$, $s_0-s_0'=0$ and $(\varphi-\varphi')/\tau=0$. The values $(\varphi-\varphi')/\tau$ are in the same horizontal row with the temperatures, θ, to which they belong. It is not necessary to distinguish $s_0-s'_0$ and $s_{20}-s'_{20}$, here.

[1] For definition of thermo-electric hardness, cf Bull. U. S. Geol. Survey, No. 14, p. 65.

TABLE 56.—*Values of* $\frac{\varphi-\varphi'}{\tau}\times 10^3$ *for consecutive times.*

Temper.	No. s_0.	s_0-s_0' / $s_{100}-s'_{20}$.	θ.	2m.	5m.	10m.	20m.	30m.	40m.	50m.
Annealed at 450°	2	·8	20	−·00	−·03	−·05	−·07	−·08	−·10	−·10
	19·4	5·9	100	+·00	+·56	+1·12	+1·88	+2·47	+2·90	+3·29
	3	·6	20	−·00	−·06	−·12	−·19	−·23	−·26	−·28
	19·2	5·6	100	+·00	+1·15	+1·97	+3·05	+3·91	.....	.....
Annealed at 360°	4	·4	22	+·00	+·14	+·34	+·57	+·69	+·75	+·81
	19·0	5·5	100	·00	1·58	3·20	4·79	5·92	6·66	7·34
	5	1·5	23	·00	·08	·17	·26	·30	.....	.....
	20·1	6·5	100	·00	1·12	1·97	3·13	4·00	.....	.....
Annealed at 190°	6	10·2	19	·00	·06	·12	·16	·16	.....	.....
	28·8	15·8	100	·00	2·99	7·48	12·72	15·84	17·95	.....
	7	12·3	20	·00	·46	·88	1·31	.....	.....	.....
	30·9	17·7	100	·00	5·10	11·08	18 02	21·90	24·82	.....
	8	12·7	20	·00	·14	·34	·57	·68	.....	.....
	31·3	17·8	100	·00	6·12	12·92	20·54	24·96	28·08	.....
Annealed at 100°	9	18·6	19	·00	·30	·68	1·20	1·56	.....	.....
	37·2	24·2	100	·00	17·48	32·98	.....	.....	.....	.....
	10	15·6	20	·00	·43	·86	1·38	1·71	1·96	.....
	34·2	21·0	100	·00	13·94	27·88	47·26	.....	.....	.....
	11	14·9	21	·00	·32	·69	1·08	1·31	1·48	1·61
	33·5	20·5	100	·00	8·84	19·38	32·64	41·82	.....	.....
Annealed at 30°.	12	20·8	20	·00	2·41	4·24	6·09	7·42	.....	.....
	39·2	.....	.....	.....	.....	.....	.....	.....	.....	.....
	13	18·4	20	·00	1·15	2·27	3·55	4·38	.....	.....
	37·0	.....	.....	.....	.....	.....	.....	.....	.....	.....
	14	22·1	20	·00	2·14	4·00	5·86	7·00	7·91	.....
	40·6	.....	.....	.....	.....	.....	.....	.....	.....	.....

86. *Data of the measurements with platinum alloys.*—Before proceeding to a discussion of these results, I will insert the following set of similar data which I found with alloys of platinum. The normals here are wires of pure platinum, and with these the alloys are to be compared. The plan of comparison and tabulation is the same as explained for steel except that in the present case the temperature, 100°, is made to act on the upper wire without untwisting. The experiment at 100° therefore joins on to the experiment at ordinary temperature in such a way that no adjustment of apparatus is made, further than the change of temperature specified.

The wires n, n' and A are of commercial platinum, purified by long continued fusion on a lime hearth before the oxyhydrogen flame. The other compositions are given. In all cases S_0 is the specific resistance, α the temperature coefficient of the alloy.

As before, like signs of τ and $\varphi-\varphi'$ show that the lower wire (pure platinum, normal) is of greater viscosity.

TABLE 57.—VISCOSITY OF PLATINUM ALLOYS.

Fixed Normal, n', $\left\{\begin{matrix} l'=26{\cdot}2^{cm} \\ \rho'=0{\cdot}0215^{cm}\end{matrix}\right\}$ $\theta=25°$.

No. of wire: l, ρ.	Alloy.	S_0 $\alpha\times10^3$	τ $(\phi+\phi)$	θ'	Time.	$\frac{\phi-\phi'}{\tau}\times10^3$
Normal		13·3	−0·0829	25	0	
n	Pt	2·50	−0·0378		2	+ ·00
l=26·2cm					14	2·26
ρ=0.0215cm					34	3·54
					54	4·16
				100	68	−3·54
					73	−4·24
					85	−4·86
					91	−5·13
					107	−5·49
					120	−5·71
A	Pt	12·0	−0·0748	25	0	
l=26·0cm		2·96	−0·0460		2	0·00
ρ=0·0215cm					17	+3·97
					29	+5·00
					50	+5·98
					75	+6·71
				100	80	+5·20
					102	+5·29
					119	+5·49
					129	+5·78
1	Pt. annealed	18·5	−0·0561	25	0	
l=26·1cm		1·84	−0·0643		2	− ·00
ρ=0·0215cm					19	− ·26
					34	− ·54
					47	− ·54
				100	54	−5·21
					69	−5·46
					91	−5·60
					104	−5·73
2	Pt. annealed	22·1	−0·0805	25	0	
l=26·1cm		1·49	−0·0404		3	− ·00
ρ=0·0214cm					16	− ·32
					39	− ·69
					55	− ·82
**				100	65	− ·00
					75	−1·18
					86	−1·55
3	Pt. annealed	24·7	−0·0905	25	0	
l=26·0cm		1·33	−0·0303		2	+ ·00
ρ=0·0215cm					6	+2·43
					21	+5·67
					45	+7·87
					57	+8·56
				100	63	+2·19
					74	+2·55
					85	+2·63
					117	+2·75
4	Pt. Ag	19·1	−0·0547	24	0	
l=26·1cm		1·87	−0·0660		3	− ·00
ρ=0·0215cm					12	−1·06
					22	−1·49
					36	−1·86
					56	−2·17
**				100	60	− ·00
					70	−7·38
5	Pt. Ag	22.3	−0·0923	25	0	
l=25·9cm		1·5	−0·0255		2	− ·00
ρ=0·0217cm					6	−2·30
					15	−4·85
					30	−7·11
					42	−8·07
					48	−8·56
				100	55	−2·75
					60	−3·02
					75	−3·20
					81	−3·24

** Scale readjusted.

TABLE 57.—VISCOSITY OF PLATINUM ALLOYS—Continued.

Fixed Normal, n', $\left\{\begin{array}{l} l' = 26{\cdot}2^{cm} \\ \rho' = 0{\cdot}0215^{cm} \end{array}\right\} \theta = 25°$.

No. of wire: l, ρ.	Alloy.	S_0 $a \times 10^3$	τ $(\phi+\phi)$	θ'	Time.	$\frac{\phi-\phi'}{\tau} \times 10^3$
6	Pt. Ag	34·0	−0·1003	25	0	
$l = 25{\cdot}3^{cm}$		1·0	−0·0203		1	+ ·00
$\rho = 0{\cdot}0220^{cm}$					6	+3·68
					30	+6·68
					42	+7·18
					52	+7·38
				100	59	+0 8
					64	−1·64
					75	−2·23
					80	−2·99
					100	−4 01
					106	−4·20
7	Pt. Pd	19·4	−0 0738	25	0	
$l = 26{\cdot}4^{cm}$		1·76	−0·0468		2	− ·00
$\rho = 0{\cdot}0216^{cm}$					23	− 8·07
					38	−14·01
					47	−14·90
					54	−15 46
**				100	59	− ·00
					74	− 3·47
					87	− 4 27
					105	− 4.87
					119	− 5·27
8	20·4	1·67	−0·0796	25	0	
$l = 26{\cdot}4^{cm}$			−0·0413		1	− ·00
$\rho = 0{\cdot}0215^{cm}$					6	− 1.84
					21	− 3·18
					30	− 3.51
					41	− 3·83
				100	52	−15·32
					62	−16·15
					79	−16·70
					84	−16·70
n.						
Normal	Pt.		−0·0867	25	0	
*	a		−0·0344		2	− ·00
$l = 26{\cdot}2^{cm}$					9	−2·28
$\rho = 0{\cdot}0215^{cm}$					15	−3·05
					27	−3·90
					33	−4·06
n.						
Normal			−0·0652	25	0	
†	Pt		−0·0556		2	− ·00
	b				17	−2·48
					44	−3·38
Normal			−0·0443	22	0	
After heating lower wire thoroughly red hot.	Pt		−0·0766		1	+ ·00
	c				4	+14·21
					14	+32·29
					28	+42·71
					44	+49·61
Normal			−0·0436	22	0	
After heating upper wire thoroughly red hot.	Pt		−0·0773		2	− ·00
	d				7	−15·32
					17	−28·57
					25	−34·64
					43	−42·37
					54	−45·40
Normal			−0 0383	22	0	
After heating lower wire thoroughly red hot.	Pt		−0 0826		3	+ ·00
	e				7	+ 3·84
					26	+13·52
					47	+19·18
					55	+20·70
Normal			−0 0396	22	0	
After heating upper wire thoroughly red hot.	Pt		−0·0813		3	− ·00
	f				8	− 7·52
					13	−12·07
$l = 26{\cdot}3^{cm}$					29	−20·15
$\rho = 0{\cdot}0212^{cm}$					37	−22·77
					46	−25·39

* Without annealing. † After careful annealing (both wires) at red heat. ** Scale readjusted.

DEDUCTIONS FROM PLATINUM ALLOYS.

87. *Viscous effect of alloying.*—The results just communicated for platinum alloys were made first in chronological order, and led to the results already communicated for steel. It will therefore be in place to discuss the purposes of the alloy work and the difficulties encountered in endeavoring to arrive at intelligible results. In Chapter II I inferred from certain results obtained by Dr. Schroeder and by myself independently that wires of small viscosity were proportionately more susceptible to the influence of temperature. It is this proposition which the alloy work was primarily intended to elucidate. Moreover the resistances and temperature coefficients of the alloys were known from earlier investigations,[1] in which the same alloys had been operated upon. Hence I hoped also to see whether any relation between viscosity and electrical resistance would be discernible, a result which in a general way obtains for tempered steel. In other words, if the temperature coefficient of viscous deformation should hold similar generic relations to the amount of deformation that the temperature coefficient of electrical conductivity holds to conductivity, then an investigation like the present might reasonably be expected to lead to results bearing directly on the mechanism of electrical resistance. This is the chief point of view from which the work is undertaken.

The normal wires N and N' were originally identical. After heating both to redness, however, I was surprised to find that the large difference registered at the beginning of Table 57 obtained. The upper wire is more viscous than the lower at 25°, but decidedly less viscous than the lower when 100° is applied to the upper wire, the temperature of the lower—as in all these experiments—being fixed. Yet these results are not sustained by the platinum wire A.

The gold alloys form a series in which mean viscosity tends to increase with electrical resistance; but the behavior in the divers cases is obscure. Again, the silver alloys form a series in which the increase of viscosity with resistance is very decidedly marked. Moreover, the effect of temperature to accelerate viscous deformatian decreases as the viscosity decreases.

Finally these deductions hold for the two palladium alloys.

88. These results are encouraging, notwithstanding the element of vagueness which it is almost impossible to eliminate. The general result is in favor of high electrical conductivity associated with small viscosity and small temperature coefficients. Platinum itself, however, is a marked exception to this observation. Hence to determine the amount of variation of the normals, as well as to enter more closely into the nature of the discrepancy, I made the investigation comprehended in the second part of Table 57. The results obtained show, in the first place, that in consequence of the twisting incident to the examinations

[1] Bulletin U. S. Geol. Survey, No. 54, 1889, p. 143.

of the wires Nos. 1 to 8, the lower wire has gained in viscosity sufficiently to exceed the upper (unused) wire in this respect. This relation is maintained even after both wires are similarly annealed. I then made the experiment of annealing only one of these wires in the way the table indicates. The results are striking. It appears that in every case the freshly annealed wire is enormously less viscous than the unannealed wire; and that this difference of viscosity produced by these mere mechanical means is so large as would wholly mask and obscure the effects of alloying investigated in the earlier part of Table 57. Having thus found how large an error in the observed viscosity may be introduced by even trifling differences of mechanical treatment, I concluded that an investigation into the true effect of alloying would necessarily be of an exceedingly refined character, and would therefore not fall properly within the scope of the present bulletin. Further attempts to study the viscosity of alloys were therefore for the present abandoned.

Whatever may be the effect of alloying on viscosity, it is clear that the enormous electrical effect produced by alloying platinum (resistance increased) and the relatively small electrical effect produced by imparting a strain are quite out of proportion and symmetry with the corresponding viscous effects in the two cases. Again, the effect of temperature in changing viscosity is much too marked to conform with the resistance effect of temperature. These results appear more clearly in the data for steel.

89. *Effect of annealing and hardening platinum.*—In this place, however, the effect of annealing at red heat is of special interest. The two wires N and N' being chemically identical, the effect of annealing on the molecular configuration must be the same in each case. If a thin wire be heated red hot and cooled in air, the result is a strain of dilatation imparted to the wire on cooling. Hence, for this reason alone, the molecular stability of the wire is of a lower order than obtains, cæteris paribus, when cooling takes place with extreme slowness. This is the case of the freshly annealed wire. Again, it is probable, if such a wire is twisted, even over small arcs, that the greater number of the more unstable configurations will be mechanically broken up; for it is clear that the prevailing tendency must be such as to cause the unstable configurations continually to fall to positions of minimum potential energy. If twisting be indefinitely repeated, therefore, the result is very perceptible hardness and elasticity. It is not necessary to recite here the many experiments I made to elucidate these questions. I will confine myself to a statement of results.

In the first place, when two wires as nearly as possible identical (chemically and physically) are compared, the question arises in how far viscosity may vary with the time which elapses *after* annealing. The experiments made showed a slight increase of viscosity with the time given to the molecules to subside after annealing in air. Two identical wires of platinum were annealed, for instance, and then al-

lowed to rest for several hours; after which only one of the wires was again annealed, and the two were at once compared after cooling. This and similar tests indicated small decrease of viscosity of the freshly annealed wire; results, however, by no means comparable with the data of the above table and often obscure.[1]

The sign of the twist of the wires N, N', of pure platinum, compared in the table is the same throughout. The question therefore arises in how far an earlier stage of viscous subsidence simply overtakes a later stage. To determine this it is sufficient to *reverse* the sign of the twist alternately without fresh annealing; or to reverse it with each of the alternate annealings of the upper and lower wire. Experiments made in some number showed that even in this case the results of the second part of Table 57 hold good, the diminution produced being in degree, not in sign. The viscous deformation of freshly annealed wire takes place at a decidedly greater rate even when the viscous motion of the other wire is intensified by an impressed latent strain.

SUMMARY.

All these results are such as follow at once from Maxwell's theory. They show that the effect of twisting is to be referred to the motion of molecules which accompanies it. The molecules, together with their component atoms, are thus placed in new relative positions; therefore unstable configurations during the course of such motion are continually broken up into configurations of smaller potential energy and greater stability. Hence, finally, the observed decrease of viscosity. By reversing the sign of the twist the original configurations can be only partially restored, even for small permanent set, such as is here in question; and the effect of prolonged and repeated twisting is stiffness and constant viscosity, because all the molecules have collapsed into configurations of minimum stability, and the intrinsic molecular energy is the potential minimum compatible with the given conditions.

DEDUCTIONS FOR STEEL.

"ACCOMMODATION" IN GLASS-HARD STEEL.

91. *The phenomena proper.*—Steel wires were used in our earlier work,[2] free from torsion strain. The hard steel wires of the present paper, employed in other researches, may contain twists stored up like residual magnetism. This produces a kind of unilateral symmetry, so far as torsions are concerned; but it is not otherwise objectionable. In critical cases wires free from latent torsion are selected.

[1] The thermal effect without annealing is so nearly negligible as to prove that in Dr. Schroeder's work (Wied. Ann., vol. 28, 1886, p. 369) the observed result is to be ascribed to annealing of hard-drawn wire. Regarding my own work I may say that I am not sure that both wires were kept quite free from tensile stress, the importance of which I did not at the time fully appreciate. In a later research I made a detailed study of the viscous effects of traction and other mechanical stress. (Cf. Phil. Mag. Feb., 1889, p. 155.)

[2] Barus and Strouhal: Am. Jour. Sci., (III), 886, vol. 32, 1p. 448; 1887, vol. 34, p. 4.

Turning to Table 55, the individual wires are found to show wide differences of viscous behavior. In No. 2 the viscous subsidence takes place at nearly the same rate for $-\tau$ and for $+\tau$, both at $\theta=20°$ and at $\theta=100°$. In No. 3 the effect of $-\tau$ and $+\tau$ is of different magnitude at 20°, and enormously different at 100°. In No. 4 the effect of $+\tau$ following $-\tau$ is even more phenomenally pronounced at 20° and particularly at 100°. In No. 5 the wires nearly identical at 20° show differences at 100°. In No. 6 this is true in even much greater degree, whereas in No. 7 wires differing considerably at 20° show relatively small differences at 100°. And so I might go through the series. Nos. 13 and 14 are wires originally free from strain (shear); but vagueness also appears in these.

Careful inspection of the tables reveals the law that viscous deformation takes place at numerically greater rates during the even twists than during the odd twists which immediately precede them respectively. Aside from these oscillations, the effect of twisting here (§ 90) is again pronounced increase of viscosity.

Maxwell's theory accounts for the stated vagueness of behavior at once. In two samples of a complex substance like steel the distributions and relations of the unstable molecular configurations will only in very rare instances be physically and chemically identical. The foregoing paragraph shows that such identity is rare even in pure homogeneous metal. (§ 94.)

The effect of twisting alternately in opposite directions is of so great importance in its bearing on Maxwell's theory that I made further special experiments. From these I select the following example, tabulating it as in case of Table 55. The normal No. 1, annealed at 450°, has been described. No. 18, annealed at 25°, or glass-hard, is carefully selected free from latent torsion, having experienced no other strain prior to the examination in Table 58 than that incident to tempering (quenching). There are twelve alternations of twist, indicated by subscripts, and the current time in hours and minutes of each is given. I also give under m the time in minutes which refers specially to the duration of each twist. No. 1 being of greater viscosity, τ and $(\varphi-\varphi')/\tau$ are alike in sign, by agreement.

Experiments made by counter-twisting two glass-hard wires gave results like this, but on a smaller scale. To compare the results of Table 58 perspicuously it is sufficient to construct the differences, $\Delta(\varphi-\varphi')/\tau$, of the respective value of $(\varphi-\varphi')/\tau$, at *two* and *four* minutes after twist is imparted. These are then to be compared in their dependence on current time, as has been done in Fig. 24. Phenomena of this kind were called "accommodation" by Streintz,[1] their discoverer, by Wiedemann,[2] Kohlrausch,[3] and others. The fact that Boltzmann's law contains them is among its chief excellencies.

[1] Streintz: Pogg. Ann., 1874, vol. 153, p. 406.

[2] Wiedemann: Wied. Ann., 1879, vol. 6, p. 512. This work is the most comprehensive of the relevant researches.

[3] Kohlrausch: Pogg. Ann., 1876, vol. 158, p. 371. Cf. Schmidt: Wied. Ann., vol. 2, 1877, p. 48.

TABLE 58.—*Viscous effects of twisting glass-hard steel alternately in opposite directions.*

Normal wire No. 1; $l=30^{cm}$ $\rho=\cdot0405^{cm}$.

Remarks.	τ	Time.	m	$\frac{\phi-\phi'}{\tau}\times10^3$
No. 18_1 Annealed at 25°.	—·102	9h 13m	0	.....
		15	2	— ·00
		17	4	—1·31
		28	15	—4·07
		35	22	—5·04
No. 18_2.....	+·102	9h 37m	0	.....
		39	2	+ ·00
		41	4	1·59
		52	15	5·17
		60	23	6·45
No. 18_3.....	—·102	10h 2m	0	.....
		4	2	— ·00
		8	6	—1·45
		19	17	—3·28
		25	23	—3·93
No. 18_4.....	+·102	10h 28m	0	.....
		30	2	+ ·00
		36	8	2·04
		41	13	3·04
		48	20	3·86
No. 18_5.....	—·102	10h 50m	0	.....
		52	2	— ·00
		59	7	—1·59
		70	20	—2·73
No. 18_6.....	+·102	11h 11m	0	.....
		13	2	+ ·00
		15	4	·76
		25	14	2·59
		33	22	3·42
No. 18_7.....	—·102	11h 35m	0	.....
		37	2	— ·00
		39	4	— ·52
		56	21	—2·38
		57	22	—2·45
No. 18_8.....	+·102	12h 0m	0	.....
		2	2	+ ·00
		4	4	·59
		20	20	2·62
No. 18_9.....	—·102	12h 24m	0	.....
		26	2	— ·00
		28	4	— ·41
		40	16	—2·38
No. 18_{10}.....	+·102	12h 42m	0	.....
		44	2	+ ·00
		46	4	·55
		54	12	1·76
		63	21	2·55
No. 18_{11}.....	—·102	1h 5m	0	.....
		7	2	— ·00
		9	4	— ·45
		21	16	—1·76
No. 18_{12}.....	+·102	1h 23m	0	.....
		25	2	·00
		27	4	·48
		36	13	1·69
After several days.				
No. 18_{13}.....	—·102	9h 47m	0	.....
		49	2	— ·00
		51	4	— ·62
		60	13	—2·07
No. 18_{14}.....	+·102	10h 2m	0	.....
		4	2	+ ·00
		6	4	·69
		15	13	4·42
No. 18_{15}.....	—·102	10h 17m	0	.....
		19	2	— ·00
		21	4	— ·62
		30	13	—2·21

TABLE 59.—*Viscous accommodation of glass-hard steel. Digest of Table* 58.

Twist No.	Time.	$\Delta\frac{\phi-\phi'}{\tau}\times10^3$	Twist No.	Time.	$\Delta\frac{\phi-\phi'}{\tau}\times10^3$	Twist No.	Time.	$\Delta\frac{\phi-\phi'}{\tau}\times10^3$
1	0m	—1·31	5	97m	— ·69	9	191m	— ·41
2	24	+1·59	6	118	+ ·76	10	209	+ ·55
3	49	— ·83	7	142	— ·52	11	232	— ·45
4	75	+ ·97	8	167	+ ·59	12	250	+ ·48

If the numerics of $\Delta(\varphi-\varphi')/\tau$ in Table 59 be regarded in their dependence on time, the results are seen to oscillate around a mean line of equilibrium. The ordinates of this mean line decrease with time at a gradually retarded rate, until a definite inferior limit is eventually reached. This is very clearly shown by the diagram, Fig. 24. It is curious to note that the largest observed ordinate (time = 0, nearly), is at least three times the limiting ordinate (time = ∞). After 12 twists oscillation has considerably subsided, but it has not ceased; in the same degree the viscosity of the glass-hard rod has reached a fixed maximum.

This complicated phenonomen is at once elucidated by Maxwell's theory. The ordinates of the line around which oscillation takes place, are an index of the degree of instability of molecular configuration, at the time given by the abscissæ. The oscillations are the result of strain

(latent shear, I may call it) imparted to the configurations by the successive twists to which the wire is subjected. Thus if τ be the impressed twist, and $\Delta\tau$ the mean strain left in the configurations at the

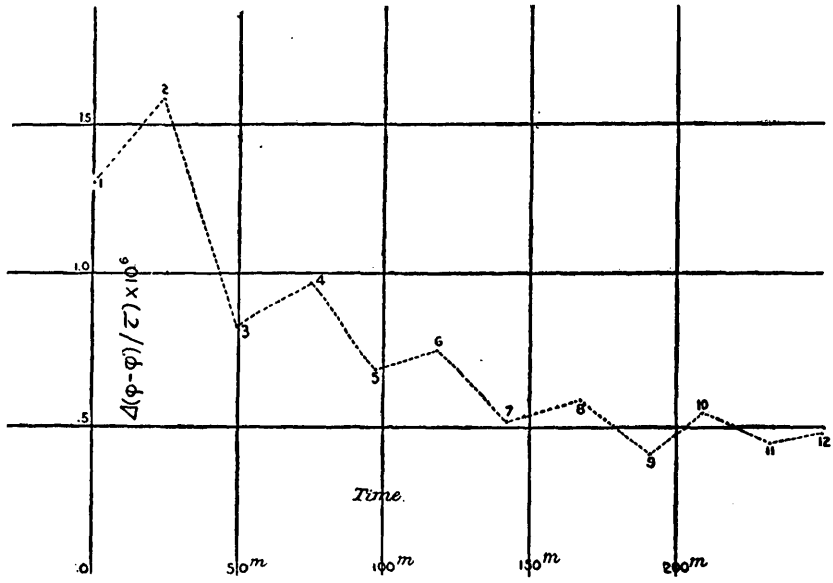

FIG. 24.—Viscous accommodation of glass-hard steel.

instant when τ is removed; and if n be the original relative number of unstable configurations, and Δn the number broken up during the period of the strain τ; then (apart from subsidiary considerations) Maxwell's theory analyzes the effects of alternate twisting in accordance with the following scheme:

Strain.	Molecular instability.
First twist .... $-\tau$	$+n$
Second twist .. $+\tau+\Delta\tau$	$+n-\Delta n$
Third twist ... $-\tau+\Delta_1\tau-\Delta'\tau$	$+n-\Delta_1 n-\Delta' n$
Fourth twist.. $+\tau+\Delta_2\tau-\Delta'_1\tau+\Delta''\tau$	$+n-\Delta_2 n-\Delta'_1 n-\Delta''' n$
Fifth twist.... $-\tau+\Delta_3\tau-\Delta'_2\tau+\Delta''_1\tau-\Delta'''\tau$	$+n-\Delta_3 n-\Delta'_2 n-\Delta''_1 n-\Delta''' n$

The variation which Δ undergoes in passing from one twist to the next is indicated by subscripts. Thus $\Delta\tau$, $\Delta_1\tau$, $\Delta_2\tau$, ... is probably a decreasing series; whereas Δn, $\Delta_1 n$, $\Delta_2 n$, ... is an increasing series because reversal of the sign of the twist must be supposed to reconstruct some of the configurations broken up by the preceding twist. The first part of the scheme indicates that the strain in the 2d, 4th, 6th ... twists is necessarily greater than the strain in the immediately preceding 1st, 3d, 5th ... twists respectively, at least at the outset of the experiments. Hence the observed oscillation. Again, the number of unstable configurations must continually decrease, according to the second half of the scheme. Hence the mean line about which the observed viscous deformations oscillate. Finally, experiment shows that

the accelerating effect of $\Delta\tau$ on viscous deformation is greater than the retarding effect of $-\Delta n$. After this, however, the accelerating effect of $\Delta_2\tau - \Delta'_1\tau + \Delta''\tau$, and the succeeding τ-quantities, is always less than the retarding effect of $-\Delta_2 n - \Delta_1 n - \Delta'' n$, and the succeeding n-quantities, respectively.

The scheme admits of simplification; but inasmuch as the period of oscillation is arbitrary, the phenomenon remains indefinitely complex.

92. *Motional effects estimated.*—The second part of Table 58 shows that the viscosity gained in virtue of consecutive alternate twisting of glass-hard steel is permanently gained. The results indicate some recuperation; but the amount is small in comparison with the havoc of configurations made by twisting. Mere molecular motion has therefore permanently broken the more unstable configurations. I will note that the viscous effect of prolonged twisting to and fro in case of glass-hard steel, is of the same order of magnitude as the effect of prolonged annealing at 100°. This indicates the importance of the motional effect in question.

If, following the analogy of steel, I consider annealing a process by which unstable configurations are broken up, I may designate the phenomenon here in question as motional annealing. Experiments for which there is no room here showed that motional annealing is relatively without electrical effect. For increasing rates of twist thick wires show viscous deformation sooner than thin wires. Hence motional break up commences at the external surface where stress is most intense, and proceeds thence toward the axis where stress is least. Thus there appears an essential dependence on the dimensions of the twisted rod. Elsewhere[1] I pointed out that the limits of torsional resilience of soft iron are reached when the obliquity of the external fiber (shear) somewhat exceeds ·003 radians. Regarding the laws of motional annealing, cf. § 101. It follows from the absence of electrical effect, that motional annealing probably presents a pure case of Maxwell's "break up" of configurations of molecules.

93. *"Accommodation" and hysteresis.*—Streintz and Wiedemann's phenomenon "accommodation" admits of representation from a different point of view. Returning to the data of Tables 58 and 59, suppose the experiment so conducted that the twelve twists are immediately consecutive. Suppose, furthermore, that time, m, instead of being reckoned positively onward from the beginning of each of the said twists, were reckoned alternately positive and negative, conformably with the sign of the deformations $(\varphi - \varphi')/\tau$. In this oscillatory march (time as abscissa), since each deformation (ordinate) now begins where the preceding deformation ceased, a continuous series of open cycles is necessarily generated. The positions of these cycles shift at a gradually retarded rate, toward a final very flat cycle, which for the constant values of time and stress is fixed in position and closed.

[1] Am. Jour. Sci. (III), 1887, vol. 34, p. 183.

Cycles here, fixed or not, are expressions of the fact that the "past histories" (in Maxwell's words) of the molecular configurations in the "stress positive" and "stress negative" phase of each cycle are not the same. Shifting is brought about by permanent molecular break up, the amount of which gradually vanishes. In the ultimate and fixed cycle as many configurations are broken during the "stress positive" as are reconstructed in the "stress negative" phase, though they need not be the same configurations.

These considerations suggest a comparison between "accommodation" and Prof. Ewing's[1] "hysteresis," for the purpose of detecting the extent to which like causes are discernible in each phenomenon. Both exhibit a static character. But such comparison would not be fruitful without special and direct experiments; for the *instantaneous* values of stress and of viscosity must be coordinated.

94. *Viscous and electrical behavior compared.*—Having thus discussed one phase of the results in Table 55, I pass to Table 56, which is a digest of the mean values of Table 55, in so far as such a digest can be made. Following the scheme at the end of the preceding paragraph, this comparison should be made after an infinite number of twists have been imparted to each wire. In such a case, however, the original number of unstable configurations has been seriously reduced; so that apart from the annoyance of so time-consuming a method as this, the original properties of the wire are not clearly present in the results. In wires perfectly free from strain, at the outset, the first twist leads to the best indications of the viscous quality. As this condition could not always be guaranteed for the wires of this chapter, I have accepted the mean viscous behavior during the first and second twists as the best available index for comparison. It is sufficient, at least for the present purposes. Again taking the mean for rods of the same nominal temper, I obtain data from which a chart is easily constructed, by representing mean viscous deformation $(\varphi-\varphi')/\tau$, varying with time, for each of the divers degrees of hardness annealed at 25°, 100°, 190°, 360°, 450°, respectively. It so happens that the normal rod, No. 1, is less viscous than the other rods, No. 2 and No. 3, of like temper. Hence the negative numbers in Table 56, which may be eliminated by increasing the other data.

Returning to Table 56, it is clear, inasmuch as viscous deformations are measured differentially, that $(\varphi-\varphi')\tau$ and $s_0-s'_0$ are to be compared. It appears that these quantities increase and decrease together. This is more easily discernible when rods free from strain are compared.[2] The exceptions of Table 56 are due to the fact that the latent strains influence $(\varphi-\varphi')/\tau$ to a relatively much greater extent than $s_0-s'_0$. Again, if $s_{100}-s'_{20}$ and $(\varphi-\varphi')/\tau$ be compared at 100°, Table 56 shows that in this case also the two quantities increase and decrease

[1] Ewing: Phil. Trans. Roy. Soc., II, 1885, p. 523; ibid., II, 1886, p. 361. Prof. Ewing's earlier papers are there given. Cf. also Cohn: Wied. Ann., 1879, vol. 6, p. 403.

[2] Barus and Strouhal: Am. Jour. Sci. (III), 1887, vol. 33, pp. 26, 27.

together. Indeed, the data for 100° are the more uniform, a result due to the fact that at 100° much of the latent torsion is made to vanish because of the annealing effect of 100°. Data of even greater uniformity, cæteris paribus, are to be looked for at higher temperatures.[1]

When a comparison is made between the mean rates at which viscous deformation and resistance increase together with temper, at 20° $((\varphi-\varphi')/\tau$ and $s_0-s'_0)$, and at 100° $((\varphi-\varphi')/\tau$ and $s_{100}-s'_{20})$, it is seen that the mean rate of increase of $(\varphi-\varphi')/\tau$ relatively to $s-s'$ is about ten times as great at 100° as at 20°. This is the phenomenon in virtue of which the viscous behavior of steel, regarded as a test of Maxwell's theory, is almost crucially important. I will endeavor to explain it.

95. *Viscosity at mean atmospheric temperature.*—Dr. Strouhal and I defined the glass-hard state of steel[2] as the stage of temper which undergoes incipient annealing at mean atmospheric temperature. Inasmuch therefore as annealing is demonstrably accompanied by chemical decomposition[3] even at temperatures slightly above mean atmospheric, the molecular configuration of glass-hard steel is always in a state of incipient change.[4] A part, though not the whole, of this change must be of a permanent kind; and I wish to keep in mind that at the temperature of incipient annealing the heat motion is such that broken configurations are sometimes reconstructed.

Inasmuch therefore as glass-hard steel contains more unstable configurations than any other state of temper, at the same temperature, it follows from Maxwell's theory that glass-hard steel, despite extreme hardness, must be the least viscous member of the whole group of tempered and soft steels. This strikingly anomalous prediction of the theory is verified both by the results of Table 56, as well as in earlier work[5] in a way so pronounced as to be irrefragable.

If glass-hard steel is annealed at 100°, the greater number of the unstable configurations are broken up in virtue of the increased molecular vibration at the higher temperature. The cold rod, after annealing, will show increased viscosity in proportion as the number of unstable configurations has decreased. Experiment proves this in a strikingly conclusive way: the increase of viscosity thus produced is marked, being nearly half the difference between the soft and hard states of steel. This, too, is an observation favorable to Maxwell's theory; for

[1] Maxwell's words are (loc. cit.); ". . . but if, on account of rise of temperature . . . the breaking up of the less stable groups is facilitated, the more stable groups may again assert their sway, and tend to restore the body to the shape it had before deformation."

[2] Wied. Ann., vol. 11, 1880, pp. 962, 963.

[3] Barus and Strouhal: Am. Jour. Sci., (III), 1886, p. 276.

[4] During the last three years I have been making experiments on the secular annealing of cold hard steel. The results are very striking, and show that mean atmospheric temperature acting on freshly quenched steel for a period of years produces a diminution of hardness nearly equal to that of 100° C. acting for a period of hours. I examined some twenty rods, the specific resistance of which, within thirty-seven months, has fallen from 46·5 to 42·5, in the case of thin rods (diameter $<0{\cdot}08^{cm}$), and from 43·7 to 35·4 in the case of thicker rods (diameter $0{\cdot}13^{cm}$). Freshly quenched pieces showed resistances as high as 50, nearly.

[5] Barus and Strouhal: Am. Jour. Sci. (III), 1887, vol. 23, pp. 25, 26.

if there be configurations with an inherent tendency to collapse at ordinary temperatures, but a small fraction of them will survive at 100°. Moreover the configurations broken up cannot be reconstructed without expenditure of fresh energy (quenching). Since no such energy is ordinarily available, the viscous properties of the annealed rod are of a permanent kind.

Again, if glass-hard steel (or steel annealed at 100°) be softened by annealing at 200°, a greater number of unstable groups will be broken up than in the foregoing case. The viscosity of the cold rod must therefore be considerably greater than that of the hard rod. Experiment proves the viscous increase to be about two-thirds of the whole viscous difference between hard and soft steel. Chemical analysis gives evidence of the occurrence of decomposition; and inasmuch as the unstable groups are permanently broken, the annealed rod shows determinate viscous properties.

If glass-hard steel be annealed at 300°, 400°, 500°, etc., effects of the same nature as those just discussed, but differing from them in the degree of thorough removal of the unstable configurations, will result.

The phenomenon as a whole must be considered continuous, both as regards temperature and time. In proportion as temperature is higher, however, Maxwell's theory predicts that the effects of the same increment of the temperature of annealing, will produce increments of viscosity successively diminishing at a very rapid rate. Supposing molecular configurations originally present in all states of stability, it follows at once that the groups which retain this quality after annealing must very soon vanish when the temperature of annealing is increased. The data prove this in a convincing way: Rods annealed at 300°, 400°, 500°, ... 1000°, show about the same viscous behavior (relatively speaking), notwithstanding the fact that chemical analysis proves that the decomposition incident to the successive application of these temperatures on glass-hard steel continues steadily to increase.[1] Indeed, chemical decomposition above 300° is more marked than below 300°; yet its bearing on Maxwell's theory is now without interest, because in none of the high annealed rods do configurations unstable at mean atmosperic temperature survive after annealing.

96. *Viscosity at* 100°.—Having analyzed the phenomena at mean atmospheric temperature, I come next to consider the conditions of mean relative viscosity at 100°. The glass-hard state must here be withdrawn for consideration in § 98; because such a rod would undergo annealing during the viscous measurements at 100°.

Hard steel annealed at 100° bears the same relations to 100° that glass-hard steel does to mean atmospheric temperature. Hence the reasoning of the preceding paragraph, mutatis mutandis, applies at once. It is merely necessary to bear in mind that 100° is now the temperature of incipient annealing, and that therefore the temperatures

[1] Am. Jour. Sci., 1886, vol. 32, pp. 277, 282.

which produce corresponding viscous effects are proportionately higher. Rods annealed at 200° now occupy about the same relative position that rods annealed at 100° did in § 95; rods annealed at 300° the same relative position as rods annealed at 200°, etc. Moreover, for equal increments of the temperature of annealing, the increment of viscosity shown by the rod at 100° diminishes rapidly as temperature increases, etc.

In one respect the present results differ from the above: the phenomena are here spread out over a scale (roughly estimated) about ten times as large. This means, following Maxwell's theory, that at 100° the number of unstable molecular configurations is relatively much larger than at mean atmospheric temperature. The reasons, though not far to seek, are exceedingly significant. In hard steel, at 100° two causes of molecular instability produce superposed effects. The first is the chemical or carbon instability already discussed; the second cause is purely thermal. Cf. § 102.

The explanation of the diagram for 100° is now clear. Viscous deformation is marked in all the rods examined from annealed at 500° to annealed at 100°; but the deformability increases at a rapid pace in proportion as we pass from softer to harder steel, because in such a march the carburation instability, superimposed upon the thermal instability, increases rapidly. Molecular configurations on the verge of instability are encountered in continually increasing numbers.

97. *Viscosity at higher temperatures.*—The line of argument followed out for 100° applies, mutatis mutandis, at 200°. Results of this kind I published elsewhere.[1] The character of the evidence bearing on all the points in question is here even more pronounced and conclusive. Steel annealed at 200° is in a state of incipient annealing at 200°. Thermal and carburation instabilities of high degree being superposed, the effects are correspondingly large.

Finally above 300° the molecular instability is largely thermal. The behavior of hard steel therefore approaches that of other metals more nearly.[2] The effect of the carburation instability ceases to predominate, and finally vanishes altogether in proportion as the march is made from lower to higher temperatures of annealing.

98. *Annealing and viscous deformations superposed.*—I have finally to touch upon the series of phenomena in which pronounced annealing occurs simultaneously with pronounced external viscous deformation. If, for instance, a glass-hard rod is twisted and then suddenly heated to 100°, the rod is both annealed and suffers deformation in virtue of the applied twist at the given temperature. Conformably with the excessively greater amount of molecular instability which characterizes these experiments, the observed viscous deformation must be proportionately large. This prediction of Maxwell's theory is fully verified by experiment. In the case of the twisted rod postulated the motion

[1] Am. Jour. Sci.: (III), 1887, vol. 34, pp. 14–16.
[2] Cf., Chapter I.

of the image across the field of the telescope is so rapid that Gauss's method of angular measurement is no longer satisfactorily available. I may say without exaggeration that during the small interval of time within which appreciable annealing occurs, a glass-hard steel rod suddenly heated to 300° is a viscous fluid. I have shown[1] that if a glass-hard and a soft rod (cæteris paribus) be identically twisted and heated to 350° the former will have lost all its strain, whereas in the soft rod only about $\frac{1}{3}$ will have vanished. Cf. § 60.

Advantage may be taken of two simultaneous causes of molecular instability in other and purely mechanical ways. Thus molecular instability is produced by drawing soft steel wire through a draw-plate, and the instability increases enormously with the intensity of the strain. Experiments which I made in some number by countertwisting soft and hard drawn steel wire at 30° and at 100° showed results quite comparable in striking interest with the behavior of tempered steel. The character of both phenomena is the same, so that as far as viscous comparisons go the drawn strain replaces the temper strain perfectly. Cf. § 102.

99. *Viscosity and stress.*—Viscosity in the above pages has been considered apart from the stress-intensity under which the deformation takes place. This is liable to lead to confusion, unless the stress relative to which the constants of viscosity are defined be kept clearly in mind, or unless the terms viscosity be applied to solids in the restricted sense of "elastische Nachwirkung." Thus if a glass-hard and a soft steel rod be subjected alike and at ordinary temperature to torsional stress of continually increasing magnitude, a stress value will be reached for which the viscosity of the hard rod will equal, and eventually overtake, the viscosity of the soft rod. I was able to exhibit this phenomenon in even a more striking way at 190°, finding that for rates of twist less than $\tau=3°$ steel rod (radius $=0{\cdot}041^{cm}$) is much less viscous and more susceptible to the influence of temperature in proportion as it permanently harder;[2] whereas for rates of twist greater than $\tau=6°$ steel cæteris paribus is less viscous and more susceptible to the influence of temperature in proportion as it is softer.

Here I may profitably advert to certain considerations postulated in the earlier chapter[3] relative to the association of hardness with resistance against ∞-forces acting through zero-time, and the association of viscosity with resistance against zero-forces acting through ∞-time, all magnitudes being regarded from a relative point of view. "We may reasonably conceive," is there further stated, "that in case of viscous motion the molecules slide into each other, or even partially through each other, so that the molecular configuration is being continually reconstructed; whereas in the other case (hardness) the molecules are

[1] Am. Jour. Sci. (III), 1887, vol. 34, pp. 4, 5. Experiments made by annealing twisted systems.

[2] At 190°, being of course the maximum hardness admissible.

[3] Am. Jour. Sci. (III), 1887, vol. 33, p. 28; or above chapter I.

urged over and across each other . . . ." In the ordinary case of scratching the action is usually accompanied by physical discontinuity of the parts tangentially strained.

Viscosity and hardness may be clearly distinguished by an experimental method as follows: Imagine a round harder body sliding tangentially along the plane of a softer body without rotation. Let there be sufficient pressure on the round body normal to the plane of the softer body to insure cohesion at the point of contact. Then the softer body is subjected to a *simple shear* along the line of scratching. If the motion is sufficiently slow, the soft body may yield viscously and there will be no scratching. If the motion is sufficiently fast, the soft body will generally be scratched. Hence hardness and viscosity are each resistances against simple shearing, according as the tangential motion of the hard body is sufficiently rapid or sufficiently slow, respectively. The conditions for the occurrence of quiescent friction may therefore be stated thus: if there is sufficient time given for the molecules of two bodies to react on each other at the point of contact, the friction is quiescent. If the time be insufficient, friction is kinetic.

The intensity of stress by which the above deformations are evoked was nearly constant and equal to 0·5 kg. on centimeter of arm. This couple, when applied to the given steel rods (radius 0·041^{cm}), is admirably adapted for the exhibition of the nearly *pure* viscous phenomenon, the "Nachwirkung" of Weber and Kohlrausch.

It is just here that certain cardinal distinctions must be made. According to Maxwell's views, viscosity is the same phenomenon in liquids and in solids, and the molecular mechanism by which it manifests itself quite the same in both cases. There is nothing in the theory to induce the reader to limit viscosity in solids to certain special changes of configuration. In solids at high temperatures, and of course in viscous fluids, there is indeed no need of distinction, and viscosity appears as the one property into which the other configuration properties of solid matter eventually merge. In solids at low temperatures, on the other hand, the case is much more complex; and whereas viscosity ("Nachwirkung") still appears as a property common to solids, whether soft or hard, plastic or brittle, these ulterior distinctions, softness, hardness, plasticity (permanent set), brittleness, etc , separate solids by very broad lines. Hence it is improbable that the whole mechanism in virtue of which viscous deformations are possible in viscous fluids is fully of the same nature as that by which viscous motion takes place in solids at ordinary temperatures. Viscosity in liquids is the mean result of divers superposed phenomena, the occurrence of any one of which in a solid would give rise to some special physical property of that solid. From this point of view, since viscosity is independent of the other physical properties above enumerated, and since viscosity (Nachwirkung) is common to solids without exception, I have ventured to refer it to such action between contiguous molecules as involves the

least amount of free motion. Viscosity in solids is the result of changes of configuration resulting from localized thermal agitation, and often superinduced by the atomic attraction of contiguous configurations in the manner explained by the Clausius-Maxwell principle.[1]

This premised, further distinctions may be made. Questions arise as to whether such action can be indefinitely repeated without rupture, as in plastic solids, or in viscous fluids; or whether it can not be indefinitely repeated, as in brittle solids, etc. The indefinite repetition of the phenomenon is equivalent to a passage of molecules over or across each other, the phraseology above used in reference to hardness.[2] § 103.

The ideas underlying this paragraph may be summarized thus: In passing from the fluid to the solid state of matter the residual or extramolecular affinities acquire an increased importance relatively to the intermolecular affinities. At the same time the conditions of action have gradually become exceedingly unfavorable. In a liquid under impressed favorable conditions chemical reaction between molecules is demonstrable (electrolysis). In a solid under impressed favorable conditions (strain of dilatation) it is also demonstrable, for instance, in the marked secular annealing of glass-hard steel. It is not necessary for the manifestation of viscosity that the integrity of the molecule be actually invaded; but as the action intensifies one may pass continuously from Maxwell's into Clausius's hypothesis without being able to define the line of transition, at least from the character of the viscous phenomena.

100. *The phenomenon of glass-hardness.*—The observations made in the above paragraphs relative to the visible viscous subsidence of a mechanical strain imposed on a steel rod apply for the complete explanation of the phenomenon of temper. With this purpose in view, it is merely necessary to conceive of hardening or quenching (sudden cooling of steel) as an operation by which a strain of dilatation is imparted to steel. This strain, once supplied, is locked up in the metal in virtue of viscosity.[3] The strained structure of hard steel is proved by the fact that massive pieces of hard steel often explode spontaneously,[4] and by the tendency to rupture during quenching exhibited by the metal. The temper strain may be studied optically and in other ways, in glass, and at low temperatures, even in resin.[5]

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[1] "Betrachten wir ferner das Verhalten der Gesammtmolecüle unter einander, so glaube ich dass es auch hier zuweilen geschicht, dass das positive Theilmolecül eines Gesammtmolecüls zu dem negativen eines anderen in eine günstigre Lage kommt, als jedes dieser beiden Theilmolecüle in Augenblicke gerade zu dem anderen Theilmolecül seines eigenen Gesammtmolecüls hat, etc." Mech. Wärmeth., 1879, vol. 2, 2 Aufl, p. 163. Again, following Maxwell: ". . . . Thus we may suppose that in a certain number of groups the ordinary agitation of the molecules is liable to accumulate so much that every now and then the configuration of one of the groups breaks up, and this whether it is in a state of strain or not. . . . ."

"But if a solid also contains . . . . groups of the first kind which break up of themselves . . . ." Maxwell, loc. cit.

[2] Cf. Am. Jour. Sci. (III), 1887, vol. 34, pp. 1, 18.

[3] Cf. Bull. U. S. Geol. Survey, No. 14, 1885, p. 88.

[4] Batchelder: Jour. Franklin Inst. (III), 1844, vol. 8, p. 133.

[5] Marangoni: N. Cim. (III), vol. 5, 1879, p. 116 (Rupert's drops of resin); De Luynes: Phil. Mag. (IV), vol. 45, 1873, p. 464 (Rupert's drops of glass); B. and S.: loc. cit.

Reckoned from the observed volume increase[1] due to quenching, the stress intensity corresponding to the observed strain may be estimated at 10^{10} dynes per square cm. in steel and 10^9 dynes per square cm. in glass. It is thus of the order of the respective tenacities of steel and of glass.

In view of the fact that the viscosity of glass-hard steel is not above that of glass,[2] exceptionally great strain intensity, though impartible, would not be permanently retained. Hence the secular changes of glass-hard steel. Cf. § 95, foot-note. At this point the function of carbon appears. Sudden cooling induces carbon and iron to remain in the combined state in a way favorable to the dilatation in question. Throughout the process of cooling, carbon and iron, at any place within the metal, are united in conformity with the given degree of carburation and with the strain there experienced. In the cold metal, at the given place, strain is to a certain extent permanent and independent of the surrounding medium of steel.[3] Hence if by gradual *secular* annealing of massive glass hard steel a sufficient number of carbon configurations are broken, stress may increase to an intensity sufficient to rupture the metal explosively.

In our earlier papers on this subject Dr. Strouhal and I were much puzzled to know whether the temper-strain, and in general the phenomena of annealing, were to be interpreted physically or chemically; whether annealing was a case of viscous subsidence of a mechanical temper-strain, or a mere case of decomposition of chemical hardness. In the light of the present advanced conceptions this distinction is superfluous. It makes no difference whether the configuration breaks up into parts chemically different, as carbon and iron (say) in steel, or into parts chemically though not structurally identical, as in homogeneous metals. Viscosity is conditioned by the degree of instability. Again, it is clear that the principles which account for the subsidence of the mechanical strain will also account at once for such chemical decomposition as is here in question, the difference in the two cases being vested in mere details of molecular mechanism. §§ 98, 99.

101. *The phenomenon of annealing.*—However complex the nature of the temper-strain in steel may be, the behavior of hard steel, when subjected to the influence of temperature, offers sufficient proof of its occurrence. The laws of annealing hard steel[4] are as follows:

(1) The annealing effect of any temperature acting on glass-hard steel increases gradually at a rate diminishing through infinite time; diminishing very slowly in case of low temperatures ($<100°$); diminishing very rapidly at first and then again slowly at high temperatures ($>200°$);

[1] Am. Jour. Sci. (III), 1886, vol. 31, pp. 441, 443; 1886, vol. 32, p. 191; 1887, vol. 33, p. 33; Bull. U. S. Geol. Survey, No. 27, 1886, pp. 30–50.

[2] Barus and Strouhal: Am. Jour. Sci. (III), 1887, vol. 33, p. 30.

[3] Bull. U. S. Geol. Survey, No. 35, 1886, p. 42. Structure studied by the density method. Shells consecutively removed by galvanic solution.

[4] Phil. Mag., V, 1879, vol. 8, p. 341; Bull. U. S. Geol. Survey, No. 14, 1885, p. 195.

so that the highest and hardest of the states of temper possible at any given temperature is approached asymtotically.

(2) The ultimate annealing effect of any temperature (time $=\infty$) decreases at a retarded rate with temperature, and practically reaches the limit of variation below 350°.

(3) The ultimate annealing effect of any temperature, $t°$, is independent of the possibly preexisting effects of the temperature $t'°$, and is not influenced by subsequent applications of $t'°$, provided $t > t'$. In case of partial annealing at $t°$ (time finite), this law applies more fully as the ultimate effect of $t°$ is more nearly reached.

Postulating the strain discussed in § 100, these laws follow at once from Maxwell's theory, and the explanation (mutatis mutandis) is identical in character with that given in §§ 95 to 98 with reference to the applied torsion strain. Inasmuch as annealing is accompanied by chemical decomposition, the conditions under which the temper-strain is reduced are those of § 98.

The third law of annealing asserts that the heat effect is analytic, but not in the same degree synthetic. The carbon configuration definitely broken up by annealing does not recombine on cooling. In pure metal, and up to a certain limiting (small) stress, configurations broken up by stress may recombine when stress is released or reversed. § 102.

102. *The temper-strain in other substances.*—In certain comparisons between the strain effect exhibited by glass and by steel,[1] we were led both by gravimetric and by polariscopic observations to this distinction: the strain in hard steel is very perceptibly affected by annealing temperature as low as 50°, whereas in the case of quenched glass (Rupert drop), perceptible annealing is incipient only at 200°. The bearing of this result on the present discussion is manifest: the difference of behavior is due to the absence in glass of anything equivalent to the unstable carbon configuration in hard steel. The case of glass is nearly that of soft steel, and the behavior as regards viscosity in these two instances is similar.

Schroeder's[2] important result has relevancy here: in the case of hard drawn wire (Ag,Fe, german silver) minimum viscosity is found associated with maximum susceptibility to temperature. This is the general deduction from steel for varying intensities both of temper strain and of drawn strain.

103. *Extremes of complex and of simple molecular structure.*—Following the suggestion of § 99, it may be inferred that in case of very complex molecular structure instability of configuration will be a more probable occurrence than in the case of simple bodies, cæteris paribus. Complex structured matter may be looked upon as a solidified mixture of homologous chemical series, with a predominating member to give the sub-

[1] Barus and Strouhal: Am. Jour. Sci. (III), 1886, vol. 32, p. 185; vol. 31, 1886, p. 451.

[2] Schroeder: Wied. Ann., 1886, vol. 28, p. 369.

stance character. Comformably with this view, the complex organic solids,[1] like silk and ebonite, show more pronounced viscous deformation than metals or mineral solids. These known facts are thus in general accordance with the present theory. Nor is it remarkable that a complex substance like glass lies somewhere between hard steel and soft steel in the scale of viscosity.

On the other hand, when the atoms of the molecule are all alike, and the structure of the substance is essentially atomic, we meet conditions favorable to permanent set. This is probably the case with many metals.

104. *Thermal stability of magnetic configuration.*—Maxwell's theory lends itself at once to the explanation of superposition (perfect or imperfect) of viscous motions, inasmuch as the interpretation given is independent of the special peculiarity of the strain to be discussed. I will adduce a few magnetic results which bear upon this point.

Considering the permanent effects of temperature on the residual magnetic induction of hard saturated steel, Dr. Strouhal and I[2] found it necessary to discriminate between two species of magnetic loss:

(1) The direct effect, due simply to thermal action on the magnetic configuration;

(2) The indirect effect, due to the action of temperature in producing mechanical annealing.

These two kinds of loss of residual induction often occur together. Considered separately, the latter, cæteris paribus, is very decidedly the greater in amount, and its character is fully typified by the concomitant phenomenon of mechanical annealing. The former (1) is not only much smaller in relative magnitude, but subsides completely within a much smaller interval of time. In general, the occurrence of permanent magnetism in hard steel, in its thermal relations, is subject to nearly the same laws of variation as those adduced, §§ 94 to 98, for ordinary mechanical strains. Instability of the carbon configuration is more seriously detrimental to magnetic permanence than is instability of thermal configuration.

If the unstable carburation configuration be removed by thorough annealing at 100°, then the cold hard resaturated magnet must show exceptionally good magnetic stability as regards the effects of mean atmospheric temperature. If the saturated magnet is again thoroughly annealed at 100°, the exceptionally good magnetic stability in question is even further enhanced, because the magnetic configurations unstable as far as 100° have now also been removed. Experiment shows the second magnetic loss to be relatively small. The rods carry the maximum of permanent hardness and the maximum of permanent magnetization as far as 100°. This process of consecutive annealing is the one

[1] Cf. Kohlrausch: Pogg. Ann., 1866, vol. 128, p. 414, and many others.

[2] Strouhal and Barus: Wied. Ann., 1883, vol. 20, p. 662.

we proposed when the magnets made are to withstand the effects of atmospheric temperature, of percussion, and of secular time.[1]

106. *Intensity of magnetic configurations.*—In the above paragraphs I have referred to thermal, carbon, and magnetic configurations, using the adjectives merely to designate the chief cause of the instability under special consideration, whereas the configurations themselves were not necessarily different. In the same way I contrasted thermal and motional instability. Thus a carbon and a thermal configuration may be one and the same grouping, considered from different points of view; so may a thermal and a magnetic configuration. The latter phrase is used advisedly, and the rod showing residual magnetic induction supposed to consist of configurations of all degrees of magnetic stability, as well as in all degrees of magnetic intensity. Stability and intensity are the qualities which in the present case correspond to stability and strain, respectively, in the above configurations.

Magnetic stability decreases from hard to soft steel and from soft steel to soft iron, following therefore the inverse order of viscosity; and its character, too, is different from viscosity, the tendency being toward sudden magnetic changes, even when the cause of such change is superinduced by heat. Cf. §104.

The mean magnetic intensity of the configuration must depend on the dimensions of the saturated rod. In the normal case of linear rods this magnetic intensity increases from hard to soft steel[2] and from steel to iron. Hence from one point of view carbon configurations interfere with the occurrence of intense magnetic configurations; from the other point of view magnetic intensity increases in the direct order of viscosity or stability of molecular configuration.

Among methods for elucidating the nature of the magnetic configuration, a comparison of the effect of a magnetic field on torsional rigidity is probably best. Extending the classic researches of G. Wiedemann,[3] I commenced a series of such measurements. In the case of a given field of great intensity, if two identical iron wires, respectively magnetic and unmagnetic, be countertwisted to the same maximum shear (obliquity of external fiber, $\omega=0{\cdot}003$, say), then the product of the detorsion due to longitudinal magnetization and the diameter of the wires is constant and proportional to the product of the shear and the magnetic coefficient of rigidity. This remarkable relation implies that the increment of the twisting couple evolved by magnetization increases as the third power of the radius, cæteris paribus.

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[1] Details in Bull. U. S. Geol. Survey, No. 14, chapter VI; or loc. cit.

[2] Strouhal and Barus: Wied. Ann., 1883, vol. 20, p. 621.

[3] Wiedemann: Electricität, 1883, 3d ed. (III), pp. 683 to 698; Barus: Am. Jour. Sci. (III), 1887, vol. 34, p. 180. I believe my researches are the first attempt to interpret these relations quantitatively, and they lead to the law expressed in the text. Other results I will communicate later. (Cf., Chapter IV.)

SUMMARY.

CLAUSIUS'S AND MAXWELL'S THEORIES COMPARED.

107. Summarizing the results of the above paragraphs, I believe the introductory statements to be fully verified. I have shown that the effect of distributing unstable molecular configurations uniformly throughout the substance of a rigid metal like steel is analogous to that of dissolving molecules of acid or of salt in a nonconductor like water. These added molecules are the unstable groups with which Clausius's theory deals. In both cases the effect produced is proportional to the number of unstable groups distributed. If the number be sufficiently increased, the medium will ultimately be a viscous fluid in the one case and an electrolytic conductor in the other. At the outset, pure water typifies the rigid solid.

The applied stress imparts a permanent strain to the solid. Viscous deformation is therefore accompanied by a residual phenomenon, which manifests itself when the applied stress is reversed or removed.[1] In liquids acted on by an electromotive force the analogous reaction is the reciprocating force of galvanic polarization.

Again, Clausius's and Maxwell's theories mutually sustain each other. For if the conception that molecular configurations are present in a solid in all degrees of stability is necessary to explain the behavior of strained solid matter, it follows that configurations of more pronounced instability will be present in electrolytic systems. Conversely, the fact that many solids can be electrolyzed points to the occurrence in these of a very advanced state of molecular instability. To take the concrete example of glass, the same molecular mechanism which at 300° promotes electrolytic conduction, when the solid is influenced by electromotive force, manifests itself at low temperatures as the viscosity of the solid under stress.

[1] Kohlrausch: Pogg. Ann., 1866, vol. 128, p. 419.

CHAPTER IV.

THE EFFECT OF MAGNETIZATION ON THE VISCOSITY AND THE RIGIDITY OF IRON AND OF STEEL.

INTRODUCTION.

108. Mr. Herbert Tomlinson[1] has recently communicated results on the changes of viscosity and of elasticity produced by magnetizing iron. As both classes of data are obtained by the vibration method, it seems not undesirable to attempt to verify them by some static method, and the one described in Chapter II, Fig. 20, is so easily applicable that I have made use of it.

My original purpose was to confine the investigation to measurements of viscosity, inasmuch as such results have an immediate bearing on Chapter III; but I found in the course of the work that the incidental data on the rigidity of iron and steel could be grouped together. Taken collectively in this way, they led to inferences which the well-known and comprehensive researches of Wiedemann[2] fail to point out. I refer to the effect of magnetization on the rigidity of iron and steel as modified by the dimensions of the metal temporarily strained. In this respect, too, the results are supplementary to Chapter III, for the chapter virtually applies a magnetic method to study the character of a strained atomic configuration. I shall show that the increment of rigidity due to magnetization increases at an accelerated rate as the soft, temporarily twisted wire becomes more nearly filimentary. Now, since the said increment is *independent* of the sign of the current in the helix, and, moreover, increases with the current intensity in a way which I have found not seriously irregular, it seems profitable to attempt to utilize this principle for the construction of electric dynamometers.[3] This is the point of view from which much of the present chapter has been written. It also contains a series of results on the rigidity of magnetized steel temporarily strained and varying in temper from extreme hard to extreme soft.

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[1] Tomlinson: Beiblätter No. 3, 1887, p. 176; original in Proc. Roy. Soc., 1886, vol. 40, p. 447, is unfortunately not at my disposal.

[2] Wiedemann: Pogg. Ann., 1858, vol. 103, p. 571; ibid., 1859, vol. 106, p. 161; Galvanismus, 3d ed. (III), 1883, p. 683–698. Less closely allied researches on the effect of magnetization on torsion, etc., are due to Matteuci: C. R., 1847, p. 301, and to Wertheim: C. R., 1852, vol. 35, p. 702, as well as to Wiedemann (loc. cit.), and more lately to Ewing (Proc. Roy. Soc., 1883, No. 228, p. 117).

[3] Cf. p. 177.

VISCOSITY OF MAGNETIZED STEEL AND IRON.

109. *Apparatus and method.*—To make the present measurements it is merely necessary to replace, by an appropriate helix, the heating apparatus described in Chapter II. The two wires to be examined are in the same vertical line, separated by a rigid piece of brass which carries the index mirror. Any desirable rate of twist may be imparted by rotating and then fastening either the upper or the lower end of the system. If both ends be rotated symmetrically in opposite directions, the mirror remains stationary. In this way any reasonable amount of twist may be stored without moving the image of the scale out of the field of the telescope.

The wires of the system were identical as regards length, diameter, and composition; and the arrangement adopted was such that at any time the upper wire might be under the influence of a powerful magnetic field. The helix used for this purpose consisted of ten layers of 230 turns each, doubly wound in such a way as to form two independent partial helices of 115 turns of wire for each layer. For the same current the fields of these partial helices were identical; by connecting them, either differentially or in series, the field obtained was either zero or the maximum for the given conditions. The length of the helix being 20^{cm}, its internal radius $1{\cdot}25^{cm}$, its external radius $2{\cdot}07^{cm}$, the magnetizing force at the center proves to be 1425 c. g. s. units of intensity per c. g. s. unit of current or 142·5 ($g^{\frac{1}{2}}/c^{\frac{1}{2}}s$) per ampère. Current was obtained from five flat Grove cells, in mean intensity of about one Ampère. Hence the iron wires were probably not far from saturation, supposing that a field of 140 c. g. s. units of intensity is sufficient to magnetically saturate soft iron.

Without elaborate precautions it is impossible to produce a strong field for great lengths of time in this way, without perceptibly heating the helix. This introduces a serious error, for the viscous detorsion actually observed is due to a heat effect superposed on a magnetic effect, both of which are of the same order of minuteness. By using a helix like the one above, in which the current may be passed through the two partial coils in the same or in opposite directions, the full heat effect may be observed either with or without the magnetic effect. For when the coils are joined differentially the field produced is zero, whereas the heat generated in the helix is, cæteris paribus, not changed in amount. This is the way in which I endeavored to eliminate the temperature discrepancy.

It is difficult to find two wires which are absolutely identical; after softening some parts of a wire yield more easily to stress than other parts. It is not until the less rigid parts have been stiffened by receiving permanent set that the rate of twist temporarily stored is the same throughout the length of the wire. All viscous motion which is due to differences in the mechanical or chemical properties of the metals may be detected by allowing a series of observations for open circuit to alternate with similar series for closed circuit.

110. *Results.*—Table 60 contains results as obtained with steel annealed about midway between soft and hard. Here θ is the temperature of the helix and of the upper wire, θ' the temperature of the lower wire, l and ρ respectively the length and radius of each. $(\varphi-\varphi')$ denotes the amount of viscous detorsion per centimeter per wire (i. e., per two centimeters of the system), in radians at the time h in hours. In other words $(\varphi-\varphi')$ has the meaning already fully defined in § 56, except that in the present case the essential difference of φ and φ' is to be induced by magnetization. τ denotes the rate of twist in degrees imparted to the system at the unmagnetic end. If $(\varphi-\varphi')$ increases or decreases according as the sign of the twist is positive or negative, then the unmagnetic wire is of greater viscosity than the magnetic wire; and vice versa. Intervals of observation corresponding to open and closed circuits are appropriately indicated. Whenever the circuit is closed differentially in the helix so as to produce a zero magnetic field, this is also stated. $(\varphi-\varphi')$ is correct to three or four units of the last place.

The three parts of this table show that the viscous differences in question are invariably minute. In part first the originally less viscous lower wire becomes perceptibly more viscous when the circuit is closed around the upper wire. The apparent effect of magnetization is here a decrement of viscosity. In the second part the wires for open circuit are equally viscous; for closed circuit the results are the same as in part first both as regards sign and amount. The apparent effect of magnetization is again a decrement of viscosity. Part third, finally, contains the interpretation of these results: it is seen that results of the same magnitude and character are obtained when the magnetic field due to the helix (other effects remaining the same) is zero. The viscous effects here observed are therefore wholly due to temperature, and it may be safely inferred that magnetized and unmagnetized steel differ in viscosity by an amount less than is produced by 5° difference of temperature.

TABLE 60.

Viscosity of steel, annealed 360°. $l=30^{cm}$; $2\rho=0{\cdot}083^{cm}$; $\theta'=20°$.

h	$(\phi-\phi')\times 10^6$	θ	Remarks.	h	$(\phi-\phi')\times 10^6$	θ	Remarks.
hours.	radians.	° C.		hours.	radians.	° C.	
0·00	— 0	..		1·10	—21	..	
2·67	—33	..	Circuit open. $\tau=+6°$.	1·12	—10	19	
4·33	—35	..		1·40	—23	23	
24·00	—50	..		1·67	—23	25	Circuit closed. $\tau=-6°$.
25·42	—50	..		1·92	—34	27	
25·60	—35	..		2·08	—38	..	
25·67	—15	..	Circuit closed. $\tau=+6°$.	2·40	—42	28	
25·83	— 3	..		2.80	—49	29	
26·00	+ 7	..		0·00	— 0	17	
26·25	+10	..		0·18	—30	21	Circuit closed; field zero. $\tau=-6°$.
26·75	+15	..		0·43	—46	26	
0·00	— 0	..	Circuit open. $\tau=-6°$.	0·60	—53	28	
1·00	— 3	..		0·85	—58	29	
				1·00	—63	30	
				4·52	— 7	..	Circuit open. $\tau=-6°$.
				23·25	—17	..	

Inasmuch as the steel wire of Table 60 is permanently a magnet after the first induction, sharper results may be looked for in experimenting with iron. As regards viscosity, iron is less susceptible to the influence of temperature than annealed steel or even soft steel; it is more magnetically permeable and it loses its magnetization completely when slightly jarred in a zero field.

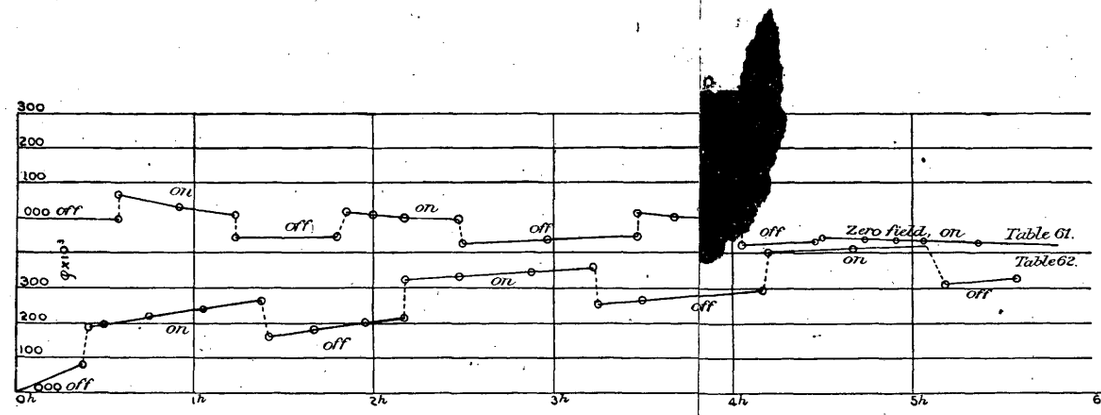
111. Table 61 contains results for soft iron, the arrangement being identical in plan with Table 60.

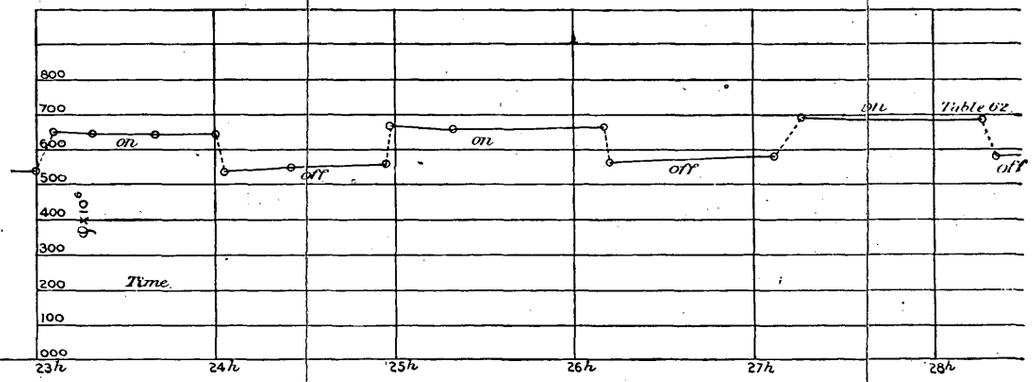
In the first part of Table 61 $(\varphi - \varphi')$ increases whenever both wires are unmagnetic and decreases at a gradually diminishing rate whenever the upper wire is magnetic. The apparent effect of magnetization is therefore a diminution of viscosity, a result in accordance with the inferences from Table 60. To interpret this apparent result the second part of Table 61 is available. It shows that when the current is passed through the helix differentially, $(\varphi - \varphi')$ increases at the same rate as before, notwithstanding the fact that the field is now zero. Hence the amount of viscous detorsion observed is a temperature discrepancy and no viscous effect due to magnetization is discernible.

TABLE 61.

Viscosity of magnetized iron. Rods Nos. 7, 8, annealed soft. $l = 30^{cm}$; $2\rho = 0{\cdot}110^{cm}$; $\theta' = 20°$; $\tau = -3{\cdot}6°$.

h	$(\phi - \phi') \times 10^6$	θ	Circuit.	h	$(\phi - \phi') \times 10^6$	θ	Circuit.
0·00	0	..	Open.	3·48	+15	26	Closed.
0·57	− 3	..		3·68	+ 4	30	
0·58	+65	..	Closed.	4·05	− 5	34	
0·92	30	33		4·05	−72	34	Open.
1·23	8	34		4·47	−62	29	
1·23	−58	33	Open.	4·52	−55	..	Closed differentially; magnetic field zero.
1·80	−53	28		4·75	−65	32	
1·85	+18	..		4·92	−63	33	
2·00	+10	29	Closed.	5·07	−70	34	
2·17	0	33		5·38	−73	..	
2·48	− 3	34		5·56	−73	..	Open.
2·50	−71	34	Open.	22·30	−50	..	
2·97	−60	29					
3·47	−52	26					





EFFECT OF MAGNETIZATION ON THE VISCOSITY AND RIGIDITY OF IRON, ϕ VARYING WITH TIME.

In Table 62, I give similar results with a thinner wire.

TABLE 62.

Viscosity of magnetized iron. $l=30^{cm}$; $2\rho=0{\cdot}083^{cm}$; $\theta'=20°$; $\tau=-3°$.

h	$(\phi-\phi')\times 10^6$	θ	Circuit.	h	$(\phi-\phi')\times 10^6$	θ	Circuit.
0·00	0	..	Open.	5·13	310	..	Open.
0·38	80	..		5·57	328	25	
0·42	189	21	Closed.	23·00	545	18	
0·50	195	21		23·10	653	..	Closed.
0·75	220	26		23·33	645	..	
1·05	245	29		23·67	643	28	
1·38	265	30		24·00	645	30	
1·42	161	30	Open.	24·05	538	30	Open.
1·68	183	28		24·42	550	26	
1·95	203	25		24·95	560	23	
2·17	218	23		24·97	670	..	Closed.
2·18	328	24	Closed.	25·33	660	28	
2·48	334	27		26·17	668	32	
2·88	348	30		26·20	561	..	Open.
3·23	360	32		27·12	580	23	
3·25	255	..	Open.	27·27	690	..	Closed.
3·50	270	..		28·28	685	31	
4·17	298	..		28·35	580	..	Open.
4·20	407	..	Closed.	48·00	740	..	
4·67	410	28					
5·08	420	30					

112. In these results the thermal and magnetic effects are superposed on the continuous viscous motion due to inequalities in the wires, the upper wire being more viscous than the lower wire. In every case, however, this normal viscous detorsion is perceptibly retarded whenever the upper wire is magnetized. It follows conformably with the above results that the viscosity of iron is apparently diminished by magnetization; that the amount of this diminution is no larger than is quite in keeping with the heating effects due to the passage of current through the helix. My results therefore fail to give any satisfactory proof that magnetized and unmagnetized iron differ in viscosity by more than the equivalent of one or two degrees centigrade, at ordinary temperatures.

In Pl. V the results of the above Tables 61 and 62 for soft iron are graphically given, time as abscissa and deformation $(\varphi-\varphi')$, as ordinate. The breaks in the curve show the time at which current was made and broken and the $(\varphi-\varphi')$-effect produced. In Table 61 the two wires are originally identical, as is shown by the "off" branches. The effect of magnetization as apparent diminution of viscosity is shown by the slope of the "on" branches; but the effect of the zero field with current "on" at the end of the curve, is the same in sense and approximate magnitude, as the effect of magnetization. Hence the latter is obscure. Similar interpretation is to be made with reference to the curve for Table 62, in which viscous motion preexists. But this motion, due to greater viscosity of the upper wire (in the helix), is invariably retarded by magnetization. In the second part of this figure the preexisting viscous motion is nil, and the effects are therefore clearer.

In the figure for Table 61, note the tendency to concavity upward in the "on" branches. This is also true of the second half of Table 62.

In Table 61, moreover, prolonged magnetic twisting stiffens the wire; hence the tendency of the "off" branches to slope upward toward the end of the curves.

I am far from wishing to assert, however, that Tomlinson's results are temperature phenomena. It is possible that the marked tendency of soft iron to assume permanent set, and consequently the relatively large viscous motion immediately after strain is imparted, makes the vibration method particularly sensitive in registering the viscous effect of magnetizing iron.[1] But by using static methods and observing in the interval of *gradual* or *purely viscous* deformation, no satisfactory evidence of such an effect is discernible. Moreover, magnetization changes the rigidity of iron and therefore necessarily jars a twisted wire. There result such changes of viscosity as are produced by any sudden vibratory disturbance; changes which of course are purely mechanical, but which may obscure the direct result of magnetic induction.

I may state, in concluding, that inasmuch as Weber's theory associates magnetism with certain rotations of molecule, while divers theories mentioned in Chapter III refer viscosity to a similar mechanism, it would be permissible to look for a marked effect of magnetization on viscosity. The vanishing results of the above paragraphs do not sustain such an inference.

RIGIDITY OF MAGNETIZED IRON.

113. *Definitions and results.*—The effect of longitudinal magnetization on iron or steel twisted within the elastic limits is marked detorsion, increasing in amount with the intensity of magnetic field, increasing also with the rate of twist, at a retarded rate in both instances, toward a maximum. If the sense of the magnetization be reversed (i. e., if the helix current be changed in direction) the amount of detorsion is in general unchanged.[2]

In Tables 60 to 62, discontinuous but perfectly regular variations of $(\varphi - \varphi')$ on passing from unmagnetized to magnetized iron or steel are strikingly apparent. Inasmuch, however, as the differences in question are now to be considered in their bearing on rigidity, and not on viscosity, I will replace $(\varphi - \varphi')$ in subsequent tables by φ. Hence φ is the deflection observed at the index placed at the junction of the two identical wires forming the system, per centimeter of length of each, immediately after one only of the wires is magnetized. Thus φ is evoked and vanishes with the magnetization.

Table 63 contains special results for large rates of twist (τ), and has been drawn up to exhibit the independence of φ of any possible irregularities in the position of the helix as well as the change of sign of φ with the sign of τ. The table shows also that the sign of φ is reversed when the helix is passed from the upper to the lower wire. It follows,

[1] See Wiedemann's remarks on the vibration method, Wied. Ann., vi, 1879, p. 485.

[2] The above phenomena have been elaborately discussed by Wiedemann (Galvanismus, pp. 683–698).

therefore, that φ is not influenced by errors which in my apparatus might result from the increase of length due to magnetization. In other words, in the mechanism adopted, magnetic elongation has no rotational effect. l and ρ denote length and radius each in centimetres. τ is in degrees, φ in radians, as above.

TABLE 63.—*Showing the effect of sign of twist and of position of helix.*

Remarks.	No.	τ	$\phi\times10^6$
Helix symmetrically on upper wire	17, 18	+6	−130
	$2\rho=0{\cdot}053^{cm}$	−4·5	+133
	$l=30^{cm}$	+6	−139
Helix symmetrically, on *upper* wire	19, 20	−3·0	+93
" +eccentrically, " "	$2\rho=0{\cdot}083^{cm}$	−3·0	95
" −eccentrically, " "	$l=30^{cm}$	−3·0	93
" +diagonally, " "		−3·0	95
" −diagonally, " "		−3·0	95
" high position, " "		−3·0	94
" low position, " "		−3·0	97
" symmetrically, on *lower* wire		−3·0	88
Helix symmetrically on upper wire	15, 16	−3·0	+66
	$2\rho=0{\cdot}110^{cm}$	+3·0	−68
	$l=30^{cm}$		

114. These sudden changes in the values of φ due to magnetization are equivalent to an increase of the rigidity[1] of steel, and I shall therefore describe them as such. If T be the torsional rigidity of a wire, i. e., the reciprocal of the amount of twist per unit length per unit moment of twisting couple; and if G be the absolute rigidity of the material, i. e., the reciprocal of the amount of shear per unit of shearing force, then

$$G=\frac{2T}{\pi\rho^4}.$$

Now if in the present apparatus τ' be the rate of twist in radians and φ the change of τ' due to magnetization; if moreover G_m and G be the rigidities of the magnetized and unmagnetized wires respectively; then since the radii (ρ) are identical

$$G_m(\tau'-\varphi)=G(\tau'+\varphi),$$

which if φ is small in comparison with τ' may be written

$$\frac{G_m}{G}=1+\frac{2\varphi}{\tau'}=1\times\mu.$$

115. A final datum of interest is the *obliquity*, ω, of the external fiber, or the angle in radians between the axis of the wire and any tangent of the helix into which a straight surface fiber, or generatrix has been twisted. This is approximately

$$\omega=\frac{\pi\rho}{180}\tau,$$

[1] The retrograde motion formerly observed when the upper wire heated to a high temperature (100°—300°), is cooled to the temperature of the lower wire (Am. Jour. Sci. (III), 1887, vol. 34, p. 17), might in like manner be used to compute the variations of rigidity due to temperature. In such a case, however, the variations of temperature from the high value to the low value must be as nearly instantaneous as possible, otherwise elastic and viscous detorsions will be erroneously confounded.

τ being the rate of twist in degrees. This variable ω is intended to have no more than geometrical significance. It is known that on twisting and again untwisting an iron wire beyond the elastic limits, the fibers do not return upon themselves to their original forms and positions, but that they become irregularly sinuous lines. Perard's[1] investigations on this subject are to the point.

TABLE 64.

Showing the effect of magnetization on the rigidity of soft iron. $l = 30^{cm}$.

τ	Nos. 1, 2. $2\rho = 0{\cdot}231$			Nos. 3, 4. $2\rho = 0{\cdot}136$			Nos. 5, 6. $2\rho = 0{\cdot}090$			Nos. 7, 8. $2\rho = 0{\cdot}110$			Nos. 9, 10. $2\rho = 0{\cdot}070$			Nos. 11, 12. $2\rho = 0{\cdot}048$			Nos. 13, 14. $2\rho = 0{\cdot}022$		
	$\omega \times 10^3$	$\phi \times 10^6$	$\mu \times 10^3$	$\omega \times 10^3$	$\phi \times 10^6$	$\mu \times 10^4$	$\omega \times 10^3$	$\phi \times 10^6$	$\mu \times 10^3$	$\omega \times 10^3$	$\phi \times 10^6$	$\mu \times 10^3$	$\omega \times 10^3$	$\phi \times 10^6$	$\mu \times 10^3$	$\omega \times 10^3$	$\phi \times 10^6$	$\mu \times 10^3$	$\omega \times 10^3$	$\phi \times 10^6$	$\mu \times 10^3$
0																					
— 0·43	0·8	12	3·2	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....
— 0·75	1·5	18	2·7	0·9	24	3·7	0.6	42	6·4	0·7	34	5·2	0.5	37	5·6	0·3	60	9.2	....	....	....
— 1·12	2·3	23	2·3	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....
— 1·50	3·1	30	2·3	1·8	43	3·3	1·2	68	5·2	1.4	54	4·1	0·9	82	6·3	0·6	107	8·2	....	....	....
— 2·25				2·7	49	2 5	1·8	75	3.8	2·2	62	3·2	....	....	....	0·9	116	5·9	....	....	....
— 3·00				3·6	51	2·0	2·3	78	3·0	2·9	65	2·5	1·8	95	3·6	1·2	131	5·0	0·6	86	3·3
— 3·75							....	....	....	3·2	66	2·0	....	....	....	....	....	....	....	....	....
— 4·50							3·5	77	2·0				2·7	104	2·7	1·9	148	3·8	....	....	....
— 5·25													3·2	103	2·3	....	....	....	....	....	....
— 6·00																2·5	156	3·0	1·2	166	3·2
— 7·50																3·1	148	2·3	....	....	....
— 9·00																			1·7	205	2·6
—12·50																			2·4	220	2.0
—15·00																			2·9	237	1·8

In Table 64, I have given values for these variables as obtained with soft iron wires of different thicknesses. The intensity of magnetic field has been stated above. Commencing at low values, the rate of twist (τ) is successively increased to the maximum value, which can be imparted, i. e., until, with the occurrence of marked permanent set, the limits of resilience of soft iron are fully reached. After this it was usual to twist the wires further, imparting several thousand degrees of permanent set, whereupon τ was decreased from the maximum to zero. In the table, ω, φ, μ, are means of the values obtained by thus successively increasing and decreasing τ. As before ρ is the radius and l the length of the wires, of which Nos. 1–6 were drawn down from a single piece of wire, Nos. 7–12 from a second piece of wire, Nos. 15–18 from a third piece of wire. A priori, comparison of data is to be made only for wires drawn to different diameters from the same sample of iron;[2] but inasmuch as iron to be ductile must be nearly pure, the results may safely be grouped in a single series. All wires are annealed at red heat in air.

[1] Perard: Revue d'univers. des Mines (2), 1884, vol. 15, p. 346; ibid., 1879.

[2] Imperfections in the wire plate prevented me from drawing all the wires from a single sample of iron.

To this may be added the following data for soft iron filaments drawn from the same wire. ω is the maximum.

TABLE 65.—*Data for filaments of iron.*

No.	2ρ	$\omega\times10^3$	$\phi\times10^6$	$\mu\times10^3$
15, 16	0·032	2·9	212	2·4
17, 18	0·025	2·8	241	2·3

DEDUCTIONS.

116. *Constant fields.*—It has been stated that the wires to which Table 64 refers were all soft. Their diameters vary over the very large interval 0·022^{cm} to 0·234^{cm}, and the rates of stored twist from 0.4° to 15°. Nevertheless the maximum obliquity, ω, of the external fiber is nearly the same for all the rods. In other words, the limits of torsional resilience of soft iron are reached when the obliquity of the external fiber exceeds 0.0032 radians.[1] After this, iron yields to the torsional couple and the result is indefinite permanent set.

Inasmuch therefore as ω is comprehended within about the same interval (0 to 0·003) for all the wires irrespective of thickness, it is expedient to discuss the variation of φ with reference to ω. Indeed, the diagram of φ as a function of ω shows the characteristic family of curves with remarkable terseness. Figure 25 shows that φ increases with ω at a rate which is greater in proportion as the thickness of the wires, 2ρ, diminishes, and which decreases and finally approaches zero when ω is a maximum. The detorsion due to magnetization ceases to increase when the sections of the rod slide on each other. Indefinite twisting beyond the elastic limits has no effect so long as the wire is not appreciably hardened. But it is the dependence of φ on ρ to which I desire principally to advert. The following series of values obtain for ω=0·003.

TABLE 66.—*Effect of dimensions.*

$2\rho\times10^3$=	234	136	110	90	70	48	22
$\phi\times10^6$=	30	50	65	80	105	150	240

To these data add the values on Table 65.

[1] The literature on torsional strains is so voluminous that I can scarcely suppose similar simplifying observations have not been made. But I have found none. Permanent and temporary torsions have been compared with regard to their relations to stress, to the mechanical condition of the material carrying stress, etc., by Wertheim (Comptes Rendus., 1855, vol. 40, p. 411; Ann. de Chim. (3), 1857, vol. 50, p. 195; more elaborately even by Wiedemann (Wied. Ann., 1879, vol. 6, p. 485; ibid., 1879, vol. 7, p. 496; Phil. Mag. (5), 1880, vol. 9, pp. 1, 97, and by Perard (Revue univ. d. Mines, 1879). Reference is also to be made to Tresca (C. R., 1871, vol. 73, p. 1104), to De Saint-Venant, who has investigated the mechanics of ductile substances, and to others. In Wiedemann's last research, correlated values of temporary and permanent torsions are discussed both as they exist immediately after imparting strain and as they exist after an indefinite lapse of time. I have no such comprehensive purpose in view; nevertheless the great convenience of the variable ω is obvious from the way in which I am able to use it.

Recent experiments made with iron wire drawn down finer emphasized the above results. In the case of maximum permanent set the following results were obtained:

TABLE 67.—*Effect of dimensions.*

$2\rho=0{\cdot}018^{cm}$	$10^6\times\phi=280$
$2\rho=0{\cdot}015^{cm}$	$10^6\times\phi=370$

The importance of these results induced me to exhibit their mean variation in a chart. This is done in Fig. 25. In this plate the curves concave to X denote the variation of $\varphi\times10^6$ and $\omega\times10^3$. They make up a family. The other line of the contour of an equilateral hyperbola denotes the variations of $\varphi\times10^6$ with $2\rho\times10$.

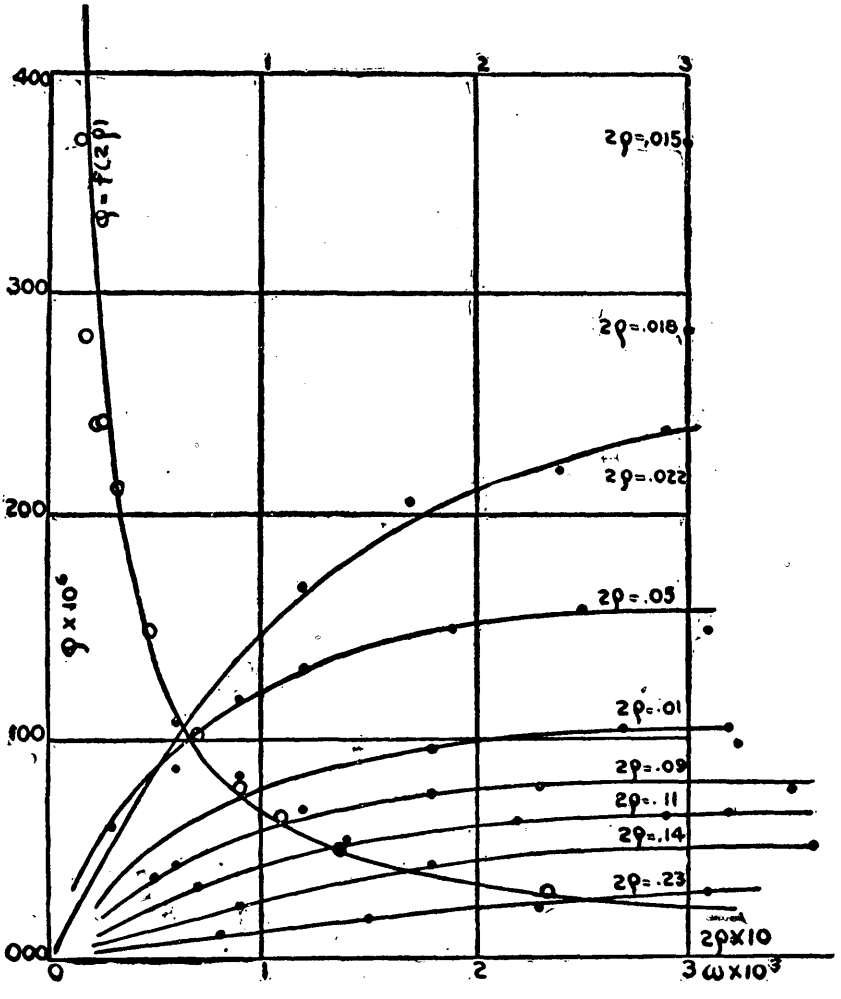


FIG. 25.—Increments of rigidity varying with obliquity of fibers. Increments of rigidity varying with diameter of wire.

The great interest which attaches to these results is the fact that the observed increase of rigidity, as registered by φ, takes place at an exceedingly rapid rate when the diameter of the wires is small. I was not able to make 2ρ smaller than $0{\cdot}015^{cm}$ with the means at my disposal; but copper and platinum wire may be had commercially in diameter as low as $2\rho=0\ 008^{cm}$ or even $2\rho=0{\cdot}004^{cm}$, and no doubt, by skilled hands, iron wire can be much further diminished in diameter than I have drawn it. Here therefore is a method of investigating the magnetics of *filamentary* wires; a method which increases in sensitiveness in proportion as thickness of wire nearly vanishes. From the importance of the properties of filamentary wires this method therefore deserves careful attention.

117. With copper no results were obtained even for $2\rho=0{\cdot}008^{cm}$. φ is zero in all fields.

118. To return again to Fig. 25, I will note that $\mu=\frac{2\varphi}{\tau'}$, τ' being in radians. Now, $\omega=\rho\tau'$, and therefore

$$\mu=\frac{2\varphi\rho}{\omega}=\frac{\varphi\cdot 2\rho}{\omega}.$$

Hence, since experiment shows the approximate constancy of ω and of μ, it follows that $\varphi=f(2\rho)$ is to the same degree an equilateral hyperbola. The mean value of μ taken from Table 64 is, $\mu=2{\cdot}1/10^3$; and the mean value of ω corresponding, $\omega=3\ 2/10^3$. Hence

$$\varphi=\frac{6{\cdot}7}{10^6}\frac{1}{2\rho},$$

and this is the hyperbola with reference to which the points of Pl. V are coordinated.

Again put $\chi=\frac{1}{\varphi}$. Then χ is the *length* of each wire of the system of two wires, which, if the upper wire only be magnetized to saturation, will show a deflection $=1$ radian. It appears therefore that $\chi=\frac{2\rho}{\mu\omega}$, an exceedingly remarkable result.

Hence in the case of a torsion system of two wires, of the same length and radius, but respectively magnetic and unmagnetic, if the twist stored produces maximum obliquity ($\omega\times10^3=3{\cdot}2$) of the external fiber, then the lengths of wire corresponding to a given fixed deflection at the point of junction will vary linearly with the thickness of the wires. These results deserve special tabulation.

TABLE 68.—*Effect of dimensions.*

Mean $\omega=3{\cdot}2/10^3$; mean $\mu=2{\cdot}1/10^3$.

$2\rho\times10$	$\phi\times10^6$	$\chi/10^3$	$2\rho\times10$	$\phi\times10^6$	$\chi/10^3$	$2\rho\times10$	$\phi\times10^6$	$\chi/10^3$
·234	30	33·3	·090	77	13·0	·022	237	4·2
·136	51	19·6	·070	103	9·7	·018	278	3·6
·110	66	15·1	·048	146	6·8	·015	370	2·7

From this table the proportionality of χ and 2ρ clearly appears, as may be seen at once by graphic construction.

119. Since the couple for a given angle φ will vary as ρ^4/l, where l is the length of the wire, it follows that in case of maximum $\omega \times 10^3 = 3{\cdot}2$, the force with which the wire is twisted in virtue of the occurrence of magnetism increases as the cube of the radius. The couple, in other words, is nearly $(.4E)\frac{\pi\rho^4}{2L}\varphi$. And since $\varphi\rho$ is constant for all values of ρ within the limits of observation, and since E the modulus of elasticity, and L the length of the iron wire are also constant, the couple in question becomes

$$L = .4E\frac{\pi\mu\omega}{4L}\rho^3 = \text{const. } \rho^3, \text{ nearly.}$$

120. There are certain curious considerations to which I can only advert here. Suppose the obliquity of the external fiber, ω, to remain the constant maximum. Suppose also the diameter of the iron filament decreases indefinitely until we have reached the diameter of a single molecule. In this case the twisted fiber would be an elementary helix, the spires of which are a continuous succession of oblique molecules. The axis of each molecule is inclined ω, by agreement.

Now it is supposable that if an elementary helix of this kind is placed in a magnetic field of great intensity, that the effect would be (proper adjustment presupposed) to rotate the molecules parallel to the axis of the filament; in other words to reduce ω to zero. In a given system of two countertwisted filaments, such as have been operated on in the above work, the elementary result would be double the rate of twist τ' of the lower unmagnetic filament, and reduce τ' of the upper magnetic filament to zero; thus retaining the same mean rate of twist. More simply the numerical result would be $\tau' = \varphi$. But this ultimate effect runs directly against the datum of experiment: for Table 64 shows that throughout a considerable range of diameter ($2\rho = {\cdot}022^{\text{cm}}$ to $2\rho = {\cdot}234^{\text{cm}}$), the final value of $\varphi/\tau' = \frac{\mu}{2}$ when $\omega = 0{\cdot}003$, is nearly constant, $\mu/2 = {\cdot}001$ nearly. It is certain that the results show no increase of μ for decreasing ρ.

It is difficult without further experiments to reconcile these conflicting deductions. Probably some light may be obtained by increasing the intensity of field. But (constancy of ω presupposed) the hypothetical result $\tau' = \varphi$ for $2\rho =$ molecular diameter, seems to me an inference of Weber's theory of magnetism, whereas $\frac{\varphi}{\tau} = \frac{\mu}{2}$ const. and independent of diameter conflicts with it.

121. *Intermittent fields.*—The accelerated rate of increase of φ with ρ as exhibited by these data has suggested the remarks already made, viz: since φ is independent of the sign of the current in the helix it is probable that ρ may be taken sufficiently small to render these phe-

nomena practically available for the measurement of induced or alternating currents. The chief difficulties encountered are the phenomena of "accommodation" already very carefully discussed in Chapter III. Twisting by magnetic means has of course the same effect on the unstable configuration, as does twisting by ordinary mechanical couples. To return, however, to the subject in hand, the actual deflection increases with the length of the iron wire. It would be expedient therefore to suspend a light mirror bifilarly from very long filaments of iron and of glass or metal, surrounded by a long cylindrical helix.[1] The rate of twist temporarily stored in the soft iron filament is the maximum within the elastic limits. I give the following example of deflections produced by the current of an ordinary induction coil, acting on a relatively thick and short iron wire.[2] The currents are alternately off and on. The two series were made about an hour apart.

TABLE 69.—*Intermittent currents.*

Nos. 5, 6; $2\rho = 0{\cdot}090^{cm}$; $l = 30^{cm}$; deflection =

30	75	40	80	40	83	40	$\phi = 20 \times 10^{-6}$
52	85	40	75	35	73	35	$\phi = 19 \times 10^{-6}$

In so thick a wire the heat effect of alternating magnetizations soon becomes apparent. The result is temporary diminution of the rigidity of the wire. Much better results are already apparent when the above thin wires are used. For instance (same coil),

TABLE 70.—*Intermittent currents.*

Nos. 15, 16; $2\rho = 0{\cdot}034^{cm}$; $l = 30^{cm}$; deflections =

50	180	50	200	70	190	70;	$\phi = 65 \times 10^{-6}$
70	200	60	200	65	200	60;	$\phi = 69 \times 10^{-6}$

Further discussion of these results (the errors of which are largely due to the apparatus) is not now expedient.

The values of φ in Table 64 are means of five observations with the circuit alternately made and broken. A small amount of permanent detorsion is always imparted to the wire after the magnetism has disappeared, as Wiedemann[3] first pointed out. The following example may be given from very many results of my own. In the case of Nos. 3, 4, the scale readings for currents alternately off and on, were as follows:

[1] It is needless to state that I am well aware of the difficulties here in the way, difficulties which Wiedemann has so carefully digested. In my apparatus, however, all pivot and pulley rotations are avoided. I have also repeated the results relating to fine wires at length with very light mirrors, by which no errors due to an asymmetrical mirror adjustments are encountered. The very small couples which come into play in operating with filamentary wires often make damping by submerged vanes objectionable. I have therefore often discarded them.

[2] The wire plates at my disposal were such that $2\rho = 0{\cdot}02^{cm}$ was the smallest diameter obtainable. But the above series of results are fully sufficient and sufficiently in accordance to justify the inferences drawn.

[3] Wiedemann: Galvanismus, p. 689.

TABLE 71.—*Experiments in cycles.*

$\tau=$	0·7°	1·5°	2·3°	2·2°	1·5°	0·7°
(off)	130	110	90	80	50	60
(on)	150	200	185	180	128	100
(off)	80	85	70	70	38	58
(on)	160	188	180	178	130	100
(off)	80	82	70	72	40	60

This is very perceptible in the first part of the measurements where τ or ω increase from zero to the maximum; it generally becomes smaller in amount when ω decreases from the maximum to zero. The method of varying ω from zero to the high value and then from the high value to zero leads to a cycle of results. In the case of Nos. 7, 8, for instance the data ran thus:

TABLE 72.—*Experiments in cycles.*

$\omega\times10^3=$	0·0	0·7	1·4	2·2	2 9	3·2	ω increasing.
$\phi\times10^6=$	5	40	57	63	66	65	
$\omega\times10^3$	3·2	2·9	2·2	1·4	0·7	0·0	ω decreasing.
$\phi\times10^6$	67	64	61	51	28	3	

These observations might perhaps be grouped with Mr. Ewing's[1] "hysteresis." But in some of the many other like experiments which I made there is an element of vagueness.

Table 64 shows finally that the influence of magnetism on the rigidity of soft iron is greatest when ω is zero. It is obvious since φ increases at a retarded rate while τ or ω increase uniformly, that μ must continually decrease. The large values of μ differ considerably, being greater as a rule for smaller diameters. The minimum values of μ obtained are of about the same magnitude, $\mu=0{\cdot}0021$. If the conditions of indefinite permanent set are such that the obliquity of the external fiber, attain a fixed value, then these conditions imply that the rigidity of steel is changed by magnetization by a fixed minimum amount.

RIGIDITY OF MAGNETIZED STEEL.

122. In the following Table 73 I give the values of φ which hold for steel for the large rate of twist $\tau=6°$. The wires are carefully annealed as described elsewhere. Diameter $2\rho=0{\cdot}083^{cm}$. A soft iron wire for which maximum ω is only 0·0021 is also tested for comparison.

The curious result of this table is this, that during the first phase[2] of annealing, the effect of magnetization on the rigidity of steel is almost nil. This effect becomes of marked importance during the second

[1] Ewing: Trans. Roy. Soc., 1885, vol. 2, p. 523-4.
[2] Am. Jour. Sci. (III), 1886, vol. 31, p. 443.

phase.[1] In other words, if we suppose the wire to pass continuously from hard to soft, *the increase of the magnetic coefficient of rigidity, μ, is particularly pronounced after the variations of the thermo-electric, the galvanic and the viscous properties of steel have practically subsided.* μ therefore shows close affinities to the induced and to the residual magnetization of an iron-carburet. Agreeing with Wiedemann's results, μ is invariably smaller for steel than for iron. In general if μ be regarded in its dependence both on ω and on hardness, it appears that the increment of rigidity of an iron-carburet produced by magnetization is greater in proportion as the metal shows greater tendency to assume permanent set—a result which applies for iron and for steel.

TABLE 73.—*Rigidity of magnetized Stubs's steel.* ω=0·0043.

Temper.	$\phi\times10^6$	$\mu\times10^3$
Glass-hard	3	0·06
Annealed 100°	4	0·08
" 190°	4	0·08
" 360°	22	0·42
" 450°	25	0·48
Soft steel	41	0·78
Soft iron	95	3·63

[1] In a letter to the Am. Jour. Sci. (III), vol. 33, p. 308, Prof. W. F. Barrett has taken exception to certain remarks made by Dr. Strouhal and myself (ibid., p. 35) on phenomena more or less connected with the second phase of annealing. We regret exceedingly to have overlooked Prof. Barrett's papers. Our object, however, was only to give an enumeration of such observations as had occurred to us *incidentally*. Of Gore's discovery we were aware, and the statement is thus made in the text and in U. S. Geol. Survey, Bull. 14. Owing to a misconstruction of the foot-note in the Journal (p. 35, ‡) "ibid." happens to refer to "Wied. Ann." instead of to "Phil. Mag." The reference is otherwise correct.

We avail ourselves of the present opportunity to state that in Prof. J. A. Ewing's paper on the "effects of stress and magnetization on the thermo-electric quality of iron" (Phil. Trans. II, 1886, p. 361), mention is made of our results only in a final note dated Sept. 17, 1886. Our work was accessibly published much earlier (Wied. Ann., 1881, vol. 14, p. 54; cf. also ibid., 1879, vol. 7, p. 408). We believe we were the first to actually measure Thomson's thermo-electric effect of magnetization as well as to point out its probable relations to the strain of a magnetized rod.

CHAPTER V.

THE CHANGE OF THE ORDER OF ABSOLUTE VISCOSITY ENCOUNTERED ON PASSING FROM FLUID TO SOLID.

INTRODUCTION.

123. Notwithstanding the abundance of the literature on viscosity, nobody has defined the difference between the solid state and the fluid state quantitatively. In case of liquids and gases viscosity can be absolutely expressed with facility, and the data may be stated with considerable rigor. This is not true for solids. The results are relative throughout, no attempt having as yet been made to coordinate their viscous behavior with that of liquids. The present chapter makes use of two methods for this purpose.

In proposing a differential method for studying the viscosity of solids, Dr. Strouhal and I[1] pointed out the way in which the viscosity of solids may be expressed in terms of the respective sectional areas, by which the motion at the junction of two countertwisted wires or rods is annulled. Suppose now that one of the wires is a true solid, whereas the other is a very viscous fluid. Then, since the viscosity of the latter is just capable of being arrived at by transpiration methods, it follows that the viscosity of the solid with which it was countertwisted may also be absolutely expressed. This indicates the first method of procedure adopted. I measured the viscosity of marine glue, by observing the excessively slow transpiration of this substance throughout many months. Knowing this datum, a very thick rod of the cement was compared in viscous behavior with a fine steel wire. §§126, 127, 130, 131.

My second method for expressing the absolute viscosity of solids is direct and dynamic. It is capable of yielding results of any desirable degree of rigor, supposing the necessary facilities for work to be given. §§ 134, 135.

GASES AND VAPORS.

124. It is well to note the viscosities η, of typical substances in passing. The viscosity of vapors and gases has been measured by many observers.[2] The data present some curious aspects: Thus, for instance, the viscosity of vapors[3] like ether, is frequently less than the viscosity

[1] Barus and Strouhal: Am. Jour. Sci., 1887, vol. 33, p. 29.

[2] O. E. Meyer: Kinetische Theorie der Gase, Breslau, 1877, p. 138, et seq.

[3] Cf. Puluj: Phil. Mag. (5), 1878, vol. 6, p. 157.

of gases. The absolute values encountered in the present case at mean atmospheric temperature are of the order of $\eta = \cdot 0001$ to $\eta = \cdot 0002$ (g/cs). Cf. § 140.

LIQUIDS.

125. Results are also available in great numbers. The literature is systematized in Landolt and Boernstein's[1] tables, and the data may easily be expressed absolutely by means of Slotte's[2] values of the viscosity of water at different temperatures. Water may be taken as the type liquid. Slotte's value for 20° is $\eta = \cdot 0100$ (g/cs). Cf. § 140.

VISCOUS FLUIDS.

126. *Apparatus.*—Data are wanting.[3]

My experiments were made with marine glue.[4] The method of procedure is simple. A fairly wide capillary tube, *a b*, Fig. 26, provided with a cylindrical open reservoir, *a*, at one end, was half filled with marine glue. The thread extended from the reservoir *a* as far down as the middle, *b*, of the tube. The other (open) end of the tube was introduced into a flask, *d d d d*, from which the air was then exhausted. By this adjustment a pressure of about one atmosphere was continually brought to bear on the reservoir *a*, of the tube *a b*, the other end being in vacuo. To facilitate exhaustion, the flask is provided with a lateral arm, *e*, closed by fusion. A small vacuum gauge, *g g*, held in place by a layer of paraffine, *h*, shows the observer to what degree the vacuum is maintained. It is necessary to cover the rubber stopper *f f* with cement. A number of tubes, *a b*, may be inserted, side by side.

127. *Computation.*—The transpiration equation due to Poiseuille and theoretically corrected by Hagenbach[5] has the form

$$\eta = \frac{\pi P r^4}{8 v l} t - \frac{m}{2^{10/3} \pi l} \frac{1}{t} \quad \ldots \ldots \quad (1)$$

where η is the absolute viscosity of the liquid, v the volume, m the mass transpiring through the capillary of length l and radius r in the time t. In the present case of excessively slow transpiration the corrective member may be neglected. P is the pressure-excess in dynes per square centimetre. Since all the magnitudes are to be expressed in *cgs* units, the dimensions of η are (g/cs).

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[1] Berlin, J. Springer, 1883, p. 153.

[2] Slotte: Wied. Ann., 1888, vol. 20, p. 267. Similar results are obtained by others. Slotte gives the references.

[3] Since this was written Prof. J. D. Everett, of Belfast, has kindly called my attention to certain experiments of v. Obermayer. Looking up this research in the Wien. Ber., (2), vol. 75, 1877, p. 665, I found important data for pitch and storax obtained by a method quite different from mine, but agreeing very well with the order of my results for marine glue, and for paraffine. V. Obermayer finds for pitch at 6·5° $\eta = 21 \times 10^8$ at 10·1°, $\eta = 5{\cdot}3 \times 10^8$, and at 12·2°, $\eta = 2{\cdot}6 \times 10^8$. His datum for storax at 15·5° is $\eta = 13 \times 10^{10}$. Attention may here also be called to some data for glycerine due to Schöttner (Wien. Ber., 1879, vol. 79.)

[4] A valuable cement, being (nominally) a specially prepared mixture of rubber and shellac. It has a pitchy consistency.

[5] Hagenbach: Pogg. Ann., 1860, vol. 109, p. 358. This reference is incidental or I should have to refer to Poisson, Navier, Stokes, Stefan, Helmholtz and others.

In the above form of apparatus l is not constant, and v is measured in terms of the increase of length of the capillary thread ab, Fig. 26. Hence the equation corresponding to (1) (since $dv = \pi r^2 dl$) is

$$2ldl = \frac{Pr^2}{4\eta} dt \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Integrating this equation between 0 and t, and between l_1 and l_2, and again solving for η, there results

$$\eta = Pr^2 t/4\ (l_2^2 - l_1^2) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

an exceedingly simple expression. Here l_1 is the original length ($t = 0$) and l_2 the final length ($t = t$) of the capillary thread. The effect of the cylinder of viscous fluid in the reservoir may obviously be neglected in virtue of its large radius.

By way of digression I may insert another similar and interesting case, viz, that of a viscous liquid like glycerine transpiring in a vertical capillary tube in virtue of the weight of its own column only. In this case $P = (L+l)\ \delta g$, where δ is the density of the liquid, g the acceleration of gravity, L the initial head of the column. Inserting these quantities in equation (2) it becomes

$$\frac{ldl}{L+l} = \frac{\delta g r^2}{8\eta} dt;$$

whence by integration

$$\eta = \frac{\delta g r^2}{8\left(l_2 - l_1 + L \ln \frac{L+l_1}{L+l_2}\right)} (t_2 - t_1)$$

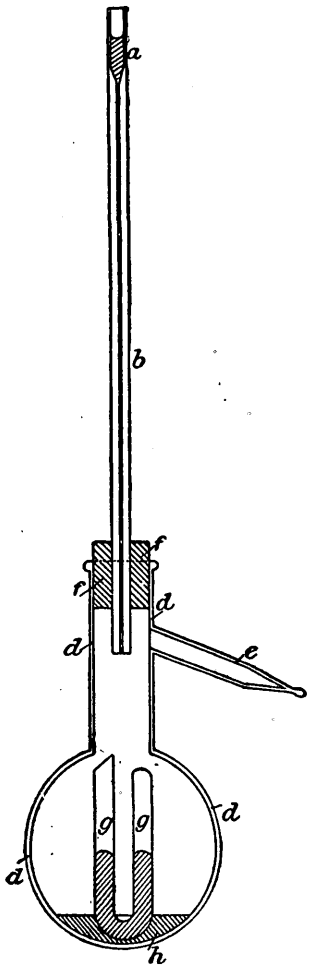

FIG. 26.—Apparatus for the secular transpiration of viscous liquids.

I give an example of the application of this equation in Table 74. Gylcerine is the liquid chosen, a substance which Graham[1] pronounced too viscous for his apparatus, and for which no data are in hand.

[1] Graham: Phil. Trans., 1861, vol. 151, p, 382. His glycerine work was done with "18-hydrates." "The liquid (glycerine) is too viscous to be transpired by means of the bulb and capillary employed in these experiments." Graham, loc. cit.

TABLE 74.—*Viscosity of glycerine.*

$r = 0505^{cm}$; $L = 1{\cdot}35^{cm}$; $\delta = 1{\cdot}26$; $\Delta t = 120^{sec.}$

t	l	$L \ln \frac{L+l_1}{L+l_2}$	η
sec.	*cm.*		*g/cs*
30	10·07	—·86	5·05
150	20·30		
60	12·72	—·73	5·07
180	22·75		
90	15·30	—·62	5·11
210	25·10		
120	17·80	—·52	5·13
240	27·43		
270	29·82	—·34	5·53
390	38·60		
300	31·95	—·32	5·44
420	40·83		
330	34·20	—·30	5·44
450	43·04		
360	36·40	—·28	5·43
480	45·22		

The observations though made upon a single descending thread at the end of each half minute are grouped in two batches. The tube was dry; perhaps a moist tube (adhering thin film of glycerine) would have been preferable. To account for the difference of η in the two sets of results is beyond the present purpose. It is noteworthy that each measurement occupies but 2 minutes, even admitting that the tube is unfavorably wide; for in the case of viscous liquids the temperature effect is of paramount importance, and this expedious method therefore has some advantages. The experiment may be varied by noting the descending upper meniscus while the lower end of the capillary is submerged in glycerine.

128. *Data for marine glue.*—Applying equation (3) to the data of the following table, η may be at once computed.

TABLE 75.—*Transpiration of marine glue.*

$P = 10^6$ dynes; $r = .0406^{cm}$; temperature 25°.

Date.	t	l	$\eta \times 10{-}6$
	sec	*cm*	*g/cm sec*
May 7, 1889, 12h ...	0	3·7	.....
Dec. 7, 1889, 12h ...	18,490,000	7·2	200

This is a remarkable result. Since P in the above apparatus was probably not much smaller than one atmosphere, it follows that η was not much smaller; and yet the substance can easily be fashioned by the fingers, and a sphere if placed on a plane will run out to a flat cake in a few months. This type of viscous fluid is nevertheless 20 billion times as viscous as water at the same temperature.

After long continued transpiration the thread of marine glue shows two strata. The advance portion is amber colored and less viscous;

the rear portion brown, clearly containing most of the shellac. For this reason the observed datum for η is probably somewhat too low. Cf. § 132.

129. *Results for paraffine.*—Paraffine (melting point 55°), treated in this way showed absolutely no transpiration. The tube was nearly of the same bore as above ($\rho = \cdot 0408^{cm}$), and the length of thread $2 \cdot 2^{cm}$. The motion of the paraffine thread must have been below $\cdot 01^{cm}$, or it would have been detected. Hence the viscosity of paraffine must be considerably greater than 2×10^{11} *g/cs*, at about 25°.

SOLIDS.

130. *Method of comparison.*—Having obtained the result of § 128, I proceeded to use it for obtaining the viscosity of hard steel by the method of comparison indicated in § 123. The apparatus was essentially like that used in my earlier experiments.[1] To secure additional safety, however, I introduced the principle of substitution, whereby the steel wire and rod of marine glue were consecutively subjected to known torsion couples in ways otherwise identical. From the enormous viscosity of steel as compared even with marine glue, a beam of this substance something like a decimeter thick would show about the same deformation, cæteris paribus, as a hard steel wire only a millimeter thick. So large a mass of marine glue would have been inconvenient to handle. I therefore used a rod about 4^{cm} thick and suitably apportioned the torsion couples in the two cases. The results are given in the following Tables, 76 and 77. Here 2ρ is the diameter, l the length of the similarly twisted rods. This twist, τ, is given in the first columns, per cm. of length. The applied couples are proportional to 77. N denotes the viscous motion at the index, at the time specified, and is given in scale parts, the distance between mirror and scale being 200^{cm}. Reduction of N is not necessary, since the present method is comparative. The steel wire was twisted alternately in opposite directions to allow for the accommodation. In the other case (Table 77), this precaution is not necessary, the substance being a viscous fluid. Three special twists are given in the table merely to show the behavior.

Other experiments which I made with the cement may be omitted.

TABLE 76.—*Viscous deformation of hard steel wire No.* 1.

$\rho = \cdot 040^{cm}$; $l = 30^{cm}$.

Twist, τ	Time.	N	Twist, τ	Time.	N	Twist, τ	Time.	N
°	*minutes.*	*cm.*	°	*minutes.*	*cm.*	°	*minutes.*	*cm.*
+5·7	0	.....	−5·7	0	.....	5·8	0	.....
	2	·00		2	— ·00		2	·00
	3	·70		3	— ·71		3	·40
	4	1·15		4	—1·25		4	·72
	6	1·87		6	—2·00		6	1·22
	8	2·35		8	—2·58		8	1·60
	10	2·78		10	—3·01		10	1·90
	12	3·15		12	—3·40		12	2·12

[1] Am. Jour. Sci. (III), 1887, vol. 34, p. 2. Pl. I, this Bulletin.

TABLE 77.—*Viscous deformation of marine glue.*

$\rho = 2^{cm}$; $l = 30^{cm}$.

Twist, τ	Time.	N	Twist, τ	Time.	N	Twist, τ	Time.	N
°	*minutes.*	*cm.*	°	*minutes.*	*cm.*	°	*minutes.*	*cm.*
*1·4	0	·0	1·4	0	·0	1·4	0	·0
	2	2·6		2	2·5		2	2·2
	3	4·8		3	4·5		3	4·2
	4	7·1		4	7·0		4	6·1
	5	9·1		5	9·0		5	8·1
							6	10·1

* This refers to the steel wire (normal). The initial twist of the bar of marine glue is practically zero.

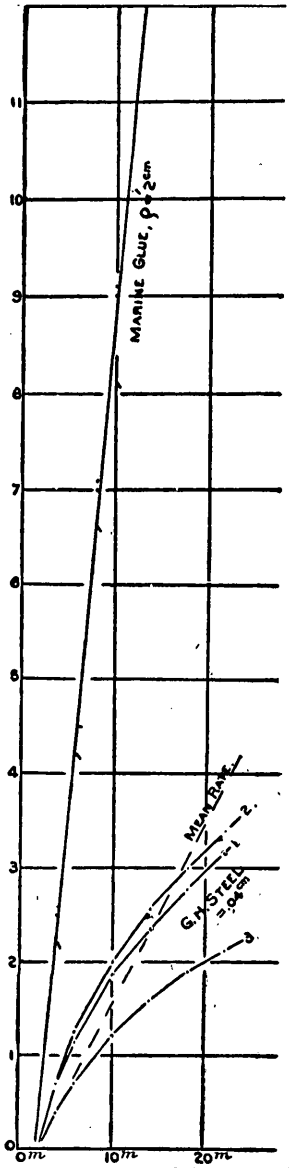

FIG. 27.—Viscous deformation of a bar of marine glue compared with a hard steel wire.

131. *Discussion of results.*—From these tables (graphically constructed in Fig. 27) it appears that the mean rate of motion in case of marine glue is 2^{cm} per minute, whereas the corresponding motion for hard steel is (say) $0{\cdot}4^{cm}$ per minute, this being the mean for the three cases. Further distinction will be made below, §133 et seq., but is here unnecessary. Again, the couple acting on the cement is $1{\cdot}4x$, while the couple in the other case is $5{\cdot}7x$. Since the cement is a liquid, the rate of viscous motion will increase as the couples. Hence for identical couples ($5{\cdot}6x$, say) the motion for marine glue would be 8^{cm}, whereas the rate for steel is $0{\cdot}4^{cm}$, as above. I will now apply the principle[1] mentioned in § 123, whereby viscosities are, cæteris paribus, inversely as the fourth power of the radii. If η' and η be the viscosities of marine glue and hard steel, respectively, then

$$\frac{\eta'}{\eta} = \left(\frac{\cdot 04}{2{\cdot}0}\right)^4 \frac{0{\cdot}4}{8{\cdot}0} = 8/10^9 \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

From Table 75, $\eta' = 200 \times 10^6$ g/cs. Hence

$$\eta = 25 \times 10^{15} g/cs \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where η is the mean viscosity of originally untwisted glass-hard steel, during three consecutive alternate twists of about 12^m duration each.

132. *Errors encountered.*—To fully interpret the result (2) it would have been necessary to compute the η for consecutive times during each of the consecutive twists. This I will do below, where the method used is direct and much more sensitive. The pres-

[1] Am. Jour. Sci., 1887, vol. 33, p. 36.

ent results subserve their chief purpose in furnishing an estimate of the order of viscosity of glass hard steel, which may then be compared with the more detailed statements. Cf. § § 134, 135.

The errors encountered in the above comparative method are these: In the first place, the thread of marine glue which transpires, § 128, is not identical with the rod of this material which is twisted, either as to temperature or composition. Cf. § 128. These difficulties are almost insuperable, though possibly better results might be obtained with pitch. Again, the rod of marine glue, while it is being twisted, is also being stretched by its own weight. This makes the angular measurement difficult, for it is hard to manage a thick rod satisfactorily. Finally, from this cause and the difficulty of obtaining and keeping a rod of the cement free from bubbles, straight and round, the sectional error is considerable. Regarding the above result, I should be surprised if hard steel were less viscous than 10^{16} or more viscous than 10^{17}, under the conditions given.

133. *General remarks on solid viscosity.*—Before giving the results of the following direct method, it is desirable to insert a word on the viscosity of solids generally. In the above case, § 130, while the steel wire is strained by the couple almost as far as the limits of elasticity permit, the rod of marine glue is initially scarcely strained at all. In the latter case, however, the unstable configurations essential to viscous motion are continually supplied in very great number by the ever changing distribution of the heat agitation within the body. In a homogeneous solid (soft steel) these configurations are supplied only in vanishing number. I doubt whether in soft steel the viscous deformation due to instantaneous exceptionally intense heat motion at different points of the solid could be recognized at ordinary temperatures. The case is different when the solid is under stress. Any twist, no matter how small, is accompanied by a proportionate amount of permanent strain. With this a similarly proportionate amount of instability is necessarily associated. Hence it must be carefully borne in mind that the viscosities expressed absolutely below refer to steel in a condition of strain just within the limits of elasticity.[1]

Now it is clear that if the stressed steel be left to itself the number of unstable configurations becomes rapidly less as the time after twisting increases. The viscosity of solids is therefore essentially a time function as well as a strain function.

134. *Apparatus for direct method.*—The apparatus with which the following absolute results were obtained is shown in Fig. 28. *AA* and *BB* are two massive torsion circles, the hollow cylindrical shafts *oo* of which are firmly clamped to supports projecting from the wall of the room. The steel wire *ww* to be tested is tensely drawn between the

[1] I can here merely allude to the recent and suggestive paper of C. H. Carns-Wilson (Nat., vol. 41, p. 213, 1890), in which the behavior of steel near the elastic limits is interpreted in analogy with the well-known circumflexed isothermals of James Thomson.

cylindrical cores $ala'l'$ of AA and BB, and clamped without torsion. To accomplish this the ends of the wire ww are bent hookshaped and inserted in a fissure at the ends of the rods al $a'l'$, the clamp screw (shown at a') passing through the loop of the wire. The middle part of the wire ww carries a pair of small brass plates bc, screwed together

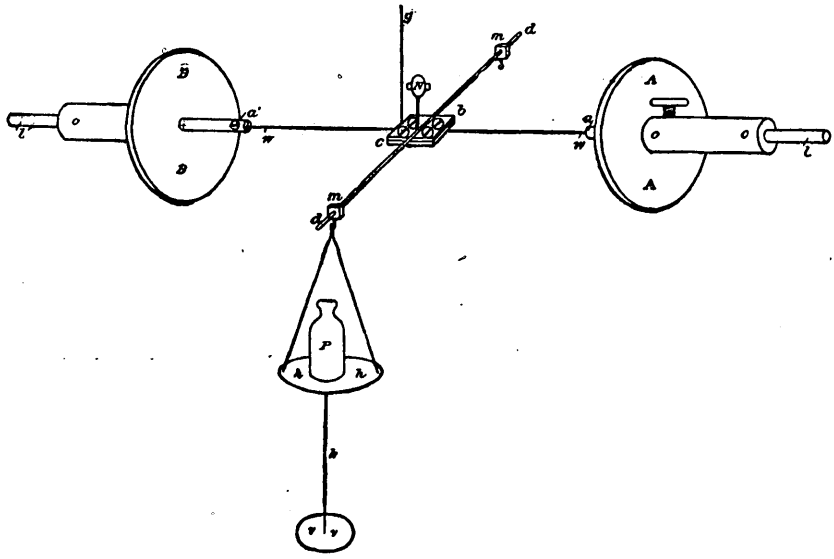

FIG. 28.—Apparatus for measuring the absolute viscosity of solids.

by four screws in such a way as to hold the wire firmly between the plates. A brass rod of screw wire dd is soldered to the upper plate, and when in adjustment is at right angles to the axis of ww. The rod dd is virtually the beam of a balance, and two small nuts mm', symmetrically placed near the ends of dd, are provided with hooks, from which a scale pan hh may be suspended. The latter carries a light horizontal disk vv, to be submerged in water (not shown), with the object of deadening vibrations when the apparatus is in adjustment. The upper plate bc also carries an adjustable mirror N for angular measurement, as well as an index fg, moving across a stationary graduated dial (not shown in figure), by which the amount of twist which holds the weight P in equilibrium is roughly registered.

In adjusting the apparatus the scale pan hh is removed, and the rods al so placed that the lever dd is vertical. After thoroughly clamping al (care being taken to avoid twisting the wire ww), the scale pan is attached at m', and additional weights P added, such as hold the lever dd horizontal. Hence the two halves of the wire ww are twisted 90° each for the whole length. Other twists may be applied by rotating the torsion circles and adding suitable weights P.

To fasten the wire between the plates ab the lower one is clamped in a vise, and the four screws then fastened securely. There is no rotational sliding here under the given stresses, as is proved by exam-

ining wires of exceedingly high viscosity. Some difficulty is experienced in originally fixing the wire *ww* in place quite free from torsional strain. There is also some flexure. But for wires of the radius given these discrepancies are negligible in comparison with other errors inherent in the wires. In examining the effect of strain on viscosity it would be desirable to connect A and B rigidly together and express the strain in terms of the weight P; but this is beyond the purpose of the present paper.

135. *Method of computation.*—To compute[1] the results obtained with the apparatus just described, let 2φ be the amount of viscous angular motion in radians of any right section of the wire relative to another right section whose distance from the first is the unit of length. Then in the given adjustment the mirror registers an angle corresponding to 2φ; since at each section, though equal angular detorsion takes place in opposite directions, both must influence the mirror in view of the constancy of the twisting couple. Furthermore, suppose that in the plane of a right section there is no shearing. Then $2d\varphi/dt=2C_t$ is the common angular velocity with which the shear is increased throughout the length of the wire. Consider any elementary cylindrical shell whose length l is the effective length of either half of the wire *ww* and whose right section is $2\pi r dr$, where r is the radius of the section. Then the amount of force distributed uniformly over $2\pi r dr$ will be

$$df=\eta\frac{2\pi r dr}{l}l\,2C_t r \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Multiplying by r and integrating both sides between 0 and ρ, the radius of the wire,

$$\frac{1}{2}P\lambda=\eta_t\frac{\pi\rho^4}{2}2C_t \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

where P is the weight at the end of the lever arm λ, by which the system is held in equilibrium. In any given adjustment $P\lambda/\pi\rho^4=A$, and is constant. Hence

$$\eta_t=A/2C_t \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

In these equations it has been tacitly assumed that $\int\eta r^3dr=\eta_t\int r^3dr$. Under any conditions, therefore, η_t is a mean value for the sections defined by these integrals.

136. *Data obtained.*—With these equations at hand it is not difficult to understand the results of the following tables, 78 to 81. Here ρ is the radius and l the length of each effective half of the steel wire. P is the total force (being the weight plus the weight of scale pan) acting at the end of the lever arm λ (see Fig. 28). The rate of twist is given under τ in degrees, and $2\varphi'$ is the total yield (in degrees) observed at the end of the experiment. N denotes the actual scale reading in the telescope at the time given, and 2φ is the viscous angular motion in radians between two right sections whose distance apart is 1^{cm}.

[1] The quantity 2ϕ is here used conformably with my earlier papers.

In the second half of the tables $2C$ the time rate of change of 2φ is inscribed, and the final column coutains the computed viscosity for the time specified. All times are reckoned from the moment at which the twist is applied. To obtain $2C_t$, I availed myself of graphic methods, these being quite in conformity with the mean accuracy of the work. Table 78 contains results for the wire No. 1, already twisted in Table 75 above. Three twists are applied alternately in opposite directions. In Table 79 the same wire is similarly examined after it has been softened by heating to redness and cooling in air. Table 80 supplies data for a new glass-hard steel wire, originally free from stored twist. Table 81, finally, contains the viscous constants of a wire tempered at 450°. This is the identical wire with which I made my earlier differential observations.

TABLE 78.—*Absolute viscosity of glass-hard steel wire No.* 1.

$R=200^{cm}$; $l=12{\cdot}9^{cm}$; $P=82\times981$ *dynes*; $\rho={\cdot}0105^{cm}$; $\lambda=9{\cdot}43^{cm}$.

FOURTH TWIST.

τ / $2\phi'$	Time.	N	$10^6\times2\phi$	Time.	$2C_t\times10^9$	$\eta_t/10^{15}$
°	*sec.*	*cm.*	*radians per cm.*	*sec.*	*radians per cm. per sec.*	*g/cs.*
+7	0	*Twisted.*	——	500	735	122
	420	·00	0	1000	462	194
	720	1·04	202	1500	310	291
	1020	1·80	349	2000	222	405
	1860	3·20	621	2500	183	491
	2210	3·60	698	3000	150	600
	2820	4·17	808			
	3480	4·65	·901			

FIFTH TWIST.

—7	0	——	——	500	910	99
·5	480	·00	0	1000	474	190
	840	1·20	233	1500	347	259
	1320	2·35	456	2000	270	333
	1860	3·29	638	2500	230	391
	2400	4·02	779	3000	210	428
	2940	4·64	900			
	3180	5·20	1008			

SIXTH TWIST.

7	0	——	——	500	467	193
·5	360	·00	0	1000	265	339
	960	1·20	233	1500	193	466
	1260	1·60	310	2000	156	577
	1800	2·10	407	2500	138	652
	2340	2·50	485	3000	126	715
	3120	3·06	593			
	3900	3·53	648			

TABLE 79.—*Absolute viscosity of soft steel, being wire No.* 1.

$R=200^{cm}$; $P=82\times981$ *dynes*; $\lambda=9{\cdot}43^{cm}$; $l=13{\cdot}0^{cm}$; $\rho={\cdot}0405^{cm}$.

HEATED TO REDNESS IN AIR. FIRST TWIST.

τ / $2\phi'$	Time.	N	$2\phi\times10^6$	Time.	$2\theta_t\times10^9$	$\eta_t/10^{15}$
°	*sec.*	*cm.*	*radians per cm.*	*sec.*	*radians per cm. per sec.*	*g/cs.*
+7	0	——	——	500	220	409
·4	420	·00	0	1000	89	1010
	900	·40	77	1500	63	1430
	1500	·63	121	2000	47	1920
	2100	·80	154	2500	34	2650
	2700	·92	177	3000	26	3460
	3300	·98	188			
	3780	1·03	198			

SOFTENED. SECOND TWIST.

—7	0	——	——	500	164	550
·6	360	.00	0	1000	116	776
	1020	·50	96	1500	59	1520
	1500	·70	135	2000	40	2250
	1980	·82	158	2500	30	3000
	2520	·93	179	3000	15	6000
	3120	1·00	192			

SOFTENED. THIRD TWIST.

+7	0	——	——	500	138	652
·4	240	·00	0	1000	70	1290
	840	·42	81	1500	40	2250
	1560	·64	123	2000	28	3210
	2100	·74	142	2500	17	5300
	2520	·76	146	3000	14	6400
	2640	·78	150			
	3120	·81	156			

TABLE 80.—*Absolute viscosity of glass-hard steel wire No.* 2.

$l=13{\cdot}0^{cm}$; $\rho={\cdot}0405^{cm}$; $R=200^{cm}$; $P=82\times981$ *dynes*; $\lambda=9{\cdot}43^{cm}$.

FIRST TWIST.

τ / 2ϕ	Time.	N	$2\phi\times10^6$	Time.	$2\theta_t\times10^9$	$\eta_t/10^{15}$
°	*sec.*	*cm.*	*radians per cm.*	*sec.*	*radians per cm. per sec.*	*g/cs.*
7·2	0	——	——	500	660	136·0
·5	120	·00	0	1000	325	277·2
	420	1·86	358	1500	230	391·5
	720	2·70	520	2000	180	500·0
	1080	3·35	644	2500	140	643·0
	1620	4·10	789			
	2100	4·58	881			
	2580	4·95	952			
	3000	5·18	996			

TABLE 80.—*Absolute viscosity of glass-hard steel wire, No. 2*—Continued.

GLASS-HARD. SECOND TWIST.

$\frac{\tau}{2\phi}$	Time.	N	$2\phi \times 10^6$	Time.	$2C_t \times 10^9$	$\eta_t/10^{15}$
°	*sec.*	*cm.*	*radians per cm.*	*sec.*	*radians per cm. per sec.*	*g/cs.*
7·3	0	——	——	500	683	131·4
·6	240	·00	0	1000	392	229·5
	600	1·44	277	1500	285	316·0
	900	2·20	423	2000	240	375·0
	1260	2·88	554	2500	205	439·0
	1800	3·65	702			
	2100	4·04	777			
	2460	4·50	865			
	2700	4·70	904			
	3000	5·02	965			

GLASS-HARD. THIRD TWIST.

——	0	——	——			
——	240	·00	0			
	600	1·17	225			
		Wire breaks spontaneously				

TABLE 81.—*Absolute viscosity of steel annealed at* 450°, *wire No.* 15.

$l = 13{\cdot}6^{cm}$; $\rho = {\cdot}0405^{cm}$; $R = 200^{cm}$; $P = 82 \times 981$ *dynes*; $\lambda = 9{\cdot}43^{cm}$.

TWISTED INDEFINITELY.

$\frac{\tau}{2\phi'}$	Time.	N	$2\phi \times 10^6$	Time.	$2C_t \times 10^9$	$\eta_t/10^{15}$
°	*sec.*	*cm.*	*radians per cm.*	*sec.*	*radians per cm. per sec.*	*g/cs.*
+7°	0	——	——	500	177	508
·1	240	·00	0	1000	69	1300
	720	·51	94	1500	40	2250
	1200	·71	131	2000	32	2810
	1860	·86	158	2500	25	3600
	2520	·95	175	3000	20	4500
	3000	1·01	186			

NEXT TWIST.

−7°	0	——	——	500	111	811
·2	240	·00	0	1000	46	1950
	600	·26	48	1500	40	2250
	1200	·43	80	2000	34	2650
	1740	·54	100	2500	31	2910
	2580	·68	125	3000	30	3000
	3240	·81	149			

NEXT TWIST.

+7°	0	——	——	500	180	500
·2	240	·00	0	1000	82	1100
	660	·48	88	1500	57	1570
	1260	·78	143	2000	48	1870
	1740	·92	169	2500	45	2000
	2520	1·12	206	3000	42	2140
	3120	1·26	232			

Some few irregularities are introduced into the above data, from the fact that vibrations could not be rigorously excluded. Inasmuch as vibration increases the rate of twist periodically, it thus supplies a greater number of instabilities than exactly correspond to the mean twist (position of equilibrium) in question. Hence viscosity is decreased by vibration.

In case of No. 15 and No. 2, etc., the rate of twist of the two halves of the wire *wow* (Fig. 28) was possibly not quite the same. Hence the viscous motion would also be slightly too large. In case of different mean lengths l, in two different wires, the rate of twist for the same load P would again be different, thus introducing small discrepancies. Finally for No. 15, which had become somewhat worn after long use, the load P probably strained the wire too near the elastic limits. This supplies fresh instabilities by which viscosity is successively decreased from twist to twist and accords with the results actually obtained. Hence it is best to accept the behavior of the softened wire No. 2 as typical of No. 15. Steel annealed at 450° and at 1000° is of equal viscosity, as I have shown in numerous experiments.

DISCUSSION.

137. *Retrospective.*—Turning first to the values φ, it is seen at once that they substantiate the results of my earlier papers throughout.[1] They need not therefore be further commented on here. Table 81 perhaps has special interest, since the φ here contained is the datum for the normal wire formerly used. With this datum in hand, the results of the earlier papers may easily be reduced to absolute values, for I have endeavored to make the angles τ of the same value in both cases, so that φ/τ may be comparable at once.

138. *Spontaneous breaking.*—The spontaneous breaking of the glass-hard wire in Table 80 is worthy of remark. It occurred during the third twist, and at a time when the wire was in no way interfered with. This corresponds to the spontaneous explosion of tempered projectiles frequently observed. It also corresponds to the spontaneous rupture of stressed glass, whether the stress be stored internally or applied externally. I have observed this interesting phenomenon in glass under a great variety of conditions. In all cases there is gradual molecular change, or slow viscous yielding to the point of rupture.

139. *Time variation of absolute viscosity.*—The chief results of Tables 78 to 81 are the values η_t, whether they be regarded as time functions or as exponents of the degree of solid state. In Pl. VI, I have therefore constructed them graphically. As an inspection of the figure shows at once, both the viscosity and its rate of increase with time are greater in case of hard steel (Tables 78, 80) than in soft steel (Tables 79, 81). After about an hour's time the final viscosity of hard steel may be quite as

[1] Am. Jour. Sci. (III), 1866, vol. 32, p. 181–192, and 1887, vol. 33, p. 20–36.

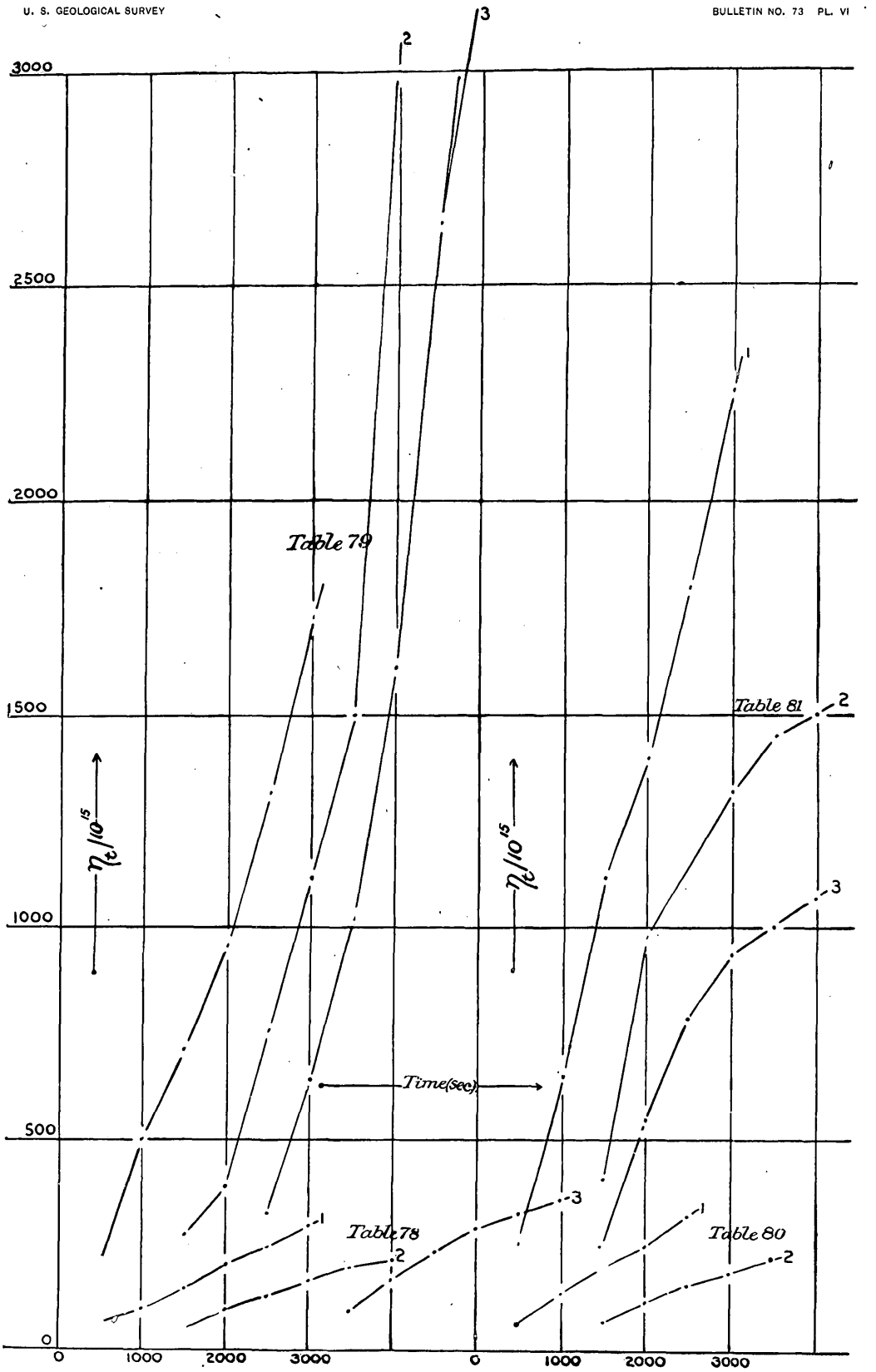

CHART SHOWING THE ABSOLUTE VISCOSITY OF HARD STEEL (TABLES 78 AND 80) AND OF SOFT STEEL (TABLES 79 AND 81) DURING THE FIRST HOUR AFTER TWISTING JUST WITHIN THE ELASTIC LIMITS

large as the initial viscosity of soft steel. For equal times after twisting, the viscosity of soft steel is enormously the greater.

The curves show no pronounced curvature, being in part convex and in part concave. This is particularly clear in the case of hard wires, where the viscosities are smaller, the viscous deformations large, and the observations therefore, as a whole, much more accurate (Tables 78, 80). The extended curves (Tables 79, 81) for soft wires, on the contrary, are less certain, because the viscous deformation is proportionately smaller. Hence I may regard the promiscuous irregularities of the curves of Pl. VI due to errors of observation and construction. It is exceedingly difficult to draw tangents correctly. Again, the observations are hampered by vibrations, etc., in the way indicated at the end of § 136. According to Kohlrausch's law,[2] $-d\varphi/dt=a\varphi/t^n$, where a and n are constants. In case of small torsions, or during short times, the simple relation $-d\varphi/dt=a\varphi/t$ suffices. Thus the character of the curves of Fig. 33 may be considered linear during the first 3000 sec. after twisting. Viscosity therefore increases proportionally to time, at the rates expressed in the following Table 82:

TABLE 82.—*Rate of increase of the absolute viscosity of steel.*

Hard steel.	Twist.	10^{-12} η/sec	Soft steel.	Twist.	10^{-12} $\eta/sec.$
		g/cs^2			g/cs^2
Tab. 4	+4	220	Tab. 5	+1	1200
	−5	140		−2	1400
	+6	240		+3	1900
Tab. 6	+1	230	Tab. 7	+1	1500
	−2	140		−2	1200
				+3	1000

Table 82 shows the mean rate $\eta/sec.$ for hard steel to be 194×10^{12} and its mean viscosity η at 500 *sec.* to be 140×10^{15}; for soft steel $\eta/sec.=1400\times10^{12}$ and mean viscosity at 500 *sec.*, $\eta=570\times10^{15}$. The general explanation of these results has been indicated in §133. In a liquid or a viscous fluid under moderate stress the instabilities are supplied by the mere thermal agitation at ordinary temperatures at the same rate in which they are used in promoting the viscous motion. Hence viscosity is constant at a given temperature. In a solid this is not the case; instabilities are here expended at a rate decidedly greater than the small rate of continuous supply. Thus viscosity decidedly increases with time. Suppose therefore a substance of initially greater viscosity to start on its viscous deformation at an instant when the viscosity of a substance of initially smaller viscosity has been increased by time to the initial value of the former case. This can be realized by twisting the soft steel about an hour after the hard steel rod has been twisted. Then in the succeeding times, even though the substance of greater and the substance of less viscosity start at a stage of equal viscosity, the vis-

[2] Kohlrausch: Pogg. Ann., vol. 128, 1866, p. 216.

cosity of the former will rapidly overtake the viscosity of the latter; for the supply of instability due to temperature alone is continuously greater in one case than in the other.

140. *Solidity of the three states of aggregation.*—Summarizing the results of the above paragraphs, the viscosities of the three states of aggregation may be expressed in terms of the absolute *g/cs* scale as follows:

TABLE 83.—*Viscosity in its variation with the state of aggregation.*

Gases and vapors.		Andrews's critical state.		Liquids. ‡		Viscous fluids.		Solids.
Substance.	η	Substance.	η	Substance.	η	Substance.	η	
*Ether, 0°	$6{\cdot}8 \times 10^{-5}$	?	?	‡ Ether, 30°	9×10^{-4}	Marine glue	4×10^{8}	Parafline, (m. p. 55°) at 20° $> 4 \times 10^{11}$
*H_2, 0°.	$8{\cdot}7 \times 10^{-5}$			‡ Ether, 10°	$1{\cdot}9 \times 10^{-3}$			††Hard steel, glass, etc. 10^{17} to 10^{18}.
*Air, 0°.	$1{\cdot}75 \times 10^{-4}$			‡ Water, 97°	$3{\cdot}0 \times 10^{-3}$			
†O_2, 0°.	$2{\cdot}12 \times 10^{-4}$			‡ Water, 20°	$1{\cdot}00 \times 10^{-2}$			††Soft steel. 6×10^{17} to 6×10^{18}.
				Glycerine**	5			
Range..	10^{-5} to ?	Range	? to ?	Range	? to 10^{2}	Range ..	10^{2} to 10^{11}	Range... 10^{11} to 10^{20}

* Puluj: loc. cit.

† O. E. Meyer: loc. cit., p. 142.

‡ Landolt and Boernstein's tables (loc. cit.) and Slotte (loc. cit.).

** Rough measurement of my own, § 127.

†† During the first hour (500 to 3,000 seconds) after twisting just within the elastic limits.

The limits here defined are of course somewhat arbitrary. They will be made more definite when a greater number of bodies lying on the boundary between the classes have been examined. Information is notably lacking in relation to Andrews's critical temperature, which, in the light of the context of Table 83, is particularly interesting.[1] It is not improbable that the critical temperature may be definable by a narrow limit of viscosity, quite independent of the substance operated on. What this limit will probably be I do not venture to assert, seeing that the viscosity of gases decreases on cooling, whereas that of liquids increases on cooling.

Table 83 gives the positively astounding range of variation of η, the chief variable of our material environment—a variable which throughout the whole enormous interval nowhere fails to appeal to our senses. Knowing these data, it will be possible to look somewhat into the insides of solids and to express the existing state of instability statistically. But this calls for an application of more searching mathematics than I am justified in attempting.

[1] Professor Warburg has had the kindness to refer me to an elaborate paper by himself and v. Babo, in which the vicosity of carbon-dioxide passing continuously through the critical temperature is elaborately considered. (Wied. Ann., vol. 17, 1882, p. 390.) Unfortunately, I can here do no more than call attention to this valuable contribution, which was inadvertently overlooked in my original study of the subject. Warburg and v. Babo's results are such as conform with the substance of the above text and I am glad to acknowledge that they were the first in the field to to approach the question.

In concluding, I may state that though I am cognizant of the reasons which induced Sir William Thomson to apply the word viscosity to the phenomena of the present paper, I wish he had chosen some other term. Viscosity is misleading. Undoubtedly a substance which is infinitely viscous is solid, for the parts of such a body are stuck together beyond the possibility of permanent yielding. This truth, however, is of the kind which Gauss[1] in referring to Lagrange's extension of the principle of least action, called "mehr witzig als wahr." Unfortunately the impressions conveyed in speaking of a very viscous body and of a solid body are different. I do not know, but I think solidity would have been a better word than viscosity.

[1] Cf. Klein: Die Principien der Mechanik, Leipzig, Teubner, 1872, p. 33.

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