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VOLUME 1

THERMAL ANALYSIS

VUILLEUMIER PROGRAM ENGINEERING NOTEBOOK

72-8416-1

August 8, 1972

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Prepared under Contract No. NAS 5-21096

for

National Aeronautics and Space Administration
Goddard Space Flight Center
Greenbelt, Maryland

GARRETT

AIRESEARCH MANUFACTURING COMPANY
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FOREWORD

Under NASA Contract NAS 5-21096, the AiResearch Manufacturing Company, a Division of The Garrett Corporation, developed a 75°K Vuilleumier (VM) cryogenic refrigerator for the NASA Goddard Space Flight Center (GSFC), Greenbelt, Maryland. During the program, thermal analysis and stress analysis notebooks were compiled for submittal to GSFC. This two-volume document contains (or references) all material compiled during the course of the thermal and stress analyses. In certain instances, copyrighted reference material was used during the analytical work and will not be reproduced in this document.

Volume 1, identified as AiResearch document 72-8416-1, presents the detailed thermal analyses that was conducted during the program.

Volume 2, identified as AiResearch document 72-8416-2, presents the detailed stress analysis that was conducted during the program on various component parts/assemblies of the VM refrigerator.

Volume 3, identified as AiResearch document 72-8416-3, presents test data, and computer program listing, computer data printouts, and calibration data employed to analyze labyrinth seals developed during the program.

CONTENTS

CONTENTS (Continued)

CONTENTS (Continued)

CONTENTS (Continued)

SECTION I

INTRODUCTION

The thermal design of a VM refrigerator is a lengthy iterative process. Initially, rough cut design calculations are made in order to establish the design approach and basic sizes of the machine's elements. After the basic design is defined, effort must be concentrated on matching the thermodynamic design with that of the heat transfer devices (heat exchangers and regenerators). Typically the configurations and volumes of the heat transfer devices are adjusted to improve their heat transfer and pressure drop characteristics. These adjustments imply that changes be made to the active displaced volumes to compensate for the influence of the heat transfer devices on the thermodynamic processes of the working fluid. In turn, once the active volumes are changed, the heat transfer devices require adjustment to account for the variations in flows, pressure levels, and heat loads. This iterative process is carried out until the thermodynamic cycle parameters match the designs of the heat transfer devices. By examining several matched designs, a near-optimum refrigerator can be selected.

In this program emphasis has not been placed on complete optimization of the refrigerator with respect to thermal performance; long operational life and reliability were considered more important. It is believed, however, that due to the care taken in the detail thermal design, a near-optimum thermal design has resulted.

A preliminary design of the refrigerator was presented in the Task I Report (Ref 1). Since publication of the Task I Report, several changes that influence thermal performance have been incorporated into the design. Most notable is an increase in the hot displaced volume to provide a greater margin in refrigeration capacity and ensure satisfaction of the required cooling of 5 w at the end of two years of operation. Additionally, refinements have been incorporated in each heat transfer device and considerable attention paid to providing uniform flow distribution in the cold end of the machine.

Detail analyses leading to the final refrigerator design configuration and performance are contained in this Volume I of the Engineering Notebook. This Thermal Analysis volume summarizes the design analyses and presents engineering notes and calculations generated during the course of this work. Topics covered are:

System description

Cycle parameters and performance

Cold-end heat exchanger

Cold-end flow distributor

Cold-end regenerator

Design of cold-end seal

Ambient sump heat exchanger

Flow distribution and pressure losses in sump region

Hot-end regenerator

Hot-end heat exchanger

Hot-end seal leakage

Hot-end insulation loss and heater temperature

Conduction losses

Sump cooling interface

Drive motor power requirements

SECTION 2

SYSTEM DESCRIPTION

INTRODUCTION

A brief description of the GSFC VM refrigerator is given here as an aid in understanding the analyses given in subsequent sections of this document.

BASIC CONFIGURATION

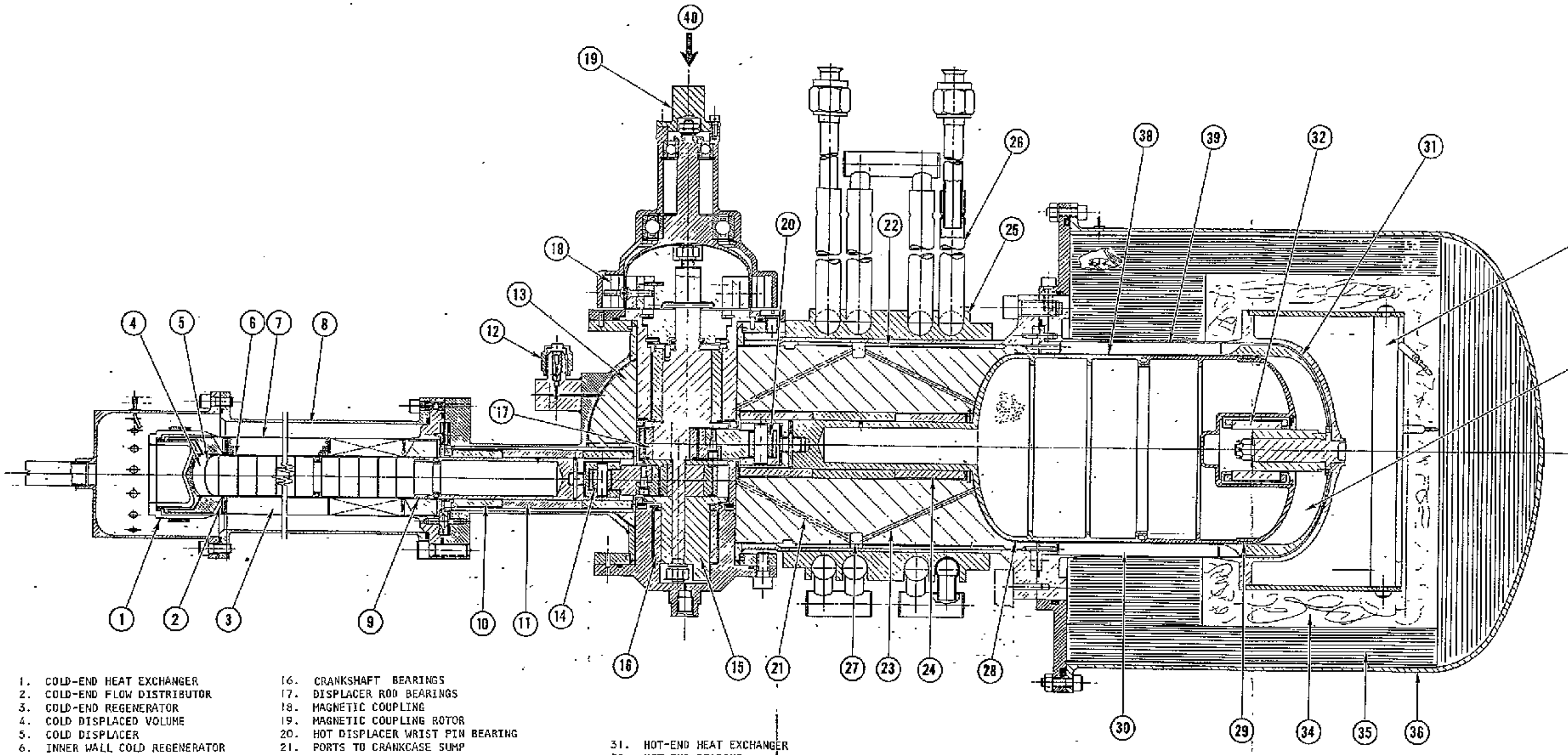
The refrigerator is a gas-cycle reciprocating machine with a nominal design speed of 400 rpm and a continuous-operation design life of five years. A cross sectional drawing of the GSFC VM refrigerator is shown in Figure 2-1.

Moving components within the refrigerator consist of a crankshaft assembly which drives two displacers (one hot, one cold) through connecting rods attached to the crankshaft. The crankshaft is driven by a magnetic coupling at one end of the shaft, with rotary motion provided by a simple laboratory motor (motor is not shown in Figure 2-1). Crankshaft throws are 90 degrees apart with the hot displacer leading. The displacers travel inside cylinders surrounded by packed-bed regenerators which are enclosed in pressure shells joined to the crankshaft housing. Thus, the entire assembly forms a pressure-tight enclosure with helium as the working fluid.

The design is an in-line configuration, with the cold displacer positioned at an angle of 180^0 from the hot displacer. This configuration was selected to simplify the mechanical design of the crankcase and sump heat exchanger, and to minimize the interaction between the hottest and coldest parts of the system.

The interface at the hot end transfers energy by radiation to the refrigerator. Figure 2-1 shows this interface as an electrical heater. The heater simulates a high temperature heat pipe which would transfer energy to the hot end of the machine from a spacecraft heat source (in the intended application). The refrigerator rejects heat to water cooling coils which interface with the crankcase and sump heat exchanger. These cooling coils replace ammonia heat pipes which would be used in a flight type system. The refrigeration load is absorbed at the cold end via the cold-end heat exchanger (Figure 2-1). For test purposes, the refrigeration load is generated by a small resistance-type heater bonded to the exterior surface of the cold end heat exchanger. In a spacecraft system, the cold-end heat exchanger would interface with a cryogenic heat pipe, providing the thermal link to the device being cooled.

A primary design feature is the absence of any organic material within the unit. Organic materials were avoided due to the outgassing characteristics of these materials, and to the potential contamination of the working fluid during extended operating periods. An additional design feature is the use of dynamic (non-contacting) seals in the hot end of the machine. Non-contacting seals are also used in the cold end. Additional sealing is provided by the



1. COLD-END HEAT EXCHANGER
2. COLD-END FLOW DISTRIBUTOR
3. COLD-END REGENERATOR
4. COLD DISPLACED VOLUME
5. COLD DISPLACER
6. INNER WALL COLD REGENERATOR
7. OUTER WALL COLD REGENERATOR
8. DEWAR OUTER WALL
9. COLD-END SEAL
10. COLD-END LINEAR BEARINGS
11. BEARING SUPPORT
12. FILL VALVE
13. SUMP FILLER BLOCK
14. COLD DISPLACER WRIST PIN BEARING
15. CRANKSHAFT ASSEMBLY
16. CRANKSHAFT BEARINGS
17. DISPLACER ROD BEARINGS
18. MAGNETIC COUPLING
19. MAGNETIC COUPLING ROTOR
20. HOT DISPLACER WRIST PIN BEARING
21. PORTS TO CRANKCASE SUMP
22. SUMP HEAT EXCHANGER
23. PORTS TO BACKSIDE OF HOT DISPLACER
24. HOT END LINEAR BEARINGS
25. COPPER COOLING COLLAR
26. WATER COOLING TUBES
27. SUMP HEAT EXCHANGER FLOW DISTRIBUTION SLOT
28. HOT DISPLACER
29. HOT-END SEAL
30. HOT-END REGENERATOR
31. HOT-END HEAT EXCHANGER
32. HOT-END BEARING
33. HOT-END HEATER
34. MIN-K INSULATION
35. FIBERGLASS INSULATION
36. HOT-END INSULATION VACUUM JACKET
37. HOT DISPLACED VOLUME
38. INNER WALL HOT REGENERATOR
39. OUTER WALL HOT REGENERATOR
40. POINT OF LABORATORY MOTOR SHAFT INPUT POWER

L- 73758

Figure 2-1. GSFC 5-Watt, 75 °K, Vuilleumier Cryogenic Refrigerator

2-2 A

contacting seals which are directly in the leakage path. Since these seals have a very low wear rate, they provide a backup to the non-contacting seals.

The thermal analysis given in the remainder of this notebook is primarily concerned with the design and performance of: (1) the heat transfer devices of the machine, (2) internal seals for both the hot and cold displacers, and (3) internal fluid passages. The heat transfer devices consist of the three heat exchangers (hot end, cold end, and sump) and the two regenerators (hot and cold) shown in Figure 2-1. The location of the internal seals as well as the majority of the internal flow passages is also shown in Figure 2-1. Detailed descriptions of elements are included with the thermal analysis.

PERFORMANCE AND DESIGN SUMMARY

A summary of major performance and design parameters for the GSFC VM refrigerator is given in Table 2-1.

TABLE 2-1

GSFC VUILLEUMIER REFRIGERATOR
PERFORMANCE AND DESIGN SUMMARY NOMINAL VALUES

WORKING FLUID	Helium
TEMPERATURES	
Interfaces	
Cold End Interface	130°R (72.2°K)
Sump Interface	600°R (333.3°K)
Hot End Interface	1660°R (922.2°K)
Gas	
Cold End	125°R (69.4°K)
Sump	620°R (344.4°K)
Hot End	1630°R (905.6°K)
PRESSURES	
Charge Gas Pressure at 535°R	550 psia (37.44 atm)
Maximum Cycle Pressure	800 psia (54.46 atm)
Minimum Cycle Pressure	682 psia (46.43 atm)
THERMAL INPUT/OUTPUT	
Net Cold End Refrigeration	23.87 Btu/hr (7.0 w)
Cold End Insulation Loss	0.375 Btu/hr (0.11 w)
Total Hot End Input Power	1011.0 Btu/hr (296.5 w)
Hot End Insulation Loss	78.43 Btu/hr (23 w)
Hot End Input Less Insulation Loss	932.6 Btu/hr (273.5 w)
Motor Input Power	32 w
Heat Rejection Rate	1041.8 Btu/hr (305.5 w)
DRIVE MOTOR POWER INPUT	
Speed	400 rpm
Shaft Power	16 w
Electrical Input Power	32 w max.
COLD DISPLACER SPECIFICATIONS	
Length	5.0 in. (12.7 cm)
Bore	0.840 in. (2.134 cm)
Stroke	0.45 in. (1.14 cm)
Displaced Volume	0.2511 in.^{3} (4.115 cm^3)
HOT DISPLACER SPECIFICATIONS	
Length	6.08 in. (15.44 cm)
Bore	3.86 in. (9.8 cm)
Stroke	0.60 in. (1.52 cm)
Displaced Volume	6.841 in.^{3} (112.1 cm^3)
COLD REGENERATOR SPECIFICATIONS	
NOTE: Cold regenerator consists of 2 sections: (1) Sump end with screens and (2) Cold end with spheres.	
Sump End	
Configuration	Annular
Matrix Material	Monel screen, 150 mesh
Inside Diameter	0.874 in. (2.22 cm)
Outside Diameter	1.554 in. (3.95 cm)
Frontal Area	1.297 in.^{2} (8.37 cm^2)
Length	1.87 in. (4.75 cm)
Matrix Hydraulic Diameter	0.00791 in. (0.0201 cm)
Matrix Surface/Volume Ratio	367 $\frac{in.^2}{in.^3}$ (144.5 $\frac{cm^2}{cm^3}$)
Void Fraction	0.725
Cold End	
Configuration	Annular
Matrix Material	Monel spheres. 0.0075 in. dia (0.01905 cm)
Inside Diameter	0.874 in. (2.22 cm)
Outside Diameter	1.554 in. (3.95 cm)
Frontal Area	1.297 in.^{2} (8.37 cm^2)
Length	2.53 in. (6.43 cm)
Matrix Hydraulic Diameter	0.003197 in. (0.00812 cm)
Matrix Surface/Volume Ratio	488.5 $\frac{in.^2}{in.^3}$ (192.3 $\frac{cm^2}{cm^3}$)
Void Fraction	0.39
HOT REGENERATOR SPECIFICATIONS	
Configuration	Annular
Matrix Material	Stainless Steel, 100 mesh
Inside Diameter	3.85 in. (9.78 cm)
HOT REGENERATOR SPECIFICATIONS--Continued	
Outside Diameter	4.37 in. (11.10 cm)
Frontal Area	3.18 in.^{2} (20.52 cm^2)
Length	3.3 in. (8.38 cm)
Matrix Hydraulic Diameter	0.01132 in. (0.0287 cm)
Matrix Surface/Volume Ratio	256 $\frac{in.^2}{in.^3}$ (100.7 $\frac{cm^2}{cm^3}$)
Void Fraction	.725
DEAD VOLUMES	
Cold End at 125°R	0.16438 in.^{3} (2.694 cm^3)
Sump at 620°R	8.804 in.^{3} (144.27 cm^3)
Hot end at 1630°R	2.050 in.^{3} (33.59 cm^3)
Cold Regenerator at 372°R	3.112 in.^{3} (51.00 cm^3)
Hot Regenerator at 1125°R	8.140 in.^{3} (133.39 cm^3)
HEAT TRANSFER CHARACTERISTICS	
Cold Heat Exchanger	
Maximum Pressure Drop	0.615 lb/in.^{2} (0.0419 atm)
Conductance (ηhA)	15.68 Btu/hr-°R (8.277 w/°K)
Hot Heat Exchanger	
Maximum Pressure Drop	0.155 lb/in.^{2} (0.0106 atm)
Conductance (ηhA)	48.84 Btu/hr-°R (25.78 w/°K)
Sump Heat Exchanger	
Maximum Pressure Drop	0.012 lb/in.^{2} (0.00082 atm)
Conductance (ηhA)	113 Btu/hr-°R (59.65 w/°K)

2-4 A

B

SECTION 3

CYCLE PARAMETERS AND PERFORMANCE

INTRODUCTION

The final refrigerator design has a hot displaced volume of 6.841 cu. in. and a cold displaced volume of 0.2511 cu. in. The hot displaced volume is significantly larger than the 4.09 cu. in. of the preliminary design of the system presented in the Task 1 Report. The increased displaced volume at the hot end provides a greater margin in refrigeration capacity and ensures satisfying the requirement of 5 w of cooling at the end of 2 years of operation. Selection of this design was by mutual agreement between GSFC and AiResearch, after a parametric study following publication of the Task 1 Report (Ref 1).

NOMINAL DESIGN CONDITIONS AND PERFORMANCE

Figure 3-1 is an output sheet from the ideal VM cycle analysis computer program for the final design configuration of the refrigerator operating at its nominal design conditions. Table 3-1 gives the nomenclature for interpretation of the output data. The detail drawings and as-built dimensions provided under Task 3 were used to compute the various input parameters for the computer output presented in Figure 3-1. Figures 3-2, 3-3, and 3-4 represent plots of the refrigerator internal volumes, pressure, and mass flow rates as functions of the crank angle position. The data for these plots were taken from the computer output given as Figure 3-1.

The gas temperatures in each hot, sump, and cold zone of the refrigerator have been computed based on the performance of the heat exchangers in these zones and the associated heat loads. The average gas temperatures are:

Cold end volume = 125°R

Sump volume = 620°R

Hot volume = 1630°R

The film temperature drop in the cold end heat exchanger is 1.5°R and the wall temperature drop is an additional 3.0°R. The specified refrigeration temperature at the cold head (external surface of the cold end heat exchanger) is 135°R (75°K), thus a margin of 5.5°R exists; that is, the actual surface temperature at the design conditions would be 129.5°R. This temperature at the cold end of the machine provides for a 5.5°R drop across the interface with the cryogenic heat pipe before the 135°R (75°K) temperature level is reached. Since the calculated temperature drop across this interface is 4°R, a slight margin in performance is provided. These figures are based on a net refrigeration load of 7 w as opposed to the specified 5-w capacity. Early in the program, by mutual agreement between GSFC and AiResearch, the 7-w capacity design value was selected to allow 2-w degradation over 2 years of operation.

The film temperature drop in the sump heat exchanger is approximately 10°R; this allows an additional 10°R temperature drop across the sump pressure vessel wall and ambient heat pipe interface clamp assembly for the specified 600°R (140°F) sump temperature. In a spacecraft application, this arrangement would correspond to an effective radiator temperature of approximately 580°R with a radiator area requirement of 12 sq ft for rejection of 300 w.

GSFC VM FINAL DESIGN PARAMETERS 3 14 72 DATE = 14 MAR 72 TIME = 13:30:17

OPERATING PARAMETERS

```
COLD VOLUME TEMP.      =    125.00 R
SUMP VOLUME TEMP.      =    620.00 R
HOT VOLUME TEMP.       =   1630.00 R
COLD REGEN. TEMP.      =    372.00 R
HOT REGEN. TEMP.       =   1125.00 R
COLD DISPLACED VOL.    =    .25110 CU-IN
HOT DISPLACED VOL.     =   6.84100 CU-IN
COLD DEAD VOL.         =    .16438 CU-IN
SUMP DEAD VOL.         =   8.80395 CU-IN
HOT DEAD VOL.          =   2.05000 CU-IN
COLD REGEN. VOL.       =   3.11210 CU-IN
HOT REGEN. VOL.        =   8.14030 CU-IN
GAS CONSTANT           =   4634.40 IN-LB/LBM-R
SPEED                  =    400.00 RPM

CHARGE PRESSURE        =    550.00 PSIA
CHARGE TEMPERATURE     =    535.00 R
MASS OF FLUID          =     .0063 LBM
TOTAL VOLUME           =  29.36283 CU-IN
```

PRESSURE - MASS - FLOW PROFILE

ANGLE DEG	PC PSIA	PA PSIA	PH PSIA	VC CU-IN	VA CU-IN	VH CU-IN	MDOTC LB/SEC	MDOTA LB/SEC	MDOTH LB/SEC	MDOTRCA LB/SEC	MDOTRHA LB/SEC
24.	777.83	777.83	777.83	.1752	11.0737	6.8615	.00319	-.02614	.01545	.00716	.01899
48.	797.09	797.09	797.09	.2059	9.8925	8.0121	.00533	-.02187	.01168	.00790	.01397
72.	806.13	806.13	806.13	.2511	9.1360	8.7233	.00643	-.01264	.00517	.00698	.00566
96.	802.67	802.67	802.67	.3030	8.9350	8.8724	.00615	-.00055	-.00254	.00453	-.00398
120.	787.61	787.61	787.61	.3527	9.3244	8.4334	.00458	.01126	-.00946	.00120	-.01246
144.	764.61	764.61	764.61	.3915	10.2366	7.4823	.00221	.02004	-.01403	-.00214	-.01790
168.	738.65	738.65	738.65	.4127	11.5142	6.1835	-.00035	.02439	-.01565	-.00479	-.01960
192.	714.49	714.49	714.49	.4128	12.9362	4.7615	-.00260	.02431	-.01454	-.00639	-.01792
216.	695.75	695.75	695.75	.3916	14.2569	3.4619	-.00427	.02060	-.01134	-.00691	-.01369
240.	684.75	684.75	684.75	.3528	15.2480	2.5096	-.00525	.01423	-.00676	-.00642	-.00781
264.	682.65	682.65	682.65	.3032	15.7382	2.0690	-.00549	.00614	-.00140	-.00509	-.00105
288.	689.66	689.66	689.66	.2512	15.6428	2.2164	-.00498	-.00287	.00421	-.00305	.00592
312.	705.06	705.06	705.06	.2060	14.9782	2.9262	-.00374	-.01190	.00948	-.00048	.01238
336.	727.11	727.11	727.11	.1753	13.8593	4.0758	-.00182	-.01981	.01370	.00237	.01744
360.	752.78	752.78	752.78	.1644	12.4796	5.4664	.00061	-.02510	.01599	.00511	.01999

IDEAL REFRIGERATION AND HEAT INPUT

```
REFRIGERATION =    17.9516 WATTS
THERMAL HEAT  =   108.5229 WATTS
MAX. PRESSURE =   806.4589 PSIA
```

Figure 3-1. Ideal VM Cycle Computer Program Output for Nominal Design Conditions

TABLE 3-1

IDEAL VUILLEUMIER CYCLE ANALYSIS
NOMENCLATURE KEY

Symbol	Definition
PC	Pressure in cold displaced volume
PA	Pressure in ambient volume
PH	Pressure in hot displaced volume
VC	Cold displaced volume
VA	Ambient displaced volume
VH	Hot displaced volume
MDOTC	Flow rate into cold volume
MDOTA	Flow rate into ambient volume
MDOTH	Flow rate into hot volume
MDOTRCA	Flow rate into cold regenerator at the end toward the sump
MDOTRHA	Flow rate into hot regenerator at end toward the sump

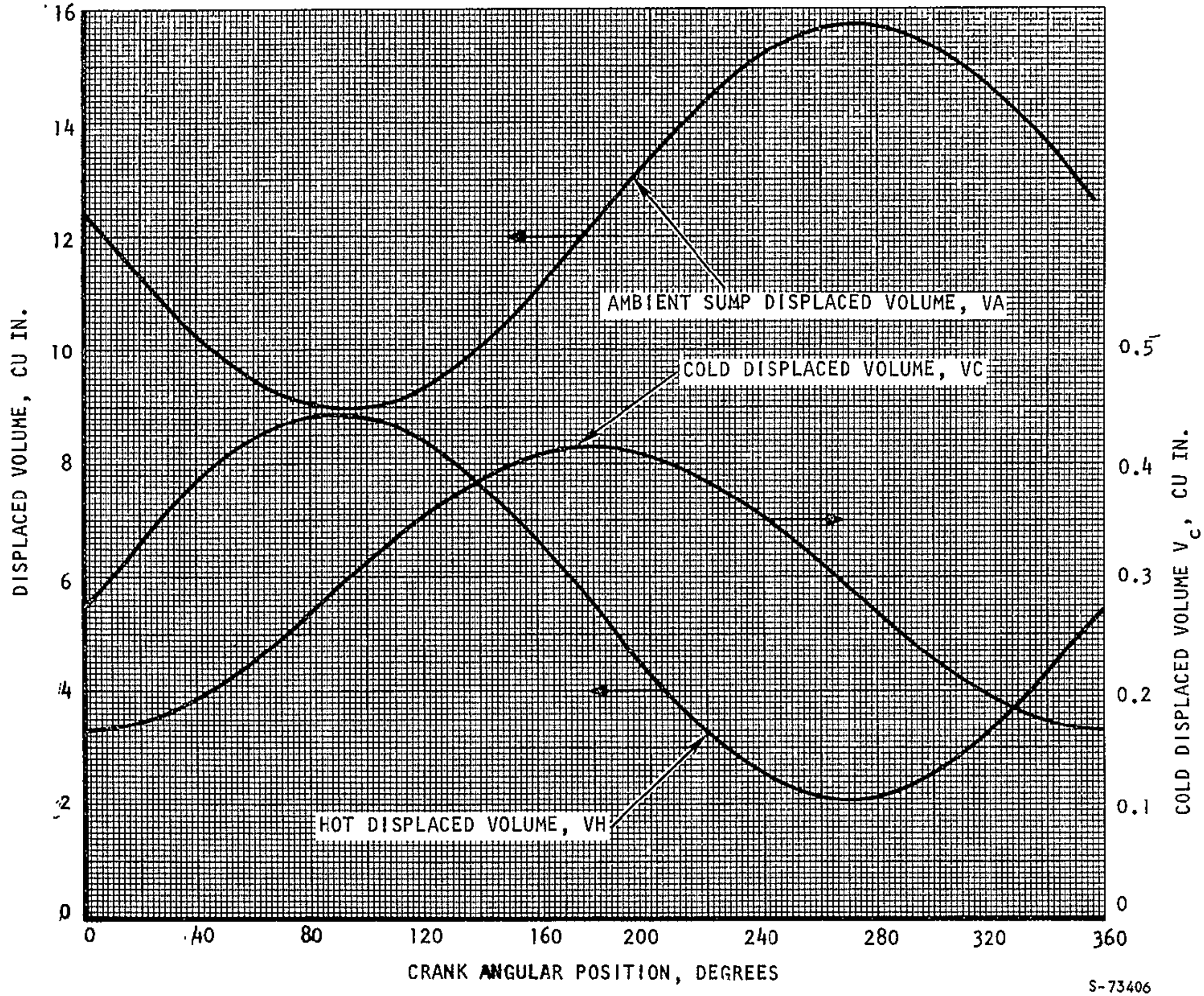


Figure 3-2. Displaced Volumes within VM Refrigerator

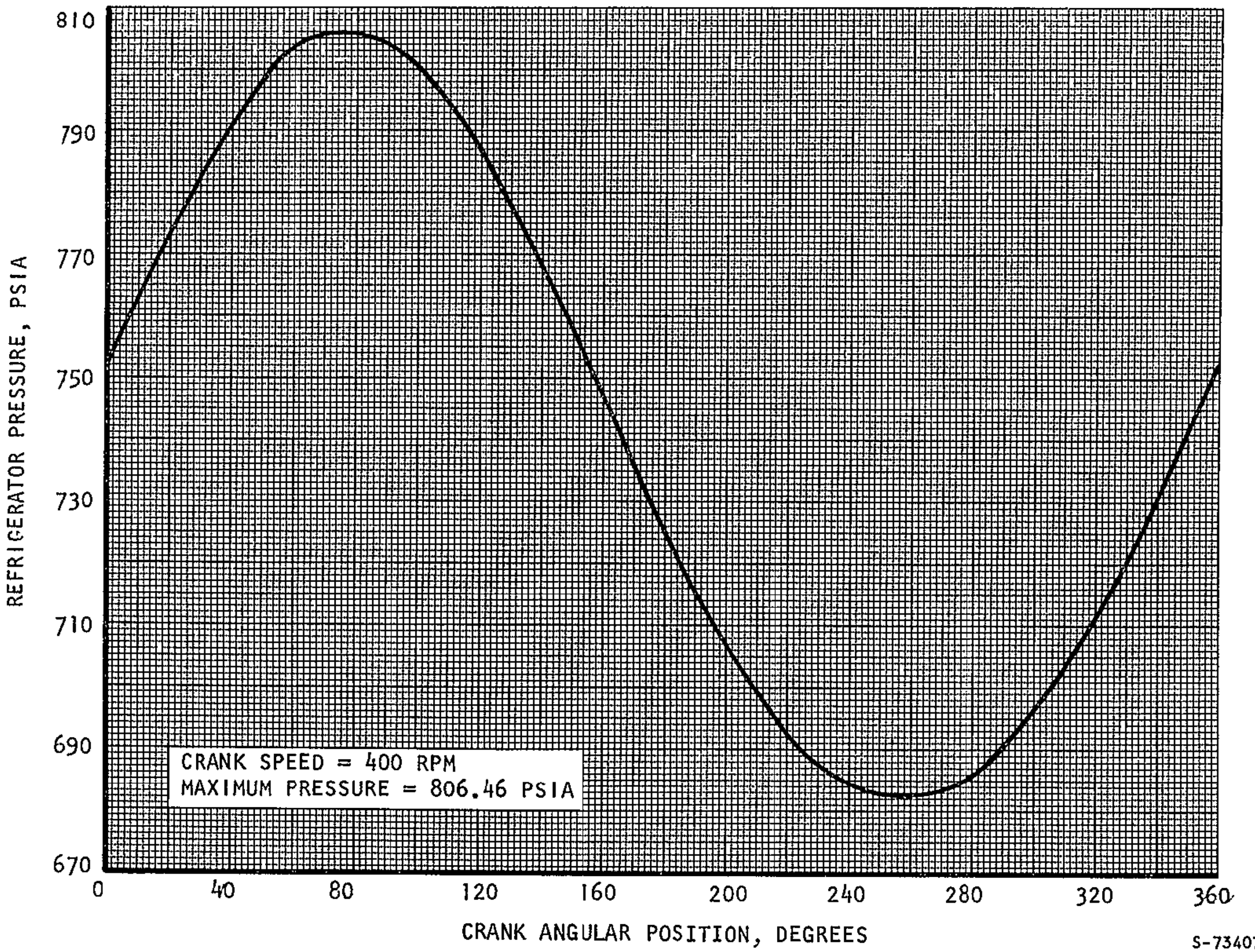


Figure 3-3. Refrigerator Pressure Characteristic at Design Crank Speed Condition

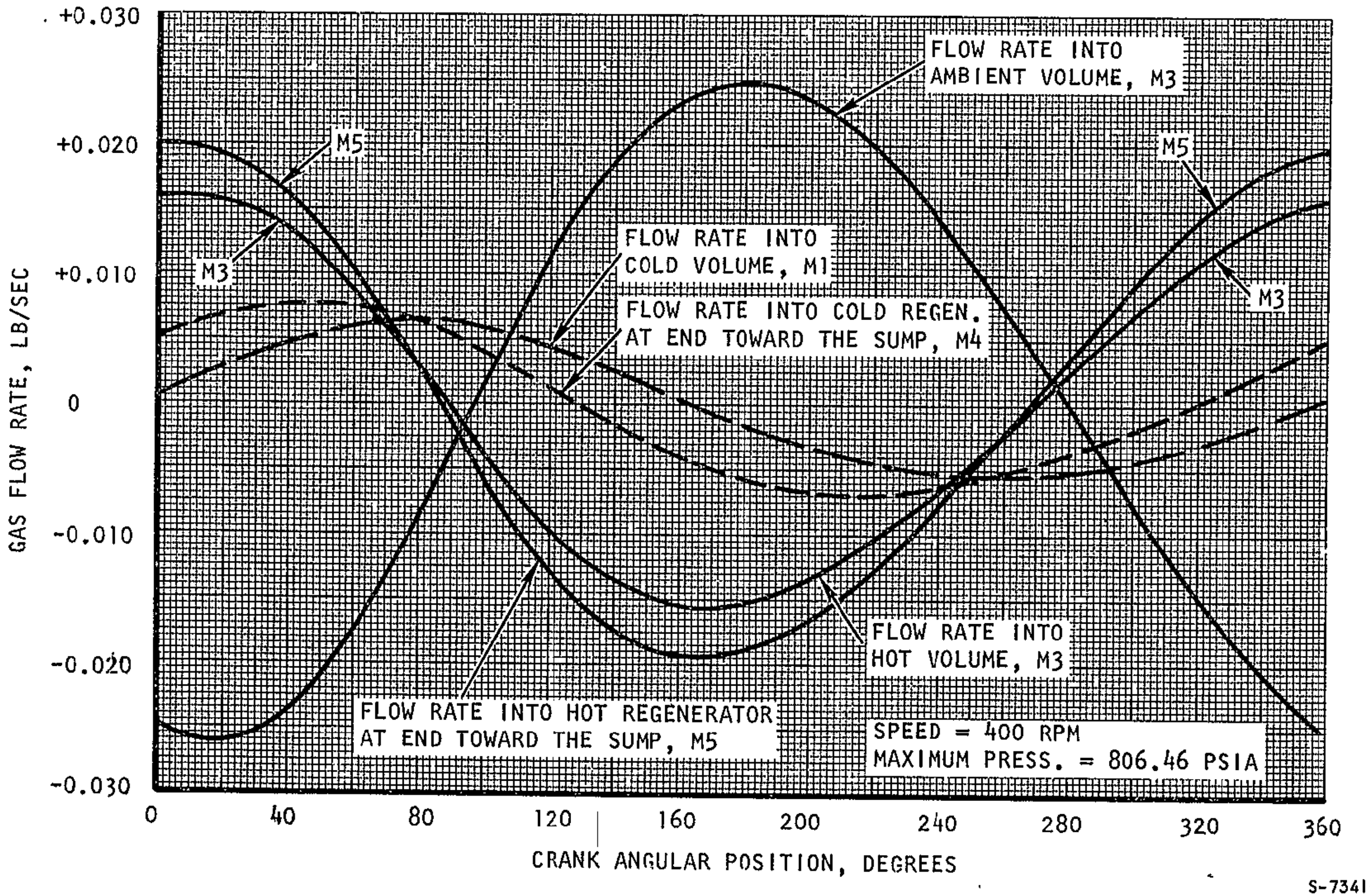


Figure 3-4. Refrigerator Gas Flow Rates at Design Crank Speed Condition

The hot end heat exchanger has a film temperature drop of approximately 20°R and a wall temperature drop of 10°R; the gas temperature of 1630°R thus results in an outer wall temperature of 1660°R (1200°F) at the nominal design conditions. The hot end heat exchanger receives its heat input via radiation from a heater that simulates a liquid metal heat pipe coupled with a radioisotope. The heater operates at approximately 1980°R (1520°F) while supplying 300 w of thermal power to the system.

The ideal refrigeration capacity of the system is 17.95 w, as given in Figure 3-1. Table 3-2 summarizes the thermal losses that are directly amenable to analysis in the cold end of the machine. Other factors contributing to the losses consist of mismatched temperature gradients along the displacer and cylinder walls, pressure drops, and nonuniform flow distribution; these losses are estimated at about 1 w.

Table 3-2 includes the estimated losses for a machine that has been operating for two years. The factor causing the indicated degradation (a loss of approximately 2 w of cooling) is increased leakage of the working fluid past the linear bearings that support the cold displacer. These bearings act as backup seals to the cold end labyrinth seals and greatly reduce the cold end leakage and associated thermal losses. The estimated degradation is believed very conservative; actual degradation is expected to be less since the worst cases of both bearing wear and pressure drop, which promote the leakage, were used in its calculation.

Subtracting the losses given in Table 3-2 from the ideal refrigeration yields net refrigeration capacities of 7.95 w and 5.95 w for the machine new and after two years of wear, respectively. These figures show a design margin of slightly less than one watt at the end of 2 years of operation.

The ideal thermal power input is 108.5 w as given in Figure 3-1. The hot end losses are summarized in Table 3-3. The total thermal power required (296.5 w) is the sum of the ideal power and the losses. This level of input power is considerably below the 370 w originally budgeted for the refrigerator; thus a design margin with respect to thermal power input is provided.

GROWTH POTENTIAL OF BASIC DESIGN

To retain flexibility in the refrigerator design and ensure that the required cooling capacity would be achieved if losses were higher than anticipated, growth potential with respect to cooling capacity has been designed into the unit. The basic design of the machine fixes the configuration of the heat transfer devices, the dead volumes, and the displaced volumes; the design requirements establish the effective gas temperatures at the sump and cold end within narrow limits. The remaining parameters that can be varied to change the performance consist of: (1) hot end temperature, (2) operating pressure, and (3) machine rotational speed. It will be shown later that increasing any of these design parameters increases the ideal cooling capacity of the refrigerator.

TABLE 3-2

COLD END LOSSES IN WATTS

Parameter	New Machine	2 Years of Operation
Conduction		
Displacer		
Walls	0.560	0.560
Packing	0.026	0.026
Regenerator		
Walls	2.642	2.642
Matrix	1.886	1.886
Dewar	0.111	0.111
Regenerator	3.90	3.90
Leakage	0.1	2.30
Other	≈1.0	≈1.0
Total losses	≈10.0	≈12.0

If the hot end temperature, operating pressure, and/or the rotational speed are to be increased, the capability to operate under these new conditions must be incorporated into the design. On the other hand, if the machine structure is designed to accommodate substantial increases in each of these parameters, the nominal design point performance will be sacrified. To establish which of the parameters should be used to provide growth potential or a performance design margin, each must be examined in view of the overall design philosophy and its influence on efficiency.

ROTATIONAL SPEED SELECTION

The achievement of a long life machine was considered the prime objective of this program. Of the above parameters, only rotational speed significantly influences the life; operational life is roughly inversely proportional to the rotational speed. Speed also affects the size and weight of the refrigerator; higher speed machines can be made smaller and lighter since more cooling capacity can be obtained per unit of displaced volume. Size and weight were not specified for the present design, yet the end use of the technology developed

TABLE 3-3

HOT END THERMAL LOSSES IN WATTS

Parameter	Loss*, w
Conduction	
Displacer	
Walls	35.86
Packing	2.49
Regenerator	
Walls	68.18
Matrix	8.00
Insulation	-
Regenerator	29.5
Leakage	1.0
Other Internal Losses	20.0
Insulation	23.0
Total	188.0

*Hot end losses are independent of operational time, since bearings are not used as seals.

is for spacecraft applications. For this reason an operation speed (nominal design value of 400 rpm) that yields a system size compatible with spacecraft applications was selected. This speed results in a life expectancy of well over 2 years and the system envelope and weight, if the design were configured as a flight unit, are not considered excessive.

With the nominal design speed selected, the dynamics of the machine can be designed to allow operation at moderately higher speeds without danger of failure or excessive wear. This can be accomplished with very minor weight and volume penalties associated with oversized bearings and stiffer rotating members. If operated at higher speeds, life is reduced but failures and excessive wear rates can be avoided through design. The final refrigerator design has an upper speed limit of 600 rpm compared to the nominal design value of 400 rpm.

Figure 3-5 gives the ideal refrigeration capacity as a function of rotational speed at various peak cycle pressures. These data were generated with the ideal VM cycle analysis computer program for the final refrigerator design configuration. The ideal cooling capacity is directly proportional to the rotational speed (double the speed, double the refrigeration capacity). Over a limited range of speed the efficiency of the refrigerator is not expected to change greatly; thus, except for the decreased operating life, increasing the rotational speed is an effective method of increasing the refrigerator's cooling capacity. Due, however, to the importance placed on life in the present design, increasing the speed is the least desirable method of increasing the performance.

HOT END TEMPERATURE SELECTION

The influence of hot end gas temperature on the ideal cooling capacity is given in Figure 3-6. Again these data are based on the final design configuration, and all other parameters such as peak cycle pressure and rotation speed are held constant. The increased cooling capacity at higher temperatures indicates the hot end temperature is an effective means of providing growth potential. Two other factors, however, influence the selection of this parameter; these are (1) the temperature dependence of the strength of the material of construction, and (2) the influence of temperature on the efficiency of the machine.

Because of its high strength at elevated temperatures and ease of fabrication into complex configurations, Inconel 718 was selected as the material of construction for the hot end of the machine. The ultimate strength of Inconel 718 as a function of temperature is given in Figure 3-7. The rapid decrease in strength above 1200°F sets this temperature as a practical upper limit in operating temperature. The film temperature drop in the hot end heat exchanger in turn sets an upper limit of hot end gas temperature of approximately 1170°F (1630°R).

Figure 3-8 gives the ideal coefficient of performance (COP) of the final design as a function of hot end gas temperature. The COP improves at higher temperatures; thus selection of the maximum allowable temperature is indicated. For the GSFC 5-w VM refrigerator, the maximum allowable temperature was selected

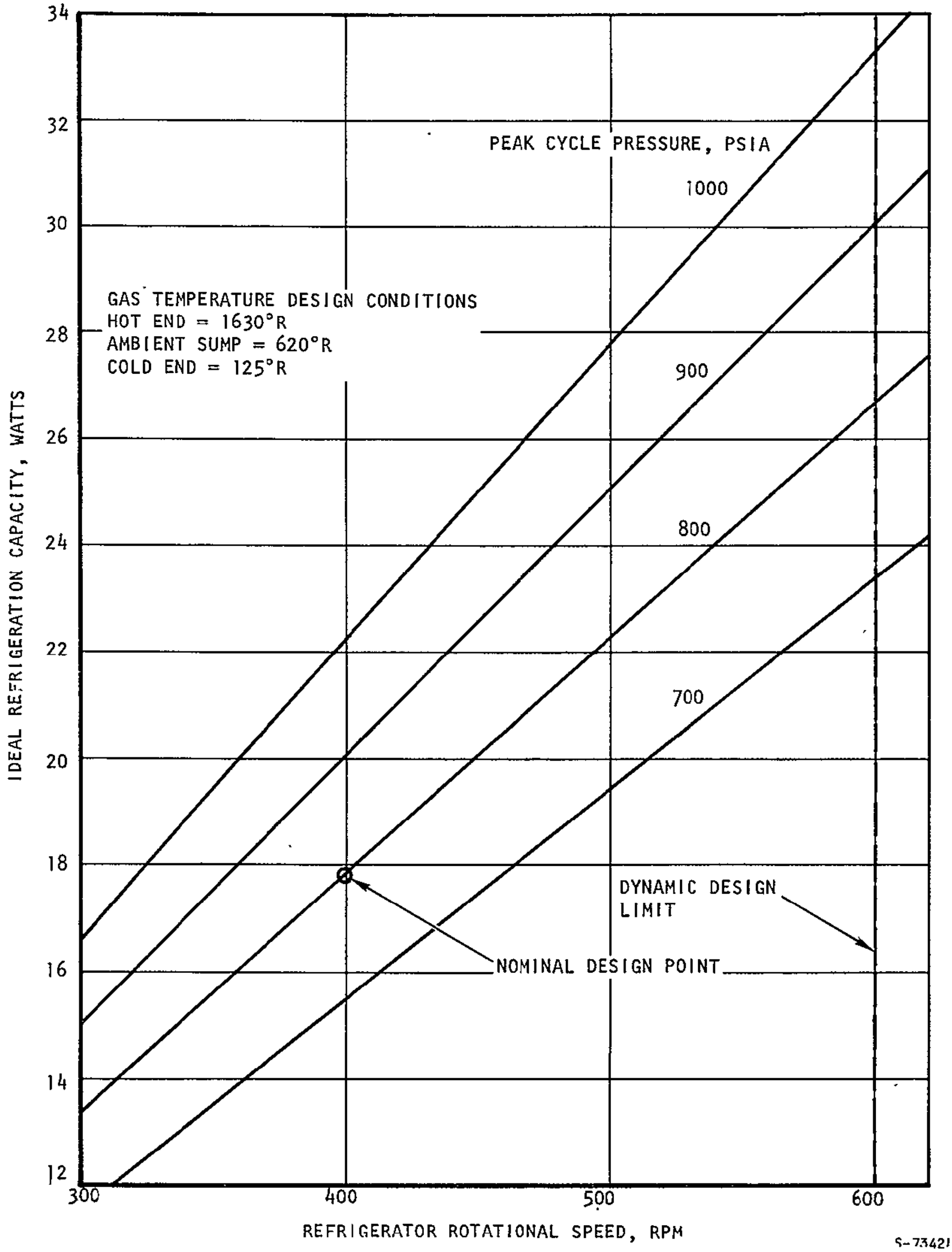


Figure 3-5. Ideal Cooling Capacity as a Function of Speed

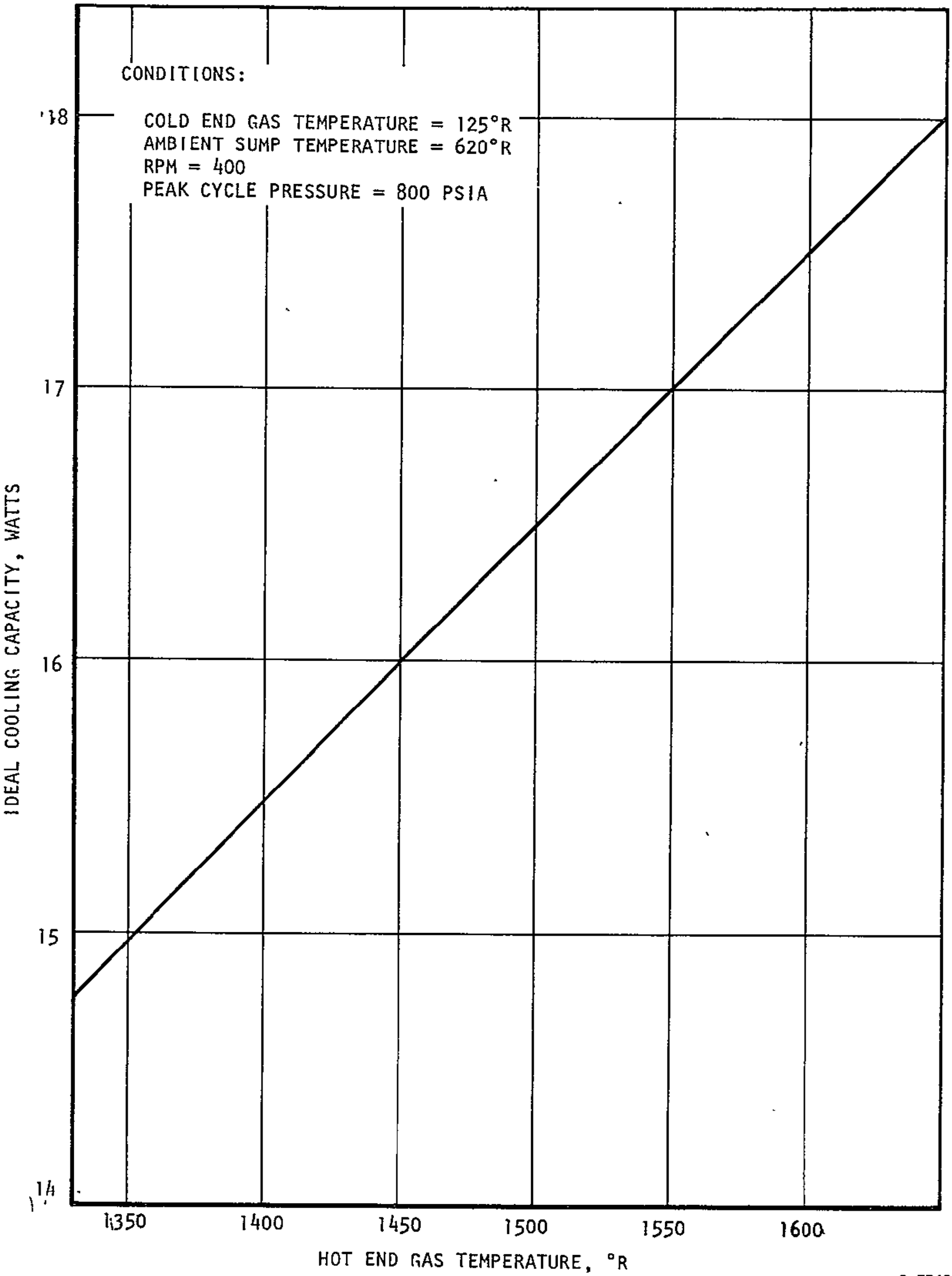


Figure 3-6. Cooling Capacity as a Function of Hot End Gas Temperature

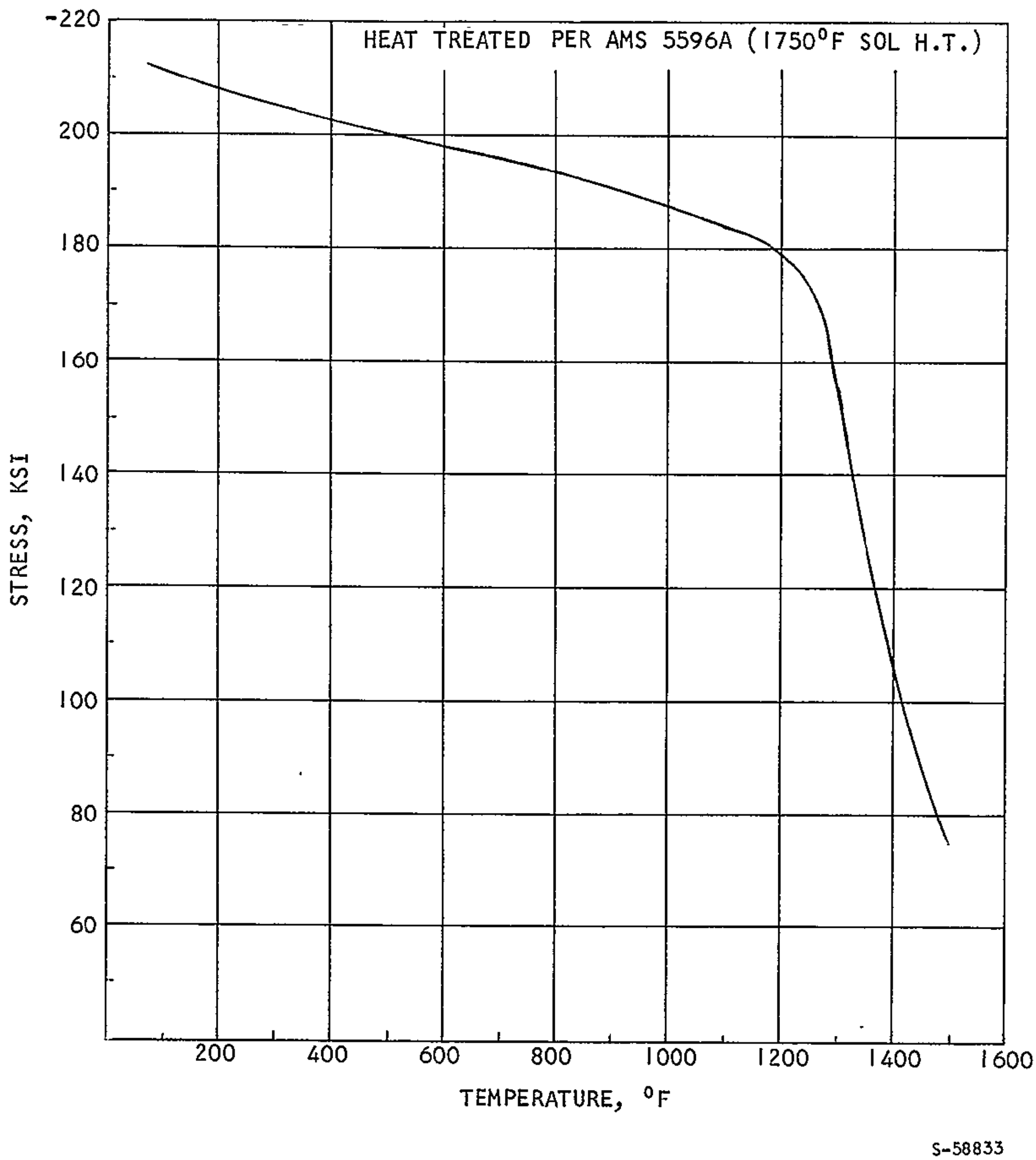


Figure 3-7. Ultimate Strength of Inconel 718

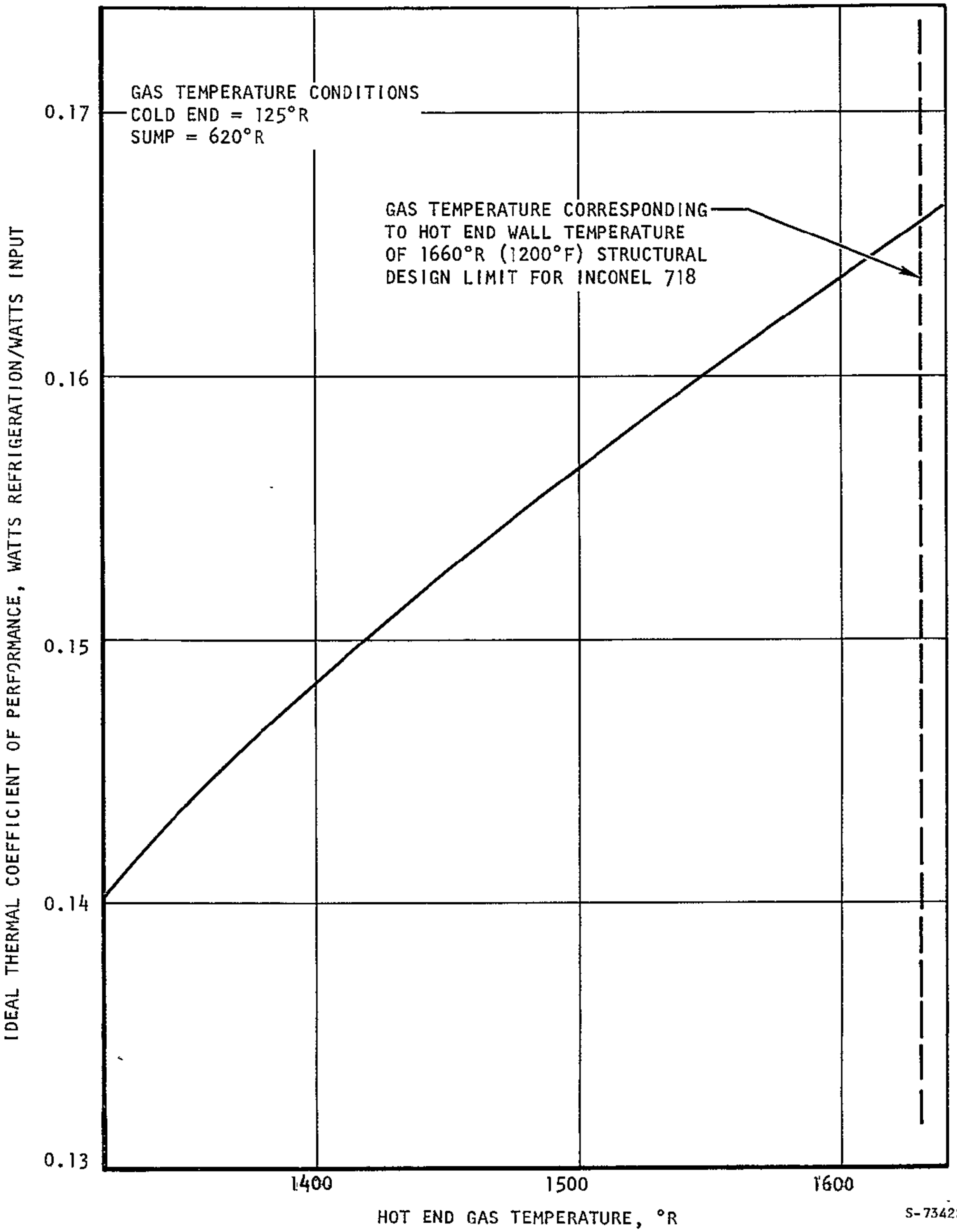


Figure 3-8. Ideal Coefficient of Performance

as a nominal design parameter due to the performance (efficiency) advantage of this selection; no design margin with respect to increasing the cooling capacity is provided by this selection.

PRESSURE INFLUENCE ON COOLING CAPACITY

Peak cycle pressure is the remaining parameter that can be used to increase the cooling capacity. Figure 3-9 gives the ideal refrigeration capacity of the final GSFC 5-w machine design as a function of peak cycle pressure for various rotational speeds up to the dynamic limit of 600 rpm. Figure 3-9 shows that increasing the peak cycle pressure is an effective method of increasing the ideal cooling capacity. For example, at the nominal design speed of 400 rpm, an increase from 800 to 1000 psia increases the ideal cooling capacity by approximately 25 percent.

This method of increasing the capacity or providing growth potential has two major advantages over using the rotational speed and/or hot end temperature: neither operating life nor thermal efficiency is greatly sacrificed for moderate increases. Increased operating pressures result in higher pressure drops in the internal flow passages of the machine; the increased pressure drops are roughly proportional to the increase in pressure (gas density). The higher pressure drops increase bearing loads and in turn somewhat decrease the operating life. The decrease in operating life, however, is substantially below that resulting from an increase in rotational speed to bring about a like increase in cooling capacity. Increasing the speed has a compounding effect due to the increase in flow velocities internal to the machine. The pressure drops go up as the square of these velocities, and combine with the higher bearing surface speeds to decrease the life expectancy at an accelerated rate.

Increasing the pressure does not greatly affect the efficiency of the refrigerator; on an ideal basis where pressure drops are neglected and perfect regenerators are assumed, efficiency of the COP is independent of pressure.

In comparison the hot end temperature has a pronounced influence on the ideal COP (Figure 3-8). If a design margin is to be provided by variation of the hot end temperature, the machine efficiency is degraded at its nominal design operating conditions.

In the real case, increased pressure drops and internal mass flow rates associated with high operating pressures do increase particular losses. The primary influence is on the performance of the heat transfer devices where the heat loads increase more rapidly than the heat transfer capability of the devices. This effect can be reduced by using conservative designs for the heat transfer devices.

In the GSFC 5-w refrigerator design, the maximum operating pressure is 1000 psia (200 psia higher than the nominal design level of 800 psia for the peak cycle pressure). These design levels are indicated in Figure 3-9. This selection of design parameters provides an approximate 25-percent growth potential or design margin in the cooling capacity.

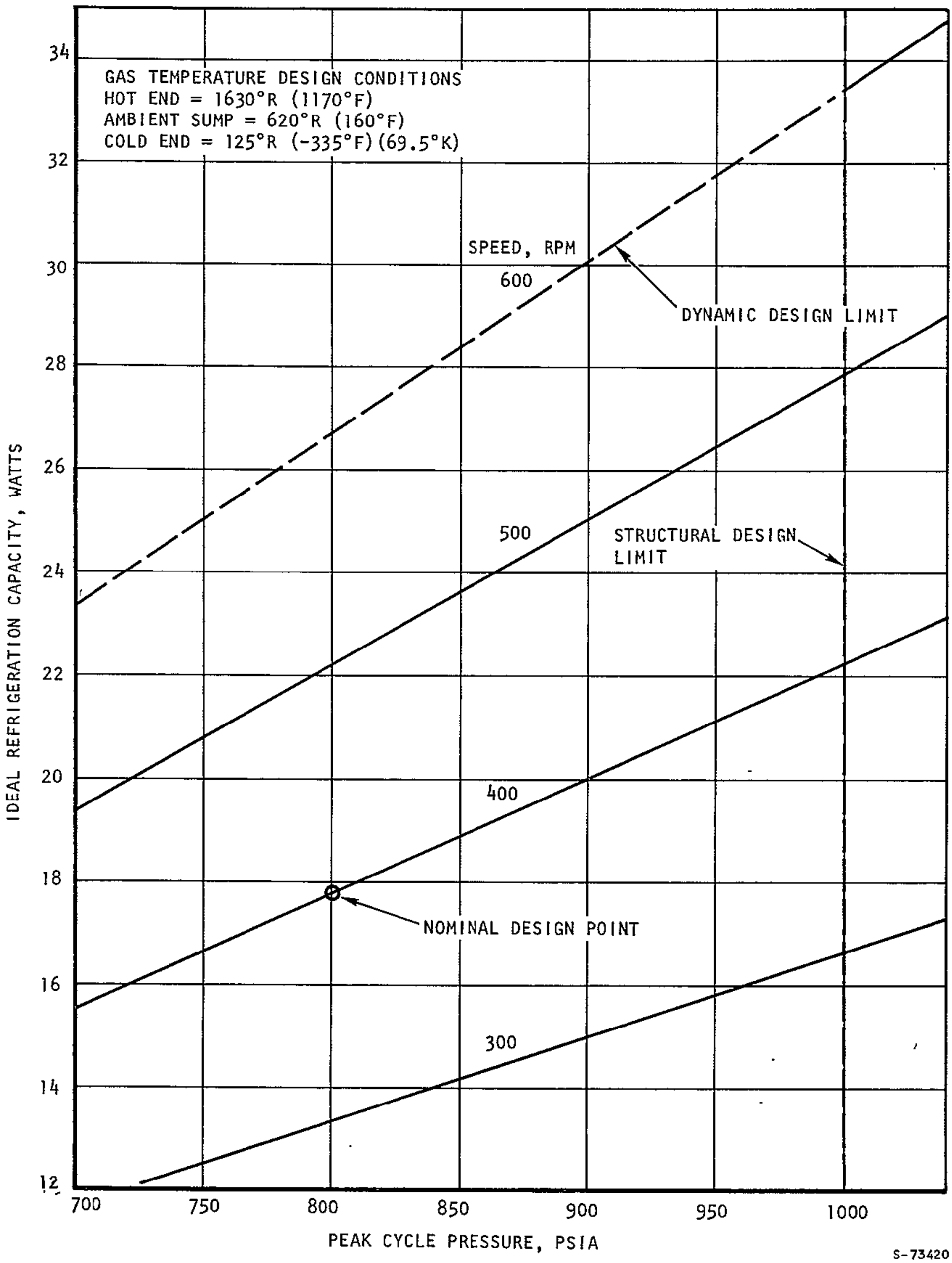


Figure 3-9. Ideal Cooling Capacity vs Peak Cycle Pressure

Figure 3-10 gives the peak cycle pressure as a function of charge pressure at 535°R (ambient). The data of Figure 3-10 are dependent on the operating temperatures given and the final design configuration of the 5-w machine. Over the current range of interest, the relation between peak cycle pressure and charge pressure is independent of rotational speed. The machine should not be charged to a nonoperating pressure in excess of 682 psia at 535°R to avoid exceeding the structural design limit when set into operation.

SUMMARY AND MAXIMUM PERFORMANCE PARAMETERS

Table 3-4 summarizes the selections of the previous discussion.

TABLE 3-4

SUMMARY OF MAJOR DESIGN PARAMETER SELECTION

Parameter	Selected Design Level	Comments
Machine rotational speed	Nominal design of 400 rpm; dynamic limit of 600 rpm	400 rpm yields a reasonable size system consistent with end application; increased speed to provide additional cooling capacity is available but is a poor or second choice since decreased life results
Hot end temperature	Nominal design set at maximum practical limit of 1200°F wall temperature	Use of lower hot end tempertures degrades efficiency significantly
Peak cycle pressure	Nominal design of 800 psia; maximum structural limit of 1000 psia	Increasing peak cycle pressure to gain cooling capacity is the most effective method; efficiency only slightly affected

Figure 3-11 is an output sheet from the Ideal VM cycle analysis computer program with each of the design parameters discussed above at its maximum level. Comparing this data with that of Figure 3-1 shows the ideal cooling capacity can almost be doubled by running the refrigerator at its maximum design levels. The net cooling under these conditions is estimated at about 12 w at 75°K. Figures 3-12 and 3-13 give the cycle pressure and flow variations for comparison with Figures 3-3 and 3-4, which give the same parameters for the nominal design conditions.

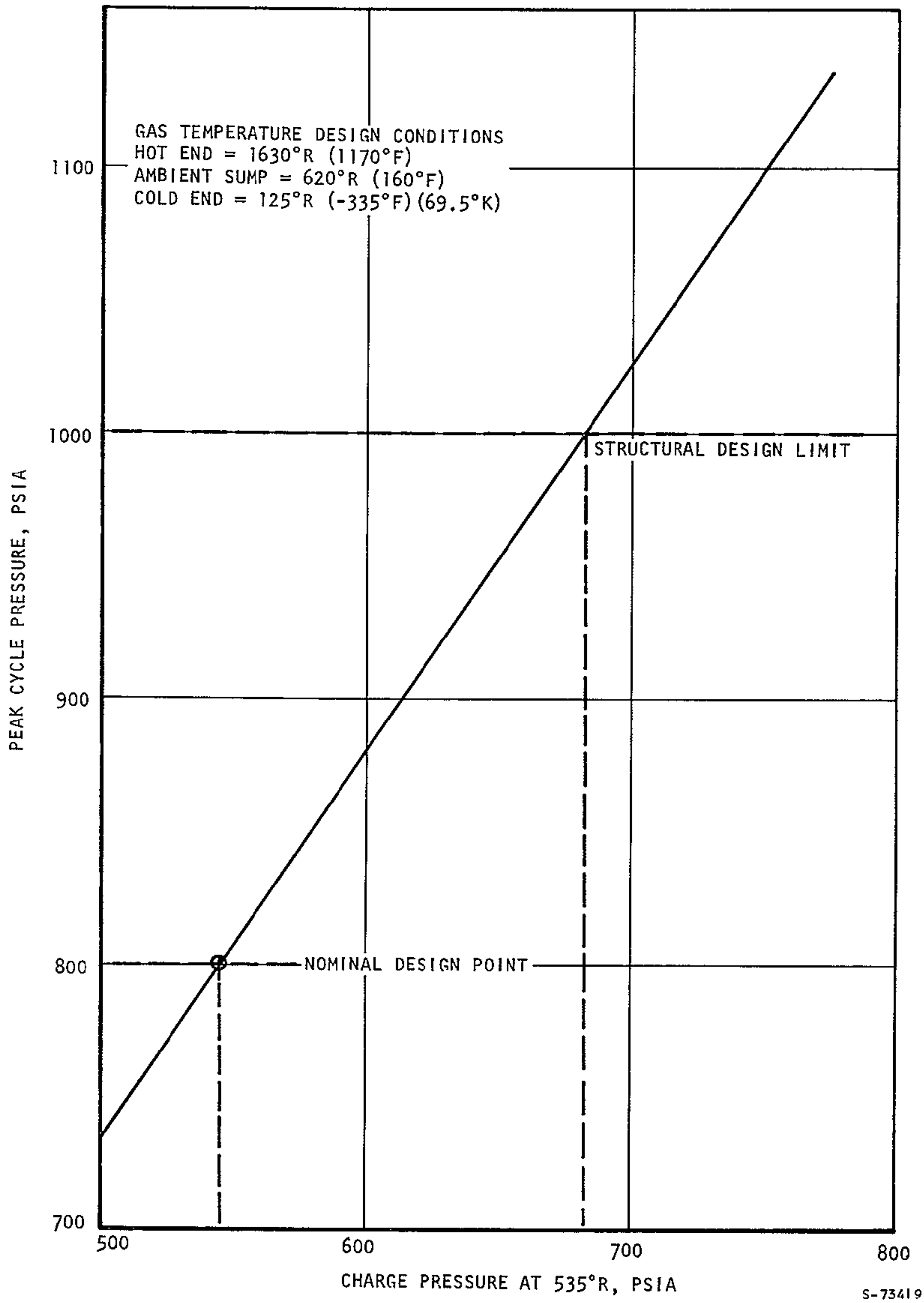


Figure 3-10. Peak Cycle Pressure vs Charge Pressure

GSFC VM FINAL DESIGN PARAMETERS 3-14-72 DATE = 14 MAR 72 TIME = 14:20:07

OPERATING PARAMETERS

COLD VOLUME TEMP.	=	125.00 R
SUMP VOLUME TEMP.	=	620.00 R
HOT VOLUME TEMP.	=	1630.00 R
COLD REGEN. TEMP.	=	372.00 R
HOT REGEN. TEMP.	=	1125.00 R
COLD DISPLACED VOL.	=	.25110 CU-IN
HOT DISPLACED VOL.	=	6.84100 CU-IN
COLD DEAD VOL.	=	.16438 CU-IN
SUMP DEAD VOL.	=	8.80395 CU-IN
HOT DEAD VOL.	=	2.05000 CU-IN
COLD REGEN. VOL.	=	3.11210 CU-IN
HOT REGEN. VOL.	=	8.14030 CU-IN
GAS CONSTANT	=	4634.40 IN-LB/LBM-R
SPEED	=	600.00 RPM
CHARGE PRESSURE	=	680.00 PSIA
CHARGE TEMPERATURE	=	535.00 R
MASS OF FLUID	=	.0078 LBM
TOTAL VOLUME	=	29.36283 CU-IN

PRESSURE - MASS - FLOW PROFILE

ANGLE DEG	PC PSIA	PA PSIA	PH PSIA	VC CU-IN	VA CU-IN	VH CU-IN	MDOTC LB/SEC	MDOTA LB/SEC	MDOTH LB/SEC	MDOTRCA LB/SEC	MDOTRHA LB/SEC
24.	961.69	961.69	961.69	.1752	11.0737	6.8615	.00592	-.04849	.02866	.01328	.03521
48.	985.49	985.49	985.49	.2059	9.8925	8.0121	.00988	-.04057	.02166	.01466	.02591
72.	996.67	996.67	996.67	.2511	9.1360	8.7233	.01193	-.02344	.00959	.01295	.01049
96.	992.40	992.40	992.40	.3030	8.9350	8.8724	.01140	-.00102	-.00470	.00840	-.00738
120.	973.78	973.78	973.78	.3527	9.3244	8.4334	.00849	.02089	-.01754	.00223	-.02311
144.	945.34	945.34	945.34	.3915	10.2366	7.4823	.00409	.03716	-.02602	-.00396	-.03319
168.	913.25	913.25	913.25	.4127	11.5142	6.1835	-.00064	.04523	-.02902	-.00887	-.03635
192.	883.37	883.37	883.37	.4128	12.9362	4.7615	-.00483	.04509	-.02697	-.01186	-.03323
216.	860.20	860.20	860.20	.3916	14.2569	3.4619	-.00793	.03820	-.02104	-.01281	-.02539
240.	846.60	846.60	846.60	.3528	15.2480	2.5096	-.00973	.02640	-.01254	-.01191	-.01448
264.	844.01	844.01	844.01	.3032	15.7382	2.0690	-.01017	.01138	-.00260	-.00944	-.00194
288.	852.67	852.67	852.67	.2512	15.6428	2.2164	-.00924	-.00532	.00780	-.00566	.01099
312.	871.71	871.71	871.71	.2060	14.9782	2.9262	-.00694	-.02208	.01758	-.00089	.02297
336.	898.97	898.97	898.97	.1753	13.8593	4.0758	-.00337	-.03674	.02541	.00440	.03234
360.	930.71	930.71	930.71	.1644	12.4796	5.4664	.00114	-.04655	.02965	.00947	.03707

IDEAL REFRIGERATION AND HEAT INPUT

REFRIGERATION = 33.2920 WATTS
THERMAL HEAT = 201.2608 WATTS
MAX. PRESSURE = 997.0765 PSIA

Figure 3-11. Ideal VM Cycle Program Output for Design Limit Conditions

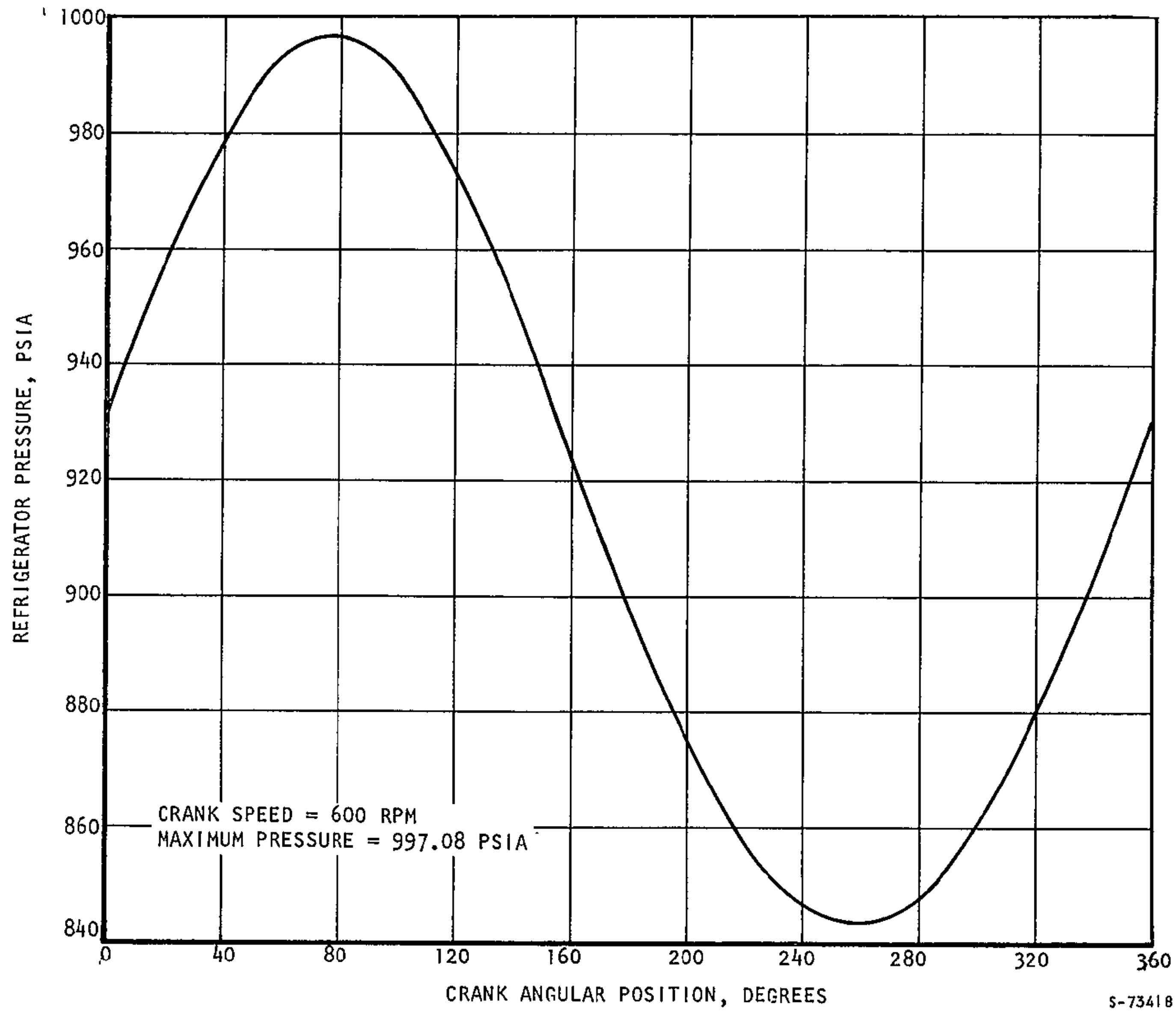


Figure 3-12. Refrigerator Pressure Characteristic at Crank Speed of 600 RPM

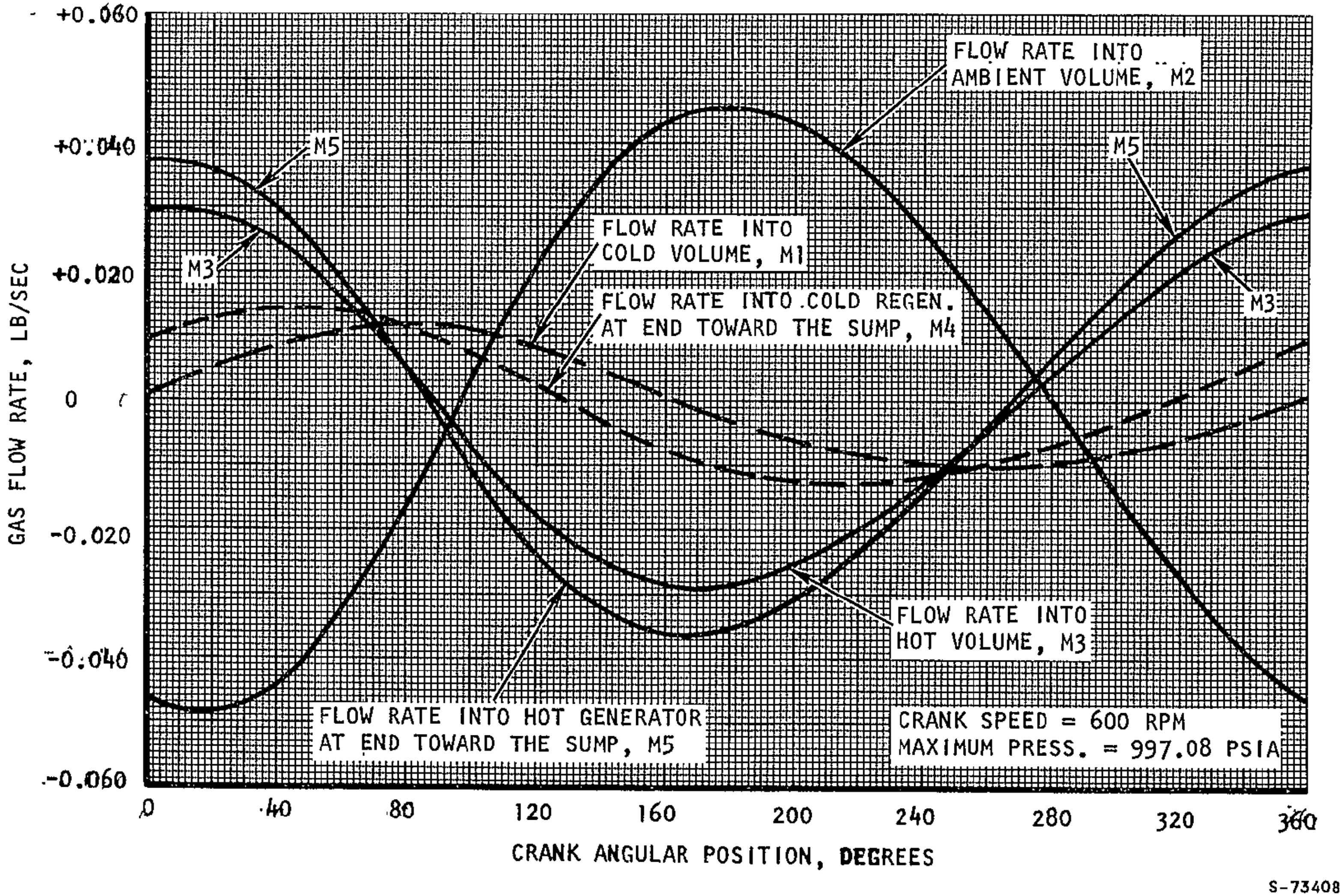


Figure 3-13. Refrigerator Gas Flow Rates at Crank Speed of 600 RPM

Figure 3-14 gives the ideal thermal power input as a function of the ideal refrigeration. The performance at various combinations of peak cycle pressure and rotational speed is shown in Figure 3-14. The slope of the curve and the ideal efficiency or COP are independent of peak pressure and rotational speed.

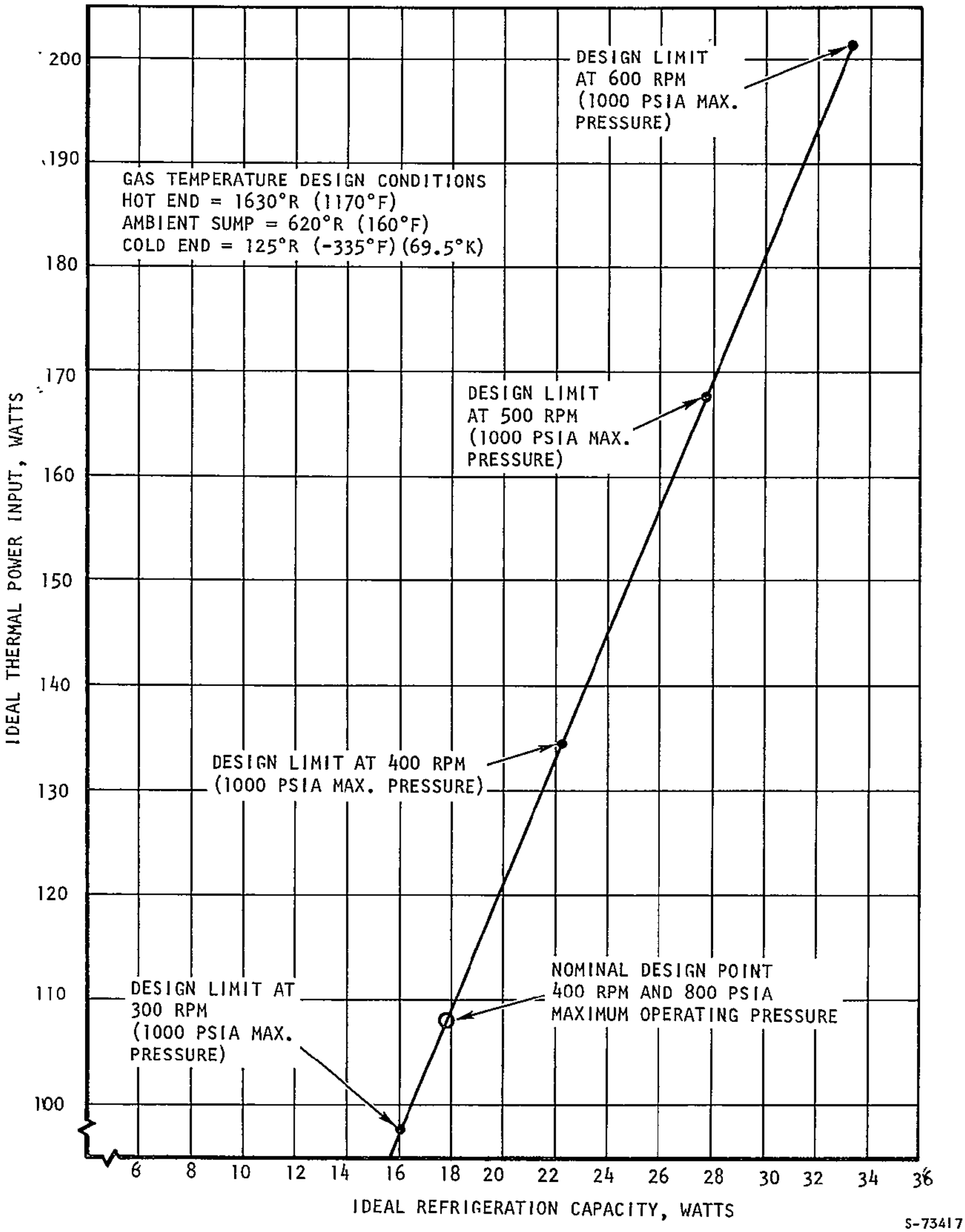


Figure 3-14. Ideal Thermal Power Input vs
Ideal Refrigeration Load

SECTION 4

COLD-END HEAT EXCHANGER

INTRODUCTION

The cold end heat exchanger's function is to transfer heat from the refrigeration load to the working fluid of the VM refrigerator. The primary design criteria for this heat exchanger consist of:

Minimization of the temperature difference between the working fluid and the refrigeration load--This is an extremely important feature since the load temperature is fixed, and the larger the temperature difference, the lower temperature the refrigerator must produce with inherent lower thermodynamic efficiencies.

Low working fluid pressure drop--The heat exchanger must provide the above thermal performance and yet not lead to an excessive pressure drop of the working fluid. The heat exchanger pressure drop subtracts from the pressure-volume variations in the cold expansion volume that produce the refrigeration.

Low void or internal volume--Void volumes at low temperatures significantly reduce the refrigeration capacity of VM refrigerators by decreasing the pressure variations or pressure ratio; minimization of the heat exchanger internal volume therefore is important.

Flow distribution--Uniform flow within each element of the heat exchanger is critical. The problem here is twofold: (1) nonuniform flow leads to reduced conductance of the heat exchanger, and (2) nonuniform flow leads to fluid elements at different temperatures. Subsequent mixing of these elements results in an increase in entropy and reduced thermodynamic efficiency of the refrigerator.

Heat exchanger interfaces--The cold end heat exchanger must interface with both the cold regenerator and cryogenic heat pipe. These interface requirements, to some extent, control the configuration of the heat exchanger. The interface with the cryogenic heat pipe is provided by the outer cylindrical surface of the cold end heat exchanger. The annular flow passage of the heat exchanger interfaces with the annular cold regenerator through a perforated plate flow distributor.

DESIGN CONFIGURATION

The configuration of the cold end heat exchanger is shown in Figure 4-1. This configuration is a refinement of the configuration evolved under Task 1.

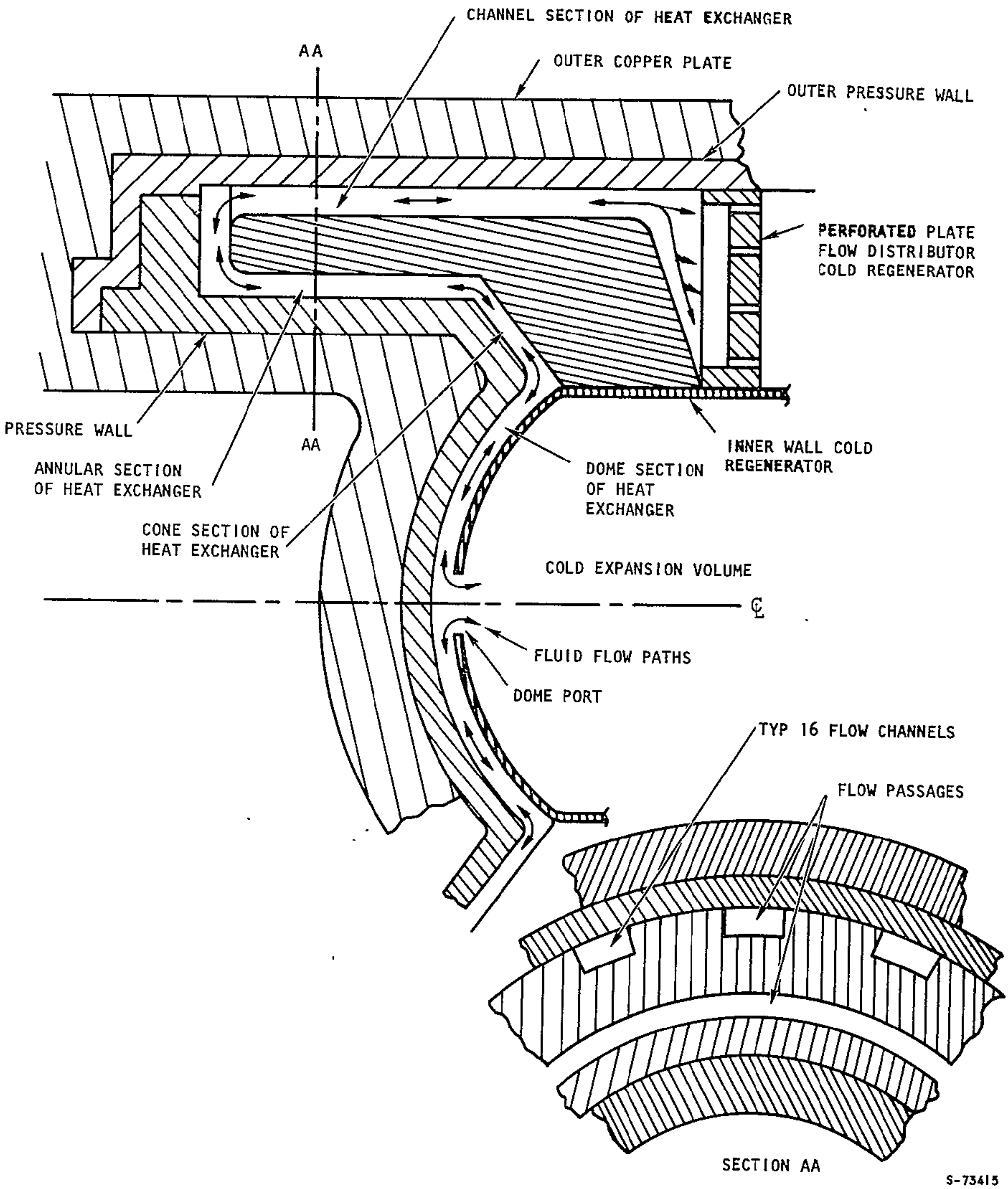


Figure 4-1. Cold End Heat Exchanger Configuration

Starting from the cold expansion volume, flow into and from the heat exchanger passes through a port in the dome of the wall that forms the expansion volume. The working fluid flow (which is actually cyclic with reversing flow) from the expansion volume outward into the heat exchanger is along the following flow paths while picking up heat from the refrigeration load. From the port in the expansion volume the flow path is primarily radial around the dome section of the heat exchanger, as shown in Figure 4-1. Heat transferred from the refrigeration load to this region is conducted from the outer cylindrical surface by a thick copper plate on the outer surface of the heat exchanger. The flow then passes from the dome section through the cone section of the heat exchanger and on to the annular section, as shown. Part of the refrigeration heat load reaches these sections by conduction through the outer copper plate in a manner similar to the dome section. The final flow path and heat transfer surface of the exchanger is provided by 16 channels or slots cut in a cylindrical surface as shown. These channels interface with a perforated plate to provide distribution of the flow to the cold regenerator.

Approximately 60 percent of the heat transfer capability (conductance) of the cold end heat is provided by the 16 channels. This section of the heat exchanger is located directly below the surface originally intended to provide the contact interface with the cryogenic heat pipe that carries the refrigeration heat load to the refrigeration cold head. Figure 4-2 shows the interface arrangement between the refrigerator and the cryogenic heat pipes. As delivered, the refrigerator will not interface with a heat pipe but will make use of a resistance heating element to simulate the refrigeration heat load. The heating element is wrapped around the outer surface of the cold end, replacing the indium foil shown in Figure 4-2, to simulate the energy distribution of the cryogenic heat pipe interface.

SUMMARY OF COLD END HEAT EXCHANGER PERFORMANCE CHARACTERISTICS

The calculated heat transfer and pressure drop characteristics of the cold end heat exchanger are summarized in Table 4-1.

The heat transfer characteristics of the cold end heat exchanger are based on the average flow during cyclic operation and therefore are believed to be conservative. The pressure drop is based on the peak or maximum flow and therefore represents the worst case conditions.

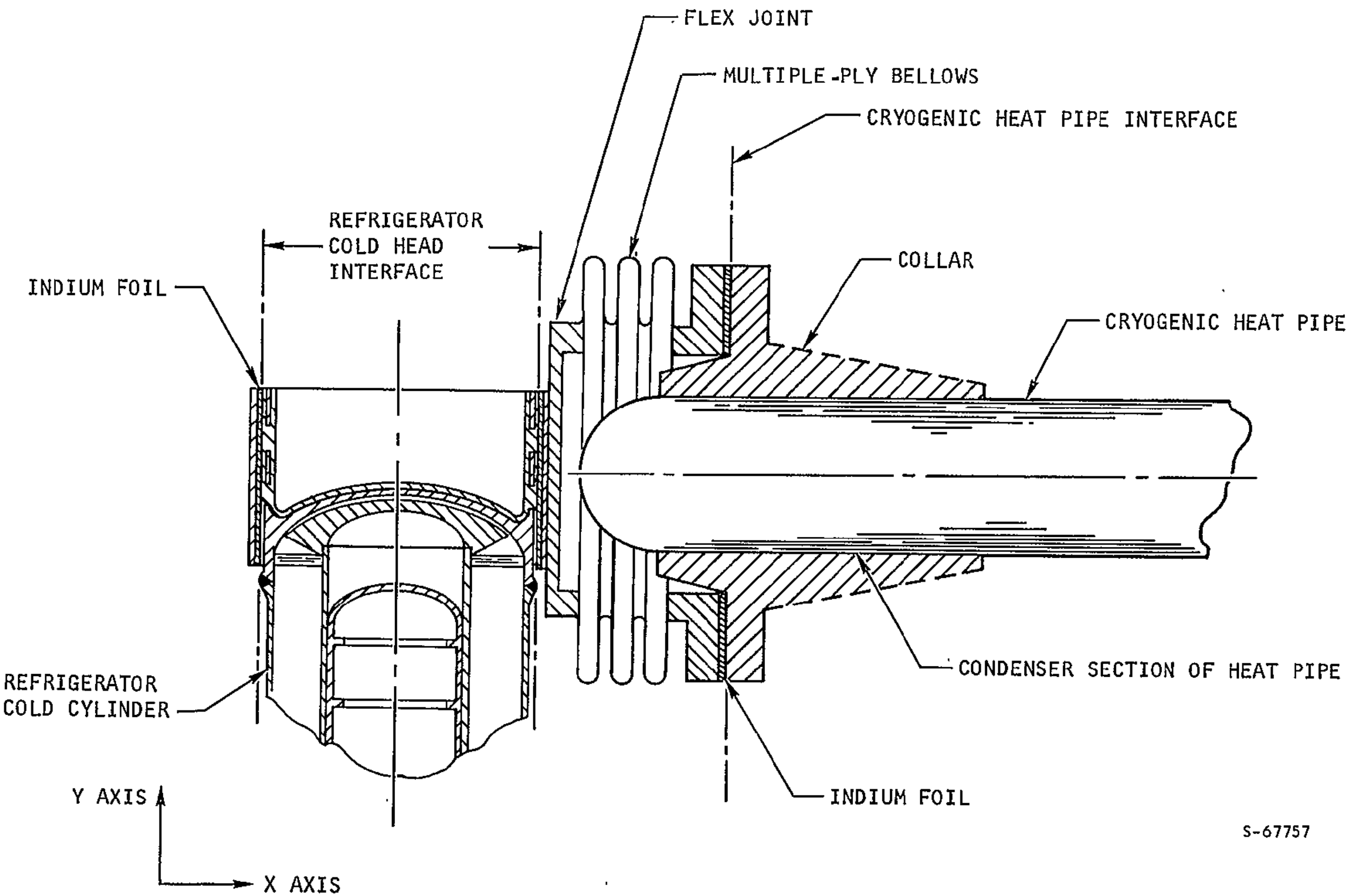


Figure 4-2. Interface Joint Design

TABLE 4-1

COLD END HEAT EXCHANGER CHARACTERISTICS

Section of Heat Exchanger	Conductance (ηhA), Btu/Hr-°R	Pressure Drop, psi
Dome port	3.27	0.1230
Dome and cone	3.16	0.1980
Inner annulus	3.16	0.0110
180° turn		0.0289
Outer slots	9.25	0.2540
Total	15.68	0.6149

COLD END HEAT EXCHANGER

1.0 HEAT TRANSFER CHARACTERISTICS

Use 10-4-71 Printout for data

Average $\dot{w} = .00377$ lb/sec $\quad \dot{w}_{max} \cong .0065$ lb/sec
$T = 125°R \quad \rho = 2.20$ lb/ft³ $\quad \mu = .0189$ lb/hr-ft
$P = 800$ PSIA $\quad C_p = 1.28$ Btu/lb °R $\quad k = .0332$ Btu/hr ft-°R

1.1 HEAT TRANSFER AREAS

1.1.1 Dome + Cone

Area assuming one side only

$A = 1.018 \text{ IN}^2$ { dome along

$A = 1.081 \text{ IN}^2$ { cone

$A = 2.099 \text{ IN}^2$ { dome and cone

1.1.2 Inner Annulus

$A = 2.236 \text{ IN}^2$

1.1.3 Outer Annulus Slots

16 slotes

$A = (.025) * (.893) \times 16 = 0.35 \text{ IN}^2$ } SUBJECT TO WATER CHANGE

1.2 DOME AND CONE

Take as a disc of equal area

$$A = 2.099 \text{ IN}^2 = \frac{\pi}{4} D_e^2$$

$$D_e = \sqrt{\frac{4}{\pi} \times 2.099} = 1.632 \text{ IN}$$

$$D_H = \frac{4A_c}{P} = \frac{4 * C * \pi d}{2\pi d} = 2C = .0250\ IN$$

where C = gap of flow passage

$A_c = \pi C d$

$C_{max} = .014\ IN$
$C_{min} = .011\ IN$ > Use ave .0125

$$\tilde{Q} = \frac{\dot{w}}{\rho} = \frac{.00377\ lb/sec}{2.20\ lb/ft^3} = .001716\ ft^3/sec$$

$$\tilde{V} = \frac{Q}{A_c} = \frac{.001716\ ft^3/sec * 1728\ IN^3/ft^3}{(\pi)(.0125)(d)\ IN^2 * 12\ IN/ft} = 6.296/d\ \ ft/sec$$

$$R_e = \frac{V d_H \rho}{\mu} = \frac{V D_H \rho}{\mu} \qquad \frac{D_H \rho}{\mu} = \frac{(.0250\ IN)(2.20\ lb/ft^3)\ ft * 3600\ sec/hr}{.0189\ lb/ft\text{-}hr * 12\ IN}$$

$$R_e = \frac{6.296}{d} * 873 = 5496\ d^{-1} \qquad \frac{D_H \rho}{\mu} = 873.0\ sec/ft$$

d	V	Re
.1		54,960
.2		27,480
.4		13,740
.6		9,160
.8		6,870
1.0		5,496
1.2		4,580
1.372		4,006
1.632		3,361

Due cyclic nature assume turbulent flow over whole reagion

$$N_U = 0.0265\,(N_{Re})^{0.8}(N_{PR})^{0.3} = \frac{h\,D_H}{k}$$

$$h = \frac{k}{D_H}\,N_U = \frac{k}{D_H} * .0265 * \left(\frac{5496}{d}\right)^{0.8}(.67)^{0.3}$$

$$h = \frac{.0352\ Btu/hr\text{-}ft\text{-}°R * .0265\left(\frac{5496}{d}\right)^{0.8}(.67)^{0.3} * 12\,IN/ft}{.025\,IN}$$

$$= .397\left(\frac{5496}{d}\right)^{0.8} = 385\left(\frac{1}{d}\right)^{0.8}\ Btu/hr\,ft^2\,°R \quad \{d\text{ in inches}$$

What we want is:

$$\int h\,dA = \tilde{h}A$$

$$A = \frac{\pi}{4}d^2 \quad dA = \frac{\pi}{2}D\,dD \qquad D = d$$

$$\therefore \quad \tilde{h}A = \int_{D_i}^{D_o} \frac{385}{144}\left(\frac{1}{D}\right)^{0.8}\frac{\pi}{2}D\,dD \qquad D = IN$$

$$= \frac{605}{144}\int_{D_i}^{D_o} D^{0.2}\,dD = \frac{605}{144}\left[\frac{D^{1.2}}{1.2}\right]_{D_i}^{D_o}$$

$$= 4.2\left[\frac{D_o^{1.2} - D_i^{1.2}}{1.2}\right] =$$

$D_o = 1.632$

$D_i = .144$

$$= 3.5\left[1.80 - .0975\right] = \underline{\underline{5.96\ Btu/hr\ °R}}$$

2.665
.021
2.644

2.665
.034
2.631

note some typical h's for this area

d	Re	$(1/d)^{0.8}$	h Btu/hr-ft-°R
.1	54,960	6.3	2428
.2	27,480		
.4	13,740	2.16	840
.6	9,160		
.8	6,870	1.195	460
1.0	5,496	1.00	385
1.2	4,580		
1.372	4,006	.778	248

$$\tilde{h} = \frac{\widetilde{hA}}{A} = \frac{5.96}{.01456} = \underline{\underline{409.9}}\ Btu/hr\ ft^2\ °R$$

$$A = \frac{\pi D^2}{4} = \frac{\pi}{4}(1.632)^2 = 2.099\ IN^2 = .01456\ ft^2$$

1.3 INNER ANNULUS

$d = 1.40$ IN

$c = 0.0125$ IN

$$A_F = \pi d c = \pi * 1.4 * (.0125) = 0.05495\ IN^2$$

$$V = \frac{Q}{A_F} = \frac{.001716\ ft^3/sec}{.05495\ IN^2} * \frac{144\ IN^2}{ft^2} = 4.4968\ ft/sec$$

$$Re = \frac{Vd\rho}{\mu} = \frac{(4.4968\ ft/sec)(.025\ IN)(2.20\ lb/ft^3)(3600\ sec/hr)}{(.0189)\ lb/hr\text{-}ft \times 12\ IN/ft}$$

$$= 3925$$

From curve

$$J = \frac{h}{C_p G} Pr^{2/3} = .00325$$

$$h = (.00325) \frac{C_p G}{(Pr)^{2/3}} =$$

$$G = \rho V = 2.20 \frac{lbm}{ft^3} \cdot 4.4968 \ ft/sec = 9.89296 \frac{lbm}{ft^2 sec}$$

$C_p = 1.26$ Btu/lb-°R $\quad Pr = .67 \quad (Pr)^{.666} = .769$

$$h = \frac{(.00325)\ 1.26\ Btu/lb_m\text{-}°R * 9.893 \frac{lbm}{ft^2 sec} * 3600\ sec/hr}{.769} = 204.2$$

$h = 204.2$ Btu/hr ft²-°R

$A_H = \pi d L = 2.236\ IN^2 = .0155\ ft^2$

$hA = 204$ Btu/hr ft²-°R $* .0155\ ft^2 = 3.16$ Btu/hr °R

(1,4) OUTER ANNULUS SLOTS

$$D_H = \frac{4A_c}{P} = \frac{4 * .025 * .025}{4 * .025} = .025 \text{ IN}$$

$$A_F = (.025)^2 \times 16 = .01 \text{ IN}^2$$

$$A_H = \pi d \times L \times N = (.025)(.893)(16) = .357 \text{ IN}^2 = .002481 \text{ ft}^2$$

$$V = \frac{Q}{A} = \frac{.001716 \text{ ft}^3/\text{sec}}{.01 \text{ IN}^2} \cdot \frac{144 \text{ IN}^2}{\text{ft}} = 24.71 \text{ ft/sec}$$

$$N_{Re} = \frac{V D_H \rho}{\mu} = \frac{(24.71 \text{ ft/sec})(.025 \text{ IN})(2.20 \text{ lb/ft}^3) \times 3600 \text{ sec/hr}}{.0189 \text{ lb/ft-hr} \times 12 \text{ IN/ft}}$$

$$= 21,572$$

$$j = \frac{h}{C_p G} P_r^{2/3} = .0025$$

$$h = 0.0025 \frac{C_p G}{P_r^{2/3}} \qquad G = \rho V = 2.2 \text{ lb/ft}^3 \cdot 24.71 \text{ ft/sec} = 54.362 \text{ lb/ft}^2\text{-sec}$$

$$h = \frac{.0025 (1.28 \frac{\text{Btu}}{\text{lb-}^\circ\text{R}})(54.362 \text{ lb/ft}^2\text{-sec}) * 3600 \text{ sec/hr}}{\underset{.7656}{(.67)^{2/3}}} = 818 \text{ Btu/hr-ft}^2\text{-}^\circ\text{R}$$

$$hA = .002481 \text{ ft}^2 * 818 \text{ Btu/hr-ft}^2\text{-}^\circ\text{R} = 2.029 \text{ Btu/hr-}^\circ\text{R}$$

note: we can actually look for some fin effect
for other sides of the slots ∴ $hA_{actual} > hA_{calc}$

1.5 TOTAL hA COLD END EXCHANGER

	V	Re	A.	h	h A
	ft/sec		in^2	Btu/hr-ft^2-°R	Btu/hr-°R
DOME AND CONE	–	–	2.029	~~409.9~~	5.96 (5.0)
INNER ANNULUS	4.997	3925	2.236	204	3.16
OUTER ANNULUS SLOTS	24.71	21,572	.357	818	2.03
TOTAL					11.15

10.19 (old)

for 7w of ref:

- heat transfer will only take place over about ½ of cycle

∴ base ΔT on $2 \times Q$

$$\Delta T = \frac{2 \times Q}{hA} = \frac{2 * 17 * 3.41}{11.15} = 4.28\ °R$$

for this ΔT plus regenerator ΔT_r of $\approx 5°R$

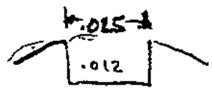
ΔT_{TOTAL} of 10°R or: for $T_{ref} = 135°R = 75°K$

use $T_{gas} = 125°R = 69.5°K$

1.6 METHODS OF INCREACING TOTAL HA

1.6.1 OUTER ANNULUS SLOTS

The actual part is made with slotes:



In the above calculations we assumed a .025 by .025 slot, thus in we use same flow cross section we can maintain same value of h and increase A by increasing slot width.

$$(.025)^2 = .012 * w$$

$$w = \frac{(.025)^2}{.012} = .052 \text{ IN width}$$

$$A_h = w * L \times N = (.052)(.893)(16) = .742 IN^2 = .00515 ft^2$$

$$hA = 818 \; Btu/hr\,ft^2\,°R \cdot 0.00515 = 4.22 \; Btu/hr\,°R$$

∴ New Total hA

	hA
Dome and Cone	5.96
Inner Annulus	3.16
Outer Annulus Slots	4.22
	13.34

This should be ok if pressure drop is not too high! Come back after pressure drop calculations; would like to have an hA of 15 Btu/hr-°R if possible

5 (4)

2.0 PRESSURE DROP CHARACTERISTICS

2.1 PRESSURE DROP WITH FLOW FROM COLD VOLUME

2.1.1 LOSS IN HOLE

$D_H = \begin{matrix} .060 \\ .065 \end{matrix}$ IN

CONTRACTION LOSS GIVEN BY:

$$\Delta P = K_c \frac{V_2^2}{2g_c} \rho_f \quad (5\text{-}91) \text{ pp } 5\text{-}30 \text{ Perry}$$

$$K_c = f\left(\frac{A_2}{A_1}\right) \qquad A_1 = \frac{\pi}{4} D_1^2$$

$$A_2 = \frac{\pi}{4} D_H^2$$

$$\frac{A_2}{A_1} = \frac{D_H^2}{D_1^2} = \left(\frac{.06}{.860}\right)^2$$

$$K_c = .5$$

$\dot{W}_{max} = .0065$ lb/sec
$\mu = .0189$ lb/hr-ft
$k = .0332$ Btu/hr ft-°R
$\rho = 2.2$ lb/ft^3
$C_p = 1.28$ Btu/lb-°R
$T = 125$°R
$P = 800$ PSI
$g_c = 32.2 \frac{lb_m\text{-}ft}{lb_f\text{-}sec^2}$

$$Q = \frac{\dot{w}}{\rho} = \frac{.0065 \text{ lb/sec}}{2.20 \text{ lb/ft}^3} = .002955 \frac{ft^3}{sec}$$

$$V_2 = \frac{Q}{A_2}$$

$$A_2 = \frac{\pi}{4} D_H^2 = \frac{\pi}{4}(.06)^2 = .00283 \text{ IN}^2$$

$$A_2 = 1.963 \times 10^{-5} ft^2$$

$\therefore$

$$V_2 = \frac{Q}{A_2} = \frac{2.955 \times 10^{-3} \frac{ft^3}{sec}}{1.963 \times 10^{-5} ft^2} = 1.502 \times 10^2 \text{ ft/sec}$$

$$V_2^2 = 2.26 \times 10^4 \text{ ft}^2/\text{sec}^2$$

Based just on the contraction loss

$$\Delta P = (0.5)\ \frac{2.26\times10^{4} * 2.20\ lbm/ft^3}{2 * 32.2\ \frac{lbm-ft}{lbf-sec^2}} = 386\ lbf/ft^2$$

$\Delta P = 2.68\ PSI$ { THIS FAR TOO HIGH }

Actually we should use about 1.5 velocity head loss to account for turning into dome part of heat exchange.

First work on the bases of only a ΔP of 0.25 psi is allowable and determine hole size

$$\Delta P = (1.5)\ \frac{V_2^2}{2g_c}\rho_f \equiv .25\ PSI$$

$$\Delta P = (1.5)\ \frac{V_2^2}{2 * 32.2} * 2.2 * \frac{1}{144}\ \frac{\frac{ft^2}{sec^2}}{\frac{lbm-ft}{sec^2}}\ \frac{lbm/ft^3 \cdot ft^2}{IN^2} = .25\ PSI$$

$$V_2^2 = \frac{(0.25)}{(1.5)} * \frac{64.4}{2.2} * 144 = 702$$

$$V_2 = 26.5\ ft/sec$$

$$V_2 = \frac{Q}{A_2} \qquad Q = 2.955\times10^{-3}\ ft^3/sec$$

$$A_2 = \frac{Q}{V_2} = \frac{2.955\times10^{-3}\ ft^3/sec}{2.65\times10^{1}\ ft/sec} = 1.115\times10^{-4}\ ft^2$$

$$A_2 = 1.608\times10^{-2}\ IN^2 = \frac{1}{4}\pi D_H^2$$

$$D_H = \sqrt{\frac{4A_2}{\pi}} = \left(2.042\times10^{-2}\right)^{1/2} = 1.429\times10^{-1}\ IN = .143\ IN$$

(11)

Get pressure drop as a function of hole diameter range .06 to .15 IN

$$\Delta P = K \frac{V_2^2}{2g_c} \rho_f = (1.5) \frac{V_2^2}{2 \times 32.2} \frac{(2.20)}{144} = 3.559 \times 10^{-4} V_2^2$$

$$V_2 = \frac{Q}{A_2} = \frac{(2.955 \times 10^{-3})}{\frac{\pi}{4} D_H^2} \times 144 \ \text{ft/sec} = \frac{.5421}{D_H^2}$$

D_H IN	D_H^2	V_2 ft/sec	V_2^2	ΔP PSI	
.06	.00360	150.6	2.268×10^4	.80700	{Better procedure later}
.08	.00640	84.7	7.175×10^3	2.550	
.10	.01000	54.21	2.939×10^3	1.046	
.12	.01440	37.64	1.417×10^3	.504	
.142	.02016	26.89	7.23×10^2	.257	
.150	.0225	24.09	5.80×10^2	.206	
.160	.0256	21.19	4.495×10^2	.159	{ 0.156
.180	.0322	16.83	2.838×10^2	.101	0.187

Tentatively select D_H = .144 IN

Check if reduction in dead volume by extending tip of cold displacer looks practical or worth the trouble

L = .005 + .031 + .011 − .005 — End clearence

Clearence, dome wall, HX gap

∴ L = .042   D = .1 IN {this leaves .02 IN on each side

potential decrease in dead volume

$$\Delta V = \frac{\pi D^2}{4} \times L = \frac{\pi}{4} (.1)^2 \times 0.042 = 3.3 \times 10^{-4} \text{IN}^3 = .00033 \text{ IN}^3$$

dead volume is ≈ .08533 IN³

$$\% \text{ decrease} = \frac{.00033}{0.08533} \times 100 = 0.26\%$$ {not worth it

2.1.2 LOSS IN DOME AND CONE

2.1.2.1 Main Surface

Take the velocity at an average diameter of 0.8 in

Later we show using ave dia is not so good, better to set up and integrate over surface

$$A_F = \pi \tilde{D} C = \pi(.8)(.0125) = .0314\ IN^2 = .000218\ ft^2$$

$$Q = \frac{w'}{\rho} = .002955\ \frac{ft^3}{sec}$$

$$V = \frac{Q}{A_F} = \frac{.002955\ ft^3/sec}{.000218\ ft^2} = 13.5\ ft/sec$$

$$Re = \frac{V D_H \rho}{\mu} \qquad \frac{D_H \rho}{\mu} = \frac{(.025\ IN)(2.20\ lb/ft^3) \times 3600\ sec/hr}{.0189\ lb/(ft\text{-}hr) \times 12\ IN/ft} = 873\ sec/ft$$

$$Re = 13.5\ ft/sec * 873\ sec/ft = 11{,}800$$

from figure $f = .0076$

$$\Delta P = \left(\frac{4 f L \rho}{D_H}\right)\frac{V^2}{2 g_c} \qquad L = 1.632\ IN$$

$$\Delta P = \frac{(4)(.0076)(1.632\ IN)(2.20\ lbm)}{(.025\ IN)\ ft^3}\ \frac{(13.5)^2\ ft^2}{2\ sec^2}\ \frac{sec^2\text{-}lbf}{32.2\ lbm\text{-}ft}$$

$$= 12.355\ lbf/ft^2 = .0858\ PSI$$

2.1.2.2 LOSS AT SHARP TURNS

- At intersection of cone and dome take 1.5 velocity heads (Like 180° turn)

$$A_F = \pi \tilde{D} C = \pi(.945)(.0125) = .0371\ IN^2 = .0002575\ ft^2$$

$$V = \frac{Q}{A} = \frac{.002955\ ft^3/sec}{.0002575\ ft^2} = 11.5\ ft/sec$$

$$\Delta P = 1.5\left(\frac{V^2}{2g_c}\right)\rho = (1.5)\left(\frac{(11.5)^2}{64.4}\right)2.2 = 6.82\ \frac{lbf}{ft^2} = \underline{.0473\ PSI}$$

• At turn into inner annulus use 0.5 velocity head

$$A_F = \pi D c = \pi(1.4)(.0125) = .055\ IN^2 = .000382\ ft^2$$

$$V = \frac{.002955\ ft^3/sec}{.000382\ ft^2} = 7.73\ ft/sec$$

$$\Delta P = (.5)\left(\frac{V^2}{2g_c}\right)\rho = (.5)\left(\frac{(7.73)^2}{64.4}\right)(2.2) = 1.02\ \frac{lbs}{ft^2} = \underline{.00897\ PSI}$$

2.1.2.3 TOTAL DOME, CONE, AND TURNS TO INNER ANNULUS

	ΔP (PSI)
DOME AND CONE	.0858
TURN (1)	.0473
TURN (2)	.0090
TOTAL	.1421 PSI

2.1.3 LOSS IN INNER ANNULUS

$\tilde{D} = 1.3875$ $\quad c = .0125$ in

$$A_F = \pi \tilde{D} c = \pi(1.3875)(.0125) = .0545\ IN^2 = .0003782\ ft^2$$

$$V = \frac{Q}{A_F} = \frac{2.955 \times 10^{-3}\ ft^3/sec}{3.782 \times 10^{-4}\ ft^2} = 7.8\ ft/sec$$

$D_H = .025$ IN

$$R_e = \frac{V D_H \rho}{\mu} = V\left(\frac{D_H \rho}{\mu}\right) = V\left(873 \text{ sec/ft}\right)$$

$$R_e = 7.8 \text{ ft/sec} \cdot 873 \text{ sec/ft} = 6{,}810$$

$\therefore$

$f = .00935$

$$\Delta P = \left(\frac{4 f L \rho}{D_H}\right)\frac{V^2}{2 g_c} \qquad L = 0.512 \text{ IN.}$$

$$\Delta P = \frac{(4)(.00935)(0.512 \text{ in})(2.2 \text{ lbm})}{(.025 \text{ IN})\;(\text{ft}^3)} \cdot \frac{(7.8)^2 \text{ ft}^2}{\text{sec}^2 \; 64.4 \frac{\text{lbm-ft}}{\text{lbf-sec}^2}} = 1.592 \frac{\text{lbf}}{\text{ft}^2}$$

$$\Delta P = 1.592 \text{ lb/ft}^2 = .0111 \text{ psi}$$

- Take 2 velocity heads loss in 180° sharp turn at end of inner annulus

$$\Delta P = 2.0\left(\frac{V^2}{2 g_c}\right)\rho = 2.0\left(\frac{7.8^2 \text{ ft}^2/\text{sec}^2}{64.4 \frac{\text{lbm-ft}}{\text{lbf-sec}^2}}\right) 2.2 \frac{\text{lbm}}{\text{ft}^3} = 4.157 \text{ lb/ft}^2$$

$$\Delta P = 4.157 \text{ lb/ft}^2 = .02887$$

TOTAL	ΔP (PSI)
ANNULUS	.01110
180° TURN	.02887
	.04997 PSI

2.1.4 LOSS IN OUTER ANNULUS SLOTS

First at one end of the slotted section we will have a contraction loss and at the other we will have an expansion loss. For worst case condition we can take 1.5 velocity head loss total for expansion and contraction loss

$$\Delta P_{ends} = 1.5 \left(\frac{V_s^2}{2g_c}\right)\rho$$

where V_s = the velocity in slots

∴ Total pressure drop across slots including end effects are:

$$\Delta P = 1.5\left(\frac{V_s^2}{2g_c}\right)\rho + \left(\frac{4fL}{D_H}\right)\frac{V^2}{2g_c}\rho$$

$$\Delta P = \left(1.5 + \frac{4fL}{D_H}\right)\left(\frac{V_s^2}{2g_c}\right)\rho$$

2.1.4.1 Look at slots 0.0125 deep by .025 wide

These slot dimensions are presently on drawings; though we expect that the flow area will have to be incresed the pressure drop will be calculated

$$A_F = (0.0125)(.025)\underset{\text{No. of slots}}{(16)} = .005\ \text{IN}^2 = .0000347\ \text{ft}^2$$

$$D_H = \frac{2 \times (.0125)(.025)}{.0125 + .025} = .0169\ \text{in}$$

$$V_s = \frac{Q}{A_F} = \frac{2.955 \times 10^{-3}\ \text{ft}^3/\text{sec}}{3.47 \times 10^{-5}\ \text{ft}^2} = 85.2\ \text{ft/sec}$$

$$Re = \frac{V D_H \rho}{\mu} = V\left(\frac{D_H \rho}{\mu}\right) \qquad \frac{D_H \rho}{\mu} = 873 * \frac{.0169}{.025} = 591\ \text{sec/ft}$$

$$Re = 85.2 * 591 = 50{,}300$$

$Re = 59{,}300 \longrightarrow f = .0052 \qquad L = .893\ IN$

$$\frac{4fL}{D_H} = \frac{(4)(.0052)(.893)}{.0169} = 1.097$$

$$\frac{V_s^2}{2g_c} = \frac{(85.2)^2\ ft^2/sec^2}{64.4\ \frac{lbm-ft}{lbf-sec^2}} = 112.6\ \frac{lbf-ft}{lbm}$$

$$\Delta P = (1.5 + 1.097)\left(112.6\ \frac{lbf-ft}{lbm}\right)\left(2.2\ \frac{lbm}{ft^3}\right) = 643\ lb/ft^2$$

$\Delta P = 643\ lb/ft^2 = \underline{\underline{4.47\ PSI}}$ TOTAL

note in slots along $\Delta P = 4.47\ \frac{1.097}{2.597} = \underline{\underline{1.89\ PSI}}$

This checks with NASA'S calc.

2.1.4.1 Look at slots of various widths while maintaining depth at .0125

(1) First try .05 IN wide

$A_F = (.0125)(.05)(16) = .010\ IN^2 = .0000694\ ft^2$

$$D_H = \frac{2 \times (.0125)(.05)}{.0125 + .05} = .02\ IN$$

$$V_S = \frac{Q}{A_P} = \frac{2.955 \times 10^{-3}\ ft^3/sec}{6.94 \times 10^{-5}\ ft^2} = 42.6\ ft/sec$$

$$Re = \frac{V D_H \rho}{\mu} = V\left(\frac{D_H \rho}{\mu}\right) \qquad \frac{D\rho}{\mu} = 591 * \frac{.02}{.0169} = 700$$

$Re = 42.6 \times 700 = 29{,}850$

$f = .00595$

$$\frac{4fL}{D_H} = \frac{(4)(.00595)(.893)}{(.02)} = 1.064$$

$$\frac{V_6^2}{2g_c} = \frac{(42.6)^2}{64.4} = 28.2 \frac{lb_f\ ft}{lbm}$$

$$\Delta P = (1.5 + 1.064)(28.2)(2.2) = 159.3\ lb/ft^2 = 1.108\ psi$$

(2) .071 width

$$A_F = (.0125)(.071)(16) = .0142\ IN^2 = .0000986\ ft^2$$

$$D_H = \frac{2(.0125)(.071)}{.0125 + .071} = .02121$$

$$V_s = \frac{2.955 \times 10^{-3}}{9.86 \times 10^{-5}} = 30\ ft/sec$$

$$Re = V\left(\frac{D_H \rho}{\mu}\right) \qquad \frac{D_H \rho}{\mu} = 700 * \frac{.02121}{.02} = 745$$

$$Re = 30 * 745 = 22{,}380$$

$$f = .0064$$

$$\therefore \frac{4fL}{D_H} = \frac{(4)(.0064)(.893)}{.02121} = 1.075$$

$$\Delta P = (1.5 + 1.075)\left(\frac{V_s^2}{2g_c}\right)(2.2)$$

$$\frac{V_2^2}{2g_c} = \frac{(30)^2}{64.4} = 14 \frac{lb_f - ft}{lbm}$$

$$\Delta P = (2.575)(14.)(2.2) = 78.2\ lb_f/ft^2 = .55\ PSI$$

(3) .062 width

$$A_F = (.0125)(.062)(16) = .0124\ IN^2 = .0000861\ ft^2$$

$$D_H = \frac{2(.0125)(.062)}{.0125 + .062} = .0208$$

$$V_S = \frac{2.955\times10^{-3}}{8.61\times10^{-5}} = 34.3\ ft/sec$$

$$R_e = V\left(\frac{D_H\rho}{\mu}\right) \qquad \frac{D_H\rho}{\mu} = 745\times\frac{.0208}{.0212} = 730$$

$$R_e = (34.3)(730) = 25{,}000$$

$$f = .0062$$

$$\frac{4fL}{D_H} = \frac{(4)(.0062)(.893)}{.0208} = 1.062$$

$$\frac{V_S^2}{2g_c} = \frac{(34.3)^2}{64.4} = 18.28\ \frac{lb_f\text{-}ft}{lb_m}$$

$$\Delta P = (1.5 + 1.062)(18.28)(2.2) = 103.1\ lb/ft^2 = .716\ PSI$$

(4) .09 width

$$A_F = (.0125)(.09)(16) = 0.018\ IN^2 = .000125\ ft^2$$

$$D_H = \frac{2(0.0125)(.09)}{.0125 + .09} = 0.02185\ IN$$

$$V_S = \frac{2.955\times10^{-3}}{1.25\times10^{-4}} = 23.6\ ft$$

$$R_e = V\left(\frac{D_H\rho}{\mu}\right) \qquad \left(\frac{D_H\rho}{\mu}\right) = 730\times\frac{.02185}{.0208} = 768$$

(29)

$Re = 23.6 \times 768 = 18110$

$f = .00678$

$$\frac{4fL}{D_H} = \frac{(4)(.00678)(.893)}{.02185} = 1.105$$

$$\frac{V_s^2}{2g_c} = \frac{(23.6)^2}{64.4} = 8.29 \frac{lb_f - ft}{lb_m}$$ 8.65 error

$$\Delta P = (1.5 + 1.105)(8.29)(2.2) = 47.5\ lb_f/ft^2 = 0.33\ psi$$

NOW LETS SUMMARIZE THE ΔP'S = f(width) IN PREPARATION FOR LOOKING AT HEAT TRANSFER CHARACTERISTICS

FOR DEPTH OF 0.0125 IN 16 SLOTS

WIDTH IN	AF ft^2	DH IN	Re	ΔP PSI	V_s
.025	.0000347	.0169	50,300	4.47	85.2
.050	.0000694	.0200	29,850	1.108	42.6
.062	.0000861	.0208	25,000	.716	34.3
.071	.0000986	.02121	22,380	.55	30.0
.090	.0001250	.02185	18,110	.33	23.6

SLOT DEAD VOLUME
DEPTH = 0.0125 IN

WIDTH (IN)	AF (IN^2)	Vol_s (IN^3)
.025	.00500	.00550
.050	.0100	.01100
.062	.0124	.01363
.071	.0142	.01560
.090	.018	.01980
.100		.022

PARAMETRIC STUDY OF HA VS SLOT WIDTH

WIDTH IN	V_s' ft/sec	$\left(\frac{D_H \rho}{\mu}\right)$	Re	j	G lbm/ft²-sec	h Btu/ft²-°R	A_H ft²	hA Btu/°R-hr
.025	49.5	591	29,210	.00240	109.0	1560	.00248	3.87
.050	24.75	700	17,310	.00260	54.4	842	.00496	4.18
.062	20.11	730	14,690	.00270	44.2	710	.00615	4.36
.071	17.39	745	12,950	.00275	38.3	628	.00705	4.42
.090	13.71	768	10,530	.00285	30.2	514	.00893	4.50

$V_s' = \frac{Q}{A_F}$ $\quad Q = .001716\ ft^3/sec$ → based on average flow

$$j = \frac{h}{C_p G} P_r^{2/3}$$

$$G = \frac{\dot{w}}{A_f} = \frac{.00377\ lbm/sec}{A_f\ ft^2} = \rho V_s'$$

$$h = \frac{j * C_p * G}{P_r^{2/3}} = \frac{j * 1.28\ Btu/lb_m{}^\circ R \cdot G\ \frac{lbm}{ft^2 sec}}{(.67)^{2/3}}$$

$$= \frac{j * 1.28 Btu/lb_m{}^\circ R \cdot G \frac{lbm}{ft^2 sec} * \frac{3600 sec}{hr}}{.7738}$$

$$h = 5.96 \times 10^3 j * G \quad \begin{cases} G \text{ in } \frac{lbm}{ft^2 sec} \\ h \text{ in } Btu/ft^2 {}^\circ R \end{cases}$$

$$A_H = W * L \times 16 = W \times \frac{(.893)(16)}{144} = W * .0993 \quad \begin{cases} W \text{ in } IN \\ A_H \text{ in } ft^2 \end{cases}$$

SUMMARY OF SLOT DATA DEPTH = 0.0125

WIDTH (IN)	ΔP PSI	hA Btu/hr-°R	Slot Dead Vol (IN³)
.025	4.47	3.87	.0055
.050	1.108	4.18	.0110
.062	.716	4.36	.01363
.071	.55	4.12	.01560
.090	.33	4.50	.01980

{select as reasonable}

2.1.5 SUMMARY OF ΔP AND HA

{Revised later pp 30}

	HA Btu/hr-°R	ΔP PSI	
DOME HOLE		.15900	D_H = .160 IN
DOME AND CONE	5.96	.08580	
TURN (1)		.04730	
TURN (2)		.00900	
INNER ANNULUS	3.16	.01100	
180° TURN		.02887	
OUTER SLOTS	4.50	.33000	Width = .09 in
TOTAL	13.62	.67097	

WOULD STILL LIKE A HIGHER HA ON THE ORDER OF 18 Btu/hr-°R

TRY AN ANNULAR HEAT EXCHANGER IN PLACE OF THE SLOTS

13.62
4.50
18.12

2.1.6 OUTER ANNULUS HEAT EXCHANGER

$D_o = 1.5537$

We know that for a $A_F \approx .018\ IN^2 = .000125\ ft^2$
we should have an acceptable pressure drop

$$A_F = \frac{\pi}{4}(D_o^2 - D_i^2)$$

$$D_i^2 = D_o^2 - \frac{4}{\pi}A_F$$

$$= (1.5537)^2 - \frac{4}{\pi}(.018)$$

$$= 2.4139837 - .02293 = 2.3911$$

$D_i = 1.5463$

$2C = D_o - D_i = .0074\ IN$

$C = .0037\ IN$

$D_H = D_o - D_i = .0074\ IN$

Dead Volume

$(.018)(.893) = .01608\ IN^3$

TO DISTRIBUTE THE FLOW AT EXIT WE WILL TAKE
A ONE VELOCITY HEAD LOSS

$$\Delta P = \left(1.0 + \frac{4fL}{D_H}\right)\left(\frac{V_c^2}{2g_c}\right)\rho$$

$A_F = 1.25 \times 10^{-4}\ ft^2 \quad Q = 2.955 \times 10^{-3}\ ft^3/sec$

$$V_c = \frac{Q}{A_F} = \frac{2.955 \times 10^{-3}}{1.25 \times 10^{-4}} = 23.6\ ft/sec$$

$$Re = V\left(\frac{D_H \rho}{\mu}\right) \qquad \left(\frac{D_H \rho}{\mu}\right) = 768 \times \frac{.0074}{.02185} = 260$$

$$Re = 23.6 \times 260 = 6150$$

$f = .0092$

$$\frac{4fL}{D_H} = \frac{(4)(.0092)(.893)}{.0074} = 4.45$$

$$\frac{V_c^2}{2g_c} = \frac{(23.6)^2}{64.4} = 8.65$$

$$\Delta P = (1 + 4.45)(8.65)(2.2) = 103.8 \text{ lbf/ft}^2 = .72 \text{ PSI}$$

Check the hA

$$V_H = \frac{Q}{A_F} = \frac{1.716 \times 10^{-3} \text{ ft}^3/\text{sec}}{1.25 \times 10^{-4} \text{ ft}^2} = 1.37 \times 10^1 \text{ ft/sec} = 13.7 \text{ ft/sec}$$

$$Re = V\left(\frac{D_H \rho}{\mu}\right) = (13.7)(260) = 3{,}560$$

$$j = .0033 = \frac{h}{C_p G} \, Pr^{2/3}$$

$$G = \rho V_H = (2.2)(13.7) \frac{\text{lbm}}{\text{ft}^3} \cdot \frac{\text{ft}}{\text{sec}} = 30.2 \frac{\text{lbm}}{\text{ft}^2\text{-sec}}$$

$$h = \frac{j * 1.28 \frac{\text{Btu}}{\text{lbm}\ ^\circ\text{R}} * 30.2 \frac{\text{lbm}}{\text{ft}^2\text{sec}} * 3600 \text{ sec/hr}}{(.67)^{2/3}}$$

$$h = 5.96 \times 10^3 \, j * G \qquad \left\{ \begin{array}{l} G \text{ in } \frac{\text{lbm}}{\text{ft}^2\text{sec}} \\ h = \text{Btu/ft}^2\ ^\circ\text{R} \end{array} \right.$$

$$h = (5.96 \times 10^3)(3.3 \times 10^{-3})(3.02 \times 10^1) = 593 \text{ Btu/hr-ft}^2$$

$$A = \frac{\pi}{4} D_o^2 = \frac{\pi}{4}(1.5537)^2 = 1.895 \text{ IN}^2 = .01316 \text{ ft}^2$$

$$hA = 5.93 \times 10^2 \frac{\text{Btu}}{\text{hr ft}^2\ ^\circ\text{R}} * 1.316 \text{ ft}^2 = 7.8 \text{ Btu/hr}\ ^\circ\text{R}$$

note:

with this approach we can get

hA (BTU/°R-Hr)

Dome & Cone	5.96
Inner Annulus	3.16
Outer Annulus	7.80
	16.92

Would like to see a little less pressure drop.

(1) Go to a clearance of .005

$D_i' = D_o - .01 = 1.5537 - .01 = 1.5437$

$A_F = \frac{\pi}{4}(D_o^2 - D_i^2) = \frac{\pi}{4}\left((1.5537)^2 - (1.5437)^2\right) = .02435 \text{ IN}^2$

$A_F = .02435 \text{ IN}^2 = .000169 \text{ ft}^2$

$D_H = .01 \text{ IN}$

Dead VoL
$V_d = .02435 \times .893 = .0217 \text{ IN}^3$

$\Delta P = \left(1.0 + \frac{4fL}{D_H}\right)\left(\frac{V_c^2}{2g_c}\right)\rho$

$Q = 2.955 \times 10^{-3} \text{ ft}^3/\text{sec}$ $\quad V_c = \frac{Q}{A_F} = \frac{2.955 \times 10^{-3} \text{ ft}^3/\text{sec}}{1.69 \times 10^{-4} \text{ ft}^2} = 17.5 \text{ ft/sec}$

$Re = V\left(\frac{D_H \rho}{\mu}\right)$ $\quad \left(\frac{D_H \rho}{\mu}\right) = 260 \times \frac{.01}{.0074} = 351$

$Re = (17.5)(351) = 6150$ $\quad f = .0092$

$\frac{4fL}{D_H} = \frac{(4)(.0092)(.893)}{.01} = 3.285$

$\frac{V_c^2}{2g_c} = \frac{(17.5)^2}{64.4} = 4.75$

$\Delta P = (1 + 3.285)(4.75)(2.2) = 44.8 \text{ lb/ft}^2 = .311 \text{ psi}$

check the hA

$$V_H = \frac{Q}{A_F} = \frac{1.716 \times 10^{-3}\ ft^3/sec}{1.69 \times 10^{-4}\ ft^2} = 10.17\ ft/sec$$

$$Re_H = (10.17)(351) = 3,560$$

$$j = .0033 = \frac{h}{C_p G} \ Pr^{2/3}$$

$$G = \rho V_H = (2.2)(10.17) = 22.3\ ft/sec$$

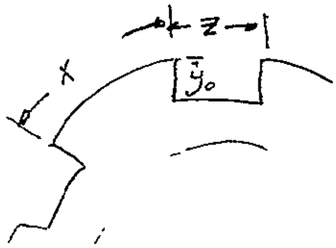
$$h = 5.96 \times 10^{-3}\ j' \times G = (5.96 \times 10^{3})(3.3 \times 10^{-3})(2.23 \times 10^{1}) = 440\ Btu/hr\text{-}ft^2$$

$$A_H = .01316\ ft^2$$

$$hA = (4.40 \times 10^{2})(1.316 \times 10^{-2}) = 5.78\ Btu/hr$$

	hA
Dome & Cone	5.96
Inner Annulus	3.16
Outer Annulus	5.78
	14.90

FINAL SELECTION OF SLOTTED ANNULUS PART OF HEAT EXCHANGER



First rough out a fin effectiveness

$$\eta = \frac{\tanh\left(\sqrt{\frac{2h}{kx}} * y_o\right)}{y_o\sqrt{\frac{2h}{kx}}}$$

We know from previous analysis that we want a flow area with $z = .09$ IN and $y_o = 0.0125$

Lets try a maximum y_o consistant with beef in present part

$y_o = .04$ IN leave .037 thickness at botton of slots

$z * 0.04 = (.0125)(.090)$

$z = .028$ $\big\{$ lets say (.03)

$C_{in} = \pi D = \pi(1.5537) = 4.88$ in

$\Sigma x_i = C - 16 * z = 4.88 - 0.48 = 4.4$ IN

$x = \frac{4.4}{16} = 0.275$ IN

error should be 1000 i.e. 2h

$k \cong 4.0$ Btu/hr ft °R $\qquad h \doteqdot 500$ Btu/hr ft² °R

$$y_o\sqrt{\frac{2h}{kx}} = \frac{.04 \text{ IN-ft}}{12 \text{ IN}}\sqrt{\frac{500 \text{ Btu/hr ft}^2 \cdot 12 \text{ IN/ft}}{4.0 \text{ Btu/hr f °R} * 0.275 \text{ IN}}} = .\cancel{245} = .347$$

$\tanh(.245) = .24$

$$\eta = \frac{.24}{.245} = .98 = \frac{.33}{.347} = .95$$

27

With $\eta = 0.98$ we can take at least three sides of the slot surface as effective heat transfer area.

$$A_H = \lambda * (z + 2*(.90)(y_0))$$

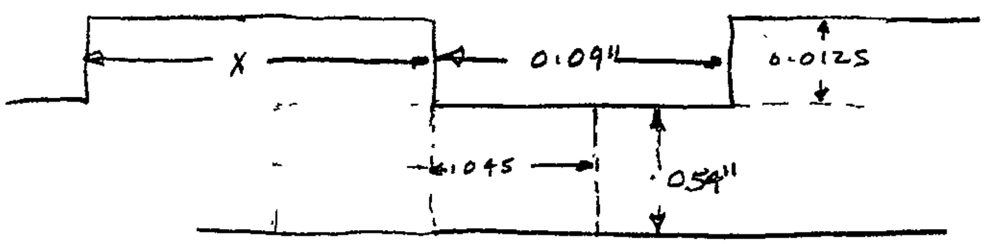
Use .90 to account for fin contact

$\lambda = .893$ in

$$A_H = (.893)(.03 + 1.80*(.04)) = (.893)(.102)$$

$$A_H = 0.0908\ in^2/slot$$

note this only slight improvement over considering only the direct contact area 0.09 wide slots. It is obvious that the maximum A_H can be obtained with wide slots if bottom of slot has a reasonable fin effectiveness



Set up a hypothetical worst case where we take a fin thickness x, = .054" and a fin length of (.0125 + .045) = .0575

$$y_0 \sqrt{\frac{h}{kx}} = \frac{.0575}{12\ IN} \sqrt{\frac{514\ Btu/hr\ °F\ ft^2\ 12\ IN/ft}{5.08\ Btu/hr\ ft\ °F * .054}} = 0.724$$

$$\eta = \frac{\tanh y_0 \sqrt{\frac{h}{kx}}}{y_0 \sqrt{\frac{h}{kx}}} = \frac{0.619}{.724} = 0.855$$

∴ bottom of slot is 85% effective

though sides of slots are effected by the flow of heat from botten lets see what the fin effectiveness is

$$y_o \sqrt{\frac{2h}{kx}}$$

$$\Sigma x_i = 4.88 - 16 * .09 = \quad 3.48$$

$$x = \frac{3.48}{16} = 0.217 \text{ IN}$$

$$y_o \sqrt{\frac{2h}{kx}} = \frac{0.0125}{12} \sqrt{\frac{(1028)(12)}{(5.0)(.217)}} = 0.111$$

$$\eta = \frac{\tanh 0.111}{0.111} = \frac{.10956}{.111} = 0.987$$

the heat flux from the bottom of the slot would not change this much.

Before assuming the effect of the contact between the top edge of the slots and the outer surface lets look at the temperature drop across a gas gap with a load of about 5 watts

$$A = \Sigma x_i * L = (3.48)(.893) = 3.10 \text{ IN}^2 = .0215 \text{ ft}^2$$

$$Q = \frac{k A \Delta T}{z}$$ assuming conduction only across the gas

$$\Delta T = \frac{Q z}{k A}$$

$k = 0.103$ Btu/hr ft-°R

$Q = 5w * 3.41$ Btu/hr w $= 17$ Btu/hr

$z_{max} = .0005''$
$z_{min} = .0002''$ } pressent design without srink fit

$$\Delta T = \frac{17 \frac{Btu}{hr} * .0005 \text{ IN} \quad \text{ft}}{0.103 \text{ Btu/hr-ft-°R} * 0.0215 \text{ ft}^2 * 12 \text{ IN}} = 0.319 \text{°R}$$

z (in)	ΔT °R	
.005	3.191	
.001	0.638	
.0005	0.319	max
.0003	0.191	
.0002	0.127	min

with good contact or small clearence there is just no problem. therefore we can use the fin effectiveness of the surfaces directly.

$$A_H = L * (z + \eta_1 z + 2 * \eta_2 y_0) * N$$

$$= L * (0.090 + 0.85 * 0.09 + (2)(0.98)(0.0125))\ N$$

$$= (0.893)(.09 + .0765 + .0245)\ N \quad = (0.893)(0.1910)\ 16$$

$$= 2.73\ \text{IN}^2 = 0.01895\ \text{ft}^2$$

h from previous calculations is 514 Btu/Hr-ft²°R

$$hA = (514)(1.895 \times 10^{-2}) = 9.75\ \text{Btu/hr}\cdot °R$$

SUMMARY OF COLD END HEAT EXCHANGER PERFORMANCE CHARACTERISTICS

	HA (Btu/hr-°R)	ΔP (PSI)
DOME HOLE		.15900
DOME AND CONE	5.96	.08580
TURN (1)		.04730
TURN (2)		.00900
INNER ANNULUS	3.16	.01100
180° TURN		.02887
OUTER SLOTS	9.75	.33000
TOTAL	18.87	0.671

SUMMARY

REWORK PER FINAL CONFIGURATION AFTER ACTUAL PARTS REWORKED

SUMMARY SEE ATTACHED CALC.

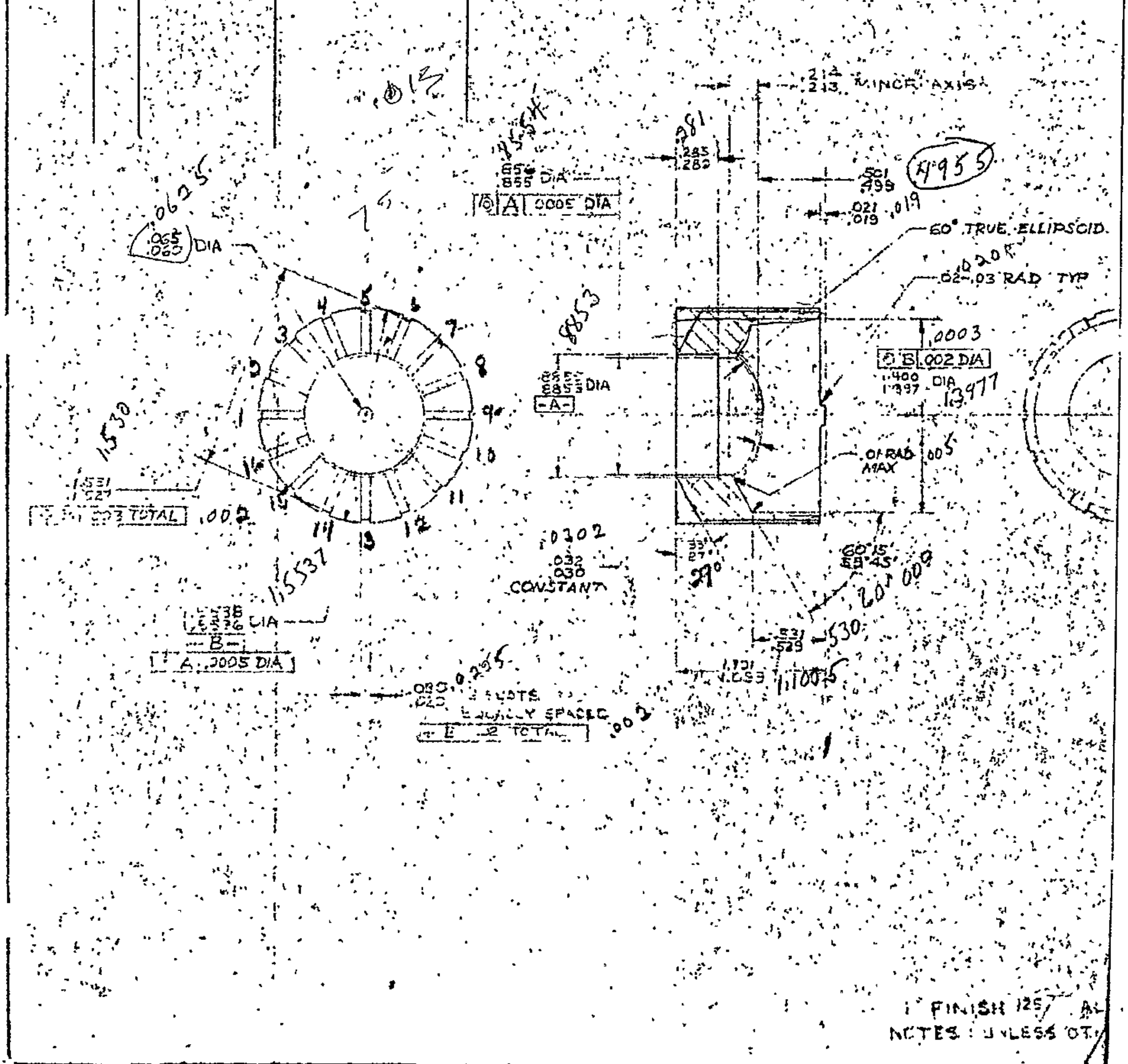
	HA Btu/hr°R	ΔP (PSI)
DOME HOLE		.123
DOME AND CONE	3.27	.198
INNER ANNULUS	3.16	.01100
180° TURN		.02887
OUTER SLOTS	9.25	.2590
TOTAL	15.68	0.6149

THESE ARE LATEST NO.S

pages which follow show where changes from the No.s at top of page come from

Preceding page blank

	WIDTH	DEPTH
1	[illegible]-.025	.0117-.0118
2	[illegible]-.024	.0118-.0117
3	.022-.024	.0121-.0120
4	.023-.025	.0120-.0120
5	.022-.025	.0120-.0120
6	.022-.025	.0116-.0120
7	.022-.024	.0116-.0121
8	.022-.024	.0109-.0113
9	.021-.024	.0114-.0115
10	.022-.024	.0111-.0110
11	.023-.024	.0121-.0114
12	[illegible]-.024	.0121-.0113
13	.021-.025	.0119-.0117
14	.023-.025	.0123-.0121
15	.023-.025	.0118-.0121
16	.023-.025	.0114-.0119



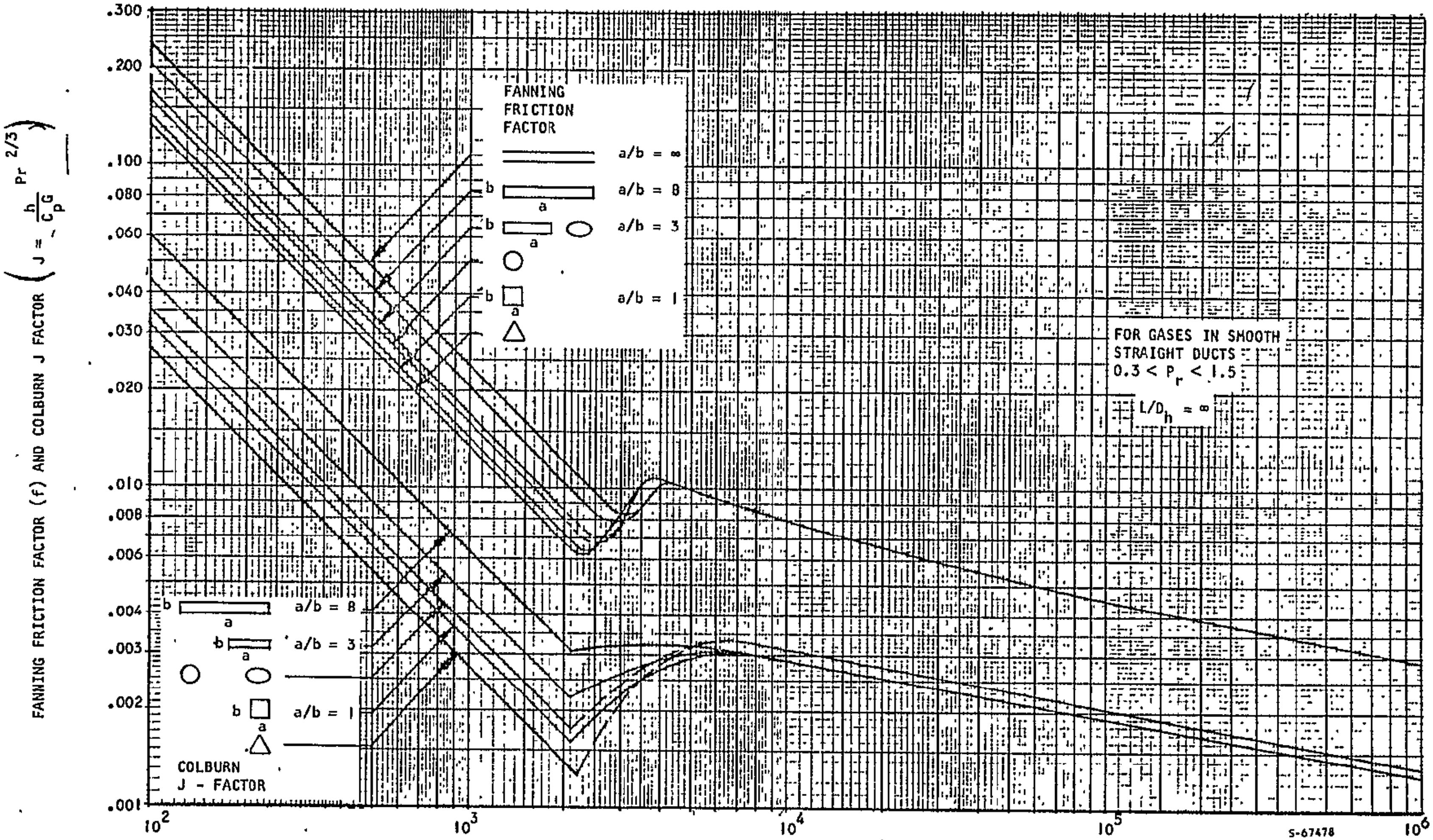


Figure 11. Fanning Friction Factor and Colburn J-Factor for Various Passage Geometries

OPERATING PARAMETERS

COLD VOLUME TEMP.	=	125.00 R
SUMP VOLUME TEMP.	=	620.00 R
HOT VOLUME TEMP.	=	1630.00 R
COLD REGEN. TEMP.	=	372.00 R
HOT REGEN. TEMP.	=	1125.00 R
COLD DISPLACED VOL.	=	.25500 CU-IN
HOT DISPLACED VOL.	=	6.80000 CU-IN
COLD DEAD VOL.	=	.08533 CU-IN
SUMP DEAD VOL.	=	5.46550 CU-IN
HOT DEAD VOL.	=	1.27200 CU-IN
COLD REGEN. VOL.	=	3.36000 CU-IN
HOT REGEN. VOL.	=	7.49000 CU-IN
GAS CONSTANT	=	4634.40 IN-LB/LBM-R
SPEED	=	400.00 RPM
CHARGE PRESSURE	=	540.00 PSIA
CHARGE TEMPERATURE	=	535.00 R
MASS OF FLUID	=	.0052 LBM
TOTAL VOLUME	=	24.72783 CU-IN

PRESSURE - MASS - FLOW PROFILE

ANGLE DEG	PC PSIA	PA PSIA	PH PSIA	VC CU-IN	VA CU-IN	VH CU-IN	MDOTC LB/SEC	MDOTA LB/SEC	MDOTH LB/SEC	MDOTRCA LB/SEC	MDOTRHA LB/SEC	DPC PSI	DPH PSI	DPCA PSI	DPHA PSI
24.	769.49	769.49	769.49	.0963	7.7268	6.0547	.00298	-.02712	.01530	.00800	.01912	.0000	.0000	.0000	.0000
48.	792.14	792.14	792.14	.1275	6.5520	7.1983	.00524	-.02265	.01165	.00852	.01413	.0000	.0000	.0000	.0000
72.	802.76	802.76	802.76	.1734	5.7991	7.9053	.00648	-.01280	.00513	.00715	.00565	.0000	.0000	.0000	.0000
96.	798.47	798.47	798.47	.2261	5.5982	8.0535	.00626	.00008	-.00263	.00415	-.00423	.0000	.0000	.0000	.0000
120.	780.47	780.47	780.47	.2765	5.9841	7.6172	.00469	.01243	-.00951	.00036	-.01279	.0000	.0000	.0000	.0000
144.	753.36	753.36	753.36	.3159	6.8901	6.6718	.00230	.02127	-.01390	-.00320	-.01807	.0000	.0000	.0000	.0000
168.	723.19	723.19	723.19	.3375	8.1596	5.3807	-.00026	.02528	-.01530	-.00579	-.01949	.0000	.0000	.0000	.0000
192.	695.51	695.51	695.51	.3376	9.5731	3.9672	-.00249	.02473	-.01405	-.00715	-.01758	.0000	.0000	.0000	.0000
216.	674.31	674.31	674.31	.3160	10.8863	2.6755	-.00414	.02062	-.01087	-.00733	-.01329	.0000	.0000	.0000	.0000
240.	662.00	662.00	662.00	.2767	11.8723	1.7289	-.00511	.01404	-.00645	-.00652	-.00752	.0000	.0000	.0000	.0000
264.	659.75	659.75	659.75	.2263	12.3606	1.2909	-.00540	.00587	-.00136	-.00489	-.00098	.0000	.0000	.0000	.0000
288.	667.75	667.75	667.75	.1735	12.2669	1.4374	-.00499	-.00315	.00397	-.00262	.00577	.0000	.0000	.0000	.0000
312.	685.28	685.28	685.28	.1276	11.6072	2.1430	-.00387	-.01226	.00906	.00015	.01211	.0000	.0000	.0000	.0000
336.	710.53	710.53	710.53	.0964	10.4958	3.2857	-.00205	-.02036	.01325	.00316	.01720	.0000	.0000	.0000	.0000
360.	740.22	740.22	740.22	.0853	9.1245	4.6680	.00035	-.02592	.01565	.00599	.01993	.0000	.0000	.0000	.0000

IDEAL REFRIGERATION AND HEAT INPUT

REFRIGERATION = 21.0761 WATTS
THERMAL HEAT = 127.4216 WATTS
MAX. PRESSURE = 803.1153 PSIA

39

REVISED DOME AND CONE SECTION HEAT TRANSFER (update)

Before $D_H = .0250$ in $C = .0125$ in

New $D_H = .0400$ in $C = .020$ in

$$h = \frac{k}{D_H} N_u$$

Before $h = 385\left(\frac{1}{d}\right)^{0.8} \frac{Btu}{hr\,ft^2\,{}^\circ R}$ {d in inches}

$$Re \neq f(D_H) \neq f(C)$$

i.e.

$$Re = \frac{V D_H \rho}{\mu} \qquad D_H = 2 * C$$

$$V = \frac{Q}{A_c} \qquad A_c = \pi d C$$

$$Re = \frac{Q\, 2 * C\, \rho}{\pi d C\, \mu} = \frac{2 Q \rho}{d \mu} \neq f(C)$$

$\therefore$

$$h_{NEW} = h_{OLD} * \frac{D_{H\,old}}{D_{H\,new}} = h_{old} * \frac{.0250}{.0400} = .625$$

$\therefore$

$$\tilde{h}A = (0.625)(4.2)\left[\frac{D_o^{1.2} - D_i^{1.2}}{1.2}\right] = 2.19\left[D_o^{1.2} - D_i^{1.2}\right]$$

$D_o = 1.632 \qquad D_o^{1.2} = 1.80$

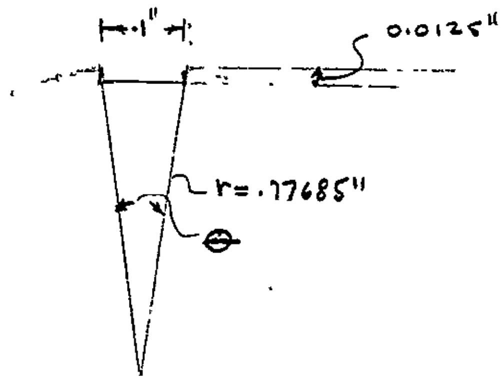
$D_i = 0.375 \qquad D_i^{1.2} = .308$

$$hA = 2.19\left[1.80 - .308\right] = 2.19\{1.492\} = 3.27\ Btu/hr\,{}^\circ R$$

FINAL DESIGN OF ANNULAR SLOTS

From previous calculations we have shown that slots 0.90" wide by .0125 deep given an acceptable solution; though pressure drop of .33 psi was a little higher than desired. Lets consider a slightly wider slot for final design: (1) this will give us a smaller pressure drop at the design conditions (400rpm) and (2) if we need to increase the speed to 600rpm the pressure drop should still be maintained in the range of .5psi or so. Lets look at a slot configuration of .1 x .0125

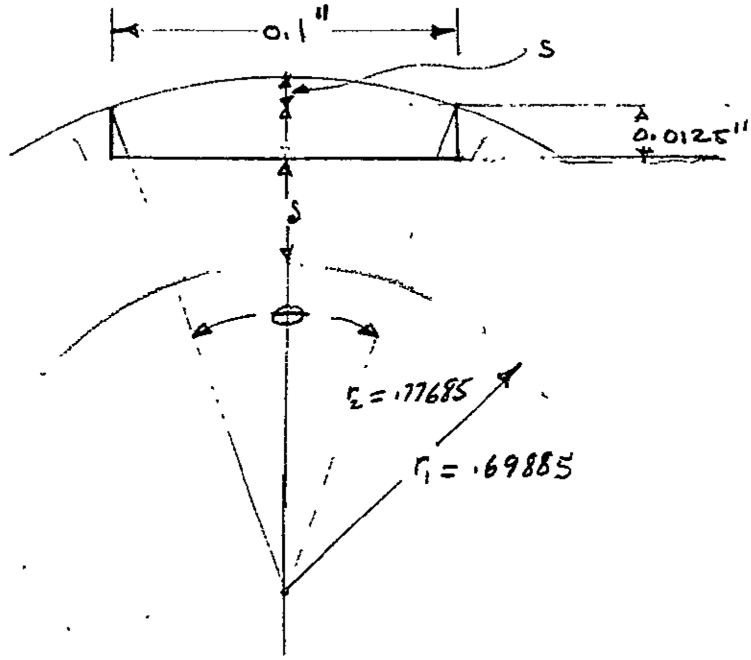
1.0 CONFIGURATION



Flow Cross Section

(36) (2)

In the previous calculations the flow area due to the curved surface has not been included



Per Slot

$A_F' = A_c + A_r$

$A_F' = .0013149 \ IN^2$

Total Cross section for 16 slots

$A_F = 16 A_F' = .021038 \ IN^2$

$A_F = .021038 IN^2 = .0001461 \ ft^2$

$A_r = (0.1 * 0.0125) = .00125 \ IN^2$

$A_c = r_2^2 \left\{ \frac{\pi \theta}{360} - \frac{\sin\theta}{2} \right\}$

$\sin\frac{\theta}{2} = \frac{.05}{.77685} = .064362$

$\frac{\theta}{2} = 3^\circ 41.35''$

$\theta = 7^\circ 22.7' = 7.378^\circ$

$\sin\theta = 0.12849$

$A_c = (.77685)^2 \left\{ \frac{(\pi)(7.378)}{360} - \frac{0.12849}{2} \right\}$

$A_c = .6034959 \left\{ 0.06435255 - .064245 \right\}$

$A_c = .0000649 \ IN^2$

$\underline{A_F = .021038 IN^3 = .0001461 \ ft^2}$

Dead volume

$$V_D = A_F * L = .021038 * 1.1005 =$$

$$V_D = 0.02318 \text{ IN}^3$$

Depth at center of slot

$$d = s + d' \qquad d' = .0125''$$

$$s = r_2 - r_2 \cos\frac{\theta}{2} = r_2\left(1 - \cos\frac{\theta}{2}\right) \qquad \cos\frac{\theta}{2} = .997925$$

$$s = .77685 * 0.002075 = .00161$$

$$d = .0141 \qquad \left\{ \text{use } .014 \text{ to } .015 \right.$$

Thickness of Material Under Slot

$$\delta = r_2 - d - r_1 = .77685 - .0145 - .69885$$

$$\delta = .06350''$$

2.0 PRESSURE DROP

ΔP @ Design Speed of 400 RPM

$A_F = .0001461\ ft^2$

$$D_H \cong \frac{2(.0125)(.1)}{0.1125} = .0222$$

$$Q = \frac{\dot{w}}{\rho} = \frac{.0065\ lb/sec}{2.2\ lb/ft^3} = .002955\ ft^3/sec$$

$$V_s = \frac{Q}{A_F} = \frac{.002955\ ft^3/sec}{.0001461\ ft^2} = 20.2\ ft/sec$$

$$\frac{V_s^2}{2g_c} = \frac{(20.2)^2}{64.4} = 6.33\ \frac{lbf\text{-}ft}{lbm}$$

$$Re = \frac{V D_H \rho}{\mu} = \frac{(20.2)\frac{ft}{sec} * 0.0222\ in * 2.2\frac{lbm}{ft^3} * ft\text{-}hr * ft * 3600\frac{sec}{hr}}{.0184\ lbm * 12\ in}$$

$$= 15,660$$

$f = .007$

End effects

$$\frac{4fL}{D_H} = \frac{(4)(.007)(.893)}{.0222} = 1.125$$

$$\Delta P = \left(1.5 + \frac{4fL}{D_H}\right)\frac{V_s^2}{2g_c}\rho$$

$$\Delta P = (1.5 + 1.125)\left(6.33\ \frac{lbf\text{-}ft}{lbm}\right) 2.2\ \frac{lbm}{ft^3} = 36.6\ lbf/ft^2$$

$\underline{\Delta P = 0.254\ PSI}$ $\left\{ \begin{array}{l} RPM = 400 \\ \dot{w}_{max} = .0065\ lb/sec \end{array} \right.$

ΔP @ 600 RPM

$$\dot{w}_{RPM=600} = (1.5)(.0065) = .00975 \text{ lb/sec}$$

check by idea VM program

$$V_s = (1.5)(20.2) = 30.3 \text{ ft/sec}$$

$$Re = (1.5)(15,660) = 23,450 \rightarrow f = .0063$$

$$\frac{4fL}{D} = \frac{.0063}{.0070} * 1.125 = 1.012$$

$$\Delta P = \left(1.5 + 1.012\right) \frac{V_2}{2g_c} \cdot \rho$$

$$= \left(2.512\right)\left(\frac{(30.3)^2}{64.4}\right)(2.2) = 78.5 \text{ lbf/ft}^2$$

$$\Delta P = 78.5 \frac{\text{lbf}}{\text{ft}^2} = 0.545 \text{ psi}$$

looks good

3.0 HEAT TRANSFER CHARACTERISTICS

$V_s' = \frac{Q}{A_F}$ $\quad Q = .001716\ ft^3/sec \rightarrow$ based on average flow @ 400RPM

$$V_s' = \frac{0.001716\ ft^3/sec}{0.0001461\ ft^2} = 11.72\ ft/sec$$

$$Re = \frac{V_s' D_H \rho}{\mu} = \left(\frac{11.72}{20.2}\right) * 15{,}660 = 9070$$

$$j = .0029 = \frac{h}{C_p G} P_r^{2/3}$$

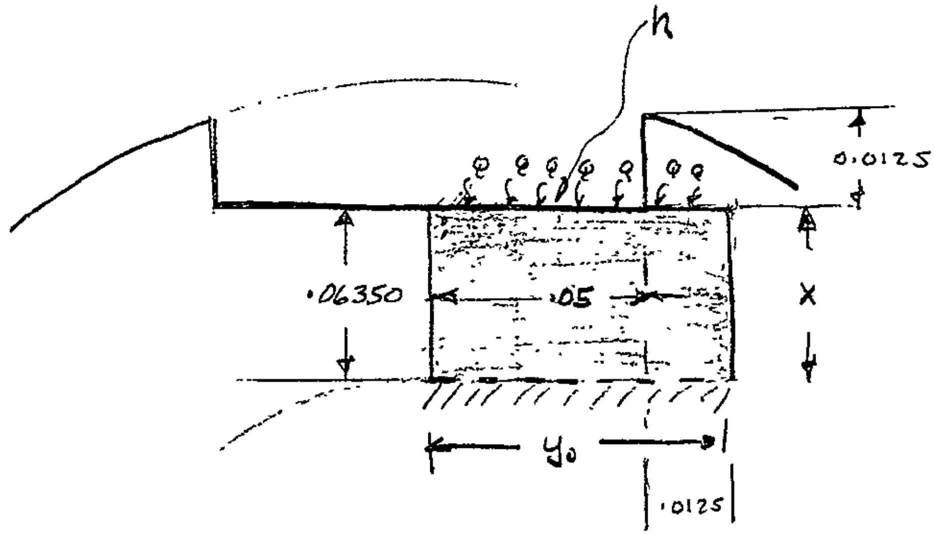
$$h = \frac{j * C_p * G}{P_r^{2/3}} = \frac{j * 1.28 * G}{(6.67)^{2/3}} * 3600\ sec/hr$$

$$h = 5.96 \times 10^3\ j * G \left\{ \begin{array}{l} G \text{ in } lbm/ft^2\text{-}sec \\ h \text{ in } Btu/ft^2\ °R \end{array} \right.$$

$$G = \rho V_s' = (2.2)(11.72) = 25.8\ \frac{lbm}{ft^2\text{-}sec}$$

$$h = (5.96 \times 10^3)(2.9 \times 10^{-3})(25.8) = \underline{446}\ Btu/hr\ ft^2\ °R$$

Now for bottom an sides of slots look at fin effectivness Take a fin shap as shown in the following figure



Equivalent Fin Configuration - one sided

$$\eta = \frac{\tanh y_0 \sqrt{\frac{h}{kX}}}{y_0 \sqrt{\frac{h}{kX}}}$$

$$y_0 \sqrt{\frac{h}{kX}} = \frac{.0625 \text{ IN} \cdot \text{ft}}{12 \text{ IN}} \sqrt{\frac{446 \text{ Btu/hr °R ft}^2 \times 12 \text{ IN/ft}}{5.0 \text{ Btu/hr ft °R} \times .06350}}$$

$$= .674$$

$$\eta = \frac{\tanh(.674)}{.674} = \frac{0.587}{.674} = 0.872$$

Therefor we can take 87% of the bottom and sides of the slots as effective heat transfer area neglecting small added area due to curved outer surface (A2) (8)

$$A_{eff} = N * L * \left(w + 0.87 (w + (2)(d')) \right)$$

$w = .1$ IN
$d' = .0125$ $L = .893$ IN {Could actually use a little larger L}
$N = 16$

$$A_{eff} = 16 * 0.893 * \left(.1 + .87 (.1 + .025) \right) = 2.99 \text{ IN}^2$$

$$A_{eff} = 2.99 \text{ IN}^2 = .02075 \text{ ft}^2$$

$$hA = (4.46)(.02075) = 9.25 \text{ Btu/hr °R}$$

4.0 SUMMARY OF FINAL SLOT DESIGN

Width $= .100 \begin{smallmatrix} +.01 \\ -.00 \end{smallmatrix}$

Depth from surface at center $= 0.014''$ / 0.015 {.0125 min at scale}

$A_F = .0001461 \text{ ft}^3$ min (16 total)

Void Volume $= .02318 \text{ IN}^3$

ΔP max @ 400 rpm = 0.254 psi
" 600 rpm = 0.595 psi

h @ average flow = 446 Btu/hr-ft^2 °R

A_{eff} for heat transfer $= .02075 \text{ ft}^2$

$hA = 9.25$ Btu/hr °R

COLD END PRESSURE DROP

LOOK AT PRESS DROP ACROSS DISC MODEL OF DOME AND CONE REGION FOR INWARD FLOW:

(RESULTS NOW SAY ~.29 psi)

$$\Delta p = \int 4 \frac{f}{D_h} \frac{\rho V^2}{2 g_c} dx$$

$$f = .084 Re^{-1/4} = .084 \left(\frac{\mu}{D_h \rho V}\right)^{1/4}$$

$$D_h = 2c$$

$$V = \frac{\dot{w}}{\rho A} = \frac{\dot{w}}{\pi \rho D c}$$

$$x = \frac{D_o - D}{2} \qquad D = D_o - 2x$$

$$V = \frac{\dot{w}}{\pi \rho c (D_o - 2x)}$$

$$f = .084 \left[\frac{\pi \mu \rho c (D_o - 2x)}{2 \rho c \dot{w}}\right]^{1/4} = .084 \left[\frac{\pi \mu (D_o - 2x)}{2 \dot{w}}\right]^{1/4}$$

$$\Delta p = \frac{.084 \rho}{c g_c} \int \left[\frac{\pi \mu (D_o - 2x)}{2 \dot{w}}\right]^{1/4} \left(\frac{\dot{w}}{\pi \rho c (D_o - 2x)}\right)^2 dx$$

$$= \frac{.0707}{g_c} \left(\frac{\dot{w}}{\pi}\right)^{1.75} \frac{\mu^{.25}}{\rho c^3} \int (D_o - 2x)^{-1.75} dx$$

$$= 2.97 \times 10^{-4} \frac{\dot{w}^{1.75} \mu^{.25}}{\rho c^3} \left[\frac{(D_o - 2x)^{-.75}}{1.5}\right]_0^{x_{max}}$$

$$= 1.98 \times 10^{-4} \frac{\dot{w}^{1.75} \mu^{.25}}{\rho c^3} \left[(D_o - 2x_{max})^{-.75} - D_o^{-.75}\right]$$

$$x_{max} = \frac{D_o - D_i}{2}$$

$$\Delta p = -1.98 \times 10^{-4} \frac{\dot{w}^{1.75} \mu^{.25}}{\rho c^3} \left(D_o^{-.75} - D_i^{-.75}\right) \qquad lb_f/ft^2$$

(AA) (2)

$$\Delta p = 1.375 \times 10^{-6} \frac{\dot{w}^{1.75} \mu^{.25}}{\rho c^3} \left(D_i^{-.75} - D_o^{-.75} \right) \quad , \text{psi}$$

$\dot{w} = .0065$ lbm/sec $\qquad \dot{w}^{1.75} = .00015$

$\mu = 5.25 \times 10^{-6}$ lbm/sec-ft $\qquad \mu^{.25} = .048$

$\rho = 2.2$ lbm/ft³

$c = \frac{.0125}{12} = .00104$ ft $\qquad c^3 = 1.126 \times 10^{-9}$

$D_o = \frac{1.632}{12} = .136$ ft $\qquad D_o^{-.75} = 4.47$

$D_i = \frac{.144}{12} = .012$ ft $\qquad D_i^{-.75} = 27.6$

$$\Delta p = (1.375 \times 10^{-6}) \frac{(.0015)(.048)}{(2.2)(1.126 \times 10^{-9})} \overset{23.13}{(27.6 - 4.47)} = .925 \text{ psi}$$

AT CENTRAL HOLE IN DOME, VELOCITY HEAD IS:

$$V = \frac{.0065}{\pi (2.2)(.012)(.00104)} = 75.2 \text{ fps}$$

$$H_v = \frac{2.2 (75.2)^2}{(2)(32.2)(144)} = 1.34 \text{ psi}$$

FOR FLOW INTO COLD REGION, THIS IS PROBABLY LOST

ASSUME 2.25 H_v FOR FLOW THRU VORTEX & HOLE AND EXPANSION INTO COLD REGION

IF TURNS CAUSE A LOSS OF .056 PSI, TOTAL LOSS FOR DOME/CONE REGION IS, FOR INWARD FLOW

$$\Delta p_{TOT} = .925 + .056 + \overset{3.02}{(2.25)(1.34)} = \underline{4.001 \text{ psi}}$$

FOR $C = .020\text{ in} = .00167\text{ ft}$ $\qquad C^3 = 4.62 \times 10^{-9}$

$D_i = .375\text{ in.} = .03125\text{ ft}$ $\qquad D_i^{-.75} = 13.5$

$$\Delta p = \left(\frac{1.126}{4.62}\right)\left(\frac{14.1}{23.13}\right)(.925) = .138\text{ psi}$$

$$H_V = \left(\frac{.012}{.03125}\right)^2\left(\frac{.00104}{.00167}\right)(1.34) = .123\text{ psi} \quad \text{AT } D_i$$

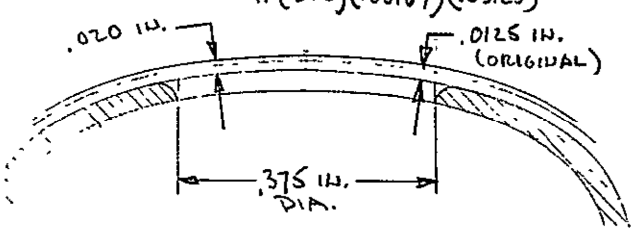
FOR FLOW THRU HOLE

$$V_H = \frac{(4)(.0065)}{\pi(2.2)(.03125)^2} = 3.9\text{ ft/sec}$$

IN DOME CHANNEL AT EDGE OF HOLE

$$V_c = \frac{.0065}{\pi(2.2)(.00167)(.03125)} = 18\text{ ft/sec}$$

$$\frac{V_H}{V_c} = .217$$



APPROX 4:1 SCALE

IF EXPANSION INTO THE LARGE HOLE IS TREATED AS A SUDDEN EXPANSION, LOSS WOULD BE EQUAL TO H_V. HOWEVER, MANIFOLD DATA AT RIGHT INDICATES THAT ROUNDING OF HOLE EDGE SHOULD REDUCE LOSS BELOW H_V.

$\therefore$ USE $\Delta p = .123$ psi FOR CONSERVATISM

Figure H3.7, Ref. 2

TAKING BEND LOSSES TO BE AS ORIGINALLY CALCULATED;

i.e., $\Delta p = .06$ psi,

TOTAL LOSSES FOR DOME/CONE SECTION IS

FLUID FRICTION	.138
BENDS	.06
EXPANSION	.123
	.321 psi

FOR FLOW OUT OF COLD REGION TO HEAT EXCHR, LOSSES AT HOLE SHOULD BE LOWER:

SAY, $\Delta p = .7 H_v = .09$ psi

$$\Delta P_{TOTAL} = \begin{cases} .32 \text{ psi} & \text{FOR INFLOW} \\ .29 \text{ psi} & \text{FOR OUTFLOW} \end{cases}$$

NOW, ASSUME THAT CLEARANCE c ON DOME SURFACE VARIES DIRECTLY WITH RADIUS AS FOLLOWS:

$C_0 = .020$ AT EDGE

$C = .030$ AT HOLE

EQUIV DISC DIA FOR DOME ALONE

$$D_D = \sqrt{4/\pi\,(1.018)} = 1.14 \text{ in.} \qquad \begin{cases} X_1 = \frac{1.632-1.14}{2} = .246 \\ X_2 = \frac{1.632-.375}{2} = .628 \end{cases}$$

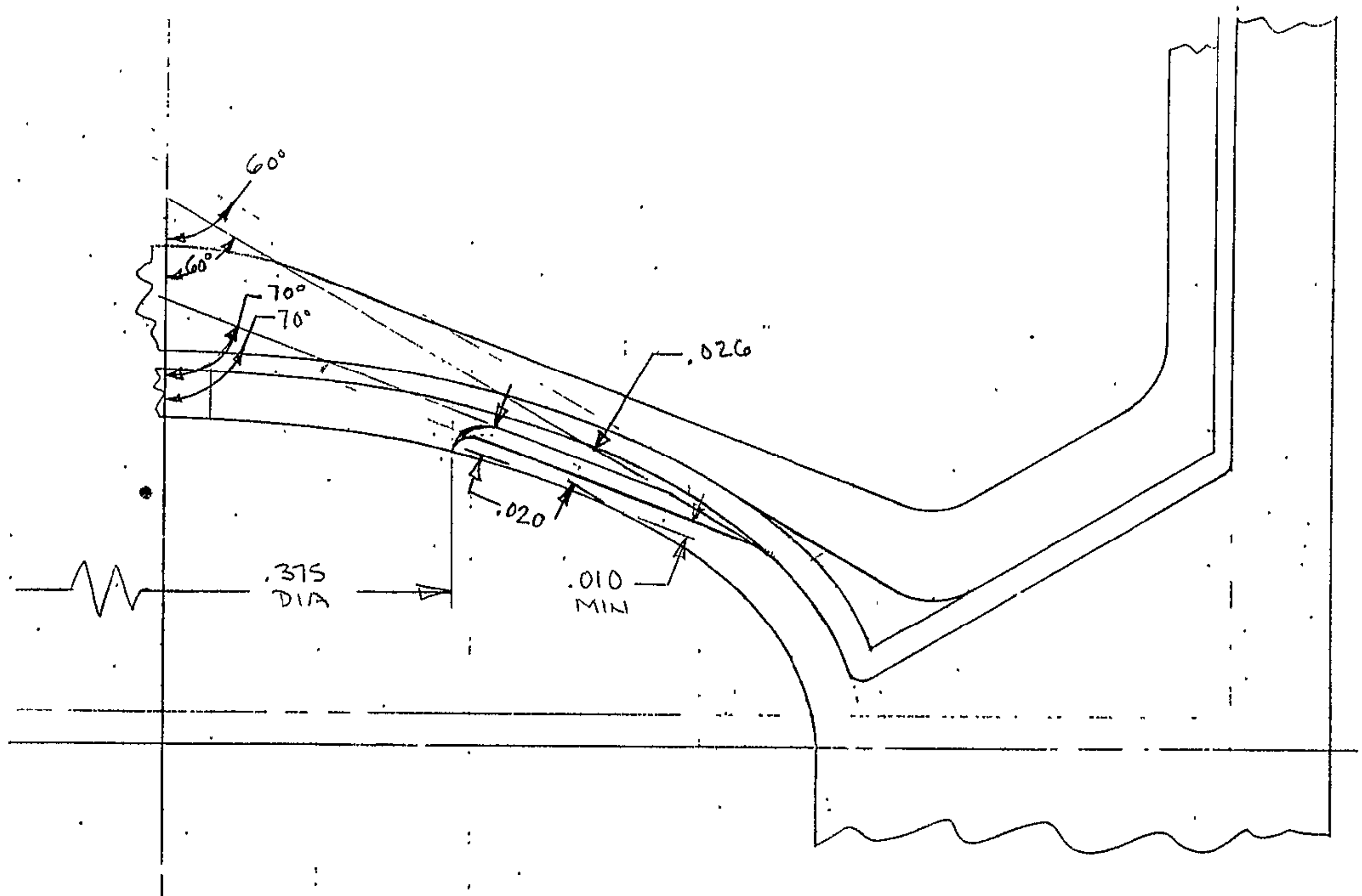
$$C = .0262x + .0135 \quad ; x > .246$$

$$\Delta p = 1.375 \times 10^{-6} \frac{\dot{w}^{1.75} \mu^{.25}}{\rho C_0^3} \left(D_D^{-.75} - D_0^{-.75} \right)$$

$$+ 2.06 \times 10^{-6} \frac{\dot{w}^{1.75} \mu^{.25}}{\rho} \int_{x_1}^{x_2} \frac{(D_0 - 2x)^{-1.75}}{(.0262x + .0135)^3} dx$$

INTEGRATION WILL BE TOO DIFFICULT TO BE WORTHWHILE

DETERMINE MODIFICATION TO DOME REGION
TO GIVE .020 IN. MIN CLEARANCE



SCALE 10:1

$V_0 = .08533 \text{ in}^3$

$\Delta V_{RED} = .01624 \text{ in}^3 = .19 V_0$

$\Delta V_{GREEN} = .01295 \text{ in}^3 = .15 V_0$

$\Delta V_{DISTR} = .0525 \text{ in}^3 = .615 V_0$

5

⑥

VOID VOLUME CALCULATIONS

P/N 852344, WITH SINGLE 70° CUT (IN RED ON LAYOUT)

ELEMENT DIMS	AREA IN^2	r IN.	Ar IN^3
(.09)(.031)	.00279	.075	.000209
(.069)(.031)	.00214	.154	.000330
(.165)(.015)	.00248	.265	.000657
(.015)$\left(\frac{.010}{2}\right)$	.00008	.193	.000015
(.061)$\left(\frac{.015}{2}\right)$	.00046	.359	.000165
(.165)(.004)	.00066	.265	.000175

$\Sigma A = .00861$ $\qquad \Sigma Ar$ .001551

$$\bar{r} = \frac{.001551}{.00861} = .180 \text{ in.}$$

$$\Delta V_{VOID} = 2\pi(.180)(.00861) = \underline{.00974 \text{ in}^3}$$

WITH 60° & 70° CUTS (IN GREEN ON LAYOUT):

(.022)$\left(\frac{.0015}{2}\right)$	.0000165	.206	.0000034
(.018)$\left(\frac{.011}{2}\right)$	.0000990	.207	.0000205
(.012)$\left(\frac{.005}{2}\right)$	.0000300	.197	.0000059
(.122)(.011)	.0013420	.271	.0003637
(.068)$\left(\frac{.011}{2}\right)$	.0003740	.348	.0001302
	.0018615		.0005236

$\bar{r} = .2813$ in. $\qquad 2\pi\bar{r}\Sigma A = .00329 \text{ in}^3$

$$\Delta V_{VOID} = .00974 - .00329 = \underline{.00645 \text{ in}^3}$$

(49) (7)

P/N 852326

ELEMENT DIMS	AREA IN^2	r IN	$A r$ IN^3
$(.055)\left(\frac{.009}{2}\right)$	.000248	.417	.0001032
$(.040)\left(\frac{.009}{2}\right)$	.000180	.435	.0000783
$(.047)\left(\frac{.070}{2}\right)$	.001645	.466	.0007666
$(.034)\left(\frac{.010}{2}\right)$	.000170	.503	.0000855
	.002243		.0010336

$$\bar{r} = .4609 \text{ in.}$$

$$\Delta V_{VOID} = 2\pi(.4609)(.002243) = .00650 \text{ in}^3$$

TOTAL VOLUME INCREASE ($V_o = .08533 \text{ in}^3$ FOR COLD END)

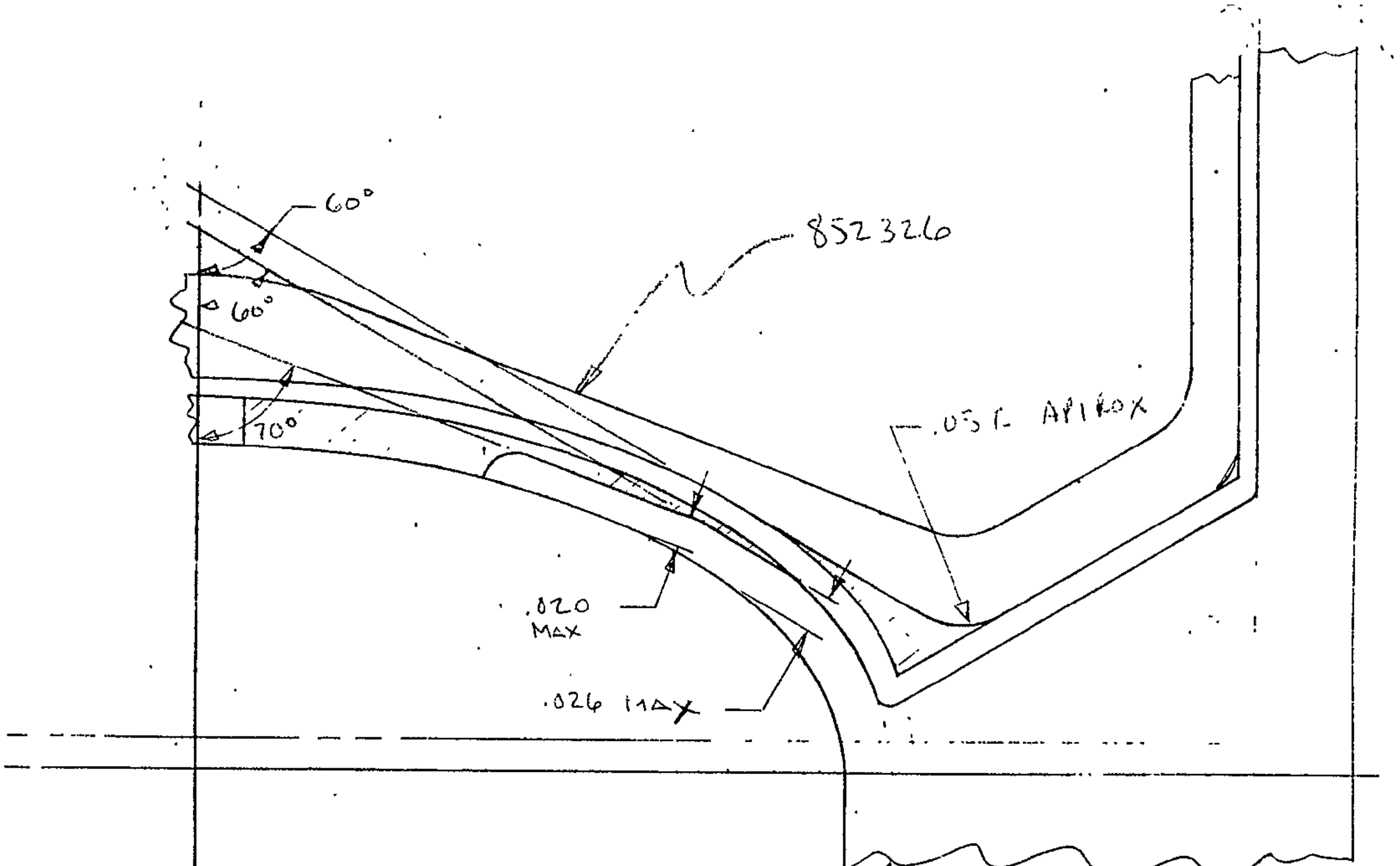
RED APPROACH : $\Delta V_{VOID} = .00974 + .00650 = \underline{.001624 \text{ in}^3}$

$= .19 V_o$

GREEN APPROACH : $\Delta V_{VOID} = .00645 + .00650 = \underline{.01295 \text{ in}^3}$

$= .152 V_o$

CHOOSE GREEN APPROACH — THIS ALSO IS SAFEST REWORK APPROACH



$V_0 = .08533\ in^3$

$\Delta V_{VOID} = .01295\ in^3 = .152 V_0$

COLD END DOME FLOW MODIFICATION

58

8

SECTION 5

COLD-END FLOW DISTRIBUTOR

INTRODUCTION

Test results from the AiResearch IR & D VM refrigerator revealed non-uniform flow in the cold end of the refrigerator as a potential problem for the 5-w GSFC VM refrigerator. The configuration of the cold end of the AiResearch VM refrigerator (cold end heat exchanger, displacer, and cold regenerator) is very similar to that of the GSFC VM refrigerator. Initial tests on the AiResearch refrigerator indicated an unbalance in flow in the cold end heat exchanger and low temperature end of the cold regenerator. To overcome this problem, a flow distributor was designed and installed in this refrigerator. As a result of the successful testing of this flow distributor, the same basic design was incorporated into the GSFC VM refrigerator.

DESIGN CONFIGURATION

The configuration of the cold end flow distributor is shown in Figure 5-1. The distributor consists of a perforated plate with standoff rings, which forms an annular cavity. The distributor is located between the cold end heat exchanger and the low temperature end of the cold regenerator. The ratio of axial-to-circumferential pressure drop is adjusted to allow circumferential flow around the annular cavity in sufficient quantity to redistribute any non-uniformity in flow entering either face of the distributor.

The design criteria for the flow distributor consist of three inter-related parameters: (1) axial-to-circumferential pressure drop ratio, (2) axial pressure drop, and (3) dead or void volume. Selecting the best combination of these parameters involves engineering judgement, since exacting tradeoffs are impractical.

Experience indicates that an axial-to-circumferential pressure drop ratio of 5 or greater will provide redistribution of the flow around the face area of the distributor for any reasonable upstream flow unbalance. Conservatively, a minimum axial-to-circumferential pressure drop ratio of 10 was used as the prime design criterion, assuming it was necessary to distribute 25 percent of the total flow one-half the distance around the annular cavity or through 180 degrees.

For maximum thermal performance of the refrigerator, both the axial pressure drop and dead volume associated with the flow distributor should be minimized. In the actual case, minimization of one of these parameters means enlarging the other. The size of the annular cavity of the flow distributor is the major factor controlling the dead volume of the distributor. The size of the cavity also controls the circumferential pressure drop; larger cavities yield lower pressure drops. Thus with the ratio of axial-to-circumferential pressure drop set as the prime design consideration, if the allowable axial pressure drop is set too low, the void volume becomes excessive in order to maintain the circumferential pressure drop at an acceptable level.

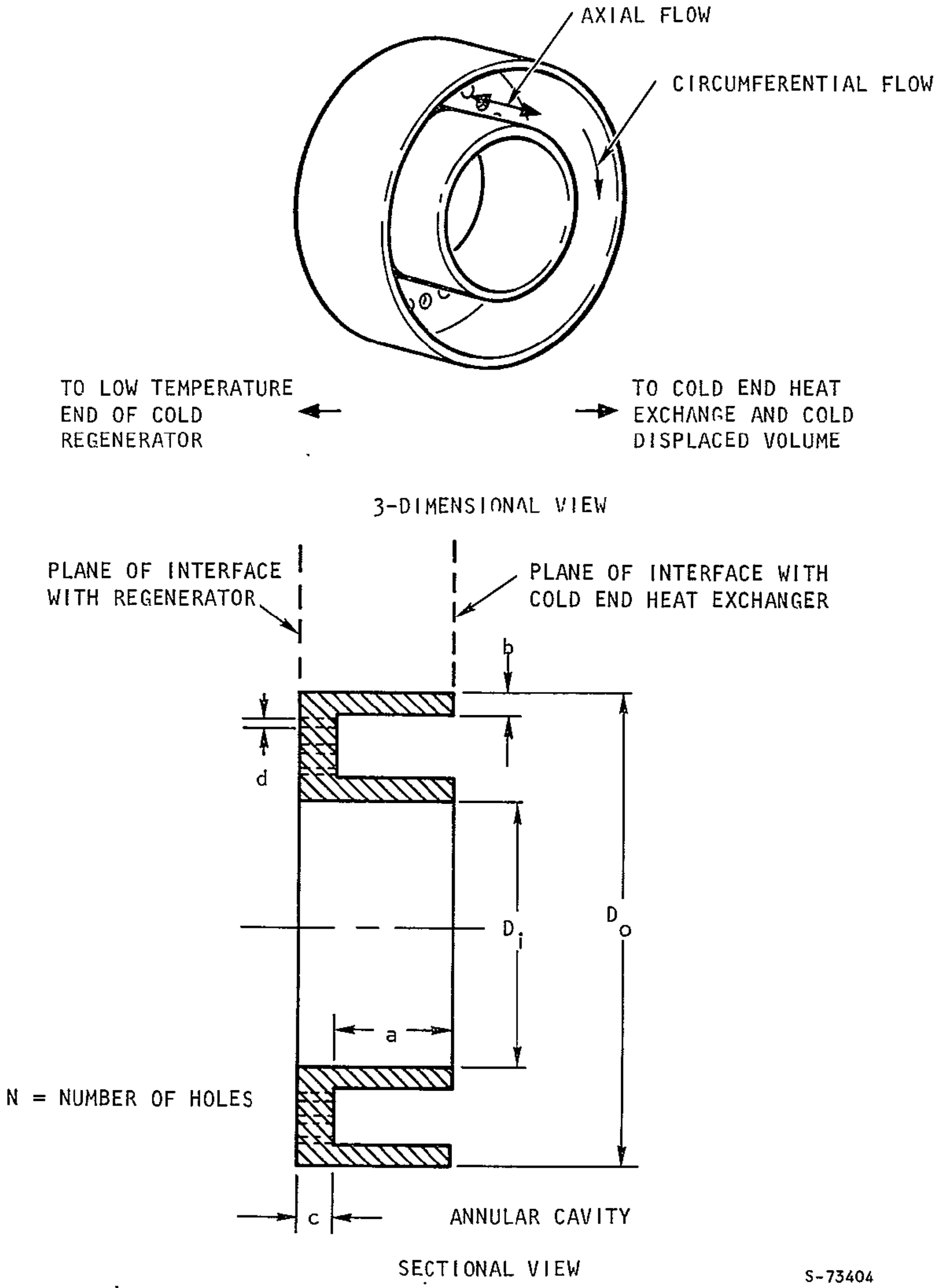


Figure 5-1. Cold End Flow Distributor Configuration

Preliminary calculations indicated that reasonable flow distributor designs could be achieved if the axial pressure drop were limited to 10 percent of the cold regenerator pressure drop and the dead volume limited to under 1 percent of the total dead volume. These values were used as guidelines in establishing the final design of the flow distributor. The characteristics of the final flow distributor are given in Table 5-1; Figure 5-1 defines the various dimensions.

TABLE 5-1

COLD END FLOW DISTRIBUTOR CHARACTERISTICS

Parameter	Value
Dimensions	
Outside diameter (D_o), in.	1.552
Inside diameter (D_i), in.	0.880
Annular cavity depth (a), in.	0.045
Ring thickness (b), in.	0.025
Hole diameter (d), in.	0.020
Plate thickness (c), in.	0.040
Number of holes	80
Void volume, cu. in.	0.055
Pressure drop ratio $\frac{\Delta P_{axial}}{\Delta P_{circ}}$	12
Maximum axial pressure drop, psi	0.1

PERFORMANCE CHARACTERISTICS

Figure 5-2 gives the effect of the flow rate of the working fluids and the dimensional tolerance of the annular cavity on the axial-to-circumferential pressure drop ratio. As the flow rate decreases the pressure drop ratio also decreases; for this reason the average flow rate was used in lieu of the maximum flow in establishing the configuration yielding a maximum pressure drop ratio of 10.

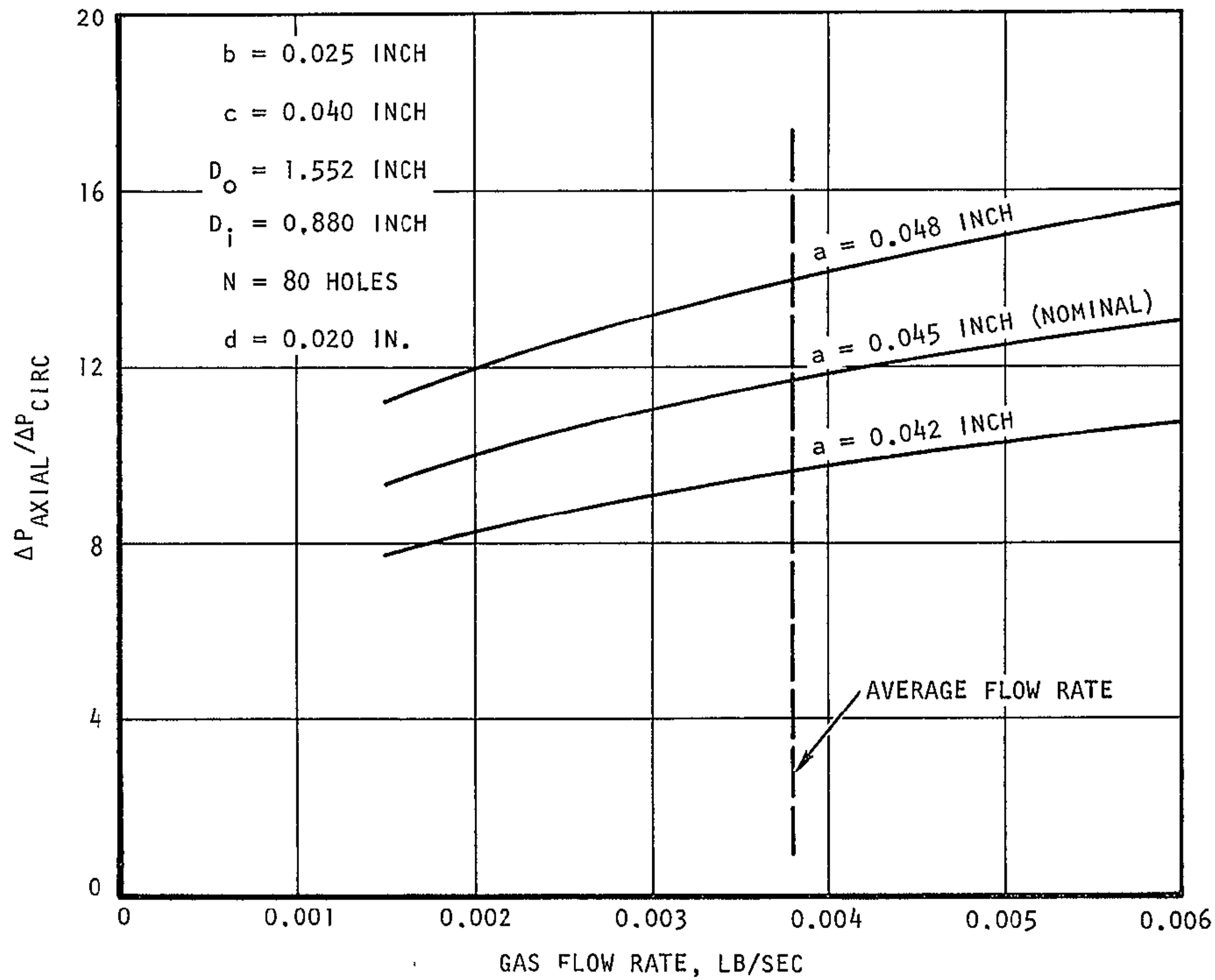


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Figure 5-2. Influence of Gas Flow Rate and Dimension Tolerance of Annular Cavity on Pressure Drop Ratio

Figure 5-3 gives the axial pressure drop as a function of flow rate for three hole sizes covering a range of tolerances on the diameter of the holes. The cold regenerator pressure drop is approximately 1.2 psi. To avoid exceeding 10 percent of this value at maximum flow conditions (0.12 psi), the -0.002-in. tolerance in hold diameter cannot be allowed; however, this does not present a problem in actual fabrication.

Figure 5-4 gives the axial pressure drop as a function of the number of holes in the distributor for various hole diameters. Several combinations of hole diameter and number of holes yield the same pressure drop. Selection of 80 holes with a nominal diameter of 0.020 in. was based on (1) ease of fabrication, and (2) providing a sufficient number of holes for a hole pattern that covers the plate surface in a uniform manner.

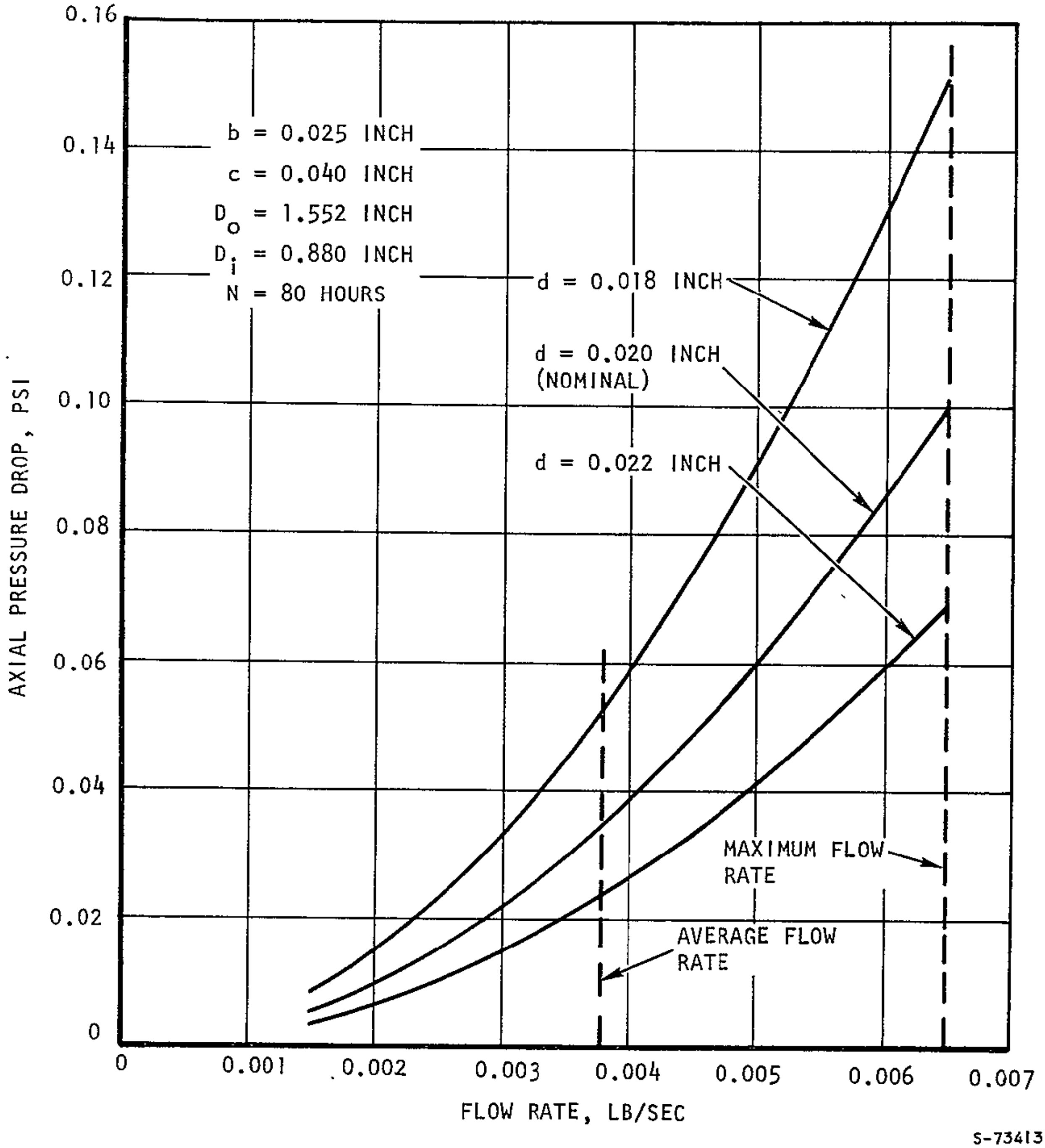

Figure 5-3. Effect of Hole Size Tolerance upon Axial Pressure Drop

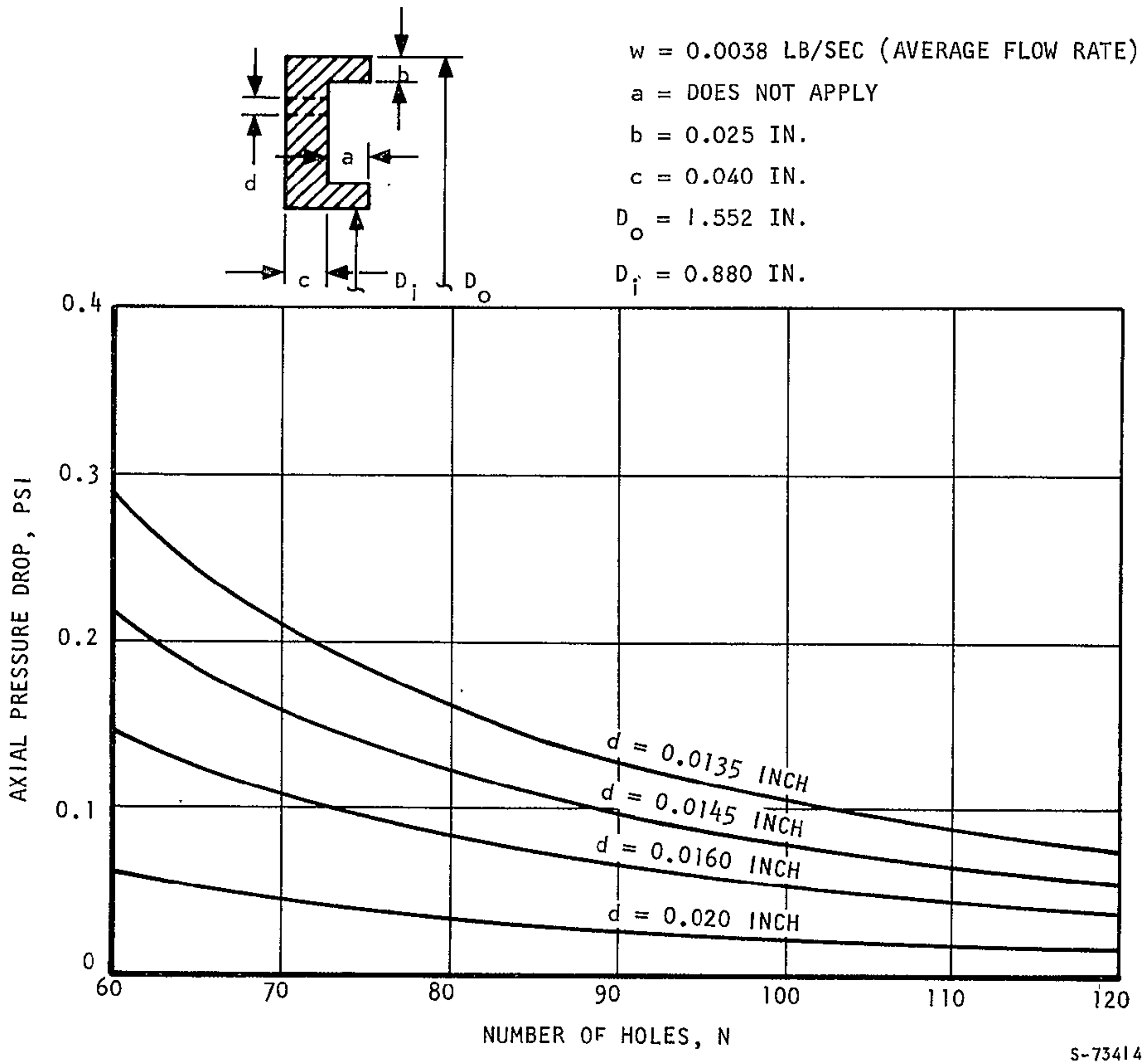

Figure 5-4. Axial Pressure Drop vs Number of Holes for Four Hole Diameters

To provide greater uniformity in gas temperature at the cold-end heat exchanger-to-regenerator interface, a flow distributor was designed for installation within the refrigerator. This flow distributor placed next to the cold regenerator is shown in Figure 5-1. Gas passes into the distributor where it enters the annular cavity of the flow distributor. If the flow is non-uniform, part of the gas travels circumferentially within the cavity before leaving.

In the selection of the dimensional characteristics of the flow distributor, the following operational requirements and constraints were considered:

(a) Ratio of axial pressure drop-to-circumferential pressure drop should be maintained between 10 and 20. As apparent by the need or partial circumferential gas flow, a ratio approaching the higher value is favored.

(b) A maximum axial pressure drop through the flow distributor holes corresponding to 10 percent of the cold regenerator pressure drop is desirable. The flow rate used for reference is 0.0038 lb/sec. A higher pressure drop is acceptable in cases where dimensional tolerances are concerned.

(c) The void volume of the flow distributor should not exceed 3 percent of the sump dead volume to minimize the drop in refrigeration capacity.

In view of the need for improved gas flow distribution, it was assumed that about 25 percent of the entire flow to the distributor will travel circumferentially within the annular cavity. Additionally, it was assumed that this circumferential gas flow will travel at least 180 degrees before leaving the distributor. For the circumferential gas flow only frictional pressure drop was considered; turning losses were ignored.

Referring to Figure 5-1, dimensions b, c, D_o, and D_i are fixed by the installation dimension of the flow distributor within the refrigerator. It was the purpose of this analysis to briefly determine the influence of the remaining unfixed dimensions, a and d, and quantity N (number of holes) upon the flow characteristics of the distributor.

Having established the flow characteristics, the values of the dimensions satisfying the requirement and constraints were then selected. As a guideline, the following dimensions were used in the analysis: (1) dimension a = 0.01 to 0.050 in., (2) dimension d = 0.0135 to 0.020 in., and (3) quantity N = 20 to 80 holes. The applicable equations are briefly described below and in the attached calculation section.

Axial pressure drop

$$\Delta P_{axial} = \frac{K_1 W_2}{N^2 d^4}$$

Ratio of axial pressure drop to circumferential pressure drop

$$\frac{\Delta P_{axial}}{\Delta P_{circ}} = \frac{K_2 a^3 W^{1/4}}{(K_3 + a) N^{2.25} d^{4.25} \psi \phi^2}$$

Void volume

$$V_{void} = K_4 a + K_5 N d^2$$

where

K_1, K_2, K_3, K_4, and K_5 are constants

W = gas flow rate, lb/sec

a, and d are shown in figure 5-1

N = number of holes

ϕ = circumferential flow fraction = 0.25

ψ = angle of travel of gas within cavity, degrees

RESULTS

Figures A, B, and C show the variation of axial pressure drop as a function of flow rate for different hole diameters and 20, 40, and 80 holes. At a reference design gas flow rate of 0.0038 lb/sec, Figure D summarizes the axial pressure drop as a function of number of holes for different hole diameters. To satisfy the axial pressure drop requirement for the flow distributor, a maximum value of 0.04 psi is allowed. At this pressure drop, Figure D indicates a hole diameter of 0.020 is acceptable. In addition, 80 holes within the annular cavity are adequate. Taking a ±0.002 in. dimensional hole tolerance into account, Figure E illustrates the effect of this tolerance upon the axial pressure drop. As noted at 0.0038 lb/hr design flow, the pressure drop will be between 0.024 and 0.053 psi.

Corresponding to the selected flow distributor hole dimensions, the calculated ratio of axial pressure drop-to-circumferential pressure drop is 12. The required annular cavity depth dimension, a, at this ratio is 0.045 in. For a typical dimensional tolerance of ±0.003 for dimension a, Figure F shows the resulting variation in pressure drop ratio. This ratio will be between 9.6 and 14. The void volume for the distributor is 0.055 in.^{3} as shown in Figure G for a = 0.045 in.

The selected dimensions and operational characteristics for the flow distributor are summarized below:

(a) Dimension a = 0.045 in.

(b) Dimension b = 0.025 in.

(c) Dimension c = 0.040 in.

(d) Dimension d = 0.020 in.

(e) Dimension D_o = 1.552 in.

(f) Dimension D_i = 0.880 in.

(g) Quantity N = 80 holes

(h) Void volume = 0.055 in.

(i) Pressure drop ratio $\frac{\Delta P_{axial}}{\Delta P_{circ}} = 12$

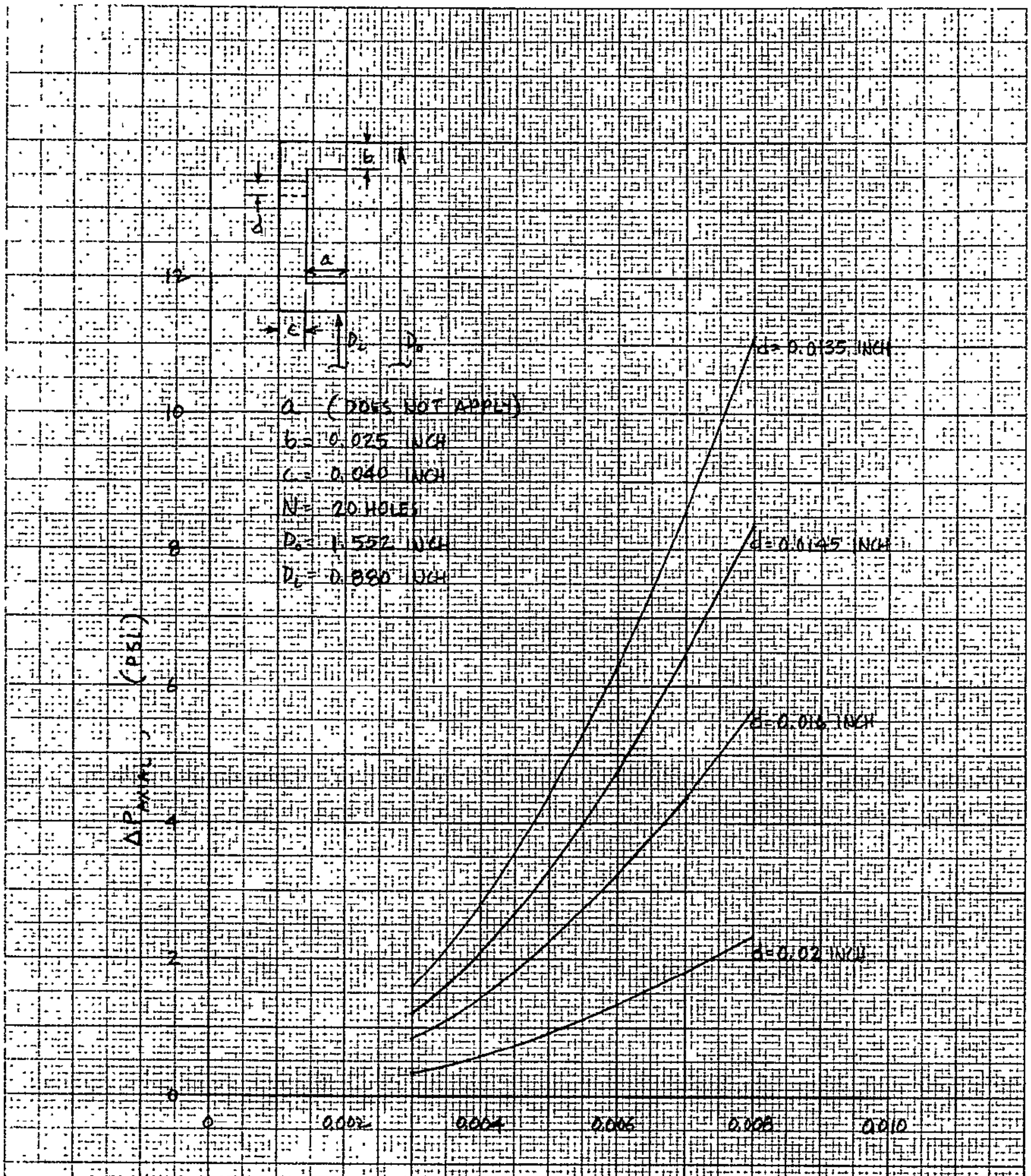

Figure A. Axial Pressure Drop - Flow Rate Characteristics For N = 20 holes

ΔP_{AXIAL} FOR N = 20 HOLES
25 OCT 71 CALCULATIONS

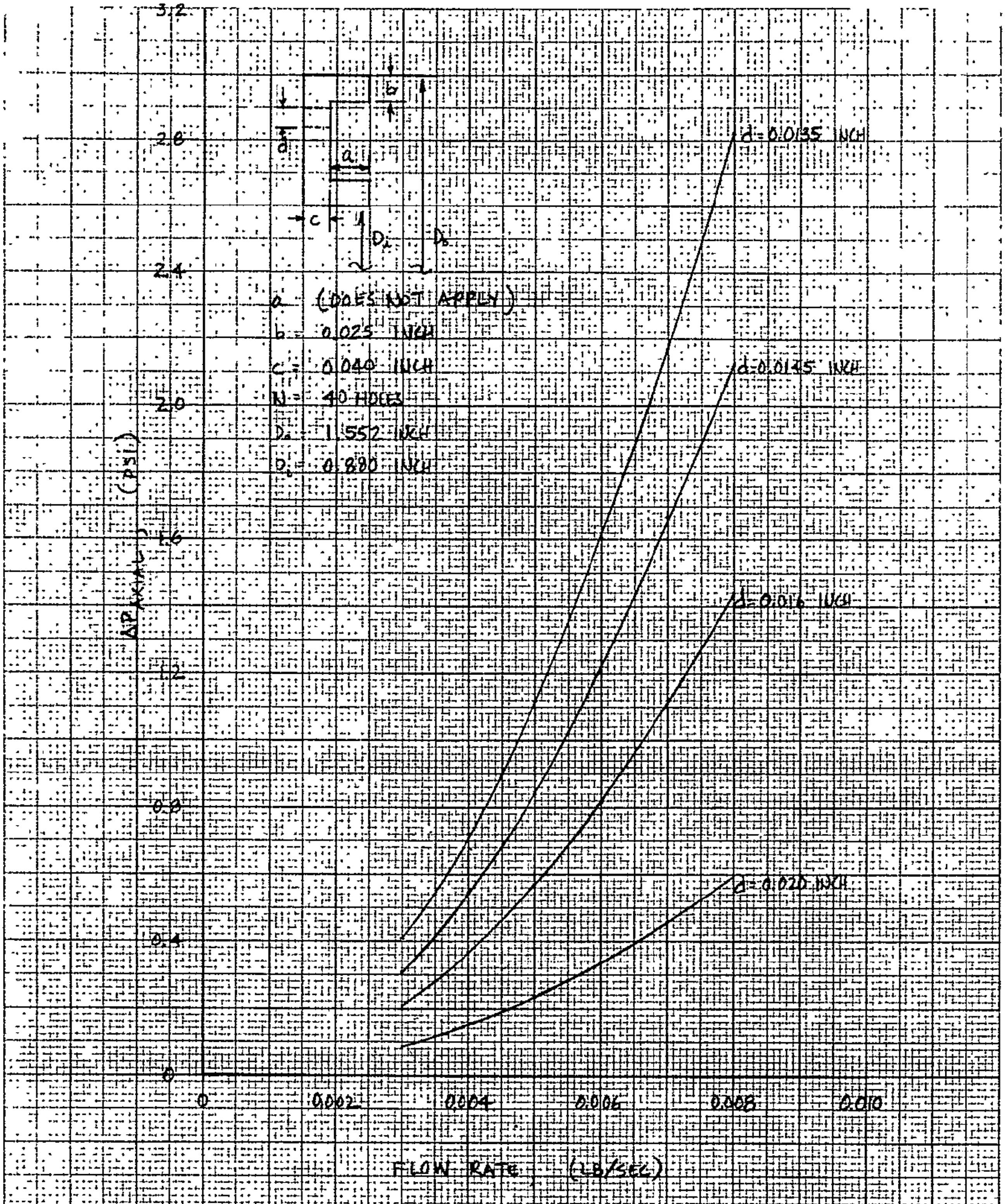

Figure 8. Axial Pressure Drop - Flow Rate Characteristics For N = 40 holes

ΔP_{AXIAL} FOR N = 40 HOLES
25 OCT 71 CALCULATIONS

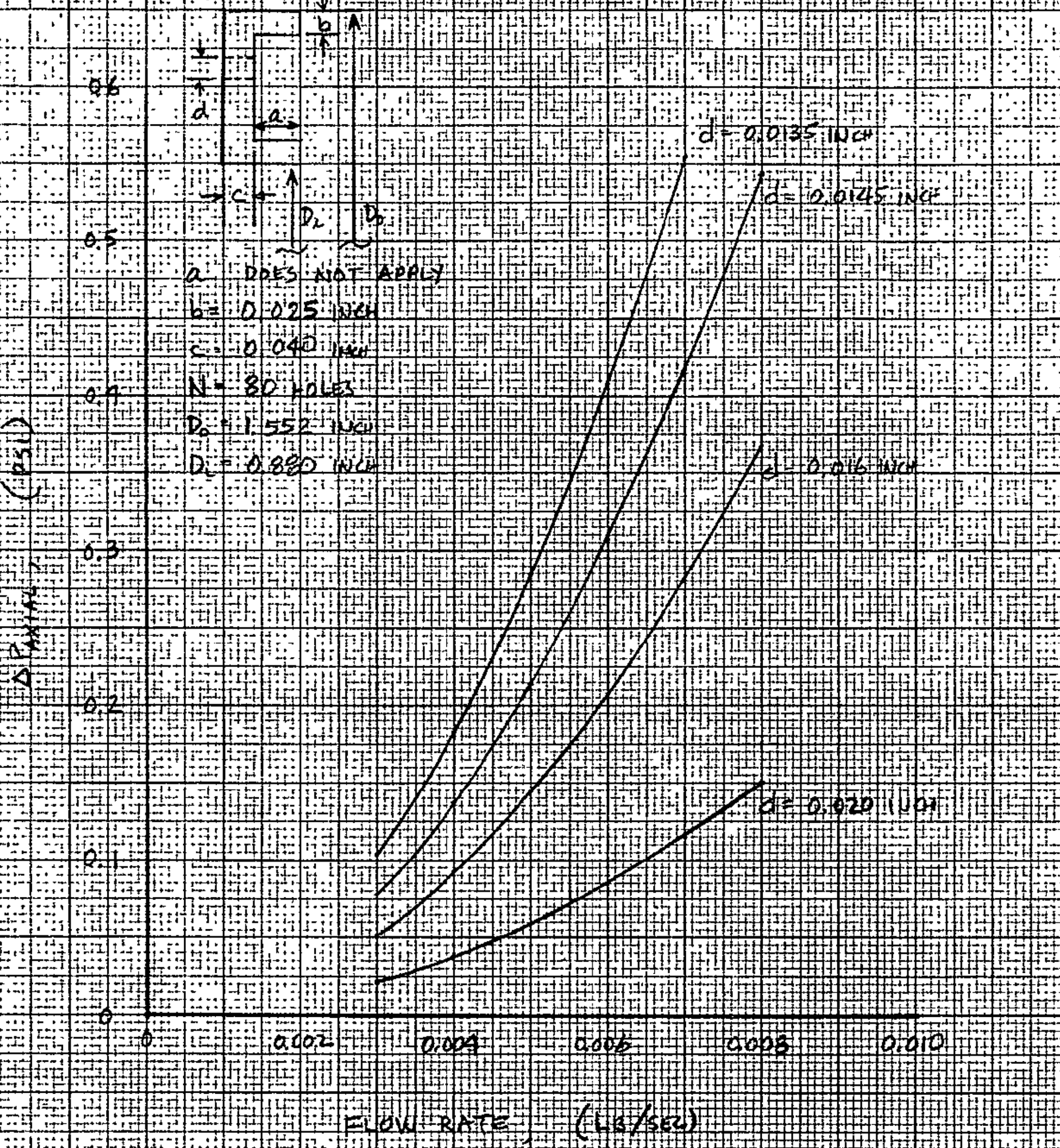

Figure G. Axial Pressure Drop - Flow Rate Characteristics For N = 80 holes

N - 80

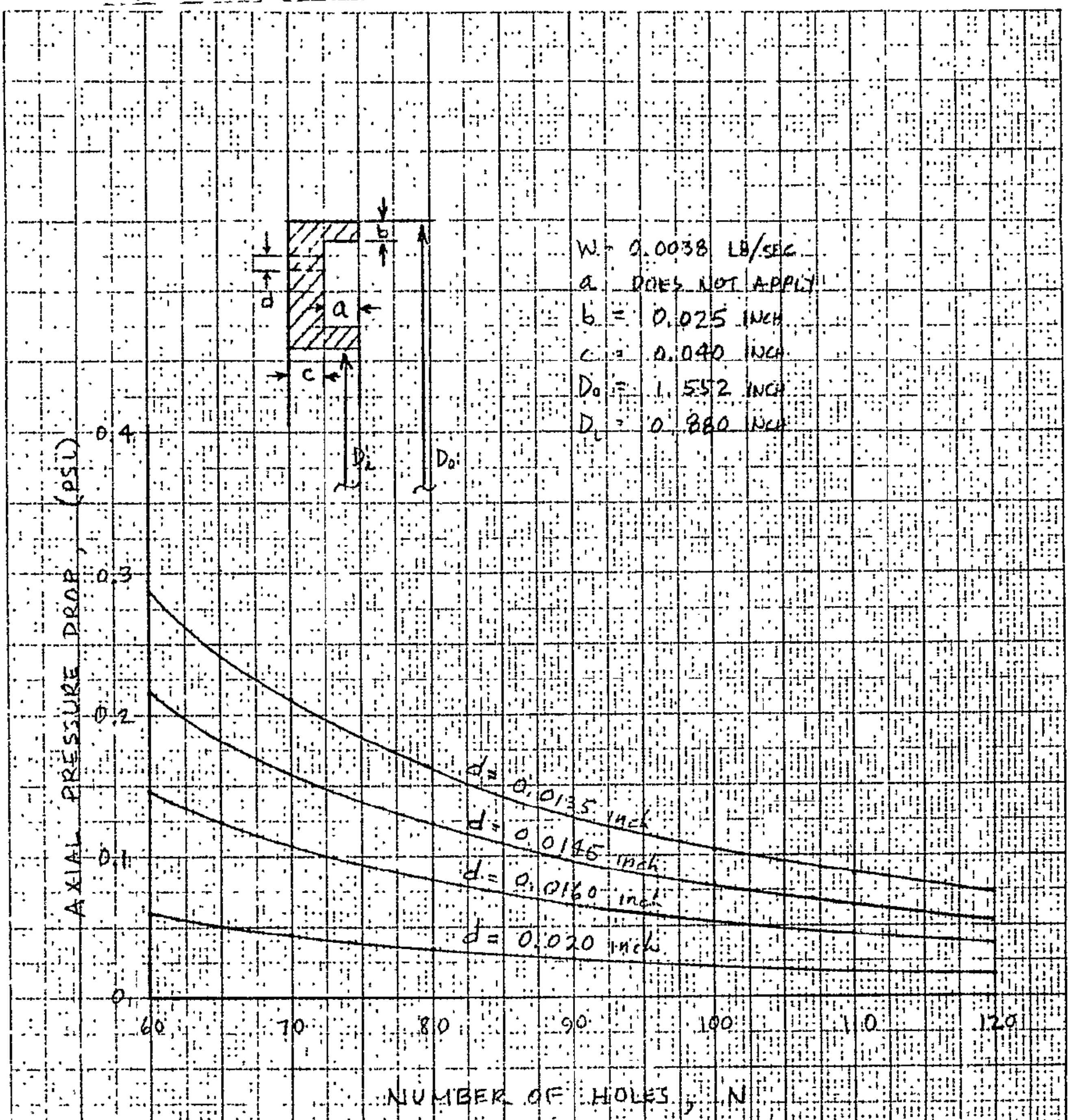

Figure D. Axial Pressure Drop VS Number of Holes For four Hole Diameters.

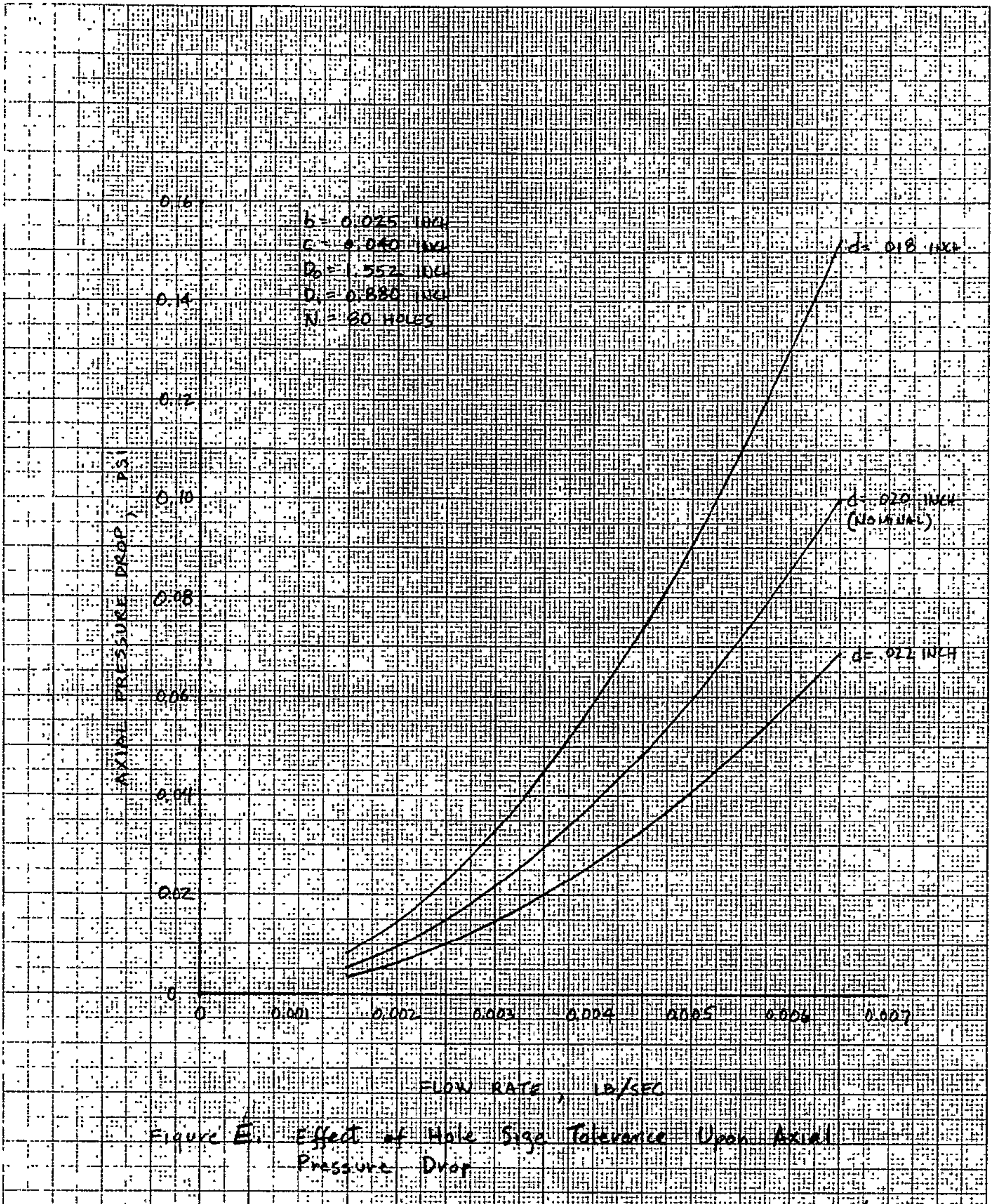

Figure E. Effect of Hole Size Tolerance Upon Axial Pressure Drop

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AXIAL PRESSURE DROP FOR N = 80 HOLES; d = .018 – .022 INCH, a = .042 – .048 INCH;
29 OCT 71 CALCULATIONS

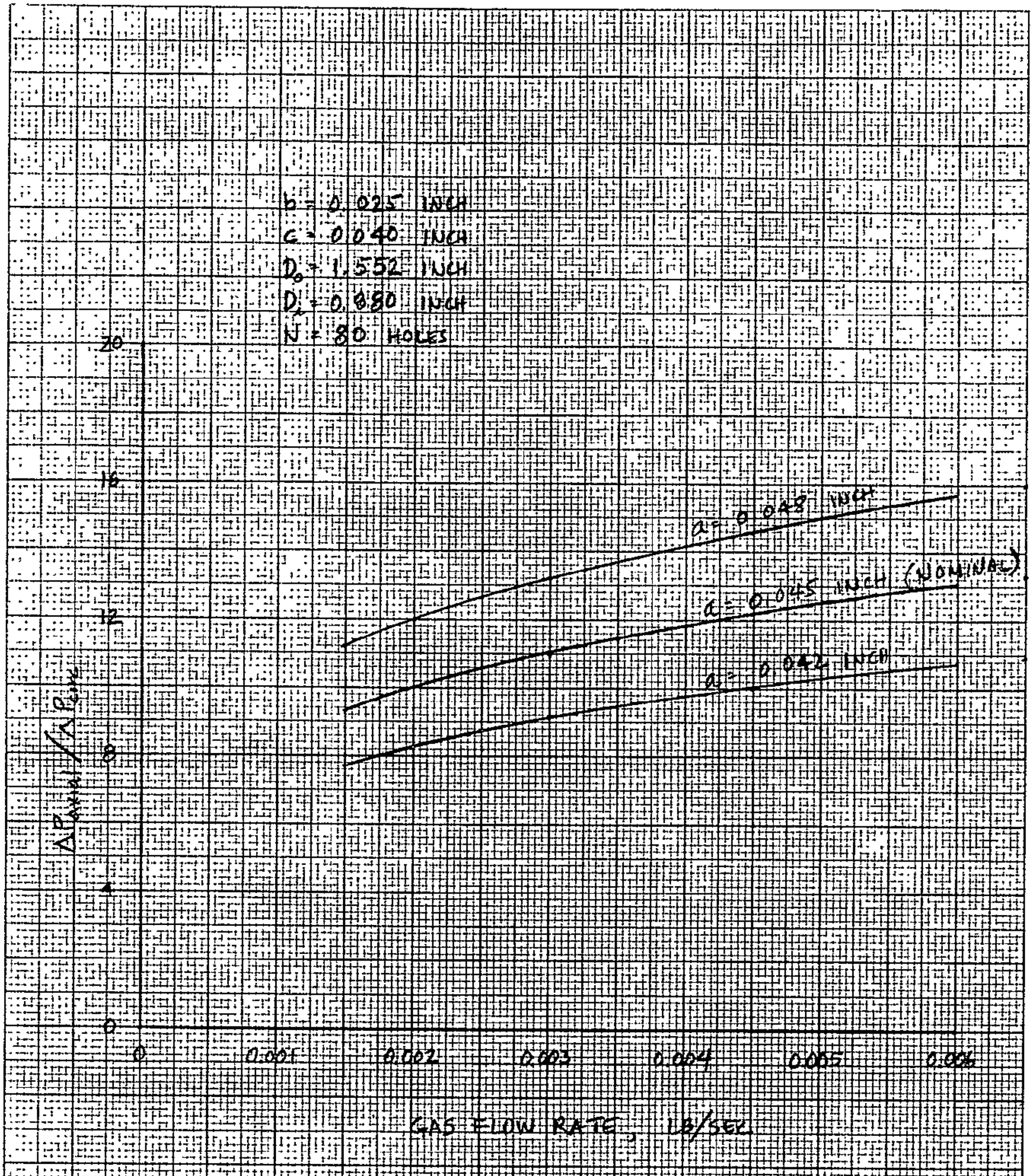

Figure F. Effect of Dimension Tolerance in Dimension a Upon Pressure Drop Ratio

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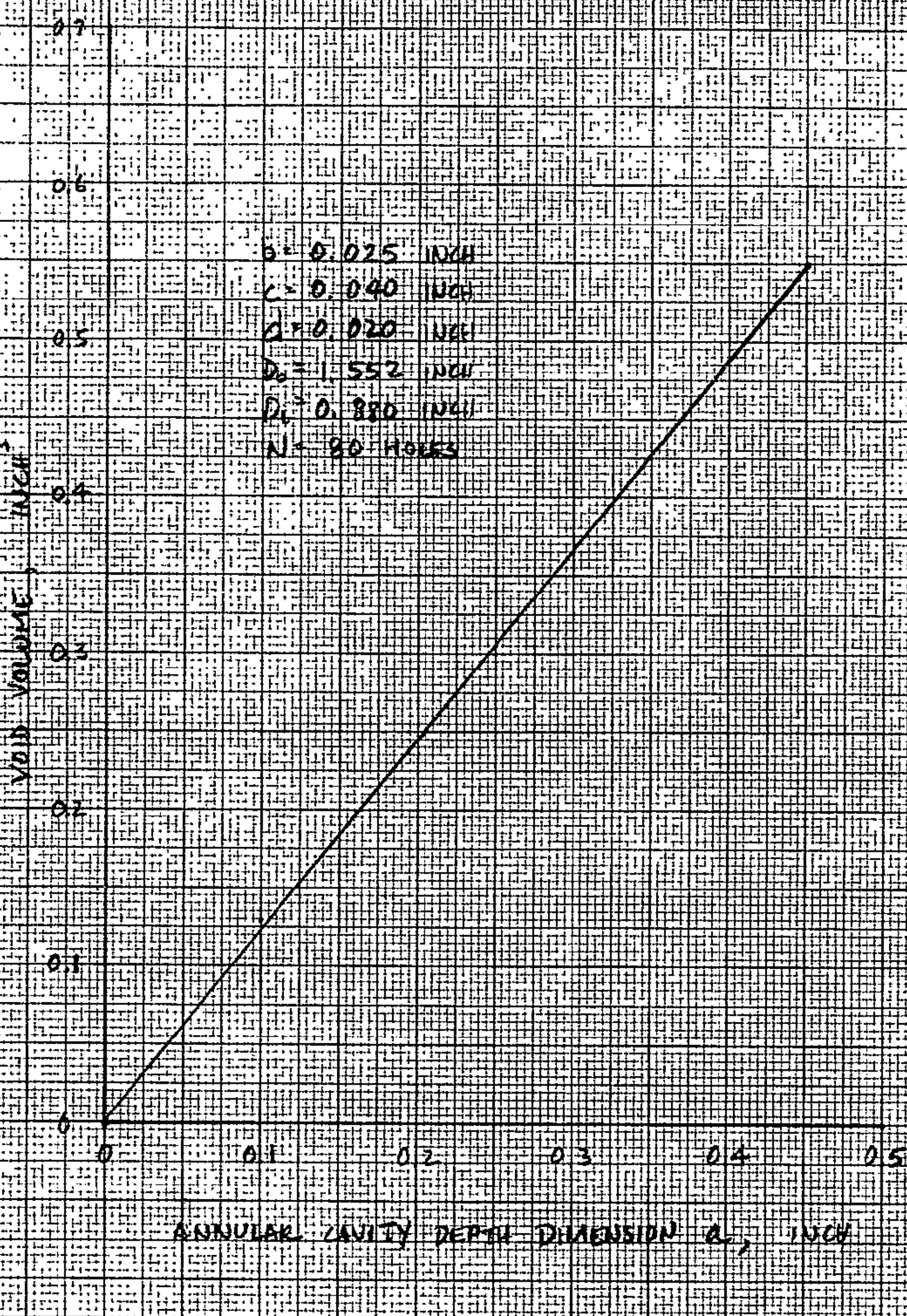


Figure 6. Void Volume – Dimension a Characteristics of 80 Hole Flow Distributor

Flow Distributor Calculations (Assume $\phi = 0.25$; $\psi = 180°$)

The axial pressure drop through the holes is:

$$\Delta P_{axial} = \rho (K_c + K_e) \frac{V^2_{holes}}{2g}$$

ρ = gas density = 2.24 $\frac{lb}{ft^3}$
K_c = contraction coeff.
K_e = expansion coeff.
V_{hole} = gas velocity thru holes, ft/sec
Δ_{holes} = ... psi

The circumferential pressure drop in the annular cavity is defined by

$$\Delta P_{circ} = \frac{f_F L}{2g D_H} \frac{(\phi W)^2}{\rho A^2_{circ}}$$

$$= 4.81788(10^{-5}) f_F \frac{P_{circ}}{A^2_{circ}} L_{circ} (\phi W)^2$$

f_F = friction factor (Darcy)
P_{circ} = cavity perimeter, ft
A_{circ} = cavity flow area, ft
L = flow length, ft

The flow distributor void volume is

$$V_{void} = \frac{a\pi}{4} \left[(D_o - b)^2 - (D_i + b)^2\right] + N c \frac{\pi}{4} d^2$$

void = in.^{3}
dimensions a, b, c, d, D_o, D_i are shown in FIGURE 5-1

Velocity of gas through each hole is

$$V_{hole} = \frac{W}{\rho A_{hole}} = \frac{W}{2.24 A_{holes}} \text{ ... ft/sec}$$

where $A_{holes} = 0.0054545 \; N d^2$ ... ft^2

The Reynolds number for the holes is

$$N_{Re_{holes}} = \frac{VD\rho}{\mu} = 15873 \frac{dW}{A_{holes}}$$

d = in
W = lb/sec
A_{holes} = ft^2
$\mu = 0.0189 \frac{lb}{ft\text{-}hr}$

The free flow to frontal area ratio is

$$\sigma = A_{holes} / A_{\substack{annulus\\axial}}$$

where $A_{\substack{annulus\\axial}} = 0.0054545 \left[(D_o - b)^2 - (D_i + b)^2 \right] \ldots ft^2$

Use N_{Re} and σ to find Kc and Ke. (See Kays and London, <u>Compact Heat Exchangers</u>, p. 93.)

Perimeter of annular cavity is

$$P_{circ} = 2 \left[a + \frac{1}{2}(D_o - D_i) - 2b \right] / 12 \ldots ft$$

Flow area of annular cavity is

$$A_{circ} = \frac{\left[\frac{1}{2} D_o - D_i - 2b \right] a}{144} \ldots ft^2$$

Annular cavity flow length is

$$L_{circ} = \frac{\psi}{360} \frac{\pi (D_i + b)}{12} = \frac{\psi}{360} (0.2618)(D_i + b) \ldots ft$$

The Reynolds number for flow in the annular cavity is

$$N_{Re} = \frac{V D_H \rho}{\mu} = 761905 \frac{\phi W}{P_{circ}}$$

ϕ = flow fraction = 0.25
W = total gas flow to distributor

$N_{Re_{circ}}$ is used to find friction factor f_F

SECTION 6

COLD-END REGENERATOR

INTRODUCTION

The cold regenerator is one of the most, if not the most, important components of the VM refrigerator. The regenerator is used to cool incoming gas as it enters the cold expansion volume of the machine. The expansion process of the gas further reduces the gas temperature in order to provide cooling at the cold temperature. After absorbing heat from the refrigeration load, the gas is exhausted through the regenerator where it regains previously stored energy. This exhaust process reestablishes the temperature profile in the regenerator for cooling the incoming gas for the next cycle.

The design requirements for an efficient regenerator demand that, as closely as possible, it must (1) absorb heat from the gas stream while at nearly the same temperature as the gas, (2) store this energy without significant temperature changes in a given locality, and (3) resupply the energy to the gas stream when the flow reverses direction--again, while as near as possible to the gas stream temperature. These requirements dictate that the regenerator packing (1) be of a material with a very large heat capacity relative to that of the gas; (2) have a high heat transfer coefficient, high thermal diffusivity, and a large heat transfer area; and (3) be of a configuration to limit the amount of axial conduction through the packing. Obtaining these features from a specific packing generally leads to increased pressure drop as the thermal characteristics improve. Since excessive pressure drop degrades the overall refrigerator performance, the basic tradeoff in the design of a regenerator is between irreversible pressure drop and heat transfer potential with a minimum void volume.

METHOD OF ANALYSIS

The analysis of regenerators for a VM refrigerator cannot be based on the classical effectiveness parameters that make use of inlet and outlet temperatures. The system pressure fluctuates and a considerable amount of gas is stored in the regenerator void volume during various parts of the flow period. The result of this characteristic is that the mass flow of gas into the regenerator is unequal to the mass flow of gas out of the regenerator at all points in time. This behavior, coupled with the basic transient nature of regenerators, dictates that a finite difference technique be used to analyze regenerators.

To analyze regenerators, a computer program making use of finite difference techniques has been developed by AiResearch. A description of this computer program is given in the Task I Report (Ref 1).

The computer program is capable of analyzing both transient performance and cyclically steady performance. The inputs required are the physical characteristics of the regenerator, heat transfer and friction loss characteristics of the matrix, initial conditions, and boundary conditions. The physical characteristics are reflected in the matrix areas, volume, length, heat capacity, thermal conductivity, and mass. The heat transfer and friction loss characteristics are read into the computer program in the form of Colburn j-factor and Fanning friction factor as a function of Reynolds number. The initial conditions must be fully described in terms of pressure and temperature of the gas and the matrix. The boundary conditions required are the time dependent characteristics of pressure, mass flow rate, and gas temperature at one end of the regenerator, and the return gas temperature at the other end (basically defining the thermodynamic process at the other end).

REGENERATOR CHARACTERISTICS

In the analyses of the regenerators, friction coefficients and the Colburn j-factor have been taken from the data presented by Kays and London (Ref 4). The data for randomly stacked screens are shown in Figure 6-1 and the data for regenerators packed with spherical shot are shown in Figure 6-2. Standard screen mesh data, shown in Table 6-1, were used to characterize the screen matrix. Figures 6-3 and 6-4 show the specific heats of monel, stainless steel, and Inconel as a function of temperature.

Area of Volume Ratio

For screens, the area-to-volume ratio can be derived as

$$\beta = \frac{4(1 - \epsilon)}{d} \tag{6-1}$$

where β = area-to-volume ratio

ϵ = porosity

d = wire diameter

For spheres, the area-to-volume ratio is

$$\beta = \frac{6(1 - \epsilon)}{d} \tag{6-2}$$

where d = sphere diameter

Hydraulic Diameter

Another required regenerator characteristic is the hydraulic diameter. Defining the hydraulic radius as being equal to the flow volume divided by the total surface area, the hydraulic radius of a stack screen is found to be

$$r_H = \frac{\epsilon d}{4 (1 - \epsilon)} \tag{6-3}$$

For a sphere, the hydraulic radius is equal to

$$r_H = \frac{\epsilon d}{6 (1 - \epsilon)} \tag{6-4}$$

The hydraulic diameter is equal to 4 times the hydraulic radius.

AIRESEARCH MANUFACTURING COMPANY
Los Angeles, California

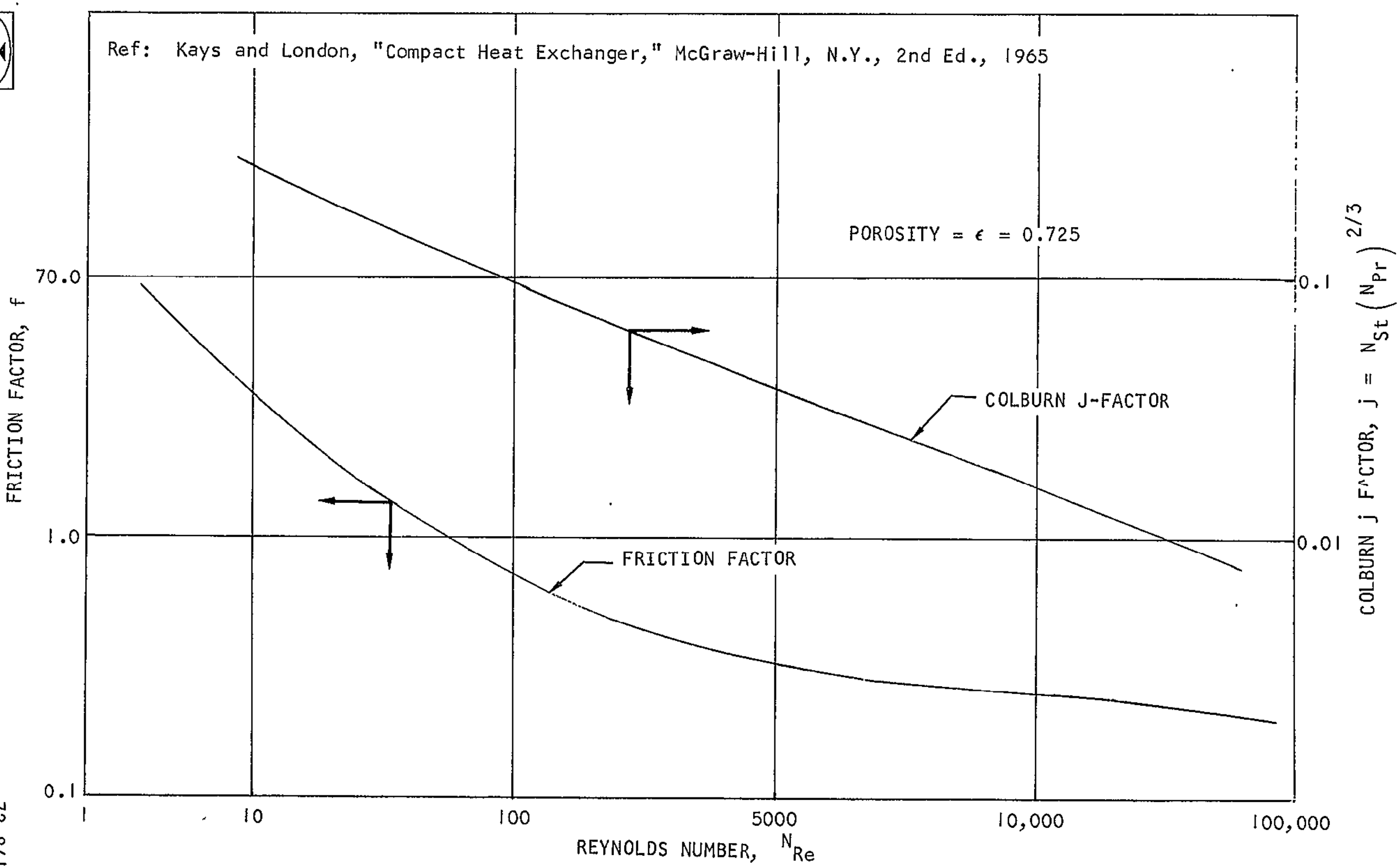
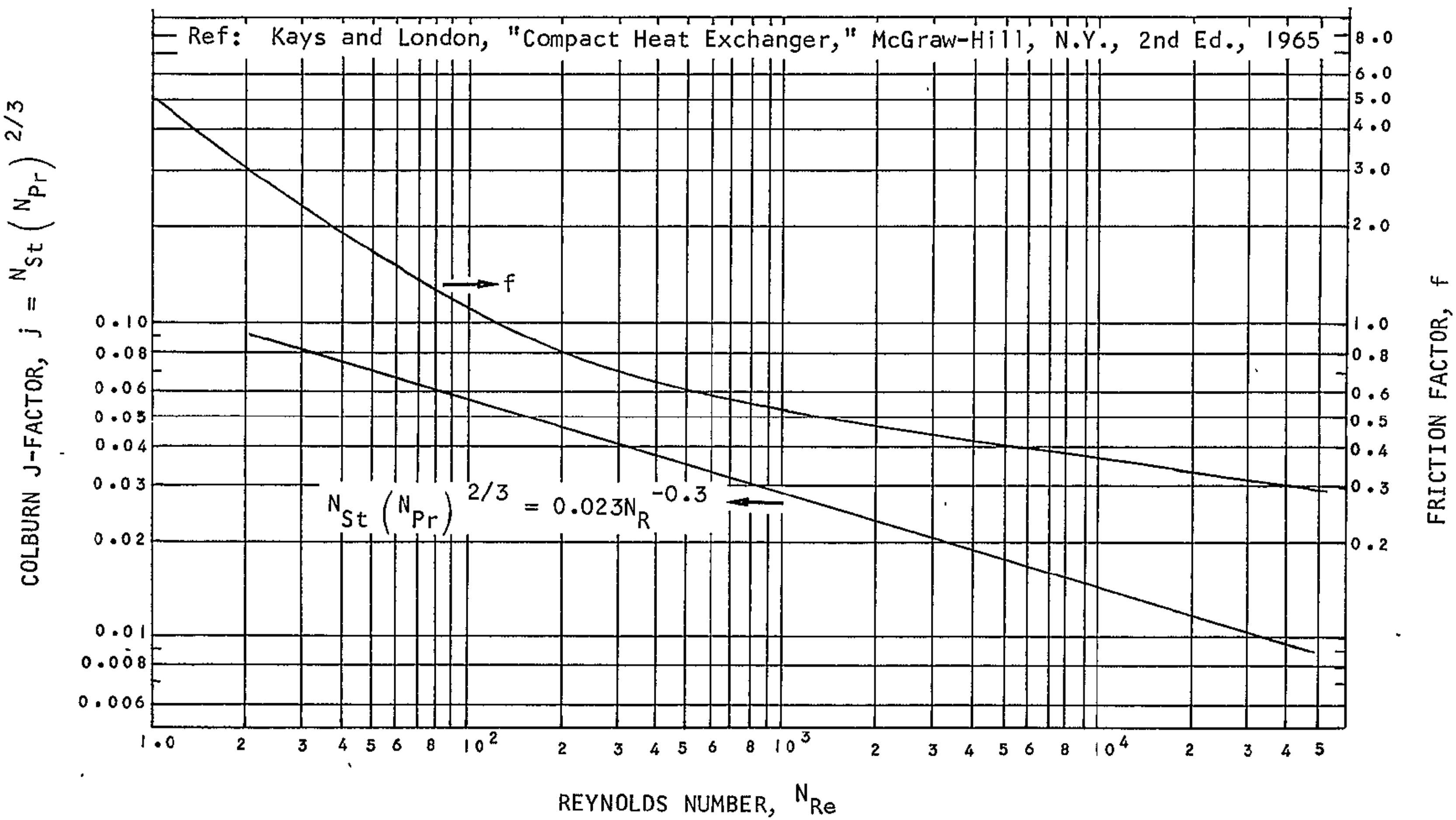


Figure 6-1. Heat Transfer and Pressure Drop Data for Stacked Screens

S-58836



S-58837

Figure 6-2. Gas Flow through an Infinite Randomly Stacked Sphere Matrix

TABLE 6-1

U.S. SIEVE SERIES AND
TYLER EQUIVALENTS

A.S.T.M. - E-11-61

Sieve Designation		Sieve Opening		Nominal Wire Diameter		
Standard	Alternate, No.	mm	Inches, approx. equiv.	mm	Inches, approx. equiv.	Tyler Equiv. Designation, mesh
1.41 mm*	14	1.41	0.0555	0.725	0.0285	12
1.19 mm	16	1.19	0.0469	0.650	0.0256	14
1.00 mm*	18	1.00	0.0394	0.580	0.0228	16
841 micron	20	0.841	0.0331	0.510	0.0201	20
707 micron*	25	0.707	0.0278	0.450	0.0177	24
595 micron	30	0.595	0.0234	0.390	0.0154	28
500 micron*	35	0.500	0.0197	0.340	0.0134	32
420 micron	40	0.420	0.0165	0.290	0.0113	35
354 micron*	45	0.354	0.0129	0.247	0.0097	42
297 micron	50	0.297	0.0117	0.215	0.0085	48
250 micron*	60	0.250	0.0098	0.180	0.0071	60
210 micron	70	0.210	0.0083	0.152	0.0060	65
177 micron*	80	0.177	0.0070	0.131	0.0052	80
149 micron	100	0.149	0.0059	0.110	0.0043	100
125 micron*	120	0.125	0.0049	0.091	0.0036	115
105 micron	140	0.105	0.0041	0.076	0.0030	150
88 micron*	170	0.088	0.0035	0.064	0.0025	170
74 micron	200	0.074	0.0029	0.053	0.0021	200
63 micron*	230	0.063	0.0024	0.044	0.0017	250
53 micron	270	0.053	0.0021	0.037	0.0015	270
44 micron*	325	0.044	0.0017	0.030	0.0012	325
37 micron	400	0.037	0.0015	0.025	0.0010	400

*These sieves correspond to those proposed as an international (ISO) standard. It is recommended that wherever possible these sieves be included in all sieve analysis data or reports intended for international publication.

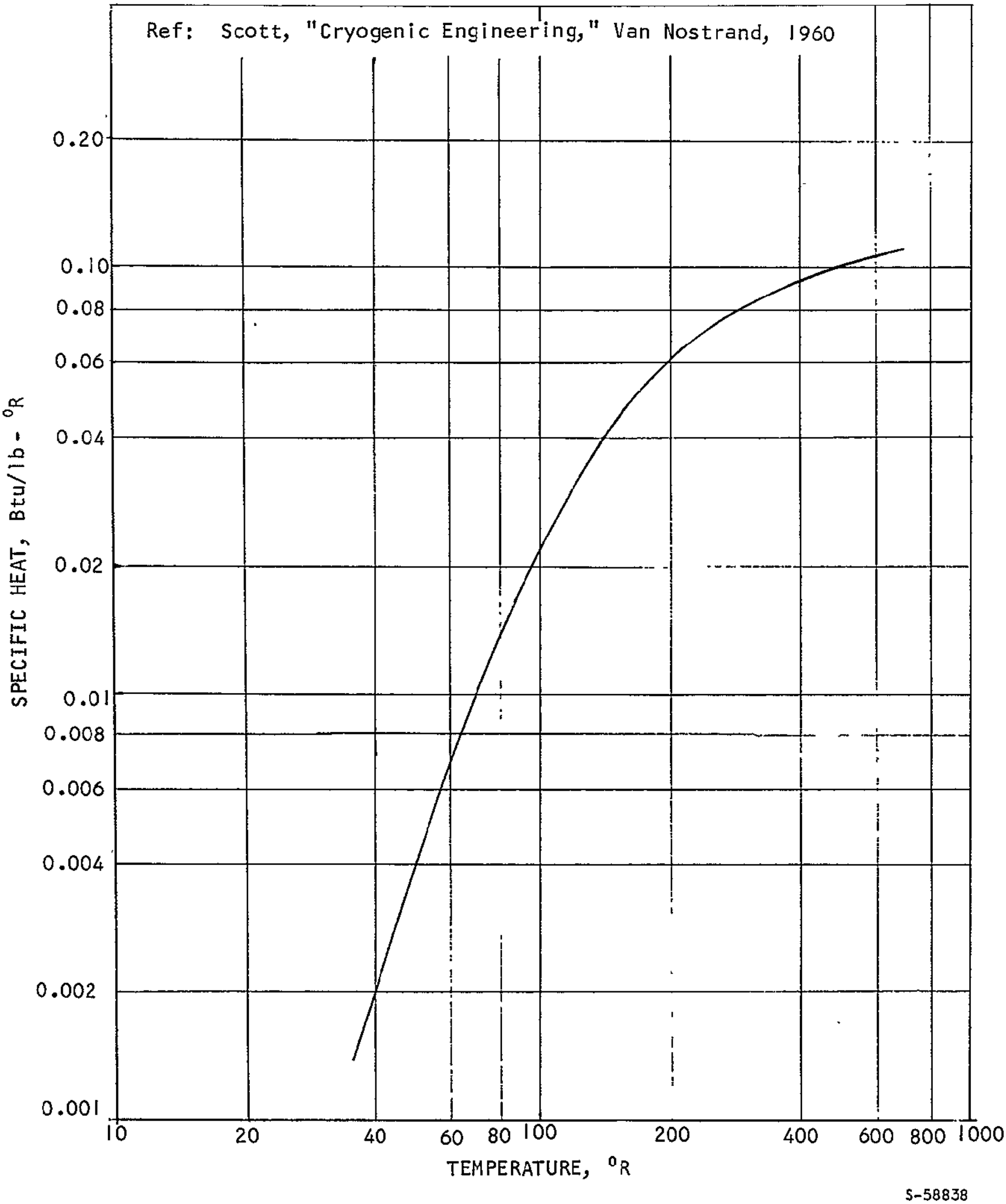


Figure 6-3. Monel Specific Heat

Ref: AiResearch Material Design Data
347 STAINLESS STEEL: ASTM STP NO. 227
INCONEL 100: G.E. HANDBOOK, SEC. 2

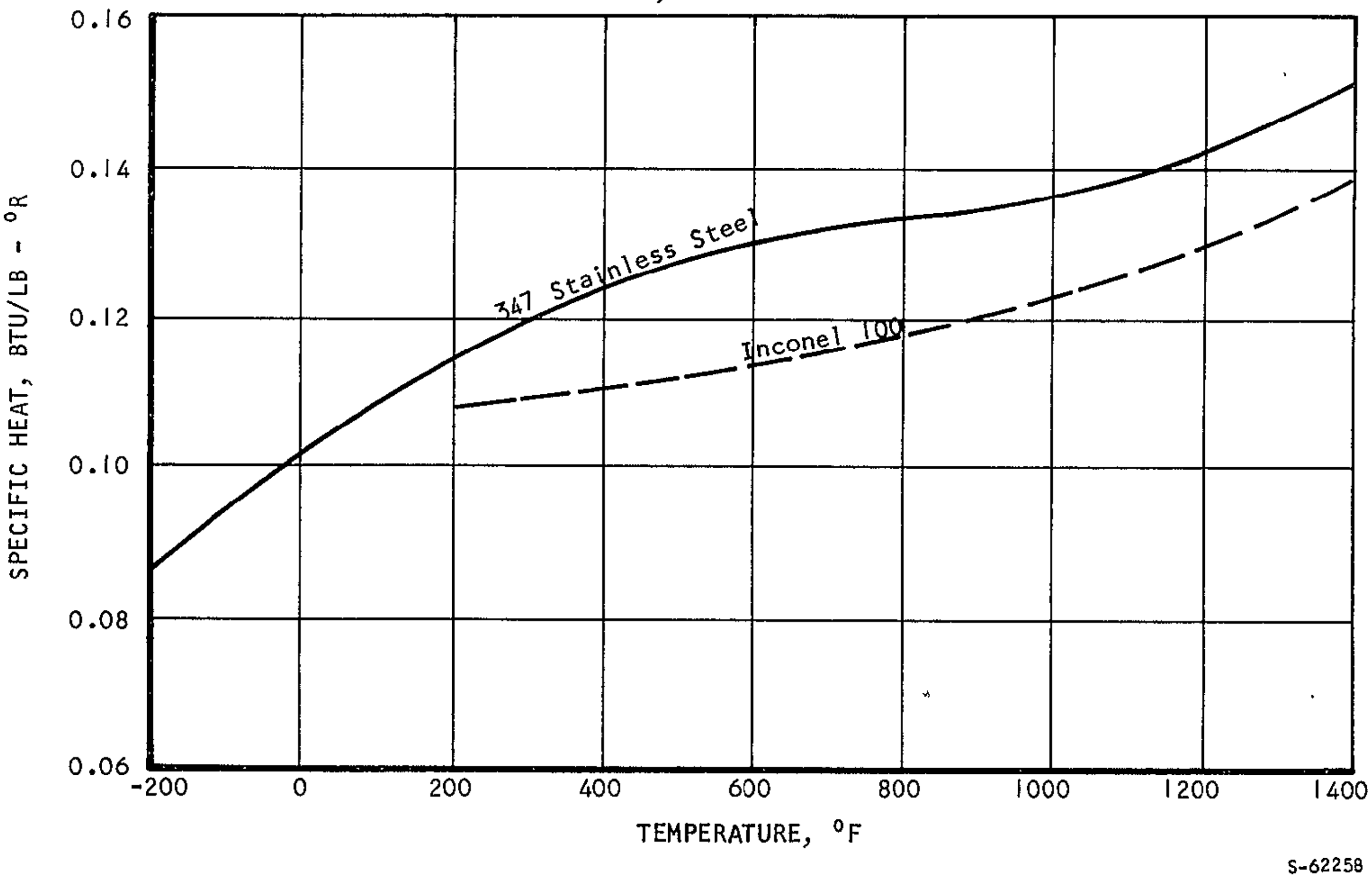


Figure 6-4. Stainless Steel and Inconel Specific Heats

DESIGN CONFIGURATION AND PERFORMANCE

During Task I of the program, several cold end regenerator configurations and matrix materials were studied. This effort resulted in the selection of an annular configuration for the cold regenerator and monel as the matrix material. This configuration yielded the best overall performance; the monel has a heat capacity superior to other candidate materials, as discussed in the Task I Final Report (Ref I). During Task 2, the same basic regenerator configuration and material have been retained and design refinements and adjustments incorporated as the whole refrigerator design was iterated into its final configuration.

The major change in the cold regenerator since the Task I effort is the selection of smaller diameter monel shot for the section of the regenerator toward the cold end heat exchanger. This shot size was reduced to 0.0075 in. diameter from the preliminary design size of 0.014 in. diameter. The smaller diameter shot provides for increased heat transfer in the cold end of the regenerator. Test results from the AiResearch VM refrigerator verified that the overall performance is increased by the use of smaller shot.

The final cold end regenerator has a frontal area of 1.297 sq in. and a total length of 4.4 in. The first 1.87 in. (from the warm end) is packed with 150-mesh monel screen. This screen has a porosity of 72.5 percent, an area-to-volume ratio of 367 sq in./cu in., and a hydraulic diameter of 0.00791 in. The last 2.53 in. of the regenerator (toward the cold end) is packed with monel spheres with an average diameter of 0.0075 in. The porosity for this section is 39 percent, the area-to-volume ratio is 488.5 sq in./cu in., and the hydraulic diameter is 0.003197 in.

The screen matrix packing is used in the warm end of the cold regenerator to reduce the pressure drop, since the gas density is relatively low in this section and the velocities correspondingly higher. Additionally, the influence of higher dead volume associated with the screen packing is less, due to the low gas density. Shot is used in the cold end to provide the maximum heat capacity per unit volume and to minimize the dead volume at low temperature.

Table 6-2 presents the detailed output from the regenerator analysis computer program for the final cold end regenerator design. It shows the node-wise pressures and temperatures of the gas, as well as the time (angular position of the crankshaft) response characteristics between the matrix and gas temperatures. In this table, the angular position, θ, is referenced to the top-dead-center position of the cold displacer, as is the data presented in Figure 3-1. The parameters listed in Table 6-2 are the matrix temperatures, gas density, gas pressure, and mass flow rate of the gas. Positive mass denotes flow toward the ambient end of the regenerator. Node 0 represents the the condition at the ambient end, and Node 16 represents that at the cold end.

Figures 6-5 to 6-7 represent plots of key parameters from the data of Table 6-2. The cold end temperature of the gas and matrix are given as functions of the crank angle position in Figure 6-5. The small difference between the gas and matrix is indicative of excellent heat transfer. The moderate temperature swing of the matrix, approximately 1.5°R, shows that the matrix has adequate heat capacity.

TABLE 6-2

FINAL DESIGN COLD REGENERATOR PERFORMANCE CHARACTERISTICS

Node No.	Matrix Temperature	Gas Temperature	Gas Density	Gas Pressure	Mass Flow Rate

θ = 0°

REGULAR PRINTOUTS

TIME(SEC.) = 4.49893-01

DATE = 24 MAR 72 TIME = 13:06:30

3197 = COUNT (NO. OF CALCULATIONS)

N	TM(N) DEG.R	TG(N) DEG.R	RG(N) LBM/CF	PG(N) PSIA	WG(N) LBM/SEC
-0	6.17517+02	6.20000+02	4.28866-01	7.52506+02	5.08070-03
1	5.95254+02	5.96896+02	4.57726-01	7.52496+02	5.00300-03
2	5.68511+02	5.70820+02	4.80748-01	7.52478+02	4.83997-03
3	5.37273+02	5.39867+02	5.08078-01	7.52462+02	4.66790-03
4	5.04653+02	5.07361+02	5.36781-01	7.52448+02	4.48634-03
5	4.71761+02	4.74537+02	5.76651-01	7.52436+02	4.29167-03
6	4.38828+02	4.41661+02	6.19737-01	7.52426+02	4.08287-03
7	4.05901+02	4.08782+02	6.62827-01	7.52417+02	3.85992-03
8	3.72961+02	3.75901+02	7.19006-01	7.52410+02	3.61869-03
9	3.40237+02	3.43238+02	7.83575-01	7.52404+02	3.35651-03
10	3.07719+02	3.10806+02	8.62996-01	7.52400+02	3.06875-03
11	2.73724+02	2.74001+02	9.77561-01	7.52388+02	2.89426-03
12	2.40698+02	2.41128+02	1.10319+00	7.52286+02	2.36465-03
13	2.07700+02	2.08153+02	1.26823+00	7.52224+02	1.75966-03
14	1.74896+02	1.75395+02	1.51030+00	7.52194+02	1.04622-03
15	1.43850+02	1.44418+02	1.80695+00	7.52184+02	2.01718-04
16	1.25000+02	1.25000+02	2.11621+00	7.52184+02	-2.86610-04

θ = 30°

TIME(SEC.) = 4.62473-01

DATE = 24 MAR 72 TIME = 13:06:42

3287 = COUNT (NO. OF CALCULATIONS)

N	TM(N) DEG.R	TG(N) DEG.R	RG(N) LBM/CF	PG(N) PSIA	WG(N) LBM/SEC
-0	6.18875+02	6.20000+02	4.54648-01	7.82593+02	7.34300-03
1	5.96544+02	5.98137+02	4.74465-01	7.82578+02	7.28483-03
2	5.70258+02	5.72395+02	4.98051-01	7.82549+02	7.16286-03
3	5.39226+02	5.41690+02	5.26188-01	7.82524+02	7.03416-03
4	5.06670+02	5.09267+02	5.55899-01	7.82501+02	6.89835-03
5	4.73793+02	4.76454+02	5.96408-01	7.82480+02	6.75290-03
6	4.40858+02	4.43565+02	6.41130-01	7.82462+02	6.59685-03
7	4.07916+02	4.10661+02	6.85872-01	7.82446+02	6.43019-03
8	3.74978+02	3.77760+02	7.43126-01	7.82433+02	6.25003-03
9	3.42250+02	3.45062+02	8.09035-01	7.82421+02	6.05439-03
10	3.09726+02	3.12580+02	8.91515-01	7.82411+02	5.83953-03
11	2.74047+02	2.74417+02	1.01330+00	7.82384+02	5.70879-03
12	2.41130+02	2.41567+02	1.14247+00	7.82119+02	5.31216-03
13	2.08151+02	2.08604+02	1.31314+00	7.81921+02	4.85901-03
14	1.75378+02	1.75849+02	1.56197+00	7.81781+02	4.32493-03
15	1.44376+02	1.44849+02	1.86537+00	7.81690+02	3.69369-03
16	1.25277+02	1.25541+02	2.18303+00	7.81660+02	3.32867-03

TABLE 6-2 (Continued)

Node No.	Matrix Temperature	Gas Temperature	Gas Density	Gas Pressure	Mass Flow Rate

$\theta = 60^0$

REGULAR PRINTOUTS

DATE = 24 MAR 72 TIME = 13:06:54

TIME(SEC.) = 4.74930-01   3386 = COUNT (NO. OF CALCULATIONS)

N	TM(N) DEG.R	TG(N) DEG.R	RG(N) LBM/CF	PG(N) PSIA	WG(N) LBM/SEC
-0	6.19529+02	6.20000+02	4.70440-01	8.01547+02	7.44646-03
1	5.97855+02	5.99234+02	4.84660-01	8.01531+02	7.41960-03
2	5.71992+02	5.73797+02	5.08504-01	8.01503+02	7.36334-03
3	5.41225+02	5.43359+02	5.37039-01	8.01477+02	7.30405-03
4	5.08774+02	5.11051+02	5.67328-01	8.01453+02	7.24151-03
5	4.75936+02	4.78278+02	6.07899-01	8.01431+02	7.17470-03
6	4.43020+02	4.45407+02	6.53617-01	8.01412+02	7.10307-03
7	4.10088+02	4.12514+02	6.99365-01	8.01395+02	7.02666-03
8	3.77175+02	3.79635+02	7.56809-01	8.01379+02	6.94426-03
9	3.44469+02	3.46957+02	8.22973-01	8.01366+02	6.85503-03
10	3.11969+02	3.14493+02	9.07254-01	8.01354+02	6.75717-03
11	2.74470+02	2.74854+02	1.03515+00	8.01321+02	6.69759-03
12	2.41596+02	2.41973+02	1.16651+00	8.00989+02	6.51743-03
13	2.08651+02	2.09044+02	1.34063+00	8.00723+02	6.31238-03
14	1.75929+02	1.76338+02	1.59278+00	8.00518+02	6.07233-03
15	1.44998+02	1.45405+02	1.89731+00	8.00366+02	5.79106-03
16	1.25691+02	1.25938+02	2.22403+00	8.00309+02	5.62942-03

$\theta = 90^0$

DATE = 24 MAR 72 TIME = 13:07:04

TIME(SEC.) = 4.87500-01   3485 = COUNT (NO. OF CALCULATIONS)

N	TM(N) DEG.R	TG(N) DEG.R	RG(N) LBM/CF	PG(N) PSIA	WG(N) LBM/SEC
-0	6.19789+02	6.20000+02	4.71567-01	8.03535+02	5.14251-03
1	5.98810+02	5.99781+02	4.85309-01	8.03526+02	5.15277-03
2	5.73201+02	5.74384+02	5.09177-01	8.03508+02	5.17428-03
3	5.42676+02	5.44134+02	5.37608-01	8.03491+02	5.19695-03
4	5.10337+02	5.11928+02	5.67877-01	8.03476+02	5.22084-03
5	4.77552+02	4.79204+02	6.08081-01	8.03462+02	5.24636-03
6	4.44671+02	4.46367+02	6.53863-01	8.03449+02	5.27372-03
7	4.11770+02	4.13509+02	6.99673-01	8.03437+02	5.30291-03
8	3.78898+02	3.80673+02	7.56594-01	8.03426+02	5.33436-03
9	3.46236+02	3.48044+02	8.22132-01	8.03417+02	5.36840-03
10	3.13784+02	3.15631+02	9.06475-01	8.03408+02	5.40573-03
11	2.74867+02	2.75177+02	1.03647+00	8.03384+02	5.42851-03
12	2.41956+02	2.42203+02	1.16817+00	8.03130+02	5.49739-03
13	2.09050+02	2.09312+02	1.34256+00	8.02914+02	5.57582-03
14	1.76385+02	1.76664+02	1.59394+00	8.02736+02	5.66756-03
15	1.45531+02	1.45813+02	1.89513+00	8.02592+02	5.77488-03
16	1.26093+02	1.26284+02	2.22358+00	8.02534+02	5.83661-03

TABLE 6-2 (Continued)

Node No.	Matrix Temperature	Gas Temperature	Gas Density	Gas Pressure	Mass Flow Rate

REGULAR PRINTOUTS

θ = 120°

DATE = 24 MAR 72 TIME = 13:07:11

TIME(SEC.) = 4.99962-01

3572 = COUNT (NO. OF CALCULATIONS)

N	TM(N) DEG.R	TG(N) DEG.R	RG(N) LBM/CF	PG(N) PSIA	WG(N) LBM/SEC
-0	6.19877+02	6.20000+02	4.58996-01	7.87667+02	1.21251-03
1	5.99129+02	5.99162+02	4.76535-01	7.87665+02	1.25789-03
2	5.73499+02	5.73167+02	5.00517-01	7.87662+02	1.35313-03
3	5.43104+02	5.42952+02	5.28391-01	7.87658+02	1.45354-03
4	5.10843+02	5.10819+02	5.58035-01	7.87655+02	1.55946-03
5	4.78104+02	4.78164+02	5.97902-01	7.87652+02	1.67274-03
6	4.45262+02	4.45398+02	6.42747-01	7.87649+02	1.79429-03
7	4.12404+02	4.12624+02	6.87601-01	7.87646+02	1.92411-03
8	3.79578+02	3.79877+02	7.43813-01	7.87643+02	2.06422-03
9	3.46968+02	3.47347+02	8.08400-01	7.87640+02	2.21612-03
10	3.14573+02	3.15036+02	8.90962-01	7.87637+02	2.38298-03
11	2.75093+02	2.75215+02	1.01698+00	7.87629+02	2.48493-03
12	2.42090+02	2.42105+02	1.14719+00	7.87534+02	2.79433-03
13	2.09214+02	2.09249+02	1.31854+00	7.87441+02	3.14776-03
14	1.76593+02	1.76651+02	1.56557+00	7.87353+02	3.56362-03
15	1.45797+02	1.45875+02	1.86073+00	7.87271+02	4.05307-03
16	1.26346+02	1.26433+02	2.18251+00	7.87233+02	4.33671-03

θ = 150°

DATE = 24 MAR 72 TIME = 13:07:20

TIME(SEC.) = 5.12364-01

3634 = COUNT (NO. OF CALCULATIONS)

N	TM(N) DEG.R	TG(N) DEG.R	RG(N) LBM/CF	PG(N) PSIA	WG(N) LBM/SEC
-0	6.19558+02	6.18092+02	4.36320-01	7.58473+02	-2.76650-03
1	5.98547+02	5.96242+02	4.61863-01	7.58477+02	-2.68783-03
2	5.72830+02	5.70313+02	4.84944-01	7.58485+02	-2.52282-03
3	5.42469+02	5.39837+02	5.12073-01	7.58492+02	-2.34880-03
4	5.10242+02	5.07516+02	5.40843-01	7.58498+02	-2.16524-03
5	4.77541+02	4.74719+02	5.80926-01	7.58502+02	-1.96846-03
6	4.44745+02	4.41818+02	6.24384-01	7.58506+02	-1.75738-03
7	4.11941+02	4.08857+02	6.67921-01	7.58509+02	-1.53196-03
8	3.79174+02	3.75874+02	7.24684-01	7.58511+02	-1.28798-03
9	3.46633+02	3.43060+02	7.90138-01	7.58512+02	-1.02270-03
10	3.14345+02	3.11604+02	8.67753-01	7.58513+02	-7.32326-04
11	2.75094+02	2.74732+02	9.82663-01	7.58515+02	-5.56321-04
12	2.41985+02	2.41894+02	1.10802+00	7.58522+02	-2.24233-05
13	2.09107+02	2.08754+02	1.27521+00	7.58518+02	5.87950-04
14	1.76506+02	1.76238+02	1.51519+00	7.58502+02	1.30629-03
15	1.45737+02	1.45537+02	1.80296+00	7.58474+02	2.15225-03
16	1.26389+02	1.26339+02	2.11084+00	7.58456+02	2.64148-03

TABLE 6-2 (Continued)

Node No.	Matrix Temperature	Gas Temperature	Gas Density	Gas Pressure	Mass Flow Rate

θ = 180°

REGULAR PRINTOUTS

TIME(SEC.) = 5.24983-01

DATE = 24 MAR 72 TIME = 13:07:32
3712 = COUNT (NO. OF CALCULATIONS)

N	TM(N) DEG.R	TG(N) DEG.R	RG(N) LBM/CF	PG(N) PSIA	WG(N) LBM/SEC
-0	6.18588+02	6.17022+02	4.11594-01	7.26611+02	-5.58727-03
1	5.97139+02	5.94935+02	4.44026-01	7.26623+02	-5.51388-03
2	5.71312+02	5.68855+02	4.66322-01	7.26644+02	-5.35992-03
3	5.40924+02	5.38352+02	4.92397-01	7.26663+02	-5.19757-03
4	5.08699+02	5.06067+02	5.19995-01	7.26679+02	-5.02634-03
5	4.76012+02	4.73339+02	5.59007-01	7.26694+02	-4.84265-03
6	4.43242+02	4.40540+02	6.00623-01	7.26706+02	-4.64570-03
7	4.10467+02	4.07733+02	6.42248-01	7.26716+02	-4.43547-03
8	3.77723+02	3.74963+02	6.96995-01	7.26725+02	-4.20793-03
9	3.45214+02	3.42447+02	7.59712-01	7.26733+02	-3.96061-03
10	3.13220+02	3.10725+02	8.34949-01	7.26738+02	-3.68975-03
11	2.74790+02	2.74398+02	9.44459-01	7.26753+02	-3.52576-03
12	2.41712+02	2.41315+02	1.06692+00	7.26895+02	-3.02774-03
13	2.08857+02	2.08451+02	1.22663+00	7.26987+02	-2.45900-03
14	1.76271+02	1.75866+02	1.45937+00	7.27041+02	-1.78917-03
15	1.45518+02	1.45287+02	1.73773+00	7.27066+02	-1.00022-03
16	1.26072+02	1.25000+02	2.05170+00	7.27070+02	-5.40598-04

θ = 210°

DATE = 24 MAR 72 TIME = 13:07:45

TIME(SEC.) = 5.37423-01

3809 = COUNT (NO. OF CALCULATIONS)

N	TM(N) DEG.R	TG(N) DEG.R	RG(N) LBM/CF	PG(N) PSIA	WG(N) LBM/SEC
-0	6.17367+02	6.15800+02	3.92759-01	7.00580+02	-6.77597-03
1	5.95488+02	5.93399+02	4.29770-01	7.00596+02	-6.71888-03
2	5.69484+02	5.67130+02	4.51468-01	7.00624+02	-6.59908-03
3	5.39036+02	5.36560+02	4.76716-01	7.00649+02	-6.47277-03
4	5.06791+02	5.04258+02	5.03393-01	7.00672+02	-6.33956-03
5	4.74104+02	4.71534+02	5.41814-01	7.00693+02	-6.19652-03
6	4.41345+02	4.38747+02	5.82016-01	7.00710+02	-6.04318-03
7	4.08586+02	4.05961+02	6.22217-01	7.00726+02	-5.87956-03
8	3.75851+02	3.73207+02	6.76013-01	7.00739+02	-5.70228-03
9	3.43365+02	3.40727+02	7.37547-01	7.00750+02	-5.50946-03
10	3.11513+02	3.08989+02	8.10337-01	7.00760+02	-5.29836-03
11	2.74404+02	2.74011+02	9.13520-01	7.00785+02	-5.17098-03
12	2.41322+02	2.40922+02	1.03278+00	7.01041+02	-4.78403-03
13	2.08453+02	2.08043+02	1.18760+00	7.01229+02	-4.34219-03
14	1.75862+02	1.75459+02	1.41443+00	7.01361+02	-3.82156-03
15	1.45261+02	1.45020+02	1.68524+00	7.01445+02	-3.20825-03
16	1.25320+02	1.25000+02	1.98595+00	7.01472+02	-2.85175-03

TABLE 6-2 (Continued)

Node No.	Matrix Temperature	Gas Temperature	Gas Density	Gas Pressure	Mass Flow Rate

REGULAR PRINTOUTS

θ = 240°

TIME(SEC.) = 5.49974-01

DATE = 24 MAR 72 TIME = 13:07:57

3917 = COUNT (NO. OF CALCULATIONS)

N	TM(N) DEG.R	TG(N) DEG.R	RG(N) LBM/CF	PG(N) PSIA	WG(N) LBM/SEC
-0	6.16168+02	6.14730+02	3.82463-01	6.84778+02	-6.42126-03
1	5.93948+02	5.92110+02	4.21358-01	6.84793+02	-6.38766-03
2	5.67749+02	5.65656+02	4.42741-01	6.84820+02	-6.31716-03
3	5.37222+02	5.35012+02	4.67508-01	6.84845+02	-6.24282-03
4	5.04946+02	5.02681+02	4.93637-01	6.84867+02	-6.16443-03
5	4.72243+02	4.69944+02	5.31878-01	6.84887+02	-6.08017-03
6	4.39477+02	4.37148+02	5.71238-01	6.84905+02	-5.98987-03
7	4.06716+02	4.04362+02	6.10588-01	6.84920+02	-5.89353-03
8	3.73970+02	3.71597+02	6.64071-01	6.84934+02	-5.78904-03
9	3.41487+02	3.39118+02	7.25195-01	6.84946+02	-5.67530-03
10	3.09670+02	3.07345+02	7.96552-01	6.84956+02	-5.55084-03
11	2.74024+02	2.73680+02	8.94950-01	6.84984+02	-5.47599-03
12	2.40929+02	2.40577+02	1.01239+00	6.85271+02	-5.24852-03
13	2.08035+02	2.07673+02	1.16427+00	6.85495+02	-4.98882-03
14	1.75427+02	1.75073+02	1.38789+00	6.85663+02	-4.68263-03
15	1.44955+02	1.44734+02	1.65485+00	6.85782+02	-4.32177-03
16	1.25068+02	1.25000+02	1.94575+00	6.85825+02	-4.11251-03

θ = 270°

DATE = 24 MAR 72 TIME = 13:08:09

TIME(SEC.) = 5.62451-01

4025 = COUNT (NO. OF CALCULATIONS)

N	TM(N) DEG.R	TG(N) DEG.R	RG(N) LBM/CF	PG(N) PSIA	WG(N) LBM/SEC
-0	6.15293+02	6.14343+02	3.82998-01	6.84368+02	-4.59001-03
1	5.92904+02	5.91876+02	4.21298-01	6.84378+02	-4.61143-03
2	5.66532+02	5.65278+02	4.42779-01	6.84396+02	-4.65639-03
3	5.35926+02	5.34556+02	4.67589-01	6.84412+02	-4.70381-03
4	5.03609+02	5.02186+02	4.93730-01	6.84428+02	-4.75381-03
5	4.70878+02	4.69411+02	5.32182-01	6.84442+02	-4.80757-03
6	4.38086+02	4.36575+02	5.71563-01	6.84455+02	-4.86519-03
7	4.05304+02	4.03757+02	6.10924-01	6.84467+02	-4.92667-03
8	3.72525+02	3.70944+02	6.64754-01	6.84478+02	-4.99338-03
9	3.40017+02	3.38404+02	7.26333-01	6.84488+02	-5.06602-03
10	3.08113+02	3.06280+02	7.98431-01	6.84497+02	-5.14558-03
11	2.73741+02	2.73538+02	8.94818-01	6.84523+02	-5.19330-03
12	2.40626+02	2.40409+02	1.01250+00	6.84801+02	-5.33838-03
13	2.07700+02	2.07466+02	1.16450+00	6.85042+02	-5.50404-03
14	1.75067+02	1.74833+02	1.38892+00	6.85244+02	-5.69942-03
15	1.44670+02	1.44496+02	1.65753+00	6.85411+02	-5.92984-03
16	1.25012+02	1.25000+02	1.94487+00	6.85481+02	-6.06321-03

TABLE 6-2 (Continued)

Node No.	Matrix Temperature	Gas Temperature	Gas Density	Gas Pressure	Mass Flow Rate

REGULAR PRINTOUTS

θ = 300°

DATE = 24 MAR 72 TIME = 13:08:18

TIME(SEC.) = 5.74961-01 4123 = COUNT (NO. OF CALCULATIONS)

N	TM(N) DEG.R	TG(N) DEG.R	RG(N) LBM/CF	PG(N) PSIA	WG(N) LBM/SEC
-0	6.14908+02	6.14680+02	3.92417-01	6.97300+02	-1.77499-03
1	5.92538+02	5.92615+02	4.28444-01	6.97303+02	-1.82198-03
2	5.66051+02	5.65941+02	4.50364-01	6.97309+02	-1.92064-03
3	5.35381+02	5.35152+02	4.75665-01	6.97314+02	-2.02470-03
4	5.03027+02	5.02730+02	5.02309-01	6.97319+02	-2.13444-03
5	4.70262+02	4.69896+02	5.41288-01	6.97324+02	-2.25242-03
6	4.37433+02	4.36989+02	5.81445-01	6.97329+02	-2.37890-03
7	4.04618+02	4.04107+02	6.21572-01	6.97334+02	-2.51385-03
8	3.71796+02	3.71215+02	6.76296-01	6.97339+02	-2.66027-03
9	3.39241+02	3.38573+02	7.39022-01	6.97343+02	-2.81976-03
10	3.07139+02	3.06012+02	8.13359-01	6.97347+02	-2.99467-03
11	2.73609+02	2.73565+02	9.10680-01	6.97360+02	-3.09949-03
12	2.40473+02	2.40408+02	1.03025+00	6.97501+02	-3.41822-03
13	2.07516+02	2.07427+02	1.18477+00	6.97635+02	-3.78214-03
14	1.74856+02	1.74756+02	1.41306+00	6.97759+02	-4.21155-03
15	1.44468+02	1.44365+02	1.68714+00	6.97872+02	-4.71824-03
16	1.25002+02	1.25000+02	1.97683+00	6.97923+02	-5.01120-03

θ = 330°

DATE = 24 MAR 72 TIME = 13:08:26

TIME(SEC.) = 5.87431-01 4199 = COUNT (NO. OF CALCULATIONS)

N	TM(N) DEG.R	TG(N) DEG.R	RG(N) LBM/CF	PG(N) PSIA	WG(N) LBM/SEC
-0	6.15456+02	6.20000+02	4.02251-01	7.21446+02	1.63786-03
1	5.92846+02	5.94389+02	4.41400-01	7.21443+02	1.57080-03
2	5.66430+02	5.68907+02	4.63021-01	7.21439+02	1.43030-03
3	5.35728+02	5.38510+02	4.88814-01	7.21435+02	1.28217-03
4	5.03340+02	5.06391+02	5.16069-01	7.21432+02	1.12599-03
5	4.70530+02	4.73946+02	5.54314-01	7.21430+02	9.58599-04
6	4.37645+02	4.41493+02	5.95198-01	7.21428+02	7.79228-04
7	4.04772+02	4.08961+02	6.36182-01	7.21426+02	5.87826-04
8	3.71891+02	3.76117+02	6.90037-01	7.21426+02	3.80689-04
9	3.39281+02	3.42851+02	7.53427-01	7.21425+02	1.55032-04
10	3.06922+02	3.07845+02	8.35884-01	7.21425+02	-9.42281-05
11	2.73664+02	2.73959+02	9.39289-01	7.21425+02	-2.44137-04
12	2.40509+02	2.40722+02	1.06218+00	7.21434+02	-6.99871-04
13	2.07530+02	2.07690+02	1.22150+00	7.21455+02	-1.22045-03
14	1.74848+02	1.74962+02	1.45609+00	7.21487+02	-1.83466-03
15	1.44397+02	1.44404+02	1.73909+00	7.21527+02	-2.55993-03
16	1.25000+02	1.25000+02	2.03752+00	7.21550+02	-2.97940-03

TABLE 6-2 (Continued)

Node No.	Matrix Temperature	Gas Temperature	Gas Density	Gas Pressure	Mass Flow Rate

$\theta = 360^0$

REGULAR PRINTOUTS

DATE = 24 MAR 72 TIME = 13:08:34

TIME(SEC.) = 5.99888-01

4264 = COUNT (NO. OF CALCULATIONS)

N	TM(N) DEG.R	TG(N) DEG.R	RG(N) LBM/CF	PG(N) PSIA	WG(N) LBM/SEC
-0	6.17379+02	6.20000+02	4.28855-01	7.52492+02	5.07929-03
1	5.93730+02	5.95447+02	4.58998-01	7.52482+02	5.00138-03
2	5.67641+02	5.69923+02	4.81532-01	7.52465+02	4.83809-03
3	5.37029+02	5.39592+02	5.08313-01	7.52449+02	4.66594-03
4	5.04665+02	5.07358+02	5.36774-01	7.52435+02	4.48437-03
5	4.71857+02	4.74629+02	5.76521-01	7.52423+02	4.28974-03
6	4.38962+02	4.41792+02	6.19554-01	7.52413+02	4.08100-03
7	4.06065+02	4.08944+02	6.62603-01	7.52404+02	3.85814-03
8	3.73167+02	3.76104+02	7.18617-01	7.52397+02	3.61703-03
9	3.40512+02	3.43508+02	7.82900-01	7.52391+02	3.35508-03
10	3.08014+02	3.11099+02	8.62263-01	7.52387+02	3.06755-03
11	2.73820+02	2.74098+02	9.77214-01	7.52375+02	2.89312-03
12	2.40769+02	2.41199+02	1.10282+00	7.52273+02	2.36368-03
13	2.07777+02	2.08230+02	1.26782+00	7.52211+02	1.75888-03
14	1.75077+02	1.75576+02	1.50880+00	7.52181+02	1.04611-03
15	1.44480+02	1.45046+02	1.79691+00	7.52171+02	2.05902-04
16	1.25000+02	1.25000+02	2.11618+00	7.52171+02	-2.82425-04

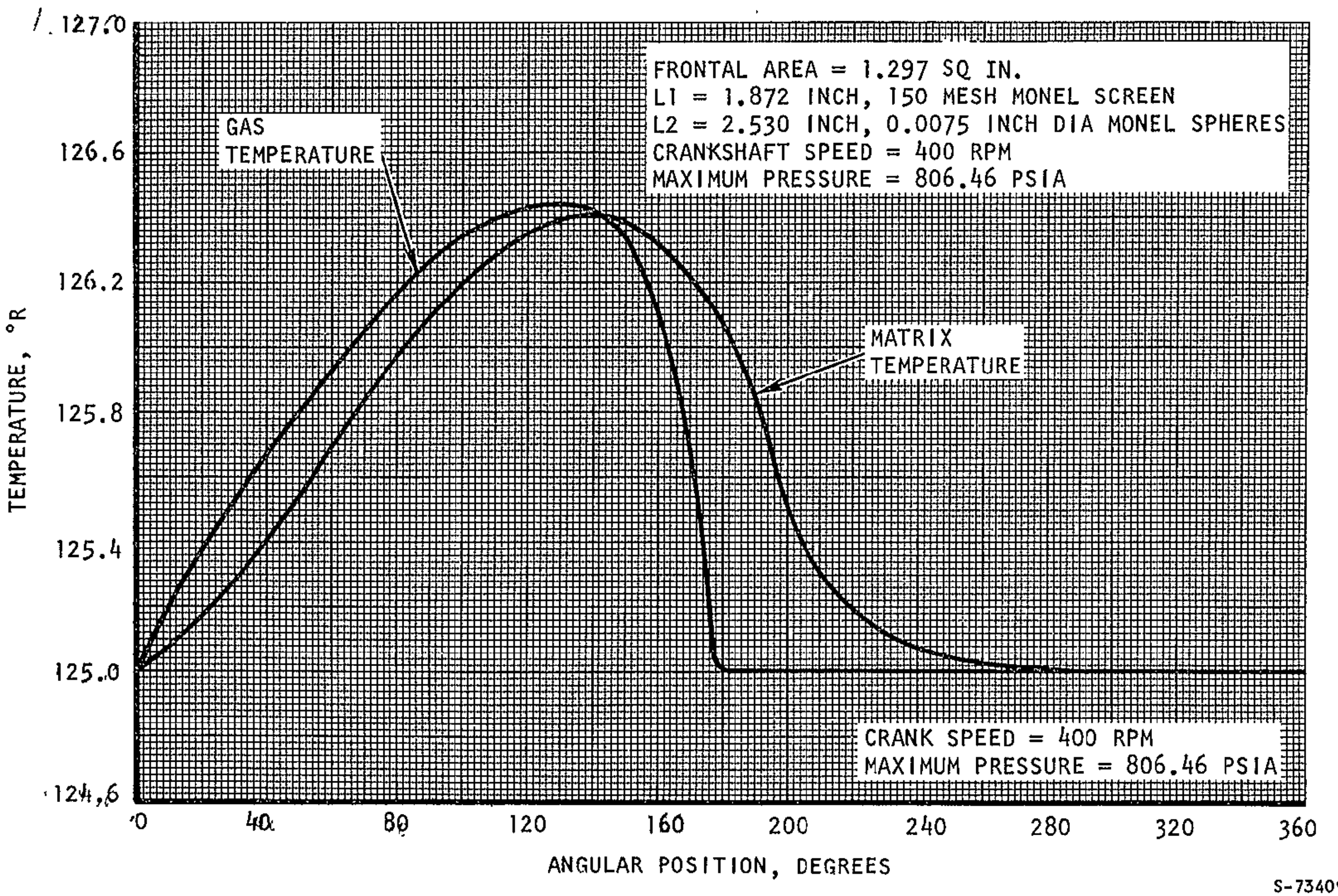


Figure 6-5. Temperature Variation at Cold End of Cold Refrigerator

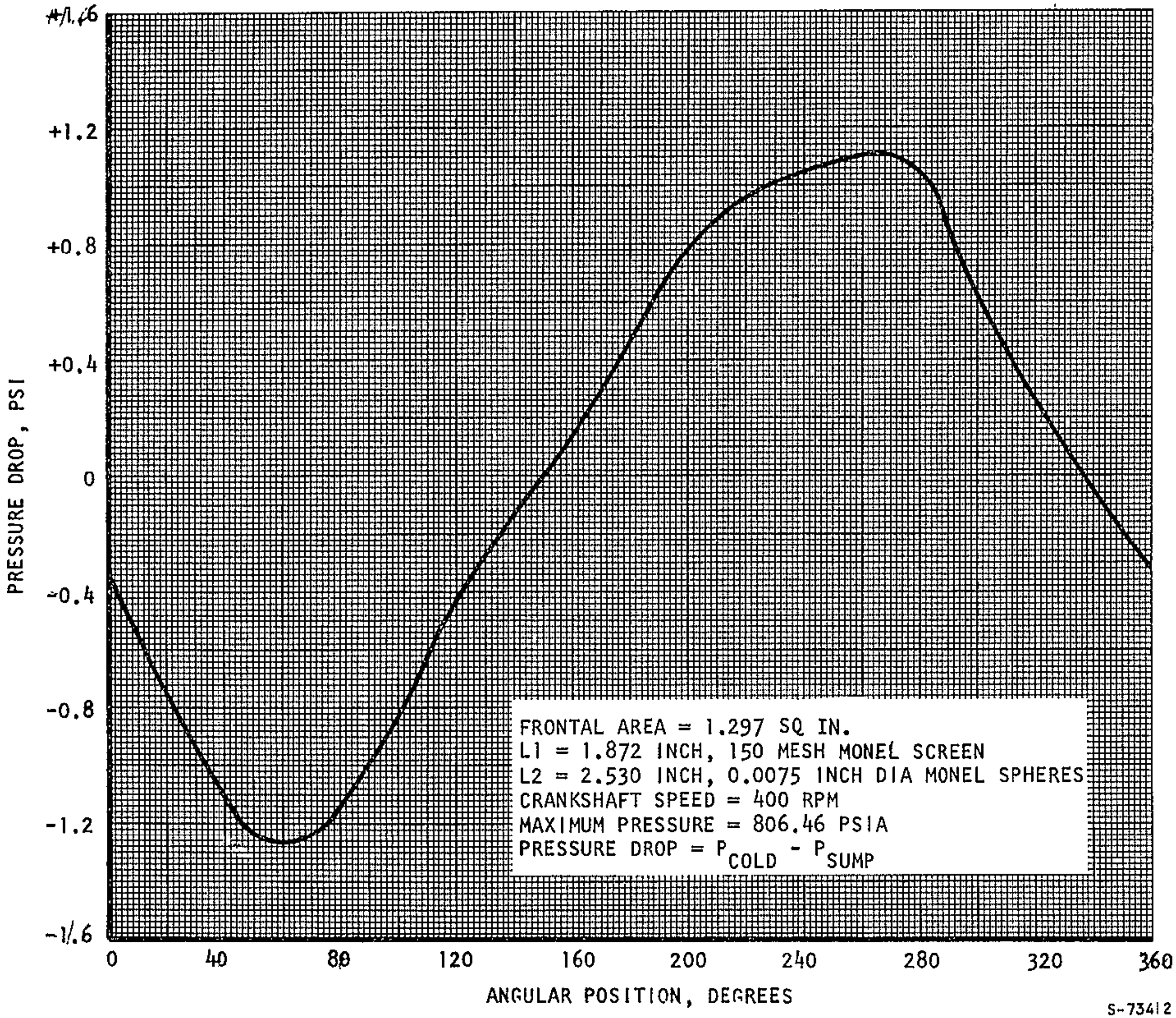


Figure 6-6. Pressure Drop for Cold Regenerator

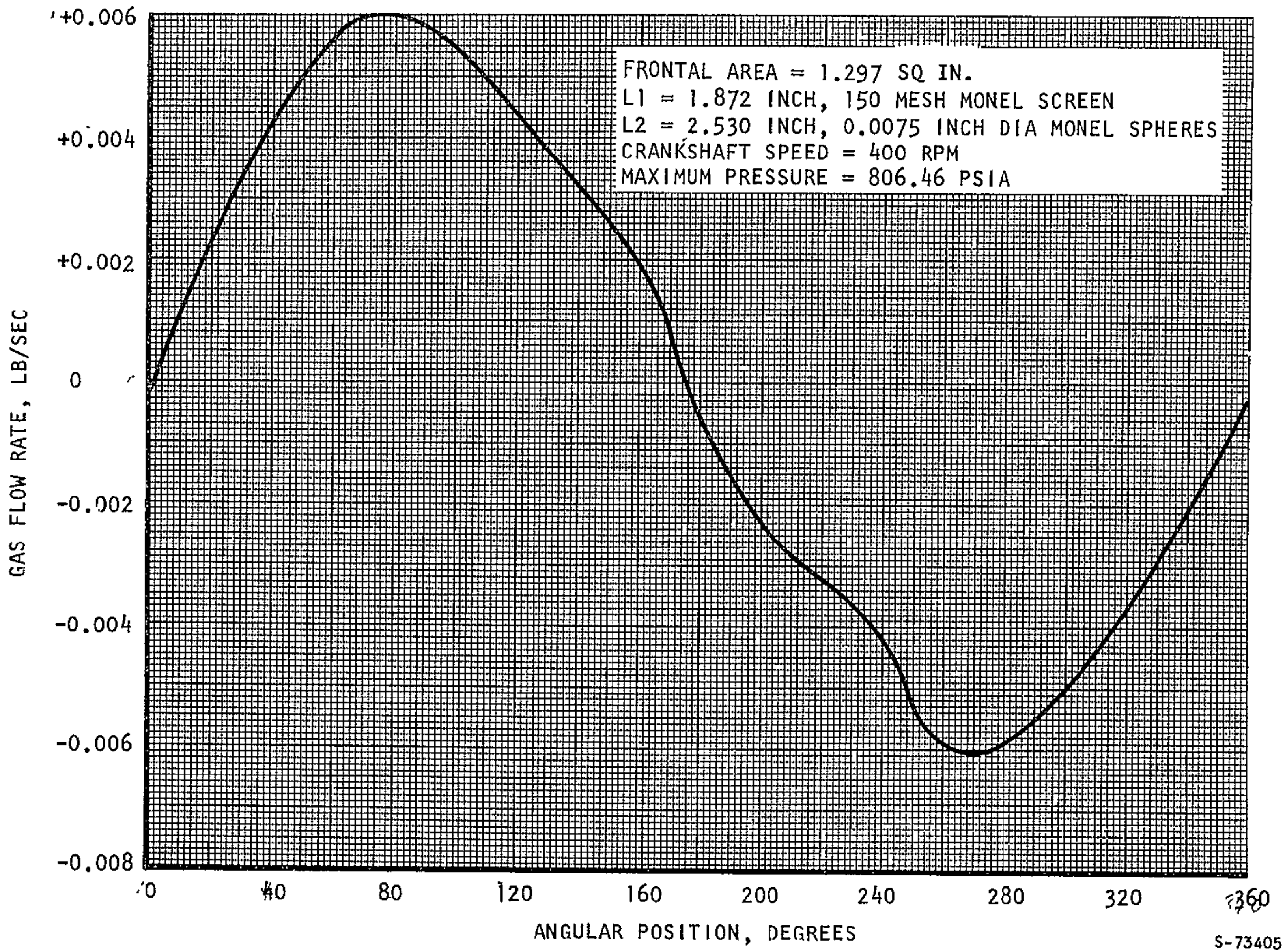


Figure 6-7. Flow Rate into Cold Volume

The data of Figure 6-6 show a maximum pressure drop across the cold regenerator of approximately 1.3 psi. This low pressure drop was intentionally designed into the system to provide for low wear ratios and compatibility with the use of dynamic seals. Figure 6-7 gives the gas flow rate into the cold displaced volume. These data, coupled with the temperature, can be used to estimate the losses associated with the performance of the cold regenerator.

For an ideal or perfect regenerator, the temperature of the gas entering the cold volume from the regenerator would be identical to the temperature of the gas returning to the regenerator subsequent to the refrigeration process. In the actual case, the temperature varies as the gas enters the cold displaced volume as a result of the performance of the regenerator (Figures 6-5 and 6-7). The loss associated with this temperature variation can be estimated by computing the excess energy in the fluid stream above the reference level associated with the ideal case. The losses per cycle can then be expressed as:

$$\dot{Q}_{loss} = \oint \dot{\omega} \left(h - h_{ref} \right) d\tau \qquad (6\text{-}5)$$

where $\dot{\omega}$ = flow rate at a point in time

h = enthalphy of the fluid entering or exiting the cold volume

h_{ref} = reference enthalpy for the ideal refrigeration gas temperature

τ = time

$\oint$ = integration over a complete cycle or revolution of the refrigerator

Figure 6-8 gives the accumulative losses due to the cold regenerator as a function of crank angle position. The loss per cycle is 0.000555 Btu, which yields a total loss of 3.90 w.

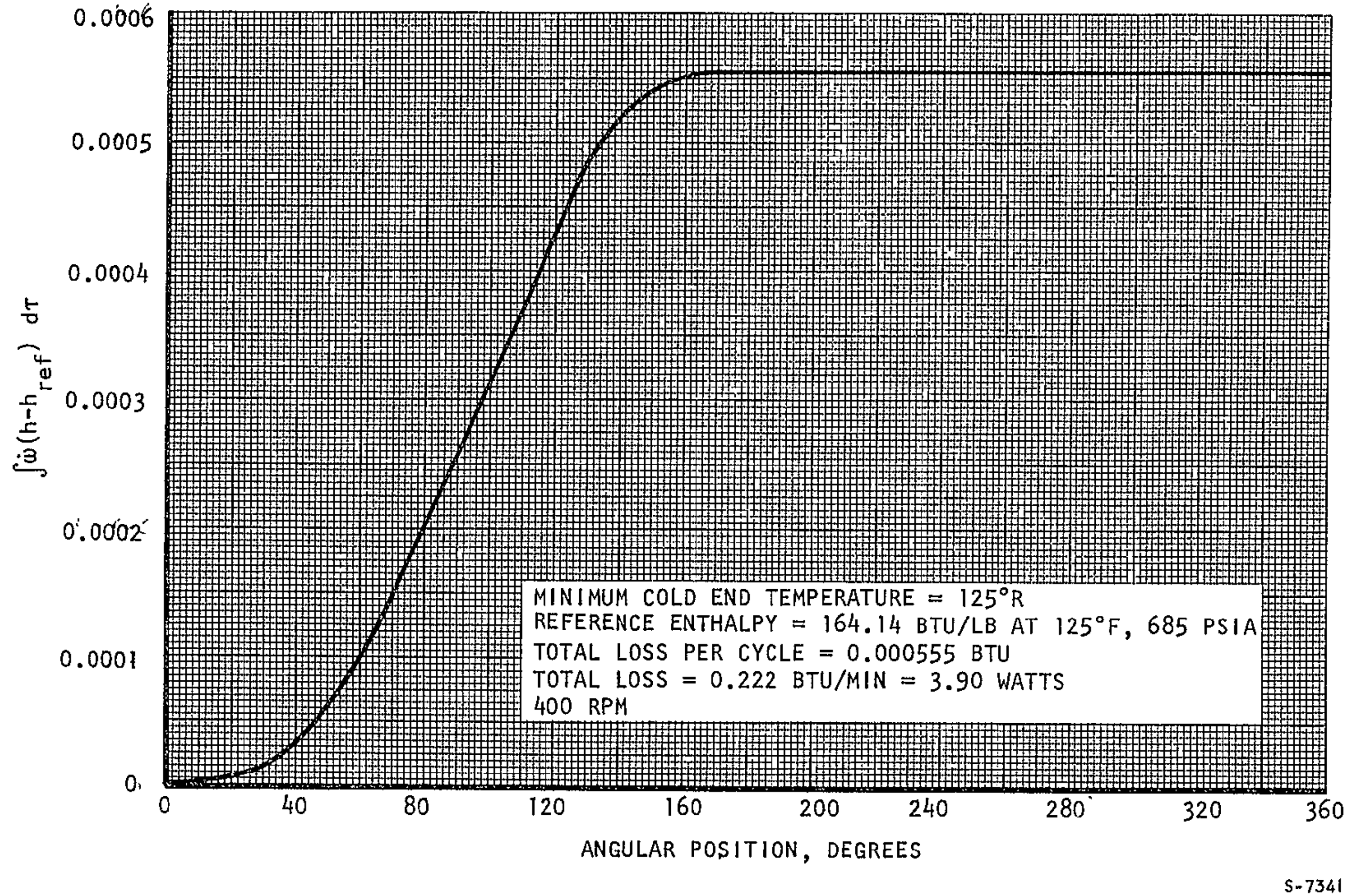


Figure 6-8. Cold Regenerator Thermal Loss per Cycle

SECTION 7

DESIGN OF COLD-END SEAL

INTRODUCTION

The cold end seals function to control the rate of leakage which bypasses the cold regenerator. This function is critical in the refrigerator design since leakage past the seals can result in a significant loss in thermal performance. Leakage past the seals bypasses the regenerator by flowing through the annular space between the cold displacer and cold cylinder walls. At low leakage rates, the displacer and cylinder walls effectively regenerate the leakage fluid temperatures and the resulting thermal losses are small. As leakage rates increase, the walls can no longer function as an effective regenerator, and significant losses in overall thermal performance occur.

DESIGN CONFIGURATION

The basic configuration of the cold end sealing system is shown in Figure 7-1. Seals incorporated in the cold end sealing system, from the cold end toward the sump end of the cold displacer, are as follows:

(a) An 0.945 in. long, close-fit annular seal with a clearance between the inner wall of the regenerator and the seal of 0.0025 in. maximum.

(b) A 5-groove labyrinth seal with a tip clearance of 0.0025 in., a groove spacing of 0.05 in., and a nominal tip width of 0.005 in.

(c) The first linear bearing which acts as an annular seal with a clearance of 0.0004 in. and a length of 1.0 in.

(d) The bearing support member which acts as an annular seal with a clearance of 0.0035 in. and a length of 1.69 in.

(e) The second linear bearing which is identical to the first.

The arrangement of the machine is such that these sealing elements are in series.

Originally, it was not intended that the linear bearings and the bearing support would be used as part of the sealing system, but the mechanical arrangement of the machine allows the use of these components as seals without penalty. The only disadvantage in their use is that the leakage rate is dependent on bearing clearance and increases as the bearing wears. However, the loading and rate of wear of these bearings is low, and will provide over two years of operation before bearing wear affects the performance (even in the worst case analysis).

METHOD OF ANALYSIS

The correlations for analyzing the leakage past both labyrinth and annular seals were developed and modified to match the seal test data in Section 4 of the final report for the Vuilleumier Cryogenic Engine Development Program (Ref. 3).

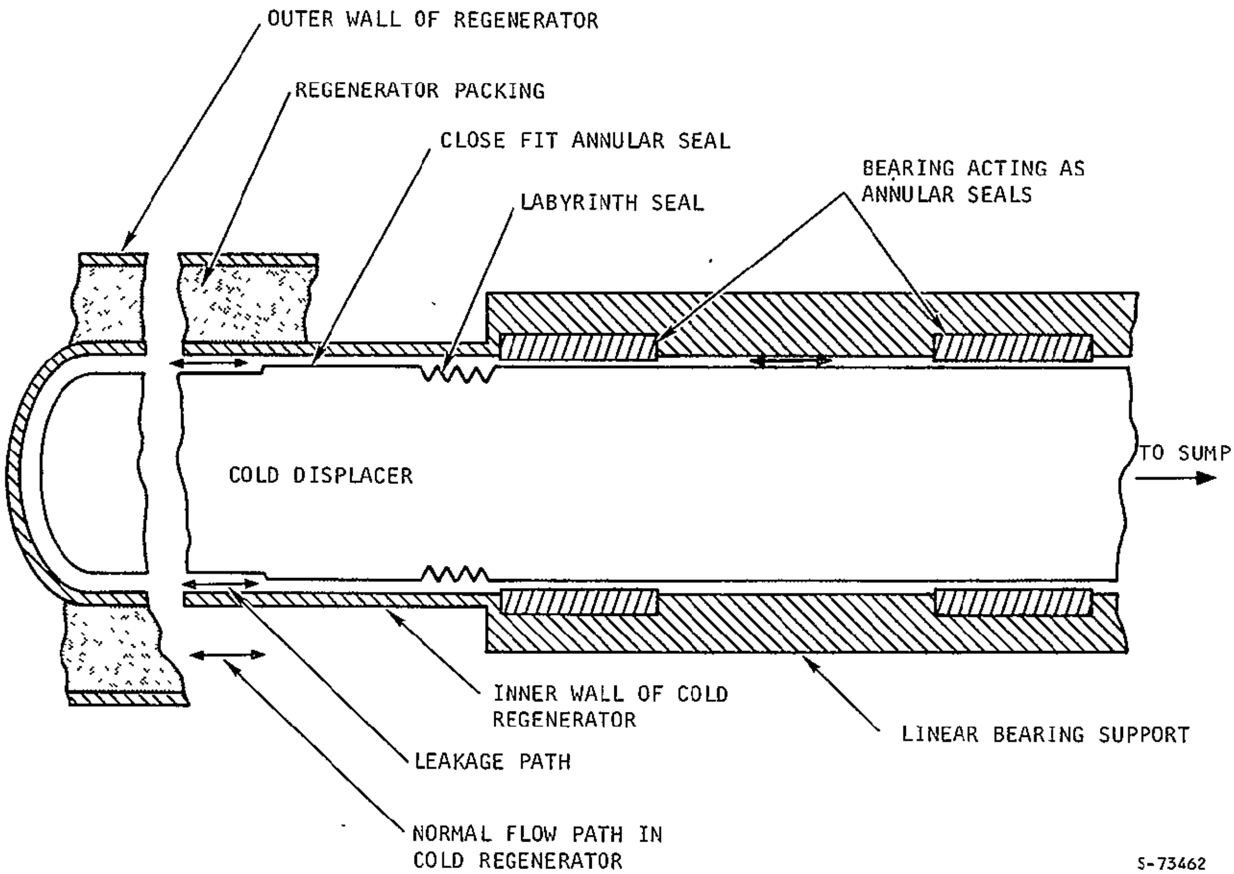


Figure 7-1. Cold Displacer Sealing Design Configuration

the final report for the Vuilleumier cryogenic engine development program (Ref 3; Equations 4-24 and 4-25). Solving these Ref 3 equations for pressure drop yields:

for Labyrinth seal

$$\Delta P_i = \frac{\dot{\omega}_L^2 T_o R \eta}{C^2 A^2 \sqrt{P_o} K_L^2 \zeta \, 2 g_c} \tag{7-1}$$

for Annular Seal

$$\Delta P_i = \frac{\dot{\omega}_A L}{K_A \rho} \left[\frac{48 \mu}{D_H^2 g_c A} \right] \tag{7-2}$$

Then noting that:

$$\Delta P_T = \sum_{0}^{N} \Delta P_i = \sum_{0}^{N} f_i(\dot{\omega}_i) \qquad (7\text{-}3)$$

where

ΔP_T = the total pressure drop across the series of sealing elements

i and N = the identity and number of elements respectively

then further noting that

$$\dot{\omega}_T = \dot{\omega}_L = \dot{\omega}_A = \dot{\omega}_i \qquad (7\text{-}4)$$

that is, the flow past each element is the same, the pressure drop can be computed as a function of leakage rate from Equation 7-3.

PERFORMANCE CHARACTERISTICS

Leakage Rate

The configuration of the cold end of the refrigerator is such that seal elements can be added in series (or subtracted) as desired by providing flow passages into the active cycle volume at different locations. Figure 7-2 gives the leakage rate as a function for pressure drop for the various combinations of seal elements. The selected design combines all possible seal elements to minimize the losses due to leakage.

Figure 7-2 presents leakage rate data for seal systems with both new bearings and bearings after 2 years of wear. As the bearings wear, the clearance of the annular seal (the seal which the bearings form with the cold displacer) increases and hence the leakage rate increases. The bearing wear rate used in the analysis is based on the bearing material tests described in Section 3 of Ref 3. The use of these wear rate data is believed very conservative due to the low loading of the linear bearings compared to the wear rate test conditions.

The dashed vertical lines in Figure 7-2 correspond to design limit levels of pressure drop across the cold end seal. Both pressure drop levels are based on the maximum cold end flow rate at a rotational speed of 400 rpm and thus represent the worst case conditions. The lower pressure drop level assumes uniform packing of perfectly spherical shot in the section of the cold regenerator containing shot type packing--approximately one half of the cold regenerator length is packed with shot while the other half is packed with screens. The higher pressure drop level assumes the shot packed section will have a distribution in shot size between 0.007 and 0.008 in. diameter (0.0075 in. ave dia) and that many of the individual shot will not be perfectly spherical.

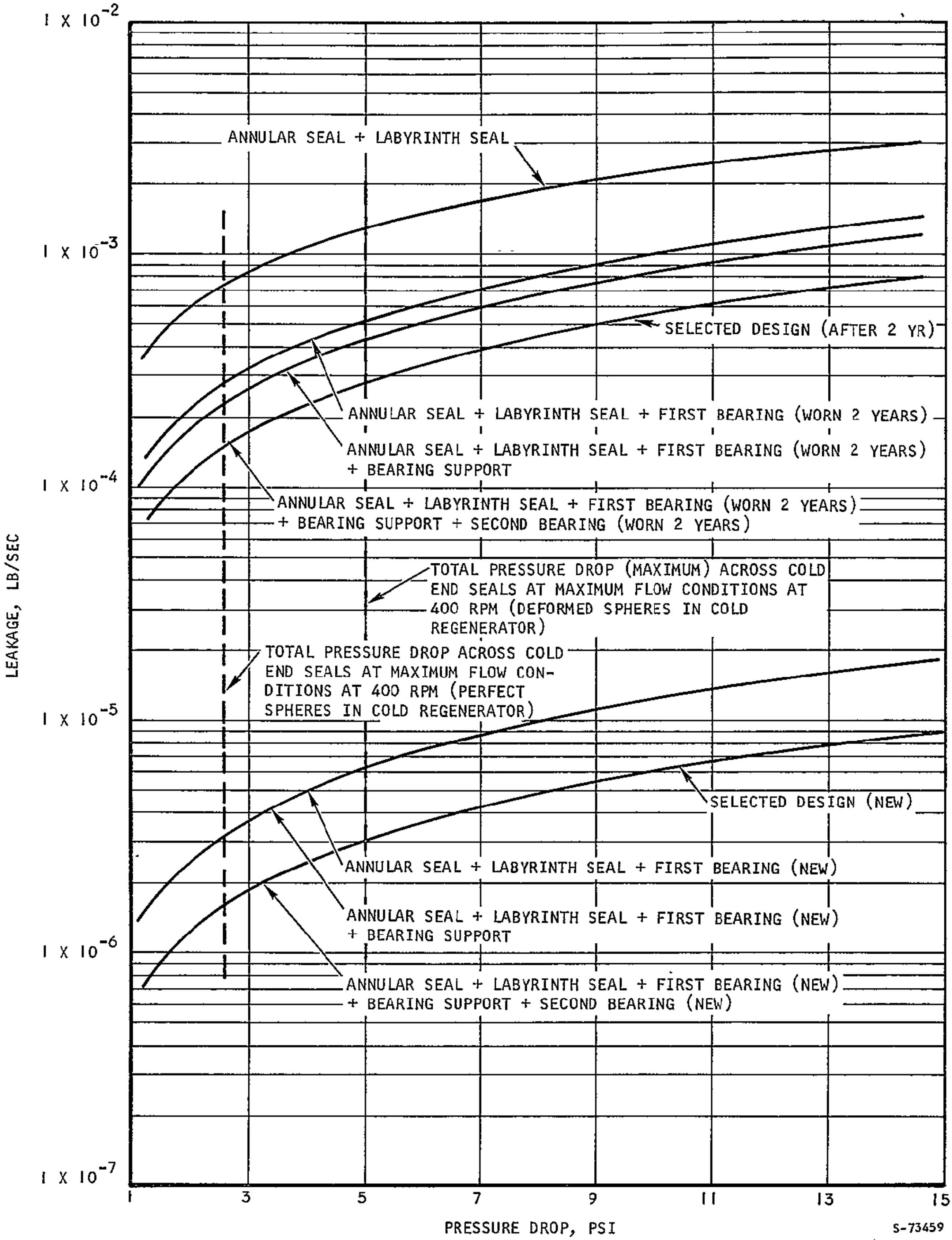


Figure 7-2. Cold End Seal Leakage--Pressure Drop Characteristics

The increased pressure drop predicted for the deformed shot packing is based on tests conducted under AiResearch sponsorship. These tests have shown that regenerator beds packed with commercially available shot can have pressure drops as large as three times that predicted for a bed with perfect spheres of uniform size. This factor of three was used in establishing the pressure drop upper limit shown in Figure 7-2.

Careful processing of commercially available shot can be used to bring the pressure drop of packed beds into closer agreement with that of a bed of perfect spheres. Multiple screening of the shot used in the GSFC VM refrigerator is expected to reduce the pressure drop in the shot packed section of the cold regenerator to 1.5 to 2.0 times that predicted for perfectly spherical shot. The actual upper limit in pressure drop will thus be between those shown in Figure 7-2.

Thermal Losses

The cold end leakage is important, in that it influences refrigerator thermal performance. A good approximation of the thermal losses associated with the cold end leakage is provided by the simple model developed below.

Figure 7-3 depicts the basic elements of the thermal loss model employed. Taking an element of length (dX) along the annular flow passage between displacer and inner regenerator wall, the following differential equation can be written:

$$\frac{dT_f}{dX} + \frac{hA_c}{\dot{\omega}C_p} T_f = \frac{hA_c}{\dot{\omega}C_p} T_\omega \qquad (7\text{-}5)$$

where

T_f = temperature of the leakage gas at any location X

T_ω = temperature of the regenerator and displacer walls at any location X

h = local heat transfer coefficient between leakage gas and the surrounding walls

A_c = heat transfer area per unit length along the leakage path

$\dot{\omega}$ = rate of leakage

C_p = heat capacity of leakage gas

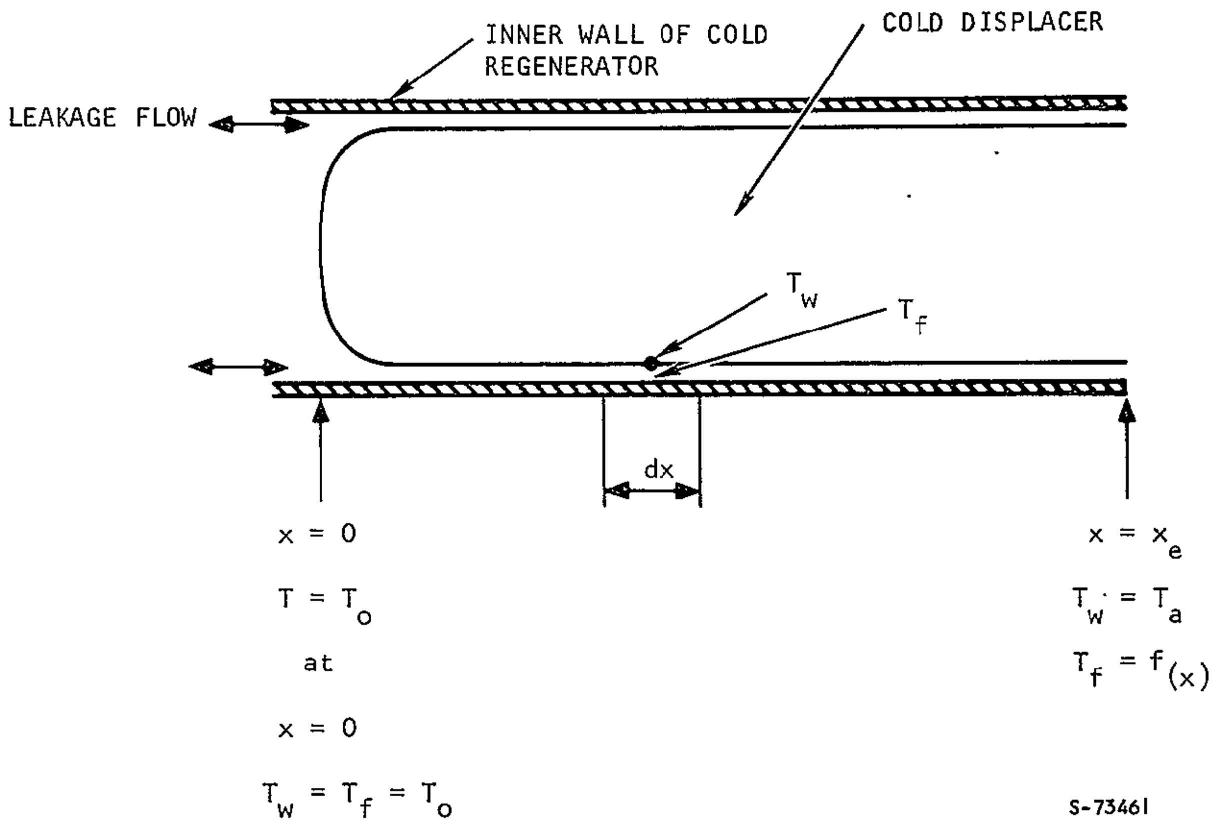


Figure 7-3. Leakage Thermal Loss Model

Then if a linear temperature distribution is assumed along the displacer and regenerator walls and ratio of heat transfer coefficient to gas heat capacity is taken as a constant, Equation 7-5 can be written as:

$$\frac{dT_f}{dX} + \alpha T_f = \alpha\,(T_a - T_o)\,\frac{X}{X_e} + \alpha T_o \tag{7-6}$$

where the new terms are:

$$\alpha = \frac{hA_c}{\dot{\omega} C_p}$$

T_a = wall temperature at the sump end of the displacer

T_o = refrigeration temperature or wall temperature at X = o

X_e = length of the displacer

Solving Equation 7-6 with the boundary conditions of $T_f = T_o$ at $X = o$ yields:

$$T_f = \frac{T_a - T_o}{X_e} \left\{ X - \frac{1}{\alpha} + \frac{1}{\alpha} e^{-\alpha X} \right\} + T_o \tag{7-7}$$

If the leakage flow were completely regenerated by the walls, the temperature of the fluid would be that of the wall at the end of the displacer; that is:

$$T_f \longrightarrow T_a \;@\; X = X_e$$

or if flow (leakage) is considered in the reverse direction

$$T_f \longrightarrow T_o \;@\; X = 0$$

In setting up the relation for T_f, this latter condition was used as a boundary condition assuming flow in the positive X direction (see Figure 7-3). Relationships for flow in the reverse direction are similar but are not developed here. In the actual case, due to the cyclic operation of the VM refrigerator, the leakage flow does reverse direction. The leakage losses can be estimated, however, by considering flow in one direction with an appropriate time span. With this consideration the thermal losses per cycle due to leakage can be expressed as:

$$Q_L = \oint \left| \dot{\omega} \right| c_p \left\{ (T_a - T_o) \frac{1}{\alpha X_e} \left(1 - e^{-\alpha X_e}\right) \right\} d\tau \tag{7-8}$$

where

$\oint$ implies integration around the cycle and

τ = time

Assuming a constant leakage rate, the leakage thermal losses can be expressed as:

$$\dot{Q}_L = \dot{\omega} c_p \left\{ (T_a - T_o) \frac{1}{\alpha X_e} \left(1 - e^{-\alpha X_e}\right) \right\} \tag{7-9}$$

Figure 7-4 gives the losses (Equation 7-9) as a function of leakage rate. The thermal losses at leakage rates corresponding to the design limit levels of pressure drop and associated cold end leakage rates (see Figure 7-2) are shown in Figure 7-4. It is noted these losses are estimated for the machine after two years of wear; thermal losses due to leakage are negligible for a new machine. In fact, the leakage thermal losses for the machine after 2 years of wear are expected to be considerably lower than indicated in Figure 7-4 for two reasons. First, the bearing wear rate used in estimating the

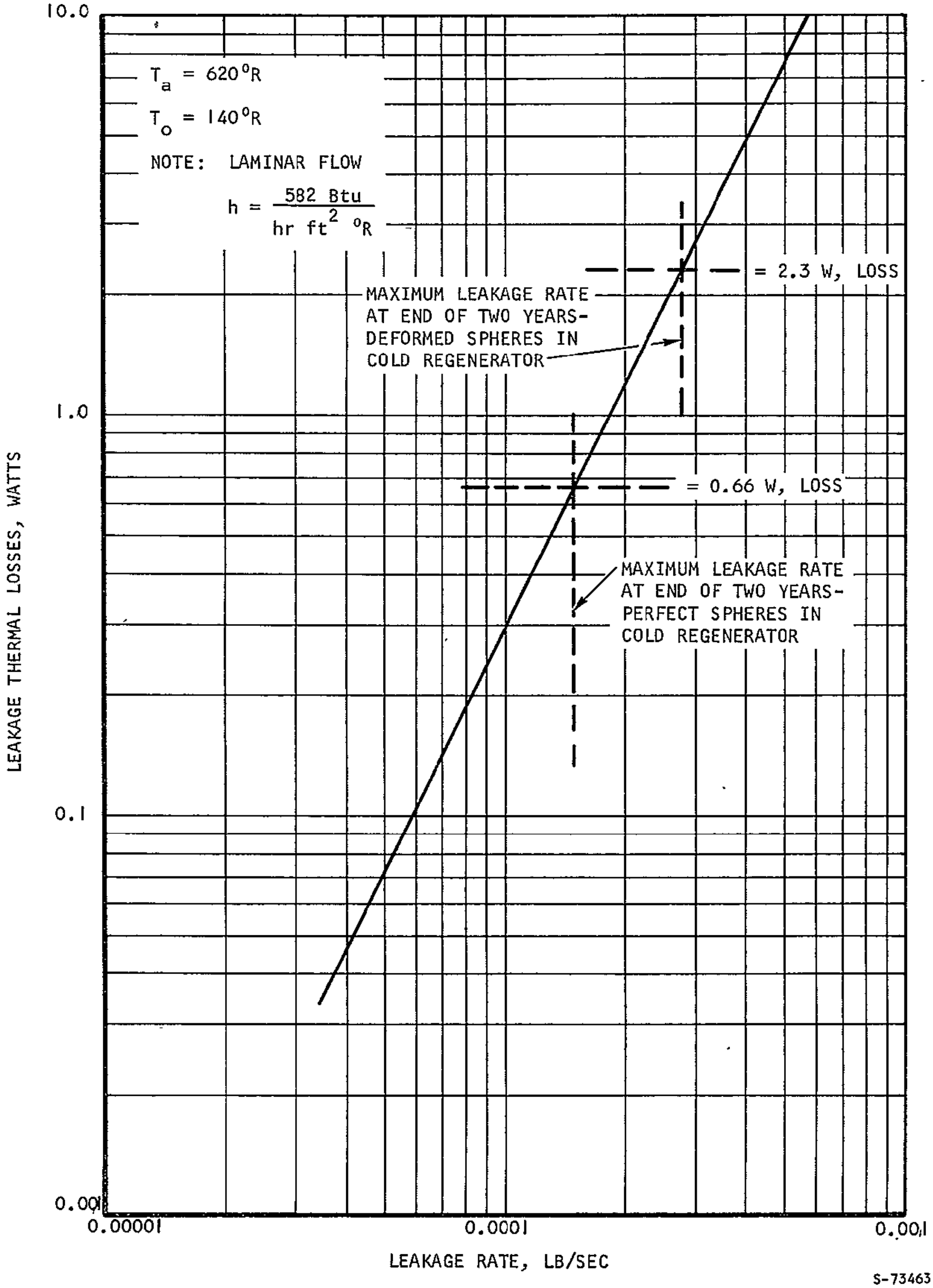
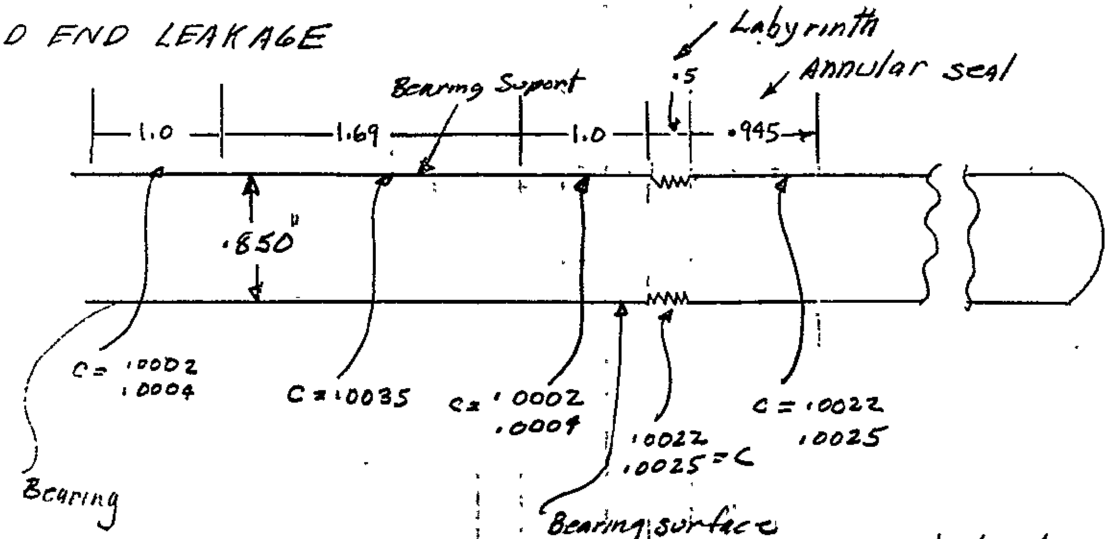


Figure 7-4. Thermal Losses Due to Leakage as a Function of Leakage Rate

leakage is considered to yield a conservative result as previously mentioned. Second, constant leakage rates at their maximum levels were used in estimating the losses by use of Equation 7-8. In the actual case, the leakage rate is a periodic function with an average value on the order of 70 percent of the maximum. Thus not only are the losses lower due to the decreased leakage flow, but the regenerator and displacer walls more effectively regenerate the leakage gas, further reducing the losses below the maximum values indicated in Figure 7-4.

COLD END LEAKAGE



c = radial clearance between displacer and adjacent surface

1.0 ANALYTICAL RELATIONS

FOR ANNULAR SECTIONS

$$\Delta P = \frac{4fL}{D_H}\left(\frac{V^2}{2g_c}\right)\rho$$ for small pressure drops

which reduces to

$$\dot{w} = A_c\sqrt{\frac{D_H \rho g_c \Delta P}{2fL}} = \frac{A_c}{\sqrt{f}}\sqrt{\frac{D_H \rho g \Delta P}{2L}} \quad (1)$$

$$f = f(Re) \quad Re = \frac{V D_H \rho}{\mu} = \frac{\dot{w} D_H}{A_c \mu}$$

for $Re < 2{,}100$ $\quad f = 24/Re$

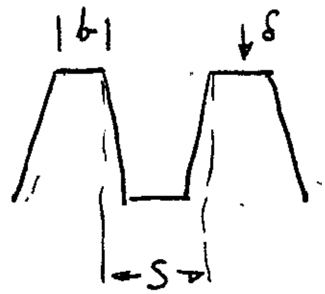
$$\dot{w} = \frac{\rho \Delta P}{L}\left[\frac{D_H^2 g A_c}{48\mu}\right] \quad (2)$$

FOR LABYRINTH SEAL

$$\dot{w} = f \frac{C A_c P_o}{\sqrt{T_o R}} \sqrt{\frac{2g_c}{n}} \left(\frac{\Delta P}{P_o} \right)^{1/2} \qquad (3)$$

f = carry over factor = $f\left(\frac{\delta}{S}\right)$

C = flow coefficient = $f\left(\frac{\delta}{b}, \frac{P_i}{P_o}\right)$



$S = .05''$
$b = .003$ to $.007''$
$\delta = .0022$ to $.0025 > .0025$

$\frac{\delta}{S} = \frac{.0025}{.05} = .05$

$f = 1.50$ {Ref Egli}

$\frac{S}{b} = \begin{cases} .835 \text{ max} \\ .357 \text{ min} \end{cases}$

$\frac{b}{\delta} = \begin{cases} 1.20 \\ 2.80 \end{cases}$

} Use $C = (.65)$

note $f * C = 1.50 * (.65) = 0.975$

$$\dot{w} = 0.975 \frac{A_c \sqrt{P_o}}{\sqrt{T_o R}} \sqrt{\frac{2g_c}{n}} \left(\Delta P \right)^{1/2}$$

This will give $\dot{w}$ a little higher than indicated by seal tests

3

(2.0) LABYRINTH SEAL LEAKAGE

$$\dot{w} = 0.975 \frac{A_c \sqrt{P_o}}{\sqrt{T_o R}} \sqrt{\frac{2g_c}{N}} (\Delta P)^{1/2}$$

P in psi

$$R = 10.73 \frac{lbf\ ft^3}{IN^2\ °R\ lbmole} = \frac{10.73}{4.0} \frac{lbf\ ft^3}{IN^2\ °R\ lbm} = 2.68 \frac{lbf\ ft^3}{IN^2\ °R\ lbm}$$

A_c in IN^2

$$\dot{w} = .975 \frac{A_c\ IN^2 \left(P_o \frac{lbf}{IN^2}\right)^{1/2}}{\left(T_o\ °R * 2.68 \frac{lbf\ ft^3}{IN^2\ °R\ lbm}\right)^{1/2}} \left(\frac{64.4 \frac{lbm-ft}{lbf-sec^2}}{N}\right)^{1/2} \left(\Delta P \frac{lbf}{IN^2}\right)^{1/2}$$

$$\dot{w} = .975\ A_c \frac{(IN)}{ft^{1/2}-sec} \frac{1}{(N)^{1/2}}\ lbm * \frac{ft}{12\ IN} * \frac{(P_o)^{1/2}}{(T_o * 2.68)^{1/2}} * (64.4)^{1/2} (\Delta P)^{1/2}$$

$$\dot{w} = .975\ A_c \sqrt{\frac{P_o}{T_o N}}\ \Delta P^{1/2}\ \frac{1}{12} * \frac{1}{(2.68)^{1/2}} (64.4)^{1/2}$$

$$\dot{w} = 0.399\ A_c \sqrt{\frac{P_o}{T_o N}} (\Delta P)^{1/2}$$

P_o, ΔP in lbf/IN^2
T_o in °R
A_c in IN^2
$\dot{w}$ in lbm/sec

$$A_c = \pi DC = \pi(.850)(.0025) = 0.00667\ IN^2$$

P_o = 800 PSIA

T_o = 620°R

N = 5

$$A_c \sqrt{\frac{P_o}{T_o N}} = .003388$$

$$\dot{w} = .0013518 (\Delta P)^{1/2} \qquad \Delta P \text{ in } \frac{lbf}{IN^2}$$

$$\dot{w} = (.399)(.003388)(\Delta P)^{1/2} = 0.0013518\, \Delta P^{1/2}$$

ΔP (PSI)	ẇ (lb/sec)
.1	.0004275
.4	.0008551
.6	.001047
.8	.001209
1.0	.001352
1.5	.001656
2.0	.001912
2.5	.002137
3.0	.002342
3.5	.002529
4.0	.002704
4.5	.002868
5.0	.003023
5.5	.003171
6.0	.003312
6.5	.003447
7.0	.003577
8.0	.003824
9.0	.004056
10.0	.004275
12.0	.004683
15.0	.005236
20.0	.006046

LABYRINTH SEAL COLD END

(3.0) ANNULAR SECTION BETWEEN LABYRINTH AND COLD END

$$\dot{w} = \frac{A_c}{\sqrt{f}}\sqrt{\frac{D_H \rho g_c \Delta P}{2L}} \quad \left\{ \text{in general} \right\}$$

for Laminar $R_e < 2{,}100$

$$\dot{w} = \frac{\rho \Delta P}{L}\left[\frac{D_H^2 g_c A_c}{48 \mu}\right]$$

$$R_e = \frac{\dot{w} D_H}{A_c \mu}$$

we are only concerned with flows below .0065 lb/sec
∴ check if we can use laminar equation throughout

$D_H = 2 * C = .005$ IN

$A_c = \pi D C = \pi (.850)(.0025) = .00668$ IN^2

$\mu = .0521$ lbm/Ft-hr

$T_0 = 620$

$\rho = .46786$ lb/ft^3

$P = 800$ PSIA

$L = .945$ in

$$R_{e_{max}} = \frac{.0065 \frac{lbm}{sec} * .005 \text{ in} * \text{ft} \cdot \text{hr} * 3600 \text{ sec/hr} * 12 \text{ in/ft}}{.00668 \text{ in}^2 * .0521 \text{ lbm}} = 4{,}030$$

so for $\dot{w} > .00349$ lb/sec

most use general equation –
won't be interested in flows above
.00349 anyway

$\therefore \quad \dot{w} < .00349 \text{ lb/sec}$

$$\dot{w} = \frac{\rho}{L}\left[\frac{D_H^2 g_c A_c}{48 \mu}\right] \Delta P$$

$$= \frac{0.46786 \frac{lbm}{ft^3}}{.945 \text{ IN}}\left[\frac{(.005)^2 \text{IN}^2\, 32.2 \frac{lbm-ft}{lbf-sec^2} * 0.00668 \text{IN}^2 \times ft * hr * \frac{lbf}{IN^2} * 3600 \frac{sec}{hr}}{(48)(.0521)\ lbm}\right] \Delta P$$

$$= \frac{(.46786)(.005)^2(32.2)(.00668)(3600)}{(.945)(48)(.0521)}\left[lbm \times \frac{1}{sec} * \frac{1}{ft} * IN * \frac{ft}{12 IN}\right] \Delta P \quad \left\{\frac{\Delta P\ IN}{lb_f/IN^2}\right\}$$

$$\dot{w} = .000319\ \Delta P$$

(ANNULAR SEAL)

ΔP (PSI)	$\dot{w}$ lb/sec
.1	.0000319
.5	.0001597
1.0	.000319
2.0	.000639
3.0	.000958
4.0	.001277
5.0	.001597
6.0	.001916
7.0	.002235
8.0	.00255

(4.0) BEARINGS

(4.1) NEW BEARINGS

We know from the above analysis of the annular section that we will have laminar flow in the bearings

$$D_H = 2*C = .0008 \quad \left\{ \text{Come back and look at wear later} \right\}$$

$$C_{max} = .0004 \text{ (NEW)}$$

$$\dot{w} = \frac{\rho}{L}\left[\frac{D_H^2 g_c A_c}{48\mu}\right]\Delta P$$

$$L = 1.0 \text{ IN}$$

$$A_c = \pi D C = \pi(.850)(.0004) = .001069 \text{ IN}^2$$

$$\dot{w} = \frac{.146786 \frac{lbm}{ft^3}}{1.0\ in}\left[\frac{(.0008)^2 in^2\ 32.2 \frac{lbm-ft}{lbf-sec^2} * ft*hr * (.001069\ in^2) * 3600 \frac{sec}{hr} \cdot ft}{48*(.0521)\ lbm\ \ 12\ in}\ \frac{lbf}{in^2}\right]\Delta P$$

$$\dot{w} = 0.000001236\ \Delta P \qquad \Delta P \text{ in } lbf/in^2$$

ΔP (PSI)	$\dot{w}$ lb/sec
1.0	.000001236
2.0	.000002473
5.0	.000006182
10.0	.00001236
20.0	.00002473
40.0	.00004945
50.0	.00006182
100.0	.0001236
200.0	.0002473

Bearings Cold End Per Bearing (New with Maximum Clearence)

very-very low with new bearings the leakage past bearings is like almost zero

4.2 BEARINGS AFTER 2YR WEAR

Lets take a look at bearings after two years of wear

Let Δm = weight loss of bearing material per unit length travel per unit contact area.

Per the $0.156\ in^2$ contact area of the LFW-1 test samples a wear rate of

$$\Delta m = 2.5 \times 10^{-6}\ mg/m \quad \text{or}$$

$$\Delta M = 1.6 \times 10^{-5}\ mg/m\text{-}IN^2$$

from bearing test data this value looks conservative ∵ measured wear rate was $1 \times 10^{-5}\ mg/m$ but most of ware took place during initial run in period and loads in the linear bearing are very much lower than bearing test loads.

$$\Delta m = 1.6 \times 10^{-5}\ mg/_{m\text{-}IN^2} = 1.6 \times 10^{-8} \frac{gr}{m\text{-}IN^2} * \frac{1m}{39.37IN} = 4.08 \times 10^{-10}\ gr/_{IN\text{-}IN^2}$$

$$= 0.9 \times 10^{-12}\ lb/_{IN\text{-}IN^2}$$

Let δ = the distance traveled during 2yr then:

$$\delta = 2S * N_S * \Theta$$

S = stroke = 0.45 IN

N_S = rpm = 400

Θ = time = 2yr = 17,500hr = 1.05×10^6 min

$$\delta = (2)(.45)(400)(1.05 \times 10^6) = 3.78 \times 10^8\ IN$$

Then let ΔW = loss in weight of linear bearing

Then

$$\Delta W = \pi D L * \Delta C * \rho$$

D = bearing diameter
ΔC = change in clearance
L = length
ρ = bearing materal density = .21 lb/IN3

also

$$\Delta W = \Delta \dot{m} * \delta * \pi D L$$

or

$$\pi D L * \Delta C * \rho = \Delta m * \delta * \pi D L$$

$$\Delta C = \frac{\Delta m * \delta}{\rho} = \frac{0.9 \times 10^{-12} \frac{lb}{IN-IN^2} * 3.78 \times 10^{8} IN}{0.21 \frac{lb}{IN^3}} = 16.2 \times 10^{-4} IN$$

$\Delta C = .00162$ IN & $C = .00202$ INCH

$D_H = 2C = .00404$ INCH

$A_c = \pi D C = \pi(.850)(.00202) = .005394$

$$\frac{\dot{W}_{NEW}}{\dot{W}_{WORN}} = \left(\frac{D_{H\,NEW}}{D_{H\,WORN}}\right)^2 \left(\frac{A_{c\,NEW}}{A_{c\,WORN}}\right) = \left(\frac{.0008}{.00404}\right)^2 \left(\frac{.001069}{.005394}\right) = \frac{1}{5.05^2}\left(\frac{1}{5.05}\right) = \frac{1}{128.8}$$

$\therefore \dot{W}_{WORN} = 128.8\, \dot{W}_{NEW}$

$= 128.8(.000001236)\Delta P = .0001592\ \Delta P$

5.0 BEARING SUPPORT

Here we have a clearance of 0.0035 IN so can expect to go turbulent at lower flow rate: Therefor check max flow where laminar equation is valid.

$$Re = \frac{\dot{w} D_H}{A_c \mu} \qquad D_H = 2 \times C = .0070 \, IN$$

$$A_c = \pi D C = \pi (.85)(.0035) = .00935 \, IN^2$$

$$Re = 2100$$

$$Re = \frac{\dot{w} \, \frac{lbm}{sec} \cdot 0.007 IN * ft{\cdot}hr * 3600 \, sec/hr * 12 \, IN/ft}{.00935 \, IN^2 * .0521 \, lb_m}$$

$$\dot{w}_{\substack{max \\ for \\ laminar}} = 2100 * \frac{(.00935)*(.0521)}{(.007)*(3600)(12)} \; lb/sec$$

$$\dot{w}_{max} = .003382 \; lb/sec$$ {This should still cover the range for interest}

Then

$$\dot{w} = \frac{\rho \Delta P}{L} \left[\frac{D_H^2 g_c A_c}{48 \mu} \right] \qquad L = 1.69$$

ratio from annular bearing equation

$$\dot{w} = .000319 \frac{(.945)}{(1.69)} \left(\frac{.007}{.005} \right)^2 \left(\frac{.00935}{.00668} \right) \Delta P$$

$$\dot{w} = 0.000490 (\Delta P) = 1.533 \, \dot{w}_{ANNULAR \; SEAL}$$

BEARING SUPPORT (FULL LENGTH)

ΔP (PSI)	$\dot{w}$ (lb/sec)
.1	.0000490
.5	.000245
1.0	.000490
2.0	.00098
3.0	.001469
4.0	.00196
5.0	.00245

6.0 TOTAL LEAKAGE AND PRESSURE DROP

For the various seals, bearing and etc the flow is in series; therefor each must pass the same leakage

$$\dot{w}_L = \dot{w}_i = \dot{w}_j = \dot{w}_k \text{ - - - - -}$$

what we are looking for is the leakage as a function of the total pressure drop across the series of sealing components

$$\Delta P_T = f(\dot{w}) \quad \text{and} \quad \Delta P_i = f_i(\dot{w})$$

$$\Delta P_T = \sum \Delta P_i = \sum f_i(\dot{w})$$

We can thus write equations for various combinations of the sealing components in series. By comparing the leakage for various combinations with the pressure drop forced of the seals by the system we can then make the best selection. The options here are made available by plumbing the leakage path back into the active cycle volume. The major options are:

6.1 Annular Seal and Labyrinth Seal In Series

Here the pressure drop is that of the cold end heat exchanger, the cold end regenerator and one of the regenerator support plates. The leakage path would be plumbed into the cycle between the two ported plates at the sump flange. For this case

$$\Delta P_{LABYRINTH} = \frac{\dot{w}^2}{(.00135/8)^2} = \frac{\dot{w}^2}{1.842 \times 10^{-6}}$$

$$\Delta P_{\substack{ANNULAR \\ SEAL}} = \frac{\dot{w}}{.000319} = \frac{\dot{w}}{3.19 \times 10^{-4}}$$

$$\Delta P_T = \left(\frac{1}{3.19 \times 10^{-4}} + \frac{\dot{w}}{1.842 \times 10^{-6}} \right) \dot{w}$$

$$\Delta P_T = \left\{ 3.135 \times 10^3 + 5.425 \times 10^5 \dot{w} \right\} \dot{w} \qquad \left\{ \begin{matrix} \dot{w} = \text{lb/sec} \\ \Delta P = \text{PSI} \end{matrix} \right\}$$

$\dot{w}$ (lb/sec)	ΔP (psi)
.00001	.0314
.0001	.3189
.00015	.4824
.00020	.6486
.00040	1.3408
.00060	2.0766
.00080	2.8544
.00100	3.678
.00200	8.44
.00300	14.295

6.2 Annular Seal, Laybrinth Seal, and First Bearing in series

Here the pressure drop would be increased from that in case 6.1 to include the second ported plate, and the loss in a 1.0 in length of the slotted bearing support. The leakage path would be plumbed into the cycle ~~volume~~ just upstream of the first bearing from the cold end.

To the ΔP_T of case 6.1 we must add

$$\Delta P_{BEARING} = \frac{\dot{w}}{1.236 \times 10^{-6}} = 8.0906 \times 10^{5} \dot{w}$$

which yields:

$$\Delta P_T = \left\{ 8.12196 \times 10^{5} + 5.425 \times 10^{5} \dot{w} \right\} \dot{w}$$

$$= \left\{ 8.12196 + 5.425 \dot{w} \right\} \dot{w} \times 10^{5}$$

note: This is for new bearing -- look at leakage after 2yr next

FOR NEW BEARING

$\dot{w}$ (lb/sec)	ΔP (psi)
.0000001	.08122
.000001	.8122
.00001	8.122
.0000012	.97463
.0000014	1.1371
.0000016	1.2995
.0000018	1.46195
.000002	1.6244
.000004	3.2488
.000006	4.8732
.000008	6.4976
.000012	9.7464
.000014	11.3708
.000016	12.995
.000018	14.620
.00002	16.244
.00003	24.3684
.00004	32.488

Now take a look at 2yr old bearings

$$\Delta P_{BEARINGS} = \frac{\dot{w}}{6.56316 \times 10^{-4}} = 1.524 \times 10^{3} \dot{w}$$

$\therefore$

$$\Delta P_T = \left\{ 9.416 \times 10^{3} + 5.425 \times 10^{5} \dot{w} \right\} \dot{w}$$

$$= \left\{ 9.416 + 542.5 \dot{w} \right\} \times 10^{3} \times \dot{w}$$

$\dot{w}$ (lb/sec)	ΔP (PSI)
.00001	.09421
.0001	.9470
.0002	1.9049
.0004	3.8532
.0006	5.845
.0008	7.88
.0010	9.9585
.0015	15.345
.0020	21.032

WORST CASE FOR BEARING AFTER 2YRS.

6.3 Annular Seal, Labyrinth Seal, First Bearing and Bearing Support in Series

Here the pressure drop would be increased from that in 6.2 above to include the loss in the bearing support between bearings. The leakage path would be plumbed into the cycle volume at a point just upstream of the second bearing.

To the ΔP_T of case 6.2 we must add.

$$\Delta P_{\substack{BEARING \\ SUPPORT}} = \frac{\dot{w}}{.00049} = 2.040 \times 10^{3}\, \dot{w}$$

FOR NEW BEARINGS

$$\Delta P_T = \left\{ 8.14235 + 5.425\, \dot{w} \right\} \times 10^5 \times \dot{w}$$

which is essentially equal to case 6.2 for new bearing as would be expected

FOR 2Yr BEARINGS

$$\Delta P_T = \left\{ 11,456 + 542.5\, \dot{w} \right\} \times 10^3 \times \dot{w}$$

$\dot{w}$ (lb/sec)	ΔP (PSI)
.00001	.11461
.0001	1.151
.0002	2.3129
.0004	4.6692
.0006	7.0689
.0008	9.512
.0010	11.9985
.0015	18.405
.0020	25.082

6.3

(18)

6.4 Annular Seal, Labyrinth Seal, First Bearing, Bearing Support and Second Bearing in Series

Here the pressure drop would be increased from that in 6.3 above to include the loss all the way to the engine's sump. The leakage path would be plumbed directly into the sump

To the ΔP_T of Case 6.3 we must add

For new bearing

$$\Delta P_{BEARINGS} = 8.0906 \times 10^5 \dot{w}$$

FOR 2yr old bearings

$$\Delta P_{BEARINGS} = 6.2811 \times 10^3 \dot{w}$$

THEN:

FOR NEW BEARINGS:

$$\Delta P_T = \left\{ 16.23296 + 5.425\,\dot{w} \right\} * 10^5 * \dot{w}$$

$\dot{w}$ (lb/sec)	ΔP (PSI)
.0000001	0.16233
.000001	1.6233
.000002	3.2466
.000004	6.4932
.000006	9.7398
.000008	12.9864
.00001	16,233
.00002	32,466
.0001	162,335

FOR 2 yr OLD BEARINGS

$$\Delta P_T = \left\{17.737 + 542.5\,\dot{\omega}\right\} * 10^3 * \dot{\omega}$$

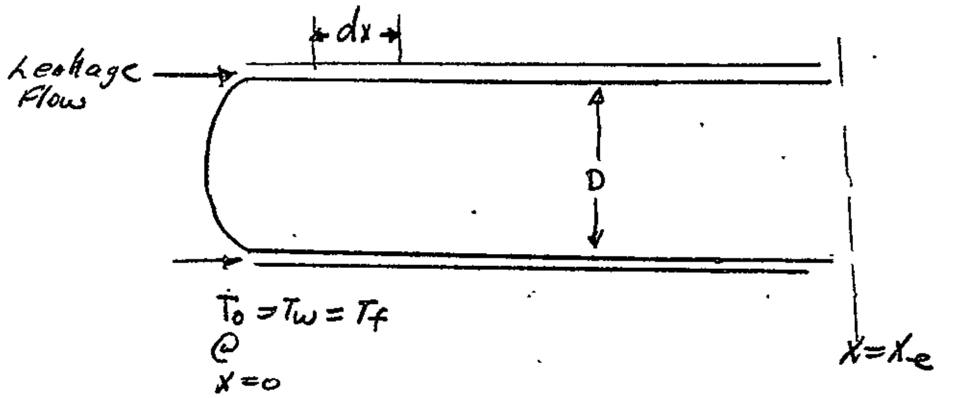
$\dot{\omega}$ (lb/sec)	ΔP (PSI)
.00001	.17742
.0001	1.7791
.0002	3.5691
.0004	7.1816
.0006	10.8375
.0008	14.5368
.0010	18.2795

A comparison between the various options is given on the following page. All this is nice but now we need the actual pressure drop for each option and an idea of what is an acceptable leakage rate. Note the maximum system flow is ≈ .0065 lb/sec and the leakage should be a small fraction of this -- try to show this in a subsequent analysis

ESTIMATION OF LOSSES AT COLD END DUE TO LEAKAGE

FIRST LETS TRY A SIMPLE MODEL

assume leakage is small and does not change the temperature gradient of regenerator or displacer walls, and assume a linear temperature distribution along these walls. We then have from taking leakage from cold to hot end:



where T_w = wall temperature

T_f = fluid temperature

$$T_w = \left(\frac{T_a - T_0}{x_e}\right)x + T_0 \quad \left\{ \text{linear wall temperature distribution} \right.$$

The differential equation for the annular flow (leakage) passage is:

$$\frac{dT_f}{dx} + \frac{hA_c}{\dot{w}c_p}T_f = \frac{hA_c}{\dot{w}c_p}T_w \qquad (1)$$

Then

taking an average film coefficient (i.e. h=const) and the same for c_p we can solve this equation without much trouble

②

Leting $\frac{hA_c}{WC_p} = \alpha$ $\Big\}$ note $A_c = 2\pi D$

and substitution of linear temperature distribution in (1) we get

$$\frac{dT_f}{dx} + \alpha T_f = \alpha (T_a - T_o)\frac{x}{x_e} + \alpha T_o \qquad (2)$$

Then by standard process:

$$T_f = e^{-\int \alpha dx} \int \left(\alpha (T_a - T_o)\frac{x}{x_e} + \alpha T_o\right) e^{\int \alpha dx} dx + C e^{-\int \alpha dx}$$

which gives

$$= e^{-\alpha x}\left[\int \left(\alpha (T_a - T_o)\frac{x}{x_e} + \alpha T_o\right) e^{\alpha x} dx\right] + C e^{-\alpha x}$$

$$= e^{-\alpha x}\left[\frac{\alpha (T_a - T_o)}{x_e}\frac{e^{\alpha x}}{\alpha^2}(\alpha x - 1) + \alpha T_o \frac{e^{\alpha x}}{\alpha}\right] + C e^{-\alpha x}$$

$$T_f = (T_a - T_o)\frac{x}{x_e} - \frac{(T_a - T_o)}{\alpha x_e} + T_o + C e^{-\alpha x}$$

Using the boundry condition of:

$$x = 0 \qquad T_f = T_o$$

$$C = \frac{T_a - T_o}{\alpha x_e}$$

and

$$\boxed{T_f = \frac{T_a - T_o}{x_e}\left\{x - \frac{1}{\alpha} + \frac{1}{\alpha} e^{-\alpha x}\right\} + T_o}$$

what we are really interested in is the ($\Delta T = T_a - T_f$) - approach of the leakage to the wall temperature. if $T_a - T_f$ ~~→ 0 the~~ losses would be small. Then ΔT is given by

$$\boxed{\Delta T = T_a - T_f = (T_a - T_o)\left\{1 - \frac{1}{X_e}\left\{X - \frac{1}{\alpha} + \frac{1}{\alpha}e^{-\alpha X}\right\}\right\}}$$

Now lets look at some numbers

$T_a = 620°R$

$T_o = 140°R$

$D \cong 0.850$

$\tilde{T} = \frac{760}{2} = 380°R$

$A_c = 2\pi D = 2\pi(.850) = 5.34 \frac{IN^2}{IN} = 0.453 ft$

$X_e = 4.5'' = 0.375 ft$

$k = .0708 \frac{Btu}{hr\text{-}ft\text{-}°R}$

$D_H = 2C \quad C = .006''$

$D_H = .012'' = .001 ft$

$N_u = \frac{hD}{k} = 8.23$

$C_p = 1.296 \frac{Btu}{lb\ °R}$

$$h = \frac{(8.23)(.0708)\ Btu/hr\ ft\text{-}°R}{(.01 \times 10^{-1})\ ft} = 582 \frac{Btu}{hr\ ft^2\ °R}$$

$$\alpha = \frac{hA_c}{\dot{w}C_p} = \frac{\left(582 \frac{Btu}{hr\ ft^2\ °R}\right)(.453\ ft)}{\dot{w}\ lb/sec\ 1.296\ Btu/lb\ °R}\ \frac{1}{3600}\frac{hr}{sec} = \frac{5.878 \times 10^{-2}}{\dot{w}}\ \frac{1}{ft}$$

$$\alpha = \frac{5.878 \times 10^{-2}}{\dot{w}}\ \frac{1}{ft}$$

$\dot{w}$	α	$\frac{1}{\alpha}$	$e^{-\alpha X_e}$	αX_e
.00001	5.878×10^3	1.7×10^{-4}	0 ~	2204.25
.0001	5.878×10^2	1.7×10^{-3}	0 ~	220.425
.001	5.878×10	1.7×10^{-2}	0 ~	22.0425
.01	$.5878 \times 10$	1.7×10^{-1}	.1103	2.20425

$$\Delta T = 480 \left\{ 1 - 2.6667 \left(\frac{1}{2.6667} - \frac{1}{\alpha} + \frac{1}{\alpha} e^{-\alpha x} \right) \right\}$$

$$\boxed{\Delta T = 480 \left\{ 2.667 \left(\frac{1}{\alpha} - \frac{1}{\alpha} e^{-\alpha x} \right) \right\}}$$

$\dot{w}$ (lb/sec)	ΔT °R	$Q = \dot{w} c_p \Delta T$ (Btu/sec)	Q watts
.00001	2.176×10^{-1}	2.72×10^{-6}	2.88×10^{-3}
.0001	21.76×10^{-1}	2.72×10^{-4}	2.88×10^{-1}
.001	193.62×10^{-1}	2.42×10^{-2}	256×10^{-1}
.00008	17.42×10^{-1}	1.742×10^{-4}	1.839×10^{-1}
.00012	26.14×10^{-1}	3.921×10^{-4}	4.238×10^{-1}
.00014	30.49×10^{-1}	5.336×10^{-4}	5.633×10^{-1}
.00018	39.198×10^{-1}	8.82×10^{-4}	9.311×10^{-1}
.00020	43.564×10^{-1}	1.089×10^{-3}	11.50×10^{-1}
.00024	52.269×10^{-1}	1.568×10^{-3}	16.55×10^{-1}

$\dot{w}$	α	$\frac{1}{\alpha}$	$e^{-\alpha X_e}$	αX_e	$e^{-\alpha X_e}$
.00008	734.75	0.001361	0 ~	275.53	0
.00012	489.83	0.002041	0 ~	183.686	
.00014	419.86	0.002382	0 ~	157.4475	
.00018	326.56	0.003062	0 ~	122.460	
.00020	293.90	0.003402	0 ~	110.2125	
.00024	244.92	0.004083	.0001	91.845	

SECTION 8

AMBIENT SUMP HEAT EXCHANGER

INTRODUCTION

The sump heat exchanger functions to transfer heat from the working fluid of the VM refrigerator for rejection from the system. The design criteria for this heat exchanger are similar to those of the cold end heat exchanger, with changed emphasis on the various items. The primary design criteria consist of:

- Low Working Fluid Pressure Drop--The heat exchanger must provide good thermal performance and yet not lead to an excessive pressure drop of the working fluid. As with the cold end heat exchanger, the pressure drop subtracts from the pressure-volume variations in the cold expansion volume thereby reducing the refrigeration capacity. In addition, the pressure drop across the sump heat exchanger can have a significant affect on the drive motor power requirements unless this pressure drop is minimized.

- Low Void or Internal Volume--Void volumes reduce the refrigeration capacity of VM refrigerators by decreasing the pressure variations or pressure ratio; minimization of the heat exchanger internal void volume is therefore important.

- Minimization of the Film Temperature Drop--The thermodynamic efficiency of the refrigerator increases as the temperature of the gas in the sump decreases. Minimization of the sump heat exchanger film temperature drop allows maximum performance for the fixed heat rejection or heat sink temperature.

- Flow Distribution--Uniform flow within the heat exchanger is important for two reasons: (1) non-uniform flow leads to reduced conductance of the heat exchanger and (2) non-uniform flow leads to fluid elements at different temperatures; subsequent mixing of these elements results in an increase in entropy and reduced thermodynamic efficiency of the refrigerator.

- Heat Exchanger Interfaces--The sump heat exchanger must interface with both the hot and cold regenerators, fluid passages into the sump volume and a cooling collar or clamp which provides the heat sink.

DESIGN CONFIGURATION

The configuration of the ambient sump heat exchanger is shown in Figure 8-1. This configuration is a refinement of the design evolved under Task I.

The annular shaped heat exchanger is divided into two sections, as shown, with th sections being identical in configuration except for length. The heat transter surface of each section is formed by brazing an offset copper plate fin to the inside surface of the cylindrical section of the sump pressure vessel wall. The cylindrical sump filler block fits inside the plate fin, thereby

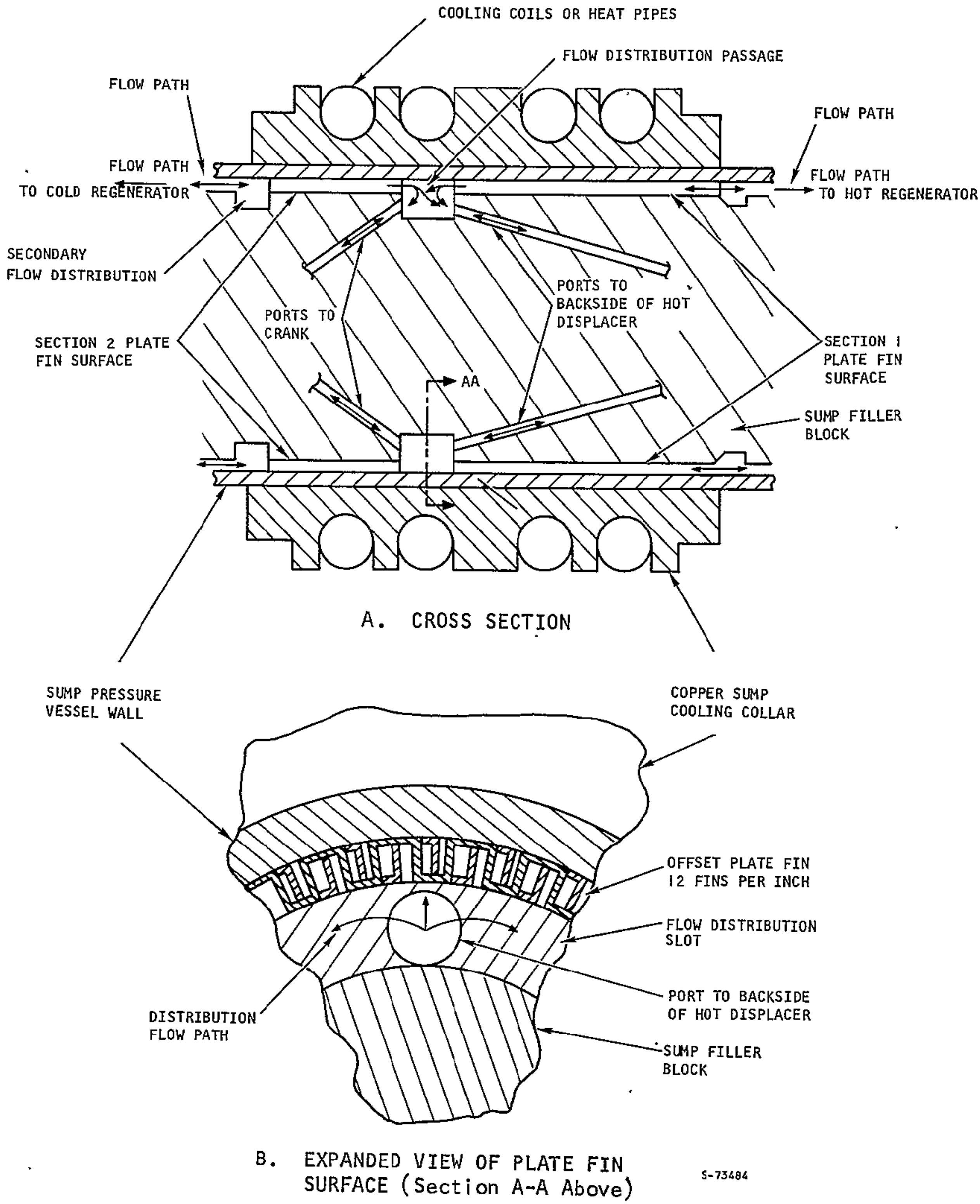


Figure 8-1. Sump Heat Exchanger Configuration

forming an annular passage forcing flow through the finned surface. At the right hand side (Figure 8-1), flow of the working fluid enters and exits Section 1 of the heat exchanger as it leaves and returns to the hot regenerator during the cyclic flow process. The average flow rate in this section of the heat exchanger is approximately five times that in Section 2; this accounts for the greater length (larger heat transfer surface) required for this section. At the other end of Section 1 of the heat exchanger, the flow enters and exits from a flow distribution passage cut into the sump filler block. This distribution process, or slot, is supplied working fluid via ports that connect to the active cycle volumes in the crank case and behind the hot displacer as shown. The distribution slot is sized to provide uniform flow across the face of the heat exchanger; sizing of this slot is discussed later in this section.

On the left hand side of Figure 8-1, flow enters and exits Section 2 of the heat exchanger as it leaves and returns to the cold regenerator. This section of the heat exchanger is pneumatically connected to the cold regenerator via channels cut into the sump filler block (not shown in Figure 8-1) and passages in the cold-end linear bearing support. To provide uniform flow distribution at the left hand face of Section 2 of the heat exchanger, a secondary flow distribution slot is cut into the sump filler block as shown in Figure 8-1. The right hand of this section of the heat exchanger interfaces and shares the central flow distribution slot with the other section of the exchanger.

The path for heat transfer from both sections of the heat exchanger is from the gas to the plate fin surface, from the fined surface through the pressure vessel wall and on into the copper sump cooling collar. Indium foil is placed between the sump pressure vessel and the cooling collar; this foil is maintained under a 100 psi interface pressure to ensure good thermal contact. Heat is finally rejected from the system to cooling coils brazed into channels cut in the cooling collar. This collar was originally designed to allow interfacing of the refrigerator with ammonia heat pipes, but is interchangeably usable with simple water cooling coils.

HEAT EXCHANGER CHARACTERIZATION

The rate of heat transfer for each section of sump heat exchanger can be expressed as:

$$\dot{Q} = h(Ap + \eta_f A_f)\, \overline{\Delta T} \qquad (8\text{-}1)$$

where

h = the average heat transfer coefficient

Ap = basic area of the plate

η_f = fin effectiveness

A_f = fin area

$\overline{\Delta T}$ = average temperature difference between the working fluid and the heat transfer surface.

Referring to Figure 8-2, the following relations can be derived:

Plate area

$$Ap = (1-N\delta)WL \tag{8-2}$$

where

N = fins per inch

W = plate width

L = plate length

Fin area

$$A_f = \{2N(b-\delta) + \frac{N}{2}(\frac{1}{N}-\delta)\}\ WL \tag{8-3}$$

Note: This neglects the fin area exposed between fin and the sump filler block--a conservative approach.

Fin effectiveness

$$\eta_f = \frac{\text{Tanh }(ML_e)}{ML_e} \tag{8-4}$$

where

$$M = \sqrt{\frac{2h}{k\delta}} \tag{8-5}$$

k = fin material thermal conductivity and the fin length L_e is given by

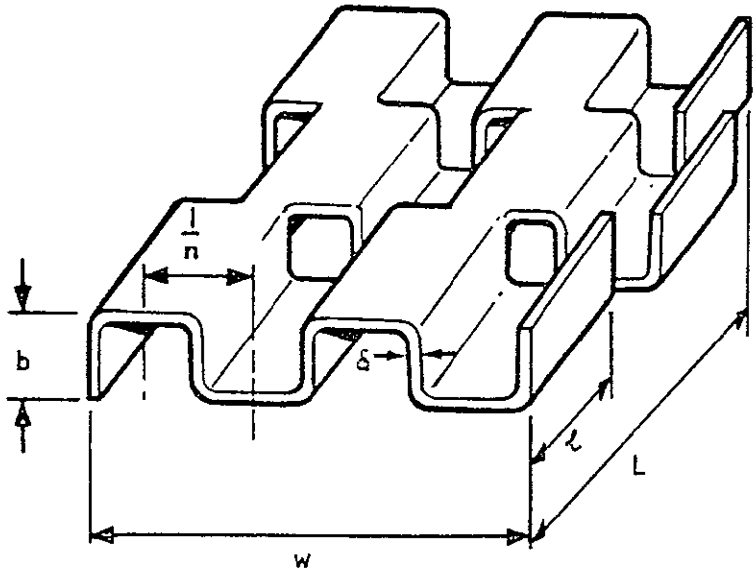
$$L_e = b + \frac{1}{2}(\frac{1}{N} - \delta) \tag{8-6}$$

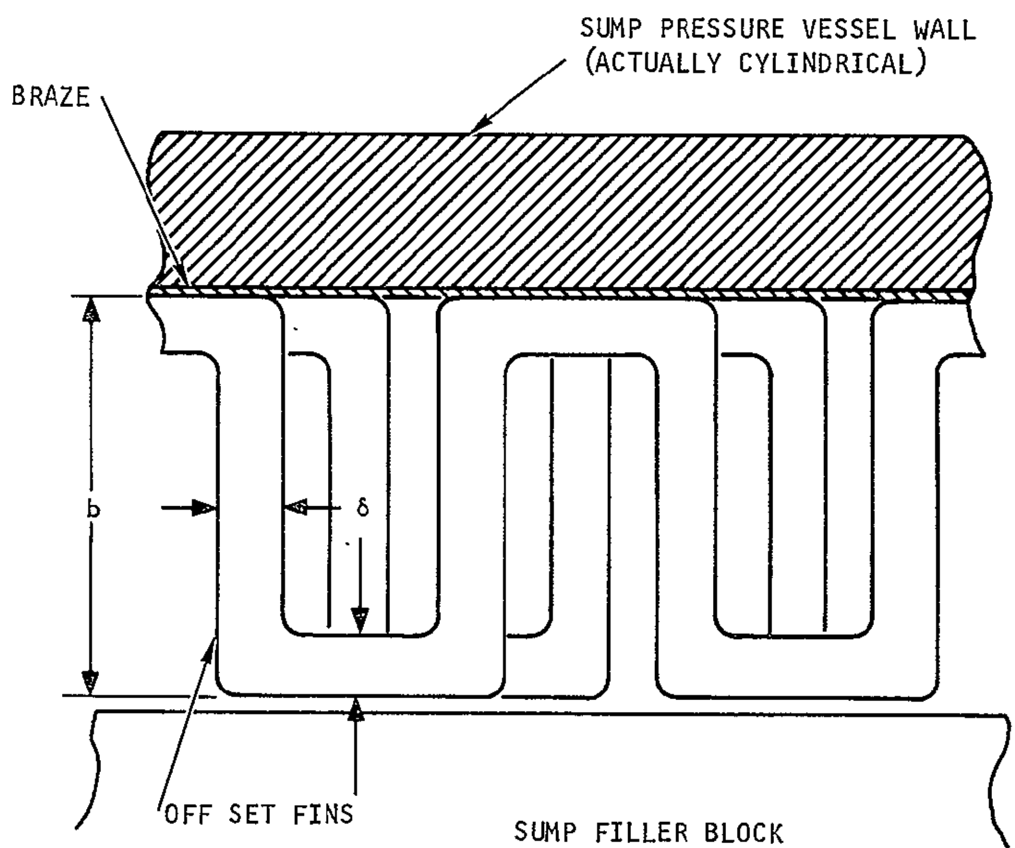
Flow cross sectional area

$$A_f = \{b-\delta(N(b-\delta) +1)\}\ W \tag{8-7}$$

Hydraulic diameter

$$D_H = \frac{2(b-\delta)(\frac{1}{N}-\delta)}{(b-\delta) + (\frac{1}{N}-\delta)} \tag{8-8}$$





b. INSTALLED PLATE-FIN SURFACE

S-73483

Figure 8-2. Rectangular Offset Plate-Fin VM Refrigerator Sump Heat Exchanger

Performance characteristics unique to a given plate fin surface are generally presented as plots of Colburn's j factor and Fanning's friction factor f as functions of Reynolds Number. Colburn's j factor is defined by:

$$j = \frac{h}{C_p G} \quad Pr^{2/3} \tag{8-9}$$

where

C_p = gas heat capacity

Pr = Prandtl number

$G = \frac{\dot{\omega}}{Ac}$

$\dot{\omega}$ = flow rate

Reynolds number is defined as

$$Re = \frac{D_H G}{\mu} \tag{8-10}$$

Figure 8-3 gives the Colburn j factor for the fin used in the GSFC VM refrigerator sump heat exchanger. This surface has 12 fins per inch, an 0.5 in. offset length, fin length of 0.075 in. and a fin thickness of 0.006 in. Figure 8-4 gives the friction factor for this surface. The pressure drop is then computed by use of

$$\Delta P = \frac{4fL}{D_H} \left(\frac{V^2}{2g_c}\right) \rho \tag{8-11}$$

where

V = gas velocity

ρ = gas density

g_c = gravitational constant

PERFORMANCE CHARACTERISTICS

The sump heat exchanger performance characteristics are summarized in Table 8-1. The heat transfer performance is based on the average flow in each section of the heat exchanger. For the pressure drop, maximum flows were assumed. The overall total conductance of the heat exchanger leads to less than a $10^{0}R$ film temperature drop at the nominal design heat load of 300 watts.

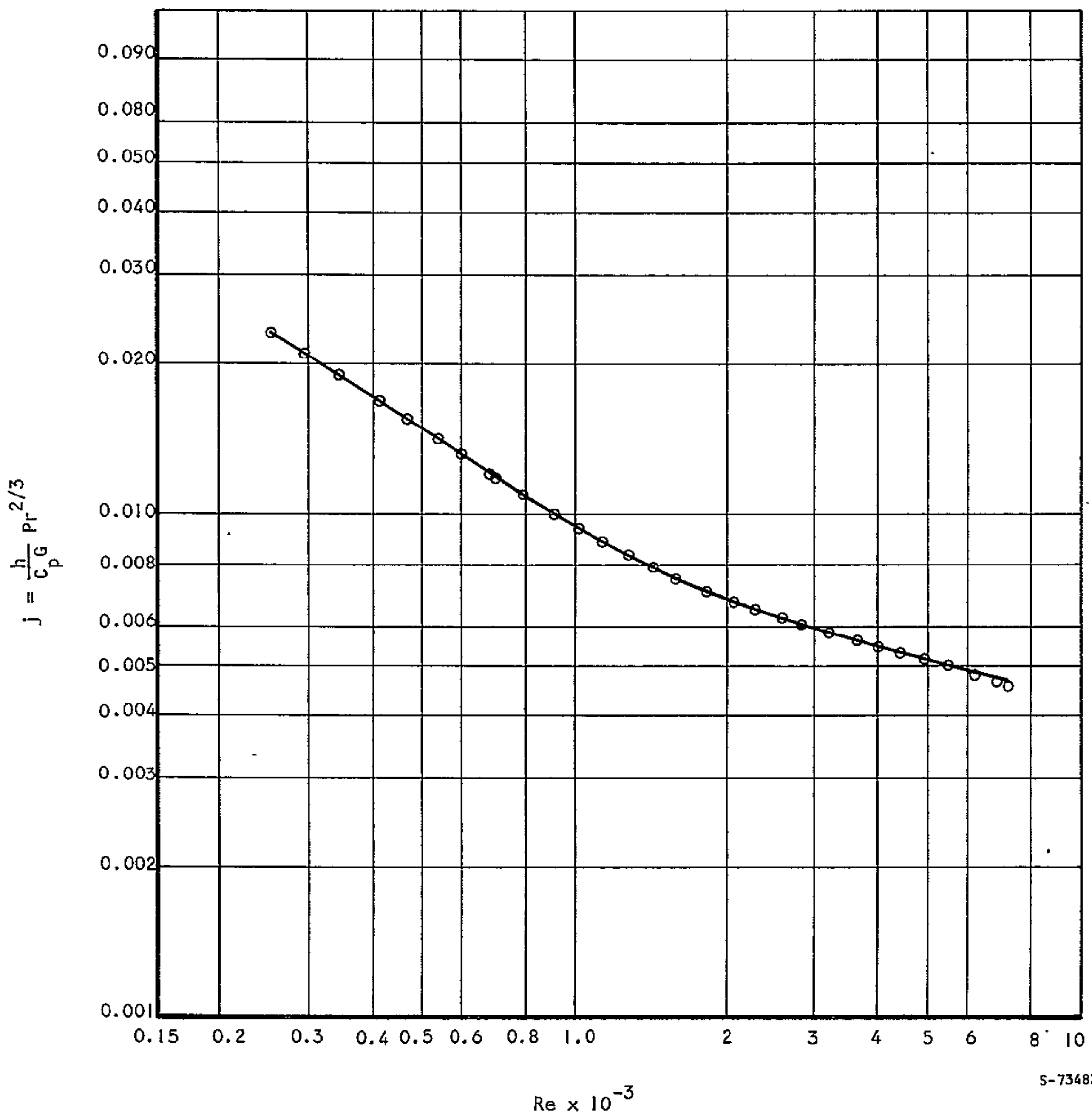


Figure 8-3. Colburn j Factor vs Reynolds Number for Sump Heat Exchanger Heat Transfer Surface

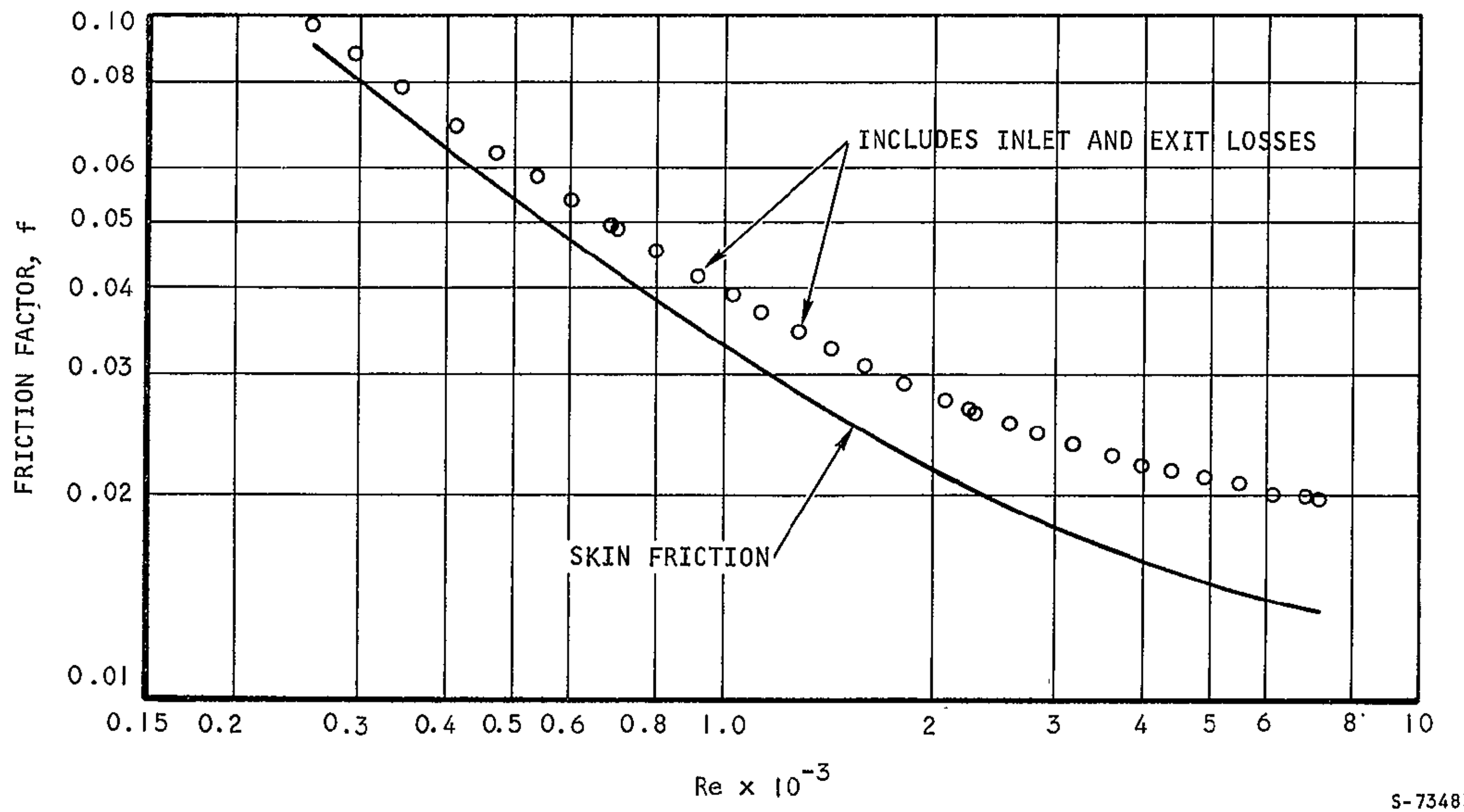


Figure 8-4. Fanning Friction Factor vs Reynolds Number for Sump Heat Exchanger Heat Transfer Surface

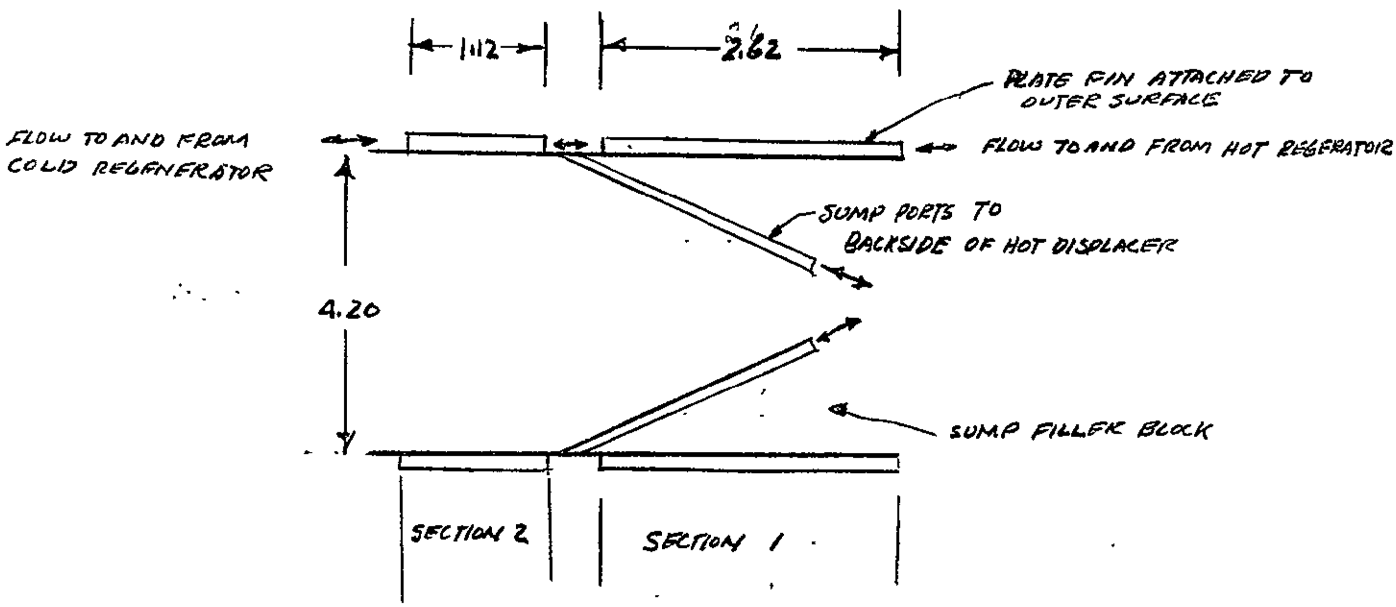
TABLE 8-1

SUMP HEAT EXCHANGER DESIGN AND PERFORMANCE SUMMARY

Parameter	Section 1	Section 2
Plate Area, ft^2	0.223	0.0952
Fin Area, ft^2	0.510	0.2183
Maximum Flow, lb/sec	0.0200	0.0086
Average Flow, lb/sec	0.01252	0.0050
Fin effectiveness	0.95	0.954
Fluid Temperature, ^oR	620.	620.
Heat Transfer Coefficient, Btu/ft^2 ^oR hr	122.	88.
Conductance (hA), Btu/^oR-hr	86.5	26.7
Pressure Drop, psi	0.011	0.0013
	Total for Sections 1 and 2	
Conductance (hA), Btu/^oR-hr	113	

SUMP HEAT EXCHANGER

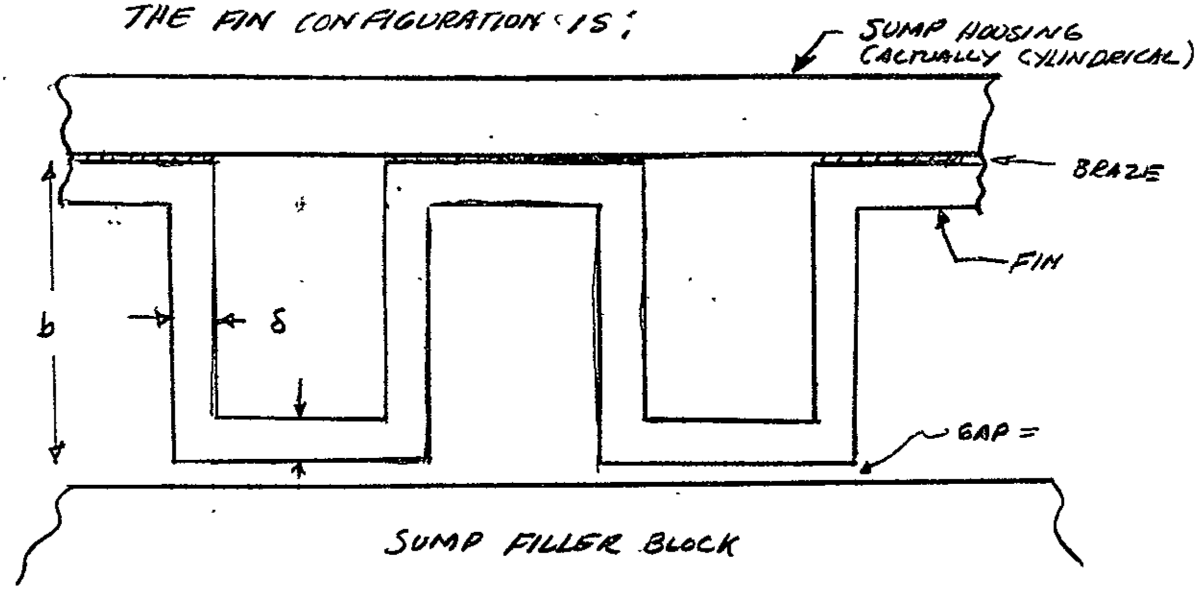
The sump heat exchanger consist of a single plate fin located in the annular space between the sump filler block and the sump housing. The exchanger is divided into two sections as shown below



The plate fin configuration is the same for both sections. The first task is to define the fin configuration and the relations yielding its pressure drop and heat tranfer characteristics

1.0 PLATE FIN RELATIONS

THE FIN CONFIGURATION IS;



Let

N = number of fins per in (12)
δ = fin thickness
b = fin height

Then the total heat transfer area can be divided into two parts; the effective area of the plate to which the plate fin is attached (plate area) and the fin area.

Let A_p = plate area
and
A_f = fin area

The $\boxed{A_p = (1 - N\delta)\, W * L}$

where W = width of the HX
$= \pi D_{SH}$

where D_{SH} = diameter sump housing

and L = length

$$\boxed{A_f = \left\{ 2N(b-\delta) + \frac{N}{2}\left(\frac{1}{N} - \delta\right) \right] W * L}$$

this neglects the fin area exposed between the fin and the sump filler block

The fin length L_e is given by:

$$\boxed{L_e = b + \frac{1}{2}\left(\frac{1}{N} - \delta\right)}$$

and the fin effectiveness by:

$$\boxed{\eta_f = \frac{\tanh(m L_e)}{m L_e}}$$

where $m = \sqrt{\frac{2h}{k\delta}}$

Where h = the heat transfer coefficient

k = the fin conductivity

The flow cross sectional area is given by:

$$A_c = \left\{ \frac{1}{N} b - \left((b-\delta)\delta + \frac{1}{N}\delta \right) \right\} W * N$$

which reduces to

$$\boxed{A_c = \left\{ b - \delta \left[N(b-\delta) + 1 \right] \right\} W}$$

Then heat transfer is given by:

$$\boxed{Q = h\,(A_p + \eta_f A_f)\,\overline{\Delta T}}$$

How lets compute the various factor that a constant for both section of the sump heat exchanger

$b = 0.075$ IN $\delta = .006$ IN $N = 12$ fins/IN

$D_{SH} = 4.20$ IN

PLATE AREA:

$$A_p = (1 - 12(.006))\,\pi(4.20) * L$$

$$= 12.24 * L \quad (IN^2)$$

FIN AREA:

$$A_f = \left\{ (2)(12)(.075 - .006) + \frac{12}{2}\left(\frac{1}{12} - .006\right) \right\} \pi(4.20) * L$$

$$A_f = 28.07 * L \quad (IN^2)$$

Fin Length

$$L_e = .075 + \frac{1}{2}\left(\frac{1}{12} - .006\right)$$

$$L_e = 0.11362 \text{ IN} = .00946 \text{ ft}$$

FLOW CROSS SECTIONAL AREA:

$$A_c = \left\{(.075) - .006\left(12(.069)+1\right)\right\} \pi(4.20)$$

$$= (.07405)(13.2) = 0.977 IN^2 = .00678 ft^2$$

2.0 AVERAGE AND MAXIMUM FLOWS

SECTION 1

$\dot{W}_{max} = .020$ lb/sec @ 900 RPM

$W_{ave} = .01252$ lb/sec

SECTION 2

$W_{max} = .0086$ lb/sec

$W_{ave} = .004999$ lb/sec

3.0 SECTION 1 HEAT EXCHANGER

3.1 HEAT TRANSFER CONDUCTANCE

USE $\dot{w} = .01252$ lb/sec

$P = 500$ PSIA
$T = 620°R$

$\rho = .44786$ lb/ft³

$\mu = .0521$ lb/ft-hr

$C_p = 1.243$ Btu/lb-°R

$P_r = .67$

$k_g = .0969 \dfrac{Btu}{hr\ ft-°R}$

$k_f = 220$ Btu/hr ft-°R

Note we forgot the D_H

$$D_H = \frac{2(b-\delta)(\frac{1}{N}-\delta)}{(b-\delta)+(\frac{1}{N}-\delta)}$$

$$D_H = .0728 \text{ in} = .00606 \text{ ft}$$

$$Re = \frac{D_H G}{\mu} \qquad j = \frac{h}{C_p G} Pr^{2/3}$$

$$G = \frac{\dot{w}}{A_c} = \frac{.01252 \text{ lb/sec}}{.00678 \text{ ft}^2} = 1.85 \frac{\text{lb}}{\text{sec-ft}^2}$$

$$Re = \frac{(.00606)\text{ft}\ 1.85 \frac{\text{lb}}{\text{sec ft}^2} \cdot \text{ft-hr} \cdot 3600 \text{ sec/hr}}{.0521 \text{ lb}} = 774.65$$

$$j = .0113$$

$$h = \frac{j\, C_p G}{Pr^{2/3}} = \frac{(.0113)(1.243 \frac{\text{Btu}}{\text{lb}°\text{R}})(1.85 \frac{\text{lb}}{\text{sec-ft}^2})(\frac{3600 \text{ sec}}{\text{hr}})}{(.67)^{2/3}}$$

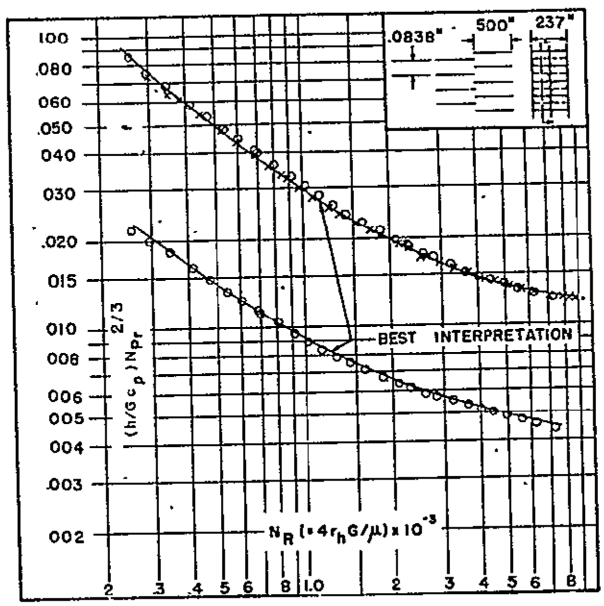
$$= \frac{(.0113)(1.243)(1.85)(3600)}{(.766)} = 122.12 \text{ Btu/hr ft}^2 °\text{R}$$

Then $$m = \sqrt{\frac{2h}{k\delta}} = \left(\frac{244}{220} \frac{\text{Btu}}{\text{hr ft}^2 °\text{R}} \frac{\text{hr ft}°\text{R}}{\text{Btu}} \frac{1}{5\times10^{-4}\text{ ft}}\right)^{1/2} = 47.1 \frac{1}{\text{ft}}$$

$$l_e m = .946 \times 10^{-2} \text{ ft} \times 4.71 \times 10^1 \frac{1}{\text{ft}} = .445$$

$$\eta = \frac{\tanh .445}{.445} = \frac{.42277}{.445} = 0.95$$

Fig. 10-56. Strip-fin plate-fin surface 1/2-11.94(D).



Fin pitch = 11.94 per in.

Plate spacing, $b = 0.237$ in.

Splitter symmetrically located

Fin length flow direction = 0.500 in.

Flow passage hydraulic diameter, $4r_h = 0.007436$ ft

Fin metal thickness = 0.006 in., aluminum

Splitter metal thickness = 0.006 in.

Total heat transfer area/volume between plates, $\beta = 461.0\ ft^2/ft^3$

Fin area (including splitter)/total area = 0.796

This f and j data is about like that used. Actual data is proprietary to AiResearch

$$HA = h(A_p + \eta_f A_f)$$

$$= (122.12)\frac{Btu}{hr\,ft^2\,°R}(A_p + \eta_f A_f)$$

$$A_p = 12.24 * 2.62 = 32.13\,IN^2 = 0.223\,ft^2$$

$$A_f = (28.07)(2.62) = 73.5\,IN^2 = 0.510\,ft^2$$

$$\eta_f A_f = (.95)(.510) = .4848\,ft^2$$

$$HA = (122.12)(.223 + .4848) = 86.5\,Btu/hr\text{-}°R$$ } SECTION 1 CONDUCTANCE

3.2 PRESSURE DROP

$$\omega_{max} = .02\,lb/sec$$

$$G = \frac{\omega}{A_c} = \frac{.02\,lb/sec}{.00678\,ft^2} = 2.95\,\frac{lb}{ft^2\,sec}$$

$$Re = \frac{D_H G}{\mu} = \frac{(.00606)\,ft\,(2.95)\frac{lb}{ft^2\text{-}sec} * 3600\,sec/hr}{.0521\,\frac{lb}{ft\text{-}hr}} = 1232$$

$$f = .035$$

$$V = \frac{G}{\rho} = \frac{2.95\,\frac{lb}{ft^2\,sec}}{.46786\,lb/ft^3} = 6.31\,ft/sec$$

$$\frac{V^2}{2g_c} = 0.619$$

$$\Delta P = \frac{4fL}{D_H}\frac{V^2}{2g_c}\rho$$

$$\frac{4fL}{D_H} = \frac{(4)(.035)(2.62)}{.0728} = 5.04$$

$$\Delta P = (5.04)(.619)(.46786) = 1.455\ lbf/ft^2 = \underline{.011\ PSI}$$

4.0 SECTION 2 HEAT EXCHANGER

4.1 HEAT TRANSFER CONDUCTANCE

USE $\dot{w}_{avex} = .004999\ lb/sec$

$$G = \frac{\dot{w}}{A_c} = \frac{.004999\ lb/sec}{.00678\ ft^2} = 0.737\ \frac{lb}{ft^2 sec}$$

$$Re = \frac{D_H G}{\mu} = \frac{(0.00606)(.737)\ ft\ \frac{lb}{ft^2 sec} \times 3600\ sec/hr}{.0521\ lb/ft\text{-}hr} = 308.1$$

$$j = .0205$$

$$h = \frac{J C_p G}{Pr^{2/3}} = \frac{(.0205)(1.243\ \frac{Btu}{lb\,°R})(.737)\frac{lb}{ft^2 sec} \times 3600\ sec/hr}{(.766)}$$

$$= 88.2\ Btu/hr\ ft^2\ °R$$

Then

$$m = \sqrt{\frac{2h}{kS}} = \left(\frac{176.4}{220 \times 5\times10^{-4}}\right)^{1/2} = 40\ \frac{1}{ft}$$

$$L_e m = .946\times10^{-2} ft \times 4.0\times10^{1} = .379$$

$$\eta_f = \frac{\tanh .379}{.379} = \frac{.36184}{.379} = .954$$

$$HA = h\,(A_P + \eta_f\, A_f)$$

$$A_P = (12.24)*(1.12) = 13.71\ in^2 = .0952\ ft^2$$

$$A_f = (28.07)*(1.12) = 31.92\ in^2 = .2183\ ft^2$$

$$\eta_f A_f = (.954)(.2183) = 0.2081\ ft$$

$$HA = (88.2)(.0952 + .2081) = \underline{26.73\ Btu/hr\text{-}°F}$$

4.2 PRESSURE DROP

$$\dot{w}_{max} = .0086\ lb/sec$$

$$G = \frac{\dot{w}}{A_c} = \frac{.0086\ lb/sec}{.00678\ ft^2} = 1.27\ \frac{lb}{sec\ ft^2}$$

$$Re = \frac{D_H G}{\mu} = \frac{(.00606)(1.27)\ ft\ \frac{lb}{sec\text{-}ft^2} * 3600\ \frac{sec}{hr}}{.0521\ \frac{lb}{ft\text{-}hr}} = 530$$

$f = .058$ $\quad\quad$ $V = \dfrac{G}{\rho} = \dfrac{1.27\ lb/sec\ ft^2}{.46786\ lb/ft^3} = 2.72\ ft/sec$

$$\frac{V^2}{2g_c} = \frac{(2.72)^2}{64.4} = 0.115$$

$$\frac{4fL}{D_H} = \frac{(4)(.058)(1.12)}{.0728} = 3.57$$

$$\Delta P = \frac{4fL}{D_H}\frac{V^2}{2g_c}\rho = (3.57)(.115)(.46786) = 0.1865\ lbf/ft^2$$

$$\underline{\Delta P = .001295\ PSI}$$

5.0 TOTAL CONDUCTANCE

	HA (Btu/hr-°R)
SECTION 1	86.5
SECTION 2	26.7
TOTAL	113.2

SECTION 9

FLOW DISTRIBUTION AND PRESSURE LOSSES IN THE SUMP REGION

INTRODUCTION

In the refrigerator sump region, flow passages must be provided to pneumatically connect the active sump volume to the hot and cold volumes through the various heat transfer devices. The following discussion covers the major elements of flow passage design.

FLOW DISTRIBUTION AROUND SUMP FILLER BLOCK AT INTERFACES WITH COLD END AND SECTION 2 OF AMBIENT HEAT EXCHANGER

Section 2 of the ambient heat exchanger receives flow from the cold end of the refrigerator via channels cut in the sump filler block. These channels, and the interface with the sump heat exchanger, are shown in Figure 9-1. The flow passages around the sump filler block are formed by chemical milling of slots in the block. The filler block is encased within the sump pressure vessel to provide enclosed flow passages as shown in Figure 9-1. Items of particular interest are:

- Flow distribution and pressure drop in the channels around the surface of the sump filler block. These channels are not of equal length; some non-uniformity in flow between channels is to be expected.

- Flow distribution in the distribution slot upstream of Section 2 of the sump heat exchanger.

- Pressure drop in the secondary distribution slot where the sump flow channels interface with flow passages in the linear bearing support.

Sump Filler Block Flow Channels

The flow rate through individual sump channels, and the total flow through all ten channels, are given as functions of pressure drop in Figure 9-2. At a total flow corresponding to the maximum cold end flow of 0.0065 lb/sec, the pressure drop is 0.0066 psi and the flow distribution between the separate channels is given by the points cut by the dashed line in Figure 9-2. This pressure drop is low enough so as not to pose a significant penalty on performance. Even though the shortest channels carry approximately 20 percent higher flows than the longest channels, the non-uniform flow between the shorter and longer channels does not pose a problem for distribution of fluid to the ambient heat exchanger. This is due to the use of a distribution slot at the interface between the channels and the heat exchanger.

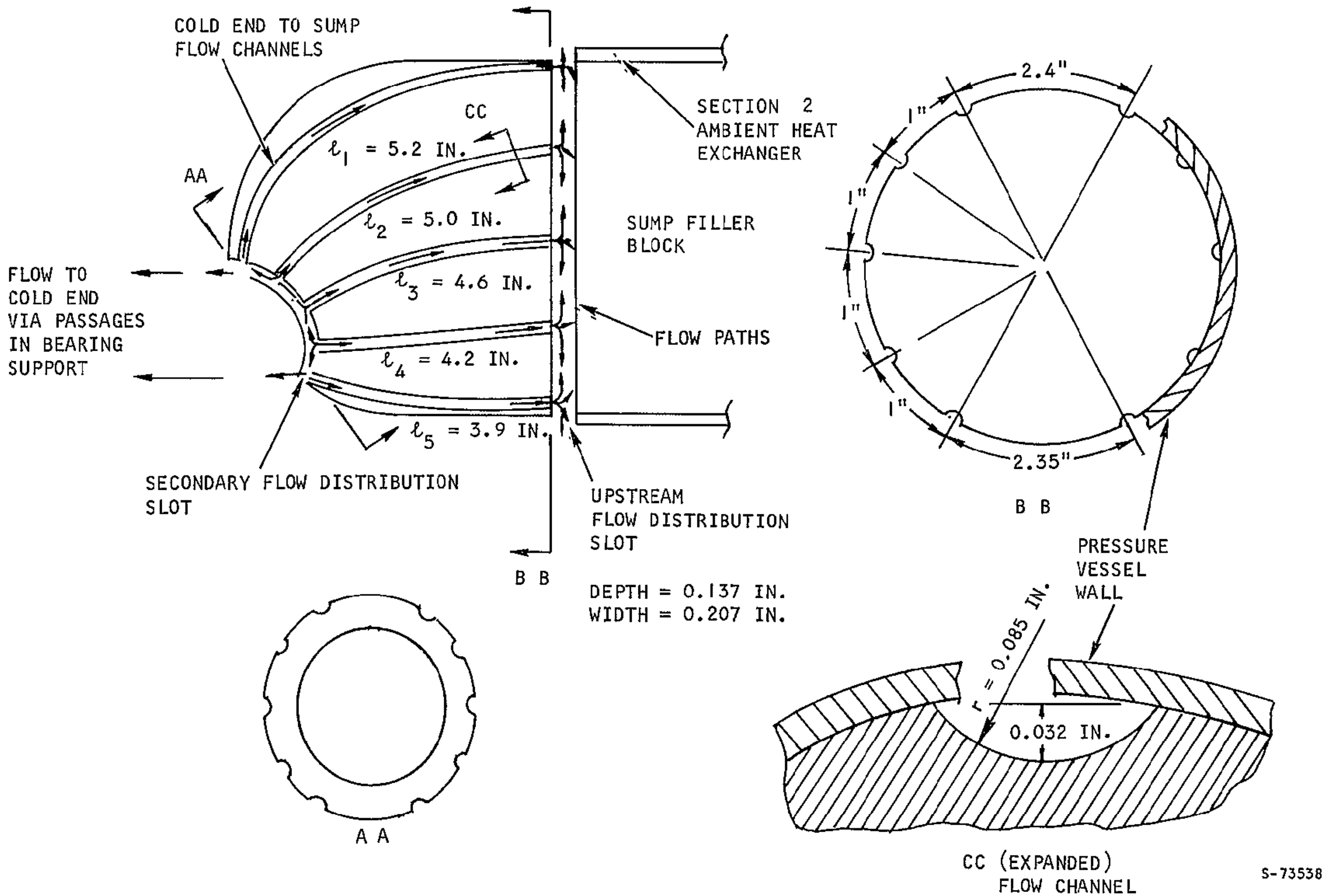


Figure 9-1. Sump Filler Block Flow Channels Toward Cold End

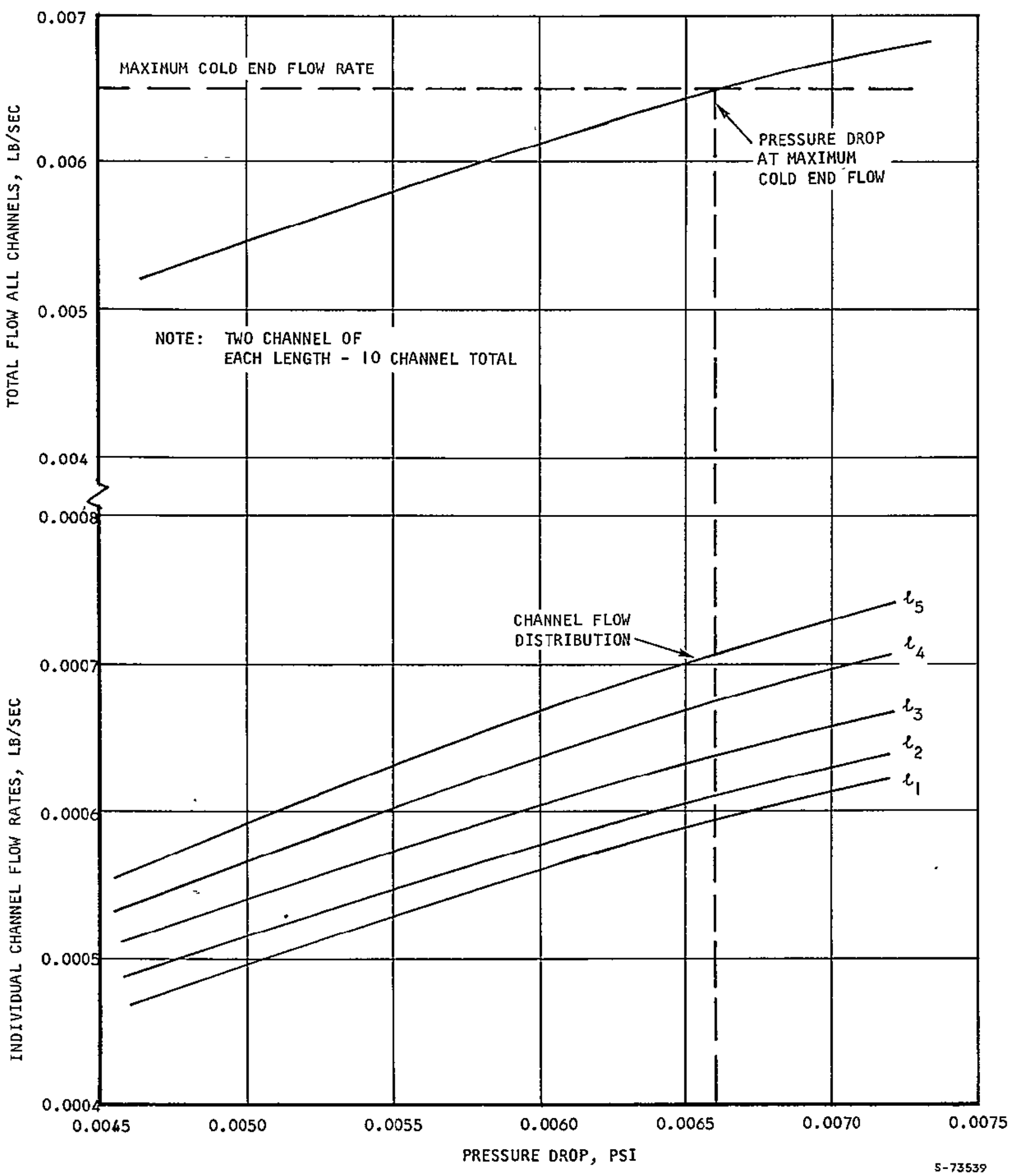


Figure 9-2. Sump Channels Pressure Drop and Flow Distribution

In operation, the actual maximum total flow rate through the channels is somewhat greater than the maximum cold end flow rate due to storage of gas in the cold regenerator at points during the cyclic operation. The maximum total channel flow rate is approximately 0.008 lb/sec, with a corresponding pressure drop of 0.01 psi. This does not significantly penalize performance and the relative distribution between the individual channels is nearly unchanged from that indicated by Figure 9-2.

Upstream Flow Distribution Slot

From the above discussion, it is noted that the distribution of the cold end flow between the sump channels is reasonably uniform--only a 20 percent difference in flow rate exists from the maximum to minimum flow between the channels. The flow is sufficiently well distributed between the channels so that segments of the ambient heat exchanger between channels (Section 2 of this heat exchanger) can be supplied fluid from the adjacent channels. This fact allows a straightforward design or sizing of the distribution slot, with a design requirement that the circumferential pressure drop around the slot be small compared to that of the sump channels or Section 2 of the sump heat exchanger.

The worst case, with respect to circumferential pressure drop, occurs if distribution of the flow from bottom channels (ℓ_5, see Figure 9-1) is considered. Here, distribution of flow around the bottom of the sump to the midpoint between the two bottom channels requires a pressure drop of 0.000168 psi (for the distribution slot dimensions selected). This is approximately one 40th of the pressure drop through the sump channels and one 7th that of Section 2 of the sump heat exchanger. Due to these much higher pressure drops in both the upstream and downstream components, the distribution slot adequately distributes the flow around the face of Section 2 of the sump heat exchanger.

Secondary Flow Distribution Slot

Here we are not overly concerned with flow distribution between the various channels since down stream--toward the cold regenerator--the flow paths along the bearing support to the cold regenerator are of unequal length. It is better to provide a distribution slot at a point along the bearing support where the downstream flow passage lengths can be made equal in length. The design of this bearing support distribution slot is discussed in the next topic below.

In the secondary distribution slot shown in Figure 9-1, it is sufficient to provide a flow passage, between the sump flow channels and the flow passages in the bearing support, which has very low pressure drop. This is accomplished by taking a 45^0 cut of the edge of the sump filler block at the interface between the sump block and the bearing support. The triangular flow passage formed by this cut has a cross sectional flow area of four times that of the sump flow channels. Due to the short distance between sump channels and bearing support flow passages, the pressure drop in the interfacing flow passage is negligible.

BEARING SUPPORT FLOW PASSAGE PRESSURE DROP AND FLOW DISTRIBUTION SLOT DESIGN

The flow passage configuration along the length of the bearing support, and the bearing support flow distribution slot, are shown in Figure 9-3. The flow passages provide the fluid connection between the cold end and the sump region. The distribution slot is included to ensure uniform distribution of flow to the cold regenerator. The design problems here are: (1) sizing the lengthwise flow passages to provide a low axial pressure drop; and (2) sizing the distribution slot such that the circumferential pressure drop is small compared to the axial pressure drop across the bearing support.

Flow Passage Sizing and Axial Pressure Drop

The depth of the flow passages is limited due to structural considerations with a maximum allowable depth of 0.05 in. To provide good distribution of flow at the ends of passages, a configuration of sixteen separate flow passages was selected. Figure 9-4 gives the axial pressure drop and void volume of the passages as functions of the passage width. The selected design is a compromise between pressure drop and void volume. The pressure drop at maximum flow is 0.34 psi; a low pressure drop is desirable but an increased dead volume is not. Also, if the axial pressure drop is made very low the flow distribution slot must be large to provide a low circumferential pressure drop compared to the axial pressure drop.

Flow Distribution Slot

Non-uniform distribution of flow in the sump flow channels is the most likely cause for non-uniform flow into the bearing support flow passages From the previous discussion however, it is seen that non-uniform distribution flow between the sump channels is not extreme. In conservatively sizing the bearing support distribution slot, it was assumed that one sixth of the total flow must be distributed around each side of the bearing support. On this basis, the circumferential pressure drop as a function of slot width is given in Figure 9-5. The slot depth has been set equal to the lengthwise flow passage depth of 0.05 in. to avoid unnecessary loss in the axial direction.

The selected design has a slot width of 0.3 in. providing a circumferential pressure drop which is one twentieth (1/20) of the axial pressure drop. This large difference in axial to circumferential pressure drop will insure uniform distribution of the flow downstream of the slot (toward the cold regenerator). A smaller slot could be used, however, since the void volume penalty is not great and it is extremely critical that the cold regenerator inlet flow be uniformly distributed, the selected design is a good choice.

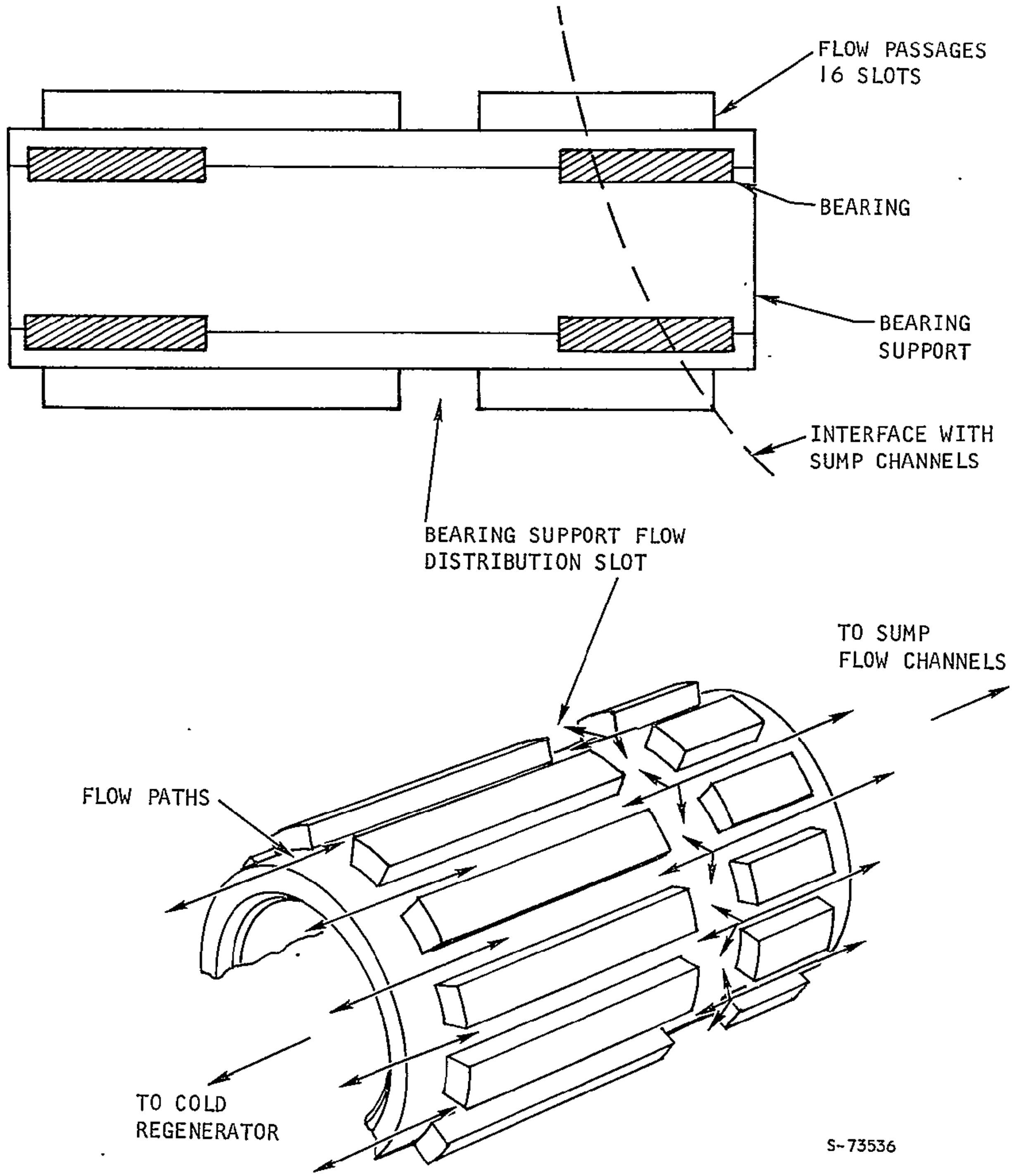


Figure 9-3. Bearing Support Flow Passages

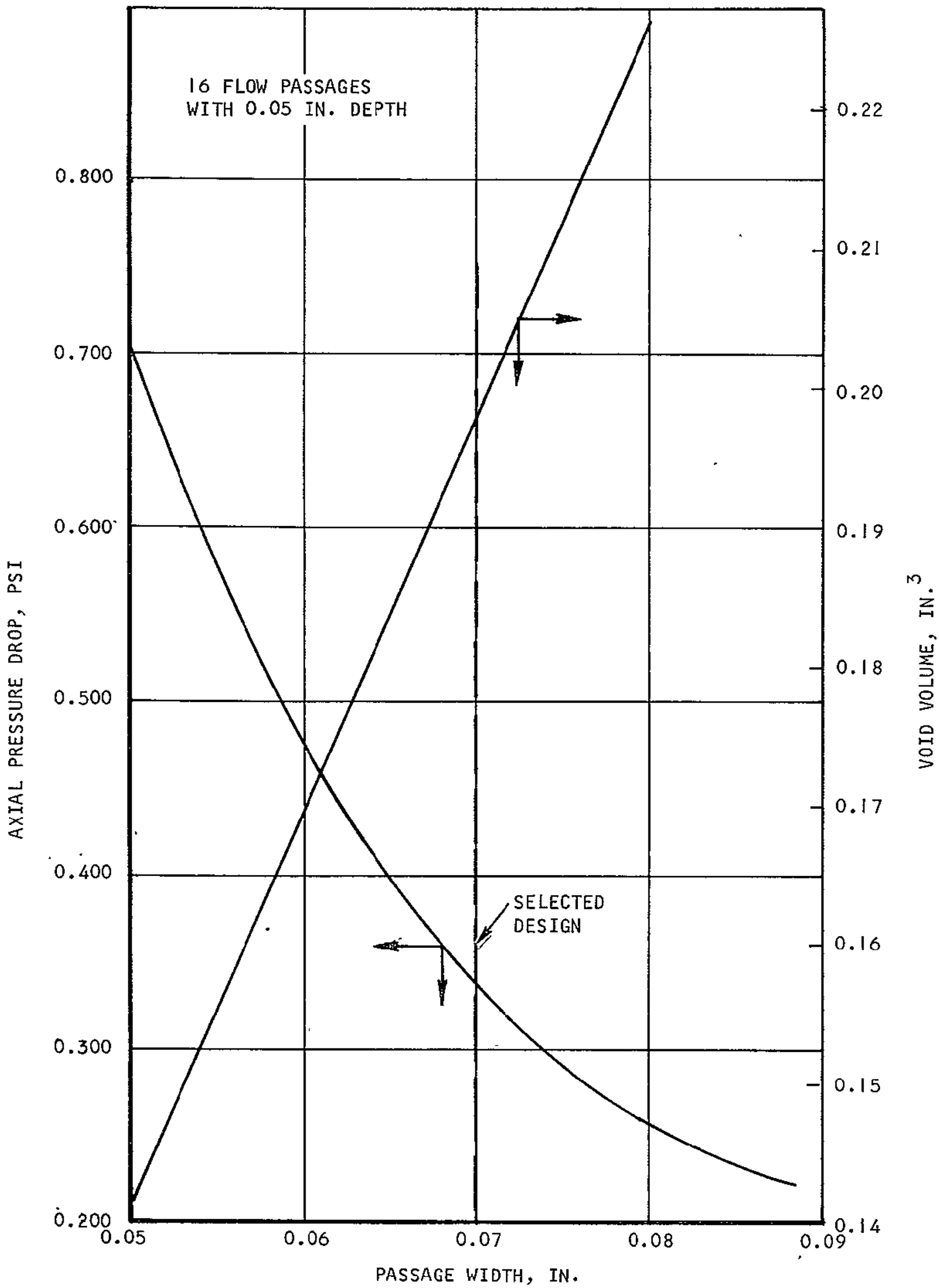


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Figure 9-4. Bearing Support Flow Passage Pressure Drop and Void Volume

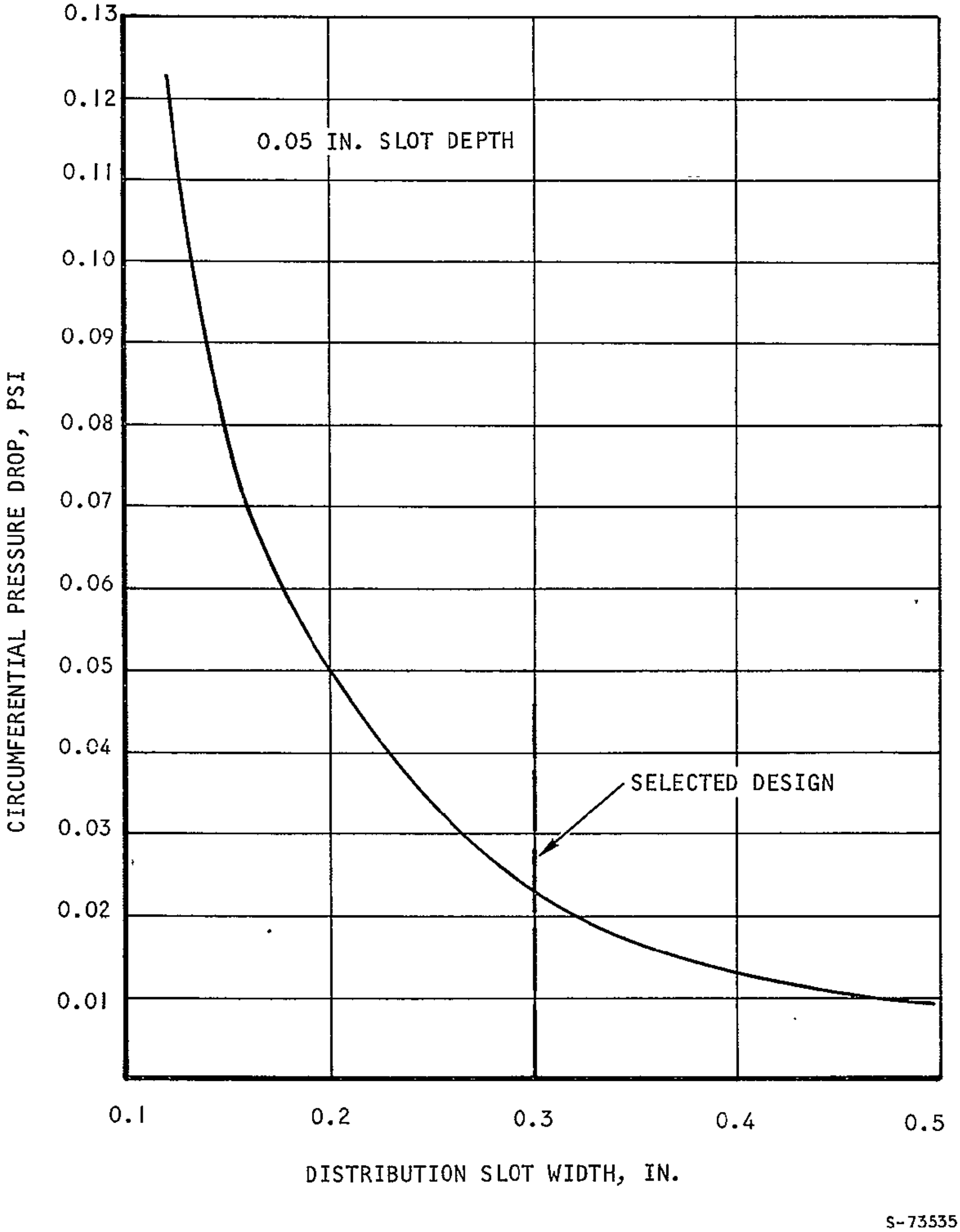

Figure 9-5. Bearing Support Flow Distribution Slot Circumferential Pressure Drop

SIZING OF PORTS TO ACTIVE SUMP VOLUMES

The basic GSFC VM refrigerator configuration has two active volumes in the sump region (Figure 9-6). These active volumes are plumbed to the hot and cold active volume, through various heat transfer devices via ports drilled into the sump filler block as shown. The design criteria for sizing these interconnecting ports consist of:

- Low pressure drop--The pressure drop in the ports must be low to minimize losses. The pressure drop in the ports to the backside of the hot displacer has a pronounced effect on the drive motor power requirements; a pressure drop much in excess of 0.5 psi cannot be tolerated in these ports. The major effect of pressure drop in ports to the crankcase sump is cold end seal leakage losses; here the port pressure drop is part of the total pressure drop imposed on the cold-end seal and it is desirable to maintain the pressure drop at something less than 0.1 psi.

- Dead volume--The pressure drop in the sump ports could be decreased by the use of either large diameter ports and/or a large number of ports, but the refrigeration capacity of the machine decreases as the port volume is increased. For this reason, the size and number of sump ports cannot be increased without limitation. The design approach minimized port volume consistent with a reasonable pressure drop.

- Interface with flow distribution slot--The sump ports interface with sump heat exchanger flow distribution slot (Figure 9-6). As the number of ports is increased, assuming they are symetrically located around the axis of the sump, the flow cross section of the distribution slot (in the circumferential direction) can be decreased while still providing distribution of the flow across the face of the sump heat exchanger. Thus, the number of ports selected must provide for a reasonably sized sump heat exchanger flow distribution slot.

- Manufacturing Ease and Tolerances--The ports to the backside of the hot displacer are nearly three inches in length. Drilling uniform, small diameter ports over this length is difficult. The practical limit in port size was thus established at 0.05 in. diameter, with even larger diameters preferred. An additional consideration is the influence of manufacturing tolerances on uniformity of the ports and hence flow distribution. A design with few ports of large diameter is less influenced by manufacturing tolerances, as far as pressure drop is concerned, than an equivalent design which has a large number of ports of small diameter.

The above criteria were applied in selecting the sump ports as described below.

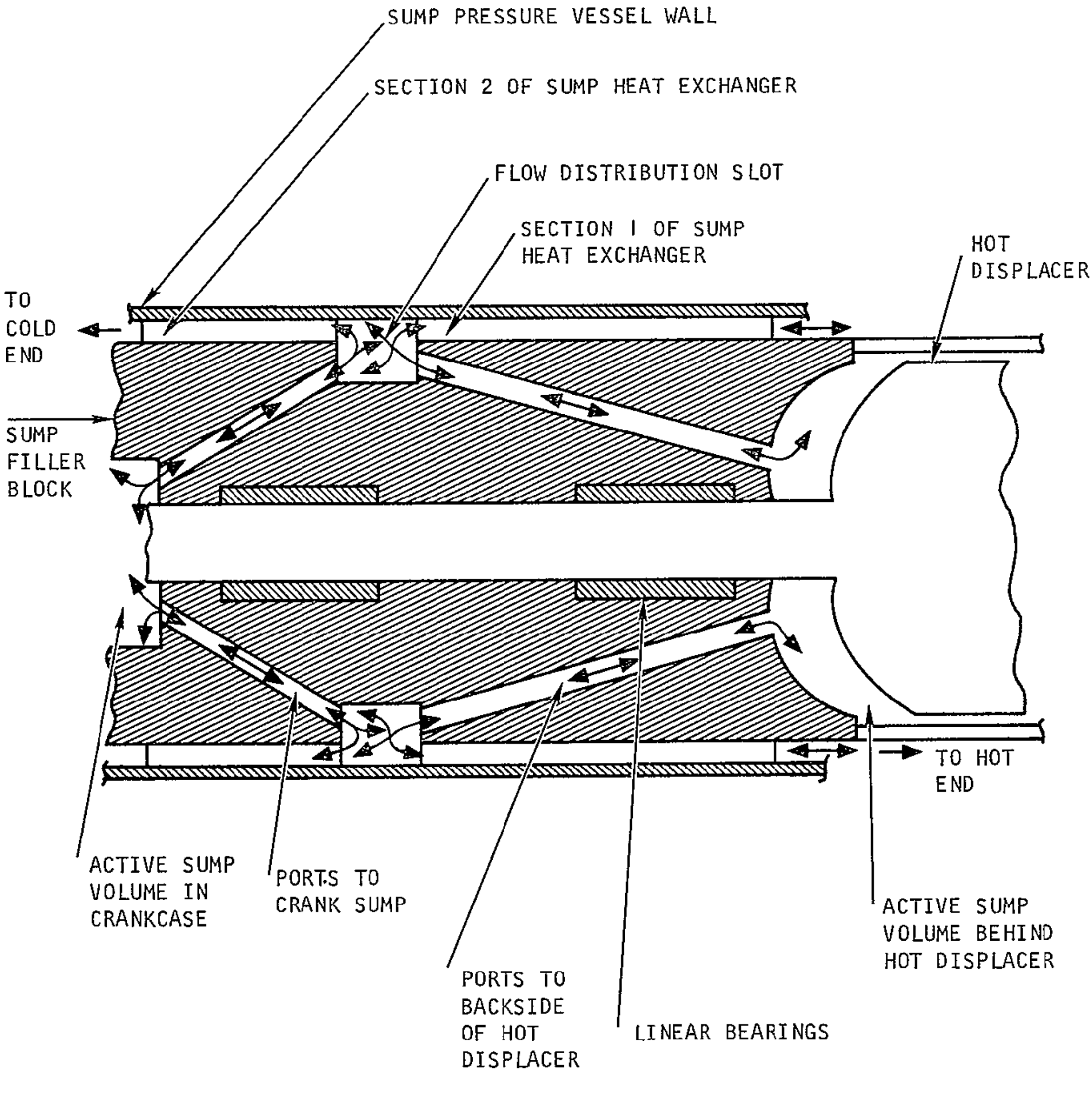

Figure 9-6. Configuration of Ports to Active Sump Volumes

Ports to Backside of the Hot Displacer

For fabrication simplicity, a six-port configuration was selected for the ports to the backside of the hot displacer. The pressure drop and volume associated with these ports as a function of port diameter is given in Figure 9-7. Since the active volume behind the hot displacer is much larger than the active volume in the crankcase, the maximum total flow (0.028 lb/sec) to the sump region was used to establish the pressure drop in these ports. The pressure drop includes the contraction and expansion losses at each end of the ports.

The port diameter selected is 0.160 in. This port diameter gives a pressure drop slightly below 0.5 psi and does not have an excessive void volume. In addition, the use of six ports of this diameter does not greatly penalize the design of the ambient heat exchanger flow distribution slot.

Ports to Crankcase Sump

In the case of the crankcase sump ports, use of the maximum total sump flow for port design is overly conservative since only a small fraction of the total flow enters the crankcase sump volume. The first problem thus consists of establishing a flow rate to use as a basis for design.

1. Crankcase Sump Flow

The flow to the crankcase sump can be established by investigating the rate of change of fluid mass stored in this volume. The mass stored in the crankcase sump can be expressed as:

$$M = \frac{PV_s}{ZRT} \qquad (9\text{-}1)$$

where

V_s = volume of crankcase sump

Z = compressibility factor of working fluid

R = gas constant of working fluid

T = absolute temperature

Then, neglecting small changes in the compressibility factor and assuming the temperature will remain relatively constant, the rate of change of mass with crank angle can be determined by differentiation of Equation 9-1 to yield:

$$\frac{dM}{d\theta} = \frac{P}{ZRT}\frac{dV_s}{d\theta} + \frac{V_s}{ZRT}\frac{dP}{d\theta} \qquad (9\text{-}2)$$

where

θ = crank angle

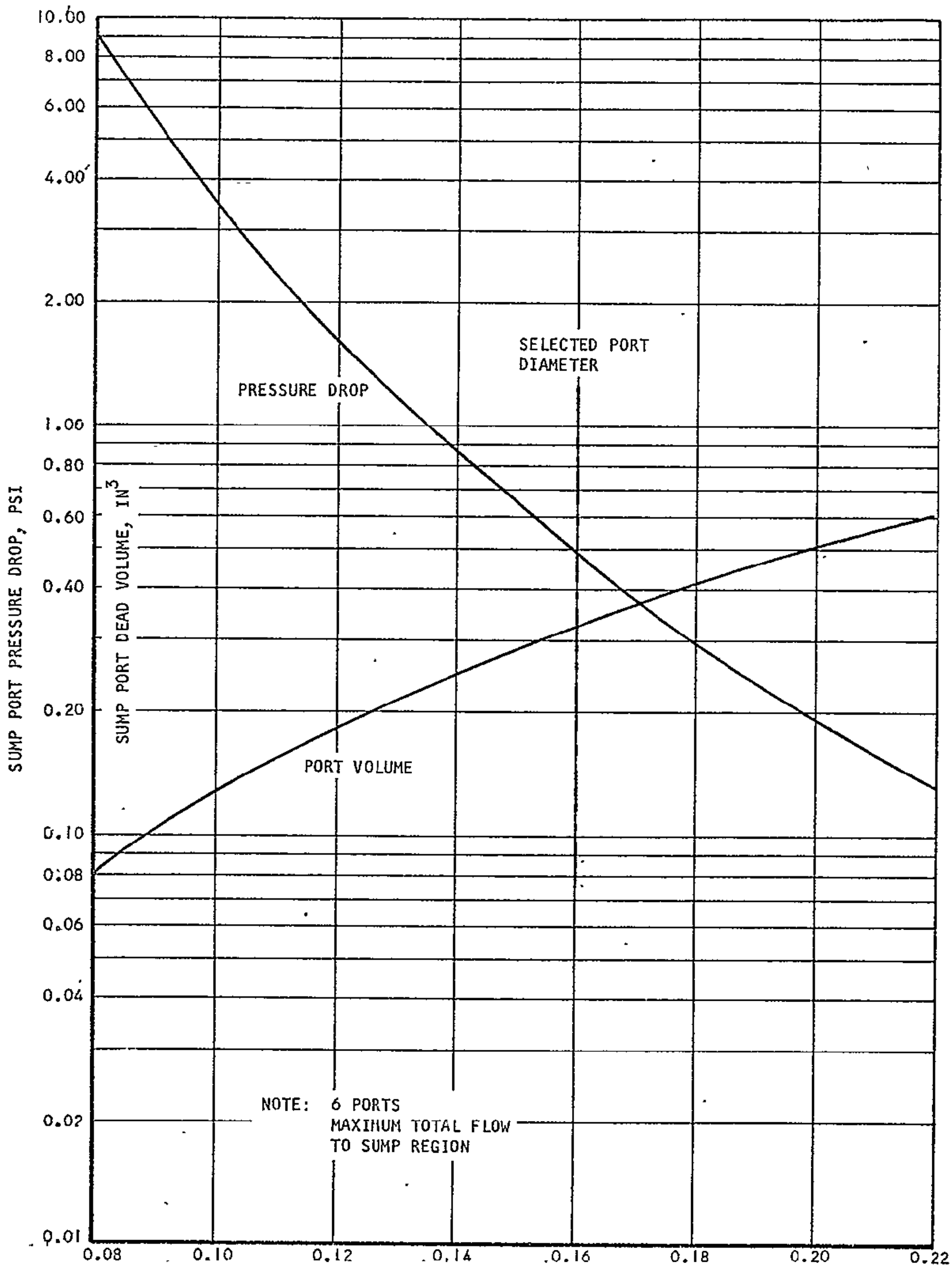

Figure 9-7. Pressure Drop and Volume of Sump Ports to Backside of Hot Displacer

From the geometry of the machine, the crankcase sump volume can be expressed as

$$V_s = 1.8921 - 0.2685 \sin (\theta - 28^0, 21') \qquad (9\text{-}3)$$

where

V_s = crankcase sump volume in in^3.

it then follows that

$$\frac{dV_s}{d\theta} = -0.2685 \cos (\theta - 28^0, 21') \qquad (9\text{-}4)$$

substitution of Equations 9-3 and 9-4 into Equation 9-2 yields:

$$\frac{dM}{d\theta} = \frac{1}{ZRT} \left\{ \underline{(1.8931 - 0.2685 \sin (\theta - 28^0, 21')} \frac{dP}{d\theta} - 0.2685 P \cos (\dot{\theta} - 28^0, 21') \right\} \qquad (9\text{-}5)$$

Then noting

$$\dot{\omega} = \frac{dM}{d\tau} = \left(\frac{dM}{d\theta}\right)\left(\frac{d\theta}{d\tau}\right) \qquad (9\text{-}6)$$

where

$\dot{\omega}$ = mass flow rate

τ = time

$\frac{d\theta}{d\tau} = \frac{2\pi \text{ (rpm)}}{60}$ in $\frac{\text{radians}}{\text{sec}}$

and combining with Equation 9-5 at a rotational speed of 400 rpm yields

$$\dot{\omega} = 1.457 \times 10^{-5} \left\{ \left[1.8931 - 0.2685 \sin (\theta - 28^0, 21')\right] \frac{dP}{d\theta} - \left[0.2685 \quad \cos (\theta - 28^0, 21')\right] \right\} \qquad (9\text{-}7)$$

where

$\dot{\omega}$ = mass flow rate in lb/sec

P = pressure in psia

$\frac{dP}{d\theta}$ = change in pressure in $\frac{lb}{in^2}$ per radian

For small pressure drops in the ports to the crankcase sump volume, the rate of change in sump pressure can be set equal to the rate of change in cycle pressure. That is

$$\frac{dP}{d\theta} = \left(\frac{dP}{d\theta}\right)_{cycle} \tag{9-8}$$

Then Equation 9-7 can be solved making use of the rate of change in cycle pressure provided for the ideal VM cycle computer program.

Figure 9-8 is a plot of the crankcase sump volume port flow rate (Equation 9-7) as a function of crank angle. The maximum flow indicated by Figure 9-8 is slightly less than 0.0022 lb/sec; this maximum flow was used in the sizing of the ports described below.

2. Port Design

Figure 9-9 gives the pressure drop and dead volume as functions of port diameter for 1, 2 and 4 ports into the crankcase sump. The selected design consists of four ports with a diameter of 0.1 in. The use of four ports in lieu of fewer ports is better from the standpoint of interfacing with the sump heat exchanger flow distribution slot. The use of more ports is ruled out since it is impractical to penetrate the sump crank volume at more than four points. It is also noted that for four ports the pressure drop rapidly increases for port diameters below 0.1 in. In addition, if operation at 600 rpm is considered, the pressure drop goes up by a factor of approximately 2.25; thus, selection of a design with a very low pressure drop (0.023 psi at 400 rpm) is indicated.

SUMP HEAT EXCHANGER FLOW DISTRIBUTION SLOT

The interface between the ambient sump heat exchanger, the ports into active sump volumes, and the flow distribution slot is shown in Figure 8-1. Figure 9-10 shows the orientation of the two sets of ports around the flow distribution slot. The function of this distribution slot, as previously discussed, is to distribute flow coming from the sump ports uniformly across the face of the sump heat exchanger.

As with other flow distribution devices (discussed previously), the sizing of this flow distribution slot will be based on providing a small circumferential pressure drop relative to the pressure drop of the adjacent flow passages--the sump ports and sump heat exchanger. Due to the relatively high flow rates in this region of the machine, particular care must be taken in sizing the flow distributor; overly conservative approaches must be avoided since they result in selection of a design with excessive dead volume.

Method of Analysis

The basic design approach assumes uniform distribution of the total flow between the sump ports and across the heat exchanger face and then sizes the distribution slot to provide a sufficiently low circumferential pressure so that nearly uniform flow distribution will occur.

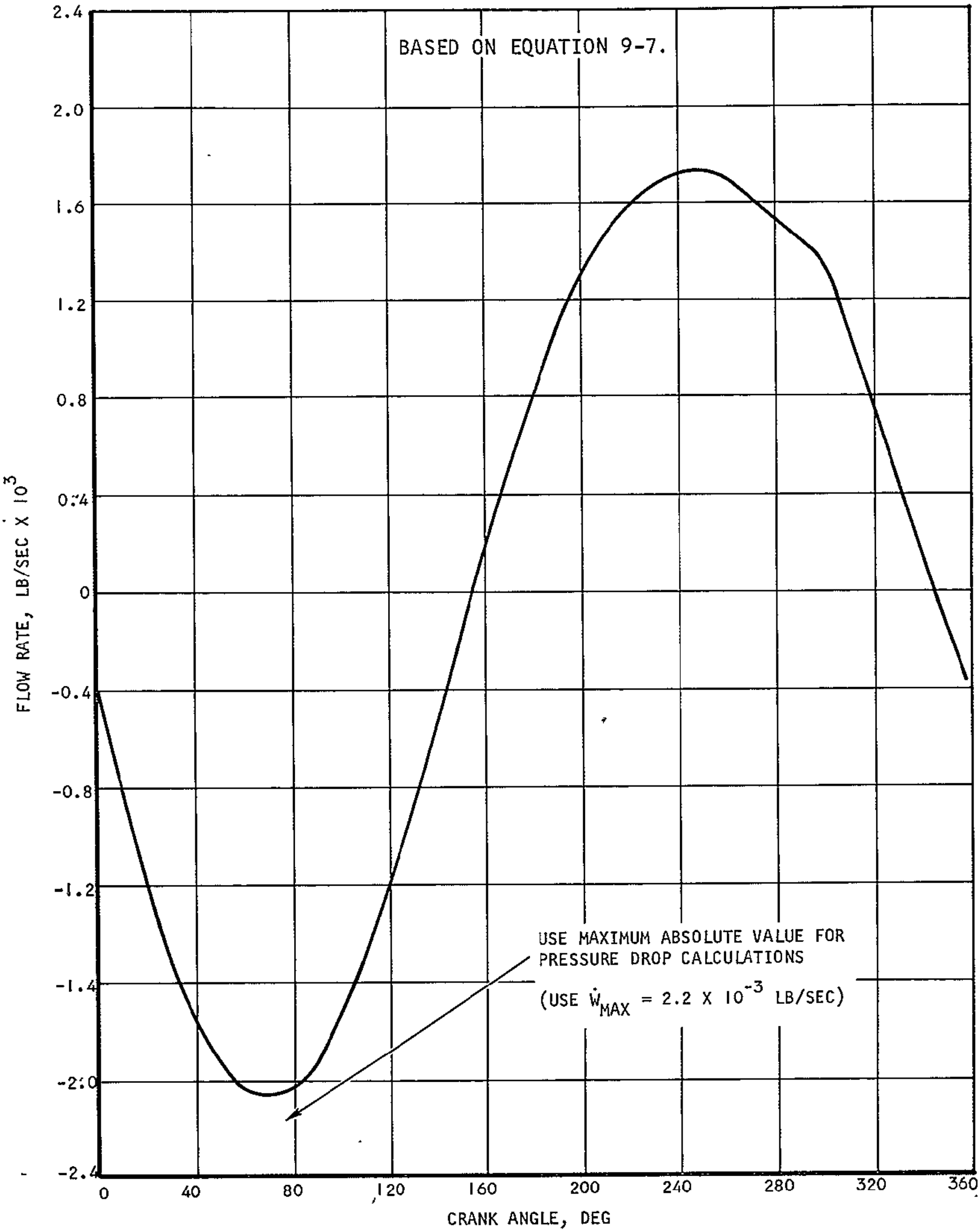

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Figure 9-8. Crankcase Sump Port Flow Rate at 400 RPM

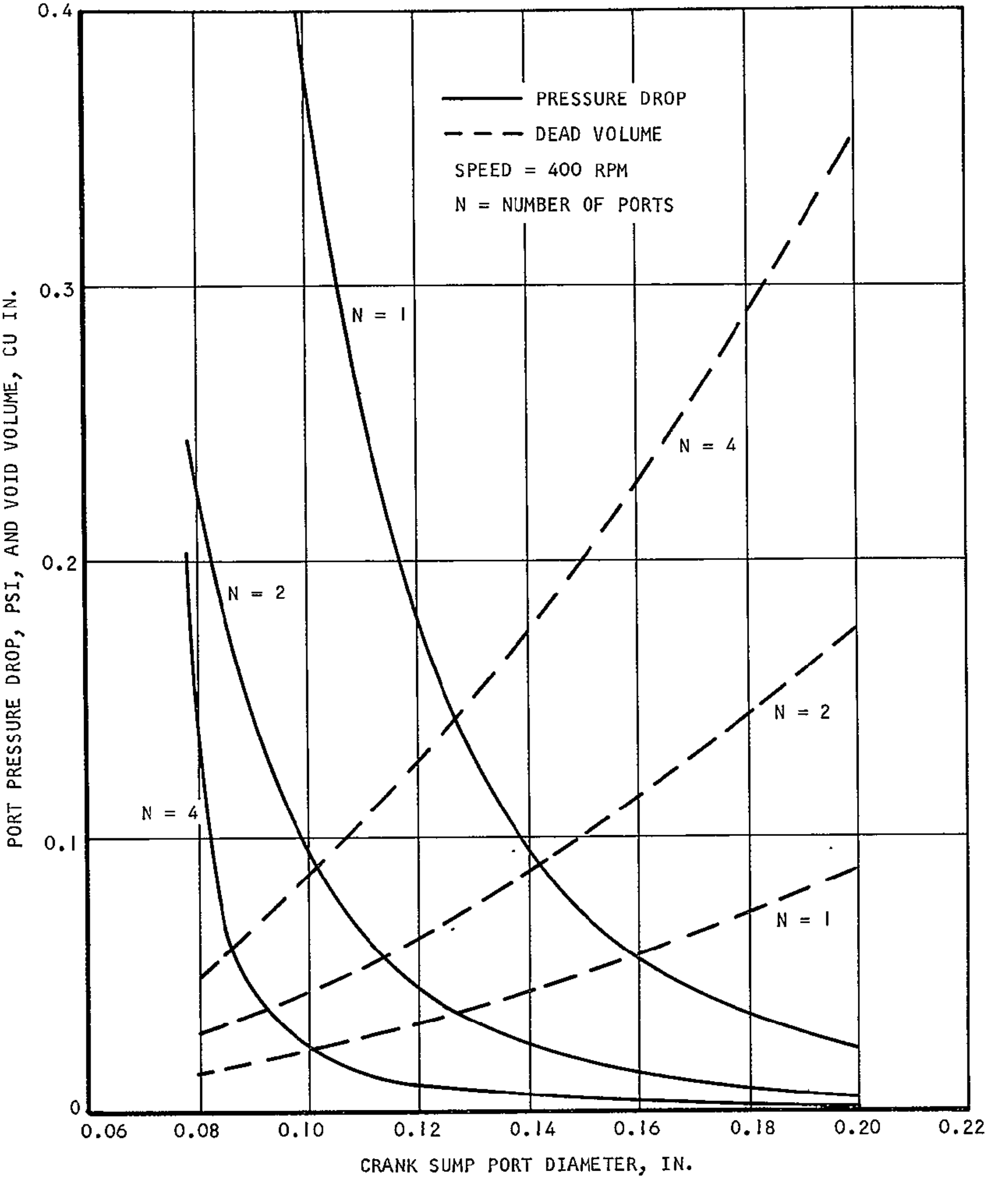


Figure 9-9. Crankcase Sump Port Pressure Drop and Dead Volume

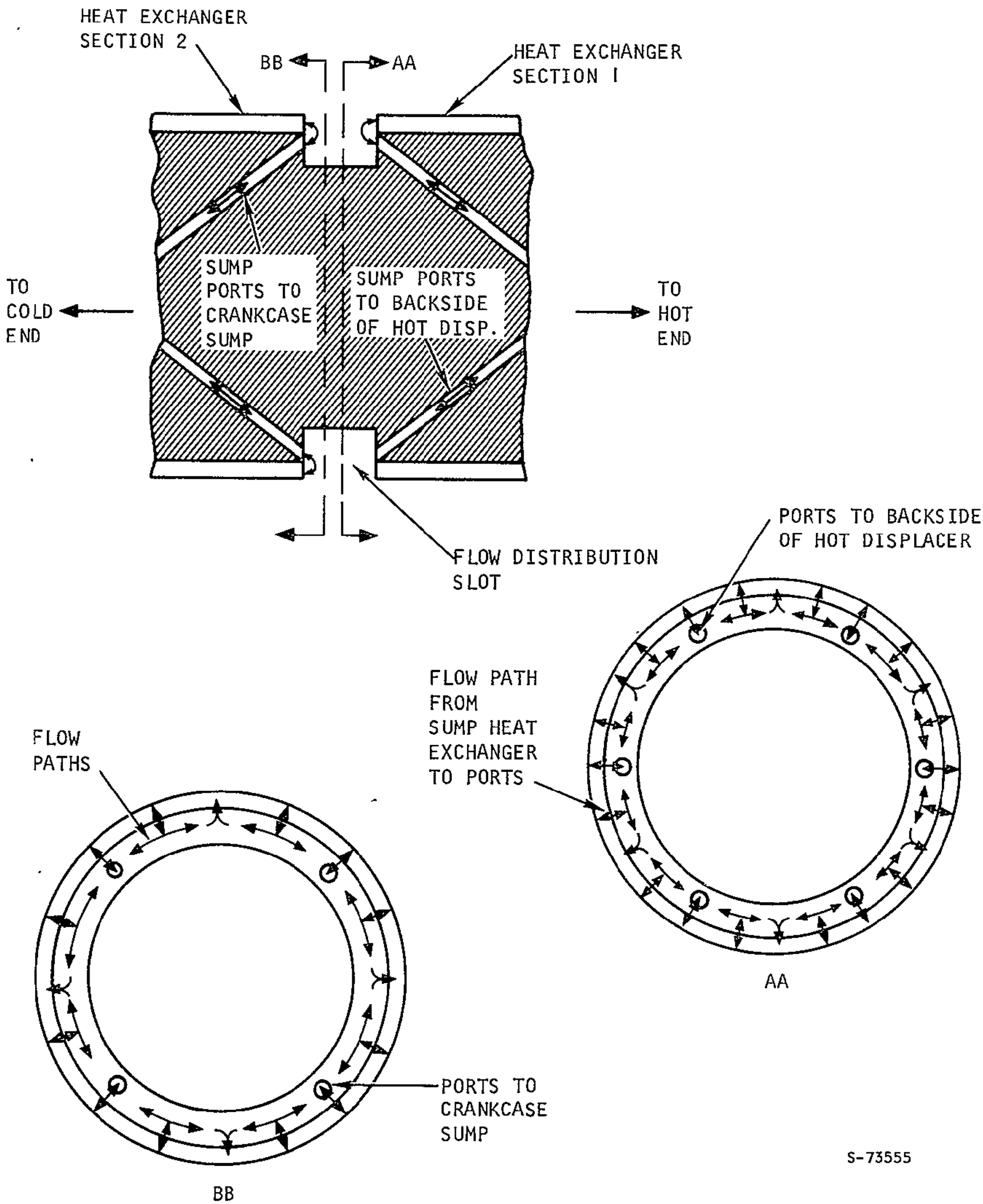


Figure 9-10. Sump Heat Exchanger Flow Distribution Slot

The first step in the analysis is to set up the relationships which define the circumferential pressure drop. Figure 9-11 represents a segment of the distribution slot. Referring to this figure, and taking flow in the X direction (circumferentially), the pressure gradient can be expressed as:

$$\frac{dP}{dX} = -\left(\frac{4f}{D_H}\right)\frac{V^2}{2g_c}\rho \qquad (9\text{-}9)$$

where

f = Fanning's friction factor

D_H = slot hydraulic diameter

V = fluid velocity

g_c = gravitational constant

ρ = fluid density

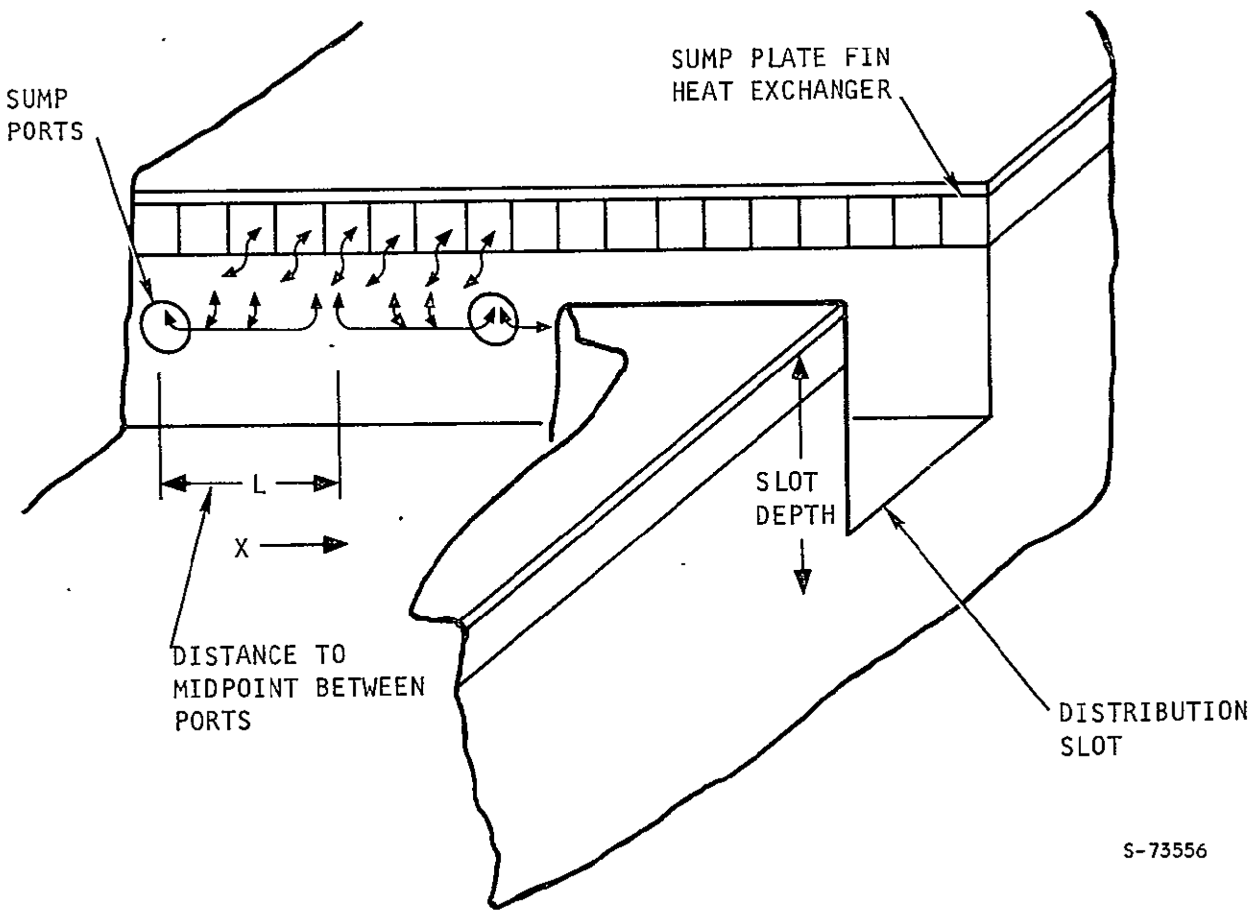


Figure 9-11. Flow Distribution Slot Model Schematic

Considering flow from a reference sump port (as flow goes in the X direction, see Figure 9-11) the rate of flow will decrease due to flow into the ambient heat exchanger. Then assuming uniform flow in the heat exchanger, the flow rate $\dot{\omega}$ can be expressed as a function of X.

$$\dot{\omega}_{(X)} = \frac{\dot{\omega}_o}{2}\left(1 - \frac{X}{L}\right) \tag{9-10}$$

where

$\dot{\omega}_{(X)}$ = circumferential flow rate as a function of X

$\dot{\omega}_o$ = total flow rate from a particular port--equal to the total from the set of ports divided by the number of ports

L = distance to midpoint between ports

Then noting

$$V = \frac{\dot{\omega}}{\rho A_F} \tag{9-11}$$

then

$$V_{(X)} = \frac{\dot{\omega}_o\left(1 - \frac{X}{L}\right)}{2\rho A_F} \tag{9-12}$$

where

$V_{(X)}$ = fluid velocity as a function of X

A_F = cross sectional area of flow distribution slot

Substitution of Equation 9-12 into Equation 9-9 yields

$$\frac{dP}{dX} = -\left(\frac{f}{2}\right)\frac{\dot{\omega}_o^{\,2}\left(1 - \frac{X}{L}\right)^2}{D_H g_c A_F^{\,2}} \tag{9-13}$$

In general, f is a function of Reynolds number (or flow) and the form of the dependency is dependent on the flow regime--whether turbulant or laminar. Since $\dot{\omega}_{(X)}$ will go from $\frac{\dot{\omega}_o}{2}$ at X = o to zero at X = L both flow regimes are likely to be encountered; thus an expression for each is developed below:

I. Turbulent Flow

For turbulent flow, assumed here for $Re \geqq 2100$, the Fanning's friction factor can be expressed as

$$f = 0.084(Re)^{-1/4} = 0.084\left(\frac{\mu}{D_H V_{(X)}\rho}\right)^{1/4} \qquad (9\text{-}14)$$

which upon combining with the expression for $V_{(X)}$ yields:

$$f = 0.084\left[\frac{2\mu A_F}{\dot{\omega}_o D_H\left(1-\frac{X}{L}\right)}\right]^{1/4} \qquad (9\text{-}15)$$

where the new terms are:

μ = fluid viscosity

substitution of Equation (9-15) into Equation (9-13) yields:

$$\frac{dP}{dX} = -\frac{0.042}{g_c\rho}(2\mu)^{1/4}\left(\frac{\dot{\omega}_o}{A_F}\right)^{1.75}\frac{1}{(D_H)^{1.25}}\left(1-\frac{X}{L}\right)^{1.75} \qquad (9\text{-}16)$$

Then assuming turbulent flow at X = o and that Re = 2100 at $X = X_i$, Equation (9-16) can be integrated to give:

$$\Delta P_I = \left[\frac{0.042}{g_c\rho}(2\mu)^{1/4}\left(\frac{\dot{\omega}_o}{A_F}\right)^{1.75}\frac{1}{(D_H)^{1.25}}\right]\frac{L}{2.75}\left\{\left(1-\frac{X_i}{L}\right)^{2.75}-1\right\}$$

(9-17)

where

ΔP_I = circumferential pressure drop in slot in turbulent flow

X_i = distance where the Reynolds (Re) number becomes equal to 2100 assuming turbulent flow at X = o, that is:

$$Re = \frac{\dot{\omega}_o\left(1-\frac{X_i}{L}\right)}{2A_F\mu} D_H \geqq 2100$$

NOTE: The choice of Re = 2100 is somewhat arbitrary since the transition between turbulent and laminar flow takes place over a range of Reynolds numbers. The use of Re = 2100 as a transition point yields conservative results in present case.

2. Laminar Flow

For laminar flow

$$f = \frac{K}{Re} = \frac{2KA_F}{\dot{\omega}_o}\left(\frac{\mu}{D_H}\right)\frac{1}{\left(1 - \frac{X}{L}\right)} \tag{9-18}$$

which upon substitution into Equation 9-13 yields:

$$\frac{dP}{dX} = - \frac{K \mu \dot{\omega}_o}{g_c D_H^2 \rho A_F}\left(1 - \frac{X}{L}\right) \tag{9-19}$$

then integrating between $X = X_i$ and $X = o$ such that $Re \leqq 2100$ at $X = X_i$ yields

$$\Delta P_2 = - \frac{K \mu \dot{\omega}_o}{2g_c D_H^2 \rho A_F}\left\{\left(1 - \frac{X_i}{L}\right)\right\}^2 \tag{9-20}$$

where

K = a constant between 13 and 24 depending on geometry of the slot

3. Circumferential Pressure Drop

The circumferential pressure drop is then the sum of the pressure drop given by Equations 9-17 and 9-20.

$$\Delta P_{cir} = \Delta P_1 + \Delta P_2 \tag{9-21}$$

Design Results

Figure 9-12 gives the major design parameters of the flow distribution slot as functions of slot depth. A selected slot width of 0.25 in. was based on providing the maximum practical ambient sump heat exchanger area within the available length; width is thus constant at this value for the data of Figure 9-12.

A slot depth of 0.30 in. was selected as indicated in Figure 9-12. This slot depth provides a circumferential pressure drop which is less than one-fortieth of the pressure drop in Section 1 of the sump heat exchanger and one-fifth the pressure drop of Section 2 of this heat exchanger. Distribution around the face of Section 1 of the heat exchanger--which is the major section of the exchanger--is therefore excellent. The distribution around Section 2 is marginal, but use of a deeper slot adds excessive dead volume to the sump region of the machine. Also, this section of the sump heat exchanger has an additional distribution slot at its other end which will aid in providing uniform flow.

It is noted that the circumferential pressure drop is very much lower than the pressure drop in either the ports going to the crankcase sump volume or those going to the backside of the hot displacer. The uniform distribution among these ports as originally assumed should therefore be valid.

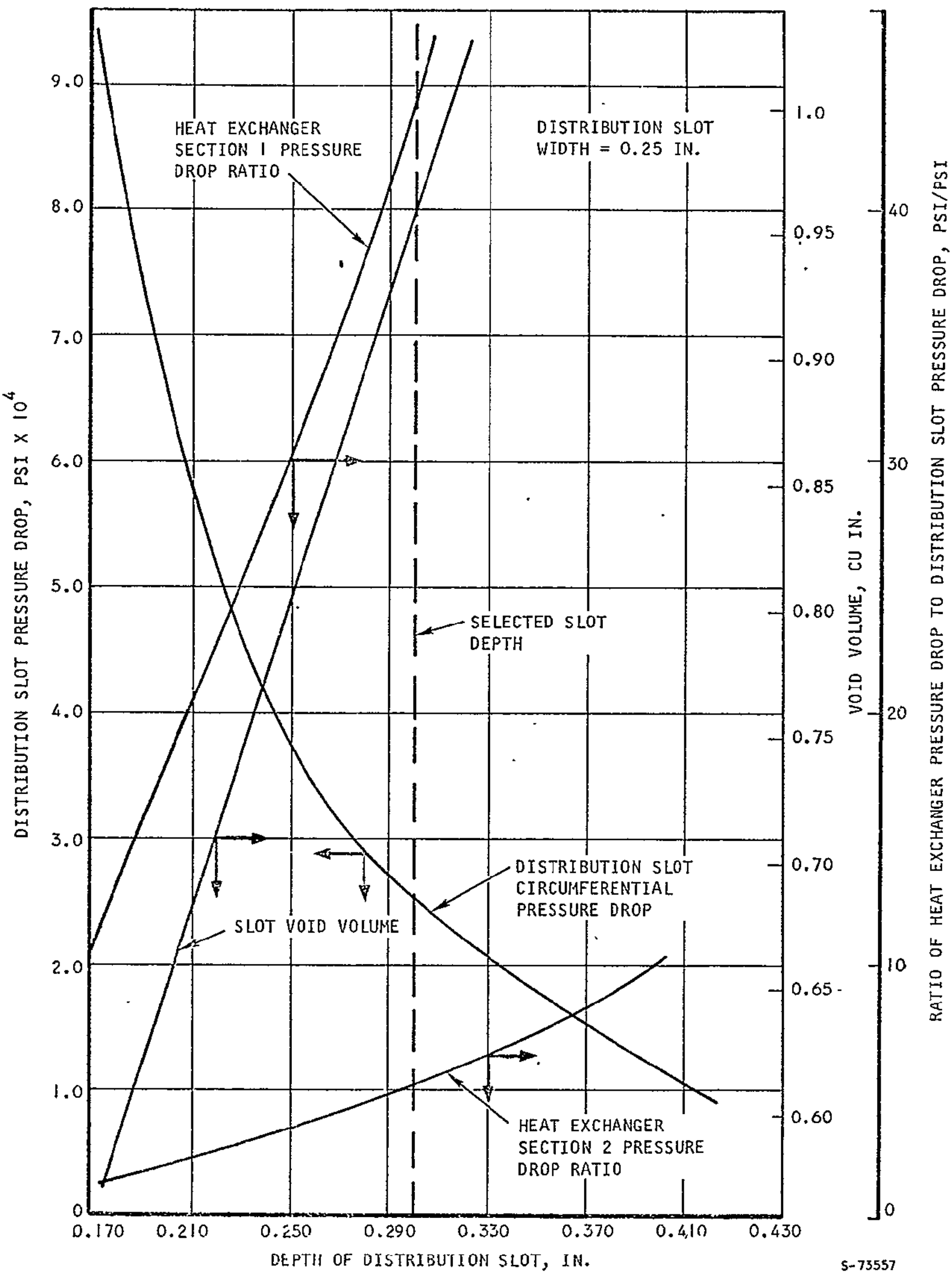
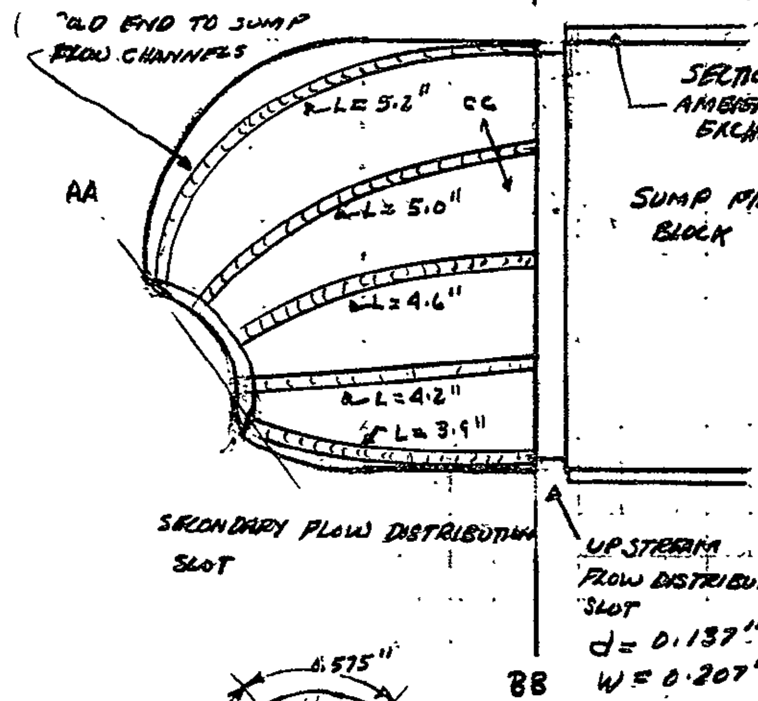
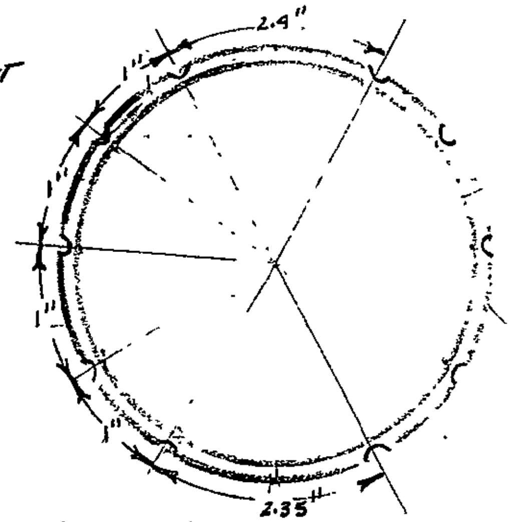


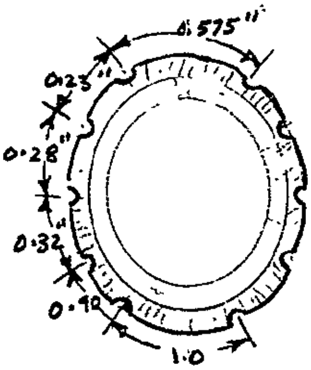
Figure 9-12. Distribution Slot Design Parameters

FLOW DISTRIBUTION AROUND SUMP FILLER BLOCK AT INTERFACES WITH COLD END AND SECTION NO. 2 OF AMBIENT HEAT EXCHANGER

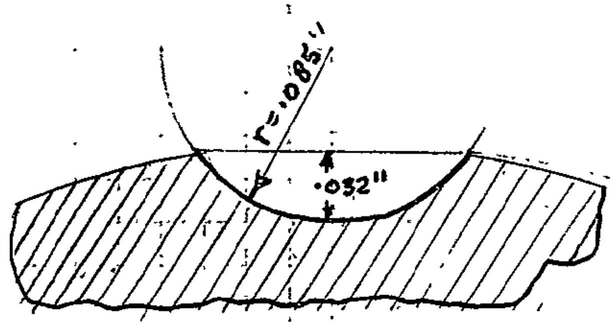




B B



A A



CC (Expanded) Flow Channel

Here we are interested in: (1) the flow distribution in the channels ~~around the~~ surface of the sump filler block -- these channels connect the cold end to the ambient heat exchanger section of the engine and since the channel are not of equal length we cannot expect uniform flow in between the channels. (2) the flow distribution in flow distribution slot upstream from Section 2 of the ambient heat exchanger and (3) the flow distribution in the secondary flow distribution slot.

1.0 FLOW DISTRIBUTION IN SUMP CHANNELS

1st lets check if the flow in channels is laminar or turbulent.

From the geometry of the channel

$$A_F = \frac{\pi r^2}{2} - \left\{ x\sqrt{r^2-x^2} + r^2 \sin^{-1}\left(\frac{x}{r}\right) \right\}$$

Where $x = r - .032'' = .053''$

$r = .085''$

$\frac{x}{r} = \frac{.053}{.085} = 0.624$

$\sin^{-1}\left(\frac{x}{r}\right) = 38°36' = 0.674 \text{ rad}$

$$A_F = \frac{\pi}{2}(.085)^2 - \left\{(.053)\sqrt{(.085)^2-(.053)^2} + (.085)^2(.674)\right\}$$

$A_F = .01134325 - .00839165 = .00295 \text{ IN}^2 = .0000205 \text{ ft}^2$

$D_H = \frac{4A_F}{W_P}$

where W_P = wetted perimeter

$W_P = 2r\sin\left(\frac{x}{r}\right) + 2r\sin^{-1}\frac{x}{r} = 2r\left(\sin\frac{x}{r} + \sin^{-1}\frac{x}{r}\right)$

$W_P = (2)(.085)(.624 + .674) = .2205 \text{ IN}$

$D_H = \frac{(4)(.00295)}{.2205} = 0.05351 \text{ IN} = .00446 \text{ ft}$

Then for the present assuming equal flow in all ten channels

$$Q_T = \frac{\dot{w}_{max}}{\rho} = \frac{.0065 \text{ lb/sec}}{.468 \text{ lb/ft}^3} = 0.0139 \text{ ft}^3/\text{sec}$$

$$Q_i = \frac{Q_T}{10} = .00139 \text{ ft}^3/\text{sec}$$

$$V = \frac{Q_i}{A_F} = \frac{.00139 \text{ ft}^3/\text{sec}}{.0000205 \text{ ft}^2} = 67.80 \text{ ft/sec}$$

$$Re = \frac{V D_H \rho}{\mu} = \frac{67.8 \text{ ft/sec} * (.00996 \text{ ft})(.468 \text{ lb/ft}^3) * 3600 \text{ sec/hr}}{0.0521 \frac{\text{lbm}}{\text{ft-hr}}} = 9778.$$

Even with a reasonable amount of non-uniform distribution between the channels it looks like we will have turbulent flow. Then the pressure drop is given by

$$\Delta P = \frac{4fL}{D_H} \frac{V^2}{2g_c} \rho$$

where $f = 0.084 \, Re^{-1/4}$

which upon substitution into the above yields

$$\Delta P = (.084) \frac{2L}{g_c \rho} \frac{\mu^{1/4}}{D_H^{1.25}} \frac{\dot{w}^{1.75}}{(A_F)^{1.75}}$$

Then

$$\Delta P = \frac{(.084)(2)(L)(IN)\ (.0521)^{1/4}\left(\frac{lbm}{ft\text{-}in}\right)^{1/4} \dot{w}^{1.75}\left(\frac{lbm}{sec}\right)^{1.75}}{(32.2)\frac{lbm\text{-}ft}{lbf\text{-}sec^2} * 0.468\frac{lbm}{ft^3}\ (0.00446)^{1.25} ft^{1.25}\ (.0000205)^{1.75} ft^{3.5}}$$

$$= \frac{(.084)(2)(.0521)^{1/4}(\dot{w})^{1.75} L}{(32.2)(.468)(.00446)^{1.25}(.0000205)^{1.75}} \ \frac{\frac{in\ lbm^2}{ft^{1/4}(in)^{1/4} sec^{1.75}}}{\frac{lbm^2\ ft^{2.75}}{lbf\text{-}sec^2}} * \frac{ft}{12 IN}\left(\frac{in}{3600 sec}\right)^{1/4}$$

$$= \frac{(0.084)(2)(.0521)^{1/4}\ L * \dot{w}^{1.75}}{(32.2)(.468)(.00446)^{1.25}(.0000205)^{1.75}(12)(3600)^{1/4}}$$

$$\Delta P = 8.05 \times 10^6\ L * \dot{w}^{1.75} \quad \left\{\begin{array}{l} L \text{ in in} \\ \dot{w} = lb/sec \end{array}\right\}$$

ΔP in lbf/ft^2

$$\boxed{\Delta P = 5.58 \times 10^4\ L * \dot{w}^{1.75}} \quad \left\{\begin{array}{l} L \text{ in IN} \\ \dot{w} \text{ in lb/sec} \\ \Delta P \text{ in PSI} \end{array}\right\}$$

interested in flows between .001 and .0003 lb/sec

Let identify the channels as

$\ell_1 = 5.2''$

$\ell_2 = 5.0'$

$\ell_3 = 4.6''$

$\ell_4 = 4.2''$

$\ell_5 = 3.9''$

Then

$$\Delta P_1 = (5.58 \times 10^4)(5.2)(\dot{w}_1)^{1.75} = 2.901 \times 10^3 (\dot{w}_1)^{1.75}$$

$$\Delta P_2 = (5.58 \times 10^4)(5.0)(\dot{w}_2)^{1.75} = 2.79 \times 10^3 (\dot{w}_2)^{1.75}$$

$$\Delta P_3 = (5.58 \times 10^4)(4.6)(\dot{w}_3)^{1.75} = 2.567 \times 10^3 (\dot{w}_3)^{1.75}$$

$$\Delta P_4 = (5.58 \times 10^4)(4.2)(\dot{w}_4)^{1.75} = 2.344 \times 10^3 (\dot{w}_4)^{1.75}$$

$$\Delta P_5 = (5.58 \times 10^4)(3.9)(\dot{w}_5)^{1.75} = 2.1762 \times 10^3 (\dot{w}_5)^{1.75}$$

$\dot{w}_i$ lb/sec	ΔP_1 PSI	ΔP_2 PSI	ΔP_3 PSI	ΔP_4 PSI	ΔP_5 PSI	$\dot{w}_i^{1.75}$
.0003	.001984	.0019018	.0017553	.0016028	.001488	.0000006838
.0004	.003282	.0031563	.002904	.0026518	.00246	.0000011313
.0005	.004850	.004664	.004292	.0039187	.003637	.0000016718
.0006	.006612	.006417	.005909	.005391	.005005	.0000023001
.0007	.008739	.008405	.0077328	.007061	.006513	.0000030124
.0008	.011039	.010616	.009767	.008919	.008280	.000003805
.0009	.0135664	.013097	.012005	.010962	.010176	.0000046765
.0010	.016313	.015689	.014435	.0131812	.012237	.0000056234

See plots on following pages

Then assuming the flow distribution slots at end ends of the channels will result in equal pressure drops across each channel we can, from the data in Figures x and 3, obtain the total flow as well as the individual channel flow rates as a function of pressure drop. Where the total flow equals the cold end flow of 0.0065 lb/sec

⑥

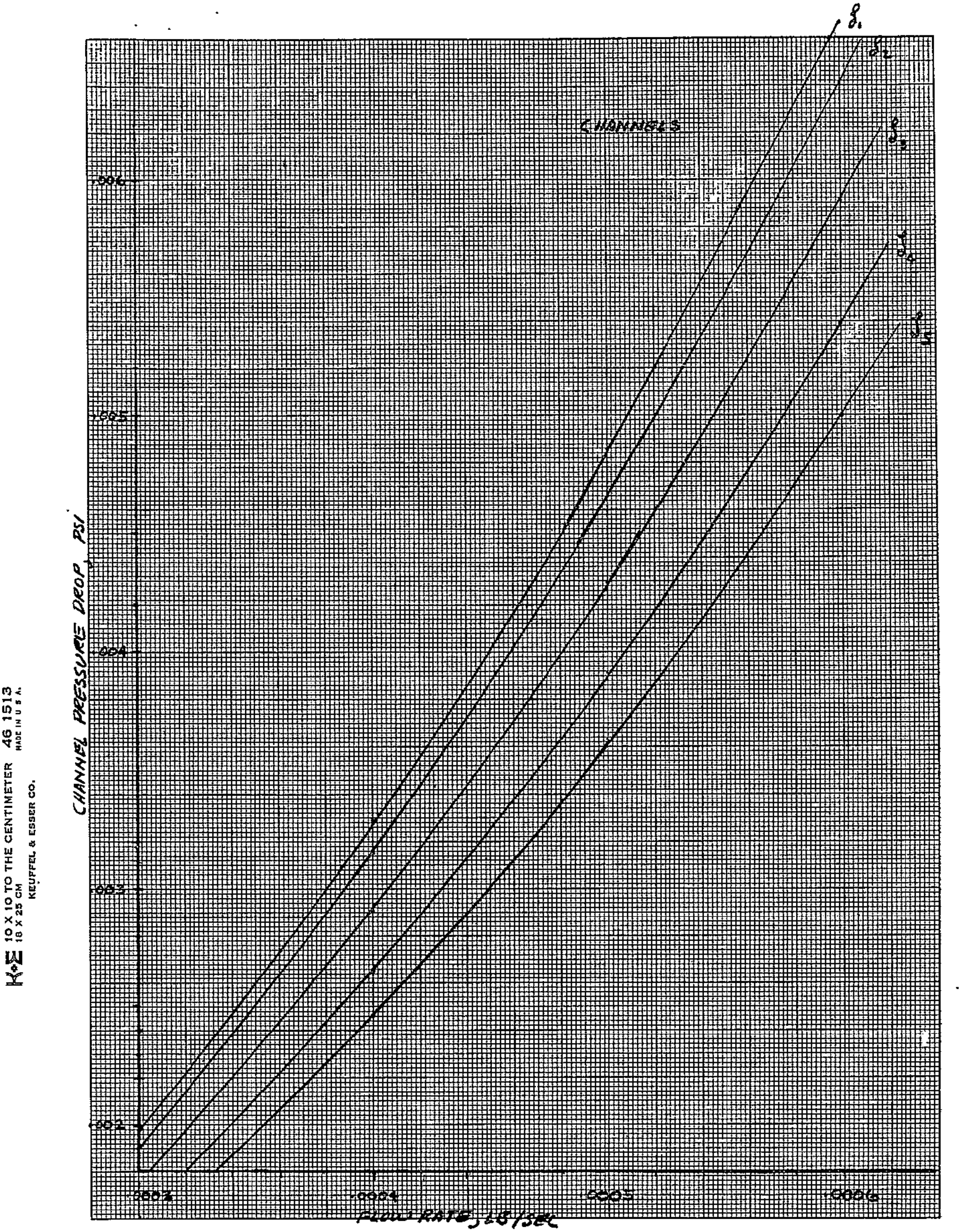


Figure X

10 X 10 TO THE CENTIMETER 18 X 25 CM 46 1513 KEUFFEL & ESSER CO. MADE IN U.S.A.

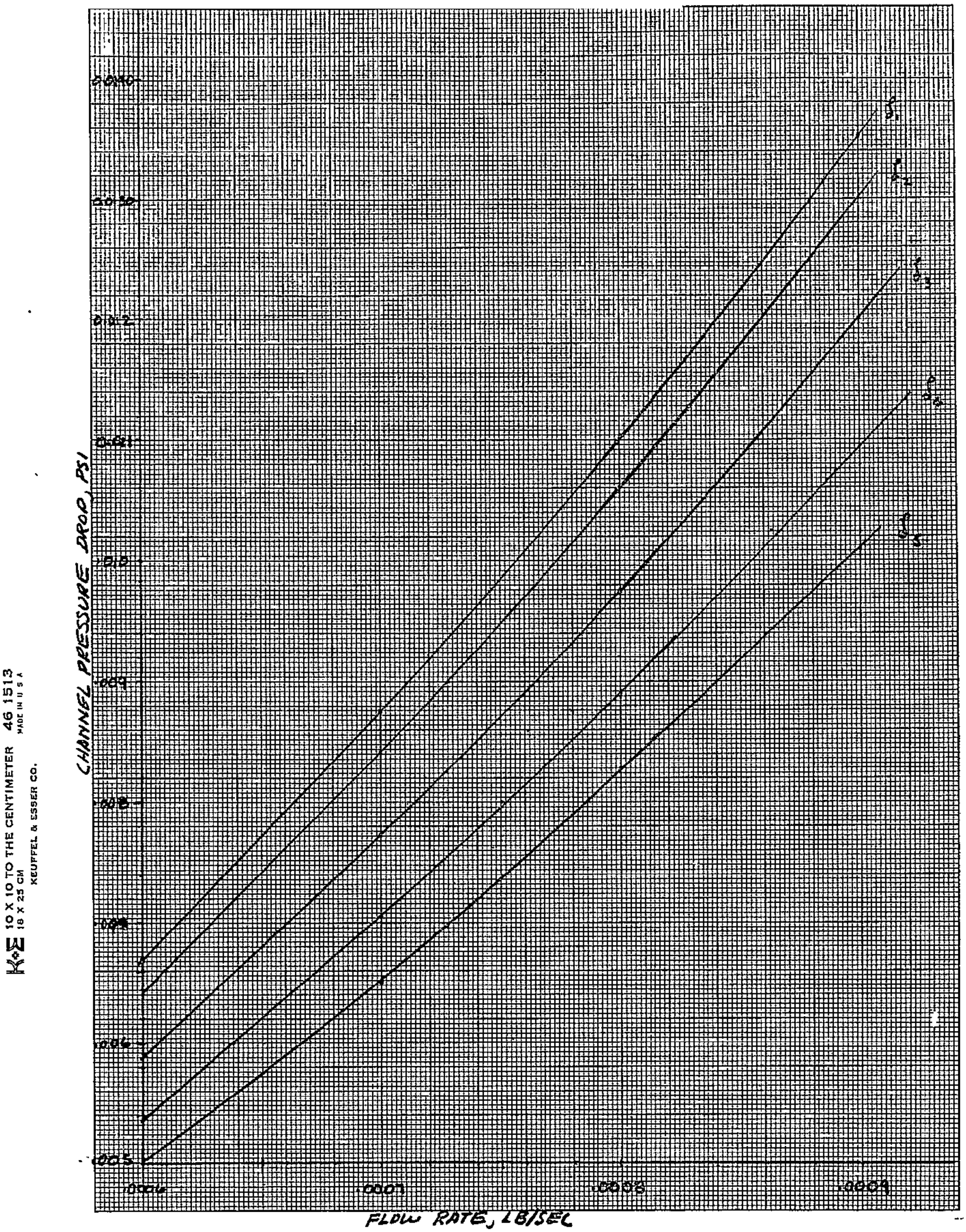

CHANNEL PRESSURE DROP, PSI
FLOW RATE, LB/SEC
.005
.006
.007
.008
.009
.010
.0006
.0007
.0008
.0009
10 X 10 TO THE CENTIMETER 46 1513
18 X 25 CM
KEUFFEL & ESSER CO.
MADE IN U.S.A.

Figure 3

we have the correct pressure drop, channel flows and total flows.

From Figures X and 3

$$\dot{w}_{TOTAL} = 2 \sum_{1}^{5} \dot{w}_{(\ell_i)}$$

ΔP psi	$\dot{w}_{(\ell_1)}$	$\dot{w}_{(\ell_2)}$	$\dot{w}_{(\ell_3)}$	$\dot{w}_{(\ell_4)}$	$\dot{w}_{(\ell_5)}$	$\dot{w}_{TOTAL}$
		← lb/sec	→			
.003	.000380	.000388	.0004065	.000429	.000449	.004105
.004	.000450	.000459	.0004810	.000517	.000530	.004874
.005	.000509	.000521	.0005460	.000577	.000600	.005506
.006	.000563	.000577	.0006050	.000688	.000667	.006100
.007	.000615	.000629	.000660	.000696	.000729	.006658
.008	.000664	.000679	.000714	.000752	.000785	.007188
.009	.000711	.000727	.000754	.000805	.000840	.007674
.010						

From Figure 4

The channel pressure drop is .0066 psi and from Figures X, and 3 the flow through each of the channels is:

$\dot{w}_{\ell_1} = .000596$

$\dot{w}_{\ell_2} = .000610$

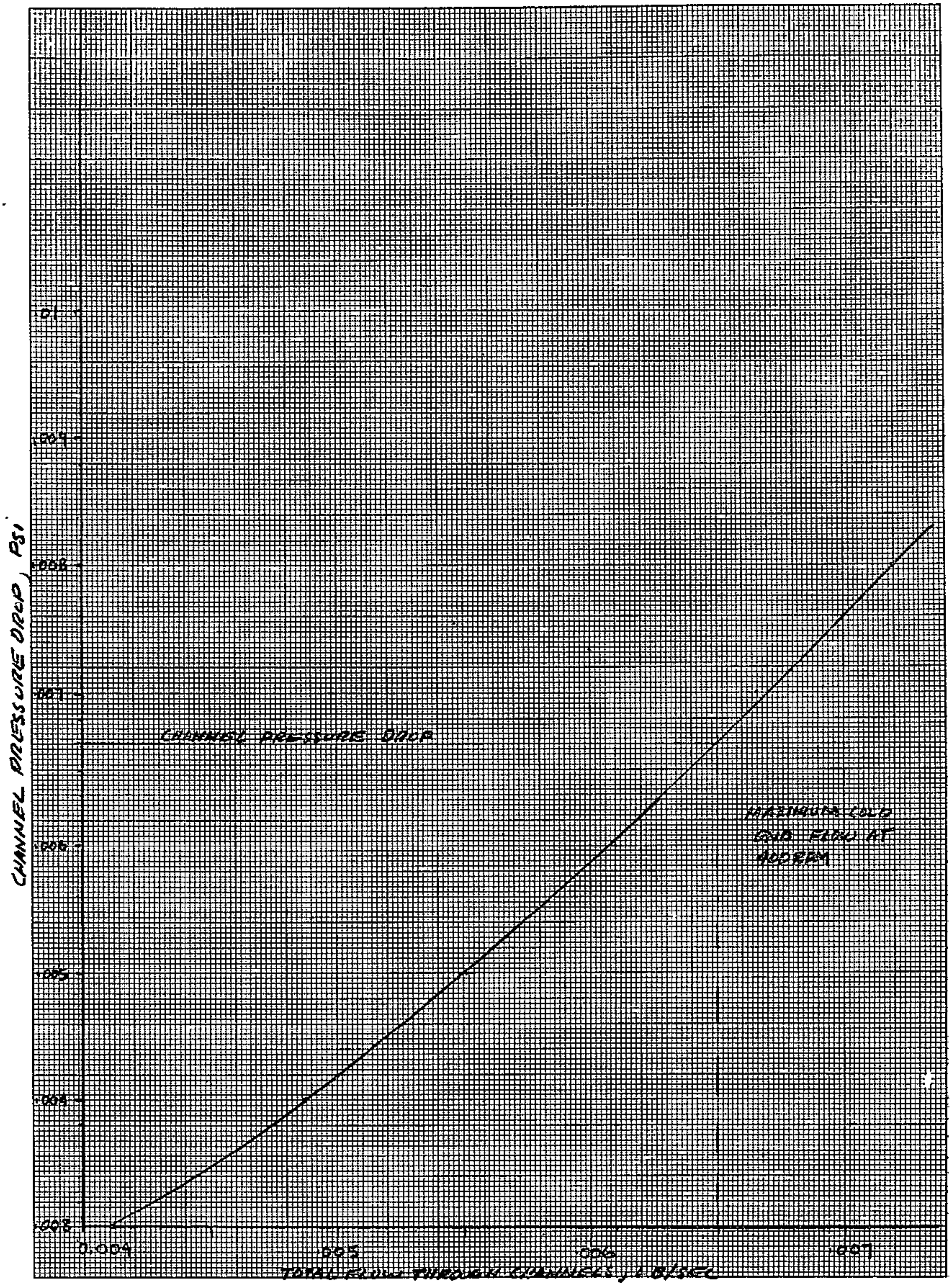
$\dot{w}_{\ell_3} = .000649$

$\dot{w}_{\ell_4} = .000673$

$\dot{w}_{\ell_5} = .000729$

$$\begin{array}{r} 729 \\ -596 \\ \hline 133 \end{array} \qquad \frac{133}{596}$$

This gives the initial data to start looking at the distribution in the slots at each end of the channels -- we can also check that the pressure drop around these slots must be small compared to the channel pressure drops for the above flow distribution in the channels to be valid.



CHANNEL PRESSURE DROP, PSI
.01
.009
.008
.007
.006
.005
.004
.003
0.004
.005
.006
.007
TOTAL FLOW THROUGH CHANNELS, LB/SEC
CHANNEL PRESSURE DROP
MAXIMUM COLD GAS FLOW AT 400RPM

Figure 4

2.0 UPSTREAM FLOW DISTRIBUTION SLOT

We note that the channel distribution is not bad and sufficiently well distribution takes places such that segments of the heat exchanger are fed by their adjcent channels. The worst case pressure drop for the distribution slot occures if we assume the most of the flow from the bottom channel ($\mathcal{L}_6$) must be distributed around the bottom to the midpoint between the two bottom channels. For uniform flow $W_o = .000599$ lb/sec and $L = 1.175$ in

assuming laminar flow:

$$\Delta P_c = \frac{-(16)(\mu)(W_o)(L)}{g_c D_H^2 \rho A_F}$$

$$D_H = \frac{2(0.137)(.207)}{.344} = 0.16488 \text{ IN} = .01374 \text{ ft}$$

$$A_F = (.137)(.207) = .02836 \text{ IN}^2 = .0001969 \text{ ft}^2$$

$$D_H^2 = .0001888 \text{ ft}^2$$

$$\Delta P = \frac{-(16)\left(0.0521 \frac{lbm}{ft\text{-}hr}\right)\left(.000599 \frac{lbm}{sec}\right)(1.175\ IN)\ ft \times \frac{1\ hr}{3600\ sec}}{32.2 \frac{lbm\text{-}ft}{lbf\text{-}sec^2} \cdot (.0001888\ ft^2)\left(.468 \frac{lbm}{ft^3}\right)(.0001969\ ft^2)(12\ IN)}$$

$$= \frac{-(16)(.0521)(.000599)(1.175)}{(32.2)(.0001888)(.468)(.0001969)(12)(3600)} \frac{lbf}{ft^2} = .0242\ lb/ft^2$$

ΔP = .000168 PSI

Note:

The ΔP through the channels is .0066 or 39 times ~~the~~ maximum ΔP in the distribution slit, therefore our previous assumption looks good.

Additionally for section 2 of the ambient heat exchanger the pressure drop is .001295 psi. This gives a core to header pressure ratio of

$$r_{CH} = \frac{.001295}{.000168} = 7.72$$

which should be adequate.

Before we run off lets check the slot Re and make sure its laminar

$$Re = \frac{V D_H \rho}{\mu}$$

$$Q = \frac{\dot{w}}{\rho} = \frac{.000599 \text{ lb/sec}}{.486 \text{ lb/ft}^3} = .0012799 \frac{ft^3}{sec}$$

$$V = \frac{Q}{A_p} = \frac{.0012799}{.0001969 \text{ ft}^2} = 6.5 \text{ ft/sec}$$

$$D_H = .01374 \text{ ft}$$

$$Re = \frac{(6.5 \text{ ft/sec})(.01374 \text{ ft})(.968 \text{ lbm/ft}^3)(3600 \text{ sec/hr})}{0.0521 \frac{lbm}{ft\text{-}hr}} = \underline{2888}$$

darn

We are in the transition range and the friction factor can be as much as 1/3 higher

∴

$$\Delta P = 1.33 \Delta P_{above} = .000168 \times 1.33 = .000224$$

still 29 times lower than channel pressure drop and

$$r_{CH} = \frac{.001295}{.000224} = 5.8$$

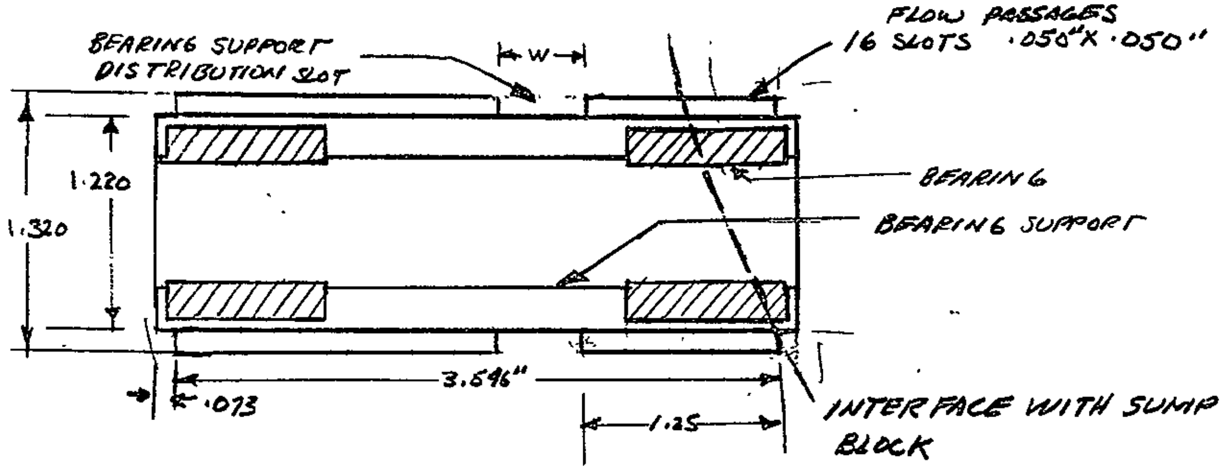
Which is not as good but adding more dead volume to slot to increase this ratio does not look good either.

3.0 SECONDARY FLOW DISTRIBUTION SLOT

Here lets not worry about distribution but provide a sufficient flow passage between the channels and the slots in the bearing support to hold the pressure drop between channels to a very small value. If we take a 45° cut off the edge of the sump block at the interface between the sump block and bearing support (making cuts .15 on an edge) we have a flow area of .01125 IN^2. This is approximately 4 times the flow cross section of each channel, thus due to the short lenght between channels the pressure drop will be very small.

The reason for not worrying about flow distribution here is that we have flow paths of unequal length from this interface along the bearing support as we go toward the cold end. Thus it is better to provide a distribution slot at a point along the bearing support where the down stream (toward cold end) flow passage lengths become equal - see below.

(4.0) BEARING SUPPORT PRESSURE DROP AND FLOW DISTRIBUTION SLOT



$\dot{W}_{max} = .0065$ lb/sec total

4.1 Axial Flow Passage Pressure Drop

First lets calculate the pressure drop without the bearing support distribution slot and see if .05 x .05 slots are adequate or if the pressure drop is too high.

$$D_H = .05\,\text{IN} = 0.0041667\,\text{ft}$$

$$A_F = (.05)(.05) = 0.0025\,\text{IN}^2 = .00001736\,\text{ft}^2$$

$$\dot{W}_i = \frac{\dot{W}_{MAX}}{16} = \frac{.0065\,\text{lb/sec}}{16} = .00040625\,\text{lb/sec}$$

$$Q_i = \frac{\dot{W}_i}{\rho} = \frac{.00040625\,\text{lb/sec}}{.468\,\text{lb/ft}^3} = .0008681$$

$$V_i = \frac{Q_i}{A_F} = \frac{.0008681\,\text{ft}^3/\text{sec}}{.00001736\,\text{ft}^2} = 50\,\text{ft/sec}$$

$$H_v = \frac{V^2}{2g_c}\rho = \frac{(50)^2\frac{\text{ft}^2}{\text{sec}^2} * .468\frac{\text{lbm}}{\text{ft}^3}}{64.4\frac{\text{lbm-ft}}{\text{lbf-sec}^2}} = 18.1677\,\frac{\text{lbf}}{\text{ft}^2} = 0.1262\,\text{PSI}$$

$$Re = \frac{V D_H \rho}{\mu} = \frac{(50\,\text{ft/sec})(.0041667\,\text{ft})(.468\,\text{lbm/ft}^3)(3600\,\text{sec/hr})}{.0521\,\frac{\text{lbm}}{\text{ft-hr}}} = 6737$$

$f = .009$

$$\frac{4fL}{D_H} = \frac{(4)(.009)(3.546)}{.05} = 2.553$$

taking a 1.5 H_v loss due to contraction and expansion at ends

$$\Delta P = (1.5 + 2.553)\,H_v = (4.053)(.1262) = .5115 \text{ psi}$$

This is not too good even when considered by it's self, but when we add the distribution slot we will pickup another 1.5 H_v loss which would then give us something like a .7 psi loss which is a little higher than we want. Would like something around .25 psi total. Keeping the same slot depth of .05 in lets look at increasing the width.

Width IN	D_H IN	D_H ft	A in²	A ft²	V_1 ft/sec	$\frac{V_1^2}{2g_c}\rho$ (psi)	Re	f
.05	.05	.004167	.0025	.00001736	50	.1262	6737	.0090
.06	.05455	.004545	.0030	.00002083	41.68	.08767	6125	.0093
.07	.05833	.004861	.0035	.0000243	35.72	.0644	5615	.0095
.08	.06154	.005128	.0040	.00002778	31.25	.0492	5182	.0098

Then neglecting length of the distribution slot.

$$\Delta P = \left(3 + \frac{4fL}{D}\right) H_v$$

Width IN	$\frac{4fL}{D_H}$	ΔP PSI	Void Vol IN³
.05	2.553	.704	.1419
.06	2.418	.475	.1702
.07	2.310	.342	.1986
.08	2.259	.259	.2270

note: It looks like we need slot width of .06 IN minimum and .07 IN ~~to .08~~ looks better. Tentatively select at width of .07 IN.

4.2 Flow Distribution Slot

Now lets look at the distribution slot around the bearing support. The thing most likely to set up non-uniform flow at this point is the non-uniform flow in the channels around the sump filler block. An examination of the flow distribution between these channels indicates the non-uniform distribution is not bad (see pp 8). To be conservative we will assume a much worse case by saying 1/6 of the total flow has to be distributed around each side of the bearing support.

Thus $\dot{W}_s = \frac{.0065}{6} = 0.00108333$ lb/sec

Then considering that the fluid must travel ½ of the distance around the support -- actually conservative since some of the fluid enters the flow passages along the support as it goes around -- the length traveled is $L = \frac{\pi D}{2}$.
The pressure drop is:

$$\Delta P = \frac{4fL}{D_H} \frac{V_s^2}{2g_c} \rho$$

The depth of the slot is fixed at .05 IN, so lets look at width required to give a pressure drop on the order of $\frac{1}{20}$ of the total pressure drop along the bearing support i.e. a pressure drop of about 0.015 to .025 psi.

$$Q_s = \frac{\dot{w}_s}{\rho} = \frac{.00108333 \text{ lb/sec}}{0.468 \text{ lb/ft}^3} = .002315 \text{ ft}^3\text{/sec}$$

$$L = \frac{\pi D}{2} = \frac{\pi(1.27)}{2} = 1.994 \text{ IN} = 0.1662 \text{ ft}$$

$$A_F = W * .05$$

$$V_s = \frac{Q_s}{A_F} \qquad R_e = \frac{V D_H \rho}{\mu}$$

$$D_H = \frac{2(W)(.05)}{.05 + W}$$

(18)

CALCULATION SUMMARY

W IN	AF IN^2	AF ft^2	V_s ft/sec	$\frac{V_s^2}{2g_c} * \rho$ lbf/ft^2	D_H IN	D_H ft	Re	f	$\frac{4fL}{D_H}$	ΔP lbf/ft^2	ΔP PSI
.1	.005	.00003472	66.76	32.31	.04667	.005556	16,993	.0079	.948	30.62	.2126
.2	.010	.00006944	33.34	8.08	.080	.006667	7,188	.0088	.880	7.109	.0493
.3	.015	.0001042	22.22	3.59	.0857	.007143	5,133	.0099	.924	3.317	.02303
.4	.020	.0001389	16.667	2.01	.0889	.007407	3,992	.0109	.972	1.953	.01356

From this it looks like a slot width of 0.3 IN is about right -- note we will locate this slot as close to the end of the support toward to sump interface as possible (about 1.25 in). This provides the best distribution down stream toward the cold end.

5.0 PRESSURE DROP THROUGH HOLES AT INTERFACE BETWEEN SUMP FILLER BLOCK AND AMBIENT END OF HOT REGENERATOR

48 Holes dia = .056
.065

Too be conservative lets take a 2 velocity head loss across these holes; 1.5 Hv for normal contraction and expansion and .5 for turning the fluid upon intering or exiting the sump heat exchanger. Then

$$\Delta P = 2 \frac{V_o^2}{2g_c}\rho = \frac{V_o^2}{g_c}\rho$$

$$\dot{w}_{max_{TOTAL}} = .01252 \text{ lb/sec} \quad @\ 400 \text{ rpm}$$

$$\rho = 0.468 \text{ lb/ft}^3$$

$$\dot{w}_i = \frac{\dot{w}_{max}}{48} = \frac{.01252 \text{ lb/sec}}{48} = .000261 \text{ lb/sec}$$

$$\dot{Q}_i = \frac{\dot{w}_i}{\rho} = \frac{.000261 \text{ lb/sec}}{0.468 \text{ lb/ft}^3} = .0005573 \text{ ft}^3/\text{sec}$$

$$V_o = \frac{Q_i}{A_F} = \frac{.0005573 \text{ ft}^3/\text{sec}}{.0000171 \text{ ft}^2} = 32.59 \text{ ft/sec}$$

$$A_F = \frac{\pi}{4} D_o^2 = \frac{\pi}{4}(.056)^2 = .002462 \text{ IN}^2 = .0000171 \text{ ft}^2$$

$$\Delta P = \frac{V_o^2}{g_c}\rho = \frac{(32.59)^2 \, ft^2/sec^2}{32.2 \, \frac{lbm-ft}{lbf-sec^2}} * .468 \frac{lbm}{ft^3} = 15.437 \frac{lbf}{ft^2}$$

$$\Delta P = 15.437 \frac{lbf}{ft^2} = 0.1072 \text{ PSI}$$

Actual hole diameter is .059 in

9.162
9.158

$$A_F = \frac{\pi}{4}(.059)^2 = .00273 \text{ in}^2 = .00001898 \text{ ft}^2$$

$$V_o = \frac{.0005573}{.00001898} = 29.36 \text{ ft/sec}$$

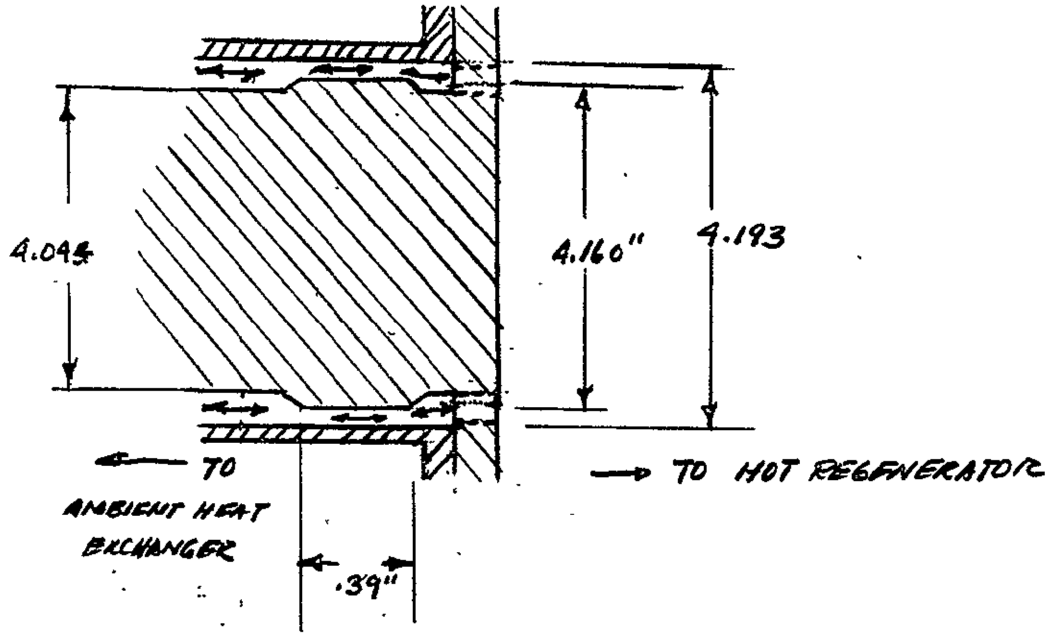
$$\Delta P = \frac{(29.36)^2}{32.2} * 0.468 = 12.53 \text{ lb/ft}^2$$

$\Delta P = 12.53 \text{ lb/ft}^2 = \underline{.087 \text{ psi}}$ This is acceptable

Note we have a similar set of holes in the regenerator retainer therefor the total pressure drop at the interface is .174 psi.

a

6.0 PRESSURE DROP IN ANNULAR ZONE OF AMBIENT HEAT EXCHANGER TOWARD HOT REGENERATOR



17.58249

$\dot{W}_{MAX} = .01252$ lb/sec @ 400 RPM

$A_F = \frac{\pi}{4}(D_o^2 - D_i^2) = \frac{\pi}{4}((4.193)^2 - (4.160)^2) = 0.21638\ IN^2$

$A_F = 0.21638\ IN = .00150\ ft^2$

$Q = \frac{\dot{W}}{\rho} = \frac{.01252\ lb/sec}{.468\ lb/ft^3} = .026752\ \frac{ft^3}{sec}$

$V_A = \frac{Q}{A_F} = \frac{.026752\ ft^3/sec}{.00150\ ft^2} = 17.83\ ft/sec$

$\frac{V_A^2}{2g_c}\rho = \frac{(17.83)^2 ft^2/sec^2}{64.4\ \frac{lbm-ft}{lbf-sec^2}} \times 0.468\ \frac{lbm}{ft^3} = 2.31\ \frac{lbf}{ft^2\ H_v} = .016\ \frac{PSI}{H_v}$

$R_e = \frac{V D_H \rho}{\mu} = \frac{17.83\ ft/sec \times (.00275 ft)(.468\ \frac{lbm}{ft^3})\ 3600\ \frac{sec}{hr}}{.0521\ \frac{lbm}{ft-hr}} = 1586$

$D_H = D_o - D_i = .033 in = .00275\ ft$

(22)

Since laminar

(2)

$$f = \frac{24}{Re} = \frac{24}{1586} = .01513$$

$$\frac{4fL}{D_H} = \frac{(4)(.01513)(.39)}{.033} = 0.715$$

17.581247

The velocity up and down stream is:

$$A_{F_i} = \frac{\pi}{4}\left((4.193)^2 - (4.044)^2\right) = .9634\ IN^2 = .00669\ ft^2$$

$$V_i = \frac{Q}{A_{F_i}} = \frac{.026752\ ft^3/sec}{.00669\ ft^2} = 3.999\ ft^2/sec$$

$$\frac{A_F}{A_{F_i}} = \frac{.21638\ IN^2}{.9634\ IN^2} = .225$$

$$K_c \cong .45$$

$$K_e \cong .60$$

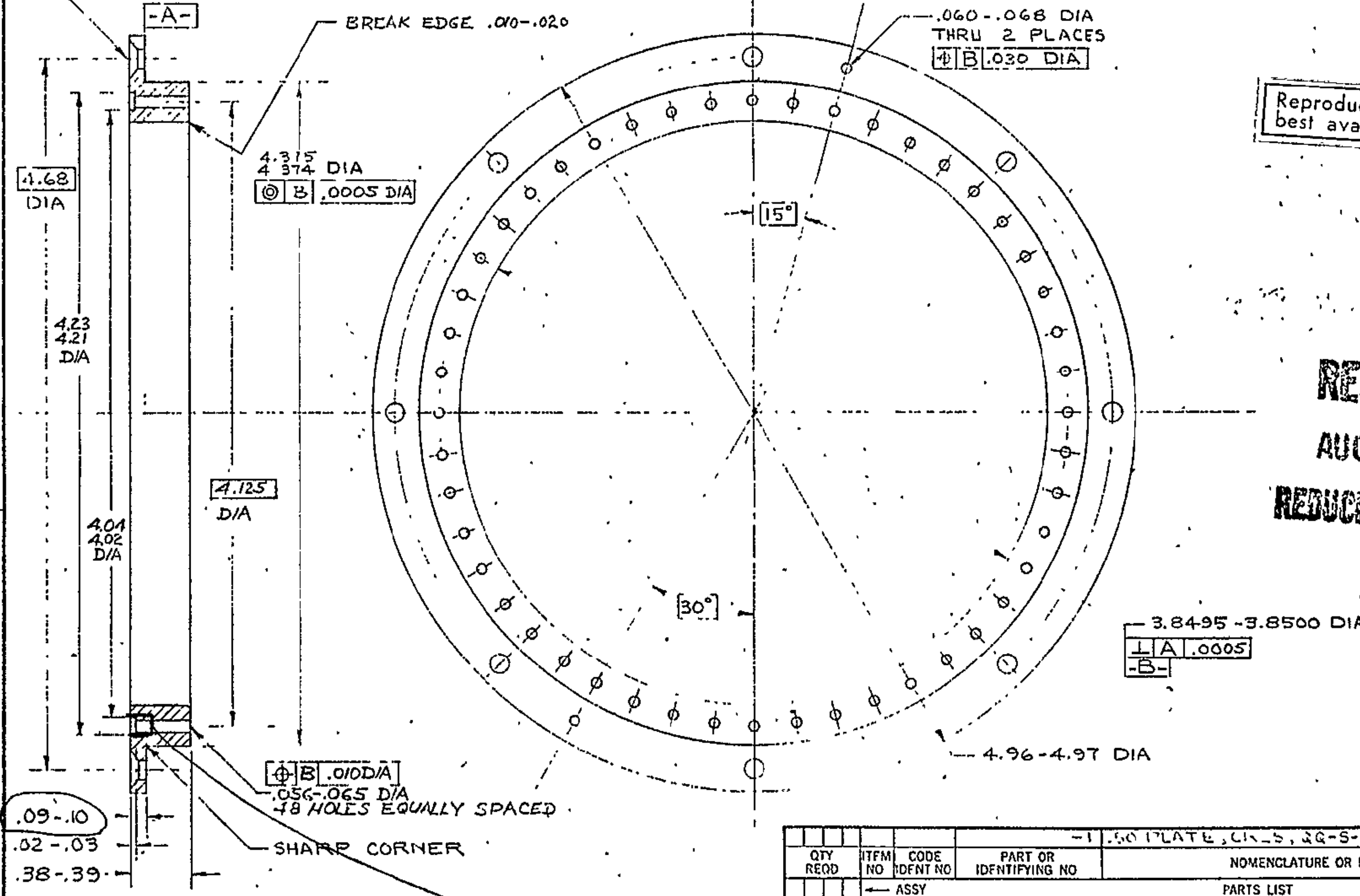
$$\Delta P = \left(K_c + K_e + \frac{4fL}{D}\right)\frac{V_i^2}{2g_c}\rho$$

$$\Delta P = (0.45 + .60 + .715)(.016\ PSI/H_v) = \underline{0.0283\ PSI}$$

low enough

AIRESEARCH MANUFACTURING COMPANY
Los Angeles, California

REVISIONS			
LTR	DESCRIPTION	DATE	APPROVED



made 2 of 3 Zimne drop area

1. ALL SURFACES 125√

NOTES UNLESS OTHERWISE SPECIFIED

QTY REQD	ITEM NO	CODE IDENT NO	PART OR IDENTIFYING NO	NOMENCLATURE OR DESCRIPTION	SYM
			-1	.50 PLATE, CRES, QQ-S-763, CL 304, COND A	
← ASSY				PARTS LIST	

REQD	NEXT ASSY	USED ON
1(-1)	851239-1	851238-1
APPLICATION		

UNLESS OTHERWISE SPECIFIED
BURR CONTROL PER SC653 CL C
IDENTIFICATION MARKING PER MC15 CL VI
STD INTERPRETATIONS PER PI85

HEAT TREATMENT	PROCESS
HARDNESS AND SPEC	NAME AND SPEC

CONTRACT NO.
DFT 1/29/71
CHK
VALUE ENGR 8-2-71
MATL
STRESS G. K. 8-3-71
APPD
APPD 8/5/71
AIRESEARCH APPD
OTHER ACTIVITY APPD

AIRESEARCH MANUFACTURING COMPANY
A DIVISION OF THE GARRETT CORPORATION
LOS ANGELES, CALIFORNIA

RETAINER, REGENERATOR

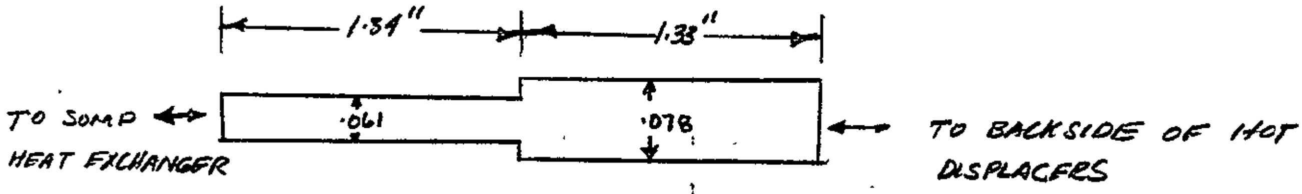
SIZE	CODE IDENT NO.	DWG NO.
C	70210	X852366

SCALE 2/1 SHEET OF

PRESSURE DROP IN SUMP FILLER BLOCK PORTS TO BACKSIDE OF HOT DISPLACER

(1) INITIAL DISIGN

Flow to the backside of the hot displacer must pass through six ports in the sump filler block. The initial configuration of the is:



$\dot{w}_{max} \cong .028$ lb/sec $T = 620°R$ $\rho = 0.468$ lbm/ft3

$\mu = .0521$ lbm/ft-hr

Lets first see which direction of flow gives greatest pressure drop -- this will be determined by expansion & contraction losses since losses along port walls are independent of direction.

Note we have two velocities which we will call Vmax in smaller port and Vmin in larger port. Then in terms of velocity head losses we have

<u>Flow toward displacer</u>

H_v (LOSS) Velocity

Inlet

$K_c = .5$ V_{max}

Change in d (EXP) V_{max}

$$K_e = \left(1 - \frac{A_1}{A_2}\right)^2 = \left(1 - \left(\frac{.061}{.1078}\right)^2\right)^2 = 0.152$$

Outlet V_{min}

$$K_e = \left(1 - \frac{A_2}{A_0}\right)^2 \cong (1)^2 = 1$$

$$\Delta P_{①} \propto .5\, V_{max}^2 + 0.152\, V_{max}^2 + V_{min}^2$$

<u>Flow toward Sump</u>

H_v LOSS Velocity

Inlet V_{min}

$K_c = .5$

Change in d (CON) V_{max}

$K_c = .21$

Outlet V_{max}

$K_e = 1$

$$\Delta P_{②} \propto .5\, V_{min}^2 + .21\, V_{max}^2 + V_{max}^2$$

$$V_{max} > V_{min}$$

$$\Delta P_{②} > \Delta P_{①}$$

$\therefore$ Flow toward sump gives maximum pressure drop.

Then: Let subscript 1 refer to smaller diameter ports
" " 2 " " larger " "

and

$$\Delta P_T = \left\{ (1.21)\frac{V_1^2}{2g_c} + (.5)\frac{V_2^2}{2g_c} + \frac{4f_1L_1}{D_1}\frac{V_1^2}{2g_c} + \frac{4f_2L_2}{D_2}\frac{V_2^2}{2g_c} \right\}\rho$$

$$= \left\{ \left(1.21 + \frac{4f_1L_1}{D_1}\right)\frac{V_1^2}{2g_c} + \left((.5) + \frac{4f_2L_2}{D_2}\right)\frac{V_2^2}{2g_c} \right\}\rho$$

$A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} (.061)^2 = .0029201N^2 = .0000202846 \ ft^2$ {per port}

$A_2 = \frac{\pi}{4} D_2^2 = \frac{\pi}{4} (.078)^2 = .00477594 \ IN^2 = .000033166 \ ft^2$ {per port}

$\varphi = \frac{\dot{w}}{\rho} = \frac{.028 \ lb/sec}{.468 \ lb/ft^3} = .059847 \ ft^3/sec$ total for 6 ports

$Q_i = \frac{\varphi}{6} = .0099745 \ ft^3/sec$ {per port}

$V_1 = \frac{Q_i}{A_1} = \frac{.0099745 \ ft^3/sec}{.0000202846 \ ft^2} = 491.75 \ ft/sec$

$V_2 = \frac{Q_i}{A_2} = \frac{.0099745}{.000033161} = 300.74 \ ft/sec$

$Re_1 = \frac{DV\rho}{\mu} = \frac{(.061 \ IN)(491.7 \ ft/sec)(.468 \ lbm/ft^3)(3600 \ sec/hr)}{(.0521 \frac{lb}{ft\text{-}hr})(12 \ IN/ft)} = 89{,}803$

$Re_2 = \frac{(.078)(300.7)(.468)(3600)}{(.0521)(12)} = 63{,}187$

$f_1 = .0047 \qquad f_2 = .005$

$\frac{4 f_1 L_1}{D_1} = \frac{(4)(.0047)(1.34)}{.061} = 0.413 \qquad \frac{4 f_2 L_2}{D_2} = \frac{(4)(.005)(1.33)}{.078} = 0.341$

$1.21 + \frac{4 f_1 L_1}{D_1} = 1.623 \qquad 0.5 + \frac{4 f_2 L_2}{D_2} = 0.841$

$\frac{V_1^2}{2g_c} = 3754 \qquad \frac{V_2^2}{2g_c} = 1404$

$\Delta P = \{(1.623)(3754) + (.841)(1404)\} * .468 = 3403 \ lbf/ft^2$

$\Delta P = \underline{23.63 \ PSI}$ much too high

(2) Parametric Port Diameter vs ΔP

For larger diameter ports we can have uniform port diameter

Thus $L = 2.67\ IN$

and

$$\Delta P = \left\{ \Sigma K + \frac{4fL}{D} \right\} \frac{V^2}{2g_c} \rho$$

$\Sigma K = 1.5$ for contraction and expansion at ends

$\dot{w} = .00467\ lb/sec$ per port

$$Re = \frac{4\dot{w}}{\pi D \mu} = \frac{(4)(.00467\ lb/sec)}{\pi (D)(IN)(.0521\ \frac{lb}{ft\text{-}hr})} * 3600\ sec/hr * 12\ \frac{IN}{ft} = \frac{4932.7}{D}$$

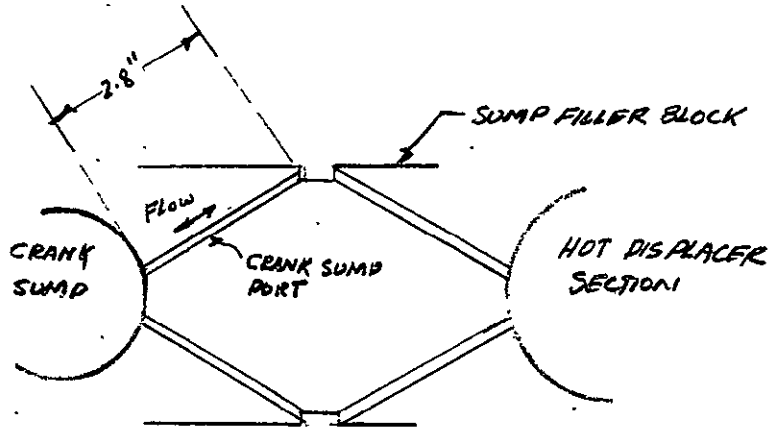
{D in inches}

D (IN)	A_F' (IN^2)	A_F' (ft^2)	V (ft/sec)	Re	f
0.160	.02009	.0001396	71.45	30,829	.0057
0.187	.02745	.0001906	52.33	26,378	.0062
0.125	.012265	.00000852	117.10	39,462	.0056
0.200	.03140	.0002181	45.734	24,663	.0063
0.100	.00785	.0000545	183.02	49,327	.0053
.250	.04906	.0003407	29.276	19,730	.0067
.08	.00502	.000034889	285.89	61,659	.0050
.300	.07065	.0004906	20.33	16,442	.0069
.140	.015386	.00010685	93.35	35,233	.0057

D(IN)	$4fL/D$	$1.5 + 4fL/D$	$V^2/2g_c$	ΔP lb/ft^2	ΔP (PSI)	Void Vol (IN^3)
.187	.3540	1.854	42.525	36.886	.256	.4397
.200	.3364	1.836	32.4776	27.898	.1937	.503
.250	.2862	1.7862	13.301	11.122	.0772	.7859
.300	.2456	1.7456	6.4186	5.24	.0364	1.1318
.160	.3938	1.894	79.274	70.225	.48767	.3218
.140	.4348	1.9348	135.32	122.50	.851	.2465
.125	.4785	1.9785	212.92	197.09	1.3687	.1965
.100	.5660	2.066	520.01	502.64	3.491	.1258
.08	.6675	2.1675	1269.16	1287.1	8.937	.0804

Best chacc looks like a port diameter between 0.160 and .170 in diameter. The main considerations here are the dead volume vs the motor power.

PORTS TO CRANK SUMP PRESSURE DROP AND SIZING



The crank sump ports can be schematically represented as above. The objective here is to determine the number and size of the ports required to yield a reasonable pressure drop. It is noted that the pressure drop allowable depends if we impose this pressure drop across the cold end seal or not. Once we have some numbers on the pressure drop and dead volume of the ports we will be in a possition to make a choice.

1.0. PORT FLOW RATE:

The step in the analysis is to determine the flow rates in the ports. This we can do by looking at the change in mess stored in the crank sump assuming zero pressure drop between the crank sump and the active system volume -- this yields the maximum flow rates. The mass stored in the sump can be expressed as

$$m = \frac{PV_s}{ZRT}$$

Then assuming the temperature will remain constant and neglecting small changes in Z (the compressibility factor) we have

$$\frac{dm}{d\theta} = \frac{P}{ZRT}\frac{dV_s}{d\theta} + \frac{V_s}{ZRT}\frac{dP}{d\theta}$$

From previous analysis we have

$$V_s = 1.8931 - 0.2685 \sin(\theta - 28°21')$$

Where V_s is in IN^3

Then

$$\frac{dV_s}{d\theta} = -0.2685 \cos(\theta - 28°21')$$

and

$$\frac{dm}{d\theta} = -0.2685\frac{P}{ZRT}\cos(\theta - 28°21') + \frac{1.8931 - 0.2685\sin(\theta - 28°21')}{ZRT}\frac{dP}{d\theta}$$

$$\frac{dm}{d\theta} = \frac{1}{ZRT}\left\{-0.2685P * \cos(\theta - 28°21') + (1.8931 - 0.2685\sin(\theta - 28°21'))\frac{dP}{d\theta}\right\}$$

P IN PSIA, T in °R V in IN³ $R = 4{,}636 \frac{lbf\ IN}{°R\ lbm}$

@ 800 to 600 & 620°R $Z \cong 1.0$

$$\frac{dm}{d\theta} = \frac{lbm}{rad}$$

$$\frac{dm}{d\theta} * \frac{d\theta}{d\tau} = \frac{dm}{d\tau} = \dot{w}$$

let R = rpm

$$\frac{d\theta}{d\tau} = 2\pi * R * \frac{1}{60}\frac{min}{sec} = \frac{2\pi R}{60} \quad \frac{rad}{sec}$$

@ 400 rpm

$$\frac{d\theta}{d\tau} = \frac{(2\pi)(400)}{60} = 41.867\frac{rad}{sec}$$

@ 600 rpm

$$\frac{d\theta}{d\tau} = \frac{(2\pi)(600)}{60} = 62.80\frac{rad}{sec}$$

@ 400 rpm

$$\dot{w} = 41.867\frac{rad}{sec}\frac{°R\ lbm}{4{,}636\ lbf\text{-}IN\ 620°R}\left\{-0.2685P\cos(\theta - 28°21')\frac{lbf\text{-}IN}{rad}\right.$$

$$\left. + (1.8931 - 0.2685\sin(\theta - 28°21'))\frac{dP}{d\theta}\frac{lbf\text{-}IN}{rad}\right\}$$

Thus at 400rpm

$$\boxed{\dot{w} = 1.957\times10^{-5}\left\{\left\{1.8931-0.2685\sin(\theta-28^\circ 21')\right\}\frac{dp}{d\theta}-\left\{0.2685P\cos(\theta-28^\circ 21')\right\}\right\}}$$

and @ 600 rpm $\dot{w} = 1.5\,\dot{w}_{400\,rpm}$

$\dot{w}$ in lb/sec and p and $\frac{dp}{d\theta}$ must come from the cycle analysis p, psia, $\frac{dp}{d\theta}$ = psi/rad

Now using the output of the idealized VM program we can estimate the flow as a function of the crank angle. What we are looking for is the maximum flow to use in the pressure drop calculations

NASA GSFC VM REFR 10-22-71, ALL SPH SHOT IN COLD REGEN, MAX DEAD VOLUME   DATE = 22 OCT 71 TIME = 11:21:20

OPERATING PARAMETERS

COLD VOLUME TEMP.	=	125.00 R
SUMP VOLUME TEMP.	=	620.00 R
HOT VOLUME TEMP.	=	1630.00 R
COLD REGEN. TEMP.	=	372.00 R
HOT REGEN. TEMP.	=	1125.00 R
COLD DISPLACED VOL.	=	.25500 CU-IN
HOT DISPLACED VOL.	=	6.80000 CU-IN
COLD DEAD VOL.	=	.08533 CU-IN
SUMP DEAD VOL.	=	7.66550 CU-IN
HOT DEAD VOL.	=	1.27200 CU-IN
COLD REGEN. VOL.	=	2.24000 CU-IN
HOT REGEN. VOL.	=	7.49000 CU-IN
GAS CONSTANT	=	4634.40 IN-LB/LBM-R
SPEED	=	400.00 RPM
CHARGE PRESSURE	=	530.00 PSIA
CHARGE TEMPERATURE	=	535.00 R
MASS OF FLUID	=	.0054 LBM
TOTAL VOLUME	=	25.84783 CU-IN

PRESSURE - MASS - FLOW PROFILE

ANGLE DEG	PC PSIA	PA PSIA	PH PSIA	VC CU-IN	VA CU-IN	VH CU-IN	MDOTC LB/SEC	MDOTA LB/SEC	MDOTH LB/SEC	MDOTRCA LB/SEC	MDOTRHA LB/SEC	DPC PSI	DPH PSI	DPCA PSI	DPHA PSI
24.	773.55	773.55	773.55	.0963	9.9268	6.0547	.00298	-.02544	.01534	.00634	.01910	.0000	.0000	.0000	.0000
48.	795.86	795.86	795.86	.1275	8.7520	7.1983	.00526	-.02156	.01167	.00745	.01411	.0000	.0000	.0000	.0000
72.	806.30	806.30	806.30	.1734	7.9991	7.9053	.00650	-.01261	.00515	.00695	.00565	.0000	.0000	.0000	.0000
96.	802.08	802.08	802.08	.2261	7.7982	8.0535	.00630	-.00070	-.00261	.00489	-.00419	.0000	.0000	.0000	.0000
120.	784.37	784.37	784.37	.2765	8.1841	7.6172	.00473	.01090	-.00951	.00184	-.01274	.0000	.0000	.0000	.0000
144.	757.66	757.66	757.66	.3159	9.0901	6.6718	.00234	.01938	-.01393	-.00134	-.01804	.0000	.0000	.0000	.0000
168.	727.90	727.90	727.90	.3375	10.3596	5.3807	-.00023	.02344	-.01536	-.00394	-.01950	.0000	.0000	.0000	.0000
192.	700.54	700.54	700.54	.3376	11.7731	3.9672	-.00248	.02323	-.01412	-.00561	-.01762	.0000	.0000	.0000	.0000
216.	679.56	679.56	679.56	.3160	13.0863	2.6755	-.00415	.01964	-.01094	-.00630	-.01334	.0000	.0000	.0000	.0000
240.	667.38	667.38	667.38	.2767	14.0723	1.7289	-.00515	.01365	-.00650	-.00609	-.00756	.0000	.0000	.0000	.0000
264.	665.15	665.15	665.15	.2263	14.5506	1.2909	-.00544	.00610	-.00137	-.00510	-.00099	.0000	.0000	.0000	.0000
288.	673.07	673.07	673.07	.1735	14.4669	1.4374	-.00503	-.00234	.00400	-.00344	.00578	.0000	.0000	.0000	.0000
312.	690.43	690.43	690.43	.1276	13.8072	2.1430	-.00390	-.01093	.00912	-.00121	.01213	.0000	.0000	.0000	.0000
336.	715.39	715.39	715.39	.0964	12.6958	3.2157	-.00207	-.01864	.01332	.00142	.01722	.0000	.0000	.0000	.0000
360.	744.70	744.70	744.70	.0853	11.3245	4.6680	.00034	-.02405	.01571	.00412	.01993	.0000	.0000	.0000	.0000

IDEAL REFRIGERATION AND HEAT INPUT

REFRIGERATION = 20.8046 WATTS
THERMAL HEAT = 125.7796 WATTS
MAX. PRESSURE = 806.6493 PSIA

From the attached printout and etc

θ °	$\bar{\theta}$ °	P PSIA	$\bar{P}$ PSIA	ΔP PSI	$\Delta\theta$ °	$\Delta P/\Delta\theta$ PSI/rad	$\sin(\bar{\theta}-28°21')$	$\cos(\bar{\theta}-28°21')$
24		773.55						
	36		784.71	22.31	24	53.10	.13312	.99110
48		795.86						
	60		801.08	10.44	24	24.95	.52473	.85127
72		806.30						
	84		804.19	-4.22	24	-10.01	.82561	.56425
96		802.08						
	108		793.23	-17.71	24	-42.3	.98373	.17966
120		784.37						
	132		771.02	-26.71	24	-63.80	.97176	-.23599
144		757.66						
	156		742.78	-29.76	24	-71.00	.79176	-.61084
168		727.90						
	180		714.22	-27.36	24	-65.40	.47486	-.88006
192		700.54						
	204		690.05	-20.99	24	-50.0	.07585	-.99712
216		679.56						
	228		673.47	-12.18	24	-29.05	-.33627	-.94176
240		667.38						
	252		666.27	-2.23	24	-5.26	-.69025	-.72357
264		665.15						
	276		669.11	7.42	24	17.90	-.92488	-.38026
288		673.07						
	300		681.75	17.36	24	41.40	-.99959	-.02879
312		690.43						
	324		702.91	24.96	24	59.60	-.9446	-.43287
336		715.39						
	348		730.05	29.31	24	70.00	-.69746	.76210
360		744.70						

.00001457

$\bar{\theta}$ °	$\{1.8931 - .2685 \sin(\theta - 28°21')\}\frac{dp}{d\theta}$ lbf-IN	$-\{.2865 P \cos(\theta - 28°21')\}$	$\dot{w}$ lb/sec
36	98.62567	208.8195	-.001605
60	43.7176	183.0996	-.00203
84	-16.7310	121.8357	-.00202
108	-68.9054	38.2694	-.00156
132	-104.1332	-48.8544	-.000805
156	-119.3169	-121.8237	.00003653
180	-115.4702	-168.7674	.0007765
204	-93.6367	-184.7448	.0013274
228	-57.6174	-170.2953	.0016417
252	-10.9326	-129.4420	.0017267
276	40.4730	-68.3160	.001585
300	89.4857	-5.2700	.001380
324	127.2545	81.6961	.0006638
348	144.686	149.385	-.00006846

2.0 PRESSURE DROP VS PORT SIZE AND NUMBER

The configuration of the sump filler block will allow up to 4 ports bored into the crank sump from the ambient heat exchanger. From the standpoint of flow distribution in the heat exchanger the more ports the better. Due to the difficulty of boring small diameter long ports the use of one or two ports may be the most practical approach. Lets get some no's and see what looks good

Using a 1.5 velocity head loss for the contraction and expansion at the end of the ports

$$\Delta P = \left\{ 1.5 + \frac{4fL}{D} \right\} \frac{V^2}{2g_c} \rho$$

The total flow is $\dot{w}_{max} = 0.0022$ lb/sec

lets look at 4, 2, and 1 port configurations

$N = 4 \qquad \dot{\omega}_{max} = \frac{.0022}{4} = .00055$ lb/sec per port

$N = 2 \qquad \dot{\omega}_{max} = \frac{.0022}{2} = .0011$ lb/sec per port

$N = 1 \qquad \dot{\omega}_{max} = \qquad = .0022$ lb/sec

.0047

$N = 4$

$$Q_4' = \frac{\dot{w}}{\rho} = \frac{.00055 \text{ lb/sec}}{.467786 \text{ lb/ft}^3} = .00117557 \text{ ft}^3/\text{sec}$$

$N = 2$

$$Q_2' = \frac{\dot{w}}{\rho} = \frac{.0011}{.46786} = .00235113 \text{ ft}^3/\text{sec}$$

$N = 1$

$$Q_1' = \frac{\dot{w}}{\rho} = \frac{.0022}{.46786} = .00470226 \text{ ft}^3/\text{sec}$$

We don't want any ports below 0.08 in in diameter and in fact we would like something 0.1 in diameter or larger

$$V = \frac{Q_i}{A_F} \qquad A_F = \frac{\pi}{4} D^2$$

D	A_F' (IN²)	A_F (ft²)	V_4 ft/sec	V_2 ft/sec	V_1 ft/sec	$\frac{V_4^2}{2gc}$	$\frac{V_2^2}{2gc}$	$V_1^2/2gc$
.08	.00502	.0000348889	33.695	67.389	134.779	17.630	70.517	282.071
.10	.00785	.0000545	21.570	43.140	86.280	7.225	28.898	115.594
.125	.012265	.00008518	13.801	27.602	55.204	2.958	11.830	47.321
.140	.015386	.000106847	11.002	22.005	44.009	1.880	7.518	30.073
.160	.02009	.00013960	8.421	16.892	33.684	1.101	4.405	17.618
.187	.02795	.0001906	6.168	12.335	24.671	.591	2.362	9.452
.200	.03140	.0002181	5.390	10.780	21.560	.451	1.804	7.218

$$Re = \frac{4\dot{w}}{\pi D \mu} = \frac{4\, \dot{w}\,(\text{lb/sec})(\text{ft-hr}) \times 3600(\text{sec/hr}) \times 12 \text{ in/ft}}{\pi D(\text{in})(.0521)(\text{lb}) \cdot (\text{ft})} = \frac{\dot{w}}{D} \times 1.05627 \times 10^6$$

$$Re_4 = \frac{580.95}{D_{in}}$$

$$Re_2 = \frac{1161.9}{D_{in}}$$

$$Re_1 = \frac{2323.8}{D_{in}}$$

D	Re_4	Re_2	Re_1	f_4	f_2	f_1
.08	7,262	14,524	29,048	.0088	.0072	.0060
.10	5,810	11,619	23,238	.0094	.0076	.0062
.125	4,648	9,295	18,590	.0102	.0082	.0067
.140	4,150	8,299	16,599	.0108	.0084	.0070
.160	3,631	7,261	14,524	.0105	.0088	.0072
.187	3,106	6,213	12,427	.0086	.0092	.0075
.200	2,905	5,810	11,619	.0072	.0094	.0077

.46786

.6

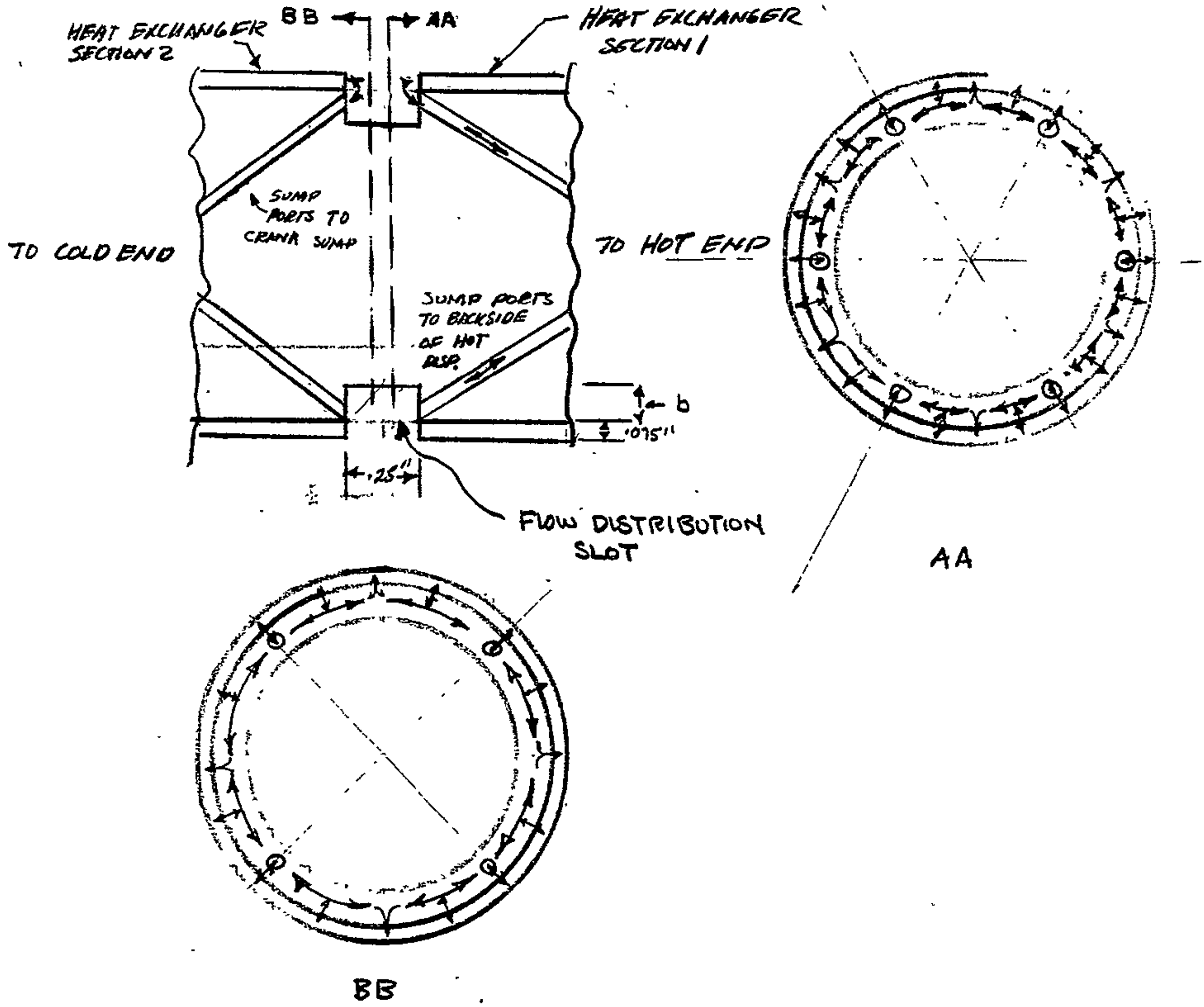
D	$\left(\frac{4fL}{D}\right)_4$	$\left(\frac{4fL}{D}\right)_2$	$\left(\frac{4fL}{D}\right)_1$	$\left(1.5+\frac{4fL}{D}\right)_4$	$\left(1.5+\frac{4fL}{D}\right)_2$	$\left(1.5+\frac{4fL}{D}\right)_1$
.08	1.232	1.008	.840	2.732	2.508	2.34
.10	1.053	.851	.694	2.553	2.351	2.194
.125	.914	.735	.6003	2.414	2.235	2.100
.140	.864	.672	.560	2.364	2.172	2.060
.160	.735	.616	.504	2.235	2.116	2.004
.187	.515	.551	.449	2.0151	2.0510	1.949
.200	.403	.526	.431	1.9032	2.0264	1.931

D	$(\Delta P\ lbf/ft^2)_4$	$(\Delta P(PSI))_4$	$(\Delta P\ lbf/ft^2)_2$	$(\Delta P(PSI))_2$	$(\Delta P\ lbf/ft^2)_1$	$(\Delta P(PSI))_1$
.08	22.53	.1565	32.992	.2291	131.9697	.916
.10	3.38	.0235	13.520	.0939	54.082	.3756
.125	1.383	.0096	5.535	.0384	22.1396	.1537
.140	.8796	.0061	3.5114	.0244	14.0960	.0977
.160	.5151	.0036	2.0609	.0143	8.2427	.05724
.187	.2765	.0019	1.1051	.00767	4.4222	.0307
.200	.2110	.0015	.8440	.00586	3.377	.02345

D	Void Volumes IN^3	V_4	V_2	V_1
.08	.05622	.05622	.02811	.01406
.10	.08792	.08792	.04396	.02198
.125	.13736	.13736	.06868	.03434
.140	.17232	.17232	.08616	.04308
.160	.2250	.2250	.1125	.05625
.187	.3130	.3130	.1565	.07825
.200	.3516	.3516	.1758	.08790

1

FLOW DISTRIBUTION AT INTERFACE OF SUMP FILTER BLOCK PORTS/ AND AMBIENT HEAT EXCHANGER



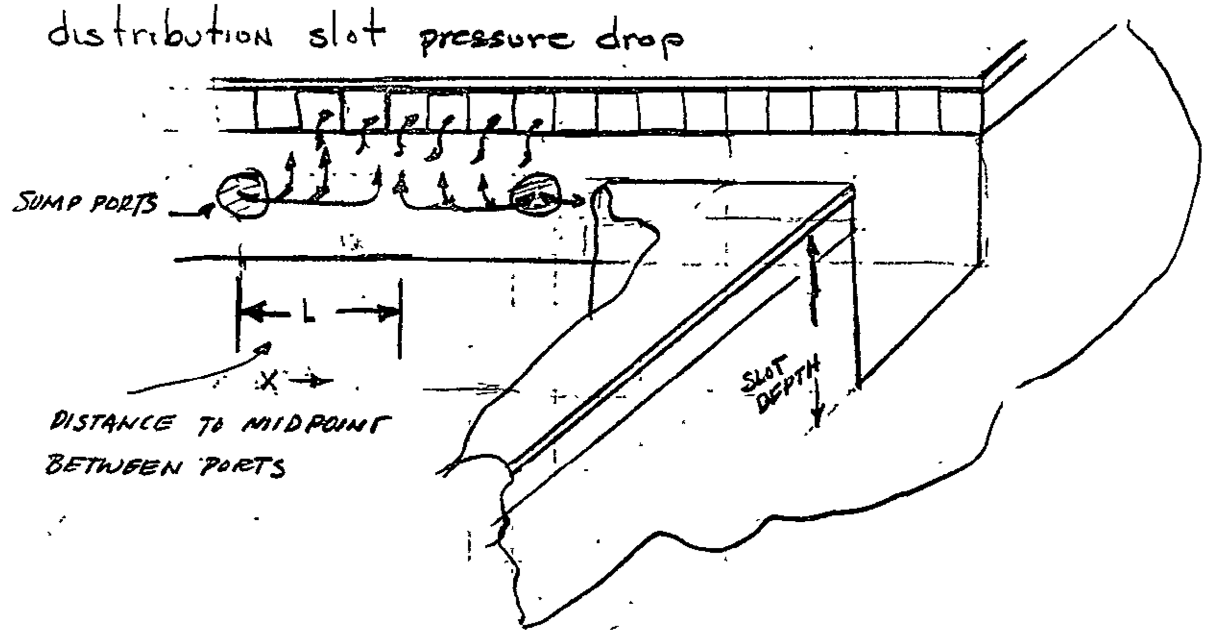
Here we have two situations we must consider:

(1) If the flow through the sump ports is uniform for each port (each port that travels in the same direction) the flow distribution slot must distribute the flow the ambient heat exchanger in a relatively uniform manner

(2) The flow distribution slot must distribute the flow around the ambient heat exchanger for a reasonable amount of non-uniform flow from the sump ports

1.0 PRESSURE AND FLOW DISTRIBUTION IN SLOTS

First lets set up a model for the flow distribution slot pressure drop



Taking flow in the direction from a reference port we can express the pressure drop as

$$\frac{dP}{dx} = -\left(\frac{4f}{D_H}\right)\frac{V^2\rho}{2g_c} \qquad (1)$$

Now as flow goes in the x direction it will decrease due to flow into the ambient heat exchange

Assuming uniform flow in the exchanger we can express $\dot{w}$ as a function x as:

$$w(x) = \frac{\dot{w}_o}{2}\left(1 - \frac{x}{L}\right) \qquad (2)$$

where
$\dot{w}_o$ = flow from the port or source of fluid
L = distance to midpoint between ports

Then noting

$$V = \frac{Q}{A_F} = \frac{\dot{w}}{\rho A_F} \qquad (3)$$

then

$$V(x) = \frac{\dot{w}_0 \left(1 - \frac{x}{L}\right)}{2 \rho A_F} \qquad (4)$$

where

A_F = cross sectional flow area of flow distribution slot

Substitution of (4) into (1) yields

$$\frac{dP}{dx} = -\left(\frac{4f}{D_H}\right) \frac{\dot{w}_0^2 \left(1 - \frac{x}{L}\right)^2}{8 \rho^2 A_F^2 g_c} \rho = -\left(\frac{f}{2 D_H}\right) \frac{\dot{w}_0^2 \left(1 - \frac{x}{L}\right)^2}{g_c \rho A_F^2} \qquad (5)$$

Now since $\dot{w}$ will go from $\frac{\dot{w}_0}{2}$ at $x=0$ to zero at $x=L$ we will need two expressions: one for turbulent and one for laminar flow.

For turbulent flow $Re > 3600$

$$f = 0.084 \left(Re\right)^{-1/4} = 0.084 \left(\frac{\mu}{D_H V \rho}\right)^{1/4} \qquad (6)$$

and for laminar

$$f = \frac{K}{Re} = K\left(\frac{\mu}{D_H V \rho}\right) \quad \text{where } 13.5 \leq K \leq 24.0 \qquad (7)$$

depending on geometry of slot

1.1 Turbulant Relation

Substitution of (4) into (1) yields

$$f = 0.084\left(\frac{\mu}{D_H\,\rho}\right)^{1/4}\left(\frac{2\rho A_F}{\dot{w}_o(1-\frac{x}{L})}\right)^{1/4} \qquad (8)$$

$$= 0.084\left(\frac{2\mu A_F}{\dot{w}_o D_H(1-\frac{x}{L})}\right)^{1/4}$$

Which upon substitution into (5) gives

$$\frac{dP}{dx} = -\frac{.042}{g_c}\left(\frac{2\mu A_F}{\dot{w}_o D_H (1-\frac{x}{L})}\right)^{1/4}\frac{\dot{w}_o^2(1-\frac{x}{L})^2}{D_H\;\rho A_F^2} \qquad (9)$$

$$= -\frac{.042}{g_c\rho}(2\mu)^{1/4}\left(\frac{\dot{w}_o(1-\frac{x}{L})}{A_F}\right)^{1.75}\frac{1}{(D_H)^{1.25}} \qquad (10)$$

$$= -\frac{.042}{g_c\rho}(2\mu)^{1/4}\left(\frac{\dot{w}_o}{A_F}\right)^{1.75}\frac{1}{(D_H)^{1.25}}\left(1-\frac{x}{L}\right)^{1.75} \qquad (11)$$

Then assume for the time that $Re > 2100$ at $x=0$

$$\Delta P = -\left[\frac{.042}{g_c\rho}(2\mu)^{1/4}\left(\frac{\dot{w}_o}{A_F}\right)^{1.75}\frac{1}{(D_H)^{1.25}}\right]\int_{x=0}^{x=L_1}\left(1-\frac{x}{L}\right)^{1.75}dx \qquad (12)$$

$$\Delta P = -\left[\frac{.042}{g_c\rho}(2\mu)^{1/4}\left(\frac{\dot{w}_o}{A_F}\right)^{1.75}\frac{1}{(D_H)^{1.25}}\right]\left[\frac{L\left(1-\frac{x}{L}\right)^{2.75}}{2.75}\right]_{x=0}^{x=x_i} \qquad (13)$$

$$\Delta P = \left[\frac{.042}{g_c\rho}(2\mu)^{1/4}\left(\frac{\dot{w}_o}{A_F}\right)^{1.75}\frac{1}{(D_H)^{1.25}}\right]\frac{L}{2.75} * \left\{\left(1-\frac{x_i}{L}\right)^{2.75}-1\right\} \qquad (14)$$

x_i such that $Re > 2100$

$$\Delta P = \left[\frac{.01815}{g_c\,\rho} (\mu)^{1/4} \left(\frac{\dot{w}_o}{A_F}\right)^{1.75} \frac{L}{(D_H)^{1.25}} \right] \cdot \left\{ L\left(1 - \frac{X_1}{L}\right)^{2.75} - 1 \right\} \qquad (15)$$

$$Re = \frac{\dot{w}_o\left(1 - \frac{X_1}{L}\right) D_H}{2\, A_F\, \mu} > 4200 \qquad (16)$$

(1.2) Laminar Relation

$$f = \frac{K}{Re} = K\left(\frac{\mu}{D_H V \rho}\right) = K\left(\frac{\mu}{D_H \rho}\right)\frac{2\rho A_F}{\dot{w}_o\left(1 - \frac{X}{L}\right)} = \frac{2K}{\dot{w}_o}\left(\frac{\mu}{D_H}\right)\frac{A_F}{\left(1 - \frac{X}{L}\right)} \qquad (17)$$

which upon substitution into (5) yields

$$\frac{dP}{dX} = -\frac{\cancel{2}K}{2 D_H \cancel{g_c} \cancel{\dot{w}_o}}\left(\frac{\mu}{D_H}\right)\frac{A_F}{\left(1 - \frac{X}{L}\right)} \; \frac{\dot{w}_o^{\cancel{2}}\left(1 - \frac{X}{L}\right)^2}{\cancel{\rho} A_F^{\cancel{2}}} \qquad (18)$$

$$\frac{dP}{dX} = -\frac{K\mu}{g_c D_H^2 \rho} \cdot \frac{\dot{w}_o}{A_F}\left(1 - \frac{X}{L}\right) \qquad (19)$$

and

$$\Delta P = -\int_{X_1}^{L} \frac{K\mu\,\dot{w}_o}{g_c D_H^2 \rho A_F}\left(1 - \frac{X}{L}\right) dX \qquad (20)$$

$$\Delta P = -\frac{K\mu\,\dot{w}_o}{g_c D_H^2 \rho A_F}\left[\frac{L\left(1 - \frac{X}{L}\right)^2}{2}\right]_{X_1}^{L} \qquad (21)$$

$$\Delta P = \frac{K\mu\,\dot{w}_o}{g_c D_H^2 \rho A_F}\left\{\frac{L}{2}\left(\left(1 - 1\right)^2 - \left(1 - \frac{X_1}{L}\right)^2\right)\right\} \qquad (22)$$

$$\Delta P = -\frac{K \mu \dot{w}_o L}{2 g_c D_H^2 \rho A_F} \left\{ \left(1 - \frac{x_1}{L}\right)^2 \right\} \quad (23)$$

$$R_e = \frac{\dot{w}_o \left(1 - \frac{x_1}{L}\right) D_H}{2 A_F \mu} < 2{,}100$$

1.3 Summary of Equations

In the transition range between R_e of 2,100 and 4200 we don't have a simple expression for f. To be conservative we will use turbulant f down to $Re = 2100$. Thus in summary

$$\Delta P_1 = -\left\{ \frac{.01815}{g_c \rho} (\mu)^{1/4} \left(\frac{\dot{w}_o}{A_F}\right)^{1.75} \frac{L}{(D_H)^{1.25}} \left(1 - \left(1 - \frac{x_1}{L}\right)^{2.75}\right) \right\}$$

$$R_e \geqq 2100 = \frac{\dot{w}_o}{2} \frac{\left(1 - \frac{x_1}{L}\right) D_H}{A_F \mu}$$

$$\Delta P_2 = -\frac{K \mu \dot{w}_o}{2 g_c D_H^2 \rho A_F} \left\{ \left(1 - \frac{x_1}{L}\right)^2 \right\}$$

$$R_e < 2100$$

and

$$\Delta P_T = \Delta P_1 + \Delta P_2$$

$x = 0$
$x = L$

2.0 CALCULATIONS FOR UNIFORM FLOW IN SUMP PORTS

Lets look at slot depths up to 0.375 IN

let $d =$ slot depth $= b + .075$

$b =$ depth of cut in sump filler block

take parametrically $b = .1'', .2'', .3''$
then $d = .175'', .275'', .375''$

d=0.175"

$$D_H = \frac{2*(.175)(.250)}{.425} = 0.20588\,IN = .01715\,ft$$

$$A_F = (0.175)(.25) = .04375\,IN^2 = .00030382\,ft^2$$

$$\mu = .0521\,\frac{lbm}{ft\text{-}hr} * \frac{hr}{3600\,sec} = 1.447\times10^{-5}\,\frac{lbm}{ft\,sec}$$

Lets check value of Re @ x=0

$$Re_{x=0} = \frac{\dot{w}_0}{2}\frac{D_H}{A_F\mu} \qquad \dot{w}_0 = .028\,lb/sec = .00467\,lb/sec$$

$$Re_{x=0} = \frac{.00467\,\frac{lb}{sec}\,(.01715\,ft)}{2\,(.00030382\,ft^2)(.00001447\,\frac{lbm}{ft\text{-}sec})} = 9109$$

Since turbulent @ x=0 we must find X where Re = 2100.

$$2100 = \frac{\dot{w}}{2}\frac{(1-\frac{X}{L})D_H}{A_F\mu} = 9109\left(1-\frac{X}{L}\right)$$

$$L = \frac{\pi D}{12} = \frac{\pi(4.20)}{12} = 1.1\,IN = .0917\,ft$$

$$\left(1-\frac{X_i}{L}\right) = \frac{2100}{9109} = 0.2309$$

$$\frac{x_1}{L} = .7691$$

$$x_i = (1.1)(.7691) = 0.845 \text{ in} = .0705 \text{ ft}$$

Then

$$\Delta P_1 = -\left\{ \frac{.01815}{g_c \rho}(\mu)^{1/4}\left(\frac{\dot{w}_o}{A_F}\right)^{1.75}\frac{L}{(D_H)^{1.25}}\left(1-(1-.7691)^{2.75}\right)\right\}$$

$$= -\left\{ \frac{.01815 \text{ lbf-sec}^2 \text{ ft}^3}{32.2 \text{ lbm-ft} \times 0.468 \text{ lbm}} \times (.1447 \times 10^{-4})^{1/4} \frac{\text{lbm}^{1/4}}{\text{ft}^{1/4}\,\text{sec}^{1/4}} \left(\frac{.00467}{.00030382}\right)^{1.75} \left(\frac{\text{lbm}}{\text{sec}}\right)^{1.75} \left(\frac{1}{\text{ft}^2}\right)^{1.75} \frac{(.0917)\text{ ft}}{(0.01715)^{1.25}\text{ ft}^{1.25}} \times \left(1-(.2309)^{2.75}\right)\right\} = -0.1286 \text{ lbf/ft}^2$$

$$\Delta P_1 = -0.1286 \frac{\text{lbf}}{\text{ft}^2} = 0.00089 \text{ PSI}$$

and

$$\Delta P_2 = -\frac{K \mu \dot{w}_o L}{2 g_c D_H^2 \rho A_F}\left\{(.2309)^2\right\} \qquad K = 16$$

$$\Delta P_2 = -\frac{(16)(1.447\times10^{-5}\frac{\text{lbm}}{\text{ft sec}})(4.67\times10^{-3}\frac{\text{lbm}}{\text{sec}})(.0917 \text{ ft})(.0532)}{(2)(32.2)\frac{\text{lbm-ft}}{\text{sec}^2/\text{lbf}}(.01715 \text{ ft})^2(.468\frac{\text{lbm}}{\text{ft}^3})(.00030382 \text{ ft}^2)} = .00196 \text{ lbf/ft}^2$$

$$\Delta P_2 = .00196 \text{ lbf/ft}^2 = .00001 \text{ PSI}$$

$$\Delta P_T = 0.0009 \text{ PSI}$$

note:

The pressure drop through section 1 of the ambient heat exchanger is 0.011 psi and section 2 is 0.001295 psi at the corresponding maximum flows. For section 1 (the main section) we have a ration core to distribution pressure drop of:

$$r_{CH} = \frac{\Delta p\ Core}{\Delta p\ Header} = \frac{0.011}{.0009} = 12$$

which should be adequate to provide flow distribution in this section. For section 2 were have a ratio of

$$r_{CH} = \frac{.001295}{.0009} = 1.44$$

This is not generally high enough

d = 0.275"

$$D_H = \frac{(2)(.275)(.250)}{.525} = 0.2619" = .02183\ ft$$

$$A_F = (.275)(.250) = .06875\ IN^2 = .0004774\ ft^2$$

$$Re_{x=0} = \frac{(.00467)(.02183)}{2\ (0.00001447)(.0004774)} = 7379$$

Since turbulent @ x=0 we must find x where Re = 2100

$$2100 = 7379\left(1 - \frac{X}{L}\right)$$

$$1 - \frac{X}{L} = 0.285$$

$$\frac{X}{L} = 0.715$$

$$x = (1.1)(.715) = 0.786\ IN = 0.0656\ ft$$

Then using ratios from previous anser

$$\Delta P_1 = -0.1286 * \left(\frac{A_F^*}{A_F}\right)^{1.75}\left(\frac{D_H^*}{D_H}\right)^{1.25} \frac{\left\{1 - (0.285)^{2.75}\right\}}{\left\{1 - (0.2309)^{2.75}\right\}}*$$

$$\Delta P_1 = -0.1286 * \left(\frac{.00030382}{.0004774}\right)^{1.75}\left(\frac{.01715}{.02183}\right)^{1.25}\left\{\frac{.9683}{.9822}\right\}$$

$$\Delta P_1 = -0.1286 \times (.4535)(.73962)(.9858)$$

$$\Delta P_1 = -.04252 = -.000295\ \text{psi}$$

and similar for ΔP_2

$$\Delta P_2 = -.00196\ \frac{D_H^{*2} A_F^*}{D_H^2 A_F}\left\{\frac{(.285)^2}{(.2309)^2}\right\}$$

$$\Delta P_2 = -.00196\left(\frac{.01715}{.02183}\right)^2\left(\frac{.00030382}{.0004774}\right)\left(\frac{.285}{.2309}\right)^2$$

$$\Delta P_2 = -0.00125(.61719)(1.523) = -0.001175\ lbf/ft^2$$

$$\Delta P_2 = .0000082\ \text{PSI}$$

$$\Delta P_T = .000303\ \text{Psi}$$

Then

Section 1

$$r_{CH} = \frac{.011}{.000303} = 36.7$$

Section 2

$$r_{CH} = \frac{.001295}{.000303} = 4.28$$

d = .375

$$D_H = \frac{(2)(.375)(.250)}{.625} = 0.300 \text{ in} = .025 \text{ ft}$$

$$A_F = (.375)(.250) = (.09375) \text{ in}^2 = .000651$$

$$Re_{x=0} = \frac{.00467\ (.025)}{2\ (.00001447)(\underbrace{.000651}_{A_F})} = .6197$$

Since turbulent @ x = 0 we must find x where $Re_x = 2100$

$$2100 = 6197\left(1 - \frac{x}{L}\right)$$

$$1 - \frac{x}{L} = .3395$$

$$\frac{x}{L} = .6605$$

$$x = (1.1)(.6605) = 0.727 \text{ in} = 0.0606 \text{ ft}$$

Then using ratio from previous anser

$$\Delta P_1 = -0.1286 * \left(\frac{A_P^*}{A_F}\right)^{1.75}\left(\frac{D_H^*}{D_H}\right)^{1.25} \frac{\left\{1-(0.3395)^{2.75}\right\}}{\left\{1-(.2309)^{2.75}\right\}^*}$$

$$\Delta P_1 = -0.1286 * \left(\frac{.00030382}{.000651}\right)^{1.75}\left(\frac{.01715}{.025}\right)^{1.25} \frac{.94873}{.9822}$$

$$\Delta P_1 = -0.1286\,(.26352)(.62432)(.96593) = -0.02044$$

$$\Delta P_1 = .000142\ PSI$$

and similer for ΔP_2

$$\Delta P_2 = -0.00196\left(\frac{D_H^*}{D_H}\right)^2 \frac{A_F^*}{A_F}\left\{\left(\frac{.3395}{.2309}\right)^2\right\}$$

$$\Delta P_2 = -0.00196\left(\frac{.01715}{.025}\right)^2\left(\frac{.00030382}{.000651}\right)\left(\frac{.3395}{.2309}\right)^2$$

$$\Delta P_2 = -(.0009197)(0.470596)(2.1619) = .0009305\ lbf/ft^2$$

$$\Delta P_2 = -.0000065\ PSI$$

and

$$\Delta P_T = .000148\ PSI$$

Then

Section 1

$$r_{CH} = \frac{.011}{.000148} = 74.3$$

Section 2

$$r_{CH} = \frac{.001295}{.000148} = 8.75$$

Summary to this point

d	ΔP_T (PSI)	r_{CH}-SECTION 1-HX	r_{CH} - SECTION 2-HX	Void Vol (IN)^3
.175	.000900	12	1.44	.5735
.275	.000303	36.7	4.28	.8797
.375	.000148	79.3	8.75	1.1701

$$\text{Void Volume} = \frac{\pi}{4}\left(D_o^2 - D_i^2\right)L = \frac{\pi}{4}\left(4.35^2 - (4.35 - 2d)^2\right)$$

$$D_i = D_o - 2d$$

Note:

Based on what we see to this point it looks like we need a distribution slot with a depth of around .3 in. This gives us a little more void volume than we like but the ratio of pressure drop in Section ~~2~~ of the ambient heat exchanger to distribution slot is marginally low at about 5:1 (8:1 to 12:1 would be better but we can't afford the increased dead volume. Lets look at some additional criterion for sizing the distribution slot before we get excited.

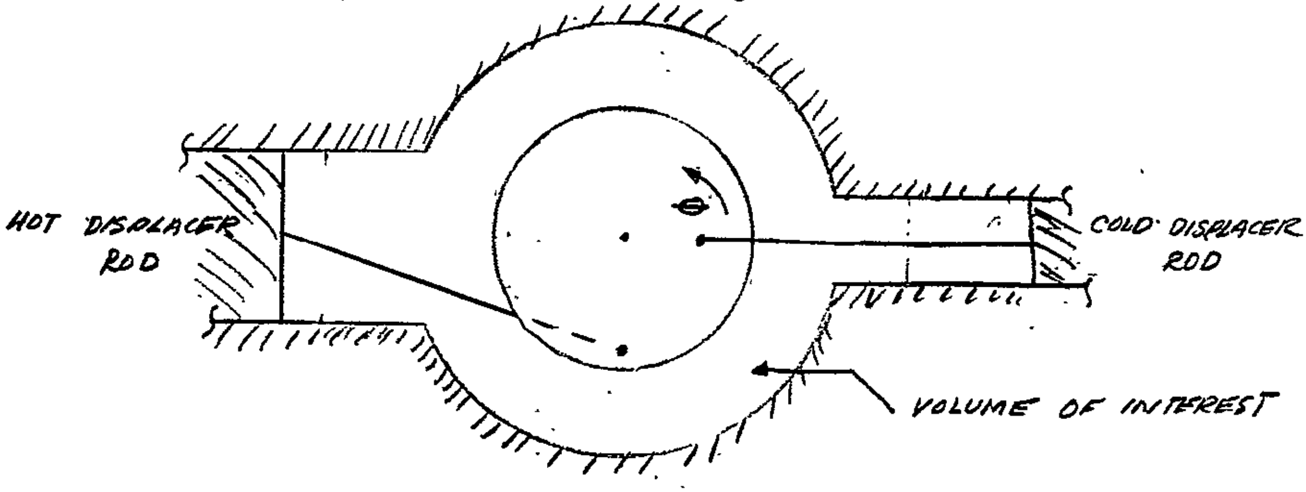
Select $d = 0.30$
$b = 0.225$

VARIATION OF CRANKCASE SUMP VOLUME AND PRESSURE

In the follow pages we are concerned with the pressure variations within the crankcase sump of the machine should we isolate that volume from the remainder of the active volume. In the final design the crankcase sump was plumbed to remainder of the refrigerator's volume via four 0.1 in. diameter ports -- the following analysis is therefore not too important.

VARIATION OF CRANKCASE SUMP VOLUME AND PRESSURE

The volume inboard of the rod bearings can be schematically represented by:



The constant volume with both rods retracted toward the crank is $1.530\ IN^3$

The volume of the sump can then be expressed as:

$$V_S = V_c + \frac{A_H S_H + A_c S_c}{2} + \frac{A_c S_c}{2}\cos\theta - \frac{A_H S_H}{2}\sin\theta$$

Where

V_c = the constant volume with both rods retracted ~~toward crank~~

A_H and A_c = cross sectional area of hot and cold displacer rods respectively

S_H and S_c = stroke of hot and cold displacer rods respectively

θ = crank angle

Then

$$A_H = \frac{\pi}{4} D_H^2 = \frac{\pi}{4}(1.0)^2 = .785 \text{ IN}^2$$

$$A_H S_H = (.785)(.600) = .472 \text{ IN}^3$$

$$A_C = \frac{\pi}{4} D_C^2 = \frac{\pi}{4}(.85)^2 = 0.566 \text{ IN}^2$$

$$A_C S_C = (.566)(.45) = .2542 \text{ IN}^3$$

and

$$V_S = f(\theta) = 1.530 + \frac{0.472 + 0.2542}{2} + \frac{.2542}{2} \cos\theta - \frac{0.472}{2} \sin\theta$$

$$V_S = 1.8931 + 0.1271 \cos\theta - 0.236 \sin\theta$$

Then noteing;

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$

$$(.1271 \cos\theta - 0.236 \sin\theta) = K \sin(x - \theta)$$

$$x = \tan^{-1} \frac{\sin x}{\cos x} = \tan^{-1}\left(\frac{.1271}{.236}\right) = \tan^{-1}(.5395) = 28^\circ 21'$$

$$.1271 \cos\theta - 0.236 \sin\theta = -K \sin(\theta - 28^\circ 21')$$

$$@ \quad \theta = 0$$

$$.1271 = -K \sin(-28^\circ 21')$$

$$K = \frac{0.1271}{0.474} = 0.2685$$

and

$$\boxed{V_S = 1.8931 - 0.2685 \sin(\theta - 28^\circ 21')} \quad \text{IN}^3$$

$V_{S_{MAX}} = 1.8931 + 0.2685 = 2.1616 \text{ IN}^3$ @ $\theta = 298°21' \ (-61°39')$

$V_{S_{MIN}} = 1.8931 - 0.2685 = 1.6246 \text{ IN}^3$ @ $\theta = 118°21'$

$$\frac{V_{S_{MAX}}}{V_{S_{MIN}}} = \frac{2.1616}{1.6246} = 1.34$$

$V_{S_{ave}} = 1.8931 \text{ IN}^3$

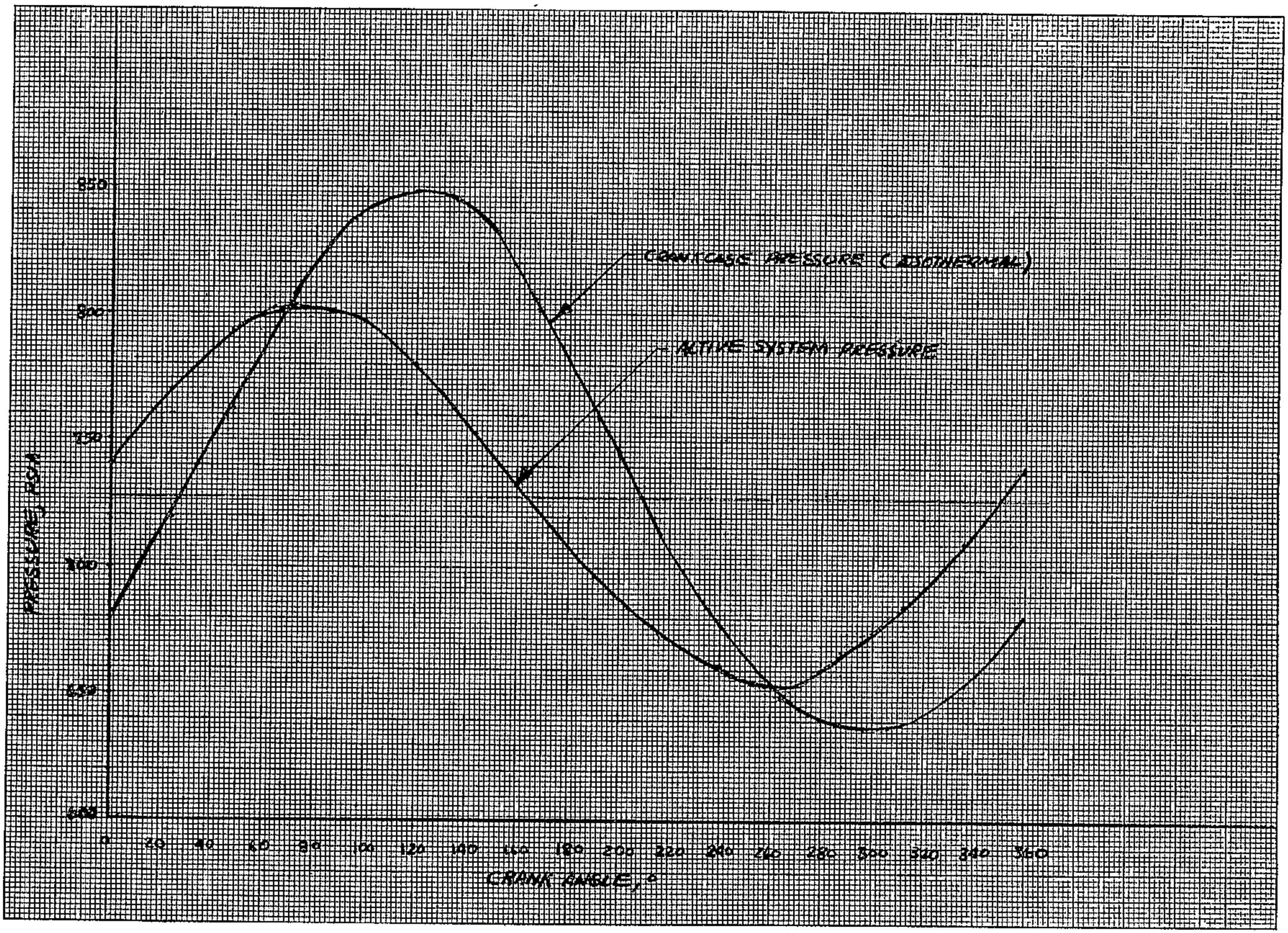
Assuming Isothermal {actually somewhere between isothermal and adiabatic}

$$P_{ave} V_{ave} = RT = C$$

$$P_S V_S = P_{ave} V_{ave}$$

$$P_S = \frac{P_{ave} V_{ave}}{V_S} = \frac{(727.68)(1.8931)}{1.8931 - 0.2685 \sin(\theta - 28°21')}$$

θ	$\theta - 28.21°$	$\sin(\theta - 28°21')$	~~1.8931 - 0.2685 sin( )~~	P_S
24	-4°21'	-0.07585	1.9135	[illegible]
48	19°39'	0.31979	1.8072	762.28
72	43°39'	0.69025	1.7078	806.65
96	67°39'	0.92488	1.6448	837.54
120	91°39'	0.99959	1.6247	847.91
168	139°39'	0.65144	1.7182	801.77
192	163°39'	0.28652	1.8162	758.51
216	187°39'	-0.13312	1.9288	714.23
240	211°39'	-0.52473	2.0390	677.29
264	235°39'	-0.82561	2.1148	651.41
288	259°39'	-0.98373	2.1572	638.61
312	283°39'	-0.97176	2.1540	639.53
336	307°39'	-0.79176	2.1057	654.22
360	331°39'	-0.47486	2.0206	681.78


850
800
750
700
650
600
PRESSURE, PSIA
0
20
40
60
80
100
120
140
160
180
200
220
240
260
280
300
320
340
360
CRANK ANGLE, °
CRANKCASE PRESSURE (ISOTHERMAL)
ACTIVE SYSTEM PRESSURE

OPERATING PARAMETERS

COLD VOLUME TEMP.	=	125.00 R
SUMP VOLUME TEMP.	=	620.00 R
HOT VOLUME TEMP.	=	1630.00 R
COLD REGEN. TEMP.	=	372.00 R
HOT REGEN. TEMP.	=	1125.00 R
COLD DISPLACED VOL.	=	.25500 CU-IN
HOT DISPLACED VOL.	=	6.80000 CU-IN
COLD DEAD VOL.	=	.08533 CU-IN
SUMP DEAD VOL.	=	5.46550 CU-IN
HOT DEAD VOL.	=	1.27200 CU-IN
COLD REGEN. VOL.	=	3.36000 CU-IN
HOT REGEN. VOL.	=	7.49000 CU-IN
GAS CONSTANT	=	4634.40 IN-LB/LBM-R
SPEED	=	400.00 RPM
CHARGE PRESSURE	=	540.00 PSIA
CHARGE TEMPERATURE	=	535.00 R
MASS OF FLUID	=	.0052 LBM
TOTAL VOLUME	=	24.72783 CU-IN

ave ≅ .004999 lb/sec

ave = .01252 lb/sec

PRESSURE - MASS - FLOW PROFILE

ANGLE DEG	PC PSIA	PA PSIA	PH PSIA	VC CU-IN	VA CU-IN	VH CU-IN	MDOTC LB/SEC	MDOTA LB/SEC	MDOTH LB/SEC	MDOTRCA LB/SEC	MDOTRHA LB/SEC	DPC PSI	DPH PSI	DPCA PSI	DPHA PSI
24.	769.49	769.49	769.49	.0963	7.7268	6.0547	.00298	-.02712	.01530	.00800	.01912	.0000	.0000	.0000	.0000
48.	792.14	792.14	792.14	.1275	6.5520	7.1983	.00524	-.02265	.01165	.00852	.01413	.0000	.0000	.0000	.0000
72.	802.76	802.76	802.76	.1734	5.7991	7.9053	.00648	-.01280	.00513	.00715	.00565	.0000	.0000	.0000	.0000
96.	798.47	798.47	798.47	.2261	5.5982	8.0535	.00626	.00008	-.00263	.00415	-.00423	.0000	.0000	.0000	.0000
120.	780.47	780.47	780.47	.2765	5.9841	7.6172	.00469	.01243	-.00951	.00036	-.01279	.0000	.0000	.0000	.0000
144.	753.36	753.36	753.36	.3159	6.8901	6.6718	.00230	.02127	-.01390	-.00320	-.01807	.0000	.0000	.0000	.0000
168.	723.19	723.19	723.19	.3375	8.1596	5.3807	-.00026	.02528	-.01530	-.00579	-.01949	.0000	.0000	.0000	.0000
192.	695.51	695.51	695.51	.3376	9.5731	3.9672	-.00249	.02473	-.01405	-.00715	-.01758	.0000	.0000	.0000	.0000
216.	674.31	674.31	674.31	.3160	10.8863	2.6755	-.00414	.02062	-.01087	-.00733	-.01329	.0000	.0000	.0000	.0000
240.	662.00	662.00	662.00	.2767	11.8723	1.7289	-.00511	.01404	-.00645	-.00652	-.00752	.0000	.0000	.0000	.0000
264.	659.75	659.75	659.75	.2263	12.3606	1.2909	-.00540	.00587	-.00136	-.00489	-.00098	.0000	.0000	.0000	.0000
288.	667.75	667.75	667.75	.1735	12.2669	1.4374	-.00499	-.00315	.00397	-.00262	.00577	.0000	.0000	.0000	.0000
312.	685.28	685.28	685.28	.1276	11.6072	2.1430	-.00387	-.01226	.00906	.00015	.01211	.0000	.0000	.0000	.0000
336.	710.53	710.53	710.53	.0964	10.4958	3.2857	-.00205	-.02036	.01325	.00316	.01720	.0000	.0000	.0000	.0000
360.	740.22	740.22	740.22	.0853	9.1245	4.6680	.00035	-.02592	.01565	.00599	.01993	.0000	.0000	.0000	.0000

IDEAL REFRIGERATION AND HEAT INPUT

REFRIGERATION = 21.0761 WATTS
THERMAL HEAT = 127.4216 WATTS
MAX. PRESSURE = 803.1153 PSIA

Max flow to back side of hot displacer

≅ .028 lb/sec

Pave = 727.68

MISC PRESSURE DROP CALCULATIONS

The remainder of this section contains pressure drop calculations for components not previously analyzed

AMBIENT END SLOTTED FLOW DISTRIBUTOR (SUPPORT RING)

(1) Per 852330

$D_o = 1.550$ SLOT DEPTH $= \frac{.04}{.03} > .035$

WIDTH $= \frac{.06}{.05} > .055$

$A_F = (.055)(.035)(16) = 3.08 \times 10^{-2} = .0308 \text{ IN}^2 = .0002139 \text{ ft}^2$

$\dot{W}_{max} = .0085$ lb/sec

$P = 800$ PSI

$T = 620°R$

$Q = \frac{\dot{\omega}}{\rho} = \frac{.0085 \text{ lb/sec}}{.46786 \text{ lb/ft}^3} = .01816 \text{ ft}^3/\text{sec}$

$\rho = .46786$ lb/ft³

$\mu = 0.0521$ lb/ft-HR

$V_S = \frac{Q}{A_F} = \frac{1.816 \times 10^{-2} \text{ ft}^3/\text{sec}}{2.139 \times 10^{-4} \text{ ft}^2} = 85.0$ ft/sec

$\frac{V_S^2}{2g_c} = \frac{85.0^2}{64.4} = 112.2$

$\frac{V_S^2}{2g_c} \times \rho = 112.2 \times (.46786) = 52.5 \frac{\text{lbf}}{\text{ft}^2} = 0.364$ PSI

$D_H = \frac{2(.055)(.035)}{.055 + .035} = .0429$ IN

$Re = \frac{V_S D_H \rho}{\mu}$ $\frac{D_H \rho}{\mu} = \frac{(.0429 \text{ IN})(.46786 \text{ lb/ft}^3)}{.0521 \text{ lb/ft-hr } (12 \text{ IN/ft})} \times 3600 \text{ sec/hr} = 115.3 \text{ sec/ft}$

$Re = 85.0 \times 115.3 = 9{,}800$ $f = .008$ $L = .437$ IN

$\frac{4fL}{D_H} = \frac{(4)(.008)(.437)}{.0429} = 0.326$

Taking 1.5 velocity heads for contraction + expansion

$\Delta P = (1.5 + .326)(.364) = 0.665$ PSI $\Big\{$ TOO LARGE

(2) LOOK AT INCREASED SLOT SIZE

WOULD LIKE TO END UP WITH ABOUT $\Delta P \cong .200$ psi

MAKE SLOTS 0.055 deep and retain .055 width

$A_F = (.055)^2 \times 16 = .0493 \ IN^2 = .000342 \ ft^2$

$Q = .01816 \ ft^3/sec$

$V_S = \dfrac{1.816 \times 10^{-2} \ ft^3/sec}{3.42 \times 10^{-4} \ ft^2} = 53.1 \ ft/sec$

$\dfrac{V_S^2}{2g_c} = \dfrac{(53.1)^2}{64.4} = 43.7$

$\dfrac{V_S^2}{2g_c} \times \rho = (43.7) \times (.46786) = 20.5 \dfrac{lbf}{ft^2} = 0.1425$ psi / velocity head

$D_H = \dfrac{2(.055)^2}{2(.055)} = .055$

$Re = V_S \left(\dfrac{D_H \rho}{\mu}\right)$ $\qquad \dfrac{D_H \rho}{\mu} = 115.3 \times \dfrac{.055}{.0429} \doteq 148$

$Re = (53.1)(148) = 7,860$

$f = .0086$

$\dfrac{4fL}{D_H} = \dfrac{(4)(.0086)(.437)}{.055} = 0.273$

$\Delta P = (1.5 + .273)(.1425) = .253$ psi $\left\{ \text{looks ok} \right\}$

Would still like smaller pressure drop

Try .095 deep by .055 wide

$A_F = (.075)(.055) * 16 = .078 \text{ IN}^2 = .00054 \text{ ft}^2$

$Q = .01816 \text{ ft}^3/\text{sec}$

$V_S = \dfrac{1.816 \times 10^{-2} \text{ ft}^3/\text{sec}}{5.4 \times 10^{-4} \text{ ft}^2} = 33.63 \text{ ft/sec}$

$\dfrac{V_S^2}{2g_c} = 17.561$

$\dfrac{V_S^2}{2g_c} * \rho = 8.216 \dfrac{\text{lbf}}{\text{ft}^2} = .0571 \text{ psi}/H_v$

$D_H = \dfrac{(2)(.075)(.055)}{.075 + .055} = .063$

$Re = V_S \left(\dfrac{D_H \rho}{\mu}\right) \quad \dfrac{D_H \rho}{\mu} = 148 \dfrac{.063}{.055} = 169.53$

$Re = (33.63)(169.53) = 5701$

$f = .0095$

$\dfrac{4fL}{D_H} = \dfrac{(4)(.0095)(.437)}{.063} = .2635$

$\Delta P = (1.5 + .2635)(.0571) = \underline{0.1007 \text{ psi}}$ better

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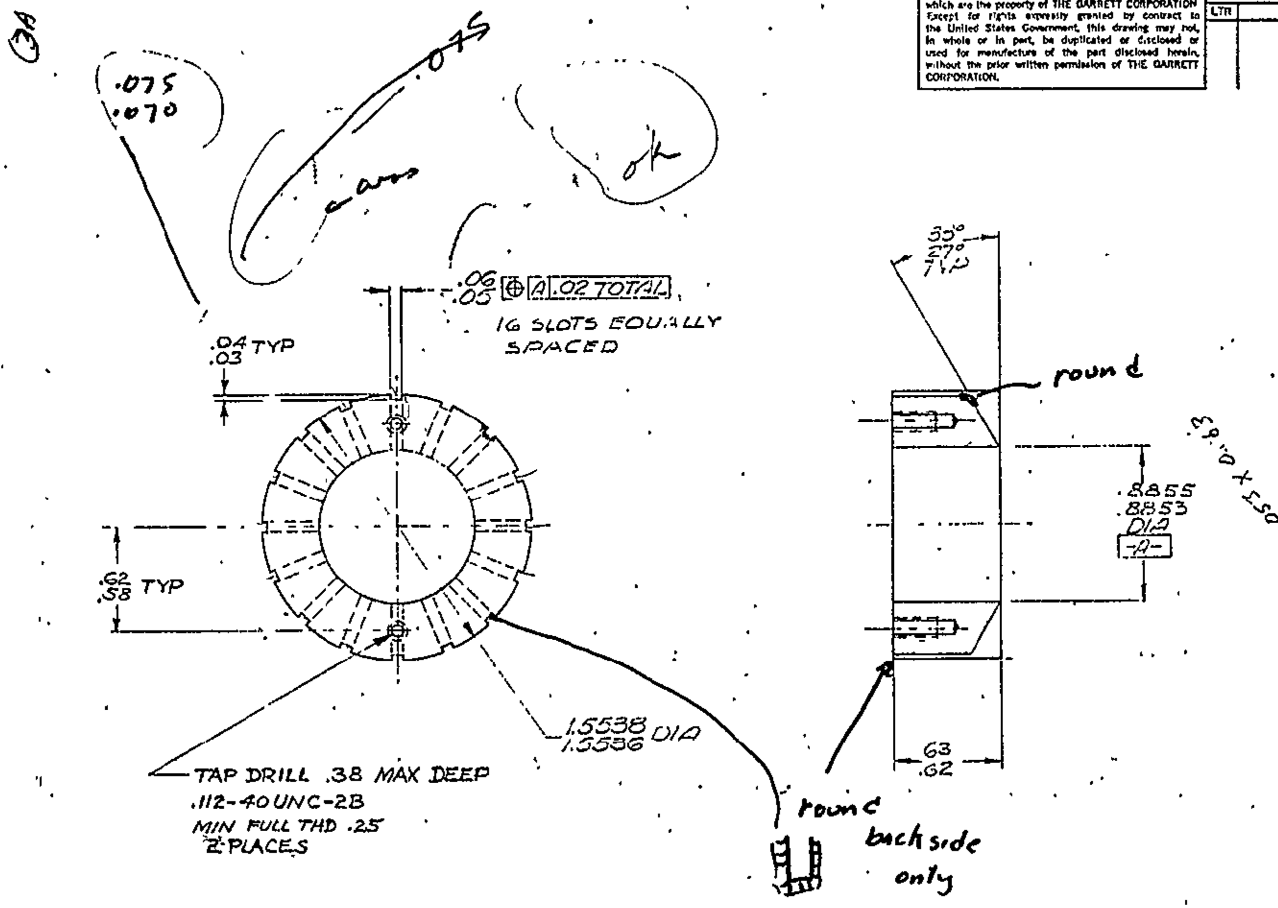
REVISIONS			
LTR	DESCRIPTION	DATE	APPROVED

GARRETT AIRESEARCH MANUFACTURING COMPANY Los Angeles, California



RESEARCH
AUG 4 1971
REDUCED DRAW

X852330

A'L SURFACES 125

QTY REQD	ITEM NO	CODE IDENT NO	PART OR IDENTIFYING NO.	NOMENCLATURE OR DESCRIPTION	SYM
			-1	CRES ...	

← ASSY — PARTS LIST

UNLESS OTHERWISE SPECIFIED: BURR CONTROL PER SC652
IDENTIFICATION MARKING PER MC16
STD INTERPRETATIONS PER PI05

HEAT TREATMENT	PROCESS
HARDNESS AND SPEC	NAME AND SPEC

DFT	
CHK	
VALUE ENGR	5.2.71
MATL	
STRESS	
APPD	
APPD	
AIRESEARCH APPD	
OTHER ACTIVITY APPD	

REQD	NEXT ASSY	USED ON
	4512...	4512...
APPLICATION		

AIRESEARCH MANUFACTURING COMPANY — A DIVISION OF THE GARRETT CORPORATION, LOS ANGELES, CALIFORNIA

RING, SUPPORT

SIZE	CODE IDENT NO.	DWG NO.
C	70210	X852330

SCALE | SHEET OF

REGENERATOR RETAINER (852329)

24 HOLE ≈ .06" in diameter.

$Q = 1.816 \times 10^{-2}\ ft^3/sec$

$A_F = N \cdot \frac{\pi}{4} D^2 = (24)\frac{\pi}{4}(.06)^2 = .06791 IN^2 = .000471 ft^2$

$V_H = \frac{Q}{A_F} = \frac{1.816 \times 10^{-2}\ ft^3/sec}{4.71 \times 10^{-4}\ ft^2} = 38.5\ ft/sec$

$\frac{V_H^2}{2g_c} = \frac{(38.5)^2}{64.4} = 23.08$

$\frac{V_H^2}{2g_c}\rho = (23.08)(.46786) = 10.75\ lb/ft^2 = .0747\ psi$

Taking 1.5 velocity head loss

$\Delta P = (1.5)\frac{V_H^2}{2g_c}\rho = (1.5)(.0747) = .1158\ psi$

OK but better if edges rounded off and diameter increased to 0.07 IN.

(a) With .07 dia

$\Delta P = .1158 * \left(\frac{.06}{.07}\right)^2 = .0852\ psi$

(b) Di = .08

$\Delta P = .1158 * \left(\frac{.06}{.08}\right)^2 = .0652\ psi$

(c) Di = .09

$\Delta P = .1158 * \left(\frac{.06}{.09}\right)^2 = .0516\ psi$

d Di = .1

$\Delta P = .1158 * \left(\frac{.06}{.1}\right)^2 = .0417\ psi$

Select .093 rounded edges

RETAINER (852332)

Same hole size as (852329) except bored on a smaller major diameter, must be assured that machined annular space between plates is large enough.

ΔP calculations same as above

Note:

Holes between these plates 852329 and 852332 will be lined up - small distribution slot between

SECTION 10

HOT-END REGENERATOR

INTRODUCTION

The hot regenerator functions to regenerate the temperature of the working fluid as it cycles in the refrigerator between the hot end and sump active volumes. Hot regenerator basic design requirements are similar to those of the cold regenerator, with changed emphasis on particular items. Due to higher operating temperatures, the void or dead volume associated with the hot regenerator is not as critical as that of the cold regenerator. At the high operating temperatures, much less gas can be stored per unit of dead volume. Therefore, dead volume in high temperature regions of the machine has a much less of an effect on the pressure variation of the cycle and refrigeration capacity than low temperature dead volumes. Also, thermal losses in the hot regenerator, though still very important, are not as detrimental to refrigerator performance as are the losses in the cold regenerator.

Pressure drop is the only design parameter of the hot regenerator which is as critical (if not more so) as that of the cold regenerator. The hot regenerator pressure drop is the major factor controlling the drive motor power input required. As such, the hot regenerator pressure drop must be controlled to yield a reasonable drive motor design.

METHOD OF ANALYSIS AND CHARACTERIZATION

The analytical methods and characterization of the hot regenerator are identical to those of the cold regenerator. Due to the transient nature of this heat transfer device, a finite difference computer program is essential to accurately predict performance.

DESIGN CONFIGURATION AND PERFORMANCE

During a study effort in Task I of the program, several hot regenerator configurations were considered; this resulted in selection of an annular configuration for the hot regenerator. Since the Task I effort, only minor changes have been made to the hot regenerator.

The hot regenerator final configuration has a frontal area of 3.18 in.^{2} and a total length of 3.3 in. This is an increase in frontal area and a decrease in length from the Task I preliminary design. These changes were needed to maintain a low regenerator pressure drop after the hot displaced volume of the preliminary refrigerator design was increased to provide a greater margin of cooling capacity. The entire length of the regenerator is packed with 100-mesh stainless steel screen that has a porosity of 72.5 percent, an area to volume ratio of 256 in.^{2}/in.^{3}, and a hydraulic diameter of 0.01132 in.

A screen matrix was selected, rather than a packed bed of spheres or a combination of screens and spheres, to minimize the pressure drop. The high operating temperature of the heat regenerator favors the selection of screens since the higher dead volume of the screens, compared to packed beds of spheres, does not greatly degrade the refrigerator's performance. The screen matrix is also more predictable as far as pressure drop through screen beds is concerned, compared to beds packed with spheres. Minor deviations in sphere sizes and the fact that individual spheres are not perfectly spherical, can greatly increase the pressure drop through beds packed with spheres. Stainless steel was selected over other candidate materials for the matrix due to it's superior heat capacity in the operating temperature range of the hot regenerator. (See Figure 6-4).

Table 10-1 is presented to show the detailed output from the regenerator analysis computer program for the selected design. This table shows the mode pressures and temperatures of the gas, and response characteristics between the matrix and gas temperatures as functions of time (angular position of the crankshaft). In the table, the angular position θ, is referenced to the top-dead-center position of the cold displacer, as is the data presented in Figure 3-1. It is noted that the data of Figure 3-1 forms a major part of the input data necessary to run the regenerator analysis computer program.

Parameters listed in Table 10-1 are the matrix temperatures, the gas temperatures, gas density, the gas pressure, and the mass flow rate of the gas. Positive mass flow rates denote flow toward the hot end of the regenerator. Node 0 represents the conditions at the ambient end, and Node 11 represents the conditions at the hot end of the regenerator.

Figures 10-1 to 10-3 represent plots of key parameters from the data of Table 10-1. The hot end temperatures of the gas and matrix are given as functions of the crank angle position in Figure 10-1. The small difference between gas and matrix is indicative of good heat transfer between the gas and matrix. The moderate temperature swing of the matrix, approximately 4.0°R shows the matrix has adequate heat capacity.

The data of Figure 10-2 shows a maximum pressure drop across the hot regenerator of 0.86 psi. This low pressure drop was intentionally designed into the system to provide a low drive motor power requirement.

Figure 10-3 gives the gas flow rate into the hot displaced volume. These data, coupled with the gas temperature, can be used to estimate the losses associated with the hot regenerator. Following the same reasoning used in the development of Equation 6-5 for the cold regenerator, the hot regenerator losses can be expressed as:

$$\dot{Q}_{loss} = \oint \dot{\omega}(h_{ref}\, h)\, d\tau \qquad (10\text{-}1)$$

where

$\dot{\omega}$ = flow rate into or from hot volume at a point in time

h = enthalpy of the fluid entering or exiting the hot volume

h_{ref} = reference enthalpy for in hot end gas temperature

τ = time

TABLE 10-1

FINAL DESIGN HOT REGENERATOR PERFORMANCE CHARACTERISTICS

NODE NO. N	MATRIX TEMP. TM(N) DEG.R	GAS TEMP. TG(N) DEG.R	GAS DENSITY RG(N) LBM/CF	GAS PRESSURE PG(N) PSIA	MASS FLOW RATE WG(N) LBM/SEC
$\theta = 0^0$					
-0	6.21728+02	6.20000+02	4.42424-01	7.52765+02	1.99886-02
1	6.78208+02	6.75573+02	4.07068-01	7.52746+02	1.96893-02
2	7.72016+02	7.68000+02	3.58879-01	7.52702+02	1.91604-02
3	8.72060+02	8.67866+02	3.18055-01	7.52651+02	1.86907-02
4	9.73085+02	9.69031+02	2.85204-01	7.52594+02	1.82687-02
5	1.07424+03	1.07035+03	2.54685-01	7.52530+02	1.78913-02
6	1.17540+03	1.17164+03	2.24177-01	7.52455+02	1.75582-02
7	1.27656+03	1.27288+03	1.93685-01	7.52368+02	1.72695-02
8	1.37764+03	1.37402+03	1.63227-01	7.52264+02	1.70252-02
9	1.47813+03	1.47455+03	1.32958-01	7.52135+02	1.68249-02
10	1.57334+03	1.56997+03	1.04231-01	7.51970+02	1.66665-02
11	1.62786+03	1.62583+03	8.74162-02	7.51867+02	1.65996-02
$\theta = 30^0$					
-0	6.21006+02	6.20000+02	4.59548-01	7.82568+02	1.77551-02
1	6.76827+02	6.74294+02	4.23696-01	7.82552+02	1.75301-02
2	7.69919+02	7.65949+02	3.73854-01	7.82516+02	1.71322-02
3	8.69890+02	8.65708+02	3.31289-01	7.82475+02	1.67790-02
4	9.71000+02	9.66938+02	2.96996-01	7.82427+02	1.64617-02
5	1.07225+03	1.06834+03	2.65281-01	7.82373+02	1.61778-02
6	1.17348+03	1.16969+03	2.33584-01	7.82310+02	1.59273-02
7	1.27467+03	1.27098+03	2.01909-01	7.82237+02	1.57101-02
8	1.37578+03	1.37216+03	1.70270-01	7.82148+02	1.55261-02
9	1.47628+03	1.47272+03	1.38827-01	7.82039+02	1.53753-02
10	1.57161+03	1.56826+03	1.08958-01	7.81899+02	1.52559-02
11	1.62682+03	1.62480+03	9.12863-02	7.81811+02	1.52055-02
$\theta = 60^0$					
-0	6.20637+02	6.20000+02	4.70445-01	8.01555+02	9.86599-03
1	6.75783+02	6.73763+02	4.34123-01	8.01547+02	9.76168-03
2	7.68296+02	7.65092+02	3.83206-01	8.01531+02	9.57685-03
3	8.68198+02	8.64824+02	3.39567-01	8.01513+02	9.41253-03
4	9.69364+02	9.66088+02	3.04395-01	8.01491+02	9.26487-03
5	1.07068+03	1.06753+03	2.71928-01	8.01466+02	9.13261-03
6	1.17195+03	1.16891+03	2.39482-01	8.01437+02	9.01573-03
7	1.27318+03	1.27022+03	2.07060-01	8.01403+02	8.91423-03
8	1.37431+03	1.37142+03	1.74673-01	8.01362+02	8.82811-03
9	1.47483+03	1.47200+03	1.42486-01	8.01311+02	8.75726-03
10	1.57025+03	1.56760+03	1.11896-01	8.01245+02	8.70092-03
11	1.62600+03	1.62440+03	9.37195-02	8.01204+02	8.67706-03
$\theta = 90^0$					
-0	6.20509+02	6.20045+02	4.71645-01	8.03726+02	-1.03718-03
1	6.75405+02	6.74218+02	4.34958-01	8.03728+02	-9.97277-04
2	7.67694+02	7.65788+02	3.83873-01	8.03734+02	-9.26581-04
3	8.67567+02	8.65543+02	3.40201-01	8.03743+02	-8.63734-04
4	9.68748+02	9.66630+02	3.05063-01	8.03760+02	-8.07246-04
5	1.07008+03	1.06807+03	2.72532-01	8.03808+02	-7.56654-04
6	1.17137+03	1.16945+03	2.40031-01	8.03893+02	-7.11952-04
7	1.27260+03	1.27058+03	2.07596-01	8.03958+02	-6.73139-04
8	1.37375+03	1.37191+03	1.75085-01	8.03988+02	-6.40211-04
9	1.47428+03	1.47263+03	1.42765-01	8.04011+02	-6.13135-04
10	1.56973+03	1.56831+03	1.12063-01	8.04034+02	-5.91615-04
11	1.62569+03	1.62499+03	9.38714-02	8.04046+02	-5.82501-04

TABLE 10-1 (Continued)

NODE NO.	MATRIX TEMP.	GAS TEMP.	GAS DENSITY	GAS PRESSURE	MASS FLOW RATE
N	TM(N) DEG.R	TG(N) DEG.R	RG(N) LBM/CF	PG(N) PSIA	WG(N) LBM/SEC
$\theta = 120^0$					
-0	6,20950+02	6,23127+02	4.60368-01	7.87611+02	-1,24596-02
1	6,76044+02	6,79351+02	4.23026-01	7.87620+02	-1,22851-02
2	7,68345+02	7,71804+02	3.73325-01	7.87643+02	-1,19763-02
3	8.68181+02	8,71525+02	3.31174-01	7.87669+02	-1,17019-02
4	9,69326+02	9,72539+02	2.97206-01	7.87699+02	-1,14553-02
5	1,07064+03	1,07375+03	2.65374-01	7.87733+02	-1,12347-02
6	1.17192+03	1,17495+03	2.33543-01	7.87772+02	-1,10401-02
7	1,27315+03	1,27613+03	2.01717-01	7.87818+02	-1,08715-02
8	1,37429+03	1,37724+03	1.69914-01	7.87872+02	-1,07290-02
9	1.47480+03	1,47754+03	1.38362-01	7.87940+02	-1,06123-02
10	1,57007+03	1,57181+03	1.08705-01	7.88027+02	-1,05198-02
11	1,62675+03	1,63000+03	9.03952-02	7.88082+02	-1,04811-02
$\theta = 150^0$					
-0	6,22091+02	6,24669+02	4.42530-01	7,58154+02	-1,83228-02
1	6,77717+02	6,81528+02	4.06127-01	7,58172+02	-1,80229-02
2	7,70056+02	7,74020+02	3.58548-01	7,58210+02	-1,74922-02
3	8,69799+02	8,73615+02	3.18212-01	7,58254+02	-1,70204-02
4	9,70858+02	9,74516+02	2.85673-01	7,58305+02	-1,65961-02
5	1,07211+03	1,07564+03	2,55028-01	7,58362+02	-1,62167-02
6	1,17335+03	1,17680+03	2,24371-01	7,58427+02	-1,58822-02
7	1,27456+03	1,27797+03	1.93709-01	7,58503+02	-1,55925-02
8	1,37569+03	1,37906+03	1.63070-01	7.58594+02	-1,53476-02
9	1,47610+03	1,47922+03	1.32706-01	7.58707+02	-1,51472-02
10	1,57090+03	1,57286+03	1.04316-01	7.58850+02	-1,49883-02
11	1,62793+03	1,63000+03	8,69896-02	7.58939+02	-1,49215-02
$\theta = 180^0$					
-0	6,23542+02	6,26258+02	4.23547-01	7.26610+02	-1,87628-02
1	6.79837+02	6,83827+02	3.88162-01	7.26628+02	-1,84846-02
2	7,72225+02	7,76368+02	3,42815-01	7,26671+02	-1,79922-02
3	8,71860+02	8,75858+02	3.04380-01	7,26719+02	-1,75542-02
4	9,72821+02	9,76662+02	2.73344-01	7,26774+02	-1,71603-02
5	1,07399+03	1,07771+03	2,43961-01	7,26836+02	-1,68080-02
6	1,17519+03	1,17883+03	2.14556-01	7,26908+02	-1,64975-02
7	1,27639+03	1,27997+03	1.85141-01	7,26992+02	-1,62287-02
8	1,37749+03	1,38101+03	1.55752-01	7,27092+02	-1,60016-02
9	1.47777+03	1,48100+03	1.26666-01	7,27216+02	-1,58158-02
10	1,57193+03	1,57393+03	9.96297-02	7,27375+02	-1,56684-02
11	1,62876+03	1,63000+03	8.33125-02	7,27474+02	-1,56062-02
$\theta = 210^0$					
-0	6,24896+02	6,27382+02	4.07973-01	7.00435+02	-1,47475-02
1	6,81803+02	6,85424+02	3.73526-01	7.00449+02	-1,45323-02
2	7,74237+02	7,77989+02	3.29967-01	7.00480+02	-1,41513-02
3	8,73778+02	8,77399+02	2.93052-01	7.00517+02	-1,38121-02
4	9,74652+02	9,78132+02	2.63223-01	7.00558+02	-1,35071-02
5	1.07576+03	1.07912+03	2,34878-01	7.00606+02	-1,32343-02
6	1,17692+03	1,18020+03	2.06507-01	7.00660+02	-1,29939-02
7	1,27809+03	1,28132+03	1.78124-01	7.00724+02	-1,27858-02
8	1,37917+03	1,38234+03	1.49767-01	7.00801+02	-1,26101-02
9	1,47931+03	1,48220+03	1.21731-01	7.00895+02	-1,24664-02
10	1,57287+03	1,57464+03	9.57754-02	7.01016+02	-1,23522-02
11	1,62924+03	1,63000+03	8,02296-02	7.01092+02	-1,23040-02

TABLE 10-1 (Continued)

NODE NO.	MATRIX TEMP.	GAS TEMP.	GAS DENSITY	GAS PRESSURE	MASS FLOW RATE
N	TM(N) DEG.R	TG(N) DEG.R	RG(N) LBM/CF	PG(N) PSIA	WG(N) LBM/SEC
θ = 240°					
-0	6.25830+02	6.27697+02	3.98882-01	6.84769+02	-7.82013-03
1	6.83161+02	6.85844+02	3.65107-01	6.84775+02	-7.69393-03
2	7.75627+02	7.78391+02	3.22543-01	6.84789+02	-7.47036-03
3	8.75107+02	8.77758+02	2.86472-01	6.84805+02	-7.27140-03
4	9.75925+02	9.78458+02	2.57314-01	6.84824+02	-7.09237-03
5	1.07699+03	1.07943+03	2.29583-01	6.84844+02	-6.93233-03
6	1.17812+03	1.18050+03	2.01824-01	6.84869+02	-6.79127-03
7	1.27928+03	1.28162+03	1.74051-01	6.84897+02	-6.66922-03
8	1.38034+03	1.38263+03	1.46306-01	6.84931+02	-6.56614-03
9	1.48038+03	1.48247+03	1.18883-01	6.84974+02	-6.48183-03
10	1.57352+03	1.57480+03	9.35201-02	6.85029+02	-6.41486-03
11	1.62949+03	1.63000+03	7.83564-02	6.85063+02	-6.38656-03
θ = 270°					
-0	6.26153+02	6.20000+02	4.03143-01	6.84402+02	6.92485-04
1	6.83682+02	6.85839+02	3.64916-01	6.84402+02	6.12077-04
2	7.76166+02	7.78971+02	3.22110-01	6.84400+02	4.69722-04
3	8.75630+02	8.78651+02	2.86000-01	6.84398+02	3.43038-04
4	9.76433+02	9.79697+02	2.56808-01	6.84372+02	2.29037-04
5	1.07749+03	1.08049+03	2.29127-01	6.84346+02	1.27123-04
6	1.17862+03	1.18139+03	2.01429-01	6.84346+02	3.73014-05
7	1.27977+03	1.28229+03	1.73728-01	6.84346+02	-4.04257-05
8	1.38082+03	1.38306+03	1.46064-01	6.84346+02	-1.06081-04
9	1.48081+03	1.48272+03	1.18705-01	6.84346+02	-1.59798-04
10	1.57377+03	1.57471+03	9.34510-02	6.84347+02	-2.02490-04
11	1.62959+03	1.63000+03	7.82729-02	6.84348+02	-2.20525-04
θ = 300°					
-0	6.24804+02	6.20000+02	4.10588-01	6.97359+02	9.14963-03
1	6.83375+02	6.81443+02	3.74264-01	6.97352+02	8.97172-03
2	7.75791+02	7.73297+02	3.30606-01	6.97335+02	8.65660-03
3	8.75245+02	8.72640+02	2.93394-01	6.97316+02	8.37637-03
4	9.76071+02	9.73567+02	2.63292-01	6.97294+02	8.12445-03
5	1.07714+03	1.07476+03	2.34996-01	6.97270+02	7.89915-03
6	1.17829+03	1.17598+03	2.06691-01	6.97242+02	7.70047-03
7	1.27944+03	1.27719+03	1.78391-01	6.97210+02	7.52841-03
8	1.38049+03	1.37826+03	1.50133-01	6.97171+02	7.38296-03
9	1.48048+03	1.47827+03	1.22172-01	6.97123+02	7.26381-03
10	1.57346+03	1.57141+03	9.61318-02	6.97061+02	7.16916-03
11	1.62938+03	1.62806+03	8.02948-02	6.97022+02	7.12929-03
θ = 330°					
-0	6.23126+02	6.20000+02	4.24463-01	7.21508+02	1.61544-02
1	6.82393+02	6.79817+02	3.87926-01	7.21493+02	1.58990-02
2	7.74504+02	7.70995+02	3.42878-01	7.21458+02	1.54464-02
3	8.73915+02	8.70213+02	3.04230-01	7.21419+02	1.50440-02
4	9.74797+02	9.71200+02	2.72923-01	7.21374+02	1.46824-02
5	1.07593+03	1.07248+03	2.43648-01	7.21323+02	1.43589-02
6	1.17712+03	1.17378+03	2.14368-01	7.21265+02	1.40736-02
7	1.27830+03	1.27504+03	1.85100-01	7.21196+02	1.38264-02
8	1.37936+03	1.37615+03	1.55879-01	7.21115+02	1.36173-02
9	1.47936+03	1.47621+03	1.26966-01	7.21014+02	1.34459-02
10	1.57243+03	1.56952+03	1.00005-01	7.20885+02	1.33097-02
11	1.62873+03	1.62687+03	8.34378-02	7.20805+02	1.32523-02

TABLE 10-1 (Continued)

NODE NO. N	MATRIX TEMP. TM(N) DEG.R	GAS TEMP. TG(N) DEG.R	GAS DENSITY RG(N) LBM/CF	GAS PRESSURE PG(N) PSIA	MASS FLOW RATE WG(N) LBM/SEC
$\theta = 360^\circ$					
-0	6.21844+02	6.20000+02	4.42388-01	7.52703+02	1.99824-02
1	6.80991+02	6.78207+02	4.05360-01	7.52683+02	1.96843-02
2	7.72600+02	7.68693+02	3.58517-01	7.52639+02	1.91558-02
3	8.71929+02	8.67777+02	3.18062-01	7.52589+02	1.86860-02
4	9.72882+02	9.68833+02	2.85241-01	7.52532+02	1.82640-02
5	1.07410+03	1.07021+03	2.54706-01	7.52467+02	1.78864-02
6	1.17535+03	1.17158+03	2.24176-01	7.52393+02	1.75533-02
7	1.27657+03	1.27289+03	1.93667-01	7.52306+02	1.72646-02
8	1.37765+03	1.37403+03	1.63212-01	7.52201+02	1.70203-02
9	1.47768+03	1.47412+03	1.33075-01	7.52073+02	1.68198-02
10	1.57088+03	1.56760+03	1.04935-01	7.51909+02	1.66604-02
11	1.62774+03	1.62563+03	8.74715-02	7.51806+02	1.65934-02

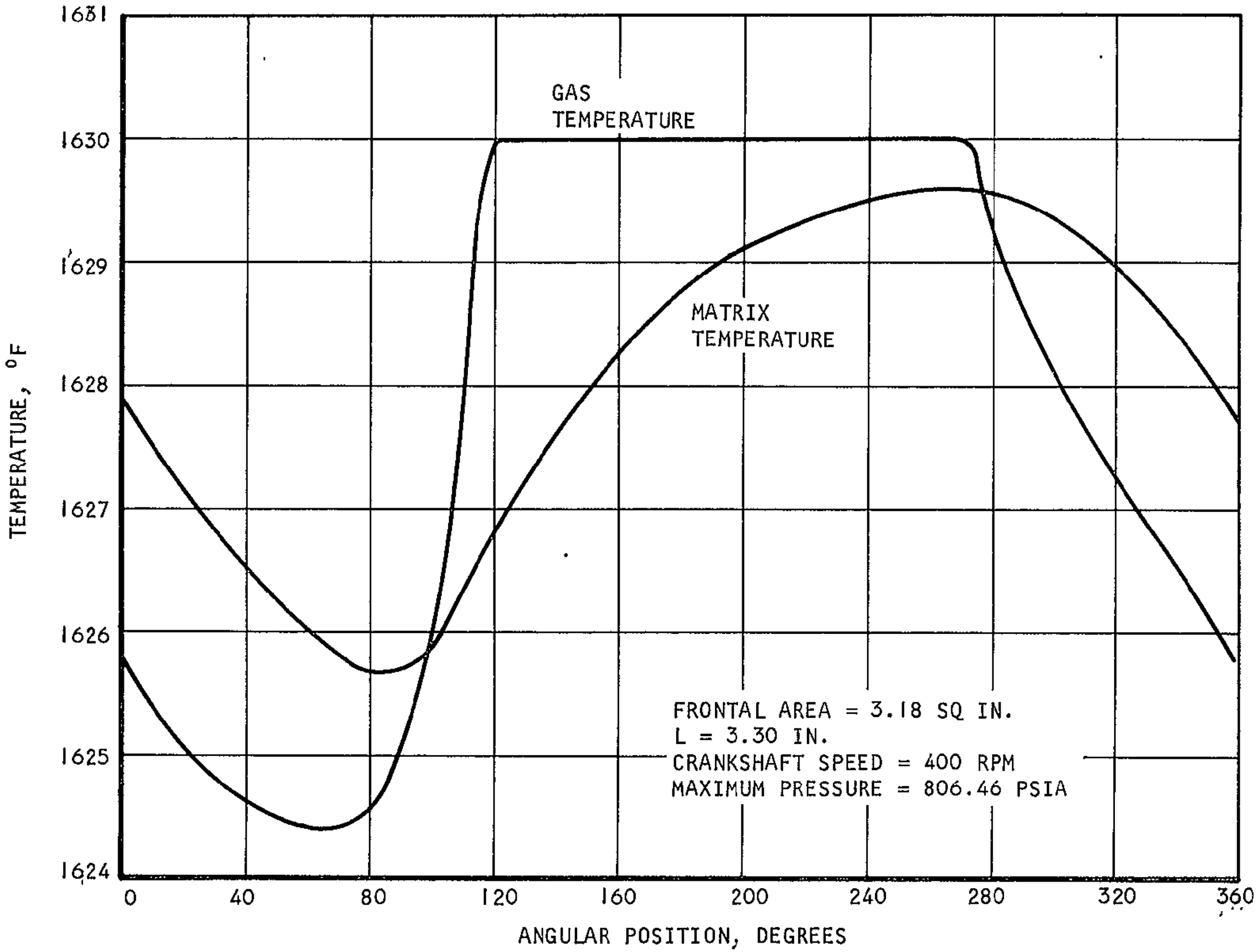


Figure 10-1. Temperature Variation at Hot End of Hot Regenerator

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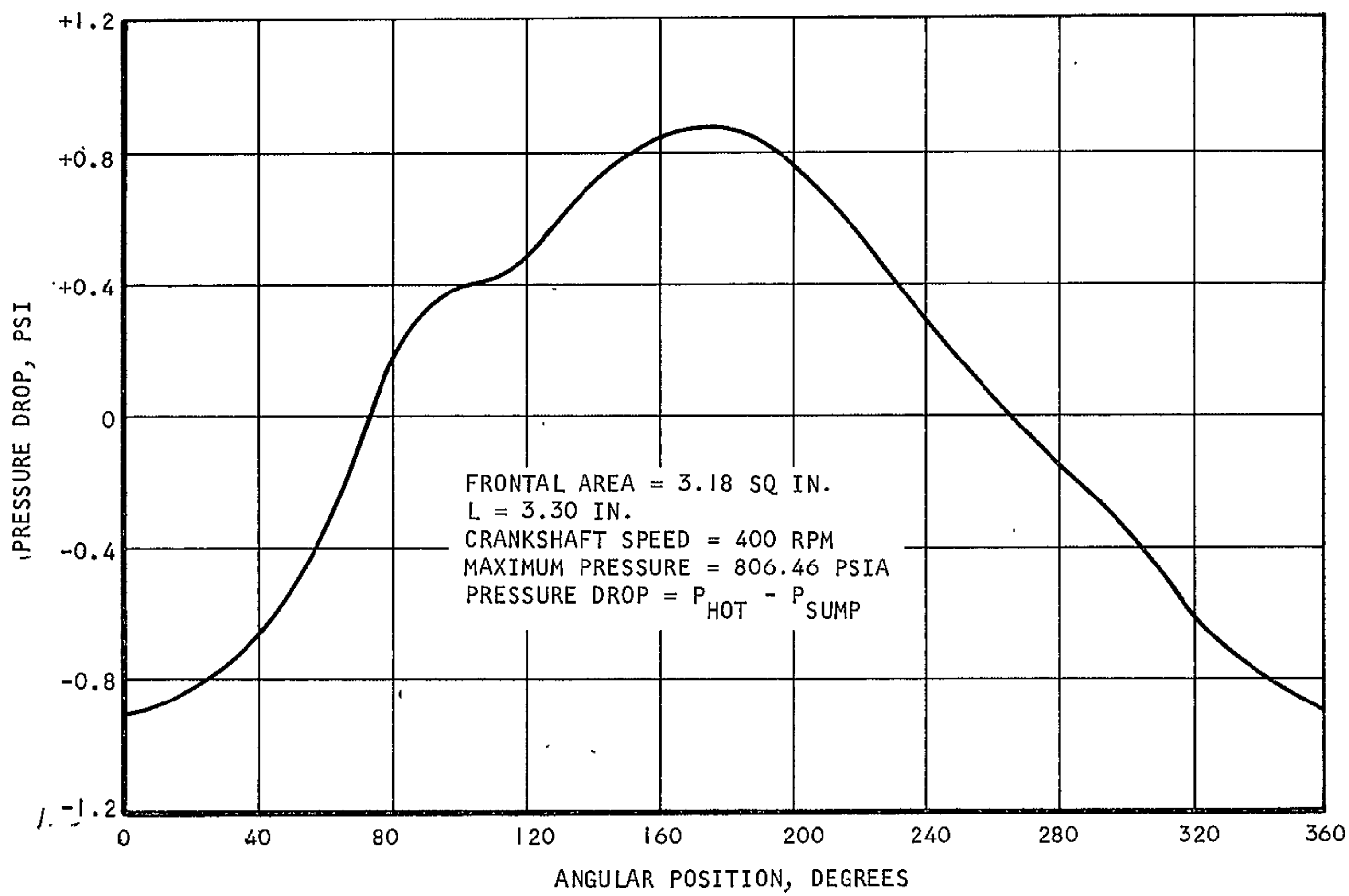

Figure 10-2. Pressure Drop for Hot Regenerator

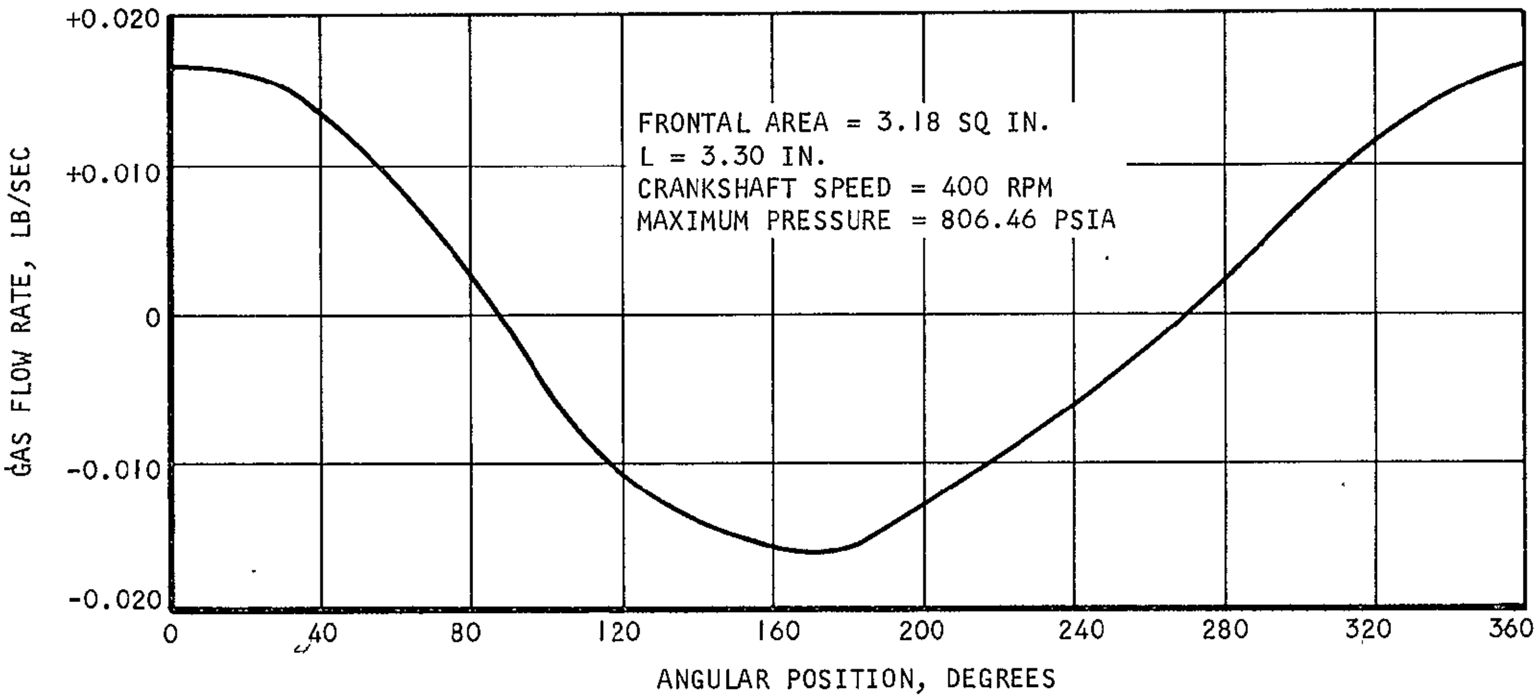

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Figure 10-3. Flow Rate into Hot Volume

Performing the integration indicated by Equation 10-1 yields a hot regenerator loss of 29.5 watts. This is an acceptable loss from the standpoint of meeting the required overall performance.

SECTION II

HOT-END HEAT EXCHANGER

INTRODUCTION

The hot-end heat exchanger functions to transfer heat to the working fluid of the VM refrigerator, providing the energy input necessary to drive the system. Design criteria for this heat exchanger are similar to those of the cold-end heat exchanger and the sump heat exchanger except for a change in emphasis on various design items.

- Low working-fluid pressure drop--As with the cold-end heat exchanger and the sump heat exchanger the pressure drop across this heat exchanger subtracts from the pressure-volume variations in the cold expansion volume thereby reducing the refrigeration capacity. In addition, similar to the sump heat exchanger, the pressure drop across the hot-end heat exchanger has a direct effect on the design of the drive motor; the motor power increases directly with this pressure drop. Due to the high operating temperature (and low working fluid density) of the hot-end heat exchanger, design for low pressure drop is more difficult than with either the sump or cold-end heat exchangers due to higher fluid velocities for similar flow rates and flow passage geometries.

- Minimization of film temperature drop--The thermodynamic efficiency of the refrigerator increases as the temperature of the gas in the hot end increases. The maximum temperature of the heat exchanger wall bounding the gas is set by structural considerations, thus, minimizing the gas film temperature drop maximizes the hot-end gas temperature and thermodynamic efficiency.

- Flow distribution--Non-uniform flow within the hot-end heat exchanger is to be avoided for the same reasons given for the cold-end and sump heat exchangers.

- Low void or internal volume--Void volumes reduce the refrigeration capacity of the machine as previously discussed. The higher the temperature of the working fluid in a given void or dead volume the less it influences the refrigeration capacity. Void volume is therefore less important in the hot-end heat exchanger design than either the cold-end or sump heat exchangers.

- Heat exchanger interfaces--The hot-end heat exchanger must provide flow transitions to both the hot regenerator and the hot displaced volume. The heat exchanger must also provide a surface for radiant heat exchange with a heater which simulates an interface with a hot heat pipe. The energy is transferred in this manner to the working fluid as it passes through the heat exchanger in it's path between the hot regenerator and the hot displaced volume.

DESIGN CONFIGURATION

The hot-end heat exchanger configuration (Figure II-I) is a refinement of the design evolved under Task I of this program.

Flow enters and exits the heat exchanger at its interface with the hot displaced volume through ports cut in the heat exchanger inner wall. Considering flow from the hot volume, flow from the ports into the heat exchanger is initially radially outward around the dome of the refrigerator in flow passages between the pressure vessel wall and the heat exchanger inner wall. The number of flow passages increases at two points as the flow progresses rapidly outward. The number and size of passages--formed by ribs machined on the inner heat exchanger wall--were selected to promote high rates of heat transfer while maintaining a low pressure drop. From the heat exchanger dome section, the flow enters flow passages between ribs in the cylindrical section (Section AA, Figure II-I) and then passes on to the hot regenerator. For flow toward the hot volume, the flow paths described above are reversed.

Heat transfer to the working fluid is accomplished by both primary and secondary heat transfer surfaces. The primary heat transfer surface consists of segments of the pressure vessel wall which form the outer boundary of the flow passages in both the dome and cylindrical sections of the heat exchanger. Heat transfers directly into the working fluid from this surface. The secondary surface consists of the heat exchanger inner wall including the ribs. Here, heat is conducted from the exterior through the ribs to the inner wall and then into the working fluid. This secondary heat transfer surface is approximately 85 percent as effective as the primary surface.

PERFORMANCE CHARACTERISTICS

The calculated heat transfer and pressure drop characteristics of the hot-end heat exchanger are summarized in Table II-I.

TABLE II-I

HOT END HEAT EXCHANGER CHARACTERISTICS

Parameter	Design Value
Conductance (ηhA), Btu/hr-^{0}R)	48.84
Pressure drop, psi	0.155

As with the other two heat exchangers, the heat transfer characteristics are based on the average flow during the cyclic operations, and maximum flow was used to calculate the pressure drop. The detail analysis of the heat exchanger, except for geometric considerations, is straightforward. Analysis is presented on the subsequent pages of this section.

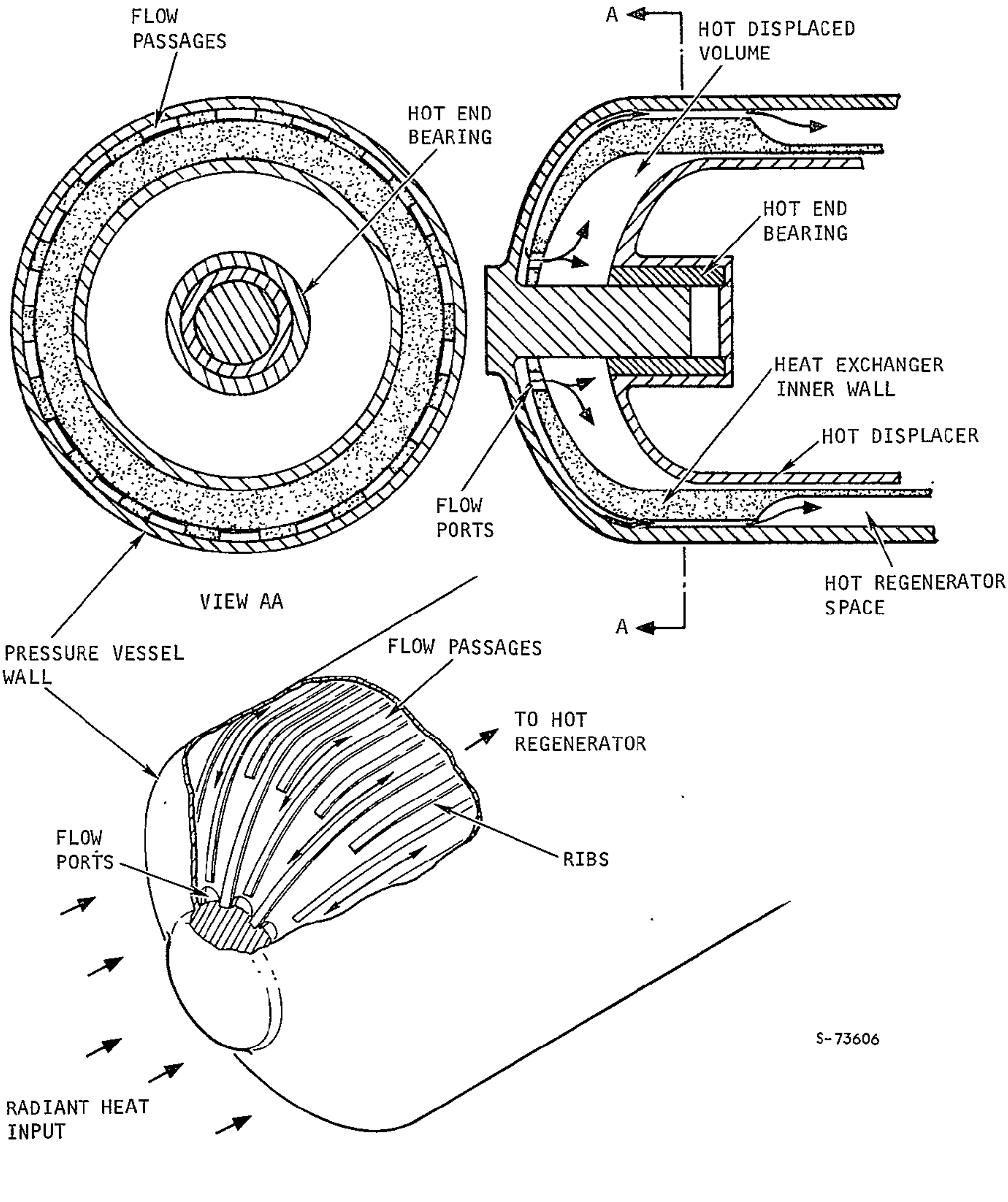


Figure II-1. Hot-End Heat Exchanger

HOT-END HEAT EXCHANGER

1. DESIGN CONDITIONS

From 10-22-71 computer run, at 400 rpm:

$\dot{w}_{MAX} = .0157$ lb/sec

$\dot{w}_{AVG} = \frac{2}{\pi}(.0157) = .010$ lb/sec

$T = 1630\ °R$ $\quad\quad p = 800$ psia

$\rho = .183$ lb/ft³

$c_p = 1.24$ B/lb °R

$\mu = .097$ lbm/ft-hr

$k = .181$ B/hr-ft-°R

Req'd heat transfer rate, $q_H = 300$ watt

2. GEOMETRIC CHARACTERISTICS

Dwgs used: 852363, 852364 & 852387

2.1 Dome Heat Transfer Area

Total area of ellipsoidal surface with $a = 2.188$ in.
$b = 1.094$ in.

$$A = 2\pi a^2 + \pi \frac{b^2}{\epsilon} \ln \frac{1+\epsilon}{1-\epsilon}$$

$$\epsilon = \frac{\sqrt{a^2-b^2}}{a} = \sqrt{1 - b^2/a^2}$$

$$\epsilon = \sqrt{1-(1/2)^2} = .866 \text{ for 2:1 ellipsoid}$$

$$\ln \frac{1.866}{.134} = 2.634$$

Thus, for 2:1 semi-ellipsoids:

$$A = \frac{\pi}{2}\left(2a^2 + \frac{2.634}{.866} b^2\right)$$

$$= \frac{\pi}{2}\left(2a^2 + 3.042\, b^2\right) = \frac{\pi}{2}a^2\left(2 + \frac{3.042}{4}\right)$$

$$A = 1.38\,\pi\, a^2$$

$$= (1.38)\,\pi\,(2.188)^2 = 20.763 \text{ in}^2$$

Assume central hole is well-approximated as a disk area - then total surface area is:

$$A_{tot} = 20.763 - \frac{\pi}{4}(1.0003)^2 = \underline{19.978 \text{ in}^2}$$

Surface area occupied by flow ribs

$$A_R = n_R w_R l_R$$

n_R = number of ribs

w_R = rib width

l_R = rib length

The rib length on the dome surface is

$$l_R = s(x_2) - s(x_1)$$

where $s(x)$ is ellipse arc length from $x = 0$ to $x = x$

The arc length is found from

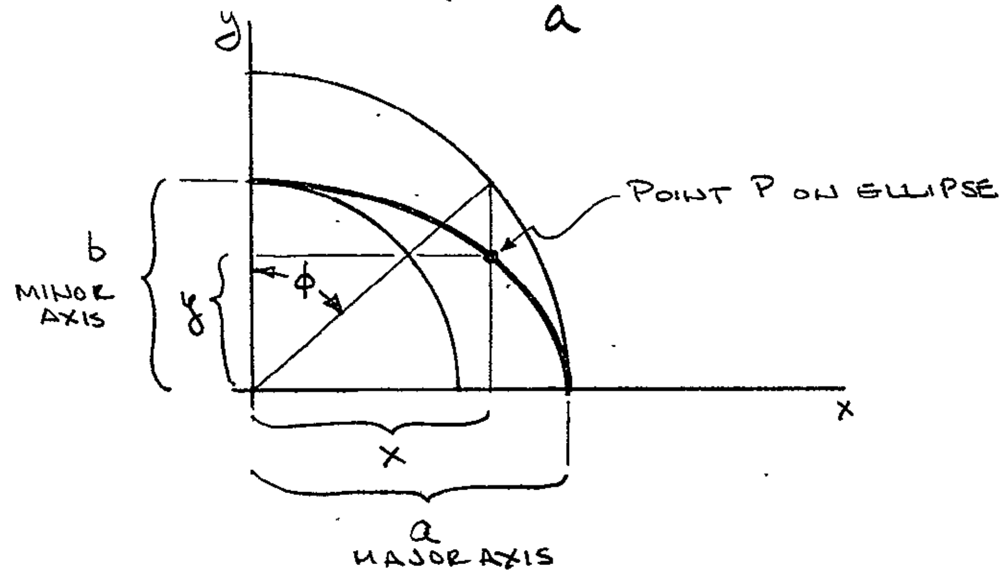
$$s = a\,E(\theta, \phi)$$

$E(\theta, \phi)$ = ellipsoidal integral of second kind

$E(\theta, \phi)$ can be found in math tables

ϕ = minor eccentric angle

$$= \sin^{-1} \frac{x}{a}$$



θ = function eccentricity of ellipse

$$= \sin^{-1} e = \sin^{-1}\sqrt{1 - b^2/a^2}$$

For 2:1 ellipse

$\theta = 60°$

The 3 rib lengths existing on dome are defined by

① $x = .50015$ in. , $x = a = 2.188$ in.

② $x = .83$ in. , $x = a$

③ $x = 1.50$ in. , $x = a$

x	ϕ, deg	$E(60°, \phi)$	$S(x)$
.5002	13.2	.229	.501
.83	22.3	.382	.836
1.50	43.3	.7045	1.540
2.188	90	1.211	2.650

Thus

$$l_{R_{D_1}} = 2.65 - .501 = 2.149 \text{ in.}$$

$$l_{R_{D_2}} = 2.65 - .836 = 1.814 \text{ in.}$$

$$l_{R_{D_3}} = 2.65 - 1.540 = 1.11 \text{ in.}$$

Total surface area occupied by ribs is

$$A_{R_D} = A_{R_{D_1}} + A_{R_{D_2}} + A_{R_{D_3}}$$

where $n_{F_1} = 12$ corresponds to l_{R_1}

$n_{F_2} = 12$ " " l_{R_2}

$n_{F_3} = 24$ " " l_{R_3}

The nominal rib width w_R is 0.085 in. :

$$A_{R_D} = .085\left[\overset{25.788}{(12)(2.149)} + \overset{21.768}{(12)(1.814)} + \overset{26.64}{(24)(1.11)}\right] = \underline{6.307 \text{ in}^2}$$

The prime dome heat transfer area is then

$$A_{H_{D_1}} = A_{TOT} - A_R = 19.978 - 6.307 = \underline{\underline{13.671}} \text{ in}^2$$

$A_{H_{D_1}}$ is the active area of the fluid side of P/N 852364. The secondary heat transfer area available is located on P/N 852363 & is separated from P/N 852364 by a gas conduction gap.

From the drawings, these gaps are:

radially, $\delta_R = .001$ in.

axially, $\delta_A = .0005$ in.

Flow rib height is uniform at $c = .040$ in.
This results in

$$a = 2.18725 - .04 = 2.147 \text{ in.}$$

$$b = 1.0936 - .04 = 1.054 \text{ in.}$$

$$\epsilon = \sqrt{1 - \left(\frac{1.054}{2.147}\right)^2} = .8713 \qquad \theta = 60.6^\circ$$

Total ellipsoidal base area for sleeve dome

$$A = \pi\left[(2.147)^2 + \frac{(1.054)^2}{(2)(.8713)} \ln\frac{1.8713}{.1287}\right] = 19.843 \text{ in.}^2$$

Rib arc lengths at the sleeve dome surface are:

X, in.	ϕ, deg	$E(60.6°, \phi)$	S(x)
.5002	13.58	.2353	.5052
.83	22.74	.3891	.8354
1.50	44.32	.7181	1.5418
2.147	90	1.2054	2.5880

$$l_{RD_1} = 2.588 - .505 = 2.083 \text{ in}$$

$$l_{RD_2} = 2.588 - .835 = 1.753 \text{ in.}$$

$$l_{RD_3} = 2.588 - 1.542 = 1.046 \text{ in.}$$

Area occupied by ribs on sleeve dome

$$A_{RD} = .085\left[(12)(2.083) + (12)(1.753) + (24)(1.046)\right] = 6.047 \text{ in.}^2$$

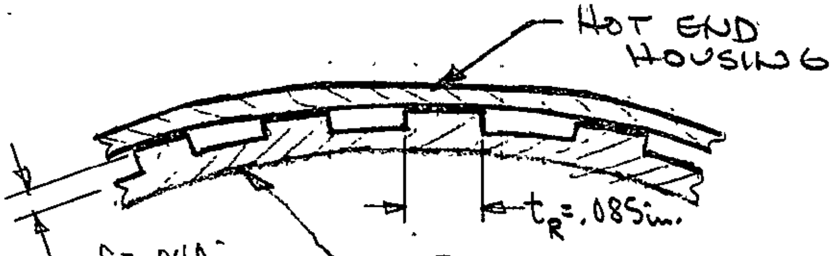
Rib wall areas

$$A_{RW} = .040\left[(24)\left(\frac{2.149+2.083}{2}\right) + (24)\left(\frac{1.814+1.753}{2}\right) + (48)\left(\frac{1.11+1.046}{2}\right)\right]$$

$$= 5.813 \text{ in}^2$$

Total secondary heat transfer area on dome:

$$A_{HD_2} = 19.843 - 6.047 + 5.813 = \underline{\underline{19.609 \text{ in}^2}}$$



2.2 Hot End Geometric Model

In analysis that follows, represent ellipsoidal dome as a disk and cylinder having the same maximum diameter as dome. Thus

$$\pi(r_o^2 - r_i^2) + 2\pi r_o L = A = 19.97 \text{ in}^2$$

where
$r_o = 2.187$ in.
$r_i = .5$ in.

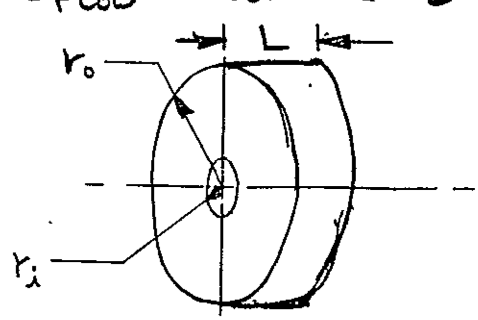
and

$$L = \frac{19.97 - \pi\left[(2.187)^2 - (.5)^2\right]}{2\pi(2.187)} = \underline{.418 \text{ in.}}$$

Check flow length:

$$L_{FLOW} = r_o - r_i + L = 2.187 - .5 + .418 = 2.105 \text{ in.}$$

Actual flow length (from p. 4) is 2.149 in.
Thus L_{FLOW} is within 2% of actual flow path.

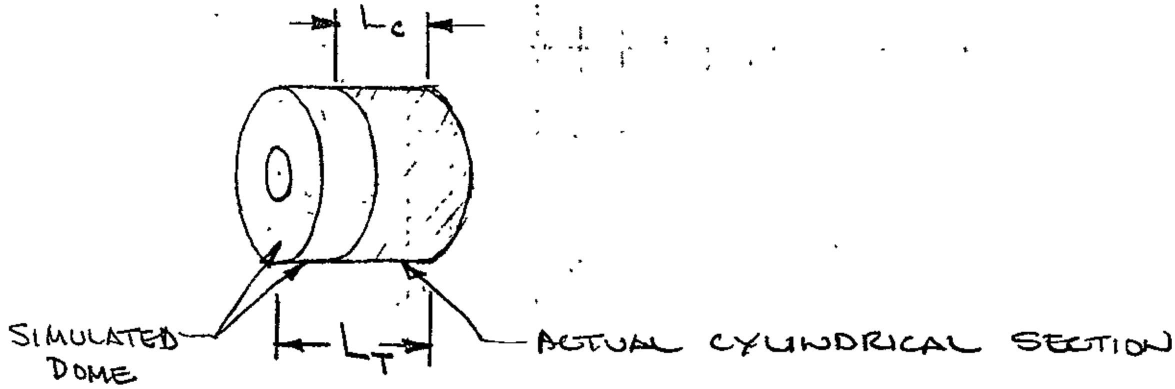


DISK & CYLINDER SIMULATION OF ELLIPSOIDAL DOME

2.3 Cylinder Heat Transfer Area

8/27

For total hot end area, consider:



Use only the portion of cylinder section from dome to center of heater support flange on hot end housing (P/N 852364). Heat transfer value of remaining housing wall decreases rapidly. Thus L_c is taken to be

$$L_c = \underline{.66 \text{ in.}}$$

Total cyl length is then

$$L_T = .66 + .42 = 1.08 \text{ in.}$$

Primary heat transfer area on actual cylinder

$$A_{H_{c_1}} = 2\pi r_o L_c - 48\, t_R L_c$$

$$= (.66)\left[2\pi(2.187) - (48)(.085)\right] = \underline{6.38 \text{ in}^2}$$

Total primary area

$$A_{H_1} = 13.67 + 6.38 = \underline{\underline{20.05 \text{ in}^2}}$$

Secondary heat transfer area on actual cylindrical section:

$$A_{H_{C_2}} = 2\pi(r_0 - c)L_c + 48 t_R L_c + (2)(48) c L_c$$

$$= 2L_c\left[\pi(r_0 - c) - 24(t_R - 2c)\right]$$

$$= (2)(.66)\left[\pi(\overset{2.147}{2.187 - .04}) - (24)(\overset{.005}{.085 - .080})\right]$$

$$= \underline{8.74 \text{ in}^2}$$

Total secondary heat transfer area:

$$A_{H_2} = 19.61 + 8.74 = \underline{\underline{28.35 \text{ in}^2}}$$

2.4 Analytical Expression of Areas

Expressions are needed to give flow & heat transfer areas and hydraulic diameter as a function of radius for use in heat transfer and pressure drop calculations for the dome region.

2.4.1 Flow Area

On disk surface

$$A_c = 2\pi r c - n_R t_R c$$

where n_R = no of flow ribs

$.501 < r < .836 \qquad n_R = 12$

$.836 < r < 1.540 \qquad n_R = 24$

$1.54 < r < 2.187 \qquad n_R = 48$

2.4.2 Heat Transfer Area

$$A_h = \pi(r^2 - r_i^2) - n_R t_R (r - r_i) + (n_R - n_R') t_R (r' - r_i)$$

where r' = point where no. of ribs changes from n_R' to n_R.

2.4.3 Hydraulic Diameter

$$D_h = \frac{4A_c}{P}$$

wetted perimeter is

$$P = 2(2\pi r - n_R t_R + n_R c)$$

$$= 2[2\pi r - n_R(t_R - c)]$$

$$D_h = \frac{2[2\pi rc - n_R t_R c]}{2\pi r - n_R(t_R - c)}$$

3. HEAT TRANSFER CALCULATIONS

Investigate Reynolds number over dome surface.

$$Re = \frac{V D_H \rho}{\mu} = \frac{\dot{w}}{\mu}\left(\frac{D_H}{A_c}\right)$$

$$D_H = \frac{4A_c}{P}$$

$$Re = \frac{\dot{w}}{\mu}\left(\frac{4}{P}\right)$$

$$\frac{4}{P} = \frac{4}{2\left[2\pi r - n_R(t_R - c)\right]}$$

$$= \frac{1}{\pi r - .5\, n_R (t_R - c)}$$

$$\frac{4}{P} = \frac{1}{\pi r - (.5)(.085 - .040) n_R} = \frac{1}{\pi r - .0225 n_R}$$

$$Re = \frac{\dot{w}/\mu}{\pi r - .0225 n_R}$$

Using $\dot{w} = \dot{w}_{AVG} \doteq .010$ lb/sec, $\mu = .097$ lb/ft-hr

$$Re = \frac{(.010)(12)(3600)}{(.097)(\pi r - .0225 n_R)}$$

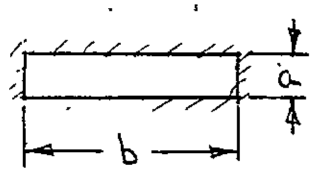
$$Re = \frac{4460}{\pi r - .0225 n_R}$$

r, in.	n_R	A_c, in^2	P/4	D_h	Re
.5	12	.0847	1.30	.0652	3430
.83	12	.1677	2.34	.0717	1905
.84	24	.1294	2.10	.0616	2125
1.54	24	.305	4.30	.0709	1037
1.55	48	.226	3.78	.0598	1180
2.19	48	.387	5.80	.0668	770

3.1 Heat Transfer Relation

Reynolds nos. indicate laminar regime — however, the flow passage geometry makes established laminar flow unlikely. For conservatism, use the limiting (established flow) Nusselt numbers.

Check variation of flow channel height/width ratio over dome.



Here, $a = c = .040$ in.

and $$2b = \frac{4(P/4)}{n_R - 1} - 2c$$

Thus $$\frac{a}{b} = \frac{2a}{2b} = \left[\frac{2(P/4)}{(n_R-1)c} - 1\right]^{-1} = \left[50\left(\frac{P/4}{n_R-1}\right) - 1\right]^{-1}$$

13/27

r	b/a	a/b
.5	4.9	.204
.83	9.6	.104
.84	3.56	.281
1.54	8.35	.12
1.55	3.02	.331
2.19	5.16	.194

For the range of a/b of interest here, it appears that the limiting Nusselt number is given by

$$Nu = 7.6 - 12(a/b)$$

Figure 13-11. (Ref 5)

On the ellipsoidal dome (prime surface only):

$$\overline{hA} = \int h\,dA_h$$

$$dA_h = d\left[\pi(r^2 - r_i^2) - n_R t_R (r - r_i) + (n_R - n_R')t_R(r' - r_i)\right]$$
$$= 2\pi r\,dr - n_R t_R\,dr$$
$$= (2\pi r - n_R t_R)\,dr$$

$$h = \frac{K}{D_h} Nu$$
$$= \frac{K\,(P/4)}{A_c} Nu$$

Since Nu varies from approx. 4 to 6.4 over the dome,

use $Nu = 5.2$

Then

$$\overline{hA} = \int_{r_i}^{r_o} \frac{5.2\,K\,(P/4)}{A_c}(2\pi r - n_R t_R)\,dr$$

$$= 5.2\,K \int_{r_i}^{r_o} \frac{\pi r - .0225\,n_R}{2\pi r c - n_R t_R c}(2\pi r - n_R t_R)\,dr$$

3.2 Prime Surface Heat Transfer

15/27

$$\overline{hA} = \frac{5.2\,K}{c}\int_{r_i}^{r_o}(\pi r - .0225\, n_R)\,dr$$

$$= \frac{5.2\,K}{c}\left[\frac{\pi}{2}r^2 - .0225\, n_R r\right]_{r_i}^{r_o}$$

$$\overline{hA} = \frac{5.2K}{c}\left[\frac{\pi}{2}(r_o^2 - r_i^2) - .0225\, n_R (r_o - r_i)\right]$$

Or, for the three regions for n_R

$$\overline{hA} = \frac{5.2\,K}{c}\left\{\frac{\pi}{2}(r_o^2 - r_i^2) - .0225\left[n_{R_3} r_o - (n_{R_3} - n_{R_2}) r_2 - (n_{R_2} - n_{R_1}) r_1 - n_{R_1} r_i\right]\right\}$$

where $r_1 = .836$ in $\quad n_{R_1} = 12$

$r_2 = 1.54$ in. $\quad n_{R_2} = 24$

$n_{R_3} = 48$

$$\overline{hA} = \overset{1.96}{\frac{(5.2)(.181)}{(.040)(12)}}\left\{\overset{7.11}{\frac{\pi}{2}}\left[\overset{4.53}{(2.187)^2 - (.5)^2}\right] - (.0225)\overset{1.17}{\left[(48)(2.187) - (48-24)(1.54) - (24-12)(.836) - (12)(.5)\right]}\right\}$$

$$= (1.96)\,(\overset{5.94}{7.11 - 1.17}) = 11.64\ B/hr\ {}^\circ R$$

For prime surface of cylindrical portion:

$$h = \frac{(5.2)(.181)(5.80)(12)}{(.387)} = 169.3\ B/hr\ ft^2\ {}^\circ R$$

Prime cyl heat trans area

$$A_h = \frac{1.08}{.66}(6.38) = 10.43 \text{ in}^2$$

$$hA = (169.3)\frac{10.43}{144} = 12.28\ B/hr\,°R$$

3.3 Secondary (Sleeve) Surface Heat Transfer

Total heat transferred to fluid is

$$q = q_1 + q_2$$

where q_1 is transferred at prime surface A_1
q_2 is transferred at secondary area A_2

$$q = h A_1 \Delta T + h \eta_2 A_2 \Delta T_2$$

Secondary surface receives heat conducted across sleeve gap δ; therefore

$$q_2 = \frac{K_g A_g}{\delta}(\Delta T - \Delta T_2) = h \eta_2 A_2 \Delta T_2$$

where K_g = gas thermal conductivity in gap
A_g = gap conduction area
δ = gap clearance

Temperature potential for heat transfer at the secondary surface is

$$\Delta T_2 = \frac{\frac{K_g A_g}{\delta}\,\Delta T}{h\eta_2 A_2 + \frac{K_g A_g}{\delta}} = \frac{1}{\frac{h\eta_2 A_2}{\left(\frac{K_g A_g}{\delta}\right)} + 1}\,\Delta T$$

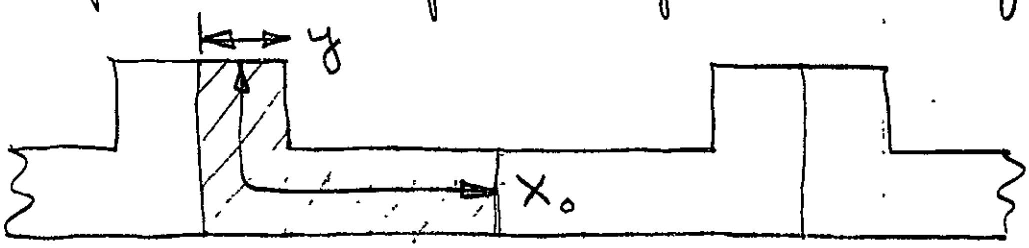
Thus, the total heat transfer is

$$q = \left[hA_1 + \frac{1}{\frac{1}{\eta_2 h A_2} + \frac{\delta}{K_g A_g}}\right]\Delta T$$

Or, for entire hot end

$$\overline{hA} = hA_1 + \frac{1}{\frac{1}{\eta_2 h A_2} + \frac{\delta}{K_g A_g}}$$

where η_2 is defined by considering sleeve surface as simple single sided fin:



$$\eta_2 = \frac{\tanh x_o\sqrt{\frac{h}{ky}}}{x_o\sqrt{\frac{h}{ky}}}$$

k = 12.2 B/hr-ft-°R
for Inco 718
@ 1630°R

Examine η_2 across disk-simulated dome

Use $x_o = b/2 + a = b/2 + .040$

$y = \frac{1}{2} t_R = \frac{1}{2}(.085) = .0425$ in.

r	x_o	h	$\sqrt{\ }$	$x_o\sqrt{\ }$	$\tanh x_o\sqrt{\ }$	η_2
.5	.138	173.3	65.3	.751	.6358	.847
.83	.232	151.8	62.3	1.204	.8349	.693
.84	.111	183.	67.1	.621	.5518	.888
1.54	.207	159.2	62.6	1.080	.7932	.734
1.55	.100	189.	68.2	.568	.5139	.905
2.19	.143	169.3	64.5	.768	.6457	.842

.770 .811 .874

Calculate area-weighted surface efficiency:

From r = .5 to .83 in. $\bar{\eta} = .770$

r = .83 to 1.54 in $\bar{\eta} = .811$

r = 1.54 to 2.19 in $\bar{\eta} = .874$

$$\bar{\eta}_2 = \frac{\overbrace{\left[(.83)^2-(.5)^2\right]}^{.44}(.770) + \overbrace{\left[(1.54)^2-(.83)^2\right]}^{1.69}(.811) + \overbrace{\left[(2.19)^2-(1.54)^2\right]}^{2.42}(.874)}{\underbrace{(2.19)^2-(.5)^2}_{4.55}}$$

$\bar{\eta}_2 = \underline{.840}$

3.4 Total Hot-End hA Product

For disk surface

$\eta_2 = .84$

$\delta = .0005$ in.

$A_g = A_{R_D} = 6.31$ in²

$(hA)_1 = 11.64$ B/hr °R $\{A_1 = 13.67$ in²

$A_2 = 19.61$ in²

For cylindrical surface

$\eta_2 = .842$

$\delta = .001$ in.

$A_g = (48)(.085)(1.08) = 4.40$ in²

$(hA)_1 = 12.28$ B/hr °R $\{A_1 = 10.43$ in² $h = 169.3$ B/hr ft² °R

$A_2 = 10.43 + (96)\overset{4.15}{(1.08)}(.040) = 14.58$ in²

Thus

$$(\overline{hA})_{disk} = 11.64 + \frac{1}{\dfrac{1}{(.84)\frac{11.64}{13.67}(19.61)} + \dfrac{(.0005)(12)}{(.181)(6.31)}} = 24.704 \text{ B/hr °R}$$

$$(\overline{hA})_{cyl} = 12.28 + \frac{1}{\dfrac{144}{(.842)(169.3)(14.58)} + \dfrac{(.001)(12)}{(.181)(4.40)}} = 24.135 \text{ B/hr °R}$$

$$(\overline{hA})_{total} = 24.704 + 24.135 = \underline{\underline{48.84 \text{ B/hr °R}}}$$

4. PRESSURE DROP CALCULATIONS

4.1 Displacer Volume Ports

Flow must pass through 12 - .094 in. dia holes.

$$A = (12)\frac{\pi}{4}(.094)^2 = .0833 \text{ in}^2$$

Flow area in dome passages at r = .5 in. is .0847 in^2 (see §3.0). Associated velocity is

$$V = \frac{\dot{w}}{\rho A} \qquad \text{Use } \dot{w} = \dot{w}_{max} = .0157 \text{ lbm/sec}$$

$$\rho = .183 \text{ lbm/ft}^3$$

$$V = \frac{(.0157)(144)}{(.183)}\frac{1}{A} = \frac{12.35}{A} \qquad A \text{ in in}^2.$$

$$V = \frac{12.35}{.0847} = 146 \text{ ft/sec}$$

For flow into displacer

$$\Delta P_{IN} = \frac{\rho V^2}{2g_c} = \frac{(.183)(146)^2}{(64.4)(144)} = .42 \text{ psi}$$

For flow out of displacer region

$$\Delta P_{out} = 0.5\frac{\rho V^2}{2g_c} = .21 \text{ psi}$$

4.2 Ellipsoidal Dome

Consider disk & cylinder approximation of dome as earlier. For disk region

$$\Delta p = \int_{x_1}^{x_2} 4 \frac{f}{D_h} \frac{\rho V^2}{2g_c} dx$$

As noted earlier, flow is in laminar regime. Use, for friction factor:

$$f = \frac{17}{Re}$$

Since

$$Re = \frac{\dot{w}}{\mu} \frac{D_h}{A_c}$$

$$V = \frac{\dot{w}}{\rho A_c}$$

$$D_h = \frac{4A_c}{P} \qquad P = \text{wetted perimeter}$$

$$\Delta p = \frac{(2)(17)}{16 g_c} \frac{\dot{w}\mu}{\rho} \int_{x_1}^{x_2} \frac{P^2}{A^3} dx$$

$$= \frac{(2)(17)(12)}{(16)(32.2)(3600)} \frac{(.0157)(.097)}{.183} \int_{r_i}^{r_o} \frac{4\left[2\pi r - n_R(t_R - c)\right]^2}{(2\pi r c - n_R t_R c)^3} dr$$

$$\Delta p = 7.32 \times 10^{-6} \left[I_1 + I_2 + I_3 \right]$$

where

$$I_1 = \int_{r_i}^{r_1} \frac{[2\pi r - n_{R_1}(t_R - c)]^2}{(2\pi r c - n_{R_1} t_R c)^3} dr$$

$$I_2 = \int_{r_1}^{r_2} \frac{[2\pi r - n_{R_2}(t_R - c)]^2}{(2\pi r c - n_{R_2} t_R c)^3} dr$$

$$I_3 = \int_{r_2}^{r_0} \frac{[2\pi r - n_{R_3}(t_R - c)]^2}{(2\pi r c - n_{R_3} t_R c)^3} dr$$

These integrals are of form:

$$\int \frac{(a+bx)^m}{(a'+b'x)^n} dx$$

By use of reduction formulas, this is found to yield

$$\int_{x_1}^{x_2} \frac{(a+bx)^2}{(a'+b'x)^3} dx = \frac{1}{b'} \left\{ \left(\frac{b}{b'}\right)^2 \ln(a'+b'x) - \left(\frac{a+bx}{a'+b'x}\right)\left[\frac{1}{2}\left(\frac{a+bx}{a'+b'x}\right) + \frac{b}{b'}\right] \right\}_{x_1}^{x_2}$$

$$= \frac{1}{b'}\left[A^2 \ln B + \frac{1}{2}(C^2 - D^2) + A(C-D)\right]$$

where

$A = b/b'$

$B = \dfrac{a'+b'x_2}{a'+b'x_1}$

$C = \dfrac{a+bx_1}{a'+b''x_1}$

$D = \dfrac{a+bx_2}{a'+b'x_2}$

For I_1:

$x_1 = .5 \qquad x_2 = .83$

$a = -n_{R_1}(t_R - c) = -(12)(.085 - .040) = -.54$

$b = 2\pi = 6.28$

$a' = -n_{R_1} t_R c = -(12)(.085)(.040) = -.041$

$b' = 2\pi c = .251$

$A = 25.02$

$B = 1.9802$

$C = 30.769$

$D = 27.923$

$$I_1 = \frac{1}{.251}\left[(25.02)^2 \ln(1.9802) + \frac{(30.769)^2 - (27.923)^2}{2} + (25.02)(30.769 - 27.923)\right]$$

$= 2320.35$

For I_2

$x_1 = .83 \qquad x_2 = 1.54$

$a = -1.08$

$b = 6.28$

$a' = -.082$

$b' = .251$

$A = 25.02$

$B = 3.709$

$C = 32.711$

$D = 28.210$

$$I_2 = \frac{1}{.251}\left[(25.02)^2 \ln(3.709) + \frac{(32.711)^2 - (28.210)^2}{2} + (25.02)(32.711 - 28.210)\right]$$

$$= 4263.97$$

25/27

For I_3:

$x_1 = 1.54$ $\qquad$ $x_2 = 2.19$

$a = -2.16$

$b = 6.28$

$a' = -.164$

$b' = .251$

$A = 25.02$

$B = 1.74$

$C = 33.752$

$D = 30.00$

$$I_3 = \frac{1}{.251}\left[(25.02)^2 \ln(1.74) + \frac{(33.752)^2 - (30.00)^2}{2} + (25.02)(33.752 - 30.00)\right]$$

$= 2231.90$

Pressure drop:

$$\Delta p = (7.32 \times 10^{-6})(\underbrace{2320.35 + 4263.97 + 2231.90}_{8816.22})$$

$= .0645$ psi

4.3 Cylindrical Section

At $r = 2.19$ in., $Re = 770$

$$f = \frac{17}{770} = .0221$$

$$A_c = .387 \text{ in}^2$$

$$V = \frac{12.35}{.387} = 31.9 \text{ fps}$$

$$D_h = .0668 \text{ in.}$$

Flow length:

L = Actual length + simulated dome cyl

= .85 + .42 = 1.27 in.

$$\Delta p = (4)\frac{(.0221)(1.27)\;(.183)(31.9)^2}{(.0668)\;\;(64.4)(144)} = .0337 \text{ psi}$$

4.4 Saw-cut Transitions

$$\Delta p_{OUT} = \frac{\rho V^2}{2g_c} = \frac{(.183)(31.9)^2}{(64.4)(144)} = .0202 \text{ psi}$$

$$\Delta p_{IN} = .5\frac{\rho V^2}{2g_c} = .0101 \text{ psi}$$

4.5 Pressure Drop Summary

For flow into displacer region

$$\Delta p_{TOT_{IN}} = .42 + .065 + .034 + .01 = \underline{.529} \text{ psi}$$

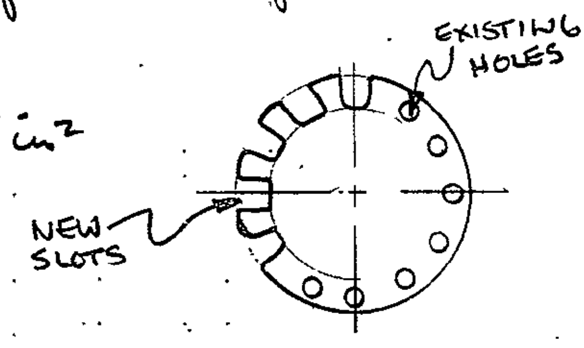
For flow out of displacer region

$$\Delta p_{TOT_{OUT}} = .21 + .065 + .034 + .02 = \underline{.329} \text{ psi}$$

The major contributor is the ports at the inlet to the displacer.

By replacing the 12 holes with .14 x .14 in. slots on the periphery of item 2 of P/N 852387, flow area is increased to

$$A_c = (12)(.14)^2 = .235 \text{ in}^2$$



Thus $V = 52.6$ fps

and $$\Delta p_{IN} = \left(\frac{52.6}{146}\right)^2 (.42) = .0545 \text{ psi}$$

$$\Delta p_{OUT} = .0272 \text{ psi}$$

Then

$$\Delta p_{TOT_{IN}} = \underline{.164 \text{ psi}}$$

$$\Delta p_{TOT_{OUT}} = \underline{.146 \text{ psi}}$$

$\underline{0.155}$ psi ave

4.6 Modification of Hot Volume Porting

Since rework of existing port provisions of P/N 852364 Housing Assy would be difficult, rework of the P/N 852363 Sleeve is preferable.

To avoid flow distribution problems while allowing support of the sleeve on the port disc of the housing assy, a slot should be milled in each of 12 flow channels at the edge of the existing 1.0003 in. central hole.

Slot width (or cutter dia) should be as wide as permitted by the flow channel ribs. Maximum rib width is .10 in. Thus, the minimum channel width at the hole periphery is

$$W_{MIN} = \frac{1.0003\pi - (12)(.10)}{12} \doteq .162 \text{ in.}$$

Therefore, use cutter dia:

$$D_{cutter} = 5/32 \text{ in.} = .156 \text{ in.}$$

From §4.5, it was seen that a port flow area of .24 in² was beneficial. For the modification considered here, assume that flow area is increased by .25 in² by the slots to be added. Since existing area provided by the 12 - .094 in. dia holes is .0833 in², the total area is now

$$A_c = .0833 + .25 = .3333 \text{ in}^2$$

Velocity is then

$$V = \frac{12.35}{.3333} = 41.2 \text{ fps}$$

Giving:

$$\Delta P_{IN} = \left(\frac{41.2}{14.6}\right)^2 (.42) = .033 \text{ psi}$$

$$\Delta P_{OUT} = .017 \text{ psi}$$

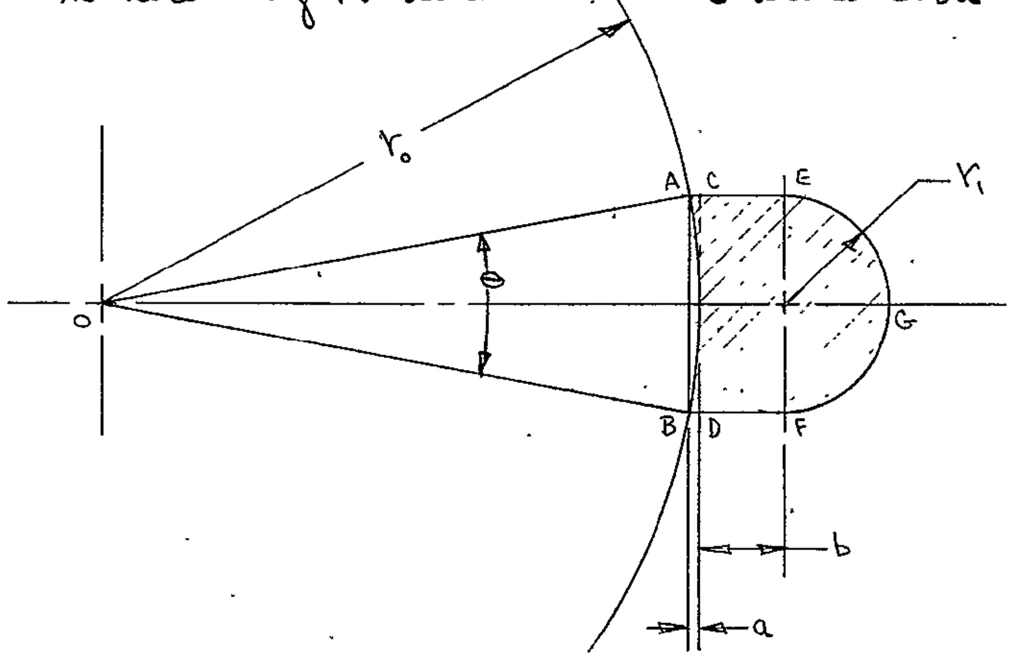
and

$$\Delta P_{TOT_{IN}} = \underline{.142 \text{ psi}}$$ for flow <u>into</u> hot vol.

$$\Delta P_{TOT_{OUT}} = \underline{.136 \text{ psi}}$$ for flow <u>out</u> of hot vol.

This is satisfactory. Now slot length must be determined to give A = .25 in².

Assume slots are cut from central hole periphery as sketched here. Since r_1 has been selected, it is necessary to determine the dimension b:



Areas:

Triangle OAB $\quad \frac{1}{2}(2r_1)(r_o \cos\frac{\theta}{2}) = r_1 r_o \cos\frac{\theta}{2}$

Rectangle ABDC $\quad (2r_1)(r_o - r_o\cos\frac{\theta}{2}) = 2r_1 r_o(1-\cos\frac{\theta}{2})$

Rectangle CDFE $\quad (2r_1)(b) = 2r_1 b$

Semicircle EFG $\quad \frac{1}{2}\pi r_1^2$

Sector OAB $\quad \pi r_o^2 \frac{\theta}{360}$

Shaded Area:

$$A_1 = r_1 r_o \cos\frac{\theta}{2} + 2r_1 r_o\left(1-\cos\frac{\theta}{2}\right) + 2r_1 b + \frac{\pi}{2}r_1^2 - \frac{\pi\theta}{360}r_o^2$$

$$= 2r_1 r_o - r_1 r_o \cos\frac{\theta}{2} + 2r_1 b + \frac{\pi}{2}r_1^2 - \frac{\pi\theta}{360}r_o^2$$

$$A_1 = r_1 r_0 \left(2 - \cos\frac{\theta}{2}\right) + 2r_1 b + \frac{\pi}{2}\left(r_1^2 - \frac{\theta}{180} r_0^2\right)$$

12 slots are req'd:

$$A_T = 12A_1 = 12\left[r_1 r_0 \left(2 - \cos\frac{\theta}{2}\right) + 2r_1 b + \frac{\pi}{2}\left(r_1^2 - \frac{\theta}{180} r_0^2\right)\right]$$

Find dimension b:

$$2r_1 b = \frac{A_T}{12} - r_1 r_0\left(2 - \cos\frac{\theta}{2}\right) - \frac{\pi}{2}\left(r_1^2 - \frac{\theta}{180} r_0^2\right)$$

$$b = \frac{1}{2r_1}\left[\frac{A_T}{12} - r_1 r_0\left(2 - \cos\frac{\theta}{2}\right) - \frac{\pi}{2}\left(r_1^2 - \frac{\theta}{180} r_0^2\right)\right]$$

Since

$$r_1 = r_0 \sin\frac{\theta}{2}$$

$$\theta = 2\sin^{-1}\left(\frac{r_1}{r_0}\right)$$

From Dwg 852363, $r_0 = .50015$ in.

If 5/32 cutter (.156 in. dia) is used, $r_1 = .078$ in.

$$\theta = 2\sin^{-1}\left(\frac{.078}{.50015}\right) = 2\sin^{-1}(.1560) = 2(8.975^\circ) = 17.95^\circ$$

$$\cos\frac{\theta}{2} = \cos 8.98^\circ = .9849$$

Thus, for $A_T = .25$ in^2:

$$b = \frac{1}{(2)(.078)}\left\{\frac{.25}{12} - (.078)(.50015)(2 - .9849) - \frac{\pi}{2}\left[(.078)^2 - \frac{17.95}{180}(.50015)^2\right]\right\}$$

$= .0696$ in. OK Use $\underline{b = .07 \text{ in}}$

As an alternative, holes may be used — this may be better for installation of the sleeve in the hot end housing. To provide a .25 in² flow area, the needed hole diameter is

$$D = \sqrt{\frac{.25}{(12)(\pi/4)}} = \sqrt{\frac{.25}{3\pi}} = .163 \text{ in.}$$

Use $D = 11/64$ in. $= .172$ in.

Then

$$A = 3\pi D^2 = 3\pi(.172)^2 = .279 \text{ in}^2$$

and

$$A_c = .279 + .083 = .362 \text{ in}^2$$

$$V = \frac{12.35}{.362} = 34.2 \text{ fps}$$

$$\Delta P_{IN} = \left(\frac{34.2}{146}\right)^2 (.42) = .023 \text{ psi}$$

$$\Delta P_{OUT} = .012 \text{ psi}$$

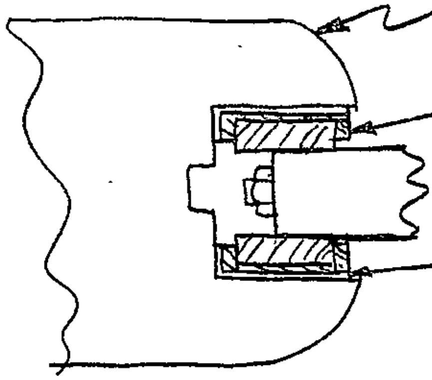
$$\left.\begin{array}{l} \Delta P_{TOT_{IN}} = .132 \text{ psi} \\ \\ \Delta P_{TOT_{OUT}} = .131 \text{ psi} \end{array}\right\} \underline{\text{OK}}$$

4.7 Investigation of Hot End Bearing Cavity Ports

Hot displacer includes a cavity into which the axial bearing surfaces are installed, as sketched here:

$T = 1630°R$
$\rho^* = .183 \text{ lb/ft}^3$
$\mu^* = .097 \text{ lb/ft-hr}$

*at $p = 800$ psia



This analysis is to verify that the peripheral grooves provided on the displacer bearing retainer periphery are adequate to permit flow into & out of the bearing cavity without large pressure losses.

4.7.1 Bearing Cavity Volume

As a first step, it is desirable to define cavity volume as a function of engine crank angle.

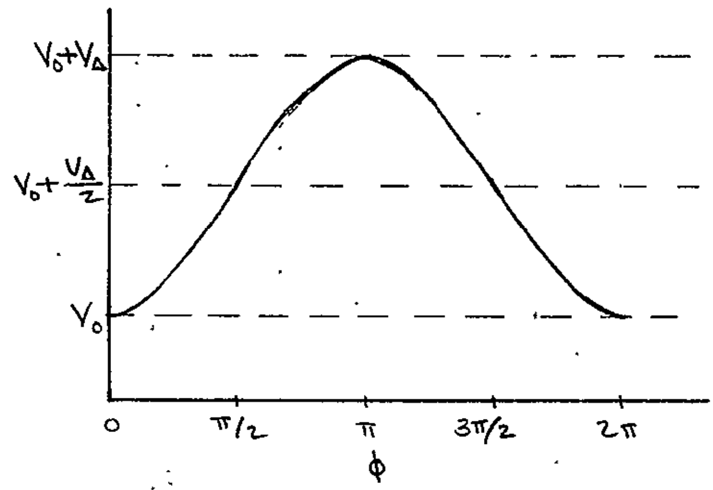
From J. Steeneken 12-15-71, the hot end bearing space dead volume is

$$V_0 = \begin{Bmatrix} .122 \ (\text{max}) \\ .104 \ (\text{min}) \end{Bmatrix} \text{in}^3$$

Displacement of bearing space is (for .60 in stroke)

$$V_\Delta = \frac{\pi}{4}(.8522)^2(.60) = .342 \text{ in}^3$$

$$V_B = V_0 + \frac{V_\Delta}{2}\left[1 + \sin\left(\phi - \frac{\pi}{2}\right)\right]$$



Since ϕ is such that $V_{B_{MIN}}$ occurs at $\phi = 0$ & $V_{B_{MAX}}$ at $\phi = \pi$, it appears that the engine crank angle θ is related by $\theta = \phi - \pi/2$

Thus

$$V_B = V_0 + \frac{V_\Delta}{2}\left[1 + \sin\theta\right]$$

Or, using the max value of V_0:

$$V_B = .122 + .171\,(1 + \sin\theta)\ ,\ in^3$$

4.7.2 Cavity Ideal Flow

Based on this cyclic volume variation, determine the ideal flow rate characteristic for the bearing space for the case of frictionless flow. The max flow thus determined can then be used to evaluate the peak pressure losses for the existing design.

$$\dot{w} = \frac{dm}{dt} = \frac{dm}{d\theta}\,\frac{d\theta}{dt} = \frac{2\pi N}{60}\,\frac{dm}{d\theta} \qquad \left\{ \begin{array}{l} N = \text{rot. speed} \\ \quad \text{rpm} \end{array} \right.$$

Since $m = \dfrac{pV_B}{ZRT}$

$$\dot{w} = \frac{2\pi N}{60}\left[\frac{p}{ZRT}\frac{dV_B}{d\theta} + \frac{V_B}{ZRT}\frac{dp}{d\theta}\right]$$

$$= \frac{\pi N}{30ZRT}\left[.171\,p\cos\theta + (.293 + .171\sin\theta)\frac{dp}{d\theta}\right]$$

The results from the AiR VM program can be used to determine $\Delta p/\Delta\theta$ as a function of θ

At $N = 400$ rpm

$T = 1630°R$ $(Z \simeq 1.0)$

$$\dot{w} = \frac{\pi(400)}{(12)(30)(386)(1630)}\left[.171\,p\cos\theta + (.293+.171\sin\theta)\frac{dp}{d\theta}\right]$$

$$= 5.55\times10^{-6}\Big[\underbrace{.171\,p\cos\theta}_{\text{TERM 1}} + \underbrace{(.293+.171\sin\theta)\frac{dp}{d\theta}}_{\text{TERM 2}}\Big], \frac{lb}{sec} \quad \left\{ p \text{ in psi} \right.$$

Using $\bar{p}$ & $\frac{\Delta p}{\Delta\theta}$ from the 22 Oct 1971 computer run, $\dot{w}$ can be calculated and plotted as on the following two pages.

From the plot, it appears that

$\dot{w}_{MAX} = .00083$ lb/sec

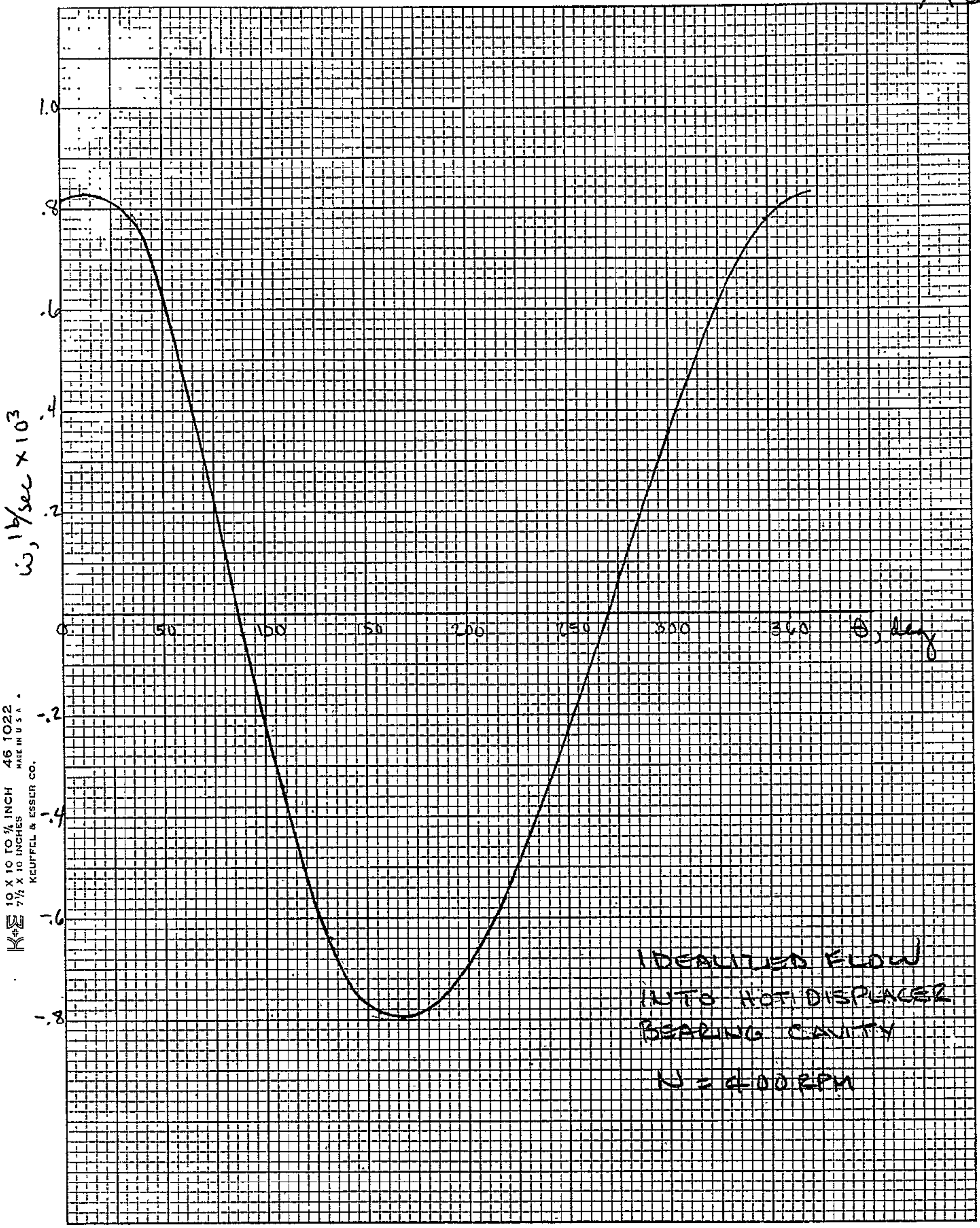
is a reasonable value to use in examining the pressure drop.

Based on VM program run dated 22 Oct 71:

θ deg	$\bar{\theta}$ deg	$\bar{p}$ psia	$\frac{\Delta p}{\Delta \theta}$	TERM 1	TERM 2	$\dot{w}$ lbm/sec
	12	759.13	68.80	126.97	22.60	.000830
24						
	36	784.71	53.10	122.58	19.25	.000787
48						
	60	801.08	24.95	68.49	11.01	.000441
72						
	84	804.19	-10.01	14.37	-4.64	.000054
96						
	108	793.23	-42.30	-41.92	-19.27	-.000340
120						
	132	771.02	-63.80	-88.22	-26.80	-.000638
144						
	156	742.78	-71.0	-116.03	-25.74	-.000787
168						
	180	714.22	-65.4	-122.13	-19.16	-.000784
192						
	204	690.05	-50.0	-107.80	-11.17	-.000660
216						
	228	673.47	-29.05	-77.06	-4.82	-.000454
240						
	252	666.27	-5.26	-35.21	-.69	-.000199
264						
	276	669.11	18.90	11.96	2.32	.000079
288						
	300	681.75	41.40	52.29	6.00	.000357
312						
	324	702.91	59.60	97.24	11.47	.000603
336						
	348	730.05	70.0	122.11	18.02	.000778
360						

$\bar{p}$ & $\frac{\Delta p}{\Delta \theta}$ taken from crank sump calcs.

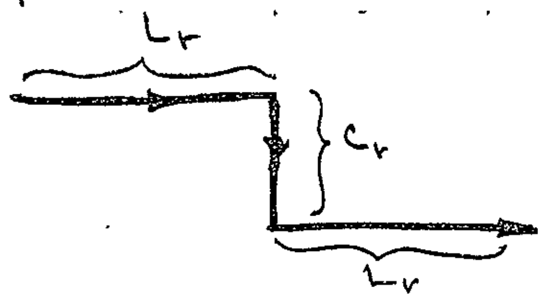
6/16



1.0
.8
.6
.4
.2
-.2
-.4
-.6
-.8
0
50
100
150
200
250
300
360
θ, deg
ẇ, lb/sec × 10³
IDEALIZED FLOW
INTO HOT DISPLACER
BEARING CAVITY
N = 400RPM
10 X 10 TO ½ INCH 46 1022
7½ X 10 INCHES MADE IN U.S.A.
KEUFFEL & ESSER CO.

4.7.3 Flow Path Model

Since the bearing retainer is a two-piece arrangement, with no angular keying, it cannot be assumed that the flow grooves in the two halves are aligned. The grooves are located at 60° intervals around periphery. For a worst case analysis, assume flow path is :



L_r = retainer length

C_r = Distance equiv to 30° increment at at retainer periphery.

$L_r = .635$ in.

$C_r = \pi D \frac{30}{360} = \frac{\pi(1.375)}{12} = .36$ in.

Slot flow area:

Min dimensions are .020 x .030 in.

$A_{slots} = (6)(.02)(.03) = .0036 \text{ in}^2$

Flow area at gap between retainer halves:

Min axial gap is .040 in.

Use radial dimension = .020 in. (as for slots)

$$A_{gap} = (.04)(.02) = .0008 \text{ in}^2$$

Flow area at bearing clearance:

Male half, max dia = .8500 in

Female half, min dia = .8521 in

$$A_{brg} = \frac{\pi}{4}\left[(.8521)^2 - (.85)^2\right] = .0028 \text{ in}^2$$

Flow area at retainer periphery clearance:

Retainer, max dia = 1.3749 in.

Displacer cavity, min dia = 1.3750 in.

$$A_{ret.} = \frac{\pi}{4}\left[(1.3750)^2 - (1.3749)^2\right] = .0002 \text{ in}^2$$

From above, it appears that the retainer and bearing clearances provide almost as much flow area as the peripheral grooves. However, the associated hydraulic diameters are much different. For the slots

$$D_h = \frac{4(.0036)}{(6)(2)(.024+.03)} = .024 \text{ in.}$$

For the bearing clearance

$$D_h = \frac{4(.0028)}{2\pi(.852)} = .0021 \text{ in.}$$

Thus, relatively little flow will pass through the clearances.

4.7.4 Original Design Pressure Drop

Conservatively neglecting the clearance flow areas & using only the 6 slots:

$$V = \frac{\dot{w}}{\rho A} = \frac{(.00083)(144)}{(.183)(.0036)} = \frac{.643}{.0036} = 181.5 \text{ fps}$$

$$R_e = \frac{(181.5)(.024)(.183)}{(12)(.097)}(3600) = (566)(181.5)(.024)$$

$$= 2460$$

Laminar flow, in transition

Say, $f = .007$

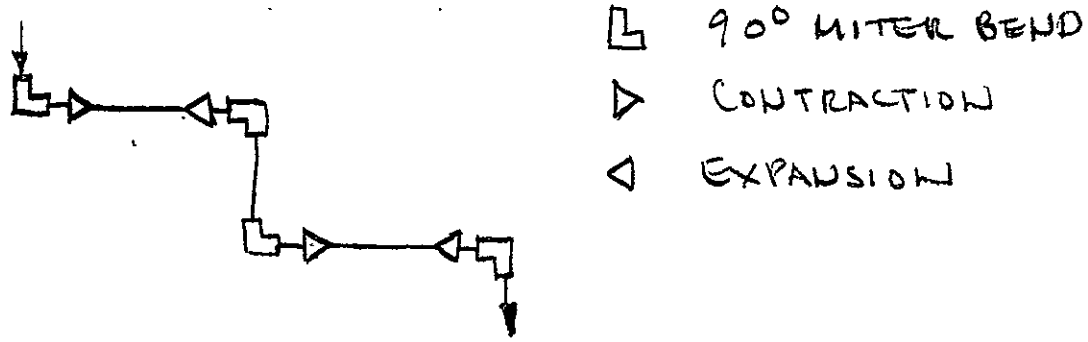
For each flow stream (of the total of six) the retainer gap flow area is of same order as slot area, but slightly larger:

i.e., $.0006 \text{ in}^2$ vs. $.0008 \text{ in}^2$

Assume same f & D_h applies to both slot and gap. Total flow length is thus

$$L = (2)(.635) + .36 = 1.63 \text{ in},$$

In addition to friction loss, turns, contractions and expansions as follows should be allowed:



Define loss coefficients, $K = \dfrac{\Delta p}{\left(\dfrac{\rho V^2}{2g_c}\right)}$

	K
Miter bends	1.25
Contractions	
$A_{MIN}/A_{MAX} = 0$	.5
$A_{MIN}/A_{MAX} = \dfrac{.0006}{.0008} = .75$	.07
Expansions	
$A_{MIN}/A_{MAX} = 0$	1.0
$A_{MIN}/A_{MAX} = .75$	.06
	1.63

$$\Sigma K = (4)(1.25) + .5 + .07 + 1.0 + .06 = 6.63$$

Total pressure drop

$$\Delta p = \left[4\frac{fL}{D_h} + \Sigma K\right]\frac{\rho V^2}{2g_c}$$

$$= \left[\frac{(4)(.007)\overset{1.9}{(1.63)}}{.024} + 6.63\right]\frac{(.183)(181.5)^2}{(64.4)(144)}$$

$$\Delta p = (8.53)(.65) = 5.5 \text{ psi}$$ TOO HIGH

4.7.5 Design Modifications

Try widening slots to, say, .060 in.

$$A_{slots} = (6)(.02)(.06) = .0072 \text{ in}^2$$

$$V = \frac{.643}{.0072} = 90.8 \text{ fps}$$

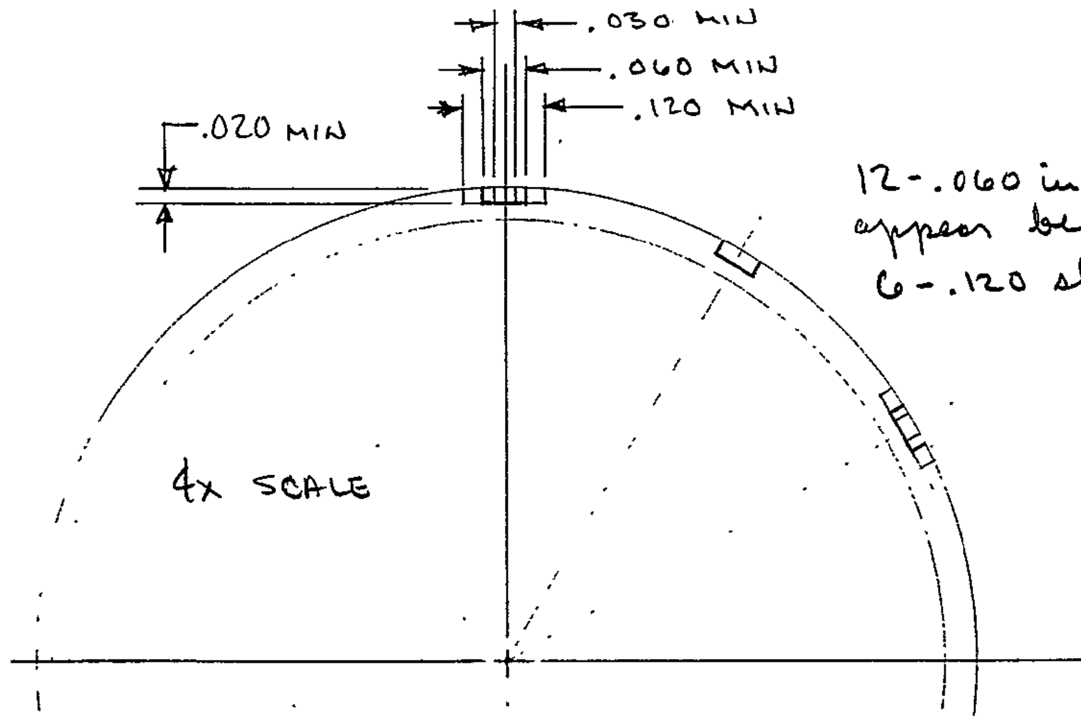
$$D_h = \frac{(4)(.0072)}{(6)(2)(.06+.02)} = .030 \text{ in.}$$

$$Re = (566)(90.8)(.030) = 1540$$

$$f = .010$$

$$\Delta p = \left[\frac{\overset{2.17}{(4)(.01)(1.63)}}{.030} + 6.63\right]\frac{\overset{.1427}{(.183)(90.8)^2}}{(144)(64.4)} = 1.43 \text{ psi}$$

Still too high – keep slot width at .060 in., but increase number of slots to 12.



12 - .060 in. slots appear better than 6 - .120 slots.

Using 12 .060 in. slots, the previous flow area is doubled:

$A_{slots} = .0144 \text{ in}^2$

$V = 45.4 \text{ fps}$

$D_h = \dfrac{(4)(.0144)}{(12)(2)(.06+.02)} = .030 \text{ in.}$

$Re = 770$

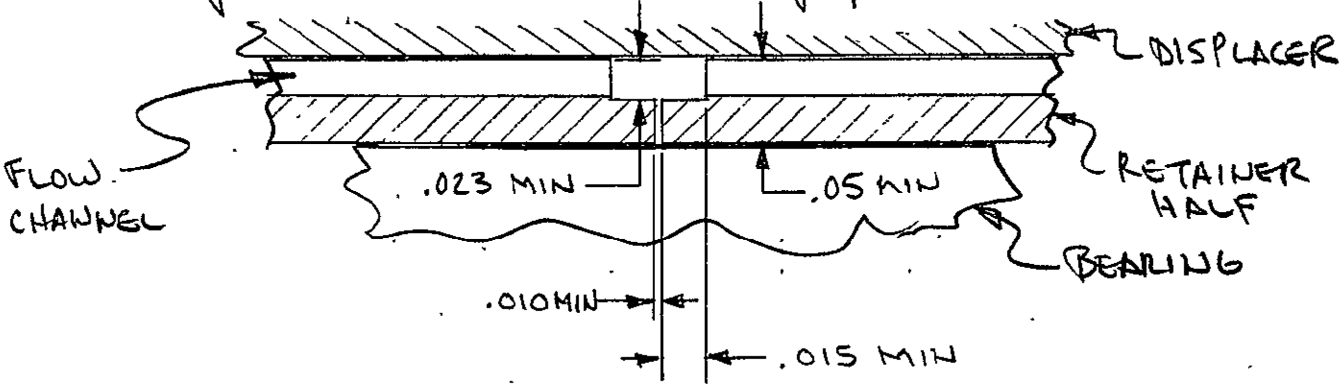
$f = .021$

$c_r = .18 \text{ in.} \longrightarrow L = 1.45 \text{ in}$

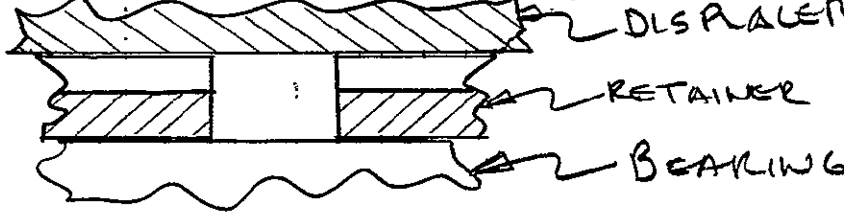
$$\Delta p = \left[\frac{\overset{4.06}{(4)}\overset{10.69}{(.021)(1.45)}}{.03} + 6.63\right]\frac{\overset{.0407}{(.183)(45.4)^2}}{(144)(64.4)} = .435 \text{ psi}$$

In the foregoing it has been assumed that the gap between retainer halves is modified to provide flow area area around the periphery equal to the cross-section of a single slot or better.

However, losses can be further reduced if the gap is large enough to be considered as a plenum — then, the two miter bend losses in this area can be removed from the loss summation. At present, the retainer gap is as below:



The simplest approach to increasing gap area would be to shorten retainer length:



Based on the .06 width by .02 depth, a single slot has

$$A_c = (.06)(.02) = .0012 \text{ in.}$$

The gap should have a cross-section about an order of magnitude greater than this; say $A = .01 \text{ in}^2$

Since the retainer thickness is .05 in., a gap of

$$L_G = \frac{.01}{.05} = .2 \text{ in.}$$

is required.

Since the existing gap is .01 in., this can be obtained by removing

$$\Delta_R = \frac{.20 - .01}{2} = .095 \text{ in.}$$

from each retainer half.

Now, at the gap, we have expansion into a passage where

$$\frac{A_{MIN}}{A_{MAX}} = \frac{.0012}{.01} = .12$$

for which $K_E = .78$

Contraction back to .0012 in^2 results in

$K_c = .42$

The summation (see 4.7.4) is now

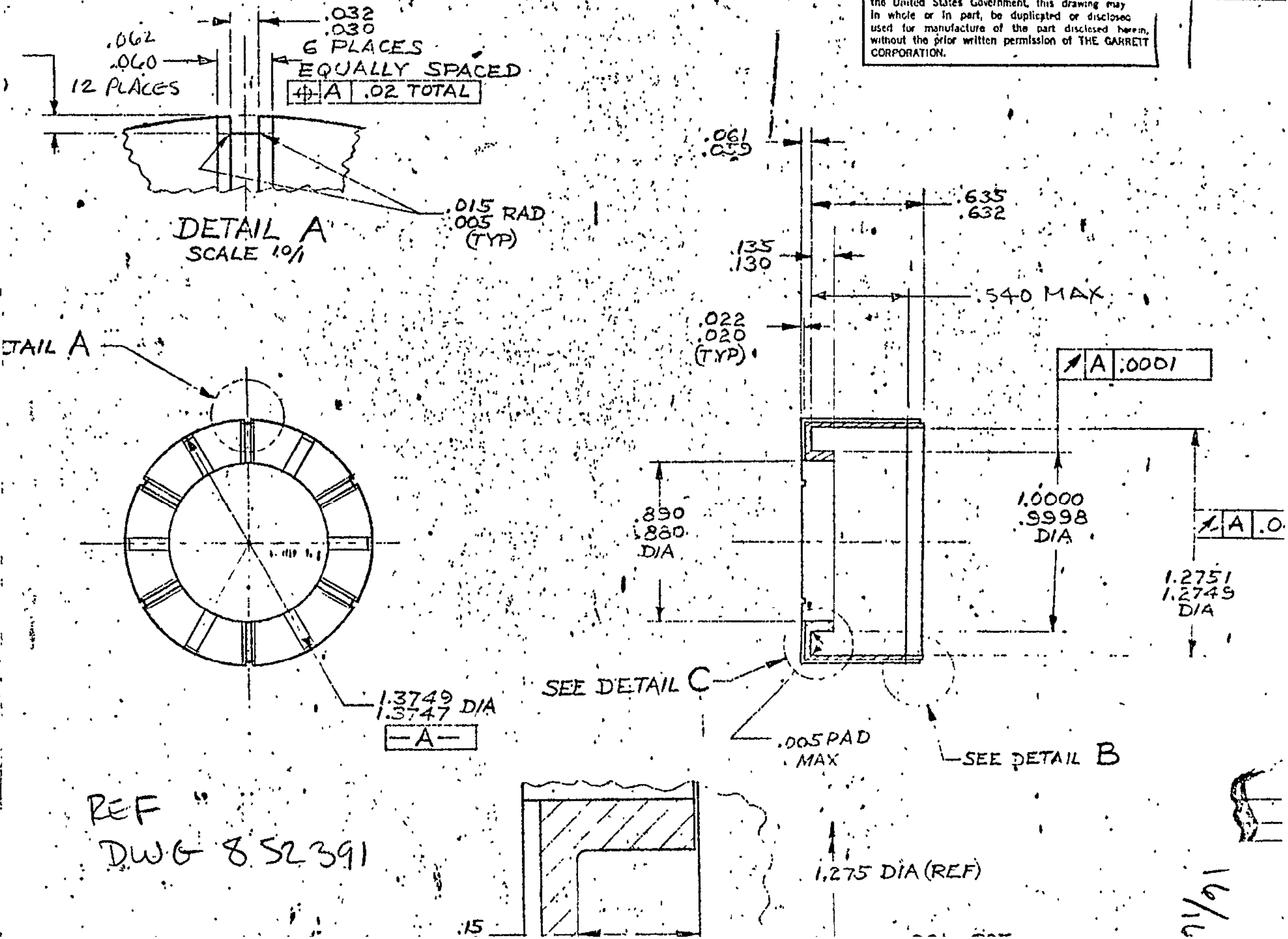
$$\Sigma K = (2)(1.25) + 1.5 + .78 + .42 = 5.2$$

Pressure drop is now

$$\Delta p = (\overset{9.26}{4.06 + 5.2})(.0407) = \underline{\underline{.377 \text{ psi}}}$$

While further reduction may be desirable, this would be difficult with present retainer & cavity design. Note, however that the above figure reflects the various clearance flow paths available.

LTR



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SECTION 12

HOT-END SEAL LEAKAGE

INTRODUCTION

The hot-end seal functions to control the leakage rate of the working fluid which bypasses the hot regenerator. Leakage bypasses the regenerator by flowing through the annular space between the hot displacer and the inner wall of the regenerator. Excessive leakage results in a loss in thermal performance; therefore hot end seal design is an important consideration.

At low leakage rates, the hot displacer and cylinder walls effectively regenerate the leakage fluid temperatures and the resulting thermal losses are small. As the leakage rates increase, the walls can no longer function as an effective regenerator; thus significant losses in overall thermal performance result.

DESIGN CONFIGURATION

The hot-end seal configuration (Figure 12-1) consists of the following two elements, from the hot end toward the sump end: (1) a 5-groove labyrinth seal with a tip clearance of 0.0025 in., a groove spacing of 0.05 in., and a nominal tip width of 0.005 in. and (2) a 1.0 in. long close fit annular seal with a clearance between the inner wall of the regenerator and the seal of 0.0025 in. maximum.

These sealing elements are similar to those used in the cold-end seal except for the diameter. One major difference is the use of linear bearings as part of the sealing system for the cold end. The bearings are not used in the hot-end seal and thus the performance of this seal is independent of wear (or operational time).

It should be noted that the seal is located in the highest temperature region possible within the machine. The leakage rate is an inverse function of the temperature; therefore, this location minimizes the leakage rate for a given seal configuration. The seal design selected provides very low losses as discussed in the following paragraphs.

METHOD OF ANALYSIS

The method of analysis is identical to that used for the cold-end seals.

PERFORMANCE CHARACTERISTICS

Figure 12-2 gives the hot-end seal leakage rate as a function of the pressure drop across the seal. The design limit pressure drop and the associated leakage rate shown in Figure 12-2 corresponds to the maximum pressure drop across the seal. This pressure drop includes the maximum pressure drops across: (1) sump ports to backside of hot displacer, (2) Section 1 of the sump heat exchanger, (3) interface between sump heat exchanger and hot regenerator, (4) hot regenerator and (5) the hot-end heat exchanger. Due to the use of maximum pressure drops--pressure drops corresponding to the maximum flow rates at a rotational speed of 400 rpm-- the indicated design leakage rate is conservative.

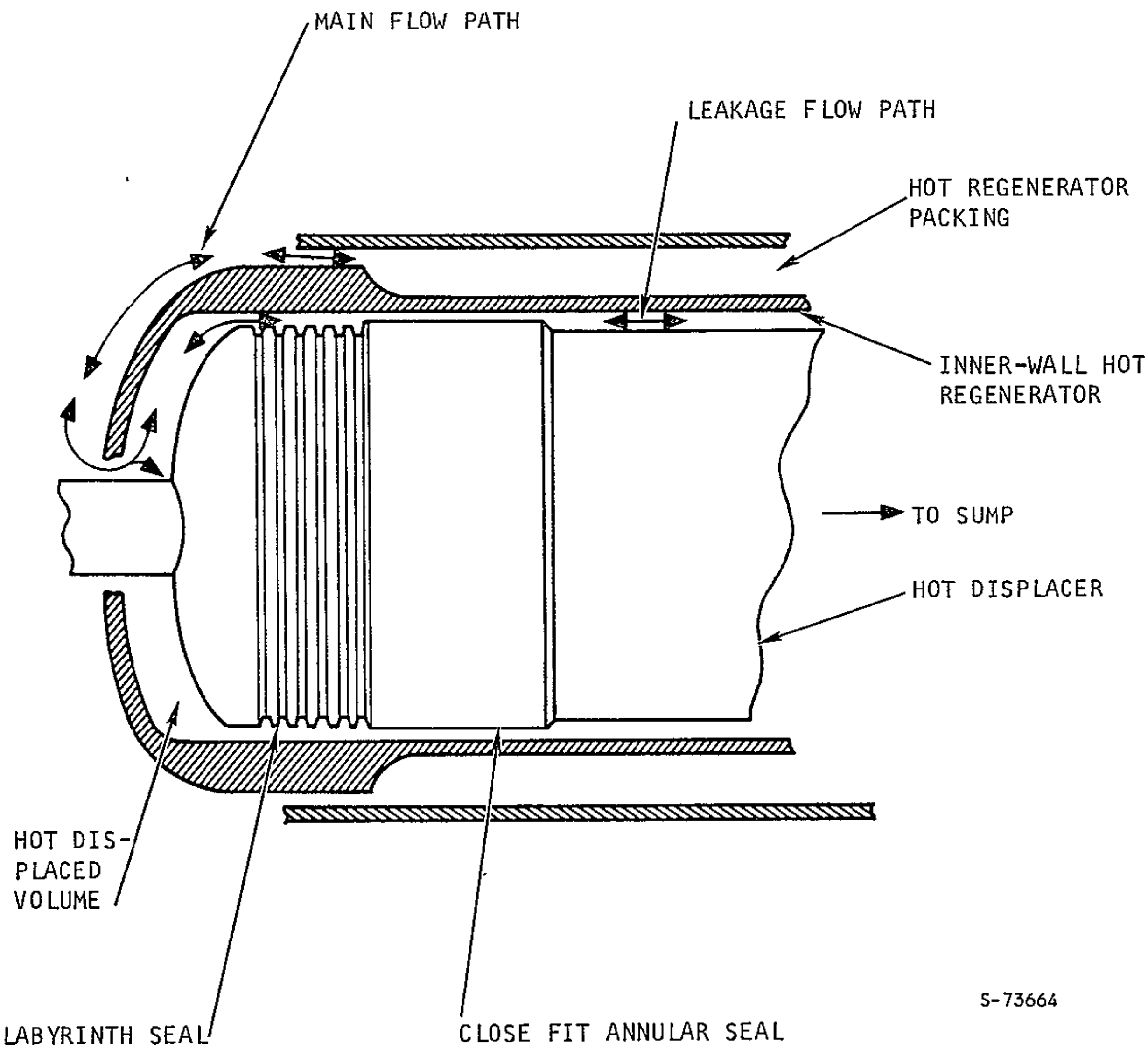


Figure 12-1. Hot-End Seal Configuration

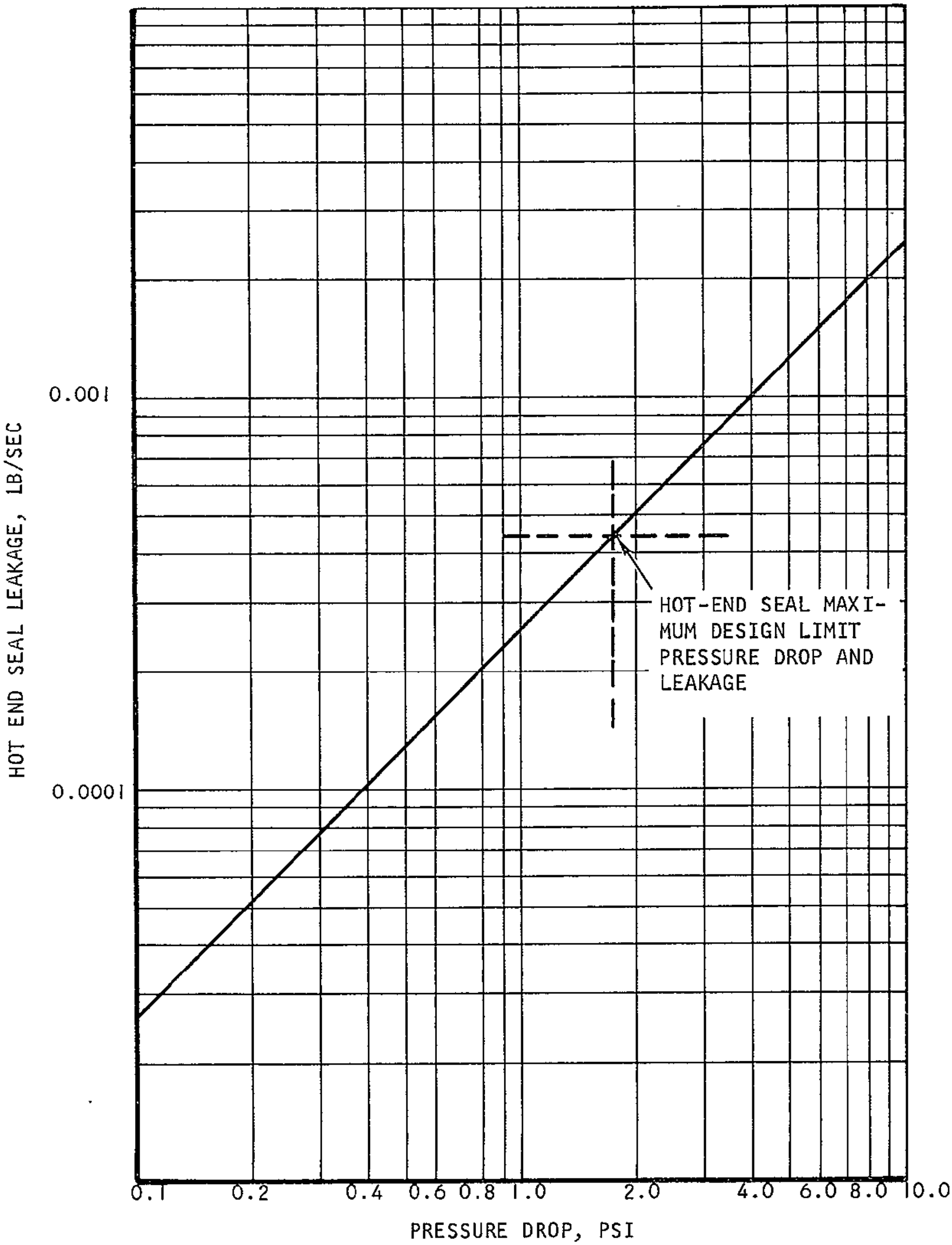


Figure 12-2. Hot-End Seal Leakage Rate vs Pressure Drop

Comparing Figure 12-2 with Figure 7-2 (Figure 7-2 gives the cold-end leakage rate vs pressure drop) two major differences are noted. First, since the pressure drop through the all-screen matrix of the hot regenerator is predictable--uniform screens can be obtained from batch to batch--a worst case pressure drop, taking into account deviations from the desired matrix geometry is not required. Secondly, the hot-end seal is completely non-contacting and is not subject to wear. The hot-end leakage is therefore independent of operational life.

Figure 12-3 gives the thermal loss for leakage past the hot-end seal as a function of leakage rate. The method of calculating this thermal loss, which takes into account the regenerative effect of the displacer and cylinder walls, is identical to that used for the cold-end seal thermal losses. The loss indicated for the design limit conditions is not significant compared to the 260 to 300 watts of power input to the hot-end required to operate the system.

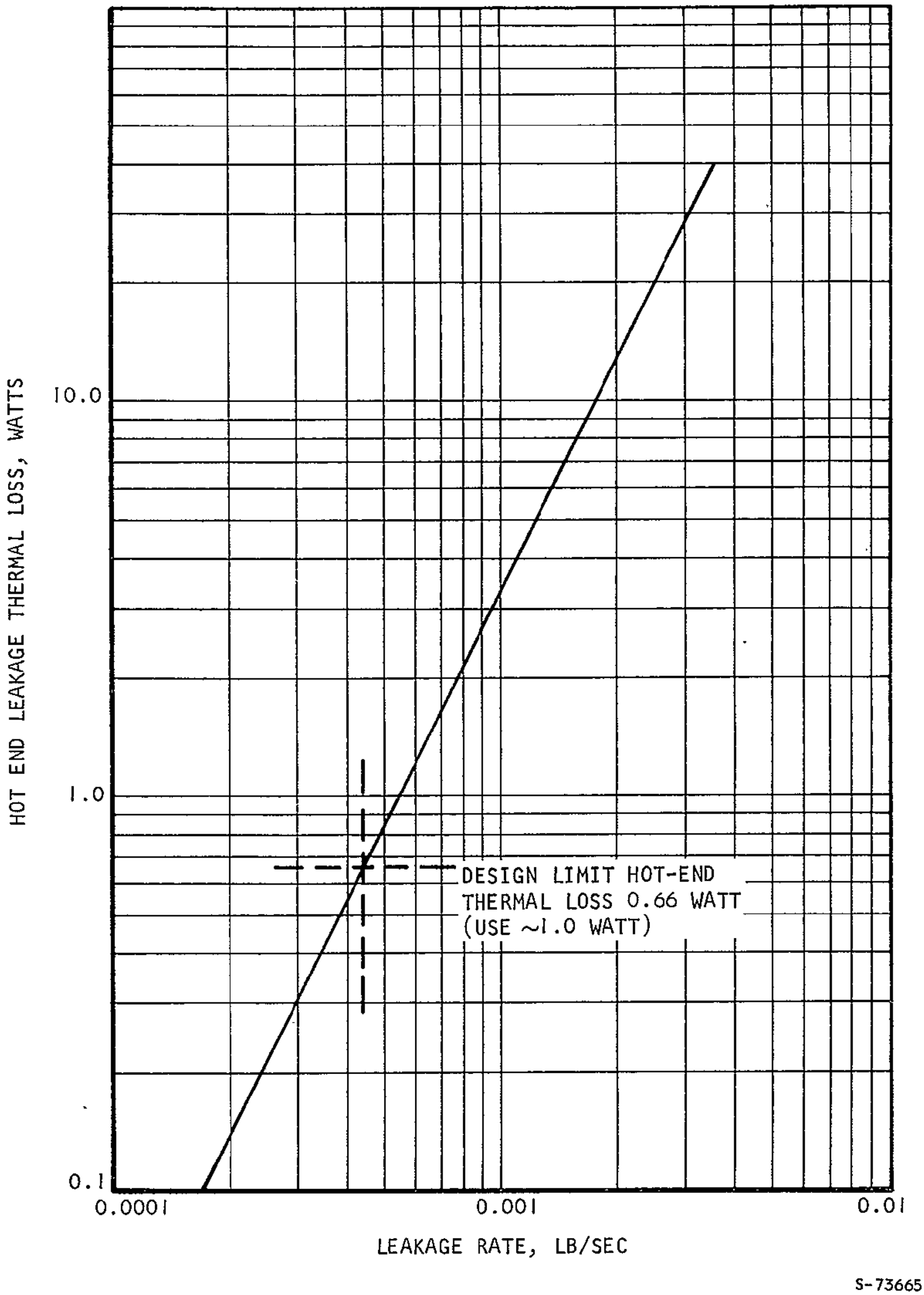
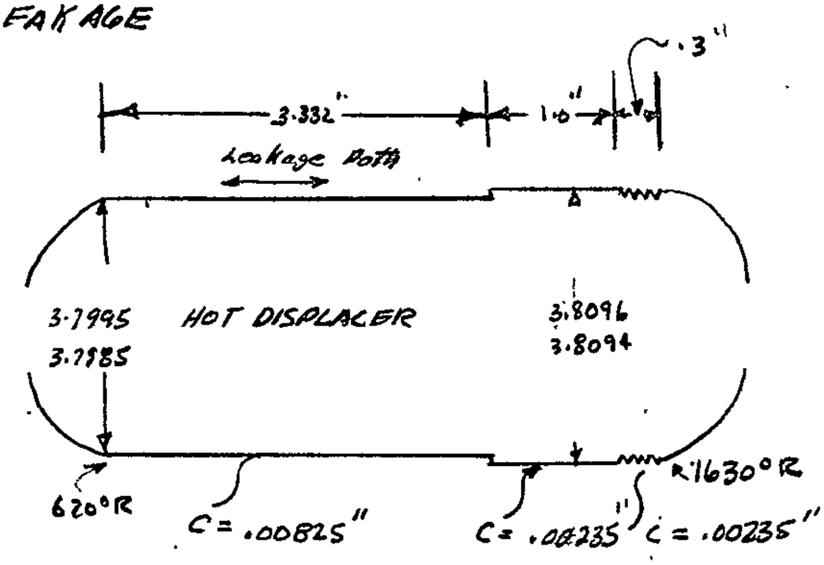


Figure 12-3. Hot-End Thermal Loss vs Leakage Rate

HOT END LEAKAGE



1.0 ANALYTICAL RELATIONS

FOR ANNULAR SECTIONS

$$\dot{w} = A_c \sqrt{\frac{D_H \rho g_c \Delta P}{2fL}}$$, for small pressure drops

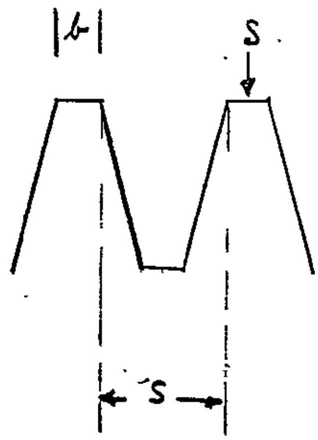
for $R_e < 2{,}100$ $\quad f = \frac{24}{Re}$

and

$$\dot{w} = \frac{\rho \Delta P}{L} \left[\frac{D_H^2 g_c A_c}{48 \mu} \right]$$

32

FOR LAYBRINTH SEAL



$s = .05$ to $.04$

$b = .007$ to $.003$

$\delta = .00235''$

$\frac{\delta}{S} = \frac{.00235}{.045} = .052$

$$\dot{w} = \frac{\phi\, C\, A_c P_o}{\sqrt{T_o R}} \sqrt{\frac{2 g_c}{n}} \left(\frac{\Delta P}{P_o}\right)^{1/2}$$

$\phi = 1.50$

$C = .65$

$$\phi * C = 1.50 * .65 = 0.975$$

$$\dot{w} = 0.975\, A_c \frac{\sqrt{P_o}}{\sqrt{T_o R}} \sqrt{\frac{2 g_c}{n}} \left(\Delta P\right)^{1/2}$$

2.0 LAYBRINTH SEAL LEAKAGE

$$\dot{w} = 0.975 \frac{A_c \sqrt{P_o}}{\sqrt{T_o R}}$$

Which from cold seal analysis reduces to

$$\dot{w} = 0.399\, A_c \sqrt{\frac{P_o}{T_o N}}\, (\Delta P)^{1/2}$$

P_o, ΔP in lbf/in^2
T_o in °R
A_c in IN^2
$\dot{w}$ in lb/sec

$A_c = \pi D C = \pi(3.8096)(.00235) = 0.028111$ IN^2

$P_o = 800$ PSI

$T_o = 1597°R$ {see below}

$N = 5$

assuming a linear temperature distribution along the hot displacer

$$T_x = \frac{1630-620}{4.636} X + 620 = 218\,X + 620 \quad \{°R\}$$

at midpoint of laybrinth seal $X = 4.486$ IN

$\therefore T_o = (218)(4.486) + 620 = 1597°R$

$$A_c \sqrt{\frac{P_o}{T_o N}} = 0.02811 \sqrt{\frac{800}{1597 * 5}} = .0088978$$

$$\boxed{\dot{w} = 0.00355\, (\Delta P)^{1/2}}$$

3.0 ANNULAR SEAL SECTION

For laminar flow

$$\dot{w} = \frac{\rho \Delta P}{L}\left[\frac{D_H^2 g_c A_c}{48 \mu}\right]$$

$$Re = \frac{\dot{w} D_H}{A_c \mu} < 2{,}100$$

$D_H = 2*C = 2*.00235 = .00470''$

$A_c = \pi D C = .028111 \; IN^2$

$$Re = \frac{\dot{w} D_H}{A_c \mu} = \frac{\dot{w}\, 2*C}{\pi D C \mu} = \frac{2\dot{w}}{\pi D \mu}$$

$P = 800$ psia @ midpoint of seal $x = 3.986$

$T = (218)(3.986) + 620 = 1490°R$

$\rho = 0.1965 \frac{lbm}{ft^3}$ $\mu = .092 \frac{lbm}{ft\text{-}hr}$ $k = 0.17 \frac{Btu}{hr\, ft\, °R}$

$$Re = \frac{(2)(\dot{w})\ lb/sec \cdot ft\text{-}hr * 3600\ sec/hr * 12\ in/ft}{\pi(3.8096)\ in\ (.092)\ lbm} = 78508.5\ \dot{w}$$

@ $Re = 2{,}100$

$$\dot{w} = \frac{2{,}100}{78508.5} = 0.026749\ lb/sec$$ for laminar flow -- this will more than cover range of interest.

then:

$$\dot{w} = \frac{(0.1965)\frac{lbm}{ft^3}\ \Delta P \frac{lbf}{IN^2}}{1.0\ IN}\left[\frac{(0.00470)^2\ IN^2 * 32.2 \frac{lbm\text{-}ft}{lbf\text{-}sec^2} * (.028111)\ IN^2 \cdot ft\text{-}hr \cdot 3600\ sec/hr \cdot ft}{48 * 0.092\ lbm * 12\ IN}\right]$$

$$\boxed{\dot{w}\left(\frac{lbm}{sec}\right) = \frac{(.1965)(.00470)^2(32.2)(.028111)(3600)}{(48)(.092)(12)}\ \Delta P\left(\frac{lbf}{IN^2}\right) = 0.00026692\ \Delta P\left(\frac{lbf}{IN^2}\right)}$$

4.0 SECTION ALONG DISPLACER WALL

$$\dot{w} = \frac{\rho \Delta P}{L}\left[\frac{D_H^2 g_c A_c}{48\mu}\right]$$

since $Re \neq C$ we will have laminar flow in this section also

$D_H = 2*C = 2*.00825 = .01650''$

$A_c = \pi DC = \pi(3.8096)(0.01650) = 0.197375 \text{ IN}^2$

The midpoint of this section is $X = 1.666$

$T = (218)(1.666) + 620°R = 984°R$

$\rho = .2988 \text{ lb/ft}^3 \quad \mu = .07 \frac{\text{lbm}}{\text{ft-hr}}$

$\therefore$

$$\dot{w} = \frac{(.2988 \text{ lb/ft}^3)(\Delta P \frac{\text{lbf}}{\text{ft}^2})}{3.332 \text{ in}}\left[\frac{(0.0165)^2 \text{in}^2\, 32.2 \frac{\text{lbm ft}}{\text{lbf-sec}^2} \times (.19738) \text{in}^2 \times 3600 \frac{\text{sec}}{\text{hr}}}{48(.07) \frac{\text{lbm}}{\text{ft-hr}} \times 12 \frac{\text{in}}{\text{ft}}}\right]$$

$$\dot{w}\left(\frac{\text{lb}}{\text{sec}}\right) = \frac{(.2988)(0.0165)^2(32.2)(.19738)(3600)}{(3.332)(48)(.07)(12)}\, \Delta P\left(\frac{\text{lbf}}{\text{in}^2}\right)$$

$$\boxed{\dot{w}\,(\text{lb/sec}) = 0.01385\, \Delta P}$$

5.0 LEAKAGE AND TOTAL PRESSURE DROP

In the same manner as in the cold end leakage analysis:

$$\boxed{\Delta P_T = \sum P_i = \sum f_i(\dot{w})}$$

For laybrinth seal

$$\dot{w} = 0.00355(\Delta P_1)^{1/2}$$

$$\Delta P_1 = \frac{\dot{w}^2}{(.00355)^2} = 79{,}349.335\,\dot{w}^2$$

For annular seal

$$\dot{w} = 0.0002669\,\Delta P_2$$

$$\Delta P_2 = \frac{\dot{w}}{.0002669} = 3{,}746.72\,\dot{w}$$

For displacer section

$$\dot{w} = .01385\,\Delta P$$

$$\Delta P_3 = \frac{\dot{w}}{.01385} = 72.20\,\dot{w}$$

Then

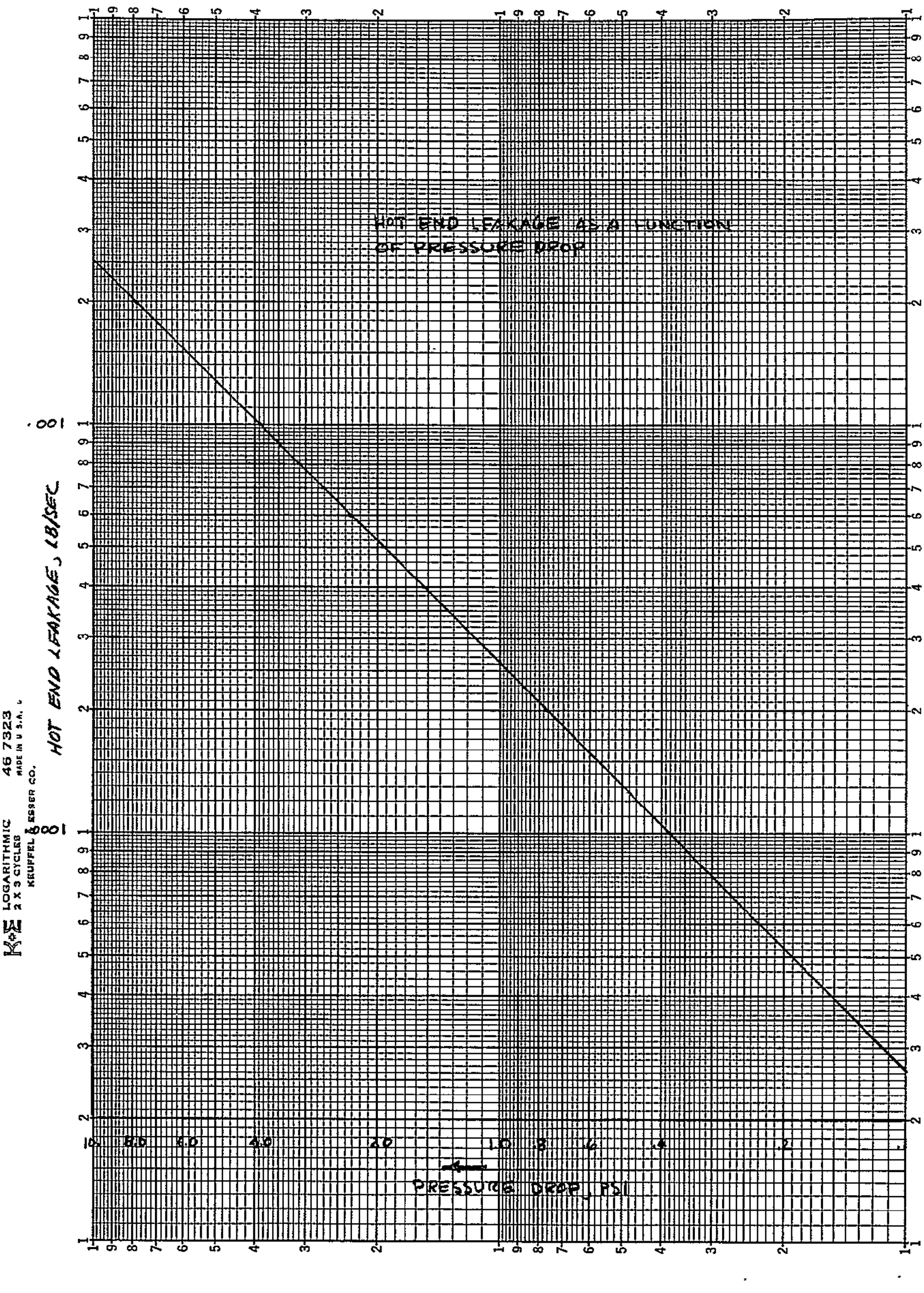
$$\Delta P_T = \sum f_i(\dot{w}) = \left\{79{,}349.3\,\dot{w} + 3{,}746.7 + 72.2\right\}\dot{w}$$

$$\boxed{\Delta P_T = \left\{79{,}349\,\dot{w} + 3{,}819\right\}\dot{w}}$$

Calculating some no's

$\dot{\omega}$	ΔP_r
.000001	.00382
.00001	.0382
.0001	.3827
.0002	.7669
.0004	1.540
.0006	2.320
.0008	3.106
.001	3.898
.0012	4.697
.0014	5.502
.0016	6.315
.0018	7.131
.002	7.955
.0022	8.786
.0024	9.626
.0026	10.465

⑧



HOT END LEAKAGE AS A FUNCTION OF PRESSURE DROP
HOT END LEAKAGE, LB/SEC
.001
10, 8.0, 6.0, 4.0, 2.0, 1.0, .8, .6, .4, .2, .1
PRESSURE DROP, PSI
K&E LOGARITHMIC 2 X 3 CYCLES KEUFFEL & ESSER CO. 46 7323 MADE IN U.S.A.

6.0 ESTIMATION OF LOSSES AT HOT END DUE TO LEAKAGE

Using the same model developed in the analysis of the cold end the temperture difference between the displacer wall and leakage gas is given by:

$$\Delta T(x) = T_H - T_f = (T_H - T_A)\left\{1 - \frac{1}{X_e}\left\{x - \frac{1}{\alpha} + \frac{1}{\alpha} e^{-\alpha x}\right\}\right\}$$

where

$$\alpha = \frac{hA_c}{\dot{w} C_p}$$

$A_c = 2\pi D$ {heat transfer areal per unit length}

T_H and T_A = the hot end wall temperature and sump temperature respectively

X_e = length of displacer

h = heat transfer coefficient in the annular space between the displacer and regenerator wall

at the hot end $x = X_e$ and the above reduces to:

$$\Delta T = \frac{(T_H - T_A)}{\alpha}\left\{1 - e^{-\alpha X_e}\right\}$$

and the losses are given by:

$$\mathcal{L} = \dot{w} C_p \Delta T = \frac{\dot{w} C_p (T_H - T_A)}{\alpha}\left\{1 - e^{-\alpha X_e}\right\}$$

The losses here mean something different than in the case of the cold end. Here the losses represent additional energy that must be supplied to the hot end and removed from the sump if the system is not heat transfer limited. On the other hand where the system is heat transfer limited (essentially the case here since we operate with fixed source and sink temperatures) the gas temperature at the hot end is reduced by mixing with the colder leakage gas and the sump gas temperature is increased by mixing with the warmer leakage gas. The net effect is a decrease in cycle performance which we will try to estimate later if it appears worth while. First lets calculate some ΔT_s and losses for the non-heat transfer limited case and see what we have.

6.1 NON-HEAT TRANSFER LIMITED LOSSES

Using

$$\Delta T = \frac{(T_H - T_a)}{\alpha}\left\{1 - e^{-\alpha X_e}\right\}$$

$$\dot{Q} = \frac{\dot{w} C_p}{\alpha}(T_H - T_a)\left\{1 - e^{-\alpha X_e}\right\}$$

$T_H = 1630°R$

$T_a = 620°R$ $\tilde{T}_g \cong \frac{2250}{2} \cong 1125°R$ $P = 800 PSIA$

$\rho_g = 0.260\ lb/ft^3$ $K_g = .142\ Btu/hr\text{-}ft\text{-}°R$

$D = 3.80\ IN$

$A_c = 2\pi D = 2\pi(3.80) = 23.95 \frac{IN^2}{IN} = \cancel{1.995}\ ft^2$

$X_e = 4.632\ IN = 0.386\ ft$

$D_H = 2\times C = 2\times(.008) = .016\ IN = 0.00132\ ft$

$C_p = 1.24\ Btu/lb_m°R$

$N_u = \frac{h D_H}{k} = 8.23$

$$h = \frac{k\,(8.23)}{D_H} = \frac{0.142\ Btu/hr\text{-}ft\text{-}°R\ (8.23)}{0.00132\ ft} = 887.5 \frac{Btu}{hr\ ft^2\ °R}$$

$$\alpha = \frac{h A_c}{\dot{w} C_p} = \frac{(887.5 \frac{Btu}{hr\ ft^2\ °R})(1.995\ ft)}{\dot{w}\ (lb_m/sec)\ 1.24 \frac{Btu}{lb_m\ °R}} \frac{1}{3600} \frac{hr}{sec} = \frac{0.396}{\dot{w}} \frac{1}{ft}$$

$$\alpha = \frac{0.396}{w\,(lb/sec)} \frac{1}{ft}$$

$$\Delta T = \frac{1010}{\alpha}\left\{1 - e^{-\alpha x_e}\right\}$$

$$\Delta T = \frac{\dot{w}(1010)}{.396}\left\{1 - e^{-\frac{(0.396)(.386)}{\dot{w}}}\right\}$$

$$\Delta T = 2554\,\dot{w}\left\{1 - e^{-\frac{0.1529}{\dot{w}}}\right\}$$

note for $\dot{w} < .01529$ $\quad e^{-\alpha x_e} \rightarrow$ zero

$\therefore \quad \dot{w} < .01529$

$$\Delta T = 2554\,\dot{w}$$

$\dot{w}$ (lb/sec)	ΔT (°R)	$\mathcal{L}$ ~~Btu~~/m	$\mathcal{L}$ watts	ΔP (PSI)
.0001	0.2554	0.114	.0334	.3827
.0005	~~1.278~~	2.85	0.865	1.920
.0010	2.554	11.40	3.343	3.898
.0015	3.830	25.646	7.521	5.890
.0020	5.100	45.532	13.350	7.900
.0025	6.385	71.257	20.896	10.000

For pressure drops ~~of interest~~ (~~below~~ 10 psi) ~~the~~ losses are not great. Also note that for a maximum hot end flow of .017 lb/sec the temperature change of the gas upon mixing with the leakage ~~at a~~ ΔP of 10 psi is below 1.0 °R (see below)

$$(.0025)C_p(T+6.385) + (0.017-0.0025)C_pT = 0.017\,C_pT_{MIX}$$

$$.0025T + 0.01594 + 0.0145T = .017\,T_{MIX}$$

$$.01594 = .017\,(T_{MIX}-T)$$

$$T_{MIX}-T_0 = \Delta T_{mixing} = .94\,°R$$

Don't think we would ever find it in our cycle calculations.

6.2 CONCLUSION

It looks like for any reasonable pressure drop we are in good shape as far a losses due to leakage in the hot end

SECTION 13

HOT-END INSULATION LOSSES AND HEATER TEMPERATURE

INTRODUCTION

The GSFC VM refrigerator hot end is designed to simulate the interface between a VM refrigerator and a thermal power source, supplying energy to the refrigerator via a hot heat pipe. In Task V of the program, a radiation heat transfer interface was selected to eliminate mechanical problems at the interface due to thermal expansion and contraction. This design was discussed in the Task I report; the design and performance of the final interface configuration is reported here. It is also essential to insulate the hot end interface and adjacent elements of the refrigerator from the surroundings to avoid excessive thermal losses from the hot end of the machine. Since this insulation system design is directly related to the hot end interface design and operating temperature levels, it is also discussed here.

CONFIGURATION AND PERFORMANCE

The configuration of the hot end insulation and heater assembly is shown in Figure 13-1. Heat is supplied to the refrigerator hot-end heat exchanger by a coiled resistance heater element brazed to the backside of an Inconel 718 plate (disc). The plate is physically separated from the hot-end heat exchanger to simulate the condenser-end of a hot heat pipe. The primary mode of heat exchange between the heater plate and hot-end heat exchanger is radiation; some heat is also transferred by conduction through the heater support.

Figure 13-2 gives the thermal input power to the hot-end heat exchanger as a function of heater temperature and is based on a constant hot-end heat exchanger surface temperature of 1660°R (120°F). In operation, the heat exchanger temperature will generally be somewhat below 1660°R and is never allowed to exceed this temperature due to structural limitations. Lower heat exchanger temperatures result in lower heater temperatures for the same input power level; thus the heater will never exceed 2000°R (1540°F). Since the heater elements can withstand continuous operation at 2060°R (1600°F), burn-out problems are avoided. Also, the heater leads are terminated on the heater plate where good thermal contact can be maintained between the leads, heater element and plate. This prevents sections of the heater from reaching elevated temperatures (above that of the plate) as a result of being insulated from the plate and reduces potential burn-out problems.

The shaded area on Figure 13-2 shows the normal operating range for the hot end heater. The lower limit of power and temperature is based on the lowest anticipated thermal losses as compared to the upper limit which includes a 20-watt input power contingency.

The insulation system consists of an inner layer of Min-K and an outer layer of fiberglass. Both insulation materials are contained within a sealed enclosure and dynamically pumped to maintain a vacuum environment.

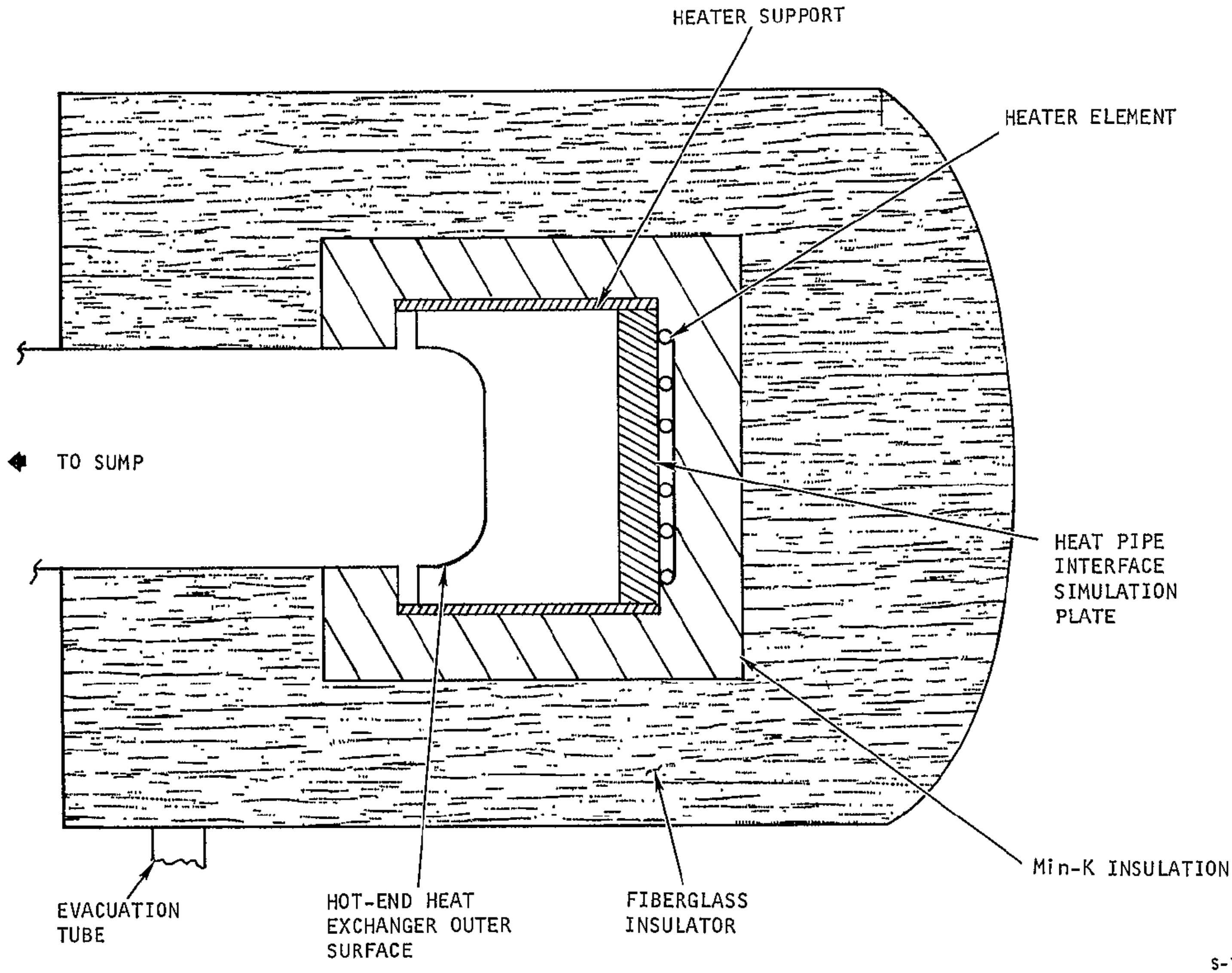


Figure 13-1. Hot-End Insulation and Heater Configuration

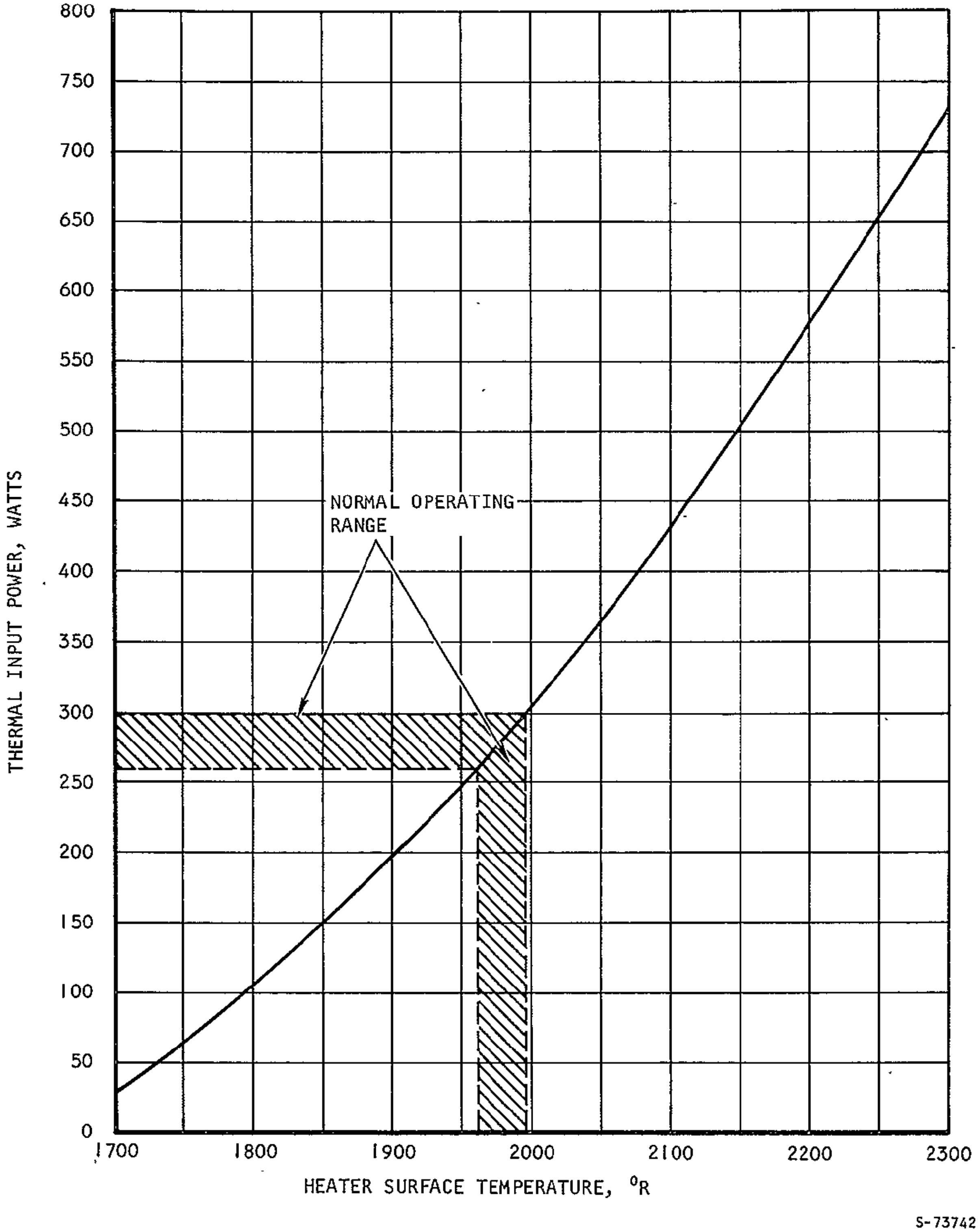


Figure 13-2. Thermal Input Power vs Heater Temperature

The Min-K is used in the high temperature region of the system due to its excellent high temperature properties. Where the temperature level of the insulation is below 1400°F, fiberglass is used due to its superior thermal performance. The insulation system has not been optimized with respect to either weight or thermal performance. The only objective used in the design was to obtain a practical insulation system that would limit the hot end insulation losses to 25 watts.

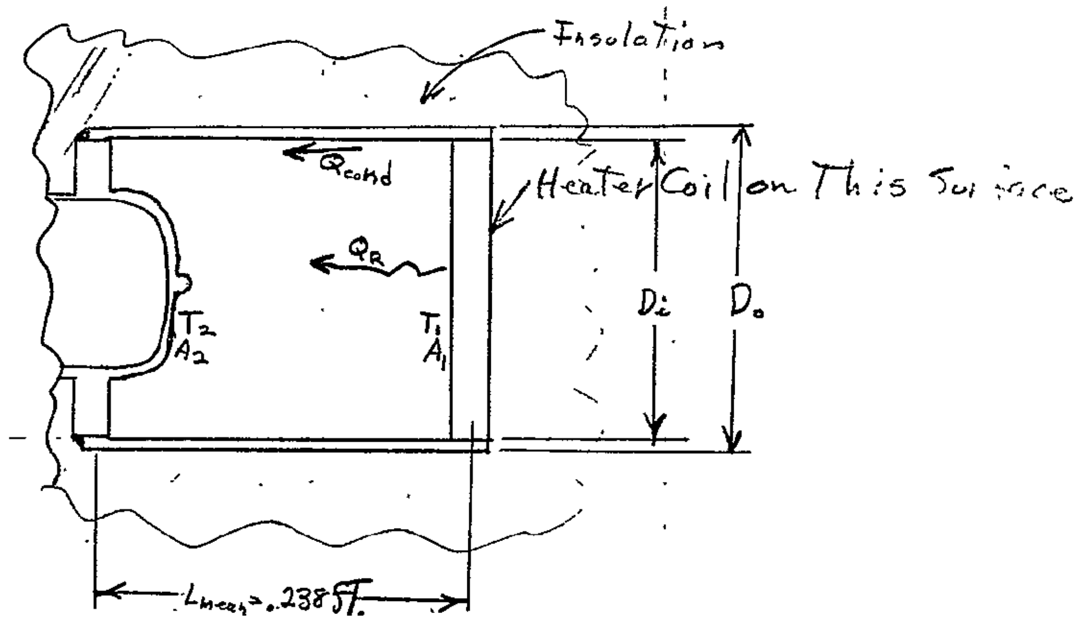
Based on the inner insulation boundary temperatures established by Figure 13-2, and the analysis associated with this figure, the estimated insulation losses range between 23 and 25 watts.

HEATER TEMPERATURE

Summary

The hot end heat added is supplied by a heater coil mounted on the back side of an Inconel 718 disc which is physically separated from the hot dome. The available modes of heat transfer are radiation and conduction down the cylinder which separates the heater disc and the hot dome. The contributions of each heat transfer mode will be calculated separately for a range of assumed heater temperatures, and then summed. This total heat flow will be plotted against heater temperature. The calculated heat input plus losses (exclusive of insulation loss) is 253.5 watts. In order to allow a small degree of conservatism, a value of 260 watts will be used to determine heater temperature. The heater temperature determined from this calculation will then be used in the calculation of hot end insulation losses.

Heater Configuration



Hot Dome Temp = 1200°F = 1660°R

Assume reradiating walls and Projected $A_2 = A_1$

Plot Heat flow ($Q_R + Q_{cond}$) as a function of Heater Disc Temperature.

Use Inconel Emissivity from Table A-23 McAdams, Type B, Surface A

I Conduction

$$Q = \frac{kA}{L}(T_1 - T_2)$$

$$A = \frac{\pi}{4}(D_o^2 - D_i^2) = .0113 \text{ ft}^2 \qquad \therefore D_o = .474 \text{ ft}, D_i = .459 \text{ ft}$$

$$A/L = .0113/.238 = .0474 \text{ ft.}$$

K from WADC Tech Report, fig IV-A-5-a

T_{Heater}, °R	T_{av}, °R	k, BTU/hr-ft-°R	Q, BTU/hr	Q, watts
1700	1680	14.2	26.9	7.89
1900	1780	14.8	168.4	49.3
2100	1880	15.2	317.0	93.0
2300	1980	15.8	479	140.5

II Radiation

$$Q_R = \sigma A_1 F_{1-2}\left(T_1^4 - T_2^4\right) = \frac{.1714 \times \cancel{10^{-8}}\ BTU}{ft^2\text{-}hr\text{-}{}^\circ R^4} A_1 F_{12}\left[\frac{\left(\frac{T_1}{100}\right)^4 - \left(\frac{T_2}{100}\right)^4}{\cancel{10^{-8}}}\right]$$

$$F_{12} = \frac{1}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1}$$

@ 1660 °R, $\epsilon_2 = .48 \Rightarrow F_{12} = \frac{1}{\frac{1}{.48} - 1 + \frac{1}{\epsilon_1}} = \frac{1}{2.08 - 1 + \frac{1}{\epsilon_1}}$

$A_1 = \frac{\pi}{4} \cdot .459^2 = .1653\ ft^2$

$(T_2/100)^4 = 16.6^4 = 75,900$

T_{Heater}, °R	$(T_1/100)^4$	$f(T_1) - f(T_2)$	ϵ_1	F_{12}	Q_R, BTU/hr	Q_R, watts
1700	83,500	7,600	.486	.318	68.5	20.06
1900	130,200	54,300	.52	.333	514	150.6
2100	194,200	118300	.554*	.347	1162	341
2300	280,000	204100	.588*	.348	2015	590

* Extrapolations of McAdams Data Range.

III Total — $Q_{Tot} = Q_R + Q_{cond}$

T_{Heater}, °R	Q_{Tot}, watts
1700	27.95
1900	199.9
2100	434
2300	730.5

} Plot on next Page

IV Heater Temp.

Thermal Power Input + Calculated Losses = 253.5 watts. To be conservative use 260 watts. From pg. 4 curve, $T_{Heater} = 1961$ °R

$\therefore T_{Heater} = 1501$ °F

use this value in insulation loss calculations

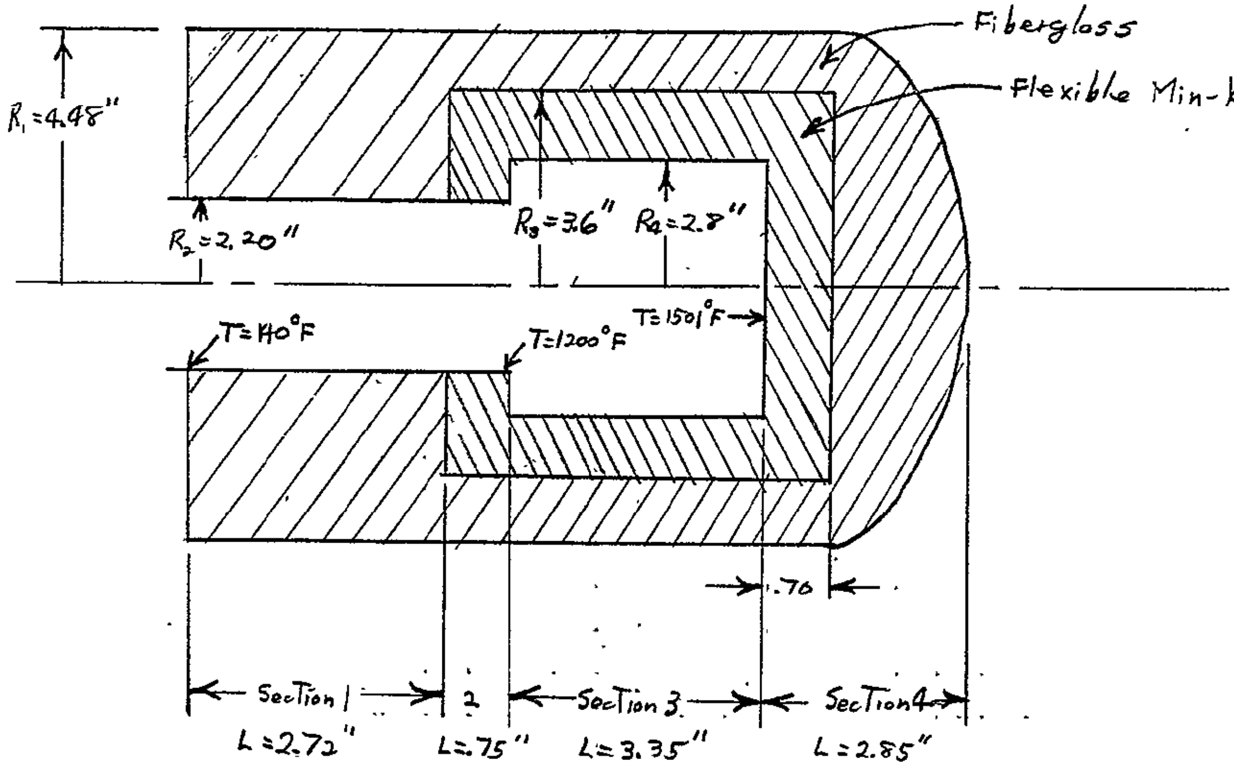
Hot End Insulation Loss

The hot end external losses must be calculated for the composite Min-K, Fiberglass batting insulation blanket. A rigorous solution would involve a two dimensional nodal analysis which is best solved by the thermal analyzer computer program. In view of the uncertainties in the thermal properties of the insulation materials in vacuum, the errors introduced by the use of a simplified one dimensional analysis will be insignificant. The natural convection coefficient at the outer surface will be evaluated at a film temperature of 100°F. The actual value is expected to be somewhat lower than this, but the change in coefficient with film temperature is not expected to influence the results since the major resistance to heat flow occurs in the insulation. The model assumed is shown on the next page.

(9 2000
14.60
1500

Hot End Insulation Loss Model

$T_{ambient} = 70°F$



1. Break up into 4 sections, Consider One Dimensional Conduction only.
2. Ignore Conduction Losses in metal
3. Estimate Air Coefficient based on Air Film Temperature = 100°F
4. Thermal Conductivity of Min-K = 0.0334 BTU/hr-ft-°F (1 atmosphere value - decrease with pressure is small and uncertain)
5. Thermal Conductivity of evacuated fiberglass = 0.0033 BTU/ft-hr-°F (based on hard vaccum data modified to account for loose packing and higher absolute pressure).
6. Assume linear Thermal Gradients in internal metal walls.

I Natural Convection Coefficient

For a short horizontal cylinder, the characteristic dimension is determined by:

$$\frac{1}{L} = \frac{1}{L_{Horizontal}} + \frac{1}{L_{Vertical}}$$

$L_{Horizontal} = 2.72 + .75 + 3.35 + 2.85 = 9.67\text{ in} = .806\text{ ft.}$

$L_{Vertical} = 2 \times R_1 = 2 \times 4.48 = 8.96\text{ in} = .747\text{ ft.}$

$$\frac{1}{L} = \frac{1}{.806} + \frac{1}{.747} = 1.24 + 1.34 = 2.58$$

$L = (2.58)^{-1} = .388\text{ ft.}$

$Nu = f(Gr\ Pr)_f = f\left(c_p \rho^2 g \beta \Delta T\, D_o^3 / \mu k\right)_f$

@ 100°F, $Pr = c_p \mu / k = 0.72$

$$\rho^2 g \beta / \mu^2 = 3.16 \times 10^6 / {}^\circ F \cdot ft^3$$

if $T_f = 100°F$, $\Delta T = (100 - T_{amb}) \times 2 = (100 - 70)2 = 60°F$

$$Gr\,Pr = \frac{3.16 \times 10^6}{°F\ ft^3} \times .72 \times 60°F \times (.388\ ft)^3 = 7.98 \times 10^6$$

from Fig. 7-3, Kreith, $\log_{10} Nu = 1.4$

$Nu = \dfrac{hD}{k} = 25.1$ at 100°F, $K = .0154$ BTU/ft-hr-°F

$h = Nu\, k/D = 25.1 \times .0154 / .388 = 0.996$ BTU/ft²-hr-°F

II Conduction

A. Section 1

$$Q = U A_o \Delta T$$

1. $A_o = \pi D_o L_1 = \pi \times .747\,ft \times \frac{2.72\,in\,ft}{12\,in} = 0.532\,ft^2$

2. To get ΔT mean, assume linear gradient from 1200° at end of section 2 to 140° at beginning of section 1.

Thus at end of section 1,

$$T = 140 + \frac{2.72}{2.72+.75}(1200-140) = 140 + 831 = 971$$

$$T_{av} = \frac{971+140}{2} = 555.5\,°F$$

$$\Delta T = 555.5 - 70\,°F = 485.5\,°F$$

3. For this section, there is only one type of insulation, Fiberglass.

Therefore, $U = \dfrac{1}{\dfrac{r_1 \ln r_1/r_2}{k_f} + \dfrac{1}{h_o}} = \dfrac{1}{\dfrac{4.48 \ln 4.48/2.2}{12 \times .0033} + \dfrac{1}{.996}}$

$$= \frac{1}{80.7 + 1.003} = \frac{1}{81.703} = .01223\ BTU/ft^2\text{-}hr\text{-}°F$$

as expected, the natural convection at the outer surface is a negligible contribution to the overall resistance, and need not be evaluated later at a different film temp.

4. Section 1 Q

$$Q = \frac{.01223 \frac{\cancel{BTU}}{\cancel{ft^2}-\cancel{hr}-\cancel{°F}} \times .532 \cancel{ft^2} \times 485.5 \cancel{°F} \times 0.293 \frac{watts}{\cancel{BTU/hr}}}{} = 0.925 \text{ watts}$$

B. Section 2

$$Q = U A_o \Delta T$$

1. $A_o = \pi D_o L_2 = \pi \times .747 \times .75/12 = .1467 \ ft^2$

2. From A2, inner wall Temp at beginning of section 2 = 971°F

$$T_{av\,Hot} = \frac{1200+972}{2} = 1086°F$$

$$\Delta T = 1086 - 70 = 1016°F$$

3. For composite wall

$$U = \frac{1}{\frac{r_1 \ln r_3/r_2}{K_{min-k}} + \frac{r_1 \ln r_1/r_3}{K_f} + \frac{1}{h_o}} = \frac{1}{\frac{4.48 \ln 3.6/2.2}{12 \times .0334} + \frac{4.48 \ln 4.48/3.6}{12 \times .0033} + \frac{1}{.996}}$$

$$= \frac{1}{5.52 + 24.6 + 1.003} = \frac{1}{31.123} = .03215 \ BTU/ft^2\text{-}hr\text{-}°F$$

4. Section 2 Q

$$Q = \frac{0.03215 \frac{\cancel{BTU}}{\cancel{ft^2}-\cancel{hr}-\cancel{°F}} \times .1467 \cancel{ft^2} \times 1016 \cancel{°F} \times 0.293 \frac{watts}{\cancel{BTU/hr}}}{} = 1.03 \text{ watts}$$

C. Section 3

$Q = U A_o \Delta T$

1. $A_o = \pi D_o L_3 = .747 \pi \times \frac{3.35}{12} = 0.656 \ ft.^2$

2. $T_{av} = \frac{1200+1501}{2} = 1350.5°F$

$\Delta T = 1350.5 - 70 = 1280.5°F$

3. $$U = \frac{1}{\frac{r_1 \ln r_3/r_4}{K_{Min-k}} + \frac{r_1 \ln r_1/r_3}{K_f} + \frac{1}{h_o}} = \frac{1}{\frac{4.48 \ln 3.6/2.8}{12 \times .0374} + \frac{4.48 \ln 4.48/3.6}{12 \times .0033} + \frac{1}{.996}}$$

$$= \frac{1}{2.8 + 24.6 + 1.003} = \frac{1}{28.403} = 0.0352 \ BTU/ft^2\text{-}hr\text{-}°F$$

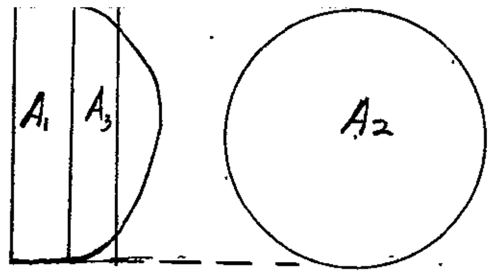
4. Section 3 Q

$$Q = \frac{0.0352 \ \cancel{BTU} \times 0.656 \ \cancel{ft^2} \times 1280.5 \cancel{°F} \times 0.293 \ watts}{\cancel{ft^2\text{-}hr\text{-}°F} \qquad \cancel{BTU/hr}} = 8.66 \ watts$$

D. Section 4

$Q = U A_o \Delta T$

1. Since this is truly a two dimensional problem, some assumptions must be made in order to treat as a one dimensional problem. Since the minimum insulation thickness occurs in the radial direction, the use of the overall heat transfer coefficient from section 3 will be conservative. The only other variable which must be determined is the area to be used.



IT will be assumed that the total surface area of section 4 is composed of 3 portions as defined below.

A_1 = Cylindrical Area $= \pi D_o L_1 = .747\pi \times .7/12 = .137\ ft^2$

A_2 = Projected Circular Area $= \frac{\pi}{4} D_o^2 = \frac{\pi}{4} .747^2 = .439\ ft^2$

A_3 = Cylindrical Area as if cylinder continued for 5/8 of the dome length. Dome $L = 2.85 - .7 = 2.15$ in.

$L_{eff} = .625 \times 2.15 = 1.344$ in, $A = \pi D L_{eff} = \pi \times .747 \times \frac{1.344}{12} = .2635\ ft^2$

$A_o = .8395\ ft^2$

2. $\Delta T = 1501 - 70 = 1431\ ^\circ F$

3. Section 4 Q

$$Q = \frac{0.0352\ BTU \times 0.8395\ ft^2 \times 1431\ ^\circ F \times .293\ watts}{ft^2\text{-}hr\text{-}^\circ F \qquad BTU/hr} = 12.39\ watts$$

III Total Insulation Heat Loss

$$Q_{TOT} = Q_1 + Q_2 + Q_3 + Q_4 = 0.925 + 1.03 + 8.66 + 12.39 =$$

$$Q_{TOT} = 23.005 \text{ watts}$$

~~IV~~ Total Heater input

① From Heater Temp calcs, Hot Dome input plus losses = 260 watts.

② Insulation loss = 23 watts

Therefore Total heat input = 260 + 23 = 283 watts

SECTION 14

CONDUCTION LOSSES

INTRODUCTION

Conduction losses are used in estimating the cold-end losses and the hot-end power requirements. These losses were included in the discussion of cycle parameters and performance in Section 3 of this report. These losses are summarized here for convenient reference.

METHOD OF ANALYSIS

The method of determining conduction losses is straightforward; details are given on the remaining pages of this section. The only conduction calculation requiring special information is the one associated with the packing (matrix) of the regenerator. Here, the properties of the packed beds were taken from Reference 6.

CONDUCTION LOSSES SUMMARY

The hot- and cold-end conduction losses are summarized in Table 14-1.

TABLE 14-1

HOT-END AND COLD-END LOSSES

Element	Hot-End Losses, Watts	Cold-End Losses, Watts
Displacer		
Walls	35.86	0.560
Packing*	2.49	0.026
Subtotal	38.35	0.586
Regenerator		
Walls	68.18	2.642
Matrix	8.00	1.886
Subtotal	76.18	4.528
Dewar	-	0.111
Total	114.53	5.225

* Each displacer contains a low conductivity packing to eliminate convective heat transfer due to gas contained within the sealed displacers.

10/6/71

SUMMARY OF CONDUCTION LOSS CALCULATIONS

HOT END

	LOSS (WATTS)
DISPLACER	
WALLS	35.86
PACKING	2.49
SUBTOTAL	38.35
REGENERATOR	
WALLS	68.18
MATRIX	8.00
SUBTOTAL	76.18
TOTAL	114.53

COLD END

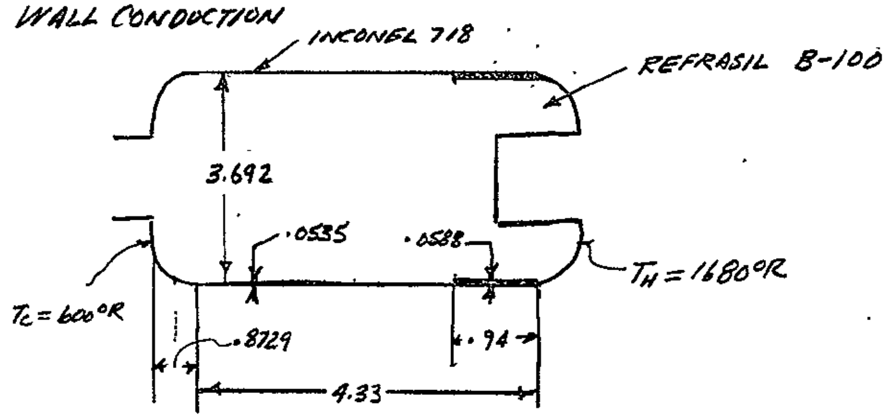
DISPLACER	
WALLS	0.560
PACKING	0.026
SUBTOTAL	0.586
REGENERATOR	
OUTER WALL	1.825
INNER WALL	.817
MATRIX	1.886
SUBTOTAL	4.528
TOTAL	5.114
INSULATION (DEWAR)	.111
TOTAL COLD END	5.225

1.825
.817
2.642

A. HOT END DISPLACER AND REGENERATOR CONDUCTION LOSSES

1. HOT DISPLACER CONDUCTION LOSSES

1.1 WALL CONDUCTION



From 10/2/71 calculations use ½ of length of dome at ambient end of displacer

Consider walls as series resistance do to change in thickness

$$L_1 = 4.33 - .94 + \frac{.8729}{2} = 3.826''$$

$$L_2 = .94''$$

$$\therefore \; Q = C(T_H - T_C)$$

$$R = \frac{1}{C} = \frac{L_1}{k_1 A_1} + \frac{L_2}{k_2 A_2} = \frac{k_2 A_2 L_1 + k_1 A_1 L_2}{k_2 k_1 A_1 A_2}$$

assume an average $\bar{k}$

$$C = \frac{\bar{k} A_1 A_2}{A_2 L_1 + A_1 L_2} \qquad \bar{k} = 10.25\ \text{Btu}/\text{ft-hr-}°R$$

$$L_1 = \frac{3.826}{12} = .31883\ \text{ft}$$

$$L_2 = \frac{.94}{12} = .07833\ \text{ft}$$

$$A_1 = \pi D t_1 = \pi(3.692)(.0535) = 0.6202\ \text{in}^2 = .004307\ \text{ft}^2$$

$$A_2 = \pi D t_2 = \pi(3.692)(.0588) = 0.68166\ \text{in}^2 = .004734\ \text{ft}^2$$

$$C = \frac{.00020899}{.001846} = .1132\ \text{Btu}/°R$$

$$Q = C(T_H - T_C) = .1132 * 1080 = 122.269\ \text{Btu/hr}$$

$$Q = 122.269\ \text{Btu/hr} = 35.856\ \text{WATTS}$$

1.2 REFRASIL B-100 PACKING

$$Q = \frac{k_f A_c (T_H - T_C)}{L} \qquad L_2 = 4.33 + \frac{.8729}{2} = 4.766'' = .3972$$

$$k \cong .042\ \text{Btu/hr ft} \cdot {}^\circ R$$

$$A_c = \frac{\pi D^2}{4} = \frac{\pi (3.692)^2}{4} = 10.700\ \text{IN}^2$$

$$A_c = 10.700\ \text{IN}^2 = .074307\ \text{ft}^2$$

$$Q = \frac{(.042)(.074307)(1080)}{.3972} = 8.4858\ \text{Btu/hr}$$

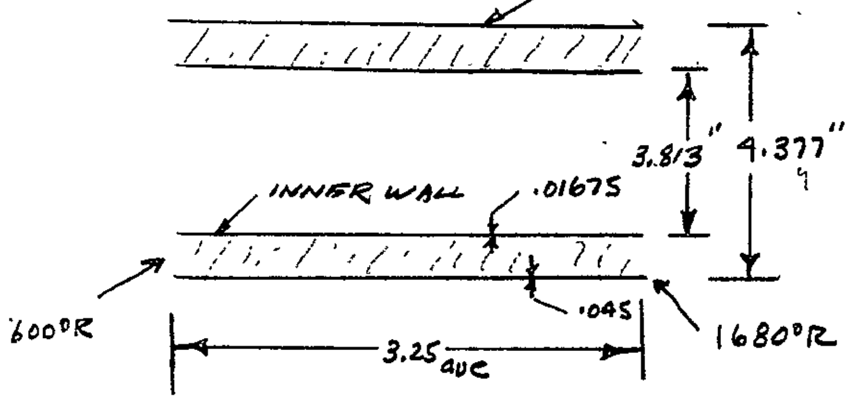
$$Q = 8.4858\ \text{Btu/hr} = 2.489\ \text{WATTS}$$

1.3 TOTAL DISPLACER CONDUCTION LOSS

Q_{WALLS} =	35.86	WATTS
$Q_{PACKING}$ =	2.49	WATTS
TOTAL	38.35	WATTS

2. HOT REGENERATOR CONDUCTION LOSSES

2.1 WALL LOSSES



$$Q = \frac{\bar{k}\,(A_i + A_o)(T_H - T_c)}{L} \qquad L = \frac{3.25}{12} = .27083$$

$$A_i = \pi D_i t = \pi (3.813)(.01675) = .2005 \text{ IN}^2 \qquad k = 10.25 \text{ Btu/hr °F-ft}$$

$$A_i = 0.001393 \text{ ft}^2$$

$$A_o = \pi D_o t_o = \pi (4.377)(.045) = .6185 \text{ IN}^2$$

$$A_o = .004295 \text{ ft}^2$$

$$A_i + A_o = .0056877$$

$$Q = \frac{(10.25)(.0056877)(1080)}{.27083} = 232.48 \text{ Btu/hr}$$

$$Q = 232.48 \text{ Btu/hr} = 68.18 \text{ WATT}$$

2.2 REGENERATOR MATRIX LOSSES

Note: Will use Fig 11-7 pp 290 Mc Adams which is for spherical packed beds – take 50% of results for void $\epsilon = .5$

$T_{ave} = \frac{1680+600}{2} = 1140$

$k_{g_{@800PSIA}} = .135 \; Btu/hr \; ft \cdot {}^\circ R$

$k_s = 10.25$

$\frac{k_s}{k_g} = \frac{10.25}{.135} = 76.5$

$L = .27083$

from Fig 11-7

$\frac{K_B}{K_g} = 4$

$\therefore k_B = 4.0 \times .135 = .54 \; Btu/hr \; ft \; {}^\circ R$

$Q = \frac{k_B A_c (T_H - T_c)}{L_1}$

$A_c = \frac{\pi}{4}\left((4.377)^2 - (3.813)^2\right) = \frac{\pi}{4}(19.1581 \quad 14.5389)$

$= 3.6261 \; IN^2 = .02518 \; ft^2$

$Q = \frac{(.54)(.02518)(1080)}{.27083} = 54.22 \; Btu/hr$

$Q = 54.25 \; Btu/hr = 15.90$ WATTS

Note use 1/2 or 8 WATTS

2.3 TOTAL REGENERATOR LOSSES

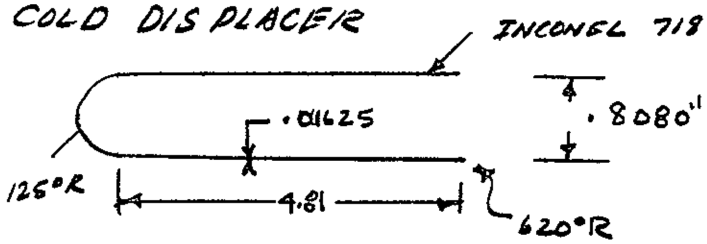
Q_{WALLS} =	68.18 WATTS
Q_{MATRIX} =	8.00 WATTS
Total	76.0 WATTS

TOTAL HOT END CONDUCTION LOSSES

DISPLACER			
	WALLS	35.86	WATTS
	PACKING	2.49	
	TOTAL	38.35	
REGENERATOR			
	WALLS	68.18	
	MATRIX	8.00	
	TOTAL	76.18	WATTS
TOTAL		$\boxed{114.53}$	WATTS

B COLD END CONDUCTION LOSSES

1.0 COLD DISPLACER



1.1 WALLS

$$Q = \frac{k A_c (T_H - T_C)}{L} \qquad k = 5.4\ Btu/hr\ ft\text{-}°R$$

$$L = \frac{4.81}{12} = .4008\ ft$$

$$A_c = \pi D t = \pi(.808)(.01625)$$

$$A_c = .041228\ IN^2 = .0002863\ ft^2$$

$$Q = \frac{(5.4)(.0002863)(495)}{.4008} = 1.909\ Btu/hr$$

$$Q = 1.909\ Btu/hr = .55995\ WATT$$

1.2 FIBERGLASS PACKING

$$A_c = \frac{\pi D^2}{4} = \frac{\pi}{4}(.8080)^2 = .5125\ in^2 \qquad k = .02\ Btu/hr\text{-}ft\text{-}°R$$

$$A_c = .003559$$

note argon @ 1atm and 373°R

$$Q = \frac{(.02)(.003559)(495)}{.4008} = .0879\ Btu/hr \qquad k = .0141\ Btu/hr\text{-}ft\text{-}°R$$

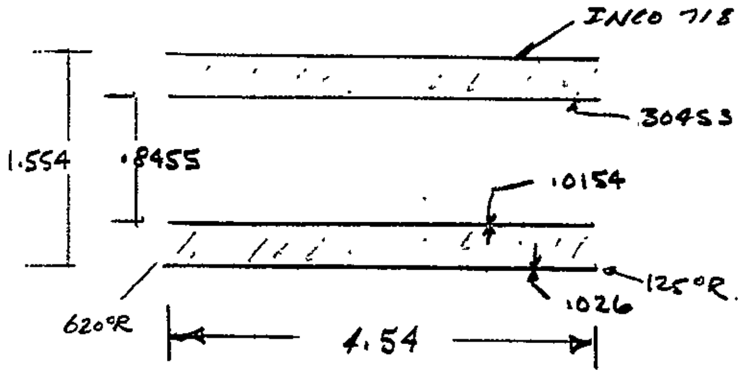
∴ .02 is conservative

$$Q = .0879\ Btu/hr = .0258\ WATTS$$

1.3 TOTAL COLD DISPLACER CONDUCTION LOSSES

Q_{WALLS}	=	.55995 WATT
$Q_{PACKING}$	=	.0258 WATT
		.5857 WATTS

2. COLD REGENERATOR CONDUCTION LOSSES



2.1 OUTER WALL

$$Q = \frac{k A_c (T_H - T_C)}{L} \qquad \bar{k} = 5.4\ Btu/hr\,ft\text{-}°R$$

$$L = 4.54'' = .37833\,ft$$

$$A_c = \pi D t = \pi(1.554)(.026) = .12687\ in^2$$

$$A_c = .12687\ in^2 = .000881\ ft^2$$

$$Q = \frac{(5.4)(.000881)(495)}{.37833} = 6.225\ Btu/hr$$

$$Q = 6.225\ Btu/hr = 1.825\ WATTS$$

2.2 INNER WALL

$$A_c = \pi D t = \pi(.8465)(.0154) \qquad k_{ss} = 7.5\ Btu/hr\,ft\text{-}°R$$

$$A_c = .040885\ in^2 = .0002839\ ft^2$$

$$Q = \frac{(7.5)(.0002839)(495)}{.37833} = 2.786\ Btu/hr$$

$$Q = 2.786\ Btu/hr = .817\ WATTS$$

2.3 REGENERATOR MATRIX

Method: Ref Fig 11-7 pp 290 McAdams

$k_{g\,@\,373°R,\ 800\,PSIA} = 0.07\ Btu/hr\text{-}ft\text{-}°R$

$k_{s\,MONEL} = 11\ Btu/hr\ ft\ °R$

$k_s/k_g = 157 \longrightarrow \text{Fig 11-7} \longrightarrow \dfrac{k_B}{k_g} = 7.2$

$\epsilon = .39$

$\therefore k_B = (.07)(7.2) = .504\ Btu/hr\ ft\text{-}°R$

$$Q = \frac{k_B A_c (T_A - T_c)}{L}$$

$$A_c = \frac{\pi}{4}\left(\left(1.554\right)^2 - \left(.8763\right)^2\right) = \frac{\pi}{4}\left(2.4149 - 0.7679\right) = 1.2929\ in^2$$

$A_c = 1.2929\ IN^2 = .0089785$

$$Q = \frac{(.504)(.0089785)(495)}{.37833} = 5.9206\ Btu/hr$$

$$Q = 5.9206\ Btu/hr = 1.886\ WATTS$$

2.4 TOTAL COLD REGENERATOR CONDUCTION LOSSES

COLD DISPLACER

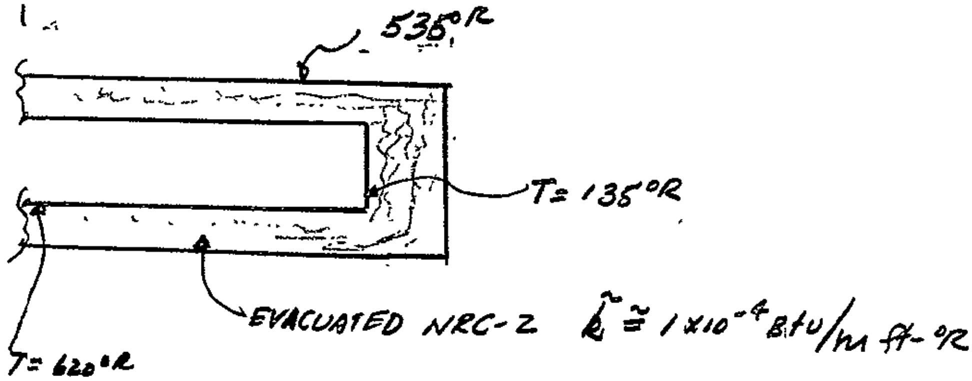
WALLS	.560	WATTS
PACKING	.026	WATTS
TOTAL	.586	WATTS

REGENERATOR

OUTER WALL	1.825	W
INNER WALL	.817	W
REG MATRIX	1.886	W
TOTAL	4.528	WATTS

.586
4.528
5.114 WATTS

3. INSULATION LOSSES



TAKE THE WORST CASE ASSUMING TOTAL AREA OF INNER ONE ENDED CYLINDER IS AT 135°R

$$Q = \frac{kA(535-135)}{t}$$

$$A = 2\pi r L + \pi r^2 = \pi D L + \frac{\pi D^2}{4}$$

$L \cong 6.0''$

$$A = \pi\left((1.95)(6.0) + \frac{(1.95)^2}{4}\right)$$

USE r_{ave} or D_{ave}

$$= \pi(11.7 + .951) = 39.72 \text{ IN}^2$$

$$D_{ave} = \frac{2.30 + 1.60}{2} = 1.95''$$

$$A = 39.72 \text{ IN}^2 = .2758 \text{ ft}^2$$

$$Q = \frac{(1\times10^{-4})(.2758)(400)}{.0292} = .3778 \text{ Btu/hr}$$

$t = .35 \text{ IN} = .0292 \text{ ft}$

$$Q = .3778 \text{ Btu/hr} = .1108 \text{ WATTS}$$

SECTION 15

SUMP COOLING INTERFACE

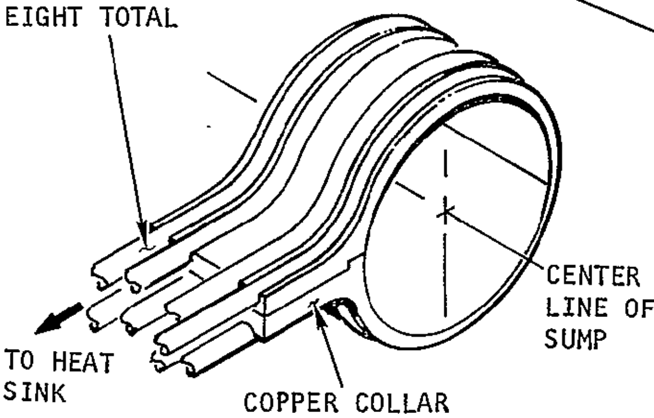
To properly function, VM refrigerators must reject heat from the sump or crankcase region. The amount of heat that must be rejected is equal to the sum of: the hot end input power; the refrigeration load; all losses; and the drive motor input power. In the GSFC 5-watt VM refrigerator, the heat rejection rate is between 325 to 370 watts.

At the beginning of the program, an ambient heat pipe assembly was designed for the purpose of rejecting heat from the refrigerator to a simulated space radiator. This design effort was accomplished under Task 6 of the program. The Task 6 effort also included fabrication and test of a module of the final heat pipe assembly design to verify performance. The module was constructed and successfully tested, clearly demonstrating that design performance could be attained. AiResearch and NASA GSFC mutually agreed that the major technical problems of rejecting heat from the VM refrigerator via a heat pipe assembly had been resolved. As a result of this agreement, Task 6 was terminated to concentrate efforts on other aspects of the refrigerator's development.

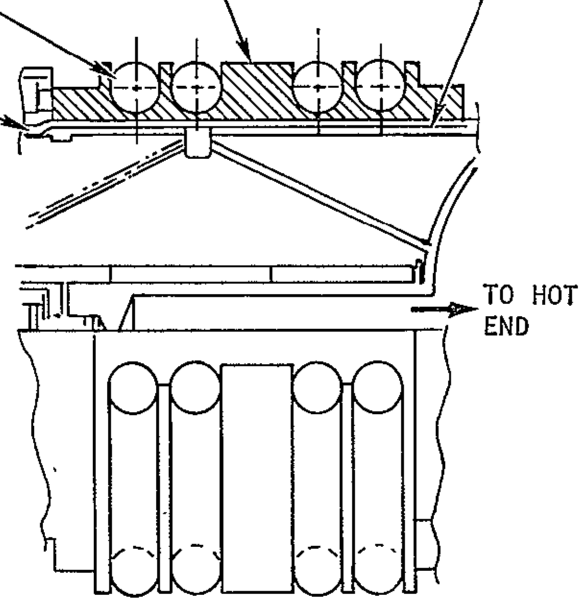
With Task 6 terminated, an alternate method of rejecting heat from the system was required so that the engineering model refrigerator could be tested. Water cooling coils to replace the original heat pipe assembly proved to be the most practical approach. In adapting the refrigerator for water cooling, as much as possible of the heat pipe assembly to refrigerator interface design was retained.

The original interface between the heat pipe assembly and refrigerator is shown in views a and b of Figure 15-1. In this design, heat is rejected from the sump heat exchanger through the sump pressure vessel wall to a copper collar which is clamped around the cylindrical section of the sump (see also Figure 15-1). This original design had four ammonia-filled heat pipes brazed to each half of the copper collar to transport the heat to a remote heat sink. Indium foil placed between the copper collar and sump pressure wall assured good thermal contact. The foil is maintained under a constant interface pressure of approximately 100 psi by Belleville spring washers placed on the bolts which hold the collar halves together, resulting in a contact conductance of 250 $Btu/In^2\text{-}Hr\text{-}^oR$

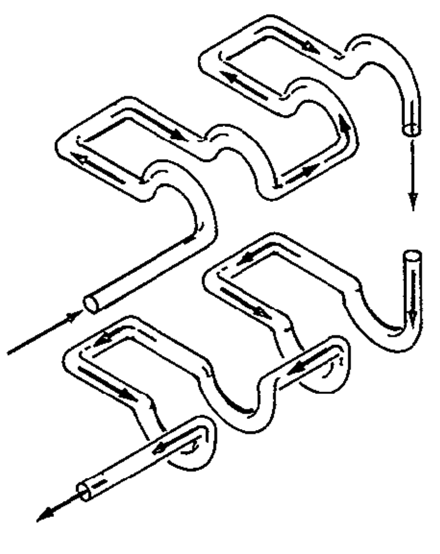
In adapting the refrigerator for water cooling, the copper collar interface was retained and the heat pipes (0.5 in. O.D.) replaced with coolant passages formed by brazing copper tubes (0.5 in. O.D.) to the copper collar. A brief thermal analysis was conducted to establish the best tube arrangement on the collar that would provide the low temperature drop between the water and collar. Water flow through the cooling system was set at 100 gal/hr (typical for laboratory water supplies). At a heat rejection rate of 400 watts



a. SUMP END OF ORIGINAL HEAT PIPE ASSEMBLY



b. CROSS SECTION OF SUMP INTERFACE



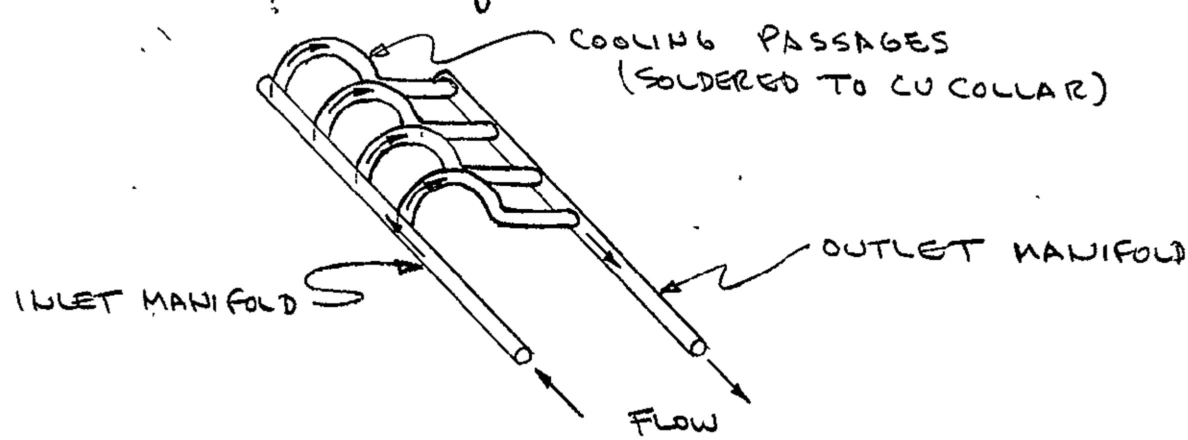
c. FLOW PATH OF WATER COOLANT TUBES TO REPLACE HEAT PIPES

Figure 15-1. Sump Cooling Interface Schematics which exceeds the maximum anticipated rejection rate, the series flow arrangement shown in Figure 15-1c leads to an average temperature difference between the water and cooling collar of slightly greater than 3.0°R. The temperature rise of the water from its inlet to its exit is only 1.64°R. The small temperature differences were entirely consistent with the testing planned for the system. Other arrangements, making use of parallel flow schemes, result in greater temperature gradients between the water and collar for the same water flow rate. Thus, the arrangement shown in Figure 15-1c was selected.

Sump Cooling Water Interface

At the sump, heat is removed by the P/N 852401 cooling jacket assy, which consists of a heavy 2-piece copper collar and 2 copper tube assys for cooling water flow. The collar, with the P/N 852400 tube assys soldered in place, is clamped to the sump region of the engine. This analysis is concerned with the heat transfer performance of the tube assys.

Each tube assy is fabricated as sketched:



The curved portion of each of the four parallel cooling passages is soldered to the copper collar.

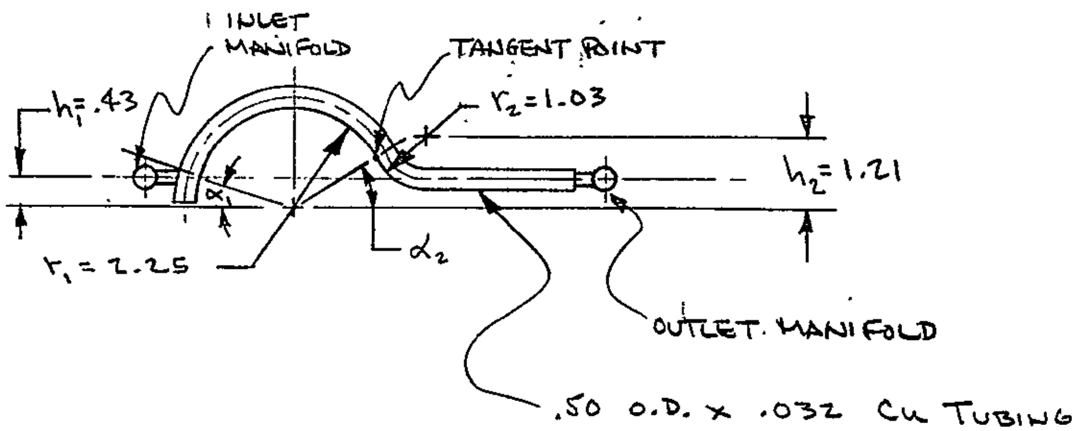
At this interface, approx. 400 w. is to be transferred to the cooling water stream, which is to be available at a flow rate of approx. 100 gph.

In the sump region, the engine working fluid temperature is approx 620 °R. Earlier calculations indicate that the temperature of the copper cooling collar can be taken as 600 °R.

As presently configured, the tube assy call for dividing the available flow between the two assy; in each assy, the flow is further divided among the 4 parallel passages. This may result in unacceptably low heat transfer coefficients in the tubes. In addition, the constant cross-section manifolds will cause flow distribution problems within each tube assy.

Heat Transfer Area

Geometry of curved cooling tube:



Portion of tube with 2.25 in. radius is soldered to collar.

For determining primary heat transfer surface, use arc defined by:

$$\beta = 180^\circ - (\alpha_1 + \alpha_2)$$

Corresponding arc length, at 2.25 in. radius, is:

$$s = \frac{\pi \beta r_2}{180}$$

$$\alpha_1 = \sin^{-1} \frac{h_1}{r_1} = \sin^{-1}\left(\frac{.43}{2.25}\right) = 11^\circ$$

$$\alpha_2 = \sin^{-1} \frac{h_2}{r_1 + r_2} = \sin^{-1}\left(\frac{1.21}{\underset{3.28}{2.25+1.03}}\right) = 21.6^\circ$$

$$\beta = 180 - (11 + 21.6) = 147.4^\circ$$

$$s = 2.25\pi\left(\frac{147.4}{180}\right) = 5.80 \text{ in.}$$

The grooves in the collar are such that the inner half of the tube circumference will be soldered to the collar. Thus, for the 2 tube assy, the prime heat transfer area is

$$A_1 = (2)(4)\,\frac{\pi D_i}{2}\,s$$

The tube inside diameter is

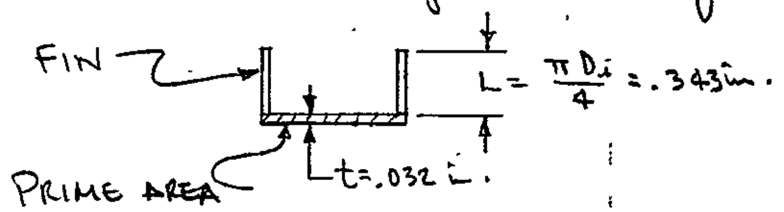
$$D_i = D_o - 2t = .5 - (2)(.032) = .436\text{ in.}$$

$$A_1 = (2)(4)\,\frac{\pi(.436)}{2}\,(5.80) = \underline{31.8\text{ in}^2}$$

$$= \underline{.221\text{ ft}^2}$$

Determine efficiency of remaining tube surface (still considering only the length within the arc β)

Consider as a single-sided fin:



$$\eta_{fin} = \frac{\tanh \sqrt{\dfrac{h}{k_f t}}\;L}{\sqrt{\dfrac{h}{k_f t}}\;L}$$

Since the surface considered as a fin represents the outer half circumference of the tube, it is equal to A_1.

Thus the total heat transfer area for the tube assy is

$$A_h = A_1 + \eta_f A_1 = \underline{\underline{(1+\eta_f) A_1}}$$

Heat Transfer Coefficients

For flow in cylindrical tubes

$$h = .023 \frac{K}{D_i} Re^{.8} Pr^{.4}$$

1. Parallel Flow — As Designed

Assume that uniform flow distribution is achieved in some manner so that flow in each passage is

$$\dot{w} = \frac{\dot{w}_T}{8} = \frac{(100)\left(8.34 \frac{lb\ H_2O}{gal}\right)}{8} = 104\ lb/hr\ H_2O$$

Take water to be supplied at $T_1 = 590°R$ (130°F)

$K = .376$ B/hr ft°R

$Pr = 3.45$

$\mu = .359 \times 10^{-3}$ lb_m/ft-sec

Reynolds number is

$$Re = \frac{4\dot{w}}{\pi D_i \mu}$$

$$Re = \frac{(4)(104)(12)}{\pi(.436)(.359\times10^{-3})(3600)}$$

$$Re = 4.25\times10^{-3}\frac{(104)}{(.436)(.359\times10^{-3})} = \underline{2820}$$ Parallel flow paths

In transition region — use $Nu = 3.66$

$$h = (3.66)\frac{(.376)(12)}{.436} = \underline{\underline{37.8\ B/hr\ ft^2\,°R}}$$

For the copper tubing, $K_f = 225\ B/hr\ ft\,°R$ @ 600°R

$$\sqrt{\frac{h}{k_f t}}\,L = \sqrt{\frac{(37.8)(12)}{(225)(.032)}}\ \frac{.343}{12} = .227$$

$$\eta_f = \frac{\tanh(.227)}{.227} = \frac{.2232}{.227} = .982$$

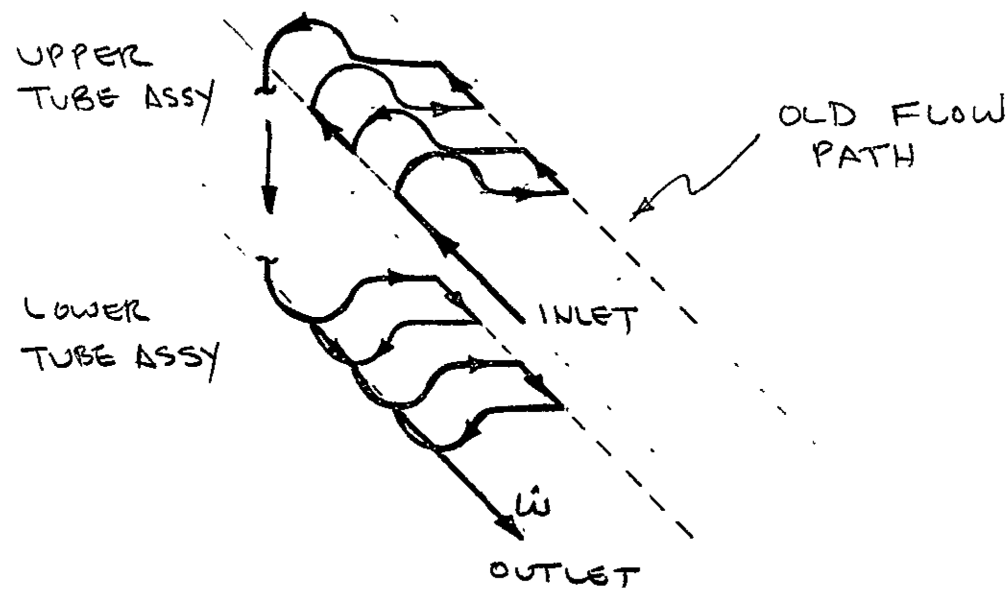
$$A_h = (1.982)(.221) = .438\ ft^2$$

Assume that the tube wall is isothermal at the collar temperature, 600°R. Then, the mean temperature difference to transfer the req'd 400 w. is

$$\Delta T_m = \frac{(400)(3.41)}{(37.8)(.438)} = \underline{\underline{82.5\ °R}}$$ TOO HIGH!

2. Series Flow – Modified Design

To improve existing situation, consider modification of tube assy as sketched to put all cooling passages in series:



Under this arrangement

$$\dot{w} = w_T = (100)(834) = 834 \text{ lb/hr } H_2O$$

and
$$Re = (8)(2820) = \underline{22,550}$$ Series flow

$$h = (.023)\frac{(.376)(12)}{(.436)}(\overset{3000}{22,550})^{.8}(\overset{1.64}{3.45})^{.4} = \underline{\underline{1170 \text{ B/hr ft}^2 \text{°R}}}$$

$$\sqrt{\frac{h}{K_f t}}\,L = \sqrt{\frac{\overset{5.56}{1170}}{37.8}}\,(.227) = 1.262$$

$$\eta_f = \frac{\tanh(1.262)}{1.262} = \frac{.852}{1.262} = .675$$

$A'_h = (1.675)(.221) = .370\ ft^2$

Mean ΔT for $q = 400$ w.

$$\Delta T_m = \frac{(400)(3.41)}{(1170)(.37)} = \underline{3.15°R}$$

Cooling water temp rise:

$$T_2 - T_1 = \frac{q}{\dot{w} c_p} = \frac{(400)(3.41)}{(834)(1.0)} = \underline{1.64°R}$$

Thus, since $T_w = 600°R$

$$T_1 = 600 - (3.15 + .82) = 596.03°R$$

$$T_2 = 597.67°R$$

This change in the flow configuration appears to adequately allow for sufficient sump heat rejection with moderate coolant water flow rates.

SECTION 16

DRIVE MOTOR POWER REQUIREMENTS

INTRODUCTION

Early in the program, considerable effort was devoted to the selection and design of an appropriate drive motor; this effort is reported in the Task I Final Report. Subsequent to the Task I effort it was decided to employ a simple laboratory motor to drive the refrigerator; this provides a more practical and flexible means for driving the refrigerator for the test program envisioned. The design point power requirements for this motor are discussed below.

POWER REQUIREMENTS

Figure 16-1 gives the upper and lower limits of the motor shaft power required with the refrigerator operating at nominal design conditions. The upper limit corresponds to the maximum pressure drop across the cold end previously discussed in relation to the cold regenerator packing and cold end leakage. This maximum pressure drop assumes that the shot used in the packed part of the cold regenerator is not perfectly spherical. The lower power requirement corresponds to a cold-end pressure drop assuming perfect spheres in the cold regenerator packing. The maximum hot displacer pressure drop was used in establishing both power limits.

Based on a laboratory motor efficiency of 50 percent, Figure 16-1 indicates that the motor can be expected to draw between 30 and 32 watts of input power. It should be noted that this is based on operation (after initial run-in of the bearings) at the nominal design temperature, a peak pressure of 800 psia, and a rotational speed of 400 rpm. To provide for operation at other conditions and bearing run-in, an oversized 1/4-hp laboratory motor will be supplied with the refrigerator.

The data of Figure 16-1 was generated by use of the dynamic analysis computer program developed under Task I of the program. This computer program and the associated analysis is described in Appendix A of the Task I Final Report.

During the Task I effort, it was shown that the coefficient of friction of the bearing materials and the resultant losses were of secondary importance when compared to the pressure drop losses in determining the motor shaft power required. In generating the data of Figure 16-1, a conservative coefficient of friction ($\mu = 0.02$) was used for all bearing surfaces. Figure 16-1 clearly indicates the importance of the hot- and cold-end pressure drops relative to motor power. The shaft power increase for each pressure drop is approximately 8 watts/psi for the hot end and 1.2 watts/psi for the cold end. This clearly indicates the reason for reducing the pressure drop through the various flow passages; particularly those associated with the hot end (discussed in previous sections of this report).

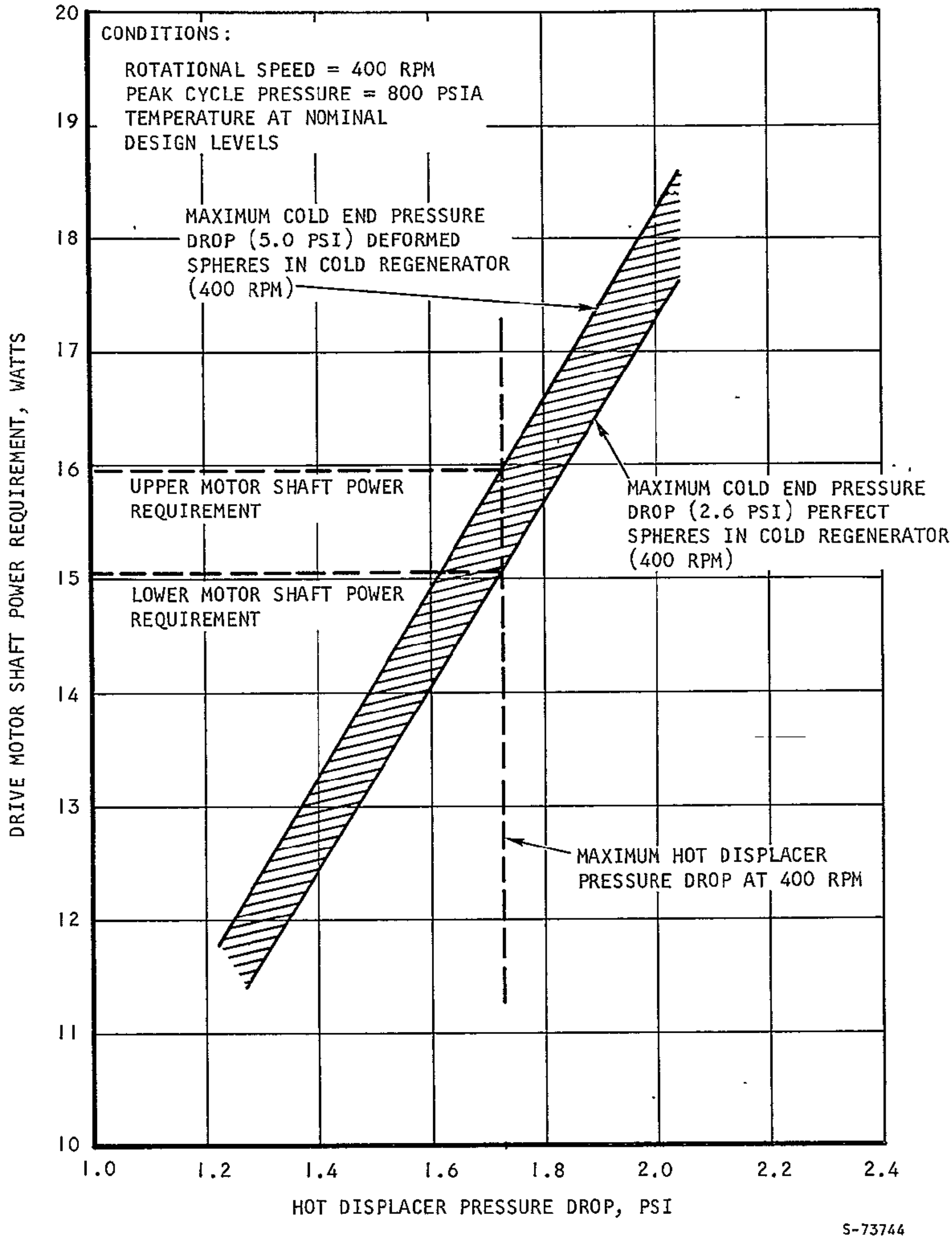


Figure 16-1. Drive Motor Shaft Power Requirements

REFERENCES

1. D. K. Yoshikawa, 75°K Miniature Vuilleumier Cryogenic Engine, Final Report for Task 1 - Preliminary Design, AiResearch Report 70-6854, October, 1970.

2. Fraas and Ozisik, Heat Exchanger Design, Wiley, 1965.

3. C. W. Browning, D. K. Yoshikawa, and V. L. Potter, 75°K Vuilleumier Cryogenic Refrigerator, Final Report for Task 2, AiResearch Report 72-8497.

4. Kays and London, Compact Heat Exchangers, McGraw Hill, Second Edition.

5. S. H. Clark and W. M. Kays, Limiting Nusselt Number for Laminar Flow in Rectangular Ducts, Trans. ASME, 75:859, 1953.

6. McAdams, Heat Transmission, McGraw Hill, New York, Third Edition.