TR-DA1508
December 1967

PROGRAM DESCRIPTION AND THEORETICAL BASIS FOR THE ORBIT DETERMINATION PROGRAM

Contract NAS5-9939

Prepared for:

GODDARD SPACE FLIGHT CENTER
GREENBELT, MARYLAND

PHILCO Ford

PHILCO-FORD CORPORATION
Space & Re-entry Systems Division
Palo Alto, California

FACILITY FORM 602

N68-33361 (ACCESSION NUMBER)
(THRU)
244 (PAGES)
1 (CODE)
CR-96322 (NASA CR OR TMX OR AD NUMBER)
30 (CATEGORY)

GPO PRICE $

CFSTI PRICE(S) $

Hard copy (HC)

Microfiche (MF)

ff 653 July 65

TR-DA1508
December 1967

PROGRAM DESCRIPTION AND THEORETICAL BASIS FOR THE ORBIT DETERMINATION PROGRAM

Contract Number NAS5-9939

Prepared by

SYSTEM DEVELOPMENT AND MISSION ANALYSIS DEPARTMENT
Space and Re-entry Systems Division
Philco-Ford Corporation
Palo Alto, California

for

GODDARD SPACE FLIGHT CENTER
Greenbelt, Maryland

PHILCO Ford
PHILCO-FORD CORPORATION

Space & Re-entry
Systems Division

FOREWORD

Contract NAS5-9939 covered development of the Orbit Determination Program system described in this report, and several modifications to the Mark II Error Propagation Program developed under an earlier contract. Documentation of the modifications to the Mark II program was completed as additions to the Mark II manuals, and was delivered as each modification was completed. Final documentation of the Orbit Determination Program system is contained in

1.	TR-DA1508	Program Description and Theoretical Basis for the Orbit Determination Program.
2.	TR-DA1509	Subroutine Descriptions and Listings for the Orbit Determination Program.
3.	TR-DA1510	Input - Output Summary for the Orbit Determination Program.

The program development was done in the Systems Development and Mission Analysis Department under the technical direction of R. E. Brown, Engineering Section Supervisor. Major technical contributors were:

W. S. Bjorkman	Senior Engineering Specialist
W. F. Colescott	Project Engineer
D. E. Ekman	Project Engineer
M. J. Brooks	Senior Programmer

ACKNOWLEDGEMENT

The contributions of Dr. Stanley F. Schmidt of Astro Consultants, Inc. are gratefully acknowledged. Dr. Schmidt is principally responsible for the filter design. He was also a major contributor in the definition of program capabilities and in the program conceptional design.

TABLE OF CONTENTS

TABLE OF CONTENTS (Continued)

TABLE OF CONTENTS (Continued)

TABLE OF CONTENTS (Continued)

TABLE OF CONTENTS (Continued)

PRECEDING PAGE BLANK NOT FILMED.

LIST OF ILLUSTRATIONS

LIST OF TABLES

SECTION 1

ORBIT DETERMINATION PROGRAM SYSTEM

1.1 INTRODUCTION

The Orbit Determination Program System (ODP) is a set of computer programs for the estimation of space vehicle trajectories from tracking data of various types. It was designed for post-flight analysis and the study of post-flight analysis techniques. The program capabilities and their allocation to the various subprograms result from this basic purpose.

The ODP was developed for the IBM 7094 computer. With the exceptions discussed in Section 4, the system was written in FORTRAN IV, Version 13, to simplify conversion to other computer systems. Consistent attempts were made to simplify future expansion of the system in several areas, including:

- addition of new types and sources,
- modification of error models, and
- relaxation of dimensioning restrictions imposed by the 32K memory of the IBM 7094.

Expansions in all these areas are discussed in Sections 2-5 and in Reference 1.

In order to avoid a protracted study of process errors, all computations except those noted in Section 4 were written in double precision.

1.2 PROGRAM ORGANIZATION

The ODP consists of four programs, each of which is discussed independently in a section of this report. Due to the size of the programs, no attempt was made to include them under a single executive routine. A brief description of each program is given below.

1.2.1 Differential Correction Program (DCP)

The Differential Correction Program is the central program of the ODP. It accepts a tape of raw data from all sources, unpacked and written in a single format, and a deck of control cards. It defines a state vector based on certain control cards, and constructs an a priori estimate of the state and its covariance matrix from:

- standard data in a block data deck,
- input data in the data deck,
- tape-stored results of previous runs,
- maximum likelihood estimates from the raw data, or
- certain combinations of the above.

The DCP then uses the raw data in any order specified by the control cards for computing differential corrections to the state estimate or for computing and outputting residuals. The program is described in detail in Section 2.

1.2.2 Tracking Data Editing Program (TDEP)

The Tracking Data Editing Program accepts raw data tapes from a number of tracking stations of various types. It discards frames of data which cannot be interpreted, unpacks the data and converts units. It divides the data into station passes and merges all data onto a single edited data tape, sorted by station on-time, for input to the DCP. The program is described in detail in Section 3.

1.2.3 Tracking Data Simulator (TDS)

The Tracking Data Simulator accepts a tape-stored history of the space vehicle trajectory, and input data describing a tracking network and measurement schedule, and generates a tape of simulated data. The data tape has the same format as that written by the TDEP, and may be used for analyses for which real data is not available or where knowledge of the true state is desirable. The program is described in detail in Section 4.

1.2.4 Residual Output Program (ROP)

The Residual Output Program is designed primarily for those analyses of the residuals which may be desirable for a given application, and which are not within the capabilities of the ODP as delivered. The ROP, then, is a skeleton program which demonstrates the recovery of estimates and residuals from the record tapes written by the DCP. The program is described in detail in Section 5.

1.3 PROGRAM OPERATION

We will consider in Sections 2 through 5 each of the details of the program operation for the specific programs. In order to avoid unnecessary repetition, we will discuss here the aspects of operation which are common to all programs which are executed under IBSYS and hence form the basis for the detailed discussions which follow. Only those operations requirements which are appropriate to the ODP are discussed.

1.3.1 The Program Deck

The program deck consists of the object decks for all subroutines which are a part of the program and the control cards necessary for proper loading of the object decks into core. The object decks may be source decks for compilation, binary decks from previous compilations, $IBLDR cards used in conjunction with an IEDIT tape, or any combination of the three. Each deck must be present in one of the three forms in the proper link. The order within a link is immaterial except as noted in the IEDIT discussion below.

1.3.1.1 Use of OVERLAY. Each of the ODP programs uses the OVERLAY feature of IBSYS. At the start of each run, the entire program is placed on the overlay tape. The segments of the program are loaded into core by IBSYS as they are required.

Clearly, this procedure can be time consuming, particularly if much changing of links is required. The link structure of the ODP programs was designed to minimize the overlay loading for the anticipated uses of the programs, within the restrictions imposed by the 32K core and expected expansions of program capabilities. Cross references are provided for each program in order that a particular user may modify the link structure for improved run times in particular applications.

The link structure for each program is shown in block diagram form in Sections 2 through 5. In the program deck, each link except the main link is preceded by the control card

```
column    1             16
          $ ORIGIN       NAME
```

and contains all the decks designated in its block. The order in which the links occur in the program deck is determined from the following rule: once a link at origin NAME1 is included, each of its sub-links must also be included before the next link at NAME1 or a higher origin is started.

For a more detailed description of OVERLAY, the reader is referred to any recent IBM publication on the subject. For example, IBM 7090/7094 IBSYS Operating System, Version 13, IBJOB Processor, File Number 7090-27, Form C28-6389-1, pp. 39-42.

1.3.1.2 Use of an IEDIT Tape. The IEDIT option is a feature of the FORTRAN monitor by which decks may be loaded from a tape unit other than the system input tape. Either source or binary decks may be loaded from the IEDIT tape, but from a practical standpoint only binary decks should be so loaded.

In order to make an IEDIT tape, a program deck consisting of binary decks is placed on tape. Control cards which are not a part of the individual binary decks should be omitted. In making the tape, it should be remembered that the loader always searches forward on the IEDIT tape as far as it can go before rewinding. It is important, therefore, that the decks are placed on tape in the same order as they appear in a properly ordered program deck. Very large loading times may result from neglecting the proper ordering of the decks on tape or their recall from tape.

To load a deck from the IEDIT tape, that deck's position in the program deck is taken by a copy of the first card of the deck as it appears on the tape. For binary decks

```
column    1       8        15
          $IBLDR  MC13XX    Comments
```

Since decks from the IEDIT tape may be interspersed with source or binary decks from the program deck, the loader must be informed at each change to or from the IEDIT tape. For example, if the IEDIT tape is mounted on tape unit SYSLB3, and contains one file of decks, each set of $IBLDR cards must be preceded by the control card

```
column    1       16
          $IEDIT   SYSLB3,SRCH1
```

and followed by

```
column    1       16
          $IEDIT   SYSIN1
```

or simply

```
column    1
          $IEDIT
```

For further information concerning the IEDIT option, see IBM 7090/7094 IBSYS Operating System, IBJOB Processor, File Number 7090-27, Form C28-6275-3.

1.3.1.3 Control Cards. Several control cards in addition to the $ORIGIN and $IEDIT cards described above must be included in the program deck.
In each program, the main link is preceded by

```
column    1         16
          $JOB
          $IBJOB     Options
```

and the final deck is followed by

```
column    1
          $DATA
```

The $JOB card contains user identification as required by the user's particular installation. The options available for the $IBJOB processor are described in IBM Form C28-6275-3, cited above.

In addition, the ODP programs use two types of $NAME cards. "Global" name cards,

```
column    1         16
          $NAME      NAME1=NAME2
```

are used to change all occurances of NAME1 in the program to NAME2. These cards are inserted following the $IBJOB card. They are used by the ODP programs only for reassigning tape units for convenience at a particular installation.

"Qualified" name cards

```
column    1         16
          $NAME      .MC13XX(NAME 1)=NAME2
```

change references to NAME1 in deck MC13XX (only) to NAME2. These cards are used only by the DCP to allow loading of a single subroutine in more than one link. They may be placed in the link containing deck MC13XX or any of its parent links.

Summaries of the $NAME cards required by each program are given in Sections 2-5.

1.3.2 Tape Unit Assignments

Each of the ODP programs uses a number of tape units. The logical unit assignments used were selected for convenience at the installation used for development of the ODP system, and these assignments may not be appropriate for the user's installation. Two techniques are used for making the ODP assignments compatible with monitor assignments.

The simplest method for changing unit assignments is the use of global name cards. Where inconsistencies do not result, the logical numbers used by the program are effectively changed by inclusion of old and new file control block pointer word names in name cards. For example, the system input and output units are assigned the standard IBSYS logical numbers 5 and 6. For an installation using units 2 and 3 for these functions, we merely include the global name cards

```
column    1          16
          $NAME      .UNO5.=.UNO2.
          $NAME      .UNO6.=.UNO3.
```

Where reassignments by name cards are confusing or lead to inconsistencies, the monitor assignments of tape mode or physical unit designations may be changed by inserting FILE subroutines into the program link. For a description of FILE subroutines, see IBM Form C28-6275-3, cited above.

SECTION 2

DIFFERENTIAL CORRECTION PROGRAM

2.1 GENERAL DESCRIPTION OF THE PROGRAM

The Differential Correction Program (DCP) is the central program of the Orbit Determination Program system. It contains the logic and computations for the use of measurement values for the differential correction of an estimate of state.

2.1.1 Capabilities of the DCP

The DCP is capable of a wide variety of processes which are useful in the post-flight analysis of space vehicle trajectories. The DCP requires a single tape containing the data to be processed, written in a common format, and information identifying the tracking station and measurement type and mode. It processes the data under control of a sequence of control cards in the data deck.

The coordinate system and time scales used by the DCP are described in Appendix A. Appendix B describes the statistical process by which the state estimate is improved.

The DCP is capable of processing measurements of four types. These are:

- C-Band
  Azimuth
  Elevation
  Range

- Goddard Range and Range Rate
  - X-Angle
  - Y-Angle
  - Range
  - Doppler

- Unified S-Band
  - X-Angle
  - Y-Angle
  - Range
  - Doppler

- DSIF (JPL)
  - Hour Angle
  - Declination
  - Doppler

The measurements are described parametrically in terms of an assumed set of error parameters, in Appendix C.

The equations of motion of the space vehicle include accelerations due to

- gravitational attraction of the central body, including zonal and tesseral harmonics.
- inverse-square gravitational attractions of other celestial bodies
- solar radiation pressure
- atmospheric drag
- venting.

The accelerations are described in terms of an assumed set of error parameters in Appendix D. No attitude-dependent accelerations are included; hence the last three of the accelerations listed above are modelled in simplified forms.

2.1.1.1 Tracking Network. The DCP may process data from up to 20 stations in a single run. Data describing these stations are obtained from the labelled common STNCOM which may contain data for up to 50 stations. The descriptive data include nominal station locations, station error parameter values, and the variances and correlation coefficients of the error parameters.

At the start of a run, those stations occurring either as transmitting or receiving stations on the edited data tape are identified, and the data describing these stations is extracted from STNCOM and placed on the scratch tape (see 2.1.4 below). These data are used at the start of each case for the construction of the measurement error portions of the *a priori* estimate of state and its covariance matrix.

2.1.1.2 Definition of State. A state vector of up to 30 components may be selected for any given case. The state elements may be chosen from the vehicle state and error parameters almost freely, subject only to the constraints:

- all six vehicle cartesian states (position and velocity components) must be included or omitted as a group,

- a maximum of six state elements may be equation of motion parameters.

The set of parameters from which the state elements may be selected include

- vehicle six-state at a prescribed epoch,
- equation of motion parameters summarized in Table D-1 of Appendix D,
- measurement parameters summarized in Table C-1 of Appendix C, for each station which occurs on the edited data tape.

The stated limits on the length of the state vector (30) and the number of equation of motion parameters (6) may easily be increased, providing space is available in a given computer. The dimensioning changes involve the labelled commons

/DCRCOM/
/DQDCOM/
/ESOCOM/
/ES1COM/
/SBFCOM/

described in Reference 1.

Each element of the state vector may either be solved for or be included as an uncertain parameter. The treatment of each type of state element is described in Appendix B, paragraphs B.2 and B.3.

2.1.1.3 Data Start. If no adequate *a priori* estimate of the vehicle six-state is known, the DCP can generate one from a small number of data points. A subset of the data points are first smoothed by least squares fitting of a polynomial to each measurement type. The smoothed measurement values are then used to obtain a starting estimate for an iterative maximum likelihood estimation. The process is described in Appendix B, paragraphs B.4.

The measurement types required for a data start include two angles and either a range or doppler measurement. Clearly, the time interval covered by the measurements must be sufficiently long to define the six-state adequately. Short time intervals will normally result in very poor estimates of velocity, particularly in the plane normal to the line of sight.

The covariance matrix of the *a priori* state is taken from block data, overlay input, or from the results of previous estimations. Some care is required to insure that the covariance matrix is realistic in light of the low accuracy of the data start procedure.

2.1.1.4 Differential Correction. The principal function of the DCP is the differential correction of an *a priori* estimate of the state. The DCP accepts data from

- C-Band
- Unified S-Band
- Goddard Range and Range Rate
- DSIF (JPL)

tracking stations, and computes differential corrections for one measurement at a time using a Kalman filter modified to

- estimate the state at a fixed epoch,
- optimally include uncertain parameters not being estimated,
- non-optimally process data to prevent ill-behaved covariance matrices.

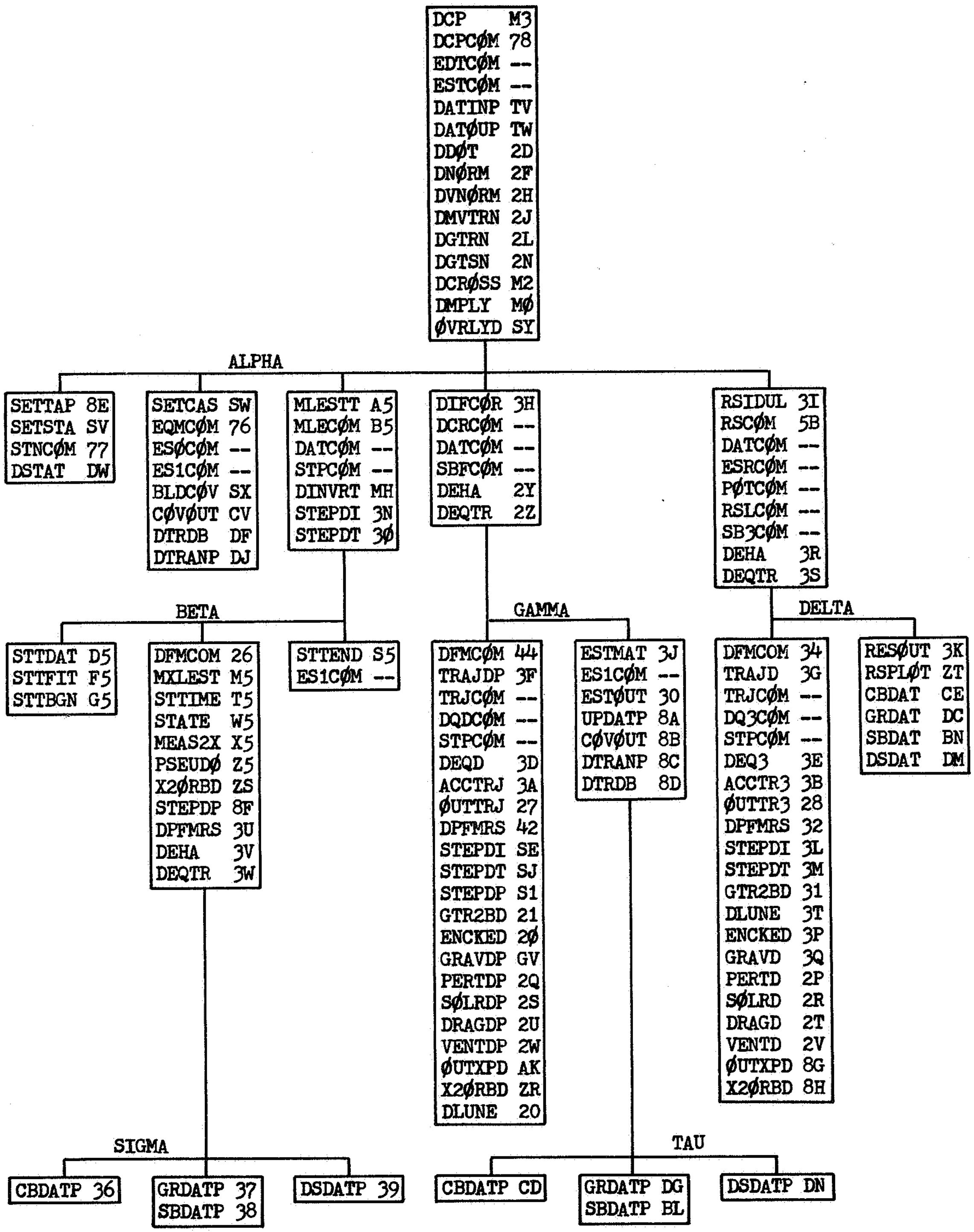
The derivation and implementation of the filter is described in Appendix B.

2.1.1.5 Special Output. The DCP can produce a variety of output useful in the evaluation of a state estimate. Residual plots on the system output tape and the tabulation of residuals of any set of measurement data relative to any computed estimate are easily obtained. In addition, two binary tapes containing any desired set of estimates and tabulations of residuals may be written for later use either in restarting the differential correction process, or for input to the Residual Output Program for any desired special purpose analyses of the residuals.

2.1.2 Program Structure

Figure 2-1 lists all the subroutines and labelled commons used by the DCP. Each deck for which a deckname is given in the figure must be present in the object deck. Those commons for which the deckname is omitted do not require block data, and hence do not require an object deck.

Note that a number of subroutine names appear in more than one link of the DCP, with differing deck names. In each case, a single source deck is compiled with each deck name required, and the binary decks are loaded as indicated.



-- Denotes a labelled common for which no block data, and hence no object deck, is required.

Figure 2-1

LINK STRUCTURE

DIFFERENTIAL CORRECTION PROGRAM

The OVERLAY section of the loader requires that a subroutine or common name appear in only one link of a program. It is necessary, then, to change all references to duplicated names by means of qualified name cards. Figure 2-2 is a list of a complete program deck for the DCP, for use with an IEDIT tape, including all $ORIGIN and $NAME cards. Each of the $NAME cards has a deck or common name in columns 73-78. These names identify the duplicate which requires the $NAME card. For example, if we were to include the card

```
Column      1                16
            $INCLUDE          STPCOM
```

in the main link, all name cards with STPCOM in columns 73-38 would no longer be required.

The link structure shown in Figure 2-1 was designed for the 32K core of the IBM 7094. It may be modified to allow special usage of the program or to take advantage of added core capacity. To simplify the bookkeeping for such a modification, a cross reference for each major link of the DCP is given in Figures 2-3 through 2-6.

In considering the removal of $ORIGIN cards or the reorganization of the program linking, the user should condider the following program operation characteristics.

- Subroutines SETTAP and SETSTA are program initialization subroutines and are called only once per run.

- Subroutine SETCAS is the case definition subroutine and is called once per case.

- Subroutine MLESTT (data start), DIFCOR (propagation and differential correction), and RSIDUL (residual output) each accomplish major data processing functions, which are best separated and performed sequentially.

```
$JOB
$IBJOB
$IBLDR MC13M3                 DCP
$IBLDR MC1378                 DCPCOM
$IBLDR MC13TV                 DATINP
$IBLDR MC13TW                 DATOUP
$IBLDR MC132D                 DDOT
$IBLDR MC132F                 DNORM
$IBLDR MC132H                 DVNORM
$IBLDR MC132J                 DMVTRN
$IBLDR MC132L                 DGTRN
$IBLDR MC132N                 DGTSN
$IBLDR MC13M2                 DCROSS
$IBLDR MC13M0                 DMPLY
$IBLDR MC13SY                 OVRLYD
$ORIGIN        ALPHA
$IBLDR MC138E                 SETTAP
$IBLDR MC13SV                 SETSTA
$IBLDR MC1377                 STNCOM
$IBLDR MC13DW                 DSTAT
$ORIGIN        ALPHA
$IBLDR MC13SW                 SETCAS
$IBLDR MC1376                 EQMCOM
$IBLDR MC13SX                 BLDCOV
$IBLDR MC13CV                 COVOUT
$IBLDR MC13DJ                 DTRANP
$IBLDR MC13DF                 DTRDB
$ORIGIN        ALPHA
$NAME          MC13G5(DATCOM)=DASCOM,MC13M5(DATCOM)=DASCOM               DATCOM
$ETC           MC13T5(DATCOM)=DASCOM,MC13W5(DATCOM)=DASCOM
$ETC           MC1336(DATCOM)=DASCOM,MC1337(DATCOM)=DASCOM
$ETC           MC1338(DATCOM)=DASCOM,MC1339(DATCOM)=DASCOM
$NAME          MC1336(CBDATP)=CBDATS,MC13T5(CBDATP)=CBDATS               MC1336
$NAME          MC1337(GRDATP)=GRDATS,MC13T5(GRDATP)=GRDATS               MC1337
$NAME          MC1338(SBDATP)=SBDATS,MC13T5(SBDATP)=SBDATS               MC1338
$ETC           MC1337(SBDATP)=SBDATS
$NAME          MC1339(DSDATP)=DSDATS,MC13T5(DSDATP)=DSDATS               MC1339
$NAME          MC1326(DFMCOM)=SFMCOM,MC13M5(DFMCOM)=SFMCOM               DFMCOM
$ETC           MC133U(DFMCOM)=SFMCOM,MC13W5(DFMCOM)=SFMCOM
$NAME          MC133U(DPFMRS)=SPFMRS,MC13M5(DPFMRS)=SPFMRS               MC133U
$ETC           MC13W5(DPFMRS)=SPFMRS
$NAME          MC133V(DEHA)=SEHA,MC13M5(DEHA)=SEHA                       MC133V
$ETC           MC13T5(DEHA)=SEHA
$NAME          MC133W(DEQTR)=SEQTR,MC13M5(DEQTR)=SEQTR                   MC133W
$NAME          MC133N(STPCOM)=SSPCOM,MC1330(STPCOM)=SSPCOM               STPCOM
$ETC           MC138F(STPCOM)=SSPCOM
$NAME          MC133N(STEPDI)=STEPSI,MC13M5(STEPDI)=STEPSI               MC133N
$ETC           MC13S5(STEPDI)=STEPSI
$NAME          MC1330(STEPDT)=STEPST,MC13W5(STEPDT)=STEPST               MC1330
$ETC           MC13S5(STEPDT)=STEPST
$NAME          MC138F(STEPDP)=STEPSP,MC13T5(STEPDP)=STEPSP               MC138F
$NAME          MC13ZS(X2ORBD)=X2ORBS,MC13M5(X2ORBD)=X2ORBS               MC13ZS
$NAME          MC13S5(ES1COM)=ES3COM                                     ES1COM
$INCLUDE       DASCOM
$IBLDR MC13A5                 MLESTT
$IBLDR MC13B5                 MLECOM
$IBLDR MC13MH                 DINVRT
$IBLDR MC133N                 STEPDI
$IBLDR MC1330                 STEPDT
$ORIGIN        BETA
$IBLDR MC13D5                 STTDAT
$IBLDR MC13F5                 STTFIT
$IBLDR MC13G5                 STTBGN
```

FIGURE 2-2

PROGRAM DECK
DIFFERENTIAL CORRECTION PROGRAM

```
$ORIGIN       BETA
$IBLDR MC1326               DFMCOM
$IBLDR MC13M5               MXLEST
$IBLDR MC13T5               STTIME
$IBLDR MC13W5               STATE
$IBLDR MC13X5               MEAS2X
$IBLDR MC13Z5               PSEUDO
$IBLDR MC13ZS               X2ORBD
$IBLDR MC138F               STEPDP
$IBLDR MC133U               DPFMRS
$IBLDR MC133V               DEHA
$IBLDR MC133W               DEQTR
$ORIGIN       SIGMA
$IBLDR MC1336               CBDATP
$ORIGIN       SIGMA
$IBLDR MC1337               GRDATP
$IBLDR MC1338               SBDATP
$ORIGIN       SIGMA
$IBLDR MC1339               DSDATP
$ORIGIN       BETA
$IBLDR MC13S5               STTEND
$ORIGIN       ALPHA
$NAME         MC133J(ES1COM)=ES2COM,MC1330(ES1COM)=ES2COM          ES1COM
$FTC          MC138A(ES1COM)=ES2COM,MC138B(ES1COM)=ES2COM
$NAME         MC1330(COVOUT)=COVOUP,MC138B(COVOUT)=COVOUP          MC138B
$NAME         MC138B(DTRANP)=DTRANC,MC138C(DTRANP)=DTRANC          MC138C
$NAME         MC138B(DTRDB)=DTRDBC,MC138D(DTRDB)=DTRDBC            MC138D
$INCLUDE      DATCOM
$IBLDR MC133H               DIFCOR
$IBLDR MC132Y               DEHA
$IBLDR MC132Z               DEQTR
$ORIGIN       GAMMA
$IBLDR MC1344               DFMCOM
$IBLDR MC133F               TRAJDP
$IBLDR MC133D               DEQD
$IBLDR MC133A               ACCTRJ
$IBLDR MC1327               OUTTRJ
$IBLDR MC1342               DPFMRS
$IBLDR MC13SE               STEDPI
$IBLDR MC13SJ               STEPDT
$IBLDR MC13S1               STEPDP
$IBLDR MC1321               GTR2BD
$IBLDR MC1320               ENCKED
$IBLDR MC13GV               GRAVDP
$IBLDR MC132Q               PERTDP
$IBLDR MC132S               SOLRDP
$IBLDR MC132U               DRAGDP
$IBLDR MC132W               VENTDP
$IBLDR MC1320               DLUNE
$IBLDR MC13AK               OUTXPD
$IBLDR MC13ZR               X2ORBD
$ORIGIN       GAMMA
$IBLDR MC133J               ESTMAT
$IBLDR MC1330               ESTOUT
$IBLDR MC138A               UPDATP
$IBLDR MC138B               COVOUT
$IBLDR MC138C               DTRANP
$IBLDR MC138D               DTRDB
$ORIGIN       TAU
$IBLDR MC13CD               CBDATP
$ORIGIN       TAU
$IBLDR MC13DG               GRDATP
$IBLDR MC13BL               SBDATP
```

FIGURE 2-2 - CONTD

PROGRAM DECK
DIFFERENTIAL CORRECTION PROGRAM

```
$ORIGIN        TAU
$IBLDR MC13DN                 DSDATP
$ORIGIN        ALPHA
$NAME          MC13CE(DATCOM)=DARCOM,MC13DC(DATCOM)=DARCOM                DATCOM
$ETC           MC13BN(DATCOM)=DARCOM,MC13DM(DATCOM)=DARCOM
$ETC           MC133K(DATCOM)=DARCOM
$NAME          MC1332(DFMCOM)=RFMCOM,MC1334(DFMCOM)=RFMCOM                DFMCOM
$ETC           MC133G(DFMCOM)=RFMCOM,MC133B(DFMCOM)=RFMCOM
$ETC           MC1328(DFMCOM)=RFMCOM,MC138G(DFMCOM)=RFMCOM
$NAME          MC1332(DPFMRS)=RPFMRS,MC133G(DPFMRS)=RPFMRS                MC1332
$ETC           MC133B(DPFMRS)=RPFMRS,MC1328(DPFMRS)=RPFMRS
$NAME          MC133G(TRJCOM)=TR3COM,MC133B(TRJCOM)=TR3COM                TRJCOM
$ETC           MC1328(TRJCOM)=TR3COM,MC138G(TRJCOM)=TR3COM
$ETC           MC1331(TRJCOM)=TR3COM
$NAME          MC1331(GTR2BD)=GTR2BR,MC133B(GTR2BD)=GTR2BR                MC1331
$ETC           MC1328(GTR2BD)=GTR2BR
$NAME          MC133R(DEHA)=REHA,MC1331(DEHA)=REHA,MC138G(DEHA)=REHA      MC133R
$ETC           MC1328(DEHA)=REHA,MC133K(DEHA)=REHA
$NAME          MC133S(DEQTR)=REQTR,MC1331(DEQTR)=REQTR                    MC133S
$ETC           MC133K(DEQTR)=REQTR
$NAME          MC133T(DLUNE)=RLUNE,MC1331(DLUNE)=RLUNE                    MC133T
$NAME          MC138G(OUTXPD)=OUTXPR,MC1328(OUTXPD)=OUTXPR                MC138G
$NAME          MC138H(X2ORBD)=X2ORBR,MC138G(X2ORBD)=X2ORBR                MC138H
$NAME          MC133L(STPCOM)=STRCOM,MC133M(STPCOM)=STRCOM                STPCOM
$NAME          MC133L(STEPDI)=STEPRI,MC133G(STEPDI)=STEPRI                MC133L
$NAME          MC133M(STEPDT)=STEPRT,MC133G(STEPDT)=STEPRT                MC133M
$ETC           MC133B(STEPDT)=STEPRT,MC1328(STEPDT)=STEPRT
$NAME          MC133P(ENCKED)=ENCKER,MC133B(ENCKED)=ENCKER                MC133P
$INCLUDE       POTCOM,RSLCOM,DARCOM
$IBLDR MC133I                 RSIDUL
$IBLDR MC135B                 RSCOM
$IBLDR MC133R                 DEHA
$IBLDR MC133S                 DEQTR
$ORIGIN        DELTA
$IBLDR MC1334                 DFMCOM
$IBLDR MC133G                 TRAJD
$IBLDR MC133E                 DEQ3
$IBLDR MC133B                 ACCTR3
$IBLDR MC1328                 OUTTR3
$IBLDR MC1332                 DPFMRS
$IBLDR MC133L                 STEPDI
$IBLDR MC133M                 STEPDT
$IBLDR MC1331                 GTR2BD
$IBLDR MC133T                 DLUNE
$IBLDR MC133P                 ENCKED
$IBLDR MC133Q                 GRAVD
$IBLDR MC132P                 PERTD
$IBLDR MC132R                 SOLRD
$IBLDR MC132T                 DRAGD
$IBLDR MC132V                 VENTD
$IBLDR MC138G                 OUTXPD
$IBLDR MC138H                 X2ORBD
$ORIGIN        DELTA
$IBLDR MC133K                 RESOUT
$IBLDR MC13ZT                 RSPLOT
$IBLDR MC13CE                 CBDAT
$IBLDR MC13DC                 GRDAT
$IBLDR MC13BN                 SBDAT
$IBLDR MC13DM                 DSDAT
$DATA
```

FIGURE 2-2 - CONTD

PROGRAM DECK
DIFFERENTIAL CORRECTION PROGRAM

Link	Calling Routine	Referenced Subroutines																										
		Link 0												α1				α2								α3	α4	α5
		DCPCØM	EDTCØM	ESTCØM	DATINP	DATØUP	DDØT	DNØRM	DVNØRM	DMVTRN	DGTSN	DCRØSS	ØVRLYD	SETTAP	SETSTA	STNCØM	DSTAT	SETCAS	EQMCØM	ES1CØM	ESØCØM	BLDCØV	CØVØUT	DTRDB	DTRANP	MLESTT	DIFCØR	RSIDUL
0	DCP	x	x	x	x									x	x			x								x	x	x
	DNØRM						x																					
	DVNØRM						x																					
	DGTRN										x																	
	DGTSN								x																			
α1	SETTAP	x		x																								
	SETSTA	x		x		x										x	x											
α2	SETCAS	x		x	x	x							x						x	x	x	x	x					
	BLDCØV																			x								
	CØVØUT									x										x				x	x			
	DTRDB							x				x																
	DTRANP									x																		

Figure 2-3

CROSS REFERENCE

DCP MAIN AND INITIALIZATION LINKS

Link	Calling Routine	Referenced Subroutines																																						
		Link 0													α3						β1			β2															β3	
		DCPCØM	EDTCØM	ESTCØM	DATØUP	DDØT	DNØRM	DVNØRM	DMVTRN	DGTRN	DGTSN	DCRØSS	DMPLY	ØVRLYD	MLECØM	DATCØM	STPCØM	DINVRT	STEPDI	STEPDT	STTDAT	STTFIT	STTBGN	DFMCØM	MXLEST	STTIME	STATE	MEAS2X	PSEUDØ	X2ØRBD	STEPDP	DPFMRS	DEHA	DEQTR	CBDATP	GRDATP	SBDATP	DSDATP	STTEND	ES1CØM
α3	MLESTT	x													x						x	x	x		x														x	
	STEPDI					x	x										x																							
	STEPDT																x																							
β1	STTDAT	x	x		x										x																									
	STTFIT	x											x		x			x																						
	STTBGN	x	x	x					x					x	x	x																								
β2	MXLEST	x	x	x			x		x	x	x		x		x	x								x		x		x	x	x		x	x	x						
	STTIME								x				x														x				x				x	x	x	x		
	STATE								x	x										x												x								
	MEAS2X								x	x		x																												
	PSEUDØ												x					x																						
	X2ØRBD					x		x				x																												
	STEPDP																x																							
	DPFMRS																							x																
	CBDATP					x	x		x			x				x																								
	GRDATP															x																					x			
	SBDATP					x	x	x	x			x				x																								
	DSDATP					x	x		x			x				x																								
β3	STTEND	x		x	x										x				x	x																				x

Figure 2-4

CROSS REFERENCE

DCP DATA START LINK

Link	Calling Routine	Referenced Subroutines																																																			
		Link 0														α4					γ1																						γ2										
		DCPCØM	EDTCØM	ESTCØM	DATINP	DATØUP	DDØT	DNØRM	DVNØRM	DMVTRN	DGTRN	DGTSN	DCRØSS	DMPLY	ØVRLYD	DCRCØM	DATCØM	SBFCØM	DEHA	DEQTR	DFMCØM	TRAJDP	TRJCØM	DQDCØM	STPCØM	DEQD	ACCTRJ	ØUTTRJ	DPFMRS	STEPDI	STEPDT	STEPDP	GTR2BD	ENCKED	GRAVDP	PERTDP	SØLRDP	DRAGDP	VENTDP	ØUTXPD	X2ØRBD	DLUNE	ESTMAT	ESTØUT	ES1CØM	UPDATP	CØVØUT	DTRANP	DTRDB	CBDATP	GRDATP	SBDATP	DSDATP
α4	DIFCØR	x													x	x		x				x																					x	x		x							
γ1	TRAJDP	x	x	x	x		x	x										x					x	x		x	x	x	x	x	x	x																					
	DEQD																							x			x	x																									
	ACCTRJ	x		x			x		x	x			x	x							x		x	x					x		x	x	x	x	x	x	x	x	x														
	ØUTTRJ	x		x		x		x		x								x	x		x		x	x					x		x	x	x							x													
	STEPDI						x	x																	x																												
	STEPDT																								x																												
	STEPDP																								x																												
	GTR2BD									x	x	x							x	x			x																														
	ENCKED						x																																														
	GRAVDP						x		x	x																																											
	PERTDP						x																																														
	SØLRDP						x		x																																												
	DRAGDP						x			x			x																																								
	VENTDP						x		x																																												
	ØUTXPD	x		x						x									x		x		x																			x												
	X2ØRBD						x		x				x																																								
	DLUNE										x																																										
γ2	ESTMAT	x	x	x			x			x	x	x		x		x	x	x	x	x																									x	x					x	x	x	x
	ESTØUT	x		x		x										x		x																												x		x						
	UPDATP	x		x										x				x																												x								
	CØVØUT									x																																				x			x	x				
	DTRANP									x																																											
	DTRDB							x					x																																								
	CBDATP							x	x		x		x				x																																				
	GRDATP																x																																			x	
	SBDATP							x	x	x	x		x				x																																				
	DSDATP							x	x		x		x				x																																				

Figure 2-5

CROSS REFERENCE

DCP DIFFERENTIAL CORRECTION LINK

Link	Calling Routine	Referenced Subroutines																																														
		Link 0													α5								δ1																						δ2					
		DCPCØM	EDTCØM	ESTCØM	DATØUP	DDØT	DNØRM	DVNØRM	DMVTRN	DGTRN	DGTSN	DCRØSS	DMPLY	ØVRLYD	RSCØM	DATCØM	ESRCØM	PØTCØM	RSLCØM	SB3CØM	DEHA	DEQTR	DFMCØM	TRAJD	TRJCØM	DQ3CØM	STPCØM	DEQ3	ACCTR3	ØUTTR3	DPFMRS	STEPDI	STEPDT	GTR2BD	DLUNE	ENCKED	GRAVD	PERTD	SØLRD	DRAGD	VENTD	ØUTXPD	X2ØRBD	RESØUT	RSPLØT	CBDAT	GRDAT	SBDAT	DSDAT
α5	RSIDUL	x		x										x			x			x				x																					x					
δ1	TRAJD	x	x			x	x										x						x		x	x		x	x	x	x	x																		
	DEQ3																									x			x	x																			
	ACCTR3	x				x											x						x		x	x					x		x	x		x	x	x	x	x	x								
	ØUTTR3	x			x		x		x								x			x	x		x		x	x					x		x	x								x							
	DPFMRS																						x																										
	STEPDI					x	x																				x																						
	STEPDT																										x																						
	GTR2BD								x	x	x										x	x			x										x														
	ENCKED					x																																											
	GRAVD					x		x																																									
	PERTD					x																																											
	SØLRD					x		x																																									
	DRAGD					x						x																																					
	VENTD					x																																											
	ØUTXPD	x		x					x												x		x		x																		x						
	X2ØRBD					x		x				x																																					
δ2	RESØUT	x	x						x	x	x		x		x	x	x	x	x	x	x	x																							x	x	x	x	x
	RSPLØT																	x																															
	CBDAT						x		x							x																																	
	GRDAT															x																																x	
	SBDAT					x	x		x			x				x																																	
	DSDAT					x	x	x	x			x				x																																	

Figure 2-6

CROSS REFERENCE

DCP RESIDUAL OUTPUT LINK

2.1.3 Tape Requirements

The DCP uses a number of tape units for input and output, temporary storage, and restart capability. The units and their assignments are described in Table 2-1.

TABLE 2-1

TAPE UNIT ASSIGNMENTS

Logical Unit No.	Mode	Function
5	BCD	System input tape
6	BCD	System output tape
8	Bin	Planetary ephemeris tape[1]
9	Bin	Temporary storage
10	Bin	Edited data tape
11	Bin	Residual output tape
12	Bin	Estimate storage tape

[1] "User's Description of JPL Ephemeris Tapes," JPL TR 32-580, P.R. Peabody, J.F. Scott, E.G. Orozco, March 2, 1964.

Each of these units must be mounted for any run. Unit 9 is always a blank tape and is never saved. Units 8 and 10 contain required input. A previously written residual tape may be mounted on unit 11, in which case new residual records are added to the existing records. A blank tape may be mounted if stacked residual output is not desired. A previously written estimate tape may be mounted to allow restart from an estimate previously generated by the DCP. If no such estimate is to be used, a blank tape may be mounted on unit 12.

Tape 9, the scratch tape, contains only two records, and is always positioned between the two. The first record contains the data describing the 20 or less stations to be used during the run, and is used to load the SEN array of SETCAS at the start of each case. The second record contains the covariance matrix for the a priori estimate. It is loaded into ES1COM and its renamed images as required.

Tapes 10, 11, 12 are written by the ODP for communication between programs and for restart capability. Each is a binary tape, and each has the following format. The first record is a summary containing an eleven-word alphanumeric header and, in the case of the edited data tape, certain data identifying the tape contents (see paragraph 3.2.2). The remaining records occur in pairs, the first containing identification data and the second containing the data for which the tape was intended.

2.2 INPUT AND OUTPUT

This section describes the formats and procedures for supplying input data and operating controls to the DCP, and the resulting output.

Data is input to the DCP in three forms: BLOCK DATA checks for labelled commons, input tapes, and the data deck. The BLOCK DATA decks contain most of the required standard or nominal data. The input tapes contain the tracking data to be processed, keys identifying the data, and the results of previous DCP runs. The data deck is primarily a sequence of control cards that defines the desired sequence of data processing operations. Certain limited modifications of BLOCK DATA are permitted in the data deck.

2.2.1 BLOCK DATA Input

The contents of each labelled common are described in Reference 1. These inputs are summarized in the paragraphs below. Changes, where appropriate, are discussed in paragraph 2.2.3.

2.2.1.1 DCPCOM. DCPCOM is the main labelled common for the DCP. It is loaded in the main link, and contains those quantities required for more than one link, or those that must be saved regardless of program overlay during a run. Quantities required in the block data include

- constants describing the celestial bodies of interest,
- constants describing the dimensions of principal arrays, and
- options controlling the equations of motion, output, and data start procedures.

Since DCPCOM is loaded in the main link, any changes through data deck input are retained until changed further.

2.2.1.2 STNCOM. STNCOM contains the library of data describing all stations (currently up to 50) which may be considered. The data for each station include

- a six-character alphanumeric name used to identify the station.
- nominal station location (latitude, longitude, altitude), and
- station location and measurement bias errors, standard deviations, and correlation coefficients.

The data for the appropriate stations are loaded into the SEN array of SETCAS and any data deck changes apply to that array. The changes, then, are retained only until the next case (next call of SETCAS).

2.2.1.3 EQMCOM. EQMCOM contains values for the equation of motion error parameters, their standard deviations, and correlation coefficients. EQMCOM is loaded in the SETCAS link, and hence data deck changes are retained only until the next case (next call of SETCAS).

2.2.1.4 RSCOM. RSCOM contains scale limits and other control parameters for residual plotting. It may be changed only by recompilation of the BLOCK DATA.

2.2.2 Input-Output Tapes

2.2.2.1 Edited Data Tapes. The edited data tape is the primary source of problem dependent input data. It contains the tracking data to be processed and data identifying the tracking stations, data types, etc. It is written by either the Tracking Data Editing Program (Section 3) or the Tracking Data Simulator (Section 4).

The first record on the tape is a summary record. The following records are written in pairs - a key record describing the contents of the data record, and a data record containing time tags, quality indicators, and measurement values. Following the last valid record pair is a dummy key record containing an end-of-tape indicator. A data arc, or station pass, consists of one or more record pairs. It contains all the data from a single receiving station from that station's on-time to its off-time, except that dimensioning limitations in the TDEP artificially end an arc at 300 time points and starts a new arc on the next point. A complete data arc is stored on the data tape in sequential record pairs. The data arcs are written on the tape in ascending order of station on-times.

The contents of the tape records are described in paragraph 3.2.2, below. In the DCP, data from the summary record are retained in DCPCOM and ESTCOM. The key and data record pair in use at any given time are stored in EDTCOM.

2.2.2.2 Residual Tape. The residual tape is primarily an output tape. A previously written tape is mounted only when it is desired to stack new residual records on that tape.

The first record on the tape is an eleven-word alphanumeric header. The remaining records are written in pairs: a key record and a data record. The contents of the key record are written from INDRSL and those of the data record from BUFRSL. Both arrays are contained in RSLCOM, and are described in Reference 1.

2.2.2.3 Estimate Tape. The estimate tape is written by the DCP for use in restarting the estimation process. Any estimate previously stored on the tape may be used to replace segments of the a priori estimate and covariance before proceeding with the processing of data. The first record on the tape is an eleven-word alphanumeric header. The remaining records are written in pairs, numberd in the order in which they are written. A given record pair may be referenced only by its number.

The first record of each pair is the array CEST, stored in ESTCOM, and contains the estimate of all error parameters and certain data related to the accumulation of that estimate. The second record is the array DEST, stored in ES1COM, and contains the covariance matrix and indices defining the composition of the state vector and the associated treatment codes. Both arrays are described in the labelled common descriptions of Reference 1.

2.2.3 Data Deck Input

2.2.3.1 Overlay Data. The replacement of data in labelled commons is permitted at a number of places in the data deck. In each case, the data are read by the subroutine OVRLYD, described in Reference 1. All overlay data are double precision floating point data, and are loaded into the double precision array C, as described below.

Each overlay card contains three location/value field pairs, with the format 3(I3, D21.16). Let k_1, x_1, k_2, x_2, k_3, x_3 denote the contents of the six fields. If $k_n > 0$, the value x_n will be stored in $C(k_n)$. If $k_n = 0$ and $x_n \neq 0$, k_n is set to $k_{n-1} + 1$, and x_n is then stored in $C(k_n)$. If $k_n = 0$ and there was not previously entered pair, k_n is set to 1 and x_n is stored in C(1). If both $k_n = 0$ and $x_n = 0$, reading is terminated when $n = 1$, and reading continues with the next card if $n \neq 1$. If $k_n < 0$, x_n is stored in $C(-k_n)$, and reading is terminated. Thus reading may be terminated by a negative value of k in any location field or by a card on which both k_1, x_1 are zero (or blank).

2.2.3.2 Input-Output Sequence. The order in which cards appear in the data deck follows closely, and in some cases defines, the order in which computations are performed by the program. For purposes of discussion we divide the data deck into a "run initialization" deck and N "case" data decks. Each of the case data decks is divided into a "case initialization" deck and M_N "process" decks. Normally, the process decks will be one card each.

The sequence of major operations by the DCP is shown schematically in Figure 2-7, below. Each of the major operations is accomplished by one of the major

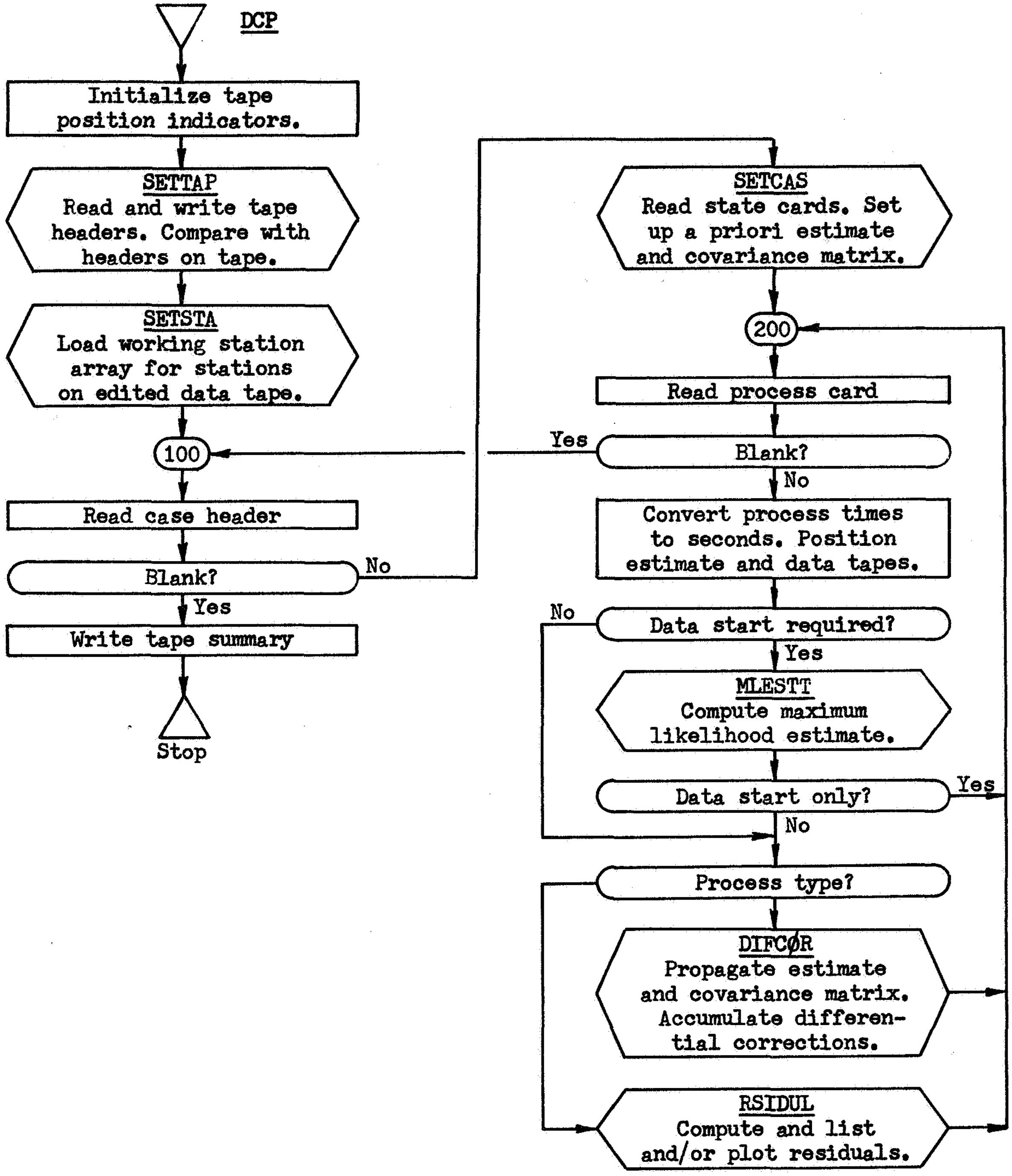


Figure 2-7

FLOW DIAGRAM
DIFFERENTIAL CORRECTION PROGRAM

links as shown in Figure 2-1.

The data deck composition is shown in Figure 2-8. In this figure, we have shown each process deck as a single "process control card." In use, each process card may optionally be followed by a deck of overlay cards, which will be read before the requested process is executed (see, for example, the data deck of Figure 2-9).

2.2.3.3 Run Initialization. The run initialization deck consists of three cards, each with the format (I6, 11A6). These cards identify the edited data tape, the residual tape, and the estimate tape, in that order.

The integer field specifies the number of record pairs previously written on the tape, and the A-field contains the eleven-word alphanumeric header written in the first tape record. If the record count is zero, the contents of the A-field are written on the tape as the header for that tape. If the record count is not zero, the header is read from the tape, and is compared with the A-field contents from the card. If the two are not identical, an error stop occurs. The record count is retained in core to locate the proper tape position for any further writing.

The edited data tape is always an input tape, and contains an end-of-tape record to eliminate the need for a record count. The integer field on the edited data tape card is therefore ignored, and the tape header is compared with the A-field. In addition, the remainder of the summary record is stored in core, and the tracking network is established.

Each station name which occurs on the data tape (now loaded in NAMSTA) is located in STNCOM, and the data describing that station are extracted and loaded into the 20-station working array, SEN. If any station name cannot be located, an error stop occurs.

When the complete set of station names has been located, the array SEN is written as the first record on the scratch tape, unit 09.

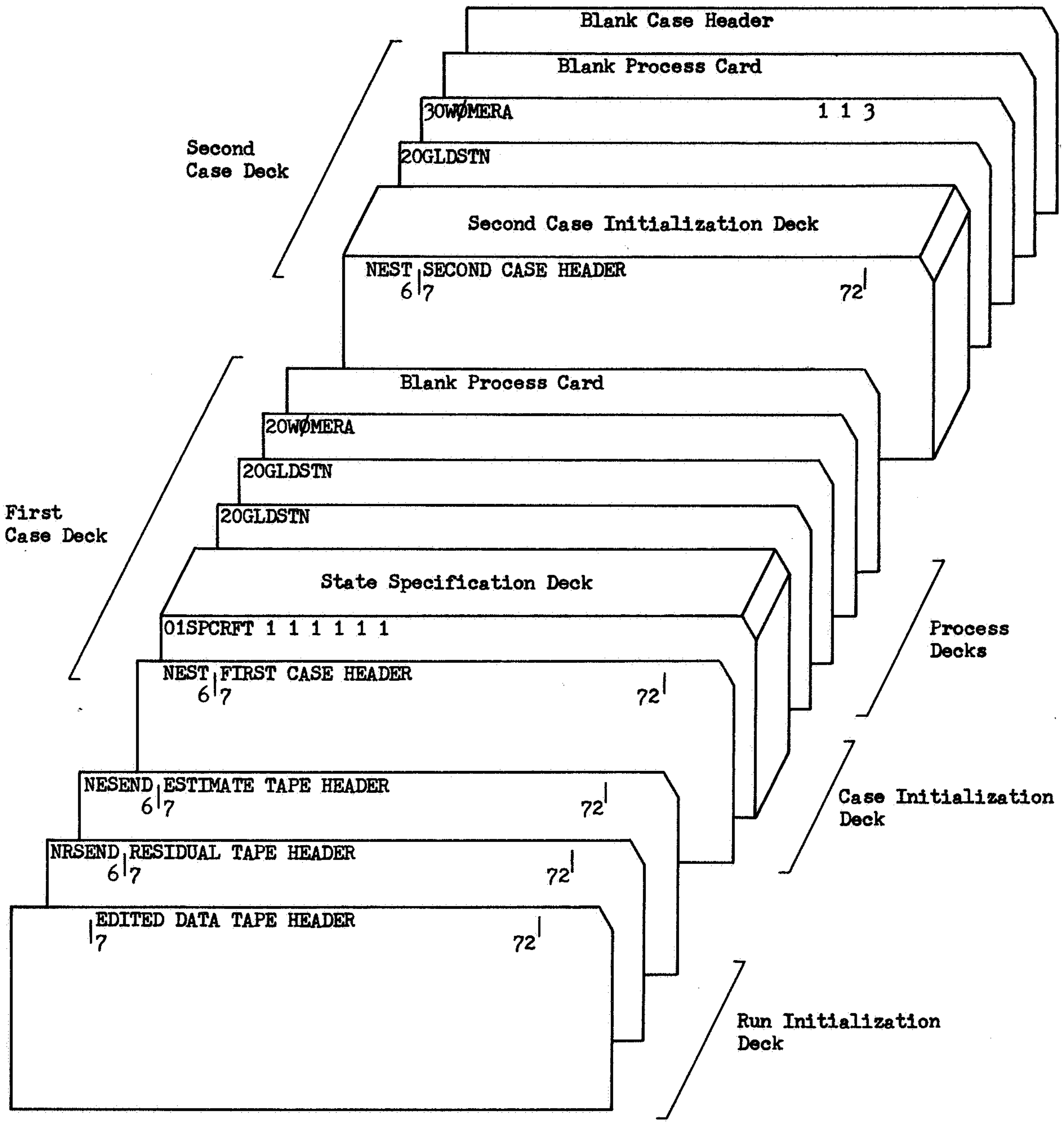


Figure 2-8

DATA DECK SETUP
DIFFERENTIAL CORRECTION PROGRAM

```
      THIS IS THE HEADER ØN THE EDITED DATA TAPE
      THIS IS THE HEADER TØ BE WRITTEN ØN THE RESIDUAL TAPE
    20THIS IS THE HEADER ØN THE ESTIMATE TAPE
    17THIS IS THE FIRST CASE HEADER - USE ESTIMATE RECØRD PAIR 17
01SPCRFT 1 1 1 1 1 1
  1 2094.7124             2 -6219.7374            3 -3.2542983
  4 6.5028253             5 2.1881183             6 3.6959842
 29 6701.                30 101200.             -31 0.
01EFEMRS
 -3 398603.24
00PRESUR 1-1-1-1
10HARM03
01GLDSTN-1-1-1
  9 .0035
          (blank card ends Overlay data for GLDSTN)
          (blank card ends state specification)
2 GLDSTN                (estimate using first GLDSTN pass)
2 GLDSTN                (include next GLDSTN pass)
          (blank card ends this case)
          (blank card ends this run)
```

Figure 2-9

SAMPLE DATA DECK

DIFFERENTIAL CORRECTION PROGRAM

Each tape identification card is written on the system output tape as it is read. The SEN array is summarized on the output tape as it is filled. Finally, the time span covered by the data tape is written.

2.2.3.4 Case Initialization. The case initialization deck contains a case header card, program option cards, and a deck defining the composition of the state and the source of the *a priori* estimate of the state.

Case Header. The case header card is always the first card of the case data deck. It has the format (I6, 11A6). The contents of the I6 field, NEST, specifies the tape-stored estimate, if any, to be used in constructing the *a priori* estimate. It is interpreted as follows:

NEST >0, NEST is the record pair number of the estimate on unit 12 to be used in building the *a priori* estimate and covariance matrix,

NEST <0, no tape-stored estimate is to be used for building the estimate,

NEST =0, no case data follows; the run is to be terminated.

Then if NEST = 0, the number of records on the residual and estimate tapes are written on the system output tape, and the run is ended. That is, a normal program stop occurs when the program finds a blank card (or card with columns 1-6 blank) when it expects a case header card.

If NEST $\neq$ 0, the 11A6 field is written on the output tape as a case identification header.

Program Option Cards. Five program option arrays are included in DCPCØM, and nominal values for their components are compiled in the BLOCK DATA deck for that common. These options may be changed by the data deck using a single card with format (2I1, A6, 24I2) for each array.

The five arrays and their dimensions are

IFPLNT (11)
IFEMPS (8)
IFOUTP (11)
KEYOUT (10)
NSCTRL (14)

Their components are listed in the description of DCPCOM, Reference 1.

The 2I1 fields of the option card are ignored. The A6 must contain the array name for the options to be changed. The numerical values for the n components of the array are contained in the first n I2 fields.

In changing options by input cards, the following rules must be observed.

- All components of a given array must appear on the card for that array.
- Any subset of array names may appear on the input cards, in any order, provided that all option cards appear before the state definition described below is begun.

2.2.3.5 Specification of State Variables

The variables to be included in the state and their mode of treatment are specified by input at the case level. During the process of specifying state variables it is possible to overlay the block data values of any of the possible state variables, their standard deviations, and normalized correlations, even for those variables not to be included in the state. This feature allows a user to alter, for example, the values of equation of motion parameters to be used in the integration, even though these parameters are not to be included in the state.

The possible state variables are arranged in logically related groups. Within each group are 24 or fewer variables in a fixed order. Each state variable group has its unique code name. The code names and the possible variables are as follows:

SPCRFT (Spacecraft cartesian position and velocity)

1	X	Spacecraft position, km
2	Y	
3	Z	
4	$\dot{X}$	Spacecraft velocity, km/sec
5	$\dot{Y}$	
6	$\dot{Z}$	

EFEMRS (Planetary ephemeris data)

1	Mercury	Gravitational constants, km^3/sec^2
2	Venus	
* 3	Earth	
4	Mars	
5	Jupiter	
6	Saturn	
7	Uranus	
8	Neptune	
9	Pluto	
**10	Sun	
11	Moon	

* When the gravitational constant of the earth is altered by differential correction as a state variable, the mean earth radius used for ephemeris scaling is also altered to keep the length of the sidereal month constant at a value determined by initial values of the earth's gravitational constant and the mean earth radius.

** When the gravitational constant of the sun is altered by differential correction as a state variable, the astronomical unit is also altered to keep the length of the tropical year constant at a value determined by initial values of the sun's gravitational constant and the astronomical unit.

PRESUR (Solar pressure, drag, and venting thrust)

1 Solar pressure constant, km^3/sec^2
2 First drag constant, km^{-1}
3 Second drag constant, km^{-1}
4 Venting thrust, km/sec^2

DELTIM (Ephemeris time to universal time correction)

1 ET-UT, sec
2 (ET-UT) rate, sec/sec

*HARMmn (Gravitational harmonic terms)

1 J2, 0
2 J3, 0
3 J4, 0
4 J5, 0
5 J6, 0
6 J7, 0

Zonal harmonics, dimensionless

7	J2, 1
8	λ2, 1
9	J2, 2
10	λ2, 2
11	J3, 1
12	λ3, 1
13	J3, 2
14	λ3, 2
15	J3, 3
16	λ3, 3
17	J4, 1
18	λ4, 1
19	J4, 2
20	λ4, 2
21	J4, 3
22	λ4, 3
23	J4, 4
24	λ4, 4

Longitudinal harmonics

Jm, n	m^{th} order coefficient of n^{th} order longitudinal harmonic, dimensionless
m, n	Associated longitude, radians

* mn refers to a body number as follows:

01	Mercury
02	Venus
03	Earth
04	Mars
05	Jupiter
06	Saturn
07	Uranus
08	Neptune
09	Pluto
10	Sun
11	Moon

Examples: HARM03 means earth harmonics;
HARM11 means moon harmonics.

*(name) (Tracking station biases)

1	North error, km
2	East error, km
3	Down error, km
4	Clock bias, sec
5	Clock bias rate, sec/sec
6	Light speed correction, km/sec
7	Angle 1 bias, radians
8	Angle 2 bias, radians
9	Range bias, km (seconds for Goddard systems)
10	Doppler bias, km

* Any valid station name appearing on the edited or simulated tracking data tape; e.g., GLDSTN, CANBRA.

Data Cards for State Variable Specification.

One card is required for each group from which the user wishes to select state variables or for which he wishes to overlay the block data. The contents of the data cards are as follows:

Columns	Contents
1	KODE
2	IOVRLY
3-8	TYPE
9-10	ITREAT (1)
11-12	ITREAT (2)

Columns	Contents
.	.
.	.
.	.
55-56	ITREAT (24)

Description of Contents of Data Cards

TYPE (columns 3-8)

The state variable group is indicated by punching its code name in columns 3-8 of the state variable card. This must be one of the 15 built-in names described above or the name of a tracking station appearing on the edited or simulated tracking data tape. An illegal code name results in an error message and a program stop. No group name may appear on more than one card.

ITREAT (I), I=1, 24 (columns 9-56)

The treatment of each variable in the group is determined by its corresponding ITREAT value. A group having only k possible variables where k < 24 uses only the first k entries in the ITREAT array. The remainder are ignored.

The treatment code is:

ITREAT = +1: Solve for this variable
= -1: Optimally consider this variable
= 0: Omit this variable from the state

KODE (column 1)

The initial values of the variables in a variable group and that portion of the covariance matrix dealing exclusively with those variables may be obtained either from a previously stored estimate on the stimation tape, or from the block data, which may be overlayed. The source is determined by the value of KODE as follows:

KODE = 0 : obtain values from block data
= 1 : obtain values from tape-stored estimate

IOVRLY (column 2)

Each state variable card may, if desired, be followed by one or more overlay cards that allow the block data values of the variables, their standard deviations, and their normalized correlations to be altered. If IOVRLY = 0, no overlay cards may follow the state variable card; if IOVRLY = 1, at least one overlay card must follow the state variable card. Only one array may be altered by the overlay data that may follow one single state variable card. The correspondence between the arrays that may be overlayed and code names appearing in the related state variable cards is shown below. For exact descriptions of the contents of these arrays, see Reference 2.

Group Code Name	Overlayable Array
SPCRFT	SPCDAT
EFEMRS	EFEDAT
PRESUR	PREDAT
DELTIM	DELDAT
HARMO3	EHADAT
HARM11	MHADAT
*HARM mn	XHADAT
**(Station name)	SEN (1, KSTA)

*It is assumed that mn is a valid body number other than 03 and 11.

**Each station appearing on the edited or simulated tracking data tape has a column of the SEN array assigned to it by the program. For the purposes of overlay, the appropriate column is automatically referenced and is considered as a singly-dimensioned, double-precision array.

Terminating the Specification of State Variables. The last state variable card must have the code name (columns 3-8) left blank. This signals the program that no more state variables are to be specified. No overlay cards may be associated with this card.

Assumptions, Restrictions, and other Observations:

1. Overlay without including State Variables

   It is often desirable to overlay the block data values in an array associated with a state variable group but not to include any of the variables from that group in the state. This may be accomplished by including a state variable card with the appropriate code name in columns 3-8, a value of 1 for IOVRLY, and zeros or blanks for all the ITREAT values. One limitation must be observed. In no case may more than one state variable card with a code name of the form HARM mn be included. Thus it is not possible to include gravitational harmonics for more than one body in the state. It is also impossible, for example, to overlay the block data values of gravitational harmonics for one body without including any of these variables in the state, and then request inclusion in the state of gravitational harmonics for some other body. Failure to observe this restriction will result in an error message and a program stop.

2. Intergroup Correlations

   When the covariance matrix is constructed at the beginning of a case, all correlations between variables in different groups are set to zeros. The only possible non-zero correlations at case initiation are between variables within the same group. (For further discussion of covariance matrix construction the reader is referred to the description of subroutine BLDCOV.) During data processing the correlations between variables in different groups will develop in the proper fashion. They will be written properly on the estimate tape if one is written. They may not, however, be recovered for use in initializing a new case.

3. Starting Values from Previous Estimates

When a state variable card appears with KODE = 1, the treatment codes on that card are ignored. The block data, even if overlayed, are also ignored. Starting values for all the variables in the group, their treatment codes, and the relevant portion of the covariance matrix are all lifted intact from the tape stored estimate.

4. Group SPCRFT

If a state variable card for this group appears and all the treatment codes are zeros or blanks, no spacecraft variables will appear in the state. If at least one treatment code is non-zero, all the remaining treatment codes set zero by the user will be reset to -1 by the program. Thus the SPCRFT group consists of no state variables or all six state variables.

5. Ordering the State Variable Cards

State variable cards, followed by their related overlay cards, if any, may appear in any order the user may desire. They are automatically reordered by the program in the same order in which they are described in this section. Those relating to tracking stations are ordered according to the ordering of the station names in the header/ summary record of the edited or simulated tracking data tape.

Example Data for State Variable Specification. Figure 2-9 shows an example of variable specification requesting the following actions:

1. Solve for all six spacecraft variables. Build the a priori estimate and the relevant portion of the covariance matrix from block data after overlaying the following values in block data

X = 2094.7124
Y = -6219.7374 km
Z = -3.2542983

$\dot{X}$ = 6.5028253
$\dot{Y}$ = 2.1881183 km/sec
$\dot{Z}$ = 3.6959842

Must be earth centered, equator and equinox of 1950.

YM = 6701.
DHM = 101200. Time of estimate, ephemeris time
SEC = 0.

2. Include no variables from the EFEMRS group, but change the earth's gravitational constant to 398603.24 km^3/sec^2.

3. Include all four variables fromthe PRESUR group. Solve for solar pressure; optimally consider the remaining three. Use block data values without alteration.

4. Find the earth's gravitational harmonic terms on the tape-stored estimate. Use the values of the variables, their treatment codes, and the relevant portion of the covariance matrix exactly as they appear on that tape-stored estimate.

5. Optimally consider the location errors for tracking station GLDSTN. Obtain all values from block data after assigning this station a range bias of .0035 km

2.2.3.6 Process Control. Following the specification of a state vector and the construction of an *a priori* estimate, a sequence of process control cards is used to define the sequence of desired data processing operations. These cards have the format (2I1, A6, 2(I3, I7, I2), I4, 18I2). Each process card causes one or more major data processing operations.

The process control card is read into the NPROC array of DCPCOM. The card columns and NPROC components are described in Reference 1, and in the discussion below.

Data Processing Operations. The processing operations of the DCP are divided into four major functions:

1. Propagate the estimate and covariance matrix from the anchor point to a specified time or event.
2. Estimate differential corrections based on data from a specified station on a specified time interval.
3. Residual computation and output for a specified state estimate, from the data from a specified station on a specified time interval.
4. Start using the data from a specified station on a specified time interval and replace the *a priori* estimate of state by the maximum likelihood estimate.

The specific functions of the process control card fields are described below.

NPROCS (column 1)

The process type is specified by the first integer field. The permitted values are:

0: no process - start reading the next case data deck.
1: propagate estimate in ESTCOM to specified end-point.
2: compute differential corrections and correct estimate in ESTCOM.
3: compute residuals from estimate in ESTCOM.
4: compute estimate from the data and replace the estimate in ESTCOM.
5: data start followed by propagation.
6: data start followed by differental correction.
7: data start followed by residual computation.

NPROVR (column 2)

Any process card for which NPROVR $\neq 0$ must be followed by at least one overlay data card. The overlay data are read into DCPCOM if NPROCS $\neq 3$, and into ESRCOM if NPROC = 3 (see References 1 and 2). Since not all data in DCPCOM is double precision, overlay should be limited to the variables summarized in Reference 2.

NPRSTA (columns 3-8)

NPRSTA is the six-character name of the receiving station whose data is to be processed. It must agree exactly with one of the station names on the data tape.

NDATE (columns 9-32)

The process start and stop times are specified in the NDATE field. The interpretation of that field is:

Columns for Start Time	Stop Time	Field Contents
9-11	21-23	3-digit year designation, years after 1900
12-14	24-26	3-digit day number (Jan 1 = 1)
15-16	27-38	hours after midnight
17-18	29-30	minutes
19-20	31-32	seconds

For example, the calendar date and time

December 15, 1967 6:30:15 PM

is punched in columns 10-20 or 22-32 as

67349183015

In a request for propagation to a given time (see NPSTOP, below) the stop time must appear on the card. For all other processes, either time may be omitted, with the times supplied by the DCP as follows.

The array STIMNX (DCPCOM) contains, for each station, a time slightly later than the time tag of the last data from that station previously used in differential correction. If the start time is omitted, the first data to be processed is the first data, for the specified station, having a time tag greater than the appropriate component of STIMNX. If the stop time is omitted, data processing is continued until the end of a data arc is reached. Thus, sequential processing, a data arc at a time, may be accomplished without input of times.

NPREST (columns 33-36)

For any process, the estimate of state may be replaced from the estimate tape before execution of the process. This is accomplished by the placing of the desired estimate record number in NPREST.

Some care is required to ensure that the estimate from tape was established using the same state composition as that defined for the case in progress. Accidental redefinition of state can occur otherwise.

NREWND (columns 37-38)

This field contains a yes/no option for rewinding the data tape before the search for the process data is begun. Searching forward on a tape is significantly faster (especially in FORTRAN) than in searching backward. The rewind option should be "yes" if backward searching through a major part of the data tape (say 1/4 to 1/3 of the distance to the beginning of tape) is expected. "Yes" is specified by any non-zero integer in the field.

NPRSTO (columns 39-40)

This is a yes/no option for storage of the final estimate (at completion of the process) on the estimate tape.

NPSKIP (columns 41-42)

A non-zero integer in this field will result in the skipping of NPSKIP data points (complete sets of measurement values with a single time tag) between processed points.

IFSKIP (columns 43-50)

This field contains four yes/no options for the suppression of all measurements of a given type. That is, a non-zero integer in the I th of the four I2 fields will cause omission of the I th measurement type from all data processing. The measurement types and the associated number (I) are

I = 1, angle 1
I = 2, angle 2
I = 3, range
I = 4, doppler

IFBADD (columns 51-52)

This field contains a yes/no option for the processing of data with "bad" quality flags set by the tracking system (see Section 3). A non-zero integer causes processing of all flagged "bad" data.

IFOUTL (columns 53-54)

This field contains a yes/no option for the processing of data flagged by the TDEP as outliers.

NRTAPE (columns 55-56)

This is a yes/no option for the writing of residual records on the residual tape.

NRPLOT (columns 57-58)

This is a yes/no option for the plotting of residuals on the system output tape.

NRLIST (columns 59-60)

This field contains an option controlling the completeness of residual tabulations on the system output tape. The levels are described in RESOUT, Reference 1.

For the propagation process, two fields are interpreted differently. These are:

NPSTOP (columns 37-38)

This is an integer from 0-3 defining the type of event at which propagation is to be ended.

NPSTOP = o stop at specified stop time,
1 patch to or from body NTARGT
2 radius of closest approach to body NTARGT
3 fixed radius from body NTARGT

NTARGT (columns 39-40)

This field contains the target body number. The body numbers are summarized in paragraph 2. 2. 3. 5, above.

2.2.4 Error Stops

The DCP writes a number of error messages for conditions which result from disallowed or ambiguous input and for recognizable failures to compute meaningful state estimates. The principal error messages are written in the form

```
*** ERROR STOP
ERROR MESSAGE
```

The error conditions, their probable source, and corrective action are described below.

IERR Message, Cause, and Corrective Action

1 TAPE HEADER DOES NOT AGREE.

The alphanumeric header on the tape identification card last written on the system output tape does not agree with the header on the corresponding input tape. Make card columns 7-72 identical with the card used to write the tape.

2 STATION CANNOT BE FOUND IN BLOCK DATA.

The first station name appearing in the NAMSTA array of the edited data tape, but not written on the system output tape, is not contained in the STNCOM block data. Add the missing station name and data to STNCOM and recompile.

3 STATE REQUESTED FROM TAPE WITH NEGATIVE ESTIMATE NUMBER.

The case header card specified no estimate number (NEST<0), but a state specification card asks for the estimate from tape (KODE>0). Either specify a positive NEST on the case header card or remove all non-zero punches from column 1 on the state specification cards.

4 STATE TYPE NAMED DOES NOT EXIST.

The last state specification card written on the system output tape is not a defined state group name. Correct the state group name.

5 STATE TYPE DUPLICATED

A state group name has appeared on more than one state specification card. Remove duplications and combine overlay decks as required.

6 INCLUSION OF HARMONICS FOR TWO BODIES IS NOT PERMITTED.

The state group name HARMmn has appeared more than once. Include only one body's harmonics in the state. If it is desired to change harmonic coefficients for other bodies, recompile the block data for EQMCOM.

7 STATE REQUESTED FROM TAPE ESTIMATE DOES NOT EXIST THEREIN.

A state group name (last written) with KODE>0 was not included in the state when the estimate NEST was stored on tape. Either correct NEST or set KODE=0 on the offending state specification card.

8 STATION PARAMETERS REQUESTED FROM TAPE, STATION NOT IN ESTIMATE.

The remarks for IERR=7 are appropriate, for the last written station name.

9 TOO MANY STATE VARIABLES.

The number of state elements appearing on the state specification cards (as non-zero ITREAT values) exceeds the maximum number permitted by program dimensioning. Eliminate enough state elements to satisfy dimensioning limits. Note the implicit state definitions resulting from the use of tape estimates (paragraphs 2. 2. 3. 5).

10 TOO MANY EQUATION OF MOTION PARAMETERS.

The number of equation of motion parameters exceeds the maximum permitted by program dimensioning. See IERR=9, above.

11 CENTRAL BODY REQUESTED FROM TAPE ESTIMATE DOES NOT AGREE WITH TAPE.

The state group HARMmn was requested from tape with mn not the same as that in the tape estimate state. Correct mn or set KODE=0.

12 OBSERVING STATION NOT ON DATA TAPE.

The last process card written on the system output tape does not occur on the data tape. Check the station name on the process card against the name in the run initialization output.

13 PROCESS INTERVAL OUTSIDE DATA TAPE RANGE.

The last read process card specifies a stop time preceding the tape start time or a start time after the tape stop time. Correct the process card time fields.

14 PROCESS INTERVAL CONTAINS NO DATA FOR REQUESTED STATION.

The station name on the last read process card occurs on the data tape only as a transmitting station. It may not be used to identify data to be processed.

15 DATA START FAILED TO CONVERGE.

The maximum likelihood estimator failed to obtain a valid starting estimate from the data. The cause of failure may be identified from the messages written immediately before the error stop massage.

16 TIME PRECEDES EPHEMERIS TAPE RANGE.

The data tape or integration results have generated a time outside the time range of the planetary ephemeris tape. Make sure the correct ephemeris tape has been mounted; if correct, trace error in the computation of internal times.

17 TIME FOLLOWS EPHEMERIS TAPE RANGE.

See IERR=16, above.

18 ESTIMATE FAILED TO CONVERGE.
The differential correction failed to satisfy convergence tests during iteration. Indications of the cause of failure must be deduced from the output during the d. c. process.

SECTION 3

TRACKING DATA EDITING PROGRAM

3.1 GENERAL DESCRIPTION OF THE PROGRAM

The Differential Correction Program (DCP) is able to process tracking data of a variety of types. Each of the types of data is coded and formatted in a different form. The Tracking Data Editing Program (TDEP) provides a convenient means for decoding the data in the various formats, converting the data to a single fundamental set of units, and rewriting the data in a single format, thus simplifying the input procedures for the DCP.

In the decoding process, the TDEP eliminates frames of data that have been garbled in transmission, with no attempt to reconstruct the data. All observable values are made positive by adding 2π to the angles as necessary. The TDEP tests strings of successive values of the observables for smoothness, and flags obvious outliers by making the value negative without changing the magnitude.

3.1.1 Capabilities of the TDEP

The (TDEP) is capable of processing the following types of tracking data:

- C-Band
    - Azimuth
    - Elevation
    - Range

- Goddard Range and Range Rate
    - X-Angle
    - Y-Angle
    - Range
    - Doppler

- Unified S-Band
    - X-Angle
    - Y-Angle
    - Range
    - Doppler

- DSIF (JPL)
    - Hour Angle
    - Declination
    - Doppler

The TDEP assumes that any one input raw data tape contains data of one and only one of these above types. Further, it is assumed that the data are sorted into proper time order and that station passes appear on the tapes in ascending order of pass start times. The TDEP is not a sorting program.

During development of the program, information defining the data format and units was not available for the DSIF data. The processing subroutine, DSTEST, is therefore a dummy which should be reprogrammed when format descriptions are available. The decoding techniques used for the other data types may be used as a guide for this reprogramming.

The program may easily be expanded to include other data types. It is necessary to provide the appropriate decoding subroutine, XXTEST, which should follow the techniques used in the existing subroutines, and the identification logic required by the main program.

3.1.2 Processing Operations

The operations performed by the TDEP fall into three broad categories, each of which is described below. The sequence of computations is shown in the flow diagram, Figure 3-1.

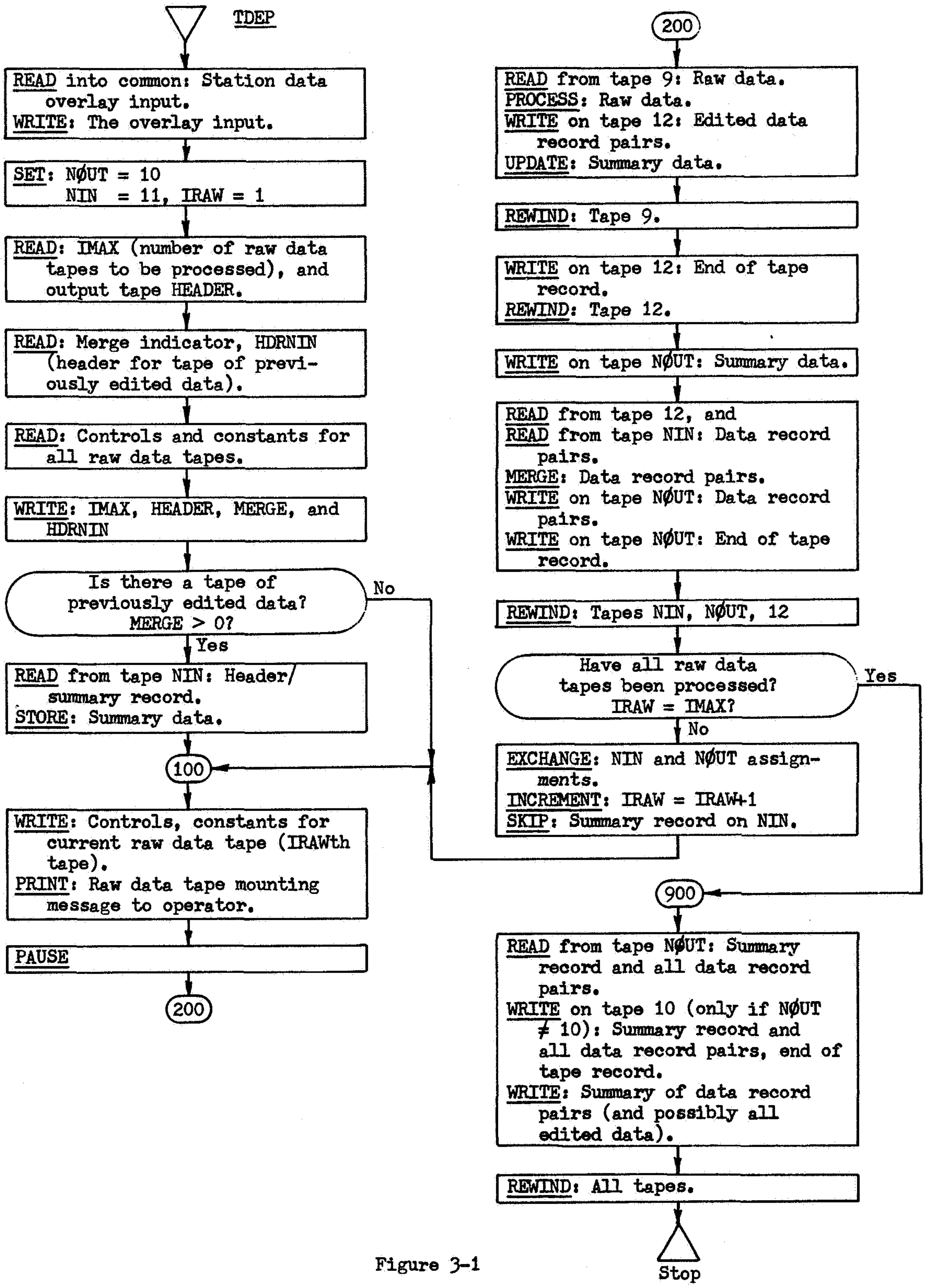


Figure 3-1

FLOW DIAGRAM

TRACKING DATA EDITING PROGRAM

3.1.2.1 Decoding and Unit Conversion. Each frame[1] is first examined for illegal characters, which are replaced with zeros. This function is performed by the standard IBSYS system library routine FCNV, not by any part of the TDEP. Next, the number in the station ID field is compared with a user-supplied list of acceptable codes. If no agreement can be found, the frame is assumed to be garbled; it will be ignored and the next frame will be read in. Next, the values in the date fields are decoded and compared with a valid date range defined by two user-supplied test dates, TLO and THI. Failure of the decoded date to lie in the test range indicates garbled data and the frame is ignored.

An acceptable frame is decoded and stored with unit conversions as indicated in Table 3-1, below, until an entire station pass is stored. The end of the station pass is indicated by a change in the station ID code or by a change in the keys describing the data. An artificial end of pass is forced when the storage arrays for decoded data are filled.

During the decoding and unit conversion process the TDEP will, if desired, ignore all the values on the raw data tape for any of the four observables. Under this option, the storage locations for the suppressed observables are filled with the code number -.12345678E20. The DCP recognizes this number as indicative of a missing or supressed observable and does not attempt to use this number as valid data.

3.1.2.2 Detecting Outliers. When an entire data arc (station pass) is decoded and loaded into temporary storage, the TDEP tests for, and flags, outliers in each of the four observables.[2] The test for outliers is based on the assumption that the trend for an observable over short intervals can be reasonably well approximated with a polynomial of low degree. The four observables are tested sequentially rather than simultaneously for reasons dictated mainly by a desire to keep the program logic fairly simple. The testing begins, for any of the observables, with

---

[1] One frame is one physical BCD record for all types except the Goddard Range and Range Rate system, for which one frame is two physical records.

[2] Testing is not performed on missing or suppressed observables.

TABLE 3-1

UNITS FOR STORAGE OF DECODED DATA

	Time	Angle 1	Angle 2	Range	Doppler
C-Band	Double-precision seconds from Jan. 0, 1950	Radians	Radians	Kilometers, (one-way)	*
GRR	Double-precision seconds from Jan. 0, 1950	Radians	Radians	Seconds (two-way transit time)	Seconds (time interval for a fixed number of counts)
USBS	Double-precision seconds from Jan. 0, 1950	Radians	Radians	Kilometers, (two-way)	Counts or Seconds
DSIF	Double-precision seconds from Jan. 0, 1950	Radians	Radians	*	Counts

*Indicates observable not present in raw data. Storage position filled with the code value -.12345678E+20.

the selection of the first k values of the observable and its time tag in the data arc. The number k is a variable controlled by user input. A polynomial of degree n is fitted to the data, using time as the independent variable, by the method of least squares. The degree of the polynomial is determined by the user via input data and may be between 1 and 5 inclusive. Residuals are computed and the sample standard deviation (from the fitted polynomial) determined. The k values of the observable are tested individually for deviation from the fitted polynomial; those lying more than r standard deviations away are tagged as outliers. The value r is set by the user, and a different value of r may be set for each of the four observables. The tagging consists merely of making the sign of the observable value negative without altering the magnitude. This is possible because range and doppler are necessarily positive, and the angle values may be initially made positive by adding 2π as necessary. After the first k values are thus tested and flagged, if necessary, the polynomial is walked by m points, where m is controlled by user input. By this is meant that the first m points are deleted from the sums used in fitting the polynomial and m new points are added to these sums. Thus the polynomial fit is still made over k points, but the fitting range has been moved forward in time by m points. New polynomial coefficients, residuals, and sample standard deviation are computed, and the k points now included in the fit are tested for outliers. It should be clear that one point may be tested several times during the fitting-walking process. To avoid being tagged as an outlier, a point must successfully pass each test to which it is subjected.

3.1.2.3 Outputting and Merging. When an entire data arc (station pass) has been decoded, unit converted, and tested for outliers, it is written in the standard TDEP output format on a temporary storage tape (unit 12). This continues until the program has finished processing the input raw data tape. At this point the merging function of the program is called into play. The edited data on unit 12 and the edited data that may exist on unit NIN are all written on unit NOUT. Station passes from these two tapes are interspersed so that they appear on unit NOUT arranged in order of station pass start times. The edited data on unit NIN are not necessarily present at the beginning of a run. If a tape of edited data is mounted on unit NIN, it is a tape of edited data resulting from a previous run of the TDEP. This feature allows a user to merge edited data from a number of raw data tapes onto one output tape and yet process only one raw data tape per run if short runs are desired. In fact, this one-at-a-time operation is necessary if the computer system being used does not permit a pause and restart operation. The variable tape assignments NIN and NOUT are used because the TDEP first assumes that NIN = 11, NOUT = 10, and then alternates those assignments with each raw data tape processed. Thus, if two or more raw data tapes are processed in one run, tapes 10 and 11 will both be written on. Raw data from a previous run should always be mounted on unit 11; the final output will always appear on unit 10. When the last raw data tape has been processed, the program insures that the final edited output is on unit 10 and simultaneously writes out a description of the contents of that tape on the system output tape.

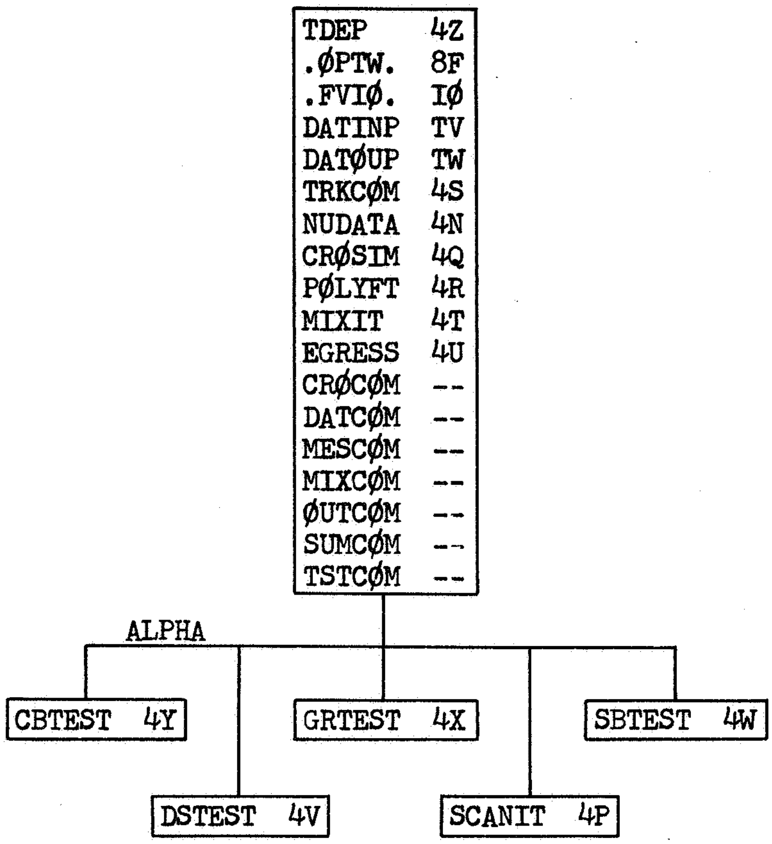
3.1.3 Program Structure

Figure 3-2 lists all the subroutines and labelled commons used by the TDEP. Each deck for which a deckname is given in the figure must be present in the object deck. Those commons for which the deckname is omitted do not require a block data, and hence do not require an object deck.

The link structure shown is optional, since the entire program may be loaded on the IBM 7094 as a single link. In the form shown, the program may be expanded to include processing of data types other than the four provided, without serious concern for core storage requirements. Clearly, any new XXTEST subroutines would be loaded as new links at origin ALPHA.

With the link structure as shown, significant expansion of the edited data buffers (DATCOM) is possible. The restriction currently imposed that data arcs are artificially ended at 300 points is the result of the dimension of the data buffers, and hence an increase in buffer size results in a like increase in the maximum number of points per arc. The changes required for an increase in buffer size are described in detail in the description of DATCOM, Reference 1.

Note that overlay is required only n + 1 times, where n is the number of raw data tapes to be processed. One decoding subroutine, XXTEST, is used for the



-- Denotes a labelled common for which no block data, and hence no object deck, is required.

Figure 3-2
LINK STRUCTURE
TRACKING DATA EDITING PROGRAM

	Labelled Commons								Referenced Subroutines											
	CROCOM	DATCOM	MESCOM	MIXCOM	OUTCOM	SUMCOM	TRKCOM	TSTCOM	CBTEST	CROSIM	DATINP	DATOUP	DSTEST	EGRESS	GRTEST	MIXIT	NUDATA	POLYFT	SBTEST	SCANIT
TDEP				x	x	x		x	x		x	x	x	x	x	x	x		x	x
CBTEST		x			x	x	x	x			x							x		
CROSIM	x																			
DSTEST		x			x	x	x	x			x							x		
EGRESS			x	x	x	x						x								
GRTEST		x			x	x	x	x			x							x		
MIXIT			x	x	x															
NUDATA							x													
POLYFT	x	x	x		x			x		x										
SBTEST		x			x	x	x	x			x							x		
SCANIT			x		x	x														

Figure 3-3

CROSS REFERENCE

TRACKING DATA EDITING PROGRAM

processing of each tape, and the subroutine SCANIT is required for output when all tapes have been processed.

Figure 3-3 summarizes all references to labelled commons and subroutines.

3.1.4 Tape Requirements

The tape units required by the TDEP are listed in Table 3-2. Note that only one unit is used for raw data tape input. If more than one tape is to be processed, they are mounted one at a time on the same unit. The input controls are described in Paragraph 3.2, below.

TABLE 3-2

TAPE UNIT ASSIGNMENTS

TRACKING DATA EDITING PROGRAM

Logical Unit No.	Mode	Function
5	BCD	System input tape
6	BCD	System output tape
9	BCD	Raw data tape input
10	Bin	Edited data tape output
11	Bin	Edited data tape input and temporary edited data storage
12	Bin	Temporary storage of edited data

Units 10 and 11 are referenced through FVIO, and entries must be available for those units. It may be necessary to replace the system FVIO to provide these entries. Each of the units listed in Table 3-2 must be mounted. This includes unit 11, even if no previously edited data is to be used.

3.2 INPUT AND OUTPUT

This section describes the formats and procedures for supplying input data to the TDEP and the resulting output. Paragraphs 3.2.1 and 3.2.2 describe formats for input and output tapes, respectively. Paragraph 3.2.3 describes the data deck composition and formats.

3.2.1 Input Raw Data Tape Formats

All input raw data tapes are assumed to be binary coded decimal, compatible with the tape drives and input-output system being used.

3.2.1.1 C-Band Data. Each physical record on the C-Band tapes is a complete data frame, consisting of data recorded at one point in time. The following format is assumed:

Columns or Characters	Contents
1-2	Station ID (Decimal)
3-4	*
5	Data ID (Octal Digit)
6-8	*
9-11	Time - Day of Year (Jan 1 = 1)
12	*
13-14	- Hours
15-16	- Minutes
17	*
18-19	- Seconds
20	· (Decimal Point)
21-23	- Fractional Seconds
24-25	*
26-28	Azimuth - Decimal Degrees
29	· (Decimal Point)
30-32	- Fractional Degrees

Columns or Characters	Contents
33-34	*
35-37	Elevation - Decimal Degrees
38	• (Decimal Point)
39-41	- Fractional Degrees
42-43	*
44-55	Range - Decimal Kilometers IIIIIIFFFFFF

where:

1. * A blank or unused column

2. Column 5 (Octal Digit)

   ≠ 0: Good Data

   = 0: Bad Data, out of lock.

3. Columns 26-32

   Azimuth is in decimal degrees. The field contains 3 digits for integer, a decimal point, and 3 digits for fraction. Range is 0-360 degrees.

4. Columns 35-41

   The elevation field is the same as for azimuth except the range is 0-180 degrees.

5. Columns 44-55

   Range is in decimal kilometers. The field contains 6 digits for integer and 6 for fraction. There is no decimal point.

3.2.1.2 Goddard Range and Range Rate Data. One data frame requires two physical records, consisting of X-angle, Y-angle, range, and doppler recorded at one time point, followed by three equally-spaced time points of range and doppler. There are two types of trackers in use: the regular systems at Rosman, Madagascar, and Carnarvon, and the hybrid systems at Santiago and Alaska. The form of the format is the same for either system. The type of system is indicated by the station ID code. The general format is:

Record 1

Columns or Characters	Contents	
1-5	X-angle, Decimal Degrees, ± IIFF.	
6	QR1 - Quality indicator for following range data. See below.	
7-14	R1, Range time interval, Decimal Microseconds, IIIIIIFF.	
15	QRD1, Quality indicator for following doppler data or extra doppler digit. See below.	
16-22	RD1, Doppler Count Interval, Decimal Microseconds, IIIIIIF.	
23	(Ignored by TDEP)	
24-26	Day or Year (Jan. 1 = 1)	
27-28	Time - Hours	At beginning of data frame.
29-30	- Minutes	
31-32	- Seconds	
33	QR2, Quality indicator for following range data. See below.	
34-41	R2, Range time interval, Decimal Microseconds, IIIIIIFF.	
42	QRD2, Quality indicator for following doppler data or extra doppler digit. See below.	
43-49	RD2, Doppler Count Interval, Decimal Microseconds, IIIIIIF.	

Record 2

Columns or Characters	Contents
1-5	Y-Angle, Decimal Degrees, ± IIFF.
6	QR3, Quality Indicator for following range data. See below.
7-14	R3, Range time interval, Decimal Microseconds, IIIIIIFF.
15	QRD3, Quality indicator for following doppler data or extra doppler digit. See below.
16-22	RD3, Doppler Count Interval, Decimal Microseconds, IIIIIIF.
23	(Ignored by TDEP).
24-26	Three digit satellite number, decimal.
27-28	Station ID, decimal.
29	IC1 Data keys. See below.
30	IC2
31	IC3
32	IC4
33	QR4, Quality Indicator for following range data. See below.
34-41	R4, Range time interval, Decimal Microseconds, IIIIIIFF.
42	QRD 4, Quality indicator for following doppler data or extra doppler digit. See below.
43-49	RD4, Doppler count interval, Decimal Microseconds, IIIIIIF.

where, for the regular systems,

1. Record 1, Columns 6, 15, 33, 42,
   Record 2, Columns 6, 15, 33, 42

The quality indicators apply to the data immediately following each indicator. A blank indicates good data. Any other character indicates faulty data.

2. Record 2, Column 29, IC1

The true range time interval is the value in the data field plus an unspecified integral multiple of a gating ambiguity. The value of the ambiguity is indicated by IC1 as follows:

IC1	Ambiguity
1, 2, 3	.125 seconds
4, 5, 6	.03125 seconds
7, 8, 9	.00625 seconds

3. Record 2, Column 30, IC2

The time tag attached to each of the four samples of range and doppler as well as the number of doppler cycles counted are indicated by IC2. Let T_{DR} denote the time increment between each sample. Then T_{DR} and the number of doppler cycles counted are indicated by IC2 as follows:

IC2	T_{DR}	S-Band Counts	VHF Counts
0, 4	1.0 second	229,263	14,328
1, 5	0.5 second	131,007	8,187
2, 6	0.25 second	65,503	4,093
3, 7	0.125 second	32,751	2,046

Let T_F be the time indicated in the data frame. Then the time for each sample is

$$T_S = T_F + K\,T_{DR} + \begin{bmatrix} 0 \text{ if IC2} \neq 7 \\ .125 \text{ sec if IC2} = 7 \end{bmatrix}$$

where K = 0, 1, 2, 3 denotes the first, second, third, and fourth samples in the frame.

4. Record 2, Column 31, IC3

The uplink carrier frequency and the doppler bias frequency are indicated by IC3 as follows:

IC3	Carrier, Mc/sec	Bias Mc/sec
0	2271.9328	.500
1	2270.9328	.500
2	2270.1328	.500
3	148.26	.030

5. Record 2, Column 32, IC4

The value of IC4 indicates the range counter frequency which determines the granularity of the range data. It has no effect on the units in the range fields and is ignored by the TDEP.

For the hybrid systems:

1. Record 1, Columns 6 and 33, QRI and QR2
Record 2, Columns 6 and 33, QR3 and QR4

For the hybrid systems, these indicators apply to all data and not just range data, as follows:

QRn	Failure	Data Affected
Blank	None	None
*	Any two or more of the following failures	All
0	Receiver carrier loop	All
1	Receiver subcarrier	Range, Doppler
2	Antenna not autotrack	All
3	Digital range tone extractor not locked	Range

2. Record 1, Columns 15 and 42, QRD1 and QRD2
Record 2, Columns 15 and 42, QRD3, and QRD4

These indicators are used only when IC2 is a 3 or a 7. In this case, these are extra doppler digits and are decimal seconds to be added to the microseconds immediately following each. This allows longer doppler count intervals for the extra slow data rate of 6 per minute used only by the hybrid system.

3. Record 2, Column 29, IC1

The uplink carrier frequency and the doppler bias frequency, are indicated by IC1 as follows:

IC1	Carrier, Mc/sec	Bias Mc/sec
0	2271.9328	.500
1	2270.9328	.500
2	2270.1328	.500
3	148.2600	.030

4. Record, Column 30, IC2

The time increment T_{DR} between each sample and the number of doppler cycles counted are indicated by IC2 as follows:

IC2	T_{DR}	S-Band Counts	VHF Counts
0, 4	1.00 sec	229,263	14,328
1, 5	0.5 sec	131,007	8,187
2, 6	0.25 sec	65,503	4,093
3, 7	10.00 sec	3,133,956	182,182

5. Record 2, Columns 31, 32; IC3 and IC4

In the hybrid system, the range ambiguity is normally a multiple of .125 seconds. The multiple N of this value to use may be indicated by IC3 and IC4. If IC3 and IC4 are blanks, the resolution system is not in use and N must be determined by the DCP. If IC3 and IC4 are asterisks (**), the system is in use, but the resolution unit is not in lock. The value of N still must be determined by the DCP. When IC3 and IC4 are integers, the value of N is given by

$$N = 10 \times IC3 + IC4$$

3.2.1.3 Unified S-Band Data. Each physical record is a complete date frame, consisting of data recorded at one point in time. The following format is assumed:

Columns or Characters			Contents
1-2	Station ID, Decimal		
3	IDC1		Data ID, See below.
4	IDC2		
5	IDC3		
6	IDC4		
7	IDC5		
8-10	Time	-	Day of Year (Jan. 1 = 1)
11-12		-	Hours
13-14		-	Minutes
15	*		
16-17		-	Seconds
18	.	-	(Decimal Point)
19		-	Tenths of Seconds
20-21	*		
22-27	X-Angle	-	6 Octal Digits
28-29	*		
30-35	Y-Angle	-	6 Octal Digits
36-37	*		
38-48	Range	-	Decimal Range Units, IIIIIIIFFFF
49-50	*		
51	IRDOP	-	Range/Doppler Quality Indicator (Most significant bit) plus 2 bits of doppler.
52-62	Doppler	-	33 bits (11 Octal Digits) of Doppler.

Where

1. * indicates a blank or unused column.

2. Column 3, IDC1

a.	MSB	Doppler destruct/non-destruct	1 = Destruct 0 = Non-destruct
b.		Fixed cycle count (destruct)	1 = 77824 cycles 0 = 778240 cycles
c.	LSB	Range/frequency indicator	1 = Range 0 = Frequency

3. Column 4, IDC2

a.	MSB	Real/test	1 = Real data 0 = Test data
b.		Auto track/other	1 = Auto track 0 = Other
c.	LSB	Time, X- and Y-Angle	1 = Good data 0 = Bad data

4. Column 5, IDC3

a. b.	MSB	Doppler mode	00 = One-way 01 = Two-way 10 = Multiple, non-coherent 11 = Multiple, coherent
c.	LSB	Frequency standard	1 = Primary 0 = Secondary

5. Column 6, IDC4

Octal Digit	6 = CSM data 7 = LEM data

6. Column 7, IDC 5

a.	MSB	Range Data Quality	1 = Good data 0 = Bad data
b.		Range Acquisition	1 = Acquired 0 = Other
c.	LSB	Doppler Data Quality	1 = Good data 0 = Bad data

7. Columns 22-27, 30-35, X- and Y-Angle fields

Each field contains a 6 octal digit number with a granularity of .00068664 degrees/bit. Of the 18 binary bits, the most significant bit is a sign bit with 1 representing plus and 0 representing minus. The remaining 17 are the magnitude with the above given granularity.

8. Columns 38-48, Range Field

This field contains range or VCO frequency according as the least significant bit of IDC1 is a 1 or a 0; VCO frequency data are ignored. Range data are converted to kilometers by using a range unit of 1. 048397 km or 1. 050694 km for CSM or LEM data respectively.

9. Columns 51-62, IRDOP and Doppler

This is a 12 octal digit field. Of the 36 binary bits, the most significant bit is a range/doppler quality bit with the following meaning:

1 = Good data (Auto track)
0 = Bad data (Manual track)

The remaining 35 bits are either doppler counts with a granularity of 1 cycle of biased doppler/bit or doppler count interval with a granularity of 10 nanoseconds/bit according as the doppler type is non-destruct or destruct.

3.2.1.4 DSIF (JPL) Data. Because neither format specifications nor sample data were available from Goddard Space Flight Center, the decoding subroutine for this data type is a dummy. The TDEP, as delivered, cannot edit this type of data.

3.2.2 Output Edited Data Tape Format

The output tape is written in binary mode and is identical in format to the tape produced by the tracking data simulator and conforms to the input requirements of the differential correction program.

The first logical record on the tape is a summary record. Thereafter the records appear in pairs - a key record describing the contents of the data record, and a data record containing time tags, quality indicators and values of the observables. Following the last data record is a dummy key record containing an end-of-tape flag. A data arc (station pass) consists of one or more key/data record pairs. The complete data arcs are written on the tape in ascending order of initial times. All the record pairs required by a data arc are written on the tape before the next data arc appears.

The format is as follows:

Record 1 (Summary Record)

Word	Variable	Dimension	Description
1	HEADER	11	Tape identification header
12	NUMSTA		Number of stations on tape
13	NAMSTA	20	Names of stations on tape
33	TSTART	d*	Start of tape, seconds from 1950
35	TSTOP	d	End of tape, seconds from 1950
37	STIMNX	20	First station on-times, seconds from 1950

Record 2n (Key Record)

1	N		Record pair number
2	NEOT		End of tape indicator; -1 for first key record on tape, +1 for dummy key record at end of tape, 0 otherwise
3	OBSNAM		Observing station name
4	TRANAM		Transmitting station name
5	NRCD		Arc record pair counter
6	NPTS		Number of data points in data record
7	KONT		Data arc continuation; 0, no, +1, yes
8	MTYPE		Measurement system type 1 = C-Band 2 = Goddard Range, Range Rate System 3 = USBS 4 = DSIF (JPL)
9	NALIGN		X-Y mount dish size (when applicable) 1 = 30-foot dish 2 = 85-foot dish
10	MODE		Doppler mode, 2 or 3 way, (when applicable)
11	DELT		Measurement time interval, seconds
12	KTAU		Destruct/Non-destruct doppler indicator when applicable: +1, Non-destruct, word 19 contains interval, data contains counts 0, Destruct; word 19 contains fixed count, data contains intervals.

---

*d denotes double-precision word.

Word	Variable	Dimension	Description
13	ONTIME	d	Start of data arc, seconds from 1950
15	TFIRST	d	Record start, seconds from 1950
17	TLAST	d	Record end, seconds from 1950
19	TAU	d	Count interval, seconds; or count in cycles
21	FTR	d	Transmitter carrier frequency for doppler, cps
23	C1	d	Refraction constants
25	C2	d	
27	DR	d	Range ambiguity, units of range data
29	BIAS	d	Doppler offset bias, cps
31	RETR	d	Transponder ratio
33	NBAD	4	Number of tagged outlines, 4 observables
37	SSD	4	Sample standard deviations, 4 observables
Record 2n+1			
1	DATA	(85, 6)	85 time points of data where one row is as follows: DATA (1, I), I = 1, 6 I = 1 IQ, quality indicator 0 = good 1 = bad angles 2 = bad range 4 = bad doppler >4 = combination of above I = 2 T, Time, seconds from ONTIME I = 3 $\alpha 1$, Azimuth, X, or hour angle, rad I = 4 $\alpha 2$, Elevation, Y, or declination, rad I = 5 R, Range, seconds for Goddard, km otherwise I = 6 $\dot{R}$, Doppler seconds or counts

3.2.3 Data Deck Requirements

The Tracking Data Editor requires a data deck that consists of:

1. One or more block-data overlay cards
2. Two run control cards
3. Six additional data cards for each raw data tape to be processed during the run.

The minimum number of cards clearly is nine. Since the program will process a maximum of ten raw data tapes sequentially, the total data deck will consist of no more than n + 62 cards where n is the (unlimited) number of block-data overlay cards.

3.2.3.1 Block Data Overlay Cards. The nine tracking station data arrays used by the tracking data editor are loaded with nominal values by a BLOCK DATA (TRKCOM) subroutine. These arrays can accommodate data for up to 50 tracking stations. An overlay subroutine, NUDATA, is provided to allow the user to alter any of these array values for the duration of the run. For overlay input purposes, each array is assigned a code name. Table 3-3 displays these code names with an indication of the type of data and a description of the data in each array. For a list of compiled values, see the description of subroutine BLOCK DATA /TRKCOM/.

Not all of the arrays are used by all of the raw data decoding subroutines. The actual use is as follows:

STANAM

- CBTEST (C-Band Data)
  Always used.
- GRTEST (Goddard Range - Range Rate Data)
  Always used.
- SBTEST (Unified S-Band Data)
  Always used.
- DSTEST (DSIF/JPL Data, Dummy Subroutine)
  Always used.

PAIR

- CBTEST

  Not used.
- GRTEST

  Not used.
- SBTEST

  Used when 3-way doppler.
- DSTEST

  Undetermined (Dummy subroutine)

KODSTA

- CBTEST

  Always used.
- GRTEST

  Always used, but subroutine reads station ID numbers from tape and reassigns new numbers for block data. This avoids duplication conflict. The reassignments are as follows:

On Tape	Reassigned
26	30
22	31
52	32
27	33
28	34

- SBTEST

  Always used.
- DSTEST

  Undetermined.

NALIGN

- CBTEST

  Not used.
- GRTEST

  Not used.

- SBTEST

  Always used.
- DSTEST

  Undetermined.

FTR

- CBTEST

  Not used.
- GRTEST

  Not used. Value is assigned by subroutine.
- SBTEST

  Always used.
- DSTEST

  Undetermined.

CFRAC1, CFRAC2

- CBTEST

  Always used.
- GRTEST

  Always used.
- SBTEST

  Always used.
- DSTEST

  Always used.

BIAS

- CBTEST

  Not used.
- GRTEST

  Not used. Value assigned by subroutine.
- SBTEST

  Always used.
- DSTEST

  Undetermined.

RATIO

- CBTEST

  Not used.
- GRTEST

  Used only for S-Band. For VHF, assigned by subroutine.
- SBTEST

  Always used.
- DSTEST

  Undetermined.

TABLE 3-3

STATION DATA ARRAYS FOR TRACKING DATA EDITOR

Code Name	Data Type	Description
STANAM	BCD	6-letter station name.
PAIR	BCD	6-letter name of associated transmitting station if three-way doppler.
KODSTA	Integer	Two digit decimal station ID
NALIGN	Integer	S-Band dish size: 1 = 30-foot 2 = 85-foot
FTR	Double Precision	Transmitter frequency, Hz (cps), used for doppler.
CFRAC1	Double Precision	Atmospheric refraction constants.
CFRAC2	Double Precision	
BIAS	Double Precision	Fixed doppler offset bias, Hz (cps).
RATIO	Double Precision	Spacecraft transponder retransmission ratio for doppler.

Formats. Overlay data for each of the nine arrays consists of two or more cards. The first card contains the code name for the array in columns 1-6 (left adjusted) as indicated by the example below.

```
|S|T|A|N|A|M|
 1 2 3 4 5 6
   Column
```

The remaining cards for an array each contain up to three pairs of numbers. Each pair consists of an integer k and a value v. The integer is used to indicate the relative location in the array C into which the contents of the value field are to be placed; i.e., C(k) = v. Three types of value data are permitted: BCD (Hollerith), integer, and double precision. The type used must be appropriate to the array name as indicated by Table 3-3. The formats for each of these three types are illustrated by the sample shown in Figure 3-4.

BCD
    3(I3, A6, 15X)
Integer
    3(I3, I2, 19X)
Double Precision
    3(I3, D21.16)

Figure 3-4 shows the following: The first array to be overlayed is STANAM, the array of station names. The first card contains the code name of this array. The overlay assignments are as follows:

$$\text{STANAM}\left\{\begin{array}{lcl} (1) & = & \text{GLDSTN} \\ (2) & = & \text{CANBRA} \\ (23) & = & \text{WOMERA} \\ (4) & = & \text{GUAM} \\ (5) & = & \text{HAWAII} \end{array}\right.$$

Notice that the location field for STANAM(5) is negative in this example. A negative location field signals the program that this is the last value to be overlayed for the current array. The program expects the next card to be another array name card.

```
STANAM
  1GLDSTN                 2CANBRA               23WØMERA
  4GUAM                  -5HAWAII
PAIR
  1GLDSTN                 2CANBRA               23WØMERA
  4GUAM                  -5HAWAII
KØDSTA
  128                     2 7                   4396
  497                     598
                   (blank card)
FTR
  1           2045.69+06 -2 2045690000.
STØP               (or blank card)
```

Figure 3-4

SAMPLE OVERLAY DATA
TRACKING DATA EDITING PROGRAM

The next array to be overlayed is PAIR, the array of associated transmitting station names. In this example, the transmitter for each receiving station is the receiving station itself. Notice again that the list is terminated by a negative location value.

The next array to be overlayed is KODSTA, an array of two digit integers that are the station ID codes. The assignments are as follows:

$$\text{KODSTA} \left\{ \begin{array}{lcl} (1) & = & 28 \\ (2) & = & 07 \\ (43) & = & 96 \\ (4) & = & 97 \\ (5) & = & 98 \end{array} \right.$$

Note that the last location field in this list is not negative. Instead, a blank card follows the last data card. This is an alternate way of terminating a data list.

The last array of data to be overlayed is FTR, the transmitter frequency array. The values are double-precision Hz (cycles per second).

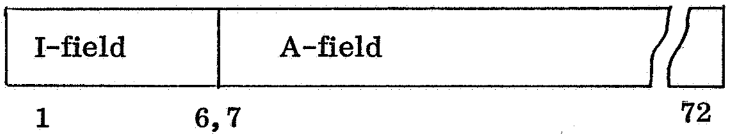
$$\text{FTR} \left\{ \begin{array}{lcl} (1) & = & 2045.69 \times 10^{6} \text{ Hz} \\ (2) & = & 2045.69 \times 10^{6} \text{ Hz} \end{array} \right.$$

Once again, the list is terminated by a negative location field.

After the end of each data list, the program looks for another array name. A blank card terminates the overlay process. An array name card with the word STOP in columns 1-4 will also end the overlay process.

If an array name card with an invalid name is encountered, the invalid name is printed out in an error message, and the program run is immediately terminated.

3.2.3.2 Control Cards. The two control cards must be the first two cards in the data deck after the overlay data. Each card is used to enter one integer and one 11-word BCD message. Each uses an (I6, 11A6) format as illustrated below.



Control Card No. 1

Columns	Contents
1-6	IMAX
7-72	HEADER

IMAX has two functions. Its magnitude (not greater than 10) is the number of raw data tapes to be processed sequentially during the run. If IMAX is negative, all of the data on the final edited data tape will be written on the system output tape at the end of the run. If IMAX is positive, only summary data will be written out. In normal use, the summary data will be adequate to allow intelligent use of the edited data tape.

HEADER is the 11-word header that will be written on the final output edited data tape. It is used for tape identification.

Control Card No. 2

Columns	Contents
1-6	MERGE
7-72	HDRNIN

MERGE is a key that, if non-zero, tells the tracking data editor that a previously edited data tape has been mounted on tape unit 11. The data arcs on this tape will be interspersed at appropriate places with the data arcs that result from editing the raw data tapes. A zero value for MERGE indicates that no such merging is necessary.

HDRNIN is used to test whether the correct edited data tape has been mounted, should MERGE be non-zero. The eleven words of HDRNDN are compared, bit by bit, with the header on the tape of previously edited data. Any disagreement will cause an error message and a program stop. When MERGE = 0, HDRNIN is not used.

Other Data Cards

These cards must be supplied in groups of six cards for each raw data tape to be processed. The number of such groups must agree with the magnitude of IMAX.

Data Card No. 1

FORMAT (6X, 11A6)

Columns	Contents
1-6	Not used
7-72	MESSGE

MESSGE is an eleven-word BCD array used in printing an on-line message to the operator. Immediately before each raw data tape is processed, the following two-line message is printed:

```
*** MOUNT THE FOLLOWING TAPE ON UNIT 9 AND HIT START
    (MESSGE (1),  I = 1, 11)
```

The program then pauses to allow the operator to mount the desired raw data tape. Thus MESSGE may be any information to assist the operator in mounting the raw data tapes in the correct sequence.

Data Card No. 2

FORMAT (5I3)

Columns	Contents
1-3	MTYPE
4-6	NYR
7-9	NPTS
10-12	NSTEP
13-15	NDEG

MTYPE indicates the type of data to be found on the raw data tape according to the following code:

$$\text{MTYPE} \begin{cases} = 1, & \text{C-Band} \\ = 2, & \text{Goddard Range and Range Rate} \\ = 3, & \text{USBS} \\ = 4, & \text{DSIF (JPL)} \end{cases}$$

NYR is the year number from 1900 for the data on the tape, (e.g., 1967 is represented as 67, 2001 as 101). This year number must be supplied because the raw data tape formats provide for no indication of the year. NPTS is the number of points to include in each fit of the polynomial used to test for outliers. NPTS must not exceed 20.

NSTEP is the number of points by which the polynomial is to be walked between fits.

NDEG is the degree of polynomial to be used. NDEG must not be less than 1 nor greater than 5.

Data Card No. 3

FORMAT (4F10.2)

Columns	Contents
1-10	SD_1
11-20	SD_2
21-30	SD_3
31-40	SD_4

The SD values are the numbers of standard deviations by which a value of an observable may deviate from the fitted polynomial without being tagged as an outlyer. Values lying more than SD standard deviations away are tagged. The four values of SD apply to the four observables, angle 1, angle 2, range, and doppler, in that order.

Data Card No. 4

FORMAT (4I1)

Columns	Contents
1	$IFOMIT_1$
2	$IFOMIT_2$
3	$IFOMIT_3$
4	$IFOMIT_4$

The IFOMIT keys are used to signal the editor to supress the observables and replace their values on the edited tape with the number -.12345678E20. The four keys apply to the four observables angle 1, angle 2, range, and doppler in that order. A non-zero value causes suppression. Data values for observables that do not exist on the raw data tape (e.g., doppler in C-Band systems) are set to -.12345678E20 regardless of the key setting.

Data Card No. 5

FORMAT (I3, 20I2)

Columns	Contents
1-3	KMAX
4-5	K_1
6-7	K_2
.	
.	
.	
(2KMAX+2) - (2KMAX+3)	K_{KMAX}

To assist the Tracking Data Editor in rejecting garbled lines of data, it is necessary to input a list of acceptable station ID codes. KMAX is the number of such two-digit codes; the K are the codes. Up to 20 codes may be entered. Any line of data with a number in the station code field that cannot be found in this list will be rejected.

Data Card No. 6

FORMAT (9F8.2)

Columns	Contents	
1-8	YM.	TLO
9-16	DHM.	
17-24	Sec.	
25-32	YM.	THI
33-40	DHM.	
41-48	Sec.	
49-56	YM.	TSTOP
57-64	DHM.	
65-72	Sec.	

Three dates must be entered on this card in the standard date format (e.g., Feb. 12, 1967, 14^h 26^m 12.025^s is represented by three numbers as 6702., 121426., 12.025). The first two of these dates, TLO and THI, are additional aids to the editor for rejecting garbled lines of data. The date field of each line is decoded and if the resultant date is earlier than TLO or later than THI, the line of data is rejected. The third date, TSTOP, is used to halt the reading of a raw data tape in the absence of an end-of-tape key record on that tape. (An end-of-tape key record is an extra record at the end of the data and has the letters ED in columns 1 and 2. This record is useful on C-Band and USBS tapes; the Goddard System tapes end with two or more special records with ** in columns 1 and 2.) As soon as a valid date equal to or later than TSTOP is reached, the editor considers this to be the end of the raw data tape. To be effective, this date must lie inside the valid date range defined by TLO and THI.

3.2.3.3 Data Deck Summary. Figure 3-5 and the following list will provide a quick check list for a data deck.

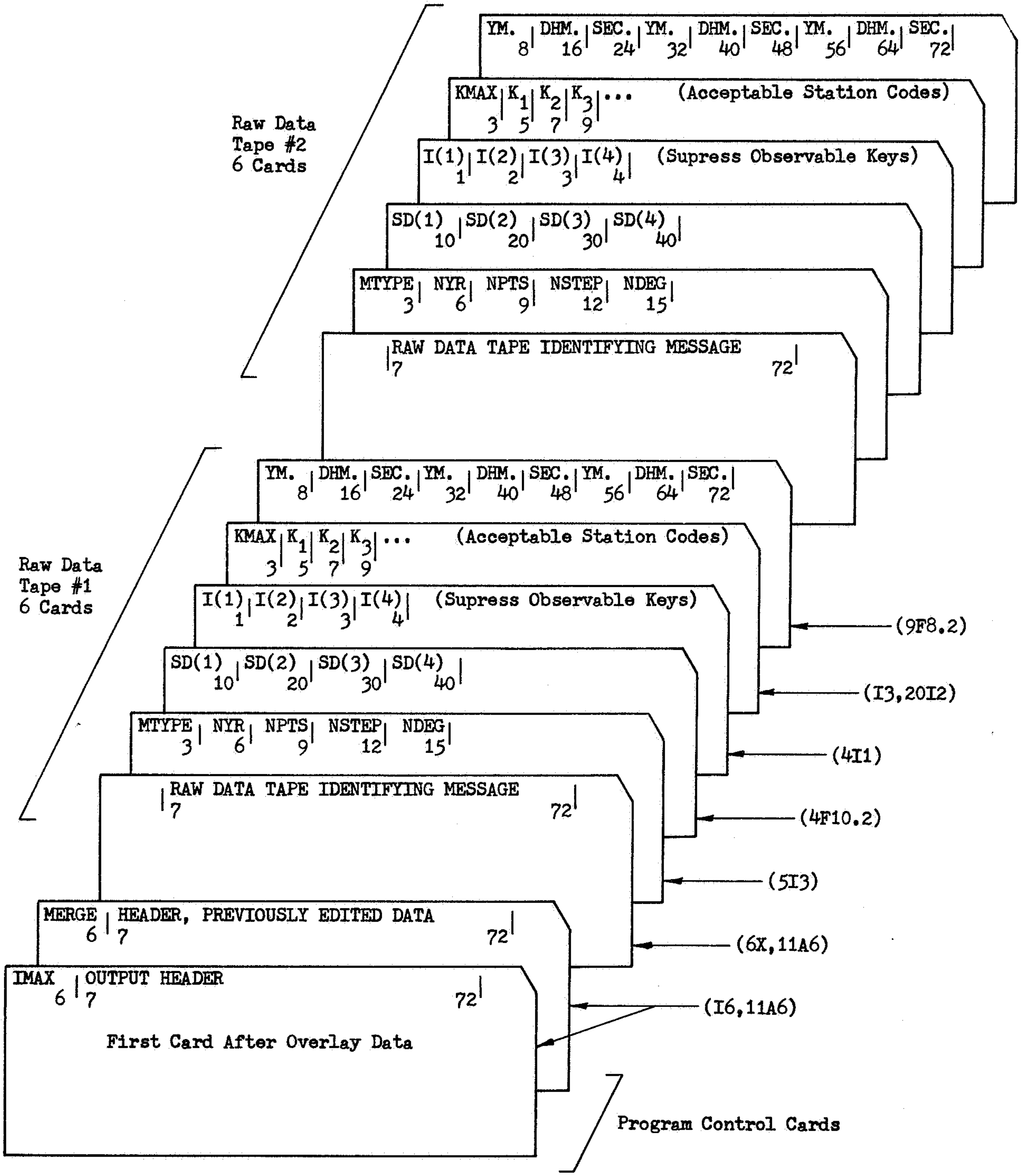


Figure 3-5

DATA DECK SETUP EXCLUSIVE OF OVERLAY CARDS

TRACKING DATA EDITING PROGRAM

Overlay Cards

Control Cards (2 required)

Card No.	Contents
1	IMAX, HEADER
2	MERGE, HDRNIN

Other Data Cards (6 per raw data tape)

Card No.	Contents
1	MESSGE
2	MTYPE, NYR, NPTS, NSTEP, NDEG
3	(SD(I), I=1, 4)
4	(IFOMIT(I), I=1, 4)
5	KMAX, (K(I), I=1, KMAX)
6	TLO, THI, TSTOP

3.3 POLYNOMIAL FITTING BY LEAST SQUARES

Polynomial fitting by least squares admits of two interpretations. On the one hand, one may have a sequence of observations or measurements known to have been made with great accuracy. It may then be desirable to find a polynomial of sufficiently high degree that passes exactly or very nearly through each point, reproducing all the minor variations in the trend of the observations. On the other hand, one may have a sequence of observations of a process known to proceed rather smoothly, but made with a noisy observing apparatus. In this case, it may be desirable to fit a low degree polynomial through the points in such a way that the polynomial describes the general trend of the data but avoids the minor variations.

It is the latter interpretation that is employed in the Tracking Data Editing Program. In order to detect outliers, a low degree polynomial is fitted through a sequence of measurements. Those measurements deviating from the fitted polynomial by more than a specified number of sample standard deviations are flagged as outliers.

The mathematics of least-squares fitting are simple and well known. Hence we merely outline the procedure. We start with a sequence of observations of the scalar quantity y

$$y\,(t_1),\quad y(t_2),\quad \ldots\; y(t_n)$$

We wish to obtain a k^{th} degree polynomial fit to the observations $(k < n)$ such that the sum of the squares of the residuals is minimized. That is, we wish to find a set of coefficients x_i of

$$\begin{aligned} y(t) &= x_0 + x_1 t + x_2 t^2 + \cdots + x_k t^k \\ &= (1,\ t,\ t^2,\ \cdots,\ t^k) \begin{bmatrix} x_0 \\ x_1 \\ \cdot \\ \cdot \\ \cdot \\ x_k \end{bmatrix} \end{aligned} \qquad (3.3\text{-}1)$$

For all the observations this becomes

$$\begin{bmatrix} y_1 \\ y_2 \\ \cdot \\ \cdot \\ \cdot \\ y_n \end{bmatrix} = \begin{bmatrix} 1 & t_1 & t_1^2 & \cdots & t_1^k \\ 1 & t_2 & t_2^2 & \cdots & t_2^k \\ \cdot & & & & \\ \cdot & \cdot & \cdot & \cdot & \\ \cdot & & & & \\ 1 & t_n & t_n^2 & \cdots & t_n^k \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ \cdot \\ \cdot \\ \cdot \\ x_k \end{bmatrix} \qquad (3.3\text{-}2)$$

or

$$y = Tx \qquad (3.3\text{-}3)$$

As a constraint, we wish to minimize

$$\begin{aligned} L &= (y - Tx)^T\,(y - Tx) \qquad (3.3\text{-}4) \\ &= y^T y - y^T\,Tx - x^T T^T y + x^T T^T Tx. \end{aligned}$$

Since $$y^T\ Tx = x^T\ Ty,$$

$$L = y^T y - 2y^T\ Tx = x^T\ T^T\ Tx. \qquad (3.3\text{-}5)$$

Taking the gradient with respect to x and setting the result zero gives

$$\nabla L = 0 = -\ 2y^T\ T + 2x^T\ T^T\ T, \qquad (3.3\text{-}6)$$

whence

$$T^T\ Tx = T^T y \qquad (3.3\text{-}7)$$

Under the stated conditions, the inverse matrix

$$(T^T\ T)^{-1}$$

exists and the unique solution is given by

$$x = (T^T\ T)^{-1}\ T^T y \qquad (3.3\text{-}8)$$

In the TDEP, the inverse is not computed explicitly. Instead, we form the augmented matrix

$$(T^T T \mid T^T y) = \left[\begin{array}{ccccc|c}
n & \sum_i^n t_i & \sum_i^n t_i^2 & \cdots & \sum_i^n t_i^k & \sum_i^n y_i \\
\sum_i^n t_i & \sum_i^n t_i^2 & \sum_i^n t_1^3 & \cdots & \sum_i^n t_i^{k+1} & \sum_i^n y_i t_i \\
\vdots & \cdot & \cdot & \cdot & \cdot & \\
\sum_i^n t_i^k & \sum_i^n t_i^{k+1} & & & \sum_i^n t_1^{2k} & \sum_i^n y_i\ t_i^k
\end{array}\right] \qquad (3.3\text{-}9)$$

and develop the required vector x in the right-most column of the augmented matrix by a reduction due to Crout[1].

Let
$$T^T T = a \qquad (3.3\text{-}10)$$
$$T^T y = c$$

then the augmented matrix M may be written

$$M = (a \mid c) \qquad (3.3\text{-}11)$$

Crout's method for systems with real coefficients proceeds from the augmented matrix of the system

$$M = \left[\begin{array}{cccc|c} a_{11} & a_{12} & \cdots & a_{1n} & c_1 \\ a_{21} & a_{22} & \cdots & a_{2n} & c_2 \\ \vdots & & & & \\ a_{n1} & a_{n2} & \cdots & a_{nn} & c_n \end{array}\right] = (a \mid c), \qquad (3.3\text{-}12)$$

which may be considered as partitioned into the coefficient matrix a and the column vector c, to an auxiliary matrix

---

[1] Crout, P. D.: "A short method for Evaluating Determinants and Solving Systems of Linear Equations with Real or Complex Coefficients," Trans. AIEE, Vol. 60, pp 1235-1241 (1941).

See also:
Hildebrand, F. B.: Methods of Applied Mathematics, Prentice-Hall, Inc., Englewood Cliffs, N.J., 1952, pp 503-507.

$$M' = \left[\begin{array}{cccc|c} a'_{11} & a'_{12} & \cdots & a'_{1n} & c'_1 \\ a'_{21} & a'_{22} & \cdots & a'_{2n} & c'_2 \\ \vdots & & & & \\ a'_{n1} & a'_{n2} & \cdots & a'_{nn} & c'_n \end{array}\right] = (a' \,\vdots\, c') \qquad (3.3\text{-}13)$$

of the same dimensions, and thence to the required solution vector

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \qquad (3.3\text{-}14)$$

The procedure for finding the elements of M' from those of M is described by the following rules:

1. The elements of M' are determined in the following order: elements of the first column, then elements of the first row to the right of the first column; elements of the second column below the first row, then elements of the second row to the right of the second column; and so on, until all elements are determined.

2. The column operations may be divided into three steps as follows:
   a. Each element a'_{ij} on or below the principal diagonal of M' is obtained by subtracting from the corresponding a_{ij} of M the sum of the products of elements in the i^{th} row and corresponding elements in the j^{th} column of M', all uncalculated elements being imagined to be zeros. That is

$$a'_{ij} = a_{ij} - \sum_{k=1}^{j-1} a'_{ik}\, a'_{kj} \quad (i \geq j). \qquad (3.3\text{-}15)$$

   For the first column this is clearly no operation at all.

b. The magnitude of the diagonal element a'_{ii} of the i^{th} column of M' is compared with the magnitudes of the elements of the i^{th} column below the diagonal a'_{ik}, $k = i + 1, n$. If the largest of these, say a'_{ik}, is greater in magnitude than a'_{ii}, then the entire i^{th} and k^{th} rows of M (and M') are interchanged. This is purely a numerical device to minimize round-off errors.

c. Each element a'_{ij} below the principal diagonal is finally determined by division by the diagonal element a'_{jj} of M'. That is

$$a'_{ij} = a'_{ij}/a'_{jj} \qquad (i > j) \qquad (3.3\text{-}16)$$

3. Row operations consist of a single step: Each element a'_{ij} to the right of the principal diagonal is obtained by subtracting from the corresponding element a_{ij} of M the sum of the products of elements in the i^{th} row and corresponding elements in the j^{th} column of M', all uncalculated elements being imagined to be zeros. That is

$$a'_{ij} = a_{ij} - \sum_{k=1}^{i-1} a'_{ik} \, a'_{kj} \qquad (i < j) \qquad (3.3\text{-}17)$$

Clearly for the first row this involves no operation at all.

The procedure for obtaining the final solution vector from the matrix a' and the vector c' into which M' is partitioned is described by the following rules:

1. The elements of x are determined in the reverse order x_n, x_{n-1}, x_{n-2}, $\cdots$, x_1 from the last element to the first.

2. The last element x_n is determined by dividing the last element c'_n of c' by the last element a'_{nn} on the principal diagonal of M'. That is

$$x_n = c'_n / a'_{nn} \qquad (3.3\text{-}18)$$

3. Each of the remaining elements x_i of x is determined by subtracting from the corresponding element c'_i of c' the sum of the products of elements in the i^{th} row of a' by corresponding elements of the column x, followed by a division by the diagonal element a'_{ii}, all uncalculated elements of x being imagined to be zero. That is

$$x_i = \frac{c_i - \sum_{k=i+1}^{n} a'_{ik} x_k}{a'_{ii}} \qquad (3.3-19)$$

SECTION 4

TRACKING DATA SIMULATOR

4.1 GENERAL DESCRIPTION OF THE PROGRAM

The Tracking Data Simulator (TDS) was developed to satisfy two basic needs. During development and checkout of the Differential Correction Program (DCP), the TDS provided a means of supplying tracking data of each of the four types which can be processed by the DCP. This simplified checkout of the DCP processing logic, since the orbit geometry and measurement parameters could be controlled as desired.

Subsequent to delivery of the entire Orbit Determination Program system, the TDS will aid in exercising the various operating options of the DCP for both testing and gaining experience in using the DCP. It will also serve as an aid in testing and evaluating any contemplated changes in the DCP.

4.1.1 Capabilities of the TDS

The Tracking Data Simulator creates a tape of simulated measurements by an earth-based tracking station network. This tape is identical in format to those produced by the Tracking Data Editing Program and conforms to the requirements of the Differential Correction Program.

4.1.1.1 Tracking Station Network. The tracking station network consists of from one to twelve earth-based tracking stations, each of which must be specified by input. Each station may be one of the following four types:

- C-Band
  - Azimuth
  - Elevation
  - Range

- Goddard Range and Range-Rate
  - X-Angle
  - Y-Angle
  - Range
  - Doppler

- Unified S-Band
  - X-Angle
  - Y-Angle
  - Range
  - Doppler

- DSIF (JPL)
  - Hour Angle
  - Declination
  - Doppler

The following parameters are specified independently for each station:

1. Station Name (BCD identifying code)
2. Observation Period
3. Location
   a. Latitude (Geodetic)
   b. Longitude
   c. Altitude
4. Artifical Horizon
5. Artifical Zenith
6. Doppler parameters, if applicable
   a. Transmitter Carrier Frequency
   b. Associated Transponder Ratio
   c. Offset Bias Frequency
   d. Doppler Count Interval
7. Antenna dish size and orientation (USBS only)

4.1.1.2 Error Sources. In addition to the above parameters, the treatments of each of the individual error sources may be specified independently for each station. The following errors sources are provided for:

Random Errors

Random errors (normally distributed) may be added to any of the observables as desired. The standard deviation must be specified separately for each observable. The random errors in the various observables are uncorrelated.

Bias Errors

Fixed bias errors of a specified size may be added independently to each observable, the three station location parameters (North, East, Down), and the station clock.

Other Error Sources

A nominal value of the speed of light is compiled into the BLOCK DATA subroutine. This value affects all of the measurements and may be altered by the block data overlay feature as an additional error source. Since the TDS requires, as one of its inputs, a spacecraft trajectory tape in the same format as these produced by the PINT portion of the Mark II Error Propagation Program (delivered to Goddard Space Flight Center under Contract NAS5-9700), no provision is made for including equation of motion errors directly in the TDS. They must appear indirectly as a result of the integration performed during the making of the spacecraft trajectory tape.

4.1.1.3 Refraction Corrections. As part of the specification data for each station, the user must indicate whether the effects of atmospheric refraction are to be included in the computation of the observables. Although it is possible to include refraction effects for some stations and not others, no further subdivision is possible; i.e., it is not possible to include the effects for only a part of the observables recorded for any one station.

All stations for which refraction effects are specified share the same two parameter refraction model, with calculations appropriate to the observables recorded by each station. The nominal values of the two parameters are compiled into the BLOCK DATA subroutine. They may be altered by use of the BLOCK DATA overlay feature of the TDS.

4.1.1.4 Tracking Station Network Changes. The configuration of the tracking station network need not remain fixed during the entire run of the TDS, but may be altered as many times as the user desires. This is possible because the input cards specifying the tracking station network are always preceeded by a control times card indicating the time interval over which measurements are to be made with the network specified on the following cards. The first set of station cards sets up the network. Subsequent sets alter the existing network, and only those stations involved in the alteration need be included in these sets. That is, once a station is specified, it remains in effect until it is altered by a subsequent station card containing the station number involved. (These numbers are the internal station numbers from 1 to 12 that must appear on the station cards.)

Two restrictions must be observed. No station name may appear in a set of station change cards unless it appears in the first set. The effect of this restriction is that new stations may not be added during a run. A station may, however, be deleted and subsequently reinstated. The second restriction is that the intervals indicated on the control times must progress monotonically. Thus if the network is specified to be effective from t_1 to t_2 and then altered from t_3 to t_4, it must be true that

$$t_1 \leq t_2 \leq t_3 \leq t_4.$$

There are no internal checks in the TDS to detect violations of these two restrictions. If the first restriction is violated, the offending station name will not appear in the list of station names contained in the first record of the simulated data tape. The Differential Correction Program, if directed to process data from this station, will stop with an error message indicating an invalid station name. A violation of the second restriction may cause tape search errors in the Differential Correction Program.

4.1.2 Program Structure

Figure 4-1 lists all the subroutines and labelled commons used by the TDS. Each deck for which a deckname is given in the figure must be present in the object deck. These commons for which the deckname is omitted do not require a block data, and hence do not require an object deck.

Figure 4-2 summarizes all reference to labelled commons and subroutines. Note that overlay at the ALPHA origin occurs twice per case; once for initialization, and once for all futher computation during the case.

4.1.3 Tape Requirements

The tape units required by the TDS are listed in Table 4-1. Units 8 and 10 are input tapes and unit 12 is the output tape.

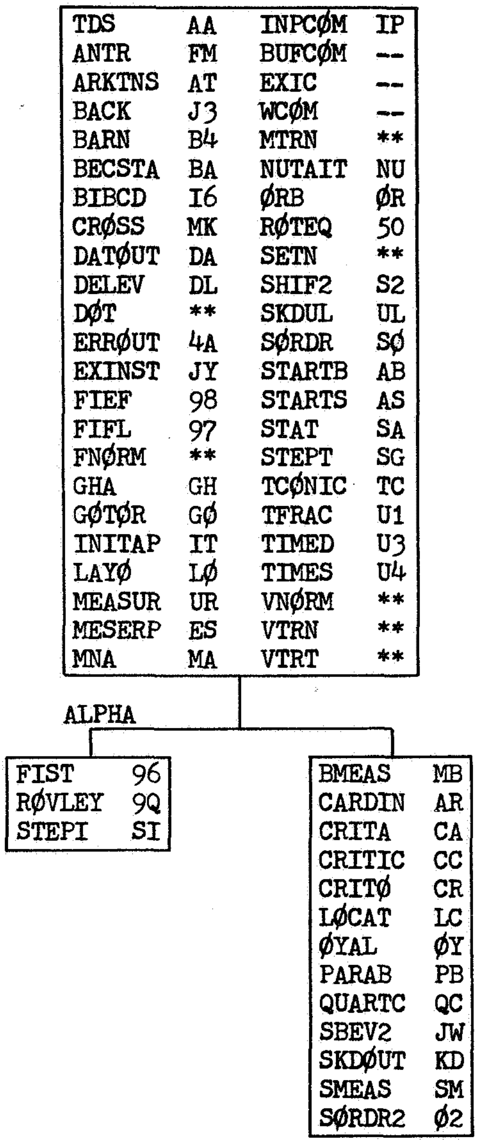
TABLE 4-1

TAPE UNIT ASSIGNMENTS

Logical Unit No.	Mode	Function
8	Bin	Single precision planetary ephemeris tape, for use by ANTR.
10	Bin	Vehicle ephemeris tape, written by the PINT subprogram of the Mark II Error Propagation Program.
12	Bin	Simulated data tape.

4.2 INPUT AND OUTPUT CONTROLS

This section describes the procedure for supplying input data to the Tracking Data Simulator. Because this program is built around basic elements of the Mark II Error Propagation Program, delivered to Goddard Space Flight Center under



TDS	AA	INPCØM	IP
ANTR	FM	BUFCØM	--
ARKTNS	AT	EXIC	--
BACK	J3	WCØM	--
BARN	B4	MTRN	**
BECSTA	BA	NUTAIT	NU
BIBCD	I6	ØRB	ØR
CRØSS	MK	RØTEQ	50
DATØUT	DA	SETN	**
DELEV	DL	SHIF2	S2
DØT	**	SKDUL	UL
ERRØUT	4A	SØRDR	SØ
EXINST	JY	STARTB	AB
FIEF	98	STARTS	AS
FIFL	97	STAT	SA
FNØRM	**	STEPT	SG
GHA	GH	TCØNIC	TC
GØTØR	GØ	TFRAC	U1
INITAP	IT	TIMED	U3
LAYØ	LØ	TIMES	U4
MEASUR	UR	VNØRM	**
MESERP	ES	VTRN	**
MNA	MA	VTRT	**

FIST	96
RØVLEY	9Q
STEPI	SI

BMEAS	MB
CARDIN	AR
CRITA	CA
CRITIC	CC
CRITØ	CR
LØCAT	LC
ØYAL	ØY
PARAB	PB
QUARTC	QC
SBEV2	JW
SKDØUT	KD
SMEAS	SM
SØRDR2	Ø2

-- Denotes a labelled common for which no block data, and hence no object deck, is required.

** Deck not in the MC13xx sequence. Any appropriate deckname may be used.

Decks FM, I6, 98 are MAP-coded decks, valid only for IBM FØRTRAN IV.

Figure 4-1

LINK STRUCTURE

TRACKING DATA SIMULATOR

Link	Calling Routine	Referenced Subroutines																																																								
		Link 0																																											Link 2												
		BUFCØM	EXIC	INPCØM	WCØM	ANTR	ARKTNS	BACK	BARN	BECSTA	BIBCD	CRØSS	DATØUT	DELEV	DØT	ERRØUT	EXINST	FIEF	FIFL	FNØRM	GHA	GØTØR	INITAP	LAYØ	MEASUR	MESERP	MNA	MTRN	NUTAIT	ØRB	RØTEQ	SETN	SHIF2	SKDUL	SØRDR	STARTB	STARTS	STAT	STEPT	TCØNIC	TFRAC	TIMED	TIMES	VNØRM	VTRN	VTRT	BMEAS	CARDIN	CRITA	CRITIC	CRITØ	LØCAT	ØYAL	PARAB	QUARTIC	SBEV2	SKDØUT	SMEAS	SØRDR2
0	TDS			x																						x																																	
	ANTR																															x																											
	BECSTA		x	x	x												x																																	x						x			
	DATØUT															x																									x																		
	DELEV			x	x																																																						
	EXINST							x																																																			
	FIFL															x		x																																									
	INITAP		x	x	x								x			x	x		x		x							x	x	x	x										x	x	x																
	LAYØ																																									x																	
	MEASUR	x	x	x	x											x	x																x								x						x											x	
	MESERP	x	x	x	x					x				x		x	x						x		x									x	x	x	x				x																		
	MNA						x																																																				
	ØRB						x					x			x					x																				x																			
	SHIF2			x	x	x																																	x																				
	SKDUL	x		x	x											x																										x		x				x									x		
	STARTB	x		x	x												x																																	x						x			
	STARTS	x		x	x												x																																	x						x			
	STEPT																					x																																					
	TIMES										x																																																
2	BMEAS			x	x		x		x			x			x												x	x	x									x						x	x														
	CARDIN			x	x																			x																																		
	CRITA																																										x																
	CRITIC			x				x																																																		x
	CRITØ						x													x																							x															
	LØCAT			x	x															x																		x																					
	SBEV2														x						x						x	x	x		x	x	x		x			x	x		x			x	x	x			x		x			x	x				
	SKDØUT			x	x																																															x	x						
	SMEAS		x	x	x		x					x			x	x	x			x													x					x			x				x	x													
1	FIST																		x																																								
	STEPI														x					x																																							

The subroutines of Link 1 (FIST, RØVLEY, STEPI) are called only by INITAP.

Figure 4-2
CROSS REFERENCE
TRACKING DATA SIMULATOR

Contract NAS5-9700, considerable similarity exists between the input controls for error propagation with the Mark II and the input controls for this program.

The simulated data option is keyed in by an option card containing a 2 in column 5. Since only this one option is currently available, a 2 is the only option code permitted with the exception of a 0 (zero), which terminates the run of the program. Only one case per run is permitted; thus the first card in the data deck has a 2 in column 5 and the last card has a zero in column 5. All the remaining cards pertain to the same single case of Tracking Data Simulation.

Figure 4-3 is a block diagram or flow chart that shows the order in which the principal subroutine (MESERP) accepts data and writes output.

4.2.1 Trajectory Tape Descriptors

First, the trajectory tape required must be specified. (This tape must be available on Unit 10 and be a standard Mark II vehicle ephemeris tape. It may be either a patched conic CONW tape or a precision integrated PINT tape.) This tape specification is communicated to MESERP via two cards. The first card contains the word HEAD in columns 61-64 indicating that the next card contains a 12-word alphanumeric header identical in columns 1 thru 72 to that which appears on the binary trajectory tape to be used.

Second, the desired case on the trajectory tape is specified by two cards; the first card contains a 377 in columns 1-3, a 377 in columns 16-18, and an I in column 61. This card indicates that integer data on the next card is to be stored in locations IW(377) to IW(377). The next card contains the trajectory case number in column 6.

4.2.2 Output Tape Descriptors

The alphanumeric header to be written on the simulated data tape is communicated to MESERP via two cards. The first card contains a 1 in column 3 and the word HEAD in columns 61-64. The following card contains an eleven-word, alphanumeric header in columns 1-66.

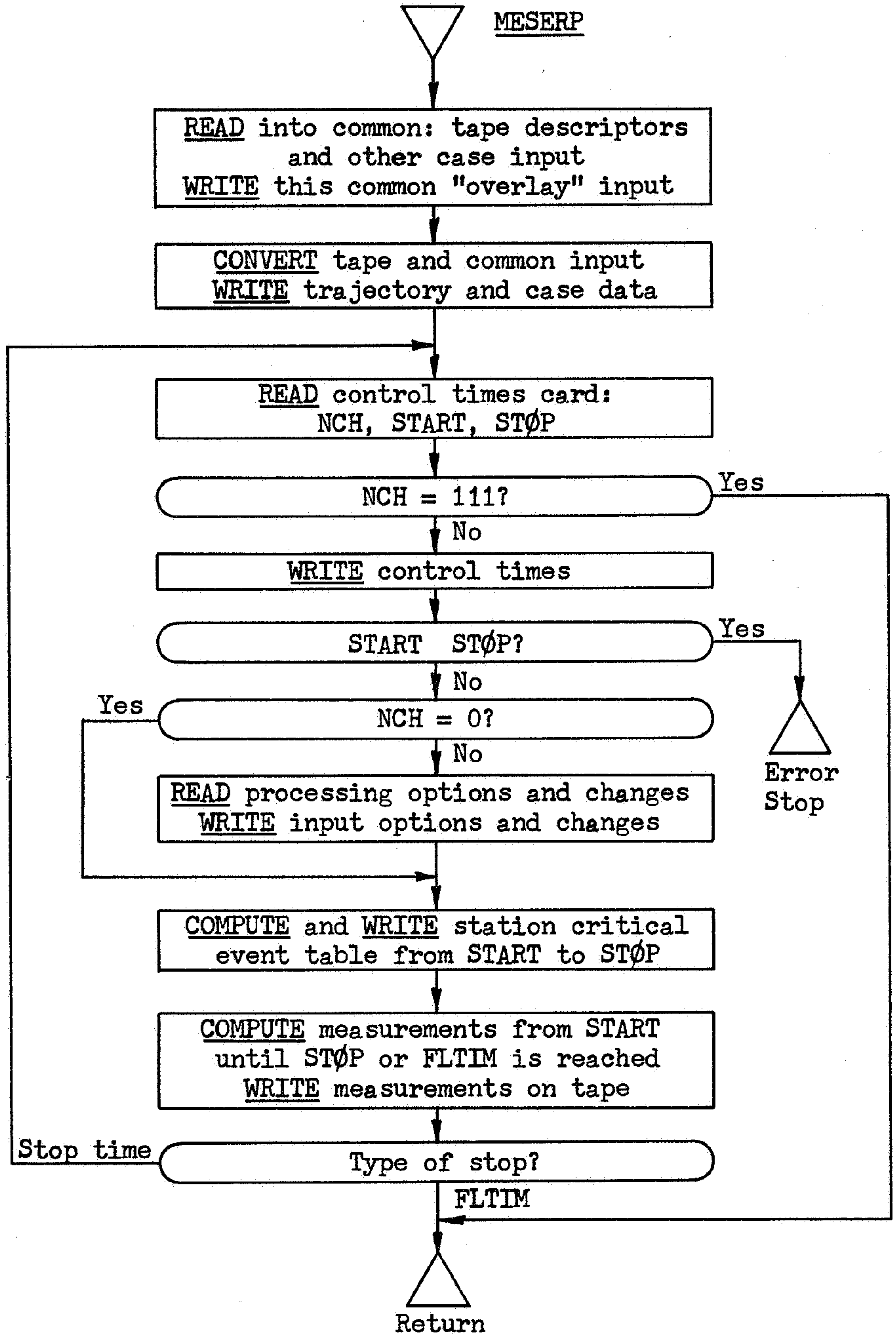


Figure 4-3

INPUT-OUTPUT SEQUENCE

TRACKING DATA SIMULATOR

4.2.3 Other Input Requirements

At this point, stored block data in the INPCOM or C array may be altered by one or more cards with a 4(I3, E12.8) format. From one to four sets of an integer k and a number x may be specified per card to be stored as C(k) = x. (The program will operate properly without any of these C-array values being altered.) This read-in section must be terminated with a blank card whether any C-array values are entered or not.

4.2.4 Control Times Input

The second READ block encountered in MESERP requires a control times card. The format for this card is identical to that described for entering C-array data, 4(I3, E12.8), although certain parts are superfluous. The values to be entered are as follows:

Columns:	1-3	NCH		
	4-15	TST(1)	=	START time
	19-30	TST(2)	=	STOP time

The TST values are times from epoch in the DH.MS format.

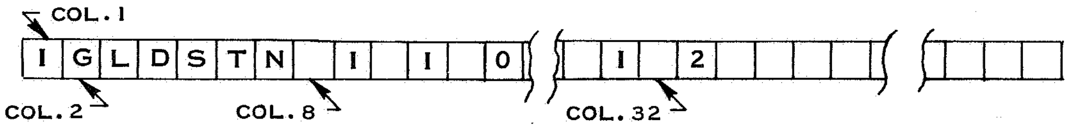
The NCH integer is a control variable whose use is best explained by referring to the flow diagram.

4.2.5 Processing Options and Changes

The remaining READ block "READ processing options and changes" is concerned with specification of measurements to be made and their treatment.

4.2.5.1 Input Data Formats. Input to be provided in this READ block is of two types. The first type consists of fixed-point option keys and indicators. The second type consists of floating-point values of station (and beacon) parameters such as locations, observation periods, and measurement biases and random error standard deviations.

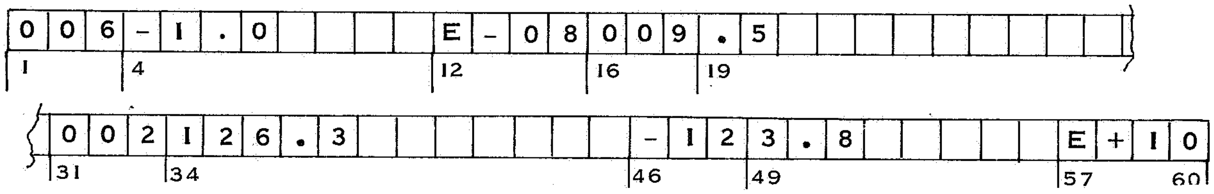
Fixed point information is supplied by a (I1, A6, 13I2) format as shown below:



The information in columns 2-7 in the figure is seen to be alphanumeric rather than fixed-point. This alphanumeric information is the tracking station name and must be identifiable as such by the Differential Correction Program. That is, it must be one of the names in the station block data of the DCP.

The significance of the remaining data will be explained later under the heading "Station Change Card."

Floating-point data are provided through a 4(I3, E12.8) format shown below or by means of a BLOCK DATA subroutine which loads information into the INPCOM common arrays. The block data subroutine is loaded only at the start of the computer run. Common information entered by the above format replaces the block data information for the remainder of the run. Significance of the data input by the above format is described below under the heading "Station Data Cards."



Floating-Point Format

4.2.5.2 Station Change Card. A 1 in the first column of the change card tells the program to consider the rest of the data on the change card and the following floating point cards to be related to a tracking station. (Since the TDS does not presently allow for beacon and on-board measurements, the only non-zero value permitted is a 1.) The station name, which is limited to a maximum of six letters, and which must be a name recognizable by the Differential Correction Program, occurs next. Columns 8-9 are for an indicator that, if non-zero, says that the station is to be considered and, if zero or blank, says that the station is to be deleted from further consideration. (Stations may not be added in the middle of a run; they may, however be deleted and subsequently reinstated.) Columns 10-11 contain the station number, which may be any number from 1 to 12. This number does not appear on the output data tape; it is used only within the Tracking Data Simulator. The remaining pairs of columns contain keys defining the type of measurements and the treatment of error sources. Table 4-2 lists the quantities that may be specified or indicated on the station change card. The error source treatment codes are specified in Table 4-3.

TABLE 4-2

STATION CHANGE CARDS

Columns	Quantity
1	Station change indicator (=1)
2-7	Station name
8-9	Consider/omit key 1: Consider 0: Omit or delete
10-11	Station ID number, internal to TDS
12-13	Measurement system type 1: C-Band 2: Goddard Range and Range Rate 3: Unified S-Band 4: DSIF (JPL)
14-15	Range treatment
16-17	Doppler treatment, ignored for C-Band
18-19	Azimuth, X-angle, or hour angle treatment
20-21	Elevation, Y-angle, or declination treatment
22-23	Northing error treatment
24-25	Easting error treatment
26-27	Down error treatment
28-29	Clock bias treatment
30-31	Antenna alignment, USBS only 1: Principal axis north, 30 foot dish 2: Principal axis east, 85 foot dish
32-33	Atmospheric refraction effects 1: Include 2: Omit

TABLE 4-3

ERROR SOURCE TREATMENT CODES

Code	Meaning	
	Measurements	Other Sources
00	Omit Measurement	Add No Error
01	Add Bias and Random Errors to Measurement	Add Bias Error
02	Add Only Random Error to Measurement	Not Permitted, Meaningless

Note: "Clean" measurements may be generated by setting the biases and random error standard deviations to 0. on the station data cards.

4.2.5.3 Station Data Cards. Information (location, measurement parameters, etc) about the station follows the station change card. This information requires one or more cards and is communicated to the program by the 4(I3, E12.8) format shown on page 4-11. Quantities read in this way are converted from input units (degrees, meters, etc.) to stored units (radians, km, etc.) by the program. Those quantities that are not to be changed from previously input values need not be read in again. Quantities to be provided for each station, with their appropriate indices and units, are shown in Table 4-4. The first location in the array is not shown in the table since it is reserved for the station name. The end of the data for each station is signaled either by making the index of the last change negative, or by adding a blank card after the changes. The program then looks for the next fixed-point change card.

The last change card must be blank to indicate no more changes.

TABLE 4-4

STATION INFORMATION

Index	Quantity	Input Units	Stored Units
2	Period of Observation	DH. MS or -Sec eg. -. 5 = . 5 Sec	Sec
3	Latitude, Geodetic	Degrees	Radians
4	Longitude	Degrees	Radians
5	Altitude	Meters	Kilometers
6	Artificial Horizon	Degrees	Radians
7	Artificial Zenith	Degrees	Radians
8	Range Error S. D.	Meters	Kilometers
9	Doppler Error S. D.	Counts/Sec	Counts/Sec
10	Angle 1 Error S. D.	Milliradians	Radians
11	Angle 2 Error S. D.	Milliradians	Radians
12	Range Bias Error	Meters	Kilometers
13	Doppler Bias Error	Counts/Sec	Counts/Sec
14	Angle 1 Bias Error	Milliradians	Radians
15	Angle 2 Bias Error	Milliradians	Radians
16	Northing Error	Meters	Kilometers
17	Easting Error	Meters	Kilometers
18	Down Error	Meters	Kilometers
19	Clock Bias	Seconds	Seconds
20	Fixed Doppler Offset	Megacycles/Sec	Cycles/Second
21	Transponder Ratio (transmit/receive)	Unitless	Unitless
22	Transmitter Carrier Frequency	Megacycles/Sec	Cycles/Second
23	Doppler Count Interval	Seconds	Seconds

SECTION 5

RESIDUAL OUTPUT PROGRAM

5.1 GENERAL DESCRIPTION OF THE PROGRAM

The Residual Output Program (ROP) is a very simple program designed to demonstrate the recovery of the state estimate and residual records from the tapes written by the DCP.

5.1.1 Capabilities of the ROP

The ROP is a much simplified version of the DCP residual output link. It is designed to locate a specified record pair on the residual tape and the associated estimate record pair on the estimate tape, if any. At the user's option, it will

- list the estimate record, including all parameters which may be included in an extended state,
- list the residuals and/or vehicle cartesian state, from the first specified record pair through all continuation records,
- plot specified residuals on the system output tape.

5.1.2 Program Structure

The ROP uses only three decks, of which one is the main program, and two are subroutines taken from the DCP. The decks required are

MC138I	ROP	Main Program
MC13SY	OVRLYD	Input Subroutine
MC13ZT	RSPLOT	Plot Subroutine

The labelled commons /ESTCOM/, /RSLCOM/, and /POTCOM/ are also used, but no block data is required.

The tape units used are listed in Table 5-1, below. Of the listed units, only unit 12 is optional.

TABLE 5-1

TAPE UNIT ASSIGNMENTS

Logical Unit No	Mode	Function
5	BCD	System input tape
6	BCD	System output tape
11	Bin	Residual tape (input)
12	Bin	Estimate tape (optional input)

5.2 INPUT AND OUTPUT

Input to the ROP consists of two tape header cards, a sequence of control cards and possible "overlay" cards. This section describes the format and contents of the cards and the associated output.

5.2.1 Tape Header Cards

Two tape header cards, one for each of units 11 and 12, in that order, are the first cards of the data deck. They have the format

(I6, 11A6)

and contain the integer IFTEST and the 11-word alphanumeric tape header. The integer IFTEST is interpreted as follows:

IFTEST = 0: Compare input header with the header on the tape. Stop if the headers do not agree.

IFTEST $\neq$ 0: Accept the mounted tape and proceed without testing the header.

If the header card for unit 12 is blank, the program assumes that no tape is mounted on unit 12.

5.2.2 Control Card

The control cards have the format

(I4, 7I2)

and contain the integers listed in Table 5-2, below:

TABLE 5-2

CONTROL CARD CONTENTS

Card Cols	Symbol	Definition
1 to 4	NRECRD	Number of first residual record pair to be output
5 to 6	NRLIST	Option for quantities to be listed
7 to 8	NROVER*	Does overlay data follow the control card?
9 to 10	NPLBAD*	Are flagged "bad" data to be plotted?
11 to 12	NPLOT1*	Are residuals of measurement type 1 (angle 1) to be plotted?
13 to 14	NPLOT2*	Are angle 2 residuals to be plotted?
15 to 16	NPLOT3*	Are range residuals to be plotted?
17 to 18	NPLOT4*	Are doppler residuals to be plotted?

*Denotes yes/no option (1 = yes; 0 or blank = no).

The integer NRLIST is

$$\text{NRLIST} = I_4 + I_2 + I_1$$

$$I_4 = \begin{cases} 4 & \text{if state parameters are to be listed} \\ 0 & \text{otherwise} \end{cases}$$

$$I_2 = \begin{cases} 2 & \text{if vehicle cartesian state is to be listed} \\ 0 & \text{otherwise} \end{cases}$$

$$I_1 = \begin{cases} 1 & \text{if residuals are to be listed} \\ 0 & \text{otherwise} \end{cases}$$

5.2.3 Overlay

A single array, PCNTRL, containing plot scale control parameters may be modified by overlay. Once modified, it stays modified throughout a run. The overlay format and procedure are described in paragraph 2.2.3.1. The array PCNTRL is described in Table 5-3 below.

TABLE 5-3

PLOT CONTROL PARAMETERS

Location	Symbol	Definition
1	PTIME	Plot time increment
2	PSCALE	Residual scale factor
3 to 6	PMAX(4)	Residual scale limits, for each residual type

In initializing the plot, the scale limits (DATMAX) are set equal to the corresponding component of PMAX. If PSCALE $\neq$ 0 and an estimate tape record has been read, the plot scale limits (DATMAX) are reset to PSCALE* (standard deviation for each measurement type).

The time increment for plotting, DT, is

$$DT = \begin{cases} \text{PTIME if PTIME} > 0 \\ \text{time interval between stored residuals if PTIME} \leq 0. \end{cases}$$

REFERENCES

1. Subroutine Descriptions and Listings for the Orbit Determination Program," Philco-Ford Corporation, TR-DA1509, Palo Alto, California, December 1967.

2. "Input-Output Summary for the Orbit Determination Program," Philco-Ford Corporation, TR-DA1510, Palo Alto, California, December 1967.

APPENDIX A

COORDINATE SYSTEMS AND TRANSFORMATIONS

A.1 INTRODUCTION

A number of coordinate systems are used by the Orbit Determination Program. Each has advantages in certain of the computations or in input or output. The use of the most natural coordinate system for each phase of the computations with transformation as required to other coordinate systems simplifies the writing and programming of the necessary equations and the interpretation of results.

Although the program system is designed primarily for Earth orbital and Earth-Moon flights, it may also be used for interplanetary trajectories. In order to admit such general application, each of the coordinate systems is so defined as to perform the same function for any central body. Since most tracking data for any flight is from Earth-based stations, some duplication is required to provide both central body reference and Earth reference.

A.2 GENERAL

A.2.1 Notation

Each of the cartesian reference frames described below is a right-handed frame. Each is given a letter designation and vectors referred to their components in the frame are subscripted with that letter. That is, X_B is a column of the components of the vector $\underline{X}$ in the B-frame.

The components of a vector in one frame are related to the components of that vector in another frame by an orthogonal transformation. If X_B, X_R are column vectors of the components of $\underline{X}$ in the B- and R-frames, respectively, we write

$$X_R = T_{B2R} X_B$$

$$X_B = T_{B2R}^T X_R$$

$$T_{R2B} = T_{B2R}^T \qquad \text{(A. 2-1)}$$

where $(\ )^T$ denotes the transpose of the matrix ( ).

A. 2. 2 Properties of the Transformation

The transformation T_{B2R} has a number of properties which are useful in its computation or use. If we denote the unit vectors of the B-frame by $\underline{i}_B$, $\underline{j}_B$, $\underline{k}_B$, etc., the transformation may be written as the row of column vectors

$$T_{B2R} = \left[i_{BR}, j_{BR}, k_{BR} \right] \qquad \text{(A. 2-2)}$$

or as the column of row vectors

$$T_{B2R} = \begin{bmatrix} i_{RB}^T \\ j_{RB}^T \\ k_{RB}^T \end{bmatrix} \qquad \text{(A. 2-3)}$$

Normally, the most natural way of describing the transformation T_{B2R} is to specify a sequence of rotations about the coordinate axes which carry the B-frame axes into the R-frame axes. We write

$$T_1(\alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{bmatrix}$$

$$T_2(\alpha) = \begin{bmatrix} \cos\alpha & 0 & -\sin\alpha \\ 0 & 1 & 0 \\ \sin\alpha & 0 & \cos\alpha \end{bmatrix}$$

$$T_3(\alpha) = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \text{(A. 2-4)}$$

for the transformations corresponding to rotation through the angle α about the X, Y, Z axes, respectively.

Consider for example the Euler angle transformation shown in Figure A-1.

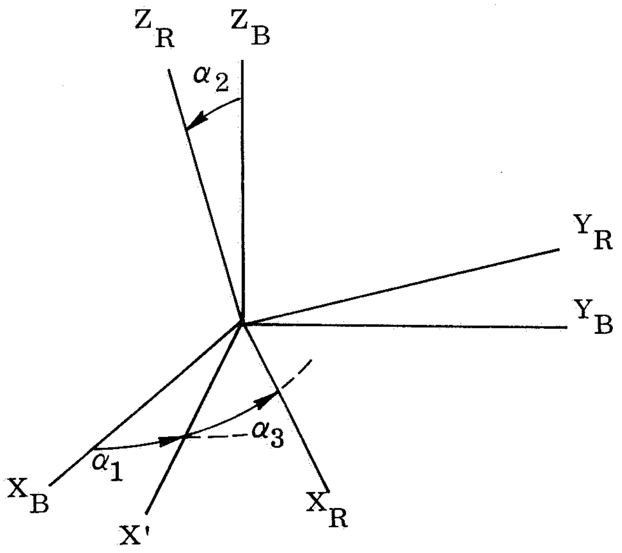


Figure A-1 Euler Angle Transformation

The rotation α_1 about the Z_B-axis carries the X_B-axis into the line of nodes, X'. The rotation α_2 about the line of nodes moves the Z-axis into the Z_R-axis, and the rotation α_3 about the Z_R-axis establishes the X_R and Y_R axes. The transformation T_{B2R} is computed from

$$T_{B2R} = T_3(\alpha_3)\, T_1(\alpha_2)\, T_3(\alpha_1) \tag{A.2-5}$$

Any sequence of rotations may be replaced by a single rotation about an appropriately chosen axis. We consider the rotation of an arbitrary vector $\underline{a}$ about the unit vector $\underline{w}$ through the angle α. We write

$$\underline{a} = \underline{w}\,(\underline{w} \cdot \underline{a}) + \underline{b} \tag{A.2-6}$$

where $\underline{b}$ is the projection of $\underline{a}$ in the plane normal to $\underline{w}$. Then if we take $\underline{a}'$ as the image of $\underline{a}$ rotated through α, we obtain

$$\underline{a}' = \underline{w}(\underline{w} \cdot \underline{a}) + \underline{b} \cos \alpha + \underline{w} \times \underline{b} \sin \alpha \tag{A.2-7}$$

since

$$\underline{w} \times \underline{a} = \underline{w} \times \underline{b}$$

$$\underline{b} = (\underline{w} \times \underline{b}) \times \underline{w} = \underline{a} - \underline{w}(\underline{w} \cdot \underline{a}) \tag{A.2-8}$$

Writing w, a in terms of B-frame components, and defining the matrix

$$W_BX = \begin{bmatrix} 0 & -W_{BZ} & W_{BY} \\ W_{BZ} & 0 & -W_{BX} \\ -W_{BY} & W_{BX} & 0 \end{bmatrix} \qquad \text{(A. 2-9)}$$

we have

$$a_B' = \left\{ \cos \alpha I + (1-\cos \alpha) W_B W_B^T + \sin \alpha (W_B X) \right\} a_B \qquad \text{(A. 2-10)}$$

Now if we rotate $\underline{i}_B$, $\underline{j}_B$, $\underline{k}_B$, where

$$i_{BB} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad j_{BB} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad k_{BB} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \qquad \text{(A. 2-11)}$$

we obtain i_R, j_R, k_R, respectively, where i_{RB}, j_{RB}, k_{RB} are the columns of the bracketted expression in (A. 2-10). We have immediately from (A. 2-3)

$$T_{B2R} = \cos \alpha I + (1-\cos \alpha) W_B W_B^T - \sin \alpha (W_B X) \qquad \text{(A. 2-12)}$$

A. 3 TIME SYSTEMS

It is necessary to differentiate between four methods of time-keeping used by the program system. Definitions of the time scales as given by References 3. (pp. 21, 22, 66-95) and 4. are included for completeness, and all simplifying assumptions used by the program system are noted.

A. 3. 1 Notation

The symbols t, d, T are used to denote intervals of time. Their units are:

t: seconds; or days, hours minutes, seconds
d: days (d = t/86400, t = 86400d)
T: Julian centuries of 36525 days (d = 36525T, T = d/36525).

Where the context of its use does not clearly indicate the time scale on which the interval is measured, a literal subscript is used to denote the scale. For example, t_E and T_U denote, respectively, values in ephemeris seconds and Julian centuries of universal time.

Epochs are writen T_j, where j is a numerical subscript. In particular, the subscript "o" will denote the fundamental epoch for definition of the time scale. That is, T_{Uo} is the fundamental epoch for definition of universal time. Where convenient, t_j will also be used to denote epochs.

The epochs of the various time scales are described by calendar date or Julian date. The accepted notation is used throughout. For example

$$\begin{aligned} &1900 \text{ January } 0^d12^h \\ &= 1900 \text{ January } 0,\ 12^h \\ &= 1900 \text{ January } 0.5 \\ &= \text{JD } 2415020.0 \end{aligned}$$

denotes the instant of noon on January 1, 1900 A. D. The symbol JD is used to denote the Julian date of an epoch, and unless otherwise noted, it will imply universal time. JED will be used to specify Julian date in ephemeris time.

A. 3. 2 Time Scales and Fundamental Epochs

A. 3. 2. 1 Ephemeris Time (ET). Ephemeris time is defined to be the uniform measure of time which is the independent variable of the equations of motion of the planets, including the effects of relativity. The standard of measure, then, is the motion of the planets, and the relationship with other time standards must be inferred from observations of planetary motion.

The fundamental epoch from which ephemeris time is measured is

$$T_{Eo} = 1900 \text{ January } 0.5 \text{ ET} = \text{JED } 2415020.0 \qquad \text{(A.3-1)}$$

A.3.2.2 Universal Time (UT). Universal time is the basis of all civil time-keeping. It is defined as 12 hours plus the Greenwich hour angle of a point on the true equator whose right ascension measured from the mean equinox of date is

$$R_U = 18^h 38^m 45\overset{s}{.}836 + 8,640,184\overset{s}{.}542 T_U + 0\overset{s}{.}0929 T_U^2 \qquad \text{(A.3-2)}$$

Irregularities in the Earth's rotation rate cause corresponding irregularities in the time scale. These irregularities are smoothed in a sequence of corrections with the following corresponding time scales:

UT0: universal time as measured by zenith transits of stars, using nominal longitude of the observing station, corrected for nutation in right ascension.

UT1: UT0 corrected for the migrations of Earth's polar axis.

UT2: UT1 corrected for predictable seasonal fluctuations.

UTC: An approximation of UT2 obtained from a frequency standard with step changes as required to maintain the approximation.

The fundamental epoch for the measurement of universal time is

$$T_{Uo} = 1900 \text{ January } 0.5 \text{ UT} = \text{JD } 2415020.0 \qquad \text{(A.3-3)}$$

For our purposes, universal time is used primarily as an intermediate time scale in determining tracking station locations and in relating tracking station clocks to ephemeris time. Unless otherwise noted, UT will denote UT1. We will sometimes use the term "civil time" for UTC, and use the subscript "C" to distinguish civil time from UT1.

A.3.2.3 Atomic Time (A.1). Atomic time is obtained from the U.S. Frequency Standard, a cesium resonator. The atomic second is defined as 9,192,631,770 cycles of cesium, the best current estimate of the length of the ephemeris second. The value of A.1 was set equal to UT2 at the epoch

$$T_{Ao} = 1958 \text{ January } 0.0 \text{ UT2} = \text{JD } 2436204.5 \qquad \text{(A.3-4)}$$

A.3.2.4 Station Time (ST). Station time is measured at each station using its own frequency standard. Differences between ST and UTC may be monitored by the individual stations.

A.3.3 Transformations Between Time Scales

Ephemeris time, by its definition, is coordinate time for the heliocentric system. The remaining time scales are all measures of Earth's proper time. We take atomic time as the basic measure of Earth's proper time, and assume the transformations.

$$\begin{aligned} t_A - t_S &= \Delta t_{AS} = C_1 + C_2 t \\ t_A - t_U &= \Delta t_{AU} = C_3 + C_4 t \\ t_A - t_C &= \Delta t_{AC} = C_5 + C_6 t \qquad \text{(A.3-5)} \end{aligned}$$

where in each equation, t may represent either of the subscripted t's on the left-hand side, and both t's on the left must be measured from the same epoch.

The coefficient C_6 is set annually by international agreement, and periodic changes in C_5 are specified by the U.S. Naval Observatory. These changes are made on the first day of the month. The coefficients C_3, C_4 may be obtained by fitting values of Δt_{AU} published by the U. S. Naval Observatory. The coefficients C_1, C_2 describe station clock errors, and differ from station to station. Either or both constants may be included in the set of uncertain biases for a given station (that is, they may be solved for in the orbit determination).

The transformation from atomic time to ephemeris time as used by the ODP is

$$t_E - t_A = \Delta T_{EA} + \frac{86,400\, d_A}{9,192,631,770} \Delta f \qquad \text{(A. 3-6)}$$

where

1 ephemeris second

$= (9,192,631,770 + \Delta f)$ cycles of cesium

$\Delta T_{EA} = \left[t_E - t_{UT2}\right] T_{Ao}$

$d_A = d_E - 21184.5$

d_E = days elapsed since T_{Eo}.

Equation (A. 3-6) neglects the difference between Earth's proper time and heliocentric coordinate time, a periodic term of amplitude 1.658×10^{-3} seconds (see Reference 3, p 37).

We may write (A. 3-6) in the form

$$t_E - t_A = \Delta t_{EA} = C_7 + C_8 t$$

$$C_8 = \Delta f / 9,192,631,770$$

$$C_7 = \Delta T_{EA} + C_8\, (86,400)(d_E - 21184.5) \qquad \text{(A. 3-7)}$$

with d_E measured to the epoch from which t_E, t_A are measured.

In the ODP, ephemeris time is used as the independent variable for the space vehicles equations of motion and for planetary position determination. Universal time (UT1) is used in the determination of Earth's orientation, and station time is used to identify observation times. The epoch used for all internal time calculations is 1950 January 0.0 ET (T_{E1}).

The transformations between time scales are used in the form

$$t_U - t_S = \Delta t_{US} = \gamma_1 + \gamma_2 t$$

$$t_E - t_U = \Delta t_{EU} = \gamma_3 + \gamma_4 t \qquad \text{(A. 3-8)}$$

The γ's may be solved for in the orbit determination, using the a priori values

$$\begin{aligned} \gamma_1 &= C_1 - C_3 \\ \gamma_2 &= C_2 - C_4 \\ \gamma_3 &= C_3 + C_7 \\ \gamma_4 &= C_4 + C_8 \end{aligned} \qquad \text{(A. 3-9)}$$

A. 4 SPATIAL COORDINATES

The coordinate systems used by the ODP are described below. All are defined as right-handed cartesian systems, though in some cases the angles defining the orientation of the coordinate axes rather than the components of a vector referred to those axes are the quantities of interest. They include the systems used computationally, those used for input or output, and those principal intermediate frames used in passing from one frame to another.

Position vectors will normally be defined as the position relative to a body center or other well-defined point rather than a coordinate frame origin. The reference frames are used primarily for the coordinatization of vectors, and hence no distinction is made between "stationary" reference frames and those with accelerating origins.

A. 4. 1 Central Coordinate Systems

The coordinate systems below have their origin at the central body center.

A. 4. 1. 1 C-Frame (Earth's Mean Equator, Equinox of 1950). The C-frame is an inertial reference frame with axes through the mean vernal equinox, summer solstice, and north pole of 1950. 0, respectively. It is the basic reference frame for all position, velocity, and orientation calculations in the ODP.

A. 4. 1. 2 D-Frame (Earth's True Equator, Equinox of Date). The D-frame is a rotating reference frame with axes through the true vernal equinox, summer solstice, and north pole of date, respectively. It is used as an intermediate reference frame for locating Earth-based tracking stations and for input and output. It is obtained from the C-frame in two steps as follows.

The precession of the Earth's mean equator, equinox of date is accounted for in the transformation

$$T_{C2D_o} = T_W(\alpha) \qquad \text{(A. 4-1)}$$

The eigenvector of the transformation, W, and the quantities sin α, cos α together with the mean obliquity of the ecliptic, $\bar{\epsilon}$, are computed as functions of t_E by DEQTR.

The nutations in longitude and obliquity, δL, $\delta\epsilon$, are computed as functions of t_E by DPFMRS, and the transformation T_{C2D} is computed from

$$T_{C2D} = T_{Do2D}\, T_{C2Do}$$

$$T_{Do2D} = T_1(-\bar{\epsilon}-\delta\epsilon)\, T_3 \;\; (-\delta L)\, T_1(\bar{\epsilon}) \qquad \text{(A. 4-2)}$$

A.4.1.3 B-Frame (Body-Fixed). The B-frame is a body-fixed reference frame with Z-axis through the north pole and X-axis through the prime meridian. It is used as an intermediate reference frame for locating tracking stations.

For Earth-based tracking, the B-frame is obtained from the D-frame by a rotation about the z-axis through the right ascension of the prime meridian, Υ. That is,

$$T_{D2B} = T_3 (\Upsilon) \qquad (A.4-3)$$

where Υ and the Earth's angular velocity, ω_E, are computed as functions of t_U by DEHA.

For tracking of lunar beacons, the transformation T_{D2B} and the moon's angular velocity, ω_M, are computed as functions of t_E by DLUNE, using the ephemerides of Sun and Moon from Reference 2 (pp 98, 107, 108).

A.4.1.4 E-Frame (Ecliptic, Equinox). The ecliptic, vernal equinox equivalents of the C and D frames are available for input and output. The transformations

$$T_{C2E50} = T_1(\epsilon_{50})$$

$$T_{C2E} = T_1(\epsilon) \qquad (A.4-4)$$

where the subscript "50" denotes the epoch $t_E = 0$, are computed from the obliquities computed by DEQTR and DPFMRS.

A.4.2 Local Coordinate Systems

The coordinate systems described below are centered at the space vehicle or tracking station.

A.4.2.1 L-Frame (Geocentric). The axes of the L-frame are taken along the geocentric radius vector, positive outward, the normal to the local meridian plane, positive eastward, and along the local meridian, positive north (see Figure A-2). The L-frame is used for input and output. The transformation defining the L-frame is

$$T_{EZL} = T_2 (-\phi) T_3 (\lambda) \qquad (A.4\text{-}5)$$

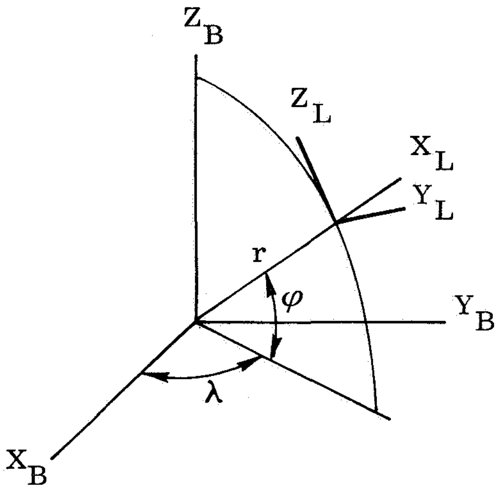


Figure A-2 Polar Coordinates

The polar coordinates of the vehicle (or stations) are

r = geocentric distance to the vehicle
ϕ = declination of the geocentric position vector
λ = longitude of the position vector, measured from the prime meridian

and the corresponding polar form of the velocity is

V = magnitude of the velocity
γ_L = flight path angle (inclination of V with the plane $X_L = 0$)
A_L = Azimuth, measured clockwise from north.

A.4.2.2 T-Frame (Topocentric). The axes of the T-frame are taken along the local meridian, positive north, normal to the local meridian, positive east, and normal to the reference spheroid, positive down, respectively. The transformation defining the T-frame is

$$T_{B2T} = T_2\ [-(\pi/2 + \phi_g)]\ T_3(\lambda) \qquad \text{(A.4-6)}$$

where the geodetic coordinates of the origin are

h = altitude, the distance from the reference spheroid, measured along the -Z-axis,

ϕ_g = declination of the local geodetic vertical (outward normal to the reference spheroid),

λ = longitude, from the prime meridian.

The T-frame is used for the specification of tracking station location errors.

REFERENCES

1. "Subroutine Descriptions and Listings for the Orbit Determination Program," Space and R-entry Systems Division, Philco-Ford Corporation, TR-DA1509, Palo Alto, California, December 1967.

2. "Input-Output Summary for the Orbit Determination Program," Space and Re-entry Systems Division, Philco-Ford Corporation, TR-DA1510, Palo Alto, California, December 1967.

3. "Explanatory Supplement to the Ephemeris," Her Majesty's Stationery Office, London, 1961.

4. Moyer, T.D., "Theoretical Basis for the DPODP: Time Transformations," JPL Space Programs Summary No. 37-39, Vol. III, pp 36-38.

APPENDIX B

ESTIMATION PROCEDURE

B. 1 INTRODUCTION

The function of the Orbit Determination Program (ODP) is the statistical estimation of the state of a spacecraft from a collection of measurements of observable quantities. That is, we wish to determine that solution of the spacecraft equations of motion which provides a "best" fit to the measurement data, where "best" is defined by some statistical criterion.

The measurement of observables at one point in time is normally inadequate for the determination of the spacecraft state at that time. The number of observables is usually less than the number of state variables, and the precision of the measurements is less than the precision required in the state determination. It becomes necessary to provide some dynamic model to relate the observables over some time interval, and to fit the collection of measurements on that interval to the selected model. The most productive dynamic model, of course, is a solution of the equations of motion of the spacecraft, containing all known appreciable effects.

The estimation of uncertain parameters of the equations of motion and of the measurements of observables is usually an integral part of the estimation of state. These uncertainties, as well as the effects which are omitted from the dynamic or observation models, cause a degradation in the accuracy of state determination and errors in the prediction of spacecraft motion. In post-flight analysis, particularly, we are interested in identifying discrepancies between predicted and observed behavior, in order to remove these discrepancies from future predictions.

The estimation procedure used by the ODP is a recursive procedure based on the method derived by Kalman (3) for linear systems. The method was applied to nonlinear systems through linearization of the equations of motion and the measurements about the current best estimate of the state by Schmidt and others (4, 5, 6). The specific procedure used is a process using modified weighting proposed by Schmidt (7), written for the estimation of state at selected "anchor points." It is derived in B. 2 below, and its mechanization in the ODP is described in B. 3.

The recursive "filter" requires a starting point; including estimates of

- initial spacecraft state
- parameters affecting spacecraft motion
- parameters affecting tracking measurements

and an initial weighting matrix reflecting the relative uncertainties in these quantities. A procedure for the determination of approximate initial estimates of the spacecraft state from a small sample of measurements is discussed in B. 4.

B. 2 DERIVATION OF THE FILTER

B. 2. 1 Notation and Definitions

We consider any system whose equations of state may be written as the vector first-order ordinary differential equation

$$\dot{X} = F(X, U, t) \tag{B. 2-1}$$

where the "state vector", X, includes all those quantities of interest whose behavior is so described and the vector U includes those constants of interest which affect the state history, X(t), except the initial conditions, $X_o = X(t_o)$. We call U the vector of "equation of motion parameters", since for the problem at hand, the equations (B. 2-1) are the equations of motion of a space vehicle.

We let G(X,V,t) be a vector of "observables;" that is; a vector of quantities related to the state for which values may be measured. The vector V contains those constants of interest which affect the observables. We write the measured values of the observables, or measurements, in the form

$$Y = G(X, V, t) + q \tag{B. 2-2}$$

where q is a random error in the measured value. We call V the vector of "measurement parameters."

We will wish to consider the effects of small changes in X, U, V. It is convenient to use an ordered array, or matrix, of partial derivatives to simplify the required equations.

Let f(X, U) be a scalar function of the scalars X, U. We may expand f(X, U) in Taylor's series about $\overline{X}, \overline{U}$, retaining only terms linear in x, u, where

$$\begin{aligned} X &= \overline{X} + x \\ U &= \overline{U} + u \end{aligned} \qquad (B.2\text{-}3)$$

obtaining

$$f(X, U) = f(\overline{X}, \overline{U}) + \frac{\partial f}{\partial X} x + \frac{\partial f}{\partial U} u \qquad (B.2\text{-}4)$$

where the partial derivatives are evaluated at $\overline{X}, \overline{U}$.

If X, U are the vectors

$$\begin{aligned} X &= \text{col}\ (X_1, X_2, \cdots, X_n) \\ U &= \text{col}\ (U_1, U_2, \cdots, U_m) \end{aligned} \qquad (B.2\text{-}5)$$

we again obtain (B. 2-4) if we use the symbols

$$\begin{aligned} \frac{\partial f}{\partial X} &= \left(\frac{\partial f}{\partial X_1}, \frac{\partial f}{\partial X_2}, \cdots, \frac{\partial f}{\partial X_n}\right) \\ \frac{\partial f}{\partial U} &= \left(\frac{\partial f}{\partial U_1}, \frac{\partial f}{\partial U_2}, \cdots, \frac{\partial f}{\partial U_m}\right) \end{aligned} \qquad (B.2\text{-}6)$$

and take the matrix product $\frac{\partial f}{\partial X} x$. We may also use the gradient symbol

$$\nabla_x f = \frac{\partial f}{\partial X} \qquad (B.2\text{-}7)$$

Now if f(X, U) is the vector

$$f = \text{col}\ (f_1, f_2, \cdots, f_\ell) \qquad \text{(B. 2-8)}$$

we may once again write (B. 2-4) for the linear variation in f if we define the gradient symbols by

$$\frac{\partial f}{\partial X} = \begin{bmatrix} \frac{\partial f_1}{\partial X_1} & \frac{\partial f_1}{\partial X_2} & \cdots & \frac{\partial f_1}{\partial X_n} \\ \frac{\partial f_2}{\partial X_1} & \frac{\partial f_2}{\partial X_2} & \cdots & \frac{\partial f_2}{\partial X_n} \\ \vdots & & & \\ \frac{\partial f_\ell}{\partial X_1} & \frac{\partial f_\ell}{\partial X_2} & \cdots & \frac{\partial f_\ell}{\partial X_n} \end{bmatrix} \qquad \text{(B. 2-9)}$$

and again use matrix products. Note that the gradient (B. 2-9) obeys the chain rule of differentiation

$$\frac{\partial f}{\partial \alpha} = \frac{\partial f}{\partial X}\ \frac{\partial X}{\partial \alpha} + \frac{\partial f}{\partial U}\ \frac{\partial U}{\partial \alpha} \qquad \text{(B. 2-10)}$$

The expected value of the scalar function f(x) is

$$E\ [f(x)] = \int_{-\infty}^{\infty} f(x)\ p(x)\ dx \qquad \text{(B. 2-11)}$$

where p(x) is the probability density function

$$\int_a^b p(x)\, dx = \text{probability that } a \le x \le b$$

$$\int_{-\infty}^{\infty} p(x)\, dx = 1$$

The gaussian or normal density function is

$$p(x) = \frac{\ell}{\sqrt{2\pi}\;\sigma}\; e^{-(x-\bar{x})^2/2\sigma^2} \tag{B. 2-12}$$

where

$\bar{x} = E[x] =$ mean value of x

$\sigma^2 = E[(x-\bar{x})^2] =$ variance of x

$\sigma = \sqrt{\sigma^2}$

For a vector

$$x = \text{col}\,(x_1, x_2, \cdots, x_n)$$

the function $p(x) = p(x_1, x_2, \cdots, x_n)$ is the joint probability density function

$$\int_{a_1}^{b_1}\int_{a_2}^{b_2}\cdots\int_{a_n}^{b_n} p(x)\,dx_n \cdots dx_2\,dx_1$$

= probability that simultaneously

$$\begin{aligned} a_1 &\le x_1 \le b_1, \\ a_2 &\le x_2 \le b_2, \\ &\;\;\vdots \\ a_n &\le x_n \le b_n \end{aligned}$$

The gaussian joint density function is

$$p(x) = (2\pi)^{-n/2}\,|P|^{-1/2}\; e^{-\frac{1}{2}(x-\bar{x})^T P^{-1}(x-\bar{x})} \tag{B. 2-13}$$

where

$\bar{x} = E[x] = \text{col}\left(E[x_1], \cdots, E[x_n]\right)$

$P = E\left[(x-\bar{x})(x-\bar{x})^T\right]$

$|P| =$ determinant of P

Since $E[(x_i-\bar{x}_i)(x_j-\bar{x}_j)]$ is the covariance of x_i and x_j, we call P the covariance matrix. It is symetric ($P_{ij} = P_{ji}$) and positive semi-definite ($a^T Pa \geq 0$ for any vector $a \neq 0$).

B.2.2 Linearized Equations

Closed-form solutions, X(t), are not available for most problems of interest. For a given set of initial conditions, $\bar{X}_o$, and parameter, $\bar{U}$, we may integrate

$$\dot{\bar{X}} = F(\bar{X}, \bar{U}, t) \tag{B.2-14}$$

numerically, obtaining a "nominal" trajectory, $\bar{X}(t)$. Let

$$\begin{aligned} X &= \bar{X} + x \\ U &= \bar{U} + u \\ V &= \bar{V} + v \end{aligned} \tag{B.2-15}$$

If we expand F(X, U, t) in Taylor's series about $\bar{X}(t), \bar{U}$, retaining only first order terms in the deviations x(t), u, v we obtain the "variational equations".

$$\begin{aligned} \dot{x} &= \frac{\partial F}{\partial X} x + \frac{\partial F}{\partial U} u \\ y &= Y(X, V, t) - Y(\bar{X}, \bar{V}, t) \\ &= \frac{\partial Y}{\partial X} x + \frac{\partial Y}{\partial V} v \end{aligned} \tag{B.2-16}$$

where the partial derivatives are evaluated at $\bar{X}(t)$, $\bar{U}, \bar{V}$. Note that the derivatives are functions of time only.

Let $\varphi(t;t_o)$ be an nxn matrix whose columns are the linearly independent solutions of the homogeneous equation

$$\dot{\varphi}(t;t_0) = \frac{\partial F}{\partial X} \varphi(t;t_0)$$

$$\varphi(t_0;t_0) = I \tag{B.2-17}$$

where I is the identity matrix. The solution for the nonhomogeneous equation is

$$x(t) = \varphi(t;t_0)\, x_0 + U(t;t_0)\, u$$

$$\dot{U}(t;t_0) = \frac{\partial F}{\partial X} U(t;t_0) + \frac{\partial F}{\partial U}$$

$$U(t_0;t_0) = 0 \tag{B.2-18}$$

for u constant on the interval (t_0, t).

If

$$\bar{x}_0 = E\,[x(t_0)]$$

$$P_0 = E\left[\left(x(t_0)-\bar{x}_0\right)\left(x(t_0)-\bar{x}_0\right)^T\right]$$

$$C_0 = E\left[\left(x(t_0)-\bar{x}_0\right)(u-\bar{u})^T\right]$$

$$Q_0 = E\left[(u-\bar{u})(u-\bar{u})^T\right] \tag{B.2-19}$$

we have immediately

$$\bar{x}(t) = \varphi(t;t_0)\, \bar{x}_0 + U(t;t_0)\, \bar{u}$$

$$\begin{bmatrix} P(t) & C(t) \\ C^T(t) & Q(t) \end{bmatrix} = \begin{bmatrix} \varphi & U \\ \phi & I \end{bmatrix} \begin{bmatrix} P_0 & C_0 \\ C_0{}^T & Q_0 \end{bmatrix} \begin{bmatrix} \varphi^T & \phi \\ U^T & I \end{bmatrix} \tag{B.2-20}$$

B.2.3 Kalman's Filter

In the orbit determination problem, we wish to use measured values of the observables at discrete values of t to determine the trajectory, x(t), and possibly certain equations of motion and measurement parameters, U, V. The equations above provide for the computation of errors and their uncertainties at time t_1 arising from errors and uncertainties at time t_0. We are left with the problem of determining corrections to $x(t_1)$, U, V (or equivalently, $x(t_0)$, U, V) from the measurements.

Let us define for convenience an extended state vector

$$Z = \begin{bmatrix} X \\ U \\ V \end{bmatrix} \qquad \text{(B. 2-21)}$$

where

$$\begin{aligned} \dot{X} &= F(Z,t) \\ Y &= G(Z,t) + q \end{aligned} \qquad \text{(B. 2-22)}$$

If $\overline{Z}$ denotes the collection of nominal values and

$$Z = \overline{Z} + z \qquad \text{(B 2-23)}$$

we have

$$z(t) = \Phi(t;t_0)z(t_0)$$

$$\Phi(t;t_0) = \begin{bmatrix} \varphi(t;t_0) & U(t;t_0) & \emptyset \\ \emptyset & I & \emptyset \\ \emptyset & \emptyset & I \end{bmatrix} \qquad \text{(B. 2-24)}$$

Let

$$\begin{aligned} \hat{Z} &= \overline{Z} + \hat{z} \\ P(t) &= E\left[(z-\hat{z})(z-\hat{z})^T\right] \end{aligned} \qquad \text{(B. 2-25)}$$

denote an estimate of the extended state and its covariance. Let

$$\hat{Y} = G(\hat{Z}, t) \qquad \text{(B. 2-26)}$$

be the value of the observable computed at this estimate. If z is the true deviation, then the observed value of the measurement is

$$\begin{aligned} Y &= \hat{Y} + H(t)(z-\hat{z}) + q \\ H(t) &= \frac{\partial G}{\partial Z} \end{aligned} \qquad \text{(B. 2-27)}$$

Let

$$\begin{aligned} E\,[q] &= 0 \\ E\left[qq^T\right] &= Q \\ E\left[zq^T\right] &= 0 \end{aligned} \qquad \text{(B. 2-28)}$$

We seek a new estimate $\hat{z}_n$ which combines the information contained in $\hat{z}$ and in Y. In particular, we seek that estimate for which

$$L = E\left[(z-\hat{z}_n)^T(z-\hat{z}_n)\right] \qquad \text{(B. 2-29)}$$

is minimized.

For any linear filter, K,

$$\begin{aligned} \hat{z}_n &= \hat{z} + K(Y-\hat{Y}) \\ &= \hat{z} + KH(z-\hat{z}) + Kq \end{aligned} \qquad \text{(B. 2-30)}$$

we may compute the variance

$$\begin{aligned} P_n &= E\,[(z-\hat{z}_n)(z-\hat{z}_n)] \\ &= E\left[\left((I-KH)(z-\hat{z}) - Kq\right)\left((I-KH)(z-\hat{z}) - Kq\right)^T\right] \\ &= (I-KH)\,E\left[(z-\hat{z})(z-\hat{z})^T\right](I-KH)^T + KE\left[qq^T\right]K^T \end{aligned} \qquad \text{(B. 2-31)}$$

since by assumption $E\left[(z-\hat{z})\, q^T\right] = 0$. Then

$$P_n = P-KHP-PH^TK^T + K(HPH^T+Q)K^T \qquad (B.2-32)$$

For a scalar K, P, H, it is easily seen that P_n is minimized by

$$K = PH^T(HPH^T+Q)^{-1} \qquad (B.2-33)$$

For the matrix equation, we may set

$$K = PH^T(HPH^T+Q)^{-1} + A \qquad (B.2-34)$$

where A can be any matrix. The resulting covariance is

$$P_n = P - PH^T(HPH^T+Q)^{-1}HP + A(HPH^T+Q)\, A^T \qquad (B.2-35)$$

Now the loss function (B.2-29) is the trace of P_n. Since HPH^T+Q is the covariance of the measurement deviation $Y-\hat{Y}$, it is positive semi-definite, and since the diagonal elements of the last term are

$$A_i\, (HPH^T + Q)\, A_i^T \geq 0 \qquad (B.2-36)$$

where A_i is the ith row of A, the trace of P_n is minimized by the choice $A = 0$. That is, of all linear filters (B.2-30) that one which minimizes the loss function (B.2-29) is the filter

$$\begin{aligned} \hat{Z}_n &= \hat{Z} + PH^T(HPH^T + Q)^{-1}\, (Y-\hat{Y}) \\ P_n &= P - PH^T\, (HPH^T + Q)^{-1}\, HP \qquad (B.2-37) \\ \hat{Y} &= G(\hat{X}, \hat{U}, t) \end{aligned}$$

If instead of assuming a linear filter, we assume that the errors are gaussian, we arrive again at (B.2-37) as the optimal filter (see, for example, the derivation of Reference 8).

We may interpret $P_n(t)$ as the covariance matrix of errors in the estimate of state, $\hat{Z}_n$, provided that

- The deviations z, $\hat{z}$ are sufficiently small that the linearizations (B. 2-16) are valid,
- The equations of state (B. 2-1) and measurement equations (B. 2-2) properly model all appreciable phenomena, and
- $E(z) = \hat{z}_n$.

B. 2. 4 Anchor Point Estimation

The equations of B. 2. 3 are written for the estimation of the current state, Z(t). As we remarked earlier, this is entirely equivalent to the estimation of state at a fixed time, $Z(t_1)$, since Z(t), $Z(t_1)$ are uniquely related by the equations of state. The estimation of $Z(t_1)$, however, has some computational advantages, and we now considered the form taken by the equations for that estimation. We will call t_1 the "anchor point".

We set

$$Z(t_1) = \overline{Z}(t_1) + z_1$$

$$\hat{Z}(t_1) = \overline{Z}(t_i) + \hat{z}_1 \qquad \text{(B. 2-38)}$$

$$P_1 = P(t_1)$$

and assume that the trajectory $\overline{X}(t)$,

$$\dot{\overline{X}}(t) = F(\overline{Z}, t)$$

$$Z(t_1) = \begin{bmatrix} \overline{X}(t_1) \\ \overline{U} \\ \overline{V} \end{bmatrix} \qquad \text{(B. 2-39)}$$

is known, as are $\varphi(t; t_1)$, $U(t;t_1)$ and hence $\Phi(t;t_1)$. From (B.2-20), (B.2-24),

$$
\begin{aligned}
z(t) &= \Phi(t;t_i)\, z_1 \\
\hat{z}(t) &= \Phi(t;t_1)\, \hat{z}_1 \qquad \text{(B. 2-40)} \\
P(t) &= \Phi(t;t_1) P_i\, \Phi^T(t;t_i)
\end{aligned}
$$

If we set

$$H_1 = \frac{\partial G\ (\overline{Z}, t)}{\partial Z\ (t_1)} \qquad \text{(B. 2-41)}$$

and use the chain rule of differentiation,

$$
\begin{aligned}
H_1 &= \frac{\partial G\ (\overline{Z}, t)}{\partial Z\ (t)} \frac{\partial Z\ (t)}{\partial Z\ (t_1)} \\
&= H(t)\ \Phi\ (t;t_1) \qquad \text{(B. 2-42)}
\end{aligned}
$$

Substituting equations (B.2-40) into the filter equations (B.2-37),

$$
\begin{aligned}
\hat{z}_{1n} &= \Phi^{-1}\ (t;t_1)\ \hat{z}_n \\
&= \Phi^{-1}\{ \hat{z} + \Phi P_1 \Phi^T H^T\ (H \Phi P_1 \Phi^T H^T + Q)^{-1}\ (Y - \hat{Y})\} \\
&= \hat{z}_1 + P_1 H_1^T\ (H_1 P_1 H_1^T + Q)^{-1}\ (Y - \hat{Y}) \\
P_{1n} &= \Phi^{-1}\{ P - PH^T\ (HPH^T + Q)^{-1}\ HP \}\, \Phi^{-1\ T} \qquad \text{(B. 2-43)} \\
&= P_1 - P_1 H_1^T\ (H_1 P_1 H_1^T + Q)^{-1}\ H_1 P_1 \\
\hat{Y} &= \overline{Y} + H \hat{z}(t) \\
&= \overline{Y} + H_1 \hat{z}_1
\end{aligned}
$$

Note that for estimation at t_1, it is not necessary to compute P(t) from (B. 2-20) at each measurement, but instead we compute H_1 from the simpler (B. 2-42). In addition, the corrections to $\hat{Z}(t_1)$ are more easily interpreted, and the convergence of the estimate is more easily examined.

One of the principal advantages of the anchor point formulation is the explicit use of the estimate as the initial conditions for integration of the equations of state. In using the Kalman, or minimum variance, filter, we normally consider that the equations are linearized about the current best estimate of the state. This approach, however, would require numerical integration of the equations of state from each observation time to the next, with the integration restarted after the observations. This is a wasteful process even for the simplest equations, and for the models used by the ODP, it becomes prohibitive.

We observe that if the linearizations (B. 2-16) adequately represent the variations in F, Y, the nominal state $\overline{\overline{Z}}(t)$ has no function other than to serve as a reference point for measuring the deviations. Then, once a nominal trajectory is integrated, there is no point in establishing a new nominal until the deviations exceed some linearity bounds.

B. 2. 5 Non-Optimal Filter

As we noted earlier, the equations (B. 2-20) and (B. 2-24) for updating $\hat{Z}$, P in time are valid only if all appreciable effects in the equations of motion are adequately modeled. Omissions or inadequate modeling will normally result in a divergence of $\hat{Z}$ from Z which is not reflected in the updated covariance matrix, P.

Suppose that a number of measurements, subject to small measurement modeling errors, have been made. The eigenvalues of the P matrix will in general be reduced below the eigenvalues of the second moment of the distribution of Z about $\hat{Z}$, due to the modeling errors and the deviation of the measurement sample mean from the population mean. The discrepancy grows with time updating. In fact, a zero eigenvalue will remain zero in spite of the errors introduced in modeling the equations of motion. As a result of the underestimation of P, the estimate $\hat{Z}$ will be undercorrected by subsequent measurements.

A number of non-optimal filters have been proposed to accommodate modeling errors. The one described below was given by Schmidt (7), and was chosen here for its computational simplicity.

The correction and its associated P_{1n}

$$\hat{Z}_{1n} = \hat{Z}_1 + (P_1 H_1^T + \epsilon H_1^T)(H_1 P_1 H_1^T + Q)^{-1} (Y-\hat{Y})$$

$$P_{1n} = P_1 - P_1 H_1^T (H_1 P_1 H_1^T + Q)^{-1} H_1 P_1 + \epsilon^2 H_1^T (H_1 P_1 H_1^T + Q)^{-1} H_1 \qquad \text{(B. 2-44)}$$

reduce to the minimum variance correction for $\epsilon = 0$. The term in ϵ is proportional to the maximum likelihood estimate if: (1) no a priori information is available ($P_1^{-1} = 0$), and (2) the probability density of q has its maximum at $q = 0$. For a large number of measurements of a single observable at a fixed time (H_1 = constant), the variance of $H_1(z_1 - \hat{z}_1)$, $H_1 P_1 H_1^T$, asymptotically approaches $\epsilon H_1 H_1^T$.

B. 2. 6 Inclusion of Effects of Parameter Uncertainties

We sometimes wish to include a set of equation of motion or measurement parameters in computing the uncertainty in the state, without improving our knowledge of these parameters. In particular, the inclusion of these parameters provides an effective alternative to the non-optimal filter described above, in that the additional uncertainties provide the desired growth of the covariance matrix, with user control which has intuitive basis.

To obtain the appropriate filter, we first note that if we include the additional parameters in the extended state vector, the time update equations (B. 2-20), (B. 2-24), and the covariance of the measurement

$$E\left[(Y-\hat{Y})(Y-\hat{Y})^T\right] = H_1 P_1 H_1^T + Q \qquad \text{(B. 2-45)}$$

properly include the additional uncertainty. Let the new extended state and its covariance be

$$W = \begin{bmatrix} Z_1 \\ \\ Z_2 \end{bmatrix}$$

$$E\left[(W - \hat{W})(W - \hat{W})^T\right] = \begin{bmatrix} P_1^* & C \\ \\ C^T & W \end{bmatrix} \qquad (B.\,2\text{-}46)$$

$$E\left[Z_2\right] = 0$$

and set

$$Y = H_1^* Z_1 + GZ_2 + q \qquad (B.\,2\text{-}47)$$

$$E\left[Z_2 q^T\right] = 0$$

We again consider the linear filter

$$\hat{Z}_{1n} = \hat{Z}_1 + K(Y - \hat{Y}) \qquad (B.\,2\text{-}48)$$

The submatrices of the covariance of W are

$$\begin{aligned} P_{1n}^* &= E\left[(Z_1 - \hat{Z}_{1n})\,(Z_1 - \hat{Z}_{1n})^T\right] \\ &= E\left[\{Z - \hat{Z}_1 = K(Y - \hat{Y})\}\,\{Z - \hat{Z}_1 - K(Y - \hat{Y})\}^T\right] \\ &= P_1^* - E\left[(Z - \hat{Z}_1)(Y - \hat{Y})^T K^T\right] - E\left[K(Y - \hat{Y})\,(Z - \hat{Z}_1)^T\right] \\ &\qquad + E\left[K\,(Y - \hat{Y})\,(Y - \hat{Y})^T K^T\right] \qquad (B.\,2\text{-}49) \\ &= P_1^* - (P_1^* H_1^{*T} + CG^T)\,K^T - K(P_1^* H_1^{*T} + CG^T)^T + K\Sigma K^T \end{aligned}$$

$$
\begin{aligned}
C_n &= E\left[(Z_1 - \hat{Z}_{1n})\,(Z_2 - \hat{Z}_2)^T\right] \\
&= E\left[\left\{Z - \hat{Z}_1 - K(Y - \hat{Y})\right\}\,(Z_2 - \hat{Z}_2)^T\right] \\
&= C - K(H_1^* C - GW)
\end{aligned}
$$

$$
W_n = W
$$

where the covariance of the measurement deviation is

$$
\begin{aligned}
\Sigma &= E\left[(Y - \hat{Y})(Y - \hat{Y})^T\right] \\
&= E\left[\left\{H_1^*\,(Z_1 - \hat{Z}_1) + G(Z_2 - \hat{Z}_2) + q\right\}\left\{H_1^*\,(Z_1 - \hat{Z}_1)\right.\right. \qquad \text{(B. 2-50)} \\
&\qquad\qquad \left.\left. + G(Z_2 - \hat{Z}_2) + q\right\}\right] \\
&= H_1^*\,P_1\,H_1^{*T} + H_1^*\,CG^T + GC^T H_1^{*T} + GWG + Q
\end{aligned}
$$

As before, we set

$$
K = (P_1^*\,H_1^{*T} + CG^T)\Sigma^{-1} + A \qquad \text{(B. 2-51)}
$$

and compute

$$
P_{1n}^* = P_1^* - (P_1^* H_1^{*T} + CG^T)\,\Sigma^{-1}\left(P_1^* H_1^{*T} + CG^T\right)^T + A\Sigma A^T \qquad \text{(B. 2-52)}
$$

Again, the trace of P_{1n}^* is minimized by the choice $A = 0$. The equations become

$$
\begin{aligned}
\hat{Z}_{1n} &= \hat{Z}_1 + (P_1^*\,H_1^{*T} + CG^T)\,\Sigma^{-1}\,(Y - \hat{Y}) \\
P_{1n}^* &= P_1^* - (P_1^* H_1^{*T} + CG^T)\Sigma^{-1}\,(P_1^*\,H_1^{*T} + CG^T)^T \qquad \text{(B. 2-53)} \\
C_n &= C - (P_1^* H_1^{*T} + CG^T)\Sigma^{-1}\,(H_1^*\,C + GW)
\end{aligned}
$$

If in addition to the additional parameters, we include the term ϵH_1,

$$
\begin{aligned}
Z_{1n} &= Z_1 + (P_1^* H_1^* + CG^T + \epsilon H_1^T) \Sigma^{-1} (Y - \hat{Y}) \\
P_{1n}^* &= P_1^* - (P_1^* H_1^{*T} + CG^T) \Sigma^{-1} (P_1^* H_1^{*T} + CG^T)^T \qquad \text{(B. 2-54)} \\
&\qquad + \epsilon^2 H_1^T \Sigma^{-1} H_1 \\
C_n &= C - (P_1^* H_1^{*T} + CG^T) \Sigma^{-1} (H_1^* C + GW) - \epsilon H_1^T \Sigma^{-1} (H_1^* C + GW)
\end{aligned}
$$

B. 3 MECHANIZATION OF THE FILTER

B. 3. 1 Definition of the State and Measurements

The independent variable, t, in the equations of motion of the space vehicle is ephemeris time (see Appendix A). For the estimation procedure, it is to be interpreted as the ephemeris time at which a signal is retransmitted by the vehicle. It will hereafter be denoted t_v. The subscript E will be omitted from all ephemeris times unless it is required for clarity.

The measurements are given at specified station times at reception of the signal. We denote reception time on the station time scale by t_{Sr}. Clearly, the gradient H_1, used by the estimator must be computed with t_{Sr} fixed.

We take the anchor point at the instant t_{v1}, setting

$$
X_1 = X(t_{v1}) = \begin{bmatrix} T_{Z2X} R_v (t_{v1}) \\ T_{Z2X} R_v (t_{v1}) \end{bmatrix}
$$

where R_v, $\dot{R}_v$ are the position and velocity of the vehicle relative to Earth in C-frame coordinates, and T_{Z2X} is an orthogonal transformation which may be selected for conditioning the P-matrix. The extended state vector is

$$Z = \begin{bmatrix} X_1 \\ U \\ V \end{bmatrix} \qquad \text{(B. 3-2)}$$

The order of the components of Z is established by the program, without regard to whether a particular component is being estimated or merely included as an uncertain parameter. The cartesian state, X_1, is always first, followed by the equation of motion parameters, U, with the measurement parameters, V, last.

The cartesian state must contain all six position and velocity components, or it must be totally omitted. The equation of motion parameters may include any of the parameters described in Appendix D, subject to the limitations of program dimensioning, and they occur in Z in the order in which they appear in Table D-1. The measurement errors may include any of the measurement errors described in Appendix C, for any of the stations from which data is obtained. They are grouped by station in the order in which they appear in Table C-1. The stations are ordered as they appear in the station name array on the edited data tape.

B. 3. 2 The Nominal Trajectory

Let

$$\overline{Z} = \begin{bmatrix} \overline{X}_1 \\ \overline{U} \\ \overline{V} \end{bmatrix} \qquad \text{(B. 3-3)}$$

be a "nominal" state. The nominal trajectory, $\overline{X}(t_r)$, the state transition matrix, $\varphi(t_v;t_{v1})$, and the sensitivity matrix, $\varphi_u(t_v;t_{v1})$, are integrated in the second order form

$$\begin{aligned}
\ddot{\overline{R}}_v(t_v) &= F(\overline{R}_v, \dot{\overline{R}}_v, \overline{U}, t_v) \\
\ddot{\varphi}(t_v;t_{v1}) &= \frac{\partial F}{\partial R_v}\,\varphi(t_v;t_{v1}) + \frac{\partial F}{\partial \dot{R}_v}\,\dot{\varphi}(t_v;t_{v1}) \\
\ddot{\varphi}_u(t_v;t_{v1}) &= \frac{\partial F}{\partial R_v}\,\varphi_u(t_v;t_{v1}) + \frac{\partial F}{\partial \dot{R}_v}\,\dot{\varphi}_u(t_v;t_{v1}) + \frac{\partial F}{\partial U}
\end{aligned} \tag{B.3-4}$$

from the initial conditions

$$\begin{aligned}
\begin{bmatrix} \overline{R}_v(t_{v1}) \\ \dot{\overline{R}}_v(t_{v1}) \end{bmatrix} &= \overline{X}_1 \\
\begin{bmatrix} \varphi(t_{v1};t_{v1}) \\ \dot{\varphi}(t_{v1};t_{v1}) \end{bmatrix} &= I_{6x6} \\
\begin{bmatrix} \varphi_u(t_{v1};t_{v1}) \\ \dot{\varphi}_u(t_{v1};t_{v1}) \end{bmatrix} &= \phi_{6xn}
\end{aligned} \tag{B.3-5}$$

where n is the number of components of $\overline{U}$. The analytical form of $F(R_v, R_v, U, t_v)$, or dynamic model, is described in Appendix D.

During integration, the time is compared with the vehicle time corresponding to the first data to be processed. At the point t_{vo}, the last integration point preceding the data, the position, $\overline{R}_{vo} = \overline{R}_v(t_{vo})$, and the matrices $\varphi(t_{vo};t_{v1})$, $\varphi_u(t_{vo};t_{v1})$ are stored. The velocities, $\dot{R}_v$, $\dot{\varphi}$, $\dot{\varphi}_u$, are then stored at the sequence of equally spaced times

$$t_n = t_{vo} + nh; \quad n = o, 1, \ldots, 7 \tag{B.3-6}$$

to cover the interval $(t_{vo}, t_{vo} + 7h)$.

The signal reception time, t_r, corresponding to each t_v is computed by an iterative solution of

$$t_r - t_v = \frac{1}{c} \left| \bar{R}_v(t_v) - R_r(t_r) \right| \tag{B3. 7}$$

where

c = effective speed of light assuming straight-line propagation,

R_r = location of the receiving station

Upon completion of the integration to t_{vo} + 7h, the stored derivatives are converted to interpolation coefficients, using

$$\begin{bmatrix} \alpha_o \\ \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \\ \alpha_5 \\ \alpha_6 \\ \alpha_7 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{363}{140} & 7 & -\frac{21}{2} & \frac{35}{3} & -\frac{27}{4} & \frac{21}{5} & -\frac{7}{6} & \frac{1}{7} \\ \frac{469}{90} & -\frac{223}{10} & \frac{879}{20} & -\frac{949}{18} & 41 & -\frac{201}{10} & \frac{1019}{180} & -\frac{7}{10} \\ -\frac{967}{120} & \frac{638}{15} & -\frac{3929}{40} & \frac{389}{3} & -\frac{2545}{24} & \frac{268}{5} & -\frac{1849}{120} & \frac{29}{15} \\ \frac{28}{3} & -\frac{111}{2} & 142 & -\frac{1219}{6} & 176 & -\frac{185}{2} & \frac{82}{3} & -\frac{7}{2} \\ -\frac{23}{3} & \frac{295}{6} & -135 & \frac{1235}{6} & -\frac{565}{3} & \frac{207}{2} & -\frac{95}{3} & \frac{25}{6} \\ 4 & -27 & 78 & -125 & 120 & -69 & 22 & -3 \\ -1 & 7 & -21 & 35 & -35 & 21 & -7 & 1 \end{bmatrix} \begin{bmatrix} \dot{A}(t_{vo}) \\ \dot{A}(t_{vo}+h) \\ \dot{A}(t_{vo}+2h) \\ \dot{A}(t_{vo}+3h) \\ \dot{A}(t_{vo}+4h) \\ \dot{A}(t_{vo}+5h) \\ \dot{A}(t_{vo}+6h) \\ \dot{A}(t_{vo}+7h) \end{bmatrix} \tag{B. 3-8}$$

for each of the derivatives $\dot{A}$, $(\dot{R}_v, \dot{\varphi}, \dot{\varphi}_u)$.

During the processing of the data points on the interval, the values of $\overline{R}_v(t_v)$, $\dot{\overline{R}}_v(t_v)$ and of the transition and sensitivity matrices are interpolated, using

$$\tau = (t_v - t_{vo})/h$$

$$A(t_v) = A(t_{vo}) + h\sum_{i=o}^{7} \alpha_i \tau^{i+1}/(i+1)! \qquad (B.3\text{-}9)$$

$$\dot{A}(t_v) = \sum_{i=o}^{7} \alpha_i \tau^i / i!$$

B. 3. 3 Accumulation of Corrections

The nominal state at the anchor point, $\overline{Z}$, is retained in storage as the reference point for all corrections. Let

$$\begin{aligned} Z &= \overline{Z} + z = \text{true state at the anchor point, } t_{v1}, \\ \hat{Z} &= \overline{Z} + \hat{z} = \text{current estimate of } Z, \\ Z_o &= \overline{Z} + \overline{z} = \underline{\text{a priori}} \text{ state at } t_{v1}. \end{aligned} \qquad (B3\text{-}10)$$

The differential corrections $\overline{z}$, $\hat{z}$ are used for accumulation of corrections during data processing, in lieu of accumulation of the totals Z_o, $\hat{Z}$.

We denote the covariance matrix at the anchor point by P_1:

$$P_1 = E\left[(Z-\hat{Z})\ (Z-\hat{Z})^t\right] \qquad (B.3\text{-}11)$$

Note that the ordering of Z is different from the partitioned form of B. 2, above, and hence the partitioning of Z, H_1, P_1 must now be accomplished in a different form. We define the diagonal matrix, J, according to

$$J_{ii} = 1 \text{ if } Z_i \text{ is being estimated}$$

$$J_{ii} = 0 \text{ if } Z_i \text{ is included as an uncertainty} \qquad (B.\ 3\text{-}12)$$

Clearly, for any matrix A (having the same number of rows as Z), we may write

$$A = JA + (I-J)A \qquad (B.\ 3\text{-}13)$$

where JA contains non-zero rows corresponding to the elements of Z being estimated and (I-J)A contains non-zero rows corresponding to the elements not being solved for. That is, JZ contains the elements of Z_1 and (I-J)Z contains the elements of Z_2 of equation (B. 2-46).

Similarily, except for the zero rows and columns the H_1^*, G of (B. 2-47) are replaced by JH_1 and $(I-J)H_1$ and the submatrices P_1^*, C, W are replaced by JP_1J, $JP_1(I-J)$, $(I-J)P_1(I-J)$, respectively.

Computationally, we write the filter of B. 2. 6 as

$$\hat{Z}_n = \hat{Z} + J(P_1H_1^T + \epsilon H_1^T)\Sigma^{-1}(Y-\hat{Y})$$

$$\begin{aligned} P_{1n} = P_1 &- J\left\{P_1H_1^T\Sigma^{-1}H_1P_1 - \epsilon^2H_1^T\Sigma^{-1}H_1\right\}J \\ &- J(P_1H_1^T + \epsilon H_1^T)H_1P_1(I-J) \qquad (B.\ 3\text{-}14) \\ &- (I-J)P_1H_1^T\Sigma^{-1}(P_1H_1^T - \epsilon H_1^T)^T J \end{aligned}$$

$$\Sigma = H_1P_1H_1^T + Q$$

The measurements to be processed are loaded from the edited data tape into a data buffer. They are processed sequentially as they appear in the buffer (assumed time-ordered). The algorithm used for the processing of the measurements is described below.

1. Initialization.

The first data to be processed is identified, and its location in the buffer is stored. Let

$$\overline{Z} = Z_0$$

$$\overline{z} = \hat{z} = 0$$

$$P_1 = P_0$$

where P_0 is the *a priori* corvariance matrix, retained on the scratch tape, unit 9.

2. Locate next data time.

a. If next data is not in the buffer, refill the buffer and reset buffer location counter. If no new data is available, go to step 6.

b. If next data time is beyond the process stop time, go to step 6.

c. Compute the vehicle time, t_v, corresponding to the data time, t_{Sr}, and nominal state, $\overline{Z}$. If t_v is beyond the interpolation tables, set

$$\overline{Z} = \hat{Z} \quad (= \overline{Z} + \hat{z})$$

$$\overline{z} = 0 \quad (Z_0 = \overline{Z})$$

$$P_0 = P_1$$

and reset buffer counter. That is, all succeeding data is to be processed with a new *a priori* estimate. Integrate new nominal and establish new interpolation tables. Recompute the vehicle time, t_v.

3. Compute Nominal State.

Interpolate for $\overline{X}(t_v)$, $\varphi(t_v; t_{v1})$, $\varphi_u(t_v; t_{v1})$. Compute the measurements $\overline{Y}_i = Y_i(\overline{X}(t_v), \overline{V}, t_{Sr})$ and their partial derivatives, H_i.

4. Accumulate Corrections.

For each measurement at the time t_{Sr}, compute the anchor point partials, H_1, and the residual

$$Y - \hat{Y} = Y - \overline{Y} + H_1 (\overline{z} - \hat{z})$$

set

$$\hat{z} \longleftarrow \hat{z} + J(P_1 H_1^T + \epsilon H_1^T) \sum^{-1} (Y - \hat{Y})$$

$$P_1 \longleftarrow P_1 - J \left\{ P_1 H_1^T \sum^{-1} H_1 P_1 + \epsilon^2 H_1^T \sum^{-1} H_1 \right\} J$$

$$- J (P_1 H_1^T + \epsilon H_1^T) \sum^{-1} H_1 P_1 (I-J)$$

$$- (I-J) P_1 H_1^T \sum^{-1} \left(P_1 H_1{}^T + \epsilon H_1^T \right)^T J$$

5. Test Linearity.

If an iteration is in process, continue from 5b if the point of failure of the linearity test has been reached, and from step 2 otherwise.

a. Test $\left[\hat{R}_v(t_v) - \overline{R}_v(t_v)\right]$ versus $\epsilon_1 \cdot$ slant range and $\epsilon_2 \cdot$ radius from the central body. If either test is failed, set

$$\overline{Z} = \hat{Z} \quad (\overline{z} = \overline{z} - \hat{z}, \; \hat{z} = 0)$$

$$P_1 = P_0$$

and reset counters to process all points since the <u>a priori</u> covariance was last stored. Continue from step 2 whether the tests were failed or passed.

b. Test $\left|\hat{R}_v(t_v) - \bar{R}_v(t_v)\right|$ versus $\epsilon_1 \cdot$ slant range, $\epsilon_2 \cdot$ radius from the central body, and its value on the previous iteration.

(1) If $\left|\hat{R}_v(t_v) - \bar{R}_v(t_v)\right|$ is larger than its previous value, stop with an error message.

(2) If either of the linearity tests are failed, continue from step 5a.

(3) Set

$$\bar{z} = \hat{z} \quad (Z_0 = \hat{Z})$$

$$P_0 = P_1$$

and reset counters to identify data points included in the a priori estimate and covariance. Continue from step 2.

6. Terminate Process.

Set

$$\bar{Z} \leftarrow \bar{Z} + \hat{z}$$

$$P_0 = P_1$$

and write the final estimate (now $\bar{Z}$) and covariance onto the estimate tape. Continue with the next process.

B.4 COMPUTATION OF INITIAL ESTIMATE FROM THE DATA

It is often the case that no adequate initial estimate is available for the differential correction procedure described above. We now consider a method for the computation of an initial estimate from a maximum likelihood estimation, using a small number of data points.

The maximum likelihood estimation, or data start, is performed in a separate link of the program. Dimensioning restrictions in that link limit the number of time point sets of data which may be used for the data start. To provide the best possible chance of a valid start, the data is first scanned, and only those time points having a complete set of apparently valid measurements are loaded for the data start.

B. 4. 1 Preliminary Smoothing.

As a first step in the data start, a small subset of measurements of each type, normally 7-10, is fit by a polynomial. For best results, the polynomial should be of a low, odd degree, say 3 or 5.

Let $\bar{t}$ denote the reference time for the polynomial, and let the measurement values y_i be given at the times t_i, i = 1, 2, ···, N. We seek the polynomial

$$y(t) = \sum_{i=0}^{M} a_i \, (t-\bar{t})^i \qquad \text{(B. 4-1)}$$

which minimizes the sum of the squares of the residuals. This problem has the familiar solution

$$A = (M^T M)^{-1} \, M^T Y \qquad \text{(B. 4-2)}$$

where

$$\begin{aligned} A &= \text{col}\,(a_0, \; a_1, \; \cdots, \; a_M) \\ Y &= \text{col}\,(y_1, \, y_2, \, \cdots, \, y_N) \\ M &= \{m_{ij}\} \\ m_{ij} &= (t_i - \bar{t})^j \end{aligned} \qquad \text{(B. 4-3)}$$

The coefficients a_0, a_1 are retained as smoothed values of $y(\bar{t})$ and $\dot{y}(\bar{t})$. When the set of measurements have been smoothed, the coefficients are used to determine a starting estimate for the maximum likelihood iteration.

If two angles and range are known, the sets $y(\bar{t})$, $\dot{y}(\bar{t})$ uniquely determine the vehicle state $x(\bar{t})$. With measurements of range at all but near-earth distances, the range is known only to within a given ambiguity, or modulus. For such data, that multiple of the ambiguity which minimizes the weighted sum of the squares of the residuals of the data from a conic section is first added to the range value, $y(\bar{t})$.

If two angles and doppler are known, the range is first estimated by a maximum likelihood estimation, with the angles, their rates, and the doppler fixed.

B. 4. 2 Maximum Likelihood Estimation.

We define the "measurement coordinates"

$$X_M = \begin{bmatrix} \text{slant range} \\ \text{angle 1} \\ \text{angle 2} \\ \text{range rate} \\ \text{angle 1 rate} \\ \text{angle 2 rate} \end{bmatrix} \quad \text{at time } \bar{t}$$

Each $X_{\hat{M}}$ is uniquely related to a cartesian state of the vehicle $X(\bar{t})$ once the angles are defined.

We assume that for purposes of the data start, only the inverse square gravitational attraction of the central body need be considered. The trajectory, $x(t)$, then, is the conic section determined by $x(\bar{t})$. Since a closed form representation of $x(t)$ is available, we need not integrate the trajectory.

For each measurement to be considered, we may compute the residual

$$Y_i - Y(X(t_i), V_, t_i) \qquad \text{(B. 4-4)}$$

and the gradients

$$H_i = \frac{\partial Y}{\partial X(t_i)}$$

$$H_{Mi} = \frac{\partial Y}{\partial X_M} = \frac{\partial Y}{\partial X(t_i)} \quad \frac{\partial X(t_i)}{\partial X(\bar{t})} \quad \frac{\partial X^{(\bar{t})}}{\partial X_M} \qquad \text{(B. 4-5)}$$

If we assume that the random errors in the measurements are gaussian with zero mean and covariance Q, the joint probability density of the Y_i may be written

$$p(Y_i) = \frac{1}{2\pi|Q|^{\frac{1}{2}}} e^{-\frac{1}{2} R^T Q^{-1} R} \qquad \text{(B. 4-6)}$$

where

$$R = \text{col}\left\{ Y_i - Y(X(t_i), V, t_i) \right\}$$

Maximizing $p(Y_i)$ as a function of X_M, we obtain the condition, linearized about $X_M^{(n)}$,

$$H_M^T Q^{-1} H_M (X_M - X_M^{(n)}) = H_M^T Q^{-1} R^{(n)}$$

$$R^{(n)} = R \Big|_{X_M = X_M^{(n)}} \qquad \text{(B. 4-7)}$$

$$H_M = \begin{bmatrix} H_{M1} \\ H_{M2} \\ \vdots \\ H_{MN} \end{bmatrix}$$

N = total number of measurements

If the "information matrix", $H_M^T Q^{-1} H_M$ has an inverse,

$$X_M = X_M^{(n)} + (H_M^T Q^{-1} H_M)^{-1} H_M^T Q^{-1} R^{(n)} \qquad \text{(B. 4-8)}$$

Now if we consider $X_M^{(n)}$ to be the estimate of X_M after n solutions of (B. 4-8), we have the iterative process

1. Compute $X^{(n)} = X(t_i; X_M^{(n)})$, $Y^{(n)} = Y(X^{(n)}(t_i), V, t_i)$

   and $H_M = \frac{\partial Y}{\partial X_M} \Big|_{X_M^{(n)}}$.

2. Accumulate the information matrix and $H_M^T Q^{-1} R^{(n)}$ contributions by each measurement.

3. Compute $X_m^{(n+1)}$ from

$$X_M^{(n+1)} = X_M^{(n)} + (H_M^T Q^{-1} H_M)^{-1} H_M^T Q^{-1} R^{(n)}$$

4. Test $\left\| R^{(n)} \right\|$ for convergence and repeat or terminate the iteration.

The data start link includes a variety of convergence control options, including the use of a pseudo inverse for ill-behaved information matrices, which are described in detail in References 1 and 2.

REFERENCES

1. "Subroutine Descriptions and Listings for the Orbit Determination Program," Philco-Ford Corporation, TR-DA1509, Palo Alto, California, December 1967.

2. "Input-Output Summary for the Orbit Determination Program," Philco-Ford Corporation, TR-DA1510, Palo Alto, California, December 1967.

3. Kalman, R. E., "A New Approach to Linear Filtering and Prediction Problems," J1 of Basic Engineering. (ASME Trans.) Vol. 82D, March 1960.

4. Smith, G. L., Schmidt, S. F., and McGee, L. A., "Application of Statistical Filter Theory to the Optional Estimation of Position and Velocity On Board a Circumlunar Vehicle," NASA TR R-135, 1962.

5. McLean, J. D., Schmidt, S. F., and McGee, L. A., "Optional Filtering and Linear Prediction Applied to a Midcourse Navigation System for the Circumlunar Mission," NASA TN D-1208, March 1962.

6. Schmidt, S. F., "State Space Techniques Applied to the Design of a Space Navigation System," JACC Conf. Paper, 1962.

7. Schmidt, S. F., "Estimation of State with Acceptable Accuracy Constraints," Philco-Ford Corp., Systems Development and Mission Analysis Department, TR-1, 20 July 1966.

8. Schmidt, S. F., "The Application of State Space Methods to Navigation Problems," Philco-Ford Corporation, Guidance and Control System Engineering Department TR-4, July 1964.

APPENDIX C

MEASUREMENT MODEL

C.1 INTRODUCTION

This appendix describes the modelling of the tracking data measurements. Analytic expressions for each of the observables are given as functions of the vehicle position and velocity and an assumed set of measurement error sources. Models are given for four measurement systems: C-band, Unified S-band, Goddard Range and Range-Rate, and DSIF. The partial derivatives of the observables with respect to vehicle state and measurement error sources are developed in the computational form used by the DCP.

C.1.1 Notation

The following notation has been adopted for this appendix:

1. Vectors are represented by upper-case letters (capitals).
2. Scalar quantities are represented by lower-case letters.
3. The magnitude (or length) of a vector is denoted either by the lower-case symbol for that vector or by the conventional straight brackets, e.g., the magnitude of R is r or $|R|$.
4. The conventional (•) and (x) are used for the vector dot and cross products respectively.
5. The symbol I denotes the identity matrix.
6. The symbol A_X denotes the skew-symmetric matrix

$$\begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix}$$

7. The total time derivative of a quantity is denoted by a dot over the symbol for that quantity.

C.1.2 Vector Operations

The following relationships for any vectors A, B, C and D are used in this appendix:

1. $A \times B = -B \times A$
2. $A \cdot B \times C = A \times B \cdot C = B \cdot C \times A$
3. $A \times A = 0$
4. $A \cdot A = a^2$
5. $A \times (B \times C) = (A \cdot C)B - (A \cdot B)C$
6. $\frac{d}{dt} (A \cdot B) = A \cdot \dot{B} + \dot{A} \cdot B$
7. $\frac{d}{dt} (A \times B) = A \times \dot{B} + \dot{A} \times B$
8. Let A, B, and C be an orthonormal set, and D be an arbitrary vector in the space spanned by A, B, and C. Then:
    - a. $A \cdot D^2 + B \cdot D^2 + C \cdot D^2 = D \cdot D$
    - b. $A \cdot D^2 + B \cdot D^2 = \left| C \times D \right|^2$
    - c. $(A \cdot D)A + (B \cdot D)B + (C \cdot D)C = D$

C.1.3 Definitions

The following symbols will be used throughout this appendix for the stated quantities except where explicitly noted in the text. All vectors are referred to the earth's equator and equinox of 1950.0 coordinate system unless otherwise noted in the text. All times are ephemeris time unless otherwise noted.

Symbol	Meaning
c, c_e	Speed of light - current estimate in the program
c_s	Fixed speed of light estimate used at the tracker in processing range data.
D	Unit down vector at the tracker at the time of the observation.

Symbol	Meaning
E	Unit east vector at the tracker at the time of the observation.
Υ	Greenwich hour angle
I_e	Unit vector along the vernal equinox of date
J_e	Unit vector in the equator of date 90° east of I_e
K_e	Unit vector along the north polar axis of date $K_e = I_e \times J_e$
λ	Longitude of the tracker
N	Unit north vector at tracker at time of observation
n	Differenced doppler count
ν_{tr}	Transmitted doppler frequency
Ω	Earth rotation vector
ω_3	Doppler bias frequency
ω_4	Doppler reception/retransmission constant at the spacecraft ($\omega_4 = 1$ for reflection)
R_v	Vehicle position vector; unless otherwise noted, at time t_v
R_r	Receiving tracker position vector; unless otherwise noted, at time t_r
R_t	Transmitting station position vector; unless otherwise noted, at time t_t
S, S_r	Slant range vector $R_v - R_r$
S_t	Slant range vector $R_v - R_t$
S_1	$R_v(t_v - \tau) - R_t(t_t - \tau)$
S_2	$R_v(t_v - \tau) - R_r(t_r - \tau)$
S_3	$R_v(t_v) - R_t(t_t) = S_t$
S_4	$R_v(t_v) - R_r(t_r) = S_r$

Symbol	Meaning
t_r	Time of reception of signal at tracker, end of doppler count interval.
t_v	Time of reception/retransmission of signal at vehicle, end of doppler count interval.
t_t	Time of transmission of signal from transmitting station, end of doppler count interval.
τ	Doppler count period.
T_{B2C}	Transformation matrix (orthogonal) from earth-fixed cartesian system to equator and equinox of 1950.0 at time t_r.
T_{B2CR}	Identical to T_{B2C}
T_{B2CT}	Transformation from earth-fixed system to equator and equinox of 1950.0 at time t_t.
y_i	Observables, defined for each tracking system.

C.2 GENERAL

Certain remarks and equations are applicable to all systems of measurement, and these are treated in this paragraph to avoid duplication.

C.2.1 Observation Times

The independent variable in the equations of motion of the spacecraft (Appendix D) is vehicle time, t_v (ET). The estimate of state, then, is computed as a function of t_v, and the anchor point for accumulating corrections to the estimate is taken as the fixed time t_{v1}. In the analytical formulation of the filter (Appendix B), however, the independent variable, t, in

$$Y = Y(X, V, t) \qquad (C.2-1)$$

is the observation time (ST), t_{Sr}. That is, the partial derivatives of Y must be computed with t_{Sr} fixed. In this appendix, the partial derivatives are shown for fixed t_r (ET), and are referred to the vehicle retransmission time, t_v.

Before use, they must be corrected for variations in t_r, and referred to the anchor point time, t_{v1}.

From (A.3-8),

$$t_r = t_{Sr} + (\gamma_1 + \gamma_3) + (\gamma_2 + \gamma_4)\, t_{Sr} \tag{C.2-2}$$

and hence

$$\left(\frac{\partial Y}{\partial \alpha}\right)_{t_{Sr}} = \left(\frac{\partial Y}{\partial \alpha}\right)_{t_r} + \dot{Y} \left(\frac{\partial t_r}{\partial \alpha}\right)_{t_{Sr}} \tag{C.2-3}$$

$$\left(\frac{\partial t_r}{\partial \alpha}\right)_{t_{Sr}} = \begin{cases} 1 & \alpha = \gamma_1 \text{ or } \gamma_3 \\ t_{Sr} & \alpha = \gamma_2 \text{ or } \gamma_4 \\ 0 & \text{otherwise} \end{cases}$$

where

$\dot{Y}$ = total time derivative of Y

Let

$$H(t_v) = \left[\left(\frac{\partial Y}{\partial X}\right)_{t_{Sr}}, \left(\frac{\partial Y}{\partial U}\right)_{t_{Sr}}, \left(\frac{\partial Y}{\partial V}\right)_{t_{Sr}}\right]$$

$$= \left(\frac{\partial Y}{\partial Z}\right)_{t_{Sr}} \tag{C.2-4}$$

U = equation of motion parameters

$$\left(\frac{\partial Y}{\partial U}\right)_{t_{Sr}} = 0$$

To obtain the partial derivatives with respect to the anchor point state, H_1, we use the chain rule

$$\frac{\partial Y}{\partial X}(t_{vl}) = \frac{\partial Y}{\partial X(t_v)} \cdot \frac{\partial X(t_v)}{\partial X(t_{v1})} \tag{C.2-5}$$

$$\frac{\partial Y}{\partial U} = \frac{\partial Y}{\partial X(t_v)} \cdot \frac{\partial X(t_v)}{\partial U}$$

That is,

$$H_1 = H \Phi (t_v ; t_{v1}) \tag{C.2-6}$$

where Φ is the transition matrix described in Appendix D.

C.2.2 Measurement Error Sources

The vector of measurement errors, V, may have as components any of the errors listed in Table C-1, below, for any of the stations whose measurements are being included. Partial derivatives of Y with respect to these error sources, as well as partials with respect to vehicle state, X, are developed for the various measurement systems below.

TABLE C-1

MEASUREMENT ERROR SOURCES

Symbol	Units	Definition
$(R_r)_e$	km	Station location errors, northing , easting , down
γ_1	sec	Station clock bias
γ_2	sec/sec	Station clock bias rate
c	km/sec	Error in effective speed of light
y_1	rad	Bias in angle 1
y_2	rad	angle 2
y_3	km/sec	range observable
ω_3	rad/sec	doppler bias frequency

C.3 C-BAND MEASUREMENT SYSTEM

C.3.1 C-Band Observables

The C-Band observables are azimuth, elevation, and slant range, with range in units of length. They are computed from the following equations.

$$\text{Azimuth:} \quad y_1 = \tan^{-1}\left(\frac{S \cdot E}{S \cdot N}\right) \quad 0 \leq y_1 < 2\pi$$

$$\text{Elevation:} \quad y_2 = \tan^{-1}\left(\frac{-S \cdot D}{|S \times D|}\right) \quad -\frac{\pi}{2} \leq y_2 \leq \frac{\pi}{2} \tag{C.3-1}$$

Range: With the gating ambiguity removed, the C-Band ranging observable is one-half the recorded two-way delay time, T_r, multipled by a fixed constant, c_s, representing the speed of light for the recording station. Letting c be the speed of light estimate internal to the Orbit Determination Program,

$$y_3 = \frac{T_r}{2} c_s = \frac{s}{c} c_s \tag{C.3-2}$$

For computation purposes

$$y_3 = s \tag{C.3-3}$$

C.3.2 Gradients

The gradients of the observables with respect to vehicle position, R_v, are

$$\frac{\partial y_1}{\partial R_v} = \frac{\partial y_1}{\partial S} = \frac{1}{1 + \frac{S \cdot E^2}{S \cdot N^2}} \frac{\partial}{\partial S}\left(\frac{S \cdot E}{S \cdot N}\right)$$

$$= \frac{S \cdot N^2}{S \cdot E^2 + S \cdot N^2} \quad \frac{(S \cdot N)E - (S \cdot E)N}{S \cdot N^2}$$

$$= \frac{S \times (E \times N)}{S \cdot N^2 + S \cdot E^2}$$

$$\frac{\partial y_1}{\partial R_v} = \frac{D \times S}{|D \times S|^{\;2}} \qquad \text{(C.3-4)}$$

$$\frac{\partial y_2}{\partial R_v} = \frac{\partial y_2}{\partial S} = \frac{1}{\left(1 - \frac{S \cdot D^2}{s^2}\right)^{1/2}} \left\{ \frac{(S \cdot D)S - (S \cdot S)D}{s^3} \right\}$$

$$= \frac{s}{(S \cdot S - S \cdot D^2)^{1/2}} \left\{ \frac{S \times (S \times D)}{s^3} \right\}$$

$$\frac{\partial y_2}{\partial R_v} = \frac{(D \times S) \times S}{s^2 \; |D \times S|} \qquad \text{(C.3-5)}$$

$$\frac{\partial y_3}{\partial R_v} = \frac{\partial y_3}{\partial S} = \frac{S}{s} \qquad \text{(C.3-6)}$$

Since vehicle velocity does not enter the formulation of the observables,

$$\frac{\partial y_i}{\partial R_v} = (0,\ 0,\ 0)\ , \qquad i = 1,\ 2,\ 3 \qquad \text{(C.3-7)}$$

C.3.3 Error Partials

Station location partials. Since both vehicle position, R_v, and station position, R_r, enter the observables only in the differences S, we have immediately

$$\frac{\partial y_i}{\partial R_r} = -\frac{\partial y_i}{\partial S} = -\frac{\partial y_i}{\partial R_v} \qquad \text{(C.3-8)}$$

The tracking station location errors, however, are accumulated in the earth-fixed B-frame. The transformation from the B-frame to the inertial C-frame is the time-dependent transformation T_{B2C}, where

$$R_r = T_{B2C} (R_r)_e \tag{C. 3-9}$$

Then

$$\frac{\partial y_i}{\partial (R_r)_e} = - \frac{\partial y_i}{\partial R_v} T_{B2C} \tag{C. 3-10}$$

Station Clock Partials. As are noted in paragraph C. 2. 1, the partial derivatives of the observables with respect to the observing station's clock errors are

$$\begin{aligned} \frac{\partial y_i}{\partial \gamma_1} &= \dot{y}_i \\ \frac{\partial y_i}{\partial \gamma_2} &= \dot{y}_i t_{Sr} \end{aligned} \tag{C. 3-11}$$

The time derivatives of the observables are

$$\frac{dy_1}{dt} = \dot{y}_1 = \frac{1}{1 + \frac{S \cdot E^2}{S \cdot N^2}} \frac{d}{dt} \left(\frac{S \cdot E}{S \cdot E} \right)$$

$$= \frac{S \cdot N^2}{S \cdot N^2 + S \cdot E^2} \left[\frac{(S \cdot N) \; (S \cdot \dot{E} + \dot{S} \cdot E) - (S \cdot E) \; (S \cdot \dot{N} + \dot{S} \cdot N)}{S \cdot N^2} \right]$$

$$= \frac{1}{|D \times S|^2} \left\{ \dot{S} \cdot \left[(S \cdot N) E - (S \cdot E) N \right] + S \cdot \left[(S \cdot N) \dot{E} - (S \cdot E) \dot{N} \right] \right\}$$

$$= \frac{1}{|D \times S|^2} \left\{ \dot{S} \cdot (D \times S) + S \cdot \left[\Omega \times \left((S \cdot N) E - (S \cdot E) N \right) \right] \right\}$$

$$= \frac{1}{|D \times S|^2} \left\{ \dot{S} \cdot (D \times S) + S \cdot \ \Omega \times (D \times S) \right\}$$

$$= \frac{1}{|D \times S|^2} \left\{ \dot{S} \cdot (D \times S) + (S \times \Omega) \cdot (D \times S) \right\}$$

$$= \frac{D \times S}{|D \times S|^2} \cdot \left\{ \dot{S} + S \times \Omega \right\}$$

$$\dot{y}_1 = \frac{\partial y_1}{\partial R_v} \cdot \left\{ \dot{S} + S \times \Omega \right\} \qquad (C.\ 3\text{-}12)$$

$$\frac{dy_2}{dt} = \dot{y}_2 = \frac{\partial y_2}{\partial R_s} \dot{S} + \frac{\partial y_2}{\partial D} \dot{D}$$

$$\dot{y}_2 = \frac{\partial y_2}{\partial R_v} \cdot \left\{ \dot{S} + S \times \Omega \right\} \qquad (C.\ 3\text{-}13)$$

$$\frac{dy_3}{dt} = \dot{y}_3 = \frac{S \cdot \dot{S}}{s} \qquad (C.\ 3\text{-}14)$$

Speed of Light Partials. The speed of light enters the formulation of range explicitly. It also enters each of the observables implicitly through the determination of t_v and hence $R_v(t_v)$. Since

$$t_v - t_r = \frac{1}{c} \left| R_r(t_v) - R_r(t_r) \right| = t_r - \frac{s}{c}$$

$$\frac{\partial t_v}{\partial c} = \frac{s}{c^2} \qquad (C.\ 3\text{-}15)$$

Measurement Bias Partials. For each of the observables, the bias errors are added to the observable and have the units of the observable.

Hence

$$\frac{\partial y_i}{\partial y_i} = 1 \qquad (C.3-17)$$

C.4 UNIFIED S-BAND SYSTEM

C.4.1 USBS Observables

The USBS observables are the X and Y angles, two-way range, and doppler. The 30-foot dish mount has its principal (X) axis north, and the 85-foot dish mount has its principal axis east. The observables are computed from the following equations.

X-Angle (y_1):

$$X_{30} = \tan^{-1}\left(\frac{-S \circ E}{S \cdot D}\right) \qquad 0 \leq y_1 \leq 2\pi$$

$$X_{85} = \tan^{-1}\left(\frac{S \circ N}{S \cdot D}\right)$$

$$(C.4-1)$$

Y-Angle (y_2):

$$Y_{30} = \tan^{-1}\left(\frac{N \cdot S}{|N \times S|}\right) \qquad -\frac{\pi}{2} \leq y_2 \leq \frac{\pi}{2}$$

$$Y_{85} = \tan^{-1}\left(\frac{E \cdot S}{|E \times S|}\right)$$

Range. The S-Band ranging observable is the two-way delay time, T_r, multiplied by a fixed constant, c_s, representing the speed of light for the observing station. Letting c be the estimate of the speed of light for the observing station,

$$y_3 = T_r\, c_s = \frac{s_r + s_t}{c}\, c_s \qquad (C.4-2)$$

For computation

$$y_3 = s_r + s_t = \left|R_v - R_r\right| + \left|R_v - R_t\right| \qquad (C.4-3)$$

Doppler. It is convenient to compute the doppler observable as counts of differenced doppler cycles. This computation lends itself readily to a range-difference method that avoids the truncation errors inherent in a truncated series expansion of the doppler integral (See Reference 6). Let

$$S_4 = R_v(t_v) - R_r(t_r)$$

$$S_3 = R_v(t_v) - R_t(t_t)$$

$$S_2 = R_v(t_v - \tau) - R_r(t_r - \tau) \qquad (C. 4-4)$$

$$S_1 = R_v(t_v - \tau) - R_t(t_t - \tau)$$

$$s_i = \left| S_i \right|, \; i = 1, \; 4$$

τ = doppler count interval

ω_3 = doppler bias frequency

ω_4 = retransmission ratio

ν_{tr} = transmitted doppler frequency

Then

$$y_4 = n = \tau\omega_3 + \frac{\omega_4 \nu_{tr}}{c} \left[s_3 + s_4 - s_1 - s_2 \right] \qquad (C. 4-5)$$

C. 4. 2 Gradients

The gradients of the observables with respect to vehicle position, R_v, are

$$\frac{\partial X_{30}}{\partial R_v} = \frac{\partial X_{30}}{\partial S} = \frac{1}{1 + \frac{S \cdot E^2}{S \cdot D^2}} \quad \frac{\partial}{\partial S} \left(\frac{-S \cdot E}{S \cdot D}\right)$$

$$= \frac{S \cdot D^2}{S \cdot D^2 + S \cdot E^2} \left\{\frac{(S \cdot E)D - (S \cdot D)E}{S \cdot D^2}\right\}$$

$$= \frac{S \times (D \times E)}{|N \times S|^2}$$

$$\frac{\partial X_{30}}{\partial R_v} = \frac{N \times S}{|N \times S|^2} \qquad (C. 4-6)$$

$$\frac{\partial X_{85}}{\partial R_v} = \frac{\partial X_{85}}{\partial S} = \frac{1}{1 + \frac{S \cdot N^2}{S \cdot D^2}} \quad \frac{\partial}{\partial S} \left(\frac{S \cdot N}{S \cdot D}\right)$$

$$= \frac{S \cdot D^2}{S \cdot D^2 + S \cdot N^2} \left\{\frac{(S \cdot D)N - (S \cdot N)D}{S \cdot D^2}\right\}$$

$$= \frac{S \times (N \times D)}{|E \times S|^2}$$

$$\frac{\partial X_{85}}{\partial R_v} = \frac{E \times S}{|E \times S|^2} \qquad (C. 4-7)$$

$$\frac{\partial Y_{30}}{\partial R_v} = \frac{\partial Y_{30}}{\partial S} = \frac{1}{\left(1 - \frac{S \cdot N^2}{s^2}\right)^{1/2}} \left\{\frac{(S \cdot S)N - (S \cdot N)S}{s^3}\right\}$$

$$= \frac{s}{|N \times S|} \left\{\frac{S \times (N \times S)}{s^3}\right\}$$

$$\frac{\partial Y_{30}}{\partial R_v} = \frac{S \times (N \times S)}{s^2 |N \times S|} \qquad \text{(C. 4-8)}$$

$$\frac{\partial Y_{85}}{\partial R_v} = \frac{\partial Y_{85}}{\partial S} = \frac{1}{\left(1 - \frac{S \cdot E^2}{s^2}\right)^{1/2}} \left\{\frac{(S \cdot S)N - (S \cdot N)S}{s^3}\right\}$$

$$\frac{\partial Y_{85}}{\partial R_v} = \frac{S \times (E \times S)}{s^2 |E \times S|} \qquad \text{(C. 4-9)}$$

The determination of t_t and therefore R_t depends on R_v. Thus the gradient of y_3 is more complex than might at first appear.

$$\frac{\partial y_3}{\partial R_v} = \frac{S_r}{s_r} + \frac{S_t}{s_t} + \frac{\partial y_3}{\partial R_t} \frac{\partial R_t}{\partial t_t} \frac{\partial t_t}{\partial R_v}$$

$$= \frac{S_r}{s_r} + \frac{S_t}{s_t} + \frac{\partial y_3}{\partial R_t} \cdot \dot{R}_t \left\{ \frac{S_r}{cs_r} + \frac{S_t}{cs_t} \right\}$$

$$\frac{\partial y_3}{\partial R_v} = \left[\frac{\partial y_3}{\partial R_t} \cdot \frac{R_t}{c} + 1 \right] \left[\frac{S_r}{s_r} + \frac{S_t}{s_t} \right] \qquad (C.4-10)$$

The quantity $\frac{\partial y_3}{\partial R_t}$ is in inertial coordinates and is $-\frac{S_t}{s_t}$

To obtain the gradient of the doppler observable, we rewrite S_2 and S_1 as

$$S_2 = R_v(t_v) - \dot{R}_v(t_v)\,\tau + 1/2\,\ddot{R}_v(t_v)\,\tau^2 - \cdots$$

$$- \left[R_r(t_r) - \dot{R}_r(t_r)\,\tau + 1/2\,\ddot{R}_r(t_r)\,\tau^2 - \cdots \right]$$

$$S_1 = R_v(t_v) - \dot{R}_v(t_v)\,\tau + 1/2\,\ddot{R}_v(t_v)\,\tau^2 - \cdots$$

$$- \left[R_t(t_t) - \dot{R}_t(t_t)\,\tau + 1/2\,\ddot{R}_t(t_t)\,\tau^2 - \cdots \right] \qquad (C.4-11)$$

We compute the gradient with respect to the vehicle position at the end of the doppler count interval, i.e., at time t_v.

$$\frac{\partial y_4}{\partial R_v(t_v)} = \frac{\omega_4 \nu_{tr}}{c}\left(\frac{S_3}{s_3} + \frac{S_4}{s_4} - \frac{S_2}{s_2} - \frac{S_1}{s_1}\right) + \frac{\partial y_4}{\partial R_t}\,\frac{\partial R_t}{\partial t_t}\,\frac{\partial t_t}{\partial R_v(t_v)}$$

$$+ \frac{\partial y_4}{\partial R_t}\,\frac{\partial R_t}{\partial (t_t - \tau)}\,\frac{\partial (t_t - \tau)}{\partial R_v(t_v)}$$

$$= \frac{\omega_4 \nu_{tr}}{c}\left(\frac{S_3}{s_3} + \frac{S_4}{s_4} - \frac{S_2}{s_2} - \frac{S_1}{s_1}\right)$$

$$+ \frac{\partial y_4}{\partial R_t} \cdot \frac{\dot{R}_t(t_t)}{c}\left(\frac{S_3}{s_3} + \frac{S_4}{s_4}\right)$$

$$- \frac{\partial y_4}{\partial R_t} \cdot \frac{\dot{R}_t(t_t - \tau)}{c}\left(\frac{S_1}{s_1} + \frac{S_2}{s_2}\right) \qquad \text{(C. 4-12)}$$

where $\frac{\partial y_4}{\partial R_t}$ is shown in (C. 4 16),

Vehicle velocity enters the formulation of the doppler observable only, and there only in the displacement of the vehicle over the count interval. Using (C. 4-11),

$$\frac{\partial y_i}{\partial \dot{R}_v} = (0,\ 0,\ 0) \qquad i = 1,\ 2,\ 3$$

$$\frac{\partial y_4}{\partial \dot{R}_v} = \frac{\omega_4 \nu_{tr} \tau}{c}\left(\frac{S_2}{s_2} + \frac{S_1}{s_1}\right) \qquad \text{(C. 4-13)}$$

C. 4. 3 Error Partials

Station Location Partials. Once again, the station location and vehicle positions enter the formulations only in the down-leg slant range, s, and hence

$$\frac{\partial y_i}{\partial (R_r)_e} = - \frac{\partial y_i}{\partial R_v} T_{B2C} \qquad i = 1, 2$$

$$\frac{\partial y_i}{\partial (R_t)_e} = (0, 0, 0) \tag{C. 4-14}$$

Similarly,

$$\frac{\partial y_3}{\partial (R_r)_e} = - \frac{S_r}{s_r} T_{B2C} \qquad \text{(C. 4-15}$$

$$\frac{\partial y_3}{\partial (R_t)_e} = - \frac{S_t}{s_t} T_{B2C} \tag{C. 4-15}$$

Using (C. 4-11),

$$\frac{\partial y_4}{\partial R_r} = \frac{\omega_4 \nu_{tr}}{c} \left\{ - \frac{S_4}{s_4} + \frac{S_2}{s_2} \left[I - (\tau \Omega x) + \frac{1}{2} (\tau \Omega x)^2 - \frac{1}{6} (\tau \Omega x)^3 + \cdots \right] \right\}$$

$$\frac{\partial y_4}{\partial R_t} = \frac{\omega_4 \nu_{tr}}{c} \left\{ - \frac{S_3}{s_3} + \frac{S_1}{s_1} \left[I - (\tau \Omega x) + \frac{1}{2} (\tau \Omega x)^2 - \frac{1}{6} (\tau \Omega x)^3 + \cdots \right] \right\}$$

(C. 4-16)

$$\frac{\partial y_4}{\partial (R_r)_e} = \frac{\partial y_4}{\partial R_r} \, T_{B2C}$$

$$\frac{\partial y_4}{\partial (R_t)_e} = \frac{\partial y_4}{\partial R_t} \, T_{B2C}$$

Note that if the transmitting and receiving stations are the same, the partials with respect to the two stations locations are summed.

Station Clock Partials. The total time derivatives of the observables are

$$\frac{dX_{30}}{dt} = \dot{X}_{30} \; \frac{1}{1 + \frac{S \cdot E^2}{S \cdot D^2}} \; \frac{d}{dt} \left(\frac{-S \cdot E}{S \cdot D} \right)$$

$$= \frac{S \cdot D^2}{|N \times S|^2} \left\{ \frac{(S \cdot E)\,(S \cdot \dot{D} + S \cdot D) - (\dot{S} \cdot D)\,(S \cdot \dot{E} + \dot{S} \cdot E)}{S \cdot D^2} \right\}$$

$$= \frac{1}{|N \times S|^2} \left\{ \dot{S} \cdot \Big[(S \cdot E)D - (S \cdot D)E \Big] + S \cdot \Big[(S \cdot \dot{D})E - (S \cdot \dot{E})D \Big] \right\}$$

$$= \frac{1}{|N \times S|^2} \left\{ \dot{S} \cdot S \times (D \times E) + S \cdot \Big[(S \cdot \Omega \times D)E - S \cdot \Omega \times E)D \Big] \right\}$$

$$= \frac{1}{|N \times S|^2} \left\{ S \cdot N \times S + S \cdot \Big[(S \times \Omega \cdot D)E - (S \times \Omega \cdot E)D \Big] \right\}$$

$$= \frac{1}{|N \times S|^2} \left\{ \dot{S} \cdot N \times S + S \times \Omega \cdot \Big[(S \cdot D)E - (S \cdot D)D \Big] \right\}$$

$$= \frac{1}{|N \times S|^2} \left\{ S. N \times S + S \times \Omega N \times S \right\}$$

$$\dot{X}_{30} = \frac{\partial X_{30}}{\partial R_v} \cdot \left\{ \dot{S} + S \times \Omega \right\} \qquad (C. 4\text{-}17)$$

Similarly

$$\dot{X}_{85} = \frac{\partial X_{85}}{\partial R_v} \cdot \left\{ \dot{S} + S \times \Omega \right\}$$

$$\dot{Y}_{30} = \frac{\partial Y_{85}}{\partial S} \cdot \dot{S} + \frac{\partial Y_{30}}{\partial N} \dot{N} \qquad (C. 4\text{-}18)$$

$$= \frac{\partial Y_{30}}{\partial R_v} \cdot \left\{ \dot{S} + S \times \Omega \right\}$$

$$\dot{Y}_{85} = \frac{\partial Y_{85}}{\partial R_v} \cdot \left\{ \dot{S} + S \times \Omega \right\}$$

The range and doppler derivatives are

$$\dot{y}_3 = \frac{S_r \cdot \dot{S}_r}{s_r} + \frac{S_t \cdot \dot{S}_t}{s_t}$$

$$\dot{y}_4 = \frac{\omega_4 \nu_{tr}}{c_2} \left(\frac{S_3 \cdot \dot{S}_3}{s_3} + \frac{S_4 \cdot \dot{S}_4}{s_4} - \frac{S_1 \cdot \dot{S}_1}{s_1} - \frac{S_2 \cdot \dot{S}_2}{s_2} \right) \qquad (C. 4\text{-}19)$$

Speed of Light Partials. The speed of light enters the angle formulations implicitly through the determination of t_v and R_v.

$$\frac{\partial X_{30}}{\partial c} = \frac{\partial X_{30}}{\partial R_v} \frac{\partial R_v}{\partial t_v} \frac{\partial t_v}{\partial c}$$

$$= \frac{\partial X_{30}}{\partial R_v} \cdot \dot{R}_v \frac{s}{c^2} \qquad (C.\ 4\text{-}20)$$

Similarly

$$\frac{\partial X_{85}}{\partial c} = \frac{\partial X_{85}}{\partial R_v} \cdot \dot{R}_v \frac{s}{c^2}$$

$$\frac{\partial Y_{30}}{\partial c} = \frac{\partial Y_{30}}{\partial R_v} \cdot \dot{R}_v \frac{s}{c^2}$$

$$\frac{\partial Y_{85}}{\partial c} = \frac{\partial Y_{85}}{\partial R_v} \cdot \dot{R}_v \frac{s}{c^2} \qquad (C.\ 4\text{-}21)$$

Noting that

$$t_V = t_t - \frac{s_r}{c}$$

$$t_t = t_r - \frac{s_r + s_t}{c} \qquad (C.\ 4\text{-}22)$$

$$\frac{\partial y_3}{\partial c} = - \frac{s_r + s_t}{c} + \frac{\partial y_3}{\partial R_v} \frac{\partial R_v}{\partial t_v} \frac{\partial t_v}{\partial c} + \frac{\partial y_3}{\partial R_t} \frac{\partial R_t}{\partial t_t} \frac{\partial t_t}{\partial c}$$

$$= - \frac{s_r + s_t}{c} + \left(\frac{S_r}{s_r} + \frac{S_t}{s_t} \right) \cdot \dot{R}_v \frac{s_R}{c^2}$$

$$- \frac{S_t}{s_t} \cdot \dot{R}_t \frac{s_r + s_t}{c^2} \qquad (C.\,4\text{-}23)$$

$$\frac{\partial y_4}{\partial c} = - \frac{\omega_3 \nu_{tr}}{c^2} \left(s_3 + s_4 - s_1 - s_2 \right) + \frac{\partial y_4}{\partial R_v} \cdot \frac{\partial R_v}{\partial t_v} \frac{\partial t_v}{\partial c}$$

$$+ \frac{\partial y_4}{\partial R_v} \frac{\partial R_v}{\partial (t_v - \tau)} \frac{\partial (t - \tau)}{\partial c}$$

$$+ \frac{\partial y_4}{\partial R_t} \frac{\partial R_t}{\partial t_t} \frac{\partial t_t}{\partial c} + \frac{\partial y_4}{\partial R_t} \frac{\partial R_t}{\partial (t_t - \tau)} \frac{\partial (t_t - \tau)}{\partial c}$$

$$= - \frac{\omega_4 \nu_{tr}}{c^2} \left(s_3 + s_4 - s_1 - s_2 \right) + \frac{\partial y_4}{\partial R_v} \cdot \frac{R_v (t_v)}{c^2} s_4$$

$$- \frac{\partial y_4}{\partial R_v} \cdot \frac{\dot{R}_v (t_v - \tau)}{c^2} s_2 + \frac{\partial y_4}{\partial R_t} \cdot \frac{\dot{R}_t (t_t)}{c^2} \left(s_3 + s_4 \right)$$

$$- \frac{\partial y_4}{\partial R_t} \cdot \frac{\dot{R}_t}{c^t} \left(s_1 + s_2 \right) \qquad (C.\,4\text{-}24)$$

Measurement Bias Partials. Since the bias errors are additive to y_1, y_2, y_3, ω_3 and have the same units,

$$\frac{\partial y_i}{\partial y_i} = 1 \qquad i = 1, 2, 3$$

$$\frac{\partial y_4}{\partial \omega_3} = \tau$$

C. 5 GODDARD RANGE AND RANGE-RATE SYSTEM

C. 5. 1 GRR Observables

The Goddard system observables are the X and Y angles, range, and doppler. The angles are identical to the X_{30}, Y_{30} of the USBS system. The ranging observable is the two-way transit time. The observable and its partials may be computed using the S-band ranging observable equations, with a subsequent division by the current speed of light estimate. The measurement bias error is assumed to be in the units of the observable, and remains 1.

The formulation of the doppler observable and its partial derivatives are nearly identical to those of the S-band system. In the Goddard system, doppler is

$$y_4 = t\omega_3 + \frac{\omega_4 \nu_{tr}}{c} \left[s_1 + s_2 - s_3 - s_4 \right] \qquad \text{(C. 5-1)}$$

where, due to the receiver mechanization, $\omega_4 = 1$. Except for the bias term, the signs are opposite of the S-band system. Then the partial derivatives are the same except that the signs are reversed for the Goddard system. Again, the exception is the measurement bias partial

$$\frac{\partial y_4}{\partial \omega_3} = \tau \qquad \text{(C. 5-2)}$$

C. 6 DSIF MEASUREMENT SYSTEM

C. 6. 1 DSIF Observables

The DSIF observables are hour-angle, declination, and doppler. They are computed from the following equations.

Hour angle: $$y_1 = \Upsilon + \lambda - \tan^{-1}\left(\frac{S\cdot J_e}{S\cdot I_e}\right) \quad 0 \leq y, < 2\pi$$

Declination: $$y_2 = \tan^{-1}\left(\frac{S\cdot K_e}{\left(S\cdot I_e{}^2 + S\cdot J_e{}^2\right)^{1/2}}\right) - \frac{\pi}{2} \leq y_2 \leq \frac{\pi}{2} \qquad \text{(C. 6-1)}$$

Doppler: The doppler observable and its partial derivatives are identical with those of the S-band system.

C. 6. 2 Gradients

The gradients of the angles with respect to vehicle position, R_v, are

$$\frac{\partial y_1}{\partial R_v} = \frac{\partial y_1}{\partial S} = \frac{-1}{1 + \dfrac{S\cdot J_e{}^2}{S\cdot I_e{}^2}}\left(\frac{(S\cdot I_e)\,J_e - (S\cdot J_e)\,I_e}{S\cdot I_e{}^2}\right)$$

$$= \frac{-S \times (J_e \times I_e)}{S\cdot I_e{}^2 + S\cdot J_e{}^2}$$

$$\frac{\partial y_1}{\partial R_v} = \frac{S \times K_e}{|S \times K_e|^2} \qquad \text{(C. 6-2)}$$

$$\frac{\partial y_2}{\partial R_v} = \frac{\partial y_2}{\partial S} = \frac{1}{\left(1 - \frac{S \cdot K_e{}^2}{S \cdot S}\right)^{1/2}} \left(\frac{(S \cdot S)K_e - (S \cdot K_e)S}{(S \cdot S)s}\right)$$

$$= \frac{(S \cdot S)K_e - (S \cdot K_e)S}{(S \cdot S - S \cdot K_e{}^2)^{1/2}(S \cdot S)}$$

$$\frac{\partial y_2}{\partial R_v} = \frac{S \times (K_e \times S)}{(S \cdot S) \mid K_e \times S \mid} \qquad (C. 6-3)$$

Again

$$\frac{\partial y_1}{\partial R_v} = \frac{\partial y_2}{\partial R_v} = (0,\ 0,\ 0) \qquad (C. 6-4)$$

C. 4. 3 Error Partials

Station Location Partials. The station position again enters the angle formulations in the slant range vector, S. In the hour angle, we have in addition the station longitude, λ. Then since

$$\lambda = \mathcal{T} + \tan^{-1}\left(\frac{R_r \cdot J_e}{R_r \cdot I_e}\right) \qquad (C. 6-5)$$

we have

$$\frac{\partial \lambda}{\partial R_r} = \frac{K_e \times R_r}{|K_e \times R_r|}$$

and hence

$$\frac{\partial y_1}{\partial (R_r)_e} = - \frac{\partial y_1}{\partial R_v} T_{B2C} + \frac{\partial y_1}{\partial \lambda} \frac{\partial \lambda}{\partial (R_r)_e}$$

$$= \left[\frac{K_e \times R_r}{|K_e \times R_r|} - \frac{\partial y_1}{\partial R_v} \right] T_{B2C}$$

$$\frac{\partial y_2}{\partial (R_r)_e} = \frac{\partial y_2}{\partial R_v} T_{B2C} \qquad \text{(C. 6-7)}$$

Station Clock Partials. The total time derivatives are

$$\dot{y}_1 = \frac{\partial y_1}{\partial \Upsilon} \dot{\Upsilon} + \frac{\partial y_1}{\partial s} \cdot \dot{s}$$

$$= \omega + \frac{\partial y_1}{\partial R_v} \cdot \dot{s}$$

$$\dot{y}_2 = \frac{\partial y_2}{\partial R_v} \cdot \dot{s} \qquad \text{(C. 6-8)}$$

where ω is the earth rotation rate.

Speed of Light Partials. Again,

$$\frac{\partial y_1}{\partial c} = \frac{\partial y_1}{\partial R_v} \frac{\partial R_v}{\partial t_v} \frac{\partial t_v}{\partial c}$$

$$= \frac{\partial y_1}{\partial R_v} \cdot \dot{R}_v \frac{s}{c^2}$$

$$\frac{\partial y_2}{\partial c} = \frac{\partial y_2}{\partial R_v} \frac{\partial R_v}{\partial t_v} \frac{\partial t_v}{\partial c}$$

$$= \frac{\partial y_2}{\partial R_v} \cdot \dot{R}_v \frac{s}{c^2} \qquad \text{(C. 6-9)}$$

C. 7 ATMOSPHERIC REFRACTION

The correction to the observables caused by bending of an electromagnetic wave in the atmosphere is the least satisfying of all the mathematical modeling in the Orbit Determination Program System because the atmosphere is neither static nor predictable and fluctuations will cause unknown variable errors in any corrective formula. Resort is made, therefore, to a fairly simple two-parameter model that provides reasonable mean corrections. Some variation is possible by altering the values of the two parameters.

The fundamental assumption underlying the model used is that the wave is confined to a plane containing the tracking station, the spacecraft, and the center of the earth. Implicit in the assumption is that the path length, and therefore range and elevation observations, is affected, while azimuth observations are not. The effects in range and elevation are computed directly from empirical formulae. Doppler effects are computed from range effects, while the effects on angles measured by X-Y mounts and equatorial mounts are computed from elevation effects and the assumption that azimuth is unaffected. The two refraction parameters are denoted in this section as a_1 and a_2. The first, a_1, is surface refractivity and has the nominal value of 0.34×10^{-3}. It is used in computing the elevation, and therefore the other angle, effects. The second, a_2, is used along with a_1 to compute range, and therefore doppler, effects. Its nominal value is 0.138771×10^{-3}.

C.7.1 Elevation Refraction Effects

Let Y_2 denote the computed elevation angle based on the current estimate of state as indicated in paragraph C.3.1. Then the correction to be added is computed as:

$$\Delta y_2 = a_1 \cot y \text{ (radians)}, \ (y_2 \geq 10°) \qquad \text{(C.7-1)}$$

$$\Delta y_2 = T - F\left[\left(a_1 + \frac{T}{2}^2\right) \cos y_2 - T \sin y_2\right] (y_2 < 10°)$$

where

$$T = \left[1.03585796 - \frac{0.01072014}{y_2} + \frac{0.1279119 \times 10^{-7}}{y_2^2} - \frac{0.1227363 \times 10^{-7}}{y_2^3}\right] a_1 \cot y_2 \qquad \text{(C.7-2)}$$

and

$$F = \frac{|R_r|}{|S|}$$

Curves showing the elevation corrections resulting from these formula are displayed in Figures C-1 and C-2.

C. 7. 2 Range Refraction Effects

Let y_3 denote the computed one-way range based on the current estimate of state as indicated in paragraph C. 3. 1. Then the correction to be added is computed as:

$$\Delta y_3 = \frac{a_1}{(a_2 \sin (y_2 + \Delta y_2))} \text{ meters} \qquad \text{(C. 7-3)}$$

This correction is displayed as a single curve in Figure C-3.

When the ranging observable is two-way range, as is the case for Unified S-Band and Goddard Range and Range Rate Systems, the effect must be added to both the up-link and down-link contributions to the ranging observable. The down-link correction is based on the corrected elevation angle at signal reception; the up-link correction is boxed on the corrected elevation angle at signal transmission.

C. 7. 3 Doppler Refraction Correction

The doppler observable is computed by a range difference technique described in paragraph C. 4. 1. Involved in the computation are four values of range denoted as s_i, $i = 1, 4$. For each of these a correction is computed by the method described in paragraph C. 7. 2, using the corrected elevation angle appropriate to each. Denoting these range corrections as Δs_i, $i = 1, 4$, the doppler correction Δy_4 is computed as

$$\Delta y_4 = \frac{\omega_4 \nu_{tr}}{c} \left[\Delta s_3 + \Delta s_4 - \Delta s_1 - \Delta s_2 \right] \qquad \text{(C. 7-4)}$$

For the Goddard Range and Range Rate System the correction must be given the opposite sign since, in that system, doppler counts are subtracted from rather than added to the fixed bias counts.

C.7.4 X- and Y-Angle Refraction Effects

The changes in X and Y due to atmosphere refraction are computed from the change in elevation with the constraint that azimuth is unchanged by refraction.

30-Foot Dish

$$X_{30} = \tan^{-1}\left(\frac{S\cdot E}{S\cdot D}\right)$$

$$= \cos^{-1}\left(\frac{-S\cdot D}{(S\cdot E^2 + S\cdot D^2)^{1/2}}\right)$$

$$Y_{30} = \tan^{-1}\left(\frac{S\cdot N}{S \times N}\right) = \tan^{-1}\left(\frac{S\cdot N}{(S\cdot E^2 + S\cdot D^2)^{1/2}}\right)$$

$$= \cos^{-1}\left(\frac{(S\cdot E^2 + S\cdot D^2)^{1/2}}{(S\cdot S)^{1/2}}\right) \qquad (C.7-5)$$

Elevation is given by

$$E = \sin^{-1}\left(\frac{-S\cdot D}{(S\cdot S)^{1/2}}\right) \qquad (C.7-6)$$

Therefore

$$\sin E = \cos X_{30} \text{ Cos } Y_{30}$$

$$\cos E = \left(1 - \cos^2 X_{30} \cos^2 Y_{30}\right)^{1/2} \qquad (C.7-7)$$

Azimuth is given by

$$A = \tan^{-1}\left(\frac{S\cdot E}{S\cdot N}\right) \qquad (C.7\text{-}8)$$

Therefore

$$\frac{\tan X_{30}}{\tan Y_{30}} = \frac{S\cdot E}{-S\cdot D} \quad \frac{(S\cdot E^2 + S\cdot D^2)^{1/2}}{S\cdot N} = \frac{S\cdot E}{S\cdot N} \quad \frac{(S\cdot E^2 + S\cdot D^2)^{1/2}}{-S\cdot D}$$

$$= \frac{\tan A}{\cos X_{30}} \qquad (C.7\text{-}9)$$

whence

$$\tan A = \frac{\sin X_{30}}{\tan Y_{30}} \qquad (C.7\text{-}10)$$

85-Foot Dish

$$X_{85} = \tan^{-1}\left(\frac{-S\cdot N}{-S\cdot D}\right)$$

$$= \cos^{-1}\left(\frac{-S\cdot D}{(S\cdot N^2 + S\cdot D^2)^{1/2}}\right)$$

$$Y_{85} = \tan^{-1}\left(\frac{S\cdot E}{|S \times E|}\right) = \tan^{-1}\left(\frac{S\cdot E}{(S\cdot N^2 + S\cdot D^2)^{1/2}}\right)$$

$$= \cos^{-1}\left(\frac{(S\cdot N^2 + S\cdot D^2)^{1/2}}{(S\cdot S)^{1/2}}\right) \qquad (C.7\text{-}11)$$

Therefore

$$\sin E = \cos X_{85} \cos Y_{85}$$

$$\cos E = (1- \cos^2 X_{85} \cos^2 Y_{85})^{1/2} \qquad (C.7-12)$$

Azimuth is related to X_{85} and Y_{85} as follows:

$$\frac{\tan Y_{85}}{\tan X_{85}} = \frac{S \cdot E}{(S \cdot N^2 + S \cdot D^2)^{1/2}} \quad \frac{S \cdot D}{S \cdot N} \quad \frac{S \cdot E}{S \cdot N} \quad \frac{S \cdot D}{(S \cdot N^2 + S \cdot D^2)^{1/2}}$$

$$= - \tan A \cos X_{85}, \qquad (C.7-13)$$

whence

$$- \cot A = \frac{\sin X_{85}}{\tan Y_{85}} \qquad (C.7-14)$$

<u>Combined Equations, Either Mount.</u> Except for the subscripts, equations (C. 7-7) and (C. 7-12) are identical. We rewrite them as

$$\sin E = \cos X \cos Y$$

$$\sin E = (1 - \cos^2 X \cos^2 Y)^{1/2} \qquad (C.7-15)$$

Differentiating the first of equation (C. 7-15) gives

$$\cos E \, dE = -\cos Y \sin X \, dX - \cos X \sin Y \, dY \qquad (C.7-16)$$

Equations (C. 7-10) and (C. 7-14) may be written in a common form as

$$f(A) = \frac{\sin X}{\tan Y} \qquad (C.7-17)$$

We take the total differential of this equation and set the result equal to zero, since azimuth is to be considered unaffected by refraction. This yields

$$\frac{df}{dA}\ dA = \frac{\cos X}{\tan\ Y}\ dX \qquad \frac{\sin X\ \sec^2\ Y}{\tan^2 Y}\ dY = 0$$

whence

$$\cos\ Y\ dX = \frac{\sin X}{\cos X\ \sin\ Y}\ dy \tag{C. 7-18}$$

Combining equations (C. 7-16) and (C. 7-18) gives

$$\cos\ E\ dE = \frac{-\sin^2\ X}{\cos X\ \sin\ Y}\ dY - \cos X\ \sin\ Y\ dY$$

$$= \frac{-\sin^2 X - \cos^2 X\ \sin^2 Y}{\cos\ X\ \sin\ Y}\ dY \tag{C. 7-19}$$

whence

$$dY = \frac{-\cos X\ \sin\ Y}{\sin^2 X + \cos^2 X\ \sin\ ^2 Y}\ \cos\ E\ dE$$

$$= \frac{-\cos\ X\ \sin\ Y}{1 - \cos^2 X\ \cos^2 Y}\ \cos\ E\ dE \tag{C. 7-20}$$

Combining this result with equation (C. 7-15) gives

$$dY = \frac{-\cos X\ \sin\ Y}{\cos\ E}\ dE \tag{C. 7-21}$$

Combining equations (C. 7-18) and (C. 7-21) gives

$$dX = \frac{-\sin X}{\cos Y \cos E} \ dE \qquad \text{(C. 7-22)}$$

Thus equations (C. 7-21) and (C. 7-22) are used to compute the refractive change in X and Y given the change in E, and the equations are identical for the 30-foot and 85-foot antennas.

C. 7. 5 Hour Angle and Declination Refraction Effects

The changes in hour angle and declination due to atmospheric refraction are computed from the change in elevation with the constraint that azimuth is unchanged by refraction.

Hour angle is measured in a plane parallel to the earth's equatorial plane from the station meridian westward to the projection of the station-to-vehicle vector S onto this plane. It is convenient at this point to define a coordinate system as follows:

The coordinate center is at the tracker. The first axis I and the second axis J define a plane parallel to the equator plane; I is in the plane of the station meridian, J is 90° east of I. The third axis K completes the right-handed system.

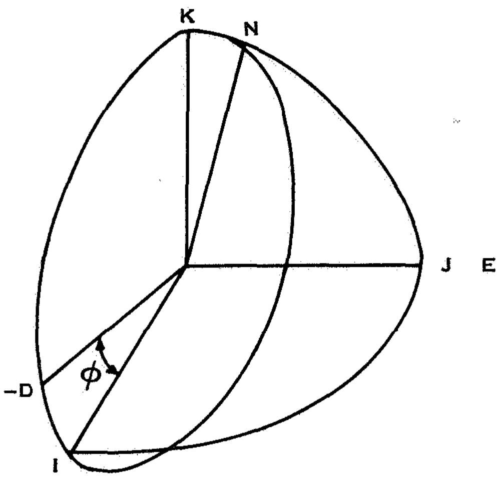
Hour angle is then defined as

$$H = \tan^{-1}\left(\frac{-S\cdot J}{S\cdot I}\right) = \sin^{-1}\left(\frac{-S\cdot J}{(S\cdot I^2 + S\cdot J^2)^{1/2}}\right)$$

$$= \cos^{-1}\left(\frac{S\cdot I}{(S\cdot I^2 + S\cdot J^2)^{1/2}}\right) \qquad \text{(C. 7-23)}$$

$$\delta = \tan^{-1} \frac{S \cdot K}{(S \cdot I^2 + S \cdot J^2)^{1/2}} = \sin^{-1} \frac{S \cdot K}{(S \cdot S)^{1/2}}$$

$$= \cos^{-1} \frac{(S \cdot I^2 + S \cdot J^2)^{1/2}}{(S \cdot S)^{1/2}} \qquad (C. 7-24)$$



The I - J - K coordinate system is easily expressed in term of the local North-East-Down system. Let ϕ be the geodetic latitude of the tracker. Then

$$I = -N \sin \phi - D \cos \phi$$

$$J = E$$

$$K = N \cos \phi - D \sin \phi \qquad (C. 7-25)$$

Hour angle and declination may now be written in terms of azimuth and elevation. Azimuth and elevation are given by

$$A = \tan^{-1}\left(\frac{S\cdot E}{S\cdot N}\right) = \sin^{-1}\left(\frac{S\cdot E}{S\cdot N^2 + S\cdot E^2)^{1/2}}\right)$$

$$= \cos^{-1}\left(\frac{S\cdot N}{(S\cdot N^2 + S\cdot E^2)^{1/2}}\right)$$

$$E = \tan^{-1}\left(\frac{-S\cdot D}{(S\cdot N^2 + S\cdot E^2)^{1/2}}\right) = \sin^{-1}\left(\frac{-S\cdot D}{(S\cdot S)^{1/2}}\right)$$

$$= \cos^{-1}\left(\frac{(S\cdot N^2 + S\cdot E^2)^{1/2}}{(S\cdot S)^{1/2}}\right) \qquad (C.\ 7\text{-}26)$$

Hour angle is given by

$$\tan H = \frac{-S\cdot J}{S\cdot I} = \frac{-S\cdot E}{-S\cdot N \sin \phi - S\cdot D \cos \phi} \qquad (C.\ 7\text{-}27)$$

whence

$$\cot H = \frac{S\cdot N}{S\cdot E} \sin \phi + \frac{S\cdot D}{S\cdot E} \cos \phi$$

or

$$\cot H = \cot A \sin \phi - \frac{\tan E}{\sin A} \cos \phi \qquad (C.\ 7\text{-}28)$$

Differentiating equation (C. 7-28) with respect to elevation while holding azimuth fixed gives the correction in hour angle in terms of the correction in elevation

$$- \csc^2 H \, dH = \frac{- \sec^2 E}{\sin A} \cos \phi \, dE$$

or

$$dH = \frac{\sin^2 H \cos \phi}{\sin A \cos^2 E} dE \qquad (C. 7-29)$$

Declination is given by

$$\sin \delta = \frac{S \cdot K}{(S \cdot S)^{1/2}} \cos \phi - \frac{S \cdot D}{(S \cdot S)^{1/2}} \sin \phi$$

or

$$\sin \delta = \cos A \cos E \cos \phi + \sin E \sin \phi \qquad (C. 7-30)$$

Differentiating equation (C. 7-30) with respect to elevation while holding azimuth fixed gives

$$\cos \delta \, d\delta = (- \cos A \sin E \cos \phi + \cos E \sin \phi) \, dE$$

or

$$d\delta = \frac{\cos E \sin \phi - \cos A \sin E \cos \phi}{\cos \delta} dE \qquad (C. 7-31)$$

Thus equations (C. 7-29) and (C. 7-30) are used to compute the refractive change in hour angle and declination given the change in elevation.

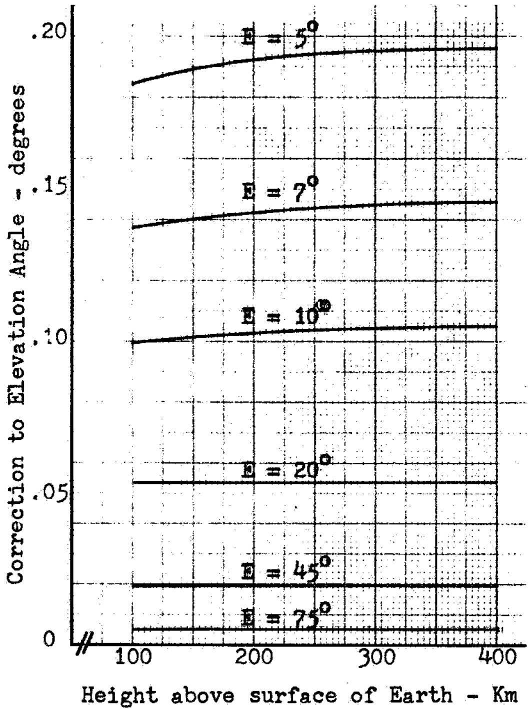


Figure C-1

ELEVATION CORRECTION

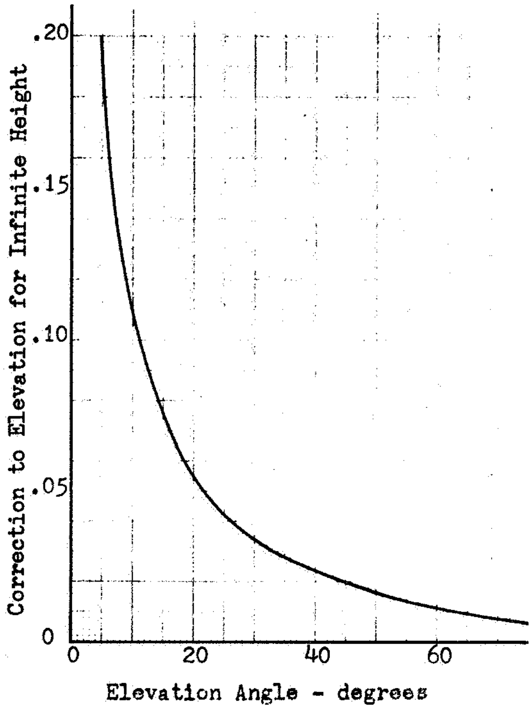


Figure C-2

ELEVATION CORRECTION

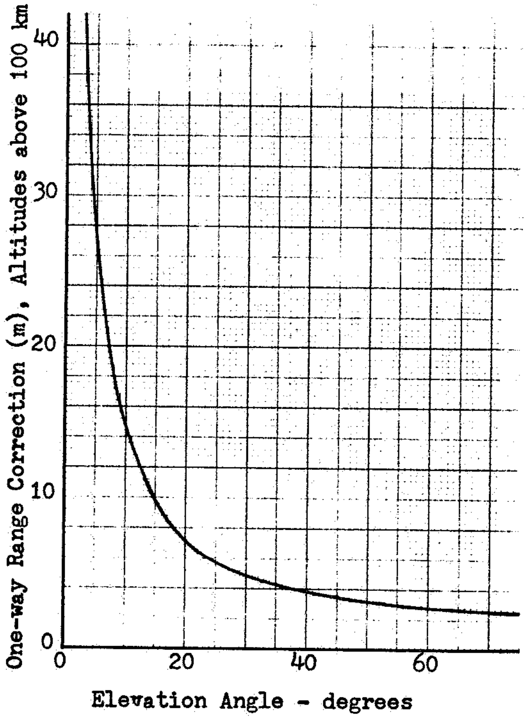


Figure C-3

ONE-WAY RANGE CORRECTION

REFERENCES

1. Proceedings of the Apollo Unified S-Band Technical Conference, NASA SP-87, National Aeronautics and Space Administration, Goddard Space Flight Center, July 14-15, 1965.

2. Basic Equations and Logic for the Apollo 503 Real Time Ground Navigation Program, MSC Internal Note No. 66-FM-, Mission Planning and Analysis Division, National Aeronautics and Space Administration, Manned Spacecraft Center, Houston, Texas, Jan. 13, 1967.

3. Goddard Range and Range Rate System Design Evaluation Report, Report No. W2719-Z-1, Revision 1, Motorola Inc., Scottsdale, Arizona, 23 November 1962.

4. The Orbit Determination Program of the Jet Propulsion Laboratory, TM 33-168, Jet Propulsion Laboratory, California Institute of Technology, Pasadena, California, March 19, 1964.

5. Theoretical Basis of the JPL Orbit Determination Program, TM 312-409, Jet Propulsion Laboratory, California Institute of Technology, Pasadena, California, March 24, 1964.

6. A Range-Difference Method for Computing the Doppler Observable, MSC Internal Note No. 67-FM-24, Mission Planning and Analysis Division, National Aeronautics and Space Administration, Manned Spacecraft Center, Houston, Texas, February 17, 1967.

7. Tracking Data Conversion and Format Program (TDCFP/7094), International Business Machines Corporation, Federal Systems Division, Garthersburg, Maryland, October, 1967.

8. Memorandum for Record: "Algonithms (sic) for Converting Range Rate Information in Unified S-Band High Speed Format to Engineering Units" File 17.0.1, Serial 234, Goddard Space Flight Center, Greenbelt, Maryland, May 16, 1966.

9. Memorandum for Record: "The Reduction of Goddard Range and Range Rate Data from the Hybrid Systems at Alaska and Santiago, Revision I," File 1.3/12.0, Serial 209, Goddard Space Flight Center, Greenbelt, Maryland, December 15, 1966.

REFERENCES (Cont)

10. Memorandum for Record: "The Reduction of Goddard Range and Range Rate Data from the Hybrid Systems at Alaska and Santiago, Revision II," File 1.3/12.0, Serial 304, Goddard Space Flight Center, Greenbelt, Maryland, January 27, 1917.

11. Undated Memorandum: "Description of the Goddard Range - Range Rate Data Processing Program for the CDC-160A," Edward R. Watkin and Douglas H. Rose, Goddard Space Flight Center, Greenbelt, Maryland.

12. Letter: "Tracking Data Conversion and Format Program," E. H. Mead, Manager. NICO Programming, to W. Kahn, NASA, Goddard Space Flight Center, Greenbelt, Maryland, February 21, 1967, with attachment.

13. : Revised attachment to above, October, 1967.

14. Telephone Conversation: With E. H. Mead, Manager, NICO Programming, Goddard Space Flight Center, Greenbelt, Maryland, July 10, 1967.

15. Telephone Conversation with Douglas H. Rose, Goddard Space Flight Center, Greenbelt, Maryland, July 10, 1967.

APPENDIX D

DYNAMIC MODEL

D.1 INTRODUCTION

This appendix describes the spacecraft equations of motion used by the ODP, and the techniques used for this solution. Each of the environmental effects included is discussed with respect to assumptions and restrictions imposed and the identification of adjustable constants in the model. The sensitivities of spacecraft position and velocity to small variations in initial conditions and equation of motion parameters are also developed.

D.2 EQUATIONS OF MOTION

The equations of motion of the spacecraft may be written in the form

$$\dot{X} = \overline{F}(X,u) \tag{D.2-1}$$

where the dot denotes the derivative with respect to the vehicle's proper time, assumed here to be ephemeris time (see Appendix A). Here u is a vector of parameters affecting the vehicle's motion. Using the more familiar form

$$\begin{aligned} \dot{R} &= V \\ \dot{V} &= F(R,V,u) \end{aligned} \tag{D.2-2}$$

we have the three-component second-order equation

$$\ddot{R} = F(R,V,u) \tag{D.2-3}$$

Equation (D.2-3) may be integrated numerically from given initial conditions

$$R_1 = R(t_1)$$
$$V_1 = V(t_1) \qquad (D.2-4)$$

to yield $R(t_n)$, $V(t_n)$ at a sequence of times t_n along the trajectory.

$F(R, V, u)$ may be written and examined as the sum of a set of functions

$$F(R, V, u) = \sum_{k=o}^{N} F_k(R, V, u) \qquad (D.2-5)$$

The acceleration which usually dominates the F_k is the gravitational acceleration of the central body. This fact, together with the existence of a closed-form solution for the inverse-square acceleration problem, admits an advantage in accuracy and computation time with the use of Encke's method for integrating equations (D.2-3). That is, we use a reference trajectory for the most significant part of the solution and integrate numerically only the perturbing accelerations.

Let $R_o(t)$, $V_o(t)$ be the position and velocity obtained from the solution of the inverse-square acceleration problem

$$\ddot{R}_o = F_o(R_o, V_o, u) \qquad (D.2-6)$$

Writing

$$R = R_o + \delta R$$
$$\delta R(t_1) = \delta\dot{R}(t_1) = o$$

we obtain the perturbation acceleration

$$\delta\ddot{R} = \left[F_o(R, V, u) - F_o(R_o, V_o, u)\right] + \sum_{k=1}^{N} F_k(R, V, u) \qquad (D.2-7)$$

D.2.1 Central Force Acceleration

The inverse-square acceleration is

$$\ddot{R}_o = F_o(R_o, V_o, u) = -\mu R_o/r_o^3 \qquad (D.2\text{-}8)$$

where:

$r_o = |R_o|$ = magnitude of R_o

μ = central body gravitational constant

Instantaneous values of R_o, V_o at any desired time are obtained from a form of Kepler's equation by the subroutines STEPDI, STEPDT (Reference 1).

The actual central force acceleration is

$$F_o(R, V, u) = -\mu R/r^3 \qquad (D.2\text{-}9)$$

and differs from $F_o(R_o, V_o, u)$ due to the deviation δR from the reference trajectory. The perturbation acceleration, then, contains the difference

$$F_o(R, V, u) - F_o(R_o, V_o, u) = \frac{\mu b}{r_o^3}(cR_o - \delta R) \qquad (D.2\text{-}10)$$

where:

$q = (2R_o + \delta R) \cdot \delta R/r_o^2$

$p = (1+q)^3 - 1$

$b = 1/\sqrt{1+p}$

$c = p/(1+\sqrt{1+p})$ (D.2-11)

The difference (D.2-10) is sometimes called the Encke acceleration.

D.2.2 Perturbing Body Acceleration

Each non-central body affecting the motion of the vehicle contributes an acceleration which may be considered inverse-square due to the distances of those bodies from the vehicle. Setting

μ_i = gravitational constant of body i

P_i = position of body i relative to the central body

$\Delta_i = R - P_i$

$p_i = |P_i|$

$\delta_i = |\Delta_i|$

the gravitational acceleration due to the non-central bodies is

$$F_1 = \sum_i -\mu_i \left(\frac{\Delta_i}{\delta_i^3} + \frac{P_i}{p_i^3} \right) \qquad \text{(D. 2-12)}$$

where the summation includes all non-central bodies desired.

D.2.3 Central Body Gravitational Perturbations

That part of the central body's gravitational attraction which is not inverse-square is obtained from the potential

$$F_2 = -\nabla u$$

$$u = \frac{\mu}{r} \sum_{n,m} J_{nm} \left(\frac{a}{r}\right)^n P_n^m \cos m (\lambda - \lambda_{nm}) \qquad \text{(D. 2-13)}$$

where:

a = equatorial radius of the central body

r = vehicle radius from the central body

δ = vehicle latitude, from the equatorial plane

λ = vehicle longitude, from the prime meridian and we have used the abbreviations

$$\sum_{n,m} \text{ for } \sum_{n=2}^{N} \sum_{m=o}^{n}$$

$P_n^m = P_n^m(s)$ = Associated Legendre function of the first kind, order n, degree m

$s = \sin \varphi$

The J_{nm}, λ_{nm} are the harmonic coefficients. The maximum value of the upper limit of the n-summation, imposed by the dimensioning of the ODP, is $N = 7$. All J_{nm} are assumed zero for $m \neq o$, $n \geq 5$.

Let

$$c = \cos \varphi$$

$$C_{nm} = J_{nm} \left(\frac{a}{r}\right)^n \cos m (\lambda - \lambda_{nm})$$

$$S_{nm} = J_{nm} \left(\frac{a}{r}\right)^n \sin m (\lambda - \lambda_{nm}) \qquad \text{(D. 2-14)}$$

and

$\underline{r}$ = unit vector along the position vector

$\underline{k}$ = unit vector along the central body polar axis

$\underline{\lambda} = \underline{k} \times \underline{r}/c$

Using the identities

$$(n-m+1)\, P_{n+1}^{m} = (2n+1)\, sP_n^m - (n+m)\, P_{n-1}^{m}$$

$$(2n+1)\, cP_n^m = P_{n+1}^{m+1} - P_{n-1}^{m+1} \qquad \text{(D. 2-15)}$$

and the gradients

$$\nabla\left(\frac{1}{r}\right) = \underline{r}^T/r^2$$

$$\nabla s = \underline{k}^T \, (I-\underline{r}\underline{r}^T)/r$$

$$\nabla\lambda = \lambda^T/rc \qquad \text{(D. 2-16)}$$

we obtain

$$F_2 = \underline{r}\left[\frac{\mu}{r^2} \sum C_{nm} \left(\frac{1}{c} P_{n+1}^{m+1} - \frac{m}{c^2} P_n{}^m\right)\right]$$

$$- \underline{k}\left[\frac{\mu}{r^2} \sum C_{nm} \left(\frac{1}{c} P_n^{m+1} - \frac{ms}{c^2} P_n{}^m\right)\right]$$

$$- \underline{\lambda}\left[\frac{\mu}{r^2} \sum S_{nm} \left(- \frac{m}{c} P_n{}^m\right)\right] \qquad \text{(D. 2-17)}$$

A somewhat simpler form is obtained by using the unit vector

$$\underline{\ell} = \underline{r} \times \underline{\lambda}$$

$$\underline{k} = c\underline{\ell} + s\underline{r} \qquad \text{(D. 2-18)}$$

Replacing $\underline{k}$ in (D. 2-17),

$$F_2 = G_r\underline{r} + G_\ell\underline{\ell} + G_\lambda\underline{\lambda}$$

$$G_r = \frac{\mu}{r^2} \sum_{n,m} C_{nm} \, (n+1) \, P_n{}^m$$

$$G_\ell = \frac{-\mu}{r^2} \sum_{n,m} C_{nm}\left(P_n^{m+1} - \frac{ms}{c} P_n{}^m\right)$$

$$G_\lambda = \frac{-\mu}{r^2} \sum_{n,m} S_{nm} \left(- \frac{m}{c} P_n{}^m\right) \qquad \text{(D. 2-19)}$$

D.2.4 Solar Radiation Pressure Acceleration

Solar radiation pressure is assumed to cause an inverse-square repulsion relative to the sun. The acceleration is

$$F_3 = C_1 R_s / |R_s|^3 \qquad \text{(D.2-20)}$$

where R_s is the position of the vehicle relative to the sun

D.2.5 Atmospheric Drag

Drag due to the atmosphere of the central body may be included. It is assumed to be simple drag; i.e., it directly opposes the velocity of the vehicle relative to the atmosphere.

The atmosphere is assumed to rotate with the central body. The velocity relative to the atmosphere and the acceleration due to drag are

$$V_a = V - \omega \times R$$

$$F_4 = - \frac{C_2 \rho}{\rho_o} |V_a| V_a \qquad \text{(D.2-21)}$$

where

ω = angular velocity of the central body

ρ = atmosphere density, assumed exponential = $\rho_o e^{-C_3 h}$

C_2, C_3 = constants

h = altitude above central body surface

D.2.6 Venting Acceleration

A constant-thrust venting parallel to the vehicle's velocity may be included. The acceleration is

$$F_5 = C_4 \frac{V}{|V|} \qquad (D.2\text{-}22)$$

D.2.7 Equation of Motion Parameters

The vector of equation of motion parameters, U, may be made up of any of the constants in the above equations. In addition, constants defining the time difference ET-UT (see Appendix A) may be included. A complete list of permissible components of U is given in Table D-1. Values of these constants are stored in the labelled common ESTCOM in the arrays indicated.

Two additional constants are implicit in the gravitational constants μ_3 (Earth) and μ_{10} (Sun). These are the astronomical unit, a.u., and Earth radius, r_E. These constants are used by the ODP in computing planetary and lunar ephemerides. The ODP uses the JPL Ephemeris Tape System (Reference 3), which provides planetary (and Earth-Moon barycenter) positions and velocities in a.u. and a.u./mean solar day and lunar position and velocity in earth radii and earth radii/day. These are scaled to km and km/sec using the given conversions a.u.-to-km and earth radii-to-km.

The sidereal periods of the barycenter orbit about the sun and the lunar orbit about the Earth are known with considerably more precision than are the conversion factors. If we fix these periods, we obtain

$$(a.u.)^3/\mu_{10} = \text{constant}$$

$$(r_E)^3/\mu_3 = \text{constant} \qquad (D.2\text{-}23)$$

TABLE D-1

EQUATION OF MOTION PARAMETERS

Symbol	Dimension	Math Symbol	Units	Equation	Description
SPCDAN	d (6)	R, V	km	D. 2-8	Initial position and velocity, equator, equinox of 1950. 0
EFEDAN	d (14)	μ_i	km^3/sec^2	D. 2-12	1-12 gravitational constants
		a. u.	km	D. 2-21	13 a. u.
		r_E	km		14 r_E
PREDAN	d (4)	C_1	km^3/sec^2	D. 2-18	Solar pressure coefficients
		C_2, C_3	1/km	D. 2-19	Drag coefficients
		C_4	km/sec^2	D. 2-20	Venting coefficient
DELDAN	d (2)	γ_3, γ_4	sec sec/sec	A. 3-8	ET-UT parameters
EHADAN	d (24)	$J_2 \ldots J_7$	--	D. 2-14	Earth harmonic coefficients
		J_{21}, λ_{21}			
		J_{22}, λ_{22}			
		J_{31}, λ_{31}			
		J_{32}, λ_{32}			
		J_{33}, λ_{33}			
		J_{41}, λ_{41}			
		J_{42}, λ_{42}			
		J_{43}, λ_{43}			
		J_{44}, λ_{44}			
MHADAN	d (24)				Moon harmonic coefficients
XHADAN	d (24)				Extra body harmonic coefficients. *

* Harmonic coefficients for one extra body may be included in the equations of motion. Components of either EHADAN or MHADAN or XHADAN may be solved for or included as uncertain.

Variations in μ_3, μ_{10}, then, must be accompanied by variations in r_E, a.u., respectively, as required by (D.2-23). That is,

$$\frac{d(a.u.)}{a.u.} = \frac{1}{3} \frac{d\mu_{10}}{\mu_{10}}$$

$$\frac{d(r_E)}{r_E} = \frac{1}{3} \frac{d\mu_3}{r_E} \tag{D.2-24}$$

D.3 VARIATIONAL EQUATIONS

To find the sensitivity of the position and velocity, R(t), V(t), to the equation of motion parameters, we consider that a trajectory $\overline{R}(t)$ corresponding to $\overline{U}$,

$$\ddot{\overline{R}} = F(\overline{R}, \overline{V}, \overline{U}) \tag{D.3-1}$$

is known. Using the chain rule of differentiation,

$$\frac{\partial \ddot{R}}{\partial \alpha} = \frac{\partial F}{\partial R} \frac{\partial R}{\partial \alpha} + \frac{\partial F}{\partial V} \frac{\partial V}{\partial \alpha} + \frac{\partial F}{\partial U} \frac{\partial U}{\partial \alpha} \tag{D.3-2}$$

where $\partial F/\partial R$ denotes the gradient of F with respect to R, evaluated on the known trajectory $\overline{R}(t)$. Interchanging the order of differentiation,

$$\frac{d^2}{dt^2}\left(\frac{\partial R}{\partial \alpha}\right) = \frac{\partial F}{\partial R} \frac{\partial R}{\partial \alpha} + \frac{\partial F}{\partial V} \frac{\partial V}{\partial \alpha} + \frac{\partial F}{\partial U} \frac{\partial U}{\partial \alpha} \tag{D.3-3}$$

Let

$$\varphi_1(t;t_1) = \frac{\partial R(t)}{\partial R(t_1)}$$

$$\varphi_2(t;t_1) = \frac{\partial R(t)}{\partial V(t_1)}$$

$$\varphi_u(t;t_1) = \frac{\partial R(t)}{\partial U} \tag{D.3-4}$$

Again interchanging the order of integration,

$$\dot{\varphi}_1(t;t_1) = \frac{\partial V(t)}{\partial R(t_1)} \qquad \text{(D. 3-5)}$$

etc. Successively setting $\alpha = R(t_1)$, $\alpha = V(t_1)$, $\alpha = U$, we obtain

$$\ddot{\varphi}_1(t;t_1) = \frac{\partial F}{\partial R}\ \varphi_1(t;t_1) + \frac{\partial F}{\partial V}\ \dot{\varphi}_1(t;t_1)$$

$$\ddot{\varphi}_2(t;t_1) = \frac{\partial F}{\partial R}\ \varphi_2(t;t_1) + \frac{\partial F}{\partial V}\ \dot{\varphi}_2(t;t_1)$$

$$\ddot{\varphi}_u(t;t_1) = \frac{\partial F}{\partial R}\ \varphi_u(t;t_1) + \frac{\partial F}{\partial V}\ \dot{\varphi}_u(t;t_1) + \frac{\partial F}{\partial U} \qquad \text{(D. 3-6)}$$

Since

$$\frac{\partial R(t_1)}{\partial R(t_1)} = I_{3x3}$$

$$\frac{\partial R(t_1)}{\partial V(t_1)} = O_{3x3} \qquad \text{(D. 3-7)}$$

etc., we have the initial conditions for equations (D. 3-6)

$$\varphi_1(t_1;t_1) = \dot{\varphi}_2(t_1;t_1) = I_{3x3}$$

$$\dot{\varphi}_1(t_1;t_1) = \varphi_2(t_1;t_1) = O_{3x3}$$

$$\varphi_u(t_1;t_1) = \dot{\varphi}_u(t_1;t_1) = O_{3x3} \qquad \text{(D. 3-8)}$$

The state transition matrix is the 6 x 6 matrix

$$\varphi(t;t_1) = \begin{bmatrix} \varphi_1(t;t_1) & \varphi_2(t;t_1) \\ \dot{\varphi}_1(t;t_1) & \dot{\varphi}_2(t;t_1) \end{bmatrix} \tag{D. 3-9}$$

It is easily seen that if

$$\begin{aligned} \ddot{R} &= F(R, \dot{R}, U) \\ \ddot{\overline{R}} &= F(\overline{R}, \dot{\overline{R}}, \overline{U}) \end{aligned} \tag{D. 3-10}$$

then for the small variations r, $\dot{r}$, u, where

$$\begin{aligned} R &= \overline{R} + r \\ \dot{R} &= \dot{\overline{R}} + \dot{r} \\ U &= \overline{U} + u \end{aligned} \tag{D. 3-11}$$

we have, to within first order terms in r, $\dot{r}$, u,

$$\begin{aligned} r(t) &= \varphi_1(t;t_1)\, r(t_1) + \varphi_2(t;t_1)\, r(t_1) + \varphi_u(t;t_1)\, u \\ \dot{r}(t) &= \dot{\varphi}_1(t;t_1)\, r(t_1) + \dot{\varphi}_2(t;t_1)\, r(t_1) + \dot{\varphi}_u(t;t_1)\, u \end{aligned} \tag{D. 3-12}$$

Clearly, the transition matrices contain the same frequencies as R(t), and in order to gain the interval size advantage offered by the Encke integration of (D. 2-3), we must likewise integrate (D. 3-6) by a perturbation method. We consider the reference conic discussed in Paragraph D. 2.

$$\begin{aligned} \ddot{R}_o &= -\frac{\mu}{r_o} R_o \\ R_o(t_1) &= R_1 \\ V_o(t_1) &= V_1 \end{aligned} \tag{D. 3-13}$$

The state transition matrix for the conic, the solution of

$$\ddot{\varphi}_{i0}(t;t_1) = -\frac{\mu}{r_o^3}\left(I - \frac{3R_oR_o^T}{r_o^2}\right)\varphi_{i0}(t;t_1)$$

$$\varphi_{10}(t_1;t_1) = \dot{\varphi}_{20}(t_1;t_1) = I$$

$$\dot{\varphi}_{10}(t_1;t_1) = \varphi_{20}(t_1;t_1) = 0 \qquad \text{(D.3-14)}$$

was given by Pines (Reference 5). It is computed in closed form by subroutine STEPDP (Reference 1). Let

$$\varphi_1(t;t_1) = \varphi_{10}(t;t_1) + \delta\varphi_1(t;t_1)$$

etc. We have immediately that

$$\delta\ddot{\varphi}_1 = -\frac{\mu}{r_o^3}\left(I - \frac{3R_oR_o^T}{r_o^2}\right)\delta\varphi_1 + \frac{\partial\delta\ddot{R}}{\partial R}\varphi_1 + \frac{\partial\delta\ddot{R}}{\partial V}\dot{\varphi}_1$$

$$\delta\ddot{\varphi}_2 = -\frac{\mu}{r_o^3}\left(I - \frac{3R_oR_o^T}{r_o^2}\right)\delta\varphi_2 + \frac{\partial\delta\ddot{R}}{\partial R}\varphi_2 + \frac{\partial\delta\ddot{R}}{\partial V}\dot{\varphi}_2$$

$$\ddot{\varphi}_u = \frac{\partial\delta\ddot{R}}{\partial R}\varphi_u + \frac{\partial\delta\ddot{R}}{\partial V}\dot{\varphi}_u + \frac{\partial F}{\partial U} \qquad \text{(D.3-15)}$$

with the initial conditions

$$\delta\varphi_1(t_1;t_1) = \delta\dot{\varphi}_1(t_1;t_1) = 0$$

$$\delta\varphi_2(t_1;t_1) = \delta\dot{\varphi}_2(t_1;t_1) = 0$$

$$\varphi_u(t_1;t_1) = \varphi_u(t_1;t_1) = 0 \qquad \text{(D.3-16)}$$

The gradients of the accelerations required for the integration of the variational equations are given below. We use the notation

I = identity matrix

A = column vector

A^T = row vector, the matrix transpose of A

AX = skew-symmetric matrix which, operating on any vector B, gives the vector product A x B.

D.3.1 Encke Acceleration

The gradients of the Encke acceleration, (D.2-10), are

$$\frac{\partial}{\partial R}\left[F_o(R, V, U) - F_o(R_o, V_o, U)\right] = -\frac{\mu}{r^3}\left(I - \frac{3RR^T}{r^2}\right)$$

$$\frac{\partial}{\partial V}\left[F_o(R, V, U) - F_o(R_o, V_o, U)\right] = 0 \qquad \text{(D.3-17)}$$

The partial derivative with respect to μ is

$$\frac{\partial F_o}{\partial \mu} = -\frac{1}{r^3} R \qquad \text{(D.3-18)}$$

D.3.2 Perturbing Body Acceleration

From (3.2-12),

$$\frac{\partial F_1}{\partial R} = \sum_i \frac{\mu}{\delta_i^3}\left(\frac{3\Delta_i\Delta_i^T}{\delta_i^2} - I\right)$$

$$\frac{\partial F_1}{\partial V} = 0 \qquad \text{(D.3-19)}$$

where the summation extends over all bodies except the central body. For the μ_i in U,

$$\frac{\partial F}{\partial \mu_i} = -\left(\frac{\Delta_i}{\delta_i^{3}} + \frac{P_i}{p_i^{3}}\right) \qquad i \neq 3,10$$

$$\frac{\partial F}{\partial \mu_3} = -\left(\frac{\Delta_3}{\delta_3^{3}} + \frac{P_3}{p_3^{3}}\right) + \frac{1}{\mu_3} \; \frac{r_E}{3} \; \frac{\partial F}{\partial r_E}$$

$$\frac{\partial F}{\partial \mu_{10}} = -\left(\frac{\Delta_{10}}{\delta_{10}^{3}} + \frac{P_{10}}{p_{10}^{3}}\right) + \frac{1}{\mu_{10}} \; \frac{a.u.}{3} \; \frac{\partial F}{\partial a.u.} \qquad \text{(D. 3-20)}$$

$$\frac{(a.u.)}{3} \; \frac{\partial F}{\partial (a.u.)} = \sum_{i \neq 3,11} \mu_i \left\{(a_i - b_i)\, P_i + b_i \, P_{12} + c_i \, \Delta_i\right\}$$

$$\frac{r_E}{3} \; \frac{\partial F}{\partial r_E} = \sum_{i \neq 3,11} \mu_i \left\{\frac{1}{p_i^{5}} \, (P_i \cdot P_{12})\, P_i + b_i \, P_{12} - \frac{1}{\delta_i^{5}} (\Delta_i \cdot P_{12})\, \Delta_i\right\}$$

$$+ \; \mu_{11} \left\{\left(\frac{1}{p_{11}^{3}} - b_{11}\right) P_{11} + \frac{1}{\delta_{11}^{5}} \, (\Delta_{11} \cdot P_{11})\, \Delta_{11}\right\}$$

$$a_i = \frac{1}{p_i^{5}} \left(p_i^{2} - P_i \cdot P_{12}\right)$$

$$b_i = \frac{1}{3} \left(\frac{1}{p_i^{3}} - \frac{1}{\delta_i^{3}}\right)$$

$$c_i = \frac{1}{\delta_i^{5}} \left(\Delta_i \cdot P_i - \Delta_i \cdot P_{12}\right)$$

where:

P_{12} = position of the Earth-Moon barycenter.

D. 3. 3 Central Body Harmonics

From equations (D. 2-15) through (D. 2-19), using

$$
\begin{aligned}
I &= \underline{rr}^T + \underline{\ell\ell}^T + \underline{\lambda\lambda}^T \\
rx &= \underline{\ell\lambda}^T - \underline{\lambda\ell}^T
\end{aligned}
\tag{D. 3-21}
$$

etc., since r, λ, ℓ form an orthonormal triad, we compute

$$
\begin{aligned}
\frac{\partial F_2}{\partial R} = \; & G_{rr}\,\underline{rr}^T + G_{r\ell}\,\underline{r\ell}^T + G_{r\lambda}\,\underline{r\lambda}^T \\
& + G_{\ell r}\,\underline{\ell r}^T + G_{\ell\ell}\,\underline{\ell\ell}^T + G_{\ell\lambda}\,\underline{\ell\lambda}^T \\
& + G_{\lambda r}\,\underline{\lambda r}^T + G_{\lambda\ell}\,\underline{\lambda\ell}^T + G_{\lambda\lambda}\,\underline{\lambda\lambda}^T
\end{aligned}
$$

$$
\begin{aligned}
G_{rr} &= \frac{\mu}{r^3}\sum_{n,m} C_{nm}\left[-(n+2)(n+1)\,P_n^m\right] \\
G_{\ell\ell} &= \frac{\mu}{r^3}\sum_{n,m} C_{nm}\left[-\left(\frac{1}{c}\,P_{n+1}^{m+1} + \frac{m(m+1)}{c^2}\right)P_n^m \right. \\
&\qquad\qquad \left. + (n+2)(n+1)\,P_n^m\right] \\
G_{\lambda\lambda} &= \frac{\mu}{r^3}\sum_{n,m} C_{nm}\left[\frac{1}{c}\,P_{n+1}^{m+1} + \frac{m(m-1)}{c^2}\,P_n^m\right] \\
G_{r\ell} &= G_{\ell r} = \frac{\mu}{r^3}\sum_{n,m} C_{nm}\left[(n+2)\left(P_n^{m+1} - \frac{ms}{c}\,P_n^m\right)\right] \\
G_{r\lambda} &= G_{\lambda r} = \frac{\mu}{r^3}\sum_{n,m} S_{nm}\left[-(n+2)\,\frac{m}{c}\,P_n^m\right]
\end{aligned}
\tag{D. 3-22}
$$

$$G_{\ell\lambda} = G_{\lambda\ell} = \frac{\mu}{r^3} \sum_{n,m} S_{nm} \left[m \left(\frac{1}{c} P_n^{m+1} - \frac{(m-1)s}{c^2} P_n^m \right) \right]$$

$$\frac{\partial F_2}{\partial V} = 0 \qquad \text{(D. 3-22)} \quad \text{(Cont'd.)}$$

To obtain $\partial F_2/\partial U$,

$$\frac{\partial G_r}{\partial J_{nm}} = \frac{\mu}{r^2} \left(\frac{a}{r}\right)^n \left[\frac{1}{c} P_{n+1}^{m+1} - \frac{m}{c^2} P_n^m \right] \cos m\,(\lambda - \lambda_{nm})$$

$$\frac{\partial G_k}{\partial J_{nm}} = -\frac{\mu}{r^2} \left(\frac{a}{r}\right)^n \left[\frac{1}{c} P_n^{m+1} - \frac{ms}{c^2} P_n^m \right] \cos m\,(\lambda - \lambda_{nm})$$

$$\frac{\partial G_\lambda}{\partial J_{nm}} = -\frac{\mu}{r^2} \left(\frac{a}{r}\right)^n \left[-\frac{m}{c^2} P_n^m \right] \sin m\,(\lambda - \lambda_{nm})$$

$$\frac{\partial G_r}{\partial \lambda_{nm}} = \frac{\mu}{r^2}\, m\, S_{nm} \left[\frac{1}{c} P_{n+1}^{m+1} - \frac{m}{c^2} P_n^m \right]$$

$$\frac{\partial G_n}{\partial \lambda_{nm}} = -\frac{\mu}{r^2}\, m\, S_{nm} \left[\frac{1}{c} P_n^{m+1} - \frac{ms}{c^2} P_n^m \right]$$

$$\frac{\partial G_\lambda}{\partial \lambda_{nm}} = \frac{\mu}{r^2}\, m\, C_{nm} \left[-\frac{m}{c^2} P_n^m \right] \qquad \text{(D. 3-23)}$$

D.3.4 Solar Pressure

From (D.2-20), since $\partial R_s / \partial R = I$,

$$\frac{\partial F_3}{\partial R} = \frac{C_1}{|R_s|^3} \left(I - \frac{3 R_s R_s^T}{R_s^2} \right)$$

$$\frac{\partial F_3}{\partial V} = 0$$

$$\frac{\partial F_3}{\partial C_1} = \frac{1}{C_1} F_3 \qquad \text{(D. 3-24)}$$

D.3.5 Atmospheric Drag

From (D.2-21),

$$\frac{\partial F_4}{\partial R} = - \frac{C_3}{r} F_4 R^T - \frac{\partial F_4}{\partial V} (\omega X)$$

$$\frac{\partial F_4}{\partial V} = \frac{1}{V_a^2} F_4 V_a^T - C_2 e^{-C_3 h} |V_a| I$$

$$\frac{\partial F_4}{\partial C_2} = \frac{1}{C_2} F_4$$

$$\frac{\partial F_4}{\partial C_3} = -hF_4 \qquad \text{(D. 3-25)}$$

D. 3. 6 Venting

From (D. 2. 22),

$$\frac{\partial F_5}{\partial R} = 0$$

$$\frac{\partial F_5}{\partial V} = \frac{C_4}{|V|} \quad I - \frac{VV^T}{V^2}$$

$$\frac{\partial F_5}{2C_4} = \frac{V}{|V|} \qquad \text{(D. 3-26)}$$

D. 4 NUMERICAL INTEGRATION

We now consider the integration of the equations of motion and variational equations for a given set of parameters, U, and given initial conditions $R(t_1)$, $V(t_1)$. All the equations may be considered as the vector equation

$$\ddot{X} = f(X, \dot{X}) \qquad \text{(D. 4-1)}$$

In any numerical integration process, we approximate the integral $X(t)$ at a sequence of points, t_i, on the integration interval, (t_o, t_n) obtaining the $X(t_i)$ from some approximation of the Taylor's series

$$X\left(t_{i+1}\right) = X(t_i) + h\dot{X}(t_i) + \frac{1}{2} h^2 \ddot{X}(t_i) + \frac{1}{6} h^3 \dddot{X}(t_i) + \ldots$$

$$h = t_{i+1} - t_i \qquad \text{(D. 4-2)}$$

At any t_i, the second derivative may be determined from the differential equation, and the higher order derivatives must be developed implicitly from the known derivative at neighboring points. The various methods differ in the way in which the series (D. 4-2) is approximated.

The ODP uses Adams' method, which approximates the series using the values of $f(X, \dot{X})$ computed at the previous integration points, t_{i-1}, t_{i-2}, etc., for long-term integration, and a generalized Kutta method for short-term integration and for starting the Adams' integration. The Kutta method uses values of $f(X, \dot{X})$ at suitably chosen points on the internal (t_i, t_{i+1}). The two methods are described below.

D. 4. 1 Adams' Method

We assume that the quantities

$$X_i = X(t_i)$$

$$\dot{X}_i = \dot{X}(t_i) \tag{D. 4-3}$$

$$f_i = f(X_i, \dot{X}_i)$$

have been determined at the sequence of equally spaced points

$$t_{n-m} = t_n - mh$$

$$m = 0, 1, \ldots, N \tag{D. 4-4}$$

We write the Taylor's series

$$f(t_n + sh) = f_n + sh\,\dot{f}(t_n) + \frac{1}{2}\, s^2 h^2 \ddot{f}(t_n) + \ldots \qquad (D.\,4\text{-}5)$$

truncating after terms in $(sh)^N$. The coefficients of the resulting Nth degree polynomial is s may be determined to satisfy the N + 1 conditions.

$$f(t_n - mh) = f_{n-m} \qquad (D.\,4\text{-}6)$$

The polynomial is usually written in terms of the backward differences

$$\begin{aligned} \nabla f_n &= f_n - f_{n-1} \\ \nabla^2 f_n &= \nabla f_n - \nabla f_{n-1} \qquad (D.\,4\text{-}7) \\ \nabla^{p+1} f_n &= \nabla^p f_n - \nabla^p f_{n-1} \end{aligned}$$

and hence

$$\begin{aligned} f^{(o)}_{n+s} &= \sum_{k=0}^{N} a_k(s)\, \nabla^k f_n \\ a_o &= 1 \qquad (D.\,4\text{-}8) \\ a_k &= \frac{1}{k!}\, s(s+1)\ldots(s+k-1),\; k \geq 1 \end{aligned}$$

The error in approximation on the interval (t_i, t_{i+1}) is

$$f(t_n + sh) - f^{(o)}_{n+s} = a_{N+1}(s)\, h^{N+1}\, f^{(N+1)}(\xi) \qquad \text{(D. 4-9)}$$

$$t_n - N \leq \xi \leq t_n + sh$$

D. 4. 1. 1 Integration Formulas. If we substitute the polynomial (D. 4-8) into the integral relationships

$$\dot{X}_{n+s} = X_n + \int_{t_n}^{t_n+sh} f\left(X(t), \dot{X}(t)\right) dt$$

$$X_{n+s} = \dot{X}_n + \int_{t_n}^{t_n+sh} \int_{t_n}^{t} f\left(X(t), \dot{X}(t)\right) dt\ dt \qquad \text{(D. 4-10)}$$

we obtain the approximations

$$\dot{X}^{(o)}_{n+s} = \dot{X}_n + h \sum_{k=0}^{N} A_k(s)\, \nabla^k f_n$$

$$X^{(o)}_{n+s} = X_n + sh\dot{X}_n + h^2 \sum_{k=0}^{N} B_k(s)\, \nabla^k f_n \qquad \text{(D. 4-11)}$$

where

$$A_k(s) = \int_o^s a_k(t)\, dt$$

$$B_k(s) = \int_o^s A_k(t)\, dt \qquad \text{(D. 4-12)}$$

with errors

$$\dot{X}_{n+s} - \dot{X}^{(o)}_{n+s} = A_{N+1}(s)\, h^{N+2} f^{(N+1)} (\xi_o)$$

$$X_{n+s} - X^{(o)}_{n+s} = B_{N+1}(s)\, h^{N+3} f^{(N+1)} (\eta o) \qquad \text{(D. 4-13)}$$

$$t_{n-N} \leq \xi_o, \eta o \leq t_n + sh$$

since $a_{N+1}(s)$, $A_{N+1}(s)$ do not change sign on (0, 1). These formulas resulting from extrapolation are termed open. An alternative form of the polynomial

$$f^{(1)}_{n+s} = \sum_{k=o}^{N} a_k(s-1)\, \nabla^k f_{n+1} \qquad \text{(D. 4-14)}$$

yields the closed formulas

$$X^{(1)}_{n+s} = X_n + h \sum_{k=o}^{N} C_k(s)\, \nabla^k f_{n+1}$$

$$X^{(1)}_{n+s} = X_n + sh\dot{X}_n + h^2 \sum_{k=o}^{N} D_k(s)\, \nabla^k f_{n+1}$$

$$C_k(s) = \int_o^s a_k\,(t-1)\, dt \qquad (D.\ 4\text{-}15)$$

$$D_k(s) = \int_o^s C_k\,(t)\, dt$$

with errors

$$\dot{X}_{n+s} - \dot{X}^{(1)}_{n+s} = C_{N+1}(s)\, h^{N+2}\, f^{(N+1)}(\xi_1)$$

$$X_{n+s} - X^{(1)}_{n+s} = D_{N+1}(s)\, h^{N+3}\, f^{(N+1)}(\eta_1) \qquad (D.\ 4\text{-}16)$$

The closed formulas require knowledge of $f(X_{n+1}, \dot{X}_{n+1})$ for the determination of X_{n+1}, $\dot{X}_{n+1}$, and hence may be used directly only in simple quadrature. For the integration of differential equations, they must be used in conjunction with formulas for the prediction of X_{n+1}, $\dot{X}_{n+1}$. The obvious solution is to use the open formulas as predictors to compute estimates $X^{(o)}_{n+1}$, $\dot{X}^{(o)}_{n+1}$, and to use the closed formulas as correctors. Evaluating the coefficients at s = 1,

$$X_{n+1} = \dot{X}^{(o)}_{n+1} + A_{n+1}\, h^{N+2} f^{(N+1)}(\xi_o)$$

$$= X^{(1)}_{n+1} + C_{N+1}\, h^{N+2}\, f^{(N+1)}(\xi_1) + \sum_{k=o}^{N} C_k\, h(f_{n+1} - f^{(o)}_{n+1})$$

$$X_{n+1} = X^{(o)}_{n+1} + B_{N+1} h^{N+3} f^{(N+1)} (\eta_o)$$

$$= X^{(1)}_{n+1} + D_{N+1} h^{N+3} f^{(N+1)}(\eta_1) + \sum_{k=o}^{N} D_k h^2 (f_{n+1} - f^{(o)}_{n+1}) \tag{D. 4-17}$$

If we assume that h is sufficiently small that $h(f_{n+1} - f^{(o)}_{n+1})$ is negligible compared with $h^{N+2} f^{(N+1)} (\xi)$, and that $f^{(N+1)} (\xi)$ varies only slowly with ξ, we may eliminate $\dot{X}_{n+1}$, X_{n+1}, obtaining

$$h^{N+2} f^{(N+1)} (\xi) = (X^{(1)}_{n+1} - X^{(o)}_{n+1}) / (A_{N+1} - C_{N+1}) \tag{D. 4-18}$$

Using the easily established relations

$$A_k(s) = A_{k+1}(s) - C_{k+1}(s)$$

$$\nabla^k f_n = \nabla^k f_{n+1} - \nabla^{k+1} f_{n+1} \tag{D. 4-19}$$

we have

$$h^{N+1} f^{(N+1)} (\xi) \nabla^{N+1} f_{n+1} \tag{D. 4-20}$$

and hence our best estimate of the integrals is

$$\dot{X}_{n+1} = \dot{X}^{(o)}_{n+1} + hA_{N+1} \nabla^{N+1} f^{(o)}_{n+1}$$

$$X_{n+1} = X^{(o)}_{n+1} + h^2 B_{N+1} \nabla^{N+1} f^{(o)}_{n+1}$$

The integration coefficients are listed through k=8 in Table D-2, below.

TABLE D-2

ADAMS' INTEGRATION COEFFICIENTS

k	A_k	B_k	C_k	D_k
0	1	$\frac{1}{2}$	1	$\frac{1}{2}$
1	$\frac{1}{2}$	$\frac{1}{6}$	$-\frac{1}{2}$	$-\frac{1}{3}$
2	$\frac{5}{12}$	$\frac{3}{24}$	$-\frac{1}{12}$	$-\frac{1}{24}$
3	$\frac{9}{24}$	$\frac{38}{360}$	$-\frac{1}{24}$	$-\frac{7}{360}$
4	$\frac{251}{720}$	$\frac{135}{1440}$	$-\frac{19}{720}$	$-\frac{17}{1440}$
5	$\frac{475}{1440}$	$\frac{863}{10080}$	$-\frac{27}{1440}$	$-\frac{82}{10080}$
6	$\frac{19087}{60480}$	$\frac{9625}{120960}$	$-\frac{863}{60480}$	$-\frac{731}{120960}$
7	$\frac{36799}{120960}$	$\frac{135812}{1814400}$	$-\frac{1375}{120960}$	$-\frac{8563}{1814400}$
8	$\frac{1070017}{3628800}$	$\frac{515529}{7257600}$	$-\frac{33953}{3628800}$	$-\frac{17719}{7257600}$

D. 4. 1. 2 Interpolation. To obtain f, $\dot{X}$, X at points other than integration points, we may use the polynominals (D. 4-8), (D. 4-11). Setting

$$t = t_n + sh$$

$$F_k = h^k \frac{d^k f(t_n)}{dt^k} = \frac{d^k f(t_n)}{ds^k} \qquad \text{(D. 4-22)}$$

we obtain

$$f(t) = \sum_{k=o}^{N} F_k s^k/k!$$

$$\dot{X}(t) = X_n + h \sum_{k=o}^{N} F_k s^{k+1}/(k+1)! \qquad \text{(D. 4-23)}$$

$$X(t) = X_n + sh\,X_n + h^2 \sum_{k=o}^{N} F_k s^{k+2}/(k+2)!$$

and for s on the interval (-1, 0), the derivatives F_k are obtained from

$$\begin{bmatrix} F_o \\ F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \\ F_6 \\ F_7 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{7} \\ 0 & 0 & 1 & 1 & \frac{11}{12} & \frac{5}{6} & \frac{137}{180} & \frac{7}{10} \\ 0 & 0 & 0 & 1 & \frac{3}{2} & \frac{7}{4} & \frac{15}{8} & \frac{29}{15} \\ 0 & 0 & 0 & 0 & 1 & 2 & \frac{17}{6} & \frac{7}{2} \\ 0 & 0 & 0 & 0 & 0 & 1 & \frac{5}{2} & \frac{25}{6} \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} f_n \\ \nabla f_n \\ \nabla^2 f_n \\ \nabla^3 f_n \\ \nabla^4 f_n \\ \nabla^5 f_n \\ \nabla^6 f_n \\ \nabla^7 f_n \end{bmatrix} \qquad \text{(D. 4-24)}$$

where all differences after the N^{th} are to be set zero.

D. 4. 1. 3 Change of Interval Size. For a set of differences $\nabla^k f_n$ for the spacing h, we may compute an equivalent set $\nabla^k \bar{f}_n$ for any spacing sh, so that the interpolation polynominals for the two sets are identical in t. Two particular changes may be made rather simply, for s = 1/2 and s = 2, and these changes provide all the spacing flexibility required.

Using (D. 4-22), we have for s = 1/2,

$$
\begin{bmatrix} \bar{f}_n \\ \nabla \bar{f}_n \\ \nabla^2 \bar{f}_n \\ \nabla^3 \bar{f}_n \\ \nabla^4 \bar{f}_n \\ \nabla^5 \bar{f}_n \\ \nabla^6 \bar{f}_n \\ \nabla^7 \bar{f}_n \end{bmatrix}
=
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 & \frac{1}{2} & \frac{1}{8} & \frac{1}{16} & \frac{5}{128} & \frac{7}{256} & \frac{21}{1024} & \frac{33}{2048} \\
 & & \frac{1}{4} & \frac{1}{8} & \frac{5}{64} & \frac{7}{128} & \frac{21}{512} & \frac{33}{1024} \\
 & & & \frac{1}{8} & \frac{3}{32} & \frac{9}{128} & \frac{7}{128} & \frac{45}{1024} \\
 & & & & \frac{1}{16} & \frac{1}{16} & \frac{7}{128} & \frac{3}{64} \\
 & & & & & \frac{1}{32} & \frac{5}{128} & \frac{10}{256} \\
 & & & & & & \frac{1}{64} & \frac{3}{128} \\
 & & & & & & & \frac{1}{128}
\end{bmatrix}
\begin{bmatrix} f_n \\ \nabla f_n \\ \nabla^2 f_n \\ \nabla^3 f_n \\ \nabla^4 f_n \\ \nabla^5 f_n \\ \nabla^6 f_n \\ \nabla^7 f_n \end{bmatrix}
\qquad \text{(D. 4-25)}
$$

and for S = 2,

$$\begin{bmatrix} \bar{f}_n \\ \nabla \bar{f}_n \\ \nabla^2 \bar{f}_n \\ \nabla^3 \bar{f}_n \\ \nabla^4 \bar{f}_n \\ \nabla^5 \bar{f}_n \\ \nabla^6 \bar{f}_n \\ \nabla^7 f_n \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & 2 & -1 & 0 & 0 & 0 & 0 & 0 \\ & & 4 & -4 & 1 & 0 & 0 & 0 \\ & & & 8 & -12 & 6 & -1 & 0 \\ & & & & 16 & -32 & 24 & -8 \\ & & & & & 32 & -80 & 80 \\ & & & & & & 64 & -192 \\ & & & & & & & 128 \end{bmatrix} \begin{bmatrix} f_n \\ \nabla f_n \\ \nabla^2 f_n \\ \nabla^3 f_n \\ \nabla^4 f_n \\ \nabla^5 f_n \\ \nabla^6 f_n \\ \nabla^7 f_n \end{bmatrix} \quad \text{(D. 4-26)}$$

D. 4. 1. 4 Ordinate Formulas. The use of difference formulas has some computational disadvantages. At each integration point, a complete set of differences must be computed, and the old set must be retained until the integration accuracy is verified. More efficient computation results from direct use of the computed ordinates. The corresponding formulas may be obtained from the relations

$$\nabla^k f_n = \sum_{m=0}^{k} \frac{(-1)^m \; m!}{k! \; (k-m)!} \; f_{n-m} \qquad \text{(D. 4-27)}$$

The various coefficients depend upon N as well as on k. For N = 5, the integration formulas are:

$$\dot{X}^{(0)}_{n+4} = \dot{X}_n + \frac{h}{10080} \Big[29939 f_n - 55461 f_{n-1} + 69874 f_{n-2} - 51086 f_{n-3} + 20139 f_{n-4} - 3325 f_{n-5} \Big] \qquad \text{(D. 4-28)}$$

$$X_{n+1}^{(0)} = X_n + h\dot{X}_n + \frac{h^2}{10080}\Big[10852f_n - 15487f_{n-1} + 18752f_{n-2}$$

$$- 13474f_{n-3} + 5260f_{n-4} - 863f_{n-5}\Big]$$

$$\dot{X}_{n+1} = \dot{X}_{n+1}^{(0)} + \frac{19087h}{60480} \nabla^6 f_{n+1}$$

$$X_{n+1} = X_{n+1}^{(0)} + \frac{9625h^2}{120960} \nabla^6 f_{n+1}$$

$$\nabla^6 f_{n+1} = f_{n+1}^{(0)} - 6f_n + 15f_{n-1} - 20f_{n-2} + 15f_{n-3} - 6f_{n-4} + f_{n-5} \qquad \text{(D. 4-28) (Contd.)}$$

The interpolation formulas are

$$\begin{bmatrix} F_o \\ F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \end{bmatrix} = \frac{1}{60} \begin{bmatrix} 60 & 0 & 0 & 0 & 0 & 0 \\ 137 & -300 & 300 & -200 & 75 & -12 \\ 225 & -770 & 1070 & -780 & 305 & -50 \\ 255 & -1065 & 1770 & -1470 & 615 & -105 \\ 180 & -840 & 1560 & -1440 & 660 & -120 \\ 60 & -300 & 600 & -600 & 300 & -60 \end{bmatrix} \begin{bmatrix} f_n \\ f_{n-1} \\ f_{n-2} \\ f_{n-3} \\ f_{n-4} \\ f_{n-5} \end{bmatrix} \qquad \text{(D. 4-29)}$$

D. 4. 2 Generalized Kutta Method

The various methods called Kutta or Runge-Kutta methods are based on a process suggested by Runge (Reference 7) and developed for first order equations by Kutta (Reference 8). Applied to second order equations, the method requires evaluation of the derivative f at the sequence of points:

$$t_n = t_o + C_n h$$

$$\dot{X}_n = \dot{X}_o + h\sum_{i=0}^{n-1} c_{ni} f_i$$

$$X_n = X_o + h\left(b_n \dot{X}_o + h\sum_{i=0}^{n-1} d_{ni} f_i\right)$$

$$C_n = \sum_{i=0}^{n-1} c_{ni} \qquad \text{(D. 4-30)}$$

where t_o is the initial point on the integration interval. The process (D. 4-30) is repeated through N substitutions (n = 0, 1, . . . , N-1), and $\dot{X}(t_o+h)$, $X(t_o+h)$ are then approximated by the N + 1 st values in the sequence. Appropriate values of the coefficients are determined by matching as many as possible of the leading terms of the series (D. 4-2) with those of the series obtained by substituting the Taylors series for $f(X,\dot{X})$ into the sequence (D. 4-30).

Several methods have been developed for integrating (D. 4-1) and for using two adjacent intervals for computing truncation error, interpolating between interval end-points, etc. Miachin (Reference 9) treated the special case $\ddot{X} = f(X)$, obtaining accuracy through terms in h^5 and an expression for the truncation error in $X(t_o+h)$ using derivatives computed on the two intervals (t_o, t_o+h) and (t_o+h, t_o+2h). In two unpublished communications, T. W. Hinton* treated the case $\ddot{X} = f(X,\dot{X})$, obtaining accuracy through h^5 for the case $\partial f/\partial\dot{X} = 0$ and through h^4 for the general case. Hinton also gave an expression for the truncation error in $X(t_o+h)$ and equations for interpolating on the interval pair (t_o, t_o+h) and (t_o+h, t_o+2h).

---

* Lockheed Missiles and Space Co., Sunnyvale, California, August 1963.

D.4.2.1 Integration Formulas. The coefficients given by Hinton are:

$$\begin{aligned}
C_1 &= b_1 = 3/10 & d_1 &= 9/200 \\
C_2 &= b_2 = 3/4 & d_2 &= 9/32 \\
C_3 &= b_3 = C_4 = b_4 = 1 & d_3 &= d_4 = 1/2 \\
C_{20} &= -21/32 & d_{20} &= 0 \\
C_{21} &= 45/32 & d_{21} &= 9/52 \\
C_{30} &= 83/27 & d_{30} &= 10/27 \\
C_{31} &= -280/81 & d_{31} &= 7/162 \\
C_{32} &= 112/81 & d_{32} &= 14/81 \\
C_{40} &= 5/54 & d_{40} &= 5/54 \\
C_{41} &= 250/567 & d_{41} &= 25/81 \\
C_{42} &= 32/81 & d_{42} &= 8/81 \\
C_{43} &= 1/14 & d_{43} &= 0
\end{aligned} \qquad \text{(D. 4-31)}$$

If we denote by $f_{i,1}$ and $f_{i,2}$ the values calculated for $f(X_i, \dot{X}_i)$ on the intervals (t_o, t_o+h) and (t_o+h, t_o+2h), respectively, the truncation error in $X(t_o+2h)$, assuming no error in $X(t_o+h)$ is

$$\begin{aligned}
T = \frac{h^2}{34020} \Big[& 81f_{3,2} + 112f_{2,2} - 550f_{1,2} - 2478f_{0,2} \\
& + 1134f_{3,4} + 3248f_{2,1} \\
& - 2450f_{1,1} + 903f_{0,1} \Big]
\end{aligned} \qquad \text{(D. 4-32)}$$

This term is of order h^5 and the error in the approximation is of order h^6 for the general case.

D. 4. 2. 2 Interpolation. Linear combinations of the f_n may be used to interpolate for $f, \dot{X}, X$ on the interval (t_o, t_o+2h). We again set

$$S = \left(t - [t_o+2h]\right)/h$$

$$F_k = \frac{d^k f}{dS^k} \qquad \text{(D. 4-33)}$$

obtaining the interpolation formulas (D. 4-23). The F_k are given by

$$\begin{bmatrix} F_o \\ F_1 \\ \frac{1}{2}F_2 \\ \frac{1}{6}F_3 \end{bmatrix} = \frac{1}{1134}\begin{bmatrix} 486 & 1344 & -1200 & 1638 & -486 & -1344 & 1200 & -504 \\ 1620 & 2912 & -8900 & 8904 & -2106 & -5600 & 5900 & -2730 \\ 1458 & 2016 & -9900 & 9828 & -1944 & -4704 & 6900 & -3654 \\ 324 & 448 & -2200 & 1428 & -324 & -448 & 2200 & -1428 \end{bmatrix} \begin{bmatrix} f_{3,2} \\ f_{2,2} \\ f_{1,2} \\ f_{0,2} \\ f_{3,1} \\ f_{2,1} \\ f_{1,1} \\ f_{0,1} \end{bmatrix} \qquad \text{(D. 4-34)}$$

The expression yields accuracy through h^3, h^4, h^5 for $f, \dot{X}, X$ respectively for the general case $f(X, \dot{X})$.

D. 4. 2. 3 Conversion to Adams' Ordinates. As we noted earlier, some starting process is required to accumulate the necessary ordinates for the Adams' integration. The necessary ordinates are computed by the ODP by interpolation on a single interval pair integrated in the Kutta mode. To avoid extrapolation beyond the interval pair in computing $\nabla^3 f_n$, the highest significant difference obtainable, we take S = 1/2 and set $\nabla^i f_n = 0$ for all $j \geq 4$. We have

$$\begin{bmatrix} f_n \\ \nabla f_n \\ \nabla^2 f_n \\ \nabla^3 f_n \end{bmatrix} = \frac{1}{756} \begin{bmatrix} 324 & 896 & -800 & 1092 & -324 & -896 & 800 & -336 \\ 324 & 672 & -1500 & 1449 & -405 & -1120 & 1000 & -420 \\ 324 & 448 & -2200 & 2562 & -486 & -1344 & 1200 & -504 \\ 162 & 224 & -1100 & 714 & -162 & -224 & 1100 & -714 \end{bmatrix} \begin{bmatrix} f_{3,2} \\ \cdot \\ \cdot \\ f_{0,1} \end{bmatrix} \tag{D. 4-35}$$

REFERENCES

1. "Subroutine Descriptions and Listings for the Orbit Determination Program," Philco-Ford Corporation, TR-DA1509, Palo Alto, California, December 1967.
2. "Input-Output Summary for the Orbit Determination Program," Philco-Ford Corporation TR-DA1510, Palo Alto, California, December 1967.
3. Peabody, P. E., Scott, J. F., Orozes, E. G., "User's Description of JPL Ephemeris Tapes," JPL TR32-580, Pasadena, California, March 2, 1964.
4. Battin, R. H., "Astronautical Guidance," McGraw-Hill, New York, 1964, pp 50-52.
5. Pines, S., "Mean Conic State Transition Matrix," Astro Consultants, Inc., Huntington, Long Island, New York, 1965.
6. Hildebrand, F. B., "Introduction to Numerical Analysis," McGraw-Hill, New York, 1956.
7. Runge, C., "Über die numerische Auflösung von Differentialgleichungen," Math. Ann., Sol. 46, pp 167-178 (1895).
8. Kutta, W., "Beitrag zun näherungsweisen Integration totaler Differentialgleichungen," Z. angew. Math. u. Phys., Vol. 46, pp 435-453 (1901).
9. Miachin, V. F., "Criterion for Changing the Integration Step in Numerical Integration of Equations of Celestial Mechanics by the Generalized Runge Method," ARS Jour., Vol. 32, No. 9, p 1479 (1962); translated from Bull. Theor. Astr., Vol. 8, No. 2, pp 132-152 (1961).