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# Computer Program for Calculation of Real Gas Turbulent Boundary Layers with Variable Edge Entropy

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# COMPUTER PROGRAM FOR CALCULATION OF REAL GAS TURBULENT BOUNDARY LAYERS WITH VARIABLE EDGE ENTROPY

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## SUMMARY

This report describes a computer program developed at NASA-LRC for the calculation of real gas turbulent boundary layers with variable edge entropy on a blunt cone or flat plate at zero angle of attack. The analytical techniques employed and results obtained using the program have been previously reported in (ref. 42) NASA TN D-6217. The details of the coding and operation of the program are discussed in the present report. The digital computer program is described in detail including the flow-charts, program code and instructions for use with sample input and output. The program is written in FORTRAN IV language for use on a CDC 6600 computer.

An integral method is used to compute turbulent boundary layer. The method includes the effect of real gas in thermodynamic equilibrium and variable edge entropy. A modified Crocco enthalpy velocity relationship is used for the enthalpy profiles and an empirical correlation of the N-power law profile is used for the velocity profile. The skin friction coefficient expression of Van Driest, corrected for axisymmetric flow by a turbulent Mangler transformation is used in the solution of the momentum equation. The value of the local coefficient of skin friction is also

calculated using Spalding-Chi and Eckert's theories. Heat-transfer predictions are obtained by use of various modified forms of Reynolds analogy.

The program is written in FORTRAN IV language for use on a CDC 6000 series computer.

Minimum machine requirements are 77000 octal locations of core storage and three input tapes.

The calculation of a typical case with 25 points on the body and four iterations required 260 seconds of CDC 6600 machine time.

## INTRODUCTION

This report describes the analytical techniques employed in the calculation of the turbulent boundary layer with variable edge entropy on a blunt cone at zero angle of attack or on a flat plate. The description of theory was previously reported in NASA TN D-6217 (ref. 42) and is repeated here for ready reference.

In the section entitled Program Description a description of the function performed by each subroutine in the program is given. Flowcharts are provided for the more complicated subroutines. The descriptions of the subroutines are intended as a guide in relating the analysis and governing equations presented in NASA TND 6217 to the program described in the present report.

The section entitled Input Description gives instructions on the preparation of input data for the computer program. Sample turbulent boundary layer flight calculations at an altitude of 60,000 ft. and flight conditions of approximately 19,000 ft./sec. for a 13 ft. long, 5° half angle cone with a nose radius of .4 inch are used to demonstrate the use of the program. In the section entitled Output Description some of the results of this calculation are used to describe the form of the output data.

## SYMBOLS

| | |
|---|---|
| $A$ | area |
| $A_R$, $B_R$ | coefficients in equations (10) and (12) |
| $a$ | speed of sound |
| $\bar{C}_F$ | average skin-friction coefficient based on conditions at edge of boundary layer |
| $C_f$ | local skin-friction coefficient based on conditions at edge of boundary layer |
| $F_c$ | functions given by equations (20), (21), and (22) |
| $F_{MT}$ | Mangler transformation factor |
| $g_c$ | a dimensional constant to convert from slug/ft$^3$ to lbm/ft$^3$ |
| $H$ | total enthalpy |
| $h$ | static enthalpy |
| $\bar{h}$ | heat-transfer coefficient |
| $M$ | Mach number |
| $N$ | exponent in velocity-profile relation |
| $N_{Pr}$ | Prandtl number |
| $N_r$ | recovery factor |
| $N_{St,e}$ | local Stanton number based on conditions at the edge of the boundary layer |
| $p$ | pressure |
| $\bar{q}$ | normalized heat-transfer rate |

| | |
|---|---|
| $q$ | heating rate |
| $R$ | unit Reynolds number |
| $R_{AF}$ | Reynolds analogy factor |
| $R_{e,x}$ | local Reynolds number based on surface distance $x/r_n$ |
| $R_{e,\theta}$ | local Reynolds number based on momentum thickness |
| $r_b/r_n$ | dimensionless body radius |
| $r_n$ | nose radius |
| $r_s/r_n$ | dimensionless radius out to shock (entering stream tube) |
| $S$ | entropy |
| $T$ | temperature |
| $u$ | velocity along $x/r_n$ |
| $v$ | velocity along $y/r_n$ |
| $x$ | surface distance |
| $x/r_n$ | normalized surface distance from stagnation point or sharp-cone apex |
| $x_{vo}$ | distance from virtual origin of turbulent boundary layer |
| $y/r_n$ | normalized coordinate normal to wall |
| $z$ | correlation parameter defined by equation (8) |
| $\alpha_R$ | exponent in equation (10) |
| $\gamma$ | ratio of specific heats |
| $\delta/r_n$ | dimensionless boundary-layer thickness |

$\delta^*/r_n$ dimensionless displacement thickness

$\theta/r_n$ dimensionless momentum thickness

$\theta_c$ cone half-angle

$\rho$ density

$\tau$ shear stress

Subscripts:

aw adiabatic wall

e edge of boundary layer

i incompressible value

max maximum

R one of three regions in boundary layer, I, II, or III

s shock conditions

t stagnation conditions

tr transition

w wall

$\infty$ free-stream value

Primes denote evaluation at reference enthalpy condition of equations (13).

## PROBLEM DISCUSSION

### Thermodynamic Properties for a Real Gas

The physical problem to be considered is that of a blunt body traveling in a homogeneous gas with known thermodynamic properties. The gas in which the body is traveling can be in any state of molecular excitation provided it is in thermodynamic equilibrium. For use in the present program the calculated values of the thermodynamic properties of several gases have been spline fitted with cubics and the coefficients of the cubics have been stored on magnetic tape (TAPE10) by NASA-Ames Research Center. The subroutine RGAS described in reference 27 reads the tape, searches for the proper coefficients, and evaluates the desired properties.

### Inviscid Flow Field and Laminar Boundary Layer

Prior to the turbulent boundary layer calculation, the inviscid flow field is determined by the Lomax and Inouye blunt-body and method of characteristics programs. (See refs. 27 and 34.) The inviscid solution gives the first-order stagnation entropy flow property distribution along the body which is used as the initial estimate of conditions at the edge of the boundary layer. In addition, the shock shape $r_s/r_n$ and entropy distribution along the shock are found from the inviscid solution. The results of the inviscid solution are first used to make a laminar boundary-layer calculation, with variable entropy, over the entire length of the body. (See ref. 35.)

The initial use of the edge conditions from the variable-entropy solution for laminar flow enables the turbulent calculation to be completed in less time than the initial use of the stagnation entropy edge conditions directly from the inviscid solution. Subroutine CRRD reads the inviscid flow field from tapes generated by the Lomax and Inouye blunt body and method of characteristics programs. The laminar boundary layer program punches on cards tables of $r_s/r_n$ and dimensionless shear as functions of $x/r_n$ which are read as input to the turbulent boundary layer program.

## Transition Region

The turbulent boundary layer calculation is initiated at the point where transition has been determined to start (XMIN). The region from the end of the laminar flow solution to the point where the solution is considered to be fully turbulent (X2REX) is defined as the transition region. The velocity profile at the end of the transition region is printed in subroutine FOFX.

## Turbulent Boundary Layer

The general method of calculation of the turbulent boundary layer is an iterative procedure which requires a calculation from the beginning of transition (XMIN) to the end of the body (XMAXTB(20)) to be repeated until the velocity at the edge of the boundary layer changes less than UERR from one iteration to the next.

The momentum integral equation (eq. 3) is solved by a variable step size fifth-order Runge Kutta numerical scheme (subroutine INT1A). The calculation of the integrals from the wall to the edge of the boundary layer (eq. 4, 5, 19) is by Gaussian quadrature (subroutine VGAUSS).

## Variable-Entropy Momentum Integral Equation

The boundary-layer equations for the conservation of mass and momentum for application to a body of revolution at zero angle of attack are

$$\frac{\partial \rho u \frac{r_b}{r_n}}{\partial \frac{x}{r_n}} + \frac{\partial \rho v \frac{r_b}{r_n}}{\partial \frac{x}{r_n}} = 0 \qquad (1)$$

$$\rho u \frac{\partial u}{\partial \frac{x}{r_n}} + \rho v \frac{\partial u}{\partial \frac{y}{r_n}} = - \frac{dp}{d \frac{x}{r_n}} + \frac{\partial \tau}{\partial \frac{y}{r_n}} \qquad (2)$$

where $p = p(x)$ and $\frac{\delta}{r_n} << \frac{r_b}{r_n}$ .

Equations (1) and (2) are combined and integrated from the wall to the edge of the boundary layer. The resulting equations are cast in an integral form amenable for numerical integration as

$$\frac{d\,\frac{\theta}{r_n}}{d\,\frac{x}{r_n}} = \frac{c_f}{2} - \frac{\theta}{r_n}\left[\left(2.0 + \frac{\delta^*/r_n}{\theta/r_n}\right)\frac{1}{u_e}\,\frac{du_e}{d\,\frac{x}{r_n}} + \frac{1}{\rho_e}\,\frac{d\rho_e}{d\,\frac{x}{r_n}} + \frac{1}{r_b/r_n}\,\frac{d\,\frac{r_b}{r_n}}{d\,\frac{x}{r_n}}\right]$$

$$+ \frac{\delta/r_n}{u_e}\left(\frac{dp_e}{d\,\frac{x}{r_n}}\,\frac{g_c}{\rho_e u_e} + \frac{du_e}{d\,\frac{x}{r_n}}\right) \tag{3}$$

where $\delta^*/r_n$ and $\theta/r_n$ are defined by

$$\frac{\delta^*}{r_n}\left(1.0 + \frac{\cos\theta_c}{2\,\frac{r_b}{r_n}}\,\frac{\delta^*}{r_n}\right) = \frac{\delta}{r_n}\int_0^{1.0}\left(1.0 - \frac{\rho u}{\rho_e u_e}\right)\left(1.0 + \frac{\frac{\delta}{r_n}\,\frac{y/r_n}{\delta/r_n}\cos\theta_c}{r_b/r_n}\right) d\,\frac{y/r_n}{\delta/r_n} \tag{4}$$

$$\frac{\theta}{r_n}\left(1.0 + \frac{\cos\theta_c}{2\,\frac{r_b}{r_n}}\,\frac{\theta}{r_n}\right) = \frac{\delta}{r_n}\int_0^{1.0}\frac{\rho u}{\rho_e u_e}\left(1.0 - \frac{u}{u_e}\right)\left(1.0 + \frac{\frac{\delta}{r_n}\,\frac{y/r_n}{\delta/r_n}\cos\theta_c}{r_b/r_n}\right) d\,\frac{y/}{\delta/} \tag{5}$$

Equation (3) is similar to the well-known momentum integral equation (eq. (42), Ch. 9 of ref. 10), with the exception of the additional term

$$\frac{\delta/r_n}{u_e}\left[\frac{dp_e}{d\,\frac{x}{r_n}}\,\frac{g_c}{\rho_e u_e} + \frac{du_e}{d\,\frac{x}{r_n}}\right] \tag{6}$$

For configurations with slight nose bluntness, a variable-entropy condition exists along the edge of the boundary layer. Therefore, the Bernoulli equation is not applicable and the previous term (eq. (6)) is added.

## Velocity Profile

The velocity profiles used in the integral parameters (eqs. (4) and (5)) were calculated from the N-power law relations:

$$\frac{u}{u_e} = \left(\frac{y/r_n}{\delta/r_n}\right)^{1/N} \tag{7}$$

The value of $N$ was calculated from a correlation of data taken from references 11 to 24 and correlate as

$$N = 6.0 \log z - 7.0$$

where

$$z = \frac{R_{e,\theta}^{\;1/3}\left(\frac{T_w}{T_e}\right)^{1/2}\left(\frac{x_{vo}/r_n}{\theta/r_n}\right)^{1/3}}{M_e^{\;1/4}} \tag{8}$$

The $x_{vo}/r_n$ used in equation (8) is the dimensionless distance from the virtual origin of turbulent flow. The expression used to find the virtual origin distance on a cone is

$$\frac{x_{vo}}{r_n} = \frac{\theta / r_n}{1.045 \frac{\bar{C}_F}{2}} \tag{9}$$

where $\bar{C}_F/2$ is the Spalding-Chi (ref. 7) average value of skin friction and $\theta/r_n$ is the local value of momentum thickness.

Density Profiles

The calculation of a density profile through the boundary layer is based on the assumption that the total enthalpy through the boundary layer varies as a function of $u/u_e$ according to a modified form of the Crocco expresssion given by

$$\frac{H - h_w}{H_e - h_w} = A_R + B_R\left(\frac{u}{u_e}\right)^{\alpha_R} \tag{10}$$

The total enthalpy at any point in the boundary layer is given by

$$H = h + \frac{u^2}{2} \tag{11}$$

When equations (10) and (11) are combined, the resulting expression for static enthalpy is

$$h = h_w + \left(H_e - h_w\right)\left[A_R + B_R\left(\frac{u}{u_e}\right)^{\alpha_R}\right] - \frac{u_e^2}{2}\left(\frac{u}{u_e}\right)^2 \tag{12}$$

where the coefficients $A_R$ and $B_R$ and the exponent $\alpha_R$ vary with three regions of calculation through the boundary layer (denoted by subscripts I, II, and III) which are as follows:

<u>Region I (wall) ($0 \leq u/u_e < 0.01$).</u> – The coefficients for the wall region as used in equation (12) are

$$\left.\begin{aligned} A_I &= 0 \\ B_I &= \frac{H_{aw} - h_w}{H_e - h_w} \frac{\left(N_{Pr}\right)_w}{\left(N'_{Pr}\right)^{2/3}} \\ \alpha_I &= 1.0 \end{aligned}\right\} \tag{13}$$

where the coefficient $B_I$ is derived from

$$\left(\frac{d \dfrac{H - h_w}{H_e - h_w}}{d \dfrac{u}{u_e}}\right)_w \tag{14}$$

combined with the energy equation, the Fourier conduction law, and the Colburn form of the Reynolds analogy. (See ref. 28.) The prime indicates that the Prandtl number is evaluated at Eckert's reference enthalpy which is

$$h' = \frac{1}{2}\left(h_w + h_e\right) + 0.22\left(h_{aw} - h_e\right) \tag{15}$$

where $h_{aw} = h_e + N_r\left(H_e - h_e\right)$ and $N_r = 0.89$ were used. The local $N'_{Pr}$ is found from table IV of reference 29 where the Prandtl number is tabulated as a function of temperature and pressures. The temperature used to determine the Prandtl number in reference 29 is found from the real-gas thermodynamic tables as a function of $h'$ (eq. (15)) and the local pressure $p_e$.

Region III (outer) $(0.1 < u/u_e \leq 1.0)$. – The coefficients for the outer region (region III) as used in equation (10) are

$$\left.\begin{aligned} A_{III} &= 0 \\ B_{III} &= 1.0 \\ \alpha_{III} &= \text{Constant at a given } x \text{ station} \end{aligned}\right\} \quad (16)$$

The value of $\alpha_{III}$ varies linearly from an initial value (ALMIN) at the start of transition to ALX at the end of the transition region.

Region II (intermediate) $(0.01 \leq u/u_e \leq 0.1)$. – Regions I and III are joined by an intermediate linear relationship which matches the region I (wall) equation at $u/u_e = 0.01$ and the region III (outer) at $u/u_e = 0.10$, where the coefficients used in equation (10) for the intermediate region are calculated from

$$\left.\begin{aligned} A_{II} &= B_{III}(0.1)^{\alpha_{III}} - 0.1B_{II} \\ B_{II} &= \frac{B_{III}(0.1)^{\alpha_{III}} - B_{I}(0.01)}{(0.09)} \\ \alpha_{II} &= 1.0 \end{aligned}\right\} \quad (17)$$

It should be noted that the boundaries of the three regions may be different from the values assumed herein. (See ref. 8.)

## Mass Flow Into Boundary Layer

The change in entropy at the edge of the boundary layer is determined from the entropy distribution along the shock. The variation in boundary-layer edge conditions due to change in entropy may be calculated by balancing the mass flow in the boundary layer with that entering the stream tube through the shock given by

$$2\pi \left(\frac{r_s}{r_n}\right)^2 \rho_\infty u_\infty = \int_0^\delta \rho u dA \qquad (18)$$

The expression for the shock radius is then given by

$$\frac{r_s}{r_n} = \sqrt{2 \frac{r_b}{r_n} \frac{\rho_e u_e}{\rho_\infty u_\infty} \frac{\delta}{r_n} \int_0^{1.0} \frac{\rho u}{\rho_e u_e} \left(1.0 + \frac{\frac{\delta}{r_n} \frac{y/r_n}{\delta/r_n} \cos \theta_c}{r_b/r_n}\right) d \frac{y/r_n}{\delta/r_n}} \qquad (19)$$

The value of the entropy at the edge of the boundary layer is found with $r_s/r_n$ (from eq. (19)) from a table of shock radius and shock entropy from the inviscid calculation.

## Skin-Friction Calculation

The calculation of the Van Driest II theory in reference 30 (described in detail in ref. 6) uses a Karman-Schoenherr incompressible skin-friction expression. The Karman-Schoenherr equation uses a transformed value of the local Reynolds number based on momentum thickness to obtain skin-friction coefficient for the incompressible plane. The skin-friction coefficient for the incompressible plane is transformed to the compressible plane by using an expression which is a function of $M_e$, and $T_w/T_e$, and is analogous to the Spalding-Chi $F_c$ function. (See ref. 7.) This analogous $F_c$ function is referred to as the "Van Driest II $F_c$ function" and is given in reference 6 as

$$F_c = \frac{0.176 M_e^{2}}{\sin^{-1}\alpha + \sin^{-1}\beta} \tag{20}$$

where $\alpha$ and $\beta$ are functions of Mach number and temperature ratio. (See ref. 6.)

The general expression given in reference 7 for $F_c$ is

$$F_c = \left[\int_0^1 \left(\frac{\rho}{\rho_e}\right)^{1/2} d\,\frac{u}{u_e}\right]^{-2} \tag{21}$$

The $F_c$ function given in equation (21) may be integrated by using a real-gas variation of equation (12); however, the correct values of $\alpha_R$ must be used. The ideal gas $F_c$ function used in reference 7 to correlate the skin-friction data is given by

$$F_c = \left(\int_0^1 \frac{d\,\frac{u}{u_e}}{\left\{\frac{T_w}{T_e} + \left[1 + \frac{N_r}{2}(\gamma - 1)M_e^{2} - \frac{T_w}{T_e}\right]\frac{u}{u_e} - \frac{N_r}{2}(\gamma - 1)M_e^{2}\left(\frac{u}{u_e}\right)^{2}\right\}^{1/2}}\right)^{-2} \tag{22}$$

where $N_r$, as used in reference 7, is equal to 0.89. Equation (22) was derived from equation (21) with the assumption of a linear Crocco relationship (that is, $\alpha_R = 1.0$). The local skin-friction coefficient obtained from the turbulent-flat-plate theories from either reference 7 or reference 6 must be corrected for application to a cone by the Mangler transformation factor $F_{MT}$. (See equations 33, 34.) The local skin friction on a cone downstream of the end of transition is found from

$$\left(\frac{C_f}{2}\right)_{Cone} = F_{MT}\left(\frac{C_f}{2}\right)_{Flat\ plate} \tag{23}$$

where for fully turbulent flow

$$F_{MT} = 1.176 \tag{24}$$

The value of the local coefficient of skin friction needed at each point of calculation in order to evaluate the right-hand side of equation (3) is determined from Van Driest II (ref. 6) theory, calculated by use of a Reynolds number based on momentum thickness. Subroutine CFA also computes skin friction coefficient by seven additional methods which are given in the output for comparison: (1) Van Driest II based on $R_{ex}$, (2) Spalding-Chi based on $R_{ex}$ and ideal gas $F_c$ function, (3) Eckert's reference enthalpy based on $R_{ex}$, (4) Eckert's reference enthalpy based on $R_{ex_{MIN}}$, (5) Eckert's reference enthalpy based on $R_{e\theta}$, (6) Spalding-Chi based on $R_{e\theta}$ and ideal gas $F_c$ function, (7) Spalding-Chi based on $R_{e\theta}$ and real gas $F_c$ function.

## Transition Region Skin Friction

The value of $C_f/2$ in the transition region is assumed to vary as

$$\frac{C_f}{2} = \left.\frac{C_f}{2}\right|_{tr} + \frac{\Upsilon}{2\tanh\psi}\left[\left(\frac{C_f}{2}\right)_{turb} - \left(\frac{C_f}{2}\right)_{tr}\right] \tag{25}$$

where

$$Y = \tanh \psi + \tanh X$$

$$X = \psi - 2\psi \left[ \frac{\left(\frac{x}{r_n}\right)_{turb} - \frac{x}{r_n}}{\left(\frac{x}{r_n}\right)_{turb} - \left(\frac{x}{r_n}\right)_{tr}} \right]$$

and $\psi$ = Constant.

The value of $N$ in the transition region is assumed to vary as

$$N = N_{tr} + \frac{Y}{2 \tanh \psi} \left(N_{turb} - N_{tr}\right) \tag{26}$$

where $Y$ is the expression used in equation (25) and $\psi = 2.0$. The value of $N_{tr}$ is taken from the theoretical laminar calculation. The value of $N$ at the end of the transition $N_{turb}$ is calculated from equation (8).

The value of $\alpha_{III}$ used in equation (12) varies linearly from an initial value (ALMIN) at the start of transition to $\alpha_{III} = 1.0$ (ALX) at the end of the transition region.

## Heat Transfer

The local Stanton number is calculated from a modified Reynolds analogy in the form

$$N_{St,e} = \frac{C_f}{2} R_{AF} \tag{27}$$

where $R_{AF}$ is the Reynolds analogy factor. The value of the $R_{AF}$ depends on which local turbulent skin-friction theory is used. If the Van Driest II (ref. 6) is used for heat transfer, the Reynolds analogy factor is a function of $h_w/H_e$ as listed in table I and for the Spalding-Chi skin-friction theory (ref. 7), $R_{AF} = \frac{1}{\left(N'_{Pr}\right)^{2/3}}$.

TABLE I. - VARIATION OF REYNOLDS ANALOGY FACTOR WITH $h_w/H_e$

| $h_w/H_e$ | $R_{AF}$ |
|---|---|
| $\frac{h_w}{H_e} < 0.2$ | 1.0 |
| $0.2 \leq \frac{h_w}{H_e} \leq 0.65$ | $0.8311 + 0.9675 \frac{h_w}{H_e} - 0.6142 \left(\frac{h_w}{H_e}\right)^2$ |
| $\frac{h_w}{H_e} > 0.65$ | 1.2 |

The value of $N'_{Pr}$ is taken from tables of Prandtl number given in reference 29 evaluated at h' (see eq. (15)) and the local static pressure.

The local heat-transfer coefficient $\bar{h}$ is calculated from

$$\bar{h} = N_{St,e} u_e \rho_e \tag{28}$$

The heating rate q is calculated from

$$q = \bar{h}\left(h_{aw} - h_w\right) \tag{29}$$

where $h_{aw} = h_e + N_r\left(H_e - h_e\right)$ where $N_r = 0.89$.

The variation of the wall shear is calculated from

$$\tau_w = \frac{C_f}{2} \rho_e u_e^2 \tag{30}$$

## Transformation of the Local and Average Turbulent Skin Friction from a Flat-Plate to a Sharp-Cone Value

### Local Skin Friction

The general form of the Mangler transformation factor $F_{MT}$ (see ref. 41) converting local flat-plate skin friction to a sharp-cone value is

$$F_{MT} = \frac{C_f\big|_{Cone}}{C_f\big|_{Flat\ plate}} = \left[\frac{\frac{x}{r_n}\left(\frac{r_b}{r_n}\right)^{\frac{n}{n-1}}}{\int_0^{x/r_n}\left(\frac{r_b}{r_n}\right)^{\frac{n}{n-1}} d\,\frac{x}{r_n}}\right]^{1/n} \tag{31}$$

When the Blasius form of the turbulent skin friction for a flat plate

$$C_f \propto R_{e,x}^{-1/n} \tag{32}$$

is used, with $n = 5.0$, along with the assumption of turbulent flow over the entire cone, the resulting Mangler transformation factor is

$$F_{MT} = 1.176 \tag{33}$$

In the region of fully turbulent flow, the $F_{MT} = 1.176$ (eq. (33)) was used for the calculations in which the Van Driest II skin-friction theory was used. When the Spalding-Chi skin-friction theory was used, an alternate variation of the $F_{MT}$ was used which took into account the transition from a laminar $F_{MT}$ $(\sqrt{3})$ to a turbulent $F_{MT}$ (1.176) in the form

$$F_{MT}^2 - 2\sqrt{3}\, F_{MT} + 3.0 = \bar{P}\left[\frac{x}{r_n} - \left(\frac{x_{v0}}{r_n}\right)_0\right] \tag{34}$$

where

$$\bar{P} = \frac{0.309136}{\left[\left(\frac{x}{r_n}\right)_{max} - \left(\frac{x_{v0}}{r_n}\right)_0\right]}$$

The $\left(x_{v0}/r_n\right)_0$ term was evaluated at the beginning of fully turbulent flow from equation (9) and the $\left(x/r_n\right)_{max}$ was evaluated at the end of the cone.

## Average Skin Friction

The ratio of the average skin friction on a sharp cone to that on a flat plate is

$$\frac{\bar{C}_F\Big|_{Cone}}{\bar{C}_F\Big|_{Flat\ plate}} = \frac{\dfrac{\int_0^L (C_f)_{Cone}\, dA}{\int_0^L dA}}{\dfrac{\int_0^L (C_f)_{Flat\ plate}\, d\frac{x}{r_n}}{\int_0^L d\frac{x}{r_n}}} \tag{35}$$

where L is a constant and is the same for the flat plate and cone. When equations (32) and (33) are substituted into equation (35) and the indicated integration is performed, the resulting ratio of the average skin friction on a cone to that on a flat plate is

$$\frac{\bar{C}_{F_{\text{Cone}}}}{\bar{C}_{F_{\text{Flat plate}}}} = 1.045 \tag{36}$$

PROGRAM DESCRIPTION

The turbulent boundary layer program is written in FORTRAN IV. The various portions of the program, the main program D3340 and subroutines, are given. The function performed by each subroutine is explained. Flow charts are provided for the more complicated subroutines. A FORTRAN listing is given for each subroutine. Library subroutines DIF for differentiation, DISCOT, UNS, DISSER, LAGRAN for interpolation, and INTIA, VGAUSS for integration are used. A description of these subroutines is included in the appendix.

An alphabetical listing of Subroutines with the calling programs written in parentheses.

D3340.- Program D3340 computes the equilibrium gas compressible turbulent boundary layer over blunt cones or flat plates including the effects of variable entropy.

AL3CAL. (D3340, DERSUB)

Subroutine AL3CAL computes coefficients $\alpha_{III}$, $B_{II}$, $A_{II}$ for use in the calculation of density profiles.

CFA. (D3340)

Subroutine CFA computes skin friction coefficient by seven methods-- Van Driest ($R_{ex}$), Spalding-Chi I ($R_{ex}$), Eckert's reference enthalpy ($R_{ex}$), Eckert's reference enthalpy ($R_{ex_{MIN}}$), Eckert's reference enthalpy ($R_{e\theta}$), Spalding-Chi I ($R_{e\theta}$), Spalding-Chi II ($R_{e\theta}$).

CFCAL. (DERSUB)

Subroutine CFCAL computes the VanDriest II $F_c$ function to correlate the skin-friction data.

CHECK. (D3340)

Subroutine to be used by INTIA to allow certain logical control. The control is not desired in this case and a dummy subroutine is inserted.

CMT. (CFA)

Subroutine CMT computes Mangler transformation factor for VanDriest and Eckert's skin-friction coefficient.

CMT1. (CFA)

Subroutine CMT1 computes Mangler transformation factor for Spalding-Chi skin-friction coefficient.

CRRD. (D3340)

Subroutine CRRD reads inviscid flow field from tapes 15, 16, 22 if CARD = 0 is input. This subroutine computes tables of $x/r_n$, $r_b/r_n$, s, p, on body and $r_s/r_n$, s on shock.

<u>DELITR</u>. (DERSUB)

Subroutine DELITR computes boundary layer thickness for cone or flat plate and displacement thickness for flat plate.

<u>DERSUB</u>. (D3340)

Subroutine DERSUB evaluates the derivative of the variable entropy momentum integral equation.

<u>DIF</u>. (D3340)

This is a function subprogram which differentiates the function at any given point in a table of supplied values.

<u>DISCOT</u>. (D3340, CFA, DERSUB, EDGE)

Single or double interpolation subroutine for continuous or discontinuous functions.

<u>DISSER</u>. (DISCOT)

Library subroutine used by DISCOT.

<u>EDGE</u>. (D3340, DERSUB)

Subroutine EDGE computes the initial conditions at the edge of the boundary layer using the results of the inviscid solution and the laminar boundary layer calculation.

<u>FOFX</u>. (DELITR, START)

Function subroutine FOFX computes integrals in the boundary layer equations for conservation of mass and momentum.

FOFZ. (CFA, CFCAL)

Function subroutine FOFZ computes the ideal gas $F_c$ function to correlate the skin-friction data.

FOFZA. (CFA)

Function subroutine FOFZA computes the real gas $F_c$ function to correlate the skin-friction data.

INPN2. (D3340)

Subroutine INPN2 reads NAMELIST $N2 if CARD = 1. is input. Given cone angle, shock angle, constant pressure and entropy this subroutine computes tables of $x/r_n$, $r_b/r_n$, s, p on body and $r_s/r_n$, s on shock.

INT1A. (D3340)

INT1A is a closed subroutine for the solution of a set of ordinary differential equations.

LAGRAN. (DISCOT)

Library subroutine used by DISCOT.

RFCAL. (CFA)

Subroutine RFCAL computes Reynolds analogy factor for VanDriest skin-friction theory.

RFCAL1. (CFA)

Subroutine RFCAL1 computes Reynolds analogy factor for Spalding-Chi and Eckert's skin-friction theories.

RGAS. (D3340, EDGE, RGASH, RGAST)

Subroutine RGAS computes the thermodynamic properties for a real gas.

RGASH. (CFA, EDGE, FOFX, FOFZA)

Subroutine RGASH computes thermodynamic properties density, speed of sound, temperature and entropy given pressure, enthalpy and an estimate of entropy.

RGAST. (D3340, EDGE)

Subroutine RGAST computes thermodynamic properties density, speed of sound, enthalpy and entropy, given pressure, temperature and an estimate of entropy.

ROLL. (RGAS)

Subroutine called by RGAS to position TAPE10 to proper file for gas properties.

SERCH. (RGAS)

Subroutine called by RGAS to locate information for gas properties.

START. (D3340)

Subroutine START computes initial values of boundary layer thickness, displacement thickness, momentum thickness, skin-friction coefficient.

UNS. (DISCOT)

Library Subroutine used by DISCOT.

VANDCF. (CFCAL)

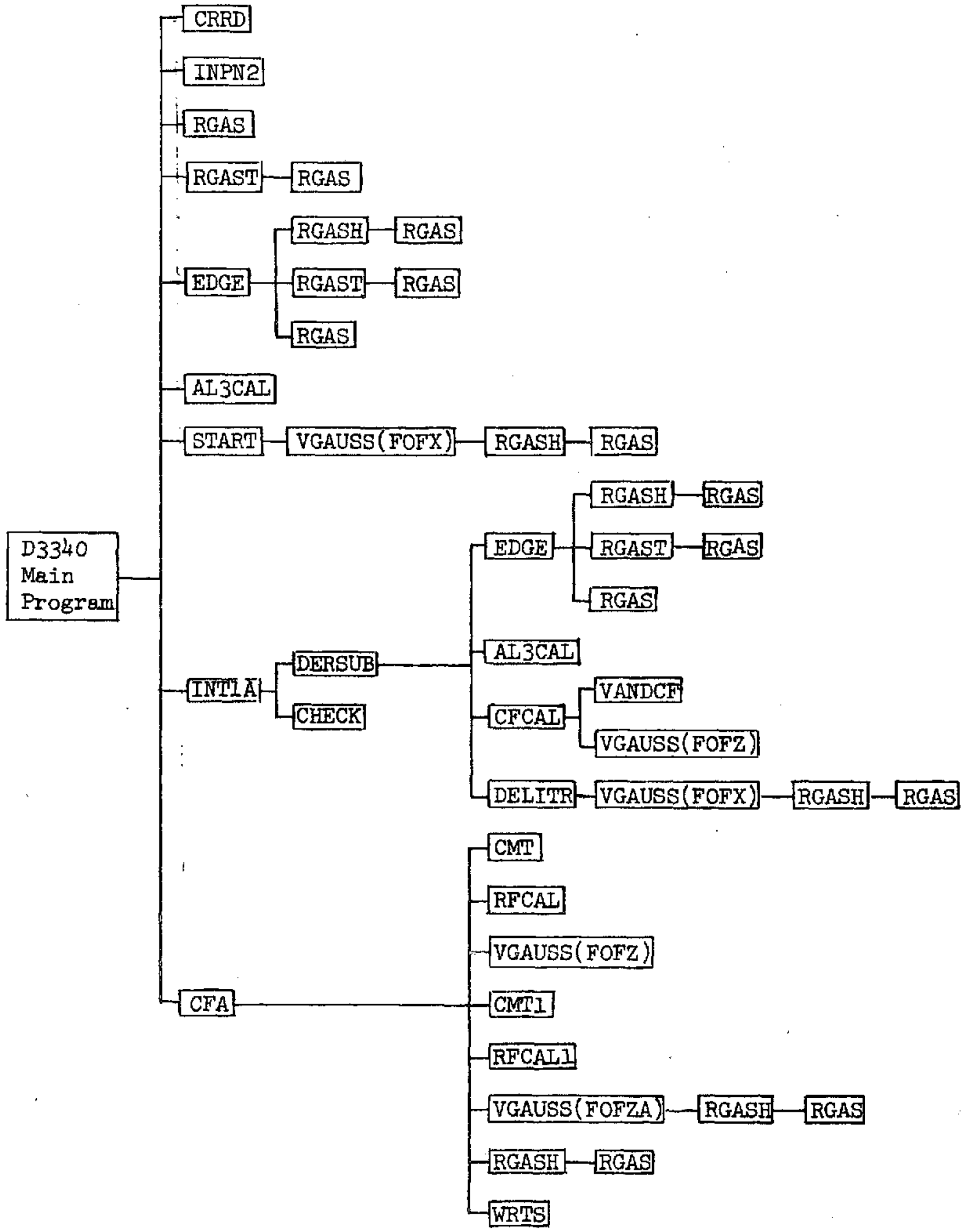
Subroutine VANDCF computes the VanDriest II skin-friction coefficient using Reynolds number based on momentum thickness.

VGAUSS. (CFA, CFACAL, DELITR, START)

To compute the integrals $\int_a^b F_i(x)\,dx$ for i = 1, 2, 3,---, number.

WRTS. (CFA)

Subroutine WRTS computes heat transfer coefficient and heating rates based on each skin-friction theory and prints output.




A Flow Diagram of D3340 by Subroutines Called

## Fortran Subroutines and their functions with flow charts

D3340.-Program D3340 computes the equilibrium gas compressible turbulent boundary layer over blunt cones or flat plates including the effects of variable entropy. The boundary-layer equations for the conservation of mass and momentum are combined and integrated from the wall to the edge of the boundary layer. Prior to the boundary-layer calculation, the inviscid flow field must be determined. The Lomax and Inouye blunt-body and method-of-characteristics programs were used to make this determination. (See refs. 27 and 34.) The inviscid solution gives the first-order stagnation entropy flow property distribution along the body which is used as the initial estimate of conditions at the edge of the boundary layer. In addition, the shock shape $r_s/r_n$ and entropy distribution along the shock are found from the inviscid solution. The results of the inviscid solution are first used to make a laminar boundary-layer calculation, with variable entropy, over the entire length of the body. (See ref. 35.) The turbulent-boundary-layer calculation is initiated at the point where transition has been determined to start (XMIN). The initial edge conditions used in the turbulent-boundary-layer calculation come from the laminar solution.

The momentum integral equation (eq. 3) is solved by a variable-step-size fifth-order Runge-Kutta numerical scheme. The iterative procedure requires a calculation from the beginning of transition to the end of the body to be repeated until the velocity at the edge of the boundary-layer changes less than UERR from one iteration to the next.

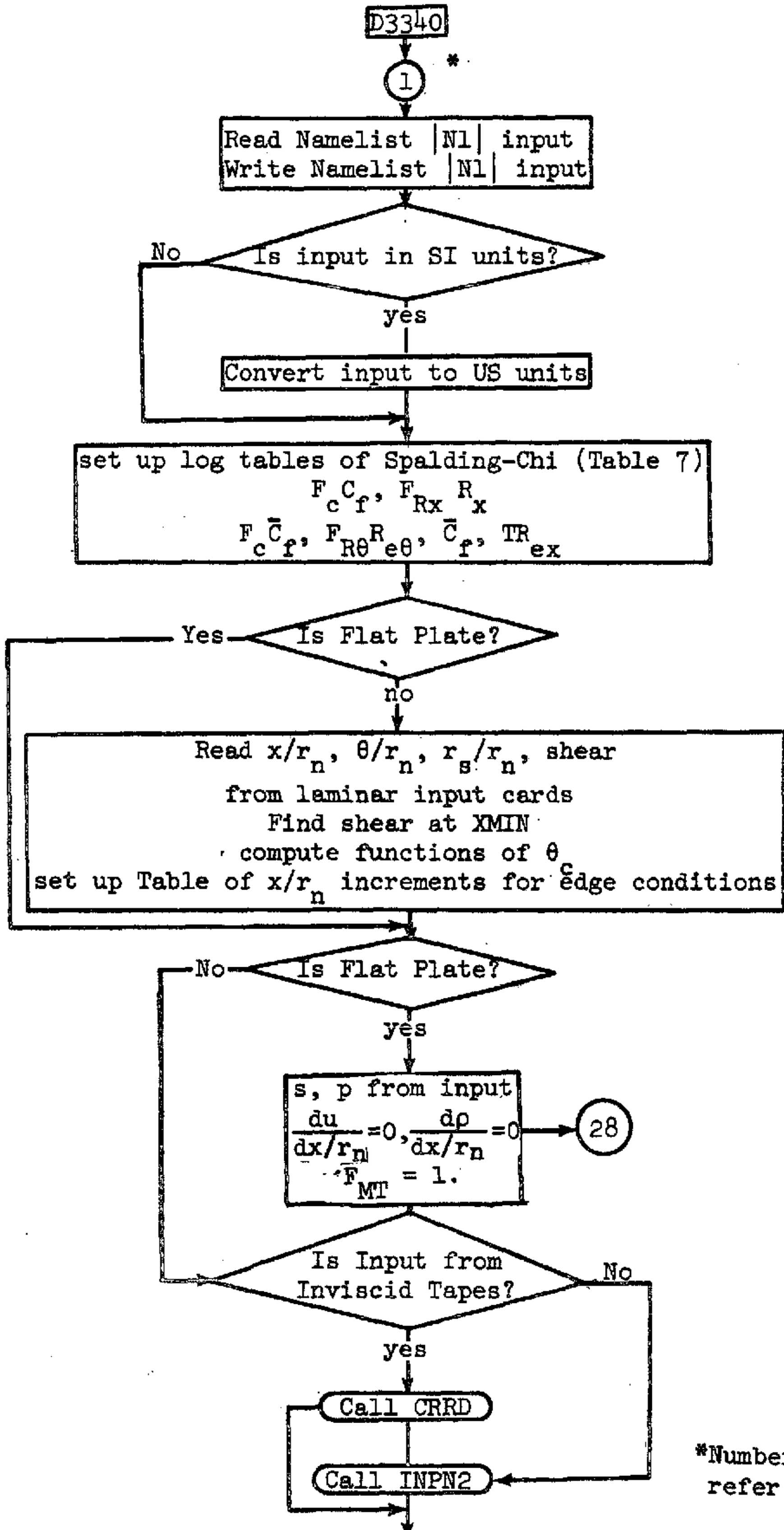
Skin friction, heat transfer coefficient and heating rate are calculated by VanDriest II theory with Reynolds number based on momentum thickness. In addition, skin-friction, heat transfer coefficient and heating rate are calculated by Spalding-Chi and Eckerts reference enthalpy theories with Reynolds number based on surface distance or momentum thickness. These additional skin-friction theories are not used in the calculation of the momentum equation but are printed output from the program.

The velocity profile through the boundary-layer at the end of transition (X2REX + .01) is printed output from the subroutine FOFX.

The thermodynamic properties for a real gas in thermodynamic equilibrium are calculated by the RGAS subroutine described by Lomax and Inouye (ref. 27). The RGAS subroutine requires the use of TAPE10 which contains on file 1 the information for Nitrogen and on file 2 the information for Air, $T < 27000^{o}R$.

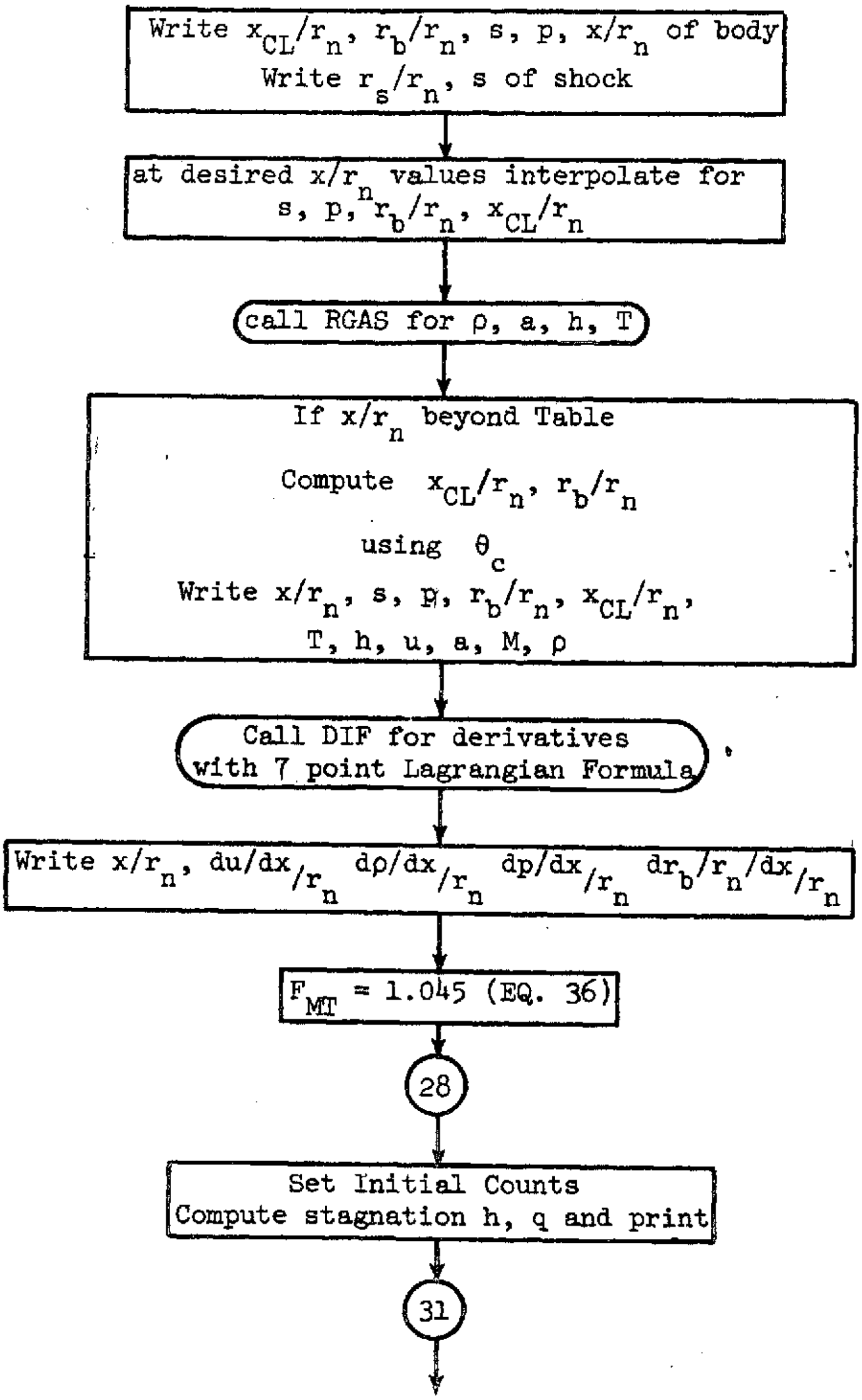
A maximum of 160 stations along the body may be retained in a block of common storage. This block is replaced in the course of each iteration. Thus, values from the final iteration are available at the end of the program and the user may call his own plotting program for plots of $X$, $N$, $R_{e\theta}$, $C_{f/2}$, $M$, $\delta$, $\delta^*$, $\theta$, $h/H$, $R_{ex}$, $S/S_T$, $u/u_\infty$, $\rho/\rho_\infty$, $q/q_t$, $r_s/r_n$.

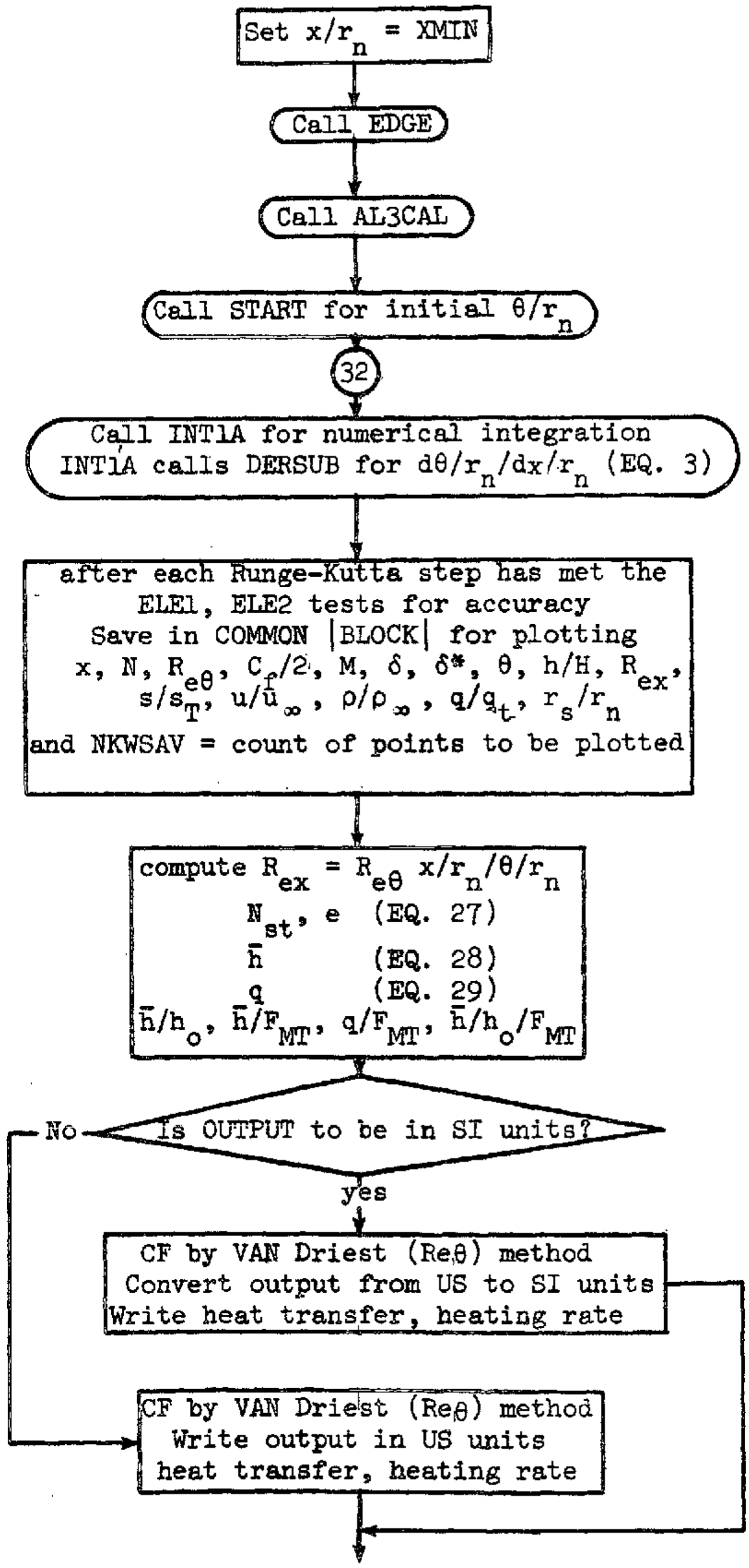
Flow Chart of D3340




*Numbers in circles refer to statement numbers

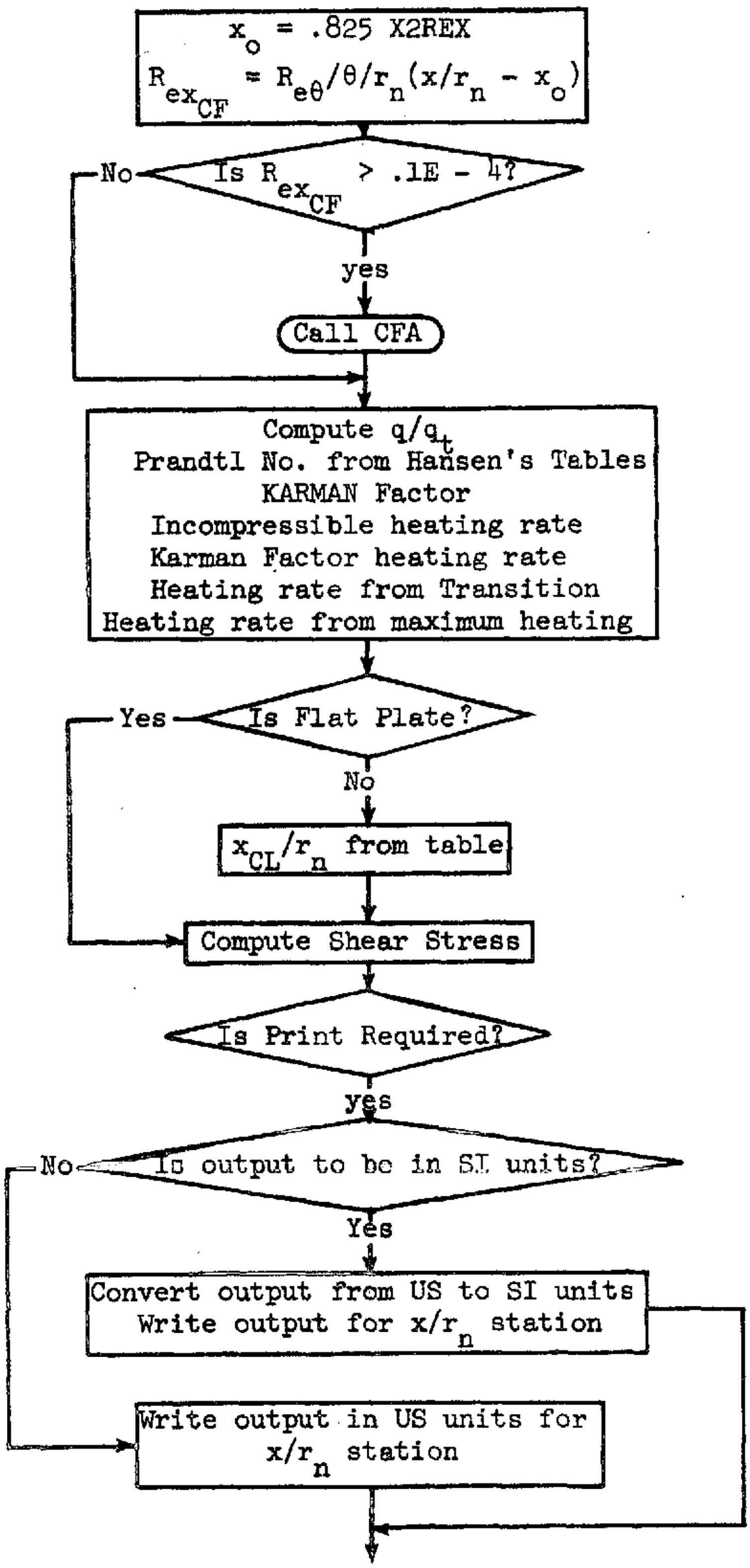
NASA-Langley Form 22 (Apr 69)

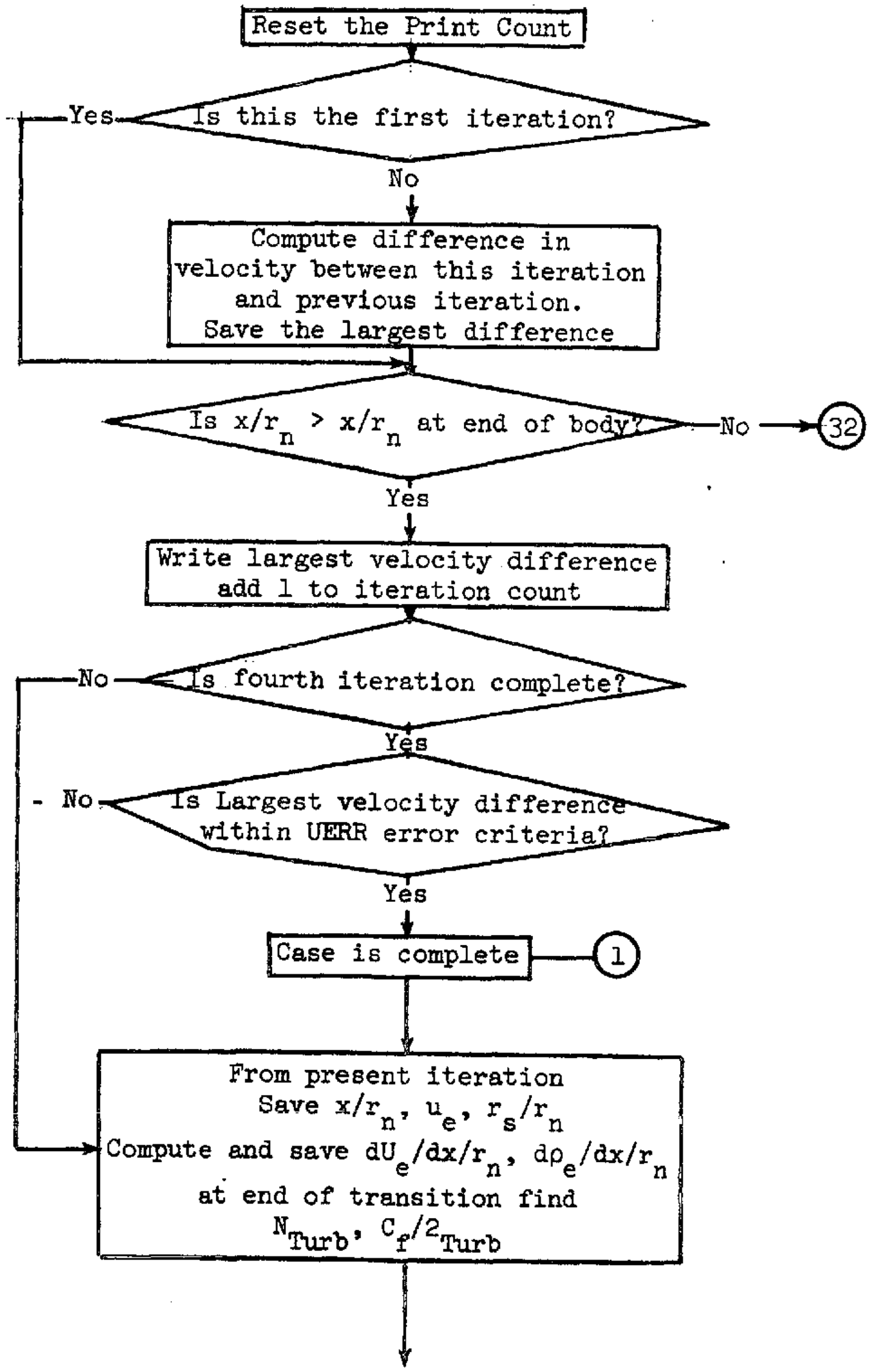

Write $x_{CL}/r_n$, $r_b/r_n$, s, p, $x/r_n$ of body
Write $r_s/r_n$, s of shock
at desired $x/r_n$ values interpolate for
s, p, $r_b/r_n$, $x_{CL}/r_n$
call RGAS for ρ, a, h, T
If $x/r_n$ beyond Table
Compute $x_{CL}/r_n$, $r_b/r_n$
using $\theta_c$
Write $x/r_n$, s, p, $r_b/r_n$, $x_{CL}/r_n$,
T, h, u, a, M, ρ
Call DIF for derivatives
with 7 point Lagrangian Formula
Write $x/r_n$, $du/dx_{/r_n}$ $d\rho/dx_{/r_n}$ $dp/dx_{/r_n}$ $dr_b/r_n/dx_{/r_n}$
$F_{MT}$ = 1.045 (EQ. 36)
28
Set Initial Counts
Compute stagnation h, q and print
31






NASA-Langley Form 22 (Apr 69)


$x_o$ = .825 X2REX
$R_{ex_{CF}} = R_{e\theta}/\theta/r_n(x/r_n - x_o)$
Is $R_{ex_{CF}}$ > .1E - 4?
No
yes
Call CFA
Compute $q/q_t$
Prandtl No. from Hansen's Tables
KARMAN Factor
Incompressible heating rate
Karman Factor heating rate
Heating rate from Transition
Heating rate from maximum heating
Is Flat Plate?
Yes
No
$x_{CL}/r_n$ from table
Compute Shear Stress
Is Print Required?
yes
Is output to be in SI units?
No
Yes
Convert output from US to SI units
Write output for $x/r_n$ station
Write output in US units for $x/r_n$ station


Reset the Print Count
Is this the first iteration?
Yes
No
Compute difference in velocity between this iteration and previous iteration. Save the largest difference
Is $x/r_n > x/r_n$ at end of body?
No
32
Yes
Write largest velocity difference add 1 to iteration count
Is fourth iteration complete?
No
Yes
Is Largest velocity difference within UERR error criteria?
No
Yes
Case is complete
1
From present iteration
Save $x/r_n$, $u_e$, $r_s/r_n$
Compute and save $dU_e/dx/r_n$, $d\rho_e/dx/r_n$
at end of transition find
$N_{Turb}$, $C_f/2_{Turb}$

Compute and print at end of iteration

$x/r_{n_{Turb}}$, $N_{Turb}$, $N_{Turb}-N_{Tran}$,

$2\psi/(x/r_{n_{Turb}}-x/r_{n_{Tran}})$, $C_f/2_{Turb}$,

$(C_f/2_{Turb}-C_f/2_{Tran})$, $\tanh\psi$, $x/r_{n_{Tran}}$,

$N_{Tran}$, $C_f/2_{Tran}$, $R_{ex_{Tran}}$

↓

(31)

```
      PROGRAM D3340(INPUT=1001,OUTPUT=1001,TAPE5=INPUT,TAPE6=OUTPUT,TAPE
     110=1001,TAPE15=1001,TAPE16=1001,TAPE17=1001,TAPE18=1001,TAPE19=100
     21,TAPE20=1001,TAPE21=1001,TAPE22=1001)
C
C     MAIN PROGRAM FOR VARIABLE ENTROPY,REAL GAS,TURBULENT BOUNDARY
C                LAYER
C     READS NAMELIST INPUT,LAMINAR INPUT CARDS, AMES TAPES FOR INVISCID
C                  SOLUTION
C          READ TAPES AND SET UP TABLES
C     AT START OF TRANSITION COMPUTES MOMENTUM THICKNESS FROM SHOCK
C                RADIUS
C     CALLS INTIA FOR VARIABLE STEP SIZE FIFTH ORDER RUNGE KUTTA TO
C                INTEGRATE
C     VARIABLE ENTROPY MOMENTUM INTEGRAL EQUATION
C     AFTER EACH STEP COMPUTES HEATING RATES,SHEAR STRESS,PRINT IF
C                DESIRED
C     IN COMMON/BLOCK/ SAVES OUTPUT TO BE PLOTTED
C     COMPARES VELOCITIES WITH THOSE FROM PREVIOUS ITERATION
C     SAVES VELOCITY,SHOCK RADIUS,DENSITY,N POWER VELOCITY PROFILE ,SKIN
C                  FRICTION
C     VELOCITY DERIVATIVE,DENSITY DERIVATIVE FOR NEXT ITERATION
C     WHEN VELOCITIES FROM TWO SUCCESSIVE ITERATIONS AGREE WITHIN ERROR
C                  CRITERIA
C     AND FINAL X/L ON BODY REACHED,CASE IS COMPLETE
C
C     $N1        I N P U T            (UNIN=1.)     (UNIN=2.)
C     XMIN       X/RN BEGIN TRANSITION
C     DXLTAB(20) TABLE OF INCREMENTS OF X/RN FOR EDGE CONDITION
C     XMAXTB(20) TABLE OF X/RN VALUES WHERE INCREMENTS CHANGE
C                XMAXTB(20) IS END OF BODY
C     FLT        X2REX+.01 FOR PRINT AT END OF TRANSITION
C     XLSH       MAXIMUM X/RN ON SHOCK
C                  FROM TAPE15 IF BODY ANGLE.NE.SHOCK ANGLE
C     XLSH1      MAXIMUM X/RN ON SHOCK
C                  FROM TAPE22 IF BODY ANGLE.NE.SHOCK ANGLE
C                  XLSH=XLSH1 IF BODY=SHOCK
C     XMHL       X/RN AT MAXIMUM HEATING
C     FL         RN NOSE RADIUS (M)                  (INCH)
C     X2REX      X/RN END TRANSITION,PROFILE PRINT AT X2REX+.01
C     RHOUI      RHOINF*UINF (KG/M2SEC)              (LBM/FT2SEC)
C     SHEAR      IF SHEAR=0 INTERPOLATE FROM LAMINAR CARDS
C     RO         RHOO DENSITY (KG/M3)                (LBM/FT3)
```

```
C       RHOI        RHOINF DENSITY (KG/M3)                  (SLUG/FT3)
C       UI          UINF VELOCITY (M/SEC)                   (FT/SEC)
C       HT          TOTAL ENTHALPY (M2/SEC2)                (FT2/SEC2)
C       ST          TOTAL ENTROPY (M2/SEC2 DEGK)            (FT2/SEC2DEGR)
C       TT11        TOTAL TEMPERATURE (DEGK)                (DEGR)
C       DEL         INITIAL DEL/RN BOUNDARY LAYER THICKNESS FROM LAMINAR
C                     FLAT PLATE INPUT,OMIT IF CONE
C       THC         CONE HALF ANGLE (RAD),OMIT IF FLAT PLATE  (DEG)
C       PR          PRANDTL NO.=.72
C       GC          DIMENSIONAL CONSTANT TO CONVERT FROM SLUG/FT3 TO
C                     LBM/FT3 =32.174
C       TRAN        =0 NO TRANSITION
C                   =1 TRANSITION
C       CARD        =0 READ INPUT FROM AMES TAPES
C                   =1 READ NAMELIST N2
C       PRINT       PRINT FREQUENCY
C       A(3)        AR COEFFICIENTS FOR ENTHALPY EQ(12) A(1)=0,A(3)=0
C       F(3)        BR COEFFICIENTS FOR ENTHALPY EQ(12) F(3)=1.
C       ALPHE(3)    ALPHAR COEFFICIENTS FOR ENTHALPY EQ(12)
C                   ALPHE(1)=ALPHE(2)=1.
C       ZMIN        =31.623 EQ(8),SETS N=2
C       ZMAX        =681.33 EQ(8),SETS N=10
C       AN          =1/3 EQ(8),RETH**AN
C       CN          =1/2 EQ(8),(TW/TE)**CN
C       DN          =1/4 EQ(8),ME**DN
C       KN          =1/3 EQ(8),(XVO/THETA)**KN
C       FRXT(20)    FRXRX FROM SPALDING CHI TABLE 7
C       FCXT(20)    FCCF FROM SPALDING CHI TABLE 7
C       FRTB(20)    FRTHRETH FROM SPALDING CHI TABLE 7
C       FCTB(20)    FCCBARF FROM SPALDING CHI TABLE 7
C       TKTAB(30)   TEMPERATURE FROM HANSEN TABLE 4 (DEGK)     (DEGK)
C       PATAB(7)    PRESSURE FROM HANSEN TABLE 4 (N/M2) (ATM)
C       PRTAB(210)  PRANDTL NO. FROM HANSEN TABLE 4
C                     STARTS WITH SMALL PRESSURE AND INCREASING TEMPERATURE
C                     ----LARGE PRESSURE AND INCREASING TEMPERATURE
C       NXINT       NO. OF VALUES IN WALL TEMPERATURE TABLE
C       XINT(99)    VALUES OF X FOR WALL TEMPERATURE TABLE (M) (INCH)
C       TWT(99)     VALUES OF WALL TEMPERATURE (DEGK)   (DEGR)
C       L           =5 NO. OF INTERVALS FOR GAUSSIAN INTEGRATION
C       NN          =4 NO. OF POINTS PER INTERVAL FOR GAUSSIAN INTEGRATION
C       NT          =1 NO. OF VALUES IN ELT BLOCK FOR RUNGE KUTTA
C       CI          INITIAL INTERVAL OF X/RN FOR RUNGE KUTTA
C       SPEC        =0 TO PRINT EVERY INTERVAL IN RUNGE KUTTA
```

```
C       CIMAX         MAXIMUM INTERVAL OF X/RN FOR RUNGE KUTTA
C       ELE1          RELATIVE ERROR IN THETA/RN
C                       IF ERROR GREATER,RK HALVES COMPUTING INTERVAL
C       ELE2          RELATIVE ZERO  IN THETA/RN
C                       WILL NOT APPLY RELATIVE ERROR IF THETA/RN BELOW THIS
C       ERR           RELATIVE ERROR IN T OR H WHEN ITERATING IN RGAS
C       UERR          RELATIVE ERROR IN VELOCITY ON SUCCESSIVE ITERATIONS
C       CFERR         RELATIVE ERROR IN SKIN FRICTION, SP I OR SP II
C       XVO           XVO/RN VIRTUAL ORIGIN FOR EQ(34), SP I OR SP II
C       PSI           FACTOR IN CF/2 EQ(25)
C       ENI           N AT BEGIN TRANSITION EQ(26)
C       ALMIN         ALPHA(3) AT BEGIN TRANSITION EQ(10)
C       ALX           ALPHA(3) AT END TRANSITION =1. EQ(10)
C       FPOPT         OPTION =1 FLAT PLATE
C                            =0 CONE
C       TP            PRESSURE FLAT PLATE (N/M2)          (LBF/FT2)
C                       OMIT IF CONE
C       TS            ENTROPY FLAT PLATE (M2/SEC2DEGK)    (FT2/SEC2DEGR)
C                       OMIT IF CONE
C       ENX           N TURBULENT EQ(26)
C       RHOT2         DENSITY AT STAGNATION POINT (KG/M3) (SLUG/FT3)
C       RNOT          RN NOSE RADIUS (M)                  (FEET)
C       PT2           PRESSURE AT STAGNATION POINT (N/M2) (LBF/FT2)
C       PINF          FREESTREAM PRESSURE (N/M2)          (LBF/FT2)
C       TWO           WALL TEMPERATURE AT STAGNATION POINT (DEGK) (DEGR)
C       IGAS          =1 NITROGEN FILE FROM AMES RGAS TAPE
C                     =2 AIR FILE FROM AMES RGAS TAPE
C       ACFT(22)      TABLE USED WITH VANDRIEST REX
C       TREXT(22)     TABLE USED WITH VANDRIEST REX
C       UNIN          =1. INPUT IN SI UNITS
C                     =2. INPUT IN US CUSTOMARY UNITS
C       UNIO          =1.OUTPUT IN SI UNITS
C                     =2.OUTPUT IN US CUSTOMARY UNITS
C       $
C
C       THE LAMINAR PROGRAM OUTPUTS CARDS WHICH ARE ALSO USED AS INPUT
C       TO THE TURBULENT PROGRAM
C       THESE CARDS CONTAIN NO. OF X/RN,X/RN,THETA/RN,RS/RN,AND
C       DIMENSIONLESS SHEAR IF SHEAR NOT GIVEN IN $N1 (LIMIT OF 100 VALUES
C       OF X/RN)
C
C
C                  IF CARD=1 THE NAMELIST N2 IS READ
```

```
C                   AND THE INVISCID FLOW FIELD TAPES ARE OMITTED          PRG 129
C                   IF CARD=0 THE FLOW FIELD TAPES ARE READ                PRG 130
C                   AND NAMELIST N2 IS OMITTED                             PRG 131
C                                                                          PRG 132
C     $N2         I N P U T                                                PRG 133
C     DELTX       INCREMENT OF X/RN                                        PRG 134
C     TP          PRESSURE (N/M2)                 (LBF/FT2)                PRG 135
C     TS          ENTROPY (M2/SEC2DEGK)           (FT2/SEC2DEGR)           PRG 136
C     THSD        SHOCK ANGLE (RAD)               (DEG)                    PRG 137
C     $                                                                    PRG 138
C                                                                          PRG 139
C     IF CARD=0                                                            PRG 140
C     THE INVISCID FLOW FIELD AS DETERMINED BY THE LOMAX AND INOUYE        PRG 141
C     BLUNT BODY AND METHOD OF CHARACTERISTICS PROGRAMS(REF27,34) IS       PRG 142
C     OUTPUT ON TAPES WHICH ARE USED AS INPUT TO THE LAMINAR AND           PRG 143
C     TURBULENT BOUNDARY LAYER PROGRAMS                                    PRG 144
C     THESE TAPES CONTAIN X/RN CENTERLINE,Y/RN,ENTROPY IN FT2/SEC2DEGR,    PRG 145
C     AND PRESSURE IN LBF/FT2 ON BODY AND SHOCK                            PRG 146
C                                                                          PRG 147
C                 T A P E S                                                PRG 148
C     TAPE5       INPUT                                                    PRG 149
C     TAPE6       OUTPUT                                                   PRG 150
C     TAPE10      AMES RGAS TAPE,FILE2=AIR,FILE1=NITROGEN                  PRG 151
C     TAPE15      INPUT AMES MOC USED FOR BODY POINTS                      PRG 152
C     TAPE16      INPUT AMES BLUNT BODY AND SHOCK POINTS                   PRG 153
C     TAPE17      INTERMEDIATE STORAGE OF INPUT BODY AND SHOCK             PRG 154
C     TAPE18      INTERMEDIATE STORAGE OF INPUT BODY AND SHOCK             PRG 155
C     TAPE19      INTERMEDIATE STORAGE OF INPUT BODY AND SHOCK             PRG 156
C     TAPE20      INTERMEDIATE STORAGE OF INPUT BODY AND SHOCK             PRG 157
C     TAPE21      INTERMEDIATE STORAGE OF INPUT BODY AND SHOCK             PRG 158
C     TAPE22      INPUT AMES MOC USED FOR SHOCK POINTS                     PRG 159
C                                                                          PRG 160
C                 O U T P U T                                              PRG 161
C                                                                          PRG 162
C                 B O D Y  A N D  S H O C K  PTS. FROM A M E S  T A P E SPRG 163
C     X           X/RN CENTERLINE                                          PRG 164
C     Y           Y/RN                                                     PRG 165
C     Q           STREAM VELOCITY                     (FT/SEC)             PRG 166
C     THETA       STREAM ANGLE                        (RAD)                PRG 167
C     M           MACH NUMBER                                              PRG 168
C                                                                          PRG 169
C     S           ENTROPY                             (FT2/SEC2DEGR)PRG 170
C     P           PRESSURE                            (LBF/FT2)            PRG 171
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C       H            ENTHALPY                                        (FT2/SEC2)     PRG 172
C       RHO          DENSITY                                         (SLUG/FT3)     PRG 173
C       PT           OMIT                                                           PRG 174
C                                                                                   PRG 175
C                    B O D Y   P O I N T S   A M E S   T A P E S                    PRG 176
C                    ( L I M I T   O F   4 5 0 )                                    PRG 177
C       XCL          X/RN CENTERLINE                                                PRG 178
C       Y            Y/RN                                                           PRG 179
C       S            ENTROPY                                         (FT2/SEC2DEGR)PRG 180
C       P            PRESSURE                                        (LBF/FT2)      PRG 181
C       X            X/RN SURFACE                                                   PRG 182
C                                                                                   PRG 183
C                    S H O C K   P O I N T S   A M E S   T A P E S                  PRG 184
C                    ( L I M I T   O F   4 5 0 )                                    PRG 185
C       YS          SHOCK RADIUS                                                    PRG 186
C       S           ENTROPY                                          (FT2/SEC2DEGR)PRG 187
C                                                                                   PRG 188
C                    T A B L E   O F   B O D Y   P O I N T S                        PRG 189
C                    I N   DXLTAB   I N C R E M E N T S                             PRG 190
C                    ( L I M I T   O F   1 6 0 )                                    PRG 191
C       X           X POINT LOCATION                                                PRG 192
C       S           ENTROPY                                          (FT2/SEC2DEGR)PRG 193
C       P           PRESSURE                                         (LBF/FT2)      PRG 194
C       RB          BODY RADIUS                                                     PRG 195
C       XC          X CENTERLINE                                                    PRG 196
C       T           TEMPERATURE                                      (DEGR)         PRG 197
C       H           ENTHALPY                                         (FT2/SEC2)     PRG 198
C       U           VELOCITY                                         (FT/SEC)       PRG 199
C       A           SPEED OF SOUND                                   (FT/SEC)       PRG 200
C       M           MACH NUMBER                                                     PRG 201
C       RHO         DENSITY                                          (LBM/FT3)      PRG 202
C       RS          SHOCK RADIUS  (OMIT)                                            PRG 203
C                                                                                   PRG 204
C                    T A B L E   O F   D E R I V A T I V E S                        PRG 205
C                    ( L I M I T   O F   1 6 0 )                                    PRG 206
C       X           X POINT LOCATION                                                PRG 207
C       DU/DX       VELOCITY DERIVATIVE                              (FT/SEC)       PRG 208
C       DRHO/DX     DENSITY DERIVATIVE                               (LBM/FT3)      PRG 209
C       DP/DX       PRESSURE DERIVATIVE                              (LBF/FT2)      PRG 210
C       DRB/DX      BODY RADIUS DERIVATIVE                                          PRG 211
C                                                                                   PRG 212
C                    S T A G N A T I O N   P O I N T                                PRG 213
C       HO          HEAT TRANSFER AT STAGNATION(KG/M2SEC)            (LBM/FT2SEC)   PRG 214
```

```
C      QO          HEATING AT STAGNATION      (WATT/M2)      (BTU/FT2SEC)  PRG 215
C                                                                          PRG 216
C                                                                          PRG 217
C                  O U T P U T   E A C H   I T E R A T I O N               PRG 218
C                                                                          PRG 219
C                   H E A T I N G   R A T E S   E A C H   M E T H O D      PRG 220
C      RAF         REYNOLDS ANALOGY FACTOR                                 PRG 221
C      CFMT        MANGLER TRANSFORMATION FACTOR                           PRG 222
C      CF2         SKIN FRICTION                                           PRG 223
C      FNST        STANTON NO.                                             PRG 224
C      HBAR        LOCAL HEAT TRANSFER COEFF  (KG/M2SEC)   (LBM/FT2SEC)    PRG 225
C      Q           HEATING RATE               (WATT/M2)    (BTU/FT2SEC)    PRG 226
C      HH          HBAR/HO LOCAL HEAT/STAGNATION HEAT                      PRG 227
C      XX1         Y/2TANH PSI (EQUATION 25)                               PRG 228
C      HBCF        HBAR/CFMT                  (KG/M2SEC)   (LBM/FT2SEC)    PRG 229
C      QXCF        Q/CFMT                     (WATT/M2)    (BTU/FT2SEC)    PRG 230
C      HHCF        HH/CFMT                                                 PRG 231
C                                                                          PRG 232
C                   E A C H  X/RN  S T A T I O N (MAX OF 160 STATIONS)     PRG 233
C      X/L         DISTANCE/NOSE RADIUS                                    PRG 234
C      X           DISTANCE                   (M)        PLOT 1  (INCH)    PRG 235
C      THETA/L     MOMENTUM THICKNESS/NOSE RADIUS                          PRG 236
C      THETA       MOMENTUM THICKNESS         (M)        PLOT 8  (INCH)    PRG 237
C      P           PRESSURE                   (N/M2)             (LBF/FT2) PRG 238
C      RB          BODY RADIUS/NOSE RADIUS                                 PRG 239
C      S           ENTROPY                    (M2/SEC2DEGK) (FT2/SEC2DEGR) PRG 240
C      S/ST        ENTROPY/TOTAL ENTROPY                 PLOT 11           PRG 241
C      T           TEMPERATURE                (DEGK)             (DEGR)    PRG 242
C      H           ENTHALPY                   (M2/SEC2)          (FT2/SEC2)PRG 243
C      H/HT        ENTHALPY/TOTAL ENTHALPY              PLOT 9             PRG 244
C      RHO         DENSITY                    (KG/M3)            (LBM/FT3) PRG 245
C                                                                          PRG 246
C      RHO/RHOI    DENSITY/DENSITY FREESTREAM            PLOT 13           PRG 247
C      U           VELOCITY                   (M/SEC)            (FT/SEC)  PRG 248
C      U/SQRT2HT   VELOCITY/SQRT(2*TOTAL ENTHALPY)                         PRG 249
C      U/UI        VELOCITY/VELOCITY FREESTREAM          PLOT 12           PRG 250
C      A           SPEED OF SOUND             (M/SEC)            (FT/SEC)  PRG 251
C      M           MACH NUMBER                          PLOT 5             PRG 252
C      MU          VISCOSITY                  (NSEC/M2)        (LBM/FTSEC) PRG 253
C      RETH        REYNOLDS NO. BASED ON MOMENTUM THICKNESS PLOT 3         PRG 254
C      REX         REYNOLDS NO. BASED ON DISTANCE       PLOT 10            PRG 255
C      TW          WALL TEMPERATURE           (DEGK)           (DEGR)      PRG 256
C      HW          WALL ENTHALPY              (M2/SEC2)        (FT2/SEC2)  PRG 257
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C       HW/HT      WALL ENTHALPY/TOTAL ENTHALPY                                  PRG 258
C                                                                                PRG 259
C       RHOW       WALL DENSITY                (KG/M3)            (LBM/FT3)      PRG 260
C       TW/T       WALL TEMPERATURE/TEMPERATURE                                  PRG 261
C       TAW        ADIABATIC WALL TEMP         (DEGK)             (DEGR)         PRG 262
C       FRTH       FRTHETA                                                       PRG 263
C       FC         FC (EQUATION 22)                                              PRG 264
C       CFMT       MANGLER TRANSFORMATION FACTOR                                 PRG 265
C       LVO        DISTANCE VIRTUAL ORIGIN                                       PRG 266
C       Z          Z EQUATION(8)                                                 PRG 267
C       CFI        SKIN FRICTION INITIAL                                         PRG 268
C       CF2        SKIN FRICTION                         PLOT 4                  PRG 269
C       HAW        ADIABATIC WALL ENTHALPY     (M2/SEC2)          (FT2/SEC2)     PRG 270
C       HAW/HT     ADIABATIC WALL ENTHALPY/TOTAL ENTHALPY                        PRG 271
C                                                                                PRG 272
C       HP         ECKERTS REFERENCE ENTHALPY (M2/SEC2)           (FT2/SEC2)     PRG 273
C       TP         ECKERTS REFERENCE TEMP      (DEGK)             (DEGR)         PRG 274
C       PRP        ECKERTS REFERENCE PRANDTL NO.                                 PRG 275
C       PRW        WALL PRANDTL NO.                                              PRG 276
C       F(1)       COEFFICIENT BI    EQUATION(10)                                PRG 277
C       F(2)       COEFFICIENT BII   EQUATION(10)                                PRG 278
C       A(2)       COEFFICIENT AII   EQUATION(10)                                PRG 279
C       ALPH(3)    COEFFICIENT ALPHAIII EQUATION(10)                             PRG 280
C       N          EXPONENT IN VELOCITY PROFILE RELATION PLOT 2                  PRG 281
C       INT1       INTEGRAL (RHOU/RHOUE-RHOUSQ/RHOUESQ)                          PRG 282
C       INT2       INTEGRAL (RHOU/RHOUE)(Y/DEL)(COSLAM/RB)                       PRG 283
C       INT3       INTEGRAL  (1.-RHOU/RHOUE)                                     PRG 284
C                                                                                PRG 285
C       INT4       INTEGRAL  (1.-RHOU/RHOUE)(Y/DEL)(COSLAM/RB)                   PRG 286
C       INT5       INTEGRAL  (RHOU/RHOUE-RHOUSQ/RHOUESQ)(Y/DEL)(COSLAM/RB)       PRG 287
C       INT6       INTEGRAL  (RHOU/RHOUE)                                        PRG 288
C       INT7       INTEGRAL  (RHOUSQ/RHOUESQ)                                    PRG 289
C       DEL/L      BOUNDARY LAYER THICKNESS/NOSE RADIUS                          PRG 290
C       DEL        BOUNDARY LAYER THICKNESS    (M)        PLOT 6    (INCH)       PRG 291
C       RSL        SHOCK RADIUS/NOSE RADIUS               PLOT 15                PRG 292
C       DELS/L     DISPLACEMENT THICKNESS/NOSE RADIUS                            PRG 293
C       DELS       DISPLACEMENT THICKNESS      (M)        PLOT 7    (INCH)       PRG 294
C       X/DELS     DISTANCE/DISPLACEMENT THICKNESS                               PRG 295
C       DSTH       DISPLACEMENT THICKNESS/MOMENTUM THICKNESS                     PRG 296
C       DU         DERIVATIVE OF VELOCITY WRT. DISTANCE (M/SEC)     (FT/SEC)     PRG 297
C                                                                                PRG 298
C       DRHO       DERIVATIVE OF DENSITY WRT. DISTANCE  (KG/M3)     (LBM/FT3)PRG 299
C       DP         DERIVATIVE OF PRESSURE WRT.DISTANCE  (N/M2)      (LBF/FT2)PRG 300
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C       DRB         DERIVATIVE OF BODY RADIUS WRT. DISTANCE                        PRG 301
C       DTHETA      DERIVATIVE OF MOMENTUM THICKNESS WRT. DISTANCE                 PRG 302
C       NST         STANTON NO.                                                    PRG 303
C       HBAR        LOCAL HEAT TRANSFER COEFFICIENT (KG/M2SEC)   (LBM/FT2SEC)PRG 304
C       Q/QO        HEATING RATE                               PLOT 14             PRG 305
C       PRE         PRANDTL NO.                                                    PRG 306
C       EKF         KARMAN FACTOR                                                  PRG 307
C       QIN         HEATING RATE INCOMPRESSIBLE (WATT/M2)        (BTU/FT2SEC)PRG 308
C       QKF         HEATING RATE KARMAN FACTOR  (WATT/M2)        (BTU/FT2SEC)PRG 309
C       QXTR        HEATING RATE FROM TRANSITION(WATT/M2)        (BTU/FT2SEC)PRG 310
C                                                                                  PRG 311
C       QXMH        HEATING RATE FROM MAX HEATING(WATT/M2)       (BTU/FT2SEC)PRG 312
C       XC          DISTANCE ON CENTERLINE/NOSE RADIUS                             PRG 313
C       RHOUSQ      DENSITY*VELOCITY**2            (N/M2)        (LBF/FT2)         PRG 314
C       TAU         SHEAR STRESS                   (N/M2)        (LBF/FT2)         PRG 315
C                                                                                  PRG 316
C                                                                                  PRG 317
C                    V E L O C I T Y   P R O F I L E  A T  END OF TRANSITION PRG 318
C       Y/D         Y/DELTA                                                        PRG 319
C       U/UE        VELOCITY RATIO                                                 PRG 320
C       H           ENTHALPY                       (M2/SEC2)     (FT2/SEC2)        PRG 321
C       RHO/RHOE    DENSITY RATIO                                                  PRG 322
C       RHOU/RHOEUE DENSITY*VELOCITY RATIO                                         PRG 323
C       HBAR        AR+BR(U/UE)**ALPHAR                                            PRG 324
C       T           TEMPERATURE                  (DEGK)          (DEGR)            PRG 325
C       M           MACH NUMBER                                                    PRG 326
C                                                                                  PRG 327
C                    E R R O R  F O R  E A C H  I T E R A T I O N                  PRG 328
C       TOLL        RELATIVE ERROR IN VELOCITY                                     PRG 329
C                                                                                  PRG 330
C                    E N D  O F  I T E R A T I O N                                 PRG 331
C       REX2        (OMIT)                                                         PRG 332
C       X2REX       X/RN END OF TRANSITION                                         PRG 333
C       EN2REX      N END OF TRANSITION                                            PRG 334
C       CEM         COEFF N EQUATION  (EN2REX-ENI)                                 PRG 335
C       CEB         COEFF CF,N EQNS.  (2PSI)/(X2REX-XMIN)                          PRG 336
C       CF2REX      CF/2 END OF TRANSITION                                         PRG 337
C       CEMP        COEFF CF EQUATION (CF2REX-CF2I)                                PRG 338
C       CEBP        COEFF CF,N EQNS.  (TANH(PSI))                                  PRG 339
C       XMIN        X BEGIN TRANSITION                                             PRG 340
C       RSISAV      (OMIT)                                                         PRG 341
C       ENI         N BEGIN TRANSITION                                             PRG 342
C       CF2I        CF/2 BEGIN TRANSITION                                          PRG 343
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C
C     REXSAV     REX BEGIN TRANSITION
C
C                 D E B U G   O U T P U T   F R O M   RGASH IF SG.GT.1.E6
C      SG(1)     ENTROPY TRIAL 1                        (FT2/SEC2DEGR)
C      SG(2)     ENTROPY TRIAL 2                        (FT2/SEC2DEGR)
C      HG(1)     ENTHALPY TRIAL 1                       (FT2/SEC2)
C      HG(2)     ENTHALPY TRIAL 2                       (FT2/SEC2)
C      SGN       NEXT ESTIMATE OF ENTROPY               (FT2/SEC2DEGR)
C      HW        DESIRED ENTHALPY                       (FT2/SEC2)
C
C
      COMMON XLSH,XLSH1,THCR,JLIM,KLIM,TXLCL(450),TYL(450),TS(450),TP(45
     10),TXL(450),TYLS(450),TSS(450)
      COMMON ENX,FPOPT,ALX,RAF,ALMIN,COPT,STHCR,T1,T2,L,XVO,PSI
      COMMON CMTOPT
      COMMON ALGN,AN,ANZ,AP,AVAR,AW,AX,CEB,CEBP,CEM,CEMP,CFBMT,CFB2,CFER
     1R,CFI,CFMT,CF2,CF2I,CN,COE1,COE2,COE3,CTHCR,DEL,DELAM,DELS,DERR,DN
     2,DPVAR,DRBVAR,DRVAR,DSTH,DUVAR,E,EK,EL,ELT,ELVO,EMU,EMVAR,EMX,EN,E
     3NGN,ENI,ERR,FC,FCCF,FCCFLG,FCF,FCFPR,FD,FDPR,FRRE,FRTH,G,GC,GX,H,H
     4AW,HHAT,HP,HT,HVAR,HW,H2,ICELL,II,IN,JJ,JJLIM,JN,K,KK,KN,LLIM,NKW,
     5NN,NO,NXINT,PATM,PIVAR,PR,PRP,PRW,RBVAR,RET,RETH,RHOUI,RHOW,RO,ROP
     6,ROVAR,ROW,RRI,RRN,RRTVAR,RRX,RSERR,RSISAV,RSLVAR,RX,SHEAR,SP,SVAR
     7,SW,SX,TAW,TEMP,TEMP10,TEMP11,TEMP12,TEMP14,TEMP2,TEMP3,TEMP4,TEMP
     85,TEMP6,TEMP7,TEMP8,TEMP9,TEP,TIVAR,TK,TPP,TRAN,TT11,TW,TX,WVAR,XI
     9N,XMIN,X2REX,Z,ZMAX,ZMIN
      REAL IN,JN,KN
      COMMON F(3),A(3),ALPHE(3),XINT(99),TWT(99),ZTABL(6),TABIN(6),TABJN
     1(6),DXLTAB(20),DELK(2),RSLG(2),ALG(2),ENG(2),AN2(7)
      COMMON FRXT(20),FRXLGT(20),FCXT(20),FCXLGT(20),FRTB(20),FRLGT(20),
     1FCTB(20),FCLGT(20),XW(100),FIN6(100),RSLW(100),SHEER(100),X(160),S
     2(160),PI(160),XC(160),RB(160),W(160),RRT(160),XMAXTB(20),DUDXT(160
     3),DRDXT(160),DPDXT(160),DRBDXT(160),VAR(2),DER(2),CUVAR(2),WSAV(16
     40),RRTSAV(160),XSAV(160),RSLSAV(160),REXSAV(160),ENSAV(160),CFSAV(
     5160),TKTAB(30),PRTAB(210),PATAB(7),XKW(160),WKW(160),RSLKW(160)
      COMMON IGAS
      COMMON XX1
      COMMON ACFT(22),TREXT(22),ACFLGT(22),TRLGT(22),KEXCF,HO,XO
      COMMON HBCF,OXCF,HHCF
      COMMON /BLK/ CV1,CV2,CV3,CV4,CV5,CV6,CV7,CV8,CV9,CV10,CV11,CV12,CV
     113,UNIN,UNIO
      COMMON /BLOCK/ PLT1(160),PLT2(160),PLT3(160),PLT4(160),PLT5(160),P
     1LT6(160),PLT7(160),PLT8(160),PLT9(160),PLT10(160),PLT11(160),PLT12
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```
     2(160),PLT13(160),PLT14(160),PLT15(160),NKWSAV
C      QUANTITY  TO CONVERT FROM US      TO SI        MULTIPLY BY
C
C      LENGTH       INCH             METER           2.54E-2
C      VELOCITY     FT/SEC           M/SEC           3.048E-1
C      ENTHALPY     FT2/SEC2         M2/SEC2         9.290304E-2
C      ENTROPY      FT2/SEC2DEGR     M2/SEC2DEGK     16.7225472E-2
C      HEATTRANSFER LBM/FT2SEC       KG/M2SEC        4.882427522E0
C      DENSITY      LBM/FT3          KG/M3           1.6018463E+1
C      DENSITY      SLUG/FT3         KG/M3           5.15379E+2
C      TEMPERATURE  DEGR             DEGK            .5555555E0
C      ANGLE        DEG              RADIAN          1.745329252E-2
C      PRESSURE     LBF/FT2          NEWTON/M2       4.7880258E+1
C      PRESSURE     ATM              NEWTON/M2       1.01325E+5
C      HEAT         BTU/FT2SEC       WATT/M2         1.1348931E+4
C      VISCOSITY    LBM/FTSEC        NEWTONSEC/M2    1.4881639E0
      DATA CV1/2.54E-2/,CV2/3.048E-1/,CV3/9.290304E-2/,CV4/16.7225472E-2
     1/,CV5/4.882427522/,CV6/16.018463/,CV7/5.15379E2/,CV8/.5555555556/,
     2CV9/1.745329252E-2/,CV10/4.7880258E1/,CV11/1.01325E5/,CV12/1.13489
     331E4/,CV13/1.4881639/
      EXTERNAL DERSUB
      EXTERNAL CHECK
      NAMELIST /N1/ XMIN,DXLTAB,XMAXTB,ELT,XLSH,XLSH1,XMHL,EL,X2REX,RHOU
     1I,SHEAR,RO,RHOI,UI,HT,ST,TT11,DEL,THC,PR,GC,TRAN,CARD,PRINT,A,F,AL
     2PHE,ZMIN,ZMAX,AN,CN,DN,KN,FRXT,FCXT,FRTB,FCTB,TKTAB,PATAB,PRTAB,NX
     3INT,XINT,TWT,L,NN,NT,CI,SPEC,CIMAX,ELE1,ELE2,ERR,UERR,CFERR,XVO,PS
     4I,ENI,ALMIN,ALX,FPOPT,TP,TS,ENX,RHOT2,RNOT,PT2,PINF,TWO,IGAS,ACFT,
     5TREXT,UNIN,UNIO
C
C     READ NAMELIST /N1/ INPUT
1     READ (5,N1)
      IF (ENDFILE 5) 2,3
2     CALL EXIT
3     WRITE (6,N1)
      IF (UNIN-1.) 4,4,7
C     CONVERT INPUT FROM SI TO US UNITS
4     EL=EL/CV1
      RHOUI=RHOUI/CV5
      RO=RO/CV6
      RHOI=RHOI/CV7
      UI=UI/CV2
      HT=HT/CV3
      ST=ST/CV4
```

```
      THC=THC/CV9
      TP=TP/CV10
      TS=TS/CV4
      RHOT2=RHOT2/CV7
      RNOT=RNOT/CV2
      PT2=PT2/CV10
      PINF=PINF/CV10
      TWO=TWO/CV8
      TT11=TT11/CV8
      DO 5 N=1,NXINT
      XINT(N)=XINT(N)/CV1
5     TWT(N)=TWT(N)/CV8
      DO 6 N=1,7
6     PATAB(N)=PATAB(N)/CV11
7     CUVAR(2)=0
      XX1=0
      HBCF=0
      OXCF=0
      HHCF=0
      RETH=0
      REX=0
      FC=0
      CFMT=0
      ELVO=0
      Z=0
      CFI=0
      TOLL=0
      REX2=0
      RSISAV=0
      REXSAV(1)=0
      CFMT=0
      CFI=0
      FC=0
C
C     SET UP LOG OF SPALDING-CHI TABLES
C
      DO 8 N=1,20
      FRXLGT(N)=ALOG10(FRXT(N))
      FCXLGT(N)=ALOG10(FCXT(N))
      FRLGT(N)=ALOG10(FRTB(N))
8     FCLGT(N)=ALOG10(FCTB(N))
      DO 9 N=1,22
      ACFLGT(N)=ALOG10(ACFT(N))
```

```
9       TRLGT(N)=ALOG10(TREXT(N))
C
C       IS FLAT PLATE
C
        IF (FPOPT) 13,10,13
C
C       READ X/L,TH/L,RS/L FROM LAMINAR INPUT CARDS
C
10      READ (5,65) LLIM
        READ (5,66) (XW(LL),LL=1,LLIM)
        READ (5,66) (FIN6(LL),LL=1,LLIM)
        READ (5,66) (RSLW(LL),LL=1,LLIM)
        IF (SHEAR) 12,11,12
11      READ (5,66) (SHEER(LL),LL=1,LLIM)
        CALL DISCOT (XMIN,XMIN,XW,SHEER,SHEER,011,LLIM,0,SHEAR)
C
C       COMPUTE FUNCTIONS OF THC
C
12      THCR=THC/57.29578
        STHCR=SIN(THCR)
        CTHCR=COS(THCR)
        TTHCR=STHCR/CTHCR
        T1=THCR-1.570796
        T2=1.-STHCR
C
C       WRITE NAMELIST INPUT
C
13      CONTINUE
        WRITE (6,67)
C
C       IS FLAT PLATE
C
        IF (FPOPT) 14,15,14
C
C       S,P FROM INPUT TS AND TP,DU/DX=DRHO/DX=0,MTF=1.
C
14      SVAR=TS(1)
        PIVAR=TP(1)
        S(1)=SVAR
        PI(1)=PIVAR
        NKW=2
        XKW(1)=XMIN
        DUDXT(1)=DUDXT(2)=0
```

```
      DRDXT(1)=DRDXT(2)=0
      XKW(2)=XMAXTB(20)
      CFBMT=1.
      GO TO 26
C
C     IS INPUT FROM AMES BLUNT BODY AND MOC TAPES
C
15    IF (CARD) 17,16,17
C
C     CALL CRRD
C
16    CALL CRRD
      GO TO 18
C
C     CALL INPN2
C
17    CALL INPN2
C
C     WRITE TABLES X/LCL,RB/L,S,P,X/L ON BODY,RS/L,S ON SHOCK
C
18    WRITE (6,71)
      WRITE (6,73) ((TXLCL(J),TYL(J),TS(J),TP(J),TXL(J)),J=1,JLIM)
      WRITE (6,72)
      WRITE (6,74) ((TYLS(K),TSS(K)),K=1,KLIM)
C
C     AT DESIRED X/L STATIONS INTERPOLATE IN TABLES FOR S,P,RB/L,X/LCL
C     CALL RGAS TO FIND RHO,A,H,T
C     FIND U,M
C     IF X/L BEYOND TABLE VALUE USE FINAL S,P,FIND X/LCL,RB/L
C
      J=1
      X(J)=XMIN
      WRITE (6,75)
19    IF (X(J)-TXL(JLIM)) 21,21,20
20    S(J)=TS(JLIM)
      PI(J)=TP(JLIM)
      XC(J)=(X(J)+T1)*CTHCR+T2
      RB(J)=CTHCR+(XC(J)-T2)*TTHCR
      GO TO 22
21    CALL DISCOT (X(J),X(J),TXL,TS,TS,011,JLIM,0,S(J))
      CALL DISCOT (X(J),X(J),TXL,TP,TP,011,JLIM,0,PI(J))
      CALL DISCOT (X(J),X(J),TXL,TYL,TYL,011,JLIM,0,RB(J))
      CALL DISCOT (X(J),X(J),TXL,TXLCL,TXLCL,011,JLIM,0,XC(J))
```

```
22    GX=1.4
      CALL RGAS (PI(J),RX,AX,HX,TX,S(J),RRX,GX,-1,5,IGAS)
      W(J)=SQRT(2.*(HT-HX))
      RRT(J)=RX*32.174
      EMX=W(J)/AX
C
C     WRITE TABLES X/L,S,P,RB/L,X/LCL,T,H,U,A,M,RHO
C
      WRITE (6,70) X(J),S(J),PI(J),RB(J),XC(J),TX,HX,W(J),AX,EMX,RRT(J)
      IF (X(J)-XMAXTB(20)) 23,24,24
23    CALL DISCOT (X(J),X(J),XMAXTB,DXLTAB,DXLTAB,011,20,0,DXL)
      J=J+1
      X(J)=X(J-1)+DXL
      GO TO 19
24    JJLIM=J
      WRITE (6,69)
C
C     WRITE X/L,DU/DX,DRHO/DX,DP/DX,DRB/DX USING 3 POINT SLOPE SUBR DIF
C
      DO 25 J=1,JJLIM
      XKW(J)=X(J)
      DUDXT(J)=DIF(J,3,JJLIM,X,W)
      DRDXT(J)=DIF(J,3,JJLIM,X,RRT)
      DPDXT(J)=DIF(J,3,JJLIM,X,PI)
      DRBDXT(J)=DIF(J,3,JJLIM,X,RB)
25    WRITE (6,70) X(J),DUDXT(J),DRDXT(J),DPDXT(J),DRBDXT(J)
C
C     MTF=1.045  EQ(36)
C
      CFBMT=1.045
C
C     SET  INITIAL COUNTS AND X/L=XMIN
C
      NKW=JJLIM
26    CEBP=TANH(PSI)
      CNT=0
      ICELL=0
      PATM=PT2*.4725E-3
      TK=TWO/1.8
      CALL DISCOT (TK,PATM,TKTAB,PRTAB,PATAB,011,210,7,PRTWO)
      RHOW=RHOT2*TT11/TWO
      EMUW=7.310615E-7*SQRT(TWO)/(1.+201.6/TWO)
      EMUT2=7.310615E-7*SQRT(TT11)/(1.+201.6/TT11)
```

```
      DUEDX=SQRT(2.*(PT2-PINF)/RHOT2)/RNOT
      CALL RGAST (PT2,RX,AX,HWO,TWO,S(1),ERR,IGAS)
      HO=.768*SQRT(32.174)*PRTWO**(-.6)*(RHOW*EMUW)**.1*(RHOT2*EMUT2)**.
     14*DUEDX**.5
      QO=HO*(HT-HWO)/2.5036E4
      IF (UNIO-1.) 27,27,28
27    O87=HO*CV5
      O88=QO*CV12
      WRITE (6,60) O87,O88
      GO TO 29
28    WRITE (6,60) HO,QO
29    TOLL=0
      II=0
      SP=6.9E4
      SW=4.8E4
      EK=.4
      E=12.
      PRCT=0
      CO=CI
      NKWSAV=0
      VAR(1)=XMIN
      CUVAR(1)=XMIN
C
C     CALL EDGE
C
      CALL EDGE
C
C     CALL AL3CAL
C
      CALL AL3CAL
C
C     CALL START TO GET INITIAL TH/L
C
      CALL START
      VAR(2)=CUVAR(2)
      WRITE (6,63)
      WRITE (6,61)
      WRITE (6,62)
      WRITE (6,63)
      WRITE (6,68)
C
C     CALL INT1A VARIABLE STEP SIZE RUNGE KUTTA
C     INT1A CALLS DERSUB TO GET DTH/DX EQ(3)
```

```
C
30    CALL INTIA (II,1,NT,CO,SPEC,CIMAX,IERR,VAR,CUVAR,DER,ELE1,ELE2,ELT
     1,ERRVAL,DERSUB,CHECK,0)
      IF (IERR-1) 31,32,31
31    CALL EXIT
C
C     AFTER EACH RK STEP HAS MET THE ELE1,ELE2 TESTS FOR ACCURACY
C     IN COMMON/BLOCK/ SAVE FOR PLOTTING
C     X,N,RETH,CF/2,M,DEL,DELST,THETA,H/HT,REX,S/ST,U/UI,RHO/RHOI,Q,RS/L
C     AND NKWSAV=COUNT OF POINTS TO BE PLOTTED
C
32    NKWSAV=NKWSAV+1
      PLT1(NKWSAV)=VAR(1)*EL
      PLT3(NKWSAV)=RETH
      PLT5(NKWSAV)=EMVAR
      PLT6(NKWSAV)=DEL*EL
      PLT7(NKWSAV)=DELS*EL
      PLT8(NKWSAV)=VAR(2)*EL
      PLT9(NKWSAV)=HVAR/HT
C
C     REYNOLDS NO,STANTON NO(EQ27),HEAT TRANSFER COEFF(EQ28)
C
      REX=RETH*VAR(1)/VAR(2)
      PLT11(NKWSAV)=SVAR/ST
      PLT12(NKWSAV)=WVAR/UI
      PLT13(NKWSAV)=ROVAR/RHOI
      ENST=CF2*RAF
      HBAR=ENST*WVAR*RRTVAR
C
C     HEATING RATE(EQ29),PRANDTL NO.FROM HANSENS TABLE,KARMAN FACTOR
C
      Q=HBAR*(HAW-HW)/2.5036E4
      HH=HBAR/HO
      WRITE (6,63)
      IF (CFMT) 33,34,33
33    HBCF=HBAR/CFMT
      QXCF=Q/CFMT
      HHCF=HH/CFMT
34    IF (UNIO-1.) 35,35,36
35    O81=HBAR*CV5
      O82=Q*CV12
      O85=HBCF*CV5
      O86=QXCF*CV12
```

```
      WRITE (6,70) RAF,CFMT,CF2,ENST,O81,O82,HH,XX1,O85,O86,HHCF
      GO TO 37
36    WRITE (6,70) RAF,CFMT,CF2,ENST,HBAR,Q,HH,XX1,HBCF,QXCF,HHCF
37    XO=.825*X2REX
      REXCF=(RETH/VAR(2))*(VAR(1)-XO)
      IF (ABS(REXCF)-.1E-4) 39,38,38
38    CALL CFA
39    Q=Q/QO
      WRITE (6,63)
      PLT14(NKWSAV)=Q
      WSAV(NKWSAV)=WVAR
      RRTSAV(NKWSAV)=RRTVAR
      XSAV(NKWSAV)=VAR(1)
      RSLSAV(NKWSAV)=PLT15(NKWSAV)=RSLVAR
      REXSAV(NKWSAV)=PLT10(NKWSAV)=REX
      ENSAV(NKWSAV)=PLT2(NKWSAV)=EN
      CFSAV(NKWSAV)=PLT4(NKWSAV)=CF2
      TK=TIVAR/1.8
      CALL DISCOT (TK,PATM,TKTAB,PRTAB,PATAB,O11,210.7,PRE)
      EKF=1./(1.+5.*SQRT(CFI)*(PRE-1.-ALOG((5.*PRE+1.)/6.)))
C
C     INCOMPRESSIBLE Q,KARMAN FACTOR Q
C
      QIN=(WVAR*RRTVAR*CF2*(HAW-HW))/2.5036E4
      QKF=EKF*QIN
      VARD=VAR(1)/DELS
      PN3=WVAR/SQRT(2.*HT)
      PN7=HAW/HT
      PN8=HW/HT
      FRX=FRTH/FC
C
C     TRANSITION HEATING
C
      RXTR=REX*(1.-XMIN/VAR(1))
      IF (RXTR-1000.) 40,41,41
40    QXTR=1.
      GO TO 42
41    FRRX=ALOG10(FRX*RXTR)
      CALL DISCOT (FRRX,FRRX,FRXLGT,FCXLGT,FCXLGT,O11,20.0,FCCXLG)
      FCCX=10.**FCCXLG
      CFP2=CFMT*FCCX/(2.*FC)
      ENSTP=CFP2*RAF
      HBARP=ENSTP*WVAR*RRTVAR
```

```
      QXTR=HBARP*(HAW-HW)/2.5036E4
C
C     MAXIMUM HEATING
C
42    RXMH=REX*(1.-XMHL/VAR(1))
      IF (RXMH-1000.) 43,44,44
43    QXMH=1.
      GO TO 45
44    FRRM=ALOG10(FRX*RXMH)
      CALL DISCOT (FRRM,FRRM,FRXLGT,FCXLGT,FCXLGT,O11,20,0,FCCMLG)
      FCCM=10.**FCCMLG
      CFPP2=CFMT*FCCM/(2.*FC)
      ENSTPP=CFPP2*RAF
      HBARPP=ENSTPP*WVAR*RRTVAR
      QXMH=HBARPP*(HAW-HW)/2.5036E4
45    IF (FPOPT) 47,46,47
46    CALL DISCOT (VAR(1),VAR(1),X,XC,XC,O11,JJLIM,0,XCVAR)
C
C     SHEAR STRESS  EQ(30)
C
47    RHOUSQ=ROVAR*WVAR*WVAR
      TAU=CF2*RHOUSQ
C
C     WRITE OUTPUT
C
      PRCT=PRCT+1
      IF (PRINT-PRCT) 48,48,52
48    IF (UNIO-1.) 49,49,50
C     CONVERT OUTPUT FROM US TO SI UNITS
49    O2=PLT1(NKWSAV)*CV1
      O4=PLT8(NKWSAV)*CV1
      O5=PIVAR*CV10
      O7=SVAR*CV4
      O9=TIVAR*CV8
      O10=HVAR*CV3
      O12=RRTVAR*CV6
      O14=WVAR*CV2
      O17=AVAR*CV2
      O19=EMU*CV13
      O22=TW*CV8
      O23=HW*CV3
      O25=RHOW*CV6
      O27=TAW*CV8
```

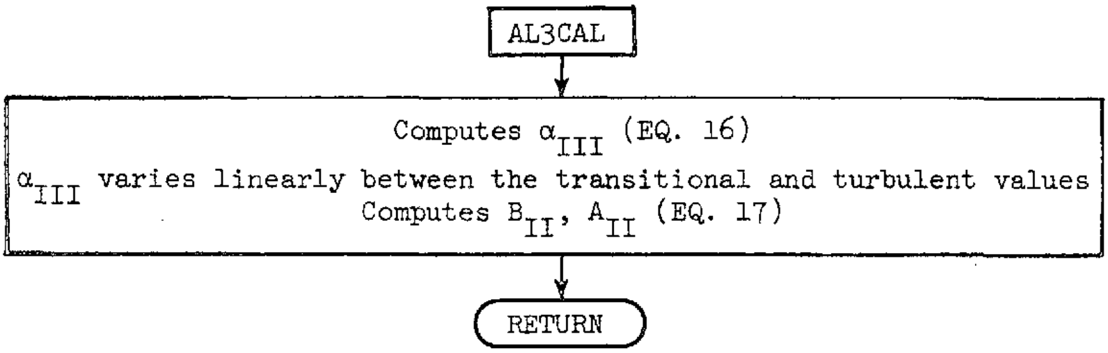
```
      035=HAW*CV3
      037=HP*CV3
      038=TPP*CV8
      054=PLT6(NKWSAV)*CV1
      057=PLT7(NKWSAV)*CV1
      060=DUVAR*CV2
      061=DRVAR*CV6
      062=DPVAR*CV10
      066=HBAR*CV5
      070=QIN*CV12
      071=QKF*CV12
      072=QXTR*CV12
      073=QXMH*CV12
      075=RHOUSQ*CV10
      076=TAU*CV10
      WRITE (6,70) VAR(1),02,VAR(2),04,05,RBVAR,07,PLT11(NKWSAV),09,010,
     1PLT9(NKWSAV),012,PLT13(NKWSAV),014,PN3,PLT12(NKWSAV),U17,PLT5(NKWS
     2AV),019,PLT3(NKWSAV),PLT10(NKWSAV),022,023,PN8,025,COE1,027,FRTH,F
     3C,CFMT,ELVO,Z,CFI,PLT4(NKWSAV),035,PN7,037,038,PRP,PRW,F(1),F(2),A
     4(2),ALPHE(3),PLT2(NKWSAV),AN2,DEL,054,PLT15(NKWSAV),DELS,057,VARD,
     5DSTH,060,061,062,DRBVAR,DER(2),ENST,066,PLT14(NKWSAV),PRE,EKF,070,
     6071,072,073,XCVAR,075,076
      GO TO 51
50    WRITE (6,70) VAR(1),PLT1(NKWSAV),VAR(2),PLT8(NKWSAV),PIVAR,RBVAR,S
     1VAR,PLT11(NKWSAV),TIVAR,HVAR,PLT9(NKWSAV),RRIVAR,PLT13(NKWSAV),WVA
     2R,PN3,PLT12(NKWSAV),AVAR,PLT5(NKWSAV),EMU,PLT3(NKWSAV),PLT10(NKWSA
     3V),TW,HW,PN8,RHOW,COE1,TAW,FRTH,FC,CFMT,ELVO,Z,CFI,PLT4(NKWSAV),HA
     4W,PNT,HP,TPP,PRP,PRW,F(1),F(2),A(2),ALPHE(3),PLT2(NKWSAV),AN2,DEL,
     5PLT6(NKWSAV),PLT15(NKWSAV),DELS,PLT7(NKWSAV),VARD,DSTH,DUVAR,DRVAR
     6,DPVAR,DRBVAR,DER(2),ENST,HBAR,PLT14(NKWSAV),PRE,EKF,QIN,QKF,QXTR,
     7QXMH,XCVAR,RHOUSQ,TAU
51    PRCT=0
C
C     IS FIRST ITERATION
C
52    IF (ICELL) 53,54,53
C
C     COMPUTE DIFFERENCE IN VELOCITY
C     SAVE LARGEST DIFFERENCE IN VELOCITY
C
53    CALL DISCOT (VAR(1),VAR(1),XKW,WKW,WKW,011,NKW,0,WOLD)
      DFF=ABS((WOLD-WVAR)/WOLD)
      TOLL=AMAX1(DFF,TOLL)
```

```
C
C     IS END OF BODY
C
54    IF (VAR(1)-XMAXTB(20)) 30,55,55
55    CONTINUE
      WRITE (6,64) TOLL
C
C     ADD TO ITERATION COUNT
C
      CNT=CNT+1
C
C     IS FOURTH ITERATION COMPLETE
C
      IF (CNT-4) 58,56,56
C
C     IS LARGEST VELOCITY DIFFERENCE WITHIN ERROR CRITERIA
C
56    IF (TOLL-UERR) 57,58,58
C
C     CASE IS COMPLETE
C
57    CONTINUE
      GO TO 1
C
C     FROM PRESENT ITERATION SAVE X/L,U,RS/L,DU/DX,DRHO/DX
C
58    DO 59 NK=1,NKWSAV
      WKW(NK)=WSAV(NK)
      XKW(NK)=XSAV(NK)
      RSLKW(NK)=RSLSAV(NK)
      DUDXT(NK)=DIF(NK,3,NKWSAV,XSAV,WSAV)
59    DRDXT(NK)=DIF(NK,3,NKWSAV,XSAV,RRTSAV)
      NKW=NKWSAV
C
C     FIND AT END OF TRANSITION N(TURB) AND CF/2(TURB)
C
      CALL DISCOT (X2REX,X2REX,XKW,ENSAV,ENSAV,111,NKW,0,EN2REX)
      CALL DISCOT (X2REX,X2REX,XKW,CFSAV,CFSAV,111,NKW,0,CF2REX)
C
C     FIND TRANSITION REGION COEFF FOR CF/2 AND N (EQ25,26)
C
      CEM=EN2REX-ENT
      CEB=(PSI+PSI)/(X2REX-XMIN)
```

```
      CEMP=CF2REX-CF2I
      WRITE (6,76)
      WRITE (6,70) REX2,X2REX,EN2REX,CEM,CEB,CF2REX,CEMP,CEBP,XMIN,RSISA
     1V,ENI,CF2I,REXSAV(1)
C
C     START NEXT ITERATION
C
      ICELL=2
      GO TO 29
C
C
C
60    FORMAT (1H 3HHO=,E11.3,4H QO=,E11.3)
61    FORMAT (21H SKIN FRICTION THEORY,/24H  (1) VAN DRIEST RETHETA,/54H
     1  (2) VAN DRIEST REX                   OMIT IN TRANS REGION,/54H  (3) S
     2PALDING CHI REX               OMIT IN TRANS REGION,/54H  (4) ECKERTS 
     3REF ENTHALPY REX     OMIT IN TRANS REGION,/54H  (5) ECKERTS REF ENT
     4HALPY REXMIN OMIT IN TRANS REGION,/21H  (6) ECKERTS RETHETA,/37H  
     5(7) SPALDING CHI I RETHETA IDEAL FC,/37H  (8) SPALDING CHI II RETH
     6ETA REAL FC)
62    FORMAT (1H ,7X,3HRAF,7X,4HCFMT,7X,3HCF2,8X,4HENST,7X,4HHBAR,7X,1HO
     1,10X,2HHH,9X,3HXX1,8X,4HHBCF,7X,4HOXCF,7X,4HHHCF,7X)
63    FORMAT (1H )
64    FORMAT (1H 5HTOLL=,E11.3)
65    FORMAT (I5)
66    FORMAT (5E15.8)
67    FORMAT (1H 14X,1HX,15X,1HY,15X,1HQ,11X,5HTHETA,15X,1HM,15H BODY+SH
     1OCK PTS/15X,1HS,15X,1HP,15X,1HH,13X,3HRHO,12X,4H  PT)
68    FORMAT (1H 7X,3HX/L,10X,1HX,4X,7HTHETA/L,6X,5HTHETA,10X,1HP,9X,2HR
     1B,10X,1HS,7X,4HS/ST,10X,1HT,10X,1HH,7X,4HH/HT,8X,3HRHO/3X,8HRHO/RH
     2OI,10X,1HU,2X,9HU/SQRT2HT,7X,4HU/UI,10X,1HA,10X,1HM,9X,2HMU,7X,4HR
     3ETH,8X,3HREX,9X,2HTW,9X,2HHW,6X,5HHW/HT/7X,4HRHOW,7X,4HTW/T,8X,3HT
     4AW,7X,4HFRTH,9X,2HFC,7X,4HCFMT,8X,3HLVO,10X,1HZ,8X,3HCFI,8X,3HCF2,
     58X,3HHAW,5X,6HHAW/HT/9X,2HHP,9X,2HTP,8X,3HPRP,8X,3HPRW,7X,4HF(1),7
     6X,4HF(2),7X,4HA(2),4X,7HALPH(3),10X,1HN,7X,4HINT1,7X,4HINT2,7X,4HI
     7NT3/7X,4HINT4,7X,4HINT5,7X,4HINT6,7X,4HINT7,6X,5HDEL/L,8X,3HDEL,8X
     8,3HRSL,5X,6HDELS/L,7X,4HDELS,5X,6HX/DELS,7X,4HDSTH,9X,2HDU/7X,4HDR
     9HO,9X,2HDP,8X,3HDRB,5X,6HDTHETA,8X,3HNST,7X,4HHBAR,10X,1HQ,8X,3HPR
     $E,8X,3HEKF,8X,3HQIN,8X,3HQKF,7X,4HQXTR/7X,4HQXMH,9X,2HXC,5X,6HRHOU
     $SQ,8X,3HTAU)
69    FORMAT (1H 9X,1HX,3X,5HDU/DX,5X,7HDRHO/DX,5X,5HDP/DX,5X,6HDRB/DX)
70    FORMAT (12E11.3)
71    FORMAT (1H 6X,3HXCL,14X,1HY,15X,1HS,15X,1HP,15X,1HX)
```

```
72     FORMAT (1H 6X,2HYS,15X,1HS)
73     FORMAT (5E16.8)
74     FORMAT (2E16.8)
75     FORMAT (1H 5X,1HX,10X,1HS,10X,1HP,9X,2HRB,9X,2HXC,10X,1HT,10X,1HH,
      110X,1HU,10X,1HA,10X,1HM,9X,3HRHO,8X,2HRS)
76     FORMAT (1H 6X,4HREX2,6X,5HX2REX,5X,6HEN2REX,8X,3HCEM,8X,3HCEB,5X,6
      1HCF2REX,7X,4HCEMP,7X,4HCEBP,7X,4HXMIN,5X,6HRSISAV,8X,3HENI,7X,4HCF
      22I/5X,6HREXSAV)
       END
```

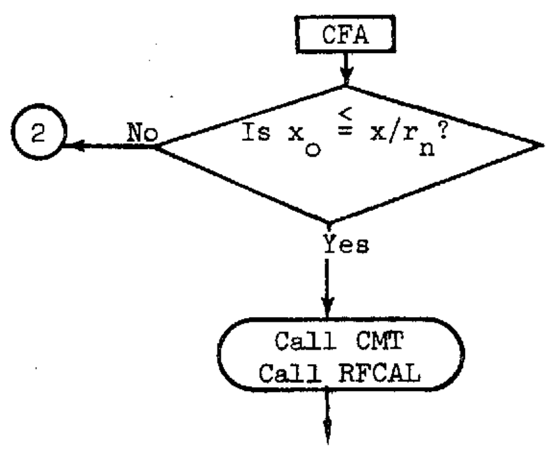
AL3CAL.- Subroutine AL3CAL computes coefficients $\alpha_{III}$, $B_{II}$, $A_{II}$ for use in the calculation of density profiles.



```
      SUBROUTINE AL3CAL
C
C     COMPUTE ALPHA(3) EXPONENT IN OUTER BOUNDARY LAYER REGION
C     FOR USE IN STATIC ENTHALPY MODIFIED CROCCO EXPRESSION
C     ALPHA(3)=ALMIN AT START OF TRANSITION
C     ALPHA(3)= LINEAR VARIATION IN TRANSITION REGION
C     ALPHA(3)=ALX    AT END OF TRANSITION
C
      COMMON XLSH,XLSH1,THCR,JLIM,KLIM,TXLCL(450),TYL(450),TS(450),TP(45
     10),TXL(450),TYLS(450),TSS(450)
      COMMON ENX,FPOPT,ALX,RAF,ALMIN,COPT,STHCR,T1,T2,L,XVO,PSI
      COMMON CMTOPT
      COMMON ALGN,AN,ANZ,AP,AVAR,AW,AX,CEB,CEBP,CEM,CEMP,CFBMT,CFB2,CFER
     1R,CFI,CFMT,CF2,CF2I,CN,COE1,COE2,COE3,CTHCR,DEL,DELAN,DELS,DERR,DN
     2,DPVAR,DRBVAR,DRVAR,DSTH,DUVAR,E,EK,EL,ELT,ELVO,EMU,EMVAR,EMX,EN,E
     3NGN,ENI,ERR,FC,FCCF,FCCFLG,FCF,FCFPR,FD,FDPR,FRRE,FRTH,G,GC,GX,H,H
     4AW,HHAT,HP,HT,HVAR,HW,H2,ICELL,II,IN,JJ,JJLIM,JN,K,KK,KN,LLIM,NKW,
     5NN,NO,NXINT,PATM,PIVAR,PR,PRP,PRW,RBVAR,RET,RETH,RHOUI,RHOW,RO,ROP
     6,ROVAR,ROW,RRI,RRN,RRTVAR,RRX,RSERR,RSISAV,RSLVAR,RX,SHEAR,SP,SVAR
     7,SW,SX,TAW,TEMP,TEMP10,TEMP11,TEMP12,TEMP14,TEMP2,TEMP3,TEMP4,TEMP
     85,TEMP6,TEMP7,TEMP8,TEMP9,TEP,TIVAR,TK,TPP,TRAN,TT11,TW,TX,WVAR,XI
     9N,XMIN,X2REX,Z,ZMAX,ZMIN
      REAL IN,JN,KN
      COMMON F(3),A(3),ALPHE(3),XINT(99),TWT(99),ZTABL(6),TABIN(6),TABJN
     1(6),DXLTAB(20),DELK(2),RSLG(2),ALG(2),ENG(2),AN2(7)
      COMMON FRXT(20),FRXLGT(20),FCXT(20),FCXLGT(20),FRTB(20),FRLGT(20),
     1FCTB(20),FCLGT(20),XW(100),FIN6(100),RSLW(100),SHEER(100),X(160),S
     2(160),PI(160),XC(160),RB(160),W(160),RRT(160),XMAXTB(20),DUDXT(160
     3),DRDXT(160),DPDXT(160),DRBDXT(160),VAR(2),DEK(2),CUVAR(2),WSAV(16
     40),RRTSAV(160),XSAV(160),RSLSAV(160),REXSAV(160),ENSAV(160),CFSAV(
     5160),TKTAB(30),PRTAB(210),PATAB(7),XKW(160),WKW(160),RSLKW(160)
      COMMON /BLOCK/ PLT1(160),PLT2(160),PLT3(160),PLT4(160),PLT5(160),P
     1LT6(160),PLT7(160),PLT8(160),PLT9(160),PLT10(160),PLT11(160),PLT12
     2(160),PLT13(160),PLT14(160),PLT15(160),NKWSAV
      IF (CUVAR(1)-XMIN) 2,1,2
C
C     X/L(TRAN),X/L(TURB),ALPHAIII(TRAN),ALPHAIII(TURB),
C     AI=0,ALPHAI=1,ALPHAII=1.,AIII=0,BIII=1. ARE INPUT
C     BI FROM SUBROUTINE EDGE
C
1     ALPHE(3)=ALMIN
      GO TO 5
```

```
2       IF (CUVAR(1)-X2REX) 3,3,4
C
C       ALPHAIII LINEAR BETWEEN TRANSITION AND TURBUENT (EQ16)
C
3       ALPHE(3)=((CUVAR(1)-XMIN)/(X2REX-XMIN))*(ALX-ALMIN)+ALMIN
        GO TO 5
4       ALPHE(3)=ALX
C
C       BII (EQ17)
C       AII
C
5       TEP=F(3)*.1**ALPHE(3)
        F(2)=(TEP-F(1)*.01)/9.E-2
        A(2)=TEP-.1*F(2)
        RETURN
        END
```

CFA.- Subroutine CFA computes skin friction coefficient by seven methods-- VanDriest ($R_{ex}$), Spalding-Chi I ($R_{ex}$), Eckert's reference enthalpy ($R_{ex}$), Eckert's reference enthalpy ($R_{ex_{MIN}}$), Eckert's reference enthalpy ($R_{e\theta}$), Spalding-Chi I ($R_{e\theta}$), Spalding-Chi II ($R_{e\theta}$).



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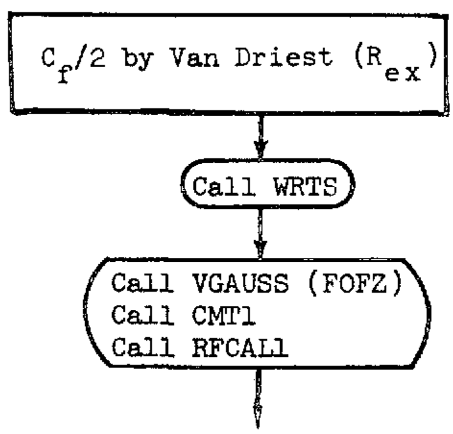
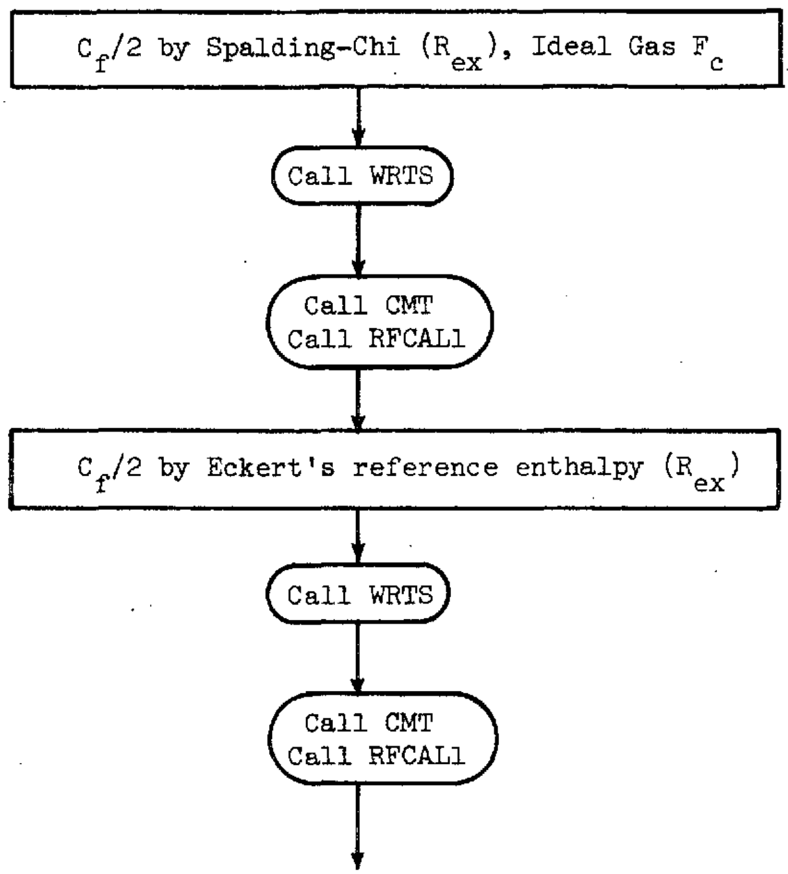
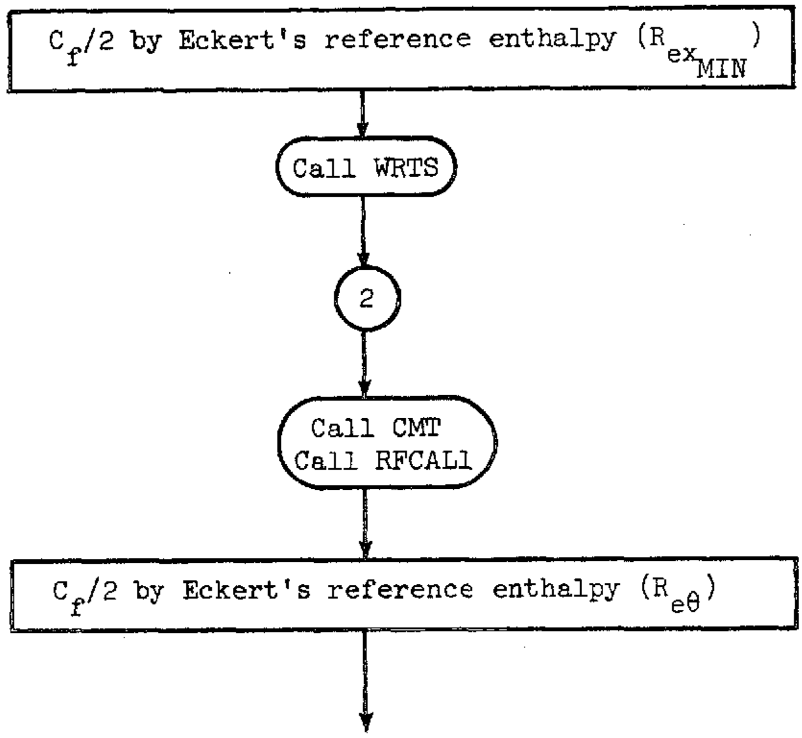
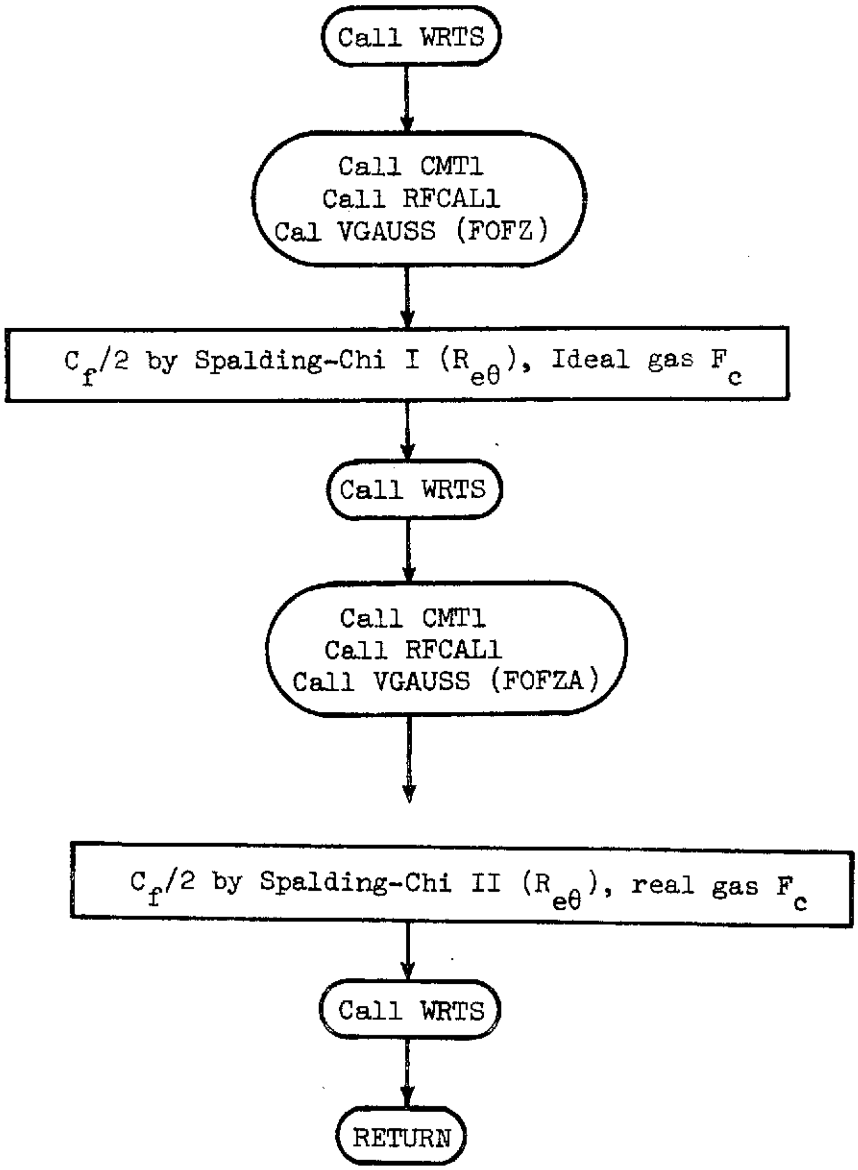

$C_f/2$ by Van Driest ($R_{ex}$)
Call WRTS
Call VGAUSS (FOFZ)
Call CMT1
Call RFCAL1


$C_f/2$ by Spalding-Chi ($R_{ex}$), Ideal Gas $F_c$
Call WRTS
Call CMT
Call RFCAL1
$C_f/2$ by Eckert's reference enthalpy ($R_{ex}$)
Call WRTS
Call CMT
Call RFCAL1


$C_f/2$ by Eckert's reference enthalpy ($R_{ex_{MIN}}$)
Call WRTS
2
Call CMT
Call RFCAL1
$C_f/2$ by Eckert's reference enthalpy ($R_{e\theta}$)


Call WRTS
Call CMT1
Call RFCAL1
Cal VGAUSS (FOFZ)
$C_f/2$ by Spalding-Chi I ($R_{e\theta}$), Ideal gas $F_c$
Call WRTS
Call CMT1
Call RFCAL1
Call VGAUSS (FOFZA)
$C_f/2$ by Spalding-Chi II ($R_{e\theta}$), real gas $F_c$
Call WRTS
RETURN

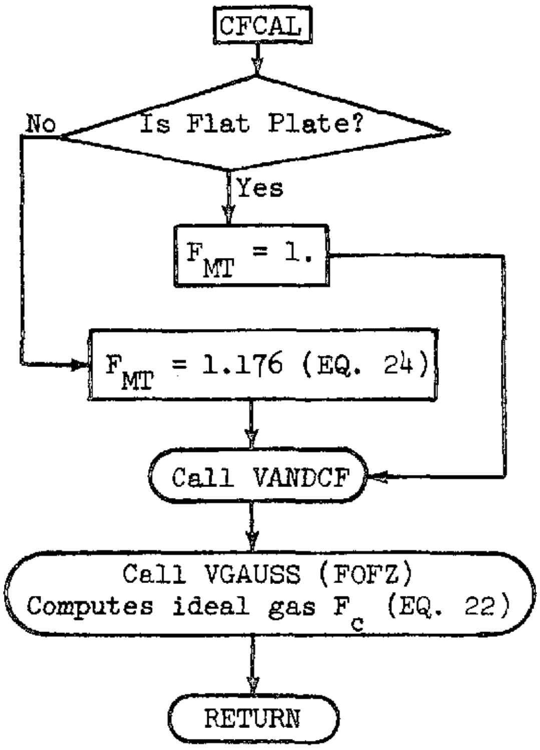
```
      SUBROUTINE CFA
C     COMPUTES HEATING RATES AND SKIN FRICTION BY
C       VAN DRIEST (REX) METHOD
C       SPALDING CHI (REX) METHOD
C       ECKERTS REFERENCE ENTHALPY (REX) METHOD
C       ECKERTS REFERENCE ENTHALPY (REXMIN) METHOD
C       ECKERTS REFERENCE ENTHALPY (RETHETA) METHOD
C       SPALDING CHI (RETHETA) METHOD  (EQ22)
C       SPALDING CHI (RETHETA) METHOD  (EQ21)
C       AND WRITES OUTPUT
      COMMON XLSH,XLSH1,THCR,JLIM,KLIM,TXLCL(450),TYL(450),TS(450),TP(45
     10),TXL(450),TYLS(450),TSS(450)
      COMMON ENX,FPOPT,ALX,RAF,ALMIN,COPT,STHCR,T1,T2,L,XVO,PSI
      COMMON CMTOPT
      COMMON ALGN,AN,ANZ,AP,AVAR,AW,AX,CEB,CEBP,CEM,CEMP,CFBMT,CFB2,CFER
     1R,CFI,CFMT,CF2,CF2I,CN,COE1,COE2,COE3,CTHCR,DEL,DELAM,DELS,DERR,DN
     2,DPVAR,DRBVAR,DRVAR,DSTH,DUVAR,E,EK,EL,ELT,ELVU,EMU,EMVAR,EMX,EN,E
     3NGN,ENI,ERR,FC,FCCF,FCCFLG,FCF,FCFPR,FD,FDPR,FRRE,FRTH,G,GC,GX,H,H
     4AW,HHAT,HP,HT,HVAR,HW,H2,ICELL,II,IN,JJ,JJLIM,JN,K,KK,KN,LLIM,NKW,
     5NN,NO,NXINT,PATM,PIVAR,PR,PRP,PRW,RBVAR,RET,RETH,RHOUI,RHOW,RO,ROP
     6,ROVAR,ROW,RRI,RRN,RRTVAR,RRX,RSERR,RSISAV,RSLVAR,RX,SHEAR,SP,SVAR
     7,SW,SX,TAW,TEMP,TEMP10,TEMP11,TEMP12,TEMP14,TEMP2,TEMP3,TEMP4,TEMP
     85,TEMP6,TEMP7,TEMP8,TEMP9,TEP,TIVAR,TK,TPP,TRAN,TIII,TW,TX,WVAR,XI
     9N,XMIN,X2REX,Z,ZMAX,ZMIN
      REAL IN,JN,KN
      COMMON F(3),A(3),ALPHE(3),XINT(99),TWT(99),ZTABL(6),TABIN(6),TABJN
     1(6),DXLTAB(20),DELK(2),RSLG(2),ALG(2),ENG(2),AN2(7)
      COMMON FRXT(20),FRXLGT(20),FCXT(20),FCXLGT(20),FRTB(20),FRLGT(20),
     1FCTB(20),FCLGT(20),XW(100),FIN6(100),RSLW(100),SHEER(100),X(160),S
     2(160),PI(160),XC(160),RB(160),W(160),RRT(160),XMAXTB(20),DUDXT(160
     3),DRDXT(160),DPDXT(160),DRBDXT(160),VAR(2),DER(2),CUVAR(2),WSAV(16
     40),RRTSAV(160),XSAV(160),RSLSAV(160),REXSAV(160),ENSAV(160),CFSAV(
     5160),TKTAB(30),PRTAB(210),PATAB(7),XKW(160),WKW(160),RSLKW(160),IG
     6AS
      COMMON XX1
      COMMON ACFT(22),TREXT(22),ACFLGT(22),TRLGT(22),REXCF,HO,XO
      EXTERNAL FOFZ
      EXTERNAL FOFZA
      IF (XO-VAR(1)) 1,1,2
    1 CALL CMT (CFMTA)
      CALL RFCAL (RAFA)
      TERM1=.2*EMVAR*EMVAR
```

```
COE1=TW/TIVAR
AVDSQ1=TERM1/COE1
BVD1=(1.+TERM1-COE1)/COE1
BVDSQ1=BVD1*BVD1
ALVD1=(AVDSQ1+AVDSQ1-BVD1)/SQRT(4.*AVDSQ1+BVDSQ1)
BEVD1=BVD1/SQRT(4.*AVDSQ1+BVDSQ1)
TREX=(2.57*(ASIN(ALVD1)+ASIN(BEVD1))**2*REXCF)/(TERM1*COE1**.76)
TRLG=ALOG10(TREX)
CALL DISCOT (TRLG,TRLG,TRLGT,ACFLGT,ACFLGT,011,22,0, AVLG)
AVCF=10.**AVLG
CF2FPA=(ASIN(ALVD1)+ASIN(BEVD1))**2*AVCF/(2.*TERM1)
CF2A=CFMTA*CF2FPA
CALL WRTS (CF2A,RAFA,CFMTA)
CALL VGAUSS (0,1.,L,ANZ,FOFZ,FZ,1,NN)
FCB=1./(ANZ*ANZ)
CALL CMT1 (CFMTB)
CALL RFCAL1 (RAFB)
FRX=FRTH/FCB
FRXRX=REXCF*FRX
FRXG=ALOG10(FRXRX)
CALL DISCOT (FRXG,FRXG,FRXLGT,FCXLGT,FCXLGT,011,20,0,FCLG)
FCFC=10.**FCLG
CF2FPB=FCFC/(2.*FCB)
CF2B=CFMTB*CF2FPB
CALL WRTS (CF2B,RAFB,CFMTB)
CALL CMT (CFMTC)
CALL RFCAL1 (RAFC)
HP=.5*(HW+HVAR)+.22*(HAW-HVAR)
CALL RGASH (PIVAR,ROP,AP,HP,TPP,SP,ERR,IGAS)
EMUT=(TIVAR/TPP)**1.5*(TPP+198.6)/(TIVAR+198.6)
CF2FPC=.370/((ALOG10(REXCF*(ROP/ROVAR)*EMUT))**2.584)*.5
CF2C=CFMTC*CF2FPC
CALL WRTS (CF2C,RAFC,CFMTC)
CALL CMT (CFMTD)
CALL RFCAL1 (RAFD)
REX=RETH*VAR(1)/VAR(2)
REXMIN=REX*(1.-XMIN/VAR(1))
HP=.5*(HW+HVAR)+.22*(HAW-HVAR)
CALL RGASH (PIVAR,ROP,AP,HP,TPP,SP,ERR,IGAS)
EMUT2=(TIVAR/TPP)**1.5*(TPP+198.6)/(TIVAR+198.6)
CF2FPD=.370/((ALOG10(REXMIN*(ROP/ROVAR)*EMUT2))**2.584)*.5
CF2D=CFMTD*CF2FPD
CALL WRTS (CF2D,RAFD,CFMTD)
```

```
2       CALL CMT (CFMTE)
        CALL RFCAL1 (RAFE)
        HP=.5*(HW+HVAR)+.22*(HAW-HVAR)
        CALL RGASH (PIVAR,ROP,AP,HP,TPP,SP,ERR,IGAS)
        EMUT3=SQRT(TIVAR/TPP)*(1.+220.*10.**(-9./TPP)/TPP)/(1.+220.*10.**(
       1-9./TIVAR)/TIVAR)
        RBARTH=RETH*EMUT3
        RBTLG=ALOG10(RBARTH)
        CFI2E=.5/((17.08*RBTLG+25.11)*RBTLG+6.012)
        CF2FPE=CFI2E/(TPP/TIVAR)
        CF2E=CFMTE*CF2FPE
        CALL WRTS (CF2E,RAFE,CFMTE)
        CALL CMT1 (CFMT1)
        CALL RFCAL1 (RAF1)
C
C       CALL VGAUSS,FOFZ
C
        CALL VGAUSS (0.,1.,L,ANZ,FOFZ,FZ,1,NN)
C
C       COMPUTE FC FROM EQ(22)OR (21)
C
        FC1=1./(ANZ*ANZ)
C
C       INITIAL ESTIMATE X
C
        CF21=.9375E-2*(FRTH*RETH)**(-.2148)
C
C       SOLVE FOR X USING NEWTONS METHOD AND CFERR FOR ERROR CRITERIA
C
3       FCF=RETH*FRTH-1./(6.*CF21)-(1./(EK*E))*((1.-2.*CF21**.5/EK)*EXP(EK
       1*CF21**(-.5))+2.*CF21**.5/EK+1.-EK**2/(6.*CF21)-EK**3*CF21**(-1.5)
       2/12.-EK**4*CF21**(-2)/40.-EK**5*CF21**(-2.5)/180.)
        FCFPR=1./(6.*CF21*CF21)-(1./(EK*E))*(EXP(EK*CF21**(-.5))*(1./CF21-
       1EK*CF21**(-1.5)/2.-CF21**(-.5)/EK)+CF21**(-.5)/EK+EK*EK/(6.*CF21*C
       2F21)+EK**3*CF21**(-2.5)/8.+EK**4*CF21**(-3)/20.+EK**5*CF21**(-3.5)
       3/72.)
        H2=-FCF/FCFPR
        IF (ABS(H2/CF21)-CFERR) 5,5,4
4       CF21=CF21+H2
        GO TO 3
C
C       SKIN FRICTION CF/2
C
```

```
5     CF21=CF21*CFMT1/FC1
      CALL WRTS (CF21,RAF1,CFMT1)
      CALL CMT1 (CFMT2)
      CALL RECAL1 (RAF2)
C
C     CALL VGAUSS,FOFZA
C
      CALL VGAUSS (0.,1.,L,ANZ,FOFZA,FZ,1,NN)
      FC2=1./(ANZ*ANZ)
      CF22=.9375E-2*(RETH*RETH)**(-.2148)
6     FCF=RETH*RETH-1./(6.*CF22)-(1./(EK*E))*((1.-2.*CF22**.5/EK)*EXP(EK
     1*CF22**(-.5))+2.*CF22**.5/EK+1.-EK**2/(6.*CF22)-EK**3*CF22**(-1.5)
     2/12.-EK**4*CF22**(-2)/40.-EK**5*CF22**(-2.5)/180.)
      FCFPR=1./(6.*CF22*CF22)-(1./(EK*E))*(EXP(EK*CF22**(-.5))*(1./CF22-
     1EK*CF22**(-1.5)/2.-CF22**(-.5)/EK)+CF22**(-.5)/EK+EK*EK/(6.*CF22*C
     2F22)+EK**3*CF22**(-2.5)/8.+EK**4*CF22**(-3)/20.+EK**5*CF22**(-3.5)
     3/72.)
      H2=-FCF/FCFPR
      IF (ABS(H2/CF22)-CFERR) 8,8,7
7     CF22=CF22+H2
      GO TO 6
8     CF22=CF22*CFMT2/FC2
      CALL WRTS (CF22,RAF2,CFMT2)
      RETURN
      END
```

CFCAL.- Subroutine CFCAL computes the VanDriest II $F_c$ function to correlate the skin-friction data.



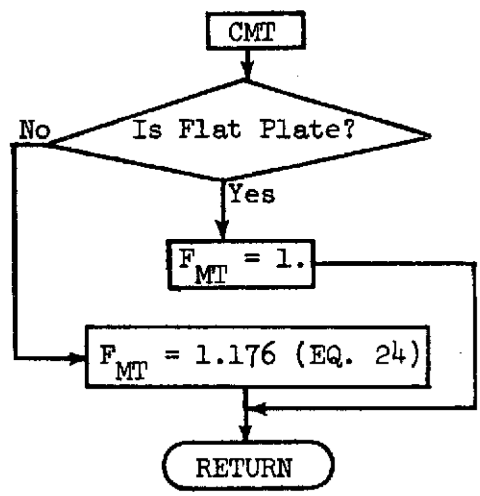
```
      SUBROUTINE CFCAL
C
C     CALCULATES SKIN FRICTION FOR FIRST ITERATION
C     OR FOR OTHER ITERATIONS BEYOND TRANSITION REGION
C     COMPUTES MANGLER TRANSFORMATION FACTOR FOR
C       VAN DRIEST II (RETHETA) METHOD
C
      COMMON XLSH,XLSH1,THCR,JLIM,KLIM,TXLCL(450),TYL(450),TS(450),TP(45
     10),TXL(450),TYLS(450),TSS(450)
      COMMON ENX,FPOPT,ALX,RAF,ALMIN,COPT,STHCR,T1,T2,L,XVO,PSI
      COMMON CMTOPT
      COMMON ALGN,AN,ANZ,AP,AVAR,AW,AX,CEB,CEBP,CEM,CEMP,CFBMT,CFB2,CFER
     1R,CFI,CFMT,CF2,CF2I,CN,COE1,COE2,COE3,CTHCR,DEL,DELAM,DELS,DERR,DN
     2,DPVAR,DRBVAR,DRVAR,DSTH,DUVAR,E,EK,EL,ELT,ELVO,EMU,EMVAR,EMX,EN,E
     3NGN,ENI,ERR,FC,FCCF,FCCFLG,FCF,FCFPR,FD,FDPR,FRKE,FRTH,G,GC,GX,H,H
     4AW,HHAT,HP,HT,HVAR,HW,H2,ICELL,II,IN,JJ,JJLIM,JN,K,KK,KN,LLIM,NKW,
     5NN,NO,NXINT,PATM,PIVAR,PR,PRP,PRW,RBVAR,RET,RETH,RHOUI,RHOW,RO,ROP
     6,ROVAR,ROW,RRI,RRN,RRTVAR,RRX,RSERR,RSISAV,RSLVAR,RX,SHEAR,SP,SVAR
     7,SW,SX,TAW,TEMP,TEMP10,TEMP11,TEMP12,TEMP14,TEMP2,TEMP3,TEMP4,TEMP
     85,TEMP6,TEMP7,TEMP8,TEMP9,TEP,TIVAR,TK,TPP,TRAN,TT11,TW,TX,WVAR,XI
     9N,XMIN,X2REX,Z,ZMAX,ZMIN
      REAL IN,JN,KN
      COMMON F(3),A(3),ALPHE(3),XINT(99),TWT(99),ZTABL(6),TABIN(6),TABJN
     1(6),DXLTAB(20),DELK(2),RSLG(2),ALG(2),ENG(2),AN2(7)
      COMMON FRXT(20),FRXLGT(20),FCXT(20),FCXLGT(20),FRTB(20),FRLGT(20),
     1FCTB(20),FCLGT(20),XW(100),FIN6(100),RSLW(100),SHEER(100),X(160),S
     2(160),PI(160),XC(160),RB(160),W(160),RRT(160),XMAXTB(20),DUDXT(160
     3),DRDXT(160),DPDXT(160),DRBDXT(160),VAR(2),DER(2),CUVAR(2),WSAV(16
     40),RRTSAV(160),XSAV(160),RSLSAV(160),REXSAV(160),ENSAV(160),CFSAV(
     5160),TKTAB(30),PRTAB(210),PATAB(7),XKW(160),WKW(160),RSLKW(160)
      COMMON /BLOCK/ PLT1(160),PLT2(160),PLT3(160),PLT4(160),PLT5(160),P
     1LT6(160),PLT7(160),PLT8(160),PLT9(160),PLT10(160),PLT11(160),PLT12
     2(160),PLT13(160),PLT14(160),PLT15(160),NKWSAV
      EXTERNAL FOFZ
      EXTERNAL FOFZA
C
C     IS FLAT PLATE
C
      IF (FPOPT) 1,2,1
C
C     MANGLER TRANSFORMATION =1.
C
```

```
1     CFMT=1.
      GO TO 3
C
C     MANGLER TRANSFORMATION EQ(24)
C
2     CFMT=1.176
3     CONTINUE
C
C     CALL VANDCF
C
      CALL VANDCF
C
C     INTEGRAL USING GAUSS 5INTERVALS,4PTS PER INTERVAL
C     CALL VGAUSS,FOFZ
C
      CALL VGAUSS (0,1.,L,ANZ,FOFZ,FZ,1,NN)
C
C     COMPUTE FC FROM EQ(22)
C
      FC=1./(ANZ*ANZ)
      RETURN
      END
```

CHECK.- Subroutine to be used by INT1A to allow certain logical control. The control is not desired in this case and a dummy subroutine is inserted.

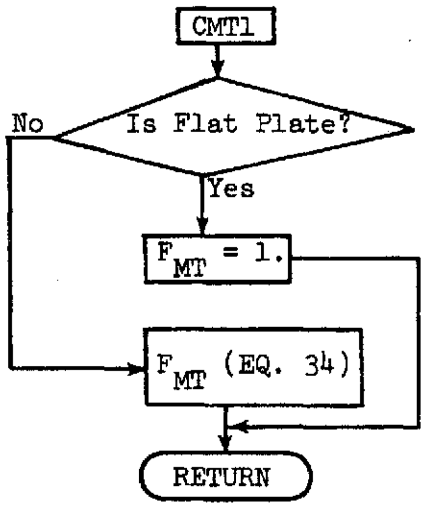
```
      SUBROUTINE CHECK
C
C     DUMMY SUBROUTINE FOR RUNGE KUTTA
C
      RETURN
      END
```

CMT.- Subroutine CMT computes Mangler transformation factor for VanDriest and Eckert's skin-friction coefficient.



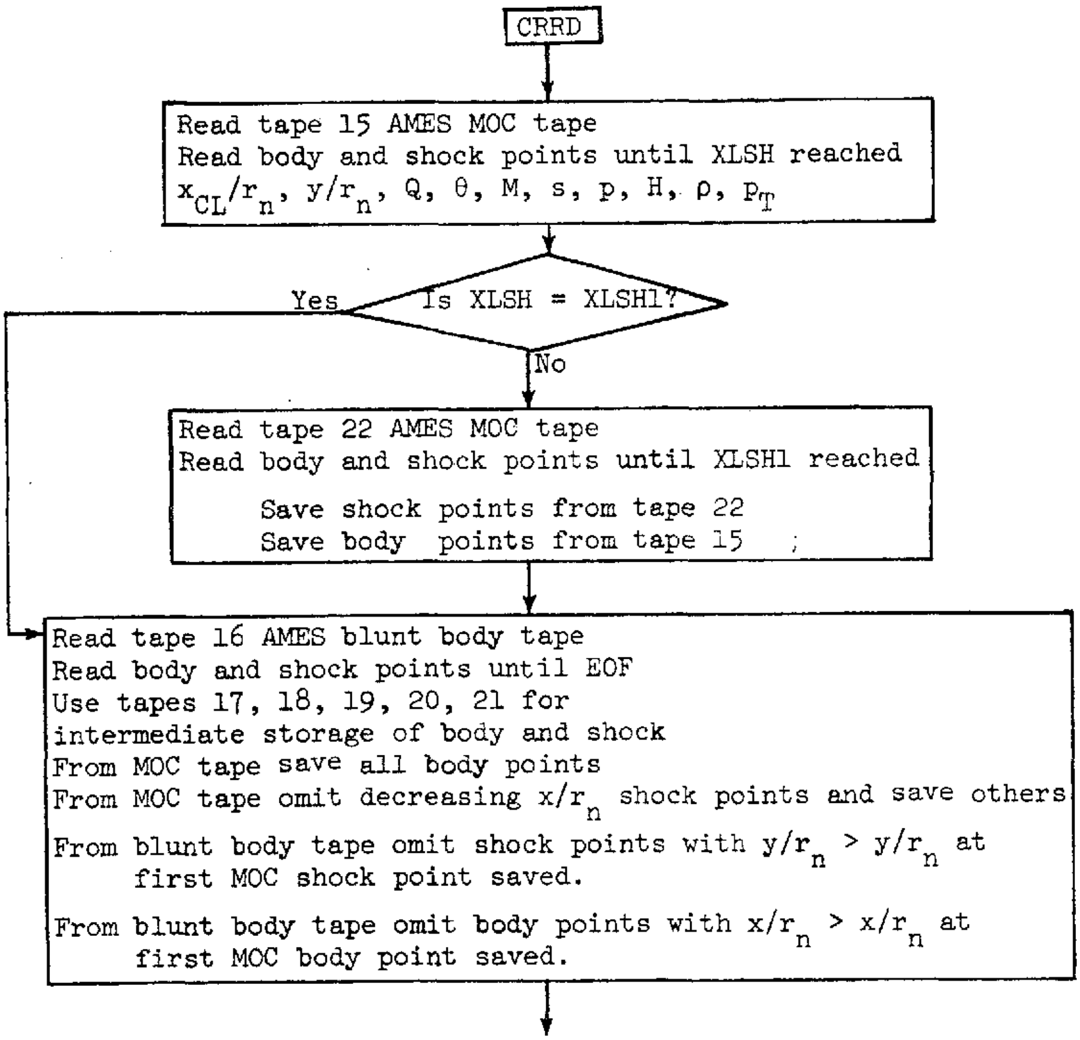
```
      SUBROUTINE CMT (CFMTX)                                            CMT
C     COMPUTES MANGLER TRANSFORMATION FACTOR FOR                        CMT
C       VAN DRIEST AND ECKERT SKIN FRICTION                             CMT
      COMMON XLSH,XLSH1,THCR,JLIM,KLIM,TXLCL(450),TYL(450),TS(450),TP(45CMT
     10),TXL(450),TYLS(450),TSS(450)                                    CMT
      COMMON ENX,FPOPT,ALX,RAF,ALMIN,COPT,STHCR,T1,T2,L,XVO,PSI         CMT
      COMMON CMTOPT                                                     CMT
      COMMON ALGN,AN,ANZ,AP,AVAR,AW,AX,CEB,CEBP,CEM,CEMP,CFBMT,CFB2,CFERCMT
     1R,CFI,CFMT,CF2,CF2I,CN,COE1,COE2,COE3,CTHCR,DEL,DELAM,DELS,DERR,DNCMT
     2,DPVAR,DRBVAR,DRVAR,DSTH,DUVAR,E,EK,EL,ELT,ELVO,EMU,EMVAR,EMX,EN,ECMT
     3NGN,ENI,ERR,FC,FCCF,FCCFLG,FCF,FCFPR,FD,FDPR,FRKE,FRTH,G,GC,GX,H,HCMT
     4AW,HHAT,HP,HT,HVAR,HW,H2,ICELL,II,IN,JJ,JJLIM,JN,K,KK,KN,LLIM,NKW,CMT
     5NN,NO,NXINT,PATM,PIVAR,PR,PRP,PRW,RBVAR,RET,RETH,RHOUI,RHOW,RO,ROPCMT
     6,ROVAR,ROW,RRI,RRN,RRTVAR,RRX,RSERR,RSISAV,RSLVAR,RX,SHEAR,SP,SVARCMT
     7,SW,SX,TAW,TEMP,TEMP10,TEMP11,TEMP12,TEMP14,TEMP2,TEMP3,TEMP4,TEMPCMT
     85,TEMP6,TEMP7,TEMP8,TEMP9,TEP,TIVAR,TK,TPP,TRAN,TT11,TW,TX,WVAR,XICMT
     9N,XMIN,X2REX,Z,ZMAX,ZMIN                                          CMT
      REAL IN,JN,KN                                                     CMT
      COMMON F(3),A(3),ALPHE(3),XINT(99),TWT(99),ZTABL(6),TABIN(6),TABJNCMT
     1(6),DXLTAB(20),DELK(2),RSLG(2),ALG(2),ENG(2),AN2(7)               CMT
      COMMON FRXT(20),FRXLGT(20),FCXT(20),FCXLGT(20),FRTB(20),FRLGT(20),CMT
     1FCTB(20),FCLGT(20),XW(100),FIN6(100),RSLW(100),SHEER(100),X(160),SCMT
     2(160),PI(160),XC(160),RB(160),W(160),RRT(160),XMAXTB(20),DUDXT(160CMT
     3),DRDXT(160),DPDXT(160),DRBDXT(160),VAR(2),DER(2),CUVAR(2),WSAV(16CMT
     40),PRTSAV(160),XSAV(160),RSLSAV(160),REXSAV(160),ENSAV(160),CFSAV(CMT
     5160),TKTAB(30),PRTAB(210),PATAB(7),XKW(160),WKW(160),RSLKW(160),IGCMT
     6AS                                                                CMT
      COMMON XX1                                                        CMT
      COMMON ACFT(22),TREXT(22),ACFLGT(22),TRLGT(22),REXCF,HO,XO        CMT
      IF (FPOPT) 1,2,1                                                  CMT
1     CFMTX=1.                                                          CMT
      GO TO 3                                                           CMT
2     CFMTX=1.176                                                       CMT
3     RETURN                                                            CMT
      END                                                               CMT
```

CMT1.- Subroutine CMT1 computes Mangler transformation factor for Spalding-Chi skin-friction coefficient.



```
      SUBROUTINE CMT1 (CFMTX)                                           CM1
C     COMPUTES MANGLER TRANSFORMATION FACTOR FOR                        CM1
C       SPALDING CHI SKIN FRICTION                                      CM1
      COMMON XLSH,XLSH1,THCR,JLIM,KLIM,TXLCL(450),TYL(450),TS(450),TP(45CM1
     10),TXL(450),TYLS(450),TSS(450)                                    CM1
      COMMON ENX,FPOPT,ALX,RAF,ALMIN,COPT,STHCR,T1,T2,L,XVO,PSI         CM1
      COMMON CMTOPT                                                     CM1
      COMMON ALGN,AN,ANZ,AP,AVAR,AW,AX,CEB,CEBP,CEM,CEMP,CFBMT,CFB2,CFERCM1
     1R,CFI,CFMT,CF2,CF2I,CN,COE1,COE2,COE3,CTHCR,DEL,DELAM,DELS,DERR,DNCM1
     2,DPVAR,DRBVAR,DRVAR,DSTH,DUVAR,E,EK,EL,ELT,ELVO,EMU,EMVAR,EMX,EN,FCM1
     3NGN,ENI,ERR,FC,FCCF,FCCFLG,FCF,FCFPR,FD,FDPR,FRRE,FRTH,G,GC,GX,H,HCM1
     4AW,HHAT,HP,HT,HVAR,HW,H2,ICELL,II,IN,JJ,JJLIM,JN,K,KK,KN,LLIM,NKW,CM1
     5NN,NO,NXINT,PATM,PIVAR,PR,PRP,PRW,RBVAR,RET,RETH,RHOUI,RHOW,RO,ROPCM1
     6,ROVAR,ROW,RRI,RRN,RRTVAR,RRX,RSERR,RSISAV,RSLVAR,RX,SHEAR,SP,SVARCM1
     7,SW,SX,TAW,TEMP,TEMP10,TEMP11,TEMP12,TEMP14,TEMP2,TEMP3,TEMP4,TEMPCM1
     85,TEMP6,TEMP7,TEMP8,TEMP9,TEP,TIVAR,TK,TPP,TRAN,TT11,TW,TX,WVAR,XICM1
     9N,XMIN,X2REX,Z,ZMAX,ZMIN                                          CM1
      REAL IN,JN,KN                                                     CM1
      COMMON F(3),A(3),ALPHE(3),XINT(99),THT(99),ZTABL(6),TABIN(6),TABJNCM1
     1(6),DXLTAB(20),DELK(2),RSLG(2),ALG(2),ENG(2),AN2(7)               CM1
      COMMON FRXT(20),FRXLGT(20),FCXT(20),FCXLGT(20),FRTB(20),FRLGT(20),CM1
     1FCTB(20),FCLGT(20),XW(100),FIN6(100),RSLW(100),SHEER(100),X(160),SCM1
     2(160),PI(160),XC(160),RB(160),W(160),RRT(160),XMAXTB(20),DUDXT(160CM1
     3),DRDXT(160),DPDXT(160),DRBDXT(160),VAR(2),DER(2),CJVAR(2),WSAV(16CM1
     40),RPTSAV(160),XSAV(160),RSLSAV(160),REXSAV(160),ENSAV(160),CFSAV(CM1
     5160),TKTAB(30),PRTAB(210),PATAB(7),XKW(160),WKW(160),RSLKW(160),IGCM1
     6AS                                                                CM1
      COMMON XX1                                                        CM1
      COMMON ACFT(22),TREXT(22),ACFLGT(22),TRLGT(22),REXCF,HO,XO        CM1
      IF (FPOPT) 1,2,1                                                  CM1
1     CFMTX=1.                                                          CM1
      GO TO 3                                                           CM1
2     CC=3.-.309136*(VAR(1)-XVO)/(XMAXTB(20)-XVO)                       CM1
      CFMTX=(3.4641016-SQRT(12.-4.*CC))/2.                              CM1
3     RETURN                                                            CM1
      END                                                               CM1
```

CRRD.- Subroutine CRRD reads inviscid flow field from tapes 15, 16, 22 if CARD = 0 is input. This subroutine computes tables of $x/r_n$, $r_b/r_n$, s, p, on body and $r_s/r_n$, s on shock.



Set up table of body points $x_{CL}/r_n$, $r_b/r_n$, s, p, $x/r_n$

IF $(x_{CL}/r_n - 1 + \sin\theta_c) \leq 0$ , $x/r_n = \cos^{-1}(1 - x_{CL}/r_n)$

IF $(x_{CL}/r_n - 1 + \sin\theta_c) > 0$ , $x/r_n = \pi/2 - \theta_c + (x_{CL}/r_n - 1 + \sin\theta_c)/\cos\theta_c$

Set up table of shock points $r_s/r_n$, s

Save count of number of body and shock points in tables

RETURN

```
      SUBROUTINE CRRD
C
C     READ TAPES FROM AMES BLUNT BODY AND CHARACTERISTICS PROGRAMS
C     SET UP TABLES OF X/L CENTERLINE,X/L BODY, BODY RADIUS,ENTROPY,
C                 PRESSURE
C     SHOCK RADIUS, SHOCK ENTROPY
C     KLIM= COUNTOF SHOCK POINTS, LIMIT OF 450
C     JLIM= COUNTOF BODY  POINTS, LIMIT OF 450
C
      COMMON XLSH,XLSH1,THCR,JLIM,KLIM,TXLCL(450),TYL(450),TS(450),TP(45
     10),TXL(450),TYLS(450),TSS(450)
      DIMENSION CHB(10), CHS(10,2), BLB(10), BLS(10), TSVSH(10), SVBD(10
     1), SVSH(10)
      REWIND 17
      REWIND 15
      REWIND 18
      J=0
C
C     READ TAPE 15 AMES MOC TAPE
C     X/LCL,Y/L,Q,THETA,M,S,P,H,RHO,PT
C     READ BODY AND SHOCK POINTS UNTIL XLSH REACHED
C     USE TAPES 17,18,19,20,21 FOR INTERMEDIATE STORAGE OF BODY,SHOCK
C
1     READ (15,37) (CHB(I),I=1,10)
      WRITE (6,37) (CHB(I),I=1,10)
      IF (ENDFILE 15) 2,3
2     JM=J-1
      GO TO 7
3     WRITE (17,37) (CHB(I),I=1,10)
      J=J+1
      READ (15,37) (CHS(I,1),I=1,10)
      WRITE (6,37) (CHS(I,1),I=1,10)
      IF (CHS(1,1)-XLSH) 4,4,5
4     IF (ENDFILE 15) 5,6
5     JM=J-2
      GO TO 7
6     WRITE (18,37) (CHS(I,1),I=1,10)
      GO TO 1
C
C     IS ANGLE BODY=ANGLE SHOCK
C
7     IF (XLSH-XLSH1) 8,15,8
```

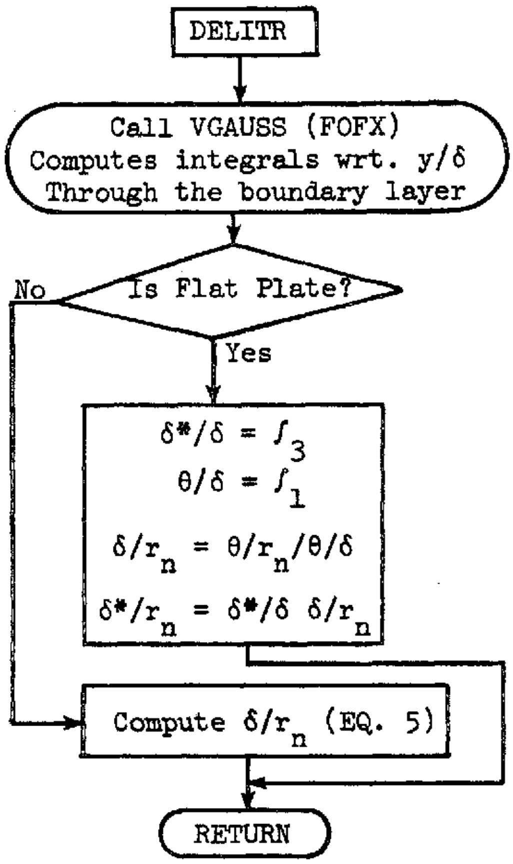
```
C
C     READ TAPE 22 AMES MOC TAPE
C     READ BODY AND SHOCK POINTS UNTIL XLSH1 REACHED
C     SAVE SHOCK FROM TAPE 22
C     SAVE BODY FROM TAPE 15
C
8     REWIND 22
      REWIND 18
      L=0
9     READ (22,37) (CHB(I),I=1,10)
      WRITE (6,37) (CHB(I),I=1,10)
      IF (ENDFILE 22) 10,11
10    LM=L-1
      GO TO 15
11    CONTINUE
      L=L+1
      READ (22,37) (CHS(I,1),I=1,10)
      WRITE (6,37) (CHS(I,1),I=1,10)
      IF (CHS(1,1)-XLSH1) 12,12,13
12    IF (ENDFILE 22) 13,14
13    LM=L-2
      GO TO 15
14    WRITE (18,37) (CHS(I,1),I=1,10)
      GO TO 9
C
C     READ TAPE 16 AMES BLUNT BODY TAPE
C     READ BODY AND SHOCK POINTS UNTIL EOF
C
15    K=0
      REWIND 16
      REWIND 19
      REWIND 20
16    READ (16,37) (BLB(I),I=1,10)
      WRITE (6,37) (BLB(I),I=1,10)
      IF (ENDFILE 16) 19,17
17    WRITE (19,37) (BLB(I),I=1,10)
      READ (16,37) (BLS(I),I=1,10)
      WRITE (6,37) (BLS(I),I=1,10)
      IF (ENDFILE 16) 19,18
18    WRITE (20,37) (BLS(I),I=1,10)
      K=K+1
      GO TO 16
C
```

```
C     FROM MOC TAPE SAVE ALL BODY POINTS
C     FROM MOC TAPE OMIT DECREASING X/L SHOCK PTS AND SAVE OTHERS
C     FROM BLUNT BODY TAPE OMIT SHOCK POINTS WITH Y/L.GT.Y/L AT
C     FIRST MOC SHOCK PT SAVED
C     FROM BLUNT BODY TAPE OMIT BODY PTS WITH X/L.GT.X/L AT
C     FIRST MOC BODY PT SAVED
C
19    REWIND 17
      READ (17,37) (CHB(I),I=1,10)
      REWIND 19
      REWIND 21
      JJ=0
      DO 21 KK=1,K
      READ (19,37) (BLB(I),I=1,10)
      IF (BLB(1)-CHB(1)) 20,20,21
20    JJ=JJ+1
      WRITE (21,37) (BLB(I),I=1,10)
21    CONTINUE
      REWIND 17
      DO 22 KK=1,J
      JJ=JJ+1
      READ (17,37) (CHB(I),I=1,10)
      WRITE (21,37) (CHB(I),I=1,10)
22    CONTINUE
      LL=0
      REWIND 17
      REWIND 18
      READ (18,37) (CHS(I,2),I=1,10)
      IF (XLSH-XLSH1) 24,23,24
23    LM=JM
24    DO 27 KK=1,LM
      DO 25 I=1,10
25    CHS(I,1)=CHS(I,2)
      READ (18,37) (CHS(I,2),I=1,10)
      IF (CHS(1,1)-CHS(1,2)) 26,26,27
26    LL=LL+1
      WRITE (17,37) (CHS(I,1),I=1,10)
27    CONTINUE
      LL=LL+1
      WRITE (17,37) (CHS(I,2),I=1,10)
      REWIND 18
      REWIND 17
      READ (17,37) (TSVSH(I),I=1,10)
```

```
      REWIND 20
      ML=0
      DO 29 KK=1,K
      READ (20,37) (BLS(I),I=1,10)
      IF (BLS(2)-TSVSH(2)) 28,28,29
28    ML=ML+1
      WRITE (18,37) (BLS(I),I=1,10)
29    CONTINUE
      REWIND 17
      DO 30 MK=1,LL
      ML=ML+1
      READ (17,37) (TSVSH(I),I=1,10)
      WRITE (18,37) (TSVSH(I),I=1,10)
30    CONTINUE
C
C     SET UP TABLE OF BODY PTS X/LCL,RB/L,S,P,X/L
C     SET UP TABLE OF SHOCK PTS RS/L,S
C     SAVE COUNT OF NO. OF BODY AND SHOCK PTS. IN TABLES
C
      REWIND 21
      STHCR=SIN(THCR)
      CTHCR=COS(THCR)
      PMTH=1.570796-THCR
      J=0
      DO 33 KJ=1,JJ
      J=J+1
      READ (21,37) (SVBD(I),I=1,10)
      TXLCL(J)=SVBD(1)
      TYL(J)=SVBD(2)
      TS(J)=SVBD(6)
      TP(J)=SVBD(7)
      XL=TXLCL(J)-1.+STHCR
      IF (XL) 31,31,32
31    TXL(J)=ACOS(1.-TXLCL(J))
      GO TO 33
32    TXL(J)=PMTH+XL/CTHCR
33    CONTINUE
      JLIM=J
      REWIND 18
      K=0
      DO 36 KJ=1,ML
      K=K+1
      READ (18,37) (SVSH(I),I=1,10)
```

```
      TYLS(K)=SVSH(2)
      IF (SVSH(1)-XLSH1) 35,35,34
34    TSS(K)=TSS(K-1)
      GO TO 36
35    TSS(K)=SVSH(6)
36    CONTINUE
      KLIM=K
      CALL EVICT (6LTAPE15)
      CALL EVICT (6LTAPE16)
      CALL EVICT (6LTAPE22)
      RETURN
C
C
C
37    FORMAT (5E16.9)
      END
```

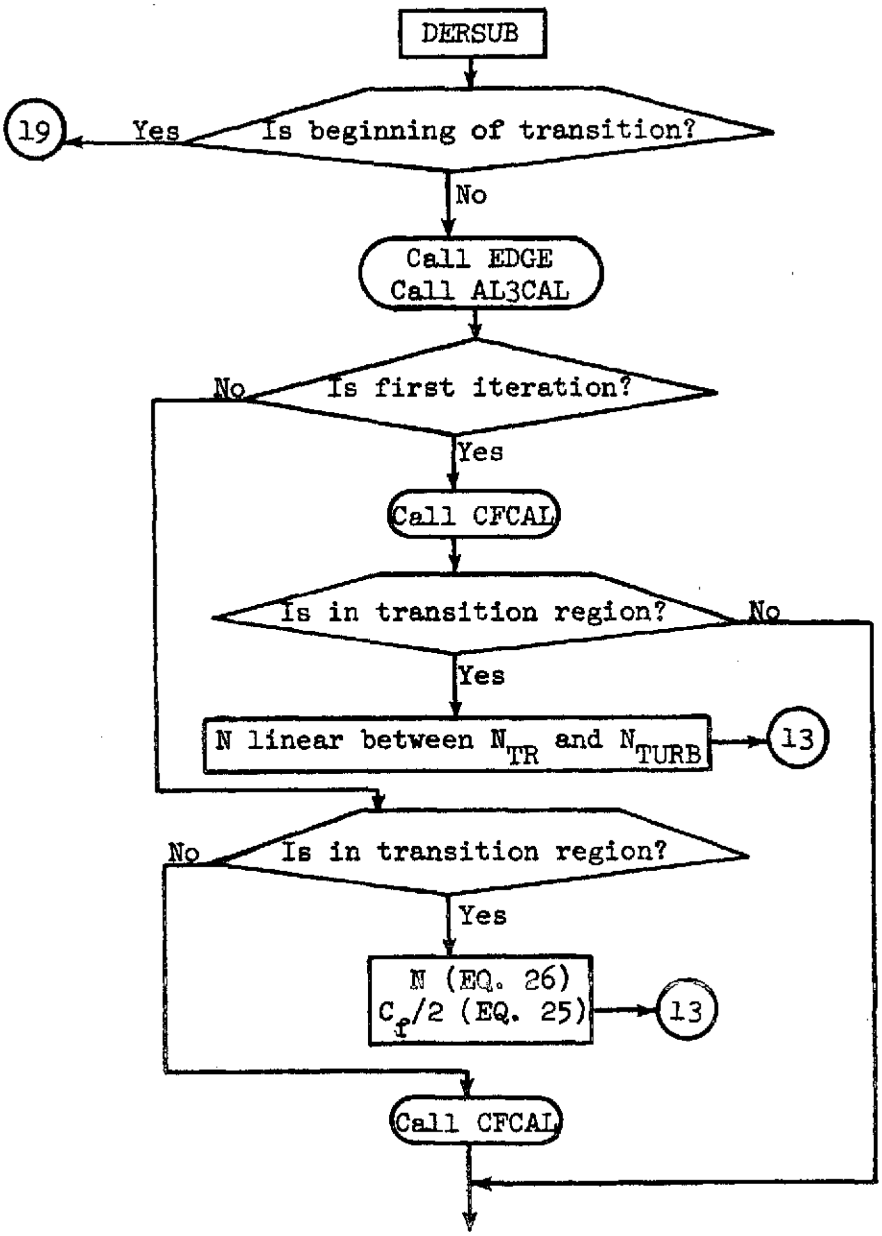
DELITR.- Subroutine DELITR computes boundary layer thickness for cone or flat plate and displacement thickness for flat plate.



```
      SUBROUTINE DELITR
C
C     CALLS VGAUSS,FOFX TO GET INTEGRALS WRT. Y/DEL THROUGH BOUNDARY
C
      COMMON XLSH,XLSH1,THCR,JLIM,KLIM,TXLCL(450),TYL(450),TS(450),TP(45
     10),TXL(450),TYLS(450),TSS(450)
      COMMON ENX,FPOPT,ALX,RAF,ALMIN,COPT,STHCR,T1,T2,L,XVO,PSI
      COMMON CMTOPT
      COMMON ALGN,AN,ANZ,AP,AVAR,AW,AX,CEB,CEBP,CEM,CEMP,CFBMT,CFB2,CFER
     1R,CFI,CFMT,CF2,CF2I,CN,COE1,COE2,COE3,CTHCR,DEL,DELAM,DELS,DERR,DN
     2,DPVAR,DRBVAR,DRVAR,DSTH,DUVAR,E,EK,EL,ELT,ELVO,EMU,EMVAR,EMX,EN,E
     3NGN,ENI,ERR,FC,FCCF,FCCFLG,FCF,FCFPR,FD,FDPR,FRRE,FRTH,G,GC,GX,H,H
     4AW,HHAT,HP,HT,HVAR,HW,H2,ICELL,II,IN,JJ,JJLIM,JN,K,KK,KN,LLIM,NKW,
     5NN,NO,NXINT,PATM,PIVAR,PR,PRP,PRW,RBVAR,RET,RETH,RHOUI,RHOW,RO,ROP
     6,ROVAR,ROW,RRI,RRN,RRTVAR,RRX,RSERR,RSISAV,RSLVAR,RX,SHEAR,SP,SVAR
     7,SW,SX,TAW,TEMP,TEMP10,TEMP11,TEMP12,TEMP14,TEMP2,TEMP3,TEMP4,TEMP
     85,TEMP6,TEMP7,TEMP8,TEMP9,TEP,TIVAR,TK,TPP,TRAN,TT11,TW,TX,WVAR,XI
     9N,XMIN,X2REX,Z,ZMAX,ZMIN
      REAL IN,JN,KN
      COMMON F(3),A(3),ALPHE(3),XINT(99),TWT(99),ZTABL(6),TABIN(6),TABJN
     1(6),DXLTAB(20),DELK(2),RSLG(2),ALG(2),ENG(2),AN2(7)
      COMMON FRXT(20),FRXLGT(20),FCXT(20),FCXLGT(20),FRTB(20),FRLGT(20),
     1FCTB(20),FCLGT(20),XW(100),FIN6(100),RSLW(100),SHEER(100),X(160),S
     2(160),PI(160),XC(160),RB(160),W(160),RRT(160),XMAXTB(20),DUDXT(160
     3),DRDXT(160),DPDXT(160),DRBDXT(160),VAR(2),DER(2),CUVAR(2),WSAV(16
     40),RRTSAV(160),XSAV(160),RSLSAV(160),REXSAV(160),ENSAV(160),CFSAV(
     5160),TKTAB(30),PRTAB(210),PATAB(7),XKW(160),WKW(160),RSLKW(160)
      COMMON /BLOCK/ PLT1(160),PLT2(160),PLT3(160),PLT4(160),PLT5(160),P
     1LT6(160),PLT7(160),PLT8(160),PLT9(160),PLT10(160),PLT11(160),PLT1
     22(160),PLT13(160),PLT14(160),PLT15(160),NKWSAV
      DIMENSION F2(7)
      EXTERNAL FOFX
      SX=SVAR
C
C     CALLS VGAUSS,FOFX TO GET INTEGRALS WRT. Y/DEL THROUGH BOUNDARY
      CALL VGAUSS (0,1.,L,AN2,FOFX,F2,7,NN)
C
C     IS FLAT PLATE
C
      IF (FPOPT) 1,2,1
C
C     DELTASTAR/DELTA =INTEGRAL(3)
```

```
C
1     DSOD=AN2(3)
C
C     THETA/DELTA=INTEGRAL(1)
C
      THOD=AN2(1)
C
C     DELTA/L
C     THETA/L  FROM  SUBROUTINE  DERSUB
C
      DEL=CUVAR(2)/THOD
C
C     DELTASTAR/L
C
      DELS=DEL*DSOD
      GO  TO  3
C
C     COMPUTE  BOUNDARY  LAYER  THICKNESS
C     IF  CONE  FIND  DELTA/L  FROM  EQ(5)
C
2     DEL=(-AN2(1)+SQRT(AN2(1)**2+4.*AN2(5)*TEMP11))/(2.*AN2(5))
3     RETURN
C
      END
```

DERSUB.- Subroutine DERSUB evaluates the derivative of the variable entropy momentum integral equation.



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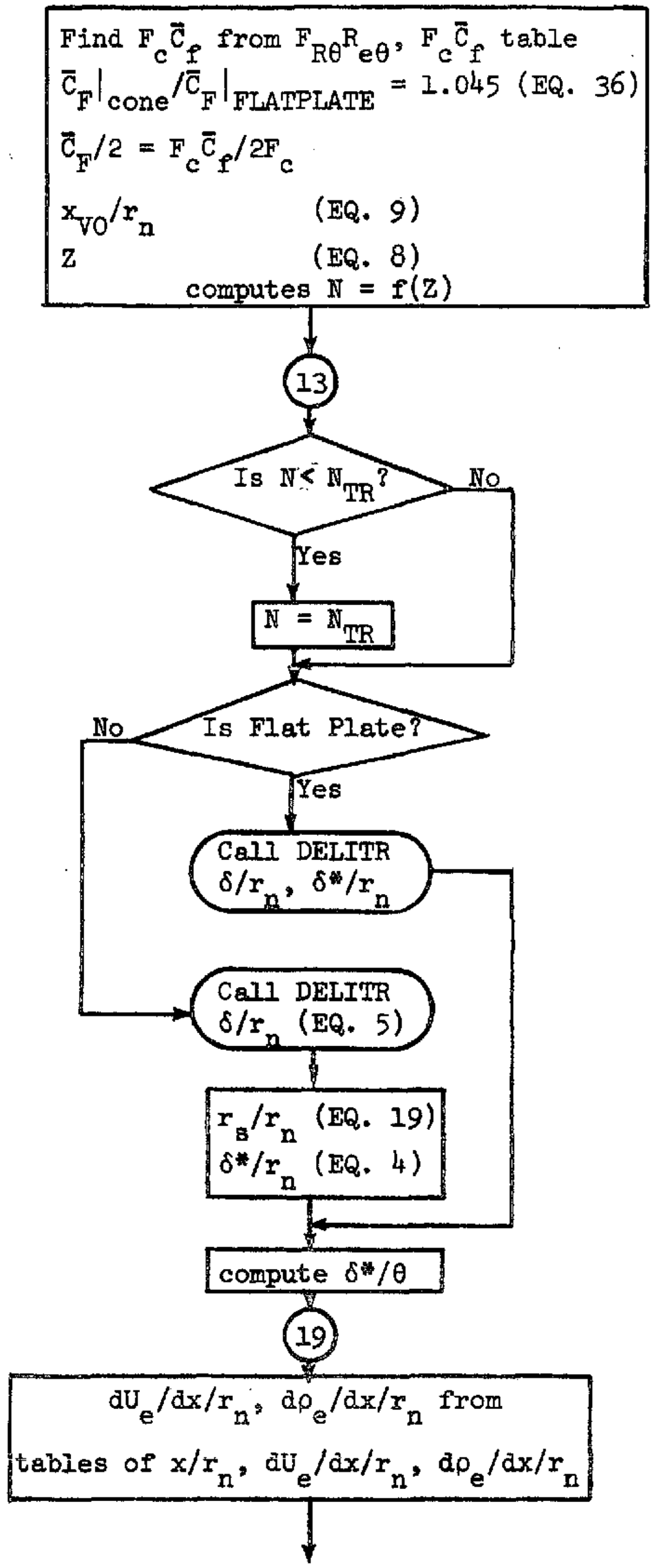
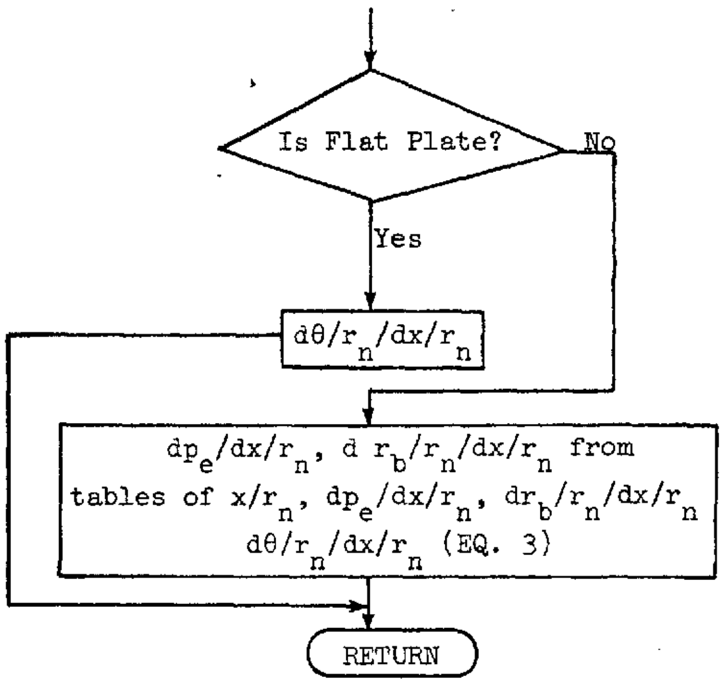


Find $F_c\bar{C}_f$ from $F_{R\theta}R_{e\theta}$, $F_c\bar{C}_f$ table
$\bar{C}_F|_{cone}/\bar{C}_F|_{FLATPLATE} = 1.045$ (EQ. 36)
$\bar{C}_F/2 = F_c\bar{C}_f/2F_c$
$x_{VO}/r_n$ (EQ. 9)
Z (EQ. 8)
computes N = f(Z)
13
Is $N < N_{TR}$?
No
Yes
$N = N_{TR}$
Is Flat Plate?
No
Yes
Call DELITR
$\delta/r_n$, $\delta^*/r_n$
Call DELITR
$\delta/r_n$ (EQ. 5)
$r_s/r_n$ (EQ. 19)
$\delta^*/r_n$ (EQ. 4)
compute $\delta^*/\theta$
19
$dU_e/dx/r_n$, $d\rho_e/dx/r_n$ from
tables of $x/r_n$, $dU_e/dx/r_n$, $d\rho_e/dx/r_n$



Is Flat Plate?
No
Yes
dθ/r_n/dx/r_n
dp_e/dx/r_n, d r_b/r_n/dx/r_n from
tables of x/r_n, dp_e/dx/r_n, dr_b/r_n/dx/r_n
dθ/r_n/dx/r_n (EQ. 3)
RETURN

```
      SUBROUTINE DERSUB
C
C     X/L=CUVAR(1), MOMENTUM THICKNESS=CUVAR(2)
C     AT X/L STATION COMPUTE  VARIABLE ENTROPY MOMENTUM INTEGRAL
C             EQUATION
C     COMPUTES DTHETA/DX AT EACH STEP IN RUNGE-KUTTA INTEGRATION BY EQ3
C
      COMMON XLSH,XLSH1,THCR,JLIM,KLIM,TXLCL(450),TYL(450),TS(450),TP(45
     10),TXL(450),TYLS(450),TSS(450)
      COMMON ENX,FPOPT,ALX,RAF,ALMIN,COPT,STHCR,T1,T2,L,XVO,PSI
      COMMON CMTOPT
      COMMON ALGN,AN,ANZ,AP,AVAR,AW,AX,CEB,CEBP,CEM,CEMP,CFBMT,CFB2,CFER
     1R,CFI,CFMT,CF2,CF2I,CN,COE1,COE2,COE3,CTHCR,DEL,DELAM,DELS,DERR,DN
     2,DPVAR,DRBVAR,DRVAR,DSTH,DUVAR,E,EK,EL,ELT,ELVO,EMU,EMVAR,EMX,EN,E
     3NGN,ENI,ERR,FC,FCCF,FCCFLG,FCF,FCFPR,FD,FDPR,FRRE,FRTH,G,GC,GX,H,H
     4AW,HHAT,HP,HT,HVAR,HW,H2,ICELL,II,IN,JJ,JJLIM,JN,K,KK,KN,LLIM,NKW,
     5NN,NO,NXINT,PATM,PIVAR,PR,PRP,PRW,RBVAR,RET,RETH,RHOUI,RHOW,RO,ROP
     6,ROVAR,ROW,RRI,RRN,RRTVAR,RRX,RSERR,RSISAV,RSLVAR,RX,SHEAR,SP,SVAR
     7,SW,SX,TAW,TEMP,TEMP10,TEMP11,TEMP12,TEMP14,TEMP2,TEMP3,TEMP4,TEMP
     85,TEMP6,TEMP7,TEMP8,TEMP9,TEP,TIVAR,TK,TPP,TRAN,TT11,TW,TX,WVAR,XI
     9N,XMIN,X2REX,Z,ZMAX,ZMIN
      REAL IN,JN,KN
      COMMON F(3),A(3),ALPHE(3),XINT(99),TWT(99),ZTABL(6),TABIN(6),TABJN
     1(6),DXLTAB(20),DELK(2),RSLG(2),ALG(2),ENG(2),AN2(7)
      COMMON FRXT(20),FRXLGT(20),FCXT(20),FCXLGT(20),FRTB(20),FRLGT(20),
     1FCTB(20),FCLGT(20),XW(100),FIN6(100),RSLW(100),SHEER(100),X(160),S
     2(160),PI(160),XC(160),RB(160),W(160),RRT(160),XMAXTB(20),DUDXT(160
     3),DRDXT(160),DPDXT(160),DRBDXT(160),VAR(2),DER(2),CUVAR(2),WSAV(16
     40),RRTSAV(160),XSAV(160),RSLSAV(160),REXSAV(160),ENSAV(160),CFSAV(
     5160),TKTAB(30),PRTAB(210),PATAB(7),XKW(160),WKW(160),RSLKW(160)
      COMMON IGAS,XX1
      COMMON /BLOCK/ PLT1(160),PLT2(160),PLT3(160),PLT4(160),PLT5(160),P
     1LT6(160),PLT7(160),PLT8(160),PLT9(160),PLT10(160),PLT11(160),PLT12
     2(160),PLT13(160),PLT14(160),PLT15(160),NKWSAV
C
C     IS START OF TRANSITION
C
      IF (CUVAR(1)-XMIN) 1,19,1
C
C     CALL EDGE
C
    1 CALL EDGE
```

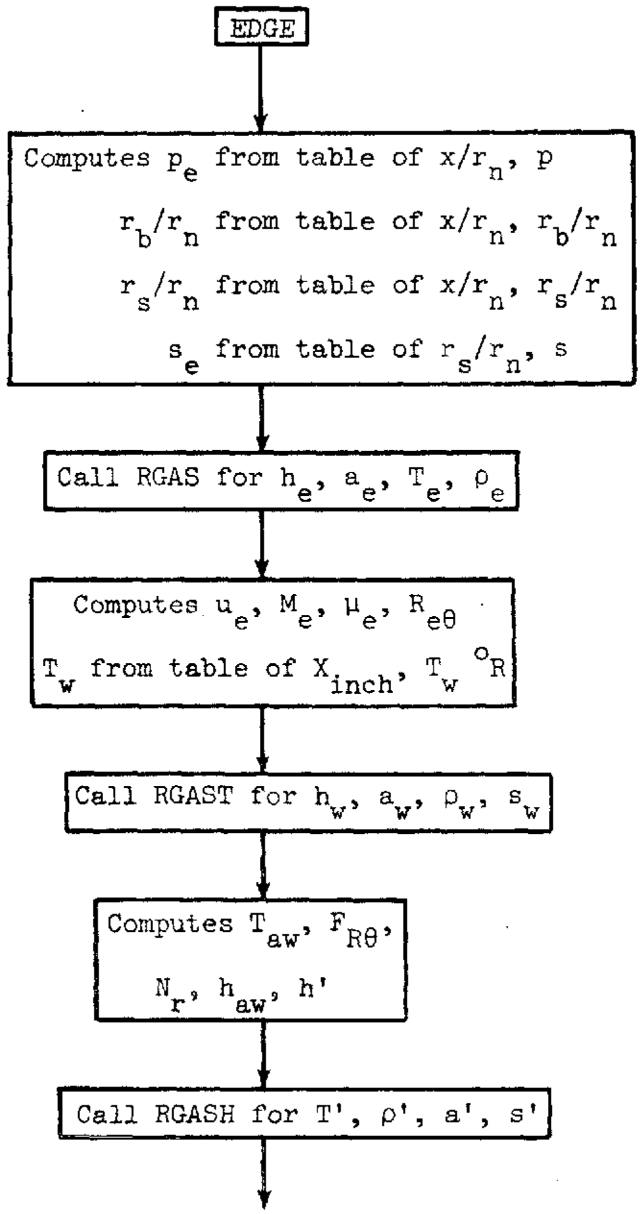
```
C
C     CALL AL3CAL
C
      CALL AL3CAL
C
C     IS FIRST ITERATION
C     TEST CODE ICELL                IF=0 FIRST ITERATION
C
      IF (ICELL) 4,2,4
C
C     CALL CFCAL
C
2     CALL CFCAL
C
C     IS TRANSITION REGION
C
      IF (CUVAR(1)-X2REX) 3,3,8
C
C     N=LINEAR BETWEEN N(TRANSITION) AND N(TURBULENT)
C
3     EN=ENI-(XMIN-CUVAR(1))*(ENI-ENX)/(XMIN-X2REX)
      GO TO 13
C
C     IS TRANSITION REGION
C     TEST CODE TRAN                IF=0 NO TRANSITION REGION
C
4     IF (TRAN) 5,7,5
5     IF (CUVAR(1)-X2REX) 6,6,7
C
C     ITERATION AFTER FIRST  IN TRANSITION REGION THE N VARIATION IS
C               NONLINEAR
C                 WITH PSI CONTROLLING THE DEGREE OF NONLINEARITY
C                 THE SKIN FRICTION COEFFICIENT ALSO NONLINEAR
C               CONTROLLED BY PSI
C     N EQ(26)
C     CF/2 EQ(25)
C
6     XX=PSI-CEB*(X2REX-CUVAR(1))
      XX1=(CEBP+TANH(XX))/(CEBP+CEBP)
      EN=ENI+XX1*CEM
      CF2=CF2I+XX1*CEMP
      GO TO 13
C
```

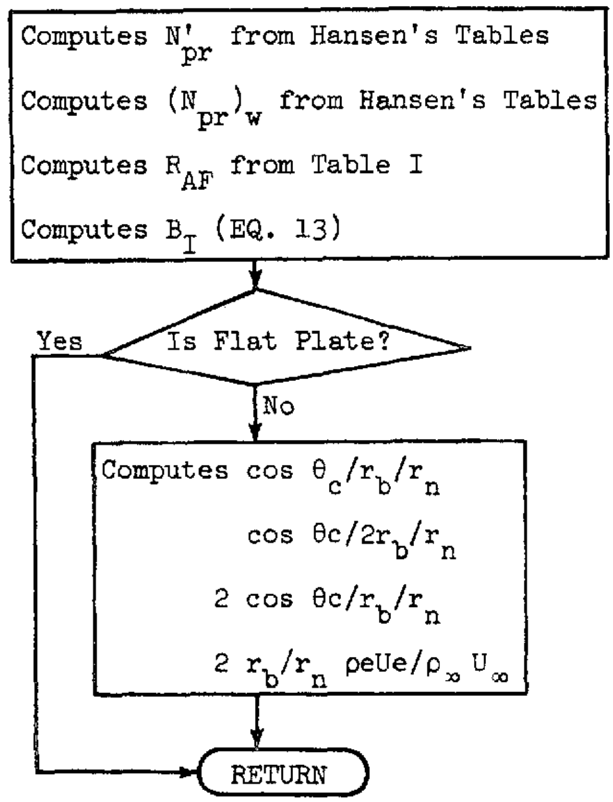
```
C       CALL CFCAL
C
7       CALL CFCAL
C
C       USING FRTHETA*RETHETA,SPALDING-CHI TABLE GIVES FCCBARF
C
8       FRRE=ALOG10(FRTH*RETH)
        CALL DISCOT (FRRE,FRRE,FRLGT,FCLGT,FCLGT,011,20,0,FCCFLG)
        FCCF=10.**FCCFLG
        CFB2=FCCF/(2.*FC)
C
C       CBFMT EQ(36)
C       XVO/LO EQ(9)
C
        ELVO=CUVAR(2)/(CFBMT*CFB2)
C
C       Z EQ(8)
C
        Z=(RETH**AN*COE1**CN*(ELVO/CUVAR(2))**KN)/(EMVAR**DN)
C
C                       BEYOND TRANSITION N=6.*LOG(Z)-7. EXCEPT
C                        N=2 BELOW ZMIN,N=10 ABOVE ZMAX
C                       WHERE Z COMPUTED USING DISTANCE FROM VIRTUAL ORIGIN.
C       Z.LT.ZMIN,N=2
C       ZMIN.LE.Z.LE.ZMAX,N(EQ8)
C       Z.GT.ZMAX,N=10
C
        IF (Z-ZMIN) 9,10,10
9       EN=2.
        GO TO 13
10      IF (Z-ZMAX) 12,12,11
11      EN=10.
        GO TO 13
12      EN=6.*ALOG10(Z)-7.
C
C       IS N.LT.N(TRANSITION)
C
13      IF (ENI-EN) 15,15,14
C
C       N=N(TRAN)
C
14      EN=ENI
15      TEMP3=1./EN
```

```
C
C     IS FLAT PLATE
C
      IF (FPOPT) 16,17,16
C
C     CALL DELITR FOR DEL/L,DELSTAR/L
C
16    CALL DELITR
      GO TO 18
C
C     GIVEN MOMENTUM THICKNESS COMPUTE BOUNDARY LAYER THICKNESS,SHOCK
C               RADIUS
C     DISPLACEMENT THICKNESS EXCEPT AT START OF TRANSITION WHEN SHOCK
C               RADIUS GIVEN
C     FIND VALUE           TEMP11   COS(THETAC)/(2RB/L)*(O/L)SQ+(O/L)
C     CALL DELITR FOR DEL/L(EQ5)
C
17    TEMP11=TEMP10*CUVAR(2)*CUVAR(2)+CUVAR(2)
      CALL DELITR
C
C     RS/L(EQ19)
C
      RSLVAR=SQRT(TEMP12*DEL*AN2(6)+TEMP12*DEL*DEL*AN2(2))
C
C     DELSTAR/L(EQ4)
C
      G=-DEL*AN2(3)-DEL*DEL*AN2(4)
C
C     DELSTAR/THETA
C
      DELS=(-1.+SQRT(1.-TEMP14*G))/TEMP9
18    DSTH=DELS/CUVAR(2)
C
C     TABLE OF VELOCITY,DENSITY,PRESSURE,BODY RADIUS DERIVATIVES FROM
C               PREVIOUS ITERATION
C     DU/DX,DRHO/DX FROM TABLES
C
19    CALL DISCOT (CUVAR(1),CUVAR(1),XKW,DUDXT,DUDXT,011,NKW,0,DUVAR)
      CALL DISCOT (CUVAR(1),CUVAR(1),XKW,DRDXT,DRDXT,011,NKW,0,DRVAR)
C
C     IS FLAT PLATE
C
      IF (FPOPT) 20,21,20
```

```
C                                                                          DER 172
C      DTHETA/DX                                                           DER 173
C                                                                          DER 174
20     DER(2)=CF2-CUVAR(2)*(((2.+DSTH)*DUVAR)/WVAR+DRVAR/RRTVAR)           DER 175
       GO TO 22                                                            DER 176
C                                                                          DER 177
C      DP/DX,DRB/DX FROM TABLES                                            DER 178
C                                                                          DER 179
21     CALL DISCOT (CUVAR(1),CUVAR(1),X,DPOXT,DPOXT,011,JJLIM,0,DPVAR)     DER 180
       CALL DISCOT (CUVAR(1),CUVAR(1),X,DRBOXT,DRBOXT,011,JJLIM,0,DRBVAR)DER 181
C                                                                          DER 182
C      DTHETA/DX = DER(2)  INCREMENT IN MOMENTUM THICKNESS FOR             DER 183
C                RUNGE KUTTA INTEGRATION                                   DER 184
C      DTHETA/DX(EQ3)                                                      DER 185
C                                                                          DER 186
       DER(2)=CF2-CUVAR(2)*(((2.+DSTH)*DUVAR)/WVAR+DRVAR/RRTVAR+DRBVAR/RBDER 187
      1VAR)+(DEL/WVAR)*(DPVAR*GC/(RRTVAR*WVAR)+DUVAR)                      DER 188
22     RETURN                                                              DER 189
C                                                                          DER 190
       END                                                                 DER 191-
```

EDGE.- Subroutine EDGE computes the initial conditions at the edge of the boundary layer using the results of the inviscid solution and the laminar boundary layer calculation.




Computes N'pr from Hansen's Tables
Computes (Npr)w from Hansen's Tables
Computes RAF from Table I
Computes BI (EQ. 13)
Is Flat Plate?
Yes
No
Computes cos θc/rb/rn
cos θc/2rb/rn
2 cos θc/rb/rn
2 rb/rn ρeUe/ρ∞ U∞
RETURN

```
      SUBROUTINE EDGE
C
C     AT X/L STATION COMPUTE PRESSURE,BODY RADIUS,SHOCK RADIUS,ENTROPY,
C              DENSITY
C     VELOCITY,MACH NO.,ENTHALPY,TEMPERATURE,
C     WALL TEMPERATURE,DENSITY,ENTHALPY,PRANDTL NO.
C     ADIABATIC WALL ENTHALPY,TEMPERATURE
C     REYNOLDS NO. BASED ON MOMENTUM THICKNESS
C     ECKERTS REFERENCE ENTHALPY,TEMPERATURE,PRANDTL NO..
C
      COMMON XLSH,XLSHI,THCR,JLIM,KLIM,TXLCL(450),TYL(450),TS(450),TP(45
     10),TXL(450),TYLS(450),TSS(450)
      COMMON ENX,FPOPT,ALX,RAF,ALMIN,COPT,STHCR,T1,T2,L,XVO,PSI
      COMMON CMTOPT
      COMMON ALGN,AN,ANZ,AP,AVAR,AW,AX,CEB,CEBP,CEM,CEMP,CFBMT,CFB2,CFE
     1R,CFI,CFMT,CF2,CF2I,CN,COE1,COE2,COE3,CTHCR,DEL,DELAM,DELS,DERR,DN
     2,DPVAR,DRBVAR,DRVAR,DSTH,DUVAR,E,EK,EL,ELT,ELVO,EMU,EMVAR,EMX,EN,E
     3NGN,ENI,ERR,FC,FCCF,FCCFLG,FCF,FCFPR,FD,FDPR,FRRE,FRTH,G,GC,GX,H,H
     4AW,HHAT,HP,HT,HVAR,HW,H2,ICELL,II,IN,JJ,JJLIM,JN,K,KK,KN,LLIM,NKW,
     5NN,NO,NXINT,PATM,PIVAR,PR,PRP,PRW,RBVAR,RET,RETH,RHOUI,RHOW,RO,ROP
     6,ROVAR,ROW,RRI,RRN,RRTVAR,RRX,RSERR,RSISAV,RSLVAR,RX,SHEAR,SP,SVAR
     7,SW,SX,TAW,TEMP,TEMP10,TEMP11,TEMP12,TEMP14,TEMP2,TEMP3,TEMP4,TEMP
     85,TEMP6,TEMP7,TEMP8,TEMP9,TEP,TIVAR,TK,TPP,TRAN,TT11,TW,TX,WVAR,XI
     9N,XMIN,X2REX,Z,ZMAX,ZMIN
      REAL IN,JN,KN
      COMMON F(3),A(3),ALPHE(3),XINT(99),TWT(99),ZTABL(6),TABIN(6),TABJN
     1(6),DXLTAB(20),DELK(2),RSLG(2),ALG(2),ENG(2),AN2(7)
      COMMON FRXT(20),FRXLGT(20),FCXT(20),FCXLGT(20),FRTB(20),FRLGT(20),
     1FCTB(20),FCLGT(20),XW(100),FIN6(100),RSLW(100),SHEER(100),X(160),S
     2(160),PI(160),XC(160),RB(160),W(160),RRT(160),XMAXTB(20),DUDXT(160
     3),DRDXT(160),DPDXT(160),DRBDXT(160),VAR(2),DER(2),CUVAR(2),WSAV(16
     40),RRTSAV(160),XSAV(160),RSLSAV(160),REXSAV(160),ENSAV(160),CFSAV(
     5160),TKTAB(30),PRTAB(210),PATAB(7),XKW(160),WKW(160),RSLKW(160)
      COMMON IGAS
      COMMON /BLOCK/ PLT1(160),PLT2(160),PLT3(160),PLT4(160),PLT5(160),P
     1LT6(160),PLT7(160),PLT8(160),PLT9(160),PLT10(160),PLT11(160),PLT12
     2(160),PLT13(160),PLT14(160),PLT15(160),NKWSAV
      IF (FPOPT) 10,1,10
C
C     FIND PRESSURE          PIVAR     LBS/FT2       BOUNDARY LAYER EDGE AT
C                 X/L STATION
C     IF X OUTSIDE TABLE USE FINAL PRESSURE FROM TABLE
```

```
C       IF X INSIDE  TABLE USE INTERPOLATED PRESSURE
C
1       IF (CUVAR(1)-TXL(JLIM)) 3,3,2
2       PIVAR=TP(JLIM)
        GO TO 4
C
C       P FROM TABLE X/L,P
C
3       CALL DISCOT (CUVAR(1),CUVAR(1),TXL,TP,TP,011,JLIM,0,PIVAR)
C
C       INTERPOLATE FOR BODY RADIUS
C       RB/L FROM TABLE X/L,RB/L
C
4       CALL DISCOT (CUVAR(1),CUVAR(1),X,RB,RB,011,JJLIM,0,RBVAR)
C
C       FIRST ITERATION INTERPOLATE FOR SHOCK RADIUS USING LAMINAR TABLE
C       ITERATION AFTER FIRST INTERPOLATE FOR SHOCK RADIUS USING PREVIOUS
C                   ITERATION
C
        IF (ICELL) 6,5,6
C
C       RS/L FROM TABLE X/L,RS/L
C
5       CALL DISCOT (CUVAR(1),CUVAR(1),XW,RSLW,RSLW,011,LLIM,0,RSLVAR)
        GO TO 7
6       CALL DISCOT (CUVAR(1),CUVAR(1),XKW,RSLKW,RSLKW,011,NKW,0,RSLVAR)
C
C       FIND ENTROPY          SVAR      FT2/SEC2DEGR
C       IF SHOCK RADIUS OUTSIDE TABLE USE FINAL ENTROPY FROM TABLE
C       IF SHOCK RADIUS INSIDE TABLE USE  INTERPOLATED ENTROPY
C
7       IF (RSLVAR-TYLS(KLIM)) 9,9,8
8       SVAR=TSS(KLIM)
        GO TO 10
C
C       S FROM TABLE RS/L,S
C
9       CALL DISCOT (RSLVAR,RSLVAR,TYLS,TSS,TSS,011,KLIM,0,SVAR)
10      GX=1.4
C
C       DENSITY,SOUND SPEED,ENTHALPY,TEMPERATURE FROM REAL GAS
C                THERMODYNAMIC TABLES
C       FIND DENSITY          ROVAR     SLUG/FT3      REAL GAS THERMODYNAMIC
```

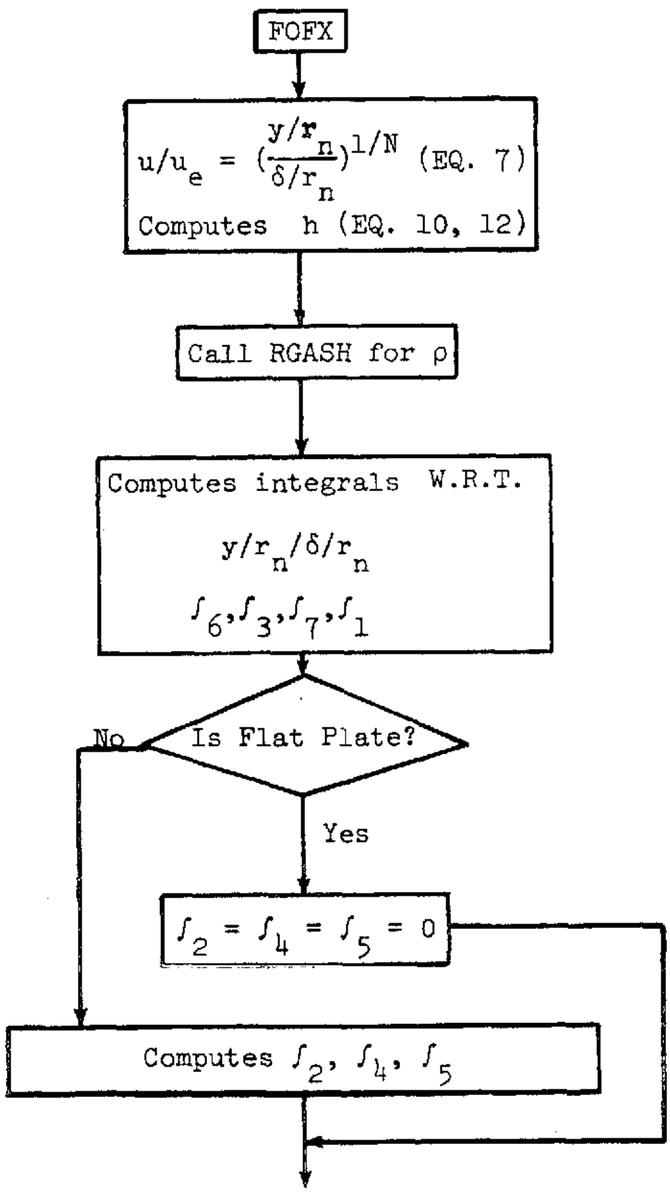
```
C                TABLES
C       FIND SOUND SPEED     AVAR      FT/SEC
C       FIND ENTHALPY        HVAR      FT2/SEC2
C       FIND TEMPERATURE     TIVAR     DEGR
C       H,A,T,RHO FROM RGAS USING P,S
C
        CALL RGAS (PIVAR,ROVAR,AVAR,HVAR,TIVAR,SVAR,RRX,GX,-1,5,IGAS)
C
C       FIND PRESSURE ATM    PATM      ATM
C
        PATM=PIVAR*.4725E-3
C
C       FIND DENSITY         RRTVAR    LBM/FT3
C
        RRTVAR=ROVAR*32.174
C
C       FIND VELOCITY        WVAR      FT/SEC
C
        WVAR=SQRT(2.*(HT-HVAR))
C
C       FIND MACH NO         EMVAR
C
        EMVAR=WVAR/AVAR
        IF (IGAS-1) 11,11,12
11      EMU=6.887E-7*SQRT(TIVAR)/(1.+180./TIVAR)
        GO TO 13
12      CONTINUE
C
C       FIND VISCOSITY       EMU
C
        EMU=7.310615E-7*SQRT(TIVAR)/(1.+201.6/TIVAR)
13      CONTINUE
        RET=(RRTVAR*WVAR)/(EMU*12.)
C
C       FIND X INCH          XIN       INCH
C
        XIN=CUVAR(1)*EL
C
C       FIND WALL TEMP       TW        DEGR          INPUT TABLE OF WALL
C                TEMPERATURE
C       TW FROM TABLE XINCH,TWDEGR
C
        CALL DISCOT (XIN,XIN,XINT,TWT,TWT,011,NXINT,0,TW)
```

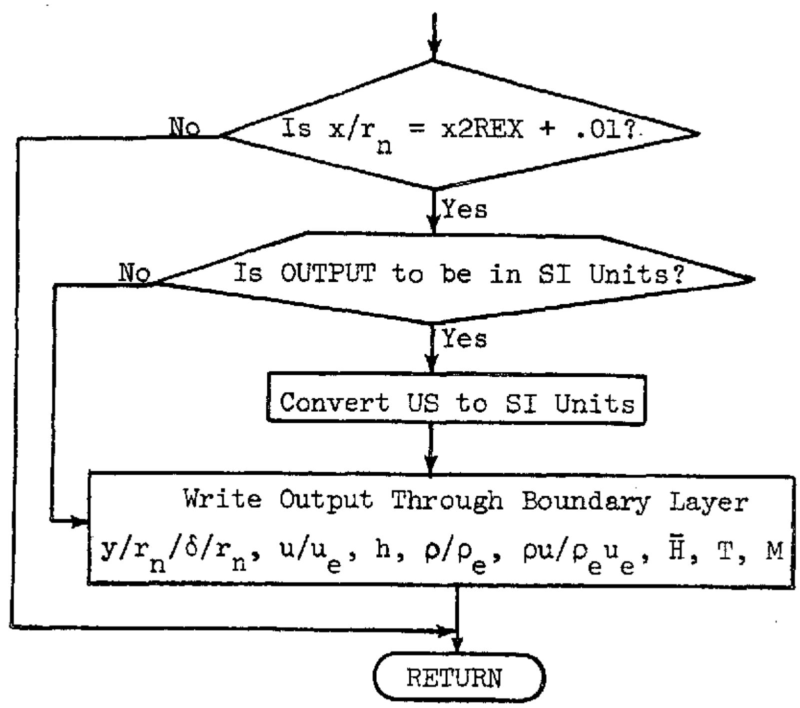
```
C
C     WALL DENSITY,SOUND SPEED,ENTHALPY,ENTROPY FROM REAL GAS
C                THERMODYNAMIC TABLES
C     START WITH ESTIMATE FOR WALL ENTROPY
C     FIND DENSITY WALL    ROW       SLUG/FT3      REAL GAS THERMODYNAMIC
C                 TABLES
C     FIND ENTHALPY WALL   HW        FT2/SEC2
C     HW,AH,RHOW,SW FROM RGAS USING P,TW
C
      CALL RGAST (PIVAR,ROW,AH,HW,TW,SW,ERR,IGAS)
C
C     FIND DENSITY WALL    RHOW      LBM/FT3
C
      RHOW=ROW*32.174
C
C     COEFFICIENTS FOR FC EQUATION
C     FIND COEFF           COE2      WHERE NR=.89 AND GAMMA=1.4
C     FIND COEFF           COE1      TW/TE
C     FIND COEFF           COE3      COEFF USED IN FC IN SPALDING CHI
C                THEORY
C
      COE2=.178*EMVAR*EMVAR
      COE1=TW/TIVAR
      COE3=1.+COE2-COE1
C
C     FIND REYNOLDS NO     RETH      REYNOLDS NO BASED ON MOMENTUM
C                THICKNESS
C
      RETH=RET*CUVAR(2)*EL
C
C     FIND ADR WALL TEMP   TAW       ADIABATIC WALL TEMP
C
      TAW=(PR**.33333333)*(TT11-TIVAR)+TIVAR
C
C     FIND FRTHETA         FRTH      FRTHETA IN SPALDING CHI THEORY  EQ 22
C
      FRTH=COE1**(-1.474)*(TAW/TIVAR)**.772
C
C     FIND VALUE           TEMP8     HT-HW
C
      TEMP8=HT-HW
C
C     FIND VALUE           TEMP7     USQ/2.
```

```
C
      TEMP7=WVAR*WVAR/2.
C
C     FIND ADBWALL ENTH   HAW      ADIABATIC WALL ENTHALPY
C
      HAW=HVAR+.89*(HT-HVAR)
C
C     ENTHALPY ECKERTS REFERENCE (EQ15)
C
      HP=.5*(HW+HVAR)+.22*(HAW-HVAR)
C
C     REFERENCE DENSITY,SOUND SPEED,TEMPERATURE,ENTROPY FROM REAL GAS
C                THERMODYNAMIC
C     TABLES,START WITH ESTIMATE FOR REFERENCE ENTROPY
C     TP,RHOP,AP,SP FROM RGAS USING P,HP
C     FIND REF TEMP        TPP      DEGR            REAL GAS THERMODYNAMIC
C                 TABLES
C
      CALL RGASH (PIVAR,ROP,AP,HP,TPP,SP,ERR,IGAS)
C
C     FIND REF TEMP        TK       DEGK
C
      TK=TPP/1.8
C
C     FIND REF PRANDTL     PRP      TABLE IV REF.30
C     NPRP FROM HANSENS TABLES USING PATM,TPDEGK
C
      CALL DISCOT (TK,PATM,TKTAB,PRTAB,PATAB,011,210,7,PRP)
C
C     FIND WALL TEMP       TK       DEGK
C
      TK=TW/1.8
C
C     FIND WALL PRANDTL    PRW      TABLE IV REF.30
C     NPRW FROM HANSENS TABLES USING PATM,TWDEGK
C
      CALL DISCOT (TK,PATM,TKTAB,PRTAB,PATAB,011,210,7,PRW)
C
C     VANDRIEST II
C     REYNOLDS ANALOGY FACTOR (TABLE I)
C
      HWT=HW/HT
      IF (HWT-.2) 14,15,15
```

```
14    RAF=1.
      GO TO 18
15    IF (HWT-.65) 16,16,17
16    RAF=.8311+.9675*HWT-.6142*HWT**2
      GO TO 18
17    RAF=1.2
C
C     FIND BI             F(1)     COEFF STATIC ENTH REG I BOUNDARY
C                  LAYER
C     BI  (EQ13)
C
18    CONTINUE
      F(1)=((HAW-HW)/TEMP8)*PRW/PRP**.66666667
      IF (FPOPT) 20,19,20
C
C     FIND VALUE          TEMP9    COS(THETAC)/(RB/L)
C
19    TEMP9=CTHCR/RBVAR
C
C     FIND VALUE          TEMP10   COS(THETAC)/(2RB/L)
C
      TEMP10=.5*TEMP9
C
C     FIND VALUE          TEMP14   2COS(THETAC)/(RB/L)
C
      TEMP14=2.*TEMP9
C
C     FIND VALUE          TEMP12   2RB/L*RHO*U/RHOI*UI
C
      TEMP12=2.*RBVAR*RRTVAR*WVAR/RHOUI
20    RETURN
      END
```

FOFX.- Function subroutine FOFX computes integrals in the boundary layer equations for conservation of mass and momentum. This subroutine is called by VGAUSS for integration.





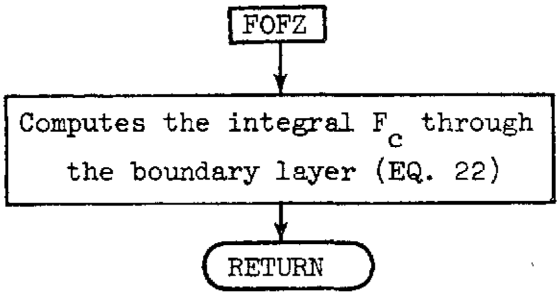
Is x/rn = x2REX + .01?
No
Yes
Is OUTPUT to be in SI Units?
No
Yes
Convert US to SI Units
Write Output Through Boundary Layer
y/rn/δ/rn, u/ue, h, ρ/ρe, ρu/ρeue, H̄, T, M
RETURN

```
      FUNCTION FOFX (V,F2)                                              FOX
C                                                                       FOX
C     FUNCTION CALLED BY VGAUSS FOR INTEGRATION THROUGH BOUNDARY LAYER  FOX
C     AR,BR,ALPHAR HAVE BEEN INPUT OR CALCULATED                        FOX
C     IN SUBROUTINES EDGE AND AL3CAL                                    FOX
C     N HAS BEEN INPUT OR CALCULATED IN SUBROUTINE DERSUB               FOX
C                                                                       FOX
      COMMON XLSH,XLSH1,THCR,JLIM,KLIM,TXLCL(450),TYL(450),TS(450),TP(45FOX
     10),TXL(450),TYLS(450),TSS(450)                                    FOX
      COMMON ENX,FPOPT,ALX,RAF,ALMIN,COPT,STHCR,T1,T2,L,XVO,PSI         FOX
      COMMON CMTOPT                                                     FOX
      COMMON ALGN,AN,ANZ,AP,AVAR,AW,AX,CEB,CEBP,CEM,CEMP,CFBMT,CFB2,CFEPFOX
     1R,CFI,CFMT,CF2,CF2I,CN,COE1,COE2,COE3,CTHCR,DEL,DELAM,DELS,DERR,DNFOX
     2,DPVAR,DRBVAR,DRVAR,DSTH,DUVAR,E,EK,EL,ELT,ELVO,EMU,EMVAR,EMX,EN,EFOX
     3NGN,ENI,ERR,FC,FCCF,FCCFLG,FCF,FCFPR,FD,FDPR,FRKE,FRTH,G,GC,GX,H,HFOX
     4AW,HHAT,HP,HT,HVAR,HW,H2,ICELL,II,IN,JJ,JJLIM,JN,K,KK,KN,LLIM,NKW,FOX
     5NN,NO,NXINT,PATM,PIVAR,PR,PRP,PRW,RBVAR,RET,RETH,RHOUI,RHOW,RO,ROPFOX
     6,ROVAR,ROW,RRI,RRN,RRTVAR,RRX,RSERR,RSISAV,RSLVAR,RX,SHEAR,SP,SVARFOX
     7,SW,SX,TAW,TEMP,TEMP10,TEMP11,TEMP12,TEMP14,TEMP2,TEMP3,TEMP4,TEMPFOX
     85,TEMP6,TEMP7,TEMP8,TEMP9,TEP,TIVAR,TK,TPP,TRAN,TT11,TW,TX,WVAR,XIFOX
     9N,XMIN,X2REX,Z,ZMAX,ZMIN                                          FOX
      REAL IN,JN,KN                                                     FOX
      COMMON F(3),A(3),ALPHE(3),XINT(99),TWT(99),ZTABL(6),TABIN(6),TABJNFOX
     1(6),DXLTAB(20),DELK(2),RSLG(2),ALG(2),FNG(2),AN2(7)               FOX
      COMMON FRXT(20),FRXLGT(20),FCXT(20),FCXLGT(20),FKT8(20),FRLGT(20),FOX
     1FCTB(20),FCLGT(20),XW(100),FIN6(100),RSLW(100),SHEER(100),X(160),SFOX
     2(160),PI(160),XC(160),RB(160),W(160),RRT(160),XMAXIB(20),DUDXT(160FOX
     3),DRDXT(160),DPDXT(160),DRBDXT(160),VAR(2),DER(2),CJVAR(2),WSAV(16FOX
     40),RRTSAV(160),XSAV(160),RSLSAV(160),REXSAV(160),ENSAV(160),CFSAV(FOX
     5160),TKTAB(30),PRTAB(210),PATAB(7),XKW(160),HKW(160),RSLKW(160)   FOX
      COMMON IGAS                                                       FOX
      COMMON /BLK/ CV1,CV2,CV3,CV4,CV5,CV6,CV7,CV8,CV9,CV10,CV11,CV12,CVFOX
     113,UNIN,UNIO                                                      FOX
      COMMON /BLOCK/ PLT1(160),PLT2(160),PLT3(160),PLT4(160),PLT5(160),PFOX
     1LT6(160),PLT7(160),PLT8(160),PLT9(160),PLT10(160),PLT11(160),PLT12FOX
     2(160),PLT13(160),PLT14(160),PLT15(160),NKWSAV                     FOX
      DIMENSION F2(7)                                                   FOX
C                                                                       FOX
C     COMPUTE INTEGRALS WRT. Y/DELTA FOR BOUNDARY LAYER THICKNESS,      FOX
C               DISPLACEMENT                                            FOX
C     THICKNESS,SHOCK RADIUS EQUATIONS,GAUSSIAN QUADRATURE              FOX
C     FO(7)                                                             FOX
```

```
C
      CON2=V**TEMP3
      IF (CON2-.01) 2,1,1
1     IF (CON2-.10) 3,4,4
C
C     R=I
C
2     K=1
      GO TO 5
C
C     R=II
C
3     K=2
      GO TO 5
C
C     R=III
C
4     K=3
C
C     EQ(10)
C
5     HHAT=A(K)+F(K)*CON2**ALPHE(K)
C
C     EQ(12)
C
      H=HW+TEMP8*HHAT-TEMP7*CON2*CON2
C
C     RHO FROM REAL GAS USING H AND P
C
      CALL RGASH (PIVAR,RX,AX,H,TX,SX,ERR,IGAS)
      RRI=RX/ROVAR
C
C     INTEGRAL 6,3,7,1
C
      F2(6)=RRI*CON2
      F2(3)=1.-F2(6)
      F2(7)=F2(6)*CON2
      F2(1)=F2(6)-F2(7)
C
C     IS FLAT PLATE
C
      IF (FPOPT) 6,7,6
C
```

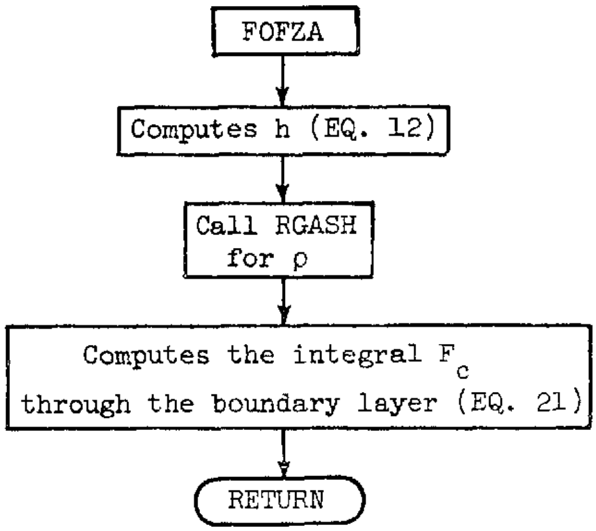
```
C      INTEGRAL 2,4,5=0
C
6      F2(2)=F2(4)=F2(5)=0
       GO TO 8
C
C      INTEGRAL 2,4,5
C
7      TEMP2=V*TEMP9
       F2(2)=F2(6)*TEMP2
       F2(4)=F2(3)*TEMP2
       F2(5)=F2(1)*TEMP2
C
C      IS X/L=X2REX+.01
C
8      IF (ABS(ELT-CUVAR(1))-.0001) 9,9,12
C
C      WRITE OUTPUT THROUGH BOUNDARY LAYER
C      Y/DEL,U/UE,H,RHO/RHOE,RHOU/RHOEUE,HHAT,T,M
C
9      EMX=CON2*WVAR/AX
       WRITE (6,14)
       IF (UNIO-1.) 10,10,11
10     O89=H*CV5
       O90=TX*CV8
       WRITE (6,13) V,CON2,O89,RRI,F2(6),HHAT,O90,EMX
       GO TO 12
11     WRITE (6,13) V,CON2,H,RRI,F2(6),HHAT,TX,EMX
12     RETURN
C
C
C
13     FORMAT (8E16.8)
14     FORMAT (1H 5X,3HY/D,13X,4HU/UE,13X,1HH,12X,8HRHO/RHOE,6X,11HRHOU/R
      1HOEUE,9X,4HHBAR,13X,1HT,15X,1HM)
       END
```

FOFZ.- Function subroutine FOFZ computes the ideal gas $F_c$ function to correlate the skin-friction data. This subroutine is called by VGAUSS for integration.



```
      FUNCTION FOFZ (V,FZ)                                              FOZ
C                                                                       FOZ
C     FUNCTION CALLED BY VGAUSS FOR INTEGRATION THROUGH BOUNDARY LAYER  FOZ
C     FOR SPALDING-CHI I SKIN FRICTION                                  FOZ
C     FIND INTEGRAL WRT. U/UE FOR EQ(22)                                FOZ
C                                                                       FOZ
      COMMON XLSH,XLSHI,THCR,JLIM,KLIM,TXLCL(450),TYL(450),TS(450),TP(45FOZ
     10),TXL(450),TYLS(450),TSS(450)                                    FOZ
      COMMON ENX,FPOPT,ALX,RAF,ALMIN,COPT,STHCR,T1,T2,L,XVO,PSI         FOZ
      COMMON CMTOPT                                                     FOZ
      COMMON ALGN,AN,ANZ,AP,AVAR,AW,AX,CEB,CEBP,CEM,CEMP,CFBMT,CFB2,CFERFOZ
     1R,CFI,CFMT,CF2,CF2I,CN,COE1,COE2,COE3,CTHCR,DEL,DELAM,DELS,DERR,DNFOZ
     2,DPVAR,DRBVAR,DRVAR,DSTH,DUVAR,E,EK,EL,ELT,ELVO,EMU,EMVAR,EMX,EN,EFOZ
     3NGN,ENI,ERR,FC,FCCF,FCCFLG,FCF,FCFPR,FD,FDPR,FRRE,FRTH,G,GC,GX,H,HFOZ
     4AW,HHAT,HP,HT,HVAR,HW,HZ,ICELL,II,IN,JJ,JJLIM,JN,K,KK,KN,LLIM,NKW,FOZ
     5NN,NO,NXINT,PATM,PIVAR,PR,PRP,PRW,RBVAR,RET,RETH,RHOUI,RHOW,RO,ROPFOZ
     6,ROVAR,ROW,RRI,RRN,RRTVAR,RRX,RSERR,RSISAV,RSLVAR,RX,SHEAR,SP,SVARFOZ
     7,SW,SX,TAW,TEMP,TEMP10,TEMP11,TEMP12,TEMP14,TEMP2,TEMP3,TEMP4,TEMPFOZ
     85,TEMP6,TEMP7,TEMP8,TEMP9,TFP,TIVAR,TK,TPP,TRAN,TT11,TW,TX,WVAR,XIFOZ
     9N,XMIN,X2REX,Z,ZMAX,ZMIN                                          FOZ
      REAL IN,JN,KN                                                     FOZ
      COMMON F(3),A(3),ALPHE(3),XINT(99),TWT(99),ZTABL(6),TABIN(6),TABJNFOZ
     1(6),DXLTAB(20),DELK(2),RSLG(2),ALG(2),ENG(2),AN2(7)               FOZ
      COMMON FRXT(20),FRXLGT(20),FCXT(20),FCXLGT(20),FRTB(20),FRLGT(20),FOZ
     1FCTB(20),FCLGT(20),XW(100),FIN6(100),RSLW(100),SHEER(100),X(160),SFOZ
     2(160),PI(160),XC(160),RB(160),W(160),RRT(160),XMAXTB(20),DUDXT(160FOZ
     3),DRDXT(160),DPDXT(160),DRBDXT(160),VAR(2),DER(2),CUVAR(2),WSAV(16FOZ
     40),RRTSAV(160),XSAV(160),RSLSAV(160),REXSAV(160),ENSAV(160),CFSAV(FOZ
     5160),TKTAB(30),PRTAB(210),PATAB(7),XKW(160),WKW(160),RSLKW(160)   FOZ
      COMMON /BLOCK/ PLT1(160),PLT2(160),PLT3(160),PLT4(160),PLT5(160),PFOZ
     1LT6(160),PLT7(160),PLT8(160),PLT9(160),PLT10(160),PLT11(160),PLT12FOZ
     2(160),PLT13(160),PLT14(160),PLT15(160),NKWSAV                     FOZ
      RRN=1./(COE1+(COE3-COE2*V)*V)                                     FOZ
      FZ=SQRT(RRN)                                                      FOZ
      RETURN                                                            FOZ
      END                                                               FOZ
```

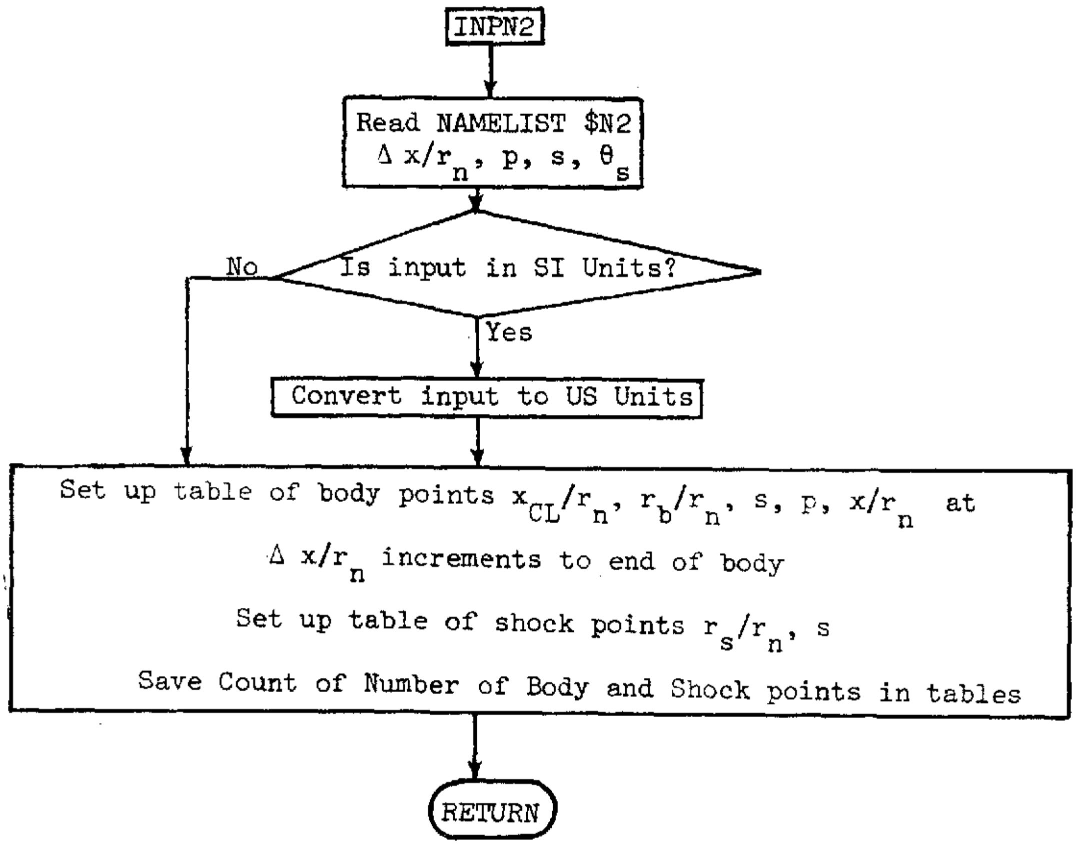
FOFZA.- Function subroutine FOFZA computes the real gas $F_c$ function to correlate the skin-friction data. This subroutine is called by VGAUSS for integration.



```
      FUNCTION FOFZA (V,FZ)
C
C     FUNCTION CALLED BY VGAUSS FOR INTEGRATION THROUGH BOUNDARY LAYER
C     FOR SPALDING-CHI II SKIN FRICTION
C     FIND INTEGRAL WRT.U/UE FOR EQ(21)
C
      COMMON XLSH,XLSH1,THCR,JLIM,KLIM,TXLCL(450),TYL(450),TS(450),TP(45
     10),TXL(450),TYLS(450),TSS(450)
      COMMON ENX,FPOPT,ALX,RAF,ALMIN,COPT,STHCR,T1,T2,L,XVO,PSI
      COMMON CMTOPT
      COMMON ALGN,AN,ANZ,AP,AVAR,AW,AX,CEB,CEBP,CEM,CEMP,CFBMT,CFB2,CFER
     1R,CFI,CFMT,CF2,CF2I,CN,COE1,COE2,COE3,CTHCR,DEL,DELAM,DELS,DERR,DN
     2,DPVAR,DRBVAR,DRVAR,DSTH,DUVAR,E,EK,EL,ELT,ELVO,EMU,EMVAR,EMX,EN,E
     3NGN,ENI,ERR,FC,FCCF,FCCFLG,FCF,FCFPR,FD,FDPR,FRRE,FRTH,G,GC,GX,H,H
     4AW,HHAT,HP,HT,HVAR,HW,H2,ICELL,II,IN,JJ,JJLIM,JN,K,KK,KN,LLIM,NKW,
     5NN,NO,NXINT,PATM,PIVAR,PR,PRP,PRW,RBVAR,RET,RETH,RHOUI,RHOW,RO,ROP
     6,ROVAR,ROW,RRI,RRN,RRTVAR,RRX,RSERR,RSISAV,RSLVAR,RX,SHEAR,SP,SVAR
     7,SW,SX,TAW,TEMP,TEMP10,TEMP11,TEMP12,TEMP14,TEMP2,TEMP3,TEMP4,TEMP
     85,TEMP6,TEMP7,TEMP8,TEMP9,TEP,TIVAR,TK,TPP,TRAN,TT11,TW,TX,WVAR,XI
     9N,XMIN,X2REX,Z,ZMAX,ZMIN
      REAL IN,JN,KN
      COMMON F(3),A(3),ALPHE(3),XINT(99),TWT(99),ZTABL(6),TABIN(6),TABJN
     1(6),DXLTAB(20),DELK(2),RSLG(2),ALG(2),ENG(2),AN2(7)
      COMMON FRXT(20),FRXLGT(20),FCXT(20),FCXLGT(20),FRTB(20),FRLGT(20),
     1FCTB(20),FCLGT(20),XW(100),FIN6(100),RSLW(100),SHEER(100),X(160),S
     2(160),PI(160),XC(160),RB(160),W(160),RRT(160),XMAXTB(20),DUDXT(160
     3),DRDXT(160),DPDXT(160),DRBDXT(160),VAR(2),DER(2),CUVAR(2),WSAV(16
     40),RRTSAV(160),XSAV(160),RSLSAV(160),REXSAV(160),ENSAV(160),CFSAV(
     5160),TKTAB(30),PRTAB(210),PATAB(7),XKW(160),WKW(160),RSLKW(160)
      COMMON IGAS
      COMMON /BLOCK/ PLT1(160),PLT2(160),PLT3(160),PLT4(160),PLT5(160),P
     1LT6(160),PLT7(160),PLT8(160),PLT9(160),PLT10(160),PLT11(160),PLT12
     2(160),PLT13(160),PLT14(160),PLT15(160),NKWSAV
      CON2=V
C
C     0.LE.U/UE.LT..01 R=I
C     .01.LE.U/UE.LE..1 R=II
C     U/UE.GT..1 R=III
C
      IF (CON2-.01) 2,1,1
1     IF (CON2-.10) 3,4,4
2     K=1
```

```
      GO TO 5
3     K=2
      GO TO 5
4     K=3
C
C     EQ(10)
C
5     HHAT=A(K)+F(K)*CON2**ALPHE(K)
C
C     EQ(12)
C
      H=HW+TEMP8*HHAT-TEMP7*CON2*CON2
C
C     RHO FROM REAL GAS USING H AND P
C
      CALL RGASH (PIVAR,RX,AX,H,TX,SX,ERR,IGAS)
      RRI=RX/ROVAR
      FZ=SQRT(RRI)
      RETURN
      END
```

__INPN2__.- Subroutine INPN2 reads NAMELIST $N2 if CARD = 1. is input. Given cone angle, shock angle, constant pressure and entropy this subroutine computes tables of $x/r_n$, $r_b/r_n$, s, p on body and $r_s/r_n$, s on shock.

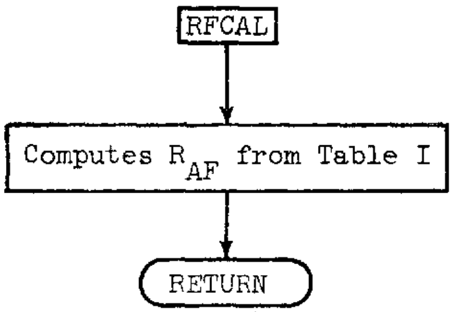


```
      SUBROUTINE INPN2
C
C     IF INPUT NOT ON TAPE,PRESSURE AND ENTROPY CONSTANT,SHOCK ANGLE
C              GIVEN
C     SETS UP TABLES AT X CENTERLINE INCREMENT,COMPUTES X CENTERLINE,
C              Y CONE, Y SHOCK
C
      COMMON XLSH,XLSH1,THCR,JLIM,KLIM,TXLCL(450),TYL(450),TS(450),TP(45
     10),TXL(450),TYLS(450),TSS(450)
      COMMON ENX,FPOPT,ALX,RAF,ALMIN,COPT,STHCR,T1,T2,L,XVO,PSI
      COMMON CMTOPT
      COMMON ALGN,AN,ANZ,AP,AVAR,AW,AX,CEB,CEBP,CEM,CEMP,CFBMT,CFB2,CFER
     1R,CFI,CFMT,CF2,CF2I,CN,COE1,COE2,COE3,CTHCR,DEL,DELAM,DELS,DERR,DN
     2,DPVAR,DRBVAR,DRVAR,DSTH,DUVAR,E,EK,EL,ELT,ELVO,EMU,EMVAR,EMX,EN,E
     3NGN,ENI,ERR,FC,FCCF,FCCFLG,FCF,FCFPR,FD,FDPR,FRKE,FRTH,G,GC,GX,H,H
     4AW,HHAT,HP,HT,HVAR,HW,H2,ICELL,II,IN,JJ,JJLIM,JN,K,KK,KN,LLIM,NKW,
     5NN,NO,NXINT,PATM,PIVAR,PR,PRP,PRW,RBVAR,RET,RETH,RHOUI,RHOW,RO,ROP
     6,ROVAR,ROW,RRI,RRN,RRTVAR,RRX,RSERR,RSISAV,RSLVAR,RX,SHEAR,SP,SVAR
     7,SW,SX,TAW,TEMP,TEMP10,TEMP11,TEMP12,TEMP14,TEMP2,TEMP3,TEMP4,TEMP
     85,TEMP6,TEMP7,TEMP8,TEMP9,TEP,TIVAR,TK,TPP,TRAN,TT11,TW,TX,WVAR,XI
     9N,XMIN,X2REX,Z,ZMAX,ZMIN
      REAL IN,JN,KN
      COMMON F(3),A(3),ALPHE(3),XINT(99),TWT(99),ZTABL(6),TABIN(6),TABJN
     1(6),DXLTAB(20),DELK(2),RSLG(2),ALG(2),ENG(2),AN2(7)
      COMMON FRXT(20),FRXLGT(20),FCXT(20),FCXLGT(20),FRTB(20),FRLGT(20),
     1FCTB(20),FCLGT(20),XW(100),FIN6(100),RSLW(100),SHEER(100),X(160),S
     2(160),PI(160),XC(160),RB(160),W(160),RPT(160),XMAXTB(20),DUDXT(160
     3),DRDXT(160),DPDXT(160),DRBDXT(160),VAR(2),DER(2),CUVAR(2),WSAV(16
     40),RRTSAV(160),XSAV(160),RSLSAV(160),REXSAV(160),ENSAV(160),CFSAV(
     5160),TKTAB(30),PRTAB(210),PATAB(7),XKW(160),WKW(160),RSLKW(160)
      COMMON /BLK/ CV1,CV2,CV3,CV4,CV5,CV6,CV7,CV8,CV9,CV10,CV11,CV12,CV
     113,UNIN,UNIO
      COMMON /BLOCK/ PLT1(160),PLT2(160),PLT3(160),PLT4(160),PLT5(160),P
     1LT6(160),PLT7(160),PLT8(160),PLT9(160),PLT10(160),PLT11(160),PLT12
     2(160),PLT13(160),PLT14(160),PLT15(160),NKWSAV
      NAMELIST /N2/ DELTX,TP,TS,THSD
C
C     READ NAMELIST/N2/
C     DX,P,S,THETAS(DEG)
C
      READ (5,N2)
      IF (ENDFILE 5) 1,2
```

```
1       CALL EXIT
2       WRITE (6,N2)
        IF (UNIN-1.) 3,3,4
3       TP=TP/CV10
        TS=TS/CV4
        THSD=THSD/CV9
4       TXLCL(1)=0
        TSS(1)=TS(1)
        THSR=THSD*.01745329
        TYL(1)=0
        TYLS(1)=0
        TXL(1)=0
C
C       SET UP TABLE OF BODY PTS X/LCL,RB/L,S,P,X/L
C       AT DX INCREMENTS TO END OF BODY
C       SET UP TABLE OF SHOCK PTS RS/L,S
C       SAVE COUNT OF NO. OF BODY AND SHOCK PTS IN TABLES
C
        DO 8 J=2,450
        TXLCL(J)=TXLCL(J-1)+DELTX
C
C       BODY RADIUS
C
        TYL(J)=TAN(THCR)*TXLCL(J)
C
C       SHOCK RADIUS
C
        TYLS(J)=TAN(THSR)*TXLCL(J)
        TP(J)=TP(J-1)
        TS(J)=TS(J-1)
        TSS(J)=TS(J)
C
C       X/L
C
        XL=TXLCL(J)-T2
        IF (XL) 5,5,6
5       TXL(J)=ACOS(1.-TXLCL(J))
        GO TO 7
6       TXL(J)=-T1+XL/CTHCR
7       IF (TXLCL(J)-XMAXTB(20)) 8,8,9
8       CONTINUE
9       JLIM=J
        KLIM=J
```

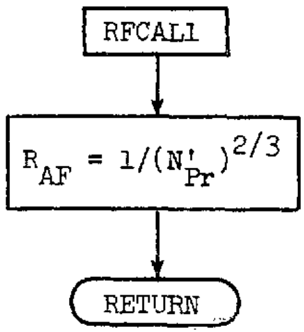
```
      RETURN                                                            TNP  86
      END                                                               TNP  87-
```

RFCAL.- Subroutine RFCAL computes Reynolds analogy factor for VanDriest skin-friction theory.



```
      SUBROUTINE RFCAL (RAFX)
C     COMPUTES REYNOLDS ANALOGY FACTOR FOR
C       VAN DRIEST SKIN FRICTION
      COMMON XLSH,XLSHI,THCR,JLIM,KLIM,TXLCL(450),TYL(450),TS(450),TP(45
     10),TXL(450),TYLS(450),TSS(450)
      COMMON ENX,FPOPT,ALX,RAF,ALMIN,COPT,STHCR,T1,T2,L,XVO,PSI
      COMMON CMTOPT
      COMMON ALGN,AN,ANZ,AP,AVAR,AW,AX,CEB,CEBP,CEM,CEMP,CFBMT,CFB2,CFER
     1R,CFI,CFMT,CF2,CF2I,CN,COE1,COE2,COE3,CTHCR,DEL,DELAM,DELS,DERR,DN
     2,DPVAR,DRBVAR,DRVAR,DSTH,DUVAR,E,EK,EL,ELT,ELVO,EMU,EMVAR,EMX,EN,E
     3NGN,ENI,ERR,FC,FCCF,FCCFLG,FCF,FCFPR,FD,FDPR,FRRE,FRTH,G,GC,GX,H,H
     4AW,HHAT,HP,HT,HVAR,HW,H2,ICELL,II,IN,JJ,JJLIM,JN,K,KK,KN,LLIM,NKW,
     5NN,NO,NXINT,PATM,PIVAR,PR,PRP,PRW,RBVAR,RET,RETH,RHOUI,RHOW,RO,ROP
     6,ROVAR,ROW,RRI,RRN,RRTVAR,RRX,RSERR,RSISAV,RSLVAR,RX,SHEAR,SP,SVAR
     7,SW,SX,TAW,TEMP,TEMP10,TEMP11,TEMP12,TEMP14,TEMP2,TEMP3,TEMP4,TEMP
     85,TEMP6,TEMP7,TEMP8,TEMP9,TEP,TIVAR,TK,TPP,TRAN,TT11,TW,TX,WVAR,XI
     9N,XMIN,X2REX,Z,ZMAX,ZMIN
      REAL IN,JN,KN
      COMMON F(3),A(3),ALPHE(3),XINT(99),THT(99),ZTABL(6),TABIN(6),TABJN
     1(6),DXLTAB(20),DELK(2),RSLG(2),ALG(2),ENG(2),AN2(7)
      COMMON FRXT(20),FRXLGT(20),FCXT(20),FCXLGT(20),FRTB(20),FRLGT(20),
     1FCTB(20),FCLGT(20),XW(100),FIN6(100),RSLW(100),SHEER(100),X(160),S
     2(160),PI(160),XC(160),RB(160),W(160),RRT(160),XMAXTB(20),DUDXT(160
     3),DRDXT(160),DPDXT(160),DRBDXT(160),VAR(2),DER(2),CUVAR(2),WSAV(16
     40),RRTSAV(160),XSAV(160),RSLSAV(160),REXSAV(160),ENSAV(160),CFSAV(
     5160),TKTAB(30),PRTAB(210),PATAB(7),XKW(160),WKW(160),RSLKW(160),IG
     6AS
      COMMON XX1
      COMMON ACFT(22),TREXT(22),ACFLGT(22),TRLGT(22),REXCF,HO,XO
      HWT=HW/HT
      IF (HWT-.2) 1,2,2
1     RAFX=1.
      GO TO 5
2     IF (HWT-.65) 3,3,4
3     RAFX=.8311+.9675*HWT-.6142*HWT**2
      GO TO 5
4     RAFX=1.2
5     RETURN
      END
```

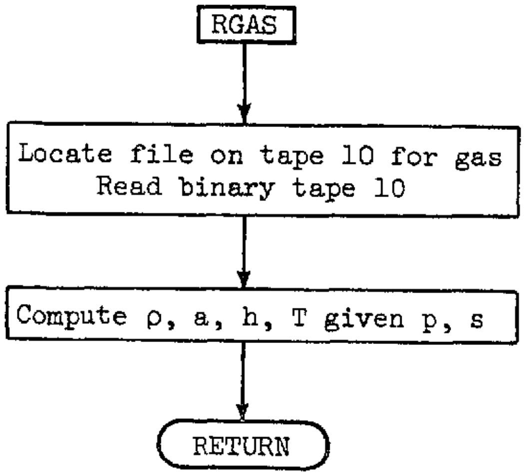
RFCAL1.- Subroutine RFCAL1 computes Reynolds analogy factor for Spalding-Chi and Eckert's skin-friction theories.



```
      SUBROUTINE RFCAL1 (RAFX)                                          RF1
C     COMPUTES REYNOLDS ANALOGY FACTOR FOR                              RF1
C       SPALDING CHI AND ECKERT SKIN FRICTION                           RF1
      COMMON XLSH,XLSH1,THCR,JLIM,KLIM,TXLCL(450),TYL(450),TS(450),TP(45RF1
     10),TXL(450),TYLS(450),TSS(450)                                    RF1
      COMMON ENX,FPOPT,ALX,RAF,ALMIN,COPT,STHCR,T1,T2,L,XVO,PSI         RF1
      COMMON CMTOPT                                                     RF1
      COMMON ALGN,AN,ANZ,AP,AVAR,AW,AX,CEB,CEBP,CEM,CEMP,CFBMT,CFB2,CFERRF1
     1R,CFI,CFMT,CF2,CF2I,CN,COE1,COE2,COE3,CTHCR,DEL,DELAM,DELS,DERR,DNRF1
     2,DPVAR,DRBVAR,DRVAR,DSTH,DUVAR,E,EK,EL,ELT,ELVO,EMU,EMVAR,EMX,EN,ERF1
     3NGN,FNI,ERR,FC,FCCF,FCCFLG,FCF,FCFPR,FD,FDPR,FRRE,FRTH,G,GC,GX,H,HRF1
     4AW,HHAT,HP,HT,HVAR,HW,H2,ICFLL,II,IN,JJ,JJLIM,JN,K,KK,KN,LLIM,NKW,RF1
     5NN,NO,NXINT,PATM,PIVAR,PR,PRP,PRW,RBVAR,RET,RETH,RHOUI,RHOW,RO,ROPRF1
     6,ROVAR,ROW,RRI,RRN,RRTVAR,RPX,RSERR,RSISAV,RSLVAR,RX,SHEAR,SP,SVARRF1
     7,SW,SX,TAW,TEMP,TEMP10,TEMP11,TEMP12,TEMP14,TEMP2,TEMP3,TEMP4,TEMPRF1
     85,TEMP6,TEMP7,TEMP8,TEMP9,TEP,TIVAR,TK,TPP,TRAN,TT11,TW,TX,WVAR,XIRF1
     9N,XMIN,X2REX,Z,ZMAX,ZMIN                                          RF1
      REAL IN,JN,KN                                                     RF1
      COMMON F(3),A(3),ALPHE(3),XINT(99),TWT(99),ZTABL(6),TABIN(6),TABJNRF1
     1(6),DXLTAB(20),DFLK(2),RSLG(2),ALG(2),ENG(2),AN2(7)               RF1
      COMMON FRXT(20),FRXLGT(20),FCXT(20),FCXLGT(20),FRTB(20),FRLGT(20),RF1
     1FCTB(20),FCLGT(20),XW(100),FIN6(100),RSLW(100),SHEER(100),X(160),SRF1
     2(160),PI(160),XC(160),RB(160),W(160),RRT(160),XMAXTB(20),DUDXT(160RF1
     3),DPDXT(160),DPDXT(160),DPBDXT(160),VAR(2),DER(2),CUVAR(2),WSAV(16RF1
     40),RRTSAV(160),XSAV(160),RSLSAV(160),REXSAV(160),ENSAV(160),CFSAV(RF1
     5160),TKTAB(30),PRTAB(210),PATAB(7),XKW(160),HKW(160),RSLKW(160),IGRF1
     6AS                                                                RF1
      COMMON XX1                                                        RF1
      COMMON ACFT(22),TREXT(22),ACFLGT(22),TRLGT(22),KEXCF,HO,XO        RF1
      RAFX=1./PRP**.66666667                                            RF1
      RETURN                                                            RF1
      END                                                               RF1
```

RGAS.- Subroutine RGAS computes the thermodynamic properties for a real gas.

The flow properties $p_e$, $\rho_e$, $T_e$, $S_e$, $h_e$, and $a_e$ for a real gas in thermodynamic equilibrium are calculated by the computer subroutine (RGAS) described by Lomax and Inouye in reference 27. The subroutine RGAS requires use of the Ames real gas TAPE10 containing information for nitrogen on file 1 (IGAS=1) and information for air on file 2 (IGAS=2). Subroutine ROLL and SERCH are used by subroutine RGAS to locate the information on TAPE10. During the calculation, thermodynamic data at a point are found by entering the RGAS subroutine with pressure and entropy. For calculation of density profiles through the boundary layer, subroutines RGASH and RGAST are used to allow thermodynamic data to be found for a given enthalpy (or temperature) and pressure. The procedure in this case is to enter the RGAS subroutine with various estimated values of entropy and the local pressure until the value of entropy is found that yields the desired value of enthalpy (or temperature).


RGAS
Locate file on tape 10 for gas
Read binary tape 10
Compute ρ, a, h, T given p, s
RETURN

```
      SUBROUTINE RGAS (PX,RX,AX,HX,TX,SX,RRX,GX,NTEST,NUMX,NGAS)
C
C     AMES PROGRAM FOR REAL GAS EQUILIBRIUM THERMODYNAMIC PROPERTIES
C     AMES TAPE MOUNTED ON UNIT 10
C     PX     PRESSURE LBS/FT2
C     RX     DENSITY  SLUGS/FT3
C     AX     SPEED OF SOUND FT/SEC
C     HX     ENTHALPY FT2/SEC2
C     TX     TEMPERATURE DEGR
C     SX     ENTROPY  FT2/SEC2 DEGR
C     RRX    GAS CONSTANT FT2/SEC2 DEGR
C     GX     RATIO OF SPECIFIC HEATS
C     NTEST =1 FOR REAL GAS,=0 FOR PERFECT GAS
C     NUMX  =5 FOR PRESSURE AND ENTROPY INPUT
C     NGAS  =2 FOR AIR ON FILE 2 OF AMES TAPE 10
C
C     AMES SUBROUTINE TO FIND RHO,A,H,T WHEN P AND S GIVEN
C     CALLS TAPE10 AMES REAL GAS TAPE (FILE2 IS AIR)
C     USES SUBROUTINES ROLL AND SERCH TO LOCATE ON TAPE DESIRED GAS DATA
C     RGASR       WALKER      TEMP CONVERTED TO RANKINE
      DIMENSION NLL(8), JXX(8), DZZ(8), TZ(3000), NDZ(89)
      DIMENSION TH(5,600), NDL(4,11), NDU(4,11), AN(4), C(7), ANR(17), B
     1N(4)
      EQUIVALENCE (TZ,TH), (NDZ,NDL), (NDZ(45),NDU)
      DATA KEY,NTIMES/0,0/
      DATA WORD1,WORD2/6HNUM HI,6HNUM LO/
      DATA NTAPE/10/
      DATA GTEST/0/
      DATA GTESTR/0/
1     KEY=KEY+1
      P=PX
      S=SX
      R=RX
      NUM=NUMX
      IF (NUM) 6,6,2
2     IF (NUM-8) 7,7,3
3     WORD=WORD1
4     WRITE (6,108) WORD
5     CALL EXIT
6     WORD=WORD2
      GO TO 4
7     IF (NTEST) 8,99,99
```

```
8     IF (NFIRST-NGAS) 9,12,9
9     NFIRST=NGAS
      IND=0
      NFILES=NGAS-1
      CALL ROLL (NTAPE,NFILES,IND)
C
C                    FOR TAPE WRITTEN BY FORTRAN 2
C
      READ (NTAPE) (NDZ(N),N=1,89)
      DO 10 N=1,88
10    NDZ(N)=NDZ(N)/(2**18)
      NMM=NDZ(89)/(2**18)
C
      READ (NTAPE) (TZ(N),N=1,NMM),WTMIX,(C(N),N=1,7)
      REWIND NTAPE
      CALL EVICT (6LTAPE10)
      DO 11 N=1,88
11    NDZ(N)=5*NDZ(N)
      CONC=WTMIX/28.966
      PO=2116.
      RO=.002498*CONC
      RRR=1716./CONC
      RRX=RRR
      RTO=RRR*493.635
      SQPORO=SQRT(RO/PO)
      B=TZ(NMM-2)
      E=TZ(NMM-1)
      D=TZ(NMM)
      FM=2.1632+.3468*CONC
      AA=D*FM
      BB=E*FM+1.
      CCC=B+FM
12    P=ALOG10(P/PO)
      GO TO (22,22,22,22,13,3,3,3), NUM
13    REAL=S/RRR
      GG=(REAL-C(1)-C(2)*P)/(C(3)+P*(C(4)+P*C(5)))
      R=C(6)*GG+C(7)*P
      RL=P-B
      CC=CCC-P
      RH=-CC*(1.+AA*CC/(BB*BB))/BB+.005
      IF (RH+7.) 14,15,15
14    RH=-7.
15    IF (R-RH) 16,17,17
```

```
16    R=RH
17    IF (3.-RL) 18,19,19
18    RL=3.
19    IF (RL-R) 20,21,21
20    R=RL
21    NUMB=0
      NIMX=0
      NUMM=5
      NBOT=9-NUM
      NUP=NBOT
      GO TO 23
22    R=ALOG10(R/RO)
      NUMM=5
      NBOT=1
      NUP=NUM
23    CONTINUE
      IF (R) 24,24,26
24    NR=R-1.
      IF (NR+7) 25,25,28
25    NR=-7
      GO TO 28
26    NR=R
      IF (NR-3) 28,27,27
27    NR=2
28    DX=R-FLOAT(NR)
      NR=NR+8
      F=(P-R-B)/(1.+R*(E+D*R))
      IF (NUMM-9+NUM) 31,29,31
29    IF (F-.000001) 105,30,30
30    IF (FM-F) 83,31,31
31    DO 42 N1=NBOT,NUP
      IF (N1-NUMM) 32,41,32
32    NER1=N1
      NER2=N1+4
      NL=NDL(N1,NR)
      IF (NLL(NER1)-NL) 35,33,35
33    J=JXX(NER1)
      DIFF2=F-TH(5,J)
      IF (DIFF2) 35,34,34
34    IF (DZZ(NER1)-ABS(DIFF2)) 35,35,36
35    NU=NDU(N1,NR)
      CALL SERCH (F,TH,NL,NU,5,J,NER)
      J=J/5
```

```
      DZZ(NER1)=ABS(TH(5,J+1)-TH(5,J))
      JXX(NER1)=J
      NLL(NER1)=NL
36    XYZ=XYZ
      NL=NDL(N1,NR+1)
      IF (NLL(NER2)-NL) 39,37,39
37    K=JXX(NER2)
      DIFF2=F-TH(5,K)
      IF (DIFF2) 39,38,38
38    IF (DZZ(NER2)-ABS(DIFF2)) 39,39,40
39    NU=NDU(N1,NR+1)
      CALL SERCH (F,TH,NL,NU,5,K,NER)
      K=K/5
      DZZ(NER2)=ABS(TH(5,K+1)-TH(5,K))
      JXX(NER2)=K
      NLL(NER2)=NL
40    Y1=TH(1,J)+F*(TH(2,J)+F*(TH(3,J)+F*TH(4,J)))
      Y2=TH(1,K)+F*(TH(2,K)+F*(TH(3,K)+F*TH(4,K)))
      AN(N1)=Y1+DX*(Y2-Y1)
      GO TO 42
41    AN(N1)=REAL
42    CONTINUE
      IF (NUM-5) 43,48,48
43    GO TO (47,46,45,44,44,44,44,44), NUM
44    SX=AN(4)*RRR
45    TX=AN(3)*1.8
46    HX=AN(2)*RTO
47    AX=AN(1)/SQPORO
      GO TO 104
48    IF (NUMM-9+NUM) 50,49,50
49    RX=RO*10.**R
      GO TO 43
50    DIFF=ABS((REAL-AN(NUP))/REAL)
      IF (DIFF-.0001) 51,51,52
51    NUMM=9-NUM
      NBOT=1
      NUP=4
      GO TO 23
52    NUMB=NUMB+1
      NIMX=NIMX+1
      IF (NIMX-20) 53,53,83
53    IF (NUMB-2) 54,61,80
54    IF (REAL-AN(NUP)) 55,51,58
```

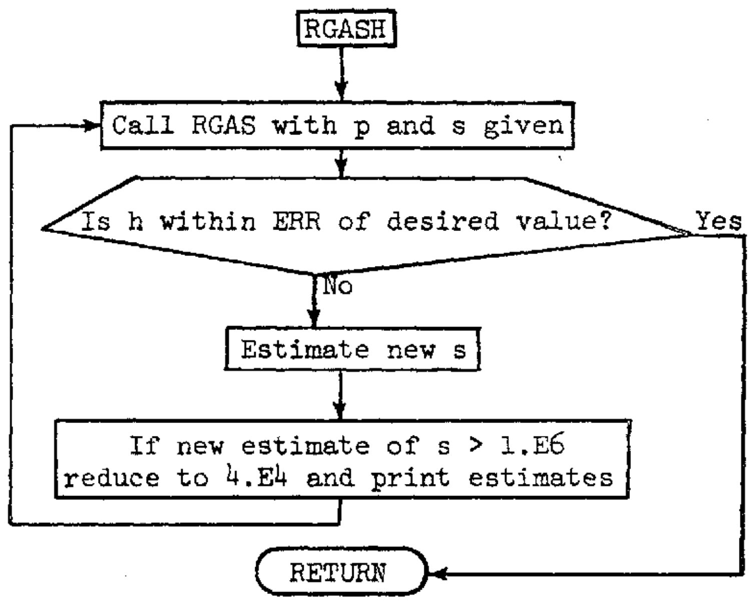
```
55    R1=R
      S1=AN(NUP)
      R=R+.3
      IF (RL-R) 56,57,57
56    R=RL
57    R2=R
      L=0
      GO TO 23
58    R2=R
      S2=AN(NUP)
      R=R-.3
      IF (R-RH) 59,60,60
59    R=RH
60    R1=R
      L=1
      GO TO 23
61    IF (L) 67,62,67
62    S2=AN(NUP)
      IF (S2-S1) 64,63,64
63    R=R2
      GO TO 65
64    R=R2-(S2-REAL)/(S2-S1)*(R2-R1)
65    IF (RL-R) 66,72,72
66    R=RL
      GO TO 72
67    S1=AN(NUP)
      IF (S2-S1) 69,68,69
68    R=R1
      GO TO 70
69    R=(REAL-S1)/(S2-S1)*(R2-R1)+R1
70    IF (R-RH) 71,72,72
71    R=RH
72    IF (R2-R) 73,51,76
73    NUMB=1
      R1=R2
      S1=S2
      L=0
      IF (R2+.3-RL) 75,74,74
74    R2=RL
      R=R2
      GO TO 23
75    R2=R2+.3
      R=R2
```

```
      GO TO 23
76    IF (R-R1) 77,51,23
77    NUMB=1
      R2=R1
      S2=S1
      L=1
      IF (RH-R1+.3) 79,78,78
78    R1=RH
      R=R1
      GO TO 23
79    R1=R1-.3
      R=R1
      GO TO 23
80    IF (REAL-AN(NUP)) 81,81,82
81    R1=R
      GO TO 67
82    R2=R
      GO TO 62
83    IF (F-.000001) 105,84,84
84    NTIMES=NTIMES+1
      WRITE (6,109)
      WRITE (6,110) PX
      IF (NUM-5) 85,86,86
85    WRITE (6,111) RX
      GO TO 87
86    WRITE (6,112) SX
87    IF (NTIMES-999) 104,88,88
88    WRITE (6,113)
      GO TO 5
89    L=0
      IF (GTEST-GX) 90,92,90
90    GTEST=GX
      L1=2
      ANR(1)=RRX
      ANR(2)=GX
      ANR(3)=ANR(1)/(ANR(2)-1.)
      ANR(4)=ANR(1)+ANR(3)
      ANR(8)=49008.609-ANR(3)*ALOG(171.6/.0001**ANR(2))
91    ANR(L+5)=1./ANR(L+2)
      ANR(L+6)=ANR(L+4)/ANR(L+1)
      ANR(L+7)=ANR(L+6)/ANR(L+2)
92    GO TO (93,93,93,93,98,99,100,102), NUM
93    QUOD=P/R**ANR(L+2)
```

```
      QUOT=P/R
      GO TO (97,96,95,94,98,99,100,102), NUM
94    S=ANR(L+8)+ANR(L+3)*ALOG(QUOD)
95    T=QUOT/ANR(L+1)
96    H=QUOT*ANR(L+6)
97    LL=L+L1
      A=SQRT(ANR(LL)*QUOT)
      GO TO 103
98    EX=S-ANR(L+8)
      EX=EXP(EX/ANR(L+3))
      R=(P/EX)**ANR(L+5)
      QUOD=P/R**ANR(L+2)
      QUOT=P/R
      GO TO 95
99    R=P/(T*ANR(L+1))
      QUOD=P/R**ANR(L+2)
      QUOT=P/R
      S=ANR(L+8)+ANR(L+3)*ALOG(QUOD)
      GO TO 96
100   ASSIGN 97 TO NJUMP
101   T=H/ANR(L+4)
      P=P/(T*ANR(L+1))
      QUOD=P/R**ANR(L+2)
      QUOT=P/R
      S=ANR(L+8)+ANR(L+3)*ALOG(QUOD)
      GO TO NJUMP, (97,103)
102   ASSIGN 103 TO NJUMP
      H=ANR(L+7)*A**2
      GO TO 101
103   AX=A
      HX=H
      TX=T
      SX=S
      RX=R
104   RETURN
105   L=8
      P=PX
      R=RX
      IF (GTESTR-GX) 106,92,106
106   GTESTR=GX
      L1=9
      Z2=R0/10.**7
      PR=-7.+B
```

```
      PR=PO*10.**PR
      Z1=PR
      DO 107 N1=1,4
      NL=NDL(N1,1)
      NU=NDU(N1,1)
      F=0.
      CALL SERCH (F,TH,NL,NU,5,J,NER)
      J=J/5
107   BN(N1)=TH(1,J)
      BN(1)=BN(1)/SQPOPO
      BN(2)=BN(2)*RTO
      BN(3)=BN(3)*1.8
      BN(4)=BN(4)*RRR
      ANR(9)=PR/(Z2*BN(3))
      RRX=ANR(9)
      ANR(12)=BN(2)/BN(3)
      ANR(10)=1.+ANR(9)/(ANR(12)-ANR(9))
      ANR(11)=ANR(12)/ANR(10)
      ANR(17)=BN(1)*BN(1)*Z2/Z1
      ANR(16)=BN(4)-ANR(11)*ALOG(Z1/Z2**ANR(10))
      GO TO 91
C
C
C
108   FORMAT (12H0 ER IN RGAS,3X,A6)
109   FORMAT (1H0,10X,36HOUTSIDE TABLES IN RGAS ENTERING WITH)
110   FORMAT (11X,2HP=,E13.6)
111   FORMAT (11X,2HR=E14.6)
112   FORMAT (11X,2HS=,E13.6)
113   FORMAT (20X,28HEXIT CALLED ON TENTH FAILURE)
      END
```

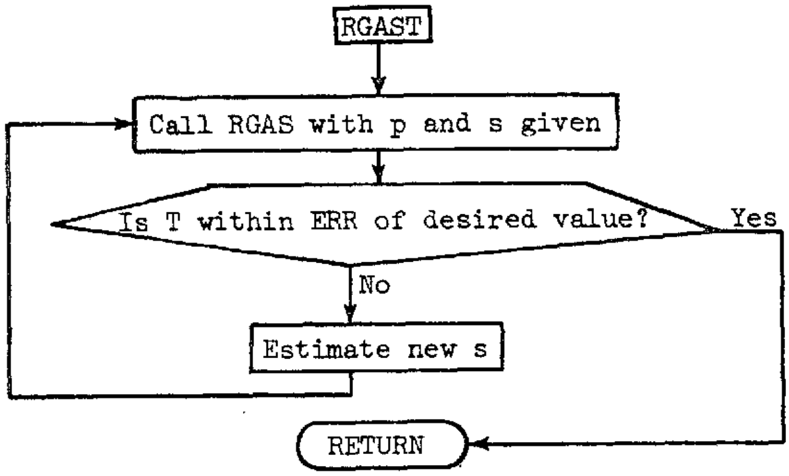
RGASH.- Subroutine RGASH computes thermodynamic properties density, speed of sound, temperature and entropy given pressure, enthalpy and an estimate of entropy.



```
      SUBROUTINE RGASH (PX,RX,AX,HX,TX,SX,ERR,IGAS)
C
C     GIVEN PRESSURE,ENTHALPY,ESTIMATE ENTROPY
C     VARY S UNTIL H WITHIN ERR OF DESIRED VALUE
C     FIND DENSITY,SPEED OF SOUND, TEMPERATURE,ENTROPY
C     REAL GAS EQUILIBRIUM THERMODYNAMIC PROPERTIES
C
      DIMENSION SG(2), HG(2)
1     HW=HX
      SG(1)=SX
      JJ=1
2     GX=1.4
C
C     RGAS GIVES RHO,A,T,S
C
      CALL RGAS (PX,RX,AX,HG(JJ),TX,SG(JJ),RRX,GX,-1.5,IGAS)
      IF (ABS(HG(JJ)-HW)-ERR*HW) 10,3,3
3     IF (JJ-1) 4,4,5
4     JJ=2
      SG(2)=SG(1)*1.01
      GO TO 2
5     IF (HG(2)-HG(1)) 7,6,7
6     WRITE (6,11) SG(1),SG(2),HG(1),HG(2),SGN,HW
      GO TO 10
7     SGN=((SG(2)-SG(1))/(HG(2)-HG(1)))*(HW-HG(1))+SG(1)
C
C     IF AN S.GT.1.E6 IS ESTIMATED,S WILL BE REDUCED TO 4.E4 AND
C     OUTPUT OF S AND H VALUES GIVEN AND ITERATION CONTINJES
C
      IF (SGN-1.E6) 9,8,8
8     SGN=4.E4
      WRITE (6,11) SG(1),SG(2),HG(1),HG(2),SGN,HW
9     HG(1)=HG(2)
      SG(1)=SG(2)
      SG(2)=SGN
      GO TO 2
10    SX=SG(JJ)
      RETURN
C
C
C
11    FORMAT (6E16.8)
```

END

RGAST.- Subroutine RGAST computes thermodynamic properties density, speed of sound, enthalpy and entropy, given pressure, temperature and an estimate of entropy.



```
      SUBROUTINE RGAST (PX,RX,AX,HX,TX,SX,ERR,IGAS)
C
C     GIVEN PRESSURE,TEMPERATURE,ESTIMATE ENTROPY
C     VARY S UNTIL T WITHIN ERR OF DESIRED VALUE
C     FIND DENSITY,SPEED OF SOUND,ENTHALPY,ENTROPY
C     REAL GAS EQUILIBRIUM THERMODYNAMIC PROPERTIES
C
      DIMENSION SG(2), TG(2)
      TW=TX
      SG(1)=SX
      JJ=1
1     GX=1.4
C
C     RGAS GIVES RHO,A,H,S
      CALL RGAS (PX,RX,AX,HX,TG(JJ),SG(JJ),RRX,GX,-1,5,IGAS)
      IF (ABS(TG(JJ)-TW)-ERR*TW) 5,2,2
2     IF (JJ-1) 3,3,4
3     JJ=2
      SG(2)=SG(1)*1.01
      GO TO 1
4     SGN=((SG(2)-SG(1))/(TG(2)-TG(1)))*(TW-TG(1))+SG(1)
      TG(1)=TG(2)
      SG(1)=SG(2)
      SG(2)=SGN
      GO TO 1
5     SX=SG(JJ)
      RETURN
C
      END
```

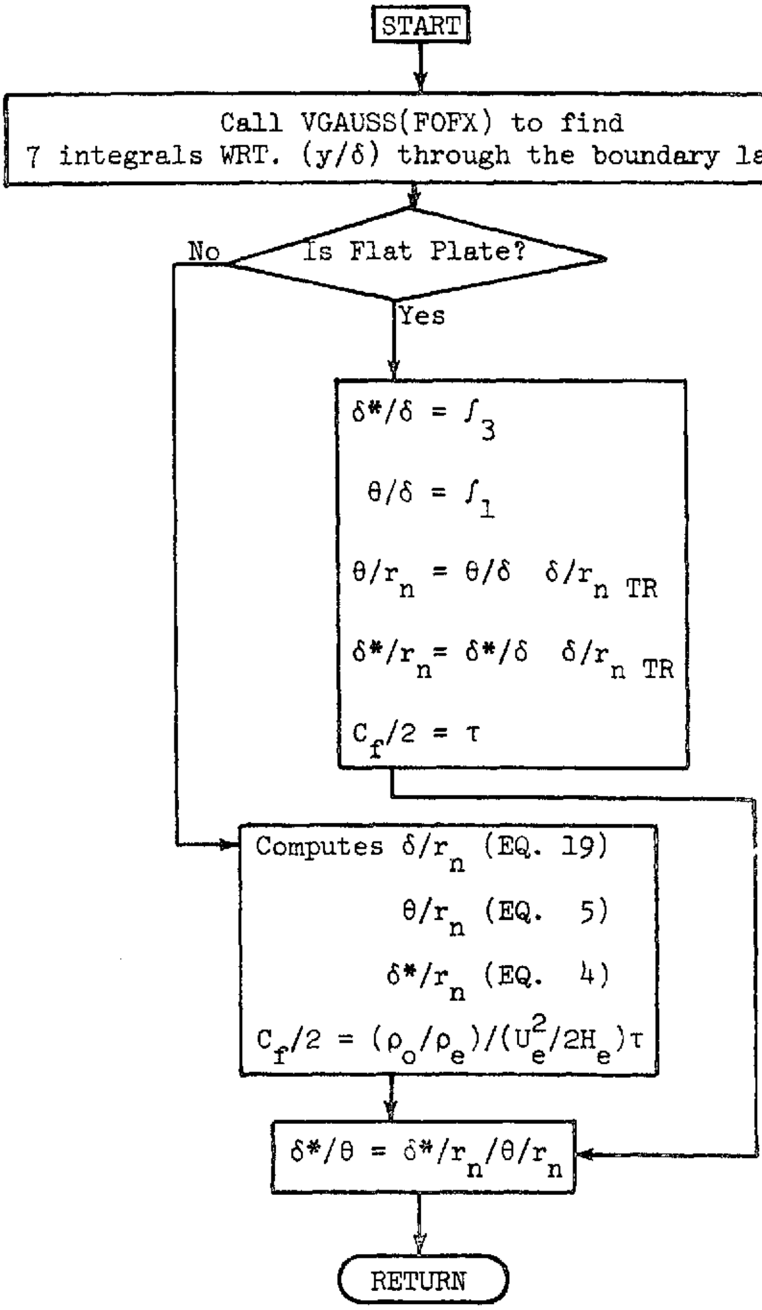
ROLL.- Subroutine called by RGAS to position TAPE10 to proper file for gas properties.

```
      SUBROUTINE ROLL (N,NFILES,IND)
C
C     POSITIONS AMES TAPE 10 TO PROPER FILE FOR AIR
C
      REWIND N
      NEOF=0
1     IF (NEOF-NFILES) 2,4,4
2     READ (N) A
      IF (ENDFILE N) 3,2
3     NEOF=NEOF+1
      GO TO 1
4     RETURN
      END
```

SERCH.- Subroutine called by RGAS to locate information for gas properties.

```
      SUBROUTINE SERCH (X,Q,NL,NU,NS,NOUT,NERR)
C
C     AMES PROGRAM USED BY RGAS
C
      DIMENSION Q(1,1)
      NERR=0
      K=NL+NS
      IF (Q(NL,1)-Q(K,1)) 1,1,4
1     DO 2 J=NL,NU,NS
      IF (X-Q(J,1)) 3,2,2
2     CONTINUE
3     NOUT=J-NS
      RETURN
4     DO 5 J=NL,NU,NS
      IF (X-Q(J,1)) 5,6,6
5     CONTINUE
6     NOUT=J
      RETURN
      END
```

START.- Subroutine START computes initial values of boundary layer thickness, displacement thickness, momentum thickness, skin-friction coefficient.

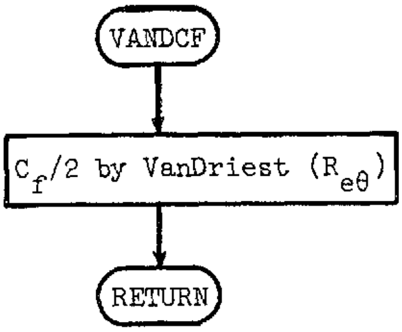


```
      SUBROUTINE START                                                  STR
C                                                                       STR
C     COMPUTE MOMENTUM THICKNESS,BOUNDARY LAYER THICKNESS, DISPLACEMENT STR
C                THICKNESS                                              STR
C     AT START OF TRANSITION GIVEN SHOCK RADIUS                         STR
C                                                                       STR
      COMMON XLSH,XLSH1,THCR,JLIM,KLIM,TXLCL(450),TYL(450),TS(450),TP(45STR
     10),TXL(450),TYLS(450),TSS(450)                                    STR
      COMMON ENX,FPOPT,ALX,RAF,ALMIN,COPT,STHCR,T1,T2,L,XVO,PSI         STR
      COMMON CMTOPT                                                     STR
      COMMON ALGN,AN,ANZ,AP,AVAR,AW,AX,CEB,CEBP,CEM,CEMP,CFBMT,CFB2,CFERSTR
     1R,CFI,CFMT,CF2,CF2I,CN,COE1,COE2,COE3,CTHCR,DEL,DELAM,DELS,DERR,DNSTR
     2,DPVAR,DRBVAR,DRVAR,DSTH,DUVAR,E,EK,EL,ELT,ELVO,EMU,EMVAR,EMX,EN,ESTR
     3NGN,ENI,ERR,FC,FCCF,FCCFLG,FCF,FCFPR,FD,FDPR,FRRE,FRTH,G,GC,GX,H,HSTR
     4AW,HHAT,HP,HT,HVAR,HW,H2,ICELL,II,IN,JJ,JJLIM,JN,K,KK,KN,LLIM,NKW,STR
     5NN,NO,NXINT,PATM,PIVAR,PR,PRP,PRW,RBVAR,RET,RETH,RHOUI,RHOW,RO,ROPSTR
     6,ROVAR,ROW,RRI,RRN,RRTVAR,RRX,RSERR,RSISAV,RSLVAR,RX,SHEAR,SP,SVARSTR
     7,SW,SX,TAW,TEMP,TEMP10,TEMP11,TEMP12,TEMP14,TEMP2,TEMP3,TEMP4,TEMPSTR
     85,TEMP6,TEMP7,TEMP8,TEMP9,TEP,TIVAR,TK,TPP,TRAN,TI11,TW,TX,WVAR,XISTR
     9N,XMIN,X2REX,Z,ZMAX,ZMIN                                          STR
      REAL IN,JN,KN                                                     STR
      COMMON F(3),A(3),ALPHE(3),XINT(99),TWT(99),ZTABL(6),TABIN(6),TABJNSTR
     1(6),DXLTAB(20),DELK(2),RSLG(2),ALG(2),ENG(2),AN2(7)               STR
      COMMON FRXT(20),FRXLGT(20),FCXT(20),FCXLGT(20),FRTB(20),FRLGT(20),STR
     1FCTB(20),FCLGT(20),XW(100),FIN6(100),RSLW(100),SHEER(100),X(160),SSTR
     2(160),PI(160),XC(160),RB(160),W(160),RRT(160),XMAXTB(20),DUDXT(160STR
     3),DRDXT(160),DPDXT(160),DRBDXT(160),VAR(2),DER(2),CUVAR(2),WSAV(16STR
     40),RRTSAV(160),XSAV(160),RSLSAV(160),REXSAV(160),ENSAV(160),CFSAV(STR
     5160),TKTAB(30),PRTAB(210),PATAB(7),XKW(160),WKW(160),RSLKW(160)   STR
      COMMON /BLOCK/ PLT1(160),PLT2(160),PLT3(160),PLT4(160),PLT5(160),PSTR
     1LT6(160),PLT7(160),PLT8(160),PLT9(160),PLT10(160),PLT11(160),PLT12STR
     2(160),PLT13(160),PLT14(160),PLT15(160),NKWSAV                     STR
      EXTERNAL FOFX                                                     STR
      DIMENSION F2(7)                                                   STR
C                                                                       STR
C     N(TRAN) IS INPUT                                                  STR
C                                                                       STR
      EN=ENI                                                            STR
      TEMP3=1./EN                                                       STR
      SX=SVAR                                                           STR
C                                                                       STR
C     CALL VGAUSS,FOFX TO FIND 7 INTEGRALS WRT.(Y/DEL) THROUGH BOUNDARY STR
```

```
C
      CALL VGAUSS (0,1.,L,AN2,FOFX,F2,7,NN)
C
C     IS FLAT PLATE
C
      IF (FPOPT) 1,5,1
C
C     DEL/L(TRAN),SHEAR ARE INPUT
C     DELST/DEL
C
1     DSOD=AN2(3)
C
C     THETA/DEL
C
      THOD=AN2(1)
      IF (ICELL) 3,2,3
2     DELI=DEL
      GO TO 4
3     DEL=DELI
4     CONTINUE
C
C     THETA/L
C
      CUVAR(2)=THOD*DEL
C
C     DELST/L
C
      DELS=DEL*DSOD
C
C     CF/2
C
      CF2=SHEAR
      GO TO 6
C
C     RS/L FROM SUBROUTINE EDGE,SHEAR(LAMINAR)IS INPUT
C
C     DEL/L (EQ19)
C
5     ADEL=TEMP12*AN2(2)
      BDEL=TEMP12*AN2(6)
      CDEL=-RSLVAR*RSLVAR
      DELP=(-BDEL+SQRT(BDEL*BDEL-4.*ADEL*CDEL))/(ADEL+ADEL)
      DELM=(-BDEL-SQRT(BDEL*BDEL-4.*ADEL*CDEL))/(ADEL+ADEL)
```

```
      DEL=DELP
C
C     THETA/L  (EQ5)
C
      GA=-DEL*DEL*AN2(5)-DEL*AN2(1)
      CUV2P=(-1.+SQRT(1.-TEMP14*GA))/TEMP9
      CUV2M=(-1.-SQRT(1.-TEMP14*GA))/TEMP9
      CUVAR(2)=CUV2P
C
C     DELST/L  (EQ4)
C
      G=-DEL*AN2(3)-DEL*DEL*AN2(4)
      DELS=(-1.+SQRT(1.-TEMP14*G))/TEMP9
C
C     CF/2
C
      CF2=((RO/RRTVAR)/(TEMP7/HT))*SHEAR
C
C     DELST/THETA
C
6     DSTH=DELS/CUVAR(2)
      CF2I=CF2
      RETURN
C
C
      END
```

VANDCF.- Subroutine VANDCF computes the VanDriest II skin-friction coefficient using Reynolds number based on momentum thickness.



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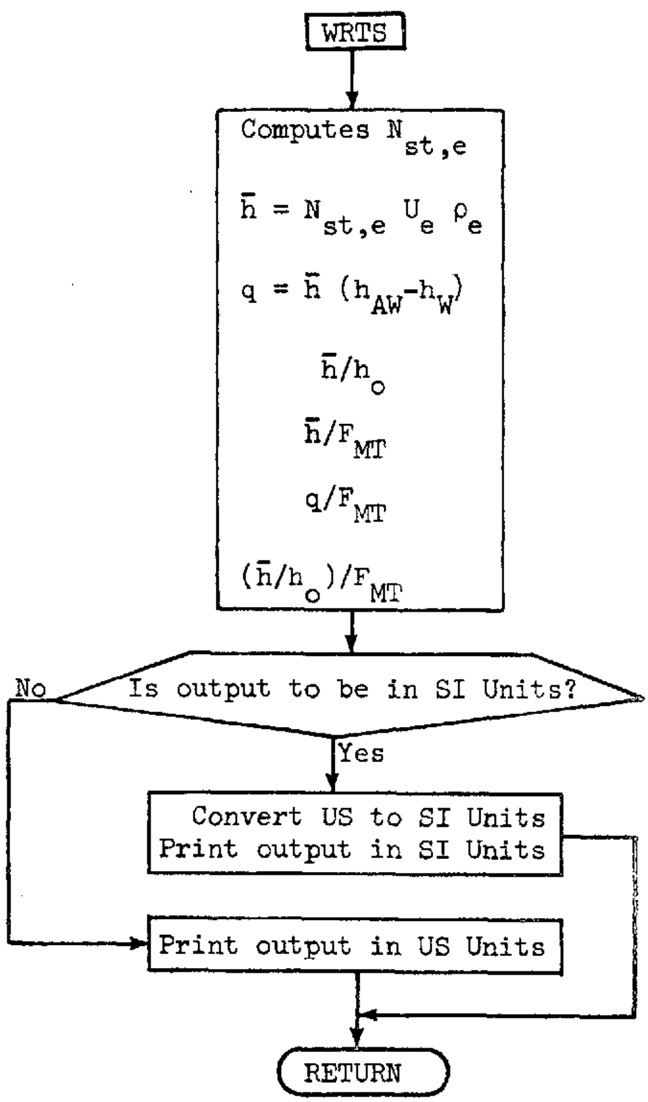
```
      SUBROUTINE VANDCF
C
C     COMPUTES HEATING RATES AND SKIN FRICTION BY
C       VAN DRIEST II (RETHETA) METHOD
C
      COMMON XLSH,XLSH1,THCR,JLIM,KLIM,TXLCL(450),TYL(450),TS(450),TP(45
     10),TXL(450),TYLS(450),TSS(450)
      COMMON ENX,FPOPT,ALX,RAF,ALMIN,COPT,STHCR,T1,T2,L,XVO,PSI
      COMMON CMTOPT
      COMMON ALGN,AN,ANZ,AP,AVAR,AW,AX,CEB,CEBP,CEM,CEMP,CFBMT,CFB2,CFER
     1R,CFI,CFMT,CF2,CF2I,CN,COE1,COE2,COE3,CTHCR,DEL,DELAM,DELS,DERR,DN
     2,DPVAR,DRBVAR,DRVAR,DSTH,DUVAR,E,EK,EL,ELT,ELVO,EMU,EMVAR,EMX,EN,E
     3NGN,ENI,ERR,FC,FCCF,FCCFLG,FCF,FCFPR,FD,FDPR,FRRE,FRTH,G,GC,GX,H,H
     4AW,HHAT,HP,HT,HVAR,HW,H2,ICELL,II,IN,JJ,JJLIM,JN,K,KK,KN,LLIM,NKW,
     5NN,NO,NXINT,PATM,PIVAR,PR,PRP,PRW,RBVAR,RET,RETH,RHOUI,RHOW,RO,ROP
     6,ROVAR,ROW,RRI,RRN,RRTVAR,RRX,RSERR,RSISAV,RSLVAR,RX,SHEAR,SP,SVAR
     7,SW,SX,TAW,TEMP,TEMP10,TEMP11,TEMP12,TEMP14,TEMP2,TEMP3,TEMP4,TEMP
     85,TEMP6,TEMP7,TEMP8,TEMP9,TEP,TIVAR,TK,TPP,TRAN,TT11,TW,TX,WVAR,XI
     9N,XMIN,X2REX,Z,ZMAX,ZMIN
      REAL IN,JN,KN
      COMMON F(3),A(3),ALPHE(3),XINT(99),TWT(99),ZTABL(6),TABIN(6),TABJN
     1(6),DXLTAB(20),DELK(2),RSLG(2),ALG(2),ENG(2),AN2(7)
      COMMON FRXT(20),FRXLGT(20),FCXT(20),FCXLGT(20),FRTB(20),FRLGT(20),
     1FCTB(20),FCLGT(20),XW(100),FIN6(100),RSLW(100),SHEEK(100),X(160),S
     2(160),PI(160),XC(160),RB(160),W(160),RRT(160),XMAXTB(20),DUDXT(160
     3),DRDXT(160),DPDXT(160),DRBDXT(160),VAR(2),DER(2),CUVAR(2),WSAV(16
     40),RRTSAV(160),XSAV(160),RSLSAV(160),REXSAV(160),ENSAV(160),CFSAV(
     5160),TKTAB(30),PRTAB(210),PATAB(7),XKW(160),WKW(160),RSLKW(160)
      COMMON /BLOCK/ PLT1(160),PLT2(160),PLT3(160),PLT4(160),PLT5(160),P
     1LT6(160),PLT7(160),PLT8(160),PLT9(160),PLT10(160),PLT11(160),PLT12
     2(160),PLT13(160),PLT14(160),PLT15(160),NKWSAV
      TERM=.176*EMVAR*EMVAR
      EMEW=SQRT(TIVAR/TW)*(1.+220.*10.**(-9./TW)/TW)/(1.+220.*10.**(-9./
     1TIVAR)/TIVAR)
      RBAR=RETH*EMEW
      RBRLG=ALOG10(RBAR)
      CFI2=.5/((17.08*RBRLG+25.11)*RBRLG+6.012)
      AVDSQ=TERM/COE1
      BVD=(1.+TERM-COE1)/COE1
      BVDSQ=BVD*BVD
      ALVD=(AVDSQ+AVDSQ-BVD)/SQRT(4.*AVDSQ+BVDSQ)
      BEVD=BVD/SQRT(4.*AVDSQ+BVDSQ)
```

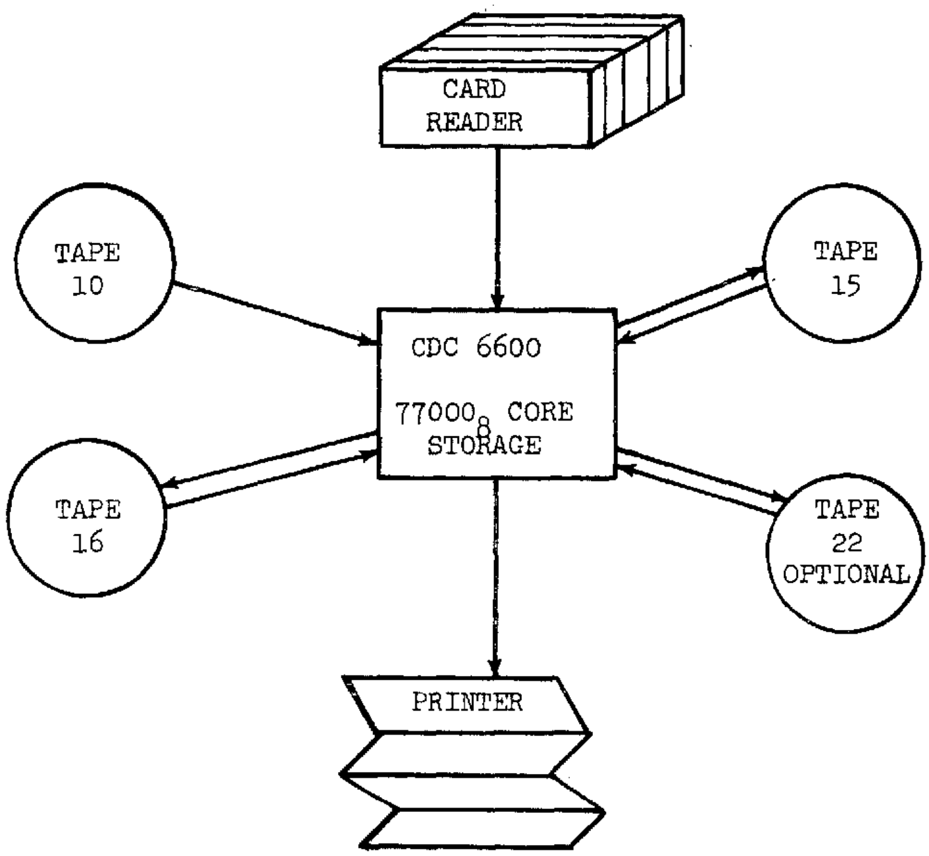
```
C
C     EQ(20)
C
      CF2FP=CFI2*((ASIN(ALVD)+ASIN(BEVD))**2)/TERM
C
C     EQ(23)
C
1     CF2=CF2FP*CFMT
      RETURN
C
      END
```

WRTS.- Subroutine WRTS computes heat transfer coefficient and heating rates based on each skin-friction theory and prints output.



```
      SUBROUTINE WRTS (CFX,RAFX,CFMTX)                                  WRT
C                                                                       WRT
C                 COMPUTES HEAT TRANSFER COEFFICIENT AND HEATING        WRT
C                 RATES BASED ON EACH SKIN FRICTION THEORY              WRT
C                                                                       WRT
      COMMON XLSH,XLSH1,THCR,JLIM,KLIM,TXLCL(450),TYL(450),TS(450),TP(45WRT
     10),TXL(450),TYLS(450),TSS(450)                                    WRT
      COMMON ENX,FPOPT,ALX,RAF,ALMIN,COPT,STHCR,T1,T2,L,XVO,PSI         WRT
      COMMON CMTOPT                                                     WRT
      COMMON ALGN,AN,ANZ,AP,AVAR,AW,AX,CEB,CEBP,CEM,CEMP,CFBMT,CFB2,CFERWRT
     1R,CFI,CFMT,CF2,CF2I,CN,COE1,COE2,COE3,CTHCR,DEL,DELAM,DELS,DERR,DNWRT
     2,DPVAR,DRBVAR,DRVAR,DSTH,DUVAR,E,EK,EL,ELT,ELVO,EMU,EMVAR,EMX,EN,EWRT
     3NGN,ENI,ERR,FC,FCCF,FCCFLG,FCF,FCFPR,FD,FDPR,FRKE,FRTH,G,GC,GX,H,HWRT
     4AW,HHAT,HP,HT,HVAR,HW,H2,ICELL,II,IN,JJ,JJLIM,JN,K,KK,KN,LLIM,NKW,WRT
     5NN,NO,NXINT,PATM,PIVAR,PR,PRP,PRW,RBVAR,RET,RETH,RHOUI,RHOW,RO,ROPWRT
     6,ROVAR,RCW,RRI,RRN,RRTVAR,RRX,RSERR,RSISAV,RSLVAR,RX,SHEAR,SP,SVARWRT
     7,SW,SX,TAW,TEMP,TEMP10,TEMP11,TEMP12,TEMP14,TEMP2,TEMP3,TEMP4,TEMPWRT
     85,TEMP6,TEMP7,TEMP8,TEMP9,TEP,TIVAR,TK,TPP,TRAN,TT11,TW,TX,WVAR,XIWRT
     9N,XMIN,X2REX,Z,ZMAX,ZMIN                                          WRT
      REAL IN,JN,KN                                                     WRT
      COMMON F(3),A(3),ALPHE(3),XINT(99),TWT(99),ZTABL(6),TABIN(6),TABJNWRT
     1(6),DXLTAB(20),DELK(2),RSLG(2),ALG(2),ENG(2),AN2(7)               WRT
      COMMON FRXT(20),FRXLGT(20),FCXT(20),FCXLGT(20),FRTB(20),FRLGT(20),WRT
     1FCTB(20),FCLGT(20),XW(100),FIN6(100),RSLW(100),SHEER(100),X(160),SWRT
     2(160),PI(160),XC(160),RB(160),W(160),RRT(160),XMAXTB(20),DUDXT(160WRT
     3),ORDXT(160),DPDXT(160),DRBDXT(160),VAR(2),DER(2),CUVAR(2),WSAV(16WRT
     40),RRTSAV(160),XSAV(160),RSLSAV(160),REXSAV(160),ENSAV(160),CFSAV(WRT
     5160),TKTAB(30),PRTAB(210),PATAB(7),XKW(160),WKW(160),RSLKW(160),IGWRT
     6AS                                                                WRT
      COMMON XX1                                                        WRT
      COMMON ACFT(22),TRFXT(22),ACFLGT(22),TRLGT(22),REXCF,HO,XO        WRT
      COMMON HBCF,QXCF,HHCF                                             WRT
      COMMON /BLK/ CV1,CV2,CV3,CV4,CV5,CV6,CV7,CV8,CV9,CV10,CV11,CV12,CVWRT
     113,UNIN,UNIO                                                      WRT
      FXX=CFX*RAFX                                                      WRT
      HB=FXX*WVAR*RRTVAR                                                WRT
      QX=HB*(HAW-HW)/2.5036E4                                           WRT
      HH=HB/HO                                                          WRT
      HBCF=HB/CFMTX                                                     WRT
      QXCF=QX/CFMTX                                                     WRT
      HHCF=HH/CFMTX                                                     WRT
      IF (UNIO-1.) 1,1,2                                                WRT
```

```
1     O81=HB*CV5
      O82=QX*CV12
      O85=HBCF*CV5
      O86=QXCF*CV12
      WRITE (6,4) RAFX,CFMTX,CFX,EXX,O81,O82,HH,XX1,O85,O86,HHCF
      GO TO 3
2     WRITE (6,4) RAFX,CFMTX,CFX,EXX,HB,QX,HH,XX1,HBCF,QXCF,HHCF
3     RETURN
C
C
C
4     FORMAT (12E11.3)
      END
```




Machine Requirements for Turbulent Boundary Layer Program D3340

## USAGE

The turbulent boundary layer program D3340 is run on the Control Data 6000 series computer under the SCOPE 3.0 operating system. Minimum machine requirements are 77000 octal locations of core storage for a minimum of 160 body points.

### Computer Calculation Time

The time required for a given variable-entropy calculation for turbulent flow depends on (1) the number of iterative calculations from the start of transition to the end of the body; (2) the number of points on the body per iteration; (3) the number of points in the integration from the wall to the edge of the boundary layer; and (4) the error criterion for various iterative solutions used in the calculation. It usually took four iterations to find a reasonable engineering solution which had a change in velocity of less than 1 percent ($\frac{\Delta u_e}{u_e} < 0.01$) from one iteration to the next. For the flight calculation, the Runge-Kutta integration was limited to maximum step size of $\frac{\Delta x}{r_n} = 16.0$; however, if calculation time on the computer is extremely critical, the step size could possibly be increased with little effect on the final solution. The number of points required in the Gaussian quadrature for the calculation of $\delta/r_n$ and $\delta^*/r_n$ from equations (5) and (4), respectively, was determined from a comparison of the values of $\delta/r_n$ and $\delta^*/r_n$ calculated for various numbers of points used in the quadrature. It was determined that a minimum number of 20 points could be used in the integration through the boundary layer. The momentum integral equation (eq. (3)) is solved by a variable-step-size fifth-order Runge-Kutta numerical scheme which uses a maximum relative error criterion of 0.001 for the value of $\theta$ for one step of integration. The maximum percent error allowed in the iterative solution for the Spalding-Chi (ref. 7) $C_f/2$ was 0.4. When an iterative procedure had to be used in the determination of thermodynamic properties from the real-gas subroutine (see ref. 27), the maximum allowable relative error was 0.001. For real-gas turbulent-boundary-layer calculations made at altitudes from 18.29 km to 25.91 km (60 000 ft to 85 000 ft) at $M \approx 20$, the time per body station was approximately 2.6 seconds on the Control Data 6600 computer system based on a single iteration from the start of transition to the end of the body and using the relative error criteria.

## Input Description

This section describes input procedures for the turbulent boundary layer program. The preparation of input tapes and cards is discussed, including a description of various options available in the program. This is followed by sample input data for the test case.

### Preparation of Input Tapes

TAPE 10 - The flow properties $p_e$, $\rho_e$, $T_e$, $S_e$, $h_e$ and $a_e$ for a real gas in thermodynamic equilibrium are calculated by the computer subroutine (RGAS) described by Lomax and Inouye in reference 27. The subroutine RGAS requires use of TAPE10 containing information for nitrogen on file 1 (IGAS = 1) and information for air on file 2 (IGAS = 2). TAPE 10 was developed by NASA-Ames Research Center and contains information applicable to real equilibrium air ($T<27000^\circ R$). The information for thermally perfect gas mixtures was taken from NBS Circular 564. Subroutines ROLL and SERCH are used by subroutine RGAS to locate the information on TAPE 10.

If CARD = 0, tapes 16, 15, 22 are used for input of the inviscid flow field. If CARD ≠ 0, as in the case of cone flow, these tapes are not used and input is taken from NAMELIST $N2 cards.

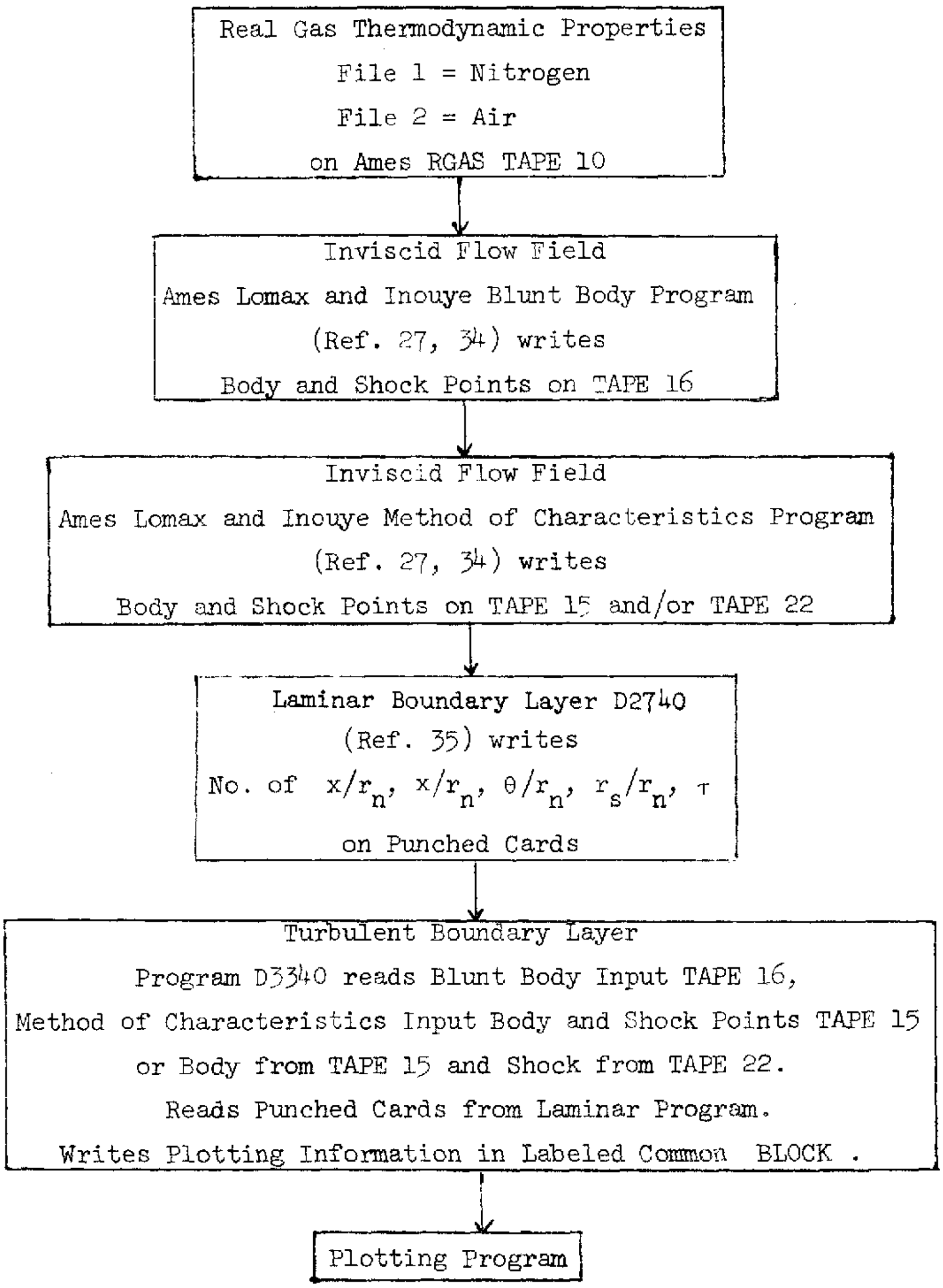
TAPE 15, 16, 22 - Prior to the turbulent boundary layer calculation the inviscid flow field is determined by the Lomax and Inouye blunt body and method of characteristics programs. (See refs. 27 and 34.) The inviscid solution gives the first-order stagnation entropy flow property distribution along the body which is used as the initial conditions at the edge of the boundary layer. In addition, the shock shape $r_s/r_n$ and entropy distribution along the shock are found from the inviscid solution. Subroutine CRRD reads the body and shock points of the inviscid flow field from tapes 15, 16, and 22.

TAPE 16 contains the body and shock points from the Lomax and Inouye blunt body program.

TAPES 15, 22 contain the body and shock points from the Lomax and Inouye method of characteristics program. TAPE 15 may be used for both body and shock points (XLSH = XLSH1) or TAPE 15 may be used for the body points and TAPE 22 for the shock points.

TAPES 16, 15, 22 contain $x/r_n$ centerline, $y/r_n$, stream velocity (ft/sec), stream angle (rad), Mach number, entropy ($ft^2/sec^2\ {}^\circ R$), pressure ($lbf/ft^2$), enthalpy ($ft^2/sec^2$), density ($slug/ft^3$), total pressure, written in (5E16.9) format with body points on odd numbered records and shock points on even numbered records. A maximum of 450 body points and 450 shock points may be read.

## Diagram of Linkage for Input Programs



## Preparation of NAMELIST N1 Input Cards

The FORTRAN NAMELIST capability is used for data input with $N1 as the NAMELIST name. The maximum allowable dimension appears following the variable name. Units are SI or US Customary depending on the option chosen for UNIN.

| | |
|---|---|
| $N1 | |
| XMIN | $x/r_n$ at beginning of transition |
| XVO | $x/r_n$ at virtual origin |
| X2REX | $x/r_n$ at end of transition |
| ELT | X2REX + .01 for print at end of transition |
| XMHL | $x/r_n$ at maximum heating |
| XLSH | maximum $x/r_n$ on shock (= 0 if flat plate) from tape 15 if body points from this MOC case |
| XLSH1 | maximum $x/r_n$ on shock (= 0 if flat plate) from tape 22 if shock points from this MOC case XLSH1 = XLSH if body and shock both from tape 15 |
| DXLTAB(20) | table of increments of $x/r_n$ for edge conditions |
| XMAXTB(20) | table of $x/r_n$ values where increments change, XMAXTB(20) is end of body. DXLTAB and XMAXTB should be chosen so that no. of $x/r_n$ points on the body will not exceed 160 |
| EL | $r_n$ nose radius (m) (inch) |
| RNOT | $r_n$ nose radius (m) (inch) |
| XINT(99) | values of X for wall temperature table (m) (inch) |
| TWT(99) | values of wall temperature ($^{o}$K) ($^{o}$R) |
| NXINT | no. of values in wall temperature table |
| THC | cone half angle (= 0 if flat plate) (rad) (deg) |

| | |
|---|---|
| DEL | initial $\delta/r_n$ boundary layer thickness from laminar flat plate input (= 0 if cone) |
| RØ | density parameter from laminar program ( = 0 if flat plate) (kg/m$^3$) (lbm/ft$^3$) |
| RHØI | density freestream (kg/m$^3$) (slug/ft$^3$) |
| RHØT2 | density stagnation point (kg/m$^3$) (slug/ft$^3$) |
| UI | velocity freestream (m/sec) (ft/sec) |
| RHØUI | density * velocity freestream (kg/m$^2$ sec) (lbm/ft$^2$ sec) |
| PT2 | pressure stagnation point (N/m$^2$) (lbf/ft$^2$) |
| PINF | pressure freestream (N/m$^2$) (lbf/ft$^2$) |
| HT | total enthalpy (m$^2$/sec$^2$) (ft$^2$/sec$^2$) |
| ST | total entropy (m$^2$/sec$^2$ °K) (ft$^2$/sec$^2$°R) |
| TT11 | total temperature (°K) (°R) |
| TWØ | wall temperature stagnation point (°K) (°R) |
| SHEAR | shear stress (if SHEAR = 0, interpolate from laminar cards) |
| A(3) | $A_R$ coefficients for enthalpy, Eq. (12), A(1) = A(3) = 0 |
| F(3) | $B_R$ coefficients for enthalpy, Eq. (12), F(3) = 1 |
| ALPHE(3) | $\alpha_R$ coefficients for enthalpy, Eq. (12), $\alpha(1) = \alpha(2) = 1$ |
| | A(2), F(1), F(2), ALPHE(3) are calculated in program |
| ALMIN | $\alpha(3)$ at beginning of transition, Eq. (12) |
| ALX | $\alpha(3)$ at end of transition, Eq. (12) |
| ENI | N at the beginning of transition, Eq. (26) |
| ENX | N at the end of transition, Eq. (26) |
| PSI | $\psi$ factor in $C_f/2$, Eq. (25) |
| TRAN | = 0, no transition region |
| | ≠ 0, transition region |

| | | |
|---|---|---|
| CARD | = 0, read input from TAPE 15, 16, 22 | omit if flat plate |
| | ≠ 0, read NAMELIST N2 | |
| FPØPT | = 0, cone | |
| | ≠ 0, flat plate | |
| UNIN | ≤ 1., input in SI units | |
| | > 1., input in US customary units | |
| UNIØ | ≤ 1., output in SI units | |
| | > 1., output in US customary units | |
| IGAS | = 1, nitrogen file from RGAS TAPE 10 | |
| | = 2, air file from RGAS TAPE 10 | |
| PRINT | print frequency | |
| ERR | relative error in T or H when iterating in RGAS | |
| UERR | relative error in velocity on successive iterations | |
| CFERR | relative error in Spalding-Chi skin friction | |
| L | = 5, no. of intervals in Gaussian integration | |
| NN | = 4, no. of points per interval for Gaussian integration | |
| NT | = 1, no. of values in ELT block for Runge Kutta | |
| CI | initial interval of $x/r_n$ for Runge Kutta | |
| SPEC | = 0, prints every interval in Runge Kutta | |
| CIMAX | maximum interval of $x/r_n$ for Runge Kutta | |
| ELE1 | relative error in $\theta/r_n$ | |
| | if error greater, Runge Kutta halves computing interval | |
| ELE2 | relative zero in $\theta/r_n$ | |
| | will not apply relative error if $\theta/r_n$ below this | |
| PR | = .72, Prandtl no. used in $T_{AW}$ | |

GC = 32.174, dimensional gravitational constant to convert from slug/ft$^3$ to lbm/ft$^3$

ZMIN = 31.623, Eq. (8), sets $N = 2$

ZMAX = 681.33, Eq. (8), sets $N = 10$

AN = .33333333, Eq. (8), $R_{e\theta}^{AN}$

CN = .5, Eq. (8), $\left(T_w/T_e\right)^{CN}$

DN = .25, Eq. (8), $M_e^{DN}$

KN = .33333333, Eq. (8), $\left(x_{v0}/r_n / \theta/r_n\right)^{KN}$

FRXT(20) $F_{RX}R_X$ from Spalding-Chi table 7

FCXT(20) $F_cC_f$ from Spalding-Chi table 7

FRTB(20) $R_{R\theta}R_{e\theta}$ from Spalding-Chi table 7

FCTB(20) $F_c\bar{C}_f$ from Spalding-Chi table 7

TKTAB(30) temperature °K from Hansen table VI

PATAB(7) pressure from Hansen table VI (N/m$^2$) (ATM)

PRTAB(210) Prandtl no. from Hansen table VI, starts with Prandtl no.'s for smallest pressure and increasing temperatures up to largest pressure and increasing temperatures.

ACFT(22) $C_f$ table to use with Van Driest skin friction

TREXT(22) transformed $R_{ex}$ table to use with Van Driest skin friction

TP pressure flat plate = PINF (N/m$^2$) (lbf/ft$^2$) ( = 0 if cone)

TS entropy flat plate = ST (m$^2$/sec$^2$ °K) (ft$^2$/sec$^2$°R) ( = 0 if cone)

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## Input from the Laminar Program

The results of the inviscid solution are first used to make a laminar boundary-layer calculation, with variable entropy, over the entire length of the body. (See ref. 35.) The initial use of the edge conditions from the variable entropy solution for laminar flow enables the turbulent calculation to be completed in less time than the initial use of the stagnation entropy edge conditions directly from the inviscid solution. The laminar boundary layer program D2740 punches on cards tables of $r_s/r_n$ and dimensionless shear as functions of $x/r_n$ which are read as input to the turbulent boundary layer program.

After the NAMELIST N1 cards are read, the turbulent program next reads the punched cards from the laminar program. These punched cards are of the following form:

| | | |
|---|---|---|
| card type 1 | FORMAT(I5) | no. of $x/r_n$ to be read, not exceeding 100 |
| card type 2 | FORMAT(5E15.8) | $x/r_n$ values, 5 per card |
| card type 3 | FORMAT(5E15.8) | $\theta/r_n$ values, 5 per card |

Card type 3 are read but not used in the present version of the turbulent program.

| | | |
|---|---|---|
| card type 4 | FORMAT(5E15.8) | $r_s/r_n$ values, 5 per card |
| card type 5 | FORMAT(5E15.8) | dimensionless shear, 5 per card |

Card type 5 are not read if SHEAR $\neq$ 0 in NAMELIST N1 .

## Preparation of NAMELIST N2 Input Cards

Following the input from the laminar program, data input with $N2 as the NAMELIST name may be read if CARD $\neq$ 0 in NAMELIST N1 and inviscid flow field tapes are omitted.

$N2

| | |
|---|---|
| DELTX | increment of $x/r_n$ |
| TP | pressure $(N/m^2)$ $(lbf/ft^2)$ |
| TS | entropy $(m^2/sec^2\ °K)$ $(ft^2/sec^2\ °R)$ |
| THSD | shock angle (rad) (deg) |

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## Input for Sample Calculation

The sample case chosen to illustrate the use of the program is the performance of turbulent boundary layer flight calculations at an altitude of 60,000 ft. and flight conditions of approximately 19,000 ft./sec. for a 13 ft. long, 5° half angle cone with a nose radius of .4 inch.

The corresponding input data cards are listed here and sample pages of the output data are shown in the ensuing section to illustrate the printout formats. The input and output for the sample case are in SI units.

## Sample Input (S I Units)

```
$N1
XMIN=15.,XVO=10.,X2REX=223.,ELT=223.01,XMHL=213.,XLSH=283.,XLSH1=283.,
DXLTAB=.1,.1,.2,.2,.5,.5,1.,1.,5.,5.,10.,10.,8*25.,
XMAXTB=15.,16.,16.,20.,20.,35.,35.,65.,65.,115.,115.,215.,8*390.,
XINT=.36576,.747014,.975868,1.280922,1.465072,1.81483,2.119884,2.475992,
2.729992,3.035046,3.340608,3.620008,
TWT=635.55556,562.77778,568.88889,577.77778,600.55556,698.88889,748.33333,
788.33333,802.22222,819.44444,838.88889,827.77778,
UNIN=1.,UNIO=1.,
EL=1.016E-2,RHOUI=710.3932045,RO=2.755175636,RHOI=1.225468186E-1,
UI=.579674736E4,HT=1.701240468E7,ST=11.2592608553,THC=8.72664626E-2,
RHOT2=1.414612279,RNOT=1.016E-2,PT2=3.94432777456,PINF=.736972931 1E4,
TWO=.1666666667E4,TT11=.0735E5,
NXINT=12,DEL=0,
          SHEAR=6.76136E-6,
A=0,0,0,F=0,0,1.,ALPHE=1.,1.,0,
ALMIN=1.26,ALX=1.,ENI=1.54,ENX=7.,PSI=2.,
TRAN=1.,CARD=0,FPOPT=0,                    1GAS=2,PRINT=1.,
ERR=.001,UERR=.001,CFERR=.004,L=5,NN=4,NT=1,CI=.0125,SPEC=0,CIMAX=16.,
ELE1=.001,ELE2=.001,PR=.72,GC=32.174,ZMIN=31.623,ZMAX=681.33,AN=.33333333,
CN=.5,DN=.25,KN=.33333333,
     FRXT  =5.826E3,6.883E3,9.253E3,1.006E4,1.251E4,1.592E4,2.078E4,
     2.796E4,3.901E4,5.679E4,8.697E4,1.417E5,2.492E5,4.828E5,1.062E6,
     2.778E6,9.340E6,4.651E7,4.610E8,5.758E10,
     FCXT  =.0105,.01,.0095,.009,.0085,.008,.0075,.007,.0065,.006,.0055,
     .005,.0045,.004,.0035,.003,.0025,.002,.0015,.001,
     FRTB=50.46,55.87,62.55,70.91,92.49,95.62,114.4,140.4,177.6,233.0,319.4,
     462.3,716.0,1208,2282,5030,1.386E4,5.425E4,3.955E5,2.878E7,
     FCTB=.01732,.01624,.01516,.01409,.01304,.012016,.011014,.010042,.009105,
     .008205,.007345,.006526,.005747,.005006,.004299,.003621,.002967,.002333,
     .001716,.001117,
TKTAB=500,1000,1500,2000,2500,3000,3500,4000,4500,5000,5500,6000,6500,
7000,7500,8000,8500,9000,9500,10000,10500,11000,11500,12000,12500,13000,
13500,14000,14500,15000,
PATAB=1.01325E1,1.01325E2,1.01325E3,1.01325E4,1.01325E5,1.01325E6,1.01325E7,
PRTAB=.738,.756,.767,.614,.771,.714,.606,.587,.764,.993,.871,.384,.348,
.337,.330,.316,.276,.1987,.1140,.0577,.0312,.0207,.0157,.0132,.0120,
.0115,.0109,.0109,.0109,.0109,.738,.756,.767,.668,.654,.745,.658,.580,
.611,.799,.989,.891,.383,.346,.334,.328,.321,.307,.273,.210,.1427,.0870,
.0503,.0321,.0213,.0166,.0142,.0130,.0119,.0114,.738,.756,.767,.724,
.611,.740,.737,.619,.578,.624,.785,.969,.955,.830,.350,.332,.324,.320,
.316,.313,.284,.246,.1945,.1409,.0949,.0634,.0416,.0293,.0202,.0119,
.738,.756,.767,.766,.645,.636,.744,.759,.610,.581,.617,.736,.906,.986,
.969,.648,.335,.321,.314,.310,.309,.303,.293,.276,.250,.215,.1733,
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.1338,.0903,.0719,.738,.756,.767,.773,.696,.627,.660,.762,.752,.611,
.583,.602,.673,.796,.927,.983,.943,.807,.330,.308,.301,.296,.295,.293,
.290,.284,.276,.263,.237,.220,.738,.756,.767,.773,.751,.680,.631,.662,
.743,.767,.620,.592,.592,.620,.688,.788,.891,.961,.966,.872,.310,.294,
.284,.277,.272,.272,.270,.269,.265,.263,.738,.756,.767,.773,.762,.740,
.678,.640,.654,.702,.748,.763,.610,.593,.595,.620,.666,.730,.806,.886,
.937,.955,.947,.908,.728,.275,.251,.245,.241,.238,
      ACFT=.02,.0175,.015,.0125,.01,.009,.008,.007,.006,.0055,.005,.0045,
      .004,.0035,.003,.0025,.00225,.002,.0019,.0018,.0017,.0016,
      TREXT=2.5714E+3,3.8575E+3,6.307E+3,1.1684E+4,2.6303E+4,3.9506E+4,
      6.3468E+4,1.1152E+5,2.2185E+5,3.3324E+5,5.2896E+5,9.0014E+5,
      1.6761E+6,3.5195E+6,8.7332E+6,2.7673E+7,5.6185E+7,1.2889E+8,
      1.8754E+8,2.8101E+8,4.354E+8,7.0126E+8,
TP=0,TS=0,
$
  52
0.              1.00000000E-01 2.00000000E-01 3.00000000E-01 4.00000000E-01
5.00000000E-01 6.00000000E-01 7.00000000E-01 8.00000000E-01 9.00000000E-01
1.00000000E+00 1.10000000E+00 1.20000000E+00 1.30000000E+00 1.40000000E+00
1.50000000E+00 1.60000000E+00 1.70000000E+00 1.80000000E+00 1.90000000E+00
2.00000000E+00 3.00000000E+00 4.00000000E+00 5.00000000E+00 6.00000000E+00
7.00000000E+00 8.00000000E+00 9.00000000E+00 1.00000000E+01 1.50000000E+01
2.00000000E+01 3.00000000E+01 3.73000000E+01 4.00000000E+01 5.00000000E+01
6.00000000E+01 6.78000000E+01 7.00000000E+01 8.00000000E+01 9.04000000E+01
1.00000000E+02 1.20500000E+02 1.38700000E+02 1.73200000E+02 2.03400000E+02
2.38500000E+02 2.63600000E+02 2.93700000E+02 3.00000000E+02 3.23800000E+02
3.51400000E+02 4.00000000E+02
7.74874187E-04 5.65139131E-04 6.10217578E-04 6.17637067E-04 6.52955756E-04
6.97554929E-04 7.59994135E-04 8.37431679E-04 9.42183903E-04 1.08141354E-03
1.26337507E-03 1.49549067E-03 1.81369757E-03 2.25208282E-03 2.87283048E-03
3.80062763E-03 4.59650658E-03 4.84963409E-03 5.05258637E-03 5.27283967E-03
5.49100524E-03 7.31011937E-03 8.73563712E-03 9.91140259E-03 1.10563112E-02
1.21445645E-02 1.31402231E-02 1.40646459E-02 1.48882199E-02 1.69085027E-02
1.94744337E-02 2.30178717E-02 2.43947375E-02 2.46461261E-02 2.50719485E-02
2.46896983E-02 2.41801884E-02 2.42371613E-02 2.42959237E-02 2.40106092E-02
2.39135596E-02 2.30470534E-02 2.20729627E-02 2.02254653E-02 1.88707917E-02
1.77369142E-02 1.70865639E-02 1.64681056E-02 1.63862858E-02 1.59776292E-02
1.55013646E-02 1.44399776E-02
0.              2.44281545E-02 4.78320812E-02 7.04198673E-02 9.21476258E-02
1.10728983E-01 1.29227740E-01 1.46239728E-01 1.61342162E-01 1.74350769E-01
1.85500919E-01 1.91767598E-01 1.99401108E-01 2.05451976E-01 2.06631376E-01
2.07587764E-01 2.06965292E-01 2.05167090E-01 2.07415444E-01 2.09306340E-01
2.11040809E-01 2.21331610E-01 2.33851832E-01 2.41151621E-01 2.52215348E-01
2.57139493E-01 2.66479662E-01 2.74888141E-01 2.83022579E-01 3.24824888E-01
```

```
3.68180137E-01 4.46402282E-01 4.99952742E-01 5.19899268E-01 5.93882270E-01
6.69018134E-01 7.28881417E-01 7.45840068E-01 8.21595024E-01 8.97335729E-01
9.64180061E-01 1.10351691E+00 1.22138486E+00 1.42816025E+00 1.59729447E+00
1.78499250E+00 1.91500818E+00 2.06595084E+00 2.09697462E+00 2.21468145E+00
2.39837410E+00 2.59003632E+00
```

## Output Description

In this section the formats for printing the results of the machine computations are described. The results of the sample calculation, for which the input data were described, are used to illustrate the printout procedure.

### NAMELIST N1 Output

For each computational case or run, the program lists the NAMELIST N1 input data. The program listing of the input data for the sample calculation is shown in Figure 1. The format for this printout is evident from the figure.

### Inviscid Solution Output

If CARD ≠ 0, the program lists the NAMELIST N2 input data. If CARD = 0, the program lists the information read from tapes 15, 22, 16. This information is printed on two lines for each body or shock point of the method of characteristics and blunt body solutions as shown in Figure 2.

The first line contains

| | |
|---|---|
| X | $x/r_n$ measured on centerline |
| Y | $y/r_n$ |
| Q | stream velocity (ft/sec) |
| THETA | stream angle (rad) |
| M | mach number |

The second line contains

| | |
|---|---|
| S | entropy ($ft^2/sec^2\ ^oR$) |
| P | pressure ($lbf/ft^2$) |

H — enthalpy ($ft^2/sec^2$)

RHØ — density ($slug/ft^3$)

PT — total pressure (not used in present version of program)

Body and shock points of the method of characteristics solution alternate until $x/r_n$ of shock point from TAPE 15 reaches XLSH and if XLSH ≠ XLSH1 $x/r_n$ of shock point from TAPE 22 reaches XLSH1. This is followed by all the body and shock points read from the blunt body solution on TAPE 16. Of these items of information only $x/r_n$, $y/r_n$, S, and P of body points and $x/r_n$, $y/r_n$, S of shock points will be used in the turbulent program.

Tables of body points and shock points are listed as shown in Figure 3. The number of points in these tables may not exceed 450. The information given for body points is

XCL — $x/r_n$ measured on centerline

Y — $y/r_n$

S — entropy ($ft^2/sec^2$ °R)

P — pressure ($lbf/ft^2$)

X — $x/r_n$ measured on surface

The information given for shock points is

YS — $y/r_n$ shock radius

S — entropy ($ft^2/sec^2$ °R)

At specified increments of $x/r_n$ on the body (XMAXTB, DXLTAB) linear interpolation in the tables shown in Figure 3 is performed and the results are listed as shown in Figure 4. The number of points in these tables may not exceed 160. The information given is

| | | |
|---|---|---|
| X | $x/r_n$ | |
| S | entropy $(ft^2/sec^2\ °R)$ | |
| P | pressure | |
| RB | $r_b/r_n$ | |
| XC | $x/r_n$ measured on centerline | |
| T | temperature (°R) | |
| H | enthalpy $(ft^2/sec^2)$ | |
| U | velocity (ft/sec) | |
| A | speed of sound (ft/sec) | |
| M | mach number | |
| RHØ | density $(lbm/ft^3)$ | |
| X | $x/r_n$ | |
| DU/DX | $du/d\frac{x}{r_n}$ | velocity derivative |
| DRHØ/DX | $d\rho/d\frac{x}{r_n}$ | density derivative |
| DP/DX | $dp/d\frac{x}{r_n}$ | pressure derivative |
| DRB/DX | $d\frac{r_b}{r_n} \Big/ d\frac{x}{r_n}$ | body radius derivative |

All the output shown in Figs. 2, 3, 4 is in U. S. customary units.

## Stagnation Point Output

The information for the stagnation point is listed as shown in Figure 5.

| | | |
|---|---|---|
| HØ | heat transfer $(kg/m^2\ sec)$ $(lbm/ft^2\ sec)$ | |

QØ heating rate $(W/m^2)$ $(Btu/ft^2 \ sec)$

These may be output in either SI units (UNIØ = 1.) or U. S. customary units (UNIØ = 2.).

## Output Each Iteration

The turbulent boundary layer calculation is initiated at the point where transition has been determined to start (XMIN) and continues to the end of the body (XMAXTB(20)). Integration of the momentum equation is performed by a variable step-size fifth-order Runge Kutta numerical scheme. After each step in the integration the heating rates are calculated. The procedure is repeated until the velocity at the edge of the boundary layer varies from one iteration to the next by less than the allowable difference (UERR). The output may be in either SI units (UNIØ = 1.) or U. S. customary units (UNIØ = 2.).

## Heating Rates Output

Heat transfer has been calculated using several different skin friction theories. There may be one, four, or eight lines of information printed as shown in Figure 6. If one line is printed, the theory used is Van Driest ($R_{e\theta}$). If four lines are printed, the theories used are Van Driest ($R_{e\theta}$), Eckert's ($R_{e\theta}$), Spalding Chi ($R_{e\theta}$, ideal gas), Spalding Chi ($R_{e\theta}$, real gas). If eight lines are printed, the theories used are Van Driest ($R_{e\theta}$), Van Driest ($R_{ex}$), Spalding Chi ($R_{ex}$, ideal gas), Eckert's ($R_{ex}$), Eckert's ($R_{ex_{min}}$), Eckert's ($R_{e\theta}$), Spalding Chi ($R_{e\theta}$, ideal gas), Spalding Chi ($R_{e\theta}$, real gas).

The values printed on a line for each skin friction theory are:

| | |
|---|---|
| RAF | Reynolds analogy factor |
| CFMT | Mangler transformation factor |
| CF2 | skin friction coefficient/2 |
| ENST | Stanton number |
| HBAR | local heat transfer coefficient (kg/m$^2$ sec) (lbm/ft$^2$ sec) |
| Q | heating rate (W/m$^2$) (Btu/ft$^2$ sec) |
| HH | local heat/stagnation heat |
| XX1 | ($Y/2 \tanh \psi$) term from Eq. (25) |
| HBCF | local heat transfer coefficient/Mangler transformation (kg/m$^2$ sec) (lbm/ft$^2$ sec) |
| QXCF | heating rate/Mangler transformation (W/m$^2$) (Btu/ft$^2$ sec) |
| HHCF | (local heat/stagnation heat)/Mangler transformation |

## Output at Each Integration Step

In addition to the heating rate information seven lines are printed at each step of the integration as shown in Figure 6. The values printed on line one are:

| | |
|---|---|
| X/L | distance/nose radius |
| X | distance (m) (inch) |
| THETA/L | momentum thickness/nose radius |
| THETA | momentum thickness (m) (inch) |

| | |
|---|---|
| P | pressure $(N/m^2)$ $(lbf/ft^2)$ |
| RB | body radius/nose radius |
| S | entropy $(m^2/sec^2\ {}^\circ K)$ $(ft^2/sec^2\ {}^\circ R)$ |
| S/ST | entropy/total entropy |
| T | temperature $({}^\circ K)$ $({}^\circ R)$ |
| H | enthalpy $(m^2/sec^2)$ $(ft^2/sec^2)$ |
| H/HT | enthalpy/total enthalpy |
| RHØ | density $(kg/m^3)$ $(lbm/ft^3)$ |

The values printed on line two are:

| | |
|---|---|
| RHØ/RHØI | density/freestream density |
| U | velocity (m/sec) (ft/sec) |
| U/SQRT2HT | velocity/(2*total enthalpy)$^{1/2}$ |
| U/UI | velocity/freestream velocity |
| A | speed of sound (m/sec) (ft/sec) |
| M | mach number |
| MU | viscosity $(N\ sec/m^2)$ (lbm/ft sec) |
| RETH | Reynolds number based on momentum thickness |
| REX | Reynolds number based on distance |
| TW | wall temperature $({}^\circ K)$ $({}^\circ R)$ |
| HW | wall enthalpy $(m^2/sec^2)$ $(ft^2/sec^2)$ |
| HW/HT | wall enthalpy/total enthalpy |

The values printed on line three are

| | |
|---|---|
| RHØW | wall density $(kg/m^3)$ $(lbm/ft^3)$ |
| TW/T | wall temperature/temperature |
| TAW | adiabatic wall temperature $({}^\circ K)$ $({}^\circ R)$ |

| | | |
|---|---|---|
| 1 | FRTH | $F_{R\theta}$ |
| 2 | FC | $F_c$, Eq. (22) |
| 3 | CFMT | Mangler transformation factor |
| 4 | LVØ | distance from virtual origin |
| 5 | Z | z, Eq. (8) |
| 6 | CFI | initial skin friction |
| 7 | CF2 | skin friction/2 |
| 8 | HAW | adiabatic wall enthalpy $(m^2/sec^2)$ $(ft^2/sec^2)$ |
| 9 | HAW/HT | adiabatic wall enthalpy/total enthalpy |
| 10 | The values printed on line four are: | |
| 11 | HP | Eckert's reference enthalpy $(m^2/sec^2)$ $(ft^2/sec^2)$ |
| 12 | TP | Eckert's reference temperature (°K) (°R) |
| 13 | PRP | Eckert's reference Prandtl number |
| 14 | PRW | wall Prandtl number |
| 15 | F(1) | $B_I$, Eq. (10) |
| 16 | F(2) | $B_{II}$, Eq. (10) |
| 17 | A(2) | $A_{II}$, Eq. (10) |
| 18 | ALPH(3) | $\alpha_{III}$, Eq. (10) |
| 19 | N | N exponent in velocity-profile relation |
| 20<br>21 | INT1 | $\int_0^1 \frac{\rho u}{\rho_e u_e} - \frac{\rho u^2}{\rho_e u_e^2} \, d\frac{y}{\delta}$ |
| 22<br>23 | INT2 | $\int_0^1 \frac{\rho u}{\rho_e u_e} \frac{y}{\delta}\left(\cos \theta_c \Big/ r_b/r_n\right) d\frac{y}{\delta}$ |
| 24<br>25 | INT3 | $\int_0^1 1 - \frac{\rho u}{\rho_e u_e} \, d\frac{y}{\delta}$ |

The values printed on line five are:

| | |
|---|---|
| INT4 | $\int_0^1 \left(1 - \frac{\rho u}{\rho_e u_e}\right) \frac{y}{\delta} \left(\cos \theta_c / r_b / r_n\right) d \frac{y}{\delta}$ |
| INT5 | $\int_0^1 \left(\frac{\rho u}{\rho_e u_e} - \frac{\rho u^2}{\rho_e u_e^2}\right) \frac{y}{\delta} \left(\cos \theta_c / r_b / r_n\right) d \frac{y}{\delta}$ |
| INT6 | $\int_0^1 \frac{\rho u}{\rho_e u_e} \, d \frac{y}{\delta}$ |
| INT7 | $\int_0^1 \frac{\rho u^2}{\rho_e u_e^2} \, d \frac{y}{\delta}$ |
| DEL/L | boundary layer thickness/nose radius |
| DEL | boundary layer thickness |
| RSL | shock radius/nose radius |
| DELS/L | displacement thickness/nose radius |
| DELS | displacement thickness |
| X/DELS | distance/displacement thickness |
| DSTH | displacement thickness/momentum thickness |
| DU | $d\, u_e / d \frac{x}{r_n}$ |

The values printed on line six are:

| | | | |
|---|---|---|---|
| DRHØ | $d\, \rho_e / d \frac{x}{r_n}$ | $(kg/m^3)$ | $(lbm/ft^3)$ |
| DP | $d\, P_e / d \frac{x}{r_n}$ | $(N/m^2)$ | $(lbf/ft^2)$ |
| DRB | $d \frac{r_b}{r_n} \Big/ d \frac{x}{r_n}$ | | |
| DTHETA | $d \frac{\theta}{r_n} \Big/ d \frac{x}{r_n}$ | | |

NST — Stanton number

HBAR — local heat transfer coefficient by Van Driest ($R_{e\theta}$) ($kg/m^2$ sec) ($lbm/ft^2$ sec)

Q — heating rate/stagnation heating rate (Q/QO)

PRE — Prandtl number

EKF — Karman factor

QIN — incompressible heating rate ($W/m^2$) ($Btu/ft^2$ sec)

QKF — Karman factor heating rate ($W/m^2$) ($Btu/ft^2$ sec)

QXTR — heating rate from transition ($W/m^2$) ($Btu/ft^2$ sec)

The values printed on line seven are:

QXMH — heating rate from maximum heating ($W/m^2$) ($Btu/ft^2$ sec)

XC — distance on centerline/nose radius

RHØUSQ — $\rho u^2$ ($N/m^2$) ($lbf/ft^2$)

TAU — shear stress ($N/m^2$) ($lbf/ft^2$)

## Output at End of Transition

In each iteration, whenever $x/r_n$ is within .0001 of ELT a velocity profile is printed. The information printed for all points in the Gaussian integration is:

Y/D — $y/r_n \,/\, \delta/r_n$

U/UE — velocity ratio

H — enthalpy ($m^2/sec^2$) ($ft^2/sec^2$)

RHØ/RHØE — density ratio

RHØU/RHØEUE — density * velocity ratio

| | |
|---|---|
| HBAR | $A_R + B_R(u/u_e)^{\alpha R}$ |
| T | temperature (°K) (°R) |
| M | Mach number |

## Output at End of Iteration

At the end of each iteration the maximum relative error between the velocities at the edge of the boundary layer of the present iteration and the previous iteration is printed (TØLL).

At the end of each iteration two additional lines of information are printed as shown in Figure 7. The values printed on line one are:

| | |
|---|---|
| REX2 | (omit in present version of program) |
| X2REX | $(x/r_n)_{turb}$, distance at end of transition |
| EN2REX | $N_{turb}$, exponent in velocity profile relation at end of transition |
| CEM | $N_{turb} - N_{tr}$ |
| CEB | $2\psi / \left[(x/r_n)_{turb} - (x/r_n)_{tr}\right]$ |
| CF2REX | $(C_f/2)_{turb}$, skin friction coefficient/2 at end of transition |
| CEMP | $(C_f/2)_{turb} - (C_f/2)_{tr}$ |
| CEBP | $\tanh \psi$ |
| XMIN | $(x/r_n)_{tr}$, distance at start of transition |
| RSISAV | (omit in present version of program) |
| ENI | $N_{tr}$, exponent in velocity profile relation at start of transition |

CF2I $(C_f/2)_{tr}$, skin friction coefficient/2 at start of transition

The value printed on line two is:

REXSAV $R_{e,x}$, local Reynolds number based on surface distance at start of transition

## Final Iteration Plotting Output

If plotting is desired, the information in labeled common BLOCK may be called for in the user's own plotting routine. This block holds all information from the final iteration. The block contains:

| | | |
|---|---|---|
| PLT1 | X | distance (inch) |
| PLT2 | N | exponent in velocity profile relation |
| PLT3 | RETH | Reynolds no. based on momentum thickness |
| PLT4 | CF2 | skin friction coefficient/2 |
| PLT5 | M | mach number |
| PLT6 | DEL | boundary layer thickness (inch) |
| PLT7 | DELS | displacement thickness (inch) |
| PLT8 | THETA | momentum thickness (inch) |
| PLT9 | H/HT | enthalpy/total enthalpy |
| PLT10 | REX | Reynolds no. based on distance |
| PLT11 | S/ST | entropy/total entropy |
| PLT12 | U/UI | velocity/velocity freestream |
| PLT13 | RHØ/RHØI | density/density freestream |
| PLT14 | Q/QØ | heating rate |

PLT15 RSL shock radius/nose radius

NKWSAV number of X stations (maximum of 160)

$N1

Figure 1

```
XMIN    =  0.15E+02,

DXLTAB  =  0.1E+00,  0.1E+00,  0.2E+00,  0.2E+00,  0.5E+00,  0.5E+00,  0.1E+01,
           0.1E+01,  0.5E+01,  0.5E+01,  0.1E+02,  0.1E+02,  0.25E+02,
           0.25E+02,  0.25E+02,  0.25E+02,  0.25E+02,  0.25E+02,  0.25E+02,
           0.25E+02,

XMAXTB  =  0.15E+02,  0.16E+02,  0.16E+02,  0.2E+02,  0.2E+02,  0.35E+02,
           0.35E+02,  0.65E+02,  0.65E+02,  0.115E+03,  0.115E+03,  0.215E+03,
           0.39E+03,  0.39E+03,  0.39E+03,  0.39E+03,  0.39E+03,  0.39E+03,
           0.39E+03,  0.39E+03,

ELT     =  0.22301E+03,

XLSH    =  0.283E+03,

XLSH1   =  0.283E+03,

XMHL    =  0.213E+03,

EL      =  0.1016E-01,

X2REX   =  0.223E+03,

RHOUI   =  0.7103932045E+03,

SHEAR   =  0.676136E-05,

RO      =  0.2755175636E+01,

RHOI    =  0.1225468186E+00,

UI      =  0.579674736E+04,

HT      =  0.1701240468E+08,

ST      =  0.1125026085E+05,

TT11    =  0.735E+04,

DEL     =  0.0,

THC     =  0.872664626E-01,

PR      =  0.72E+00,

GC      =  0.32174E+02,
```

```
TRAN    =  0.1E+01,

CARD    =  0.0,

PRINT   =  0.1E+01,

A       =  0.0,  0.0,  0.0,

E       =  0.0,  0.0,  0.1E+01,

ALPHE   =  0.1E+01,  0.1E+01,  0.0,

ZMIN    =  0.31623E+02,

ZMAX    =  0.68133E+03,

AN      =  0.33333333E+00,

CN      =  0.5E+00,

DN      =  0.25E+00,

KN      =  0.33333333E+00,

FRXT    =  0.5826E+04,  0.6883E+04,  0.8253E+04,  0.1006E+05,  0.1251E+05,
           0.1592E+05,  0.2078E+05,  0.2796E+05,  0.3901E+05,  0.5679E+05,
           0.8697E+05,  0.1417E+06,  0.2492E+06,  0.4828E+06,  0.1062E+07,
           0.2778E+07,  0.934E+07,  0.4651E+08,  0.461E+09,  0.5758E+11,

FCXT    =  0.105E-01,  0.1E-01,  0.95E-02,  0.9E-02,  0.85E-02,  0.8E-02,
           0.75E-02,  0.7E-02,  0.65E-02,  0.6E-02,  0.55E-02,  0.5E-02,
           0.45E-02,  0.4E-02,  0.35E-02,  0.3E-02,  0.25E-02,  0.2E-02,
           0.15E-02,  0.1E-02,

FRTB    =  0.5046E+02,  0.5587E+02,  0.6255E+02,  0.7091E+02,  0.9249E+02,
           0.9562E+02,  0.1144E+03,  0.1404E+03,  0.1776E+03,  0.233E+03,
           0.3194E+03,  0.4623E+03,  0.716E+03,  0.1208E+04,  0.2283E+04,
           0.503E+04,  0.1386E+05,  0.5425E+05,  0.3955E+06,  0.2878E+08,

FCTB    =  0.1732E-01,  0.1624E-01,  0.1516E-01,  0.1409E-01,  0.1304E-01,
           0.12016E-01,  0.11014E-01,  0.10042E-01,  0.9105E-02,  0.8205E-02,
           0.7345E-02,  0.6526E-02,  0.5747E-02,  0.5006E-02,  0.4299E-02,
           0.3621E-02,  0.2967E-02,  0.2333E-02,  0.1716E-02,  0.1117E-02,

TKTAB   =  0.5E+03,  0.1E+04,  0.15E+04,  0.2E+04,  0.25E+04,  0.3E+04,
           0.35E+04,  0.4E+04,  0.45E+04,  0.5E+04,  0.55E+04,  0.6E+04,
           0.65E+04,  0.7E+04,  0.75E+04,  0.8E+04,  0.85E+04,  0.9E+04,
           0.95E+04,  0.1E+05,  0.105E+05,  0.11E+05,  0.115E+05,  0.12E+05,
           0.125E+05,  0.13E+05,  0.135E+05,  0.14E+05,  0.145E+05,  0.15E+05,

PATAB   =  0.101325E+02,  0.101325E+03,  0.101325E+04,  0.101325E+05,
```

```
          0.101325E+06,  0.101325E+07,  0.101325E+08,

PRTAB   = 0.738E+00,  0.756E+00,  0.767E+00,  0.614E+00,  0.771E+00,
          0.714E+00,  0.606E+00,  0.587E+00,  0.764E+00,  0.993E+00,
          0.871E+00,  0.384E+00,  0.348E+00,  0.337E+00,  0.33E+00,
          0.316E+00,  0.276E+00,  0.1987E+00,  0.114E+00,  0.577E-01,
          0.312E-01,  0.207E-01,  0.157E-01,  0.132E-01,  0.12E-01,
          0.115E-01,  0.109E-01,  0.109E-01,  0.109E-01,  0.109E-01,
          0.738E+00,  0.756E+00,  0.767E+00,  0.668E+00,  0.654E+00,
          0.745E+00,  0.658E+00,  0.58E+00,  0.611E+00,  0.799E+00,
          0.989E+00,  0.891E+00,  0.383E+00,  0.346E+00,  0.334E+00,
          0.328E+00,  0.321E+00,  0.307E+00,  0.273E+00,  0.21E+00,
          0.1427E+00,  0.87E-01,  0.503E-01,  0.321E-01,  0.213E-01,
          0.166E-01,  0.142E-01,  0.13E-01,  0.119E-01,  0.114E-01,
          0.738E+00,  0.756E+00,  0.767E+00,  0.724E+00,  0.611E+00,
          0.74E+00,  0.737E+00,  0.619E+00,  0.578E+00,  0.624E+00,
          0.785E+00,  0.969E+00,  0.955E+00,  0.83E+00,  0.35E+00,  0.332E+00,
          0.324E+00,  0.32E+00,  0.316E+00,  0.313E+00,  0.284E+00,
          0.246E+00,  0.1945E+00,  0.1409E+00,  0.949E-01,  0.634E-01,
          0.416E-01,  0.293E-01,  0.202E-01,  0.119E-01,  0.738E+00,
          0.756E+00,  0.767E+00,  0.766E+00,  0.645E+00,  0.636E+00,
          0.744E+00,  0.759E+00,  0.61E+00,  0.581E+00,  0.617E+00,
          0.736E+00,  0.906E+00,  0.986E+00,  0.969E+00,  0.648E+00,
          0.335E+00,  0.321E+00,  0.314E+00,  0.31E+00,  0.309E+00,
          0.303E+00,  0.293E+00,  0.276E+00,  0.25E+00,  0.215E+00,
          0.1733E+00,  0.1338E+00,  0.903E-01,  0.719E-01,  0.738E+00,
          0.756E+00,  0.767E+00,  0.773E+00,  0.696E+00,  0.627E+00,
          0.66E+00,  0.762E+00,  0.752E+00,  0.611E+00,  0.583E+00,
          0.602E+00,  0.673E+00,  0.796E+00,  0.927E+00,  0.983E+00,
          0.943E+00,  0.807E+00,  0.33E+00,  0.308E+00,  0.301E+00,
          0.296E+00,  0.295E+00,  0.293E+00,  0.29E+00,  0.284E+00,
          0.276E+00,  0.263E+00,  0.237E+00,  0.22E+00,  0.738E+00,
          0.756E+00,  0.767E+00,  0.773E+00,  0.751E+00,  0.68E+00,
          0.631E+00,  0.662E+00,  0.743E+00,  0.767E+00,  0.62E+00,
          0.592E+00,  0.592E+00,  0.62E+00,  0.688E+00,  0.788E+00,
          0.891E+00,  0.961E+00,  0.966E+00,  0.872E+00,  0.31E+00,
          0.294E+00,  0.284E+00,  0.277E+00,  0.272E+00,  0.272E+00,
          0.27E+00,  0.269E+00,  0.265E+00,  0.263E+00,  0.738E+00,
          0.756E+00,  0.767E+00,  0.773E+00,  0.762E+00,  0.74E+00,
          0.678E+00,  0.64E+00,  0.654E+00,  0.702E+00,  0.748E+00,
          0.763E+00,  0.61E+00,  0.593E+00,  0.595E+00,  0.62E+00,  0.666E+00,
          0.73E+00,  0.806E+00,  0.886E+00,  0.937E+00,  0.955E+00,
          0.947E+00,  0.908E+00,  0.728E+00,  0.275E+00,  0.251E+00,
          0.245E+00,  0.241E+00,  0.238E+00,

NXINT   = 12,

XINT    = 0.36576E+00,  0.747014E+00,  0.975868E+00,  0.1280922E+01,
          0.1465077E+01,  0.181483E+01,  0.2119884E+01,  0.2475992E+01,
          0.2729992E+01,  0.3035046E+01,  0.3340608E+01,  0.3620008E+01,
```

```
TWT      =  0.63555556E+03,  0.56277778E+03,  0.56888889E+03,  0.57777778E+03,
            0.60055556E+03,  0.69888889E+03,  0.74833333E+03,  0.78833333E+03,
            0.80222222E+03,  0.81944444E+03,  0.83888889E+03,  0.82777778E+03,


L        =  5,

NN       =  4,

NT       =  1,

CI       =  0.125E-01,

SPEC     =  0.0,

CIMAX    =  0.16E+02,

ELF1     =  0.1E-02,

ELF2     =  0.1E-02,

FRR      =  0.1E-02,

UEPR     =  0.1E-02,

CFFPR    =  0.4E-02,

XVO      =  0.1E+02,

PSI      =  0.2E+01,

FNI      =  0.154E+01,

ALMIN    =  0.126E+01,

ALX      =  0.1E+01,

FPOPT    =  0.0,

TP       =  0.0,
```

```
TS       =  0.0,

ENX      =  0.7E+01,
RHOT2    =  0.1414612279E+01,
RNOT     =  0.1016E-01,
PT2      =  0.3944327774E+07,
PINF     =  0.7369729311E+04,
TWO      =  0.1666666667E+04,
IGAS     =  2,
```

```
ACFT     =  0.2E-01,  0.175E-01,  0.15E-01,  0.125E-01,  0.1E-01,  0.9E-02,
            0.8E-02,  0.7E-02,  0.6E-02,  0.55E-02,  0.5E-02,  0.45E-02,
            0.4E-02,  0.35E-02,  0.3E-02,  0.25E-02,  0.225E-02,  0.2E-02,
            0.19E-02,  0.18E-02,  0.17E-02,  0.16E-02,

TREXT    =  0.25714E+04,  0.38575E+04,  0.63075E+04,  0.11684E+05,  0.26303E+05,
            0.39506E+05,  0.63468E+05,  0.11152E+06,  0.22185E+06,  0.33324E+06,
            0.52896E+06,  0.90014E+06,  0.16761E+07,  0.35195E+07,  0.87332E+07,
            0.27673E+08,  0.56185E+08,  0.12889E+09,  0.18754E+09,  0.28101E+09,
            0.4354E+09,  0.70126E+09,

UNIN     =  0.1E+01,

UNIO     =  0.1E+01,

$END
```

Figure 2

BODY+SHOCK PTS

| X | Y | Q | THETA | M |
|---|---|---|---|---|
| S | P | H | RHO | PT |
| 2.350939232E-01 | 6.441418273E-01 | 5.954901872E+03 | 8.767613229E-01 | 1.049999554E+00 |
| 6.642657948E+04 | 4.437314223E+04 | 1.662669132E+08 | 1.619327190E-03 | 0. |
| 2.279971312E-01 | 6.500534248E-01 | 6.332075186E+03 | 8.679162273E-01 | 1.120991082E+00 |
| 6.620885998E+04 | 4.489474022E+04 | 1.639497359E+08 | 1.653346467E-03 | 0. |
| 2.373942435E-01 | 6.468635561E-01 | 5.954596207E+03 | 8.673318994E-01 | 1.052433683E+00 |
| 6.642657948E+04 | 4.307798325E+04 | 1.653881849E+08 | 1.579747821E-03 | 8.009311401E+04 |
| 2.214822293E-01 | 6.554803102E-01 | 6.686270488E+03 | 8.611436674E-01 | 1.188470421E+00 |
| 6.599363039E+04 | 4.540485199E+04 | 1.616648377E+08 | 1.687554989E-03 | 0. |
| 2.404488724E-01 | 6.504476032E-01 | 5.976382038E+03 | 8.626227390E-01 | 1.056666045E+00 |
| 6.642657948E+04 | 4.287087365E+04 | 1.652582217E+08 | 1.573318208E-03 | 8.009311401E+04 |
| 2.154718238E-01 | 6.604869524E-01 | 7.020471474E+03 | 8.558793623E-01 | 1.252970121E+00 |
| 6.577916222E+04 | 4.590722703E+04 | 1.593937141E+08 | 1.722232315E-03 | 0. |
| 2.439180499E-01 | 6.544769550E-01 | 6.003453024E+03 | 8.573056970E-01 | 1.061997743E+00 |
| 6.642657948E+04 | 4.250707843E+04 | 1.650960688E+08 | 1.561575743E-03 | 8.009311400E+04 |
| 2.098895729E-01 | 6.651369436E-01 | 7.337298558E+03 | 8.516774494E-01 | 1.314936374E+00 |
| 6.556517087E+04 | 4.640386127E+04 | 1.571342015E+08 | 1.757495314E-03 | 0. |
| 2.477014425E-01 | 6.588223436E-01 | 6.048674568E+03 | 8.515440520E-01 | 1.070810443E+00 |
| 6.642657948E+04 | 4.207747553E+04 | 1.648235609E+08 | 1.548202512E-03 | 8.009311400E+04 |
| 2.046816801E-01 | 6.694750961E-01 | 7.638437330E+03 | 8.482877610E-01 | 1.374641639E+00 |
| 6.535193062E+04 | 4.689511543E+04 | 1.548897253E+08 | 1.793363177E-03 | 0. |
| 2.517016128E-01 | 6.633622869E-01 | 6.097979528E+03 | 8.454932208E-01 | 1.080441570E+00 |
| 6.642657948E+04 | 4.160948977E+04 | 1.645241157E+08 | 1.533608768E-03 | 8.009311401E+04 |
| 1.999038734E-01 | 6.735382882E-01 | 7.925335375E+03 | 8.455068675E-01 | 1.432333859E+00 |
| 6.513933586E+04 | 4.738123330E+04 | 1.526582961E+08 | 1.829895321E-03 | 0. |
| 2.558596891E-01 | 6.680233511E-01 | 6.149720477E+03 | 8.392470023E-01 | 1.090574203E+00 |
| 6.642657948E+04 | 4.111894555E+04 | 1.642072619E+08 | 1.518282480E-03 | 8.009311401E+04 |
| 1.952110754E-01 | 6.773640694E-01 | 8.199614766E+03 | 8.431620506E-01 | 1.488310276E+00 |
| 6.492690394E+04 | 4.786233707E+04 | 1.504340909E+08 | 1.867186783E-03 | 0. |
| 2.601415062E-01 | 6.727625206E-01 | 6.202888511E+03 | 8.328599980E-01 | 1.101013972E+00 |
| 6.642657948E+04 | 4.061558855E+04 | 1.638788799E+08 | 1.502524565E-03 | 8.009311401E+04 |
| 1.908788331E-01 | 6.809728087E-01 | 8.462134734E+03 | 8.411773470E-01 | 1.542723228E+00 |
| 6.471453052E+04 | 4.833851778E+04 | 1.482159090E+08 | 1.905300957E-03 | 0. |
| 2.645150471E-01 | 6.775410571E-01 | 6.257126759E+03 | 8.263821575E-01 | 1.111693075E+00 |

Figure 3

| XCL | Y | S | P | X |
|---|---|---|---|---|
| 0. | 0. | 6.63771505E+04 | 8.23788913E+04 | 0. |
| 3.05617151E-04 | 2.38022945E-02 | 6.63768433E+04 | 8.23217647E+04 | 2.47237829E-02 |
| 1.15244400E-03 | 4.76045890E-02 | 6.63759072E+04 | 8.21504048E+04 | 4.80138611E-02 |
| 2.56585963E-03 | 7.14068835E-02 | 6.63743315E+04 | 8.18648376E+04 | 7.16513455E-02 |
| 4.54886810E-03 | 9.52091780E-02 | 6.63721301E+04 | 8.14651308E+04 | 9.54182476E-02 |
| 7.10554445E-03 | 1.19011472E-01 | 6.63693186E+04 | 8.09514154E+04 | 1.19280973E-01 |
| 1.02409998E-02 | 1.42813767E-01 | 6.63658954E+04 | 8.03239180E+04 | 1.43237756E-01 |
| 1.39614534E-02 | 1.66616062E-01 | 6.63618328E+04 | 7.95829928E+04 | 1.67296515E-01 |
| 1.82743294E-02 | 1.90418356E-01 | 6.63570640E+04 | 7.87291525E+04 | 1.91469375E-01 |

| | | | | |
|---|---|---|---|---|
| 3.46786637E+02 | 3.12562876E+01 | 6.64265795E+04 | 8.73135277E+02 | 3.48678504E+02 |
| 3.48493199E+02 | 3.14055925E+01 | 6.64265795E+04 | 8.73121593E+02 | 3.50391584E+02 |
| 3.50163579E+02 | 3.15517319E+01 | 6.64265795E+04 | 8.73108332E+02 | 3.52068345E+02 |
| 3.52330323E+02 | 3.17412976E+01 | 6.64265795E+04 | 8.73095413E+02 | 3.54243366E+02 |
| 3.54336586E+02 | 3.19168229E+01 | 6.64265795E+04 | 8.73086818E+02 | 3.56257292E+02 |
| 3.56840722E+02 | 3.21359066E+01 | 6.64265795E+04 | 8.73079509E+02 | 3.58770994E+02 |
| 3.59173091E+02 | 3.23399624E+01 | 6.64265795E+04 | 8.73074999E+02 | 3.61112272E+02 |

| YS | S |
|---|---|
| 0. | 6.63550253E+04 |
| 4.76045890E-02 | 6.63365946E+04 |
| 9.52091780E-02 | 6.62813295E+04 |
| 1.42813767E-01 | 6.61892724E+04 |
| 1.90418356E-01 | 6.60605121E+04 |
| 2.38022945E-01 | 6.58952153E+04 |
| 2.85627534E-01 | 6.56936475E+04 |
| 3.33232123E-01 | 6.54561844E+04 |
| 3.80836712E-01 | 6.51833386E+04 |
| 4.28441301E-01 | 6.48758421E+04 |
| 4.76045890E-01 | 6.45327507E+04 |
| 5.23650479E-01 | 6.41584196E+04 |
| 5.71255068E-01 | 6.37554612E+04 |
| 6.18859657E-01 | 6.33284298E+04 |
| 6.66464246E-01 | 6.28735615E+04 |
| 7.10765960E-01 | 6.24235728E+04 |
| 7.17789191E-01 | 6.23499531E+04 |
| 7.24930764E-01 | 6.22746193E+04 |
| 7.32145333E-01 | 6.21976852E+04 |
| 7.39455866E-01 | 6.21190523E+04 |
| 7.46848360E-01 | 6.20387883E+04 |
| 7.54322201E-01 | 6.19569352E+04 |

Figure 4

| X | S | P | RB | XC | T | H | U | A | M | RHO |
|---|---|---|---|---|---|---|---|---|---|---|
| 1.500E+01 | 6.643E+04 | 8.373E+02 | 2.174E+00 | 1.438E+01 | 7.110E+03 | 8.515E+07 | 1.400E+04 | 4.185E+03 | 3.345E+00 | 1.875E-03 |
| 1.510E+01 | 6.643E+04 | 8.340E+02 | 2.183E+00 | 1.448E+01 | 7.106E+03 | 8.509E+07 | 1.400E+04 | 4.183E+03 | 3.347E+00 | 1.869E-03 |
| 1.520E+01 | 6.643E+04 | 8.310E+02 | 2.192E+00 | 1.458E+01 | 7.101E+03 | 8.504E+07 | 1.401E+04 | 4.181E+03 | 3.349E+00 | 1.863E-03 |
| 1.530E+01 | 6.643E+04 | 8.280E+02 | 2.200E+00 | 1.468E+01 | 7.097E+03 | 8.499E+07 | 1.401E+04 | 4.180E+03 | 3.352E+00 | 1.858E-03 |
| 1.540E+01 | 6.643E+04 | 8.250E+02 | 2.209E+00 | 1.478E+01 | 7.093E+03 | 8.494E+07 | 1.401E+04 | 4.178E+03 | 3.354E+00 | 1.852E-03 |
| 1.550E+01 | 6.643E+04 | 8.219E+02 | 2.218E+00 | 1.488E+01 | 7.089E+03 | 8.489E+07 | 1.402E+04 | 4.177E+03 | 3.356E+00 | 1.847E-03 |
| 1.560E+01 | 6.643E+04 | 8.189E+02 | 2.227E+00 | 1.498E+01 | 7.085E+03 | 8.483E+07 | 1.402E+04 | 4.175E+03 | 3.358E+00 | 1.841E-03 |
| 1.951E+02 | 6.643E+04 | 8.866E+02 | 1.787E+01 | 1.938E+02 | 7.177E+03 | 8.598E+07 | 1.394E+04 | 4.209E+03 | 3.311E+00 | 1.965E-03 |
| 2.051E+02 | 6.643E+04 | 8.846E+02 | 1.874E+01 | 2.039E+02 | 7.174E+03 | 8.595E+07 | 1.394E+04 | 4.208E+03 | 3.313E+00 | 1.961E-03 |
| 2.151E+02 | 6.643E+04 | 8.825E+02 | 1.961E+01 | 2.137E+02 | 7.171E+03 | 8.592E+07 | 1.394E+04 | 4.207E+03 | 3.314E+00 | 1.957E-03 |
| 2.251E+02 | 6.643E+04 | 8.907E+02 | 2.049E+01 | 2.237E+02 | 7.169E+03 | 8.568E+07 | 1.395E+04 | 4.207E+03 | 3.315E+00 | 1.954E-03 |
| 2.360E+02 | 6.643E+04 | 8.791E+02 | 2.143E+01 | 2.345E+02 | 7.167E+03 | 8.586E+07 | 1.395E+04 | 4.206E+03 | 3.316E+00 | 1.951E-03 |
| 2.478E+02 | 6.643E+04 | 9.777E+02 | 2.246E+01 | 2.463E+02 | 7.165E+03 | 8.583E+07 | 1.395E+04 | 4.205E+03 | 3.317E+00 | 1.948E-03 |
| 2.606E+02 | 6.643E+04 | 8.764E+02 | 2.358E+01 | 2.590E+02 | 7.163E+03 | 8.581E+07 | 1.395E+04 | 4.205E+03 | 3.318E+00 | 1.946E-03 |
| 2.745E+02 | 6.643E+04 | 8.754E+02 | 2.479E+01 | 2.729E+02 | 7.162E+03 | 8.580E+07 | 1.395E+04 | 4.204E+03 | 3.319E+00 | 1.944E-03 |
| 2.896E+02 | 6.643E+04 | 8.747E+02 | 2.611E+01 | 2.879E+02 | 7.161E+03 | 8.579E+07 | 1.395E+04 | 4.204E+03 | 3.319E+00 | 1.943E-03 |
| 3.060E+02 | 6.643E+04 | 8.740E+02 | 2.753E+01 | 3.043E+02 | 7.160E+03 | 8.577E+07 | 1.395E+04 | 4.203E+03 | 3.320E+00 | 1.942E-03 |
| 3.239E+02 | 6.643E+04 | 8.735E+02 | 2.909E+01 | 3.220E+02 | 7.159E+03 | 8.577E+07 | 1.395E+04 | 4.203E+03 | 3.320E+00 | 1.941E-03 |
| 3.431E+02 | 6.643E+04 | 8.732E+02 | 3.077E+01 | 3.412E+02 | 7.159E+03 | 8.576E+07 | 1.395E+04 | 4.203E+03 | 3.320E+00 | 1.940E-03 |
| 3.641E+02 | 6.643E+04 | 8.731E+02 | 3.260E+01 | 3.621E+02 | 7.159E+03 | 8.576E+07 | 1.395E+04 | 4.203E+03 | 3.320E+00 | 1.940E-03 |
| 3.869E+02 | 6.643E+04 | 8.731E+02 | 3.458E+01 | 3.848E+02 | 7.159E+03 | 8.576E+07 | 1.395E+04 | 4.203E+03 | 3.320E+00 | 1.940E-03 |
| 4.116E+02 | 6.643E+04 | 8.731E+02 | 3.674E+01 | 4.095E+02 | 7.159E+03 | 8.576E+07 | 1.395E+04 | 4.203E+03 | 3.320E+00 | 1.940E-03 |

| X | DU/DX | DRHO/DX | DP/DX | DRB/DX |
|---|---|---|---|---|
| 1.500E+01 | 4.644E+01 | -6.855E-05 | -3.755E+01 | 9.715E-02 |
| 1.510E+01 | 3.800E+01 | -5.603E-05 | -3.067E+01 | 8.716E-02 |
| 1.520E+01 | 3.732E+01 | -5.498E-05 | -3.007E+01 | 8.716E-02 |
| 1.530E+01 | 3.758E+01 | -5.530E-05 | -3.022E+01 | 8.716E-02 |
| 1.540E+01 | 3.759E+01 | -5.524E-05 | -3.016E+01 | 9.716E-02 |
| 1.550E+01 | 3.768E+01 | -5.533E-05 | -3.019E+01 | 8.716E-02 |
| 1.560E+01 | 3.781E+01 | -5.546E-05 | -3.023E+01 | 9.716E-02 |
| 1.570E+01 | 3.526E+01 | -5.167E-05 | -2.814E+01 | 8.716E-02 |
| 1.590E+01 | 3.321E+01 | -4.862E-05 | -2.646E+01 | 8.716E-02 |
| 1.590E+01 | 3.374E+01 | -4.935E-05 | -2.683E+01 | 8.716E-02 |
| 1.600E+01 | 3.375E+01 | -4.930E-05 | -2.679E+01 | 8.716E-02 |
| 1.610E+01 | 3.380E+01 | -4.933E-05 | -2.678E+01 | 8.716E-02 |
| 1.630E+01 | 3.194E+01 | -4.653E-05 | -2.522E+01 | 8.716E-02 |
| 1.650E+01 | 2.985E+01 | -4.340E-05 | -2.349E+01 | 8.716E-02 |

Figure 5

HO=   6.131E+00 OO=   9.285E+07

Figure 6

SKIN FRICTION THEORY
(1) VAN DRIEST RETHETA
(2) VAN DRIEST REX OMIT IN TRANS REGION
(3) SPALDING CHI REX OMIT IN TRANS REGION
(4) ECKERTS REF ENTHALPY REX OMIT IN TRANS REGION
(5) ECKERTS REF ENTHALPY REXMIN OMIT IN TRANS REGION
(6) ECKERTS RETHETA
(7) SPALDING CHI I RETHETA IDEAL FC
(8) SPALDING CHI II RETHETA REAL FC

| RAF | CFMT | CF2 | ENST | HBAR | Q | HH | XX1 | HBCF | QXCF | HHCF | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| X/L | X | THETA/L | THETA | P | RB | S | S/ST | T | H | H/HT | RHO |
| RHO/RHOI | U | U/SORT2HT | U/UI | A | M | MU | RETH | REX | TW | HW | HW/HT |
| RHOW | TW/T | TAW | FRTH | FC | CFMT | LVO | Z | CFI | CF2 | HAW | HAW/HT |
| HP | TP | PRP | PRW | F(1) | F(2) | A(2) | ALPH(3) | N | INT1 | INT2 | INT3 |
| INT4 | INT5 | INT6 | INT7 | DEL/L | DEL | RSL | DELS/L | DELS | X/DELS | DSTH | DU |
| ORHD | OP | ORB | OTHETA | NST | HBAR | O | PRE | EKF | OIN | OKF | OXTR |
| OXMH | XC | RHOUSO | TAU | | | | | | | | |
| 1.000E+00 | 0. | 1.017E-03 | 1.017E-03 | 1.439E-01 | 2.194E+06 | 2.348E-02 | 0. | 0. | 0. | 0. | |
| 1.500E+01 | 1.524E-01 | 3.831E-02 | 3.893E-04 | 4.009E+04 | 2.174E+00 | 1.095E+04 | 9.736E-01 | 3.753E+03 | 7.316E+06 | 4.300E-01 | 3.214E-02 |
| 2.623E-01 | 4.404E+03 | 7.550E-01 | 7.597E-01 | 1.223E+03 | 3.601E+00 | 8.683E-05 | 0. | 0. | 6.763E+02 | 6.944E+05 | 4.082E-02 |
| 2.046E-01 | 1.802E-01 | 6.977E+03 | 2.018E+01 | 0. | 0. | 0. | 0. | 0. | 1.017E-03 | 1.595E+07 | 9.373E-01 |
| 5.904E+06 | 3.380E+03 | 6.964E-01 | 7.443E-01 | 8.855E-01 | 5.122E-01 | 3.733E-03 | 1.260E+00 | 1.540E+00 | 2.270E-01 | 1.869E-01 | 2.950E-01 |
| 4.227E-02 | 3.976E-02 | 7.050E-01 | 4.780E-01 | 1.655E-01 | 1.601E-03 | 3.248E-01 | 4.942E-02 | 5.021E-04 | 3.035E+02 | 1.290E+00 | 1.415E+01 |
| -1.098E-03 | -1.798E+03 | 8.715E-02 | 4.395E-04 | 1.017E-03 | 1.439E-01 | 2.363E-02 | 7.385E-01 | 1.000E+00 | 2.194E+06 | 2.194E+06 | 1.135E+04 |
| 1.135E+04 | 1.438E+01 | 6.233E+05 | 6.338E+02 | | | | | | | | |
| 1.000E+00 | 1.176E+00 | 2.576E-03 | 2.576E-03 | 3.645E-01 | 5.555E+06 | 5.945E-02 | 0. | 3.099E-01 | 4.724E+06 | 5.055E-02 | |
| 1.273E+00 | 1.176E+00 | 3.058E-03 | 3.892E-03 | 5.507E-01 | 8.394E+06 | 8.983E-02 | 0. | 4.683E-01 | 7.138E+06 | 7.638E-02 | |
| 1.273E+00 | 1.668E+00 | 2.441E-03 | 3.107E-03 | 4.396E-01 | 6.700E+06 | 7.170E-02 | 0. | 2.635E-01 | 4.016E+06 | 4.298E-02 | |
| 1.273E+00 | 1.668E+00 | 3.321E-03 | 4.227E-03 | 5.980E-01 | 9.115E+06 | 9.754E-02 | 0. | 3.585E-01 | 5.464E+06 | 5.847E-02 | |
| 1.501E+01 | 1.525E-01 | 3.834E-02 | 3.895E-04 | 4.007E+04 | 2.175E+00 | 1.095E+04 | 9.736E-01 | 3.753E+03 | 7.315E+06 | 4.300E-01 | 3.213E-02 |
| 2.622E-01 | 4.404E+03 | 7.550E-01 | 7.597E-01 | 1.223E+03 | 3.601E+00 | 8.682E-05 | 6.347E+02 | 2.486E+05 | 6.763E+02 | 6.943E+05 | 4.081E-02 |
| 2.046E-01 | 1.802E-01 | 6.977E+03 | 2.018E+01 | 9.633E-01 | 1.176E+00 | 0. | 0. | 0. | 2.576E-03 | 1.595E+07 | 9.373E-01 |
| 5.903E+06 | 3.380E+03 | 6.964E-01 | 7.443E-01 | 8.855E-01 | 5.122E-01 | 3.732E-03 | 1.260E+00 | 1.540E+00 | 2.269E-01 | 1.868E-01 | 2.950E-01 |
| 4.220E-02 | 3.973E-02 | 7.050E-01 | 4.780E-01 | 1.656E-01 | 1.683E-03 | 3.250E-01 | 4.946E-02 | 5.025E-04 | 3.035E+02 | 1.290E+00 | 1.383E+01 |
| -1.073E-03 | -1.757E+03 | 8.715E-02 | 1.978E-03 | 2.576E-03 | 3.645E-01 | 5.983E-02 | 7.384E-01 | 1.000E+00 | 5.555E+06 | 5.555E+06 | 1.135E+04 |
| 1.135E+04 | 1.439E+01 | 6.231E+05 | 1.605E+03 | | | | | | | | |
| 1.000E+00 | 1.176E+00 | 2.575E-03 | 2.575E-03 | 3.641E-01 | 5.550E+06 | 5.939E-02 | 0. | 3.096E-01 | 4.719E+06 | 5.050E-02 | |
| 1.273E+00 | 1.176E+00 | 3.057E-03 | 3.892E-03 | 5.502E-01 | 8.387E+06 | 8.975E-02 | 0. | 4.679E-01 | 7.131E+06 | 7.632E-02 | |
| 1.273E+00 | 1.668E+00 | 2.440E-03 | 3.106E-03 | 4.391E-01 | 6.693E+06 | 7.162E-02 | 0. | 2.632E-01 | 4.012E+06 | 4.294E-02 | |
| 1.273E+00 | 1.668E+00 | 3.320E-03 | 4.225E-03 | 5.974E-01 | 9.106E+06 | 9.744E-02 | 0. | 3.582E-01 | 5.459E+06 | 5.842E-02 | |

Figure 6 (cont.)

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1.894E+02 | 1.924E+00 | 1.132E-01 | 1.150E-03 | 4.249E+04 | 1.737E+01 | 9.006E+03 | 8.005E-01 | 1.659E+03 | 1.849E+06 | 1.086E-01 | 8.851E-02 |
| 7.223E-01 | 5.507E+03 | 9.441E-01 | 9.500E-01 | 7.896E+02 | 6.975E+00 | 5.568E-05 | 1.007E+04 | 1.684E+07 | 7.166E+02 | 7.377E+05 | 4.337E-02 |
| 2.047E-01 | 4.321E-01 | 6.760E+03 | 1.019E+01 | 1.935E+00 | 1.176E+00 | 0. | 0. | 0. | 7.349E-04 | 1.534E+07 | 9.020E-01 |
| 4.262E+06 | 2.957E+03 | 6.354E-01 | 7.458E-01 | 9.056E-01 | 9.080E-01 | -2.413E-05 | 1.042E+00 | 6.118E+00 | 6.305E-02 | 2.050E-02 | 3.966E-01 |
| 8.168E-03 | 1.235E-03 | 6.034E-01 | 5.404E-01 | 1.742E+00 | 1.770E-02 | 5.152E+00 | 7.015E-01 | 7.127E-03 | 2.700E+02 | 6.196E+00 | 2.321E-02 |
| -1.873E-06 | -3.095E+00 | 8.716E-02 | 1.706E-04 | 7.348E-04 | 3.582E-01 | 5.631E-02 | 7.675E-01 | 1.000E+00 | 5.228E+06 | 5.228E+06 | 4.030E+06 |
| 1.135E+04 | 1.881E+02 | 2.684E+06 | 1.972E+03 | | | | | | | | |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 1.000E+00 | 1.176E+00 | 6.703E-04 | 6.703E-04 | 3.578E-01 | 5.206E+06 | 5.837E-02 | 0. | 3.043E-01 | 4.427E+06 | 4.963E-02 |
| 1.000E+00 | 1.176E+00 | 8.862E-04 | 8.862E-04 | 4.731E-01 | 6.883E+06 | 7.717E-02 | 0. | 4.023E-01 | 5.853E+06 | 6.562E-02 |
| 1.352E+00 | 1.333E+00 | 7.962E-04 | 1.076E-03 | 5.747E-01 | 8.361E+06 | 9.373E-02 | 0. | 4.310E-01 | 6.270E+06 | 7.030E-02 |
| 1.352E+00 | 1.176E+00 | 2.237E-03 | 3.024E-03 | 1.615E+00 | 2.349E+07 | 2.633E-01 | 0. | 1.373E+00 | 1.997E+07 | 2.239E-01 |
| 1.352E+00 | 1.176E+00 | 1.520E-03 | 2.054E-03 | 1.097E+00 | 1.596E+07 | 1.789E-01 | 0. | 9.326E-01 | 1.357E+07 | 1.521E-01 |
| 1.352E+00 | 1.176E+00 | 8.279E-04 | 1.119E-03 | 5.976E-01 | 8.694E+06 | 9.747E-02 | 0. | 5.081E-01 | 7.393E+06 | 8.288E-02 |
| 1.352E+00 | 1.333E+00 | 5.755E-04 | 7.780E-04 | 4.153E-01 | 6.043E+06 | 6.774E-02 | 0. | 3.115E-01 | 4.532E+06 | 5.081E-02 |
| 1.352E+00 | 1.333E+00 | 7.112E-04 | 9.615E-04 | 5.133E-01 | 7.468E+06 | 8.372E-02 | 0. | 3.850E-01 | 5.601E+06 | 6.279E-02 |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 2.054E+02 | 2.087E+00 | 1.158E-01 | 1.176E-03 | 4.235E+04 | 1.877E+01 | 8.894E+03 | 7.905E-01 | 1.517E+03 | 1.673E+06 | 9.832E-02 | 9.639E-02 |
| 7.865E-01 | 5.539E+03 | 9.496E-01 | 9.555E-01 | 7.580E+02 | 7.300E+00 | 5.295E-05 | 1.186E+04 | 2.104E+07 | 7.430E+02 | 7.667E+05 | 4.507E-02 |
| 1.967E-01 | 4.896E-01 | 6.745E+03 | 9.063E+00 | 2.088E+00 | 1.176E+00 | 0. | 0. | 0. | 6.703E-04 | 1.533E+07 | 9.009E-01 |
| 4.223E+06 | 2.944E+03 | 6.362E-01 | 7.467E-01 | 9.047E-01 | 9.557E-01 | -5.100E-04 | 1.022E+00 | 6.538E+00 | 5.680E-02 | 1.857E-02 | 4.145E-01 |
| 7.975E-03 | 1.035E-03 | 5.855E-01 | 5.237E-01 | 1.973E+00 | 2.305E-02 | 5.885E+00 | 8.308E-01 | 8.441E-03 | 2.472E+02 | 7.176E+00 | 7.691E-02 |
| -6.200E-06 | -1.024E+01 | 8.716E-02 | 1.459E-04 | 6.703E-04 | 3.578E-01 | 5.607E-02 | 7.671E-01 | 1.000E+00 | 5.206E+06 | 5.206E+06 | 4.059E+06 |
| 1.135E+04 | 2.040E+02 | 2.957E+06 | 1.982E+03 | | | | | | | | |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 1.000E+00 | 1.176E+00 | 6.163E-04 | 6.163E-04 | 3.573E-01 | 5.184E+06 | 5.827E-02 | 0. | 3.039E-01 | 4.409E+06 | 4.955E-02 |
| 1.000E+00 | 1.176E+00 | 7.544E-04 | 7.544E-04 | 4.373E-01 | 6.345E+06 | 7.132E-02 | 0. | 3.718E-01 | 5.396E+06 | 6.065E-02 |
| 1.351E+00 | 1.317E+00 | 6.827E-04 | 9.223E-04 | 5.346E-01 | 7.758E+06 | 8.720E-02 | 0. | 4.058E-01 | 5.889E+06 | 6.619E-02 |
| 1.351E+00 | 1.176E+00 | 2.012E-03 | 2.719E-03 | 1.576E+00 | 2.287E+07 | 2.570E-01 | 0. | 1.340E+00 | 1.945E+07 | 2.186E-01 |
| 1.351E+00 | 1.176E+00 | 1.498E-03 | 2.024E-03 | 1.173E+00 | 1.702E+07 | 1.913E-01 | 0. | 9.974E-01 | 1.447E+07 | 1.627E-01 |
| 1.351E+00 | 1.176E+00 | 7.516E-04 | 1.015E-03 | 5.886E-01 | 8.541E+06 | 9.600E-02 | 0. | 5.005E-01 | 7.263E+06 | 8.164E-02 |
| 1.351E+00 | 1.317E+00 | 5.265E-04 | 7.113E-04 | 4.123E-01 | 5.983E+06 | 6.724E-02 | 0. | 3.130E-01 | 4.541E+06 | 5.104E-02 |
| 1.351E+00 | 1.317E+00 | 6.395E-04 | 8.639E-04 | 5.008E-01 | 7.267E+06 | 8.168E-02 | 0. | 3.801E-01 | 5.516E+06 | 6.200E-02 |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 2.214E+02 | 2.249E+00 | 1.179E-01 | 1.198E-03 | 4.220E+04 | 2.016E+01 | 8.793E+03 | 7.816E-01 | 1.399E+03 | 1.520E+06 | 8.982E-02 | 1.042E-01 |
| 8.500E-01 | 5.565E+03 | 9.540E-01 | 9.600E-01 | 7.297E+02 | 7.626E+00 | 5.055E-05 | 1.373E+04 | 2.579E+07 | 7.629E+02 | 7.888E+05 | 4.637E-02 |
| 1.908E-01 | 5.453E-01 | 6.733E+03 | 8.222E+00 | 2.238E+00 | 1.176E+00 | 0. | 0. | 0. | 6.163E-04 | 1.531E+07 | 8.999E-01 |
| 4.190E+06 | 2.934E+03 | 6.368E-01 | 7.475E-01 | 9.038E-01 | 1.006E+00 | -1.017E-03 | 1.002E+00 | 6.958E+00 | 5.156E-02 | 1.694E-02 | 4.302E-01 |
| 7.761E-03 | 8.794E-04 | 5.698E-01 | 5.183E-01 | 2.210E+00 | 2.245E-02 | 6.644E+00 | 9.653E-01 | 9.809E-03 | 2.293E+02 | 8.189E+00 | 6.397E-02 |
| -5.144E-06 | -8.495E+00 | 8.716E-02 | 1.183E-04 | 6.163E-04 | 3.573E-01 | 5.583E-02 | 7.648E-01 | 1.000E+00 | 5.184E+06 | 5.184E+06 | 4.078E+06 |
| 6.379E+06 | 2.200E+02 | 3.226E+06 | 1.988E+03 | | | | | | | | |

Figure 6 (cont.)

| Y/D | U/UE | H | RHO/RHOE | RHOU/RHOEUE | HBAR | T | M |
|---|---|---|---|---|---|---|---|
| 6.60018940E-02 | 6.26710054E-01 | 2.55947834E+08 | 4.19331041E-01 | 2.62799979E-01 | 6.26710054E-01 | 3.12916985E+03 | 3.30731544E+00 |
| Y/D | U/UE | H | RHO/RHOE | RHOU/RHOEUE | HBAR | T | M |
| 1.38863688E-02 | 4.79388116E-01 | 2.63066796E+08 | 4.13044031E-01 | 1.98008400E-01 | 4.79388116E-01 | 3.16312940E+03 | 2.51117634E+00 |
| Y/D | U/UE | H | RHO/RHOE | RHOU/RHOEUE | HBAR | T | M |
| 1.33998106E-01 | 7.07843269E-01 | 2.36931426E+08 | 4.38322410E-01 | 3.10263567E-01 | 7.07843269E-01 | 3.02993246E+03 | 3.81499676E+00 |
| Y/D | U/UE | H | RHO/RHOE | RHOU/RHOEUE | HBAR | T | M |
| 1.86113631E-01 | 7.48971926E-01 | 2.23196113E+08 | 4.53241882E-01 | 3.39465445E-01 | 7.48971926E-01 | 2.95258900E+03 | 4.09952956E+00 |
| Y/D | U/UE | H | RHO/RHOE | RHOU/RHOEUE | HBAR | T | M |
| 2.66001894E-01 | 7.96398322E-01 | 2.03937095E+08 | 4.77743294E-01 | 3.80473958E-01 | 7.96398322E-01 | 2.83162413E+03 | 4.45897654E+00 |
| Y/D | U/UE | H | RHO/RHOE | RHOU/RHOEUE | HBAR | T | M |
| 2.13886369E-01 | 7.67096353E-01 | 2.16268638E+08 | 4.61819446E-01 | 3.54260012E-01 | 7.67096353E-01 | 2.90962549E+03 | 4.23270990E+00 |
| Y/D | U/UE | H | RHO/RHOE | RHOU/RHOEUE | HBAR | T | M |
| 3.33998106E-01 | 8.28181470E-01 | 1.88990153E+08 | 4.99916701E-01 | 4.14021748E-01 | 8.28181470E-01 | 2.72671260E+03 | 4.72447388E+00 |
| Y/D | U/UE | H | RHO/RHOE | RHOU/RHOEUE | HBAR | T | M |
| 3.86113631E-01 | 8.49084770E-01 | 1.78246239E+08 | 5.18577720E-01 | 4.40316444E-01 | 8.49084770E-01 | 2.64165892E+03 | 4.91475862E+00 |
| Y/D | U/UE | H | RHO/RHOE | RHOU/RHOEUE | HBAR | T | M |
| 4.66001894E-01 | 8.76983870E-01 | 1.62811066E+08 | 5.50425066E-01 | 4.82713705E-01 | 8.76983870E-01 | 2.50378662E+03 | 5.19420470E+00 |
| Y/D | U/UE | H | RHO/RHOE | RHOU/RHOEUE | HBAR | T | M |
| 4.13886369E-01 | 8.59284484E-01 | 1.72750278E+08 | 5.29337656E-01 | 4.54851634E-01 | 8.59284484E-01 | 2.59393880E+03 | 5.01371878E+00 |
| Y/D | U/UE | H | RHO/RHOE | RHOU/RHOEUE | HBAR | T | M |
| 5.33998106E-01 | 8.97760712E-01 | 1.50492614E+08 | 5.82132951E-01 | 5.22616003E-01 | 8.97760712E-01 | 2.37654087E+03 | 5.43131813E+00 |
| Y/D | U/UE | H | RHO/RHOE | RHOU/RHOEUE | HBAR | T | M |
| 5.86113631E-01 | 9.12248401E-01 | 1.41486862E+08 | 6.09739949E-01 | 5.56234292E-01 | 9.12248401E-01 | 2.27349248E+03 | 5.61721069E+00 |
| Y/D | U/UE | H | RHO/RHOE | RHOU/RHOEUE | HBAR | T | M |
| 6.66001894E-01 | 9.32509294E-01 | 1.28317226E+08 | 6.59230298E-01 | 6.14738380E-01 | 9.32509294E-01 | 2.10704742E+03 | 5.92122034E+00 |
| Y/D | U/UE | H | RHO/RHOE | RHOU/RHOEUE | HBAR | T | M |
| 6.13886369E-01 | 9.19537846E-01 | 1.36824227E+08 | 6.26165281E-01 | 5.75782673E-01 | 9.19537846E-01 | 2.21574246E+03 | 5.72058792E+00 |
| Y/D | U/UE | H | RHO/RHOE | RHOU/RHOEUE | HBAR | T | M |
| 7.33998106E-01 | 9.48224643E-01 | 1.17643539E+08 | 7.08697521E-01 | 6.72004455E-01 | 9.48224643E-01 | 1.96135507E+03 | 6.20642508E+00 |
| Y/D | U/UE | H | RHO/RHOE | RHOU/RHOEUE | HBAR | T | M |
| 7.86113631E-01 | 9.59472624E-01 | 1.09757033E+08 | 7.52656180E-01 | 7.22153000E-01 | 9.59472624E-01 | 1.84674885E+03 | 6.44690878E+00 |
| Y/D | U/UE | H | RHO/RHOE | RHOU/RHOEUE | HBAR | T | M |
| 8.66001894E-01 | 9.75570564E-01 | 9.81114578E+07 | 8.29528611E-01 | 8.09263694E-01 | 9.75570564E-01 | 1.67453481E+03 | 6.84873954E+00 |
| Y/D | U/UE | H | RHO/RHOE | RHOU/RHOEUE | HBAR | T | M |
| 8.13886369E-01 | 9.65216564E-01 | 1.05650187E+08 | 7.78223223E-01 | 7.51153946E-01 | 9.65216564E-01 | 1.78576716E+03 | 6.58301608E+00 |
| Y/D | U/UE | H | RHO/RHOE | RHOU/RHOEUE | HBAR | T | M |
| 9.33998106E-01 | 9.88330269E-01 | 8.95809415E+07 | 9.08761076E-01 | 8.98156078E-01 | 9.88330269E-01 | 1.52786259E+03 | 7.23603037E+00 |
| Y/D | U/UE | H | RHO/RHOE | RHOU/RHOEUE | HBAR | T | M |
| 9.86113631E-01 | 9.97598912E-01 | 8.14916930E+07 | 9.79035946E-01 | 9.76685195E-01 | 9.97598912E-01 | 1.41814742E+03 | 7.56264640E+00 |

Figure 6 (cont.)

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1.000E+00 | 1.176E+00 | 6.115E-04 | 6.115E-04 | 3.572E-01 | 5.182E+06 | 5.826E-02 | 0. | 3.037E-01 | 4.407E+06 | 4.954E-02 | |
| 1.000E+00 | 1.176E+00 | 7.443E-04 | 7.443E-04 | 4.348E-01 | 6.308E+06 | 7.091E-02 | 0. | 3.697E-01 | 5.364E+06 | 6.030E-02 | |
| 1.351E+00 | 1.316E+00 | 6.737E-04 | 9.102E-04 | 5.317E-01 | 7.714E+06 | 8.672E-02 | 0. | 4.041E-01 | 5.863E+06 | 6.591E-02 | |
| 1.351E+00 | 1.176E+00 | 1.997E-03 | 2.698E-03 | 1.576E+00 | 2.280E+07 | 2.570E-01 | 0. | 1.340E+00 | 1.944E+07 | 2.186E-01 | |
| 1.351E+00 | 1.176E+00 | 1.496E-03 | 2.021E-03 | 1.190E+00 | 1.713E+07 | 1.925E-01 | 0. | 1.004E+00 | 1.456E+07 | 1.637E-01 | |
| 1.351E+00 | 1.176E+00 | 7.446E-04 | 1.006E-03 | 5.876E-01 | 8.526E+06 | 9.585E-02 | 0. | 4.997E-01 | 7.250E+06 | 8.150E-02 | |
| 1.351E+00 | 1.316E+00 | 5.220E-04 | 7.052E-04 | 4.120E-01 | 5.977E+06 | 6.719E-02 | 0. | 3.131E-01 | 4.542E+06 | 5.107E-02 | |
| 1.351E+00 | 1.316E+00 | 6.331E-04 | 8.553E-04 | 4.996E-01 | 7.248E+06 | 8.149E-02 | 0. | 3.797E-01 | 5.509E+06 | 6.193E-02 | |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 2.230E+02 | 2.266E+00 | 1.181E-01 | 1.200E-03 | 4.218E+04 | 2.030E+01 | 8.784E+03 | 7.808E-01 | 1.388E+03 | 1.515E+06 | 8.907E-02 | 1.049E-01 |
| 8.562E-01 | 5.567E+03 | 9.544E-01 | 9.604E-01 | 7.271E+02 | 7.657E+00 | 5.033E-05 | 1.392E+04 | 2.630E+07 | 7.647E+02 | 7.900E+05 | 4.644E-02 |
| 1.904E-01 | 5.508E-01 | 6.732E+03 | 8.148E+00 | 2.253E+00 | 1.176E+00 | 2.446E+02 | 1.368E+02 | 0. | 6.115E-04 | 1.531E+07 | 8.998E-01 |
| 4.187E+06 | 2.934E+03 | 6.368E-01 | 7.475E-01 | 9.038E-01 | 1.011E+00 | -1.069E-03 | 1.000E+00 | 5.817E+00 | 5.559E-02 | 1.605E-02 | 4.632E-01 |
| 8.481E-03 | 9.631E-04 | 5.368E-01 | 4.812E-01 | 2.057E+00 | 2.090E-02 | 6.256E+00 | 9.659E-01 | 9.814E-03 | 2.309E+02 | 8.181E+00 | 6.245E-02 |
| 5.020E-06 | -8.290E+00 | 8.716E-02 | 1.146E-04 | 6.115E-04 | 3.572E-01 | 5.581E-02 | 7.645E-01 | 1.000E+00 | 5.182E+06 | 5.182E+06 | 4.090E+06 |
| 6.221E+06 | 2.216E+02 | 3.252E+06 | 1.969E+03 | | | | | | | | |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1.000E+00 | 1.176E+00 | 3.459E-04 | 3.459E-04 | 3.713E-01 | 5.338E+06 | 6.056E-02 | 0. | 3.157E-01 | 4.539E+06 | 5.150E-02 | |
| 1.346E+00 | 1.171E+00 | 2.915E-04 | 3.922E-04 | 4.210E-01 | 6.053E+06 | 6.868E-02 | 0. | 3.597E-01 | 5.171E+06 | 5.866E-02 | |
| 1.346E+00 | 1.176E+00 | 1.477E-03 | 1.987E-03 | 2.133E+00 | 3.067E+07 | 3.479E-01 | 0. | 1.814E+00 | 2.608E+07 | 2.958E-01 | |
| 1.346E+00 | 1.176E+00 | 1.345E-03 | 1.810E-03 | 1.943E+00 | 2.794E+07 | 3.170E-01 | 0. | 1.653E+00 | 2.376E+07 | 2.695E-01 | |
| 1.346E+00 | 1.176E+00 | 3.706E-04 | 4.987E-04 | 5.353E-01 | 7.696E+06 | 8.731E-02 | 0. | 4.552E-01 | 6.544E+06 | 7.425E-02 | |
| 1.346E+00 | 1.171E+00 | 2.606E-04 | 3.507E-04 | 3.764E-01 | 5.412E+06 | 6.140E-02 | 0. | 3.215E-01 | 4.623E+06 | 5.245E-02 | |
| 1.346E+00 | 1.171E+00 | 3.059E-04 | 4.116E-04 | 4.419E-01 | 6.353E+06 | 7.208E-02 | 0. | 3.775E-01 | 5.427E+06 | 6.157E-02 | |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 3.974E+02 | 4.037E+00 | 1.265E-01 | 1.265E-03 | 4.180E+04 | 3.550E+01 | 8.100E+03 | 7.199E-01 | 7.651E+02 | 7.910E+05 | 4.650E-02 | 1.885E-01 |
| 1.538E+00 | 5.696E+03 | 9.765E-01 | 9.826E-01 | 5.494E+02 | 1.039E+01 | 3.522E-05 | 3.916E+04 | 1.231E+08 | 8.112E+02 | 8.421E+05 | 4.950E-02 |
| 1.777E-01 | 1.060E+00 | 6.667E+03 | 4.860E+00 | 3.714E+00 | 1.176E+00 | 4.681E+02 | 3.013E+02 | 0. | 3.333E-04 | 1.523E+07 | 8.951E-01 |
| 3.993E+06 | 2.870E+03 | 6.406E-01 | 7.492E-01 | 8.969E-01 | 1.011E+00 | -1.145E-03 | 1.000E+00 | 7.874E+00 | 3.136E-02 | 7.933E-03 | 5.585E-01 |
| 6.098E-03 | 3.253E-04 | 4.415E-01 | 4.101E-01 | 3.884E+00 | 3.946E-02 | 1.403E+01 | 2.194E+00 | 2.229E-02 | 1.812E+02 | 1.735E+01 | 1.756E-04 |
| 1.405E-08 | -2.318E-02 | 8.715E-02 | 2.284E-05 | 3.333E-04 | 3.577E-01 | 5.539E-02 | 7.475E-01 | 1.000E+00 | 5.143E+06 | 5.143E+06 | 4.200E+06 |
| 4.603E+06 | 3.953E+02 | 6.114E+06 | 2.038E+03 | | | | | | | | |

TOLL= 0.

Figure 7

| REX2 | X2REX | EN2REX | CEM | CEB | CF2REX | CEMP | CEBP | XMIN | RSISAV | ENI | CF2I |
|---|---|---|---|---|---|---|---|---|---|---|---|
| REXSAV | | | | | | | | | | | |
| 0. | 2.230E+02 | 5.824E+00 | 4.284E+00 | 1.923E-02 | 6.115E-04 | -4.054E-04 | 9.640E-01 | 1.500E+01 | 0. | 1.540E+00 | 1.017E-03 |
| 0. | | | | | | | | | | | |

# APPENDIX

## LANGLEY LIBRARY SUBROUTINES

# FUNCTION DIF

LANGUAGE: FORTRAN

PURPOSE: This is a function subprogram which differentiates the function at any given point in a table of supplied values.

USE: R = DIF (L, M, NP, VARI, VARD)

L An integer, the point at which the function is differentiated.

M An integer from 1 - 5 to determine the point formula, N, where N = 2M + 1. If M is not in this range, the derivative is set to INDEFINITE.

NP An integer, the number of points in the table.

VARI A one-dimensional array of the independent variable.

VARD A one-dimensional array of the dependent variable.

R The result.

RESTRICTIONS: The maximum N for the formula (see Method below) is 11, that is, M may not exceed 5. The differentiation is indefinite for an invalid M.

The answer will be I, indefinite, for M out of range or N > NP.

The two arrays must be dimensioned in the calling program as indicated: VARI(NP), VARD(NP).

METHOD: The Lagrangian N-point formula (N always odd) is used.

DIF

$$\frac{dY}{dX} = \prod_{j=0}^{j=n} (x_k - x_j)_{j \neq k} \left[ \sum_{i=0}^{i=n} \frac{Y_i}{D_{ik}} \right]_{i \neq k} +$$

$$Y_k \sum_{j=0}^{j=n} (x_k - x_j)^{-1}{}_{j \neq k}$$

where

$$D_{ik} = (x_k - x_i) P_i(x_i) \qquad (i \neq k)$$

$$P_i(x_i) = \prod_{j=0}^{j=n} (x_i - x_j) \qquad (j \neq i)$$

N points are required to differentiate where N is odd. N = 2*M +1, M = (N - 1)/2

The following table shows the value of the discrete point, L, the value of k, and the points used for the differentiation.

NP = last point of the table.

| L | K | POINTS FROM TABLE |
|---|---|---|
| 1, 2, . . . M+1 | L | 1, 2, . . . N |
| M+2, M+3, . . . NP-M | M+1 | L-M, . . . L+M |
| NP-M+1, NP-M+2, . . . NP | L-(NP-N) | NP-N+1, NP-N+2, . . . NP |

ACCURACY: The accuracy is a function of the chosen M and the number of points in the supplied tables.

REFERENCE: K. L. Neilson, Methods in Numerical Analysis, pp. 150-154

STORAGE: DIF $217_8$ locations

SOURCE: NASA, LRC, Vivian P. Adair

RESPONSIBLE PERSON: Vivian P. Adair

```
      FUNCTION DIF (L,M,NP,VARI,VARD)
C
C        THIS FUNCTION SUBPROGRAM FINDS THE DERIVATIVE AT A GIVEN POINT,
C        L, FOR THE DESIRED X AND Y IN A GIVEN TABLE.  THE N-POINT
C        LAGRANGIAN FORMULA IS USED WHERE N IS ODD.
C
C        L = INTEGER, THE POINT OF X AND Y AT WHICH DERIVATIVE IS FOUND
C        M = INTEGER, 1-5, TO DETERMINE THE POINT FORMULA, N.  N=2*M+1
C        NP= INTEGER, THE NUMBER OF POINTS IN TABLE OF VARIABLES
C        VARI = ARRAY OF INDEPENDENT VARIABLE, X.  VARI(NP)
C        VARD = ARRAY OF DEPENDENT VARIABLE, Y.    VARD(NP)
C
      DIMENSION VARI(NP), VARD(NP), X(11), Y(11)
C
      DIF=01777000000000000000
      IF (M.LT.1) RETURN
      N=2*M+1
      IF (M.GT.5.OR.N.GT.NP) RETURN
      M1=M+1
      M2=NP-M+1
      K=L
      IF (L.LE.M1.OR.N.EQ.NP) GO TO 1
      K=M1
      IF (L.LT.M2) GO TO 1
      K=L-(NP-N)
1     MX=L-K
      DO 2 J=1,N
      MJ=MX+J
      X(J)=VARI(MJ)
2     Y(J)=VARD(MJ)
      A=1.
      B=0.
      C=0.
      DO 4 J=1,N
      IF (J.EQ.K) GO TO 4
      P=1.
      DO 3 I=1,N
      IF (I.EQ.J) GO TO 3
      P=P*(X(J)-X(I))
3     CONTINUE
      T=X(K)-X(J)
      B=B+Y(J)/(P*T)
```

```
      A=A*T
      C=C+1./T
    4 CONTINUE
      DIF=A*B+Y(K)*C
      RETURN
      END
```

# SUBROUTINE DISCOT

LANGUAGE: FORTRAN

PURPOSE: SINGLE OR DOUBLE INTERPOLATION SUBROUTINE FOR CONTINUOUS OR DISCONTINUOUS FUNCTIONS

Given some function with two independent variables, x and z, this subroutine performs $K_x$th and $K_z$th order interpolation to calculate the dependent variable. In this subroutine all single line functions are read in as two separate arrays and all multi-line functions are read is as three separate arrays, i.e.

$X_i$ $i = 1, 2, \ldots L$

$Y_j$ $j = 1, 2, \ldots M$

$Z_k$ $k = 1, 2, \ldots N$

USE: CALL DISCOT (XA, ZA, TABX, TABY, TABZ, NC, NY, NZ, ANS)

XA - The X argument.

ZA - The Z argument (may be the same name as X on single lines).

TABX - A one-dimensional array of X's.

TABY - A one-dimensional array of Y's.

TABZ - A one-dimensional array of Z's.

NC - A control word that consists of a sign and three digits. The control word is formed as follows:

(1) If NX = NY, the sign is -. If NX ≠ NY, NX is computed by DISCOT as NX = NY/NZ. The sign is + and may be omitted if desired.

(2) A 1 in the hundreds position of the word indicates that no extrapolation occurs

above Z max. With a zero in this position extrapolation occurs when Z > Z max. The zero may be omitted if desired.

(3) 1-7 in the tens position of the word indicates the order of interpolation in the X direction.

(4) 1-7 in the units position of the word indicates the order of interpolation in the Z direction.

NY - The number of points in the Y array.

NZ - The number of points in the Z array.

ANS - The dependent variable Y.

The following programs will illustrate various ways to use DISCOT.

Case I. Given Y = f(x)

NY = 50
NX (number of points in X array) = NY
Extrapolation when Z > Z max
Second order interpolation in X direction
No interpolation in Z direction
Control word = -020

1.
```
        DIMENSION TABX (50), TABY (50)
      1 FORMAT (8E 9.5)
        READ (5, 1) TABX, TABY
        READ (5, 1) XA
        CALL DISCOT (XA, XA, TABX, TABY, TABY,
        -020, 50, 0, ANS)
```

Case II Given Y = f(x, z)

NY = 800
NZ = 10
NX = NY/NZ (computed by DISCOT)
Extrapolation when Z > Z max
Linear interpolation in X direction
Linear interpolation in Z direction
Control word = 11

```
  DIMENSION TABX (800), TABY (800), TABZ (10)
1 FORMAT (8E 9.5)
  READ (5, 1) TABX, TABY, TABZ
  READ (5, 1) XA, ZA
  CALL DISCOT ( XA, ZA, TABX, TABY, TABZ,
  11, 800, 10, ANS)
```

Case III. Given Y = f(x, z)

NY = 800
NZ = 10
NX = NY
Extrapolation when Z > Z max
Seventh order interpolation in X direction
Third order interpolation in Z direction
Control word = -73

```
  DIMENSION TABX (800), TABY(800), TABZ (10)
1 FORMAT (8E 9.5)
  READ (5, 1) TABX, TABY, TABZ
  READ (5, 1) XA, ZA
  CALL DISCOT (XA, ZA, TABX, TABY, TABZ,
  -73, 800, 10, ANS)
```

Case IV. Same as Case III with no extrapolation above Z max. Control word = -173

```
  CALL DISCOT (XA, ZA, TABX, TABY, TABZ, -173,
  800, 10, ANS)
```

RESTRICTIONS: See 4c of METHOD for restrictions on tabulating arrays and discontinuous functions. The order of interpolation in the X and Z directions may be from 1-7.

The following subprograms are used by DISCOT: UNS, DISSER, LAGRAN.

METHOD: Lagrange's interpolation formula is used in both the X and Z direction for interpolation. This method is explained in detail in Methods in Numerical Analysis by Nielsen. The search in both the X and Z direction observe the following rules:

1. $X < X_1$ the routine chooses the following points for extrapolation.

$X_1, X_2, \ldots X_{k+1} \quad Y_1, Y_2, \ldots Y_{k+1}$

2. $X > X_n$ the routine chooses the following points for extrapolation.

   $X_{n-k}, X_{n-k+1}, \ldots X_n \quad Y_{n-k}, Y_{n-k+1}, \ldots Y_n$

3. $X \leq X_n$ the routine chooses the following points for interpolation.

   k is odd $X_{i-\frac{k+1}{2}}, X_{i-\frac{k+1}{2}+1}, \ldots X_{i-\frac{k+1}{2}+k}$

   $Y_{i-\frac{k+1}{2}}, Y_{i-\frac{k+1}{2}+1}, \ldots Y_{i-\frac{k+1}{2}+k}$

   k is even $X_{i-\frac{k}{2}}, X_{i-\frac{k}{2}+1}, \ldots X_{i-\frac{k}{2}+k}$

   $Y_{i-\frac{k}{2}}, Y_{i-\frac{k}{2}+1}, \ldots Y_{i-\frac{k}{2}+k}$

4. If any of the subscripts in Rule 3 become negative or greater than n (number of points), Rules 1 and 2 apply. When discontinuous functions are tabulated, the independent variable at the point of discontinuity is repeated, i.e.

   $k = 2 \; (X_1, X_2, X_3, X_3, X_4, X_5, Y_1, Y_2, Y_3, Y_4, Y_5, Y_6)$ .

   The subroutine will automatically examine the points selected before interpolation and if there is a discontinuity, the following rules apply. Let $X_d$ and $X_{d+1}$ be the point of discontinuity.

a. $X \leq X_d$ points previously chosen are modified for interpolation as shown

$X_{d-k}, X_{d-k+1}, \ldots . X_d$

$Y_{d-k}, Y_{d-k+1}, \ldots . Y_d$

b. $X > X_d$ points previously chosen are modified for interpolation as shown

$X_{d+1}, X_{d+2}, \ldots . X_{d+k}$

$Y_{d+1}, Y_{d+2}, \ldots . Y_{d+k}$

c. When tabulating discontinuous functions, there must always be k+1 points above and below the discontinuity in order to get proper interpolation.

When tabulating arrays for this subroutine, both independent variables must be in ascending order.

In some engineering programs with many tables, it is quite desirable to read in one array of x's that could be used for all lines of a multi-line function or different functions.

The above not only saves much time in preparing tabular data, but can also save many locations previously used when every y coordinate had to have a corresponding x coordinate. Even though the above is not always applicable, the subroutine has been written to handle this situation.

Another additional feature that may be useful is the possibility of a multi-line function with no extrapolation above the top line.

DISCOT

| | |
|---|---|
| ACCURACY: | A function of the order of interpolation used. |
| REFERENCE: | Nielsen, K.L.; Methods in Numerical Analysis |
| STORAGE: | DISCOT - $555_8$ locations |
| SUBPROGRAMS USED: | UNS $40_8$ locations<br>DISSER $110_8$ locations<br>LAGRAN $55_8$ locations |
| OTHER CODING INFORMATION: | NONE |
| SOURCE: | SHARE Library, General Motors Corp., Allison Div. |
| RESPONSIBLE PERSON: | Vivian P. Adair |

```
      SUBROUTINE DISCOT (XA,ZA,TABX,TABY,TABZ,NC,NY,NZ,ANS)
C     *** DOCUMENT DATE 08-01-68   SUBROUTINE REVISED 08-01-68 *********
C     THE DIMENSIONS IN THIS SUBROUTINE ARE ONLY DUMMY DIMENSIONS.
      DIMENSION TABX(2), TABY(2), TABZ(2), NPX(8), NPY(8), YY(8)
C     DIMENSION TABX(2),TABY(2),TABZ(2),NPX(8),NPY(8),YY(8)
      CALL UNS (NC,IA,IDX,IDZ,IMS)
      IF (NZ-1) 1,1,2
1     CALL DISSER (XA,TABX(1),1,NY,IDX,NN)
      NNN=IDX+1
      CALL LAGRAN (XA,TABX(NN),TABY(NN),NNN,ANS)
      GO TO 12
2     ZARG=ZA
      IP1X=IDX+1
      IP1Z=IDZ+1
      IF (IA) 3,5,3
3     IF (ZARG-TABZ(NZ)) 5,5,4
4     ZARG=TABZ(NZ)
5     CALL DISSER (ZARG,TABZ(1),1,NZ,IDZ,NPZ)
      NX=NY/NZ
      NPZL=NPZ+IDZ
      I=1
      IF (IMS) 6,6,8
6     CALL DISSER (XA,TABX(1),1,NX,IDX,NPX(1))
      DO 7 JJ=NPZ,NPZL
      NPY(I)=(JJ-1)*NX+NPX(1)
      NPX(I)=NPX(1)
7     I=I+1
      GO TO 10
8     DO 9 JJ=NPZ,NPZL
      IS=(JJ-1)*NX+1
      CALL DISSER (XA,TABX(1),IS,NX,IDX,NPX(I))
      NPY(I)=NPX(I)
9     I=I+1
10    DO 11 LL=1,IP1Z
      NLOC=NPX(LL)
      NLOCY=NPY(LL)
11    CALL LAGRAN (XA,TABX(NLOC),TABY(NLOCY),IP1X,YY(LL))
      CALL LAGRAN (ZARG,TABZ(NPZ),YY(1),IP1Z,ANS)
12    RETURN
      END
```

DISSER.- Library subroutine used by DISCOT.

```
      SUBROUTINE DISSER (XA,TAB,I,NX,ID,NPX)
      DIMENSION TAB(2)
C     DIMENSION TAB(2)
      NPT=ID+1
      NPB=NPT/2
      NPU=NPT-NPB
      IF (NX-NPT) 2,1,2
1     NPX=I
      RETURN
2     NLOW=I+NPB
      NUPP=I+NX-(NPU+1)
      DO 3 II=NLOW,NUPP
      NLOC=II
      IF (TAB(II)-XA) 3,4,4
3     CONTINUE
      NPX=NUPP-NPB+1
      RETURN
4     NL=NLOC-NPB
      NU=NL+ID
      DO 5 JJ=NL,NU
      NDIS=JJ
      IF (TAB(JJ)-TAB(JJ+1)) 5,6,5
5     CONTINUE
      NPX=NL
      RETURN
6     IF (TAB(NDIS)-XA) 8,7,7
7     NPX=NDIS-ID
      RETURN
8     NPX=NDIS+1
      RETURN
      END
```

# SUBROUTINE INT1A

LANGUAGE: FORTRAN

PURPOSE: INT1A is a closed subroutine for the solution of a set of ordinary differential equations.

USE: The calling sequence is identical to INT1 except that the VAR and CUVAR arrays are single precision.

A general guideline for selecting the better subroutine is to use INT1A when the ELE1 values are set to $.1 \times 10^{-8}$ or larger. In those cases the word length of the CDC 6000 series computers affords adequate control of the rounding error without the "partial double precision mode of operation."

RESTRICTIONS: See INT1.

METHOD: This subroutine is identical to INT1 except that the "partial double precision mode of operation" has been eliminated.

ACCURACY: See INT1.

REFERENCES: See INT1.

STORAGE: INT1A $2557_8$ locations

SOURCE: NASA, LRC, Jules J. Lambiotte

RESPONSIBLE PERSON: Jules J. Lambiotte

# SUBROUTINE INT1

LANGUAGE: FORTRAN

PURPOSE: INT1 is a closed subroutine for the solution of a set of ordinary differential equations.

USE: CALL INT1 (II,N,NT,CI,SPEC,CIMAX,IERR,VAR,CUVAR, DER,ELE1,ELE2,ELT,ERRVAL,DERSUB,CHSUB,ITEXT)

II - INT1 is composed of an initialization section and an integration section. The user is required to enter the initialization section before he starts his first integration step. The above calling sequence is used for both initialization and integration with the value of the code word II determining which of the two sections of INT1 will be entered.

The user must set II = 0 in order to initialize.

During initialization the derivatives will be evaluated using the initial values of the variables but no integration will occur and control will be returned to the calling program. When INT1 is called with II > 0, entry is made to the integration section. Upon each entry to INT1, the subroutine stores a 1 in II so that the users need not supply a value of II > 0 for repetitive integration.

Besides serving as a means for specifying the entry point to INT1 from the calling program, II can also be set to specified values in CHSUB to accomplish the following:

2 The user will store the integer 2 in II if the answers in CHSUB are not acceptable to him and he wishes to recompute the answers using a shorter interval. This shorter interval must be stored by the user in CI. It must be smaller than the computing interval just used.

3 The user will store the integer 3 in II if he wishes to return to the calling program. The answers for the interval are considered acceptable to the user and will be transferred to the VAR array (explained below) by INT1.

In DERSUB II may be set to:

4 The user will store the integer 4 in II if he wishes to discontinue calculation of the present interval and return to the calling program. On return to the calling program, the answers at the beginning of the interval will still be in the VAR array.

If the user does not set II to a value in either CHSUB or DERSUB, II will always be 1 upon the return to the calling program.

N - An integer value supplied by the user which is the number of differential equations to be solved. INT1 is compiled to solve a maximum of 20 equations but may be recompiled for larger values of N if necessary.

NT - An integer value supplied by the user which is the number of values in the ELT block described below. INT1 is compiled with a maximum of 10 values in the ELT block but may be recompiled for more values if necessary.

CI - A floating point value supplied by the user which is the computing interval INT1 will use initially. CI must be a signed value, positive if integrating forward, negative if integrating backwards. Upon entry to CHSUB, CI will contain the computing interval that INT1 will use for the next step unless it has to take a short interval to hit an ELT value or a SPEC value described below. The computing interval used on the present step is available in CHSUB as the algebraic difference between CUVAR (1) and VAR (1). Since the subroutine is used on a binary computer and the interval variation is a halving and doubling process, CI should be a power of 2.

SPEC - A floating point value supplied by the user which specifies how often he wishes INT1 to return control to the calling program so that the user may print his results.

SPEC = 0.0 - Control will be returned after every acceptable integration step.

SPEC > 0.0 - SPEC is the absolute value of the specified increment of the independent variable for which the user desires control returned to the calling program.

The first printout is made at the initial value of the independent variable. The next return is at the nonzero integer multiple of SPEC closest to the initial value of the independent variable. The remaining returns occur at values which have been updated from this point by the increment given in SPEC. The return times generated by the increment given in SPEC are not altered by an intervening return due to an ELT value (explained below).

CIMAX - A floating point value supplied by the user which is the absolute value of the maximum computing interval that will be used. This value will be used if the doubling process would extend the computing interval to a value larger than

CIMAX. CIMAX should be set to 0.0 if there is no desired maximum.

IERR - An integer value supplied by INT1 as an error code. It must be checked at every return to the calling program. It may have the following values:

1 A normal return, no error.

2 The ELT block is not monotonic in the direction of integration.

3 The variables have failed to meet the local truncation error requirements nine consecutive times. The answers at the beginning of the interval are still in the VAR array.

4 The variables have failed to meet the local truncation error requirements at least nine times over the last three intervals. An acceptable answer has been reached, however, and is in the VAR array.

VAR - A double precision one-dimensional array containing the independent variable followed by the N dependent variables. The user must store the N+1 initial values (in the double precision mode) in the array for initialization. INT1 will store the new values of the variables in VAR after each integration step when they are accepted by the user in CHSUB. The elements of the VAR block can be printed out in the calling program in accordance with the user's specification in SPEC.

CUVAR - A double precision one-dimensional array which is given values by INT1 for two purposes. INT1 will store in the same order as the VAR array the values of the independent variable and N dependent variables at which it wishes the derivatives to be evaluated in the DERSUB subroutine. Although CUVAR must be in a double precision array in INT1 to maintain the "partial double precision mode" of computation, the evaluation of the derivatives should be in single precision. Two suggested ways of doing this are as follows: (1) Consider CUVAR as a single precision array of 2(N+1) elements in the DERSUB subroutine and when using the ith element in a computation subscript it with the value (2i-1). (2) At the begining of the DERSUB subroutine, transfer from CUVAR to some newly defined single precision array and evaluate the derivatives using the latter.

INT1 will also store the tentative answers after each integration in the CUVAR array before calling CHSUB so that the user can check these values to decide to accept or reject the answers. If accepted, the CUVAR values will then be transferred to the VAR array. The decision as to whether the computation in the CHSUB subroutine should be done in single or double precision is a function of the individual application. In most cases single precision is adequate and can be accomplished by applying the above suggestions to the VAR and CUVAR arrays.

No values need to be initially stored in CUVAR.

DER - An N+1 single precision array in which the user will store the derivatives evaluated in DERSUB. The derivatives should be arranged by the user in DERSUB in the same order as the VAR block so that DER (2) will be the derivative of the variable stored in VAR (2), etc. DER (1) will be unused. The derivatives must be computed using values of the variables which have been stored in CUVAR (not VAR) by INT1. To avoid unnecessary double precision computation the user should apply the suggestion described under CUVAR.

ELE1 - A one-dimensional array of N values supplied by the user each of which is the upper bound of local relative truncation error for the respective dependent variables. If the error for any variable exceeds its respective ELE1 value, the computing interval is halved and the integration restarted at the beginning of the present interval. If the error for all of the variables is less than 1/128 of their respective ELE1 values, the computing interval is doubled for the next integration step.

ELE2 - A one-dimensional array of N values supplied by the user which represents a small value or "relative zero" for the respective dependent variables. If the absolute value of any of the variables is less than its respective ELE2 value, the relative error criteria for that variable will not be applied.

ELT - A one-dimensional array of NT values supplied by the user which are values of the independent variable at which the user specifically desires control returned to his program. The values in the ELT block must be monotomic in the direction of integration or an error return will be given by INT1.

ERRVAL - A one-dimensional array of N elements in which INT1 stores an estimate of the local truncation error for each of the N dependent variables. The relative errors are computed from these values and compared with the specified ELE1 values.

DERSUB - The name of a subroutine written by the user which will be called by INT1 to evaluate the derivatives. The derivatives must be stored in the DER array. INT1 will call DERSUB to evaluate the derivatives with the values of the variable it has stored in the CUVAR array.

These evaluations should be done using single precision arithmetic. The name given to the DERSUB subroutine must appear in an EXTERNAL statement in the calling program. The user may return to the calling program by setting II to 4.

CHSUB - The name of a subroutine written by the user to allow certain logical control. After each integration step, INT1 will make available to the user in CHSUB the tentative answers in the CUVAR array. The VAR array will contain the last accepted answer (that is, the value of the variables at the beginning of the interval). Whenever the user specifies the answers are acceptable, the values in the CUVAR block are transferred to the VAR block. In CHSUB the DER block will contain the values of the derivatives evaluated with the present CUVAR block. The user has three options:

1. Not change II. II = 1 is considered by INT1 to denote that the user has accepted the answers in the CUVAR block. II always equals 1 upon entry to CHSUB from INT1.

2. Set II = 2. The user does not accept the answers and wishes to recompute the interval using a new computing interval which he stores in CI. This computing interval must be smaller than the computing interval just used.

This new value of CI will now be stored by INT1 as the normal computing interval for the subsequent integration steps.

3. Set II = 3. The user accepts the answer but wishes to denote a condition that he can test in the calling program. Control will be returned to the calling program with the answers in the CUVAR array transferred to the VAR array.

The name given to the CHSUB subroutine must appear in an EXTERNAL statement in the calling program.

ITEXT - An integer code word supplied by the user which gives him the option to have INT1 print out a time history of the computing interval and the reasons for its variation. This print out should be requested only for problems which must be rerun due to unsatisfactory results the first time.

ITEXT = 0  No printout requested
ITEXT = 1  A printout requested

RESTRICTIONS: See arguments listed under CALL statement.

METHOD: INT1, written in coordination with the other integration subroutines in the INT(x) common usage series, is a fifth-order integration subroutine. The classical fourth-order Runge-Kutta formula is applied in conjunction with Richardson's Extrapolation to the Limit Theory. INT1 is a variable interval size routine in which the interval is varied to meet a specified local relative truncation error.

ACCURACY: The variable interval size mode of logic is used to make available an estimate of the local relative truncation error which is then controlled as explained in the ELE1 block discussion.

Roundoff error is controlled by use of the "partial double precision mode of computation" as explained in Reference (1).

REFERENCE: Henrici, Peter (1962): DISCRETE VARIABLE METHODS IN ORDINARY DIFFERENTIAL EQUATIONS, John Wiley and Sons, New York.

STORAGE: INT1 $2703_8$ locations

SOURCE: NASA, LRC, Jules J. Lambiotte

RESPONSIBLE PERSON: Jules J. Lambiotte

```
      SUBROUTINE INT1A (II,N,NT,CI,SPEC,CIMAX,IERR,VAR,CUVAR,DER,ELE1,EL
     1E2,ELT,ERRVAL,DERSUB,CHSUB,ITEXT)
C     *** DOCUMENT DATE 08-01-68   SUBROUTINE REVISED 08-01-68 *********
      DIMENSION S1VAR(20), SELE1(20), ELE1(20), ELE2(20), DER(21), FDERV
     1(21), SDY(20), SDY1(20), YINCR(20), ERRVAL(20), ERVDVH(20), ELT(10
     2), SELT(13), RELMIN(20), STEP(3)
      DIMENSION VAR(21), CUVAR(21)
      INTEGER TEX(15)
      INTEGER CODE,TPSH,SUMHAF,STEP,TEST,DCODE
      REAL K1
C     BEGIN INITIALIZATION
      IF (II.GT.0) GO TO 19
      TP=0
      SSPEC=SIGN(SPEC,CI)
      SCIMAX=SIGN(CIMAX,CI)
      VAR1=VAR(1)
      IF (CI.EQ.0.0) GO TO 18
      IF (SSPEC.EQ.0.0) GO TO 4
      IF (ABS(SCIMAX).GT.ABS(SSPEC).OR.SCIMAX.EQ.0.0) SCIMAX=SSPEC
C     TEST TO SEE IF VAR IS ZERO
      IF (ABS(VAR1).GT.1.0E-11) GO TO 1
      TP=SSPEC
      GO TO 4
1     IF ((VAR1/SSPEC).GT.1.E-13) GO TO 2
      K1=0.0
      GO TO 3
2     K1=1.0
3     TP=VAR1-AMOD(VAR1,SSPEC)
      IF (ABS(TP-VAR1).LT.1.E-12) K1=1.0
      TP=TP+K1*SSPEC
      IF (ABS((TP-VAR1)/VAR1).LT.1.E-11) TP=TP+SPEC
C     TEST FOR DIRECTION OF INTEGRATION
4     K1=1.0
      IF (CI.LT.0.0) K1=-1.0
      CIK=CI*K1
      CIMAXK=SCIMAX*K1
      TPK=TP*K1
      VARK=VAR1*K1
C     SET UP STORAGE FOR INTERNAL USE
      NP1=N+1
      NELT=1
      REMAIN=0.0
```

```
      NHAF=0
      NTS=NT
      SUMHAF=0
      LOOP=0
      DO 5 I=1,3
5     STEP(I)=0
      IERR=1
      DO 6 I=1,NP1
6     CUVAR(I)=VAR(I)
      DO 7 I=1,N
7     SELE1(I)=ELE1(I)
      IF (NT.EQ.0) GO TO 11
      IF (NT.EQ.1) GO TO 9
      NTM1=NT-1
      ELTK=K1*ELT(1)
      DO 8 I=1,NTM1
      ELTK2=K1*ELT(I+1)
      IF (ELTK.LT.ELTK2) GO TO 8
      GO TO 97
8     ELTK=ELTK2
9     CONTINUE
      ELTK=K1*ELT(NELT)
      IF (VARK.LT.ELTK) GO TO 10
      IF (NELT.EQ.NT) GO TO 11
      NELT=NELT+1
      GO TO 9
10    NELTL=NT-NELT+1
      GO TO 12
11    NELTL=0
12    DO 13 I=1,N
13    RELMIN(I)=SELE1(I)/128.0
      IF (NT.EQ.0) GO TO 15
      DO 14 I=1,NT
14    SELT(I)=ELT(I)
15    CALL DERSUB
      IF (II.EQ.4) GO TO 87
      DO 16 I=1,N
16    FDERV(I)=DER(I+1)
      II=1
      TEST=0
      DO 17 I=1,15
17    TEX(I)=0
      TEX(1)=1
```

```
      TEX(2)=1
      KK3=1
      IF (ITEXT) 93,83,93
18    PRINT 100
      STOP
C     END OF INITIALIZATION
19    II=1
      TPSH=0
      LTSH=0
      VARK=VAR(1)*K1
      CIK=CI*K1
      S1=VARK+CIK
      IF (SSPEC.EQ.0.0) GO TO 28
      KK=1
      IF (NELTL.EQ.0) GO TO 21
      IF (ELTK-TPK) 20,20,21
20    CV=ELTK
      CODE=1
      GO TO 22
21    CV=TPK
      CODE=2
22    IF (ABS(CV).LT.1.E-12) GO TO 27
      IF (CV-S1) 24,24,23
23    IF (ABS((CV-S1)/CV).GE..1E-11) GO TO 36
24    IF (NELTL.EQ.0) GO TO 25
      IF (ABS((ELTK-TPK)/CV).LT..1E-11) GO TO 26
      IF (CODE.EQ.1) GO TO 32
25    DX=TP-VAR(1)
      TEX(5)=1
      TP=TP+SSPEC
      TPK=TP*K1
      TPSH=1
      GO TO 37
C     SHORT INTERVAL DUE TO BOTH
26    TP=TP+SSPEC
      TEX(6)=1
      TPK=TP*K1
      TPSH=1
      GO TO 32
C     IF HERE CV IS LIKELY ZERO
27    IF (S1.LT.-1.0E-12) GO TO 36
      IF (CODE.EQ.1) GO TO 26
      IF (NELTL.EQ.0) GO TO 25
```

```
      IF (ABS(ELTK).LT.1.0E-12) GO TO 26
      GO TO 25
C     SPEC IS ZERO
28    IF (ABS(REMAIN).GT..1E-11) GO TO 34
      IF (NELTL.EQ.0) GO TO 33
29    IF (ABS(ELTK).GE.1.E-12) GO TO 30
      IF (S1.LT.-1.0E-12) GO TO 33
      GO TO 32
30    S2=ELTK-S1
      IF (S2) 32,32,31
31    IF (ABS(S2/ELTK).LT.1.0E-12) GO TO 32
      GO TO 33
C     SHORTINTERVAL IS DUE TO ELT BLOCK
32    DELT=SELT(NELT)
      TEX(4)=1
      DX=DELT-VAR(1)
      REMAIN=CI-DX
      REMAIK=REMAIN*K1
      LTSH=1
      NELT=NELT+1
      NELTL=NELTL-1
      IF (NELTL.EQ.0) GO TO 37
      ELTK=K1*SELT(NELT)
      GO TO 37
33    DX=CI
      TEX(3)=1
      GO TO 37
34    IF (NELTL.EQ.0) GO TO 35
      IF (ELTK.LT.(VARK+REMAIK)) GO TO 29
35    DX=REMAIN
      TEX(7)=1
      REMAIN=0.0
      GO TO 37
36    DX=CI
      TEX(3)=1
      TEST=1
      GO TO 38
C
C     BEGIN RUNGE-KUTTA
C
37    TEST=0
38    DO 39 I=1,N
39    S1VAR(I)=VAR(I+1)
```

```
40    CUVAR(1)=VAR(1)
41    DO 42 I=1,N
      SDY(I)=DER(I+1)
42    CUVAR(I+1)=S1VAR(I)+(DX*DER(I+1))/2.0
      CUVAR(1)=CUVAR(1)+DX/2.0
      CALL DERSUB
      IF (II.EQ.4) GO TO 87
      DO 43 I=1,N
      SDY(I)=SDY(I)+2.0*DER(I+1)
43    CUVAR(I+1)=S1VAR(I)+(DX*DER(I+1))/2.0
      CALL DERSUB
      IF (II.EQ.4) GO TO 87
      DO 44 I=1,N
      SDY(I)=SDY(I)+2.0*DER(I+1)
44    CUVAR(I+1)=S1VAR(I)+DX*DER(I+1)
      CUVAR(1)=CUVAR(1)+DX/2.0
      CALL DERSUB
      IF (II.EQ.4) GO TO 87
      DO 45 I=1,N
      SDY(I)=(SDY(I)+DER(I+1))/6.0
45    CONTINUE
      IF (LOOP) 46,46,48
46    DO 47 I=1,N
      SDY1(I)=SDY(I)
      YINCR(I)=0.0
47    DER(I+1)=FDERV(I)
      DX=DX/2.0
      LOOP=1
      GO TO 40
C
C     LOOP WAS NOT ZERO
C
48    DO 49 I=1,N
49    YINCR(I)=YINCR(I)+SDY(I)
      IF (LOOP.EQ.2) GO TO 51
      DO 50 I=1,N
      S1VAR(I)=VAR(I+1)+DX*YINCR(I)
50    CUVAR(I+1)=S1VAR(I)
      CUVAR(1)=VAR(1)+DX
      LOOP=2
      CALL DERSUB
      IF (II.EQ.4) GO TO 87
      GO TO 41
```

```
51    LOOP=0
      H=2.0*DX
      DO 52 I=1,N
      ERVOVH(I)=(YINCR(I)/2.0-SDY1(I))/15.0
      ERRVAL(I)=H*ERVOVH(I)
52    S1VAR(I)=S1VAR(I)+DX*SDY(I)+ERRVAL(I)
C
C     S1VAR HOLD THE APPROXIMATE ANSWERS
C
      IF (SCIMAX) 53,54,53
53    IF (ABS(SCIMAX-CI).LT.1.0E-12) GO TO 55
54    IF (ABS(H-CI).GT.1.0E-12) GO TO 55
      DCODE=0
      GO TO 56
55    DCODE=1
56    CONTINUE
      I=0
57    I=I+1
      IF (I.GT.N) GO TO 58
      IF (ABS(S1VAR(I)).LT.ELE2(I)) GO TO 57
      RELER=ABS(ERRVAL(I)/S1VAR(I))
      IF (RELER.GT.SELE1(I)) GO TO 63
      IF (RELER.GT.RELMIN(I)) DCODE=1
      GO TO 57
58    CONTINUE
      IF (DCODE-1) 59,78,59
59    CONTINUE
      IF (SSPEC) 62,60,62
60    IF (SCIMAX) 62,61,62
61    CI=2.0*CI
      TEX(8)=1
      NHAF=NHAF-1
      GO TO 78
62    IF (2.0*ABS(CI).LE.ABS(SCIMAX)) GO TO 61
      CI=SCIMAX
      TEX(8)=1
      GO TO 78
C
C     HALF INTERVAL
63    NHAF=NHAF+1
      TEX(9)=1
      NVAR=I+1
      IF (NHAF-8) 64,64,98
```

```
64    IF (LTSH.EQ.0) GO TO 65
      TEST=1
      LTSH=0
      NELT=NELT-1
      NELTL=NELTL+1
      ELTK=K1*SELT(NELT)
      REMAIN=0.0
65    IF (TPSH.EQ.0) GO TO 66
      TEST=1
      TP=TP-SSPEC
      TPK=K1*TP
      TPSH=0
66    IF (SSPEC.NE.0.0) GO TO 67
      TEST=0
      IF (ABS(CI-2.0*DX).GT.1.E-12) GO TO 71
67    CI=DX
68    DX=DX/2.0
      CIK=K1*CI
      DO 69 I=1,N
      S1VAR(I)=VAR(I+1)
      DER(I+1)=FDERV(I)
      SDY1(I)=YINCR(I)-SDY(I)
69    YINCR(I)=0.0
      KK3=2
      IF (ITEXT.EQ.1) GO TO 94
70    LOOP=1
      GO TO 40
71    CONTINUE
      IF (NHAF.GT.1) GO TO 68
      NTS=NTS+1
      IF (NTS.GT.13) GO TO 67
      ACV=VAR(1)+CI
      ACVK=ACV*K1
      IF (NELTL.EQ.0) GO TO 73
      NLT=NELT
72    ELTK1=SELT(NLT)*K1
      IF (ACVK.LT.ELTK1) GO TO 74
      NLT=NLT+1
      IF (NLT.EQ.NTS) GO TO 76
      GO TO 72
73    SELT(NELT)=ACV
      GO TO 77
74    NLTP1=NLT+1
```

```
      I=NTS
75    SELT(I)=SELT(I-1)
      IF (I.EQ.NLTP1) GO TO 76
      I=I-1
      GO TO 75
76    SELT(NLT)=ACV
77    NELTL=NELTL+1
      TEX(9)=0
      TEX(10)=1
      ELTK=K1*SELT(NELT)
      GO TO 68
C
C     DOUBLE PRECISIION UPDATING
C
78    LOOP=0
      DH=H
      DO 79 I=1,N
      PHI=ERVOVH(I)+YINCR(I)/2.0
      DPHI=PHI
79    CUVAR(I+1)=VAR(I+1)+DH*DPHI
      CUVAR(1)=VAR(1)+DH
      CALL DERSUB
      IF (II.EQ.4) GO TO 87
      CALL CHSUB
      IF (II-2) 81,88,80
80    TEST=0
81    DO 82 I=1,N
82    FDERV(I)=DER(I+1)
      SUMHAF=SUMHAF+NHAF-STEP(1)
      STEP(1)=STEP(2)
      STEP(2)=STEP(3)
      STEP(3)=NHAF
      NHAF=0
      IERR=1
      IF (SUMHAF-8) 83,83,99
83    DO 84 I=1,NP1
84    VAR(I)=CUVAR(I)
      TEX(12)=1
85    KK3=4
      IF (ITEXT.EQ.1) GO TO 94
86    IF (TEST.EQ.1) GO TO 19
87    RETURN
C
```

```
C     RECOMPUTE INTERVAL
C
88    TEST=0
      NHAF=0
      II=1
      DX=CI
      TEX(11)=1
      KK3=3
      IF (ITEXT.EQ.1) GO TO 92
89    CIK=CI*K1
      DO 90 I=1,N
      DER(I+1)=FDERV(I)
90    CUVAR(I)=VAR(I)
      CUVAR(N+1)=VAR(N+1)
      IF (TPSH.EQ.0) GO TO 91
      TP=TP-SPEC
      TPK=TP*K1
      TPSH=0
91    IF (LTSH.EQ.0) GO TO 38
      NELT=NELT-1
      REMAIN=0.0
      NELTL=NELTL+1
      ELTK=SELT(NELT)*K1
      GO TO 38
92    PRINT 113, VAR(1),DX
      GO TO 95
93    IF (TEX(1).EQ.1) PRINT 101, VAR(1)
      IF (TEX(2).EQ.1) PRINT 102, CI,CIMAX,SPEC
94    IF (TEX(3).EQ.1) PRINT 103
      IF (TEX(4).EQ.1) PRINT 104, H
      IF (TEX(5).EQ.1) PRINT 105, H
      IF (TEX(6).EQ.1) PRINT 106, H
      IF (TEX(7).EQ.1) PRINT 114, H
      IF (TEX(8).EQ.1) PRINT 107, CI
      IF (TEX(9).EQ.1) PRINT 108, NVAR,CI
      IF (TEX(10).EQ.1) PRINT 115, NVAR,DX
      IF (TEX(11).EQ.1) PRINT 113, VAR(1),DX
      IF (TEX(12).EQ.1) PRINT 109, VAR(1)
      IF (TEX(13).EQ.1) PRINT 110
      IF (TEX(14).EQ.1) PRINT 111
      IF (TEX(15).EQ.1) PRINT 112
95    DO 96 I=3,13
96    TEX(I)=0
```

```
      GO TO (87,70,89,86), KK3
97    IERR=2
      TEX(13)=1
      TEST=0
      GO TO 83
98    IERR=3
      TEX(15)=1
      TEST=0
      GO TO 85
99    IERR=4
      TEST=0
      TEX(14)=1
      GO TO 83
C
C
100   FORMAT (//11H CI IS ZERO)
101   FORMAT (33H INITIALIZATION STARTS AT VAR(1)=,E16.8/)
102   FORMAT (4H CI=,E15.8,9H    CIMAX=,E15.8,8H    SPEC=,E15.8/)
103   FORMAT (37H DX IS THE FULL COMPUTING INTERVAL CI/)
104   FORMAT (28H DX IS A SHORTENED INTERVAL ,E15.8,25H  DUE TO A CRITIC
     1AL VALUE/)
105   FORMAT (28H DX IS A SHORTENED INTERVAL ,E15.8,21H  DUE TO A SPEC V
     1ALUE/)
106   FORMAT (28H DX IS A SHORTENED INTERVAL ,E15.8,39H  DUE TO BOTH A S
     1PEC AND CRITICAL VALUE/)
107   FORMAT (27H CI HAS BEEN LENGTHENED TO ,E16.8/)
108   FORMAT (5H VAR(,I2,32H) HAS CAUSED CI TO BE HALVED TO ,E16.8/)
109   FORMAT (27H VAR(1) HAS BEEN UPDATED TO,E16.8,/)
110   FORMAT (31H ERROR RETURN-ELT NOT MONOTONIC/)
111   FORMAT (55H ERROR RETURN-HAVE HALVED 9 TIMES OVER LAST 3 INTERVALS
     1/)
112   FORMAT (45H ERROR RETURN-HAVE HALVED 9 CONSECUTIVE TIMES/)
113   FORMAT (31H INTERVAL RECOMPUTED AT VAR(1)=,E16.8,9H WITH DX=,E16.8
     1/)
114   FORMAT (25H DX IS SHORTENED INTERVAL,E16.8,28H DUE TO A PREVIOUS E
     1LT VALUE/)
115   FORMAT (5H VAR(,I2,32H) HAS CAUSED DX TO BE HALVED TO ,E16.8,38H B
     1UT NOT CI SINCE CI ALREADY SHORTENED/)
      END
```

LAGRAN.- Library subroutine used by DISCOT.

```
      SUBROUTINE LAGRAN (XA,X,Y,N,ANS)
      DIMENSION X(2), Y(2)
C     DIMENSION X(2),Y(2)
      SUM=0.0
      DO 3 I=1,N
      PROD=Y(I)
      DO 2 J=1,N
      A=X(I)-X(J)
      IF (A) 1,2,1
1     B=(XA-X(J))/A
      PROD=PROD*B
2     CONTINUE
3     SUM=SUM+PROD
      ANS=SUM
      RETURN
      END
```

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Identification: LF-VGAUSS

FORTRAN IV Coded

Purpose: To compute the integrals $\int_a^b F_i(x)\,dx$ for i = 1, 2, 3, ----, number.

Restrictions: An EXTERNAL statement for the name of the subprogram, FUNC (x, FOFX), must appear in the calling program. If values other than x and FOFX are needed in FUNC, they may be transmitted via COMMON. FOFX (NOFX) and SUM(NOFX) must be dimensioned in the calling program.

Usage: Call VGAUSS (A, B, N, SUM, FUNC, FOFX, NOFX, K)

where A - Lower Limit of integration

B - upper limit of integration

N - An integer used to divide the interval (a, b)

The interval (a, b) is divided into N equal intervals and a K-point quadrature is performed on each of the intervals.

SUM(NOFX) - One dimensional array for answers

FUNC(x,FOFX) - Name of subprogram which evaluates $F_i(x)$- (Only two arguments in list.)

FOFX(NOFX) - One-dimensional array for the functions evaluated in FUNC.

NOFX - The number of functions to be evaluated

K - An integer which determines the quadrature formula the routine will use. K may equal 3, 4, ---10, 16, 32.

Accuracy: The routine is at least as accurate as Simpson's rule. The accuracy depends on N and K.

Method: Gauss Quadrature Method: This technique gives the most accurate quadrature formula for a given number of ordinates. The interval (a, b) is subdivided into K * N intervals. Thus, if N is 5 and K is 10, the number of points used to calculate the integral is 50.

Storage: 364 Octal

```
      SUBROUTINE VGAUSS (A,B,N,SUM,FUNC,FOFX,NOFX,K)
C
C        SUBROUTINE TO USE VARIABLE GAUSSIAN WEIGHTING VALUES
C          A = LOWER LIMIT OF INTEGRATION
C          B = UPPER LIMIT OF INTEGRATION
C          N = INTEGER TO GIVE NO. OF INTERVALS, I.   I = N * K
C          SUM(NOFX) = ANSWER ARRAY, ON RETURN = 0. IF A=B.
C          FUNC - NAME OF ROUTINE, TYPED EXTERNAL, TO SOLVE F(X)
C          FOFX(NOFX) = ARRAY OF FUNCTIONS EVALUATED AT X IN FUNC
C          NOFX = THE NUMBER OF FUNCTIONS TO BE INTEGRATED
C          K = INTEGER TO CHOOSE WEIGHTING TABLE
C            = 0,4,5,6,7,8,9,10,16, OR 32
C
      DIMENSION U(52), R(52), ITAB(8), FOFX(NOFX), SUM(NOFX)
C
      DATA (U(I),I=1,52)/0.5,.11270166,.33000947,.06943184,0.5,.2307653
     14,.046910077,.38069040,.16939530,.033765242,0.5,.29707742,.1292344
     20,.025446043,.40828267,.23723379,.10166676,.019855071,0.5,.3378732
     38,.19331428,.081984446,.015919880,.42556283,.28330230,.16029521,.0
     467468316,.013046735,.45249374,.35919822,.27099161,.19106187,.12229
     5779,.067184398,.027712488,.0052995325,.47584016,.42776401,.3803563
     61,.33406569,.28932436,.24655004,.20614212,.16847786,.13390894,.102
     775810,.075316193,.051839422,.032546962,.017618872,.0071942442,.001
     83680690/
      DATA (R(I),I=1,52)/.22222222,.27777777,.32607257,.17392742,.142222
     122,.23931433,.11846344,.23395696,.18038078,.085662246,.10448979,.1
     29091502,.13985269,.064742483,.18134189,.15685332,.11119051,.050614
     3268,.082559838,.15617353,.13030534,.090324080,.040637194,.14776211
     4,.13463335,.10954318,.074725674,.033335672,.094725305,.091301707,.
     5084578259,.074797994,.062314485,.047579255,.031126761,.013576229,.
     6048270044,.047819360,.046922199,.045586939,.043826046,.041655962,.
     7039096947,.036172897,.032911111,.029342046,.025499029,.021417949,.
     8017136931,.012696032,.0081371973,.0035093050/,(ITAB(I),I=1,8)/0,2,
     94,7,10,14,18,23/
C
      DO 1 L=1,NOFX
    1 SUM(L)=0.0
      IF (A.EQ.B) RETURN
      NN=(K+1)/2
      IF (K-16) 2,3,4
    2 J=ITAB(K-2)
      GO TO 5
```

```
3       J=28
        GO TO 5
4       J=36
5       FINE=N
        DELTA=FINE/(B-A)
        DO 7 KK=1,N
        XI=KK-1
        FINF=A+XI/DELTA
        DO 6 I=1,NN
        LRB=I+J
        UU=U(LRB)/DELTA+FINE
        CALL FUNC (UU,FOFX)
        DO 6 JB=1,NOFX
6       SUM(JB)=R(LRB)*FOFX(JB)+SUM(JB)
        DO 7 JJ=1,NN
        LRB=JJ+J
        UU=(1.0-U(LRB))/DELTA+FINE
        CALL FUNC (UU,FOFX)
        DO 7 JB=1,NOFX
7       SUM(JB)=R(LRB)*FOFX(JB)+SUM(JB)
        DO 8 NX=1,NOFX
8       SUM(NX)=SUM(NX)/DELTA
        RETURN
        END
```

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