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THE MAGNETIC STATE OF THE EARTH AT EPOCH 1885.0

Adolf Schmidt

Gotha

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THE MAGNETIC STATE OF THE EARTH AT EPOCH 1885.0

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The general theoretical developments on which the investigations presented here are based, were presented in an earlier issue of this journal (XII, 2, 1889). A report of the most important results provided by the application of these developments to the state of the earth's magnetism in the year 1885 has also been published (Abhandlungen der k. bayer. Akademie der Wissensch., II.Kl., XIX Vol., I Abth) On the following pages the fundamentals and the results of this investigation will be presented in greater detail and in addition, the calculations themselves will be presented. This is done with the intent of creating a reliable and comfortable basis for future investigations of a similar nature, but not because the results themselves might have a conclusive significance. These results doubtless require significant improvement and it has long been my intention to perform a definitive recalculation for this period of time as soon as the needed information becomes available. The presentation here is based entirely on the values of the earth's magnetic force components at 1800 points of the earth's surface derived by Dr. Neumayer. The observations on which these values are based, extend back to about 1887; the vast majority come from the time before 1885, to which the derived values presented in the earth atlas of magnetism pertain. The determination of these latter values had to be by extrapolation in most cases; this necessarily reduces their validity. This circumstance was unavoidable since the atlas naturally was to provide a representation for a short time segment. But in addition to this problem, there is the deficiency of the observation material for wide regions, as Dr. Neumayer discussed in detail in the notes on his atlas. It is clear that progress has been achieved in two ways through the incorporation of recent observations--which it is hoped will be expanded considerably in coming years. The scope of valuable material has increased, both in uniformity of geographic distribution, and the use of observations symmetrical to the normal epoch increases the reliability of application to the earth. If these considerations should make a future repitition of the present calculations seem expedient, then in addition it should be noted that observations made above 60° N-latitute have been entirely excluded.

In spite of the described, generally unavoidable deficiencies, it is hoped that the reporting of provisional results will not be thought unjustified, not only because of the simplification this means for a final working, but also because the anticipated observations from the South Polar regions will have to be delayed for several years, and also because no signficantly better results are likely to be available for some time.

The two papers mentioned above--which will be referenced below as A and B, contain such a detailed presentation of everything not relating exclusively to the performance of the numerical calculation, that I can limit this discussion almost entirely to an exposition of these calculations. Thus, rep.titions have been prevented, except where absolutely necessary for the cohesion of this presentation.

Survey of Mathematic Aids in the Expansion

The empirical basis of the entire investigation is formed by the maps of the geomagnetic elements H, δ, i constructed by Dr. Neumayer for the beginning of the year 1885, or rather, by the values of these quantities taken by him for 1800 points where the meridians of 0°, 5°, 10°....355° East longitude from Greenwich and the parallel circles of 0°, 5°, 10°....60° North and Sough geographic latitude intersect. The mentioned, detailed text of the atlas of geomagnetism (the 4th part of Berghaus' Physical Atlas) provides information about the materials used in construction of the maps and about the applied methods of map-making; it is thus unnecessary to discuss these matters any further here.

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From the values of the elements H, δ, i, those of the components X, Y, Z were derived. These latter are presented in table III. 'X' means North, 'Y' means East, 'Z' means the downward, positive-measured component of force, so that the arrangement of positive semi-axes agrees with the present standard. The unit of measure used here and in all numbers in the present report, is $0.1^5\ cm^{-\frac{1}{2}}g^{\frac{1}{2}}s^{-1}$, i.e. the unit of the last place which still has some relevance in variation observations. Prof. Eschenhagen suggested the designation γ as a remembrance of Gauss (see Terrestrial Magnetism, Vol. I, p. 57, note 2). I will use this designation hereafter.

The values of the components were then presented on each latitude by means of trigonometric series as functions of geographic longitude λ. The coefficients of this series developed to 4th order terms are presented in table IVa, b, c. They formed the starting data for my own calculation.

$$X = k_0 + k_1 \cos\lambda + K_1 \sin\lambda + \dots\dots\dots + K_4 \sin 4\lambda$$
$$Y = l_0 + l_1 \cos\lambda + L_1 \sin\lambda + \dots\dots\dots + L_4 \sin 4\lambda$$
$$Z = m_0 + m_1 \cos\lambda + M_1 \sin\lambda + \dots\dots\dots + M_4 \sin 4\lambda$$

If the flattening of the earth is ignored, then the obtained numbers could be expressed by spherical functions of geographic latitude. But this is directly possible only for Z, since X and Y are inconstant at the poles because they approach equivocal expressions of the form

$c_1 \cos(\lambda - \alpha_1)$ and $c_1 \sin(\lambda - \alpha_1)$ at the North pole
$-c_2 \cos(\lambda - \alpha_2)$ and $c_2 \sin(\lambda - \alpha_2)$ at the South pole.

So whereas Z can be developed without change, X and Y must be represented by expressions formed in connection with Z, or by themselves, and these expressions must be free of all discontinuity. This can be done in a variety of ways. A limitation is introduced by the requirement that the selected expressions should permit a simple and a closed series development leading to a derivation of the potential on the earth's surface. The simplest possible values in this case, which are sufficient for a unique definition

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of the force vector, are X sin u, Y sin u and Z, if u denotes the complement of the geographic latitude. Besides these, the following might also be taken into consideration:

$$X\cos\lambda + Y\cos u\sin\lambda, \quad X\sin\lambda - Y\cos u\cos\lambda, \quad Z$$

and $-X\cos u\cos\lambda - Y\sin\lambda - Z\sin u\cos\lambda, \quad -X\cos u\sin\lambda + Y\cos\lambda - Z\sin u\sin\lambda, \quad X\sin u - Z\cos u.$

The second group represents the components of force in three fixed axes, i.e. rectified at all points; these axes are parallel to the earth radii to the equatorial points of 0° and 90° East longitude and to the North pole.

Now if the deviation of the earth's surface from the spherical is to be taken into account, as is the case here, then this necessitates a modification of the calculation (see A, p. 13; B, p. 4). First, the geographic latitude has to be replaced by the geocentric latitude. Its complement, called v, is defined by the equation:

$$tg\, v = \sqrt{1+\epsilon^2}\, tg\, u = [0.0014542]\, tg\, u$$

where the bracketed figure is the usual abbreviation for num log. The value of ϵ^2 used here is 0.00671922; it corresponds to the Bessel factor for flattening, 1:299.1528. The computed values of v belonging to u = 0°, 5°, 10°...90° have been rounded off to whole seconds of degrees (see B, p. 5):

0° 0′ 0″ 5° 1′ 0″ 10° 1′ 58″ 15° 2′ 53″ 20° 3′ 42″ 25° 4′ 25″ 30° 4′ 59″ 35° 5′ 25″ 40° 5′ 40″
45° 5′ 45″ 50° 5′ 40″ 55° 5′ 24″ 60° 4′ 59″ 65° 4′ 24″ 70° 3′ 42″ 75° 2′ 52″ 80° 1′ 58″ 85° 1′ 0″
90° 0′ 0″.

For $u_1 = 180^{\circ} - u$, we have $v_1 = 180^{\circ} - v$. All other calculations are based on the rounded values given here, and not on the equation presented above.

Another deviation of the calculation from that of a sphere is that instead of the force components X, Y, Z, we have to use the slightly different quantities αX, βY, γZ where:

$$\alpha = \sqrt{1+\epsilon^2 \cos v^2} \qquad \beta = \sqrt{1+\epsilon^2} \qquad \gamma = \sqrt{1+\epsilon^2 \cos v^2} : \sqrt{1+\epsilon^2}$$
$$= \frac{\sin v}{\sin u} \qquad = \frac{tg\, v}{tg\, u} \qquad = \frac{\cos v}{\cos u}$$

The quantities to be developed according to spherical functions of the argument v are now:

$$\alpha X \sin v \qquad \beta Y \sin v \qquad \gamma Z.$$

For the parallel circle of u = 0°, 5°, 10°....90°....180°, the logarithms of the coefficients contained herein are (their numerical values have been given in B, p. 47, table II, in addition to those for α, β and γ):

u	$\log \alpha \sin v$	$\log \beta \sin v$	$\log \gamma$		u	$\log \alpha \sin v$	$\log \beta \sin v$	$\log \gamma$	
0°	$-\infty$	$-\infty$	0.0000000	180°	45°	9.8509361	9.8510644	9.9992717	135°
5	8.9431807	8.9431918	9.9999889	175	50	9.8854533	9.8863078	9.9991455	130
10	9.2424871	9.2425311	9.9999560	170	55	9.9143187	9.9152956	9.9990231	125
15	9.4157098	9.4158075	9.9999023	165	60	9.9382561	9.9393477	9.9989084	120
20	9.5366174	9.5367880	9.9998294	160	65	9.9577935	9.9589687	9.9988048	115
25	9.6283366	9.6285970	9.9997396	155	70	9.9733253	9.9746099	9.9987154	110
30	9.7011483	9.7015128	9.9996355	150	75	9.9851378	9.9864949	9.9986429	105
35	9.7605416	9.7610211	9.9995205	145	80	9.9934389	9.9948494	9.9985895	100
40	9.8097714	9.8103734	9.9993980	140	85	9.9983663	9.9996095	9.9985568	95
45	9.8509361	9.8516644	9.9992717	135	90	0.0000000	0.0014542	9.9985458	90

Since X, Y and Z have already been developed by λ, the only problem left to solve is the representation of the coefficients

$$\alpha k_m \sin v,\ \alpha K_m \sin v; \qquad \beta l_m \sin v,\ \beta L_m \sin v; \qquad \gamma m_m,\ \gamma M_m$$

by spherical functions of m-th rank $(P_m^m, P_m^{m+1} \ldots\ldots)$. For each of these coefficients, 25 values are known which belong to the parallel circles of geographic North-pole distances u = 30°, 35°.. ..150°, which I will denote by a second, lower index i = 1,2,...25. For $i+i' = 26$, we have $v_i + v_{i'} = 180°$, thus sin v_i = sin $v_{i'}$, and $\alpha_i = \alpha_{i'}$ etc. Now it is clear that the sums:

$$\alpha_i k_{m,i} \sin v_i + \alpha_{i'} k_{m,i'} \sin v_{i'}, \ldots\ldots\ldots\ldots \gamma_i M_{m,i} + \gamma_{i'} M_{m,i'}$$

i.e.

$$\alpha_i \sin v_i (k_{m,i} + k_{m,i'}), \ldots\ldots\ldots\ldots \gamma_i (M_{m,i} + M_{m,i'})$$

are even functions of cos v, and that the corresponding differences:

$$\alpha_i \sin v_i (k_{m,i} - k_{m,i'}), \ldots\ldots\ldots\ldots \gamma_i (M_{m,i} - M_{m,i'})$$

are uneven functions of cos v. The former depend only on spherical functions P_m^n with even difference (n-m) of the two indices; the latter depend on those with uneven difference (n-m), and since the observation data can be expressed completely by those sums and differences (due to their symmetrical distribution about the equator), the unknowns--the coefficients of spherical functions--break down into two separately determined groups. With regard to this circumstance--which considerably simplifies the numerical calculation--I do not intend to report the quantities $a_i k_{m,i} \sin v_i \ldots\ldots \gamma_i M_{m,i}$ (in table Va, b, c), rather only the cited sums and differences.

The spherical function P^n_m (or $P^{n,m}$ in Gaussian notation) is defined by the equation:

$$P^n_m(\cos v) = \sin v^m \left[\cos v^{n-m} - \frac{(n-m)(n-m-1)}{2(2n-1)} \cos v^{n-m-2} + \frac{(n-m)(n-m-1)(n-m-2)(n-m-3)}{2.4.(2n-1)(2n-3)} \cos v^{n-m-4} - \ldots\ldots\right]$$

For the functions up to 7th order used below, i.e. for those in which $n \leq 7$, I intend to compile the resultant series since it is convenient for many purposes to have the numerical values of the coefficients at hand. As abbreviation, I write P^n_m instead of $P^n_m(\cos v)$, furthermore, c instead of $\cos v$ and s instead of $\sin v$.

$$P^0_0 = 1 \qquad P^1_0 = c \qquad P^2_0 = c^2 - \frac{1}{3} \qquad P^3_0 = c^3 - \frac{3}{5}c$$

$$P^1_1 = s \qquad P^2_1 = sc \qquad P^3_1 = s\left(c^2 - \frac{1}{5}\right)$$

$$P^2_2 = s^2 \qquad P^3_2 = s^2 c$$

$$P^3_3 = s^3$$

$$P^4_0 = c^4 - \frac{6}{7}c^2 + \frac{3}{35} \qquad P^5_0 = c^5 - \frac{10}{9}c^3 + \frac{5}{21}c$$

$$P^4_1 = s\left(c^3 - \frac{3}{7}c\right) \qquad P^5_1 = s\left(c^4 - \frac{2}{3}c^2 + \frac{1}{21}\right)$$

$$P^4_2 = s^2\left(c^2 - \frac{1}{7}\right) \qquad P^5_2 = s^2\left(c^3 - \frac{1}{3}c\right)$$

$$P^4_3 = s^3 c \qquad P^5_3 = s^3\left(c^2 - \frac{1}{9}\right)$$

$$P^4_4 = s^4 \qquad P^5_4 = s^4 c$$

$$P^5_5 = s^5$$

$$P^6_0 = c^6 - \frac{15}{11}c^4 + \frac{5}{11}c^2 - \frac{5}{231} \qquad P^7_0 = c^7 - \frac{21}{13}c^5 + \frac{105}{143}c^3 - \frac{35}{429}$$

$$P^6_1 = s\left(c^5 - \frac{10}{11}c^3 + \frac{5}{33}c\right) \qquad P^7_1 = s\left(c^6 - \frac{15}{13}c^4 + \frac{45}{143}c^2 - \frac{5}{429}\right)$$

$$P^6_2 = s^2\left(c^4 - \frac{6}{11}c^2 + \frac{1}{33}\right) \qquad P^7_2 = s^2\left(c^5 - \frac{10}{13}c^3 + \frac{15}{143}c\right)$$

$$P^6_3 = s^3\left(c^3 - \frac{3}{11}c\right) \qquad P^7_3 = s^3\left(c^4 - \frac{6}{13}c^2 + \frac{3}{143}\right)$$

$$P^6_4 = s^4\left(c^2 - \frac{1}{11}\right) \qquad P^7_4 = s^4\left(c^3 - \frac{3}{13}c\right)$$

$$P^6_5 = s^5 c \qquad P^7_5 = s^5\left(c^2 - \frac{1}{13}\right)$$

$$P^6_6 = s^6 \qquad P^7_6 = s^6 c$$

$$P^7_7 = s^7$$

For a numerical calculation of the functional values and in particular of their most-frequently used logarithms, a representation by products is preferred to this one using sums. P^n_m is a whole function of $\cos v^2$, except for a factor $\sin v^m$ or $\sin v^m \cos v$. If we break this down into its real, linear factors, we then obtain:

$$P^n_m(\cos v) = \sin v^m (\cos v - a_1)(\cos v - a_2) \ldots\ldots\ldots (\cos v - a_{n-m})$$

with $$a_1 = -a_{n-m}, \quad a_2 = -a_{n-m-1} \ldots..$$

In this form the equation is valid for all cases. Now if n-m is uneven, then the independent value $a_{\frac{1}{2}(n-m+1)} = 0$ appears and P^n_m obtains the factor $\cos v$, as it must be.

Now if we set: $a_1 = \cos\alpha_1,\ a_2 = \cos\alpha_2,\ \ldots.. \quad a_{n-m} = \cos\alpha_{n-m} = -\cos\alpha_1,$

and note that: $(\cos v - \cos\alpha)(\cos v + \cos\alpha) = -\sin(v+\alpha)\sin(v-\alpha) = \sin(v+\alpha)\sin(v+180-\alpha)$

then we find: $P^n_m(\cos v) = \sin^m . \sin(v+\alpha_1)\sin(v+\alpha_2) \ldots\ldots\ldots\ldots \sin(v+\alpha_{n-m})$

with $\alpha_1 + \alpha_{n-m} = 180°, \quad \alpha_2 + \alpha_{n-m-1} = 180° \ldots..$

and for uneven n-m: $\alpha_{\frac{1}{2}(n-m+1)} = 90°.$

In order to make this convenient formula applicable for the numerical calculation, it is sufficient to compute the occurring, constant angles $\alpha_1, \alpha_2 \ldots \alpha_{n-m}$ for the various spherical functions one time only. I have done this and present the results in the overview below. Naturally it was expedient to carry the computation out so that the results will be useful for all future applications. Therefore, although a much less stringent computation would be sufficient for present purposes, the calculation was performed with 10-place logarithms (from Vega's Thesaurus) and the results are presented to 4 decimal places of seconds of arc. Their uncertainty is approximately 0".0002. It hardly need be stated that in the representation of P_o^n, the roots of the equation $P_o^n = 0$ already computed by Gauss could be used. The appropriate angles α_i are found in the tables published by Prof. Seeliger for the Neumann method of coefficient computation of spherical function series; the results are accurate to tenths of an arc-second (Sitzungsberichte der math.-phys. Klasse d. k. Akademie d. Wissenschaft zu Muenchen, 1890. page 499.)

$$P_m^n(\cos v) = \sin v^m \prod_{\nu=1}^{\nu=n-m} \sin(v+\alpha_\nu)$$

P_0^2:	$\alpha_1 = 54°44'\ 8''.1971$		P_0^3:	$\alpha_1 = 39°13'53''.4787$	$\alpha_2 = 90°$
	$\alpha_2 = 125°15'51''.8029$			$\alpha_3 = 140°46'\ 6''.5213$	
P_1^3:	$\alpha_1 = 63°26'\ 5''.8158$		P_0^4:	$\alpha_1 = 30°33'20''.1802$	$\alpha_2 = 70°\ 7'27''.4111$
	$\alpha_2 = 116°33'54''.1842$			$\alpha_4 = 149°26'39''.8198$	$\alpha_3 = 109°52'32''.5889$
P_1^4:	$\alpha_1 = 49°\ 6'23''.7792$	$\alpha_2 = 90°$	P_2^4:	$\alpha_1 = 67°47'32''.4446$	
	$\alpha_3 = 130°53'36''.2208$			$\alpha_2 = 112°12'27''.5554$	
P_0^5:	$\alpha_1 = 25°\ 1'\ 2''.4232$	$\alpha_2 = 57°25'13''.8042$		$\alpha_3 = 90°$	
	$\alpha_5 = 154°58'57''.5768$	$\alpha_4 = 122°34'46''.1958$			
P_1^5:	$\alpha_1 = 40°\ 5'17''.1091$	$\alpha_2 = 73°25'38''.3284$			
	$\alpha_4 = 139°54'42''.8909$	$\alpha_3 = 106°34'21''.6716$			
P_2^5:	$\alpha_1 = 54°44'\ 8''.1971$	$\alpha_2 = 90°$	P_3^5:	$\alpha_1 = 70°31'43''.6057$	
	$\alpha_3 = 125°15'51''.8029$			$\alpha_2 = 109°28'16''.3943$	
P_0^6:	$\alpha_1 = 21°10'36''.8445$	$\alpha_2 = 48°36'28''.1779$		$\alpha_3 = 76°11'41''.7914$	
	$\alpha_6 = 158°49'23''.1555$	$\alpha_5 = 131°23'31''.8221$		$\alpha_4 = 103°48'18''.2086$	
P_1^6:	$\alpha_1 = 33°52'41''.7201$	$\alpha_2 = 62°\ 2'25''.4575$		$\alpha_3 = 90°$	
	$\alpha_5 = 146°\ 7'18''.2799$	$\alpha_4 = 117°57'34''.5425$			
P_2^6:	$\alpha_1 = 45°59'34''.7020$	$\alpha_2 = 75°29'21''.0527$			
	$\alpha_4 = 134°\ 0'25''.2980$	$\alpha_3 = 104°30'38''.9473$			
P_3^6:	$\alpha_1 = 58°31'\ 4''.2452$	$\alpha_2 = 90°$	P_4^6:	$\alpha_1 = 72°27'\ 5''.7578$	
	$\alpha_3 = 121°28'55''.7548$			$\alpha_2 = 107°32'54''.2422$	
P_0^7:	$\alpha_1 = 18°21'28''.2940$	$\alpha_2 = 42°\ 8'16''.7533$		$\alpha_3 = 66°\ 3'21''.2416$	$\alpha_4 = 90°$
	$\alpha_7 = 161°38'31''.7060$	$\alpha_6 = 137°51'43''.2467$		$\alpha_5 = 113°56'38''.7584$	
P_1^7:	$\alpha_1 = 29°20'13''.6361$	$\alpha_2 = 53°43'20''.0954$		$\alpha_3 = 77°55'\ 7''.3639$	
	$\alpha_6 = 150°39'46''.3639$	$\alpha_5 = 126°16'39''.9046$		$\alpha_4 = 102°\ 4'52''.6361$	
P_2^7:	$\alpha_1 = 39°41'41''.9905$	$\alpha_2 = 65°\ 6'27''.5168$		$\alpha_3 = 90°$	
	$\alpha_5 = 140°18'18''.0095$	$\alpha_4 = 114°53'32''.4832$			
P_3^7:	$\alpha_1 = 50°\ 9'36''.9108$	$\alpha_2 = 76°55'59''.3598$			
	$\alpha_4 = 129°50'23''.0892$	$\alpha_3 = 103°\ 4'\ 0''.6402$			
P_4^7:	$\alpha_1 = 61°17'22''.1467$	$\alpha_2 = 90°$	P_5^7:	$\alpha_1 = 73°53'52''.3905$	
	$\alpha_3 = 118°42'37''.8533$			$\alpha_2 = 106°\ 6'\ 7''.6095$	

In this overview the functions P_n^n and P_{n-1}^n have been omitted because they already appear in the original formulas as products ($\sin v^n$ and $\sin v^{n-1}\cos v$).

The various spherical functions P_m^n do not differ considerably from each other's average values. This is a disadvantage for numerical expansions which is noticed all the more, the farther the series is carried. Therefore, I have added to the functions P_m^n (see B, p. 6, 7) constant factors r_m^n of such magnitude that the quadratic average of the product $r_m^n P_m^n$, which I call R_m^n, taken over the entire spherical surface, is equal to 1 for all values of m and n. For this purpose we must set:

$$r_m^n = 1.3.5 \ldots (2n-1)\sqrt{\frac{\epsilon_m(2n+1)}{(n+m)!\,(n-m)!}} \quad \text{with } \epsilon_0 = 1,\ \epsilon_1 = \epsilon_2 = \ldots = 2$$

(this results from the known properties of the functions P_m^n).
For the factors of the 7th order functions using this formula, we obtain the following expressions, whose values have been reported in B, p. 47, table III to 7 significant figures.

$r_0^0 = 1$	$r_0^1 = \sqrt{3}$	$r_0^2 = \frac{3}{2}\sqrt{5}$	$r_0^3 = \frac{5}{2}\sqrt{7}$	$r_0^4 = \frac{105}{8}$	$r_0^5 = \frac{63}{8}\sqrt{11}$	$r_0^6 = \frac{231}{16}\sqrt{13}$	$r_0^7 = \frac{429}{16}\sqrt{15}$
	$r_1^1 = \sqrt{3}$	$r_1^2 = \sqrt{15}$	$r_1^3 = \frac{5}{4}\sqrt{42}$	$r_1^4 = \frac{21}{4}\sqrt{10}$	$r_1^5 = \frac{21}{8}\sqrt{165}$	$r_1^6 = \frac{33}{8}\sqrt{273}$	$r_1^7 = \frac{429}{32}\sqrt{105}$
		$r_2^2 = \frac{1}{2}\sqrt{15}$	$r_2^3 = \frac{1}{2}\sqrt{105}$	$r_2^4 = \frac{21}{4}\sqrt{5}$	$r_2^5 = \frac{3}{4}\sqrt{1155}$	$r_2^6 = \frac{33}{32}\sqrt{2730}$	$r_2^7 = \frac{429}{32}\sqrt{70}$
			$r_3^3 = \frac{1}{4}\sqrt{70}$	$r_3^4 = \frac{3}{4}\sqrt{70}$	$r_3^5 = \frac{9}{16}\sqrt{770}$	$r_3^6 = \frac{11}{16}\sqrt{2730}$	$r_3^7 = \frac{429}{32}\sqrt{35}$
				$r_4^4 = \frac{3}{8}\sqrt{35}$	$r_4^5 = \frac{3}{8}\sqrt{385}$	$r_4^6 = \frac{33}{16}\sqrt{91}$	$r_4^7 = \frac{39}{16}\sqrt{385}$
					$r_5^5 = \frac{3}{16}\sqrt{154}$	$r_5^6 = \frac{3}{16}\sqrt{2002}$	$r_5^7 = \frac{39}{32}\sqrt{385}$
						$r_6^6 = \frac{1}{32}\sqrt{6006}$	$r_6^7 = \frac{3}{32}\sqrt{10010}$
							$r_7^7 = \frac{3}{32}\sqrt{715}$

The logarithms of the functions $R_0^7, R_1^7 \ldots R_7^7$ for the values of v coming into consideration here, are found in table I; the function values themselves are presented in B, p. 48/50, table IV. The reported numbers (in whose computation I have not yet applied the products stated for P_m^n) have been calculated with 7-place

The logarithms of these numbers are:*

m; n:	0	1	2	3	4	5	6	7
0	0.0000000	0.2385607	0.5255763	0.8204890	1.1160993	1.4169469	1.7164637	2.0163830
1		0.2385607	0.5880457	0.9085347	1.2201593	1.5278713	1.8335053	2.1379030
2			0.2870157	0.7093647	1.0696443	1.4063523	1.7314452	2.0498563
3				0.3204890	0.7976103	1.1933679	1.5553540	1.8993413
4					0.3460653	0.8667617	1.2939146	1.6796750
5						0.3667617	0.9237334	1.3786450
6							0.3841427	0.9721884
7								0.3991243

logs while omitting the last digit in the key values attainable only through the use of multi-place tables. This place will thus sometimes be inaccurate by somewhat more than half a unit; in the 7-place values, sometimes even by one whole unit.

The table of functions R_m^n is indeed sufficient for the derivation of $\alpha k_m \sin v, \ldots \gamma M_m$ for the values of v contained therein; but it is convenient in many regards to be able to determine the coefficients $k_m \ldots M_m$ directly from the applicable series for $\alpha X \sin v$, $\beta Y \sin v$, γZ. In order to do this, tables of the logs of $R_m^n : \alpha \sin v$, $R_m^n : \beta \sin v$, $R_m^n : \gamma_m^n$ are needed. Since these tables can be used repeatedly, I have prepared them for inclusion (as table II) at the end of the work. It seemed sufficient to cite values of these figures rounded to 4 decimal places since in future calculations of potential, only the deviations from the values determined here will come into consideration--that is, relatively small values. In this severe rounding, $\log(R_m^n : \alpha \sin v)$ and $\log(R_m^n : \beta \sin v)$ differ at most by 15 units in the last place. Therefore, I have reported only the compilation of values of the former function and specified the (dependent on the angle v, identical with $\log(1:\gamma)$) difference $(\log \sin \beta - \log \sin \alpha)$, which is to be subtracted from it in order to obtain $\log(R_m^n : \beta \sin v)$. The pile-up of rounding errors occurring in many numbers, which could be eliminated by addition of +1 or -1 to the last decimal, is of no importance to the purpose of the table. I thus felt justified in omitting any reference to this

*Line 3, 4 and 5. In $\log r_0^1$, $\log r_1^1$, $\log r_1^2$ & $\log r_2^2$, in the last place write a "6" instead of a "7".

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problem in the table.

Those figures under the functions $(R_m^n : \alpha \sin v)$ and $(R_m^n : \beta \sin v)$ whose lower index m is equal to zero, become infinite for $v = 0$ and $v = 180°$, i.e. at both poles. Thus, the part of the expansion of X and Y dependent on them must be transformed, and it is expedient to use this transformation for the other values of v. The expansion for X follows easily from the other reported conditional equations for the coefficients of the spherical function series, according to the infinite expressions:

$$\frac{R_v^{1n} - \sqrt{4r+1}\,R_v^1}{\alpha \sin v} \quad \text{and} \quad \frac{R_n^{1n+1} - \sqrt{\tfrac{1}{4}(4n+8)}\,R_n^1}{\alpha \sin v}$$

where only α is to be replaced by β for this representation (see also B, p. 25, 26). Thus, the logs of these values have been incorporated into table IIa.

The coefficients of the series used in the representation of $\alpha X \sin v$, $\beta Y \sin v$ and γZ are subject to certain conditions, some of which have already been discussed. Since these will be discussed in detail in the coming sections, this passing mention will suffice at this time.

The expansion of the force components in fixed directions is simpler in an analytical respect--since only one very simple conditional equation has to be taken into account--it was already discussed above (p. 3). Therefore, it may be permissible to go into this in brief, even though no use is to be made of the appertinent series. I will call those components Ξ, H, Z and write them in the following form:

$$\begin{aligned}
\Xi &= -X \sin u\,.\,ctg\,u \cos\lambda - Y \sin u\,.\,cosec\,u \sin\lambda - Z\,.\,\sin u \cos\lambda \\
\mathrm{H} &= -X \sin u\,.\,ctg\,u \sin\lambda + Y \sin u\,.\,cosec\,u \cos\lambda - Z\,.\,\sin u \sin\lambda \\
\mathrm{Z} &= X \sin u - Z\,.\,\cos u
\end{aligned}$$

from which follows:

$$\begin{aligned}
X \sin u &= -\cos u\,(\Xi\,.\,\sin u \cos\lambda + \mathrm{H}\,.\,\sin u \sin\lambda) + \mathrm{Z}\,.\,(1 - \cos u^2) \\
Y \sin u &= \qquad -\Xi\,.\,\sin u \sin\lambda + \mathrm{H}\,.\,\sin u \cos\lambda \\
Z &= \qquad -(\Xi\,.\,\sin u \cos\lambda + \mathrm{H}\,.\,\sin u \sin\lambda) - \mathrm{Z}\,.\,\cos u
\end{aligned}$$

These formulas are generally valid, but can only be used for a sphere since the expansion for the ellipsoid is performed by r and not by u. From this we see that for Ξ, H, Z we obtain limited spherical function series when values for $X \sin u$, $Y \sin u$, Z are given, and that under certain conditions--the ones mentioned just above--the reverse will apply. In order to perform the real transformation, a number of identities has to be used through which the products

$$R_m^n(\cos u).\cos u, \quad R_m^n(\cos u).\sin u, \quad R_m^n(\cos u).\operatorname{cosec} u, \quad R_m^n(\cos u).\operatorname{ctg} u$$

are converted into sums of spherical functions, to which, however, for the last two products, expressions of another form appear, which consequently enable the conditional equations in Ξ and H applicable for X sin u and Y sin u to disappear. As can be derived from the known properties of spherical functions, we have an abbreviated notation with $\epsilon_0 = 1, \epsilon_1 = \epsilon_2 = \ldots = 2$.

$$R_m^n.\cos u = \frac{r_m^n}{r_m^{n+1}} R_m^{n+1} + \frac{r_m^{n-1}}{r_m^n} R_m^{n-1}$$

$$R_m^n.\sin u = -\frac{r_m^n}{r_{m-1}^{n+1}} R_{m-1}^{n+1} + \frac{2}{\epsilon_{m-1}} \cdot \frac{r_{m-1}^{n-1}}{r_m^n} R_{m-1}^{n-1} \qquad \text{for } m > 0$$

$$= \frac{r_m^n}{r_{m+1}^{n+1}} R_{m+1}^{n+1} - \frac{\epsilon_m}{2} \cdot \frac{r_{m+1}^{n-1}}{r_m^n} R_{m+1}^{n-1}$$

These formulas generally apply if we specify that r_m^n is zero for $n < m$. From this follows the recursive formulas below, which provide the solution for the last two problems:

$$R_m^n.\operatorname{cosec} u = \frac{r_m^n}{r_{m-1}^{n-1}} R_{m-1}^{n-1} + \frac{\epsilon_{m-1}}{2} \cdot \frac{r_m^n r_m^{n-2}}{(r_{m-1}^{n-1})^2} R_m^{n-2}.\operatorname{cosec} u$$

$$= -\frac{r_m^n}{r_{m+1}^{n-1}} R_{m+1}^{n-1} + \frac{2}{\epsilon_m} \cdot \frac{r_m^n r_m^{n-2}}{(r_{m+1}^{n-1})^2} R_m^{n-2}.\operatorname{cosec} u$$

$$R_m^n.\operatorname{ctg} u = \frac{r_m^n}{r_{m-1}^n} R_{m-1}^n + \frac{2n+1}{n+m} \cdot \frac{r_m^{n-1}}{r_m^n} R_m^{n-1}.\operatorname{cosec} u$$

$$= -\frac{r_m^n}{r_{m+1}^n} R_{m+1}^n \quad \frac{n+1}{i-m} \cdot \frac{r_m^{n-1}}{r_m^n} R_m^{n-1}.\operatorname{cosec} u$$

The doubled solutions appearing everywhere except at $R_m^n.\cos u$ are necessary because from the factor $\cos m\lambda$ or $\sin m\lambda$ always combined with R_m^n, through the factor $\cos\lambda$ or $\sin\lambda$ occurring together with $\sin u$, $\operatorname{cosec} u$ or $\operatorname{ctg} u$, simultaneous functions of $(m+1)\lambda$ and of $(m-1)\lambda$ always appear, which necessarily require spherical functions with the corresponding lower indices (m+1) and (m-1) as factors.

It has already been pointed out that the coefficients of the series pertaining to $X\sin v$ and $Y\sin v$ are subject to certain conditions of a purely analytical nature, which is not the case for those of the series for Ξ, H, Z. Now there is another condition due to physical considerations, which naturally must be expressed in both representations; that is, that the integral taken over the entire earth's surface of the force component perpendicular to this surface, must disappear. If we call the coefficients of $R_m^n(\cos v)\cos m\lambda$ and $R_m^n(\cos v)\sin m\lambda$ in the spherical function series applicable for any function f of v and λ, with

$$C_m^n(f) \text{ and } S_m^n(f),$$

then that condition is expressed in the simple equation:

$$C_0^0(Z) = 0$$

If we introduce the components Ξ, H, Z, then by substitution of the expression specified for Z, we immediately obtain the slightly more complicated equation

$$C_1^1(\Xi) + S_1^1(H) + C_0^1(Z) = 0.$$

All these comments apply essentially when taking into account the flattening of the earth. The formulas to be applied in this case (which can be easily derived from the foregoing by introduction of ε), thus are:

$$\begin{aligned}
\alpha\gamma\Xi &= -\alpha X\sin v\,.\,ctg\, v\cos\lambda - \beta Y\sin v\,.\,\gamma^2\operatorname{cosec} v\sin\lambda - \gamma Z\,.\,\sin v\cos\lambda \\
\alpha\gamma H &= -\alpha X\sin v\,.\,ctg\, v\sin\lambda + \beta Y\sin v\,.\,\gamma^2\operatorname{cosec} v\cos\lambda - \gamma Z\,.\,\sin v\sin\lambda \\
\alpha\gamma Z &= \beta^{-1}.\,\alpha X\sin v - \gamma Z\,.\,\beta\cos v
\end{aligned}$$

$$\begin{aligned}
\alpha X\sin v &= -\beta\cos v(\Xi\,.\,\sin v\cos\lambda + H\,.\,\sin v\sin\lambda) + Z\,.\,(1-\cos v^2) \\
\beta Y\sin v &= \beta(-\Xi\,.\,\sin v\sin\lambda + H\,.\,\sin v\cos\lambda) \\
\gamma Z &= -\beta^{-1}(\Xi\,.\,\sin v\cos\lambda + H\,.\,\sin v\sin\lambda) - Z\,.\,\cos v
\end{aligned}$$

The conditional equation to be fulfilled is thus:

$$C_1^1(\Xi) + S_1^1(H) + \beta C_0^1(Z) = 0.$$

Since $\beta(=\sqrt{1+\varepsilon^2})$ is a constant, then the derivative of $\alpha X\sin v, \beta Y\sin v, Z$ from Ξ, H, Z is no different and no more complicated than that of a sphere. The converse, but practically unimportant problem, undergoes an important modification inasmuch as a closed expansion does not result for Ξ, H, Z, but for the products of these quantities with αr, that is, $\beta^{-1}(1+\epsilon^2 \cos v^2)$. The elimination of this factor is of course possible, but generally leads to infinite series. However, the factor γ^2 occurring in connection with $\beta Y \sin v$, which can be used in the form $(1-\epsilon^2\beta^{-2}\sin v^2)$, causes only a small expansion of the calculation, without changing anything about its nature.

Set-up and General Solution to the Normal Equations

Everything is now ready to derive the normal equations. If $f_{m,i}$ is one of the quantities belonging to the value v_i:

$$\alpha_i k_{m,i} \sin v_i\ ;\quad \alpha_i K_{m,i} \sin v_i\ ;\quad \beta_i l_{m,i} \sin v_i\ ;\quad \beta_i L_{m,i} \sin v_i\ ;\quad \gamma_i m_{m,i}\ ;\quad \gamma_i M_{m,i}$$

and if F^n_m (mit $n = m, m+1, m+2 \ldots$) denotes the corresponding coefficients:

$$B^n_m\ ;\quad C^n_m\ ;\quad D^n_m\ ;\quad E^n_m\ ;\quad j^n_m\ ;\quad k^n_m\ ,$$

then the system of error equations runs:

$$f_{m,i} = \sum_{n=m}^{n=m+\nu} F^n_m \cdot R^n_m(\cos v_i) \qquad i = 1, 2, 3 \ldots 35$$

For reasons presented in detail in B, p. 22/24, I have given equal weight to all these equations. In the case (not initially treated here) that no secondary conditions are to be met, we then have the following normal equations:

$$\sum_{i=1}^{i=35} f_{m,i} R^p_m(\cos v_i) = \sum_{n=m}^{n=m+\nu} F^n_m \cdot \sum_{i=1}^{i=35} R^n_m(\cos v_i) R^p_m(\cos v_i) \qquad p = m, m+1, \ldots m+\nu$$

or, in standard, abbreviated notation:

$$[f_m R^p_m] = \sum_{n=m}^{n=m+\nu} F^n_m \cdot [R^n_m R^p_m]$$

Here, ν depends on the expansion of the series of spherical functions. For reasons given below, in the series for $\alpha X \sin v$, the expansion shall be carried one step farther, i.e. ν is to be set greater by 1 than in the series for $\beta Y \sin v$ and γZ.

As mentioned earlier (p. 6), the system of normal equations now breaks down into two completely separate systems, one of which contains only $F^m_m, F^{m+2}_m \ldots$, and the other contains only $F^{m+1}_m, F^{m+3}_m \ldots$ as unknowns, because in general:

$$R^n_m(\cos v_{36-i}) = (-1)^{n-m} R^n_m(\cos v_i)$$

and since consequently the sum $[R^n_m R^p_m]$, i.e. disappears for uneven values of (n-p) through an easily understood abbreviation:

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$$\sum_{i=1}^{i=12} R^{n}_{m,i} R^{p}_{m,i} [1+(-1)^{n+p-2m}] + R^{n}_{m,13} R^{p}_{m,13}$$

Let us write the two groups of equations in the form:

$$\sum_{\mu=0,1,2\ldots} F_m^{m+2\mu} \cdot [R_m^{m+2\mu} R_m^{m+2\pi}] = [f_m R_m^{m+2\pi}] = q_m^{m+2\pi} \qquad \pi = 0, 1, 2 \ldots.$$

$$\sum_{\mu=0,1,2\ldots} F_m^{m+2\mu+1} \cdot [R_m^{m+2\mu+1} R_m^{m+2\pi+1}] = [f_m R_m^{m+2\pi+1}] = q_m^{m+2\pi+1} \qquad \pi = 0, 1, 2 \ldots.$$

or in shortened form, omitting the lower index m:

$$(A) \qquad \sum_{\mu} a_{2\mu,2\pi} F_{2\mu} = q_{2\pi}; \quad \sum_{\mu} a_{2\mu+1,2\pi+1} F_{2\pi+1} = q_{2\pi+1} \qquad \pi = 0, 1, 2 \ldots.$$

For the numerical calculation on which tables I and V are based, we naturally set:

$$a_{2\mu,2\pi} = 2\sum_{i=1}^{i=12} R_{m,i}^{m+2\mu} R_{m,i}^{m+2\pi} + R_{m,13}^{m+2\mu} R_{m,13}^{m+2\pi}; \quad a_{2\mu+1,2\pi+1} = 2\sum_{i=1}^{i=12} R_{m,i}^{m+2\mu+1} \cdot R_{m,i}^{m+2\pi+1}$$

$$q_{2\pi} = \sum_{i=1}^{i=12} (f_{m,i} + f_{m,26-i}) R_{m,i}^{m+2\pi} + f_{m,13} R_{m,13}^{m+2\pi}; \quad q_{2\pi+1} = \sum_{i=1}^{i=12} (f_{m,i} - f_{m,26-i}) R_{m,i}^{m+2\pi+1}$$

The coefficients 'a' do not depend on the observation data, but only on the selection of the parallel circle to which these data relate. Thus, they can be used for every other calculation which is based on the same parallel circle. Naturally, its application is by no means limited to geomagnetic problems. Primarily for this reason, I have computed the solution of the individual equation systems, even though a frequent application of the obtained formulas (presented in the following pages) seems unlikely. In general, for similar problems, the Neumann method or a graphic derivation will be preferred (see A, p. 25, 26, and due to the reasons which induced me to select this method, B, p. 21, 22).

Regarding the coefficients 'a' presented in the following table, it should be noted that they have been derived from the values of $[P_m^{m+\mu} P_m^{m+\nu}]$ computed initially by me and rounded to 8 decimal places, through multiplication with $r_m^{m+\mu} r_m^{m+\nu}$. The reason for this is that after some delay and after a considerable part of the numerical calculations had been completed, I decided on the deviation from the usual method in the introduction of the functions R. I mention this because in a direct calculation of 'a' from R, the last decimal places were found not always to agree with those given here.

The differences are practically meaningless, which is why I omitted the time-consuming re-calculation.

Coefficients of the Normal Equations (A)

$$a_{\mu\nu} = [E_m^{m+\mu} \cdot E_m^{m+\nu}]$$

$m = 0$

$a_{00} = 25.00000$	$a_{20} = -1.87202$	$a_{40} = -7.14895$	$a_{60} = -4.52886$
	$a_{22} = 17.62068$	$a_{42} = -9.58886$	$a_{62} = -8.44497$
		$a_{44} = 18.88245$	$a_{64} = -4.58558$
			$a_{66} = 22.75595$
$a_{11} = 23.82561$	$a_{31} = -7.87997$	$a_{51} = -10.15889$	$a_{71} = -8.75241$
	$a_{33} = 16.18116$	$a_{53} = -7.51818$	$a_{73} = -5.58246$
		$a_{55} = 21.88857$	$a_{75} = -8.12799$
			$a_{77} = 21.95444$

$m = 1$

$a_{00} = 51.67489$	$a_{20} = 4.93221$	$a_{40} = -4.55287$	$a_{60} = -6.18189$
	$a_{22} = 52.73554$	$a_{42} = -7.64619$	$a_{62} = -17.60242$
		$a_{44} = 88.15086$	$a_{64} = -15.60996$
			$a_{66} = 89.05169$
	$a_{11} = 56.96852$	$a_{31} = 0.89181$	$a_{51} = -11.89867$
		$a_{33} = 44.68674$	$a_{53} = -18.99684$
			$a_{55} = 86.42590$

$m = 2$

$a_{00} = 50.85086$	$a_{20} = 5.59272$	$a_{40} = -0.47484$
	$a_{22} = 58.53060$	$a_{42} = 2.90692$
		$a_{44} = 49.55171$
$a_{11} = 56.80878$	$a_{31} = 6.22129$	$a_{51} = -8.85854$
	$a_{33} = 55.62099$	$a_{53} = -8.18960$
		$a_{55} = 48.08946$

$m = 3$

$a_{00} = 49.27458$	$a_{20} = 4.69178$	$a_{40} = 0.59175$
	$a_{22} = 58.52248$	$a_{42} = 7.04828$
		$a_{44} = 57.85657$
	$a_{11} = 54.98297$	$a_{31} = 6.92004$
		$a_{33} = 59.46148$

$m = 4$

$a_{00} = 48.56728$	$a_{20} = 8.79217$
	$a_{22} = 56.99944$
$a_{11} = 58.27085$	$a_{31} = 6.16905$
	$a_{33} = 59.45207$

$m = 5$

$a_{00} = 48.09689$	$a_{20} = 8.11497$
	$a_{22} = 55.48200$
	$a_{11} = 52.08705$

$m = 6$

$a_{00} = 47.76858$

$a_{11} = 51.18567$

$m = 7$

$a_{00} = 47.52800$

The expansions of the normal equations then run:

$$F_{2\mu} = \sum_{n} \alpha_{2\mu,2n} q_{2n}; \quad F_{2\mu+1} = \sum_{n} \alpha_{2\mu+1,2n+1} q_{2n+1} \qquad \mu = 0, 1, 2 \ldots..$$

The coefficients α appearing here are functions of the reported quantities 'a' formed in the known manner. Their logarithms rounded to 6 decimal places (which could be shortened even more for most applications) are found in the following table, as series shown for different limitations on the series expansion.

Logarithms of the Coefficients of the Solutions of Normal Equations (A)*

$m = 0$

$\log \alpha_{00} = 9.007269$	$\log \alpha_{20} = 9.08881\bar{5}$	$\log \alpha_{40} = 9.100947$	$\log \alpha_{60} = 8.958866$
	$\log \alpha_{22} = 9.455399$	$\log \alpha_{42} = 9.382101$	$\log \alpha_{62} = 9.251214$
		$\log \alpha_{44} = 9.433688$	$\log \alpha_{64} = 9.227006$
			$\log \alpha_{66} = 9.209016$
$\log \alpha_{00} = 8.704756$	$\log \alpha_{20} = 8.850648$	$\log \alpha_{40} = 8.497846$	
	$\log \alpha_{22} = 8.948616$	$\log \alpha_{42} = 8.741774$	
		$\log \alpha_{44} = 8.980707$	
$\log \dot{\alpha}_{00} = 8.605516$	$\log \alpha_{20} = 7.680816$		
	$\log \alpha_{22} = 8.755960$		
$\log \alpha_{00} = 8.602060$			
$\log \alpha_{11} = 9.642880$	$\log \alpha_{31} = 9.700518$	$\log \alpha_{51} = 9.627701$	$\log \alpha_{71} = 9.420154$
	$\log \alpha_{33} = 9.819725$	$\log \alpha_{53} = 9.715881$	$\log \alpha_{73} = 9.515889$
		$\log \alpha_{55} = 9.673406$	$\log \alpha_{75} = 9.484078$
			$\log \alpha_{77} = 9.827872$
$\log \alpha_{11} = 9.055500$	$\log \alpha_{31} = 8.983011$	$\log \alpha_{51} = 8.944128$	
	$\log \alpha_{33} = 9.191057$	$\log \alpha_{53} = 9.001887$	
		$\log \alpha_{55} = 9.098638$	
$\log \alpha_{11} = 8.710228$	$\log \alpha_{31} = 8.397743$		
	$\log \alpha_{33} = 8.869052$		
$\log \alpha_{11} = 8.682167$			

$m = 1$

$\log \alpha_{00} = 8.808738$	$\log \alpha_{20} = 6.755958$	$\log \alpha_{40} = 7.676051$	$\log \alpha_{60} = 7.780408$
	$\log \alpha_{22} = 8.419957$	$\log \alpha_{42} = 8.087296$	$\log \alpha_{62} = 8.226187$
		$\log \alpha_{44} = 8.579862$	$\log \alpha_{64} = 8.331509$
			$\log \alpha_{66} = 8.629620$
$\log \alpha_{00} = 8.294080$	$\log \alpha_{20} = 7.191095_n$	$\log \alpha_{40} = 7.309080$	
	$\log \alpha_{22} = 8.293420$	$\log \alpha_{42} = 7.574440$	
		$\log \alpha_{44} = 8.434688$	

*Lines 9-12, 19-22: The values of $\log \alpha_{00} \ldots \log \alpha_{66}$ and $\log \alpha_{11} \ldots \log \alpha_{77}$ belonging to m = 0 (not used in further calculations and presented here only for the sake of completeness) are somewhat inaccurate because they were computed from provisional normal equations whose coefficients $\alpha_{00} \ldots \alpha_{66}$ and $\alpha_{11} \ldots \alpha_{77}$ deviate sometimes by several units in the last place, from the final values presented on p. 16.

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$\log \alpha_{00} = 8.290651$	$\log \alpha_{20} = 7.263346_n$	
	$\log \alpha_{22} = 8.281823$	
$\log \alpha_{00} = 8.28672\bar{5}$		
$\log \alpha_{11} = 8.278682$	$\log \alpha_{31} = 7.305811$	$\log \alpha_{51} = 7.343344$
	$\log \alpha_{33} = 8.409722$	$\log \alpha_{53} = 8.022462$
		$\log \alpha_{55} = 8.529641$
$\log \alpha_{11} = 8.244391$	$\log \alpha_{31} = 6.187220_n$	
	$\log \alpha_{33} = 8.350334$	
$\log \alpha_{11} = 8.244365$		

$m = 2$

$\log \alpha_{00} = 8.302727$	$\log \alpha_{20} = 7.286395_n$	$\log \alpha_{40} = 6.485218$
	$\log \alpha_{22} = 8.238579$	$\log \alpha_{42} = 7.014794_n$
		$\log \alpha_{44} = 8.306308$
$\log \alpha_{00} = 8.302627$	$\log \alpha_{20} = 7.282367_n$	
	$\log \alpha_{22} = 8.237251$	
$\log \alpha_{00} = 8.297993$		
$\log \alpha_{11} = 8.25315\bar{5}$	$\log \alpha_{31} = 7.283283_n$	$\log \alpha_{51} = 7.164920$
	$\log \alpha_{33} = 8.261529$	$\log \alpha_{53} = 7.071828$
		$\log \alpha_{55} = 8.369694$
$\log \alpha_{11} = 8.250937$	$\log \alpha_{31} = 7.299579_n$	
	$\log \alpha_{33} = 8.260114$	
$\log \alpha_{11} = 8.24558\bar{5}$		

$m = 3$

$\log \alpha_{00} = 8.310706$	$\log \alpha_{20} = 7.214404_n$	$\log \alpha_{40} = 4.9915_n$
	$\log \alpha_{22} = 8.242418$	$\log \alpha_{42} = 7.328176_n$
		$\log \alpha_{44} = 8.247883$
$\log \alpha_{00} = 8.310705$	$\log \alpha_{20} = 7.214716_n$	
	$\log \alpha_{22} = 8.236006$	
$\log \alpha_{00} = 8.307378$		
$\log \alpha_{11} = 8.266581$	$\log \alpha_{31} = 7.332454_n$	
	$\log \alpha_{33} = 8.232179$	
$\log \alpha_{11} = 8.260167$		

$m = 4$

$\log \alpha_{00} = 8.315919$	$\log \alpha_{20} = 7.188986_n$
	$\log \alpha_{22} = 8.246391$
$\log \alpha_{00} = 8.313657$	
$\log \alpha_{11} = 8.278761$	$\log \alpha_{31} = 7.294812_n$
	$\log \alpha_{33} = 8.231068$
$\log \alpha_{11} = 8.278510$	

In several cases, conditional equations have to be taken into account. The resultant changes in the solutions will be given now.

In the Z-series (as I will call the series used for the expansion of ${}_{\gamma}Z$), only the condition $j_0^0 = 0$ has to be met. A modification of the computation only occurs here for the first of the two equation systems characterized by m = 0. The coefficients $\alpha_{00}, \alpha_{20}, \alpha_{40} \ldots$ do not come into consideration or are set equal to zero;

the others (which I will call β) obtain values whose logarithms are:

$m = 0$

$\log \beta_{22} = 9.188859$	$\log \beta_{42} = 8.949891$	$\log \beta_{62} = 8.887790$
	$\log \beta_{44} = 9.060862$	$\log \beta_{64} = 8.747660$
		$\log \beta_{66} = 8.906588$
$\log \beta_{22} = 8.897198$	$\log \beta_{42} = 8.615490$	
	$\log \beta_{44} = 8.881470$	
$\log \beta_{22} = 8.752502$		

The problem for the two equation systems belonging to m = 0 is only a little more complicated for the expansion of $\alpha X \sin v$ and $\beta Y \sin v$. In this case, if F is again to be set in series for B, C, D, E, in all 4 cases the conditional equations (B, p. 11) apply:

$$a_0^0 F_0^0 + a_0^2 F_0^2 + a_0^4 F_0^4 + \ldots\ldots = 0$$

$$a_0^1 F_0^1 + a_0^3 F_0^3 + a_0^5 F_0^5 + \ldots\ldots = 0$$

where

$$a_0^n = 2^n \frac{n!\,n!}{(2n)!}\, r_0^n = \sqrt{2n+1}$$

denotes the value of the function R_0^n at the North pole (i.e. for $v = 0$). The coefficients of the sought solutions computed with respect to these conditional equations will be called γ. Their logarithms are:

$m = 0$

$\log \gamma_{00} = 8.555270$	$\log \gamma_{20} = 7.829511_n$	$\log \gamma_{40} = 6.878826_n$	$\log \gamma_{60} = 7.711288_n$
	$\log \gamma_{22} = 8.493678$	$\log \gamma_{42} = 7.928054_n$	$\log \gamma_{62} = 8.017281_n$
		$\log \gamma_{44} = 8.423535$	$\log \gamma_{64} = 8.220089_n$
			$\log \gamma_{66} = 8.886278$
$\log \gamma_{00} = 8.540271$	$\log \gamma_{20} = 7.964750_n$	$\log \gamma_{40} = 7.671407_n$	
	$\log \gamma_{22} = 8.417877$	$\log \gamma_{42} = 8.215791_n$	
		$\log \gamma_{44} = 8.140841$	
$\log \gamma_{00} = 8.519842$	$\log \gamma_{20} = 8.170857_n$		
	$\log \gamma_{22} = 7.820872$		
$\log \gamma_{11} = 8.514664$	$\log \gamma_{31} = 7.516348_n$	$\log \gamma_{51} = 7.097879_n$	$\log \gamma_{71} = 8.058581_n$
	$\log \gamma_{33} = 8.519360$	$\log \gamma_{53} = 7.963791_n$	$\log \gamma_{73} = 8.120704_n$
		$\log \gamma_{55} = 8.416568$	$\log \gamma_{75} = 8.189589_n$
			$\log \gamma_{77} = 8.486642$
$\log \gamma_{11} = 8.447558$	$\log \gamma_{31} = 7.941971_n$	$\log \gamma_{51} = 7.884080_n$	
	$\log \gamma_{33} = 8.426269$	$\log \gamma_{53} = 8.228190_n$	
		$\log \gamma_{55} = 8.288924$	
$\log \gamma_{11} = 8.391718$	$\log \gamma_{31} = 8.207724_n$		
	$\log \gamma_{33} = 8.028786$		
$\log \gamma_{11} = -\infty$			

The solution for the X and Y-series is much more complex, provided $m > 0$, because in the conditional equations appearing here, the coefficients of these two series are not separate. If we state in general:

$$a_m^m F_m^m + a_m^{m+2} F_m^{m+2} + a_m^{m+4} F_m^{m+4} + \dots\dots\dots = {}^0F_m$$

$$a_m^{m+1} F_m^{m+1} + a_m^{m+3} F_m^{m+3} + a_m^{m+5} F_m^{m+5} + \dots\dots = {}^1F_m$$

with

$$a_m^n = 2^{n-m} \frac{n!(n+m)!}{m!(2n)!} r_m^n$$

then the conditional equations are (see B. p. 11):

$${}^0B_m - {}^1E_m = 0, \quad {}^1B_m - {}^0E_m = 0, \quad {}^0C_m + {}^1D_m = 0, \quad {}^1C_m + {}^0D_m = 0.$$

The importance of a_m^n is that $a_m^n \sin v^m$ represents the value of R_m^n for infinitely small values of v. The logarithms of the a_m^n coming into consideration here are:

$\log a_1^1 = 0.288561$	$\log a_1^3 = 0.81162\overline{5}$	$\log a_1^5 = 1.108742$	$\log a_1^7 = 1.81162\overline{5}$
$\log a_2^2 = 0.287016$	$\log a_2^4 = 1.002698$	$\log a_2^6 = 1.417051$	
$\log a_3^3 = 0.820489$	$\log a_3^5 = 1.142215$	$\log a_3^7 = 1.647086$	
$\log a_4^4 = 0.846065$	$\log a_4^6 = 1.252522$		
$\log a_1^2 = 0.588046$	$\log a_1^4 = 0.977121$	$\log a_1^6 = 1.218081$	
$\log a_2^3 = 0.70956\overline{5}$	$\log a_2^5 = 1.230261$	$\log a_2^7 = 1.575762$	
$\log a_3^4 = 0.797610$	$\log a_3^6 = 1.417051$		
$\log a_4^5 = 0.866762$	$\log a_4^7 = 1.565782$		

The simplest method would be to determine the coefficients of both expansions linked by a conditional equation, by means of a common balancing. Accordingly, I have done this by suggesting a detour which permits a certain judgement about the reliability of the final results (see B, p. 36, 37). The cited conditional equations are of a purely analytical nature; they tell us that the horizontal force at both poles is unequivocally determined in its magnitude and direction. They would have to be inherently fulfilled if the coefficients of the series for X and Y were computed on the basis of our knowledge of the force distribution over the entire earth's surface and if this distribution were expressed without remainder. Now this knowledge is missing for the calculation to be performed here, for the two polar spherical indentations (the other side of 60° N. and S. latitude) and related to this,

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the computed, first coefficients of the series expansion are not dependent on the others, which are neglected. Consequently, the numbers obtained in the independent calculation of the coefficients of both series need not necessarily satisfy the conditional equations, and they will also in general not satisfy them. The amount of the remaining error apparently permits a view of the level of reliability of the results.

Consequently, I first computed the values B,C,D,E independently and used the general solutions with the found coefficients •. To the found values (printed in B, p. 54, 55 under II), which I will call B', C', D', E', I have added those corrections (after calculation of the remaining error of the conditional equations) which make these errors disappear and the sum of the error-squares of the original equations are brought to a minimum. (The final results thus agree completely with those which would result from a direct, joint balancing using the least squares method--this must be stated in order to prevent any misunderstanding).

For the sake of brevity, I shall be satisfied with a statement of the numerical results without their somewhat cumbersome, but easy derivation.

By insertion of the corresponding values of B' and E', let:

$${}^0B'_m - {}^1E'_m = \Delta_m, \quad {}^1B'_m - {}^0E'_m = E_m$$

Then we have, depending on the expansion of the series, the following equations:

$$B_1^1 = B_1^{1\prime} - [7.54483]\,\Delta_1 \qquad E_1^2 = E_1^{2\prime} + [7.54077]\,\Delta_1$$
$$B_1^3 = B_1^{3\prime} - [8.05062]\,\Delta_1 \qquad E_1^4 = E_1^{4\prime} + [7.85145]\,\Delta_1$$
$$B_1^5 = B_1^{5\prime} - [8.22918]\,\Delta_1 \qquad E_1^6 = E_1^{6\prime} + [8.05825]\,\Delta_1$$
$$B_1^7 = B_1^{7\prime} - [8.32545]\,\Delta_1$$

$$B_1^1 = B_1^{1\prime} - [7.78087]\,\Delta_1 \qquad E_1^2 = E_1^{2\prime} + [7.90274]\,\Delta_1$$
$$B_1^3 = B_1^{3\prime} - [8.31750]\,\Delta_1 \qquad E_1^4 = E_1^{4\prime} + [8.40597]\,\Delta_1$$
$$B_1^5 = B_1^{5\prime} - [8.65646]\,\Delta_1$$

$$B_1^1 = B_1^{1\prime} - [8.30608]\,\Delta_1 \qquad E_1^2 = E_1^{2\prime} + [8.79728]\,\Delta_1$$
$$B_1^3 = B_1^{3\prime} - [9.04700]\,\Delta_1$$

$$B_1^2 = B_1^{2\prime} - [7.97157]\,E_1 \qquad E_1^2 = E_1^{2\prime} + [7.36397]\,E_1$$
$$B_1^3 = B_1^{3\prime} - [8.28226]\,E_1 \qquad E_1^3 = E_1^{3\prime} + [7.89110]\,E_1$$
$$B_1^4 = B_1^{4\prime} - [8.48906]\,E_1 \qquad E_1^4 = E_1^{4\prime} + [8.29007]\,E_1$$

$$B_1^3 = B_1^{3\prime} - [8.38312]\,E_1 \qquad E_1^3 = E_1^{3\prime} + [7.85182]\,E_1$$
$$B_1^4 = B_1^{4\prime} - [8.83685]\,E_1 \qquad E_1^4 = E_1^{4\prime} + [8.58839]\,E_1$$

$$B_1^4 = B_1^{4\prime} - [9.32542]\,E_1 \qquad E_1^4 = E_1^{4\prime} + [9.01829]\,E_1$$

$$B_2^2 = B_2^{2\prime} - [7.12758]\,\Delta_2 \qquad E_2^2 = E_2^{2\prime} + [7.44082]\,\Delta_2$$
$$B_2^3 = B_2^{3\prime} - [7.84655]\,\Delta_2 \qquad E_2^3 = E_2^{3\prime} + [8.16544]\,\Delta_2$$
$$B_2^4 = B_2^{4\prime} - [8.40487]\,\Delta_2$$

$$B_2^3 = B_2^{3\prime} - [7.94705]\,\Delta_2 \qquad E_2^3 = E_2^{3\prime} + [8.61066]\,\Delta_2$$
$$B_2^4 = B_2^{4\prime} - [8.88597]\,\Delta_2$$

$$B_2^2 = B_2^{2\prime} - [7.32148]\,E_2 \qquad E_2^2 = E_2^{2\prime} + [6.69167]\,E_2$$
$$B_2^3 = B_2^{3\prime} - [7.79147]\,E_2 \qquad E_2^3 = E_2^{3\prime} + [7.41069]\,E_2$$
$$B_2^4 = B_2^{4\prime} - [8.21259]\,E_2 \qquad E_2^4 = E_2^{4\prime} + [7.96901]\,E_2$$

$$B_2^3 = B_2^{3\prime} - [7.90686]\,E_2 \qquad E_2^3 = E_2^{3\prime} + [7.48868]\,E_2$$
$$B_2^4 = B_2^{4\prime} - [8.62296]\,E_2 \qquad E_2^4 = E_2^{4\prime} + [8.37775]\,E_2$$

$$B_2^4 = B_2^{4\prime} - [9.22554]\,E_2 \qquad E_2^4 = E_2^{4\prime} + [8.85540]\,E_2$$

$$B_3^2 = B_3^{2\prime} - [6.61810]\,\Delta_3 \qquad E_3^2 = E_3^{2\prime} + [7.10808]\,\Delta_3$$
$$B_3^3 = B_3^{3\prime} - [7.48569]\,\Delta_3 \qquad E_3^3 = E_3^{3\prime} + [7.96152]\,\Delta_3$$
$$B_3^4 = B_3^{4\prime} - [8.20898]\,\Delta_3$$

$$B_3^3 = B_3^{3\prime} - [7.69679]\,\Delta_3 \qquad E_3^3 = E_3^{3\prime} + [8.45298]\,\Delta_3$$
$$B_3^4 = B_3^{4\prime} - [8.76710]\,\Delta_3$$

$$B_3^3 = B_3^{3\prime} - [7.60068]\,E_3 \qquad E_3^3 = E_3^{3\prime} + [7.12612]\,E_3$$
$$B_3^4 = B_3^{4\prime} - [8.46087]\,E_3 \qquad E_3^4 = E_3^{4\prime} + [8.19642]\,E_3$$

$$B_3^4 = B_3^{4\prime} - [9.15167]\,E_3 \qquad E_3^4 = E_3^{4\prime} + [8.72176]\,E_3$$

$$B_4^3 = B_4^{3\prime} - [7.50326]\,\Delta_4 \qquad E_4^3 = E_4^{3\prime} + [8.31789]\,\Delta_4$$
$$B_4^4 = B_4^{4\prime} - [8.67180]\,\Delta_4$$

$$B_4^3 = B_4^{3\prime} - [7.87086]\,E_4 \qquad E_4^3 = E_4^{3\prime} + [6.87118]\,E_4$$
$$B_4^4 = B_4^{4\prime} - [8.32968]\,E_4 \qquad E_4^4 = E_4^{4\prime} + [8.08772]\,E_4$$

$$B_4^4 = B_4^{4\prime} - [9.09196]\,E_4 \qquad E_4^4 = E_4^{4\prime} + [8.61141]\,E_4$$

A glance at the conditional equations teaches that the coefficients B and E can be replaced in the reported formulas by C and -D. This substitution naturally must also be performed in the quantities Δ and E, so that these can be defined by the equations:

$${}^0C_m' + {}^1D_m = \Delta_m, \quad {}^1C_m' + {}^0D_m = E_m$$

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Numerical Resolution of the Normal Equations to Derive the Series Expansion of $\alpha X \sin v$, $\beta Y \sin v$ and γZ.

Now in order to obtain the coefficients of the series for the three force components, one only has to insert the values in table V, which express the state of the geomagnetic force at the earth's surface, into the formulas reported above.

First determine the values of φ. For any value of m, we have:

$$\varphi_\nu = \varphi_m^{m+\nu} = [f_m E_m^{m+\nu}],$$

where $f_{m,i}$ stands for the earlier cited 6 quantities ($\alpha_i k_{m,i} \sin v_i$ etc., see p. 11). The computation provides the following numbers:

f :	$\alpha k \sin v$	$\alpha K \sin v$	$\beta l \sin v$	$\beta L \sin v$	γm	γM
$[f_0, E_0]$	546744		− 1221		8017	
$[f_0, E_0]$	−226880		844		24985	
$[f_0, E_0]$	− 40649		− 58		− 29870	
$[f_0, E_0]$	− 16184		88		− 1062	
$[f_0, E_0]$	10264		1789		856948	
$[f_0, E_0]$	− 22077		− 1848		−311774	
$[f_0, E_0]$	14668		− 807		−852686	
$[f_0, E_0]$	2584					
$[f_1, E_1]$	− 96828	19809	−176276	−69808	159809	−855690
$[f_1, E_1]$	29022	−22969	− 9957	−84124	118922	− 14570
$[f_1, E_1]$	9822	128	17922	15882	− 50622	47557
$[f_1, E_1]$	− 4518	8418				
$[f_1, E_1]$	11829	101860	18765	72567	−217583	57284
$[f_1, E_1]$	− 44449	10272	− 5278	8967	− 40228	− 18878
$[f_1, E_1]$	20798	−29197	− 2662	−17889	58594	− 12069
$[f_2, E_2]$	− 87484	− 2444	− 62197	26828	− 52887	− 98111
$[f_2, E_2]$	16907	2514	− 1511	24107	− 51502	9642
$[f_2, E_2]$	12467	− 822	− 1996	− 288	6479	− 1801
$[f_2, E_2]$	− 7592	88561	1266	61877	−127010	− 2485
$[f_2, E_2]$	20810	− 2584	23	19540	− 36416	4917
$[f_2, E_2]$	44	694				
$[f_3, E_3]$	8258	7948	− 36870	25364	− 28841	− 54065
$[f_3, E_3]$	− 3743	− 2241	− 2366	1648	− 4464	− 9907
$[f_3, E_3]$	− 122	− 288				
$[f_3, E_3]$	7478	12899	11468	−18298	31299	24947
$[f_3, E_3]$	3798	1898	3871	− 9586	28741	18806
$[f_4, E_4]$	8860	2219	12894	8072	− 8669	6801
$[f_4, E_4]$	− 2602	116	− 5077	4472	7881	11900
$[f_4, E_4]$	5480	− 961	6900	− 4816	− 4786	6171
$[f_4, E_4]$	− 49	− 175				

By substitution of these values into the formulas of the preceeding section, we obtain the desired coefficients of the series of spherical functions needed for representation of $\alpha X \sin r, \beta Y \sin r, \gamma Z$. Their values are found in table VI for three different limits of these series. A comparison of the values of the first, common coefficients in these three cases permits a certain view of the attained approximation of the true values of these coefficients. The occurring differences are a result of the fact that the valuable observation material does not include the entire earth's surface. If this were the case, then the individual normal equations would contain only one of the coefficients and these would then not be interdependent; the computed ones could thus not be affected by the ones neglected in the expansion, and change with alternating extension of the expansion. It would thus also be formally possible to reduce the mentioned differences by taking into account the unused observations in the polar regions. As long as our knowledge of the geomagnetic force distribution in these regions remains so deficient, particularly in the Southern hemisphere as is presently the case, only little more than an apparent increase in accuracy would be attained by this.

In order to have a completely well-defined analytical expression for further calculations, I rounded off the computed coefficients where possible, to whole units of the introduced unit (γ). This is the case without exception for those of γZ. For those belonging to X and Y which are linked by a number of conditional equations with irrational factors, one of those appearing in the same equation as a function of the others (all of which were rounded off) would always have to be computed and thus would not permit any arbitrary modification. These coefficients are given in the table to two decimal places. Originally I had selected those of the lowest order (see B, p. 25, 26). The values computed under this assumption form table VIII of my previous report (B, p. 57). Later it seemed better to choose one of the latter coefficients of each conditional equation belonging to the highest order, as a function of the others. On the one hand, the corrections occurring in the rounding are smaller in this case than for the first method (as a glance at the conditional equations will show); on the other hand, the coefficients of the first orders are those which analytically define the represented state, whereas the very first one cannot be given accurately as a function of the following one and it is of no importance for the definition of the status.

Thus, the coefficients of the X and Y-series compiled here in table VI_1 do deviate in part from those of the earlier report (B), and the same applies for the computed coefficients of the series for U, W, V and i. (However, due to an untimely discovered failure in B, table XIa, b, p. 60, 61, those values of the computed coefficients $k_1 \ldots K_4$, $l_1 \ldots L_4$ were printed which result from the quantities B, C, D, E rounded off by the second method and reported here. Only for k_o and l_o are the coefficients B and D based on the first type. Let us take this opportunity to correct another minor error which showed up in a repeated, thorough examination of all figures in tables XIa, b, c. The values of M_1 belonging to $v = 80°$ and $v = 100°$ should be changed to -11000.7 and -12784.3).

In the following tables VIIa, b, c we now find the values of the coefficients of the trigonometric series computed from the numbers of VI_1 which represent X, Y, Z, for u = 0°, 5°, 10°, 15°... 180°. The k_o, l_o, m_o independent of the geographic location, are given directly; all others whose numerical values are found in B, are given by their logarithms. They were computed to hundredths of γ and then rounded off to tenths of this unit.

The tables VIIIa, b, c based on this, finally give the values of the force components X, Y, Z in whole units γ for all points separated by 5° longitude (in the calculation, tenths of a unit were carried over).

The three tables VI_1, VIIa, b, c, VIIIa, b, c contain the main result of the present investigation. They present the same distribution of geomagnetic forces in three different forms. However, only VI_1 (which contains the coefficients of a series of spherical functions) defines the distribution without additional information, because accordingly the calculation can be carried out for each point of the earth's surface. The numbers of VII, the coefficients of the trigonometric series belonging to 35 parallel circles (except the poles), define the state of the force initially only for all points of these parallel circles (and for the poles). In VIII finally, this state is shown for 2522 separate, regularly distributed points. All three representations are theoretically entirely equivalent if we add the condition to VII and VIII that the force distribution be expressed by a 6th order (for X, 7th order) series terminated with spherical functions, and for VIII the added condition that the trigonometric development along the geographic longitude be terminated with the functions of the 4-fold angle.

The state defined in three ways is also characterized by the fact that of all possible states, it most closely approximates the state observed by the numbers illustrated in tables III (or IV)--a statement that has a certain, though practically insignificant incongruity that it depends on a specific setting of the weightings.

The main value of the figures reported here lies in the fact that they form a convenient starting point for any future computation of potential based on new material. In order to perform such a computation, one will calculate that value of the measured element for each location where a valuable observation is made; said element results from the analytical representation provided here, and then the difference between observation and calculation--which naturally also contains the secular revision--will be selected as the basis for a refined calculation. (See the discussion by E. Schering in Geogr. Jahrbuch, XV, 1891, p. 143/146, which is still accurate if we proceed in a non-Gaussian manner by using graphic methods or Neumann's formulas).

In order to determine the computed values of X, Y, Z (and thus also those of H, δ, i) for the individual, randomly distributed observation points, it would be best to proceed so that first the tables VIII are expanded for each full degree of longitude and latitude by interpolation, and then to pass to the observation point through a second interpolation. If no greater accuracy is needed for the computed value than about 5 to 10 γ--which should be sufficient with regard to the accuracy of most observations in general--then in the second operation one can get by everywhere with linear interpolations; if more accurate values are desired for individual locations where particularly accurate observations were made, then under all circumstances it is sufficient to take the second differences into account. The first operation, the expansion of table VIII to degree-intervals, does require an interpolation with fourth, and sometimes even with fifth differences if the values are to be obtained to approximately 1γ accuracy. This rather complicated method for a table with double entries will be difficult to apply to the interpolation of a single value and is thus subject to a significant simplification such that the computation is always to be performed for the same, very convenient interval of $\pm$ 1/5, $\pm$ 2/5. The pertinent formula will be derived below and a practical example of its application will be given.

Let Δ_0 be the value of a component belonging to the tabular value of u, let Δ_2 and Δ_4 be the 2nd and 4th difference of the values standing in the same line, Δ_1', Δ_3', Δ_5' and Δ_1'', Δ_3'', Δ_5'' are those of the 1st, 3rd and 5th differences in the preceeding and following spaces. Then we obtain the values belonging to $(u-2^\circ)$, $(u-1^\circ)$, $(u+1^\circ)$, $(u+2^\circ)$ of the quantity to be interpolated by substituting in the expression

$$\Delta_0 + \frac{n}{1} \cdot \frac{\Delta_1' + \Delta_1''}{2} + \frac{n^2}{1.2} \Delta_2 + \frac{n(n^2-1)}{1.2.3} \cdot \frac{\Delta_3' + \Delta_3''}{2} + \frac{n^2(n^2-1)}{1.2.3.4} \Delta_4 + \frac{n(n^2-1)(n^2-4)}{1.2.3.4.5} \cdot \frac{\Delta_5' + \Delta_5''}{2}$$

for n, the series using the values -2/5, -1/5, +1/5, +2/5. We thus obtain the four functional values:

$$A-(B'+B''), \quad C-(D'+D''), \quad C+(D'+D''), \quad A+(B'+B'')$$

if we use as an abbreviation:

$$A = \Delta_0 + \frac{2}{25}\Delta_2 - \frac{7}{1250}\Delta_4, \quad B = \frac{1}{5}\Delta_1 - \frac{7}{250}\Delta_3 + \frac{84}{15625}\Delta_5$$

$$C = \Delta_0 + \frac{1}{50}\Delta_2 - \frac{1}{625}\Delta_4, \quad D = \frac{1}{10}\Delta_1 - \frac{2}{125}\Delta_3 + \frac{99}{31250}\Delta_5$$

(The factors of Δ_4 and Δ_5 can be rounded off without causing any notable error if one does not wish to prepare a few small secondary tables. Equivalences are: $\frac{7}{1250}$, $\frac{84}{15625}$, $\frac{99}{31250}$ with $\frac{1}{180}$, $\frac{1}{190}$, $\frac{1}{320}$.)

As an arbitrary example, let us use the calculation of Z for a few points of the 15° East meridian. The difference outline and the compilation of the auxiliary quantities obtained from the differences then look as follows:

u	Δ_0	Δ_1	Δ_2	Δ_3	Δ_4	Δ_5	A	B	$\mp(B'+B'')$	C	D	$\mp(D'+D'')$
20°	50406											
		−1771										
25°	48635		−12									
		−1783		−184								
30°	46852		−196		−34							
		−1979		−218		50		−389.4			−194.2	
35°	44873		−414		16		44839.8		± 862.1	44864.7		±430.2
		−2393		−202		43		−472.7			−236.0	
40°	42480		−616		59		42430.4		±1070.3	42467.6		±534.5
		−3009		−143		41		−597.6			−298.5	
45°	39471		−759		100							
		−3768		−43								
50°	35703		−802									
		−4570										
55°	31133											

From this, we have for u = 33°, 34°...42°, the following values of Z:

u	Z	u	Z
33°	44839.8+862.1 = 45702	38°	42430.4+1070.3 = 43501
34°	44864.7+430.2 = 45295	39°	42467.6+ 534.5 = 43002
35°	44873	40°	42480
36°	44864.7−430.2 = 44435	41°	42467.6− 534.5 = 41933
37°	44839.8−862.1 = 43978	42°	42430.4−1070.3 = 41360

To check the calculation it is simplest to use the difference series of the found number series. In the present case it turns out that the second differences--which alone have any significant value--are sufficiently regular. The small anomalies in their profile can be attributed to rounding errors. (It may be important to note that the irregularities caused in some cases in the fifth differences of the original series, are intensified somewhat by a special circumstance. Strictly speaking, the numbers cited in the tables are not exactly equidistant because they (see p. 4) are derived from the values of v rounded to whole arc-seconds; their attendant u differs from the round numbers used as arguments for the tables. This difference is a few tenths of an arc-second. Practically speaking, this inaccuracy is meaningless; for none of the computed values of X, Y or Z does it cause an error of 0.5γ).

Perhaps even more convenient, though it contains three successive interpolations, is another method which is somewhat better than the above method, at least for regions with numerous and accurate observations. It consists in first finding the functional values for the middle of the 5°-intervals by interpolation, and then interpolating for every $\frac{1}{2}^{\circ}$. The first operation where the differences of uneven ordering numbers drop out, need only be carried out to the fourth difference, the second operation where the above-developed formulas are used, is carried out only to the third difference. The last interpolation within the $\frac{1}{2}^{\circ}$-interval which leads to the values for the individual observation points, is performed linearly only. Through the repeated interpolations (longitude and latitude) a pile-up of rounding errors occurs, in addition to the errors due to neglect of the higher differences; the total uncertainty of the final values will then not generally exceed 2 to 3 γ.

If Δ_0 and Δ'_0 are two sequential functional values (belonging to u, λ and $u+5^{\circ}, \lambda$ or to u, λ and $u, \lambda+5^{\circ}$) and $\Delta_2, \Delta'_2, \Delta_4, \Delta'_4$ are the second and fourth differences standing in the same line with them, then the interpolation for the middle of the interval gives the value:

$$\frac{\Delta_0+\Delta'_0}{2} - \frac{1}{8}\cdot\frac{\Delta_2+\Delta'_2}{2} + \frac{8}{128}\cdot\frac{\Delta_4+\Delta'_4}{2}$$

or when

$$\Delta_0 - \frac{1}{8}\left(\Delta_2 - \frac{8}{16}\Delta_4\right) = P, \quad \Delta'_0 - \frac{1}{8}\left(\Delta'_2 - \frac{8}{16}\Delta'_4\right) = P'$$

is used, $\frac{1}{2}(P + P')$.

The next interpolation occurs, as mentioned, by using the earlier formulas which are simplified by elimination of Δ_1. We thus have:

$$A=\Delta_0+\frac{2}{25}\Delta_2,\quad B=\frac{1}{5}\Delta_1-\frac{7}{250}\Delta_3,\quad C=\Delta_0+\frac{1}{50}\Delta_2,\quad D=\frac{1}{10}\Delta_1-\frac{2}{125}\Delta_3$$

and the following values must be formed:

$$A-(B'+B''),\quad C-(D'+D''),\quad C+(D'+D''),\quad A+(B'+B'')$$

An example may serve to illustrate the method. The values of Z reported above give the following values of P for u = 30°, 35°, 40° and 45°:

46875.7 44925.1 42558.4 39568.2

The average values of the sequential $\frac{1}{2}(P+P')$ are:

45900 43742 41063

and denote the values of Z for 32.5°, 37.5°, 42.5°.

The following outline contains an additional interpolation for all intermediate points from ½° to ½°.

u	Δ_0	Δ_1	Δ_2	Δ_3	A	B	$\mp(B'+B'')$	C	D	$\mp(D'+D'')$
30°0	46852									
		−952								
32°5	45900		−75							
		−1027		−29		−204.6			−102.3	
35°0	44873		−104		44864.7		±430.0	44870.9		±215.0
		−1131		−27		−225.4			−112.7	
37°5	43742		−131		43731.5		±477.1	43739.4		±238.5
		−1262		−24		−251.7			−125.8	
40°0	42480		−155		42467.6		±534.5	42476.9		±267.2
		−1417		−20		−282.6			−141.4	
42°5	41063		−175							
		−1592								
45°0	39471									

The following table results from this; it fully agrees with the results of the earlier calculation:

u	Z	u	Z	u	Z
34°0	45295	36°5	44209	39°0	43002
34°5	45086	37°0	43978	39°5	42744
35°0	44873	37°5	43742	40°0	42480
35°5	44656	38°0	43501	40°5	42210
36°0	44435	38°5	43254	41°0	41938

The differences formed to check the calculation exhibit a quite regular profile.

Calculation of the Coefficients of the Potential

From the series computed above for $\alpha X \sin v$ and $\beta Y \sin v$, the potential of the horizontal force, provided one exists, can be determined and illustrated likewise in a closed form. For this purpose we have to calculate the functions:

$$U = \int_0^v \alpha X dv = U_0 + f(v, \lambda), \qquad W = \psi(v) - \int_0^\lambda \beta Y \sin v \, d\lambda = W_0 + \chi(v) . \lambda$$

which generally each contain a part ($f(v, \lambda)$ and $\chi(v) . \lambda$) which cannot be represented by a finite series of spherical functions. $\psi(v)$ denotes the λ-independent part of U_0 which can be expressed by spherical functions (see B, p. 9).

If it turns out that U = W, in which case f and χ always disappear, then the entire magnetic horizontal force at the earth's surface can be defined by a potential which is determined by

$$V = bU = bW$$

with b = 6,365 $\cdot 10^8$ cm as the polar radius of the earth. In fact, we then have:

$$X = \frac{1}{\alpha}\frac{\partial U}{\partial v} = \frac{1}{\alpha b}\frac{\partial V}{\partial v}, \quad Y = -\frac{1}{\beta \sin v}\frac{\partial W}{\partial \lambda} = -\frac{1}{\beta b \sin v}\frac{\partial V}{\partial \lambda}$$

as it must be according to the definition of the potential (see A,p. 7).

If U and W are not equal, then that part of the force to which a potential is ascribed, remains undefined to a certain extent (see A, p. 17). If we take it to be as large as possible--which is evidently useful, even though not entirely sufficient for an unequivocal definition--then in the simplest case we set:

$$V = \frac{b}{2}(U_0 + W_0)$$

To characterize that part f the horizontal force to which no potential will correspon, we use the statement of the difference (W-U), through which ($W_0 - U_0$) is given with consideration to the 'a priori' specified form of f and χ.

The determination of U now takes place by means of the following formulas (see A, p. 20; B, p. 12).

As abbreviation, we write:

$$\alpha X \sin v = \sum A_m^n R_m^n, \text{ i.e. } A_m^n = B_m^n \cos m\lambda + C_m^n \sin m\lambda$$

and let:

$$\lambda_m^0 = 1, \quad \lambda_m^1 = 1, \quad \lambda_m^p = \lambda_m^{p-2} \frac{(m+p)(2m+p-1)(p-1)}{(m+p-1)(2m+2p-3)(2m+2p-1)}$$

$$\mu_m^p = \lambda_m^p r_m^{m+p}, \quad \nu_m^p = (m+p-1)\lambda_m^p r_m^{m+p-1}$$

$$\Pi_m = \int_0^v \frac{P_m^m(\cos v)}{\sin v} dv$$

so that:

$$\Pi_1 = v \qquad \Pi_2 = 1 - \cos v$$

$$\Pi_3 = \frac{1}{2}v - \frac{1}{4}\sin 2v \qquad \Pi_4 = \frac{2}{3} - \frac{3}{4}\cos v + \frac{1}{12}\cos 3v$$

$$\Pi_5 = \frac{3}{8}v - \frac{1}{4}\sin 2v + \frac{1}{32}\sin 4v \qquad \Pi_6 = \frac{8}{15} - \frac{5}{8}\cos v + \frac{5}{48}\cos 3v - \frac{1}{80}\cos 5v$$

Then: $$U = f(v,\lambda) + U_0 = \sum \pi_m \Pi_m + \sum F_m^n R_m^n$$

if we compute the quantities:

$$\pi_m = \eta_m \cos m\lambda + \zeta_m \sin m\lambda, \quad F_m^n = G_m^n \cos m\lambda + H_m^n \sin m\lambda$$

from the following equations:

$$\begin{aligned}
\pi_m &= \mu_m^0 A_m^m + \mu_m^2 A_m^{m+2} + \mu_m^4 A_m^{m+4} + \dots \\
\nu_m^2 F_m^{m+1} &= \mu_m^2 A_m^{m+2} + \mu_m^4 A_m^{m+4} + \dots \\
\nu_m^4 F_m^{m+3} &= \mu_m^4 A_m^{m+4} + \dots \\
&\dots\dots\dots \\
\nu_m^1 F_m^m &= \mu_m^1 A_m^{m+1} + \mu_m^3 A_m^{m+3} + \mu_m^5 A_m^{m+5} + \dots \\
\nu_m^3 F_m^{m+2} &= \mu_m^3 A_m^{m+3} + \mu_m^5 A_m^{m+5} + \dots \\
\nu_m^5 F_m^{m+4} &= \mu_m^5 A_m^{m+5} + \dots \\
&\dots\dots\dots
\end{aligned}$$

For m = 0 it is easy to see that μ_m^p equals the corresponding constant α_0^p introduced above (p. 14) which appears in the conditional equations for the coefficients B_0^n and C_0^n. By virtue of these conditional equations, we evidently obtain:

$$\pi_0 = 0, \quad F_0^0 = 0.$$

The other equations run as follows after introduction of the numerical values of μ and ν.

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$$F_0^1 = [0.2870156]\,A_0^2 + [0.4146518]\,A_0^4 + [0.4945023]\,A_0^6 + \ldots.$$
$$F_0^3 = [9.8204890]\,A_0^4 + [9.9008995]\,A_0^6 + \ldots.$$
$$F_0^5 = [9.6005468]\,A_0^6 + \ldots.$$
$$F_0^2 = [9.9988827]\,A_0^3 + [0.0920806]\,A_0^5 + [0.1598794]\,A_0^7 + \ldots.$$
$$F_0^4 = [9.6967881]\,A_0^5 + [9.7641869]\,A_0^7 + \ldots.$$
$$F_0^6 = [9.5217680]\,A_0^7 + \ldots.$$

$$\pi_1 = [0.2385607]\,A_1^1 + [0.3856560]\,A_1^3 + [0.4786538]\,A_1^5 + [0.5443627]\,A_1^7 + \ldots.$$
$$F_1^2 = [0.0194590]\,A_1^3 + [0.1124568]\,A_1^5 + [0.1781657]\,A_1^7 + \ldots.$$
$$F_1^4 = [9.7056520]\,A_1^5 + [9.7718614]\,A_1^7 + \ldots.$$
$$F_1^6 = [9.5262454]\,A_1^7 + \ldots.$$
$$F_1^1 = [0.3494850]\,A_1^2 + [0.4655598]\,A_1^4 + [0.5426626]\,A_1^6 + \ldots.$$
$$F_1^3 = [9.8845038]\,A_1^4 + [9.9116066]\,A_1^6 + \ldots.$$
$$F_1^5 = [9.6066640]\,A_1^6 + \ldots.$$

$$\pi_2 = [0.2870156]\,A_2^2 + [0.3494850]\,A_2^4 + [0.4170518]\,A_2^6 + \ldots.$$
$$F_2^3 = [9.8829583]\,A_2^4 + [9.9505246]\,A_2^6 + \ldots.$$
$$F_2^5 = [9{,}6261229]\,A_2^6 + \ldots.$$
$$F_2^2 = [0.1215190]\,A_2^3 + [0.1950573]\,A_2^5 + [0.2553220]\,A_2^7 + \ldots.$$
$$F_2^4 = [9.7846480]\,A_2^5 + [9.7949127]\,A_2^7 + \ldots.$$
$$F_2^6 = [9.5402598]\,A_2^7 + \ldots.$$

$$\pi_3 = [0.3204890]\,A_3^3 + [0.3360354]\,A_3^5 + [0.3849838]\,A_3^7 + \ldots.$$
$$F_3^4 = [9.7936976]\,A_3^5 + [9.8426455]\,A_3^7 + \ldots.$$
$$F_3^6 = [9.5658860]\,A_3^7 + \ldots.$$
$$F_3^3 = [0.0000000]\,A_3^4 + [0.0454098]\,A_3^6 + \ldots.$$
$$F_3^5 = [9.6630161]\,A_3^6 + \ldots.$$

$$\pi_4 = [0.3460658]\,A_4^4 + [0.3817082]\,A_4^6 + \ldots.$$
$$F_4^5 = [9.7281829]\,A_4^6 + \ldots.$$
$$F_4^4 = [9.9186864]\,A_4^5 + [9.9441904]\,A_4^7 + \ldots.$$
$$F_4^6 = [9.6076091]\,A_4^7 + \ldots.$$

[See note below*]

*The numerical formulas compiled here are not given in the form used in the general presentation in order to simplify the usual logarithmic computation. But they can naturally be converted into this form immediately and this would be an advantage for a numerical calculation when using addition logarithms.

These equations whose coefficients were given as more highly accurate, permanently valid values than would be necessary for present purposes, show that the series for $\alpha X \sin \beta$ has to be developed down to 7th order terms if those for U are to be obtained to the 6th order (equal to those for Y, Z, W).

Now if we set the numerical values reported in table VI into the above, general formulas for the coefficients A^n_m (i.e. $B^n_m \cos m\lambda + C^n_m \sin m\lambda$) , then we obtain the coefficients of U specified in IX. For the case of the farthest expansion of the series, we also find these values printed in B, p. 58, table IX. The small differences existing in the last decimal place between the two statements are due to the different rounding of B^n_m and C^n_m (see p. 18).

The calculation of the coefficients of $W-\psi(v)$ from those of $\beta Y \sin v$ is very simple--in contrast to the derivation of U above. Evidently, since:

$$W-\psi(v) = -\int_0^\lambda \beta Y \sin v \, d\lambda$$

and

$$\beta Y \sin v = \sum_{n=0}^{n=\nu} \sum_{m=0}^{m=n} (C^n_m \cos m\lambda + D^n_m \sin m\lambda) E^n_m$$

then:

$$W-\psi(v) = -\lambda \sum_{n=0}^{n=\nu} C^n_0 E^n_0 + \sum_{n=1}^{n=\nu} \sum_{m=1}^{m=n} \frac{1}{m} (D^n_m \cos m\lambda - C^n_m \sin m\lambda) E^n_m$$

The function $\psi(v)$ which appears as an integration constant with respect to λ, is as already remarked, the part in U_0 free of λ:

$$\psi(v) = \sum_{n=1}^{n=\nu} F^n_0 E^n_0$$

By means of these expressions, W breaks down, like U, into a finite series of spherical functions W_0 and into a part which cannot be represented in this form:

$$\chi(v) . \lambda = -\lambda \sum_{n=0}^{n=\nu} C^n_0 E^n_0$$

These latter, as well as the coefficients of W_0, are also found in table IX, which further contains the coefficients of $\frac{1}{2}(U_0 + W_0)$, i.e. of V:b.

The two functions U and W or the equivalent expressions:

$$V:b = \sum (g_m^n \cos m\lambda + h_m^n \sin m\lambda) R_m^n = \tfrac{1}{2}(U_0 + W_0)$$

and

$$W - U = (W_0 - U_0) + \chi(v).\lambda - f(v, \lambda)$$

together determine the state of the magnetic field in the earth's surface uniquely and completely, as is done by $\alpha X \sin v$ and $\beta Y \sin v$ together.

V, the potential of the horizontal forces at the earth's surface, can be broken down into two parts, V_i and V_a, by means of the expression found for γZ; these parts represent potentials originating from agents in the interior of the earth and those in outer space. This is done by means of the formulas (see A, p. 23; B, p. 13):

$$V_i = b \sum (c_m^n \cos m\lambda + s_m^n \sin m\lambda) R_m^n$$

$$V_a = b \sum (\gamma_m^n \cos m\lambda + \sigma_m^n \sin m\lambda) R_m^n$$

with

$$c_m^n = \epsilon_m^n g_m^n - \delta_m^n j_m^n \qquad s_m^n = \epsilon_m^n h_m^n - \delta_m^n k_m^n$$

$$\gamma_m^n = g_m^n - c_m^n \qquad \sigma_m^n = h_m^n - s_m^n$$

The constants ϵ_m^n and δ_m^n appearing herein, depend on the flattening of the earth. For the case of a sphere, we have:

$$\epsilon_m^n = \frac{n}{2n+1} \qquad \delta_m^n = \frac{1}{2n+1}$$

The only slightly different values which result from using the flattening derived by Bessel (1:299.1528), are found in B, p. 46, 47. Their logarithms are contained in the following compilation.

m; n:	0	1	2	3	4	5	6
				$\log \mathfrak{E}_m^n$			
0	0.0019384	9.5240411	9.3024156	9.1563229	9.0471927	8.9600494	8.8875019
1		9.5252064	9.3020905	9.1564525	9.0472680	8.9600987	8.8875372
2			9.3035222	9.1568398	9.0474942	8.9602518	8.8876427
3				9.1574867	9.0478716	8.9604954	8.8878186
4					9.0484010	8.9608434	8.8880650
5						8.9612914	8.8883828
6							8.8887707
				$\log e_m^n$			
0	$-\infty$	9.5240411	9.6024739	9.6322799	9.6480059	9.6577263	9.6648309
1		9.5222937	9.6022689	9.6322169	9.6479780	9.6577112	9.6648222
2			9.6016439	9.6320244	9.6478933	9.6576707	9.6642961
3				9.6316994	9.6477521	9.6575926	9.6642523
4					9.6475528	9.6574881	9.6641908
5						9.6573532	9.6641110
6							9.6640138

The resulting series derived for V_i:b and V_a:b have the coefficients specified in table X. The same table also contains the presentation of a function $\alpha\beta bi$ obtained from (W-U) and which conversely is sufficient for the determination of (W-U) so that the data contained in X provides a new, complete and unique representation of the geomagnetic field. According to B, p. 13, we have:

$$\alpha\beta bi = \frac{1}{4\pi \sin v} \cdot \frac{\partial^2 (W-U)}{\partial v \, \partial \lambda} = -\frac{1}{4\pi b \sin v} \left[\frac{\partial \alpha X}{\partial \lambda} + \frac{\partial \beta Y \sin v}{\partial v} \right]$$

The function i introduced here, means that an electrical current penetrating perpendicularly into the earth's surface having surface density i, would generate just that part of the magnetic, horizontal force (also expressed in (W-U)) which is not generated from the potential V. Since the unit of the numbers given in X is equal to $0.1^4 cm^{-\frac{1}{2}} g^{\frac{1}{2}} s^{-1}$, i.e. 0.1^4 Ampere $\cdot$ cm^{-1} and furthermore, b = $6.356 \cdot 10^8$cm, then we obtain from these figures the current strength (or rather the $\alpha\beta$-fold multiple of it which differs insignificantly) in the unit Ampere:km^2, if it is multiplied with $0.1^4 . 10^{10} : (6.356 . 10^8)$, i.e. with 1:635.6 or 0.001573. Note also that positive values of i would indicate a downward-directed flow.

To derive $\alpha\beta bi$ from (W-U) we use the following formulas whose numerical coefficients which are again given to 7 places for the sake of overall accuracy, even though at the present at most 4 places are needed.

$$\frac{1}{\sin v}\frac{dR_0^1}{dv} = -[0.2385607]^{*} R_0^2$$
$$\frac{1}{\sin v}\frac{dR_0^3}{dv} = -[0.4225490] R_0^2 - [0.7720840] R_0^4$$
$$\frac{1}{\sin v}\frac{dR_0^5}{dv} = -[0.5206968] R_0^2 - [0.8701818] R_0^4 - [0.9978176] R_0^6$$

$$\frac{1}{\sin v}\frac{dR_0^2}{dv} = -[0.5880456] R_0^1$$
$$\frac{1}{\sin v}\frac{dR_0^4}{dv} = -[0.7156819] R_0^1 - [0.8996702]^{*} R_0^3$$
$$\frac{1}{\sin v}\frac{dR_0^6}{dv} = -[0.7955828] R_0^1 - [0.9795207] R_0^3 - [1.0776680] R_0^5$$

$$\frac{1}{\sin v}\frac{d\Pi_1}{dv} = [0.0000000] \sin v^{-1}$$
$$\frac{1}{\sin v}\frac{dR_1^2}{dv} = [0.5880456] \sin v^{-1} - [0.6505150] R_1^1$$
$$\frac{1}{\sin v}\frac{dR_1^4}{dv} = [0.9771218] \sin v^{-1} - [0.9560445] R_1^1 - [0.9186846] R_1^3$$
$$\frac{1}{\sin v}\frac{dR_1^6}{dv} = [1.2160614] \sin v^{-1} - [1.1758158] R_1^1 - [1.0596692] R_1^3 - [1.0887858] R_1^5$$

$$\frac{1}{\sin v}\frac{dR_1^1}{dv} = [0.2385607]^{*} \cos v \sin v^{-1}$$
$$\frac{1}{\sin v}\frac{dR_1^3}{dv} = [0.8116247] \cos v \sin v^{-1} - [0.7976108] R_1^2$$
$$\frac{1}{\sin v}\frac{dR_1^5}{dv} = [1.1067420] \cos v \sin v^{-1} - [0.9978176] R_1^2 - [1.0066820] R_1^4$$

$$\frac{1}{\sin v}\frac{d\Pi_2}{dv} = [0.0000000]$$
$$\frac{1}{\sin v}\frac{dR_2^3}{dv} = [1.0105947] - [0.8996708] R_2^2$$
$$\frac{1}{\sin v}\frac{dR_2^5}{dv} = [1.5812910] - [1.3584196] R_2^2 - [1.0856780] R_2^4$$

$$\frac{1}{\sin v}\frac{dR_2^2}{dv} = [0.5880456] \cos v$$
$$\frac{1}{\sin v}\frac{dR_2^4}{dv} = [1.3087275] \cos v - [0.9621896] R_2^3$$
$$\frac{1}{\sin v}\frac{dR_2^6}{dv} = [1.7160818] \cos v - [1.2815178] R_2^3 - [1.1082442] R_2^5$$

$$\frac{1}{\sin v}\frac{d\Pi_3}{dv} = [0.0000000] \sin v$$
$$\frac{1}{\sin v}\frac{dR_3^4}{dv} = [1.2747816] \sin v - [1.0791818] R_3^3$$
$$\frac{1}{\sin v}\frac{dR_3^6}{dv} = [1.9341725] \sin v - [1.6457700] R_3^3 - [1.1401878] R_3^5$$

$$\frac{1}{\sin v}\frac{dR_3^3}{dv} = [0.7976108] \cos v \sin v$$
$$\frac{1}{\sin v}\frac{dR_3^5}{dv} = [1.6193866] \cos v \sin v - [1.0947276] R_3^4$$

*Change 0.2385607 to read 0.2385606 and change 0.8996702 to read 0.8996703.

$$\frac{1}{\sin v}\frac{dD_4^1}{dv} = [0.0000000] \sin v^4$$
$$\frac{1}{\sin v}\frac{dR_4^1}{dv} = [1.4688217] \sin v^4 - [1.2196664] E_4^1$$
$$\frac{1}{\sin v}\frac{dR_5^1}{dv} = [0.9481255] \cos v \sin v^4$$
$$\frac{1}{\sin v}\frac{dE_5^1}{dv} = [1.8548619] \cos v \sin v^4 - [1.2068042] E_4^1$$

It is easy to see that when substituting these expressions into $\frac{1}{\sin v}\frac{\partial(W-U)}{\partial v}$, the coefficients of $\sin v^{-7}$, $\cos v \sin v^{-1}$, ... $\cos v \sin v^3$ become aggregates of B_m^n, C_m^n, D_m^n, E_m^n which disappear due to the identical conditional equations valid for the expansion of $\alpha X \sin v$ and $\beta Y \sin v$. For $\alpha\beta bi$ we thus have a finite series of spherical functions comprising the first five orders here. That the coefficient of R_o^o is equal to zero therein (likewise as in the series for V_i:b and V_a:b) expresses the fact that the algebraic sum of all the currents penetrating the earth's surface, i.e. the integral of $i\,dw$ taken over the entire surface, disappears. The surface element dw is equal to $\alpha\beta b^2 \sin v\, dv\, d\lambda$. This explains that the expansion is obtained not for i, but for αi, i.e. $i\sqrt{1+e^2\cos v^2}$ --except for constant factors.

Due to the results reported above, the problem to be solved has been completed. However, we still have to investigate the validity of the obtained results. Since this has already been done in sufficient detail in my preliminary report (see B, p. 34-43), I do not believe it necessary to go into detail again here, but I will limit the discussion to a brief presentation of results.

It turns out that the presentation of the force distribution defined by $\alpha X \sin v$, $\beta Y \sin v$, γZ can be viewed as generally satisfactory, but that the derived functions V_a:b, V_i:b and $\alpha\beta bi$ are affected by such considerable uncertainty that only the first, absolutely large coefficient of V_i:b can be viewed as determined with sufficient accuracy. V_a:b and $\alpha\beta bi$ however, which assume small values only, can hardly be viewed with enough certainty to believe in their existance at all. The reason for this considerable uncertainty lies mainly in the fact that the individual series coefficients cannot be calculated independently of each other; this would require a knowledge of the force distribution over the entire earth's surface which we do not now have. Even though I still felt justified in giving a positive result in my preliminary report, this was done in the final analysis only through assuming very large and broad systematic errors in the observations of quantities to be explained and regular distribution of the value of l_o (see B, p. 43). Thus this decision is based primarily on the same principles used by L.A. Bauer (Terr. Magn. Vol. II, p. 11) and v. Bezold (Berlin Sitz.-Ber. f. 1887, p. 414) in their discussions. The investigations since published by Schuster, Rucker et al. reinforce the weight of the counterarguments so much that the real existance of V_a and of i have to be designated as at least quite doubtful, and that the reliability of the empirical fundamentals to be evaluated in any new calculation of potential appear in fact to be significantly affected by systematic errors. This applies in particular for i which can be determined independently for each geomagnetically accurately investigated part of the earth's surface, and its calculation using the Rucker method now permits a check of the results obtained here (see A, p. 16). Just this possibility for a mutual verification--which would lose its significance only if we had a complete empirical knowledge of the distribution of the earth's magnetism--indicates the desirability of refining the expansions given here for the entire earth's surface; this would also be needed for a more accurate determination of V_a.

The most important task of future research related to the determination of the spatial force distribution of the earth's magnetism, must deal with filling the large gaps, especially in the polar regions, and then on the oceans and continental interiors.

I. Logarithms of the Functions $R_n^m(\cos v)$.

$(tg\,v = \sqrt{1+\epsilon^2}\,tg\,u = [0.0014342]\,tg\,u)$.

u	$\log R_0^1$	$\log R_1^1$	$\log R_0^2$	$\log R_1^2$	$\log R_2^2$	$\log R_0^3$	$\log R_1^3$	
0°	0.2385607	$-\infty$	0.3494850	$-\infty$	$-\infty$	0.4225490	$-\infty$	180°
5	0.2368939	9.1802983	0.3444748	9.5281164	8.1704908	0.4124994	9.7491911	175
10	0.2318683	9.4796376	0.3292498	9.8224302	8.7691695	0.3815877	0.0359051	170
15	0.2234067	9.6529141	0.3031911	9.9872451	9.1157224	0.3272477	0.1877507	165
20	0.2113761	9.7738945	0.2651457	0.0961949	9.3576833	0.2440608	0.2778627	160
25	0.1955758	9.8657035	0.2131723	0.1722036	9.5413013	0.1207326	0.3283553	155
30	0.1757273	9.9386193	0.1440536	0.2252708	9.6871329	9.9300928	0.3479629	150
35	0.1514453	9.9981276	0.0521680	0.2604972	9.8061496	9.5755371	0.3397636	145
40	0.1222130	0.0474799	9.9267475	0.2806172	9.9048541	8.8744021$_n$	0.3031405	140
45	0.0873181	0.0887709	9.7430448	0.2870133	9.9874361	9.6764504$_n$	0.2334380	135
50	0.0457736	0.1234144	9.4188925	0.2801123	0.0567230	9.9028911$_n$	0.1188076	130
55	9.9961761	0.1524021	8.2904097$_n$	0.2595025	0.1146985	0.0134475$_n$	9.9279364	125
60	9.9364385	0.1764542	9.4528835$_n$	0.2238170	0.1628027	0.0639885$_n$	9.5343645	120
65	9.8633149	0.1960952	9.7178833$_n$	0.1703344	0.2020847	0.0709396$_n$	9.2157973$_n$	115
70	9.7713260	0.2117164	9.8621291$_n$	0.0939667	0.2333270	0.0378423$_n$	9.8044843$_n$	110
75	9.6502031	0.2236014	9.9517010$_n$	9.9847288	0.2570970	9.9592078$_n$	0.0188256$_n$	105
80	9.4768195	0.2319559	0.0075561$_n$	9.8196997	0.2738061	9.8146544$_n$	0.1324854$_n$	100
85	9.1774103	0.2369160	0.0385108$_n$	9.5252506	0.2837262	9.5319934$_n$	0.1912176$_n$	95
90	$-\infty$	0.2385607	0.0484550$_n$	$-\infty$	0.2870157	$-\infty$	0.2095647$_n$	90
	$\log(-R_0^1)$	$\log R_1^1$	$\log R_0^2$	$\log(-R_1^2)$	$\log R_2^2$	$\log(-R_0^3)$	$\log R_1^3$	u

(Continuation of I)

u	$\log R_2^3$	$\log R_3^3$	$\log R_0^4$	$\log R_1^4$	$\log R_2^4$	$\log R_3^4$	$\log R_4^4$	
0°	$-\infty$	$-\infty$	0.477121	$-\infty$	$-\infty$	$-\infty$	$-\infty$	180°
5	8.5913730	7.1457017	0.460306	9.911341	8.882261	7.621156	6.113016	175
10	9.1850261	8.0437198	0.407704	0.187804	9.469195	8.514149	7.310373	170
15	9.5231175	8.5635491	0.311442	0.321808	9.795833	9.025516	8.003479	165
20	9.7530477	8.9264903	0.151270	0.385133	0.009235	9.376427	8.487400	160
25	9.9208654	9.2019174	9.862996	0.397423	0.154872	9.636054	8.854637	155
30	0.0468484	9.4206648	8.776192	0.362769	0.252153	9.834953	9.146300	150
35	0.1415832	9.5991898	9.719059$_n$	0.274551	0.310316	9.989196	9.384333	145
40	0.2110554	9.7472467	9.983953$_n$	0.107603	0.333242	0.108020	9.581742	140
45	0.2587425	9.8711196	0.087018$_n$	9.762709	0.320868	0.196998	9.746906	135
50	0.2864849	9.9750500	0.107859$_n$	9.14314$\bar{5}_n$	0.268613	0.259384	9.885480	130
55	0.2948629	0.0620132	0.061330$_n$	9.896182$_n$	0.163694	0.2967$\bar{5}$0	0.001431	125
60	0.2832294	0.1341695	9.935148$_n$	0.110783$_n$	9.970280	0.309169	0.097639	120
65	0.2493879	0.1930924	9.661993$_n$	0.202026$_n$	9.525913	0.294968	0.176203	115
70	0.1886413	0.2399560	7.759087$_n$	0.220631$_n$	9.440352$_n$	0.249843	0.238688	110
75	0.0912884	0.2756110	9.637733	0.175552$_n$	9.922172$_n$	0.16437$\bar{5}$	0.286228	105
80	9.9346139	0.3006746	9.902953	0.05236$\bar{5}_n$	0.109124$_n$	0.01605$\bar{5}$	0.319646	100
85	9.6451248	0.3155548	0.017392	9 781673$_n$	0.197669$_n$	9.731526	0.339486	95
90	$-\infty$	0.3204890	0.051153	$-\infty$	0.224546$_n$	$-\infty$	0.346065	90
	$\log(-R_2^3)$	$\log R_3^3$	$\log R_0^4$	$\log(-R_1^4)$	$\log R_2^4$	$\log(-R_3^4)$	$\log R_4^4$	u

(Continuation of I)

u	log R^5_0	log R^5_1	log R^5_2	log R^5_3	log R^5_4	log R^5_5	log R^6_0	
0°	0.520697	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	0.55697	180°
5	0.49535$\bar{5}$0	0.038766	9.107059	7.963676	6.632045	5.0754$\bar{5}$0	0.52127	175
10	0.414265	0.302221	9.685487	8.850359	7.824377	6.572146	0.40378	170
15	0.256769	0.412958	9.997520	9.351026	8.509021	7.438529	0.15378	165
20	9.948884	0.439608	0.18940$\bar{5}$	9.686547	8.980912	8.043431	9.38983	160
25	7.975588$_n$	0.394940	0.305249	9.925625	9.332348	8.502476	9.87869$_n$	155
30	9.875583$_n$	0.265649	0.362113	0.098114	9.604163	8.86705$\bar{5}$	0.13188$_n$	150
35	0.089929$_n$	9.982352	0.364963	0.219028	9.817914	9.164596	0.17071$_n$	145
40	0.143633$_n$	7.078336$_n$	0.309014	0.295986	9.986091	9.411358	0.06836$_n$	140
45	0.093537$_n$	9.937618$_n$	0.172999	0.331952	0.116360	9.617813	9.71711$_n$	135
50	9.921477$_n$	0.171438$_n$	9.876589	0.325689	0.213389	9.791030	9.33603	130
55	9.442768$_n$	0.244200$_n$	8.756484$_n$	0.270157	0.279743	9.935969	9.92288	125
60	9.487472	0.216558$_n$	9.907324$_n$	0.145776	0.316213	0.056229	0.0674$\bar{5}$	120
65	9.900740	0.079218$_n$	0.138483$_n$	9.888895	0.321654	0.154434	0.05241	115
70	0.037531	9.715417$_n$	0.221982$_n$	8.826804	0.2921$\bar{5}$0	0.232540	9.87317	110
75	0.055342	9.397713	0.214201$_n$	9.797241$_n$	0.218567	0.291965	9.17378	105
80	9.968453	9.976786	0.113381$_n$	0.082857$_n$	0.078601	0.333738	9.68141$_n$	100
85	9.717087	0.156100	9.854847$_n$	0.203648$_n$	9.799032	0.358538	9.97874$_n$	95
90	$-\infty$	0.205652	$-\infty$	0.239125$_n$	$-\infty$	0.366762	0.05182$_n$	90
	log $(-R^5_0)$	log R^5_1	log $(-R^5_2)$	log R^5_3	log $(-R^5_4)$	log R^5_5	log R^6_0	u

(Continuation of I)

u	log R^6_1	log R^6_2	log R^6_3	log R^6_4	log R^6_5	log R^6_6	log R^7_0	
0°	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	0.58805	180°
5	0.14305	9.29051	8.23601	7.01559	5.63075	4.03457	0.54011	175
10	0.39060	9.85865	9.11508	8.20208	7.12243	5.83060	0.3763$\bar{5}$	170
15	0.47184	0.15271	9.60271	8.87648	7.9803$\bar{5}$	6.87026	9.97165	165
20	0.44853	0.31737	9.91922	9.33366	8.57322	7.5961$\bar{5}$	9.63345$_n$	160
25	0.31397	0.39376	0.13233	9.66551	9.01646	8.14700	0.12758$_n$	155
30	9.96843	0.39286	0.27032	9.91227	9.36119	8.584$\bar{5}$0	0.20059$_n$	150
35	9.45431$_n$	0.30509	0.34551	0.09458	9.63445	8.9415$\bar{5}$	0.07449$_n$	145
40	0.10615$_n$	0.07814	0.36058	0.22358	9.85198	9.23766	9.57110$_n$	140
45	0.25822$_n$	9.2655$\bar{5}$	0.30808	0.30480	0.02354	9.48540	9.70528	135
50	0.25188$_n$	9.89569$_n$	0.15960	0.33935	0.15521	9.69326	0.04642	130
55	0.09558$_n$	0.17271$_n$	9.79317	0.32331	0.25056	9.86719	0.09108	125
60	9.57454$_n$	0.24574$_n$	9.44679$_n$	0.24370	0.31108	0.01150	9.93175	120
65	9.75000	0.19099$_n$	0.03089$_n$	0.06216	0.33616	0.1293$\bar{5}$	9.18237	115
70	0.1126$\bar{5}$	9.97009$_n$	0.20194$_n$	9.59099	0.32228	0.22308	9.76588$_n$	110
75	0.20986	8.89930$_n$	0.23632$_n$	9.62037$_n$	0.26058	0.29439	0.02537$_n$	105
80	0.16269	9.89024	0.15899$_n$	0.05248$_n$	0.12897	0.34451	0.0400$\bar{5}_n$	100
85	9.93121	0.14719	9.91281$_n$	0.20831$_n$	9.85436	0.37427	9.83682$_n$	95
90	$-\infty$	0.21293	$-\infty$	0.25252$_n$	$-\infty$	0.38414	$-\infty$	90
	log $(-R^6_1)$	log R^6_2	log $(-R^6_3)$	log R^6_4	log $(-R^6_5)$	log R^6_6	log $(-R^7_0)$	u

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(Continuation of I)

u	$\log R_1^1$	$\log R_2^1$	$\log R_3^1$	$\log R_4^1$	$\log R_5^1$	$\log R_6^1$	$\log R_7^1$	
0°	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	180°
5	0.23068	9.44531	8.46312	7.32668	6.04696	4.6209$\bar{5}$	2.99129	175
10	0.45923	0.00133	9.33325	8.50586	7.5347$\bar{5}$	6.41196	5.08666	170
15	0.50353	0.27376	9.80543	9.16817	8.38272	7.44315	6.29960	165
20	0.41062	0.40443	0.09891	9.60779	8.96134	8.15701	7.14646	160
25	0.10921	0.42813	0.27963	9.9158$\bar{5}$	9.38563	8.69206	7.78912	155
30	9.33543$_n$	0.33923	0.37247	0.13189	9.70621	9.10971	8.29953	150
35	0.15439$_n$	0.06328	0.38280	0.27305	9.9492$\bar{5}$	9.44247	8.71609	145
40	0.29394$_n$	8.99844$_n$	0.29619	0.34854	0.12934	9.70936	9.06156	140
45	0.23421$_n$	0.07765$_n$	0.04450	0.35669	0.25440	9.92221	9.35060	135
50	9.91683$_n$	0.25320$_n$	8.16390	0.28346	0.32748	0.08852	9.59310	130
55	9.49452	0.2347$\bar{5}_n$	0.00100$_n$	0.07829	0.34682	0.21285	9.79601	125
60	0.10188	0.01399$_n$	0.21752$_n$	9.38384	0.30319	0.29743	9.96488	120
65	0.22385	7.86753$_n$	0.2410$\bar{5}_n$	9.86001$_n$	0.16936	0.3421$\bar{5}$	0.10187	115
70	0.14881	9.99083	0.10141$_n$	0.16380$_n$	9.83952	0.34389	0.21121	110
75	9.77378	0.19589	9.57991$_n$	0.24685$_n$	9.31886$_n$	0.29407	0.29441	105
80	9.64134$_n$	0.19266	9.78530	0.19480$_n$	0.01739$_n$	0.17082	0.35289	100
85	0.10687$_n$	9.98170	0.13677	9.96069$_n$	0.21164$_n$	9.90117	0.38761	95
90	0.20438$_n$	$-\infty$	0.22113	$-\infty$	0.26470$_n$	$-\infty$	0.39912	90
	$\log R_1^7$	$\log(-R_2^7)$	$\log R_3^7$	$\log(-R_4^7)$	$\log R_5^7$	$\log(-R_6^7)$	$\log R_7^7$	u

IIa. Logarithms of the Functions* used to Compute X and Y

u	$\log \frac{\beta}{\alpha}$	$\log \frac{R_0^2 - \sqrt{5} R_0^0}{\alpha \sin v}$	$\log \frac{R_0^3 - \sqrt{\frac{7}{3}} R_0^1}{\alpha \sin v}$	$\log \frac{R_0^4 - \sqrt{9} R_0^0}{\alpha \sin v}$	$\log \frac{R_0^5 - \sqrt{\frac{11}{3}} R_0^1}{\alpha \sin v}$	$\log \frac{R_0^6 - \sqrt{13} R_0^0}{\alpha \sin v}$	$\log \frac{R_0^7 - \sqrt{\frac{15}{3}} R_0^1}{\alpha \sin v}$	
0°	0.0000	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	180°
5	0	9.4659$_n$	9.7591$_n$	0.113$\bar{5}_n$	0.3007$_n$	0.5110$_n$	0.6478$_n$	175
10	0	9.7652$_n$	0.053$\bar{5}_n$	0.4041$_n$	0.5837$_n$	0.7877$_n$	0.9144$_n$	170
15	1	9.9386$_n$	0.2183$_n$	0.5627$_n$	0.7294$_n$	0.9229$_n$	1.0320$_n$	165
20	2	0.0596$_n$	0.3274$_n$	0.6630$_n$	0.8110$_n$	0.9897$_n$	1.0728$_n$	160
25	3	0.1515$_n$	0.403$\bar{5}_n$	0.7278$_n$	0.8507$_n$	1.010$\bar{5}_n$	1.0574$_n$	155
30	4	0.2245$_n$	0.4566$_n$	0.7672$_n$	0.8577$_n$	0.9944$_n$	0.9924$_n$	150
35	5	0.2842$_n$	0.4920$_n$	0.786$\bar{5}_n$	0.8354$_n$	0.9459$_n$	0.8786$_n$	145
40	6	0.3336$_n$	0.5122$_n$	0.7883$_n$	0.7845$_n$	0.8681$_n$	0.7124$_n$	140
45	7	0.3751$_n$	0.5187$_n$	0.7746$_n$	0.7031$_n$	0.7647$_n$	0.4967$_n$	135
50	9	0.4098$_n$	0.5120$_n$	0.7462$_n$	0.5862$_n$	0.6446$_n$	0.2518$_n$	130
55	10	0.4389$_n$	0.491$\bar{5}_n$	0.7039$_n$	0.4232$_n$	0.5279$_n$	0.0783$_n$	125
60	11	0.4631$_n$	0.4559$_n$	0.6485$_n$	0.1911$_n$	0.4487$_n$	0.0940$_n$	120
65	12	0.4829$_n$	0.4025$_n$	0.5812$_n$	9.8219$_n$	0.4362$_n$	0.2125$_n$	115
70	13	0.4986$_n$	0.3262$_n$	0.5046$_n$	8.6367$_n$	0.4829$_n$	0.3063$_n$	110
75	14	0.5105$_n$	0.2171$_n$	0.4241$_n$	9.4623	0.543$\bar{5}_n$	0.3286$_n$	105
80	14	0.5189$_n$	0.0521$_n$	0.3490$_n$	9.5579	0.6178$_n$	0.2538$_n$	100
85	14	0.5239$_n$	9.7577$_n$	0.2937$_n$	9.3694	0.6604$_n$	0.0116$_n$	95
90	15	0.5256$_n$	$-\infty$	0.2730$_n$	$-\infty$	0.6751$_n$	$-\infty$	90

*To compute X, use the numbers of the table; to compute Y, use the numbers derived by subtraction of $\log \frac{\beta}{\alpha}$.

(Continuation of IIa)

u	$\left(\log \frac{\beta}{\alpha}\right)$	$\log \frac{R_1^{2*)}}{\alpha \sin v}$	$\log \frac{R_1^3}{\alpha \sin v}$	$\log \frac{R_1^4}{\alpha \sin v}$	$\log \frac{R_1^5}{\alpha \sin v}$	$\log \frac{R_1^6}{\alpha \sin v}$	$\log \frac{R_1^7}{\alpha \sin v}$	$\log \frac{R_2^2}{\alpha \sin v}$	$\log \frac{R_2^3}{\alpha \sin v}$	$\log \frac{R_2^4}{\alpha \sin v}$	
0°	0	0.5866	0.8102	0.9757	1.1073	1.2166	1.3102	$-\infty$	$-\infty$	$-\infty$	180°
5	0	0.5849	0.8060	0.9682	1.0956	1.1999	1.287$\bar{5}$	9.2273	9.6482	9.939[illegible]	175
10	0	0.5799	0.7934	0.9453	1.0597	1.1481	1.2167	9.5267	9.9425	0.2267	170
15	1	0.5715	0.7720	0.9061	0.9972	1.0561	1.0878	9.7000	0.1074	0.3801	165
20	2	0.5596	0.7412	0.8485	0.9030	0.9119	0.8740	9.8211	0.2164	0.4726	160
25	3	0.5439	0.7000	0.7691	0.7666	0.6856	0.4609	9.9130	0.2925	0.5265	155
30	4	0.5241	0.6468	0.6616	0.5645	0.2673	9.6543$_n$	9.9860	0.3457	0.5510	150
35	5	0.$\bar{5}$000	0.5792	0.5140	0.2218	9.6938$_n$	0.3939$_n$	0.0456	0.3810	0.5498	145
40	6	0.4708	0.4934	0.2978	7.2686$_n$	0.2964$_n$	0.4842$_n$	0.0951	0.4018	0.528$\bar{5}$	140
45	7	0.4361	0.3825	9.9118	0.0867$_n$	0.4073$_n$	0.3833$_n$	0.1365	0.4078	0.4699	135
50	9	0.3947	0.2334	9.2577$_n$	0.2860$_n$	0.3664$_n$	0.0334$_n$	0.1713	0.4010	0.3832	130
55	10	0.3452	0.0136	9.9819$_n$	0.3299$_n$	0.1813$_n$	9.5802	0.2004	0.3805	0.2494	125
60	11	0.2856	9.5961	0.1725$_n$	0.2783$_n$	9.6363$_n$	0.1636	0.2245	0.34$\bar{5}$0	0.0320	120
65	12	0.2125	9.2580$_n$	0.2442$_n$	0.1214$_n$	9.7922	0.2661	0.2443	0.2916	9.5681	115
70	13	0.1206	9.8312$_n$	0.2473$_n$	9.7421$_n$	0.1392	0.175$\bar{5}$	0.2600	0.2153	9.4670$_n$	110
75	14	9.9996	0.0337$_n$	0.1904$_n$	9.4126	0.2247	9.7887	0.2720	0.1062	9.9370$_n$	105
80	14	9.8263	0.1390$_n$	0.0589$_n$	9.9833	0.1692	9.6479$_n$	0.2804	9.9412	0.1157$_n$	100
85	14	9.5269	0.1929$_n$	9.7833$_n$	0.157[illegible]	9.9328	0.1085$_n$	0.2854	9.6468	0.1993$_n$	95
90	15	$-\infty$	0.2096$_n$	$-\infty$	0.205[illegible]	$-\infty$	0.2044$_n$	0.2870	$-\infty$	0.2245$_n$	90
		$\log \frac{-R_1^2}{\alpha \sin v}$	$\log \frac{R_1^3}{\alpha \sin v}$	$\log \frac{-R_1^4}{\alpha \sin v}$	$\log \frac{R_1^5}{\alpha \sin v}$	$\log \frac{-R_1^6}{\alpha \sin v}$	$\log \frac{R_1^7}{\alpha \sin v}$	$\log \frac{R_2^2}{\alpha \sin v}$	$\log \frac{-R_2^3}{\alpha \sin v}$	$\log \frac{R_2^4}{\alpha \sin v}$	u

*We have $\log \frac{R_1^1}{\beta \sin v} = 0.2371$, $\log \frac{R_1^1}{\alpha \sin v} = 0.2371 + \log \frac{\beta}{\alpha}$.

(Continuation of IIa)

u	$\left(\log \frac{\beta}{\alpha}\right)$	$\log \frac{R_2^5}{\alpha \sin v}$	$\log \frac{R_2^6}{\alpha \sin v}$	$\log \frac{R_2^7}{\alpha \sin v}$	$\log \frac{R_3^3}{\alpha \sin v}$	$\log \frac{R_3^4}{\alpha \sin v}$	$\log \frac{R_3^5}{\alpha \sin v}$	$\log \frac{R_3^6}{\alpha \sin v}$	$\log \frac{R_3^7}{\alpha \sin v}$	$\log \frac{R_4^4}{\alpha \sin v}$	
0°	0	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	180°
5	0	0.1639	0.3473	0.5021	8.2025	8.6780	9.020$\bar{5}$	9.2928	9.5199	7.1698	175
10	0	0.4430	0.6162	0.7588	8.8012	9.2717	9.6079	9.8726	0.0908	8.0679	170
15	1	0.5818	0.7370	0.8580	9.1478	9.6098	9.9353	0.1870	0.3897	8.5878	165
20	2	0.6528	0.7818	0.8678	9.3899	9.8398	0.1499	0.3826	0.5623	8.9508	160
25	3	0.6769	0.7654	0.7998	9.5736	0.0077	0.2973	0.5040	0.6513	9.2263	155
30	4	0.6610	0.6917	0.6381	9.7195	0.1338	0.3970	0.5692	0.6713	9.4452	150
35	5	0.6044	0.5446	0.3027	9.8386	0.2287	0.458$\bar{5}$	0.58$\bar{5}$0	0.6223	9.6238	145
40	6	0.4992	0.2684	9.1887$_n$	9.937$\bar{5}$	0.2982	0.4662	0.5508	0.4864	9.7720	140
45	7	0.3221	9.4146	0.2267$_n$	0.0202	0.3461	0.4810	0.4571	0.1936	8.8960	135
50	9	9.9911	0.0102$_n$	0.3677$_n$	0.0896	0.3739	0.4402	0.2741	8.278$\bar{5}$	0.0000	130
55	10	8.8422$_n$	0.2584$_n$	0.3204$_n$	0.1477	0.3824	0.3558	9.8789	0.0867$_n$	0.0871	125
60	11	9.9691$_n$	0.307$\bar{5}$$_n$	0.0757$_n$	0.[illegible]959	0.3709	0.2075	9.5085$_n$	0.2793$_n$	0.1594	120
65	12	0.1807$_n$	0.2332$_n$	7.9097$_n$	0.2353	0.3372	9.9311	0.0731$_n$	0.2833$_n$	0.2184	115
70	13	0.2487$_n$	9.9968$_n$	0.0175	0.2666	0.2765	8.853$\bar{5}$	0.2286$_n$	0.1281$_n$	0.2654	110
75	14	0.2291$_n$	8.9142$_n$	0.2108	0.290$\bar{5}$	0.2792	9.8121$_n$	0.2512$_n$	9.5948$_n$	0.3011	105
80	14	0.1199$_n$	9.8968	0.1992	0.3072	0.0226	0.0894$_n$	0.1656$_n$	9.7919	0.3262	100
85	14	9.856$\bar{5}$$_n$	0.1488	9.9833	0.3172	9.7332	0.2053$_n$	9.9144$_n$	0.1404	0.3411	95
90	15	$-\infty$	0.2129	$-\infty$	0.320$\bar{5}$	$-\infty$	0.2391$_n$	$-\infty$	0.2211	0.3461	90
		$\log \frac{-R_2^5}{\alpha \sin v}$	$\log \frac{R_2^6}{\alpha \sin v}$	$\log \frac{-R_2^7}{\alpha \sin v}$	$\log \frac{R_3^3}{\alpha \sin v}$	$\log \frac{-R_3^4}{\alpha \sin v}$	$\log \frac{R_3^5}{\alpha \sin v}$	$\log \frac{-R_3^6}{\alpha \sin v}$	$\log \frac{R_3^7}{\alpha \sin v}$	$\log \frac{R_4^4}{\alpha \sin v}$	u

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(Continuation of IIa)

u	$\left(\log\frac{\beta}{\alpha}\right)$	$\log\frac{R_4^5}{\alpha\sin v}$	$\log\frac{R_4^6}{\alpha\sin v}$	$\log\frac{R_4^7}{\alpha\sin v}$	$\log\frac{R_5^5}{\alpha\sin v}$	$\log\frac{R_5^6}{\alpha\sin v}$	$\log\frac{R_5^7}{\alpha\sin v}$	$\log\frac{R_6^6}{\alpha\sin v}$	$\log\frac{R_6^7}{\alpha\sin v}$	$\log\frac{R_7^7}{\alpha\sin v}$	
0°	0	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	180°
5	. 0	7.6889	8.0724	$8.383\overline{5}$	6.1323	6.6876	7.1058	5.0914	5.6778	4.0481	175
10	0	8.5819	8.9596	9.2634	7.3297	7.8799	8.2923	6.5881	$7.169\overline{5}$	5.8442	170
15	1	9.0933	9.4608	$0.752\overline{5}$	8.0228	8.5646	8.9670	7.4546	8.0274	6.8839	165
20	2	9.4443	.9.7970	0.0712	8.5068	9.0366	9.4247	8.0595	8.6204	7.6098	160
25	3	9.7040	0.0372	0.2875	8.8741	9.3881	9.7573	8.5187	9.0637	8.1608	155
30	4	9.9030	0.2111	0.4302	9.1659	9.6600	0.0051	8.8833	9.4086	8.5984	150
35	5	0.0574	0.3340	0.5125	9.3041	9.8739	0.1887	9.1810	9.6819	8.9556	145
40	6	0.1763	0.4138	0.5388	9.6016	0.0422	0.3196	9.4279	9.8996	9.2518	140
45	7	0.2654	0.4539	0.5058	9.7669	0.1726	$0.403\overline{5}$	$9.634\overline{5}$	0.0713	9.4997	135
50	9	0.3279	0.4539	0.3980	9.9056	0.2698	0.4420	9.8076	0.2031	9.7077	130
55	10	0.3654	0.4090	0.1630	0.0217	0.3362	0.4325	9.9529	0.2985	9.8817	125
60	11	0.3780	0.3054	9.4456	0.1180	0.3728	0.3649	0.0732	0.3592	0.0261	120
65	12	0.3639	0.1044	9.9022_n	0.1966	0.3784	0.2116	0.1716	0.3844	0.1441	115
70	13	0.3188	9.6176	$0.190\overline{5}_n$	0.2592	0.3490	9.8662	0.2498	0.3706	0.2379	110
75	14	0.2334	9.6352_n	0.2617_n	0.3068	0.2754	9.3337	0.3092	0.3089	0.3093	105
80	14	0.0852	0.0590_n	0.2009_n	0.3403	0.1355	0.0240_n	0.3511	0.1774	$0.359\overline{5}$	100
85	14	9.8007	0.2099_n	9.9623_n	0.3602	9.8560	0.2133_n	0.3759	9.9028	0.3892	95
90	15	$-\infty$	0.2525_n	$-\infty$	0.3668	$-\infty$	0.2647_n	0.3841	$-\infty$	0.3991	90
		$\log\frac{-R_4^5}{\alpha\sin v}$	$\log\frac{R_4^6}{\alpha\sin v}$	$\log\frac{-R_4^7}{\alpha\sin v}$	$\log\frac{R_5^5}{\alpha\sin v}$	$\log\frac{-R_5^6}{\alpha\sin v}$	$\log\frac{R_5^7}{\alpha\sin v}$	$\log\frac{R_6^6}{\alpha\sin v}$	$\log\frac{-R_6^7}{\alpha\sin v}$	$\log\frac{R_7^7}{\alpha\sin v}$	u

IIb. Logarithms of the Functions Used to Compute Z

u	$\log\frac{R_0^1}{\gamma}$	$\log\frac{R_0^2}{\gamma}$	$\log\frac{R_0^3}{\gamma}$	$\log\frac{R_0^4}{\gamma}$	$\log\frac{R_0^5}{\gamma}$	$\log\frac{R_0^6}{\gamma}$	$\log\frac{R_0^7}{\gamma}$	$\log\frac{R_1^1}{\gamma}$	
0°	0.2386	$0.349\overline{5}$	0.4225	0.4771	0.5207	0.5570	0.5881	$-\infty$	180°
5	0.2369	$0.344\overline{5}$	0.4125	0.4603	0.4954	0.5213	0.5401	9.1803	175
10	0.2319	0.3293	0.3816	0.4077	0.4143	0.4038	0.3764	9.4797	170
15	0.2236	0.3033	0.3273	0.3115	0.2569	0.1538	9.9717	9.6530	165
20	0.2116	0.2653	0.2442	0.1514	9.9491	9.3900	9.6336_n	9.7741	160
25	0.1959	0.2134	0.1210	9.8633	7.9759_n	9.8740_n	0.1278_n	9.8660	155
30	0.1762	0.1444	$9.930\overline{5}$	8.7766	9.8759_n	0.1322_n	0.2010_n	9.9390	150
35	0.1520	0.0526	9.5760	9.7195_n	0.0905_n	0.1712_n	0.0750_n	9.9986	145
40	0.1229	9.9273	8.8750_n	9.9846_n	0.1442_n	0.0640_n	9.5717_n	0.0481	140
45	0.0882	9.9438	9.6772_n	0.0877_n	0.0943_n	9.7178_n	9.7060	$0.089\overline{5}$	135
50	0.0468	9.4197	9.9037_n	0.1087_n	9.9223_n	9.3369	0.0473	0.1243	130
55	9.9973	8.2914_n	0.0144_n	0.0623_n	9.4437_n	9.9239	0.0921	0.1534	125
60	9.9376	9.4540_n	0.0651_n	9.9362_n	9.4886	0.0685	9.9328	0.1775	120
65	9.8646	9.7191_n	0.0721_n	9.6632_n	9.9019	0.0536	9.1836	0.1973	115
70	9.7727	9.8634_n	0.0391_n	7.7604_n	0.0388	$9.874\overline{5}$	9.7672_n	0.2130	110
75	9.6516	9.9531_n	9.9606_n	9.6391_n	0.0567	9.1751	0.0267_n	$0.22\overline{5}0$	105
80	9.4783	0.0090_n	9.8161_n	9.9044_n	9.9699	9.6828_n	$0.041\overline{5}_n$	0.2334	100
85	9.1789	0.0400_n	9.5334_n	0.0188_n	9.7185	9.9802_n	9.8383_n	0.2384	95
90	$-\infty$	0.0499_n	$-\infty$	0.0526_n	$-\infty$	0.0533_n	$-\infty$	0.2400	90
	$\log\frac{-R_0^1}{\gamma}$	$\log\frac{R_0^2}{\gamma}$	$\log\frac{-R_0^3}{\gamma}$	$\log\frac{R_0^4}{\gamma}$	$\log\frac{-R_0^5}{\gamma}$	$\log\frac{R_0^6}{\gamma}$	$\log\frac{-R_0^7}{\gamma}$	$\log\frac{R_1^1}{\gamma}$	u

(Continuation of IIb)

u	$\log \frac{R_1^2}{\gamma}$	$\log \frac{R_1^3}{\gamma}$	$\log \frac{R_1^4}{\gamma}$	$\log \frac{R_1^5}{\gamma}$	$\log \frac{R_1^6}{\gamma}$	$\log \frac{R_1^7}{\gamma}$	$\log \frac{R_2^2}{\gamma}$	$\log \frac{R_2^3}{\gamma}$	$\log \frac{R_2^4}{\gamma}$	
0°	−∞	−∞	−∞	−∞	−∞	−∞	−∞	−∞	−∞	180°
5	9.5281	9.7492	9.9114	0.0388	0.1431	0.2307	8.1705	8.5914	8.8823	175
10	9.8225	0.0359	0.1678	0.3023	0.3906	0.4593	8.7692	9.1651	9.4692	170
15	9.9873	0.1878	0.3219	0.4131	0.4719	0.5036	9.1158	9.5232	9.7959	165
20	0.0964	0.2780	0.3853	0.4398	0.4487	0.4108	9.3579	9.7532	0.0094	160
25	$0.172\bar{5}$	0.3286	0.3977	0.3952	0.3142	$0.109\bar{5}$	9.5416	9.9211	0.1551	155
30	0.2256	0.3483	0.3631	0.2660	0.9688	9.3358_n	$9.687\bar{5}$	0.0472	0.2525	150
35	0.2610	0.3402	0.2750	9.9828	9.4548_n	0.1549_n	9.8066	0.1421	0.3108	145
40	0.2812	0.3037	0.1082	7.0789_n	0.1068_n	0.2945_n	$9.905\bar{5}$	0.2117	0.3338	140
45	0.2877	0.2342	9.7634	9.9383_n	0.2589_n	0.2349_n	9.9882	$0.259\bar{5}$	0.3216	135
50	0.2810	0.1197	9.1440_n	0.1723_n	0.2527_n	9.9197_n	0.0576	0.2878	$0.269\bar{5}$	130
55	$0.260\bar{5}$	9.9289	9.8972_n	0.2452_n	0.0966_n	9.4955	0.1157	0.2958	0.1647	125
60	0.2249	$9.535\bar{5}$	0.1119_n	0.2176_n	9.5756_n	0.1030	0.1639	0.2843	9.9714	120
65	0.1715	9.2170_n	0.2032_n	0.0804_n	9.7512	0.2250	0.2033	0.2506	9.5272	115
70	0.0953	9.8058_n	0.2219_n	9.7167_n	0.1139	0.1501	0.2346	0.1899	9.4416_n	110
75	9.9861	0.0202_n	0.1769_n	9.3991	0.2112	9.7751	$0.258\bar{5}$	0.0926	9.9235_n	105
80	9.8211	0.1339_n	0.0538_n	9.9782	0.1641	9.6428_n	0.2752	9.9350	0.1105_n	100
85	9.5267	0.1927_n	9.7831_n	0.1575	9.9327	0.1083_n	0.2852	9.6466	0.1991_n	95
90	−∞	0.2110_n	−∞	0.2071	−∞	0.2058_n	0.2885	−∞	0.2260_n	90
	$\log \frac{-R_1^2}{\gamma}$	$\log \frac{R_1^3}{\gamma}$	$\log \frac{-R_1^4}{\gamma}$	$\log \frac{R_1^5}{\gamma}$	$\log \frac{-R_1^6}{\gamma}$	$\log \frac{R_1^7}{\gamma}$	$\log \frac{R_2^2}{\gamma}$	$\log \frac{-R_2^3}{\gamma}$	$\log \frac{R_2^4}{\gamma}$	u

(Continuation of IIb

u	$\log \frac{R_2^5}{\gamma}$	$\log \frac{R_2^6}{\gamma}$	$\log \frac{R_2^7}{\gamma}$	$\log \frac{R_3^3}{\gamma}$	$\log \frac{R_3^4}{\gamma}$	$\log \frac{R_3^5}{\gamma}$	$\log \frac{R_3^6}{\gamma}$	$\log \frac{R_3^7}{\gamma}$	$\log \frac{R_4^4}{\gamma}$	
0°	−∞	−∞	−∞	−∞	−∞	−∞	−∞	−∞	−∞	180°
5	9.1071	9.2905	9.4453	7.1457	7.6212	7.9037	8.2360	8.4631	6.1130	175
10	9.6855	9.8587	0.0014	8.0838	8.5142	8.8504	9.1151	9.3333	7.3104	170
15	9.9976	0.1528	0.2739	8.5636	9.0256	9.3511	9.6028	9.8055	8.0036	165
20	0.1896	0.3175	0.4046	8.9267	9.3766	9.6867	9.9194	0.0991	8.4876	160
25	0.3055	0.3940	0.4284	9.2022	9.6363	9.9259	0.1326	0.2799	8.8549	155
30	$0.362\bar{5}$	0.3932	0.3396	9.4210	9.8353	$0.098\bar{5}$	0.2707	0.3728	9.1467	150
35	0.3654	0.3056	0.0638	9.5997	9.9897	0.2195	0.3460	0.3833	9.3848	145
40	0.3096	0.0787	8.9990_n	9.7478	0.1086	0.2966	0.3612	0.2968	9.5823	140
45	0.1737	9.2663	0.0784_n	9.8718	0.1977	0.3327	0.3088	0.0452	9.7476	135
50	9.8774	9.8965_n	0.2541_n	9.9759	0.2602	0.3265	$0.160\bar{5}$	8.1648	9.8863	130
55	$8.757\bar{5}_n$	0.1737_n	0.2357_n	0.0630	0.2977	0.2711	9.7941	0.0020_n	0.0015	125
60	9.9084_n	0.2468_n	0.0151_n	0.1353	0.3103	0.1469	9.4479_n	0.2186_n	0.0987	120
65	0.1397_n	0.1922_n	7.8687_n	0.1943	0.2962	9.8901	0.0321_n	0.2422_n	0.1774	115
70	0.2233_n	9.9714_n	9.9921	0.2412	0.2511	8.8281	0.2032_n	0.1027_n	0.2400	110
75	0.2156_n	8.9007_n	0.1972	0.2770	0.1657	9.7986_n	0.2377_n	9.5813_n	0.2876	105
80	0.1146_n	9.6917	0.1941	0.3021	$0.017\bar{5}$	0.0843_n	0.1604_n	9.7867	0.3211	100
85	9.8563_n	0.1486	9.9831	0.3170	9.7330	0.2051_n	9.9143_n	0.1402	0.3409	95
90	−∞	0.2144	−∞	0.3219	−∞	0.2406_n	−∞	0.2226	0.3475	90
	$\log \frac{-R_2^5}{\gamma}$	$\log \frac{R_2^6}{\gamma}$	$\log \frac{-R_2^7}{\gamma}$	$\log \frac{R_3^3}{\gamma}$	$\log \frac{-R_3^4}{\gamma}$	$\log \frac{R_3^5}{\gamma}$	$\log \frac{-R_3^6}{\gamma}$	$\log \frac{R_3^7}{\gamma}$	$\log \frac{R_4^4}{\gamma}$	u

(Continuation of IIb)

u	$\log \frac{R_4^5}{\gamma}$	$\log \frac{R_4^6}{\gamma}$	$\log \frac{R_4^7}{\gamma}$	$\log \frac{R_5^5}{\gamma}$	$\log \frac{R_5^6}{\gamma}$	$\log \frac{R_5^7}{\gamma}$	$\log \frac{R_6^6}{\gamma}$	$\log \frac{R_6^7}{\gamma}$	$\log \frac{R_7^7}{\gamma}$	
0°	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	180°
5	6.6321	7.0156	7.3267	$5.075\bar{5}$	5.6308	6.0490	4.0346	4.6210	2.9918	175
10	7.8244	8.2021	8.5059	6.5722	$7.122\bar{5}$	7.5348	5.8306	6.4120	5.0867	170
15	8.5091	8.8766	9.1683	7.4386	7.9804	8.3828	6.8704	7.4432	6.2997	165
20	8.9811	9.3337	9.6080	8.0436	8.5734	8.9615	7.5963	8.1572	7.1466	160
25	9.3326	9.6658	9.9161	8.5027	9.0167	9.3859	8.1478	8.6923	7.7894	155
30	9.6045	9.9126	0.1318	8.8674	9.3616	9.7066	8.5849	9.1101	8.2999	150
35	9.8184	0.0951	0.2735	9.1651	9.6349	9.9497	8.9420	9.4429	8.7166	145
40	9.9867	0.2242	0.3491	9.4120	9.8526	0.1299	9.2383	9.7100	9.0622	140
45	0.1171	0.3055	0.3574	9.6185	0.0243	0.2551	9.4861	9.9229	9.3518	135
50	0.2142	0.3402	0.2843	9.7919	0.1561	0.3283	9.6941	0.0894	9.5940	130
55	0.2807	0.3243	0.0793	9.9369	0.2515	0.3478	9.8682	0.2138	9.7970	125
60	0.3173	0.2448	9.3849	0.0573	0.3122	0.3043	0.0126	0.2985	$9.965\bar{5}$	120
65	0.3228	0.0634	9.8612_n	0.1556	0.3374	0.1706	0.1305	0.3433	0.1031	115
70	0.2934	9.5929	0.1651_n	0.2338	0.3236	9.8408	0.2244	0.3452	$0.212\bar{5}$	110
75	0.2199	9.6217_n	0.2482_n	0.2933	0.2619	9.3202_n	0.2957	0.2954	0.2958	105
80	0.0800	0.0539_n	0.1957_n	0.3351	0.1304	0.0188_n	0.3459	0.1722	0.3548	100
85	$9.800\bar{5}$	0.2098_n	9.9621_n	0.3600	9.8558	0.2131_n	0.3757	9.9026	0.3891	95
90	$-\infty$	0.2540_n	$-\infty$	0.3682	$-\infty$	0.2662_n	0.3856	$-\infty$	0.4006	90
	$\log \frac{-R_4^5}{\gamma}$	$\log \frac{R_4^6}{\gamma}$	$\log \frac{-R_4^7}{\gamma}$	$\log \frac{R_5^5}{\gamma}$	$\log \frac{-R_5^6}{\gamma}$	$\log \frac{R_5^7}{\gamma}$	$\log \frac{R_6^6}{\gamma}$	$\log \frac{-R_6^7}{\gamma}$	$\log \frac{R_7^7}{\gamma}$	u

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III. Observed Values of the Force Components

u:	λ:	0°	5°	10°	15°	20°	25°	30°	35°	40°
30°	X	14031	14578	15043	15560	15961	16260	16498	16649	16740
	Y	− 5129	− 4373	− 3611	− 2744	− 2031	− 1137	− 240	601	1415
	Z	46564	46155	45791	46061	46379	46809	47470	48202	49022
35°	X	15901	16477	17019	17444	17706	18132	18550	18838	19081
	Y	− 5423	− 4647	− 3903	− 2997	− 2268	− 1454	− 647	263	1052
	Z	45013	44406	44136	43993	43813	44205	44809	45410	46067
40°	X	17808	18443	18918	19402	19775	20140	20605	21000	21291
	Y	− 5643	− 4965	− 4298	− 3526	− 2633	− 1797	− 1008	− 153	632
	Z	42791	42400	41994	41813	41360	41305	41376	41966	42418
45°	X	20016	20571	21095	21633	22059	22468	22895	23372	23760
	Y	− 5942	− 5288	− 4645	− 3892	− 3061	− 2243	− 1434	− 598	173
	Z	40541	39948	39507	38982	38443	38080	38179	38655	39155
50°	X	21992	22686	23286	23828	24405	24955	25477	26057	26418
	Y	− 6133	− 5530	− 4921	− 4230	− 3473	− 2696	− 1931	− 1100	− 307
	Z	37500	36889	36187	35322	34880	34759	34740	35245	35596
55°	X	24140	24830	25478	26128	26774	27349	27893	28242	28708
	Y	− 6423	− 5808	− 5184	− 4544	− 3858	− 3156	− 2440	− 1645	− 835
	Z	34277	33132	32203	31234	30755	30486	30557	30515	30888
60°	X	25943	26658	27328	27974	28635	29230	29808	30372	31030
	Y	− 6725	− 6114	− 5519	− 4899	− 4262	− 3589	− 2914	− 2177	− 1373
	Z	29764	27994	26845	25873	25538	25375	25355	25475	25756
65°	X	27382	28039	28686	29368	30037	30737	31418	32089	32795
	Y	− 7150	− 6576	− 5967	− 5337	− 4686	− 4028	− 3394	− 2676	− 1910
	Z	23747	21702	20263	19323	18812	18561	18076	18779	19222
70°	X	28462	29092	29821	30572	31303	31926	32538	33151	33832
	Y	− 7680	− 7119	− 6539	− 5869	− 5191	− 4487	− 3803	− 3143	− 2445
	Z	17423	15370	13860	12894	12445	12268	12249	12340	12570
75°	X	29403	30066	30653	31378	31914	32488	33050	33630	34178
	Y	− 8293	− 7757	− 7124	− 6479	− 5704	− 4933	− 4234	− 3574	− 2890
	Z	11270	8903	7265	6228	5619	5401	5426	5558	5689
80°	X	29857	30421	30952	31505	32021	32516	32901	33380	34232
	Y	− 8920	− 8361	− 7717	− 6985	− 6224	− 5393	− 4663	− 4000	− 3377
	Z	5355	3084	1393	282	− 332	− 575	− 696	− 684	− 600
85°	X	29667	29940	30224	30697	31154	31543	31880	32256	32644
	Y	− 9497	− 8850	− 8136	− 7417	− 6622	− 5846	− 5068	− 4438	− 3767
	Z	91	− 2046	− 3843	− 5096	− 6047	− 6819	− 7156	− 7318	− 7335
90°	X	28143	28426	28689	28989	29307	29659	30051	30471	30975
	Y	− 9800	− 9145	− 8425	− 7722	− 6946	− 6196	− 5497	− 4826	− 4215
	Z	− 4056	− 6438	− 8668	−10574	−12017	−12810	−13283	−13521	−13701
95°	X	26407	26583	26762	26986	27239	27479	27779	28114	28627
	Y	−10066	− 9414	− 8730	− 8011	− 7299	− 6555	− 5905	− 5253	− 4637
	Z	− 7660	−10637	−13277	−15337	−16777	−17998	−18738	−19170	−19622
100°	X	24412	24539	24593	24739	24941	25209	25548	25931	26365
	Y	−10362	− 9683	− 9033	− 8318	− 7625	− 6991	− 6370	− 5749	− 5188
	Z	−11257	−14224	−16799	−18789	−20499	−21886	−22887	−23776	−24550
105°	X	22389	22342	22302	22406	22591	22882	23271	23684	24192
	Y	−10480	− 9908	− 9238	− 8601	− 7985	− 7435	− 6893	− 6346	− 5808
	Z	−14368	−16954	−19260	−21109	−22671	−23990	−25427	−27322	−28875
110°	X	20900	20787	20737	20778	20910	21137	21409	21751	22129
	Y	−10634	−10213	− 9670	− 9142	− 8576	− 8057	− 7546	− 7067	− 6485
	Z	−17039	−19433	−21150	−22899	−24635	−26363	−28453	−30350	−31707
115°	X	19813	19495	19352	19313	19342	19474	19677	19818	20185
	Y	−10535	−10366	−10038	− 9629	− 9226	− 8752	− 8285	− 7607	− 7346
	Z	−19335	−21260	−23000	−24898	−26781	−28818	−30719	−32181	−33760

(Continuation of III)

ω;	λ:	0°	5°	10°	15°	20°	25°	30°	35°	40°
115°*,	X	19813	19495	19352	19313	19342	19474	19677	19818	20185
	Y	−10535	−10366	−10038	− 9629	− 9236	− 8752	− 8285	− 7807	− 7346
	Z	−19335	−21260	−23000	−24898	−26781	−28818	−30719	−32181	−33760
120°	X	19054	18574	18277	18084	18026	18009	18126	18303	18495
	Y	−10217	−10326	−10200	−10011	− 9720	− 9397	− 9051	− 8665	− 8222
	Z	−21558	−23258	−25018	−26777	−28748	−30398	−32048	−33970	−35484
125°	X	18530	18020	17531	17187	17006	16885	16826	16859	16966
	Y	− 9818	−10044	−10155	−10090	− 9984	− 9814	− 9617	− 9313	− 8945
	Z	−23494	−25250	−26886	−28569	−30136	−31912	−33567	−35107	−36533
130°	X	18429	17830	17168	16701	16341	16060	15881	15768	15744
	Y	− 9492	− 9829	− 9965	−10068	−10081	−10026	− 9873	− 9713	− 9485
	Z	−25829	−27515	−28792	−30259	−31745	−33328	−34730	−36348	−37883
135°	X	18546	17777	17052	16446	15913	15482	15149	14917	14750
	Y	− 9193	− 9572	− 9845	− 9947	−10024	−10054	−10015	− 9948	− 9875
	Z	−28491	−29746	−31007	−32306	−33588	−34914	−36291	−37874	−39251
140°	X	18730	17874	17052	16315	15713	15156	14685	14341	14037
	Y	− 8814	− 9239	− 9582	− 9777	− 9896	− 9965	−10019	−10029	−10043
	Z	−31275	−32283	−33429	−34323	−35723	−36920	−38362	−39616	−41060
145°	X	18897	17963	17103	16283	15605	14906	14456	13947	13486
	Y	− 8203	− 8748	− 9158	− 9496	− 9689	− 9743	− 9898	− 9997	−10052
	Z	−34172	−35029	−36030	−37209	−38284	−39080	−40454	−41600	−42883
150°	X	18966	18070	17178	16239	15515	14895	14275	13664	13103
	Y	− 7375	− 7970	− 8533	− 8989	− 9322	− 9562	− 9720	− 9838	− 9946
	Z	−37016	−37994	−38979	−39803	−40654	−41699	−42745	−43870	−44992

*Repeated in order to simplify the interpolation

(Continuation of III)

ω;	λ:	45°	50°	55°	60°	65°	70°	75°	80°	85°
30°	X	16722	16646	16492	16367	16077	15840	15580	15379	15238
	Y	2152	2560	3405	3794	4053	4170	4175	4035	3611
	Z	49907	50872	51573	52326	52852	53465	53878	54550	54733
35°	X	19084	19036	18941	18814	18654	18486	18306	18187	18097
	Y	1810	2506	3141	3572	3863	4030	4058	3899	3490
	Z	46858	47521	48330	49372	50149	50829	51514	52043	52777
40°	X	21482	21577	21679	21590	21556	21518	21468	21485	21533
	Y	1427	2109	2726	3227	3575	3794	3902	3808	3526
	Z	43025	43643	44634	45339	46329	46858	48629	49396	50183
45°	X	24061	24291	24492	24642	24708	24866	24944	24956	24971
	Y	966	1699	2301	2793	3176	3421	3580	3641	3465
	Z	39814	40526	41350	42102	43012	43767	44541	45188	45793
50°	X	26788	27079	27378	27628	27873	28070	28217	28244	28316
	Y	374	1087	1691	2255	2667	2909	3132	3260	3243
	Z	36094	36624	37297	38037	38895	39685	40544	41110	41857
55°	X	29400	29846	30184	30563	30938	31194	31441	31626	31719
	Y	− 154	434	984	1495	1965	2318	2613	2804	2887
	Z	31528	32010	32480	33200	34028	34944	35870	36522	37408
60°	X	31643	32219	32797	33290	33727	34081	34293	34407	34439
	Y	− 663	− 112	363	823	1237	1637	1997	2255	2508
	Z	26167	26639	27360	28025	28910	29748	30569	31228	32003
65°	X	33379	34045	34600	35199	35714	36165	36492	36638	36604
	Y	− 1204	− 545	− 151	236	623	1031	1434	1760	2085
	Z	19544	19990	20381	20672	21675	22389	23191	23972	24650

(Continuation of III)

ω:	λ:	45°	50°	55°	60°	65°	70°	75°	80°	85°
65°	X	33379	34045	34600	35199	35714	36165	36492	36638	36604
	Y	−1204	−545	−151	256	623	1031	1434	1760	2005
	Z	19544	19990	20381	20872	21675	22389	23191	23772	24690
70°	X	34378	35024	35516	36080	36599	37195	37685	38001	37981
	Y	−1701	−1060	−558	−210	192	595	1040	1416	1714
	Z	12812	13101	13339	13670	14110	14654	15336	16013	16531
75°	X	34652	35127	35606	36026	36460	37030	37643	38165	38471
	Y	−2170	−1534	−984	−566	−191	237	712	1110	1478
	Z	5810	5989	6281	6569	6813	7254	7717	8232	8888
80°	X	34099	34403	34771	35288	35696	36260	36899	37541	38130
	Y	−2634	−198	−1417	−924	−519	−105	343	819	1221
	Z	−497	−40	−202	−154	0	158	322	655	1110
85°	X	33008	33394	33801	34224	34748	35277	35840	36497	37130
	Y	−3062	−2393	−1821	−1345	−910	−461	−31	446	885
	Z	−7248	−7116	−7041	−6813	−6757	−6698	−6697	−6545	−6215
90°	X	31442	31922	32380	32950	33474	34088	34549	35320	36000
	Y	−3536	−2868	−2274	−1804	−1344	−892	−422	21	503
	Z	−13758	−13770	−13667	−13613	−13535	−13375	−13205	−13088	−12867
95°	X	29098	29658	30351	31132	31747	32430	33236	33876	34550
	Y	−4055	−3379	−2789	−2341	−1849	−1397	−948	−498	0
	Z	−19817	−19882	−19794	−19889	−19671	−19762	−19716	−19692	−19481
100°	X	26873	27492	28148	28857	29499	30283	31090	31978	32791
	Y	−4577	−3986	−3373	−2880	−2443	−2011	−1584	−1173	−763
	Z	−25126	−25381	−25601	−25732	−25731	−25845	−25891	−25836	−25703
105°	X	24668	25217	25799	26473	27149	27870	28606	29442	30308
	Y	−5244	−4582	−3948	−3470	−3053	−2700	−2318	−1844	−1588
	Z	−29967	−30545	−30830	−31078	−31152	−31372	−31412	−31635	−31759
110°	X	22527	23061	23633	24251	24842	25520	26208	26970	27770
	Y	−5952	−5381	−4722	−4131	−3727	−3435	−3179	−2851	−2478
	Z	−32868	−33610	−34207	−34809	−35217	−35660	−35895	−36208	−36554
115°	X	20533	20954	21430	21967	22518	23032	23566	24197	24920
	Y	−6771	−6194	−5642	−5139	−4684	−4407	−4155	−3963	−3613
	Z	−35052	−36126	−37125	−37795	−38153	−38518	−38796	−39240	−39908
120°	X	18754	19071	19431	19825	20255	20751	21257	21742	22275
	Y	−7692	−7130	−6596	−6093	−5694	−5431	−5267	−5046	−4769
	Z	−36719	−37972	−39140	−40126	−41057	−41738	−42129	−42578	−43297
125°	X	17138	17344	17626	17923	18241	18570	18900	19234	19773
	Y	−8545	−8118	−7664	−7241	−6893	−6576	−6354	−6158	−5920
	Z	−38076	−39614	−40906	−42091	−43121	−43904	−44611	−45017	−45926
130°	X	15775	15884	16054	16252	16448	16676	16938	17232	17544
	Y	−9255	−8926	−8596	−8281	−8022	−7776	−7565	−7315	−7090
	Z	−39462	−40668	−42148	−43498	−44731	−45850	−46891	−47850	−48811
135°	X	14640	14585	14581	14626	14728	14825	14963	15066	15327
	Y	−9751	−9580	−9348	−9141	−8885	−8721	−8523	−8302	−8024
	Z	−40714	−42128	−43597	−45094	−46415	−47688	−48627	−49751	−51210
140°	X	13758	13524	13347	13219	13126	13101	13078	13043	12989
	Y	−10020	−9916	−9786	−9604	−9404	−9219	−9044	−8881	−8641
	Z	−42482	−43802	−45144	−46356	−47441	−48725	−50023	−51613	−52948
145°	X	13045	12660	12310	11946	11587	11216	10863	10590	10342
	Y	−10071	−10059	−9968	−9877	−9838	−9778	−9724	−9535	−9257
	Z	−44078	−45453	−46531	−47330	−48700	−49703	−51127	−52448	−50163
150°	X	12524	11965	11399	10831	10269	9693	9007	8453	7991
	Y	−10021	−10088	−10115	−10159	−10180	−10185	−10110	−10074	−9780
	Z	−45931	−46863	−47849	−49083	−49989	−51372	−52355	−53043	−53816

(Continuation of III)

u;	λ:	90°	95°	100°	105°	110°	115°	120°	125°	130°
30°	X	15199	15215	15306	15384	15515	15679	15866	16060	16289
	Y	3037	2383	1699	1053	406	− 159	− 656	− 1142	− 1581
	Z	55143	55530	56037	56237	56281	56413	55994	55658	55231
35°	X	18047	17985	18040	18185	18380	18610	18851	19028	19371
	Y	2912	2208	1473	741	27	− 623	− 1208	− 1765	− 2236
	Z	53090	53380	53577	54133	54668	55012	55016	54570	54313
40°	X	21601	21634	21652	21691	21829	21981	22142	22396	22585
	Y	2940	2242	1438	599	− 191	− 908	− 1600	− 2157	− 2733
	Z	50137	50624	51122	51749	52271	52464	52300	51953	51098
45°	X	25063	25192	25287	25373	25419	25458	25478	25488	25531
	Y	2989	2337	1473	591	− 281	− 1052	− 1782	− 2394	− 2946
	Z	46487	47250	47807	48240	48659	48773	48432	47810	46683
50°	X	28369	28403	28537	28522	28500	28401	28324	28259	28160
	Y	2940	2360	1562	664	− 182	− 1025	− 1773	− 2348	− 2794
	Z	42549	43060	43570	44073	44309	44042	43701	42886	41885
55°	X	31816	31808	31804	31758	31700	31540	31243	30928	30700
	Y	2812	2429	1713	878	55	− 780	− 1528	− 2109	− 2488
	Z	38404	38812	39331	39622	39734	39487	38743	37717	36598
60°	X	34362	34312	34228	34120	33928	33648	33284	32881	32394
	Y	2604	2460	1693	1172	395	− 342	− 1017	− 1608	− 1887
	Z	32512	33123	33394	33549	33537	33261	32819	32070	31010
65°	X	36450	36250	36044	35830	35541	35199	34847	34366	33878
	Y	2251	2408	2015	1460	776	154	− 456	− 1000	− 1232
	Z	25256	25754	26228	26616	26789	26847	26452	25720	25009
70°	X	37810	37517	37277	37020	36782	36445	36100	35658	35236
	Y	1937	2185	2171	1724	1135	615	105	− 311	− 513
	Z	17254	18059	18685	19292	19773	19997	19874	19604	19267
75°	X	38361	38249	38094	37900	37651	37367	37096	36750	36300
	Y	1731	1982	2052	1964	1479	1003	561	235	105
	Z	9574	10262	11059	11723	12243	12628	12956	12954	12795
80°	X	38347	38658	38745	38545	38312	38125	37887	37594	37296
	Y	1585	1801	2076	2065	1729	1376	992	667	564
	Z	1564	2367	3224	4147	4822	5361	5777	5956	6130
85°	X	37657	38064	38358	38653	38838	38668	38626	38336	38068
	Y	1293	1573	1809	1913	1810	1576	1371	1026	975
	Z	− 5743	− 5129	− 4375	− 3556	− 2720	− 2028	− 1462	− 1060	− 776
90°	X	36590	37138	37568	37963	38144	38329	38573	38680	38779
	Y	905	1297	1553	1680	1665	1561	1459	1306	1286
	Z	−12602	−12254	−11616	−10896	−10052	− 9443	− 9029	− 8816	− 8602
95°	X	35208	35711	36412	36975	37320	37590	37820	37956	38023
	Y	410	779	1166	1356	1390	1420	1450	1348	1439
	Z	−19184	−18860	−18562	−17913	−17412	−16750	−16519	−16318	−16151
100°	X	33558	34299	34998	35543	36085	36482	36832	37031	37117
	Y	− 312	99	428	744	1029	1146	1157	1185	1367
	Z	−25596	−25380	−25042	−24586	−24350	−23855	−23552	−23152	−23440
105°	X	31235	31888	32796	33499	34160	34755	35240	35605	35890
	Y	− 1255	− 854	− 496	− 97	248	586	820	1036	1483
	Z	−31810	−31349	−31399	−31239	−31308	−31024	−31187	−31146	−31042
110°	X	28571	29405	30065	31014	31643	32330	32996	33488	33971
	Y	− 2123	− 1798	− 1454	− 993	− 498	19	480	677	1404
	Z	−36891	−37258	−37529	−37979	−38298	−38533	−38752	−38990	−38884
115°	X	25598	26411	27172	27932	28667	29372	30000	30544	31016
	Y	− 3219	− 2815	− 2417	− 1953	− 1377	− 663	44	622	1218
	Z	−40550	−41371	−42142	−42843	−43698	−44388	−44898	−45406	−45503

ORIGINAL PAGE IS
OF POOR QUALITY

(Continuation of III)

φ:	λ:	90°	95°	100°	105°	110°	115°	120°	125°	130°
115°	X	25598	26411	27172	27932	28667	29372	30000	30544	31016
	Y	− 3219	− 2815	− 2417	− 1953	− 1377	− 663	44	600	1218
	Z	−40550	−41371	−42142	−42843	−43692	−44388	−44898	−45406	−45503
120°	X	22852	23482	24180	24859	25555	26180	26774	27339	27883
	Y	− 4442	− 4000	− 3399	− 2832	− 2206	− 1448	− 584	278	957
	Z	−439[illegible]	−44800	−45767	−46793	−47904	−48900	−49840	−50530	−51032
125°	X	20245	20779	21382	21967	22555	23149	23707	24150	24540
	Y	− 5583	− 5117	− 4545	− 3840	− 3050	− 2161	− 1242	− 155	678
	Z	−46787	−47506	−48528	−49620	−50801	−52017	−53238	−54043	−54905
130°	X	17906	18258	18709	19146	19592	20076	20322	20909	21297
	Y	− 6647	− 6257	− 5601	− 4845	− 4016	− 3012	− 1778	− 681	309
	Z	−49757	−50808	−51867	−53201	−54213	−55774	−56560	−58108	−59056
135°	X	15534	15776	16075	16401	16769	17064	17342	17576	17749
	Y	− 7661	− 7245	− 6659	− 5872	− 4861	− 3793	− 2489	− 1229	− 103
	Z	−52524	−53962	−55298	−57457	−59580	−61773	−63533	−65152	−66043
140°	X	13014	13046	13189	13419	13660	13918	14186	14391	14538
	Y	− 8291	− 7917	− 7311	− 6545	− 5565	− 4411	− 3145	− 1776	− 900
	Z	−54530	−56295	−58170	−60480	−62641	−64262	−65722	−67066	−67485
145°	X	10004	10204	10225	10261	10266	10320	10466	10610	10827
	Y	− 8870	− 8263	− 7497	− 6600	− 5613	− 4505	− 3384	− 2223	− 1074
	Z	−55743	−57792	−59656	−61334	−62620	−63860	−64352	−64777	−65016
150°	X	7735	7704	7664	7568	7449	7262	7143	7123	7064
	Y	− 9240	− 8457	− 7467	− 6421	− 5330	− 4221	− 3755	− 2318	− 1412
	Z	−57514	−59038	−60170	−61474	−63174	−63804	−64712	−64167	−64930

(Continuation of III)

φ:	λ:	135°	140°	145°	150°	155°	160°	165°	170°	175°
30°	X	16547	16802	17026	17268	17496	17686	17753	17783	17533
	Y	− 1759	− 1815	− 1589	− 1056	− 366	401	1293	2124	3013
	Z	54597	53873	52369	51310	50114	49497	49577	49580	49550
35°	X	19593	19851	20136	20466	20690	20858	20887	20562	20727
	Y	− 2406	− 2437	− 2237	− 1491	− 650	273	1339	2408	3496
	Z	53032	51652	50195	48736	47683	47222	47010	47167	47540
40°	X	22754	22928	23172	23378	23493	23516	23345	23143	22806
	Y	− 2996	− 3052	− 2436	− 1566	− 601	445	1598	2705	3854
	Z	49974	48493	46731	45169	44198	43469	43158	43213	43540
45°	X	25628	25734	25818	26664	25818	25618	25268	24814	24329
	Y	− 3147	− 2932	− 2297	− 1356	− 300	805	1915	3047	4144
	Z	45483	43824	42298	41049	40072	39467	[illegible]	38867	38969
50°	X	28149	28108	28037	27886	27629	27234	26601	26065	25631
	Y	− 2925	− 2624	− 1879	− 893	161	1205	2305	3365	4366
	Z	40417	39094	37562	36470	35578	34788	34430	34408	[illegible]
55°	X	30339	30083	29824	29558	29192	28791	28249	27680	27112
	Y	− 2476	− 2016	− 1259	− 343	663	1677	2720	3726	4600
	Z	35120	33880	32575	31514	30680	30014	[illegible]	30067	29721
60°	X	31909	31533	31144	30779	30438	30008	29532	29050	28533
	Y	− 1812	− 1303	− 616	− 197	1152	2168	3156	4040	4775
	Z	29630	28417	27398	26461	25741	25315	25441	25873	26265
65°	X	33424	32934	32480	32081	31614	31230	30796	30391	29978
	Y	− 1050	− 610	472	790	1703	2683	3354	4343	4907
	Z	24147	23279	22351	21469	20757	20418	20518	21034	21937

(Continuation of III)

φ:	λ:	135°	140°	145°	150°	155°	160°	165°	170°	175°
65°	X	33424	32934	32480	32021	31614	31230	30796	30391	29978
	Y	−1050	−610	478	792	1703	2627	3554	4343	4987
	Z	24147	23279	22351	21469	20757	20418	20518	21034	21937
70°	X	34778	34300	33814	33352	32872	32462	32109	31734	31424
	Y	−354	50	590	1378	2251	3168	3942	4630	5069
	Z	18623	17919	17232	16643	16213	16032	16247	16695	17517
75°	X	35880	35494	35050	34643	34186	34209	33463	33143	32882
	Y	261	680	1224	1997	2791	3606	4307	4875	5557
	Z	12530	12281	11791	11555	11311	11288	11289	11918	12694
80°	X	36912	36481	36070	35713	35349	34971	35185	33906	34570
	Y	751	1168	1785	[illegible]	3279	4005	4718	5047	5448
	Z	6123	5945	5721	5670	5623	5785	6059	6662	7852
85°	X	37812	37464	37031	36679	36211	35955	35756	35604	35461
	Y	1188	1658	2265	2994	3717	4373	4972	5322	5596
	Z	− 770	− 818	− 756	− 642	− 485	105	683	1519	2510
90°	X	38718	38190	37730	37283	36836	36603	36278	36078	36423
	Y	1578	2113	2748	3426	4069	4664	5206	5551	5791
	Z	− 8413	− 8247	− 8041	− 7731	− 7096	− 6306	− 5260	− 4290	− 3207
95°	X	38030	37819	37501	37172	36793	36521	36454	36594	36788
	Y	1927	2479	3116	3743	4355	4916	5394	5796	5881
	Z	−16033	−15828	−15523	−15095	−14282	−13291	−11973	−10799	− 9691
100°	X	37124	37060	36928	36714	36413	36158	36150	36212	36003
	Y	2054	2722	3361	3988	4579	5125	5596	5843	5898
	Z	−23007	−22772	−22427	−21898	−20906	−19529	−18771	−17234	−15949
105°	X	36015	36065	36003	35933	35827	35704	35630	35594	35549
	Y	2182	2691	3575	4221	4823	5336	5749	5903	5987
	Z	−30816	−30539	−30019	−29038	−27990	−26713	−25585	−24032	−22666
110°	X	34252	34453	34361	34601	34554	34530	34483	34421	34338
	Y	2175	2954	3714	4402	5041	5530	5822	5945	5952
	Z	−38679	−38404	−37494	−36330	−34717	−33497	−32327	−31085	−29941
115°	X	31433	31740	31863	31993	32050	32060	32123	32112	32118
	Y	2060	2870	3724	4439	5076	5512	5838	6068	6146
	Z	−45548	−44956	−44154	−43124	−41658	−40196	−38567	−36722	−35169
120°	X	28240	28647	28771	28866	29070	29137	29314	29480	29724
	Y	1651	2758	3635	4400	5081	5458	5893	6177	6282
	Z	−51055	−51146	−50230	−48918	−47672	−46096	−44890	−43283	−41433
125°	X	24849	25057	25268	25452	25725	26001	26344	26616	26913
	Y	1592	2486	3439	4221	4861	5369	5824	6161	6337
	Z	−54850	−54494	−53946	−53890	−52130	−51001	−49691	−48288	−46784
130°	X	21424	21770	21897	22095	22430	22823	23307	23603	24040
	Y	1248	2192	3129	3976	4618	5164	5714	6109	6092
	Z	−58695	−58521	−57624	−56510	−55287	−54032	−53081	−52525	−51253
135°	X	17950	18114	18499	18803	19215	19652	20137	20646	21144
	Y	836	1771	2737	3687	4348	5009	5597	5998	6183
	Z	−66445	−65038	−63130	−60614	−59452	−57803	−56806	−55431	−54299
140°	X	14716	14941	15252	15601	16068	16630	17278	17890	18505
	Y	364	1325	2270	3221	4006	4768	5393	5778	5989
	Z	−67797	−67475	−66791	−65348	−63880	−60737	−60460	−59147	−58130
145°	X	11050	11394	11877	12325	12814	13372	13947	14577	15261
	Y	− 90	780	1712	2714	3568	4280	4692	5267	5494
	Z	−64872	−64768	−64746	−64424	−63943	−63770	−63180	−62167	−61246
150°	X	7558	7906	8376	8867	9481	10068	10715	11421	12151
	Y	− 572	254	1115	2047	2917	3661	4311	4809	5158
	Z	−64852	−65536	−63897	−62761	−63107	−62935	−64198	−63793	−63481

(Continuation of III)

φ:	λ:	180°	185°	190°	195°	200°	205°	210°	215°	220°
30°	X	17314	16976	16555	16025	15460	14814	14143	13309	12471
	Y	3865	4719	5399	5965	5882	6751	7000	7176	7200
	Z	50096	50932	51783	52690	54046	54960	56065	57093	58113
35°	X	20457	20115	19603	19157	18619	18038	17384	16631	15875
	Y	4432	5296	5968	6596	7023	7410	7619	7755	7889
	Z	48136	49002	49598	50782	51841	53000	54097	55377	56711
40°	X	22455	22078	21732	21354	21001	20592	20155	19617	19019
	Y	4739	5532	6129	6699	7197	7631	7906	8092	8171
	Z	44046	44668	45788	46746	47971	49383	50595	52065	53692
45°	X	23860	23478	23177	22913	22708	22505	22266	22110	21698
	Y	4912	5520	5994	6440	6836	7240	7579	7830	7983
	Z	39366	40143	41051	42067	43386	44932	46160	47719	49207
50°	X	25078	24712	24380	24192	24118	24091	24060	24007	23897
	Y	5041	5316	5838	6144	6387	6681	6975	7225	7443
	Z	35100	35938	36706	37710	38789	40009	41417	42845	44390
55°	X	26539	26093	25782	25643	25667	25816	25907	25994	26007
	Y	5183	5467	5676	5842	6004	6198	6379	6600	6796
	Z	31382	32055	32835	33866	35086	36431	37636	38781	40132
60°	X	28075	27676	27412	27281	27304	27462	27594	27800	28041
	Y	5187	5413	5511	5551	5638	5712	5815	5943	6114
	Z	27156	28004	28953	29854	30964	32268	33508	34653	35824
65°	X	29658	29381	29211	29086	29057	29115	29254	29526	29861
	Y	5141	5269	5256	5216	5211	5221	5246	5313	5427
	Z	23087	24172	25349	26297	27129	28270	29036	30000	31008
70°	X	31167	31008	30888	30844	30851	30838	30963	31137	31449
	Y	5216	5189	5058	4830	4701	4609	4581	4638	4719
	Z	18614	19646	20782	21928	23087	23925	24823	25660	26448
75°	X	32727	32672	32635	32672	32605	32591	32590	32660	32813
	Y	5281	5175	4897	4640	4389	4194	4040	3962	3951
	Z	14071	15250	16334	17423	18237	18972	19473	20003	20585
80°	X	34174	34114	34108	34069	33999	33905	33850	33824	33913
	Y	5413	5149	4844	4536	4225	3915	3674	3456	3365
	Z	9056	10220	11194	12115	12753	13260	13630	11852	14417
85°	X	35399	36146	35284	35170	35036	34903	34670	34419	34208
	Y	5460	5174	4875	4526	4147	3790	3338	3011	2893
	Z	3607	4647	5452	6146	6592	6876	7086	7239	7297
90°	X	36414	36260	36053	35884	35657	35376	35091	34720	34490
	Y	5659	5333	5014	4618	4168	3614	3117	2980	2850
	Z	− 2093	− 1226	− 423	157	522	827	1005	1217	1359
95°	X	36795	36690	36437	36097	35786	35450	35095	34680	34262
	Y	5806	5505	5228	4838	4352	[illegible]	3194	3030	2900
	Z	− 8486	− 8456	− 7604	− 6821	− 6149	− 5608	− 5057	− 4685	− 4427
100°	X	36192	36094	35963	35773	35414	35190	34798	34330	34039
	Y	5840	5631	5354	5070	4663	4196	3760	3427	3327
	Z	−14812	−14083	−13294	−12676	−12163	−11858	−11378	−10767	−10456
105°	X	35456	35291	35095	34860	34628	34340	33990	33633	33317
	Y	5880	5716	5475	5231	4949	4622	4294	4060	3943
	Z	−21382	−20827	−19420	−18743	−18339	−17910	−17457	−17015	−16545
110°	X	34220	34060	33907	33750	33521	33254	32992	32681	32375
	Y	5931	5802	5623	5406	5189	4950	4735	4574	4434
	Z	−28714	−27319	−26094	−25446	−24645	−24129	−23410	−23107	−22671
115°	X	32180	32210	32170	32098	31968	31777	31539	31276	30986
	Y	6081	5921	5750	5583	5397	5204	5042	4916	4834
	Z	−34104	−33133	−32396	−31738	−31126	−30700	−30098	−29524	−28905

(Continuation of III)

φ;	λ:	180°	185°	190°	195°	200°	205°	210°	215°	220°
115°	X	32180	32210	32170	32008	31968	31777	31539	31276	30986
	Y	6081	5921	5750	5583	5397	5204	5042	4916	4834
	Z	−34104	−33133	−32396	−31736	−31126	−30700	−30098	−29524	−28905
120°	X	29914	30068	30186	30190	30137	30065	29916	29729	29556
	Y	6249	6099	5913	5732	5585	5410	5293	5197	5141
	Z	−39947	−38918	−38099	−37389	−36744	−36087	−35361	−34923	−34309
125°	X	27275	27645	27806	28054	28127	28134	28101	28032	27931
	Y	6330	6196	6031	5861	5705	5571	5479	5415	5412
	Z	−45395	−44413	−43255	−42225	−41370	−40582	−39891	−39296	−38565
130°	X	24561	25061	25446	25762	25991	26100	26169	26188	26209
	Y	6314	6217	6070	5948	5841	5723	5642	5598	5603
	Z	−50131	−49180	−48181	−47212	−46297	−45002	−44261	−43417	−42834
135°	X	21733	22255	22727	23176	23594	23764	23769	24089	24155
	Y	6198	6116	6054	5958	5897	5793	5705	5768	5836
	Z	−53460	−52454	−51413	−50624	−49863	−48775	−47623	−47247	−46737
140°	X	19071	19604	20144	20633	21027	21375	21646	21874	22069
	Y	6025	5993	5935	5865	5831	5608	5841	5916	6037
	Z	−57005	−55718	−54803	−53901	−53115	−52182	−51563	−51054	−50674
145°	X	15995	16666	17285	17857	18325	18731	19085	19406	19716
	Y	5611	5657	5700	5716	5731	5774	5847	5958	6141
	Z	−60101	−59417	−58795	−58166	−57494	−56923	−56627	−56283	−55725
150°	X	12950	13748	14415	15019	15578	16200	16558	16964	17356
	Y	5320	5425	5476	5516	5619	5779	5907	6107	6403
	Z	−63853	−64020	−63576	−63542	−63273	−63820	−63033	−62877	−62454

(Continuation of III)

φ;	λ:	225°	230°	235°	240°	245°	250°	255°	260°	265°
30°	X	11592	10693	9783	8909	8032	7248	6539	5826	5177
	Y	7150	6944	6598	6104	5330	4441	3356	2121	589
	Z	59390	60416	61662	63181	63558	63847	63687	62857	61616
35°	X	15120	14255	13290	12494	11569	10831	10175	9359	8692
	Y	7771	7527	7126	6643	5929	5166	4180	2906	1377
	Z	57885	58791	59162	60488	61164	62906	64016	63672	62615
40°	X	18299	17540	16856	16180	15523	14777	14022	13359	12802
	Y	8071	7883	7575	7176	6632	5871	4920	3768	2257
	Z	54950	55848	57158	58197	59853	61114	61502	61927	62500
45°	X	21279	20753	20192	19605	19171	18762	19000	17508	16788
	Y	7991	7883	7670	7330	6915	6326	5475	4392	3036
	Z	50853	52300	53686	55488	57551	59177	60566	61066	61172
50°	X	23749	23509	23228	22879	22646	22340	21994	21339	20880
	Y	7549	7578	7473	7250	6924	6406	5702	4698	3557
	Z	45898	47448	49118	50698	52364	54093	55674	56675	57411
55°	X	26022	26059	26043	25886	25752	25615	25466	25081	24660
	Y	6932	7064	7125	7017	6780	6387	5763	4875	3796
	Z	41205	42654	44304	45835	47390	48961	50696	51810	52505
60°	X	28362	28529	28662	28704	28593	28462	28379	28285	28109
	Y	[illegible]	6456	6547	6583	6487	6154	5619	4860	3909
	Z	37183	38465	39612	40909	42264	43581	44891	46229	47232
65°	X	30238	30596	30002	31089	31193	31158	31056	31009	31018
	Y	5587	5744	5848	5931	5922	5728	5271	4680	4038
	Z	32122	33383	34522	35773	36956	38250	39838	40020	40888

(Continuation of III)

φ:	λ:	225°	230°	235°	240°	245°	250°	255°	260°	265°
65°	X	30238	30396	30902	31089	31193	31158	31056	31009	31018
	Y	5587	5744	5848	5931	5922	5728	5271	4680	4038
	Z	32122	33383	34522	35773	36956	38250	39838	40080	40888
70°	X	31852	32167	32600	32962	33233	33390	33379	33289	33108
	Y	4855	4980	5124	5270	5343	5249	4989	4560	4095
	Z	27228	27965	28889	29793	30934	31962	33166	33993	34747
75°	X	33077	33491	34019	34502	34885	35160	35220	35158	34950
	Y	4022	4172	4378	4542	4696	4754	4689	4545	4188
	Z	21159	21709	22488	23253	24042	25029	25893	27039	28167
80°	X	34035	34407	34989	35521	36055	36538	36681	36529	36233
	Y	3357	3465	3657	3943	4214	4379	4450	4464	4406
	Z	14400	14738	15419	16037	16927	17684	18490	19361	20308
85°	X	34380	34820	35345	35751	36202	36576	36540	36294	35907
	Y	2887	3026	3092	3450	3794	4060	4379	4564	4748
	Z	7596	7908	8409	8976	9583	10437	11368	12239	13183
90°	X	34490	34780	35050	35306	35436	35468	35424	35270	35026
	Y	2730	2890	3050	3193	3620	4062	4434	4831	5131
	Z	1561	1829	1998	2479	3012	3752	4384	5162	6083
95°	X	34280	34370	34448	34525	34508	34465	34413	34268	34040
	Y	2880	2990	3014	3061	3830	4293	4755	5203	5615
	Z	− 4122	− 3880	− 3584	− 3185	− 2783	− 2175	− 1517	− 756	− 201
100°	X	33866	33750	33692	33657	33629	33590	33479	33399	33041
	Y	3340	3398	3561	3884	4278	4721	5203	5689	6104
	Z	−10080	− 9727	− 9343	− 8815	− 8348	− 7831	− 7098	− 6330	− 6025
105°	X	33110	32942	32864	32755	32654	32568	32417	32210	31844
	Y	3919	3967	4113	4409	4764	5197	5619	6086	6498
	Z	−16082	−15708	−15152	−14485	−13782	−13047	−12356	−11394	−10874
110°	X	32095	31887	31724	31559	31411	31251	31066	30795	30441
	Y	4435	4482	4628	4857	5209	5604	6001	6434	6842
	Z	−22129	−21720	−21153	−20472	−19639	−18702	−17599	−16261	−14994
115°	X	30754	30512	30332	30179	29986	29815	29580	29290	28952
	Y	4825	4878	5058	5303	5576	5931	6332	6762	7219
	Z	−28277	−27660	−27126	−26324	−25292	−24037	−22657	−21244	−19751
120°	X	29351	29175	29018	28890	28744	28595	28409	28135	27849
	Y	5158	5232	5422	5615	5935	6296	6750	7276	7723
	Z	−33683	−32918	−32122	−31285	−30393	−29110	−27629	−26090	−24250
125°	X	27833	27712	27614	27538	27487	27440	27384	27317	27211
	Y	5444	5537	5719	5979	6304	6757	7337	7876	8406
	Z	−38094	−37502	−36862	−36177	−35347	−34181	−32708	−31026	−29235
130°	X	26190	26176	26149	26174	26227	26271	26374	26459	26529
	Y	5686	5843	6053	6380	6824	7351	7963	8597	9265
	Z	−42068	−41617	−41016	−40447	−39678	−38720	−37460	−36039	−34291
135°	X	24230	24314	24428	24572	24761	24960	25179	25397	2556[illegible]
	Y	5951	6152	6431	6815	7319	7066	8670	9369	9968
	Z	−46016	−45495	−44890	−44316	−43544	−42894	−41935	−40692	−39067
140°	X	22251	22447	22675	22941	23170	23486	23832	24158	24500
	Y	6205	6465	6825	7321	7963	8626	9348	10089	10738
	Z	−50107	−49906	−49572	−49300	−48502	−47656	−46760	−45345	−45716
145°	X	20023	20351	20673	21042	21430	21802	22218	22648	23106
	Y	6429	6809	7287	7882	8571	9344	10086	10819	11478
	Z	−55508	−55042	−54943	−54472	−53937	−53276	−52326	−51010	−49881
150°	X	17748	18148	18558	19002	19379	19991	20476	20949	21418
	Y	6784	7271	7852	8460	9078	9837	10584	11296	11938
	Z	−62145	−62322	−61287	−60636	−59332	−58806	−57630	−56070	−54560

(Continuation of III)

ϑ;	λ:	270°	275°	280°	285°	290°	295°	300°	305°	310°
30°	X	4384	3894	4009	4487	4817	5166	5639	6146	6343
	Y	− 636	− 1573	− 2603	− 3923	− 5257	− 6304	− 7218	− 7914	− 8325
	Z	57355	55427	54636	62855	59208	59241	58943	57810	56096
35°	X	8000	7502	7320	7495	7755	8020	8253	8508	8870
	Y	0	− 1390	− 2668	− 3902	− 5133	− 6172	− 7090	− 7796	− 8308
	Z	60766	59287	59246	60510	61614	61020	59183	57405	55415
40°	X	12227	11678	11308	11129	11096	11125	11209	11383	11575
	Y	748	− 714	− 2096	− 3545	− 4825	− 5915	− 6780	− 7534	− 8105
	Z	63022	62620	62045	62012	61296	60138	58287	56494	54636
45°	X	16210	15800	15441	15064	14803	14594	14397	14204	14235
	Y	1513	69	− 1351	− 2792	− 4245	− 5456	− 6385	− 7056	− 7305
	Z	60758	60948	60661	59957	59547	58146	56474	54413	52333
50°	X	20394	19988	19690	19101	18497	17878	17322	16898	16640
	Y	2084	698	− 630	− 1951	− 3362	− 4557	− 5517	− 6346	− 6995
	Z	57624	58084	58595	57943	56736	55140	53404	51688	50045
55°	X	24371	24164	23780	23148	22363	21496	20635	19939	19378
	Y	2518	1323	60	− 1213	− 2482	− 3629	− 4638	− 5561	− 6471
	Z	53145	54355	55128	54830	53875	52544	50851	48964	47174
60°	X	28069	28155	27392	26877	26550	25003	24036	23190	22475
	Y	2917	2315	677	− 391	− 1624	− 2716	− 3771	− 4789	− 5757
	Z	48390	50272	50117	50377	50917	49182	47750	46474	44949
65°	X	30991	30865	30352	29628	28742	27847	26964	26107	25340
	Y	3239	2294	1307	345	− 669	− 1719	− 2715	− 3785	− 4849
	Z	42239	43124	43711	44067	44131	43794	43370	42770	41558
70°	X	32964	32690	32146	31483	30700	29991	29129	28375	27558
	Y	3503	2688	1872	1026	196	− 741	− 1714	− 2666	− 3808
	Z	35548	36428	37042	37209	37352	37603	37572	37480	36919
75°	X	34596	34086	33411	32638	31717	30920	30342	29948	29336
	Y	3759	3132	2434	1653	923	180	− 706	− 1762	− 2911
	Z	29115	29931	30429	31103	30910	31191	31612	31984	31891
80°	X	35557	35005	34170	33241	32281	31418	30780	30487	30320
	Y	4156	3700	2959	2305	1579	568	134	− 905	− 2209
	Z	21228	22497	23281	23982	24532	25001	25448	25973	26348
85°	X	35494	34887	34208	33355	32377	31467	30794	30498	30411
	Y	4673	4294	3635	2889	2169	1447	627	− 355	− 1727
	Z	14162	15041	16041	17059	17988	18740	19433	20188	20675
90°	X	34677	34272	33766	32906	32001	31115	30482	30230	29971
	Y	5306	5040	4395	3575	2762	2012	1064	88	− 1308
	Z	7297	8423	9549	10595	11533	12387	13315	14097	14794
95°	X	33709	33340	32810	32041	31219	30447	29901	29497	29182
	Y	5893	5679	5079	4237	3373	2503	1523	429	− 1019
	Z	1195	2266	3489	4638	5820	6866	7836	8644	9300
100°	X	32702	32217	31593	30868	30192	29551	29035	28590	28352
	Y	6386	6262	5684	4871	3957	2975	1945	749	− 701
	Z	− 4337	− 3064	− 1776	− 433	886	2077	3187	3977	4492
105°	X	31444	30867	30222	29579	29012	28540	28120	27808	27498
	Y	6741	6674	6195	5411	4509	3471	2378	1117	− 384
	Z	− 9171	− 7923	− 6652	− 5302	− 3865	− 2347	− 1150	− 364	200
110°	X	29921	29404	28927	28430	28049	27631	27395	27078	26730
	Y	7091	7104	6732	5974	5047	4006	2815	1514	0
	Z	−13584	−12119	−10810	− 9159	− 7726	− 6446	− 5228	− 4295	− 3638
115°	X	28596	28186	27851	27703	27348	27091	26799	26512	26097
	Y	7529	7553	7332	6692	5771	4549	3290	1900	379
	Z	−18121	−16398	−14674	−13139	−11463	−10044	− 8773	− 7706	− 7034

(Continuation of III)

ω:	λ:	270°	275°	280°	285°	290°	295°	300°	305°	310°
115°	X	28596	28186	27851	27703	27348	27091	26799	26512	26097
	Y	7529	7553	7332	6692	5771	4549	3290	1900	379
	Z	−18121	−16398	−14674	−13139	−11463	−10044	− 8773	− 7706	− 7034
120°	X	27553	27399	27446	27459	27080	26770	26497	26199	25825
	Y	8031	8203	8043	7443	6501	5252	3881	2407	872
	Z	−22585	−20779	−18930	−17207	−15332	−13552	−12063	−10853	−10005
125°	X	27106	27344	27601	27489	27191	26912	26511	26230	25864
	Y	8807	9043	8897	8247	7286	6032	4556	2989	1356
	Z	−27283	−25555	−23765	−21627	−19468	−17514	−15792	−14584	−13687
130°	X	26688	27040	27102	27122	27050	26962	26697	26440	26125
	Y	9713	9913	9597	8917	7970	6772	5310	3716	1980
	Z	−32384	−30615	−28418	−25933	−23454	−21396	−19476	−18179	−17341
135°	X	25807	26113	26525	27106	27156	27074	26939	26712	26424
	Y	10392	10515	10288	9732	8649	7525	6096	4454	2684
	Z	−37031	−34929	−32960	−31028	−28500	−26051	−24009	−22456	−21380
140°	X	24851	25313	25913	26533	27284	27256	27071	26904	26711
	Y	11107	11200	10999	10407	9528	8333	6917	5295	3580
	Z	−41915	−41141	−37539	−35406	−33481	−30922	−28681	−27103	−26026
145°	X	23585	24244	24832	25756	26607	26985	26920	26822	26682
	Y	11862	11929	11618	11110	10213	9012	7702	6159	4505
	Z	−47140	−44969	−42223	−40259	−38050	−35724	−33465	−31658	−30372
150°	X	21945	22583	23388	24229	24963	25648	25822	25891	25874
	Y	12349	12475	12245	11687	10751	9293	8224	6921	5343
	Z	−52203	−49738	−47627	−45125	−42528	−40141	−38040	−36395	−35274

(Continuation of III)

ω:	λ:	315°	320°	325°	330°	335°	340°	345°	350°	355°
30°	X	7219	7888	8565	9401	10256	11095	11931	12729	13442
	Y	− 8603	− 8709	− 8716	− 8539	− 8231	− 7865	− 7287	− 6626	− 5901
	Z	54547	53368	52100	51254	50544	49884	49023	48187	47424
35°	X	9355	9883	10541	11229	11955	12784	13655	14497	15225
	Y	− 8648	− 8795	− 8845	− 8773	− 8554	− 8092	− 7543	− 6889	− 6151
	Z	53754	52413	51354	50360	49206	48084	47307	46394	45525
40°	X	11854	12180	12600	13002	13791	14539	15375	16285	17028
	Y	− 8455	− 8635	− 8714	− 8665	− 8462	− 8114	− 7599	− 6935	− 6254
	Z	53101	51222	49340	47611	46550	45745	44677	43809	43260
45°	X	14094	14289	14573	15062	15761	16583	17404	18307	19134
	Y	− 7947	− 8250	− 8459	− 8510	− 8380	− 8088	− 7617	− 7003	− 6476
	Z	50395	48607	46803	45463	44180	43465	42343	41559	40761
50°	X	16481	16554	16799	17281	17945	18703	19613	20473	21249
	Y	− 7482	− 7848	− 8145	− 8366	− 8368	− 8145	− 7726	− 7284	− 6768
	Z	47774	45726	44162	43124	42142	41066	39982	39068	38240
55°	X	19061	18972	19101	19454	19985	20864	21740	22570	23321
	Y	− 7127	− 7633	− 8108	− 8425	− 8456	− 8289	− 7898	− 7552	− 7041
	Z	45175	43360	41923	40581	39013	38240	37257	36417	35334
60°	X	21941	21561	21520	21642	22157	22848	23619	24375	25164
	Y	− 6729	− 7508	− 8118	− 8525	− 8691	− 8505	− 8194	− 7826	− 7295
	Z	43163	41471	39571	37833	36649	35517	34096	32767	31745
65°	X	24774	24158	23898	23819	24072	24651	25348	26029	26685
	Y	− 6039	− 7171	− 8089	− 8764	− 9000	− 8875	− 8547	− 8165	− 7652
	Z	40467	38559	36825	35147	33493	31972	29708	27599	25887

(Continuation of III)

u:	λ:	315°	320°	325°	330°	335°	340°	345°	350°	355°
65°	X	24774	24158	23808	23819	24072	24651	25348	26029	26685
	Y	− 6039	− 7171	− 8089	− 8764	− 9000	− 8875	− 8547	− 8165	− 7652
	Z	40467	38559	36825	35147	33493	31972	29708	27599	25887
70°	X	26865	26317	25908	25703	25763	26154	26659	27224	27830
	Y	− 5182	− 6594	− 7880	− 8901	− 9419	− 9305	− 9006	− 8601	− 8156
	Z	36046	34725	33243	31382	29160	26808	24533	22039	19561
75°	X	28729	28170	27674	27309	27143	27313	27643	28140	28729
	Y	− 4379	− 5988	− 7606	− 8891	− 9746	− 9896	− 9563	− 9180	− 8783
	Z	31254	29997	28368	26317	24057	21693	19116	16464	13796
80°	X	29915	29485	28925	28447	28090	28177	28345	28853	29463
	Y	− 3761	− 5535	− 7373	− 8879	−10040	−10301	−10130	− 9888	− 9479
	Z	25903	24951	23532	21651	19372	16973	14036	11101	8197
85°	X	30200	29945	29405	29024	28571	28436	28443	28755	29323
	Y	− 3307	− 5235	− 7154	− 8874	−10211	−10640	−10726	−10466	−10059
	Z	20621	19931	18599	16823	14417	11764	8717	5763	2803
90°	X	29646	29267	28782	28281	27848	27609	27530	27659	27887
	Y	− 3029	− 4880	− 6821	− 8601	− 9971	−10672	−11011	−10821	−10334
	Z	14750	14047	13016	11446	9138	6698	3860	951	− 1558
95°	X	28863	28421	27916	27360	26837	26628	26143	26090	26208
	Y	− 2821	− 4654	− 6574	− 8261	− 9592	−10044	−11097	−11119	−10678
	Z	9423	8805	7774	6467	4514	2323	− 330	− 2731	− 5118
100°	X	27986	[illegible]	26941	26297	25710	25181	24698	24406	24367
	Y	− 2531	− 4405	− 6220	− 7873	− 9062	−10071	−10910	−11191	−10917
	Z	4576	4253	3477	2201	634	− 1303	− 3514	− 6035	− 8547
105°	X	27057	26488	25867	25241	24612	23991	23350	22797	22495
	Y	− 2248	− 4077	− 5853	− 7317	− 8475	− 9491	−10396	−10972	−10907
	Z	355	[illegible]	− 463	− 1377	− 2736	− 4549	− 6769	− 9084	−11658
110°	X	26235	25663	25087	24432	23764	23091	22406	21725	21135
	Y	− 1850	− 3645	− 5348	− 6760	− 7890	− 8910	− 9836	−10479	−10768
	Z	− 3424	− 3489	− 3910	− 4813	− 6050	− 7567	− 9804	−12114	−14441
115°	X	25642	25087	24482	23807	23087	22387	21624	20927	20300
	Y	− 1381	− 3117	− 4759	− 6194	− 7353	− 8311	− 9253	− 9922	−10381
	Z	− 6681	− 6695	− 7034	− 7874	− 9180	−10841	−12859	−14897	−17233
120°	X	25337	24699	24097	23361	22618	21832	21071	20324	19601
	Y	− 811	− 2996	− 4141	− 5501	− 6736	− 7731	− 8656	− 9370	− 9901
	Z	− 9562	− 9592	−10127	−11064	−12329	−13779	−15608	−17590	−19600
125°	X	25359	24738	24024	23228	22393	21560	20772	19981	19225
	Y	− 258	− 2019	− 3519	− 4902	− 6140	− 7200	− 8113	− 8868	− 9446
	Z	−13202	−13197	−13784	−14453	−15514	−16922	−18602	−20266	−21861
130°	X	25637	25024	24310	23510	22676	21781	20896	20006	19167
	Y	410	− 1348	− 2877	− 4301	− 5570	− 6742	− 7674	− 8458	− 9060
	Z	−16864	−16797	−17194	−18010	−18909	−20196	−21497	−22955	−24388
135°	X	26019	25453	24753	23953	23099	22174	21243	20302	19375
	Y	1045	− 592	− 2195	− 3651	− 4980	− 6184	− 7232	− 8052	− 8728
	Z	−20987	−20888	−21199	−21817	−22686	−23672	−24849	−26028	−27199
140°	X	26337	25859	25211	24443	23564	22632	21642	20612	19625
	Y	1826	226	− 1409	− 2857	− 4226	− 5538	− 6617	− 7504	− 8229
	Z	−25316	−25119	−25250	−25783	−26588	−27410	−28146	−29257	−30297
145°	X	26399	25996	25395	24649	23865	22913	21908	20867	19861
	Y	2736	1135	− 488	− 1943	− 3354	− 4648	− 5788	− 6746	− 7558
	Z	−29736	−29549	−29600	−30049	−30571	−31215	−31965	−32677	−33356
150°	X	25734	25419	24966	24340	23590	22736	21841	20897	19915
	Y	3708	2038	479	− 992	− 2285	− 3601	− 4742	− 5697	− 6599
	Z	−34399	−34086	−34160	−34363	−34592	−34956	−35491	−35930	−36338

IVa. Coefficients of the Trigonometric Series for the North Component X

u	k_0	k_1	K_1	k_2	K_2	k_3	K_3	k_4	K_4
30°	12930	-1590	5180	2820	1110	-50	-70	-130	-10
35	15690	-2290	4970	2470	1240	-100	140	-90	-50
40	18530	-2650	4820	1590	1250	120	230	-50	-60
45	21300	-2640	4720	590	1280	360	440	20	-70
50	23930	-2360	4570	-440	1330	510	690	110	-100
55	26520	-2010	4410	-1420	1320	560	790	270	10
60	28830	-1710	3970	-2210	1350	360	830	340	70
65	30770	-1550	3500	-2660	1320	190	770	350	40
70	32290	-1620	3030	-2820	1190	140	710	330	60
75	33400	-1960	2470	-2650	960	70	820	350	70
80	34070	-2470	1930	-2400	640	440	620	400	90
85	34160	-3240	1630	-1980	150	460	460	370	110
90	33700	-4220	1380	-1720	-280	310	340	200	160
95	32730	-5110	1100	-1500	-610	140	170	230	160
100	31540	-5830	660	-1380	-890	110	40	120	160
105	30110	-6380	70	-1140	-1320	50	30	70	40
110	28600	-6430	-570	-780	-1340	-40	-10	-190	-20
115	26850	-5900	-1530	-530	-1350	-140	-130	-260	-10
120	25160	-5080	-2600	-420	-1290	-250	-230	-270	60
125	23580	-3970	-3760	-470	-1260	-360	-320	-210	170
130	22090	-2620	-4800	-440	-1220	-420	-460	-170	180
135	20600	-1070	-5730	-340	-1260	-500	-580	-140	200
140	19140	490	-6580	-130	-1270	-630	-610	-170	200
145	17480	2210	-7350	150	-1130	-740	-600	-230	160
150	15720	3760	-7710	420	-800	-720	-600	-230	140

IVb. Coefficients of the Trigonometric Series for the East Component Y

u	l_0	l_1	L_1	l_2	L_2	l_3	L_3	l_4	L_4
30°	40	-4350	1190	-1070	4910	-90	-510	300	-210
35	60	-4620	770	-1090	5190	-180	-630	440	-210
40	110	-4780	350	-1230	5220	-220	-650	590	-300
45	150	-4850	20	-1420	4960	-300	-550	700	-380
50	150	-4920	-230	-1580	4500	-400	-360	780	-380
55	120	-5050	-410	-1710	3980	-480	-150	850	-300
60	100	-5180	-520	-1850	3390	-560	90	870	-150
65	50	-5300	-570	-1980	2740	-710	310	780	20
70	0	-5400	-630	-2160	2110	-930	500	700	170
75	-50	-5530	-670	-2320	1500	-1190	680	620	310
80	-110	-5670	-750	-2500	1000	-1420	820	570	450
85	-150	-5850	-870	-2600	580	-1600	1030	520	530
90	-130	-6010	-1130	-2650	240	-1700	1170	510	570
95	-110	-6170	-1500	-2650	-10	-1750	1270	490	570
100	-80	-6370	-2040	-2610	-130	-1720	1320	390	530
105	-60	-6520	-2650	-2460	-260	-1670	1340	270	440
110	-80	-6670	-3300	-2310	-450	-1630	1320	170	330
115	-150	-6770	-4030	-2110	-690	-1580	1310	130	250
120	-210	-6820	-4810	-1910	-940	-1530	1330	160	200
125	-250	-6780	-5650	-1710	-1170	-1460	1390	190	190
130	-280	-6690	-6490	-1540	-1340	-1390	1510	210	220
135	-290	-6520	-7270	-1400	-1450	-1350	1600	190	210
140	-200	-6230	-7940	-1360	-1530	-1330	1630	190	180
145	-60	-5830	-8520	-1390	-1620	-1190	1600	170	200
150	140	-5390	-8980	-1360	-1700	-950	1470	150	260

IVc. Coefficients of the Trigonometric Series for the Vertical Component Z

u	m_0	m_1	M_1	m_2	M_2	m_3	M_3	m_4	M_4
30°	53950	−2530	−3250	−4910	220	900	−720	−990	−110
35	52470	−2380	−4540	−5770	−500	850	−280	−510	320
40	50010	−1750	−5800	−6820	−1120	970	170	−60	750
45	46810	−830	−6590	−7300	−1290	1130	500	180	590
50	42940	170	−6950	−7210	−1670	1280	−20	400	700
55	38850	740	−7510	−6840	−2110	520	−60	540	920
60	34270	1160	−8430	−6320	−2580	0	−640	250	810
65	28750	800	−9450	−5350	−3190	−280	−1270	−250	540
70	22920	240	−10230	−4200	−3600	−460	−1530	−720	400
75	16760	−160	−10810	−3200	−4060	−860	−1690	−850	270
80	10430	−10	−11040	−2040	−4360	−1220	−1850	−1040	170
85	4050	340	−11280	−1110	−4670	−1440	−1980	−990	10
90	−2250	860	−11660	−340	−4580	−1460	−2260	−710	−70
95	−8220	1820	−12280	170	−4280	−1250	−2480	−490	−140
100	−13770	3180	−12810	700	−4200	−1090	−2440	−350	−150
105	−19050	5060	−13450	1220	−3780	−1000	−2360	−60	20
110	−24030	7380	−13850	1400	−3200	−1230	−2220	140	20
115	−28430	9560	−13590	1390	−2840	−1630	−2170	450	60
120	−32450	11520	−13180	1310	−2450	−1690	−2240	380	290
125	−35980	12820	−12180	1360	−2100	−1560	−2240	150	510
130	−39380	13870	−11150	1590	−1680	−1650	−2030	−180	410
135	−43030	14800	−10680	2100	−350	−2360	−2200	240	90
140	−46370	15220	−9140	2260	60	−2360	−2370	70	280
145	−49030	14830	−6330	2020	−950	−2000	−1730	−320	200
150	−52210	14780	−3710	2120	−2160	−1920	−700	−390	−170

Va. Coefficients of the Trigonometric Series for $\alpha X \sin v$.

f:	$\alpha k_0 \sin v$	$\alpha k_1 \sin v$	$\alpha K_1 \sin v$	$\alpha k_2 \sin v$	$\alpha K_2 \sin v$	$\alpha k_3 \sin v$	$\alpha K_3 \sin v$	$\alpha k_4 \sin v$	$\alpha K_4 \sin v$
f_1+f_{25}	14397.0	1090.5	−1271.4	1628.2	155.8	−386.9	−336.7	−180.9	65.8
f_2+f_{24}	19111.2	−46.1	−1371.3	1509.5	63.4	−484.0	−265.0	−184.4	63.4
f_3+f_{23}	24309.0	−1393.9	−1135.8	942.1	−12.9	−329.1	−245.2	−142.0	90.3
f_4+f_{22}	29726.9	−2632.1	−716.6	177.4	14.2	−99.3	−99.8	−85.1	92.2
f_5+f_{21}	35350.8	−3825.4	−176.7	−676.0	84.5	69.1	176.7	−46.1	61.4
f_6+f_{20}	41129.8	−4909.3	533.6	−1551.6	49.3	164.2	385.9	49.8	147.6
f_7+f_{19}	46834.8	−5890.1	1188.4	−2281.5	52.0	95.4	520.5	60.7	112.8
f_8+f_{18}	52283.8	−6760.1	1787.6	−2894.5	−27.2	45.4	580.7	81.7	27.2
f_9+f_{17}	57262.6	−7570.4	2313.4	−3385.6	−141.1	94.0	658.3	131.7	37.6
$f_{10}+f_{16}$	61373.4	−8059.4	2454.6	−3662.5	−347.9	116.0	821.4	405.9	106.3
$f_{11}+f_{15}$	64626.3	−8175.6	2551.2	−3723.3	−246.3	541.8	650.1	512.2	246.3
$f_{12}+f_{14}$	66638.8	−8318.6	2719.7	−3467.0	−458.3	597.8	647.6	597.8	269.0
f_{13}	33700.0	−4220.0	1360.0	−1720.0	−280.0	310.0	380.0	200.0	160.0
f_1-f_{25}	−1402.0	−2688.5	6477.4	1206.0	959.8	336.7	266.8	50.8	−75.4
f_2-f_{24}	−1031.8	−2592.7	7098.3	1336.7	1365.5	368.7	426.4	80.7	−121.0
f_3-f_{23}	−393.6	−2026.8	7356.6	1109.9	1626.2	484.0	542.1	77.4	−167.8
f_4-f_{22}	496.6	−1113.9	7414.0	659.8	1802.1	610.2	723.7	113.5	−191.6
f_5-f_{21}	1413.4	199.7	7197.7	0.0	1958.8	714.3	883.4	215.1	−215.1
f_6-f_{20}	2413.6	1609.1	6707.2	−779.9	2118.1	755.3	911.8	394.1	−131.4
f_7-f_{19}	3183.6	2923.4	5699.3	−1552.8	2290.1	529.3	919.5	529.3	8.7
f_8-f_{18}	3557.0	3947.1	4564.2	−1932.7	2422.7	299.4	816.6	553.5	45.4
f_9-f_{17}	3470.2	4523.5	3385.5	−1918.5	2379.3	169.3	677.1	489.0	75.2
$f_{10}-f_{16}$	3179.8	4271.3	2319.3	−1459.2	2203.3	19.3	768.4	270.6	29.0
$f_{11}-f_{15}$	2492.1	3309.6	1251.0	−1004.7	1507.1	325.1	571.3	275.8	−69.0
$f_{12}-f_{14}$	1424.6	1863.0	526.0	−478.3	757.1	318.8	808.8	139.5	−49.8

Vb. Coefficients of the Trigonometric Series for $\beta Y \sin v$.

f:	$\beta l_0 \sin v$	$\beta l_1 \sin v$	$\beta L_1 \sin v$	$\beta l_2 \sin v$	$\beta L_2 \sin v$	$\beta l_3 \sin v$	$\beta L_3 \sin v$	$\beta l_4 \sin v$	$\beta L_4 \sin v$
$f_1 + f_{25}$	90.5	−4898.6	−3917.9	−1222.1	1614.4	−523.1	482.8	226.2	25.1
$f_2 + f_{24}$	0.0	−6027.5	−4470.3	−1430.4	2059.2	−790.2	559.5	351.8	−5.8
$f_3 + f_{23}$	−58.2	−7114.8	−4904.7	−1673.7	2384.5	−1000.2	633.3	504.0	−77.5
$f_4 + f_{22}$	−99.5	−8080.2	−5152.8	−2004.1	2494.4	−1172.6	746.2	632.5	−120.8
$f_5 + f_{21}$	−100.0	−8935.0	−5172.2	−2401.4	2432.2	−1377.7	885.1	762.0	−123.1
$f_6 + f_{20}$	−107.0	−9733.8	−4986.2	−2814.1	2312.1	−1596.3	1020.3	855.7	−90.5
$f_7 + f_{19}$	−95.7	−10435.9	−4635.3	−3269.9	2130.7	−1817.6	1234.9	895.8	43.5
$f_8 + f_{18}$	−91.0	−10982.4	−4185.5	−3721.4	1865.3	−2083.7	1474.0	828.0	245.7
$f_9 + f_{17}$	−75.5	−11384.6	−3706.8	−4216.2	1565.8	−2414.6	1716.4	820.6	471.6
$f_{10} + f_{16}$	−106.6	−11681.1	−3218.3	−4633.6	1202.0	−2772.4	1958.1	862.7	727.0
$f_{11} + f_{15}$	−187.8	−11898.0	−2757.1	−5049.8	859.7	−3103.0	2114.8	948.7	968.5
$f_{12} + f_{14}$	−259.9	−12014.7	−2368.9	−5247.8	569.7	−3348.5	2299.0	1009.6	1099.5
f_{13}	−130.5	−6030.2	−1133.8	−2658.9	240.8	−1705.7	1173.9	511.7	571.9
$f_1 - f_{25}$	−50.3	523.1	5114.9	145.9	3324.4	432.5	−995.8	75.4	−236.4
$f_2 - f_{24}$	69.2	697.9	5358.4	173.0	3928.0	582.6	−1286.2	155.7	−236.5
$f_3 - f_{23}$	200.8	937.0	5357.1	84.0	4361.8	717.3	−1473.3	258.5	−310.2
$f_4 - f_{22}$	312.7	1186.6	5180.7	−14.2	4555.8	746.2	−1527.9	362.4	−419.3
$f_5 - f_{21}$	331.0	1362.8	4818.2	−30.8	4494.9	762.0	−1439.3	488.7	−461.8
$f_6 - f_{20}$	304.4	1423.5	4311.5	0.0	4237.5	806.4	−1267.1	548.1	−408.2
$f_7 - f_{19}$	269.6	1426.2	3730.8	52.2	3765.7	843.6	−1078.4	617.5	−304.4
$f_8 - f_{18}$	182.0	1337.5	3148.2	118.3	3120.9	791.6	−909.9	591.4	−209.3
$f_9 - f_{17}$	75.5	1197.9	2518.4	141.5	2414.6	660.3	−773.4	499.9	−150.9
$f_{10} - f_{16}$	9.7	959.7	1919.4	135.7	1706.1	465.2	−639.8	339.2	−126.0
$f_{11} - f_{15}$	−29.6	691.7	1274.8	108.7	1116.7	296.5	−494.1	177.9	−79.1
$f_{12} - f_{14}$	−20.0	319.9	629.7	50.0	589.7	149.9	−239.[illegible]	30.0	−40.0

Vc. Coefficients of the Trigonometric Series for γZ.

f:	γm_0	γm_1	γM_1	γm_2	γM_2	γm_3	γM_3	γm_4	γM_4
$f_1 + f_{25}$	1738.5	12239.7	−6954.2	−2787.7	−1938.4	−1019.2	−1418.8	−1378.9	−279.8
$f_2 + f_{24}$	3436.2	12436.8	−10858.0	−3745.8	−1448.4	−1148.7	−2007.8	−829.1	519.4
$f_3 + f_{23}$	3635.0	13451.8	−14919.8	−4553.7	−1058.5	−1388.1	−2197.0	10.0	1028.6
$f_4 + f_{22}$	3773.7	13946.6	−17241.1	−5191.8	−1637.2	−1227.9	−1697.2	419.3	678.9
$f_5 + f_{21}$	3553.0	14012.4	−18064.5	−5609.0	−3343.4	−369.3	−2046.0	219.6	1107.8
$f_6 + f_{20}$	2863.6	13529.5	−19595.9	−5467.6	−4200.5	−1037.6	−2294.8	688.5	1426.8
$f_7 + f_{19}$	1815.4	12648.2	−21555.8	−4997.4	−5017.4	−1685.8	−2872.8	628.4	1097.2
$f_8 + f_{18}$	319.1	10331.5	−22976.7	−3949.1	−6013.4	−1904.[illegible]	−3430.5	199.4	598.3
$f_9 + f_{17}$	−1106.7	7597.5	−24008.8	−2791.8	−6780.0	−1685.0	−3738.9	−578.3	418.8
$f_{10} + f_{16}$	−2282.9	4884.7	−24184.8	−1973.8	−7815.6	−1854.2	−4037.4	−907.2	289.1
$f_{11} + f_{15}$	−3329.2	3159.7	−23772.7	−1335.6	−6539.2	−2302.6	−4276.1	−1385.5	19.9
$f_{12} + f_{14}$	−4156.2	2152.8	−23481.8	−936.9	−8920.4	−2681.1	−4445.2	−1475.1	−129.6
f_{13}	−2242.5	857.1	−11621.1	−338.9	−4514.9	−1455.1	−2252.5	−707.7	−69.8
$f_1 - f_{25}$	106071.0	−17295.5	459.6	−7024.3	2378.1	2817.7	−20.0	−599.5	60.0
$f_2 - f_{24}$	101388.1	−17191.0	1788.0	−7781.4	449.5	2846.8	1448.4	−189.8	119.9
$f_3 - f_{23}$	96246.5	−16946.5	3335.4	−9067.6	−1178.4	3325.4	2536.5	−129.8	469.4
$f_4 - f_{22}$	89689.6	−15603.8	4083.1	−9384.8	−938.4	3484.2	2695.4	−59.9	499.2
$f_5 - f_{21}$	82158.2	−13673.1	4191.7	−8782.8	10.0	2924.8	2006.1	578.9	289.4
$f_6 - f_{20}$	74661.8	−12052.9	4609.6	−8181.4	−10.0	2075.2	2175.1	389.1	409.1
$f_7 - f_{19}$	66552.6	−10334.0	4738.1	−7610.8	−129.7	1685.8	1596.0	−129.7	518.7
$f_8 - f_{18}$	57022.9	−8735.8	4128.6	−6721.4	−349.0	1346.8	897.5	−698.1	478.7
$f_9 - f_{17}$	46811.3	−7119.0	3609.3	−5583.5	−398.8	767.7	688.0	−857.5	378.9
$f_{10} - f_{16}$	35698.3	−5203.7	2631.7	−4406.3	−279.1	139.6	667.9	−787.5	249.2
$f_{11} - f_{15}$	24121.6	−3179.6	1764.8	−2731.1	−159.5	−129.6	588.1	−687.8	319.0
$f_{12} - f_{14}$	12229.3	−1475.1	996.7	−1275.8	−388.7	−189.4	498.3	−498.2	149.5

VI. Coefficients of the Series for Representation of $\alpha X \sin v$, $\beta Y \sin v$, γZ.

1.

$\alpha X \sin v$

m; n:	0	1	2	3	4	5	6	7
0	21279	361	−10235	−933	695	535	−132.58	17.77
1		−1921 438	258 1736	789 −442	−924 129	227 10	273.81 −221.57	55.08 95.78
2			− 783 − 54	−179 606	396 50	372 −114	252.18 − 19.12	−20.66 52.14
3				71 166	125 226	− 85 − 57	14.68 − 0.69	−68.50 −28.56
4					84 43	107 − 14	− 44.96 − 49.76	14.50 30.46

$\beta Y \sin v$

m; n:	0	1	2	3	4	5	6
0	−50	67	34	− 38	− 9	− 4.87	0.27
1		−3421 −1277	330 1261	132 −496	−121 166	61 170	− 7 −38
2			−1247 483	23 1051	95 372	− 1 216	−64 − 2
3				−749 518	202 −312	12 6	17 −81
4					275 160	108 − 84	−91 54

γZ

m; n:	0	1	2	3	4	5	6
0	0	36388	701	−1412	−1273	291	− 40
1		2847 −6881	−3841 966	1900 445	− 909 − 483	−607 515	−132 −202
2			− 959 −1990	−2191 − 54	− 797 362	−410 95	168 − 77
3				− 582 −1089	517 431	− 30 − 82	423 182
4					− 190 114	− 89 116	141 201

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VI. Coefficients of the Series for Representation of $\alpha X \sin v$, $\beta Y \sin v$, γZ.

2.

$\alpha X \sin v$

m; n:	0	1	2	3	4	5
0	21247	368	−10299	−924	594.00	544.00
1		−1913 443	271 1703	817 −424	−790.11 − 52.02	309.05 84.08
2			− 761 − 58	−184 606	633.23 10.14	341.09 −100.05
3				66 162	167.47 247.00	−144.27 −106.74
4					0 0	54.27 − 74.47

$\beta Y \sin v$

m; n:	0	1	2	3	4
0	−49	64	34	− 41.02	− 9.01
1		−3452 −1312	353 1250	− 19 −644	− 50 117
2			−1245 485	2 956	100 390
3				−741 503	182 −297
4					247 182

γZ

m; n:	0	1	2	3	4
0	0	36182	736	−1648	−1245
1		2893 −6919	−3813 1008	1984 374	− 868 − 420
2			− 962 −1988	−2236 − 44	− 788 358
3				− 585 −1097	570 454
4					− 179 130

3.

$\alpha X \sin v$

0	1	2	3
21449	609	−9592.79	− 398.68
	−1872 450	− 455.71 1572.85	1128.41 − 356.92
		0 0	151.56 464.17

$\beta Y \sin v$

0	1	2
−53	0	23.70
	−3517 −1019	396 1051
		−1281 401

γZ

0	1	2
0	30738	1410
	3083 −6883	−3819 1005
		−1049 −1949

VIIa. Numerical Values or Logarithms of the Computed Coefficients k and K in the Series Expansion of X

	k_0	$\log k_1$	$\log K_1$	$\log k_2$	$\log K_2$	$\log k_3$	$\log K_3$	$\log k_4$	$\log K_4$
0°	0.0	3.411536	3.629155	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$
5	1907.0	3.375041	3.630784	3.06830	2.41631	1.3365.$_n$	0.842..$_n$	9.699.	9.000.
10	3869.8	3.251322	3.635695	3.34286	2.69992	1.9042.$_n$	0.785..$_n$	0.556.	0.000.
15	5937.9	2.96156.	3.643808	3.47360	2.84844	2.19893$_n$	0.690..$_n$	1.0682	0.447.
20	8147.5	1.9504..$_n$	3.654667	3.53203	2.93797	2.36399$_n$	1.0253.	1.4183	0.672.
25	10517.9	3.029384$_n$	3.667117	3.53732	2.99480	2.43981$_n$	1.6902.	1.6756	0.699.
30	13046.8	3.274481$_n$	3.679192	3.49241	3.03310	2.43489$_n$	2.06371	1.8745	0.279.
35	15710.0	3.383959$_n$	3.688322	3.38780	3.06213	2.33203$_n$	2.32408	2.0316	0.792.$_n$
40	18460.8	3.422459$_n$	3.691647	3.18794	3.08743	2.03623$_n$	2.51614	2.1578	1.2856$_n$
45	21233.4	3.411232$_n$	3.686359	2.71584	3.11099	1.4654.	2.65906	2.2598	1.5551$_n$
50	23946.2	3.361652$_n$	3.670023	2.69966$_n$	3.13123	2.24279	2.76320	2.3424	1.7168$_n$
55	26507.3	3.285985$_n$	3.640779	3.14879$_n$	3.14370	2.48173	2.83474	2.4094	1.8000$_n$
60	28821.6	3.206718$_n$	3.597553	3.32564$_n$	3.14214	2.59528	2.87749	2.4623	1.8021$_n$
65	30798.4	3.162803$_n$	3.540342	3.41047$_n$	3.11886	2.63909	2.89409	2.5016	1.6893$_n$
70	32358.9	3.191088$_n$	3.470484	3.44185$_n$	3.06348	2.63215	2.88615	2.5271	1.2504$_n$
75	33443.5	3.287130$_n$	3.390970	3.43463$_n$	2.95856	2.56377	2.85425	2.5367	1.4456
80	34018.8	3.412326$_n$	3.306017	3.39632$_n$	2.76245	2.50010	2.79810	2.5272	1.9175
85	34079.2	3.533925$_n$	3.218798	3.33214$_n$	2.28012	2.38846	2.71550	2.4935	2.1405
90	33653.8	3.636418$_n$	3.126294	3.24765$_n$	2.34163$_n$	2.25959	2.60043	2.4262	2.2658
95	32797.2	3.714665$_n$	3.010003	3.15088$_n$	2.78604$_n$	2.12516	2.43727	2.3073	2.3259
100	31591.7	3.767542$_n$	2.81003.	3.05385$_n$	2.97557$_n$	1.9814.	2.17609	2.0878	2.3294
105	30133.0	3.794920$_n$	2.13001.	2.97095$_n$	3.07715$_n$	1.7634.	1.4713.	1.4886	2.2721
110	28522.1	3.796200$_n$	2.75020.$_n$	2.91185$_n$	3.12882$_n$	0.756..	1.9410.$_n$	1.8089$_n$	2.1303
115	26851.9	3.769289$_n$	3.166282$_n$	2.87216$_n$	3.14672$_n$	1.8645.$_n$	2.30211$_n$	2.1875$_n$	1.8096
120	25196.1	3.709092$_n$	3.406574$_n$	2.83296$_n$	3.14085$_n$	2.26305$_n$	2.48869$_n$	2.3600$_n$	1.1835$_n$
125	23598.9	3.603740$_n$	3.572802$_n$	2.76597$_n$	3.11945$_n$	2.50786$_n$	2.60842$_n$	2.4504$_n$	1.9395$_n$
130	22070.4	3.421801$_n$	3.692036$_n$	2.62818$_n$	3.09068$_n$	2.67348$_n$	2.68744$_n$	2.4883$_n$	2.1611$_n$
135	20585.7	3.033062$_n$	3.775421$_n$	2.29248$_n$	3.06221$_n$	2.78483$_n$	2.73456$_n$	2.4849$_n$	2.2541$_n$
140	19090.4	2.76448.	3.828615$_n$	1.9465.	3.03926$_n$	2.85101$_n$	2.75220$_n$	2.4436$_n$	2.2739$_n$
145	17510.8	3.352877	3.854974$_n$	2.59528	3.02247$_n$	2.87541$_n$	2.73965$_n$	2.3636$_n$	2.2368$_n$
150	15765.8	3.586452	3.856898$_n$	2.82763	3.00698$_n$	2.85794$_n$	2.69390$_n$	2.2416$_n$	2.1455$_n$
155	13786.8	3.726613	3.836792$_n$	2.94096	2.98367$_n$	2.79463$_n$	2.60927$_n$	2.0686$_n$	1.9952$_n$
160	11526.8	3.819965	3.798029$_n$	2.97941	2.94067$_n$	2.67642$_n$	2.47524$_n$	1.8293$_n$	1.7781$_n$
165	8973.7	3.883463	3.746424$_n$	2.95027	2.86231$_n$	2.48416$_n$	2.27161$_n$	1.4942$_n$	1.4502$_n$
170	6153.8	3.925013	3.691850$_n$	2.83860	2.72173$_n$	2.17260$_n$	1.9523.$_n$	0.996.$_n$	0.959.$_n$
175	3131.6	3.948638	3.648740$_n$	2.57496	2.44279$_n$	1.5944.$_n$	1.3692.$_n$	0.114.$_n$	0.079.$_n$
180	0.0	3.956317	3.632103$_n$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$

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VIIb. Numerical Values or Logarithms of the Computed Coefficients l and L in the Series Expansion of Y.

	l_0	$\log l_1$	$\log L_1$	$\log l_2$	$\log L_2$	$\log l_3$	$\log L_3$	$\log l_4$	$\log L_4$
0°	0.0	3.629155$_n$	3.411536	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$
5	32.0	3.630163$_n$	3.403189	2.41747$_n$	3.07295	0.362..	1.3404.$_n$	9.000.$_n$	9.699.
10	62.2	3.633084$_n$	3.377670	2.70432$_n$	3.36186	0.698..	1.9217.$_n$	0.000.$_n$	0.556.
15	89.0	3.637770$_n$	3.333629	2.85763$_n$	3.51767	1.1303.	2.23955$_n$	0.362.$_n$	1.0569
20	110.7	3.643906$_n$	3.268344	2.95255$_n$	3.61402	1.1399.	2.44012$_n$	0.380.$_n$	1.3945
25	126.4	3.651171$_n$	3.177017	3.01414$_n$	3.67386	0.491..	2.56808$_n$	0.255.	1.6365
30	135.3	3.659203$_n$	3.050650	3.05572$_n$	3.70720	1.3945.$_n$	2.64157$_n$	1.1790	1.8142
35	137.0	3.667696$_n$	2.87005.	3.08629$_n$	3.71924	1.8733.$_n$	2.66717$_n$	1.6284	1.9440
40	131.8	3.676392$_n$	2.58024.	3.11301$_n$	3.71279	2.17522$_n$	2.64404$_n$	1.9479	2.0354
45	120.2	3.685159$_n$	1.7917..	3.14139$_n$	3.68946	2.39967$_n$	2.55967$_n$	2.1942	2.0952
50	102.6	3.693938$_n$	2.29798.$_n$	3.17461$_n$	3.64997	2.57634$_n$	2.37420$_n$	2.3902	2.1291
55	81.2	3.702810$_n$	2.59472.$_n$	3.21336$_n$	3.59448	2.71925$_n$	1.8591.$_n$	2.5472	2.1427
60	56.2	3.711908$_n$	2.71917.$_n$	3.25592$_n$	3.52255	2.83620$_n$	2.06408	2.6719	2.1436
65	29.3	3.721448$_n$	2.77931.$_n$	3.29927$_n$	3.43307	2.93252$_n$	2.49471	2.7686	2.1421
70	1.5	3.731621$_n$	2.81137.$_n$	3.33995$_n$	3.32391	3.01157$_n$	2.70191	2.8400	2.1514
75	− 25.9	3.742576$_n$	2.83916.$_n$	3.37493$_n$	3.19114	3.07599$_n$	2.83136	2.8875	2.1833
80	− 52.0	3.754341$_n$	2.88275.$_n$	3.40192$_n$	3.02082	3.12775$_n$	2.91950	2.9116	2.2418
85	− 76.2	3.766807$_n$	2.95434.$_n$	3.41936$_n$	2.81218	3.16844$_n$	2.98236	2.9119	2.3214
90	− 98.1	3.779676$_n$	3.053655$_n$	3.42646$_n$	2.48742	3.19926$_n$	3.02918	2.8867	2.4108
95	−117.4	3.792469$_n$	3.170262$_n$	3.42295$_n$	1.4683.	3.22110$_n$	3.06595	2.8327	2.4989
100	−134.0	3.804589$_n$	3.291702$_n$	3.40919$_n$	2.29776$_n$	3.23467$_n$	3.09677	2.7434	2.5777
105	−148.0	3.815312$_n$	3.408596$_n$	3.38612$_n$	2.59428$_n$	3.24037$_n$	3.12388	2.6056	2.6426
110	−159.5	3.823885$_n$	3.515596$_n$	3.35532$_n$	2.75511$_n$	3.23830$_n$	3.14799	2.3860	2.6908
115	−168.4	3.829567$_n$	3.610287$_n$	3.31904$_n$	2.86841$_n$	3.22832$_n$	3.16844	1.9499	2.7209
120	−174.6	3.831678$_n$	3.691894$_n$	3.28015$_n$	2.95799$_n$	3.20994$_n$	3.18353	1.6503$_n$	2.7318
125	−178.6	3.829632$_n$	3.760573$_n$	3.24175$_n$	3.03201$_n$	3.18239$_n$	3.19086	2.1666$_n$	2.7224
130	−179.6	3.823018$_n$	3.816924$_n$	3.20642$_n$	3.09276$_n$	3.14445$_n$	3.18766	2.3286$_n$	2.6914
135	−177.4	3.811602$_n$	3.861809$_n$	3.17551$_n$	3.14004$_n$	3.09451$_n$	3.17102	2.3818$_n$	2.6370
140	−171.8	3.795428$_n$	3.896256$_n$	3.14820$_n$	3.17272$_n$	3.03028$_n$	3.13767	2.3713$_n$	2.5565
145	−162.5	3.774853$_n$	3.921421$_n$	3.12133$_n$	3.18907$_n$	2.94866$_n$	3.08383	2.3090$_n$	2.4462
150	−149.3	3.750640$_n$	3.938590$_n$	3.08976$_n$	3.18670$_n$	2.84516$_n$	3.00458	2.1976$_n$	2.2997
155	−132.1	3.724063$_n$	3.949185$_n$	3.04681$_n$	3.16221$_n$	2.71299$_n$	2.89298	2.0310$_n$	2.1079
160	−111.1	3.696941$_n$	3.954749$_n$	2.98372$_n$	3.10998$_n$	2.54083$_n$	2.73767	1.7959$_n$	1.8543
165	− 86.7	3.671617$_n$	3.956864$_n$	2.88750$_n$	3.01974$_n$	2.30814$_n$	2.51799	1.4624$_n$	1.5079
170	− 59.5	3.650754$_n$	3.957047$_n$	2.73320$_n$	2.86841$_n$	1.9685.$_n$	2.18724	0.964.$_n$	1.0000
175	− 30.3	3.636949$_n$	3.956577$_n$	2.44576$_n$	2.58218$_n$	1.3729.$_n$	1.5988.	0.079.$_n$	0.114.
180	0.0	3.632103$_n$	3.956317$_n$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$

VIIc. Numerical Values or Logarithms of the Computed Coefficients m and M in the Series Expansion of Z.

	m_0	$\log m_1$	$\log M_1$	$\log m_2$	$\log M_2$	$\log m_3$	$\log M_3$	$\log m_4$	$\log M_4$
0°	57859.5	−∞	−∞	−∞	−∞	−∞	−∞	−∞	−∞
5	57788.7	3.141951_n	$2.76200._n$	2.25575_n	$0.883.._n$	0.924..	0.431..	9.000.	9.477.
10	57563.7	3.410271_n	3.064196_n	2.84763_n	$1.4502._n$	1.8028.	1.2989.	0.114.	0.623.
15	57147.6	3.580443_n	3.243112_n	3.18276_n	$1.8228._n$	2.29292	1.7810.	0.763.	1.3010
20	56483.8	3.573834_n	3.373390_n	3.40619_n	2.10483_n	2.61342	2.08600	1.2068	1.7684
25	55499.1	3.559272_n	3.478898_n	3.56964_n	2.34025_n	2.63225	2.28061	1.5182	2.1011
30	54116.0	3.489523_n	3.569947_n	3.68702_n	2.54814_n	2.97836	2.38810	1.7235	2.3560
35	52255.5	3.355356_n	3.651307_n	3.77048_n	2.73608_n	3.06502	2.40722	1.8500	2.5498
40	49848.8	3.124504_n	3.724865_n	3.82558_n	2.90650_n	3.09686	2.30081	1.8582	2.6928
45	46848.5	$2.65820._n$	3.790932_n	3.85585_n	3.05994_n	3.07111	1.7993.	1.7931	2.7937
50	43210.9	2.84850.	3.849211_n	3.86122_n	3.19623_n	2.97230	2.19340_n	1.3201	2.8559
55	38951.2	2.79141.	3.899290_n	3.84344_n	3.31507_n	2.74780	2.64670_n	1.8021	2.8809
60	34095.7	2.85582.	3.941074_n	3.80113_n	3.41637_n	1.9978.	2.86784_n	2.2445_n	2.8675
65	28709.1	2.76125.	3.974963_n	3.73216_n	3.50014_n	2.56608_n	3.04556_n	2.4912_n	2.8116
70	22884.9	2.49318.	4.001924_n	3.63249_n	3.56656_n	2.86745_n	3.15400_n	2.6531_n	2.7080
75	16743.8	1.8331..	4.023458_n	3.49449_n	3.61581_n	3.02457_n	3.22850_n	2.7597_n	2.5177
80	10422.6	0.041...	4.041420_n	3.30216_n	3.64813_n	3.06142_n	3.27839_n	2.8226_n	2.1781
85	4067.7	2.38489.	4.057704_n	3.01220_n	3.66375_n	3.08962_n	3.31057_n	2.8459_n	$0.415._n$
90	− 2178.1	2.94503	4.073788_n	2.39270_n	3.66293_n	3.06867_n	3.33098_n	2.8300_n	2.0294_n
95	− 8187.8	3.289728	4.090293_n	2.51121	3.64602_n	3.03898_n	3.34473_n	2.7700_n	2.1785_n
100	−13858.1	3.533658	4.106677_n	2.85028	3.61351_n	3.02477_n	3.35585_n	2.6538_n	2.1082_n
105	−19123.1	3.716429	4.121254_n	2.97978	6.56619_n	3.04548_n	3.36655_n	2.4469_n	1.7451_n
110	−23953.8	3.856723	4.131467_n	3.05092	3.50530_n	3.10188_n	3.37681_n	2.0004_n	1.6875
115	−28361.6	3.965004	4.134244_n	3.10541	3.43279_n	3.17676_n	3.38444_n	1.8096	2.2044
120	−323[illegible].8	4.047660	4.126323_n	3.15933	3.35153_n	3.24937_n	3.38586_n	2.2878	2.4072
125	−36107.0	4.108852	4.104374_n	3.21476	3.26515_n	3.30524_n	3.37674_n	2.4414	2.5015
130	−39589.5	4.151302	4.064971_n	3.26614	3.17759_n	3.33616_n	3.35261_n	2.4887	2.5285
135	−42908.5	4.176702	4.004317_n	3.30542	3.09174_n	3.33746_n	3.30903_n	2.4701	2.5024
140	−46113.0	4.185874	3.917826_n	3.32517	3.00779_n	3.30561_n	3.24135_n	2.3978	2.4280
145	−49219.4	4.176791	3.799292_n	3.31906	2.92200_n	3.23674_n	3.14417_n	2.2742	2.3043
150	−52204.6	4.154400	3.639297_n	3.28108	2.82653_n	3.12519_n	3.01047_n	2.0952	2.1261
155	−55005.8	4.110223	3.421555_n	3.20385	2.70995_n	2.96185_n	2.83001_n	1.8506	1.8825
160	−57528.6	4.041341	3.111800_n	3.07613	2.55642_n	2.73094_n	2.58614_n	1.5198	1.5527
165	−59659.7	3.937869	$2.60423._n$	2.87743	2.34143_n	2.40209_n	2.24778_n	1.0645	1.0969
170	−61284.2	3.776941	1.5899..	2.56170	2.01576_n	$1.9063._n$	$1.7459._n$	0.380.	0.431.
175	−62303.4	3.484954	2.07700.	1.9814.	$1.4298._n$	$1.0253._n$	$0.857.._n$	9.301.	9.301.
180	−62651.0	−∞	−∞	−∞	−∞	−∞	−∞	−∞	−∞

VIII. Computed VAlues of the Geomagnetic Force Components

φ:	λ:	0°	5°	10°	15°	20°	25°	30°	35°	40°
0°	X	2580	2941	3380	3594	3880	4137	4363	4555	4713
	Y	− 4258	− 4016	− 3745	− 3445	− 3119	− 2768	− 2397	− 2006	− 1603
	Z	57860	57860	57860	57860	57860	57860	57860	57860	57860
5°	X	5428	5519	6154	6431	6649	6807	6926	6950	6939
	Y	− 4495	− 4054	− 3581	− 3064	− 2570	− 2048	− 1526	− 1012	− 511
	Z	56230	56187	56160	56148	56152	56172	56206	56254	56315
10°	X	7779	8004	8549	8811	8969	9086	9204	9047	8922
	Y	− 4733	− 4182	− 3477	− 2809	− 2133	− 1461	− 806	− 179	468
	Z	54358	54271	54237	54219	54246	54308	54403	54529	54684
15°	X	9683	10143	10513	10783	10959	11037	11023	10983	10746
	Y	− 4963	− 4217	− 3434	− 2630	− 1822	− 1009	− 263	453	1116
	Z	52435	52322	52267	52268	52323	52432	52591	52798	53050
20°	X	11258	11761	12167	12473	12676	12778	12780	12696	12528
	Y	− 5179	− 4336	− 3453	− 2549	− 1645	− 761	83	872	1588
	Z	50602	50464	50399	50406	50481	50623	50830	51099	51428
25°	X	12666	13215	13673	14033	14294	14456	14523	14499	14394
	Y	− 5381	− 4479	− 3533	− 2567	− 1602	− 661	236	1069	1821
	Z	48874	48711	48633	48635	48713	48866	49092	49391	49764
30°	X	14076	14673	15194	15625	15966	16215	16373	16449	16446
	Y	− 5574	− 4648	− 3675	− 2681	− 1688	− 720	201	1056	1885
	Z	47169	46973	46872	46852	46912	47048	47262	47556	47935
35°	X	15624	16275	16861	17372	17804	18155	18425	18618	18739
	Y	− 5768	− 4547	− 3879	− 2856	− 1894	− 926	− 4	851	1622
	Z	45326	45083	44935	44873	44891	44985	45160	45421	45774
40°	X	17392	18093	18744	19335	19859	20314	20698	21013	21264
	Y	− 5973	− 5063	− 4143	− 3176	− 2209	− 1262	− 338	483	1243
	Z	43152	42829	42609	42480	42434	42468	42585	42794	43105
45°	X	19387	20130	20838	21499	22106	22654	23142	23569	23940
	Y	− 6203	− [illegible]	− 4466	− 3544	− 2616	− 1706	− 833	− 14	734
	Z	40461	[illegible]	39694	39471	39341	39300	39352	39504	39768
50°	X	21541	22313	23062	23777	24448	25069	25639	26158	26627
	Y	− 6466	− 5683	− 4846	− 3979	− 3102	− 2236	− 1399	− 606	187
	Z	37124	36517	36048	35703	35472	35349	35335	35434	35656
55°	X	23727	24508	25277	26021	26729	27398	28023	28604	29142
	Y	− 6769	− 6051	− 5277	− 4468	− 3645	− 2826	− 2005	− 1257	− 533
	Z	33093	32276	31627	31133	30784	30572	30493	30547	30735
60°	X	25779	26546	27309	28054	28771	29456	30104	30717	31294
	Y	− 7114	− 6461	− 5749	− 4998	− 4227	− 3451	− 2682	− 1931	− 1210
	Z	28411	27350	26489	25821	25337	25028	24853	24897	25064
65°	X	27523	28251	28981	29699	30395	31066	31706	32322	32908
	Y	− 7497	− 6907	− 6252	− 5553	− 4827	− 4086	− 3341	− 2600	− 1873
	Z	23211	21884	20790	19928	19294	18874	18657	18625	18766
70°	X	28806	29470	30142	30807	31457	32087	32697	33286	33858
	Y	− 7912	− 7376	− 6771	− 6115	− 5424	− 4709	− 3978	− 3240	− 2500
	Z	17684	16090	14754	13686	12853	12339	12031	11930	10039
75°	X	29514	30095	30689	31281	31863	32433	32990	33537	34077
	Y	− 8344	− 7856	− 7290	− 6666	− 5999	− 5299	− 4575	− 3832	− 3077
	Z	12056	10212	8640	7368	6382	5691	5269	5089	5180
80°	X	29596	30081	30581	31086	31588	32085	32579	33074	33574
	Y	− 8781	− 8332	− 7793	− 7192	− 6537	− 5844	− 5119	− 4369	− 3606
	Z	6548	4488	2708	1219	52	− 604	− 1367	− 1667	− 1748
85°	X	29068	29445	29843	30255	30671	31092	31520	31961	32420
	Y	− 9205	− 8788	− 8271	− 7679	− 7009	− 6335	− 5606	− 4851	− 4077
	Z	1351	− 876	− 2846	− 4518	− 5872	− 6906	− 7636	− 8090	− 8305
90°	X	28004	[illegible]	28563	28880	29210	29557	29923	30313	30740
	Y	− 9601	− 9210	− 8708	− 8120	− 7468	− 6771	− 6040	− 5286	− 4517
	Z	− 3391	− 5734	− 7847	− 9663	− 11215	− 12433	− 13347	− 13981	− 14368

(Continuation of VIII)

u:	λ:	0°	5°	10°	15°	20°	25°	30°	35°	40°
90°	X	28004	28270	28563	28880	29210	29557	29923	30313	30740
	Y	− 9601	− 9210	− 8708	− 8120	− 7468	− 6771	− 6040	− 5286	− 4517
	Z	− 3391	− 5734	− 7847	− 9683	−11215	−12433	−13347	−13981	−14368
95°	X	26534	26683	26873	27096	27346	27626	27938	28290	28688
	Y	− 9950	− 9585	− 9093	− 8509	− 7856	− 7157	− 6429	− 5688	− 4941
	Z	− 7597	−10002	−12215	−14184	−15873	−17270	−18373	−19199	−19776
100°	X	24823	24855	24940	25070	25245	25465	25733	26054	26434
	Y	−10239	− 9903	− 9427	− 8847	− 8196	− 7503	− 6790	− 6074	− 5369
	Z	−11242	−13661	−15931	−17996	−19816	−21369	−22648	−23664	−24436
105°	X	23050	22960	22936	22976	23079	23244	23474	23770	24136
	Y	−10453	−10153	− 9700	− 9135	− 8498	− 7822	− 7138	− 6463	− 5818
	Z	−14354	−16746	−19030	−21130	−23062	−24741	−26172	−27357	−28310
110°	X	21393	21173	21036	20982	21011	21122	21313	21583	21931
	Y	−10580	−10329	− 9912	− 9378	− 8769	− 8128	− 7488	− 6873	− 6303
	Z	−17004	−19333	−21593	−23723	−25687	−27449	−28994	−30316	−31423
115°	X	20001	19645	19388	19233	19181	19229	19373	19608	19926
	Y	−10610	−10422	−10061	− 9576	− 9017	− 8430	− 7853	− 7315	− 6834
	Z	−19299	−21536	−23731	−25831	−27797	−29596	−31209	−32626	−33850
120°	X	18984	18487	18103	17837	17692	17664	17745	17926	18192
	Y	−10534	−10428	−10143	− 9731	− 9243	− 8732	− 8236	− 7786	− 7408
	Z	−21368	−23482	−25575	−27602	−29523	−31312	−32944	−34411	−35713
125°	X	18396	17759	17243	16858	16606	16484	16480	16580	16766
	Y	−10348	−10342	−10155	− 9840	− 9449	− 9032	− 8632	− 8280	− 7996
	Z	−23362	−25320	−27273	−29187	−31022	−32751	−34357	−35831	−37171
130°	X	18225	17458	16814	16303	15934	15699	15583	15576	15651
	Y	−10049	−10159	−10091	− 9894	− 9620	− 9316	− 9025	− 8778	− 8592
	Z	−25437	−27204	−28985	−30744	−32451	−34082	−35621	−37062	−38403
135°	X	18396	17520	16760	16131	15636	15272	15027	14882	14816
	Y	− 9640	− 9877	− 9944	− 9883	− 9743	− 9566	− 9393	− 9251	− 9157
	Z	−27747	−29292	−30865	−32437	−33981	−35480	−36919	−38293	−39603
140°	X	18773	17822	16971	16234	15618	15121	14731	14433	14203
	Y	− 9129	− 9500	− 9710	− 9796	− 9800	− 9759	− 9706	− 9668	− 9657
	Z	−30428	−31723	−33061	−34419	−35775	−37114	−38426	−39706	−40958
145°	X	19176	18193	17284	16464	15743	15118	14583	14124	13723
	Y	− 8532	− 9032	− 9386	− 9622	− 9773	− 9868	− 9933	− 9994	−10058
	Z	−33578	−34606	−35693	−36820	−37969	−39129	−40292	−41455	−42620
150°	X	19402	18433	17506	16638	15837	15106	14439	13829	13263
	Y	− 7868	− 8489	− 8978	− 9356	− 9646	− 9871	−10051	−10201	−10329
	Z	−37233	−37996	−38830	−39720	−40655	−41623	−42623	−43646	−44695
155°	X	19248	18338	17437	16558	15713	14906	14137	13401	12693
	Y	− 7167	− 7887	− 8494	− 8998	− 9413	− 9755	−10037	−10267	−10452
	Z	−41363	−41877	−42472	−43133	−43858	−44632	−45454	−46318	−47224
160°	X	18543	17729	16890	16040	15190	14346	13513	12690	11879
	Y	− 6461	− 7253	− 7952	− 8557	− 9076	− 9516	− 9884	−10184	−10421
	Z	−45843	−46130	−46536	−46993	−47516	−48100	−48738	−49427	−50166
165°	X	17176	16473	15723	14937	14119	13280	12427	11563	10694
	Y	− 5786	− 6622	− 7379	− 8054	− 8649	− 9164	− 9600	− 9959	−10242
	Z	−50479	−50631	−50851	−51136	−51481	−51884	−52340	−52846	−53398
170°	X	15099	14516	13867	13160	12405	11608	10777	9920	9042
	Y	− 5177	− 6023	− 6804	− 7517	− 8157	− 8721	− 9207	− 9613	− 9939
	Z	−55013	−55068	−55172	−55324	−55521	−55763	−56045	−56367	−56724
175°	X	12351	11871	11319	10702	10025	9297	8523	7711	6867
	Y	− 4669	− 5492	− 6264	− 6980	− 7634	− 8221	− 8738	− 9180	− 9544
	Z	−59163	−59172	−59206	−59263	−59347	−59453	−59581	−59730	−59898
180°	X	9043	8635	8162	7626	7032	6384	5688	4949	4172
	Y	− 4287	− 5058	− 5792	− 6481	− 7121	− 7707	− 8234	− 8698	− 9096
	Z	−62651	−62651	−62651	−62651	−62651	−62651	−62651	−62651	−62651

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OF POOR QUALITY

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u;	λ:	45°	50°	55°	60°	65°	70°	75°	80°	85°
0°	X	4834	4919	4967	4977	4949	4883	4780	4641	4466
	Y	−1187	−761	−329	105	539	968	1390	1801	2199
	Z	57860	57860	57860	57860	57860	57860	57860	57860	57860
5°	X	6880	6776	6633	6456	6251	6023	5783	5530	5272
	Y	−30	423	850	1242	1599	1919	2203	2451	2664
	Z	56389	56473	56567	56670	56780	56896	57016	57139	57263
10°	X	8737	8501	8224	7915	7587	7249	6912	6586	6279
	Y	949	1433	1857	2216	2505	2733	2894	2994	3039
	Z	54863	55069	55293	55534	55786	56047	56312	56577	56837
15°	X	10501	10200	9856	9482	9093	8701	8323	7969	7652
	Y	1706	2218	2642	2975	3213	3362	3420	3396	3298
	Z	53343	53673	54036	54423	54830	55248	55669	56083	56487
20°	X	12289	11991	11648	11276	10889	10504	10136	9800	9508
	Y	2217	2749	3175	3490	3690	3778	3757	3635	3423
	Z	51813	52254	52739	53264	53818	54389	54966	55532	56073
25°	X	14217	13981	13699	13387	13059	12731	12418	12136	11898
	Y	2476	3022	3449	3751	3923	3972	3896	3706	3414
	Z	50209	50724	51304	51939	52620	53329	54048	54756	55427
30°	X	16373	16243	16067	15857	15629	15393	15169	14966	14797
	Y	2492	3045	3473	3769	3930	3956	3851	3622	3283
	Z	48400	48954	49592	50307	[illegible]	51908	52749	53579	54365
35°	X	18795	18796	18751	18669	[illegible]	18443	18320	18205	18110
	Y	2292	2848	3279	3577	3738	3762	3651	3413	3059
	Z	46229	46788	47452	48214	49060	49966	50901	51827	52704
40°	X	21455	21593	21685	21738	21759	21755	21735	21706	21675
	Y	1912	2470	2908	3218	3393	3436	3343	3121	2782
	Z	43528	44071	44737	45519	46402	47360	48357	49348	50284
45°	X	24256	24523	24743	24920	25059	25161	25232	25273	25294
	Y	1395	1959	2412	2746	2953	3030	2976	2794	2493
	Z	40154	40672	41324	42107	43001	43981	45005	46026	46987
50°	X	27047	27420	27748	28030	28266	28457	28601	28699	28753
	Y	788	1363	1842	2213	2467	2597	2600	2477	2233
	Z	36010	36504	37138	37906	38791	39764	40783	41797	42750
55°	X	29638	30091	30499	30861	31174	31432	31633	31772	31848
	Y	134	731	1246	1667	1981	2179	2254	2203	2031
	Z	31070	31544	32159	32904	33762	34702	35685	36662	37578
60°	X	31833	32339	32802	33221	33589	33899	34144	34317	34414
	Y	−529	100	662	1143	1529	1803	1961	1992	1899
	Z	25380	25839	26432	27147	27962	28849	29772	30685	31540
65°	X	33467	33996	34492	34930	35359	35712	35999	36209	36336
	Y	−1170	−502	114	661	1123	1482	1724	1839	1827
	Z	19064	19504	20070	20743	21502	22319	23162	23993	24777
70°	X	34412	34947	35461	35943	36391	36787	37121	37383	37560
	Y	−1770	−1061	−390	223	760	1198	1523	1721	1788
	Z	12304	12711	13236	13853	14543	15276	16028	16770	17472
75°	X	34613	35143	35666	36173	36657	37104	37500	37832	38087
	Y	−2320	−1574	−855	−184	417	925	1323	1593	1735
	Z	5327	5676	6138	6683	7286	7923	8578	9226	9849
80°	X	34084	34603	35135	35669	36196	36703	37179	37603	37961
	Y	−2824	−2051	−1299	−587	60	621	1076	1409	1614
	Z	−1628	−1367	−996	−543	−42	493	1042	1594	2136
85°	X	32905	33418	33958	34522	35100	35680	36243	36777	37259
	Y	−3292	−2510	−1746	−1019	−350	239	730	1108	1366
	Z	−8322	−8185	−7932	−7598	−7211	−6790	−6349	−5899	−5442
90°	X	31203	31714	32269	32866	33498	34151	34810	35456	36068
	Y	−3744	−2976	−2227	−1514	−853	−263	241	645	943
	Z	−14549	−14565	−14458	−14261	−14003	−13709	−13387	−13046	−12690

(Continuation of VIII)

u;	λ:	45°	50°	55°	60°	65°	70°	75°	80°	85°
90°	X	31205	31714	32269	32866	33498	34151	34810	35456	36068
	Y	−3744	−2976	−2227	−1514	−853	−263	241	645	945
	Z	−14549	−14565	−14458	−14261	−14005	−13709	−13387	−13046	−12690
95°	X	29141	29651	30222	30852	31532	32252	32994	33741	34472
	Y	−4200	−3473	−2769	−2100	−1478	−915	−423	−11	319
	Z	−20142	−20335	−20393	−20353	−20243	−20085	−19897	−19684	−19453
100°	X	26879	27392	27976	28629	29343	30109	30910	31730	32548
	Y	−4682	−4021	−3390	−2795	−2240	−1731	−1273	−869	−519
	Z	−24994	−25373	−25609	−25737	−25787	−25786	−25750	−25693	−25620
105°	X	24573	25086	25672	26330	27053	27832	28654	29502	30359
	Y	−5207	−4634	−4100	−3601	−3136	−2701	−2293	−1910	−1548
	Z	−29053	−29617	−30034	−30338	−30562	−30734	−30876	−31005	−31129
110°	X	22355	22853	23423	24060	24758	25510	26303	27126	27963
	Y	−5785	−5317	−4894	−4507	−4144	−3794	−3447	−3091	−2722
	Z	−32331	−33067	−33660	−34143	−34550	−34912	−35256	−35600	−35958
115°	X	20320	20784	21311	21895	22529	23206	23919	24658	25415
	Y	−6416	−6060	−5754	−5481	−5224	−4961	−4675	−4350	−3975
	Z	−34895	−35782	−36538	−37198	−37793	−38355	−38920	−39502	−40118
120°	X	18532	18934	19387	19882	20411	20969	21551	22155	22774
	Y	−7090	−6844	−6650	−6485	−6324	−6140	−5908	−5609	−5228
	Z	−36861	−37874	−38777	−39603	−40386	−41156	−41944	−42771	−43650
125°	X	17022	17329	17673	18043	18430	18831	19244	19669	20110
	Y	−7785	−7640	−7543	−7467	−7385	−7262	−7072	−6790	−6400
	Z	−38387	−39498	−40527	−41504	−42460	−43427	−44431	−45491	−46617
130°	X	15788	15967	16170	16385	16601	16818	17035	17258	17495
	Y	−8472	−8411	−8390	−8380	−8350	−8267	−8100	−7823	−7421
	Z	−39656	−40835	−41964	−43068	−44176	−45313	−46504	−47762	−49092
135°	X	14805	14827	14863	14899	14927	14945	14957	14972	15001
	Y	−9116	−9119	−9148	−9176	−9172	−9103	−8940	−8657	−8239
	Z	−40862	−42079	−43275	−44473	−45695	−46963	−48293	−49694	−51166
140°	X	14021	13865	13717	13564	13398	13219	13033	12849	12683
	Y	−9679	−9726	−9782	−9821	−9817	−9740	−9563	−9263	−8827
	Z	−42188	−43409	−44635	−45884	−47173	−48515	−49922	−51395	−52929
145°	X	13361	13019	12683	12342	11990	11628	11262	10904	10569
	Y	−10130	−10203	−10265	−10296	−10272	−10160	−9965	−9642	−9189
	Z	−43791	−44977	−46189	−47438	−48734	−50056	−51496	−52963	−54477
150°	X	12727	12208	11694	11179	10659	10137	9620	9118	8646
	Y	−10440	−10528	−10584	−10594	−10539	−10402	−10167	−9821	−9356
	Z	−45773	−46883	−48032	−49224	−50465	−51756	−53097	−54479	−55894
155°	X	12006	11331	10664	10001	9341	8688	8050	7437	6861
	Y	−10596	−10694	−10741	−10727	−10641	−10472	−10210	−9846	−9377
	Z	−48173	−49163	−50198	−51277	−52399	−53564	−54766	−55996	−57245
160°	X	11077	10283	9496	8718	7951	7200	6472	5776	5123
	Y	−10595	−10706	−10749	−10720	−10614	−10424	−10147	−9778	−9318
	Z	−50951	−51781	−52654	−53567	−54516	−55497	−56505	−57530	−58564
165°	X	9823	8954	8090	7235	6396	5577	4786	4030	3319
	Y	−10430	−10580	−10633	10606	−10498	−10308	−10034	−9678	−9242
	Z	−53993	−54629	−55301	−56006	−56739	−57495	−58269	−59054	−59843
170°	X	8150	7252	6354	5462	4583	3724	2891	2093	1335
	Y	−10184	−10346	−10425	−10422	−10336	−10170	−9924	−9602	−9206
	Z	−57114	−57535	−57983	−58454	−58945	−59452	−59970	−60496	−61025
175°	X	6000	5116	4224	3330	2441	1566	710	−120	−917
	Y	−9830	−10035	−10159	−10202	−10165	−10048	−9855	−9587	−9247
	Z	−60084	−60286	−60504	−60734	−60975	−61225	−61482	−61743	−62008
180°	X	3363	2529	1676	809	−63	−935	−1800	−2651	−3482
	Y	−9425	−9683	−9866	−9975	−10007	−9964	−9844	−9650	−9382
	Z	−62651	−62651	−62651	−62651	−62651	−62651	−62651	−62651	−62651

(Continuation of VIII)

…:	λ:	90°	95°	100°	105°	110°	115°	120°	125°	130°
0°	X	4258	4016	3745	3445	3119	2768	2397	2008	1603
	Y	2580	2941	3280	3594	3880	4137	4363	4555	4713
	Z	57860	57860	57860	57860	57860	57860	57860	57860	57860
5°	X	5013	4757	4506	4264	4030	3805	3589	3380	3176
	Y	2846	2999	3127	3233	3323	3399	3466	3527	3585
	Z	57388	57512	57634	57753	57869	57980	58086	58187	58284
10°	X	5999	5753	5544	5376	5247	5158	5105	5082	5083
	Y	3037	2996	2925	2834	2735	2636	2549	2481	2441
	Z	57090	57331	57558	57769	57961	58134	58269	58425	58544
15°	X	7382	7168	7014	6925	6900	6938	7034	7178	7362
	Y	3137	2926	2670	2414	2144	1888	1660	1475	1345
	Z	56866	57216	57530	57804	58033	58218	58358	58457	58519
20°	X	9274	9106	9012	8906	9060	9199	9409	9681	10001
	Y	3135	2787	2398	1988	1578	1190	844	559	354
	Z	56575	57023	57406	57714	57942	58088	58154	58146	58074
25°	X	11717	11602	11563	11603	11725	11927	12203	12544	12937
	Y	3034	2587	2092	1575	1060	572	137	− 221	− 480
	Z	56041	56573	57006	57323	57516	57580	57521	57348	57078
30°	X	14676	14611	14611	14682	14827	15044	15328	15673	16066
	Y	2849	2340	1780	1195	612	60	− 431	− 834	− 1125
	Z	55074	55673	56136	56439	56569	56521	56302	55927	55421
35°	X	18043	18015	18034	18105	18231	18414	18651	18936	19260
	Y	2605	2072	1484	867	254	− 327	− 842	− 1262	− 1559
	Z	53486	54131	54604	54875	54926	54752	54361	53775	53029
40°	X	21651	21642	21654	21694	21766	21873	22014	22186	22384
	Y	2339	1813	1229	614	2	− 577	− 1087	− 1498	− 1780
	Z	51111	51780	52246	52474	52444	52149	51601	50829	49874
45°	X	25206	25286	25270	25254	25244	25245	25257	25283	25319
	Y	2086	1593	1039	452	− 135	− 687	− 1170	− 1550	− 1799
	Z	47831	48500	48947	49132	49033	48645	47983	47079	45984
50°	X	28765	28739	28680	28596	28491	28373	28246	28114	27979
	Y	1882	1440	934	392	− 152	− 662	− 1102	− 1438	− 1641
	Z	43582	44234	44655	44808	44670	44236	43524	42570	41427
55°	X	31862	31815	31713	31560	31365	31135	30879	30604	30314
	Y	1747	1369	921	434	− 57	− 516	− 904	− 1188	− 1339
	Z	38374	38997	39397	39539	39401	38982	38299	37389	36306
60°	X	34433	34373	34239	34037	33774	33460	33106	32721	32314
	Y	1689	1380	996	568	133	− 271	− 605	− 834	− 931
	Z	32287	32877	33267	33426	33336	32996	32423	31651	30730
65°	X	36376	36325	36188	35968	35675	35320	34914	34469	33998
	Y	1694	1455	1136	768	389	40	− 240	− 416	− 460
	Z	25467	26027	26423	26628	26630	26426	26032	25475	24798
70°	X	37647	37639	37536	37343	37069	36723	36320	35872	35394
	Y	1730	1561	1306	998	676	381	154	29	35
	Z	18107	18644	19060	19335	19459	19430	19258	18964	18580
75°	X	38253	38326	38302	38185	37979	37697	37351	36956	36526
	Y	1748	1647	1457	1211	948	709	536	463	515
	Z	10429	10948	11391	11746	12002	12159	12220	12198	12113
80°	X	38241	38431	38527	38529	38439	38268	38026	37729	37392
	Y	1693	1659	1536	1356	1158	942	868	850	951
	Z	2661	3159	3623	4045	4418	4738	5006	5226	5405
85°	X	37675	38010	38256	38409	38469	38443	38340	38173	37956
	Y	1506	1540	1490	1386	1264	1162	1118	1162	1317
	Z	− 4982	− 4521	− 4061	− 3607	− 3163	− 2732	− 2318	− 1924	− 1547
90°	X	36628	37119	37528	37847	38073	38208	38259	38236	38153
	Y	1141	1245	1275	1259	1228	1216	1257	1377	1595
	Z	−12316	−11925	−11514	−11083	−10635	−10173	− 9700	− 9220	− 8736

(Continuation of VIII)

ϑ;	λ:	90°	95°	100°	105°	110°	115°	120°	125°	130°
90°	X	36628	37119	37528	37847	38073	38205	38259	38236	38153
	Y	1141	1243	1275	1259	1228	1216	1257	1377	1595
	Z	−12316	−11923	−11514	−11063	−10635	−10173	−9700	−9220	−8736
95°	X	35165	35802	36365	36844	37231	37523	37728	37848	37893
	Y	567	744	864	950	1028	1126	1269	1481	1776
	Z	−19200	−18923	−18619	−18284	−17917	−17515	−17081	−16615	−16119
100°	X	33342	34092	34780	35392	35916	36346	36683	36929	37094
	Y	−222	31	251	454	658	863	1148	1468	1853
	Z	−25532	−25425	−25293	−25132	−24930	−24681	−24380	−24021	−23599
105°	X	31204	32019	32784	33482	34102	34634	35073	35423	35684
	Y	−1204	−872	−546	−216	127	495	897	1341	1830
	Z	−31252	−31371	−31475	−31554	−31593	−31574	−31482	−31300	−31016
110°	X	28799	29616	30397	31128	31793	32383	32890	33311	33646
	Y	−2334	−1923	−1487	−1023	−533	−15	533	1110	1712
	Z	−36332	−36720	−37109	−37480	−37809	−38070	−38233	−38271	−38162
115°	X	26177	26933	27669	28372	29030	29632	30168	30632	31021
	Y	−3543	−3057	−2513	−1922	−1286	−616	79	791	1508
	Z	−40770	−41453	−42150	−42834	−43473	−44028	−44459	−44727	−44799
120°	X	23406	24042	24677	25300	25902	26473	27004	27484	27910
	Y	−4759	−4200	−3559	−2846	−2076	−1267	−434	403	1230
	Z	−44584	−45560	−46558	−47543	−48474	−49304	−49981	−50459	−50696
125°	X	20567	21041	21531	22033	22541	23048	23544	24020	24467
	Y	−5895	−5276	−4551	−3737	−2854	−1926	−977	−32	890
	Z	−47806	−49043	−50303	−51547	−52722	−53776	−54652	−55299	−55669
130°	X	17754	18040	18361	18716	19106	19525	19966	20419	20874
	Y	−6885	−6217	−5429	−4539	−3572	−2556	−1521	−493	501
	Z	−50489	−51931	−53387	−54814	−56161	−57369	−58381	−59142	−59609
135°	X	15057	15152	15296	15497	15758	16076	16447	16862	17309
	Y	−7678	−6977	−6148	−5212	−4193	−3127	−2040	−964	74
	Z	−52696	−54263	−55832	−57359	−58791	−60074	−61154	−61978	−62506
140°	X	12550	12466	12445	12497	12629	12843	13134	13495	13916
	Y	−8248	−7530	−6686	−5735	−4704	−3622	−2520	−1431	−379
	Z	−54510	−56113	−57703	−59239	−60674	−61957	−63041	−63881	−64444
145°	X	10274	10035	9869	9788	9800	9909	10113	10406	10778
	Y	−8602	−7883	−7047	−6110	−5097	−4037	−2957	−1887	−852
	Z	−56022	−57573	−59099	−60564	−61927	−63147	−64183	−65006	−65584
150°	X	8220	7858	7572	7377	7280	7257	7396	7603	7904
	Y	−8769	−8066	−7258	−6360	−5394	−4383	−3354	−2331	−1337
	Z	−57324	−58748	−60139	−61467	−62702	−63812	−64768	−65545	−66125
155°	X	6336	5876	5493	5203	5007	4914	4923	5030	5231
	Y	−8803	−8129	−7364	−6523	−5623	−4683	−3724	−2766	−1829
	Z	−58498	−59736	−60940	−62088	−63158	−64127	−64977	−65691	−66257
160°	X	4523	3988	3528	3151	2863	2670	2570	2564	2643
	Y	−8768	−8133	−7424	−6630	−5823	−4964	−4083	−3198	−2323
	Z	−59595	−60610	−61596	−62537	−63420	−64230	−64956	−65588	−66120
165°	X	2660	2062	1532	1076	699	404	191	61	10
	Y	−8728	−8143	−7494	−6789	−6040	−5256	−4449	−3632	−2813
	Z	−60627	−61399	−62150	−62871	−63554	−64191	−64776	−65302	−65767
170°	X	625	−32	−629	−1164	−1631	−2028	−2353	−2608	−2791
	Y	−8740	−8210	−7623	−6984	−6301	−5583	−4837	−4071	−3293
	Z	−61552	−62072	−62581	−63073	−63545	−63994	−64414	−64804	−65161
175°	X	−1676	−2391	−3057	−3670	−4226	−4722	−5155	−5523	−5828
	Y	−8841	−8370	−7842	−7260	−6630	−5959	−5251	−4512	−3750
	Z	−62272	−62536	−62796	−63050	−63297	−63535	−63762	−63977	−64178
180°	X	−4287	−5058	−5792	−6481	−7121	−7707	−8234	−8698	−9096
	Y	−9043	−8635	−8162	−7626	−7032	−6384	−5688	−4949	−4172
	Z	−62651	−62651	−62651	−62651	−62651	−62651	−62651	−62651	−62651

(Continuation of VIII)

φ:	λ:	135°	140°	145°	150°	155°	160°	165°	170°	175°
0°	X	1187	761	329	− 105	− 539	− 968	− 1390	− 1801	− 2199
	Y	4834	4919	4967	4977	4949	4883	4780	4641	4466
	Z	57860	57860	57860	57860	57860	57860	57860	57860	57860
5°	X	2974	2770	2563	2347	2120	1878	1619	1342	1045
	Y	3642	3699	3757	3816	3872	3926	3973	4010	4033
	Z	58375	58461	58542	58619	58691	58758	58822	58881	58936
10°	X	5099	5121	5140	5145	5125	5071	4976	4830	4630
	Y	2434	2465	2535	2644	2790	2969	3173	3393	3621
	Z	58649	58741	58823	58899	58972	59044	59119	59197	59280
15°	X	7572	7794	8012	8211	8373	8484	8530	8498	8379
	Y	1280	1288	1372	1533	1768	2070	2428	2829	3258
	Z	58551	58560	58556	58546	58541	58548	58575	58627	58710
20°	X	10354	10723	11087	11427	11722	11951	12097	12145	12081
	Y	243	234	336	548	867	1284	1786	2354	2968
	Z	57951	57793	57617	57442	57286	57165	57093	57083	57143
25°	X	13365	13808	14246	14656	15015	15301	15493	15575	15532
	Y	− 624	− 639	− 516	− 255	141	660	1287	1999	2771
	Z	56734	56342	55931	55531	55171	54879	54677	54583	54610
30°	X	16491	16929	17360	17761	18109	18383	18563	18630	18573
	Y	− 1282	− 1291	− 1143	− 835	− 373	232	959	1785	2676
	Z	54818	54156	53478	52827	52245	51769	51430	51254	51255
35°	X	19608	19965	20312	20631	20899	21098	21210	21220	21117
	Y	− 1712	− 1703	− 1524	− 1172	− 654	17	817	1718	2687
	Z	52166	51239	50303	49416	48629	47991	47540	47305	47302
40°	X	22596	22812	23017	23195	23329	23405	23410	23331	23162
	Y	− 1911	− 1873	− 1658	− 1265	− 703	13	856	1796	2797
	Z	48792	47647	46505	45435	44500	43753	43239	42986	43011
45°	X	25360	25400	25427	25432	25404	25332	25209	25027	24785
	Y	− 1893	− 1817	− 1562	− 1132	− 537	204	1061	2002	2990
	Z	44759	43479	42218	41052	40050	39269	38755	38536	38624
50°	X	27841	27696	27541	27369	27177	26958	26709	26430	26122
	Y	− 1689	− 1567	− 1272	− 808	− 190	557	1403	2313	3249
	Z	40164	38856	37585	36427	35453	34721	34274	34136	34315
55°	X	30014	29706	29391	29067	28734	28391	28041	27686	27331
	Y	− 1335	− 1166	− 830	− 337	293	1032	1847	2699	3551
	Z	35117	33899	32729	31683	30829	30220	29896	29875	30160
60°	X	31894	31465	31032	30599	30169	29744	29329	28930	28554
	Y	− 875	− 659	− 286	230	864	1583	2351	3127	3875
	Z	29723	28698	27728	26882	26220	25791	25626	25740	26130
65°	X	33511	33017	32523	32035	31560	31100	30663	30255	29885
	Y	− 355	− 96	310	844	1476	2167	2879	3570	4202
	Z	24054	23301	22601	22014	21589	21367	21375	21622	22101
70°	X	34899	34398	33900	33412	32941	32494	32074	31689	31346
	Y	185	482	916	1461	2055	2745	3398	4001	4519
	Z	18144	17705	17309	17002	16828	16816	16988	17350	17895
75°	X	36076	35619	35165	34722	34298	33897	33523	33182	32879
	Y	708	1038	1494	2046	2658	3285	3881	4403	4816
	Z	11991	11864	11766	11732	11792	11971	12285	12740	13309
80°	X	37029	36653	36278	35910	35556	35221	34909	34621	34361
	Y	1184	1545	2018	2574	3173	3768	4313	4765	5089
	Z	5560	5708	5872	6074	6336	6677	7109	7638	8258
85°	X	37705	37434	37153	36873	36599	36335	36085	35848	35624
	Y	1592	1984	2474	3031	3617	4184	4686	5081	5337
	Z	− 1183	− 823	− 455	− 66	360	834	1367	1963	2618
90°	X	38024	37861	37678	37483	37284	37084	36886	36690	36494
	Y	1919	2344	2851	3411	3985	4528	4997	5351	5560
	Z	− 8247	− 7752	− 7245	− 6718	− 6163	− 5573	− 4942	− 4268	− 3555

(Continuation of VIII)

α;	λ:	135°	140°	145°	150°	155°	160°	165°	170°	175°
90°	X	38024	37861	37678	37483	37284	37084	36886	36690	36494
	Y	1019	2344	2851	3411	3985	4528	4997	5351	5560
	Z	− 8247	− 7752	− 7243	− 6718	− 6163	− 5573	− 4942	− 4268	− 3555
95°	X	37882	37823	37728	37609	37472	37324	37166	37000	36823
	Y	2158	2622	3150	3714	4279	4804	5248	5575	5758
	Z	−15593	−15036	−14447	−13823	−13161	−12459	−11717	−10939	−10132
100°	X	37185	37216	37108	37142	37056	36948	36822	36678	36519
	Y	2305	2817	3371	3944	4504	5015	5443	5755	5928
	Z	−23110	−22553	−21927	−21231	−20470	−19647	−18771	−17853	−16910
105°	X	35864	35973	36021	36020	35980	35908	35812	35695	35560
	Y	2362	2929	3517	4103	4663	5166	5584	5890	6065
	Z	−30619	−30103	−29467	−28712	−27849	−26890	−25856	−24768	−23654
110°	X	33899	34077	34188	34242	34251	34224	34169	34092	33999
	Y	2333	2963	3589	4194	4757	5256	5668	5974	6159
	Z	−37885	−37431	−36796	−35987	−35019	−33916	−32710	−31437	−30139
115°	X	31334	31576	31753	31872	31944	31978	31984	31970	31942
	Y	2220	2918	3587	4214	4784	5282	5692	6001	6199
	Z	−44649	−44261	−43633	−42774	−41706	−40466	−39098	−37653	−36184
120°	X	28276	28582	28832	29029	29182	29300	29391	29465	29530
	Y	2031	2794	3506	4158	4738	5237	5647	5961	6173
	Z	−50662	−50337	−49720	−48825	−47683	−46337	−44846	−43272	−41681
125°	X	24878	25249	25578	25866	26117	26339	26538	26722	26898
	Y	1769	2591	3344	4019	4611	5114	5525	5844	6070
	Z	−55731	−55465	−54873	−53972	−52799	−51407	−49861	−48232	−46595
130°	X	21321	21752	22161	22545	22903	23239	23557	23861	24158
	Y	1440	2309	3095	3791	4394	4903	5318	5644	5885
	Z	−59749	−59545	−59001	−58139	−56999	−55639	−54127	−52539	−50951
135°	X	17777	18256	18735	19206	19665	20111	20542	20961	21371
	Y	1052	1951	2760	3472	4085	4601	5024	5361	5619
	Z	−62710	−62578	−62116	−61348	−60315	−59075	−57695	−56248	−54806
140°	X	14385	14888	15411	15946	16482	17012	17534	18045	18543
	Y	611	1521	2340	3062	3684	4211	4647	5000	5282
	Z	−64707	−64662	−64318	−63699	−62844	−61806	−60644	−59426	−58214
145°	X	11218	11712	12244	12803	13375	13952	14524	15087	15637
	Y	126	1029	1847	2572	3203	3744	4199	4579	4893
	Z	−65904	−65961	−65764	−65337	−64712	−63933	−63052	−62121	−61193
150°	X	8252	8727	9224	9758	10317	10888	11462	12030	12586
	Y	− 392	490	1296	2020	2661	3221	3706	4122	4482
	Z	−66498	−66662	−66626	−66408	−66035	−65540	−64960	−64337	−63707
155°	X	5515	5871	6286	6747	7240	7755	8280	8807	9327
	Y	− 929	− 79	710	1433	2088	2674	3197	3664	4082
	Z	−66670	−66930	−67043	−67022	−66886	−66656	−66359	−66019	−65662
160°	X	2807	3041	3337	3684	4071	4487	4924	5373	5827
	Y	− 1472	− 655	119	844	1517	2140	2713	3241	3730
	Z	−66547	−66870	−67094	−67226	−67276	−67256	−67182	−67066	−66923
165°	X	35	128	283	493	749	1045	1373	1727	2102
	Y	− 2005	− 1214	− 448	289	992	1660	2294	2895	3456
	Z	−66168	−66505	−66779	−66993	−67152	−67260	−67324	−67350	−67343
170°	X	− 2906	− 2955	− 2940	− 2868	− 2741	− 2564	− 2341	− 2076	− 1772
	Y	− 2510	− 1729	− 955	− 195	550	1276	1980	2662	3322
	Z	−65483	−65769	−66019	−66234	−66413	−66558	−66669	−66750	−66800
175°	X	− 6066	− 6237	− 6343	− 6383	− 6360	− 6274	− 6128	− 5921	− 5658
	Y	− 2969	− 2175	− 1374	− 570	231	1025	1807	2574	3322
	Z	−64365	−64535	−64689	−64825	−64944	−65043	−65124	−65186	−65229
180°	X	− 9425	− 9683	− 9566	− 9975	−10007	− 9964	− 9844	− 9650	− 9382
	Y	− 3363	− 2529	− 1676	− 809	63	935	1800	2651	3482
	Z	−62651	−62651	−62651	−62651	−62651	−62651	−62651	−62651	−62651

(Continuation of VIII)

φ:	λ:	180°	185°	190°	195°	200°	205°	210°	215°	220°
0°	X	− 2580	− 2941	− 3280	− 3594	− 3880	− 4137	− 4363	− 4555	− 4713
	Y	4258	4016	3745	3445	3119	2768	2397	2008	1603
	Z	57860	57860	57860	57860	57860	57860	57860	57860	57860
5°	X	728	392	39	− 329	− 706	− 1089	− 1471	− 1845	− 2206
	Y	4036	4014	3963	3873	3755	3589	3379	3121	2815
	Z	58987	59033	59075	59111	59141	59164	59180	59188	59186
10°	X	4372	4055	3680	3250	2772	2253	1704	1137	563
	Y	3843	4049	4226	4363	4446	4467	4416	4288	4076
	Z	59369	59465	59565	59670	59775	59879	59977	60067	60144
15°	X	8168	7861	7460	6068	6393	5747	5044	4300	3533
	Y	3696	4123	4521	4871	5152	5350	5449	5439	5312
	Z	58826	58975	59155	59363	59594	59841	60095	60348	60588
20°	X	11898	11593	11165	10620	9961	9222	8401	7523	6611
	Y	3603	4232	4830	5370	5827	6179	6408	6499	6442
	Z	57277	57487	57769	58117	[illegible]	58966	59441	59927	60408
25°	X	15357	15043	14594	14015	13318	12517	11632	10685	9700
	Y	3571	4369	[illegible]	5826	6425	6900	7230	7399	7394
	Z	54765	55047	55451	[illegible]	56576	57262	58001	58768	59539
30°	X	18383	18056	17596	17008	16306	15505	14624	13686	12713
	Y	3601	4522	5402	6206	6902	7459	7856	8075	8104
	Z	51440	51808	52350	53048	53879	54819	55834	56895	57968
35°	X	20806	20554	20095	19529	18866	18123	17317	16468	15596
	Y	3687	4678	5622	6481	7222	7817	8242	8482	8527
	Z	47536	48004	48688	49567	50611	51786	53055	54381	55726
40°	X	22900	22546	22107	21590	21008	20375	19707	19016	18319
	Y	3820	4825	5774	6632	7365	7949	8364	8597	8642
	Z	43316	43890	44711	45750	46971	48335	49600	51325	52872
45°	X	24484	24127	23721	23279	22809	22325	21837	21355	20884
	Y	3986	4951	5850	6649	7323	7852	8220	8421	8452
	Z	39016	39695	40633	41794	43136	44617	46194	47827	49478
50°	X	25790	25441	25084	24731	24391	24073	23785	23530	23307
	Y	4173	5050	5847	6540	7110	7543	7834	7981	7987
	Z	34801	35573	36594	37824	39219	40734	42327	43962	45608
55°	X	26984	26655	26355	26094	25882	25726	25629	25589	25600
	Y	4368	5117	5775	6324	6753	7061	7250	7327	7302
	Z	30736	31575	32637	33879	35254	36720	38238	39777	41313
60°	X	28211	27912	27668	27491	27388	27367	27425	27560	27758
	Y	4560	5158	5651	6030	6297	6460	6532	6530	6469
	Z	26777	27650	28708	29906	31200	32548	33917	35282	36627
65°	X	29562	29298	29103	28989	28963	29031	29191	29436	29755
	Y	4746	5181	5499	5700	5796	5806	5756	5670	5571
	Z	22793	23666	24680	25791	26958	28143	29318	30464	31573
70°	X	31054	30822	30662	30583	30593	30696	30893	31176	31534
	Y	4923	5200	5347	5374	5305	5169	4999	4831	4692
	Z	18605	19449	20391	21391	22411	23418	24358	25306	26172
75°	X	32621	32415	32270	32196	32199	32285	32455	32706	33028
	Y	5094	5229	5223	5096	4879	4611	4334	4090	3914
	Z	14036	14836	15697	16584	17463	18306	19091	19808	20458
80°	X	34132	33941	33792	33695	33657	33682	33776	33937	34161
	Y	5263	5280	5151	4899	4563	4186	3821	3513	3302
	Z	8958	9717	10508	11302	12069	12784	13429	13994	14482
85°	X	35417	35288	35061	34923	34821	34761	34748	34787	34876
	Y	5433	5366	5147	4806	4383	3930	3498	3140	2899
	Z	3325	4064	4816	5555	6255	6894	7454	7928	8318
90°	X	36299	36105	35914	35730	35560	35410	35287	35198	35145
	Y	5606	5487	5215	4823	4352	3853	3382	2992	2727
	Z	− 2811	− 2051	− 1295	− 564	119	733	1264	1705	2059

(Continuation of VIII)

u;	λ:	180°	185°	190°	195°	200°	205°	210°	215°	220°
90°	X	36290	36105	35914	35730	35560	35410	35287	35198	35145
	Y	5606	5487	5213	4823	4352	3853	3382	2996	2727
	Z	− 2811	− 2051	− 1295	− 564	119	733	1264	1705	2059
95°	X	36635	36436	36225	36005	35782	35560	35347	35149	34975
	Y	5780	5639	5351	4944	4459	3947	3463	3060	2782
	Z	− 9306	− 8480	− 7671	− 6901	− 6191	− 5558	− 5013	− 4562	− 4198
100°	X	36342	36146	35932	35700	35452	35193	34929	34665	34411
	Y	5948	5812	5536	5145	4679	4185	3713	3316	3037
	Z	−15959	−15021	−14118	−13273	−12504	−11825	−11244	−10761	−10365
105°	X	35407	35234	35043	34830	34598	34347	34081	33805	33523
	Y	6098	5987	5745	5396	4974	4521	4085	3713	3446
	Z	−22543	−21463	−20442	−19502	−18663	−17933	−17315	−16800	−16371
110°	X	33890	33767	33629	33472	33296	33100	32883	32647	32394
	Y	6215	6141	5948	5657	5297	4906	4524	4195	3957
	Z	−28855	−27625	−26452	−25452	−24552	−23766	−23149	−22623	−22178
115°	X	31905	31859	31804	31736	31653	31550	31423	31271	31092
	Y	6282	6250	6114	5890	5604	5287	4974	4704	4510
	Z	−34746	−33386	−32145	−31051	−30118	−29346	−28719	−28209	−27777
120°	X	29588	29644	29696	29740	29773	29787	29776	29735	29661
	Y	6283	6294	6215	6062	5854	5619	5387	5189	5056
	Z	−40137	−38696	−37403	−36288	−35362	−34619	−34035	−33572	−33180
125°	X	27071	27243	27414	27580	27736	27874	27986	28065	28107
	Y	6206	6258	6235	6150	6022	5873	5727	5611	5551
	Z	−45020	−43568	−42286	−41203	−40327	−39647	−39133	−38738	−38403
130°	X	24451	24741	25028	25310	25579	25830	26054	26244	26395
	Y	6046	6137	6167	6148	6098	6034	5977	5950	5972
	Z	−49435	−48053	−46849	−45851	−45064	−44473	−44042	−43722	−43447
135°	X	21773	22168	22535	22931	23292	23632	23944	24223	24465
	Y	5807	5936	6017	6063	6088	6108	6141	6203	6312
	Z	−53439	−52203	−51139	−50270	−49599	−49109	−48762	−48507	−48281
140°	X	19029	19502	19960	20402	20824	21223	21594	21935	22246
	Y	5502	5673	5807	5917	6015	6117	6237	6388	6582
	Z	−57069	−56040	−55160	−54449	−53906	−53513	−53236	−53025	−52825
145°	X	16170	16685	17181	17656	18107	18536	18939	19319	19677
	Y	5155	5373	5566	5742	5915	6096	6298	6531	6804
	Z	−60315	−59526	−58852	−58307	−57888	−57578	−57348	−57157	−56957
150°	X	13126	13647	14147	14625	15082	15519	15937	16340	16733
	Y	4795	5075	5332	5578	5826	6085	6364	6670	7009
	Z	−63105	−62559	−62086	−61695	−61383	−61136	−60931	−60736	−60514
155°	X	9837	10333	10812	11276	11724	12160	12588	13012	13439
	Y	4461	4811	5143	5466	5790	6123	6472	6840	7230
	Z	−65309	−64978	−64679	−64417	−64189	−63984	−63786	−63574	−63323
160°	X	6281	6733	7180	7623	8063	8503	8949	9403	9874
	Y	4187	4620	5035	5440	5841	6245	6654	7070	7493
	Z	−66764	−66599	−66432	−66265	−66096	−65919	−65723	−65496	−65224
165°	X	2493	2898	3316	3747	4192	4653	5134	5637	6168
	Y	4011	4532	5036	5524	6001	6468	6927	7377	7816
	Z	−67309	−67249	−67168	−67064	−66936	−66782	−66596	−66372	−66106
170°	X	− 1432	− 1058	− 651	− 211	260	764	1301	1873	2480
	Y	3958	4571	5162	5730	6276	6797	7294	7763	8202
	Z	−66820	−66812	−66775	−66710	−66616	−66491	−66336	−66148	−65925
175°	X	− 5339	− 4967	− 4542	− 4068	− 3546	− 2977	− 2365	− 1711	− 1018
	Y	4048	4747	5417	6055	6657	7220	7739	8213	8637
	Z	−65251	−65255	−65238	−65202	−65147	−65071	−64977	−64862	−64729
180°	X	− 9043	− 8635	− 8162	− 7626	− 7032	− 6384	− 5688	− 4949	− 4172
	Y	4287	5058	5792	6481	7121	7707	8234	8698	9096
	Z	−62651	−62651	−62651	−62651	−62651	−62651	−62651	−62651	−62651

(Continuation of VIII)

φ	λ:	225°	230°	235°	240°	245°	250°	255°	260°	265°
0°	X	−4834	−4919	−4967	−4977	−4949	−4883	−4780	−4641	−4466
	Y	1187	761	329	−105	−539	−968	−1390	−1801	−2199
	Z	57860	57860	57860	57860	57860	57860	57860	57860	57860
5°	X	−2545	−2856	−3130	−3361	−3543	−3669	−3735	−3736	−3671
	Y	2460	2060	1616	1131	613	65	−504	−1087	−1675
	Z	59175	59153	59120	59075	59018	58948	58866	58772	58667
10°	X	−3	−547	−1055	−1515	−1914	−2240	−2484	−2638	−2697
	Y	3779	3398	2935	2395	1786	1118	401	−350	−1123
	Z	60204	60242	60255	60241	60195	60115	60001	59851	59667
15°	X	2762	2008	1289	623	29	−480	−889	−1189	−1372
	Y	5064	4695	4208	3611	2914	2131	1277	371	−569
	Z	60807	60995	61142	61240	61280	61258	61169	61011	60783
20°	X	5688	4777	3901	3082	2339	1690	1148	725	499
	Y	6232	5869	5358	4709	3935	3053	2083	1048	−30
	Z	60865	61281	61640	61926	62125	62227	62223	62109	61882
25°	X	8700	7712	6757	5858	5035	4302	3676	3165	2779
	Y	7212	6852	6321	5632	4800	3846	2792	1663	487
	Z	60286	60985	61611	62141	62555	62838	62975	62958	62781
30°	X	11729	10755	9812	8920	8094	7348	6693	6138	5690
	Y	7940	7584	7045	6336	5475	4484	3387	2210	980
	Z	59019	60017	60931	61731	62393	62894	63215	63343	63269
35°	X	14717	13850	13010	12210	11460	10768	10142	9587	9107
	Y	8375	8029	7498	6796	5942	4957	3865	2669	1457
	Z	57052	58323	59504	60564	61473	62204	62733	63043	63120
40°	X	17625	16945	16285	15650	15043	14465	13917	13401	12919
	Y	8498	8170	7669	7007	6200	5268	4230	3108	1922
	Z	54402	55880	57271	58543	59666	60610	61350	61860	62123
45°	X	20429	19990	19563	19144	18726	18303	17870	17423	16963
	Y	8316	8018	7570	6980	6262	5430	4496	3475	2383
	Z	51113	52701	54213	55620	56895	58008	58933	59642	60109
50°	X	23111	22933	22760	22577	22369	22121	21822	21464	21045
	Y	7860	7605	7231	6745	6153	5461	4674	3797	2838
	Z	47236	48825	50352	51799	53143	54361	55425	56306	56975
55°	X	25648	25716	25783	25826	25820	25744	25580	25317	24950
	Y	7185	6984	6703	6345	5909	5389	4782	4081	3283
	Z	42828	44311	45752	47140	48465	49709	50850	51859	52703
60°	X	28003	28271	28533	28762	28925	28996	28953	28779	28468
	Y	6363	6221	6045	5830	5567	5241	4833	4328	3709
	Z	37946	39238	40506	41751	42975	44168	45316	46395	47371
65°	X	30125	30521	30912	31265	31546	31724	31772	31672	31415
	Y	5475	5394	5324	5257	5172	5044	4842	4538	4104
	Z	32648	33697	34737	35780	36839	37917	39008	40095	41148
70°	X	31948	32390	32832	33238	33574	33809	33912	33864	33653
	Y	4605	4579	4610	4682	4766	4827	4824	4714	4459
	Z	26994	27789	28582	29397	30258	31178	32163	33204	34277
75°	X	33404	33813	34228	34616	34945	35184	35303	35281	35102
	Y	3831	3849	3965	4157	4391	4630	4793	4858	4768
	Z	21054	21619	22184	22780	23441	24190	25044	26002	27049
80°	X	34437	34747	35071	35381	35649	35846	35944	35920	35758
	Y	3216	3266	3445	3731	4062	4447	4768	4982	5035
	Z	14907	15295	15678	16095	16585	17179	17903	18766	19762
85°	X	35011	35182	35373	35564	35732	35851	35898	35849	35688
	Y	2805	2871	3090	3438	3872	4337	4769	5101	5271
	Z	8639	8917	9184	9483	9853	10335	10959	11742	12686
90°	X	35129	35147	35189	35243	35290	35312	35258	35197	35022
	Y	2621	2688	2924	3303	3784	4312	4819	5235	5496
	Z	2341	2574	2795	3043	3361	3791	4366	5109	6005

(Continuation of VIII)

ω:	λ:	225°	230°	235°	240°	245°	250°	255°	260°	265°
90°	X	35129	35147	35189	35243	35290	35312	35288	35197	35022
	Y	2621	2658	2924	3303	3764	4312	4819	5235	5496
	Z	2341	2574	2795	3043	3361	3792	4366	5109	6005
95°	X	34826	34705	34609	34530	34458	34379	34277	34134	33935
	Y	2663	2721	2953	3337	3835	4390	4939	5410	5736
	Z	− 3907	− 3662	− 3428	− 3166	− 2832	− 2388	− 1797	− 1038	− 103
100°	X	34169	33947	33743	33557	33384	33216	33041	32847	32621
	Y	2909	2953	3169	3539	4029	4586	5149	5649	6018
	Z	−10035	− 9744	− 9455	− 9128	− 8722	− 8200	− 7528	− 6689	− 5676
105°	X	33242	32967	32701	32448	32206	31974	31745	31511	31264
	Y	3319	3351	3547	3894	4361	4903	5461	5971	6365
	Z	−15999	−15651	−15257	−14865	−14347	−13697	−12890	−11912	−10764
110°	X	32127	31853	31576	31301	31031	30768	30512	30261	30009
	Y	3842	3871	4052	4375	4816	5334	5877	6384	6791
	Z	−21778	−21382	−20943	−20418	−19769	−18964	−17985	−16827	−15498
115°	X	30888	30662	30420	30168	29911	29657	29408	29168	28936
	Y	4424	4465	4642	4950	5369	5864	6389	6886	7296
	Z	−27373	−26950	−26452	−25834	−25057	−24093	−22932	−21576	−20045
120°	X	29552	29410	29240	29049	28844	28636	28431	28237	28057
	Y	5014	5083	5273	5580	5987	6466	6973	7459	7864
	Z	−32800	−32372	−31837	−31147	−30262	−29159	−27830	−26289	−24562
125°	X	28107	28068	27994	27891	27770	27642	27517	27404	27309
	Y	5570	5685	5903	6224	6633	7104	7599	8071	8468
	Z	−38062	−37649	−37101	−36365	35404	−34195	−32738	−31049	−29165
130°	X	26504	26572	26603	26606	26590	26569	26553	26553	26575
	Y	6063	6237	6499	6847	7268	7739	8224	8682	9063
	Z	−43150	−42760	−42213	−41457	−40455	−39186	−37654	−35877	−33897
135°	X	24669	24837	24973	25087	25189	25291	25405	25541	25705
	Y	6482	6722	7035	7418	7858	8331	8807	9247	9606
	Z	−48016	−47645	−47107	−46350	−45341	−44061	−42514	−40722	−38722
140°	X	22526	22779	23013	23236	23459	23693	23948	24233	24550
	Y	6829	7135	7500	7918	8374	8847	9308	9722	10049
	Z	−52574	−52210	−51678	−50934	−49946	−48700	−47199	−45463	−43528
145°	X	20016	20343	20664	20991	21331	21695	22091	22524	22994
	Y	7121	7485	7893	8336	8799	9262	9697	10074	10358
	Z	−56695	−56323	−55794	−55070	−54127	−52953	−51550	−49937	−48143
150°	X	17121	17512	17915	18337	18787	19273	19799	20368	20976
	Y	7382	7789	8223	8675	9128	9564	9958	10284	10513
	Z	−60227	−59836	−59308	−58613	−57734	−56662	−55399	−53960	−52368
155°	X	13876	14330	14809	15320	15870	16464	17102	17785	18507
	Y	7641	8069	8508	8946	9368	9757	10093	10352	10513
	Z	−63007	−62599	−62078	−61425	−60628	−59682	−58590	−57361	−56013
160°	X	10367	10889	11445	12041	12680	13364	14093	14861	15663
	Y	7922	8349	8768	9169	9537	9859	10118	10297	10379
	Z	−64892	−64486	−63992	−63401	−62707	−61907	−61006	−60008	−58927
165°	X	6730	7327	7963	8639	9356	10113	10906	11731	12580
	Y	8241	8646	9024	9365	9660	9898	10066	10154	10148
	Z	−65788	−65415	−64981	−64451	−63915	−63282	−62586	−61831	−61025
170°	X	3123	3803	4519	5269	6053	6865	7701	8555	9419
	Y	8606	8970	9289	9556	9765	9908	9977	9967	9871
	Z	−65666	−65371	−65037	−64666	−64257	−63813	−63336	−62828	−62295
175°	X	− 289	474	1265	2081	2917	3767	4626	5487	6342
	Y	9007	9320	9571	9758	9875	9921	9892	9786	9601
	Z	−64577	−64407	−64220	−64016	−63797	−63564	−63319	−63062	−62797
180°	X	− 3363	− 2539	− 1676	− 809	63	935	1800	2651	3482
	Y	9425	9653	9866	9975	10007	9964	9844	9650	9382
	Z	−62651	−62651	−62651	−62651	−62651	−62651	−62651	−62651	−62651

(Continuation of VIII)

φ:	λ:	270°	275°	280°	285°	290°	295°	300°	305°	310°
0°	X	−4258	−4016	−3745	−3445	−3119	−2768	−2397	−2008	−1603
	Y	−2580	−2941	−3280	−3594	−3880	−4137	−4363	−4555	−4713
	Z	57860	57860	57860	57860	57860	57860	57860	57860	57860
5°	X	−3539	−3337	−3069	−2737	−2344	−1896	−1398	−858	−283
	Y	−2259	−2830	−3380	−3899	−4378	−4810	−5189	−5507	−5760
	Z	58550	58423	58287	58144	57994	57841	57684	57526	57370
10°	X	−2657	−2518	−2280	−1946	−1523	−1017	−437	205	899
	Y	−1902	−2673	−3420	−4128	−4786	−5379	−5896	−6328	−6668
	Z	59448	59198	58919	58615	58290	57949	57598	57241	56884
15°	X	−1435	−1374	−1192	−842	−481	33	638	1322	2071
	Y	−1522	−2469	−3389	−4264	−5075	−5806	−6444	−6976	−7391
	Z	60487	60187	59708	59237	58723	58175	57605	57003	56439
20°	X	263	233	338	570	924	1391	1962	2622	3358
	Y	−1126	−2216	−3276	−4283	−5221	−6065	−6802	−7417	−7898
	Z	61544	61099	60558	59930	59289	58472	57677	56861	56042
25°	X	2522	2396	2400	2532	2787	3158	3636	4211	4870
	Y	−712	−1905	−3068	−4175	−5203	−6136	−6952	−7636	−8177
	Z	62447	61960	61331	60575	59712	58763	57753	56713	55666
30°	X	5353	5129	5021	5028	5148	5380	5717	6154	6684
	Y	−273	−1527	−2752	−3923	−5017	−6012	−6890	−7634	−8230
	Z	62993	62517	61853	61018	60036	58934	57747	56507	55252
35°	X	8707	8391	8164	8028	7989	8049	8210	8472	8833
	Y	193	−1075	−2323	−3523	−4638	−5699	−6629	−7428	−8082
	Z	62957	62555	61922	61075	60039	58845	57531	56139	54712
40°	X	12473	12076	11729	11445	11233	11105	11071	11137	11311
	Y	697	−546	−1783	−2989	−4142	−5217	−6193	−7049	−7765
	Z	62126	61861	61333	60553	59542	58332	56962	55477	53928
45°	X	16493	16026	15567	15141	14759	14442	14209	14077	14057
	Y	1237	55	−1143	−2333	−3492	−4596	−5619	−6535	−7323
	Z	60315	60244	59890	59257	58357	57218	55873	54372	52762
50°	X	20569	20048	19498	18941	18403	17912	17494	17173	16974
	Y	1805	712	−425	−1585	−2743	−3874	−4946	−5931	−6801
	Z	57403	57563	57443	57028	56322	55340	54111	52671	51072
55°	X	24483	23929	23309	22650	21983	21344	20767	20284	19922
	Y	2389	1405	343	−779	−1934	−3092	−4220	−5281	−6241
	Z	53348	53759	53904	53761	53316	52569	51533	50240	48726
60°	X	28024	27459	26796	26066	25303	24553	23852	23238	22744
	Y	2968	2103	1125	47	−1103	−2291	−3478	−4621	−5679
	Z	48203	48654	49276	49432	49291	48837	48063	46966	45630
65°	X	31003	30446	29769	29004	28188	27364	26577	25868	25273
	Y	3522	2783	1890	857	−287	−1304	−2751	−3978	−5137
	Z	42126	42981	43661	44114	44293	44161	43695	42888	41751
70°	X	33276	32746	32081	31314	30481	29625	28792	28024	27360
	Y	4032	3416	2609	1623	488	−757	−2061	−3370	−4628
	Z	35344	36356	37254	37976	38460	38653	38511	38006	37131
75°	X	34763	34269	33639	32898	32062	31230	30386	29592	28888
	Y	4486	3986	3163	2327	1205	−60	−1414	−2798	−4149
	Z	28154	29268	30331	31273	32026	32517	32687	32489	31897
80°	X	35451	35000	34417	33724	32930	32130	31305	30514	29794
	Y	4881	4490	3848	2964	1863	588	−805	−2253	−3688
	Z	20866	22034	23209	24318	25284	26027	26473	26562	26248
85°	X	35404	34994	34464	33831	33116	32349	31505	30798	30062
	Y	5227	4933	4370	3541	2470	1197	−221	−1719	−3226
	Z	13772	14963	16206	17429	18552	19492	20163	20499	20437
90°	X	34730	34375	33899	33329	32682	31979	31249	30521	29825
	Y	5543	5335	4847	4078	3046	1788	360	−1174	−2741
	Z	7102	8310	9596	10695	12128	13209	14052	14580	14727

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OF POOR QUALITY

(Continuation of VIII)

φ:	λ:	270°	275°	280°	285°	290°	295°	300°	305°	310°
90°	X	34750	34375	33909	33329	32682	31979	31249	30521	29825
	Y	5543	5335	4647	4078	3046	1788	360	− 1174	− 2741
	Z	7102	8510	9596	10895	12128	13209	14052	14580	14787
95°	X	33666	33319	32800	32382	31805	31172	30504	29825	29158
	Y	5855	5782	5306	4509	3614	2384	959	− 396	− 2011
	Z	999	2238	3169	4927	6037	7416	8379	9048	9354
100°	X	32350	32025	31639	31190	30681	30119	29518	28892	28261
	Y	6193	6123	5773	5130	4198	3004	1595	30	− 1602
	Z	− 4308	− 3194	− 1799	− 377	999	2250	3297	4064	4484
105°	X	30904	30690	30345	29953	29512	29022	28489	27921	27330
	Y	6580	6563	6274	5691	4814	3663	2278	712	− 985
	Z	− 9462	− 8041	− 6546	− 5038	− 3588	− 2269	− 1157	− 320	180
110°	X	29749	29475	29176	28845	28475	28060	27599	27092	26545
	Y	7034	7058	6820	6293	5470	4364	3006	1449	− 244
	Z	−14024	−12445	−10813	− 9190	− 7645	− 6248	− 5070	− 4173	− 3610
115°	X	28709	28480	28241	27982	27691	27358	26975	26536	26037
	Y	7556	7610	7414	6934	6160	5096	3770	2226	527
	Z	−18373	−16609	−14811	−13046	−11383	− 9891	− 8634	− 7670	− 7043
120°	X	27890	27731	27573	27402	27205	26966	26671	26306	25864
	Y	8132	8205	8038	7595	6861	5836	4542	3019	1324
	Z	−2[illegible]93	−20743	−18772	−16853	−15058	−13454	−12102	−11054	−10349
125°	X	27234	27174	27120	27060	26974	26844	26649	26372	25997
	Y	8733	8813	8662	8245	7542	6552	52[illegible]1	3796	2118
	Z	−27135	−25023	−22900	−20840	−18916	−17196	−15739	−14592	−13790
130°	X	26621	26688	26764	26834	26877	26871	26791	26614	26321
	Y	9317	9392	9245	8842	8163	7303	5979	4519	2874
	Z	−31766	−29551	−27325	−25163	−23139	−21319	−19760	−18508	−17594
135°	X	25896	26110	26335	26551	26738	26868	26914	26850	26653
	Y	9837	9895	9741	9342	8680	7750	6566	5154	3559
	Z	−36571	−34332	−32076	−29877	−27804	−25924	−24289	−22943	−21917
140°	X	24898	25270	25651	26022	26357	26628	26807	26866	26783
	Y	10248	10279	10107	9705	9036	8138	7020	5670	4144
	Z	−41445	−39271	−37073	−34917	−32868	−30966	−29322	−27916	−26797
145°	X	23497	24023	24556	25074	25551	25960	26271	26457	26496
	Y	10514	10508	10311	9901	9263	8401	7[illegible]19	6042	4604
	Z	−46211	−44191	−42139	−40113	−38170	−36361	−34731	−33318	−32148
150°	X	21618	22279	22944	23590	24192	24723	25155	25464	25628
	Y	10615	10564	10336	9916	9293	8468	7451	6260	4926
	Z	−50657	−48866	−47039	−45224	−43465	−41805	−40283	−38929	−37769
155°	X	19258	20025	20770	21534	22232	22859	23392	23809	24089
	Y	10552	10449	10187	9756	9148	8366	7419	6324	5103
	Z	−54570	−53062	−51519	−49977	−48470	−47009	−45684	−44461	−43381
160°	X	16488	17323	18151	18953	19710	20401	21005	21504	21882
	Y	10347	10187	9889	9445	8854	8116	7242	6243	5139
	Z	−57779	−56582	−55358	−54129	−52918	−51749	−50640	−49612	−48680
165°	X	13441	14304	15154	15975	16752	17469	18108	18656	19101
	Y	10040	9821	9484	9026	8448	7752	6943	6039	5046
	Z	−60177	−59399	−58602	−57792	− 56610	−55741	− 54908	−54122	−53395
170°	X	10283	11139	11974	12778	13538	14241	14878	15437	15909
	Y	9685	9403	9024	8549	7977	7314	6564	5737	4840
	Z	−61741	−61172	−60595	−60016	−59448	−58880	−58337	−57819	−57333
175°	X	7185	8007	8820	9556	10267	10905	11522	12052	12509
	Y	9336	8991	8567	8066	7491	6845	6134	5364	4542
	Z	−62526	−62250	−61972	−61695	−61421	−61133	−60893	−60645	−60409
180°	X	4287	5058	5792	6481	7121	7707	8234	8698	9096
	Y	9043	8635	8160	7606	7032	6384	5688	4949	4172
	Z	−62651	−62651	−62651	−62651	−62651	−62651	−62651	−62651	−62651

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(Continuation of VIII)

φ:	λ:	315°	320°	325°	330°	335°	340°	345°	350°	355°
0°	X	−1187	−761	−399	105	539	968	1390	1801	2199
	Y	−4834	−4919	−4967	−4977	−4949	−4883	−4780	−4641	−4468
	Z	57860	57860	57860	57860	57860	57860	57860	57860	57860
5°	X	318	936	1561	2185	2799	3394	3961	4494	4984
	Y	−5944	−6056	−6096	−6062	−5938	−5783	−5546	−5248	−4895
	Z	57816	57067	56984	56790	56665	56551	56430	56363	56289
10°	X	1631	2380	3157	3924	4673	5397	6079	6710	7279
	Y	−6909	−7048	−7084	−7016	−6848	−6585	−6233	−5860	−5397
	Z	56533	56193	55868	55564	55283	55033	54812	54604	54470
15°	X	2870	3702	4552	5402	6238	7043	7801	8500	9133
	Y	−7684	−7851	−7888	−7798	−7584	−7853	−6613	−6076	−5658
	Z	55866	55313	54788	54301	53857	53466	53120	52834	52605
20°	X	4155	4995	5861	6736	7603	8445	9245	9990	10664
	Y	−8240	−8435	−8483	−8385	−8148	−7772	−7374	−6666	−5960
	Z	55238	54466	53739	53069	52466	51936	51483	51110	50817
25°	X	5600	6386	7211	8058	8911	9751	10560	11328	12033
	Y	−8565	−8795	−8865	−8776	−8532	−8142	−7616	−6969	−6218
	Z	54637	53651	52727	51883	51130	50476	49925	49476	49187
30°	X	7295	7976	8714	9495	10296	11106	11906	12678	13406
	Y	−8669	−8943	−9048	−8965	−8759	−8377	−7852	−7197	−6431
	Z	54015	52828	51718	50707	49810	49036	48390	47867	47463
35°	X	9290	9833	10453	11137	11868	12631	13405	14174	14919
	Y	−8577	−8905	−9061	−9043	−8855	−8505	−8004	−7368	−6616
	Z	53293	51922	50635	49459	48416	47516	46768	46153	45679
40°	X	11592	11979	12465	13040	13689	14394	15136	15897	16655
	Y	−8327	−8722	−8942	−8986	−8854	−8555	−8100	−7504	−6788
	Z	52364	50832	49378	48036	46832	45784	44897	44170	43594
45°	X	14160	14387	14736	15198	15760	16405	17111	17858	18623
	Y	−7963	−8438	−8738	−8857	−8798	−8564	−8170	−7631	−6968
	Z	51099	49438	47829	46317	44936	43709	42651	41760	41037
50°	X	16906	16980	17197	17549	18006	18607	19273	20000	20763
	Y	−7530	−8097	−8489	−8696	−8719	−8563	−8241	−7769	−7169
	Z	49367	47615	45873	44192	42617	41183	39910	38810	37884
55°	X	19702	19638	19732	19980	20372	20889	21507	22203	22951
	Y	−7069	−7739	−8232	−8536	−8650	−8578	−8334	−7935	−7425
	Z	47043	45250	43406	41571	39799	38135	36614	35260	34085
60°	X	22395	22210	22193	22344	22652	23098	23661	24312	25087
	Y	−6614	−7393	−7993	−8400	−8610	−8626	−8463	−8140	−7682
	Z	44037	42260	40360	38400	36444	34549	32763	31130	29673
65°	X	24822	24534	24419	24478	24701	25071	25564	26155	26817
	Y	−6182	−7075	−7786	−8300	−8608	−8714	−8634	−8388	−8000
	Z	40311	38612	36712	34674	32569	30665	28826	26509	24760
70°	X	26831	26459	26258	26229	26365	26651	27063	27578	28167
	Y	−5782	−6789	−7613	−8235	−8643	−8842	−8845	−8673	−8353
	Z	35897	34336	32494	30437	28830	25969	23713	21540	19514
75°	X	28304	27867	27590	27477	27525	27719	28030	28460	28960
	Y	−5409	−6587	−7484	−8195	−8700	−8998	−9085	−8987	−8731
	Z	30904	29527	27803	25789	23558	21192	18777	16399	14136
80°	X	29177	28689	28340	28154	28112	28209	28429	28745	29145
	Y	−5047	−6273	−7322	−8162	−8777	−9165	−9337	−9313	−9218
	Z	25510	24347	22785	20873	18679	16236	13785	11269	8830
85°	X	29449	28923	28523	28257	28125	28122	28234	28440	28727
	Y	−4673	−6004	−7162	−8113	−8836	−9323	−9585	−9632	−9497
	Z	19948	19000	17630	15864	13766	11414	8904	6322	3776
90°	X	29189	28637	28186	27802	27630	27544	27524	27613	27780
	Y	−4871	−5696	−6960	−8005	−8858	−9449	−9800	−9915	−9880
	Z	14447	13717	12541	10946	8985	6730	4469	1698	−887

(Continuation of VIII)

α:	λ:	315°	320°	325°	330°	335°	340°	345°	350°	355°
90°	X	29189	28637	28186	27849	27630	27524	27524	27613	27780
	Y	− 4271	− 5696	− 6962	− 8025	− 8858	− 9449	− 9800	− 9925	− 9850
	Z	14447	13717	12541	10946	8985	6730	4269	1698	− 887
95°	X	28528	27957	27464	27059	26751	26541	26423	26391	26432
	Y	− 3812	− 5329	− 6699	− 7873	− 8820	− 9518	− 9963	−10170	−10156
	Z	9248	8701	7797	6286	4483	2361	0	− 2509	− 5072
100°	X	27644	27060	26525	26053	25657	25338	25099	24938	24848
	Y	− 3284	− 4884	− 6356	− 7643	− 8703	− 9511	−10057	−10347	−10398
	Z	4509	4107	3269	2010	366	− 1608	− 3840	− 6248	− 8744
105°	X	26730	26136	25562	25023	24533	24103	23735	23436	23208
	Y	− 2679	− 4355	− 5921	− 7316	− 8489	− 9409	−10058	−10436	−10559
	Z	298	6	− 705	− 1824	− 3321	− 5147	− 7238	− 9522	−11921
110°	X	25963	25363	24752	24146	23561	23011	22509	22066	21691
	Y	− 2000	− 3741	− 5392	− 6887	− 8168	− 9197	− 9950	−10422	−10623
	Z	− 3420	− 3626	− 4235	− 5236	− 6600	− 8283	−10234	−12384	−14664
115°	X	25482	24875	24229	23557	22877	22206	21566	20974	20447
	Y	− 1258	− 3051	− 4774	− 6356	− 7736	− 8868	− 9723	−10290	−10578
	Z	− 6785	− 6914	− 7432	− 8323	− 9566	−11114	−12921	−14929	−17076
120°	X	25341	24738	24063	23331	22561	21773	21000	20260	19581
	Y	− 475	− 2302	− 4078	− 5730	− 7194	− 8418	− 9369	−10033	−10413
	Z	−10013	−10056	−10476	−11257	−12370	−13778	−15436	−17290	−19286
125°	X	25517	24929	24240	23452	22617	21731	20833	19960	19138
	Y	322	− 1517	− 3324	− 5023	− 6530	− 7851	− 8889	− 9648	−10128
	Z	−13353	−13289	−13590	−14239	−15207	−16457	−17945	−19626	−21448
130°	X	25900	25346	24663	23863	22972	22013	21024	20039	19093
	Y	1103	− 724	− 2533	− 4253	− 5818	− 7176	− 8289	− 9138	− 9719
	Z	−17036	−16839	−16993	−17481	−18273	−19336	−20627	−22104	−23722
135°	X	26313	25817	25170	24383	23482	22492	21430	20394	19364
	Y	1836	49	− 1733	− 3443	− 5021	− 6413	− 7553	− 8513	− 9194
	Z	−21227	−20877	−20859	−21156	−21741	−22582	−23643	−24884	−26265
140°	X	26541	26131	25554	24823	23956	22983	21942	20867	19798
	Y	2493	774	− 952	− 2623	− 4184	− 5587	− 6797	− 7792	− 8567
	Z	−25983	−25477	−25276	−25364	−25719	−26313	−27116	−28092	−29207
145°	X	26371	26072	25601	24965	24185	23286	22300	21262	20209
	Y	3048	1422	− 219	− 1823	− 3340	− 4729	− 5957	− 7003	− 7860
	Z	−31240	−30599	−30225	−30107	−30229	−30569	−31101	−31799	−32634
150°	X	25630	25463	25123	24623	23972	23195	22319	21374	20391
	Y	3482	1972	438	− 1074	− 2524	− 3875	− 5100	− 6178	− 7101
	Z	−36819	−36089	−35579	−35256	−35198	−35301	−35576	−36003	−36564
155°	X	24219	24191	24002	23657	23167	22548	21823	21014	20148
	Y	3785	2404	994	− 410	− 1773	− 3067	− 4267	− 5357	− 6323
	Z	−42458	−41702	−41118	−40706	−40460	−40372	−40432	−40626	−40941
160°	X	22126	22229	22188	22003	21686	21242	20689	20043	19322
	Y	3951	2704	1425	141	− 1123	− 2344	− 3502	− 4580	− 5569
	Z	−47856	−47150	−46568	−46111	−45780	−45571	−45480	−45500	−45624
165°	X	19432	19642	19727	19688	19527	19250	18865	18383	17816
	Y	3982	2866	1716	552	− 607	− 1744	− 2842	− 3889	− 4873
	Z	−52735	−52151	−51646	−51225	−50891	−50642	−50480	−50400	−50401
170°	X	16287	16564	16737	16804	16764	16620	16376	16037	15609
	Y	3887	2887	1856	806	− 230	− 1297	− 2324	− 3320	− 4274
	Z	−56883	−56475	−56112	−55798	−55534	−55324	−55166	−55062	−55012
175°	X	12886	13180	13388	13506	13534	13471	13320	13080	12756
	Y	3673	2771	1841	891	− 67	− 1023	− 1972	− 2901	− 3803
	Z	−60189	−59986	−59802	−59640	−59499	−59382	−59290	−59222	−59180
180°	X	9425	9683	9866	9975	10007	9964	9844	9650	9382
	Y	3363	2329	1676	809	− 63	− 935	− 1800	− 2651	− 3482
	Z	−62651	−62651	−62651	−62651	−62651	−62651	−62651	−62651	−62651

VIIIa. Deviations of Computed Values of the Components from the Observations*

φ:	λ:	15°	45°	75°	105°	135°	165°	195°	225°	255°	285°	315°	345°
30°	ΔX	−65	349	411	702	56	−810	−983	−137	−154	−541	−76	25
	ΔY	−63	−340	324	−142	−477	334	−241	−790	−31	0	66	565
	ΔZ	−791	1507	1129	−202	−221	−1853	−158	371	472	1887	532	633
40°	ΔX	67	27	−267	−3	158	−65	−236	674	105	−316	262	239
	ΔY	−350	−485	559	−15	−1085	742	67	−427	690	−556	−128	501
	ΔZ	−667	−503	272	−725	1182	−81	996	548	152	1459	737	−206
50°	ΔX	51	−259	−384	−74	308	92	−539	638	172	160	−425	340
	ΔY	−251	−414	532	272	−1236	902	−396	−311	1028	−366	48	515
	ΔZ	−381	84	−239	−735	253	156	−114	−1338	249	915	−1593	72
60°	ΔX	−80	−192	149	83	15	203	−210	359	−574	811	454	−42
	ΔY	99	−134	36	604	−937	805	−479	−75	786	−438	−115	269
	ΔZ	52	787	797	123	−93	−181	−52	−763	−425	945	−874	1331
70°	ΔX	−235	−34	504	−323	−121	35	261	−96	−533	169	34	−404
	ΔY	246	69	−483	726	−539	544	−544	250	165	−597	600	−161
	ΔZ	−792	508	−692	−43	479	−956	537	234	1003	767	149	820
80°	ΔX	419	15	−280	16	−117	276	374	−402	737	483	738	−84
	ΔY	207	190	−733	709	−433	403	−363	141	−318	−659	1286	−793
	ΔZ	−937	1131	−720	102	563	−850	813	−507	587	−336	393	251
90°	ΔX	109	237	−261	116	694	−608	154	−629	130	−423	457	6
	ΔY	398	208	−663	421	−341	209	−205	109	[illegible]	−503	1242	−1211
	ΔZ	−891	791	182	−813	−166	−318	721	−780	[illegible]	−300	303	−409
100°	ΔX	−331	−6	180	151	−61	−672	73	−303	438	−278	342	−401
	ΔY	−529	105	−311	290	−251	153	−75	431	54	−259	753	−853
	ΔZ	−793	−132	−141	546	103	400	597	−45	430	44	67	326
110°	ΔX	−204	172	−95	−114	353	314	278	−32	554	−415	270	−103
	ΔY	236	−167	268	32	−168	154	−251	593	124	−319	150	114
	ΔZ	826	−537	−639	−499	−794	383	6	−351	388	31	−4	430
120°	ΔX	247	222	−294	−441	−36	−77	450	−201	−22	57	−4	71
	ΔY	−280	−602	641	14	−180	246	−330	144	−223	−152	−336	713
	ΔZ	825	142	−185	750	−393	−44	−1101	−883	201	−354	451	−174
130°	ΔX	396	−13	−97	430	103	−350	452	−314	−179	288	263	−128
	ΔY	−174	−783	535	−306	−192	396	−200	−423	251	75	−693	615
	ΔZ	485	194	−387	1613	1054	446	−1361	1082	194	−770	172	−870
140°	ΔX	81	−263	45	922	331	−256	231	−275	−116	511	−204	−300
	ΔY	19	−341	519	−810	−247	746	−52	−624	40	702	−667	180
	ΔZ	96	−294	−101	−1241	−3090	184	548	2467	439	−489	667	−1030
150°	ΔX	−399	−203	−613	185	−724	−747	394	627	677	−639	134	−478
	ΔY	367	419	57	−61	−180	605	−62	−598	626	1771	226	358
	ΔZ	−83	−158	742	−7	1646	768	−1847	−1918	−2231	99	2420	85

*This table (not mentioned in the text) has been added to provide an overview for the next two tables (IX and X). It contains the differences of the computed and observed values of X, Y, Z, i.e. the values in VIII and those in III, formed as (observed − computed). The corresponding differences for the intermediate points (λ = 0°, 30°, 60°...330°) of the same parallel circles are found (according to provisional computation) in B, table XIVa, b, c, p. 64-66.

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IX. Coefficients of the Series Representing U, W, $V:b$. ($b = 6.856 \cdot 10^8$ cm.)

1.

U_0

m; n:	0	1	2	3	4	5	6
0	0	−18428.2	− 233.0	354.4	276.5	− 52.8	5.9
1		− 1167.0 3485.7	1202.8 − 304.9	−407.8 − 92.7	147.8 61.7	110.7 − 89.6	18.5 32.2
2			308.9 716.9	527.4 21.1	189.1 − 29.4	106.6 − 8.1	− 7.3 18.1
3				141.2 225.3	−100.6 − 55.3	6.7 − 0.8	−25.2 −10.5
4					101.5 15.2	− 24.1 − 26.6	5.9 12.8

$$U = U_0 + \Pi_1(-533.5\cos\lambda + 50.0\sin\lambda) + \Pi_2(27.9\cos 2\lambda - 42.7\sin 2\lambda) + \Pi_3(-202.2\cos 3\lambda + 154.9\sin 3\lambda) + \Pi_4(89.3\cos 4\lambda - 11.4\sin 4\lambda)$$

W_0

m; n:	0	1	2	3	4	5	6
0	0	−18428.2	− 233.0	354.4	276.5	− 52.8	5.9
1		− 1277.0 3421.0	1261.0 − 330.0	−496.0 −132.0	166.0 121.0	170.0 − 61.0	−38.0 7.0
2			241.5 623.5	525.5 − 11.5	186.0 − 47.5	108.0 0.5	− 1.0 32.0
3				172.7 249.7	−104.0 − 67.8	2.0 − 4.0	−27.0 − 5.7
4					40.0 − 68.8	− 21.0 − 27.0	13.5 22.8

$$W = W_0 + \lambda(50 - 67R_0^1 - 34R_0^2 + 38R_0^3 + 9R_0^4 + 4.67R_0^5 - 0.27R_0^6)$$

$V:b$

m; n:	0	1	2	3	4	5	6
0	0	−18428.2	− 233.0	354.4	276.5	− 52.8	5.9
1		− 1222.0 3453.3	1231.6 − 317.5	−451.9 −112.8	156.9 91.8	140.8 − 75.3	− 9.8 19.6
2			275.2 670.2	526.5 4.8	187.5 − 38.4	107.8 − 3.8	− 4.1 25.0
3				157.0 237.4	−102.3 − 61.3	4.4 − 2.2	−26.1 − 8.1
4					70.7 − 26.8	− 22.5 − 26.8	9.7 17.6

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IX. Coefficients of the Series Representing U, W, $V:b$. ($b = 6.356 \cdot 10^8$ cm.)

2.

U_0

m; n:	0	1	2	3	4
0	0	−18400.4	− 237.6	393.0	271.1
1		− 1702.1 3656.1	1254.8 − 333.7	−539.7 − 35.5	156.9 43.0
2			292.0 644.9	483.6 7.7	185.5 − 54.8
3				167.7 247.0	− 89.7 − 66.4
4					45.5 − 61.8

$$U = U_0 + \Pi_1(-397.4\cos\lambda - 8.2\sin\lambda) + \Pi_2(-57.7\cos 2\lambda - 89.7\sin 2\lambda) + \Pi_3(-174.7\cos 3\lambda + 107.5\sin 3\lambda)$$

W_0

m; n:	0	1	2	3	4
0	0	−18400.4	− 237.6	393.0	271.1
1		− 1312.0 3452.0	1250.0 − 353.0	−644.0 19.0	117.0 50.0
2			242.5 622.5	478.0 − 1.0	195.0 − 50.0
3				167.7 247.0	− 99.0 − 60.7
4					45.5 − 61.8

$$W = W_0 + \lambda(49 - 64R_0^1 - 34R_0^2 + 41.92R_0^3 + 9.01R_0^4)$$

$V:b$

m; n:	0	1	2	3	4
0	0	−18400.4	− 237.6	393.0	271.1
1		− 1507.0 3554.0	1252.4 − 343.4	−591.9 − 8.8	137.0 46.5
2			267.3 633.7	480.8 3.4	190.2 − 52.3
3				167.7 247.0	− 94.4 − 63.5
4					45.5 − 61.8

3.

U_0

0	1	2
0	−18618.2	− 393.1
	− 1019.0 3517.0	1180.1 − 378.3
		200.5 640.5

$$U = U_0 + \Pi_1(-499.7\cos\lambda - 88.0\sin\lambda)$$

W_0

0	1	2
0	−18618.2	− 393.1
	− 1019.0 3517.0	1051.0 − 396.0
		200.5 640.5

$$W = W_0 + \lambda(53 - 23.70R_0^2)$$

$V:b$

0	1	2
0	−18618.2	− 393.1
	− 1019.0 3517.0	1115.6 − 384.6
		200.5 640.5

X. Coefficients of the Series Representing $V_i:b$, $V_a:b$, $\alpha\beta bi$ ($b = 6.356 \cdot 10^8$ cm.)*

1.

$V_i:b$

m; n:	0	1	2	3	4	5	6
0	0	−18321.0	− 238.9	354.4	364.9	− 50.6	5.8
1		− 1360.9 3455.5	1264.0 − 321.0	−466.2 −112.0	171.1 94.4	119.2 − 81.2	5.7 24.6
2			302.9 668.1	540.0 9.8	172.3 − 57.5	86.2 − 10.4	−14.9 17.5
3				151.0 258.2	−103.2 − 75.2	4.7 6.5	−44.3 −17.8
4					52.7 − 24.7	− 2.1 − 22.8	− 6.4 − 7.4

$V_a:b$

m; n:	0	1	2	3	4	5	6
0	0	−107.2	0.9	0.0	[illegible]	− 2.2	0.1
1		138.9 − 2.2	−32.4 3.5	14.3 − 0.8	[illegible] [illegible]	21.1 5.9	−15.5 − 5.0
2			−27.7 2.1	−13.5 − 5.0	15.2 19.1	21.1 6.6	10.8 7.5
3				6.0 −20.8	0.9 14.0	− 0.8 − 8.7	18.7 9.7
4					18.0 − 2.1	−20.4 − 4.0	16.1 25.0

$\alpha\beta bi$

m; n:	0	1	2	3	4	5
0	0	6.8	−20.7	− 5.7	− 3.7	0.0
1		− 3.7 −33.4	− 3.0 2.9	−15.7 −39.7	−23.1 47.9	34.3 −54.5
2			10.3 2.6	−15.9 14.4	−14.9 2.4	−28.1 12.5
3				−16.6 −28.3	11.0 −14.0	−15.9 − 5.9
4					2.1 16.4	−52.2 *8.8

*The coefficients of the series expansion of $\alpha\beta bi$ differ strikingly at $m = 3$ and $m = 4$ from those reported in B, p. 59. This is due to the fact that the factor m had been omitted accidentally during the calculation.

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X. Coefficients of the Series Representing $V_i:b$, $V_e:b$, $\alpha\beta bi$ ($b = 6.356 \cdot 10^8$ cm.)*

2.

$V_i:b$

m; n:	0	1	2	3	4
0	0	−18242.9	− 242.8	404.7	259.3
1		− 1470.6 3496.5	1266.7 − 339.8	−538.2 − 57.2	157.7 67.5
2			300.3 653.1	526.9 7.8	172.5 − 63.2
3				155.5 263.4	−105.6 − 78.9
4					40.2 − 42.0

3.

$V_i:b$

0	1	2
0	−18501.5	− 440.3
	− 1857.8 3477.4	1218.2 − 355.7
		291.1 648.8

$V_e:b$

m; n:	0	1	2	3	4
0	0	−157.5	5.2	−1[illegible].7	−11.8
1		− 36.9 57.5	−14.8 − 3.6	−53.7 46.9	−20.7 −21.0
2			−38.0 −19.4	−46.1 − 4.4	17.7 11.0
3				12.2 −16.4	11.3 15.4
4					5.8 −19.8

$V_e:b$

0	1	2
0	−116.7	47.2
	338.2 39.6	−97.6 −28.9
		−90.6 − 7.8

$\alpha\beta bi$

m; n:	0	1	2	3
0	0	6.8	−19.7	− 5.7
1		1.8 −27.0	−28.2 −52.1	− 4.6 −26.0
2			11.0 − 7.1	− 6.5 13.9
3				−16.8 −26.6

$\alpha\beta bi$

0	1
0	7.3
	8.1 −10.4

*As coefficient of $R_1^1 \sin\lambda$ in the series expansion of $\alpha\beta bi$, insert the value-45.9 instead of -10.4.