STATIC AND DYNAMIC ASPECTS OF THE DEMAND FOR GOODS IN KOREA

by
JI-CHUL RYU

CORRECTIONS

1. p.32, line 9: where F is *not* where f is.
2. p.33, line 12: *Insert* "only if the estimated $\beta_i \geq 0$, for all $i=1,\ldots,m$ [see (4.2) below]" *after* "the utility function".
3. p.35, line 8: restrictions *not* restictions.
4. p.41, eq. (5.14): $\Delta\bar{M}_t$ *not* $\Delta\bar{M}$.
5. p.41, line 14: *Insert*

   "$\Delta\bar{M}_t = \Sigma_{i=1}^{m} W_{it}^{*} \Delta \ell n\, q_{it}$"

   below line 14.
6. p.58, lines 16-17: the linearly approximated almost ideal demand system *not* the almost ideal demand system.
7. p.61: *Insert the following paragraph after the last line.*

   "Since all proper demand systems are singular, Σ is usually a singular matrix in the demand systems. However, a digression on SUR theory with a nonsingular Σ matrix can be made without loss of generality in the discussion of estimation of the demand systems. Details of singularity problems will be discussed in Section 3.5 of this Chapter."
8. p.70, line 6: m(m-1)/2 *not* m(m-2)/2.
9. p.80, line 2: statistics *not* statitics.
10. p.80, last line: /(m-1)(T-k)] *not* /m(T-k)].
11. p.81, lines 6 and 10: (m-1)] *not* m].
12. p.179, lines 3 and 20: transportation *not* trasportation.
13. p.293, line 7: restrictions *not* restricitons.
14. p.304, line 14: (4.10) *not* (4.9).
15. p.313, line 7: single *not* sigle.
16. p.363, line 10 and p.370, line 5: asymptotically *not* asympototically.

17. p.387, last line: Dhrymes *not* Dhryme.

18. p.401, line 8: homogeneity *not* homogeniety.

19. p.401, line 23: Lagrange *not* Lagrage.

20. p.404: *Insert the following sentence below the last line:* "As Σ is always unknown, the restricted estimate of Σ, $\tilde{\Sigma} = \tilde{u}'\tilde{u}/T$, is updated with newly obtained $\tilde{B}$, $\tilde{C}$ or $\tilde{\Gamma}$ at each step to secure the ML estimation."

21. p.407, line 3: *Insert* "$Q = [Z, Z_{-1}]$," *after* "where".

22. p.407, eq. (2.30): *Should be*

$$\text{"}var(\beta^*_{ij}) = var(\beta_{oij}) + var(\beta_{1ij}) + 2cov(\beta_{oij}, \beta_{1ij})\text{"}.$$

23. p.407, line 12: $var(\beta_{1ij})$, $var(\beta_{oij})$ and $cov(\beta_{oij}, \beta_{1ij})$ *not* $var(\beta_{ij})$, $var(\beta_{ij})$ and $cov(\beta_{ij}, {}_{ij})$.

24. p.409, line 12: $Y^o_{-1}\sim$ *not* $Y^o_{-1}i$.

25. p.449, line 11: necessity *not* necessicity.

- END -

STATIC AND DYNAMIC ASPECTS OF THE DEMAND FOR GOODS IN KOREA

JI-CHUL RYU

A thesis submitted for the degree of Doctor of Philosophy
of The Australian National University.

November, 1986.

DECLARATION

The contents of this thesis are my own work, except where otherwise indicated.

Ji-Chul Ryu

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Canberra
November 1986

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ABSTRACT

This thesis is concerned with demand analysis for Korea in the context of complete systems of demand equations. Two broad categories of demand analysis - static and dynamic - are considered. This thesis is not only concerned with empirical work but also with the investigation of economic theory relating to the specification of static and dynamic demand systems and also with the development of statistical techniques relating to empirical demand analysis. The scope of this thesis is outlined in the introductory chapter. Chapter 2 sketches the static theory of demand, and reviews various specifications of static demand system. Statistical theories relating to the empirical (static) demand model are considered in Chapter 3, which includes estimation and tests of hypothesis. In respect of estimation, a new algorithm of estimation of a symmetric system (based on analytic solutions of the Lyapunov equation) is developed, and the technique is subsequently extended to the estimation of dynamic demand systems under nonlinear long run equilibrium conditions in Chapter 7. The use of separate induced tests of hypothesis is also considered in Chapter 3. The application of various static demand systems to Korean quarterly household expenditure data is carried out in Chapter 4. Stochastic dynamisation of the disturbances of static demand systems is also considered in Chapter 4. Some aspects of dynamic demand theory are examined in Chapter 5. A new specification of a dynamic demand system, based on the dynamic

equilibrium assumption, is derived in the context of the Rotterdam model approximation, and applied to the Korean data in the conjunction with dynamic demand analysis. Chapter 6 examines the effect of misspecification of a demand system on tests of demand restrictions, analytically as well as using Monte Carlo simulation. Two specific types of misspecification, (one the inclusion of irrelevant explanatory variables, and the other the omission of the relevant explanatory variables) are considered. In Chapter 7, problems relating to the estimation of dynamic demand system under long run equilibrium conditions are considered, and a new algorithm for ML estimation is developed and applied to the examination of the long run demand patterns in Korea. In the last chapter of the thesis, a summary and conclusion are provided.

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CHAPTER 1

INTRODUCTION

1.1 Current Trends in Demand Analysis

For the last two decades, the scope of demand analysis has expanded greatly both empirically and theoretically. The expansion has mainly been due to the generalisation of demand theory, the increasing variety of specifications of complete systems of demand equations, the development of econometric theory, the availability of data, and the improvement of the computing facilities. As pointed out by Deaton (1986, p.1768), "it is not possible to study applied demand analysis without keeping statistics and economic theory simultaneously in view." Synthesising the related economic and statistical theory, demand analysis is in a central position in econometrics, as it is concerned with the modelling of consumer behaviour and the construction of a theoretical framework for testing the empirical validity of economic theory.

Nowadays, it is quite normal to perform demand analysis in the context of the complete system of the demand equations.

In applied demand analysis, the complete system of demand equations plays an important role; for example, the system-wide modelling of the consumer's allocation of total expenditure over the goods, the quantifying of the impact of changes in prices and income (total expenditure), the measurement of welfare gains or losses associated with policy changes, and so on. Theories of the formation and properties of demand equations are generally based on the classical economic theory, such as the utility maximisation or cost (expenditure) minimisation. However, a variety of specifications of estimable systems of demand equations have been developed. Earlier attempts to specify complete systems of demand equations were based on particular types of utility functions, such as the Stone-Geary and addilog functions, which are defined under separable and additive preference structures. Recent developments in the specification of complete demand systems have been more concerned with derivation of the flexible functional forms of demand equations, using the various mathematical approximation methods and allowing non-separable preference structures. The specification of demand systems will be examined in Chapter 2.

The properties of demand equations derived from the utility maximisation or cost minimisation postulate the restrictions to be imposed on the parameters of complete demand systems. The restrictions include adding up, homogeneity, symmetry and negativity conditions. Consequently, the estimation of a demand system (or flexible functional form) generally involves the imposition of restrictions and tests of the validity of the restrictions. Since the development of the SUR estimation method by Zellner (1962), demand equations have been jointly estimated subject to these restrictions to increase the (asymptotic) efficiency of the parameter estimates. For a demand system which is nonlinear in parameters, the nonlinear FIML method, based on the Gauss-Newton iterative method, is used. The restrictions are also jointly tested. The econometric theory used in the applied demand studies will be examined in Chapter 3.

However, in many empirical studies, demand restrictions are often rejected by the data [e.g., Barten (1969), Byron (1970), Deaton (1972), Christensen, *et al.* (1975), Deaton and Muellbauer (1980b)]. As listed in Muellbauer (1982), the suspicious source of the rejection of demand restrictions are related to violations of separability, problems of aggregation over consumers and commodities, changing tastes, uncertainty about price or budget changes, incorrect functional form of demand equations, ignorance of durability and habit formation, and measurement errors. Thus, a "catch-all" solution for the rejection problem does not seem to exist.

Much economic and econometric research has been done to identify the source of the repeated rejection of demand restrictions and to improve the compatibility of demand theory with the sample information. For example, attention has been paid to the local and global properties of flexible functional forms by mathematical demand theorists to ensure flexible functional forms have the ability to mimic the properties of actual preferences [e.g., Caves and Christensen (1980) and Barnett and Lee (1985)]. Econometric issues, such as the problems of over-rejection of demand restrictions, have been addressed. Laitinen (1978) attributed the repeated rejection of the homogeneity restriction to the incorrect use of the asymptotic χ^2 test instead of the exact Hotelling T^2 test and showed that asymptotic χ^2 tests are biased towards rejection [see also Meisner (1979) and Bera, *et al.* (1981)]. To handle the over-rejection problem, Byron and Rosalsky (1985) considered the application of Edgeworth small sample corrections to the χ^2 tests. Anderson and Blundell (1983, 1984) attempted to test the validity of demand restrictions on the long run structure which stems from a generalised dynamic specification of the demand system. Stapleton (1984) considered the problem of tests of the demand restrictions in relation to the errors-in-variable problem in demand systems, and Attfield (1985) considered the simultaneity of demand systems.

The necessity for the dynamic generalisation of demand theory has also been emphasised in the applied demand analysis during the last two decades. The rejection of static demand restrictions has often been attributed to the lack of dynamic features in the static demand theory [e.g., Anderson and Blundell (1983, 1984) and Muellbauer and Pashardes (1982)]. The dynamic generalisation of demand theory has been achieved in various ways; for example, incorporating taste change and the habit formation hypothesis [Brown (1952), Peston (1967), Gorman (1967), Fisher and Shell (1968), Von Weizsäcker (1971), Krelle (1973), Hammond (1976), El-Safty (1976a, 1976b), Pollak (1976a), Muellbauer (1975), Kapteyn, *et al.* (1980), and Spinnewyn (1981)], intertemporal theory [Diewert (1974b), Lluch (1973) and Spinnewyn (1981)], optimal control theory [Lluch (1973, 1974) and Klijn (1977)] and rational expectation hypothesis in a life cycle context [Attfield and Browing (1985)]. Many dynamic demand systems were derived on the basis of the above generalised demand theory and the related empirical studies are available. For example, under the habit formation hypothesis, Pollak and Wales (1969), Pollak (1970), and Phlips (1972) considered the dynamic specification of the linear expenditure system, Manser (1976) specified and estimated the dynamic translog model, and Boyce (1975) estimated the dynamic Gorman polar form. Demand systems incorporating the intertemporal theory have been derived by Lluch (1973) in the application of optimal control theory to the linear expenditure system, and by Muellbauer and Pashardes (1982) in the context of the dynamic version of the almost ideal demand system. Recently, Attfield and Browing (1985) derived a demand system that explicitly incorporates intertemporal theory with the rationality hypothesis in a life cycle context.

Apart from the dynamic generalisation, the static demand system has been generalised by incorporating sociodemographic effects [Pollak and Wales (1980)], the S-branch utility tree [Gorman (1971), Brown and Heien (1972) and Blackorby, *et al.* (1978)], and so on.

Current demand analysis covers a great many other areas, such as Engel curve analysis [Leser (1963), Gorman (1981), Bewley (1982a) and Aasness and Rødseth (1983)], the interdependence preference hypothesis [Pollak (1976b), Kapteyn, *et al.* (1980)], price dependent preference hypothesis [Pollak (1977)], aggregation over consumers [Deaton and Muellbauer (1980a, Chapter 6)], the imposition of curvature conditions [Wales and Woodland (1983) and Diewert and Wales (1984)], and so on. The comparative study of the empirical performance of different demand systems is also an important area in applied demand studies [Park (1969), Deaton (1974), Theil (1975), Klevmarken (1981), and Bewley (1982b)].

In this thesis, we do not intend to deal with all of the problems mentioned above. Obviously, the scope of current demand analysis is too large to cover in one thesis, given the limited time and space and the lack of available data. The scope and structure of this thesis will be outlined in the following section.

1.2 The Scope and Structure of the Thesis

The main purpose of this study is to perform a demand analysis for Korea in the context of complete systems of demand equations using quarterly household expenditure data series covering the period 1965 -1981. Broadly, two aspects of demand analysis are carried out in this study; one is concerned with a static demand model, and the other with a dynamic demand model. As the Korean economy has changed rapidly during the last two decades, structural changes in the demand pattern can be observed from the actual household expenditure data. Consequently, the necessity for dynamic analysis arises inevitably. When using static analysis, we apply the linear expenditure system, the Rotterdam model, and the almost ideal demand system, and compare their performance on the Korean data. Stochastic dynamisation of the disturbances of demand systems are also considered in the context of static demand systems. For the purpose of (structural) dynamic analysis, we derive a flexible dynamic demand system, using the Rotterdam model approximation, in the context of taste change and the dynamic equilibrium model. Furthermore, a new algorithm for estimation of dynamic demand systems under long run equilibrium conditions is developed. As an attempt to examine the over-rejection problem of demand restrictions, we analyse the effect of model misspecification on tests of hypothesis, analytically as well as using Monte Carlo simulations. This study not only consists of empirical work but also develops the economic and statistical theory related to the demand models used.

The structure of this thesis is as follows:

In Chapter 2, we review the static demand theory and survey the problems relating to the specification and properties of various existing complete demand systems: the linear expenditure system, the Rotterdam model, the translog model, and the almost ideal demand system. The discussion will be confined to the basic problems as they relate to the present study.

Chapter 3 examines the statistical problems relating to the system-wide estimation of complete (static) demand systems and tests of related hypotheses in connection with the linear and nonlinear multivariate model. The discussion will include issues on the restricted estimation of demand systems, tests of hypothesis, the singularity of systems and the implications for restricted estimation, the estimation of systems of nonlinear demand equations and problems in estimation and inference for systems of equations with autocorrelated residuals. A new GLS type estimation procedure of the linear symmetry restricted system will be developed as part of this chapter. The result is derived from the solution of a Lyapunov equation, based on the work of Bowden (1973) and Byron (1982). As a complement to direct χ^2 tests of demand restrictions, the use of the separate induced test [Seber (1964b) and Savin (1980, 1984)], will be examined, and discussed in the context of tests of demand restrictions. A condition for the autoregressive error process to be stationary in a singular system will be extended to the higher order AR(p) case from Berndt and Savin's (1975) AR(1) case.

In Chapter 4, we analyse consumption patterns in Korea in the context of systems of static demand equations using quarterly household expenditure data for the period 1966-1981. We will apply three static demand systems, the linear expenditure system, the Rotterdam model and the almost ideal dmeand system, to the Korean data covering with various specifications of seasonal effects. We perform a comparative study of the performance of various demand systems on the data and an analysis of the effects of commodity aggregation on the estimates and tests of hypothesis. In particular, we test and analyse the structural changes in consumption patterns in Korea over the sample period.

In Chapter 5, the dynamic generalisation of demand theory and specification of dynamic demand systems will be discussed in the context of the dynamic equilibrium assumption and the taste change hypothesis. The Rotterdam model approximation will be applied to dynamic Marshallian demand functions to obtain an estimable specification of the dynamic demand equations. The statistical aspects of the estimation of (singular) demand systems will be considered and their empirical application to the Korean household expenditure data will be discussed.

In Chapter 6, we examine the effect of misspecification of demand systems on tests of hypothesis. We will consider two types of misspecification; one is the incorrect inclusion of irrelevant explanatory variables, and the other is the omission of relevant explanatory variables. The discussion will be analytical in the context of testing the general linear restrictions in a linear multivariate model. Experimental Monte Carlo simulation results will also be provided.

In Chapter 7, we consider problems associated with estimation of the dynamic (short run) demand systems under long run equilibrium conditions. A maximum likelihood procedure for estimating the dynamic (short run) demand system subject to the long run equilibrium restrictions from demand theory will be proposed. This procedure is based on an iterative algorithm using the Lyapunov solution. An empirical application to the Korean household expenditure data is provided.

Finally, a summary and conclusion are given in Chapter 8.

CHAPTER 2

ECONOMIC THEORY FOR CONSUMER'S BEHAVIOUR AND SYSTEMS OF DEMAND EQUATIONS: A SURVEY

2.1 Introduction

In this chapter, various specifications of a complete system of demand equations will be examined on the basis of static demand theory. In general, a complete system of demand equations, or simply a demand system, is defined as a set of demand functions which are derived from the postualtes of demand theory, such as utility maximisation or the cost (expenditure) minimisation, and are expressed in terms of present prices and total expenditure with the same functional form for all goods. Since the work of Stone (1954) on the estimation of the linear expenditure system, there has been continuing research on alternative specifications of demand systems [Deaton and Muellbauer (1980b)]. A large number of excellent surveys of the theory of consumer behaviour and demand systems are already available; notably, Brown and Deaton (1972), Phlips (1974), Barten (1977), Deaton and Muellbauer (1980a), Bewley (1984) and Deaton (1986).

Demand systems fall into three major categories: (1) arbitrary systems, (2) exact systems, and (3) approximate systems [Barnett (1981,p.43)]. An *arbitrary* demand system is defined as one which is specified without being based on the consumer's optimisation problem; the double logarithmic (double log) system falls into this category, being specified simply for the direct computation of price and total expenditure elasticities. A demand system with expressions derived from the optimisation of a well-defined objective function in terms of the consumer's utility or expenditure (cost) function is called an *exact* demand system. The linear expenditure system is an example of such an exact system. If the unknown demand function is approximated adequately, the resulting demand system is called an *approximate* demand system. Given that the true forms of utility and expenditure functions are always unknown, recent developments in specification of demand systems have emphasised approximate systems much more.

There may well be innumerable ways of generating approximate demand systems. We can directly approximate to an unknown demand function [Theil's (1965) Rotterdam model], or we can indirectly derive demand functions from approximate direct and indirect utility functions or from approximate expenditure function [e.g., Christensen, *et al.*'s (1975) translog model and Deaton and Muellbauer's (1980b) Almost Ideal Demand System]. An approximate demand system is usually derived as a first order approximation to the unknown demand functions, or derived from a second order approximation to the consumer's objective functions to obtain greater flexibility and a more informative parameterisation of the model. The principal examples of mathematical approximation methods are the differential approximation method [Theil's (1965) Rotterdam model], the Taylor series expansion [Christensen, *et al.*'s (1975) translog model], the Fourier series expansion [Gallant's (1981) Fourier flexible system], and the Laurent expansion [Barnett's (1983a) Miniflex Laurent demand system]. However, while exact systems are globally integrable and satisfy the theoretical restrictions of demand theory, approximate system are not integrable, (or are only locally integrable), and hence they do not satisfy such restrictions automatically. The global properties of approximate demand systems are generally unknown and have attracted the attentions of demand theorists [e.g., see Caves and Christensen (1980) and Barnett and Lee (1985) for theoretical approach, and Wales (1977) and Byron (1984) for Monte Carlo study].

These are the general problems in the specification of demand systems, and will be discussed briefly in this chapter. First we will consider the theory of demand and then we will examine the derivation and properties of four demand systems; the linear expenditure system (an exact demand system), the Rotterdam model, the translog model and the almost ideal demand system (three flexible functional demand systems).

The scheme of the chapter is as follows: In Section 2, the theory of demand will be sketched out, and in Section 3, implications of separable and homothetic utility functions will be examined. We will consider the linear expenditure system in Section 4, the Rotterdam model in Section 5, and flexible functional demand systems (the translog model and the almost ideal demand system) in Section 6. Finally, concluding remarks will be provided in Section 7.

2.2 The Theory of Demand

2.2.1 Preference Ordering and Utility

In classical economics, the theory of consumer behaviour is developed by postulating a preference ordering for each consumer, which is representable by a real-valued "utility" function.[1] Each consumer is assumed to choose the most preferred bundle among all available bundles of goods to maximise a utility function subject to the budget constraint. Accordingly, we assume that there exists a real-valued function u(q) defined on the consumption set $\mathbb{Q}$ such that

$$u = u(q) \tag{2.1}$$

for any nonnegative consumption bundle $q = (q_1,\dots,q_m)' \in \mathbb{Q}$ of all available m goods in market and $u \in \mathbb{R}^1$, the real number space, where a consumption set $\mathbb{Q}$ is a closed and convex nonnegative orthant in $\mathbb{R}^m$. A function u(q) in (2.1) is called the *utility function* and its image, a real number u, is called the utility index or simply *utility*, which reflects the consumer's satisfaction obtained from consuming q.

The theoretical foundations for the existence and properties of utility functions are provided by axioms, which postulate that the consumer's preference (taste) is represented by an ordering (binary relation) on a consumption set $\mathbb{Q}$ satisfying the relations: (i) completeness, (ii) reflexivity, (iii) transitivity, (iv) continuity, (v) strong monotonicity, (vi) local nonsatiation, and (vii) strict convexity. Axioms (i) - (iii) are sufficient for the existence of the utility function, and the axioms (iv), (v) and (vii) characterise the utility function as a continuous, monotonic increasing, and strictly quasi-concave function of $q \in \mathbb{Q}$. We also assume that the utility function $u = u(q)$ is twice differentiable with respect to q_i's for all $i = 1, \ldots, m$. [Comprehensive discussions will be found in Varian (1978, Chapter 3), Deaton and Muellbauer (1980a, Chapter 2), and Phlips (1974, Chapter 1). For a more mathematical discussion see Debreu (1959, Chapter 4).]

2.2.2 Utility Maximisation and Marshallian Demand Functions

Since the utility function is assumed to be monotonic and increasing in q, the marginal utilities, $\partial u/\partial q_i$'s, are always positive for all goods. This means that the maximisation of the utility function cannot be achieved unconditionally without restriction, so that the consumer's choice will be limitless without having any constraint imposed on utility maximisation [Theil (1975, p.1)]. However, it is also assumed that each consumer always encounters a *budget constraint* in the decision making process, and utility maximisation is then constrained. Writing the budget constraint in vector notation as

$$p'q = \mu, \tag{2.2}$$

the consumer's problem becomes

$$max \;\; \bar{u}(q,\lambda) = u(q) - \lambda(p'q - \mu), \tag{2.3}$$

where $p = (p_1,\ldots,p_m)'$ is an m x 1 vector of prices, μ is the consumer's total expenditure and λ is a scalar Lagrange multiplier.

The first order conditions for maximising (2.3) are

$$u_q = \lambda p \tag{2.4}$$

and

$$p'q = \mu, \tag{2.5}$$

where $u_q = \partial u/\partial q = [\partial u/\partial q_1, \ldots, \partial u/\partial q_m]'$. Since the Hessian matrix, $U = \partial^2 u/\partial q \partial q'$, is negative definite from the strict quasi-concavity of u(q), a global maximum of u(q) can be achieved conditional on the budget constraint [Theil (1975, p.3-4)]. Solutions of (2.4) and (2.5) for q and λ lead to a set of m *Marshallian* demand functions

$$q = q(p,\mu), \tag{2.6}$$

and the *marginal utility of income*

$$\lambda = \lambda(p,\mu). \tag{2.7}$$

The m Marshallian demand functions in (2.6), which relate the individual's demand for goods to the prices of all commodities and total expenditure (or "income"), comprise of a system of demand equations. From (2.4) and the fact that $\partial u/\partial q_i > 0$, $\lambda(p,\mu)$ in (2.7) is a positive scalar-valued function of p and μ. The properties of Marshallian demand functions will be discussed later in this section in the context of restrictions on demand systems.

2.2.3 The Indirect Utility Function

The substitution of Marshallian demand functions (2.6) into the utility function (2.1) gives the maximum utility attainable for given p and μ. The resulting utility function can then be expressed in terms of p and μ

$$u^* = \upsilon(p,\mu) \tag{2.8}$$

and is called the *indirect utility function*. This utility function is continuous at all $p > 0$ and $\mu > 0$; it is nonincreasing in p but nondecreasing in μ, it is also quasi-convex in p, and is homogeneous of degree 0 in (p,μ) [see for details Varian (1978, p.89-90)].

An important application of the indirect utility function $\upsilon(p,\mu)$ is Roy's identity

$$q_i(p,\mu) = - [\partial\upsilon(p,\mu)/\partial p_i] \big/ [\partial\upsilon(p,\mu)/\partial\mu] \tag{2.9}$$

for all $i = 1,...,m$, which is a useful formula for derivation of the Mashallian demand functions $q_i(p,\mu)$'s from the indirect utility function [Varian (1978 p.93)].

2.2.4 Expenditure Minimisation and Hicksian Demand Functions

The *dual* of the consumer's utility maximisation problem is his minimisation of the cost (or expenditure $p'q$) of a given level of utility, $u = u_o$; that is,

$$\textit{min} \;\; p'q \;\; \text{subject to } u(q) = u_o. \qquad (2.10)$$

The demand functions obtained from optimisation of (2.10) can then be expressed in terms of p and u_o

$$q^\circ = h(p,u_o) \qquad (2.11)$$

for given p and u_o and are called *Hicksian* or *compensated* demand functions. The minimum expenditure attained at $q = q^\circ$ can be written as

$$\mu^* = p'h(p,u_o) = e(p,u_o), \qquad (2.12)$$

and is called the *expenditure* or *cost* function. The expenditure function, $e(p,u_o)$, is nondecreasing, homogeneous of degree one, concave and continuous in prices p. Moreover, the expenditure function, $e(p,u_o)$, is the inverse function of the indirect utility function $\upsilon(p,\mu)$, in the sense that the equalities

$$e[p,\nu(p,\mu)] = \mu \tag{2.13}$$

and

$$\nu[p,e(p,u_o)] = u_o \tag{2.14}$$

hold [Varian (1978, p.91-92)].

The Hicksian demand functions can be obtained directly from the partial derivatives of the expenditure function

$$q_i^o = h_i(p,u_o) = \partial e(p,u_o)/\partial p_i, \tag{2.15}$$

for all $i = 1,\ldots,m$. The relation (2.15) is called *Shephard's lemma.* There are important relations between Hicksian demand functions and Marshallian demand functions in that

$$h_i(p,u_o) = q_i[p,e(p,u_o)], \tag{2.16}$$

that is, the Hicksian demand function at utility u_o is the same as the Marshallian demand function at total expenditure $e(p,u_o)$, and

$$q_i(p,\mu) = h_i[p,\nu(p,\mu)], \tag{2.17}$$

that is, the Marshallian demand function at total expenditure μ is the same as the Hicksian demand function, at utility $\nu(p,\mu)$ [see Varian (1978, p.92)].

The relations (2.16) and (2.17) ensure that at any given equilibrium position, the consumer's behaviour can be explained equally well by the assumption of utility maximisation or expenditure minimisation. Thus, the specification of an expenditure function is equivalent to the specification of a well-behaved utility function. Since the indirect utility function is simply the inverse of the expenditure function, the same duality holds for the indirect utility function [Varian (1978, p.99)]. Therefore, specifications of demand functions derived from direct and indirect utility functions and the expenditure function are theoretically equivalent.

2.2.5 The Fundamental Matrix Equation and the Slutsky Matrix

As Marshallian demand functions are derived from maximisation of a utility function subject to the budget constraint, their properties also result from the axioms on the consumer's preference ordering and relate to the budget constraint. These properties take the form of mathematical restrictions on the derivatives of the demand functions which can be obtained by totally differentiating the first order conditions, (2.4) and (2.5), as well as the solutions, (2.6) and (2.7).

Total differentiation of (2.4) and (2.5) gives the "fundamental matrix equation of the theory of consumer demand" [Barten (1964)]

$$\begin{bmatrix} U & p \\ p' & 0 \end{bmatrix} \begin{bmatrix} dq \\ -d\lambda \end{bmatrix} = \begin{bmatrix} 0 & \lambda I \\ 1 & -q' \end{bmatrix} \begin{bmatrix} d\mu \\ dp \end{bmatrix}, \qquad (2.18)$$

where U is the Hessian matrix such that $U = \partial^2 u/\partial q \partial q'$. Then, total differentiation of (2.6) and (2.7) gives

$$\begin{bmatrix} dq \\ -d\lambda \end{bmatrix} = \begin{bmatrix} q_\mu & q_p \\ -\lambda_\mu & -\lambda_p' \end{bmatrix} \begin{bmatrix} d\mu \\ dp \end{bmatrix}, \tag{2.19}$$

where $q_\mu = \partial q/\partial\mu$ is an $m \times 1$ vector, $\lambda_p = \partial\lambda/\partial p$ an $m \times 1$ vector, $q_p = \partial q/\partial p$ an $m \times m$ matrix, and $\lambda_\mu = \partial\lambda/\partial\mu$ a scalar. Substitution of (2.19) into (2.18) gives the fundamental matrix equation expressed in terms of partial derivatives as

$$\begin{bmatrix} U & p \\ p' & 0 \end{bmatrix} \begin{bmatrix} q_\mu & q_p \\ -\lambda_\mu & -\lambda_p' \end{bmatrix} = \begin{bmatrix} 0 & \lambda I \\ 1 & -q' \end{bmatrix} \tag{2.20}$$

[Phlips (1974, p.47-48)]. Then, using the inversion result for a partitioned matrix,

$$\begin{bmatrix} U & p \\ p' & 0 \end{bmatrix}^{-1} = (p'U^{-1}p)^{-1} \begin{bmatrix} (p'U^{-1}p)U^{-1} - U^{-1}pp'U^{-1} & U^{-1}p \\ p'U^{-1} & 1 \end{bmatrix},$$

the solutions of (2.20) are given by

$$q_\mu = (p'U^{-1}p)^{-1}U^{-1}p, \tag{2.21}$$

$$q_p = \lambda U^{-1} - \lambda(p'U^{-1}p)^{-1}U^{-1}pp'U^{-1} - (p'U^{-1}p)U^{-1}pq' \tag{2.22}$$

$$\lambda_\mu = (p'U^{-1}p)^{-1} \tag{2.23}$$

$$\lambda_p = -[\lambda(p'U^{-1}p)^{-1}U^{-1}p + (p'U^{-1}p)^{-1}q], \tag{2.24}$$

which implies that

$$q_\mu = \lambda_\mu U^{-1}p, \qquad (2.25)$$

$$q_p = [\lambda U^{-1} - (\varphi\mu)q_\mu q_\mu'] - q_\mu q', \qquad (2.26)$$

$$\lambda_\mu = \lambda(\varphi\mu)^{-1} \qquad (2.27)$$

$$\lambda_p = -\lambda[q_\mu + (\varphi\mu)^{-1}q], \qquad (2.28)$$

where φ is the inverse of the *income flexibility* (elasticity) of the marginal utility of money and is given by

$$\varphi = [\partial\, ln\, \lambda / \partial\, ln\, \mu]^{-1} = (\lambda/\mu)(p'U^{-1}p)^{-1} \qquad (2.29)$$

[Brown and Deaton (1972, p.1161-1162)]. Equation (2.22) or (2.26) is called the *Slutsky equation* and can be written as

$$q_p = K - q_\mu q', \qquad (2.30)$$

where K is an $m \times m$ *substitution* matrix, called the *Slutsky* matrix, such that

$$K = \lambda U^{-1} - \lambda(p'U^{-1}p)^{-1}U^{-1}pp'U^{-1} = \lambda U^{-1} - (\varphi\mu)q_\mu q_\mu' \qquad (2.31)$$

and $-q_\mu q'$ is the matrix of income effects.[2] Rearranging (2.30), the Slutsky equation can be written as

$$K = q_p + q_\mu q', \tag{2.32}$$

and the (i,j)'th term of K is then

$$k_{ij} = \partial q_i/\partial p_j + q_j \partial q_i/\partial \mu \tag{2.33}$$

for all $i,j = 1,\ldots,m$. The k_{ij}'s are called the *Slutsky* coefficients, while the term $\partial q_i/\partial p_j$ is called the *uncompensated* price effect. The Slutsky equation is unaffected by transformations of the utility function since the right hand side of (2.31) will be unaffected.

2.2.6 The Slutsky Equation for a Hicksian Demand Function

The Slutsky matrix can be obtained by partial differentiation of Hicksian demand functions h(p,u) with respect to p. Partial differentiation of equation (2.16) with respect to p_j gives

$$\partial h_i(p,u_o)/\partial p_j = \partial q_i/\partial p_j + q_j \partial q_i/\partial \mu \tag{2.34}$$

by the Envelope theorem and Shephard's lemma (2.15), so that

$$k_{ij} = \partial h_i(p,u_o)/\partial p_j \tag{2.35}$$

[Varian (1978, p.96)]. Thus, the Slutsky coefficients k_{ij} can be thought as the substitution effect of a change in price p_j on the demand for good i with utility held constant. In this sense, the Slutsky coefficients k_{ij}'s in (2.33) are also called the *substitution effect*, or *the compensated price effect*. The diagonal terms of K, k_{ii} for i = 1,...m, are called *own-substitution* effects, while the off-diagonals, k_{ij}, $i \neq j$, are *cross-substitution* effects. Returning to equation (2.30), we can see that the uncompensated price effect, $q_p = [\partial q_i/\partial p_j]$, can be decomposed initially into two separate effects: the substitution effect $K = [k_{ij}]$ and the income effect $q_\mu q' = [q_j \partial q_i/\partial \mu]$.

2.2.7 The Restrictions on a System of Demand Equations

The properties of demand functions can be expressed in terms of the restrictions on a system of demand equations derived from equations (2.21) to (2.31) [Phlips (1974, p.51-53)].

Premultiplication of (2.21) by p′ gives the *adding-up* restriction that

$$p'q_\mu = 1, \tag{2.36}$$

that is, the sum of the marginal propensities to consume is exactly equal to one.

Postmultiplying the transpose of (2.22) by p gives the *'Cournot aggregation'* restriction that

$$q_p'p = -q. \tag{2.37}$$

Postmultiplying K in (2.31) by p, we have the *homogeneity* restrictions

$$Kp = q_p p + \mu q_\mu = 0, \tag{2.38}$$

since

$$Kp = \lambda U^{-1}p - \lambda(p'U^{-1}p)^{-1}U^{-1}p(p'U^{-1}p)$$

$$= \lambda U^{-1}p - \lambda U^{-1}p = 0. \tag{2.39}$$

The homogeneity restriction can be written, in summation notation, as

$$\sum_{j=1}^{m} p_j \partial q_i/\partial p_j + \mu \partial q_i/\partial \mu = 0, \tag{2.40}$$

for each equation $i = 1,\ldots m$, and implies that each demand equation is homogeneous of degree zero in income and prices, so that proportional changes in prices and total expenditure will leave the level of demand unchanged.

The *symmetry* restriction on the Slutsky matrix K, that is, $K = K'$, follows directly from the definition of K in (2.31), since U^{-1} and $q_\mu q_\mu'$ are symmetric. This implies

$$k_{ij} = k_{ji}. \tag{2.41}$$

for all $i,j = 1,\ldots,m$, where k_{ij} is the Slutsky coefficient defined in (2.33).

The *negativity* restriction that the Slutsky matrix is negative semi-definite follows from concavity of the expenditure function e(p,u), since

$$k_{ij} = \partial h_i(p,u)/\partial p_j = \partial e^2(p,u)/\partial p_i \partial p_j < 0.$$

[Varian (1978, p.99)]. An implication of this restriction is that all own substitution effects k_{ii} are negative, since

$$\iota_i' K \iota_i = k_{ii} < 0 \qquad (2.42)$$

where ι_i is an $m \times 1$ unit vector having one at the i'th position and zero elsewhere.

These are the five demand restrictions that a demand system should satisfy. However, it can easily be seen that the Cournot aggregation restriction is automatically satisfied given the other restrictions, so that it is redundant and can be ignored [Phlips (1974, p.51)].

2.2.8 Some Informative Measurements and the Characterisation of Goods

The *marginal propensity to consume* a good i (mpc_i) or *marginal budget share* of a good i is defined as

$$mpc_i = p_i \partial q_i/\partial \mu = \partial p_i q_i/\partial \mu, \qquad (2.43)$$

and the *total expenditure (income) elasticity,* e_i, is given by the logarithmic derivatives of the Marshallian demands

$$e_i = \partial \ln q_i / \partial \ln \mu = [\partial q_i/\partial \mu][\mu/q_i], \qquad (2.44)$$

for $i = 1,\ldots,m$.

A good having a positive mpc is called a *normal* good, while a negative mpc denotes an *inferior* good. Classified by total expenditure elasticity, a good having a total expenditure elasticity greater than 1, so that the marginal propensity to consume is greater than its average budget share, is called a *luxury* good, and otherwise is called a *necessity*.

The *uncompensated price elasticities*, e_{ij}'s, are given by the logarithmic derivatives of the Marshallian demands

$$e_{ij} = \partial \ln q_i / \partial \ln p_j = [\partial q_i/\partial p_j][p_j/q_i], \qquad (2.45)$$

while the *compensated price elasticities*, e^*_{ij}'s, are given by

$$e^*_{ij} = k_{ij}[p_j/q_i], \qquad (2.46)$$

where k_{ij} is the (i,j)'th element of the Slutsky matrix K. From (2.33), it is obvious that the uncompensated price elasticity, e_{ij}, can be expressed in terms of the compensated price elasticity, e^*_{ij}, and the total expenditure elasticity, e_i, as

$$e_{ij} = e^*_{ij} - e_i w_j, \qquad (2.47)$$

where w_j is the expenditure share for a good j [see Deaton and Muellbauer (1980a, p.62)].

These price elasticities cannot determine complementarity, substitutability or independence between goods, since it is quite possible to have both $\partial q_i/\partial p_j > 0$ and $\partial q_j/\partial p_i < 0$ and in general $e_{ij} \neq e_{ji}$ and $e^*_{ij} \neq e^*_{ji}$. Instead, the Slutsky coefficient k_{ij} must be used to identify such relations.[3] According to Hicks' definition [see Deaton and Muellbauer (1980,p.46)], any pair of commodities (i,j) for which $k_{ij} > 0$ are called *substitutes*, while any pair for which $k_{ij} < 0$ are *complements*. If $k_{ij} = 0$, goods i and j are called *independent*.[4]

2.3 Separable and Homothetic Utility Functions

In this section, we will review separable and homothetic utility functions. For a more detailed discussion on these subjects, see Phlips (1974, Chapter 3) or Deaton and Muellbauer (1980a, Chapter 5).

2.3.1 Separable Utility

From the practical point of view, it is often more useful to hypothesise that a consumer's choice is dependent on groupwise preferences within broad commodity groups rather than overall preferences on individual commodities in one big grouping. For example, the choice between butter and margarine may be independent of that between T-shirts and blouses. Both butter and margarine (or T-shirts and blouses) can be categorised into the same commodity group, say food (or clothing) and can be thought of as a single good. We assume that all m commodities can be categorised into M broader commodity groups, such as food, housing, clothing, and so on,[5] and that each commodity group has a sub-utility function defined by goods *within* that group. If an overall utility function can be expressed in terms of these sub-utilities, it is said to be *weakly separable*. Thus, a weakly separable utility function u(q) can formally be written as

$$u(q) = f[u^1(\bar{q}^1),\ldots,u^M(\bar{q}^M)], \tag{3.1}$$

where f is a differentiable increasing function, while $\bar{q}^1,\ldots,\bar{q}^{M-1}$, and $\bar{q}^M$ are the M commodity groups and $u^K(\bar{q}^K)$ is a sub-utility function for a group $\bar{q}^K$ for $K = 1,\ldots,M$.

The fundamental implication of weak separability of a utility function is that the marginal rate of substitution between any two commodities belonging to the same group, say G, is independent of the consumption of a commodity in any other group, that is,

$$\partial(u_i/u_j)/\partial q_k = 0 \tag{3.2}$$

for all $i,j \in G$ and $k \notin G$ [Gorman (1959) and Phlips (1974, p.69]. Separability implies that the maximisation of the overall utility function u(q) subject to the budget constaint (2.2) reduces to

$$\max u^I(\bar{q}^I) \text{ subject to } p^{I\prime}q^I = \mu^I \tag{3.3}$$

for all I = 1,...,M, where $p^{I\prime}q^I = \mu^I$ is the intragroup budget constraint, μ^I is the total expenditure on group I and p^I is a vector of prices of goods within group I.[6] Therefore, demand functions within group I can be written as a function of μ^I and prices p^I only:

$$q^I = q(p^I, \mu^I). \tag{3.4}$$

However, this does not mean that demands in one group are independent of prices of goods in other groups. Since prices of goods in other groups affect demands in the group through their effect on the group expenditure μ^I, demand functions in (3.4) possess all the usual properties of demand functions [Phlips (1974, p.73)]. However, for $i \in q^I$, $j \in q^J$, and $q^I \neq q^J$, the uncompensated price effect, $\partial q_i/\partial p_j$, and the substitution effects, k_{ij}, are just

$$\partial q_i/\partial p_j = \chi^{IJ}[\partial q_i/\partial \mu^I] \tag{3.5}$$

and

$$k_{ij} = \xi^{IJ}[\partial q_i/\partial \mu][\partial q_j/\partial \mu], \tag{3.6}$$

respectively, where $\chi^{IJ} = [\partial \mu^I/\partial p_j]$ and ξ^{IJ} are common terms applying to all goods in q^I and q^J [Phlips (1974,p.74) and Deaton and Muellbauer (1980a, p.128)]. However, within each group, the general Slutsky equation (2.33) holds.

A utility function u(q) is said to be *strongly separable,* if it can be written as

$$u = f[u^1(q^1) + u^2(q^2) + \ldots + u^M(q^M)] \quad (3.7)$$

so that the utility function takes an explicitly additive form under some monotone transformation. Strong separability assumes that consumers' preferences between groups are strictly independent of each other. However, interactions between commodities *within* groups are assumed to hold.

A utility function is said to be *additive*, if the q^I's in (3.7) are individual commodities, that is, if the strong separability is pointwise. The important implication of additivity is that the marginal utility of commodity i is independent of the consumption of any other commodity, that is,

$$\partial^2 u/\partial q_i \partial q_j = 0 \text{ for } i \neq j. \quad (3.8)$$

Thus, the uncompensated (cross) price effect is proportional to the income derivatives,

$$\partial q_i/\partial p_j = -\chi^j(\partial q_i/\partial \mu), \quad (3.9)$$

where $\chi^j = (q_j + \varphi \partial q_j/\partial \mu)$ and $\varphi = \lambda/(\partial\lambda/\partial\mu)$, but the cross substitution effect is given by

$$k_{ij} = -\varphi[\partial q_i/\partial \mu][\partial q_j/\partial \mu] \quad (3.10)$$

and the own substitution effect is

$$k_{ii} = [\varphi/p_i][\partial q_i/\partial\mu][1 - p_i\partial q_i/\partial\mu] \qquad (3.11)$$

[Houthakker (1960, p.248)]. The relation (3.9) is the necessary and sufficient condition for a utility function to be additive.

2.3.2 Homothetic Utility Functions

A utility function is said to be *homothetic* if u(q) can be written, for an arbitrary scalar $\tau > 0$

$$u = F[v(q)], \qquad (3.12)$$

where f is monotonic and v(q) is homogeneous of degree one; $v(\tau q) = \tau v(q)$ [Deaton and Muellbauer (1980a, p.143)]. This means that utility obeys the equivalence of constant returns to scale and thus doubling quantities doubles utility. Homotheticity has an important implication for the form of the demand equations: demand functions derived from a homothetic utility function can be written as

$$q_i = \Phi_i(p_1, p_2, \ldots, p_m)\mu \qquad (3.13)$$

where $\Phi_i(p)$ is homogeneous of degree -1 in prices [Katzner (1970, p.76) and Phlips (1974, p.87)]. It is clear from (3.13) that the budget share is independent of total expenditure and all total expenditure elasticities are unity since

$$e_i = (\partial q_i/\partial \mu)(\mu/q_i) = \Phi_i(p)[\Phi_i(p)]^{-1} = 1. \qquad (3.14)$$

The Engel curve derived from a homothetic utility function is a straight line through the origin. The proposition that all expenditure elasticities should be unity contradicts all known household budget studies [Philips (1974, p.88)].

2.4 The Linear Expenditure System

The linear expenditure system (LES) is an exact demand system derived from the additive Stone-Geary or Klein-Rubin utility function [Klein and Rubin (1948) and Stone (1954)]. Therefore, the linear expenditure system automatically satisfies the restrictions of demand theory as well as those stemming from additivity of the utility function.

The underlying Stone-Geary utility function is given by

$$u(q) = \prod_{i=1}^{m} (q_i - \gamma_i)^{\beta_i} \qquad (4.1)$$

with parameters γ_i's and β_i's. For (4.1) to act as a utility function, it is sufficient to require that for all $i=1,\ldots,m$,

$$0 < \beta_i < 1, \quad \sum_{i=1}^{m} \beta_i = 1, \text{ and } \quad \gamma_i < q_i. \qquad (4.2)$$

Then, the demand equations are obtained from the maximisation of the logarithm of (4.1)

$$\ln u(q) = \sum_{i=1}^{m} \beta_i \ln (q_i - \gamma_i) \qquad (4.3)$$

subject to the budget constraint $\mu = \sum_{j=1}^{m} p_j q_j$, and are given by

$$p_i q_i = p_i \gamma_i + \beta_i(\mu - \sum_{j=1}^{m} p_j \gamma_j), \quad i = 1, \ldots, m. \qquad (4.4)$$

The marginal utility of income is

$$\lambda = (\mu - \sum_{j=1}^{m} p_j \gamma_j)^{-1}. \qquad (4.5)$$

The term $p_i \gamma_i$ may be interpreted as the subsistence level of expenditure on good i, while the sum of $\sum_{j=1}^{m} p_j \gamma_j$ is usually interpreted as total subsistence expenditure. $(\mu - \sum_{j=1}^{m} p_j \gamma_j)$ is interpreted as supernumerary expenditure and $\beta_i(\mu - \sum_{j=1}^{m} p_j \gamma_j)$ as supernumerary expenditure spent on i′th good. Substitution (4.4) divided by p_i into (4.1) leads to the indirect utility function

$$v(p,\mu) = \prod_{i=1}^{m}[(\mu - \sum_{j=1}^{m} p_j \gamma_j)(\beta_i / p_i)]^{\beta_i}, \qquad (4.6)$$

from which we can confirm that the demand function (4.4) can be obtained using Roy′s identity.

It is obvious from (4.1) that $\partial^2 u / \partial q_i \partial q_j = 0$ for $i \neq j$, since the underlying utility function is additive. The relations (3.9) to (3.11) in the previous section are satisfied by the demand functions in (4.4), as is demonstrated below. From (4.4) and (4.5), we have

$$\partial q_i / \partial \mu = \beta_i / p_i, \qquad (4.7)$$

$$\partial q_i/\partial p_j = -(\beta_i/p_i)\gamma_j \text{ for } i \neq j \quad (4.8)$$

$$= -(\beta_i/p_i)[\gamma_i + 1/p_i\lambda] \text{ for } i = j, \quad (4.9)$$

and

$$k_{ij} = (1/\lambda)(\beta_i\beta_j/p_ip_j) \text{ for } i \neq j \quad (4.10)$$

$$= (1/\lambda)(\beta_i/p_i^2)(\beta_i-1) \text{ for } i = j. \quad (4.11)$$

From (4.10), we see that symmetry restictions, $k_{ij} = k_{ji}$, for $i \neq j$, are automatically satisfied. In addition, from (4.7) and (4.11), the adding up and negativity restictions are also satisfied, providing the conditions in (4.2) hold. From (4.10) and (4.11), it follows that the homogeneity restrictions are satisfied, since for all goods

$$\sum_{j=1}^{m} p_j k_{ij} = \sum_{j \neq i} p_j(\beta_i\beta_j/\lambda p_i p_j) + p_i(\beta_i/\lambda p_i^2)(\beta_i-1)$$

$$= (\beta_i/\lambda p_i)\sum_{j \neq i}\beta_j + (\beta_i/\lambda p_i)(\beta_i-1)$$

$$= (\beta_i/\lambda p_i)(1-\beta_i) + (\beta_i/\lambda p_i)(\beta_i-1) = 0.$$

However, the linear expenditure system has the deficiency that complementarity is infeasible since all substitution effects are positive. Hence, all goods are substitutes in the linear expenditure system.

With the analogous concept of the translog approximation in mind, the logarithm of the Stone-Geary utility function (4.2) can be regarded as a first order Taylor series expansion of logarithm of any utility function around a vector of subsistence quantities γ_i's with the parameters

$$\beta_i = \partial\, ln\, u/\partial\, ln\, q_i \Big|_{q_i=\gamma_i} \tag{4.12}$$

and $ln\, u(\gamma_i) = 0$. However, despite the fact that the utility function in (4.2) is a first order approximation to any utility function, the demand equations in (4.4) cannot be regarded as an approximation to general demand equations. The reason is that an order of approximation is lost in the step from the utility function to the demand functions when solving first order conditions (4.2) [see Deaton (1986, p.1789)]. For approximate demand equations to be first order approximations, the utility function should be approximated at least up to the second order. In this sense, the linear expenditure system is not 'flexible'. In fact, the linear expenditure system is over-restrictive in the sense that it has too few parameters to give it a reasonable flexibility. Deaton (1986, p.1788) points out that "the linear expenditure system does little more than fit bivariate regressions between individual expenditures and their total." However, the linear expenditure system can remain as an exact demand system derived from a specific utility function (4.1).

Moreover, demand equations in the linear expenditure system are nonlinear in parameters, γ_i and β_i, so that estimation is computationally expensive, as a nonlinear procedure has to be adopted. Two-step iterative estimation procedures or the Gauss-Newton method are the most commonly used in empirical such application [see, for example, Stone (1954), Pollak and Wales (1969), Parks (1969, 1971), Pollak and Wales (1978) and Klevmarken (1981)]. However, additivity of the underlying utility function, inflexibility, over-restrictiveness, nonlinearity of the model, and the inability to represent complementarity are severe costs paid for the exactness of the linear expenditure system.

2.5 The Rotterdam Model

The Rotterdam model is an approximate demand system, proposed by Theil (1965). The model is a direct first order approximation to any arbitrary demand function based on the total differentiation of the demand function $q = q(p,\mu)$:

$$dq = q_\mu d\mu + q_p dp. \tag{5.1}$$

Two different versions of the Rotterdam model are available; the relative price version and the absolute price version. The difference between these two versions lies in the different substitutions of q_p into (5.1). Substitution of $q_p = [\lambda U^{-1} - (\varphi\mu)q_\mu q_\mu'] - q_\mu q'$ yields the relative price version, whilst using $q_p = K - q_\mu q'$ yields the absolute price version [see Theil (1975, Chapter 2)]. However, the absolute price version of the Rotterdam model has advantages over the relative price version. For example, it is capable of estimating the Slutsky matrix directly, and is linear in parameters to be estimated. We will discuss the derivation of the relative price version and then focus on the absolute price version and its properties.

2.5.1 The Relative Price Version

Substitution of $q_p = [\lambda U^{-1} - (\varphi\mu)q_\mu q_\mu'] - q_\mu q'$ into (5.1) gives

$$dq = q_\mu d\mu + [\lambda U^{-1} - (\varphi\mu)q_\mu q_\mu' - q_\mu q'] \, dp \tag{5.2}$$

$$= q_\mu(d\mu - q'dp) + \lambda U^{-1}dp - (\varphi\mu)q_\mu(q_\mu'dp),$$

where $\varphi = (\lambda/\mu)(p'U^{-1}p)^{-1}$ is the inverse of the income flexibility of the marginal utility of money. Since $(\varphi\mu)q_\mu = (\lambda/\lambda_\mu)\lambda_\mu U^{-1}p = \lambda U^{-1}p$ from (2.25) and (2.27), (5.2) can be written as

$$dq = q_\mu[d\mu - q'dp] + \lambda U^{-1}dp - \lambda U^{-1}p[q_\mu'dp]. \tag{5.3}$$

Then, using the relation $dx = x\,d\ln x$ and multiplying p_i/μ on both sides of (5.3), (5.3) can be expressed in terms of changes in logarithms, the i'th equation being

$$w_i\, d\ln q_i = b_i d\bar{M} + \sum_{j=1}^{m} G_{ij}[d\ln p_j - \sum_{k=1}^{m} b_k\, d\ln p_k] \tag{5.4}$$

where b_i is the marginal budget share of good i

$$b_i = p_i(\partial q_i/\partial \mu) = w_i e_i, \tag{5.5}$$

and $G_{ij} = \lambda u^{ij} p_i p_j/\mu$, u^{ij} is the (i,j)'th element of the inverse of the Hessian U, while e_i is the total expenditure (income) elasticity on good i. The rate of change in real income (total expenditure) flow $d\overline{M}$ is defined as

$$d\overline{M} = d\ln \mu - \sum_{k=1}^{m} w_k d\ln p_k. \tag{5.6}$$

Equation (5.4) is the relative price version of the Rotterdam model and is nonlinear in parameters b_i and G_{ij}.

2.5.2 The Absolute Price Version

As mentioned, the derivation of the absolute price version of the Rotterdam model is the same as that of the relative price version except for the substitution of the Slutsky equation, $q_p = K - q_\mu q'$, into (5.1). After substitution, (5.1) can be written as

$$dq = q_\mu d\mu + (K - q_\mu q')dp, \tag{5.7}$$

so that the i'th equation in (5.7) can be written as

$$dq_i = (\partial q_i/\partial \mu)[d\mu - \sum_{k=1}^{m} q_k dp_k] + \sum_{j=1}^{m} k_{ij} dp_j. \tag{5.8}$$

Again, using the relation $dx = x\,d\ln x$ and multiplying p_i/μ on both sides of (5.8), we have the absolute price version of the

Rotterdam model

$$w_i \, d\ln q_i = b_i d\bar{M} + \sum_{j=1}^{m} C_{ij} \, d\ln p_j, \tag{5.9}$$

where b_i and $d\bar{M}$ are defined as in (5.5) and (5.6), and C_{ij} are the Slutsky coefficients are expressed as compensated cross price elasticities weighted by expenditure shares such that, for all i and j,

$$C_{ij} = k_{ij}p_i p_j/\mu = w_i e^*_{ij}, \tag{5.10}$$

and the e^*_{ij} are the compensated cross price elasticities between goods i and j. We have derived the absolute price version of the Rotterdam model as an approximation to a Marshallian demand function. However, it can also be derived for a first order approximation to a Hicksian demand function [Deaton (1986, p.1789)].

Total differentiation of the Hicksian demand function $q_i = h_i(u_o, p)$ gives

$$dq_i = (\partial h/\partial u_o)du_o + \sum_{j=1}^{m} k_{ij} dp_j, \tag{5.11}$$

where k_{ij} is the (i,j)'th Slutsky coefficient. Then, using the relation $dx = x \, d\ln x$ and multiplying (5.11) by p_i/μ, we have

$$w_i \, d\ln q_i = (u_o p_i/\mu)(\partial h_i/\partial u_o) \, d\ln u_o + \sum_{j=1}^{m} C_{ij} \, d\ln p_j, \tag{5.12}$$

where C_{ij} is defined as in (5.10). Total differentiation of the expenditure function $\mu = c(u_o, p)$ gives

$$d\,ln\, u_o = (\partial\, ln\, \mu / \partial\, ln\, u_o)^{-1} d\overline{M} \tag{5.13}$$

where $d\overline{M}$ is defined as in (5.6). The first term on right hand side of (5.12) then becomes

$$(u_o p_i/\mu)(\partial h_i/\partial u_o)(\partial\, ln\, \mu/\partial\, ln\, u_o)^{-1} d\overline{M}$$

$$= (u_o p_i/\mu)(q_i/u_o)[\partial\, ln\, h_i/\partial\, ln\, u_o](\partial\, ln\, \mu/\partial\, ln\, u_o)^{-1} d\overline{M}$$

$$= (p_i q_i/\mu)(\partial\, ln\, h_i/\partial\, ln\, \mu) d\overline{M} = w_i e_i d\overline{M} = b_i d\overline{M},$$

which proves that the equation (5.12) reduces to (5.9).

The discrete analogue of the absolute version of Rotterdam model (5.9), (hereafter simply called the Rotterdam model), at time t can be given by

$$w^*_{it} \Delta\, ln\, q_{it} = b_i \Delta\overline{M} + \textstyle\sum_{j=1}^{m} C_{ij} \Delta\, ln\, p_{jt} \tag{5.14}$$

where Δ is log difference operator such that

$$\Delta\, ln\, X_{it} = ln\, X_{it} - ln\, X_{i(t-1)}$$

and w^*_{it} is given by

$$w^*_{it} = \tfrac{1}{2}[w_{it} + w_{i(t-1)}]. \tag{5.15}$$

Since the model (5.14) is linear in coefficients and the explanatory variable are identical in each equation, OLS estimation produces maximum likelihood estimates. However, since the Rotterdam model is not an exact demand system, these OLS estimates do not satisfy the restrictions of demand theory automatically. The restrictions have to be imposed using a constrained version of Zellner's (1962) iterative SUR estimator, to produce maximum likelihood estimates of the b_i's and C_{ij}'s. [Full details are given in Chapter 3.]

The restrictions imposed on the Rotterdam model (5.9) are as follows [Theil (1975, p.49]:

Adding up: $\sum_{i=1}^{m} b_i = 1$ and (5.16)

$\sum_{i=1}^{m} C_{ij} = 0$, for all $j = 1,\ldots,m$,

Homogeneity: $\sum_{j=1}^{m} C_{ij} = 0$, for all $i = 1,\ldots,m$, (5.17)

Symmetry: $C_{ij} = C_{ji}$, for all $i,j = 1,\ldots,m$, (5.18)

Negativity: The matrix $C = [C_{ij}]$ is negative semi-definite. (5.19)

The adding-up restrictions are automatically satisfied in estimation, but the homogeneity and symmetry restrictions need to be be imposed and tested.

The Rotterdam model is "flexible" in the sense that it has sufficient parameters for a first order local approximation to any demand function. It has the ability to estimate marginal budget shares and income compensated substitution effects directly, as well as identifying substitutes and complements. The estimate of the Slutsky matrix can be obtained from the restricted estimates under (5.16)-(5.19). However, satisfaction of these restrictions does not imply integrablity of the Rotterdam model.[7] Integrablity requires much more severe conditions. Yoshihara (1969) and Barnett (1979) have shown that the Rotterdam model is integrable if and only if the underlying utility function is of Cobb-Douglas form.[8] This means that the utility function underlying the Rotterdam model must be a linear logarithmic utility function which is not only additive but homothetic. Therefore, if integrability is required *a priori*, the Rotterdam model must be subject to additivity and homotheticity, so that the model is parallel with linear Engel curves passing the origin. This seriously damages the model's abilities as an arbitrary first order approximation and also destroys the flexibility of the model [Barnett (1979)]. However, as Barnett (1979) points out, the theoretical properties of the Rotterdam model provided by the restrictions (5.16)-(5.19) do not depend for their validity on integrability of the system (5.9). The usefulness of the Rotterdam model as an approximation to a true demand systems is not affected by the model's integrability [see also Barten (1977)]. Recently, Byron (1984) showed through a Monte Carlo simulation that the Rotterdam model is a fairly good approximation to demand systems, such as the linear expenditure system, the translog model and a demand system based on a quadratic utility function.

2.6 Flexible Functional Forms

We have seen that the Rotterdam model is a direct first order approximation to an arbitrary demand system. Alternatively, it is also possible to approximate to the demand system through a second order approximation to a utility function or an expenditure (cost) function. For example, the *translog* model [Christensen, *et al.* (1975)] is derived from a second order approximation to a general, direct or indirect utility function. The *Almost Ideal Demand System* (AIDS) [Deaton and Muellbauer (1980b)] is derived from a second order approximation to an expenditure function defined by the PIGLOG preference class. Such a procedure is called the *flexible functional form* approach.

Besides the translog model and the almost ideal demand system, there are many flexible functional forms available, for example, the generalised Leontief system, the generalised Cobb-Douglas system [Berndt, *et al.* (1977)], the generalised Box-Cox system [Berndt and Khaled (1979)], the Fourier flexible system [Gallant (1981)], the generalised addilog system [Bewley (1982b)], the Flexible Laurent system [Barnett (1983a)], the Müntz-Szatz system [Barnett and Jonas (1983)], and so on. However, it is not intended to cover all these systems but only to consider the two best known, the translog model and the almost ideal demand system. The reasons for choosing these two models for consideration are: firstly, there are many empirical applications of these models; secondly, they are duals of one another in the sense that the translog model is derived from utility maximisation, while the almost ideal demand system is derived from cost minimisation; and thirdly, that derivations of other flexible demand systems are not very different from those of these two models. We now present a brief review of the flexible functional form approach.

Formally, a functional form $S^*(x)$ is said to be *flexible* if it can provide a second order approximation to an arbitrary twice continuously differentiable function $S(x)$ at any arbitrary point x_o [Diewert(1982)]. *A second order (local) approximation* S^* to S at the point x_o is defined as one such that

$$[S^*(x) - S(x)]/\|x - x_o\|^2 \longrightarrow 0 \qquad (6.1)$$

as $x \longrightarrow x_o$ [see Barnett (1983b)].[9] In practice, flexible function forms consistent with these definitions can be obtained by mathematical approximation techniques, such as a Taylor series expansion, a Fourier series expansion, a Laurent expansion, the Müntz-Szatz theorem, and so on. The flexible functional forms in the translog model and the almost ideal demand system are based on second order Taylor series expansions to utility and expenditure functions, respectively.

Once the flexible functional form of the utility or expenditure function is obtained and if it is consistent with economic theory (e.g., quasi-concavity of utility function, concavity and homogeneity for expenditure function), then demand functions can be derived by Roy's identity or by Shephard's lemma. This is the usual technique in the flexible functional form approach. As mentioned in Section 2.4, an order of approximation is lost by passing from the objective function to the demand system, so that a second order approximation should be attained for the objective function in order to guarantee a first order approximation for the demand functions [see Deaton (1986, p.1789)]. In this sense, the translog model and the almost ideal demand system are first order flexible (approximate) demand systems consistent with demand theory.

The advantages of using flexible functional forms are well documented in the literature [Wales (1977), Berndt, *et al.* (1977), Christensen and Manser (1977), Caves and Christensen (1980), and Barnett and Lee (1985)]. In summary, the advantages are the capability of providing a reasonable approximation to any unknown function at any base point without any information about the exact form of the underlying function and the possession of enough parameters for a reasonable approximation to be independent of the true functional form. Neverthless, in application the performance of flexible forms has usually been poor and they often reject the restrictions of demand theory. Their local and global properties are generally unknown except in special cases [Caves and Christensen (1980), and Barnett and Lee (1985)]. A more detailed discussion of the translog demand system and the almost ideal demand system now follows.

2.6.1 The Translog Model

There are two versions of the translog (transcendental logarithmic) model: one is derived from an approximation to a direct utility function and the other from an approximation to the indirect utility function.

The direct translog approximate demand function is derived from a second-order Taylor series expansion of the logarithmic of reciprocal of a general direct utility function around a base point $q = \bar{q}$ which can be written as

$$ln\, u(q) = \alpha_o + \sum_{i=1}^{m} \alpha_i \, ln\, q_i + \tfrac{1}{2}\sum_{i=1}^{m}\sum_{j=1}^{m} \beta_{ij} \, ln\, q_i \, ln\, q_j, \quad (6.1)$$

where $\beta_{ij} = \beta_{ji}$ for all i and j. If we write $z_i = ln\, q_i$ and $\psi(z) = ln\, u(q)$, the parameters α_o, α_i and β_{ij} can be interpreted as

$$\alpha_o = \psi(\bar{z}) - \sum_i (\partial\bar{\psi}/\partial z_i)\bar{z}_i + \tfrac{1}{2}\sum_i \sum_j (\partial^2\bar{\psi}/\partial z_i \partial z_j)\bar{z}_i \bar{z}_j$$

$$\alpha_i = \partial\bar{\psi}/\partial z_i - \sum_j (\partial^2\bar{\psi}/\partial z_i \partial z_j)\bar{z}_j \quad (6.2)$$

$$\beta_{ij} = \partial^2\bar{\psi}/\partial z_i \partial z_j,$$

where $\partial\bar{\psi}/\partial z_i$ and $\partial^2\bar{\psi}/\partial z_i \partial z_j$ are the first and second derivatives of $\psi(z)$ evaluated at an arbitrary base point $z_i = \bar{z}_i$ (= $ln\, \bar{q}_i$) and $z_j = \bar{z}_j$, respectively [see Simmons and

Weiserbs (1979)]. The resulting demand functions are given in budget share form by,

$$w_i = [\alpha_i + \sum_{j=1}^{m} \beta_{ij} \, ln \, q_j] \big/ [\alpha_m + \sum_{k=1}^{m} \beta_{mk} \, ln \, q_k] \qquad (6.3)$$

where

$$\alpha_m = \sum_{k=1}^{m} \alpha_k \text{ and } \beta_{mk} = \sum_{j=1}^{m} \beta_{jk} \qquad (6.4)$$

[for derivation, see Christensen, *et al.* (1975)]. Since the demand equation are homogeneous of degree zero in the parameters α_i and β_{ij}; it is necessary to impose a normalisation condition such that

$$\alpha_m = \sum_{k=1}^{m} \alpha_k = 1. \qquad (6.5)$$

The indirect translog utility function is given by

$$ln \, v(P,\mu) = \alpha_o + \sum_{i=1}^{m} \alpha_i \, ln \, (p_i/\mu)$$

$$+ \tfrac{1}{2} \sum_{i=1}^{m} \sum m_{j=1} \beta_{ij} \, ln \, (p_i/\mu) \, ln \, (p_i/\mu) \quad (6.6)$$

where the parameters α_o, α_i's and β_{ij}'s are obtained in a similar manner to (6.2) but with $z_i = ln \, (p_i/\mu)$. From Roy's identity, the demand equations can be expressed as

$$w_i = [\alpha_i + \sum_{j=1}^{m} \beta_{ij} \, ln \, (p_i/\mu)] \big/ [\alpha_m + \sum_{k=1}^{m} \beta_{mk} \, ln \, (p_k/\mu)],$$

$$\text{for } i=1,\dots,m, \qquad (6.7)$$

where α_m and β_{mk} are defined as (6.4). A normalisation rule

(6.5) is also required to identify the parameters.

It has been claimed that the demand equations defined in (6.3) and (6.7) can be thought as first order approximations to any demand function. However, as McLaren (1982) points out, the obvious problem with the direct translog system is that the share equations in (6.3) are written as functions of the endogeneous variables, q_i's [Bewley (1984)]. There is only one exogeneous variable, the constant 1. Therefore, maximum likelihood estimation of (6.3) leads to biased and inconsistent parameter estimates. However, this problem does not occur in the indirect translog model.

It has also been claimed that restrictions of demand theory can be tested directly in terms of the parameters α_i and β_{ij} in the direct and indirect translog demand functions (6.3) and (6.7). Christensen, *et al.* (1975) argue that the symmetry of the Hessian is equivalent to

$$\beta_{ij} = \beta_{ji}, \quad i > j = 1,\ldots,m. \tag{6.8}$$

However, the use of a test of (6.8) to support the symmetry restriction is misleading in Christensen, *et al.* (1975), in that rejection of (6.8) implies the rejection of symmetry of the substitution matrix. Simmons and Weiserbs (1979) show that symmetry restrictions of the substitution effect can be expressed in terms of β_{ij}'s and w_i's as

$$\beta_{ij} - w_i\beta_{mj} - w_j\beta_{im} = \beta_{ji} - w_j\beta_{mi} - w_i\beta_{jm}. \tag{6.9}$$

It is clear that (6.8) implies (6.9) but (6.9) does not imply (6.8). This means that if (6.8) is accepted, (6.9) can be accepted. However, rejection of (6.8) cannot imply the rejection of (6.9). When (6.8) is rejected, one should test explicitly (6.9) for the symmetry of the substituition matrix [McLaren (1982)]. Simmons and Weiserbs (1979) identify three different utility functions producing the same share equations as (6.6) and having the symmetric Hessian but they show that (6.8) does not hold for the utility functions.

The global approximation properties of the translog system as a flexible functional form are known only in some special cases, such as the homothetic utility function and some two or three good situations [Caves and Christensen (1980) and Barnett and Lee (1985)]. Caves and Christensen (1980) examined the regional properties of the translog and the generalised Leontief (GL) models for homothetic preferences with two and three goods and the nonhomothetic case with two goods, and found that the translog model has good regional properties when preferences are near homothetic and all substitution elasticities between goods are near one. Recently, Barnett and Lee (1985) found that the translog model has an unpredictable regular region's shape, location and size without prior knowledge of the model's parameters. [See also Wales' (1977) Monte Carlo study.] Thus, the case for the translog model as a second order approximation to an arbitrary utility function is not totally convincing.

The indirect and direct translog models are very nonlinear in parameters, so that they are not particulary attractive models for large demand systems. Empirical applications of the translog model to more than three commodity situations are rare. For example, Christensen, *et al.* (1975) apply the translog model to American 3 commodity data, Berndt, *et al.* (1977) to Canadian 3 commodity data, Simmons and Weiserbs (1979) to American 3 commodity data, and McLaren (1982) to Australian 3 commodity data.

2.6.2 The Almost Ideal Demand System

Deaton and Muellbauer (1980b) derived the Almost Ideal Demand System (AIDS) from a flexible functional form of the expenditure function defined by the PIGLOG preference class with

$$ln\ c(u,p) = (1-u)\, ln\ a(p) + u\, ln\ b(p). \qquad (6.10)$$

where u is a given utility level and a(p) and b(p) can be regarded as the costs of subsistence and bliss. For the expenditure function (6.10) to be flexible, second order approximations to *ln* a(p) and *ln* b(p) are chosen as

$$ln\ a(p) = \alpha_o + \sum_{k=1}^{m} \alpha_k\, ln\ p_k + \tfrac{1}{2}\sum_{k=1}^{m}\sum_{l=1}^{m} \gamma^*_{kl}\, ln\ p_k\, ln\ p_l \qquad (6.11)$$

and

$$ln\ b(p) = ln\ a(p) + \beta_o \prod_{k=1}^{m} p_k^{\beta_k} \qquad (6.12)$$

where α_k, β_k, and γ^*_{kl} are parameters. For c(u,p) to be homogeneous of degree 1 in p, as required by demand theory, it is necessary that

$$\sum_{k=1}^{m} \alpha_k = 1 \tag{6.13}$$

and

$$\sum_{k=1}^{m} \gamma_{kl}^{*} = \sum_{l=1}^{m} \gamma_{kl}^{*} = \sum_{k=1}^{m} \beta_k = 0. \tag{6.14}$$

The demand functions can be obtained directly from (6.10) using Shephard's lemma

$$\partial\, ln\, c(u,p)/\partial\, ln\, p_i = p_i q_i / c(u,p) = w_i, \tag{6.15}$$

and are then expressed with the budget shares as a function of prices and utility

$$w_i = \alpha_i + \sum_{j=1}^{m} \gamma_{ij}\, ln\, P_j + \beta_i u \beta_o \prod_{k=1}^{m} p_k^{\beta_k}, \tag{6.16}$$

where $\gamma_{ij} = \frac{1}{2}(\gamma_{ij}^{*} + \gamma_{ji}^{*})$. Since the expenditure function is the inverse of the indirect utility function, u in (6.10) can be written as a function p and μ by using (6.11) and (6.12). Then, rearranging (6.10) and substituting into (6.16), we have demand functions in the almost ideal demand system expressed in budget share form:

$$w_i = \alpha_i + \sum_{j=1}^{m} \gamma_{ij}\, ln\, p_j + \beta_i\, ln\, (\mu/P_o), \tag{6.17}$$

where P_o is an overall price index such that

$$ln\, P_o = \alpha_o + \sum_k \alpha_k\, ln\, p_k + \tfrac{1}{2}\sum_k \sum_l \gamma_{kl}\, ln\, p_k\, ln\, p_l \tag{6.18}$$

which is, however, well approximated by

$$\ln P^* = \sum_{j=1}^{m} w_j \ln p_j \tag{6.19}$$

to avoid the nonlinearity due to (6.18) [see Deaton and Muellbauer (1980b, p.316)].

The coefficients (γ_{ij} and β_i) in (6.17) are not directly related to the Slutsky matrix or the marginal budget shares. Each γ_{ij} in (6.17) measures the change in the i'th budget share following a one percentage change in p_j with μ/P_o held constant, while β_i represents $\partial w_i/\partial \ln \mu$ [Deaton and Muellbauer (1980b, p.314)]. Since $\partial w_i/\partial \ln \mu = \partial p_i q_i/\partial\mu - p_i q_i/\mu$; that is, β_i is the marginal budget share minus the average budget share of good i, the coefficient β_i indicates whether a good is a luxury or necessity. If $\beta_i > 0$, then good i is a luxury, because w_i increases with μ, while if $\beta_i < 0$, then good i is a necessity. Consequently, the marginal budget share and the total expenditure elasticity of good i can be given as

$$\partial p_i q_i/\partial\mu = \beta_i + w_i \tag{6.20}$$

and

$$e_i = \beta_i/w_i + 1, \tag{6.21}$$

respectively. The Slutsky coefficients ($C_{ij} = k_{ij} p_i p_j/\mu$) can be expressed as

$$C_{ij} = \gamma_{ij} + \beta_i \beta_j \ln (\mu/P_o) - w_i \delta_{ij} + w_i w_j, \tag{6.22}$$

where δ_{ij} is a Kronecker delta [Deaton and Muellbauer (1980b, p.316)]. The compensated price elasticity, e^*_{ij}, can then be obtained as

$$e^*_{ij} = C_{ij}/w_i. \tag{6.23}$$

However, the demand restrictions (other than negativity) can be imposed directly on the coefficients, α_i, β_i, and γ_{ij}. The restrictions on the coefficients of the almost ideal demand system (6.17) are as follows:

Adding up: $\sum_{i=1}^{m} \alpha_i = 1$, $\sum_{i=1}^{m} \beta_i = 0$ and
$\sum_{i=1}^{m} \gamma_{ij} = 0$, for all $j = 1,\ldots,m$. (6.24)

Homogeneity: $\sum_{j=1}^{m} \gamma_{ij} = 0$, for all $i = 1,\ldots,m$. (6.25)

Symmetry: $\gamma_{ij} = \gamma_{ji}$, for all $i,j = 1,\ldots,m$. (6.26)

Negativity: The matrix $C = [C_{ij}]$ is negative semi-definite (6.27)

where C_{ij} is defined as in (6.22). Thus, even though the Slutsky coefficients (6.22) vary over the sample space, the homogeneity and symmetry restrictions are linear and constant for all observations, as can be seen from (6.25) and (6.26). However, the test of negativity varies with the data.

The almost ideal demand system in (6.17) are flexible and give an arbitrary first order approximation to any demand system, as the expenditure function is a second order approximation. However, their global and regional properties still remain unknown.

2.7 Concluding Remarks

In this chapter, we presented an overview of demand theory and the problems relating to the specification and properties of existing complete demand systems: the linear expenditure system, the Rotterdam model, the translog model, and the almost ideal demand system. The discussion in this chapter does not cover all topics in contemporary demand analysis; it is confined to the basic problems underlying the present study. One might point out the omission of important topics, such as Engel curve analysis, aggregation over consumers, the S-branch utility, the generalisations of the linear expenditure system, and so on. However, as such topics are not directly related to our empirical work, we are forced to curtail the discussion at this point.

The dynamic generalisation of demand theory and the specification of dynamic demand systems will be discussed elsewhere (in Chapter 5). Finally, the statistical aspects of the estimation of demand systems considered in this chapter and their application to Korean household expenditure data will be discussed in Chapters 3 and 4, respectively.

FOOTNOTES:

1. The other approach, due to Samuelson (1947), is that of revealed preference theory, in which neither the utility function nor the preference ordering is pre-assumed. This approach goes directly to the demand for commodities, but will not be discussed in this chapter.

2. Decomposing the matrix K into two matrices, λU^{-1} and $\lambda(p'U^{-1}p)^{-1}U^{-1}pp'U^{-1}$, $\lambda U^{-1} = [\lambda u^{ij}]$ is called the *specific* substitution effect matrix and $\lambda(p'U^{-1}p)^{-1}U^{-1}pp'U^{-1} = [-(\lambda/\lambda_{\mu})(\partial q_i/\partial \mu)(\partial q_j/\partial \mu)]$ is called the *general* substitution effect matrix, where u^{ij} is the (i,j)'th term of U^{-1} [Houthakker (1960)].

3. The matrix of uncompensated price effects, $q_P = [\partial q_i/\partial p_j]$, is not necessarily symmetric.

4. Alternative definitions for these relations are expressed in terms of the second order derivatives of utility, $u_{ij} = \partial^2 u_i/\partial q_i \partial q_j$. That is, if $u_{ij} > 0$, the goods, i and j, are called complements, if $u_{ij} < 0$, substitutes, and if $u_{ij} = 0$, independent goods [see Phlips (1974, p.77-78)].

5. The composite commodity theorem asserts that if a group of prices move in parallel, then the corresponding group of commodities can be treated as a single commodity [Deaton and Muellbauer (1980a, p.121)].

6. Thus, the second stage of two-stage budgeting is both necessary and sufficient for weak separability.

7. As Barnett notes, it has widely been asserted that tests of the classical restrictions with the Rotterdam model implicitly test for the existence of a double log aggregate utility function [see for example Phlips (1974) and Christensen *et al.* (1975)].

8. Yoshihara (1969) uses the relative price version of Rotterdam model and Barnett's (1979) result is based on the aggregate (over consumers) Rotterdam model.

9. Equation (6.1) can be written as $S^*(x)-S(x) = o(\|x-x_o\|^2)$, equivalently, which means that $S^*(x)-S(x)$ converges to zero faster than $\|x-x_o\|^2$. Different definitions are available in Diewert (1971). Barnett (1983) shows that Diewert's (1971) definition is equivalent to (6.1).

CHAPTER 3

STATISTICAL ASPECTS ON THE ESTIMATION OF COMPLETE SYSTEMS OF DEMAND EQUATIONS AND TESTS OF HYPOTHESES

3.1 Introduction

The object of this chapter is to examine statistical problems relating to the system-wide estimation of complete (static) demand systems and tests of related hypotheses. Statistically, demand systems are characterised by the multivariate model, as they typically have identical regressors in each equation. Demand systems are also characterised by the restrictions derived from demand theory, such as adding up, homogenity and symmetry conditions. A slight complication is that the adding up conditions result in demand systems which are singular.

More specifically, demand systems which are linear in parameters, such as the Rotterdam model or the almost ideal demand system, can be expressed in the form of the *linear* multivariate model. Systems which are nonlinear in parameters, such as the linear expenditure system and the translog model, can be expressed in the form of the *nonlinear* multivariate model. Consequently, the estimation of demand systems and tests of related hypotheses are carried out in the context of multivariate analysis using OLS and restricted Aitkin GLS methods in the application of SUR [Zellner (1962), Byron (1968, 1970), Barten (1969), and Deaton (1972)]. For nonlinear multivariate demand systems, iterative step-wise procedures and nonlinear estimation techniques, such as the Newton-Raphson and Gauss-Newton methods are adopted.

In this chapter, statistical issues relating to these problems will be surveyed and examined. Therefore, this chapter contains many subjects already discussed in the literature, for which it is intended to provide summaries without the encumbrance of detailed proofs.

A new GLS type estimation procedure for the linear symmetry restricted system will be developed. The result is derived from the solution of a Lyapunov equation, based on the work of Bowden (1973) and Byron (1982). As a complement to direct χ^2 tests of demand restrictions, the use of the separate induced test [Seber (1964b) and Savin (1980 ,1984)], will be examined.

The design of this chapter is as follows: in Section 2, the statistical model for the estimation of linear systems of demand equations will be presented and aspects relating to unrestricted estimation will be discussed. Issues on the restricted estimation of demand systems subject to linear restrictions will follow in Section 3. Section 4 will discuss tests of hypothesis; in particular, direct tests of hypothesis and separate induced tests of individual hypotheses in the context of simultaneous tests. Section 5 will consider the singularity of demand systems and the implications for estimation subject to homogeneity and symmetry restrictions. In Section 6, the estimation of symmetric systems, using the Lyapunov solution, will be developed. In Section 7, the estimation of systems of nonlinear equations will be examined, while in Section 8, problems of estimation and inference for systems of equations with autocorrelated residuals will be discussed. A condition for the autoregressive error process to be stationary in a singular system will be extended from Berndt and Savin's (1975) AR(1) case to the higher order AR(p) case. Finally, the chapter will close with some concluding remarks in Section 9.

3.2 Unrestricted Estimation of a Complete System of Demand Equations

In this section, we will present the statistical model for unrestricted estimation of systems of demand equations which are linear in coefficients. A system of m demand equations, with identical explanatory variables in each equation, can be written in the form of the linear multivariate model,

$$Y = XB + U, \tag{2.1}$$

where $Y = (y_1, \ldots, y_m)$ is a $T \times m$ matrix of m dependent variables, X a $T \times k$ matrix of k non-stochastic explanatory variables[1] (usually defined in terms of prices and total expenditure), with rank(X) = k, $B = (\beta_1,\ldots,\beta_m)$ an m x k matrix of unknown coefficients, $U = (u_1,\ldots,u_m)$ a T x m matrix of disturbances, and T is the number of observations. We assume that each row of U has an identically independent multivariate normal distribution with $E(u_{.t}) = \underset{\sim}{0}_m'$ and $E(u_{.t}'u_{.t}) = \Sigma$ for t = 1,...,T, where $u_{.t}$ is the t'th row of U, $\underset{\sim}{0}_m$ is an m x 1 null vector. Σ is an m x m positive definite symmetric matrix, termed the contemporaneous covariance matrix of the disturbances, and is given by

$$\Sigma = \begin{bmatrix} \sigma_{11} & \ldots & \sigma_{1m} \\ . & \ldots & . \\ . & \ldots & . \\ \sigma_{m1} & \ldots & \sigma_{mm} \end{bmatrix}. \tag{2.2}$$

Thus, E(U) = 0 and $E(U'U) = \Sigma$, and $U \sim N(0,\Sigma)$. We assume no serial correlation in each u_i and that there exists a finite nonsingular k x k matrix Q such that $Q = \lim_{T\to\infty} X'X/T$.

Using the Kronecker product and vectorising (2.1), we can write (2.1) in the familiar SUR form[2]

$$y = (I_m \otimes X)\beta + u = Z\beta + u \tag{2.3}$$

where y = vec(Y) is a Tm x 1 vector, $Z = (I_m \otimes X)$ is a Tm x mk matrix, and β = vec(B) is an mk x 1 vector, u = vec(U) is a Tm x 1 vector, and I_m is an identity matrix of order m. From the assumption of multivariate normality of U, it follows that $u \sim N(0,\Omega)$, where $\Omega = E(uu') = \Sigma \otimes I_T$.

It is well known that in (unrestricted) estimation of the linear multivariate model, the OLS estimator of β in (2.3) [or B in (2.1)] is fully efficient, BLU, and identical to the GLS and maximum likelihood (ML) estimators. The OLS estimator of β in (2.3) is obtained from the minimisation of $u'u = (y - Z\beta)'(y - Z\beta)$ with respect to β, and is given by

$$\hat{\beta}_{OLS} = (Z'Z)^{-1}Z'y = (I \otimes (X'X)^{-1}X')y. \qquad (2.4)$$

The GLS and ML estimators are obtained from the minimisation of $u'(\Sigma^{-1} \otimes I_T)u = T\, tr\, (\Sigma^{-1}U'U)$ and $T\, ln\, |\Sigma| + u'(\Sigma^{-1} \otimes I_T)u$ with respect to β, and are both given by

$$\hat{\beta}_{GLS} = \hat{\beta}_{ML} = (Z'\Omega^{-1}Z)^{-1}Z'\Omega^{-1}y. \qquad (2.5)$$

Note that the hat ^ indicates the unrestricted estimator.

However, since $Z = I \otimes X$, the GLS (or ML) estimator of β becomes

$$\hat{\beta}_{GLS} = (Z'\Omega^{-1}Z)^{-1}Z'\Omega^{-1}y \qquad (2.6)$$

$$= (\Sigma^{-1} \otimes X'X)^{-1}(\Sigma^{-1} \otimes X')\, y$$

$$= (I \otimes (X'X)^{-1}X')y = \hat{\beta}_{OLS}$$

[see Goldberger (1970) and Breusch (1978)]. Thus, the identity of $\hat{\beta}_{OLS} = \hat{\beta}_{GLS} = \hat{\beta}_{ML}$ holds. Therefore, there is no gain in efficiency from joint estimation of the equations. Similarly, the BLU estimator of the coefficient matrix B in (2.1) is given by $\hat{B}_{OLS} = (X'X)^{-1}X'Y$.

Under the normality assumption of U, $\hat{\beta}_{OLS}$ in (2.4) is normally distributed with mean β and covariance matrix

$$\mathrm{cov}(\hat{\beta}_{OLS}) = \Sigma \otimes (X'X)^{-1}. \qquad (2.7)$$

An unbiased and consistent OLS estimator of the contemporaneous covariance matrix, Σ, is given by

$$\hat{\Sigma}_{OLS} = \hat{U}'\hat{U}/(T-k) = Y'M_XY/(T-k), \qquad (2.8)$$

where $\hat{U}$ is the OLS residual matrix given by $\hat{U} = Y - X\hat{B}_{OLS}$. However, the ML estimator of Σ is given by

$$\hat{\Sigma}_{ML} = \hat{U}'\hat{U}/T = Y'M_XY/T, \qquad (2.9)$$

which is also consistent but biased. Laitinen (1978) has shown that the sufficient condition for nonsingularity of $\hat{\Sigma}$ is that $T-k \geqslant m$.

3.3 Linear Restrictions and Restricted Estimation

In this section, we will consider restricted estimation of a system of equations given by (2.3) subject to s linear independent constraints,

$$R\beta = r, \qquad (3.1)$$

where R is an s x mk restriction matrix with rank(R) = s < mk and r is an s x 1 known constant vector. Since the homogeneity and symmetry restrictions on the demand system are linear in coefficients, they can be expressed as (3.1). For these two types of restrictions the constant vector r in (3.1) is a null vector.

There are two special types of restrictions for which (3.1) can be expressed in the matrix mutiplicative form. One is the uniform mixed linear restriction, the other is the uniform within equation restriction. If the restriction matrix R in (3.1) takes on a Kronecker product form such that $R = (G' \otimes F)$ for suitable matrices F and G, the restrictions (3.1) are called uniform mixed linear restrictions [see Berndt and Savin (1977) and Bewley (1983)]. These restrictions (3.1) can be expressed in terms of the coefficient matrix B in (2.1) as FBG = S, where S is such that vec(S) = r in (3.1). On the other hand, uniform within equation restrictions are defined as a special case of uniform mixed linear restrictions when G is an identity matrix. In that case, uniform within equation restrictions can be expressed as $R\beta = (I \otimes F)\beta = vec(S)$ or as FB = S. The homogeneity restrictions in demand systems are imposed on the price coefficients within an equation and uniformly on all equations. Thus, they fall into this category with $G = I_m$ and $F = (\iota_m',0)$, when the first m rows of B refer to price coefficients and ι_m is an m x 1 vector of unit element [Bera (1982) and Bewley (1983)]. The adding up restriction is also a special case of a uniform mixed linear restriction, in which F is an m x m identity matrix and G is an m x 1 vector of unit elements. However, since symmetry restrictions are imposed across equations in a demand system, they can only be expressed in the form of (3.1).

Full efficiency in restricted estimation of the system (2.3) subject to the restrictions (3.1) is achieved by joint estimation of the equations, even though the explanatory variables are the same in each equation. Thus, GLS (or ML) estimation produces a fully efficient restricted estimator [Byron (1970) and Deaton (1972)]. The restricted GLS estimator of β in (2.3) is obtained from minimisation of the Lagrange function, $u'(\Sigma^{-1} \otimes I_T)u + \lambda'(R\beta - r)$, with respect to β and λ, where λ is an $s \times 1$ Lagrange multiplier vector. The solution is

$$\tilde{\beta}_{GLS} = \hat{\beta}_{OLS} - (\Sigma \otimes (X'X)^{-1})R'[R(\Sigma \otimes (X'X)^{-1})R']^{-1} \times (R\hat{\beta}_{OLS} - r), \qquad (3.2)$$

and the GLS estimator of λ is given by

$$\tilde{\lambda}_{GLS} = -[R(\Sigma \otimes (X'X)^{-1})R']^{-1}(r - R\hat{\beta}_{OLS}), \qquad (3.3)$$

where $\hat{\beta}_{OLS}$ is the unrestricted OLS estimator of β as in (2.4). Note that the tilde ~ refers to the restricted estimator. Under the restrictions (3.1), $\tilde{\beta}_{GLS}$ is fully efficient, consistent, and BLU for β [see for proof Deaton (1972)]; furthermore, $E(\tilde{\lambda}_{GLS}) = \underset{\sim}{0}_s$, where $\underset{\sim}{0}_s$ is an $s \times 1$ null vector. Under the normality assumption, $\tilde{\beta}_{GLS}$ and $\tilde{\lambda}_{GLS}$ are normally distributed with covariance matrices,

$$\text{cov}(\tilde{\beta}_{GLS}) = (\Sigma \otimes (X'X)^{-1}) - [\Sigma \otimes (X'X)^{-1}]R' \times[R(\Sigma \otimes (X'X)^{-1})R']^{-1}R[\Sigma \otimes (X'X)^{-1}] \quad (3.4)$$

and

$$\text{cov}(\tilde{\lambda}_{GLS}) = [R(\Sigma \otimes (X'X)^{-1})R']^{-1}. \quad (3.5)$$

However, since Σ is usually unknown in empirical situations, the direct application of (3.2) - (3.5) is infeasible. Hence, a consistent estimate of Σ, $\hat{\Sigma}_{OLS}$ (or $\hat{\Sigma}_{ML}$), obtained by unrestricted OLS (or ML) estimation, is substituted in the place of Σ in (3.2) - (3.5). This procedure refers to the restricted Zellner's two-stage estimation (ZEF) [Zellner (1962) and Byron (1968, 1970)]. We shall denote the estimators of β and λ obtained from this procedure by $\tilde{\beta}_{ZEF}$ and $\tilde{\lambda}_{ZEF}$. Since $\hat{\Sigma}_{OLS}$ (or $\hat{\Sigma}_{ML}$) is a consistent estimate of Σ, $\tilde{\beta}_{ZEF}$ is also consistent, and $\tilde{\beta}_{ZEF}$ and $\tilde{\lambda}_{ZEF}$ have the same asympototic distribution as $\tilde{\beta}_{GLS}$ and $\tilde{\lambda}_{GLS}$, respectively. Thus, under (3.1), $\tilde{\beta}_{ZEF}$ and $\tilde{\lambda}_{ZEF}$ are asymptotically normally distributed with their asymptotic covariance matrices given by (3.4) and (3.5), respectively. In addition, they are asymptotically efficient and unbiased. The restricted estimator of a covariance matrix, Σ, is

$$\tilde{\Sigma}_{ZEF} = \tilde{U}'\tilde{U}/(T-k), \quad (3.6)$$

where $\tilde{U}$ is the restricted residual matrix formed using $\tilde{\beta}_{ZEF}$. $\tilde{\Sigma}_{ZEF}$ in (3.6) is consistent under the restrictions (3.1).

The restricted maximum likelihood (MLR) estimators of β

and Σ are obtained by maximising the joint likelihood function,

$$L(\beta,\Sigma) = (2\pi)^{-mT/2}|\Sigma|^{-T/2} exp -[u'(\Sigma^{-1} \otimes I_T)u/2], \quad (3.7)$$

or equivalently, by minimising $T\,ln\,|\Sigma| + u'(\Sigma^{-1} \otimes I_T)u$, subject to the restictions (3.1). $\tilde{\beta}_{ML}$ and $\tilde{\lambda}_{ML}$ are the same as in (3.2) and (3.3), respectively. On the other hand, the restricted ML estimator of a covariance matrix, Σ, is given as

$$\tilde{\Sigma}_{ML} = \tilde{U}'\tilde{U}/T. \quad (3.8)$$

where $\tilde{U}$ is the restricted residual matrix formed with $\tilde{\beta}_{ML}$. Computationally, $\tilde{\beta}_{ML}$ and $\tilde{\Sigma}_{ML}$ can be obtained by iterative restricted Zellner estimation (IZEF) procedure [Kmenta and Gilbert (1968)]. At the k'th iteration, $\tilde{\Sigma}_{ML}$ in (3.8) is updated by the restricted estimate of β, $\tilde{\beta}_{ML}$, formed in the (k-1)'th iteration. The restricted estimate $\tilde{\beta}_{ML}$ is recalculated with a newly updated covariance estimate, $\tilde{\Sigma}_{ML}$, and so on. The convergence and equivalence of the IZEF method to ML estimation have been shown by Kmenta and Gilbert (1968) using a Monte Calro simulation on the unrestricted SUR model. Analytical proofs of convergence are provided by Oberhofer and Kmenta (1974), Dhrymes (1971), and Phillips (1976) [see also Malinvaud (1980, Chapter 9)].

A minor point is that $\tilde{\beta}_{ML}$ is not identical to $\tilde{\beta}_{ZEF}$ because $\tilde{\beta}_{ML}$ depends on an updated estimate of Σ from iteration to iteration, while $\tilde{\beta}_{ZEF}$ is obtained at the first iteration.

However, $\tilde{\beta}_{ML}$ has the same asymptotic distribution as $\tilde{\beta}_{GLS}$, so that $\tilde{\beta}_{ZEF}$ and $\tilde{\beta}_{ML}$ are asymptotically equivalent. This implies that there is no gain in asymptotic efficiency through iteration.[3]

However, an important question in the practical use of these estimators is their relative performance in small samples. Unfortunately, little is known about their exact distributions in the general context.[4] Kmenta and Gilbert (1968) demonstrated in a Monte Carlo simulation with the unrestricted SUR model that the efficiency of the ZEF can carry over to small samples, and that there is little difference in efficiency between the ZEF and the ML (IZEF) estimates in small samples. However, Klevmarken (1975) found, in another Monte Carlo simulation using the restricted Rotterdam model, that the estimates of covariance matrix from the IZEF shows a higher positive bias than the ZEF and he suggested (ad hoc) the use of a small sample correction factor $(T-k)/[T - k +(s/m)]$ to reduce this bias.

Higher order approximations to the distribution of the ZEF estimator of the SUR model have been worked out by Phillips (1977) and Rothenberg (1984a) using Edgeworth expansions. They found that the expansion of the distribution of the standardised ZEF estimator is normal to order $T^{-3/2}$, and that correcting the variance of the limiting distribution is all that is required to approximate to the normal distribution.

Finally, the restricted OLS estimator of β, $\tilde{\beta}_{OLS}$, is given by

$$\tilde{\beta}_{OLS} = \hat{\beta}_{OLS} - [I_m \otimes (X'X)^{-1}]R'[R(I_m \otimes (X'X)^{-1})R']^{-1} \times(R\hat{\beta}_{OLS} - r). \qquad (3.9)$$

Under general restrictions (3.1), $\tilde{\beta}_{OLS}$ is consistent and unbiased, but is neither fully efficient nor identical to $\tilde{\beta}_{GLS}$ even with the identical regressors. However, subject to the uniform within-equation and uniform mixed linear restrictions, $\tilde{\beta}_{OLS}$ is identical to the $\tilde{\beta}_{GLS}$ (and $\tilde{\beta}_{ML}$) and is fully efficient [see for proof Deaton (1972) and Bewley (1983)]. Therefore, there is no gain in efficiency from the joint estimation of equations and the iterative estimates (IZEF) are convergent at the first step. This procedure is applicable to the estimation of demand systems subject to homogeneity only.[5] Since $\tilde{\beta}_{OLS}$ in (3.9) no longer involves Σ, it is unaffected by the singularity of Σ [Deaton (1972, p.403)].

3.4 Tests of Hypothesis

Now, we consider tests of s linear restrictions given by (3.1). For simplicity, we can set the constant vector r in (3.1) to be null without loss of generality. Then, the null hypothesis to be tested can be written as

$$H_0: R\beta = \underset{\sim}{0}_s, \qquad (4.1)$$

where R is of the same form as (3.1), β is defined in (2.3) and $\underset{\sim}{0}_s$ is an s x 1 null vector. Thus, both the null hypothesis to be tested and the model on which tests are performed are linear in the coefficient vector β.

In a demand system with m equations, such as the Rotterdam model and the almost ideal demand system, there are m(m-2)/2 *independent* restrictions to be tested, that is, (m-1) homogeneity and (m-1)(m-2)/2 symmetry restrictions [see Section 5 of this Chapter]. It is a tradition in demand analysis to treat these restrictions as a single null hypothesis and to test them by using one test, for example, χ^2 test. Such a test procedure is called the direct test [Savin (1980)] and will be considered in the first subsection. However, since the H_0 in (4.1) consists of s independent individual hypotheses, tests of the H_0 can also be carried out by testing the individual hypotheses simultaneously. In other words, we can separately test the individual null hypotheses in (4.1) and then infer the validity of the null hypothesis H_0 from the separate individual tests. This test procedure is called the separate induced test [Seber (1964b) and Savin (1980)], and will be discussed in the second subsection.

3.4.1 The Direct Test of Hypothesis

We will consider three testing principles for the direct test of the null hypothesis H_0 in (4.1); the Wald (W), likelihood ratio (LR), and Lagrange multiplier (LM) principles. Byron (1968) initially introduced the use of this terminology into econometric practice, and Berndt and Savin

(1977) discussed in detail their application to the testing of uniform mixed linear restrictions as well as to general linear restrictions in the context of a linear multivariate model.

Although these three testing principles are used to test the same null hypothesis H_0, their basic approaches are somewhat different. The Wald procedure tests whether the unrestricted estimates of β satisfy the null hypothesis H_0, that is, $R\hat{\beta} = \underset{\sim}{0}_s$. The LR procedure tests for the equality of the unrestricted and the restricted estimates, that is, $\hat{\beta} = \tilde{\beta}$. Finally, the LM approach tests whether the Lagrange multipliers associated with the null hypothesis are nearly equal to zero under the null hypothesis H_0, that is, $\tilde{\lambda} = \underset{\sim}{0}_s$ [Seber (1964a, p.264)]. Consequently, their test statistics have different definitions.

If we let $\xi = R\beta$, testing the null hypothesis H_0: $R\beta = \underset{\sim}{0}_s$ is equivalent to testing whether ξ is equal to the null vector. Taking this point, the Wald test statistic is given in a general form

$$W = \hat{\xi}'\{cov(\hat{\xi})\}^{-1}\hat{\xi}, \tag{4.2}$$

where $\hat{\xi}$ is an s x 1 vector of unrestricted estimate of ξ, that is, $\hat{\xi} = R\hat{\beta}$, and $cov(\hat{\xi})$ is an s x s covariance matrix of $\hat{\xi}$. Using the results in the previous section, we can then obtain the Wald statistic (4.2) for testing the H_0 in (4.1) as

$$W = (R\hat{\beta}_{OLS})'[R(\Sigma \otimes (X'X)^{-1})R']^{-1}(R\hat{\beta}_{OLS}), \tag{4.3}$$

since $\hat{\xi} = R\hat{\beta}_{OLS}$ and the covariance of $\hat{\xi}$ is given by

$$\mathrm{cov}(\hat{\xi}) = [R(\Sigma \otimes (X'X)^{-1})R']. \qquad (4.4)$$

Since $\{\mathrm{cov}(\hat{\xi})\}$ is positive definite, so is $\{\mathrm{cov}(\hat{\xi})\}^{-1}$, and there exists an s x s nonsingular matrix H such that $H'H = \{\mathrm{cov}(\hat{\xi})\}^{-1}$. Thus, W in (4.3) can be written as $W = \hat{\varphi}'\hat{\varphi}$, where $\hat{\varphi} = H\hat{\xi}$ is an s x 1 vector. Under the null hypothesis H_0 and the normality condition, it is clear that $\hat{\xi} \sim N[\underset{\sim}{0}_s, \mathrm{cov}(\hat{\xi})]$, when Σ is known, so that $\hat{\varphi} \sim N(\underset{\sim}{0}_s, I_s)$. Therefore, it follows that $W = \hat{\varphi}'\hat{\varphi}$ has an exact central χ^2 distribution with s degrees of freedom, when Σ is known. Hence, the confidence interval for the Wald test is defined by the central χ^2 distribution as

$$\{W \mid W = \hat{\xi}'(\mathrm{cov}(\hat{\xi}))^{-1}\hat{\xi} \leqslant \chi^2_s(\alpha)\}, \qquad (4.5)$$

where $\chi^2_s(\alpha)$ is an α % critical value of central χ^2 distribution with s degrees of freedom. Geometrically, the confidence region (4.5) defines the ellipsoid on the ξ-space

$$\{\ \hat{\xi} \mid \hat{\xi}'[\mathrm{cov}(\hat{\xi})]^{-1}\hat{\xi}/\ \chi^2_s(\alpha) \leqslant 1\},$$

which is inscribed to the box defined as $\{\xi_i \mid \xi_i < \omega_i,\ i=1,..,s\}$, where ω_i's are the eigenvalues of $[\mathrm{cov}(\hat{\xi})]^{-1}/\chi^2_s(\alpha)$ [Scheffé (1959, Appendix III)].

However, when Σ is unknown, a consistent estimate $\hat{\Sigma}_{ML}$

given in (2.9) (or $\hat{\Sigma}_{OLS}$ in (2.8)) replaces Σ in (4.4), so that the Wald statistic (4.3) becomes

$$W = (R\hat{\beta}_{OLS})'[R(\hat{\Sigma}_{ML} \otimes (X'X)^{-1})R']^{-1}(R\hat{\beta}_{OLS}). \qquad (4.6)$$

Then, W in (4.6) has an asymptotic central χ^2 distribution with s degrees of freedom under the null hypothesis H_0 [see for proof Berndt and Savin (1977, p.1267)].[6]

The LM test statistic [Silvey (1959) and Aitchinson and Silvey (1960)], to test if $\lambda = \underset{\sim}{0}_s$ under H_0, is given by

$$LM = \tilde{\lambda}'\{cov(\tilde{\lambda})\}^{-1}\tilde{\lambda}, \qquad (4.7)$$

where $\tilde{\lambda}$ is an s x 1 vector of the ML estimates of the Lagrange multipliers under the null hypothesis H_0 and $cov(\tilde{\lambda})$ is the covariance matrix of $\tilde{\lambda}$. From (3.5), the LM statistic for testing H_0 in (4.1) can be written as

$$LM = \tilde{\lambda}'[R(\Sigma \otimes (X'X)^{-1})R']\tilde{\lambda}. \qquad (4.8)$$

If we substitute $\tilde{\lambda}_{ML}$ (= $\tilde{\lambda}_{GLS}$) in (3.3) in (4.8) with $r = \underset{\sim}{0}_s$, we can see that LM in (4.8) reduces to W in (4.3), when Σ is known [Byron (1970)]. Hence, the LM in (4.8) is also exactly distributed as a central χ^2 with s degrees of freedom.

However, when Σ is unknown, the equality between the LM and the Wald test statistics does not hold, since the LM statistic and its covariance matrix are computed using $\tilde{\Sigma}_{ML}$

(given in (3.8)) at the convergence point of the IZEF method. Therefore, the LM test in (4.8) becomes

$$LM = \tilde{\lambda}'[R(\tilde{\Sigma}_{ML} \otimes (X'X)^{-1})R']\tilde{\lambda}. \tag{4.9}$$

$$= (R\hat{\beta}_{OLS})'[R(\tilde{\Sigma}_{ML} \otimes (X'X)^{-1})R']^{-1}(R\hat{\beta}_{OLS}),$$

which also has an aymptotic central χ^2 distribution with s degrees of freedom under the null hypothesis H_0 [see for proof Berndt and Savin (1977, p.1267-1269)].

The LR test criterion to test whether the unrestricted estimate of β is sufficiently close to the restricted estimate of β is given by

$$LR = - 2\,ln\,\Lambda, \tag{4.10}$$

where $\Lambda = L(\tilde{\beta})/L(\hat{\beta})$ is the Wilks' statistic, and $L(\tilde{\beta})$ and $L(\hat{\beta})$ are the maximum likelihood values evaluated at $\beta = \tilde{\beta}$ and at $\beta = \hat{\beta}$, respectively.

When Σ is known, $L(\hat{\beta})$ and $L(\tilde{\beta})$ are given as

$$L(\hat{\beta}) = (2\pi)^{-mT/2}|\Sigma|^{-T/2} exp\,\{-T\,tr\,\Sigma^{-1}\hat{U}'\hat{U}/2\}$$

and

$$L(\tilde{\beta}) = (2\pi)^{-mT/2}|\Sigma|^{-T/2} exp\,\{-T\,tr\,\Sigma^{-1}\tilde{U}'\tilde{U}/2\},$$

where $\hat{U}$ and $\tilde{U}$ are the unrestricted and restricted residual matrices from ML estimation. Hence, the LR statistic becomes

$$LR = T\,tr\,\Sigma^{-1}(\tilde{U}'\tilde{U} - \hat{U}'\hat{U}), \tag{4.11}$$

since Λ is given as

$$\Lambda = exp\,\{-T\,tr\,\Sigma^{-1}\tilde{U}'\tilde{U}/2\}/exp\,\{-T\,tr\,\Sigma^{-1}\hat{U}'\hat{U}/2\}, \tag{4.12}$$

The right hand side of (4.11) can be written as

$$\tilde{u}'(\Sigma^{-1} \otimes I_T)\tilde{u} - \hat{u}'(\Sigma^{-1} \otimes I_T)\hat{u}$$

$$= u'(I \otimes X(X'X)^{-1})R'[R(\Sigma \otimes (X'X)^{-1})R']^{-1}R(I \otimes (X'X)^{-1}X')u,$$

since $\hat{u}'(\Sigma^{-1} \otimes I_T)\hat{u} = u'(\Sigma^{-1} \otimes M_X)u$ and

$$\tilde{u}'(\Sigma^{-1} \otimes I_T)\tilde{u} = u'(\Sigma^{-1} \otimes M_X)u + u'(I \otimes X(X'X)^{-1})R' \times[R(\Sigma \otimes (X'X)^{-1})R']^{-1}R(I \otimes (X'X)^{-1}X')u$$

under the H_0, and

$$\begin{aligned} &rank\{(\Sigma \otimes X(X'X)^{-1})R'[R(\Sigma \otimes (X'X)^{-1})R']^{-1}R(\Sigma \otimes (X'X)^{-1}X')\} \\ &= rank\{R(\Sigma \otimes (X'X)^{-1})R'[R(\Sigma \otimes (X'X)^{-1})R']^{-1}\} \\ &= rank\{I_s\} = s. \end{aligned}$$

Therefore, using (17.40) in Pollock (1979, p. 319)[7], we can see that the LR statistic has an exact central χ^2 distribution with s degrees of freedom, when Σ is known.

However, when Σ is unknown, $\{-T\,tr\,\Sigma^{-1}\hat{U}'\hat{U}/2\}$ in $L(\hat{\beta})$ and

$\{-T\, tr\, \Sigma^{-1}\tilde{U}'\tilde{U}/2\}$ in $L(\tilde{\beta})$ are concentrated out using their corresponding ML estimates. $L(\hat{\beta})$ and $L(\tilde{\beta})$ are then given as

$$L(\hat{\beta}) = (2\pi)^{-mT/2}|\hat{\Sigma}_{ML}|^{-T/2}\, exp\, \{T^2/2\}$$

and

$$L(\tilde{\beta}) = (2\pi)^{-mT/2}|\tilde{\Sigma}_{ML}|^{-T/2}\, exp\, \{T^2/2\}.$$

Hence, the Wilks' statistic Λ becomes the ratio of a generalised covariance of the restricted residuals to that of the unrestricted residuals, that is,

$$\Lambda = \{|\tilde{\Sigma}_{ML}|/|\hat{\Sigma}_{ML}|\}^{-T/2}, \tag{4.13}$$

so that the LR statistic becomes

$$LR = T\, ln\, \{|\tilde{\Sigma}_{ML}|/|\hat{\Sigma}_{ML}|\}. \tag{4.14}$$

The restricted estimate of Σ, $\tilde{\Sigma}_{ML}$, in (4.14) is obtained at the point of convergence of the IZEF method. The LR statistic in (4.14) has an aymptotic central χ^2 distribution when Σ is unknown [see Berndt and Savin (1977, p.1266)].

Convenient expressions for W in (4.6) and LM in (4.9) can be obtained from trivial algebric manipulations on (4.6) and (4.9), and are given by

$$W = T\, tr\, \hat{\Sigma}_{ML}^{-1}(\tilde{\Sigma}_{ML} - \hat{\Sigma}_{ML}) \tag{4.15}$$

and

$$LM = T\, tr\, \tilde{\Sigma}_{ML}^{-1}(\tilde{\Sigma}_{ML} - \hat{\Sigma}_{ML}), \tag{4.16}$$

when $\hat{\Sigma}_{ML}$ and $\tilde{\Sigma}_{ML}$ are the unrestricted and restricted ML estimates of Σ given in (2.9) and (3.8) [Berndt and Savin (1977)]. However, it should be noted that the relations in (4.15) and (4.16) hold only when $\tilde{\Sigma}_{ML}$ in W is obtained at the first step of iteration and $\tilde{\Sigma}_{ML}$ in LM is obtained at convergence of the IZEF method. When Σ is known, the W in (4.3) and LM in (4.8) identically reduce to the LR statistic in (4.11) [see for proof Byron (1970)], so that the three tests are exactly equivalent.

Even when Σ is unknown, the three tests, Wald, LR and LM, are asymptotically (but locally) equivalent, having test statistics identically distributed as an aymptotic central χ^2 with s degrees of freedom [Berndt and Savin (1977)]. However, the three tests can yield conflicting inferences when the same critical value is used, since the three tests have different sizes [Rothenberg (1984b)] and there exists a systematic numerical inequality between the test statistics such that

$$W \geqslant LR \geqslant LM \tag{4.17}$$

[Berndt and Savin (1977) and Breusch (1979)]. The inequality in (4.17) holds whether the null or the alternative hypothesis is true. Recently, Rothenberg (1984b) has shown that to a second order of approximation under local alternatives, the LR test statistic is approximated by a simple average of the Wald statistic and the LM statistic [Proposition 1 in Rothenberg (1984b)]. Furthermore, when s > 1, that is, when the null hypothesis is multidimensional, the three tests are not functions of each other, the power functions of the three tests differ, and no one test is uniformly more powerful than any other.

However, in testing homogeneity restrictions in a demand system, there is an exact relationship between the three test statistics such that[8]

$$LM = W/(1 + W/T) \tag{4.18}$$

and

$$LR = T\,ln\,(1 + W/T) \tag{4.19}$$

[see for proof Bera (1982)]. Moreover, the Wald statistic W in (4.6) (or in (4.15)) has a distribution function proportional to that of Hotelling's generalised T^2, even in the situation when Σ is unknown, the proportional factor being (T-k)/T [Hotelling (1931) and Laitinen (1978)]. Therefore, the three tests for homogeneity restrictions on a demand system can be made exact by using the T^2 criterion and the relations (4.18) and (4.19). The three tests then have the same power.

More specifically, when testing (m-1) independent homogeneity restrictions in a demand system with m equations and k regressors, the Wald test statistic multiplied by (T-k)/T

$$W^* = [(T-k)/T]W = (T-k)\,tr\,\hat{\Sigma}_{ML}^{-1}(\tilde{\Sigma}_{ML} - \hat{\Sigma}_{ML}),$$

is exactly distributed as T^2. Furthermore, T^2 is distributed as a multiple (m-1)(T-k)/(N-(m-1)-k) of F-distribution with (m-1) and (N-(m-1)-k) degrees of freedom, so that the critical region for the Wald test in (4.6) can be determined by

$$z_w = T(m-1)F_\alpha[(m-1),(N-(m-1)-k)]/(N-(m-1)-k), \qquad (4.20)$$

where $F_\alpha[(m-1),(N-(m-1)-k)]$ is the α % critical region of the F-distribution with (m-1) and (N-(m-1)-k) degrees of freedom [Laitinen (1978)].[9]

Recently, the problem of the validity of asymptotic tests in finite sample situations has attracted considerable attention in the demand literature. Laitinen (1978) and Meisner (1979) showed in a series of simulations that the Wald test is biased toward rejection of the null hypothesis in small samples when asymptotic χ^2 critical values are used. The same results were observed by Bera, *et al.* (1981) for the LR and LM tests. The authors commonly find that the problem of over-rejection is more serious the larger the number of equations in the system. However, Laitinen (1978) also showed that there is no small sample bias in the Wald test for homogeneity restrictions when using Hotelling's T^2 critical value.

As an attempt to correct or reduce the bias of asymptotic tests in small samples, the need for size corrections of the χ^2 tests has been suggested [e.g., Anderson (1958), Klevmarken (1975), Böhm, *et al.* (1980), Wales (1984) and Byron and

Rosalsky (1985)]. It is commonly accepted that the exact distributions of the asymptotic test statitics have fatter right hand tails than a true χ^2 distribution, and the exact distributions may differ from the χ^2 by a scalar multiple which can vary with size [Böhm, *et al.* (1980, p.132)]. The size correction factors used in the demand studies are often arbitrary. For example, Klevmarken (1975), in an *ad hoc* guess, used a correction factor, [T-k+s/(m-1)]/T for the Wald test, Böhm, *et al.* (1980) used (T-k)/T for the Wald, LR, and LM tests, and Wales (1984) used Anderson's (1958) correction factor[10] modified as

$$[T-k-\tfrac{1}{2}(m-s/m+1)]/T. \qquad (4.21)$$

for the LR test. Recently, Rothenberg (1984b) has derived size corrections for the Wald, LR and LM tests using an second order Edgeworth expansion [see Proposition 2 in Rothenberg (1984b) and also Phillips (1984)]. Byron and Rosalsky (1985) applied the Edgeworth corrections to the three asymptotic tests for symmetry restrictions in a demand system, and found through Monte Carlo simulations that the Edgeworth corrections work reasonably well for the moderate size system, such as 5, 8, 11 equation systems, but are not satisfactory for the larger system of 14 equations.

To handle the problem of over-rejection of the Wald test, Deaton (1972) proposed a test based on

$$[\,tr\,\tilde{\Sigma}_{h-1}{}^{-1}(\tilde{\Sigma}_h - \hat{\Sigma}_{OLS})/s]\big/[\,tr\,\tilde{\Sigma}_{h-1}{}^{-1}\hat{\Sigma}_{OLS}/m(T - k)],$$

which he argues is approximately distributed as F[s,m(T-k)] under the null hypothesis, where $\hat{\Sigma}_{OLS}$ is given in (2.8) and $\tilde{\Sigma}_h$ = $\tilde{U}'\tilde{U}/(T-k)$ is formed at the h-th iteration of the IZEF procedure. As an alternative, Deaton also formed a statistic which is asymptotically distributed as χ^2_s

$$D_h = (T - k)\, tr\, \tilde{\Sigma}_{h-1}^{-1}(\tilde{\Sigma}_h - \hat{\Sigma}_{OLS}) \big/ [\, tr\, \tilde{\Sigma}_{h-1}^{-1}\hat{\Sigma}_{OLS}/m].$$

Clearly, $D_1 = W^* = W(T-k)/T$, since $\tilde{\Sigma}_{h-1} = \hat{\Sigma}_{OLS}$ for h = 1. Bewley (1983) shows that as h approaches infinity, the limiting value of D_h is

$$D^* = LM^* \big/ (1 - [LM^*/(T - k)m]),$$

with $LM^* = LM(T-k)/T$, and that the inequality $W^* \geqslant D_h \geqslant LM^*$ holds. Bewley (1983) also demonstrates in a Monte Carlo simulation that the performance of D_2 is remarkably good, but D^* is biased towards acceptance of the null hypothesis when testing symmetry restrictions.

3.4.2 The Separate Induced Test of Hypothesis

In this subsection, we will discuss problems of testing of individual hypotheses in connection with the simultaneous test of multiple hypotheses. As the null hypothesis H_0 in (4.1) consists of s > 1 independent hypotheses, it can be decomposed into s separate hypotheses, $H_1, H_2, \ldots, H_s$. Each separate null hypothesis, H_i, for $i = 1,..,s$, can be written as

$$H_i : \theta_i = R_i\beta = 0, \tag{4.22}$$

where R_i is an 1 x mk i'th row vector of the restriction matrix R defining each i'th hypothesis.

Separate inferences on the validity of each individual hypothesis, H_i, may be of importance in situations in which the direct χ^2 test rejects the null hypothesis H_0. In these cases, one may want to find out why H_0 is rejected and which individual hypothesis is responsible for the rejection of H_0. Byron (1970) suggests the use of the t-ratio of the Lagrange multiplier estimate to identify the source of rejection of H_0, in which the test of each separate hypothesis is discussed conditionally given that the other hypotheses are correct. Deaton and Muellbauer (1980b) also consider t-ratios of the sum of the unrestricted estimates of price coefficients for each equation to check the validity of the individual homogeneity restrictions in the almost ideal demand system. However, there has been no systematic analytic work for the test of separate individual hypotheses in the context of demand systems.

In testing individual hypotheses, the separate test of each H_i in (4.22) should bear upon the test of H_0. It is logically certain that H_0 is true if and only if all the separate hypotheses H_i's are true, so that H_0 is accepted if and only if all the H_i's are accepted. Therefore, testing individual hypotheses H_i's should be carried out simultaneously in order that it might induce the test of H_0. Such a procedure for testing the separate hypotheses H_i's is termed the separate induced test [Seber (1964b) and Savin (1980)] in the sense that it induces a test of H_0. Procedures for constructing simulataneous confidence intervals are often called multiple comparison procedures. Miller (1966, 1977) presents an excellent survey of induced tests and multiple comparison procedures. Applications of the separate induced tests in econometrics are discussed in Savin (1980, 1984).

The Bonferroni and Scheffé procedures have been widely used for separate induced tests in the construction of simultaneous confidence intervals. Their derivations, properties and relations with the direct test are reviewed by Savin (1980). Therefore, the present discussion will be brief. However, the distributional properties of the test statistics for the individual hypothesis in demand models, such as individual homogeneity and symmetry restrictions, will be emphasised.

Tests of separate hypotheses can also be constructed, based on either the Wald, the LR, or the LM principle [Savin (1984)]. For the Wald test, we test if $\hat{\theta}_i$ is equal to zero, where $\hat{\theta}_i = R_i\hat{\beta}_{OLS}$ is the BLU estimate of θ_i in (4.22). For the LM test, we test if $\tilde{\lambda}_i$ is equal to zero, where $\tilde{\lambda}_i$ is the ML estimate Lagrage multiplier corresponding to the separate null hypothesis, H_i [Byron (1970)]. The LR test statistic of the separate hypothesis H_i can be given by $LR(H_i) = -2\,ln\,\Lambda(H_i)$, where $\Lambda(H_i)$ is Wilks' statistic for testing each H_i. Note, each $LR(H_i)$ has an asymptotic χ^2 distribution with 1 degree of freedom. Darroch and Silvey (1963) show that the LR statistic for the direct test of H_0 can be expressed as the sum of the $LR(H_i)$'s,

$$LR(H_0) = \sum_{i=1}^{s} LR(H_i),$$

which can be used for identifing the source of rejection of H_0 by the direct LR test procedure.[11] However, the LR separate tests may be computationally expensive for the large s, since it requires the calculation of $LR(H_i)$ for all i.

The Wald statistic for H_i is a t-ratio of $t(\hat{\theta}_i) = \hat{\theta}_i/\sigma(\hat{\theta}_i)$, where $\sigma^2(\hat{\theta}_i) = R_i(\Sigma \otimes (X'X)^{-1})R_i'$ is the scalar variance of $\hat{\theta}_i$. When Σ is known, $t(\hat{\theta}_i)$ is N[0,1] since $\hat{\theta}_i$ is $N[0,\sigma^2(\hat{\theta}_i)]$ under the normality assumption and $\sigma(\hat{\theta}_i)$ is a constant scalar. However, when Σ is unknown, the exact distribution of $t(\hat{\theta}_i)$ can be determined as Student's central t-distribution only for the case when the estimate of $\sigma^2(\hat{\theta}_i)$ has an exact distribution proportional to χ^2. Otherwise, we have to rely on the asymptotic normality of the distribution of $t(\hat{\theta}_i)$.

For individual homogeneity restrictions, the form of R_i is $R_i = e_i \otimes a'$, where e_i is the i'th row vector of an m x m identity matrix, $a'=(\iota_m', 0)$ and ι_m' is an m x 1 vector of unit elements. Therefore, the estimate of the variance of $\hat{\theta}_i$ is given by

$$\hat{\sigma}^2(\hat{\theta}_i) = (e_i \otimes a')(\hat{\Sigma} \otimes (X'X)^{-1})(e_i \otimes a)$$

$$= [a'(X'X)^{-1}a][e_i\hat{\Sigma}e_i'] = [a'(X'X)^{-1}a]\hat{\sigma}_{ii},$$

which is proportional to a $\chi^2_{(T-k)}$ distribution where $\hat{\Sigma}$ is given by $\hat{\Sigma}_{OLS} = (T-k)^{-1}\hat{U}'\hat{U}$ and $\hat{\sigma}_{ii}$ is the i'th diagonal term of $\hat{\Sigma}$. Therefore, $t(\hat{\theta}_i)$ has an exact central t-distribution in testing individual homogeneity restrictions.

However, for the individual symmetry hypothesis, $\beta_{pq} = \beta_{qp}$, for all p, q = 1,...,m, R_i can be written as $R_i = (0,..,0,1,0,..,0, -1,0,..,0)$ with a 1 on the [(p-1)k+q]'th position and -1 on the [(q-1)k+p]'th position. Then, the estimate of variance of $\hat{\theta}_i = \hat{\beta}_{pq} - \hat{\beta}_{qp}$ is

$$\hat{\sigma}^2(\hat{\theta}_i) = R_i(\hat{\Sigma} \otimes (X'X)^{-1})R_i'$$

$$= x_{pp}\hat{\sigma}_{pp} + x_{qq}\hat{\sigma}_{pp} - 2x_{pq}\hat{\sigma}_{pq},$$

where x_{pq} and $\hat{\sigma}_{pq}$ are the (p,q)'th elements of matrices $(X'X)^{-1}$ and $\hat{\Sigma}$, respectively. Since $\hat{\sigma}_{pq} = \hat{u}_p'\hat{u}_q/(T-k)$ (or $\hat{u}_p'\hat{u}_q/T$), where $\hat{u}_p$ is the residual vector of the p'th equation, $\hat{\sigma}^2(\hat{\theta}_i)$ is a linear combination of the quadratics of two different normally distributed variables, $\hat{u}_p$ and $\hat{u}_q$, for which the exact distribution is not known. Therefore, we cannot claim exactness of the t-test for an individual symmetry restriction, but can rely on its asymptotic normality, as $\hat{\sigma}^2(\hat{\theta}_i)$ is a consistent estimate of $\sigma^2(\hat{\theta}_i)$.

The ML estimator of λ, $\tilde{\lambda} = [R(\Sigma \otimes (X'X)^{-1})R']^{-1}R\hat{\beta}$, is $N(\underset{\sim}{0}_s,[R(\Sigma \otimes (X'X)^{-1})R']^{-1})$ under the H_0 when Σ is known, so that the LM test statistic for H_i given by a ratio of $t(\tilde{\lambda}_i) = \tilde{\lambda}_i/\sigma(\tilde{\lambda}_i)$ is then N(0,1), where $\sigma^2(\tilde{\lambda}_i)$ is the scalar variance of $\tilde{\lambda}_i$ obtained from the i'th diagonal term of $[R(\Sigma \otimes(X'X)^{-1})R']^{-1}$. However, when Σ is unknown and $\hat{\Sigma}$ is used, the exact distribution of $t(\tilde{\lambda}_i)$ cannot be identified, since the distribution of the diagonal element of $\hat{Q}^{-1}$ is not known.[12] Therefore, we rely on asymptotic normality for the distribution of $t(\tilde{\lambda}_i)$.

Now, consider the construction of confidence intervals for testing the separate hypothesis, H_i, using the Bonferonni and the Scheffé procedures. First, to define notation, we let A_i denote a confidence interval for testing H_i ($i = 1,\ldots,s$), C_i denotes a critical region for testing H_i, and $\alpha_i = P(C_i)$ is the significance level for testing H_i. The same notation with the suffix o will be used for the simultaneous test of H_0. The simultaneous confidence interval for testing H_0 is $A_0 = \bigcap_{i=1}^{s} A_i$[13] and an overall significant level α_0 for the test of H_0 is defined by $1 - \alpha_0 = P(\bigcap_{i=1}^{s} A_i)$. We assume that the overall significant level α_0 is given and fixed. Only the construction of a confidence interval for the Wald statistic will be considered, as the construction of the LM test interval is the similar to the Wald test.

The Bonferonni inequality

$$P(A_0 = \bigcap_{i=1}^{s} A_i) \geqslant 1 - \sum_{i=1}^{s} P(A_i'),$$

(where A_i' is the complement of A_i, i.e., $A_i' = C_i$) provides us with the boundary relation between α_0 and α_i's such that

$$\alpha_0 \leqslant \sum_{i=1}^{s} \alpha_i . \tag{4.23}$$

Hence, when α_0 is fixed, if we set $\alpha_i = \bar{\alpha}$ for all $i = 1,\ldots,s$, we have the lower bound of the α_i's such that $\alpha_0/s \leqslant \alpha_i$, since (4.23) becomes $\alpha_0 \leqslant s\bar{\alpha}$. Therefore, for a given overall significant level α_0, we can test the H_i's with significant level $\alpha_i = \alpha_0/s$. A test using this method is called a Bonferroni induced test.

The confidence intervals corresponding to each of the Bonferroni tests of the Wald type are:

$$A_i = \{B_i \mid \hat{B}_i - M(\bar{\alpha}/2,\nu)\hat{\sigma}(\hat{B}_i) \leqslant B_i \leqslant \hat{B}_i + M(\bar{\alpha}/2,\nu)\hat{\sigma}(\hat{B}_i)\}, \tag{4.24}$$

for $i = 1,\ldots,s$, where $M(\bar{\alpha}/2,\nu)$ is the upper $\bar{\alpha}/2$ critical point either from the t-distribution with $\nu = T-k$ degrees of freedom or from the standard normal distribution for the asymptotic test. We shall call these B-intervals. Since $\bar{\alpha}$ is the lower bound of the α_i's, the B-intervals are the upper bound of the confidence intervals for a given α_0. The simultaneous confidence region induced by the B-intervals in (4.24), denoted by $\bigcap_{i=1}^{s} A_i$, is of the shape of a polyhedron. The Bonferroni separate induced tests accept H_0 if and only if all the separate hypotheses H_i's are accepted, that is, the simultaneous confidence polyhedron covers the origin. However, the confidence region of the direct test is of the shape of ellipsiod, so that there can be conflicting results between the direct test and the Bonferroni separate tests, as shown in Figure 1 in Savin (1980).

In the Scheffé procedure the critical value, $M(\cdot,\cdot)$, is uniformly obtained from the distribution of the test statistics for the direct test of H_0. When the direct test of H_0 is an asypmtotic χ^2 test, $M(\cdot,\cdot)$ is the square root of the upper α_0 point of the χ^2_s distribution. Scheffé (1953) proved, in the context of an F-test, that the set of individual confidence intervals is the infinity of orthogonal projections of the ellipsoid (the confidence region for the direct test) on the coordinate axes and all linear combinations of the coordinate axes.[14] Thus, the Scheffé intervals have the form:

$$A_i = \{B_i \mid \hat{B}_i - M(\alpha_0,s)\hat{\sigma}(\hat{B}_i) \leqslant B_i \leqslant \hat{B}_i + M(\alpha_0,s)\hat{\sigma}(\hat{B}_i)\}, \quad (4.25)$$

where $M(\alpha_0,s)$ is the square root of the upper α_0 point of the χ^2 distribution with s degrees of freedom. This interval is called the S-interval and has a computational advantage over the B-interval, since it can be obtained directly from the critical value of the direct test. The relation between the direct test of H_0 and the Scheffé induced test is that the direct test accepts H_0 if and only if all the S-intervals cover the origin.

In conclusion, it should be noted that there is no theortical ground in demand theory for the validity of an individual restriction by itself. The demand restrictions are postulated collectively under utility maximisation subject to a budget constraint. Therefore, statistical testing of an individual restriction using separate induced tests appears meaningless in an economic sense. However, one should also note that separate induced tests are not intended as tests of the validity of each individual hypothesis, but rather as a test of the validity of a set of hypotheses. There is no logical disharmony between the direct test and separate induced tests. However, it is not our intention to suggest the use of the separate induced test as a substitute for existing direct tests. Instead, separate induced tests can be used as a complement to direct tests. In empirical situations when demand restrictions are rejected by direct tests, separate induced tests can be used to identify which restrictions are responsible for the rejection. What we suggest in this subsection is that separate induced tests in demand analysis should be carried out with the B-intervals in (4.24) or the S-interval in (4.25), so that the sizes of tests are compatible with the size of direct test, $1-\alpha_o$.

3.5 The Estimation of Singular Equation Systems

Singularity in a system of equations occurs when the sum of the dependent variables at each observation is equal to a linear combination of certain explanatory variables in the system [Berndt and Savin (1975, p.937)]. Demand systems are usually singular due to the adding up conditions. If the dependent variables are expenditures, their sum at each observation is total expenditure; in the case of share equations the sum is the constant term.

Singular systems are characterised by a linear dependence of the disturbances across the equations. Consequently, the covariance matrix of disturbances (Σ) is singular, so the inverse of Σ does not exist. Hence, neither Aitken's objective function for GLS estimators, $u'(\Sigma^{-1} \otimes I_T)u$, nor the likelihood function of the disturbances, can be defined. The standard treatment of the singularity problem has been a g-inverse solution. For example, Rao (1973) suggests the likelihood function with a generalised inverse of the covariance matrix and Anderson (1980) derives the necessary conditions for the maximum likelihood estimation of singular system using Rao's likelihood function and the Moore-Penrose generalised inverse [see also Khatri (1968), Theil (1971), and Pollock (1979)].

However, it is clear from the adding up conditions that all the information in one of the equations of the system can be obtained from the other equations, so that one of the equations in the system is redundant. Consequently, the elimination of one equation from the system does not result in any loss of information on the complete system. Barten (1969) showed that the ML estimates of the coefficients in the complete m equation system can be recovered from those of the m-1 equation subsystem, and that the results are invariant to which equation is deleted. He shows that the likelihood function of the disturbances in the reduced system,

$$L = (2\pi)^{-(m-1)T/2}|\Sigma_m|^{-T/2}\,exp\,\{-u_m'(\Sigma_m^{-1}\otimes I_T)u_m/2\},$$

can be written as

$$L = m^{T/2}(2\pi)^{-(m-1)T/2}|\Sigma+ii'|^{-T/2}\,exp\,\{-u'((\Sigma^{-1}+ii')\otimes I_T)u/2\},$$

where Σ_m is the covariance matrix Σ with the last row and column deleted, u_m the disturbance vector u with its elements of the m'th equation deleted, and $i = (1/\sqrt{m})(1,\ldots,1)'$ an $m \times 1$ column vector of unit elements multiplied by $(1/\sqrt{m})$ [Barten (1969, p.24-27)]. In other words, the likelihood function of u_m can be written in terms of the complete vector of disturbances, u, and the complete covariance matrix, Σ, and is entirely independent of the equation deleted. Obviously, this implies that the ML estimates of a singular system are invariant to the equation deleted.

The equivalence between the g-inverse approach and the method of deleting one equation from the system has been proved by Powell (1969) for Aitken's GLS estimation procedure. He considered the minimisation of the "pseudo Aitken objective function", which is defined as $u'(\Sigma^{+} \otimes I_T)u$, with the Moore-Penrose generalised inverse of Σ (Σ^{+}) and showed that the relation

$$u'(\Sigma^{+} \otimes I_T)u = u_k'(\Sigma_k^{-1} \otimes I_T)u_k \qquad (5.1)$$

holds for k=1,...,m, where Σ_k is the matrix Σ with the k'th column and row deleted, and u_k is the disturbance vector u with the k′th equation terms deleted [see also McGuire, *et al.* (1968)]. From (5.1), it is apparent that the g-inverse approach reduces to GLS estimation on m-1 equations.

Now, we consider how the elimination of one equation from a demand system affects restricted estimation of the coefficients under homogeneity and symmetry. To this end, we consider a system of linear demand equations without a constant term[15]

$$Y = X\Pi + \mu\gamma + U, \tag{5.2}$$

where Y is a T × m matrix of m dependent variables, X is a T × m matrix of price variables, μ is a T × 1 vector of total expenditure variable, Π is an m × m price coefficient matrix on which homogeneity and symmetry restrictions are imposed, γ is an 1 × m total expenditure coefficient vector and U is a T × m disturbance matrix. We suppose that μ in (5.2) is defined as $\mu = Y\iota$ due to the adding up condition, where ι is an m × 1 unit vector.

Post multiplying ι on both sides of (5.2), we have $Y\iota = X\Pi\iota + \mu\gamma\iota + U\iota = \mu$, which necessarily requires that

$$\Pi\iota = \underset{\sim}{0}_m,\ \gamma\iota = 1,\ \text{and}\ U\iota = \underset{\sim}{0}_T \tag{5.3}$$

hold identically, where $\underset{\sim}{0}_k$ is a k × 1 null vector. The last condition $U\iota = \underset{\sim}{0}_T$ in (5.3) implies the singularity of the covariance matrix of disturbances. Partitioning the system into an m-1 equation subsystem together with the last equation, (5.2) becomes

$$Y_o = X_o\Pi_o + x_{m.}\pi_{.m} + \mu\gamma_o + U_o \tag{5.4}$$

and

$$y_m = X_o\pi_{m.} + x_{m.}\pi_{mm} + \mu\gamma_m + u_m,$$

where $Y = [Y_o : y_m]$, $X = [X_o : x_{m.}]$, $U = [U_o : u_m]$, $\gamma = [\gamma_o : \gamma_m]$ and

$$\Pi = \begin{bmatrix} \Pi_o & \pi_{m.} \\ \pi_{.m} & \pi_{mm} \end{bmatrix}.$$

However, the condition $\Pi\iota = \underset{\sim}{0}_m$ in (5.3) implies that $\pi_{m.} = -\Pi_o\iota$ and $\pi_{mm} = -\pi_{.m}\iota$. Therefore, the symmetry restriction $\Pi = \Pi'$ which is imposed on the subsystem (5.4) can be expressed as

$$\Pi_o = \Pi_o' \text{ and } \pi_{.m} = -\iota'\Pi_o'. \tag{5.5}$$

The symmetry condition $\pi_{.m} = -\iota'\Pi_o'$ in (5.5) can be written under the condition $\Pi o = \Pi_o'$ as

$$\pi_{.m} = -\iota'\Pi_o. \tag{5.6}$$

The condition (5.6) is identical to the homogeneity restrictions on Π. Therefore, it has been proved that the adding up and homogeneity restrictions are equivalent under symmetry. However, substituting (5.6) into (5.4), we can write the subsystem (5.4) as

$$Y_o = X_o\Pi_o - x_{m.}\iota'\Pi_o + \mu\gamma_o + U_o$$

$$= \overline{X}\Pi_o + \mu\gamma_o + U_o, \qquad (5.7)$$

where $\overline{X} = (X_o - x_{m.}\iota')$ is the reduced X matrix whose column vectors are subtracted from the last column vector of X of order T x (m-1). Thus, the homogeneity restriction can be substituted out by transforming the explanatory variables, so that unrestricted estimation of (5.7) automatically produces homogeneity restricted estimates on the full system. This confirms again that restricted estimation subject to homogeneity is independent of the covariance matrix [see the discussion on $\tilde{\beta}_{OLS}$ in Section 3 of this Chapter]. The restricted estimates of the reduced system (5.7) subject to symmetry satisfy both the homogeneity and adding up conditions on the full system (5.2).

A more general approach to restricted estimation of a singular system can be found in Breusch (1978). He showed that any $s < k(m-1)$ independent linear restrictions on a singular system of m equations can be reformulated so as to apply to the coefficients of a (m-1) equation subsystem, provided they are consistent with the adding-up conditions. Partitioning the coefficient matrix B in the model (2.1) into $B = [B_*: \beta_m]$ and the restriction matrix R into $R = [R_1, R_2]$ and letting $\beta_* = vec(B_*)$, we can decompose the restriction $R\beta = r$ in (3.1) as

$$r = R\beta = R_1\beta_* + R_2\beta_m. \qquad (5.8)$$

The subscript * refers to the m-1 equation subsystem and m to the last equation in the system. If we assume that the n′th explanatory variable is equal to the sum of the dependent variables at each observation, we can write the adding up condition as $B\iota = e_n$, where e_n is a unit vector with 1 at the n′th position and 0 elsewhere. The restrictions in (5.8) can then be expressed as

$$r = R_1\beta_* + R_2\beta_m = R_1\beta_* + R_2[e_n - (\iota' \otimes I)\beta_*], \quad (5.9)$$

since the vectorisation of the adding up condition

$$\text{vec}(B\iota) = (\iota' \otimes I)\text{vec}(B) = (\iota' \otimes I)\text{vec}[B_*: \beta_m] = e_n$$

implies that $\beta_m = e_n - (\iota' \otimes I)\beta_*$. Therefore, the restrictions $R\beta = r$ can be reformulated as

$$R_*\beta_* = r_*, \quad (5.10)$$

where $R_* = [R_1 - R_2(i' \otimes I)]$ and $r_* = r - R_2e_m$ [see Proposition 12 in Breusch (1978)]. Breusch (1978) also proved the invariance of the restricted ML and GLS estimates to the equation deleted [see Proposition 13 in Breusch (1978)].

In testing restrictions, it is important to deduce the correct number of *independent* restrictions. For a demand system with m equations, there are m(m-1)/2 symmetry and m homogeneity restrictions. However, it is obvious from (5.5) and (5.6) that (m-1) symmetry restrictions are redundant under both adding up and homogeneity restrictions, so that the number of independent symmetry restrictions in the complete system reduces to (m-1)(m-2)/2, which is the number of symmetry restrictions in the reduced subsystem. By similar reasoning, the number of independent homogeneity restrictions reduces to (m-1) under the adding up conditions. Therefore, there are $m(m-1)/2 = (m-1) + (m-1)(m-2)/2$ independent restrictions in an m equation demand system.

3.6 Estimation of Symmetric Systems using the Lyapunov Equation

Bowden (1973) and Byron (1982) showed that the estimation of the symmetry coefficient matrix in a linear multivariate model is best achieved by the solution of a Lyapunov equation, $A'B + BA = C$.[16] Byron (1982) demonstrated that the estimation of symmetric systems using the Lyapunov equation is computationally much more efficient than restricted SUR estimation, particularly for large systems of equations.

However, despite the fact that the Lyapunov equation is linear in matrix terms, the solution of a Lyapunov equation is not straightforward, since matrix multiplication is not commutative. Hence, both Bowden (1973) and Byron (1982) rely on the numerical solution of the equation which suffers from the disadvantage of neither providing an analytic solution for the restricted estimator nor directly computed standard errors. In this section, it will be shown that the solution of the Lyapunov equation can be obtained analytically in terms of the vectorised matrix variables. The standard errors of the symmetry restricted estimator emerges directly from the solution. It will also be shown that the Lyapunov solution is identical to the symmetry restricted SUR estimator given in (3.2). For its mathematics, the approach in this section owes much to Jameson (1968) and MacRae (1974).

For simplicity of exposition, let us consider the estimation of a system of m linear multivariate equations

$$Y = XB + U \tag{6.1}$$

with m regressors and T observations, subject to symmetry restrictions on the coefficient matrix B, such that $B = B'$. We assume that $\Sigma = E(U'U)$ is known. The restricted GLS estimator of B in (6.1) subject to $B = B'$ is then obtained as a solution for B which minimises the Lagrange function

$$L = tr\,\Sigma^{-1}U'U + tr\,\Lambda(B - B'),$$

where Λ is an $m \times m$ skew-symmetric matrix of the Lagrange multipliers.[17] Differentiating the Lagrange function with respect to B and Λ, we have the first order conditions,

$$X'X\tilde{B}\Sigma^{-1} - X'Y\Sigma^{-1} = (\tilde{\Lambda} - \tilde{\Lambda}')/2 \tag{6.2}$$

and

$$\tilde{B} = \tilde{B}'. \tag{6.3}$$

Since Λ is skew-symmetric, we also have

$$\tilde{\Lambda} = -\tilde{\Lambda}'. \tag{6.4}$$

The solution of the above first order conditions (6.2) - (6.4) ultimately requires the solution of the Lyapunov equations. From (6.4), (6.2) can be written as

$$\tilde{\Lambda} = X'X\tilde{B}\Sigma^{-1} - X'Y\Sigma^{-1}, \tag{6.5}$$

or, equivalently

$$\tilde{B} = (X'X)^{-1}X'Y + (X'X)^{-1}\tilde{\Lambda}\Sigma. \tag{6.6}$$

Solving (6.4) and (6.5), we have a Lyapunov equation for $\tilde{B}$,

$$A'\tilde{B} + \tilde{B}A' = C, \tag{6.7}$$

and solving (6.3) and (6.6), we have a Lyapunov equation for $\tilde{\Lambda}$,

$$A\,\tilde{\Lambda} + \tilde{\Lambda}\,A = F, \tag{6.8}$$

where $A = X'X\Sigma$, $C = \Sigma X'Y + Y'X\Sigma$, $F = X'X(\hat{B}' - \hat{B})X'X$, and $\hat{B} = (X'X)^{-1}X'Y$ is the unrestricted OLS estimator of B. We have two Lyapunov equations (6.7) and (6.8), solutions of which determine separately the GLS estimators of B and Λ, respectively. The Lyapunov equations cannot be solved directly for the matrix variables $\tilde{B}$ and $\tilde{\Lambda}$, since matrix multiplication is not commutative, but can be solved for their vectors,

$vec(\tilde{B})$ and $vec(\tilde{\Lambda})$, by applying the vec operator to both sides of (6.7) and (6.8).

Since $vec(PQ) = (I_q \otimes P)vec(Q) = (Q' \otimes I_p)vec(P)$ for a $p \times n$ matrix P and an $n \times q$ matrix Q[18], (6.7) and (6.8) can be written in terms of $vec(\tilde{B})$ and $vec(\tilde{\Lambda})$, respectively, as

$$G'vec(\tilde{B}) = vec(C) \quad \text{and} \quad Gvec(\tilde{\Lambda}) = vec(F), \tag{6.9}$$

where $G = (I_m \otimes A + A \otimes I_m)$ and I_m is an identity matrix of order m. The equations in (6.9) have a unique solution for $vec(\tilde{B})$ and $vec(\tilde{\Lambda})$, respectively,

$$vec(\tilde{B}) = G'^{-1}vec(C) \quad \text{and} \quad vec(\tilde{\Lambda}) = G^{-1}vec(F), \tag{6.10}$$

if and only if $rank[G':vec(\tilde{B})] = rank[G:vec(\tilde{\Lambda})] = rank(G)$ and G is nonsingular.[19] Then, the estimators for B and Λ can be obtained from (6.10).

However, given the equations (6.5) and (6.6), one of two solutions in (6.10) is computationally redundant, since $\tilde{B}$ can be obtained by substitution of $\tilde{\Lambda}$ into (6.6), or vice versa. This substitution reduces the computational burden, as the tranpose of an $mm \times mm$ matrix G and the associated matrix multiplication can be avoided. However, since A is an $m \times m$ matrix, solving (6.10) requires the inversion of an $mm \times mm$ matrix G, which will be computationally troublesome when m is large. Several approaches have been suggested to find the inverse of G without direct inversion.[20] The most attractive and easily accessible method is Jameson's (1968). Let V be an $m \times m$ matrix of eigenevectors of A and D_A an $m \times m$ diagonal matrix of the eigenvalues of A. Since $V^{-1}AV = D_A$, so that $(V^{-1} \otimes V^{-1})G(V \otimes V) = D$, where $D = (I_m \otimes D_A + D_A \otimes I_m)$ is an $mm \times mm$ diagonal matrix, whose j'th diagonal term in i'th diagonal block matrix is $d_i + d_j$ and d_i's denote the eigenvalues of A, the inverse of G can be obtained by

$$G^{-1} = (V \otimes V)D^{-1}(V^{-1} \otimes V^{-1}). \tag{6.11}$$

Clearly, the inverse of G using (6.11) is much simpler than the direct inverse of the $mm \times mm$ matrix G itself, since D is diagonal and V is an $m \times m$ matrix. This procedure is obviously more computationally efficient than the restricted SUR estimation procedure.

The covariance matrices of $\text{vec}(\tilde{\Lambda})$ and $\text{vec}(\tilde{B})$ can be obtained directly from (6.10). $\text{Vec}(\tilde{\Lambda})$ can be written as

$$\begin{aligned} \text{vec}(\tilde{\Lambda}) &= W\text{vec}(\hat{B}' - \hat{B}) \qquad (6.12) \\ &= W[(Q \otimes I_m)\text{vec}(Y') - (I_m \otimes Q)\text{vec}(Y)] \\ &= W[P_{mm}(I_m \otimes Q)P_{Tm}P_{mT}\text{vec}(Y) - (I_m \otimes Q)\text{vec}(Y)] \\ &= W(P_{mm} - I_{mm})(I_m \otimes Q)\text{vec}(Y), \end{aligned}$$

where $W = G^{-1}[(X'X) \otimes (X'X)]$, $Q = (X'X)^{-1}X'$, and P_{mm} is an $mm \times mm$ permuted identity matrix which transforms $\text{vec}(S)$ into $\text{vec}(S')$

for an arbitrairy m x m matrix S, since

$$(Q \otimes I_m) = P_{mm}(I_m \otimes Q)P_{Tm} \text{ and } P_{Tm}P_{mT} = I_{mT}$$

[see MacRae (1974) and Appendix 3.1]. Applying the vec operator to (6.6) and substituting (6.12) into it, vec($\tilde{B}$) can be written as

$$\text{vec}(\tilde{B}) = \text{vec}(\hat{B}) + (\Sigma \otimes (X'X)^{-1})\text{vec}(\tilde{\Lambda}) \quad (6.13)$$

$$= \text{vec}(\hat{B}) + (\Sigma \otimes (X'X)^{-1})W(P_{mm}-I_{mm})(I_m \otimes Q)\text{vec}(Y)$$

Since, under the restriction $B = B'$, (6.12) and (6.13) can be written as

$$\text{vec}(\tilde{\Lambda}) = W(P_{mm} - I_{mm})(I_m \otimes Q)\text{vec}(U) \quad (6.14)$$

and

$$\text{vec}(\tilde{B}) = \text{vec}(B) + (I_m \otimes Q)\text{vec}(U) + (\Sigma \otimes (X'X)^{-1})W(P_{mm}- I_{mm})(I_m \otimes Q)\text{vec}(U)$$

$$= \text{vec}(B) + [I_{mm} + (\Sigma \otimes (X'X)^{-1})W(P_{mm}- I_{mm})] \times(I_m \otimes Q)\text{vec}(U), \quad (6.15)$$

we can see that $E[\text{vec}(\tilde{\Lambda})] = \underset{\sim}{0}$ and vec($\tilde{B}$) is an unbiased estimator of vec(B). Since $E[\text{vec}(U)\text{vec}(U)'] = \Sigma \otimes I_T$ and $Q'Q = (X'X)^{-1}$, and from (6.14), the covariance matrix of vec($\tilde{\Lambda}$) is given by

$$W(P_{mm}- I_{mm})(\Sigma \otimes (X'X)^{-1})(P_{mm} - I_{mm})W'. \quad (6.16)$$

From (6.15), the covariance matrix of $vec(\tilde{B})$ is given by

$$\Sigma \otimes (X'X)^{-1} + (\Sigma \otimes (X'X)^{-1})W(P_{mm} - I_{mm})(\Sigma \otimes (X'X)^{-1}). \quad (6.17)$$

Furthermore, using the relations (i) - (iv) in Appendix 3.1, it can be shown that covariance matrices of $vec(\tilde{\Lambda})$ and $vec(\tilde{B})$ reduce to more compact forms[21]

$$(I_{mm} - P_{mm})W \quad (6.16')$$

and

$$(\Sigma \otimes \Sigma)G^{-1}(I_{mm} + P_{mm}) \quad (6.17')$$

(see for proof Appendix 3.2).

For the general model $Y = XB + Z\Gamma + U$, the same procedure with the matrix $M_Z X$ replacing X (where $M_Z = I_T - Z(Z'Z)^{-1}Z'$) can be applied. For example, the matrix A becomes $A = (X'M_Z X)\Sigma$, $\hat{B} = (X'M_Z X)^{-1}X'M_Z Y$, $C = \Sigma X'M_Z Y + Y'M_Z X\Sigma$, $F = X'M_Z X(\hat{B}' - \hat{B})X'M_Z X$, and so on. The estimate of Γ is given by

$$\tilde{\Gamma} = (Z'Z)^{-1}Z'Y - (Z'Z)^{-1}Z'X\tilde{B}$$

with the covariance matrix of $vec(\tilde{\Gamma})$,

$$\Sigma \otimes (Z'Z)^{-1} + (I_m \otimes (Z'Z)^{-1}Z'X)[var(vec(\tilde{B}))] \times (I_m \otimes X'Z(Z'Z)^{-1}),$$

where $\tilde{B}$ is given by $(X'M_Z X)^{-1}X'M_Z Y + (X'M_Z X)\tilde{\Lambda}\Sigma$ and $cov\{vec(\tilde{B})\}$

is given by (6.17′) but with $A = (X'M_2X)\Sigma$ in G. For the estimation of a symmetric singular system, we eliminate one equation, estimate the reduced system and apply the above result to the reduced system (5.7).

The restricted SUR estimator of vec(B) is given in (3.2) with $\beta = \mathrm{vec}(B)$ and the restriction $B = B'$ transformed into $R\mathrm{vec}(B) = \underset{\sim}{0}$, where R is a restriction matrix of order $m(m-1)/2 \times mm$, and can be written as

$$\mathrm{vec}(\tilde{B}) = \mathrm{vec}(\hat{B}) - (\Sigma \otimes (X'X)^{-1})R'\tilde{\lambda}. \qquad (6.18)$$

$\tilde{\lambda}$ is the estimator of the Lagrange multiplier vector λ associated with the symmetry restrictions defined by $R\mathrm{vec}(B) = \underset{\sim}{0}$, and is given by

$$\tilde{\lambda} = [R(\Sigma \otimes (X'X)^{-1})R']^{-1}R\mathrm{vec}(\hat{B}), \qquad (6.19)$$

and $\mathrm{vec}(\hat{B}) = \hat{\beta}$ is the unrestricted OLS estimator of vec(B) in (2.4). Comparing (6.19) with the first equation in (6.13), we can see that it is sufficient to show that $-R'\tilde{\lambda} = \mathrm{vec}(\tilde{\Lambda})$ in order to prove the identity of SUR estimator in (6.18) and the solution of the Lyapunov equation in (6.13).

Using the permutation identity matrix P_{mm}, we can write the symmetry restriction, $B = B'$, as

$$\mathrm{vec}(\Delta B) = (I_{mm} - P_{mm})\mathrm{vec}(B) = \underset{\sim}{0}, \qquad (6.20)$$

where $\Delta B = B - B'$. However, since ΔB is skew-symmeric,[22] there are $m(m-1)/2$ distinct elements in ΔB. Therefore, (6.20) can be written as

$$\text{veck}(\Delta B) = K\text{vec}(\Delta B) = K(I_{mm} - P_{mm})\text{vec}(B) = \underset{\sim}{0}, \qquad (6.20')$$

where veck is a vec operator choosing the lower triangular part of a skew-symmetric matrix with excluding the zero diagonal terms and stacking them in a vector of order $m(m-1)/2$, and K is an $m(m-1)/2 \times mm$ matrix such that $\text{veck}(S) = K\text{vec}(S)$ for an $m \times m$ skew-symmetric matrix S [see also Appendix 3.1]. From (6.20′), we can see that the restriction matrix R used in the SUR estimation procedure can be written as

$$R = K(I_{mm} - P_{mm}). \qquad (6.21)$$

From (6.21) and using the relations (v)-(vii) in Appendix 3.1, it can be shown that

$$-\tilde{\lambda} = K\text{vec}(\tilde{\Lambda}), \qquad (6.22)$$

where $\tilde{\lambda}$ is the estimator of the Lagrange multiplier vector defined in (6.19). Then, from (6.21) and (6.22) and using the relations in Appendix 3.1, it is easily shown that $-R'\tilde{\lambda} = \text{vec}(\tilde{\Lambda})$. Therefore, the identity of (6.13) to (6.18) is proved.

The direct use of the above results for testing symmetry leads to some minor difficulties. For example, the Lagrange multiplier (LM) test, which is used to test whether $\tilde{\Lambda}$ is zero and is given by $vec(\tilde{\Lambda})'\{var(vec(\tilde{\Lambda}))\}^{-1}vec(\tilde{\Lambda})$, cannot be defined, since $\tilde{\Lambda}$ is skew-symmetric and $(I_{mm}- P_{mm})$ is singular, so that variance matrix of $vec(\tilde{\Lambda})$ in (6.16′) is also singular. However, this problem can simply be overcome by taking $veck(\tilde{\Lambda})$ instead of $vec(\tilde{\Lambda})$ in the expression for the LM test statistic. This results in test statistic

$$LM = veck(\tilde{\Lambda})'\{var(veck(\tilde{\Lambda}))\}^{-1}veck(\tilde{\Lambda}). \tag{6.23}$$

The Wald statistic, which is used to test $\Delta\hat{B} = \underset{\sim}{0}$, where $\Delta\hat{B} = \hat{B} - \hat{B}'$, is given by $veck(\Delta\hat{B})'\{var(veck(\Delta\hat{B})\}^{-1}veck(\Delta\hat{B})$ but cannot be defined, because $\Delta\hat{B}$ is skew-symmetric and the covariance matrix of $vec(\Delta\hat{B})$ is given by

$$(I_{mm}- P_{mm})(\Sigma \otimes (X'X)^{-1})(I_{mm}- P_{mm})$$

which is singular. Similarly, taking $veck(\Delta\hat{B}) = Kvec(\Delta\hat{B})$ instead of $vec(\Delta\hat{B})$, we have the Wald test statistic

$$W = veck(\Delta\hat{B})'\{var(veck(\Delta\hat{B})\}^{-1}veck(\Delta\hat{B}). \tag{6.24}$$

However, these two test statistics, the LM in (6.23) and the W in (6.24), can eventually be expressed as (4.9) and (4.6), respectively, when Σ is unknown, and are asymptotically distributed as the central χ^2 with $\dim\{veck(\Lambda)\} = \dim\{veck(\Delta\hat{B})\} = m(m-1)/2$ degrees of freedom.

In summary, we have shown that the symmetry constrained SUR estimator can be obtained as the solution of a Lyapunov equation using the vec operator, and that standard errors can be computed directly using (6.17′). The estimation of the Lagrange multipliers and their standard errors is also straightforward. We have also shown that the solution of the Lyapunov equation is algebraically identical to the standard restricted SUR estimator. This solves a problem left unresolved by Bowden (1973) and Byron (1982).

3.7 The Estimation of Nonlinear Equation Systems

The estimation of a system of nonlinear equations is much more difficult than that of linear systems. In most nonlinear models, exact algebraic expressions for the coefficient estimators are not available, and statistical inference is usually based on asymptotic results.[23] Even if the estimators can be derived analytically, they may be too complicated for practical use. Therefore, the estimation of nonlinear models usually depends on numerical solutions, computed by numerical optimisation procedures[24]. In this section, we will review briefly numerical optimisation procedures associated with the estimation of nonlinear equation systems. For excellent surveys on the application of numerical optimisation procedures in econometrics, see, for example, Goldfeld and Quandt (1972), Chambers (1973), Malinvaud (1980, Chapter 9) and Harvey (1981a, Chapter 4). For the nonlinear multivariate model, see Gallant (1975a), Barnett (1976) and Gallant and Holly (1980).

A system of m nonlinear equations can be written as

$$y = f(X;\beta) + u, \tag{7.1}$$

where y and u are defined in (2.3) of section 2, $f(X;\beta)$ is a nonlinear (vector) function in a $p_o \times 1$ coefficient vector β and X is a set of k_o regressors in the system. If we write the i'th equation in (7.1) as $y_i = f_i(X_i;\beta_i) + u_i$, where $f_i(X_i;\beta_i)$ is nonlinear in a $p_i \times 1$ coefficient vector, β_i, X_i a set of k_i explanatory variables for the i'th equation, then $f(X;\beta)$ in (7.1) can be written as $f(X;\beta) = vec[f_1(X_1;\beta_1), \ldots, f_m(X_m;\beta_m)]$ with $X = \cup_{i=1}^m X_i$ and $\beta = \cup_{i=1}^m \beta_i$.[23] We assume that $f(X;\beta)$ is continuous in $(X;\beta)$, and the first and second partial derivatives, $\partial f(X;\beta)/\partial\beta$ and $\partial^2 f(X;\beta)/\partial\beta\partial\beta'$, exist and are continuous in $(X;\beta)$. We also assume that the disturbances $u = vec(u_1,u_2,\ldots,u_m)$ have a multivariate normal distribution with zero mean and covariance matrix $\Omega = \Sigma \otimes I_T$, where $\Sigma = E(U'U)$ with $U = Y - F(X;\beta) = Y - [f_1(X_1;\beta_1), \ldots, f_m(X_m;\beta_m)]$. The model (7.1) is a nonlinear multivariate model which can represent nonlinear demand systems, such as the linear expenditure system and the translog model.

The ML estimation of β and Σ in (7.1) can be obtained by minimising the -2 log likelihood function

$$S(\beta,\Sigma) = \kappa + T\,ln\,|\Sigma| + (y-f(X;\beta))'(\Sigma^{-1}\otimes I_T)(y-f(X;\beta)) \tag{7.2}$$

where κ is a constant. However, to illustrate the problem in general terms, consider the minimisation of a continuous scalar valued function $S(\psi)$ of a q-dimensional vector ψ with respect to ψ; while maximisation can simply be taken as the minimisation of the negative objective function, i.e., $-S(\psi)$. It is assumed that the first partial derivatives $S_\psi(\psi) = \partial S(\psi)/\partial\psi$ (often called the *gradient*), and the Hessian $S_{\psi\psi}(\psi) = \partial^2 S(\psi)/\partial\psi\partial\psi'$ exist and are continuous in ψ. We assume that $S_{\psi\psi}$ is positive definite and nonsingular.

In numerical optimisation procedures, the optimal solution ψ^* of the first order conditions for minimising $S(\psi)$

$$S_\psi(\psi) = \partial S(\psi)/\partial\psi = \underset{\sim}{0}. \tag{7.3}$$

is usually solved by an iterative technique. The iteration is carried out under a given algorithm and continues, as long as it produces an improvement or until convergence[26] [Harvey (1981a, p.119)]. There are a wide range of iterative algorithms. For example, stepwise optimisation, the steepest descent method, and the Newton-Raphson and Gauss-Newton methods [see for details Goldfeld and Quandt (1972), Malinvaud (1980) and Harvey (1981a)]. The choice of algorithm depends, to some extent, on the type of objective function $S(\psi)$ or the first order conditions, S_ψ, to be solved in (7.3).

For example, when the parameter vector ψ can be partitioned as $\psi = (\psi_1, \psi_2)$ and S_ψ is linear in ψ_1 for given ψ_2 (and also linear in ψ_2 for given ψ_1), the solution $\psi^* = (\psi_1{}^*,$

ψ_2^*) of (7.3) can be obtained by an iterative stepwise optimisation procedure. That is, a solution for ψ_1 is obtained by solving S_ψ with a given value of ψ_2, then a solution for ψ_2 is obtained by solving S_ψ with newly updated values of ψ_1. This iterative process is continued until a certain convergence criterion is met. This procedure consists of iterative conditional optimisations which search out the conditional minimum of $S(\psi_i | \bar{\psi}_j)$ given $\psi_j = \bar{\psi}_j$, $i \neq j$, at each iteration, and is shown to always converge to solution of the first order minimising conditions [see a proof in Sargan (1964) and Oberhofer and Kmenta (1974)]. This procedure has been extensively adopted in ML estimation of the linear expenditure system in demand studies [e.g., Stone (1954) and Parks (1969)] and in estimating linear models with autoregressive disturbances [e.g., the Cochran-Orcutt iterative method]. One practical deficiency of this procedure is that it does not produce a valid covariance matrix of the coefficient estimates [Parks (1971)].

In general, most iterative schemes used in numerical optimisation algorithms are of the form

$$\psi^{(n+1)} = \psi^{(n)} + \rho \, d(\psi^{(n)}) \qquad (7.4)$$

where $\psi^{(n)}$ is the current approximation to the solution at the n′th iteration, $\psi^{(n+1)}$ is the updated approximation at the (n+1)′th iteration, ρ is a scalar "optimum" step-length, and $d(\psi)$ is the direction vector of order q [Berndt, *et al.*

(1974) and Harvey (1981a, p.121)]. Different algorithms have different ways of defining the direction vector $d(\psi)$. For example, the steepest descent method defines the direction vector as $d(\psi) = -S_\psi$, and the Newton-Raphson method as $d(\psi) = -S_{\psi\psi}^{-1}S_\psi$.

The choice of the direction vector in the Newton-Raphson method is based on a Taylor series expansion of $S(\psi)$ to the second order around the optimum $\psi = \psi^*$,

$$S(\psi) = S(\psi^*) + \Delta\psi' S_\psi(\psi^*) + (\tfrac{1}{2})\Delta\psi' S_{\psi\psi}(\psi^*)\Delta\psi \qquad (7.5)$$

where $\Delta\psi = \psi - \psi^*$. Differentiating (7.5) with respect to ψ yields

$$S_\psi(\psi) = S_\psi(\psi^*) + (\psi - \psi^*)' S_{\psi\psi}(\psi^*). \qquad (7.6)$$

Since $S_\psi = \underset{\sim}{0}$ must be satisfied at the optimum $\psi = \psi^*$, we can rearrange (7.6) and the iterative scheme becomes

$$\psi^{(n+1)} = \psi^{(n)} - [S_{\psi\psi}^{(n)}]^{-1} S_\psi^{(n)} \qquad (7.7)$$

where $S_{\psi\psi}^{(n)}$ and $S_\psi^{(n)}$ are $S_{\psi\psi}$ and S_ψ evaluated at $\psi = \psi^{(n)}$, respectively. Thus, the direction vector is obtained as $d(\psi) = -S_{\psi\psi}(\psi)^{-1}S_\psi(\psi)$.

If $S(\psi)$ is quadratic (like the Aitken objective fuction $(y-X\beta)'\Omega^{-1}(y-X\beta)$ for the linear model), the approximation

(7.5) is exact and the Newton-Raphson method (7.7) yields the optimum in a single iteration. However, when $S(\psi)$ is not quadratic, so that the approximation (7.5) may be poor, and if the initial values of ψ are far from the optimum, the Newton-Raphson method may even diverge. To reduce such instability in the algorithm and to speed up convergence, the Newton-Raphson iterative scheme (7.7) is modified by introducing a variable step length, ρ, into the scheme

$$\psi^{(n+1)} = \psi^{(n)} - \rho[S_{\psi\psi}{}^{(n)}]^{-1}S_{\psi}{}^{(n)}, \qquad (7.8)$$

which is called "the extended Newton-Raphson method". An optimal step length ρ^* can be chosen so as to minimise $S(\psi)$ with respect to ρ.[27]

The extended Newton-Raphson method [or the Newton-Raphson method] can be applied directly to the joint ML estimation of both β and Σ in (7.1) by using (7.8) [or (7.7)] with $S(\psi) = S(\beta,\Sigma)$ and $\psi = (\beta,\Sigma)$. However, in nonlinear optimisation, the estimation of a large number of coefficients may have a heavy penalty relating to the inversion of a large Hessian matrix at each iteration and to the slowness or failure of convergence. To avoid such problems, step-wise optimisation procedures are often incorporated in the nonlinear procedure for ML estimation of β and Σ. This reduces the number of coefficients being estimated by the nonlinear estimation routine [Harvey (1981a, p.137)]. In fact, at each iteration, a covariance matrix of the disturbances Σ can be estimated conditionally on the given values of β and then β can be estimated conditionally on the updated values of Σ. Iteration of this procedure eventually results in ML estimates of β and Σ at convergence [see Oberhofer and Kmenta (1974), Phillips (1976) and Malinvaud (1980, p.357)]. The ML estimator of covariance matrix Σ conditional on β is given by

$$\hat{\Sigma} = \hat{U}'\hat{U}/T,$$

as in (2.9), where the residual matrix U is updated by the newly estimated β after each iteration, that is, $\hat{U} = Y - F(X;\hat{\beta}^{(n)})$ after the n′th iteration. Conditional on $\Sigma = \hat{\Sigma}$, $S(\beta,\Sigma)$ in (7.2) reduces to the Aitken objective function,[28]

$$S(\beta|\Sigma) = u(\beta)'(\hat{\Sigma}^{-1} \otimes I_T)u(\beta) \tag{7.9}$$

where $u(\beta) = y - f(X;\beta)$. Therefore, at each iteration, nonlinear optimisation routines can be applied only to the ML estimation of β conditional on $\Sigma = \hat{\Sigma}$, $\hat{\Sigma}$ is then updated by the newly estimated β, and so on.[29]

The gradient and the Hessian matrix of (7.9) with respect to β are given as

$$S_\beta = 2u_\beta'\Omega^{-1}u \tag{7.10}$$

and

$$S_{\beta\beta} = 2u_\beta'\Omega^{-1}u_\beta + 2(u_{\beta\beta})'[I_P \otimes \Omega^{-1}u], \tag{7.11}$$

where $u_\beta = \partial u/\partial\beta$ is a Tm x $p_\circ$ matrix, $u_{\beta\beta} = (\partial^2 u/\partial\beta\partial\beta')$ is a (Tm x $p_\circ$) x $p_\circ$ matrix[30] and I_p is an identity matrix of order $p_\circ$. Hence, the extended Newton-Raphson iterative scheme in (7.8) becomes

$$\beta^{(n+1)} = \beta^{(n)} - \rho[S_{\beta\beta}{}^{(n)}]^{-1}S_\beta{}^{(n)}, \qquad (7.12)$$

where $S_\beta{}^{(n)}$ and $S_{\beta\beta}{}^{(n)}$ are S_β in (7.10) and $S_{\beta\beta}$ in (7.11) evaluated at $\beta = \beta^{(n)}$. However, the extended Newton-Raphson procedure in (7.12) is relatively inefficient, since it involves considerable computation of a (Tm x $p_\circ$) x $p_\circ$ matrix $u_{\beta\beta}$ in $S_{\beta\beta}$. Such computational effort is absent in the Gauss-Newton method, in which $u_{\beta\beta}$ is dropped from $S_{\beta\beta}$, so that $S_{\beta\beta}$ may be approximated by $S_{\beta\beta} \simeq 2u_\beta'\Omega^{-1}u_\beta$.

The Gauss-Newton method is based on a first order Taylor series expansion of $u(\beta) = y - f(X;\beta)$ around $\beta = \beta^*$

$$u(\beta) = y - f(X;\beta^*) - (\beta^* - \beta)'u_\beta(\beta^*). \qquad (7.13)$$

Since $u(\beta)$ in (7.13) is linear in β, we obtain the newly approximated $S(\beta|\Sigma)$ in (7.9) as a quadratic function in β with $u_{\beta\beta} = 0$. Therefore, $S_{\beta\beta}$ in (7.11) can be approximated by $S_{\beta\beta} = 2u_\beta'\Omega^{-1}u_\beta$. The iterative scheme for Gauss-Newton method is given by

$$\beta^{(n+1)} = \beta^{(n)} - \rho[u_\beta'\Omega^{-1}u_\beta]^{-1}[u_\beta'\Omega^{-1}u], \qquad (7.14)$$

where $u_\beta'\Omega^{-1}u_\beta$ and $u_\beta'\Omega^{-1}u$ are evaluated at $\beta = \beta^{(n)}$. The Gauss- Newton method has a computational advantage over Newton-Raphson method, as it requires only first derivatives.[31] Phillips (1976) proved that, given a sufficiently large number of observations (T), the estimates of β obtained by the Gauss-Newton method are the (quasi) ML estimates of β on convergence. He also proved that the iteration (7.14) is numerically stable, provided the initial values of β at the start of the iteration are sufficiently close to the true β.

The statistical properties of the nonlinear estimates obtained using the above procedure were investigated by Gallant (1975a, 1975b), Phillips (1976), Barnett (1976) and Malinvaud (1980) using large sample theory. Malinvaud (1980) showed that the estimates obtained by minimisation of (7.9) with respect to β are consistent and asymptotically efficient, and that if a consistent estimate of Σ is used, $\sqrt{T}(\hat{\beta} - \beta)$ is asymptotically normally distributed with the asymptotic covariance of the estimates, $[u_\beta'\Omega^{-1}u_\beta]^{-1}$. Strong consistency of the ML estimates of β (and Σ) has been proved by Gallant (1975b), Barnett (1976) and Phillips (1976). Thus, in using Gauss-Newton methods, an estimator of the asymptotic covariance matrix of β is directly available

$$\mathrm{var}(\hat{\beta}) = [\hat{u}_\beta'\hat{\Omega}^{-1}\hat{u}_\beta]^{-1}, \qquad (7.15)$$

which is the weighting matrix computed in iteration. Strong consistency of $var(\hat{\beta})$ in (7.15) has been shown in Barnett (1976).

In restricted estimation of the nonlinear model, the restrictions can simply be handled by substitution or by transformation of the parameters. Therefore, there is no need to generate the restriction matrix. The theoretical approach for nonlinear estimation subject to restrictions on parameters can be found in Gallant (1975b). Statistical tests for restrictions on parameters were considered by Gallant and Holly (1980). In testing the null hypothesis $h(\beta) = \underset{\sim}{0}$, where $h(\beta)$ is an s x 1 vector valued function of β with Jacobian matrix $H(\beta) = \partial h/\partial\beta$, the Wald test statistic is found to be

$$W = T\,[h'(\hat{\beta})\{H(\hat{\beta})[var(\hat{\beta})]H'(\hat{\beta})\}^{-1}h(\hat{\beta})], \qquad (7.16)$$

the Lagrange multiplier test[32] is

$$LM = T\,[S_{\tilde{\beta}}'[var(\tilde{\beta})]S_{\tilde{\beta}}'] \qquad (7.17)$$

and the likelihood ratio test statistic is

$$LR = -2T\,ln\,[S(\tilde{\beta};\hat{\Sigma})/(S(\hat{\beta};\hat{\Sigma})], \qquad (7.18)$$

which are all asymptotically distributed as χ^2 with s degrees of freedom, where S_β is given in (7.10) [Gallant and Holly

(1980, p.712)].

The estimation of a singular system of nonlinear equations can be carried out analogously to that of the linear system discussed in Section 5. We eliminate one equation from the system and then apply the nonlinear estimation procedure to the reduced system. However, there may be an identification problem for the coefficients of the full system, unless all the information on the deleted equation is contained in the other equations. For example, to meet such a requirement, normalisation of coefficients is necessary when estimating the translog model. However, in the linear expenditure system, the adding up conditions are sufficiently informative for the identification of all the coefficients. The invariance of the coefficient estimates to the equation deleted from the system follows from the discussion in Barten (1969).

3.8 The Estimation of Systems of Equations with Autocorrelated Errors

The nonlinear estimation procedure described in the previous section can be extended to the estimation of a system of equations having autocorrelated disturbances [Harvey (1981a, Chapter 6.8)]. Even when the system is linear in parameters, the joint estimation of structural and autocorrelation parameters reduces to a nonlinear estimation problem.

We consider the case when the disturbance matrix in a system of nonlinear equations, $U = Y - F(X;\beta)$, follows a vector AR(p) process

$$U = U_{-1}\Psi_1 + \ldots + U_{-p}\Psi_p + E, \tag{8.1}$$

where $E \sim NID(0,\Sigma_E)$ is a $(T-p) \times m$ white noise matrix, Ψ_i is an $m \times m$ parameter matrix for all $i = 1,\ldots,p$, and minus integer subscripts denote the lag order [see for a more general exposition of the ARMA(p,q) process Harvey (1981a, Chapter 6.8)]. Vectorising and rearranging (8.1), we have

$$\varepsilon = \varepsilon(\beta,\Psi_1, \ldots, \Psi_p)$$

$$= y - f(X;\beta) - (\Psi_1' \otimes I)u_{-1} - \ldots -(\Psi_p' \otimes I)u_{-p} \tag{8.2}$$

where y, $f(X;\beta)$, β and X are defined as in (7.1), $\varepsilon = vec(E)$ with $E(\varepsilon) = \underset{\sim}{0}$ and $E(\varepsilon\varepsilon') = \Omega_E = \Sigma_E \otimes I$, and the identity matrix I is of order $(T-p)$. The joint ML estimates of β and $\Psi_1, \ldots, \Psi_p$ can be obtained by the Gauss-Newton procedure for minimising

$$S(\beta,\Psi) = \varepsilon(\beta,\Psi)'\Omega_E^{-1}\varepsilon(\beta,\Psi) \tag{8.3}$$

with respect to β and Ψ, where $\Psi = (\Psi_1, \ldots, \Psi_p)$. Then, letting $\alpha = (\beta',vec(\Psi)')'$, we can apply the Gauss-Newton method (7.14) with $\varepsilon_\alpha = \partial\varepsilon/\partial\alpha$ and Ω_E^{-1} instead of $u_\beta = \partial u/\partial\beta$ and Ω^{-1} in (7.14). An estimate of the asymptotic covariance matrix of estimate α, $\hat{\alpha}$, is given by

$$\text{Avar}(\hat{\alpha}) = [\hat{\varepsilon}_{\alpha}'\Omega_{E}^{-1}\hat{\varepsilon}_{\alpha}]^{-1} \tag{8.4}$$

As discussed in Pagan and Byron (1977), the treatment of the pre-period disturbances, $U^* = [U_1, \ldots, U_p]$ as fixed terms or as stochastic ones is an important practical issue in estimation, since the likelihood function is different in each case. If U^* is assumed to be nonstochastic, it can be excluded in the likelihood function, and the Gauss-Newton method discussed previously can be used.

Stationarity of the autoregressive error process is an important question in any empirical study. For AR(p) in (8.1) to be stationary, it is sufficient that the roots of determinantal polynomials

$$|\lambda^{p}I - \lambda^{p-1}\Psi_1 - \ldots - \Psi_p| = 0 \tag{8.5}$$

be all less than one in absolute value [Harvey (1981b, p.50-51)]. For the AR(1) process, the roots of the determinantal equation (8.5) can be obtained as the eigenvalues of Ψ_1. However, for the higher order AR(p) process, the solution of (8.5) for λ can be extremely complicated. Nevertheless, the stationarity of the higher order AR(p) process can be checked from the eigenvalues of the *companion* matrix A_p with the form

$$A_p = \begin{bmatrix} \Psi_1 & I_m & 0 & 0 & . & 0 \\ \Psi_2 & 0 & I_m & 0 & . & 0 \\ \Psi_3 & 0 & 0 & I_m & . & 0 \\ . & . & . & . & . & . \\ \Psi_{p-1} & 0 & 0 & 0 & . & I_m \\ \Psi_p & 0 & 0 & 0 & . & 0 \end{bmatrix}. \tag{8.6}$$

That is, if eigenvalues of the companion matrix A_p in (8.6) are all less than one in absolute value, the AR(p) process can be said to be stationary [Chow (1975, p.23-25)].

When a system is singular, so that the error process in (8.1) is subject to adding up conditions that $U\iota = \underset{\sim}{0}$; then $\Psi_i \iota = \kappa_i \iota$, and $E\iota = \underset{\sim}{0}$, where $\kappa_i \iota$ is a constant vector for $i = 1,\ldots,p$, and the full coefficient matrices Ψ_i's in (8.1) cannot be recovered from estimation of the reduced system. Berndt and Savin (1975) showed that under the adding up conditions, the full-system first order autoregressive error process in disturbances

$$U = U_{-1} \Psi_1 + E \tag{8.7}$$

reduces to

$$U^{(m)} = U_{-1}^{(m)} \overline{\Psi}_1 + E^{(m)} \tag{8.8}$$

in the reduced system. The superscript (m) indicates the system has the last equation deleted and $\overline{\Psi}_1$ is defined as an (m-1) x (m-1) matrix such that

$$\overline{\Psi}_1 = \Psi_1 - \psi_1^{(m)} \iota'. \tag{8.9}$$

with the last row and column deleted, while $\psi_1^{(m)}$ is the last column of Ψ_1. Thus, the (i,j)'th element $\overline{\psi}_{ij}$ in $\overline{\Psi}_1$ in the reduced system does not refer to that of ψ_{ij} in Ψ, but to $\overline{\psi}_{ij} = \psi_{ij} - \psi_{im}$. Therefore, it is impossible to recover estimates of the full Ψ_1 matrix using estimates of $\bar{\Psi}_1$ from the reduced system without some degree of prior information on Ψ_1 [Berndt and Savin (1975)]. However, the ML estimates of the structural parameters, β and Σ, of the full system can be recovered from the estimates of the reduced system and are invariant to the equation deleted. This result for the AR(1) process can obviously be generalised to the case of the AR(p) process. Moreover, it is obvious that $\bar{\Psi}_i = 0$ does not imply $\Psi_i = 0$, so that statistical inference on Ψ_i, including a significance test of Ψ_i, cannot be inferred from $\bar{\Psi}_i$ in the reduced sytem [Berndt and Savin (1975)].

In empirical situations, it is common to assume that the disturbances are serially correlated within equation but not across equations. For example, the first order autoregressive process for the i'th equation can be of the form

$$u_{it} = \psi_i u_{it-1} + \varepsilon_{it} \text{ for } i = 1,\ldots,m, \tag{8.10}$$

but $E(u_{it}u_{jt-1}) = 0$ for $i \neq j$. Consequently, the autoregressive coefficient matrix Ψ_1 in (8.7) is diagonal. However, when the system is singular, the coefficient matrix Ψ_1 is also subject to the adding up condition that $\Psi_1 \iota = \kappa_1$ [Berndt and Savin (1975)]. Therefore, Ψ_1 should be equal to a scalar matrix $\kappa_1 I_m$ when Ψ_1 is diagonal, where I_m is an identity matrix of order m. In other words, all ψ_i's in (8.10) are the same to κ_1, which is unlikely in the context of a demand system. Thus, the diagonality of autoregressive error structure may not be a sensible specification for singular demand system [Deaton (1986, p.1782)].

Berndt and Savin (1975) showed that stationarity of the AR(1) process in a singular system can be checked using the eigenvalues of the reduced autoregressive coefficient matrix $\bar{\Psi}_1$. They showed that eigenvalues of $\bar{\Psi}_1$ such that $|\lambda I - \bar{\Psi}_1| = 0$ are also those of Ψ_1 with the possible exception of a multiple root $\lambda = \kappa_1$, where κ_1 is a constant such that $\Psi_1 \imath = k_1 \imath$ [Berndt and Savin (1975, p.944)]. A similar discussion can be developed for the stationarity of a higher order AR(p) process in a singular system. It can be seen that the eigenvalues of the reduced companion matrix $\bar{A}_p$, defined as in (8.6) with Ψ_i's and I_m replaced by the reduced coefficient matrices $\bar{\Psi}_i$ and I_{m-1}, respectively, are also eigenvalues of the complete companion matrix A_p. In this case, there may be p multiple eigenvalues. A simple theoretical exposition is given in Appendix 3.3 for an AR(2) process with 3 equation system. The result in Appendix 3.3 can obviously generalise to the case of arbitrary p and m.

3.9 Concluding Remarks

In this chapter, we surveyed the statistical problems relating to the estimation of demand systems and tests of hypotheses. However, many topics have been omitted. Important omissions include the simultaneous equation approach to demand systems [Attfield (1985)], the error-in-variable problem in demand systems [Stapleton (1984)], the imposition of negativity restrictions in the estimation of demand systems [e.g., Wales and Woodland (1983)], tests of the negativity using the eigenvalues of the Slutsky matrix [e.g., Böhn, *et al*. (1980) and Bewley (1982)], and so on. The scope of demand analysis is becoming much too large for a one chapter survey.

However, additional statistical problems will be considered in the subsequent chapters as they arise. For example, tests of structural change in demand equations will be discussed in the next chapter, and the estimation of dynamic demand systems will be considered in Chapter 5.

APPENDIX 3.1

MacRae (1974) defines a permuted identity matrix as P_{pq} of order pq, partitioned into q rows and p columns of submatrices of order p x q, such that the (i,j)′th submatrix has unity as its (j,i)′th element and zero elsewhere. This permuted identity matrix P_{pq} transforms vec(S) to vec(S′), for a q x p matrix S in such a way that $P_{pq}vec(S_{q \times p}) = vec(S'_{p \times q})$. Some useful and important properties of permuted identity matrices are

$$(i)\ P_{p1} = P_{1p} = I_p,\quad (ii)\ (P_{pq})' = P_{qp},\quad (iii)\ P_{pq}P_{qp} = I_{pq},$$

with an identity matrix of order k, I_k, and for their use with Kronecker products

$$(iv)\ B_{r \times s} \otimes A_{p \times q} = P_{pr}(A_{p \times q} \otimes B_{r \times s})P_{sp}.$$

We use the terminology P_{pq} for this permuted matrix instead of MacRae′s original $I_{(p,q)}$, to avoid confusion with the identity matrix I_{pq}.

The operator *veck* is defined as a vec operator choosing the lower triangular part of a skew-symmetric matrix excluding the zero diagonal terms and stacking them in a vector of order m(m-1)/2. Then, we can define an m(m-1)/2 x mm matrix matrix K such that veck(S) = Kvec(S) for an m x m skew-symmetric matrix S, which transforms vec(S) to veck(S). The expression of the matrix K is not unique. For a 3 x 3 skew-symmetric matrix, K can be defined as follows:

$$K_3 = \begin{bmatrix} 0 & \frac{1}{2} & 0 & : & -\frac{1}{2} & 0 & 0 & : & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & : & 0 & 0 & 0 & : & -\frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & : & 0 & 0 & \frac{1}{2} & : & 0 & -\frac{1}{2} & 0 \end{bmatrix}$$

However, the K matrix defined as the above has the following important properties;

(v) $KK' = (1/2)I_{m(m-1)/2}$, (vi) $K'K = (1/4)(I_{mm} - P_{mm})$,

and

(vii) $vec(S) = 2K'veck(S)$.

APPENDIX 3.2

1. The Covariance matrix of $\text{vec}(\tilde{\Lambda})$.

$$W(P_{mm} - I_{mm})(\Sigma \otimes (X'X)^{-1})(P_{mm} - I_{mm})W' \tag{A2.1}$$

$$= (I_{mm} - P_{mm})W \tag{A2.2}$$

Proof: Expanding the term $(P_{mm} - I_{mm})(\Sigma \otimes (X'X)^{-1})(P_{mm} - I_{mm})$ in (A2.1) gives us

$$(P_{mm} - I_{mm})[\Sigma \otimes (X'X)^{-1}](P_{mm} - I_{mm})$$

$$= P_{mm}[\Sigma \otimes (X'X)^{-1}]P_{mm} - P_{mm}[\Sigma \otimes (X'X)^{-1}] - [\Sigma \otimes (X'X)^{-1}]P_{mm} + [\Sigma \otimes (X'X)^{-1}]$$

$$= [(X'X)^{-1} \otimes \Sigma] - [(X'X)^{-1} \otimes \Sigma]P_{mm} - [\Sigma \otimes (X'X)^{-1}]P_{mm} + [\Sigma \otimes (X'X)^{-1}]$$

$$= [(X'X)^{-1} \otimes \Sigma](I_{mm} - P_{mm}) - [\Sigma \otimes (X'X)^{-1}](I_{mm} - P_{mm})$$

$$= [(X'X)^{-1} \otimes \Sigma - \Sigma \otimes (X'X)^{-1}](I_{mm} - P_{mm})$$

$$= W^{-1}(I_{mm} - P_{mm}),$$

since $W = G^{-1}[(X'X) \otimes (X'X)]$, $G = (I \otimes (X'X)\Sigma + (X'X)\Sigma \otimes I)$, and

$$(X'X)^{-1} \otimes \Sigma + \Sigma \otimes (X'X)^{-1}$$

$$= [(X'X)^{-1} \otimes (X'X)^{-1}][I \otimes (X'X)\Sigma + (X'X)\Sigma \otimes I]$$

$$= [(X'X)^{-1} \otimes (X'X)^{-1}]G = W^{-1}.$$

Therefore, (A2.1) reduces to (A2.2), since

$$W[(X'X)^{-1} \otimes \Sigma - \Sigma \otimes (X'X)^{-1}](I - P_{mm})W'$$

$$= WW^{-1}(I - P_{mm})W' = (I - P_{mm})W$$

and $W = W'$. Q.E.D.

2. The Covariance matrix of $vec(\tilde{B})$

$$\Sigma \otimes (X'X)^{-1} + (\Sigma \otimes (X'X)^{-1})W(P_{mm} - I_{mm})(\Sigma \otimes (X'X)^{-1}) \qquad (A2.3)$$

$$= (\Sigma \otimes \Sigma)G^{-1}(I_{mm} + P_{mm}). \qquad (A2.4)$$

Proof: Write $I = I_{mm}$. Since $W = G^{-1}[(X'X) \otimes (X'X)]$ and $P_{mm}[\Sigma \otimes (X'X)^{-1}] = P_{mm}P_{mm}[(X'X)^{-1} \otimes \Sigma]P_{mm} = [(X'X)^{-1} \otimes \Sigma]P_{mm}$, the right hand term in (A2.3) can be written as

$$(\Sigma \otimes (X'X)^{-1})G^{-1}\{[(X'X) \otimes (X'X)][(X'X)^{-1} \otimes \Sigma]P_{mm} - [(X'X) \otimes (X'X)](\Sigma \otimes (X'X)^{-1})\}$$

$$= (\Sigma \otimes (X'X)^{-1})G^{-1}[(I \otimes A)P_{mm} - (A \otimes I)], \qquad (A2.5)$$

where $A = (X'X)\Sigma$. Using the relations (iii) and (iv) in

Appendix 3.1, we can write (A2.5) as

$$\Sigma \otimes (X'X)^{-1})G^{-1}[P_{mm}(A \otimes I) - (A \otimes I)]$$

$$= (\Sigma \otimes (X'X)^{-1})G^{-1}(P_{mm} - I)[A \otimes I] \qquad \text{(A2.6)}$$

Then, from (A2.6) and postmultiplying $I = G^{-1}G$ on the first term in (A2.3), (A2.3) can be written as

$$[\Sigma \otimes (X'X)^{-1}]G^{-1}[G + (P_{mm} - I)(A \otimes I)]$$

$$= [\Sigma \otimes (X'X)^{-1}]G^{-1}[I \otimes A + P_{mm}(A \otimes I)\ ]$$

$$= [\Sigma \otimes (X'X)^{-1}]G^{-1}(I \otimes A)(I + P_{mm}), \qquad \text{(A2.7)}$$

since $G = I \otimes A + A \otimes I$ and $P_{mm}(A \otimes I) = (I \otimes A)P_{mm}$. However, since $\Sigma \otimes (X'X)^{-1} = (\Sigma \otimes \Sigma)(I \otimes A^{-1})$, (A2.7) becomes

$$(\Sigma \otimes \Sigma)(I \otimes A^{-1})(I \otimes A + A \otimes I)^{-1}(I \otimes A)(I + P_{mm})$$

$$= (\Sigma \otimes \Sigma)[(I \otimes A^{-1})(I \otimes A + A \otimes I)(I \otimes A)]^{-1}(I + P_{mm})$$

$$= (\Sigma \otimes \Sigma)[(I \otimes I + A \otimes A^{-1})(I \otimes A)]^{-1}(I + P_{mm})$$

$$= (\Sigma \otimes \Sigma)[I \otimes A + A \otimes I]^{-1}(I + P_{mm})$$

$$= (\Sigma \otimes \Sigma)G^{-1}(I + P_{mm}),$$

which shows that (A2.3) = (A2.4). Q.E.D.

APPENDIX 3.3

Consider the case of the AR(2) error process in a singular system of 3 equations,

$$U = U_{-1}\,\Psi + U_{-2}\,\Pi + E. \qquad \text{(A3.1)}$$

Then, the stationarity of the process (A3.1) can be checked with eigenvalues of a companion matrix of the form

$$A_2 = \begin{bmatrix} \Psi & I_m \\ \Pi & 0 \end{bmatrix}. \qquad \text{(A3.2)}$$

For the case m = 3, the characteristic equation of A_2 is given by

$$|A_2 - \lambda I_m| = \begin{vmatrix} \psi_{11}-\lambda & \psi_{12} & \psi_{13} & 1 & 0 & 0 \\ \psi_{21} & \psi_{22}-\lambda & \psi_{23} & 0 & 1 & 0 \\ \psi_{31} & \psi_{32} & \psi_{33}-\lambda & 0 & 0 & 1 \\ \pi_{11} & \pi_{12} & \pi_{13} & -\lambda & 0 & 0 \\ \pi_{21} & \pi_{22} & \pi_{23} & 0 & -\lambda & 0 \\ \pi_{31} & \pi_{32} & \pi_{33} & 0 & 0 & -\lambda \end{vmatrix}$$

Taking the following procedure,

(1) subtract the 3rd row from the 1st and 2nd rows,

(2) subtract the 6th row from the 4th and 5th rows,

(3) add the 1st and 2nd columns to the 3rd column,

(3) add the 4th and 5th columns to the 6th column,

we have

$$|A_2-\lambda I_3| = \begin{vmatrix} \psi_{11}-\psi_{13}-\lambda & \psi_{12}-\psi_{13} & 0 & 1 & 0 & 0 \\ \psi_{21}-\psi_{23} & \psi_{22}-\psi_{23}-\lambda & 0 & 0 & 1 & 0 \\ \psi_{31} & \psi_{32} & \kappa_1-\lambda & 0 & 0 & 1 \\ & & & & & \\ \pi_{11}-\pi_{13}-\lambda & \pi_{12}-\pi_{13} & 0 & -\lambda & 0 & 0 \\ \pi_{21}-\pi_{23} & \pi_{22}-\pi_{23}-\lambda & 0 & 0 & -\lambda & 0 \\ \pi_{31} & \pi_{32} & \kappa_2 & 0 & 0 & -\lambda \end{vmatrix}$$

since the adding up conditions imply that $\Psi_1\iota = \kappa_1$ and $\Psi_2\iota = \kappa_2$, where κ_1 and κ_2 are real valued constants. Moving the 3rd column into the 5th column, so that the 4th and 5th columns become the 3rd and 4th columns, respectively, and similarly moving the 3rd row into the 5th row, we obtain

$$|A_2-\lambda I_3| = \begin{vmatrix} \psi_{11}-\psi_{13}-\lambda & \psi_{12}-\psi_{13} & 1 & 0 & 0 & 0 \\ \psi_{21}-\psi_{23} & \psi_{22}-\psi_{23}-\lambda & 0 & 1 & 0 & 0 \\ & & & & & \\ \pi_{11}-\pi_{13}-\lambda & \pi_{12}-\pi_{13} & -\lambda & 0 & 0 & 0 \\ \pi_{21}-\pi_{23} & \pi_{22}-\pi_{23}-\lambda & 0 & -\lambda & 0 & 0 \\ & & & & & \\ \psi_{31} & \psi_{32} & 0 & 0 & \kappa_1-\lambda & 1 \\ \pi_{31} & \pi_{32} & 0 & 0 & \kappa_2 & -\lambda \end{vmatrix}$$

$$= \begin{vmatrix} \bar{A}_2 - \lambda I_2 & L_{12}' \\ & \\ L_{21} & L_{22} \end{vmatrix}, \qquad \text{(A3.3)}$$

where $\bar{A}_2 = \begin{bmatrix} \bar{\Psi} & I_2 \\ \bar{\Pi} & 0 \end{bmatrix}$, $L_{12} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$L_{21} = \begin{bmatrix} \pi_{31} & \pi_{32} & 0 & 0 \\ \psi_{31} & \psi_{32} & 0 & 0 \end{bmatrix}$, and $L_{22} = \begin{bmatrix} \kappa_1 - \lambda & 1 \\ \kappa_2 & -\lambda \end{bmatrix}$.

Obviously, $\bar{A}_2$ is the companion matrix of the AR(2) coefficient matrix of the reduced reduced system. Then, using the determinant of the partioned matrix [Dhrymes (1978, Proposition 30, p.457)], we can see that

$$
\begin{aligned}
|A_2 - \lambda I_m| &= |L_{22}|\,|(\bar{A}_2 - \lambda I_2) - L_{12}L_{22}^{-1}L_{21}| \\
&= (\lambda^2 - \kappa_1\lambda - \kappa_2)\ |(\bar{A}_2 - \lambda I_2)|. \qquad \text{(A3.4)}
\end{aligned}
$$

Therefore, it has been shown that eigenvalues of the reduced companion matrix are also those of the full companion matrix.

FOOTNOTES:

1. Recently, Attfield (1985) treated total expenditure as an endogenous variable, as it is defined as the sum of dependent variables, and considered (2.1) as a simultaneous equation system.

2. The model (2.3) is a special case of the SUR model when the set of explanatory variables are identical for each equation.

3. In empirical demand studies, the iterative estimation procedure is often rejected on the basis of the computing expense and that the first step ZEF is already consistent and asymptotically efficient [Barten (1969) and Byron (1970)].

4. For the unrestricted SUR model, Zellner (1963) derived the exact sampling distribution of $\hat{\beta}_{ZEF}$ for a simple special case when m = 2 and the set of explanatory variables for each equation are orthogonal. Kakwani (1967) showed that the ZEF estimate is unbiased, provided its mean exists and the disturbances follow a continuous symmetric probability law.

5. Technically, computation of the restricted OLS is identical to that of GLS but with $\Sigma = I_m$ [Deaton (1972)].

6. This asymptotic distributional property holds even if X includes stochastic variables.

7. When $\varepsilon \sim N(0,\Sigma_\varepsilon)$ and A is symmetric, $\varepsilon' A\varepsilon \sim \chi^2_p$ if and only if $\Sigma_\varepsilon A\Sigma_\varepsilon A\Sigma_\varepsilon = \Sigma_\varepsilon A\Sigma_\varepsilon$ and $\mathrm{rank}(\Sigma_\varepsilon A\Sigma_\varepsilon) = p$.

8. Evans and Savin (1982) showed that the same relations hold in the context of single equation for a general linear restriction.

9. Hotelling (1931) also showed that the α % critical region of T^2 distributed statistic can be determined by the beta distribution as

$$T^2_\alpha = (T - k)(1 - b_\alpha)/b_\alpha,$$

where b_α is the upper α % point of the beta distribution with parameters (T-k-s+1)/2 and s/2 [see for detail Hotelling (1931, p.376-377) and Wilks (1962, p.594)].

10. Anderson's (1958, p. 208) correction factor is

$$[T-k-\tfrac{1}{2}(m-s_1+1)]/T,$$

where s_1 is the number of zero restrictions in each equation, which ensures that the approximation is to order 1/T. Wales (1984) simply averaged out the restrictions across equations in the sense that s_1 is replaced by s/m.

11. From this equality, we can see that there can be a conflict between the direct LR test and the separate induced LR tests, since the critical values of χ^2 distribution is not additive over degrees of freedom. Darroch and Silvey (1963) showed that this equality does not hold in small samples, even if H_i's are independent.

12. It is known that when Q has a Wishart distribution, Q^{-1} has the inverted Wishart distribution, when $\hat{\Sigma} = U'U/(T-k)$ is used. But the distribution of the diagonal element of Q^{-1} is not known.

13. If we regard the A_i as a one-dimensional space defined on the θ_i axis in the θ-space, A_o should be expressed as a Cartesian product of A_i's, that is, $A_o = \prod_{i=1}^{s} A_i$. However, the A_i is an s-dimensional subspace in the θ-space, so that the expression $A_o = \bigcap_{i=1}^{s} A_i$ for A_o is authentic. For example, when s = 2, A_i is a band shaped subspace along with the θ_i axis in the $\mathbb{R}^2$ space and A_o is the square of which sides are defined by the A_i's.

14. The large sample analogue of the Bonferroni and the Scheffé procedures is discussed in Savin (1980) based on the Wald test of H_o.

15. The Rotterdam model can be written in this form, for example.

16. See the equation (4) in Byron (1982).

17. The skew-symmetry of Λ is strictly required for the symmetric restriction on B [Bowden (1973)].

18. See Theorem 1 in Neudecker (1969).

19. The nonsingularity of G is guaranteed by the condition that $d_i + d_j \neq 0$, for all i, j, where d_i's are the eigenvalues of A [Neudecker (1969)]. For the generalised inverse approach, see Hartwig (1975).

20. See, for example, Hartwig (1972,1975) and Jameson (1968).

21. The symmetry of (6.16′) as well as that of (6.17′) can be shown using the identity of (6.16) and (6.16′), and the identity of (6.17) and (6.17′), respectively. The symmetry of $W(P_{mm} - I_{mm})$ in (6.17) can easily be seen by viewing the relations (6.16) and (6.16′).

22. ΔB is a null matrix under the symmetry restriction.

23. We face the same situation, when the nonlinear restrictions are imposed on the linear model.

24. Numerical optimisation procedures were originally designed to solve the optimisation problem for which analytical solutions were difficult or impossible to obtain.

25. Since one coefficient may appear in more than one equation, p_0 is not necessarily equal to $\sum_{i=1}^{m} p_i$ but $p_0 \leqslant \sum_{i=1}^{m} p_i$. For example, one parameter appears in two equations of the symmetry-resticted demand system.

26. In practice, convergence is often checked using the norm

$$(\psi^{(n+1)}-\psi^{(n)})'(\psi^{(n+1)}-\psi^{(n)}) = \|\psi^{(n+1)}-\psi^{(n)}\| < \kappa,$$

or, alternatively, by finding whether $S(\psi^{(n)})$ is close to $S(\psi^{(n+1)})$ with the norm

$$|S(\psi^*) - S(\hat{\psi})| < \kappa,$$

where κ is a sufficiently small positive scalar and $\psi^{(n)}$ is the current approximation to the solution at the n′th iteration. [Harvey (1981a, p.123)]

27. The most common procedure to find the optimum ρ is the quadratic search routine in which $S(\psi)$ is assumed to be a quadratic function of ρ around $\rho = \rho^*$ and the interpolation method is used to find the minimum ρ^*. Since $\psi = \psi(\rho)$ from (7.8), $S(\psi)$ can also be expressed implicitly as a function of ρ, that is, $S(\psi) = S(\psi(\rho)) = G(\rho)$, where $G(\cdot)$ is a composite function $S(\psi(\cdot))$. Substituting three arbitrary values of ρ, say, $\{\rho_o, \rho_1, \rho_2\}$ (for example, $\rho_o = 0$, $\rho_1 = -\frac{1}{2}$ and $\rho_2 = \frac{1}{2}$) in $G(\rho)$, we have three corresponding values of $S(\psi)$, say $\{S_0, S_1, S_2\}$, respectively. Then, $\{S_0, S_1, S_2\}$ and $\{\rho_0, \rho_1, \rho_2\}$ may be used as data points to fit a quadratic function from which the minimum to S may be interpolated. The ρ corresponding to this minimum is chosen to give the optimal step length.

28. This is equivalent to concentrated likelihood estimation which minimises the concentrated likelihood function of $\bar{L} = \ln |U'U|$.

29. Strictly speaking, the separation of β and Σ in the iterative scheme is based on the block diagonality of the information matrix.

30. In general terminology,

$$\partial Y/\partial X = \partial U/\partial X\ (V \otimes I_n) + \partial Y/\partial X\ (U \otimes I_m)$$

when U and V are the matrix function of a matrix variable X and if $Y = UV$ [Nel (1980, p.152, (5.2.5))]. Since u is a Tm x 1 vector, u_β is a $Tm \times p_o$ matrix, and thus $u_{\beta\beta}$ is a $(Tm \times p_o) \times p_o$ matrix.

31. For convenience of computation, the first derivatives $\partial u/\partial\beta$ at $\beta = \beta^*$ can be calculated numerically as

$$\partial u/\partial\beta|_{\beta=\beta^*} = [u(\beta^* + h) - u(\beta^* - h)]/2h$$

where h is a real value scalar chosen so as to be small enough for the evaluated derivatives to be close to the true derivatives, for example, $h = 10^{-2}\beta^*$. This numerical approximation has the advantage of extreme simplicity and appears to be fairly accurate in approximating the true derivatives.

32. This test statistic is expressed as the form of Rao's efficient score test [Gallant and Holly (1980, p.712)].

CHAPTER 4

ESTIMATION OF STATIC DEMAND SYSTEMS FOR KOREA

4.1 Introduction

The purpose of this chapter is to analyse consumption patterns in Korea in the context of system-wide modelling of static demand equations using quarterly household expenditure data for the period 1965-1981. This chapter consists of three major empirical studies which have all arisen from the estimation of static demand systems. The first is a comparative study of the performance of various demand systems on the Korean data, the second is an empirical analysis of effects of commodity aggregation on estimates and tests of hypothesis, and the third involves the testing and analysis of structural change in Korean consumption patterns over the sample period. For these studies, two data sets are used; one is an aggregated five commodity group data, and the other is a disaggregated twelve commodity group data.

Given the great variety of specifications for demand systems [see Chapter 2], a comparative study of their empirical performance over a particular data set is indispensible if one is selecting a system which explains the observed demand patterns best. To this end, we choose three different systems, the linear expenditure system expressed in terms of the expenditure share (LES-W), the absolute price version of the Rotterdam model (RDAM), and the almost ideal demand system (AIDS).[1,2] Earlier comparative studies on the empirical performance of these systems indicate that the Rotterdam model dominates the linear expenditure system [e.g., see Parks (1969), Deaton (1974), and Theil (1975)], while no significant difference in performance between the Rotterdam model and the almost ideal demand system was observed [Bewley (1982b)].[3] However, as Barten (1977, p.45) suggested, the repetition of such a comparative study is worthwhile for other data sets. This will now be done using Korean data.

Since the seasonally unadjusted quarterly data are used in this study, the modelling of seasonal patterns in demand equations is also an important feature in the estimation of such systems. Therefore, the three demand systems incorporate various specifications of seasonal effects, such as seasonal dummies, the introduction of vector autoregressive disturbances, and deseasonalisation of the quarterly data by applying fourth order differences in the application of the Rotterdam model. Consequently, eleven different variants of the three demand systems are used for the comparative study. Comparisons are carried out not only between different demand systems but also between different seasonal variants of the same demand systems. The specifications of the eleven models will be provided in Section 2. In the interests of economy, comparisons of the eleven models will be performed using the aggregated five commodity group data.

However, since the five commodity data set is used for comparison, the aggregation level chosen may effect the estimates and tests of hypothesis. One can expect different results in these areas, when more diaggregated data are used. Klevmarken's (1981) result indicates that estimates of total expenditure and price elasticities, as well as goodness of fit of models, are not independent of the level of aggregation of commodity groups. To examine such effects, we will re-estimate, compare and analyse three selected demand systems using the disaggregated twelve commodity data set.

When a model is fitted to time series data, an important feature is the structural stability of the estimated model. In the Korean case, which is one of rapid change, the data covers the period from 1965 to 1981, and attention should be focused on the stability of the demand systems as well as on the structural change in the consumption patterns over the sample period. In this chapter, we test and analyse the change in consumption patterns over the sample period in the context of static demand systems.

These are the main issues to be discussed in this chapter. The analytical framework for the specification of demand systems and the statistical methods used in this chapter were discussed in Chapters 2 and 3, respectively. The additional statistical issues necessary for comparisons and tests of structural change will be discussed in Section 4 and in part of Section 7.

The design of this chapter is as follows. In the following section, specifications for the eleven models will be provided. The data and the seasonal and dynamic consumption patterns in Korea will be reviewed in Section 3. In Section 4, the methodology necessary for the comparison of the empirical performance of the different demand systems will be discussed. The estimation results on the eleven demand systems will be presented in Section 5. In Section 6, an empirical analysis on the effect of aggregation will be given using the Rotterdam system. In Section 7, the evidence on structural change will be examined. Finally, the chapter has some concluding remarks in Section 8.

4.2 The Models

4.2.1 Basic Demand Systems in Estimation

As seen in Chapter 2, the functional forms of the three demand systems to be considered in this study, the linear expenditure system expressed in terms of expenditure share (LES-W), the absolute price version of the Rotterdam model (RDAM), and the almost ideal demand system (AIDS), are given, respectively, as

LES-W: $w_{it}=(p_{it}/\mu_t)\gamma_i + \beta_i - \beta_i \sum_{j=1}(p_{jt}/\mu_t)\gamma_j + u_{it},$ (2.1)

RDAM: $w_{it}\Delta_1 \ln q_{it} = b_i\Delta_1\overline{M}_t + \sum_{j=1} C_{ij}\Delta_1 \ln p_{jt} + u_{it},$ (2.2)

AIDS: $w_{it} = \alpha_i + \sum_{j=1}\gamma_{ij} \ln p_{jt} + \beta_i \ln(\mu_t/P_{ot}) + u_{it},$ (2.3)

for $i = 1,\ldots,m$, and $t = 1,\ldots,T$, where the notation is the same as in Chapter 2, but the time subscript is t and a disturbance term u_{it} is introduced. No details on the demand systems will be discussed here, since their derivation and properties were considered in Chapter 2.

4.2.2 Specifications of Seasonal Effects in Demand Systems

When quarterly data are used in estimation, one can remove the seasonal variation in the data by using a seasonal adjustment technique, and then estimate the demand systems using the adjusted data. However, it has been argued that there exists a risk in that the seasonal adjustment procedure can introduce considerable distortions into data series [Harvey (1981b, p.177-178)], and the use of seasonally adjusted data can produce systematic biases in the estimated models when data series are adjusted separately [Wallis (1974) and Deaton and Muellbauer (1980a, p.334)]. Moreover, there is no guarantee that the adjusted series are free from seasonal variation [Harvey (1981b, p.177-178)]. Therefore, seasonally unadjusted data are used in this study. We incorporate the modelling of seasonal effects into the estimation of the demand sytem to distinguish the seasonal effects from the other systematic movements in the demand equations.

Assuming that seasonal effects are exogenously determined and fixed, we introduce seasonal dummy variables D_j for the four quarters as explanatory variables. For the demand systems with a constant term, such as LES-W and AIDS models, only the first three seasonal dummies, D_j, $j = 1,2,3$, are introduced to avoid perfect collinearity. Being tagged with 'SD', the resultant demand equations can be written as

$$\text{LES-W-SD: } w_{it} = (p_{it}/\mu_t)\gamma_i + \beta_i - \beta_i \sum_{j=1}(p_{jt}/\mu_t)\gamma_j + \sum_{j=1} D_{jt}\varphi_j + u_{it}, \tag{2.4}$$

$$\text{RDAM-SD: } w_{it}\Delta \ln q_{it} = b_i \Delta \bar{M}_t + \sum_{j=1} C_{ij} \Delta \ln p_{jt} + \sum_{j=1} D_{jt}\varphi_j + u_{it}, \tag{2.5}$$

$$\text{AIDS-SD: } w_{it} = \alpha_i + \sum_{j=1}\gamma_{ij} \ln p_{jt} + \beta_i \ln(\mu_t/P_{ot}) + \sum_{j=1} D_{jt}\varphi_j + u_{it}. \tag{2.6}$$

Since the introduction of the seasonal variables into the Rotterdam model is equivalent to using a constant term, the estimates of coefficients b_i's and C_{ij}'s in RDAM-SD will be identical to those resulting from the inclusion of a constant term.

Next, we consider what happens when we introduce a fourth order autoregressive vector error process into the disturbances. We specify the error process of the disturbance matrix in the demand systems as

$$U = U_{-4}\Psi_4 + E, \tag{2.7}$$

where Ψ_4 is an m x m coefficient matrix, U is a (T-4) x m disturbance matrix, U_{-4} is the matrix U consisting of the fourth order lagged elements of U, and E is a (T-4) x m white noise matrix [Harvey (1981b, p.172)]. This is a special case of an AR(4) process shown in (8.1) in Chapter 3, with $\Psi_1 = \Psi_2 = \Psi_3 = 0$. This approach is applied to the Rotterdam model and the almost ideal demand system. The resultant models are tagged with 'AR(4)' and are given as[4]

RDAM-AR(4): RDAM in (2.2) with $u_{.t} = u_{.(t-4)}\Psi_4 + \varepsilon_{.t}$ (2.8)

AIDS-AR(4): AIDS in (2.3) with $u_{.t} = u_{.(t-4)}\Psi_4 + \varepsilon_{.t}$, (2.9)

where $u_{.t}$, $u_{.(t-4)}$ and $\varepsilon_{.t}$ in (2.8) and (2.9) refer to the t'th row vectors of the matrices, U, U_{-4} and E.

The specification of the autoregressive error process in (2.7) may be too restrictive since the lower order autocorrelations in the disturbances are set to zero. To relax this restriction in (2.7), we combine the first order autoregressive error process with (2.7) in an additive manner such that[4]

$$U = U_{-1}\Psi_1 + U_{-4}\Psi_4 + E, \tag{2.10}$$

where Ψ_1 is an m x m coefficient matrix, and U_{-1} is the first order lagged matrix of U. The error process (2.10) is also applied to the Rotterdam model and the almost ideal demand system, and the resultant models are tagged with 'AR(1,4)',

RDAM-AR(1,4): RDAM in (2.2) with

$$u_{.t} = u_{.(t-1)}\Psi_1 + u_{.(t-4)}\Psi_4 + \varepsilon_{.t} \qquad (2.11)$$

AIDS-AR(1,4): AIDS in (2.3) with

$$u_{.t} = u_{.(t-1)}\Psi_1 + u_{.(t-4)}\Psi_4 + \varepsilon_{.t}. \qquad (2.12)$$

In the application of the Rotterdam model to seasonally unadjusted quarterly data, we can deseasonalise the data by taking fourth order differences Δ_4 such that

$$\Delta_4 \ln X_{it} = \ln X_{it} - \ln X_{i(t-4)}$$

instead of first order differences Δ_1 in (2.2) [Attfield and Browning (1985, p.41)].[5] The resultant version of the Rotterdam model is then tagged with 'D4' and written as

$$\text{RDAM-D4:} \quad w^*_{4it}\Delta_4 \ln q_{it} = b_i\Delta_4\overline{M}_t + \sum_{j=1} C_{ij}\Delta_4 \ln p_{jt} + u_{it} \qquad (2.13)$$

where w_{it} in (2.2) is now given by $w^*_{4it} = ½[w_{it} + w_{i(t-4)}]$. No seasonal effects are incorporated into the model (2.13).

We have generated eight variants of the three demand systems for modelling the seasonal effects. When quarterly data are used, the estimation of systems without seasonal effects, (2.1) - (2.3), may be unnecessary, since the models are likely to be dynamically misspecified. However, we include these models in our study to examine the effect of dynamic misspecification of demand systems on tests of demand restrictions as well as on the coefficient estimates. It has been argued that the misspecification of the dynamic structure of demand systems may be responsible for the rejection of demand restrictions [e.g., Deaton and Muellbauer (1980b)], so that a comparison of their performance with the other seasonal effect versions will be interesting. Moreover, the demand systems (2.1) - (2.3) are estimated in practice, since they are the maintained models in testing the significance of seasonal dummies and autoregressive processes.

Hence, there are eleven models involved in the study, two versions of the linear expenditure system, five versions of the Rotterdam model, and four versions of the almost ideal demand system. However, since the Rotterdam model and the almost ideal demand system require unrestricted estimation, as well as homogeneity and symmetry restricted estimation, the total number of models considered is 29.

When modelling the seasonal effect in a singular system, the choice between the seasonal dummy approach and autoregressive disturbance methods involves a number of serious issues. As discussed in Section 3.8, the full coefficient matrix of the autoregressive process in the singular system cannot be recovered from the estimates of the reduced system. Consequently, the seasonal effect in each demand equation cannot be identified from the estimation of models with autoregressive disturbances. Moreover, the estimation of autoregressive error process models is much more computationally expensive than seasonal dummy models. Even when the system is linear, the introduction of the autoregressive error process causes a nonlinear problem in estimation. However, introducing seasonal dummies into the model does not cause these problems and all the coefficients of the seasonal effects can be recovered by linear estimation of the reduced system. Thus, if they perform reasonably, seasonal dummies may be more desirable than autoregressive coefficients.

4.3 The Data

4.3.1 Household Expenditure Data and Consumer Price Index

The series of household expenditure data in Korea were compiled from *Toshi Kakye Yonbo* [Annual Report on the Family Income and Expenditure Survey], the National Bureau of Statistics, Seoul, Korea, individual years, 1965 - 1981. It is reported that the household expenditure series are estimated from a sample survey, which is conducted monthly using a family account-book method. Design of the sample survey is based on a stratified two-phase sampling method covering all urban households in all cities in Korea. Farmers', fishermen's, single person's, and foreigners' households are excluded from the survey. The expenditure series are published in terms of the monthly sample mean of expenditure per household in all cities and Seoul, with components of salary and wage earners' households (and others), on both quarterly and annual bases. However, we concentrated our attention on the "all households in all cities" category.

For the classification of commodities, five major commodity groups, (1) food, (2) housing, (3) fuel and light, (4) clothing, and (5) miscellaneous commodities, were consistently taken for the period 1965-1981. However, the highest level of disaggregation was 23 commodities for the period of 1963-1971. This was altered to 35 commodities for 1972-1981.[6] In order that the disaggregated commodity series be consistent over the sample period, the 35 commodity groups in 1972-1981 were aggregated into 23 commodity groups by addition. However, since the 'rental value of owner-occupied housing' is double-counted in household income and housing expenditure, that item is excluded from both total and housing expenditure to calculate the household's net expenditure series.[7] Consequently, the most disaggregated commodity grouping in this study covers 22 commodities.

In order to minimise problems for the estimation of a large demand system, such as the collinearity between price series, loss of degrees of freedom, and the costs of computation, a medium-sized 12 commodity group was generated for the disaggregated data. The aggregation and commodity groups are shown in Table 4.1.

The consumer price indices were taken from *Hankook Tongkye Yon-gam* [the Korea Statistical Yearbook], compiled by the National Bureau of Statistics, Seoul, Korea. The price series cover all urban areas in Korea, and the base is 1975.

The classification of price series for the aggregated commodity groups is consistent with that of the expenditure series, but the classification for the disaggregated price group is even more disaggregated than that of the expenditure series. Therefore, some price series are aggregated using the weighted mean method to be consistent with the expenditure series. The weights to be used for aggregation of price indices are presented in parentheses in Table 4.1.

A complete listing of expenditure series and the consumer price series for the five and twelve commodity groups is given in Tables 4.A.1 - 4.A.4 of the Appendix to this chapter.

4.3.2 Preview of the Data and Consumption Patterns in Korea

When discussing consumption patterns in Korea, it is important to understand the country's natural conditions as well as the performance of the Korean economy over the last two decades. Although they are not explicitly involved in the specification of demand systems, these two features may be important factors shaping the traditional consumption patterns and the seasonal and dynamic influences in Korea.

Korea is a relatively small and densely populated country, with a total land area of about 99,000 square km. The population is about 38.72 million and the population density was 391 persons per square km in 1981. As it is geographically located in the continental climate zone of northeast Asia, Korea has four very distinctive seasons over a year: moderate spring and autumn, tropical Monsoon hot summer, and harshly cold winter. About 67% of the total land is unproductive mountain slopes, and mineral resources are very limited.

Such topographical and climatic conditions influence the economic activity in Korea. For example, agriculture is restricted by the cold winters and the limited cultivated area, which is only 23% of the total land. Manufacturing industry in Korea had traditionally been primitive until the 1950's due to the scarcity of mineral resources. Agricultural production consists mainly of grains, such as rice, barley, and vegetables, which can mostly be cultivated during the summer season. Stock farming is not popular due to topographical and climatic conditions. Affected by such production restrictions, food consumption in Korea has traditionally consisted of grains and vegetables, while dairy products have been regarded as luxury goods.

Seasonal trends in the consumption pattern in Korea can also be best viewed in the context of its climatic conditions. The seasonal average expenditure shares are given in Table 4.2, from which we can see that the seasonal consumption patterns in Korea depend more or less on temperature. For example, the shares for fuel and light and clothing all increase during the first and fourth quarters which include the cold winter, and decline during the second and third quarters. The large share for vegetable and fruit consumption during the fourth quarter reflects the traditional storage purchases of vegetables for the winter, while the small share of vegetables and fruit in the first quarter reflects the effect of stocking as well as the the lack of availability of those products during the winter. The share of education appears to relate to the academic calendar which starts in the first and third quarters. The share of "other housing", which consists largely of household durable goods, shows an increase in the second quarter during which the household seems to have less pressure from expenditure on the fuel and education sectors.

Over the last two decades, the Korean economy has experienced rapid growth and changes in the industrial structure. Like the other East Asian countries, agriculture was traditionally the major industry in Korea. However, given the poor endowment of natural resources and the pressure from a large population, the strategy for development of the economy in Korea has mainly concentrated on the expansion of the manufacturing sectors aiming at export. The most important implications of industrialisation for consumption may be the change in income and changes in production and availability of commodities. Over the sample period 1965-1981, the gross national product (GNP) for Korea has maintained a geometric growth rate of 9.2 %, and per capita income has increased from US$491.4 in 1962 to US$1657.5 in 1982 in constant (1980) US dollars. The share of agricultural production in GNP has decreased from 37.6 % in 1965 to 17.9 % in 1981, while that of the manufacturing sector has increased from 19.9 % in 1965 to 31.1 % in 1981.

The overall change in consumption over the sample period can be seen from Table 4.3, which presents the annual average shares of household expenditures for the five and twelve commodity group breakdowns over the sample period 1965 - 1981. Geometric means of annual changes of expenditure shares are also reported to help identify the changing patterns. Clearly, there have been some substantial expenditure changes during the 17 year period. In the aggregate five commodity grouping, the share for food declined significantly, while the shares for the housing and the aggregated miscellaneous sectors showed steady increases from mid-1970 and from the late 1960's, respectively. The changes in expenditure on fuel and light and clothes were steady over time with minor fluctations.

In the twelve commodity grouping, the share of cereals declined drastically, while the shares for miscellaneous foods, rent, other housing, medical care, and the transportation and communication sectors increased steadily over the sample period. These changes in consumption patterns can be expected, as household incomes increased with economic growth, and should be integrated into any model of Korean demand patterns.

4.4 Methodology in Comparative Studies of Demand Systems

As Parks (1969) and Klevmarken (1981) have mentioned, comparisons of the empirical performances of different demand systems present a number of difficulties. The substantial problem is that different demand systems have different functional forms and different dependent variables. Hence, the demand systems have different stochastic structures making inference difficult [Parks (1969)]. Moreover, the demand systems have parameter sets which are restricted in different ways and have different interpretations. For example, in the Rotterdam model, the dependent variables are defined as log changes in expenditure shares and the parameter estimates are directly informative on the Slutsky matrix. On the other hand, in the almost ideal demand system, the dependent variables are expenditure shares and the parameter estimates are not directly related to the Slutsky matrix [Deaton and Muellbauer (1980b, p.317)]. In econometric language, such different demand systems are said to be non-nested, in the sense that neither can be obtained as a special case of the other.

Recent developments of the Cox test procedures [Cox (1961, 1962)] for non-nested models seem to provide a solution for the choice between non-nested demand systems. However, the application of the Cox-type test is limited only to the case where the dependent variables are identical or when one can be expressed as a Box-Cox transformation of the other [e.g., see Pesaran (1974), Deaton (1978), Pesaran and Deaton (1978), Breusch and Pagan (1980) and Godfrey and Wickens (1981)]. Pesaran and Deaton (1978), in a general application of the Cox test, considered the case where the competing systems are of the form

$$H_0:\ y = f(\beta_0;X) + u_0$$

and

$$H_1:\ y = g(\beta_1;Z) + u_1,$$

that is, where the dependent variables in H_0 and H_1 are the same. Godfrey and Wickens (1981) considered a single equation case of testing linear and log-linear regressions such that

$$H_0:\ y = \sum_{i=1}\beta_i X_i + \sum_{j=1}\alpha_j Z_j + u_0$$

and

$$H_1:\ \ln y = \sum_{i=1}\beta_i \ln X_i + \sum_{j=1}\alpha_j Z_j + u_1,$$

that is, when the dependent variable $\ln y$ in H_1 is the Box-Cox transformation of y in H_0, $\ln y = y(\lambda) = (y^{\lambda} - 1)/\lambda$ with $\lambda = 0$. Pesaran and Deaton's (1978) Cox test can be applied to testing the linear expenditure system in share terms (LES-W) and the almost ideal demand system, since they have the same dependent variables, and also to testing between the linear expenditure system and the Rotterdam model on the basis of Deaton's (1974) reparameterisation of the linear expenditure system[8]. However, it still appears to be true that there is no standard rule for choosing between different demand systems, unless there is an exact functional transformation between the dependent variables of the competing systems. Conseqently, there is no stringent rule for comparing the performance of the Rotterdam model and the almost ideal demand system, since their dependent variables are different and neither can be expressed as a function of the other [Deaton and Muellbauer (1980b, p.317)].

Faced with such problems, empirical comparisons of demand systems have been carried out on the basis of comparisons of goodness of fit, using the adjusted R^2 and the measure of information inaccuracy [Theil (1971)], the predictive ability, and the randomness of residuals in the estimated systems [e.g., Parks (1969), Yoshihara (1969), Theil (1975), Klevmarken (1981) and Bewley (1982b)]. Intuitive comparisons of parameter estimates and the implied income (or total expenditure) and price elasticities of different demand systems have also been used [e.g. Klevmarken (1981)].

When the competing systems have different dependent variables and different functional forms, the direct comparison of R^2's is statistically meaningless, since their residuals are estimated from different stochastic structures [Parks (1969, p.646)]. Instead, Theil (1971) suggested the use of the measure of average information inaccuracy, which is defined as

$$I_t = \sum_{i=1} w_{it}\, ln\,(w_{it}/\hat{w}_{it}),$$

where $\hat{w}_{it}$ is the predicted value of the average budget share. It is common to use the arithmetic means of I_t for the whole sample period such that $\bar{I} = \sum_{t=1} I_t$ or various subsamples [Parks (1969), Theil (1975), Klevmarken (1981) and Bewley (1982b)].

Checking the randomness of residuals plays an important role in comparing the performance of demand systems. If a model is reasonably specified, the residuals should be approximately random. Therefore, the departure of residuals from randomness is regarded as an indication of specification error in the model [Harvey (1981a, p.148)]. The Durbin-Watson statistic (DW) is extensively used for checking the randomness of residuals as well as for testing the misspecification of the model. Although this statistic was originally constructed for testing a first order autoregressive process in the disturbances, the DW statistic appears to be sensitive to structural change and to functional misspecification. When the model is fitted using quarterly data, the randomness of residuals should be checked by the generalisation of Durbin-Watson statistic which is given by

$$DW4 = \sum_{t=5}(\hat{u}_t - \hat{u}_{t-4})^2/\sum_{t=1}\hat{u}_t^2, \qquad (4.2)$$

where $\hat{u}_t$ is the residual at t. The test procedure of using DW4 in (4.2) is essentially the same as in the first-order case [Wallis (1972)]. The upper and lower significance points for a bounds test based on DW4 are available in Wallis (1972), in which two tables are presented for DW4, depending on whether or not seasonal dummies are included in the model. Visual examination of plots of residuals or fitted values can also be exploited in checking the randomness of residuals [e.g., Yoshihara (1969)].

4.5 Static Demand Systems for Korea: Estimation and Performance

The above mentioned eleven demand systems were estimated using the five commodity data set. Since the first four observations for 1965 are reserved for the lag structure, the sample covers the period 1966-1981, and the number of observations is 64.

Demand systems which are linear in coefficients (i.e., RDAM, RDAM-D4, RDAM-SD, AIDS, and AIDS-SD models) are estimated by OLS for the unrestricted model, and by restricted ML for the homogeneity and symmetry restricted models. The demand systems which are nonlinear in coefficients, such as two linear expenditure systems, LES-W and LES-W-SD, and the Rotterdam model with autoregressive disturbances and the almost ideal demand system with autoregressive disturbances, RDAM-AR(4), RDAM-AR(1,4), AIDS-AR(4), and AIDS-AR(1,4), are estimated by the Gauss-Newton iterative nonlinear optimisation procedure. No difficulty was experienced in obtaining convergence. The ML estimate of the covariance matrix was used. Parameter estimates for all eleven models are provided but the discussion in this section mainly concentrates on comparisons of performance over the whole sample period using the criteria discussed in the previous section.

Coefficient estimates of the eleven models are presented in Tables 4.4 - 4.14. For all the versions of the Rotterdam model and the almost ideal demand system, the unrestricted estimates, and the homogeneity retricted and homogeneity and symmetry restricted estimates of coefficients are presented. Asymptotic standard errors are given in parentheses for each coefficient. For the convenience of direct comparison between the models, the adjusted R^2's, the average information inaccuracies, and Durbin-Watson statistics, DW and DW4, are presented separately in Tables 4.15, 4.16, 4.17, and 4.18, respectively. The results of testing the significance of seasonal dummy and autoregressive error processes are summarised in Table 4.19. Maximum and minimum absolute values of the eigenvalues of the companion matrix of coefficients of the autoregressive error process are presented in Table 4.20. The results of direct and separate induced tests of the demand restrictions are presented in Tables 4.21 and 4.22, respectively. The Slutsky matrices implied by versions of the linear expenditure system and the almost ideal demand system are presented in Table 4.23. However, the Slutsky matrices in the Rotterdam model versions relate directly to the price coefficients from the symmetry and homogeneity restricted models. Complementarity and substitutability relations between commodity groups indicated by the Slutsky matrix are summarised in Table 4.24. Finally, total expenditure and compensated own price elasticities are presented in Tables 4.25 and 4.26, respectively.

4.5.1 Coefficient Estimates

Only passing reference to the coefficient estimates of the models will be necessary here, since extensive discussions on the estimated Slutsky matrices and total expenditure and price elasticities of the models will be provided below. Therefore, in this subsection, we will concentrate on *peculiar* estimation results.

In LES-W and LES-W-SD models, the estimate of subsistence expenditure for the housing sector was negative and significantly different from zero, while subsistence expenditures on the clothing and the miscellaneous sectors were not significantly different from zero.[9]

In the Rotterdam model without seasonal effects (RDAM), the estimates of the marginal budget shares of the housing and miscellaneous sectors were negative but not significant. In other versions of the Rotterdam model with seasonal effects, the estimated marginal budget shares were all positive and generally were highly significant. The significance of own and cross price responses and of seasonal effects will be examined below.

In the almost ideal demand system without seasonal effects, i.e., AIDS model, the income effects for the food and the fuel and light sectors were found to be statistically insignificant in the unrestricted model, while those for the fuel and light and the miscellaneous sectors were insignificant in the restricted models. The constant term for the miscellaneous sector was found to be insignificant in most of versions of the almost ideal demand system.

The estimates of coefficients of the models generally appeared to differ with alternative specifications of the seasonal effect. This will be seen in detail when the Slutsky matrix and the estimated elasticities are discussed.

4.5.2 Goodness of Fit

Comparing the adjusted R^2's shown in Table 4.15, we can see that introduction of seasonal dummies and autoregressive errors into the specification of the demand systems improved goodness of fit in each case, as expected. Remarkably, the deseasonalised version of the Rotterdam model, RDAM-D4, yielded the closest fit for the housing and miscellaneous sectors among the five versions of the Rotterdam model. No significant difference in goodness of fit between the seasonal dummy, the AR(4) and AR(1,4) models was observed. Comparing the linear expenditure systems and the almost ideal demand systems, all the almost ideal demand systems were found to be superior to the linear expenditure systems. Note, there is no meaning in comparing the R^2's of the Rotterdam model with those of the almost ideal demand system and the linear expenditure system.

On comparison of average information inaccuracies, the demand systems without seasonal effects appeared to be inferior to the models with seasonal effects. The AID-SD model showed the best fit in all subsamples, while the RDAM-SD and RDAM-D4 models were superior to the LES-W-SD model. In addition, the earlier subsample always showed the higher measure of information inaccuracy, which may be taken as evidence for the structural instablity of models [see Section 7 of this Chapter]. As expected, imposition of demand restrictions increases the measure of information inaccuracy. Unfortunately, average information inaccuracy measures for the autoregressive error process versions were not obtained.

4.5.3 Autocorerelation and Randomness of Residuals

As expected when using quarterly data, demand systems without seasonal effects exhibited strong positive fourth order autocorrelation in their residuals. The Durbin-Watson DW4 statistics in the LES-W, ADAM and AIDS models were very low for all demand equations [see Table 4.18]. However, according to the DW tests [see Table 4.17], significant positive first order autocorrelation was only seen in the fuel and light and the clothing sectors of the LES-W model. Negative autocorrelation was observed in the miscellaneous sector of the ADAM model.

Introducing seasonal dummy variables improved the fourth order autocorrelation of residuals, but did not completely eliminate the autocorrelation in the demand systems. In the LES-W-SD model, all the equations exhibited positive fourth order autocorrelated residuals, while in the ADAM-SD and AIDS-SD models only the housing sector rejected fourth order autocorrelation in the residuals. On the other hand, no evidence of fourth order autocorrelation was seen in the ADAM-AR(4), ADAM-AR(1,4), AIDS-AR(4), and AIDS-AR(1,4) models.

Thus, the AR(4) and AR(1,4) versions appeared to be superior to the seasonal dummy versions in terms of autocorrelation structure. The deseasonalised RDAM-D4 model did not exhibit fourth order autocorrelation and showed the best performance among the Rotterdam model versions.

However, some demand systems appeared to suffer from first order autocorrelated residuals after introducing seasonal effects, as is often encountered in econometrics. The fuel and light and the miscellaneous sectors in RDAM-D4 exhibited some positive first order autocorrelation, while the clothing sector in RDAM-SD and the clothing and miscellaneous sectors in RDAM-AR(4) also exhibited slightly negative first order autocorrelation.

In terms of randomness of the estimated residuals, the linear expenditure systems were inferior to either the Rotterdam models or the almost ideal demand systems. In fact, randomness of residuals in the linear expenditure system was rejected by the bounds test of the DW and DW4 statistics. Roughly speaking, the Rotterdam models showed more randomness of residuals than the almost ideal demand systems, given that the bounds tests of the DW statistics in the almost ideal demand systems were less conclusive than in the Rotterdam model. In general, randomness of residuals in the Rotterdam models and the almost ideal demand systems can be achieved by introducing AR(1,4) process.

It has been argued in the literature that the imposition of homogeneity restrictions increases the autocorrelation in residuals, so that the rejection of homogeneity is often attributed to misspecification of the dynamic structure of the model [e.g., Deaton and Muellbauer (1980b)]. In this study, a deterioration of autocorrelation in residuals can be seen only in the AIDS-SD model which is caused by imposing the symmetry restrictions, but not by homogeneity.

Serial independence between residuals was also rejected by testing the significance of the coefficient matrix of the autoregressive error process.[10] Table 4.19 shows likelihood ratio test results that the reduced coefficient matrices of autoregressive error processes in the RDAM-AR(4), RDAM-AR(1,4), AIDS-AR(4) and AIDS-AR(1,4) models were all significantly different from zero.[11] From Table 4.19, it can also be seen that the coefficient matrix of the seasonal dummy variables were significantly different from zero in the LES-W-SD, RDAM-SD and AIDS-SD models. Therefore, it is again confirmed that the three demand systems without seasonal effects, the LES, RDAM and AIDS models, suffer from misspecification in the dynamic structure.

The coefficient matrices $\bar{\Psi}_i$ of the AR(4) and AR(1,4) processes estimated from the reduced demand systems are presented in Tables 4.9, 4.10, 4.13 and 4.14 for the RDAM-AR(4), RDAM- AR(1,4), AIDS-AR(4) and AIDS-AR(1,4) models. Since all the coefficients of the full matrices Ψ_i are not recoverable from the reduced matrices $\bar{\Psi}_i$, we considered two ways of eliminating one equation from the system. One is when the last equation (the miscellaneous sector) is deleted, and the other is when the first equation (the food sector) is deleted. In both of RDAM-AR(4) and AIDS-AR(4) models, estimates of all the diagonal terms of $\bar{\Psi}_4$ were positive, whether the first or the last equation was deleted. When AR(1) specification was added to AR(4), all the diagonal terms of $\bar{\Psi}_1$ appeared to be negative in the RDAM-AR(1,4) model, but all were positive in the AIDS-AR(1,4) model, except for $\bar{\psi}_{11}$. However, the diagonal terms of $\bar{\Psi}_4$ remained positive, except for $\bar{\psi}_{22}$ in the RDAM-AR(4). However, as discussed previously, in singular systems, the (i,j)'th element of $\bar{\Psi}$ is not associated with that of the full matrix Ψ but refers to $\bar{\psi}_{ij} = \psi_{ij} - \psi_{in}$, where n refers to the equation deleted from the system [see Section 3.8, and Berndt and Savin (1975)]. Therefore, such findings are not directly related to the full matrices Ψ_1 and Ψ_4.

The stationarity of the AR(4) and AR(1,4) processes was checked by the modulus of the eigenvalues of the reduced companion matrices, $\bar{A}_1$ and $\bar{A}_{14}$ as given in (8.6) in Chapter 3 [see Section 3.8 and Appendix 3.3 in Chapter 3]. Table 4.20 presents the minimum and maximum absolute values of eigenvalues of A_4 and A_{14}, which confirm the stationarity of the AR(4) and AR(1,4) versions of the Rotterdam model and the almost ideal demand system. Some of the eigenvalues appeared to be complex which implies oscillatory movement.

4.5.4 Seasonal Patterns in Dummy Variable Models

Seasonal patterns for individual commodities were captured adequately by the seasonal dummy variables in the RDAM-SD and AIDS-SD models. In particular, the seasonal dummy coefficients in RDAM-SD exhibited plausible patterns in Korean terms. For example, the demand equation for food was found to have a significant negative coefficient for the first quarter and a positive coefficient for the fourth quarter. The demand equation for housing had significant positive coefficients for the first and second quarters but negative for the third and fourth quarters. For the fuel and light and clothing sectors, the seasonal coefficients were positive for the first and fourth quarters but negative for the second and third quarters. The miscellaneous sector had seasonal coefficients which were significantly positive for the first and third quarters, but negative elsewhere. All these results held irrespective of whether the model was unrestriced or restricted.

However, not all the seasonal coefficients of the AIDS-SD model coincided with those of the RDAM-SD model. For example, the second and third quarter's seasonal coefficients for the food sector were negative in AIDS-SD model while they were positive in RDAM-SD model. Such an inconsistency between the seasonal effects in the AIDS-SD and RDAM-SD models may be attributed to the existence of a constant term in the AIDS-SD model. From a close inspection of coefficient estimates for the seasonal dummy variables and the constant term in AIDS-SD, we can see that the seasonal dummy coefficients in AIDS-SD overshoot in the equations with a negative estimated intercept (like food and fuel and light) but undershoot in the equations with a positive intercept. The seasonal dummy coefficients which were insignificantly positive (negative) in the RDAM-SD model became significantly negative (positive) in the AIDS-SD model. Seasonal dummies in the LES-W-SD model worked out reasonably well for the food sector but failed to capture the expected seasonal movement for the other commodities.

4.5.5 Tests of Demand Restrictions

The demand restrictions, homogeneity and symmetry, were tested on versions of the Rotterdam model and almost ideal demand system using the Wald, LR and LM test statistics as well as by separate induced tests. Exact Hotelling's T^2 tests were also employed for the direct test of homogeneity. The critical value for the T^2 test was computed using the relation in Footnote 9 of Chapter 3 and the IMSL subroutine MDBETI. Symmetry restrictions were tested subject to the imposition of homogeneity. All direct tests were performed at the 5 % significance level. For separate induced tests, the Bonferroni and Scheffė tests were applied under Wald and LM procedures. In the Bonferroni test, the size of a separate test of an individual restriction was set as $\bar{\alpha}$ = 5%/s, where s is the number of restrictions, and the critical value for individual separate tests was obtained from $\bar{\alpha}$/2 % upper point of standard normal distribution by using the IMSL subroutine MDNOR. Thus, the Bonferroni procedure is a two-tailed asymptotic standard normal test. On the other hand, the critical value of the Scheffé test was taken as the square root of χ^2 critical value for the direct test. Separate induced tests were considered only for the demand systems which are linear in coefficients, since the Wald and LM statistics for individual separate tests could not be obtained in the nonlinear estimation algorithm. Since the LR separate induced test requires estimation of the models restricted by every individual restriction, it was omitted for computational reasons.

The direct test results are summarised in Table 4.21. The critical values and results of the separate induced tests are presented in Table 4.22. Remarkably, the demand restrictions were all accepted in all the versions of the Rotterdam model, except for RDAM-D4. A minor conflict between Wald, LR and LM tests was shown in RDAM-SD when testing homogeneity and symmetry restrictions together, in which the Wald test marginally rejected the restrictions, while the LR and LM tests accepted the restrictions. However, the conflict disappeared in the test at 2.5 % significant level. The rejection of demand restrictions in the RDAM-D4 model was attributed to homogeneity restrictions which were rejected by all the asymptotic χ^2 tests as well as by the exact T^2 test. This result may imply that using fourth order differences Δ_4 may not be adequate for the finite approximation to the differential variables in the Rotterdam model. In view of the fact that the demand restrictions are derived from differential properties of static demand functions and the homogeneity restrictions are associated with the consumer's current budget constraint, it is intuitively plausible that the use of Δ_4 could result in biased slope estimates, since Δ_4 covers too wide an interval for an adequate approximation. Our empirical results appear to favour the use of first differences for finite approximations even when quarterly data are used.[12]

On the other hand, in all the versions of the almost ideal demand system, homogeneity restrictions were rejected by all the direct tests, while symmetry restrictions were accepted, except in the case of the AIDS-SD model. Since homogeneity restrictions were also rejected by the exact T^2 test, there is no need to consider a small sample correction for the asymptotic tests. The introduction of the autoregressive error process into the almost ideal demand system made the test statistics less significant, but did not significantly improve the rejection of the homogeneity restrictions.

The results of the direct tests were all confirmed by the separate induced tests. Homogeneity restrictions in the RDAM-D4, AIDS, AIDS-SD models and symmetry restrictions in the AIDS-SD model were also rejected by the separate induced tests. On the other hand, all t statistics for testing the individual restrictions in RDAM, RDAM-SD were found to be insignificant. In the identification of which restrictions were responsible for the rejection, we can see that individual homogeneity restrictions on the housing, fuel and light, and the miscellaneous sectors in the RDAM-D4 and AIDS-SD models and those on the food and the miscellaneous sector in the AIDS model were rejected. The rejection of symmetry in the AIDS-SD model appeared to be due to $\beta_{14} = \beta_{41}$, the equality of cross substitution effects between food and clothing. Remarkably, the RDAM-D4 and AIDS-SD gave similar results for the Wald separate tests on the individual homogeneity restrictions. The LM separate tests were generally similar to the Wald tests. However, one exception can be observed in testing homogeneity on the fuel and light sector in the RDAM-D4 and AIDS-SD models, i.e., the Wald test significantly rejected the restriction while the LM type test did not.

The use of the S-interval for testing both symmetry and homogeneity restrictions appeared to be not all consistent with the Bonferroni tests, as its critical value was larger than the Bonferroni critical value. To compare their performance with the direct test, the Bonferroni tests were found more consistent with the direct test than Scheffė's tests.

4.5.6 The Slutsky Matrix, Complementarity and Substitutability

Using average expenditure shares for each commodity sector [see Table 4.3], the Slutsky matrices implied by the linear expenditure systems and the almost ideal demand systems were computed from the compensated price elasticities weighted by the expenditure shares, so that they are directly comparable with the Slutsky matrix estimated for the Rotterdam models [see Table 4.23]. For versions of the almost ideal demand system, the Slutsky matrix was computed from the homogeneity and symmetry restricted estimates of the price coefficients to maintain the restrictions on the matrix.[13] The marginal budget shares in the almost ideal demand systems were computed and presented in the last column of Table 4.23.2.

In view of the sign of the diagonal terms of the estimated Slutsky matrix, we can see that the necessary condition for negative semi-definiteness of the Slutsky matrix was generally satisfied in all the versions of the Rotterdam model and the almost ideal demand system, with the exception of the AIDS-SD model.[14] The own substitution effect of fuel and light in the AIDS-SD model was found to be positive and significant. However, in other models, positive estimates of the own substitution effect (for example, the miscellaneous sector in RDAM-SD model) were not significantly different from zero. No positive estimate of the own substitution effect was found in the RDAM, RDAM-D4, RDAM-AR(4) and RDAM-AR(1,4) models.

Using Hicks' definition, complementarity and substitutability relationships between commodity groups can be identified by the sign of the cross substitution effect between two goods, that is, by the sign of the corresponding off-diagonal term of the Slusky matrix. If the estimated cross substitution effect is significantly positive, the pair of commodity groups can be said to be substitutes, while if the estimated cross substitution effect is significantly negative, the pair can be said to be complements. Any two commodity groups having their cross substitution effect not significantly different from zero can be viewed as independent goods [see Section 2 in Chapter 2]. In LES-W and LES-W-SD, no complements were seen, as implied by the nature of the linear expenditure system. The relations observed in versions of the Rotterdam model and the almost ideal demand system are summarised in Table 4.24. Roughly speaking, the same patterns of complementarity and substitutability between commodity groups were seen in all models. Strikingly, the RDAM-D4, RDAM-SD, RDAM-AR(1,4) and AIDS-AR(4) models showed exactly the same sign pattern of cross substitution effects, and the RDAM and RDAM-AR(4) as well as the AIDS-SD and AIDS-AR(1,4) also showed similar patterns with only one exception. Thus, the identification of complementarity, substitutability, and independent relations appeared to be insensitive to the specification of the model.

The food and fuel and light sectors were found to be substitutes in all the models, and the fuel and light and the miscellaneous sectors were shown to be complements, except in the case of the AIDS and AIDS-SD models. The cross substitution effect between the fuel and light and the clothing sectors was found to be negative in all models. However, the fuel and light and the clothing sectors emerged as complements only in the AIDS models. The clothing and miscellaneous sectors were independent in all models, except the AIDS-AR(1,4) in which they were complements.

4.5.7 Total Expenditure Elasticities

Total expenditure elasticities were computed using marginal budget shares obtained from the unrestricted and restricted models and the average budget share for each sector[15] [see for the results Table 4.25]. Estimates of total expenditure elasticities were, however, found to be sensitive to the specification of the demand sytems, the modelling of seasonal effects, and the imposition of demand restrictions. For example, the total expenditure elasticities of the food and housing sectors were found to vary with the specification of demand systems, the elasticities of the housing and the fuel and light sectors were found to vary with the modelling of seasonal effects in the category of the Rotterdam model, and the elasticity of fuel and light was found to vary with the imposition of demand restrictions in the seasonal effect version of the almost ideal demand system.

More specifically, the total expenditure elasticity of the food sector was found to be significantly greater than unity in RDAM, and not significantly different from unity in RDAM-SD, RDAM-AR(4), RDAM-AR(1,4) and the unrestricted AIDS model. It was found to be significantly less than unity in the RDAM-D4 model, other versions of the almost ideal demand system and the linear expenditure systems. Thus, the food sector was classified as a luxury in the original and seasonal effect versions of the Rotterdam model, but as a necessity in RDAM-D4 and all the versions of the linear expenditure system and the almost ideal demand system, except for the unrestricted AIDS model.

Roughly speaking, the opposite situation occurred for the total expenditure elasticity of the housing sector. In all versions of the linear expenditure system and the almost ideal demand system as well as in the RDAM-D4 model, the housing sector was established as a luxury, as the estimated total expenditure elasticity was greater than unity; however, it emerged as a necessity in the RDAM, RDAM-AR(4) and RDAM-AR(1,4) models. Moreover, in RDAM and RDAM-AR(4), housing appeared to be inelastic to the total expenditure as the estimated elasticities were not significantly different from zero.

The fuel and light sector was classified as a necessity in the RDAM-D4, AIDS-SD, AIDS-AR(4) and AIDS-AR(1,4) models, but as luxury in RDAM, RDAM-SD, RDAM-AR(4), RDAM-AR(1,4) and AIDS. The imposition of demand restrictions appeared to increase the total expenditure elasticities of fuel and light in all the models, particularly, in RDAM-D4 and seasonal effect versions of the almost ideal demand system.

The clothing sector was classified as a luxury in all models, as the total expenditure elasticity was always greater than unity, whether the models were restricted or unrestricted.

The total expenditure elasticity of the miscellaneous sector appeared to be equal to or greater than unity throughout the linear expenditure systems and the seasonal effect versions of the Rotterdam model and the almost ideal demand system, so that the miscellaneous sector might be regarded as a luxury.

4.5.8 Compensated Own Price Elasticities

The compensated own price elasticities were computed from the diagonal terms of the Slutsky matrices using the average expenditure shares, and are presented in Table 4.26. Although the compensated own price elasticities also appeared to vary with the specification of the demand system and the modelling of seasonal effects, there were common findings in all versions of the models. The housing sector was found to be elastic to own price changes, while other sectors were not.

The compensated own price elasticity of the food sector was negative and significant, except in the case of the AIDS-SD model. The price elasticity for the fuel and light sector was significantly different from zero only for RDAM-SD, but was not significantly different from zero in other cases. However, in all versions of the almost ideal demand system, the compensated own price elasticity of the fuel and light sector was positive. Comparing the t-ratios of individual price elasticities, we see that, except for the AIDS-SD model, the positive price elasticities were not significantly different from zero. Own price elasticity of the clothing sector was significantly different from zero in RDAM-D4 and in all versions of the almost ideal demand system. The compensated own price elasticity, for the miscellaneous sector, was significantly different from zero only in RDAM-AR(4), but was found to be positive, but not significant, in RDAM-SD and AIDS-SD.

4.5.9 Conclusions

An important task in concluding this section is to determine which of the eleven models performs best and is the most appropriate for explaining demand patterns in Korea. However, as experienced in other studies [e.g., Klevmarken (1981)], the ranking process of our eleven demand models involves trade-offs between models, since the rankings differ with the criteria used. For example, the AIDS-SD model performs best in terms of goodness of fit, the RDAM-D4 best achieves randomness of residuals, and so on. Neverthless, a few conclusions emerge.

It is apparent that demand systems without seasonal effects can be rejected, since they exihibited serious fourth order autocorrelation in the estimated residuals. Of those demand systems with seasonal effects, models with autoregressive error processes were found to be superior to those with seasonal dummy variables in terms of the randomness of the estimated residuals. The RDAM-D4, RDAM-AR(1,4) and AIDS-AR(1,4) models achieved the best results according to this criterion.

The linear expenditure systems (LES-W and LES-W-SD) were inferior to the Rotterdam models and the almost ideal demand systems in terms of goodness of fit and randomness of residuals in terms of the seasonal effect models. In addition, the estimated coefficients of the linear expenditure systems were unacceptable, so that we can reject linear expenditure system in favour of more flexible demand systems. The rejection of the linear expenditure system is not surprising, given that it is an over-restrictive and underparameterised model compared to the flexible systems.

If goodness of fit was taken as a criterion, versions of the almost ideal demand system were slightly superior to the Rotterdam model versions. However, this superiority of the almost ideal demand system was offset by their inability of achieving compatibility of demand theory and the data. The demand restrictions were rejected in all versions of the almost ideal demand system, while they were accepted in the seasonal effect versions of the Rotterdam model. Similarly, the superiority of the deseasonalised Rotterdam model, RDAM-D4, in terms of randomness of residuals, was also offset by its rejection of demand restrictions.

Given these results, the Rotterdam models with seasonal effects, such as RDAM-SD, RDAM-AR(4) and RDAM-AR(1,4), can be taken as the most reasonable of the eleven demand systems. They achieved reasonable goodness of fit and randomness of residuals. In terms of choice between RDAM-SD and RDAM-AR(4) or RDAM-AR(1,4), we were faced with a trade off between identification of seasonal effects and randomness of residuals. The RDAM-SD model was found to have fourth order autocorrelation in the residuals, while the RDAM-AR(4) and RDAM-AR(1,4) models exhibited no autocorrelation. However, the RDAM-AR(4) and RDAM-AR(1,4) models suffered from the identification of the autoregressive coefficient matrices.

However, there was one peculiar result in the Rotterdam model versions; food was classified as a luxury in the Rotterdam model versions, while it was found to be a necessity elsewhere. In general, the total expenditure and price elasticities were found to be sensitive to the specification of demand systems as well as the specification of seasonal effects.

Misspecification of the dynamic structure of the demand systems appeared to have no effect on the tests of restrictions, although it did affect the estimates of coefficients. Our results imply that the correct specification of dynamic structure of demand systems may be less crucial for the success of tests of demand restrictions than the proper specification of those systems. Our results favour the use of the Rotterdam model to achieve the compatability of demand theory and data rather than the almost ideal demand system.

4.6 The Estimation of Demand Systems Using Disaggregated Data

The analysis of the previous section was confined to the five commodity data set. However, this aggregation to five sectors is not based on aggregation theory, such as the separability of the commodity groups, but was prescribed by the classification of the data series. This situation is far from ideal, but must often be faced in practice. In this section, we increased the number of equations to twelve and examined the effect of commodity aggregation on the estimates.

The disaggregated 12 commodity sectors are shown in Table 4.1. The food sector consisted of four subsectors, (1) cereal, (2) fish and meat, (3) vegetable and fruit, and (4) other foods including processed food and "meals away from home". The housing sector was disaggregated into (5) rent and (6) other housing, which includes household durable goods. Two sectors, (7) the fuel and light" and (8) the clothing sectors, were not disaggregated. The miscellaneous sector was disaggregated into four subsectors, (9) medical care, (10) education and recreation, (11) transportation and communication, and (12) other miscellaneous.

Naturally, we might expect that the eleven demand systems considered in the previous section would perform differently with the disaggregated data. Therefore, a comparison of their performance on the twelve commodity data set will be necessary. However, the increase in the number of equations leads to a significant increase in the number of parameters to be estimated, so that a comparison of the eleven models becomes difficult. We select only three demand linear systems, the ADAM-D4, ADAM-SD, and AIDS-SD models for comparison.

Table 4.27 presents results of tests of restrictions on the three demand system. As in the five commodity case, the demand restrictions were rejected in the RDAM-D4 and AIDS-SD models but were accepted by the RDAM-SD model. Homogeneity restrictions were rejected by the exact T test in the RDAM-D4 and AIDS-SD models. Only RDAM-SD model achieved the compatibility with demand theory on disaggregated twelve commodity data and thus was selected for further analysis.

Parameter estimation results for the RDAM-SD model are presented in Table 4.28. Randomness of residuals in the estimated twelve demand equations were generally satisfied, with a few exceptions. The exceptions were observed in cereals, clothing, and the education and recreation sector, (which suffered slightly from negative first order serial correlation). Fourth order autocorrelation was found in the unrestricted equations for the fuel and light and the clothing sectors.[16] The imposition of symmetry restrictions appeared to introduce fourth order autocorrelation in the residuals of the cereal and transportation and communication sectors.

Seaonal dummy variables again model quarterly consumption patterns in the twelve commodity groups quite well. The fuel and light and the clothing sectors showed the same seasonal patterns exhibited in the five commodity case. A significant positive seasonal effect was observed in the cereal sector for the fourth quarter, for the rent sector in the first quarter, for the "other housing" sector in the second quarter, for the medical care sector in the first quarter, for the education and recreation sector in the first and third quarters, and for the miscellaneous sector in the fourth quarter. Significant negative effects were found in the fish and meat sector in the second quarter, the vegetable and fruit and the other food sectors in the first quarter, the rent sector in the fourth quarter, the other housing sector in the third quarter, the medical care sector in the fourth quarter, and the education and recreation sector in the second and fourth quarters.

To examine the effect of aggregation on the estimates of the income responses, we generated the marginal budget shares of the aggregated five commodity groups by adding those of the corresponding subgroups in the twelve commodity groups and comparing them with those obtained previously. The two series of marginal budget shares [see Table 4.29] demonstrate that the estimates of the marginal budget shares were not seriously affected by aggregation. Only the marginal budget share of the fuel and light sector was found to be significantly affected in the restricted model.

The estimated total expenditure elasticities of the twelve groups are presented in Table 4.30. As in the case of the five goods, the clothing sector was classified as a luxury, and the fuel and light sector was found to be income (total expenditure) inelastic. Among the food commodity sectors, only the cereal sector was classified as a necessity, while the other food items were classified as luxuries. In particular, the vegetable and fruit sector was found to be highly income elastic. The rent sector also appeared to be highly income elastic, and the other housing displayed a unitary income elasticity. Among the miscellaneous sectors, only "education and recreation" was classified as a luxury, while the medical care, the trasportation and communication, and the miscellaneous sectors were classified as necessities.

The Slutsky matrix for the disaggregated data subject to homogeneity and symmetry is given in Table 4.28. The estimates of the diagonal terms of the Slutsky matrix were all negative, so that the necessary conditions for negativity of the Slutsky matrix were satisfied. Eight diagonal terms of the matrix were found to be statistically significant. Recalling the results of the previous section that one of the diagonal terms of the Slutsky matrix in the ADAM-SD model (the miscellaneous sector) was positive, we can see that the negativity of the Slutsky matrix was improved by disaggregation.

However, of the 66 off-diagonal terms in the Slutsky matrix only six were found to have t-ratios greater than 2. Strong complementarity relations were found between the cereal and the fuel and light sectors, between the rent and the fuel and light sectors, and between the education and recreation and the trasportation and communication sectors. On the other hand, strong substitutability were found between the vegetable and fruit and the medical care sectors, between the rent and the medical care sectors, and between the fuel and light and the education and recreation sectors. Other commodities appeared to be independent goods, as their estimated cross substitution effects were not significantly different from zero.

A close examination of the price elasticities of the fuel and light and clothing sectors found in Table 4.30 with those obtained in the previous section [see RDAM-SD in Table 4.26] indicates that some price elasticities were affected by aggregation. The price elasticity of the fuel and light and the clothing sectors were estimated as -.51084 and -.97034 in the case of the five commodities, but as -.69646 and -1.02383 in this section. As to the compensated own price elasticities of the twelve commodity groups, all items in food and miscellaneous commodity groups were found to be price inelastic, while the rent and other housing sectors were found to be price elastic. However, from the inspection of t-ratios of the individual elasticities, we can see that the price elasticities for other food, clothing, medical care, and the education and recreation sectors were not significantly different from zero.

In summary, our empirical results imply that the aggregation level of the commodity groups does not affect the tests of demand restrictions. The RDAM-D4 and AIDS-SD models which rejected the demand restrictions with the aggregated commodity group data also rejected the restrictions with the disagregated data. On the other hand, the RDAM-SD model which accepted the demand restrictions on the aggregated data appeared again accepted the restrictions on the disaggregated data. However, the negativity of the Slutsky matrix was improved by disaggregation. It was also observed that the estimates of the marginal budget shares and total expenditure and price elasticities can be affected by the level of aggregation.

4.7 Structural Change in Consumption Patterns in Korea

The estimates and discussion in the last two sections were based on the assumption that the coefficients of demand systems are constant over time. No structural change over the sample period was assumed or modelled. Such an assumption about parameter constancy may be too restrictive for demand systems fitted to data collected from an economy experiencing rapid change over time, as in the case of Korea. Consequently, the question of the structural stability of the estimated demand systems should be examined.

Although it is of importance for correct statistical inference, the structural stability of estimated demand systems has tended to be been overlooked in empirical demand analysis. Exceptions can be seen in Byron and Terrell (1982) and Anderson and Blundell (1984).[17] Recently, Byron (1984) emphasised the importance of tests of parameter contancy of demand systems in the application of the Rotterdam model to time series data [see also Barnett (1984)]. In this section, the Chow (1960) type prediction and parameter constancy tests were employed for testing the structural and parameter stability of three demand systems, namely the RDAM-SD, RDAM-D4, and AIDS-SD.

The simultaneous equation analogues of the Chow prediction and parameter constancy tests were derived by Anderson and Mizon (1983). The likelihood ratio (LR) test statistic for the prediction test is of the form

$$2[(T_0/T_1)\hat{L}_1 - \hat{L}_0] + mT_0 \ ln (T_0/T_1), \qquad (7.1)$$

where the subscript 0 refers to the whole sample of size T_0, the subscript 1 to the subsample with the first T_1 observations; while $\hat{L}_i$, i = 0,1, refers to the value of the log likelihood function, and m is the number of equations in the system.[18,19] This test is an asymptotic test and the test criterion is χ^2 with $m(T_0 - T_1)$ degrees of freedom. On the other hand the LR statistic for testing parameter constancy is given by

$$-2(\hat{L}_0 - \hat{L}_1 - \hat{L}_2), \qquad (7.2)$$

where the subscript 2 refers to the second subsample with the T_2 observations [see Table 2 in Anderson and Mizon (1983)]. The LR test statistic in (7.2) is asymptotically distributed as χ^2 with p+m(m+1)/2 degrees of freedom, where p is the number of parameters in the model. This statistic is used to test not only the constancy of the structural parameters, β, but also the constancy of the error covariance matrix, Σ, so that the null hypothesis is H_0: $\{\beta_1 = \beta_2\} \cap \{\Sigma_1 = \Sigma_2\}$.[20]

To apply these tests, we split the whole sample into two sub-periods, 1966 - 1973 and 1974 - 1981. The number of observations in the two subsamples is the same (i.e. 32). The selection of the switchpoint as the mid-point of the whole sample was arbitrary. However, if we take the estimates of total subsistence expenditure in LES-W-SD model as the borderline between the poverty and non-poverty levels of total expenditure, the selection of the switch point can be partially justified in the sense that the estimated value of total subsistence expenditure 50220.3 Won in LES-W-SD lies between the average total expenditures, 47397 Won in 1973 and 58033 Won in 1974. Therefore, the first subsample covering the period for 1966-1973 can be regarded as a sub-subsistence period relative to the second subsample.

The results of the LR tests for structural stability are presented in Table 4.31. Tests were carried out for the systems with five and twelve goods as well as for the unrestricted and homogeneity and symmetry restricted models. The null hypothesis of structural stability was rejected for all models by the asymptotic χ^2 criterion, except in the case of the restricted five commodity RDAM-D4 model. A small sample size adjustment of $T_0/(T_0-k)$ was used, where k is the number of parameters in each equation.[21] The adjusted χ^2 critical values are shown in the last column in the table. Only the unrestricted RDAM-D4 model for five commodity groups accepted the null hypothesis.

Parameter constancy in the two sub-samples was also rejected in all models. The results are presented in Table 4.32. In this case, two small sample size adjustments were considered; one is as used in the previous test of stability, and the other is the Wales' (1984) correction factor,

$$T_0/[T_0 - k - ½((m-1) - s/(m-1) + 1)]$$

[see also Anderson (1958, p.208)] with $s = (m-1)k + m(m-1)/2$. The adjusted critical values are shown in the last two columns of Table 4.31. Nevertheless, it appeared that the null hypothesis of parameter constancy is still rejected for all models. Thus, from these results, we conclude that the systems estimated in previous sections suffer from structural instability and that there exists a structural change in consumption patterns in Korea, at least for the sample period 1966-1981.

As the null hypotheses of stability and parameter constancy is rejected by the data, one could re-model the demand system, for example, by introducing structural dummy variables in the second subsample to reduce the instability of the estimated demand system. However, a more important task in such a situation may be to examine the changing patterns in consumption over the sample period, since consumers' income and price responses have probably changed. Therefore, it would be more desirable to observe the effect of structural change on slope coefficients in demand systems rather than on intercept term.

First, we examined the validity of the demand restrictions, homogeneity and symmetry, on each subsample with the three demand systems; the RDAM-SD, RDAM-D4 and AIDS-SD models. The results with the five and twelve good data sets are presented in Table 4.33. In the five good case, the homogeneity restrictions were accepted by the exact T^2 test in the RDAM-SD model but were rejected in both RDAM-D4 and AIDS-SD models. However, with the twelve good data set, the homogeneity restrictions were accepted by the exact T^2 test in both RDAM-SD and AIDS-SD models but rejected by the RDAM-D4 model. These results hold for both subsamples. However, as the sample size in each subsample reduced to 32, the asymptotic χ^2 tests clearly exhibited the small sample bias toward over-rejection of the restrictions. For example, in the RDAM-SD model for twelve goods, the Wald and LR tests rejected homogeneity on the asymptotic χ^2 tests, but the exact Hotelling's T^2 test accepted homogeneity in both subsamples. Therefore, to account for the small sample bias in tests of symmetry, the same two small sample adjustments were considered as previously. The adjusted critical values of the χ^2 statistic are shown in the last two columns of Table 33. After adjustment, the symmetry restrictions which were rejected by the asymptotic tests on the five good data were accepted by the AIDS-SD model for both subsamples and by the RDAM-SD model for the second subsample. However, conflict exists between the Wald, LR and LM tests in testing symmetry using the RDAM-D4 model on the five good data in the second subsample. Furthermore, the conflict between the three tests occurred more often in testing symmetry for the twelve goods in the second subsample. For example, in the second subsample of the twelve commodity data, the symmetry restriction was rejected by Wald test in all models, but was accepted by both

LR and LM tests in the RDAM-SD model and only by LM test in the RDAM-SD and AIDS-SD models.

Comparing the test results between the subsamples, the first subsample appeared to have less significant results than the second, except for the five commodity the AID-SD model. It was also observed that tests of demand restrictions on the subsamples produced less significant results than those on the whole sample in the RDAM-D4 and AIDS-SD models for five commodity groups. For example, the symmetry restrictions in AIDS-SD with five commodity groups were rejected on the whole sample, but it was accepted on both subsamples. This result implies that testing the demand restrictions is not independent of the sample interval on which tests of demand restrictions are performed. The sample interval which suffers less from structural changes in demand patterns may yield a more successful test of (static) demand restrictions.

Generally, the RDAM-SD model achieved compatibility with demand theory in the two subsamples better than the RDAM-D4 and AIDS-SD models for both data sets. Hence, the RDAM-SD model was again used for comparison of the differences in consumption patterns in the two sample periods. Parameter estimates for the RDAM-SD model using the five good data are given in Table 4.34 for the first subsample and in Table 4.35 for the second subsample. The results using the twelve commodity data are given in Table 4.36 for the first subsample and in Table 4.37 for the second subsample. The parameter estimates indicate clear differences in the consumption pattern for the periods 1966-1973 and 1974-1981.

Using five commodities, it was found that about 60% of consumer's marginal income was spent on food in the period for 1966-1973, compared with 30% in 1974-1981. On the other hand, marginal budget shares for all other sectors increased from the first sample period to the second. Most notably, the marginal budget share for the miscellaneous sector increased from 12% in 1966-1973 to 35% in 1974-1981. The changes in the marginal budget shares appeared to be larger than those in the average budget shares, which are shown in Table 4.39.

The estimated total expenditure elasticities display a pattern similar to the marginal budget shares. In the period 1966-1974, the food, housing and clothing sectors showed a tendency to be luxury goods, while the fuel and light and miscellaneous sectors were necessities. However, in the period 1974-1981, the food sector became a necessity while the miscellaneous sector became a luxury. The clothing sector showed a greater tendency to be a luxury good in the period 1974-1984 than in the period 1966-1973.

Comparisons of the price coefficients in the two subsamples reveal the diagonal terms of the Slutsky matrix for the food and miscellaneous sectors were (insignificantly) positive for the period 1966-1973. On the other hand, all the diagonal terms of the Slutsky matrix were negative for the period 1974-1981. The compensated own price elasticities, which are shown in Table 4.38, indicate that the food, clothing[22] and miscellaneous sectors were inelastic in both subsamples. While the housing sector was highly price elastic in the first subsample ,it had unit elasticity in the second subsample. Fuel and light was elastic in the period 1966-1973 but inelastic in the period 1974-1981.

Complementarity and substitutability relations were also found to change between the two periods. In the period 1966-1973, substitutability was displayed by the food and the fuel and light sectors, the housing and the fuel and light sectors, and the housing and the miscellaneous sectors. Complementarity was only observed between the fuel and light and miscellaneous sectors. However, in the period 1974-1981, only the food sector was found to be a complement to the housing, clothing, and miscellaneous sectors. However, no significant substitutability relation was found in the period 1974-1981 between the five aggregated sectors.

Significant changes in seasonal consumption patterns over the subsamples were noticed in the food and miscellaneous sectors. In the unrestricted model, the second, third and fourth quarters' seasonal dummy coefficients for the food sector were all (insignificantly) negative in the period 1966-1973, but they were all positive and significant in the period 1974-1981. Change in the seasonal pattern in the miscellaneous sector was also observed in the second and third quarters in the unrestricted model. However, the seasonal patterns in the housing, the fuel and light and the clothing sectors, which depend on climatic conditions, remained unchanged.

The twelve commodity results were similar to those of the five commodities. The marginal budget shares of four food commodity groups, the cereal, meat and fish, vegetable and fruit, the other food sector and the rent sector were found to decline. The other sector shares were found to increase from 1966-1973 to 1974-1981. From the comparisons of the total expenditure elasticities in Table 4.40, it can be seen that the cereal, fuel and light, and the transportation and communication sectors remained as necessities in both samples, while meat and fish, other housing, and the clothing sectors were luxuries. The medical care and the education and recreation sectors changed from necessities to luxuries, while movement from luxury to necessity was seen in the vegetable and fruit, miscellaneous food, and the rent sectors. The most significant change from luxury to necessity was observed in the rent category. In the period 1966-1973, rent was classified as the most luxurious good, but in the period 1974-1981, it was classified as a necessity, or even as an inferior good in the restricted model. This result may reflect the fact that the poor paid more rent than the rich in the period 1974-1981, as the rich tended to possess their own houses as income and supply of housing increased with the development of the Korean economy.

To discuss the price coefficient estimates on the twelve commodity data set for the two subsamples, the diagonal terms of the Slutsky matrix for meat and fish, medical care and the other miscellaneous sectors in the period 1966-1973 were (insignificantly) positive. On the other hand, no positive diagonal terms of the Slutsky matrix was observed in the period 1974-1981. The compensated own price elasticities, given in Table 4.41, reveal that all four food sectors, clothing[23], education, and the other miscellaneous sectors were price inelastic in both subsamples, while rent and the fuel and light sectors were highly elastic in 1966-1973 but inelastic in 1974-1981. For the other housing, clothing, medical care, and the education and recreation sectors, the compensated own price elasticity was not significantly different from zero in 1966-1973, but became significantly different from zero and negative in 1974-1981. For the education and recreation sector, the compensated own price elasticity was not significantly different from zero in either period.

Not all the complementarity and substitutability relations carried over from the first subsample period to the second. There were 15 complementarity and 11 substitutability relations in 1966-1973 and only 9 complementarity and 9 substitutability relations in 1974-1981. Only two complementarity relations (between the other food and the rent sectors, and between the rent and the clothing sectors), and one substitutability relation (between the rent and the household durable sectors) carried over from the first subsample to the second. Reversion from complementarity to substitutability was observed in several sectors (the vegetable and fruit and the fuel and light sectors, the education and recreation and the transportation and communication sectors).

The seasonal dummy coefficients on the twelve commodity data set were constant only for the sectors which are dependent on climatic conditions or the academic calendar; for example, the vegetable and fruit, other housing, fuel and light, clothing, and the education and recreation sectors. However, the cereal, meat and fish, other food, rent and the medical care sectors exibited significant changes in seasonal patterns. In the period for 1966-1973, the cereal and meat and fish sectors were found to have a significant positive seasonal effect for the first quarter. On the other hand, in 1974-1981, the cereal sector appeared to have significant positive seasonal effect for the fourth quarter, and the meat and fish sector appeared to have a positive effect in the third quarter. The seasonal effects on rent were significantly negative in the fourth quarter and positive in the first and second quarter for 1966-1973, but they were insignificant in 1974-1981.

Thus, we have seen that consumers' responses to changes in price and total expenditure as well as seasonal consumption patterns in the period 1966-1973 were different from 1974-1981. The analysis of demand patterns in Korea by estimating static demand systems with subsample data obviously led to clear differences from the previous results using the complete sample. Examined closely, the parameter estimates using the whole sample were found to lie between the corresponding estimates obtained from the first and second subsamples. However, by the nature of the estimation procedure, the estimates obtained from the whole sample were not represented by the arithmetic averages of the corresponding estimates of the two subsamples. The demand equations were found to fit the subsamples better than the whole sample, except in the case of the rent sector in the symmetry restricted model for the second subsample.

Our empirical results in this section strongly suggest that consumption patterns in Korea have experienced significant structural changes over the last two decades.

4.8 Concluding Remarks

In this chapter, we have analysed demand patterns in Korea in the context of static demand theory. We applied three static demand systems, the linear expenditure system, the Rotterdam model and the almost ideal demand system, to quarterly household expenditure data with various specifications of the seasonal effects. The performance of these systems was assessed on a pragmatic basis rather than using stringent statistical methods. Like the other comparative studies, the Rotterdam model appeared superior to the linear expenditure system, and little difference between the Rotterdam model and the almost ideal demand system was observed in terms of goodness of fit and randomness of the estimated residuals. However, the Rotterdam model exhibited more favourable results than the almost ideal demand system in testing the demand restrictions on the Korean data as well as in modelling seasonal effects. On the other hand, the deseasonalisation of quarterly data in the application of the Rotterdam model appeared unsuccessful in achieving the compatibility of demand theory and sample information.

The parameter estimates appeared to vary not only with the specification of the demand system and the seasonal effects but also with the aggregation level of the commodity groups. In particular, the parameter estimates were found to be sensitive to the specification of dynamic structure. However, tests of the demand restrictions were found to be unaffected by the dynamic specification of models as well as by the aggregation level chosen.

The results from Section 4.7 strongly suggest that consumption patterns in Korea have changed significantly in the last two decades. The structural stability of the estimated static demand systems and the parameter constancy hypothesis were rejected by the data. Consequently, static analysis risks ignoring significant dynamic changes in Korean consumption characteristics. However, the analyses attempted in Section 4.7 can be viewed as a partial treatment of the dynamics of change, in the context of modelling a static demand system. In the subsequent chapters, the dynamic versions of the Rotterdam model, embedding the taste change hypothesis, will be developed and applied.

Unfortunately, there are a limited number of demand studies in the context of complete demand systems for Korea. Only one demand study, to be found in Lluch, Powell and Williams (1977), has been published. In that case, the analysis was limited to the extended linear expenditure system. To the author's knowledge, no demand study on Korea has been carried out with the Rotterdam model or the almost ideal demand system. Therefore, a comparison of our results, with others, is not possible.

FOOTNOTES:

1. At the outset of this study, we attempted to estimate the direct and indirect translog demand systems but excluded them because the nonlinear estimation procedure converged to unacceptable values or failed to achieve convergence. Failure of convergence on the translog model has been reported elsewhere [e.g., McLaren (1982) and Klevmarken (1981)]. As seen in Section 2.6.1, most of the empirical work with the translog model has been done with three demand equation systems. In our experiments, the three equation system achieved convergence, although the results are not presented here.

2. The linear expenditure system in terms of expenditures was estimated, but was later excluded, because its performance was slightly inferior to that of LES-W.

3. Using the Swedish eight commodity data for the period 1861-1955, Parks (1969) found that the Rotterdam model was superior to the indirect addilog model or the linear expenditures system. A similar result was observed by Theil (1975) using a British four commodity data set for the period 1900-1938. Applying RDAM to a British nine commodity data set for the period 1900-1970, Deaton (1974) also found that the Rotterdam model dominated the direct addilog model, the linear expenditure system, and the additive variant of the Rotterdam model. Bewley (1982b) compared the performance of the Rotterdam model, the almost ideal demand system and the generalised addilog model, using an Australian seven commodity data set for the period 1959-60 to 1975-1976. The comparisons of Parks (1969), Yoshihara (1969), Theil (1975), and Bewley (1982b)] were based on average information inaccuracy measures, while Deaton (1974) used the estimated likelihood function. Other comparative studies of the empirical performance of different functional demand systems are available in Yoshihara (1969) and Klevmarken (1981).

4. In econometrics, the notation 'AR(4)' is usually used for the autoregressive error process given as (8.1) in Chapter 3, in which the lower order autoregressive effects are not

necessarily null. However, we adopt the notation 'AR(4)' for the error process in (2.7) for convenience of presentation in this Chapter.

5. For examples of the use of fourth order differences in general econometric models, see Davidson *et al.* (1978) and Hendry and Richard (1983, p.131-132).

6. The classifications of both the aggregated and disaggregated commodity groups were changed in 1982, and the series since then are not consistent with the previous series, so that the data after 1982 are not included in this study.

7. The exclusion of rental value of owner-occupied housing from expenditure follows from the definition of the housing expenditure group, modified in 1982.

8. Deaton (1974) reparameterised the linear expenditure system and the direct addilog model into the context of the Rotterdam model, so that they have identical dependent variables and this enables the possibility of a direct comparison.

9. The same results are obtained from the estimation of the linear expenditure system in expenditures, although they are not presented here.

10. Even though the full coefficient matrix Ψ of autogressive error process cannot be recovered from the reduced coefficient matrix $\overline{\Psi}$ in the singular system, Berndt and Savin (1975) showed that to test $\overline{\Psi} = 0$ is sufficient for testing the serial independence of residuals [Berndt and Savin (1975, p.945)].

11. As seen in Section 3.8, $\overline{\Psi} \neq 0$ does not necessarily imply that $\Psi \neq 0$.

12. In Attfield and Browning (1985, p.43 and p.45), the rational expection hypothesis is also rejected in the use of deseasonalised quarterly data.

13. The (i,j)'th term of the Slutsky matrix implied by the almost ideal demand system is obtained as $C_{ij} = \gamma_{ij} + \beta_i\beta_j \, ln(\overline{\mu}/\overline{P}_o) + \overline{w}_i\delta_{ij} + \overline{w}_i\overline{w}_j$, where the notation in (6.22) of Chapter 2 is used and the bar refers to the sample mean. The sample

mean $\overline{P}_o$ is obtained as $\overline{P}_o = exp(\sum_{j=1} \overline{w}_j \, ln \, \overline{P}_j)$. The standard error of C_{ij} is taken as that of γ_{ij} with the assumption that the average expenditure shares $\overline{w}_i$'s are fixed, and $\beta_i \beta_j \simeq 0$ [Deaton and Muellbauer (1980a, p.84)].

14. Negative semi-definiteness of the Slutsky matrix is usually checked with the necessary condition that diagonal terms of the matrix are equal to or less than zero. The fulfilment of this necessary condition does not guarantee the negative semi-definiteness of the matrix. However, violation of this necessary condition affirms that the Slutsky matrix is not negative semi-definite, but can be indefinite or positive semi-definite. The condition that the eigenvalues of the Slutsky matrix are equal to or less than zero is also the necessary condition for negative semi-definiteness. Therefore, the same reasoning can be applied to the use of significance test of eigenvalues of the Slutsky matrix for testing the negative semi-definiteness of the matrix [e.g., Böhm, *et al.* (1980) and Bewley (1982b)].

15. The total expenditure elasticity is the marginal budget share divided by the expenditure share, by definition. However, the average expenditure share over the sample period is used here.

16. Since the number of explanatory variables in each equation is seventeen including the seasonal dummy varaibles, the bounds for the Durbin-Watson statistics, DW, are given as d_l = 1.016 and d_u = 2.276 with 65 observations. However, since the bounds for the DW4 statistic are not available for the case when k'' = 13, we approximate the bounds $d4_l$ = 1.429 and $d4_u$ = 1.776 for k'' = 5 with 64 observations, where k'' is the number of explanatory variables excluding the seasonal dummy variables.

17. Byron and Terrell (1982) used the Wald type of Chow test for parameter constancy of the Rotterdam model applied to monthly Japanese data, and Anderson and Blundell (1984) used the Chow type prediction test for the structural stability of a dynamic version of the almost ideal demand system applied to quarterly U.K. data.

18. The null hypothesis for the Chow's prediction test can be

written as

$$H_0: \{\beta_1 = \beta_2\} \cap \{\Sigma_1 = \Sigma_2\} \cap \{\varphi = 0\},$$

where β is a vector of structural parameters, Σ the error covariance matrix, and φ is an $mT_2 \times 1$ vector of dummy variable coefficients introduced into all m equations in the second sub-sample [see the null hypothesis $H_4^{*s} \cap H_2^s$ in Anderson and Mizon (1983, p.10 and Table 2)].

19. The LR test statistic for the Chow type prediction test and the parameter constancy test can be written as

$$LR = T_0 \; ln\,(|\hat{\Sigma}_0|/|\hat{\Sigma}_0|),$$

and the Wald and LM statistics can be given as

$$W = T_0[(T_0/T_1) \; tr\,(\hat{\Sigma}_1^{-1}\tilde{\Sigma}_0) - m]$$

and

$$LM = T_0[m - (T_0/T_1) \; tr\,(\tilde{\Sigma}_0^{-1}\hat{\Sigma}_1)],$$

where $\hat{\Sigma}_1 = \hat{U}_1'\hat{U}_1/T_1$ and $\hat{\Sigma}_0 = \hat{U}_0'\hat{U}_0/T_0$ are the estimates of error covariance matrix from subsample 1 and the whole sample, respectively, $\tilde{\Sigma}_0$ is given as $\tilde{\Sigma}_0 = (T_1/T_0)\hat{\Sigma}_1$, $\hat{U}_i$, i=1,0, refers to the residual matrix, and m is the number of equations in the system [see equations (4) - (5) in Anderson and Mizon (1983)]. The Wald and LM statistics can be expressed in the usual forms

$$W = T_0 \; tr\,\hat{\Sigma}_0^{-1}(\tilde{\Sigma}_0 - \hat{\Sigma}_0)$$

and

$$LM = T_0 \; tr\,\tilde{\Sigma}_0^{-1}(\tilde{\Sigma}_0 - \hat{\Sigma}_0)$$

resepectively. The three test statistics are asymptotically distributed under the null hypothesis as χ^2 with degrees of freedom mT_2.

20. See the null hypothesis H in Anderson and Mizon (1983, p.9).

21. In this case, Wales' (1984) correction factor $T_0/[T_0 - k - ½((m-1) - s/(m-1) +1)]$ [see Anderson (1958, p.208)] cannot be applied as the number of restrictions

$s = mT_2$ is too large. Consequently, the calculated correction factor appears to be less than 1.

22. Although the estimated compensated own price elasticity of clothing is greater than one, its t-ratio is insignificant, so that the estimated elasticity is not significantly different from zero.

23. See Footnote 22.

TABLES IN CHAPTER 4

Table 4.1

Commodity Sectors and Aggregation

5 Commodity Sectors	12 Commodity Sectors	22 Commodity Sectors
1. Food and Beverages (458.0)	1. Cereals (204.5)	1. Cereals (204.5)
	2. Meat, Dairy Food and Fish (80.4)	2. Meat and Fish (65.4)
		3. Milk and Egg (15.0)
	3. Vegetable and Fruit (72.4)	4. Fruits and Vegetables (72.4)
	4. Other Foods (100.7)	5. Condiments (40.6)
		6. Processed Foods (16.3)
		7. Beverages and Confectionery (22.4)
		8. Alcoholic Drinks (9.3)
		9. Meals Outside (21.1)
2. Housing (110.1)	5. Rent Paid (49.3)	10. Rent Paid (49.3)
	6. Other Housing (60.8)	11. Other Housing (60.8)
3. Fuel and Light (56.0)	7. Fuel and Light (56.0)	12. Fuel (41.0)
		13. Light and Other Fuel (15.0)
4. Clothing (92.5)	8. Clothing (92.5)	14. Clothes (54.2)
		15. Other Clothing (38.3)
5. Miscellaneous (283.4)	9. Medical Care (49.0)	16. Medical Care (49.0)
	10. Education and Recreations (99.0)	17. Education and Stationery (78.4)
		18. Reading and Recreations (20.6)
	11. Transportation & Communication (53.3)	19. Transportation & Communication (53.3)
	12. Miscellaneous (82.1)	20. Personal Care (28.2)
		21. Cigarette (52.6)
		22. Miscellaneous (1.3)

Table 4.2
Average Expenditure Shares (by Season)

5 Aggregated Commodity Sectors

Quarter	1	2	3	4	5	Average Temperature
1	42.6	8.0	7.1	11.3	31.0	-0.6
2	44.9	9.8	5.7	10.8	28.8	16.1
3	46.7	8.8	5.8	9.5	29.2	21.7
4	48.2	7.8	7.3	11.1	25.5	5.9

12 Disaggregated Commodity Sectors

Quarter	1	2	3	4	5	6	7	8	9	10	11	12
1	18.5	9.4	5.8	8.9	2.4	5.6	7.1	11.3	5.1	11.1	5.2	9.6
2	18.1	9.0	7.3	10.4	2.6	7.3	5.7	10.8	5.4	8.4	5.3	9.7
3	17.1	9.7	8.6	11.2	2.5	6.4	5.8	9.5	5.4	9.6	5.5	8.8
4	18.3	9.6	9.1	11.3	2.3	5.5	7.3	11.1	4.8	6.7	5.0	9.0

Note: Average temperature is taken from 'Hankook Tong-key Yon-bo' [Korean Statistical Year Book], 1981, Seoul, Korea.

Table 4.3

Annual Expenditure Shares (in %)

5 Aggregated Commodity Sectors

Year	1	2	3	4	5
65	63.6	3.7	6.5	7.2	19.1
66	56.9	3.8	7.3	9.1	23.0
67	51.2	5.4	6.8	11.9	24.7
68	48.3	5.7	5.9	12.3	27.8
69	46.9	6.7	5.8	12.3	28.3
70	46.7	5.9	6.4	11.6	29.5
71	47.6	5.7	6.3	10.9	29.4
72	47.8	5.5	5.9	9.9	30.9
73	48.2	5.8	5.7	10.6	29.7
74	49.8	6.2	6.4	9.7	28.0
75	48.8	7.6	6.0	9.8	27.8
76	48.2	8.1	5.4	10.0	28.3
77	48.1	9.1	5.8	10.7	26.3
78	45.9	10.6	5.7	11.4	26.5
79	42.8	10.7	5.8	12.0	28.7
80	43.2	9.4	7.3	10.9	29.2
81	43.2	9.0	8.0	9.7	30.2
$\bar{w}$	45.7	8.6	6.5	10.7	28.5
$\dot{w}$	-2.39	5.71	1.30	1.90	2.90
$\bar{p}$	95.0	98.8	104.1	100.1	96.9

12 Disaggregated Commodity Sectors

Year	1	2	3	4	5	6	7	8	9	10	11	12
65	38.2	9.0	8.6	7.8	1.4	2.3	6.5	7.2	1.2	6.5	2.2	9.1
66	31.1	9.1	9.0	7.8	1.3	2.5	7.3	9.1	1.5	7.5	3.1	10.9
67	23.4	9.5	9.9	8.4	1.6	3.8	6.8	11.9	2.0	8.0	3.6	11.1
68	21.6	10.4	9.2	7.1	1.6	4.0	5.9	12.3	3.3	10.2	4.2	10.1
69	21.1	10.0	7.7	8.0	1.6	5.1	5.8	12.3	3.4	9.3	4.9	10.6
70	20.0	9.6	8.7	8.3	1.7	4.2	6.4	11.6	3.5	9.7	5.5	10.7
71	20.5	10.0	8.0	9.1	1.8	3.8	6.3	10.9	3.0	10.2	5.0	11.2
72	23.5	9.0	6.7	8.6	1.7	3.8	5.9	9.9	3.1	11.6	5.3	10.9
73	21.6	9.7	7.8	9.1	1.9	3.9	5.7	10.6	3.1	11.8	4.9	9.9
74	22.6	8.7	7.9	10.5	2.2	3.9	6.4	9.7	3.0	9.7	5.0	10.3
75	22.6	8.2	7.5	10.5	1.8	5.9	6.0	9.8	4.4	9.2	4.6	9.6
76	21.9	8.7	7.0	10.5	2.1	6.0	5.4	10.0	4.9	9.3	4.4	9.7
77	20.2	9.1	8.2	10.6	2.4	6.7	5.8	10.7	4.8	8.8	5.1	7.7
78	16.6	9.9	7.8	11.6	2.4	8.1	5.7	11.4	5.1	8.1	5.0	8.2
79	14.7	9.8	7.4	11.0	2.9	7.8	5.8	12.0	6.0	8.8	5.3	8.6
80	14.7	9.6	7.9	11.1	3.0	6.4	7.3	10.9	6.3	8.1	5.8	9.0
81	14.8	9.5	7.8	11.1	2.8	6.2	8.0	9.7	6.7	8.3	6.0	9.2
$\dot{w}$	-5.8	.39	-.61	2.2	4.3	6.4	1.3	1.9	11.1	1.5	6.3	.10
$\bar{w}$	18.0	9.4	7.6	10.4	2.5	6.2	6.5	10.7	5.2	9.0	5.3	9.3

Note: $\dot{w}$ = the geometric mean of annual change rates of expenditure share.
$\bar{w}$ = the average of expenditure shares over the sample period.
$\bar{p}$ = the average of price indices over the sample period.

Table 4.4
LES-W Model[a]

	1	2	3	4	5
β_i	.31853	.14741	.06064	.13287	.34055
	(.0235)	(.0080)	(.0061)	(.0063)	(.0176)
γ_i	152.529	-17.885	10.160	5.759	17.988
	(22.427)	(4.950)	(3.576)	(4.737)	(15.537)

[a] Estimated total subsistence expenditure at average price level is 49665.8 Won.

Table 4.5
LES-W-SD Model[a]

	1	2	3	4	5
β_i	.33731	.13829	.06618	.13779	.32043
	(.0136)	(.0054)	(.0049)	(.0054)	(.0125)
γ_i	167.947	-22.556	10.277	3.055	4.583
	(14.381)	(2.664)	(2.681)	(3.216)	(9.855)
D_1	-6.8579	.9650	.2801	.3973	5.2155
	(.6621)	(.3284)	(.2733)	(.3341)	(.7603)
D_2	-6.1394	2.8597	-1.0342	.2609	4.0530
	(.6687)	(.3303)	(.2762)	(.3355)	(.7656)
D_3	-4.2872	1.8134	-1.3497	-.8078	4.6313
	(.6579)	(.3273)	(.2731)	(.3327)	(.7573)

[a] Estimated total subsistence expenditure at average price level is 50220.3 Won.

Table 4.6
RDAM Model

Table 4.6.1 Unrestricted RDAM Model

	$dlnP_1$	$dlnP_2$	$dlnP_3$	$dlnP_4$	$dlnP_5$	dQ
1	-.24086	-.42001	.26895	.07773	.15417	.76793
	(.1213)	(.2632)	(.0970)	(.2618)	(.1935)	(.0586)
2	-.05839	-.10698	.00823	.07233	.13925	-.01097
	(.0453)	(.0984)	(.0363)	(.0979)	(.0723)	(.0219)
3	.13232	-.06183	-.02529	.07746	-.15006	.11784
	(.0392)	(.0850)	(.0313)	(.0846)	(.0625)	(.0189)
4	.05049	-.03930	-.02108	-.04028	.02588	.14594
	(.0450)	(.0977)	(.0360)	(.0972)	(.0718)	(.0218)
5	.11644	.62812	-.23081	-.18724	-.16924	-.02074
	(.1090)	(.2365)	(.0871)	(.2352)	(.1739)	(.0527)

Table 4.6.2 Homogeneity Restricted RDAM Model

	$dlnP_1$	$dlnP_2$	$dlnP_3$	$dlnP_4$	$dlnP_5$	dQ
1	-.26144	-.28814	.21665	.16891	.16402	.74796
	(.1219)	(.2464)	(.0896)	(.2558)	(.1960)	(.0574)
2	-.05139	-.15183	.02601	.04132	.13589	-.00418
	(.0455)	(.0919)	(.0334)	(.0954)	(.0731)	(.0214)
3	.12880	-.03925	-.03425	.09307	-.14837	.11443
	(.0390)	(.0788)	(.0286)	(.0818)	(.0627)	(.0184)
4	.04737	-.01929	-.02901	-.02644	.02738	.14291
	(.0447)	(.0904)	(.0329)	(.0939)	(.0719)	(.0211)
5	.13667	.49851	-.17940	-.27685	-.17892	-.00112
	(.1098)	(.2220)	(.0807)	(.2304)	(.1766)	(.0517)

Table 4.6.3 Homogeneity and Symmetry Restricted RDAM Model

	$dlnP_1$	$dlnP_2$	$dlnP_3$	$dlnP_4$	$dlnP_5$	dQ
1	-.29436	-.06469	.14740	.06075	.15090	.73152
	(.1175)	(.0440)	(.0350)	(.0423)	(.1081)	(.0539)
2	-.06469	-.17033	.02430	.05350	.15722	-.00468
	(.0440)	(.0721)	(.0308)	(.0567)	(.0635)	(.0212)
3	.14740	.02430	-.01667	-.00371	-.15131	.11953
	(.0350)	(.0308)	(.0262)	(.0267)	(.0441)	(.0177)
4	.06075	.05350	-.00371	-.11832	.00779	.14660
	(.0423)	(.0567)	(.0267)	(.0700)	(.0653)	(.0208)
5	.15090	.15722	-.15131	.00779	-.16461	.00702
	(.1081)	(.0635)	(.0441)	(.0653)	(.1438)	(.0494)

Table 4.7
RDAM-D4 Model

Table 4.7.1 Unrestricted RDAM-D4 Model

	$dlnP_1$	$dlnP_2$	$dlnP_3$	$dlnP_4$	$dlnP_5$	dY
1	-.21658	-.16521	.05157	.15550	.12545	.38483
	(.0342)	(.0733)	(.0317)	(.0615)	(.0564)	(.0248)
2	.06406	-.26924	.03693	.07740	.04463	.17480
	(.0191)	(.0410)	(.0177)	(.0343)	(.0315)	(.0139)
3	.03285	.10538	-.02182	-.02884	-.05511	.01088
	(.0113)	(.0243)	(.0105)	(.0203)	(.0187)	(.0082)
4	.03524	.08745	.00257	-.11263	-.02413	.16851
	(.0157)	(.0338)	(.0146)	(.0283)	(.0260)	(.0114)
5	.08444	.24162	-.06924	-.09143	-.09083	.26098
	(.0299)	(.0643)	(.0278)	(.0539)	(.0494)	(.0218)

Table 4.7.2 Homogeneity Restricted RDAM-D4 Model

	$dlnP_1$	$dlnP_2$	$dlnP_3$	$dlnP_4$	$dlnP_5$	dY
1	-.21665	-.07191	.01854	.16631	.10371	.35062
	(.0356)	(.0636)	(.0295)	(.0638)	(.0579)	(.0207)
2	.06400	-.18173	.00595	.08754	.02424	.14271
	(.0212)	(.0380)	(.0176)	(.0381)	(.0346)	(.0124)
3	.03289	.04395	-.00008	-.03596	-.04080	.03341
	(.0131)	(.0233)	(.0108)	(.0234)	(.0212)	(.0076)
4	.03522	.10923	-.00514	-.11010	-.02920	.16052
	(.0159)	(.0284)	(.0132)	(.0285)	(.0259)	(.0093)
5	.08454	.10046	-.01927	-.10778	-.05794	.31274
	(.0335)	(.0599)	(.0277)	(.0600)	(.0545)	(.0195)

Table 4.7.3 Homogeneity and Symmetry Restricted RDAM-D4 Model

	$dlnP_1$	$dlnP_2$	$dlnP_3$	$dlnP_4$	$dlnP_5$	dY
1	-.19946	.05541	.02909	.03749	.07746	.33198
	(.0334)	(.0188)	(.0118)	(.0145)	(.0330)	(.0192)
2	.05541	-.19606	.01469	.09713	.02883	.14325
	(.0188)	(.0297)	(.0130)	(.0218)	(.0288)	(.0118)
3	.02909	.01469	-.00332	-.00874	-.03171	.03794
	(.0118)	(.0130)	(.0107)	(.0108)	(.0162)	(.0071)
4	.03749	.09713	-.00874	-.09583	-.03005	.16273
	(.0145)	(.0218)	(.0108)	(.0247)	(.0225)	(.0088)
5	.07746	.02883	-.03171	-.03005	-.04454	.32410
	(.0330)	(.0288)	(.0162)	(.0225)	(.0499)	(.0180)

Table 4.8
RDAM-SD Model

Table 4.8.1 Unrestricted RDAM-SD Model

	$d\ln P_1$	$d\ln P_2$	$d\ln P_3$	$d\ln P_4$	$d\ln P_5$	dM
1	-.09452	-.20416	.11252	.34105	.14613	.49910
	(.0814)	(.1627)	(.0588)	(.1600)	(.1172)	(.0590)
2	.02781	-.17194	.04465	.00394	.01142	.10524
	(.0352)	(.0705)	(.0255)	(.0693)	(.0507)	(.0256)
3	.06782	.06581	-.04891	.05808	-.08348	.03676
	(.0246)	(.0492)	(.0178)	(.0484)	(.0354)	(.0178)
4	.02974	.03791	-.01832	-.15473	-.01839	.13242
	(.0405)	(.0811)	(.0293)	(.0797)	(.0584)	(.0294)
5	-.03085	.27238	-.08994	-.24833	-.05568	.22647
	(.0690)	(.1380)	(.0499)	(.1357)	(.0994)	(.0501)

	D_1	D_2	D_3	D_4
1	-.07126	-.00160	.00395	.01469
	(.0075)	(.0068)	(.0065)	(.0085)
2	.00783	.02020	-.00598	-.01150
	(.0033)	(.0030)	(.0028)	(.0037)
3	.00002	-.01476	-.00504	.01384
	(.0023)	(.0021)	(.0020)	(.0026)
4	.00884	.00403	-.00786	.01171
	(.0037)	(.0034)	(.0032)	(.0042)
5	.05457	-.00786	.01492	-.02874
	(.0064)	(.0058)	(.0055)	(.0072)

Table 4.8.2 Homogeneity Restricted RDAM-SD Model

	$d\ln P_1$	$d\ln P_2$	$d\ln P_3$	$d\ln P_4$	$d\ln P_5$	dM
1	-.13441	-.34225	.15150	.21264	.11252	.50141
	(.0819)	(.1543)	(.0578)	(.1532)	(.1201)	(.0611)
2	.03896	-.13335	.03376	.03982	.02081	.10460
	(.0348)	(.0655)	(.0245)	(.0651)	(.0510)	(.0259)
3	.05996	.03860	-.04123	.03277	-.09010	.03721
	(.0243)	(.0457)	(.0171)	(.0454)	(.0356)	(.0181)
4	.04615	.09470	-.03436	-.10192	-.00457	.13147
	(.0404)	(.0761)	(.0285)	(.0755)	(.0592)	(.0301)
5	-.01066	.34230	-.10968	-.18331	-.03866	.22530
	(.0680)	(.1280)	(.0480)	(.1272)	(.0997)	(.0507)

Table 4.8.2 (continued)

	D_1	D_2	D_3	D_4
1	-.06063 (.0058)	.00847 (.0050)	.01199 (.0054)	.02296 (.0078)
2	.00486 (.0025)	.01738 (.0021)	-.00823 (.0023)	-.01381 (.0033)
3	.00212 (.0017)	-.01278 (.0015)	-.00345 (.0016)	.01547 (.0023)
4	.00447 (.0029)	-.00011 (.0025)	-.01116 (.0027)	.00831 (.0038)
5	.04919 (.0048)	-.01296 (.0042)	.01085 (.0045)	-.03293 (.0064)

Table 4.8.3 Homogeneity and Symmetry Restricted RDAM-SD Model

	$d\ln P_1$	$d\ln P_2$	$d\ln P_3$	$d\ln P_4$	$d\ln P_5$	dM
1	-.16866 (.0814)	.02724 (.0341)	.07128 (.0224)	.05841 (.0384)	.01174 (.0678)	.44932 (.0584)
2	.02724 (.0341)	-.18719 (.0595)	.04381 (.0211)	.06286 (.0504)	.05328 (.0442)	.10850 (.0257)
3	.07128 (.0224)	.04381 (.0211)	-.03330 (.0166)	-.00618 (.0231)	-.07561 (.0267)	.04396 (.0176)
4	.05841 (.0384)	.06286 (.0504)	-.00618 (.0231)	-.10362 (.0666)	-.01147 (.0515)	.14380 (.0292)
5	.01174 (.0678)	.05328 (.0442)	-.07561 (.0267)	-.01147 (.0515)	.02205 (.0852)	.25442 (.0484)

	D_1	D_2	D_3	D_4
1	-.06037 (.0061)	.01071 (.0052)	.01252 (.0055)	.02940 (.0076)
2	.00495 (.0025)	.01681 (.0021)	-.00795 (.0023)	-.01434 (.0033)
3	.00200 (.0017)	-.01289 (.0015)	-.00386 (.0016)	.01496 (.0023)
4	.00465 (.0029)	-.00043 (.0025)	-.01171 (.0027)	.00690 (.0037)
5	.04877 (.0050)	-.01421 (.0043)	.01101 (.0046)	-.03692 (.0063)

Table 4.9
RDAM-AR(4) Model

Table 4.9.1 Unrestricted RDAM-AR(4) Model

	$dlnP_1$	$dlnP_2$	$dlnP_3$	$dlnP_4$	$dlnP_5$	dM
1	-.21762	-.07272	.09946	.07467	.25436	.51449
	(.0639)	(.1332)	(.0513)	(.1427)	(.0767)	(.0471)
2	.01234	-.25312	.05718	-.03166	.11207	.04770
	(.0380)	(.0858)	(.0320)	(.0854)	(.0523)	(.0268)
3	.06198	.07478	-.04016	.10852	-.13880	.06244
	(.0229)	(.0486)	(.0186)	(.0511)	(.0285)	(.0165)
4	.03017	.05574	-.02243	-.06344	-.03975	.12092
	(.0312)	(.0687)	(.0258)	(.0684)	(.0424)	(.0216)
5	.11313	.19532	-.09405	-.08809	-.18787	.25445
	(.0578)	(.1205)	(.0464)	(.1296)	(.0690)	(.0428)

AR(4) Coefficient Estimates when the fifth equation is deleted

	$u_1(-4)$	$u_2(-4)$	$u_3(-4)$	$u_4(-4)$
1	.89590	.34852	.34878	-.40152
	(.0746)	(.2449)	(.3028)	(.2175)
2	-.04309	.20572	-.59930	.22594
	(.0442)	(.1277)	(.1606)	(.1125)
3	.02725	.07939	.82990	.13275
	(.0269)	(.0876)	(.1078)	(.0777)
4	.03920	.21487	.46102	.55875
	(.0366)	(.1122)	(.1392)	(.0988)

AR(4) Coefficient Estimates when the First Equation is Deleted

	$u_2(-4)$	$u_3(-4)$	$u_4(-4)$	$u_5(-4)$
2	.24879	-.55623	.26904	.04309
	(.1222)	(.1518)	(.1141)	(.0442)
3	.05214	.80266	.10549	-.02725
	(.0826)	(.1043)	(.0784)	(.0269)
4	.17568	.42183	.51955	-.03920
	(.1066)	(.1332)	(.0999)	(.0366)
5	.07078	-.12113	.40334	.91926
	(.2087)	(.2649)	(.1985)	(.0672)

Table 4.9.2 Homogeneity Restricted RDAM-AR(4) Model

	$dlnP_1$	$dlnP_2$	$dlnP_3$	$dlnP_4$	$dlnP_5$	dM
1	-.23037	-.11995	.11613	-.00786	.24204	.52177
	(.0627)	(.1245)	(.0502)	(.1270)	(-)[a]	(.0478)
2	.01817	-.20911	.04027	.02927	.12141	.03798
	(.0389)	(.0792)	(.0299)	(.0813)	(.0536)	(.0267)
3	.05949	.04965	-.03021	.06689	-.14582	.06806
	(.0234)	(.0465)	(.0184)	(.0474)	(.0297)	(.0169)
4	.02589	.07241	-.03120	-.03329	-.03380	.11319
	(.0315)	(.0631)	(.0240)	(.0638)	(.0431)	(.0213)
5	.12682	.20700	-.09498	-.05501	-.18383	.25900
	(-)[a]	(.1112)	(.0450)	(.1137)	(.0682)	(.0431)

AR(4) Coefficient Estimates when the Fifth Equation is Deleted

	$u_{1(-4)}$	$u_{2(-4)}$	$u_{3(-4)}$	$u_{4(-4)}$
1	.88912	.37276	.29737	-.35503
	(.0752)	(.2515)	(.3041)	(.2181)
2	-.03399	.18188	-.54850	.18169
	(.0447)	(.1308)	(.1615)	(.1126)
3	.02118	.09646	.79782	.16023
	(.0277)	(.0903)	(.1092)	(.0779)
4	.04689	.18723	.48667	.53745
	(.0369)	(.1131)	(.1382)	(.0973)

AR(4) Coefficient Estimates when the First Equation is Deleted

	$u_{2(-4)}$	$u_{3(-4)}$	$u_{4(-4)}$	$u_{5(-4)}$
2	.21589	-.51451	.21569	.03400
	(.1249)	(.1534)	(.1137)	(.0447)
3	.07526	.77663	.13905	-.02118
	(.0861)	(.1053)	(.0792)	(.0277)
4	.14035	.43979	.49055	-.04689
	(.1079)	(.1323)	(.0985)	(.0369)
5	.08489	-.11012	.39884	.92320
	(.2151)	(.2637)	(.1991)	(.0673)

[a] Standard error cannot be obtained in the case when the first or fifth equation is deleted in the estimation of a reduced system.

Table 4.9.3 Homogeneity and Symmetry Restricted RDAM-AR(4) Model

	$dlnP_1$	$dlnP_2$	$dlnP_3$	$dlnP_4$	$dlnP_5$	dM
1	-.24270	-.00687	.07698	.01827	.15431	.49717
	(.0625)	(.0375)	(.0217)	(.0303)	(-)[a]	(.0420)
2	-.00687	-.24943	.06260	.05568	.13802	.03154
	(.0375)	(.0668)	(.0247)	(.0501)	(.0466)	(.0255)
3	.07698	.06260	-.02765	-.00249	-.10945	.08316
	(.0217)	(.0247)	(.0186)	(.0209)	(.0246)	(.0166)
4	.01827	.05568	-.00249	-.01884	-.05262	.12082
	(.0303)	(.0501)	(.0209)	(.0573)	(.0396)	(.0207)
5	.15431	.13801	-.10945	-.05262	-.13026	.26731
	(-)[a]	(.0466)	(.0246)	(.0396)	(.0639)	(.0378)

AR(4) Coefficient Estimates when the Fifth Equation is Deleted

	$u_{1(-4)}$	$u_{2(-4)}$	$u_{3(-4)}$	$u_{4(-4)}$
1	.88577	.41590	.46290	-.44695
	(.0755)	(.2546)	(.3021)	(.2144)
2	-.03451	.14146	-.53940	.15743
	(.0446)	(.1328)	(.1623)	(.1125)
3	.02052	.12192	.75847	.18963
	(.0284)	(.0919)	(.1098)	(.0777)
4	.04340	.17776	.45850	.56210
	(.0366)	(.1152)	(.1389)	(.0974)

AR(4) Coefficient Estimates when the First Equation is Deleted

	$u_{2(-4)}$	$u_{3(-4)}$	$u_{4(-4)}$	$u_{5(-4)}$
2	.17598	-.50488	.19193	.03451
	(.1264)	(.1545)	(.1128)	(.0446)
3	.10139	.73794	.16911	-.02052
	(.0874)	(.1053)	(.0784)	(.0284)
4	.13436	.41510	.51870	-.04340
	(.1097)	(.1331)	(.0979)	(.0366)
5	.05814	-.22529	.45298	.91518
	(.2153)	(.2614)	(.1921)	(.0672)

[a] See foonote (a) in Table 4.9.2.

Table 4.10
RDAM-AR(1,4) Model

Table 4.10.1 Unrestricted RDAM-AR(1,4) Model

	$dlnP_1$	$dlnP_2$	$dlnP_3$	$dlnP_4$	$dlnP_5$	dM
1	-.20633	-.04132	.05333	.03150	.20169	.48351
	(.0596)	(.1274)	(.0477)	(.1323)	(.0767)	(.0429)
2	.04480	-.27405	.06637	-.00756	.10072	.08511
	(.0325)	(.0692)	(.0254)	(.0677)	(.0453)	(.0225)
3	.05944	.04611	-.03841	.09462	-.12313	.04958
	(.0199)	(.0417)	(.0158)	(.0429)	(.0264)	(.0139)
4	.03435	.10711	-.01253	-.08432	-.05516	.14334
	(.0252)	(.0537)	(.0197)	(.0517)	(.0348)	(.0177)
5	.06774	.16213	-.06876	-.03424	-.12413	.23846
	(.0511)	(.1092)	(.0407)	(.1092)	(.0685)	(.0363)

AR(1,4) Coefficient Estimates when the Fifth Equation is Deleted

	$u_1(-1)$	$u_2(-1)$	$u_3(-1)$	$u_4(-1)$	$u_1(-4)$	$u_2(-4)$	$u_3(-4)$	$u_4(-4)$
1	-.28975	-.43097	-.92511	-.21953	.62273	.13764	.51118	-.53979
	(.1097)	(.2900)	(.4061)	(.2856)	(.1307)	(.2720)	(.3342)	(.2531)
2	-.11135	-.42469	-.10512	-.03211	-.10008	-.00837	-.78974	.04831
	(.0556)	(.1443)	(.2053)	(.1424)	(.0661)	(.1365)	(.1709)	(.1265)
3	.11821	-.07982	-.14187	-.03246	.06143	.21068	.67231	.19611
	(.0357)	(.0932)	(.1312)	(.0913)	(.0422)	(.0874)	(.1076)	(.0802)
4	-.08150	.21395	.50197	-.38021	-.00736	.07344	.50022	.38988
	(.0437)	(.1125)	(.1586)	(.1103)	(.0517)	(.1067)	(.1327)	(.0988)

AR(1,4) Coefficient Estimates when the First Equation is Deleted

	$u_2(-1)$	$u_3(-1)$	$u_4(-1)$	$u_5(-1)$	$u_2(-4)$	$u_3(-4)$	$u_4(-4)$	$u_5(-4)$
2	-.31332	.00624	.07925	.11134	.09171	-.68966	.14839	.10006
	(.1251)	(.1880)	(.1225)	(.0556)	(.1276)	(.1703)	(.1187)	(.0661)
3	-.19803	-.26008	-.15067	-.11822	.14925	.61088	.13468	-.06143
	(.0809)	(.1191)	(.0778)	(.0357)	(.0821)	(.1084)	(.0762)	(.0422)
4	.29546	.58347	-.29870	.08149	.08079	.50757	.39723	.00735
	(.0978)	(.1451)	(.0952)	(.0437)	(.0995)	(.1327)	(.0931)	(.0517)
5	.35716	.30579	.29994	-.36439	.16332	-.31726	.48221	.57672
	(.2155)	(.3158)	(.2087)	(.0943)	(.2179)	(.2891)	(.2058)	(.1119)

Table 4.10.2 Homogeneity Restricted RDAM-AR(1,4) Model

	$d\ln P_1$	$d\ln P_2$	$d\ln P_3$	$d\ln P_4$	$d\ln P_5$	dM
1	-.20343 (.0603)	-.04376 (.1168)	.05323 (.0451)	-.00022 (.1236)	.19418 (-)[a]	.48196 (.0427)
2	.03570 (.0334)	-.22238 (.0633)	.04681 (.0235)	.03259 (.0659)	.10727 (.0469)	.07051 (.0216)
3	.06089 (.0203)	.02311 (.0389)	-.02978 (.0149)	.07105 (.0395)	-.12528 (.0273)	.05440 (.0135)
4	.02715 (.0252)	.12220 (.0481)	-.01929 (.0179)	-.07651 (.0493)	-.05355 (.0351)	.13717 (.0171)
5	.07969 (-)[a]	.12083 (.0993)	-.05097 (.0379)	-.02691 (.1046)	-.12263 (.0689)	.25596 (.0358)

AR(1,4) Coefficient Estimates when the Fifth Equation is Deleted

	$u_1(-1)$	$u_2(-1)$	$u_3(-1)$	$u_4(-1)$	$u_1(-4)$	$u_2(-4)$	$u_3(-4)$	$u_4(-4)$
1	-.30266 (.1074)	-.47798 (.2916)	-1.02588 (.4013)	-.28306 (.2837)	.59515 (.1294)	.14538 (.2709)	.49444 (.3249)	-.53694 (.2459)
2	-.09624 (.0558)	-.37593 (.1491)	-.05208 (.2085)	.01515 (.1452)	-.07222 (.0670)	-.00752 (.1393)	-.70347 (.1685)	.05010 (.1265)
3	.11981 (.0354)	-.08653 (.0946)	-.14584 (.1317)	-.03024 (.0916)	.05632 (.0420)	.22657 (.0879)	.64833 (.1055)	.20943 (.0789)
4	-.07689 (.0431)	.22511 (.1139)	.51320 (.1584)	-.36969 (.1104)	.00048 (.0514)	.06500 (.1070)	.52142 (.1284)	.39040 (.0976)

AR(1,4) Coefficient Estimates when the First Equation is Deleted

	$u_2(-1)$	$u_3(-1)$	$u_4(-1)$	$u_5(-1)$	$u_2(-4)$	$u_3(-4)$	$u_4(-4)$	$u_5(-4)$
2	-.27969 (.1301)	.04416 (.1904)	.11139 (.1247)	.09624 (.0558)	.06470 (.1305)	-.63125 (.1703)	.12232 (.1183)	.07222 (.0670)
3	-.20634 (.0827)	-.26565 (.1190)	-.15005 (.0779)	-.11981 (.0354)	.17025 (.0830)	.59201 (.1076)	.15311 (.0749)	-.05632 (.0420)
4	.30200 (.0996)	.59009 (.1447)	-.29280 (.0953)	.07689 (.0431)	.06453 (.1001)	.52095 (.1301)	.38993 (.0919)	-.00048 (.0514)
5	.35936 (.2195)	.35463 (.3131)	.31186 (.2084)	-.35598 (.0928)	.15029 (.2192)	-.38100 (.2846)	.46674 (.2020)	.57972 (.1114)

[a] See foonote (a) in Table 4.9.2.

Table 4.10.3 Homogeneity and Symmetry Restricted RDAM-AR(1,4) Model

	$dlnP_1$	$dlnP_2$	$dlnP_3$	$dlnP_4$	$dlnP_5$	dY
1	-.19804 (.0600)	.02231 (.0326)	.06203 (.0191)	.02431 (.0253)	.08938 (-)[a]	.46515 (.0384)
2	.02231 (.0326)	-.27447 (.0504)	.05221 (.0203)	.10443 (.0379)	.09552 (.0412)	.06795 (.0214)
3	.06203 (.0191)	.05221 (.0203)	-.02420 (.0150)	-.00107 (.0154)	-.08897 (.0217)	.06257 (.0133)
4	.02431 (.0253)	.10443 (.0379)	-.00107 (.0154)	-.08213 (.0423)	-.04555 (.0327)	.14830 (.0165)
5	.08938 (-)[a]	.09552 (.0412)	-.08898 (.0217)	-.04555 (.0327)	-.05037 (.0649)	.25602 (.0337)

AR(1,4) Coefficient Estimates when the Fifth Equation is Deleted

	$u_{1(-1)}$	$u_{2(-1)}$	$u_{3(-1)}$	$u_{4(-1)}$	$u_{1(-4)}$	$u_{2(-4)}$	$u_{3(-4)}$	$u_{4(-4)}$
1	-.33451 (.1090)	-.41522 (.2919)	-1.05742 (.4063)	-.26016 (.2808)	.56889 (.1290)	.13578 (.2712)	.65604 (.3170)	-.53245 (.2423)
2	-.08628 (.0578)	-.32980 (.1526)	.03329 (.2164)	.06879 (.1465)	-.05721 (.0681)	-.03232 (.1412)	-.67365 (.1686)	.04689 (.1269)
3	.11058 (.0356)	-.12589 (.0953)	-.19030 (.1337)	-.08213 (.0904)	.04548 (.0420)	.23364 (.0877)	.60955 (.1033)	.20325 (.0784)
4	-.06386 (.0447)	.23719 (.1179)	.51041 (.1660)	-.35385 (.1141)	.00453 (.0531)	.08799 (.1092)	.47765 (.1298)	.42425 (.0987)

AR(1,4) Coefficient Estimates when the First Equation is Deleted

	$u_{2(-1)}$	$u_{3(-1)}$	$u_{4(-1)}$	$u_{5(-1)}$	$u_{2(-4)}$	$u_{3(-4)}$	$u_{4(-4)}$	$u_{5(-4)}$
2	-.24352 (.1313)	.11957 (.1955)	.15507 (.1244)	.08628 (.0578)	.02490 (.1301)	-.61644 (.1697)	.10411 (.1206)	.05721 (.0681)
3	-.23647 (.0826)	-.30089 (.1201)	-.19272 (.0767)	-.11058 (.0356)	.18815 (.0817)	.56407 (.1045)	.15777 (.0755)	-.04548 (.0420)
4	.30104 (.1021)	.57426 (.1499)	-.29000 (.0975)	.06386 (.0447)	.08346 (.1011)	.47312 (.1310)	.41971 (.0946)	-.00453 (.0531)
5	.25965 (.2174)	.32993 (.3152)	.25328 (.2045)	-.37406 (.0949)	.13661 (.2161)	-.50787 (.2802)	.41976 (.2011)	.56170 (.1115)

[a] See foonote (a) in Table 4.9.2.

Table 4.11
AIDS Model

Table 4.11.1 Unrestricted AIDS Model

	lnP_1	lnP_2	lnP_3	lnP_4	lnP_5	$ln\ \mu/P$	C
1	.04914	-.52468	.08368	.23162	.02199	.02819	.93703
	(.0577)	(.1581)	(.0688)	(.1059)	(.1298)	(.0434)	(.1904)
2	.02473	-.09411	.01803	.03498	.00818	.05364	-.22733
	(.0270)	(.0741)	(.0323)	(.0497)	(.0608)	(.0204)	(.0893)
3	-.01897	-.00419	.04636	.00728	-.02530	-.01586	.13851
	(.0206)	(.0565)	(.0246)	(.0378)	(.0464)	(.0155)	(.0680)
4	-.02318	.05962	-.01890	-.01139	-.01534	.07026	-.30608
	(.0191)	(.0524)	(.0228)	(.0351)	(.0430)	(.0144)	(.0631)
5	-.03173	.56335	-.12917	-.26248	.01046	-.13624	.45787
	(.0432)	(.1185)	(.0516)	(.0794)	(.0973)	(.0325)	(.1427)

Table 4.11.2 Homogeneity Restricted AIDS Model

	lnP_1	lnP_2	lnP_3	lnP_4	lnP_5	$ln\ \mu/P$	C
1	.00971	-.09488	-.08229	.28028	-.11282	-.09675	1.09083
	(.0629)	(.1238)	(.0594)	(.1167)	(.1386)	(.0319)	(.2065)
2	.02240	-.06867	.00821	.03786	.00020	.04625	-.21822
	(.0266)	(.0524)	(.0251)	(.0494)	(.0587)	(.0135)	(.0874)
3	-.01749	-.02030	.05258	.00545	-.02025	-.01118	.13275
	(.0203)	(.0399)	(.0191)	(.0376)	(.0447)	(.0103)	(.0666)
4	-.02580	.08817	-.02992	-.00816	-.02429	.06197	-.29587
	(.0189)	(.0371)	(.0178)	(.0350)	(.0416)	(.0096)	(.0620)
5	.01117	.09568	.05143	-.31543	.15715	-.00029	.29051
	(.0518)	(.1019)	(.0489)	(.0961)	(.1141)	(.0263)	(.1701)

Table 4.11.3 Homogeneity and Symmetry Restricted AIDS Model

	lnP_1	lnP_2	lnP_3	lnP_4	lnP_5	$ln\ \mu/P$	C
1	.00499	.02645	-.01394	-.01199	-.00551	-.10188	1.12624
	(.0348)	(.0189)	(.0126)	(.0131)	(.0360)	(.0290)	(.1885)
2	.02645	-.08774	-.00401	.09897	-.03367	.04665	-.22141
	(.0189)	(.0274)	(.0148)	(.0185)	(.0358)	(.0127)	(.0821)
3	-.01394	-.00401	.06243	-.02760	-.01688	-.01417	.15242
	(.0126)	(.0148)	(.0130)	(.0111)	(.0204)	(.0097)	(.0628)
4	-.01199	.09897	-.02760	.00685	-.06623	.05718	-.26507
	(.0131)	(.0185)	(.0111)	(.0201)	(.0267)	(.0087)	(.0566)
5	-.00551	-.03367	-.01688	-.06623	.12230	.01222	.20782
	(.0360)	(.0358)	(.0204)	(.0267)	(.0659)	(.0247)	(.1601)

Table 4.12
AIDS-SD Model

Table 4.12.1 Unrestricted AIDS-SD Model

	lnP_1	lnP_2	lnP_3	lnP_4	lnP_5	$ln\ \mu/P$	C
1	.08063	-.31198	.03752	.22691	-.10207	-.10734	1.52562
	(.0322)	(.0912)	(.0383)	(.0591)	(.0733)	(.0279)	(.1214)
2	.02454	-.20479	.02364	.06495	.04485	.12772	-.54045
	(.0138)	(.0390)	(.0164)	(.0253)	(.0313)	(.0119)	(.0519)
3	-.01909	.07341	.04842	-.02377	-.05020	-.05923	.31626
	(.0112)	(.0318)	(.0134)	(.0206)	(.0256)	(.0097)	(.0423)
4	-.02259	.05389	-.01217	-.02082	-.01401	.08500	-.37491
	(.0154)	(.0437)	(.0183)	(.0283)	(.0351)	(.0134)	(.0581)
5	-.06350	.38946	-.09741	-.24727	.12144	-.04614	.07348
	(.0290)	(.0820)	(.0344)	(.0531)	(.0659)	(.0251)	(.1091)

	D_1	D_2	D_3
1	-.06428	-.05475	-.03635
	(.0056)	(.0059)	(.0058)
2	.01159	.03284	.02256
	(.0024)	(.0025)	(.0025)
3	-.00043	-.01634	-.01868
	(.0020)	(.0021)	(.0020)
4	.00898	.01030	-.00220
	(.0027)	(.0028)	(.0028)
5	.04415	.02796	.03466
	(.0050)	(.0053)	(.0052)

Table 4.12.2 Homogeneity Restricted AIDS-SD Model

	lnP_1	lnP_2	lnP_3	lnP_4	lnP_5	$ln\ \mu/P$	C
1	.06590	-.09600	-.03783	.24237	-.17445	-.17815	1.66127
	(.0344)	(.0671)	(.0328)	(.0635)	(.0753)	(.0187)	(.1229)
2	.01455	-.05825	-.02748	.07543	-.00426	.07967	-.44841
	(.0162)	(.0316)	(.0155)	(.0299)	(.0355)	(.0088)	(.0579)
3	-.01295	-.01666	.07984	-.03022	-.02002	-.02970	.25969
	(.0124)	(.0241)	(.0118)	(.0228)	(.0271)	(.0067)	(.0442)
4	-.02594	.10302	-.02931	-.01730	-.03048	.06889	-.34406
	(.0156)	(.0303)	(.0148)	(.0287)	(.0340)	(.0085)	(.0555)
5	-.04157	.06788	.01478	-.27028	.22919	.05929	-.12849
	(.0345)	(.0674)	(.0329)	(.0638)	(.0756)	(.0188)	(.1233)

Table 4.12.2 (continued)

	D_1	D_2	D_3
1	-.06875 (.0059)	-.06081 (.0061)	-.04222 (.0059)
2	.00856 (.0028)	.02873 (.0029)	.01857 (.0028)
3	.00143 (.0021)	-.01382 (.0022)	-.01623 (.0021)
4	.00796 (.0026)	.00892 (.0027)	-.00353 (.0027)
5	.05079 (.0059)	.03697 (.0061)	.04340 (.0059)

Table 4.12.3 Homogeneity and Symmetry Restricted AIDS-SD Model

	$\ln P_1$	$\ln P_2$	$\ln P_3$	$\ln P_4$	$\ln P_5$	$\ln \mu/P$	C
1	.06191 (.0239)	.01992 (.0142)	-.01250 (.0087)	-.01311 (.0116)	-.05621 (.0283)	-.16743 (.0189)	1.59424 (.1245)
2	.01992 (.0142)	-.06055 (.0232)	-.02381 (.0109)	.09533 (.0171)	-.03089 (.0300)	.07575 (.0083)	-.42319 (.0543)
3	-.01250 (.0087)	-.02381 (.0109)	.07950 (.0092)	-.02606 (.0101)	-.01712 (.0162)	-.03182 (.0063)	.27375 (.0416)
4	-.01311 (.0116)	.09533 (.0171)	-.02606 (.0101)	.01844 (.0195)	-.07459 (.0240)	.05991 (.0079)	-.28557 (.0521)
5	-.05621 (.0283)	-.03089 (.0300)	-.01712 (.0162)	-.07459 (.0240)	.17882 (.0558)	.06360 (.0182)	-.15923 (.1196)

	D_1	D_2	D_3
1	-.06699 (.0072)	-.06012 (.0073)	-.04242 (.0071)
2	.00821 (.0028)	.02844 (.0029)	.01832 (.0028)
3	.00121 (.0021)	-.01414 (.0022)	-.01643 (.0021)
4	.00704 (.0028)	.00810 (.0029)	-.00421 (.0028)
5	.05053 (.0065)	.03773 (.0067)	.04473 (.0065)

Table 4.13
AIDS-AR(4) Model

Table 4.13.1 Unrestricted AIDS-AR(4) Model

	lnP_1	lnP_2	lnP_3	lnP_4	lnP_5	$ln\ \mu/P$	C
1	.00242 (.0368)	-.16001 (.0784)	.03831 (.0345)	.03287 (.0644)	.05247 (.0621)	-.08511 (.0281)	1.17703 (.1319)
2	.01641 (.0188)	-.15165 (.0434)	.04464 (.0184)	.00444 (.0337)	.04892 (.0345)	.09348 (.0149)	-.35149 (.0688)
3	.01207 (.0125)	.05384 (.0281)	.03465 (.0123)	.01764 (.0222)	-.09570 (.0223)	-.05685 (.0098)	.32517 (.0453)
4	.01015 (.0146)	.08837 (.0349)	-.02569 (.0152)	-.00642 (.0254)	-.07386 (.0282)	.04823 (.0119)	-.17109 (.0536)
5	-.04105 (.0314)	.16944 (.0694)	-.09192 (.0301)	-.04851 (.0562)	.06817 (.0547)	.00025 (.0246)	.02037 (.1144)

AR(4) Coefficient Estimates when the Fifth Equation is Deleted

	$u_1(-4)$	$u_2(-4)$	$u_3(-4)$	$u_4(-4)$
1	.67452 (.0969)	.05072 (.2767)	.43124 (.3233)	-.91044 (.2619)
2	-.01931 (.0490)	.31451 (.1341)	-.73347 (.1559)	.20200 (.1281)
3	.04524 (.0353)	.01726 (.1008)	.80932 (.1185)	.24727 (.0956)
4	.07423 (.0428)	.24987 (.1205)	.27207 (.1391)	.62252 (.1141)

AR(4) Coefficient Estimates when the First Equation is Deleted

	$u_2(-4)$	$u_3(-4)$	$u_4(-4)$	$u_5(-4)$
2	.33380 (.1185)	-.71420 (.1518)	.22134 (.1180)	.01932 (.0490)
3	-.02798 (.0898)	.76410 (.1161)	.20203 (.0898)	-.04525 (.0353)
4	.17562 (.1071)	.19782 (.1366)	.54830 (.1059)	-.07423 (.0428)
5	.14242 (.2098)	-.00437 (.2691)	.61326 (.2077)	.77468 (.0833)

Table 4.13.2 Homogeneity Restricted AIDS-AR(4) Model

	lnP_1	lnP_2	lnP_3	lnP_4	lnP_5	$ln\ \mu/P$	C
1	-.01434	-.06217	.01117	.01264	.05271	-.10438	1.14470
	(.0371)	(.0617)	(.0308)	(.0649)	(-)[a]	(.0202)	(.1292)
2	.01188	-.05580	.01381	-.00017	.03028	.05110	-.25220
	(.0200)	(.0334)	(.0167)	(.0355)	(.0366)	(.0112)	(.0716)
3	.01668	-.00455	.05439	.02261	-.08914	-.03597	.29442
	(.0136)	(.0230)	(.0115)	(.0240)	(.0239)	(.0075)	(.0478)
4	.00898	.10625	-.03048	-.00605	-.07871	.03530	-.12415
	(.0145)	(.0244)	(.0127)	(.0254)	(.0277)	(.0080)	(.0514)
5	-.02320	.01627	-.04889	-.02903	.08485	.05396	-.06277
	(-)[a]	(.0553)	(.0277)	(.0590)	(.0571)	(.0183)	(.1171)

AR(4) Coefficient Estimates when the Fifth Equation is Deleted

	$u_1(-4)$	$u_2(-4)$	$u_3(-4)$	$u_4(-4)$
1	.69068	-.06600	.42532	-.92223
	(.0888)	(.2790)	(.3266)	(.2588)
2	.01119	.29740	-.72594	.18100
	(.0468)	(.1430)	(.1668)	(.1341)
3	.03105	.04430	.79926	.26573
	(.0338)	(.1062)	(.1246)	(.0982)
4	.08710	.28298	.26695	.62813
	(.0374)	(.1173)	(.1378)	(.1103)

AR(4) Coefficient Estimates when the First Equation is Deleted

	$u_2(-4)$	$u_3(-4)$	$u_4(-4)$	$u_5(-4)$
2	.28620	-.73714	.16981	-.01119
	(.1295)	(.1632)	(.1266)	(.0468)
3	.01325	.76821	.23468	-.03105
	(.0969)	(.1222)	(.0941)	(.0338)
4	.19587	.17984	.54103	-.08710
	(.1076)	(.1357)	(.1046)	(.0374)
5	.26135	.05445	.66738	.82002
	(.2238)	(.2813)	(.2169)	(.0788)

[a] See foonote (a) in Table 4.9.2.

Table 4.13.3 Homogeneity and Symmetry Restricted AIDS-AR(4) Model

	lnP_1	lnP_2	lnP_3	lnP_4	lnP_5	$ln\ \bar{\mu}/P$	C
1	.02568	.00113	.00630	-.00986	-.02325	-.10731	1.16650
	(.0307)	(.0172)	(.0120)	(.0113)	(-)[a]	(.0191)	(.1220)
2	.00113	-.10850	.00819	.07849	.02069	.05565	-.28400
	(.0172)	(.0245)	(.0125)	(.0171)	(.0299)	(.0105)	(.0671)
3	.00630	.00819	.06282	-.02023	-.05709	-.03582	.29493
	(.0120)	(.0125)	(.0107)	(.0101)	(.0168)	(.0072)	(.0455)
4	-.00986	.07849	-.02023	.00694	-.05534	.03728	-.13730
	(.0113)	(.0171)	(.0101)	(.0195)	(.0224)	(.0076)	(.0489)
5	-.02325	.02069	-.05709	-.05534	.11499	.05020	-.04015
	(-)[a]	(.0299)	(.0168)	(.0224)	(.0522)	(.0174)	(.1107)

AR(4) Coefficient Estimates when the Fifth Equation is Deleted

	$u_1(-4)$	$u_2(-4)$	$u_3(-4)$	$u_4(-4)$
1	.67269	-.08474	.45892	-.85924
	(.0926)	(.2880)	(.3244)	(.2618)
2	.01880	.33067	-.67050	.17822
	(.0501)	(.1515)	(.1713)	(.1399)
3	.01402	-.03059	.75012	.24990
	(.0357)	(.1107)	(.1270)	(.1015)
4	.08431	.25690	.24934	.62768
	(.0392)	(.1211)	(.1375)	(.1110)

AR(4) Coefficient Estimates when the First Equation is Deleted

	$u_2(-4)$	$u_3(-4)$	$u_4(-4)$	$u_5(-4)$
2	.31183	-.68931	.15945	-.01879
	(.1352)	(.1683)	(.1314)	(.0501)
3	-.04458	.73610	.23587	-.01403
	(.0996)	(.1251)	(.0963)	(.0357)
4	.17257	.16502	.54338	-.08430
	(.1087)	(.1354)	(.1045)	(.0392)
5	.31762	.00197	.59321	.78980
	(.2223)	(.2750)	(.2143)	(.0805)

[a] See foonote (a) in Table 4.9.2.

Table 4.14
AIDS-AR(1,4) Model

Table 4.14.1 Unrestricted AIDS-AR(1,4) Model

	lnP_1	lnP_2	lnP_3	lnP_4	lnP_5	$ln\ \mu/P$	C
1	.02570	−.09923	.00934	.00681	.02486	−.09224	1.21862
	(.0314)	(.0702)	(.0299)	(.0587)	(.0590)	(.0261)	(.1224)
2	.00621	−.20219	.04610	.03214	.07618	.10240	−.38542
	(.0179)	(.0420)	(.0176)	(.0332)	(.0365)	(.0149)	(.0699)
3	.00769	.06123	.03550	−.00629	−.07000	−.06551	.35210
	(.0119)	(.0269)	(.0115)	(.0240)	(.0223)	(.0096)	(.0451)
4	.02851	.05460	−.03712	.06511	−.11730	.04087	−.12982
	(.0169)	(.0377)	(.0166)	(.0324)	(.0326)	(.0137)	(.0632)
5	−.06810	.18561	−.05382	−.09780	.08627	.01447	−.05548
	(.0294)	(.0656)	(.0288)	(.0529)	(.0568)	(.0240)	(.1140)

AR(1,4) Coefficient Estimates when the Fifth Equation is Deleted

	$u_1(-1)$	$u_2(-1)$	$u_3(-1)$	$u_4(-1)$	$u_1(-4)$	$u_2(-4)$	$u_3(-4)$	$u_4(-4)$
1	−.19244	−.05510	−.87679	−.52380	.44194	.29945	.96421	−.85596
	(.1006)	(.2471)	(.3205)	(.2225)	(.1004)	(.2792)	(.3194)	(.2360)
2	.02140	.00990	.47743	.15314	.04493	.11632	−.89943	.02167
	(.0560)	(.1340)	(.1738)	(.1215)	(.0555)	(.1518)	(.1790)	(.1294)
3	.12305	.04631	.05591	.06671	.10367	.06921	.65140	.28003
	(.0406)	(.1021)	(.1345)	(.0937)	(.0408)	(.1144)	(.1318)	(.0990)
4	.00500	.15449	.43701	.12873	.10775	.15196	.34486	.50930
	(.0509)	(.1245)	(.1597)	(.1131)	(.0510)	(.1427)	(.1586)	(.1191)

AR(1,4) Coefficient Estimates when the First Equation is Deleted

	$u_2(-1)$	$u_3(-1)$	$u_4(-1)$	$u_5(-1)$	$u_2(-4)$	$u_3(-4)$	$u_4(-4)$	$u_5(-4)$
2	−.01148	.45609	.13178	−.02138	.07136	−.94433	−.02329	−.04493
	(.1278)	(.1850)	(.1219)	(.0560)	(.1417)	(.1892)	(.1191)	(.0555)
3	−.07674	−.06715	−.05635	−.12306	−.03446	.54772	.17636	−.10367
	(.0984)	(.1425)	(.0922)	(.0406)	(.1087)	(.1416)	(.0922)	(.0408)
4	.14950	.43205	.12375	−.00499	.04420	.23713	.40154	−.10775
	(.1191)	(.1706)	(.1127)	(.0509)	(.1338)	(.1709)	(.1093)	(.0510)
5	−.19862	−.13668	.13215	−.04303	.06144	−.36280	.74332	.69827
	(.2252)	(.3201)	(.2094)	(.0949)	(.2524)	(.3259)	(.2087)	(.0952)

Table 4.14.2 Homogeneity Restricted AIDS-AR(1,4) Model

	lnP_1	lnP_2	lnP_3	lnP_4	lnP_5	$ln\ \mu/P$	C
1	.01900	-.01851	-.01020	.01692	-.00721	-.12399	1.27024
	(.0328)	(.0524)	(.0284)	(.0574)	(-)[a]	(.0189)	(.1216)
2	-.01527	-.06720	.03662	-.02566	.07151	.05794	-.29584
	(.0208)	(.0327)	(.0168)	(.0354)	(.0375)	(.0118)	(.0758)
3	.01705	-.02600	.04820	.01651	-.05576	-.04087	.32525
	(.0136)	(.0231)	(.0115)	(.0255)	(.0254)	(.0078)	(.0502)
4	.02097	.09958	-.03268	.02507	-.11294	.02818	-.07888
	(.0165)	(.0272)	(.0142)	(.0297)	(.0312)	(.0093)	(.0595)
5	-.04175	.01212	-.04194	-.03284	.10441	.07874	-.22080
	(-)[a]	(.0511)	(.0275)	(.0566)	(.0593)	(.0182)	(.1168)

AR(1,4) Coefficient Estimates when the Fifth Equation is Deleted

	$u_{1(-1)}$	$u_{2(-1)}$	$u_{3(-1)}$	$u_{4(-1)}$	$u_{1(-4)}$	$u_{2(-4)}$	$u_{3(-4)}$	$u_{4(-4)}$
1	-.13672	.02031	-.97182	-.55001	.51225	.49295	1.03784	-.71557
	(.0819)	(.2473)	(.3265)	(.2294)	(.0896)	(.2531)	(.3362)	(.2272)
2	.10018	.14662	.29527	.16499	.03193	.15938	-.92538	.10621
	(.0458)	(.1415)	(.1848)	(.1287)	(.0515)	(.1439)	(.1911)	(.1273)
3	.07411	-.07638	.13495	.03988	.10688	.04863	.59940	.25400
	(.0331)	(.1059)	(.1393)	(.0980)	(.0379)	(.1073)	(.1406)	(.0957)
4	.03868	.17939	.32990	.09789	.10775	.21196	.30383	.56597
	(.0386)	(.1190)	(.1565)	(.1109)	(.0439)	(.1211)	(.1605)	(.1093)

AR(1,4) Coefficient Estimates when the First Equation is Deleted

	$u_{2(-1)}$	$u_{3(-1)}$	$u_{4(-1)}$	$u_{5(-1)}$	$u_{2(-4)}$	$u_{3(-4)}$	$u_{4(-4)}$	$u_{5(-4)}$
2	.04645	.19505	.06478	-.10020	.12749	-.9573	.0743	-.03194
	(.1416)	(.1868)	(.1257)	(.0458)	(.1435)	(.2038)	(.1282)	(.0515)
3	-.15050	.06087	-.03422	-.07410	-.05827	.4925	.1471	-.10687
	(.1075)	(.1420)	(.0954)	(.0331)	(.1084)	(.1515)	(.0957)	(.0379)
4	.14071	.29121	.05921	-.03868	.10422	.1961	.4582	-.10775
	(.1203)	(.1590)	(.1085)	(.0386)	(.1220)	(.1734)	(.1084)	(.0439)
5	-.19370	.28800	.32352	.07626	-.15416	-.2569	.5482	.75881
	(.2471)	(.3256)	(.2208)	(.0775)	(.2533)	(.3528)	(.2218)	(.0868)

[a] See foonote (a) in Table 4.9.2.

Table 4.14.3 Homogeneity and Symmetry Restricted AIDS-AR(1,4) Model

	lnP_1	lnP_2	lnP_3	lnP_4	lnP_5	$ln\ \mu/P$	C
1	.02470	-.00064	.00175	.01120	-.03701	-.12010	1.24619
	(.0281)	(.0164)	(.0121)	(.0134)	(-)[a]	(.0192)	(.1205)
2	-.00064	-.11541	.00392	.06597	.04616	.05228	-.25879
	(.0164)	(.0245)	(.0123)	(.0165)	(.0319)	(.0114)	(.0730)
3	.00175	.00392	.07144	-.03828	-.03883	-.02993	.25388
	(.0121)	(.0123)	(.0100)	(.0110)	(.0180)	(.0077)	(.0493)
4	.01120	.06597	-.03828	.04912	-.08801	.02939	-.08729
	(.0134)	(.0165)	(.0110)	(.0191)	(.0227)	(.0089)	(.0566)
5	-.03701	.04616	-.03883	-.08801	.11769	.06836	-.15399
	(-)[a]	(.0319)	(.0180)	(.0227)	(.0554)	(.0162)	(.1043)

AR(1,4) Coefficient Estimates when the Fifth Equation is Deleted

	$u_1(-1)$	$u_2(-1)$	$u_3(-1)$	$u_4(-1)$	$u_1(-4)$	$u_2(-4)$	$u_3(-4)$	$u_4(-4)$
1	-.12141	.11228	-.96600	-.53388	.46912	.49337	1.11011	-.70021
	(.0737)	(.2427)	(.3107)	(.2295)	(.0939)	(.2396)	(.3080)	(.2099)
2	.13541	.12513	.20178	.20101	.08324	.41600	-.87707	.26379
	(.0439)	(.1392)	(.1680)	(.1295)	(.0531)	(.1324)	(.1744)	(.1179)
3	.02133	-.07584	.28475	.03236	.07459	-.19020	.59767	.07372
	(.0298)	(.1059)	(.1249)	(.0990)	(.0390)	(.0970)	(.1318)	(.0893)
4	.05592	.15348	.28759	.11891	.12806	.27003	.27037	.64982
	(.0348)	(.1161)	(.1433)	(.1096)	(.0448)	(.1149)	(.1494)	(.0998)

AR(1,4) Coefficient Estimates when the First Equation is Deleted

	$u_2(-1)$	$u_3(-1)$	$u_4(-1)$	$u_5(-1)$	$u_2(-4)$	$u_3(-4)$	$u_4(-4)$	$u_5(-4)$
2	.16117	.49114	.17710	-.10213	.12225	-.75500	-.10324	-.07934
	(.1404)	(.1828)	(.1345)	(.0520)	(.1498)	(.1967)	(.1152)	(.0553)
3	-.21423	-.10369	-.08689	-.07247	-.04989	.41127	.22956	-.06688
	(.1080)	(.1417)	(.1045)	(.0389)	(.1150)	(.1493)	(.0906)	(.0415)
4	.13516	.30621	.10650	-.04830	.10264	.20172	.45615	-.12992
	(.1175)	(.1538)	(.1160)	(.0421)	(.1251)	(.1646)	(.0994)	(.0456)
5	-.31715	-.10376	.13352	.01530	.08923	-.39529	.66853	.76142
	(.2373)	(.3077)	(.2269)	(.0828)	(.2513)	(.3272)	(.1967)	(.0877)

[a] See foonote (a) in Table 4.9.2.

Table 4.15
Coefficient of Determination: The Adjusted R^2

Model	Commodity sector				
	1	2	3	4	5
LES-W	.2756	.5699	.0063	.2113	-.1628
LES-W-SD	.7505	.7986	.4051	.3530	.3453
Unrestricted Model					
RDAM	.7686	.0096	.4528	.4056	.1149
RDAM-D4	.8659	.6765	.3940	.8096	.8071
RDAM-SD	.9159	.5208	.8268	.6185	.7143
RDAM-AR(4)	.9154	.3321	.7558	.6862	.6711
RDAM-AR(1,4)	.9253	.4503	.8286	.8008	.7435
AIDS	.4954	.6494	.1341	.4564	.4539
AIDS-SD	.8370	.9059	.7331	.6330	.7460
AIDS-AR(4)	.8205	.8697	.6746	.6630	.7489
AIDS-AR(1,4)	.8752	.8860	.6990	.6898	.7740
Homogeneity Restricted Model					
RDAM	.7664	.0052	.4494	.4092	.1084
RDAM-D4	.8607	.6231	.2153	.8465	.7808
RDAM-SD	.9099	.5066	.8215	.6001	.7071
RDAM-AR(4)	.9134	.2987	.7480	.6817	.6703
RDAM-AR(1,4)	.9250	.4227	.8262	.7961	.7426
AIDS	.3793	.6481	.1319	.4513	.1888
AIDS-SD	.8102	.8669	.6706	.6194	.6318
AIDS-AR(4)	.8125	.8460	.6289	.6636	.7197
AIDS-AR(1,4)	.8649	.8636	.6569	.6886	.7419
Homogeneity and Symmetry Restricted Model					
RDAM	.7598	.0041	.4430	.3919	.0668
RDAM-D4	.8466	.6223	.1813	.8459	.7694
RDAM-SD	.9003	.4980	.8182	.5935	.6830
RDAM-AR(4)	.9112	.2729	.7379	.6686	.6668
RDAM-AR(1,4)	.9230	.3983	.8225	.7814	.7397
AIDS	.2895	.6332	.1190	.4349	.0847
AIDS-SD	.7134	.8619	.6688	.5760	.5449
AIDS-AR(4)	.8058	.8306	.6050	.6598	.7230
AIDS-AR(1,4)	.8569	.8536	.6260	.6819	.7520

Table 4.16
Average Information Inaccuracies x 10^{-4}

Model	Time Period				
	66-81	66-69	70-73	74-77	78-81
LES-W	56.01	89.85	38.84	43.99	51.36
LES-W-SD	29.62	56.96	26.80	10.50	24.22
Unrestricted Model					
RDAM	57.35	80.91	34.31	52.24	63.42
RDAM-D4	22.99	56.87	15.83	11.52	9.85
RDAM-SD	19.28	31.98	21.54	9.12	15.26
AIDS	36.15	47.88	22.29	32.63	41.78
AIDS-SD	12.64	20.68	13.95	7.40	8.53
Homogeneity Restricted Model					
RDAM	60.70	92.34	34.64	55.10	62.70
RDAM-D4	26.76	62.36	18.17	10.97	17.76
RDAM-SD	20.23	34.33	21.87	8.79	16.82
AIDS	41.31	57.11	25.86	38.09	44.17
AIDS-SD	16.08	24.11	17.73	8.69	13.80
Homogeneity and Symmetry Restricted Model					
RDAM	62.16	95.27	35.39	57.97	62.08
RDAM-D4	27.13	64.07	18.44	11.61	16.72
RDAM-SD	21.38	38.67	21.17	10.07	16.70
AIDS	44.68	65.37	29.75	41.56	42.04
AIDS-SD	18.91	30.95	22.00	9.53	13.16

Table 4.17
Durbin-Watson *DW* statistic[a]

Model	Commodity sector				
	1	2	3	4	5
LES-W	1.918	1.412	1.229	1.198	1.711
LES-W-SD	1.148	.815	.570	.986	.590
Unrestricted Model					
RDAM	2.366	2.038	1.879	2.585	3.045
RDAM-D4	1.741	1.824	1.320	1.804	1.360
RDAM-SD	2.243	2.674	2.211	2.797	2.511
RDAM-AR(4)	2.372	2.291	2.146	2.842	2.730
RDAM-AR(1,4)	2.195	1.980	2.149	2.043	2.130
AIDS	1.833	1.557	1.593	1.906	2.228
AIDS-SD	1.625	2.096	1.360	1.902	1.367
AIDS-AR(4)	1.750	1.811	1.604	1.733	1.650
AIDS-AR(1,4)	1.956	1.886	1.670	1.618	1.943
Homogeneity Restricted Model					
RDAM	2.376	2.090	1.894	2.586	3.051
RDAM-D4	1.850	1.452	1.105	1.765	1.413
RDAM-SD	2.331	2.628	2.150	2.794	2.554
RDAM-AR(4)	2.338	2.199	2.111	2.790	2.757
RDAM-AR(1,4)	2.184	1.910	2.133	2.025	2.160
AIDS	1.970	1.506	1.569	1.862	2.081
AIDS-SD	1.624	1.375	1.242	1.772	1.081
AIDS-AR(4)	1.752	1.415	1.386	1.772	1.607
AIDS-AR(1,4)	1.923	1.630	1.514	1.719	2.034
Homogeneity and Symmetry Restricted Model					
RDAM	2.383	2.059	1.852	2.643	3.081
RDAM-D4	1.876	1.440	1.063	1.741	1.459
RDAM-SD	2.351	2.656	2.137	2.761	2.599
RDAM-AR(4)	2.376	2.157	2.061	2.759	2.759
RDAM-AR(1,4)	2.147	1.910	2.054	2.097	2.135
AIDS	1.756	1.512	1.612	1.800	1.917
AIDS-SD	1.060	1.298	1.240	1.563	.867
AIDS-AR(4)	1.757	1.385	1.403	1.738	1.596
AIDS-AR(1,4)	1.958	1.757	1.486	1.720	1.997

[a] Critical points of *DW* statistic at 5 %, when n = 65:

d_l = 1.438, d_u = 1.767, for LES-W with k' = 5.

d_l = 1.404, d_u = 1.805, for RDAM, RDAM-D4, RDAM-AR(4), RDAM-AR(1,4), AIDS, AIDS-AR(4), AIDS-AR(1,4) with k' = 6.

d_l = 1.336, d_u = 1.882, for LES-W-SD with k' = 8.

d_l = 1.301, d_u = 1.923, for RDAM-SD and AIDS-SD with k' = 9.

The critical points of *DW* statistics at 5 % for k' ∈ [1,20] and n ∈ [6,200] are avaiable in *Statistical Tables for Students,* Department of Statistics, Australian National University.

Table 4.18
Durbin Watson $\mathcal{DW}$ test statistic[a]

Model	Commodity sector				
	1	2	3	4	5
LES-W	.237	.572	.524	.802	.321
LES-W-SD	.781	1.299	.935	.967	.637
Unrestricted Model					
RDAM	.558	1.318	.729	.745	.549
RDAM-D4	1.794	2.547	2.271	2.096	2.032
RDAM-SD	1.031	1.941	1.182	1.127	1.086
RDAM-AR(4)	1.709	1.820	1.708	1.759	1.828
RDAM-AR(1,4)	1.630	1.875	1.923	1.548	1.703
AIDS	.475	.612	.444	.747	.781
AIDS-SD	1.109	2.082	1.139	1.189	1.223
AIDS-AR(4)	1.593	1.864	1.659	1.786	1.705
AIDS-AR(1,4)	1.668	1.732	1.752	1.865	1.793
Homogeneity Restricted Model					
RDAM	.517	1.242	.687	.743	.507
RDAM-D4	1.842	2.497	2.155	2.046	2.064
RDAM-SD	.951	1.926	1.205	1.123	1.030
RDAM-AR(4)	1.718	1.849	1.672	1.748	1.814
RDAM-AR(1,4)	1.611	1.900	1.896	1.561	1.688
AIDS	.384	.645	.473	.746	.574
AIDS-SD	1.126	1.806	1.266	1.128	1.079
AIDS-AR(4)	1.653	1.880	1.686	1.807	1.692
AIDS-AR(1,4)	1.609	1.754	1.795	1.826	1.772
Homogeneity and Symmetry Restricted Model					
RDAM	.516	1.283	.664	.694	.471
RDAM-D4	1.959	2.478	2.109	2.074	2.081
RDAM-SD	1.009	1.916	1.233	1.075	1.090
RDAM-AR(4)	1.756	1.805	1.580	1.656	1.805
RDAM-AR(1,4)	1.683	1.785	1.807	1.573	1.629
AIDS	.312	.659	.454	.683	.413
AIDS-SD	.706	1.777	1.275	.970	.766
AIDS-AR(4)	1.743	1.800	1.742	1.770	1.652
AIDS-AR(1,4)	1.693	1.606	1.642	1.979	1.650

[a] Since critical points of $\mathcal{DW}$ statistic for k' > 5 and k'' > 5 are not avaiable, critical points for k' = 5 and k'' = 5 when n = 64 are taken for approximates. Note that k' is the number of regressors excluding the constant in the model without the seasonal dummy, and k'' is the number of regressors excluding the constant and the seasonal dummies in the model with seasonal dummies.
Critical points at 5 % for model without seasonal dummy variable:
d_l = 1.337, d_u = 1.664, for k' = 5
Critical points at 5 % for model with seasonal dummy variable:
d_l = 1.429, d_u = 1.776, for k'' = 5.

Table 4.19
Tests of Significance of Seasonal Dummies and AR Structures[a]

Zero Restriction on	Model	LR testing statistic Unrest.	H only	H & S	(5% χ^2-C.V.)
Seasonal Dummy					(26.30)
	LES-W-SD	121.59			
	RDAM-SD	216.47	211.92	208.26	
	AIDS-SD	199.80	183.94	164.89	
AR(4)					(26.30)
	RDAM-AR(4)	166.56	166.22	162.73	
	AIDS-AR(4)	168.22	178.01	179.68	
AR(1,4)					(46.19)
	RDAM-AR(1,4)	244.76	244.63	240.85	
	AIDS-AR(1,4)	210.05	219.47	222.83	
AR(1) subject to AR(1,4)					(26.30)
	RDAM-AR(1,4)	78.20	78.41	78.12	
	AIDS-AR(1,4)	41.83	41.46	43.15	

[a] Test statistics are shwon to be invariant to the equation deleted.

Table 4.20
Maximum and Minimum Absolute Values of the Eigenvalues of the Reduced Companion Matrix of the Autoregressive Error Process[a]

Model		Maximum	Minimum
RDAM-AR(4)	Unrest	.9738	.6085
	H only	.9732	.6318
	H & S	.9732	.6481
RDAM-AR(1,4)	Unrest	.9761	.5822
	H only	.9807	.5904
	H & S	.9839	.5957
AIDS-AR(4)	Unrest	.9672	.5651
	H only	.9643	.5507
	H & S	.9692	.5289
AIDS-AR(1,4)	Unrest	.9849	.4041
	H only	.9691	.5159
	H & S	.9540	.4524

[a] The eigenvalues are shown to be invariant to the equation deleted.

Table 4.21
Direct Tests of Demand Restrictions

Model	Restriction	Testing statistic [a] W	LR	LM	T^2	C.V. of T^2 at 5 %
RDAM	H	3.806	3.695	3.589	3.443	(10.737)
	S	5.183	5.060	4.941	-	
	H & S	9.110	8.755	8.419	-	
RDAM-D4	H	50.494	37.083	28.029	45.685	(10.737)
	S	6.429	6.169	5.923	-	
	H & S	58.423	43.252	33.065	-	
RDAM-SD	H	8.807	8.244	7.727	7.409	(10.842)
	S	9.087	8.726	8.388	-	
	H & S	18.396	16.969	15.693	-	
RDAM-AR(4)	H	4.200	4.037	3.881	-	
	S	8.766	8.556	8.352	-	
	H & S	13.047	12.593	12.163	-	
RDAM-AR(1,4)	H	3.943	3.821	3.703	-	
	S	9.157	8.850	8.556	-	
	H & S	13.278	12.671	12.103	-	
AIDS	H	36.106	28.630	23.084	32.157	(10.737)
	S	11.048	10.334	9.685	-	
	H & S	49.869	38.964	30.995	-	
AIDS-SD	H	64.693	44.707	32.172	54.584	(10.814)
	S	36.463	29.472	24.232	-	
	H & S	103.549	74.179	55.137	-	
AIDS-AR(4)	H	21.762	18.838	16.420	-	
	S	9.504	8.984	8.494	-	
	H & S	32.933	27.822	23.763	-	
AIDS-AR(1,4)	H	22.618	19.209	16.396	-	
	S	7.909	7.292	6.696	-	
	H & S	31.807	26.501	22.310	-	

[a] Critical Value of x^2 at 5 %: 9.492 for homogeneity restrictions (H), 12.596 for symmetry restrcitions (S), 18.311 for both restrcitions (H & S).

Table 4.22

Separate Induced Tests of Demand Restrictions

Table 4.22.1

Bonferroni and Scheffe Confidence Intervals at 5% Level

Confidence	Hypothesis to be tested		
interval	H only	S only	H & S
Bonferroni	2.848	2.995	3.164
Scheffe	3.081	3.549	4.279

Table 4.22.2

The Wald-Type Separate Tests (t-ratios)

Individual	Model				
Restriction	RDAM	RDAM-D4	RDAM-SD	AIDS	AIDS-SD
Indivudual Symmetry					
$\beta_{12} = \beta_{21}$	-1.328	-2.754	-1.287	-2.824	-2.738
$\beta_{13} = \beta_{31}$	1.251	.516	.645	1.223	1.075
$\beta_{14} = \beta_{41}$	.101	1.773	1.737	2.059	3.218
$\beta_{15} = \beta_{51}$	.207	.793	1.467	.442	-.497
$\beta_{23} = \beta_{32}$	.788	-2.092	-.384	.365	-1.288
$\beta_{24} = \beta_{42}$	.809	-.193	-.317	-.318	.188
$\beta_{25} = \beta_{52}$	-1.916	-2.545	-1.679	-3.184	-3.162
$\beta_{34} = \beta_{43}$	1.019	-1.147	1.336	.573	-.393
$\beta_{35} = \beta_{53}$	.731	.371	.102	1.273	.872
$\beta_{45} = \beta_{54}$	.848	1.018	1.492	2.224	2.884
Individual Homogeneity					
$\sum\beta_{1j} = 0$	-1.280	-2.130	1.943	-3.235	-2.444
$\sum\beta_{2j} = 0$	1.177	-3.457	-1.333	-.474	-4.248
$\sum\beta_{3j} = 0$	-.683	3.908	1.347	.399	3.478
$\sum\beta_{4j} = 0$	-.528	-1.150	-1.692	-.755	-1.432
$\sum\beta_{5j} = 0$	1.383	3.499	-1.199	4.305	4.008

Table 4.22.3

The LM-Type Separate Test (t-ratios) [a]

Individual Restriction	Model				
	RDAM	RDAM-D4	RDAM-SD	AIDS	AIDS-SD
Individual Symmetry					
$\beta_{12} = \beta_{21}$	-.741	-.641	-1.80	1.094	2.016
$\beta_{13} = \beta_{31}$	.469	-.167	.238	.935	2.818
$\beta_{14} = \beta_{41}$	.367	.637	-.169	2.678	4.707
$\beta_{23} = \beta_{32}$	.191	-.826	-.389	-.619	-1.590
$\beta_{24} = \beta_{42}$	1.272	-.392	.717	.028	-1.393
$\beta_{34} = \beta_{43}$	1.186	-.258	1.278	1.006	1.041
Individual Homogeneity					
$\sum\beta_{1j} = 0$	-.693	-1.745	1.740	-3.408	-2.023
$\sum\beta_{2j} = 0$	.352	-3.249	-.866	-1.910	-3.603
$\sum\beta_{3j} = 0$	-.529	.933	.296	-.972	-.630
$\sum\beta_{4j} = 0$	-.593	-1.264	-.998	-2.154	-2.436

[a] T-ratios of Lagrange multiplier estimates are obtained from the reduced subsystem with the fifth equation deleted.

Table 4.23
The Slutsky Matrix Implied by the LES and AIDS Models

Table 4.23.1 L.E.S. Model

Model		Commodity sector				
		1	2	3	4	5
LES-W						
	1	-.16394	.03546	.01459	.03196	.08193
	2	.03546	-.09492	.00675	.01479	.03791
	3	.01459	.00675	-.04302	.00609	.01560
	4	.03196	.01479	.00609	-.08702	.03417
	5	.08193	.03791	.01560	.03417	-.16961
LES-W-SD						
	1	-.17071	.03562	.01705	.03549	.08254
	2	.03562	-.09100	.00699	.01455	.03384
	3	.01705	.00699	-.04720	.00696	.01619
	4	.03549	.01455	.00696	-.09073	.03372
	5	.08254	.03384	.01619	.03372	-.16630

Table 4.23.2 A.I.D.S. model[a]

Model & Equation		Commodity sector					Marginal budget share
		1	2	3	4	5	
AIDS							
	1	-.17722	.03548	.02505	-.00017	.11686	.35551
		(.0347)	(.0189)	(.0125)	(.0131)	(.0359)	(.0290)
	2	.03548	-.15234	-.00262	.12509	-.00561	.13244
		(.0189)	(.0274)	(.0148)	(.0185)	(.0358)	(.0127)
	3	.02505	-.00262	.00277	-.02579	.00059	.05102
		(.0125)	(.0148)	(.0130)	(.0111)	(.0204)	(.0097)
	4	-.00017	.12509	-.02579	-.06775	-.03137	.16397
		(.0131)	(.0185)	(.0111)	(.0200)	(.0267)	(.0087)
	5	.11686	-.00561	.00059	-.03137	-.08046	.29706
		(.0359)	(.0358)	(.0204)	(.0267)	(.0659)	(.0247)

Table 4.23.2 (continued)

AIDS-SD							
	1	-.00810	-.02145	.05118	-.02802	.00639	.28996
		(.0239)	(.0142)	(.0087)	(.0116)	(.0283)	(.0189)
	2	-.02145	-.10251	-.03354	.13334	.02417	.16154
		(.0142)	(.0232)	(.0109)	(.0171)	(.0300)	(.0083)
	3	.05118	-.03354	.02500	-.03122	-.01141	.03337
		(.0087)	(.0109)	(.0092)	(.0101)	(.0162)	(.0063)
	4	-.02802	.13334	-.03122	-.05413	-.01996	.16670
		(.0116)	(.0171)	(.0101)	(.0195)	(.0240)	(.0079)
	5	.00639	.02417	-.01141	-.01996	.00082	.34844
		(.0283)	(.0300)	(.0162)	(.0240)	(.0558)	(.0182)
AIDS-AR(4)							
	1	-.14931	.00242	.06055	.01356	.07280	.35008
		(.0307)	(.0172)	(.0120)	(.0112)	(-)[b]	(.0191)
	2	.00242	-.16725	.00111	.10084	.06288	.14144
		(.0172)	(.0245)	(.0125)	(.0171)	(.0299)	(.0105)
	3	.06055	.00111	.01005	-.02176	-.04995	.02937
		(.0120)	(.0125)	(.0107)	(.0100)	(.0168)	(.0072)
	4	.01356	.10084	-.02176	-.07961	-.01303	.14407
		(.0112)	(.0171)	(.0100)	(.0194)	(.0224)	(.0076)
	5	.07281	.06288	-.04995	-.01303	-.07270	.33504
		(-)[b]	(.0299)	(.0168)	(.0224)	(.0522)	(.0173)
AIDS-AR(1,4)							
	1	-.13181	-.00131	.05441	.03761	.04109	.33729
		(.0281)	(.0164)	(.0121)	(.0134)	(-)[b]	(.0192)
	2	-.00131	-.17647	-.00043	.08490	.09331	.13807
		(.0164)	(.0245)	(.0123)	(.0164)	(.0319)	(.0113)
	3	.05441	-.00043	.01620	-.03691	-.03327	.03526
		(.0121)	(.0123)	(.0100)	(.0110)	(.0180)	(.0077)
	4	.03761	.08490	-.03691	-.04077	-.04482	.13618
		(.0134)	(.0164)	(.0110)	(.0191)	(.0227)	(.0088)
	5	.04109	.09331	-.03327	-.04482	-.05632	.35320
		(-)[b]	(.0319)	(.0180)	(.0227)	(.0554)	(.0162)

[a] Standard errors of the substitution effects are simply taken as those of coefficients, γ_{ij}'s, of the AIDS system with the assumption that average expenditure shares are fixed.

[b] See footnote (a) in Table 4.9.2.

Table 4.24
Summary of Complementary, Substitute and Independent Relations in the 5 Commodity Models

Table 4.24.1 By Sign of Cross Substitution Effect

	RDAM	RDAM-D4	RDAM-SD	RDAM-AR(4)	RDAM-AR(1,4)	AIDS	AIDS-SD	AIDS-AR(4)	AIDS-AR(1,4)
FD:HS	(-)	(+)	(+)	(-)	(+)	(+)	(-)	(+)	(-)
FD:FL	(+)	(+)	(+)	(+)	(+)	(+)	(+)	(+)	(+)
FD:CL	(+)	(+)	(+)	(+)	(+)	(-)	(-)	(+)	(+)
FD:MS	(+)	(+)	(+)	(+)	(+)	(+)	(+)	(+)	(+)
HS:FL	(+)	(+)	(+)	(+)	(+)	(-)	(-)	(+)	(-)
HS:CL	(+)	(+)	(+)	(+)	(+)	(+)	(+)	(+)	(+)
HS:MS	(+)	(+)	(+)	(+)	(+)	(-)	(+)	(+)	(+)
FL:CL	(-)	(-)	(-)	(-)	(-)	(-)	(-)	(-)	(-)
FL:MS	(-)	(-)	(-)	(-)	(-)	(-)	(-)	(-)	(-)
CL:MS	(+)	(-)	(-)	(-)	(-)	(-)	(-)	(-)	(-)

Table 4.24.1 By Classifications after Test of Significance of Cross Substitution Effect

	RDAM	RDAM-D4	RDAM-SD	RDAM-AR(4)	RDAM-AR(1,4)	AIDS	AIDS-SD	AIDS-AR(4)	AIDS-AR(1,4)
FD:HS	i	s	i	i	i	s	i	i	i
FD:FL	s	s	s	s	s	s	s	s	s
FD:CL	i	s	i	i	i	i	c	i	s
FD:MS	i	s	i	x	x	s	i	x	x
HS:FL	i	i	s	s	s	i	c	i	i
HS:CL	i	s	i	i	s	s	s	s	s
HS:MS	s	i	i	s	s	i	i	s	s
FL:CL	i	i	i	i	i	c	c	c	c
FL:MS	c	c	c	c	c	i	i	c	c
CL:MS	i	i	i	i	i	i	i	i	c

Note: i = independence, s = substitutability, c = complementarity.
x = not identifiable, as standard errors of the cross substitution effect cannot be obtained in estimation of the reduced system. See footnote (a) in Table 4.9.2.
FD = foods, HS = housing, FL = fuel and light, CL = clothing, MS = miscellaneous commodity sector.

Table 4.25
Total Expenditure Elasticity

Model	Commodity sector				
	1	2	3	4	5
LES-W	.69641	1.71819	.93025	1.24425	1.19557
LES-W-SD	.73747	1.61188	1.01523	1.29033	1.12493
RDAM					
Unrestr'ed	1.67894	-.12786	1.80772	1.36665	-.07281
	(.1282)	(.2555)	(.2904)	(.2038)	(.1849)
Restricted	1.59934	-.05455	1.83365	1.37283	.02465
	(.1179)	(.2476)	(.2714)	(.1945)	(.1733)
RDAM-D4					
Unrestr'ed	.84136	2.03744	.16690	1.57800	.91622
	(.0543)	(.1618)	(.1261)	(.1071)	(.0764)
Restricted	.72582	1.66970	.58202	1.52387	1.13782
	(.0419)	(.1371)	(.1083)	(.0828)	(.0632)
RDAM-SD					
Unrestr'ed	1.09119	1.22666	.56392	1.24004	.79507
	(.1291)	(.2980)	(.2737)	(.2754)	(.1758)
Restricted	.98236	1.26466	.67437	1.34661	.89319
	(.1278)	(.2996)	(.2708)	(.2735)	(.1701)
RDAM-AR(4)					
Unrestr'ed	1.12484	.55598	.95786	1.13235	.89330
	(.1031)	(.3123)	(.2537)	(.2021)	(.1501)
Restricted	1.08697	.36762	1.27571	1.13141	.93844
	(.0919)	(.2977)	(.2540)	(.1939)	(.1327)
RDAM-AR(1,4)					
Unrestr'ed	1.05711	.99203	.76058	1.34230	.83716
	(.0938)	(.2617)	(.2143)	(.1660)	(.1275)
Restricted	1.01697	.79213	.95985	1.38884	.89881
	(.0840)	(.2497)	(.2040)	(.1545)	(.1181)
AIDS					
Unrestr'ed	1.06163	1.62522	.75670	1.65795	.52170
	(.0949)	(.2372)	(.2379)	(.1347)	(.1142)
Restricted	.77726	1.54374	.78263	1.53546	1.04290
	(.0634)	(.1477)	(.1486)	(.0817)	(.0866)
AIDS-SD					
Unrestr'ed	.76532	2.48868	.09138	1.79598	.83802
	(.0610)	(.1391)	(.1494)	(.1252)	(.0882)
Restricted	.63394	1.88293	.51187	1.56102	1.22328
	(.0413)	(.0963)	(.0973)	(.0743)	(.0638)
AIDS-AR(4)					
Unrestr'ed	.81392	2.08959	.12789	1.45165	1.00088
	(.0614)	(.1738)	(.1497)	(.1116)	(.0862)
Restricted	.76539	1.64865	.45050	1.34911	1.17624
	(.0418)	(.1224)	(.1100)	(.0711)	(.0609)
AIDS-AR(1,4)					
Unrestr'ed	.79833	2.19356	-.00495	1.38272	1.05080
	(.0570)	(.1740)	(.1476)	(.1286)	(.0844)
Restricted	.73742	1.60937	.54086	1.27522	1.23999
	(.0420)	(.1323)	(.1180)	(.0829)	(.0569)

Table 4.26
Compensated Own Price Elasticity

Model	Commodity sector				
	1	2	3	4	5
LES-W	-.35843	-1.10637	-.65996	-.81486	-.59545
LES-W-SD	-.37322	-1.06074	-.72400	-.84962	-.58381
RDAM	-.64357	-1.98534	-.25573	-1.10800	-.57790
	(.2569)	(.8407)	(.4016)	(.6558)	(.5050)
RDAM-D4	-.43608	-2.28524	-.05093	-.89739	-.15637
	(.0730)	(.3461)	(.1635)	(.2312)	(.1752)
RDAM-SD	-.36875	-2.18185	-.51084	-.97034	.07741
	(.1781)	(.6931)	(.2545)	(.6238)	(.2993)
RDAM-AR(4)	-.53062	-2.90731	-.42416	-.17643	-.45730
	(.1368)	(.7790)	(.2859)	(.5370)	(.2245)
RDAM-AR(1,4)	-.43298	-3.19917	-.37124	-.76910	-.17683
	(.1312)	(.5872)	(.2304)	(.3958)	(.2277)
AIDS	-.38747	-1.77567	.04247	-.63447	-.28247
	(.0760)	(.3195)	(.1999)	(.1878)	(.2314)
AIDS-SD	-.01772	-1.19488	.38348	-.50691	.00288
	(.0523)	(.2708)	(.1419)	(.1824)	(.1959)
AIDS-AR(4)	-.32645	-1.94944	.15413	-.74551	-.25523
	(.0671)	(.2857)	(.1646)	(.1821)	(.1834)
AIDS-AR(1,4)	-.28818	-2.05692	.24845	-.38182	-.19771
	(.0615)	(.2856)	(.1533)	(.1790)	(.1946)

Table 4.27
Test of Demand Restrictions (12 Sectors)

Model	Restri-ction	Testing statistics [a] W	LR	LM	T^2	C.V. of T^2 at 5 %
RDAM-D4	H	118.808	66.768	41.169	94.292	(28.017)
	S	92.633	79.710	69.438	-	
	H & S	217.035	146.479	107.759	-	
RDAM-SD	H	13.947	12.599	11.419	10.183	(29.047)
	S	61.468	55.231	49.949	-	
	H & S	76.635	67.829	60.347	-	
AIDS-SD.	H	152.377	77.961	45.070	111.902	(28.017)
	S	176.244	133.889	106.184	-	
	H & S	338.835	211.850	146.543	-	

[a] Critical values of x^2 at 5 %: 19.681 for homogeneity restrictions (H),
73.308 for symmetry restrcition (S),
85.961 for both restrictions (H & S).

Table 4.28
RDAM-SD Model (12 Sectors)

Table 4.28.1 Unrestricted RDAM-SD Model (12 Sectors)

	$dlnP_1$	$dlnP_2$	$dlnP_3$	$dlnP_4$	$dlnP_5$	$dlnP_6$	$dlnP_7$
1	-.06775	-.06910	.01676	.04899	-.02944	.07917	.01162
	(.0464)	(.0670)	(.0161)	(.0489)	(.0724)	(.1163)	(.0519)
2	.04402	-.09083	.00337	.01538	-.02119	-.19435	.04846
	(.0203)	(.0293)	(.0071)	(.0214)	(.0316)	(.0508)	(.0226)
3	.00867	.05052	-.05180	-.05891	-.04172	-.02728	.00369
	(.0397)	(.0573)	(.0138)	(.0419)	(.0620)	(.0995)	(.0444)
4	.00704	.04933	.01731	-.07581	-.05776	-.04031	.07755
	(.0308)	(.0445)	(.0107)	(.0325)	(.0481)	(.0772)	(.0344)
5	.01386	-.02212	.00610	.00245	-.05265	-.00867	.04331
	(.0143)	(.0207)	(.0050)	(.0151)	(.0224)	(.0359)	(.0160)
6	-.00536	.00465	-.00281	.04767	.01734	-.06354	.00786
	(.0211)	(.0305)	(.0073)	(.0223)	(.0330)	(.0529)	(.0236)
7	.04595	.01998	.00393	.01331	.03976	-.00407	-.04938
	(.0173)	(.0249)	(.0060)	(.0182)	(.0269)	(.0432)	(.0193)
8	.00206	.00157	.00528	-.00036	-.00331	.05326	-.02778
	(.0291)	(.0420)	(.0101)	(.0307)	(.0454)	(.0729)	(.0325)
9	-.00720	.02262	-.00967	.01660	.01323	.02810	-.02952
	(.0129)	(.0186)	(.0045)	(.0136)	(.0201)	(.0322)	(.0144)
10	-.01481	.01752	.00667	-.02153	.03402	-.01293	-.01116
	(.0315)	(.0455)	(.0110)	(.0332)	(.0492)	(.0789)	(.0352)
11	-.00318	-.01096	-.00150	.00988	.03351	.00091	-.00214
	(.0118)	(.0171)	(.0041)	(.0125)	(.0185)	(.0296)	(.0132)
12	-.02332	.02684	.00635	.00235	.06819	.18970	-.07251
	(.0287)	(.0414)	(.0100)	(.0303)	(.0448)	(.0719)	(.0321)

	$dlnP_8$	$dlnP_9$	$dlnP_{10}$	$dlnP_{11}$	$dlnP_{12}$	dY
1	.10204	-.15398	-.03672	-.01925	.05096	.01953
	(.1632)	(.0869)	(.0727)	(.0329)	(.0595)	(.0511)
2	.16474	.02932	.01499	.00880	-.03426	.14269
	(.0713)	(.0379)	(.0318)	(.0144)	(.0260)	(.0223)
3	.18561	.03766	.08051	.05239	-.04021	.17664
	(.1396)	(.0743)	(.0622)	(.0282)	(.0509)	(.0437)
4	.10599	-.03074	.04944	.01683	-.01871	.17499
	(.1084)	(.0577)	(.0483)	(.0219)	(.0395)	(.0339)
5	.02547	-.03914	-.00800	-.00073	.02781	.05545
	(.0504)	(.0268)	(.0225)	(.0102)	(.0184)	(.0158)

Table 4.28.1 (continued)

6	-.08534	.02650	-.00163	-.02440	.03514	.05571
	(.0742)	(.0395)	(.0331)	(.0150)	(.0271)	(.0232)
7	.05370	-.03021	-.05163	-.00188	.00367	.03438
	(.0607)	(.0323)	(.0270)	(.0122)	(.0221)	(.0190)
8	-.18970	.07464	-.01965	-.00404	-.01143	.12733
	(.1023)	(.0545)	(.0456)	(.0206)	(.0373)	(.0320)
9	-.01924	-.01716	-.01331	.00191	.01464	.03108
	(.0453)	(.0241)	(.0202)	(.0091)	(.0165)	(.0142)
10	-.06335	.02808	-.02811	.00626	.00969	.11088
	(.1108)	(.0590)	(.0494)	(.0224)	(.0404)	(.0347)
11	-.08230	.02425	.02107	-.03143	.01093	.03450
	(.0416)	(.0222)	(.0185)	(.0084)	(.0152)	(.0130)
12	-.19761	.05077	-.00696	-.00446	-.04824	.03682
	(.1009)	(.0537)	(.0450)	(.0204)	(.0368)	(.0316)

	D_1	D_2	D_3	D_4	R^2	DW	DW4
1	.00372	-.00540	-.00381	.01893	.5007	2.818	1.575
	(.0065)	(.0069)	(.0054)	(.0078)			
2	.00102	-.00558	.00146	-.00510	.7589	2.456	1.935
	(.0028)	(.0030)	(.0023)	(.0034)			
3	-.04580	.00439	.00710	-.00222	.8747	2.328	1.841
	(.0056)	(.0059)	(.0046)	(.0066)			
4	-.02817	.00617	.00104	-.00158	.8489	2.590	1.652
	(.0043)	(.0046)	(.0036)	(.0051)			
5	.00466	.00132	-.00123	-.00410	.3123	2.002	1.915
	(.0020)	(.0021)	(.0017)	(.0024)			
6	.00385	.01880	-.00498	-.00802	.7000	2.377	1.934
	(.0030)	(.0032)	(.0024)	(.0035)			
7	.00125	-.01393	-.00529	.01357	.8654	1.985	1.385
	(.0024)	(.0026)	(.0020)	(.0029)			
8	.00780	.00417	-.00737	.01411	.6901	2.775	1.126
	(.0041)	(.0044)	(.0034)	(.0049)			
9	.00646	.00285	.00073	-.00434	.4200	2.557	2.049
	(.0018)	(.0019)	(.0015)	(.0022)			
10	.04166	-.02433	.01217	-.02601	.8829	2.806	2.098
	(.0044)	(.0047)	(.0036)	(.0053)			
11	.00215	.00419	.00298	-.00222	.5197	2.343	1.439
	(.0017)	(.0018)	(.0014)	(.0020)			
12	.00139	.00734	-.00280	.00698	.3883	2.370	1.844
	(.0048)	(.0040)	(.0043)	(.0033)			

Table 4.28.2 Homogeneity Restricted RDAM-SD Model (12 Sectors)

	$dlnP_1$	$dlnP_2$	$dlnP_3$	$dlnP_4$	$dlnP_5$	$dlnP_6$	$dlnP_7$
1	-.06390	-.05937	.01595	.05188	-.01620	.08257	.00183
	(.0459)	(.0646)	(.0161)	(.0487)	(.0682)	(.1163)	(.0486)
2	.04469	-.08915	.00323	.01588	-.01889	-.19376	.04676
	(.0200)	(.0281)	(.0070)	(.0212)	(.0297)	(.0507)	(.0212)
3	-.00284	.02148	-.04937	-.06754	-.08125	-.03742	.03295
	(.0403)	(.0566)	(.0141)	(.0427)	(.0597)	(.1020)	(.0426)
4	.00125	.03472	.01853	-.08015	-.07764	-.04541	.09227
	(.0308)	(.0433)	(.0108)	(.0327)	(.0457)	(.0780)	(.0325)
5	.01458	-.02033	.00595	.00298	-.05021	-.00805	.04150
	(.0142)	(.0199)	(.0050)	(.0150)	(.0210)	(.0359)	(.0150)
6	-.00282	.01105	-.00335	.04957	.02606	-.06131	.00140
	(.0209)	(.0294)	(.0073)	(.0222)	(.0311)	(.0530)	(.0221)
7	.04346	.01369	.00446	.01144	.03120	-.00626	-.04305
	(.0172)	(.0241)	(.0060)	(.0182)	(.0255)	(.0435)	(.0181)
8	.00897	.01899	.00383	.00481	.02040	.05934	-.04533
	(.0293)	(.0411)	(.0103)	(.0310)	(.0434)	(.0741)	(.0309)
9	-.00726	.02247	-.00966	.01656	.01303	.02805	-.02937
	(.0127)	(.0179)	(.0045)	(.0135)	(.0189)	(.0322)	(.0134)
10	-.01194	.02476	.00606	-.01938	.04388	-.01040	-.01846
	(.0312)	(.0439)	(.0109)	(.0331)	(.0463)	(.0790)	(.0330)
11	-.00139	-.00645	-.00188	.01122	.03966	.00249	-.00669
	(.0118)	(.0165)	(.0041)	(.0125)	(.0175)	(.0298)	(.0124)
12	-.02281	.02813	.00624	.00273	.06996	.19016	-.07382
	(.0283)	(.0398)	(.0099)	(.0301)	(.0421)	(.0718)	(.0300)

	$dlnP_8$	$dlnP_9$	$dlnP_{10}$	$dlnP_{11}$	$dlnP_{12}$	dlt
1	.13801	-.15354	-.02382	-.02120	.04781	.01903
	(.1488)	(.0871)	(.0687)	(.0328)	(.0594)	(.0512)
2	.17096	.02940	.01722	.00846	-.03481	.14260
	(.0648)	(.0379)	(.0299)	(.0143)	(.0259)	(.0223)
3	.07820	.03634	.04200	.05823	-.03078	.17814
	(.1304)	(.0763)	(.0602)	(.0288)	(.0520)	(.0449)
4	.05196	-.03140	.03007	.01977	-.01397	.17574
	(.0997)	(.0583)	(.0460)	(.0220)	(.0398)	(.0343)
5	.03212	-.03905	-.00562	-.00109	.02723	.05536
	(.0459)	(.0268)	(.0212)	(.0101)	(.0183)	(.0158)
6	-.06165	.02679	.00687	-.02569	.03307	.05538
	(.0678)	(.0397)	(.0313)	(.0150)	(.0271)	(.0234)

Table 4.28.2 (continued)

7	.03043 (.0556)	-.03050 (.0325)	-.05997 (.0257)	-.00061 (.0122)	.00572 (.0222)	.03470 (.0191)
8	-.12527 (.0947)	.07543 (.0554)	.00345 (.0437)	-.00755 (.0209)	-.01709 (.0378)	.12643 (.0326)
9	-.01978 (.0412)	-.01716 (.0241)	-.01351 (.0190)	.00194 (.0091)	.01469 (.0164)	.03109 (.0142)
10	-.03657 (.1010)	.02841 (.0591)	-.01851 (.0467)	.00481 (.0223)	.00733 (.0403)	.11051 (.0348)
11	-.06560 (.0381)	.02445 (.0223)	.02706 (.0176)	-.03234 (.0084)	.00946 (.0152)	.03427 (.0131)
12	-.19281 (.0918)	.05083 (.0537)	-.00524 (.0424)	-.00473 (.0202)	-.04866 (.0366)	.03675 (.0316)

	D_1	D_2	D_3	D_4	R^2	DW	DW4
1	.00139 (.0048)	-.00826 (.0044)	-.00559 (.0042)	.01690 (.0068)	.4984	2.833	1.556
2	.00062 (.0021)	-.00607 (.0019)	.00116 (.0018)	-.00545 (.0029)	.7587	2.456	1.944
3	-.03885 (.0042)	.01294 (.0038)	.01241 (.0037)	.00383 (.0059)	.8679	2.403	1.747
4	-.02467 (.0032)	.01047 (.0029)	.00371 (.0028)	.00147 (.0045)	.8454	2.590	1.619
5	.00423 (.0015)	.00079 (.0014)	-.00155 (.0013)	-.00447 (.0021)	.3112	1.997	1.912
6	.00232 (.0022)	.01692 (.0020)	-.00615 (.0019)	-.00936 (.0031)	.6971	2.360	1.939
7	.00276 (.0018)	-.01208 (.0016)	-.00414 (.0016)	.01488 (.0025)	.8636	1.994	1.448
8	.00363 (.0031)	-.00095 (.0028)	-.01056 (.0027)	.01047 (.0043)	.6788	2.760	1.154
9	.00650 (.0013)	.00289 (.0012)	.00076 (.0012)	-.00431 (.0019)	.4200	2.556	2.049
10	.03993 (.0033)	-.02646 (.0030)	.01084 (.0028)	-.02752 (.0046)	.8822	2.810	2.109
11	.00107 (.0012)	.00286 (.0011)	.00215 (.0011)	-.00317 (.0017)	.5126	2.331	1.404
12	.00108 (.0030)	.00695 (.0027)	-.00304 (.0026)	.00671 (.0042)	.3882	2.371	1.838

Table 4.28.3 Homogeneity and Symmetry Restricted RDAM-SD Model (12 Sectors)

	$dlnP_1$	$dlnP_2$	$dlnP_3$	$dlnP_4$	$dlnP_5$	$dlnP_6$	$dlnP_7$
1	-.09704	.03306	.01308	-.01784	.01356	.00539	.03219
	(.0395)	(.0182)	(.0142)	(.0233)	(.0113)	(.0187)	(.0145)
2	.03306	-.07797	.00665	.01480	-.01555	-.02634	.02098
	(.0182)	(.0252)	(.0069)	(.0176)	(.0135)	(.0213)	(.0146)
3	.01308	.00665	-.04873	.01326	.00625	-.00208	.00488
	(.0142)	(.0069)	(.0131)	(.0101)	(.0046)	(.0070)	(.0055)
4	-.01784	.01480	.01326	-.03707	.01061	.03704	.01135
	(.0233)	(.0176)	(.0101)	(.0287)	(.0116)	(.0190)	(.0140)
5	.01356	-.01555	.00625	.01061	-.04546	-.00570	.03669
	(.0113)	(.0135)	(.0046)	(.0116)	(.0166)	(.0186)	(.0105)
6	.00539	-.02634	-.00208	.03704	-.00570	-.10002	.01837
	(.0187)	(.0213)	(.0070)	(.0190)	(.0186)	(.0414)	(.0169)
7	.03219	.02098	.00488	.01135	.03669	.01837	-.04527
	(.0145)	(.0146)	(.0055)	(.0140)	(.0105)	(.0169)	(.0159)
8	.02474	.03107	.00497	-.01721	.01580	.01451	.00342
	(.0249)	(.0286)	(.0097)	(.0253)	(.0261)	(.0440)	(.0224)
9	-.00313	.01964	-.00847	.00438	-.03400	.01174	-.01895
	(.0121)	(.0137)	(.0044)	(.0122)	(.0126)	(.0201)	(.0106)
10	-.01798	.00153	.00411	-.02295	-.01025	.03196	-.05504
	(.0261)	(.0216)	(.0101)	(.0237)	(.0149)	(.0240)	(.0170)
11	.00695	-.00504	.00002	.00602	-.00522	-.01456	.00375
	(.0107)	(.0098)	(.0040)	(.0099)	(.0071)	(.0115)	(.0079)
12	.00702	-.00284	.00607	-.00240	.03328	.02969	-.01238
	(.0233)	(.0208)	(.0094)	(.0211)	(.0129)	(.0210)	(.0151)

	$dlnP_8$	$dlnP_9$	$dlnP_{10}$	$dlnP_{11}$	$dlnP_{12}$	dY
1	.02474	-.00313	-.01798	.00695	.00702	.00183
	(.0249)	(.0121)	(.0261)	(.0107)	(.0233)	(.0449)
2	.03107	.01964	.00153	-.00504	-.00284	.13741
	(.0286)	(.0137)	(.0216)	(.0098)	(.0208)	(.0214)
3	.00497	-.00847	.00411	.00002	.00607	.17702
	(.0097)	(.0044)	(.0101)	(.0040)	(.0094)	(.0375)
4	-.01721	.00438	-.02295	.00602	-.00240	.13687
	(.0253)	(.0122)	(.0237)	(.0099)	(.0211)	(.0306)
5	.01580	-.03400	-.01025	-.00522	.03328	.05434
	(.0261)	(.0126)	(.0149)	(.0071)	(.0129)	(.0141)
6	.01451	.01174	.03196	-.01456	.02969	.06709
	(.0440)	(.0201)	(.0240)	(.0115)	(.0210)	(.0217)

Table 4.28.3 (continued)

7	.00342 (.0224)	-.01895 (.0106)	-.05504 (.0170)	.00375 (.0079)	-.01238 (.0151)	.02894 (.0174)
8	-.10955 (.0722)	.03811 (.0264)	.03195 (.0331)	-.00964 (.0152)	-.02816 (.0297)	.14628 (.0298)
9	.03811 (.0264)	-.03294 (.0194)	.00077 (.0156)	.00999 (.0075)	.01285 (.0132)	.03942 (.0137)
10	.03195 (.0331)	.00077 (.0156)	-.01549 (.0369)	.02895 (.0119)	.02245 (.0238)	.10298 (.0305)
11	-.00964 (.0152)	.00999 (.0075)	.02895 (.0119)	-.02752 (.0077)	.00632 (.0106)	.04304 (.0121)
12	-.02816 (.0297)	.01285 (.0132)	.02245 (.0238)	.00632 (.0106)	-.07189 (.0305)	.06479 (.0283)

	D_1	D_2	D_3	D_4	R^2	DW	DW4
1	-.00073 (.0047)	-.01167 (.0042)	-.00798 (.0042)	.01952 (.0060)	.4065	2.874	1.293
2	.00032 (.0022)	-.00403 (.0020)	.00228 (.0019)	-.00215 (.0028)	.7034	2.700	1.937
3	-.03712 (.0041)	.01458 (.0035)	.01224 (.0036)	.00433 (.0050)	.8457	2.284	1.620
4	-.02470 (.0033)	.01271 (.0029)	.00490 (.0029)	.00747 (.0041)	.8124	2.697	1.433
5	.00417 (.0014)	.00089 (.0013)	-.00160 (.0012)	-.00409 (.0018)	.3000	1.997	1.932
6	.00287 (.0021)	.01680 (.0020)	-.00577 (.0019)	-.01147 (.0029)	.6747	2.425	1.848
7	.00206 (.0017)	-.01258 (.0015)	-.00400 (.0015)	.01594 (.0023)	.8580	2.006	1.292
8	.00437 (.0030)	-.00235 (.0027)	-.01177 (.0027)	.00673 (.0039)	.6543	2.738	1.010
9	.00629 (.0014)	.00268 (.0012)	.00109 (.0012)	-.00568 (.0018)	.3319	2.673	2.060
10	.03940 (.0032)	-.02510 (.0029)	.01264 (.0029)	-.02642 (.0041)	.8685	2.882	2.125
11	.00080 (.0012)	.00284 (.0011)	.00224 (.0011)	-.00455 (.0016)	.4412	2.445	1.249
12	.00229 (.0030)	.00523 (.0027)	-.00426 (.0027)	.00036 (.0038)	.2779	2.428	1.631

Table 4.29
Comparison between the Marginal Budget Shares Obtained from Aggregated (5) and Disaggregated (12) Sectors

Commodity	Unrestricted		Restricted	
sector	Aggregated	Disaggregated	Aggregated	Disaggregated
Food	.49910	.51385	.44932	.45313
Housing	.10524	.11116	.10850	.12143
Fuel & Light	.03676	.03438	.04396	.02894
Clothing	.13242	.12733	.14380	.14628
Miscellaneous	.22647	.21328	.25442	.25023

Table 4.30
Total Expenditure and Compensated Own Price Elasticities:
RDAM-SD (12 Sectors)

	Comensated own price elasticity	Total expenditure elasticity	
		Unrestricted	Restricted
Cereals	-.53911	.10850	.01016
	(.2194)	(.2839)	(.0249)
Meat & Fish	-.82947	1.51798	1.46181
	(.2681)	(.2372)	(.2277)
Vegetable & Fruits	-.63290	2.29403	2.29896
	(.1701)	(.5675)	(.4870)
Other Foods	-.35305	1.66657	1.30352
	(.2733)	(.3229)	(.2914)
Rent	-1.89417	2.31042	2.26417
	(.6917)	(.6583)	(.5875)
Other Housing	-1.61323	.89855	1.08210
	(.6677)	(.3742)	(.3500)
Fuel & Light	-.69646	.52892	.44523
	(.2446)	(.2923)	(.2677)
Clothing	-1.02383	1.19000	1.36710
	(.6748)	(.2991)	(.2785)
Medical Care	-.63346	.59769	.75808
	(.3731)	(.2731)	(.2635)
Education & Culture	-.17404	1.24584	1.15708
	(.4146)	(.3899)	(.3427)
Trasportat'n & Communicat'n	-.52923	.66346	.82769
	(.1481)	(.2500)	(.2327)
Miscellaneous	-.77301	.39591	.69667
	(.3280)	(.3398)	(.3043)

Table 4.31
Test of Structural Stability, 1966-1973 vs 1974-1981 (LR Test)

Model	LR test statistic: Unrest'ed model	LR test statistic: Restricted model	Critical value at 5%: Asymptotic x^2 (df)	Critical value at 5%: Adjusted with T/(T-k)
5 Sectors		155.40 (128)		
RDAM-SD	242.88	212.15		226.04
RDAM-D4	167.21	151.80		191.27
AID-SD	280.72	268.98		226.04
12 Sectors		396.75 (352)		
RDAM-SD	1059.48	895.58		846.40
RDAM-D4	1063.33	837.99		668.21
AIDS-SD	1165.12	993.62		846.40

Table 4.32
Test of Parameter Constancy, 1966-1973 vs 1974-1981 (LR Test)

Model	LR test statistic: Unrest'ed model	LR test statistic: Restricted model	Critical value at 5%: Asymptotic x^2 (df)	Critical value at 5%: Adjusted I[a]	Critical value at 5%: Adjusted II[b]
5 Sectors					
RDAM-SD	168.03	158.54	67.05 (50)	83.89	98.13
RDAM-D4	101.12	74.32	48.61 (34)	56.01	59.83
AIDS-SD	220.06	216.10	67.50 (209)	83.89	98.13
12 Sectors					
RDAM-SD	861.90	641.86	291.10 (253)	454.40	621.01
RDAM-D4	768.41	478.36	243.73 (209)	346.63	410.49
AIDS-SD	1000.15	727.35	291.10 (253)	454.40	621.01

[a] The correction factor is T/{T-k-[(m-1)-s/(m-1)+1]/2} with s = (m-1)k+m(m-1)/2.
[b] The correction factor is T/(T-k).

Table 4.33
Comparison of Tests of Demand Restrictions between Two Subsamples for the Periods 1966-1973 and 1974-1981

Table 4.33.1 Aggregated 5 Sectors

Model	Restriction	Testing statistic				Adjusted χ^2 C.V.[a,b]	
		W	LR	LM	T^2 (5 % C.V.)	I[c]	II[d]
(1966-1973)							
RDAM-SD	H	2.41	2.33	2.24	1.64 (13.41)		
	S	6.99	6.38	5.84		19.90	18.32
	H & S	9.81	8.71	7.76		28.24	26.63
RDAM-D4	H	34.60	23.24	16.35	27.90 (12.64)		
	S	5.99	5.63	5.31		16.62	15.50
	H & S	43.06	28.87	20.23		23.67	22.54
AID-SD	H	39.30	25.64	17.64	27.02 (13.41)		
	S	22.38	17.33	13.74		19.90	18.32
	H & S	64.79	42.96	30.06		28.24	26.63
(1974-1981)							
RDAM-SD	H	3.13	2.99	2.86	2.16 (13.41)		
	S	17.47	14.76	12.63		19.90	18.32
	H & S	20.54	17.75	15.42		28.24	26.63
RDAM-D4	H	32.75	22.55	16.19	26.61 (12.64)		
	S	23.18	18.61	15.29		16.62	15.50
	H & S	56.97	41.16	30.81		23.67	22.54
AIDS-SD	H	28.49	20.38	15.07	19.59 (13.41)		
	S	17.33	14.77	12.75		19.90	18.32
	H & S	48.97	35.14	26.46		28.24	26.63

Table 4.33.2 Disaggregated 12 Sectors

Model	Restriction	Testing statistic W	LR	LM	T^2 (5 % C.V.)	Adjusted x^2 C.V.[a,e] I[c]	II[d]
(1966-1973)							
RDAM-SD	H	32.51	22.23	15.87	14.68 (155.23)		
	S	147.18	91.80	63.57		203.99	156.59
	H & S	177.74	114.03	78.56		229.23	183.38
RDAM-D4	H	131.43	51.34	25.08	76.31 (72.04)		
	S	261.27	131.61	83.10		151.35	123.47
	H & S	422.78	182.96	103.06		171.92	144.78
AIDS-SD	H	141.66	54.12	26.10	66.40 (155.23)		
	S	272.47	137.55	84.10		203.99	156.59
	H & S	436.37	191.68	105.84		229.23	183.38
(1974-1981)							
RDAM-SD	H	128.96	51.70	25.64	60.45 (155.23)		
	S	198.42	121.04	82.35		203.99	156.59
	H & S	327.33	172.73	106.26		229.23	183.38
RDAM-D4	H	123.49	50.59	25.41	73.32 (72.04)		
	S	559.10	198.54	103.95		151.35	123.47
	H & S	716.89	249.13	124.18		171.92	144.78
AIDS-SD	H	304.89	75.33	28.96	142.92 (155.23)		
	S	565.91	211.39	114.83		203.99	156.59
	H & S	906.56	286.73	138.69		229.23	183.38

[a] Critical values at 5 % are adjusted.

[b] The unadjusted x^2 critical values are 9.49 for testing H, 12.60 for testing S, and 18.31 for H & S.

[c] The correction factor is T/{T-k-[(m-1)-s/(m-1)+1]/2} where s is the number of restrictions.

[d] The correction factor is T/(T-k).

[e] The unadjusted x^2 critical values are 19.68 for testing H, 73.31 for testing S, and 85.96 for H & S.

Table 4.34
RDAM-SD Model (5 Sectors), 1966-1973

Table 4.34.1 Unrestricted RDAM-SD Model (5 Sectors), 1966-1973

	$dlnP_1$	$dlnP_2$	$dlnP_3$	$dlnP_4$	$dlnP_5$	dM
1	.06984 (.1152)	-.53624 (.2627)	.46950 (.1735)	.33810 (.2843)	.11413 (.1770)	.61011 (.0771)
2	-.00987 (.0661)	-.22246 (.1508)	.10735 (.0996)	.03760 (.1632)	.07341 (.1017)	.08978 (.0443)
3	.10770 (.0271)	.15041 (.0619)	-.10483 (.0409)	.01730 (.0670)	-.12526 (.0417)	.03895 (.0182)
4	.01953 (.0734)	.00626 (.1674)	-.01354 (.1105)	-.18671 (.1811)	.06399 (.1128)	.14019 (.0491)
5	-.18720 (.0913)	.60202 (.2083)	-.45848 (.1376)	-.20629 (.2254)	-.12628 (.1404)	.12098 (.0611)

	D_1	D_2	D_3	D_4	R^2	DW	DW4
1	-.05401 (.0135)	-.00446 (.0104)	-.00598 (.0080)	-.00066 (.0144)	.9465	2.461	1.463
2	.00550 (.0078)	.01694 (.0060)	-.00391 (.0046)	-.01577 (.0083)	.4674	2.809	2.107
3	.00053 (.0032)	-.01363 (.0025)	-.00835 (.0019)	.01475 (.0034)	.9068	2.733	1.442
4	.01029 (.0086)	.00104 (.0066)	-.00506 (.0051)	.00487 (.0092)	.4657	2.688	1.150
5	.03770 (.0107)	.00012 (.0083)	.02329 (.0063)	-.00320 (.0114)	.7220	2.743	1.180

Table 4.34.2 Homogeneity Restricted RDAM-SD Model (5 Sectors), 1966-1973

	$dlnP_1$	$dlnP_2$	$dlnP_3$	$dlnP_4$	$dlnP_5$	dM
1	.06431 (.1178)	-.55698 (.2683)	.45533 (.1772)	.06020 (.1732)	-.02284 (.1399)	.64701 (.0726)
2	-.00970 (.0661)	-.22183 (.1505)	.10778 (.0994)	.04613 (.0972)	.07761 (.0785)	.08865 (.0407)
3	.10715 (.0272)	.14835 (.0620)	-.10624 (.0410)	-.01036 (.0400)	-.13889 (.0323)	.04262 (.0168)
4	.02087 (.0736)	.01130 (.1676)	-.01010 (.1107)	-.11930 (.1082)	.09722 (.0874)	.13124 (.0453)
5	-.18262 (.0936)	.61917 (.2132)	-.44677 (.1408)	.02332 (.1376)	-.01310 (.1111)	.09049 (.0576)

Table 4.34.2 (continued)

	D_1	D_2	D_3	D_4	R^2	DW	DW4
1	-.04054 (.0079)	.00512 (.0070)	-.00061 (.0068)	.01044 (.0114)	.9440	2.380	1.534
2	.00508 (.0044)	.01665 (.0039)	-.00407 (.0031)	-.01611 (.0064)	.4673	2.806	2.112
3	.00187 (.0018)	-.01268 (.0016)	-.00782 (.0023)	.01586 (.0026)	.9060	2.702	1.461
4	.00702 (.0049)	-.00129 (.0044)	-.00636 (.0036)	.00218 (.0071)	.4620	2.662	1.131
5	.02657 (.0063)	-.00780 (.0056)	.01886 (.0065)	-.01236 (.0091)	.7076	2.699	1.275

Table 4.34.3 Homogeneity and Symmetry Restricted RDAM-SD Model (5 Sectors), 1966-1973

	$dlnP_1$	$dlnP_2$	$dlnP_3$	$dlnP_4$	$dlnP_5$	dY
1	.02185 (.1246)	-.01412 (.0639)	.10777 (.0265)	.00899 (.0667)	-.12449 (.0968)	.59274 (.0736)
2	-.01412 (.0639)	-.33639 (.1071)	.17078 (.0497)	.04959 (.0811)	.13015 (.0653)	.09737 (.0392)
3	.10777 (.0266)	.17078 (.0497)	-.11690 (.0365)	-.01857 (.0357)	-.14308 (.0311)	.04166 (.0165)
4	.00899 (.0667)	.04959 (.0811)	-.01857 (.0357)	-.13393 (.1012)	.09393 (.0727)	.12625 (.0425)
5	-.12449 (.0968)	.13015 (.0653)	-.14308 (.0311)	.09393 (.0727)	.04350 (.1065)	.14198 (.0581)

	D_1	D_2	D_3	D_4	R^2	DW	DW4
1	-.04438 (.0082)	.00038 (.0066)	-.00157 (.0071)	.02474 (.0099)	.9345	2.126	1.581
2	.00596 (.0044)	.01713 (.0037)	-.00359 (.0041)	-.01856 (.0056)	.4496	2.853	2.122
3	.00178 (.0018)	-.01286 (.0016)	-.00794 (.0021)	.01626 (.0025)	.9056	2.715	1.443
4	.00715 (.0048)	-.00146 (.0039)	-.00650 (.0043)	.00294 (.0058)	.4605	2.718	1.123
5	.02949 (.0066)	-.00318 (.0054)	.01960 (.0057)	-.02538 (.0080)	.6494	2.391	1.297

Table 4.35
RDAM-SD Model (5 Sectors), 1974-1981

Table 4.35.1 Unrestricted RDAM-SD Model (5 Sectors), 1974-1981

	$dlnP_1$	$dlnP_2$	$dlnP_3$	$dlnP_4$	$dlnP_5$	dM
1	-.30994	-.03360	-.00865	-.20244	.49092	.29176
	(.0582)	(.1251)	(.0371)	(.1671)	(.1071)	(.0576)
2	.07503	-.19855	.02995	.07254	-.06518	.11384
	(.0286)	(.0616)	(.0182)	(.0822)	(.0527)	(.0283)
3	.01693	.02504	-.01759	.05802	-.01930	.07310
	(.0357)	(.0767)	(.0227)	(.1024)	(.0656)	(.0353)
4	.09772	.05335	-.00025	.02484	-.18608	.16749
	(.0278)	(.0597)	(.0177)	(.0797)	(.0511)	(.0274)
5	.12026	.15376	-.00345	.04704	-.22036	.35382
	(.0463)	(.0994)	(.0295)	(.1328)	(.0851)	(.0457)

	D_1	D_2	D_3	D_4	R^2	DW	DW4
1	-.07893	.02383	.03148	.03656	.9809	1.993	1.148
	(.0055)	(.0063)	(.0059)	(.0076)			
2	.00969	.02409	-.00954	-.01106	.8589	2.075	2.101
	(.0027)	(.0035)	(.0029)	(.0038)			
3	.00042	-.01848	-.00249	.01197	.8810	2.218	1.327
	(.0034)	(.0041)	(.0038)	(.0047)			
4	.00599	.00053	-.01710	.01021	.9400	2.398	2.037
	(.0026)	(.0029)	(.0033)	(.0036)			
5	.06283	-.02997	-.00234	-.04768	.9630	1.779	1.452
	(.0044)	(.0050)	(.0045)	(.0061)			

Table 4.35.2 Homogeneity Restricted RDAM-SD Model (5 Sectors), 1974-1981

	$dlnP_1$	$dlnP_2$	$dlnP_3$	$dlnP_4$	$dlnP_5$	dM
1	-.29360	-.01190	-.00656	-.17102	.48308	.30458
	(.0511)	(.1200)	(.0371)	(.1588)	(.1068)	(.0534)
2	.09713	-.16918	.03278	.11507	-.07579	.13118
	(.0260)	(.0610)	(.0189)	(.0807)	(.0543)	(.0271)
3	.00075	.00354	-.01966	.02690	-.01153	.06041
	(.0316)	(.0741)	(.0229)	(.0981)	(.0660)	(.0330)
4	.10039	.05690	.00009	.02998	-.18736	.16958
	(.0243)	(.0569)	(.0176)	(.0754)	(.0507)	(.0253)
5	.09533	.12064	-.00665	-.00093	-.20839	.33425
	(.0412)	(.0967)	(.0299)	(.1280)	(.0860)	(.0430)

Table 4.35.2 (continued)

	D_1	D_2	D_3	D_4	R^2	DW	DW4
1	-.08111	.02091	.02860	.03359	.9807	1.970	1.156
	(.0041)	(.0038)	(.0032)	(.0057)			
2	.00675	.02013	-.01344	-.01508	.8478	2.128	2.045
	(.0021)	(.0019)	(.0027)	(.0029)			
3	.00258	-.01558	.00036	.01491	.8777	2.223	1.333
	(.0025)	(.0031)	(.0029)	(.0035)			
4	.00563	.00005	-.01757	.00972	.9400	2.402	2.044
	(.0019)	(.0018)	(.0015)	(.0027)			
5	.06615	-.02551	.00205	-.04315	.9616	1.766	1.483
	(.0033)	(.0029)	(.0032)	(.0046)			

Table 4.35.3 Homogeneity and Symmetry Restricted RDAM-SD Model (5 Sectors), 1974-1981

	$d\ln P_1$	$d\ln P_2$	$d\ln P_3$	$d\ln P_4$	$d\ln P_5$	dY
1	-.24087	.06677	-.01697	.06961	.12147	.26505
	(.0523)	(.0252)	(.0234)	(.0247)	(.0432)	(.0526)
2	.06677	-.15653	.02598	.05211	.01167	.12917
	(.0252)	(.0432)	(.0183)	(.0376)	(.0441)	(.0281)
3	-.01697	.02598	-.02557	-.00272	.01928	.05487
	(.0234)	(.0183)	(.0193)	(.0161)	(.0257)	(.0300)
4	.06961	.05211	-.00272	-.05798	-.06102	.17531
	(.0247)	(.0376)	(.0161)	(.0527)	(.0409)	(.0262)
5	.12147	.01167	.01928	-.06102	-.09140	.37560
	(.0432)	(.0441)	(.0257)	(.0409)	(.0767)	(.0423)

	D_1	D_2	D_3	D_4	R^2	DW	DW4
1	-.07804	.02170	.02866	.03533	.9734	2.351	1.194
	(.0046)	(.0042)	(.0039)	(.0060)			
2	.00614	.02010	-.01268	-.01441	.8335	2.215	2.006
	(.0021)	(.0025)	(.0019)	(.0030)			
3	.00239	-.01549	.00094	.01564	.8759	2.223	1.277
	(.0024)	(.0022)	(.0037)	(.0033)			
4	.00466	-.00011	-.01706	.00988	.9280	2.701	1.718
	(.0021)	(.0019)	(.0020)	(.0028)			
5	.06485	-.02620	.00013	-.04644	.9557	1.647	1.606
	(.0034)	(.0031)	(.0032)	(.0047)			

Table 4.36
RDAM-SD Model (12 Sectors), 1966-1973

Table 4.36.1 Unrestriced RDAM-SD Model (12 Sectors), 1966-1973

	$dlnP_1$	$dlnP_2$	$dlnP_3$	$dlnP_4$	$dlnP_5$	$dlnP_6$	$dlnP_7$
1	-.05726 (.0626)	-.09227 (.0859)	.03618 (.0159)	.04844 (.0859)	.00208 (.0845)	.25873 (.1386)	.08766 (.1250)
2	.05627 (.0290)	-.00559 (.0398)	-.00292 (.0074)	.05553 (.0398)	-.08409 (.0392)	-.25711 (.0642)	.15713 (.0579)
3	.06204 (.0668)	.10931 (.0916)	-.06125 (.0170)	.04396 (.0916)	-.15020 (.0902)	-.06495 (.1479)	.20147 (.1334)
4	.02920 (.0418)	.12277 (.0574)	.02951 (.0106)	-.12370 (.0574)	-.10708 (.0565)	-.04092 (.0927)	.05815 (.0836)
5	.04510 (.0231)	-.05587 (.0317)	.00775 (.0059)	.06711 (.0317)	-.10151 (.0312)	-.08541 (.0512)	.14852 (.0462)
6	-.00182 (.0400)	-.11571 (.0549)	-.00402 (.0102)	.01135 (.0548)	.06482 (.0540)	-.12572 (.0886)	-.04418 (.0799)
7	.06652 (.0204)	.07813 (.0280)	.01133 (.0052)	.05625 (.0280)	.08667 (.0276)	.01836 (.0452)	-.14274 (.0408)
8	.00045 (.0522)	-.00688 (.0717)	-.00206 (.0133)	-.13413 (.0717)	-.00802 (.0706)	.08226 (.1157)	.01390 (.1043)
9	-.04307 (.0207)	-.00319 (.0285)	-.01154 (.0053)	.00405 (.0285)	.00124 (.0280)	-.02353 (.0460)	-.01652 (.0414)
10	-.07566 (.0556)	.08136 (.0763)	-.00114 (.0141)	.00326 (.0762)	.04611 (.0751)	-.12725 (.1231)	-.16207 (.1110)
11	-.00516 (.0146)	-.06354 (.0201)	.00023 (.0037)	.02609 (.0201)	.04501 (.0198)	-.01694 (.0324)	-.03908 (.0292)
12	-.07660 (.0440)	-.04853 (.0604)	-.00208 (.0112)	-.05820 (.0604)	.20498 (.0595)	.38247 (.0975)	-.26224 (.0880)

	$dlnP_8$	$dlnP_9$	$dlnP_{10}$	$dlnP_{11}$	$dlnP_{12}$	dY
1	-.27947 (.2132)	-.00291 (.1540)	-.08398 (.1250)	-.02114 (.0401)	-.12854 (.0852)	.09712 (.0531)
2	.00947 (.0988)	-.05438 (.0713)	-.02375 (.0579)	-.01375 (.0186)	-.01892 (.0395)	.17267 (.0246)
3	.51446 (.2275)	-.07463 (.1643)	.06529 (.1334)	.07280 (.0428)	.02918 (.0909)	.18470 (.0567)
4	-.00803 (.1425)	.07345 (.1029)	.02815 (.0836)	-.01466 (.0268)	.01406 (.0570)	.17212 (.0355)
5	.19186 (.0787)	-.13400 (.0569)	-.05515 (.0462)	.01215 (.0148)	.13858 (.0315)	.07090 (.0196)

Table 4.36.1 (continued)

6	-.07549 (.1362)	-.01993 (.0984)	-.00090 (.0798)	-.00994 (.0256)	.05959 (.0544)	.05267 (.0339)
7	.05751 (.0696)	-.05845 (.0502)	-.04768 (.0408)	-.01462 (.0131)	-.00952 (.0278)	.03112 (.0173)
8	-.14336 (.1779)	.04437 (.1285)	.22484 (.1043)	.03283 (.0334)	-.17202 (.0711)	.13604 (.0443)
9	.12090 (.0707)	.01968 (.0510)	.01285 (.0414)	-.00426 (.0133)	.04318 (.0283)	-.00903 (.0176)
10	-.21635 (.1893)	.08775 (.1367)	-.18466 (.1110)	.01221 (.0356)	.03208 (.0757)	.07149 (.0472)
11	-.01314 (.0499)	.04001 (.0360)	-.00399 (.0292)	-.04057 (.0094)	.04500 (.0199)	.02525 (.0124)
12	-.15836 (.1500)	.07903 (.1083)	.06899 (.0879)	-.01106 (.0282)	-.03266 (.0600)	-.00505 (.0374)

	D_1	D_2	D_3	D_4	R^2	DW	DW4
1	.02416 (.0092)	-.00545 (.0090)	-.00798 (.0070)	.00581 (.0117)	.6608	2.769	2.027
2	.00873 (.0043)	-.00119 (.0042)	-.00213 (.0032)	-.00210 (.0054)	.8709	2.476	1.772
3	-.05793 (.0098)	-.00564 (.0096)	.00232 (.0075)	-.01812 (.0124)	.9129	2.560	1.530
4	-.02937 (.0061)	.00652 (.0060)	-.00239 (.0047)	.01234 (.0078)	.9348	2.825	2.039
5	.00065 (.0034)	.00291 (.0033)	-.00102 (.0026)	-.01419 (.0043)	.7066	2.608	2.040
6	.00680 (.0059)	.02288 (.0058)	.00454 (.0045)	-.00442 (.0075)	.6975	2.826	2.130
7	-.00083 (.0030)	-.01726 (.0029)	-.00974 (.0023)	.01386 (.0038)	.9266	2.397	1.373
8	.01067 (.0077)	-.00741 (.0075)	-.00743 (.0058)	-.00046 (.0097)	.6234	2.479	1.362
9	.00008 (.0030)	.00226 (.0030)	.00150 (.0023)	-.00408 (.0039)	.4989	2.574	2.202
10	.04438 (.0081)	-.01214 (.0080)	.01713 (.0062)	.00108 (.0104)	.8477	2.423	1.734
11	-.00299 (.0021)	.00799 (.0021)	.00573 (.0016)	-.00043 (.0027)	.8416	2.161	2.063
12	-.00434 (.0065)	.00651 (.0063)	-.00053 (.0049)	.01071 (.0082)	.6359	2.183	1.691

Table 4.36.2 Homogeneity Restricted RDAM-SD Model (12 Sectors), 1966-1973

	$dlnP_1$	$dlnP_2$	$dlnP_3$	$dlnP_4$	$dlnP_5$	$dlnP_6$	$dlnP_7$
1	-.05987	-.07579	.03394	.05375	-.00352	.27858	.10879
	(.0631)	(.0840)	(.0158)	(.0864)	(.0850)	(.1375)	(.1231)
2	.05422	.00732	-.00468	.05969	-.08848	-.24155	.17368
	(.0297)	(.0396)	(.0074)	(.0407)	(.0401)	(.0648)	(.0580)
3	.07045	.05633	-.05403	.02689	-.13220	-.12880	.13352
	(.0722)	(.0962)	(.0181)	(.0989)	(.0974)	(.1575)	(.1410)
4	.02989	.11845	.03010	-.12509	-.10561	-.04612	.05262
	(.0418)	(.0557)	(.0105)	(.0573)	(.0564)	(.0912)	(.0817)
5	.04711	-.06856	.00947	.06302	-.09720	-.10071	.13223
	(.0240)	(.0320)	(.0060)	(.0329)	(.0324)	(.0524)	(.0469)
6	-.00477	-.09714	-.00655	.01733	.05851	-.10335	-.02036
	(.0411)	(.0547)	(.0103)	(.0563)	(.0554)	(.0896)	(.0802)
7	.06766	.07092	.01231	.05392	.08912	.00967	-.15199
	(.0207)	(.0276)	(.0052)	(.0284)	(.0280)	(.0452)	(.0405)
8	-.00031	-.00208	-.00271	-.13258	-.00966	.08805	.02007
	(.0522)	(.0695)	(.0131)	(.0715)	(.0704)	(.1138)	(.1019)
9	-.04195	-.01026	-.01057	.00177	.00364	-.03206	-.02559
	(.0210)	(.0280)	(.0053)	(.0288)	(.0284)	(.0459)	(.0411)
10	-.08134	.11711	-.00601	.01477	.03397	-.08417	-.11622
	(.0586)	(.0780)	(.0147)	(.0802)	(.0789)	(.1277)	(.1143)
11	-.00545	-.06169	-.00002	.02668	.04438	-.01471	-.03671
	(.0147)	(.0195)	(.0037)	(.0201)	(.0197)	(.0319)	(.0286)
12	-.07564	-.05460	-.00125	-.06016	.20704	.37515	-.27003
	(.0441)	(.0587)	(.0110)	(.0604)	(.0594)	(.0961)	(.0860)

	$dlnP_8$	$dlnP_9$	$dlnP_{10}$	$dlnP_{11}$	$dlnP_{12}$	dY
1	-.16585	.01556	-.04686	-.02139	-.11734	.08512
	(.1555)	(.1535)	(.1164)	(.0404)	(.0848)	(.0512)
2	.09848	-.03991	.00533	-.01395	-.01014	.16327
	(.0733)	(.0724)	(.0549)	(.0191)	(.0400)	(.0242)
3	.14914	-.13401	-.05406	.07361	-.00685	.22329
	(.1781)	(.1758)	(.1333)	(.0463)	(.0971)	(.0587)
4	-.03779	.06861	.01843	-.01459	.01112	.17526
	(.1031)	(.1018)	(.0772)	(.0268)	(.0562)	(.0340)
5	.10432	-.14823	-.08375	.01234	.12995	.08015
	(.0592)	(.0585)	(.0443)	(.0154)	(.0323)	(.0195)

Table 4.36.2 (continued)

6	.05253	.00087	.04092	−.01022	.07221	.03915
	(.1013)	(.1000)	(.0758)	(.0263)	(.0552)	(.0334)
7	.00778	−.06653	−.06393	−.01451	−.01442	.03638
	(.0511)	(.0505)	(.0383)	(.0133)	(.0279)	(.0168)
8	−.11022	.04976	.23566	.03276	−.16875	.13254
	(.1287)	(.1271)	(.0964)	(.0335)	(.0702)	(.0424)
9	.07213	.01176	−.00308	−.00415	.03837	−.00388
	(.0519)	(.0512)	(.0388)	(.0135)	(.0283)	(.0171)
10	.03014	.12782	−.10413	.01167	.05639	.04546
	(.1444)	(.1426)	(.1081)	(.0376)	(.0787)	(.0476)
11	−.00039	.04208	.00017	−.04060	.04625	.02390
	(.0361)	(.0357)	(.0270)	(.0094)	(.0197)	(.0119)
12	−.20026	.07221	.05530	−.01097	−.03679	−.00062
	(.1086)	(.1073)	(.0814)	(.0283)	(.0592)	(.0358)

	D_1	D_2	D_3	D_4	R^2	DW	DW4
1	.01842	−.01064	−.01174	.00021	.6543	2.848	1.983
	(.0054)	(.0060)	(.0050)	(.0092)			
2	.00423	−.00525	−.00508	−.00649	.8638	2.575	1.735
	(.0026)	(.0029)	(.0024)	(.0043)			
3	−.03949	.01105	.01440	−.00011	.8977	2.562	1.669
	(.0062)	(.0069)	(.0058)	(.0105)			
4	−.02787	.00788	−.00141	.01381	.9346	2.810	2.066
	(.0036)	(.0040)	(.0033)	(.0061)			
5	.00507	.00691	.00188	−.00988	.6821	2.641	2.031
	(.0021)	(.0023)	(.0019)	(.0035)			
6	.00034	.01703	.00031	−.01074	.6795	2.745	2.248
	(.0035)	(.0039)	(.0033)	(.0060)			
7	.00168	−.01499	−.00810	.01631	.9241	2.406	1.429
	(.0018)	(.0020)	(.0017)	(.0030)			
8	.00900	−.00892	−.00852	−.00210	.6226	2.454	1.394
	(.0045)	(.0050)	(.0042)	(.0076)			
9	.00254	.00449	.00312	−.00168	.4828	2.514	2.206
	(.0018)	(.0020)	(.0017)	(.0031)			
10	.03195	−.02340	.00898	−.01107	.8303	2.412	1.670
	(.0050)	(.0056)	(.0047)	(.0085)			
11	−.00364	.00741	.00531	−.00106	.8409	2.185	2.074
	(.0013)	(.0014)	(.0012)	(.0021)			
12	−.00223	.00842	.00086	.01278	.6339	2.227	1.654
	(.0038)	(.0042)	(.0035)	(.0064)			

Table 4.36.3 Homogeneity and Symmetry Restricted RDAM-SD Model (12 Sectors), 1966-1973

	$dlnP_1$	$dlnP_2$	$dlnP_3$	$dlnP_4$	$dlnP_5$	$dlnP_6$	$dlnP_7$
1	-.05145 (.0471)	.05009 (.0258)	.02306 (.0145)	.02590 (.0323)	.02714 (.0136)	.02887 (.0354)	.05848 (.0183)
2	.05009 (.0258)	.00858 (.0333)	-.00010 (.0080)	.03249 (.0268)	-.07714 (.0139)	-.09527 (.0305)	.08039 (.0211)
3	.02306 (.0145)	-.00010 (.0080)	-.05408 (.0161)	.02575 (.0109)	.01064 (.0049)	.00017 (.0101)	.01229 (.0042)
4	.02590 (.0323)	.03249 (.0268)	.02575 (.0109)	-.05306 (.0443)	.04838 (.0157)	-.01135 (.0341)	.03554 (.0205)
5	.02714 (.0136)	-.07714 (.0139)	.01064 (.0049)	.04838 (.0157)	-.10048 (.0133)	-.06817 (.0188)	.12183 (.0149)
6	.02887 (.0354)	-.09527 (.0305)	.00017 (.0101)	-.01135 (.0341)	-.06817 (.0188)	-.05893 (.0526)	.06835 (.0259)
7	.05848 (.0183)	.08039 (.0211)	.01229 (.0042)	.03554 (.0205)	.12183 (.0149)	.06835 (.0259)	-.22323 (.0273)
8	-.02140 (.0409)	.04849 (.0433)	-.00589 (.0109)	-.07349 (.0469)	.05439 (.0269)	.06326 (.0608)	-.02518 (.0358)
9	-.03788 (.0178)	.02760 (.0185)	-.00824 (.0046)	-.03601 (.0225)	-.06988 (.0138)	-.04226 (.0243)	-.01760 (.0187)
10	-.03461 (.0417)	.02310 (.0336)	.00110 (.0143)	.00514 (.0434)	-.07031 (.0194)	.04512 (.0409)	-.06363 (.0274)
11	.01754 (.0139)	-.04451 (.0133)	.00232 (.0041)	-.01786 (.0150)	-.00108 (.0073)	-.00036 (.0155)	-.01699 (.0115)
12	-.08575 (.0409)	-.05372 (.0402)	-.00702 (.0131)	.01857 (.0396)	.12467 (.0178)	.07057 (.0386)	-.03026 (.0263)

	$dlnP_8$	$dlnP_9$	$dlnP_{10}$	$dlnP_{11}$	$dlnP_{12}$	$d\pi$
1	-.02140 (.0409)	-.03788 (.0178)	-.03461 (.0417)	.01754 (.0139)	-.08575 (.0409)	.05635 (.0478)
2	.04849 (.0433)	.02760 (.0185)	.02310 (.0336)	-.04451 (.0133)	-.05372 (.0402)	.16098 (.0258)
3	-.00589 (.0109)	-.00824 (.0046)	.00110 (.0143)	.00232 (.0041)	-.00702 (.0131)	.21645 (.0482)
4	-.07349 (.0469)	-.03601 (.0225)	.00514 (.0434)	-.01786 (.0150)	.01857 (.0396)	.17630 (.0354)
5	.05439 (.0269)	-.06988 (.0138)	-.07031 (.0194)	-.00108 (.0073)	.12467 (.0178)	.07757 (.0168)

Table 4.36.3 (continued)

6	.06326 (.0608)	-.04226 (.0243)	.04512 (.0409)	-.00036 (.0155)	.07057 (.0386)	.06260 (.0336)
7	-.02518 (.0358)	-.01760 (.0187)	-.06363 (.0274)	-.01699 (.0115)	-.03026 (.0263)	.02539 (.0169)
8	-.17090 (.1023)	.09540 (.0327)	.14950 (.0615)	.02792 (.0219)	-.14210 (.0528)	.11565 (.0393)
9	.09540 (.0327)	.05293 (.0272)	.01283 (.0261)	-.00454 (.0093)	.02763 (.0205)	-.00305 (.0180)
10	.14950 (.0615)	.01283 (.0261)	-.11064 (.0688)	.05327 (.0183)	-.01088 (.0507)	.06644 (.0450)
11	.02792 (.0219)	-.00454 (.0093)	.05327 (.0183)	-.04887 (.0099)	.03315 (.0187)	.03489 (.0135)
12	-.14210 (.0528)	.02763 (.0205)	-.01088 (.0507)	.03315 (.0187)	.05515 (.0725)	.01043 (.0424)

	D_1	D_2	D_3	D_4	R^2	DW	DW4
1	.01730 (.0054)	-.01541 (.0047)	-.01257 (.0046)	-.00136 (.0068)	.5684	2.665	1.760
2	.00317 (.0029)	-.00662 (.0028)	-.00613 (.0027)	-.00137 (.0037)	.7956	2.507	2.207
3	-.03752 (.0061)	.00799 (.0050)	.01335 (.0052)	.00788 (.0070)	.8720	2.163	1.542
4	-.02570 (.0040)	.00802 (.0036)	-.00045 (.0035)	.01278 (.0050)	.9019	2.519	2.116
5	.00526 (.0020)	.00653 (.0018)	.00152 (.0017)	-.00742 (.0025)	.6400	2.445	1.795
6	-.00011 (.0036)	.02015 (.0034)	.00080 (.0032)	-.01499 (.0048)	.5864	2.494	2.255
7	.00092 (.0019)	-.01644 (.0019)	-.00877 (.0017)	.02007 (.0026)	.9086	2.541	1.568
8	.00869 (.0044)	-.00893 (.0043)	-.01028 (.0040)	.00244 (.0057)	.5843	2.658	1.248
9	.00156 (.0020)	.00421 (.0019)	.00270 (.0018)	-.00153 (.0026)	.2475	2.299	2.168
10	.02995 (.0053)	-.01934 (.0050)	.01217 (.0046)	-.01636 (.0065)	.7736	2.378	2.008
11	-.00362 (.0015)	.00684 (.0014)	.00517 (.0013)	-.00378 (.0019)	.7447	2.009	1.788
12	.00011 (.0047)	.01298 (.0047)	.00249 (.0043)	.00364 (.0061)	.2847	2.144	2.225

Table 4.37
RDAM-SD Model (12 Sectors), 1974-1981

Table 4.37.1 Unrestriced RDAM-SD Model (12 Sectors), 1974-1981

	$dlnP_1$	$dlnP_2$	$dlnP_3$	$dlnP_4$	$dlnP_5$	$dlnP_6$	$dlnP_7$
1	-.05149 (.0373)	.04942 (.0668)	-.00471 (.0195)	.00152 (.0389)	-.18070 (.1141)	-.06352 (.1196)	.03371 (.0405)
2	.00319 (.0169)	-.08665 (.0303)	.01756 (.0088)	-.00240 (.0177)	-.00876 (.0517)	-.03159 (.0542)	.00930 (.0184)
3	-.01369 (.0320)	-.12047 (.0573)	-.03872 (.0167)	-.02996 (.0334)	.11277 (.0978)	-.07914 (.1025)	-.05211 (.0348)
4	-.03855 (.0299)	.12719 (.0535)	-.02756 (.0156)	.00452 (.0312)	.13953 (.0914)	.03547 (.0958)	-.01527 (.0325)
5	.00660 (.0072)	-.00200 (.0130)	-.00131 (.0038)	.01299 (.0076)	.01623 (.0221)	-.02728 (.0232)	.00543 (.0079)
6	-.01717 (.0174)	.05683 (.0311)	.02158 (.0091)	.02628 (.0181)	-.10195 (.0531)	-.01088 (.0557)	.01687 (.0189)
7	.03313 (.0197)	.03934 (.0354)	-.02232 (.0103)	-.03838 (.0206)	-.02739 (.0604)	.01466 (.0633)	-.03291 (.0215)
8	.02474 (.0185)	-.00427 (.0332)	.02822 (.0097)	.00822 (.0193)	.03533 (.0566)	.04006 (.0593)	.01040 (.0201)
9	.01538 (.0122)	-.00448 (.0218)	-.00684 (.0064)	.00000 (.0127)	-.00846 (.0372)	.07556 (.0390)	.00498 (.0132)
10	.01259 (.0216)	-.05985 (.0387)	.02763 (.0113)	.01382 (.0225)	.13815 (.0660)	.19321 (.0692)	.03349 (.0235)
11	-.00132 (.0101)	-.00466 (.0181)	-.00158 (.0053)	-.00177 (.0105)	-.03602 (.0309)	-.03337 (.0324)	.00523 (.0110)
12	.02659 (.0214)	.00961 (.0383)	.00805 (.0112)	.00517 (.0223)	-.07872 (.0654)	-.11319 (.0685)	-.01911 (.0232)

	$dlnP_8$	$dlnP_9$	$dlnP_{10}$	$dlnP_{11}$	$dlnP_{12}$	dY
1	.48446 (.2062)	-.15411 (.0627)	.10398 (.0621)	-.01405 (.0328)	.00661 (.0556)	.08705 (.0618)
2	.01155 (.0935)	.05304 (.0284)	.00621 (.0282)	-.00358 (.0149)	-.02070 (.0252)	.10249 (.0280)
3	-.27276 (.1768)	.14386 (.0537)	.10908 (.0533)	.09277 (.0282)	.01936 (.0477)	.04782 (.0530)
4	.08548 (.1651)	-.09933 (.0502)	.03036 (.0497)	-.00498 (.0263)	-.03168 (.0445)	.07057 (.0495)
5	.04159 (.0400)	-.01006 (.0122)	.00588 (.0121)	-.00367 (.0064)	-.01834 (.0108)	.00429 (.0120)

Table 4.37.1 (continued)

6	-.07851	.03168	-.03538	-.03361	.03448	.09839
	(.0960)	(.0292)	(.0289)	(.0153)	(.0259)	(.0287)
7	.00454	.00391	-.08951	.03502	.02810	.04997
	(.1091)	(.0332)	(.0329)	(.0174)	(.0294)	(.0327)
8	-.14480	.03775	-.07697	-.02989	-.00787	.17186
	(.1023)	(.0311)	(.0308)	(.0163)	(.0276)	(.0306)
9	-.15761	-.02357	-.01755	.01152	.04805	.08643
	(.0673)	(.0205)	(.0203)	(.0107)	(.0181)	(.0202)
10	-.28005	-.07909	.04276	-.05523	.07465	.16434
	(.1193)	(.0363)	(.0359)	(.0190)	(.0322)	(.0357)
11	.06117	.01842	-.02707	-.02548	-.01054	.03850
	(.0558)	(.0170)	(.0168)	(.0089)	(.0151)	(.0167)
12	.24495	.07751	-.05179	.03118	-.12213	.07829
	(.1181)	(.0359)	(.0356)	(.0188)	(.0319)	(.0354)

	D_1	D_2	D_3	D_4	R^2	DW	DW4
1	-.02087	-.02600	-.01220	.01315	.8779	2.195	1.496
	(.0075)	(.0099)	(.0081)	(.0096)			
2	-.00030	-.00074	.01046	-.00075	.9087	2.971	2.340
	(.0034)	(.0045)	(.0037)	(.0044)			
3	-.03568	.03169	.01456	.01638	.9512	2.422	2.481
	(.0064)	(.0085)	(.0070)	(.0083)			
4	-.03391	-.00101	.00555	-.00849	.9059	2.070	2.366
	(.0060)	(.0079)	(.0065)	(.0077)			
5	.00018	-.00113	-.00169	-.00133	.2478	2.137	1.635
	(.0015)	(.0019)	(.0016)	(.0019)			
6	.00617	.02336	-.00527	-.00815	.8999	2.167	1.739
	(.0035)	(.0046)	(.0038)	(.0045)			
7	.00594	-.01088	.00104	.01720	.9355	2.220	1.874
	(.0040)	(.0052)	(.0043)	(.0051)			
8	.00833	.00608	-.01543	.01449	.9529	2.101	2.121
	(.0037)	(.0049)	(.0040)	(.0048)			
9	.01351	.00770	.00122	-.00560	.7823	2.628	1.823
	(.0024)	(.0032)	(.0026)	(.0031)			
10	.04465	-.03054	.01124	-.03820	.9807	2.005	1.429
	(.0043)	(.0057)	(.0047)	(.0056)			
11	.00737	.00252	.00294	-.00199	.7197	2.891	2.459
	(.0020)	(.0027)	(.0022)	(.0026)			
12	.00462	-.00105	-.01241	.00327	.8018	1.261	2.444
	(.0043)	(.0057)	(.0046)	(.0055)			

Table 4.37.2 Homogeneity Restricted RDAM-SD Model (12 Sectors), 1974-1981

	$dlnP_1$	$dlnP_2$	$dlnP_3$	$dlnP_4$	$dlnP_5$	$dlnP_6$	$dlnP_7$
1	-.06211 (.0373)	.00995 (.0610)	-.00453 (.0200)	-.02040 (.0360)	-.26773 (.0946)	-.05071 (.1223)	.03402 (.0416)
2	.00585 (.0166)	-.07677 (.0272)	.01752 (.0089)	.00309 (.0160)	.01301 (.0421)	-.03479 (.0545)	.00922 (.0185)
3	-.00721 (.0316)	-.09635 (.0517)	-.03882 (.0169)	-.01657 (.0305)	.16595 (.0801)	-.08697 (.1035)	-.05230 (.0352)
4	-.04886 (.0302)	.08882 (.0495)	-.02739 (.0162)	-.01679 (.0291)	.05495 (.0767)	.04792 (.0991)	-.01496 (.0337)
5	.00529 (.0071)	-.00687 (.0117)	-.00129 (.0038)	.01028 (.0069)	.00550 (.0181)	-.02570 (.0234)	.00546 (.0080)
6	-.01266 (.0173)	.07362 (.0283)	.02150 (.0093)	.03560 (.0167)	-.06494 (.0438)	-.01633 (.0567)	.01674 (.0193)
7	.03574 (.0194)	.04902 (.0317)	-.02237 (.0104)	-.03300 (.0187)	-.00604 (.0491)	.01151 (.0634)	-.03298 (.0216)
8	.02871 (.0183)	.01052 (.0299)	.02815 (.0098)	.01643 (.0176)	.06794 (.0464)	.03526 (.0600)	.01028 (.0204)
9	.01855 (.0121)	.00731 (.0198)	-.00689 (.0065)	.00654 (.0117)	.01751 (.0307)	.07174 (.0397)	.00489 (.0135)
10	.00946 (.0212)	-.07146 (.0347)	.02768 (.0113)	.00737 (.0204)	.11256 (.0537)	.19697 (.0694)	.03359 (.0236)
11	.00155 (.0101)	.00599 (.0165)	-.00162 (.0054)	.00415 (.0097)	-.01253 (.0256)	-.03682 (.0331)	.00514 (.0113)
12	.02568 (.0208)	.00622 (.0341)	.00806 (.0112)	.00329 (.0201)	-.08619 (.0529)	-.11209 (.0683)	-.01909 (.0232)

	$dlnP_8$	$dlnP_9$	$dlnP_{10}$	$dlnP_{11}$	$dlnP_{12}$	dY
1	.38847 (.1974)	-.12993 (.0614)	.07011 (.0578)	.00411 (.0305)	.02875 (.0543)	.05942 (.0595)
2	.03557 (.0879)	.04699 (.0274)	.01468 (.0258)	-.00813 (.0136)	-.02623 (.0242)	.10940 (.0265)
3	-.21410 (.1671)	.12908 (.0520)	.12978 (.0489)	.08167 (.0258)	.00583 (.0460)	.06470 (.0504)
4	-.00781 (.1599)	-.07583 (.0497)	-.00256 (.0468)	.01268 (.0247)	-.01017 (.0440)	.04372 (.0482)
5	.02976 (.0377)	-.00708 (.0117)	.00170 (.0111)	-.00144 (.0058)	-.01561 (.0104)	.00088 (.0114)

Table 4.37.2 (continued)

6	-.03768	.02139	-.02098	-.04134	.02507	.11014
	(.0915)	(.0285)	(.0268)	(.0141)	(.0252)	(.0276)
7	.02810	-.00202	-.08120	.03057	.02267	.05675
	(.1024)	(.0319)	(.0300)	(.0158)	(.0282)	(.0309)
8	-.10884	.02869	-.06428	-.03670	-.01616	.18221
	(.0969)	(.0301)	(.0284)	(.0150)	(.0266)	(.0292)
9	-.12896	-.03079	-.00744	.00610	.04145	.09468
	(.0641)	(.0199)	(.0188)	(.0099)	(.0176)	(.0193)
10	-.30827	-.07198	.03280	-.04989	.08116	.15622
	(.1121)	(.0349)	(.0328)	(.0173)	(.0308)	(.0338)
11	.08708	.01189	-.01793	-.03038	-.01651	.04596
	(.0535)	(.0166)	(.0157)	(.0083)	(.0147)	(.0161)
12	.23671	.07959	-.05470	.03274	-.12023	.07592
	(.1103)	(.0343)	(.0323)	(.0170)	(.0303)	(.0332)

	D_1	D_2	D_3	D_4	R^2	DW	DW4
1	-.01274	-.01414	-.00329	.02332	.8714	2.124	1.425
	(.0042)	(.0038)	(.0044)	(.0057)			
2	-.00234	-.00371	.00823	-.00330	.9072	2.971	2.247
	(.0019)	(.0017)	(.0020)	(.0026)			
3	-.04066	.02445	.00911	.01017	.9498	2.356	2.527
	(.0035)	(.0032)	(.0037)	(.0049)			
4	-.02600	.01052	.01421	.00139	.8987	2.082	2.163
	(.0034)	(.0031)	(.0036)	(.0047)			
5	.00118	.00033	-.00059	-.00008	.2319	2.026	1.785
	(.0008)	(.0007)	(.0008)	(.0011)			
6	.00271	.01831	-.00906	-.01248	.8955	2.188	1.869
	(.0019)	(.0018)	(.0020)	(.0027)			
7	.00395	-.01379	-.00115	.01471	.9348	2.195	1.832
	(.0022)	(.0020)	(.0023)	(.0030)			
8	.00528	.00164	-.01877	.01068	.9515	2.093	2.012
	(.0020)	(.0019)	(.0022)	(.0028)			
9	.01108	.00416	-.00144	-.00863	.7727	2.507	1.859
	(.0014)	(.0012)	(.0014)	(.0019)			
10	.04704	-.02705	.01386	-.03521	.9805	2.009	1.473
	(.0024)	(.0022)	(.0025)	(.0033)			
11	.00518	-.00068	.00053	-.00473	.7051	2.877	2.509
	(.0011)	(.0010)	(.0012)	(.0016)			
12	.00532	-.00004	-.01164	.00415	.8016	1.258	2.473
	(.0023)	(.0021)	(.0025)	(.0032)			

Table 4.37.3 Homogeneity and Symmetry Restricted RDAM-SD Model (12 Sectors), 1974-1981

	$dlnP_1$	$dlnP_2$	$dlnP_3$	$dlnP_4$	$dlnP_5$	$dlnP_6$	$dlnP_7$
1	-.08169	-.00994	-.00067	-.02288	.00185	-.00718	.03569
	(.0387)	(.0156)	(.0173)	(.0221)	(.0065)	(.0171)	(.0160)
2	-.00994	-.02665	.01778	.00226	.00822	.02096	.00864
	(.0156)	(.0207)	(.0081)	(.0135)	(.0101)	(.0192)	(.0145)
3	-.00067	.01778	-.01816	-.02041	-.00718	.01021	-.02455
	(.0173)	(.0081)	(.0168)	(.0127)	(.0039)	(.0089)	(.0085)
4	-.02288	.00226	-.02041	-.02435	.01252	.02418	-.02952
	(.0221)	(.0135)	(.0127)	(.0252)	(.0020)	(.0130)	(.0130)
5	.00185	.00822	-.00718	.01252	-.01332	-.05910	-.00477
	(.0065)	(.0101)	(.0039)	(.0020)	(.0089)	(.0102)	(.0064)
6	-.00718	.02096	.01021	.02418	-.05910	-.08241	-.00430
	(.0171)	(.0192)	(.0089)	(.0130)	(.0102)	(.0294)	(.0153)
7	.03569	.00864	-.02455	-.02952	-.00477	-.00430	-.03137
	(.0160)	(.0145)	(.0085)	(.0130)	(.0064)	(.0153)	(.0170)
8	.01831	-.00765	.01568	.00950	.08081	.04503	.01000
	(.0155)	(.0206)	(.0098)	(.0107)	(.0147)	(.0248)	(.0211)
9	.01659	.02952	-.00934	.01296	-.00981	.04552	-.00140
	(.0123)	(.0148)	(.0042)	(.0096)	(.0072)	(.0157)	(.0123)
10	.01454	-.02007	.01590	.02189	.00289	.02546	-.01312
	(.0209)	(.0175)	(.0100)	(.0162)	(.0083)	(.0197)	(.0157)
11	.00889	-.00311	.00637	.01670	.01395	-.02237	.03324
	(.0099)	(.0092)	(.0049)	(.0084)	(.0042)	(.0112)	(.0085)
12	.02648	-.01997	.01436	-.00285	-.02607	.00399	.02146
	(.0179)	(.0138)	(.0087)	(.0147)	(.0068)	(.0161)	(.0133)

	$dlnP_8$	$dlnP_9$	$dlnP_{10}$	$dlnP_{11}$	$dlnP_{12}$	dY
1	.01831	.01659	.01454	.00889	.02648	.01817
	(.0155)	(.0123)	(.0209)	(.0099)	(.0179)	(.0504)
2	-.00765	.02952	-.02007	-.00311	-.01997	.10965
	(.0206)	(.0148)	(.0175)	(.0092)	(.0138)	(.0231)
3	.01568	-.00934	.01590	.00637	.01436	.11466
	(.0098)	(.0042)	(.0100)	(.0049)	(.0087)	(.0442)
4	.00950	.01296	.02189	.01670	-.00285	.04031
	(.0107)	(.0096)	(.0162)	(.0084)	(.0147)	(.0375)
5	.08081	-.00981	.00289	.01395	-.02607	-.00717
	(.0147)	(.0072)	(.0083)	(.0042)	(.0068)	(.0112)

Table 4.37.3 (continued)

6	.04503 (.0248)	.04552 (.0157)	.02546 (.0197)	-.02237 (.0112)	.00399 (.0161)	.08903 (.0254)
7	.01000 (.0211)	-.00140 (.0123)	-.01312 (.0157)	.03324 (.0085)	.02146 (.0133)	.05832 (.0266)
8	-.16294 (.0396)	-.01303 (.0208)	.02308 (.0222)	-.01272 (.0093)	-.00608 (.0158)	.18470 (.0279)
9	-.01303 (.0208)	-.05860 (.0187)	-.03948 (.0151)	.00345 (.0083)	.02362 (.0114)	.08802 (.0180)
10	.02308 (.0222)	-.03948 (.0151)	-.02590 (.0271)	-.02181 (.0106)	.01662 (.0170)	.11756 (.0311)
11	-.01272 (.0093)	.00345 (.0083)	-.02181 (.0106)	-.03552 (.0080)	.01293 (.0087)	.07107 (.0146)
12	-.00608 (.0158)	.02362 (.0114)	.01662 (.0170)	.01293 (.0087)	-.06449 (.0196)	.11566 (.0272)

	D_1	D_2	D_3	D_4	R^2	DW	DW4
1	-.01306 (.0044)	-.00940 (.0042)	-.00621 (.0042)	.03176 (.0058)	.7866	1.957	1.697
2	-.00332 (.0019)	-.00374 (.0018)	.00873 (.0018)	-.00314 (.0025)	.8862	3.023	2.175
3	-.03861 (.0038)	.02252 (.0036)	.01053 (.0036)	.00301 (.0051)	.9059	2.611	2.132
4	-.02561 (.0033)	.01262 (.0031)	.01433 (.0031)	.00377 (.0042)	.8632	2.615	2.548
5	.00041 (.0009)	-.00044 (.0008)	-.00057 (.0009)	-.00010 (.0012)	-.2446	2.579	1.585
6	.00318 (.0020)	.01801 (.0018)	-.00863 (.0020)	-.01099 (.0027)	.8549	1.893	1.810
7	.00355 (.0021)	-.01463 (.0021)	-.00027 (.0021)	.01455 (.0029)	.9203	2.159	1.430
8	.00465 (.0023)	.00025 (.0020)	-.01740 (.0021)	.00974 (.0030)	.9156	2.577	1.327
9	.01141 (.0015)	.00327 (.0014)	-.00147 (.0014)	-.00939 (.0020)	.7042	2.612	1.942
10	.04504 (.0027)	-.02931 (.0025)	.01325 (.0025)	-.03450 (.0034)	.9682	2.141	1.300
11	.00596 (.0012)	.00023 (.0012)	-.00121 (.0012)	-.00635 (.0016)	.5763	2.373	2.729
12	.00640 (.0024)	.00062 (.0022)	-.01108 (.0022)	.00165 (.0030)	.7345	1.829	1.709

Table 4.38
Comaparison of Compensated Own Price Elasticities between Two Subsamples for the Periods 1966-1973 and 1974-1981 (5 Sectors)

	1966-1973	1974-1981
1	.04506 (.2570)	-.53562 (.1163)
2	-5.92236 (1.8856)	-1.69222 (.4670)
3	-1.90391 (.5945)	-.38742 (.2924)
4	-1.21204 (.9158)	-.54544 (.4958)
5	.15189 (.3719)	-.32104 (.2687)

Table 4.39
Comparison of Total Expenditure Elasticities between Two Subsamples for the Periods 1966-1973 and 1974-1981 (Aggregated 5 Sectors)

	Average budget share		Total expendenditure elasticity			
			Unrestricted model		Restricted model	
	66-73	74-81	66-73	74-81	66-73	74-81
1	.4849	.4497	1.25822 (.1590)	.64879 (.1281)	1.22240 (.1518)	.58939 (.1170)
2	.0568	.0925	1.58063 (.7799)	1.23070 (.3059	1.71426 (.6901))	1.39643 (.3038)
3	.0614	.0660	.63436 (.2964)	1.10758 (.5348)	.67850 (.2687)	.83136 (.4545)
4	.1105	.1063	1.26869 (.4443)	1.57564 (.2578)	1.14253 (.3846)	1.64920 (.2465)
5	.2864	.2855	.42242 (.2133)	1.23930 (.1601)	.49574 (.2029)	1.31559 (.1482)

Table 4.40
Comparison of Total Expenditure Elasticities between two Subsamples for the Periods 1966-1973 and 1974-1981 (12 Sectors)

	Average budget share		Total expendenditure elasticity			
			Unrestricted model		Restricted model	
	66-73	74-81	66-73	74-81	66-73	74-81
1	.2221	.1690	.43728 (.2391)	.51509 (.3659)	.25371 (.2152)	.10751 (.2982)
2	.0967	.0939	1.78563 (.2544)	1.09148 (.2982)	1.66474 (.2668)	1.16773 (.2460)
3	.0817	.0770	2.26071 (.6940)	.62104 (.6883)	2.64933 (.5900)	1.48910 (.5740)
4	.0845	.1098	2.03692 (.4201)	.64271 (.4508)	2.08639 (.4189)	.36712 (.3415)
5	.0170	.0261	4.17118 (1.1529)	.16437 (.4598)	4.56294 (.9882)	-.27271 (.4291)
6	.0397	.0665	1.32670 (.8539)	1.47955 (.4316)	1.57683 (.8463)	1.33880 (.3820)
7	.0614	.0660	.50684 (.2818)	.75712 (.4955)	.41352 (.2752)	.88364 (.4030)
8	.1105	.1063	1.23113 (.4009)	1.61675 (.2879)	1.04661 (.3557)	1.73754 (.2625)
9	.0300	.0566	.30100 (.5867)	1.52703 (.3569)	.10167 (.6000)	1.55512 (.3180)
10	.1023	.0857	.69883 (.4614)	1.91762 (.4166)	.64946 (.4399)	1.37176 (.3629)
11	.0479	.0537	.52714 (.2589)	.71695 (.3110)	.72839 (.2818)	1.32346 (.2719)
12	.1063	.0894	-.04751 (.3518)	.87573 (.3960)	.09812 (.3989)	1.29374 (.3043)

Table 4.41
Comparison of Compensated Own Price Elasticities between Two Subsamples for the Periods 1966-1973 and 1974-1981 (12 Sectors)

Sector	1966-1973	1974-1981
1	-.23165 (.2121)	-.48337 (.2290)
2	.08873 (.3444)	-.28381 (.2204)
3	-.66193 (.1971)	-.23584 (.2182)
4	-.62793 (.5243)	-.22177 (.2295)
5	-5.91059 (.7824)	-.51034 (.3410)
6	-1.48438 (1.3249)	-1.23925 (.4421)
7	-3.63567 (.4528)	-.47530 (.2576)
8	-1.54661 (.9258)	-1.53283 (.3725)
9	1.76433 (.9067)	-1.03534 (.3304)
10	-1.08155 (.6725)	-.30222 (.3162)
11	-1.02025 (.2067)	-.66145 (.1490)
12	.51881 (.6820)	-.72136 (.2192)

Appendix

Quarterly Consumer Price Index and Household Expenditure Survey Data for 1965-1981.

Table 4.A.1
Consumer Price Index, Aggregated 5 Sectors (1975 = 100.0)

Quarter	Food (1)	Housing (2)	Fuel & Light (3)	Clothing (4)	Miscell. (5)
651	23.2	31.7	27.8	34.9	29.8
652	24.1	33.9	27.8	35.1	30.8
653	24.7	34.4	28.5	35.4	30.8
654	23.3	35.9	29.1	37.6	31.5
661	24.4	37.4	30.9	38.9	32.4
662	25.5	39.6	33.5	39.2	35.2
663	26.1	41.4	34.4	40.4	35.8
664	26.3	45.7	38.2	43.1	35.6
671	26.5	47.1	38.3	44.7	36.2
672	28.2	48.5	38.0	44.8	37.5
673	28.0	50.3	41.4	45.2	38.0
674	28.4	51.6	43.1	47.1	41.0
681	29.6	52.4	43.2	48.3	42.7
682	30.0	53.1	42.0	48.1	44.3
683	30.2	54.3	42.8	48.5	44.8
684	32.3	55.4	45.9	49.6	45.1
691	33.9	55.8	46.0	50.9	46.6
692	34.3	56.7	46.0	52.4	48.2
693	35.1	56.9	46.1	52.5	49.5
694	38.4	57.8	46.9	53.0	52.5
701	41.4	59.8	49.0	54.4	53.4
702	43.0	60.9	49.4	54.7	54.7
703	42.6	62.7	49.6	55.1	55.5
704	45.1	64.0	52.2	56.3	58.5
711	51.0	64.6	52.4	56.8	59.8
712	50.4	65.3	52.6	57.7	60.9
713	51.9	66.8	54.2	58.8	61.6
714	51.3	69.2	57.7	61.0	62.9
721	55.1	71.2	58.5	62.1	66.0
722	58.1	71.8	58.2	63.3	68.6
723	60.7	73.2	59.5	64.4	68.8
724	57.9	73.4	62.5	65.2	69.0
731	57.8	73.6	62.8	67.5	68.4
732	58.7	74.4	62.5	69.9	67.9
733	59.7	76.4	62.3	71.8	68.1
734	61.6	77.8	64.9	76.8	69.0
741	70.4	81.9	74.3	82.1	76.4
742	73.5	85.7	90.2	87.9	81.5
743	78.6	89.2	91.7	89.3	82.9
744	80.9	90.9	92.0	90.4	85.7

Table 4.A.1 (continued)

751	88.9	96.0	93.0	92.2	88.9
752	96.5	99.1	102.1	99.3	97.0
753	104.8	101.6	102.2	103.0	104.2
754	109.8	103.3	102.4	105.5	109.6
761	111.5	106.1	102.7	107.1	113.3
762	115.6	110.1	104.0	111.7	116.1
763	123.1	113.0	105.1	115.0	116.8
764	121.2	115.4	105.1	118.5	117.0
771	125.0	118.2	117.0	121.0	120.2
772	128.3	120.1	123.0	124.9	123.8
773	136.5	120.4	122.6	126.3	124.1
774	136.1	124.0	126.0	129.3	126.1
781	144.1	127.3	153.6	133.6	130.6
782	148.0	132.3	153.6	138.7	135.4
783	157.8	137.0	153.7	141.7	139.3
784	163.6	140.7	153.8	147.8	141.0
791	169.4	147.4	154.2	156.1	149.2
792	175.0	163.5	194.2	171.8	162.4
793	174.1	175.5	210.1	183.8	169.1
794	179.8	183.5	210.8	194.1	175.2
801	200.7	195.5	214.1	206.1	196.9
802	212.1	210.4	256.4	222.6	213.3
803	220.7	217.0	277.4	234.6	225.6
804	250.9	221.2	281.4	240.8	231.9
811	273.7	225.3	285.3	247.9	241.8
812	281.0	233.5	326.0	259.8	256.6
813	296.5	241.1	354.3	267.4	265.9
814	290.4	244.7	369.3	274.3	268.9

Table 4.A.2
Monthly Average of Household Expenditure by Level, Aggregated 5 Sectors (Won)

Quarter	Total expenditure	Food (1)	Housing (2)	Fuel & Light (3)	Clothing (4)	Miscell. (5)
651	7860.	4790.	250.	600.	490.	1730.
652	7900.	5100.	320.	470.	550.	1460.
653	9080.	5860.	350.	480.	660.	1730.
654	10090.	6450.	370.	710.	810.	1750.
661	9840.	5440.	310.	820.	830.	2440.
662	10300.	5840.	530.	660.	930.	2340.
663	11930.	6800.	470.	710.	1160.	2790.
664	14330.	8320.	460.	1180.	1280.	3090.
671	16140.	8230.	590.	1260.	1890.	4170.
672	16060.	7650.	1070.	1080.	1990.	4270.
673	16740.	8150.	990.	1070.	2070.	4460.
674	21660.	12120.	1180.	1360.	2440.	4560.
681	20290.	9620.	1500.	1400.	2300.	5470.
682	19540.	9040.	1170.	1090.	2500.	5740.
683	19890.	9180.	1030.	940.	2450.	6290.
684	21620.	11470.	910.	1400.	2720.	5120.
691	20500.	9650.	1130.	1490.	2570.	5660.
692	21280.	9810.	1470.	1140.	2820.	6040.
693	22960.	10390.	1910.	1000.	2520.	7140.
694	26250.	12810.	1560.	1690.	3270.	6920.
701	24470.	11190.	1310.	1890.	2930.	7150.
702	24360.	11320.	1680.	1450.	2740.	7170.
703	25700.	11770.	1620.	1360.	2660.	8290.
704	29430.	14220.	1530.	1950.	3700.	8030.
711	28650.	13310.	1290.	2110.	3290.	8650.
712	29170.	13420.	2130.	1860.	3070.	8690.
713	29250.	13790.	1690.	1490.	3170.	9110.
714	33290.	16810.	1710.	2150.	3630.	8990.
721	31180.	14590.	1370.	2110.	3460.	9650.
722	31270.	15020.	1730.	1860.	3050.	9610.
723	33700.	16280.	2040.	1790.	3170.	10420.
724	35180.	16950.	2060.	1940.	3300.	10930.
731	33680.	15780.	1810.	1940.	3950.	10200.
732	34290.	16040.	2250.	1850.	3740.	10410.
733	35170.	16840.	2230.	1840.	3310.	10950.
734	39050.	19880.	1960.	2480.	4050.	10680.
741	39300.	18700.	2080.	2510.	3760.	12250.
742	41140.	19820.	3050.	2570.	3990.	11710.
743	43950.	21750.	2910.	2550.	4130.	12610.
744	49710.	26420.	2680.	3480.	4940.	12190.
751	52090.	23260.	3360.	3790.	5670.	16010.
752	54310.	25310.	5120.	3070.	5790.	15020.
753	60390.	30100.	4680.	3210.	4950.	17450.
754	66470.	35050.	4670.	3970.	6370.	16410.

Table 4.A.2 (continued)

761	68230.	30240.	5300.	4330.	7000.	21360.
762	69060.	32670.	6510.	3360.	7090.	19430.
763	72280.	35700.	5990.	3180.	6330.	21080.
764	81080.	41400.	5770.	4790.	8760.	20360.
771	76130.	33590.	5720.	5070.	8060.	23690.
772	77090.	36280.	7890.	4110.	8170.	20640.
773	83680.	41590.	8150.	4150.	8260.	21530.
774	98160.	49630.	8770.	6090.	11310.	22360.
781	95270.	41260.	9550.	6510.	10920.	27030.
782	103270.	46190.	11790.	5930.	11690.	27670.
783	114460.	53830.	11980.	5410.	11800.	31440.
784	134900.	64140.	13950.	7820.	16490.	32500.
791	137835.	54619.	14889.	7690.	17346.	43291.
792	139953.	59021.	16801.	6436.	16878.	40817.
793	145553.	64427.	16042.	8216.	14859.	42009.
794	167146.	74605.	15390.	11964.	21854.	43333.
801	169630.	66390.	14993.	12952.	20841.	54454.
802	167445.	70698.	18387.	9659.	18730.	49971.
803	180842.	80954.	17332.	11850.	16980.	53726.
804	199166.	91945.	16609.	17734.	21493.	51385.
811	204706.	80572.	17216.	17007.	21918.	67993.
812	199351.	85904.	20990.	13029.	19388.	60040.
813	221135.	98250.	20188.	16323.	18372.	68002.
814	235903.	107077.	18909.	22120.	23912.	63885.

Table 4.A.3
Consumer Price Index, Disggregated 12 Sectors (1975 = 100.0)

Quarter	Cereal (1)	Meat & Fish (2)	Fruit & Veg. (3)	Other Food (4)	Rent (5)	Other Housing (6)	Fuel & Light (7)	Clothing (8)
651	20.5	22.2	35.0	24.7	26.2	40.5	27.8	34.9
652	20.9	23.5	40.0	25.2	29.9	41.3	27.8	35.1
653	21.8	25.3	36.8	26.1	31.2	41.3	28.5	35.4
654	20.2	25.4	27.7	27.9	33.0	42.1	29.1	37.6
661	20.4	25.1	34.4	29.2	35.0	43.0	30.9	38.9
662	21.3	25.7	35.2	31.7	38.9	43.7	33.5	39.2
663	22.7	28.0	30.6	31.5	41.3	44.8	34.4	40.4
664	22.5	29.7	36.8	29.3	50.6	45.2	38.2	43.1
671	21.5	30.1	41.7	29.0	51.4	47.4	38.3	44.7
672	24.3	29.9	40.3	29.9	54.1	47.9	38.0	44.8
673	23.7	33.2	38.4	29.8	57.7	48.3	41.4	45.2
674	23.2	34.8	41.3	30.1	57.7	49.7	43.1	47.1
681	24.2	35.4	43.8	31.9	57.7	51.0	43.2	48.3
682	24.9	37.2	39.4	32.6	57.7	51.5	42.0	48.1
683	25.6	38.6	35.8	32.8	58.1	55.4	42.8	48.5
684	29.0	39.0	34.1	33.0	59.6	56.0	45.9	49.6
691	30.8	38.9	36.5	33.6	59.8	56.7	46.0	50.9
692	30.4	39.4	40.7	34.2	61.3	57.1	46.0	52.4
693	29.9	41.7	44.0	35.7	61.3	56.9	46.1	52.5
694	31.4	42.9	53.4	41.5	62.2	57.5	46.9	53.0
701	33.5	44.2	63.7	43.4	62.8	59.6	49.0	54.4
702	34.4	47.1	66.8	45.1	64.4	60.1	49.4	54.7
703	35.5	48.1	54.3	46.9	67.3	60.4	49.6	55.1
704	37.4	49.7	64.7	45.6	69.2	61.0	52.2	56.3
711	39.9	52.6	91.8	47.9	69.8	61.5	52.4	56.8
712	42.9	55.4	69.1	50.5	70.7	61.9	52.6	57.7
713	44.0	56.5	75.2	50.1	73.2	62.4	54.2	58.8
714	48.3	56.6	53.7	51.2	76.5	64.0	57.7	61.0
721	55.4	56.3	53.9	53.5	77.7	65.7	58.5	62.1
722	58.8	57.9	61.4	54.3	77.8	66.6	58.2	63.3
723	58.7	59.3	76.4	55.7	79.2	67.5	59.5	64.4
724	57.2	58.3	63.5	54.8	79.8	67.2	62.5	65.2
731	57.3	59.9	60.2	54.8	79.9	67.3	62.8	67.5
732	56.2	63.3	65.0	55.6	80.7	68.2	62.5	69.9
733	54.3	65.1	74.4	56.4	82.0	71.1	62.3	71.8
734	55.9	68.5	79.1	55.5	82.7	73.9	64.9	76.8
741	62.6	74.5	100.8	63.8	84.6	81.0	74.3	82.1
742	66.8	81.6	87.0	73.6	86.9	86.9	90.2	87.9
743	73.5	80.4	97.8	77.0	91.9	88.5	91.7	89.3
744	80.4	82.9	80.7	81.6	93.7	89.6	92.0	90.4
751	90.1	89.3	83.2	90.1	95.5	96.4	93.0	92.2
752	97.5	97.1	91.1	97.8	99.2	98.9	102.1	99.3
753	104.2	103.4	108.4	104.4	101.7	101.6	102.2	103.0
754	108.1	110.2	117.0	107.8	103.6	103.0	102.4	105.5

Table A.I.3 (continued)

Quarter	Cereal (1)	Meat & Fish (2)	Fruit & Veg. (3)	Other Food (4)	Rent (5)	Other Housing (6)	Fuel & Light (7)	Clothing (8)
761	108.8	121.2	106.1	112.3	106.7	105.7	102.7	107.1
762	112.0	131.9	108.7	114.7	111.4	109.0	104.0	111.7
763	123.8	134.6	118.4	115.7	117.0	109.9	105.1	115.0
764	121.9	138.1	109.5	114.9	120.5	111.4	105.1	118.5
771	121.2	147.3	121.1	117.6	124.1	113.4	117.0	121.0
772	123.3	158.8	118.4	121.1	126.7	114.7	123.0	124.9
773	126.7	167.2	147.3	124.2	133.2	111.1	122.6	126.3
774	123.1	171.1	149.0	125.3	139.1	111.8	126.0	129.3
781	127.7	189.7	161.1	128.6	143.2	114.3	153.6	133.6
782	129.6	192.9	164.3	137.8	148.9	118.8	153.6	138.7
783	134.3	198.9	192.2	148.2	157.7	120.1	153.7	141.7
784	134.6	205.9	184.0	174.2	159.2	125.7	153.8	147.8
791	143.9	210.0	181.3	180.4	164.6	133.4	154.2	156.1
792	149.2	223.9	178.4	185.9	181.8	148.5	194.2	171.8
793	150.9	228.6	180.4	173.4	199.0	156.3	210.1	183.8
794	158.0	231.3	205.9	164.2	211.6	160.6	210.8	194.1
801	168.3	253.4	263.4	179.8	222.4	173.7	214.1	206.1
802	171.4	274.7	265.6	206.5	234.6	190.6	256.4	222.6
803	179.0	289.5	260.0	222.5	248.3	191.6	277.4	234.6
804	215.8	319.5	301.9	230.8	256.0	192.9	281.4	240.8
811	224.7	340.3	367.1	253.0	260.3	196.7	285.3	247.9
812	229.3	366.5	358.0	262.3	265.3	207.7	326.0	259.8
813	249.1	386.6	367.5	270.2	271.3	216.5	354.3	267.4
814	237.3	383.9	345.5	284.2	275.7	219.5	369.3	274.3

Table 4.A.3 (continued)
Consumer Price Index, Disggregated 12 Sectors (1975 = 100.0)

Quarter	Medical Care (9)	Education (10)	Transp. & Comm. (11)	Miscell. (12)
651	31.4	24.1	22.2	53.8
652	31.6	25.9	22.2	54.1
653	32.3	25.8	22.2	54.2
654	32.8	25.9	24.6	54.4
661	34.6	27.2	26.9	54.9
662	36.5	29.4	32.3	55.2
663	38.5	29.1	34.8	55.3
664	39.3	28.0	34.8	55.5
671	40.1	28.3	34.8	56.0
672	41.3	30.9	34.8	57.1
673	42.2	31.5	34.8	58.8
674	42.6	32.6	40.0	63.3

Table 4.A.3 (continued)
Consumer Price Index, Disggregated 12 Sectors (1975 = 100.0)

Quarter	Medical Care (9)	Education (10)	Transp. & Comm. (11)	Miscell. (12)
681	43.4	33.8	43.4	66.6
682	44.9	36.2	44.8	66.9
683	46.0	36.3	44.8	67.0
684	47.2	36.3	44.8	67.6
691	48.8	36.8	44.8	73.9
692	51.6	38.8	44.8	74.8
693	53.6	40.9	44.9	75.2
694	54.9	46.6	45.2	75.8
701	55.6	46.4	46.5	76.8
702	55.9	48.9	46.5	77.3
703	57.0	49.6	48.1	77.6
704	57.7	50.9	59.8	78.1
711	58.7	52.4	59.8	79.6
712	60.2	53.9	59.8	80.2
713	61.0	55.1	59.8	80.5
714	62.0	57.6	59.8	81.1
721	66.7	60.6	65.8	81.3
722	68.0	65.4	68.8	81.4
723	68.2	65.6	68.8	81.6
724	68.1	65.9	68.8	81.9
731	68.0	65.7	68.8	78.4
732	67.6	66.0	68.8	75.3
733	67.5	66.2	68.8	76.3
734	67.9	66.5	68.8	77.0
741	70.4	73.4	83.2	79.8
742	79.2	79.0	89.7	81.3
743	83.4	80.8	90.6	81.4
744	83.9	86.0	92.3	81.9
751	93.9	89.6	92.0	83.5
752	98.9	94.7	92.0	102.1
753	102.6	100.9	107.9	106.9
754	104.5	114.7	108.1	107.6
761	105.4	122.6	108.4	109.7
762	106.9	128.9	108.4	111.1
763	107.0	128.9	108.4	113.3
764	106.9	129.2	108.4	114.0
771	107.7	136.0	111.4	114.2
772	110.5	145.7	111.4	113.2
773	113.4	146.0	111.3	112.4
774	113.8	146.5	119.1	113.3
781	121.8	154.0	121.3	113.6
782	122.7	164.1	126.8	114.0
783	122.7	164.8	146.3	113.7
784	124.6	167.0	146.3	115.9

Table 4.A.3 (continued)

Quarter	Medical Care (9)	Education (10)	Transp. & Comm. (11)	Miscell. (12)
791	140.0	178.8	146.3	120.5
792	149.6	193.9	168.2	128.1
793	159.2	197.6	172.3	138.3
794	165.7	200.7	172.3	148.5
801	173.1	219.4	233.7	160.1
802	197.7	242.0	237.1	172.5
803	207.2	245.3	249.1	197.7
804	206.6	247.5	259.4	210.0
811	214.4	266.6	262.2	214.8
812	221.3	297.8	274.5	216.4
813	223.6	301.0	315.3	216.7
814	219.1	303.0	318.4	225.2

Table 4.A.4
Monthly Average of Household Expenditure by Level, Disggregated 12 Sectors (Won)

Quarter	Total Expenditure	Cereal (1)	Meat & Fish (2)	Fruit & Veg. (3)	Other Food (4)	Rent (5)	Other Housing (6)	Fuel & Light (7)
651	7860.	3250.	640.	450.	450.	120.	130.	600.
652	7900.	3380.	660.	570.	490.	110.	210.	470.
653	9080.	3400.	880.	950.	630.	120.	230.	480.
654	10090.	3300.	950.	1030.	1170.	140.	230.	710.
661	9840.	3390.	890.	530.	630.	140.	170.	820.
662	10300.	3510.	920.	710.	700.	150.	380.	660.
663	11930.	3700.	1050.	1160.	890.	150.	320.	710.
664	14330.	3810.	1350.	1780.	1380.	150.	310.	1180.
671	16140.	4370.	1540.	1150.	1170.	170.	420.	1260.
672	16060.	4230.	1330.	1070.	1020.	150.	920.	1080.
673	16740.	3920.	1540.	1480.	1210.	180.	810.	1070.
674	21660.	3970.	2290.	3310.	2550.	620.	560.	1360.
681	20290.	4210.	2210.	2030.	1170.	720.	780.	1400.
682	19540.	4300.	2040.	1360.	1340.	210.	960.	1090.
683	19890.	4110.	1740.	1940.	1390.	190.	840.	940.
684	21620.	4970.	2450.	2180.	1870.	210.	700.	1400.
691	20500.	5220.	2090.	1070.	1270.	380.	750.	1490.
692	21280.	4870.	1950.	1430.	1560.	400.	1070.	1140.
693	22960.	4310.	2360.	1920.	1800.	380.	1530.	1000.
694	26250.	4840.	2660.	2620.	2690.	290.	1270.	1690.
701	24470.	5770.	2330.	1440.	1650.	390.	920.	1890.
702	24360.	5090.	2330.	1880.	2020.	450.	1230.	1450.
703	25700.	4850.	2520.	2360.	2040.	450.	1170.	1360.
704	29430.	5130.	2800.	3320.	2970.	450.	1080.	1950.
711	28650.	6280.	2760.	1970.	2300.	470.	820.	2110.
712	29170.	5740.	2950.	2240.	2490.	630.	1500.	1860.
713	29250.	5870.	2800.	2450.	2670.	560.	1130.	1490.
714	33290.	6840.	3510.	2920.	3540.	550.	1160.	2150.
721	31180.	7650.	3070.	1460.	2410.	470.	900.	2110.
722	31270.	7830.	2710.	1940.	2540.	520.	1210.	1860.
723	33700.	7700.	2930.	2650.	3000.	590.	1450.	1790.
724	35180.	7680.	3130.	2770.	3370.	630.	1430.	1940.
731	33680.	7720.	3480.	2080.	2500.	630.	1180.	1940.
732	34290.	7490.	3420.	2270.	2860.	680.	1570.	1850.
733	35170.	7790.	3230.	2680.	3140.	690.	1540.	1840.
734	39050.	7670.	3700.	4120.	4390.	770.	1190.	2480.
741	39300.	9380.	3470.	2430.	3420.	820.	1260.	2510.
742	41140.	9380.	3300.	2910.	4230.	910.	2140.	2570.
743	43950.	9000.	4080.	3980.	4690.	1090.	1820.	2550.
744	49710.	11670.	4330.	4410.	6010.	1070.	1610.	3480.
751	52090.	11410.	4380.	2550.	4920.	970.	2390.	3790.
752	54310.	12410.	3990.	3360.	5550.	1020.	4100.	3070.
753	60390.	13150.	5240.	4770.	6940.	1040.	3640.	3210.
754	66470.	15720.	5500.	6700.	7130.	1120.	3550.	3970.

Table 4.A.4 (continued)

Quarter	Total Expenditure	Cereal (1)	Meat & Fish (2)	Fruit & Veg. (3)	Other Food (4)	Rent (5)	Other Housing (6)	Fuel & Light (7)
761	68230.	15190.	5680.	3360.	6010.	1400.	3900.	4330.
762	69060.	15280.	5560.	4610.	7220.	1490.	5020.	3360.
763	72280.	15100.	6640.	5820.	8140.	1600.	4390.	3180.
764	81080.	18090.	7430.	6640.	9240.	1510.	4260.	4790.
771	76130.	16300.	6460.	3750.	7080.	1750.	3970.	5070.
772	77090.	16170.	6470.	5250.	8390.	1950.	5940.	4110.
773	83680.	16650.	8110.	7320.	9510.	2020.	6130.	4150.
774	98160.	18670.	9300.	11150.	10510.	2230.	6540.	6090.
781	95270.	17860.	9510.	5150.	8740.	2480.	7070.	6510.
782	103270.	17390.	9660.	7840.	11300.	2520.	9270.	5930.
783	114460.	17430.	11850.	11140.	13410.	2720.	9260.	5410.
784	134900.	21500.	13210.	10820.	18610.	3230.	10720.	7820.
791	137835.	20489.	13707.	7418.	13005.	3753.	11136.	7690.
792	139953.	20060.	13346.	10092.	15523.	4438.	12363.	6436.
793	145553.	20024.	13782.	12103.	18518.	4304.	11738.	8216.
794	167146.	26090.	16830.	13908.	17777.	4637.	10753.	11964.
801	169630.	24117.	15717.	10598.	15958.	4848.	10145.	12952.
802	167445.	22966.	15464.	13649.	18619.	5232.	13155.	9659.
803	180842.	25118.	18417.	14929.	22490.	5378.	11954.	11850.
804	199166.	32953.	19046.	17338.	22608.	5731.	10878.	17734.
811	204706.	29450.	19128.	12396.	19598.	5666.	11550.	17007.
812	199351.	29130.	18532.	15189.	23053.	5894.	15096.	13029.
813	221135.	30885.	21864.	19461.	26040.	6151.	14037.	16323.
814	235903.	37856.	22660.	20035.	26526.	6109.	12800.	22120.

Table 4.A.4 (continued)
Monthly Average of Household Expenditure by Level, Disggregated 12 Sectors (Won)

Quarter	Clothing (8)	Medical Care (9)	Education (10)	Transp. & Comm. (11)	Miscell. (12)
651	490.	110.	750.	160.	710.
652	550.	100.	390.	190.	780.
653	660.	110.	610.	210.	800.
654	810.	110.	530.	220.	890.
661	830.	150.	960.	250.	1080.
662	930.	150.	630.	340.	1220.
663	1160.	200.	900.	420.	1270.
664	1280.	190.	970.	440.	1490.
671	1890.	320.	1730.	430.	1690.
672	1990.	330.	1160.	650.	2130.
673	2070.	400.	1400.	680.	1980.
674	2440.	330.	1390.	770.	2070.

Table 4.A.4 (continued)

Quarter	Clothing (8)	Medical Care (9)	Education (10)	Transp. & Comm. (11)	Miscell. (12)
681	2300.	630.	2250.	760.	1830.
682	2500.	720.	2200.	880.	1940.
683	2450.	680.	2200.	880.	2530.
684	2720.	620.	1680.	930.	1890.
691	2570.	580.	2220.	830.	2030.
692	2820.	800.	1690.	1120.	2430.
693	2520.	950.	2450.	1230.	2510.
694	3270.	790.	2130.	1280.	2720.
701	2930.	830.	2750.	1080.	2490.
702	2740.	900.	2240.	1360.	2670.
703	2660.	1080.	2670.	1650.	2890.
704	3700.	880.	2450.	1650.	3050.
711	3290.	760.	3320.	1440.	3130.
712	3070.	960.	2790.	1490.	3450.
713	3170.	980.	3170.	1550.	3410.
714	3630.	930.	3010.	1580.	3470.
721	3460.	840.	3920.	1560.	3330.
722	3050.	1060.	3240.	1660.	3650.
723	3170.	1060.	4030.	1880.	3450.
724	3300.	1110.	4100.	1900.	3820.
731	3950.	1020.	4310.	1640.	3230.
732	3740.	1010.	4450.	1670.	3280.
733	3310.	1120.	4440.	1900.	3490.
734	4050.	1200.	3640.	1800.	4040.
741	3760.	1220.	4980.	1950.	4100.
742	3990.	1380.	3380.	2240.	4710.
743	4130.	1250.	4720.	2260.	4380.
744	4940.	1360.	3830.	2270.	4730.
751	5670.	2350.	6030.	2550.	5080.
752	5790.	2520.	4640.	2440.	5420.
753	4950.	2910.	5860.	3020.	5660.
754	6370.	2580.	4820.	2750.	6260.
761	7000.	3550.	8210.	3050.	6550.
762	7090.	3300.	6090.	3080.	6960.
763	6330.	3700.	7190.	3290.	6900.
764	8760.	3730.	5570.	3280.	7780.
771	8060.	3830.	8990.	3980.	6890.
772	8170.	4020.	6530.	4030.	6060.
773	8260.	3990.	7550.	4240.	5750.
774	11310.	4320.	6260.	4690.	7090.
781	10920.	4660.	9410.	4850.	8110.
782	11690.	5520.	8260.	5180.	8710.
783	11800.	6270.	10120.	6050.	9000.
784	16490.	6560.	8480.	6430.	11030.

Table 4.A.4 (continued)

Quarter	Clothing (8)	Medical Care (9)	Education (10)	Transp. & Comm. (11)	Miscell. (12)
791	17346.	8376.	14859.	7264.	12792.
792	16878.	9178.	11528.	7603.	12508.
793	14859.	8820.	14275.	7888.	11026.
794	21854.	9186.	11035.	8777.	14335.
801	20841.	10478.	17617.	9965.	16394.
802	18730.	10973.	13428.	9720.	15850.
803	16980.	11881.	15874.	11025.	14946.
804	21493.	11534.	11317.	10967.	17567.
811	21918.	13614.	22359.	12030.	19990.
812	19388.	13732.	15163.	11939.	19206.
813	18372.	14999.	20619.	13654.	18730.
814	23912.	14951.	13610.	13635.	21689.

CHAPTER 5

SPECIFICATION AND ESTIMATION OF DYNAMIC DEMAND SYSTEMS FOR KOREA

5.1 Introduction

An implicit assumption underlying static demand theory is that the consumer maximises utility and achieves equilibrium instantaneously whenever there is a price or income change. Static theory is also based on an assumption that consumer's preference is only dependent on current consumption and is fixed over time. However, it can be argued that these assumptions suffer from a degree of the 'lack of realism' [e.g., see Pollak (1970) and Phlips (1974, p.149-151)]. In the real world, consumers may react with delay to changes in prices and income due to imperfect foresight, their preferences (or tastes) may change over time due to habit formation and to social contacts, their choice may be intertemporal due to the durability of goods, and so on. Such views on realism were catalysts for the development of the specification of dynamic demand systems from the static model. Moreover, many empirical studies with static demand systems have led to disappointing results because of the repeated rejection of static demand restrictions and to the frequent presence of serially correlated residuals which may be taken as evidence of dynamic misspecification in the model. Recent studies by Anderson and Blundell (1983, 1984) also recognised that the static demand system may be appropriate only for the explanation of the consumer's long run situation rather than his short run behaviour. They observed that static demand restrictions are not rejected on data with a long run structure derived from dynamic specifications of the almost ideal demand system. Therefore, because it can be argued that the consumer's full static equilibrium is difficult to verify, (particularly in the short run situations), dynamic generalisations of demand systems have been performed in an attempt to examine the consumer's dynamic and short run behaviour.[1]

In general, dynamics can be viewed in two ways; (i) as the searching process for equilibrium, and (ii) as the movement of equilibrium from one period to the next. The distinction between these two forms of dynamics is important for understanding the treatment of equilibrium in the dynamic model, particularly relating to the existence of equilibrium in the short run. According to the first view, an equilibrium may not exist in the short run, so that the economy may be constantly adjusting towards equilibrium. However, the second view, characterises dynamics as a sequence of the short run equilibria over time, and preserves the existence of equilibrium in the short run. These two different views ultimately categorise two groups of dynamic models: the first results in the dynamic (or short run) disequilibrium model, and the second results in the dynamic equilibrium model.

An implication of the first view for demand theory is that consumers are out of equilibrium in the short run, responding to changes in prices and income, and an equilibrium is only the consumer's long run objective. Thus, dynamic demand systems based on the first view need not be theoretically plausible, i.e., they can be inconsistent with utility maximisation. Only the implied long run system should be theoretically plausible, since the equilibrium refers to the long run situation. It is often assumed that the long run situation can be explained by static (demand) equilibrium theory. Such a point has been adopted in the tradition of econometrics for specification of dynamic models; for example, distributed lag models allowing for the delayed response and autoregressive models allowing for partial adjustment processes. Such dynamic demand systems will be referred as *dynamic disequilibrium demand systems.* The dynamic demand system considered by Anderson and Blundell (1982, 1983, 1984) falls into this category.

However, it is widely agreed in economic analysis that observed data can be viewed realisations of equilibrium [e.g., Debreu, (1983, p.217)]. If this enigmatic view on data is adopted as a standpoint in dynamic analysis, the empirical use of the dynamic disequilibrium model will face a serious contradiction between the model and data used in the analysis. That is, the model employed is a disequilibrium model while data is viewed as equilibrium data. Such a contradiction will be resolved in the *dynamic equilibrium model* which is formulated on the basis of the second view of dynamics. According to this view, equilibrium can be achieved even in the short run. Thus, the specification of a dynamic equilibrium demand system relates back to the theoretical formulation of dynamic theory for consumer behaviour in the short run. In accordance with the existence of a short run equilibrium, it is usually assumed that there exists a dynamic (or short run) utility function representing the consumer′s short run preferences. The dynamic demand system is derived from the solution maximising a dynamic utility function subject to a budget constraint. Unlike the dynamic disequilibrium system, the dynamic equilibrium demand system is theoretically plausible by itself and embodies short run equilibrium conditions. The habit formation model, in which consumer′s short run preferences are assumed to be dependent on his past consumption experience (and thus changing over time), clearly falls into this category of a dynamic equilibrium demand system.

What has been mentioned so far is the main problem associated with the specification of dynamic demand systems. Dynamisation of the demand system may be carried out either way. However, this chapter will focus on the specification of dynamic demand systems based on the second category, i.e., the dynamic equilibrium demand system. The discussion on the dynamic disequilibrium demand system will be delayed until Chapter 7.

The design of this chapter is as follows: Section 2 provides a brief review of the treatment of dynamics in economic theory. In Section 3, the derivation and implications of dynamic equilibrium demand systems will be considered on the basis of the taste change model. In section 4, the specification of dynamic equilibrium demand systems will be developed in the context of the Rotterdam model embodying a taste change hypothesis. In Section 5, some statistical aspects relating to the estimation of dynamic demand systems will be reviewed; and in Section 6, the dynamic version of the Rotterdam model will be applied to the five commodity Korean data set. Finally, concluding remarks will be presented in Section 7.

5.2 Some Preminary Considerations[2]

Dynamisation of consumer demand theory has been achieved by separating consumers from the other market mechanisms, such as producer's behaviour and the price adjustment process to a market clearing equilibrium level. As a result, dynamic demand theory can be simplified as prices are treated as given and the interaction with dynamics in the other markets can be ignored. However, general ideas on dynamics in economic theory also underly dynamic demand theory as well as the specification of dynamic demand systems. Therefore, it is worthwhile to review the treatment of dynamics in general economic theory.

It cannot be disputed that equilibrium and dynamics are indissolubly related in the real world situation. Neverthless, as Duménil and Lévy (1985, p.340-341) point out, the implications of the theoretical dependency between equilibrium and dynamics have been overlooked in the exposition of much of economic theory. It is generally held that any dynamic process in an economy can be adapted to equilibrium theory with minor modifications [Duménil and Lévy (1985)]. Hence, the formulation of the equilibrium model has been emphasised in classical as well as neoclassical economic theory without incorporating dynamics.

More specifically, in classical economics, dynamics is viewed as a disequilibrium situation which temporally occurs in the adjustment towards equilibrium; while equilibrium is viewed as the asymptotic position of the adjustment process and as the long run (characterising) feature. Rationality of the behaviour governing a dynamic adjustment process in classical economics is often interpreted by the notion of period-after-period 'inertia', which means that period t inherits from period t-1. In other words, information derived from the previous market and past experience determines the present operating mechanism for the dynamic process. However, dynamics remains as the disequilibrium situation in the short run. Stability of the dynamic process is a main concern in classical dynamic analysis. Such a view is represented in the Cobweb model for the market dynamic process, and is also clearly adopted in the specification of dynamic disequilibrium demand system.

In neoclassical economic theory, dynamics is also excluded in the formulation of general equilibrium theory by an assumption of the tatonnement process[3], which is somewhat unrealistic in the real world [Duménil and Lévy (1985)]. Thus the modelling of the economic system becomes purely static and timeless [Bannock, Baxter and Rees (1978, p.325-326)]. According to the assumptions of a tatonnement process, an equilibrium is only achieved at prices and quantities determined by the market 'auctioneer', and no transaction takes place until the auction reaches the equilibrium level of prices and quantities [Takayama (1974, p.340)]. The so-called Walrasian equilibrium (by which it is meant that an equilibrium is achieved by the tatonnement process) is a situation reached through a single transaction programmed by the tatonnement process. There is no room for disequilibrium in the general neoclassical model, since all forms of disequilibrium are rejected by the tatonnement process [Duménil and Lévy (1985)]. Thus, a Walrasian equilibrium has different features from the classical equilibrium which is the outcome of a disequilibrium adjustment process.

Such a difference between the classical and neoclassical approaches also relates to the views of those schools on the role of data in empirical analysis. In the classical view, the observed data can represent a disequilibrium situation. However, in the neoclassical view, all data must be viewed as an equilibrium situation, since a transaction occuring out of equilibrium is not theoretically possible. The specification of dynamic equilibrium demand systems is in accordance with the neoclassical view.

However, in neoclassical economics, the generalisation of static theory tends to incorporate more general assumptions on the individual's behaviour. For example, the consumer is assumed not only to be a wage-earner but also a capitalist possessing assets with involvement in the financial market. Such an assumption charaterises the so-called intertemporal model for consumer's behaviour [Deaton and Muellbauer (1980a) Chapter 4 and 13]. Unlike a pure wage-earner[4], a capitalist consumer would expect a future change in income from holding or selling assets and delay his current consumption in order to consume more later. His demand for durable goods and stocks of (illiquid) assets is dependent on rates of depreciation and current and expected future financial market developments. Such a consumer is often termed 'non-myopic', in the sense that he looks ahead to future effects when making his present decisions. The consumer's choice is determined by the maximisation of an intertemporal utility function subject to the (life-cycle) wealth constraint. The intertemporal utility function is generally expressed in terms of present and future (life-cycle) purchase and stock variables, and the relevant prices are usually discounted by interest rates to correspond to contracts negotiated at the base period.

The intertemporal model has also been extensively employed in dynamic demand studies, particularly, those concerned with the demand for durable goods as well as the rational habit formation model [Phlips (1974), Muellbauer (1982), and Muellbauer and Pashardes (1982)]. Alhough it provides the right direction for theoretical analysis of the demand for durable goods or stocks of assets, intertemporal models are not easy to use in empirical work, due to the complex (and highly nonlinear) functional forms of the resulting demand or stock equations, and the absence of data for analysis. The data required in intertemporal analysis, (such as stocks of the assets, wealth and individual interest rates) are rarely available, and thus are often generated in an artifical and restrictive manner in empirical studies.

Like the dynamic equilibrium model, the intertemporal approach also attempts to interpret consumer behaviour in the short run as an equilibrium situation. However, the equilibrium in the intertemporal model is obviously different from the dynamic (short run) equilibrium. The former is basically subject of future consumption plans, while the latter is subject to past consumption experience [Phlips (1974), p.236]. Recently, Spinnewyn (1981) and Muellbauer and Pashardes (1982) have developed simple intertemporal models integrating habit formation and the demand for durable goods.[5] They define habit formation in intertemporal terms using the 'rational' habit formation, by arguing that a 'rational' consumer takes into account the effect of his current purchases on his future habit formation. Their interpretation of habit formation is fundamentally different from the 'conventional dynamic' habit formation hypothesis which defines habit formation as an inertia phenomenon.

An important recent evolution of dynamic analysis in economic theory is the application of optimal control theory. This theory, which was originally developed by mathematicians involved in space research in order to solve the problem of choosing a trajectory for a rocket from a point on earth to a point on the moon, is applied to dynamic economic theory in an attempt to search for optimum time path of a dynamic process. Examples include the optimum growth path of an economy or the optimum growth of the capital stock of an economy [Takayama (1974), Chapter 8]. The mathematical method involved in the theory is Pontryagin's maximisation principle in the Hamiltonian system formulation. Lluch (1974), Phlips (1974) and Kljin (1977) have applied optimal control theory to the consumer's intertemporal model[6].

Given the limited amount of data available for this study, the intertemporal and optimum control approaches cannot be attempted, and the subject will not be pursued further.

5.3 Dynamic Equilibrium Demand Systems

5.3.1 Taste Change and the Dynamic Utility Function

As discussed in Section 1 of this chapter, the dynamic equilibrium demand system refers to the consumer's equilibrium situation in the short run. Therefore, the system should be specified on the basis of the economic theory involved in the formulation of the dynamic (or short run) utility function underlying a consumer's short run behaviour. By definition, the dynamic utility function changes over time [Phlips (1974) p.151], so that it can be expressed as a function of time, t, such that

$$u_t = u(t) \text{ and } \dot{u}(t) = \partial u_t/\partial t \neq 0. \quad (3.1)$$

However, the specification of the dynamic utility function is carried out in an indirect manner rather than as an explicit function of a time variable. This ensures that the utility function can be theoretically plausible in relation to the consumer's short run equilibrium behaviour. It is usually assumed that consumer's preferences depend on tastes that are changing *over time*. Hence, the dynamic utility function can be formulated in the manner of a taste changing utility function.

Following Gorman's (1967) notation, the short run utility function, allowing for taste change, can be written as

$$u_t = u_t[q_t, \alpha(s_t)], \quad (3.2)$$

where q_t is an m x 1 commodity vector in the current period t and $\alpha(s_t)$ is a vector valued function of taste parameters which are dependent on the predetermined "state variables", s_t.[7] The function $\alpha(s_t)$ is called the habit or taste function. The utility function in (3.2), being defined on the Cartesian product of the current commodity set, $\mathbb{Q}_t = \{q_t\}$ and the taste parameter set, $A_t = \{\alpha_t\}$, (that is, $\mathbb{Q}_t \times A_t$, rather than $\mathbb{Q}_t$ alone), is a generalised version of the static utility function. In our notation, we let $\mathcal{S}_t = \{s_t\}$ be a set of state variables influencing tastes in the current period.

Two distinct specifications of taste change are common in the literature; one is exogenous taste change where the state variables, s_t's, are exogenous; for example, dummy variables, time trends, and climate conditions. The other is the endogenous taste change situation, where the s_t's are endogeneous variables [see Pollak (1978)]. In the dynamic case, the state variable must be explicitly time-dependent (or contain variables whose values change over time) such that $\dot{s}_t = ds_t/dt \neq 0$, in order for the specification of (3.2) to be consistent with (3.1). For example, $\mathcal{S}_t = \{t\}$, $\mathcal{S}_t = \{q_{t-1}\}$, or $\mathcal{S}_t = \{p_{t-1}, \mu_{t-1}\}$ could be state variables for dynamic utility function, where t is a time trend. It will be seen in the next section that the constant term in the Rotterdam model results from the dynamic specification of $s_t = exp(t)$.

5.3.2 The Habit Formation Model

In the special case when the state variable is $s_t = q_{t-k}$ for $k > 0$, (when a consumer's tastes depend on consumption in a past period), the resulting dynamic model is called the habit formation model. It is common to set $k = 1$. The governing mechanism of the habit formation model is the 'inertia' notion which means that behaviour in period t inherits from experiences in period t-1. Stone (1954) originated the idea of habit formation, and there have been a large number of empirical applications of the habit formation model [e.g., Pollak and Wales (1969), Pollak (1970), Houthakker and Taylor (1966), Phlips (1972, 1974), Boyce (1975), Manser (1976), Lamm (1982) and Okamura (1983)]. The theoretical exposition of the habit formation model is basically identical to that of the taste change utility model given in (3.2); the only difference is that $\mathcal{S}_t = q_{t-k}$ in the habit formation model.

5.3.3 Maximisation of Short Run Utility and the Short Run Demand Function

Since taste parameters are expressed as a function of state variables, the short run utility function in (3.2) can also be written as

$$u_t = u_t[q_t, \mathcal{S}_t]. \tag{3.3}$$

That is, the short run utility function can be directly defined on the Cartesian product, $\mathbb{Q}_t \times \mathcal{S}_t$. There is no loss of generality in (3.3) compared to (3.2) due to the Implicit Function Theorem. However, the utility function expressed as (3.3) has the advantage of directly incorporating the effect of state variables on preferences without involving the specification of a taste function.

In the dynamic formulation, the consumer is assumed to be myopic and purely a wage earner and to participate in a competitive labor market. In other words, the consumer does not recognise the effect of the current expenditure allocation on future utility nor can he control his income by holding assets or by managing the labor market. Thus, his income (or total expenditure) is given and fixed. The consumer's problem in the short run simply reduces to the maximisation of a dynamic utility function (3.3) subject to the current budget constraint,

$$\mu_t = p_t' q_t, \tag{3.4}$$

conditional on the predetermined state variables, S_t. The consumer's short run demand equations can be obtained from the maximisation of a Lagrangian

$$\mathcal{L}(q_t, s_t, \lambda) = u(q_t, s_t) - \lambda_t(p_t' q_t - \mu_t),$$

and are given as

$$q_t = q_t(p_t, \mu_t, s_t) \tag{3.5}$$

and the marginal utility of income may be expressed as

$$\lambda_t = \lambda_t(p_t, \mu_t, s_t). \tag{3.6}$$

This results from solving the first order conditions,

$$u_{q_t} = \lambda_t p_t \tag{3.7}$$

with the budget constraint (3.4), where u_{q_t} is an m x 1 vector of marginal utilities. It is obvious that the short run demand equations given in (3.5) are the generalised form of the static Marshallian demand functions, $q = q(p,\mu)$. For simplicity of notation, the subscript t will be dropped from here on.

5.3.4 Equilibrium Conditions on the Short Run Demand System

Phlips (1974, p.180-183) derived the general demand restricitons on a system of short run demand equations, and showed that the short run equilibrium restrictions are the same as in static demand theory. By total differentiation of (3.4) and (3.7), we have the fundamental matrix equation of the short run consumer

$$\begin{bmatrix} dq \\ -d\lambda \end{bmatrix} = \begin{bmatrix} U & p \\ p' & 0 \end{bmatrix}^{-1} \begin{bmatrix} 0 & \lambda I & -U_{qs} \\ 1 & -q' & 0 \end{bmatrix} \begin{bmatrix} d\mu \\ dp \\ ds \end{bmatrix}, \qquad (3.8)$$

where U is the Hessian matrix such that $U = \partial^2 u/\partial q \partial q'$ and U_{qs} is an m x l matrix such that $U_{qs} = \partial^2 u/\partial q \partial s'$. Then, using partitioned inversion on (3.8), we obtain the partial derivatives of q with respect to μ, p, and s in a matrix form

$$q_\mu = (p'U^{-1}p)^{-1}U^{-1}p, \qquad (3.9)$$

$$q_p = \lambda U^{-1} - \lambda(p'U^{-1}p)^{-1}U^{-1}pp'U^{-1} - q_\mu q' \qquad (3.10)$$

$$q_s = -[U^{-1} - U^{-1}p(p'U^{-1}p)^{-1}p'U^{-1}]U_{qs}. \qquad (3.11)$$

From (3.9) and (3.10), the Slutsky equation can also be written as

$$K = q_P + q_\mu q', \tag{3.12}$$

Thus, the differential relations (3.9) to (3.10) and the Slutsky matrix (3.12) are expressed in the same way as in static theory, while only (3.11) is added in relation to consumer's short run behaviour. The theoretical equilibrium restrictions on the short run demand system can also be derived in a similar manner to that of static theory from the relations (3.9) to (3.12).

Premultiplying of (3.9) by p′, we have adding-up restrictions on the marginal budget shares such that

$$p'q_\mu = 1, \tag{3.13}$$

Postmultiplying the transpose of (3.10) by p, the 'Cournot aggregation' restriction results

$$q_P'p = -q. \tag{3.14}$$

Next, postmultiplying K in (3.12) by p, we have the homogeneity restriction

$$Kp = 0. \tag{3.15}$$

Finally, the symmetry and negativity restrictions on the Slutsky matrix K follow directly from (3.10). Thus, the usual restrictions on static demand system - adding up, homogeneity and symmetry - still hold and characterise the equilibrium conditions in the short run.

Furthermore, we note that only the adding-up restrictions can be imposed on the parameters of the state variables in the short run demand system. From (3.10), (3.11) and (3.12), it can be seen that

$$q_s = -[U^{-1} - (p'U^{-1}p)^{-1}U^{-1}pp'U^{-1}]U_{qs}$$

$$= -\lambda^{-1}KU_{qs}. \qquad (3.16)$$

Then, premultiplying p′ on (3.16) and in view of the homogeneity and symmetry restrictions, we have

$$p'q_s = 0. \qquad (3.17)$$

The restriction (3.17) is the adding up restrictions on the dynamic specification of the short run demand equations.

5.3.5 Long Run Demand and the Utility Function

We have referred to dynamics in the dynamic equilibrium demand system as a sequence of short run equilibrium situations. If the sequence is convergent and there exists a unique limit point, the dynamic system is called ′stable′ and the system corresponding to the limit point is referred to as the long run demand system. Technically, the long run demand system is derived by imposing steady state conditions on the short run system. The steady state condition is characterised by the situation that the state variables no longer change over time, that is, s = $ds_t/dt = 0$ for all t. Since the long run demand system is not derived from the utility maximisation, the implied system may be theoretically implausible. As Pollak (1970) pointed out, the long run utility function, unlike the demand system, cannot be the limit point obtained from the short run utility function by imposing steady state conditions. He argued that if we obtain the long run utility function by imposing the steady state condition on the short-run utility functions, we end up with a demand system different from the long run system implied from the short run. The existence of a utility function which can rationalise the long run system derived from the short run has been a major concern of dynamic demand theorists [see Gorman (1967), Peston (1967), Von Weizsäcker (1971), Pollak (1976a), and El-Safty (1976a, 1976b)].

Von Weizsäcker (1971), based on the integrability theorem and revealed preference theory, showed the existence of the long run utility function from which the long run demand function can be derived, providing that the long run Slutsky matrix is symmetric and negative definite. However, Pollak (1976a) showed, in a paper critical of Von Weizsäcker, that there exists a utility function which rationalises the long run demand functions if and only if they are the steady state solution to a system of short run demand functions generated by an additive utility function. El-Safty (1976b) also claimed that Von Weizsacher's result is not valid in general and showed that the long run demand functions can be rationalised by a utility function, different from the short run dynamic utility function, if and only if the dynamic utility function is such that past consumption of any good (or taste changes) is separable from all other goods. However, from a completely different standpoint, Phlips (1974) argued that the long run utility functions are of the same form as the corresponding static functions in the sense that long run implies full adjustment just as in static theory, and he concluded that whereas the short run utility function represents changes in tastes or preferences, the long run utility function represents an unchanging and therefore static preference [see Phlips (1974, p.176-180)].

5.3.6 Examples of Empirical Dynamic Equilibrium Demand Systems

There have been a wide range of applications of the dynamic equilibrium demand system in the context of the linear expenditure system and the translog model, mostly using the habit formation approach. Pollak (1970) dynamised the linear expenditure system, adopting the habit formation hypothesis. He assumed that the subsitence expenditures, γ_i's, in the Stone-Geary utility function $ln\, u(q) = \sum_{i=1}\beta_i\, ln\, (q_i - \gamma_i)$ linearly depend on their own past consumption expenditure levels such that

$$\gamma_i = \gamma_i^* + \delta_i q_{i_{t-1}}, \tag{3.19}$$

for all $i = 1,\ldots,m$, where γ_i^* is interpreted as a 'physiologically necessary' component of γ_i and $\delta_i q_{i_{t-1}}$ as the 'psychologically necessary' component. The resulting short run demand functions are of the form

$$q_i = \gamma_i^* + \delta_i q_{i_{t-1}} - \beta_i/p_i[\mu - \sum p_k(\gamma_k^* + \delta_i q_{i_{t-1}})], \tag{3.20}$$

for all $i = 1,\ldots,m$.

Phlips (1972) derived a different dynamic version of the linear expenditure system, introducing state variables, s_i's, which are assumed to change continuously over time according to

$$\dot{s}_i = q_i + \delta_i s_i \tag{3.21}$$

and to follow the partial adjustment process

$$\dot{s} = \kappa(s^* - s) \tag{3.22}$$

in the short run, (where δ_i is a constant rate of deprecation and κ is an adjustment coefficient). The dynamic Stone-Geary utility function considered is of the form $u(q) = \sum_i \alpha_i \, ln \, (q_i - \gamma_i^* - \beta_i s_i)$. The value of s at equilibrium is given by the solution of the steady state condition, $\dot{s} = 0$, which results in $s^* = \delta_i^{-1} q_i$. After elimination of the unobservable state variables through some algebraic manipulation, the resulting estimable equations are

$$q_{it} = \kappa_{io} + \kappa_{i1}q_{it-1} + \kappa_{i2}/\lambda_t p_{it} + \kappa_{i3}/\lambda_{t-1}p_{it-1} \qquad (3.23)$$

from which the coefficients of the dynamic utility function, δ_i, α_i, β_i, and γ_i^* are obtained in the terms of the structural coefficients

$$\delta_i = 2(\kappa_{i2} + \kappa_{i3})/(\kappa_{i2} - \kappa_{i3}),$$

$$\alpha_i = (\kappa_{i2} - \kappa_{i3})/(1 + \kappa_{i1}),$$

$$\beta_i = 2(\kappa_{i2} + \kappa_{i3})/(\kappa_{i2} - \kappa_{i3}) - 2(1 - \kappa_{i1})/(1 + \kappa_{i1}),$$

$$\gamma_i^* = \kappa_{io}(\kappa_{i2} - \kappa_{i3})/(1 + \kappa_{i1})(\kappa_{i2} + \kappa_{i3}).$$

Manser (1976) dynamises the translog demand system, specifying the α_i's in (6.2) in Section 2.6.1 to be

$$\alpha_i = a_i + d_i q_{i(t-1)}, \text{ for } i = 1,\ldots,m. \qquad (3.24)$$

Then, the resulting budget share equations are

$$w_{i(t)} = \frac{a_i + d_i q_{it-1} + \sum_{j=1}\beta_{ij}\, ln\,(p_{jt}/\mu)}{\sum_{k=1}a_k + \sum_{k=1}d_k q_{kt-1} + \sum_{k=1}\sum_{j=1}\beta_{kj}\, ln\,(p_{jt}/\mu)}. \qquad (3.25)$$

Assuming the time-varying indirect translog utility function, Jorgenson and Lau (1979) consider the dynamic indirect translog system

$$w_{i(t)} = \frac{a_i + \sum_{j=1}\beta_{ij}\, ln\,(p_{j(t)}/\mu) + b_{i(t)}t}{-1 + \sum_{k=1}\sum_{j=1}\beta_{Mj}\, ln\,(p_{j(t)}/\mu) + b_{Mt}t} \qquad (3.26)$$

for all $i = 1,\ldots,m$, where t is time trend. In this case, tastes are assumed to be exogeneous. [See also Conrad and Jorgenson (1979).]

The dynamic systems shown in the above may be reasonable dynamic versions of their corresponding static systems. In econometric parlance, their static counterparts are nested in the dynamic systems when the dynamic parameters are zero. However, these dynamic systems have not remedied the disadvantages of their static cousins. As seen in Chapter 2, these two static models are not attractive for empirical studies, in that the linear expenditure system is overrestrictive and the global properties of the translog model are unknown. Furthermore, these dynamic models are incapable of tracing interdependences between different goods in different periods. The dynamic specifications given by (3.19), (3.21), and (3.24) are simply related to single-commodity dynamics, but ignore the effect of past consumption of a good on present consumption of other goods. It is quite possible to imagine, that a consumer's past taste changes from grains to dairy products inducing the purchase of a refrigerator, for example. In this case, the past consumption of dairy products may positively affect the current purchase of a refrigerator. The identification of such relations can be an important aspect of dynamic demand analysis and can only be possible through the system-wide dynamic specification of the demand system. This will be formulated in the context of the dynamisation of the Rotterdam model in the following section.

5.4 The Dynamic Rotterdam Model

5.4.1 Derivation of the Dynamic Rotterdam Model

In this section, a flexible functional form of the dynamic demand equations given in (3.5) will be derived in the context of the Rotterdam approximation. The procedure used in deriving the static Rotterdam model will be applied to the dynamic demand equations in (3.5), and the resulting system of dynamic demand equations will be termed the *dynamic Rotterdam demand system*, consequently. Even though the derivation is rather simple and the resulting system has many advantages over the other existing dynamic demand systems, there has been no attempt to make use of the Rotterdam differential approximation in the specification of estimable dynamic demand equations, which is surprising.

In a similar manner to the static Rotterdam model, a first order approximated flexible functional form of the dynamic demand equation can be obtained by total differentiation of the general funational form of dynamic Marshallian demand equations $q = q(p,\mu,s)$ in (3.5);

$$dq = q_\mu d\mu + q_p dp + q_s ds. \qquad (4.1)$$

After the substitution of the Slutsky equation, $q_p = K - q_\mu q'$ in (3.12), into (4.1), we obtain (4.1) as

$$dq = q_\mu d\mu + (K - q_\mu q')dp + q_s ds. \qquad (4.2)$$

Assuming that there are l state variables influencing the consumer's current tastes, that is, $S_t = \{s_k,\ k = 1,\ldots,l\}$, the i'th equation in (4.2) can be written as

$$dq_i = (\partial q_i/\partial\mu)[d\mu - \textstyle\sum_k q_k dp_k] + \sum_j k_{ij} dp_j + \sum_k^l f_{ik} ds_k, \qquad (4.3)$$

for $i = 1,\ldots,m$, where k_{ij} is (i,j)'th term of the short run Slutsky substitution matrix, $f_{ik} = (\partial q_i/\partial s_k)$ is (i,k)'th term of the matrix q_s and m is the number of commodities in the system. Using the relation $dx = x\,dln\,x$ and mutiplying p_i/μ on both side of (4.3), we have the Rotterdam dynamic demand equations as

$$w_i\,dln\,q_i = b_i d\bar{M} + \textstyle\sum_{j=1}^m C_{ij}\,dln\,p_j + \sum_{k=1}^l D_{ik}\,dln\,s_k, \qquad (4.4)$$

for $i,j = 1,\ldots,m$, where

$$d\bar{M} = dln\,\mu - \textstyle\sum_{k=1}^m w_k\,dln\,p_k, \qquad (4.5)$$

$$b_i = p_i(\partial q_i/\partial \mu) = w_i e_i, \text{ for } i = 1,\ldots,m \tag{4.6}$$

$$C_{ij} = k_{ij}p_i p_j/\mu = w_i e^*_{ij}, \text{ for } i,j = 1,\ldots,m \tag{4.7}$$

and

$$D_{ik} = f_{ik}p_i s_k/\mu = (\partial q_i/\partial s_k)p_i s_k/\mu, \tag{4.8}$$

$$= w_i \eta_{ij}, \text{ for } i = 1,\ldots,m \text{ and } k = 1,\ldots,l.$$

where η_{ik} is the elasticity of a good i with respect to change in the state variable s_k. The remaining notation is the same as in the static Rotterdam model (see Section 2.4). The coefficients, b_i's and C_{ij}'s, now correpond to the consumer's short run responses, but their interpretation is identical to the static model. In other words, the b_i's refer to the short run marginal propensities to consume, and C_{ij}'s to the short run Slutsky matrix. In terms of (4.8), the dynamic parameter matrix, D_{ij} is the elasticity of good i to changes in the state variable s_k expressed in terms of budget shares. Obviously, the dynamic Rotterdam system given by (4.4) is the generalised version of the static Rotterdam system, in the sense that the static system is nested in the dynamic system, as a special case when $D_{ik} = 0$.

The short run equilibrium conditions are then charaterised by the following restrictions on the dynamic Rotterdam system:

Adding up: $\sum_{i=1}^{m} b_i = 1$ and (4.9)

$\sum_{i=1}^{m} C_{ij} = 0$, for all $j = 1,\ldots,m$,

Homogeneity: $\sum_{j=1}^{m} C_{ij} = 0$, for all $i = 1,\ldots,m$, (4.10)

Symmetry: $C_{ij} = C_{ji}$, for all $i,j = 1,\ldots,m$, (4.11)

Negativity: The matrix $C = [C_{ij}]$ is negative semi-definite. (4.12)

Restrictions on state variables:

$\sum_{i=1}^{m} D_{ij} = 0$, for all $j = 1,\ldots,l$. (4.13)

The first four restrictions (4.9) to (4.12) are obviously identical to those of the static model, while the last (4.13) are just the adding up restrictions on D_{ij} which are automatically satisfied in estimation. Clearly, the validity of the dynamic (short run) equilibrium hypothesis can be tested using restrictions (4.9) to (4.12) on the dynamic Rotterdam demand system as specified in (4.4).

The dynamic Rotterdam model (4.4) is linear in parameters (like the static model) and the effect of state variables on short run consumption can be evaluated directly in terms of elasticities without explicitly specifying the taste (habit) functions $\alpha(s_t)$. Obviously, these features give the dynamic Rotterdam model advantages over other existing dynamic demand systems.

5.4.2 Selection of State Variables and the Statistical Specification of the Dynamic Rotterdam Model

The choice of the dynamic state variables is fundamentally related to the specification of the dynamic demand equations in the context of (4.14). Changing the notation in (4.4), we can write the dynamic Rotterdam demand system as the linear multivariate model

$$Y = X\,B + Z\,\Gamma, \tag{4.14}$$

while the static Rotterdam demand system is

$$Y = X\,B \tag{4.15}$$

where $Y = [w_{ti}\, d\ln q_{ti}]$ is a T x m matrix of dependent variables, $X = [\,d\ln p_{t1}, \ldots d\ln p_{tm},\ d\bar{M}_t]$ is a T x k matrix of the current price and total expenditure variables, $Z = [\,d\ln s_{t1}, \ldots, d\ln s_{tl}]$ is a T x l matrix of state variables, B is a k x m coefficient matrix for C_{ij} and b_i, and Γ is an l x m coefficient matrix for D_{ij}, with k = (m+1) and the number of observations T. Comparing (4,14) with (4.15), we can see that the selected state variables are included in the dynamic Rotterdam demand system as simple additional explanatory variables.

If we assume that consumer's tastes are exogeneously determined and dependent only on time trends such that $s_t = exp\,(t)$, where t is time trend variable; then $D_i\, d\ln s$ in

(4.4) becomes

$$D_i \, dln \, s = D_i \, dln \, exp(t) = D_i \, dt, \tag{4.16}$$

which reduces to D_i in discrete time since $dt \simeq t - (t-1) = 1$. Thus, the model with the state varaible $s_t = exp(t)$ reduces to the static Rotterdam model with a constant term. Note, this is often regarded as an ad hoc dynamic (taste change) version of the Rotterdam model.

On the other hand, if we assume the habit formation model and choose the lagged consumption as the dynamic variables such that $S_t = \{q_{t-1}, \ldots, q_{t-p}\}$, where q_{t-i} is an m x 1 vector of m goods consumed at the period t-i, then Z in (4.14) becomes $Z = \{Y_{-1}, \ldots, Y_{-p}\}$ and the dynamic Rotterdam system (4.14) reduces to the p′th order autoregressive AR(p) version of the static Rotterdam system (4.15).

Similarly, if we assume a delayed response of consumption to changes in prices and income, then $Z = \{X_{-1}, \ldots, X_{-q}\}$ and the model (4.17) reduces to a q′th order distributed lag model. For a more general case, if we choose the dynamic state variables as $S_t = \{q_{t-1}, \ldots, q_{t-p}, p_{t-1}, \mu_{t-1}, \ldots, p_{t-q}, \mu_{t-q}\}$, the dynamic Rotterdam system in (4.17) can be expressed in the form of a generalised autoregressive-distributed lag model AD(p,q) such that

$$Y = Y\,\Gamma(L) + X\,B(L). \tag{4.17}$$

Note A(L) and B(L) are polynomials of the lag operator L with order p and q, repectively, such that

$$\Gamma(L) = \sum_{i=1}^{p} \Gamma_i L^i \tag{4.18}$$

with an m x m coefficient matrix Γ_i for i = 1,...,p and

$$B(L) = \sum_{j=0}^{q} B_{0j} L^j \tag{4.19}$$

with a k_0 x m coefficient matrix B_{0j} i for j = 1,...,q. Thus, the dynamic Rotterdam demand system can be expressed as a general dynamic variant of the static model using dynamic state variables.

Given the dynamic demand system specified as (4.17), we can test the validity of various dynamic state variables in the context of a test of specification of the dynamic linear multivariate model. Moreover, from the estimation of the dynamic Rotterdam demand system, we can simultaneously identify the time-domained interdependence between the consumption of different goods in different periods and the lagged reponse to changes in prices. Thus, the dynamic Rotterdam demand system allows a considerable generality and flexibility in terms of modelling impact effects and short run dynamic responses.

5.5 Statistical Aspects in the Estimation of Dynamic Demand Systems

Since the dynamic Rotterdam demand system derived in the previous section and the short run (demand) equilibrium restrictions are linear in coefficients, the estimation of the system and the tests of restrictions can be carried out in the context of the linear multivariate model. The results discussed for the estimation of static system in Chapter 3 are also applicable to the estimation of the dynamic demand system. However, when stochastic variables, (such as lagged dependent variables in the habit formation model), are included as explanatory variables in the dynamic system (4.17), the statistical properties of the model change. For example, the unrestricted OLS estimator of the system is no longer the BLUE, and the exactness of the Wald test as a generalised Hotelling's T^2 is no longer valid.

Adding a T x m disturbance matrix U to (4.17), letting W = [X: Z] and Π' = [B': Γ'], we can write the dynamic Rotterdam model (4.17) as

$$\begin{aligned} Y &= X B + Z \Gamma + U \\ &= W\Pi + U, \end{aligned} \qquad (5.1)$$

where W is a T x (k+l) matrix of explanatory variables, Π is a (k+l) x m coefficient matrix. We assume that the disturbance matrix U in (5.1) has a multivariate normal distribution with mean E(U) = 0 and covariance E(U'U) = Σ. Allowing for stochastic explanatory variables in Z (and thus in W), we assume the following asymptotic properties

$$\text{plim } T^{-1}[W'U] = 0, \tag{5.2}$$

and

$$\text{plim } T^{-1}W'W = D, \tag{5.3}$$

where D is a finite and nonsingular matrix, and

$$\text{plim } T^{-1}U'U = \Sigma \tag{5.4}$$

[see for the assumptions Lemma 5.5.2 of Anderson (1971, p.194 in Chapter 5)]. The unrestricted ML estimator of $\pi = \text{vec}(\Pi)$ given as the OLS estimator

$$\hat{\pi} = \text{vec}(\hat{\Pi}) = [I \otimes (W'W)^{-1}W']\text{vec}(Y). \tag{5.5}$$

is no longer an unbiased estimate of π, since W involves the stochastic explanatory variables and so $E[W'U] \neq 0$. However, under the assumptions of (5.2) to (5.4),

$$\begin{aligned} \text{plim } \hat{\pi} &= \text{plim } [I \otimes (W'W)^{-1}W']\text{vec}(Y) \\ &= \pi + (I \otimes D^{-1}) \text{ plim } (I \otimes W') \text{ vec}(U) \\ &= \pi, \end{aligned} \tag{5.6}$$

i.e., $\hat{\pi}$ in (5.5) is a consistent estimator of π, and $\sqrt{T}(\hat{\pi} - \pi)$ has limiting normal distribution with mean 0 and covariance matrix $\Sigma \otimes D^{-1}$ [see for assumptions and proof Anderson (1971, p.194-203)].

As in the static model, the short run equilibrium (demand) restrictions - homogeneity and symmetry - can be expressed linearly as

$$C\pi = \underset{\sim}{0}, \tag{5.7}$$

where C is an s x m(k+l) restriction matrix with rank(C) = s < m(k+l), $\underset{\sim}{0}$ is an s x 1 zero vector, s is the number of independent restrictions. Therefore, the restricted ML estimator of $\pi = \text{vec}(\Pi)$ subject to the short run equilibrium restrictions in (5.7) can be given as

$$\tilde{\pi} = \hat{\pi} - (\Sigma \otimes (W'W)^{-1})C'[C(\Sigma \otimes (W'W)^{-1})C']^{-1}C\hat{\pi}. \tag{5.8}$$

The asymptotic covariance matrix is

$$\text{cov}(\tilde{\pi}) = (\Sigma \otimes D^{-1}) - (\Sigma \otimes D^{-1})C'[C(\Sigma \otimes D^{-1})C']^{-1} \times C(\Sigma \otimes D^{-1}). \tag{5.9}$$

Since $E[\hat{\pi}] \neq \pi$, then $E[\tilde{\pi}] = \pi$ does *not* follow. However, under the assumptions (5.2) - (5.4) as well as the restrictions in (5.7), it follows from (5.6) that

$$\text{plim } \tilde{\pi} = \pi,$$

so that $\sqrt{T}(\tilde{\pi} - \pi)$ has limiting normal distribution with mean 0 and the asymptotic covariance matrix given by (5.9).

For testing the restriction $C\pi = 0$ in (5.7), the Wald, LR, and LM test statistics are,

$$W = T\ tr\,\hat{\Sigma}^{-1}(\tilde{\Sigma} - \hat{\Sigma}),$$

$$LR = T\ ln\,(|\tilde{\Sigma}|/|\hat{\Sigma}|),$$

and

$$LM = T\ tr\,\tilde{\Sigma}^{-1}(\tilde{\Sigma} - \hat{\Sigma}).$$

They are aymptotically equivalent and aymptotically distributed as χ^2 with s degrees of freedom under the null hypothesis H_0, even when Z contains stochastic variables [See for proof Berndt and Savin (1977, p.1269)]. However, the exactness of the Wald statistic testing homogeneity alone based on Hotelling's T^2 distribution cannot be claimed when stochastic variables are included in W.

The dynamic Rotterdam demand system is also singular due to the adding up conditions. As seen in the section 3.5, the usual solution of the singularity problem is to eliminate one equation from the system. This solution can also be applied to the estimation of dynamic demand systems, and all the parameters of the full system can be recovered from the estimates of the reduced system with one exception, which occurs when Z contains both lagged dependent and independent variables of the same orders, for example, when $Z = \{Y_{-1}, X_{-1}\}$.

Decomposing X in (5.1) into $X = [X^0 : \mu]$ and B conformably into B^0 and b such that $B' = [B^{0\prime} : b']$, the dynamic demand system (5.1), when $Z = \{Y_{-1}, X_{-1}\}$, can be written as

$$Y = X^0B^0 + \mu b^0 + X^0_{-1}B^1 + \mu_{-1}b^1 + Y_{-1}\Gamma + U. \qquad (5.10)$$

where X^0 is a T x m matrix of exogeneous variables defined by the current prices and μ a T x 1 vector defined by total expenditure. The adding up condition implies that $\mu = Y\iota$, where $\iota = (1,\ldots,1)'$ is an m x 1 column vector of unit elements. Thus, the adding up conditions with the lag, $\mu_{-1} = Y_{-1}\iota$ implies perfect multicollinearity between μ_{-1} and Y_{-1} in (5.10). The usual solution to avoid multicollinearity in estimation of (5.22) is to eliminate μ_{-1} or one of variables in Y_{-1} from the set of explanatory variables in the reduced system of (5.22). However, the dynamic singular system (5.22) is observationally equivalent to

$$Y = X^0B^0 + \mu b^0 + X^0_{-1}B^1 + Y_{-1}(\iota b^1 + \Gamma) + U, \qquad (5.11)$$

which is (5.10) with μ_{-1} eliminated. It can be clearly seen from (5.11) that the coefficients b^1 and Γ in (5.10) cannot be identified from estimation of the reduced system (5.11). Therefore, an identification problem exists for the estimated coefficients of the lagged variables, when Z contains both lagged dependent and independent variables with the same order [Anderson and Blundell (1982)].

When Z is chosen such that $Z = \{Y_{-1}\}$ or $Z = \{X^0_{-1}\}$, but not both, (so that Γ is square), the zero restrictions on all off-diagonal terms in Γ imply a zero coefficient matrix Γ due to adding up condition that $\Gamma\iota = \underset{\sim}{0}$, where $\underset{\sim}{0}$ is the zero column vector. Therefore, it is not sensible to impose the diagonality of Γ in the context of the dynamic singular system, when Γ is square. Thus, this singularity rules out the dynamisation of demand systems on a sigle equation basis as is embodied in the diagonality of the dynamic coefficient matrix, so that a flexible specification of the dynamic structure of demand systems is also required.

5.6 The Empirical Application of the Dynamic Rotterdam Demand System to Korean Data

In this section, we present the results of estimating and testing the dynamic Rotterdam demand system using quarterly Korean household expenditure data. The data series used for estimating static demand systems in Chapter 4 were again employed here. The data were seasonally unadjusted and cover the period for 1965 to 1981, giving a total of 68 observations. Since the first observation is used for differencing and the next four observations are used by the lag structure, only 63 observations are available for estimation. To achieve parametric parsimony and to avoid problems caused by the system-wide flexible dynamisation of the system,[8] (such as overparameterisation, multicollinearity and the large reduction in degrees of freedom), we applied the dynamic Rotterdam demand system to the five commodity data. This data set consisted of (1) food, (2) housing, (3) fuel and light, (4) clothing, and (5) miscellaneous categories.

Before proceeding, it is worthwhile to recall from the results of the previous chapter that the static Rotterdam demand system suffered from serial correlation in the estimated residuals. Since serially correlated residuals can indicate a specification error of the model (due possibly to the dynamic specification of disturbances,or the incorrect specification of the structural dynamics or to the omission of relevant explanatory variables), the necessity of investigating a dynamic demand system arises.

The first consideration is to select dynamic state variables which plausibly explain the consumer's short run behaviour. To accord with various conjectures on the consumer's short run situation, (such as habit formation, stock adjustment, and delayed response to changes in prices and income), we consider the introduction of lagged consumption levels as well as lagged prices and total expenditure as possible dynamic state variables. To be consistent with the use of quarterly data, lags of up to the fourth order were used. Dummies variables were also incorporated into the system to capture regular seasonal movements.

Since the selected state variables are incorporated into the dynamic Rotterdam demand system as additional explanatory variables, the selection of appropriate state variables from among the above reduces to the selection of the proper dynamic specification of demand system in the context of the autoregressive distributed lag model with fourth order lags. Thus, the selection of appropriate state variables can be carried out by means of tests of the corresponding dynamic specification of the demand system.

In terms of notation, we refer to A(k,..,l) as the structural autoregressive coefficitents, to D(p,...q,) as the distributed lag specification, and to AD[(k,...,l),(p,...,q)] as the autoregressive distributed specification, where numbers in the bracket (k,...,l) refer to autoregressive lag structures and those in (p,...,q) to the distributed lag structure. For example, A(1,4) refers to the autoregressive dynamic system

$$Y = XB_0 + Y_{-1}\Gamma_1 + Y_{-4}\Gamma_4,$$

D(1,2) refers to the distributed lag dynamic system

$$Y = XB_0 + X_{-1}B_1 + X_{-2}B_2.$$

and AD[(1,4),(1)] refers to the autoregressive distributed lag dynamic system

$$Y = XB_0 + X_{-1}B_1 + Y_{-1}\Gamma_1 + Y_{-4}\Gamma_4.$$

The most appropriate methodology for determining the dynamic specification of econometric models is a sequential testing procedure, in which a very general dynamic structure with relatively higher order of lags is specified at the outset and then sequential tests for reducing the order of the dynamics are applied until a specific model is obtained [see Mizon (1977)]. However, given the available number of observations, the inclusion of all the lagged dependent and independent variables up to the fourth order results in overfitting problems as well as the serious small sample bias of the asymptotic tests towards to over-rejection. To avoid such problems, we first tested the validity of inclusion of each individual lagged variable against the static model and applied the sequential testing procedure separately to autoregressive and distributed lag structures. Then, we applied the sequential testing procedure to the general dynamic specification with the selected lagged variables from the two initial search steps. The likelihood ratio (LR) tests were performed on unrestricted models, and their results are presented in three subtables of Table 5.1. Anderson's small sample correction was applied to the critical values of asymptotic χ^2 in the sequential test, which are shown in the last column in the subtable.

From Table 5.1.1, the effect of lagged price and total expenditure on current consumption was shown to diminish as the lag order increased, and the inclusion of lagged dependent variables of the order 3 was found to be insignificant. In Table 5.1.2, the lag of order 3 in the autoregressive model and the lags of order 3 and 4 in the distributed lag model were found to be insignificant, if Anderson's small sample correction was applied. Thus, it is clear from these two initial searches that the lagged dependent and independent variables of order 3 can be omitted from the dynamic specification of the demand system.

Consequently, we started the sequential testing procedure with AD[(1,2,4),(1,2,4)] model, and present the test results in Table 5.1.3. At the first stage, the omission of the lagged variables, X_{-4} and Y_{-2}, in AD[(1,2,4),(1,2,4)] model were insignificant, so that the AD[(1,2,4),(1,2,4)] model reduced to the AD[(1,4),(1,2)] model. At the next stage, the AD[(1,4),(1,2)] model reduced to AD[(1,4),(1)], which finally reduces to the A(1,4) model, as the coefficient matrices for X_{-2} in AD[(1,4),(1,2)] model and for X_{-1} in AD[(1,4),(1)] model were also found to be insignificantly different from zero. The static model was rejected in favour of A(1,4) model. Thus, through the sequential testing procedure, we ended up with the dynamic Rotterdam demand system as

$$Y = XB_0 + Y_{-1}\Gamma_1 + Y_{-4}\Gamma_4,$$

which supports the habit formation and stock adjustment hypotheses for short run consumer behaviour.

However, in the diagnostic checking on the econometric model, the validity of the economic theory on the estimated model is also important. Therefore, we included various plausible dynamic specifications of the demand system in the tests of short run dynamic equilibrium conditions in addition to the selected A(1,4) model. To incorporate lagged responses to changes in prices and income (total expenditure) in the dynamic demand system, we also considered the AD[(1,4),(1)] model and the AD[(4),(1)] model (in which Y_{-1} is excluded to avoid the multicollinearity with X_{-1}). A dynamic demand system involving a single set of dynamic state variables, similar to the A(1), A(4) and D(1) models, were also employed to compare their test performance with the other models. Consequently, six dynamic models were used for tests of short run equilibrium behaviour. However, as A(1,4) was chosen as the most suitable dynamic Rotterdam demand system by the sequential tests of dynamic structure, the AD[(1,4),(1)] model may be an unnecessarily over-parameterised model, while the A(1), A(4) and D(1) models may be underparameterised models. The model AD[(4),(1)] is non-nested in A(1,4) model. Thus, comparison of their performance in testing short run equilibrium conditions may be an interesting feature in light of the effect of model misspecification in testing hypotheses. The test results, using the three usual χ^2 tests, Wald, LR and LM tests, are shown in Table 5.2.

The short run symmetry restrictions were accepted in all models, while the short run homogeneity restrictions were accepted in AD[(1,4),(1)], AD[(4),(1)] and A(4) models, but were rejected in A(1,4) and A(1) and D(1) models. No regular effect of over- or under-parameterisation of the model with respect to A(1,4) model on the testing of short run restrictions was found. However, tests of the homogeneity restrictions appeared to vary with the different dynamic state variables included in the model. Adding X_{-1} to A(1,4) model as well omitting Y_{-1} from A(1,4) significantly improved the test of the homogeneity restrictions, while omitting Y_{-4} from AD[(4),(1)] model caused the test to deteriorate. On the other hand, omission of Y_{-1} from AD[(1,4),(1)] model and that of X_{-1} from AD[(4),(1)] did not significantly affect the test of the homogeneity restrictions. Compared with the results when testing the static equilibrium restrictions, only the A(4) model showed a slight improvement, the AD[(1,4),(1)] and AD[(4),(1)] models showed similar testing results, while the A(1,4), A(1) and D(1) models were worse.

Thus, in testing the short run equilibrium conditions, A(1,4) model selected by the sequential testing procedure appeared inconsistent with the dynamic equilibrium hypothesis, while A(4) and AD[(1,4),(1)] and AD[(4),(1)] models appeared consistent. Consequently, on the basis of their compatability with the dynamic equilibrium hypothesis, the three models, A(4) and AD[(1,4),(1)] and AD[(4),(1)], can be chosen as plausible dynamic demand systems. However, the choice of A(4) models bears a risk of underparameterisation of the true dynamic model, while that of AD[(1,4),(1)] model bears a risk of overparameterisation as well as the possibility of the multicollinearity between Y_{-1} and X_{-1}. Consequently, the AD[(4),(1)] model can be selected the most plausible dynamic demand system on the basis of the consistency of the model with the dynamic equilibrium hypothesis. The AD[(4),(1)] model, however, validiates habit formation on an annual basis and delayed responses to changes in prices and total expenditure in the previous quarter.

In addition, in order to discriminate between A(1,4) and AD[(4),(1)] models, which are non-nested, we computed Akaike's information criterion, given by

$$AIC = -2\,ln(L) + 2k,$$

where k is the total number of parameters and L is the likelihood, and we also tested the structural stability of the models. The A(1,4) has an AIC of 1667.98 and the AD[(4),(1)] has an AIC of 1642.42, so that the AD[(4),(1)] appeared to be the preferred specification, as it had a smaller AIC value. The structural stability of these two models are accepted by a Chow prediction test. When the sample is restricted by removing the last 8 observations, the application of the Chow test yields $\chi^2(32)$ test statistics of 40.24 for the A(1,4) model and 42.28 for the AD[(4),(1)] model, given a critical value of 46.20 at 5%. Thus, the A(1,4) model was found to be slight more stable than AD[(4),(1)] model. Results of significance tests on the coefficient matrices for Y_{-4} and X_{-1} in AD[(4),(1)] model are given at the bottom of Table 5.1.3. Both Y_{-4} and X_{-1} were found to be significant in AD[(4),(1)] model.

Next, to consider the parameter estimates on the dynamic (short run) Rotterdam demand systems, we first selected the AD[(4),(1)] model as it was shown to be consistent with the dynamic equilibrium hypothesis, and we present the results in Table 5.3. The estimates of the short run Slutsky matrix and the marginal budget shares appeared roughly identical to those of the static model RDAM-SD in the previous chapter. Comparing the results of the restricted static and short run models, the short run marginal budget shares for the food and miscellaneous sectors appeared slightly larger than the static ones, while those of the housing, fuel and light, and clothing sectors were slightly smaller than the static ones. As in the static model, the diagonal terms of the short run Slutsky matrix were not all negative, (a positive, but significant substitution effect was found in the miscellaneous sector). Hence, the necessary condition for concavity of the short run utility function was not significantly violated. The food, housing, and fuel and light sectors were shown to have more elastic own substitution effects in the short run, while the clothing and miscellaneous sectors were inelastic (as in the static model). The short run substitutability and complementarity relations were found to be identical to the relations observed in the static model.

Total expenditure in the previous quarter was shown to have a significant negative effect on current consumption of the food sector, but a positive effect on that of the housing sector. The latter result may reflect the fact that the income effect on the housing sector is spread over time, as the sector includes some durable goods, such as furniture and household equipment, purchases which are partially due to saving effects.

From inspection of the effect of past price changes on current consumption, all sectors, except the fuel and light sector, react negatively to previous quarter own price changes. As to the cross effect, the housing sector appeared to be positively affected by past price changes in the food and fuel and light sectors, the clothing sector to be negatively affected by past price changes in the food sector, and the miscellaneous sector to be postively affected by past price changes in the clothing sector.

The habit formation hypothesis, which we assume here is determined on an annual basis, was confirmed by the data for all sectors, except housing. The fourth order lagged consumption terms for food, fuel and light, clothing and the miscellaneous group appeared to have significant positive effects on current own consumption; while the housing sector had an insignificant negative effect, (which may be due to the stock adjustment process in that sector).

As to the interdependence of sectoral consumption between current and previous periods; current consumption of food appeared to be negatively related with consumption in the clothing and miscellaneous sectors for the same quarter of the previous year. This result may reflect changing consumption patterns in Korea due to the declining share of food expenditure and the increasing clothing and miscellaneous expenditure shares during the last two decades. The housing sector was found to be negatively affected by fuel and light consumption but positively by the previous year's clothing consumption; while the fuel and light sector was found to be positively affected by consumption in the housing sector during the previous year. The effect of fourth order lagged consumption in other sectors on the miscellaneous category appeared to be insignificant.

To examine the effects of quarterly first order lagged consumption on current expenditure within the framework of the stock adjustment and habit formation model, we also estimated the A(1,4) model and present the results in Table 5.4. In the A(1,4) model, the estimated Slutsky matrix and marginial budget shares and the lag structure coefficients for Y_{-4} model were almost identical to those of the AD[(4),(1)] model. The estimated own substitution effect of the miscellaneous sector was again shown to be positive but insignificant, and the significance and sign of terms in the estimated coefficient matrix for Y_{-4} were the same to those in the AD[(4),(1)] model with a some minor exceptions.

All the diagonal terms of the estimated coefficient matrix for Y_{-1} in the A(1,4) model were negative. The negative estimates for the short term lagged own dependent variables may reflect the dominance of the stock demand and stock adjustment features in the short run rather than the declining habit formation hypothesis. As Wohlgenant and Hahn (1982, p.553) point out, in the short period (e.g., month and quarter), almost any commodity can be considered a durable which provides a stream of services over time, and stock (inventory) effect may dominate the habit formation effect.[9] The clothing sector showed the most significant stock effect on a quarterly basis, while the housing sector showed insignificant stock effects. Another plausible interpretation of the negative effect of first order lagged own consumption may be seasonal consumption patterns. For example, the expenditure share of fuel and light (and clothing) increases in the first quarter but decrease in the second quarter of the year.

As for the off-diagonal terms of the coefficient matrix for Y_{-1}; the expenditure on the housing sector is posively related to that on food in the previous quarter, and the clothing and miscellaneous sectors are positively related with lagged consumption of the other sectors.

Thus, from estimation of AD[(4),(1)] and A(1,4) models, we have confirmed various conjectures on short run consumer behaviour; the presence of habit formation, stock adjustment, and delayed responses to changes in prices and total expenditure. Our empirical results imply that stock adjustment and delayed responses to changes in prices and total expenditure are the dominant features in short run consumer behaviour on a quarterly basis. However, habit formation was shown to be determined on an annual basis rather than quarter by quarter.

To incorporate these effects into a single dynamic demand system, we also estimated the AD[(1,4),(1)] model. However, since the results of AD[(1,4),(1)] model were found to be nearly identical to those of AD[(4),(1)] and A(1,4) models, the discussion of the AD[(1,4),(1)] model is skipped. For reference,the results are presented in Table 5.5. One thing to be noted in analysing the results from the AD[(1,4),(1)] model is that since the Y_{-1} and X_{-1} were included together as explanatory variables in AD[(1,4),(1)] model, estimates of coefficient matrix for Y_{-1} and the lagged total expenditure in X_{-1} cannot be identified, while the other structural coefficients are. Therefore, two sets of estimates of coefficient matrix for Y_{-1} and the lagged total expenditure were presented in Table 5.5; one is from the deletion of the last (fifth) equation from the system, the other from the deletion of the first equation.

In concluding this section, it may be worthwhile to summarise the above results on an equation-by-equation basis by briefly reviewing the dynamic pattern of the individual sectors. Such an equation by equation analysis of dynamic patterns should be an ultimate task in dynamic demand analysis, together with a final stage of diagnostic checking of the estimated models.

Apart from stock adjustment and habit formation, the predominant dynamic feature in the food sector was its negative (and significant) interdependence with the clothing and miscellaneous sectors at the fourth lag. These features may imply, in dynamic sense, that consumers in Korea increase the consumption of clothing and miscellaneous items at the expense of the food sector on an annual basis. No significant delayed response to changes in own and other prices was seen in the food sector. All the seasonal dummies were shown to be insignificant, so that no significant seasonal movement was observed in the dynamic food demand equation.

Even with a fourth order lag structure, the housing sector did not exhibit a habit formation pattern but displayed a slight stock adjustment pattern, as the estimated coefficient for own lagged consumption at the fourth lag was negative. Moreover, the housing sector exhibited a significant positive delayed response to total expenditure. Given that the housing sector includes some household durable goods, stock demand as well as stock adjustment can be viewed as the main dynamic features in the housing sector. In addition, the housing sector exhibits significant positive delayed responses to other sector price changes, such as the food and fuel and light sectors. It also exhibits significant interdependencies with the fuel and light sector and the clothing sector at the fourth lag and with the food sector at the first lag. The seasonal dummy for the second quarter was significantly positive, as in the static model.

In the fuel and light sector, no significant dynamic feature, apart from habit formation and stock adjustment, was observed; except for an interdependence with the housing sector at the fourth lag. However, regular seasonal movements were observed to be significant from the coefficients of the seasonal dummies. This sector appeared to react positively to own lagged price change, but the reaction was not statistically significant.

In the clothing sector, significant delayed responses to own price change, as well as food sector prices was observed. Additionally, the clothing sector exhibited significant negative interdependence with the food sector at the fourth lag, and positive interdependence with the miscellaneous sector at the first lag. The interpretion of the interdependece with the food sector at the fourth lag could be the same as one given for the food sector. However, since the miscellaneous sector also exhibited a positive interdependence with the clothing sector at the first lag, it could be that the consumer increases the consumption of clothing at the expense of miscellaneous items on a quarterly basis, and vice versa. Another dynamic feature in the miscellaneous sector was the significant positive delayed reaction to changes in clothing prices. However, the miscellaneous sector did not show a significant response to own current price or one period lagged price. Regular seasonal movements in the miscellaneous sectors were found to be significant. Significant evidence of stock adjustment and habit formation was also observed in both the clothing and miscellaneous sectors.

Thus the empirical results from the estimation of the dynamic Rotterdam system are not counter-intuitive. The performance of the dynamic Rotterdam demand system on the Korean data was generally satisfactory. In particular, the flexible nature of the dynamic system enables us analyze and quantify the complex dynamic features between different commodity sectors and at different lag lengths.

5.7 Concluding Remarks

The necessity for dynamisation of demand systems has long been recognised, particularly in the context of time series data. However, given the lack of the dynamic features in economic theory, dynamisation of demand systems has been often carried out on the basis of dynamic disequilibrium assumptions. Alternatively, it has been confined to the case of additive preferences with very restrictive habit formation assumptions. The main features of such dynamisations of demand systems are the abandonment of theoretical economic rationale for the consumer's dynamic behaviour and the inflexibility of the resulting dynamic demand system in terms of impact and dynamic responses.

In this chapter, we proposed a dynamic generalisation of the Rotterdam system to overcome these problems in specifing dynamic demand systems. The dynamic equilibrium assumption as well as the taste change hypothesis were adopted for the economic theoretical framework to derive the consumer's dynamic demand functions. The flexible nature of the Rotterdam model approximation allows a considerable degree of generality of the resulting estimable dynamic demand equations. In particular, given the independence of the functional specification from the taste parameters and the flexibility in the nature of the dynamic response, state variables for defining taste parameter can be incorporated directly in the dynamic demand equations without specifying taste functions.

The flexibility of the dynamic demand system allows direct estimation of the effects of state variables, such as interdependence between different commodities in different time periods and delayed responses to price and total expenditure changes. It also allows us to test a host of nested hypotheses related to the existence and nature of dynamic behaviour and the validity of dynamic equilibirum hypotheses in conjuction with the structure of the short run Slutsky matrix. Moreover, the dynamic Rotterdam system is a first order approximation to any dynamic demand system admitting non-separable dynamic preferences.

In the application to quarterly five commodity Korean data, the dynamic Rotterdam demand system performed remarkably in modelling changing consumption patterns, lagged resonses, habit formation, as well as the short run stock adjustment. The validity of the dynamic equilibirum hypothesis is not rejected by the data.

In concluding this chapter, it should be noted that the dynamic demand system derived from the taste change hypothesis does not embody a varying coefficient model. In the taste change hypothesis, changing preferences are assumed to depend on taste parameters which are defined on dynamic state variables but do not necessarily change over time. The dynamic demand system with varying coefficients may be worthwhile for a future dynamic demand study, if the economic rationale for the (time-dominant) varying nature of parameters of demand system is considered sufficiently strong. However, an efficient comptuting algorithm will be required before this can be contemplated [Byron (1984)].

FOOTNOTES:

1. The dynamic demand system is often termed the short run demand system in the sense that it is designed to identify the consumer's short run behaviour.

2. Discussion in the section is mostly based on reading Duménil and Lévy (1985), Varian (1978, Chapter 5 and 6), Takayama (1974), Bannock, Baxter and Rees (1978) and Deaton and Muellbauer (1980, Chapter 4 and 13).

3. This unrealistic assumption, suggested by Walras, assumes that all the traders gather in the one place and that there exists a 'market manager' who quotes a price for the commodity (say, i). Each trader writes the amount of that commodity that he wishes to buy or sell on a ticket. If there is an excess demand for i, the market manager raises the price of i, and if there is an excess supply of i, he lowers the price of i. Each time he quotes a new prices, the tickets are again collected. This process continues until the excess demand becomes zero, that is, until an equilibrium price is called. Until then, no actual transaction takes place. This process is called the tatonnement process.

4. Wage-earners possess labour capability, receive wages at the beginning of the period, and spend these wages on goods available at the same time. Wage-earners do not make intertemporal choices.

5. They assume that an intertemporal utility function is separable over time.

6. See also Lluch (1973) for the extended linear expenditure system.

7. The preference ordering defining such a utility funtion is "conditional" on the predetermined state S_t. The word "conditional" means that consumer's preferences depend on his tastes which are already established by the predetermined state variables. For details of the conditional preference ordering, see Katzner (1970, p.28) and Pollak (1978).

8. The required flexibility as well as singularity of the demand system implies the system-wide selection of dynamic state variables and thus system-wide dynamic specification of demand system, which results in a large increase in the number of explanatory variables and significant redunction of available degrees of freedom for a given sample size [see Sections 4 and 5 of this Chapter].

9. The stock adjustment feature is that the larger the physical stock, the smaller will be the consumer's demand in the short run. Conversely, if habits predominate, the larger (psychological) stock of habit and the greater will be the demand in the short run.

TABLES IN CHAPTER 5

Table 5.1
Tests of Significance of Dynamic Structure (Taste Change Coefficient) with the Unrestricted Model

Table 5.1.1 Tests of Significance of Coefficient of Individual Lagged Explanatory and Dependent Variables Against the Static Model

Null Model	Alternative Model	Zero Restriction Imposed on	LR Test Statistic	C.V. of χ^2 at 5 % (df)
Static	D(1)	X_{-1}	63.58	36.42 (24)
Static	D(2)	X_{-2}	47.09	36.42 (24)
Static	D(3)	X_{-3}	24.40	36.42 (24)
Static	D(4)	X_{-4}	31.64	36.42 (24)
Static	A(1)	Y_{-1}	79.42	31.41 (20)
Static	A(2)	Y_{-2}	74.73	31.41 (20)
Static	A(3)	Y_{-3}	31.76	31.41 (20)
Static	A(4)	Y_{-4}	70.68	31.41 (20)

Table 5.1.2 Tests of Significance of Coefficient of Individual Lagged Variables in the General Dynamic Structure Model

Null Model	Alternative Model	Zero Restriction Imposed on	LR Test Statistic	C.V. of χ^2 at 5 % (df)	Adjusted C.V. at 5 %
Distributed Lag Model					
D(2,3,4)	D(1,2,3,4)	X_{-1}	83.76	36.42 (24)	77.78
D(1,3,4)	D(1,2,3,4)	X_{-2}	84.89	36.42 (24)	77.78
D(1,2,4)	D(1,2,3,4)	X_{-3}	48.75	36.42 (24)	77.78
D(1,2,3)	D(1,2,3,4)	X_{-4}	71.68	36.42 (24)	77.78
Autoregressive Lag of Dependent Variables					
A(2,3,4)	A(1,2,3,4)	Y_{-1}	94.84	31.41 (20)	67.08
A(1,3,4)	A(1,2,3,4)	Y_{-2}	55.50	31.41 (20)	67.08
A(1,2,4)	A(1,2,3,4)	Y_{-3}	37.20	31.41 (20)	67.08
A(1,2,3)	A(1,2,3,4)	Y_{-4}	72.55	31.41 (20)	67.08

Table 5.1.3 Tests of Significance of Coefficients of Individual Lagged Explanatory and Dependent Variables in the General Dynamic Structure Model

Model	Zero Restriction Imposed on	LR Test Statistic	C.V. of x^2 at 5 % (df)	Adjusted C.V. at 5 %
AD[(1,2,4),(1,2,4)]	X_{-1}	112.22	31.41 (20)	86.04
	X_{-2}	101.36	31.41 (20)	86.04
	X_{-4}	72.09	31.41 (20)	86.04
	Y_{-1}	96.08	26.30 (16)	73.63
	Y_{-2}	56.46	26.30 (16)	73.63
	Y_{-4}	87.37	26.30 (16)	73.63
AD[(1,4),(1,2)]	X_{-1}	64.54	31.41 (20)	61.38
	X_{-2}	62.50	36.42 (24)	70.59
	Y_{-1}	68.02	26.30 (16)	52.59
	Y_{-4}	74.82	31.41 (20)	61.38
AD[(1,4),(1)]	X_{-1}	44.01	31.41 (20)	52.07
	Y_{-1}	61.73	26.30 (16)	44.18
	Y_{-4}	60.33	31.41 (20)	52.07
A(1,4)	Y_{-1}	74.01	31.41 (20)	46.02
	Y_{-4}	65.35	31.41 (20)	46.02
	Y_{-1} & Y_{-4}	144.77	55.75 (40)	77.20
AD[(4),(1)]	X_{-1}	57.91	36.42 (24)	53.99
	Y_{-4}	63.47	31.41 (20)	47.12
	X_{-1} & Y_{-4}	127.05	60.48 (44)	84.67

Table 5.2

Tests of the Short Run Demand Restrictions

Model	Restriction tested	Test statistic: Wald	LR	LM
RDAM-SD-D(1)				
	H	16.540	14.687	13.101
	S	10.671	10.273	9.898
	H & S	28.261	24.960	22.212
RDAM-SD-A(1)				
	H	15.561	13.906	12.479
	S	6.208	5.990	5.783
	H & S	22.056	19.896	18.030
RDAM-SD-A(4)				
	H	8.214	7.721	7.267
	S	8.206	7.881	7.575
	H & S	16.509	15.602	14.763
RDAM-SD-AD(4,1)				
	H	9.522	8.868	8.272
	S	11.677	11.035	10.444
	H & S	21.408	19.903	18.544
RDAM-SD-A(1,4)				
	H	18.510	16.228	14.306
	S	6.769	6.519	6.282
	H & S	25.376	22.747	20.511
RDAM-SD-AD[(1,4),(1)]				
	H	8.977	8.392	7.857
	S	11.728	11.023	10.379
	H & S	20.919	19.415	18.057

Table 5.3
RDAM-SD-AD(4,1) Model

Table 5.3.1 Unrestricted RDAM-SD-AD(4,1) Model

	dlnP1	dlnP2	dlnP3	dlnP4	dlnP5	dM
1	-.17991 (.0676)	-.04346 (.1425)	-.00445 (.0518)	.44025 (.1417)	.21877 (.0938)	.43134 (.0555)
2	.02582 (.0298)	-.25435 (.0627)	.05795 (.0228)	.03692 (.0624)	.02041 (.0413)	.11303 (.0244)
3	.06884 (.0209)	.07880 (.0440)	-.04374 (.0160)	.02968 (.0438)	-.10254 (.0290)	.03354 (.0171)
4	.06382 (.0358)	-.04319 (.0755)	.00751 (.0275)	-.11038 (.0751)	-.05322 (.0497)	.14247 (.0294)
5	.02143 (.0588)	.26221 (.1237)	-.01726 (.0450)	-.39647 (.1231)	-.08342 (.0815)	.27962 (.0482)

	$dlnP1_{-1}$	$dlnP2_{-1}$	$dlnP3_{-1}$	$dlnP4_{-1}$	$dlnP5_{-1}$	dM_{-1}
1	-.10243 (.0658)	.11168 (.1422)	.05132 (.0563)	-.37693 (.1602)	-.00322 (.1023)	-.09546 (.0575)
2	.10154 (.0290)	-.03744 (.0625)	.02974 (.0247)	.00444 (.0705)	-.04719 (.0450)	.08835 (.0209)
3	-.01613 (.0203)	-.04320 (.0439)	.03278 (.0174)	.03207 (.0495)	.03084 (.0316)	-.01366 (.0147)
4	-.07795 (.0349)	.18686 (.0753)	-.01275 (.0298)	-.10320 (.0848)	.05687 (.0542)	-.01057 (.0252)
5	.09496 (.0572)	-.21791 (.1235)	-.10109 (.0489)	.44363 (.1391)	-.03730 (.0889)	.03134 (.0413)

	$y1_{-4}$	$y2_{-4}$	$y3_{-4}$	$y4_{-4}$	$y5_{-4}$
1	.18597 (.0654)	-.30223 (.2251)	.20226 (.3239)	-.53914 (.1727)	-.16180 (.1099)
2	.04246 (.0288)	.01125 (.0990)	-.43117 (.1425)	.15476 (.0760)	.01463 (.0484)
3	-.02860 (.0202)	.12896 (.0695)	.30748 (.1000)	.03421 (.0533)	-.01482 (.0339)
4	-.10917 (.0346)	-.03140 (.1192)	-.09862 (.1716)	.36882 (.0914)	-.08811 (.0582)
5	-.09065 (.0568)	.19342 (.1955)	.02005 (.2814)	-.01865 (.1500)	.25010 (.0955)

Table 5.3.1 (continued)

	D_1	D_2	D_3	D_4	R^2	DW	DW4
1	-.03462	.00389	.00698	.01957	.9590	2.615	1.651
	(.0122)	(.0089)	(.0069)	(.0105)			
2	.00240	.01579	-.00671	-.01521	.7590	2.539	1.809
	(.0054)	(.0039)	(.0030)	(.0046)			
3	-.00149	-.01339	-.00477	.01074	.9118	2.548	1.894
	(.0038)	(.0027)	(.0021)	(.0032)			
4	.00302	.00273	-.00369	.01174	.7899	2.664	1.719
	(.0065)	(.0047)	(.0037)	(.0056)			
5	.03068	-.00903	.00819	-.02684	.8539	2.577	1.675
	(.0106)	(.0077)	(.0060)	(.0091)			

Table 5.3.2 Homogeneity Restricted RDAM-SD-AD(4,1) Model

	dlnP1	*dlnP2*	*dlnP3*	*dlnP4*	*dlnP5*	*dM*
1	-.22350	-.22495	.03681	.25479	.15684	.46881
	(.0689)	(.1302)	(.0519)	(.1284)	(.0954)	(.0563)
2	.03727	-.20668	.04711	.08563	.03668	.10319
	(.0294)	(.0555)	(.0221)	(.0547)	(.0406)	(.0240)
3	.06571	.06573	-.04077	.01633	-.10700	.03624
	(.0203)	(.0384)	(.0153)	(.0378)	(.0281)	(.0166)
4	.07751	.01382	-.00546	-.05211	-.03376	.13070
	(.0353)	(.0668)	(.0266)	(.0659)	(.0489)	(.0289)
5	.04301	.35208	-.03769	-.30464	-.05276	.26107
	(.0579)	(.1094)	(.0436)	(.1079)	(.0801)	(.0473)

	dlnP1₋₁	*dlnP2₋₁*	*dlnP3₋₁*	*dlnP4₋₁*	*dlnP5₋₁*	*dM₋₁*
1	-.11809	.20978	-.01130	-.15859	.06776	-.10582
	(.0689)	(.1440)	(.0533)	(.1429)	(.1036)	(.0598)
2	.10566	-.06321	.04619	-.05291	-.06584	.09107
	(.0294)	(.0614)	(.0227)	(.0609)	(.0442)	(.0212)
3	-.01726	-.03613	.02827	.04779	.03595	-.01440
	(.0203)	(.0424)	(.0157)	(.0421)	(.0305)	(.0147)
4	-.07303	.15604	.00692	-.17180	.03458	-.00731
	(.0354)	(.0739)	(.0274)	(.0733)	(.0531)	(.0256)
5	.10272	-.26648	-.07008	.33551	-.07245	.03647
	(.0579)	(.1210)	(.0448)	(.1201)	(.0870)	(.0418)

Table 5.3.2 (continued)

	$y1_{-4}$	$y2_{-4}$	$y3_{-4}$	$y4_{-4}$	$y5_{-4}$
1	.23174	-.18416	.19020	-.59288	-.24646
	(.0662)	(.2317)	(.3406)	(.1802)	(.1103)
2	.03043	-.01976	-.42801	.16888	.03687
	(.0282)	(.0988)	(.1452)	(.0768)	(.0470)
3	-.02531	.13746	.30661	.03034	-.02092
	(.0195)	(.0682)	(.1003)	(.0531)	(.0325)
4	-.12355	-.06850	-.09483	.38570	-.06151
	(.0340)	(.1189)	(.1747)	(.0924)	(.0566)
5	-.11332	.13496	.02602	.00796	.29202
	(.0556)	(.1947)	(.2861)	(.1514)	(.0926)

	D_1	D_2	D_3	D_4	R^2	DW	DW4
1	-.01867	.00568	.00707	.01824	.9547	2.633	1.651
	(.0110)	(.0093)	(.0073)	(.0111)			
2	-.00178	.01533	-.00673	-.01486	.7499	2.515	1.811
	(.0047)	(.0040)	(.0031)	(.0047)			
3	-.00034	-.01326	-.00477	.01064	.9113	2.551	1.946
	(.0032)	(.0027)	(.0021)	(.0033)			
4	-.00199	.00217	-.00372	.01216	.7821	2.613	1.719
	(.0057)	(.0048)	(.0037)	(.0057)			
5	.02278	-.00992	.00815	-.02618	.8489	2.636	1.713
	(.0093)	(.0078)	(.0061)	(.0093)			

Table 5.3.3 Homogeneity and Symmetry Restricted RDAM-SD-AD(4,1) Model

	$dlnP1$	$dlnP2$	$dlnP3$	$dlnP4$	$dlnP5$	dY
1	-.20833	.02682	.05912	.08392	.03848	.46675
	(.0652)	(.0282)	(.0184)	(.0332)	(.0564)	(.0525)
2	.02682	-.22134	.05071	.06229	.08152	.10507
	(.0282)	(.0514)	(.0183)	(.0441)	(.0364)	(.0235)
3	.05912	.05071	-.03892	.00225	-.07316	.03772
	(.0184)	(.0183)	(.0141)	(.0214)	(.0221)	(.0161)
4	.08392	.06229	.00225	-.08371	-.06475	.13185
	(.0332)	(.0441)	(.0214)	(.0607)	(.0440)	(.0278)
5	.03848	.08152	-.07316	-.06475	.01792	.25861
	(.0564)	(.0364)	(.0221)	(.0440)	(.0720)	(.0457)

Table 5.3.3 (continued)

	dlnP1₋₁	*dlnP2₋₁*	*dlnP3₋₁*	*dlnP4₋₁*	*dlnP5₋₁*	*dM₋₁*
1	-.11406 (.0714)	.16347 (.1483)	-.01054 (.0555)	-.10074 (.1453)	.06818 (.1071)	-.12247 (.0713)
2	.10437 (.0296)	-.06609 (.0619)	.04733 (.0230)	-.05084 (.0610)	-.07029 (.0445)	.09290 (.0214)
3	-.01831 (.0205)	-.03666 (.0427)	.02905 (.0159)	.04815 (.0420)	.03262 (.0308)	-.01290 (.0147)
4	-.07243 (.0356)	.14631 (.0740)	.00681 (.0276)	-.15967 (.0734)	.03533 (.0535)	-.01101 (.0256)
5	.10044 (.0615)	-.20704 (.1274)	-.07265 (.0477)	.26310 (.1252)	-.06583 (.0923)	.05348 (.0442)

	$y1_{-4}$	$y2_{-4}$	$y3_{-4}$	$y4_{-4}$	$y5_{-4}$
1	.22020 (.0656)	-.22040 (.2263)	.20206 (.3473)	-.61480 (.1803)	-.22939 (.1133)
2	.02918 (.0280)	-.00536 (.0980)	-.44201 (.1456)	.15854 (.0767)	.03876 (.0472)
3	-.02590 (.0196)	.14826 (.0674)	.29595 (.1006)	.02326 (.0526)	-.01945 (.0325)
4	-.12721 (.0336)	-.07209 (.1163)	-.09475 (.1749)	.38571 (.0915)	-.05838 (.0569)
5	-.09627 (.0568)	.14960 (.1959)	.03875 (.2993)	.04729 (.1565)	.26846 (.0974)

	D_1	D_2	D_3	D_4	R^2	DW	DW4
1	-.01845 (.0114)	.00615 (.0094)	.00402 (.0074)	.02071 (.0113)	.9509	2.592	1.786
2	-.00208 (.0047)	.01476 (.0040)	-.00633 (.0031)	-.01419 (.0047)	.7448	2.540	1.830
3	-.00058 (.0033)	-.01367 (.0027)	-.00446 (.0021)	.01107 (.0032)	.9092	2.492	2.028
4	-.00199 (.0057)	.00219 (.0048)	-.00439 (.0037)	.01254 (.0057)	.7787	2.651	1.640
5	.02310 (.0098)	-.00943 (.0081)	.01116 (.0064)	-.03014 (.0097)	.8283	2.589	1.670

Table 5.4 RDAM-SD-A(1,4) Model

Table 5.4.1 Unrestricted RDAM-SD-A(1,4) Model

	dlnP1	*dlnP2*	*dlnP3*	*dlnP4*	*dlnP5*	*dM*
1	-.15561	-.09946	-.01155	.37112	.11049	.46105
	(.0757)	(.1478)	(.0530)	(.1348)	(.0993)	(.0505)
2	.00453	-.34065	.08319	.02611	.08221	.14301
	(.0320)	(.0625)	(.0224)	(.0570)	(.0420)	(.0214)
3	.08207	.12997	-.04532	.01234	-.09629	.02645
	(.0209)	(.0408)	(.0146)	(.0372)	(.0274)	(.0139)
4	.09216	.06806	-.00797	-.12817	-.10272	.14213
	(.0356)	(.0694)	(.0249)	(.0633)	(.0467)	(.0237)
5	-.02315	.24208	-.01835	-.28141	.00630	.22736
	(.0675)	(.1319)	(.0473)	(.1202)	(.0886)	(.0451)

	$y1_{-1}$	$y2_{-1}$	$y3_{-1}$	$y4_{-1}$	$y5_{-1}$
1	-.16062	-.22451	-.32740	-.09543	.16439
	(.0639)	(.2147)	(.3673)	(.1756)	(.1152)
2	.10887	-.16689	.33110	.09269	-.01909
	(.0270)	(.0908)	(.1553)	(.0743)	(.0487)
3	.02748	-.04755	-.36609	-.01593	-.05636
	(.0176)	(.0592)	(.1014)	(.0485)	(.0318)
4	-.01949	.18717	-.05244	-.32759	.17367
	(.0300)	(.1009)	(.1726)	(.0825)	(.0541)
5	.04376	.25178	.41483	.34626	-.26261
	(.0570)	(.1915)	(.3278)	(.1567)	(.1027)

	$y1_{-4}$	$y2_{-4}$	$y3_{-4}$	$y4_{-4}$	$y5_{-4}$
1	.23013	-.22137	.51981	-.64576	-.11564
	(.0726)	(.2200)	(.3296)	(.1750)	(.1162)
2	.04188	.08151	-.50089	.14727	-.06025
	(.0307)	(.0930)	(.1394)	(.0740)	(.0491)
3	-.06548	.11869	.31666	.06272	-.01840
	(.0200)	(.0607)	(.0910)	(.0483)	(.0321)
4	-.07579	-.01892	.13674	.29209	-.00810
	(.0341)	(.1034)	(.1548)	(.0822)	(.0546)
5	-.13073	.04010	-.47231	.14368	.20240
	(.0648)	(.1963)	(.2941)	(.1561)	(.1037)

Table 5.4.1 (continued)

	D_1	D_2	D_3	D_4	R^2	DW	DW4
1	-.02260 (.0119)	-.01056 (.0095)	.00280 (.0091)	.00368 (.0108)	.9572	2.396	1.570
2	.00111 (.0050)	.02261 (.0040)	.00278 (.0039)	-.01337 (.0045)	.7677	2.286	2.174
3	-.00289 (.0033)	-.00800 (.0026)	-.00837 (.0025)	.01190 (.0030)	.9265	2.193	1.901
4	.01231 (.0056)	-.00227 (.0045)	-.00783 (.0043)	.00248 (.0051)	.8275	2.241	1.850
5	.01207 (.0106)	-.00178 (.0085)	.01062 (.0081)	-.00469 (.0096)	.8392	2.159	1.690

Table 5.4.2 Homogeneity Restricted RDAM-SD-A(1,4) Model

	dlnP1	*dlnP2*	*dlnP3*	*dlnP4*	*dlnP5*	*dY*
1	-.19769 (.0740)	-.21798 (.1364)	.01197 (.0528)	.30010 (.1325)	.10360 (.1019)	.46723 (.0517)
2	.03283 (.0325)	-.26094 (.0599)	.06738 (.0232)	.07388 (.0582)	.08685 (.0447)	.13885 (.0227)
3	.06587 (.0209)	.08434 (.0385)	-.03627 (.0149)	-.01500 (.0374)	-.09894 (.0288)	.02883 (.0146)
4	.10755 (.0344)	.11141 (.0635)	-.01657 (.0245)	-.10219 (.0616)	-.10019 (.0474)	.13986 (.0240)
5	-.00856 (.0646)	.28316 (.1191)	-.02650 (.0461)	-.25679 (.1156)	.00869 (.0889)	.22522 (.0451)

	$y1_{-1}$	$y2_{-1}$	$y3_{-1}$	$y4_{-1}$	$y5_{-1}$
1	-.14899 (.0653)	-.24859 (.2199)	-.19446 (.3696)	-.09967 (.1802)	.16107 (.1182)
2	.10105 (.0287)	-.15070 (.0965)	.24169 (.1622)	.09554 (.0791)	-.01685 (.0519)
3	.03195 (.0184)	-.05682 (.0621)	-.31491 (.1044)	-.01756 (.0509)	-.05764 (.0334)
4	-.02375 (.0304)	.19598 (.1023)	-.10106 (.1719)	-.32604 (.0838)	.17488 (.0550)
5	.03973 (.0570)	.26013 (.1920)	.36874 (.3226)	.34773 (.1573)	-.26145 (.1031)

Table 5.4.2 (continued)

	$y1_{-4}$	$y2_{-4}$	$y3_{-4}$	$y4_{-4}$	$y5_{-4}$
1	.26571 (.0718)	-.19890 (.2255)	.55577 (.3377)	-.66383 (.1793)	-.14995 (.1177)
2	.01795 (.0315)	.06639 (.0990)	-.52508 (.1482)	.15942 (.0787)	-.03718 (.0517)
3	-.05178 (.0203)	.12734 (.0637)	.33050 (.0954)	.05577 (.0507)	-.03161 (.0333)
4	-.08881 (.0334)	-.02714 (.1049)	.12358 (.1570)	.29870 (.0834)	.00444 (.0547)
5	-.14307 (.0627)	.03231 (.1968)	-.48478 (.2947)	.14994 (.1565)	.21429 (.1027)

	D_1	D_2	D_3	D_4	R^2	DW	DW4
1	-.01356 (.0111)	-.00296 (.0088)	.01074 (.0082)	.00544 (.0110)	.9549	2.479	1.661
2	-.00497 (.0049)	.01750 (.0039)	-.00256 (.0036)	-.01455 (.0048)	.7363	2.179	2.207
3	.00059 (.0031)	-.00507 (.0025)	-.00532 (.0023)	.01257 (.0031)	.9189	2.073	1.931
4	.00900 (.0052)	-.00505 (.0041)	-.01074 (.0038)	.00184 (.0051)	.8219	2.279	1.861
5	.00893 (.0097)	-.00441 (.0077)	.00787 (.0072)	-.00530 (.0096)	.8379	2.192	1.717

Table 5.4.3 Homogeneity and Symmetry Restricted RDAM-SD-A(1,4) Model

	dlnP1	*dlnP2*	*dlnP3*	*dlnP4*	*dlnP5*	*dM*
1	-.16673 (.0739)	.01857 (.0314)	.05484 (.0192)	.10850 (.0326)	-.01518 (.0640)	.46698 (.0466)
2	.01857 (.0314)	-.29816 (.0533)	.06616 (.0189)	.09675 (.0433)	.11669 (.0393)	.14013 (.0220)
3	.05484 (.0192)	.06616 (.0189)	-.03456 (.0140)	-.01375 (.0197)	-.07270 (.0229)	.03050 (.0140)
4	.10850 (.0326)	.09675 (.0433)	-.01375 (.0197)	-.09496 (.0552)	-.09655 (.0424)	.14211 (.0226)
5	-.01518 (.0640)	.11669 (.0393)	-.07270 (.0229)	-.09655 (.0424)	.06774 (.0788)	.22027 (.0409)

Table 5.4.3 (continued)

	$y1_{-1}$	$y2_{-1}$	$y3_{-1}$	$y4_{-1}$	$y5_{-1}$
1	-.15351 (.0659)	-.16296 (.2248)	-.33831 (.3719)	-.17718 (.1830)	.13437 (.1211)
2	.10418 (.0285)	-.15822 (.0970)	.29017 (.1604)	.10942 (.0787)	-.02278 (.0516)
3	.03574 (.0183)	-.06086 (.0618)	-.28082 (.1017)	-.01125 (.0502)	-.06487 (.0333)
4	-.02305 (.0302)	.19243 (.1015)	-.09100 (.1679)	-.32302 (.0831)	.17446 (.0543)
5	.03665 (.0577)	.18960 (.1944)	.41996 (.3225)	.40204 (.1586)	-.22118 (.1044)

	$y1_{-4}$	$y2_{-4}$	$y3_{-4}$	$y4_{-4}$	$y5_{-4}$
1	.23407 (.0717)	-.21252 (.2194)	.50882 (.3377)	-.65899 (.1789)	-.12577 (.1207)
2	.02231 (.0311)	.07511 (.0978)	-.53164 (.1467)	.16025 (.0781)	-.04821 (.0514)
3	-.05015 (.0202)	.13608 (.0626)	.31786 (.0948)	.05413 (.0503)	-.03903 (.0329)
4	-.08915 (.0330)	-.01994 (.1017)	.11848 (.1552)	.30268 (.0820)	.00267 (.0543)
5	-.11708 (.0624)	.02127 (.1913)	-.41351 (.2940)	.14192 (.1564)	.21034 (.1039)

	D_1	D_2	D_3	D_4	R^2	DW	DW4
1	-.01217 (.0089)	-.00166 (.0080)	.00459 (.0111)	.01005 (.0111)	.9512	2.448	1.789
2	-.00558 (.0048)	.01719 (.0039)	-.00128 (.0035)	-.01495 (.0048)	.7332	2.176	2.248
3	-.00002 (.0031)	-.00526 (.0025)	-.00456 (.0022)	.01261 (.0030)	.9177	2.117	1.983
4	.00874 (.0051)	-.00526 (.0040)	-.01046 (.0037)	.00161 (.0050)	.8218	2.261	1.877
5	.00903 (.0097)	-.00502 (.0077)	.01171 (.0070)	-.00932 (.0096)	.8271	2.282	1.691

Table 5.5
RDAM-SD-AD[(1,4),(1)] Model

Table 5.5.1 Unrestricted RDAM-SD-AD[(1,4),(1)] Model

	dlnF1	*dlnF2*	*dlnF3*	*dlnF4*	*dlnF5*	*dM*
1	-.17087	-.11003	-.01229	.46576	.17577	.44761
	(.0739)	(.1550)	(.0521)	(.1401)	(.1015)	(.0570)
2	.01918	-.28186	.06645	.04743	.07568	.12057
	(.0289)	(.0605)	(.0203)	(.0547)	(.0396)	(.0223)
3	.08670	.13279	-.04816	.00908	-.10004	.01582
	(.0207)	(.0434)	(.0146)	(.0392)	(.0284)	(.0160)
4	.08029	.00713	-.00074	-.12573	-.10218	.14517
	(.0326)	(.0684)	(.0230)	(.0618)	(.0448)	(.0251)
5	-.01530	.25195	-.00527	-.39654	-.04924	.27084
	(.0623)	(.1307)	(.0439)	(.1181)	(.0856)	(.0481)

	$dlnF1_{-1}$	$dlnF2_{-1}$	$dlnF3_{-1}$	$dlnF4_{-1}$	$dlnF5_{-1}$
1	-.12019	.01095	.06950	-.26865	.06568
	(.0751)	(.1545)	(.0565)	(.1706)	(.1132)
2	.11100	-.09061	.04311	-.04265	-.05628
	(.0293)	(.0603)	(.0221)	(.0666)	(.0442)
3	.01849	.00527	.02442	-.00624	-.02612
	(.0211)	(.0433)	(.0158)	(.0478)	(.0317)
4	-.08400	.18650	-.02075	-.08816	.07131
	(.0332)	(.0682)	(.0249)	(.0753)	(.0499)
5	.07471	-.11211	-.11628	.40570	-.05458
	(.0634)	(.1303)	(.0476)	(.1439)	(.0954)

	$y1_{-4}$	$y2_{-4}$	$y3_{-4}$	$y4_{-4}$	$y5_{-4}$
1	.22228	-.27705	.33762	-.61671	-.11530
	(.0720)	(.2212)	(.3303)	(.1734)	(.1221)
2	.02921	.01569	-.46142	.16506	.01057
	(.0281)	(.0864)	(.1290)	(.0677)	(.0477)
3	-.06693	.09597	.28294	.07637	.00213
	(.0202)	(.0620)	(.0926)	(.0486)	(.0342)
4	-.07519	-.00820	.02358	.30595	-.06788
	(.0318)	(.0976)	(.1457)	(.0765)	(.0539)
5	-.10936	.17359	-.18272	.06933	.17048
	(.0607)	(.1865)	(.2786)	(.1463)	(.1030)

Table 5.5.1 (continued)

When the Fifth Equation is Deleted

	dM_{-1}	$y1_{-1}$	$y2_{-1}$	$y3_{-1}$	$y4_{-1}$
1	.07126	-.23920	-.25404	.02203	-.16439
	(.1305)	(.1423)	(.2883)	(.4754)	(.2325)
2	.08435	.07622	-.31398	.05820	.00784
	(.0510)	(.0556)	(.1126)	(.1857)	(.0908)
3	-.03479	.06468	-.01511	-.35323	.04565
	(.0366)	(.0399)	(.0808)	(.1332)	(.0651)
4	.07802	-.13178	.20191	.03647	-.38697
	(.0576)	(.0628)	(.1272)	(.2098)	(.1026)

When the First Equation is Deleted

	dM_{-1}	$y2_{-1}$	$y3_{-1}$	$y4_{-1}$	$y5_{-1}$
2	.16057	-.39020	-.01801	-.06838	-.07622
	(.0290)	(.0924)	(.1680)	(.0818)	(.0556)
3	.02989	-.07979	-.41791	-.01903	-.06468
	(.0208)	(.0663)	(.1205)	(.0587)	(.0399)
4	-.05376	.33369	.16825	-.25520	.13178
	(.0328)	(.1044)	(.1898)	(.0924)	(.0628)
5	.03124	.15113	.00645	.26779	-.23008
	(.0627)	(.1996)	(.3627)	(.1765)	(.1200)

	D_1	D_2	D_3	D_4	R^2	DW	DW4
1	-.02856	-.00569	.01061	.01410	.9612	2.466	1.603
	(.0128)	(.0105)	(.0097)	(.0111)			
2	-.00493	.02246	.00138	-.01681	.8205	1.938	2.138
	(.0050)	(.0041)	(.0038)	(.0043)			
3	-.00494	-.00917	-.00901	.01255	.9314	2.142	1.942
	(.0036)	(.0029)	(.0027)	(.0031)			
4	.01415	-.00297	-.00803	.00724	.8622	2.062	1.795
	(.0056)	(.0046)	(.0043)	(.0049)			
5	.02428	-.00462	.00505	-.01708	.8699	2.176	1.753
	(.0108)	(.0089)	(.0082)	(.0094)			

Table 5.5.2 Homogeneity Restricted RDAM-SD-AD[(1,4),(1)] Model

	dlnP1	dlnP2	dlnP3	dlnP4	dlnP5	dM
1	-.20972	-.27680	.01708	.34815	.12128	.48068
	(.0736)	(.1349)	(.0515)	(.1312)	(.1008)	(.0563)
2	.02734	-.24685	.06029	.07211	.08712	.11363
	(.0281)	(.0515)	(.0197)	(.0501)	(.0385)	(.0215)
3	.07773	.09431	-.04138	-.01805	-.11261	.02345
	(.0204)	(.0374)	(.0143)	(.0364)	(.0280)	(.0156)
4	.09604	.07474	-.01264	-.07805	-.08008	.13176
	(.0323)	(.0592)	(.0226)	(.0576)	(.0443)	(.0247)
5	.00861	.35459	-.02334	-.32416	-.01570	.25048
	(.0612)	(.1121)	(.0428)	(.1091)	(.0838)	(.0468)

	$dlnP1_{-1}$	$dlnP2_{-1}$	$dlnP3_{-1}$	$dlnP4_{-1}$	$dlnP5_{-1}$
1	-.14862	.06029	.02987	-.07127	.14808
	(.0761)	(.1573)	(.0546)	(.1436)	(.1087)
2	.11696	-.10096	.05143	-.08408	-.07358
	(.0291)	(.0601)	(.0208)	(.0549)	(.0416)
3	.01193	.01665	.01528	.03931	-.00711
	(.0211)	(.0437)	(.0151)	(.0399)	(.0302)
4	-.07248	.16650	-.00469	-.16817	.03790
	(.0334)	(.0691)	(.0240)	(.0631)	(.0478)
5	.09221	-.14248	-.09189	.28422	-.10529
	(.0633)	(.1308)	(.0454)	(.1194)	(.0904)

	$y1_{-4}$	$y2_{-4}$	$y3_{-4}$	$y4_{-4}$	$y5_{-4}$
1	.26973	-.18456	.33573	-.66922	-.17824
	(.0701)	(.2231)	(.3407)	(.1768)	(.1217)
2	.01926	-.00372	-.46102	.17609	.02378
	(.0268)	(.0853)	(.1302)	(.0676)	(.0465)
3	-.05598	.11730	.28250	.06425	-.01239
	(.0195)	(.0619)	(.0945)	(.0491)	(.0338)
4	-.09443	-.04569	.02435	.32724	-.04237
	(.0308)	(.0980)	(.1496)	(.0777)	(.0534)
5	-.13857	.11667	-.18155	.10165	.20921
	(.0583)	(.1855)	(.2832)	(.1470)	(.1012)

Table 5.5.2 (continued)

When the Fifth Equation is Deleted

	dM_{-1}	$y1_{-1}$	$y2_{-1}$	$y3_{-1}$	$y4_{-1}$
1	.07655 (.1346)	-.28995 (.1444)	-.25707 (.2974)	.13305 (.4870)	-.10149 (.2376)
2	.08324 (.0514)	.08687 (.0552)	-.31334 (.1136)	.03490 (.1861)	-.00536 (.0908)
3	-.03357 (.0373)	.05297 (.0401)	-.01580 (.0825)	-.32762 (.1351)	.06016 (.0659)
4	.07588 (.0591)	-.11120 (.0634)	.20314 (.1306)	-.00853 (.2139)	-.41247 (.1043)

When the First Equation is Deleted

	dM_{-1}	$y2_{-1}$	$y3_{-1}$	$y4_{-1}$	$y5_{-1}$
2	.17011 (.0279)	-.40021 (.0928)	-.05196 (.1665)	-.09223 (.0794)	-.08687 (.0552)
3	.01940 (.0203)	-.06877 (.0674)	-.38059 (.1209)	.00719 (.0577)	-.05297 (.0401)
4	-.03533 (.0321)	.31434 (.1067)	.10267 (.1913)	-.30127 (.0913)	.11120 (.0634)
5	.05922 (.0607)	.12176 (.2019)	-.09311 (.3622)	.19785 (.1728)	-.26131 (.1201)

	D_1	D_2	D_3	D_4	R^2	DW	DW4
1	-.01594 (.0115)	-.00803 (.0108)	.01176 (.0100)	.01328 (.0114)	.9588	2.427	1.634
2	-.00758 (.0044)	.02295 (.0041)	.00114 (.0038)	-.01664 (.0044)	.8172	1.932	2.199
3	-.00203 (.0032)	-.00971 (.0030)	-.00875 (.0028)	.01236 (.0032)	.9284	2.199	1.995
4	.00904 (.0050)	-.00203 (.0047)	-.00850 (.0044)	.00757 (.0050)	.8547	1.968	1.710
5	.01652 (.0096)	-.00319 (.0090)	.00434 (.0083)	-.01657 (.0095)	.8654	2.222	1.785

Table 5.5.3 Homogeneity and Symmetry Restricted RDAM-SD-AD[(1,4),(1)] Model

	dlnP1	dlnP2	dlnP3	dlnP4	dlnP5	dM
1	-.16803 (.0750)	.01528 (.0272)	.06416 (.0188)	.09673 (.0308)	-.00815 (.0631)	.48055 (.0526)
2	.01528 (.0272)	-.27077 (.0456)	.06419 (.0166)	.07317 (.0376)	.11812 (.0346)	.11672 (.0210)
3	.06416 (.0188)	.06419 (.0166)	-.03795 (.0133)	-.01334 (.0180)	-.07707 (.0219)	.02676 (.0152)
4	.09673 (.0308)	.07317 (.0376)	-.01334 (.0180)	-.07513 (.0488)	-.08143 (.0383)	.13160 (.0233)
5	-.00815 (.0631)	.11812 (.0346)	-.07707 (.0219)	-.08143 (.0383)	.04852 (.0750)	.24437 (.0445)

	$dlnP1_{-1}$	$dlnP2_{-1}$	$dlnP3_{-1}$	$dlnP4_{-1}$	$dlnP5_{-1}$
1	-.12494 (.0788)	.06399 (.1663)	.01341 (.0575)	-.00972 (.1503)	.10861 (.1126)
2	.11205 (.0290)	-.10463 (.0604)	.05269 (.0209)	-.09021 (.0549)	-.07130 (.0414)
3	.00631 (.0211)	.01270 (.0442)	.01689 (.0153)	.03179 (.0400)	-.00412 (.0300)
4	-.07236 (.0329)	.16674 (.0690)	-.00461 (.0239)	-.16834 (.0627)	.03819 (.0474)
5	.07895 (.0660)	-.13881 (.1388)	-.07838 (.0480)	.23648 (.1258)	-.07138 (.0940)

	$y1_{-4}$	$y2_{-4}$	$y3_{-4}$	$y4_{-4}$	$y5_{-4}$
1	.23156 (.0718)	-.18842 (.2208)	.32135 (.3499)	-.67015 (.1815)	-.14817 (.1270)
2	.02131 (.0265)	.00843 (.0843)	-.47848 (.1297)	.17431 (.0673)	.01500 (.0463)
3	-.05319 (.0195)	.13041 (.0612)	.26392 (.0947)	.06223 (.0492)	-.02221 (.0337)
4	-.09423 (.0305)	-.04642 (.0950)	.02581 (.1478)	.32760 (.0765)	-.04198 (.0529)
5	-.10546 (.0603)	.09600 (.1858)	-.13261 (.2934)	.10601 (.1527)	.19736 (.1057)

Table 5.5.3 (continued)

When the Fifth Equation is Deleted

	dM_{-1}	$y1_{-1}$	$y2_{-1}$	$y3_{-1}$	$y4_{-1}$
1	.05592	-.27170	-.14609	-.12564	-.17859
	(.1396)	(.1476)	(.3121)	(.4885)	(.2482)
2	.07200	.10241	-.30809	.10041	.01024
	(.0511)	(.0544)	(.1140)	(.1827)	(.0906)
3	-.04529	.06939	-.01202	-.25356	.07801
	(.0374)	(.0397)	(.0832)	(.1303)	(.0659)
4	.07705	-.11273	.20143	-.01099	-.41289
	(.0582)	(.0621)	(.1299)	(.2054)	(.1034)

When the First Equation is Deleted

	dM_{-1}	$y2_{-1}$	$y3_{-1}$	$y4_{-1}$	$y5_{-1}$
2	.17441	-.41050	-.00200	-.09216	-.10241
	(.0278)	(.0932)	(.1636)	(.0788)	(.0544)
3	.02410	-.08141	-.32294	.00862	-.06939
	(.0203)	(.0675)	(.1162)	(.0573)	(.0397)
4	-.03568	.31417	.10174	-.30016	.11273
	(.0319)	(.1060)	(.1845)	(.0906)	(.0621)
5	.05295	.05213	.07715	.29059	-.21264
	(.0638)	(.2111)	(.3648)	(.1792)	(.1242)

	D_1	D_2	D_3	D_4	R^2	DW	DW4
1	-.01386	-.00733	.00273	.01674	.9537	2.403	1.732
	(.0119)	(.0113)	(.0098)	(.0119)			
2	-.00815	.02317	.00249	-.01628	.8151	1.973	2.246
	(.0044)	(.0041)	(.0037)	(.0044)			
3	-.00268	-.00949	-.00716	.01270	.9263	2.259	2.058
	(.0032)	(.0030)	(.0026)	(.0032)			
4	.00906	-.00206	-.00851	.00749	.8547	1.965	1.710
	(.0050)	(.0047)	(.0042)	(.0050)			
5	.01563	-.00429	.01045	-.02065	.8476	2.238	1.733
	(.0100)	(.0095)	(.0083)	(.0100)			

CHAPTER 6

THE EFFECT OF MODEL MISSPECIFICATION ON TESTS OF HYPOTHESIS

6.1 Introduction

Given that the true specification of a demand system is always unknown, the choice of a successful model is purely hypothetical. For example, one expects that successful modelling of the equilibrium demand system, whether static or dynamic, is accompanied by the acceptance of theoretical equilibrium restrictions on data, as well as by the randomness of the residuals of the estimated system. It has been conjectured in the literature that the success of tests of equilibrium demand restrictions can be attributed to correct specification of the demand system, including specification of its dynamic structure [e.g., see Deaton and Muellbauer (1980b) and Anderson and Blundell (1982)]. However, the empirical experience of the last two chapters suggests that demand systems which achieved randomness of residuals as well as acceptable dynamics did not always result in successful tests of demand restrictions. [See, for example, RDAM-D4 model in Chapter 4 and the dynamic RDAM-SD-A(1,4) model in Chapter 5.] The conjecture that invalid specification of the demand system negatively affects tests of restrictions was not fully verified. Accordingly, a more stringent statistical analysis of the effects of misspecification on tests of equilibrium restrictions will be necessary, and will be done in this chapter for particular types of misspecification of demand systems.

There are many possible sources of misspecification of econometric models. Examples include the use of incorrect functional form, incorrect specification of structural and stochastic dynamic structure and incorrect specification of the error covariance matrix. However, the analysis in this chapter is focused on two particular cases of the model misspecification; one is caused by the omission of relevant explanatory variables, the other is caused by inclusion of irrelevant variables. These two types of misspecification are partially related to the use of incorrect functional form as well as to incorrect specification of dynamic structure of demand system in the context of the Rotterdam demand system. As derived in the last chapter, the dynamic Rotterdam demand system nests its static versions as a special case when the dynamic state variables are omitted. Therefore, we examine the static and dynamic Rotterdam demand systems for the conflicting results.

Consequently, to illustrate the problem associated with the inclusion of irrelevant explanatory variables, we first maintain the static system as true and the dynamic system as the incorrectly specified model. Next, we maintain the dynamic system as true and the static system as misspecified, to illustrate the case of omission of relevant explanatory variables. These two types of misspecification are often termed the incorrect overparameterisation and underparameterisation of the model. Since both the static and dynamic Rotterdam demand systems are expressed in terms of the linear multivariate model, our theoretical discussion will be carried out in the context of tests of general linear restrictions on the linear multivariate model. The theoretical validity of demand restrictions on the dynamic demand system was supported by the results in the last chapter using the dynamic equilibrium hypothesis.

The effects of these two types of misspecifications on the estimation of parameters have been examined in the literature in the context of the OLS [Theil (1957) and P. Rao (1972)] as well as the SUR estimation [Rao (1974)].[2] However, little attention has been paid on their effect on tests of hypothesis. Bera and Byron (1983) discussed the effect of model misspecification on testing hypotheses, in the context of a single equation linear approximation to a nonlinear model. They showed that ignoring the remainder term in a linear approximation, which we can therefore refer to as an underparameterisation of model, leads to higher Type I errors and possibly affects the power of the test [see also Sawyer and Rosalsky (1984)]. A more thorough study on the effect of misspecification on tests of hypothesis has been done by White (1982). He derived a robust test to detect model misspecification, and showed that the standard asymptotic χ^2 tests are invalid under misspecification. A similar conclusion will be found for the underparameterised case in this chapter. However, the overparameterised case was not considered in White (1982)[1].

A notable study of misspecification analysis in the context of demand models is Kiefer and MacKinnon (1976)'s Monte Carlo simulation which showed that the use of an incorrect functional form of the demand equations would result in a biased estimates of the demand system. However, no study has been done on the effect of the misspecification of demand system in testing equilibrium restrictions.

The design of this chapter is as follows: in Section 2, the effect of inclusion of irrelevant explanatory variables in a linear multivariate model on tests of hypothesis will be examined. In Section 3, the effect of omission of relevant explanatory variables will also be discussed analytically. Experimental Monte Carlo simulation results will be presented in Section 4. As a by-product, a Monte Carlo study on the over-rejection of asymptotic χ^2 tests under the correct specification of a dynamic demand system will also be presented. Finally, Section 5 draws some conclusions from the analysis of this chapter.

6.2 Effect of Overparameterisation of the Model: Some Analytical Remarks

For analysis of the effect of inclusion of irrelevant explanatory variables, we consider the linear multivariate model with m equations and N observations, and maintain the true model as

$$Y = XB + U, \tag{2.1}$$

and the adopted model as

$$Y = XB + Z\Gamma + U_F, \tag{2.2}$$

where Y is an N x m matrix of dependent variables, X is an N x k matrix of non-stochastic explanatory variables, B is a k x m coefficient matrix, and U is an N x m matrix of disturbances having a multivariate normal distribution with $E(U) = 0$ and $E(U'U) = \Sigma$, Z is an N x l matrix of irrelevant explanatory variables, Γ is an l x m coefficient matrix, and U_F is an N x m matrix of incorrectly defined disturbances such that $U_F = U - Z\Gamma$. We allow stochastic variables, such as lagged dependent variables, in Z. We also assume that the estimation of (2.1) and (2.2) is subject to s linear independent restrictions on B, but no restriction is imposed on the coefficient matrix Γ. The asymptotic assumptions are that

$$\text{plim } N^{-1}X'U = 0, \tag{2.3}$$

$$\text{plim } N^{-1}Z'U = 0, \tag{2.4}$$

and

$$\text{plim } N^{-1}U'U = \Sigma. \tag{2.5}$$

From the true model, the ML estimators of B and Σ are given by

$$\hat{B}_T = (X'X)^{-1}X'Y \tag{2.6}$$

and

$$\hat{\Sigma}_T = \hat{U}_T'\hat{U}_T/N = Y'M_XY/N, \tag{2.7}$$

respectively, where $\hat{U}_T = M_XY$ is the residual matrix, $M_X = I_N - X(X'X)^{-1}X'$, and suffix T refers to the true model. Then, $\hat{B}_T$ is an unbiased and consistent estimate of B in (2,1), and $\hat{\Sigma}_T$ is a consistent estimate of Σ under the assumptions (2.3) and (2.5). The covariance matrix of $vec(\hat{B}_T) = \hat{\beta}_T$ is given by

$$cov(\hat{\beta}_T) = \Sigma \otimes (X'X)^{-1}. \tag{2.8}$$

If we let $W = [X:Z]$ and $\Pi' = [B':\Gamma']$ and write the incorrect model (2.2) as

$$Y = W\,\Pi + U_F, \tag{2.9}$$

then the ML estimators of Π and Σ from the incorrect model (2.9) are given by

$$\hat{\Pi}_F = (W'W)^{-1}W'Y \tag{2.10}$$

and

$$\hat{\Sigma}_F = \hat{U}_F'\hat{U}_F/N = Y'M_WY/N, \tag{2.11}$$

respectively, where W is an N x (k+l) matrix, Π a (k+l) x m coefficient matrix, $\hat{U}_F = M_WY$ a matrix of residuals with

$M_W = I_N - W(W'W)^{-1}W'$, and suffix F refers to the incorrect model. Using the partitioned inverse of $(W'W)^{-1}$, the estimator $\hat{B}_F$ in $\hat{\Pi}_F$ can be given as

$$\hat{B}_F = (X'M_ZX)^{-1}X'M_ZY \qquad (2.12)$$

where $M_Z = I_N - Z(Z'Z)^{-1}Z'$ (see for proof Appendix 6.1). It is clear that $\hat{B}_T \neq \hat{B}_F$, unless X and Z are orthogonal, i.e., if $X'Z \neq 0$. However, substituting (2.1) into (2.12), $\hat{B}_F$ can be written as

$$\hat{B}_F = (X'M_ZX)^{-1}X'M_Z(XB + Z\Gamma + U)$$

$$= B + (X'M_ZX)^{-1}X'M_ZU, \qquad (2.13)$$

since $M_ZZ = 0$. Therefore, it follows that $\hat{B}_F$ in (2.12) is consistent estimator of B under the assumptions (2.3) and (2.4) and the assumption that there exists a finite matrix F such that plim $N^{-1}X'M_ZX = F$. Unless Z contains stochastic variables, $\hat{B}_F$ in (2.12) is also unbiased. [see also Rao (1972)].

The unnecessary inclusion of irrelevant explanatory variables leads to less efficient estimate of B in (2.1). For example, when Z is non-stochastic, the covariance matrix of $\text{vec}(\hat{B}_F) = \hat{\beta}_F$ is then given by

$$\text{cov}(\hat{\beta}_F) = \Sigma \otimes (X'M_ZX)^{-1}, \qquad (2.14)$$

since $\mathrm{cov}[\mathrm{vec}(\hat{\Pi}_F)] = \Sigma \otimes (W'W)^{-1}$ using the partitioned inverse of $(W'W)^{-1}$. However, it can be seen that

$$(X'M_ZX)^{-1} - (X'X)^{-1} = (X'X)^{-1}X'Z(Z'M_XZ)^{-1}Z'X(X'X)^{-1}, \quad (2.15)$$

so that $(X'M_ZX)^{-1} - (X'X)^{-1}$ is positive definite unless Z is orthogonal to X (see for proof Appendix 6.2). Therefore, it follows that

$$\mathrm{cov}(\hat{\beta}_F) - \mathrm{cov}(\hat{\beta}_T) = \Sigma \otimes [(X'M_ZX)^{-1} - (X'X)^{-1}] \quad (2.16)$$

is also positive definite, implying that $\hat{B}_F$ is less efficient than $\hat{B}_T$ [see also Rao (1972)].

From the fact that

$$M_W = M_X - M_XZ(Z'M_XZ)^{-1}Z'M_X, \quad (2.17)$$

$$= M_Z - M_ZX(X'M_ZX)^{-1}X'M_Z,$$

(see for proof Appendix 6.3), it follows that $X'M_W = Z'M_W = 0$ and $M_WX = M_WZ = 0$. Hence, $\hat{\Sigma}_F$ in (2.11) can be written as

$$\hat{\Sigma}_F = (XB + U)'M_W\ (XB + U)/N = U'M_WU/N, \quad (2.18)$$

after the substitution of (2.1) into (2.11). This shows that $\hat{\Sigma}_F$ is also a consistent estimator of Σ, that is,

$$\mathrm{plim}\ \hat{\Sigma}_F = \Sigma, \quad (2.19)$$

under the assumptions (2.3), (2.4) and (2.5). However, since $M_X - M_W$ is positive definite from (2.17),

$$\hat{\Sigma}_T - \hat{\Sigma}_F = Y'(M_X - M_W)Y/N$$

is also positive definite. Consequently, we can expect, in the small sample situation, $\hat{\Sigma}_T - \hat{\Sigma}_F$ is positive definite. Even if X and Z are orthogonal, (2.17) becomes

$$M_W = M_X - Z(Z'Z)^{-1}Z', \tag{2.20}$$

so that $M_X - M_W$ is still positive definite.

If s independent linear restrictions are imposed on the coefficient matrix B in the true model (2.1), they can be expressed as

$$R\beta = 0, \tag{2.21}$$

where $\beta = \text{vec}(B)$ and R is an s x mk matrix with rank(R) = s. Furthermore, if the same restrictions are imposed on the incorrect model (2.2), they can equivalently be expressed as

$$C\pi = 0, \tag{2.22}$$

where C is an s x m(k+l) matrix with rank(C) = s, $\pi = \text{vec}(\Pi)$ such that $\pi' = (\beta_1', \gamma_1', \ldots, \beta_m', \gamma_m')$ and $\gamma = \text{vec}(\Gamma)$, and β_i's and γ_i's are the coefficient vectors in B and Γ for the i'th equation, respectively. However, since we assume that no restriction is imposed on the coefficient matrix Γ, the equivalence between the expressions in (2.21) and (2.22) can be established by defining an mk x m(k+q) selection matrix C_1, which selects β from π, such that

$$\beta = C_1 \pi. \tag{2.23}$$

Then, the restriction matrix C in (2.22) can be written as

$$C = R\, C_1, \tag{2.24}$$

since $R\beta = RC_1\pi = C\pi = 0$ (see Appendix 6.4). Similar distributional results for the unrestricted estimators of β and Σ can also be derived for the restricted estimators of β and Σ. For example, the restricted estimator of β from the incorrect model, $\tilde{\beta}_F$, is consistent under the restrictions (2.21), and unbiased if X and Z are orthogonal. However, direct algebraic comparison of the restricted estimators of covariance matrix, Σ, obtained from the true model and incorrect model is impossible, since their standard forms are

$$\tilde{\Sigma} = \tilde{U}'\tilde{U}/N,$$

where $\tilde{U}$ is given as $\tilde{U}_T = Y - X\tilde{B}_T$ for the true model while $\tilde{U}_F = Y - X\tilde{B}_F - Z\tilde{\Gamma}_F$ for the incorrect model, and the algebraic forms of the restricted estimators of B and Γ, $\tilde{B}_T$ and $\tilde{B}_F$, $\tilde{\Gamma}_F$, can only be given in vectorised form, such as vec($\tilde{B}_T$) and vec($\tilde{B}_F$), vec($\tilde{\Gamma}_F$).

The Wald, LR and LM testing procedure are widely used for testing the restrictions (2.21). However, the direct algebraic comparison between the LR and LM test statistics obtained from the true model and the incorrect model is impossible, since the LR and LM test statistics contain the restricted estimates of Σ. Therefore, we confine our analytical exposition to the Wald statistic as this only requires the unrestricted estimates of β and Σ.

The Wald test statistic obtained from the true model (2.1) for testing (2.21) can be written as

$$W_T = \hat{\beta}_T' R' \{R\,(\hat{\Sigma}_T \otimes (X'X)^{-1})R'\}^{-1} R\hat{\beta}_T, \qquad (2.25)$$

and is asymptotically distributed as χ^2 with s degrees of freedom. When Σ is known, $\hat{\Sigma}_T$ in (2.25) is replaced by Σ and the resultant Wald test statistic is then exactly distributed as χ^2 with s degrees of freedom [see Section 3.4]. On the other hand, the Wald statistic obtained from the incorrect model (2.2) is given by

$$W_F = \hat{\pi}_F' C' \{C(\hat{\Sigma}_F \otimes (W'W)^{-1})C'\}^{-1} C\hat{\pi}_F \qquad (2.26)$$

when $\hat{\pi}_F = vec(\hat{\Pi}_F) = (I \otimes (W'W)^{-1}W')y$. However, $\hat{\beta}_F$ and $C[\Sigma \otimes (W'W)^{-1}]C'$ in (2.26) can be written as $\hat{\beta}_F = C_1\hat{\pi}_F$ and

$$C[\Sigma \otimes (W'W)^{-1}]C' = R[\Sigma \otimes (X'M_2X)^{-1}]R' \qquad (2.27)$$

(see Appendix 6.4). The Wald statistic in (2.26) can be written as

$$W_F = \hat{\pi}_F' C_1' R' \{R C_1 (\Sigma \otimes (W'W)^{-1}) C_1' R'\}^{-1} R C_1 \hat{\pi}_F$$

$$= \hat{\beta}_F' R' \{R(\Sigma \otimes (X'M_Z X)^{-1}) R'\}^{-1} R \hat{\beta}_F. \qquad (2.28)$$

The Wald statistic W_F in (2.28) is the reduced form of (2.26), which is used to only the restriction $R\beta = 0$ in (2.21).

The effect of this type of misspecification on the test of hypothesis arises from the presence in the test statistic of W_F instead of W_T in (2.22). To analyse this we will first consider the effect on Type I error and then examine the effect on power.

After substitution of $\hat{\beta}_F = (I_m \otimes (X'M_Z X)^{-1} X'M_Z)y$ into (2.28), where $y = \mathrm{vec}(Y)$, the Wald statistic W_F can be written as

$$W_F = y'(I \otimes M_Z X(X'M_Z X)^{-1}) R' \{R(\hat{\Sigma}_F \otimes (X'M_Z X)^{-1}) R'\}^{-1} R$$
$$\times (I \otimes (X'M_Z X)^{-1} X'M_Z) y. \qquad (2.29)$$

Under the null hypothesis H_0: $R\beta = 0$, W_F becomes

$$W_F = u'(I \otimes M_Z X(X'M_Z X)^{-1}) R' \{R(\hat{\Sigma}_F \otimes (X'M_Z X)^{-1}) R'\}^{-1} R$$
$$\times (I \otimes (X'M_Z X)^{-1} X'M_Z) u. \qquad (2.30)$$

using the fact that $M_Z Z$ is a null matrix, where $u = \mathrm{vec}(U)$. Since plim $\hat{\beta}_F = \beta$ from (2.12) and plim $\hat{\Sigma}_F = \Sigma$ from (2.18), it is easily seen that

$$\sqrt{N}R(\hat{\beta}_F - \beta) = R[I \otimes N^{-1}(X'M_ZX)^{-1}][(I \otimes X'M_Z)/\sqrt{N}]u \qquad (2.31)$$

has a limiting normal distribution with mean vector zero and covariance matrix $\{R(\Sigma \otimes F^{-1})R'\}$ under the assumption that there exists a finite matrix F such that

$$\text{plim } N^{-1}X'M_ZX = F.$$

Therefore, it can be shown that W_F also has a limiting χ^2 distribution, under the null hypothesis H_0: $R\beta = 0$, with s degrees of freedom [Berndt and Savin (1977, p.1270-1271)]. Consequently, W_F is asympototically unbiased under the null hypothesis H_0, whether Z is stochastic or non-stochastic. However, in the small sample situation, it is highly possible that $\Sigma - \hat{\Sigma}_F$ is positive definite, which circumstances

$$\{R(\hat{\Sigma}_F \otimes (X'M_ZX)^{-1})R'\}^{-1} - \{R(\Sigma \otimes (X'M_ZX)^{-1})R'\}^{-1} \qquad (2.32)$$

is positive definite, so that W_F may be biased towards over-rejection in small samples.[3] Therefore, we can expect that the inclusion of irrelevant explanatory variable affects the size of the test and leads to higher Type I errors. However, when Σ is known and Z is non-stochastic, this small sample bias disappears, since $\hat{\Sigma}_F$ is replaced with Σ and W_F becomes

$$W_F = \hat{\beta}_F'R'\{R\,(\Sigma \otimes (X'M_ZX)^{-1})R'\}^{-1}R\hat{\beta}_F$$

which has the exact χ^2 distribution with s degrees of freedom under the null hypothesis.

The power of the Wald test will not be affected by the use of W_F, unless $W_F \leqslant W_T$. Consequently, to analyse the effect of misspecification on the power of the test, it is sufficient to compare W_T in (2.22) and W_F in (2.28). Since $\hat{\beta}_T = (I_m \otimes (X'X)^{-1}X')y$ from the true model, the (true) Wald statistic W_T can be written as

$$W_T = y'(I \otimes X(X'X)^{-1})R'\{R(\hat{\Sigma}_T \otimes (X'X)^{-1})R'\}^{-1}R \times(I \otimes(X'X)^{-1}X')y. \tag{2.33}$$

From (2.33) and (2.29), it clearly follows that $W_T \neq W_F$. However, the comparison of W_T and W_F is not straightforward, since the difference between them depends not only on $(X'X)^{-1}$ and $(X'M_ZX)^{-1}$ but also $\hat{\Sigma}_T$ and $\hat{\Sigma}_F$. From (2.17), it obviously follows that $\hat{\Sigma}_T - \hat{\Sigma}_F$ is positive definite. On the other hand, $(X'M_ZX)^{-1} - (X'X)^{-1}$ is positive definite, unless X and Z are orthogonal, since

$$(X'M_ZX)^{-1} = (X'X)^{-1} + (X'X)^{-1}X'Z(Z'M_XZ)^{-1}Z'X(X'X)^{-1} \tag{2.34}$$

and $(X'X)^{-1}X'Z(Z'M_XZ)^{-1}Z'X(X'X)^{-1}$ is positive definite (See for details Appendix 6.2). However, due to the complexity of the quadratic expression of W_T and W_F, the direct algebraic comparison of values of W_T and W_F appears impossible without additional assumptions.

If we assume that X and Z are orthogonal, so that $(X'M_ZX) = (X'X)$ and $\hat{\beta}_T = \hat{\beta}_F$ but $\hat{\Sigma}_T - \hat{\Sigma}_F$ is still positive definite from (2.20), we can see that $W_F > W_T$.[4] Therefore, the power of test is not affected by the inclusion of the orthogonality Z to X. In addition to the orthogonality of X and Z, if we assume that $\Sigma = \Sigma_0$ is known, we can see that $W_F = W_T$. Thus, when X and Z are orthogonal, the exact χ^2 test is not affected by overparameterisation. When X and Z are not orthogonal, the comparison of W_F and W_T is less conclusive. For example, when Σ is known and X and Z are not orthogonal,

$$[R(\Sigma \otimes (X'X)^{-1})R']^{-1} - [R(\Sigma \otimes (X'M_ZX)^{-1})R']^{-1} \qquad (2.35)$$

is positive definite from (2.15), so that it can be expected that $W_T > W_F$, if $\hat{\beta}_F$ is close to $\hat{\beta}_T$. However, this result may be reversed as the inequality between W_T and W_F depends on the direction of non-orthogonality between X and Z. Thus, we cannot exlude the possiblity that the power of the test is affected by the inclusion of irrelevant explanatory variables which are not orthogonal to X.

6.3 Effect of Underparameterisation of the Model: Some Analytical Remarks

We consider the underparameterised case which occurs when the relevant explanatory variables are omitted from the model. For this analysis, we assume that the true model is

$$Y = XB + Z\Gamma + U = W\Pi + U, \tag{3.1}$$

but the misspecified model

$$Y = XB + U_F, \tag{3.2}$$

is incorrectly adopted in estimation. The notation and assumptions on U are the same as given in the previous section, but Z is now a set of incorrectly omitted explanatory variables, Γ is assumed to be a non-zero coefficient matrix, and U_F is an N x m matrix of disturbances incorrectly defined such that $U_F = U + Z\Gamma$ and $E(U_F) = Z\Gamma$. For the asymptotic properties of (3.1) and (3.2), we assume that $\text{plim}\, N^{-1}X'U = 0$, $\text{plim}\, N^{-1}Z'U = 0$, and $\text{plim}\, N^{-1}U'U = \Sigma$, as in (2.3) to (2.5) in the previous section.

The ML estimators of B and Σ obtained from the incorrect model (3.2) are given by

$$\hat{B}_F = (X'X)^{-1}X'Y \tag{3.3}$$

and

$$\hat{\Sigma}_F = \hat{U}_F'\hat{U}_F/N = Y'M_XY/N, \tag{3.4}$$

respectively, where $\hat{U}_F = M_XY$ is the unrestricted residual matrix. After substitution of the true Y in (3.1) into (3.3), $\hat{B}_F$ can be written as

$$\begin{aligned}\hat{B}_F &= (X'X)^{-1}X'(XB + Z\Gamma + U)\\ &= B + (X'X)^{-1}X'Z\Gamma + (X'X)^{-1}X'U,\end{aligned} \tag{3.5}$$

which implies that

$$E(\hat{B}_F) \neq B, \text{ unless } X'Z = 0 \tag{3.6}$$

and

$$\text{plim } \hat{B}_F = B, \text{ unless plim } N^{-1}X'Z = 0. \tag{3.7}$$

Thus, $\hat{B}_F$ is biased and inconsistent, unless X and Z are orthogonal. Since

$$\text{plim } \hat{\Sigma}_F = \Sigma + \text{plim } N^{-1}\Gamma'Z'M_XZ\Gamma \neq \Sigma, \tag{3.8}$$

under the assumptions (2.3) - (2.5), $\hat{\Sigma}_F$ is also inconsistent, unless plim $N^{-1}\Gamma'Z'M_XZ\Gamma = 0$. There exists an upward asymptotic bias in $\hat{\Sigma}_F$, since $\Gamma'Z'M_XZ\Gamma$ is positive semi-definite. In a small sample, since $M_W - M_X$ is negative definite from (2.17), so is $\hat{\Sigma}_T - \hat{\Sigma}_F = Y'(M_X - M_W)Y/N$. However, since the covariance matrix of $\hat{\beta}_T = \text{vec}(\hat{B}_T)$ from the true model (3.1) is given by $\text{cov}(\hat{\beta}_T) = \Sigma \otimes (X'M_ZX)^{-1}$, while

$$\text{cov}(\hat{\beta}_F) = \Sigma \otimes (X'X)^{-1} \tag{3.9}$$

and $(X'X)^{-1} - (X'M_ZX)^{-1}$ is negative semi-definite, we anticipate that $\hat{\beta}_F$ will have smaller variances than $\hat{\beta}_T$.

To simplify the analysis first suppose that Σ is known ($\Sigma = \Sigma_0$). Then, from a similar discussion to (2.28), the Wald statistic obtained from the true model (3.1) for testing the restriction H_0: $R\beta = 0$ is given by

$$W_T^{\,0} = \hat{\beta}_T'R'\{R(\Sigma_0 \otimes (X'M_ZX)^{-1})R'\}^{-1}R\hat{\beta}_T \qquad (3.10)$$

and is exactly distributed as χ^2 with s degrees of freedom, when Z is non-stochastic. This holds even when Σ is unknown or Z is stochastic.

However, the Wald statistic obtained from the incorrect model (3.2) for testing the restriction H_0: $R\beta = 0$ is given by

$$W_F^{\,0} = \hat{\beta}_F'R'\{R(\Sigma_0 \otimes (X'X)^{-1})R'\}^{-1}R\hat{\beta}_F. \qquad (3.11)$$

Since $\hat{\beta}_F = \beta + (I \otimes (X'X)^{-1}X')u_F$ where $u_F = vec(U_F)$ $= (I \otimes Z)\gamma + u$, W_F under the null hypothesis H_0 can be written as

$$W_F^{\,0} = u_F'(I \otimes X(X'X)^{-1})R'D_0^{-1}R(I \otimes (X'X)^{-1}X')u_F, \quad (3.12)$$

where $D_0 = \{R(\Sigma_0 \otimes (X'X)^{-1})R'\}$. When Z is non-stochastic and under the normality assumption on U, u_F has the multivariate normal distribution with $E(u_F) = (I \otimes Z)\gamma$ and $E(u_Fu_F') = (\Sigma_0 \otimes I)$. Therefore, $R(I \otimes (X'X)^{-1}X')u_F$ also has a normal distribution with *non-zero* mean $R(I \otimes X(X'X)^{-1}X'Z)\gamma$ and covariance $D_0 = \{R(\Sigma_0 \otimes (X'X)^{-1})R'\}$. This implies that W_F in (3.11) can be written as

$$W_F = \varepsilon_F'\{cov(\varepsilon_F)\}^{-1}\varepsilon_F, \qquad (3.13)$$

where $\varepsilon_F = R(I \otimes (X'X)^{-1}X')u_F$ and $\{cov(\varepsilon_F)\} = D_0$, so that under the null hypothesis H_0, W_F in (3.11) is exactly distributed as the *non-central* χ^2 with s degrees of freedom and the noncentrality parameter $\xi = \gamma'(I \otimes Z'X(X'X)^{-1})D_0^{-1}(I \otimes (X'X)^{-1}X'Z)\gamma$, when Σ is known and Z is non-stochastic [see Giri (1977, p.107 -113)]. Since D is positive definite, the noncentrality parameter ξ is strictly positive, unless X and Z are orthogonal. It follows that the central χ^2 statistic for testing H_0 is invalid and tends to over-reject the null hypothesis, H_0. However, when Z is orthogonal to X, the noncentrality parameter ξ becomes zero, so that the central χ^2 test for testing H_0 is valid.

When Z is stochastic, u_F also has the multivariate normal distribution but with $E(u_F) = (I \otimes \mu_Z)\gamma$ and

$$E(u_F u_F') = E(uu') + cov(z) + 2cov(z,u), \qquad (3.14)$$

where $\mu_Z = E(Z)$, $z = (I \otimes Z)\gamma$ and $E(uu') = (\Sigma \otimes I)$. It follows that the covariance matrix of $\varepsilon_F = R(I \otimes (X'X)^{-1}X')u_F$ is not D $= \{R(\Sigma_0 \otimes (X'X)^{-1})R'\}$. Therefore, the Wald statistic $W^o{}_F$ in (3.11) cannot be expressed as (3.13), and $W^o{}_F$ is not a valid χ^2 test of the null hypothesis when Z is stochastic.

When Σ is unknown, Σ_0 in D is replaced by the estimate $\hat{\Sigma}_F$ in (3.11), the resultant D matrix can be written as $\hat{D}_F = \{R(\hat{\Sigma}_F \otimes (X'X)^{-1})R'\}$ and the Wald statistic as

$$W_F = \hat{\beta}_F{}'R'\{R(\hat{\Sigma}_F \otimes (X'X)^{-1})R'\}^{-1}R\hat{\beta}_F. \qquad (3.15)$$

However, since plim $\hat{\Sigma}_F \neq \Sigma$ from (3.8), so that plim $\hat{D}_F \neq D$, W_F in (3.15) is no longer distributed as the non-central χ^2 in either a small sample or asymptotically. However, since plim $\hat{\Sigma}_F - \Sigma$ is positive semi-definite from (3.8), so that $\hat{D}_F - D$ is also positive semi-definite and $\hat{D}_F^{-1} - D^{-1}$ is negative semi-definite, we can expect that $W_F \leqslant W_F^0$ holds asympototically. What is clear is that the central χ^2 test of H_0 is invalid in finites sample as well as asymptotically, when relevant explanatory variables are omitted.

The discussion on the effect of underparameterisation of the model on the power of the Wald test can be carried out analogously to that in the overparameterisation case. For example, if we wish to show that the power of the Wald test will be not affected by the use of W_F, it is sufficient to show that $W_F \leqslant W_T$. When $\Sigma = \Sigma_0$ is known and Z is orthogonal to X, it is obvious that $W_F = W_T$, since $\hat{\beta}_T = \hat{\beta}_F$ and $X'X = X'M_ZX$. Therefore, the exact χ^2 test is not affected at all by the omission of relevant explanatory variables orthogonal to X. However, when Σ is unknown but Z is orthogonal to X, it follows that $W_F < W_T$, since $\hat{\Sigma}_T - \hat{\Sigma}_F$ is still positive definite from (2.15), which implies that the power of test is weakened by the exclusion of orthogonal Z to X. When X and Z are not orthogonal, the comparison of W_F and W_T is also less conclusive, as in the overparameterised case.

6.4 Monte Carlo Simulation and Results

To examine the consequences of misspecification of demand systems on tests of hypothesis, we considered the effect on the test of homogeneity as well as the test of both homogeneity and symmetry restrictions in the context of the static and the dynamic Rotterdam demand system. In the simulations below, we examine the performance of the asymptotic Wald, LR and LM test statistics as well as that of the exact χ^2 test. When considering homogeneity alone, Hotelling's generalised T^2 test is also considered. The standard algebraic expressions of the LR and LM test statistics are given by

$$LR = N \; ln(|\tilde{\Sigma}|/|\hat{\Sigma}|),$$

and

$$LM = \tilde{\lambda}'\{R(\tilde{\Sigma} \otimes (W'W)^{-1})R'\}\tilde{\lambda},$$

where λ is the Lagrangian multiplier. When the model is correctly specified, the LM and LR test statistics are asymptotically equivalent to the Wald statistic and also aymptotically distributed as χ^2 with s degrees of freedom under the null hypothesis (see for details Section 3.4.1). When $\Sigma = \Sigma_0$ is known, the LR and LM statistics are identical to the Wald statistic, which reduces to $N\,tr\,\Sigma_0^{-1}(\tilde{\Sigma} - \hat{\Sigma})$, and has an exact χ^2 distribution with s degrees of freedom under H_0. For homogeneity restrictions, the LM and LR statistics can be expressed as function of the Wald statistic [see Bera (1982)], so that they all have the same power. However, White (1982, p.8) showed that the asymptotic equivalence of the LR test to the Wald and LM tests breaks down when the models are misspecified.

The design of the simulation is as follows. Dutch data for five commodities were obtained from data for 14 sectors with 31 observations [see, for details in the data, Theil (1975, p.264-265)]. The true parameters, β_P and Σ_P, and the dependent variables, Y_P, in the true model were generated by restricted estimation under the true specification of the model, using the actual data. Multivariate normal random numbers U_P with zero mean and covariance matrix Σ_P were generated by a random number generator and added to y_P to obtain the dependent variable in simulation. To reduce the sampling errors in synthetic generation of U_P, we tested that $U_P'U_P/N$ is not significantly different from Σ_P on each replication, using the LR test [see T. W. Anderson (1958, Chapter 10.8)] with a 5% significance level. Those samples which did not satisfy the test were excluded from the experiments. To calculate power, the true parameters β_P were perturbed in the manner of Bera, *et al.* (1981, footnote 3), and the parameters under the alternative hypothesis H_1: $R\beta \neq 0$ were thus defined.

The theoretical exact χ^2 critical values were used to obtain the effect of misspecification on Type I errors. However, as for power, the empirical critical values, obtained from the computation of Type I errors and shown in the tables, were used to obtain the percentage of rejections of H_0 under H_1. The number of replications is 500.

Our first Monte Carlo simulation was to examine the over-rejection problem of the asymptotic χ^2 tests under the correct specification of the dynamic demand system. If the demand system involves more explanatory variables, as in the dynamic case, there could be a deterioration in the test of hypothesis due to the resultant loss in degrees of freedom. Therefore, one can claim that the test of hypothesis may be misleadingly biased toward over-rejection regardless of the validity of specification of the model. This is highly likely, particularly in using the asymptotic χ^2 tests when the sample is small; as is clearly indicated in the static demand studies for larger systems [Laitinen (1978), Meisner (1979), and Bera *et al.* (1981)]. Table 6.1 confirms the suspicion that the asymptotic χ^2 tests are biased towards over-rejection (increasingly with the number of the explanatory variables), in spite of the correct specification of the model. However, no bias was found in exact tests, such as Hotelling's T^2 test for the homogeneity restrictions as well as the exact χ^2 tests when Σ is assumed to be known.

In the simulations of the incorrectly overparameterised model, the original version of the Rotterdam model is used to generate the data, and the following six misspecified models were considered

(F1). Rotterdam model with intercept (c).

(F2). Rotterdam model with first order lagged independent variables (X_{-1}).

(F3). Rotterdam model with second order lagged independent variables (X_{-2}).

(F4). Rotterdam model with first order lagged dependent variables (Y_{-1}).

(F5). Rotterdam model with both first order lagged dependent and independent variables (X_{-1} and Y_{-1}).

(F6). Model (F5) with intercepts (c, X_{-1} and Y_{-1}).

These models were designed to ascertain the effect of incorrect specification of the dynamic structure of the system when the true model is static.

To gain an information on the degree of orthogonality between variables (X) in the true model and the variables incorrectly included (or omitted), we computed correction and moment matrices between X_{-1} and X_{-2} and X (given in Table 6.2). X_{-2} shows less of a tendancy to orthogonalty with X than X_{-1}.

The Monte Carlo simulation results for the above six overparameterised models are summarised in Table 6.3.1 and Table 6.3.2 when testing homogeneity and both symmetry and homogeneity. The results seem to be fairly straightforward and to confirm the analytical results of Section 2. The inclusion of irrelevant explanatory variables appeared to cause the over-rejection in the asymptotic χ^2 test for all cases of the Wald, LR and LM tests. Type I error and the empirical critical value steadily increased with the number of incorrectly included explanatory variables. The power of the test appeared to diminish in all cases (increasingly so, with the overparameterisation of the model).

However, compared with the results in Table 6.1 and Table 6.3, for example, DM1 and F1 as well as DM2 and F2, we can see no effects from the misspecification of the model. The type I errors of the overparameterised models were shown to be almost identical to those obtained when the model was correctly specified. This result may imply that the over-rejection problem in overparameterised models is a finite sample problem due to the loss of degree of freedom rather than a result of the incorrect specification of the model. Furthermore, it was confirmed that over-rejection bias disappears in the exact χ^2 test, where the true covariance matrix Σ_P is used, as well as in Hotelling's T^2 statistic for testing homogeneity. Type I errors of the exact χ^2 test and Hotelling's T^2 statistic appeared to be around 5 %, which is the significant level set for the test, and thus type I errors of the exact tests were not affected by over-specification of the model.

Comparing models F2 and F3 in Table 6.3, we can see that the orthogonalty of the incorrectly included variables to the explanatory variables X in the true model do not significantly effect type I errors. However, power was diminished more by including X_{-1} than X_{-2}. Note that X_{-1} is more orthogonal to X. Inclusion of the intercept term, which has zero correlation with X, does not change the power of test.

In simulations for the incorrectly underparameterised models, the first order distributed lag model of the Rotterdam system with intercept was taken as the true model, and the following three misspecified models were considered in order to ascertain the effect of omission of relevant explanatory variables from the model.

(F1). Rotterdam model with intercept, which incorrectly omits the first order lagged independent variables from the true model.

(F2). Rotterdam model with the first order lagged independent variables, which incorrectly omits the intercept from the true model.

(F3). Static Rotterdam model, which incorrectly omits both the first order lagged independent variables and the intercept from the true model.

The misspecified model F2 was designed to test the effect of omission of explanatory variables which are not orthogonal to X, and the model F1 to examine the effect of omission of variables which are orthogonal to X.

The results of the Monte Carlo simulations for the effect of underparameterisation are summarised in Tables 6.4.1 and 6.4.2. The analytical results outlined in the previous section tend to be confirmed in the simulations. Omission of explanatory variables orthogonal to X does not appear to not affect type I error in the exact central χ^2 test and Hotelling's generalised T^2 test, but slightly decreases the power of the exact tests. However, omission of explanatory variables which are not orthogonal to X significantly increased type I error in the exact tests and also increased the power of the exact tests. This is expected due to the noncentrality of the χ^2 distribution of the test statistics.

In the previous section, it has been argued that the asymptotic χ^2 tests are invalid, since the estimate of covariance matrix obtained from the under-specificed model is inconsistent. However, the simulation results with asymptotic tests showed similar characteristics to the exact tests. Type I errors and the power of the asymptotic χ^2 tests were decreased by the omission of explanatory variables orthogonal to X, but were increased by the omission of explanatory variables which were not orthogonal to X.

6.5 Concluding Remarks

In summary, we have shown analytically that the inclusion of irrelevant explanatory variables in a demand system does not affect type I error in the exact as well as the asymptotic tests of demand restrictions. However, from the Monte Carlo simulations, it was shown that overparameterisation, whether it is correctly or incorrectly specified, caused a serious over-rejection of the demand restrictions. The over-rejection in over-parameterised models appeared to be due to the finite sample problem caused by the loss of degrees of freedom, rather than to the effect of misspecification. However, as mentioned, the power of the tests can diminish due to overparameterisation.

On the other hand, the omission of relevant explanatory variables has severe effects on tests of hypothesis, unless the omitted variables are orthogonal to the included variables, since the central χ^2 tests are invalid. Furthermore, the asymptotic χ^2 test are invalid due to the inconsistency of the estimate of the residual covariance matrix, (even when the omitted variables are orthogonal to the included variables).

In conclusion, it can be said that underparameterisation is less desirable than overparameterisation in terms of the consequences for hypothesis testing. However, in using an overparameterised model, we have to take account of the small sample bias in the asymptotic χ^2 tests. The small sample correction of the asymptotic test statistic may be the appropriate treatment for this problem.

APPENDIX 6.1

Partition $\hat{\Pi}$ into $\hat{B}$ and $\hat{\Gamma}$.

When W is given as $W = [X : Z]$, $W'W$ can be written as

$$W'W = \begin{bmatrix} X'X & X'Z \\ Z'X & Z'Z \end{bmatrix} \tag{A1.1}$$

Then, using the partitioned inverse, $(W'W)^{-1}$ can be expressed as

$$\begin{bmatrix} (X'M_ZX)^{-1} & -(X'M_ZX)^{-1}X'Z(Z'Z)^{-1} \\ -(Z'Z)^{-1}Z'X(X'M_ZX)^{-1} & (Z'Z)^{-1} + (Z'Z)^{-1}Z'X \times(X'M_ZX)^{-1}X'Z(Z'Z)^{-1} \end{bmatrix}. \tag{A1.2}$$

Or, symmetrically, $(W'W)^{-1}$ can be written as

$$\begin{bmatrix} (X'X)^{-1} + (X'X)^{-1}X'Z \times(Z'M_XZ)^{-1}Z'X(X'X)^{-1} & -(Z'M_XZ)^{-1}Z'X(X'X)^{-1} \\ -(X'X)^{-1}X'Z(Z'M_XZ)^{-1} & (Z'M_XZ)^{-1} \end{bmatrix}. \tag{A1.3}$$

$W'Y$ can be written as

$$W'Y = \begin{bmatrix} X'Y \\ Z'Y \end{bmatrix}. \tag{A1.4}$$

Then, using (A1.2), (A1.3) and (A1.4), we can easily see that the estimator $\hat{B}$ can be written as

$$\hat{B} = (X'M_ZX)^{-1}X'M_ZY$$
$$= (X'X)^{-1}X'(Y - Z\hat{\Gamma})$$

and $\hat{\Gamma}$ as

$$\hat{\Gamma} = (Z'Z)^{-1}Z'(Y - X\hat{B}),$$
$$= (Z'M_XZ)^{-1}Z'M_XY,$$

where $M_Z = I_T - Z(Z'Z)^{-1}Z'$ and $M_X = I_T - X(X'X)^{-1}X'$.

APPENDIX 6.2

Proof of

$$(X'M_ZX)^{-1} = (X'X)^{-1}+(X'X)^{-1}X'Z(Z'M_XZ)^{-1}Z'X(X'X)^{-1}. \quad (A2.1)$$

Proof: Premultiplying $(X'M_ZX)$ on both sides of (A2.1), we have

$$I = (X'M_ZX)[(X'X)^{-1}+(X'X)^{-1}X'Z(Z'M_XZ)^{-1}Z'X(X'X)^{-1}], \quad (A2.2)$$

so that the proof of (A2.1) is equivalent to that of (A2.2). Now,

$$(X'M_ZX)[(X'X)^{-1}+(X'X)^{-1}X'Z(Z'M_XZ)^{-1}Z'X(X'X)^{-1}]$$

$$= (X'X - X'Z(Z'Z)^{-1}Z'X)$$
$$\times[(X'X)^{-1}+(X'X)^{-1}X'Z(Z'M_XZ)^{-1}Z'X(X'X)^{-1}]$$

$$= I - X'P_ZX(X'X)^{-1} + X'Z(Z'M_XZ)^{-1}Z'X(X'X)^{-1}$$
$$- X'P_ZP_XZ(Z'M_XZ)^{-1}Z'X(X'X)^{-1}.$$

However,

$$X'P_ZP_XZ(Z'M_XZ)^{-1}Z'X(X'X)^{-1}$$

$$= X'P_Z(I - M_X)Z(Z'M_XZ)^{-1}Z'X(X'X)^{-1}$$

$$= X'P_ZZ(Z'M_XZ)^{-1}Z'X(X'X)^{-1}$$
$$- X'Z(Z'Z)^{-1}Z'M_XZ(Z'M_XZ)^{-1}Z'X(X'X)^{-1}$$

$$= X'Z(Z'M_XZ)^{-1}Z'X(X'X)^{-1} - X'Z(Z'Z)^{-1}Z'X(X'X)^{-1}$$

$$= X'Z(Z'M_XZ)^{-1}Z'X(X'X)^{-1} - X'P_ZX(X'X)^{-1}$$

Then, (A2.2) becomes

$$I - X'P_ZX(X'X)^{-1} + X'Z(Z'M_XZ)^{-1}Z'X(X'X)^{-1} - X'P_ZP_XZ(Z'M_XZ)^{-1}Z'X(X'X)^{-1}$$

$$= I - X'P_ZX(X'X)^{-1} + X'Z(Z'M_XZ)^{-1}Z'X(X'X)^{-1} - [\, X'Z(Z'M_XZ)^{-1}Z'X(X'X)^{-1} - X'P_ZX(X'X)^{-1}]$$

$$= I.$$

APPENDIX 6.3

When $W = [X, Z]$, $M_W = I_N - W(W'W)^{-1}W'$ can be written as

$$M_W = M_X - M_X Z(Z'M_X Z)^{-1}Z'M_X$$

or

$$M_W = M_Z - M_Z X(X'M_Z X)^{-1}X'M_Z.$$

Proof: Write $(W'W)^{-1}$ as

$$(W'W)^{-1} = \begin{bmatrix} W^{11} & W^{12} \\ W^{21} & W^{22} \end{bmatrix},$$

where

$$W^{11} = (X'M_Z X)^{-1} \text{ is } k \times k,$$

$$W^{12} = -(X'M_Z X)^{-1}X'Z(Z'Z)^{-1} \text{ is } k \times l,$$

$$W^{21} = -(Z'Z)^{-1}Z'X(X'M_Z X)^{-1} \text{ is } l \times k,$$

and

$$W^{22} = (Z'Z)^{-1} + (Z'Z)^{-1}Z'X(X'M_Z X)^{-1}X'Z(Z'Z)^{-1} \text{ is } l \times l.$$

Then, $P_W = W(W'W)^{-1}W'$ can be written as

$$P_W = [X\ Z] \begin{bmatrix} W^{11} & W^{12} \\ W^{21} & W^{22} \end{bmatrix} \begin{bmatrix} X' \\ Z' \end{bmatrix}$$

$$= [XW^{11}+ZW^{21} \quad XW^{12}+ZW^{22}] \begin{bmatrix} X' \\ Z' \end{bmatrix}$$

$$= XW^{11}X' + ZW^{21}X' + XW^{12}X' + ZW^{22}Z'$$

$$= X(X'M_Z X)^{-1}X' - P_Z X(X'M_Z X)^{-1}X' - X(X'M_Z X)^{-1}X'P_Z$$
$$+ P_Z + P_Z X(X'M_Z X)^{-1}X'P_Z$$

$$= M_Z X(X'M_Z X)^{-1}X' + P_Z - (I - P_Z)X(X'M_Z X)^{-1}X'P_Z$$

$$= P_Z + M_Z X(X'M_Z X)^{-1}X' - M_Z X(X'M_Z X)^{-1}X'P_Z$$

$$= P_Z + M_Z X(X'M_Z X)^{-1}X'M_Z.$$

Therefore, $M_W = I_N - P_W$ becomes

$$\begin{aligned} M_W &= I_N - P_W \\ &= I_N - P_Z - M_Z X(X'M_Z X)^{-1}X'M_Z \\ &= M_Z - M_Z X(X'M_Z X)^{-1}X'M_Z. \end{aligned}$$

If we write $W = [Z, X]$, then M_W can be written as

$$M_W = M_Z - M_Z X(X'M_Z X)^{-1}X'M_Z,$$

since in this case, W^{11}, W^{12}, W^{21}, and W^{22} become

$$W^{11} = (Z'M_X Z)^{-1} \text{ is } l \times l,$$

$$W^{12} = -(Z'M_X Z)^{-1}Z'X(X'X)^{-1} \text{ is } l \times k,$$

$$W^{21} = -(X'X)^{-1}X'Z(Z'M_X Z)^{-1} \text{ is } k \times l,$$

and

$$W^{22} = (X'X)^{-1} + (X'X)^{-1}X'Z(Z'M_X Z)^{-1}Z'X(X'X)^{-1} \text{ is } k \times k,$$

respectively.

APPENDIX 6.4

Selection Matrix for β from Π such that $\beta = C_1\Pi$

Consider the case of selecting vec(B) from vec(Π) when Π is given as $\Pi' = [B' \ \Gamma']$ for coefficient matrices B(k×m) and Γ(l×m). In general, the selection matrix C_1 such that $\beta = C_1\pi$ can be defined by

$$C_1 = I_m \otimes [I_k : 0], \qquad \text{(A4.1)}$$

where I_m and I_k: are the identity matrices of order m and k, respectively and 0 is an k×l zero matrix. For example, when k = 3, m = 2 and l = 2, β can be selected from vec(Π)in such a way that,

$$\beta = \begin{bmatrix} \beta_{11} \\ \beta_{12} \\ \beta_{13} \\ \hline \beta_{21} \\ \beta_{22} \\ \beta_{23} \end{bmatrix} = \left[\begin{array}{ccccc|ccccc} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{array}\right] \begin{bmatrix} \beta_{11} \\ \beta_{12} \\ \beta_{13} \\ \gamma_{11} \\ \gamma_{12} \\ \hline \beta_{21} \\ \beta_{22} \\ \beta_{23} \\ \gamma_{21} \\ \gamma_{22} \end{bmatrix}$$

In this case, C_1 is defined by $C_1 = I_2 \otimes [I_3 : 0]$.

If $(W'W)^{-1}$ can be partitioned as

$$(W'W)^{-1} = \begin{bmatrix} W^{11} & W^{12} \\ W^{21} & W^{22} \end{bmatrix}, \qquad \text{(A4.2)}$$

where W^{11} is k×k, W^{12} is k×l, W^{21} is a l×k matrix, and W^{11}

$l \times l$, then

$$C_1[\Sigma \otimes (W'W)^{-1}]\, C_1' = \Sigma \otimes W^{11} \tag{A4.3}$$

for an m×m matrix Σ, since

$$\begin{aligned} & C_1[\Sigma \otimes (W'W)^{-1}]\, C_1' \\ &= [I_m \otimes (I_k\colon 0)][\Sigma \otimes (W'W)^{-1}][I_m \otimes (I_k\colon 0)]' \\ &= \Sigma \otimes (I_k\colon 0)(W'W)^{-1}\begin{bmatrix} I_k \\ 0' \end{bmatrix} \\ &= \Sigma \otimes (I_k\colon 0)\begin{bmatrix} W^{11} & W^{12} \\ W^{21} & W^{22} \end{bmatrix}\begin{bmatrix} I_k \\ 0' \end{bmatrix} \\ &= \Sigma \otimes [\, W^{11} \quad W^{12}\,]\begin{bmatrix} I_k \\ 0' \end{bmatrix} = \Sigma \otimes W^{11} \end{aligned}$$

Thus, it can easily be seen that

$$C[\Sigma \otimes (W'W)^{-1}]C' = R[\Sigma \otimes (X'M_2X)^{-1}]R', \tag{A4.4}$$

when $(W'W)^{-1}$ is given as (A4.1) and $C = RC_1$, since $W^{11} = (X'M_2X)^{-1}$in (A1.2) in Appendix 6.1 and

$$\begin{aligned} & C[\Sigma \otimes (W'W)^{-1}]C' \\ &= RC_1[\Sigma \otimes (W'W)^{-1}]C_1'R' \\ &= R[\Sigma \otimes W^{11}]R' = R[\Sigma \otimes (X'M_2X)^{-1}]R'. \end{aligned}$$

Moreover, if we premultiply C_1 on the restricted estimator of π, we have $\tilde{\beta}$ as

$$\tilde{\beta} = C_1\tilde{\pi}$$

$$= C_1\hat{\pi} - C_1(\Sigma \otimes (W'W)^{-1})C'[C(\Sigma \otimes (W'W)^{-1})C']^{-1}C\hat{\pi}$$

$$= \hat{\beta} - C_1(\Sigma \otimes (W'W)^{-1})C_1'R'[RC_1(\Sigma \otimes (W'W)^{-1})C_1'R']^{-1}RC_1\hat{\pi}$$

$$= \hat{\beta} - [\Sigma \otimes (X'M_2X)^{-1}]R'[R(\Sigma \otimes (X'M_2X)^{-1})R']^{-1}R\hat{\beta},$$

which is also the restricted estimator of β under the restiction $R\beta = 0$, when the model is given as (2.2).

FOOTNOTES:

1. See Assumption A3 in White (1982, p.3-4).

2. In summary, it has been shown that omission of relevant variables results in biased and inconsistent estimates in both the OLS and SUR estimations unless the omitted variables and the explanatory variables of the estimated model are not orthogonal. When irrelevant variables are included, the resultant estimation of the model produces unbiased but inefficient estimates of the true model and has higher mean sqare errors. The bias of the SUR estimator is larger than the bias of the ordinary least square estimator. The SUR estimation procedure leads to less efficient estimates than the ordinary least sqares when the contemporaneous correlation in the error terms is caused by misspecification of the model. The true efficiency of these estimates is not reflected in the computed standard errors. He also shows that the SUR procedure is more sensitive than the ordinary least squares, when the relevant variables are omitted.

3. Note that

$$\text{plim}\{R(\hat{\Sigma}_F \otimes (X'M_2X)^{-1})R'\}^{-1} - \{R(\Sigma \otimes (X'M_2X)^{-1})R'\}^{-1}$$

is the zero matrix under the assumptions (2.3) to (2.5).

4. The positive definiteness of $A - B$ implies that $B^{-1} - A^{-1}$ is positive definite [See Dhryme (1978,p.494)].

TABLES IN CHAPTER 6

Table 6.1 Comparison of Type I errors under the True Specifications of Models:

Models:

Static. $Y = XB$.
DM1. $Y = XB + c$
DM2. $Y = XB + X_{-1}\Gamma$
DM3: $Y = XB + X_{-1}\Gamma + c$, where c is intercept term.

Table 6.1.1 Homogeneity and Symmetry

Comparison of Type I errors (in %).

Model	W	LR	LM	χ^2
Static	26.4	16.2	7.6	4.6
DM1	30.4	19.6	9.0	5.6
DM2	50.2	39.0	26.0	6.6
DM3	57.0	44.0	30.2	6.4

Comparison of Empirical CV (χ^2 = 18.31).

Model	W	LR	LM	χ^2
Static	32.17	25.15	20.23	17.94
DM1	32.56	26.15	20.97	18.64
DM2	40.81	30.81	24.36	19.16
DM3	43.96	32.55	25.64	19.15

Table 6.1.2 Homogeneity Only

Comparison of Type I errors (in %).

Model	W	LR	LM	χ^2	T^2
Static	22.0	14.0	8.0	5.2	5.6
DM1	23.4	16.2	10.6	5.4	6.0
DM2	33.4	25.8	17.8	6.4	4.6
DM3	38.6	29.8	21.6	6.4	5.4

Comparison of Empirical CV.

Model	W	LR	LM	χ^2	T^2	T^{2o} [a]
Static	16.55	13.26	10.79	9.74	13.35	12.80
DM1	17.96	14.17	11.37	9.71	13.91	12.98
DM2	23.87	17.56	13.29	9.96	14.32	14.67
DM3	27.04	19.28	14.22	10.07	15.32	15.12

[a]: T^{2o} is the theoretical critical value of Hotelling's generalised T^2 distribution.

Table 6.2 Information on Orthogonality between Explanatory Variables

Correlation coefficient between X and X_{-1}

	X_{-1}'s					
X's	.39	.55	.40	.18	.18	.06
	.20	.38	.30	.19	.33	.15
	.23	.32	.27	.27	.08	.30
	.26	.21	.54	.06	.36	.20
	.28	.42	.58	.02	.24	.04
	.19	-.10	.10	.25	.24	.16

Correlation coefficient between X and X_{-2}

	X_{-2}'s					
	-.02	.11	.04	.04	-.05	.30
	.01	.04	.12	.00	-.04	.19
X's	.23	.23	.20	.22	.06	.15
	.24	.18	.24	.04	.07	.12
	-.03	.07	.15	.33	-.10	.24
	.50	.34	.46	.29	.52	-.01

Value of $X'X_{-1}$

318.86	392.10	379.52	267.18	252.16	16.49
139.40	246.17	266.77	259.13	413.29	16.12
229.26	288.67	331.63	488.92	143.16	45.58
389.74	283.94	977.53	170.13	934.42	48.47
396.00	532.61	1002.30	53.33	605.29	9.97
28.76	-9.36	11.52	59.38	49.20	6.65

Value of $X'X_{-2}$

-5.00	84.64	32.61	53.77	-72.15	43.86
3.68	26.59	102.13	-1.70	-47.81	19.59
216.41	207.00	247.82	384.99	100.15	21.27
350.93	243.06	433.67	110.65	181.68	28.66
-41.98	86.27	255.02	808.00	-250.74	52.34
62.26	36.93	62.32	58.43	105.30	3.51

Table 6.3 Effect of Overparameterization of the Model

True Model: $Y = XB + U$

False Models:
F1: $Y = XB + c + U_F$,
F2: $Y = XB + X_{-1}\Gamma + U_F$,
F3: $Y = XB + X_{-2}\Gamma + U_F$,
F4: $Y = XB + Y_{-1}\Gamma + U_F$,
F5: $Y = XB + Y_{-1}\Gamma_1 + X_{-1}\Gamma_2 + U_F$,
F6: $Y = XB + Y_{-1}\Gamma_1 + X_{-1}\Gamma_2 + c + U_F$,

where c is intercept term.

Table 6.3.1 Homogeneity and Symmetry

Comparison of Type I errors (in %)

Model	W	LR	LM	x^2
True	26.4	16.2	7.6	4.6
F1	30.4	19.6	9.0	5.6
F2	55.0	40.6	27.2	6.6
F3	55.6	42.8	27.0	3.4
F4	44.8	32.6	20.2	6.0
F5	75.2	63.4	49.4	5.6
F6	79.4	72.0	56.4	3.6

Comparison of Empirical CV's (x^2 = 18.31)

Model	W	LR	LM	x^2
True	32.17	25.15	20.23	17.94
F1	32.37	26.11	20.97	18.69
F2	47.86	34.49	26.00	18.77
F3	46.53	33.09	25.41	17.54
F4	39.07	29.36	23.57	19.02
F5	60.35	41.86	36.53	18.77
F6	67.01	43.87	32.41	17.27

Comparison of Powers (in %)

Model	W	LR	LM	x^2
True	49.4	44.4	40.0	67.8
F1	49.0	43.6	39.6	64.0
F2	12.0	11.0	9.6	64.8
F3	21.8	20.0	15.2	78.4
F4	31.2	28.6	21.4	40.8
F5	18.4	15.0	9.6	19.6
F6	17.2	17.0	11.6	27.0

Table 6.3.2 Homogeneity Only

Comparison of Type I errors (in %)

Model	W	LR	LM	x^2	T^2
True	22.0	14.0	8.0	5.2	5.6
F1	23.4	16.2	10.6	5.4	6.0
F2	38.6	30.0	20.6	5.2	4.6
F3	40.0	30.2	20.6	4.0	3.0
F4	32.2	24.0	14.0	5.0	4.6
F5	54.0	47.2	36.0	5.2	4.0
F6	58.2	51.6	42.0	4.0	4.0

Comparison of Empirical CV's (x^2 = 9.49)

Model	W	LR	LM	x^2	T^2	T^{2o}
True	16.55	13.26	10.79	9.74	13.35	12.80
F1	17.96	14.17	11.37	9.71	13.91	12.98
F2	24.72	17.88	13.34	9.50	14.49	15.12
F3	23.40	17.16	12.95	8.63	13.72	15.12
F4	20.35	15.53	12.12	9.43	13.56	13.95
F5	34.27	22.86	16.00	9.52	15.99	17.09
F6	37.15	21.17	16.60	8.87	16.10	18.09

Comparison of Powers (in %).

Model	W	LR	LM	x^2	T^2
True	65.2	65.2	65.2	73.6	65.2
F1	61.2	61.2	61.2	71.8	61.2
F2	22.8	22.8	22.8	62.0	22.8
F3	35.2	35.2	35.2	79.0	35.2
F4	48.4	48.4	48.4	60.8	48.4
F5	24.4	24.4	24.4	30.2	24.4
F6	25.0	25.0	25.0	32.0	25.0

Table 6.4 Underparameterized Case

True Model: $Y = XB + X_{-1}\Gamma + c + U$, where c is intercept term.

False Models: F1: $Y = XB + c + U_F$,

F2: $Y = XB + X_{-1}\Gamma + U_F$,

F3: $Y = XB + U_F$.

Table 6.4.1 Homogeneity and Symmetry

Comparison of Type I errors (in %).

Model	W	LR	LM	χ^2
True	57.0	44.0	30.2	6.4
F1	82.4	70.4	49.4	89.2
F2	48.2	36.4	24.8	6.8
F3	82.0	68.6	48.8	91.4

Comparison of Empirical CV (χ^2 = 18.31).

Model	W	LR	LM	χ^2
True	43.96	32.55	25.64	19.15
F1	50.24	35.63	27.10	46.99
F2	39.49	29.53	24.16	19.16
F3	47.97	35.30	27.26	49.07

Comparison of Powers (in %).

Model	W	LR	LM	χ^2
True	16.4	15.0	13.2	59.2
F1	38.8	34.8	31.0	88.0
F2	13.6	15.0	12.2	51.8
F3	42.2	34.8	26.6	82.6

Table 6.4.2 Homogeneity Only

Comparison of Type I errors (in %)

Model	W	LR	LM	χ^2	T^2
True	38.6	29.8	21.6	6.4	5.4
F1	85.0	76.0	62.2	96.2	45.6
F2	31.6	24.6	18.0	6.2	5.0
F3	83.0	73.6	61.2	96.0	47.8

Comparison of Empirical CV

Model	W	LR	LM	χ^2	T^2	T^{2o}
True	27.04	19.28	14.22	10.07	15.32	15.12
F1	34.16	22.81	15.97	39.19	26.29	13.18
F2	23.43	17.31	13.15	10.37	14.06	14.67
F3	32.13	21.84	15.51	39.08	25.71	12.98

Comparison of Powers (in %)

Model	W	LR	LM	χ^2	T^2
True	14.0	14.0	14.0	54.0	14.0
F1	49.8	49.8	49.8	91.8	49.8
F2	13.0	13.0	13.0	45.4	13.0
F3	54.4	54.4	54.4	90.4	54.4

CHAPTER 7

ESTIMATION OF SHORT RUN DEMAND SYSTEMS UNDER LONG RUN EQUILIBRIUM CONDITIONS

7.1 Introduction

In this chapter, we will consider problems associated with the estimation of the dynamic (short run) demand system under long-run equilibrium conditions. Adopting the dynamic disequilibrium hypothesis that equilibrium is a situation in the long run rather than in the short run [c.f., Sections 5.1 and 5.2], we assume that consumers are out of equilibrium in the short run, but behave so as to achieve equilibrium in the long run. The implication of this assumption for the estimation of dynamic demand system is that the impact (short run) responses in the dynamic demand system do not necessarily satisfy the equilibrium conditions. However, the long run structure implied by the dynamic demand system, (which are usually derived by imposing steady state conditions), is bound to satisfy the equilibrium conditions derived from utility maximisation, such as the homogeneity and symmetry restrictions. Thus, the dynamic demand system to be considered in this chapter corresponds to the dynamic *disequilibrium* model subject to long run equilibrium conditions [c.f., Sections 5.1 and 5.2]. Consequently, unlike the dynamic equilibrium demand system [c.f., Chapter 5], the restrictions imposed on the dynamic demand sytem are not the dynamic equilibrium restrictions imposed on the impact multiplier matrix in the dynamic system, but those needed to ensure that the implied long run equilibrium multipliers satisfy the equilibrium conditions.

However, even when the dynamic demand system is linear in coefficients, estimation subject to the long run equilibrium conditions is not straightforward. The difficulties are due to the fact that the implied long run response coefficients are expressed nonlinearly in terms of the impact and dynamic multipliers. In particular, when the dynamic demand system involves lagged dependent variables, the symmetry restrictions on the long run equilibrium response matrix are nonlinear expressions. Therefore, the estimation of the restricted dynamic demand system reduces to a nonlinear estimation problem.

As an attempt to avoid this nonlinear problem in the estimation of a dynamic demand system subject to long run equilibrium conditions, Bewley (1979) and Anderson and Blundell (1982) proposed a reparameterisation of the dynamic system which allows the long run coefficient matrix to be included as a structural coefficient matrix in the dynamic system which enables the equilibrium multiplier matrix to be estimated directly.[1] However, in this chapter, it will be shown that it is possible to estimate dynamic system subject to the long run equilibrium conditions without reparameterisation. Furthermore, a maximum likelihood (ML) type estimation procedure will be derived directly from trace minimisation subject to long run equilibrium restrictions expressed in terms of impact and dynamic multipliers. Due to the nonlinear nature of the long run symmetry restriction, an iterative procedure will be exploited, based on the solution of the Lyapunov equation [see Section 3.6]. The methodology introduced in this chapter is different from Bewley's and Anderson and Blundell's, in the sense that we do not need to reparameterise the dynamic model in estimation, but we estimate the dynamic demand system directly subject to the long run equilibrium conditions without changing the original form of the dynamic demand system.

The design of this chapter is as follows: in Section 2, we consider ML estimation for the dynamic demand system subject to long run equilibrium conditions, under two simple dynamic situations; one is the structural autogressive case, and the other is the distributed lag case. In Section 3, the application of the estimation procedure to a singular system will be considered. In Section 4, the empirical application of ML estimation of a dynamic demand system under the long run equilibrium condition to quarterly Korean data will be examined. Finally, some concluding remarks will be presented in Section 5.

7.2 The Estimation of Short Run Demand Systems Subject to Long Run Equilibrium Conditions

We consider a case when demand relations at a long run equilibrium are given by a system of m linear equations,

$$Y = Z\Pi^*, \tag{2.1}$$

where Y is a T x m matrix of dependent variables, Z a T x k matrix of nonstochastic explanatory variables, and Π^* a k x m coefficient matrix of equilibrium response with m equations and T observations. In a demand system, Z can be partitioned into X and W, i.e., $Z = [X:W]$, where X is a T x m matrix of explanatory variables defined by prices and W is a T x (k-m) matrix of other explantory variables, such as the total expenditure variable, intercept, and seasonal dummies. The coefficient matrix Π^* in (2.1) is conformably partitioned as $\Pi^{*\prime} = [B^{*\prime}:\Gamma^{*\prime}]$, so that (2.1) can be written as

$$Y = XB^* + W\Gamma^*. \tag{2.2}$$

The Rotterdam demand system and the almost ideal demand system can be written in the form of (2.1) or (2.2), and the equilibrium conditions, such as the homogeneity and symmetry restrictions derived from static demand theory[2], are expressed in terms of B^* only; that is,

$$i'B^* = 0$$

and (2.3)

$$B^* = B^{*\prime},$$

respectively. We will adopt these two restrictions [in (2.3)] as the equilibrium restrictions imposed on the long run demand system (2.1) and (2.2).

For a given long run equilibrium relation (2.1) and (2.2), we consider a case where the dynamic (short run) demand system is given by a simple autoregressive distributed lag model

$$Y = Z\Pi_o + Y_{-1}C + Z_{-1}\Pi_1 + U, \tag{2.4}$$

where U is a T x m disturbance matrix with zero mean and covariance Σ, while the subscript -1 denotes a set of first order lagged variables. The coefficient matrix Π_o in (2.4) is often termed the impact response; Π_1 is the lagged response, and C is the adjustment multiplier matrix. After the steady state conditions, i.e., $Y = Y_{-1}$ and $Z = Z_{-1}$ are imposed, the short run demand system (2.4) reduces to (2.1) or (2.2) with the long run equilibrium response $\Pi^* = (\Pi_o+\Pi_1)(I-C)^{-1}$.

For simplicity, we will discuss two straightforward cases, which are nested within (2.4); one is when $\Pi_1 = 0$, and the other is when C = 0. In the former case, (2.4) reduces to the partial (stock) adjustment model, and in the latter case, (2.4) reduces to a simple finite distributed lag model. The estimation of (2.4) is the extension and combination of these two cases.

When $\Pi_1 = 0$, the dynamic (short run) demand system (2.4) can be written as

$$Y = Z\Pi_o + Y_{-1}C + U \qquad (2.5)$$

$$= XB + W\Gamma + Y_{-1}C + U.$$

Under the steady state condition such that $Y = Y_{-1} = Y^*$, (2.5) reduces to the long run system (2.2) but with $B^* = B(I-C)^{-1}$, $\Gamma^* = \Gamma(I-C)^{-1}$, if $(I-C)^{-1}$ exists. Therefore, the the long run equilibrium restrictions in (2.3) can be expressed in terms of B and C as follows;

$$i'B^* = 0 \Leftrightarrow i'B(I-C)^{-1} = 0 \text{ and} \qquad (2.6.1)$$

$$B^* = B^{*\prime} \Leftrightarrow B(I-C)^{-1} = (I-C')^{-1}B'. \qquad (2.6.2)$$

However, postmultiplying (I-C) on (2.6.1) and (2.6.2) and premultiplying (I-C′) on (2.6.2), we can express the restrictions in (2.6.1) and (2.6.2) on the short run demand system for the long equilibrium as

$$i'B = 0, \text{ for homogeneity and} \qquad (2.7.1)$$

$$(I-C')B = B'(I-C), \text{ for symmetry.} \qquad (2.7.2)$$

The restrictions in (2.7.1) and (2.7.2) are equivalent to (2.6.1) and (2.6.2), respectively, and are to be imposed on the short run demand system (2.5) for its implied long run structure to be consistent with the equilibrium conditions.

However, from (2.6.1) and (2.7.1), it is obvious that if homogeneity is satisfied on the short run effect, B, it is necessarily satisfied on the long run effect, $B^* = B(I-C)^{-1}$. Hence, the estimation of (2.5) under the short run homogeneity restriction (2.7.1) is equivalent to that under long run homogeniety (2.6.1). Thus, since the long run restrictions can be expressed linearly in terms of B, estimation of the short run demand system under the long run homogeneity restrictions can be carried out in the context of the linearly restricted SUR regression, and usual asymptotic χ^2-tests, such as Wald, LR and LM apply.

However, estimation of the short run demand system (2.5) under the long run symmetry restriction is not straightforward. The long run symmetric restriction in (2.6.2) or (2.7.2) is not linear in B and C, so that the estimation of (2.5) subject to long run symmetry becomes a nonlinear estimation problem. To tackle this, we consider trace minimisation of $tr\,\Sigma^{-1}U'U$ subject to the restriction (2.7.2). The Lagrange function to be minimised is

$$L = tr\,\Sigma^{-1}U'U + tr\,G[(I-C')B - B'(I-C)], \qquad (2.8)$$

where G is an m x m skew-symmetric matrix of Lagrage multipliers.[3] Solutions for B, Γ, and C from the first order conditions produce the corresponding restricted GLS (ML under the normality assumptions) estimator subject to (2.7.2). Using the matrix differentiation, the first order conditions for the minimisation of (2.8) are obtained as

$$[-X'Y + X'X\tilde{B} + X'W\tilde{\Gamma} + X'Y_{-1}\tilde{C}]\Sigma^{-1} - \tfrac{1}{2}(I - \tilde{C})(\tilde{G} - \tilde{G}') = 0, \quad (2.9)$$

$$[-Y_{-1}'Y + Y_{-1}'Y_{-1}\tilde{C} + Y_{-1}'X\tilde{B} + Y_{-1}'W\tilde{\Gamma}]\Sigma^{-1} - \tfrac{1}{2}\tilde{B}(\tilde{G} - \tilde{G}') = 0, \quad (2.10)$$

$$[-W'Y + W'W\tilde{\Gamma} + W'X\tilde{B} + W'Y_{-1}\tilde{C}]\Sigma^{-1} = 0, \quad (2.11)$$

$$(I - \tilde{C}')\tilde{B} - \tilde{B}'(I - \tilde{C}) = 0, \quad (2.12)$$

and from the skew-symmetry of G,

$$\tilde{G} = -\tilde{G}'. \quad (2.13)$$

From (2.11), we have

$$\tilde{\Gamma} = (W'W)^{-1}W'[Y - X\tilde{B} - Y_{-1}\tilde{C}], \quad (2.14)$$

Using (2.13) and after substitution of (2.14) into (2.9), we have

$$\tilde{B} = (X'M_WX)^{-1}X'M_W[Y - Y_{-1}\tilde{C}] + (X'M_WX)^{-1}[I - \tilde{C}]\tilde{G}\Sigma. \quad (2.15)$$

Substituting (2.14) into (2.10),

$$\tilde{C} = (Y_{-1}'M_WY_{-1})^{-1}Y_{-1}'M_W[Y - X\tilde{B}] + (Y_{-1}'M_WY_{-1})^{-1}\tilde{B}\tilde{G}\Sigma. \quad (2.16)$$

Note that equations (2.12), (2.15) and (2.16) are not linear in the matrix variables, $\tilde{B}$, $\tilde{C}$, and $\tilde{G}$. Thus the first order conditions are in the form of a simultaneous nonlinear matrix equation system. Analytical solution appears impossible, therefore, we rely on a numerical solution, which can be obtained by iteration on equations (2.12)-(2.16).

At each iteration, linearised conditional solutions are calculated for given values of other variables. For example, if $\tilde{B}$ is regarded as a fixed value, (2.12) can be expressed linearly in term of $\tilde{C}$ alone and (2.15) becomes a linear fuction of $\tilde{G}$. Hence, solutions for $\tilde{C}$ and $\tilde{G}$ for a given $\tilde{B}$ can be obtained by linearised system of equations (2.12), (2.13) and (2.15). Then, a conditional solution $\tilde{C}$ for given $\tilde{B}$ can be obtained from (2.16), once a solution $\tilde{G}$ is obtained. A solution Lagrange multiplier matrix $\tilde{G}$ for a given $\tilde{B}$ can be obtained by solving a Lyapunov equation. That is, solving (2.12) and (2.16) for a given $\tilde{B}$ and taking account of (2.13), we have a Lyapunov equation for $\tilde{G}$,

$$A(B)\tilde{G}(B) + \tilde{G}(B)A(B)' = F(B), \qquad (2.17)$$

where

$$A(B) = [\tilde{B}'(Y_{-1}'M_WY_{-1})^{-1}\tilde{B}]^{-1}\Sigma, \qquad (2.18)$$

$$F(B\ )= [\tilde{B}'(Y_{-1}'M_WY_{-1})^{-1}\tilde{B}]^{-1}[K - K'] \times [\tilde{B}'(Y_{-1}'M_WY_{-1})^{-1}\tilde{B}]^{-1}, \qquad (2.19)$$

and

$$K = K(B) = \tilde{B}'[I - (Y_{-1}'M_W Y_{-1})^{-1} Y_{-1}'M_W (Y - X\tilde{B})]. \tag{2.20}$$

Similarly, for a given $\tilde{C}$ and $\tilde{G}$, a conditional solution for $\tilde{B}$ can be obtained using (2.15). Solving (2.12) and (2.14), we have a Lyapunov equation for $\tilde{G}$, for a given $\tilde{C}$,

$$A(C)\tilde{G}(C) + \tilde{G}(C)A(C)' = F(C), \tag{2.21}$$

where

$$A(C) = [(I - \tilde{C}')(X'M_W X)^{-1}(I - \tilde{C})]^{-1}\Sigma, \tag{2.22}$$

$$F(C) = [(I - \tilde{C}')(X'M_W X)^{-1}(I - \tilde{C})]^{-1}[K - K'] \times [(I-\tilde{C}')(X'M_W X)^{-1}(I-\tilde{C})]^{-1}, \tag{2.23}$$

and

$$K = K(\tilde{C}) = (Y - Y_{-1}\tilde{C})'M_W X(X'M_W X)^{-1}(I - \tilde{C}). \tag{2.24}$$

Each iteration consists of several steps to find these conditional solutions, the steps are as follows:

Step 1. We solve a Lyapunov equation (2.17) for a given B and obtain $\tilde{G}(B)$.

Step 2. We update $\tilde{C}$ by substituting $\tilde{G}(B)$ into (2.16) and update $\tilde{\Gamma}$ with the new $\tilde{C}$.

Step 3. We solve a Lyapunov equation (2.21) with updated $\tilde{C}$ and obtain $\tilde{G}(C)$.

Step 4. We update $\tilde{B}$ by substituting $\tilde{G}(C)$ into (2.15) and update $\tilde{\Gamma}$ with the new $\tilde{B}$. Then, we return to Step 1.

Iteration is continued until the convergence criterion is satisfied; for example, G(B) is sufficiently close to G(C). The convergence of such an iterative procedure has been proven by Sargan (1964) and Oberhofer and Kmenta (1974). The estimated values of B, C and G obtained at convergence are the ML estimates which asymptotically efficient and consistent under the regular conditions [c.f., Section 3.7].

The asymptotic covariance matrix of the estimates can be obtained by taking the inverse of the information matrix $\mathfrak{s}$. However, since G is skew-symmetric, the second derivative matrix of the likelihood function with respect to vec(G) produces a singular information matrix. Therefore, the information matrix is obtained by partial devatives of veck(G) instead of vec(G). For detailed discussion and derivation, see Appendix 7.1. The covariance matrices of the estimates are given by

$$\mathrm{cov}(\mathrm{veck}(\tilde{G})) = \{R[\Sigma \otimes D'(Q'Q)^{-1}D]R'\}^{-1}, \qquad (2.25)$$

and

$$\mathrm{cov}\{\mathrm{vec}(\tilde{\beta}',\tilde{c}',\tilde{\gamma}')\} = \Sigma \otimes (Q'Q)^{-1} + [\Sigma \otimes (Q'Q)^{-1}D]R' \times \{R[\Sigma \otimes D'(Q'Q)^{-1}D]R'\}^{-1}R[\Sigma \otimes D'(Q'Q)^{-1}], \qquad (2.26)$$

where $\beta = \mathrm{vec}(B)$, $c = \mathrm{vec}(C)$, $\gamma = \mathrm{vec}(\Gamma)$, $Q = [X, Y_{-1}, Z]$, $D' = [(I - C'), B', 0]$, R is an $m(m-1)/2 \times mm$ matrix defined as $R = K(I - P_{mm})$ to eliminate the singularity of the information matrix due to the skew-symmetry of G, and P_{mm} is a permuted identity matrix [see also Appendix 3.1 in Chapter 3.].

However, when the dynamic demand system has a distributed lag form, the problem is much simpler, because the restrictions for a long run equilibrium are linear on the coefficient matrices of the dynamic demand system. The dynamic demand demand system of the finite distributed lag model (when C = 0 in (2.5)) can be written as

$$Y = Z\Pi_o + Z_{-1}\Pi_1 + U \qquad (2.27)$$

$$= XB_o + W\Gamma + X_{-1}B_1 + W_{-1}\Gamma_1 + U.$$

Imposing steady state conditions such as $Z = Z_{-1} = Z^*$, that is, $X = X_{-1} = X^*$ and $W = W_{-1} = W^*$; we have the long run equilibrium response, $\Pi^* = \Pi + \Pi_1$, and so $B^* = B_o + B_1$ and $\Gamma^* = \Gamma + \Gamma_1$. The restrictions in (2.3) can then be expressed in terms of B and B_1, as follows,

$$i'B^* = 0 \Leftrightarrow i'(B_o + B_1) = 0 \qquad (2.28.1)$$

$$B^* = B^{*\prime} \Leftrightarrow B_o + B_1 = B_o' + B_1'. \qquad (2.28.2)$$

From (2.28.1) and (2.28.2), we can see that the long run homogeneity and symmetry restrictions imposed on a dynamic demand system with distributed lags can be expressed linearly in terms of the impact and delayed response coefficients. Therefore, we can apply the standard restricted SUR estimation procedure to a vectorised version of (2.27) to obtain restricted estimates of the coefficients.

$$\tilde{\pi} = \hat{\pi} + (\Sigma \otimes (Q'Q)^{-1})R'[R(\Sigma \otimes (Q'Q)^{-1}R']R\hat{\pi} ,$$

where $\hat{\pi} = (I \otimes (Q'Q)^{-1}Q')vec(Y)$ is an unrestricted estimate of π, and its covariance matrix given by

$$(\Sigma \otimes (Q'Q)^{-1})-(\Sigma \otimes (Q'Q)^{-1})R'[R(\Sigma \otimes (Q'Q)^{-1})R']^{-1} \times R(\Sigma\otimes(Q'Q)^{-1}). \tag{2.29}$$

Since the long run coefficient matrix is simply the sum of B_o and B_1, that is, $B^* = B_o + B_1$, its variance can be obtained from the relation,

$$var(\beta_{ij}) = var(\beta_{ij})+var(\beta_{ij})+2cov(\beta_{ij},\beta_{ij}), \tag{2.30}$$

where β_{ij}'s are the ij'th element of the corresponding B's. $var(\beta_{ij})$, $var(\beta_{ij})$, and $cov(\beta_{ij},\beta_{ij})$ can directly be obtained from (2.29).

7.3 The Estimation of Singular Systems

The usual solution of the singularity problem, which is to eliminate one equation from the system, can be applied directly to the dynamic (distributed lag) demand system, since the long run equilibrium restrictions are linear.

However, in the case of estimation of the autoregressive model of the singular system, the application of the results of the previous section is not straightforward, because of the nonlinear nature of the long run symmetry restrictions and the existence of lagged dependent variables in the model. These two problems can be solved by reformulating the reduced subsystem in the manner suggested by Breusch (1978) for lagged dependent variables in the dynamic model. McKenzie (1979) showed that under the adding up conditions, the long run equilibrium responses derived from the reformulated (reduced) subsystem are identical to the corresponding submatrix derived from the full system. In what follows, we will review their results and then derive the reformulation of the long run equilibrium restrictions for the reduced subsytem.

Now, we consider the first order autoregressive dynamic system,

$$Y = XB + \mu\gamma + Y_{-1}C + U, \qquad (3.1)$$

which is a special case of (2.5), where W consists of μ alone. By the adding up condition, μ is usually defined as the sum of the dependent variables at each observation, i.e.,

$$\mu = Y\iota$$

where $\iota = (1,..,1)'$ is an m x 1 column vector of unit elements. After elimination of the last (m'th) equation, the reduced subsystem can be written as

$$Y^{\circ} = X^{\circ}B_{\circ} + x^{m}\beta_{m.} + \mu\gamma^{\circ} + Y^{\circ}_{-1}C_{\circ} + y^{m}_{-1}c_{m.} + U^{\circ} \qquad (3.2)$$

and the elimimated m′th equation as

$$y^m = X^\circ\beta_{.m} + x^m\beta_{mm} + \mu\gamma^m + Y^\circ_{-1}c_{.m} + y^m_{-1}c_{mm} + u^m. \quad (3.3)$$

The variable and coefficient matrices in (3.1) are partitioned conformably as

$$Y = [Y^\circ:y^m],\ X = [X^\circ:x^m],\ U = [U^\circ:u^m], \quad (3.4)$$

$$B = \begin{bmatrix} B_o & \beta_{.m} \\ \beta_{m.} & \beta_{mm} \end{bmatrix},\ \gamma = [\gamma^\circ:\gamma^m], \text{ and } C = \begin{bmatrix} C_o & c_{.m} \\ c_{m.} & c_{mm} \end{bmatrix}.$$

The adding up conditions on the coefficient matrix imply

$$\beta_{.m} = -B_o\iota \text{ and } \beta_{mm} = -\beta_{m.}\iota, \quad (3.5)$$

$$\gamma^m = 1 - \gamma^\circ\iota, \quad (3.6)$$

$$c_{.m} = -C_o\iota \text{ and } c_{mm} = -c_{m.}\iota. \quad (3.7)$$

However, if we substitute the adding up condition at the lag, i.e., $y^m_{-1} = \mu_{-1} - Y^\circ_{-1}i$, in (3.2), we can also substitute out the lagged values of the eliminated dependent variable, y^m_{-1}, and the reduce system (3.2) can be written as

$$Y^\circ = X^\circ B_o + x^m\beta_{m.} + \mu\gamma_o + Y^\circ_{-1}(C_o - \iota c_{m.}) + \mu_{-1}c_{m.} + U^\circ. \quad (3.8)$$

[Breusch (1978)]. Then, (3.8) is the model in which the singularity problem has been removed and can be used for estimation. Breusch (1978) showed the the eigenvalues of $(C_o - ic_{m.})$ are identical to those of C with the exception of a zero eigenvalue which is due to the adding up conditions on C. It is obvious that estimates of all coefficients in the original m equation system can be recovered using the adding up conditions (3.5) - (3.7).

The long run equilibrium responses obtained from the reduced subsystem (3.8) are then given as

$$D_o = \begin{bmatrix} B_o \\ \beta_{m.} \end{bmatrix} [I - (C_o - \iota c_{m.})]^{-1} \qquad (3.9)$$

and

$$d_{\mu o} = (\gamma_o + c_{m.})[I - (C_o - \iota c_{m.})]^{-1}. \qquad (3.10)$$

Using partitioned matrix algebra, McKenzie (1979) showed that D_o and $d_{\mu o}$ are identical to the corresponding submatrix of the long run equilibrium responses derived from the full m equation system (3.1),

$$D = B(I - C)^{-1} \text{ and } d_\mu = \gamma(I - C)^{-1}, \qquad (3.11)$$

respectively. [Details of the proof are sketched in Appendix 7.2.] The long run effects also satisfy the adding up constraints (see Appendix 7.3),

$$D\iota = B(I-C)^{-1}\iota = 0 \text{ and } d_\mu\iota = \gamma(I - C)^{-1}\iota = 1 \quad (3.12)$$

Therefore, the long run effect for the eliminated equation can be derived the subsystem, (3.8), and the adding up conditions, (3.12).

Now, we consider how the long run equilibrium restriction (2.7.2), $(I-C')B = B'(I-C)$, can be reformulated for the reduced subsystem (3.13). From (3.4), (3.5) and (3.7), B and (I-C) can be written as

$$B = \begin{bmatrix} B_o & -B_o\imath \\ \beta_{m.} & -\beta_{m.}\imath \end{bmatrix} \text{ and } (I - C) = \begin{bmatrix} I-C_o & -C_o\imath \\ c_{m.} & -c_{m.}\imath \end{bmatrix}, \quad (3.13)$$

Therefore, the right hand side of the restriction (2.7.2) can be written as

$$(I-C')B = \begin{bmatrix} I-C_o' & -c_{m.} \\ \imath'C_o' & 1+c_{m.}\imath \end{bmatrix} \begin{bmatrix} B_o & -B_o\imath \\ \beta_{m.} & -\beta_{m.}\imath \end{bmatrix} \quad (3.14)$$

$$= \begin{bmatrix} B_o-C_o'B_o-c_{m.}'\beta_{m.} & -B_o\imath+C_o'B_o\imath+c_{m.}'\beta_{m.}\imath \\ \imath'C_o'B_o+\beta_{m.}+c_{m.}\imath\beta_{m.} & -\imath'C_o'B_o\imath-\beta_{m.}\imath-c_{m.}\imath\beta_{m.}\imath \end{bmatrix}$$

and the left hand side of that as

$$B'(I-C) = \begin{bmatrix} B_o' & -\beta_{m.}' \\ -\imath'B_o & -\beta m.\imath \end{bmatrix} \begin{bmatrix} I-C_o & -C_o\imath \\ -c_{m.} & 1+c_{m.}\imath \end{bmatrix} \quad (3.15)$$

$$= \begin{bmatrix} B_o'-B_o'C_o-\beta_{m.}'c_{m.} & B_o'C_o\imath+\beta_{m.}'+\beta_{m.}'c_{m.}\imath \\ -\imath'B_o'+\imath'B_o'C_o+\beta_{m.}\imath c_{m.} & -\imath'B_o'C_o\imath-\beta_{m.}\imath-\beta_{m.}\imath c_{m.}\imath \end{bmatrix}.$$

Equating (3.14) and (3.15), we have the following four relations for the long run symmetry restrictions (2.7.2),

$$B_o - C_o'B_o - c_{m.}'\beta_{m.} = B_o' - B_o'C_o - \beta_{m.}'c_{m.} \qquad (3.16)$$

$$-B_o\iota + C_o'B_o\iota + c_{m.}'\beta_{m.}\iota = B_o'C_o\iota + \beta_{m.}' + \beta_{m.}'c_{m.}\iota \qquad (3.17)$$

$$\iota'C_o'B_o + \beta_{m.} + c_{m.}\iota\beta_{m.} = -\iota'B_o' + \iota'B_o'C_o + \beta_{m.}\iota c_{m.} \qquad (3.18)$$

$$-\iota'C_o'B_o\iota - \beta_{m.}\iota - c_{m.}\iota\beta_{m.}\iota = -\iota'B_o'C_o\iota - \beta_{m.}\iota - \beta_{m.}\iota c_{m.}\iota \qquad (3.19)$$

From (3.16), (3.17) becomes

$$\beta_{m.} = -\iota'B_o, \qquad (3.20)$$

and substituting (3.20) into (3.16), (3.16) becomes

$$[I - (C_o - \iota c_{m.})]'B_o = B_o'[I - (C_o - \iota c_{m.})]. \qquad (3.21)$$

It can easily be seen that (3.18) and (3.19) are automatically satisfied under (3.20) and (3.21), and are therefore redundant.

Thus, we have two restrictions for long run symmetry, (3.20) and (3.21), on the reduced m-1 subsystem. The restriction (3.21) is equivalent to that imposing the symmetry on (3.8). However, (3.20) is the additional restriction which reflects the elimination of the m'th equation from the system. Note that after eliminating one equation, the matrices are no longer square. However, we can see that (3.20) is equivalent to the homogeneity restriction on B. This reflects the well-known fact that the adding up conditions are equivalent to the homogeneity restriction under symmetry.

Therefore, the estimation of the subsystem (3.8) under long run symmetry should be carried out subject to (3.20) and (3.21) simultaneously. However, since the restrictions in (3.20) are linear, we can substitute them out and reformulate the subsystem. Substituting (3.20) into (3.8), the subsystem can be written as

$$Y^{\circ} = (X^{\circ} - x^{m}\iota')B_{\circ} + \mu\gamma_{\circ} + Y^{\circ}_{-1}(C_{\circ} - \iota c_{m.}) + \mu_{-1}c_{m.} + U^{\circ} \qquad (3.22)$$

$$= \overline{X}B_{\circ} + \mu\gamma_{\circ} + Y^{\circ}_{-1}(C_{\circ} - \iota c_{m.}) + \mu_{-1}c_{m.} + U^{\circ}$$

where $\overline{X} = (X_{\circ} - x_{m.}\iota')$ is the reduced X matrix whose column vectors are those subtracted from the last column vector of X. Then, the approach previously discussed can be applied to the reduced subsystem (3.22), referring $B_{\circ}$ to B, $(C_{\circ} - \iota c_{m.})$ to C, $(\gamma_{\circ}, c_{m.})$ to Γ, and so on.

7.4 Empirical Results

Using quarterly five commodity Korean data, covering the period 1965 to 1981 and consisting of the food, housing, fuel and light, clothing, and miscellaneous sectors, we applied the ML estimation procedure derived in Section 2 to the following four dynamic (short run) demand systems of the Rotterdam model:

D(1) = Distributed Lag Model with first order lagged independent variables.

D(1,2) = Distributed Lag Model with first and second order lagged independent variables.

A(1) = Autoregressive Model with first order lagged dependent variables.

A(4) = Autoregressive Model with fourth order lagged dependent variables.

The choice of these four dynamic demand system is somewhat arbitrary. However, from the results of the diagnostic checking procedures carried out in Chapter 5 [see Table 5.1], we have seen that these four dynamic systems represented consumers' short run behaviour, best based on the quarterly data series. Because of the complex nature of the computing procedure, the more general specifications of dynamic demand systems, such as the AD[(4),(1)] and AD[(1,4),(1)] models, considered in Chapter 5, were not examined further here.

The results of testing the long run equilibrium restrictions are summarised in Table 7.1 together with the results of testing the short run equilibrium restrictions on the dynamic systems. Test statistics for the long run homogeneity restrictions on A(1) and A(4) models were found to be the same, since the long and short run homogeneity restrictions are identical in those models. Consequently, the AR(4) model appeared to reject long run homogeneity, as it did the short run restriction. However, in the distributed lag dynamic demand systems, such as the D(1) and D(1,2) models, the homogeneity restrictions on the implied long run responses appeared to be accepted, while the homogeneity restrictions on the (short run) impact responses were rejected. In all the above models, the test results of the long run symmetry restrictions were found to be less significant that those of the short run symmetry restrictions, and were accepted. Moreover, the testing results of the long run demand restrictions were also less significant than the test results on the static system.

Parameter estimation results of the above four models are presented in Tables 7.2, 7.3, 7.4, and 7.5, respectively. Each table consists of three subtables, for the unrestricted and restricted models subject to the long run homogeneity and symmetry restrictions. Impact reponses exhibited by the above four dynamic models were found to be similar to those obtained for the static model. Consequently, we will not discuss the results on the impact responses. However, the implied long run responses from the restricted models were found to have different phases from the static models.

In all four models, the long run marginal budget share of the food sector was found to be smaller than the short run impact reponse to income. However, the long run marginal budget shares of the housing and miscellaneous sectors were invariably higher than short run impact responses. Thus, the food sector was classified as the necessity in the long run, as the estimated long run marginal budget share was less than the average budget share; while it was classified as a luxury in the static and dynamic equilibrium models. The housing and clothing sectors were also classified as luxuries in terms of long run responses, as in the static model.

Considering the long run substitution effect, all the diagonal terms of the Slutsky matrix appeared negative in the A(4) model. On the other hand, in the D(1) and D(1,2) models, the long run own substitution effect of the fuel and light sector was found to be positive and significant, while in the D(1,2) and A(1) models, the long run own substitution effect of the miscellaneous sector was positive, as in the static model. In the D(1) and A(4) model, the food and housing sectors appeared to be more own price elastic in the long run than in the short run.

For long run substitutability and complementarity relations, the fuel and light and the miscellaneous sectors appeared to be long run substitutes, but the other pairs of sectors were insignificant.

7.5 Concluding Remarks

In this chapter, we proposed a maximum likelihood procedure for estimating the dynamic (short run) demand system subject to the long run equilibrium restrictions from demand theory. Since the long run symmetry restrictions on the autogressive dynamic demand system are expressed as nonlinear matrix equations, an iterative estimation procedure using the Lyapunov equation solution was proposed.

In the application to quarterly Korean data, the long run equilibrium restrictions appeared to be less compatable with data than the short run dynamic equilibrium restrictions. The estimated long run response were shown to be different from the static and the short run equilibrium demand systems.

APPENDIX 7.1

Derivation of the Covariance Matrices of the Estimates using the Information Matrix.

Since we assume that $U \sim N(0,\Sigma)$, the log likelihood function is given by

$$L(B,C,\Gamma,\Sigma) = -(mT/2)\,ln\,(2\pi)-(T/2)\,ln\,|\Sigma|\,-\tfrac{1}{2}\,tr\,\Sigma^{-1}U'U \quad (A1.1)$$

Here, we assume that Σ is known. The ML estimates are identical to the GLS estimates and are consistent and asymptotically efficient. The asymptotic covariance matrix of the restricted ML estimates subject to the restriction $(I-C')B = B'(I-C)$ can be obtained as ϑ^{-1}, the inverse of the information matrix ϑ which is minus expectation of the second derivatives matrix, $-E[\partial^2\tilde{L}/\partial\theta\partial\theta']$, of

$$\tilde{L} = -(1/2)\,tr\,\Sigma^{-1}U'U + \,tr\,G[(I-C')B - B'(I-C)], \quad (A1.2)$$

with respect to θ, where θ is given as $\theta' = [vec(B)',vec(C)',vec(\Gamma)',vec(G)']$. The solution to the maximisation of (A1.2) is obviously identical to the restricted one derived from the minimisation of (2.8). Vectorising the variable matrices, $\beta = vec(B)$, $c = vec(C)$, $\gamma = vec(\Gamma)$, and $g = vec(G)$, the first derivatives of $\tilde{L}$ with respect to β, c, γ, and g, are given by

$$\partial\tilde{L}/\partial\beta = vec(X'Y\Sigma^{-1}) - (\Sigma^{-1}\otimes X'X)\beta - (\Sigma^{-1}\otimes X'W)\gamma$$
$$- (\Sigma^{-1}\otimes X'Y_{-1})c + vec[(I-C)(G'-G)], \quad (A1.3)$$

$$\partial\tilde{L}/\partial c = vec(Y_{-1}'Y\Sigma^{-1}) - (\Sigma^{-1}\otimes Y_{-1}'Y_{-1})c - (\Sigma^{-1}\otimes Y_{-1}'X)\beta - (\Sigma^{-1}\otimes Y_{-1}'W)\gamma + vec[B(G'-G)], \tag{A1.4}$$

$$\partial\tilde{L}/\partial\gamma = vec(W'Y\Sigma^{-1}) - (\Sigma^{-1}\otimes W'W)\gamma - (\Sigma^{-1}\otimes W'X)\beta - (\Sigma^{-1}\otimes W'Y_{-1})c, \tag{A1.5}$$

$$\partial\tilde{L}/\partial g = vec[B' - B'C - B - C'B], \tag{A1.6}$$

It is easy to prove that $E[\partial\tilde{L}/\partial\beta] = 0$, $E[\partial\tilde{L}/\partial c] = 0$, $E[\partial L/\partial\gamma] = 0$, and $E[\partial\tilde{L}/\partial g] = 0$ at the point $(\beta,c,\gamma,g) = (\tilde{\beta},\tilde{c},\tilde{\gamma},\tilde{g})$, since $E(\tilde{G}) = 0$ under the restrictions $(I-C')B=B'(I-C)$. Using a permuted identity matrix P_{mm}, such that, $vec(A')=P_{mm}vec(A)$ for an $m \times m$ matrix A [MacRae (1974)], we can write the terms $vec[(I-C)(G'-G)]$ in (A1.3), $vec[B(G'-G)]$ in (A1.4), and (A1.6), respectively,

$$vec[(I-C)(G'-G)] = -[I \otimes (I-C)](I-P_{mm})g, \text{ or,} \tag{A1.7}$$

$$= -(I-P_{mm})g + (G' \otimes I - G \otimes I)c. \tag{A1.8}$$

$$vec[B(G'-G)] = - [I \otimes B](I-P_{mm})g, \text{ or,} \tag{A1.9}$$

$$= (G \otimes I - G' \otimes I)\beta. \tag{A1.10}$$

$$vec[B'-B'c-B-c'B] = P_{mm}\beta-\beta-P_{mm}(I\otimes C')\beta+(I\otimes C')\beta, \tag{A1.11}$$

$$= P_{mm}\beta-\beta-(I\otimes B')c+P_{mm}(I\otimes B')c. \tag{A1.12}$$

Using (A1.7) - (A1.12), we have the second partial derivatives of $\tilde{L}$ with respect to β, c, γ,

$$\partial^2\tilde{L}/\partial\beta\partial\beta' = -(\Sigma^{-1} \otimes X'X),$$

$$\partial^2\tilde{L}/\partial c\partial\beta' = -(\Sigma^{-1} \otimes Y_{-1}'X)-(G' \otimes I-G \otimes I),$$

$$\partial^2\tilde{L}/\partial\gamma\partial\beta' = -(\Sigma^{-1} \otimes W'X),$$

$$\partial^2\tilde{L}/\partial g\partial\beta' = -\ (I - P_{mm})[I \otimes (I-C')],$$

$$\partial^2\tilde{L}/\partial\beta\partial c' = -(\Sigma^{-1} \otimes X'Y_{-1}) - (G \otimes I-G' \otimes I),$$

$$\partial^2\tilde{L}/\partial c\partial c' = -(\Sigma^{-1} \otimes Y_{-1}'Y_{-1}),$$

$$\partial^2\tilde{L}/\partial\gamma\partial c' = -(\Sigma^{-1} \otimes W'Y_{-1}),$$

$$\partial^2\tilde{L}/\partial g\partial c' = -(I-P_{mm})(I \otimes B'),$$

$$\partial^2\tilde{L}/\partial\beta\partial\gamma' = -(\Sigma^{-1} \otimes X'W),$$

$$\partial^2\tilde{L}/\partial c\partial\gamma' = -(\Sigma^{-1} \otimes Y_{-1}'W),$$

$$\partial^2\tilde{L}/\partial\gamma\partial\gamma' = -(\Sigma^{-1} \otimes W'W),$$

$$\partial^2\tilde{L}/\partial g\partial\gamma' = 0,$$

$$\partial^2\tilde{L}/\partial\beta\partial g' = -[I \otimes (I-C)](I-P_{mm}),$$

$$\partial^2\tilde{L}/\partial c\partial g' = -(I \otimes B)(I-P_{mm}),$$

$$\partial^2\tilde{L}/\partial\gamma\partial g' = 0, \text{ and } \partial^2\tilde{L}/\partial g\partial g' = 0.$$

Since $E(\tilde{G}) = 0$ under the restrictions, the information matrix $\vartheta = -E[\partial^2\tilde{L}/\partial\theta\partial\theta']$ calculated at the point $(\beta,c,\gamma,g) = (\tilde{\beta},\tilde{c},\tilde{\gamma},\tilde{g})$ becomes

$$\vartheta = \begin{bmatrix} \Sigma^{-1}\otimes X'X & \Sigma^{-1}\otimes X'Y_{-1} & \Sigma^{-1}\otimes X'W & [I\otimes(I-C)](I-P_{mm}) \\ \Sigma^{-1}\otimes Y_{-1}'X & \Sigma^{-1}\otimes Y_{-1}'Y_{-1} & \Sigma^{-1}\otimes Y_{-1}'W & (I\otimes B)(I-P_{mm}) \\ \Sigma^{-1}\otimes W'X & \Sigma^{-1}\otimes W'Y-1 & \Sigma^{-1}\otimes W'W & 0 \\ (I-P_{mm}) \times[I\otimes(I-C')] & (I-P_{mm})(I\otimes B') & 0 & 0 \end{bmatrix}$$

Letting $Q = [X,Y_{-1},Z]$, $D'=[(I-C'),B',0]$, we can write ϑ as

$$\vartheta(\beta,c,\gamma,g) = \begin{bmatrix} \Sigma^{-1}\otimes Q'Q & (I \otimes D)(I-P_{mm}) \\ (I-P_{mm})(I \otimes D') & 0 \end{bmatrix}$$

However, since $(I-P_{mm})$ is singular, this information matrix ϑ is also singular. Therefore, ϑ^{-1} cannot exist and solution may be via the g-inverse approach. However, this singularity is caused by the skew-symmetry of G, that is, $G = -G'$. The diagonal elements of G are zero and its upper triangular elements equal minus the lower triangular elements. There are only $m(m-1)/2$ independent elements among mm elements in G. As discussed in Section 2.6, this problem can be treated by taking the veck operator on G rather that the vec operator. Defining K as a m(m-1)/2 x mm matrix such that veck(A) = Kvec(A) for a m x m skew symmetric matrix A, we can select the lower diagonal terms of G such that λ = veck(G) = Kvec(G). Submatrices corresponding to $(I \otimes D)(I-P_{mm})$ and $(I-P_{mm})(I \otimes D)$ in $\vartheta(\tilde{\beta},\tilde{c},\tilde{\gamma},\tilde{g})$, become, repectively,

$$[I \otimes D](I-P_{mm})K' \text{ and } K(I-P_{mm})[I \otimes D'].$$

Letting $A = K(I-P_{mm})$, the information matrix for β, c, γ, and λ, $\vartheta_o(\tilde{\beta},\tilde{c},\tilde{\gamma},\tilde{\lambda})$ can be written as,

$$\vartheta_o(\tilde{\beta},\tilde{c},\tilde{\gamma},\tilde{\lambda}) = \begin{bmatrix} \Sigma^{-1} \otimes Q'Q & (I \otimes D)A' \\ A(I \otimes D') & 0 \end{bmatrix},$$

in which the singularity of $\vartheta(\beta,c,\gamma,g)$ is removed and the inverse of $\vartheta_o(\beta,c,\gamma,\lambda)$ provide the asymptotic covariance matrix of β, c, γ, and λ.

Writing $\vartheta_o^{-1} = \begin{bmatrix} A^{11} & A^{12} \\ A^{21} & A^{22} \end{bmatrix}$ and using the inverse of a partitioned matrix, we have

$$\text{cov}(\lambda) = A^{22} = [A(I \otimes D')(\Sigma \otimes (Q'Q)^{-1})(I \otimes D)A']^{-1}$$

$$= [A(\Sigma \otimes D'(Q'Q)-1D)A']^{-1}$$

$$\text{cov}[(\beta',c',\gamma')'] = A^{11}$$

$$= \Sigma \otimes (Q'Q)^{-1} + (\Sigma \otimes (Q'Q)^{-1})(I \otimes D)A'[A(\Sigma \otimes D'(Q'Q)^{-1}D)A']^{-1}$$

$$\times\ R(I \otimes D')[\Sigma \otimes (Q'Q)^{-1}]$$

$$= \Sigma \otimes (Q'Q)^{-1} + (\Sigma \otimes (Q'Q)^{-1}D)R'[R(\Sigma \otimes D'(Q'Q)^{-1}D)R']^{-1}$$

$$\times\ R[\Sigma \otimes D'(Q'Q)^{-1}].$$

Q.E.D.

APPENDIX 7.2

Using a partitioned matrix, write the inverse of (I - C) as E,

$$(I - C)^{-1} = E = \begin{bmatrix} E^{11} & E^{12} \\ E^{21} & E^{22} \end{bmatrix}, \tag{A2.1}$$

Then, the submatrices of E can be written as

$$E^{11} = (I-C_o)^{-1}(I-C_o\iota E^{22}c_{m.}(I-C_o)^{-1}) \tag{A2.2}$$

$$E^{12} = -(I-C_o)^{-1}C_o\iota E^{22} \tag{A2.3}$$

$$E^{21} = E^{22}c_{m.}(I-C_o)^{-1} \tag{A2.4}$$

$$E^{22} = \varphi^{-1} = \{1+ c_{m.}\iota + c_{m.}(I-C_o)^{-1}C_o\iota\} \tag{A2.5}$$

Since $(I-C)\iota=\iota$ implies $(I-C)^{-1}\iota=\iota$ [(see Appendix 7.3), we have the relations

$$E^{12} = (I-E^{11})\iota \text{ and } E^{22} = 1 - E^{21}\iota. \tag{A2.6}$$

Since φ is a scalar, from (A2.5) and (A2.6), φ can be obtained as

$$\varphi = 1 + c_{m.}(I-C_o)^{-1}\iota. \tag{A2.7}$$

Write $D = B(I-C)^{-1}$, the long run effect of X, as

$$D = \begin{bmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{bmatrix} . \tag{A2.8}$$

Sinc $B(I-C)^{-1} = \begin{bmatrix} B_o & -B_o\iota \\ \beta_{m.} & -\beta_{m.}\iota \end{bmatrix}\begin{bmatrix} E^{11} & E^{12} \\ E^{21} & E^{22} \end{bmatrix}$ (A2.9)

$$= \begin{bmatrix} B_oE^{11}-B_o\iota E^{21} & B_oE^{12}-B_o\iota E^{22} \\ \beta_{m.}E^{11} -\beta_{m.}\iota E^{21} & \beta_{m.}E^{12} -\beta_{m.}\iota E^{22} \end{bmatrix}$$

D_{11} and D_{21}, in which we are only interested, can be written as,

$$D_{11} = B_o(E^{11}- \iota E^{21}) \text{ and } D_{21} = \beta_{m.}(E^{11} - \iota E^{21}). \tag{A2.10}$$

However, $(E^{11} - \iota E^{21})$ can be written as

$$(E^{11} - \iota E^{21}) = (I-C_o)^{-1} - [(I-C_o)^{-1}C_o + I]\iota\varphi^{-1}c_{m.}(I-C_o)^{-1}$$

$$= (I-C_o)^{-1}- (I-C_o)^{-1}\iota\varphi^{-1}c_{m.}(I-C_o)^{-1}, \tag{A2.11}$$

since

$$(I-C_o)^{-1}C_o+I$$

$$= (I-C_o)^{-1}C_o-(I-C_o)^{-1}+(I-C_o)^{-1}+I$$

$$= -(I-C_o)^{-1}(I-C_o)+(I-C_o)^{-1}+I$$

$$= -I+(I-C_o)^{-1}+I = (I-C_o)^{-1}.$$

Then, using Proposition 33, p.459 in Dhrymes (1978), we can see that (A2.11) becomes

$$(E^{11} - \iota E^{21}) = [I - (C_o + \iota\varphi^{-1}c_{m.})]^{-1}, \tag{A2.12}$$

implying that

$$D_{11} = B_o[I - (C_o + \iota\varphi^{-1}c_{m.})]^{-1}. \tag{A2.13}$$

and

$$D_{21} = \beta_{m.}[I - (C_o + \iota\varphi^{-1}c_{m.})]^{-1}. \tag{A2.14}$$

The long run effect of μ can then be written as

$$d = [\gamma_o : \gamma_m]\begin{bmatrix} E^{11} & E^{12} \\ E^{21} & E^{22} \end{bmatrix} = [\gamma_o E^{11} + \gamma_m E^{21} : \gamma_o E^{12} + \gamma_m E^{22}]$$

$$= [\gamma_o(E^{11} - \iota E^{21}) : \gamma_o(E^{12} - E^{22})].$$

We are also interested in $d_1 = \gamma_o(E^{11} - \iota E^{21})$, which can be written as

$$d_1 = \gamma_o[I - (C_o + \iota\varphi^{-1}c_{m.})]^{-1}. \tag{A2.15}$$

APPENDIX 7.3

$$C\iota = 0 \Rightarrow (I - C)\iota = \iota$$

$$\Rightarrow (I - C)^{-1}(I - C)\iota = (I - C)^{-1}\iota$$

$$\Rightarrow (I - C)^{-1}\iota = \iota.$$

Therefore,

$$\gamma(I - C)^{-1}\iota = \gamma\iota = 1$$

and

$$B(I - C)^{-1}\iota = B\iota = 0.$$

Footnotes:

1. The reparameterisation of a dynamic system for direct estimation of the long run responses was achieved by Bewley (1979) in the context of the portfolio model with a general partial stock adjustment model and a general linear dynamic system. A similar approach was applied to a general dynamic specification of a singular equation system by Anderson and Blundell. One of the main points in such an approach is the observationally equivalent reparameterisation of the general dynamic model to achieve direct estimation of the equilibrium response. The advantage of the reparameterisation approach is that the long run equilibrium restrictions can be directly and linearly imposed on the coefficient matrix for subsequent estimation.

2. We assume that the long run equilibrium situation can be described by the static demand theory [Phlips (1974, p.180) and Anderson and Blundell (1983, 1984)].

3. See Footnote 17 in Chapter 3.

TABLES IN CHAPTER 7

Table 7.1
Comparison of Tests of the Short Run (SR) and Long Run (LR) Demand Restrictions

Homogeneity Only

Model		Test Statistic			
		Wald	LR	LM	T^2 (5% CV)
Static		8.807	8.244	7.727	7.409 (10.842)
RDAM-SD-D(1)	SR	16.540	14.687	13.101	12.340 (11.039)
	LR	7.002	6.640	6.302	5.224 (11.039)
RDAM-SD-D(1,2)	SR	16.855	14.936	13.298	10.969 (11.303)
	LR	2.088	2.054	2.021	1.359 (11.303)
RDAM-SD-A(1)	SR	15.561	13.906	12.479	11.856 (11.003)
	LR	—	13.906	—	—
RDAM-SD-A(4)	SR	8.214	7.721	7.267	6.259 (11.003)
	LR	—	7.721	—	—

Asymptotic CV (5%): $\chi^2_4 = 9.492$.

Symmetry Only

Model		Test Statistic		
		Wald	LR	LM
Static		9.087	8.726	8.388
RDAM-SD-D(1)	SR	10.671	10.273	9.898
	LR	7.975	7.691	7.422
RDAM-SD-D(1,2)	SR	10.036	9.702	9.383
	LR	6.496	6.281	6.076
RDAM-SD-A(1)	SR	6.208	5.990	5.783
	LR	—	4.635	—
RDAM-SD-A(4)	SR	8.206	7.881	7.575
	LR	—	5.804	—

Asymptotic CV (5%): $\chi^2_6 = 12.596$.

Table 7.1 (continued)

Homogeneity and Symmetry

Model		Test Statistic		
		Wald	LR	LM
Static		18.396	16.969	15.693
RDAM-SD-D(1)	SR	28.261	24.960	22.212
	LR	15.284	14.331	13.453
RDAM-SD-D(1,2)	SR	27.441	24.638	22.243
	LR	8.613	8.335	8.068
RDAM-SD-A(1)	SR	22.056	19.896	18.030
	LR	—	18,541	—
RDAM-SD-A(4)	SR	16.509	15.602	14.763
	LR	—	13.525	—

Asymptotic CV (5%): χ^2_{10} = 18.311.

Table 7.2
RDAM-SD-D(1) Model

Table 7.2.1 Unrestricted RDAM-SD-D(1) Model

	dlnP1	dlnP2	dlnP3	dlnP4	dlnP5	dY
1	-.07020	-.06449	.06109	.51234	.20909	.43231
	(.0724)	(.1535)	(.0534)	(.1574)	(.1073)	(.0578)
2	.01320	-.20823	.04577	.03745	.03005	.09990
	(.0309)	(.0655)	(.0228)	(.0672)	(.0458)	(.0247)
3	.06142	.08119	-.04986	.01732	-.10002	.03365
	(.0216)	(.0458)	(.0159)	(.0470)	(.0320)	(.0172)
4	.03807	-.02676	-.01295	-.17528	-.03475	.13852
	(.0389)	(.0826)	(.0287)	(.0847)	(.0577)	(.0311)
5	-.04249	.21829	-.04406	-.39183	-.10437	.29562
	(.0597)	(.1266)	(.0440)	(.1298)	(.0885)	(.0476)

	$dlnP1_{-1}$	$dlnP2_{-1}$	$dlnP3_{-1}$	$dlnP4_{-1}$	$dlnP5_{-1}$	dY_{-1}
1	-.09250	.06752	.12050	-.53632	-.04990	-.09776
	(.0740)	(.1596)	(.0565)	(.1662)	(.1135)	(.0536)
2	.09418	-.04571	.01523	-.00394	-.04476	.09775
	(.0316)	(.0682)	(.0241)	(.0710)	(.0484)	(.0229)
3	-.01616	-.03397	.04115	.03036	.04162	-.01257
	(.0221)	(.0477)	(.0169)	(.0496)	(.0339)	(.0160)
4	-.08309	.14125	-.03067	-.01292	.07451	-.01003
	(.0398)	(.0859)	(.0304)	(.0894)	(.0610)	(.0289)
5	.09757	-.12908	-.14621	.52282	-.02147	.02260
	(.0610)	(.1316)	(.0466)	(.1370)	(.0936)	(.0442)

	D_1	D_2	D_3	D_4	R^2	DW	DW4
1	-.06105	-.00275	.01071	.02686	.9452	2.328	1.051
	(.0090)	(.0070)	(.0069)	(.0080)			
2	-.00314	.02245	-.00349	-.01606	.6969	2.512	1.872
	(.0038)	(.0030)	(.0029)	(.0034)			
3	.00106	-.01582	-.00779	.01488	.8899	2.436	1.555
	(.0027)	(.0021)	(.0021)	(.0024)			
4	.01080	.00609	-.00908	.01214	.7105	2.690	1.153
	(.0048)	(.0038)	(.0037)	(.0043)			
5	.05233	-.00997	.00965	-.03782	.8242	2.574	1.266
	(.0074)	(.0058)	(.0057)	(.0066)			

Table 7.2.2 Long Run Homogeneity Restricted RDAM-SD-D(1) Model

	dlnP1	dlnP2	dlnP3	dlnP4	dlnP5	dM
1	-.08594 (.0710)	-.10868 (.1483)	.07448 (.0520)	.48244 (.1552)	.20861 (.1073)	.43234 (.0578)
2	.01988 (.0303)	-.18945 (.0633)	.04008 (.0222)	.05015 (.0662)	.03026 (.0458)	.09989 (.0247)
3	.05411 (.0212)	.06064 (.0443)	-.04364 (.0155)	.00342 (.0463)	-.10025 (.0320)	.03366 (.0172)
4	.05035 (.0382)	.00772 (.0798)	-.02340 (.0280)	-.15196 (.0835)	-.03438 (.0577)	.13850 (.0311)
5	-.03840 (.0585)	.22978 (.1223)	-.04754 (.0429)	-.38406 (.1279)	-.10424 (.0885)	.29561 (.0476)

	$dlnP1_{-1}$	$dlnP2_{-1}$	$dlnP3_{-1}$	$dlnP4_{-1}$	$dlnP5_{-1}$	dM_{-1}
1	-.11178 (.0720)	.02647 (.1554)	.13018 (.0559)	-.56140 (.1647)	-.05439 (.1134)	-.09610 (.0536)
2	.10237 (.0307)	-.02827 (.0663)	.01111 (.0239)	.00672 (.0703)	-.04286 (.0484)	.09705 (.0229)
3	-.02513 (.0215)	-.05305 (.0464)	.04565 (.0167)	.01870 (.0492)	.03953 (.0339)	-.01180 (.0160)
4	-.06804 (.0387)	.17326 (.0836)	-.03822 (.0301)	.00665 (.0886)	.07801 (.0610)	-.01132 (.0288)
5	.10258 (.0593)	-.11842 (.1281)	-.14873 (.0461)	.52933 (.1358)	-.02030 (.0935)	.02217 (.0442)

	D_1	D_2	D_3	D_4	R^2	DW	DW4
1	-.05614 (.0078)	.00273 (.0050)	.01555 (.0054)	.03123 (.0070)	.9442	2.348	1.012
2	-.00522 (.0033)	.02012 (.0022)	-.00555 (.0023)	-.01791 (.0030)	.6909	2.484	1.824
3	.00335 (.0023)	-.01328 (.0015)	-.00554 (.0016)	.01691 (.0021)	.8846	2.352	1.571
4	.00697 (.0042)	.00182 (.0027)	-.01285 (.0029)	.00873 (.0038)	.6985	2.666	1.181
5	.05105 (.0064)	-.01139 (.0042)	.00840 (.0044)	-.03896 (.0058)	.8239	2.581	1.254

Table 7.2.2 (continued)
Long Run Effect Derived from Long Run Homogeneity Restricted D(1) Model.

	P1	P2	P3	P4	P5	μ
1	-.19773	-.08221	.20467	-.07896	.15423	.33624
	(.0953)	(.1808)	(.0677)	(.1729)	(.1426)	(.0750)
2	.12226	-.21773	.05120	.05687	-.01260	.19694
	(.0407)	(.0772)	(.0289)	(.0738)	(.0609)	(.0320)
3	.02898	.00760	.00201	.02212	-.06071	.02186
	(.0285)	(.0540)	(.0202)	(.0516)	(.0426)	(.0224)
4	-.01770	.18098	-.06161	-.14530	.04363	.12718
	(.0513)	(.0973)	(.0364)	(.0930)	(.0767)	(.0403)
5	.06418	.11136	-.19627	.14527	-.12454	.31778
	(.0785)	(.1491)	(.0558)	(.1426)	(.1176)	(.0619)

Table 7.2.3 Long Run Homogeneity and Symmetry Restricted RDAM-SD-D(1) Model

	dlnP1	*dlnP2*	*dlnP3*	*dlnP4*	*dlnP5*	*dY*
1	-.11097	-.03866	.00666	.50022	.17041	.40747
	(.0724)	(.1295)	(.0453)	(.1420)	(.0938)	(.0573)
2	.01227	-.20764	.04100	.06557	.03884	.10341
	(.0301)	(.0626)	(.0215)	(.0643)	(.0443)	(.0249)
3	.06840	.08125	-.03859	-.02037	-.11119	.03065
	(.0210)	(.0395)	(.0158)	(.0433)	(.0298)	(.0171)
4	.04673	-.02986	-.00779	-.13770	-.01464	.14835
	(.0380)	(.0734)	(.0264)	(.0820)	(.0560)	(.0309)
5	-.01643	.19491	-.00128	-.40772	-.08343	.31012
	(.0593)	(.1064)	(.0382)	(.1167)	(.0853)	(.0465)

	$dlnP1_{-1}$	$dlnP2_{-1}$	$dlnP3_{-1}$	$dlnP4_{-1}$	$dlnP5_{-1}$	dY_{-1}
1	-.15308	.14623	.05156	-.52095	-.05142	-.14025
	(.0723)	(.1290)	(.0461)	(.1442)	(.0969)	(.0529)
2	.09529	-.05109	.01013	.02424	-.02861	.09810
	(.0304)	(.0652)	(.0226)	(.0671)	(.0464)	(.0230)
3	-.01018	-.03013	.05439	-.00992	.01634	-.00930
	(.0213)	(.0403)	(.0169)	(.0448)	(.0311)	(.0162)
4	-.06746	.11967	-.02250	.01793	.09562	-.00033
	(.0384)	(.0749)	(.0282)	(.0877)	(.0567)	(.0288)
5	.13542	-.18468	-.09357	.48870	-.03193	.05178
	(.0591)	(.1070)	(.0394)	(.1199)	(.0889)	(.0430)

Table 7.2.3 (continued)

	D_1	D_2	D_3	D_4	R^2	DW	DW4
1	-.05094 (.0079)	.00363 (.0053)	.01691 (.0056)	.03592 (.0069)	.9395	2.297	.948
2	-.00507 (.0034)	.02017 (.0022)	-.00565 (.0023)	-.01815 (.0030)	.6889	2.473	1.826
3	.00262 (.0024)	-.01342 (.0016)	-.00551 (.0017)	.01682 (.0021)	.8810	2.309	1.631
4	.00602 (.0043)	.00167 (.0028)	-.01328 (.0030)	.00741 (.0037)	.6915	2.691	1.166
5	.04738 (.0064)	-.01205 (.0043)	.00752 (.0045)	-.04200 (.0056)	.8137	2.516	1.194

Long Run Effect Derived from Long Run Symmetry Restricted D(1) Model

	P1	P2	P3	P4	P5	μ
1	-.26405 (.09191)	.10757 (.0392)	.05822 (.0264)	-.02073 (.0484)	.11899 (.0766)	.26723 (.0697)
2	.10757 (.0392)	-.25873 (.0723)	.05112 (.0251)	.08981 (.0610)	.01024 (.0545)	.20151 (.0323)
3	.05822 (.0264)	.05112 (.0251)	.01579 (.0198)	-.03029 (.0286)	-.09485 (.0318)	.02135 (.0215)
4	-.02073 (.0484)	.08981 (.0610)	-.03029 (.0286)	-.11977 (.0804)	.08098 (.0635)	.14802 (.0387)
5	.11899 (.0766)	.01024 (.0545)	-.09485 (.0319)	.08098 (.0635)	-.11536 (.1019)	.36190 (.0570)

Table 7.3
RDAM-SD-D(1,2)

Table 7.3.1 Unrestricted RDAM-SD-D(1,2) Model

	dlnP1	*dlnP2*	*dlnP3*	*dlnP4*	*dlnP5*	*dM*
1	-.07260 (.0729)	.04354 (.1647)	.02249 (.0596)	.38968 (.1667)	.25180 (.1068)	.42061 (.0563)
2	.00307 (.0284)	-.22545 (.0643)	.06053 (.0233)	-.00033 (.0651)	.00207 (.0417)	.11618 (.0220)
3	.04959 (.0211)	.02955 (.0478)	-.02803 (.0173)	.04889 (.0484)	-.08995 (.0310)	.03731 (.0163)
4	.07046 (.0343)	-.02600 (.0776)	-.02715 (.0281)	-.08121 (.0785)	-.06987 (.0503)	.13420 (.0265)
5	-.05052 (.0626)	.17837 (.1416)	-.02784 (.0513)	-.35703 (.1433)	-.09405 (.0918)	.29170 (.0484)

	dlnP1$_{-1}$	*dlnP2*$_{-1}$	*dlnP3*$_{-1}$	*dlnP4*$_{-1}$	*dlnP5*$_{-1}$	*dM*$_{-1}$
1	-.09485 (.0722)	.17220 (.1678)	.09595 (.0553)	-.33380 (.1796)	-.00263 (.1124)	-.11385 (.0570)
2	.11872 (.0282)	-.04679 (.0655)	.00933 (.0216)	-.00409 (.0701)	-.04073 (.0439)	.10531 (.0222)
3	-.01490 (.0209)	-.08750 (.0487)	.04286 (.0161)	.02193 (.0521)	.04071 (.0326)	-.00838 (.0165)
4	-.09973 (.0340)	.11659 (.0790)	-.00025 (.0261)	-.18816 (.0846)	.05050 (.0529)	-.00152 (.0268)
5	.09076 (.0621)	-.15449 (.1443)	-.14789 (.0476)	.50411 (.1544)	-.04786 (.0966)	.01845 (.0490)

	dlnP1$_{-2}$	*dlnP2*$_{-2}$	*dlnP3*$_{-2}$	*dlnP4*$_{-2}$	*dlnP5*$_{-2}$	*dM*$_{-2}$
1	.00965 (.0716)	-.19913 (.1728)	.02982 (.0574)	-.26136 (.1639)	.00456 (.1168)	.02035 (.0543)
2	-.05689 (.0279)	.14640 (.0675)	-.05525 (.0224)	.04024 (.0640)	-.01757 (.0456)	-.08520 (.0212)
3	-.05449 (.0208)	.05504 (.0501)	.00628 (.0167)	-.02654 (.0476)	.04746 (.0339)	.01204 (.0157)
4	.08319 (.0337)	-.07015 (.0814)	.00488 (.0271)	.31272 (.0772)	-.09611 (.0550)	.04446 (.0256)
5	.01854 (.0615)	.06783 (.1485)	.01428 (.0494)	-.06506 (.1409)	.06165 (.1004)	.00835 (.0467)

Table 7.3.1 (continued)

	D_1	D_2	D_3	D_4	R^2	DW	DW4
1	-.05938	-.00196	.01271	.03625	.9508	2.312	1.207
	(.0093)	(.0092)	(.0078)	(.0092)			
2	.03156	-.00700	-.01902	.00081	.7722	2.446	1.857
	(.0036)	(.0036)	(.0030)	(.0036)			
3	-.01643	-.00441	.01451	.00179	.9066	2.221	1.535
	(.0027)	(.0027)	(.0023)	(.0027)			
4	.00546	-.00158	-.01036	.00814	.8006	2.620	1.355
	(.0044)	(.0043)	(.0037)	(.0043)			
5	.05131	-.01159	.00906	-.03987	.8282	2.620	1.326
	(.0080)	(.0079)	(.0067)	(.0079)			

Table 7.3.2 Long Run Homogeneity Restricted RDAM-SD-D(1,2) Model

	$dlnP_1$	$dlnP_2$	$dlnP_3$	$dlnP_4$	$dlnP_5$	dM
1	-.07905	.03348	.02642	.37539	.25478	.42075
	(.0708)	(.1624)	(.0587)	(.1622)	(.1065)	(.0563)
2	.01086	-.21332	.05579	.01690	-.00152	.11601
	(.0276)	(.0634)	(.0229)	(.0633)	(.0416)	(.0220)
3	.04481	.02212	-.02513	.03833	-.08774	.03742
	(.0205)	(.0471)	(.0170)	(.0471)	(.0309)	(.0163)
4	.07282	-.02232	-.02859	-.07597	-.07096	.13415
	(.0333)	(.0765)	(.0276)	(.0764)	(.0502)	(.0265)
5	-.04944	.18004	-.02849	-.35465	-.09455	.29167
	(.0608)	(.1397)	(.0505)	(.1395)	(.0916)	(.0484)

	$dlnP1_{-1}$	$dlnP2_{-1}$	$dlnP3_{-1}$	$dlnP4_{-1}$	$dlnP5_{-1}$	dM_{-1}
1	-.09933	.16251	.09744	-.33111	-.00001	-.11453
	(.0712)	(.1658)	(.0552)	(.1795)	(.1122)	(.0569)
2	.12412	-.03511	.00754	-.00733	-.04388	.10612
	(.0278)	(.0647)	(.0215)	(.0701)	(.0438)	(.0222)
3	-.01822	-.09467	.04396	.02392	.04265	-.00888
	(.0207)	(.0481)	(.0160)	(.0521)	(.0325)	(.0165)
4	-.09808	.12014	-.00079	-.18914	.04954	-.00127
	(.0335)	(.0781)	(.0260)	(.0845)	(.0528)	(.0268)
5	.09150	-.15288	-.14814	.50366	-.04830	.01856
	(.0612)	(.1425)	(.0474)	(.1543)	(.0965)	(.0489)

Table 7.3.2 (continued)

	$dlnP1_{-2}$	$dlnP2_{-2}$	$dlnP3_{-2}$	$dlnP4_{-2}$	$dlnP5_{-2}$	dM_{-2}
1	.00274 (.0691)	-.20332 (.1724)	.03278 (.0569)	-.27582 (.1592)	.00311 (.1167)	.01999 (.0543)
2	-.04856 (.0270)	.15145 (.0673)	-.05883 (.0222)	.05769 (.0622)	-.01581 (.0456)	-.08476 (.0212)
3	-.05959 (.0201)	.05195 (.0500)	.00847 (.0165)	-.03724 (.0462)	.04638 (.0339)	.01177 (.0157)
4	.08573 (.0325)	-.06861 (.0812)	.00379 (.0268)	.31802 (.0750)	-.09558 (.0550)	.04460 (.0256)
5	.01969 (.0594)	.06853 (.1482)	.01378 (.0489)	-.06265 (.1369)	.06189 (.1004)	.00841 (.0467)

	D_1	D_2	D_3	D_4	R^2	DW	DW4
1	-.00019 (.0079)	.01448 (.0062)	.03802 (.0079)	-.05756 (.0080)	.9507	2.316	1.195
2	.02942 (.0031)	-.00913 (.0024)	-.02116 (.0031)	-.00138 (.0031)	.7674	2.436	1.844
3	-.01512 (.0023)	-.00310 (.0018)	.01582 (.0023)	.00314 (.0023)	.9053	2.174	1.559
4	-.00223 (.0037)	-.01101 (.0029)	.00749 (.0037)	.00480 (.0037)	.8003	2.606	1.350
5	-.01188 (.0068)	.00877 (.0053)	-.04017 (.0068)	.05101 (.0068)	.8282	2.622	1.325

Long Run Effect Derived from Long Run Homogeneity Restricted D(1,2) Model

	P1	P2	P3	P4	P5	μ
1	-.17564 (.1144)	-.00732 (.2227)	.15664 (.0941)	-.23155 (.2049)	.25787 (.1750)	.32621 (.0886)
2	.08642 (.0447)	-.09698 (.0869)	.00450 (.0367)	.06727 (.0800)	-.06121 (.0683)	.13737 (.0346)
3	-.03300 (.0332)	-.02060 (.0646)	.02730 (.0273)	.02501 (.0595)	.00129 (.0508)	.04031 (.0257)
4	.06047 (.0539)	.02921 (.1049)	-.02559 (.0443)	.05291 (.0965)	-.11699 (.0824)	.17748 (.0417)
5	.06175 (.0983)	.09569 (.1915)	-.16285 (.0809)	.08637 (.1762)	-.08095 (.1505)	.31864 (.0762)

Table 7.3.3 Long Run Homogeneity and Symmetry Restricted RDAM-SD-D(1,2) Model

	$dlnP1$	$dlnP2$	$dlnP3$	$dlnP4$	$dlnP5$	dM
1	-.07034	.08269	-.03125	.38908	.15489	.41933
	(.0731)	(.1496)	(.0518)	(.1463)	(.0948)	(.0562)
2	.00468	-.21896	.05866	.01404	.01583	.11676
	(.0278)	(.0632)	(.0222)	(.0620)	(.0405)	(.0221)
3	.04941	.03343	-.02287	.02410	-.08997	.03466
	(.0203)	(.0437)	(.0170)	(.0429)	(.0289)	(.0160)
4	.06145	-.02422	-.02320	-.08002	-.05396	.13312
	(.0327)	(.0712)	(.0256)	(.0717)	(.0487)	(.0259)
5	-.04520	.12706	.01866	-.34720	-.02679	.29613
	(.0626)	(.1293)	(.0453)	(.1260)	(.0879)	(.0481)

	$dlnP1_{-1}$	$dlnP2_{-1}$	$dlnP3_{-1}$	$dlnP4_{-1}$	$dlnP5_{-1}$	dM_{-1}
1	-.10975	.21902	.05439	-.22131	-.00052	-.12262
	(.0725)	(.1442)	(.0509)	(.1790)	(.1060)	(.0556)
2	.12004	-.03976	.00764	-.01456	-.03337	.10682
	(.0278)	(.0639)	(.0212)	(.0699)	(.0432)	(.0224)
3	-.01123	-.08183	.04760	.01325	.03372	-.01114
	(.0204)	(.0431)	(.0159)	(.0518)	(.0318)	(.0159)
4	-.10446	.11834	-.00033	-.19813	.05810	-.00325
	(.0330)	(.0704)	(.0246)	(.0839)	(.0509)	(.0259)
5	.10540	-.21577	-.10930	.42076	-.05793	.03019
	(.0623)	(.1250)	(.0444)	(.1535)	(.0950)	(.0476)

	$dlnP1_{-2}$	$dlnP2_{-2}$	$dlnP3_{-2}$	$dlnP4_{-2}$	$dlnP5_{-2}$	dM_{-2}
1	.00503	-.23102	-.03773	-.13420	-.06896	-.01543
	(.0710)	(.1680)	(.0476)	(.1372)	(.1041)	(.0527)
2	-.05404	.15459	-.05501	.04017	.00004	-.08339
	(.0270)	(.0673)	(.0211)	(.0590)	(.0449)	(.0213)
3	-.05278	.05970	.01305	-.05104	.03547	.01313
	(.0198)	(.0482)	(.0163)	(.0415)	(.0310)	(.0155)
4	.07657	-.05448	.00984	.29527	-.08079	.04454
	(.0319)	(.0783)	(.0247)	(.0721)	(.0508)	(.0247)
5	.02521	.07122	.06986	-.15021	.11424	.04115
	(.0610)	(.1435)	(.0421)	(.1215)	(.0952)	(.0453)

Table 7.3.3 (continued)

	D_1	D_2	D_3	D_4	R^2	DW	DW4
1	-.05444	.00560	.01285	.04053	.9467	2.300	1.230
	(.0080)	(.0077)	(.0063)	(.0075)			
2	-.00137	.02919	-.00900	-.02137	.7656	2.417	1.843
	(.0031)	(.0031)	(.0024)	(.0031)			
3	.00289	-.01555	-.00318	.01610	.9041	2.193	1.540
	(.0023)	(.0023)	(.0018)	(.0022)			
4	.00518	-.00246	-.01079	.00746	.7988	2.637	1.317
	(.0037)	(.0036)	(.0029)	(.0035)			
5	.04773	-.01677	.01012	-.04280	.8165	2.591	1.382
	(.0068)	(.0066)	(.0054)	(.0065)			

Long Run Effect Derived from Long Run Symmetry Restricted D(1,2) Model.

	P1	P2	P3	P4	P5	μ
1	-.17506	.07068	-.01460	.03357	.08541	.28129
	(.1156)	(.0437)	(.0312)	(.0506)	(.1005)	(.0763)
2	.07068	-.10414	.01130	.03965	-.01750	.14019
	(.0437)	(.0808)	(.0314)	(.0648)	(.0628)	(.0346)
3	-.01460	.01130	.03778	-.01370	-.02078	.03665
	(.0312)	(.0314)	(.0262)	(.0331)	(.0393)	(.0227)
4	.03357	.03965	-.01370	.01713	-.07665	.17440
	(.0506)	(.0648)	(.0331)	(.0812)	(.0696)	(.0373)
5	.08541	-.01750	-.02078	-.07665	.02952	.36747
	(.1005)	(.0628)	(.0393)	(.0696)	(.1280)	(.0659)

Table 7.4
RDAM-SD-A(1) Model

Table 7.4.1 Unrestricted RDAM-SD-A(1) Model

	dlnP1	dlnP2	dlnP3	dlnP4	dlnP5	dM
1	-.02624 (.0771)	-.11861 (.1632)	.06261 (.0553)	.37052 (.1528)	.04879 (.1116)	.48778 (.0545)
2	.01787 (.0319)	-.28281 (.0675)	.05703 (.0229)	.02329 (.0632)	.05804 (.0462)	.11503 (.0226)
3	.06181 (.0217)	.12844 (.0460)	-.05063 (.0156)	.00259 (.0431)	-.07736 (.0315)	.02904 (.0154)
4	.06058 (.0350)	.06996 (.0742)	-.01779 (.0251)	-.16313 (.0694)	-.06329 (.0507)	.14322 (.0248)
5	-.11403 (.0632)	.20302 (.1338)	-.05122 (.0454)	-.23327 (.1253)	.03382 (.0915)	.22494 (.0447)

	$y1_{-1}$	$y2_{-1}$	$y3_{-1}$	$y4_{-1}$	$y5_{-1}$
1	-.16289 (.0722)	-.30762 (.2477)	-.93041 (.3524)	-.02707 (.2012)	.15304 (.1149)
2	.10681 (.0299)	-.21119 (.1025)	.14480 (.1458)	.09499 (.0832)	.07782 (.0475)
3	.02787 (.0203)	-.01182 (.0698)	-.23620 (.0993)	-.02688 (.0567)	-.09781 (.0324)
4	-.01103 (.0328)	.21735 (.1125)	.13731 (.1601)	-.37010 (.0914)	.16136 (.0522)
5	.03924 (.0592)	.31328 (.2030)	.88452 (.2889)	.32906 (.1649)	-.29441 (.0942)

	D_1	D_2	D_3	D_4	R^2	DW	DW4
1	-.04207 (.0112)	-.02079 (.0097)	-.00369 (.0092)	.01353 (.0088)	.9413	2.182	1.039
2	-.00328 (.0046)	.02834 (.0040)	.00299 (.0038)	-.01627 (.0037)	.6950	2.253	2.003
3	.00143 (.0031)	-.00877 (.0027)	-.01096 (.0026)	.01550 (.0025)	.8949	2.074	1.443
4	.01702 (.0051)	-.00332 (.0044)	-.00847 (.0042)	.00649 (.0040)	.7787	2.219	1.263
5	.02690 (.0092)	.00453 (.0079)	.02013 (.0075)	-.01924 (.0072)	.8138	1.949	1.289

Table 7.4.2 Long Run (and Short Run) Homogeneity Restricted RDAM-SD-A(1) Model

	dlnP1	dlnP2	dlnP3	dlnP4	dlnP5	dQ
1	-.07130	-.29377	.10824	.23306	.02378	.49329
	(.0751)	(.1485)	(.0524)	(.1432)	(.1112)	(.0545)
2	.03479	-.21704	.03990	.07491	.06744	.11296
	(.0311)	(.0614)	(.0217)	(.0593)	(.0460)	(.0225)
3	.05314	.09474	-.04185	-.02386	-.08218	.03010
	(.0212)	(.0418)	(.0148)	(.0404)	(.0313)	(.0154)
4	.07578	.12902	-.03317	-.11677	-.05485	.14136
	(.0341)	(.0674)	(.0238)	(.0651)	(.0505)	(.0247)
5	-.09241	.28704	-.07311	-.16733	.04582	.22229
	(.0616)	(.1217)	(.0430)	(.1174)	(.0911)	(.0447)

	$y1_{-1}$	$y2_{-1}$	$y3_{-1}$	$y4_{-1}$	$y5_{-1}$
1	-.13836	-.37544	-.89322	-.01687	.17037
	(.0715)	(.2463)	(.3521)	(.2012)	(.1147)
2	.09760	-.18572	.13083	.09116	.07131
	(.0296)	(.1019)	(.1457)	(.0832)	(.0474)
3	.03259	-.02487	-.22905	-.02492	-.09447
	(.0202)	(.0694)	(.0992)	(.0567)	(.0323)
4	-.01930	.24022	.12476	-.37354	.15552
	(.0325)	(.1119)	(.1600)	(.0914)	(.0521)
5	.02747	.34581	.86667	.32416	-.30272
	(.0587)	(.2019)	(.2886)	(.1649)	(.0940)

	D_1	D_2	D_3	D_4	R^2	DW	DW4
1	-.03294	-.00797	.00738	.02122	.9351	2.248	.959
	(.0106)	(.0083)	(.0081)	(.0083)			
2	-.00670	.02353	-.00117	-.01916	.6684	2.201	2.008
	(.0044)	(.0034)	(.0034)	(.0034)			
3	.00318	-.00630	-.00883	.01698	.8897	2.009	1.476
	(.0030)	(.0023)	(.0023)	(.0023)			
4	.01394	-.00764	-.01220	.00389	.7658	2.257	1.219
	(.0048)	(.0038)	(.0037)	(.0038)			
5	.02252	-.00162	.01482	-.02293	.8071	1.984	1.220
	(.0087)	(.0068)	(.0067)	(.0068)			

Table 7.4.3 Long Run Homogeneity and Symmetry Restricted RDAM-SD-A(1) Model

	dlnP1	*dlnP2*	*dlnP3*	*dlnP4*	*dlnP5*	*dM*
1	-.08055	-.03137	.05925	.09269	-.04200	.46369
	(.0791)	(.1073)	(.0382)	(.1178)	(.0949)	(.2737)
2	.02665	-.24560	.05524	.06992	.09390	.11824
	(.0321)	(.0608)	(.0202)	(.0546)	(.0445)	(.1101)
3	.05901	.05917	-.03315	-.02258	-.06245	.03699
	(.0207)	(.0299)	(.0147)	(.0327)	(.0282)	(.0738)
4	.08125	.11201	-.01825	-.11818	-.05678	.14774
	(.0344)	(.0589)	(.0221)	(.0619)	(.0488)	(.1189)
5	-.08619	.10589	-.06313	-.02183	.06542	.23333
	(.0632)	(.0887)	(.0353)	(.0952)	(.0860)	(.1084)

When the Fifth Equation is Deleted [a]

	$y1_{-1}$	$y2_{-1}$	$y3_{-1}$	$y4_{-1}$	dM_{-1}
1	-.32802	-.65837	-1.20728	-.23576	.17998
	(.0555)	(.1376)	(.3184)	(.4219)	(.1229)
2	.04287	-.22278	.09442	.02832	.06095
	(.0234)	(.0554)	(.1294)	(.1715)	(.0496)
3	.13085	.08554	-.11327	.07763	.14889
	(.0154)	(.0372)	(.0861)	(.1152)	(.0535)
4	-.16629	.11075	-.00509	-.52522	.14889
	(.0249)	(.0600)	(.1392)	(.1848)	(.0535)

When the First Equation is Deleted [a]

	$y2_{-1}$	$y3_{-1}$	$y4_{-1}$	$y5_{-1}$	dM_{-1}
2	-.26558	.05210	-.01435	-.04331	.10389
	(.0234)	(.1156)	(.1549)	(.0963)	(.0310)
3	-.04527	-.24392	-.05317	-.13107	.03429
	(.0155)	(.0769)	(.1038)	(.0644)	(.0207)
4	.27706	.16158	-.35879	.16606	-.01737
	(.0250)	(.1242)	(.1666)	(.1041)	(.0334)
5	.36425	.91016	.33432	-.32056	.02738
	(.0444)	(.2234)	(.3003)	(.1875)	(.0604)

[a] Coefficients of $y1_{-1}$, $y2_{-1}$, $y3_{-1}$, $y4_{-1}$ and $y5_{-1}$ and coefficient of dM_{-1} refer to $(C_0 - ic_{m.})$'s and $c_{m.}$ in equation (3.8) in Section 7.3, respectively.

Table 7.4.3 (continued)

	D_1	D_2	D_3	D_4	R^2	DW	DW4
1	-.02898	-.00762	.00691	.02439	.9313	2.267	1.032
	(.0111)	(.0089)	(.0085)	(.0087)			
2	-.00751	.02392	-.00140	-.01955	.6633	2.276	1.969
	(.0046)	(.0036)	(.0035)	(.0036)			
3	.00249	-.00635	-.00891	.01637	.8872	2.011	1.556
	(.0030)	(.0024)	(.0023)	(.0024)			
4	.01357	-.00744	-.01265	.00335	.7643	2.224	1.213
	(.0049)	(.0039)	(.0037)	(.0038)			
5	.02044	-.00250	.01604	-.02455	.7994	2.065	1.257
	(.0088)	(.0070)	(.0067)	(.0069)			

Long Run Effect Derived from Long Run Symmetry Restricted A(1) Model.

	P1	P2	P3	P4	P5
1	-.12469	.02247	.04485	.06835	-.01097
2	.02247	-.19530	.04473	.05666	.07144
3	.04485	.04473	-.02201	-.01353	-.05405
4	.06835	.05666	-.01353	-.08078	-.03068
5	-.01097	.07144	-.05405	-.03068	.02424

	μ	D1	D2	D3	D4
1	.36168	-.00954	.01416	.01193	-.02049
2	.16414	.01874	-.00143	-.01442	-.00661
3	.01321	-.00556	-.00714	.01498	.00006
4	.16693	-.00246	-.00992	-.00020	.01065
5	.29404	-.00117	.00432	-.01228	.01637

Table 7.5
RDAM-SD-A(4)

Table 7.5.1 Unrestricted RDAM-SD-A(4) Model

	dlnP1	dlnP2	dlnP3	dlnP4	dlnP5	dM
1	-.22383	-.13479	.02081	.32502	.19546	.47288
	(.0729)	(.1442)	(.0548)	(.1366)	(.0990)	(.0526)
2	.04406	-.23796	.06287	.02354	.01105	.12735
	(.0341)	(.0674)	(.0256)	(.0639)	(.0463)	(.0246)
3	.06655	.06901	-.04897	.06554	-.09305	.03064
	(.0228)	(.0450)	(.0171)	(.0427)	(.0309)	(.0164)
4	.05328	.01116	.00689	-.11221	-.04745	.14714
	(.0384)	(.0760)	(.0289)	(.0720)	(.0522)	(.0277)
5	.05994	.29259	-.04161	-.30189	-.06602	.22198
	(.0656)	(.1299)	(.0494)	(.1231)	(.0892)	(.0473)

	$y1_{-4}$	$y2_{-4}$	$y3_{-4}$	$y4_{-4}$	$y5_{-4}$
1	.22392	-.20722	.29680	-.63871	-.22379
	(.0694)	(.2341)	(.3296)	(.1828)	(.1097)
2	.03782	.06427	-.46064	.16786	.00388
	(.0325)	(.1094)	(.1541)	(.0855)	(.0513)
3	-.02925	.11777	.44287	.02317	-.03720
	(.0217)	(.0731)	(.1029)	(.0571)	(.0342)
4	-.09214	-.02362	-.03749	.36920	-.06858
	(.0366)	(.1233)	(.1737)	(.0963)	(.0578)
5	-.14034	.04879	-.24154	.07848	.32570
	(.0625)	(.2108)	(.2969)	(.1647)	(.0988)

	D_1	D_2	D_3	D_4	R^2	DW	DW4
1	-.03832	.00427	.00419	.00790	.9503	2.565	1.849
	(.0108)	(.0086)	(.0065)	(.0105)			
2	.01318	.01335	-.00755	-.01081	.6701	2.654	1.965
	(.0051)	(.0040)	(.0030)	(.0049)			
3	-.00222	-.01074	-.00177	.00758	.8908	2.558	1.879
	(.0034)	(.0027)	(.0020)	(.0033)			
4	.00249	.00237	-.00392	.00857	.7480	2.836	1.757
	(.0057)	(.0045)	(.0034)	(.0055)			
5	.02487	-.00926	.00904	-.01324	.8096	2.652	1.779
	(.0098)	(.0077)	(.0059)	(.0095)			

Table 7.5.2 Long Run and Short Run Homogeneity Restricted RDAM-SD-A(4) Model

	dlnP1	*dlnP2*	*dlnP3*	*dlnP4*	*dlnP5*	*dM*
1	-.25297	-.21988	.03827	.25726	.17732	.47412
	(.0702)	(.1324)	(.0536)	(.1288)	(.0983)	(.0525)
2	.05945	-.19305	.05366	.05931	.02063	.12670
	(.0328)	(.0619)	(.0250)	(.0602)	(.0460)	(.0246)
3	.05713	.04149	-.04332	.04363	-.09892	.03105
	(.0219)	(.0413)	(.0167)	(.0402)	(.0307)	(.0164)
4	.06737	.05230	-.00155	-.07945	-.03867	.14654
	(.0370)	(.0697)	(.0282)	(.0679)	(.0518)	(.0277)
5	.06903	.31913	-.04706	-.28075	-.06036	.22159
	(.0632)	(.1193)	(.0482)	(.1160)	(.0885)	(.0473)

	$y1_{-4}$	$y2_{-4}$	$y3_{-4}$	$y4_{-4}$	$y5_{-4}$
1	.24766	-.19461	.32620	-.64841	-.23962
	(.0676)	(.2339)	(.3290)	(.1827)	(.1091)
2	.02529	.05762	-.47616	.17298	.01223
	(.0316)	(.1094)	(.1539)	(.0854)	(.0510)
3	-.02158	.12185	.45238	.02003	-.04232
	(.0211)	(.0730)	(.1027)	(.0570)	(.0341)
4	-.10362	-.02972	-.05170	.37389	-.06093
	(.0356)	(.1232)	(.1734)	(.0962)	(.0575)
5	-.14775	.04486	-.25071	.08150	.33064
	(.0609)	(.2107)	(.2964)	(.1646)	(.0983)

	D_1	D_2	D_3	D_4	R^2	DW	DW4
1	-.02903	.01032	.00884	.01056	.9486	2.627	1.896
	(.0089)	(.0075)	(.0057)	(.0104)			
2	.00828	.01015	-.01000	-.01222	.6553	2.594	2.010
	(.0041)	(.0035)	(.0027)	(.0048)			
3	.00079	-.00878	-.00026	.00844	.8867	2.454	1.935
	(.0028)	(.0024)	(.0018)	(.0032)			
4	-.00200	-.00055	-.00617	.00729	.7406	2.834	1.777
	(.0047)	(.0040)	(.0030)	(.0055)			
5	.02197	-.01114	.00759	-.01407	.8088	2.672	1.792
	(.0080)	(.0068)	(.0051)	(.0093)			

Table 7.5.3 Long Run Homogeneity and Symmetry Restricted RDAM-SD-A(4) Model

	dlnP1	*dlnP2*	*dlnP3*	*dlnP4*	*dlnP5*	*dM*
1	-.23268	-.01523	.08694	.04084	.11957	.48289
	(.0600)	(.0281)	(.0563)	(.1903)	(.0419)	(.2236)
2	.04884	-.21928	.04286	.08129	.04631	.12367
	(.0316)	(.0567)	(.0178)	(.0328)	(.0376)	(.1105)
3	.05156	.07512	-.04009	-.00176	-.08483	.03067
	(.0172)	(.0111)	(.0141)	(.0426)	(.0148)	(.0714)
4	.07545	.04675	.00536	-.07413	-.05341	.15003
	(.0235)	(.0386)	(.0150)	(.0609)	(.0313)	(.1221)
5	.05683	.11268	-.09510	-.04627	-.02809	.21276
	(.0555)	(.0297)	(.0421)	(.1722)	(.0580)	(.0996)

When the Fifth Equation is Deleted

	$y1_{-4}$	$y2_{-4}$	$y3_{-4}$	$y4_{-4}$	dM_{-4}
1	.45522	-.04170	.50479	-.38707	-.23127
	(.0410)	(.1138)	(.2768)	(.3587)	(.1098)
2	.01783	.05156	-.49867	.14112	.01388
	(.0239)	(.0575)	(.1328)	(.1736)	(.0519)
3	.01482	.14528	.45969	.04559	-.03697
	(.0153)	(.0367)	(.0882)	(.1134)	(.0336)
4	-.04247	.04731	.01717	.45369	-.06495
	(.0236)	(.0639)	(.1429)	(.1925)	(.0581)

	D_1	D_2	D_3	D_4	R^2	DW	DW4
1	-.03074	.01123	.00718	.01263	.9448	2.564	1.937
	(.0084)	(.0068)	(.0058)	(.0102)			
2	.00869	.00973	-.00984	-.01187	.6514	2.599	2.010
	(.0042)	(.0035)	(.0027)	(.0049)			
3	.00064	-.00907	-.00061	.00958	.8840	2.444	1.881
	(.0027)	(.0023)	(.0017)	(.0032)			
4	-.00215	-.00065	-.00609	.00676	.7400	2.828	1.785
	(.0046)	(.0037)	(.0030)	(.0055)			
5	.02358	-.01124	.00937	-.01712	.7907	2.702	1.734
	(.0074)	(.0062)	(.0051)	(.0092)			

Table 7.5.3 (continued)

Long Run Effect Derived from Long Run Symmetry Restricted A(4) Model

	P1	P2	P3	P4	P5
1	-.46054	.01573	.10207	.17848	.16052
2	.01573	-.26069	.07478	.06412	.10607
3	.10207	.07478	-.05072	.00675	-.13287
4	.17848	.06412	.00675	-.14380	-.10552
5	.16052	.10607	-.13287	-.10552	-.03178

	μ	D1	D2	D3	D4
1	.39862	.00867	.01958	.03172	-.05620
2	.14668	.01668	-.00974	-.01821	.00737
3	.05045	-.01213	-.00436	.01445	.00172
4	.13903	-.00080	-.01365	.00879	.00113
4	.26526	-.01241	.00818	-.03675	.04598

CHAPTER 8

CONCLUSIONS

The conclusion of this thesis has three aspects: the first results from the empirical analysis of demand patterns in Korea, the second covers some implications of our empirical analysis on various economic hypothesis and econometric methods, and the third draws attention to areas for further research.

In this thesis, we have performed a demand analysis for Korea in the context of static and dynamic demand systems. The necessicity for a dynamic demand analysis on Korea arose from an intuition on household behaviour and from the estimation results on the static demand systems. In the application to quarterly household expenditure data, static demand systems exhibited serial correlation in the residuals even after the introduction of seasonal dummies. Moreover, they rejected both structural stability and parameter constancy hypotheses.

Two dynamic approaches were considered; one is based on stochastic dynamisation of the disturbance vector in the static system, and the other on the structural dynamisation of the demand system. The former approach was achieved in the context of static demand systems by introducing an autoregressive error process into the disturbance terms in the static Rotterdam model and the almost ideal demand system; while the later was achieved by the structural dynamic generalisation of the demand system. Two distinct views on dynamics were incorporated into the dynamic generalisation of the Rotterdam model; the dynamic equilibrium assumption and the dynamic disequilibrium assumption. A flexible dynamic demand system to represent the first view has been derived under the Rotterdam approximation. For a dynamic demand system embedding the second view, maximum likelihood estimation subject to the long run equilibrium conditions was applied using the Lyapunov equation solution.

In terms of empirical performance of the static demand systems, the Rotterdam model and the almost ideal demand system appeared superior to the linear expenditure system because of their flexibility. Furthermore, the Rotterdam model appeared superior to the almost ideal demand system in achieving compatability of demand theory and data. The parameter estimates of the demand systems were found to vary with the specification of the demand systems, the modelling of seasonal effects, and the level of commodity aggregation.

Perhaps the major point emerging from this study is that the prior information postulated by the economic demand theory is informative on (and consistent with) the sample information in the Korean demand data set. In the context of the Rotterdam model, the static demand restrictions were not rejected by the data on either the static model or the long run structure in the dynamic framework. Moreover, neither were the dynamic (short run) equilibrium demand restrictions rejected in the dynamic equilibrium context.

The structual stability of the estimated demand system appeared to be an important factor in terms of the success of the tests of the demand restrictions. In the (static) almost ideal demand system, the demand restrictions were accepted when applied to the subset of the sample exhibiting less significant structural changes, while they were rejected on the whole sample. However, neither the dynamic specification of stochastic structure of the static demand system nor the aggregation level of commodity groups had serious effects on the tests of the demand restrictions.

The direct tests and the separated induced tests produced identical inferences in testing the demand restrictions. The use of the separate induced test as a complement to direct χ^2 tests of demand restrictions is recommended as a means of identifying the source of any rejection.

Structural change in consumption patterns in Korea was identified in both static and dynamic demand systems. However, in the application of the dynamic demand system, trend effects; such as changing consumption patterns, lagged responses, habit forming patterns, and short run stock adjustment, were captured. The results are not counter-intuitive. The stock adjustment process was the predominant dynamic feature on a quarterly basis, while habit formation predominates on an annual basis (except for the housing sector). The long run demand characteristics derived from the dynamic demand system appeared to differ from the static and short run features.

From an analysis of the effects of misspecification on tests of the demand restrictions, it was shown that, in the situation when relevant explanatory variables are omitted, the asymptotic central χ^2 tests are invalid due to the inconsistency of the estimate of the residual covariance matrix. However, when the model is incorrectly overparameterized, the asymptotic central χ^2 tests remain valid, but may cause serious over-rejection of correct restrictions due to the loss in degrees of freedom.

In this study, the dynamic analysis was based on a flexible form of Marshallian demand equations, derived under the Rotterdam approximation. A potential area for future dynamic demand analysis is the area of true cost of living indices, incorporating the methodology of flexible functional systems from cost minimization; for example, using the AIDS model. The intertemporal approach could also be combined with the flexible dynamic demand systems approach used here. A dynamic demand system with varying coefficients may also be worthwhile for future study, if the economic rationale for the varying nature of parameters of a demand system is considered sufficiently strong.

The size of the available data set limited the scope of this study, the dynamic analysis was confined to a five commodity data set. Dynamic analysis for a more disaggregated data will be feasible when longer series become available.

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