

A Self-Calibrating Instrument for Measuring Conductance at Radio Frequencies

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Accuracies obtainable in measuring resistance (conductance) at radio frequencies have lagged behind those obtainable in measuring reactance (susceptance), because frequency-dependent changes in the values of resistors are not as well known as are such changes in capacitors, and because no satisfactory method has existed for comparing resistors to capacitors at radio frequencies. An instrument based on the twin T null circuit can be made self-calibrating at a given frequency, allowing conductances in the proper range to be measured directly in terms of a change in capacitance, without any need for accurate knowledge of the values of the circuit elements. Analysis of a practical circuit shows that such an instrument is potentially capable of measuring conductance to within a few parts in 10^6 at frequencies below about 15 MHz, and that accuracies actually obtained will be limited only by the accuracy to which changes in effective capacitance are known. Conductances between 0.01 and 0.0001 mho were measured at 10^7 radians/sec with an estimated error of 0.05 percent, which is the error associated with the measurement of changes in effective capacitance at that frequency.

1. Introduction

It has long been known that capacitors are superior to either inductors or resistors for use as impedance standards at high frequencies, because changes in their effective values with increasing frequency can be more accurately and more easily evaluated than can such changes in either resistors or inductors. For this reason, the High Frequency Impedance Group in the Electronic Calibration Center at NBS-Boulder, has decided to refer all its measurements of impedance at high frequencies to standards of capacitance [1].¹ While either null or resonance techniques can be used to compare capacitors and inductors to derived standards of capacitance, no satisfactory technique has existed for comparing resistors to capacitors at radio frequencies. The existing null techniques for measuring resistance require prior knowledge of the effective values of at least one resistor at the frequency at which the measurement is to be made. The resonance techniques, while theoretically capable of measuring resistance directly in terms of a change in capacitance, are at present capable of high accuracy only when measuring very high or very low values of resistance. Because of the lack of a satisfactory measurement technique, the accuracies obtainable in resistance measurements have lagged behind those obtainable in the measurement of reactance.

An investigation of the twin T null circuit has shown that it is possible to construct an instrument capable of measuring resistance directly in terms of a change of capacitance, without prior knowledge of the values of any of the circuit elements.

This paper describes an instrument which was constructed for this purpose, and presents results which indicate that resistances, in the proper range, can be measured to an accuracy which is limited only by the accuracy to which the change in capacitance is known.

2. Circuit Equations

2.1. Ideal Case (No Residual Impedances)

The circuit to be considered (fig. 1) is adapted from one which was described by Tuttle [2], and applied to a commercial instrument by Sinclair [3], in 1940. The circuit is essentially the one which Woods [4] used in his VHF Dual Admittance Bridge in 1950. Tuttle has shown

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¹ Figures in brackets indicate the literature references at the end of this paper.

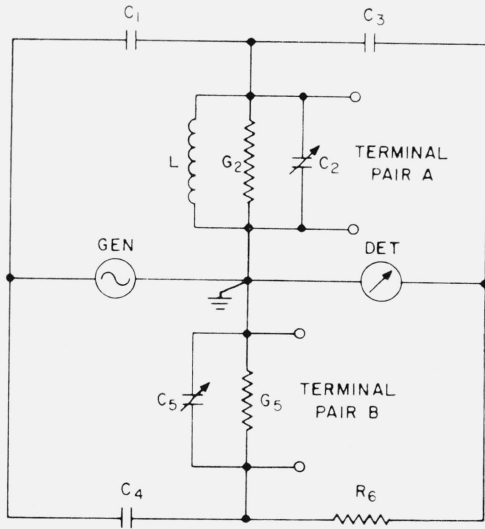


FIGURE 1. *Twin T circuit using pure components.*

that the condition for zero transmission through the twin T (null condition) is given by

$$Z_1 + Z_3 + \frac{Z_1 Z_3}{Z_2} + Z_4 + Z_6 + \frac{Z_4 Z_6}{Z_5} = 0. \quad (1)$$

In the present case, $Z_1 = \frac{1}{j\omega C_1}$, $Z_3 = \frac{1}{j\omega C_3}$, etc., and the real and imaginary parts of (1) give the two null equations:

$$C_1 + C_2 + C_3 + \frac{C_1 C_3}{C_4} (1 + R_6 G_5) = \frac{1}{\omega^2 L} \quad (2)$$

and

$$\omega^2 R_6 C_1 C_3 \left(1 + \frac{C_5}{C_4} \right) = G_2. \quad (3)$$

As in the Woods Bridge, a measurement can be made by connecting an unknown admittance $Y_x = G_x + j\omega C_x$, either to terminal-pair A , or to terminal-pair B . When Y_x is connected to terminal-pair A , C_2 must be replaced by $C_2' + C_x$, G_2 by $G_2' + G_x$, and C_5 by C_5' , in the null equations. Similar changes (indicated by double primes) must be made when Y_x is connected to terminal-pair B . Subtracting the initial null equations from the final ones yields the four expressions

$$C_{xA} = -\Delta C_2 \quad (4)$$

$$C_{xB} = -\Delta C_5' \quad (5)$$

$$G_{xA} = \omega^2 \left(R_6 \frac{C_1 C_3}{C_4} \right) \Delta C_5 \quad (6)$$

$$G_{xB} = \frac{-\Delta C_2'}{\left(R_6 \frac{C_1 C_3}{C_4} \right)}, \quad (7)$$

where

$$\Delta C_2 = C_2' - C_2 \quad \Delta C_2' = C_2'' - C_2$$

$$\Delta C_5 = C_5' - C_5 \quad \Delta C_5' = C_5'' - C_5,$$

and subscripts A and B denote the terminal-pair to which Y_x is connected.

If P is defined as

$$P \equiv R_6 \frac{C_1 C_3}{C_4},$$

then

$$G_{xA} = \omega^2 P \Delta C_5 \quad (8)$$

and

$$G_{xB} = -\frac{1}{P} \Delta C_2'. \quad (9)$$

If a conductance is measured first on terminal-pair A , then on terminal-pair B ,

$$G_{xA} = G_{xB}$$

and

$$\omega^2 P \Delta C_5 = -\frac{1}{P} \Delta C_2'$$

$$P^2 = -\frac{1}{\omega^2} \frac{\Delta C_2'}{\Delta C_5}.$$

If P is adjustable so that it can be made equal to $\frac{1}{\omega}$,

$$\frac{1}{\omega^2} = -\frac{1}{\omega^2} \frac{\Delta C_2'}{\Delta C_5},$$

and

$$\Delta C_5 = -\Delta C_2'.$$

If P is adjusted to make the same change in capacitance necessary to null the instrument when G_x is measured on terminal-pair A , as when it is measured on terminal-pair B , then $P = \frac{1}{\omega}$, and

$$G_{xA} = \omega \Delta C_5 \quad (10)$$

$$G_{xB} = -\omega \Delta C_2'. \quad (11)$$

While P has not been eliminated from the calculation of G_x , it is possible to evaluate it with great accuracy, allowing conductance to be measured with accuracies limited only by the accuracy to which a change in capacitance is known.

It should be mentioned that a conductance may be measured in terms of a change in capacitance without making P equal to $1/\omega$. If $G_{xA} = G_{xB} = G_x$, multiplying eq (8) by eq (9) yields the expression

$$G_x^2 = -\omega^2 \Delta C_2' \Delta C_5,$$

and P is eliminated from the calculation of G_x . Because the number of measurements needed to determine G_x is doubled, and because accuracy decreases as the value of P departs from $1/\omega$, this method is less effective than the other, especially when more than a few measurements are to be made at a given frequency.

2.2. Realistic Circuit (Including Residual Impedances)

In general, residual impedances cause the effective values of the circuit elements to change with frequency and cause losses in the fixed capacitors, variable losses in the variable capacitors, and some reactance to be associated with the fixed resistors.

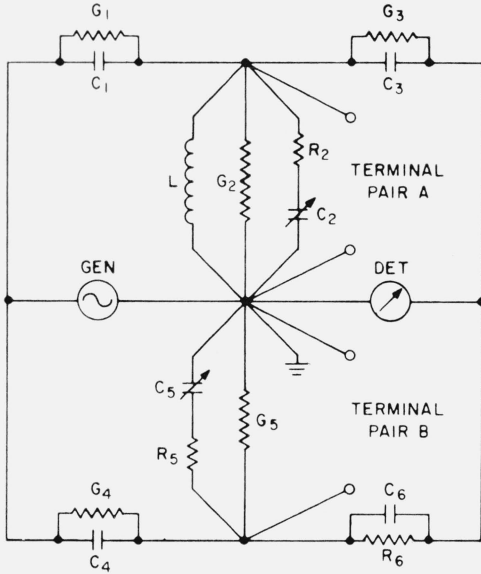


FIGURE 2. *Twin T circuit using impure components.*

The circuit of figure 2 is a more realistic representation of the actual circuit. In the figure, the value of each circuit element is taken as its effective value at the frequency at which the instrument is used. If R_2 is lumped with G_2 for the present, and R_5 with G_5 , the null equations for the circuit of figure 2 are

$$G_1 G_3 \left(G_4 + G_5 + \frac{1}{R_6} \right) - \omega^2 (C_1 G_3 + C_3 G_1) (C_4 + C_5 + C_6) - \omega^2 C_1 C_3 \left(G_4 + G_5 + \frac{1}{R_6} \right) + \frac{G_4}{R_6} (G_1 + G_2 + G_3) - \omega^2 \left(\frac{C_4}{R_6} + C_6 G_4 \right) \left(C_1 - \frac{1}{\omega^2 L} + C_2 + C_3 \right) - \omega^2 C_4 C_6 (G_1 + G_2 + G_3) = 0, \quad (12)$$

and

$$G_1 G_3 (C_4 + C_5 + C_6) + (C_1 G_3 + C_3 G_1) \left(G_4 + G_5 + \frac{1}{R_6} \right) - \omega^2 C_1 C_3 (C_4 + C_5 + C_6) + \frac{G_4}{R_6} \left(C_1 - \frac{1}{\omega^2 L} + C_2 + C_3 \right) + \left(\frac{C_4}{R_6} + C_6 G_4 \right) (G_1 + G_2 + G_3) - \omega^2 C_4 C_6 \left(C_1 - \frac{1}{\omega^2 L} + C_2 + C_3 \right) = 0. \quad (13)$$

In the appendix, these equations are reduced to give the four expressions

$$G_{xA} \approx \omega^2 \left(R_6 \frac{C_1 C_3}{C_4} \right) \Delta C_5 - \omega^2 (R_4 C_4 - R_6 C_6) (\Delta C_2 + C_{xA}) - \Delta G_2, \quad (14)$$

$$G_{xB} \approx \frac{-\Delta C_2'}{\left(R_6 \frac{C_1 C_3}{C_4} \right)} [1 - \omega^2 R_2 (C_2 + C_2') (R_4 C_4 - R_6 C_6)] - \omega^2 (R_1 C_1 + R_3 C_3) (\Delta C_5' + C_{xB}) - \Delta G_5, \quad (15)$$

$$C_{xA} \approx -\Delta C_2 [1 - \omega^2 R_2 (C_2 + C_2') (R_4 C_4 - R_6 C_6)] - [R_1 C_1 + R_3 C_3 - R_4 C_4 + R_6 C_6 + R_5 (C_5 + C_5')] G_{xA}, \quad (16)$$

and

$$C_{xB} \approx -\Delta C_5' + [R_1 C_1 + R_3 C_3 - R_4 C_4 + R_6 C_6 - R_2 (C_2 + C_2')] G_{xB}. \quad (17)$$

For the instrument whose construction is described later, these equations are accurate to within a few parts in 10^6 . If the instrument is to be self-calibrating for conductance

measurements, eqs (14) and (15) must be reduced to the form

$$G_{xA} + \Delta G_2 = \omega^2 P \Delta C_5,$$

and

$$G_{xB} + \Delta G_5 = \frac{-\Delta C'_2}{P}.$$

In order to accomplish this, define

$$K \equiv R_1 C_1 + R_3 C_3 - R_4 C_4 + R_6 C_6. \quad (18)$$

Equations (16) and (17) then become

$$C_{xA} \approx -\Delta C_2 [1 - \omega^2 R_2 (C_2 + C'_2) (R_4 C_4 - R_6 C_6)] - [K + R_5 (C_5 + C'_5)] G_{xA} \quad (19)$$

and

$$C_{xB} \approx -\Delta C'_5 + [K - R_2 (C_2 + C'_2)] G_{xB}. \quad (20)$$

When measuring a nearly pure conductance, sufficient accuracy is obtained for C_{xA} if the term $\omega^2 R_2 (C_2 + C'_2) (R_4 C_4 - R_6 C_6)$ is neglected. Then (19) becomes

$$C_{xA} \approx -\Delta C_2 - [K + R_5 (C_5 + C'_5)] G_{xA}. \quad (21)$$

If a nearly pure conductance is measured on terminal-pair A , then on terminal-pair B , using the same capacitor operated over an identical range for both measurements, then

$$G_{xA} = G_{xB} = G_x$$

and

$$R_5 (C_5 + C'_5) = R_2 (C_2 + C'_2) = R (C + C').$$

Since $C_{xA} = C_{xB}$,

$$-\Delta C_2 - [K + R (C + C')] G_x = -\Delta C'_5 + [K - R (C + C')] G_x$$

from which is obtained

$$K = \frac{\Delta C'_5 - \Delta C_2}{2G_x}, \quad (22)$$

which can be evaluated.

Now, $R_1 C_1$, $R_3 C_3$, and $R_4 C_4$ are likely to be at least an order of magnitude smaller than $R_6 C_6$. When this is true, $K \approx R_6 C_6$,

$$C_{xA} + \Delta C_2 \approx -[K + R_5 (C_5 + C'_5)] G_{xA},$$

and

$$C_{xB} + \Delta C'_5 \approx [K - R_2 (C_2 + C'_2)] G_{xB}.$$

Equations (14) and (15) can now be rewritten

$$G_{xA} \approx \omega^2 P \Delta C_5 - \omega^2 K [K + R_5 (C_5 + C'_5)] G_{xA} - \Delta G_2$$

and

$$G_{xB} \approx \frac{-\Delta C'_2}{P} [1 + \omega^2 K R_2 (C_2 + C'_2)] - \omega^2 (R_1 C_1 + R_3 C_3) [K - R_2 (C_2 + C'_2)] G_{xB} - \Delta G_5.$$

If the terms $\omega^2 K [K + R_5 (C_5 + C'_5)]$, $\omega^2 K R_2 (C_2 + C'_2)$, and $\omega^2 (R_1 C_1 + R_3 C_3) [K - R_2 (C_2 + C'_2)]$ are insignificant compared to unity, the expressions for the conductance reduce to the desired form.

If $\Delta G_2 = \Delta G_5$, or if both are insignificant compared to G_x , P can be determined as it was in the ideal case. It is possible to insure that $\Delta C_2 = \Delta C_5$, and that $\Delta G_2 = \Delta G_5$ in the two measure-

ments, by using the same variable capacitors operated over an identical range to measure G_x and C_x in both cases. This requires both G_x and the capacitors which measure G_x and C_x to be switched from terminal to terminal.

It is possible to adjust P to within the limits of stability and resettability of the capacitor used to measure G_x . In a good two-terminal air capacitor equipped with a precision coaxial connector, the error is no more than a few parts in 10^5 of the maximum capacitance. Since changes in effective capacitance are not known to better than several parts in 10^4 at present, adjusting P in this way allows one to measure conductance to the same accuracy as that to which a change in effective capacitance is known.

With the constants of the instrument known, it is only necessary to evaluate the residual impedances associated with the external variable capacitors, perhaps by the method of Field and Sinclair [5], in order to make quite good measurements of either conductance or reactance.

3. Construction of the Instrument

In converting the twin T circuit to a working instrument, several facts must be considered. Since it is necessary to move the standard capacitors from one pair of terminals to the other when evaluating the constants of the instrument, the standard capacitors cannot be made an integral part of the instrument. If the capacitors are not to be built in, more than one pair of terminals must be provided to allow paralleling the unknown admittance and the capacitor which measures C_x . Multiple terminals are also desirable because they simplify calibration of the variable capacitors used with the instrument. If multiple terminals are to be used, it is desirable for each pair to connect to a common point, the "bridge corner," and for the conductors to be mutually perpendicular in order to reduce mutual impedances as much as possible. The fact that it is possible to connect external variable capacitors of different ranges for C_2 and C_5 requires changing the values of L and G_2 necessary to provide initial balance. Since multiple terminals are already necessary, it is just as well to provide for the attachment of external standards for L and G_2 . As a result, the actual instrument becomes very simple. It is only necessary to provide for mounting the four fixed elements, the series elements of the T , in a base to which are attached the two sets of multiple terminals.

As actually constructed, the instrument takes the form shown in figures 3, 5, and 6. The four series elements are mounted in holes bored parallel to the four sides of a 2 in. by 6 in. brass block. The holes are extended through the edges of the block to provide access to the components from one end, and to form the outer conductor of a short section of 3/4 in. coaxial line on the other. Holes bored through diagonally opposite corners from face to face provide the outer conductors for two more short sections of line. When center conductors are added, six short sections of coaxial line, whose center conductors meet in a point, are formed at two diagonally opposite corners of the block. Two of these line sections connect to the series components, and the other four are equipped with NBS-Woods [6] precision coaxial connectors to provide the bridge terminals. The junction point of the six line sections forms the "bridge corner."

Capacitors C_1 and C_3 are coaxial capacitors with nominal values of 10 pF. C_4 is a parallel plate capacitor whose nominal value is 1 pF. R_6 is a commercial metal film resistor whose value is selected to give the proper value of P at the frequency at which the instrument is to be used.

Since it is necessary to adjust P , one or more of the series elements must be adjustable. C_4 is provided with two adjustments. Each plate has one half its surface cut about $\frac{1}{32}$ in. lower than the other half, so that rotating one plate with respect to the other causes a change of about 10 percent in its capacitance. Final adjustment of P is accomplished by means of a trimming screw which extends into the space between the two plates. Gross changes in P can be accomplished by replacing the resistor R_6 with another of different value.

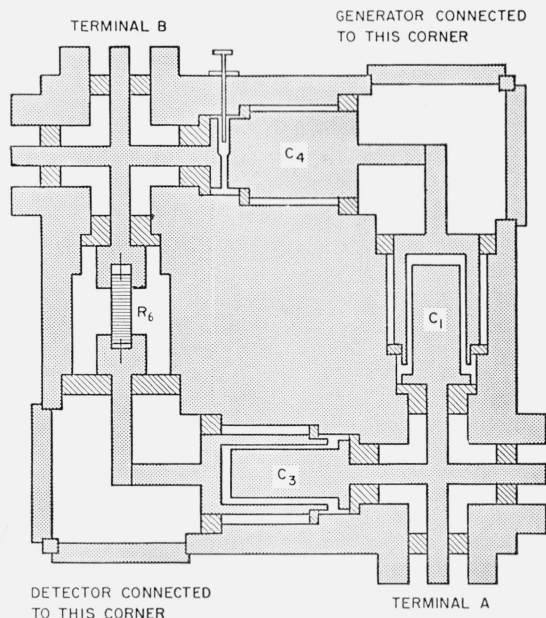


FIGURE 3. Cross section through the brass block showing construction of the instrument.

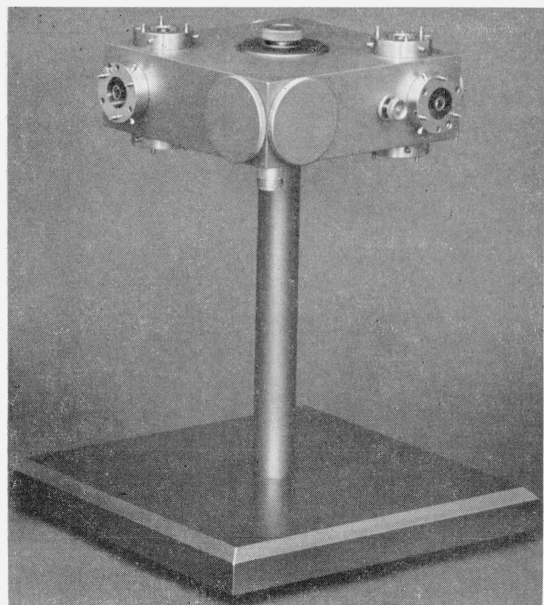


FIGURE 5. The basic instrument showing connector details and C_4 trimmer.

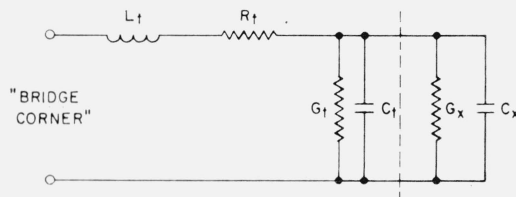


FIGURE 4. Equivalent circuit used in calculating the effect of terminal impedances on measured Y_x .

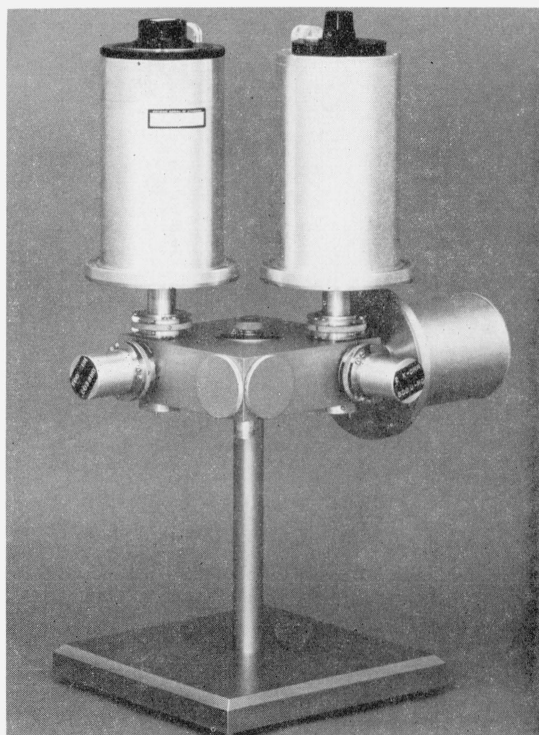


FIGURE 6. The basic instrument with standards attached for measuring a 200-ohm resistor.

4. Experimental Results

P was adjusted to equal $\frac{1}{\omega}$, at $\omega = 10^7$ radians/sec, by measuring a 1000 Ω resistor using a 10 to 110 pF variable standard capacitor. This resistor, as were those used later, is a high quality commercial metal-film resistor mounted in a shielding can which is equipped with a NBS-Woods Precision Coaxial Connector. The three standard capacitors used are equipped with similar connectors.

When P had been adjusted to the point that the capacitance change on one terminal was within a few parts in 10^5 of that on the other, over an identical capacitance range, P was considered to be adjusted to within a few parts in 10^5 . The adjustment was checked by measuring a 2000 Ω resistor using the same capacitor, a 20,000 Ω resistor using a 7 to 17 pF capacitor, and 100, 200, and 500 Ω resistors using a 100 to 1100 pF capacitor. The results are summarized in table 1. Since the error in resetting the capacitors is a few parts in 10^5 of their maximum capacitance, the adjustment of P is probably correct to within a few parts in 10^5 .

With the values obtained for ΔC_5 , or $\Delta C_2'$, in checking the adjustment of P , and knowing that P is $10^{-7} \pm$ a few parts in 10^5 , it is possible to calculate the effective conductance of each of the resistors at 10^7 radians/sec. The results of the calculations appear in table 2. The measurements of C_x obtained in checking the adjustment of P may be used to calculate a value for K . The results of the calculations appear in table 3.

TABLE 1. Verification of the adjustment of P

Resistors measured	Terminal used	Capacitor used	ΔC	% Diff. $= \left(\frac{\Delta C}{\Delta C_2} - 1 \right) 100$
Ω		pF	pF	
100	A	100-1100	999.40	+0.003
100	B	100-1100	999.37	
200	A	100-1100	499.00	+0.012
200	B	100-1100	498.94	
500	A	100-1100	192.92	.000
500	B	100-1100	192.92	
1000	A	10-110	100.050	-0.001
1000	B	10-110	100.051	
2000	A	10-110	50.206	-0.004
2000	B	10-110	50.208	
20,000	A	7-17	4.9994	+0.006
20,000	B	7-17	4.9991	
20,000	A	7-17	4.9976	-0.030

Percentage error in P is $\frac{1}{2} \times (\% \text{ diff.})$.

TABLE 2. Comparison of measured resistance to $d-c$ resistance

ΔC	$G = \omega \Delta C$	$R_f = \frac{1}{G}$	R_{dc}	% diff. $= \left(\frac{R_f}{R_{dc}} - 1 \right) 100$
pF	Mho			
1000.44	1.00044×10^{-2}	99.956	99.971	-0.015
499.71	4.9971×10^{-3}	200.12	200.155	-0.018
199.84	1.9984×10^{-3}	500.40	500.428	-0.006
100.031	1.00031×10^{-3}	999.69	999.853	-0.016
50.029	5.0029×10^{-4}	1998.8	1999.52	-0.036
5.0180	5.0180×10^{-5}	19928	19998.4	-0.353

TABLE 3. Calculation of K from measured capacitance

G_x	ΔC_5	ΔC_2	$K = \frac{\Delta C_5 - \Delta C_2}{2G_x}$
Mho	pF	pF	
0.01	3.503	-5.942	4.72×10^{-10}
.005	-0.125	-4.821	4.70×10^{-10}
.002	-2.440	-4.308	4.67×10^{-10}
.001	-2.3145	-3.2559	4.71×10^{-10}
.0005	-2.4132	-2.8826	4.69×10^{-10}

The admittance measured by the instrument is the effective admittance at the "bridge corners." In order to calculate Y_x accurately, it is necessary to know the effect of the residual impedances between the terminal and the "bridge corner." The equivalent circuit of figure 4 is a reasonable representation of the residual impedances associated with the terminal.

When the instrument is nulled initially with the terminal open, the effective admittance at the "bridge corner" is given by

$$Y_e = \frac{G_t(1 + R_t G_t) + \omega^2 R_t C_t^2 + j\omega[C_t - L_t G_t^2 - \omega^2 L_t C_t^2]}{(1 + R_t G_t - \omega^2 L_t C_t)^2 + \omega^2(R_t C_t + L_t G_t)^2}$$

For the purpose of estimating the magnitudes of the residual impedances, the terminal can be considered to be $\frac{1}{8}$ in. of 50 Ω , $\frac{3}{4}$ in., brass air line. Since the series resistance and inductance in similar sections of line have been determined to be 0.8 milliohm/in. and 4.4×10^{-9} H/in. at 10^7 radians/sec, R_t is about 1.1 milliohms, and L_t about 6.2×10^{-9} H. C_t is about 2.5 pF, and G_t is less than 1 μ mho. Using these values for the residual impedances, the equation can be reduced to

$$Y_e \approx G_t + j\omega C_t, \text{ at } \omega = 10^7 \text{ radians/sec.}$$

When an unknown admittance, $Y_x = G_x + j\omega C_x$, is connected to the terminal, the effective admittance at the "bridge corner" is given by

$$Y'_e = \frac{(G_t + G_x)[1 + R_t(G_t + G_x)] + \omega^2 R_t(C_t + C_x)^2 + j\omega[C_t + C_x - L_t(G_t + G_x)^2 - \omega^2 L_t(C_t + C_x)^2]}{[1 + R_t(G_t + G_x) - \omega^2 L_t(C_t + C_x)]^2 + \omega^2[R_t(C_t + C_x) + L_t(G_t + G_x)]^2}$$

For $G_x \leq 10^{-2}$ and $C_x < 10^{-11}$, this reduces to

$$Y'_e \approx G_t + G_x + j\omega(C_t + C_x - L_t G_x^2)$$

and

$$\Delta Y_e = Y'_e - Y_e \approx G_x + j\omega(C_x - L_t G_x^2).$$

The error in the measured conductance is insignificant, and the measured capacitance is in error by the term $(-L_t G_x^2)$. Before the reactance associated with a nearly pure conductance can be accurately measured, it is necessary to evaluate this inductance, as well as the series resistance associated with the capacitor used to measure the conductance. It should be mentioned that these residual impedances cause no error in evaluating the constants of the instrument if care is taken to make all the terminals as nearly identical as possible.

5. Discussion

The measurements of P , which are summarized in table 1, indicate that it is possible to adjust P to within a few parts in 10^5 of $1/\omega$. The limit on the accuracy of adjustment of P is the limit of stability and resettability of the standard capacitors used in making the adjustment.

Comparison of resistance measured at 10^7 radians/sec with that measured at zero frequency (table 2) indicates that the instrument is capable of measuring resistance to within a few hundredths of a percent. The very close agreement between rf and d-c values must be considered fortuitous at present because changes in capacitance are not known to much better than ± 0.05 percent.

The values calculated for K (table 3) have a scatter of less than 1 percent of K . This indicates that it will be possible to make quite good measurements of the reactance associated with a nearly pure conductance. Apparently, the accuracy of such measurements will be limited by the accuracy to which the series resistance associated with the standard capacitors and the series inductance associated with the terminal can be determined.

While the instrument has been proved only at a frequency of 10^7 radians/sec, there is no apparent reason why the same principle can not be used at much higher, or at much lower, frequencies. The main deficiency of the instrument lies in the range of conductance which can be measured accurately. At present the range is limited, by the relationship $G_x = \omega \Delta C$, to two or three decades of conductance by the range of capacitance changes which is easily obtainable. Since the magnitude of the conductance measurable by a given capacitor depends on the frequency, the range of conductances measurable by the instrument is frequency dependent. The deficiency need not be too serious if it proves feasible to use the instrument to measure conductances for use as standards when calibrating other instruments capable of covering a greater conductance range.

The instrument has demonstrated its ability to make conductance measurements limited in accuracy only by the accuracy to which changes in capacitance are known. This being the case, it is possible to immediately realize an order of magnitude increase in the accuracy of those conductance measurements which are within the range of the instrument. Of greater significance is the fact that the instrument demonstrates a principle which makes it possible to refer resistance measurements directly to standards of capacitance, thus placing the measurement of resistance and reactance on a common basis.

6. Appendix

The null equations for the circuit of figure 2 are

$$G_1 G_3 \left(G_4 + G_5 + \frac{1}{R_6} \right) - \omega^2 (C_1 G_3 + C_3 G_1) (C_4 + C_5 + C_6) - \omega^2 C_1 C_3 \left(G_4 + G_5 + \frac{1}{R_6} \right) \\ + \frac{G_4}{R_6} (G_1 + G_2 + G_3) - \omega^2 \left(\frac{C_4}{R_6} + C_6 G_4 \right) \left(C_1 - \frac{1}{\omega^2 L} + C_2 + C_3 \right) - \omega^2 C_4 C_6 (G_1 + G_2 + G_3) = 0, \quad (A1)$$

and

$$G_1 G_3 (C_4 + C_5 + C_6) + (C_1 G_3 + C_3 G_1) \left(G_4 + G_5 + \frac{1}{R_6} \right) - \omega^2 C_1 C_3 (C_4 + C_5 + C_6) \\ + \frac{G_4}{R_6} \left(C_1 - \frac{1}{\omega^2 L} + C_2 + C_3 \right) + \left(\frac{C_4}{R_6} + C_6 G_4 \right) (G_1 + G_2 + G_3) - \omega^2 C_4 C_6 \left(C_1 - \frac{1}{\omega^2 L} + C_2 + C_3 \right) = 0. \quad (\text{A2})$$

A measurement can be made by connecting an unknown admittance, $Y_x = G_x + j\omega C_x$, either to terminal-pair A , or to terminal-pair B . When Y_x is connected to terminal-pair A , C_2 must be replaced by $C'_2 + C_x$, G_2 by $G'_2 + G_x$, G_5 by G'_5 , and C_5 by C'_5 in the null equations. Similar changes must be made when the unknown admittance is connected to terminal-pair B . If $\Delta C_2 = C'_2 - C_2$, $\Delta G_2 = G'_2 - G_2$, $\Delta C_5 = C'_5 - C_5$, and $\Delta G_5 = G'_5 - G_5$, and subscripts A and B denote the terminal-pair to which the unknown admittance is connected, subtracting the initial null equations from the final ones yields the four expressions

$$C_{xA} \left[1 + \frac{R_6 G_4 C_6}{C_4} \right] = - \left[1 + \frac{R_6 G_4 C_6}{C_4} \right] \Delta C_2 + \left[\frac{R_6 G_1 G_3}{\omega^2 C_4} - \frac{R_6}{C_4} C_1 C_3 \right] \Delta G_5 \\ + \left[\frac{G_4}{\omega^2 C_4} - R_6 C_6 \right] (\Delta G_2 + G_x) - \frac{R_6}{C_4} (C_1 G_3 + C_3 G_1) \Delta C_5, \quad (\text{A3})$$

$$C_{xB} \left[\frac{\omega^2 R_6 C_1 C_3}{C_4} - \frac{R_6 G_1 G_3}{C_4} \right] = - \left[\frac{\omega^2 R_6 C_1 C_3}{C_4} - \frac{R_6 G_1 G_3}{C_4} \right] \Delta C_5 + \left[1 + \frac{R_6 G_4 C_6}{C_4} \right] \Delta G_2 \\ + \frac{R_6}{C_4} (C_1 G_3 + C_3 G_1) (\Delta G_5 + G_x) + \left(\frac{G_4}{C_4} - \omega^2 R_6 C_6 \right) \Delta C_2, \quad (\text{A4})$$

$$G_{xA} \left[1 + \frac{R_6 G_4 C_6}{C_4} \right] = \left[\frac{\omega^2 R_6 C_1 C_3}{C_4} - \frac{R_6 G_1 G_3}{C_4} \right] \Delta C_5 + \left[\omega^2 R_6 C_6 - \frac{G_4}{C_4} \right] (\Delta C_2 + C_x) \\ - \frac{R_6}{C_4} (C_1 G_3 + C_3 G_1) \Delta G_5 - \left(1 + \frac{R_6 G_4 C_6}{C_4} \right) \Delta G_2, \quad (\text{A5})$$

and

$$G_{xB} \left[\frac{R_6 C_1 C_3}{C_4} - \frac{R_6 G_1 G_3}{\omega^2 C_4} \right] = - \left(1 + \frac{R_6 G_4 C_6}{C_4} \right) \Delta C_2 - \frac{R_6}{C_4} (C_1 G_3 + C_3 G_1) (\Delta C_5 + C_x) \\ + \left[\frac{R_6 G_1 G_3}{\omega^2 C_4} - \frac{R_6 C_1 C_3}{C_4} \right] \Delta G_5 + \left[\frac{G_4}{\omega^2 C_4} - R_6 C_6 \right] \Delta G_2. \quad (\text{A6})$$

If C_1 , C_3 , and C_4 are good three-terminal capacitors, their losses will be due entirely to series resistance, in which case,

$$G_1 \approx \omega^2 R_1 C_1^2, \quad G_3 \approx \omega^2 R_3 C_3^2,$$

and

$$G_4 \approx \omega^2 R_4 C_4^2.$$

If C_2 and C_5 are good two-terminal capacitors, changes in their conductance will be due entirely to series resistance, and

$$\Delta G_2 \approx \omega^2 R_2 (C_2'^2 - C_2^2) = \omega^2 R_2 (C_2 + C_2') \Delta C_2,$$

$$\Delta G_5 \approx \omega^2 R_5 (C_5 + C_5') \Delta C_5.$$

In the instrument described,

$$C_1 \approx C_3 \approx 10 \text{ pF}$$

$$C_4 \approx 1 \text{ pF}$$

$$C_2 \leq 1000 \text{ pF}$$

$$C_5 \leq 1000 \text{ pF}$$

$$R_6 \approx 1000\Omega \text{ at } 10^7 \text{ radians/sec}$$

and

$$R_6 \approx 100\Omega \text{ at } 10^8 \text{ radians/sec.}$$

It is not unreasonable to assume $R_1 \approx R_2 \approx R_3 \approx R_4 \approx R_5 < 0.1\Omega$ and

$$C_6 \approx 1 \text{ pF.}$$

The null equations can then be reduced by using the following approximations:

$$\left[1 + \frac{G_4 R_6 C_6}{C_4}\right] \approx \left[1 + \frac{\omega^2 R_4 C_4^2 R_6 C_6}{C_4}\right] \approx 1 + \omega^2 R_4 R_6 C_4 C_6 \approx 1, \quad (\text{a})$$

$$\left[\frac{R_6 G_1 G_3}{\omega^2 C_4} - \frac{R_6 C_1 C_3}{C_4}\right] \approx \frac{R_6 C_1 C_3}{C_4} [\omega^2 R_1 R_3 C_1 C_3 - 1] \approx -\frac{R_6 C_1 C_3}{C_4}, \quad (\text{b})$$

$$\left[\frac{\omega^2 R_6 C_1 C_3}{C_4} - \frac{R_6 G_1 G_3}{C_4}\right] = \omega^2 \left[\frac{R_6 C_1 C_3}{C_4} - \frac{R_6 G_1 G_3}{\omega^2 C_4}\right] \approx \omega^2 \frac{R_6 C_1 C_3}{C_4}, \quad (\text{c})$$

$$\left[\frac{G_4}{\omega^2 C_4} - R_6 C_6\right] \approx \left[\frac{\omega^2 R_4 C_4^2}{\omega^2 C_4} - R_6 C_6\right] \approx R_4 C_4 - R_6 C_6, \quad (\text{d})$$

$$\left[\frac{G_4}{C_4} - \omega^2 R_6 C_6\right] = \omega^2 \left[\frac{G_4}{\omega^2 C_4} - R_6 C_6\right] \approx \omega^2 [R_4 C_4 - R_6 C_6], \quad (\text{e})$$

$$[C_1 G_3 + C_3 G_1] \approx [\omega^2 R_3 C_3^2 C_1 + \omega^2 R_1 C_1^2 C_3] \approx \omega^2 C_1 C_3 (R_1 C_1 + R_3 C_3). \quad (\text{f})$$

Using expressions (a), (b), (d), and (f), eq (A3) is reduced to

$$C_{xA} \approx -\Delta C_2 - \frac{R_6 C_1 C_3}{C_4} \Delta G_5 + (R_4 C_4 - R_6 C_6) (\Delta G_2 + G_{xA}) - \frac{\omega^2 R_6 C_1 C_3}{C_4} (R_1 C_1 + R_3 C_3) \Delta C_5.$$

Since

$$\Delta G_5 \approx \omega^2 R_5 (C_5 + C'_5) \Delta C_5,$$

and

$$\Delta G_2 \approx \omega^2 R_2 (C_2 + C'_2) \Delta C_2,$$

$$\omega^2 R_6 \frac{C_1 C_3}{C_4} \Delta C_5 \approx G_{xA},$$

$$C_{xA} \approx -\Delta C_2 [1 - \omega^2 R_2 (C_2 + C'_2) (R_4 C_4 - R_6 C_6)] - [R_1 C_1 + R_3 C_3 - R_4 C_4 + R_6 C_6 + R_5 (C_5 + C'_5)] G_{xA}, \quad (\text{A7})$$

which is eq (16).

Using (a), (c), (e), and (f), eq (A4) is reduced to

$$C_{xB} \approx -\Delta C_5 + (R_1 C_1 + R_3 C_3) (\Delta G_5 + G_{xB}) + \frac{\Delta C_2 (R_4 C_4 - R_6 C_6)}{\left(R_6 \frac{C_1 C_3}{C_4}\right)} + \frac{\Delta G_2}{\omega^2 \left(R_6 \frac{C_1 C_3}{C_4}\right)}.$$

Since

$$\frac{\Delta C_2}{R_6 \frac{C_1 C_3}{C_4}} \approx -G_{xB},$$

and

$$\omega^2 (R_1 C_1 + R_3 C_3) R_5 (C_5 + C'_5) \ll \ll 1,$$

this can be further reduced to

$$C_{xB} \approx -\Delta C_5 + [R_1 C_1 + R_3 C_3 - R_4 C_4 + R_6 C_6 - R_2(C_2 + C_2')] G_{xB}, \quad (\text{A8})$$

which is eq (17).

Using (a), (c), (e), and (f), eq (A5) is reduced to

$$G_{xA} \approx \frac{\omega^2 R_6 C_1 C_3}{C_4} \Delta C_5 - \omega^2 (R_4 C_4 - R_6 C_6) (\Delta C_2 + C_{xA}) - \frac{\omega^2 R_6 C_1 C_3}{C_4} (R_1 C_1 + R_3 C_3) \Delta G_5 - \Delta G_2.$$

Since

$$\Delta G_5 \approx \omega^2 R_5 (C_5 + C_5') \Delta C_5$$

and

$$\omega^2 (R_1 C_1 + R_3 C_3) R_5 (C_5 + C_5') \ll 1,$$

this reduces to

$$G_{xA} \approx \frac{\omega^2 R_6 C_1 C_3}{C_4} \Delta C_5 - \omega^2 (R_4 C_4 - R_6 C_6) (\Delta C_2 + C_{xA}) - \Delta G_2 \quad (\text{A9})$$

which is eq (14).

Using expressions (a), (b), (d), and (f), eq (A6) is reduced to

$$G_{xB} \approx \frac{-\Delta C_2}{R_6 \frac{C_1 C_3}{C_4}} - \omega^2 (R_1 C_1 + R_3 C_3) (\Delta C_5 + C_{xB}) - \Delta G_5 + \frac{(R_4 C_4 - R_6 C_6) \Delta G_2}{R_6 \frac{C_1 C_3}{C_4}}.$$

Since

$$\Delta G_2 \approx \omega^2 R_2 (C_2 + C_2') \Delta C_2,$$

$$G_{xB} \approx -\frac{\Delta C_2}{R_6 \frac{C_1 C_3}{C_4}} [1 - \omega^2 R_2 (C_2 + C_2') (R_4 C_4 - R_6 C_6)] - \omega^2 (R_1 C_1 + R_3 C_3) (\Delta C_5 + C_{xB}) - \Delta G_5, \quad (\text{A10})$$

eq (15). Equations (A7), (A8), (A9), and (A10) should be accurate to within a few parts in 10^6 at frequencies up to 10^8 radians/sec for the instrument described.

7. References

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