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A Recurrence Matrix Method for the Analysis of Longitudinal and Torsional Vibrations in Non-Uniform Multibranch beams with Variable Boundary Conditions

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16. Abstract

An approximate method for calculating the longitudinal and torsional natural frequencies and associated modal data of a beamlike, variable cross section multibranch structure is presented in this paper. The procedure described in this paper is the numerical integration of the first order differential equations that characterize the beam element in longitudinal motion and that satisfy the appropriate boundary conditions.

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SUMMARY

An approximate method for calculating the longitudinal and torsional natural frequencies and associated modal data of a beamlike, variable cross section multibranch structure is presented in this paper.

Natural vibration characteristics of systems having discontinuous physical properties are usually most conveniently solved by means of the digital computer. The procedure described in this paper is the numerical integration of the first order differential equations that characterize the beam element in longitudinal motion and that satisfy the appropriate boundary conditions.

Numerical examples included in this paper are an application to a solid fuel launch vehicle and an idealized beam. A complete description and discussion of the computer program is also provided.

INTRODUCTION

It is often necessary to determine the longitudinal and/or torsional natural frequencies and related modal data of structures since they provide basic dynamic information. Typical structures which require knowledge of the longitudinal or torsional vibration characteristics include piles, turbines, machine shafts, bridges, and towers. Natural mode characteristics are also a valuable tool in analyzing the responses of a structure due to disturbing forces.

In all but the most elementary beams, approximate methods must be utilized to determine the longitudinal or torsional natural frequencies. There are numerous references of approximate methods to calculate the longitudinal natural frequencies for variable cross section beams (see references 1, 2, and 3). However, there is a scarcity of available references to calculate the longitudinal frequencies of branched beams; in particular when the branches are partially constrained to the main member.

The purpose of this paper is to present an approximate method for the calculation of the longitudinal natural frequencies and mode shapes for a variable cross section, multibranch beam. This method is derived for the general case in which four arbitrary members intersect at a joint. Also, the branches may be fully or partially constrained to the main member.

The calculation of the longitudinal natural frequencies for a variable cross section beam has been treated by both a stiffness and flexibility matrix approach (references 1 and 4). In the stiffness matrix approach, the beam is idealized as a number of point masses connected by springs. The simultaneous equations of harmonic motion for the point masses when written in matrix notation, yield an eigenvalue problem which is solvable by standard methods.

One disadvantage of both the flexibility and stiffness matrix methods is that considerable time may be spent averaging the properties. Another disadvantage is that it may require the storage of very large matrices in the computer. In fact, for some cases, the number of masses required for accuracy may be greater than the computer storage capability.

The method presented in this paper utilizes a finite difference approach. The station properties are computed from one end of the beam to the other by numerical integration. This offers an important advantage for continuous structures. Through a recurrence equation, a very large number of stations may be utilized without storing a large matrix in the computer.

SYMBOLS

$A(x)$	stressed cross-sectional area, inch^2 (meter^2)
A	matrix (see equation (10))
B	matrix (see equation (19))
$E(x)$	modulus of elasticity, pounds/inch^2 (newton/meter^2)
$G(x)$	modulus of shear, pound/inch^2 (newton/meter^2)
$Ip(x)$	polar mass moment of inertia, pound-second^2 (newton - second^2)
$J(x)$	polar moment of inertia, inch^4 (meters^4)
L	overall length of main beam (see sketch 11)
$m(x)$	mass per unit length, $\frac{\text{pound - second}^2}{\text{inch}^2}$ $(\frac{\text{newton - second}^2}{\text{meter}^2})$
N	total number of structural members
$P(x)$	axial force, pounds (newtons)
$T(x)$	torque, inch-pounds (meter-newtons)
t	time, sec.
U	modal displacement in the axial direction, inches (meters)
$U(s,n)$	modal deflection of the nth integration station of the sth member, inches (meters)
$\ddot{U}$	modal acceleration in the axial direction, inches/second^2 (meters/second^2)
W	matrix (see equation (8))
x	longitudinal coordinate along beam centerline, inches (meters)
x_n	longitudinal coordinate at the nth integration station, inches (meters)
Δx_n	increment in recurrence solution, $x_{n+1}-x_n$, inches (meters)
Y_n	matrix (see equation (15))
$Y(s,n)$	the state vector at the nth integration station of the sth member

$Y'(s,n)$	the X derivative of $Y(s,n)$, $\frac{dY}{dx}(s,n)$
Z	the combination of all the state vectors at the boundary
$\alpha(s)$	translational flexibility constant between members s and i, inch/pounds (meter/newtons)
$\beta(s)$	spring constant between member s and ground, pounds/inch (newtons/meter)
η	constant (see equation (54))
$\gamma(\omega)$	matrix (see equation (37))
$\zeta(x)$	amplitude of mode shape, unitless
$\theta(x,t)$	cross-section rotation (radians)
ω	circular frequency

Subscripts

i	index denoting the ith beam or branch
j	index denoting the jth beam or branch
k	index denoting the kth branch
ℓ	index denoting the ℓth branch
s	index denoting a general beam or branch
r(i)	rth station of the ith beam or branch
r(j)	rth station of the jth beam or branch
r(k)	rth station of the kth branch
$r(\ell)$	rth station of the ℓth branch
r(s)	rth station of the sth beam or branch

Matrix notation

{ }	column matrix
[ ]	square or rectangular matrix
⌊ ⌋	row matrix
⌈1⌋	identity matrix

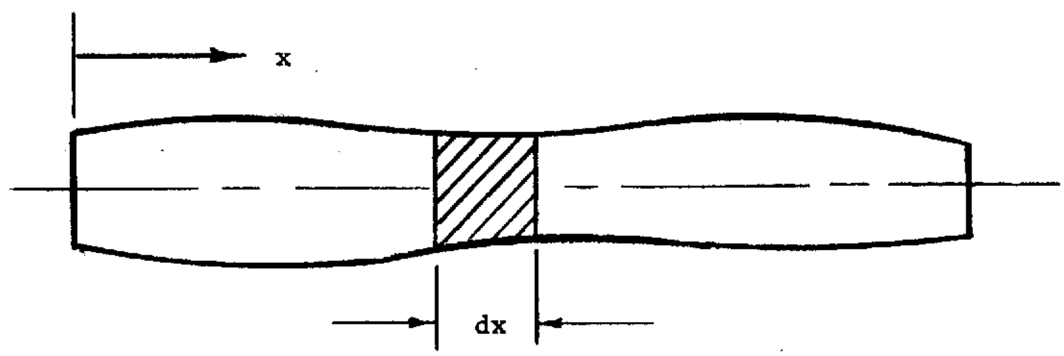
ANALYSIS

Two first order differential equations are developed for a beam element in longitudinal motion. The two equations are the elastic equation which relates the displacement function to the axial force and axial stiffness and the dynamic equation (D'Alembert's Principle) in which the inertial forces are equated to the applied forces. These equations can be put in matrix notation and integrated along the beam by a numerical procedure.

Each end of a beam is considered a joint. A method is developed for determining the boundary value equations at each joint. By combining all boundary value equations into products of a coefficient and station property matrix, it is demonstrated how the natural frequencies and corresponding mode shapes can be calculated. The derivations are based on one dimensional beam theory; therefore to apply this method to structures for which one dimensional beam theory is not applicable, one must proceed with reservations.

Derivation of Equations for a Single Beam

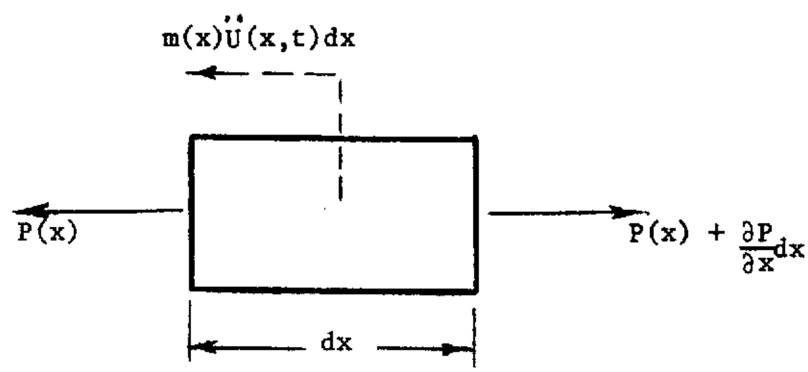
Undergoing Longitudinal Motion



Variable cross section beam.

Sketch 1.

Consider a general non-uniform beam of sketch 1 which is vibrating in the longitudinal direction. If an element of width dx is removed from the beam then we have the following inertial and applied forces of sketch 2.



Beam element in longitudinal motion.

Sketch 2.

Summing forces we get

$$P(x) + \frac{\partial P(x)}{\partial x}\,dx - P(x) - m(x)\,\ddot{U}\,(x,t)\,dx = 0 \qquad (1)$$

$$\frac{\partial P(x)}{\partial x} = m(x)\,\ddot{U}\,(x,t) \qquad (2)$$

Assuming that the element vibrates in simple harmonic motion of frequency ω and amplitude $\zeta(x)$, then, $U(x,t) = \zeta(x)e^{i\omega t}$ or

$$\ddot{U}\,(x,t) = -\omega^2\,U\,(x,t) \qquad (3)$$

Substituting equation (3) into equation (2), the equation of longitudinal motion for the beam element becomes

$$\frac{\partial P}{\partial x}(x) = -m(x)\,\omega^2\,U(x,t) \qquad (4)$$

From elementary beam theory the equation relating the displacement function to the axial force and axial stiffness is

$$\frac{\partial U}{\partial x}(x) = \frac{P(x)}{A(x)E(x)} \qquad (5)$$

If the beam is assumed to vibrate longitudinally in simple harmonic motion then we can characterize the axial force and displacement as

$$\begin{aligned} P(x,t) &= P(x)e^{i\omega t} \\ U(x,t) &= U(x)e^{i\omega t} \end{aligned} \qquad (6)$$

where $i = \sqrt{-1}$

Equation (4) and (5) can be written in matrix notation as

$$\frac{d}{dx}\begin{Bmatrix} P \\ U \end{Bmatrix} = \begin{bmatrix} 0 & -m\omega^2 \\ 1/AE & 0 \end{bmatrix}\begin{Bmatrix} P \\ U \end{Bmatrix} \qquad (7)$$

or

$$\{Y'(x)\} = [W(x)]\{Y(x)\} \qquad (8)$$

where $Y' = \frac{dY(x)}{dx}$ and $Y(x) = \begin{Bmatrix} P \\ U \end{Bmatrix}$ (9)

Method of Integration

We can integrate this matrix along the beam by a variety of numerical procedures. For example, the second order Runge-Kutta integration of this matrix is given by the following

$$\{Y_{n+1}\} = \{Y_n\} + \frac{1}{2}\left[\{k_1\} + \{k_2\}\right] \tag{10}$$

where $\{Y_n\} = \{Y(x_n)\}$ and $\Delta x_n = x_{n+1} - x_n$

$$\{k_1\} = \Delta x_n \, [W_n] \, \{Y_n\} \tag{11}$$

$$\{k_2\} = \Delta x_n \, [W_{n+1}][\{Y_n\} + \{k_1\}] \tag{12}$$

Replacing the k vectors leads to

$$\{Y_{n+1}\} = [A] \, \{Y_n\} \tag{13}$$

The transfer matrix has the form

$$[A_n] = [\diagdown 1 \diagdown] + \frac{\Delta x_n}{2}\left[[W_{n+1}] + [W_n]\right] + \frac{\Delta x_n^2}{2}[W_{n+1}][W_n] \tag{14}$$

Combining these matrices, A becomes

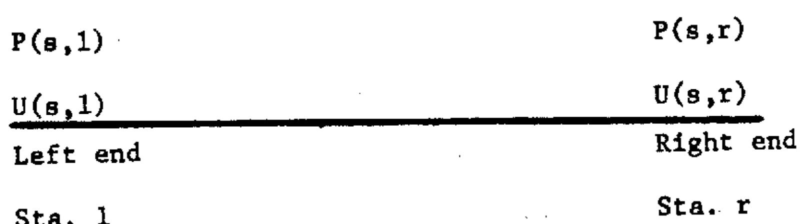
$$[A] = \begin{bmatrix} 1 - \dfrac{m_{n+1}\omega^2 \Delta x_n^2}{2 A_n E_n} & \dfrac{-(m_{n+1} + m_n)\,\Delta x_n\,\omega^2}{2} \\ \dfrac{\Delta x_n}{2}\left(\dfrac{1}{A_{n+1}E_{n+1}} + \dfrac{1}{A_n E_n}\right) & 1 - \dfrac{m_n \omega^2 \Delta x_n^2}{2A_{n+1}E_{n+1}} \end{bmatrix} \tag{15}$$

Therefore (13) may be written as:

$$\{Y_{n+1}\} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \{Y_n\} \tag{16}$$

Derivation of Member Influence Matrix

The member influence matrix relates the right end boundary conditions to the left end boundary conditions. For example, in sketch 3, the left end boundary conditions are P(s,1) and U(s,1) and the right end boundary conditions are P(s,r(s)) and U(s,r(s)).



Typical structural member.

Sketch 3.

The right end boundary conditions can be related to the left boundary conditions by the influence matrix [B] and takes the form of

$$\begin{Bmatrix} P(s,r(s)) \\ \\ U(s,r(s)) \end{Bmatrix} = [B(s)] \begin{Bmatrix} P(s,1) \\ \\ U(s,1) \end{Bmatrix} \tag{17}$$

The member influence matrix [B(s)] is computed by means of successive multiplication of the previously developed interval transfer matrix, [A(s)]. For the typical member of sketch 3

$$\begin{Bmatrix} P(s,n+1) \\ \\ U(s,n+1) \end{Bmatrix} = [A(s,n)] \begin{Bmatrix} P(s,n) \\ \\ U(s,n) \end{Bmatrix} \tag{18}$$

Each interval transfer matrix relates the station properties at the two ends of an interval. For example, again referring to sketch 3

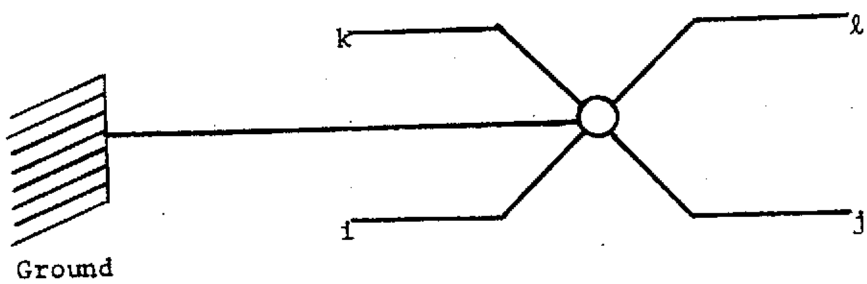
$$[B(s)] = [A(s,(r(s)-1))]\ [A(s,(r(s)-2))]\ .\ .\ .\ [A(s,1)] \tag{19}$$

Elements of the interval transfer matrix are functions of the frequency and physical characteristics which mark the boundary of the interval. Equation (17) can now be written in terms of the state vector for the sth beam with integrating stations $n = 1$ to $n = r(s)$.

$$\{Y(s,(r(s))\} = [B(s)]\ \{Y(s,1)\} \tag{20}$$

Derivation of the Boundary Equations of a Joint

Having from One to Four Structural Members



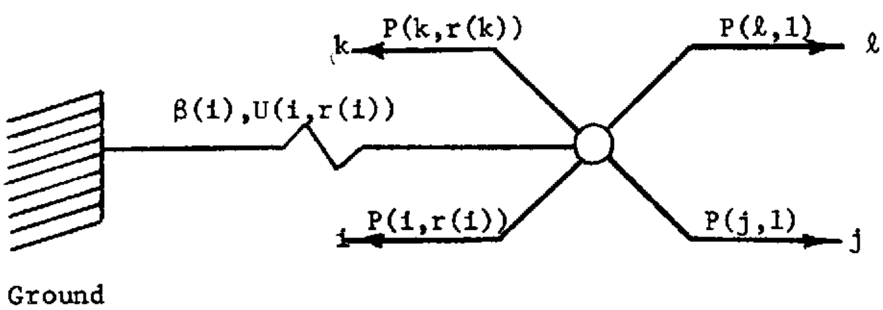
Typical joint of a multibranch beam.

Sketch 4.

A multibranch beam can be considered as a truss-like structure which consists of several members tied together by a series of joints. At each joint the boundary values must satisfy a set of equilibrium and compatibility equations.

A general set of boundary equations is derived for four arbitrary members which are designated by the indices i, j, k, and ℓ (see sketch 4). The joint representation allows for a general flexibility constraint between the members i or j and each of the other members. Also included are provisions for a general elastic constraint between members i or j and an external ground. A ground constraint can be placed on member j only when member i does not exist.

<u>Equilibrium equation.</u>- If the joint in sketch 4 is removed, and a freebody diagram is made of the axial forces



Freebody diagram of axial forces on joint.

Sketch 5.

The equilibrium equation for all the forces acting in the longitudinal direction is

$$P(i,r(i)) + P(k,r(k)) + \beta(i)\ U(i,r(i)) - P(\ell,1) - P(j,1) = 0 \qquad (21)$$

Equation (21) is derived on the assumption that four members are present, however this equation is valid for less than four members at the joint. Also the joint may be free of the ground constraint by setting $\beta(i)$ equal to zero. Then equation (21) may be rewritten as

$$P(i,r(i)) + \delta(k)\ P(k,r(k)) + \beta(i)\ U(i,r(i)) - \delta(\ell)\ P(\ell,1) - \delta(j)\ P(j,1) = 0 \qquad (22)$$

where $\delta(\ ) = 0$ for $(\ ) = 0$

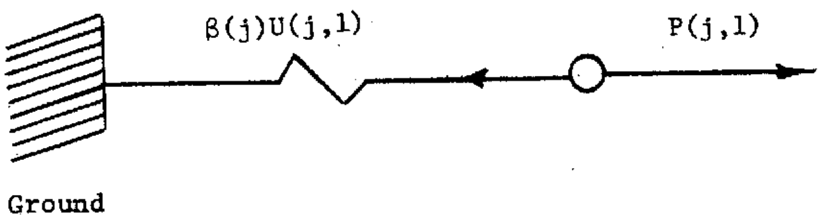
$\delta(\ ) = 1$ for $(\ ) \neq 0$

and i, j, k, or l = 0 if not present at joint

In the special care where member i is not present then the equilibrium equation is

$$\beta(j)U(j,1) - P(j,1) = 0 \quad (23)$$

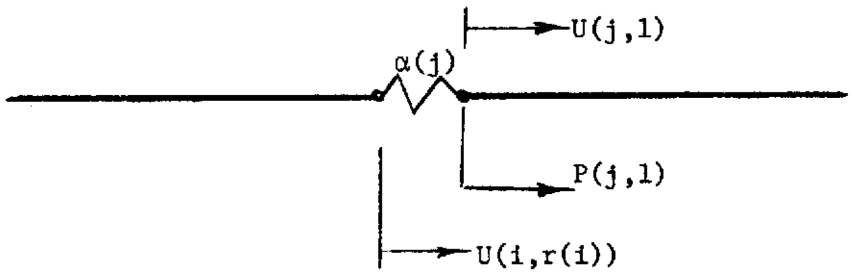
A freebody diagram of the axial forces on the joint of sketch 4 when members i, k, and l are absent is illustrated in sketch 6.



Freebody diagram of axial forces on joint with member i missing.

Sketch 6.

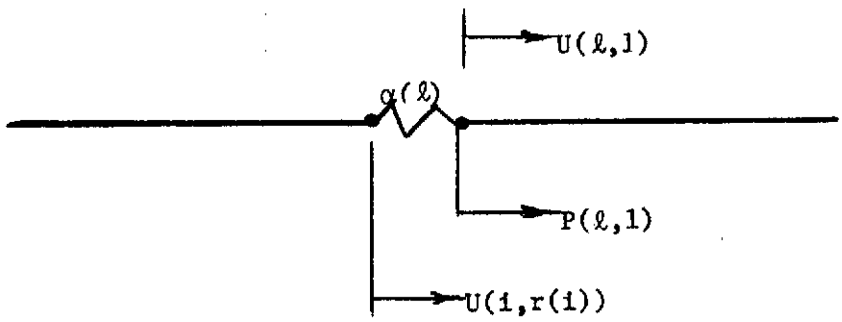
Compatability equations.- There is a compatibility relationship between the main member (i or j) and each branch member. The compatibility equations are derived on the assumption that there is no relative displacement between members at a joint except through spring deformations.



Compatibility relationship with member i and j present.

Sketch 7.

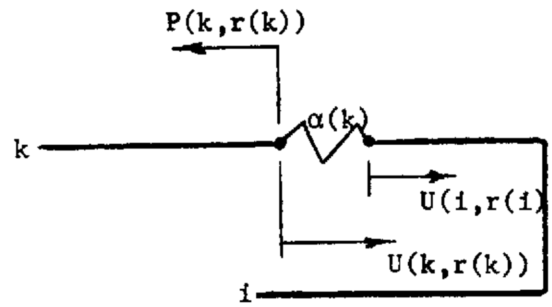
$$U(i,r(i)) + \alpha(j)\ P(j,1) - U(j,1) = 0 \tag{24}$$



Compatibility relationship with members i and ℓ present.

Sketch 8.

$$U(i,r(i)) + \alpha(\ell)\, P(\ell,1) - U(\ell,1) = 0 \tag{25}$$



Compatibility relationship with members i and k present.

Sketch 9.

$$U(i,\ r(i)) - \alpha(k)P(k,\ r(k)) - U(k,\ r(k)) \tag{26}$$

The state vector (equation 9) can be put in the form

$$\{Y(i,r(s))\} = \begin{Bmatrix} P(i,r(s)) \\ U(i,r(s)) \end{Bmatrix} \tag{27}$$

Utilizing (27), the equilibrium equations (22) and (23) can be combined to give

$$\delta(i)\lfloor 1 \;\; \beta(i)\rfloor\{Y(i,r(i))\} - \delta(j)\lfloor 1 \;\; (\delta(i) - 1) \;\; \beta(j)\rfloor\{Y(j,1)\}$$

$$+ \; \delta(i) \; \delta(k)\lfloor 1 \;\; 0\rfloor\{Y(k,r(k))\} - \delta(i) \; \delta(\ell)\lfloor 1 \;\; 0\rfloor\{Y(\ell,1)\} = 0 \qquad (28)$$

The three compatibility equations, (24) through (26), can be written as a function of the state vector.

$$\delta(i)\delta(j)\lfloor 0 \;\; 1\rfloor\{Y(i,r(i))\} - \delta(i)\delta(j)\lfloor -\alpha(j) \;\; 1\rfloor\{Y(j,1)\} = 0 \qquad (29)$$

$$\delta(i)\delta(k)\lfloor 0 \;\; 1\rfloor\{Y(i,r(i))\} - \delta(i)\delta(k)\lfloor \alpha(k) \;\; 1\rfloor\{Y(k),r(k)\} = 0 \qquad (30)$$

$$\delta(i)\delta(\ell)\lfloor 0 \;\; 1\rfloor\{Y(i,r(i))\} - \delta(i)\delta(\ell)\lfloor -\alpha(\ell) \;\; 1\rfloor\{Y(\ell,1)\} = 0 \qquad (31)$$

From equation (20), we have the relationship for the sth beam at the last integration station to the sth beam at the first integration station.

$$\{ Y(s,r(s)) \} = [B(s)]\{Y(s,1)\} \qquad (32)$$

By substituting equation (32) into equations (28), thru (31) the final form of the four boundary value equations is obtained.

$$\delta(i)\lfloor 1\ \beta(i)\rfloor[B(i)]\{Y(i,1)\} - \delta(j)\lfloor 1\ (\delta(i) - 1)\ \beta(j)\rfloor\{Y(j,1)\}$$

$$+ \delta(i)\delta(k)\lfloor 1\ 0\rfloor[B(k)]\{Y(k,1)\} - \delta(i)\delta(\ell)\lfloor 1\ 0\rfloor\{Y(\ell,1)\} = 0 \quad (33)$$

$$\delta(i)\delta(j)\lfloor 0\ 1\rfloor[B(i)]\{Y(i,1)\} - \delta(i)\delta(j)\lfloor -\alpha(j)\ 1\rfloor\{Y(j,1)\} = 0 \quad (34)$$

$$\delta(i)\delta(k)\lfloor 0\ 1\rfloor[B(i)]\{Y(i,1)\} - \delta(i)\delta(k)\lfloor \alpha(k)\ 1\rfloor[B(k)]\{Y(k,1)\} = 0 \quad (35)$$

$$\delta(i)\delta(\ell)\lfloor 0\ 1\rfloor[B(i)]\{Y(i,1)\} - \delta(i)\delta(\ell)\lfloor -\alpha(\ell)\ 1\rfloor\{Y(\ell,1)\} = 0 \quad (36)$$

When applying these boundary value equations to a particular joint, any member and it's respective equation can be left out by setting it's index equal to zero. There are more unknowns than there are equations at each joint. The boundary values at each joint can only be found by solving all of the equations simultaneously. The complete set of boundary value equations for the structure can therefore be written collectively as a product of the beam boundary conditions column matrix and the coefficient square matrix.

$$[\gamma(\omega)]\{Z\} = 0 \quad (37)$$

Calculation of the Natural Frequencies

The nontrivial solution to equation (37) requires the vanishing of the coefficient determinant

$$[\gamma(\omega)] = 0 \quad (38)$$

The expansion of the determinant of (38) yields the characteristic equation which is polynomial in ω. It is necessary to conduct a trial search by successive approximation of the eigenvalue ω to find the characteristic roots (or natural frequencies) which satisfy eq. (38).

For each characteristic root of equation (38) there is a corresponding eigenvector $\{Z\}$ where

$$\{Z\} = \begin{Bmatrix} Y(1,1) \\ \vdots \\ Y(N,1) \end{Bmatrix} \tag{39}$$

where

$$\{Y(s,n)\} = \begin{Bmatrix} P(s,n) \\ U(s,n) \end{Bmatrix} \tag{40}$$

The value of $\{Z\}$ can be determined by setting an arbitrary non-zero element of $Y(1,1)$, equal to unity and solving for the remaining elements in terms of the unit element from $\{Z\}$, creating $\{Z_{mod}\}$. The corresponding row is removed from $\left[\gamma(\omega)\right]$ the corresponding column is moved to the right side to convert (37) to the following

$$[\gamma_{mod}] \cdot \{Z_{mod}\} = -\{\text{column}\} \tag{41}$$

This set of simultaneous equations is solved for $\{Z_{mod}\}$. That element which was previously removed is then returned to $\{Z_{mod}\}$, forming the solution vector $\{Z\}$. After the left end boundary conditions of a beam are determined, then the $\{Y(s,n)\}$ modal data can be determined at each integration station along a segment by successive use of the recurrence equation (13).

$$\{Y(s,n+1)\} = [\,A\,]\{Y(s,n)\} \tag{42}$$

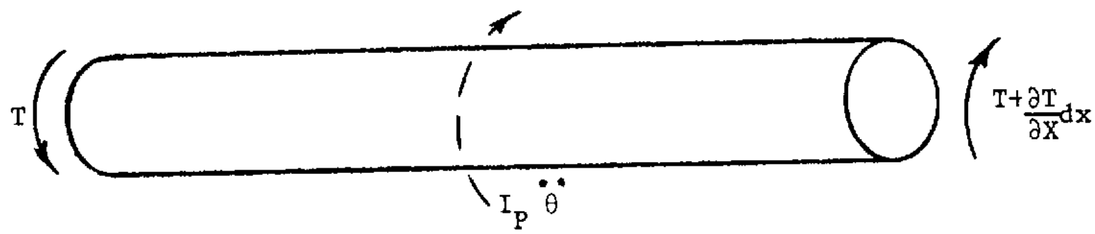
If the rate of change of the modal properties are desired, then from equation (8)

$$\{Y'(s,n)\} = [\,W\,]\{Y(s,n)\} \tag{43}$$

It should be noted that this method is applicable for determining the modal data corresponding to any frequency.

Application of the Method to Torsional Vibrations

If the elastic axis of all beams and branches are everywhere concentric, it is also possible to derive the two first order differential equations for torsional vibrations analogous to the longitudinal vibration equations.



Beam element in torsional motion.

Sketch 10.

By D'Alembert's Principle, the dynamic equation for torsional motion is found to be

$$-T(x) + T(x) + \frac{\partial T(x)}{\partial x} \quad dx - I_p(x) \frac{\partial^2 \theta}{\partial t^2} dx = 0 \tag{44}$$

or

$$\frac{\partial T(x)}{\partial x} = I_p(x) \frac{\partial^2 \theta}{\partial t^2} \tag{45}$$

If simple harmonic motion is assumed, then

$$\ddot{\theta} = -\omega^2 \theta(x,t) \tag{46}$$

Substituting equation (46) into equation (45)

$$\frac{\partial T(x)}{\partial x} = -I_p(x) \omega^2 \theta(x,t) \tag{47}$$

For longitudinal vibrations, the corresponding analogy is

$$\frac{\partial P(x)}{\partial x} = -m(x)\, \omega^2\, U(x,t) \tag{48}$$

The elastic equation for the cross-sectional rotational function for a concentric member is

$$\frac{\partial \theta(x)}{\partial x} = \frac{T(x)}{J(x)G(x)} \tag{49}$$

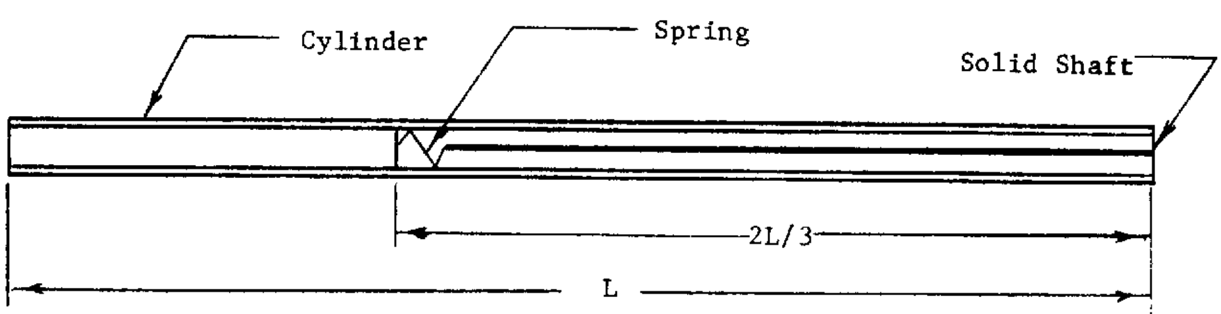
The corresponding equation for the longitudinal displacement function is

$$\frac{\partial U(x)}{\partial x} = \frac{P(x)}{A(x)E(x)} \tag{50}$$

If equation (48) is compared with equation (47) and equation (50) is compared with equation (49), it is seen that JG is analogous to AE, I_p is analogous to m, and θ is analogous to U. Therefore by replacing JG, I_p, and θ with AE, m, and U, respectively, all of the previously derived equations for the longitudinal vibrations of beams can be applied directly to torsional vibration analysis of beams whose elastic equation and equation of motion are characterized by equations (47) and (49).

NUMERICAL EXAMPLE FOR IDEALIZED BEAM

For the purpose of illustrating the procedure to calculate the longitudinal and torsional vibrations, an idealized beam is shown in sketch 11. This example consists of a solid uniform circular shaft attached by a spring to the inside of a cylindrical shell. The spring is assumed to elongate only in the longitudinal direction. The physical characteristics of the idealized beam are provided in the computer program discussion.



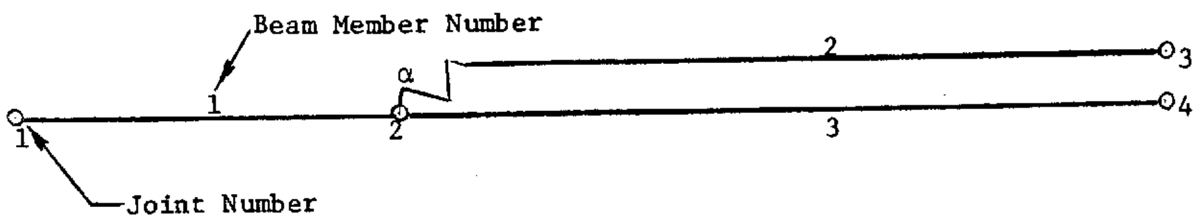
Idealized beam.

Sketch 11.

Matrix Formulation for Idealized Beam

The procedure, as outlined in the analysis, is to write the boundary value equations for each joint; put the equations in matrix notation, and arrange as a product of the coefficient matrix and station property matrix.

The mathematical model of the idealized beam is illustrated in sketch 12.



Numbering of joints and members.

Sketch 12.

$$[\gamma(\omega)]\{Z\} = 0 \tag{50}$$

Beam Member

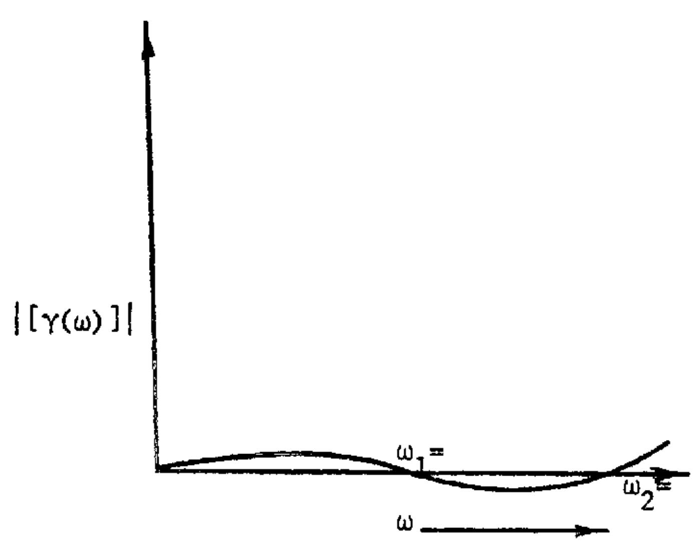
$$\begin{array}{c} \text{Joint Number} \\ (1) \\ \\ (2) \\ \\ (3) \\ (4) \end{array}
\left[\begin{array}{c:c:c}
1 & 2 & 3 \\
\lfloor 1 \quad 0 \rfloor & & \\ \hdashline
\lfloor 1 \quad 0 \rfloor[B(1)] & \lfloor 1 \quad 0 \rfloor[B(2)] & -\lfloor 1 \quad 0 \rfloor \\
\lfloor 0 \quad 1 \rfloor[B(1)] & -\lfloor -\alpha(\ell) \quad 1 \rfloor[B(2)] & -\lfloor 0 \quad 1 \rfloor \\
\lfloor 0 \quad 1 \rfloor[B(1)] & & \\ \hdashline
 & -\lfloor 1 \quad 0 \rfloor & \\ \hdashline
 & & \lfloor 1 \quad 0 \rfloor[B(3)]
\end{array}\right]
\left\{\begin{array}{c} P_1 \\ U_1 \\ P_2 \\ U_2 \\ P_3 \\ U_3 \end{array}\right\} = 0 \tag{51}$$

The vanishing of the coefficient matrix of eq. 51 is necessary to determine the natural frequencies, i.e.

$$[\gamma(\omega)] = 0 \tag{52}$$

$$\left[\begin{array}{c|c|c}
-\lfloor 1 \quad 0 \rfloor & & \\
\hline
\lfloor 1 \quad 0 \rfloor [B(1)] & \lfloor 1 \quad 0 \rfloor [B(2)] & -\lfloor 1 \quad 0 \rfloor \\
\lfloor 0 \quad 1 \rfloor [B(1)] & -\lfloor -\alpha(\ell) \quad 1 \rfloor [B(2)] & \\
\lfloor 0 \quad 1 \rfloor [B(1)] & & -\lfloor 0 \quad 1 \rfloor \\
\hline
 & -\lfloor 1 \quad 0 \rfloor & \\
\hline
 & & \lfloor 1 \quad 0 \rfloor [B(3)]
\end{array}\right] = 0 \tag{53}$$

Sketch (13) illustrates the plot of $|[\gamma(\omega)]|$ vs. ω. The natural frequencies are obtained when $|[\gamma(\omega)]| = 0$.



Natural frequencies of idealized beam.

Sketch 13.

Calculation of the Modal Data

The longitudinal mode shapes and natural frequencies are illustrated in figure 1. The first mode shape corresponds to the spring mode. The second mode shape corresponds to the first natural longitudinal frequency of the cylinder. The third mode shape corresponds to the first natural longitudinal frequency of the shaft, and the fourth mode shape corresponds to the second longitudinal frequency of the cylinder.

The mode shapes associated with the natural torsional frequencies of the idealized beam are illustrated in figure 2. The first mode shape corresponds to the first natural torsional frequency of the shaft. The second mode shape corresponds to the first natural torsional frequency of the cylinder. The third mode shape corresponds to the second natural torsional frequency of the shaft, and the fourth mode shape corresponds to the second natural torsional frequency of the cylinder.

In order to demonstrate the plotting capability of the program the Calcomp plotting instruction card is illustrated in figure 3. The modal data obtained from the Calcomp plot are illustrated in figures 4-8.

This simple numerical example demonstrates the versitility of the program by illustrating that it will calculate the spring modes, branch modes, and beam modes.

TYPICAL LAUNCH VEHICLE APPLICATION

In order to demonstrate the application of the matrix solution to a more realistic structure, a numerical example of an application to a solid fuel booster system is presented. The following assumptions apply to the analysis:

(1) The solid-fuel mass is considered to adhere to the sides of the vehicle along the length and has has no motion relative to the vehicle.

(2) The contribution of the fuel stiffness to the vehicle axial stiffness is negligible.

(3) Damping is considered to be negligible.

(4) All deformations are one-dimensional and no consideration is given to bending or breathing effects of the cylindrical shell wall.

The vehicle physical characteristics were taken from reference 1 and are tabulated in tables I and II. These data are shown graphically in figures 9 and 10. The payload parameters were selected for a typical payload.

Utilizing the method outlined in this paper the natural mode shapes and frequencies were obtained. The longitudinal natural mode shapes for the vehicle and payload are given in figure 11. There is good correlation between the natural frequencies calculated by the lumped mass method of reference 1 and those calculated by the method discussed in this paper. The frequency comparisons are illustrated in table III. The finite difference method is believed to be more accurate for two reasons. The first reason is that there were approximately ten times as many integrating stations in the finite difference method as there were lumped masses. The other reason the finite difference approach should be more accurate is because the discontinuities were accurately input in the program.

DISCUSSION OF METHOD APPLICATION

Computational Accuracy

Some of the points of interest to the user in the application of this method are discussed in this section.

Choice of the number of integration stations.- Usually the choice of the number of stations will present no problem because linear interpolation between the input physical characteristics will provide the number of stations necessary for sufficient accuracy. The computer program developed for this analysis has an upper limit of 20 members (main beam and branches) and up to a total of 600 integrating stations. However, in order to optimize the computer processing time, consistent with the required accuracy it is not usually desirable to utilize the maximum number of stations available in the program.

In order to examine the accuracy of the method discussed, comparisons were made between exact and approximate solution for a beam of exponentially varying cross section. For the approximate solution, the beam was divided into n equally spared intervals with the station properties input at each interval.

If the cross section varies as

$$A(x) = A_o e^{\frac{2\eta x}{L}} \tag{54}$$

The theoretical solution for the natural longitudinal vibrations of the exponentially varying beam is derived in reference 1. The parameter used for frequency comparison (Appendix A of reference 1) is the percent error.

$$\text{Percent error} = \frac{\omega_{exact} - \omega_{approx.}}{\omega_{exact}} \times 100$$

The percent error for the first five elastic modes as a function of the total number of integration stations is given in figure 12 for the natural frequencies calculated by the method discussed in this paper vs. the theoretical natural frequencies. For this error analysis, the cross-sectional ratio of the beam ends in approximately 50. Even with this large variation of the cross-sectional area, it is found that the accuracy of the natural frequencies is primarily dependent on the number of integration stations rather than on the variation of the cross-sectional area. The first five natural longitudinal frequencies of the exponential beam were determined to be within one percent of their theoretical values when only 75 integrating stations were utilized. It has been found that a good rule of thumb for determining the minimum number of integration stations is L/100.

Advantages of the Recurrence Method for Longitudinal and Torsional Vibration Analysis

The prime advantage of the theoretical method over the lumped mass approach for determining the longitudinal vibration of beamlike structures is that the physical characteristics may be input directly, rather than resorting to a finite element representation and averaging the properties. Therefore, the input time is greatly reduced. Another important advantage is that the method is appropriate to highly discontinuous structures. Other advantages of the method are that it is appropriate to any boundary condition at the end point of a branch or beam and that a very large number of stations may be utilized without the storage or inversion of large matrices.

CONCLUDING REMARKS

A finite difference method for the analysis of longitudinal and torsional vibrations of nonuniform multibranch beams is presented. The end of each beam or branch may be fully or partially constrained to the main member. The equations have been programmed for the CDC 6600 Series Computer Systems and have given excellent agreement when compared with numerical examples and exact solutions.

A numerical example of the procedure to calculate the longitudinal and torsional characteristics of an idealized beam along with an application of the method to a launch vehicle are provided. Comparisons of the method with exact solutions indicate that the accuracy of the solution is practically independent of cross-sectional variation, but is primarily dependent on the number of integration stations.

The primary advantage of this method is that highly discontinuous physical characteristics may be input directly. Another important advantage of the program is that the boundary condition may vary from fixed to free at each joint or end.

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TABLE I

PHYSICAL CHARACTERISTICS OF LAUNCH VEHICLE

x, in.	x, meter	m, lb-sec²/in²	m, Newton-sec²/meter²	AE lb x10⁻⁶	AE Newton x10⁻¹
0.000	0.0000	0.036808	253.782	160.00	71.168
15.800	0.4013	0.046624	321.461	295.00	131.266
15.800	0.4013	0.046624	321.461	295.00	131.266
17.276	0.4388	0.046624	321.461	295.00	131.266
17.276	0.4388	0.046624	321.461	280.30	124.683
22.270	0.5658	0.046624	321.461	280.30	124.683
22.270	0.5658	0.046624	321.461	309.40	137.627
24.876	0.6318	0.046624	321.461	309.40	137.627
24.876	0.6318	0.046624	321.461	1201.70	534.540
27.176	0.6903	0.046624	321.461	1201.70	534.540
27.176	0.6903	0.046624	321.461	504.40	224.367
29.776	0.7563	0.046624	321.461	504.40	224.367
29.776	0.7563	0.046624	321.461	613.60	272.942
31.776	0.8071	0.046624	321.461	613.60	272.942
31.776	0.8071	0.046624	321.461	699.90	311.329
33.776	0.8579	0.046624	321.461	699.90	311.329
33.776	0.8579	0.046624	321.461	2350.40	1045.504
34.900	0.8864	0.046624	321.461	2350.40	1045.504
34.900	0.8864	0.071398	492.272	2350.40	1045.504
36.576	0.9290	0.071398	492.272	2350.40	1045.504
36.576	0.9290	0.071398	492.272	841.88	374.485
39.976	1.0153	0.071398	492.272	841.88	374.485
39.976	1.0153	0.071398	492.272	689.30	306.614
41.176	1.0458	0.071398	492.272	689.30	306.614
41.176	1.0458	0.071398	492.272	316.70	104.874
41.576	1.0560	0.071398	492.272	316.70	104.874
41.576	1.0560	0.071398	492.272	576.40	256.394
44.276	1.1245	0.071398	492.272	576.40	256.394
44.276	1.1245	0.071398	492.272	316.70	140.874
45.400	1.1531	0.071398	492.272	316.70	140.874
45.400	1.1531	0.049883	343.931	316.70	140.874
48.176	1.2236	0.049883	343.931	316.70	140.874
48.176	1.2236	0.049883	343.931	576.40	256.394
53.176	1.3506	0.049883	343.931	576.40	256.394
53.176	1.3506	0.049883	343.931	316.70	140.874
116.576	2.9610	0.049883	343.931	316.70	140.874
116.576	2.9610	0.049883	343.931	576.40	256.394
118.000	2.9971	0.049883	343.931	576.40	256.394
118.000	2.9971	0.059360	410.486	576.40	256.394
119.576	3.0372	0.059360	410.486	576.40	256.394
119.576	3.0372	0.059360	410.486	316.70	140.874
177.076	4.4977	0.059360	410.486	316.70	140.874

TABLE I (Continued)

PHYSICAL CHARACTERISTICS OF LAUNCH VEHICLE

x, in.	x, meter	m, lb-sec²/in²	m, Newton-sec²/meter²	AE lb x10⁻⁶	AE Newton x10⁻¹
177.076	4.4977	0.059360	410.486	576.40	256.394
180.076	4.5739	0.059360	410.486	576.40	256.394
180.076	4.5739	0.059360	410.486	316.70	140.874
185.176	4.7034	0.059360	410.486	316.70	140.874
185.176	4.7034	0.059360	410.486	841.90	374.494
187.676	4.7669	0.059360	410.486	841.90	374.494
187.676	4.7669	0.059360	410.486	316.70	140.874
187.776	4.7694	0.059360	410.486	316.70	140.874
187.776	4.7694	0.059360	410.486	1041.30	463.191
188.776	4.7948	0.059360	410.486	1041.30	463.191
188.776	4.7948	0.059360	410.486	2449.20	1089.453
189.100	4.8031	0.059360	410.486	2449.20	1089.453
189.100	4.8031	0.100380	692.096	2449.20	1089.453
192.776	4.8964	0.100380	692.096	2449.20	1089.453
192.776	4.4951	0.100380	692.096	39.00	17.348
193.100	4.9047	0.100380	692.096	39.00	17.348
193.100	4.9047	0.007668	52.869	39.00	17.348
201.156	5.1093	0.007668	52.869	39.00	17.348
201.156	5.1093	0.007668	52.869	78.00	34.696
282.200	5.1358	0.007668	52.869	78.00	34.696
282.200	5.1358	0.018549	127.891	78.00	34.696
203.756	5.1753	0.018549	127.891	78.00	34.696
203.756	5.1753	0.018549	127.891	112.80	50.176
204.200	5.1866	0.018549	127.891	112.80	50.176
204.200	5.1866	0.012378	85.343	112.80	50.176
209.382	5.3182	0.012378	85.343	112.80	50.176
209.382	5.3182	0.012378	85.343	135.20	60.140
211.700	5.3771	0.012378	85.343	135.20	60.140
211.700	5.3771	0.025331	174.651	135.20	60.140
214.300	5.4432	0.025331	174.651	135.20	60.140
214.300	5.4432	0.025331	174.651	369.20	164.228
215.100	5.4635	0.025331	174.651	369.20	164.228
215.100	5.4635	0.025331	174.651	130.00	57.827
219.556	5.5767	0.025331	174.651	130.00	57.827
219.556	5.5767	0.025331	174.651	118.60	52.756
221.716	5.6333	0.025331	174.651	118.60	52.756
221.716	5.6333	0.025331	174.651	169.80	75.174
223.200	5.6692	0.025331	174.651	169.80	75.174
223.200	5.6692	0.023840	164.330	169.80	75.174
224.500	5.7022	0.023840	164.330	169.80	75.174
224.500	5.7022	0.023840	164.330	442.00	196.610
225.356	5.7230	0.023840	164.330	442.00	196.610
225.356	5.7230	0.023840	164.330	280.00	124.905

TABLE I (Continued)

PHYSICAL CHARACTERISTICS OF LAUNCH VEHICLE

x, in.	x, meter	m, lb-sec^2/in^2	m, Newton-sec^2/meter^2	AE lb x10^{-6}	AE Newton x10^{-1}
228.056	5.7926	0.023840	164.330	280.00	124.905
228.056	5.7926	0.023840	164.330	730.10	324.763
229.456	5.8281	0.023840	164.330	730.10	324.763
229.456	5.8281	0.023840	164.330	137.80	61.296
230.700	5.8597	0.023840	164.330	137.80	61.296
230.700	5.8597	0.024559	169.328	137.80	61.296
231.356	5.8764	0.024559	169.328	137.80	61.296
231.356	5.8764	0.024559	169.328	70.50	31.360
331.356	8.4164	0.024559	169.328	70.50	31.360
331.356	8.4164	0.024559	169.328	130.00	57.827
332.900	8.4556	0.024559	169.328	130.00	57.827
332.900	8.4556	0.044689	308.120	431.00	191.984
335.956	8.5332	0.044689	308.120	431.60	191.984
335.956	8.5332	0.044689	308.120	213.20	94.836
337.900	8.5840	0.044689	308.120	213.20	94.836
337.900	8.5840	0.013808	95.203	71.42	31.769
340.500	8.6486	0.013808	95.203	98.60	43.877
340.500	8.6486	0.013808	95.203	98.60	43.877
347.200	8.8188	0.013808	95.203	132.64	58.998
347.200	8.8188	0.025466	175.528	132.64	58.998
348.000	8.8391	0.025466	175.528	136.83	60.865
348.000	8.8391	0.025466	175.528	136.83	60.865
351.000	8.9153	0.025466	175.582	425.60	189.315
351.000	8.9153	0.025466	175.582	425.60	189.315
351.800	8.9357	0.025466	175.582	136.83	60.865
351.800	8.9357	0.025466	175.582	136.83	60.865
357.800	9.0881	0.025466	175.582	112.60	50.087
357.800	9.0881	0.025466	175.582	112.60	50.087
361.200	9.1744	0.025466	175.582	199.10	88.564
361.200	9.1744	0.022530	155.339	612.00	272.230
362.300	9.2024	0.022530	155.339	199.10	88.564
362.300	9.2024	0.022530	155.339	199.10	88.564
364.800	9.2659	0.022530	155.339	336.70	149.771
364.800	9.2659	0.022530	155.339	730.10	324.763
366.200	9.3014	0.022530	155.339	730.10	324.763
366.200	9.3014	0.024559	169.328	195.00	86.740
368.100	9.3497	0.024559	169.328	70.50	31.360
368.100	9.3497	0.024559	169.328	70.50	31.360
467.700	11.8795	0.024559	169.328	70.50	31.360
467.700	11.8795	0.044689	308.120	70.50	31.360
468.100	11.8897	0.044689	308.120	70.50	31.360
468.100	11.8897	0.044689	308.120	70.50	31.360
469.700	11.9303	0.044689	308.120	184.50	82.069

TABLE I (Continued)

PHYSICAL CHARACTERISTICS OF LAUNCH VEHICLE

x, in.	x, meter	m, lb-sec²/in²	m, Newton-sec²/meter²	AE lb x10⁻⁶	AE Newton x10⁻¹
469.700	11.9303	0.044689	308.120	184.50	82.069
471.800	11.9837	0.044689	308.120	529.10	235.354
471.800	11.9837	0.004788	308.120	529.10	235.354
472.700	12.0065	0.004788	308.120	529.10	235.354
472.700	12.0065	0.004788	308.120	187.80	83.537
474.600	12.0548	0.004788	308.120	187.80	83.537
474.600	12.0548	0.004788	308.120	75.40	33.539
477.200	12.1208	0.004788	308.120	75.40	33.539
477.200	12.1208	0.004788	308.120	707.20	314.577
477.500	12.1285	0.004788	308.120	707.20	314.577
477.500	12.1285	0.004788	308.120	75.40	33.539
481.400	12.2275	0.004788	308.120	75.40	33.539
481.400	12.2275	0.004788	308.120	75.40	33.539
483.400	12.2783	0.004788	308.120	216.60	93.679
483.400	12.2783	0.004788	308.120	190.90	84.916
484.700	12.3113	0.004788	33.012	452.40	201.236
484.700	12.3113	0.023784	163.378	452.40	201.236
484.749	12.3380	0.023784	163.378	870.70	387.305
484.749	12.3380	0.023784	163.378	870.70	387.305
485.811	12.3396	0.023784	163.378	870.70	387.305
485.811	12.3396	0.028338	193.605	162.80	72.417
486.749	12.3634	0.028338	193.605	130.00	57.829
486.749	12.3634	0.034161	235.532	130.00	57.829
487.624	12.3856	0.034161	234.642	98.50	43.815
487.624	12.3856	0.017728	122.230	33.46	14.884
488.120	12.3982	0.017728	122.230	33.46	14.884
488.120	12.3982	0.017728	122.230	33.46	14.884
490.749	12.4650	0.017728	122.230	33.46	14.884
490.749	12.4650	0.012320	77.442	33.46	14.884
491.999	12.4967	0.012320	76.376	33.46	14.884
491.999	12.4967	0.008230	73.540	33.46	14.884
494.085	12.6513	0.008230	56.774	33.46	14.884
494.085	12.5513	0.014880	102.594	33.46	14.884
498.549	12.6631	0.014880	102.594	33.46	14.884
498.549	12.6631	0.011827	87.432	33.46	14.884
504.085	12.8037	0.011827	81.544	33.46	14.884
504.085	12.8037	0.005952	41.038	33.46	14.884
504.585	12.8164	0.018245	43.540	33.46	14.884
504.585	12.8164	0.018245	125.795	33.46	14.884
505.499	12.8396	0.018245	125.795	33.46	14.884
505.499	12.8396	0.018245	125.795	182.16	81.028
506.599	12.8676	0.018245	125.795	182.16	81.028
506.835	12.8736	0.045031	310.478	25.74	11.450
507.800	12.8981	0.045031	310.478	25.74	11.450

TABLE I (Continued)

PHYSICAL CHARACTERISTICS OF LAUNCH VEHICLE

x, in.	x, meter	m, lb-sec²/in²	m, Newton-sec²/meter²	AE lb x10⁻⁶	AE Newton x10⁻¹
507.800	12.8981	0.045031	310.478	4.45	1.979
511.899	13.0022	0.045031	310.478	4.20	1.294
511.899	13.0022	0.051756	356.845	4.20	1.294
513.099	13.0372	0.051756	356.845	4.15	1.673
513.099	13.0372	0.045316	312.443	4.15	1.673
536.589	13.6293	0.045316	312.443	3.40	1.512
536.589	13.6293	0.046558	321.006	3.40	1.512
537.909	13.6651	0.046558	321.006	3.35	1.490
537.909	13.6651	0.004710	32.740	3.35	1.490
542.589	13.7817	0.004710	32.740	3.20	1.423
542.589	13.7817	0.015931	109.846	3.20	1.423
546.899	13.8912	0.015931	109.846	3.15	1.401
546.899	13.8912	0.027685	190.881	3.15	1.401
584.849	13.9407	0.027685	190.991	3.10	1.378
584.849	13.9407	0.005435	37.473	3.10	1.378
553.069	14.0479	0.005435	37.473	2.91	1.294
553.069	14.0479	0.005435	40.148	2.91	1.294
554.609	14.0870	0.005435	40.148	2.71	1.205
554.609	14.0870	0.006185	42.644	2.71	1.205
555.999	14.1223	0.005900	40.679	2.41	1.071
555.999	14.1223	0.005900	40.679	2.41	1.071
559.068	14.2003	0.005176	35.687	1.91	0.849
559.068	14.2003	0.005176	35.687	1.91	0.849
560.999	14.2493	0.005176	35.687	1.61	0.716
560.999	14.2493	0.005176	35.687	1.61	0.716
564.109	14.3283	0.004790	33.026	1.42	0.631
564.109	14.3283	0.008851	61.025	1.42	0.631
565.068	14.3527	0.008800	60.674	1.37	0.609
565.068	14.3527	0.008800	60.674	1.37	0.609
567.800	14.4221	0.008800	60.674	1.00	0.448
567.800	14.4221	0.008800	60.674	1.00	0.448
569.427	14.4634	0.008644	59.598	0.782	0.347
569.427	14.4634	0.008644	59.598	0.782	0.347
569.857	14.4744	0.001941	13.383	0.60	0.266
569.857	14.4744	0.001941	13.383	0.60	0.266
575.749	14.6240	0.001579	10.887	0.398	0.177
575.749	14.6240	0.000924	6.371	0.398	0.177
577.800	14.6761	0.000700	4.826	0.36	0.160
577.800	14.6761	0.000700	4.826	0.26	0.115
580.999	14.7573	0.000655	4.516	0.210	0.0934
580.999	14.7573	0.000655	4.516	0.210	0.0934
581.800	14.7781	0.000500	3.447	0.200	0.0098
581.800	14.7781	0.000500	3.447	0.200	0.0088

TABLE I (Concluded)

PHYSICAL CHARACTERISTICS OF LAUNCH VEHICLE

x, in.	x, meter	m, $\frac{lb-sec^2}{in^2}$	m, $\frac{Newton-sec^2}{meter^2}$	AE lb $\times 10^{-6}$	AE N $\times 10^{-1}$
587.420	14.9204	0.000466	3.447	0.169	0.0751
587.420	14.9204	0.000466	3.447	0.161	0.0716
589.260	14.9672	0.001633	10.811	2.000	0.8896
589.260	14.9672	0.001633	10.811	2.000	0.8896
590.127	14.9892	0.001633	10.811	4.620	2.0549
590.127	14.9892	0.001633	10.811	4.620	2.0549
593.429	15.0713	0.001633	0.000	0.100	0.0444

TABLE II

PHYSICAL CHARACTERISTICS OF PAYLOAD

(a) U.S. Customary Units

x, in	m, $lb\text{-}sec^2/in^2$	AE $x10^{-6}$ lb
488.320	0.02	2.0000
548.120	0.02	2.0000

(b) SI Units

x, meter	m, $\frac{Newton\text{-}sec^2}{meter^2}$	AE $x10^{-7}$ Newton
12.3983	137.8951	0.8896
13.9223	137.8951	0.8896

TABLE III

A COMPARISON OF THE NATURAL LONGITUDINAL FREQUENCIES CALCULATED BY THE LUMPED MASS METHOD OF REFERENCE 1 AND THE FINITE DIFFERENCE PROCEDURE FOR THE ROCKET-VEHICLE CONFIGURATION

Mode	Lumped Mass (Hz)	Finite Difference (Hz)	Percent Increase
1	36.6	38.5	5.2
2	47.7	49.7	4.2
3	72.6	75.0	4.6

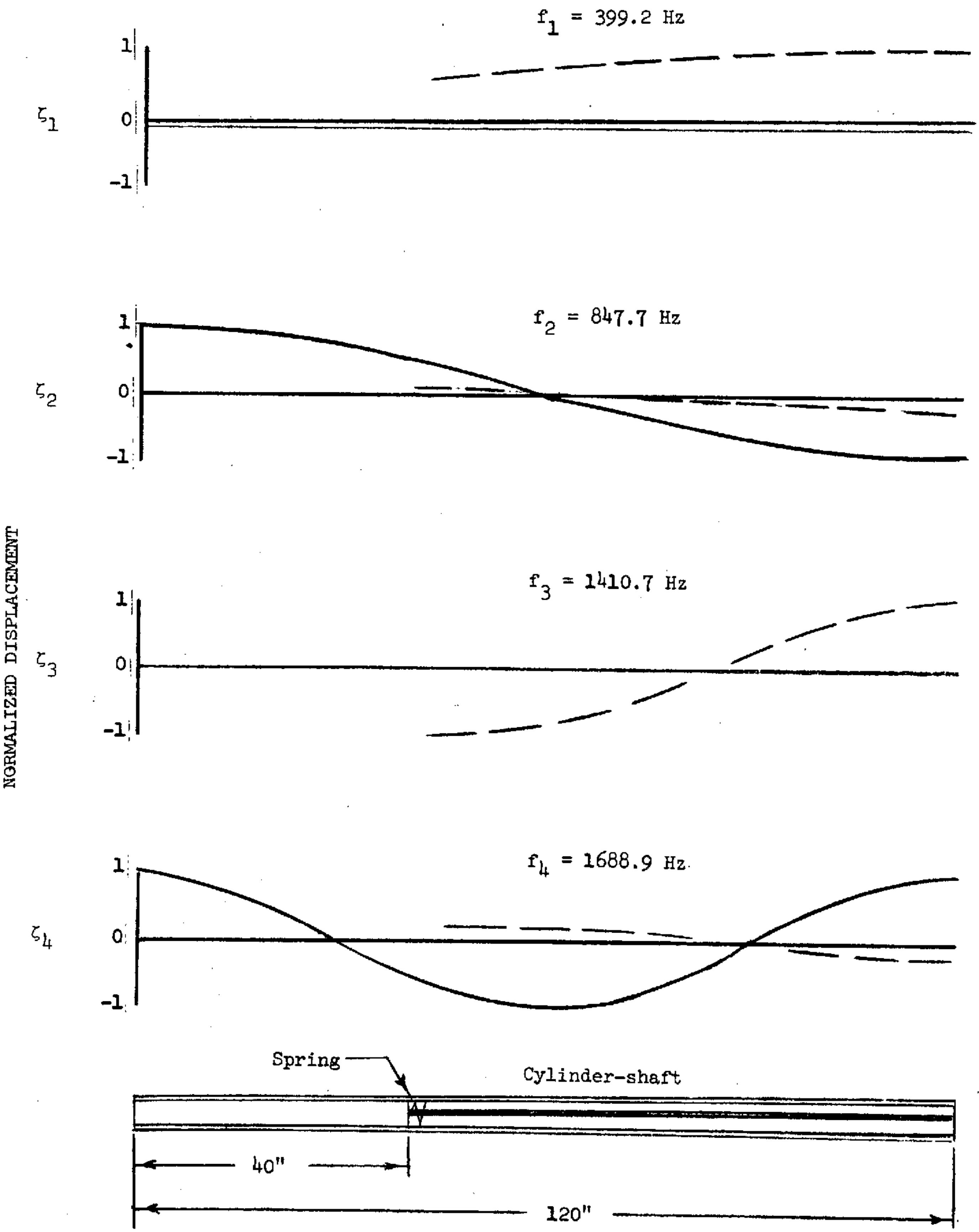


Figure 1.- Numerical example natural longitudinal mode shapes.

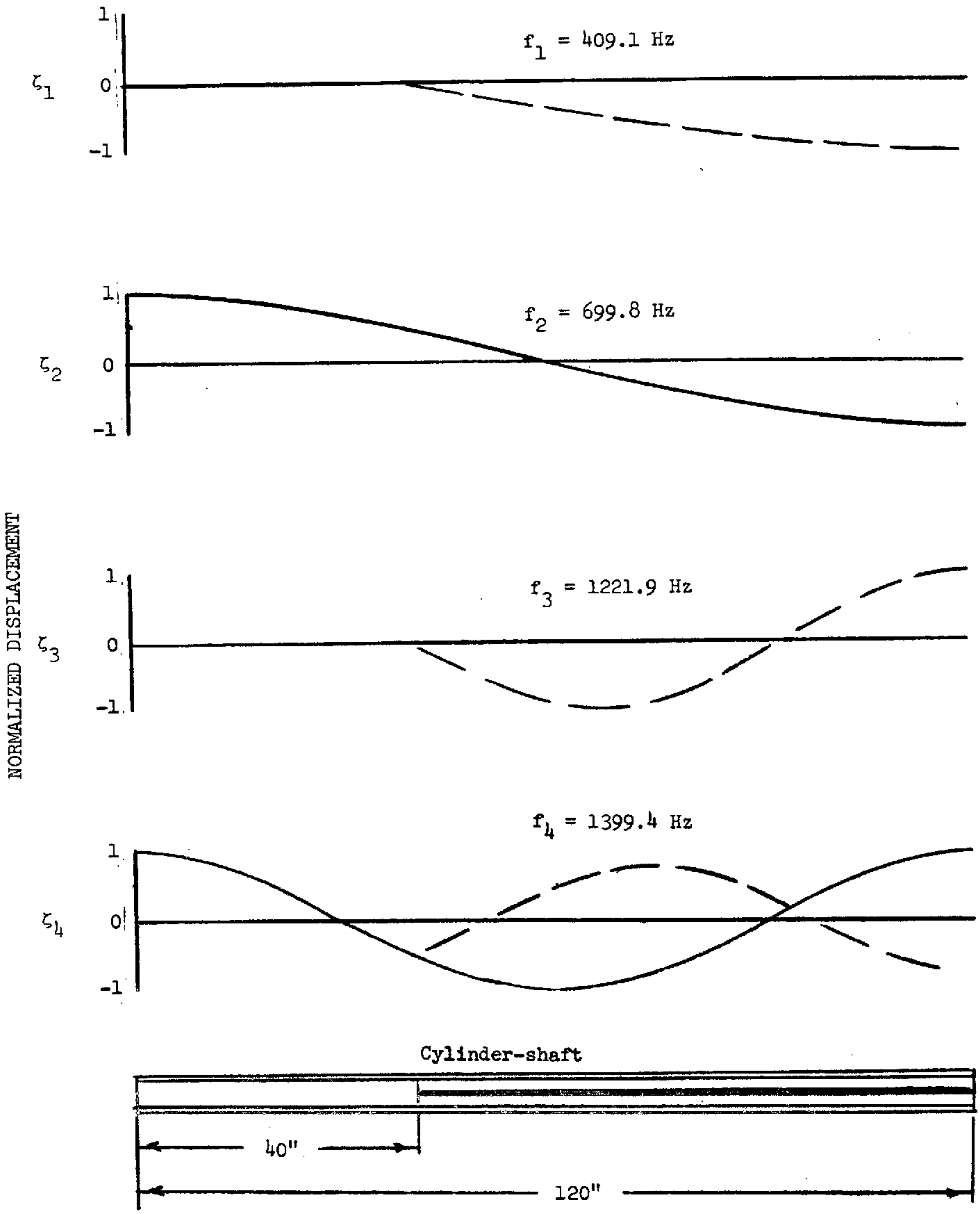


Figure 2 .- Numerical example natural torsional mode shapes.

CALCOMP PLOTTING INSTRUCTION CARD

NAME *Maria Stephens* DIV. *SEA* BLDG. NO. *586* PHONE *2512*

MAIL STOP *317* BIN NO. J.O. NO. *R2959* DATE *8-30-73*

NO. OF PLOTS *9* PLOT STOP ADDRESS *999* PLOT MODE ☐ Single ☑ Multiple PEN TYPE ☐ Ballpoint ☑ Leroy, Size ____

INK COLOR ☑ Black ☐ Blue ☐ Red ☐ Green PAPER NO. *400* TYPE PAPER *Ragbond*

STARTING LOCATION X Inches *1.0* Y Inches *1.0*

SPECIAL INSTRUCTIONS *Return to B586, R-209 TUB*

PROCESS NO. *6810* TAPE NO. *CAL 303* ON *2237* OFF *2247*

NASA Langley (Feb. 1970) ACD-OCO

Figure 3.- Calcomp plotting instruction card.

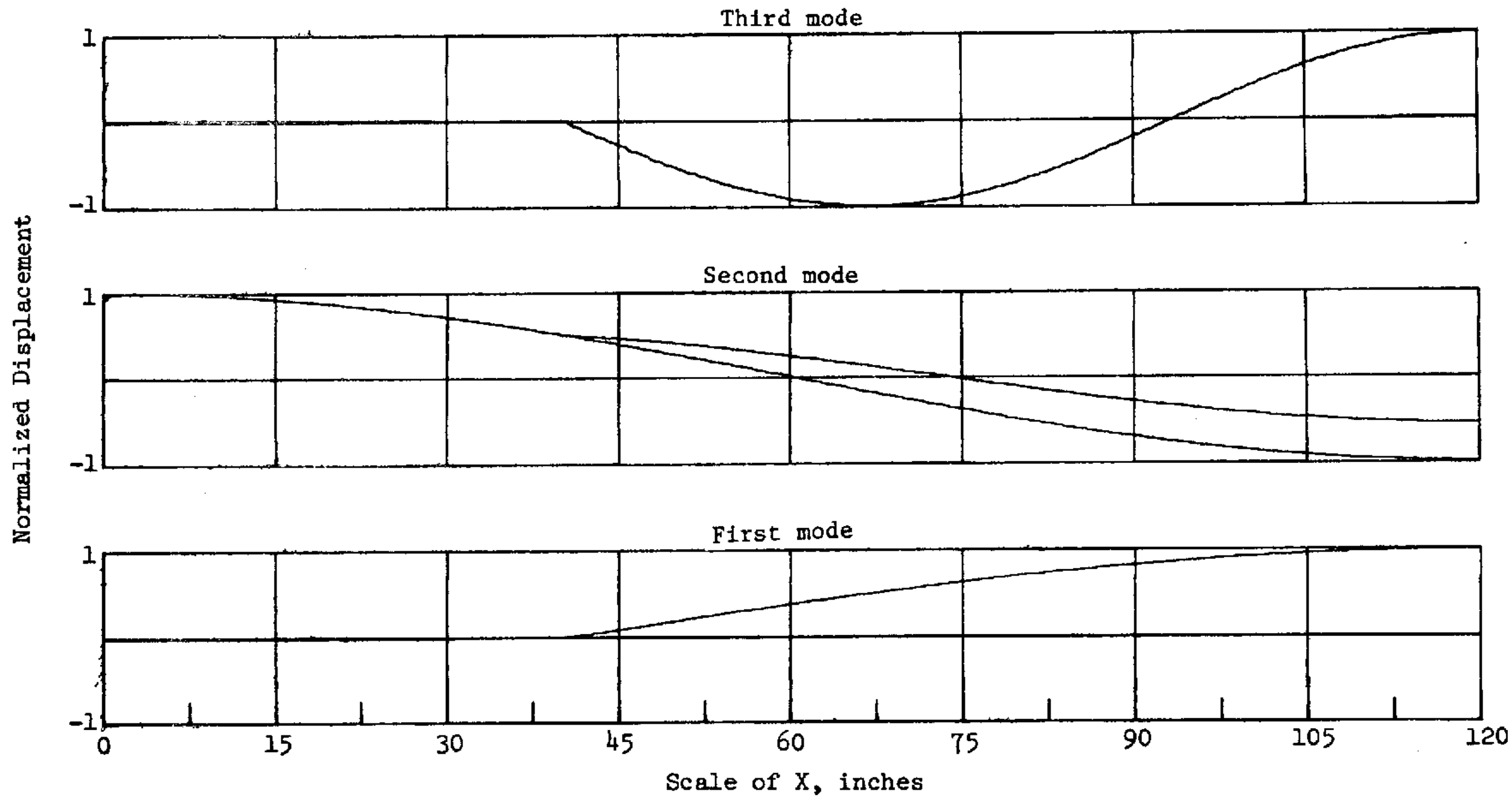


Figure 4 .- Numerical example plot of torsional mode shapes.

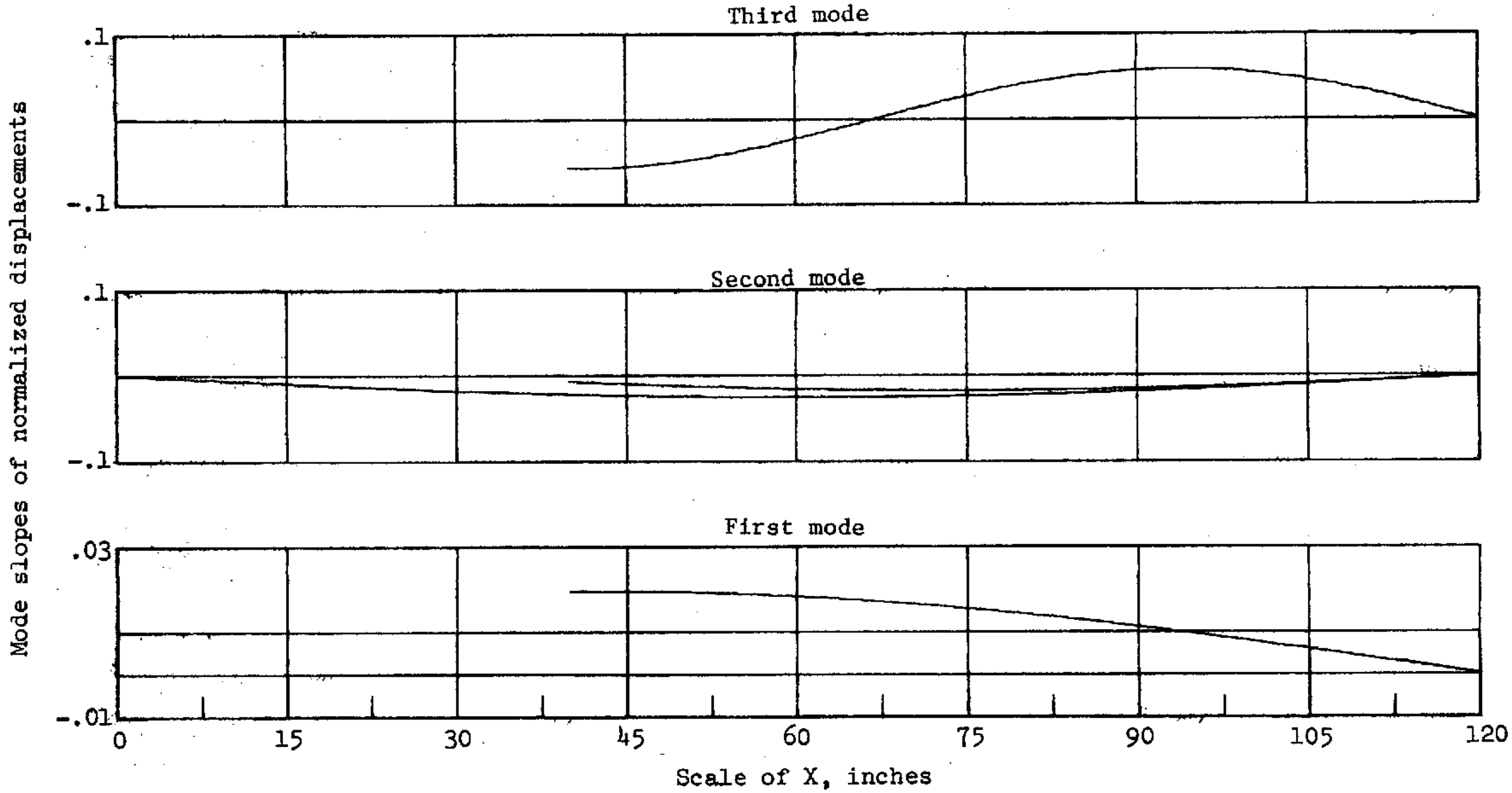


Figure 5.- Numerical example plot of torsional mode slopes.

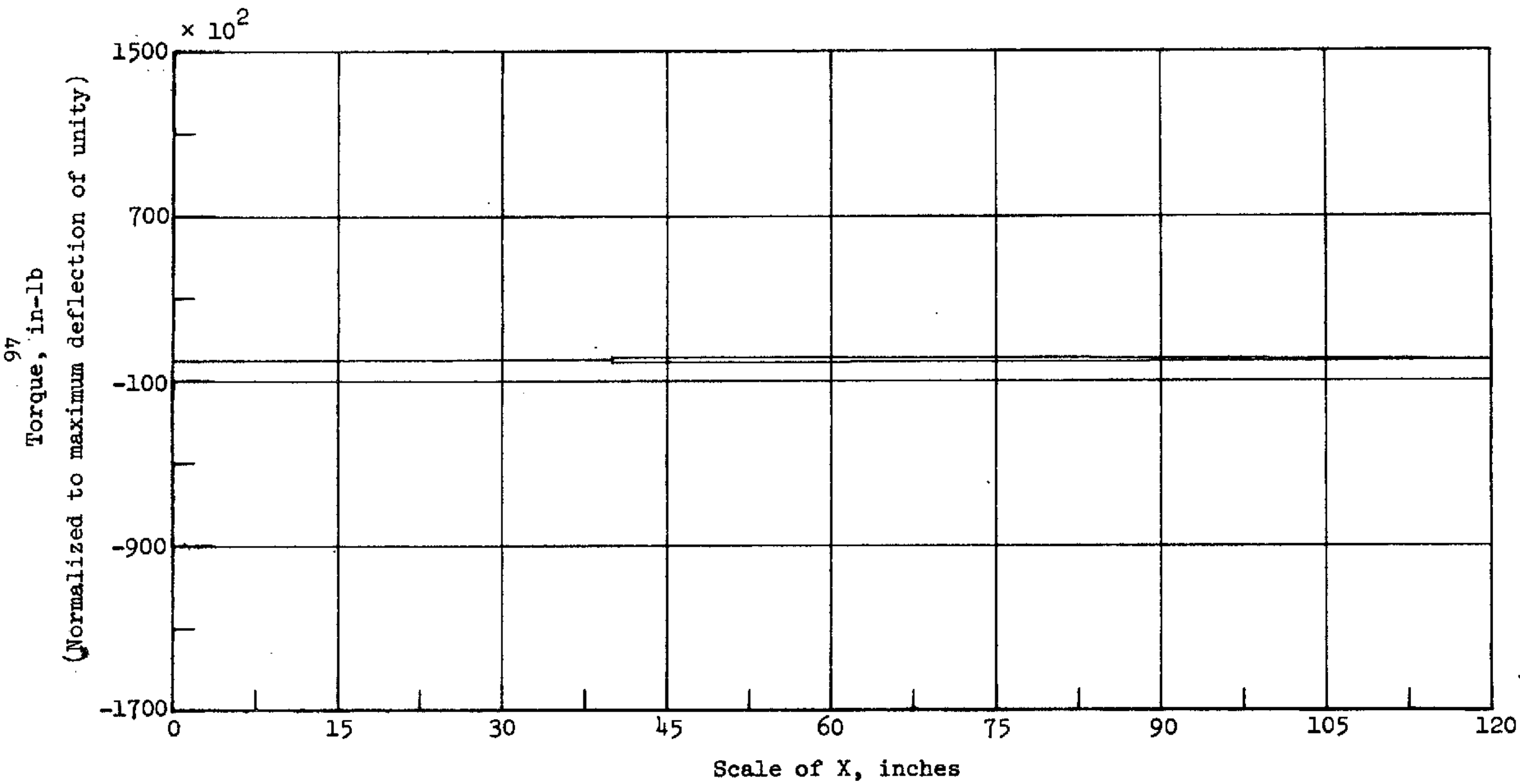


Figure 6.- Numerical example plot of torque distribution for first mode.

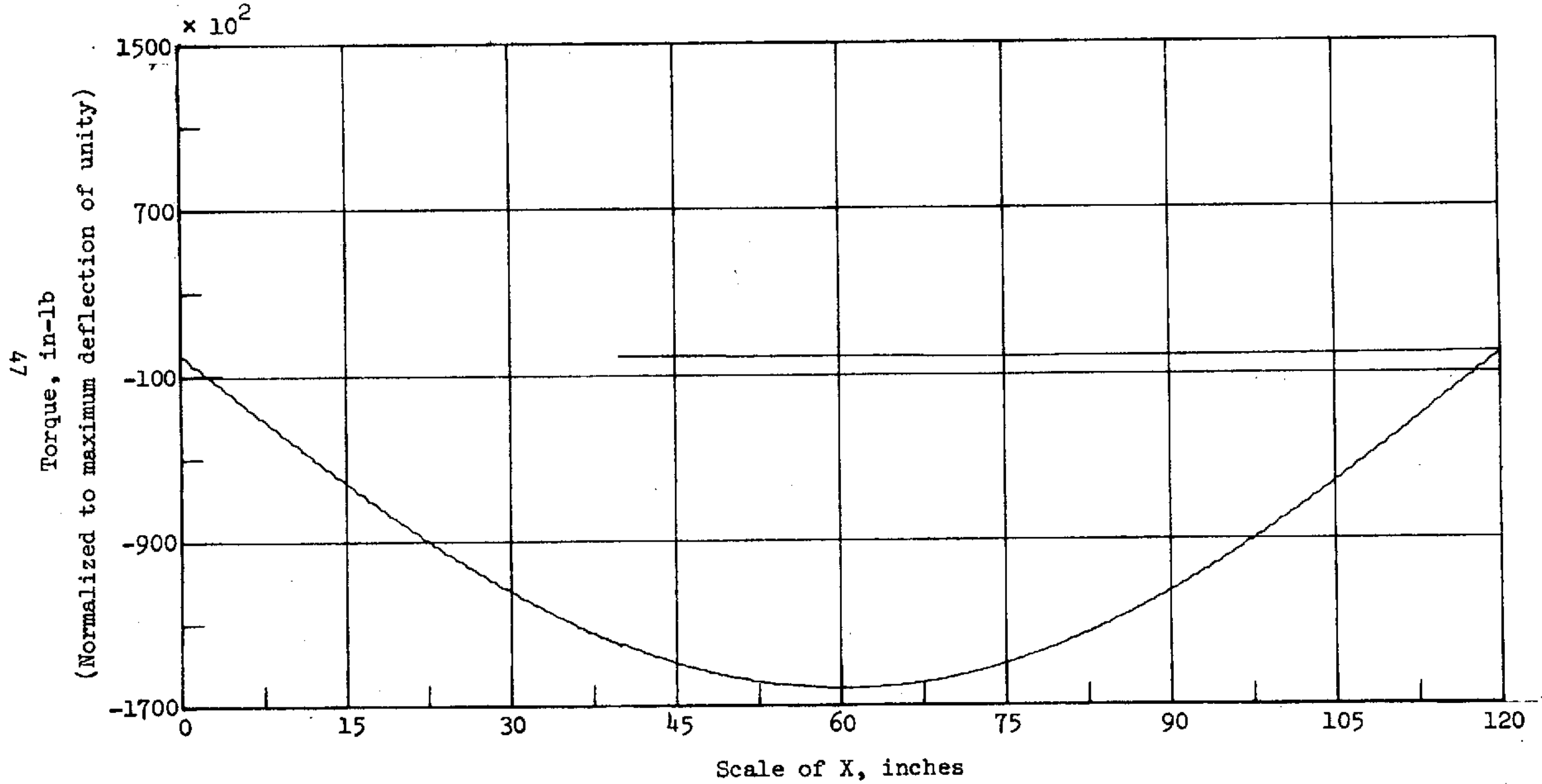


Figure 7 .- Numerical example plot of torque distribution for second mode.

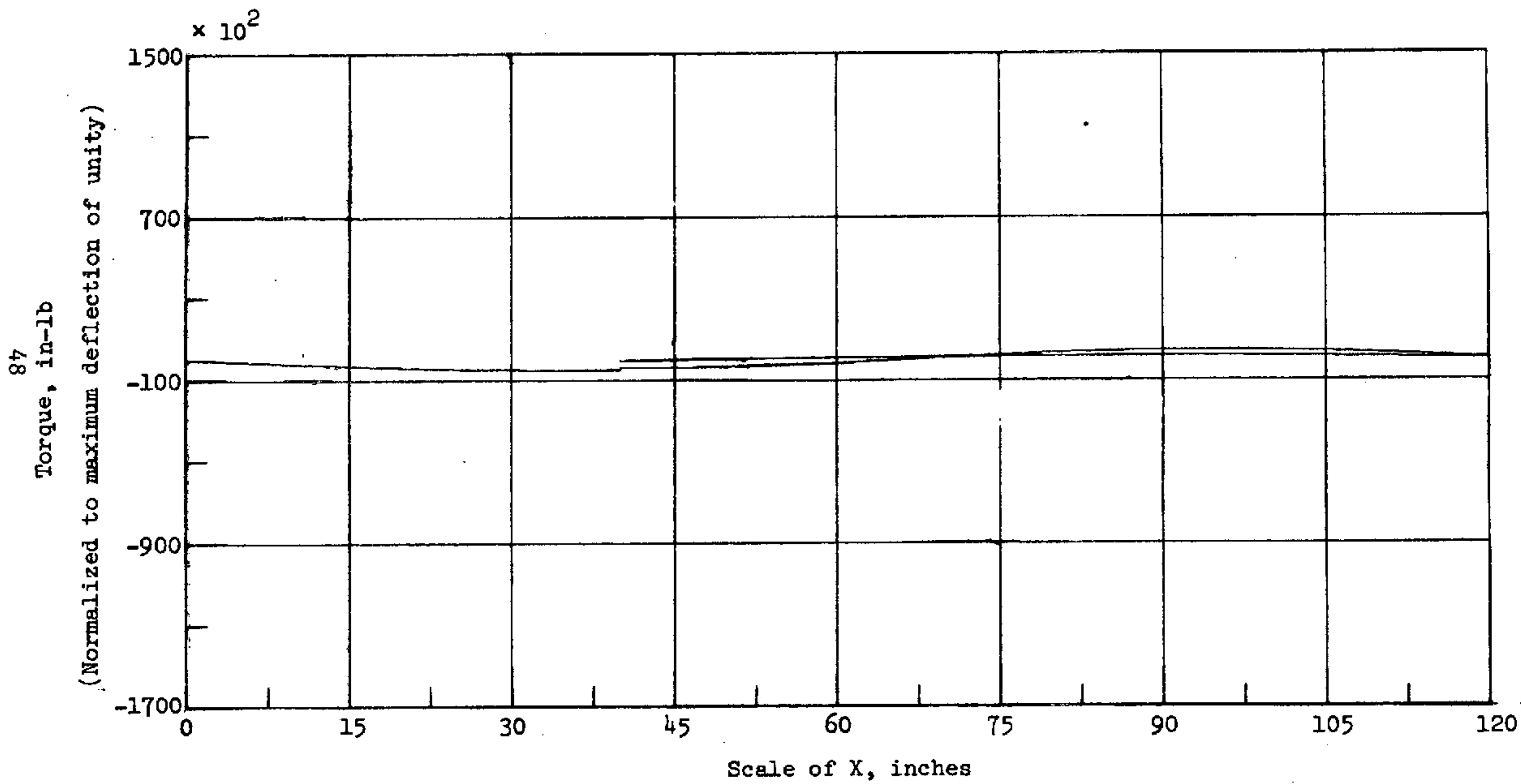


Figure 8.- Numerical example plot of torque distribution for third mode.

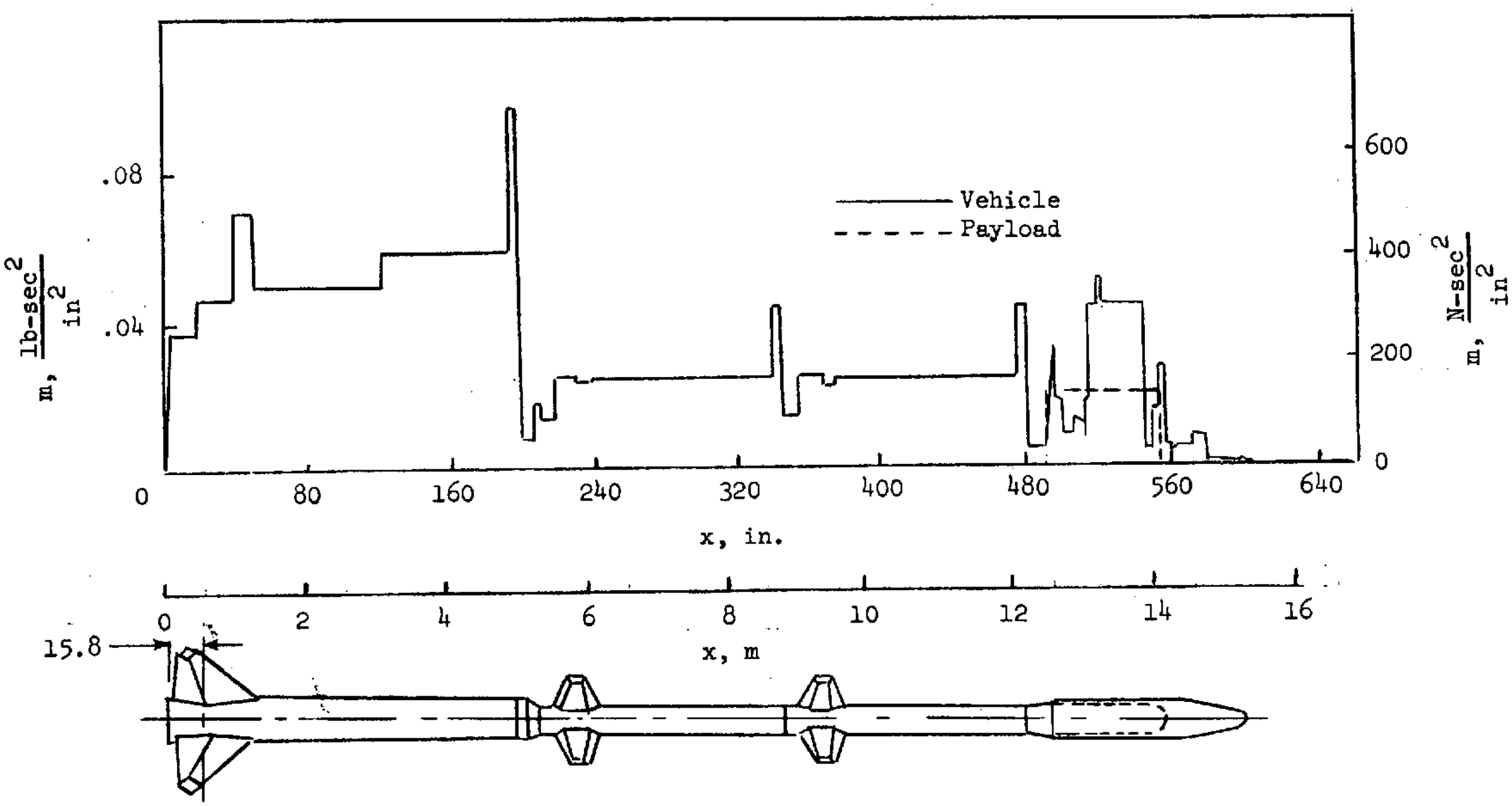


Figure 9.- Rocket-vehicle mass per inch. m values from tables I and II.

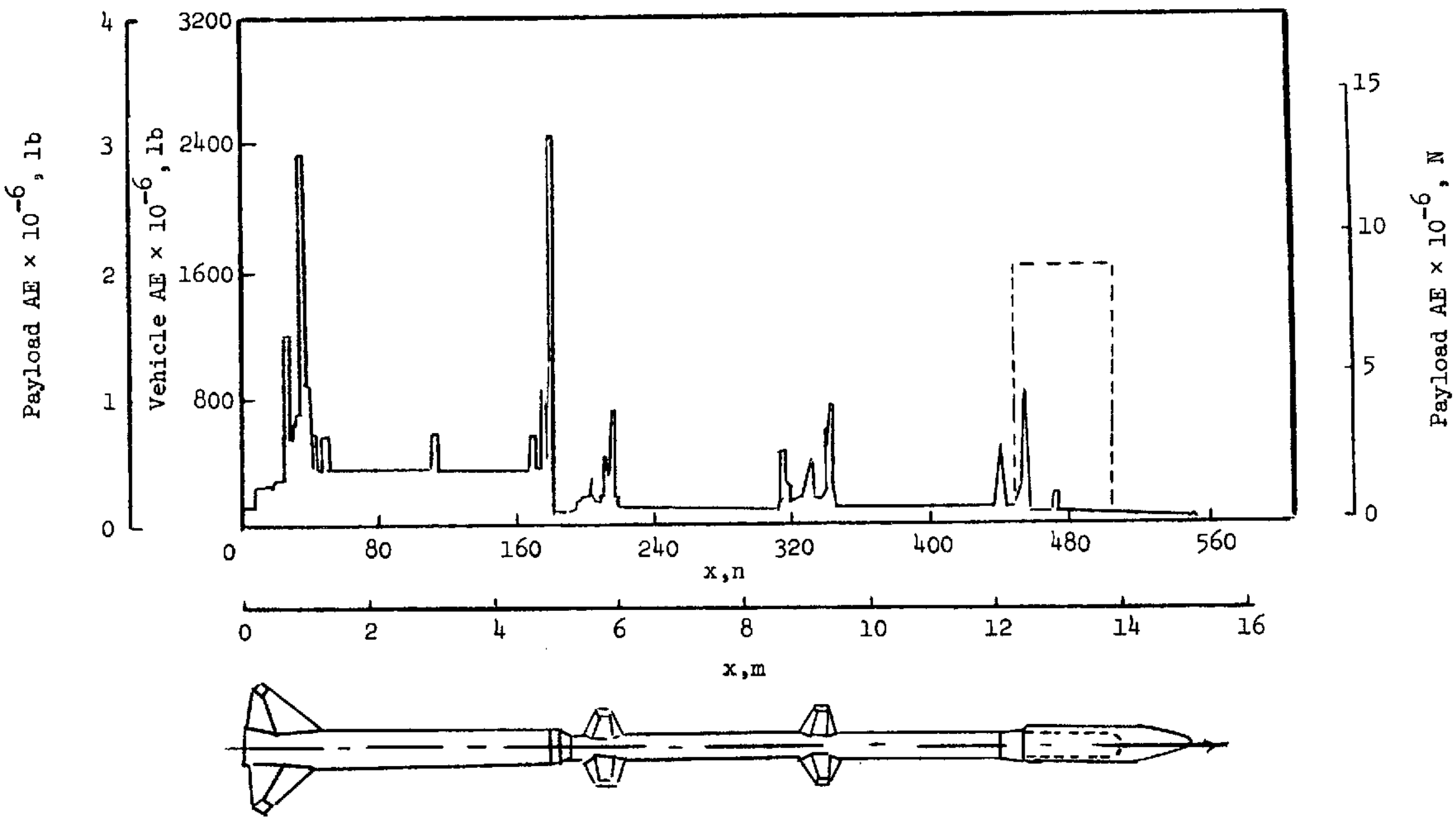


Figure 10.- Rocket-vehicle axial extension coefficient. AE values from tables I and II.

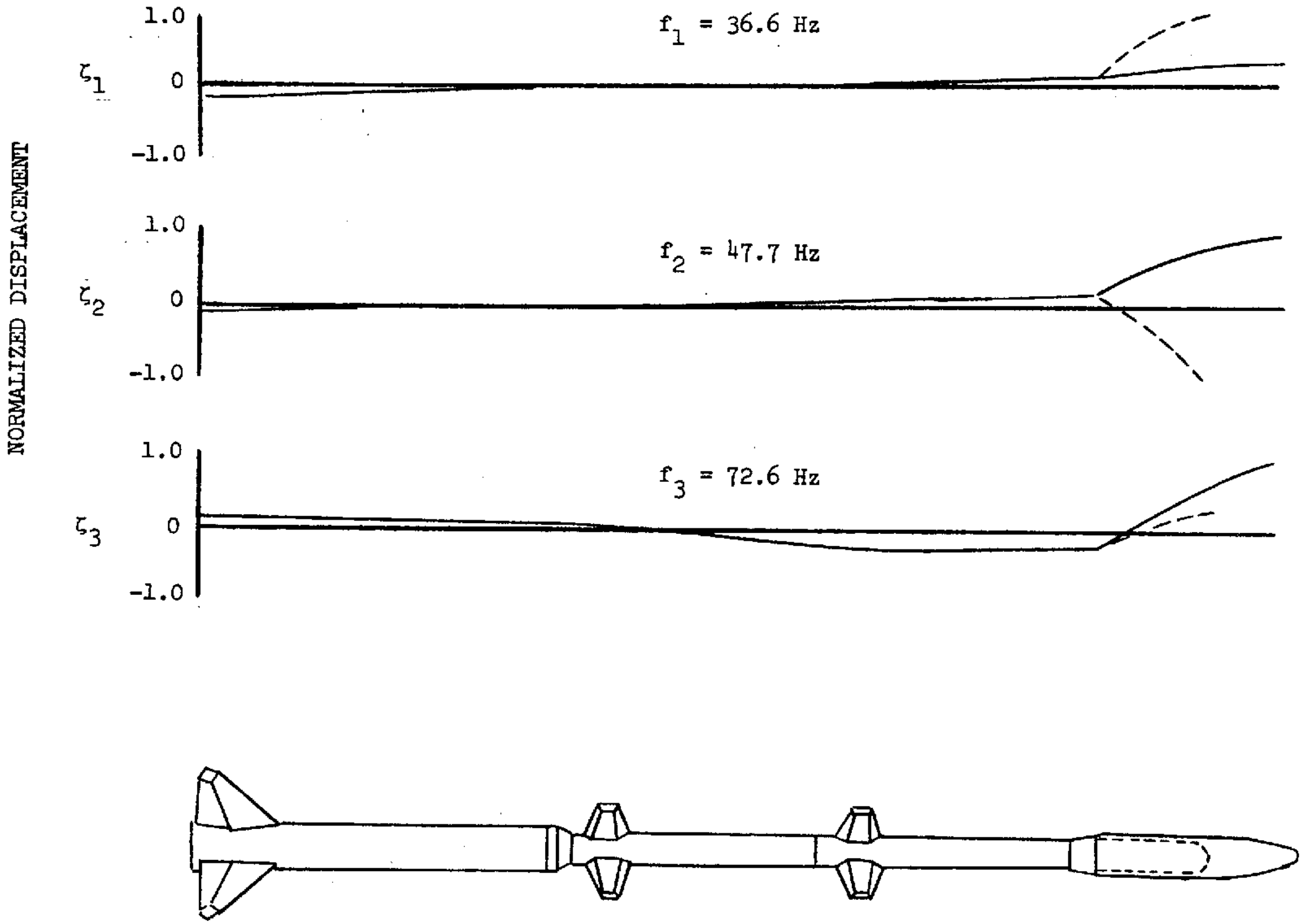


Figure 11.- Rocket-vehicle longitudinal natural mode shapes.

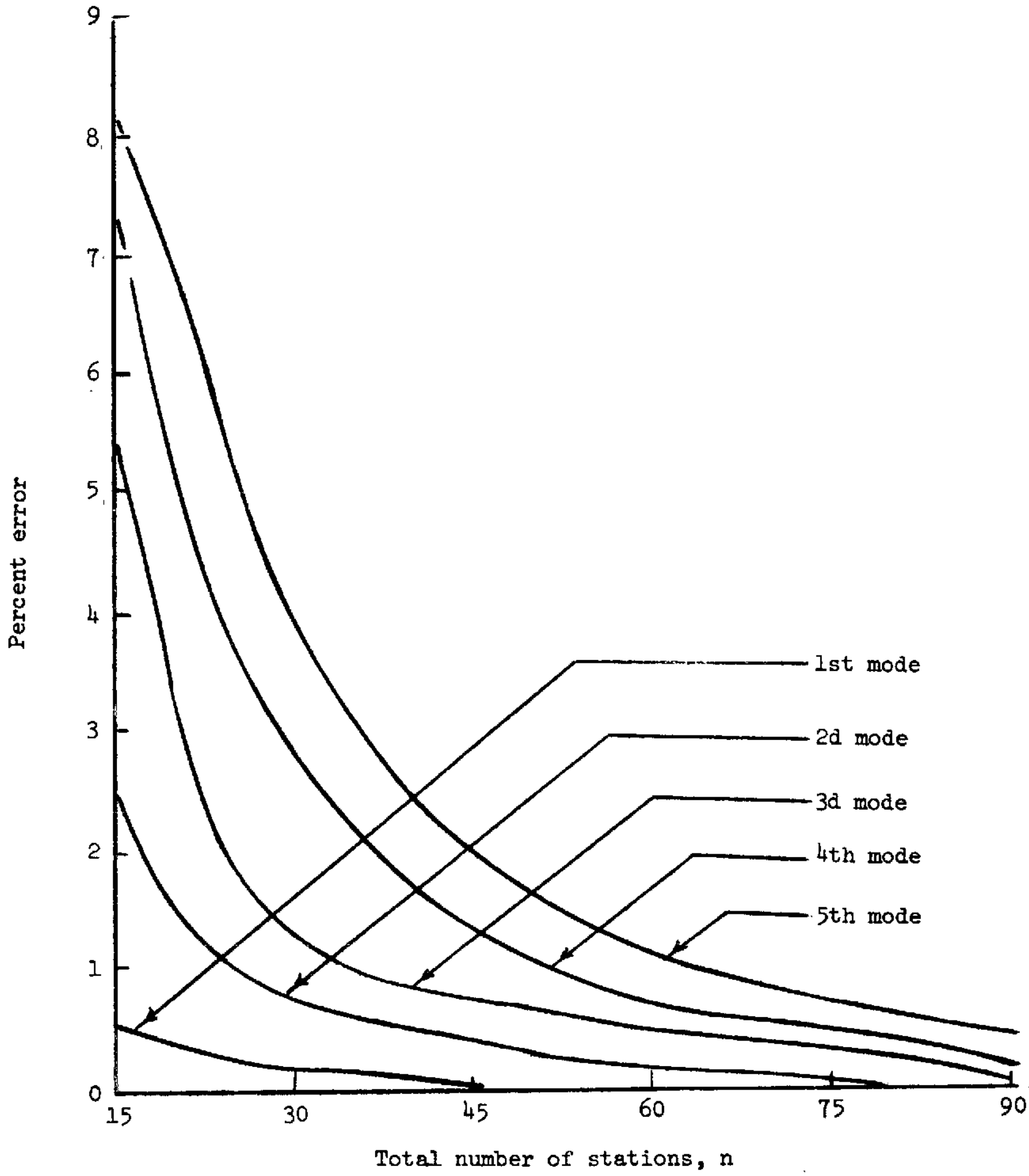
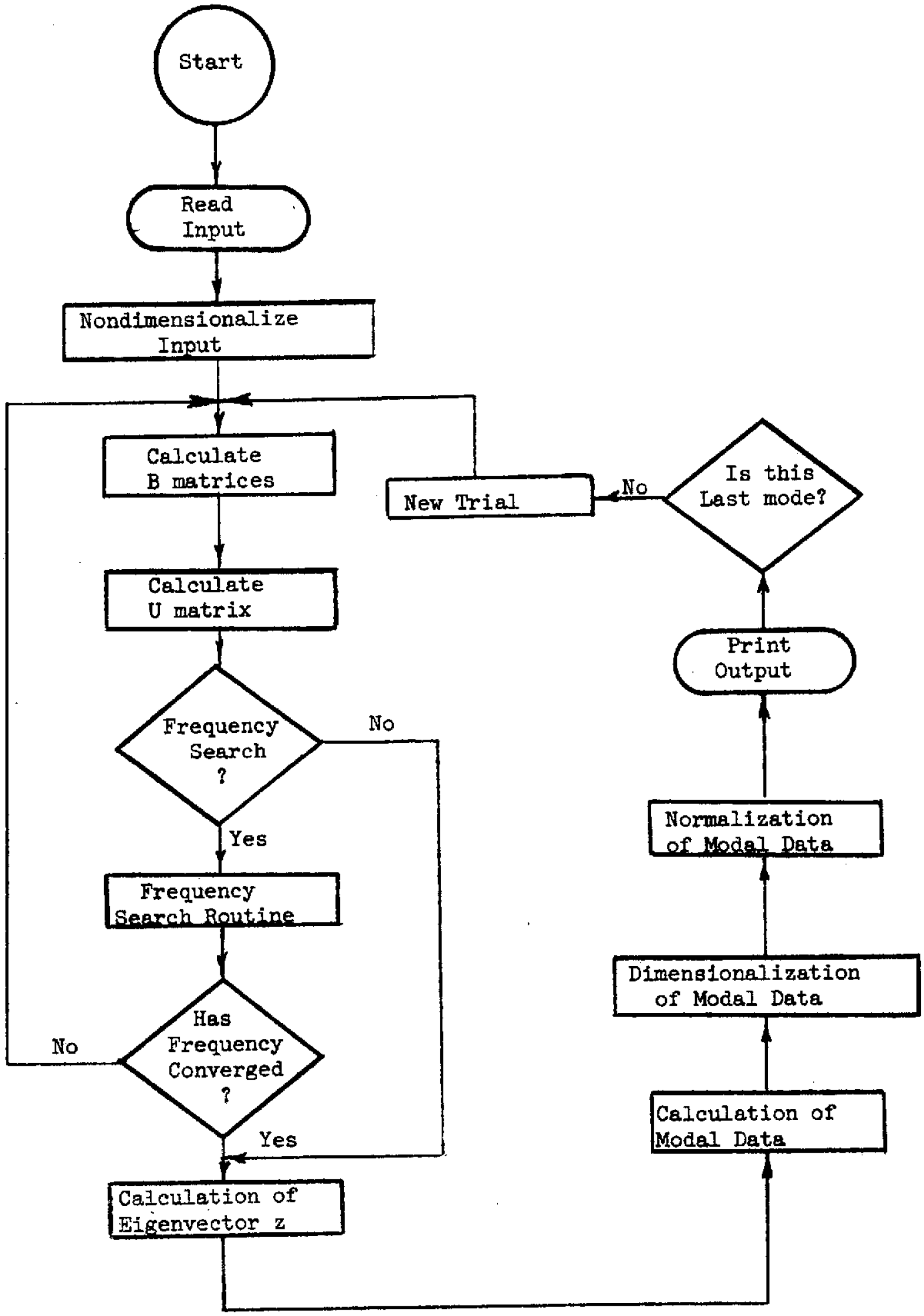


Figure 12.- Frequency percent error as a total number of stations.



Modular Flow Chart

Figure 13.- Computer program flow diagram.

APPENDIX A

DESCRIPTION OF COMPUTER PROGRAM

METHOD OF INPUT

1. One or two title cards are allowed. Columns 1 to 80 may be used on each card. Both title cards will be printed at the beginning of output. Thereafter only the first title card will be used as a header.

2. The following namelists may be read in any order.

   a. Namelist name: $LONVIB

      Variable list: X, MASS, AE, NON, TIME, BEAM

      Remarks: Each beam member is assigned a number (by the program) which depends upon the order in which the $LONVIB namelist groups are input. $LONVIB for beam member #3 must follow $LONVIB for beam member #2, for instance.

      Note: Beam is never used in the program, so need not be input. The last value read in for TIME will be printed in the output for each mode. This name list is applicable when longitudinal vibration analysis is desired.

   b. Namelist name: $TORVIB

      Variable list: X, ZR, JG, NON, TIME, BEAM

      Remarks: Each beam member is assigned a number (by the program) in the same manner as described for $LONVIB. This namelist is applicable when torsional vibration analysis is desired.

   c. Namelist name: $INPFRQ

      Variable list: OMEG

      Remarks: This namelist group is input only if the natural frequencies are known. No frequency search will be conducted when this group is input.

   d. Namelist name: $MODES

      Variable list: OMEGA, DELOMG, OMGTOL, NOMODE, FRSTMD, NORM, NORMBR, NORDIS, NOREQM

      Remarks: In every run, this group must be entered once. If entered when $INPFRQ is entered, the first four variables in this namelist group will be ignored.

      Note that OMGTOL is a <u>relative</u> difference in frequencies computed by this formula, OMGTOL = $\left|\frac{\omega_1}{\omega_2} - 1\right|$.

Note that NOMODE is not affected by the value of FRSTMD. FRSTMD is set to one (1) internally, so need be entered only when the set of numbers assigned to the modes should not begin with unity; for example when a run is to start with other than the first mode.

To normalize to a particular station on one beam member, set NORM equal to that station number and set NORMBR equal to that beam member number. To normalize to the station of maximum modal displacement, omit NORM and NORMBR and set NORDIS equal to "T". To normalize to the generalized mass, omit NORM, NORMBR and NORDIS and set NOREQM equal to "T". To obtain non-normalized data, omit NORM , NORMBR, NORDIS, and NOREQM.

To obtain a printed list of the nondimensionalized frequency and $|U|$ values calculated during the frequency search, insert a minus sign in front of the value for NOMODE.

e. Namelist name: $JOINT

Variable list: MBRI(J), MBRJ(J), MBRK(J), MBRL(J), IOJTRN(J), TRNS(J), IJTRN(J), IKTRN(J), ILTRN(J), TRNFIJ(J), TRNFIK(J), TRNFIL(J).

Remarks: This group must be input once for each run. It describes which members are constrained at each joint. If the constraint is partial, the flexibility of the partial constraint must be entered. MBRI(J), MBRJ(J), MBRK(J), MBRL(J), and IOJTRN(J) must always be entered.

The constraint codes are:

1 = Full constraint
2 = Partial constraint
3 = No constraint

f. Namelist name: $CONTRL

Variable list: RFX, RFMASS, RFAE, XMOD, MASMOD, AEMOD, DELX, JGMOD, ZRMOD, RFJG, RFZR.

Remarks: This namelist group controls the accuracy of the solution. Only those variables not equal to one must be entered, since these variables are initially set equal to one by the program.

g. Namelist name: $OUTPUT

Variable list: ZTAC, DZTAC, TENS, DTENS, TORQ, DTORQ, ISID, ISOD, PLOT.

Remarks: This group must be input once in each run. Four of the first six variables listed above must be input. ISID, ISOD, and PLOT are set to F internally so need be entered only if T. If a Calcomp plot is desired, PLOT = T, and items 3 and 4 of INPUT must follow the $END card.

h. Namelist name: $END

Variable list: None.

Remarks: Last card input of section 2 of INPUT must contain $END. If PLOT = T, in input namelist g of INPUT, then items 3 and 4 must follow.

3. Header card for Calcomp plot identification. Use columns 1 to 80 on card.

4. Namelist name: $NAM1

Variable list: INCHX, DX, XMIN, PLTZ, PLTZPR, NZAPR, MPLT, PLTT, NPLTT, TMIN, ZPMIN1, DZP1, ZPMIN2, DZP2, ZPMIN3, DZP3, DTT, PLTM, NMZR, MZRMIN, MZRMIN, DMZR, PLTZR, PLTAE, PLTGJ, NMAEJG, AJMIN, DAEJG.

Remarks: The header card and $NAM1 namelist must be read for every case separately if plots are desired.

SYMBOLS

Input Namelist Names

$CONTRL	control. Input of numerical control data.
$END	end. Required last item of input.
$INPFRQ	input frequency. Input of known modal frequencies.
$JOINT	joint. Input of joint description data.
$LONVIB	longitudinal vibration. Input data characteristics of mass and axial stiffness.
$MODES	modes. Input of data affecting selection of modes and normalization.
$NAM1	plot information, if plots are desired.
$OUTPUT	output. Input of data controlling the output of program results.
$TORVIB	torsional vibration. Input data characteristics of torsional stiffness and polar mass moment of inertia.

NOMENCLATURE

A. Input-Output Variable Names of Main Program

In the following list, fixed point variables (no decimal point allowed) are designated by "Integer". Floating point variables (decimal point required) are designated by "Decimal". The use of letters rather than numbers is designated by "Logical".

Name	Type	Description
AE(N)	Decimal	AE. Axial stiffness at the Nth station, lb/in^2.
AEMOD	Decimal	AE modification. Constant by which each value of AE(N) will be multiplied, unitless.
BEAM	Integer	beam. In input, identification number of beam assigned by Physical Characteristics Program, unitless. It indicates the order in which the beams were input to (and output from) this program.
DELOMG	Decimal	delta omega. Frequency interval used in the search for modal frequencies, rad./sec.
DELX	Decimal	delta X. Maximum length of integration interval along vehicle center line, in.
DTENS	Integer	derivative of tension column number. Identification number for the column in the output in which DTEN(N) will be printed, unitless.
DTORQ	Integer	derivative of torque column number. Identification number for the column in the output in which torque-prime will be printed, unitless.
DZETA(N)	Decimal	derivative of zeta. Lengthwise derivative of modal deflection at the Nth station, in./in. (before normalization).
DZTAC	Integer	derivative of zeta column. Identification number for the column in the output in which DZETA(N) will be printed, unitless.
FRSTMD	Integer	first mode. Number to be assigned to the first mode calculated, unitless.

Name	Type	Description
IJTRN(J)	Integer	i - j translation. Code number for the translational constraint between beam members in the "i" and "j" positions at the J^{th} joint, unitless. The code is identical to that shown in the description of IJROT(J).
IKTRN(J)	Integer	i - k translation. Code number for the translational constraint between beam members in the "i" and "k" positions at the J^{th} joint, unitless. The code is identical to that shown in the description of IJROT(J).
ILTRN(J)	Integer	i - l translation. Code number for the translational constraint between beam members in the "i" and "l" positions at the J^{th} joint, unitless. The code is identical to that shown in the description of IJROT(J).
IOJTRN(J)	Integer	i or j translation. Code number for the translational constraint between the ground and beam member in either the "i" or "j" position at the J^{th} joint, unitless. The code is identical to that shown in the description IJROT(J).
ISID	Logical	input stations for input data? True-false indicator to call for printing the physical characteristics at the stations which were input. T = Print only at input stations. F = Print at all stations.
ISOD	Logical	input stations for output data? True-false indicator to call for output of the results of the program. T = Output and plots results at input stations. F = Output and plots results at all stations.
JG	Decimal	Torsional stiffness at the N^{th} station, lb/in^2.
JGMOD	Decimal	JG modification. Constant by which each value of JG(N) will be multiplied, unitless.
MASMOD	Decimal	mass modification. Constant by which each value of MASS(N) will be multiplied, unitless.

Name	Type	Description
MASS(N)	Decimal	mass. Mass per unit length at N^{th} station, $lb.sec^2/in.^2$.
MBRI(J)	Integer	member i. Identification number of the beam member in position "i" at the J^{th} joint, unitless.
MBRJ(J)	Integer	member j. Identification number of the beam member in position "j" at the J^{th} joint, unitless.
MBRK(J)	Integer	member k. Identification number of the beam member in position "k" at the J^{th} joint, unitless.
MBRL(J)	Integer	member l. Identification number of the beam member in position "l" at the J^{th} joint, unitless.
NOMODE	Integer	no. of modes. Total number of modes to be computed, unitless. If negative, the values of the frequency and U determinant from the search routine will be printed.
NON	Integer	no. of N's. Total number of stations on a beam member, unitless.
NORDIS	Logical	normalize to displacement? True-false indicator to call for normalization of each eigenvector to its station of maximum modal displacement. T = Normalize to maximum displacement. F = Do not normalize to maximum displacement.
NOREQM	Logical	normalize to equivalent mass? True-false indicator to call for normalization or each eigenvector to its associated generalized mass. T = Normalize to unit generalized mass. F = Do not normalize to unit generalized mass.
NORMBR	Integer	normalize on member. Identification number of beam member on which the normalization station is located, unitless.
NORM	Integer	normalize on station N. Subscript of input station on beam member NORMBR at which to normalize the eigenvectors of every mode, unitless.

Name	Type	Description
OMEG(I)	Decimal	omega. Array of frequencies accepted by the program as known modal frequencies, rad./sec.
OMEGA	Decimal	omega. First trial frequency to be used in the search for natural frequencies, rad./sec. OMEGA should be less than the expected first frequency.
OMEGA SUBR	Decimal	$omega_r$. Reference value of frequency. rad./sec.
OMGTOL	Decimal	omega tolerance. Relative accuracy criteria for the natural frequency convergence, unitless.
PLOT	Logical	plot. True-false indicator to call for Calcomp plot. Initialized as F. T = Plot. F = Do not plot.
RFAE	Decimal	reference of AE. Reference value of axial stiffness, lb. in.^{2}.
RFJG	Decimal	reference of JG, lb. in.^{2}.
RFMASS	Decimal	reference of mass. Reference value of mass per unit length, lb. sec.^{2}/in.
RFX	Decimal	reference of X. Reference value of lengthwise coordinate, in.
RFZR	Decimal	reference of ZR. Reference value of mass moment of inertia. lb.sec.^{2}
TENS	Integer	tension column. Identification number for the column in the output in which the TENSN(N) will be printed out.
TENSION	Decimal	tension. Axial force at a station, lbs.
TENSION-PRIM	Decimal	longitudinal strain at a station lb./in.
TENSN(N)	Decimal	tension, axial force at the Nth station, lb. (before normalization).
TIME	Decimal	time. Value of time associated with each group of output data, seconds. (TIME is not used in the program).

Name	Type	Description
TRNFIJ(J)	Decimal	translational flexibility constant between i and j. Translational flexibility constant between beam members in positions "i" and "j" at the J^{th} joint, in./lb.
TRNFIK(J)	Decimal	translational flexibility constant between i and k. Translational flexibility constant between beam members in positions "i" and "k" at the J^{th} joint, in./lb.
TRNFIL(J)	Decimal	translational flexibility constant between i and j. Translational flexibility constant between beam members in positions "i" and "l" at the J^{th} joint, in./lb.
TRNS(J)	Decimal	translational spring constant. Translational spring constant between the ground and beam member in either the "i" or "j" position at the J^{th} joint, lb./in.
X(N)	Decimal	X. Lengthwise position coordinate of the N^{th} station, in.
XMOD	Decimal	X modification. Constant by which each value of X(N) will be multiplied, unitless.
ZETA(N)	Decimal	zeta. Modal displacement at N^{th} station, in. (before normalization).
ZETA	Decimal	longitudinal or torsional deflection at a station, unitless.
ZETA PRIME	Decimal	slope of longitudinal or torsional deflection at a station, unitless.
ZR	Decimal	mass moment of inertia at N^{th} station, lb.sec.^{2}
ZTAC	Integer	zeta column. Identification number for the column in the output in which ZETA(N) will be printed, unitless.
TORQ	Integer	torque column. Identification number for the column in the output in which the torque will be printed, unitless.
TORQUE	Decimal	torque. Torsional force at a station, in./lb.
TORQUE-PRIME	Decimal	torque-prime. Rate of change of torsional force at a station, lb.

B. Input Plot Variable Names

Name	Type	Description
INCHX	Integer	Number of inches for the horizontal scale (x).
XMIN	Decimal	Minimum value of X, to appear at lower left hand corner of graph.
DX	Decimal	Δx, per inch of graph.

The following variables are set to 0 in the program and need only be entered if their value should be 1.

Name	Type	Description
PLTM	Integer	= 1, plot mass = 0, no mass plot
PLTZR	Integer	= 1, plot roll inertia = 0, no roll inertia plot
PLTAE	Integer	= 1, plot axial stiffness coefficient = 0, no axial stiffness coefficient plot
PLTJG	Integer	= 1, plot torsional stiffness coefficient = 0, no torsional stiffness coefficient plot
PLTZ	Integer	= 1, plot ζ = 0, no ζ plot
PLTZPR	Integer	= 1, plot ζ' = 0, no ζ' plot
PLTT	Integer	= 1, plot tension or torque curves = 0, no tension or torque plots
MPLT	Integer	= 1, torque (tension) plots for each of 3 modes will be drawn on separate grids = 0, all 3 modes of the torque (tension) plots will be drawn on the same grid.

The following variables are set to 1 in the program and need only be entered if their value should be 2.

Name	Type	Description
NMZR	Integer	= 1, use ascale subroutine to find appropriate scale values for mass or roll inertia plot = 2, read in mass minimum and Δmass or roll inertia minimum and Δroll inertia

Name	Type	Description
NMAEJG	Integer	= 1, use Ascale subroutine to find appropriate scale values for axial or torsional stiffness coefficient plot = 2, read in axial or torsional stiffness scale values
NZAPR	Integer	= 1, use Ascale subroutine to find appropriate scale values for ζ' plots = 2, read in ζ' scale values
NPLTT	Integer	= 1, use Ascale subroutine to find appropriate scale values for torque (or tension) plots = 2, read in torque (or tension) scale values

If NMZR = 2, input the following:

MZRMIN	Decimal	minimum scale value for mass (or roll inertia)
DMZR	Decimal	Δmass (or Δroll inertia) value, per inch, total 4 inch grid

If NMAEJG = 2, input the following:

AJMIN	Decimal	minimum scale value of the axial (or torsional) stiffness coefficient
DAEJG	Decimal	Δaxial (or torsional) stiffness coefficient, per inch, total 4 inch grid

If NZAPR = 2, input the following:

ZPMIN1	Decimal	minimum scale value of ζ_1'
DZP1	Decimal	$\Delta\zeta_1'$, scale value per 1/2 inch, total 1 inch grid
ZPMIN2	Decimal	minimum scale value of ζ_2'
DZP2	Decimal	$\Delta\zeta_2'$, scale value per 1/2 inch, total 1 inch grid
ZPMIN3	Decimal	minimum scale value of ζ_3'
DZP3	Decimal	$\Delta\zeta_3'$, scale value per 1/2 inch, total, 1 inch grid

If NPLTT = 2, input the following:

Name	Type	Description
TMIN	Decimal	minimum scale value of torque (tension)
DTT	Decimal	Δtorque (Δtension) scale value per inch, total 4 inch grid

OUTPUT

Printed

After the title cards are printed, the input controls data is listed. If ISID = T, the physical characteristic at only the input stations are printed. If ISID = F, the physical characteristics at all stations are printed. The total mass, static moment, and center of gravity for the branched beam are printed for both the longitudinal and torsional vibration analysis.

An option has been made available to print the non-dimensionalized frequency and the corresponding determinant values for [U] throughout the iteration process. This print is requested by inputting a minus sign in front of the desired nomode value, i.e., NOMODE = -3,

IF ISOD = T, the modal frequency, generalized mass, and station properties are printed at each input station only. If ISOD = F, the modal frequency, generalized mass, and station properties are printed at all stations.

Plotted

If PLOT = F, or is not entered, no plots are made. One or more variables may be plotted against X on the Calcomp plotter if PLOT = T. A header card and $NAM1 namelist must be read into program if plot = T. See figure 7 for a sample of the request card for the plots. Also, if PLOT = T, a statement is printed as each plot is completed.

DIMENSIONALIZATION

The following equations are used in the nondimensionalization and modification of the input values.

$$X = (X)(XMOD)/(RFX)$$
$$MASS = (MASS)(MASMOD)/(RFMASS)$$
$$AE = (AE)(AEMOD)/(RFAE)$$
$$JG = (JG)(JGMOD)/(RFJG)$$
$$ZR = (ZR)(ZRMOD)/(RFZR)$$
$$OMEGA = (OMEGA)(RFX)(RFMASS)^{1/2}/(RFAE)^{1/2}$$
$$DELOMG = (DELOMG)(RFX)(RFMASS)^{1/2}/(RFAE)^{1/2}$$
$$TRNS = (TRNS)(RFX)/(RFAE)$$
$$TRNFIJ = (TRNFIJ)(RFAE)/(RFX)$$
$$TRNFIK = (TRNFIK)(RFAE)/(RFX)$$
$$TRNFIL = (TRNFIL)(RFAE)/(RFX)$$

DISCUSSION OF COMPUTER PROGRAM

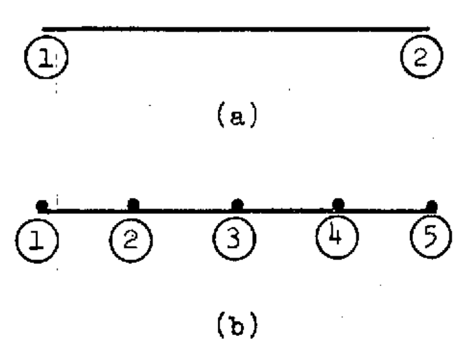
As described in the Application of the Method to Torsional Vibrations, the equations for the torsional vibration analysis are analogous to the equations for longitudinal vibrations.

The namelist method of input is used in this program, see CDC 6600 Computer Systems Fortran Reference Manual for a description. The namelist $LONVIB is used to input the modal data for the longitudinal analysis. The namelist $TORVIB is used to input the modal data for the torsional analysis. Since the two analyses are analogous, the corresponding inputs of the two namelists are equivalenced.

The input to this program consists of the necessary physical characteristics of the structure, definition of the boundary constraints, and controls to define options. In order to assist the user, it is necessary to elaborate on some of these terms.

Beam.- The vehicle or structure under analysis is composed of a system of beams or "members". Because of the program's ability to analyze branched beam problems, the basic structure characterized by a straight continuous elastic axis is referred to as a main beam; appendages which are attached to the main beam having elastic axes parallel to the main beam are then referred to as branches.

Joint.- The ends of the members are called joints. The word "joint" describes the beginning and the end of a member between which the number of stations are used to describe the physical characteristics of a member. A uniform continuous beam may be described by a single member having two ends and therefore, "two joints", or it may be described by any number of members having the appropriate number of joints, with the relationship between joints described by translational constraints. For example, two identical beams are represented in sketch 14(a) and (b).



Sketch 14.- Beam-joint arrangement.

Beam (a) consists of a single member with joints (1) and (2), the boundary conditions at the end are considered to be free-free. Joint 1 then has only a "j" member referring to sketch 14. Joint 2 has only an "i" member. In sketch 14(b) the same beam may be represented by four members with the extremes of the beam having the same boundary conditions. Joint 1 again has only a "j" member. Joint 2 has two members, 1 and 2, with member 1 the "i" member and member 2 the "j" member. Because (a) and (b) are identical uniform beams, members "i" and "j" at joint 2 are fully constrained as well as joints 3 and 4 with joint 5 identical to joint 2 of (a). It can be seen that the characteristics of both beams are identical. There are many uses for dividing beams into several members. For example if a change in the physical characteristics were required in the second quarter of the beam in figure 14 it could only be necessary to describe member 2 in sketch 14(b). If, however, the beam as described in sketch 14(a) were utilized then the entire beam input would have to be reentered.

Physical characteristics.-The physical characteristics necessary to compute the natural longitudinal modal data are entered as a continuous system and include mass and axial stiffness. The physical characteristics necessary to compute the natural torsional modal data include the torsional stiffness and the polar mass moment of inertia.

Output.- First the input data is printed, then the total mass, center of gravity, and the rigid body static moment of the beam being analyzed are printed. Also the generalized mass for each mode is printed for the longitudinal case. Next, the modal frequencies, both in cycles per second and radians per second are printed. Finally, at each station, four of the modal characteristics are printed.

Test problem.- An idealized beam, as shown in sketches 11 and 12 will be used to exercize the subject program. The necessary physical characteristics are provided. Two separate cases will be run. First the longitudinal modal data will be computed. The data required will be the first five longitudinal natural frequencies, mode shapes, axial forces, and the first derivatives of the axial forces, and mode shapes. Also, the static moment, center of gravity and total mass of the beam are required. In addition, the generalized mass is required for each mode.

In the second case, the torsional modal data will be calculated. The data required will be the first five torsional modes, torques, and their derivatives. The static moment, center of gravity, and total mass of the beam are also required. The generalized mass is not output for the torsional modes.

Identification.- Any number of cards may be used to identify or describe the problem with a minimum of one. However, the first card will be used as a title on each page of the printout. The contents of all of the title cards will be printed at the beginning of the printout.

Data.- Immediately following the identification cards are cards containing the physical characteristics of the problem. For the longitudinal case each set of data for each beam member begins and ends with $LONVIB and contains; first, TIME, which is an identifying time; second, NON, the number of stations of input for the member; third, BEAM, a number identifying the member; and fourth, the characteristics of the member X(N), MASS(N), AE(N). The data for each member is entered in the order established in sketch 12. Although the beam is numbered in the data, the program numbers the members in the order in which they are entered.

The input for the torsional vibration data is identical to the longitudinal vibration case with the exception that $TORVIB is substituted for $LONVIB, ZR(N) for MASS(N), and JG(N) for AE(N).

Examples: or

```
$LONVIB
BEAM = 1, TIME = 0.0, NON = 2,
X(1) = 0.0, 40.,
MASS(1) = 0.0009, 0.0009,
AE(1) = 36815625., 36815625.,$
$LONVIB
BEAM = 2, TIME = 0.0, NON = 2,
X(1) = 40., 120.,
MASS(1) = 0.000144, 000144
AE(1) = 5890500., 5890500., $
$LONVIB
BEAM = 3, TIME = 0.0, NON = 2,
X(1) = 40., 120.,
MASS(1) = 0.0009, 0.0009
AE(1) = 36815625., 36815625., $
```

```
$TORVIB
BEAM = 1, TIME = 0.0, NON = 2,
X(1) = 0.0, 40.
ZR(1) = 0.00220106, 0.00220106
JG(1) = 6200000., 6200000., $
$TORVIB
BEAM = 2, TIME = 0.0, NON = 2
X(1) = 40., 120.,
ZR(1) = 0.0000045, 0.0000045
JG(1) = 76699., 76699., $
$TORVIB
BEAM = 3, TIME = 0.0, NON = 3,
X(1) = 40., 120.,
ZR(1) = 0.00022, 0.00022,
JG(1) = 6200000., 6200000.,$
```

Joint description.- The next group of data informs the program of the proper assembly of members that have been input, describes the constraint units at each joint, and essentially builds the mathematical model of the beamlike structure. For the test problem, the relation holds as shown in sketch 12. The test problem member and the joint relationship is shown with joints listed in the first column and the associated members tabulated as illustrated in Table IV.

JOINT	MBRI	MBRJ	MBRK	MBRL
1	0	1	0	0
2	1	3	0	2
3	2	0	0	0
4	3	0	0	0

TABLE IV

Joint Arrangement

The next relationship to be shown is the listing of the constraints between members at their joints. The constraints for the test problem are illustrated in Table V.

JOINT	IOJTRN	IJTRN	IKTRN	ILTRN
1	3	0	0	0
2	3	1	0	2
3	3	0	0	0
4	3	0	0	0

TABLE V

Joint Constraints

The codes for constraints are; 1 = fully constrained or fixed, 2 = partially constrained, 3 = no constraint or free. If no relationship exists a zero is entered. The second column describes the constraint between either the i or j member and the ground. The test beam has no constraint to ground at any joint, and therefore code 3 is entered. Column 3 describes the translational constraint between members i and j at each joint; similarly the fourth column describes the constraint between i and k and the fifth column describes the constraint between i and ℓ. If no relationship exists, then zero is entered.

The remaining data necessary to complete the joint description are the values for the partial constraints to indicate the degree of the constraint. The tabulation is shown in Table VI for the test beam.

JOINT	TRNS	TRNFIJ	TRNFIK	TRNFIL
1	0	0	0	0
2	0	0	0	$1x10^{-5}$
3	0	0	0	0
4	0	0	0	0

TABLE VI

Joint Partial Constraint Coefficients

This Data group is entered as $JOINT and is input once for each problem. All data shown in Table I must be entered. All data shown in Tables IV and V must also be entered except where the data is zero.

Example:

```
$JOINT
MBRI(1) = 0, 1, 2, 3,
MBRJ(1) = 1, 3, 0, 0,
MBRK(1) = 4*0,
MBRL(1) = 4*0,
IOTRN(1) = 4*3,
IBTRN(1) = 0, 1, 0, 0,
IKTRN(1) = 4*0,
ILTRN(1) = 0, 2, 0, 0,
TRNS(1) = 4*0,
TRNFIJ(1) = 4*0,
TRNFIK(1) = 4*0,
TRNFIL(1) = 0, 1.x10^-5,0, 0,
```

Controls.- Further input is necessary to control the computer in its solution of the modal data. The first part of the control input is called $CONTRL. It modifies the input parameters. The input $LONVIB or $TORVIB which is originally dimensional becomes non-dimensional through controls in the program.

The reference mass, RFMASS, the reference length, RFX, and the reference axial stiffness coefficient, RFAE should be set = to 1.0. Non-dimensionalization of the physical characteristics was necessary in the lateral vibration program because of the combination of very large and very small terms in the frequency determinant. However, this situation does not exist for the longitudinal and torsional vibration program.

The second function of the $CONTRL group is to provide parameter modifiers. This group contains a group of variable multipliers that operate on the input physical characteristics before they are used in the program. These modifiers may be used to convert the units of input to the desired units. Also, since tables of stiffness coefficients are occasionally premultiplied by a constant to simplify the tabulations, these may also be entered and the proper modifier entered to correct the tabulated characteristics. These modifiers are entered only when not equal to one.

The final item of input in this group is DELX. This variable introduces additional stations in areas where the distance between stations of input is greater than desired for accuracy. The equations in the program are based on linear change of modal characteristics between stations along the length of the beam. Also the lengthwise integrations require sufficient stations in order to be able to assume linearity between steps. A value for DELX equal to 1/100 of the total length of the beam is usually adequate, and stations will be added only when the value of $X_{n+1} - X_n$ > DELX, with linear interpolation of physical characteristics for the added station values. This completes the $CONTRL group, and an example follows:

Example:

```
$CONTRL
RFX = 1.0, RFMASS = 1.0, MASMOD = .01
RFAE = 1.0, AEMOD = 1.E8, DELX = 2.0, $
```

The next group of input is called $MODES and provides the information necessary to calculate the natural frequencies. The frequency at which the search routine will start is OMEGA; the increment of ω in the routine, DELOMG; the tolerance at which the routine is considered to have solved for a natural frequency, OMGTOL; the number of modes to be solved, NOMODE; the mode number to be assigned to the first mode of input, FRSTMD; and finally the normalizing criteria, NORN, NORMBR, NOREQM.

The use of OMEGA to start the frequency search routine serves two purposes. First, the routine should be started somewhat lower than an estimated fundamental frequency to make optimum use of the computer time, especially in very stiff beams where the fundamental frequency might be high. Second, it allows the frequency search routine to start at a higher mode than the fundamental without calculating lower modes. OMGTOL is the relative difference in trial frequencies, used in the frequency search routine. OMGTOL is normally set at $1.x10^{-6}$, giving excellent results with optimum use of computer time. FRSTMD is used only when the frequency search routine is asked to start at other than the fundamental mode. Normally, FRSTMD is set internally equal to 1 and titles the first mode of output as "Mode 1" with arithmetic progression of successive modes of output.

The program has the capability of normalizing to unity at either the station of maximum deflection or at any particular station on the beam. Also it can normalize the mode shape so that the generalized mass is equal to unity. To normalize at maximum displacement equal to unity set NORDIS = T; at generalized mass equal to unity, NOREQM = T; at a particular station equal to unity, NORMBR = member containing the station, and NORN = number of the station in the member. The data for the test case is shown in the following example.

Example:

```
$MODES
OMEGA = 25., DELOMG = 200., OMGTOL = 1.E-7,
NOMODE = 5, NORDIS = T, $
```

The final group of input describes the output desired. The output will contain a complete listing of input, complete in such detail that the program may be reconstructed from this data alone. The physical characteristics are arranged in the proper size with headings for direct integration into reports. The stations are listed in a double set of integers with the first integer identifying the member and the second the station of the member. Immediately following the physical characteristics, the center of gravity, the total mass, and the rigid body static moment of the problem are printed.

The last item of output contains the modal characteristics as computed in the subject program. Each page of this modal data contains the contents of the first identification card, and also the number of the mode. The output, on the first page for each mode, also lists the natural longitudinal or torsional frequencies of the mode in both cycles per second and radians per second. In addition, the generalized mass for the normalized mode shape is printed for the longitudinal case.

Following this information are six columns of modal data. The first two are always the station number and its lengthwise location with respect to the established origin, respectively. The next four columns are the modal characteristics. This type of output is repeated for the required number of modes. All output may be printed at only the points of input or may include the additional stations provided by DELX. Additional outputs are a plot option which gives Calcomp plots.

This final group of data is called $OUTPUT. Four of the modal characteristics will be output. ISID calls for printing the input characteristics at either the original stations or including the stations added by DELX, and similarly, ISOD controls the output print. Both are set to F internally and need to be entered if only the input stations are to be printed. If only stations of input are desired set ISID = T, and if all the stations are desired for output set ISOD = F. In the same manner, if plots are desired, set PLOT = T. The identification for the column of output assigned to the modal characteristics does not include the first two columns which are always "station" and "X". The following example applies to the test problem.

Example:

```
$OUTPUT
ZTAC = 1, DZTAC = 2, TENS = 3, DTENS = 4,
ISID = T, ISOD = F, $
```

This data states that after each station and its appropriate "X" location the first column will be deflection, zeta (ZTAC = 1); the second column will be the first derivative of the deflection, (DZTAC = 2); the third column will be the axial force, (TENS = 3) or for torsional vibration the torque, (TORQ = 3); and the fourth column will be the first derivative of the axial force (DTENS = 4) or the first derivative of the torque, (DTORQ = 4). The input is terminated with $END.

This concludes the formal presentation of input required for this portion of the program. The second part of the program is designed to plot upon request any or all of the following depending upon the analyses: mass or roll inertia; stiffness coefficient, deflection or rotation; axial force or torque; and the corresponding first derivatives. The plots will be drawn for the first modes of the analysis. If plot = T in the $OUTPUT group then $NAM1 must be read. This group includes all of the plot options and the appropriate scales to be used in the plots.

<u>Concluding remarks on computer program</u>.- It should be emphasized that due to the assumption of linearity of the system variables over the length X_n to X_{n+1}, this increment must be small. Though studies conducted of comparing the frequency accuracy as a function of station separation, it has been found that 1/100 of the length of the beam gives very good results for the first five modes. Another benefit derived from the additional stations inserted in spans of constant parameters is in the quality of the plots of output characteristics with the additional points. The total number of added stations due to DELX cannot exceed 600. The total number of branches and beams cannot exceed 20.

A misleading situation may occur when values of AE or JG are near zero. This situation may occur at the free end of beams. The problem manifests itself in a radical variation of the mode shapes in the area approaching the upper boundary of the span. This variation is a combined result of near zero stiffness of AE and/or JG and the failure to achieve numerically the absolute zero. This condition is readily corrected by avoiding near the free ends, AE and JG values of less than 0.0001 of their respective average over the total span.

The output will be only as good as the math model representing the problem. With a proper input, the user can expect frequencies of the lower modes of vibration to be accurate within less than 1/2 of one percent. Some feeling for the accuracy may be gained by investigating the modal characteristics at the boundaries. The absolute value of zero for some boundary conditions of free-free beams cannot be achieved by digital computers, therefore the analyst should accept finite boundary values that are a fraction of 1 percent of the peak absolute value of their respective functions over the length of the beam. Another suggestion for gaining confidence in the modal data is to observe the continuity of the displacements and the axial forces or torques in areas where branches are attached to main beams.

APPENDIX B

COMPUTER PROGRAM LISTING

```
                 OVERLAY(LINK,0,0)
                 PROGRAM TORLONG(INPUT=201,OUTPUT=201,TAPE5=INPUT,TAPE6=OUTPUT,TAPE
                12=201,TAPE3,TAPE4=201)
000003           COMMON NPLOT,ENDFIL5
000003           INTEGER ENDFIL5
000003           NSUM=0
000004           ENDFIL5=1
000005           NPLOT=0
000006           LINK=4LLINK
000010           CALL CALCOMP
000011           CALL LEROY
000012      1    CALL OVERLAY (LINK,1,0,0)
000015           IF (ENDFIL5.EQ.0) 2,3
000021      2    IF (NSUM.GT.0) WRITE (6,4) NSUM
000030           CALL CALPLT (0.,0.,999)
000033           STOP
000035      3    CALL OVERLAY (LINK,2,0,0)
000040           NSUM=NSUM+NPLOT
000042           GO TO 1
            C
000042      4    FORMAT (/24H TOTAL NUMBER OF PLOTS =,I5)
000042           END
```

```
              OVERLAY(LINK,1,0)
              PROGRAM VIBRAT
         C    TITLE     BRANCH BEAM TORSIONAL AND LONGITUDINAL VIBRATIONS ANALYS
000003        DIMENSION TITLE(28), HEADNG(2,4), TITLES(2,6), EA(600), STN(600),
             1SSAM(600), HI(600), DY(2,600), Y(2,600), TITL(12), FORM(8), VARNO(
             24), MASS(600), NUM(20), MBRK(30), IOJTRN(30), MBRI(30), LV(6), MBR
             3L(30), IJTRN(30), MBRJ(30), MV(6), SAM(10), ILTRN(30), IKTRN(30),
             4OMEG(12), FORMA(26), OMFORM(7)
000003        DIMENSION AE(600), ZR(600), JG(600), ARRY(3), LARRY(3), TARRY(3)
000003        COMMON NPLOT,ENDFIL5
000003        INTEGER ENDFIL5
000003        COMMON /BLK1/ JC(30,8),TRNS(30),TRNFIJ(30),TRNFIK(30),TRNFIL(30),J
             1NT
000003        COMMON /BLK2/ X(600),HH(600),MAS(600),NSTA(30)
000003        COMMON /BLK3/ U(100,60),B(2,2,30),OMEGA,NOMBR,TEMP(2,30),GMASS.
000003        COMMON /BLK4/ UDET(20),NU,DELOMG,OMGTOL,IT,ITER,NUCT
000003        COMMON /BLK5/ NFRQ,NTROL(13),FRSTMD
000003        INTEGER ZTAC,DZTAC,TENS,DTENS,FRSTMD,TORQ,DTORQ
000003        REAL MASS,MCMR,MAS,MASMOD,JG,JGMOD,LARRY
000003        LOGICAL ISID,ISOD,NORDIS,NOREQM,PLCT,TORVB
000003        EQUIVALENCE (EA,HH,AE,JG), (RFAE,RFEA,RFJG), (EAMOD,AEMOD,JGMOD),
             1(RFMASS,RFZR), (MASMOD,ZRMOD), (TENS,TORQ), (DTENS,DTORQ), (MASS,M
             2AS,ZR)
000003        EQUIVALENCE (MBRI(1),JC(1,1)), (MBRJ(1),JC(1,2)), (MBRK(1),JC(1,3)
             1), (MBRL(1),JC(1,4)), (IOJTRN(1),JC(1,5)), (IJTRN(1),JC(1,6)), (IK
             2TRN(1),JC(1,7)), (ILTRN(1),JC(1,8))
000003        EQUIVALENCE (U,Y), (U(1,13),DY), (U(1,25),STN), (U(1,31),SSAM), (U
             1(1,37),HI)
000003        DATA DOLLAR/2H $/,F/3H  F/,T/3H  T/
         C
000003        DATA (TITL(I),I=1,8)/6HLONVIB,6HTORVIB,6HJOINT ,6HCONTRL,6HINPFRQ,
             16HMODES ,6HOUTPUT,6HEND   /
000003        DATA FORM/30H(5X,I3,1H-,I3,    0PF10.3,0P   /,FORM(5)/6HE14.5)/,(VA
             1RNO(I),I=1,4)/2H 1,2H 2,2H 3,2H 4/,BLANK/6H      /,RFJ/6HRFJG =/,R
             2FZ/8H  RFZR =/,RFA/6HRFAE =/,RFM/8HRFMASS =/,RNORE/9HNOREQM = /,ZR
             3M/8H ZRMOD =/,ASM/8HMASMOD =/,AEM/7HAEMOD =/,GJM/7HJGMOD =/,LARRY/
             430H FOR LONGITUDINAL VIBRATIONS  /,TARRY/30H FOR TORSIONAL VIBRATI
             5ONS     /,ZRIN/4H  ZR/,GJIN/2HJG/,RMINP/4HMASS/,RAE/2HAE/
         C
000003        DATA TITLES(1,1)/6H   TEN/,TITLES(1,2)/6H     Z/,TITLES(1,3)/6HTEN
             1SIO/,TITLES(1,4)/6H ZETA-/,TITLES(2,1)/4HSION/,TITLES(2,2)/3HETA/,
             2TITLES(2,3)/6HN-PRIM/,TITLES(2,4)/5HPRIME/,TITLES(1,5)/6H   TOR/,T
```

```
              3ITLES(2,5)/3HQUE/,TITLES(1,6)/6HTORQUE/,TITLES(2,6)/6H-PRIME/
        C
000003         DATA FORMA/260H(IH 54X,8HTABLE II
              1                                           //40XA2,13A6//50X3HT =F8.2,4H
              2 SEC                            //56X,4HMODEI3//5X,28HFREQUENCY  CYC
              3LES PER SECOND F12.3//5X,28HFREQUENCY RADIANS PER SECOND F12.5,
              4    /       /
        C
000003         DATA OMFORM/70HF12.8     ,F12.7     ,F12.6     ,F12.5     ,F12.4     ,F
              112.3     ,E16.8     ,/
        C
000003         NAMELIST /LONVIB/ X,MASS,AE,NON,TIME,BEAM
000003         NAMELIST /INPFRC/ OMEG/MODES/NOMODF,DELOMG,OMGTOL,NORN,NORMBR,NORD
              1IS,NOREQM,OMEGA,FRSTMD
000003         NAMELIST /JOINT/ TRNS,MBRI,MBRJ,MBRK,MBRL,ICJTRN,IJTRN,IKTRN,ILTRN
              1,TRNFIJ,TRNFIK,TRNFIL
000003         NAMELIST /CONTRL/ RFX,RFMASS,RFAE,XMOD,MASMOD,AEMOD,CELX,JGMOD,ZRM
              1OD,RFJG,RFZR
000003         NAMELIST /OUTPUT/ ZTAC,DZTAC,TENS,DTENS,ISID,ISOD,PLOT,TORQ,DTORQ
000003         NAMELIST /TORVIB/ X,ZR,JG,NON,TIME,BEAM
000003         REWIND 2
000005         REWIND 4
000007   1     CONTINUE
000007         DO 2 I=1,30
000011         TRNS(I)=0.
000012         TRNFIJ(I)=0.
000013         TRNFIK(I)=0.
000014         TRNFIL(I)=0.
000015   2     CONTINUE
000017         DO 3 I=1,12
000020         OMEG(I)=0.
000021   3     CONTINUE
000023         DO 4 I=1,8
000024         DO 4 J=1,30
000025   4     JC(J,I)=0
000036         K=0
000037         M=0
000040         KK=0
000041         MM=0
000042         IFRQ=0
000043         ZTAC=0
000044         DZTAC=0
000045         TENS=0
```

```
000046        DTENS=0
000047        NORN=0
000050        NORMBR=0
000051        FRSTMD=1
000052        PLOT=.FALSE.
000053        ISID=.FALSE.
000054        ISOD=.FALSE.
000055        NORDIS=.FALSE.
000056        NOREQM=.FALSE.
000057        RFEA=1.
000060        RFX=1.
000061        RFMASS=1.
000062        EAMOD=1.
000063        XMOD=1.
000064        MASMOD=1.
000065        DELX=1.
         C    .... READ TITLE CARDS
000066        READ (5,100) (TITLE(I),I=1,14)
000077        IF (ENDFILE 5) 5,6
000102   5    ENDFIL5=0
000103        RETURN
000105   6    WRITE (6,101) (TITLE(I),I=1,14)
000117   7    READ (5,100) (TITLE(I),I=15,28)
000131        IF (DOLLAR.EQ.TITLE(15)) GO TO 8
000133        WRITE (6,102) (TITLE(I),I=15,28)
000145        GO TO 7
000146   8    BACKSPACE 5
000150        DO 9 I=1,8
000152        IF (TITLE(16).EQ.TITL(I)) GO TO (10,13,14,18,19,22,23,24), I
000170   9    CONTINUE
000172        WRITE (6,103) TITLE(15),TITLE(16)
000202        STOP
         C    .... READ STATION DATA
000204   10   READ (5,LONVIB)
000207        TOPVB=.FALSE.
000210   11   M=M+1
000212        NUM(M)=NON
         C    .... MOVE DATA TO ARRAY BEHIND LAST ONE READ.
000214        DO 12 L=1,NON
000215        K=K+1
000217        STN(K)=X(L)
000221        HI(K)=EA(L)
000223        SSAM(K)=MASS(L)
```

```
000225    12    CONTINUE
000230          NOMBR=M
000231          GO TO 7
000232    13    READ (5,TORVIB)
000235          TORVB=.TRUE.
000236          GO TO 11
000237    14    READ (5,JOINT)
000242          DO 16 I=1,30
000244          DO 15 J=1,4
000245          IF (JC(I,J).NE.0) GO TO 16
000250    15    CONTINUE
000252          GO TO 17
000252    16    CONTINUE
          C     .... JNT = NO. OF JOINTS
000254          JNT=30
000255          GO TO 7
000256    17    JNT=I-1
000260          GO TO 7
000261    18    READ (5,CONTRL)
000264          GO TO 7
000265    19    READ (5,INPFRQ)
000270          DO 20 I=1,12
000272          IF (OMEG(I)) 20,21,20
000273    20    CONTINUE
          C     .... IFRQ = NO. OF INPUT FREQUENCIES
000275          IFRQ=12
000276          GO TO 7
000277    21    IFRQ=I-1
000301          GO TO 7
000302    22    READ (5,MODES)
000305          GO TO 7
000306    23    READ (5,OUTPUT)
000311          GO TO 7
          C     .... ALL INPUT IN, BEGIN CALCULATION
          C     .... READ TO SKIP $END CARD
000312    24    READ (5,100) TITLE(15)
000320          NPRNT=0
000321          IF (NOMODE.GE.0) GO TO 25
000323          NOMODE=-NOMODE
000323          NPRNT=1
000324    25    MN=0
000325          DO 26 II=1,NOMBR
000327    26    MN=MN+NUM(II)
```

```
000333          DO 27 II=1,NN
000334          STN(II)=STN(II)*XMOD
000336          HI(II)=HI(II)*EAMOD
000341          SSAM(II)=SSAM(II)*MASMOD
000343    27    CONTINUE
000345          L=0
000346          K=0
000347          I=1
          C     .... EXPAND STATION DATA ARRAYS IF NECESSARY
000350          DO 32 M=1,NOMBR
000351          L=NUM(M)+L
000353    28    I=I+1
000355          K=K+1
000356          IF (K.GT.600) GO TO 30
000361          X(K)=STN(I-1)
000362          HH(K)=HI(I-1)
000364          MAS(K)=SSAM(I-1)
000366          IF (I-L) 29,29,31
000370    29    IF ((STN(I)-X(K)).LE.DELX) GO TO 28
          C     .... INSERT EXTRA POINT
000375          K=K+1
000376          IF (K.GT.600) GO TO 30
000401          X(K)=X(K-1)+DELX
          C     .... LINEAR INTERPOLATION FOR ADDITIONAL VALUES
000403          PART=(X(K)-STN(I-1))/(STN(I)-STN(I-1))
000411          HH(K)=HI(I-1)+(HI(I)-HI(I-1))*PART
000416          MAS(K)=SSAM(I-1)+(SSAM(I)-SSAM(I-1))*PART
000424          GO TO 29
000424    30    WRITE (6,104)
000430          GO TO 1
          C     .... NSTA ARRAY HAS CUMULATIVE NO. OF STATIONS PER BEAM
000431    31    NSTA(M)=K
000433    32    CONTINUE
          C
          C     .... CALCULATE THE CENTER OF GRAVITY
000436    33    CALL CGRAV (NOMBR,TOTM,TOTS,CG)
          C     .... PRINT OUT INPUT
000441    34    IF (.NOT.ISOD) GO TO 35
          C     .... INPUT REQUIRED FOR FINAL OUTPUT, SAVE ON SCRATCH TAPE
000443          REWIND 2
000445          WRITE (2) K,(STN(I),SSAM(I),HI(I),I=1,K)
000465          END FILE 2
000467    35    WRITE (6,105) (TITLE(I),I=1,14)
```

```
        C      .... COMPUTE REFERENCE VALUE FOR FREQUENCY
000501         OMR=SQRT(RFEA/(RFMASS*RFX**2))
        C
        C      SET PRINT STATEMENTS
        C
000507         PRINT1=F
000510         PRINT2=F
000511         IF (NORDIS) PRINT1=T
000514         IF (TORVB) 36,38
000516  36     RE=RFJ
000520         RM=RFZ
000521         PRNT2=BLANK
000523         PRINT2=BLANK
000524         RMMOD=ZRM
000525         RAMOD=GJM
000527         RMAS=ZRIN
000530         RAEP=GJIN
000532         DO 37 I=1,3
000533  37     ARRY(I)=TARRY(I)
000536         GO TO 40
000537  38     IF (NOREQM) PRINT2=T
000542         RE=RFA
000544         RM=RFM
000545         PRNT2=PNORE
000547         RMMOD=ASM
000550         RAMOD=AEM
000552         RMAS=RMINP
000553         RAEP=RAE
000555         DO 39 I=1,3
000556  39     ARRY(I)=LARRY(I)
000561  40     WRITE (6,106) RE,RFEA,NOMODE,RM,RFMASS,NORMBR,RFX,NORN,OMR,PRINT1,
              1DELX,PRNT2,PRINT2,OMEGA,RMMOD,MASMOD,DELOMG,RAMOD,EAMOD,OMGTOL
000635         WRITE (6,107) (J,(JC(J,I),I=1,8),J=1,JNT)
000656         WRITE (6,108) (J,(JC(J,I),I=1,4),TRNS(J),TRNFIJ(J),TRNFIK(J),TRNFI
              1L(J),J=1,JNT)
000707         LINE=6
000710         N=0
000711         NN=0
000712         WRITE (6,119)
000715         WRITE (6,109) ARRY,(TITLE(I),I=1,14),RMAS,RAEP
000735         I=0
000736         LMAX=34
000737         DO 46 M=1,NOMBR
```

```
000741   41   LINE=LINE+1
000743        IF (LINE.LT.LMAX) GO TO 42
000745        LINE=0
000746        WRITE (6,119)
000751        WRITE (6,110) RMAS,RAEP
000761   42   N=N+1
000763        I=I+1
000764        IF (.NOT.ISID) GO TO 43
         C    .... PRINT OUT INPUT FOR INPUT STATIONS ONLY
000765        WRITE (6,111) M,N,STN(I),SSAM(I),HI(I)
001003        IF (N-NUM(M)) 41,45,45
         C    .... PRINT OUT INPUT FOR ALL STATIONS
001007   43   WRITE (6,112) M,N,X(I),MAS(I),HH(I)
001025        IF (N-NSTA(M)+NN) 41,44,44
001031   44   NN=NSTA(M)
001033   45   N=0
001034   46   CONTINUE
         C
001037   47   WRITE (6,113) CG,TOTM,TOTS
         C    .... SET UP TITLES FOR TABLE II OUTPUT
         C    .... CAN PRINT FROM 1 TO 6 COLUMNS OF DATA (NVAR)
         C    .... SUBSCRIPTS ARE CHOSEN SO THAT ALL OUTPUT IS PRINTED IN
         C    ....   REFERENCE TO THE U ARRAY (SEE EQUIVALENCE STATEMENTS)
001051   48   DO 49 I=1,4
001053        HEADNG(1,I)=BLANK
001055        HEADNG(2,I)=BLANK
001057   49   CONTINUE
001060        NVAR=TENS
001062        IF (TENS) 54,54,50
001063   50   IF (TORVB) 51,52
001065   51   HEADNG(1,TENS)=TITLES(1,5)
001070        HEADNG(2,TENS)=TITLES(2,5)
001072        GO TO 53
001072   52   HEADNG(1,TENS)=TITLES(1,1)
001075        HEADNG(2,TENS)=TITLES(2,1)
001077   53   LV(TENS)=1
001101        MV(TENS)=1
001103   54   IF (ZTAC) 56,56,55
001105   55   HEADNG(1,ZTAC)=TITLES(1,2)
001110        HEADNG(2,ZTAC)=TITLES(2,2)
001112        LV(ZTAC)=2
001113        MV(ZTAC)=1
001115        IF (ZTAC.LT.NVAR) GO TO 56
```

```
001117          NVAR=ZTAC
001117    56    IF (DTENS) 61,61,57
001121    57    IF (TOPVB) 58,59
001123    58    HEADNG(1,DTENS)=TITLES(1,6)
001126          HEADNG(2,DTENS)=TITLES(2,6)
001130          GO TO 60
001130    59    HEADNG(1,DTENS)=TITLES(1,3)
001133          HEADNG(2,DTENS)=TITLES(2,3)
001135    60    LV(DTENS)=1
001137          MV(DTENS)=13
001141          IF (DTENS.LT.NVAR) GO TO 61
001143          NVAR=DTENS
001143    61    IF (DZTAC) 63,63,62
001145    62    HEADNG(1,DZTAC)=TITLES(1,4)
001150          HEADNG(2,DZTAC)=TITLES(2,4)
001152          LV(DZTAC)=2
001153          MV(DZTAC)=13
001155          IF (DZTAC.LT.NVAR) GO TO 63
001157          NVAR=DZTAC
          C     .... PUT BCD FORM OF NVAR IN PRINT FORMAT
001157    63    FORM(4)=VARNO(NVAR)
          C     .... PREPARE TO NON-DIMENSIONALIZE ALL STATION DATA
001161          XH=RFX/RFEA
001163          X3H=XH*RFX**2
001165          HX1=RFEA/RFX
001166          HX2=HX1/RFX
001167          HX3=HX2/RFX
001170          DO 64 I=1,JNT
001172          TRNS(I)=TRNS(I)*XH
001174          TRNFIJ(I)=TRNFIJ(I)*HX1
001176          TRNFIK(I)=TRNFIK(I)*HX1
001200          TRNFIL(I)=TRNFIL(I)*HX1
001201    64    CONTINUE
001203          HCHR=1./RFEA
001205          MCMR=1/REMASS
001207          XCXR=1/RFX
001212          DO 65 I=1,K
001213          HH(I)=HH(I)*HGHR
001215    65    CONTINUE
001220          DELOMG=DELOMG/OMR
001221          IF (IFRQ.NE.0) NOMODE=IFRQ
001223          NOMODE=NOMODE+FRSTMD-1
001226          ICOUNT=0
```

```
001226          IF (PLOT) IXPL=0
001231          DO 99 NFRQ=FRSTMD,NOMODE
001233          IF (NPRNT.NE.0) WRITE (6,114)
001237          MSTA=NSTA(NOMBR)
001241          DO 66 I=1,MSTA
001243          X(I)=X(I)*XCXR
001245          MAS(I)=MAS(I)*MCMR
001247     66   CONTINUE
001252          IT=1
001253          IF (IFRQ.EQ.0) GO TO 67
           C    .... USE INPUT FREQUENCIES
001254          IT=2
001255          OMEGA=OMEG(NFRQ)
001257     67   ITER=1
001260          NU=1
001261          NUCT=0
001262          GMASS=0.
           C    .... NON-DIMENSIONALIZE FREQUENCY
001263          OMEGA=OMEGA/OMR
           C    .... CALCULATE A MATRICES
001264     68   CALL AMATRX (1)
           C    .... CREATE U MATRIX
001266     69   CALL UMATRX (IERR)
001270          IF (NPRNT.NE.0) WRITE (6,115) OMEGA,UDET(NU)
           C    .... IF IERR = -1, DETERMINANT VALUE IS NO GOOD, GO TO NEXT OMEGA
           C    .... IF IERR =  0, EVERYTHING IS OK
           C    .... IF IERR =  1, 10 BAD DETERMINANTS, GO TO NEXT PROBLEM
001301          IF (IERR) 68,70,1
001303     70   GO TO (71,72), IT
001311     71   CALL ITERAT (IERR)
001313          IF (IERR.NE.0) GO TO 1
           C    .... IT = 2 IF FREQUENCY ITERATION HAS CONVERGED
001314          GO TO (68,69), IT
001322     72   CONTINUE
           C    .... RECALCULATE A MATRICES FOR MODAL DATA
001322          CALL AMATRX (2)
           C    .... DIMENSIONALIZE MODAL DATA
001324          DO 73 I=1,MSTA
001326          X(I)=X(I)/XCXR
001330          MAS(I)=MAS(I)/MCMR
001332          Y(1,I)=Y(1,I)*RFEA
001335          Y(2,I)=Y(2,I)*RFX
001340          DY(1,I)=DY(1,I)*XH
```

```
001342    73     CONTINUE
001345           OMEGA=OMEGA*OMR
001346           GMASS=GMASS*RFMASS*RFX**3
001351           VALUE=1.
001353           IF (NORN) 80,80,74
          C      .... NORMALIZE ON A SPECIFIC STATION (ZETA)
          C      .... NORN IS THE DESIRED STATION IN REFERENCE TO THE INPUT.  MUST
          C      ....   LOOK FOR THE SAME STATION IN THE EXPANDED ARRAY.
001354    74     MTOT=0
001355           LIM=NORMBR-1
001357           IF (LIM.EQ.0) GO TO 76
001360           DO 75 M=1,LIM
001361           MTOT=MTOT+NUM(M)
001363    75     CONTINUE
001365           MTOT=MTOT+NORN
001366           L1=NSTA(LIM)+NORN
001370           L2=NSTA(NORMBR)
001372           GO TO 77
001372    76     L1=NORN
001374           L2=NSTA(1)
001375           MTOT=NORN
001376    77     DO 78 M=L1,L2
001400           IF (ABS(STN(MTOT)-X(M)).LE..0001) GO TO 79
001405    78     CONTINUE
001410    79     VALUE=Y(2,M)
001413           GO TO 83
001413    80     IF (.NOT.NORDIS) GO TO 82
          C      .... NORMALIZE ON MAXIMUM ZETA
001415           ALUE=ABS(Y(2,1))
001417           VALUE=Y(2,1)
001420           DO 81 I=1,MSTA
001421           IF (ALUE.GE.ABS(Y(2,I))) GO TO 81
001425           ALUE=ABS(Y(2,I))
001430           VALUE=Y(2,I)
001431    81     CONTINUE
001434           GO TO 83
          C      .... NORMALIZE ON GENERALIZED MASS
001434    82     IF (.NOT.NOREQM) GO TO 83
001436           VALUE=SQRT(GMASS)
001440    83     DO 85 I=1,MSTA
001442           DO 84 K=1,2
001443           Y(K,I)=Y(K,I)/VALUE
001446           DY(K,I)=DY(K,I)/VALUE
```

```
                                                                                   B 429
001451   84    CONTINUE                                                            B 430
001453   85    CONTINUE                                                            B 431
001455         GMASS=GMASS/VALUE**2                                                B 432
001457         ROMEGA=OMEGA/6.2831853                                              B 433
001461         IF (.NOT.ISOD) GO TO 86                                             B 434
         C     .... READ INPUT BACK IN                                             B 435
001462         REWIND 2                                                            B 436
001464         READ (2) K,(STN(I),SSAM(I),HI(I),I=1,K)                             B 437
001504   86    FORMA(25)=OMFORM(7)                                                 B 438
001506         WRITE (6,119)                                                       B 439
001511         DO 87 I=1,6                                                         B 440
001513         IF (OMEGA.GT.10.**(I-1)) GO TO 87                                   B 441
001523         FORMA(25)=OMFORM(I)                                                 B 442
001525         GO TO 88                                                            B 443
001525   87    CONTINUE                                                            B 444
001527   88    WRITE (6,FORMA) (TITLE(I),I=1,14),TIME,NFRQ,ROMEGA,OMEGA            B 445
001551         IF (.NOT.TORV8) WRITE (6,116) GMASS                                 B 446
001560         WRITE (6,117) (HEADNG(1,I),HEADNG(2,I),I=1,NVAR)                    B 447
001577         WRITE (6,118)                                                       B 448
001603         LINE=9                                                              B 449
001604         N=0                                                                 B 450
001605         NN=0                                                                B 451
001606         I=0                                                                 B 452
001607         NS=1                                                                B 453
001610         DO 97 M=1,NOMBR                                                     B 454
001611   89    LINE=LINE+1                                                         B 455
001613         IF (LINE.LT.LMAX) GO TO 90                                          B 456
001615         LINE=0                                                              B 457
001616         WRITE (6,119)                                                       B 458
001621         WRITE (6,120) NFRQ,(TITLE(L),L=1,14),(HEADNG(1,II),HEADNG(2,II),II  B 459
              1=1,NVAR)                                                            B 460
001650         WRITE (6,118)                                                       B 461
001654   90    N=N+1                                                               B 462
001656         I=I+1                                                               B 463
001657         IF (.NOT.ISOD) GO TO 93                                             B 464
         C     .... WRITE OUTPUT FOR INPUT STATIONS ONLY                           B 465
001660   91    IF (ABS(STN(NS)-X(I)).LE..0001) GO TO 93                            B 466
001666         I=I+1                                                               B 467
001667         GO TO 91                                                            B 468
001667   92    NS=NS+1                                                             B 469
001671         IF (N-NUM(M)) 89,96,96                                              B 470
001674   93    DO 94 II=1,NVAR                                                     B 471
001676         LL=LV(II)+2*(I-1)
```

```
001701           MM=MV(II)
001703           SAM(II)=U(LL,MM)
001707   94      CONTINUE
001711           WRITE (6,FORM) M,N,X(I),(SAM(II),II=1,NVAR)
001732           IF (ISOD) GO TO 92
001734           IF (N-NSTA(M)+NN) 89,95,95
001737   95      NN=NSTA(M)
001741   96      N=0
001742   97      CONTINUE
001745           IF (IFRQ.EQ.0) OMEGA=OMEGA+DELOMG*OMR
001751           IF (PLOT) 98,99
001753   98      ICOUNT=ICOUNT+1
001755           IF (ICOUNT.GT.3) GO TO 99
001760           CALL STORPLT (NSUM,NUM,ISOD,TORVB,IXPL)
001763   99      CONTINUE
001766           IF (PLOT.AND.ICOUNT.GE.3) RETURN
001775           GO TO 1
         C
001776   100     FORMAT (A2,13A6)
001776   101     FORMAT (1H19X,A2,13A6)
001776   102     FORMAT (1H09X,A2,13A6)
001776   103     FORMAT (5X,A2,A6,5X,42H---THIS CARD IS IN ERROR.  JOB TERMINATED.)
001776   104     FORMAT (5X,42HMORE THAN  600 STATIONS.   RUN TERMINATED.)
001776   105     FORMAT (1H15X,20HPARAMETER CONTROLS -A2,13A6///)
001776   106     FORMAT (19X,A6,0PE13.5,12X,8HNOMODE =I4//17X,A8,E13.5,12X,8HNORMBR
                1 =I4//20X,5HREX =E13.5,14X,6HNOPN =I4//13X,12HOMEGA SUBR =E13.5,12
                2X,9HNORDIS = A3//19X,6HDELX =E13.5,12X,A9,A3///13X,7HOMEGA =E13.5/
                3/17X,A8,E13.5,12X,8HDELOMG =E13.5//18X,A7,E13.5,12X,8HOMGTOL =E13.
                45//)
001776   107     FORMAT (1H05X,19HBOUNDARY CONDITIONS//6X,5HJOINT4X,53HMBRI   MBRJ
                1  MBRK    MBRL IOJTRN  IJTRN  IKTRN  ILTRN//(6X,I3,2X,8I7/))
001776   108     FORMAT (1H15X,32HSPRING AND FLEXIBILITY CONSTANTS//6X,5HJOINT4X,63
                1HMBRI   MBRJ  MBRK   MBRL   TRNS     TRNFIJ    TRNFIK    TRNFIL//
                2(6X,I3,2X,4I7,0P4E10.3))
001776   109     FORMAT (1H 50X,7HTABLE I//43X,24HPHYSICAL CHARACTERISTICS/43X,3A10
                1,//36X,A2,13A6//7X,7HSTATION7X,1HX12X,A4,13X,A2//)
001776   110     FORMAT (1H 40X,19HTABLE I (CONTINUED)//7X,7HSTATION7X,1HX12X,A4,14
                1X,A2//)
001776   111     FORMAT (I8,1H-I3,0PF12.3,0P2E16.5)
001776   112     FORMAT (I8,1H-I3,0PF12.3,0P2E16.5)
001776   113     FORMAT (////20X,22HCENTER OF GRAVITY X = F10.5//20X,22HTOTAL MASS
                1        = F10.5//20X,22HS                         = 0PE14.5)
001776   114     FORMAT (1H1)
```

```
001776   115   FORMAT (5X,16HNON-DIM. FREQ = 1PE16.7,5X,8HDETERM =E16.7)          B 515
001776   116   FORMAT (4X,17H GENERALIZED MASS,E16.7)                               B 516
001776   117   FORMAT (/5X,7HSTATION6X,1HX,2X,4(2X,2A6))                            B 517
001776   118   FORMAT (//)                                                          B 518
001776   119   FORMAT (1H1/////)                                                    B 519
001776   120   FORMAT (1H 42X,14HTABLE II, MODEI3,//36X,A2,13A6//5X,7HSTATION6X,1   B 520
              1HX,2X,4(2X,2A6))                                                     B 521
001776         END                                                                  B 522-
```

```
               SUBROUTINE STORPLT (NSUM,NUM,ISOD,TORVB,IXPL)
000010         DIMENSION NUM(30), Y(2,600), DY(2,600), STN(600), SSAM(600), NS(30
              1), HI(600)
000010         LOGICAL TORVB,ISOD
000010         REAL MAS
000010         COMMON /BLK2/ X(600),HH(600),MAS(600),NSTA(30)
000010         COMMON /BLK3/ U(100,60),B(2,2,30),OMEGA,NOMBR,TEMP(2,30),GMASS
000010         EQUIVALENCE (U,Y), (U(1,13),DY), (U(1,25),STN), (U(1,31),SSAM), (U
              1(1,37),HI)
000010         IF (ISOD) GO TO 5
000011         NS(1)=NSTA(1)
000012         NI=NOMBR-1
000014         DO 1 I=1,NI
000016    1    NS(I+1)=NSTA(I+1)-NSTA(I)
000023         NSUM=NSTA(NOMBR)
000024         WRITE (6,11) NSUM
000032         IF (IXPL.EQ.0) 2,3
000041    2    CALL RECOUT (3,1,0,TORVB,NOMBR)
000044         CALL RECOUT (3,2,0,NS,1,NOMBR,1)
000053         IXPL=1
000057         CALL RECOUT (3,1,0,NSUM)
000063         WRITE (4) (X(I),I=1,NSUM),(MAS(I),I=1,NSUM),(HH(I),I=1,NSUM)
000132    3    DO 4 K=1,NSUM
000137         CALL RECOUT (3,1,0,Y(2,K),DY(2,K),Y(1,K))
000152    4    CONTINUE
000160         RETURN
          C    PLOT DATA FOR INPUT STATIONS ONLY
000161    5    NSUM=0
000162         DO 6 I=1,NOMBR
000163    6    NSUM=NSUM+NUM(I)
000167         WRITE (6,11) NSUM
000174         IF (IXPL.EQ.0) 7,8
000203    7    IXPL=1
000204         CALL RECOUT (3,1,0,TORVB,NOMBR)
000207         CALL RECOUT (3,2,0,NUM,1,NOMBR,1)
000221         CALL RECOUT (3,1,0,NSUM)
000230         WRITE (4) (STN(I),I=1,NSUM),(SSAM(I),I=1,NSUM),(HI(I),I=1,NSUM)
000277    8    NS=0
000300         I=1
000301         DO 10 J=1,NOMBR
000306         NU=NUM(J)
000310         DO 10 II=1,NU
```

```
000311            NS=NS+1
000313    9       IF (ABS(STN(NS)-X(I)).LE..0001) GO TO 10
000321            I=I+1
000322            GO TO 9
000322   10       CALL RECOUT (3,1,0,Y(2,I),DY(2,I),Y(1,I))
000345            RETURN
          C
000346   11       FORMAT (/21H TOTAL NO. STATIONS =,I5)
000346            END
```

```
                SUBROUTINE STORE (A,N,M,I)
000007          DIMENSION A(1,2)
000007          COMMON /BLK3/ U(100,60),B(2,2,30),OMEGA,NOMBR,TEMP(2,30),GMASS
000007          JJ=M
000010          DO 4  J=1,2
000011          IF (I) 1,2,2
000012    1     U(N,JJ)=-A(1,J)
000017          GO TO 3
000020    2     U(N,JJ)=A(1,J)
000025    3     JJ=JJ+1
000027    4     CONTINUE
000031          RETURN
000032          END
```

```
                SUBROUTINE CGRAV (NOMBR,TOTM,S,CG)
000007          COMMON /BLK2/ X(600),HH(600),MAS(600),NSTA(30)
000007          REAL MAS
000007          SUM1=0.
000007          SUM2=0.
000010          L1=2
000011          L2=NSTA(1)
000013          DO 2 M=1,NOMBR
000015          DO 1 I=L1,L2
000017          SUM1=SUM1+.5*(MAS(I)+MAS(I-1))*(X(I)-X(I-1))
000026          SUM2=SUM2+.25*(MAS(I)+MAS(I-1))*(X(I)**2-X(I-1)**2)
000036    1     CONTINUE
000040          L1=NSTA(M)+2
000042          L2=NSTA(M+1)
000044    2     CONTINUE
000046          TOTM=SUM1
000047          S=SUM2
000050          CG=S/TOTM
000051          RETURN
000052          END
```

```
                SUBROUTINE MOVE (A,C,N)
000006          DIMENSION C(N,2), A(N,2)
000006          DO 2 I=1,N
000007          DO 1 J=1,2
000010          C(I,J)=A(I,J)
000021    1     CONTINUE
000022    2     CONTINUE
000024          RETURN
000025          END
```

```
                  SUBROUTINE MATMPY (A,B,C,N,M)
 000010           DIMENSION A(N,2), B(2,M), C(N,M)
           C      .... N = NO. OF ROWS IN A AND C
           C      .... M = NO. OF COLUMNS IN B AND C
 000010           DO 2 I=1,N
 000011           DO 1 J=1,M
 000012           C(I,J)=A(I,1)*B(1,J)+A(I,2)*B(2,J)
 000043    1      CONTINUE
 000046    2      CONTINUE
 000050           RETURN
 000051           END
```

```
               SUBROUTINE INTERP (XX,YY,I)
000006         DIMENSION X(4), Y(4), XX(20), YY(20)
000006         IF (I.LT.3) GO TO 6
000010         X(1)=XX(I-2)
000011         X(2)=XX(I-1)
000013         X(3)=XX(I)
000014         Y(1)=YY(I-2)
000016         Y(2)=YY(I-1)
000017         Y(3)=YY(I)
000021         S1=(X(1)-X(2))/(Y(1)-Y(2))
000025         S2=(X(2)-X(3))/(Y(2)-Y(3))
000030         IF (ABS(S1/S2-1.).LE.1.E-6) GO TO 5
000035         XMAX=AMAX1(X(1),X(2),X(3))+1.E-7
000044         XMIN=AMIN1(X(1),X(2),X(3))-1.E-7
000053         DENOM=(X(1)**2-X(3)**2)*(X(2)-X(3))-(X(2)**2-X(3)**2)*(X(1)-X(3))
000065         A=((Y(1)-Y(3))*(X(2)-X(3))-(Y(2)-Y(3))*(X(1)-X(3)))/DENOM
000077         B=((X(1)**2-X(3)**2)*(Y(2)-Y(3))-(X(2)**2-X(3)**2)*(Y(1)-Y(3)))/DE
              1NOM
000112         C=Y(3)-A*X(3)**2-B*X(3)
000120         IF (A.LT.1.E18.AND.B.LT.1.E18.AND.C.LT.1.E18) GO TO 1
000133         A=A/1.E18
000134         B=B/1.E18
000135         C=C/1.E18
000136    1    SQR=B**2-4.*A*C
000142         IF (SQR) 7,3,2
000146    2    SQR=SQRT(SQR)
000147    3    R1=(-B+SQR)/(2.*A)
000153         R2=(-B-SQR)/(2.*A)
000157         XX(I+1)=0.
000162         IF (R1.GT.XMIN.AND.R1.LT.XMAX) XX(I+1)=R1
000174         IF (R2.GT.XMIN.AND.R2.LT.XMAX) XX(I+1)=R2
000206         IF (XX(I+1).EQ.0.) GO TO 7
000210    4    CONTINUE
000210         RETURN
000211    5    XX(I+1)=X(2)-Y(2)*S2
000215         GO TO 4
000216    6    XX(3)=XX(1)-YY(1)*(XX(1)-XX(2))/(YY(1)-YY(2))
000225         GO TO 4
000225    7    J=I
000226    8    J=J-1
000230         IF (YY(J)*YY(I)) 9,9,8
000233    9    XX(I+1)=XX(J)-YY(J)*(XX(J)-XX(I))/(YY(J)-YY(I))
```

```
      GO TO 4                                                          H  43
      END                                                              H  44-
```

```
                SUBROUTINE ITERAT (IERR)
000003          DIMENSION OM(20)
000003          COMMON /BLK3/ U(100,60),B(2,2,30),OMEGA,NOMBR,TEMP(2,30),GMASS
000003          COMMON /BLK4/ UDET(20),NU,DELOMG,OMGTOL,IT,ITER,NUCT
000003          IERR=0
000004          GO TO (1,5), ITER
000011     1    IF (NU.NE.1) GO TO 2
000013          OM(1)=OMEGA
000015          GO TO 3
           C    .... TEST FOR SIGN CHANGE
000015     2    IF (UDET(NU-1)*UDET(NU)) 4,9,3
           C    .... INCREMENT FREQUENCY AND TRY AGAIN
000020     3    NU=NU+1
000022          IF (NU.GT.20) GO TO 14
000025          OM(NU)=OM(NU-1)+DELOMG
000027          GO TO 7
           C    .... SIGN HAS CHANGED.  BEGIN ITERATION.
000030     4    ITER=2
000031          IF (NU.LE.3) GO TO 6
000034          UDET(1)=UDET(NU-2)
000035          UDET(2)=UDET(NU-1)
000037          UDET(3)=UDET(NU)
000040          OM(1)=OM(NU-2)
000042          OM(2)=OM(NU-1)
000043          OM(3)=OM(NU)
000045          NU=3
000046          GO TO 6
           C    .... TEST FOR CONVERGENCE
           C    .... IS DETERMINANT NEARLY ZERO
000046     5    IF (ABS(UDET(NU)).LE.1.E-6) GO TO 9
           C    .... IS CHANGE IN DETERMINANT NEARLY ZERO
000052          IF (ABS(UDET(NU)/UDET(NU-1)-1.).LE.1.E-6) GO TO 9
           C    .... IS CHANGE IN FREQUENCY LESS THAN TOLERANCE
000057          IF (ABS(OM(NU)/OM(NU-1)-1.).LE.OMGTOL) GO TO 9
000065          IF (NU.EQ.20) GO TO 10
000067     6    CALL INTERP (OM,UDET,NU)
000072          NU=NU+1
000074          IT=1
000075     7    OMEGA=OM(NU)
000077     8    RETURN
           C    .... GOOD FREQUENCY. RETURN TO CALCULATE MODAL DATA.
000100     9    IT=2
```

```
                  SUBROUTINE UMATRX (IERR)                                              J   1
000003            DIMENSION C(1,2), D(1,2), E(1,2), F(1,2), G(1,2), H(1,2), P(1,2),     J   2
                 1Q(1,2), R(1,2), S(1,2), BASE1(1,2), BASE2(1,2), ERASE(100)            J   3
000003            DIMENSION COL(120)                                                    J   4
000003            COMMON /BLK1/ JC(30,8),TRNS(30),TRNFIJ(30),TRNFIK(30),TRNFIL(30),J    J   5
                 1NT                                                                    J   6
000003            COMMON /BLK3/ U(100,60),B(2,2,30),OMEGA,NOMBR,TEMP(2,30),GMASS        J   7
000003            COMMON /BLK4/ UDET(20),NU,DELOMG,OMGTOL,IT,ITER,NUCT                  J   8
000003            EQUIVALENCE (C,D,E,F), (G,P,R), (H,Q,S), (COL,TEMP)                   J   9
000003            DATA BASE1/1.,0./,BASE2/0.,1./                                        J  10
000003            IERR=0                                                                J  11
000004            LIM=2*NOMBR                                                           J  12
000005            DO 2 J=1,LIM                                                          J  13
000006            DO 1 I=1,LIM                                                          J  14
000007            U(I,J)=0.                                                             J  15
000013      1     CONTINUE                                                              J  16
000015      2     CONTINUE                                                              J  17
000017            JR1=1                                                                 J  18
000020            JR2=2                                                                 J  19
000021            DO 32 M=1,JNT                                                         J  20
            C     .... IS THERE AN -I- BEAM                                             J  22
000023            ITRIG=1                                                               J  21
000024            IF (JC(M,1)) 3,27,3                                                   J  23
000025      3     CALL MOVE (BASE1,C,1)                                                 J  24
000030            KK=2*(JC(M,1)-1)+1                                                    J  25
000033            LL=JR1                                                                J  26
000035            IF (JC(M,5)-2) 4,5,6                                                  J  27
000040      4     C(1,1)=0.                                                             J  28
000041            C(1,2)=1.                                                             J  29
000043            ITRIG=2                                                               J  30
000044            GO TO 6                                                               J  31
000044      5     C(1,2)=TRNS(M)                                                        J  32
000046      6     N=JC(M,1)                                                             J  33
000050            CALL MATMPY (C,B(1,1,N),TEMP,1,2)                                     J  34
000055            CALL STORE (TEMP,LL,KK,1)                                             J  35
            C     .... IS THERE A -J- BEAM                                              J  36
000060            IF (JC(M,2)) 7,13,7                                                   J  37
000063      7     CALL MOVE (BASE1,D,1)                                                 J  38
000066            CALL MOVE (BASE2,G,1)                                                 J  39
000071            CALL MOVE (BASE2,H,1)                                                 J  40
000074            GO TO (9,8), ITRIG                                                    J  41
000103      8     D(1,1)=0.                                                             J  42
```

```
000261          CALL MOVE (BASE2,S,1)
000264          GO TO (22,23), ITRIG
000273    22    F(1,1)=0.
000274    23    KKK=2*(JC(M,4)-1)+1
000277          CALL STORE (F,LL,KKK,-1)
000302          IF (JC(M,8)-2) 26,24,25
000307    24    S(1,1)=-TRNFIL(M)
000311          GO TO 26
000312    25    R(1,2)=0.
000313          S(1,2)=0.
000314          S(1,1)=1.
000315    26    LLL=JR2
000317          JR2=JR2+1
000320          N=JC(M,1)
000322          CALL MATMPY (R,B(1,1,N),TEMP,1,2)
000327          CALL STORE (TEMP,LLL,KK,1)
000332          CALL STORE (S,LLL,KKK,-1)
000335          GO TO 31
000337    27    CALL MOVE (BASE1,D,1)
000342          KK=2*(JC(M,2)-1)+1
000345          LL=JR1
000347          IF (JC(M,5)-2) 28,29,30
000352    28    D(1,1)=0.
000353          D(1,2)=-1.
000355          GO TO 30
000355    29    D(1,2)=-TRNS(M)
000357    30    CALL STORE (D,LL,KK,-1)
000362    31    JR1=JR2
000364          JR2=JR2+1
000365    32    CONTINUE
000371          IF (IT.EQ.2) GO TO 37
000373          ISCALE=0
000374          CALL SIMEQ (U,LIM,0,0,UDET(NU),TEMP,100,ISCALE)
000404          IF (ISCALE.EQ.0) GO TO 35
000406          CALL OVERFL (J)
000410          DET=UDET(NU)*1.E18**ISCALE
000416          CALL OVERFL (J)
000417          GO TO (33,34), J
000426    33    ISCALE=18*ISCALE
000430          WRITE (6,50) OMEGA,UDET(NU),ISCALE
000442          IERR=-1
000444          NU=1
000445          LARGE=LARGE+1
```

```
000104    9    KKK=2*(JC(M,2)-1)+1
000107         CALL STORE (D,LL,KKK,-1)
000112         IF (JC(M,6)-2) 12,10,11
000117   10    H(1,1)=-TRNFIJ(M)
000121         GO TO 12
000122   11    G(1,2)=0.
000123         H(1,2)=0.
000124         H(1,1)=1.
000125   12    LLL=JR2
000127         JR2=JR2+1
000130         CALL STORE (H,LLL,KKK,-1)
000133         N=JC(M,1)
000135         CALL MATMPY (G,B(1,1,N),TEMP,1,2)
000142         CALL STORE (TEMP,LLL,KK,1)
          C    .... IS THERE A -K- BEAM
000145   13    IF (JC(M,3)) 14,20,14
000150   14    CALL MOVE (BASE1,E,1)
000153         CALL MOVE (BASE2,P,1)
000156         CALL MOVE (BASE2,Q,1)
000161         GO TO (16,15), ITRIG
000170   15    E(1,1)=0.
000171   16    KKK=2*(JC(M,3)-1)+1
000174         N=JC(M,3)
000176         CALL MATMPY (E,B(1,1,N),TEMP,1,2)
000203         CALL STORE (TEMP,LL,KKK,1)
000206         IF (JC(M,7)-2) 19,17,18
000213   17    Q(1,1)=TRNFIK(M)
000215         GO TO 19
000216   18    P(1,2)=0.
000217         Q(1,2)=0.
000220         Q(1,1)=1.
000221   19    LLL=JR2
000223         JR2=JR2+1
000224         N=JC(M,1)
000226         CALL MATMPY (P,B(1,1,N),TEMP,1,2)
000233         CALL STORE (TEMP,LLL,KK,1)
000236         N=JC(M,3)
000240         CALL MATMPY (Q,B(1,1,N),TEMP,1,2)
000245         CALL STORE (TEMP,LLL,KKK,-1)
          C    .... IS THERE AN -L- BEAM
000250   20    IF (JC(M,4)) 21,31,21
000253   21    CALL MOVE (BASE1,F,1)
000256         CALL MOVE (BASE2,R,1)
```

```
000447          OMEGA=OMEGA+DELOMG
000451          IF (LARGE.GT.10) IERR=1
000454          GO TO 36
000455    34    UDET(NU)=DET
000457    35    LARGE=0
000460    36    RETURN
000461    37    CONTINUE
          C     .... CHECK FOR NON-ZERO ELEMENT IN FIRST BEAM
000461          DO 38 M=1,JNT
000463          IF (JC(M,2).EQ.1) GO TO 39
000465    38    CONTINUE
000467    39    IF (JC(M,5)-2) 40,41,41
000472    40    II=1
000473          GO TO 42
000474    41    II=2
000475    42    CONTINUE
000475          DO 43 J=1,LIM
000477          COL(J)=-U(J,II)
000504    43    CONTINUE
          C     .... REMOVE SELECTED ROW AND COLUMN
000507          LLIM=II+1
000511          DO 45 J=LLIM,LIM
000512          DO 44 I=1,LIM
000513          U(I,J-1)=U(I,J)
000522    44    CONTINUE
000524    45    CONTINUE
000526          N=LIM-1
000530          DO 47 I=LLIM,LIM
000532          DO 46 J=1,N
000533          U(I-1,J)=U(I,J)
000542    46    CONTINUE
000544          COL(I-1)=COL(I)
000546    47    CONTINUE
          C     .... SOLVE FOR MODAL.MATRIX
000551          CALL SIMEQ (U,LIM-1,COL,1,DET,ERASE,100,ISCALE)
          C     .... INSERT 1. WHERE ROW AND COLUMN REMOVED
000562          I=LIM
000564    48    IF (I.LE.II) GO TO 49
000570          COL(I)=COL(I-1)
000571          I=I-1
000573          GO TO 48
000573    49    COL(II)=1.
000575          GO TO 36
```

```
      C
000576    50    FORMAT (//5X,16HNON-DIM. FREQ = 1PE16.7,5X,8HDETERM =E16.7,7H * 10
               1**I3)
000576          END
```

```
                SUBROUTINE AMATRX (IX)
000003          DIMENSION A(2,2), W(2,2), Y(2,600), DY(2,600)
000003          DIMENSION EA(600)
000003          COMMON /BLK2/ X(600),HH(600),MAS(600),NSTA(30)
000003          COMMON /BLK3/ U(100,60),B(2,2,30),OMEGA,NOMBR,TEMP(2,30),GMASS
000003          EQUIVALENCE (U,Y), (U(1,13),DY), (HH,EA)
000003          REAL MAS
000003          FRQ=OMEGA
000005          FRQ2=FRQ**2
000006          L=-1
000007          DO 8 M=1,NOMBR
000010          K=L+2
000012          L=NSTA(M)-1
000014          DO 7 I=K,L
000016          DX=X(I+1)-X(I)
000020          IF (DX.EQ.0.) GO TO (7,3), IX
000027          DX2=DX*DX
          C     .... CALCULATE A MATRIX
000031          A(1,1)=1.-(FRQ2*MAS(I+1)*DX2)/(2.*EA(I))
000040          A(1,2)=-(MAS(I+1)+MAS(I))*DX*FRQ2/2.
000045          A(2,1)=DX/2.*(1./EA(I+1)+1./EA(I))
000052          A(2,2)=1.-MAS(I)*FRQ2*DX2/(2.*EA(I+1))
000061          GO TO (5,1), IX
          C     .... CALCULATE MODAL DATA FOR EACH STATION
000066    1     IF (I.GT.K) GO TO 2
          C     .... MOVE TEMP TO Y FOR FIRST STATION ON BEAM
000072          Y(1,I)=TEMP(1,M)
000075          Y(2,I)=TEMP(2,M)
000100          W(1,1)=W(2,2)=0.
000102          W(1,2)=-MAS(I)*FRQ2
000104          W(2,1)=1./EA(I)
          C     .... MULTIPLY W X Y TO FIND DY FOR FIRST STATION ON BEAM
000106          CALL MATMPY (W(1,1),Y(1,I),DY(1,I),2,1)
          C     .... MULTIPLY A(I) X Y(I) TO FIND Y(I+1)
000114    2     CALL MATMPY (A(1,1),Y(1,I),Y(1,I+1),2,1)
000123          W(1,1)=W(2,2)=0.
000126          W(1,2)=-MAS(I+1)*FRQ2
000130          W(2,1)=1./EA(I+1)
          C     .... MULTIPLY W(I+1) X Y(I+1) TO FIND DY(I+1)
000132          CALL MATMPY (W(1,1),Y(1,I+1),DY(1,I+1),2,1)
000140          GMASS=GMASS+(1./6.)*DX*(MAS(I)*(2.*Y(2,I)**2+Y(2,I+1)**2)+MAS(I+1)
               1*(Y(2,I)**2+2.*Y(2,I+1)**2))
```

```
000101         ICK=0                                                          I
000102         GO TO 7                                                        I
000104    10   CONTINUE                                                       I
000104         IF (ICK.EQ.1) GO TO 13                                         I
          C    .... TRY ANOTHER 20 ITERATIONS AVOIDING BAD SPOT IF POSSIBLE   I
000106         ICK=1                                                          I
000107         III=3                                                          I
000110    11   IF (UDET(III)*UDET(20).LT.0.) GO TO 12                         I
000113         III=III-1                                                      I
000114         GO TO 11                                                       I
000115    12   UDET(2)=UDET(III)                                              I
000117         UDET(1)=UDET(20)                                               I
000121         OM(2)=OM(III)                                                  I
000122         OM(1)=OM(20)                                                   I
000124         OM(3)=-(OM(1)-OM(2))/2.+OM(1)                                  I
000127         NU=3                                                           I
000130         GO TO 7                                                        I
000131    13   WRITE (6,16)                                                   I
000135         WRITE (6,18) (OM(I),UDET(I),I=1,NU)                            I
000152         IERR=1                                                         I
000154         GO TO 8                                                        I
000155    14   NUCT=NUCT+1                                                    I
000157         IF (NUCT.EQ.10) GO TO 15                                       I
000161         UDET(1)=UDET(NU-2)                                             I
000163         UDET(2)=UDET(NU-1)                                             I
000164         OM(1)=OM(NU-2)                                                 I
000166         OM(2)=OM(NU-1)                                                 I
000167         OM(3)=OM(2)+DELOMG                                             I
000171         NU=3                                                           I
000172         GO TO 7                                                        I
000172    15   WRITE (6,17)                                                   I
000176         NU=NU-1                                                        I
000200         WRITE (6,18) (OM(I),UDET(I),I=1,NU)                            I
000214         IERR=1                                                         I
000216         GO TO 8                                                        I
          C                                                                   I
000217    16   FORMAT (5X,35HNO CONVERGENCE AFTER 20 ITERATIONS.)             I
000217    17   FORMAT (5X,80H200 FREQUENCY CHANGES WITH NO SIGN CHANGE IN THE U D I
              1ETERMINANT.  RUN TERMINATED.)                                  I
000217    18   FORMAT (/(5X,16HNON-DIM. FREQ = 0PE16.7,5X,8HDETERM =E16.7))   I
000217         END                                                            I
```

```
000162            GO TO 7
           C      .... REPEAT VALUES FOR DUPLICATE STATIONS
000164     3      DO 4 II=1,2
000166            Y(II,I+1)=Y(II,I)
000172            DY(II,I+1)=DY(II,I)
000176     4      CONTINUE
000177            GO TO 7
           C      .... CALCULATE B MATRICES
000200     5      IF (I.GT.K) GO TO 6
           C      .... MOVE A TO B FOR FIRST STATION ON BEAM
000204            CALL MOVE (A(1,1),B(1,1,M),2)
000210            GO TO 7
           C      .... MULTIPLY A X B, STORE IN TEMP
000212     6      CALL MATMPY (A(1,1),B(1,1,M),TEMP(1,1),2,2)
           C      .... MOVE TEMP TO B
000220            CALL MOVE (TEMP(1,1),B(1,1,M),2)
000225     7      CONTINUE
000231     8      CONTINUE
000233            RETURN
000234            END
```

```
                SUBROUTINE SIMEQ (A,N,B,M,DETERM,IPIVOT,NMAX,ISCALE)
            C   SOLUTION OF SIMULTANEOUS LINEAR EQUATIONS
            C
000013          DIMENSION IPIVOT(N), A(NMAX,N), B(NMAX,M)
000013          EQUIVALENCE (IROW,JROW), (ICOLUM,JCOLUM), (AMAX,T,SWAP)
000013          DATA R1,R2/1.E18,1.E-18/
            C
            C   INITIALIZATION
            C
000013      1   ISCALE=0
000014          DETERM=1.0
000015          DO 2 J=1,N
000017      2   IPIVOT(J)=0
000022          DO 37 I=1,N
            C
            C   SEARCH FOR PIVOT ELEMENT
            C
000024          AMAX=0.0
000025          DO 7 J=1,N
000027          IF (IPIVOT(J)-1) 3,7,3
000032      3   DO 6 K=1,N
000034          IF (IPIVOT(K)-1) 4,6,38
000037      4   IF (ABS(AMAX)-ABS(A(J,K))) 5,6,6
000051      5   IROW=J
000053          ICOLUM=K
000054          AMAX=A(J,K)
000061      6   CONTINUE
000064      7   CONTINUE
000067          IF (AMAX) 9,8,9
000070      8   DETERM=0.0
000071          ISCALE=0
000072          GO TO 38
000073      9   IPIVOT(ICOLUM)=IPIVOT(ICOLUM)+1
            C
            C   INTERCHANGE ROWS TO PUT PIVOT ELEMENT ON DIAGONAL
            C
000076          IF (IROW-ICOLUM) 10,14,10
000100     10   DETERM=-DETERM
000101          DO 11 L=1,N
000103          SWAP=A(IROW,L)
000110          A(IROW,L)=A(ICOLUM,L)
000120     11   A(ICOLUM,L)=SWAP
```

```
000124          IF (M) 14,14,12
000125    12    DO 13 L=1,M
000127          SWAP=B(IROW,L)
000134          B(IROW,L)=B(ICOLUM,L)
000144    13    B(ICOLUM,L)=SWAP
000150    14    PIVOT=A(ICOLUM,ICOLUM)
          C
          C     SCALE THE DETERMINANT
          C
000155          PIVOTI=PIVOT
000156          IF (ABS(DETERM)-R1) 17,15,15
000161    15    DETERM=DETERM/R1
000163          ISCALE=ISCALE+1
000164          IF (ABS(DETERM)-R1) 20,16,16
000167    16    DETERM=DETERM/R1
000171          ISCALE=ISCALE+1
000172          GO TO 20
000173    17    IF (ABS(DETERM)-R2) 18,18,20
000176    18    DETERM=DETERM*R1
000200          ISCALE=ISCALE-1
000201          IF (ABS(DETERM)-R2) 19,19,20
000204    19    DETERM=DETERM*R1
000206          ISCALE=ISCALE-1
000207    20    IF (ABS(PIVOTI)-R1) 23,21,21
000213    21    PIVOTI=PIVOTI/R1
000215          ISCALE=ISCALE+1
000217          IF (ABS(PIVOTI)-R1) 26,22,22
000221    22    PIVOTI=PIVOTI/R1
000223          ISCALE=ISCALE+1
000225          GO TO 26
000225    23    IF (ABS(PIVOTI)-R2) 24,24,26
000231    24    PIVOTI=PIVOTI*R1
000233          ISCALE=ISCALE-1
000235          IF (ABS(PIVOTI)-R2) 25,25,26
000240    25    PIVOTI=PIVOTI*R1
000242          ISCALE=ISCALE-1
000244    26    DETERM=DETERM*PIVOTI
          C
          C     DIVIDE PIVOT ROW BY PIVOT ELEMENT
          C
000246          DO 28 L=1,N
000247          IF (IPIVOT(L)-1) 27,28,38
000252    27    A(ICOLUM,L)=A(ICOLUM,L)/PIVOT
```

```
000260   28    CONTINUE
000263         IF (M) 31,31,29
000264   29    DO 30 L=1,M
000266   30    B(ICOLUM,L)=B(ICOLUM,L)/PIVOT
         C
         C     REDUCE NON-PIVOT ROWS
         C
000276   31    DO 37 L1=1,N
000300         IF (L1-ICOLUM) 32,37,32
000302   32    T=A(L1,ICOLUM)
000310         DO 34 L=1,N
000311         IF (IPIVOT(L)-1) 33,34,38
000314   33    A(L1,L)=A(L1,L)-A(ICOLUM,L)*T
000325   34    CONTINUE
000330         IF (M) 37,37,35
000331   35    DO 36 L=1,M
000333   36    B(L1,L)=B(L1,L)-B(ICOLUM,L)*T
000351   37    CONTINUE
000356   38    RETURN
000357         END
*FOLLOWING VARIABLES EQUIVALENCED BUT NOT REFERENCED
JROW
JCOLUM
```

```
                    OVEPLAY(LINK,2,0)
                    PROGRAM PLOT
000003              COMMON NPLOT
000003              INTEGER PLTZ,PLTZPR,PLTT
000003              INTEGER PLTM,PLTZR,PLTAE,PLTJG
000003              LOGICAL TORVB
000003              REAL MAS,MZRMIN
          C
000003              EQUIVALENCE (ZPMIN(1),ZPMIN1), (ZPMIN(2),ZPMIN2), (ZPMIN(3),ZPMIN3
                   1), (DZP(1),DZP1), (DZP(2),DZP2), (DZP(3),DZP3)
          C
000003              DIMENSION NS(12), X(600), MAS(600), Y(600), Y1(600,3), DY(600,3),
                   1XP(602), YP(1802), ZPMIN(3), DZP(3), HEADER(8)
          C
000003              NAMELIST /NAM1/ INCHX,DX,XMIN,PLTZ,PLTZPR,NZAPR,MPLT,PLTT,NPLTT,TM
                   1IN,ZPMIN1,DZP1,ZPMIN2,DZP2,ZPMIN3,DZP3,DTT,PLTM,NMZR,MZRMIN,DMZR,P
                   2LTZP,PLTAE,PLTJG,NMAEJG,AJMIN,DAEJG
          C
000003              REWIND 3
000005              REWIND 4
000007              PLTZ=PLTZPP=PLTT=MPLT=0
000013              PLTM=PLTZR=PLTAE=PLTJG=0
000017              NZAPR=NPLTT=1
000021              NMZR=NMAEJG=1
000023              READ (5,28) HEADER
000030              READ (5,NAM1)
000033              WRITE (6,NAM1)
000036              CALL NOTATE (0.,0.,.14,HEADER,90.,80)
000042              CALL CALPLT (5.,0.,-3)
000045              CALL RECIN (3,1,2,TORVB,NOMBR)
000051              CALL RECIN (3,2,NOMBR,NS,1,NOMBR,1)
000060              CALL RECIN (3,1,1,NSUM)
000063              READ (4) (X(I),I=1,NSUM),(MAS(I),I=1,NSUM),(Y(I),I=1,NSUM)
000111              XINCH=INCHX
000113              IF (PLTM.EQ.0.AND.PLTZP.EQ.0) GO TO 6
          C
          C         PLOT MASS OR ROLL INERTIA
          C
000121              IF (TORVB) 1,2
000122    1         CALL NOTATE (0.,1.5,0.14,17HROLL INERTIA PLOT,90.,17)
000126              GO TO 3
000127    2         CALL NOTATE (0.,1.5,0.14,9HMASS PLOT,90.,9)
```

```
000133    3      CALL CALPLT (5.,0.,-3)
000136           CALL GRID (0.,0.,1.,1.,INCHX,4)
000142           CALL AXES (0.,0.,0.,XINCH,XMIN,DX,1.,2.,18HSCALE OF X, INCHES,.125
                1,-18)
000155           IF (NMZP.EQ.1) 4,5
000162    4      CALL ASCALE (MAS,4.,NSUM,1,20.)
000166           MZRMIN=MAS(NSUM+1)
000170           DMZR=MAS(NSUM+2)
000172    5      CALL AXES (0.,0.,90.,4.,MZRMIN,DMZR,1.,2.,1H ,.125,1)
000205           CALL LINEX (X,MAS,NOMBR,NS,XMIN,DX,MZRMIN,DMZR,XP,YP)
000217           CALL CALPLT (12.,0.,-3)
000222           NPLOT=NPLOT+1
000224    6      IF (PLTAE.EQ.0.AND.PLTJG.EQ.0) GO TO 9
          C
          C      PLOT STIFFNESS COEFFICIENT
          C
000232           CALL NOTATE (0.,1.5,0.14,20HSTIFFNESS COEF. PLOT,90.,20)
000236           CALL CALPLT (5.,0.,-3)
000241           CALL GRID (0.,0.,1.,1.,INCHX,4)
000245           CALL AXES (0.,0.,0.,XINCH,XMIN,DX,1.,2.,18HSCALE OF X, INCHES,.125
                1,-18)
000260           IF (NMAEJG.EQ.1) 7,8
000265    7      CALL ASCALE (Y,4.,NSUM,1,20.)
000271           AJMIN=Y(NSUM+1)
000273           DAEJG=Y(NSUM+2)
000275    8      CALL AXES (0.,0.,90.,4.,AJMIN,DAEJG,1.,2.,1H ,.125,1)
000310           CALL LINEX (X,Y,NOMBP,NS,XMIN,DX,AJMIN,DAEJG,XP,YP)
000322           CALL CALPLT (12.,0.,-3)
000325           NPLOT=NPLOT+1
000327    9      IF (PLTZ.EQ.0) GO TO 10
          C
          C      ZETA PLOTS
          C
000330           CALL NOTATE (0.,1.5,.14,10HZETA PLOTS,90.,10)
000334           CALL CALPLT (5.,0.,-3)
000337           CALL AXES (0.,0.,0.,XINCH,XMIN,DX,1.,2.,18HSCALE OF X, INCHES,.125
                1,-18)
000352    10     DO 12 II=1,3
000354           DO 11 J=1,NSUM
000355    11     CALL RECIN (3,1,3,Y(J),Y1(J,II),DY(J,II))
000370           IF (PLTZ.EQ.0) GO TO 12
000371           CALL GRID (0.,0.,1.,.5,INCHX,2)
000375           CALL AXES (0.,0.,90.,1.,-1.,2.,1.,2.,1H ,.125,1)
```

```
000410          CALL LINEX (X,Y,NOMBR,NS,XMIN,DX,-1.,2.,XP,YP)
000422          CALL CALPLT (0.,1.5,-3)
000425          NPLOT=NPLOT+1
000427   12     CONTINUE
000431          IF (PLTZ.NE.0) CALL CALPLT (12.,-4.5,-3)
000435          IF (PLTZPR.EQ.0) GO TO 17
         C
         C      ZETA PRIME PLOTS
         C
000436          CALL NOTATE (0.,1.,.14,16HZETA PRIME PLOTS,90.,16)
000442          CALL CALPLT (5.,0.,-3)
000445          CALL AXES (0.,0.,0.,XINCH,XMIN,DX,1.,2.,18HSCALE OF X, INCHES,.125
               1,-18)
000460          DO 16 II=1,3
000462          CALL GRID (0.,0.,1.,.5,INCHX,2)
000466          IF (NZAPP.EQ.1) 13,15
000473   13     DO 14 I=1,NSUM
000475   14     YP(I)=Y1(I,II)
000504          CALL ASCALE (YP,1.,NSUM,1,20.)
000510          ZPMIN(II)=YP(NSUM+1)
000513          DZP(II)=YP(NSUM+2)
000515   15     CALL AXES (0.,0.,90.,1.,ZPMIN(II),DZP(II),1.,2.,1H ,.125,1)
000530          CALL LINEX (X,Y1(1,II),NOMBR,NS,XMIN,DX,ZPMIN(II),DZP(II),XP,YP)
000547          CALL CALPLT (0.,1.5,-3)
000552          NPLOT=NPLOT+1
000554   16     CONTINUE
000556          CALL CALPLT (12.,-4.5,-3)
000560   17     IF (PLTT.EQ.0) RETURN
         C
         C      TENSION OR TORQUE PLOT
         C
000563          IF (TORVB) 18,19
000565   18     CALL NOTATE (0.,1.,.14,12HTORQUE PLOTS,90.,12)
000571          GO TO 20
000572   19     CALL NOTATE (0.,1.,.14,13HTENSION PLOTS,90.,13)
000576   20     CALL CALPLT (5.,0.,-3)
000601          IF (NPLTT.EQ.1) 21,23
000606   21     K=0
000607          DO 22 II=1,3
000611          DO 22 I=1,NSUM
000612          K=K+1
000614          YP(K)=DY(I,II)
000620   22     CONTINUE
```

```
000624         CALL ASCALE (YP,4.,K,1,20.)
000627         TMIN=YP(K+1)
000631         DTT=YP(K+2)
000633   23    DO 27 II=1,3
000635         IF (II.EQ.1.OR.MPLT.NE.0) 24,25
000643   24    CALL GRID (0.,0.,1.,1.,INCHX,4)
000647         CALL AXES (0.,0.,0.,XINCH,XMIN,DX,1.,2.,18HSCALE OF X, INCHES,.125
              1,-18)
000662         CALL AXES (0.,0.,90.,4.,TMIN,DTT,1.,2.,1H ,.125,1)
000675   25    CALL LINEX (X,DY(1,II),NOMBR,NS,XMIN,DX,TMIN,DTT,XP,YP)
000711         IF (MPLT.NE.0) 26,27
000715   26    CALL CALPLT (12.,0.,-3)
000720         NPLOT=NPLOT+1
000722   27    CONTINUE
000724         IF (MPLT.NE.0) RETURN
000727         CALL CALPLT (12.,0.,-3)
000732         NPLOT=NPLOT+1
000734         RETURN
         C
000736   28    FORMAT (8A10)
000736         END
```

```
                 SUBROUTINE LINEX (X,Y,NOMBR,NS,XMIN,DX,YMIN,DY,XP,YP)
000015           DIMENSION X(1), Y(1), NS(1), XP(1), YP(1)
000015           K=1
000015           M=0
000017           DO 2 I=1,NOMBR
000020           N=NS(I)
000022           L=0
000022           M=M+N
000024           DO 1 J=K,M
000026           L=L+1
000030           XP(L)=X(J)
000033           YP(L)=Y(J)
000036    1      CONTINUE
000040           XP(N+1)=XMIN
000042           XP(N+2)=DX
000044           YP(N+1)=YMIN
000046           YP(N+2)=DY
000051           CALL LINPLT (XP,YP,N,1,0,0,0,0)
000061           K=M+1
000063    2      CONTINUE
000071           RETURN
000072           END
```

APPENDIX C

COMPUTER PRINTOUT OF TORSIONAL MODAL DATA

FOR NUMERICAL EXAMPLE

TORSIONAL MODES OF CYLINDER-SHAFT CONFIGURATION FOR NUMERICAL EXAMPLE

PARAMETER CONTROLS - TORSIONAL MODES OF CYLINDER-SHAFT CONFIGURATION FOR NUMERICAL EXAMPLE

RFJG =	1.00000E+07	NOMODE =	4
RFZR =	1.00000E-03	NORMBR =	0
RFX =	1.00000E+02	NORN =	0
OMEGA SUBR =	1.00000E+03	NORDIS =	T
DELX =	1.00000E+00		

OMEGA =	5.00000E+00		
ZRMOD =	1.00000E+00	DELOMG =	2.50000E+01
JGMOD =	1.00000E+00	OMGTOL =	1.00000E-07

BOUNDARY CONDITIONS

JOINT	MBRI	MBRJ	MBRK	MBRL	IOJTRN	IJTRN	IKTRN	ILTRN
1	0	1	0	0	3	0	0	0
2	1	2	0	3	3	1	0	1
3	3	0	0	0	3	0	0	0
4	2	0	0	0	3	0	0	0

SPRING AND FLEXIBILITY CONSTANTS

JOINT	MBRI	MBRJ	MBRK	MBRL	TRNS	TRNFIJ	TRNFIK	TRNFIL
1	0	1	0	0	3.000E+00	0.	0.	0.
2	1	2	0	3	3.000E+00	0.	0.	0.
3	3	0	0	0	3.000E+00	0.	0.	0.
4	2	0	0	0	3.000E+00	0.	0.	0.

TABLE I

PHYSICAL CHARACTERISTICS
FOR TORSIONAL VIBRATIONS

TORSIONAL MODES OF CYLINDER-SHAFT CONFIGURATION FOR NUMERICAL EXAMPLE

STATION	X	ZR	JG
1- 1	0.000	2.20000E-04	6.20000E+06
1- 2	40.000	2.20000E-04	6.20000E+06
2- 1	40.000	2.20000E-04	6.20000E+06
2- 2	120.000	2.20000E-04	6.20000E+06
3- 1	40.000	4.50000E-06	7.66990E+04
3- 2	120.000	4.50000E-06	7.66990E+04

CENTER OF GRAVITY X = 60.26906

TOTAL MASS = .02676

S = 1.61280E+00

TABLE II

TORSIONAL MODES OF CYLINDER-SHAFT CONFIGURATION FOR NUMERICAL EXAMPLE

T = 0.00 SEC

MODE 1

FREQUENCY CYCLES PER SECOND 409.141

FREQUENCY RADIANS PER SECOND 2570.7072

STATION	X	ZETA	ZETA-PRIME	TORQUE	TORQUE-PRIME
1- 1	0.000	-5.58777E-03	0.	0.	8.12394E-10
1- 2	1.000	-5.58712E-03	1.31031E-06	8.12394E+00	8.12298E-10
1- 3	2.000	-5.58515E-03	2.62032E-06	1.62460E+01	8.12013E-10
1- 4	3.000	-5.58188E-03	3.92971E-06	2.43642E+01	8.11537E-10
1- 5	4.000	-5.57729E-03	5.23818E-06	3.24767E+01	8.10870E-10
1- 6	5.000	-5.57140E-03	6.54542E-06	4.05816E+01	8.10013E-10
1- 7	6.000	-5.56420E-03	7.85112E-06	4.86770E+01	8.08967E-10
1- 8	7.000	-5.55570E-03	9.15499E-06	5.67609E+01	8.07730E-10
1- 9	8.000	-5.54589E-03	1.04567E-05	6.48316E+01	8.06305E-10
1- 10	9.000	-5.53478E-03	1.17560E-05	7.28870E+01	8.04690E-10
1- 11	10.000	-5.52239E-03	1.30525E-05	8.09254E+01	8.02886E-10
1- 12	11.000	-5.50868E-03	1.43459E-05	8.89448E+01	8.00895E-10
1- 13	12.000	-5.49369E-03	1.56360E-05	9.69433E+01	7.98715E-10
1- 14	13.000	-5.47741E-03	1.69224E-05	1.04919E+02	7.96348E-10
1- 15	14.000	-5.45984E-03	1.82049E-05	1.12870E+02	7.93794E-10
1- 16	15.000	-5.44100E-03	1.94831E-05	1.20795E+02	7.91055E-10
1- 17	16.000	-5.42098E-03	2.07567E-05	1.28691E+02	7.88129E-10
1- 18	17.000	-5.39948E-03	2.20254E-05	1.36558E+02	7.85019E-10
1- 19	18.000	-5.37683E-03	2.32890E-05	1.44392E+02	7.81725E-10
1- 20	19.000	-5.35291E-03	2.45471E-05	1.52192E+02	7.78247E-10
1- 21	20.000	-5.32773E-03	2.57995E-05	1.59957E+02	7.74587E-10
1- 22	21.000	-5.30131E-03	2.70458E-05	1.67684E+02	7.70745E-10
1- 23	22.000	-5.27364E-03	2.82857E-05	1.75372E+02	7.66723E-10
1- 24	23.000	-5.24474E-03	2.95191E-05	1.83018E+02	7.62521E-10

TABLE II, MODE 1

TORSIONAL MODES OF CYLINDER-SHAFT CONFIGURATION FOR NUMERICAL EXAMPLE

STATION	X	ZETA	ZETA-PRIME	TORQUE	TORQUE-PRIME
1- 25	24.000	-5.21460E-03	3.07455E-05	1.90622E+02	7.58139E-10
1- 26	25.000	-5.18325E-03	3.19647E-05	1.98181E+02	7.53580E-10
1- 27	26.000	-5.15067E-03	3.31764E-05	2.05694E+02	7.48845E-10
1- 28	27.000	-5.11689E-03	3.43803E-05	2.13158E+02	7.43934E-10
1- 29	28.000	-5.08191E-03	3.55762E-05	2.20572E+02	7.38848E-10
1- 30	29.000	-5.04574E-03	3.67637E-05	2.27935E+02	7.33589E-10
1- 31	30.000	-5.00838E-03	3.79428E-05	2.35244E+02	7.28158E-10
1- 32	31.000	-4.96986E-03	3.91126E-05	2.42498E+02	7.22558E-10
1- 33	32.000	-4.93016E-03	4.02736E-05	2.49695E+02	7.16789E-10
1- 34	33.000	-4.88931E-03	4.10240E-05	2.56834E+02	7.10846E-10
1- 35	34.000	-4.84721E-03	4.25659E-05	2.63912E+02	7.04780E-10
1- 36	35.000	-4.80410E-03	4.36932E-05	2.70929E+02	6.98468E-10
1- 37	36.000	-4.75991E-03	4.48196E-05	2.77801E+02	6.92093E-10
1- 38	37.000	-4.71494E-03	4.59299E-05	2.84769E+02	6.85496E-10
1- 39	38.000	-4.66805E-03	4.70707E-05	2.91590E+02	6.78478E-10
1- 40	39.000	-4.62048E-03	4.81190E-05	2.98362E+02	6.71761E-10
1- 41	40.000	-4.57181E-03	4.91974E-05	3.05029E+02	6.64586E-10
2- 1	40.000	-4.57101E-03	-1.94391E-04	-1.20522E+03	6.64686E-10
2- 2	41.000	-4.76497E-03	-1.93296E-04	-1.19844E+03	6.92870E-10
2- 3	42.000	-4.95841E-03	-1.92156E-04	-1.19137E+03	7.20892E-10
2- 4	43.000	-5.14998E-03	-1.90971E-04	-1.18402E+03	7.48744E-10
2- 5	44.000	-5.34039E-03	-1.89741E-04	-1.17639E+03	7.76421E-10
2- 6	45.000	-5.52946E-03	-1.88466E-04	-1.16849E+03	8.03916E-10
2- 7	46.000	-5.71728E-03	-1.87147E-04	-1.16031E+03	8.31222E-10
2- 8	47.000	-5.90376E-03	-1.85785E-04	-1.15186E+03	8.58334E-10
2- 9	48.000	-6.08885E-03	-1.84378E-04	-1.14315E+03	8.85244E-10
2- 10	49.000	-6.27251E-03	-1.82929E-04	-1.13416E+03	9.11947E-10
2- 11	50.000	-6.45471E-03	-1.81437E-04	-1.12491E+03	9.38435E-10
2- 12	51.000	-6.63539E-03	-1.79902E-04	-1.11539E+03	9.64704E-10
2- 13	52.000	-6.81451E-03	-1.78325E-04	-1.10561E+03	9.90746E-10
2- 14	53.000	-6.99204E-03	-1.76706E-04	-1.09558E+03	1.01656E-09
2- 15	54.000	-7.16792E-03	-1.75046E-04	-1.08528E+03	1.04213E-09
2- 16	55.000	-7.34213E-03	-1.73344E-04	-1.07473E+03	1.06746E-09
2- 17	56.000	-7.51461E-03	-1.71602E-04	-1.06393E+03	1.09253E-09

TABLE II, MODE 1

TORSIONAL MODES OF CYLINDER-SHAFT CONFIGURATION FOR NUMERICAL EXAMPLE

STATION	X	ZETA	ZETA-PRIME	TORQUE	TORQUE-PRIME
2- 18	57.000	-7.68533E-03	-1.69820E-04	-1.05288E+03	1.11735E-09
2- 19	58.000	-7.85425E-03	-1.67998E-04	-1.04159E+03	1.14191E-09
2- 20	59.000	-8.02133E-03	-1.66136E-04	-1.03004E+03	1.16620E-09
2- 21	60.000	-8.18652E-03	-1.64236E-04	-1.01826E+03	1.19022E-09
2- 22	61.000	-8.34980E-03	-1.62297E-04	-1.00624E+03	1.21396E-09
2- 23	62.000	-8.51112E-03	-1.60320E-04	-9.93983E+02	1.23741E-09
2- 24	63.000	-8.67044E-03	-1.58305E-04	-9.81492E+02	1.26058E-09
2- 25	64.000	-8.82773E-03	-1.56253E-04	-9.68771E+02	1.28344E-09
2- 26	65.000	-8.98255E-03	-1.54165E-04	-9.55823E+02	1.30601E-09
2- 27	66.000	-9.13606E-03	-1.52041E-04	-9.42651E+02	1.32827E-09
2- 28	67.000	-9.28703E-03	-1.49880E-04	-9.29258E+02	1.35022E-09
2- 29	68.000	-9.43582E-03	-1.47685E-04	-9.15647E+02	1.37185E-09
2- 30	69.000	-9.58240E-03	-1.45455E-04	-9.01821E+02	1.39316E-09
2- 31	70.000	-9.72673E-03	-1.43191E-04	-8.87783E+02	1.41415E-09
2- 32	71.000	-9.86878E-03	-1.40893E-04	-8.73538E+02	1.43480E-09
2- 33	72.000	-1.00085E-02	-1.38562E-04	-8.59087E+02	1.45512E-09
2- 34	73.000	-1.01459E-02	-1.36199E-04	-8.44436E+02	1.47509E-09
2- 35	74.000	-1.02809E-02	-1.33804E-04	-8.29586E+02	1.49472E-09
2- 36	75.000	-1.04135E-02	-1.31378E-04	-8.14541E+02	1.51400E-09
2- 37	76.000	-1.05437E-02	-1.28920E-04	-7.99306E+02	1.53292E-09
2- 38	77.000	-1.06714E-02	-1.26433E-04	-7.83883E+02	1.55148E-09
2- 39	78.000	-1.07965E-02	-1.23915E-04	-7.68276E+02	1.56968E-09
2- 40	79.000	-1.09192E-02	-1.21369E-04	-7.52489E+02	1.58752E-09
2- 41	80.000	-1.10393E-02	-1.18794E-04	-7.36526E+02	1.60498E-09
2- 42	81.000	-1.11568E-02	-1.16192E-04	-7.20390E+02	1.62206E-09
2- 43	82.000	-1.12717E-02	-1.13562E-04	-7.04085E+02	1.63876E-09
2- 44	83.000	-1.13839E-02	-1.10906E-04	-6.87514E+02	1.65508E-09
2- 45	84.000	-1.14935E-02	-1.08223E-04	-6.70983E+02	1.67101E-09
2- 46	85.000	-1.16003E-02	-1.05515E-04	-6.54194E+02	1.68655E-09
2- 47	86.000	-1.17045E-02	-1.02783E-04	-6.37252E+02	1.70169E-09
2- 48	87.000	-1.18059E-02	-1.00026E-04	-6.20160E+02	1.71643E-09
2- 49	88.000	-1.19046E-02	-9.72457E-05	-6.02923E+02	1.73078E-09
2- 50	89.000	-1.20004E-02	-9.44427E-05	-5.85545E+02	1.74471E-09
2- 51	90.000	-1.20934E-02	-9.16176E-05	-5.68029E+02	1.75824E-09

TABLE II, MODE 1

TORSIONAL MODES OF CYLINDER-SHAFT CONFIGURATION FOR NUMERICAL EXAMPLE

STATION	X	ZETA	ZETA-PRIME	TORQUE	TORQUE-PRIME
2- 52	91.000	-1.21836E-02	-8.87710E-05	-5.50380E+02	1.77135E-09
2- 53	92.000	-1.22710E-02	-8.59036E-05	-5.32602E+02	1.78405E-09
2- 54	93.000	-1.23554E-02	-8.30160E-05	-5.14599E+02	1.79633E-09
2- 55	94.000	-1.24370E-02	-8.01090E-05	-4.96676E+02	1.80819E-09
2- 56	95.000	-1.25157E-02	-7.71831E-05	-4.78535E+02	1.81962E-09
2- 57	96.000	-1.25914E-02	-7.42392E-05	-4.60283E+02	1.83063E-09
2- 58	97.000	-1.26641E-02	-7.12779E-05	-4.41923E+02	1.84121E-09
2- 59	98.000	-1.27339E-02	-6.82998E-05	-4.23459E+02	1.85136E-09
2- 60	99.000	-1.28007E-02	-6.53057E-05	-4.04896E+02	1.86107E-09
2- 61	100.000	-1.28645E-02	-6.22964E-05	-3.86237E+02	1.87035E-09
2- 62	101.000	-1.29253E-02	-5.92724E-05	-3.67489E+02	1.87919E-09
2- 63	102.000	-1.29831E-02	-5.62345E-05	-3.48654E+02	1.88758E-09
2- 64	103.000	-1.30378E-02	-5.31834E-05	-3.29737E+02	1.89554E-09
2- 65	104.000	-1.30895E-02	-5.01198E-05	-3.10742E+02	1.90305E-09
2- 66	105.000	-1.31389E-02	-4.70445E-05	-2.91676E+02	1.91011E-09
2- 67	106.000	-1.31835E-02	-4.39582E-05	-2.72541E+02	1.91673E-09
2- 68	107.000	-1.32260E-02	-4.08615E-05	-2.53342E+02	1.92289E-09
2- 69	108.000	-1.32653E-02	-3.77553E-05	-2.34083E+02	1.92861E-09
2- 70	109.000	-1.33015E-02	-3.46402E-05	-2.14769E+02	1.93387E-09
2- 71	110.000	-1.33345E-02	-3.15170E-05	-1.95406E+02	1.93868E-09
2- 72	111.000	-1.33645E-02	-2.83864E-05	-1.75996E+02	1.94304E-09
2- 73	112.000	-1.33913E-02	-2.52492E-05	-1.56545E+02	1.94693E-09
2- 74	113.000	-1.34150E-02	-2.21060E-05	-1.37057E+02	1.95038E-09
2- 75	114.000	-1.34355E-02	-1.89576E-05	-1.17537E+02	1.95336E-09
2- 76	115.000	-1.34529E-02	-1.58048E-05	-9.79899E+01	1.95589E-09
2- 77	116.000	-1.34671E-02	-1.26483E-05	-7.84195E+01	1.95796E-09
2- 78	117.000	-1.34782E-02	-9.48883E-06	-5.88307E+01	1.95957E-09
2- 79	118.000	-1.34861E-02	-6.32712E-06	-3.92281E+01	1.96072E-09
2- 80	119.000	-1.34909E-02	-3.16393E-06	-1.96164E+01	1.96141E-09
2- 81	120.000	-1.34925E-02	-2.00118E-20	-1.24073E-13	1.96164E-09
3- 1	40.000	-4.57181E-03	1.96906E-02	1.51025E+03	1.35958E-11
3- 2	41.000	1.51197E-02	1.96885E-02	1.51009E+03	-4.49635E-11
3- 3	42.000	3.48053E-02	1.96789E-02	1.50935E+03	-1.03505E-10
3- 4	43.000	5.44774E-02	1.96616E-02	1.50802E+03	-1.62007E-10

TABLE II, MODE 1

TORSIONAL MODES OF CYLINDER-SHAFT CONFIGURATION FOR NUMERICAL EXAMPLE

STATION	X	ZETA	ZETA-PRIME	TORQUE	TORQUE-PRIME
3- 5	44.000	7.41284E-02	1.96366E-02	1.50511E+03	-2.20446E-10
3- 6	45.000	9.37507E-02	1.96041E-02	1.50361E+03	-2.78800E-10
3- 7	46.000	1.13337E-01	1.95639E-02	1.50053E+03	-3.37045E-10
3- 8	47.000	1.32879E-01	1.95162E-02	1.49687E+03	-3.95160E-10
3- 9	48.000	1.52369E-01	1.94609E-02	1.49263E+03	-4.53121E-10
3- 10	49.000	1.71800E-01	1.93980E-02	1.48781E+03	-5.10907E-10
3- 11	50.000	1.91165E-01	1.93277E-02	1.48241E+03	-5.68494E-10
3- 12	51.000	2.10456E-01	1.92498E-02	1.47644E+03	-6.25862E-10
3- 13	52.000	2.29665E-01	1.91645E-02	1.46990E+03	-6.82986E-10
3- 14	53.000	2.48785E-01	1.90717E-02	1.46278E+03	-7.39846E-10
3- 15	54.000	2.67808E-01	1.89715E-02	1.45510E+03	-7.95419E-10
3- 16	55.000	2.86728E-01	1.88640E-02	1.44685E+03	-8.52682E-10
3- 17	56.000	3.05536E-01	1.87492E-02	1.43804E+03	-9.08616E-10
3- 18	57.000	3.24226E-01	1.86271E-02	1.42868E+03	-9.64197E-10
3- 19	58.000	3.42790E-01	1.84978E-02	1.41876E+03	-1.01940E-09
3- 20	59.000	3.61222E-01	1.83613E-02	1.40829E+03	-1.07422E-09
3- 21	60.000	3.79512E-01	1.82177E-02	1.39728E+03	-1.12861E-09
3- 22	61.000	3.97657E-01	1.80670E-02	1.38572E+03	-1.18257E-09
3- 23	62.000	4.15647E-01	1.79093E-02	1.37363E+03	-1.23607E-09
3- 24	63.000	4.33476E-01	1.77447E-02	1.36100E+03	-1.28909E-09
3- 25	64.000	4.51136E-01	1.75732E-02	1.34784E+03	-1.34161E-09
3- 26	65.000	4.68622E-01	1.73948E-02	1.33417E+03	-1.39361E-09
3- 27	66.000	4.85926E-01	1.72098E-02	1.31997E+03	-1.44507E-09
3- 28	67.000	5.03041E-01	1.70180E-02	1.30527E+03	-1.49597E-09
3- 29	68.000	5.19962E-01	1.68197E-02	1.29005E+03	-1.54628E-09
3- 30	69.000	5.36681E-01	1.66148E-02	1.27434E+03	-1.59600E-09
3- 31	70.000	5.53192E-01	1.64035E-02	1.25813E+03	-1.64510E-09
3- 32	71.000	5.69488E-01	1.61858E-02	1.24144E+03	-1.69357E-09
3- 33	72.000	5.85563E-01	1.59619E-02	1.22426E+03	-1.74137E-09
3- 34	73.000	6.01412E-01	1.57318E-02	1.20661E+03	-1.78850E-09
3- 35	74.000	6.17027E-01	1.54955E-02	1.18849E+03	-1.83494E-09
3- 36	75.000	6.32403E-01	1.52533E-02	1.16991E+03	-1.88067E-09
3- 37	76.000	6.47533E-01	1.50051E-02	1.15088E+03	-1.92566E-09
3- 38	77.000	6.62413E-01	1.47511E-02	1.13140E+03	-1.96991E-09

TABLE II, MODE 1

TORSIONAL MODES OF CYLINDER-SHAFT CONFIGURATION FOR NUMERICAL EXAMPLE

STATION	X	ZETA	ZETA-PRIME	TORQUE	TORQUE-PRIME
3- 39	78.000	6.77036E-01	1.44914E-02	1.11148E+03	-2.01340E-09
3- 40	79.000	6.91356E-01	1.42261E-02	1.09113E+03	-2.05610E-09
3- 41	80.000	7.05488E-01	1.39553E-02	1.07036E+03	-2.09801E-09
3- 42	81.000	7.19307E-01	1.36791E-02	1.04917E+03	-2.13910E-09
3- 43	82.000	7.32846E-01	1.33975E-02	1.02758E+03	-2.17937E-09
3- 44	83.000	7.46102E-01	1.31108E-02	1.00558E+03	-2.21879E-09
3- 45	84.000	7.59068E-01	1.28189E-02	9.83200E+02	-2.25735E-09
3- 46	85.000	7.71740E-01	1.25221E-02	9.60436E+02	-2.29503E-09
3- 47	86.000	7.84112E-01	1.22205E-02	9.37299E+02	-2.33182E-09
3- 48	87.000	7.96181E-01	1.19141E-02	9.13799E+02	-2.36771E-09
3- 49	88.000	8.07940E-01	1.16031E-02	8.89945E+02	-2.40269E-09
3- 50	89.000	8.19387E-01	1.12876E-02	8.65746E+02	-2.43673E-09
3- 51	90.000	8.30515E-01	1.09677E-02	8.41211E+02	-2.46982E-09
3- 52	91.000	8.41322E-01	1.06435E-02	8.16349E+02	-2.50196E-09
3- 53	92.000	8.51803E-01	1.03153E-02	7.91171E+02	-2.53313E-09
3- 54	93.000	8.61953E-01	9.98301E-03	7.65687E+02	-2.56331E-09
3- 55	94.000	8.71769E-01	9.64687E-03	7.39905E+02	-2.59250E-09
3- 56	95.000	8.81246E-01	9.30699E-03	7.13837E+02	-2.62069E-09
3- 57	96.000	8.90383E-01	8.96350E-03	6.87492E+02	-2.64786E-09
3- 58	97.000	8.99174E-01	8.61654E-03	6.60880E+02	-2.67400E-09
3- 59	98.000	9.07616E-01	8.26623E-03	6.34012E+02	-2.69910E-09
3- 60	99.000	9.15705E-01	7.91272E-03	6.06898E+02	-2.72316E-09
3- 61	100.000	9.23441E-01	7.55614E-03	5.79548E+02	-2.74617E-09
3- 62	101.000	9.30818E-01	7.19663E-03	5.51974E+02	-2.76811E-09
3- 63	102.000	9.37834E-01	6.83433E-03	5.24186E+02	-2.78897E-09
3- 64	103.000	9.44487E-01	6.46938E-03	4.96195E+02	-2.80875E-09
3- 65	104.000	9.50773E-01	6.10192E-03	4.68011E+02	-2.82745E-09
3- 66	105.000	9.56691E-01	5.73209E-03	4.39646E+02	-2.84505E-09
3- 67	106.000	9.62238E-01	5.36005E-03	4.11110E+02	-2.86154E-09
3- 68	107.000	9.67411E-01	4.98592E-03	3.82415E+02	-2.87693E-09
3- 69	108.000	9.72209E-01	4.60986E-03	3.53572E+02	-2.89120E-09
3- 70	109.000	9.76631E-01	4.23201E-03	3.24591E+02	-2.90434E-09
3- 71	110.000	9.80673E-01	3.85253E-03	2.95485E+02	-2.91637E-09
3- 72	111.000	9.84336E-01	3.47154E-03	2.66264E+02	-2.92726E-09

TABLE II, MODE 1

TORSIONAL MODES OF CYLINDER-SHAFT CONFIGURATION FOR NUMERICAL EXAMPLE

STATION	X	ZETA	ZETA-PRIME	TORQUE	TORQUE-PRIME
3- 73	112.000	9.87617E-01	3.08922E-03	2.36940E+02	-2.93701E-09
3- 74	113.000	9.90514E-01	2.70569E-03	2.07524E+02	-2.94563E-09
3- 75	114.000	9.93028E-01	2.32111E-03	1.78027E+02	-2.95311E-09
3- 76	115.000	9.95157E-01	1.93564E-03	1.48461E+02	-2.95944E-09
3- 77	116.000	9.96899E-01	1.54941E-03	1.18838E+02	-2.96462E-09
3- 78	117.000	9.98255E-01	1.16258E-03	8.91691E+01	-2.96865E-09
3- 79	118.000	9.99225E-01	7.75307E-04	5.94653E+01	-2.97153E-09
3- 80	119.000	9.99806E-01	3.87729E-04	2.97384E+01	-2.97326E-09
3- 81	120.000	1.00000E+00	-1.29413E-17	-9.92588E-13	-2.97384E-09

TOTAL NO. STATIONS = 203

TABLE II

TORSIONAL MODES OF CYLINDER-SHAFT CONFIGURATION FOR NUMERICAL EXAMPLE

T = 0.00 SEC

MODE 2

FREQUENCY CYCLES PER SECOND 699.821

FREQUENCY RADIANS PER SECOND 4397.1067

STATION	X	ZETA	ZETA-PRIME	TORQUE	TORQUE-PRIME
1- 1	0.000	1.00000E+00	0.	0.	-4.25360E-07
1- 2	1.000	9.99657E-01	-6.86065E-04	-4.25360E+03	-4.25214E-07
1- 3	2.000	9.98628E-01	-1.37166E-03	-8.50428E+03	-4.24776E-07
1- 4	3.000	9.96914E-01	-2.05631E-03	-1.27491E+04	-4.24047E-07
1- 5	4.000	9.94515E-01	-2.73955E-03	-1.69852E+04	-4.23027E-07
1- 6	5.000	9.91435E-01	-3.42092E-03	-2.12097E+04	-4.21717E-07
1- 7	6.000	9.87674E-01	-4.09993E-03	-2.54196E+04	-4.20117E-07
1- 8	7.000	9.83235E-01	-4.77613E-03	-2.96120E+04	-4.18229E-07
1- 9	8.000	9.78122E-01	-5.44906E-03	-3.37841E+04	-4.16054E-07
1- 10	9.000	9.72337E-01	-6.11824E-03	-3.79331E+04	-4.13593E-07
1- 11	10.000	9.65885E-01	-6.78323E-03	-4.20560E+04	-4.10849E-07
1- 12	11.000	9.58771E-01	-7.44356E-03	-4.61501E+04	-4.07823E-07
1- 13	12.000	9.50998E-01	-8.09879E-03	-5.02125E+04	-4.04517E-07
1- 14	13.000	9.42573E-01	-8.74845E-03	-5.42404E+04	-4.00933E-07
1- 15	14.000	9.33501E-01	-9.39212E-03	-5.82311E+04	-3.97074E-07
1- 16	15.000	9.23789E-01	-1.00293E-02	-6.21819E+04	-3.92943E-07
1- 17	16.000	9.13443E-01	-1.06597E-02	-6.60900E+04	-3.88542E-07
1- 18	17.000	9.02470E-01	-1.12827E-02	-6.99528E+04	-3.83875E-07
1- 19	18.000	8.90878E-01	-1.18980E-02	-7.37675E+04	-3.78944E-07
1- 20	19.000	8.78674E-01	-1.25051E-02	-7.75316E+04	-3.73753E-07
1- 21	20.000	8.65867E-01	-1.31036E-02	-8.12426E+04	-3.68305E-07
1- 22	21.000	8.52467E-01	-1.36932E-02	-8.48978E+04	-3.62605E-07
1- 23	22.000	8.38481E-01	-1.42733E-02	-8.84947E+04	-3.56656E-07
1- 24	23.000	8.23920E-01	-1.48437E-02	-9.20309E+04	-3.50463E-07

TABLE II, MODE 2

TORSIONAL MODES OF CYLINDER-SHAFT CONFIGURATION FOR NUMERICAL EXAMPLE

STATION	X	ZETA	ZETA-PRIME	TORQUE	TORQUE-PRIME
1- 25	24.000	8.08794E-01	-1.54039E-02	-9.55040E+04	-3.44029E-07
1- 26	25.000	7.93113E-01	-1.59535E-02	-9.89115E+04	-3.37358E-07
1- 27	26.000	7.76887E-01	-1.64921E-02	-1.02251E+05	-3.30457E-07
1- 28	27.000	7.60128E-01	-1.70195E-02	-1.05521E+05	-3.23328E-07
1- 29	28.000	7.42848E-01	-1.75351E-02	-1.08718E+05	-3.15978E-07
1- 30	29.000	7.25058E-01	-1.80387E-02	-1.11840E+05	-3.08411E-07
1- 31	30.000	7.06771E-01	-1.85300E-02	-1.14886E+05	-3.00632E-07
1- 32	31.000	6.87998E-01	-1.90085E-02	-1.17853E+05	-2.92647E-07
1- 33	32.000	6.68754E-01	-1.94740E-02	-1.20739E+05	-2.84461E-07
1- 34	33.000	6.49050E-01	-1.99261E-02	-1.23542E+05	-2.76080E-07
1- 35	34.000	6.28902E-01	-2.03646E-02	-1.26261E+05	-2.67510E-07
1- 36	35.000	6.08321E-01	-2.07891E-02	-1.28892E+05	-2.58756E-07
1- 37	36.000	5.87324E-01	-2.11993E-02	-1.31436E+05	-2.49824E-07
1- 38	37.000	5.65923E-01	-2.15950E-02	-1.33889E+05	-2.40721E-07
1- 39	38.000	5.44134E-01	-2.19758E-02	-1.36250E+05	-2.31453E-07
1- 40	39.000	5.21971E-01	-2.23416E-02	-1.38518E+05	-2.22026E-07
1- 41	40.000	4.99451E-01	-2.26920E-02	-1.40691E+05	-2.12446E-07
2- 1	40.000	4.99451E-01	-2.25924E-02	-1.40073E+05	-2.12446E-07
2- 2	41.000	4.76687E-01	-2.29273E-02	-1.42149E+05	-2.02764E-07
2- 3	42.000	4.53596E-01	-2.32464E-02	-1.44128E+05	-1.92942E-07
2- 4	43.000	4.30194E-01	-2.35497E-02	-1.46008E+05	-1.82987E-07
2- 5	44.000	4.06497E-01	-2.38367E-02	-1.47788E+05	-1.72908E-07
2- 6	45.000	3.82521E-01	-2.41074E-02	-1.49466E+05	-1.62709E-07
2- 7	46.000	3.58282E-01	-2.43616E-02	-1.51042E+05	-1.52399E-07
2- 8	47.000	3.33798E-01	-2.45990E-02	-1.52514E+05	-1.41984E-07
2- 9	48.000	3.09084E-01	-2.48196E-02	-1.53882E+05	-1.31472E-07
2- 10	49.000	2.84158E-01	-2.50232E-02	-1.55144E+05	-1.20870E-07
2- 11	50.000	2.59038E-01	-2.52095E-02	-1.56299E+05	-1.10184E-07
2- 12	51.000	2.33739E-01	-2.53786E-02	-1.57347E+05	-9.94234E-08
2- 13	52.000	2.08281E-01	-2.55302E-02	-1.58287E+05	-8.85942E-08
2- 14	53.000	1.82679E-01	-2.56644E-02	-1.59119E+05	-7.77043E-08
2- 15	54.000	1.56952E-01	-2.57809E-02	-1.59842E+05	-6.67611E-08
2- 16	55.000	1.31117E-01	-2.58797E-02	-1.60454E+05	-5.57720E-08
2- 17	56.000	1.05192E-01	-2.59608E-02	-1.60957E+05	-4.47446E-08

TABLE II, MODE 2

TORSIONAL MODES OF CYLINDER-SHAFT CONFIGURATION FOR NUMERICAL EXAMPLE

STATION	X	ZETA	ZETA-PRIME	TORQUE	TORQUE-PRIME
2- 18	57.000	7.91955E-02	-2.60241E-02	-1.61349E+05	-3.36866E-08
2- 19	58.000	5.31443E-02	-2.60695E-02	-1.61631E+05	-2.26054E-08
2- 20	59.000	2.70565E-02	-2.60970E-02	-1.61801E+05	-1.15088E-08
2- 21	60.000	9.50263E-04	-2.61066E-02	-1.61861E+05	-4.04204E-10
2- 22	61.000	-2.51567E-02	-2.60983E-02	-1.61810E+05	1.07006E-08
2- 23	62.000	-5.12464E-02	-2.60721E-02	-1.61647E+05	2.17982E-08
2- 24	63.000	-7.73009E-02	-2.60280E-02	-1.61374E+05	3.28807E-08
2- 25	64.000	-1.03302E-01	-2.59660E-02	-1.60989E+05	4.39407E-08
2- 26	65.000	-1.29223E-01	-2.58863E-02	-1.60495E+05	5.49705E-08
2- 27	66.000	-1.55075E-01	-2.57887E-02	-1.59890E+05	6.59627E-08
2- 28	67.000	-1.80810E-01	-2.56735E-02	-1.59176E+05	7.69095E-08
2- 29	68.000	-2.06422E-01	-2.55406E-02	-1.58352E+05	8.78036E-08
2- 30	69.000	-2.31892E-01	-2.53902E-02	-1.57419E+05	9.86374E-08
2- 31	70.000	-2.57202E-01	-2.52224E-02	-1.56379E+05	1.09404E-07
2- 32	71.000	-2.82337E-01	-2.50373E-02	-1.55231E+05	1.20095E-07
2- 33	72.000	-3.07277E-01	-2.48350E-02	-1.53977E+05	1.30703E-07
2- 34	73.000	-3.32007E-01	-2.46157E-02	-1.52617E+05	1.41222E-07
2- 35	74.000	-3.56509E-01	-2.43795E-02	-1.51153E+05	1.51644E-07
2- 36	75.000	-3.80766E-01	-2.41265E-02	-1.49585E+05	1.61963E-07
2- 37	76.000	-4.04762E-01	-2.38570E-02	-1.47914E+05	1.72169E-07
2- 38	77.000	-4.28480E-01	-2.35712E-02	-1.46141E+05	1.82258E-07
2- 39	78.000	-4.51904E-01	-2.32691E-02	-1.44268E+05	1.92222E-07
2- 40	79.000	-4.75018E-01	-2.29511E-02	-1.42297E+05	2.02054E-07
2- 41	80.000	-4.97806E-01	-2.26173E-02	-1.40227E+05	2.11747E-07
2- 42	81.000	-5.20253E-01	-2.22680E-02	-1.38062E+05	2.21295E-07
2- 43	82.000	-5.42342E-01	-2.19035E-02	-1.35802E+05	2.30691E-07
2- 44	83.000	-5.64060E-01	-2.15239E-02	-1.33448E+05	2.39928E-07
2- 45	84.000	-5.85390E-01	-2.11295E-02	-1.31003E+05	2.49002E-07
2- 46	85.000	-6.06319E-01	-2.07206E-02	-1.28468E+05	2.57904E-07
2- 47	86.000	-6.26831E-01	-2.02976E-02	-1.25845E+05	2.66629E-07
2- 48	87.000	-6.46914E-01	-1.98606E-02	-1.23135E+05	2.75171E-07
2- 49	88.000	-6.66553E-01	-1.94099E-02	-1.20341E+05	2.83525E-07
2- 50	89.000	-6.85734E-01	-1.89460E-02	-1.17465E+05	2.91684E-07
2- 51	90.000	-7.04445E-01	-1.84690E-02	-1.14508E+05	2.99643E-07

TABLE II, MODE 2

TORSIONAL MODES OF CYLINDER-SHAFT CONFIGURATION FOR NUMERICAL EXAMPLE

STATION	X	ZETA	ZETA-PRIME	TORQUE	TORQUE-PRIME
2- 52	91.000	-7.22672E-01	-1.79794E-02	-1.11472E+05	3.07396E-07
2- 53	92.000	-7.40404E-01	-1.74774E-02	-1.08360E+05	3.14938E-07
2- 54	93.000	-7.57627E-01	-1.69634E-02	-1.05173E+05	3.22264E-07
2- 55	94.000	-7.74330E-01	-1.64378E-02	-1.01915E+05	3.29369E-07
2- 56	95.000	-7.90503E-01	-1.59010E-02	-9.85860E+04	3.36248E-07
2- 57	96.000	-8.06133E-01	-1.53532E-02	-9.51897E+04	3.42897E-07
2- 58	97.000	-8.21209E-01	-1.47949E-02	-9.17281E+04	3.49310E-07
2- 59	98.000	-8.35722E-01	-1.42264E-02	-8.82035E+04	3.55483E-07
2- 60	99.000	-8.49662E-01	-1.36481E-02	-8.46184E+04	3.61412E-07
2- 61	100.000	-8.63019E-01	-1.30605E-02	-8.09752E+04	3.67094E-07
2- 62	101.000	-8.75783E-01	-1.24640E-02	-7.72766E+04	3.72523E-07
2- 63	102.000	-8.87947E-01	-1.18588E-02	-7.35248E+04	3.77697E-07
2- 64	103.000	-8.99501E-01	-1.12456E-02	-6.97226E+04	3.82612E-07
2- 65	104.000	-9.10438E-01	-1.06246E-02	-6.58726E+04	3.87264E-07
2- 66	105.000	-9.20750E-01	-9.99635E-03	-6.19774E+04	3.91650E-07
2- 67	106.000	-9.30431E-01	-9.36123E-03	-5.80396E+04	3.95768E-07
2- 68	107.000	-9.39473E-01	-8.71968E-03	-5.40620E+04	3.99614E-07
2- 69	108.000	-9.47870E-01	-8.07215E-03	-5.00473E+04	4.03186E-07
2- 70	109.000	-9.55617E-01	-7.41908E-03	-4.59983E+04	4.06481E-07
2- 71	110.000	-9.62709E-01	-6.76092E-03	-4.19177E+04	4.09498E-07
2- 72	111.000	-9.69139E-01	-6.09812E-03	-3.78083E+04	4.12233E-07
2- 73	112.000	-9.74905E-01	-5.43114E-03	-3.36730E+04	4.14686E-07
2- 74	113.000	-9.80002E-01	-4.76042E-03	-2.95146E+04	4.16854E-07
2- 75	114.000	-9.84426E-01	-4.08645E-03	-2.53360E+04	4.18735E-07
2- 76	115.000	-9.88175E-01	-3.40967E-03	-2.11399E+04	4.20330E-07
2- 77	116.000	-9.91245E-01	-2.73054E-03	-1.69294E+04	4.21636E-07
2- 78	117.000	-9.93636E-01	-2.04955E-03	-1.27072E+04	4.22653E-07
2- 79	118.000	-9.95344E-01	-1.36715E-03	-8.47632E+03	4.23380E-07
2- 80	119.000	-9.96370E-01	-6.83809E-04	-4.23961E+03	4.23816E-07
2- 81	120.000	-9.96712E-01	3.04416E-16	1.88738E-09	4.23962E-07
3- 1	40.000	4.99451E-01	-8.05632E-03	-6.17911E+02	-4.34549E-09
3- 2	41.000	4.91111E-01	-8.61831E-03	-6.61016E+02	-4.27293E-09
3- 3	42.000	4.82214E-01	-9.17053E-03	-7.03370E+02	-4.19553E-09
3- 4	43.000	4.72770E-01	-9.71234E-03	-7.44927E+02	-4.11336E-09

TABLE II, MODE 2

TORSIONAL MODES OF CYLINDER-SHAFT CONFIGURATION FOR NUMERICAL EXAMPLE

STATION	X	ZETA	ZETA-PRIME	TORQUE	TORQUE-PRIME
3- 5	44.000	4.62790E-01	-1.02431E-02	-7.85638E+02	-4.02652E-09
3- 6	45.000	4.52284E-01	-1.07623E-02	-8.25457E+02	-3.93512E-09
3- 7	46.000	4.41265E-01	-1.12693E-02	-8.64340E+02	-3.83925E-09
3- 8	47.000	4.29746E-01	-1.17634E-02	-9.02243E+02	-3.73902E-09
3- 9	48.000	4.17739E-01	-1.22442E-02	-9.39121E+02	-3.63455E-09
3- 10	49.000	4.05257E-01	-1.27112E-02	-9.74934E+02	-3.52596E-09
3- 11	50.000	3.92316E-01	-1.31637E-02	-1.00964E+03	-3.41337E-09
3- 12	51.000	3.78930E-01	-1.36012E-02	-1.04320E+03	-3.29690E-09
3- 13	52.000	3.65114E-01	-1.40234E-02	-1.07558E+03	-3.17669E-09
3- 14	53.000	3.50883E-01	-1.44296E-02	-1.10674E+03	-3.05288E-09
3- 15	54.000	3.36255E-01	-1.48194E-02	-1.13664E+03	-2.92560E-09
3- 16	55.000	3.21245E-01	-1.51925E-02	-1.16525E+03	-2.79500E-09
3- 17	56.000	3.05870E-01	-1.55483E-02	-1.19254E+03	-2.66124E-09
3- 18	57.000	2.90148E-01	-1.58864E-02	-1.21847E+03	-2.52445E-09
3- 19	58.000	2.74097E-01	-1.62066E-02	-1.24303E+03	-2.38480E-09
3- 20	59.000	2.57735E-01	-1.65083E-02	-1.26517E+03	-2.24244E-09
3- 21	60.000	2.41081E-01	-1.67913E-02	-1.28788E+03	-2.09753E-09
3- 22	61.000	2.24153E-01	-1.70552E-02	-1.30812E+03	-1.95025E-09
3- 23	62.000	2.06970E-01	-1.72998E-02	-1.32688E+03	-1.80075E-09
3- 24	63.000	1.89553E-01	-1.75248E-02	-1.34414E+03	-1.64922E-09
3- 25	64.000	1.71921E-01	-1.77299E-02	-1.35987E+03	-1.49580E-09
3- 26	65.000	1.54093E-01	-1.79149E-02	-1.37405E+03	-1.34070E-09
3- 27	66.000	1.36091E-01	-1.80795E-02	-1.38668E+03	-1.18407E-09
3- 28	67.000	1.17934E-01	-1.82236E-02	-1.39773E+03	-1.02609E-09
3- 29	68.000	9.96439E-02	-1.83471E-02	-1.40720E+03	-8.66956E-10
3- 30	69.000	8.12403E-02	-1.84497E-02	-1.41507E+03	-7.06835E-10
3- 31	70.000	6.27445E-02	-1.85314E-02	-1.42134E+03	-5.45911E-10
3- 32	71.000	4.41775E-02	-1.85921E-02	-1.42599E+03	-3.84368E-10
3- 33	72.000	2.55604E-02	-1.86316E-02	-1.42903E+03	-2.22389E-10
3- 34	73.000	6.91424E-03	-1.86501E-02	-1.43044E+03	-6.01577E-11
3- 35	74.000	-1.17397E-02	-1.86473E-02	-1.43023E+03	1.02142E-10
3- 36	75.000	-3.03804E-02	-1.86234E-02	-1.42840E+03	2.64326E-10
3- 37	76.000	-4.89866E-02	-1.85784E-02	-1.42495E+03	4.26210E-10
3- 38	77.000	-6.75372E-02	-1.85123E-02	-1.41987E+03	5.87611E-10

TABLE II, MODE 2

TORSIONAL MODES OF CYLINDER-SHAFT CONFIGURATION FOR NUMERICAL EXAMPLE

STATION	X	ZETA	ZETA-PRIME	TORQUE	TORQUE-PRIME
3- 39	78.000	-8.60112E-02	-1.84252E-02	-1.41319E+03	7.48345E-10
3- 40	79.000	-1.04388E-01	-1.83172E-02	-1.40491E+03	9.08229E-10
3- 41	80.000	-1.22646E-01	-1.81884E-02	-1.39503E+03	1.06708E-09
3- 42	81.000	-1.40764E-01	-1.80389E-02	-1.38357E+03	1.22473E-09
3- 43	82.000	-1.58723E-01	-1.78690E-02	-1.37054E+03	1.38098E-09
3- 44	83.000	-1.76502E-01	-1.76788E-02	-1.35595E+03	1.53567E-09
3- 45	84.000	-1.94081E-01	-1.74686E-02	-1.33982E+03	1.68861E-09
3- 46	85.000	-2.11440E-01	-1.72385E-02	-1.32218E+03	1.83964E-09
3- 47	86.000	-2.28558E-01	-1.69889E-02	-1.30303E+03	1.98858E-09
3- 48	87.000	-2.45417E-01	-1.67200E-02	-1.28240E+03	2.13527E-09
3- 49	88.000	-2.61958E-01	-1.64321E-02	-1.26032E+03	2.27953E-09
3- 50	89.000	-2.78282E-01	-1.61256E-02	-1.23681E+03	2.42120E-09
3- 51	90.000	-2.94249E-01	-1.58007E-02	-1.21190E+03	2.56013E-09
3- 52	91.000	-3.09883E-01	-1.54580E-02	-1.18561E+03	2.69615E-09
3- 53	92.000	-3.25166E-01	-1.50977E-02	-1.15798E+03	2.82912E-09
3- 54	93.000	-3.40079E-01	-1.47203E-02	-1.12903E+03	2.95887E-09
3- 55	94.000	-3.54606E-01	-1.43262E-02	-1.09880E+03	3.08527E-09
3- 56	95.000	-3.68731E-01	-1.39158E-02	-1.06733E+03	3.20816E-09
3- 57	96.000	-3.82438E-01	-1.34896E-02	-1.03464E+03	3.32742E-09
3- 58	97.000	-3.95711E-01	-1.30481E-02	-1.00078E+03	3.44290E-09
3- 59	98.000	-4.08534E-01	-1.25918E-02	-9.65781E+02	3.55447E-09
3- 60	99.000	-4.20894E-01	-1.21213E-02	-9.29689E+02	3.66201E-09
3- 61	100.000	-4.32777E-01	-1.16369E-02	-8.92541E+02	3.76540E-09
3- 62	101.000	-4.44168E-01	-1.11394E-02	-8.54381E+02	3.86451E-09
3- 63	102.000	-4.55056E-01	-1.06292E-02	-8.15251E+02	3.95923E-09
3- 64	103.000	-4.65427E-01	-1.01070E-02	-7.75196E+02	4.04947E-09
3- 65	104.000	-4.75270E-01	-9.57329E-03	-7.34262E+02	4.13511E-09
3- 66	105.000	-4.84574E-01	-9.02873E-03	-6.92495E+02	4.21606E-09
3- 67	106.000	-4.93328E-01	-8.47392E-03	-6.49941E+02	4.29222E-09
3- 68	107.000	-5.01522E-01	-7.90950E-03	-6.06650E+02	4.36351E-09
3- 69	108.000	-5.09147E-01	-7.33610E-03	-5.62671E+02	4.42985E-09
3- 70	109.000	-5.16194E-01	-6.75437E-03	-5.18054E+02	4.49117E-09
3- 71	110.000	-5.22656E-01	-6.16498E-03	-4.72848E+02	4.54739E-09
3- 72	111.000	-5.28524E-01	-5.56860E-03	-4.27106E+02	4.59845E-09

TABLE II, MODE 2

TORSIONAL MODES OF CYLINDER-SHAFT CONFIGURATION FOR NUMERICAL EXAMPLE

STATION	X	ZETA	ZETA-PRIME	TORQUE	TORQUE-PRIME
3- 73	112.000	-5.33793E-01	-4.96589E-03	-3.80879E+02	4.64429E-09
3- 74	113.000	-5.38456E-01	-4.35756E-03	-3.34220E+02	4.68486E-09
3- 75	114.000	-5.42508E-01	-3.74427E-03	-2.87182E+02	4.72012E-09
3- 76	115.000	-5.45945E-01	-3.12674E-03	-2.39818E+02	4.75002E-09
3- 77	116.000	-5.48762E-01	-2.50566E-03	-1.92182E+02	4.77453E-09
3- 78	117.000	-5.50956E-01	-1.88174E-03	-1.44327E+02	4.79362E-09
3- 79	118.000	-5.52526E-01	-1.25568E-03	-9.63094E+01	4.80727E-09
3- 80	119.000	-5.53468E-01	-6.28197E-04	-4.81821E+01	4.81547E-09
3- 81	120.000	-5.53782E-01	-2.71407E-17	-2.08167E-12	4.81821E-09

TOTAL NO. STATIONS = 203

TABLE II

TORSIONAL MODES OF CYLINDER-SHAFT CONFIGURATION FOR NUMERICAL EXAMPLE

T = 0.00 SEC

MODE 3

FREQUENCY CYCLES PER SECOND 1221.945

FREQUENCY RADIANS PER SECOND 7677.7046

STATION	X	ZETA	ZETA-PRIME	TORQUE	TORQUE-PRIME
1- 1	0.000	1.93881E-02	0.	0.	-2.51432E-08
1- 2	1.000	1.93678E-02	-4.05535E-05	-2.51432E+02	-2.51169E-08
1- 3	2.000	1.93070E-02	-8.10221E-05	-5.02337E+02	-2.50380E-08
1- 4	3.000	1.92058E-02	-1.21321E-04	-7.52192E+02	-2.49067E-08
1- 5	4.000	1.90644E-02	-1.61367E-04	-1.00047E+03	-2.47234E-08
1- 6	5.000	1.88831E-02	-2.01074E-04	-1.24566E+03	-2.44882E-08
1- 7	6.000	1.86622E-02	-2.40361E-04	-1.49024E+03	-2.42019E-08
1- 8	7.000	1.84023E-02	-2.79145E-04	-1.73070E+03	-2.38649E-08
1- 9	8.000	1.81040E-02	-3.17345E-04	-1.96754E+03	-2.34779E-08
1- 10	9.000	1.77677E-02	-3.54880E-04	-2.20026E+03	-2.30418E-08
1- 11	10.000	1.73942E-02	-3.91673E-04	-2.42838E+03	-2.25575E-08
1- 12	11.000	1.69844E-02	-4.27647E-04	-2.65141E+03	-2.20259E-08
1- 13	12.000	1.65389E-02	-4.62725E-04	-2.86890E+03	-2.14483E-08
1- 14	13.000	1.60589E-02	-4.96835E-04	-3.08028E+03	-2.08258E-08
1- 15	14.000	1.55453E-02	-5.29906E-04	-3.28542E+03	-2.01597E-08
1- 16	15.000	1.49991E-02	-5.61867E-04	-3.48358E+03	-1.94514E-08
1- 17	16.000	1.44216E-02	-5.92653E-04	-3.67445E+03	-1.87024E-08
1- 18	17.000	1.38138E-02	-6.22198E-04	-3.85763E+03	-1.79143E-08
1- 19	18.000	1.31772E-02	-6.50442E-04	-4.03274E+03	-1.70887E-08
1- 20	19.000	1.25130E-02	-6.77324E-04	-4.19941E+03	-1.62273E-08
1- 21	20.000	1.18226E-02	-7.02788E-04	-4.35729E+03	-1.53319E-08
1- 22	21.000	1.11074E-02	-7.26782E-04	-4.50605E+03	-1.44045E-08
1- 23	22.000	1.03690E-02	-7.49255E-04	-4.64538E+03	-1.34469E-08
1- 24	23.000	9.60891E-03	-7.70160E-04	-4.77499E+03	-1.24612E-08

TABLE II, MODE 3

TORSIONAL MODES OF CYLINDER-SHAFT CONFIGURATION FOR NUMERICAL EXAMPLE

STATION	X	ZETA	ZETA-PRIME	TORQUE	TORQUE-PRIME
1- 25	24.000	8.82870E-03	-7.89454E-04	-4.89461E+03	-1.14494E-08
1- 26	25.000	8.03001E-03	-8.07095E-04	-5.00399E+03	-1.04136E-08
1- 27	26.000	7.21452E-03	-8.23047E-04	-5.10289E+03	-9.35606E-09
1- 28	27.000	6.38393E-03	-8.37276E-04	-5.19111E+03	-8.27891E-09
1- 29	28.000	5.53997E-03	-8.49754E-04	-5.26847E+03	-7.18444E-09
1- 30	29.000	4.68443E-03	-8.60453E-04	-5.33481E+03	-6.07494E-09
1- 31	30.000	3.81907E-03	-8.69351E-04	-5.38998E+03	-4.95272E-09
1- 32	31.000	2.94573E-03	-8.76430E-04	-5.43387E+03	-3.82013E-09
1- 33	32.000	2.06622E-03	-8.81675E-04	-5.46639E+03	-2.67955E-09
1- 34	33.000	1.18238E-03	-8.85075E-04	-5.48746E+03	-1.53335E-09
1- 35	34.000	2.96069E-04	-8.86622E-04	-5.49706E+03	-3.83953E-10
1- 36	35.000	-5.90863E-04	-8.86315E-04	-5.49515E+03	7.66254E-10
1- 37	36.000	-1.47656E-03	-8.84152E-04	-5.48174E+03	1.91486E-09
1- 38	37.000	-2.35917E-03	-8.80139E-04	-5.45686E+03	3.05946E-09
1- 39	38.000	-3.23684E-03	-8.74283E-04	-5.42056E+03	4.19765E-09
1- 40	39.000	-4.10774E-03	-8.65599E-04	-5.37291E+03	5.32707E-09
1- 41	40.000	-4.97004E-03	-8.57100E-04	-5.31402E+03	6.44533E-09
2- 1	40.000	-4.97004E-03	-1.29683E-04	-8.04036E+02	6.44533E-09
2- 2	41.000	-5.09452E-03	-1.19152E-04	-7.38742E+02	6.60677E-09
2- 3	42.000	-5.20835E-03	-1.08371E-04	-6.71901E+02	6.75438E-09
2- 4	43.000	-5.31127E-03	-9.73637E-05	-6.03655E+02	6.88786E-09
2- 5	44.000	-5.40308E-03	-8.61524E-05	-5.34145E+02	7.00692E-09
2- 6	45.000	-5.48358E-03	-7.47608E-05	-4.63517E+02	7.11132E-09
2- 7	46.000	-5.55261E-03	-6.32128E-05	-3.91919E+02	7.20083E-09
2- 8	47.000	-5.61001E-03	-5.15324E-05	-3.19501E+02	7.27528E-09
2- 9	48.000	-5.65568E-03	-3.97442E-05	-2.46414E+02	7.33450E-09
2- 10	49.000	-5.68951E-03	-2.78728E-05	-1.72811E+02	7.37837E-09
2- 11	50.000	-5.71143E-03	-1.59431E-05	-9.88470E+01	7.40680E-09
2- 12	51.000	-5.72140E-03	-3.97995E-06	-2.46757E+01	7.41973E-09
2- 13	52.000	-5.71940E-03	7.99152E-06	4.95474E+01	7.41713E-09
2- 14	53.000	-5.70542E-03	1.99463E-05	1.23667E+02	7.39901E-09
2- 15	54.000	-5.67951E-03	3.18593E-05	1.97528E+02	7.36540E-09
2- 16	55.000	-5.64171E-03	4.37057E-05	2.70975E+02	7.31638E-09
2- 17	56.000	-5.59211E-03	5.54606E-05	3.43856E+02	7.25205E-09

TABLE II, MODE 3

TORSIONAL MODES OF CYLINDER-SHAFT CONFIGURATION FOR NUMERICAL EXAMPLE

STATION	X	ZETA	ZETA-PRIME	TORQUE	TORQUE-PRIME
2- 18	57.000	-5.53080E-03	6.70994E-05	4.16016E+02	7.17254E-09
2- 19	58.000	-5.45791E-03	7.85979E-05	4.87307E+02	7.07803E-09
2- 20	59.000	-5.37361E-03	8.99318E-05	5.57577E+02	6.96870E-09
2- 21	60.000	-5.27806E-03	1.01078E-04	6.26681E+02	6.84478E-09
2- 22	61.000	-5.17146E-03	1.12012E-04	6.94474E+02	6.70654E-09
2- 23	62.000	-5.05404E-03	1.22712E-04	7.60813E+02	6.55427E-09
2- 24	63.000	-4.92604E-03	1.33155E-04	8.25560E+02	6.38827E-09
2- 25	64.000	-4.78773E-03	1.43319E-04	8.88579E+02	6.20891E-09
2- 26	65.000	-4.63941E-03	1.53184E-04	9.49739E+02	6.01656E-09
2- 27	66.000	-4.48137E-03	1.62728E-04	1.00891E+03	5.81161E-09
2- 28	67.000	-4.31396E-03	1.71931E-04	1.06597E+03	5.59450E-09
2- 29	68.000	-4.13752E-03	1.80775E-04	1.12080E+03	5.36568E-09
2- 30	69.000	-3.95241E-03	1.89240E-04	1.17329E+03	5.12564E-09
2- 31	70.000	-3.75904E-03	1.97309E-04	1.22332E+03	4.87486E-09
2- 32	71.000	-3.55780E-03	2.04965E-04	1.27079E+03	4.61389E-09
2- 33	72.000	-3.34911E-03	2.12193E-04	1.31560E+03	4.34325E-09
2- 34	73.000	-3.13342E-03	2.18976E-04	1.35765E+03	4.06353E-09
2- 35	74.000	-2.91116E-03	2.25301E-04	1.39687E+03	3.77531E-09
2- 36	75.000	-2.68282E-03	2.31155E-04	1.43316E+03	3.47918E-09
2- 37	76.000	-2.44886E-03	2.36525E-04	1.46645E+03	3.17577E-09
2- 38	77.000	-2.20977E-03	2.41399E-04	1.49668E+03	2.86572E-09
2- 39	78.000	-1.96606E-03	2.45769E-04	1.52377E+03	2.54966E-09
2- 40	79.000	-1.71824E-03	2.49624E-04	1.54767E+03	2.22827E-09
2- 41	80.000	-1.46682E-03	2.52957E-04	1.56834E+03	1.90222E-09
2- 42	81.000	-1.21232E-03	2.55761E-04	1.58572E+03	1.57219E-09
2- 43	82.000	-9.55295E-04	2.58029E-04	1.59978E+03	1.23886E-09
2- 44	83.000	-6.96267E-04	2.59758E-04	1.61050E+03	9.02945E-10
2- 45	84.000	-4.35781E-04	2.60942E-04	1.61784E+03	5.65137E-10
2- 46	85.000	-1.74383E-04	2.61581E-04	1.62180E+03	2.26147E-10
2- 47	86.000	8.73798E-05	2.61672E-04	1.62237E+03	-1.13317E-10
2- 48	87.000	3.48960E-04	2.61216E-04	1.61954E+03	-4.52545E-10
2- 49	88.000	6.09811E-04	2.60212E-04	1.61332E+03	-7.90826E-10
2- 50	89.000	8.69386E-04	2.58665E-04	1.60372E+03	-1.12745E-09
2- 51	90.000	1.12714E-03	2.56576E-04	1.59077E+03	-1.46172E-09

TABLE II, MODE 3

TORSIONAL MODES OF CYLINDER-SHAFT CONFIGURATION FOR NUMERICAL EXAMPLE

STATION	X	ZETA	ZETA-PRIME	TOPQUE	TOPQUE-PRIME
2- 52	91.000	1.38254E-03	2.53950E-04	1.57449E+03	-1.79293E-09
2- 53	92.000	1.63504E-03	2.50792E-04	1.55491E+03	-2.12038E-09
2- 54	93.000	1.88412E-03	2.47110E-04	1.53208E+03	-2.44340E-09
2- 55	94.000	2.12926E-03	2.42911E-04	1.50605E+03	-2.76131E-09
2- 56	95.000	2.36995E-03	2.38203E-04	1.47686E+03	-3.07344E-09
2- 57	96.000	2.60567E-03	2.32997E-04	1.44458E+03	-3.37913E-09
2- 58	97.000	2.83594E-03	2.27303E-04	1.40928E+03	-3.67776E-09
2- 59	98.000	3.06028E-03	2.21133E-04	1.37103E+03	-3.96869E-09
2- 60	99.000	3.27821E-03	2.14501E-04	1.32991E+03	-4.25131E-09
2- 61	100.000	3.48929E-03	2.07420E-04	1.28600E+03	-4.52504E-09
2- 62	101.000	3.69306E-03	1.99904E-04	1.23941E+03	-4.78929E-09
2- 63	102.000	3.88910E-03	1.91970E-04	1.19022E+03	-5.04353E-09
2- 64	103.000	4.07700E-03	1.83635E-04	1.13854E+03	-5.28721E-09
2- 65	104.000	4.25637E-03	1.74915E-04	1.08447E+03	-5.51982E-09
2- 66	105.000	4.42684E-03	1.65829E-04	1.02814E+03	-5.74089E-09
2- 67	106.000	4.58804E-03	1.56396E-04	9.69658E+02	-5.94994E-09
2- 68	107.000	4.73963E-03	1.46636E-04	9.09144E+02	-6.14653E-09
2- 69	108.000	4.88131E-03	1.36569E-04	8.46728E+02	-6.33027E-09
2- 70	109.000	5.01278E-03	1.26216E-04	7.82540E+02	-6.50076E-09
2- 71	110.000	5.13375E-03	1.15599E-04	7.16714E+02	-6.65764E-09
2- 72	111.000	5.24398E-03	1.04740E-04	6.49388E+02	-6.80059E-09
2- 73	112.000	5.34324E-03	9.36617E-05	5.80703E+02	-6.92931E-09
2- 74	113.000	5.43131E-03	8.23875E-05	5.10802E+02	-7.04353E-09
2- 75	114.000	5.50802E-03	7.09408E-05	4.39832E+02	-7.14300E-09
2- 76	115.000	5.57320E-03	5.93456E-05	3.67943E+02	-7.22753E-09
2- 77	116.000	5.62671E-03	4.75262E-05	2.95283E+02	-7.29693E-09
2- 78	117.000	5.66846E-03	3.58072E-05	2.22005E+02	-7.35107E-09
2- 79	118.000	5.69823E-03	2.39132E-05	1.48262E+02	-7.38981E-09
2- 80	119.000	5.71629E-03	1.19691E-05	7.42086E+01	-7.41310E-09
2- 81	120.000	5.72228E-03	-1.45815E-18	-9.04053E-12	-7.42087E-09
3- 1	40.000	-4.97004E-03	-5.88011E-02	-4.50999E+03	1.31836E-10
3- 2	41.000	-6.37626E-02	-5.85822E-02	-4.50087E+03	1.69138E-09
3- 3	42.000	-1.22335E-01	-5.83602E-02	-4.47617E+03	3.24507E-09
3- 4	43.000	-1.80483E-01	-5.78362E-02	-4.43598E+03	4.78754E-09

TABLE II, MODE 3

TORSIONAL MODES OF CYLINDER-SHAFT CONFIGURATION FOR NUMERICAL EXAMPLE

STATION	X	ZETA	ZETA-PRIME	TORQUE	TORQUE-PRIME
3- 5	44.000	-2.38007E-01	-5.71120E-02	-4.38043E+03	6.31343E-09
3- 6	45.000	-2.94708E-01	-5.61901E-02	-4.30973E+03	7.81748E-09
3- 7	46.000	-3.50388E-01	-5.50737E-02	-4.22410E+03	9.29448E-09
3- 8	47.000	-4.04856E-01	-5.37667E-02	-4.12385E+03	1.07393E-08
3- 9	48.000	-4.57923E-01	-5.22735E-02	-4.00932E+03	1.21470E-08
3- 10	49.000	-5.09404E-01	-5.05994E-02	-3.88092E+03	1.35126E-08
3- 11	50.000	-5.59123E-01	-4.87501E-02	-3.73909E+03	1.48314E-08
3- 12	51.000	-6.06906E-01	-4.67321E-02	-3.58431E+03	1.60989E-08
3- 13	52.000	-6.52589E-01	-4.45523E-02	-3.41712E+03	1.73107E-08
3- 14	53.000	-6.96012E-01	-4.22183E-02	-3.23810E+03	1.84626E-08
3- 15	54.000	-7.37027E-01	-3.97382E-02	-3.04788E+03	1.95505E-08
3- 16	55.000	-7.75491E-01	-3.71204E-02	-2.84710E+03	2.05708E-08
3- 17	56.000	-8.11270E-01	-3.43742E-02	-2.63647E+03	2.15199E-08
3- 18	57.000	-8.44242E-01	-3.15090E-02	-2.41671E+03	2.23945E-08
3- 19	58.000	-8.74291E-01	-2.85347E-02	-2.18859E+03	2.31916E-08
3- 20	59.000	-9.01314E-01	-2.54617E-02	-1.95289E+03	2.39084E-08
3- 21	60.000	-9.25217E-01	-2.23005E-02	-1.71042E+03	2.45425E-08
3- 22	61.000	-9.45917E-01	-1.90621E-02	-1.46204E+03	2.50916E-08
3- 23	62.000	-9.63344E-01	-1.57577E-02	-1.20860E+03	2.55539E-08
3- 24	63.000	-9.77435E-01	-1.23987E-02	-9.50968E+02	2.59277E-08
3- 25	64.000	-9.88144E-01	-8.99682E-03	-6.90047E+02	2.62117E-08
3- 26	65.000	-9.95432E-01	-5.56378E-03	-4.26737E+02	2.64050E-08
3- 27	66.000	-9.99274E-01	-2.11148E-03	-1.61948E+02	2.65070E-08
3- 28	67.000	-9.99658E-01	1.34815E-03	1.03401E+02	2.65171E-08
3- 29	68.000	-9.96581E-01	4.80311E-03	3.68394E+02	2.64355E-08
3- 30	69.000	-9.90055E-01	8.24147E-03	6.32112E+02	2.62624E-08
3- 31	70.000	-9.80101E-01	1.16513E-02	8.93643E+02	2.59984E-08
3- 32	71.000	-9.66755E-01	1.50208E-02	1.15208E+03	2.56444E-08
3- 33	72.000	-9.50062E-01	1.83383E-02	1.40653E+03	2.52016E-08
3- 34	73.000	-9.30081E-01	2.15924E-02	1.65612E+03	2.46715E-08
3- 35	74.000	-9.06881E-01	2.47717E-02	1.89997E+03	2.40561E-08
3- 36	75.000	-8.80541E-01	2.78653E-02	2.13724E+03	2.33574E-08
3- 37	76.000	-8.51153E-01	3.08625E-02	2.36712E+03	2.25779E-08
3- 38	77.000	-8.18818E-01	3.37528E-02	2.58881E+03	2.17201E-08

TABLE II, MODE 3

TORSIONAL MODES OF CYLINDER-SHAFT CONFIGURATION FOR NUMERICAL EXAMPLE

STATION	X	ZETA	ZETA-PRIME	TORQUE	TORQUE-PRIME
3- 39	78.000	-7.83649E-01	3.65263E-02	2.80153E+03	2.07873E-08
3- 40	79.000	-7.45768E-01	3.91734E-02	3.00456E+03	1.97824E-08
3- 41	80.000	-7.05305E-01	4.16849E-02	3.19719E+03	1.87091E-08
3- 42	81.000	-6.62401E-01	4.40521E-02	3.37875E+03	1.75710E-08
3- 43	82.000	-6.17203E-01	4.62668E-02	3.54862E+03	1.63721E-08
3- 44	83.000	-5.69869E-01	4.83214E-02	3.70620E+03	1.51165E-08
3- 45	84.000	-5.20562E-01	5.02087E-02	3.85096E+03	1.38085E-08
3- 46	85.000	-4.69453E-01	5.19222E-02	3.98238E+03	1.24528E-08
3- 47	86.000	-4.16719E-01	5.34560E-02	4.10003E+03	1.10540E-08
3- 48	87.000	-3.62542E-01	5.48048E-02	4.20348E+03	9.61688E-09
3- 49	88.000	-3.07111E-01	5.59679E-02	4.29238E+03	8.14649E-09
3- 50	89.000	-2.50616E-01	5.69293E-02	4.36642E+03	6.64789E-09
3- 51	90.000	-1.93253E-01	5.76976E-02	4.42535E+03	5.12627E-09
3- 52	91.000	-1.35221E-01	5.82662E-02	4.46896E+03	3.58691E-09
3- 53	92.000	-7.67214E-02	5.86331E-02	4.49710E+03	2.03513E-09
3- 54	93.000	-1.79557E-02	5.87970E-02	4.50967E+03	4.76296E-10
3- 55	94.000	4.08724E-02	5.87574E-02	4.50664E+03	-1.08419E-09
3- 56	95.000	9.95591E-02	5.85145E-02	4.48800E+03	-2.64093E-09
3- 57	96.000	1.57901E-01	5.80690E-02	4.45383E+03	-4.18853E-09
3- 58	97.000	2.15697E-01	5.74224E-02	4.40424E+03	-5.72164E-09
3- 59	98.000	2.72747E-01	5.65772E-02	4.33941E+03	-7.23494E-09
3- 60	99.000	3.28852E-01	5.55360E-02	4.25956E+03	-8.72321E-09
3- 61	100.000	3.83820E-01	5.43027E-02	4.16496E+03	-1.01813E-08
3- 62	101.000	4.37459E-01	5.28813E-02	4.05595E+03	-1.16041E-08
3- 63	102.000	4.89584E-01	5.12770E-02	3.93289E+03	-1.29868E-08
3- 64	103.000	5.40014E-01	4.94951E-02	3.79622E+03	-1.43245E-08
3- 65	104.000	5.88575E-01	4.75418E-02	3.64641E+03	-1.56127E-08
3- 66	105.000	6.35099E-01	4.54241E-02	3.48398E+03	-1.68468E-08
3- 67	106.000	6.79425E-01	4.31490E-02	3.30949E+03	-1.80226E-08
3- 68	107.000	7.21399E-01	4.07246E-02	3.12354E+03	-1.91360E-08
3- 69	108.000	7.60876E-01	3.81593E-02	2.92678E+03	-2.01832E-08
3- 70	109.000	7.97720E-01	3.54618E-02	2.71988E+03	-2.11605E-08
3- 71	110.000	8.31802E-01	3.26416E-02	2.50358E+03	-2.20646E-08
3- 72	111.000	8.63005E-01	2.97084E-02	2.27860E+03	-2.28923E-08

TABLE II, MODE 3

TORSIONAL MODES OF CYLINDER-SHAFT CONFIGURATION FOR NUMERICAL EXAMPLE

STATION	X	ZETA	ZETA-PRIME	TORQUE	TORQUE-PRIME
3- 73	112.000	8.91221E-01	2.66723E-02	2.04574E+03	-2.36407E-08
3- 74	113.000	9.16253E-01	2.35439E-02	1.80579E+03	-2.43074E-08
3- 75	114.000	9.38312E-01	2.03340E-02	1.55960E+03	-2.48899E-08
3- 76	115.000	9.57023E-01	1.70537E-02	1.30800E+03	-2.53862E-08
3- 77	116.000	9.72422E-01	1.37144E-02	1.05188E+03	-2.57947E-08
3- 78	117.000	9.84455E-01	1.03275E-02	7.92112E+02	-2.61139E-08
3- 79	118.000	9.93080E-01	6.90496E-03	5.29604E+02	-2.63427E-08
3- 80	119.000	9.98268E-01	3.45847E-03	2.65261E+02	-2.64803E-08
3- 81	120.000	1.00000E+00	1.01032E-15	7.74903E-11	-2.65262E-08

TOTAL NO. STATIONS = 203

TABLE II

TORSIONAL MODES OF CYLINDER-SHAFT CONFIGURATION FOR NUMERICAL EXAMPLE

T = 0.00 SEC

MODE 4

FREQUENCY CYCLES PER SECOND 1399.407

FREQUENCY RADIANS PER SECOND 8792.7318

STATION	X	ZETA	ZETA-PRIME	TORQUE	TORQUE-PRIME
1- 1	0.000	9.91408E-01	0.	0.	-1.68625E-06
1- 2	1.000	9.90049E-01	-2.71976E-03	-1.68625E+04	-1.68394E-06
1- 3	2.000	9.85971E-01	-5.43207E-03	-3.36788E+04	-1.67701E-06
1- 4	3.000	9.79186E-01	-8.12946E-03	-5.04027E+04	-1.65547E-06
1- 5	4.000	9.69714E-01	-1.08045E-02	-6.69882E+04	-1.64935E-06
1- 6	5.000	9.57579E-01	-1.34500E-02	-8.33898E+04	-1.62871E-06
1- 7	6.000	9.42816E-01	-1.60585E-02	-9.95626E+04	-1.60360E-06
1- 8	7.000	9.25464E-01	-1.86229E-02	-1.15462E+05	-1.57409E-06
1- 9	8.000	9.05572E-01	-2.11362E-02	-1.31065E+05	-1.54026E-06
1- 10	9.000	8.83193E-01	-2.35915E-02	-1.46267E+05	-1.50219E-06
1- 11	10.000	8.58390E-01	-2.59821E-02	-1.61089E+05	-1.46001E-06
1- 12	11.000	8.31231E-01	-2.83013E-02	-1.75468E+05	-1.41381E-06
1- 13	12.000	8.01789E-01	-3.05428E-02	-1.89365E+05	-1.36374E-06
1- 14	13.000	7.70147E-01	-3.27005E-02	-2.02743E+05	-1.30992E-06
1- 15	14.000	7.36390E-01	-3.47684E-02	-2.15564E+05	-1.25250E-06
1- 16	15.000	7.00611E-01	-3.67409E-02	-2.27793E+05	-1.19165E-06
1- 17	16.000	6.62910E-01	-3.86125E-02	-2.39397E+05	-1.12752E-06
1- 18	17.000	6.23388E-01	-4.03781E-02	-2.50344E+05	-1.06030E-06
1- 19	18.000	5.82155E-01	-4.20329E-02	-2.60604E+05	-9.90167E-07
1- 20	19.000	5.39323E-01	-4.35723E-02	-2.70148E+05	-9.17317E-07
1- 21	20.000	4.95011E-01	-4.49920E-02	-2.78951E+05	-8.41948E-07
1- 22	21.000	4.49340E-01	-4.62883E-02	-2.86987E+05	-7.64268E-07
1- 23	22.000	4.02435E-01	-4.74575E-02	-2.94236E+05	-6.84489E-07
1- 24	23.000	3.54426E-01	-4.84964E-02	-3.00678E+05	-6.02831E-07

TABLE II, MODE 4

TORSIONAL MODES OF CYLINDER-SHAFT CONFIGURATION FOR NUMERICAL EXAMPLE

STATION	X	ZETA	ZETA-PRIME	TORQUE	TORQUE-PRIME
1- 25	24.000	3.05443E-01	-4.94022E-02	-3.06294E+05	-5.19519E-07
1- 26	25.000	2.55622E-01	-5.01724E-02	-3.11069E+05	-4.34779E-07
1- 27	26.000	2.05099E-01	-5.08048E-02	-3.14990E+05	-3.48847E-07
1- 28	27.000	1.54013E-01	-5.12978E-02	-3.18046E+05	-2.61956E-07
1- 29	28.000	1.02504E-01	-5.16499E-02	-3.20230E+05	-1.74346E-07
1- 30	29.000	5.07135E-02	-5.18603E-02	-3.21534E+05	-8.62570E-08
1- 31	30.000	-1.21631E-03	-5.19283E-02	-3.21955E+05	2.06878E-09
1- 32	31.000	-5.31429E-02	-5.18537E-02	-3.21493E+05	9.03890E-08
1- 33	32.000	-1.04924E-01	-5.16368E-02	-3.20148E+05	1.78461E-07
1- 34	33.000	-1.56417E-01	-5.12781E-02	-3.17924E+05	2.65044E-07
1- 35	34.000	-2.07480E-01	-5.07787E-02	-3.14828E+05	3.52896E-07
1- 36	35.000	-2.57974E-01	-5.01398E-02	-3.10867E+05	4.38780E-07
1- 37	36.000	-3.07760E-01	-4.93634E-02	-3.06053E+05	5.23459E-07
1- 38	37.000	-3.56701E-01	-4.84514E-02	-3.00398E+05	6.06702E-07
1- 39	38.000	-4.04664E-01	-4.74064E-02	-2.93919E+05	6.88279E-07
1- 40	39.000	-4.51515E-01	-4.62312E-02	-2.86632E+05	7.67967E-07
1- 41	40.000	-4.97127E-01	-4.49291E-02	-2.78561E+05	8.45546E-07
2- 1	40.000	-4.97127E-01	-4.54417E-02	-2.81738E+05	8.45546E-07
2- 2	41.000	-5.41887E-01	-4.40156E-02	-2.72897E+05	9.21677E-07
2- 3	42.000	-5.85159E-01	-4.24686E-02	-2.63305E+05	9.95277E-07
2- 4	43.000	-6.26825E-01	-4.08051E-02	-2.52992E+05	1.06615E-06
2- 5	44.000	-6.66770E-01	-3.90295E-02	-2.41983E+05	1.13409E-06
2- 6	45.000	-7.04885E-01	-3.71468E-02	-2.30310E+05	1.19892E-06
2- 7	46.000	-7.41065E-01	-3.51621E-02	-2.18005E+05	1.26045E-06
2- 8	47.000	-7.75211E-01	-3.30809E-02	-2.05102E+05	1.31853E-06
2- 9	48.000	-8.07228E-01	-3.09089E-02	-1.91635E+05	1.37299E-06
2- 10	49.000	-8.37030E-01	-2.86520E-02	-1.77642E+05	1.42368E-06
2- 11	50.000	-8.64534E-01	-2.63164E-02	-1.63162E+05	1.47046E-06
2- 12	51.000	-8.89664E-01	-2.39086E-02	-1.48233E+05	1.51320E-06
2- 13	52.000	-9.12353E-01	-2.14352E-02	-1.32898E+05	1.55179E-06
2- 14	53.000	-9.32536E-01	-1.89029E-02	-1.17198E+05	1.58612E-06
2- 15	54.000	-9.50160E-01	-1.63187E-02	-1.01176E+05	1.61610E-06
2- 16	55.000	-9.65175E-01	-1.36897E-02	-8.48763E+04	1.64163E-06
2- 17	56.000	-9.77541E-01	-1.10231E-02	-6.83435E+04	1.66267E-06

TABLE II, MODE 4

TORSIONAL MODES OF CYLINDER-SHAFT CONFIGURATION FOR NUMERICAL EXAMPLE

STATION	X	ZETA	ZETA-PRIME	TORQUE	TORQUE-PRIME
2- 18	57.000	-9.87224E-01	-8.32630E-03	-5.16231E+04	1.67914E-06
2- 19	58.000	-9.94196E-01	-5.60660E-03	-3.47609E+04	1.69099E-06
2- 20	59.000	-9.98439E-01	-2.87149E-03	-1.78033E+04	1.69821E-06
2- 21	60.000	-9.99941E-01	-1.28506E-04	-7.96736E+02	1.70077E-06
2- 22	61.000	-9.98697E-01	2.61484E-03	1.62120E+04	1.69865E-06
2- 23	62.000	-9.94713E-01	5.35101E-03	3.31763E+04	1.69187E-06
2- 24	63.000	-9.87997E-01	8.07250E-03	5.00495E+04	1.68045E-06
2- 25	64.000	-9.78570E-01	1.07718E-02	6.67854E+04	1.66442E-06
2- 26	65.000	-9.66456E-01	1.34416E-02	8.33380E+04	1.64381E-06
2- 27	66.000	-9.51688E-01	1.60745E-02	9.96618E+04	1.61870E-06
2- 28	67.000	-9.34308E-01	1.86632E-02	1.15712E+05	1.58913E-06
2- 29	68.000	-9.14364E-01	2.12007E-02	1.31445E+05	1.55521E-06
2- 30	69.000	-8.91909E-01	2.36801E-02	1.46816E+05	1.51702E-06
2- 31	70.000	-8.67005E-01	2.60944E-02	1.61785E+05	1.47466E-06
2- 32	71.000	-8.39722E-01	2.84371E-02	1.76310E+05	1.42825E-06
2- 33	72.000	-8.10133E-01	3.07017E-02	1.90351E+05	1.37793E-06
2- 34	73.000	-7.78320E-01	3.28821E-02	2.03869E+05	1.32382E-06
2- 35	74.000	-7.44370E-01	3.49722E-02	2.16827E+05	1.26607E-06
2- 36	75.000	-7.08377E-01	3.69662E-02	2.29191E+05	1.20485E-06
2- 37	76.000	-6.70439E-01	3.88588E-02	2.40925E+05	1.14033E-06
2- 38	77.000	-6.30650E-01	4.06448E-02	2.51998E+05	1.07267E-06
2- 39	78.000	-5.89151E-01	4.23191E-02	2.62379E+05	1.00207E-06
2- 40	79.000	-5.46023E-01	4.38773E-02	2.72039E+05	9.28713E-07
2- 41	80.000	-5.01397E-01	4.53151E-02	2.80953E+05	8.52810E-07
2- 42	81.000	-4.55394E-01	4.66284E-02	2.89096E+05	7.74565E-07
2- 43	82.000	-4.08141E-01	4.78138E-02	2.96445E+05	6.94194E-07
2- 44	83.000	-3.59768E-01	4.88678E-02	3.02981E+05	6.11917E-07
2- 45	84.000	-3.10406E-01	4.97878E-02	3.08684E+05	5.27960E-07
2- 46	85.000	-2.60193E-01	5.05710E-02	3.13540E+05	4.42553E-07
2- 47	86.000	-2.09265E-01	5.12155E-02	3.17536E+05	3.55932E-07
2- 48	87.000	-1.57762E-01	5.17193E-02	3.20660E+05	2.68333E-07
2- 49	88.000	-1.05827E-01	5.20811E-02	3.22903E+05	1.79997E-07
2- 50	89.000	-5.36003E-02	5.23000E-02	3.24260E+05	9.11670E-08
2- 51	90.000	-1.22678E-03	5.23753E-02	3.24727E+05	2.08659E-09

TABLE II, MODE 4

TORSIONAL MODES OF CYLINDER-SHAFT CONFIGURATION FOR NUMERICAL EXAMPLE

STATION	X	ZETA	ZETA-PRIME	TORQUE	TORQUE-PRIME
2- 52	91.000	5.11502E-02	5.23068E-02	3.24302E+05	-8.69997E-08
2- 53	92.000	1.03387E-01	5.20948E-02	3.22988E+05	-1.75847E-07
2- 54	93.000	1.55340E-01	5.17397E-02	3.20786E+05	-2.64212E-07
2- 55	94.000	2.06867E-01	5.12426E-02	3.17704E+05	-3.51852E-07
2- 56	95.000	2.57825E-01	5.06048E-02	3.13750E+05	-4.38527E-07
2- 57	96.000	3.08076E-01	4.98281E-02	3.08934E+05	-5.23997E-07
2- 58	97.000	3.57482E-01	4.89146E-02	3.03270E+05	-6.08029E-07
2- 59	98.000	4.05906E-01	4.78658E-02	2.96774E+05	-6.90392E-07
2- 60	99.000	4.53216E-01	4.66876E-02	2.89463E+05	-7.70860E-07
2- 61	100.000	4.99282E-01	4.53802E-02	2.81357E+05	-8.49212E-07
2- 62	101.000	5.43978E-01	4.39483E-02	2.72479E+05	-9.25233E-07
2- 63	102.000	5.87180E-01	4.23957E-02	2.62853E+05	-9.98714E-07
2- 64	103.000	6.28770E-01	4.07267E-02	2.52506E+05	-1.06945E-06
2- 65	104.000	6.68634E-01	3.89459E-02	2.41465E+05	-1.13726E-06
2- 66	105.000	7.06663E-01	3.70582E-02	2.29761E+05	-1.20194E-06
2- 67	106.000	7.42752E-01	3.50688E-02	2.17426E+05	-1.26332E-06
2- 68	107.000	7.76802E-01	3.29830E-02	2.04495E+05	-1.32124E-06
2- 69	108.000	8.08719E-01	3.08068E-02	1.91002E+05	-1.37552E-06
2- 70	109.000	8.38417E-01	2.85459E-02	1.76985E+05	-1.42604E-06
2- 71	110.000	8.65813E-01	2.62067E-02	1.62482E+05	-1.47263E-06
2- 72	111.000	8.90832E-01	2.37956E-02	1.47532E+05	-1.51519E-06
2- 73	112.000	9.13405E-01	2.13191E-02	1.32178E+05	-1.55358E-06
2- 74	113.000	9.33472E-01	1.87841E-02	1.16461E+05	-1.58771E-06
2- 75	114.000	9.50975E-01	1.61975E-02	1.00424E+05	-1.61748E-06
2- 76	115.000	9.65868E-01	1.35664E-02	8.41117E+04	-1.64281E-06
2- 77	116.000	9.78110E-01	1.08981E-02	6.75682E+04	-1.66363E-06
2- 78	117.000	9.87666E-01	8.19987E-03	5.08392E+04	-1.67989E-06
2- 79	118.000	9.94511E-01	5.47912E-03	3.39705E+04	-1.69153E-06
2- 80	119.000	9.98626E-01	2.74333E-03	1.70086E+04	-1.69853E-06
2- 81	120.000	1.00000E+00	1.39183E-15	8.62937E-09	-1.70087E-06
3- 1	40.000	-4.97127E-01	4.14331E-02	3.17788E+03	1.72953E-08
3- 2	41.000	-4.54566E-01	4.35941E-02	3.34362E+03	1.58146E-08
3- 3	42.000	-4.09941E-01	4.55571E-02	3.49418E+03	1.42620E-08
3- 4	43.000	-3.63454E-01	4.73132E-02	3.62888E+03	1.26447E-08

TABLE II, MODE 4

TORSIONAL MODES OF CYLINDER-SHAFT CONFIGURATION FOR NUMERICAL EXAMPLE

STATION	X	ZETA	ZETA-PRIME	TORQUE	TORQUE-PRIME
3- 5	44.000	-3.15317E-01	4.88546E-02	3.74710E+03	1.09700E-08
3- 6	45.000	-2.65747E-01	5.01740E-02	3.84830E+03	9.24546E-09
3- 7	46.000	-2.14970E-01	5.12657E-02	3.93202E+03	7.47892E-09
3- 8	47.000	-1.63217E-01	5.21245E-02	3.99790E+03	5.67840E-09
3- 9	48.000	-1.10723E-01	5.27466E-02	4.04561E+03	3.85209E-09
3- 10	49.000	-5.77248E-02	5.31292E-02	4.07496E+03	2.00827E-09
3- 11	50.000	-4.46464E-03	5.32706E-02	4.08580E+03	1.55327E-10
3- 12	51.000	4.88160E-02	5.31700E-02	4.07809E+03	-1.69833E-09
3- 13	52.000	1.01975E-01	5.28280E-02	4.05185E+03	-3.54429E-09
3- 14	53.000	1.54472E-01	5.22461E-02	4.00722E+03	-5.37416E-09
3- 15	54.000	2.06368E-01	5.14269E-02	3.94439E+03	-7.17964E-09
3- 16	55.000	2.57327E-01	5.03742E-02	3.86365E+03	-8.95252E-09
3- 17	56.000	3.07117E-01	4.90927E-02	3.76536E+03	-1.06848E-08
3- 18	57.000	3.55514E-01	4.75883E-02	3.64997E+03	-1.23685E-08
3- 19	58.000	4.02296E-01	4.58677E-02	3.51801E+03	-1.39960E-08
3- 20	59.000	4.47251E-01	4.39389E-02	3.37007E+03	-1.55601E-08
3- 21	60.000	4.90175E-01	4.18105E-02	3.20683E+03	-1.70534E-08
3- 22	61.000	5.30874E-01	3.94923E-02	3.02902E+03	-1.84694E-08
3- 23	62.000	5.69163E-01	3.69947E-02	2.83746E+03	-1.98014E-08
3- 24	63.000	6.04866E-01	3.43291E-02	2.63301E+03	-2.10436E-08
3- 25	64.000	6.37824E-01	3.15076E-02	2.41660E+03	-2.21902E-08
3- 26	65.000	6.67885E-01	2.85430E-02	2.18922E+03	-2.32360E-08
3- 27	66.000	6.94913E-01	2.54487E-02	1.95189E+03	-2.41763E-08
3- 28	67.000	7.18786E-01	2.22389E-02	1.70570E+03	-2.50069E-08
3- 29	68.000	7.39394E-01	1.89281E-02	1.45176E+03	-2.57239E-08
3- 30	69.000	7.56645E-01	1.55313E-02	1.19123E+03	-2.63240E-08
3- 31	70.000	7.70451E-01	1.20639E-02	9.25291E+02	-2.68047E-08
3- 32	71.000	7.80777E-01	8.54177E-03	6.55146E+02	-2.71636E-08
3- 33	72.000	7.87548E-01	4.98082E-03	3.82024E+02	-2.73992E-08
3- 34	73.000	7.90743E-01	1.39722E-03	1.07166E+02	-2.75103E-08
3- 35	74.000	7.90347E-01	-2.19273E-03	-1.68180E+02	-2.74965E-08
3- 36	75.000	7.86361E-01	-5.77275E-03	-4.42764E+02	-2.73579E-08
3- 37	76.000	7.78805E-01	-9.32657E-03	-7.15339E+02	-2.70950E-08
3- 38	77.000	7.67712E-01	-1.28381E-02	-9.84666E+02	-2.67091E-08

TABLE II, MODE 4

TORSIONAL MODES OF CYLINDER-SHAFT CONFIGURATION FOR NUMERICAL EXAMPLE

STATION	X	ZETA	ZETA-PRIME	TORQUE	TORQUE-PRIME
3- 39	78.000	7.53133E-01	-1.62913E-02	-1.24952E+03	-2.62018E-08
3- 40	79.000	7.35134E-01	-1.96705E-02	-1.50871E+03	-2.55756E-08
3- 41	80.000	7.13756E-01	-2.29604E-02	-1.76104E+03	-2.48333E-08
3- 42	81.000	6.89217E-01	-2.61461E-02	-2.00538E+03	-2.39782E-08
3- 43	82.000	6.61507E-01	-2.92131E-02	-2.24062E+03	-2.30141E-08
3- 44	83.000	6.30794E-01	-3.21474E-02	-2.46567E+03	-2.19456E-08
3- 45	84.000	5.97216E-01	-3.49358E-02	-2.67954E+03	-2.07774E-08
3- 46	85.000	5.60926E-01	-3.75655E-02	-2.88124E+03	-1.95149E-08
3- 47	86.000	5.22088E-01	-4.00246E-02	-3.06985E+03	-1.81637E-08
3- 48	87.000	4.80879E-01	-4.23020E-02	-3.24452E+03	-1.67300E-08
3- 49	88.000	4.37487E-01	-4.43874E-02	-3.40447E+03	-1.52204E-08
3- 50	89.000	3.92107E-01	-4.62711E-02	-3.54895E+03	-1.36416E-08
3- 51	90.000	3.44947E-01	-4.79448E-02	-3.67731E+03	-1.20009E-08
3- 52	91.000	2.96220E-01	-4.94007E-02	-3.78898E+03	-1.03056E-08
3- 53	92.000	2.46147E-01	-5.06323E-02	-3.88345E+03	-8.56357E-09
3- 54	93.000	1.94956E-01	-5.16340E-02	-3.96027E+03	-6.78262E-09
3- 55	94.000	1.42880E-01	-5.24012E-02	-4.01912E+03	-4.97087E-09
3- 56	95.000	9.01551E-02	-5.29304E-02	-4.05971E+03	-3.13654E-09
3- 57	96.000	3.70202E-02	-5.32193E-02	-4.08187E+03	-1.28795E-09
3- 58	97.000	-1.62831E-02	-5.32666E-02	-4.08549E+03	5.66498E-10
3- 59	98.000	-6.95128E-02	-5.30719E-02	-4.07056E+03	2.41838E-09
3- 60	99.000	-1.22427E-01	-5.26362E-02	-4.03715E+03	4.25929E-09
3- 61	100.000	-1.74786E-01	-5.19615E-02	-3.98540E+03	6.08087E-09
3- 62	101.000	-2.26351E-01	-5.10508E-02	-3.91555E+03	7.87484E-09
3- 63	102.000	-2.76888E-01	-4.99083E-02	-3.82792E+03	9.63307E-09
3- 64	103.000	-3.26169E-01	-4.85392E-02	-3.72291E+03	1.13476E-08
3- 65	104.000	-3.73968E-01	-4.69496E-02	-3.60099E+03	1.30105E-08
3- 66	105.000	-4.20069E-01	-4.51468E-02	-3.46272E+03	1.46144E-08
3- 67	106.000	-4.64264E-01	-4.31390E-02	-3.30872E+03	1.61519E-08
3- 68	107.000	-5.06350E-01	-4.09353E-02	-3.13970E+03	1.76161E-08
3- 69	108.000	-5.46136E-01	-3.85457E-02	-2.95641E+03	1.90003E-08
3- 70	109.000	-5.83443E-01	-3.59810E-02	-2.75970E+03	2.02983E-08
3- 71	110.000	-6.18101E-01	-3.32529E-02	-2.55046E+03	2.15040E-08
3- 72	111.000	-6.49952E-01	-3.03738E-02	-2.32964E+03	2.26121E-08

TABLE II, MODE 4

TORSIONAL MODES OF CYLINDER-SHAFT CONFIGURATION FOR NUMERICAL EXAMPLE

STATION	X	ZETA	ZETA-PRIME	TORQUE	TORQUE-PRIME
3- 73	112.000	-6.78852E-01	-2.73567E-02	-2.09823E+03	2.36176E-08
3- 74	113.000	-7.04669E-01	-2.42154E-02	-1.85730E+03	2.45158E-08
3- 75	114.000	-7.27286E-01	-2.09641E-02	-1.60793E+03	2.53026E-08
3- 76	115.000	-7.46601E-01	-1.76176E-02	-1.35126E+03	2.59746E-08
3- 77	116.000	-7.62525E-01	-1.41911E-02	-1.08845E+03	2.65286E-08
3- 78	117.000	-7.74987E-01	-1.07001E-02	-8.20591E+02	2.69622E-08
3- 79	118.000	-7.83930E-01	-7.16056E-03	-5.49208E+02	2.72733E-08
3- 80	119.000	-7.89312E-01	-3.58844E-03	-2.75230E+02	2.74605E-08
3- 81	120.000	-7.91110E-01	2.33199E-16	1.78861E-11	2.75231E-08

```
$NAM1
INCHX    =  8,
DX       =  0.15E+02,
XMIN     =  0.0,
PLTZ     =  1,
PLTZPR   =  1,
NZAPR    =  1,
MPLT     =  0,
PLTT     =  1,
NPLTT    =  1,
TMIN     =  0.43164351072729-124,
ZPMIN1   = I,
DZP1     = I,
ZPMIN2   = I,
DZP2     = I,
ZPMIN3   = I,
DZP3     = I,
DTT      =  0.22373683230271-217,
PLTM     =  1,
NMZR     =  1,
MZRMIN   = I,
DMZR     =  0.11279098072731E-47,
PLTZR    =  1,
PLTAE    =  0,
PLTJG    =  1,
```

```
NMAEJG  =  1,
AJMIN   = -0.46812858247686+248,
DAEJG   =  0.22452527121716-217,
$END
TOTAL NUMBER OF PLOTS =    9
```