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COMPUTER PROGRAMS
IN
EARTHQUAKE SEISMOLOGY
VOLUME 2: SURFACE WAVE PROGRAM

EDITED BY

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16. Abstract (Limit: 200 words)

This volume contains twelve documented copies of computer programs dealing with surface wave propagation in a plane layered medium, to be used in earthquake seismology. The programs have been designed so that the output of one is compatible with the input of another program. These programs are presently operating on a Honeywell 6023 and are free of machine dependent peculiarities, with a few exceptions listed in the introduction. Although most programs require an offline CALCOMP drum plotter, in more recent programs plotting calls are performed in modular subroutine units to facilitate conversion to other plotting systems. The CALCOMP subroutines are listed. Most of the programs consist of a statement of purpose, input/output, program units, description or theory, the actual program, sample data, and sample output. The program titles are: SURFACE, REIGEN, LEIGEN, EIGEN, MODEL, NONPRP, EXCIT, SRFWVPLT, SRFWVSRC, RADPAT, QUESTION, and WIGGLE.

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Introduction to Volume 2

The purpose of this volume is to provide documented copies of computer programs which are useful in earthquake seismology. This volume contains programs dealing with surface wave propagation in a plane layered medium. As with any computer program, there is always room for improvement. The program REIGEN could be made more accurate by using compound matrix techniques for computing Rayleigh wave eigenfunctions. The program WIGGLE could be modified slightly so that a Q model could also be entered. The programs have been designed so that the output of one is compatible with the input of another program. To help understand the theory used, a translation of a Russian article is included in Appendix A.

The author has had programming experience with an IBM 1620, a CDC 3300, a CDC 6400, and most recently on a Honeywell 6023 on which these programs are presently operating. The programs should be free of machine dependent peculiarities, with a few exceptions:

1. there are six alphanumeric characters to a word;
2. a floating or fixed point number occupies one word;
3. the largest and smallest floating point numbers on the Honeywell 6023 are about 1.0 E + 39 and 1.0 E - 39 respectively;
4. there is no PROGRAM statement;
5. the arcsine and arccosine functions are ARSIN and ARCOS, respectively;
6. multiple files can be used for input or output.

Many programs contain calls for an offline CALCOMP drum plotter. In the more recent programs, plotting calls are performed in modular subroutine units so that conversion to other plotting routines can be facilitated. The CALCOMP subroutines used are as follow:

PLOTS	initiates plot tape
PLOT	move pen
FACTOR	scales entire plot
SYMBOL	plots character string and special symbols
NUMBER	plots decimal equivalent of a floating point number
SCALE	determines starting value and scale for an array of data to be plotted on a graph
AXIS	draws an annotated axis line for a graph
LINE	scales and plots a set of data points defined by X and Y coordinate arrays.

The author is interested in any comments, corrections or improvements to the programs. The computer programs hopefully will be of use to other researchers. An acknowledgment of their source is all that is requested.

I. SURFACE

PROGRAMMER: R. B. HERRMANN / Nov 1971

PURPOSE:

This program solves the Rayleigh and Love wave period equations in order to find the dispersion curves for phase and group velocity, the surface wave amplitude factors, and the Rayleigh wave ellipticity. The phase velocity is determined to an accuracy of 0.00000001. One liquid layer is allowed at the surface.

INPUT/OUTPUT

No tapes are required. Input is on card. Output is on printer with option for punched card output of dispersion curve data.

PROGRAM DESCRIPTION

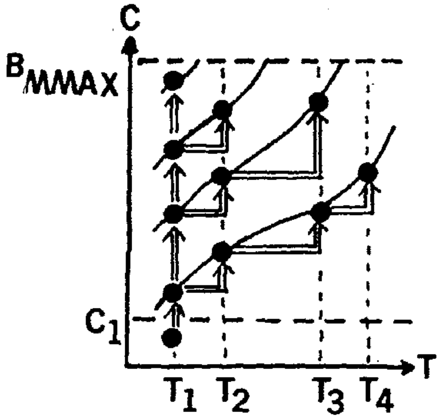
PROGRAM SURFACE: This is the main control link. The model and control cards are read in here.

SUBROUTINE GPHDIS: This subroutine evaluates the Love and/or Rayleigh wave period equations and plots the sign of the period equation. The true phase velocity is the curve for which the period equation is zero, e.g., the curve between the positive and negative values of the period equation. The ordinate is a set of phase velocities between C1 and C2 in increments of DC. This plot is helpful in determining which mode a particular solution of the period equation belongs. This is important because the modes might lie very close together and the parameter DC inputted into SUBROUTINE DISPER may be too large, in which case a particular mode may be jumped over in the root finding process. The plot also has a number to the right of the sign of the period equations. This number indicates the number of layers used in the root finding process. The number is the units value of the number of layers used (e.g., if 23 layers were used and the sign of the period equation was negative, then -3 would be plotted). A layer dropping procedure is used to increase the speed of computation and to decrease roundoff error. The abscissa of the graph is an array of up to 59 specified periods.

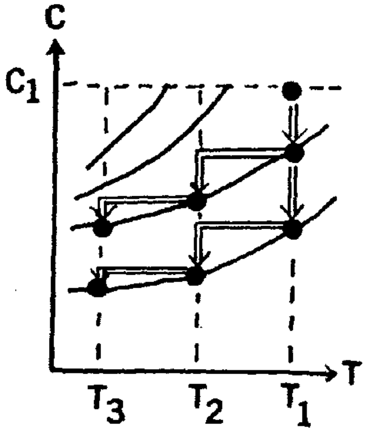
```
C2    -5-5-5 5 5 5-6-6 6
      -5-5-5 5 5-5-6-6 6
      -5-5-5 5 5-5-5 6 6
      -4-5 5 5-5-5 5 6 6
      -4-5 5 5-5-5 5 5 5
      -4 4 4-4-4 4 4 4 4
       4 4-4-4 4 4 4 4 4
       4-4-4 4 4 4 4 4 4
C1    -4-4 4 4 4 4 4 4 4
```

SUBROUTINE CUTOFF: This determines the periods corresponding to a given phase velocity by an interval halving technique to an accuracy of 0.001 sec. The routine starts with an initial period T1 at velocity C1. T2 = T1 + DT is generated and the period equation is again evaluated at phase velocity C1. The signs of the period equation values are compared and the period varies until a zero crossing is found. At this point interval halving is used to determine the period of the zero crossing. The procedure starts over again and searches for the next zero crossing until KMAX zero crossings, or cutoffs, are found.

SUBROUTINE DISPER: This routine determines the phase velocities of up to ten modes at period T(1) and then follows these modes until the phase velocities have been determined for all periods. A mode is followed until its phase velocity becomes greater than B(MMAX), the cutoff phase velocity. The routine brackets a phase velocity and calls SUBROUTINE NEVILLE to refine the value. This routine also determines the group velocity, amplitude factors and Rayleigh wave ellipticity.



The diagram at the upper left shows the operation of the mode following technique for DT positive, C1 an underestimate, and DC positive. There will be no mode jumping if DC is small enough and if C increases monotonically.



The diagram at lower left shows the operation of the mode following technique for DT negative, C1 an overestimate, and DC negative. Mode jumping may be a problem if DC or DT are too large. This is not a problem for DT and DC positive, since the program has been written to watch for this.

SUBROUTINE NEVILLE: The desired root of the period equation has been bracketed by DISPER. This subroutine refines it to an accuracy of 0.0000001. The method is as follows. The root refining proceeds by interval halving until a suitable region is found where the Neville iteration method can be used. The Neville iteration method starts out as a linear interpolation, then in the next iteration becomes quadratic interpolation, and proceeds up to a 10 degree interpolation. If at any time the Neville method becomes unsuitable, e. g. when it would lead to slow or improper convergence to the correct solution, the interval halving procedure is returned to. This combination of interval halving and polynomial interpolation has been designed for maximum efficiency.

FUNCTION DLTAR: The layer dropping procedure is performed here. Layer dropping will not occur in the middle of root refinement by Neville, since layer dropping affects the magnitude of the period equation but not its sign. This function also serves to call the period equation, ellipticity, and amplitude response routines.

FUNCTION DLTAR1: This is the Love wave formulation. Haskell's matrix is used, but working up from the bottom layer to the free surface. This routine is valid for C = B(MMAX).

FUNCTION DLTAR4: This is the Rayleigh wave function. The Dunkin-Thrower algorithm is used because it is not affected by roundoff and truncation error as much as the Haskell formulation. Ellipticity is positive for retrograde elliptical motion because the coordinate system has been chosen such that radial = positive and z-axis = positive downward. The function is valid at C = B(MMAX).

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Dunkin, J. W. (1965). Computation of modal solutions in layered, elastic media at high frequencies, Bull. Seism. Soc. Am. 55, 335-358.

Harkrider, D. G. (1970). Surface waves in multilayered elastic media. II. Higher mode spectra and spectral ratios from point sources in plane layered earth models, Bull. Seism. Soc. Am. 60, 1937-1988.

Haskell, N. A. (1953). The dispersion of surface waves on multi-layered media, Bull. Seism. Soc. Am. 43, 17-34, Isaacson, I., and H. B. Keller (1966). Analysis of Numerical Methods, John Wiley, New York.

Schwab, F., and L. Knopoff (1970). Surface-wave dispersion computations, Bull. Seism. Soc. Am. 60, 321-344.

Watson, T. H. (1970). A note on fast computation of Rayleigh wave dispersion in the multi-layered elastic halfspace, Bull. Seism. Soc. Am. 60,161-166.

INPUT DATA

Card Sequence	Column	Name	Format	Explanation
A.	1-4	MMAX	I4	LT.0 end program EQ.0 use previously inputted model with new options GT.0 number of layers in model including halfspace
	5-8	MODE	I4	Number of modes for which dispersion curves are desired
B. Earth Model (MMAX cards)				
	1-10	D(I)	F10.4	Layer thickness in km
	11-20	A(I)	F10.4	P wave velocity in km/sec
	21-30	B(I)	F10.4	S wave velocity in km/sec
	31-40	RHO(I)	F10.4	Density in gm/sec

C. Control Card

1-4	IGPHL	I4	GT.0 calls Love wave period equation plot in SUBROUTINE GPHDIS LE.0 skip Love plot
11-14	IGPHR	I4	GT.0 calls Rayleigh wave period equation plot in SUBROUTINE GPHDIS LE.0 skip Rayleigh plot
21-24	ICUT	I4	GT.0 calls CUTOFF to determine the modal cutoff periods of Love and Rayleigh waves. LE.0 skip CUTOFF
31-34	IDISPL	I4	GT.0 calls DISPER and determines phase and group velocity and amplitude factor for Love waves. LE.0 skip Love wave dispersion determination
41-44	IDISPR	I4	GT.0 calls DISPER to determine phase and group velocity, ellipticity and amplitude factor for Rayleigh waves LT.0 skip Rayleigh wave dispersion determination

C. Control Card

51-54	IPUNCH	I4	GE.1 output of DISPER also on cards LT.1 no card output from DISPER
61-70	FACT	F10.5	If zero or negative program sets FACT = 4.0.

D. If IGPHL or IGPHR GT 0, SUBROUTINE GPHDIS is called which requires the following data cards at this point

a.	1-4	KK	I4	number of periods along abscissa. KK .LE. 59.
	11-20	C1	F10.5	lower phase velocity limit
	21-30	C2	F10.5	upper phase velocity limit of ordinate
	31-40	DC	F10.5	phase velocity increment along ordinate
b.	1-80	T(I)	16F5.0	I = 1, KK. These periods form the abscissa of the plot

E. If ICUT GT 0, SUBROUTINE CUTOFF is called which requires the following data cards at this point

1-5	KMAX	I5	number of cutoff periods to be found for phase velocity C1
11-10	T1	F10.4	initial period of search
21-30	DT	F10.4	period increment (negative if going from long period toward short period)
31-40	C1	F10.4	phase velocity for which KMAX cutoffs are desired; can equal B(MMAX)

F. If IDISPL or IDISPR GT 0, SUBROUTINE DISPER is called which uses the following data:

1-4	KMAX	I4	number of periods for which phase velocity is to be determined

F.

11-20	T1	F10.4	initial period. If T1 is zero or negative, the array T(I) I = 1, MMAX are read in on the next set of cards. If positive T(I) are generated by T(I) = T1 + (I - 1)*DT. (I) = 1, MMAX.
21-30	DT	F10.4	period increment. Only has meaning if T1 is positive.
31-40	C1	F10.4	initial phase velocity guess. May not be less than 0.8 B(1). For T(I) increasing with I, C1 must be an underestimate.
41-50	DC	F10.4	increment to phase velocity for determining roots. If C1 is an underestimate, DC must be positive. Next guess is C2 = C1 + DC. DC must be small enough that modes are not skipped.

(This set of cards is read next if T1 is not positive.)

1-80	T(I)	8F10.0	I = 1. KMAX array of periods at which phase velocities are to be determined.

If all routines are to be used, the data are assembled in the above order. If a particular option is to be skipped, just skip the data cards listed under that subroutine.

At this point the program returns to Point A to terminate program, run other options, or to consider a new earth model.

```
C       PROGRAM SURFACE
C       THIS PROGRAM WILL ACCEPT ONE LIQUID LAYER AT THE SURFACE.
C       PROGRAM DEVELOPED BY ROBERT B HERRMANN SAINT LOUIS UNIVERSITY
C       NOV 71 IN WHICH CASE ELLIPTICITY OF RAYLEIGH WAVE IS THAT AT THE T
C       TOP OF SOLID ARRAY.  LOVE WAVE COMMUNICATIONS IGNORE LIQUID LAYER
        COMMON D(100),A(100),B(100),RHO(100),NMAX,MMAX,IDROP,FACT,IPUNCH
        COMMON T(100),C(100,10),U(100,10),RATIO(100,10),MODE
      4 FORMAT(1H ,40X,4F10.4)
     11 FORMAT(I4,I4,2X,F10.5)
      5 FORMAT(1H ,50X,3F10.4)
     44 FORMAT(4F10.4)
   1212 FORMAT(I4,6X,4F10.4)
     55 FORMAT(10X,3F10.4)
    801 READ 11,MMAX,MODE
C       MMAX = NUMBER OF LAYERS TO BE READ IN, INCLUDING HALFSPACE
C       MODE = NUMBER OF MODES FOR WHICH DISPERSION CURVES ARE DESIRED
        IF(MMAX) 777,779,778
    778 PRINT 20
     20 FORMAT(1H1,/,1H ,54X,13HCRUSTAL MODEL  ,/1H )
        PRINT 21
     21 FORMAT(1H ,42X,40H   THICK     P  VEL     S  VEL     DENSITY    /1H0)
        L=MMAX-1
C       D = THICKNESS OF LAYER IN KILOMETERS
C       A = COMPRESSIONAL WAVE VELOCITY IN KM/SEC
C       B = TRANSVERSE WAVE VELOCITY IN KM/SEC
C       RHO = DENSITY IN GM/CC
        DO 3 I=1,L
        READ 44,      D(I),A(I),B(I),RHO(I)
      3 PRINT 4,      D(I),A(I),B(I),RHO(I)
        READ 55,      A(MMAX),B(MMAX),RHO(MMAX)
        PRINT 5,      A(MMAX),B(MMAX),RHO(MMAX)
    779 IF(MMAX.EQ.0) MMAX = NMAX
C       IGPHL  GT 0 CALLS LOVE WAVE DISPERSION PLOT
C       IGPHR GT 0 CALLS RAYLEIGH WAVE DISPERSION PLOT
C       ICUT  GT 0 CALLS SEARCH FOR LOVE AND RAYLEIGH HIGHER MODE CUTOFF
C       IDISPL GT 0 LOVE WAVE DISPERSION CURVE
C       IDISPR GT 0 RAYL WAVE DISPERSION CURVE
C       IPUNCH .GE. 1  PUNCHED OUTPUT OF DISPERSION CURVES ALSO
C       FACT = NUMBER OF WAVELENGHTS BELOW FIRST LAYER WHERE PHASE
C               VELOCITY IS LESS THAN SHEAR VELOCITY THAT WE MAY CONSIDER
C               TO BE EFFECTIVE HALFSPACE.  SEE KNOPOFF SCHWAB-SURFACE WAVE
C               COMPUTATIONS- BSSA PP321-344 VOL 60 NO 2 APR 70
        READ 12,      IGPHL,IGPHR,ICUT,IDISPL,IDISPR,IPUNCH,FACT
     12 FORMAT(6(I4,6X),F10.5)
        IF(FACT)100,100,101
    100 FACT = 4.0
    101 CONTINUE
        NMAX = MMAX
        IF(IGPHL.GT.0.OR.IGPHR.GT.0) CALL GPHDIS(IGPHL,IGPHR)
        IF(ICUT.GT.0) CALL CUTOFF
```

```
      IF(IDISPL.GT.0.OR.IDISPR.GT.0) CALL DISPER(IDISPL,IDISPR)
      GO TO 801
  777 CONTINUE
      END

      SUBROUTINE GPHDIS(IGPHL,IGPHR)
      COMMON D(100),A(100),B(100),RHO(100),NMAX,MMAX,IDROP,FACT,IPUNCH
      COMMON T(100),C(100,10),U(100,10),RATIO(100,10),MODE
      DIMENSION KA(60)
      DIMENSION KB(60)
C     THIS SUBROUTINE GRAPHICALLY DISPLAYS THE SIGN OF THE LOVE OR
C     RAYLEIGH WAVE PERIOD EQUATION IN THE C-T PLANE. THE DISPERSION
C     CURVE IS THE LINE OF ZEROES.
   10 FORMAT(16F5.0)
C     GPHDIS READS IN UP TP 59 DIFFERENT PERIODS TO FORM  ABSCISSA OF
C     PLOT
C     THE ORDINATE VARIES FROM C1 TO C2 IN INCREMENTS OF DC
C     KK IS THE NUMBER OF ABSCISSA VALUES
C     C1 IS LESS THAN C2
C     DC IS POSITIVE
      READ 9,     KK,C1,C2,DC
    9 FORMAT(I4,6X,3F10.5)
      READ 10,    (T(I),I=1,KK)
      IF(KK.GT.59) KK = 59
      IF(DC.LT.0.0) DC = - DC
      IF(C1.LT.C2) GO TO 2000
      DUM = C1
      C1 = C2
      C2 = DUM
 2000 CONTINUE
      PRINT 9998
 9998 FORMAT(1H1,40X,40HPERIOD FOR ABSCISSA OF FOLLOWING GRAPH      /1H0)
      PRINT 9999,((I,T(I)),I=1,KK)
 9999 FORMAT(1H ,12(I3,F7.2))
      DO 801 IFUNC = 1,2
      IF(DC.EQ.0.0) GO TO 801
      GO TO (811,812),IFUNC
  811 IF(IGPHL.LE.0) GO TO 801
      PRINT 7
    7 FORMAT(1H1, 54X,25HPLOT OF   LOVE   FUNCTION/1H0)
      GO TO 813
  812 IF(IGPHR.LE.0) GO TO 801
      PRINT 8
    8 FORMAT(1H1, 54X,25HPLOT OF RAYLEIGH FUNCTION/1H0)
  813 CC = C2
 1004 DO 1010 I=1,KK
      IDROP = 0
      DEL = DLTAR(CC,T(I),IFUNC)
      L = MMAX
      L10 = L/10
```

```
      L = L - L1)*10
      KX(I) = IABS(L)
      IF(DEL)1011,1012,1013
 1011 KS(I) = 1H-
      GO TO 1010
 1012 KS(I) = 1H
      GO TO 1010
 1013 KS(I) = 1H
 1010 CONTINUE
      PRINT 1100,     CC,((KS(L),KX(L)),L=1,KK)
 1100 FORMAT(1H ,F7.3,59(A1,I1))
      CC = CC - DC
      IF(CC - C1) 801,1004,1004
  801 CONTINUE
      RETURN
      END

      SUBROUTINE CUTOFF
      COMMON D(100),A(100),B(100),RHO(100),NMAX,MMAX,IDROP,FACT,IPUNCH
      COMMON T(100),C(100,10),U(100,10),RATIO(100,10),MODE
C     KMAX = NUMBER OF CUTOFF PERIODS TO BE FOUND FOR PHASE VELOCITY C1
C     T1 = INITIAL PERIOD IN SEARCH
C     DT = PERIOD INCREMENT  NEGATIVE IF  STARTING AT HIGH PERIOD AND
C              AND GOING TOWARD SHORTER PERIOD
C     C1 = PHASE VELOCITY FOR WHICH KMAX CUTOFFS ARE BEING FOUND
C     THIS ROUTINE FINDS BOTH LOVE AND RAYLEIGH CUTOFFS
      READ 2000,KMAX,T1,DT,C1
 2000 FORMAT(I5,5X,3F10.4)
      PRINT 996
  996 FORMAT(1H1/1H0/1H0)
      PRINT 998
  998 FORMAT(1H0/1H0,51X,26HHIGHER MODE CUTOFF PERIODS /1H0,34X,
     1 10H   LOVE     ,40X,10H RAYLEIGH  /1H0,2(28X,6HPERIOD,10X,6HPH VEL
     2 )///)
      LL = 0
      TT = T1
      DO 47 IFUNC = 1,2
      K = 1
      IDROP = 0
      T1 = TT
  999 DEL1 = DLTAR(C1,T1,IFUNC)
   80 T2 = T1 + DT
      IDROP = 0
      IF(T2)3000,3000,3001
 3000 J = K + LL
      T(J) = 0.0
      KMAX = K
 3001 CONTINUE
   11 DEL2=DLTAR(C1,T2,IFUNC)
      IF(ABS (DEL2-DEL1)-ABS (DEL2+DEL1))81,81,54
```

```
   61 T1 = T2
      DEL1 = DEL2
      GO TO 80
   54 IF(ABS (T1 - T2) - 0.0001) 120,120,130
  130 T3 = (T1 + T2) * 0.5
      DEL3 = DLTAR(C1,T3,IFUNC)
      IF(ABS (DEL3 - DEL1) - ABS (DEL3 + DEL1))101,120,102
  101 T1 = T3
      DEL1 = DEL3
      GO TO 54
  102 T2 = T3
      DEL2 = DEL3
      GO TO 54
  120 TF = (T1 + T2) * 0.5
      J = K + LL
      T(J) = TF
      T1 = TF + DT
      IF(K-KMAX)76,46,46
   76 K = K + 1
      IDROP = 0
      GO TO 999
   46 LL = 50
   47 CONTINUE
      PRINT 997,    ((T(J),C1,T(J+50),C1),J=1,KMAX)
  997 FORMAT(1H ,24X,F10.4,6X,F10.4,24X,F10.4,6X,F10.4)
      RETURN
      END

      SUBROUTINE DISPER(IDISPL,IDISPR)
      COMMON D(100),A(100),B(100),RHO(100),NMAX,MMAX,IDROP,FACT,IPUNCH
      COMMON T(100),C(100,10),U(100,10),RATIO(100,10),MODE
      DIMENSION IMAX(10)
      DIMENSION AR(100,10)
C     THE ROOT DETERMINATION SECTION IS ONE OF INTERVAL HALVING ONCE A
C     ZERO CROSSING HAS BEEN FOUND.
C     TO FOLLOW MODES THE PROGRAM INITIALLY FINDS THE PHASE VELOCITIES
C     OF THE MODE NUMBER OF MODES FOR PERIOD T1.  THEN IT FINDS THE
C     PHASE VELOCITIES FOR PERIOD T1 + DT USING PREVIOUS RESULTS TO STAY
C     ON THE SAME MODE.  IF STARTING AT SHORT PERIODS DC MUST BE SMALL
C     ENOUGH SO MODE IS NOT JUMPED.  IT IS PREFERABLE TO HAVE DC POS
C     SINCE THE PROGRAM WILL FOLLOW A MODE UP TO ITS CUTOFF VALUE.  FOR
C     DC NEG THE PROGRAM CANNOT PICK UP ANY MODES.
 1000 FORMAT(I4,6X,4F10.4)
 1001 FORMAT(8F10.0)
C     T1 = INITIAL STARTING PERIOD
C     KMAX = NUMBER OF PERIOD S FOR WHICH PHASE VELOCITY IS TO BE
C     DETERMINED
C     IF T1 = 0 PROGRAM READS IN ARRAY OF T(I) PERIODS INSTEAD OF
C     COMPUTING THEM
C     DT = PERIOD INCREMENT. NEXT PERIOD T2 = T1 + DT
```

```
C       C1 = INITIAL PHASE VELOCITY GUESS.  MAKE SURE IT IS OUTSIDE
C       DESIRED RESULT
C       DC = PHASE VELOCITY INCREMENT = POS IF GOING FROM LOWER TO HIGHER
C              NEXT PHASE VELOCITY GUESS IS C2 = C1 + DC
        READ 1000,KMAX,T1,DT,C1,DC
        IF(T1)300,300,301
  300   READ 1001,(T(J),J=1,KMAX)
        GO TO 302
  301   T(1) = T1
        DO 303 I =2,KMAX
  303   T(I) = T(I-1) + DT
  302   CONTINUE
        CC = C1
        DO 9999 IFUNC = 1,2
        GO TO (10,20),IFUNC
   10   IF(IDISPL.LE.0) GO TO 9999
        GO TO 30
   20   IF(IDISPR.LE.0) GO TO 9999
   30   CONTINUE
        C1 = CC
        PRINT 1
    1   FORMAT(1H1)
        KMODE = MODE
        DO 2 I=1,10
    2   IMAX(I)=0
        DO 9998 K = 1,KMAX
        T1 = T(K)
        DO 9997 IQ = 1,KMODE
        IF(K-1)605,605,599
  599   IF(IQ - 2)600,601,601
  600   C1 = C(K-1,1) - 0.01 * ABS (DC)/DC
        GO TO 605
  601   IF(DC*(C(K-1,IQ) - C(K,IQ-1)))602,603,604
  602   C1 = C(K,IQ-1) + 0.01*ABS (DC)/DC
        GO TO 605
  603   C1 = C(K,IQ-1) + 0.01*ABS (DC)/DC
        GO TO 605
  604   C1 = C(K-1,IQ)
  605   CONTINUE
        IDROP = 0
  999   DEL1 = DLTAR(C1,T1,IFUNC)
   80   C2=C1+DC
        IDROP = 0
        DEL2 = DLTAR(C2,T1,IFUNC)
        IF (SIGN(1.,DEL1).NE.SIGN(1.,DEL2)) GO TO 54
   81   C1=C2
        DEL1=DEL2
C       CHECK THAT C1 IS IN REGION OF SOLUTIONS
        IF(C1-0.8*B(1))250,251,251
  251   IF(C1-(B(MMAX)+0.3))252,250,250
```

```
  252 GO TO 80
   54 CALL NEVILLE(T1,C1,C2,DEL1,DEL2,IFUNC,CN)
      C1 = CN
      IF(C1 - B(MMAX))121,121,250
  121 C(K,IQ) = C1
      P = DLTAR(C1,T1+0.001,IFUNC)
      Q = DLTAR(C1,T1-0.001,IFUNC)
      R = DLTAR(C1+0.001,T1,IFUNC)
      S = DLTAR(C1-0.001,T1,IFUNC)
      DCDT = -(P-Q)/(R-S)
      U(K,IQ) = C1/(1. + T1*DCDT/C1)
      GO TO (6001,6002),IFUNC
 6002 CONTINUE
C     RATIO = RAYLEIGH WAVE ELLIPTICITY
      RATIO(K,IQ) = DLTAR(C1,T1,3)
C     AR = RAYLEIGH WAVE AMPLITUDE RESPONSE. SEE HARKRIDER ESSA DEC 70.
C     AR = (2.*U*C*IQ)**(-1)
      AR(K,IQ) = ABS (DLTAR(C1,T1,4)*0.01256637/(C1*C1*T1*(R-S)))
      GO TO 6003
C     HERE AR = LOVE WAVE AMPLITUDE RESPONSE
 6001 CONTINUE
      AR(K,IQ) = ABS (DLTAR(C1,T1,5)*0.01256637/(C1*C1*T1*(R-S)))
 6003 CONTINUE
      C1 = C1 + 0.01 * ABS(DC)/DC
      IMAX(IQ) = K
      GO TO 9997
  250 IF(K*IQ-1)256,256,255
  256 PRINT 258
  258 FORMAT(1H ,40H IMPROPER INITIAL VALUE  NO ZERO FOUND     )
      GO TO 9999
 9997 CONTINUE
      GO TO 9998
  255 KMODE = IQ - 1
      IF(KMODE) 9996,9996,9998
 9998 CONTINUE
 9996 DO 9995 IQ = 1,MODE
      J=IMAX(IQ)
      IF(J)9995,9995,9994
 9994 GO TO (7001,7002),IFUNC
 7001 PRINT 105
  105 FORMAT(1H0,55X,10HLOVE WAVE /1H ,45X,30H PERIOD  PHASE VEL  GROUP
     1VEL    ,7X,2HAL /1H )
      PRINT 106,    ((T(I),C(I,IQ),U(I,IQ),AR(I,IQ)),I=1,J)
  106 FORMAT(1H ,45X,0PF7.3,1X,0PF10.6,2X,0PF8.4,4X,1PE12.5)
      IF(IPUNCH.LT.1) GO TO 9995
      PUNCH 1106,    ((T(I),C(I,IQ),U(I,IQ),AR(I,IQ),IQ),I=1,J)
 1106 FORMAT(4(1PE14.7,1X),15X,3HLOV,I2)
      GO TO 9995
 7002 PRINT 103
  103 FORMAT(1H0,53X,14HRAYLEIGH  WAVE /1H ,30X,
```

```
     131H PERIOD   PHASE VEL  GROUP VEL ,6X,5HUR/UZ, 12X,2HAR/1H )
      PRINT 104,   ((T(I),C(I,IQ),U(I,IQ),RATIO(I,IQ),AR(I,IQ)),I=1,J)
  104 FORMAT(1H ,30X,0PF7.3,1X,0PF10.6,2X,0PF8.4,4X,1PE12.5,4X,1PE12.5)
      IF(IPUNCH.LT.1) GO TO 9995
      PUNCH 1104,   ((T(I),C(I,IQ),U(I,IQ),AR(I,IQ),RATIO(I,IQ),IQ),
     1 I=1,J)
 1104  FORMAT(5(1PE14.7,1X),3HRAY,I2)
 9995 CONTINUE
 9999 CONTINUE
      RETURN
      END

      SUBROUTINE NEVILLE(T,C1,C2,DEL1,DEL2,IFUNC,CC)
C     HYBRID METHOD FOR REFINING ROOT ONCE IT HAS BEEN BRACKETTED
C     BETWEEN C1 AND C2.  INTERVAL HALVING IS USED WHERE OTHER SCHEMES
C     WOULD BE INEFFICIENT.  ONCE SUITABLE REGION IS FOUND NEVILLE S
C     ITERATION METHOD IS USED TO FIND ROOT.
C     THE PROCEDURE ALTERNATES BETWEEN THE INTERVAL HALVING AND NEVILLE
C     TECHNIQUES USING WHICHEVER IS MOST EFFICIENT
      COMMON D(100),A(100),B(100),RHO(100),NMAX,MMAX,IDROP,FACT,IPUNCH
      COMMON D1(100),D2(100,10),D3(100,10),D4(100,10),MODE
      DIMENSION X(20),Y(20)
      C3 = 0.5*(C1 + C2)
      DEL3 = DELTAR(C3,T,IFUNC)
      NEV = 1
 1310 IF(C1 - C3)1320,1320,1330
 1320 IF(C2 - C3) 1344,1344,1000
 1330 IF(C2 - C3) 1000,1344,1344
 1000 S13 = DEL1 - DEL3
      S32 = DEL3 - DEL2
      IF(SIGN(1.,DEL3)*SIGN(1.,DEL1)) 1441,1441,1443
 1441 C2 = C3
      DEL2 = DEL3
      GO TO 1444
 1443 C1 = C3
      DEL1 = DEL3
      GO TO 1444
 1444 CONTINUE
      IF(ABS (C1 - C2) -0.0000001) 20,20,22
   22 CONTINUE
      IF(SIGN (1.,S13).NE.SIGN (1.,S32)) NEV = 0
      SS1=ABS(DEL1)
      S1=0.1*SS1
      SS2=ABS(DEL2)
      S2=0.1*SS2
      IF(S1.GT.SS2.OR.S2.GT.SS1) GO TO 1344
      IF(NEV.EQ.0) GO TO 1344
      IF(NEV.EQ.2) GO TO 1350
      X(1) = C1
      Y(1) = DEL1
```

```
      X(2) = C2
      Y(2) = DEL2
      M = 1
      GO TO 1355
 1344 C3 = 0.5*(C1 + C2)
      DEL3 = DLTAR(C3,T,IFUNC)
      NEV = 1
      M = 1
      GO TO 1310
 1350 X(M+1) = C3
      Y(M+1) = DEL3
      GO TO 1355
 1355 DO 1360 KK=1,M
      J = M-KK+1
      IF (ABS(Y(M+1)-Y(J)).LE.1.E-10) GO TO 1344
      X(J) =(-Y(J)*X(J+1) + Y(M+1)*X(J))/(Y(M+1)-Y(J))
 1360 CONTINUE
   21 CONTINUE
      C3 = X(1)
      DEL3 = DLTAR(C3,T,IFUNC)
      NEV = 2
      M = M+1
      IF(M.GT.10) M = 10
      GO TO 1310
   20 CONTINUE
      CC = C3
      RETURN
      END

      FUNCTION DLTAR(CC,TT,KK)
      COMMON D(100),A(100),B(100),RHO(100),NMAX,MMAX,IDROP,FACT,IPUNCH
      COMMON T(100),C(100,10),L(100,10),RATIO(100,10),MODE
C     LAYER DROPPING PROCEDURE--HALFSPACE DEFINED AS FACT WAVELENGTHS
C     BELOW REGION WHERE C IS LESS THAN B
      IF(IDROP)899,899,905
  899 CONTINUE
      DMAX = FACT * CC * TT
      MMAX = NMAX
      SUM = 0
      DO 900 II = 1,NMAX
      IF(CC-B(II))901,900,900
  901 SUM = SUM + D(II)
      IF(SUM - DMAX) 900,900,902
  902 MMAX = II
      GO TO 904
  900 CONTINUE
  904 IDROP = 1
      IF(MMAX.LT.2) MMAX = 2
  905 CONTINUE
      GO TO (1,2,3,4,5),KK
```

```
C     LOVE WAVE PERIOD EQUATION
    1 DLTAR = DLTAR1(CC,TT,1)
      RETURN
C     RAYLEIGH WAVE PERIOD EQUATION
    2 DLTAR = DLTAR4(CC,TT,1)
      RETURN
C     RAYLEIGH WAVE ELLIPTICITY
    3 DLTAR = DLTAR4(CC,TT,2)
      RETURN
C     RAYLEIGH WAVE AMPLITUDE RESPONSE COMPONENT
    4 DLTAR =DLTAR4(CC,TT,3)
      RETURN
C     LOVE WAVE AMPLITUDE RESPONSE COMPONENT
    5 DLTAR = DLTAR1(CC,TT,2)
      RETURN
      END

      FUNCTION DLTAR1(C,T,MLP)
C     HASKELL-THOMPSON LOVE WAVE FORMULATION FROM HALFSPACE TO SURFACE
      COMMON D(100),A(100),B(100),RHO(100),NMAX,MMAX,IDROP,FACT,IPUNCH
      COMMON D1(100),D2(100,10),D3(100,10),D4(100,10),MODE
      WVNO = 6.2831853/(C*T)
      COVB = C/B(MMAX)
      H = RHO(MMAX)*B(MMAX)*B(MMAX)
      RB = SQRT (ABS (COVB**2-1.))
      UT = 1
      TT = H * RB
      MMM1 = MMAX - 1
      DO 1340 K = 1,MMM1
      M = MMAX - K
      IF(B(M).EQ.0.0) GO TO 1340
      COVB = C/B(M)
      RB = SQRT (ABS (COVB**2-1.))
      H = RHO(M)*B(M)*B(M)
 7001 Q =-WVNO*D(M)*RB
      IF(C-B(M))1209,1221,1231
 1231 SINQ = SIN(Q)
      Y = SINQ/RB
      Z = RB*SINQ
      COSQ = COS(Q)
      GO TO 1242
 1221 Y =-WVNO*D(M)
      Z = 0
      COSQ = 1
      GO TO 1242
 1209 EXQP = EXP(Q)
      EXQM = 1./EXQP
      Y = (EXQP - EXQM)/(2.*RB)
      Z = -RB*RB*Y
      COSQ = (EXQP + EXQM)/2.
```

```
 1242 EUT = COSQ*UT - Y*TT/H
      ETT = H*Z*UT + COSQ*TT
      UT = EUT
      TT = ETT
 1340 CONTINUE
      GO TO (1,2),MUP
    1 DLTAR1 =-ETT
      RETURN
    2 DLTAR1 = UT
      RETURN
      END

      FUNCTION DLTAR4(C,T,MUP)
C     MUP = 1  DLTAR4 = RAYLEIGH WAVE PERIOD EQUATION
C     MUP = 2  DLTAR4 = RAYLEIGH WAVE ELLIPTICITY
C     MUP = 3  DLTAR4 = RAYLEIGH WAVE AMPLITUDE FACTOR
C     UP TO ONE LIQUID LAYER IS ALLOWED AT THE SURFACE
C     MODIFIED DUNKIN-THROWER ALGORITHM FOR RAYLEIGH WAVES
C     C.F.  J. DUNKIN BSSA VOL 55 NO 2 PP 335-358 APR 65
C     C.F.  C. WATSON BSSA VOL 60 NO 1 PP 161-166 FEB 70
C     C.F.  D. HARKRIDER BSSA VOL 60 NO 6 PP 1937-1987 DEC 70
C     HIGHER MODE CUTOFF MODIFICATION BY R HERRMANN
      COMMON D(100),P(100),S(100), RHO(100),NMAX,MMAX,IDROP,FACT,IPUNCH
      COMMON D1(100),D2(100,10),D3(100,10),D4(100,10),MODE
   99 WVNO =6.28318531/(C*T)
      CSQ = C*C
      JUMP=1
      IF(MUP.GT.1) JUMP=2
 1000 B1 = 0.0
      B2 = 0.0
      B3 = 0.0
      B4 = 0.0
      B5 = 0.0
      IF(JUMP - 2)1,2,3
    1 B1 = 1.0
      GO TO 4
    2 B2 = 1.0
      GO TO 4
    3 B3 = 1.0
      GO TO 4
    4 CONTINUE
      DO 50 M=1,MMAX
  100 ARGA = 1.-CSQ/P(M)**2
      RA = SQRT(ABS(ARGA))
      IF(ARGA)202,202,201
  201 RA =-RA
  202 IF(S(M))101,103,101
C     LIQUID SURFACE LAYER
  103 PA=WVNO*RA*D(M)
      IF(MUP.GT.1) GO TO 50
```

```
      RHOC=RHO(M)*CSQ
      IF(RA)313,312,314
  312 SINPR = WVNO*D(M)
      RSINP = 0.0
      COSP = 1.0
      GO TO 315
  313 SINPR = (EXP(PM)-EXP(-PM))/(2.*RA)
      RSINP = -RA * RA * SINPR
      COSP = 0.5*(EXP(PM)+EXP(-PM))
      GO TO 315
  314 SINPR = SIN(PM)/RA
      RSINP = RA * SIN(PM)
      COSP = COS(PM)
  315 CONTINUE
      A11 = COSP
      A21 = RHOC*SINPR
      A31 = 0.0
      A41 = 0.0
      A51 = 0.0
      A12 = 0.0
      A22 = 0.0
      A32 = 0.0
      A42 = 0.0
      A13 = 0.0
      A23 = 0.0
      A33 = 0.0
      A14 = 0.0
      A24 = 0.0
      A15 = 0.0
      GO TO 1001
  101 ARGB=1.-CSQ/S(M)**2
      RB = SQRT(ABS(ARGB))
      IF(ARGB)204,204,203
  203 RB = -RB
  204 CONTINUE
      G=2.*S(M)**2/CSQ
      G1=G-1.0
      IF(MMAX-M)40,52,40
   40 RHOC=RHO(M)*CSQ
      PM = WVNO*RA*D(M)
      QM = WVNO * RB * D(M)
      IF(RA)213,212,214
  212 RSINP = 0.0
      SINPR = WVNO*D(M)
      COSP = 1.0
      GO TO 215
  213 RSINP = -RA*0.5*(EXP(PM)-EXP(-PM))
      SINPR=-RSINP/(RA**2)
      COSP = 0.5*(EXP(PM)+EXP(-PM))
      GO TO 215
```

```
  214 RSINP = RA*SIN(PM)
      SINPR=RSINP/(RA**2)
      COSP = COS(PM)
  215 IF(RB) 216,218,217
  216 RSINQ = -RB*0.5*(EXP(QM)-EXP(-QM))
      SINQR=-RSINQ/(RB**2)
      COSQ = 0.5*(EXP(QM)+EXP(-QM))
      GO TO 219
  217 RSINQ = RB*SIN(QM)
      SINQR=RSINQ/(RB**2)
      COSQ = COS(QM)
      GO TO 219
  218 RSINQ=0.0
      SINQR = WVNO*C(M)
      COSQ = 1.0
      GO TO 219
  219 RR=RSINP*RSINQ
      SS=SINPR*SINQR
      CC=COSP*COSQ
      A11       =-2.*G*G1+(2.*G**2-2.*G+1.)*CC-G**2*RR-G1**2*SS
      A12       =-(1./RHOC    )*(RSINP*COSQ+SINQR*COSP)
      A13       =-(2./RHOC)*((2.*G-1.)*(1.-CC)+G1*SS+G*RR)
      A14       = ( 1.0/RHOC )*(SINPR*COSQ+RSINQ*COSP)
      A15       =(RHOC**(-2))*(2.*(1.-CC)+RR+SS)
      A21       =RHOC*(G1**2*SINPR*COSQ+G**2*RSINQ*COSP)
      A22       =CC
      A23       =2.0*(G*RSINQ*COSP+G1*SINPR*COSQ)
      A24       =SINPR*SINQ
      A31       =RHOC*(G*G1*(2.*G-1.)*(1.-CC)+G1**3*SS+G**3*RR)
      A32       = G1*SINQR*COSP+G*RSINP*COSQ
      A33       =1.0+2.0*(2.*G*G1*(1.-CC)+G**2*RR+G1**2*SS)
      A41       =-RHOC*(G**2*RSINP*COSQ+G1**2*SINQR*COSP)
      A42       =RSINP*SINQR
      A51       =RHOC**2*((2.*G**2*G1**2)*(1.-CC)+G1**4*SS+G**4*RR)
 1001 CONTINUE
C     MATRIX MULTIPLICATION
C     EFFECT OF IMAGINARY ELEMENTS INCLUDED
      BB1 = A11*B1 + A12*B2 +  A13*B3 +   A14*B4 +   A15*B5
      BB2 = A21*B1 + A22*B2 +  A23*B3 +   A24*B4 - A14*B5
      BB3 = A31*B1 + A32*B2 +  A33*B3 -.5*A23*B4 +.5*A13*B5
      BB4 = A41*B1 + A42*B2 -2.*A32*B3 +  A22*B4 -   A12*B5
      BB5 = A51*B1 - A41*B2 +2.*A31*B3 -  A21*B4 +   A11*B5
      B1 = BB1
      B2 = BB2
      B3 = BB3
      B4 = BB4
      B5 = BB5
   50 CONTINUE
C     THE FOLLOWING EXPRESSION IS VALID FOR RB = 0
   52 A11       =-2.*RB*(S(M)/P(M))**2*(CSQ*G1**2)/(P(M)**2*G*RA)
```

```
      A12       =-( RHO(M) *P(M)**2*G    )**(-1)
      A13       =-RB/(RHO(M)*P(M)*P(M))+G1/(RHO(M)*P(M)*P(M)*G*RA)
      A14       =RB/(G*RHO(M)*P(M)*P(M)*RA)
      A15       =((RHO(M)*P(M))**2*CSQ*G)**(-1)*(RB-1./RA)
      BB1 = A11*B1 + A12*B2 + 2.*A13*B3 + A14*B4 + A15*B5
      GO TO (501,502,503),MUP
  501 DLTAR4=-BB1
      RETURN
  502 IF(JUMP.EQ.2) R12 = BB1
      JUMP = JUMP + 1
      IF(JUMP.EQ.3) GO TO 1000
      DLTAR4 = 0.5*BB1/R12
      RETURN
  503 DLTAR4 = ABS(BB1)
      IF(S(1).GT.0.0) RETURN
      RA = C/P(1)
      RAD = WVNO * D(1) * SQRT(ABS(RA*RA - 1.))
      DLTAR4 = ABS(BB1 * COS(RAD))
      RETURN
      END
```

```
DATA     PAGE  1


  5    2
1.0         5,0         2.89        2.5
9.0         6,1         3,52        2,7
10,         6,4         3,7         2.9
20,         6,7         3.87        3.0
            8,15        4.7         3,4
  1           1           1           1           1           0         3,0
 24         2,6         4,7         0.1
2.0   3,0   4.0   5.0   6,0   7.0   8,0   9.0   10,   11.   12,   13.   14,   15.   16.   17
18,   19.   20.   22,   24,   26,   28,   30.
    4       18.        -0,2         4,7
 19         2,0         1,0         2,6         0,07
-1
```

```
                    CRUSTAL MODEL

   THICK       P   VEL       S   VEL       DENSITY


  1,0000      5,0000      2,8900      2,5000
  9,0000      6,1000      3,5200      2,7000
 10,0000      6,4000      3,7000      2,9000
 20,0000      6,7000      3,8700      3,0000
              8,1500      4,7000      3,4000
```

```
                                              PERIOD FOR ABSCISSA OF FOLLOWING GRAPH

  1    2.00  2    3.00  3    4.00  4    5.00  5    6.00  6    7.00  7    8.00  8    9.00
13   14.00 14   15.00 15   16.00 16   17.00 17   18.00 18   19.00 19   20.00 20   22.00


                                                       9   10.00 10   11.00 11   12.00 12   13.00
                                                    21   24.00 22   26.00 23   28.00 24   30.00
```

PLOT OF   LOVE   FUNCTION

```
4,700 5 5-5 5 5-5-5-5-5-5-5 5 5 5 5 5 5 5 5 5 5 5 5
4,600 5 5 5 5-5-5-5-5-5 5 5 5 5 5 5 5 5 5 5 5 5 5 5
4,500-5-5 5 5-5-5-5-5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5
4,400-5-5 5 5-5-5-5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5
4,300-5-5 5-5-5-5-5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5
4,200 5 5 5-5-5-5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5
4,100 5 5-5-5-5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5-5-5
4,000-5 5-5-5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5-5+5-5+5
3,900 5-5-5 5 5 5 5 5 5 5 5 5 5 5 5 5-5-5-5-5+5-5+5
3,800-5 5 5 5 5 5 5 5 5 5 5 5 5-5-5-5-5-5-5-5-5+5-5+5
3,700 4 5 5 5 5 5 5 5-5-5-5-5-5*5-5-5-5-5-5-5-5+5-5+5
3,600 4 5 5 5 5-5-5-5-5-5-5-5-5*5-5-5-5-5-5-5-5+5-5+5
3,500 4 4-5-5-5-5-5-5-5-5-5-5-5*5-5-5-5-5-5-5-5+5-5+5
3,400-4-4-5-5-5-5-5-5-5-5-5-5-5*5-5-5-5-5-5-5-5+5-5+5
3,300-4-4-5-5-5-5-5-5-5-5-5-5-5*5-5-5-5-5-5-5-5+5-5+5
3,200-4-4-4-5-5-5-5-5-5-5-5-5-5*5-5-5-5-5-5-5-5+5-5+5
3,100-3-4-4-5-5-5-5-5-5-5-5-5-5*5-5-5-5-5-5-5-5+5-5+5
3,000-3-4-4-5-5-5-5-5-5-5-5-5-5*5-5-5-5-5-5-5-5+5-5+5
2,900-3-4-4-5-5-5-5-5-5-5-5-5-5*5-5-5-5-5-5-5-5+5-5+5
2,800-3-4-4-5-5-5-5-5-5-5-5-5-5*5-5-5-5-5-5-5-5+5-5+5
2,700-3-4-4-5-5-5-5-5-5-5-5-5-5*5-5-5-5-5-5-5-5+5-5+5
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```
4,700 5 5-5 5 5 5-5-5-5-5-5-5-5+5-5 5 5 5 5 5 5 5 5 5
4,600 5 5-5 5 5-5-5-5-5-5 5 5 5 5 5 5 5 5 5 5 5 5 5 5
4,500-5-5 5 5-5-5-5-5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5
4,400-5-5 5-5-5-5-5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5
4,300 5-5 5-5-5-5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5
4,200 5 5 5-5-5-5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5
4,100-5 5-5-5-5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5
4,000-5 5-5-5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5
3,900 5-5-5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5*5
3,800-5-5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5*5-5*5
3,700 4 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5-5-5*5-5*5
3,600 4 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5-5-5-5-5*5-5*5
3,500 4 4 5 5 5 5 5 5 5 5 5 5 5 5-5-5-5-5-5-5-5*5-5*5
3,400 4 4 5 5 5 5 5 5 5 5-5-5-5+5-5-5-5-5-5-5-5*5-5*5
3,300 4 4 5 5 5 5-5-5-5-5-5-5-5*5-5-5-5-5-5-5-5*5-5*5
3,200 4 4 4-5-5-5-5-5-5-5-5-5-5-5-5-5-5-5-5-5-5*5-5-5
3,100-3-4-4-5-5-5-5-5-5-5-5-5-5-5-5-5-5-5-5-5-5*5-5*5
3,000-3-4-4-5-5-5-5-5-5-5-5-5-5*5-5-5-5-5-5-5-5*5-5*5
2,900-3-4-4-5-5-5-5-5-5-5-5-5-5-5-5-5-5-5-5-5-5*5-5*5
2,800-3-4-4-5-5-5-5-5-5-5-5-5-5*5-5-5-5-5-5-5-5*5-5*5
2,700-3-4-4-5-5-5-5-5-5-5-5-5-5-5-5-5-5-5-5-5-5*5-5*5
```

```
                         HIGHER MODE CUTOFF PERIODS

        LOVE                                                     RAYLEIGH

 PERIOD           PH VEL                                  PERIOD           PH VEL


12,9806           4,7000                                 16.4834           4.7000
 6.5574           4,7000                                  7,4149           4,7000
 4.3681           4,7000                                  4,7535           4,7000
 3,2668           4,7000                                  3,4268           4.7000
```

```
            LOVE WAVE
PERIOD   PHASE VEL   GROUP VEL          AL

 2.000    3.407477     3.2085      7.38827E-03
 3.000    3.480797     3.3257      3.89512E-03
 4.000    3.526896     3.3727      2.68719E-03
 5.000    3.563732     3.4016      2.07591E-03
 6.000    3.595662     3.4234      1.69963E-03
 7.000    3.624460     3.4409      1.44338E-03
 8.000    3.651184     3.4547      1.25884E-03
 9.000    3.676586     3.4650      1.12094E-03
10.000    3.701213     3.4725      1.01479E-03
11.000    3.725450     3.4776      9.30848E-04
12.000    3.749553     3.4811      8.62696E-04
13.000    3.773685     3.4835      8.05973E-04
14.000    3.797935     3.4854      7.57680E-04
15.000    3.822342     3.4872      7.15653E-04
16.000    3.846911     3.4893      6.78389E-04
17.000    3.871615     3.4920      6.44754E-04
18.000    3.896413     3.4955      6.13988E-04
19.000    3.921246     3.5001      5.85475E-04
20.000    3.946051     3.5060      5.58787E-04

            LOVE WAVE
PERIOD   PHASE VEL   GROUP VEL          AL

 2.000    3.706053     3.4703      9.98171E-04
 3.000    3.810811     3.5588      6.44528E-04
 4.000    3.893335     3.6029      4.79195E-04
 5.000    3.973622     3.5934      4.32769E-04
 6.000    4.063835     3.5554      4.32677E-04
 7.000    4.167551     3.5162      4.37390E-04
 8.000    4.283090     3.5002      4.27207E-04
 9.000    4.404134     3.5341      3.91930E-04
10.000    4.519581     3.6480      3.27520E-04
11.000    4.615236     3.8675      2.36907E-04
12.000    4.678198     4.2091      1.27248E-04
```

```
                              RAYLEIGH  WAVE
PERIOD     PHASE VEL   GROUP VEL        UR/UZ               AR

  2.000    3.114265     3.0052     7.75331E-01       6.65303E-03
  3.000    3.151168     3.0684     8.00347E-01       4.06700E-03
  4.000    3.179362     3.0665     7.96662E-01       3.04409E-03
  5.000    3.210016     3.0588     7.86004E-01       2.44811E-03
  6.000    3.242074     3.0607     7.75376E-01       2.02923E-03
  7.000    3.273481     3.0722     7.66864E-01       1.71251E-03
  8.000    3.303160     3.0884     7.60712E-01       1.46746E-03
  9.000    3.330947     3.1049     7.56531E-01       1.27642E-03
 10.000    3.357235     3.1182     7.53791E-01       1.12696E-03
 11.000    3.382662     3.1262     7.52007E-01       1.00926E-03
 12.000    3.407898     3.1287     7.50802E-01       9.15566E-04
 13.000    3.433525     3.1260     7.49919E-01       8.39774E-04
 14.000    3.459986     3.1195     7.49199E-01       7.77146E-04
 15.000    3.487566     3.1104     7.48571E-01       7.24097E-04
 16.000    3.516395     3.1007     7.48024E-01       6.77879E-04
 17.000    3.546462     3.0922     7.47594E-01       6.36441E-04
 18.000    3.577626     3.0865     7.47349E-01       5.98293E-04
 19.000    3.609635     3.0848     7.47373E-01       5.62368E-04
 20.000    3.642155     3.0884     7.47756E-01       5.27983E-04

                              RAYLEIGH  WAVE
PERIOD     PHASE VEL   GROUP VEL        UR/UZ               AR

  2.000    3.702188     3.5109     6.22385E-01       2.04879E-04
  3.000    3.791024     3.5610     6.16642E-01       1.52631E-04
  4.000    3.869216     3.5827     5.86135E-01       1.40361E-04
  5.000    3.951656     3.5479     5.54225E-01       1.50242E-04
  6.000    4.052316     3.4753     5.19288E-01       1.73177E-04
  7.000    4.175212     3.4192     4.77813E-01       1.91032E-04
  8.000    4.309079     3.4664     4.30754E-01       1.81464E-04
  9.000    4.427527     3.6621     3.86174E-01       1.41061E-04
 10.000    4.513492     3.9042     3.51858E-01       9.87478E-05
 11.000    4.572207     4.0913     3.28316E-01       7.09548E-05
 12.000    4.614150     4.2182     3.13487E-01       5.43378E-05
 13.000    4.645877     4.3102     3.06065E-01       4.33073E-05
 14.000    4.670371     4.3906     3.05473E-01       3.40924E-05
 15.000    4.688310     4.4814     3.11537E-01       2.38601E-05
 16.000    4.698593     4.6170     3.24455E-01       8.87479E-06
```

II. REIGEN

PROGRAMMER: R. B. HERRMANN / DEC 1971

PURPOSE:

This program accepts the card output for Rayleigh wave dispersion curves determined by the program SURFACE. The output of SURFACE consists of the period, phase velocity, group velocity, amplitude factor, and Rayleigh wave ellipticity. The program REIGEN will determine the Rayleigh wave eigenfunctions as a function of depth as well as phase velocity partial derivatives as a function of layer compressional and shear velocities, as well as density. The program will accept one liquid layer at the surface.

INPUT/OUTPUT

Input is from card. The eigenfunctions and phase velocities are listed on the printer. The eigenfunctions and phase velocity partial derivatives may also be listed on FILE 1. This file can then be used by other programs to generate synthetic seismograms and amplitude spectra.

THEORY

The coordinate system used is taken such that the z-coordinate is positive downwards, and the radial coordinate is positive away from the center of a cylindrical coordinate system. In this coordinate system, the ellipticity of Rayleigh waves at the surface of an elastic halfspace will be a positive quantity.

The system of differential equations to be solved is

$$\begin{vmatrix} \frac{d}{dz}\,UZ \\ \frac{d}{dz}\,TZ \\ \frac{d}{dz}\,UR \\ \frac{d}{dz}\,TR \end{vmatrix} = \begin{vmatrix} 0 & \frac{1}{\lambda+2\mu} & \frac{k\lambda}{\lambda+2\mu} & 0 \\ -\rho\omega^2 & 0 & 0 & k \\ -k & 0 & 0 & \frac{1}{\mu} \\ 0 & \frac{-k\lambda}{\lambda+2\mu} & -\rho\omega^2 + \frac{4k^2\mu(\lambda+\mu)}{\lambda+2\mu} & 0 \end{vmatrix} \begin{vmatrix} UZ \\ TZ \\ UR \\ TR \end{vmatrix}$$

where $k = 2\pi/CT$ is the wavenumber, λ and μ are the Lame constants, $\omega = 2\pi/T$ is the natural frequency, and ρ is the density. Now T, C, and the ellipticity e_o are available from SURFACE.

The boundary conditions to the above system are that $UZ = 1$, $UR = e_o$, $TZ = TR = 0$ at $z = 0$ and that UZ and UR tend to zero as z goes to positive infinity.

The above system of equations can be derived using variational techniques and the following relation for the system Lagrangian L :

$$L = \omega^2 I_o - k^2 I_1 - 2kI_2 - I_3 \quad ,$$

where

$$I_o = \int_o^\infty \rho \, [UZ^2 + UR^2] \, dz$$

$$I_1 = \int_o^\infty [(\lambda+2\mu)UR^2 + \mu UZ^2] \, dz$$

$$I_2 = \int_o^\infty [\mu \, UZ \frac{d\,UR}{dz} - \lambda \, UR \frac{d\,UZ}{dz}] \, dz$$

$$I_3 = \int_o^\infty [(\lambda+2\mu) \left(\frac{d\,UZ}{dz}\right)^2 + \mu \left(\frac{d\,UR}{dz}\right)^2] \, dz$$

The partials of the phase velocity with respect to layer shear velocity, compressional velocity, and density at constant frequency for the m'th layer are defined as

$$\left(\frac{\partial c}{\partial \beta_m}\right)_{\omega,\rho,\alpha} = 2\,\rho_m \beta_m \left(\left(\frac{\partial c}{\partial \mu_m}\right)_{\rho,\lambda,\omega} - 2 \left(\frac{\partial c}{\partial \lambda_m}\right)_{\rho,\mu,\omega} \right)$$

$$\left(\frac{\partial c}{\partial \alpha_m}\right)_{\omega,\rho,\beta} = 2\,\rho_m \alpha_m \left(\frac{\partial c}{\partial \lambda_m}\right)_{\omega,\rho,\mu}$$

$$\left(\frac{\partial c}{\partial \rho_m}\right)_{\omega,\alpha,\beta} = \left(\frac{\partial c}{\partial \rho_m}\right)_{\omega,\lambda,\mu} + (\alpha_m^2 - 2\beta_m^2) \left(\frac{\partial c}{\partial \lambda_m}\right)_{\omega,\rho,\mu} + \beta_m^2 \left(\frac{\partial c}{\partial \mu_m}\right)_{\omega,\rho,\lambda}$$

where

$$\left(\frac{\partial c}{\partial \mu_m}\right)_{\omega,\rho,\lambda} = \frac{c}{k} \; \frac{\left(\frac{\partial L}{\partial \mu_m}\right)_{\omega,\rho,\lambda,k}}{\left(\frac{\partial L}{\partial k}\right)_{\omega,\rho,\lambda,\mu}}$$

$$\left(\frac{\partial c}{\partial \lambda_m}\right)_{\omega,\rho,\mu} = \frac{c}{k} \; \frac{\left(\frac{\partial L}{\partial \lambda_m}\right)_{\omega,\rho,\mu,k}}{\left(\frac{\partial L}{\partial k}\right)_{\omega,\rho,\mu,\lambda}}$$

$$\left(\frac{\partial c}{\partial \rho_m}\right)_{\omega,\lambda,\mu} = - \frac{c}{k} \; \frac{\left(\frac{\partial L}{\partial \rho_m}\right)_{\omega,\mu,\lambda,k}}{\left(\frac{\partial L}{\partial k}\right)_{\omega,\mu,\lambda,\rho}}$$

with

$$\left(\frac{\partial L}{\partial k}\right)_{\omega,\rho,\lambda,\mu} = - 2kI_1 - 2I_2$$

$$\left(\frac{\partial L}{\partial \omega}\right)_{k,\rho,\mu,\lambda} = 2\omega I_0$$

$$\left(\frac{\partial L}{\partial \rho_m}\right)_{\omega,k,\mu,\lambda} = \omega^2 \int_{z_m}^{z_{m+1}} \left(\mathbb{UZ}^2 + \mathbb{UR}^2 \right) dz$$

$$\left(\frac{\partial L}{\partial \mu_m}\right)_{\omega,k,\rho,\lambda} = - \int_{z_m}^{z_{m+1}} \Bigg(k^2(2\,\mathbb{UR}^2 + \mathbb{UZ}^2) + 2k\,\mathbb{UZ}\frac{d\,\mathbb{UR}}{dz} + \left(2\frac{d\,\mathbb{UZ}}{dz}\,\frac{d\,\mathbb{UZ}}{dz} + \frac{d\,\mathbb{UR}}{dz}\,\frac{d\,\mathbb{UR}}{dz} \right) \Bigg) dz$$

$$\left(\frac{\partial L}{\partial \lambda_m}\right)_{\omega,\rho,k,\mu} = - \int_{z_m}^{z_{m+1}} \left(k^2\, UR^2 - 2k\frac{dUZ}{dz}\, UR + \frac{dUZ}{dz}\,\frac{dUZ}{dz} \right) dz$$

z_m and z_{m+1} are the depths to the top and bottom of layer m. The group velocity U = U(ENER) can be defined in terms of the energy integrals as

$$U = \frac{d\omega}{dk} = \frac{(\partial L/\partial k)_\omega}{(\partial L/\partial \omega)_k} = \frac{kI_1 + I_2}{\omega I_0}$$

The amplitude factor of the Rayleigh wave A_R = ARE can be defined in terms of the energy integrals as

$$ARE = \frac{1}{2\ C\ U\ I_0}$$

The numerical integration performed in REIGEN uses a fourth order Runge-Kutta technique. Since T and C are already known, two independent solutions are integrated upward from the halfspace to the free surface and are combined in such a way that the surface conditions that UZ = 1 and UR = e_o are met. If no numerical error occurs and the T and C pair are exact, then TZ and TR must equal zero at the free surface. When the proper combination of the two independent solutions is found, then the eigenfunctions are determined as a function of depth. Simultaneously, the energy integrals and phase velocity partial derivatives are formed.

Several parameters are available to test the numerical accuracy of the results. The group velocity U = U(DC/DT), determined by SURFACE, can be compared to U = U(ENER) determined in REIGEN. The Lagrangian L should be zero (this is usually a poor parameter to use for testing goodness of fit). The two amplitude factors AR (from SURFACE) and ARE (from REIGEN) should be equal. At the bottom of the listing, under M = 0, the values of TZ and TR, determined at the free surface in the process of satisfying the surface boundary conditions on UZ and UR, are displayed. These would be equal to zero if no numerical error were encountered.

The partial derivatives for the last row are for changes in the parameters of the halfspace.

PROGRAM DESCRIPTION

PROGRAM REIGEN : This program performs the desired numerical integration and lists the results on the printer or tape.

SUBROUTINE TAPEIN : This subroutine makes it possible to update the eigenfunction tape.

INPUT DATA

There are two slightly different arrangements to the input data for the cases when the output is or is not written on FILE 01.

Case 1 - No output written on FILE 01.

Card Sequence	Column	Name	Format	Explanation
A.	1-4	MMAX	I4	LE 0 End program GT 0 Number of layers in earth model including the halfspace
	11-14	ITAPE	I4	LE 0 No tape output (In this case, ITAPE must be LE 0)
B. Earth model (MMAX cards).				
	1-10	D(I)	F10.5	Layer thickness in km (Leave blank for halfspace, e.g. I=MMAX)
	11-20	A(I)	F10.5	P velocity of layer in km/sec

Card Sequence	Column	Name	Format	Explanation
	21-30	B(I)	F10.5	S velocity of layer in km/sec
	31-40	RHO(I)	F10.5	Density of layer in gm/cm^3.
C. Period data				
	1-14	T	E14.7	T LT 0 read new MMAX, ITAPE card (Point A). EQ 0 does nothing in this case except read in a new T, C, U, AR, RATIO card GT 0 Period in seconds
	15-29	C	E14.7	Phase velocity in km/sec
	31-44	U	E14.7	Group velocity in km/sec
	46-59	AR	E14.7	Rayleigh wave amplitude factor
	51-74	RATIO	E14.7	Rayleigh wave ellipticity

At this point the program will determine the eigenfunctions corresponding to the T,C pair and then return to Point C to read a new T,C,U,AR,ELLIP card.

The program is terminated by reading a MMAX card with MMAX. LT.0. The last data card is followed by the system end of file card.

Case 2 - Output written on tape

A.	1-4	MMAX	I4	LE 0 End program GT 0 Number of layers including halfspace of the earth model

Card Sequence	Column	Name	Format	Explanation
	11-14	ITAPE	I4	LE 0 No tape output EQ 1 Earth model not written on FILE 01; eigenfunctions are. GT 1 Earth model written as file header followed by eigenfunctions.
	21-30	FACT	F10.0	LE 0.0 use FACT = 4 GT 0.0 vertical wave lengths for layer dropping. See SURFACE.
B. Earth model (MMAX cards)				
	1-10	D(I)	F10.5	Layer thickness in km
	11-20	A(I)	F10.5	P velocity in km/sec
	21-30	B(I)	F10.5	S velocity in km/sec
	31-40	RHO(I)	F10.5	Density of layer in gm/cm^3.
C. Tape update				
	1-10	TMAX	F10.5	Last eigenfunction to be read when adding information to previously written eigenfunction FILE.
	11-15	KMODMX	I5	Number of earth models written on FILE to skip. KMODMX = 0 when writing new tape.

Card Sequence	Column	Name	Format	Explanation
D. Period Sequence				
a. Header				
	1-4	IFUNC	I4	LT 0 Write IFUNC on tape to denote end of eigenfunction list. Then terminate program.
				EQ 2 for all other cases IFUNC must equal 2. This signifies the FILE as containing the Rayleigh wave eigenfunctions.
	11-14	MODE	I4	Number of modes to be placed on tape for a given period.
	22-25 27-20 . . 67-70	AMODE(I)	A4	Identification code for individual modes.
b. Period data (MODE cards)				
	1-14	T	E14.7	Period in seconds
	16-29	C	E14.7	Phase velocity in km/sec
	31-44	U	E14.7	Group velocity in km/sec
	46-59	AR	E14.7	Rayleigh wave amplitude factor
	61-74	RATIO	E14.7	Rayleigh wave ellipticity
c. End period data				
	1-80	blank card		

At this point the program returns to Point D to read a new IFUNC, MODE card. When the program finds an IFUNC.LT.0 , the program terminates.

NOTE: When the tape created here is to be used by other programs such as WIGGLE, SRFWVPLT, QUESTION, or RADPAT, the order of data is as is in the accompanying sample data set. The data are arranged in order of increasing period. Within each period set, the data are arranged in order of increasing mode, i.e., FUND first, 1 ST second, etc.

When using QUESTION and some of the other programs, both a LOVE and RAYLEIGH eigenfunction tape is used. Both tapes must have the same earth model for these other programs to work.

```
CREIGEN
C       PROGRAM REIGEN
C       THE PROGRAM WILL ACCEPT UP TO ONE LIQUID LAYER AT THE SURFACE IN
C       WHICH CASE THE ELLIPTICITIES ARE THOSE AT THE TOP OF THE SOLID
C       LAYER ARRAY.
C       THIS PROGRAM COMPUTES VALUES OF DISPLACEMENT, STRESS AT MIDPOINT
C       OF EACH LAYER TOGETHER WITH VALUES OF DC/DB FOR THAT LAYER .
C       THE METHOD IS BASED ON A RUNGE-KUTTA INTEGRATION WITHIN EACH LAYER
C       THE INTEGRATION INTERVAL IS DDZ=D(I)/12., GOOD RESULTS WILL BE
C       OBTAINED WHEN DDZ IS LESS THAN C*T/2 PI, THUS, D(I)= 5 KM WILL
C       GIVE RESULTS TO 1 SEC FOR SEDIMENTARY LAYERS.
C       THE PROGRAM ALSO YIELDS VALUES OF VARIATIONAL INTEGRALS.
C       NUMERICAL ACCURACY IMPROVEMENT BY CHIUNG-CHUAN CHENG AUG 78
        DOUBLE PRECISION YY1,YY2,YY3,YY4,YZ1,YZ2,YZ3,YZ4,AUR1,AUZ1,
      1 ATZ1,ATR1,AUR2,AUZ2,ATZ2,ATR2,AA,BB,DAUR1,DAUZ1,DATZ1,DATR1,
      1 DAUR2,DAUZ2,DATZ2,DATR2,EAUR1,EAUZ1,EATZ1,EATR1,EAUR2,EAUZ2,
      1 EATZ2,EATR2,SAUR1,SAUZ1,SATZ1,SATR1,SAUR2,SAUZ2,SATZ2,SATR2,XNORM
        COMMON D(100),A(100),B(100),RHO(100),DEPTH(100),AMPUR(100)
        COMMON AMPUZ(100),STRESR(100),STRESZ(100),DCDB(100)
        COMMON C,U,T,FLAGR,SUMI0,SUMI1,SUMI2
        COMMON SUMI3,UGR,WVNUM,DMR(5),DMZ(5),SMR(5),SMZ(5)
        COMMON DMRSMZ,RATIO,DMZSMR,XLAMB(100),XMU(100)
        COMMON AR,ARE,DCDA(100),DCDR(100)
        COMMON WWT(4),WT(4)
        COMMON YY1,YY2
        COMMON AMODE(10)
        DIMENSION YY1(100,5),YY2(100,5),YY3(100,5),YY4(100,5)
        DIMENSION YZ1(100,5),YZ2(100,5),YZ3(100,5),YZ4(100,5)
        DIMENSION DMRSMZ(5),DMZSMR(5)
     11 FORMAT (2(I4,6X),10(1X,A4))
     20 FORMAT (1H1)
     21 FORMAT (1H0,13X,2HM ,8X,7HDEPTH  ,10X,1HD,12X,5HALPHA,11X,4HBETA,
       1  11X,3HRHO,13X,2HMU,11X,5HLAMDA/1H  )
     23 FORMAT (1H ,10X,I5,7(5X,F10.4))
     25 FORMAT (1H ,10X,I5,30X,5(5X,F10.4))
     44 FORMAT (4F10.5)
   5000 FORMAT (1H1,13X,10H T(SEC) = ,F7.2,13H C(KM/SEC) =  ,F7.4)
   5001 FORMAT (1H0,13X,12H U(DC/DT) = ,F7.4,11H  U(VAR) =  ,F7.4,3X,
       1 6H E  =   ,E11,4)
   5002 FORMAT (1H0,13X,6H I0 = ,E11.4,3X,6H I1 = ,E11.4,3X,6H I2 = ,
       1 E11.4)
   5003 FORMAT (1H ,13X,6H I3 = ,E11.4,3X,6H L  = ,E11.4,3X,6H K  = ,
       1 E11.4)
   5004 FORMAT (1H0,3H  M,5X,5HDEPTH,5X,2HUR,10X,2HUZ,10X,2HTZ,10X,2HTR,
       1 9X,4HDCDA,8X,4HDCDB,8X,4HDCDR,/1H0)
   5005 FORMAT (1H ,I3,F10.2, 7(1X,E11.4))
   5007 FORMAT (1H ,13X,6H AR = ,E11.4,3X,6HARE = ,E11.4)
   6010 FORMAT (1H ,A4,3F10,5,5E11.4)
   6011 FORMAT (1H ,7(4X,E11.4))
   6012 FORMAT (1H ,2(I4,6X))
```

```
 6013 FORMAT (1H ,I4,5X,7F10.4)
      XXMIN = 1.0E-15
      WWT(1)=0.0
      WWT(2)=0.5
      WWT(3)=0.5
      WWT(4)=1.0
      WT(1)=1./6.
      WT(2)=1./3.
      WT(3)=1./3.
      WT(4)=1./6.
  801 READ (60,12) MMAX,ITAPE,FACT
   12 FORMAT (2(I4,6X),F10.0)
      IF (FACT.LE.0.) FACT=4.
      NMAX=MMAX
      IF (MMAX) 777,777,778
  778 WRITE (61,20)
      WRITE (61,21)
      BASE = 0.0
      DO 22 I=1,MMAX
      READ (60,44) D(I),A(I),B(I),RHO(I)
      BASE = BASE + D(I)
      DEPTH(I) = BASE - D(I)*0.5
      XMU(I) = RHO(I)*B(I)*B(I)
      XLAMB(I) = RHO(I)*(A(I)*A(I)-2.*B(I)*B(I))
      IF (I - MMAX) 26,24,24
   26 WRITE (61,23) I,DEPTH(I),D(I),A(I),B(I),RHO(I),XMU(I),XLAMB(I)
   22 CONTINUE
   24 WRITE (61,25) I,A(I),B(I),RHO(I),XMU(I),XLAMB(I)
      DEPTH(MMAX) = BASE - D(MMAX)
      IF (ITAPE.GT.0) CALL TAPEIN
      IF (ITAPE.LE.1) GO TO 504
      LMAX= MMAX+1
      IF (B(1).LE.0.0) LMAX = LMAX - 1
      WRITE (01,6012) LMAX
      DPTH = 0.0
      IF (B(1).LE.0.0) DPTH = D(1)
      DD = 0.0
      K = 1
      IF (B(1).LE.0.0) K = 2
      I = K - 1
      WRITE (01,6013) I,DPTH,DD,A(K),B(K),RHO(K),XMU(K),XLAMB(K)
      DO 200 I=1,MMAX
      IF (B(I).LE.0.0) GO TO 200
      WRITE (01,6013) I,DEPTH(I),D(I),A(I),B(I),RHO(I),XMU(I),XLAMB(I)
  200 CONTINUE
  504 IF (ITAPE.LE.0) GO TO 503
      READ (60,11) IFUNC,MODE,(AMODE(I),I=1,10)
      WRITE (01,6012) IFUNC,MODE
      IF (IFUNC.LT.0) GO TO 777
      IIK = 0
```

```
  503 CONTINUE
      DO 500 J=1,MMAX
      DCDA(J)=0.0
      DCDB(J)=0.0
      DCDR(J)=0.0
      AMPUR(J)=0.0
      AMPUZ(J)=0.0
      STRESR(J)=0.0
      STRESZ(J)=0.0
  500 CONTINUE
      READ (60,5) T,C,U,AR,RATIO
    5 FORMAT (5(E14.7,1X))
      IF (T) 801,504,499
  499 CONTINUE
      MMAX = NMAX
C
C     LAYER DROPPING PROCEDURE--HALFSPACE DEFINED AS FACT WAVELENGTHS
C     BELOW REGION WHERE C IS LESS THAN B.
C
      DMAX = FACT*T*C
  899 SUM = 0.0
      DO 900 II=1,MMAX
      MAX = II
      IF (C-B(II)) 901,900,900
  901 SUM= SUM + D(II)
      IF (SUM.LE.DMAX) GO TO 900
      IF (II.EQ.MMAX) GO TO 902
      IF (A(II+1)-A(II)) 902,903,902
  903 IF (B(II+1)-B(II)) 902,900,902
  900 CONTINUE
  902 MMAX = MAX
      SUMI0 = 0.0
      SUMI1 = 0.0
      SUMI2 = 0.0
      SUMI3 = 0.0
      WVNO = 6.2831853072/(C*T)
      WVNOSQ=WVNO*WVNO
      WVNUM = WVNO
      OMEGA = 6.2831853072/T
      OMEGSQ = OMEGA*OMEGA
      IF (B(1).GT.0.0) GO TO 4000
      RA = C/A(1)
      RA = WVNO * SQRT(RA*RA - 1.)
      IF (RA) 4001,4001,4002
 4001 SUMI0 = RHO(1) * D(1)
      SUMI1 = 0.0
      SUMI2 = 0.0
      SUMI3 = 0.0
      TZZ= 0.0
      GO TO 4000
```

```
 4002 SIN2RA = SIN(2.*RA*D(1))/(4.*RA)
      COSRA = COS(RA*D(1))
      COS2RM = 1./(COSRA*COSRA)
      FAC1 = (0.5*D(1) + SIN2RA) * COS2RM
      FAC3 = WVNO * ( 0.5 * D(1) - SIN2RA) * COS2RM
      FAC2 = WVNO * FAC3 / (RA*RA)
      FAC4 = RA * RA * FAC1
      SUMI0 = RHO(1) * (FAC1+FAC2)
      SUMI1 = XLAMB(1) * FAC2
      SUMI2 = XLAMB(1) * FAC3
      SUMI3 = XLAMB(1) * FAC4
      TZZ = - RHO(1) * OMEGSQ * SIN(RA*D(1))/(RA*COSRA)
 4000 CONTINUE
      COVA = C/A(MMAX)
      COVB = C/B(MMAX)
      GAM = 2./COVB**2
      GAMM1 = GAM - 1.
      RA = WVNO*SQRT(ABS(COVA**2-1.))
      RB = WVNO*SQRT(ABS(COVB**2-1.))
      DET = WVNOSQ - RA * RB
      H = RHO(MMAX) * OMEGSQ
      BRKT = - GAMM1*WVNO + GAM*RA*RB/WVNO
C     THE SUBSCRIPTS .1. AND .2. REFER TO TWO INDEPENDENT SOLUTIONS
      ITER = 0
      AUR1 = 1.D 00
      AUZ1 = 0.D 00
      ATZ1 = -H*BRKT/DET
      ATR1 = -H*RA/DET
      MMM1 = MMAX - 1
      DO 1346 MM=1,MMM1
      M = MMAX - MM
      IF (B(M).LE.0.0) GO TO 1346
C     CHOICE OF EFFICIENT VALUE OF DDZ FOR R-K INTEGRATION.  WVNO*DDZ
C     SHOULD GIVE INTEGRATION ERROR LESS THAN 0.01 PERCENT FOR ALL
C     PERIODS.
      XDIV=1.
      NDIV = 1
      DDZ = - D(M) / (4.*XDIV)
      A12 = 1./(XLAMB(M)+2.*XMU(M))
      A13 = WVNO * XLAMB(M) *A12
      A21 = - OMEGSQ * RHO(M)
      A24 = WVNO
      A31 = -WVNO
      A34 = 1./ XMU(M)
      A42 = -A13
      A43 = A21 + 4.*WVNOSQ*XMU(M)*(XLAMB(M)+XMU(M))*A12
      YY3(M,5) = AUR1
      YY1(M,5) = AUZ1
      YY2(M,5) = ATZ1
      YY4(M,5) = ATR1
```

```
      DO 1338 KK=2,5
      K = 6 - KK
C     RUNGE-KUTTA INTEGRATION
      DO 1400 JJ=1,NDIV
      EAUR1 = AUR1
      EAUZ1 = AUZ1
      EATZ1 = ATZ1
      EATR1 = ATR1
      DAUR1 = 0.
      DAUZ1 = 0.
      DATZ1 = 0.
      DATR1 = 0.
      DO 1401 LL = 1,4
C     CLASSIC RUNGE KUTTA
      SAUR1 = AUR1 + WWT(LL)*DDZ*DAUR1
      SAUZ1 = AUZ1 + WWT(LL)*DDZ*DAUZ1
      SATZ1 = ATZ1 + WWT(LL)*DDZ*DATZ1
      SATR1 = ATR1 + WWT(LL)*DDZ*DATR1
      DAUR1 =  A31*SAUZ1 + A34*SATR1
      DAUZ1 =  A12*SATZ1 + A13*SAUR1
      DATZ1 =  A21*SAUZ1 + A24*SATR1
      DATR1 =  A42*SATZ1 + A43*SAUR1
      EAUR1 =  EAUR1 + WT(LL)*DDZ*DAUR1
      EAUZ1 =  EAUZ1 + WT(LL)*DDZ*DAUZ1
      EATZ1 =  EATZ1 + WT(LL)*DDZ*DATZ1
      EATR1 =  EATR1 + WT(LL)*DDZ*DATR1
 1401 CONTINUE
      AUR1 = EAUR1
      AUZ1 = EAUZ1
      ATZ1 = EATZ1
      ATR1 = EATR1
 1400 CONTINUE
      YY1(M,K) = AUZ1
      YY2(M,K) = ATZ1
      YY3(M,K) = AUR1
      YY4(M,K) = ATR1
 1338 CONTINUE
 1346 CONTINUE
      IF (B(1).GT.0.0) GO TO 1347
      YY1(1,1) = YY1(2,1)
      YY2(1,1) = YY2(2,1)
      YY3(1,1) = YY3(2,1)
      YY4(1,1) = YY4(2,1)
 1347 CONTINUE
 4003 AUR2 = 0.D 00
      AUZ2 = 1.D 00
      ATZ2 = -H*RB/DET
      ATR2 = -H*BRKT/DET
      IF (ITER.EQ.0) GO TO 4004
      AUR1 = 1.D 00
```

```
      AUZ1 = 0.D 00
      ATZ1 = -H*BRKT/DET
      ATR1 = -H*RA/DET
      AUR2 = XNORM*AUR1 + AUR2
      AUZ2 = XNORM*AUZ1 + AUZ2
      ATZ2 = XNORM*ATZ1 + ATZ2
      ATR2 = XNORM*ATR1 + ATR2
 4004 DO 2346 MM=1,MMM1
      M = MMAX - MM
      IF (B(M).LE.0.0) GO TO 2346
      DDZ = - D(M) / (4.*XDIV)
      A12 = 1./(XLAMB(M)+2.*XMU(M))
      A13 = WVNO * XLAMB(M) *A12
      A21 = - OMEGSQ * RHO(M)
      A24 = WVNO
      A31 = -WVNO
      A34 = 1./ XMU(M)
      A42 = -A13
      A43 = A21 + 4.*WVNOSQ*XMU(M)*(XLAMB(M)+XMU(M))*A12
      YZ3(M,5) = AUR2
      YZ1(M,5) = AUZ2
      YZ2(M,5) = ATZ2
      YZ4(M,5) = ATR2
      DO 2338 KK=2,5
      K = 6 - KK
      DO 2400 JJ=1,NDIV
      EAUR2 = AUR2
      EAUZ2 = AUZ2
      EATZ2 = ATZ2
      EATR2 = ATR2
      DAUR2 = 0.
      DAUZ2 = 0.
      DATZ2 = 0.
      DATR2 = 0.
      DO 2401 LL = 1,4
      SAUR2 = AUR2 + WWT(LL)*DDZ*DAUR2
      SAUZ2 = AUZ2 + WWT(LL)*DDZ*DAUZ2
      SATZ2 = ATZ2 + WWT(LL)*DDZ*DATZ2
      SATR2 = ATR2 + WWT(LL)*DDZ*DATR2
      DAUR2 =  A31*SAUZ2 + A34*SATR2
      DAUZ2 =  A12*SATZ2 + A13*SAUR2
      DATZ2 =  A21*SAUZ2 + A24*SATR2
      DATR2 =  A42*SATZ2 + A43*SAUR2
      EAUR2 =  EAUR2 + WT(LL)*DDZ*DAUR2
      EAUZ2 =  EAUZ2 + WT(LL)*DDZ*DAUZ2
      EATZ2 =  EATZ2 + WT(LL)*DDZ*DATZ2
      EATR2 =  EATR2 + WT(LL)*DDZ*DATR2
 2401 CONTINUE
      AUR2 = EAUR2
      AUZ2 = EAUZ2
```

```
      ATZ2 = EATZ2
      ATR2 = EATR2
 2400 CONTINUE
      YZ1(M,K) = AUZ2
      YZ2(M,K) = ATZ2
      YZ3(M,K) = AUR2
      YZ4(M,K) = ATR2
 2338 CONTINUE
 2346 CONTINUE
      IF (B(1).GT.0.0) GO TO 2347
      YZ1(1,1) = YZ1(2,1)
      YZ2(1,1) = YZ2(2,1)
      YZ3(1,1) = YZ3(2,1)
      YZ4(1,1) = YZ4(2,1)
 2347 CONTINUE
      AA = (YZ3(1,1) - RATIO*YZ1(1,1))
      BB = (RATIO*YY1(1,1) - YY3(1,1))
      XNORM = AA/BB
      BB = XNORM*YY1(1,1) + YZ1(1,1)
C     THESE PARAMETERS ARE SAVED FOR USE AS A NUMERICAL ANALYSIS
C     DIAGNOSTIC
      AMPUR(100)  = (XNORM*YY3(1,1) + YZ3(1,1))/BB
      AMPUZ(100)  = (XNORM*YY1(1,1) + YZ1(1,1))/BB
      STRESZ(100) = (XNORM*YY2(1,1) + YZ2(1,1))/BB
      STRESR(100) = (XNORM*YY4(1,1) + YZ4(1,1))/BB
      ITER = ITER + 1
      IF (ITER.GT.1) GO TO 2011
C      IF ROUNDOFF ERROR IS SIGNIFICANT, MODIFY INITIAL CHOICE AT BASE
C     OF LAYERS OF ONE OF THE INDEPENDENT SOLUTIONS
      XTEST = ABS(AMPUR(100)/RATIO-1.)
      IF (XTEST.GE.0.00001) GO TO 4003
 2011 DCDB(99)   = ITER
      DCDA(100) = 0.0
      DCDB(100) = 0.0
      DCDR(100) = 0.0
      AMPUR(99)   = YY3(1,1)
      AMPUR(98)   = YZ3(1,1)
      AMPUZ(99)   = YY1(1,1)
      AMPUZ(98)   = YZ1(1,1)
      STRESZ(99)  = YY2(1,1)
      STRESZ(98)  = YZ2(1,1)
      STRESR(99)  = YY4(1,1)
      STRESR(98)  = YZ4(1,1)
      DCDB(98)    = BB
      DCDA(98)    = 0.0
      DCDA(99)    = 0.0
      DCDR(98)    = 0.0
      DCDR(99)    = 0.
C
C     COMBINATION OF TWO INDEPENDENT SOLUTIONS SUBJECT TO SURFACE B. C.
C     THAT UR/UZ = RATIO (I)
```

```
C
      DO 7000 M=1,MMAX
      IF (B(M).LE.0.0) GO TO 7000
      IF (M-MMAX) 7001,7002,7002
 7001 DZ = D(M)/4.
      DO 1339 KK=1,5
      AUR = (XNORM*YY3(M,KK)       + YZ3(M,KK)      )/BB
      AUZ = (XNORM*YY1(M,KK)       + YZ1(M,KK)      )/BB
      ATZ = (XNORM*YY2(M,KK)       + YZ2(M,KK)      )/BB
      ATR = (XNORM*YY4(M,KK)       + YZ4(M,KK)      )/BB
      DURDZ = ATR/XMU(M) - WVNO*AUZ
      DUZDZ = (ATZ + WVNO*XLAMB(M)*AUR)/(XLAMB(M)+2.*XMU(M))
      DMR(KK) = AUR*AUR
      DMZ(KK) = AUZ*AUZ
      SMR(KK) = DURDZ*DURDZ
      SMZ(KK) = DUZDZ*DUZDZ
      DMRSMZ(KK) = AUR*DUZDZ
      DMZSMR(KK) = AUZ*DURDZ
      IF (KK - 3) 1339,1301,1339
C     EIGENFUNCTIONS AT MIDPOINT OF LAYER ARE SAVED
 1301 AMPUR(M) = AUR
      AMPUZ(M) = AUZ
      STRESZ(M) = ATZ
      STRESR(M) = ATR
 1339 CONTINUE
C     INTEGRATION BY BODE S RULE
      DMMR = (DZ/22.5)*(7.*(DMR(1)+DMR(5))+32.*(DMR(2)+DMR(4))+12.
     1 *DMR(3))
      DMMZ = (DZ/22.5)*(7.*(DMZ(1)+DMZ(5))+32.*(DMZ(2)+DMZ(4))+12.
     1 *DMZ(3))
      SMMZ = (DZ/22.5)*(7.*(SMZ(1)+SMZ(5))+32.*(SMZ(2)+SMZ(4))+12.
     1 *SMZ(3))
      SMMR = (DZ/22.5)*(7.*(SMR(1)+SMR(5))+32.*(SMR(2)+SMR(4))+12.
     1 *SMR(3))
      DRSZ = (DZ/22.5)*(7.*(DMRSMZ(1)+DMRSMZ(5))+32.*(DMRSMZ(2)
     1 +DMRSMZ(4))+12.*DMRSMZ(3))
      DZSR = (DZ/22.5)*(7.*(DMZSMR(1)+DMZSMR(5))+32.*(DMZSMR(2)
     1 +DMZSMR(4))+12.*DMZSMR(3))
      SUMI0 = SUMI0 + RHO(M)*(DMMR+DMMZ)
      SUMI1 = (XLAMB(M)+2.*XMU(M))*DMMR+XMU(M)*DMMZ + SUMI1
      SUMI2 = XMU(M)*DZSR - XLAMB(M)*DRSZ + SUMI2
      SUMI3 = (XLAMB(M)+2.*XMU(M))*SMMZ + XMU(M)*SMMR + SUMI3
      DLDL = -WVNOSQ*DMMR + 2.*WVNO*DRSZ -SMMZ
      DLDM = -WVNOSQ*(2.*DMMR+DMMZ) - 2.*WVNO*DZSR - (2.*SMMZ+SMMR)
      DLDR = OMEGSQ*(DMMR+DMMZ)
      DCDB(M) = 2.*RHO(M)*B(M)*C * (DLDM-2.*DLDL)/WVNO
      DCDA(M) = 2.*RHO(M)*A(M)*C * DLDL/WVNO
      DCDR(M) = (C/WVNO)*(DLDR+XLAMB(M)*DLDL/RHO(M)+XMU(M)*DLDM/RHO(M))
      IF (ABS(AUZ)+ABS(AUR)-XXMIN) 7002,7002,7000
 7000 CONTINUE
```

```
C      EXACT HALFSPACE CONTRIBUTION
 7002 AMPUR(M) = AUR
      AMPUZ(M) = AUZ
      STRESR(M) = ATR
      STRESZ(M) = ATZ
      AP = -RHO(M)*(WVNO*AUR + RB*AUZ)/DET
      BP = -RHO(M)*(-RA*AUR/WVNO - AUZ)/DET
      A1 = -WVNO*AP/RHO(M)
      A2 = -WVNO*RB*BP/RHO(M)
      A3 = RA*AP/RHO(M)
      A4 = WVNOSQ*BP/RHO(M)
      IF (RB) 7005,7006,7005
 7005 DMMR = A1*A1/(2.*RA)+2.*A1*A2/(RA+RB)+A2*A2/(2.*RB)
      DMMZ = A3*A3/(2.*RA)+2.*A3*A4/(RA+RB)+A4*A4/(2.*RB)
      SMMZ = RA*A3*A3/2.+2.*RA*RB*A3*A4/(RA+RB)+RB*A4*A4/2.
      SMMR = RA*A1*A1/2.+2.*RA*RB*A1*A2/(RA+RB)+RB*A2*A2/2.
      DRSZ = -A1*A3/2.-(A1*A4*RB+A2*A3*RA)/(RA+RB)-A2*A4/2.
      DZSR = -A1*A3/2.-(A1*A4*RA+A2*A3*RB)/(RA+RB)-A2*A4/2.
      GO TO 7010
 7006 UGR = B(M)
      FLAGR = 0.0
      SUMI0 = RHO(M)*10.**(25)
      SUMI1 = XMU(M)*10.**(25)
      SUMI2 = 0.0
      SUMI3 = 0.0
      ARE = 0.0
      DCDB(M) = -2.*WVNO*10.**(25)
      GO TO 531
 7010 CONTINUE
      SUMI0 = SUMI0 + RHO(M)*(DMMR+DMMZ)
      SUMI1 = (XLAMB(M)+2.*XMU(M))*DMMR+XMU(M)*DMMZ + SUMI1
      SUMI2 = XMU(M)*DZSR - XLAMB(M)*DRSZ + SUMI2
      SUMI3 = (XLAMB(M)+2.*XMU(M))*SMMZ + XMU(M)*SMMR + SUMI3
      DLDR = OMEGSQ*(DMMR+DMMZ)
      DLDM = -WVNOSQ*(2.*DMMR+DMMZ)-2.*WVNO*DZSR-(2.*SMMZ+SMMR)
      DLDL = -WVNOSQ*DMMR + 2.*WVNO*DRSZ - SMMZ
      DCDA(M) = 2.*RHO(M)*A(M)*C*DLDL/WVNO
      DCDB(M) = 2.*RHO(M)*B(M)*C*(DLDM-2.*DLDL)/WVNO
      DCDR(M) = (C/WVNO)*(DLDR+XLAMB(M)*DLDL/RHO(M)+XMU(M)*DLDM/RHO(M))
 7011 CONTINUE
      UGR = (WVNO*SUMI1 + SUMI2)/(OMEGA*SUMI0)
      FLAGR = OMEGSQ*SUMI0 - WVNOSQ*SUMI1 - 2.*WVNO*SUMI2 - SUMI3
      ARE = 1./(2.*C*UGR*SUMI0)
  531 CONTINUE
  501 CONTINUE
      WRITE (61,5000) T,C
      WRITE (61,5001)U,UGR,RATIO
      WRITE (61,5002) SUMI0,SUMI1,SUMI2
      WRITE (61,5003) SUMI3,FLAGR,WVNUM
      WRITE (61,5007) AR,ARE
```

```
      WRITE (61,5004)
      AU = RATIO
      AZ = 1.0
      TR = 0.0
      TZ = 0.0
      IF (B(1).LE.0.0) TZ=TZZ
      DEPTH(100)=0.0
      IF (B(1).LE.0.0) DEPTH(100)=D(1)
      M = 0
      DUMM1 = 0.0
      DUMM2 = 0.0
      DUMM3 = 0.0
      WRITE (61,5005) M,DEPTH(100),AU,AZ,TZ,TR,DUMM1,DUMM2,DUMM3
      N = 1
      IF (B(1).LE.0.0) N = 2
      DO 5010 M = N,MMAX
      DLDK = -2.*(WVNUM * SUMI1 + SUMI2)
      DCDR(M) = DCDR(M)/DLDK
      DCDA(M) = DCDA(M)/DLDK
 5009 DCDB(M) = DCDB(M)/DLDK
 5010 WRITE (61,5005) M,DEPTH(M),AMPUR(M),AMPUZ(M),STRESZ(M),STRESR(M),
     1 DCDA(M),DCDB(M),DCDR(M)
      WRITE (61,19)
   19 FORMAT (1H0)
      M = 100
      WRITE (61,5005) M,DEPTH(M),AMPUR(M),AMPUZ(M),STRESZ(M),STRESR(M),
     1 DCDA(M),DCDB(M),DCDR(M)
      WRITE (61,19)
      M = 98
      WRITE (61,5005) M,DEPTH(M),AMPUR(M),AMPUZ(M),STRESZ(M),STRESR(M),
     1 DCDA(M),DCDB(M),DCDR(M)
      M = 99
      WRITE (61,5005) M,DEPTH(M),AMPUR(M),AMPUZ(M),STRESZ(M),STRESR(M),
     1 DCDA(M),DCDB(M),DCDR(M)
      IF (ITAPE.LE.0) GO TO 503
      MMAX = NMAX
      K = IIK +1
      WRITE (01,6010) AMODE(K),T,C,U,SUMI0,WVNUM,AR,RATIO,DUMM1
      WRITE (01,6011) AU,AZ,TZ,TR,DUMM1,DUMM2,DUMM3
      N = 1
      IF (B(1).LE.0.0) N = 2
      DO 6002 M= N,MMAX
      WRITE (01,6011) AMPUR(M),AMPUZ(M),STRESZ(M),STRESR(M),DCDA(M),
     1 DCDB(M),DCDR(M)
 6002 CONTINUE
      IIK = IIK+ 1
      GO TO 503
  777 CONTINUE
      STOP
      END
```

```
      SUBROUTINE TAPEIN
      READ (60,1) TMAX,KMODMX
    1 FORMAT(F10.5,I5)
      KMODEL = 0
 3002 CONTINUE
      IF (KMODEL.EQ.KMODMX) RETURN
      READ (01,6012) MMAX
 6012 FORMAT (1H ,2(I4,6X))
      DO 200 I = 1,MMAX
  200 READ (01,6013) IDUM
 6013 FORMAT (1H ,I4,5X,7F10.4)
 2000 READ (01,6012) IFUNC,MODE
      IF (IFUNC.LT.0) GO TO 3000
      DO 2999 IM = 1,MODE
      READ (01,6010) IDUM,T
 6010 FORMAT (1H ,A4,3F10.5,5E11.4)
      DO 2100 IN = 1,MMAX
 2100 READ (01,6011) XDUM
 6011 FORMAT (1H ,7(4X,E11.4))
 2999 CONTINUE
      IF (T.GE.TMAX) RETURN
      GO TO 2000
 3000 KMODEL = KMODEL + 1
      GO TO 3002
      END
```

```
DATA    PAGE  1

 31          2
1.0         5.00       2,89       2.5
1.0         6,1       3,52       2,7
1.0         6,1       3,52       2.7
1.0         6.1       3,52       2,7
1.0         6,1       3,52       2,7
1.0         6.1       3,52       2.7
1.0         6,1       3,52       2.7
1.0         6,1       3.52       2.7
1.0         6.1       3,52       2.7
1.0         6,1       3,52       2.7
1.0         6.4       3,7        2.9
1.0         6.4       3,7        2,9
1.0         6.4       3,7        2.9
1.0         6,4       3,7        2,9
1.0         6.4       3,7        2,9
1.0         6.4       3,7        2,9
1.0         6.4       3.7        2,9
1.0         6.4       3.7        2,9
1.0         6.4       3,7        2.9
1.0         6,4       3,7        2,9
2,0         6,7       3,87       3.0
2.0         6,7       3,87       3.0
2.0         6,7       3,87       3.0
2.0         6.7       3,87       3,0
2.0         6,7       3.87       3.0
2.0         6,7       3,87       3,0
2,0         6,7       3,87       3.0
2.0         6.7       3,87       3.0
2.0         6.7       3,87       3,0
2.0         6,7       3,87       3.0
            8,15      4.7        3,4

  2          3         FUND 1 ST 2 ND
2.0000000E 00  3.1142651E 00  3,0052558E 00  6,6519809E-03  7,7533051E-01
2.0000000E 00  3,7021885E 00  3,5109269E 00  2,0487942E-04  6,2238460E-01
2.0000000E 00  3.8652486E 00  3.6090615E 00  1,4074941E-04  5,6639548E-01

  2          3         FUND 1 ST 2 ND
2.5000000E 00  3.1358773E 00  3,0523695E 00  4,9933391E-03  7,9398409E-01
2.5000000E 00  3,7487547E 00  3,5414947E 00  1,6791463E-04  6,2641879E-01
2.5000000E 00  3,9224432E 00  3.6695645E 00  8,6686953E-05  5,6405590E-01

  2          3         FUND 1 ST 2 ND
3.0000000E 00  3.1511676E 00  3,0684696E 00  4,0669670E-03  8,0034681E-01
3.0000000E 00  3,7910240E 00  3,5610245E 00  1,5263218E-04  6,1663947E-01
3.0000000E 00  3,9830012E 00  3.6056537E 00  9,9571947E-05  5.4424362E-01

  2          3         FUND 1 ST 2 ND
3.5000000E 00  3.1650985E 00  3.0703514E 00  3,4696805E-03  8,0017772E-01
```

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DATA     PAGE  2


3.5000000E 00   3.8306909E 00   3.5773676E 00   1.4386220E-04   6.0192247E-01
3.5000000E 00   4.0642072E 00   3.4999735E 00   1.3432320E-04   5.0977601E-01

  2           3          FUND 1 ST 2 ND
4.0000000E 00   3.1793620E 00   3.0665456E 00   3.0437626E-03   7.9666199E-01
4.0000000E 00   3.8692162E 00   3.5826476E 00   1.4036134E-04   5.8613508E-01
4.0000000E 00   4.1702411E 00   3.4017723E 00   1.7067032E-04   4.6229274E-01

  2           3          FUND 1 ST 2 ND
5.0000000E 00   3.2100163E 00   3.0587665E 00   2.4481107E-03   7.8600404E-01
5.0000000E 00   3.9516559E 00   3.5479480E 00   1.5024156E-04   5.5422457E-01
5.0000000E 00   4.4235663E 00   3.4890510E 00   1.6888031E-04   3.4403332E-01

  2           3          FUND 1 ST 2 ND
6.0000000E 00   3.2420740E 00   3.0607984E 00   2.0292056E-03   7.7537596E-01
6.0000000E 00   4.0523164E 00   3.4753120E 00   1.7317689E-04   5.1928876E-01
6.0000000E 00   4.5971306E 00   3.9840677E 00   8.6218279E-05   2.6458797E-01

  2           3          FUND 1 ST 2 ND
7.0000000E 00   3.2734813E 00   3.0721896E 00   1.7125091E-03   7.6686421E-01
7.0000000E 00   4.1752123E 00   3.4192479E 00   1.9103228E-04   4.7781309E-01
7.0000000E 00   4.6848377E 00   4.3021607E 00   4.7082233E-05   2.3372602E-01

  2           2          FUND 1 ST 2 ND
8.0000000E 00   3.3031600E 00   3.0884357E 00   1.4674579E-03   7.6071229E-01
8.0000000E 00   4.3090794E 00   3.4664272E 00   1.8146382E-04   4.3075433E-01

  2           2          FUND 1 ST 2 ND
9.0000000E 00   3.3309471E 00   3.1049636E 00   1.2764350E-03   7.5653140E-01
9.0000000E 00   4.4275274E 00   3.6620983E 00   1.4106098E-04   3.8617375E-01

  2           2          FUND 1 ST 2 ND
1.0000000E 01   3.3572350E 00   3.1180076E 00   1.1269633E-03   7.5379098E-01
1.0000000E 01   4.5134921E 00   3.9041931E 00   9.8747813E-05   3.5185762E-01

  2           2          FUND 1 ST 2 ND
1.1000000E 01   3.3826617E 00   3.1261866E 00   1.0092459E-03   7.5200669E-01
1.1000000E 01   4.5722072E 00   4.0912724E 00   7.0955873E-05   3.2831591E-01

  2           2          FUND 1 ST 2 ND
1.2000000E 01   3.4078975E 00   3.1287233E 00   9.1556588E-04   7.5080240E-01
1.2000000E 01   4.6141496E 00   4.2182069E 00   5.4338796E-05   3.1348770E-01

  2           2          FUND 1 ST 2 ND
1.3000000E 01   3.4335251E 00   3.1259828E 00   8.3977392E-04   7.4991863E-01
1.3000000E 01   4.6458775E 00   4.3101667E 00   4.3387340E-05   3.0606461E-01

  2           2          FUND 1 ST 2 ND
1.4000000E 01   3.4599863E 00   3.1193709E 00   7.7717032E-04   7.4919921E-01
1.4000000E 01   4.6703708E 00   4.3906449E 00   3.4092357E-05   3.0547285E-01
```

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DATA    PAGE  3


  2          2         FUND 1 ST 2 ND
1.5000000E 01  3.4875658E 00  3.1104207E 00  7.2408858E-04  7.4857095E-01
1.5000000E 01  4.6883101E 00  4.4814298E 00  2.3859875E-05  3.1153613E-01

  2          2         FUND 1 ST 2 ND
1.6000000E 01  3.5163950E 00  3.1007019E 00  6.7787876E-04  7.4802391E-01
1.6000000E 01  4.6985934E 00  4.6169775E 00  8.8747948E-06  3.2445508E-01

  2          1         FUND 1 ST 2 ND
1.7000000E 01  3.5464619E 00  3.0924978E 00  6.3644259E-04  7.4759430E-01

  2          1         FUND 1 ST 2 ND
1.8000000E 01  3.5776256E 00  3.0865258E 00  5.9829302E-04  7.4734917E-01

  2          1         FUND 1 ST 2 ND
1.9000000E 01  3.6096355E 00  3.0847227E 00  5.6235467E-04  7.4737325E-01

  2          1         FUND 1 ST 2 ND
2.0000000E 01  3.6421553E 00  3.0883168E 00  5.2798860E-04  7.4775627E-01

  2          1         FUND 1 ST 2 ND
2.2000000E 01  3.7071341E 00  3.1146846E 00  4.6254308E-04  7.4991790E-01

  2          1         FUND 1 ST 2 ND
2.4000000E 01  3.7693573E 00  3.1656269E 00  4.0148728E-04  7.5427168E-01

  2          1         FUND 1 ST 2 ND
2.6000000E 01  3.8262323E 00  3.2363947E 00  3.4616044E-04  7.6084514E-01

  2          1         FUND 1 ST 2 ND
2.8000000E 01  3.8763158E 00  3.3190566E 00  2.9794579E-04  7.6928231E-01

  2          1         FUND 1 ST 2 ND
3.0000000E 01  3.9192900E 00  3.4042079E 00  2.5727347E-04  7.7900857E-01

  2          1         FUND 1 ST 2 ND
3.5000000E 01  3.9994231E 00  3.5978714E 00  1.8459469E-04  8.0515936E-01

  2          1         FUND 1 ST 2 ND
4.0000000E 01  4.0513421E 00  3.7414352E 00  1.4075543E-04  8.2907604E-01

  2          1         FUND 1 ST 2 ND
4.5000000E 01  4.0861130E 00  3.8427759E 00  1.1322909E-04  8.4821653E-01

  2          1         FUND 1 ST 2 ND
5.0000000E 01  4.1105629E 00  3.9130282E 00  9.4880703E-05  8.6238892E-01

  2          1         FUND 1 ST 2 ND
6.0000000E 01  4.1425549E 00  3.9985003E 00  7.2337643E-05  8.7866213E-01
```

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DATA    PAGE  4


  2            1        FUND 1 ST 2 ND
7.0000000E 01  4.1629857E 00  4.0475750E 00  5,9053952E-05  8,8405416E-01

  2            1        FUND 1 ST 2 ND
8,0000000E 01  4.1776894E 00  4.0773227E 00  5,0216392E-05  8,8319131E-01

  2            1        FUND 1 ST 2 ND
9.0000000E 01  4.1891447E 00  4.0990825E 00  4,3848633E-05  8.7890215E-01

  2            1        FUND 1 ST 2 ND
1.0000000E 02  4.1985384E 00  4.1142765E 00  3,8996830E-05  8.7283818E-01

  2            1        FUND 1 ST 2 ND
1.5000000E 02  4.2297525E 00  4.1626366E 00  2,5310945E-05  8.3814402E-01

  2            1        FUND 1 ST 2 ND
2.0000000E 02  4.2482172E 00  4.1896077E 00  1,8762922E-05  8.1026544E-01

 -2            1        FUND
```

```
 M        DEPTH          C          ALPHA        BETA        RHO         MU         LAMDA

  1       0,5000      1,0000       5,0000      2,8900      2,5000     20,8802     20,7395
  2       1,5000      1,0000       6,1000      3,5200      2,7000     33,4541     33,5588
  3       2,5000      1,0000       6,1000      3,5200      2,7000     33,4541     33,5588
  4       3,5000      1,0000       6,1000      3,5200      2,7000     33,4541     33,5588
  5       4,5000      1,0000       6,1000      3,5200      2,7000     33,4541     33,5588  R
  6       5,5000      1,0000       6,1000      3,5200      2,7000     33,4541     33,5588
  7       6,5000      1,0000       6,1000      3,5200      2,7000     33,4541     33,5588
  8       7,5000      1,0000       6,1000      3,5200      2,7000     33,4541     33,5588
  9       8,5000      1,0000       6,1000      3,5200      2,7000     33,4541     33,5588
 10       9,5000      1,0000       6,1000      3,5200      2,7000     33,4541     33,5588  5E 00
 11      10,5000      1,0000       6,4000      3,7000      2,9000     39,7010     39,3820  1E-01
 12      11,5000      1,0000       6,4000      3,7000      2,9000     39,7010     39,3820
 13      12,5000      1,0000       6,4000      3,7000      2,9000     39,7010     39,3820  0E-01
 14      13,5000      1,0000       6,4000      3,7000      2,9000     39,7010     39,3820  1E-01
 15      14,5000      1,0000       6,4000      3,7000      2,9000     39,7010     39,3820  2E-01
 16      15,5000      1,0000       6,4000      3,7000      2,9000     39,7010     39,3820  5E-01
 17      16,5000      1,0000       6,4000      3,7000      2,9000     39,7010     39,3820  2E-02
 18      17,5000      1,0000       6,4000      3,7000      2,9000     39,7010     39,3820  7E-02
 19      18,5000      1,0000       6,4000      3,7000      2,9000     39,7010     39,3820
 20      19,5000      1,0000       6,4000      3,7000      2,9000     39,7010     39,3820  1E-02
 21      21,0000      2,0000       6,7000      3,8700      3,0000     44,9307     44,8086  1E-03
 22      23,0000      2,0000       6,7000      3,8700      3,0000     44,9307     44,8086  7E-03
 23      25,0000      2,0000       6,7000      3,8700      3,0000     44,9307     44,8086  9E-04
 24      27,0000      2,0000       6,7000      3,8700      3,0000     44,9307     44,8086  6E-04
 25      29,0000      2,0000       6,7000      3,8700      3,0000     44,9307     44,8086
 26      31,0000      2,0000       6,7000      3,8700      3,0000     44,9307     44,8086  3E-05
 27      33,0000      2,0000       6,7000      3,8700      3,0000     44,9307     44,8086  37E-05
 28      35,0000      2,0000       6,7000      3,8700      3,0000     44,9307     44,8086  2E-06
 29      37,0000      2,0000       6,7000      3,8700      3,0000     44,9307     44,8086  66E-06
 30      39,0000      2,0000       6,7000      3,8700      3,0000     44,9307     44,8086  29E-07
 31                                8,1500      4,7000      3,4000     75,1060     75,6245  02E-07
                                                                                           37E-07
```

II-25

```
        U(DC/DT) =  3.0053  U(VAR) =  3.0052    E  =  0.7753E 00

        I0 =  0.8030E 01     I1 =  0.1003E 03     I2 = -0.2537E 02
        I3 =  0.2837E 02     L  =  0.1017E-03     K  =  0.1009E 01
        AR =  0.6652E-02    ARE =  0.6653E-02

  M     DEPTH     UR           UZ          TZ          TR          DCDA        DCDB        DCDR


  0     0.     0.7753E 00  0.1000E 01  0.          0.          0.          0.          0.
  1     0.50   0.3791E 00  0.1050E 01 -0.1012E 02  0.9842E 01  0.9434E-01  0.9364E-01 -0.1095E 00
  2     1.50  -0.1022E-01  0.9107E 00 -0.1896E 02  0.2170E 02  0.2507E-01  0.1569E 00 -0.3621E-01
  3     2.50  -0.1482E 00  0.6907E 00 -0.1768E 02  0.2178E 02  0.4424E-02  0.2114E 00  0.4170E-01
  4     3.50  -0.1522E 00  0.4825E 00 -0.1350E 02  0.1705E 02  0.7808E-03  0.1451E 00  0.4311E-01
  5     4.50  -0.1203E 00  0.3225E 00 -0.9478E 01  0.1208E 02  0.1380E-03  0.7710E-01  0.2602E-01
  6     5.50  -0.8599E-01  0.2098E 00 -0.6376E 01  0.8132E 01  0.2460E-04  0.3614E-01  0.1295E-01
  7     6.50  -0.5852E-01  0.1337E 00 -0.4198E 01  0.5296E 01  0.4600E-05  0.1577E-01  0.5862E-02
  8     7.50  -0.3882E-01  0.8348E-01 -0.2743E 01  0.3363E 01  0.1103E-05  0.6564E-02  0.2517E-02
  9     8.50  -0.2517E-01  0.5059E-01 -0.1795E 01  0.2089E 01  0.6648E-06  0.2634E-02  0.1051E-02
 10     9.50  -0.1507E-01  0.2932E-01 -0.1165E 01  0.1301E 01  0.1604E-05  0.1016E-02  0.4351E-03
 11    10.50  -0.8234E-02  0.1707E-01 -0.7122E 00  0.8368E 00  0.1685E-06  0.3333E-03  0.1377E-03
 12    11.50  -0.5084E-02  0.1017E-01 -0.4290E 00  0.5069E 00  0.2892E-07  0.1238E-03  0.5189E-04
 13    12.50  -0.3076E-02  0.6009E-02 -0.2554E 00  0.3029E 00  0.4963E-08  0.4457E-04  0.1886E-04
 14    13.50  -0.1837E-02  0.3531E-02 -0.1509E 00  0.1794E 00  0.8516E-09  0.1572E-04  0.6693E-05
 15    14.50  -0.1087E-02  0.2067E-02 -0.8865E-01  0.1056E 00  0.1462E-09  0.5465E-05  0.2337E-05
 16    15.50  -0.6398E-03  0.1207E-02 -0.5189E-01  0.6187E-01  0.2508E-10  0.1882E-05  0.8072E-06
 17    16.50  -0.3748E-03  0.7034E-03 -0.3029E-01  0.3615E-01  0.4304E-11  0.6435E-06  0.2766E-06
 18    17.50  -0.2190E-03  0.4093E-03 -0.1765E-01  0.2108E-01  0.7382E-12  0.2190E-06  0.9429E-07
 19    18.50  -0.1276E-03  0.2380E-03 -0.1027E-01  0.1227E-01  0.1267E-12  0.7430E-07  0.3202E-07
 20    19.50  -0.9740E-04  0.1814E-03 -0.7831E-02  0.9358E-02  0.2616E-13  0.3796E-07  0.1637E-07


100     0.     0.7753E 00  0.1000E 01  0.9053E-04  0.9526E-04  0.          0.          0.


 98     0.     0.2207E 08 -0.1802E 08  0.5718E 09 -0.7375E 09  0.          0.1245E 10  0.
 99     0.     0.4110E 08 -0.3358E 08  0.1065E 10 -0.1374E 10  0.          0.1000E 01  0.
```

```
          U(DC/DT) =  3.5109  U(VAR) =  3.5108     E  =  0.6224E 00

          I0 =  0.1876E 03    I1  =  0.2653E 04     I2 = -0.1816E 03
          I3 =  0.2499E 03    L   =  0.3397E-03     K  =  0.8486E 00
          AR =  0.2049E-03    ARE =  0.2050E-03

  M     DEPTH     UR           UZ           TZ           TR          DCDA         DCDB         DCDR


  0     0.      0.6224E 00  0.1000E 01  0.          0.          0.          0.          0.
  1     0.50    0.2420E 00  0.1012E 01 -0.1165E 02  0.4102E 01  0.2837E-02  0.7371E-03 -0.8030E-02
  2     1.50   -0.2633E 00  0.7461E 00 -0.2878E 02  0.7869E 01  0.7100E-03  0.7643E-02 -0.7651E-03
  3     2.50   -0.5523E 00  0.2860E 00 -0.3835E 02  0.1572E 01  0.1814E-03  0.2012E-01  0.9118E-02
  4     3.50   -0.6773E 00 -0.2952E 00 -0.4190E 02 -0.1048E 02  0.4427E-04  0.3000E-01  0.1366E-01
  5     4.50   -0.6907E 00 -0.9060E 00 -0.4074E 02 -0.2469E 02  0.8925E-05  0.3922E-01  0.1184E-01
  6     5.50   -0.6250E 00 -0.1478E 01 -0.3581E 02 -0.3872E 02  0.6987E-06  0.4958E-01  0.5430E-02
  7     6.50   -0.5061E 00 -0.1959E 01 -0.2802E 02 -0.5103E 02  0.2213E-05  0.6116E-01 -0.2648E-02
  8     7.50   -0.3613E 00 -0.2314E 01 -0.1841E 02 -0.6077E 02  0.1667E-04  0.7312E-01 -0.9189E-02
  9     8.50   -0.2257E 00 -0.2529E 01 -0.8340E 01 -0.6793E 02  0.7460E-04  0.8490E-01 -0.1142E-01
 10     9.50   -0.1512E 00 -0.2618E 01  0.3841E 00 -0.7366E 02  0.2984E-03  0.9778E-01 -0.7310E-02
 11    10.50   -0.3557E-01 -0.2619E 01  0.7734E 01 -0.7987E 02  0.3078E-03  0.9080E-01 -0.1287E-01
 12    11.50    0.1170E 00 -0.2515E 01  0.1296E 02 -0.8052E 02  0.7735E-04  0.9393E-01 -0.5848E-02
 13    12.50    0.1942E 00 -0.2349E 01  0.1564E 02 -0.7699E 02  0.1962E-04  0.8765E-01 -0.1809E-02
 14    13.50    0.2334E 00 -0.2150E 01  0.1704E 02 -0.7135E 02  0.5168E-05  0.7674E-01  0.3054E-03
 15    14.50    0.2529E 00 -0.1934E 01  0.1777E 02 -0.6465E 02  0.1565E-05  0.6427E-01  0.1386E-02
 16    15.50    0.2613E 00 -0.1711E 01  0.1811E 02 -0.5745E 02  0.7364E-06  0.5195E-01  0.1953E-02
 17    16.50    0.2621E 00 -0.1484E 01  0.1819E 02 -0.5010E 02  0.8213E-06  0.4065E-01  0.2273E-02
 18    17.50    0.2548E 00 -0.1258E 01  0.1799E 02 -0.4288E 02  0.2011E-05  0.3083E-01  0.2474E-02
 19    18.50    0.2351E 00 -0.1040E 01  0.1737E 02 -0.3617E 02  0.6972E-05  0.2268E-01  0.2605E-02
 20    19.50    0.1926E 00 -0.8378E 00  0.1597E 02 -0.3060E 02  0.2683E-04  0.1631E-01  0.2694E-02
 21    21.00    0.1457E 00 -0.6038E 00  0.1250E 02 -0.2381E 02  0.1241E-04  0.1720E-01  0.2875E-02
 22    23.00    0.1035E 00 -0.3804E 00  0.8232E 01 -0.1545E 02  0.7332E-06  0.7566E-02  0.1533E-02
 23    25.00    0.6673E-01 -0.2349E 00  0.5171E 01 -0.9649E 01  0.4333E-07  0.2999E-02  0.6449E-03
 24    27.00    0.4161E-01 -0.1439E 00  0.3193E 01 -0.5935E 01  0.2560E-08  0.1143E-02  0.2514E-03
 25    29.00    0.2566E-01 -0.8771E-01  0.1961E 01 -0.3624E 01  0.1528E-09  0.4281E-03  0.9524E-04
 26    31.00    0.1581E-01 -0.5316E-01  0.1207E 01 -0.2198E 01  0.1045E-10  0.1587E-03  0.3570E-04
 27    33.00    0.9824E-02 -0.3182E-01  0.7500E 00 -0.1317E 01  0.4354E-11  0.5801E-04  0.1334E-04
 28    35.00    0.6235E-02 -0.1844E-01  0.4780E 00 -0.7645E 00  0.3972E-10  0.2056E-04  0.5005E-05
 29    37.00    0.4050E-02 -0.9764E-02  0.3189E 00 -0.4122E 00  0.6451E-09  0.6816E-05  0.1929E-05
 30    39.00    0.2268E-02 -0.4033E-02  0.2142E 00 -0.2012E 00  0.1089E-07  0.1943E-05  0.7640E-06
 31    40.00    0.9364E-03 -0.2182E-02  0.1551E 00 -0.1600E 00  0.9673E-09  0.3133E-06  0.1514E-06


100     0.      0.6224E 00  0.1000E 01 -0.1659E-03 -0.6581E-04  0.          0.          0.


 98     0.      0.1294E 13 -0.9593E 12  0.2059E 14 -0.3308E 14  0.          0.7308E 13  0.
 99     0.      0.3015E 13 -0.2235E 13  0.4797E 14 -0.7707E 14  0.          0.1000E 01  0.
```

[redacted]

```
          U(DC/DT) =  3.6091  U(VAR) =  3.6090     E  =  0.5664E 00

          I0 =  0.2546E 03    I1 =  0.3938E 04    I2 = -0.3146E 03
          I3 =  0.4223E 03    L  =  0.2283E-02    K  =  0.8128E 00
          AR =  0.1407E-03   ARE =  0.1408E-03

  M    DEPTH     UR          UZ          TZ          TR          DCDA        DCDB        DCDR

  0    0.     0.5664E 00  0.1000E 01  0.          0.          0.          0.          0.
  1    0.50   0.1927E 00  0.1002E 01 -0.1185E 02  0.3066E 01  0.1824E-02  0.4090E-03 -0.6412E-02
  2    1.50  -0.3235E 00  0.7134E 00 -0.3000E 02  0.5752E 01  0.4415E-03  0.6917E-02 -0.2162E-04
  3    2.50  -0.6193E 00  0.2243E 00 -0.4024E 02 -0.3448E 00  0.1237E-03  0.1773E-01  0.8022E-02
  4    3.50  -0.7232E 00 -0.3808E 00 -0.4273E 02 -0.1107E 02  0.3332E-04  0.2441E-01  0.1033E-01
  5    4.50  -0.6737E 00 -0.9819E 00 -0.3826E 02 -0.2287E 02  0.7763E-05  0.2654E-01  0.5680E-02
  6    5.50  -0.5066E 00 -0.1479E 01 -0.2807E 02 -0.3309E 02  0.9457E-06  0.2606E-01 -0.2817E-02
  7    6.50  -0.2629E 00 -0.1796E 01 -0.1392E 02 -0.3990E 02  0.5916E-06  0.2513E-01 -0.1019E-01
  8    7.50   0.9870E-02 -0.1890E 01  0.1969E 01 -0.4236E 02  0.6063E-05  0.2517E-01 -0.1238E-01
  9    8.50   0.2576E 00 -0.1756E 01  0.1708E 02 -0.4053E 02  0.2722E-04  0.2610E-01 -0.8388E-02
 10    9.50   0.4198E 00 -0.1429E 01  0.2879E 02 -0.3560E 02  0.1021E-03  0.2613E-01 -0.8458E-03
 11   10.50   0.5208E 00 -0.1006E 01  0.3625E 02 -0.2735E 02  0.1625E-04  0.2169E-01  0.3365E-02
 12   11.50   0.6179E 00 -0.5226E 00  0.4123E 02 -0.1418E 02  0.4267E-05  0.2055E-01  0.7650E-02
 13   12.50   0.6590E 00  0.8003E-02  0.4318E 02  0.8801E 00  0.9977E-06  0.2061E-01  0.9440E-02
 14   13.50   0.6514E 00  0.5472E 00  0.4227E 02  0.1647E 02  0.1395E-06  0.2305E-01  0.8657E-02
 15   14.50   0.6015E 00  0.1060E 01  0.3872E 02  0.3146E 02  0.3839E-07  0.2806E-01  0.5722E-02
 16   15.50   0.5167E 00  0.1514E 01  0.3289E 02  0.4492E 02  0.4995E-06  0.3503E-01  0.1514E-02
 17   16.50   0.4074E 00  0.1885E 01  0.2528E 02  0.5615E 02  0.2411E-05  0.4292E-01 -0.2794E-02
 18   17.50   0.2879E 00  0.2155E 01  0.1656E 02  0.6484E 02  0.9459E-05  0.5072E-01 -0.5955E-02
 19   18.50   0.1794E 00  0.2319E 01  0.7682E 01  0.7112E 02  0.3522E-04  0.5790E-01 -0.6884E-02
 20   19.50   0.1148E 00  0.2388E 01 -0.5564E-01  0.7598E 02  0.1293E-03  0.6505E-01 -0.4739E-02
 21   21.00  -0.3073E-01  0.2363E 01 -0.8060E 01  0.8174E 02  0.1632E-03  0.1249E 00 -0.9418E-02
 22   23.00  -0.1387E 00  0.2157E 01 -0.1167E 02  0.7769E 02  0.1147E-04  0.1161E 00 -0.1007E-02
 23   25.00  -0.1617E 00  0.1895E 01 -0.1222E 02  0.6902E 02  0.8065E-06  0.9303E-01  0.1126E-02
 24   27.00  -0.1628E 00  0.1626E 01 -0.1200E 02  0.5944E 02  0.5718E-07  0.6981E-01  0.1485E-02
 25   29.00  -0.1590E 00  0.1363E 01 -0.1164E 02  0.4989E 02  0.4585E-08  0.4989E-01  0.1436E-02
 26   31.00  -0.1545E 00  0.1108E 01 -0.1130E 02  0.4057E 02  0.1288E-08  0.3377E-01  0.1320E-02
 27   33.00  -0.1499E 00  0.8605E 00 -0.1099E 02  0.3153E 02  0.7172E-08  0.2126E-01  0.1216E-02
 28   35.00  -0.1442E 00  0.6198E 00 -0.1067E 02  0.2280E 02  0.9446E-07  0.1211E-01  0.1140E-02
 29   37.00  -0.1316E 00  0.3896E 00 -0.1014E 02  0.1469E 02  0.1337E-05  0.6013E-02  0.1055E-02
 30   39.00  -0.9057E-01  0.1858E 00 -0.8600E 01  0.8421E 01  0.1901E-04  0.2412E-02  0.8244E-03
 31   40.00  -0.3907E-01  0.1099E 00 -0.6709E 01  0.7171E 01  0.2351E-05  0.5642E-03  0.2465E-03

100    0.     0.5664E 00  0.1000E 01 -0.2713E-03 -0.1792E-03  0.          0.          0.

 98    0.     0.1648E 12 -0.1183E 12  0.2213E 13 -0.3907E 13  0.         -0.3729E 11  0.
 99    0.     0.4633E 12 -0.3326E 12  0.6222E 13 -0.1099E 14  0.          0.1000E 01  0.
```

[redacted]

III. LEIGEN

PROGRAMMER: R.B. HERRMANN / DEC 1971

PURPOSE:

This program accepts the card output of the program SURFACE. This output consists of the period, phase velocity pairs of the Love wave dispersion in plane layered solid media. The program LEIGEN determines the Love wave eigenfunctions, partial of phase velocity with respect to shear velocity and density as a function of depth within the medium. One liquid layer may be placed at the surface.

INPUT/OUTPUT

Input is from card. The eigenfunctions and phase velocity partial derivatives are listed on the printer. The eigenfunctions and phase velocity partial derivatives may also be listed on FILE 01. This file can then be used by other programs to generate synthetic surface wave seismograms and amplitude spectra.

THEORY

The differential equation to be solved is

$$\frac{d}{dz}\left\{\mu \frac{dV}{dz}\right\} + (\rho\omega^2 - k^2\mu)\, V = 0$$

with the boundary conditions $V(0) = 0$, $\frac{dV}{dz} = 0$ at $z = 0$. $k = 2\pi/(CT)$ where C is the phase velocity and T is the period. ρ is the density and μ is the Lame constant $\mu = \rho\beta^2$, where β is the shear velocity in the layer. The other restriction on the solution is that $V(z) \rightarrow 0$ as $z \rightarrow \infty$.

The equation of motion can be derived from the following expression for the Lagrangian and variational methods. The Lagrangian of the system is

$$K = \omega^2 I_0 - k^2 I_1 - I_2 \ ,$$

where

$$I_0 = \int_0^\infty \rho\, V^2\, dz,\quad I_1 = \int_0^\infty \mu\, V^2\, dz, \text{ and } I_2 = \int_0^\infty \mu\, \frac{dV^2}{dz}\, dz.$$

The group velocity U can be defined in terms of the energy integrals as

$$U = \frac{kI_1}{\omega I_0}$$

The amplitude factor of Harkrider is defined as

$$A_L = \frac{1}{2\,C\,U\,I_0} \quad .$$

For a system of constant velocity layers, the partial of the phase velocity with respect to changes in the shear velocity or density in the m'th layer are determined from the following relations:

$$\left(\frac{\partial c}{\partial \beta_m} \right)_{\rho,\omega} = 2\,\rho_m\,\beta_m \left(\frac{\partial c}{\partial \mu_m} \right)_{\rho,\omega}$$

$$\left(\frac{\partial c}{\partial \rho_m} \right)_{\beta,\omega} = \left(\frac{\partial c}{\partial \rho_m} \right)_{\mu,\omega} + \beta_m^2 \left(\frac{\partial c}{\partial \mu_m} \right)_{\rho,\omega}$$

where

$$\left(\frac{\partial c}{\partial \rho_m} \right)_{\mu,\omega} = \frac{-c^3}{2I_1} \int_{z_{m-1}}^{z_m} V^2(z)\,dz$$

$$\left(\frac{\partial c}{\partial \mu_m} \right)_{\rho,\omega} = \frac{c}{2k^2 I_1} \int_{z_{m-1}}^{z_m} \{ k^2 V^2(z) + (dV/dz)^2 \}\,dz$$

where z_{m-1} and z_m are the lower and upper depth limits of the m'th layer (note that z increases downward).

The numerical integration performed in LEIGEN uses the Haskell formulation of the Love wave problem. The integration is conducted from the halfspace to the free surface.

Several parameters are calculated which aid in judging the numerical error of the integration process. From the program SURFACE, the following parameters are available T, C, U, and AL . The group velocity U is determined by implicit differentiation in SURFACE. Experience has shown that the U and A_L parameters determined by SURFACE are very reliable.

LEIGEN lists several parameters which can be used as goodness of fit parameters. The group velocity U = U(ENER), which has been determined by using the energy integrals, can be compared to U(DC/DT), which was determined by SURFACE. The A_L = ALE, the amplitude factor determined using the energy integrals, can be compared to the A_L = AL, which was determined by SURFACE using differentiation of the period equation. The Lagrangian = L should be zero if there are no numerical errors; this has been found to be a poor guide to goodness of fit. At the bottom of each printer listing, there is a line headed by M = 0. This contains the values of UT (V) and TT (μ dV/dz) as computed at the free surface. If there is no numerical error, TT = 0.

The partial derivatives for the last row in the listing of the quantities as a function of depth are for changes in the parameters of the halfspace.

PROGRAM DESCRIPTION

PROGRAM LEIGEN : This program performs the desired numerical integration and lists the results on the printer.

SUBROUTINE TAPEIN : This subroutine makes it possible to add more eigenfunctions at different periods or for different models on the same tape. This allows expansion of the tape.

INPUT DATA

There are two slightly different arrangements to the data for the cases when the output is and is not written on FILE 01.

Case 1 - No output written on FILE 01.

Card Sequence	Column	Name	Format	Explanation
A.	1-4	MMAX	I4	LE 0 END program GT 0 Number of layers including halfspace in the model.

Card Sequence	Column	Name	Format	Explanation
A.	1-4	MMAX	I4	LE 0 End program GT 0 Number of layers including halfspace in the model
	11-14	ITAPE	I4	LE 0 No tape output (In this case must be LE 0)
B. Earth model (MMAX cards)				
	1-10	D(I)	F10.5	Layer thickness in km.
	11-20	A(I)	F10.5	P velocity of layer in km/sec
	21-30	B(I)	F10.5	S velocity of layer in km/sec
	31-40	RHO(I)	F10.5	Density of layer in gm/cm^3.
C. Period data				
	1-14	T	E14.7	LT 0 read new MMAX, ITAPE card (Point A) EQ 0 does nothing in this case GT 0 Period in seconds
	16-29	C	E14.7	Phase velocity in km/sec
	31-44	U	E14.7	Group velocity in km/sec
	46-59	AL	E14.7	Amplitude factor

At this point the program will determine the appropriate eigenfunctions and return to Point C to read a new T, C, U, AL data card.

The program is terminated by reading a new MMAX card with MMAX.LE. 0. The last system card is an end of file.

Case 2 - Output written on FILE 01

Card Sequence	Column	Name	Format	Explanation
A.	1-4	MMAX	I4	LE 0 End program GT 0 Number of layers including halfspace contained in the model
	11-14	ITAPE	I4	LE 0 No tape output EQ 1 Eigenfunctions written on FILE 01 but velocity-density model is not written again GT 1 Velocity-density model written on FILE as header as well as eigenfunctions
B. Earth model (MMAX cards)				
	1-10	D(I)	F10.5	Layer thickness in km
	11-20	A(I)	F10.5	P velocity of layer in km/sec
	21-30	B(I)	F10.5	S velocity of layer in km/sec
	31-40	RHO(I)	F10.5	Density of layer in gm/cm^3

Card Sequence	Column	Name	Format	Explanation
C. Tape update				
	1-10	IMAX	F10.5	Skip model already on tape and read through the eigenfunctions with T = TMAX. Tape now ready to write after T = TMAX.
	11-15	KMODMX	I5	Number of earth models to skip. KMODMX = 0 if tape is new and has nothing written on it.

Card Sequence	Column	Name	Format	Explanation
D. Period sequence				
a. Header				
	1-4	IFUNC	I4	LT 0 Signifies end of eigenfunction written on FILE. This is essential when reading the FILE. EQ 1 Otherwise, IFUNC=1. This identifies the FILE as containing Love wave eigenfunctions. See program REIGEN where IFUNC=2 indicates Rayleigh wave eigenfunctions.
	11-14	MODE	I4	Number of modes to be placed on tape for given period.
	22-25 26-30 . . . 66-70	AMODE(I)	A4	Identification code for mode
b. Period data (MODE cards)				
	1-14	T	E14.7	Period in seconds
	16-29	C	E14.7	Phase velocity in km/sec
	31-34	U	E14.7	Group velocity in km/sec
	46-59	AL	E14.7	Amplitude factor

c. End period data

1-80 blank card

At this point, program returns to Point D to read in a new IFUNC, MODE card. When the program finds an IFUNC LT 0, the program terminates.

Note, when the FILE created here is to be used by other programs such as WIGGLE, SRFWVPLT, QUESTION, or RADPAT, the order of data is as in the example, i.e., short periods first; within each period set, the modes are listed in increasing order, FUND first, 1'ST second, etc.

```
C     THIS PROGRAM COMPUTES VALUES OF DISPLACEMENTS, STRESS AT THE
C     MIDPOINT OF EACH LAYER TOGETHER WITH THE VALUE OF DC/DB FOR THAT
C     LAYER.  THE METHOD IS BASED ON THE HASKELL MATRIX WITH SPECIAL
C     CONSIDERATION GIVEN TO THE HIGH FREQUENCIES.  THE PROGRAM ALSO
C     GIVES THE VARIOUS VALUES OF THE INTEGRALS MAKING UP THE
C     LAGRANGIAN.
C     MMAX = MINUS OR ZERO = END PROGRAM, = PLUS = NUMBER OF LAYERS
C     D = LAYER THICKNESS, A= P VEL, B = S VEL, RHO = DENSITY
C     DEPTH = DEPTH TO MIDPOINT OF LAYER  XMU, XLAMB = LAME S CONSTANTS
C     T = PERIOD  C = PHASE VELOCITY  U = GROUP VELOCITY (DC/DT DERIVED)
C     THE PROGRAM READS IN ONE SET OF T,C,U,AL OBSERVATIONS AT A TIME
C     T = NEG READ IN NEW MODEL
C     T = ZERO  CHECK TO SEE WHETHER TO ENTER NEW PARAMETERS FOR
C     TAPE WRITE
C
C     A ONE LAYER OCEANIC MODEL WILL BE ACCEPTED, IN WHICH CASE THE
C     LIQUID LAYER IS IGNORED
C
      DIMENSION D(100),A(100),B(100),DEPTH(100),AMP(100),STRESS(100)
      DIMENSION DCDB(100),DCDR(100),DMM(5),SMM(5),RHO(100)
      DIMENSION AMODE(10)
      DIMENSION XMU(100),XLAMB(100)
      XXMIN=1.0E-20
    5 FORMAT(4(E14.7,1X))
   11 FORMAT(2(I4,6X),10(1X,A4))
   20 FORMAT(1H1)
   21 FORMAT(1H0,13X,2HM ,8X,7HDEPTH  ,10X,1HD,12X,5HALPHA,11X,4HBETA,
     1 11X,3HRHO,13X,2HMU,11X,6HLAMBDA /1H )
   23 FORMAT(1H ,10X,I5,7(5X,F10.4))
   25 FORMAT(1H ,10X,I5,30X,5(5X,F10.4))
   44 FORMAT(4F10.5)
 5000 FORMAT(1H1,16X,1(10H T(SEC) =  ,F7.2,13H C(KM/SEC) =  ,F7.5,1X))
 5001 FORMAT(1H0,16X,1(12H U(DC/DT) =  ,F7.4,11H U(ENER) =  ,F7.4,2X))
 5002 FORMAT(1H0,16X,1(8H   I0 =  ,E11.4,7H  I1 =  ,E11.4,2X))
 5003 FORMAT(1H ,16X,1(8H   I2 =  ,E11.4,7H  L  =  ,E11.4,2X))
 5004 FORMAT(1H0,3H  M,8X,5HDEPTH,1(6X,4HDISP,8X,6HSTRESS,8X,5HDC/DB,8X,
     15HDC/DR )/1H0)
 5005 FORMAT(1H ,I3, F13.2,4(2X,E11.4))
 5006 FORMAT(1H ,16X,1(8H WVNC =  ,E11.4,20X)/1H )
 5007 FORMAT(1H ,16X,1(8H   AL =  ,E11.4,7H ALE =  ,E11.4,2X))
 6010 FORMAT(1H ,A4,3F10.5,5E11.4)
 6011 FORMAT(1H ,7(4X,E11.4))
 6012 FORMAT(1H ,2(I4,6X))
 6013 FORMAT(1H ,I4,5X,7F10.4)
   12 FORMAT(I4,6X,I4,6X,F10.4)
  801 READ(60,12) MMAX,ITAPE,ANY
      NMAX = MMAX
      IF(MMAX)777,777,778
  778 WRITE(61,20)
      WRITE(61,21)
```

```
      BASE=0.0
      DO 22 I=1,MMAX
      READ(60,44) D(I),A(I),B(I),RHO(I)
      BASE=BASE+D(I)
      DEPTH(I)=BASE - D(I)*0.5
      XMU(I) = RHO(I)*B(I)*B(I)
      XLAMB(I) = RHO(I)*(A(I)*A(I)-2.*B(I)*B(I))
      IF(I-MMAX)26,24,24
   26 WRITE(61,23)I,DEPTH(I),D(I),A(I),B(I),RHO(I),XMU(I),XLAMB(I)
   22 CONTINUE
   24 WRITE(61,25)I,A(I),B(I),RHO(I),XMU(I),XLAMB(I)
      DEPTH(MMAX) = BASE - D(MMAX)
      IF(ITAPE.GT.0) CALL TAPEIN
      IF(ITAPE.LE.1) GO TO 504
      LMAX = MMAX + 1
      IF(B(1).EQ.0.0) LMAX = LMAX - 1
      WRITE(01,6012) LMAX
      DPTH = 0.0
      IF(B(1).EQ.0.0) DPTH = D(1)
      DD = 0.0
      K = 1
      IF(B(1).EQ.0.0) K = 2
      I = K - 1
      WRITE(01,6013) I,DPTH,DD,A(K),B(K),RHO(K),XMU(K),XLAMB(K)
      DO 200 I=1,MMAX
      IF(B(I).LE.0.0) GO TO 200
      WRITE(01,6013) I,DEPTH(I),D(I),A(I),B(I),RHO(I),XMU(I),XLAMB(I)
  200 CONTINUE
  504 IF(ITAPE.LE.0) GO TO 503
      READ(60,11) IFUNC,MODE,(AMODE(I),I=1,10)
      WRITE(01,6012) IFUNC,MODE
      IF(IFUNC.LT.0) GO TO 777
      IIK = 0
  503 CONTINUE
      DO 500 J = 1,100
      AMP(J) = 0.0
      STRESS(J) = 0.0
      DCDR(J) = 0.0
  500 DCDB(J) = 0.0
      READ(60,5) T,C,U,AL
      IF(T) 801,504,499
  499 CONTINUE
      MMAX = NMAX
C
C     LAYER DROPPING PROCEDURE--HALFSPACE DEFINED AS ANY WAVELENGTHS
C     BELOW REGION WHERE C IS LESS THAN B
C     UNLESS OTHERWISE SPECIFIED, ANY=7.0
C
      IF (ANY.LE.0) ANY=7.0
      DMAX=ANY*C*T
```

```
  899 SUM = 0
      DO 900 II=1,MMAX
      MAX = II
      IF(C - B(II)) 901,900,900
  901 SUM = SUM + D(II)
      IF(SUM - DMAX) 900,900,902
  900 CONTINUE
  902 MMAX = MAX
      WVNO = 6.2831853/(C*T)
      WVNUM = WVNO
      WVNOSQ=WVNO*WVNO
      OMEGSQ =(6.2831853/T)**2
      COVB = C/B(MMAX)
      H = RHO(MMAX)*B(MMAX)*B(MMAX)
      RB = WVNO*SQRT(ABS(COVB**2 - 1.))
      UT = 1
      TT = - H * RB
      AMP(MMAX) = UT
      STRESS(MMAX) = TT
      IF(RB)1001,1000,1001
 1000 DM = 1.0E25
      SM = 0
      GO TO 1002
 1001 DM = 0.5/RB
      SM = 0.5*RB
 1002 CONTINUE
      DLDR = OMEGSQ * DM
      DLDM = -(WVNOSQ*DM + SM)
      DCDB(MMAX) = 2.*RHO(MMAX)*B(MMAX)*C*DLDM/WVNO
      DCDR(MMAX) = (C/WVNO)*(DLDR + B(MMAX)*B(MMAX)*DLDM)
      MMM1 = MMAX - 1
      SUMI0 = RHO(MMAX) * DM
      SUMI1 = H * DM
      SUMI2 = H * SM
      DO 1340 K=1,MMM1
      M = MMAX - K
      IF(B(M).EQ.0.0) GO TO 1340
      COVB = C/B(M)
      RB = WVNO*SQRT(ABS(COVB**2 - 1.))
      H=RHO(M)*B(M)*B(M)
      DZ = D(M)/4.
      DMM(1) = UT * UT
      SMM(1) = (TT/H)**2
      DO 1339 KK=2,5
      XKK=KK-1
      Q=RB*DZ*XKK
      IF(C-B(M))1207,1221,1231
 1231 SINQ = SIN(Q)
      Y=SINQ/RB
      Z=-RB*SINQ
```

```
      COSQ = COS(Q)
      GO TO 1242
 1221 Y=DZ*XKK
      Z=0.0
      COSQ=1.0
      GO TO 1242
 1207 EXQP = EXP(Q)
      EXQM=1./EXQP
      Y=(EXQP-EXQM)/(2.*RB)
      Z=RB*RB*Y
      COSQ=(EXQP+EXQM)/2.
 1242 EUT = COSQ*UT - Y*TT/H
      ETT = -H*Z*UT + COSQ*TT
      DMM(KK) = EUT*EUT
      SMM(KK) = (ETT*ETT)/(H*H)
      IF(KK-3)1339,1301,1339
 1301 AMP(M) = EUT
      STRESS(M) = ETT
 1339 CONTINUE
      UT = EUT
      TT = ETT
      DM=(DZ/22.5)*(7.*(DMM(1)+DMM(5))+32.*(DMM(2)+DMM(4))+12.*DMM(3))
      SM=(DZ/22.5)*(7.*(SMM(1)+SMM(5))+32.*(SMM(2)+SMM(4))+12.*SMM(3))
      DLDM = -(WVNOSQ*DM + SM)
      DLDR = OMEGSQ * DM
      DCDB(M) = 2.*RHO(M)*B(M)*C*DLDM/WVNO
      DCDR(M) = (C/WVNO)*(DLDR+B(M)*B(M)*DLDM)
      SUMI0 = SUMI0 + RHO(M)*DM
      SUMI1 = SUMI1 + H*DM
      SUMI2 = SUMI2 + H*SM
 1340 CONTINUE
      AMP(100) = 1.0
      STRESS(100) = TT/UT
      DCDB(100) = 0.0
      DCDR(100) = 0.0
      DEPTH(100) = 0.0
      IF(B(1).EQ.0.0) DEPTH(100) = D(1)
      DLDK = -2.*WVNO*SUMI1
      DO 1500 L=1,MMAX
      IF(B(L).EQ.0.0) GO TO 1500
      AMP(L) = AMP(L)/UT
      STRESS(L) = STRESS(L)/UT
      DCDB(L) = DCDB(L)/DLDK
      DCDR(L) = DCDR(L)/DLDK
C     EXCLUSION OF LOW AMPLITUDES FROM FINAL OUTPUT
      IF(B(L) - B(MMAX))1506,1507,1507
 1507 CONTINUE
      IF(ABS(AMP(L))-XXMIN) 1501,1502,1502
 1501 AMP(L) = 0.0
      STRESS(L) = 0.0
```

```
      DCDB(L) = 0.0
      DCDR(L) = 0.0
      GO TO 1500
 1502 CONTINUE
 1506 CONTINUE
 1500 CONTINUE
      SUMI0 = SUMI0/(UT*UT)
      SUMI1 = SUMI1/(UT*UT)
      SUMI2 = SUMI2 /(UT*UT)
      UGR = SUMI1/(C*SUMI0)
      FLAGR=OMEGSQ*SUMI0-WVNOSQ*SUMI1-SUMI2
      ALE = 1./(2.*C*UGR*SUMI0)
  501 CONTINUE
      WRITE(61,5000) T,C
      WRITE(61,5001) U,UGR
      WRITE(61,5002) SUMI0,SUMI1
      WRITE(61,5003) SUMI2,FLAGR
      WRITE(61,5007) AL,ALE
      WRITE(61,5006) WVNUM
      WRITE(61,5004)
      M = 0
      DPH = DEPTH(100)
      UT = 1.0
      TT = 0.0
      DCB = 0.0
      DCR = 0.0
      WRITE(61,5005) M,DPH,UT,TT,DCB,DCR
      N = 1
      IF(B(1).EQ.0.0) N = 2
      DO 5010 M = N,MMAX
 5010 WRITE(61,5005)M,DEPTH(M),AMP(M),STRESS(M),DCDB(M),DCDR(M)
      WRITE(61,45)
   45 FORMAT(1H0)
      M = 0
      WRITE(61,5005) M,DEPTH(100),AMP(100),STRESS(100),DCDB(100),DCDR(10
     10)
      IF(ITAPE.LE.0) GO TO 503
      MMAX = NMAX
      DUMM1=0.0
      DUMM2=0.0
      DUMM3=0.0
      K = IIK + 1
      WRITE(01,6010) AMODE(K),T,C,U,SUMI0,WVNUM,AL,DUMM1,DUMM2
      WRITE(01,6011)UT,TT,DUMM1,DUMM2,DUMM3,DCB,DCR
      DO 6002 M = N,MMAX
      WRITE(01,6011)AMP(M),STRESS(M),DUMM1,DUMM2,DUMM3,DCDB(M),DCDR(M)
 6002 CONTINUE
      IIK = IIK + 1
      GO TO 503
  777 CONTINUE
```

```
      END

      SUBROUTINE TAPEIN
      READ(60,1) TMAX,KMODMX
    1 FORMAT(F10.5,I5)
      KMODEL = 0
 3002 CONTINUE
      IF(KMODEL,EQ,KMODMX) RETURN
 3001 READ(01,6012) MMAX
 6012 FORMAT(1H ,2(I4,6X))
      DO 200 I = 1,MMAX
  200 READ(01,6013) IDUM
 6013 FORMAT(1H ,I4,5X,7F10.4)
 2000 READ(01,6012) IFUNC,MODE
      IF(IFUNC.LT.0) GO TC 3000
      DO 2999 IM = 1,MODE
      READ(01,6010) IDUM,T
 6010 FORMAT(1H ,A4,3F10.5,5E11.4)
 6011 FORMAT(1H ,7(4X,E11.4))
      DO 2100 IN = 1,MMAX
 2100 READ(01,6011) XDUM
 2999 CONTINUE
      IF(T.EQ.TMAX) RETURN
      GO TO 2000
 3000 KMODEL = KMODEL + 1
      GO TO 3002
      END
```

```
DATA     PAGE  1

 31            2
1.0           5.00         2.89        2.5
1.0           6.1         3.52        2.7
1.0           6.1         3.52        2.7
1.0           6.1         3.52        2.7
1.0           6.1         3.52        2.7
1.0           6.1         3.52        2.7
1.0           6.1         3.52        2.7
1.0           6.1         3.52        2.7
1.0           6.1         3.52        2.7
1.0           6.1         3.52        2.7
1.0           6.4         3.7         2.9
1.0           6.4         3.7         2.9
1.0           6.4         3.7         2.9
1.0           6.4         3.7         2.9
1.0           6.4         3.7         2.9
1.0           6.4         3.7         2.9
1.0           6.4         3.7         2.9
1.0           6.4         3.7         2.9
1.0           6.4         3.7         2.9
1.0           6.4         3.7         2.9
2.0           6.7         3.87        3.0
2.0           6.7         3.87        3.0
2.0           6.7         3.87        3.0
2.0           6.7         3.87        3.0
2.0           6.7         3.87        3.0
2.0           6.7         3.87        3.0
2.0           6.7         3.87        3.0
2.0           6.7         3.87        3.0
2.0           6.7         3.87        3.0
2.0           6.7         3.87        3.0
              6.15        4.7         3.4

  1            3          FUND 1 ST 2 ND
2.0000000E 00  3.4074768E 00  3.2084552E 00  7,3882933E-03
2.0000000E 00  3.7060527E 00  3.4703516E 00  9,9816568E-04
2.0000000E 00  3.8574009E 00  3.5638103E 00  5,3670618E-04

  1            3          FUND 1 ST 2 ND
2.5000000E 00  3.4501979E 00  3.2832408E 00  5,1014991E-03
2.5000000E 00  3.7627710E 00  3.5214867E 00  7,9472811E-04
2.5000000E 00  3,9238278E 00  3.6557350E 00  3,6684530E-04

  1            3          FUND 1 ST 2 ND
3.0000000E 00  3.4807968E 00  3.3257429E 00  3,8951203E-03
3.0000000E 00  3.8108110E 00  3.5587629E 00  6,4451305E-04
3.0000000E 00  3.9836080E 00  3.6276105E 00  3,4717436E-04

  1            3          FUND 1 ST 2 ND
3.5000000E 00  3.5055192E 00  3.3529931E 00  3,1705755E-03
```

```
DATA    PAGE  2

3.5000000E 00  3.8535174E 00  3.5874673E 00  5,4247159E-04
3.5000000E 00  4.0572187E 00  3.5507559E 00  4,0134764E-04

  1          3        FUND 1 ST 2 ND
4.0000000E 00  3.5268961E 00  3.3727236E 00  2,6871940E-03
4.0000000E 00  3.8933347E 00  3.6028673E 00  4,7919983E-04
4.0000000E 00  4.1492589E 00  3.4714303E 00  4,5483497E-04

  1          3        FUND 1 ST 2 ND
5.0000000E 00  3.5637321E 00  3.4015544E 00  2,0759147E-03
5.0000000E 00  3.9736222E 00  3.5934040E 00  4,3275715E-04
5.0000000E 00  4.3830053E 00  3.3874595E 00  4,8021588E-04

  1          3        FUND 1 ST 2 ND
6.0000000E 00  3.5956625E 00  3.4234237E 00  1,6996357E-03
6.0000000E 00  4.0638349E 00  3.5553564E 00  4,3267716E-04
6.0000000E 00  4.6303347E 00  3.6631601E 00  3,2393162E-04

  1          2        FUND 1 ST 2 ND
7.0000000E 00  3.6244602E 00  3.4409496E 00  1,4433825E-03
7.0000000E 00  4.1675513E 00  3.5162368E 00  4,3739184E-04

  1          2        FUND 1 ST 2 ND
8.0000000E 00  3.6511638E 00  3.4547122E 00  1,2588173E-03
6.0000000E 00  4.2830896E 00  3.5001658E 00  4,2720734E-04

  1          2        FUND 1 ST 2 ND
9.0000000E 00  3.6765957E 00  3.4650219E 00  1,1209179E-03
9.0000000E 00  4.4041342E 00  3.5340773E 00  3,9193020E-04

  1          2        FUND 1 ST 2 ND
1.0000000E 01  3.7012129E 00  3.4724826E 00  1,0147778E-03
1.0000000E 01  4.5195807E 00  3.6479562E 00  3,2751996E-04

  1          2        FUND 1 ST 2 ND
1.1000000E 01  3.7254500E 00  3.4776014E 00  9,3083309E-04
1.1000000E 01  4.6152358E 00  3.8675161E 00  2,3690678E-04

  1          2        FUND 1 ST 2 ND
1.2000000E 01  3.7495535E 00  3.4810791E 00  8,6269161E-04
1.2000000E 01  4.6781983E 00  4.2090951E 00  1,2724813E-04

  1          1        FUND 1 ST 2 ND
1.3000000E 01  3.7736849E 00  3.4834946E 00  8,0597303E-04

  1          1        FUND 1 ST 2 ND
1.4000000E 01  3.7979346E 00  3.4853888E 00  7,5768043E-04

  1          1        FUND 1 ST 2 ND
1.5000000E 01  3.8223424E 00  3.4872092E 00  7,1565338E-04
```

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DATA    PAGE  3


  1          1         FUND 1 ST 2 ND
1.6000000E 01  3.8469107E 00  3.4893358E 00  6.7838894E-04

  1          1         FUND 1 ST 2 ND
1.7000000E 01  3.8716151E 00  3.4919915E 00  6.4476916E-04

  1          1         FUND 1 ST 2 ND
1.8000000E 01  3.8964126E 00  3.4955367E 00  6.1398530E-04

  1          1         FUND 1 ST 2 ND
1.9000000E 01  3.9212464E 00  3.5001182E 00  5.8547207E-04

  1          1         FUND 1 ST 2 ND
2.0000000E 01  3.9460508E 00  3.5059884E 00  5.5878564E-04

  1          1         FUND 1 ST 2 ND
2.2000000E 01  3.9952857E 00  3.5217619E 00  5.0968754E-04

  1          1         FUND 1 ST 2 ND
2.4000000E 01  4.0435336E 00  3.5433432E 00  4.6502086E-04

  1          1         FUND 1 ST 2 ND
2.6000000E 01  4.0902402E 00  3.5708577E 00  4.2393653E-04

  1          1         FUND 1 ST 2 ND
2.8000000E 01  4.1349285E 00  3.6036696E 00  3.8603595E-04

  1          1         FUND 1 ST 2 ND
3.0000000E 01  4.1772286E 00  3.6409709E 00  3.5111261E-04

  1          1         FUND 1 ST 2 ND
3.5000000E 01  4.2711602E 00  3.7480045E 00  2.7625661E-04

  1          1         FUND 1 ST 2 ND
4.0000000E 01  4.3478094E 00  3.8616224E 00  2.1787531E-04

  1          1         FUND 1 ST 2 ND
4.5000000E 01  4.4086597E 00  3.9699129E 00  1.7342323E-04

  1          1         FUND 1 ST 2 ND
5.0000000E 01  4.4570596E 00  4.0673087E 00  1.3987656E-04

  1          1         FUND 1 ST 2 ND
6.0000000E 01  4.5254651E 00  4.2225180E 00  9.5084958E-05

  1          1         FUND 1 ST 2 ND
7.0000000E 01  4.5695616E 00  4.3327979E 00  6.8241050E-05

  1          1         FUND 1 ST 2 ND
```

```
DATA     PAGE  4

8.0000000E 01  4,5991852E 00  4.4111944E 00  5,1209016E-05

  1          1       FUND 1 ST 2 ND
9.0000000E 01  4,6198920E 00  4.4680420E 00  3,9813745E-05

  1          1       FUND 1 ST 2 ND
1.0000000E 02  4,6348776E 00  4.5099175E 00  3,1839386E-05

  1          1       FUND 1 ST 2 ND
1,5000000E 02  4,6708521E 00  4,6136025E 00  1,3670287E-05

  1          1       FUND 1 ST 2 ND
2,0000000E 02  4,6835724E 00  4,6510571E 00  7,5841452E-06

 -1          1       FUND
```

```
T(SEC) =    2.00 C(KM/SEC) = 3,40748

U(DC/DT) =  3.2085 U(ENER) =  3.2084

   I0 =  0,6190E 01  I1 =  0,6767E 02
   I2 =  0,3568E 01  L  = -0,4387E-05
   AL =  0,7388E-02 ALE =  0,7389E-02
 WVNO =  0,9220E 00

 M      DEPTH         D          ALPHA       BETA        RHO         MU        LAMBDA

 1      0,5000     1,0000      5,0000     2.8900     2.5000     20,8802     20,7395
 2      1,5000     1,0000      6,1000     3.5200     2.7000     33,4541     33,5588
 3      2,5000     1,0000      6,1000     3.5200     2.7000     33,4541     33,5588
 4      3,5000     1,0000      6,1000     3.5200     2.7000     33,4541     33,5588
 5      4,5000     1,0000      6,1000     3.5200     2.7000     33,4541     33,5588
 6      5,5000     1,0000      6,1000     3.5200     2.7000     33,4541     33,5588
 7      6,5000     1,0000      6,1000     3.5200     2.7000     33,4541     33,5588
 8      7,5000     1,0000      6,1000     3.5200     2.7000     33,4541     33,5588
 9      8,5000     1,0000      6,1000     3.5200     2.7000     33,4541     33,5588
10      9,5000     1,0000      6,1000     3.5200     2.7000     33,4541     33,5588
11     10,5000     1,0000      6,4000     3.7000     2.9000     39,7010     39,3820
12     11,5000     1,0000      6,4000     3.7000     2.9000     39,7010     39,3820
13     12,5000     1,0000      6,4000     3.7000     2.9000     39,7010     39,3820
14     13,5000     1,0000      6,4000     3.7000     2.9000     39,7010     39,3820
15     14,5000     1,0000      6,4000     3.7000     2.9000     39,7010     39,3820
16     15,5000     1,0000      6,4000     3.7000     2.9000     39,7010     39,3820
17     16,5000     1,0000      6,4000     3.7000     2.9000     39,7010     39,3820
18     17,5000     1,0000      6,4000     3.7000     2.9000     39,7010     39,3820
19     18,5000     1,0000      6,4000     3.7000     2.9000     39,7010     39,3820
20     19,5000     1,0000      6,4000     3.7000     2.9000     39,7010     39,3820
21     21,0000     2,0000      6,7000     3.8700     3.0000     44,9307     44,8086
22     23,0000     2,0000      6,7000     3.8700     3.0000     44,9307     44,8086
23     25,0000     2,0000      6,7000     3.8700     3.0000     44,9307     44,8086
24     27,0000     2,0000      6,7000     3.8700     3.0000     44,9307     44,8086
25     29,0000     2,0000      6,7000     3.8700     3.0000     44,9307     44,8086
26     31,0000     2,0000      6,7000     3.8700     3.0000     44,9307     44,8086
27     33,0000     2,0000      6,7000     3.8700     3.0000     44,9307     44,8086
28     35,0000     2,0000      6,7000     3.8700     3.0000     44,9307     44,8086
29     37,0000     2,0000      6,7000     3.8700     3.0000     44,9307     44,8086
30     39,0000     2,0000      6,7000     3.8700     3.0000     44,9307     44,8086
31                             8,1500     4.7000     3.4000     75,1060     75,6245
```

```
          T(SEC) =     2.00 C(KM/SEC) = 3.70605

          U(DC/DT) =  3.4704 U(ENER) =  3.4703

            I0 =  0.3894E 02  I1 =  0.5008E 03
            I2 =  0.2445E 02  L  = -0.3946E-04
            AL =  0.9982E-03 ALE =  0.9984E-03
          WVNO =  0.8477E 00

 M         DEPTH      DISP         STRESS        DG/DB         DC/DR


 0         0.       0.1000E 01   0.            0.            0.
 1         0.50     0.9427E 00  -0.4742E 01    0.5079E-01   -0.1431E-01
 2         1.50     0.6365E 00  -0.9864E 01    0.3729E-01    0.3486E-02
 3         2.50     0.3208E 00  -0.1112E 02    0.1859E-01    0.6454E-02
 4         3.50    -0.1974E-01  -0.1152E 02    0.1224E-01    0.7460E-02
 5         4.50    -0.3587E 00  -0.1102E 02    0.2018E-01    0.6200E-02
 6         5.50    -0.6700E 00  -0.9669E 01    0.4080E-01    0.3056E-02
 7         6.50    -0.9293E 00  -0.7569E 01    0.6567E-01   -0.1017E-02
 8         7.50    -0.1117E 01  -0.4883E 01    0.8939E-01   -0.4780E-02
 9         8.50    -0.1217E 01  -0.1819E 01    0.1040E 00   -0.7092E-02
10         9.50    -0.1224E 01   0.1386E 01    0.1049E 00   -0.7248E-02
11        10.50    -0.1154E 01   0.3018E 01    0.1064E 00    0.1865E-03
12        11.50    -0.1076E 01   0.3122E 01    0.9271E-01    0.2437E-03
13        12.50    -0.9965E 00   0.3219E 01    0.7961E-01    0.2986E-03
14        13.50    -0.9143E 00   0.3308E 01    0.6718E-01    0.3507E-03
15        14.50    -0.8299E 00   0.3390E 01    0.5554E-01    0.3995E-03
16        15.50    -0.7436E 00   0.3463E 01    0.4479E-01    0.4446E-03
17        16.50    -0.6555E 00   0.3529E 01    0.3504E-01    0.4854E-03
18        17.50    -0.5659E 00   0.3586E 01    0.2638E-01    0.5217E-03
19        18.50    -0.4750E 00   0.3634E 01    0.1889E-01    0.5531E-03
20        19.50    -0.3829E 00   0.3674E 01    0.1264E-01    0.5793E-03
21        21.00    -0.2636E 00   0.2692E 01    0.1345E-01    0.1329E-02
22        23.00    -0.1618E 00   0.1775E 01    0.5066E-02    0.5006E-03
23        25.00    -0.9926E-01   0.1089E 01    0.1907E-02    0.1885E-03
24        27.00    -0.6088E-01   0.6690E 00    0.7176E-03    0.7101E-04
25        29.00    -0.3730E-01   0.4113E 00    0.2696E-03    0.2675E-04
26        31.00    -0.2279E-01   0.2535E 00    0.1008E-03    0.1007E-04
27        33.00    -0.1383E-01   0.1573E 00    0.3726E-04    0.3795E-05
28        35.00    -0.8221E-02   0.9946E-01    0.1335E-04    0.1432E-05
29        37.00    -0.4615E-02   0.6575E-01    0.4387E-05    0.5473E-06
30        39.00    -0.2132E-02   0.4803E-01    0.1143E-05    0.2268E-06
31        40.00    -0.1116E-02   0.4371E-01    0.1948E-06    0.7389E-07


 0         0.       0.1000E 01  -0.2034E-04    0.            0.
```

```
               T(SEC) =     2.00 C(KM/SEC) = 3.85740

               U(DC/DT) =  3.5638 U(ENER) =  3.5638

                 I0 =  0.6776E 02  I1 =  0.9315E 03
                 I2 =  0.5091E 02  L  = -0.2338E-04
                 AL =  0.5367E-03 ALE =  0.5368E-03
               WVNO =  0.8144E 00


 M         DEPTH       DISP         STRESS         DG/DB        DC/DR


 0         0.      0.1000E 01   0.             0.           0.
 1         0.50    0.9359E 00  -0.5296E 01     0.2890E-01  -0.9306E-02
 2         1.50    0.5920E 00  -0.1141E 02     0.2088E-01   0.2637E-02
 3         2.50    0.2193E 00  -0.1324E 02     0.1158E-01   0.5685E-02
 4         3.50   -0.1823E 00  -0.1333E 02     0.1113E-01   0.5835E-02
 5         4.50   -0.5598E 00  -0.1165E 02     0.1974E-01   0.3010E-02
 6         5.50   -0.8636E 00  -0.8446E 01     0.3304E-01  -0.1349E-02
 7         6.50   -0.1054E 01  -0.4124E 01     0.4425E-01  -0.5020E-02
 8         7.50   -0.1105E 01   0.7408E 00     0.4764E-01  -0.6133E-02
 9         8.50   -0.1010E 01   0.5508E 01     0.4150E-01  -0.4119E-02
10         9.50   -0.7827E 00   0.9550E 01     0.2895E-01  -0.5789E-05
11        10.50   -0.4835E 00   0.1176E 02     0.1652E-01   0.3145E-02
12        11.50   -0.1762E 00   0.1252E 02     0.8371E-02   0.4134E-02
13        12.50    0.1412E 00   0.1256E 02     0.7925E-02   0.4188E-02
14        13.50    0.4505E 00   0.1188E 02     0.1528E-01   0.3295E-02
15        14.50    0.7340E 00   0.1052E 02     0.2877E-01   0.1657E-02
16        15.50    0.9754E 00   0.8554E 01     0.4535E-01  -0.3559E-03
17        16.50    0.1161E 01   0.6098E 01     0.6127E-01  -0.2288E-02
18        17.50    0.1280E 01   0.3292E 01     0.7292E-01  -0.3703E-02
19        18.50    0.1325E 01   0.2981E 00     0.7768E-01  -0.4281E-02
20        19.50    0.1294E 01  -0.2713E 01     0.7446E-01  -0.3890E-02
21        21.00    0.1161E 01  -0.3938E 01     0.1311E 00   0.1265E-02
22        23.00    0.9951E 00  -0.3521E 01     0.9645E-01   0.9761E-03
23        25.00    0.8465E 00  -0.3164E 01     0.6989E-01   0.7549E-03
24        27.00    0.7126E 00  -0.2863E 01     0.4962E-01   0.5860E-03
25        29.00    0.5910E 00  -0.2611E 01     0.3423E-01   0.4578E-03
26        31.00    0.4795E 00  -0.2403E 01     0.2265E-01   0.3613E-03
27        33.00    0.3764E 00  -0.2238E 01     0.1408E-01   0.2899E-03
28        35.00    0.2797E 00  -0.2111E 01     0.7926E-02   0.2386E-03
29        37.00    0.1879E 00  -0.2020E 01     0.3759E-02   0.2039E-03
30        39.00    0.9934E-01  -0.1965E 01     0.1289E-02   0.1833E-03
31        40.00    0.5579E-01  -0.1950E 01     0.2936E-03   0.9988E-04


 0         0.      0.1000E 01   0.1078E-04     0.           0.
```

IV. SUBROUTINE EIGEN

PROGRAMMER : R. B. HERRMANN / MAY 1973

PURPOSE:

This subroutine reads the eigenfunctions from the tape on Unit ITAPE and stores the eigenfunctions for a set of up to 15 focal depths labeled COMMON. This subroutine requires the subroutines MODEL prior to use and it uses the subroutine NONPRP for linear interpolation to obtain the eigenfunctions at various focal depths.

INPUT/OUTPUT

The eigenfunction tape is loaded onto the tape Unit ITAPE. Input and output parameters are stored in COMMON.

```
      SUBROUTINE EIGEN(ITAPE,TT,T)
      COMMON/ONE/AY(100,4),CY(100,4),N,DEPTH(100),D(100)
      COMMON/TWO/LAYER(100),A(100),B(100),RHO(100),XMU(100),XLAMB(100)
      COMMON/THREE/AMODE(10),WVNO(10),ELIP(10),ABSORP(10),DENOM(10)
      COMMON/FORE/NUMEPT,AAA(15),PYY1(15,10),PYY2(15,10),PYY3(15,10),
     1PYY4(15,10),IFUNC
      DIMENSION PYR(4)
 2000 READ(ITAPE,1) IFUNC,MODE
    1 FORMAT(1H ,2(I4,6X))
      MAXY = 2 * IFUNC
      IF(IFUNC.LT.0) TT = - TT
      IF(IFUNC.LT.0) GO TO 9999
      DO 2999 IM = 1,MODE
      READ(ITAPE,5) AMODE(IM),T,C,U,SUMIO,WVNO(IM),AL,ELIP(IM),
     1ABSORP(IM)
    5 FORMAT(1H ,A4,3F10.5,5E11.4)
C     THE SPECTRA ARE COMPUTED AT A DISTANCE OF 1000 KILOMETERS
      DENOM(IM) = SQRT(WVNO(IM))*0.3989423*1.0E+20
      DENOM(IM) = DENOM(IM)/AL
      DENOM(IM) = DENOM(IM) * 6.2831853 * 31.622776
      DO 2100 IN=1,N
 2100 READ(ITAPE,6)AY(IN,1),AY(IN,2),AY(IN,3),AY(IN,4),DCDA,DCDB,DCDR
    6 FORMAT(1H ,7(4X,E11.4))
      IF(TT.NE.T) GO TO 2999
      WVNO2 = WVNO(IM)**2
      GO TO (2400,2200),IFUNC
C     RAYLEIGH WAVES
C     INITIALLY AY(IN,1) = UR, AY(IN,2) = UZ, AY(IN,3) = TZ,
C                  AY(IN,4) = TR
C     NOW AY(IN, 1) = UR, AY(IN,2) = UZ, AY(IN,3) = DURDZ,
C          AY(IN,4) = DUZDZ
C     AND CY(IN,1) = DURDZ, CY(IN,2) = DUZDZ, CY(IN,3) = D2URDZ2,
C          CY(IN,4) = D2UZDZ2.
 2200 DO 2300 IN = 1,N
       H = RHO(IN)*C*C*WVNO2
      XL2M = XLAMB(IN) + 2.*XMU(IN)
      CY(IN,1) = -WVNO(IM)*AY(IN,2) + AY(IN,4)/XMU(IN)
      CY(IN,2) = (WVNO(IM)*XLAMB(IN)*AY(IN,1)+AY(IN,3))/XL2M
      CY(IN,3) = (-H/XMU(IN) + WVNO2*(3.*XL2M-2.*XMU(IN))/XL2M)*AY(IN,1)
     1 -WVNO(IM)*(1.+XLAMB(IN)/XMU(IN))*AY(IN,3)/XL2M
      CY(IN,4) = (-(H+WVNO2*XLAMB(IN))*AY(IN,2)+WVNO(IM)*(1.+XLAMB(IN)/
     1 XMU(IN))*AY(IN,4))/XL2M
      AY(IN,3) = CY(IN,1)
      AY(IN,4) = CY(IN,2)
 2300 CONTINUE
       GO TO 2501
C     LOVE WAVES
C     INITIALLY AY(IN,1) = UT, AY(IN,2) = TT
C     NOW AY(IN,1) = UT, AY(IN,2) = DUTDZ
```

```
C       AND CY(IN,1) = DUTDZ, CY(IN,2) = D2UTDZ2
 2400 DO 2500 IN=1,N
      CY(IN,1) = AY(IN,2)/XMU(IN)
      CY(IN,2) = WVNC2*(XMU(IN)- C*C*RHO(IN))*AY(IN,1)/XMU(IN)
      AY(IN,2) = CY(IN,1)
 2500 CONTINUE
 2501 CONTINUE
      DO 2555 ID = 1,NUMDPT
      AA = AAA(ID)
      NPER = ID
      IF(AA.LT.0) AA = 0.0
      CALL NONPRP(AA,NAXY,PYR)
      GO TO(2551,2552),IFUNC
 2551 CONTINUE
      PYY1(NPER,IM) = WVNC(IM)*PYR(1)/DENOM(IM)
      PYY2(NPER,IM) = PYR(2)/DENOM(IM)
      GO TO 2555
 2552 CONTINUE
      PYY1(NPER,IM) = WVNC(IM)*PYR(1)/DENOM(IM)
      PYY2(NPER,IM) = WVNC(IM)*PYR(2) / DENOM(IM)
      PYY3(NPER,IM) = PYR(3)/DENOM(IM)
      PYY4(NPER,IM) = PYR(4)/DENOM(IM)
 2555 CONTINUE
 2999 CONTINUE
      IF(IT.NE.I) GO TO 2000
 9999 CONTINUE
      RETURN
      END
```

V. SUBROUTINE MODEL

PROGRAMMER : R. B. HERRMANN / MAY 1973

PURPOSE:

This subroutine reads the earth model which is stored on the eigenfunction tape immediately in front of the eigenfunctions. The earth model is placed in labeled COMMON.

INPUT/OUTPUT

The earth model is read off the eigenfunction tape which is stored on tape Unit ITAPE.

```
      SUBROUTINE MODEL(ITAPE)
      COMMON/ONE/AY(100,4),CY(100,4),N,DEPTH(100),D(100)
      COMMON/TWO/LAYER(100),A(100),B(100),RHO(100),XMU(100),XLAMB(100)
      READ(ITAPE,1) N
    1 FORMAT(1H ,2(I4,6X))
C     READ IN MODEL
      DO 100 I=1,N
  100 READ(ITAPE,2) LAYER(I),DEPTH(I),D(I),A(I),B(I),RHO(I),XMU(I),
     1XLAMB(I)
    2 FORMAT(1H ,I4,5X,7F10.4)
      RETURN
      END
```

VI. SUBROUTINE NONPRP

PROGRAMMER : R. B. HERRMANN / MAY 1972

PURPOSE:

This subroutine is called by the subroutine EIGEN in order to obtain the eigenfunctions at a specified depth by linear interpolation of known eigenfunctions at depths above and below the desired value.

INPUT/OUTPUT

All values needed are in labeled COMMON or are in the subroutine argument.

```
      SUBROUTINE NONPRP(A,MAXY,PYR)
      COMMON/ONE/AY(100,4),CY(100,4),N,DEPTH(100),D(100)
      DIMENSION PYR(4)
      DO 100 I=1,N
      IFOCA = I
      D1=DEPTH(I)-0.5*D(I)
      D2=DEPTH(I)+0.5*D(I)
      IF(A.GE.D1.AND.A.LE.D2) GO TO 101
  100 CONTINUE
  101 DO 110 JJ = 1,MAXY
      AA = AY(IFOCA,JJ)
      CC = CY(IFOCA,JJ)
      AA = AA - CC * DEPTH(IFCCA)
  110 PYR(JJ) = AA + CC*A
      RETURN
      END
```

VII. SUBROUTINE EXCIT

PROGRAMMER: R. B. HERRMANN / MAY 1972

PURPOSE:

This subroutine calculates the theoretical surface-wave amplitude spectra at a given period from the eigenfunctions of that period for a given set of source orientation factors. The source can be either a dislocation source specified by the strike, dip, and slip, or it can be an explosive source.

INPUT/OUTPUT

All information needed for the functioning of the subroutine is taken from the argument of the subroutine.

```
      SUBROUTINE EXCIT(FAC1,FAC2,FAC3,FAC4,FAC5,FAC6,FAC7,FAC8,FAC9,
     1 PY1,PY2,PY3,PY4,IFUNC,SAZ,CAZ,AMP,IEQEX)
      GO TO (100,200),IEQEX
  100 CONTINUE
      S2AZ = 2.*SAZ*CAZ
      C2AZ = CAZ*CAZ - SAZ*SAZ
      GO TO (1,2),IFUNC
    1 CONTINUE
      XRL = PY1 *((FAC1-FAC4)*C2AZ + (FAC2+FAC3)*S2AZ )
      XIM = - PY2 *((FAC5+FAC8)*CAZ + (FAC6-FAC7)*SAZ)
      AMP = SQRT(XRL * XRL + XIM * XIM )
      RETURN
    2 CONTINUE
      XRL = PY1*((FAC3+FAC2)*C2AZ + (FAC4-FAC1)*S2AZ) + FAC9 * PY4
     1 + 0.5*FAC9*PY1
      XIM = (PY3+PY2)*((FAC7-FAC6)*CAZ+(FAC8+FAC5)*SAZ)
      AMP = SQRT(XRL * XRL + XIM * XIM )
      RETURN
  200 CONTINUE
      GO TO (10,20), IFUNC
   10 AMP = 0
      RETURN
   20 AMP = PY4 - PY1
      RETURN
      END
```

VIII. SRFWVPLT

PROGRAMMER: R. B. HERRMANN / Jun 72

PURPOSE:

This program calculates the theoretical surface-wave amplitude spectra for each individual mode for a choice of two source spectrums of the dislocation source. The two source spectrums are $S(\omega) = 1$, which corresponds to the delta-response of the system, and $S(\omega) = 1/i\omega$, which corresponds to the response of the system to a step dislocation. Depending on the surface-wave eigenfunction tape used, the program will plot the Love or Rayleigh wave amplitude spectra. The theoretical amplitude spectra are generated at a reference distance of 1000 km or 9° from the source. To specify the double-couple source, the orientations of the pressure and tension axes are used. As an aid in identifying the plots, the focal mechanism specified by the pressure and tension axes is plotted. The spectra are plotted on a 3 x 3 log-log scale, of approximately 2.5 inches per cycle.

INPUT/OUTPUT

The specification of the focal depth, focal mechanism orientation, and plotting options is taken from card on FILE 60. Printer output is on FILE 61. The eigenfunctions are read from the tape on FILE 01 (this is the tape written by REIGEN or LEIGEN using exactly the same order of input as shown in the sample input for these programs). FILE 10 is used for the CALCOMP plotting.

THEORY

The theory of this program is taken from Levshin and Yanson (1971). The formulation in terms of pressure and tension axes is used.

PROGRAM DESCRIPTION

PROGRAM SRFWVPLT : This program reads in the data and controls the plotting.

SUBROUTINE NONPRP: This subroutine performs a linear interpolation on the eigenfunctions to arrive at the values for a specified depth.

SUBROUTINE ALOGAXES: This subroutine establishes the 3 x 3 cycle log-log scale for the plot.

SUBROUTINE LIND: This is a subroutine for plotting the amplitude spectra for the various modes. Any amplitude information which is less than three orders of magnitude of the largest amplitude plotted, is not plotted. This allows a neater plot by eliminating plotting below the y-axis or along the y-axis.

SUBROUTINE DIPDD: This is a subroutine used by the subroutine NODPL. It takes a vector and redefines the vector in terms of two angles.

SUBROUTINE NODPL; This subroutine plots a focal mechanism on an equal area projection for a visual representation of the focal mechanism specified by the given orientations of the pressure and tension axes.

Card Sequence	Column	Name	Format	Explanation
A.	1-10	SIZE	F10.5	CALCOMP reduction factor between 0.0 and 1.0 .
B.	1-10	DEL	F10.5	Azimuth of the T axis measured from North
	11-20	BET	F10.5	The angle the T axis makes with the downward vertical axis
	21-30	ALP	F10.5	Azumuth of the P axis measured clockwise from North
	31-40	GAM	F10.5	The angle the P axis makes with the downward vertical axis
C.	1-10	AA	F10.5	.LT. - 10 read in new mechanism parameters .GT. - 10 and .LT. 0 end program .GT. 0 focal depth in km

Card Sequence	Column	Name	Format	Explanation
D.	1-10	PHI	F10.5	.LT.0 read new AA .GE.0 receiver azimuth
	11-20	RAD	F10.5	.GT.0 plot radial spectra of Rayleigh wave .LT.0 no radial plot
	21-30	VER	F10.5	.GT.0 plot vertical spectra of Rayleigh wave .LE.0 no vertical plot
	31-40	SRC	F10.5	.GT.0 spectra for $1/i\omega$ source-step response .LE.0 spectra for 1.0 source-impulse response (when using the LOVE tape VER and RAD have no meaning)

After performing the operations specified by the PHI, VER, RAD, SRC card, the program will read another PHI, VER, RAD, SRC card.

The example given on the following page is for the case DEL = 45 , BET = 90. , ALP = 135 , and GAM = 90. This corresponds to a left lateral vertical strike slip fault striking north. Since the plots given here are only of amplitude spectra and not the phase spectra, the plots will also correspond to a right lateral strike slip fault striking north. Basically, an interchange of the P and T axes will not affect the amplitude spectra.

```
C      PROGRAM SRFWVPLT
C      THIS PROGRAM ACCEPTS ANY ARBITRARY ORIENTATION FOR A DOUBLE COUPLE
C      FOCAL MECHANISM.  THE INPUT IS THE ORIENTATION OF THE PRESSURE AND
C      TENSION AXES.  THE PROGRAM PLOTS THEORETICAL LOVE OR RAYL WAVE
C      SPECTRA AT A DISTANCE OF 9 DEGREES OR 1000 KILOMETERS ON A
C      STANDARD 3 X 3 LOG-LOG SCALE.  THERE IS AN OPTION FOR RADIAL AND
C      OR VERTICAL DISPLACEMENT SPECTRUM FOR RAYLEIGH WAVES.  USING THE
C      EIGENFUNCTION TAPE FOR LOVE WAVES WILL GIVE THE LOVE WAVE SPECTRA.
       COMMON/ONE/AY(100,4),CY(100,4),N,DEPTH(100),D(100),
     1        LAYER(100),A(100),B(100),RHO(100),XMU(100),XLAMB(100),
     1 IBUF(1026), ELIP
       DIMENSION ANODE(10),WVNC(10),ELIP(60,10),ABSORP(10),DENOM(10)
       DIMENSION LABEL(3),LABLE(4),XX(60),YY1(60,10),NPERM(10),YY2(60,10)
       DIMENSION LABE(7),LAB(3)
       DIMENSION X(62),Y(62),PYR(4),PYI(4)
       DIMENSION PYY1(60,10),PYY2(60,10),PYY3(60,10),PYY4(60,10)
       DIMENSION TT(60)
       CHARACTER LAB,LABE,LABEL,LABLE
       CALL PLOTS(IBUF,1026,10)
       CALL PLOT(0.0,-11.0,-3)
       CALL PLOT(0.0,2.0,-3)
       READ(60,4) SIZE
C      SIZE = REDUCTION FACTOR USED IN CALL FACTOR OF CALCOMP LIBRARY
C      IF SIZE = 1.0 PLOT IS 3 CYCLE LOG-LOG , 2.5 INCHES PER CYCLE
C      HERE SIZE MUST BE LE. 1.0
       IF(SIZE.LE.0.OR.SIZE.GT.1.0) SIZE = 1.0
       CALL FACTOR(SIZE)
       LAB(1) = 6H UZ
       LAB(2) = 6H UR
       LAB(3) = 6H UT
       LABE(1) = 6HDELT
       LABE(2) = 6H  BETA
       LABE(3) = 6H    AL
       LABE(4) = 6HPH
       LABE(5) = 6HGAM
       LABEL(1) = 6HPERIOD
       LABEL(2) = 6H (SEC)
       LABLE(1) = 6H
       LABLE(2) = 6HAMP (C
       LABLE(3) = 6HM-SEC)
     1 FORMAT(1H ,2(I4,6X))
     2 FORMAT(1H ,I4,5X,7F10.4)
     4 FORMAT(8F10.5)
     5 FORMAT(1H ,A4,3F10.5,5E11.4)
     6 FORMAT(1H ,7(4X,E11.4))
       DEGRAD = 0.017452329
  4321 READ(60,4) DEL,BET,ALP,GAM
C      DEL IS THE AZIMUTH OF THE T AXIS
C      BET IS THE ANGLE THE T AXIS MAKES WITH THE DOWNWARD VERTICAL
C      ALP IS THE AXIMUTH OF THE P AXIS
```

```
C      GAM IS THE ANGLE THE P AXIS MAKES WITH THE DOWNWARD VERTICAL
       WRITE(61,7) DEL,BET,ALP,GAM
     7 FORMAT(1H0, 4F10.3)
       CALL SYMBOL(1.35,5.5,0.14,1HT,0.0,1)
       CALL SYMBOL(3.59,5.5,0.14,1HP,0.0,1)
       CALL SYMBOL(0.56,5.0,0.14,LABE,0.0,28)
       CALL NUMBER(0.56,4.7,0.14,DEL,0.0,-1)
       CALL NUMBER(1.68,4.7,0.14,BET,0.0,-1)
       CALL NUMBER(2.80,4.7,0.14,ALP,0.0,-1)
       CALL NUMBER(3.92,4.7,0.14,GAM,0.0,-1)
       DEL = DEGRAD * DEL
       BET = DEGRAD * BET
       ALP = DEGRAD * ALP
       GAM = DEGRAD * GAM
       FACT1 = (COS(BET)**2) - (COS(GAM)**2)
       FACT2 = (SIN(BET))**2
       FACT3 = (SIN(GAM))**2
       FACT4 = SIN(2.*BET)
       FACT5 = SIN(2.*GAM)
       CALL NODPL(DEL,BET,ALP,GAM)
       CALL PLOT(9.0,0.0,-3)
  2001 READ(60,4) AA
       IF(AA.LE.-10.) GO TO 4321
       IF(AA.LT.0) GO TO 9998
       DO 37 I=1,50
       DO 37 J=1,10
       PYY1(I,J) = 0
       PYY2(I,J) = 0
       PYY3(I,J) = 0
    37 PYY4(I,J) = 0
       READ(01,1)N
C      READ IN MODEL
       DO 100 I=1,N
   100 READ(01,2)  LAYER(I),DEPTH(I),D(I),A(I),B(I),RHO(I),XMU(I),XLAMB(I
      1)
       NPER = 0
       DO 1313 I = 1,10
  1313 NPERM(I) = 0
  2000 READ(01,1) IFUNC, MODE
       MAXY = 2 * IFUNC
       IF(IFUNC.LT.0) GO TO 9999
       IF(NPER.EQ.0) MODEMX = MODE
       IF(NPER.EQ.0) IFUN = IFUNC
       DO 1314 IZ = 1,MODE
  1314 NPERM(IZ) = NPERM(IZ) + 1
       NPER = NPER + 1
       DO 2999 IM = 1,MODE
       READ(01,5) AMODE(IM),T,C,U,SUMI0,WVNO(IM),AL,ELIP(NPER,IM),ABSORP(
      1IM)
       XX(NPER) = T
```

```
      TT(NPER) = T
C     THE SPECTRA ARE COMPUTED AT A DISTANCE OF 1000 KILOMETERS
      DENOM(IM) = SQRT (WVNO(IM))*0.3989423*1.0E+20
      DENOM(IM) = DENOM(IM)/AL
      DENOM(IM) = DENOM(IM) * 6.2831853 * 31.622776
      DO 2100 IN=1,N
 2100 READ(01,6) AY(IN,1),AY(IN,2),AY(IN,3),AY(IN,4),DCDA,DCDB,DCDR
      WVNO2 = WVNO(IM)**2
      GO TO (2400,2200),IFUNC
C     RAYLEIGH WAVES
C     INITIALLY AY(IN,1) = UR, AY(IN,2) = UZ, AY(IN,3) = TZ
C               AY(IN,4) = TR
C     NOW AY(IN,1) = UR, AY(IN,2) = UZ, AY(IN,3) = DURDZ,
C          AY(IN,4) = DUZDZ
C     AND CY(IN,1) = DURDZ, CY(IN,2) = DUZDZ, CY(IN,3) = D2URDZ2,
C          CY(IN,4) = D2UZDZ2.
 2200 DO 2300 IN = 1,N
      H = RHO(IN)*C*C*WVNO2
      XL2M = XLAMB(IN) + 2.*XMU(IN)
      CY(IN,1) = -WVNO(IM)*AY(IN,2) + AY(IN,4)/XMU(IN)
      CY(IN,2) = (WVNO(IM)*XLAMB(IN)*AY(IN,1)+AY(IN,3))/XL2M
      CY(IN,3) = (-H/XMU(IN) + WVNO2*(3.*XL2M-2.*XMU(IN))/XL2M)*AY(IN,1)
     1 -WVNO(IM)*(1.+XLAMB(IN)/XMU(IN))*AY(IN,3)/XL2M
      CY(IN,4) = (-(H+WVNO2*XLAMB(IN))*AY(IN,2)+WVNO(IM)*(1.+XLAMB(IN)/
     1 XMU(IN))*AY(IN,4))/XL2M
      AY(IN,3) = CY(IN,1)
      AY(IN,4) = CY(IN,2)
 2300 CONTINUE
      GO TO 2501
C     LOVE WAVES
C     INITIALLY AY(IN,1) = UT, AY(IN,2) = TT
C     NOW AY(IN,1) = UT, AY(IN,2) = DUTDZ
C     AND CY(IN,1) = DUTDZ, CY(IN,2) = D2UTDZ2
 2400 DO 2500 IN=1,N
      CY(IN,1) = AY(IN,2)/XMU(IN)
      CY(IN,2) = WVNO2*(XMU(IN)- C*C*RHO(IN))*AY(IN,1)/XMU(IN)
      AY(IN,2) = CY(IN,1)
 2500 CONTINUE
 2501 CONTINUE
      CALL NONPRP(AA,MAXY,PYR)
      GO TO(2551,2552),IFUNC
 2551 PYY1(NPER,IM) = WVNO(IM)*PYR(1)/DENOM(IM)
      PYY2(NPER,IM) = PYR(2) / DENOM(IM)
      GO TO 2555
 2552 CONTINUE
      PYY1(NPER,IM) = WVNO(IM) * PYR(1) / DENOM(IM)
      PYY2(NPER,IM) = WVNO(IM) * PYR(2) / DENOM(IM)
      PYY3(NPER,IM) = PYR(3)/DENOM(IM)
      PYY4(NPER,IM) = PYR(4)/DENOM(IM)
 2555 CONTINUE
```

```
 2999 CONTINUE
      GO TO 2000
 9999 CONTINUE
      REWIND 1
      DO 3160 I = 1,NPER
 3160 XX(I) = ALOG10(XX(I))
      XMAX = XX(NPER)
      LMAX = XMAX
      IF(XMAX - LMAX) 115,115,116
  116 LMAX = LMAX + 1
  115 CONTINUE
      LMIN = LMAX - 3
      X1 = LMIN
 9997 CONTINUE
      READ(60,4) PHI,RAD,VER,SRC
      IF(PHI.LT.0) GO TO 2001
C     PHI = AZIMUTH AT WHICH SPECTRA IS DESIRED
C     RAD -- GT 0 PLOT RADIAL SPECTRA OF RAYLEIGH WAVE
C         -- LE 0 NO RADIAL SPECTRA PLOT
C     VER -- GT 0 PLOT VERTICAL SPECTRA OF RAYLEIGH WAVE
C         -- LE 0 NO VERTICAL SPECTRA PLOT
C     (WHEN USING LOVE TAPE, RAD AND VER HAVE NO MEANING)
C     SRC -- GT 0  SPECTRA MULTIPLIED BY SOURCE SPECTRUM S(OMEGA) =
C                  1.0/OMEGA
C                  E.G. STEP RESPONSE OF SYSTEM   SEISMIC MOMENT = 1.0
C                  DYNE-CM
C         -- LE 0  SPECTRA MULTIPLIED BY SOURCE SPECTRUM S(OMEGA) = 1.0
C                  E.G. IMPULSE RESPONSE OF SYSTEM
      PHIR = DEGRAD * PHI
      SINA = SIN(ALP - PHIR)
      COSA = COS(ALP - PHIR)
      SIND = SIN(DEL - PHIR)
      COSD = COS(DEL - PHIR)
      SINDP = SIN(2.*(DEL-PHIR))
      SINAP = SIN(2.*(ALP-PHIR))
      IFUNC = IFUN
      YMAXR = 1.0E-38
      YMAXV = 1.0E-38
      DO 9995 J=1,MODEMX
      NP = NPERM(J)
      DO 9995 I=1,NP
      PYR(1) = PYY1(I,J)
      PYR(2) = PYY2(I,J)
      PYR(3) = PYY3(I,J)
      PYR(4) = PYY4(I,J)
      GO TO (2710,2720),IFUNC
 2710 CONTINUE
      XREAL = 0.5 * PYR(1) *(FACT2*SINDP - FACT3*SINAP)
      XIMAG = -0.5*PYR(2)*(FACT4*SIND - FACT5*SINA)
      GO TO 2730
```

```
 2720 CONTINUE
      XREAL = FACT1*PYR(4) -PYR(1)*(FACT2*COSD*COSD - FACT3*COSA*COSA)
      XIMAG = 0.5*(PYR(2)+PYR(3))*(FACT4*COSD-FACT5*COSA)
 2730 CONTINUE
      XREAL = 1.0E 25 * XREAL
      XIMAG = 1.0E 25 * XIMAG
      DUM = SQRT(XREAL**2 + XIMAG**2)
      DUM = 1.0E-25 * DUM
      IF(SRC.GT.0) DUM = DUM * TT(I) / 6.2831853
      IF(DUM.EQ.0) DUM = 1.0E-38
      IF(IFUNC.EQ.1) GO TO 9988
      IF(RAD.LE.0) GO TO 9989
      YY2(I,J) = DUM * ABS(ELIP(I,J))
      IF(YY2(I,J) .GT.YMAXR) YMAXR = YY2(I,J)
      YY2(I,J) = ALOG10(YY2(I,J))
 9989 IF(VER.LE.0) GO TO 9995
 9988 YY1(I,J) = DUM
      IF(YY1(I,J) .GT.YMAXV) YMAXV = YY1(I,J)
      YY1(I,J) = ALOG10(YY1(I,J))
 9995 CONTINUE
      YMAXV = ALOG10(YMAXV)
      YMAXR = ALOG10(YMAXR)
      DO 9996 IC = 1,IFUNC
      IF(IFUNC.EQ.1) GO TO 9993
      IF(IC.EQ.1.AND.VER.LE.0) GO TO 9996
      IF(IC.EQ.2.AND.RAD.LE.0) GO TO 9996
 9993 CONTINUE
      IF(IC.EQ.1) YMAX = YMAXV
      IF(IC.EQ.2) YMAX = YMAXR
      IF(IFUNC.EQ.1) LABLE(1) = LAB(3)
      IF(IFUNC.EQ.2.AND.IC.EQ.1) LABLE(1) = LAB(1)
      IF(IFUNC.EQ.2.AND.IC.EQ.2) LABLE(1) = LAB(2)
      XAXLEN = 7.535
      YAXLEN = 7.507
      DELTAX = 3./7.535
      DELTAY = 3./7.507
      SCP = 0.03 * YAXLEN
      LMAX = YMAX
      IF(YMAX - LMAX) 110,110,111
  111 LMAX = LMAX + 1
  110 CONTINUE
      LMIN = LMAX - 3
      Y1 = LMIN
      CALL ALOGAXES(XAXLEN,YAXLEN,3,3,LABEL,LABLE,12,18,X1,Y1,DELTAX,
     1 DELTAY)
      DO 3171 IM = 1,MODEMX
      NP = NPERM(IM)
      DO 3172 IZ = 1,NP
      X(IZ) = XX(IZ)
      GO TO (3168,3169),IC
```

```
 3168 Y(IZ) = YY1(IZ,IM)
      GO TO 3170
 3169 Y(IZ) = YY2(IZ,IM)
 3170 CONTINUE
      IF(Y(IZ).LT.Y1) Y(IZ) = Y1
 3172 CONTINUE
      X(NP+1)=X1
      X(NP+2) = DELTAX
      Y(NP+1) = Y1
      Y(NP+2) = DELTAY
 3181 CALL LIND(X,Y,NP,IM)
 3171 CONTINUE
      CALL SYMBOL( 2.6    ,YAXLEN+0.1,SCP,4HH = ,0.0,4)
      CALL NUMBER( 4.1    ,YAXLEN+0.1,SCP,AA,0.0,1)
      CALL SYMBOL( 2.1,YAXLEN + 0.6,SCP,4HPHI=,0.0,4)
      CALL NUMBER(4.35,YAXLEN+0.6,SCP,PHI,0.0,1)
      MMM = MODEMX - 1
      DO 3300 IM= 1,MMM
      FPN = IM
      IT = IM + 1
      XQ = XAXLEN - 1.
      YY = YAXLEN - 1. - (FPN - 1.) * 0.3
      CALL SYMBOL(XQ,YY+0.1,0.2,IT,0.0,-1)
      CALL SYMBOL(XQ+0.3,YY,0.2,1H=,0.0,1)
 3300 CALL NUMBER(XQ+0.6,YY,0.2,FPN,0.0,-1)
      CALL PLOT(11.0,0.0,-3)
      CALL FACTOR(1.0)
      CALL PLOT(0.0,-11.0,-3)
      CALL PLOT(0.0,2.0,-3)
      CALL FACTOR(SIZE)
 9996 CONTINUE
      GO TO 9997
 9998 CONTINUE
      CALL PLOT(10.5,0.0,999)
      STOP
      END

      SUBROUTINE NONPRP(A,MAXY,PYR)
      COMMON/ONE/AY(100,4),CY(100,4),N,DEPTH(100),D(100),
     1      LAYER(100),AJ(100),B(100),RHO(100),XMU(100),XLAMB(100),
     1 IBUF(1026), ELIP(60,10)
      DIMENSION PYR(4)
      DO 100 I=1,N
      IFOCA = I
      D1 = DEPTH(I) - 0.5*D(I)
                                          D2 = DEPTH(I) + 0.5*D(I)
      IF(A.GE.D1.AND.A.LE.D2) GO TO 101
  100 CONTINUE
  101 DO 110 JJ = 1,MAXY
      AA = AY(IFOCA,JJ)
```

```
      CC = CY(IFCCA,JJ)
      AA = AA - CC * DEPTH(IFCCA)
  110 PYR(JJ) = AA + CC*A
      RETURN
      END

      SUBROUTINE ALOGAXES(XAXLEN,YAXLEN,NOCX,NOCY,TTLX,TTLY,MTX,MTY,X1,
     1Y1,DELTAX,DELTAY)
      CHARACTER TTLX, TTLY
      DIMENSION TTLX(10),TTLY(10)
      SLT=.02*YAXLEN
                     SST=.01*YAXLEN
                                    SP=-.06*YAXLEN
                                                   SS=.035*YAXLEN
      SSP=SP+SS-.06
                    TTLP=-.11*YAXLEN-.1
                                        STTL=.035*YAXLEN
      XNUM = 1
      YL = Y1
      YU = Y1+ NOCY
      IF(ABS(YL) .GE. 10. .OR. ABS(YU) .GE. 10.) XNUM = XNUM + 1
      IF(ABS(YL).GE.100. .OR. ABS(YU).GE.100.) XNUM = XNUM + 1.
      IF( Y1 .LT. 0 ) XNUM = XNUM + 1.0
      CALL PLOT(-SLT,0.0,2)
                            CALL PLOT(0.0,-SLT,3)
                                                  CALL PLOT(0.0,0.0,2)
      XPO=X1
             YPO=Y1
                    IF(NOCX.EQ.0)GO TO 4
      ANOCX=NOCX
                 FACTX=XAXLEN/ANOCX
                                    CALL SYMBOL(-.6*SS,SP,SS,2H10,0.0,2)
      CALL NUMBER(999., SSP,0.5*SS,X1,0.0,-1)
                                           CALL PLOT(0.0,0.0,3)
      DO 3 J=1,NOCX
                    DO 2 I=1,10
                                X=I
                                    X=ALOG10(X)*FACTX+(J-1)*FACTX
      IF(I.EQ.1)GO TO 2
      CALL PLOT(X,0.0,2)
                         CALL PLOT(X,-SST,2)
    2 CALL PLOT(X,0.0,3)
                         CALL PLOT(X,-SLT,2)
      CALL SYMBOL(X-.6*SS,SP,SS,2H10,0.0,2)
      XPO=XPO+1.0
                  CALL NUMBER(999., SSP,0.5*SS,XPO,0.0,-1)
    3 CALL PLOT(X,0.0,3)
                         XTL=MTX
                                 XTL=(XAXLEN-XTL*STTL)/2.0
      CALL SYMBOL(XTL,TTLP,STTL,TTLX,0.0,MTX)
```

```
                                                    GO TO 6
   4 CALL AXIS(0.0,0.0,TTLX,-MTX,XAXLEN,0.0,X1,DELTAX)
   6 CALL PLOT(0.0,0.0,3)
                                 IF(NOCY.EQ.0)GO TO 10
                                                       ANOCY=NOCY
     SP = SP - (XNUM - 1.5) * 0.5 * SS
     TTLP = TTLP - (XNUM -1)*0.5*SS
     FACTY=YAXLEN/ANOCY
                               CALL SYMBOL(SP-.4,-.5*SS,SS,2H10,0.0,2)
     CALL NUMBER(999.,.5*SS-.06,.5*SS,Y1,0.0,-1)
                                                      CALL PLOT(0.0,0.0,3)
     DO 9 J=1,NOCY
                          DO 8 I=1,10
                                      Y=I
                                          Y=ALOG10(Y)*FACTY+(J-1)*FACTY
     IF(I.EQ.1)GO TO 8
     CALL PLOT(0.0,Y,2)
                               CALL PLOT(-SST,Y,2)
   8 CALL PLOT(0.0,Y,3)
                               CALL PLOT(-SLT,Y,2)
     CALL SYMBOL(SP-.4,Y-.5*SS,SS,2H10,0.0,2)
     YPO=YPO+1.0
                         CALL NUMBER(999.,Y+.5*SS-.06,.5*SS,YPO,0.0,-1)
   9 CALL PLOT(0.0,Y,3)
                               YTL=MTY
                                       YTL=(YAXLEN-YTL*STTL)/2.0
     CALL SYMBOL(TTLP-.2,YTL,STTL,TTLY,90.,MTY)
                                                       RETURN
  10 CALL AXIS(0.0,0.0,TTLY,MTY,YAXLEN,90.,Y1,DELTAY)
     RETURN
                                                             END

     SUBROUTINE LIND(X,Y,NP,IM)
     DIMENSION X(1),Y(1)
     X1 = X(NP+1)
     Y1 = Y(NP+1)
     DELTAX = X(NP+2)
     DELTAY= Y(NP+2)
     DO 100 I=1,NP
     XX = (X(I) - X1 )/DELTAX
     YY = (Y(I) - Y1)/DELTAY
     IF( I.EQ.1 ) GO TO 96
     IF( YY .LE. 0 ) GO TO 95
     IF( IM .EQ. 1 ) CALL PLOT(XX,YY,2)
     IF( IM .EQ. 1 ) GO TO 100
     CALL SYMBOL(XX,YY,0.14,IM,0.0,-2)
     GO TO 100
  95 IF(Y(I-1) .LE. Y1 ) GO TO 105
     XXX = X(I) - (X(I) - X(I-1))/(Y(I)-Y(I-1))*Y(I)
     IF(XXX .LT. X(I-1) .OR. XXX .GT. X(I)) XXX = 0.5*(X(I)+X(I-1))
```

```
      XXX = (XXX-X1)/DELTAX
      CALL PLOT(XXX,0.0,2)
  105 CONTINUE
      CALL PLOT(XX,0,2)
      IF( I.EQ.NP ) GO TO 100
  102 IF(Y(I+1) .LE. Y1 ) GO TO 100
      XX = X(I) - (X(I)-X(I+1))/(Y(I)-Y(I+1))*Y(I)
      IF(XX .LT. X(I) .OR. XX.GT. X(I+1))XX = 0.5*(X(I)+X(I+1))
      XX = (XX-X1)/DELTAX
      CALL PLOT(XX,0.0,2)
      GO TO 100
   96 IF(YY .LE. 0) GO TO 97
      IF( IM.EQ. 1 ) CALL PLOT(XX,YY,3)
      IF(IM .EQ. 1 ) GO TO 100
      CALL SYMBOL(XX,YY,0.14,IM,0.0,-1)
      GO TO 100
   97 CALL PLOT(XX,0,3)
      GO TO 102
  100 CONTINUE
      RETURN
      END

      SUBROUTINE DIPDD(X,Y,Z,PHI,DELTA)
      TWPI = 6.2831853
      CCON = 57.29578
      VNORM = SQRT(X*X+Y*Y+Z*Z)
      IF(Z.GT.0) GO TO 10
      X = -X
      Y = -Y
      Z = -Z
   10 IF(ABS(X).LE.0.0001.AND.ABS(Y).LT.0.001) GO TO 20
      DELTA =ARSIN (Z/VNORM)
      IF(ABS(X).LT.0.0001) GO TO 21
      PHI = ATAN(Y/X)
      IF(X.LT.0) PHI = PHI + TWPI/2.
      IF(PHI.LT.0) PHI = PHI + TWPI
      GO TO 30
   21 IF(Y.LT.0) PHI = -TWPI/4. + TWPI
      IF(Y.GT.0) PHI = TWPI/4
      GO TO 30
   20 DELTA = TWPI/4.
      PHI = 0
   30 PHI = PHI * CCON
      DELTA = DELTA*CCON
      DELTA = 90. - DELTA
      RETURN
      END


      SUBROUTINE NODPL(DEL,BET,ALP,GAM)
```

```
      DIMENSION X(365),Y(365),DELTA(2),GAMMA(2)
      CALL SYMBOL(2,5,3,25,0.1,1HN,0.0,1)
      CALL PLOT(2,5,3,13,3)
      CALL PLOT(2.5,2,93,2)
      CALL PLOT(2.5,1.13,3)
      CALL PLOT(2.5,0,93,2)
      CALL PLOT(1,4,2,03,3)
      CALL PLOT(1.6,2,03,2)
      CALL PLOT(3,4,2,03,3)
      CALL PLOT(3,6,2,03,2)
      CALL SYMBOL(2,5,2,03,0,1,3,0,0,-1)
      DEGRAD = 0.01745232°
      DO 111 I=1,361
      ANGI = DEGRAD * ( I - 1 )
      X(I) = 2,5 + COS(ANGI)
  111 Y(I) = 2,03 + SIN(ANGI)
      X(362) = 0.0
      Y(362) = 0,0
      X(363) = 1,0
      Y(363) = 1,0
      CALL LINE(X,Y,361,1,0,0)
C     PLOT P, T SYMBOLS
      SQTWR = 1,414214
      R = SQTWR * SIN(GAM/2,)
      X1= 2.5 + R * SIN(ALP) - 0.03
      Y1= 2,03 + R * COS(ALP) - 0,05
      CALL SYMBOL(X1,Y1,0.1,1HP,0,0,1)
      R = SQTWR * SIN(BET/2,)
      X1= 2,5 + R * SIN(DEL) - 0,03
      Y1= 2.03 + R * COS(DEL) - 0.05
      CALL SYMBOL(X1,Y1,0,1,1HT,0,0,1)
      TWPI = 6,2831853
      DINC = 1.
      NP = 179
      X1 = SIN(BET)*COS(DEL)
      Y1 = SIN(BET) * SIN(DEL)
      Z1 = COS(BET)
      X2 = SIN(GAM)*COS(ALP)
      Y2 = SIN(GAM)*SIN(ALP)
      Z2 = COS(GAM)
      CALL DIPDD(X1+X2,Y1+Y2,Z1+Z2,GAMMA(1),DELTA(1))
      CALL DIPDD(X1-X2,Y1-Y2,Z1-Z2,GAMMA(2),DELTA(2))
      WRITE(61,1) ((GAMMA(L),DELTA(L)),L=1,2)
    1 FORMAT(1H ,4F10.3)
      DO 2 I = 1,2
      GAMMA(I) = GAMMA(I) - 180,
      IF(GAMMA(I),LT.0) GAMMA(I) = GAMMA(I) + 360,
    2 CONTINUE
      DO 100 K = 1,2
      IF(DELTA(K).GE.1.) GO TO 50
```

```
      NPOINT = NP+2
      DO 40 I=1,NPOINT
      THETA =(GAMMA(K)-90.+(I-1)*DINC)*DEGRAD
      IF(THETA.LT.0) THETA = THETA + TWPI
      IF(THETA.GT.TWPI) THETA = THETA-TWPI
      X(I) = SIN(THETA) + 2.5
      Y(I) = COS(THETA) + 2.03
   40 CONTINUE
      GO TO 90
   50 IF(ABS(DELTA(K)-90.).GE.1.) GO TO 70
      NPOINT = 2
      DO 60 I=1,2
      THETA =(GAMMA(K)-90.+(I-1)*180.)*DEGRAD
      IF(THETA.LT.0) THETA = THETA + TWPI
      IF(THETA.GT.TWPI) THETA = THETA - TWPI
      X(I) = SIN(THETA) + 2.5
      Y(I) = COS(THETA) + 2.03
   60 CONTINUE
      GO TO 90
   70 THETA = (GAMMA(K)-90.) * DEGRAD
      IF(THETA.LT.0) THETA = THETA + TWPI
      X(1) = SIN(THETA) + 2.5
      Y(1) = COS(THETA) + 2.03
      THETA=(GAMMA(K)+90.) * DEGRAD
      IF(THETA.GT.TWPI) THETA = THETA - TWPI
      X(NP+2) = SIN(THETA) + 2.5
      Y(NP+2) = COS(THETA) + 2.03
      NPOINT = NP + 2
      ANG = (90.-DELTA(K)) * DEGRAD
      TANI = SIN(ANG) / COS(ANG)
      DO 80 I=1,NP
      THETA = GAMMA(K)-90. + I*DINC
      IF(THETA.LT.0) THETA = THETA + 360.
      IF(THETA.GT.360.) THETA = THETA - 360.
      ALPHA = (THETA-GAMMA(K)) * DEGRAD
      XJ = ATAN(TANI/COS(ALPHA))
      R = SQTWR * SIN(XJ/2.)
      X(I+1) = R*SIN(THETA*DEGRAD) + 2.5
      Y(I+1) = R*COS(THETA*DEGRAD) + 2.03
   80 CONTINUE
   90 CONTINUE
      X(NPOINT+1) = 0.0
      Y(NPOINT+1) = 0.0
      X(NPOINT+2) = 1.0
      Y(NPOINT+2) = 1.0
      CALL LINE(X,Y,NPOINT,1,0,0)
  100 CONTINUE
      RETURN
      END
```

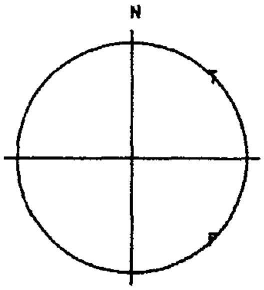
```
DATA    PAGE  1

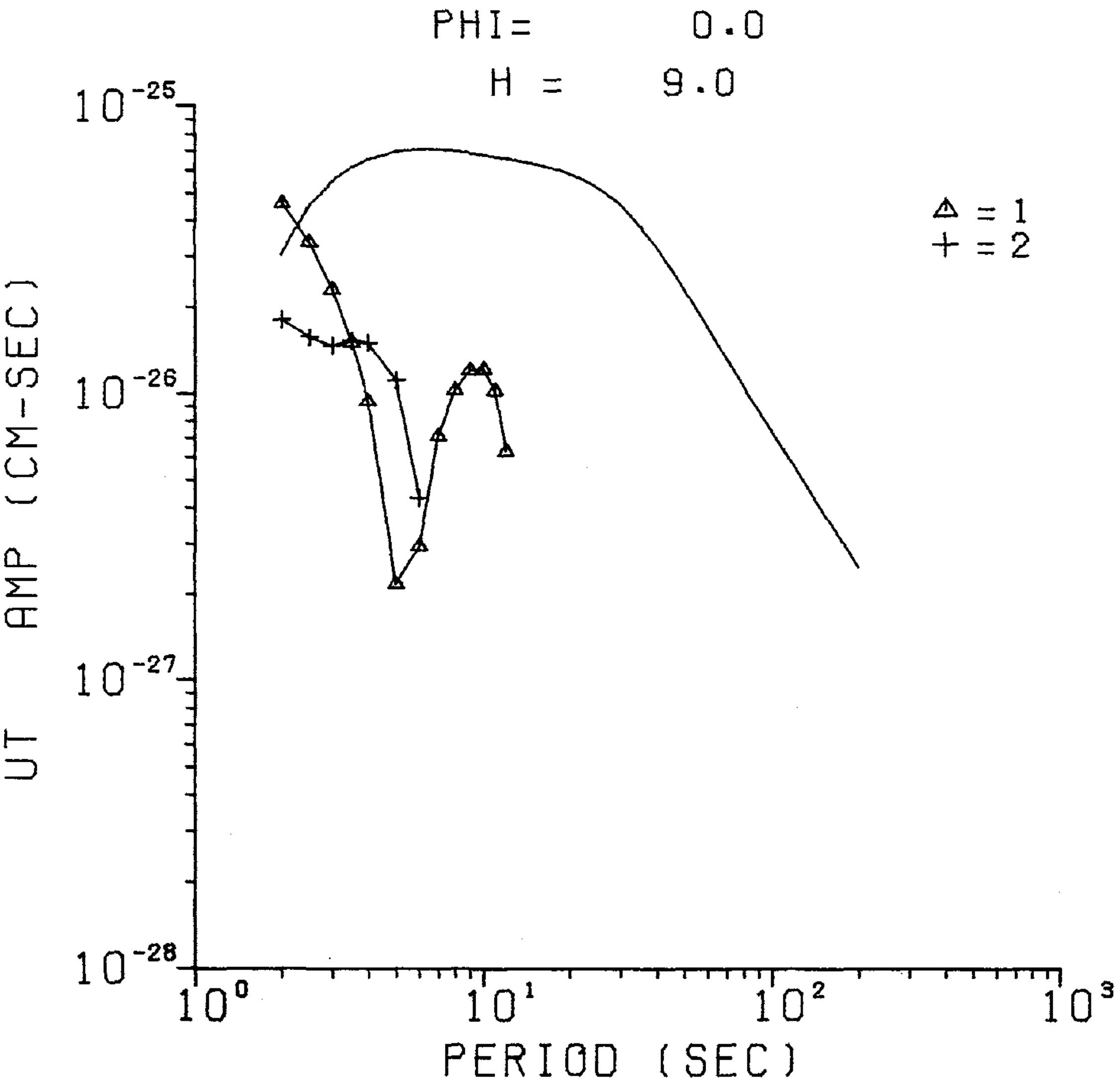
0.6
45.        90.        135.        90.
9.
0.0        1.0        1.0         1.0
-1.0
-1.0
```

```
      45.000     90.000    135.000     90.000
      89.995     89.993    179.995     90.000
```

```
       T                   P

DELT      BETA      ALPH      GAM
45        90        135       90
```




PHI= 0.0
H = 9.0
10^-25
10^-26
10^-27
10^-28
UT AMP (CM-SEC)
10^0
10^1
10^2
10^3
PERIOD (SEC)
△ = 1
+ = 2

IX. SRFWVSRC

PROGRAMMER: R. B. HERRMANN / Jan 1973

PURPOSE:

This program accepts a focal mechanism and focal depth and compares theoretical amplitude spectra for an impulsive source with observed spectral amplitudes. The source spectrum at each period as well as the anelastic attenuation coefficients are determined.

INPUT/OUTPUT

The surface wave amplitude spectra are read from card on FILE 60. One eigenfunction tape is used on FILE 01. Output is on the printer, FILE 61.

THEORY

The source spectrum S and attenuation coefficient g are determined by a least squares process in order to fit the observations to the model

$$y = S\ A\ \exp(-gr),$$

where r is the distance from the source and A is the theoretical amplitude spectra at a particular station for the focal mechanism and focal depth assumed.

Herrmann (1974) and Herrmann and Mitchell (1975) established a criteria for accepting the quality of data for determining S and g. A correlation coefficient R is determined for each fit to the data. Also output is the critical R value -- RCRIT. The g and S determinations are accepted if R is greater than RCRIT. 95% confidence limits are given to G and S.

PROGRAM DESCRIPTION

PROGRAM SRFWVSRC : This is the main input routine.

SUBROUTINE LINLSQ: This subroutine determines g and S together with confidence levels.

SUBROUTINE RCOEF: This determines the correlation coefficient between the Y vs A values.

SUBROUTINE NONPRP: This performs linear interpolation of the eigenfunctions with respect to depth.

SUBROUTINE FISHER; This determines the values of the Student "t" coefficient at a 95% confidence level.

REFERENCES

Herrmann, R. B. (1974). Surface-wave generation by central United States earthquakes, Ph. D. dissertation, Saint Louis University.

Herrmann, R. B. and B. J. Mitchell (1975). Statistical analysis and interpretation of surface-wave anelastic attenuation data for the stable interior of North America, Bull. Seism. Soc. Am. 65, 1115-1128.

INPUT DATA

Card Sequence	Column	Name	Format	Explanation
A.	1-10	DISTMX	F10.5	Only data at distances less than DISTMX are used to determine S and g
	11-20	EQEX	F10.5	.EQ.0 Earthquake data .NE.0 Explosion data
B.	2-5	NUMSRC	I4	Number of focal mechanisms to be entered (LE.10)
	12-15	NUMDPT	I4	Number of focal depths to be searched through (LE.10)
C. Focal mechanism parameters (NUMSRC cards)				
	1-10	XDEL	F10.5	Azimuth of T axis measured from north
	11-20	XBET	F10.5	Angle T axis makes with respect to downward vertical axis
	21-30	XALP	F10.5	AZ of P axis measured from north
	31-40	XGAM	F10.5	Angle P axis makes with respect to downward vertical axis
D. Focal depth (NUMDPT cards)				
	1-10	AAA	F10.5	Focal depth
E. Data header				
	1-10	TT	F10.5	Period of data

Card Sequence	Column	Name	Format	Explanation
Data header (cont'd)				
	11-15	MODAL	I5	Mode of data Fund = 1, 1'st = 2 etc.
	16-20	NUMSTA	I5	dummy set to 0 or blank
	21-25	IURUZ	I5	1 UZ data 2 UR data (has no meaning for Love wave data)
	30-40	ATTEN	E11.4	LT 0 search through admissible region then do least squares EQ 0 Do least squares only GT 0 Obtain source spectrum for given known attenuation value
F. Spectral amplitude data				
	2-5	ISTA	A4	Station identifier
	11-20	AZ	F10.5	Station azimuth from source
	21-30	DIST	F10.5	Station distance from source in km
	31-40	WEIGHT	F10.5	Observational weight to data
	41-51	AMPL	E11.4	Amplitude spectrum value normalized for geometrical spreading to 1000 km
	61-70	GPV		Group velocity of amplitude arrival

The program now returns to Point F to read another amplitude observation. When a card is found for which ISTA = 'HALT' , then the program stops reading in data and proceeds with the least squares analysis.

When the least squares analysis is completed, the program returns to Point E to read in new period data. The program terminates when a value of TT is found which is negative.

The data must be arranged in order of increasing period which is the way the eigenfunctions are placed on tape.

For an explosion the XDEL, XBET, XALP, XGAM quantities have no meaning, so these values can be read to zero.

```
C     PROGRAM SRFWVSRC
C     THIS PROGRAM  COMPUTES TEORETICAL SURFACE WAVE TRANSMISSION
C     FACTORS FOR RAYLEIGH OR LOVE WAVES AT A DISTANCE OF 1000
C     KILOMETERS.  OBSERVED SPECTRA NORMALIZED TO A DISTANCE OF 1000
C     KILOMETERS ARE READ IN.  A LINEAR LEAST SQUARES FIT IS MADE TO THE
C     DATA TO OBTAIN THE SOURCE SPECTRUM AND ATTENUATION FACTORS
C     SIMULTANEOUSLY.  COMPUTATIONS CAN ALSO BE PERFORMED FOR SOURCE
C     SPECTRUM ONLY WHEN ATTENUATION IS INPUTTED.
C     RAYLEIGH EIGENFUNCTION TAPE ENABLES USE OF UZ OR UR
C     LOVE EIGENFUNCTION TAPE ENEABLES USE OF UT
C     THE PROGRAM WILL ALSO DETERMINE THE SOURCE SPECTRUM AND THE
C     GAMMA COEFFICIENTS FOR EXPLOSION DATA IN WHICH CASE THE DEL,BET,
C     ALP,GAM PARAMETERS HAVE NO MEANING AND MAY ALL BE SET TO ZERO
      COMMON AY(100,4).CY(100,4),N,DEPTH(100),D(100)
      COMMON LAYER(100),A(100),B(100),RHO(100),XMU(100),XLAMB(100)
      COMMON ELIP
      DIMENSION AMODE(10),WVNC(10),ELIP(10),   ABSORP(10),DENOM(10)
      DIMENSION XDEL(10),XBET(10),XALP(10),XGAM(10),FACT1(10),FACT2(10)
      DIMENSION FACT3(10),FACT4(10),FACT5(10),AAA(10),ISTA(50),AZ(50)
      DIMENSION DIST(50),AMPL(50),WEIGHT(50),YY(50)
      DIMENSION PYR(4),PYI(4)
      DIMENSION PYY1(10,10),PYY2(10,10),PYY3(10,10),PYY4(10,10)
      DIMENSION GPV(50)
      DIMENSION UU(10)
      IHALT = 4HHALT
    1 FORMAT(1H ,2(I4,6X))
    2 FORMAT(1H ,I4,5X,7F10.4)
    4 FORMAT(8F10.5)
    5 FORMAT(1H ,A4,3F10.5,5E11.4)
    6 FORMAT(1H ,7(4X,E11.4))
    7 FORMAT(F10.5,3I5,4X,E11,4)
    8 FORMAT(1H1,20X,5HFOCAL,11X,4HDEL ,6X,4HBET ,6X,4HALP ,6X,4HGAM    )
    9 FORMAT(1H ,20X,I5,5X,4(4X,F6.1))
   10 FORMAT(1X,A4,5X,3F10.5,E11.4,9X,F10.5)
      DEGRAD = 0.017452329
       READ(60,4) DISTMX, EQEX
C     EQEX  .EQ. 0.0  EARTHQUAKE DATA
C     EQEX  .NE. 0.0  EXPLOSION DATA
      READ(60,1) NUMSRC,NUMDPT
      DO 25 I = 1,NUMSRC
   25 READ(60,4) XDEL(I),XBET(I),XALP(I),XGAM(I)
      WRITE(61,8)
      DO 26 I = 1,NUMSRC
   26 WRITE(61,9) I,XDEL(I),XBET(I),XALP(I),XGAM(I)
      DO 27 I = 1,NUMSRC
      XDEL(I) = DEGRAD * XDEL(I)
      XBET(I) = DEGRAD * XBET(I)
      XALP(I) = DEGRAD * XALP(I)
      XGAM(I) = DEGRAD * XGAM(I)
      FACT1(I) = (COS(XBET(I))**2) - (COS(XGAM(I))**2)
```

```
      FACT2(I) = (SIN(XBET(I)))**2
      FACT3(I) = (SIN(XGAM(I)))**2
      FACT4(I) = SIN(2.*XBET(I))
   27 FACT5(I) = SIN(2.*XGAM(I))
      DO 28 I = 1,NUMDPT
   28 READ(60,4) AAA(I)
C     TT = PERIOD OF DATA
C     MODAL = MODE OF DATA  F = 1, 1ST = 2, ETC
C     NUMSTA = DUMMY
C     IURUZ  1 = UZ DATA  2 = UR DATA
C     ATTEN  LT 0  SEARCH
C            EQ 0  LINEAR LEAST SQUARES ONLY
C            GT 0  OBTAIN SOURCE FOR GIVEN ATTEN
      XXMAX = 0.0
      READ(60,7) TT,MODAL,NUMSTA,IURUZ,ATTEN
      IF(TT.LE.0) GO TO 9999
      DO 29 I = 1,100
      NUMSTA = I
      READ(60,10) ISTA(I),AZ(I),DIST(I),WEIGHT(I),AMPL(I),GPV(I)
      IF(ISTA(I).EQ.IHALT) GO TO 291
      IF(AMPL(I).GT.XXMAX) XXMAX = AMPL(I)
   29 CONTINUE
      GO TO 292
  291 NUMSTA = NUMSTA - 1
  292 CONTINUE
C     DELETION OF AMPLITUDES NEAR NODES OF RADIATION PATTERN
      XLIM = 0.2 * XXMAX
      J = 1
      DO 4000 I = 1,NUMSTA
C     IF(AMPL(I).LT.XLIM) GO TO 4000
C     IF(DIST(I).GT.DISTMX) GO TO 4000                    CHANGE MADE HERE
      ISTA(J) = ISTA(I)
      AZ(J) = AZ(I)
      DIST(J) = DIST(I)
      WEIGHT(J) = WEIGHT(I)
      AMPL(J) = AMPL(I)
      GPV(J) = GPV(I)
      J = J + 1
 4000 CONTINUE
      NUMSTA = J - 1
      DO 37 I = 1,10
      DO 37 J = 1,10
      PYY1(I,J) = 0
      PYY2(I,J) = 0
      PYY3(I,J) = 0
   37 PYY4(I,J) = 0
      READ(01,1)N
C     READ IN MODEL
      DO 100 I=1,N
  100 READ(01,2)  LAYER(I),DEPTH(I),D(I),A(I),B(I),RHO(I),XMU(I),XLAMB(I
```

```
     1)
 2000 READ(01,1) IFUNC, MODE
      MAXY = 2 * IFUNC
      IF(IFUNC.LT.0) GO TO 9999
      DO 2999 IM = 1,MODE
      READ(01,5) AMODE(IM),T,C,U,SUMI0,WVNO(IM),AL,ELIP(IM),ABSORP(IM)
      UU(IM) = U
C     THE SPECTRA ARE COMPUTED AT A DISTANCE OF 1000 KILOMETERS
      DENOM(IM) = SQRT (WVNO(IM))*0.3989423
      DENOM(IM) = DENOM(IM)/AL
      DENOM(IM) = DENOM(IM) * 6.2831853 * 31.622776
      DO 2100 IN=1,N
 2100 READ(01,6) AY(IN,1),AY(IN,2),AY(IN,3),AY(IN,4),DCDA,DCDB,DCDR
      IF(TT.NE.T) GO TO 2999
      WVNO2 = WVNO(IM)**2
      IF(IFUNC-2)2400,2200,2200
C     RAYLEIGH WAVES
C     INITIALLY AY(IN,1) = UR, AY(IN,2) = UZ, AY(IN,3) = TZ,
C                  AY(IN,4) = TR
C     NOW AY(IN,1) = UR, AY(IN,2) = UZ, AY(IN,3) = DURDZ,
C         AY(IN,4) = DUZDZ
C     AND CY(IN,1) = DURDZ, CY(IN,2) = DUZDZ, CY(IN,3) = D2URDZ2,
C         CY(IN,4) = D2UZDZ2.
 2200 DO 2300 IN = 1,N
       H = RHO(IN)*C*C*WVNO2
      XL2M = XLAMB(IN) + 2.*XMU(IN)
      CY(IN,1) = -WVNO(IM)*AY(IN,2) + AY(IN,4)/XMU(IN)
      CY(IN,2) = (WVNO(IM)*XLAMB(IN)*AY(IN,1)+AY(IN,3))/XL2M
      CY(IN,3) = (-H/XMU(IN) + WVNO2*(3.*XL2M-2.*XMU(IN))/XL2M)*AY(IN,1)
     1 -WVNO(IM)*(1.+XLAMB(IN)/XMU(IN))*AY(IN,3)/XL2M
      CY(IN,4) = (-(H+WVNO2*XLAMB(IN))*AY(IN,2)+WVNO(IM)*(1.+XLAMB(IN)/
     1 XMU(IN))*AY(IN,4))/XL2M
      AY(IN,3) = CY(IN,1)
      AY(IN,4) = CY(IN,2)
 2300 CONTINUE
      GO TO 2501
C     LOVE WAVES
C     INITIALLY AY(IN,1) = UT, AY(IN,2) = TT
C     NOW AY(IN,1) = UT, AY(IN,2) = DUTDZ
C     AND CY(IN,1) = DUTDZ, CY(IN,2) = D2UTDZ2
 2400 DO 2500 IN=1,N
      CY(IN,1) = AY(IN,2)/XMU(IN)
      CY(IN,2) = WVNO2*(XMU(IN)- C*C*RHO(IN))*AY(IN,1)/XMU(IN)
      AY(IN,2) = CY(IN,1)
 2500 CONTINUE
 2501 CONTINUE
      DO 2555 ID = 1,NUMDPT
      AA = AAA(ID)
      NPER = ID
      IF(AA.LT.0) AA = 0.0
```

```
      CALL NONPRP(AA,MAXY,PYR)
      GO TO(2551,2552),IFUNC
 2551 PYY1(NPER,IM) = WVNO(IM)*PYR(1)/DENOM(IM)
      PYY2(NPER,IM) = PYR(2) / DENOM(IM)
      GO TO 2555
 2552 CONTINUE
      PYY1(NPER,IM) = WVNO(IM) * PYR(1) / DENOM(IM)
      PYY2(NPER,IM) = WVNO(IM) * PYR(2) / DENOM(IM)
      PYY3(NPER,IM) = PYR(3)/DENOM(IM)
      PYY4(NPER,IM) = PYR(4)/DENOM(IM)
 2555 CONTINUE
 2999 CONTINUE
      IF(TT.NE.T) GO TO 2000
 2998 CONTINUE
      IF(MODAL.LE.0.OR.MODAL.GT.MODE) GO TO 2997
      RAD = 0
      VER = 0
      IF(IFUNC.EQ.1) GO TO 2633
      IF(IURUZ-2)2631,2632,2632
 2631 VER = 1
      GO TO 2633
 2632 RAD = 1.
 2633 CONTINUE
      DO 3000 I = 1,NUMDPT
      DO 3001 J = 1,NUMSRC
      WRITE(61,11) AAA(I),J,TT,UU(MODAL),AMODE(MODAL)
   11 FORMAT(/1H ,10X,8HDEPTH =  ,F10.3,10X,9HSOURCE =  , I3,10X,4HT = ,
     1 F10.2,10X,4HU =  ,F10.5,10X,A4/)
      DEL = XDEL(J)
      ALP = XALP(J)
      BET = XBET(J)
      GAM = XGAM(J)
      DO 3002 K = 1,NUMSTA
      PYR(1) = PYY1(I,MODAL)
      PYR(2) = PYY2(I,MODAL)
      PYR(3) = PYY3(I,MODAL)
      PYR(4) = PYY4(I,MODAL)
      IF(EQEX.NE.0.0) GO TO 2725
      PHIR = AZ(K) * DEGRAD
      SINA = SIN(ALP - PHIR)
      COSA = COS(ALP - PHIR)
      SIND = SIN(DEL - PHIR)
      COSD = COS(DEL - PHIR)
      SINDP = SIN(2.*(DEL-PHIR))
      SINAP = SIN(2.*(ALP-PHIR))
      IF(IFUNC-2)2710,2720,2720
 2710 CONTINUE
      XREAL = 0.5 * PYR(1) *(FACT2(J)*SINDP-FACT3(J)*SINAP)
      XIMAG = -0.5*PYR(2)*(FACT4(J)*SIND-FACT5(J)*SINA)
      GO TO 2730
```

```
 2720 CONTINUE
      XREAL=FACT1(J)*PYR(4)-PYR(1)*(FACT2(J)*COSD*COSD-FACT3(J)*COSA*COS
     1A)
      XIMAG = 0.5*(PYR(2)+PYR(3))*(FACT4(J)*COSD-FACT5(J)*COSA)
 2730 CONTINUE
      GO TO 2731
 2725 XREAL = PYR(4) - PYR(1)
      XIMAG = 0.0
      IF(IFUNC.EQ.1) XREAL = 0.0
 2731 CONTINUE
      DUM = SQRT(XREAL**2 + XIMAG**2)
      IF(DUM.EQ.0) WEIGHT(K) = 0.0
      IF(IFUNC.EQ.1) GO TO 9988
      IF(RAD.LE.0) GO TO 9989
      YY(K) = DUM * ABS(ELIP(MODAL))
 9989 IF(VER.LE.0) GO TO 9995
 9988 YY(K) = DUM
 9995 CONTINUE
 3002 CONTINUE
C
      CALL LINLSQ(NUMSTA,WEIGHT,DIST,YY,AMPL,AZ,ISTA,ATTEN,GPV,TT)
 3001 CONTINUE
 3000 CONTINUE
      XXMAX = 0.0
 2997 READ(60,7) TT,MODAL,NUMSTA,IURUZ,ATTEN
      IF(TT.LE.0) GO TO 9999
      DO 31 I = 1,100
      NUMSTA = I
      READ(60,10) ISTA(I),AZ(I),DIST(I),WEIGHT(I),AMPL(I),GPV(I)
      IF(ISTA(I).EQ.IHALT) GO TO 311
      IF(AMPL(I).GT.XXMAX) XXMAX = AMPL(I)
   31 CONTINUE
      GO TO 312
  311 NUMSTA = NUMSTA - 1
  312 CONTINUE
C     DELETION OF AMPLITUDES NEAR NODES OF RADIATION PATTERN
      XLIM = 0.2 * XXMAX
      J = 1
      DO 5000 I = 1,NUMSTA
C     IF(AMPL(I).LT.XLIM) GO TO 5000
      IF(DIST(I).GT.DISTMX) GO TO 5000
      ISTA(J) = ISTA(I)
      AZ(J) = AZ(I)
      DIST(J) = DIST(I)
      WEIGHT(J) = WEIGHT(I)
      AMPL(J) = AMPL(I)
      GPV(J) = GPV(I)
      J = J + 1
 5000 CONTINUE
      NUMSTA = J - 1
```

```
      IF(TT.EQ.T) GO TO 2998
      GO TO 2000
 9999 CONTINUE
      REWIND 1
      STOP
      END

      SUBROUTINE LINLSQ(N,W,XX,YY,A,AZ,ISTA,ATEN,GPV,TT)
C     THIS SUBROUTINE SEARCHES FOR MINIMUM OF STATISTIC SHOWN AND GIVES
C     95 PERCENT CONFIDENCE LEVELS
      DIMENSION W(1),XX(1),YY(1),A(1),AZ(1),ISTA(1),Y(52),X(52),ERR(52)
      DIMENSION ISIGN(2)
      DIMENSION IAST(20)
      DIMENSION GPV(1),RAD(52)
      DIMENSION Q(52)
      ISIGN(1) = 4H +-
      ISIGN(2) = 4H X/
  200 FORMAT(1H      ,4HS=  ,E11.4,A4,E11.4,10X,4HK=  ,E11.4,4H +- ,E11.4
     1,10X,8HSIGMA = ,E11.4,10X,4HR =   ,E11.4)
  201 FORMAT(1H ,A4,5X,F10.2,5X,F10.2,4(4X,E11.4),2F10.4,4X,E11.4)
  202 FORMAT(1H ,4H STA,11X,4HAZ  ,9X,6HDIST  ,4X,
     1 11HRAD PATTERN,4X,11H OBSERVED  ,4X,11H PREDICTED ,4X,2X,A4,
     2 5HDEV   ,4X,6HWEIGHT ,4X,6HGP VEL  ,3X,12HCORR OBS RAD  /)
  203 FORMAT(1H0,8HNUMSTA =,I5,5X,18HESTIMATED MOMENT =,E11.4,5X,7HRCRIT
     1 = ,E11.4/)
      ATTEN = ATEN
      IMP = 0
      JUMP = 1
      IF(ATTEN.EQ.0) GO TO 5000
      IF(ATTEN.LT.0) JUMP = 2
      IF(ATTEN.LT.0) ATTEN = -0.00004
 5001 CONTINUE
      GO TO (301,302,301),JUMP
  302 IMP = IMP + 1
      ATTEN = ATTEN + 0.00004
      IF(IMP.EQ.27) JUMP = 1
      IF(IMP.EQ.27) GO TO 5000
  301 CONTINUE
C     W*(A - S*Y)**2 = W * (A- S*YY*EXP(-ATTEN*XX))**2 = MIN
      SUM1 = 0
      SUM2 = 0
      ISIG = 1
      DO 10 I = 1,N
      Y(I) = YY(I) * EXP(-ATTEN*XX(I))
      SUM1 = SUM1 + W(I) * A(I) * Y(I)
   10 SUM2 = SUM2 + W(I) * Y(I) * Y(I)
      S = SUM1/SUM2
      SUM0 = 0
      SUM1 = 0
      SUM2 = 0
```

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      SUM3 = 0.0
      DO 20 I = 1,N
      SUM0 = SUM0 + W(I)
      SUM1 = SUM1 + W(I) * Y(I)
      SUM2 = SUM2 + W(I)*Y(I)*Y(I)
      Y(I) = S * Y(I)
      ERR(I) = A(I) - Y(I)
   20 SUM3 = SUM3 + W(I) * ERR(I) * ERR(I)
      T = 1.960
C     TO PREVENT DIVISION BY ZERO
      IF(N.EQ.1) N = 100
      SIGMA = SQRT(SUM3/(N-1))
      IF(N.EQ.100) N = 1
      DS = T * SIGMA *(SQRT(SUM0/ABS(SUM0*SUM2-SUM1*SUM1)))
      DS=DS*1.0E20
      SLOPE = ATTEN
      DSLOPE = 0.0
      IF(JUMP.EQ.1) GO TO 6000
      SS=S*1.0E20
      WRITE(61,200)SS,ISIGN(ISIG),DS,SLOPE,DSLOPE,SIGMA
      B1 = - SLOPE
      B0 = ALOG(S)
 5000 SUM1 = 0
      SUM2 = 0
      SUM3 = 0
      SUM4 = 0
      SUM5 = 0
      SUM6 = 0
C     W * (ALOG(A/YY) - B0 - B1*XX) ** 2 = MIN
      DO 100 I = 1,N
      Q(I) = ALOG(A(I)/YY(I))
      SUM1 = SUM1 + W(I) * XX(I) * XX(I)
      SUM2 = SUM2 + W(I) * XX(I)
      SUM3 = SUM3 + W(I)
      SUM4 = SUM4 + W(I) * Q(I)
      SUM5 = SUM5 + W(I) * Q(I) * XX(I)
  100 SUM6 = SUM6 + W(I) * Q(I) * Q(I)
      DET = SUM3 * SUM1 - SUM2 * SUM2
      ISIG = 2
      GO TO (401,402,402),JUMP
  401 CONTINUE
      B0 = (SUM4 * SUM1 - SUM2 * SUM5)/DET
      B1 = (-SUM4*SUM2 + SUM3 * SUM5)/DET
  402 CONTINUE
C     TO PREVENT DIVISION BY ZERO
      IF(N.LE.2) N = 200
      SIGMA2 = (SUM6 - B0*SUM4 - B1*SUM5)/(N-2)
      IF(N.EQ.200) N = 2
      T = 100.
      IF(N.GT.2) CALL FISHER(N-2,T)
```

```
      T2 = T * T
      XC = 100000.
      IF(N.GT.2) XC = T2/(N-2)
      RCRIT = SQRT(XC)/SQRT(1.+XC)
      S = EXP(B0)
      SIGMA = SQRT(ABS(SIGMA2))
      SEB0 = T * SQRT(ABS(SUM1*SIGMA2/DET))
      DS = EXP(SEB0)
      GO TO (501,502,502),JUMP
  501 CONTINUE
      DO 101 I = 1,N
      ERR(I) = EXP(Q(I)-B0-B1*XX(I))
  101 Y(I) = YY(I)*S*EXP(B1*XX(I))
  502 CONTINUE
      SEB1 = T * SQRT(ABS(SUM3*SIGMA2/DET))
      SLOPE = - B1
      DSLOPE = SEB1
 6000 CONTINUE
      CALL RCOEF(N,W,Y,A,RCOEFF)
      SS=S*1.0E20
      WRITE(61,200)SS , ISIGN(ISIG),DS,SLOPE,DSLOPE,SIGMA,RCOEFF
      IF(JUMP.EQ.2) GO TO 5001
      IF(JUMP.EQ.2.OR.JUMP.EQ.3) ISIG = 1
      ESTMOM =SS * 6.2831853 / TT
      WRITE(61,203) N,ESTMOM,RCRIT
      WRITE(61,202) ISIGN(ISIG)
      DO 5999 I = 1,N
 5999 RAD(I) = A(I) * YY(I) / Y(I)
C     BECAUSE OF THE NORMILIZATION USED YY(I) AND RAD(I) ARE
C     1.0E20 TOO LARGE
      DO 6001 I = 1,N
 6001 WRITE(61,201) ISTA(I),AZ(I),XX(I),YY(I),A(I),Y(I),ERR(I),W(I),
     1 GPV(I),RAD(I)
      IF(SLOPE.LT.0.OR.SLOPE.GT.0.01) ATTEN = 0.0
      IF(SLOPE.LT.0.OR.SLOPE.GT.0.01) JUMP = 3
      IF(SLOPE.LT.0.OR.SLOPE.GT.0.01) GO TO 5001
      RETURN
      END

      SUBROUTINE RCOEF(N,W,Y,A,RCOEFF)
      DIMENSION W(1),Y(1),A(1)
      IF(N.EQ.1) RCOEFF = 1.0
      IF(N.EQ.1) RETURN
      SUM0 = 0
      SUM1 = 0
      SUM2 = 0
      SUM3 = 0
      SUM4 = 0
      SUM5 = 0
      DO 601 I = 1,N
```

```
      SUM0 = SUM0 + W(I)
      SUM1 = SUM1 + W(I) * Y(I)
      SUM2 = SUM2 + W(I) * Y(I) * Y(I)
      SUM3 = SUM3 + W(I) * A(I)
      SUM4 = SUM4 + W(I) * A(I) * A(I)
  601 SUM5 = SUM5 + W(I) * Y(I) * A(I)
      RCOEFF = (SUM0*SUM5 - SUM1*SUM3)/SQRT((SUM0*SUM2 - SUM1*SUM1)*
     1 ( SUM4 * SUM0 - SUM3 * SUM3))
      RETURN
      END

      SUBROUTINE NONPRP(A,MAXY,PYR)
      COMMON AY(100,4),CY(100,4),N,DEPTH(100),D(100)
      COMMON LAYER(100),AJ(100),B(100),RHO(100),XMU(100),XLAMB(100)
      COMMON IBUF(1026)
      COMMON ELIP(50,10)
      DIMENSION PYR(4)
      DO 100 I=1,N
      IFOCA = I
      D1 = DEPTH(I) - 0.5*D(I)
      D2 = DEPTH(I) + 0.5*D(I)
      IF(A.GE.D1.AND.A.LE.D2) GO TO 101
  100 CONTINUE
  101 DO 110 JJ = 1,MAXY
      AA = AY(IFOCA,JJ)
      CC = CY(IFOCA,JJ)
      AA = AA - CC * DEPTH(IFOCA)
  110 PYR(JJ) = AA + CC*A
      RETURN
      END

      SUBROUTINE FISHER(N,T)
      DIMENSION FT(34)
      FT(1) = 12.706
      FT(2) = 4.303
      FT(3) = 3.182
      FT(4) = 2.776
      FT(5) = 2.571
      FT(6) = 2.447
      FT(7) = 2.365
      FT(8) = 2.306
      FT(9) = 2.262
      FT(10) = 2.228
      FT(11) = 2.201
      FT(12) = 2.179
      FT(13) = 2.160
      FT(14) = 2.145
      FT(15) = 2.131
      FT(16) = 2.120
      FT(17) = 2.110
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      FT(18) = 2.101
      FT(19) = 2.093
      FT(20) = 2.086
      FT(21) = 2.080
      FT(22) = 2.074
      FT(23) = 2.069
      FT(24) = 2.064
      FT(25) = 2.060
      FT(26) = 2.056
      FT(27) = 2.052
      FT(28) = 2.048
      FT(29) = 2.045
      FT(30) = 2.042
      FT(31) = 2.021
      FT(32) = 2.000
      FT(33) = 1.980
      FT(34) = 1.960
      I = N - 2
      IF(I.LE.0) I = 1
      IF(I.GT.30) GO TO 100
      T = FT(I)
      RETURN
  100 IF(I.GT.30.AND.I.LE.40) T = (I-30)*(FT(31)-FT(30))/10.+FT(30)
      IF(I.GT.40.AND.I.LE.60) T = (I-40)*(FT(32)-FT(31))/20. + FT(31)
      IF(I.GT.60.AND.I.LE.120) T= (I-60)*(FT(33)-FT(32))/60. + FT(32)
      IF(I.GT.120) T = FT(34)
      RETURN
      END
```

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DATA    PAGE  1


3500.     0.0
   1         1
181.      52.       272.       89.
12.
16.          1    0    1
SLMT        3.2     335.3      1.0       8.4891E-04          3.3989     16.
LHCT        3.6     1425.30    1.0       6.8459E-04          3.4802     16.
AAMT       36.2     941.4      1.0       5.1925E-03          3.4276     16.
BLOT       40.3     526.4      1.0       4.4153E-03          3.4484     16.
OTTT       44.4     1650.      1.0       4.9422E-03          3.4836     16.
MNTT       47.2     1793.      1.0       4.5981E-03          3.4970     16.
CLET        47.4    1014.30    1.0       4.4010E-03          3.3878     16.
STJT       55.1    3574.1      1.0       4.2669E-03          3.4486     16.
WEST       59.8    1615.0      1.0       5.1962E-03          3.4810     16.
BLAT       76.0     917.1      1.0       2.3541E-03          3.5017     16.
ATLT      111.6     612.0      1.0       3.4088E-03          3.4991     16.
LUBT      261.1    1069.7      1.0       2.9095E-03          3.2037     16.
TUCT       264.6    1916.8     1.0       1.6224E-03          3.2604     16.
ALQT       271.7    1455.8     1.0       1.9990E-03          3.3494     16.
DUGT      291.2    2024.7      1.0       3.2214E-03          3.3583     16.
NEWT      303.7    2489.9      1.0       5.5236E-03          3.2151     16.
LONT       305.1    2881.8     1.0       3.3148E-03          3.3513     16.
MSOT      309.7    2316.3      1.0       5.3376E-03          3.3697     16.
PNTT      311.8    2613.9      1.0       5.7971E-03          3.3968     16.
EDMT      324.7    2651.9      1.0       7.7983E-03          3.3830     16.
FFCT      340.9    2307.3      1.0       2.4308E-03          3.4970     16.
MBCT      350.2    4758.5      1.0       1.7778E-03          3.4853     16.
HALT
-1.
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                FOCAL          DEL     BET     ALP      GAM
                  1           181.0    52.0   272.0     89.0

       DEPTH =      12.000        SOURCE =   1          T =        16.00         U =     3.48934          FUND
S=  0.3902E 24 X/  0.1409E 01        K=   0.1168E-03 +-  0.1610E-03        SIGMA =  0.3694E 00           R =  0.8267E 00

NUMSTA =   22    ESTIMATED MOMENT = 0.1532E 24    RCRIT = 0.4252E 00
 STA         AZ          DIST       RAD PATTERN     OBSERVED       PREDICTED        X/ DEV        WEIGHT     GP VEL    CORR OBS RAD
SLMT        3.20        335.30      0.1046E-06     0.8489E-03     0.3924E-03     0.2163E 01     1.0000     3.3987     0.2262E-06
LHCT        3.60       1425.30      0.1302E-06     0.6846E-03     0.4302E-03     0.1591E 01     1.0000     3.4802     0.2072E-06
AAMT       36.20        941.40      0.1732E-05     0.5193E-02     0.6054E-02     0.8578E 00     1.0000     3.4276     0.1485E-05
BLOT       40.30        526.40      0.1809E-05     0.4415E-02     0.6636E-02     0.6653E 00     1.0000     3.4464     0.1203E-05
OTTT       44.40       1650.00      0.1849E-05     0.4942E-02     0.5950E-02     0.8307E 00     1.0000     3.4836     0.1536E-05
MNTT       47.20       1793.00      0.1855E-05     0.4598E-02     0.5870E-02     0.7833E 00     1.0000     3.4970     0.1453E-05
CLET       47.40       1014.30      0.1854E-05     0.4401E-02     0.6428E-02     0.6847E 00     1.0000     3.3878     0.1270E-05
STJT       55.10       3374.10      0.1778E-05     0.4267E-02     0.4678E-02     0.9121E 00     1.0000     3.4486     0.1621E-05
WEST       59.80       1815.00      0.1668E-05     0.5196E-02     0.5267E-02     0.9866E 00     1.0000     3.4810     0.1646E-05
BLAT       76.00        917.10      0.9898E-06     0.2354E-02     0.3470E-02     0.6784E 00     1.0000     3.5017     0.6715E-06
ATLT      111.60        612.00      0.1210E-05     0.3409E-02     0.4395E-02     0.7756E 00     1.0000     3.4991     0.9383E-06
LUBT      261.10       1069.70      0.7115E-06     0.2910E-02     0.2450E-02     0.1187E 01     1.0000     3.2037     0.8448E-06
TUCT      264.60       1916.80      0.5186E-06     0.1622E-02     0.1618E-02     0.1003E 01     1.0000     3.2604     0.5201E-06
ALQT      271.70       1455.80      0.2627E-06     0.1999E-02     0.8647E-03     0.2312E 01     1.0000     3.3494     0.6072E-06
DUGT      291.20       2024.70      0.1190E-05     0.3221E-02     0.3666E-02     0.8787E 00     1.0000     3.3583     0.1046E-05
NEWT      303.70       2489.90      0.1674E-05     0.5524E-02     0.4885E-02     0.1131E 01     1.0000     3.2151     0.1893E-05
LONT      305.10       2881.80      0.1711E-05     0.3315E-02     0.4768E-02     0.6952E 00     1.0000     3.3513     0.1189E-05
MSOT      309.70       2316.30      0.1802E-05     0.5338E-02     0.5366E-02     0.9948E 00     1.0000     3.3697     0.1793E-05
PNTT      311.80       2813.90      0.1829E-05     0.5797E-02     0.5138E-02     0.1128E 01     1.0000     3.3968     0.2063E-05
EDMT      324.70       2651.90      0.1778E-05     0.7798E-02     0.5090E-02     0.1532E 01     1.0000     3.3830     0.2724E-05
FFCT      340.90       2307.30      0.1224E-05     0.2431E-02     0.3647E-02     0.6665E 00     1.0000     3.4970     0.8156E-06
MBCT      350.20       4758.50      0.7173E-06     0.1778E-02     0.1606E-02     0.1107E 01     1.0000     3.4853     0.7941E-06
```

X. RADPAT

PROGRAMMER: R. B. HERRMANN / May 1973

PURPOSE:

This program accepts observed surface-wave amplitude spectra for a given period, corrects for amplitude decrease due to anelastic attenuation, and plots the amplitudes as a function of receiver azimuth. For a given value of the source spectrum at the same period, orientation of the pressure and tension axes, and focal depth, the theoretical surface wave radiation pattern is plotted.

The radiation patterns are scaled to fit in a square 2 3/4 inches on a side. Beneath each radiation pattern, the period is written, as well as a scale relating the plot size to units of dyne-cm.

The amplitude apectra used by the program are values at a reference distance of 9°.

The spectra can be aligned on the page by specifying an (II,JJ) coordinate for the plot location. The (II,JJ) specifications are

(1,1)	(1,2)	(1,3)
(2,1)	(2,2)	(2,3)
(3,1)	(3,2)	(3,3)

To plot more than nine patterns, or to plot another page, just make the next II or JJ in the (II,JJ) pair negative. The program recognizes the negative quantity, moves the origin of the plotting system 13 inches, and plots the radiation pattern in the location (ABS(II), ABS(JJ)).

INPUT / OUTPUT

The eigenfunction tape are read from FILE 01. The observed surface-wave amplitude spectra are read from card on FILE 60. The FILE 10 is used for CALCOMP plots. If the card data is for Love waves, the eigenfunction tape must be the one containing the Love wave eigenfunctions. The program also requires the subroutines NONPRP, EIGEN, MODEL, These subroutines are stored separately on disk. Printer output is on FILE 61.

INPUT DATA

Card Sequence	Column	Name	Format	Explanation
A.				
	13-72	(IFORM(I), I=1,15)	15A4	Format for spectral amplitude data input
B.	1-10	XDEL	F10.5	Azimuth of T axis measured clockwise from North
	11-20	XBET	F10.5	The angle the T axis makes with the downward vertical z-axis
	21-30	XALP	F10.5	Azimuth of P axis measured clockwise from North
	31-40	XGAM	F10.5	The angle the P axis makes with the downward vertical z-axis
C.				
	1-10	AAA(1)	F10.5	Focal depth in km
D. Data header				
	1-10	TT	F10.5	.LE.0 end program .GT.0 period in seconds
	11-15	MODAL	I5	MODAL Mode of surface wave data
	16-20	IDUMP	I5	Dummy variable-not used-leave blank
	21-25	IURUZ	I5	.EQ. 1 UZ for Rayl .EQ. 2 IR fpr Rayl (has meaning only for Rayleigh wave data)

D. Data header (cont'd)

Card Sequence	Column	Name	Format	Explanation
	30-40	ATTEN	E11.4	Anelastic attenuation coefficient for surface wave in units of km^{-1}
		SRC	E11.4	Source spectrum at period TT in units of dyne-cm
E.	2-5	II	I5	Coordinates of radiation pattern positioning
F.	12-15	JJ	I5	" "
IFORM - Format read in from first data card above		ISTA		Station code
		AZ		Azimuth of station from source measured clockwise from North in degrees
		DIST		Distance from source to station in kilometers
		WEIGHT		Weighting factor on data, not used by this program
		AMPL		Spectral amplitude observed at the station at period TT in units of cm-sec
		GPV		Group velocity of this arrival at period TT--informative only-not used by this program.

The program returns to Point F to read in more ISTA, AZ, DIST, WEIGHT, AMPL, GPV data until it finds one with ISTA = HALT. The HALT indicates the end of a data set. The radiation pattern is now plotted. The program then returns to Point D to read in a new TT, MODAL, IDUMP, IURUZ, ATTEN, SRC card to see if the plotting should continue or whether the program should be terminated.

***** NOTE -- The data must be arranged in order of increasing period. Also no period interpolation is performed. Hence the periods used must agree exactly with those written on the tape.
*****

```
C       PROGRAM RADPAT
C       THIS PROGRAM ACCEPTS OBSERVED AMPLITUDES OF LOVE OR RAYLEIGH WAVES
C       AND USING SOURCE SPECTRUM AND ATTENUATION OF A GIVEN PERIOD YIELDS
C       A PLOT OF THE OBSERVED AND THEORETICAL RADIATION PATTERN.
C       THE PROGRAM YIELDS AMPLITUDES AT A DISTANCE OF 1000 KILOMETERS
        COMMON/ONE/AY(100,4),CY(100,4),N,DEPTH(100),D(100)
        COMMON/TWO/LAYER(100),A(100),B(100),RHO(100),XMU(100),XLAMB(100)
        COMMON/THREE/AMODE(10),WVNO(10),ELIP(10),ABSORP(10),DENOM(10)
        COMMON/FORE,NUMBPT,AAA(15),PYY1(15,10),PYY2(15,10),PYY3(15,10),
       1PYY4(15,10),IFUNC
        DIMENSION ISTA(100),AZ(100),DIST(100),WEIGHT(100),AMPL(100),GPV(10
       10),IBUF(1000)
        DIMENSION SINAZ(361),COSAZ(361)
        DIMENSION XO(100),YO(100),XT(363),YT(363),SA(361),CA(361),SD(361)
        DIMENSION CD(361),SDP(361),SAP(361)
        DIMENSION PYR(4)
        DIMENSION IFORM(15)
        CALL PLOTS(IBUF,1000,10)
        CALL PLOT(0.0,-11.0,-3)
        CALL PLOT(0.0,1.0,-3)
        DEGRAD = 0.017452329
        IHALT = 4HHALT
      1 FORMAT(1H ,2(I4,6X))
      4 FORMAT(8F10.5)
     15 FORMAT(1H ,2(E11.4,4X,E11.4,9X))
     16 FORMAT(1H ,5(E11.4,4X),2F10.1,2I5)
     17 FORMAT(F10.5,3I5,4X,E11.4,9X,E11.4)
     18 FORMAT(12X,10A6)
     19 FORMAT(1H0)
     20 FORMAT(1H ,A4,5X,7HDEPTH = , F10.2)
        CALL MODEL(61)
        READ(60,18) (IFORM(I),I=1,10)
        WRITE(61,18)(IFORM(I),I=1,10)
        READ(60,4) XDEL,XBET,XALP,XGAM
        WRITE(61,4) XDEL,XBET,XALP,XGAM
        XDEL = DEGRAD * XDEL
        XBET = DEGRAD * XBET
        XGAM = DEGRAD * XGAM
        XALP = DEGRAD * XALP
        FACT1 = (COS(XBET)**2) - (COS(XGAM)**2)
        FACT2 = (SIN(XBET))**2
        FACT3 = (SIN(XGAM))**2
        FACT4 = SIN(2.*XBET)
        FACT5 = SIN(2.*XGAM)
        DO 4004 K = 1,361
        XJ = K - 1
        ANG = DEGRAD * XJ
        SA(K) = SIN(XALP-ANG)
        CA(K) = COS(XALP-ANG)
        SD(K) = SIN(XDEL-ANG)
```

```
      CD(K) = COS(XDEL-ANG)
      SDP(K) = SIN(2.*(XDEL-ANG))
      SAP(K) = SIN(2.*(XALP-ANG))
      SINAZ(K) = SIN(ANG)
 4004 COSAZ(K) = COS(ANG)
      NUMDPT = 1
      T = 0
      READ(60,4) AAA(1)
C     IF WE WISH TO  TAKE AVERAGE  OVER FAULT PLANE USE FOLLOWING.
C     SMALL EFFECT
C     NUMDPT = 9
C     DEEP = AAA(1)
C     DO 4010 I = 1,NUMDPT
C     XJ = I - 5
C     THIS HAS FAULT PLANE EXTENDING FROM AAA(1) +-2.8 KM
C     AAA(I) = DEEP + 0.7 * XJ
C4010 CONTINUE
 2999 CONTINUE
      WRITE(61,10)
      READ(60,17) TT,MODAL,IDUMP,IURUZ,ATTEN,SRC
      IF(TT.LE.0) GO TO 9999
      WRITE(61,17) TT,MODAL,IDUMP,IURUZ,ATTEN,SRC
      READ(60,1) II,JJ
      CALL POSIT(II,JJ,XX,YY)
      OBMAX = 0.0
      DO 29 I = 1,100
      NUMSTA = I
      READ(60,IFORM)  ISTA(I),AZ(I),DIST(I),WEIGHT(I),AMPL(I),GPV(I)
      IF(ISTA(I).EQ.IHALT) GO TO 291
   29 CONTINUE
      GO TO 292
  291 NUMSTA = NUMSTA - 1
  292 CONTINUE
C     CORRECTION FOR ANELASTIC ATTENUATION
      DO 300 I = 1,NUMSTA
      PHI = DEGRAD * AZ(I)
      DUM = AMPL(I) * EXP(ATTEN*DIST(I))
      AMPL(I) = DUM
      YO(I) = DUM * COS(PHI)
      XO(I) = DUM * SIN(PHI)
      IF(DUM.GT.OBMAX) OBMAX = DUM
      WRITE(61,IFORM) ISTA(I),AZ(I),DIST(I),WEIGHT(I),AMPL(I),GPV(I)
  300 CONTINUE
      IF(T.EQ.TT) GO TO 2961
      CALL EIGEN(J1,TT,T)
 2961 CONTINUE
      SUM1 = 0
      SUM2 = 0
      SUM3 = 0
      SUM4 = 0
```

```
      DO 3000 I = 1,NUMDPT
      SUM1 = SUM1 + PYY1(I,MODAL)
      SUM2 = SUM2 + PYY2(I,MODAL)
      SUM3 = SUM3 + PYY3(I,MODAL)
      SUM4 = SUM4 + PYY4(I,MODAL)
 3000 CONTINUE
      THMAX = 0.0
      PYR(1) = SUM1 / NUMDPT
      PYR(2) = SUM2 / NUMDPT
      PYR(3) = SUM3 / NUMDPT
      PYR(4) = SUM4 / NUMDPT
      DO 3002 K = 1,361
      SINA = SA(K)
      COSA = CA(K)
      SIND = SD(K)
      COSD = CD(K)
      SINDP = SDP(K)
      SINAP = SAP(K)
      GO TO (2710,2720),IFUNC
 2710 CONTINUE
      XREAL = 0.5 * PYR(1) *(FACT2*SINDP - FACT3*SINAP)
      XIMAG = -0.5*PYR(2)*(FACT4*SIND - FACT5*SINA)
      GO TO 2730
 2720 CONTINUE
      XREAL = FACT1*PYR(4) -PYR(1)*(FACT2*COSD*COSD - FACT3*COSA*COSA)
      XIMAG = 0.5*(PYR(2)+PYR(3))*(FACT4*COSD-FACT5*COSA)
 2730 CONTINUE
C     THE OUTPUT OF EIGEN IS IN CGS UNITS THIS STEP IS TAKEN SO THAT
C     THEORETICAL AMPLITUDES ARE NOT TOO SMALL FOR MACHINE .LT. 1E-19
      XREAL = SRC * XREAL
      XIMAG = SRC * XIMAG
      DUM = SQRT(XREAL**2 + XIMAG**2)
      IF(IFUNC.EQ.2.AND.IURUZ.GE.2) DUM = DUM * ABS(ELIP(MODAL))
      XT(K) =          DUM * SINAZ(K)
      YT(K) =          DUM * COSAZ(K)
      IF(DUM.GT.THMAX) THMAX = DUM
 3002 CONTINUE
C     RADIATION PATTERNS ARE GIVEN WITHIN A 1 INCH RADIUS CIRCLE
      VALMAX = OBMAX
      IF(THMAX.GT.OBMAX) VALMAX = THMAX
      DO 3500 K = 1,361
      YT(K) =(YT(K) / VALMAX)+ YY
      XT(K) =(XT(K) / VALMAX)+ XX
 3500 CONTINUE
      WRITE(61,20) AMODE(MODAL),AAA(1)
      WRITE(61,15) XT(1),YT(1),XT(46),YT(46),XT(91),YT(91),XT(136),YT(13
     16)
      XT(362) = 0.0
      YT(362) = 0,0
      XT(363) = 1.0
```

```
      YT(363) = 1.0
      CALL LINE(XT,YT,361,1,0,0)
      DO 3501 IN = 1,NUMSTA
      X =(XO(IN)/VALMAX)+ XX
      Y =(YO(IN) / VALMAX)+ YY
      CALL SYMBOL(X,Y,0.035,0,0.0,-1)
 3501 CONTINUE
      Z = ALOG10(VALMAX)
      MM = Z
      FT = Z - MM
      FT = 10.**FT
      XXX = 1./FT
      IF(XXX.GT.1.) MM = MM - 1
      IF(XXX.GT.1.) XXX = XXX / 10.
      FT = MM
      WRITE(61,16) VALMAX,OBMAX,THMAX,XXX,FT,XX,YY,II,JJ
      CALL SYMBOL(XX-0.028,YY+1.0,0.10,1HN,0.0,1)
      CALL PLOT(XX,YY+0.95,3)
      CALL PLOT(XX,YY+0.82,2)
      CALL SYMBOL(XX,YY,0.03,3,0.0,-1)
      CALL PLOT(XX+XXX,YY-1.20,3)
      CALL PLOT(XX+XXX,YY-1.30,2)
      CALL PLOT(XX+XXX,YY-1.25,2)
      CALL PLOT(XX-XXX,YY-1.25,2)
      CALL PLOT(XX-XXX,YY-1.20,3)
      CALL PLOT(XX-XXX,YY-1.30,2)
      CALL SYMBOL(XX-0.35,YY-1.15,0.10,4H2 X ,0.0,4)
      CALL SYMBOL(XX+0.05,YY-1.15,0.10,2H10,0.0,2)
      CALL NUMBER(XX+0.26,YY-1.07,0.07,FT,0.0,-1)
      CALL SYMBOL(XX-0.4,YY-1.50,0.10,4HT = ,0.0,4)
      CALL NUMBER(XX,YY-1.50,0.10,TT,0.0,1)
      GO TO 2999
 9999 CONTINUE
      REWIND 01
      CALL PLOT(15.0,0.0,999)
      STOP
      END

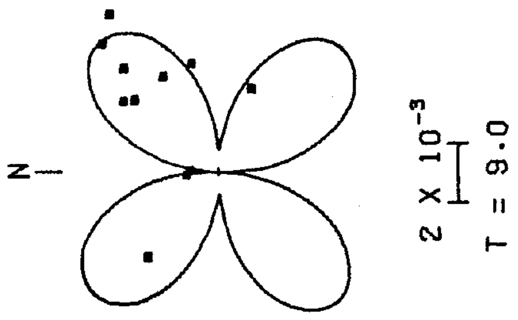
      SUBROUTINE POSIT(I,J,XX,YY)
      DIMENSION X(3),Y(3)
      X(1) = 1.5
      X(2) = 4.25
      X(3) = 7.00
      Y(1) = 7.25
      Y(2) = 4.50
      Y(3) = 1.75
      IF(I.GT.0.AND.J.GT.0) GO TO 100
      CALL PLOT(13.0,0.0,-3)
      CALL PLOT(0.0,-11.0,-3)
      CALL PLOT(0.0,1.0,-3)
```

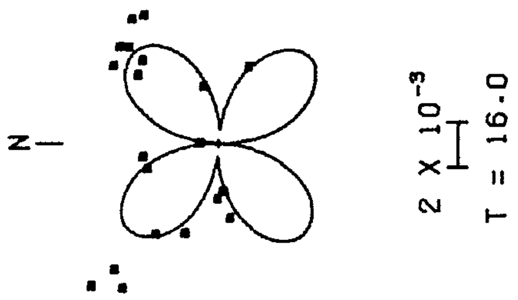
```
      I = IABS(I)
      J = IABS(J)
  100 CONTINUE
      XX = X(J)
      YY = Y(I)
      RETURN
      END
```

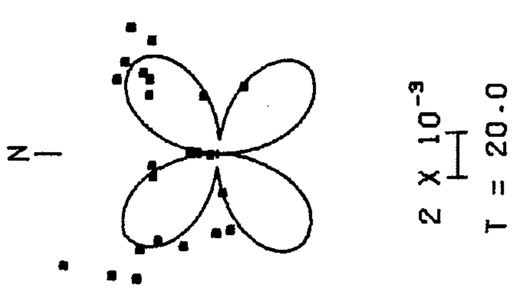
```
DATA    PAGE  1

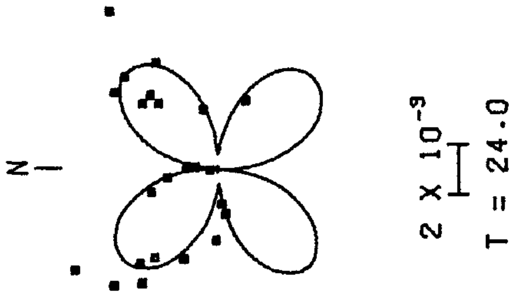

            (1X,A4,5X,3F10.2,E11.4,9X,F10.4)
181,         52,        272,       89,
12,
16,            1     0     1      1.4000E-04             2,2918E 23
   2           1
SLMT         3,2        335,3                  8,4891E-04          3,3989      16,
LHCT         3,6        1425,30                6,8459E-04          3,4802      16,
AAMT        36,2        941.4                  5,1925E-03          3,4276      16,
BLOT        40,3        526,4                  4,4153E-03          3,4484      16,
OTTT        44,4        1650,                  4,9422E-03          3,4836      16,
MNTT        47,2        1793,                  4,5981E-03          3,4970      16,
CLET         47,4       1014,30                4,4010E-03          3,3878      16,
STJT        55,1       3374.1                  4,2669E-03          3,4486      16,
WEST        59,8       1815,0                  5,1962E-03          3,4810      16,
BLAT        76,0        917,1                  2,3541E-03          3,5017      16,
ATLT       111,6        612,0                  3,4088E-03          3,4991      16,
LUBT       261.1       1069,7                  2,9095E-03          3,2037      16,
TUCT        264.6       1916,8                 1,6224E-03          3,2604      16,
ALQT        271,7       1455,8                 1,9990E-03          3,3494      16,
DUGT       291,2       2024,7                  3,2214E-03          3,3583      16,
NEWT       303,7       2489,9                  5,5236E-03          3,2151      16,
LONT        305,1       2881,8                 3,3148E-03          3,3513      16,
MSOT       309,7       2316,3                  5,3376E-03          3,3697      16,
PNTT       311,8       2813,9                  5,7971E-03          3,3968      16,
EDMT       324,7       2651,9                  7,7983E-03          3,3830      16,
FFCT       340,9       2307,3                  2,4308E-03          3,4970      16,
MBCT       350,2       4758,5                  1,7778E-03          3,4853      16,
HALT
-1,
```

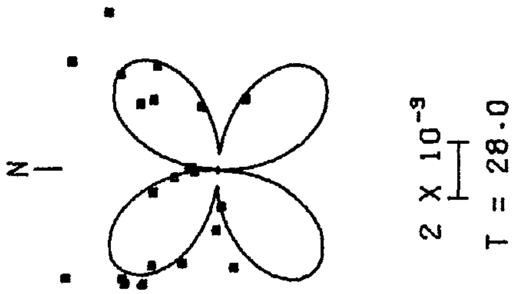
```
 16.00000    1    0    1     0.1400E-03          0.2292E 24
SLMT           3.20    335.30        0.    0.8897E-03                 3.3989
LHCT           3.60   1425.30        0.    0.8358E-03                 3.4802
AAMT          36.20    941.40        0.    0.5924E-02                 3.4276
BLOT          40.30    526.40        0.    0.4753E-02                 3.4484
OTTT          44.40   1650.00        0.    0.6226E-02                 3.4836
MNTT          47.20   1793.00        0.    0.5910E-02                 3.4970
CLET          47.40   1014.30        0.    0.5073E-02                 3.3878
STJT          55.10   3374.10        0.    0.6843E-02                 3.4466
WEST          59.80   1815.00        0.    0.6699E-02                 3.4810
BLAT          76.00    917.10        0.    0.2677E-02                 3.5017
ATLT         111.60    612.00        0.    0.3714E-02                 3.4991
LUBT         261.10   1069.70        0.    0.3380E-02                 3.2037
TUCT         264.60   1916.80        0.    0.2122E-02                 3.2604
ALQT         271.70   1455.80        0.    0.2451E-02                 3.3494
DUGT         291.20   2024.70        0.    0.4277E-02                 3.3583
NEWT         303.70   2489.90        0.    0.7827E-02                 3.2151
LONT         305.10   2881.80        0.    0.4962E-02                 3.3513
MSOT         309.70   2316.30        0.    0.7382E-02                 3.3697
PNTT         311.80   2813.90        0.    0.8596E-02                 3.3968
EDMT         324.70   2651.90        0.    0.1130E-01                 3.3830
FFCT         340.90   2307.30        0.    0.3358E-02                 3.4970
MBCT         350.20   4758.50        0.    0.3461E-02                 3.4853
FUND     DEPTH =     12.00
 0.1500E 01     0.4521E 01          0.1765E 01     0.4765E 01
0.1557E 01     0.4500E 01          0.1765E 01     0.4235E 01
 0.1130E-01     0.1130E-01     0.4251E-02     0.8846E 00    -0.2000E 01          1.5       4.5     2    1
```

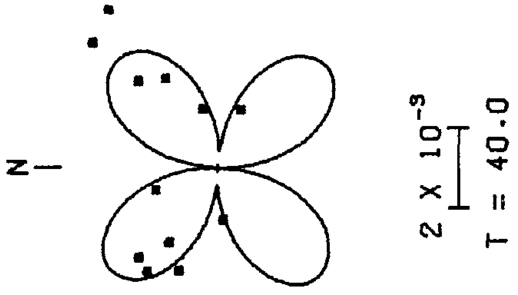

N
2 X 10^{-3}
T = 9.0


N
2 X 10^{-3}
T = 16.0


N
2 X 10^{-3}
T = 20.0


N
2 X 10^{-3}
T = 24.0


N
2 X 10^{-3}
T = 28.0


N
2 X 10^{-3}
T = 40.0

XI. QUESTION

PROGRAMMER: R. B. HERRMANN / May 1973

PURPOSE:

This program accepts observed Love and Rayleigh wave amplitude spectra for an earthquake. The program searches through a parameter space of focal mechanism orientations and focal depth. For each particular combination, the program computes several goodness of fit characteristics which are used by the user to determine the best focal mechanism and focal depth combination which satisfies the observations. $S(\omega) = M_o/i\omega$.

INPUT/OUTPUT

The surface wave amplitude spectra are read from card on FILE 60. Two tapes are used. The Love wave eigenfunction tape is placed on FILE 01 and the Rayleigh wave eigenfunction tape is placed on FILE 02. Printer output is on FILE 61. Both tapes must have the same earth model specifications. The sample data for LEIGEN and REIGEN are consistent with the periods on eigenfunction tapes.

THEORY

The theory of Levshin and Yanson is used to specify the theoretical amplitude spectra. The formulation in terms of strike, dip, and slip is used. Since only the amplitudes will be considered, the strike need vary only over 180^o, the dip over 90^o, and the slip over 180^o. Because of the symmetry of the theoretical surface wave amplitude spectra to a 180^o rotation in strike, the strike is only known to within 180^o. The slip varies over 180^o instead of 360^o since the amplitudes only are used. The choice of the proper strike (or strike + 180^o) is made from the best fit to observed P wave first motion.

For each combination of strike, dip, slip, and focal depth the following parameters are computed.

MOMENT -	Seismic moment from average of Love and Rayleigh wave estimates.
R-RAYL -	Correlation coefficient between the observed and theoretical Rayleigh wave amplitude spectra for the totality of data from all azimuths and periods.

RES-RAYL - Sum of square residuals between observed and theoretical Rayleigh wave amplitude spectra using the average seismic moment estimate

R-LOVE - Correlation coefficient between the observed and theoretical Love wave amplitude spectra for the totality of data from all azimuths and periods

RES-LOVE - Sum of square residuals betwen observed and theoretical Love wave amplitude spectra using the average seismic moment estimate

RES-L+R - Square root of sum of Love and Rayleigh square residuals.

MOMENT-L - Seismic moment estimate from only Love wave data

MOMENT-R - Seismic moment estimate from only Rayleigh wave data

If either correlation coefficient is less than zero, none of the above information is listed. This saves considerable printer time during the program execution.

The last column of the printout is flagged by a ** if the two independent seismic moment estimates are within a multiplicative factor of 1.66 .

After the computations for a given focal depth, a summary is given of the various slip, dip, and strike combinations which gave the largest RR, RL, and least RES, RESR and RESL, as well as the largest value of BEST. BEST is defined as RR*RL*(ratio of seismic moment estimates chosen to be less than 1.0).

The best focal mechanism estimate has been found to be the one with the largest values of the correlation coefficients RR, RL and for which the two independent seismic moment estimates are as equal as possible. The output of all these parameters permits a robust search.

When beginning a search for the best focal mechanism, the following search range is used:

DIP 30° (15°) 90°

SLIP 90° (20°) 70°

STRIKE 0° (20°) 160°

DEPTH 5 (5) 20 km

The spectra of SLIP = -90° equals the spectra for SLIP = $+ 90^{\circ}$. Likewise STRIKE = 0° and 180° yield the same spectra. Hence the choice above.

After performing this initial search over a wide grid, an appropriate region is found for the solution and the program is rerun using a finer grid.

PROGRAM DESCRIPTION

PROGRAM QUESTION : This is the main input/output routine.

SUBROUTINE PRESS: This subroutine determines the correlation coefficients, residuals, and seismic moments.

(The following subroutines are required for operation but are found in the User Library. They are described elsewhere in this volume).

SUBROUTINE EIGEN

SUBROUTINE MODEL

SUBROUTINE NONPRP

SUBROUTINE EXCIT

REFERENCES

Herrmann, R. B. (1974). Surface-wave generation by central United States earthquakes, Ph. D. Dissertation, Saint Louis University.

Card Sequence	Column	Name	Format	Explanation
A.	1-10	DISTMX	F10.5	Data for distances greater than DISTMX km are not used in the computations
B.	2-5	NUMDPT	I4	Number of focal depths to be considered
C. Focal depth cards (NUMDPT cards)				
	1-10	AAA(I)	F10.5	Focal depths in km
D.	1-5	ND	I5	Number of dips to be used
	11-20	DI	F10.5	First dip in degrees
	21-30	DINC	F10.5	Dip increment in degrees
E.	1-5	NS	I5	Number of slips to be used
	11-20	SI	F10.5	Initial slip angle
	21-30	SINC	F10.5	Slip angle increment
F.	1-5	NF	I5	Number of strikes to be used
	11-20	FI	F10.5	Initial strike angle
	21-30	FINC	F10.5	Strike angle increment
(Love wave data first, arranged in order of increasing period.)				
G.	1-10	TT	F10.5	.LT.0 end of Love wave data .GT.0 period in seconds
	11-15	MODAL	I5	Mode of Love wave data at period TT
	16-20	NUMSTA	I5	Dummy variable-not used

G. (cont'd)

Card Sequence	Column	Name	Format	Explanation
	21-25	IURUZ	I5	Not used for Love waves
	30-40	ATTEN	E11.4	Anelastic attenuation coefficient in km^{-1} for Love wave of period TT and mode MODAL
H.	2-5	ISTA	A4	Station identifier code
	11-20	AZ	F10.5	Azimuth from source to station in degrees, clockwise from North
	21-30	DIST	F10.5	Distance from source to station in km
	31-40	DUM1	F10.5	Dummy variable - no use in this program
	41-51	AMP	E11.4	Spectral amplitude
	61-70	DUM2	F10.5	DUM2 variable - not used by this program

The program corrects the amplitude for anelastic attenuation and returns to Point H to read another ISTA set. When it finds ISTA = HALT, the program then goes to Point G to read another TT header and corresponding ISTA data. All the Love wave data is read in until a TT = -1.0 (negative) is found. (This signifies the end of the Love wave data).

(The Rayleigh wave data is now read, arranged in order of increasing period.)

Card Sequence	Column	Name	Format	Explanation
I.	1-10	TT	F10.5	.LT.0 end of Rayleigh wave data .GT.0 period in seconds
	11-15	MODAL	I5	Mode of Rayleigh wave data Fundamental--MODAL = 1
	16-20	NUMSTA	I5	Dummy variable-not used
	21-25	IURUZ	I5	.EQ.1 Vertical component spectra used .EQ.2 Radial component. Rayleigh wave spectra used
	30-40	ATTEN	E11.4	Anelastic attenuation coefficient in km^{-1} for Rayleigh waves of period TT and mode MODAL
J.	2-5	ISTA	A4	Station identification code
	11-20	AZ	F10.5	Azimuth from source to station in degrees,measured clockwise, from N
	21-30	DIST	F10.5	Distance from source to station in km
	31-40	DUM1	F10.5	Dummy variable-not used by program
	41-51	AMP	E11.4	Spectral amplitude in cm-sec
	61-70	DUM2	F10.5	Dummy variable - not used by program

The program corrects the spectral amplitude for anelastic attenuation and returns to Point J to read another ISTA set. When it finds an ISTA = HALT, the program recognizes the end of data for period TT and mode MODAL and proceeds to POINT I to read another TT header and the corresponding ISTA data. A value of TT = negative indicates the end of Rayleigh wave data. At this point the search process begins for the best solution.

```
C       PROGRAM QUESTION
C       THIS PROGRAM ACCEPTS RAYLEIGH AND LOVE SPECTRAL VALUES FOR VARIOUS
C       PERIODS AND MODES AND COMPUTES SEISMIC MOMENT ASSUMING
C       1./OMEGA SOURCE.  THE GOODNESS OF FIT AND THE MOMENT ARE
C       DETERMINED FOR ANY FOCAL MECHANISM
        COMMON/ONE/AY(100,4),CY(100,4),N,DEPTH(100),D(100)
        COMMON/TWO/LAYER(100),A(100),B(100),RHO(100),XMU(100),XLAMB(100)
        COMMON/THREE/AMODE(10),WVNO(10),ELIP(10),ABSORP(10),DENOM(10)
        COMMON/FORE/NUMDPT,AAA(15),PYY1(15,10),PYY2(15,10),PYY3(15,10),
       1PYY4(15,10),IFUNC
        DIMENSION STRK(15),CST(15),SST(15),S2ST(15),C2ST(15)
        DIMENSION DIP(15),SD(15),CD(15),C2D(15),SLIP(15),SS(15),CS(15)
        DIMENSION SINAZ(2,550),COSAZ(2,550),XL(550),XR(550),YL(550),YR(550
       1),PYL1(10,30),PYL2(10,30),PYR1(10,30),PYR2(10,30),PYR3(10,30)
        DIMENSION PYR4(10,30),NNUML(30),NNUMR(30)
        DIMENSION IS(6),ID(6),IP(6),IBEST(6)
        DIMENSION XBEST(6)
        M1 = 4H
        M2 = 4H **
        IBEST(1) = 4H  RR
        IBEST(2) = 4H  RL
        IBEST(3) = 4H RES
        IBEST(4) = 4HRESR
        IBEST(5) = 4HRESL
        IBEST(6) = 4HBEST
      1 FORMAT(1H ,2(I4,6X))
      2 FORMAT(10X,5H NR =  ,I5,10X,5H NL = , I5   )
      3 FORMAT(1H1,8HDISTMX = ,F10.2/)
      4 FORMAT(8F10,5)
      7 FORMAT(F10.5,3I5,4X,E11.4)
     10 FORMAT(1X,A4,5X,3F10.5,E11.4,9X,F10,5)
     11 FORMAT(I5,5X,2F10.5)
     12 FORMAT(1H0,1X,5HDEPTH,2X,4HSLIP,3X,3HDIP,6H  STRK,2X,8H  MOMENT ,
       1 4X,8H  R-RAYL,4X,8HRES-RAYL,4X,8H  R-LOVE,4X,8HRES-LOVE,4X,
       2 8HRES--L+R,4X,8HMOMENT-L,4X,8HMOMENT-R       )
     13 FORMAT(1H ,3H***,A4,3(F5.0,1X),1X,E10.3)
     14 FORMAT(1H ,4F6.1,8(1X,E10.3,1X),4X,A4)
C       LOGIC UNIT 01 = LOVE TAPE
C       LOGIC UNIT 02 = RAYL TAPE
        CALL MODEL(01)
        CALL MODEL(02)
        READ(60,4) DISTMX
        WRITE(61,3) DISTMX
        READ(60,1) NUMDPT
        DO 28 I = 1,NUMDPT
     28 READ(60,4) AAA(I)
        IHALT = 4HHALT
        XLOW = 1.0E-30
        DEGRAD = 0.017452329
C           ND = NUMBER DIPS    DI = INITIAL DIP    DINC = DIP INCREMENT
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```
      READ(60,11) ND, DI, DINC
C        NS = NUMBER SLIPS   SI = INITIAL SLIP   SINC = SLIP INCREMENT
      READ(60,11) NS,SI, SINC
C        NF = NUMBER STRIKES FI = INITIAL STRIKE FINC = STRIKE INCREMENT
      READ(60,11) NF, FI, FINC
      DO 4001 J = 1,ND
      XJ = J
      DIP(J) = (DI + (XJ-1.)*DINC)
      RDIP = DEGRAD * DIP(J)
      SD(J) = SIN(RDIP)
      CD(J) = COS(RDIP)
      C2D(J) = COS(2.*RDIP)
 4001 CONTINUE
      DO 4002 K = 1,NS
      XJ = K
      SLIP(K) = (SI + (XJ-1.) * SINC)
      RSLIP = DEGRAD * SLIP(K)
      SS(K) = SIN(RSLIP)
      CS(K) = COS(RSLIP)
 4002 CONTINUE
      DO 4003 L = 1,NF
      XJ = L
      STRK(L) = (FI + (XJ-1.) * FINC)
      RSTRK = DEGRAD * STRK(L)
      CST(L) = COS(RSTRK)
      SST(L) = SIN(RSTRK)
      S2ST(L) = SIN(2.*RSTRK)
      C2ST(L) = COS(2.*RSTRK)
 4003 CONTINUE
C     SOURCE SPECTRUM IS ASSUMED TO BE 1./OMEGA
C     TT = PERIOD OF DATA
C     MODAL = MODE OF DATA   1 = FUND, 2 = 1ST, ETC
C     NUMSTA = DUMMY
C     IURUZ   1 = UZ DATA , 2 = UR DATA
C     ATTEN = APPROXIMATE VALUE OF ATTENUATION COEFFICIENT GAMMA
      T = 0
      NL = 0
C     LOVE WAVE DATA
      DO 293 J = 1,30
      READ(60,7) TT,MODAL,NUMSTA,IURUZ,ATTEN
      IF(TT.LT.0) GO TO 7997
      NUMSTA = 0
      DO 29 I = 1, 100
      READ(60,10) ISTA,AZ,DIST,DUM1,AMP,DUM2
      IF(DIST.GE.DISTMX) GO TO 29
      NUMSTA = NUMSTA + 1
      IF(ISTA.EQ.IHALT) GO TO 291
      NL = NL + 1
      IF(NL.GT.550) GO TO 9999
      XL(NL)=AMP*EXP(ATTEN*DIST)
```

```
      AZ = DEGRAD * AZ
      SINAZ(1,NL) = SIN(AZ)
      COSAZ(1,NL) = COS(AZ)
   29 CONTINUE
      GO TO 292
  291 NUMSTA = NUMSTA - 1
  292 CONTINUE
      NNUML(J) = NUMSTA
      FACT = TT/6.2831853
      IF(T.EQ.TT) GO TO 296
      CALL EIGEN(61,TT,T)
  296 DO 297 IZ = 1,NUMDPT
      PYL1(IZ,J) = PYY1(IZ,MODAL) * FACT
  297 PYL2(IZ,J) = PYY2(IZ,MODAL) * FACT
  293 CONTINUE
      GO TO 7987
 7997 J = J - 1
 7987 JLMX = J
      T = 0
      NR = 0
C     RAYLEIGH WAVE DATA
      DO 313 J = 1,30
      READ(60,7) TT,MODAL,NUMSTA,IURUZ,ATTEN
      IF(TT.LT.0) GO TO 8998
      NUMSTA = 0
      DO 31 I = 1,100
      READ(60,10) ISTA,AZ,DIST,DUM1,AMP,DUM2
      IF(DIST.GE.DISTMX) GO TO 31
      NUMSTA = NUMSTA + 1
      IF(ISTA.EQ.IHALT) GO TO 311
      NR = NR + 1
      IF(NR.GT.550) GO TO 9999
      XR(NR)=AMP*EXP(ATTEN*DIST)
      AZ = DEGRAD * AZ
      SINAZ(2,NR) = SIN(AZ)
      COSAZ(2,NR) = COS(AZ)
   31 CONTINUE
      GO TO 312
  311 NUMSTA = NUMSTA - 1
  312 CONTINUE
      NNUMR(J) = NUMSTA
      FACT = TT/6.2831853
      IF(T.EQ.TT) GO TO 315
      CALL EIGEN(62,TT,T)
  315 CONTINUE
      IF(IURUZ.EQ.2) FACT = FACT * ABS(ELIP(MODAL))
  316 DO 317 IZ = 1,NUMDPT
      PYR1(IZ,J) = PYY1(IZ,MODAL) * FACT
      PYR2(IZ,J) = PYY2(IZ,MODAL) * FACT
      PYR3(IZ,J) = PYY3(IZ,MODAL) * FACT
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  317 PYR4(IZ,J) = PYY4(IZ,MODAL) * FACT
  313 CONTINUE
      GO TO 8988
 8998 J = J - 1
 8988 JRMX = J
      WRITE(61,2) NR,NL
      DO 3000 I = 1,NUMDPT
      WRITE(61,12)
      SRESR=1.0/XLOW
      SRESL=1.0/XLOW
      SRES =1.0/XLOW
      GRR=0.0
      GRL=0.0
      PMRR = 0.0
      DO 3001 J = 1,ND
      SIND = SD(J)
      COSD = CD(J)
      COS2D = C2D(J)
      DO 3002 K = 1,NS
      SINS = SS(K)
      COSS = CS(K)
      DO 3003 L = 1,NF
      COSST = CST(L)
      SINST = SST(L)
      SIN2ST = S2ST(L)
      COS2ST = C2ST(L)
      FAC1 = SIND*COSS*COS2ST
      FAC2 = SIND*COSS*SIN2ST
      FAC3 = -SIND*COSD*SINS*COS2ST
      FAC4 = -SIND*COSD*SINS*SIN2ST
      FAC5 = COS2D*SINS*CCSST
      FAC6 = COS2D*SINS*SINST
      FAC7 = -COSD*COSS*CCSST
      FAC8 = -COSD*COSS*SINST
      FAC9 = 2.*SIND*COSD*SINS
      ML = 0
      DO 3008 KL = 1,JLMX
C     THE OUTPUT OF EIGEN IS IN CGS UNITS THIS STEP IS TAKEN SO THAT
C     THEORETICAL AMPLITUDES ARE NOT TOO SMALL FOR MACHINE ,LT. 1E-19
      PY1 = PYL1(I,KL)*1.0E20
      PY2 = PYL2(I,KL)*1.0E20
      NUM = NNUML(KL)
      DO 3006 M = 1,NUM
      ML = ML + 1
      SAZ = SINAZ(1,ML)
      CAZ = COSAZ(1,ML)
      CALL EXCIT(FAC1,FAC2,FAC3,FAC4,FAC5,FAC6,FAC7,FAC8,FAC9,
     1 PY1,PY2,PY3,PY4,1,SAZ,CAZ,AMP,1)
 3006 YL(ML) = AMP
 3008 CONTINUE
```

```
      MR = 0
      DO 3009 KR = 1,JRMX
C     THE OUTPUT OF EIGEN IS IN CGS UNITS THIS STEP IS TAKEN SO THAT
C     THEORETICAL AMPLITUDES ARE NOT TOO SMALL FOR MACHINE .LT. 1E-19
      PY1 = PYR1(I,KR)*1.0E20
      PY2 = PYR2(I,KR)*1.0E20
      PY3 = PYR3(I,KR)*1.0E20
      PY4 = PYR4(I,KR)*1.0E20
      NUM = NNUMR(KR)
      DO 3007 M = 1,NUM
      MR = MR + 1
      SAZ = SINAZ(2,MR)
      CAZ = COSAZ(2,MR)
      CALL EXCIT(FAC1,FAC2,FAC3,FAC4,FAC5,FAC6,FAC7,FAC8,FAC9,
     1 PY1,PY2,PY3,PY4,2,SAZ,CAZ,AMP,1)
 3007 YR(MR) = AMP
 3009 CONTINUE
      CALL RRESS(AL,NR,XL,XR,YL,YR,RL,RR,S,RESL,RESR,RES,SL,SR)
C     THE OUTPUT OF EIGEN IS IN CGS UNITS THIS STEP IS TAKEN SO THAT
C     THEORETICAL AMPLITUDES ARE NOT TOO SMALL FOR MACHINE .LT. 1E-19
      S = S * 1.0E20
      SL = SL * 1.0E20
      SR = SR*1.0E20
      ICROAK = M1
      IF(RL.LT.0.OR.RR.LT.0) GO TO 421
      SLDSR = SL/SR
      IF(SLDSR.GT.0.6.AND.SLDSR.LT.1.7) ICROAK = M2
      WRITE(61,14) AAA(I),SLIP(K),DIP(J),STRK(L),S,RR,RESR,RL,RESL,RES
     1 ,SL,SR,ICROAK
      IF(SLDSR.GT.1) SLDSR = 1./SLDSR
      PROD = RR * RL * SLDSR
      IF(RR.LE.GRR) GO TO 402
      GRR=RR
      IS(1)=K
      ID(1)=J
      IP(1)=L
  402 IF(RL.LE.GRL) GO TO 403
      GRL=RL
      IS(2)=K
      ID(2)=J
      IP(2)=L
  403 IF(RES.GE.SRES) GO TO 404
      SRES=RES
      IS(3)=K
      ID(3)=J
      IP(3)=L
  404 IF(RESR.GE.SRESR) GO TO 405
      SRESR=RESR
      IS(4)=K
      ID(4)=J
```

```
      IP(4)=L
  405 IF(RESL.GE.SRESL) GO TO 410
      SRESL=RESL
      IS(5)=K
      ID(5)=J
      IP(5)=L
  410 IF(PROD.LE.PMRR) GO TO 411
      PMRR=PROD
      IS(6)=K
      ID(6)=J
      IP(6)=L
  411 CONTINUE
  421 CONTINUE
 3003 CONTINUE
 3002 CONTINUE
 3001 CONTINUE
      XBEST(1) = SRR
      XBEST(2) = SRL
      XBEST(3) = SRES
      XBEST(4) = SRESR
      XBEST(5) = SRESL
      XBEST(6) = PMRR
      DO 420 NN = 1,6
      KK=IS(NN)
      JJ=ID(NN)
      LL=IP(NN)
  420 WRITE(61,13) IBEST(NN),SLIP(KK),DIP(JJ),STRK(LL),XBEST(NN)
 3000 CONTINUE
 9999 CONTINUE
      WRITE(61,2) NR,NL
      REWIND 1
      REWIND 2
      END

      SUBROUTINE RRESS(NL,NR,XL,XR,YL,YR,RL,RR,S,RESL,RESR,RES,SL,SR)
      DIMENSION XL(1),XR(1),YL(1),YR(1)
      SUML0 = 0
      SUML1 = 0
      SUML2 = 0
      SUML3 = 0
      SUML4 = 0
      SUML5 = 0
      SUMR0 = 0
      SUMR1 = 0
      SUMR2 = 0
      SUMR3 = 0
      SUMR4 = 0
      SUMR5 = 0
      DO 601 I = 1,NL
      SUML0 = SUML0 + 1.
```

```
      SUML1 = SUML1 + XL(I)
      SUML2 = SUML2 + XL(I) * XL(I)
      SUML3 = SUML3 + YL(I)
      SUML4 = SUML4 + YL(I) * YL(I)
  601 SUML5 = SUML5 + XL(I) * YL(I)
      DO 602 I = 1,NR
      SUMR0 = SUMR0 + 1.
      SUMR1 = SUMR1 + XR(I)
      SUMR2 = SUMR2 + XR(I) * XR(I)
      SUMR3 = SUMR3 + YR(I)
      SUMR4 = SUMR4 + YR(I) *YR(I)
  602 SUMR5 = SUMR5 + XR(I) * YR(I)
      RL = (SUML0*SUML5-SUML1*SUML3)/ SQRT((SUML0*SUML2-SUML1*SUML1) *
     1 (SUML0*SUML4-SUML3*SUML3))
      RR = (SUMR0*SUMR5-SUMR1*SUMR3)/SQRT((SUMR0*SUMR2-SUMR1*SUMR1) *
     1 (SUMR0*SUMR4-SUMR3*SUMR3))
      SL = SUML5/SUML4
      SR = SUMR5/SUMR4
      S = (SUML0 * SL + SUMR0 * SR)/(SUML0 + SUMR0)
      RESL = SUML2 - 2.*S*SUML5 + S*S*SUML4
      RESR = SUMR2 - 2.*S*SUMR5 + S*S*SUMR4
      RES = SQRT(ABS(RESL + RESR))
      RETURN
      END
```

```
DATA    PAGE  1

3500.
   3
5.
10.
15.
   3      45.       15.
   6     -90.       30.
   6      0.0       30.
6.0         1     0    1    6.0000E-04           0.8594E 23
BLAT     76.0       917.1                 2.7991E-03          3.4329        6.
MBCT    350.2      4758.5                 4.8066E-04          3.4202        6.
HALT
7.0         1     0    1    5.0000E-04           1.0027E 23
AAMT     36.2       941.4                 3.4337E-03          3.3160        7.
BLOT     40.3       526.4                 2.2932E-03          3.1493        7.
MNTT     47.2       1793.                 8.9092E-04          3.5107        7.
CLET      47.4      1014.30               2.5317E-03          3.3267        7.
WEST     59.8      1815.0                 1.3002E-03          3.3263        7.
BLAT     76.0       917.1                 2.3604E-03          3.4752        7.
MBCT    350.2      4758.5                 9.1541E-04          3.4288        7.
HALT
8.0         1     0    1    4.0000E-04           1.1459E 23
AAMT     36.2       941.4                 3.0654E-03          3.2870        8.
BLOT     40.3       526.4                 2.6165E-03          3.2615        8.
MNTT     47.2       1793.                 1.8601E-03          3.4800        8.
CLET      47.4      1014.30               3.7731E-03          3.3321        8.
WEST     59.8      1815.0                 1.8485E-03          3.3680        8.
BLAT     76.0       917.1                 2.4649E-03          3.4458        8.
ATLT    111.6       612.0                 1.6026E-03          3.5968        8.
MBCT    350.2      4758.5                 1.5448E-03          3.4338        8.
FCCT     355.2      2591.60               4.6747E-04          3.4472        8.
HALT
9.0         1     0    1    3.0000E-04           1.2892E 23
SLMT      3.2       335.3                 9.0595E-04          3.2824        9.
AAMT     36.2       941.4                 3.0412E-03          3.3368        9.
BLOT     40.3       526.4                 3.2108E-03          3.4093        9.
MNTT     47.2       1793.                 2.7919E-03          3.4834        9.
CLET      47.4      1014.30               4.3481E-03          3.0390        9.
STJT     55.1      3374.1                 2.3679E-03          3.3829        9.
WEST     59.8      1815.0                 2.1790E-03          3.3947        9.
BLAT     76.0       917.1                 2.8796E-03          3.4169        9.
ATLT    111.6       612.0                 2.5249E-03          3.5345        9.
MSOT    309.7      2316.3                 1.8667E-03          3.2355        9.
MBCT    350.2      4758.5                 2.0619E-03          3.4425        9.
FCCT     355.2      2591.60               5.1259E-04          3.4472        9.
HALT
10.0        1     0    1    2.0000E-04           1.4324E 23
SLMT      3.2       335.3                 9.9339E-04          3.3480       10.
CHCT     3.6        1425.30               6.4216E-04          3.4528       10.
AAMT     36.2       941.4                 3.4497E-03          3.3937       10.
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DATA    PAGE  2

BLOT     40.3      526.4                     3.6821E-03        3.4260     10.
OTTT     44.4      1650.                     2.1226E-03        3.4473     10.
MNTT     47.2      1793.                     3.4781E-03        3.4868     10.
CLET      47.4      1014.30                  4.7529E-03        3.3486     10.
STJT     55.1     3374.1                     3.8204E-03        3.3931     10.
WEST     59.8     1815.0                     2.9500E-03        3.4300     10.
BLAT     76.0      917.1                     2.8878E-03        3.4297     10.
ATLT    111.6      612.0                     3.1447E-03        3.5396     10.
LONT     305.1      2881.8                   1.1006E-03        3.2420     10.
MSOT    309.7     2316.3                     3.2504E-03        3.2698     10.
PNTT    311.8     2813.9                     1.6218E-03        3.3011     10.
EDMT    324.7     2651.9                     6.9380E-03        3.2265     10.
MBCT    350.2     4758.5                     2.5056E-03        3.4462     10.
FCCT     355.2      2591.60                  6.2815E-04        3.4244     10.
HALT
12.0         1     0    1    1.8000E-06            1.7189E 23
SLMT      3.2      335.3                     8.7832E-04        3.4162     12.
LHCT     3.6       1425.30                   6.7490E-04        3.5058     12.
AAMT     36.2      941.4                     4.6993E-03        3.4152     12.
BLOT     40.3      526.4                     4.7110E-03        3.3874     12.
OTTT     44.4      1650.                     3.5122E-03        3.4437     12.
CLET      47.4     1014.30                   4.8991E-03        3.3821     12.
MNTT     47.2      1793.                     4.5247E-03        3.4800     12.
STJT     55.1     3374.1                     4.6678E-03        3.4051     12.
WEST     59.8     1815.0                     4.7602E-03        3.4512     12.
BLAT     76.0      917.1                     2.3993E-03        3.4426     12.
ATLT    111.6      612.0                     3.3696E-03        3.5092     12.
LUBT    261.1     1069.7                     3.5037E-03        3.0814     12.
ALQT     271.7      1455.8                   1.6857E-03        3.3038     12.
DUGT    291.2     2024.7                     2.2743E-03        3.2426     12.
NEWT    303.7     2489.9                     3.3736E-03        3.1914     12.
LONT     305.1     2881.8                    2.7395E-03        3.2791     12.
MSOT    309.7     2316.3                     3.9906E-03        3.3071     12.
PNTT    311.8     2813.9                     2.1071E-03        3.4174     12.
EDMT    324.7     2651.9                     7.7152E-03        3.3050     12.
FFCT    340.9     2307.3                     7.0549E-04        3.5590     12.
MBCT    350.2     4758.5                     2.4532E-03        3.4663     12.
FCCT     355.2      2591.60                  6.5769E-04        3.4438     12.
HALT
14.          1     0    1    1.6000E-04            2.0054E 23
SLMT      3.2      335.3                     8.2858E-04        3.4603     14.
LHCT     3.6       1425.30                   8.4011E-04        3.4217     14.
AAMT     36.2      941.4                     5.1475E-03        3.4339     14.
BLOT     40.3      526.4                     4.8098E-03        3.4204     14.
OTTT     44.4      1650.                     4.2765E-03        3.4727     14.
MNTT     47.2      1793.                     4.8216E-03        3.4868     14.
CLET      47.4      1014.30                  4.8599E-03        3.3934     14.
STJT     55.1     3374.1                     4.5359E-03        3.4224     14.
WEST     59.8     1815.0                     5.1680E-03        3.4644     14.
BLAT     76.0      917.1                     2.2099E-03        3.4297     14.
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DATA     PAGE  3


ATLT      111.6      612.0                  3.1774E-03         3.4892     14.
LUBT      261.1     1069.7                  3.4543E-03         3.1287     14.
TUCT       264.6     1916.8                 2.0117E-03         3.2384     14.
ALQT       271.7     1455.8                 1.2076E-03         3.3513     14.
DUGT      291.2     2024.7                  3.0225E-03         3.3143     14.
NEWT      303.7     2489.9                  4.9861E-03         3.2213     14.
LONT       305.1     2881.8                 3.4397E-03         3.3185     14.
MSOT      309.7     2316.3                  4.7667E-03         3.3599     14.
PNTT      311.8     2813.9                  4.2536E-03         3.3906     14.
EDMT      324.7     2651.9                  7.8316E-03         3.3615     14.
FFCT      340.9     2307.3                  1.7451E-03         3.4838     14.
MBCT      350.2     4758.5                  2.0222E-03         3.4789     14.
FCCT       355.2     2591.60                4.7894E-04         3.4575     14.
HALT
16.          1     0     1     1.4000E-04          2.2918E 23
SLMT       3.2       335.3                  8.4891E-04         3.3989     16.
LHCT      3.6        1425.30                6.8459E-04         3.4802     16.
AAMT      36.2       941.4                  5.1925E-03         3.4276     16.
BLOT      40.3       526.4                  4.4153E-03         3.4484     16.
OTTT      44.4       1650.                  4.9422E-03         3.4836     16.
MNTT      47.2       1793.                  4.5981E-03         3.4970     16.
CLET       47.4      1014.30                4.4010E-03         3.3878     16.
STJT      55.1      3374.1                  4.2669E-03         3.4486     16.
WEST      59.8      1815.0                  5.1962E-03         3.4810     16.
BLAT      76.0       917.1                  2.3541E-03         3.5017     16.
ATLT      111.6      612.0                  3.4088E-03         3.4991     16.
LUBT      261.1     1069.7                  2.9095E-03         3.2037     16.
TUCT       264.6     1916.8                 1.6224E-03         3.2604     16.
ALQT       271.7     1455.8                 1.9990E-03         3.3494     16.
DUGT      291.2     2024.7                  3.2214E-03         3.3583     16.
NEWT      303.7     2489.9                  5.5236E-03         3.2151     16.
LONT       305.1     2881.8                 3.3148E-03         3.3513     16.
MSOT      309.7     2316.3                  5.3376E-03         3.3697     16.
PNTT      311.8     2813.9                  5.7971E-03         3.3968     16.
EDMT      324.7     2651.9                  7.7983E-03         3.3830     16.
FFCT      340.9     2307.3                  2.4308E-03         3.4970     16.
MBCT      350.2     4758.5                  1.7778E-03         3.4853     16.
HALT
18.          1     0     1     1.2000E-04          2.5783E 23
SLMT       3.2       335.3                  9.0823E-04         3.3564     18.
LHCT      3.6        1425.30                8.1978E-04         3.4908     18.
AAMT      36.2       941.4                  5.2242E-03         3.4623     18.
BLOT      40.3       526.4                  3.9577E-03         3.4826     18.
OTTT      44.4       1650.                  4.9760E-03         3.4984     18.
MNTT      47.2       1793.                  4.2965E-03         3.5004     18.
CLET       47.4      1014.30                3.9117E-03         3.4163     18.
STJT      55.1      3374.1                  4.5650E-03         3.4645     18.
WEST      59.8      1815.0                  5.1315E-03         3.4927     18.
BLAT      76.0       917.1                  2.4508E-03         3.5253     18.
ATLT      111.6      612.0                  3.2956E-03         3.5345     18.
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DATA    PAGE  4


LUBT     261.1     1069.7                     2.7728E-03        3.1941     18.
TUCT      264.6     1916.8                    1.4905E-03        3.2480     18.
ALQT      271.7     1455.8                    2.7499E-03        3.3075     18.
DUGT     291.2     2024.7                     3.3274E-03        3.3807     18.
NEWT     303.7     2489.9                     5.5151E-03        3.2370     18.
LONT      305.1     2881.8                    3.3145E-03        3.3670     18.
MSOT     309.7     2316.3                     4.9811E-03        3.3820     18.
PNTT     311.8     2813.9                     5.5888E-03        3.4174     18.
EDMT     324.7     2651.9                     7.1691E-03        3.4096     18.
FFCT     340.9     2307.3                     2.4461E-03        3.5076     18.
MBCT     350.2     4758.5                     1.8372E-03        3.4802     18.
HALT
20.         1     0    1     1.0000E-04               2.8648E 23
SLMT      3.2       335.3                     9.1387E-04        3.4603     20.
LHCT     3.6        1425.30                   1.0844E-03        3.4444     20.
AAMT     36.2       941.4                     5.0996E-03        3.4655     20.
BLOT     40.3       526.4                     3.8283E-03        3.4769     20.
OTTT     44.4       1650.                     4.9274E-03        3.5021     20.
MNTT     47.2       1793.                     4.0911E-03        3.5072     20.
CLET      47.4      1014.30                   4.0533E-03        3.4395     20.
STJT     55.1      3374.1                     4.8742E-03        3.4770     20.
WEST     59.8      1815.0                     4.8487E-03        3.5198     20.
BLAT     76.0       917.1                     2.4357E-03        3.4984     20.
ATLT     111.6      612.0                     3.0189E-03        3.5602     20.
LUBT     261.1     1069.7                     3.0454E-03        3.1917     20.
TUCT      264.6     1916.8                    1.4179E-03        3.2425     20.
ALQT      271.7     1455.8                    3.0171E-03        3.3094     20.
DUGT     291.2     2024.7                     3.5706E-03        3.3920     20.
NEWT     303.7     2489.9                     5.1862E-03        3.2592     20.
LONT      305.1     2881.8                    3.5135E-03        3.3788     20.
MSOT     309.7     2316.3                     4.3631E-03        3.4043     20.
PNTT     311.8     2813.9                     5.4352E-03        3.4257     20.
EDMT     324.7     2651.9                     6.5454E-03        3.4245     20.
FFCT     340.9     2307.3                     2.4413E-03        3.4996     20.
MBCT     350.2     4758.5                     1.8751E-03        3.4904     20.
FCCT      355.2     2591.60                   2.6374E-04        3.3416     20.
HALT
22.         1     0    1     0.8000E-04               3.1513E 23
SLMT      3.2       335.3                     9.0117E-04        3.4964     22.
LHCT     3.6        1425.30                   1.1741E-03        3.4074     22.
AAMT     36.2       941.4                     4.5990E-03        3.4370     22.
BLOT     40.3       526.4                     3.8602E-03        3.4093     22.
MNTT     47.2       1793.                     3.8788E-03        3.5107     22.
CLET      47.4      1014.30                   3.8511E-03        3.4221     22.
STJT     55.1      3374.1                     5.2060E-03        3.4860     22.
WEST     59.8      1815.0                     4.4698E-03        3.5387     22.
BLAT     76.0       917.1                     2.3462E-03        3.4818     22.
ATLT     111.6      612.0                     2.8630E-03        3.5706     22.
LUBT     261.1     1069.7                     2.3160E-03        3.1681     22.
TUCT      264.6     1916.8                    1.2224E-03        3.3168     22.
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DATA    PAGE  5


ALQT     271.7     1455.8             2.8285E-03        3.3417    22.
DUGT    291.2     2024.7              3.5712E-03        3.4092    22.
NEWT    303.7     2489.9              4.7496E-03        3.2753    22.
LONT     305.1     2881.8             3.5530E-03        3.4108    22.
MSOT    309.7     2316.3              4.0858E-03        3.4245    22.
PNTT    311.8     2813.9              5.3097E-03        3.4404    22.
EDMT    324.7     2651.9              5.9631E-03        3.4355    22.
FFCT    340.9     2307.3              2.4430E-03        3.5049    22.
MBCT    350.2     4758.5              1.6628E-03        3.5110    22.
FCCT     355.2     2591.60            2.5214E-04        3.3666    22.
HALT
24.         1    0    1    0.8000E-04         3.4377E 23
SLMT      3.2      335.3              8.9556E-04        3.5519    24.
LHCT     3.6       1425.30            1.1334E-03        3.4114    24.
AAMT     36.2      941.4              4.8287E-03        3.4433    24.
BLOT     40.3     526.4               3.8452E-03        3.3874    24.
OTTT     44.4      1650.              4.6535E-03        3.5321    24.
MNTT     47.2      1793.              3.4910E-03        3.5559    24.
CLET      47.4     1014.30            3.2497E-03        3.4163    24.
STJT     55.1     3374.1              5.8823E-03        3.4770    24.
WEST     59.8     1815.0              4.2839E-03        3.5422    24.
BLAT     76.0      917.1              2.2883E-03        3.5017    24.
ATLT    111.6      612.0              2.8225E-03        3.5758    24.
LUBT    261.1     1069.7              1.6599E-03        3.1241    24.
TUCT     264.6     1916.8             1.1884E-03        3.4327    24.
ALQT     271.7     1455.8             2.5445E-03        3.3668    24.
DUGT    291.2     2024.7              3.3038E-03        3.4440    24.
NEWT    303.7     2489.9              4.5242E-03        3.2861    24.
LONT     305.1     2881.8             3.4819E-03        3.4455    24.
MSOT    309.7     2316.3              4.1010E-03        3.4474    24.
PNTT    311.8     2813.9              5.0413E-03        3.4594    24.
EDMT    324.7     2651.9              5.7177E-03        3.4490    24.
FFCT    340.9     2307.3              2.3631E-03        3.5076    24.
MBCT    350.2     4758.5              1.4003E-03        3.5280    24.
FCCT     355.2     2591.60            2.5572E-04        3.2730    24.
HALT
26.         1    0    1    0.8000E-04         3.7242E 23
SLMT      3.2      335.3              9.1227E-04        3.6288    26.
LHCT     3.6       1425.30            9.6413E-04        3.4951    26.
AAMT     36.2      941.4              5.3947E-03        3.6014    26.
BLOT     40.3      526.4              3.6288E-03        3.4149    26.
OTTT     44.4      1650.              4.4517E-03        3.5473    26.
MNTT     47.2      1793.              3.1529E-03        3.6060    26.
CLET      47.4     1014.30            3.0750E-03        3.5602    26.
STJT     55.1     3374.1              6.0512E-03        3.4842    26.
WEST     59.8     1815.0              4.0293E-03        3.5613    26.
BLAT     76.0      917.1              2.2262E-03        3.5423    26.
ATLT    111.6      612.0              2.7307E-03        3.5758    26.
LUBT    261.1     1069.7              1.9948E-03        3.1751    26.
TUCT     264.6     1916.8             1.1783E-03        3.4889    26.
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DATA     PAGE  6

ALQT      271.7     1455.8                            2.2749E-03             3.3824        26.
DUGT     291,2     2024.7                            3,0513E-03             3,4675        26.
NEWT     303,7     2489.9                            4,3957E-03             3.3025        26,
LONT      305,1     2881,8                            3,4277E-03             3.4729        26,
MSOT     309.7     2314.3                            4.2757E-03             3,4654        26.
PNTT     311,8     2813.9                            4,6407E-03             3,4765        26.
EDMT     324,7     2651,9                            5,7280E-03             3,4715        26,
FFCT     340,9     2307.3                            2,2307E-03             3,5183        26,
MBCT     350,2     4758.5                            1.2157E-03             3.5398        26,
HALT
28,          1       0     1      0.8000E-04                4,0107E 23
SLMT        3,2      335,3                            1,0200E-03             3.4692        28.
LHCT       3.6        1425.30                          8.3968E-04             3,5606        28.
AAMT      36,2      941,4                             6,0470E-03             3,8362        28,
BLOT      40,3      526,4                             3,4863E-03             3,4428        28,
OTTT      44,4      1650.                             4,2973E-03             3,5664        28,
MNTT      47,2      1793,                             2.9387E-03             3,6169        28,
STJT      55,1     3374,1                             5.2375E-03             3,5427        28,
WEST      59,8     1815,0                             3,7179E-03             3,6055        28.
BLAT      76,0      917,1                             2.1691E-03             3.5944        28.
ATLT     111,6      612,0                             2.6015E-03             3,6074        28,
LUBT     261,1     1069,7                             3,2476E-03             3,3154        28,
TUCT      264,6     1916,8                            1,1281E-03             3,4984        28,
ALQT      271,7     1455,8                            1,9144E-03             3,3610        28,
DUGT     291,2     2024.7                            3,0604E-03             3,4675        28,
NEWT     303,7     2489,9                            4,8622E-03             3,3312        28,
LONT      305,1     2881,8                            3.3213E-03             3,4978        28.
MSOT     309,7     2316,3                            4,4054E-03             3,4863        28,
PNTT     311,8     2813,9                            4,1707E-03             3,5025        28,
EDMT     324,7     2651,9                            5.4612E-03             3,4967        28,
FFCT     340,9     2307.3                            2,0361E-03             3,5535        28,
MBCT     350,2     4758.5                            1,0819E-03             3,5623        28,
FCCT      355,2     2591,60                           6,8163E-04             3,4541        28,
HALT
30,          1       0     1      0.8000E-04                4,2972E 23
LHCT       3,6        1425,30                          8,0098E-04             3.5606        30.
AAMT      36,2      941,4                             6,4815E-03             3,8998        30.
BLOT      40,3      526,4                             3,3159E-03             3,4484        30.
OTTT      44,4      1650,                             4,2371E-03             3,5858        30,
MNTT      47,2      1793,                             2,8852E-03             3,6023        30.
STJT      55,1     3374,1                             4,3225E-03             3,6071        30,
WEST      59,8     1815,0                             3,4065E-03             3,6435        30.
BLAT      76,0      917,1                             2,0655E-03             3,6335        30.
ATLT     111,6      612,0                             2,4118E-03             3,6779        30,
LUBT     261,1     1069,7                             4,5324E-03             3,3283        30.
TUCT      264,6     1916,8                            1,8736E-03             3,4905        30.
DUGT     291,2     2024,7                            3,2708E-03             3,4469        30.
NEWT     303,7     2489,9                            3,6209E-03             3,3661        30.
LONT      305,1     2881,8                            3,2187E-03             3,5170        30.
MSOT     309,7     2316,3                            4,3618E-03             3,5021        30,
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DATA     PAGE  7


PNTT     311,8     2813,9                    3,8042E-03          3,5444      30,
EDMT     324,7     2651,9                    4,7444E-03          3,4990      30,
FFCT     340,9     2307,3                    1,8617E-03          3,6063      30,
MBCT     350,2     4758,5                    9,2581E-04          3,5959      30,
HALT
35,          1     0     1    0.6000E-04              5,0134E 23
LHCT       3,6      1425,30                  9,1639E-04          3,6471      35,
AAMT      36,2      941,4                    7,0043E-03          4,1444      35,
BLOT      40,3      526,4                    2,7832E-03          3,6902      35,
OTTT      44,4      1650,                    4,2554E-03          3,7191      35,
MNTT      47,2      1793,                    2,8678E-03          3,6132      35,
STJT      55,1     3374,1                    3,7171E-03          3,7021      35,
WEST      59,8     1815,0                    2,8168E-03          3,6860      35,
BLAT      76,0      917,1                    1,7757E-03          3,7525      35,
ATLT     111,6      612,0                    1,9882E-03          3,9006      35,
LUBT     261,1     1069,7                    4,6987E-03          3,6029      35,
TUCT      264,6     1916,8                   1,0858E-03          3,5986      35,
DUGT     291,2     2024,7                    2,7073E-03          3,5969      35,
NEWT     303,7     2489,9                    2,9423E-03          3,3638      35,
LONT      305,1     2881,8                   2,9365E-03          3,5386      35,
MSOT     309,7     2316,3                    3,1867E-03          3,4346      35,
PNTT     311,8     2813,9                    3,2986E-03          3,6525      35,
EDMT     324,7     2651,9                    2,5629E-03          3,5222      35,
FFCT     340,9     2307,3                    1,7760E-03          3,7317      35,
HALT
40,          1     0     1    0,6000E-04              5,7296E 23
OTTT      44,4      1650,                    3,9975E-03          3,6777      40,
MNTT      47,2      1793,                    2,6279E-03          3,7027      40,
STJT      55,1     3374,1                    3,9163E-03          3,8065      40,
WEST      59,8     1815,0                    2,3149E-03          3,7919      40,
BLAT      76,0      917,1                    1,4325E-03          3,7991      40,
ATLT     111,6      612,0                    1,5001E-03          4,0157      40,
TUCT      264,6     1916,8                   1,1493E-03          3,5290      40,
DUGT     291,2     2024,7                    2,4258E-03          3,6033      40,
NEWT     303,7     2489,9                    1,9112E-03          3,3672      40,
LONT      305,1     2881,8                   2,6314E-03          3,5604      40,
PNTT     311,8     2813,9                    2,5364E-03          3,7201      40,
FFCT     340,9     2307,3                    1,4313E-03          3,7621      40,
HALT
45,          1     0     1    0,6000E-04              6,4458E 23
OTTT      44,4      1650,                    3,9628E-03          3,7066      45,
MNTT      47,2      1793,                    2,1168E-03          3,8008      45,
STJT      55,1     3374,1                    3,5916E-03          3,8966      45,
WEST      59,8     1815,0                    1,8029E-03          3,8749      45,
ATLT     111,6      612,0                    1,1483E-03          4,3281      45,
LONT      305,1     2881,8                   2,2098E-03          3,7165      45,
PNTT     311,8     2813,9                    1,7006E-03          3,8952      45,
FFCT     340,9     2307,3                    1,2329E-03          3,7837      45,
HALT
50,          1     0     1    0,6000E-04              7,1620E 23
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DATA     PAGE  8

MNTT      47,2     1793.                 1,6703E-03        3,6837      50.
STJT      55,1    3374,1                 3,2756E-03        4.0389      50,
PNTT     311,8    2813.9                 1,5283E-03        3.8605      50.
FFCT     340.9    2307.3                 1.2582E-03        3.8988      50.
HALT
-1,
5.0          1    0    1   7,0000E-04           0,7162E 23
SLMZ        3,2     335,3                1,0161E-03        3.0099       5,
ATLZ     111,6      611,2                1,4279E-03        2,8878       5,
ROLZ     334,2      284,2                1,0276E-03        3,0299       5,
HALT
6.0          1    0    1   6,0000E-04           0,8594E 23
SLMZ        3,2     335,3                2.7808E-03        3.0166       6,
BLOZ      40,3      526,4                1,5474E-03        2,8585       6,
ATLZ     111,6      611.20               2,3801E-03        2,9829       6,
EDMZ     324,7    2651.9                 8.3150E-04        2.8230       6,
ROLZ     334,2      284,2                1,4814E-03        2,9744       6,
HALT
7,0          1    0    1   5,0000E-04           1,0027E 23
SLMZ        3,2     335,3                3,9518E-03        3.0166       7,
LHCZ       3.6      1425,30              1.0498E-03        3.0847       7.
AAMZ      36,2      941.4                2,5370E-03        2.9590       7,
BLOZ      40,3      526.4                2,4028E-03        2,8979       7,
WES1      59,8    1815.0                 1.0318E-03        2.8929       7,
BLAZ      76,0      917.1                1,3893E-03        3,0504       7.
ATLZ     111,6      611.2                2,7269E-03        3.0235       7,
MSOZ     309,7    2316.3                 1,2152E-03        2.9231       7,
EDMZ     324,7    2651.9                 1,4487E-03        2.8320       7,
ROLZ     334,2      284.2                2,3577E-03        3,0875       7,
MBCZ     350,2    4758.5                 1.0460E-03        3.0724       7,
FCCZ      355,2     2591,60              7.7257E-04        3.1497       7.
HALT
8,0          1    0    1   4.0000E-04           1,1459E 23
SLMZ        3.2     335,3                4,5646E-03        3.0861       8,
LHCZ       3,6      1425,30              1.6264E-03        3,0998       8,
AAMZ      36,2      941.4                3,2005E-03        3.0014       8,
BLOZ      40,3      526.4                2,5102E-03        2,9631       8,
OTTZ      44,4      1650,9               1,3664E-03        3,1096       8.
MNTZ      47,2      1793.6               9,9463E-04        3,0074       8,
STJZ      55,1    3374,1                 7,2545E-04        3.0527       8,
WES1      59,8    1815.0                 1,3461E-03        2,9894       8,
BLAZ      76,0      917.1                1,6706E-03        3,0889       8.
ATLZ     111,6      611.2                3,1515E-03        3.0768       8,
MSOZ     309,7    2316.3                 1.6655E-03        2,9213       8,
EDMZ     324,7    2651,9                 1,4166E-03        2,8672       8,
ROLZ     334,2      284.2                2,4695E-03        3.1128       8,
MBCZ     350,2    4758.5                 1.9471E-03        3.0813       8,
FCCZ      355,2     2591,60              1,5107E-03        3.1345       8,
HALT
9.0          1    0    1   3.0000E-04           1.2892E 23
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 DATA     PAGE  9


 SLMZ        3.2      335.3                       4.3259E-03        3.1075       9.
 LHCZ       3.6       1425.30                     2.0393E-03        3.1066       9.
 AAMZ       36.2      941.4                       3.8213E-03        3.0353       9.
 BLOZ       40.3      526.4                       2.6020E-03        3.0270       9.
 OTTZ       44.4      1650.                       1.0704E-03        3.1482       9.
 MNTZ       47.2      1793.                       1.0225E-03        3.1253       9.
 CLER        47.4     1014.30                     1.0802E-03        2.9929       9.
 STJZ       55.1     3374.1                       1.6571E-03        3.0496       9.
 WES1       59.8     1815.0                       1.6408E-03        3.0205       9.
 BLAZ       76.0      917.1                       1.8356E-03        3.1418       9.
 ATLZ      111.6      611.2                       3.2410E-03        3.1319       9.
 LUBZ      261.1     1069.7                       2.0061E-03        2.8667       9.
 TUCZ       264.6     1916.8                      1.3115E-03        2.8069       9.
 ALQZ       271.7     1455.8                      8.1349E-04        3.0085       9.
 MSOZ      309.7     2316.3                       2.0064E-03        2.9398       9.
 EDMZ      324.7     2651.9                       1.6904E-03        2.9097       9.
 ROLZ      334.2      284.2                       2.1941E-03        3.0543       9.
 FFCZ      340.9     2307.3                       6.8574E-04        3.1508       9.
 MBCZ      350.2     4758.5                       2.9412E-03        3.0954       9.
 FCCZ       355.2     2591.60                     1.9887E-03        3.1392       9.
 HALT
10.0         1     0     1     2.0000E-04               1.4324E 23
 SLMZ        3.2      335.3                       3.5582E-03        3.0932      10.
 LHCZ       3.6       1425.30                     2.1355E-03        3.1066      10.
 AAMZ       36.2      941.4                       2.8828E-03        3.0775      10.
 BLOZ       40.3      526.4                       2.3525E-03        3.0270      10.
 OTTZ       44.4      1650.                       1.3934E-03        3.1422      10.
 MNTZ       47.2      1793.                       1.1206E-03        3.1473      10.
 CLER        47.4     1014.30                     1.4342E-03        2.9929      10.
 STJZ       55.1     3374.1                       2.2226E-03        3.0538      10.
 WES1       59.8     1815.0                       1.8628E-03        3.0458      10.
 BLAZ       76.0      917.1                       1.9044E-03        3.1418      10.
 ATLZ      111.6      611.2                       2.9250E-03        3.1360      10.
 LUBZ      261.1     1069.7                       2.5717E-03        2.8705      10.
 TUCZ       264.6     1916.8                      1.1871E-03        2.8048      10.
 ALQZ       271.7     1455.8                      1.1165E-03        3.0131      10.
 LONZ       305.1     2881.8                      1.2889E-03        2.9712      10.
 MSOZ      309.7     2316.3                       2.0530E-03        2.9662      10.
 EDMZ      324.7     2651.9                       1.3119E-03        2.9338      10.
 ROLZ      334.2      284.2                       2.1952E-03        3.0218      10.
 FFCZ      340.9     2307.3                       8.8492E-04        3.1572      10.
 FCCZ       355.2     2591.60                     1.8552E-03        3.1488      10.
 HALT
12.0         1     0     1     1.8000E-04               1.7189E 23
 SLMZ        3.2      335.3                       2.8389E-03        3.1220      12.
 LHCZ       3.6       1425.30                     1.9254E-03        3.1168      12.
 AAMZ       36.2      941.4                       2.2893E-03        3.0699      12.
 BLOZ       40.3      526.4                       1.7311E-03        3.0183      12.
 OTTZ       44.4      1650.                       1.9867E-03        3.1038      12.
 MNTZ       47.2      1793.                       1.4572E-03        3.1280      12.
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DATA     PAGE 10


CLER       47.4     1014.30                       1.3160E-03          2.9973       12.
STJZ      55.1     3374.1                        2.5086E-03          3.0565       12.
WES1      59.8     1815.0                        2.3676E-03          3.0561       12.
BLAZ      76.0      917.1                        2.2352E-03          3.0811       12.
ATLZ     111.6      611.2                        2.4221E-03          3.1808       12.
TUCZ      264.6     1916.8                        9.1576E-04          2.9642       12.
ALQZ      271.7     1455.8                        2.1401E-03          2.9401       12.
DUGZ     291.2     2024.7                        2.1988E-03          2.9736       12.
LONZ      305.1     2881.8                        8.0948E-04          3.0420       12.
MSOZ     309.7     2316.3                        2.0729E-03          3.0027       12.
EDMZ     324.7     2651.9                        1.9007E-03          3.0036       12.
ROLZ     334.2      284.2                        2.3067E-03          3.1214       12.
FFCZ     340.9     2307.3                        1.2413E-03          3.1572       12.
MBCZ     350.2     4758.5                        3.4356E-03          3.0863       12.
FCCZ      355.2     2591.60                       1.1791E-03          3.1430       12.
HALT
14.          1     0     1     1.6000E-04             2.0054E 23
SLMZ        3.2     335.3                        2.6050E-03          3.1513       14.
LHCZ      3.6       1425.30                       1.8692E-03          3.1100       14.
AAMZ      36.2      941.4                        1.9791E-03          3.0329       14.
BLOZ      40.3      526.4                        1.6319E-03          3.0667       14.
OTTZ      44.4      1650.                        1.8574E-03          3.0720       14.
MNTZ      47.2      1793.                        1.4416E-03          3.1226       14.
CLER       47.4     1014.30                       7.0880E-04          3.0196       14.
STJZ      55.1     3374.1                        2.1843E-03          3.0469       14.
WES1      59.8     1815.0                        2.0088E-03          3.0433       14.
BLAZ      76.0      917.1                        1.9824E-03          3.0811       14.
ATLZ     111.6      611.2                        1.4747E-03          3.0729       14.
LUBZ     261.1     1069.7                        5.4938E-04          3.1402       14.
TUCZ      264.6     1916.8                        7.1416E-04          3.0298       14.
ALQZ      271.7     1455.8                        2.3197E-03          2.9416       14.
DUGZ     291.2     2024.7                        2.7392E-03          2.9692       14.
LONZ      305.1     2881.8                        5.4073E-04          3.0163       14.
MSOZ     309.7     2316.3                        2.0042E-03          2.9969       14.
EDMZ     324.7     2651.9                        2.0220E-03          2.9918       14.
ROLZ     334.2      284.2                        2.1265E-03          3.0875       14.
FFCZ     340.9     2307.3                        1.3113E-03          3.0979       14.
MBCZ     350.2     4758.5                        2.8563E-03          3.0615       14.
HALT
16.          1     0     1     1.4000E-04             2.2918E 23
SLMZ        3.2     335.3                        2.0586E-03          3.1662       16.
LHCZ      3.6       1425.30                       1.8304E-03          3.0583       16.
AAMZ      36.2      941.4                        2.1194E-03          3.0600       16.
BLOZ      40.3      526.4                        1.5042E-03          3.0097       16.
OTTZ      44.4      1650.                        1.6590E-03          3.0692       16.
MNTZ      47.2      1793.                        1.1696E-03          3.0535       16.
CLER       47.4     1014.30                       9.6655E-04          3.0514       16.
STJZ      55.1     3374.1                        1.7497E-03          3.0277       16.
WES1      59.8     1815.0                        1.7155E-03          3.0612       16.
BLAZ      76.0      917.1                        1.5719E-03          3.0811       16.
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DATA    PAGE 11


ATLZ     111,6     611.2                      1.1358E-03       3.1603     16.
LUBZ     261,1    1069.7                      4.4781E-04       3.2400     16,
TUCZ      264.6    1916,8                     4.7992E-04       3.0467     16,
ALQZ      271,7    1455,8                     1,7883E-03       2.9446     16,
DUGZ     291.2    2024.7                      2.0440E-03       2.9390     16.
LONZ      305.1    2881,8                     7.9251E-04       2,9774     16,
MSOZ     309,7    2316.3                      1.8302E-03       2,9892     16,
PNTZ     311,8    2813.9                      4,6592E-04       2.9514     16.
EDMZ     324,7    2651.9                      2,1366E-03       3.0002     16,
ROLZ     334,2     284.2                      1,8834E-03       3,0379     16,
FFCZ     340.9    2307.3                      1.4200E-03       3.0752     16.
HALT
18,          1      0     1     1,2000E-04                2,5783E 23
SLMZ        3.2    335.3                      1.3914E-03       3.0440     18.
LHCZ       3.6      1425,30                   1.4748E-03       3.0681     18,
AAMZ      36.2     941,4                      1.9852E-03       3.0901     18.
BLOZ      40,3     526.4                      1.4003E-03       3.0622     18.
OTTZ      44,4     1650.                      1,5572E-03       3,0578     18,
MNTZ      47,2     1793.                      1,3002E-03       3,0587     18,
CLER       47.4    1014,30                    1.9039E-03       3.1707     18.
STJZ      55,1    3374.1                      1,5582E-03       3.0250     18.
WES1      59,8    1815,0                      1,2590E-03       3,0625     18,
BLAZ      76,0     917.1                      1,1238E-03       3.0428     18.
ATLZ     111,6     611.2                      9,5085E-04       3,2399     18,
TUCZ      264,6    1916,8                     2,8667E-04       3,0576     18,
ALQZ      271,7    1455,8                     1.2206E-03       2,9268     18.
DUGZ     291.2    2024.7                      1.2671E-03       2.9137     18.
MSOZ     309,7    2316.3                      1,7215E-03       2,9795     18,
PNTZ     311,8    2813,9                      6.7301E-04       2.9654     18,
EDMZ     324,7    2651.9                      2.2259E-03       3.0070     18.
ROLZ     334,2     284.2                      1,6847E-03       3,1915     18,
FFCZ     340,9    2307,3                      1.4597E-03       3,0528     18.
MBCZ     350,2    4758.5                      2,0057E-03       3.0265     18,
FCCZ      355,2    2591,60                    1.3393E-03       3.0713     18,
HALT
20,          1      0     1     1.0000E-04                2,8648E 23
SLMZ        3,2    335.3                      1.3198E-03       2.9831     20.
LHCZ       3.6      1425.30                   1.3614E-03       3.0599     20,
AAMZ      36,2     941.4                      1,7744E-03       3.0926     20,
BLOZ      40.3     526.4                      1.1678E-03       3.0445     20.
OTTZ      44,4     1650.                      1,2368E-03       3,0353     20.
MNTZ      47,2     1793.                      1,5036E-03       3.0639     20.
STJZ      55.1    3374.1                      1.7832E-03       3.0662     20.
WES1      59,8    1815,0                      9,8851E-04       3,0017     20.
BLAZ      76,0     917.1                      9,1581E-04       2,9593     20.
ATLZ     111,6     611.2                      6,5929E-04       3,3281     20.
TUCZ      264,6    1916,8                     2,3236E-04       3.0430     20.
ALQZ      271,7    1455,8                     8,4961E-04       2,9006     20.
DUGZ     291,2    2024.7                      7,2643E-04       2,9221     20.
MSOZ     309,7    2316.3                      1.5100E-03       2,9950     20.
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DATA     PAGE 12


PNTZ      311.8    2813.9                       7.9996E-04        2.9811      20
EDMZ      324,7    2651.9                       1.9193E-03        2.9867      20
ROLZ      334,2     284.2                       1.4237E-03        3.2186      20
FFCZ      340,9    2307.3                       1.1418E-03        3.0568      20
MBCZ      350,2    4758.5                       1.5975E-03        3.0429      20
FCCZ       355,2    2591,60                     1.3285E-03        3.0677      20
HALT
22,          1    0    1     0.8000E-04               3,1513E 23
SLMZ        3.2     335.3                       1.1943E-03        3.4425
LHCZ       3.6       1425,30                    1,4126E-03        3.0948
AAMZ       36,2      941.4                      1,5003E-03        3.0575
BLOZ       40.3      526.4                      8,9760E-04        2.9631
OTTZ       44,4      1650.                      1.0495E-03        3,0465
MNTZ       47,2      1793.                      1,4961E-03        3,0850
STJZ       55.1     3374.1                      2.1183E-03        3.1058
WES1       59,8     1815.0                      7,9566E-04        3.0306
BLAZ       76,0      917.1                      8,6200E-04        2.9475
TUCZ       264.6     1916.8                     3.6588E-04        3,0262
ALQZ       271,7     1455.8                     6.0200E-04        2,8991
MSQZ      309,7    2316.3                       1,3027E-03        3.0263
PNTZ      311,8    2813.9                       8,2263E-04        3.0018
EDMZ      324,7    2651,9                       1,5745E-03        2,9750
ROLZ      334,2     284.2                       1,1667E-03        3,1128
FFCZ      340,9    2307.3                       9,9161E-04        3,0793
MBCZ      350,2    4758,5                       1.3398E-03        3,0773
FCCZ       355,2    2591.60                     1.2300E-03        3,0943
HALT
24,          1    0    1     0.8000E-04               3,4377E 23
LHCZ       3,6       1425,30                    1.4842E-03        3,1617
AAMZ       36,2      941.4                      1,2350E-03        3.0476
OTTZ       44,4      1650.                      9.6488E-04        3.1155
MNTZ       47,2      1793.                      1,5752E-03        3.1390
STJZ       55,1     3374,1                      1,7616E-03        3,2031
WES1       59,8     1815.0                      8,2929E-04        3,1792
BLAZ       76,0      917,1                      9,3695E-04        3,0227
LUBZ      261,1    1069,7                       4,0991E-04        3,1611
TUCZ       264,6     1916,8                     6,1767E-04        3,0576
ALQZ       271,7     1455,8                     4,7773E-04        2.9298
MSQZ      309,7    2316,3                       1,2286E-03        3,1095
PNTZ      311,8    2813,9                       7,0261E-04        3.0342
EDMZ      324,7    2651,9                       1.2166E-03        3,0644      24
ROLZ      334,2     284.2                       1,0908E-03        2,9744
FFCZ      340,9    2307.3                       1,0533E-03        3.1551      24
MBCZ      350,2    4758,5                       9,7179E-04        3,1372      24
FCCZ       355,2    2591,60                     1,2148E-03        3,1555      24
HALT
26,          1    0    1     0.8000E-04               3,7242E 23
LHCZ       3,6       1425,30                    1,4377E-03        3,2062
AAMZ       36,2      941.4                      9,7922E-04        3,1628      24
OTTZ       44,4      1650.                      8,6854E-04        3,1663
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MNTZ      47,2      1793.                          1.6563E-03            3.1839
STJZ      55.1     3374.1                          1.2198E-03            3.2889
WES1      59.8     1815,0                          8.5719E-04            3.2504
BLAZ      76,0      917.1                          1.0799E-03            3,0915
ATLZ     111,6      611.2                          5.7008E-04            3.6403
LUBZ     261,1     1069.7                          5.2211E-04            3.2013
TUCZ      264,6     1916,8                         7,3327E-04            3,1021
ALQZ      271.7     1455,8                         4.3560E-04            3.0320
DUGZ     291.2     2024.7                          6.7091E-04            3.1178
MSOZ     309,7     2316.3                          1.1163E-03            3.2064
PNTZ     311,8     2813.9                          5.1329E-04            3.0606
EDMZ     324,7     2651,9                          1,1945E-03            3.1706
ROLZ     334,2      284.2                          1.2399E-03            2.9900
FFCZ     340,9     2307,3                          1.0958E-03            3.2144
MBCZ     350,2     4758,5                          8.6732E-04            3.2200
FCCZ      355,2     2591,60                        1.2869E-03            3.2172
HALT
28.           1      0    1      0,8000E-04               4,0107E 23
LHCZ      3.6       1425,30                        1.3009E-03            3.2170
OTTZ      44,4      1650.                          7.9199E-04            3.2125
MNTZ      47,2      1793.                          1.5658E-03            3.2381
STJZ      55.1     3374.1                          7,9186E-04            3.2970
WES1      59,8     1815.0                          7.9624E-04            3.2650
BLAZ      76,0      917.1                          1.2243E-03            3.1338
ATLZ     111,6      611.2                          7.9483E-04            3.4212
LUBZ     261.1     1069.7                          6.9404E-04            3.2949
TUCZ      264,6     1916,8                         6,3387E-04            3,1992
ALQZ      271,7     1455,8                         4,8914E-04            3,1897
MSOZ     309,7     2316.3                          8,3720E-04            3.2514
PNTZ     311,8     2813.9                          3,9470E-04            3,1234
EDMZ     324,7     2651,9                          1,1529E-03            3,1974
ROLZ     334,2      284.2                          1,4247E-03            3,0138
FFCZ     340,9     2307,3                          1,0975E-03            3,2807
MBCZ     350,2     4758.5                          7.6243E-04            3,2630
FCCZ      355,2     2591.60                        1,3687E-03            3.3022
HALT
30,           1      0    1      0,8000E-04               4,2972E 23
LHCZ      3.6       1425,30                        1,1474E-03            3,2044
OTTZ      44,4      1650.                          7,5288E-04            3,2441
MNTZ      47,2      1793.                          1,2955E-03            3.3129
WES1      59,8     1815,0                          7.1008E-04            3.2547
BLAZ      76,0      917,1                          1,2940E-03            3.1526
ATLZ     111,6      611,2                          9.0719E-04            3.3509
LUBZ     261,1     1069,7                          8,4603E-04            3.3439
TUCZ      264,6     1916,8                         6,0935E-04            3,3284
ALQZ      271,7     1455,8                         5,8920E-04            3,2722
DUGZ     291.2     2024.7                          6,4633E-04            3,1496
MSOZ     309,7     2316,3                          5.9334E-04            3,2560
EDMZ     324,7     2651,9                          1,0920E-03            3,1820
ROLZ     334,2      284,2                          1,5339E-03            2,9209
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DATA    PAGE 14


FFCZ     340.9    2307.3                      1.0937E-03          3.3400
MBCZ     350.2    4758.5                      5.9855E-04          3.3153
FCCZ      355.2    2591.60                    1.4307E-03          3.3798
HALT
35.          1     0    1     0.6000E-04               5.0134E 23
LHCZ       3.6     1425.30                    1.0099E-03          3.3751
OTTZ      44.4     1650.                      6.1376E-04          3.1602
WES1      59.8    1815.0                      6.5948E-04          3.4252
BLAZ      76.0     917.1                      1.1449E-03          3.3120
ATLZ     111.6     611.2                      9.4786E-04          3.2834
FFCZ     340.9    2307.3                      8.8108E-04          3.4218
HALT
40.          1     0    1     0.6000E-04               5.7296E 23
OTTZ      44.4     1650.                      3.8902E-04          3.2377
BLAZ      76.0     917.1                      9.1741E-04          3.5491
ATLZ     111.6     611.2                      9.9368E-04          3.4164
DUGZ     291.2    2024.7                      2.2214E-03          3.1082
EDMZ     324.7    2651.9                      5.1449E-04          3.5890
FFCZ     340.9    2307.3                      4.8586E-04          3.7775
HALT
45.          1     0    1     0.6000E-04               6.4458E 23
OTTZ      44.4     1650.                      2.5450E-04          3.4545
BLAZ      76.0     917.1                      8.0422E-04          3.4523
ATLZ     111.6     611.2                      8.7968E-04          3.6621
DUGZ     291.2    2024.7                      2.0624E-03          3.2426
EDMZ     324.7    2651.9                      6.6968E-04          3.6991
FFCZ     340.9    2307.3                      4.1238E-04          3.8150
HALT
-1.
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DISTMX =    3500.00
          NR =  272            NL =  294

 DEPTH  SLIP   DIP  STRK    MOMENT       R-RAYL     RES*RAYL     R-LOVE     RES-LOVE    RES--L+R    MOMENT-L    MOMENT-R
  15.0 -90.0  45.0    0.   0.164E 24   0.609E 00  0.841E-03   0.755E 00  0.105E-02  0.435E-01  0.230E 24  0.936E 23
  15.0 -90.0  45.0  90.0   0.165E 24   0.674E 00  0.795E-03   0.755E 00  0.104E-02  0.429E-01  0.230E 24  0.949E 23
  15.0 -60.0  45.0    0.   0.148E 24   0.625E 00  0.601E-03   0.253E 00  0.187E-02  0.497E-01  0.199E 24  0.925E 23
  15.0 -60.0  45.0  30.0   0.152E 24   0.633E 00  0.619E-03   0.715E 00  0.103E-02  0.406E-01  0.206E 24  0.942E 23
  15.0 -60.0  45.0  90.0   0.149E 24   0.646E 00  0.554E-03   0.242E 00  0.188E-02  0.493E-01  0.200E 24  0.949E 23
  15.0 -60.0  45.0 120.0   0.152E 24   0.662E 00  0.593E-03   0.696E 00  0.106E-02  0.406E-01  0.205E 24  0.948E 23
  15.0 -30.0  45.0  30.0   0.130E 24   0.521E 00  0.504E-03   0.730E 00  0.936E-03  0.379E-01  0.169E 24  0.876E 23
  15.0 -30.0  45.0 120.0   0.132E 24   0.499E 00  0.457E-03   0.711E 00  0.940E-03  0.374E-01  0.168E 24  0.921E 23
  15.0    0.  45.0  30.0   0.121E 24   0.423E 00  0.549E-03   0.455E 00  0.143E-02  0.445E-01  0.156E 24  0.825E 23
  15.0    0.  45.0  60.0   0.121E 24   0.290E 00  0.689E-03   0.556E 00  0.125E-02  0.440E-01  0.160E 24  0.781E 23
  15.0    0.  45.0 120.0   0.122E 24   0.322E 00  0.545E-03   0.446E 00  0.142E-02  0.444E-01  0.156E 24  0.862E 23
  15.0    0.  45.0 150.0   0.124E 24   0.413E 00  0.541E-03   0.536E 00  0.121E-02  0.418E-01  0.158E 24  0.878E 23
  15.0  30.0  45.0  60.0   0.130E 24   0.465E 00  0.561E-03   0.768E 00  0.873E-03  0.379E-01  0.171E 24  0.855E 23
  15.0  30.0  45.0 150.0   0.133E 24   0.542E 00  0.439E-03   0.768E 00  0.808E-03  0.353E-01  0.170E 24  0.938E 23
  15.0  60.0  45.0    0.   0.151E 24   0.673E 00  0.536E-03   0.312E 00  0.168E-02  0.470E-01  0.202E 24  0.968E 23
  15.0  60.0  45.0  60.0   0.149E 24   0.657E 00  0.577E-03   0.579E 00  0.133E-02  0.437E-01  0.201E 24  0.938E 23
  15.0  60.0  45.0  90.0   0.149E 24   0.619E 00  0.619E-03   0.292E 00  0.178E-02  0.489E-01  0.201E 24  0.921E 23
  15.0  60.0  45.0 150.0   0.150E 24   0.619E 00  0.592E-03   0.598E 00  0.127E-02  0.432E-01  0.201E 24  0.940E 23
  15.0 -90.0  60.0    0.   0.171E 24   0.488E 00  0.183E-02   0.726E 00  0.123E-02  0.525E-01  0.255E 24  0.813E 23
  15.0 -90.0  60.0  90.0   0.173E 24   0.665E 00  0.136E-02   0.772E 00  0.111E-02  0.497E-01  0.254E 24  0.854E 23
  15.0 -60.0  60.0  30.0   0.147E 24   0.609E 00  0.578E-03   0.763E 00  0.919E-03  0.387E-01  0.197E 24  0.930E 23
  15.0 -60.0  60.0 120.0   0.145E 24   0.570E 00  0.698E-03   0.767E 00  0.958E-03  0.407E-01  0.198E 24  0.873E 23
  15.0 -30.0  60.0  30.0   0.128E 24   0.669E 00  0.203E-03   0.680E 00  0.835E-03  0.322E-01  0.145E 24  0.111E 24        **
  15.0 -30.0  60.0  60.0   0.127E 24   0.643E 00  0.214E-03   0.125E 00  0.180E-02  0.449E-01  0.143E 24  0.109E 24        **
  15.0 -30.0  60.0 120.0   0.126E 24   0.547E 00  0.261E-03   0.653E 00  0.925E-03  0.344E-01  0.144E 24  0.107E 24        **
  15.0 -30.0  60.0 150.0   0.126E 24   0.597E 00  0.229E-03   0.117E 00  0.192E-02  0.463E-01  0.141E 24  0.110E 24        **
  15.0    0.  60.0  30.0   0.121E 24   0.450E 00  0.323E-03   0.449E 00  0.126E-02  0.398E-01  0.130E 24  0.112E 24        **
  15.0    0.  60.0  60.0   0.121E 24   0.337E 00  0.405E-03   0.546E 00  0.100E-02  0.375E-01  0.133E 24  0.108E 24        **
  15.0    0.  60.0 120.0   0.124E 24   0.364E 00  0.353E-03   0.445E 00  0.126E-02  0.401E-01  0.130E 24  0.117E 24        **
  15.0    0.  60.0 150.0   0.127E 24   0.446E 00  0.384E-03   0.539E 00  0.986E-03  0.366E-01  0.132E 24  0.120E 24        **
  15.0  30.0  60.0  30.0   0.124E 24   0.563E 00  0.259E-03   0.451E-01  0.199E-02  0.474E-01  0.141E 24  0.105E 24        **
  15.0  30.0  60.0  60.0   0.127E 24   0.543E 00  0.272E-03   0.750E 00  0.688E-03  0.310E-01  0.147E 24  0.106E 24        **
  15.0  30.0  60.0 120.0   0.126E 24   0.609E 00  0.289E-03   0.542E-01  0.194E-02  0.465E-01  0.141E 24  0.110E 24        **
  15.0  30.0  60.0 150.0   0.131E 24   0.692E 00  0.189E-03   0.759E 00  0.640E-03  0.288E-01  0.147E 24  0.113E 24        **
  15.0  60.0  60.0    0.   0.142E 24   0.543E 00  0.630E-03   0.418E-01  0.218E-02  0.530E-01  0.191E 24  0.883E 23
  15.0  60.0  60.0  60.0   0.145E 24   0.605E 00  0.574E-03   0.689E 00  0.108E-02  0.407E-01  0.195E 24  0.921E 23
  15.0  60.0  60.0  90.0   0.142E 24   0.656E 00  0.519E-03   0.446E-01  0.219E-02  0.521E-01  0.189E 24  0.920E 23
  15.0  60.0  60.0 150.0   0.144E 24   0.576E 00  0.662E-03   0.705E 00  0.108E-02  0.417E-01  0.196E 24  0.879E 23
  15.0 -90.0  75.0    0.   0.207E 24   0.293E 00  0.492E-02   0.504E 00  0.182E-02  0.821E-01  0.340E 24  0.641E 23
  15.0 -90.0  75.0  90.0   0.206E 24   0.590E 00  0.412E-02   0.682E 00  0.149E-02  0.749E-01  0.331E 24  0.707E 23
  15.0 -60.0  75.0  30.0   0.143E 24   0.399E 00  0.100E-02   0.720E 00  0.112E-02  0.461E-01  0.202E 24  0.780E 23
  15.0 -60.0  75.0 120.0   0.142E 24   0.448E 00  0.117E-02   0.738E 00  0.113E-02  0.480E-01  0.209E 24  0.733E 23
  15.0 -30.0  75.0  30.0   0.127E 24   0.531E 00  0.257E-03   0.586E 00  0.977E-03  0.351E-01  0.133E 24  0.121E 24        **
  15.0 -30.0  75.0  60.0   0.130E 24   0.675E 00  0.177E-03   0.329E 00  0.136E-02  0.392E-01  0.134E 24  0.125E 24        **
  15.0 -30.0  75.0 120.0   0.123E 24   0.480E 00  0.284E-03   0.571E 00  0.105E-02  0.365E-01  0.133E 24  0.112E 24        **
  15.0 -30.0  75.0 150.0   0.124E 24   0.450E 00  0.293E-03   0.313E 00  0.144E-02  0.417E-01  0.133E 24  0.113E 24        **
  15.0    0.  75.0  30.0   0.147E 24   0.504E 00  0.313E-03   0.444E 00  0.159E-02  0.436E-01  0.117E 24  0.179E 24        **
  15.0    0.  75.0  60.0   0.150E 24   0.464E 00  0.341E-03   0.541E 00  0.130E-02  0.406E-01  0.120E 24  0.183E 24        **
  15.0    0.  75.0 120.0   0.149E 24   0.455E 00  0.333E-03   0.443E 00  0.165E-02  0.445E-01  0.117E 24  0.184E 24        **
  15.0    0.  75.0 150.0   0.156E 24   0.525E 00  0.334E-03   0.540E 00  0.145E-02  0.422E-01  0.120E 24  0.196E 24        **
  15.0  30.0  75.0  30.0   0.124E 24   0.463E 00  0.273E-03   0.261E 00  0.158E-02  0.431E-01  0.132E 24  0.115E 24        **
  15.0  30.0  75.0  60.0   0.129E 24   0.517E 00  0.259E-03   0.689E 00  0.728E-03  0.314E-01  0.136E 24  0.121E 24        **
  15.0  30.0  75.0 120.0   0.125E 24   0.620E 00  0.207E-03   0.270E 00  0.156E-02  0.421E-01  0.132E 24  0.118E 24        **
  15.0  30.0  75.0 150.0   0.127E 24   0.546E 00  0.249E-03   0.705E 00  0.710E-03  0.310E-01  0.137E 24  0.116E 24        **
  15.0  60.0  75.0  60.0   0.143E 24   0.454E 00  0.975E-03   0.756E 00  0.106E-02  0.451E-01  0.203E 24  0.789E 23
  15.0  60.0  75.0 150.0   0.142E 24   0.374E 00  0.126E-02   0.774E 00  0.109E-02  0.485E-01  0.206E 24  0.715E 23
***  RR  30.   60.  150.    0.692E 00
***  RL  60.   75.  150.    0.774E 00
*** RES  30.   60.  150.    0.288E+01
***RESR  30.   75.   60.    0.177E+03
***RESL  30.   60.  150.    0.640E+03
***BEST  30.   60.  150.    0.405E 00
```

XII. WIGGLE

PROGRAMMER: R. B. HERRMANN / May 73

PURPOSE:

This program uses the eigenfunction tapes generated by the programs LEIGEN or REIGEN to generate synthetic seismograms and also the spectra of the ground motion. If the LEIGEN tape is used, transverse ground motion is generated. If the REIGEN tape is used, the vertical or radial components of ground motion can be generated for earthquakes or an explosion. The spectra can be written on tape and are normalized for geometrical spreading to a distance of 1000 km. This tape can then be read by the programs DATAPLT or FILTER and plotted. Card output of the time series in a format for direct input to EXSPEC is also available.

This program accepts an arbitrary source spectrum and medium attenuation model. There is an option to generate source spectrum by inputting M_o, $S(\omega) = M_o/i\omega$.

Spectra and or seismograms of the following can be generated: ground velocity, ground acceleration, or displacement; velocity or acceleration traces through a 15-100, 30-100 WWSSN systems, or two LP LRSM systems.

Note: the computed ground displacements are positive for UZ when motion is up, UR when motion is away from the source, and UT when motion is clockwise from source. The instrument magnification routines are such that a positive ground impulse causes an initial upward motion of the instrument output.

THEORY

See Herrmann (1974) or Herrmann and Nuttli (1975a,b).

INPUT/OUTPUT

Card input is on FILE 60. Printer output is on FILE 61 and 42. CALCOMP plotting is on FILE 10. Punched cards are on FILE 43. Eigenfunctions are read from FILE 01. Spectra may be written on FILE 03.

PROGRAM DESCRIPTION

PROGRAM WIGGLE : This is the main routine.

SUBROUTINE EIGENF : This reads the eigenfunction tape and yields the eigenfunctions for the proper depth.

SUBROUTINE TAPEWR : This writes the ground spectra on FILE 03 in a format compatible with DATAPLT and FILTER.

SUBROUTINE SEISMAG1 : Response of the 15-100 or 30-100 WWSSN long period seismograph system.

SUBROUTINE SEISM3 : Response of LRSM LP system with filter 6824-2.

SUBROUTINE SEISM7 : Response of LRSM LP system with filter 6823-13.

SUBROUTINE RESP : This subroutine corrects the spectra of the ground motion for the instrument response.

SUBROUTINE VELRES : This multiplies the spectra by iw, which is the frequence domain equivalent of time domain differentiation.

SUBROUTINE FOUR2 : This is the fast Fourier transform routine.

SUBROUTINE EXCITT : This accounts for the force system at the source, either double-couple or explosive.

SUBROUTINE SRCIN : This reads in the real and imaginary components of the source spectrum or generates real and imaginary parts by reading in seismic moment at $S(\omega) = M_o/i\omega$.

SUBROUTINE GAMMA : This reads in the medium attenuation coefficients.

SUBROUTINE INTER : This establishes the complex spectra of the ground motion by linear interpolation.

SUBROUTINE PNCH : Punches time history of seismic trace on FILE 43 in a format for direct input to EXSPEC. A copy of output is placed on FILE 42.

REFERENCES

Herrmann, R. B. (1974). Surface-wave generation by central United States earthquakes, Ph.D. Dissertation, Saint Louis University.

Herrmann, R. B. and O. W. Nuttli (1975a). Ground motion modeling at regional distances for earthquakes in a continental interior, I. Theory and observations, Int. J. Earthq. Engr. and Struc.Dyn. 4, 49-58.

Herrmann, R. B. and O. W. Nuttli (1975b). Ground motion modeling at regional distances for earthquakes in a continental interior, II. Effect of focal depth, azimuth and attenuation, *Int. J. Earthq. Engr. and Struc. Dyn.*, *4*, 59-72.

INPUT DATA

Card Sequence	Column	Name	Format	Explanation
A.	1-5	IEQEX	I5	.EQ.1 Earthquake .EQ.2 Explosion
	6-10	JSRC	I5	.LE.0 read in source spectrum .GT.0 read in seismic moment
	11-20	DT	F10.5	Digitizing interval of seismogram
B.	1-10	DIP	F10.3	Dip of one nodal plane measured from horizontal toward downward pointing positive z-axis
	11-20	SLIP	F10.3	Direction of slip vector on nodal plane measured counterclockwise from the horizontal pointing in strike direction.
	21-30	STRK	F10.3	Strike of fault plane measured clockwise from north
C.	1-10	AA	F10.3	.GE.0 focal depth in km .LE.-10 go to Point B. .GT.-10 .and. LT.0 end program

D. Source spectrum data

1. If JSRC.LE.0 read source spectrum (up to 49 periods)

Card Sequence	Column	Name	Format	Explanation
	1-10	T	F10.2	Period of source spectrum value

Card Sequence	Column	Name	Format	Explanation
D. (cont'd)	11-21	XR	E11.4	Real part of source spectrum
	31-41	XI	E11.4	Imaginary part of source spectrum

(The source spectrum values are read in until a card is reached for which T.LE 0)

2. If JSRC .GT.0 read seismic moment (one card)

	1-10	XMOM	E10.3	Seismic moment in dyne-cm

E. Anelastic attenuation coefficients (up to 41 sets, 2 cards per set) (NOTE: two data cards are required per period since 11 sets of values are placed in 8F10.5 format)

	1-10	T(I)	F10.5	Period in seconds .LE.0 end of gamma values
	11-20 ... 71-80 1-10 11-20 21-30	ATEN(I,J), J = 1, 10		attenuation values in km^{-1}. J = 1 corresponds to fundamental mode.
F.	1-10	PHI	F10.5	Azimuth of receiver from source .LT.-400 go to Point C for AA
	12-15	STA	A4	Station identification
G.	1-10	R	F10.3	.GT.0 epicentral distance in km .LE.0 go to Point F for new PHI
	11-20	TO	F10.3	Time seismogram begins
	21-25	ID(1)	I5	Ground Disp LE 0 skip LE 2 seismogram GE 2 spectra

Card Sequence	Column	Name	Format	Explanation
G. (cont'd)	26-30	ID(2)	I5	Ground Vel LE 0 skip LE 2 seismogram GE 2 spectra
	31-35	ID(3)	I5	Ground Accl LE 0 skip LE 2 seismogram GE 2 spectra
	36-40	ID(4)	I5	WWSSN Disp LE 0 skip LE 2 seismogram GE 2 spectra
	41-45	ID(5)	I5	WWSSN Vel LE 0 skip LE 2 seismogram GE 2 spectra
	46-50	ID(6)	I5	WWSSN Accl LE 0 skip LE 2 seismogram GE 2 spectra
	51-55	IURUZ	I5	.EQ.1 Vertical component of Rayleigh wave .EQ.2 Radial component of Rayleigh wave
	56-60	NPY	I5	Number of time steps to plot on seismograms. NPY.LE.1020. If NPY.LE.0, NPY = 1020
	61-70	YAXLEN	F10.3	Length of time axis in inches LE 9. If YAXLEN .LE.0, YAXLEN = 5.12

Card Sequence	Column	Name	Format	Explanation
G. (cont'd)	71-75	INS	I5	Instrument response 0 15-100 WWSSN Resp 1 30-100 WWSSN Resp 2 LRSM 6824-13 Resp 3 LRSM 6824-2 Resp
	76-80	IPNCH		GT.0 Card output of seismogram LE.0 No card output

The program now returns to Point G to read in a new set of R, etc.

```
C      PROGRAM WIGGLE
C      THIS PROGRAM READS EIGENFUNCTION TAPES AND CALCULATES SYNTHETIC
C       SURFACE WAVE TIME HISTORIES
C      GROUND MOTION CONVENTION US  VERT + = UP
C                                   RADIAL + = AWAY FROM SOURCE
C                                   TRANS + = CLOCKWISE FROM SOURCE
       COMMON/ONE/ELIP(42,10),WVNUM(42,10)
       COMMON/TWO/IMODE(50),N
       COMMON/THREE/TT(50),ZIP(1050)
       DIMENSION PYY1(42,10),PYY2(42,10),PYY3(42,10),PYY4(42,10)
       DIMENSION SR(50),SI(50),ATTEN(42,10),XRL(42,10),XIM(42,10)
       DIMENSION DATA(2,1024)
       DIMENSION IBUF(800),ID(6),X(1024),Y(1024)
       DIMENSION DATM(2048)
       EQUIVALENCE (DATA(1,1),DATM(1))
       EQUIVALENCE(DATM(1),X(1)),(DATM(1025),Y(1))
       EQUIVALENCE (ZIP(1),SR(1)),(ZIP(51),SI(1)),(ZIP(101),XRL(1)),
      1 (ZIP(521),XIM(1))
       CHARACTER IDATE(3),ITYP(2),IEX(2),IST,ICOM,IA
       CHARACTER STA
       IEX(1) = 4HEQ
       IEX(2) = 4HEXPL
       ITYP(1) = 4HLOVE
       ITYP(2) = 4HRAYL
       CALL PLOTS(IBUF,800,10)
       CALL PLOT(0,0,-11,0,-3)
       CALL PLOT(0.0,1,0,-3)
       DEGRAD = 0.017452329
       NWRITE = 1
     3 FORMAT(1H ,8E15.8)
     4 FORMAT(8F10.3)
     5 FORMAT(2F10.3,8I5,F10.3,2I5)
     6 FORMAT(2I5,F10.5)
     7 FORMAT(10X,4HMAX ,A4,2H =,E11.4)
     8 FORMAT(F10.5,1X,A4)
    11 FORMAT(1H1,A4,6X,4HDT =,F10.3)
    12 FORMAT(1H0,7X,3HDIP,6X,4HSLIP,4X,6HSTRIKE)
    13 FORMAT(1H0,7HDEPTH = ,F10.3)
    14 FORMAT(1H0,6HPHI =  ,F10.3 ,1X,A4)
    15 FORMAT(1H ,A4)
C      IEQEX = 1 EARTHQUAKE  = 2 EXPLOSION AND DIP SLIP STRK HAVE NO
C      MEANING
C      JSRC .GT. 0 READ IN SEISMIC MOMENT IN SRCIN AND NOT SOURCE
C      SPECTRUM
C      DT = DIGITIZING INTERVAL FOR RESULTANT SEISMOGRAM
       READ(60,6) IEQEX,JSRC,DT
       WRITE(61,11) IEX(IEQEX),DT
       DO 500 K = 1,4
       DO 501 I = 1,2
       DO 501 J = 1,1024
```

```
  501 DATA(I,J) = 0.0
      DATA(1,2) = 1./(4.*DT)
      DATA(1,3) = 1./(2.*DT)
      DATA(1,4) = 1./(4.*DT)
      CALL FOUR(DATA,1024,-1,DT,DF)
      INS = K - 1
      CALL RESP(DATA,DF,INS)
      DO 502 I = 513,1024
      J = 1026 - I
      DATA(1,I) = DATA(1,J)
  502 DATA(2,I)=-DATA(2,J)
      DATA(2,513) = 0.0
      CALL FOUR(DATA,1024,+1,DT,DF)
C     SPECIAL STEPS ARE TAKEN BECAUSE OF EQUIVALENCED ARRAYS
      DO 504 I = 1,1020
      J = 2*I - 1
  504 DATM(I) = DATM(J)
      DO 505 I = 1,1020
  505 Y(I) = DATM(I)
      DO 506 I = 1,1020
  506 X(I) = (I-1)*DT
      CALL SCALE(Y,1.0,1020,1)
      CALL SCALE(X,5.10,1020,1)
      CALL AXIS(0.25,0.0,4HSEC ,-4,5.10,90.0,X(1021),X(1022))
      CALL AXIS(0.0,-0.25,4HINST,4,1.0,180.0,Y(1021),Y(1022))
      Y(1022) = - Y(1022)
      CALL LINE(Y,X,1020,1,0,0)
      CALL PLOT(3.0,0.0,-3)
  500 CONTINUE
      DF = 1./(1024.*DT)
 4321 CONTINUE
      READ(60,4) DIP,SLIP,STRK
      WRITE(61,12)
      WRITE(61,4) DIP,SLIP,STRK
C     DIP      DIP OF FAULT PLANE MEASURED FROM HORIZONTAL CLOCKWISE
C              Z DOWN
C     SLIP     DIRECTION OF SLIP VECTOR MEASURED CCL FROM HORIZONTAL
C     STRK     STRIKE OF FAULT PLANE MEASURED FROM NORTH
      XDIP = DEGRAD * DIP
      XSLP = DEGRAD * SLIP
      XSTR = DEGRAD * STRK
      SIND = SIN(XDIP)
      COSD = COS(XDIP)
      COS2D = COS(2.*XDIP)
      SINS = SIN(XSLP)
      COSS = COS(XSLP)
      COSST = COS(XSTR)
      SINST = SIN(XSTR)
      SIN2ST = SIN(2.*XSTR)
      COS2ST = COS(2.*XSTR)
```

```
      FAC1 = SIND*COSS*COS2ST
      FAC2 = SIND*COSS*SIN2ST
      FAC3 = -SIND*COSD*SINS*COS2ST
      FAC4 = -SIND*COSD*SINS*SIN2ST
      FAC5 = COS2D*SINS*COSST
      FAC6 = COS2D*SINS*SINST
      FAC7 = -COSD*COSS*COSST
      FAC8 = -COSD*COSS*SINST
      FAC9 = 2.*SIND*COSD*SINS
 2001 READ(60,4) AA
      WRITE(61,13) AA
      IF(AA.LE.-10.) GO TO 4321
      IF(AA.LT.0) GO TO 9998
      CALL EIGENF(PYY1,PYY2,PYY3,PYY4,IFUNC,AA,NPER)
      WRITE(61,15) ITYP(IFUNC)
      DO 1001 I = 1,1050
 1001 ZIP(I) = 0.0
      CALL SRCIN(SR,SI,TT,NPER,JSRC)
      CALL GAMMA(ATTEN,TT,NPER)
      CALL SYMBOL(0.0,2.0,0.14,ITYP(IFUNC),90.0,4)
      CALL SYMBOL(0.0,4.0,0.14,4HH =  ,90.0,4)
      CALL NUMBER(0.0,4.5,0.14,AA,90.0,1)
      CALL PLOT(3.0,0.0,-3)
 2112 CONTINUE
      READ(60,8) PHI,STA
C     PHI     STATION AZIMUTH MEASURED FROM NORTH
      IF(PHI.LT.-400.) GO TO 2001
      WRITE(61,14) PHI,STA
      SAZ = SIN(DEGRAD * PHI)
      CAZ = COS(DEGRAD * PHI)
      DO 1000 I = 1,NPER
      MODE = IMODE(I)
      DO 1000 J = 1,MODE
      PY1 = PYY1(I,J)
      PY2 = PYY2(I,J)
      PY3 = PYY3(I,J)
      PY4 = PYY4(I,J)
      CALL EXCITT(FAC1,FAC2,FAC3,FAC4,FAC5,FAC6,FAC7,FAC8,FAC9,PY1,
     1 PY2,PY3,PY4,IFUNC,SAZ,CAZ,XR,XI,IEQEX)
      XRL(I,J) = SR(I)*XR - SI(I)*XI
      XIM(I,J) = SR(I)*XI + SI(I)*XR
 1000 CONTINUE
 3113 CONTINUE
C     READ IN EPICENTRAL DISTANCE AND STARTING TIME OF SEISMOGRAM TRACE
      READ(60,5) R,T0,ID(1),ID(2),ID(3),ID(4),ID(5),ID(6),IURUZ
     1,NPY,YAXLEN,INS,IPNCH
      IF(R.LE.0) GO TO 2112
      IF(NPY.LE.0) NPY = 1020
      IF(YAXLEN.LE.0) YAXLEN = 5.12
      YAXLE=YAXLEN
```

```
      NOT=YAXLEN
      WRITE(61,5)R,T0,ID(1),ID(2),ID(3),ID(4),ID(5),ID(6),IURUZ
     1,NPY,YAXLEN,INS,IPNCH
C     R  EPICENTRAL DISTANCE IN KILOMETERS
C     T0 TIME SEISMOGRAM BEGINS
C     ID CONTROLS THE OUTPUT OPTIONS
C     ID(1)    GROUND DISP     LE 0 SKIP  LE 2 SEISMOGRAM   GE 2 SPECTRA
C     ID(2)    GROUND VEL      LE 0 SKIP  LE 2 SEISMOGRAM   GE 2 SPECTRA
C     ID(3)    GROUND ACCL     LE 0 SKIP  LE 2 SEISMOGRAM   GE 2 SPECTRA
C     ID(4)    SEISMO DISP     LE 0 SKIP  LE 2 SEISMOGRAM   GE 2 SPECTRA
C     ID(5)    SEISMO VEL      LE 0 SKIP  LE 2 SEISMOGRAM   GE 2 SPECTRA
C     ID(6)    SEISMO ACCL     LE 0 SKIP  LE 2 SEISMOGRAM   GE 2 SPECTRA
C     IURUZ  EQ 1  VERTICAL COMP OF RAYL  EQ 2 RADIAL COMPONENT OF RAYL
C     NPY = NUMBER OF TIME STEPS TO PLOT FOR SEISMOGRAM  NPY.LE.1024
C     YAXLEN = LENGTH OF TIME AXIS IN INCHES
C     INS = 0 15-100 WWSSN RESP
C         = 1 30-100 WWSSN RESP
C         = 2 LRSM 6824-13 RESP
C         = 3 LRSM 6824-2 RESP
C     IPNCH GT 0 CARD OUTPUT OF SEISMOGRAM
      CALL SYMBOL(0.0,0.0,0.14,4HDIST,90.0,4)
      CALL NUMBER(0.0,0.7,0.14,R,90.0,-1)
      CALL SYMBOL(0.0,2.0,0.14,4H PHI,90.0,4)
      CALL NUMBER(0.0,2.7,0.14,PHI,90.0,-1)
      CALL SYMBOL(0.0,3.5,0.14,STA,90.0,4)
      CALL PLOT(1.5,0.0,-3)
      CALL INTER(TT,DATA,NPER,DF,IMODE,XRL,XIM,WVNUM,ATTEN,IFUNC,
     1 R,IURUZ,ELIP,T0)
      WRITE(54,3) (DATM(I),I=1,1026)
      REWIND 54
      DO 5000 INK = 1,6
      IF(ID(INK).LE.0) GO TO 5000
      READ(54,3)  (DATM(I),I=1,1026)
      REWIND 54
      GO TO (5001,5001,5001,5002,5002,5002),INK
 5002 CALL RESP(DATA,DF,INS)
 5001 CONTINUE
      GO TO(5011,5012,5013,5011,5012,5013),INK
 5013 CALL VELRES(DATA,DF)
 5012 CALL VELRES(DATA,DF)
 5011 CONTINUE
      IF(ID(INK).LT.2) GO TO 4115
      IST = 4HSYNT
      ICOM = 4HHETI
      IDATE(1) = 4HC SE
      IDATE(2) = 4HISMO
      IDATE(3) = 4HGRAM
      CALL TAPEWR(DATA,128,NWRITE,IST,ICOM,R,PHI,0.0,T0,DT,IDATE,1024)
 4115 CONTINUE
      IF(ID(INK).GT.2) GO TO 5100
```

```
      DO 5500 I = 513,1024
      J = 1026 - I
      DATA(1,I) = DATA(1,J)
 5500 DATA(2,I) = - DATA(2,J)
      DATA(2,513) = 0.0
      CALL FOUR(DATA,1024,+1,DT,DF)
      FACC = SQRT(1000./R)
      DO 5600 I = 1,1024
      J = 2 * I - 1
 5600 DATM(I) = DATM(J) * FACC
      Y(1) = T0
      DO 5700 I = 2,1024
 5700 Y(I) = Y(I-1) + DT
      GO TO (5701,5702,5703,5701,5702,5703),INK
 5701 IA = 4HAMPL
      GO TO 5704
 5702 IA = 4HVEL
      GO TO 5704
 5703 IA = 4HACCL
 5704 CONTINUE
      CALL SCALE(X,1.0,NPY,1)
      CALL SCALE(Y,YAXLEN,NPY,1)
      CALL AXIS(0.25,0.0,4HSEC ,-4,YAXLE,90.0,Y(NPY+1),Y(NPY+2))
      CALL AXIS(0.0,-0.25,IA,4,1.0,180.0,X(NPY+1),X(NPY+2))
      NPY2 = NPY+2
      X(NPY2) = - X(NPY2)
      CALL LINE(X,Y,NPY,1,0,0)
      CALL PLOT(3.0,0.0,-3)
      XMAX = 0
      DO 5800 I = 1,NPY
      IF(XMAX.LT.ABS(X(I))) XMAX = ABS(X(I))
 5800 CONTINUE
      WRITE(61,7) IA,XMAX
      IF(INK.LT.4) INSTM = 6
      IF(INK.GT.3.AND.INS.EQ.0) INSTM = 1
      IF(INK.GT.3.AND.INS.GT.0) INSTM = 2
      IF(IPNCH.GT.0) CALL PNCH(X,NPY,DT,XMAX,PHI,R,T0,INSTM)
 5100 CONTINUE
 5000 CONTINUE
      GO TO 3113
 9998 CONTINUE
      CALL PLOT(10.0,0.0,999)
      STOP
      END

      SUBROUTINE EIGENF(PYY1,PYY2,PYY3,PYY4,IFUNC,AA,NPER)
      COMMON/ONE/ELIP(42,10),WVNUM(42,10)
      COMMON/TWO/IMODE(50),N
      COMMON/THREE/TT(50),ZIP(1050)
      DIMENSION AY(70,4),CY(70,4),DEPTH(70),D(70)
```

```
      DIMENSION A(70),B(70),RHO(70),XMU(70),XLAMB(70)
      DIMENSION PYY1(42,10),PYY2(42,10),PYY3(42,10),PYY4(42,10)
      DIMENSION PYR(4)
    1 FORMAT(1H ,2(I4,6X))
    2 FORMAT(1H ,I4,5X,7F10.4)
    5 FORMAT(1H ,A4,3F10.5,5E11.4)
    6 FORMAT(1H ,7(4X,E11.4))
      DO 37 I = 1,42
      IMODE(I) = 1
      DO 37 J=1,10
      ELIP(I,J) = 0
      WVNUM(I,J) = 0.0
      PYY1(I,J) = 0
      PYY2(I,J) = 0
      PYY3(I,J) = 0
   37 PYY4(I,J) = 0
      READ(01,1)N
C     READ IN MODEL
      DO 100 I=1,N
  100 READ(01,2) LAYER,DEPTH(I),D(I),A(I),B(I),RHO(I),XMU(I),XLAMB(I)
      NPER = 0
 2000 READ(01,1) IFUNC, MODE
      IF(NPER.EQ.0) IFUN = IFUNC
      MAXY = 2 * IFUNC
      IF(IFUNC.LT.0) GO TO 9999
      NPER = NPER + 1
      IMODE(NPER) = MODE
      DO 2999 IM = 1,MODE
      READ(01,5) AMODE,T,C,U,SUMIO,WVNUM(NPER,IM),AL,ELIP(NPER,IM),ARSOR
      TT(NPER) = T
C     THE SPECTRA ARE COMPUTED AT A DISTANCE OF 1000 KILOMETERS
      WVNO = WVNUM(NPER,IM)
      DENOM = SQRT(WVNO)*0.3989423*1.0E+20
      DENOM = DENOM * 6.2831853 * 31.622776 / AL
      DO 2100 IN=1,N
 2100 READ(01,6) AY(IN,1),AY(IN,2),AY(IN,3),AY(IN,4),DCDA,DCDB,DCDR
      WVNO2 = WVNO * WVNO
C     RAYLEIGH WAVES
C     INITIALLY AY(IN,1) = UR, AY(IN,2) = UZ, AY(IN,3) = TZ,
C               AY(IN,4) = TR
C     NOW AY(IN,1) = UR, AY(IN,2) = UZ, AY(IN,3) = DURDZ,
C         AY(IN,4) = DUZDZ
C     AND CY(IN,1) = DURDZ, CY(IN,2) = DUZDZ, CY(IN,3) = D2URDZ2,
C         CY(IN,4) = D2UZDZ2
      GO TO (2400,2200),IFUNC
 2200 DO 2300 IN = 1,N
       H = RHO(IN)*C*C*WVNO2
      XL2M = XLAMB(IN) + 2.*XMU(IN)
      CY(IN,1) = -WVNO    *AY(IN,2) + AY(IN,4)/XMU(IN)
      CY(IN,2) = (WVNO    *XLAMB(IN)*AY(IN,1)+AY(IN,3))/XL2M
```

```
      CY(IN,3) = (-H/XMU(IN) + WVNO2*(3.*XL2M-2.*XMU(IN))/XL2M)*AY(IN,1)
     1 -WVNO    *(1.+XLAMB(IN)/XMU(IN))*AY(IN,3)/XL2M
      CY(IN,4) = (-(H+WVNO2*XLAMB(IN))*AY(IN,2)+WVNO    *(1.+XLAMB(IN)/
     1 XMU(IN))*AY(IN,4))/XL2M
      AY(IN,3) = CY(IN,1)
      AY(IN,4) = CY(IN,2)
 2300 CONTINUE
      GO TO 2501
C     LOVE WAVES
C     INITIALLY AY(IN,1) = UT, AY(IN,2) = TT
C     NOW AY(IN,1) = UT, AY(IN,2) = DUTDZ
C     AND CY(IN,1) = DUTDZ, CY(IN,2) = D2UTDZ2
 2400 DO 2500 IN=1,N
      CY(IN,1) = AY(IN,2)/XMU(IN)
      CY(IN,2) = WVNO2*(XMU(IN)- C*C*RHO(IN))*AY(IN,1)/XMU(IN)
      AY(IN,2) = CY(IN,1)
 2500 CONTINUE
 2501 CONTINUE
      DO 103 I = 1,N
      IFOCA = I
      D1=DEPTH(I)-0.5*D(I)
      D2=DEPTH(I)+0.5*D(I)
      IF(AA.GE.D1.AND.AA.LE.D2) GO TO 101
  103 CONTINUE
  101 DO 110 JJ = 1,MAXY
      AAA = AY(IFOCA,JJ)
      CC = CY(IFOCA,JJ)
      AAA = AAA - CC * DEPTH(IFOCA)
  110 PYR(JJ) = AAA + CC * AA
      GO TO(2551,2552),IFUNC
 2551 PYY1(NPER,IM) = WVNO    *PYR(1)/DENOM
      PYY2(NPER,IM) = PYR(2) / DENOM
      GO TO 2555
 2552 CONTINUE
      PYY1(NPER,IM) = WVNO     * PYR(1) / DENOM
      PYY2(NPER,IM) = WVNO     * PYR(2) / DENOM
      PYY3(NPER,IM) = PYR(3)/DENOM
      PYY4(NPER,IM) = PYR(4)/DENOM
 2555 CONTINUE
 2999 CONTINUE
      GO TO 2000
 9999 CONTINUE
      IFUNC = IFUN
      REWIND 1
      RETURN
      END

      SUBROUTINE TAPEWR(X,M,NWRITE,IST,ICOM,DIST,DEG,BACKAZ,T0,DT,
     1 IDATE,NPTS)
      DIMENSION X(1),IDATE(3)
```

```
      WRITE(03,310) M,NPTS,IST,ICOM,DIST,DEG,BACKAZ,TO,DT,
     1 IDATE(1),IDATE(2),IDATE(3)
  310 FORMAT(2I5,A4,1X,A4,1X,5F10,2,3A4)
      DO 312 I = 1,M
      K = I * 8
      J = K - 7
  312 WRITE(03,311) NWRITE,(X(L),L=J,K)
  311 FORMAT(I5,8E15.6)
      NWRITE = NWRITE + 1
      RETURN
      END

      SUBROUTINE SEISMAG1(FREQ,PEAK,XR,XI,INS)
C     INS EQ 0  15-100 WWSSN
C     PEAK MAGNIFICATIONS ARE 350,700,1400,2800,5600
C     INS EQ 1 30-100 WWSSN
      WE=6.2831853*FREQ
      INDEX=(PEAK+1)/375
      IF(INS.GT,0) GO TO 200
  100 CONTINUE
      GO TO(1,2,2,3,3,3,3,4,4,4,4,4,4,4,4,5),INDEX
    1 FMAG=278.
      SIGMA=0.003
      GO TO 6
    2 FMAG=556.0
      SIGMA=0,013
      GO TO 6
    3 FMAG=1110,
      SIGMA=0,047
      GO TO 6
    4 FMAG=2190.
      SIGMA=0,204
      GO TO 6
    5 FMAG=3950.
      SIGMA=0,805
    6 ZETA=0.93
      ZETA1=1.
      WN=.418879
      WN1=.062831853
      GO TO 300
  200 CONTINUE
      GO TO(10,20,20,30,30,30,30,40,40,40,40,40,40,40,40,50),INDEX
   10 FMAG = 251.9
      SIGMA = 0,003
      GO TO 60
   20 FMAG = 503,1
      SIGMA = 0,012
      GO TO 60
   30 FMAG = 1001.5
      SIGMA = 0,044
```

```
      GO TO 60
   40 FMAG = 1941.9
      SIGMA = 0.195
      GO TO 60
   50 FMAG = 2241.8
      SIGMA = .767
   60 ZETA = 1.5
      ZETA1 = 1.0
      WN = .2094395
      WN1 = .062831853
  300 CONTINUE
      AR= (WE*WE-WN*WN)*(WE*WE-WN1*WN1)-4.*ZETA*ZETA1*WN*WN1*(1.-SIGMA)
     1*WE*WE
      AI=2.*WE*(ZETA1*WN1*(WN*WN-WE*WE)+ZETA*WN*(WN1*WN1-WE*WE))
      FACTOR = FMAG*WE*WE*WE / (AI*AI + AR*AR)
      XR =-AI * FACTOR
      XI =-FACTOR * AR
      RETURN
      END

      SUBROUTINE SEISM3(FREQ,PEAK,XR,XI)
C     LRSM RESPONSE FOR LP SYSTEM WITH FILTER 6824-2
C     PHASE RESPONSE OBTAINED FROM HILBERT TRANSFORM OF AMPLITUDE
C     RESPONSE.  GAIN NORMALIZED TO 1.0 AT 25 SECONDS.
      DIMENSION FRE(28),P(28),PHI(28)
      DATA FRE/.001,.002,.003,.004,.005,.006,.007,.008,.009,.01,.02,.03,
     1.04,.05,.06,.07,.08,.09,.1,.2,.3,.4,.5,.6,.7,.8,.9,1./
      DATA PHI/263.9,257.9,251.9,245.6,239.3,233.1,227.1,221.1,214.6,
     1208.3,134.9,73.2,15.3,-33.2,-71.2,-100.1,-122.4,-140.0,-153.8,-213
     2.1,-232.6,-242.8,-249.2,-253.0,-256.9,-259.3,-261.2,-262.9/
      DATA P/.00005,.00040,.00135,.00321,.00625,.01077,.01706,.02546,
     1.03631,.05006,.34117,.73904,1.0000,.97633,.79807,.61417,.46105,
     2.34464,.26315,.03583,.01097,.00468,.00240,.00139,.00087,.00058,
     3.00040,.00029/
      M = 28
      DEGRAD = 0.01745329
      IF(FREQ.GT.0.5) FREQ = 0.5
      IF(FREQ.LT.0.005) FREQ = 0.005
      DO 200 I = 1,M
      IF(FREQ.GE.FRE(I).AND.FREQ.LE.FRE(I+1)) GO TO 160
  200 CONTINUE
  160 PF= PHI(I)+(PHI(I+1)-PHI(I))/(FRE(I+1)-FRE(I))*(FREQ-FRE(I))
      PH = P(I) + (P(I+1) - P(I))/(FRE(I+1)-FRE(I))*(FREQ-FRE(I))
      PF = PF * DEGRAD
      XR = COS(PF) * PH * PEAK
      XI = SIN(PF) * PH * PEAK
      RETURN
      END

      SUBROUTINE SEISM7(FREQ,PEAK,XR,XI)
```

```
C     LRSM RESPONSE FOR LP SYSTEM WITH FILTER 6824-13
C     PHASE RESPONSE OBTAINED FROM HILBERT TRANSFORM OF AMPLITUDE
C     RESPONSE.  GAIN NORMALIZED TO 1.0 AT 25 SECONDS.
      DIMENSION FRE(28),P(28),PHI(28)
      DATA FRE/.001,.002,.003,.004,.005,.006,.007,.008,.009,.01,.02,.03,
     1.04,.05,.06,.07,.08,.09,.1,.2,.3,.4,.5,.6,.7,.8,.9,1./
      DATA P/.000015,.000391,.00131,.00310,.00609,.01068,.01713,.02556,
     1.03818,.04848,.28030,.67553,1.0,1.09448,1.0044,.86969,.74511,.6355
     27,.54818,.12153,.04630,.02741,.01827,.01328,.01017,.008052,.006545
     3,.005404/
      DATA PHI/265.0,259.1,253.4,248.1,242.8,236.9,230.3,223.4,216.4,
     1209.7,153.4,102.0,54.6,14.4,-16.8,-40.4,-58.2,-73.3,-85.4,-146.4,
     2-153.9,-155.3,-156.4,-157.7,-159.2,-160.6,-162.3,-163.8/
      M = 28
      DEGRAD = 0.01745329
      IF(FREQ.GT.0.5) FREQ = 0.5
      IF(FREQ.LT.0.005) FREQ = 0.005
      DO 200 I = 1,M
      IF(FREQ.GE.FRE(I).AND.FREQ.LE.FRE(I+1)) GO TO 160
  200 CONTINUE
  160 PF= PHI(I)+(PHI(I+1)-PHI(I))/(FRE(I+1)-FRE(I))*(FREQ-FRE(I))
      PH = P(I) + (P(I+1) - P(I))/(FRE(I+1)-FRE(I))*(FREQ-FRE(I))
      PF = PF * DEGRAD
      XR = PH*PEAK*COS(PF)
      XI = PH*PEAK*SIN(PF)
      RETURN
      END

      SUBROUTINE RESP(DATA,DF,INS)
      DIMENSION DATA(1)
      DATA(1)=0.0
      DATA(2) = 0.0
      DO 10 I = 2,513
      XI = I - 1
      FREQ = XI * DF
      J = 2 * I - 1
      K = 2 * I
      IF(INS.LT.2) CALL SEISMAG1(FREQ,3000.,XR,XI,INS)
      IF(INS.EQ.2) CALL SEISM7(FREQ,3000.,XR,XI)
      IF(INS.EQ.3) CALL SEISM3(FREQ,3000.,XR,XI)
      TEMPR = XR*DATA(J) - XI*DATA(K)
      TEMPI = XR*DATA(K) + XI*DATA(J)
      DATA(J) = TEMPR
   10 DATA(K) = TEMPI
      RETURN
      END

      SUBROUTINE VELRES(X,DF)
      DIMENSION X(1)
C     THIS IS FREQUENCY DOMAIN EQUIVALENT OF TIME DOMAIN DIFFERENTIATION
```

```
      DOM = 6.2831853 * DF
      DO 10 I = 1,513
      XI = I - 1
      FREQ = XI * DOM
      J = 2 * I - 1
      K = 2 * I
      TEMPR = - FREQ * X(K)
      TEMPI =   FREQ * X(J)
      X(J) = TEMPR
   10 X(K) = TEMPI
      RETURN
      END

      SUBROUTINE FOUR(DATA,NN,ISIGN,DT,DF)
C     THE COOLEY-TOUKEY FAST FOURIER TRANSFORM IN USASI BASIC FORTRAN
C     TRANSFORM(J) = SUM(DATA(I)*W**((I-1)(J-1)), WHERE I AND J RUN
C     FROM 1 TO NN AND W = EXP(ISIGN*2*PI*SQRT(-1)/NN).  DATA IS A ONE-
C     DIMENSIONAL COMPLEX ARRAY (I.E., THE REAL AND IMAGINARY PARTS OF
C     DATA ARE LOCATED IMMEDIATELY ADJACENT IN STORAGE, SUCH AS
C     FORTRAN IV PLACES THEM) WHOSE LENGTH NN IS A POWER OF TWO,  ISIGN
C     IS +1 OR -1, GIVING THE SIGN OF THE TRANSFORM,  TRANSFORM VALUES
C     ARE RETURNED IN ARRAY DATA, REPLACING THE INPUT DATA,  THE TIME IS
C     PROPORTIONAL TO N*LOG2(N), RATHER THAN THE USUAL N**2
C     RMS RESOLUTION ERROR BEING BOUNDED BY 6*SQRT(I)*LOG2(NN)*2**(-B),
C     WHERE B IS THE NUMBER OF BITS IN THE FLOATING POINT FRACTION.
C     PROGRAM AUTOMATICALLY DIVIDES TRANSFORM BY NN FOR INVERSE
C     TRANSFORM
      DIMENSION DATA(1)
      N = 2 * NN
      IF(DT.EQ.0.0) DT=1./(NN*DF)
      IF(DF.EQ.0.0) DF=1./(NN*DT)
      IF(DT.NE.(NN*DF)) DF = 1./(NN*DT)
      J = 1
      DO 5 I=1,N,2
      IF(I-J)1,2,2
    1 TEMPR = DATA(J)
      TEMPI = DATA(J+1)
      DATA(J) = DATA(I)
      DATA(J+1)=DATA(I+1)
      DATA(I) = TEMPR
      DATA(I+1) = TEMPI
    2 M = N/2
    3 IF(J-M) 5,5,4
    4 J = J-M
      M = M/2
      IF(M-2)5,3,3
    5 J=J+M
      MMAX = 2
    6 IF(MMAX-N)7,10,10
    7 ISTEP= 2 *MMAX
```

```
      THETA = 6.283185307/FLOAT(ISIGN*MMAX)
      SINTH=SIN(THETA/2.)
      WSTPR=-2.*SINTH*SINTH
      WSTPI=SIN(THETA)
      WR=1.0
      WI=0.0
      DO 9 M=1,MMAX,2
      DO 8 I=M,N,ISTEP
      J=I+MMAX
      TEMPR=WR*DATA(J)-WI*DATA(J+1)
      TEMPI=WR*DATA(J+1)+WI*DATA(J)
      DATA(J)=DATA(I)-TEMPR
      DATA(J+1)=DATA(I+1)-TEMPI
      DATA(I)=DATA(I)+TEMPR
    8 DATA(I+1) = DATA(I+1)+TEMPI
      TEMPR = WR
      WR = WR*WSTPR-WI*WSTPI + WR
    9 WI = WI*WSTPR+TEMPR*WSTPI + WI
      MMAX = ISTEP
      GO TO 6
   10 CONTINUE
      IF(ISIGN.LT.0) GO TO 1002
C     FREQUENCY TO TIME DOMAIN
      DO 1001 IIII=1,N
 1001 DATA(IIII)=DATA(IIII)*DF
      RETURN
 1002 CONTINUE
C     TIME TO FREQUENCY DOMAIN
      DO 1003 IIII=1,N
 1003 DATA(IIII)=DATA(IIII)*DT
      RETURN
      END

      SUBROUTINE EXCITT(FAC1,FAC2,FAC3,FAC4,FAC5,FAC6,FAC7,FAC8,FAC9,
     1 PY1,PY2,PY3,PY4,IFUNC,SAZ,CAZ,XR,XI,IEQEX)
      GO TO (100,200),IEQEX
  100 CONTINUE
      S2AZ = 2.*SAZ*CAZ
      C2AZ = CAZ*CAZ - SAZ*SAZ
      GO TO (1,2),IFUNC
    1 CONTINUE
      XR   = PY1 *((FAC1-FAC4)*C2AZ + (FAC2+FAC3)*S2AZ )
      XI   = - PY2 *((FAC5+FAC8)*CAZ + (FAC6-FAC7)*SAZ)
      RETURN
    2 CONTINUE
      XR   = PY1*((FAC3+FAC2)*C2AZ + (FAC4-FAC1)*S2AZ) + FAC9 * PY4
     1 + 0.5*FAC9*PY1
      XI   = (PY3+PY2)*((FAC7-FAC6)*CAZ+(FAC8+FAC5)*SAZ)
      RETURN
  200 CONTINUE
```

```
      GO TO (10,20),IFUNC
   10 XR = 0
      XI = 0
      RETURN
   20 XR = PY4 - PY1
      XI = 0
      RETURN
      END

      SUBROUTINE SRCIN(SR,SI,TT,NPER,JSRC)
      DIMENSION SR(1),SI(1),TT(1)
      DIMENSION XR(50),XI(50),T(50)
      IF(JSRC.GT.0) GO TO 400
C     THIS READS IN THE VALUES OF THE SOURCE SPECTRUM AT PERIODS T WHICH
C     ARE ALIGNED IN ASCENDING ORDER.   THE SOURCE VALUES CORRESPONDING
C     TO THE PERIODS TT OF THE EIGENFUNCTIONS ARE FOUND BY
C     INTERPOLATION
      DO 100 I = 1,50
      READ(60,2) T(I),XR(I),XI(I)
    2 FORMAT(F10.2,E11.4,9X,E11.4)
      IF(T(I).LE.0) GO TO 101
      N = I
  100 CONTINUE
  101 CONTINUE
      N = N - 1
      DO 300 K = 1,NPER
      PER = TT(K)
      DO 210 I = 1,N
      IF(PER.GE.T(I).AND.PER.LE.T(I+1)) GO TO 260
  210 CONTINUE
  260 SR(K) = XR(I) + (XR(I+1) - XR(I))/(T(I+1)-T(I))*(PER-T(I))
      SI(K) = XI(I) + (XI(I+1) - XI(I))/(T(I+1)-T(I))*(PER-T(I))
  300 CONTINUE
      GO TO 401
  400 CONTINUE
C     THIS READS IN THE SEISMIC MOMENT, ASSUMES A STEP DISLOCATION
C     AND COMPUTES THE SOURCE SPECTRUM
      READ(60,10) XMOM
   10 FORMAT(E10.3)
      DO 200 K = 1,NPER
      SR(K) = 0.0
      SI(K) = - XMOM*TT(K)/6.2831853
  200 CONTINUE
  401 CONTINUE
      WRITE(61,3)
    3 FORMAT(1H0,4X,6HPERIOD,4X,7HS(REAL),13X,7HS(IMAG)    )
      WRITE(61,1)((TT(I),SR(I),SI(I)),I=1,NPER)
    1 FORMAT(1H ,F9.2,E11.4,9X,E11.4)
      RETURN
      END
```

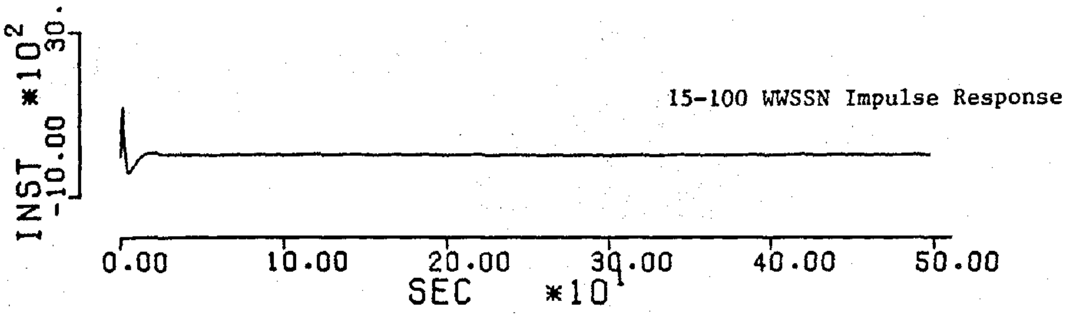
```
      SUBROUTINE GAMMA(ATTEN,TT,NPER)
      DIMENSION ATTEN(42,10),TT(50),ATEN(42,10),T(42)
    1 FORMAT(8F10.5)
      DO 100 I = 1,42
      READ(60,1) T(I),(ATEN(I,J),J=1,10)
      IF(T(I).LE.0) GO TO 101
      N = I
  100 CONTINUE
  101 CONTINUE
      N = N - 1
      DO 200 K = 1,NPER
      PER = TT(K)
      DO 210 I = 1,N
      IF(PER.GE.T(I).AND.PER.LE.T(I+1)) GO TO 260
  210 CONTINUE
  260 DO 270 J = 1,10
  270 ATTEN(K,J) = ATEN(I,J) + (ATEN(I+1,J)-ATEN(I,J))/(T(I+1)-T(I))
     1 * (PER - T(I))
  200 CONTINUE
      WRITE(61,3)
    3 FORMAT(1H0,3X,6HPERIOD,10(5X,5HGAMMA )   )
      WRITE(61,2) ((TT(I),(ATTEN(I,J),J=1,10)),I=1,NPER)
    2 FORMAT(1H ,F9.2,10F10.5)
      RETURN
      END

      SUBROUTINE INTER(TT,DATA,NPER,DF,IMODE,XRL,XIM,WVNUM,ATTEN,IFUNC,
     1 R,IURU4,ELIP,TJ)
      DIMENSION DATA(2,1024),XRL(42,10),XIM(42,10),IMODE(50),ATTEN(42,10
     1),WVNUM(42,10),TT(50),ELIP(42,10)
      PI4 = 0.7853982
      TLW = TT(1)
      TMX = TT(NPER)
      DATA(1,1) = 0.0
      DATA(2,1) = 0.0
      DO 200 II = 2,513
      DATA(1,II)=0.0
      DATA(2,II)=0.0
      XK = II - 1
      PER = 1./(XK*DF)
      IF(PER.GT.TMX) GO TO 200
      IF(PER.LT.TLW) GO TO 200
  202 DO 210 I = 1,NPER-1
      IF(PER.GE.TT(I).AND.PER.LE.TT(I+1)) GO TO 260
  210 CONTINUE
C     THIS USES FACT THAT IMODE(I+1) IS LE TO IMODE(I)
C
  260 MODE = IMODE(I+1)
      DO 270 J = 1,MODE
```

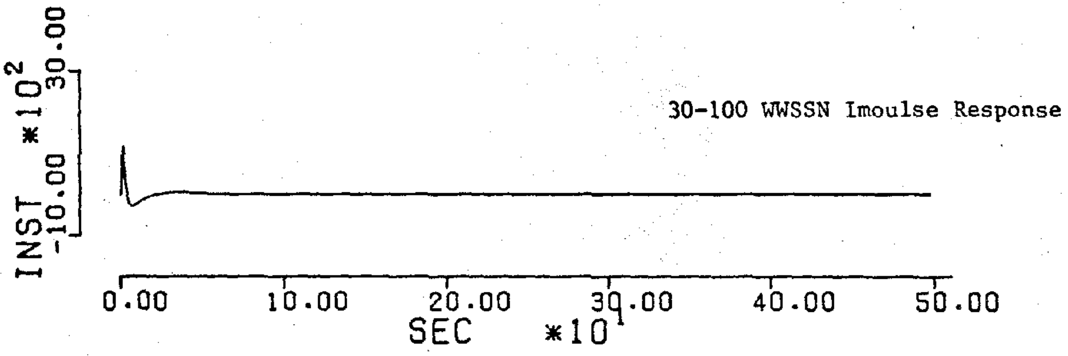
```
      XR = XRL(I,J) + (XRL(I+1,J)-XRL(I,J))/(TT(I+1)-TT(I))*(PER-TT(I))
      XI = XIM(I,J) + (XIM(I+1,J)-XIM(I,J))/(TT(I+1)-TT(I))*(PER-TT(I))
      WVNO = WVNUM(I,J)+(WVNUM(I+1,J)-WVNUM(I,J))/(TT(I+1)-TT(I))*(PER-
     1 TT(I))
      ATEN = ATTEN(I,J)+(ATTEN(I+1,J)-ATTEN(I,J))/(TT(I+1)-TT(I))*(PER
     1 - TT(I))
      FACT = (6.2831853*T0/PER) - WVNO * R
      GO TO (301,302),IFUNC
  301 FACT = FACT + PI4
      GO TO 305
  302 GO TO (303,304),ILRUZ
  303 FACT = FACT - PI4
      GO TO 305
  304 FACT = FACT - 2,3561945
      ELLIP = ELIP(I,J) + (ELIP(I+1,J)-ELIP(I,J))/(TT(I+1)-TT(I))*(PER
     1 - TT(I))
  305 CONTINUE
      CF = COS(FACT)
      SF = SIN(FACT)
      FACT = 0.0
      ATN = ATEN*R
      IF(ATN.LT.80.0) FACT = EXP(-ATN)
      IF(IFUNC.EQ.2.AND.ILRUZ.EQ.2) FACT = FACT * ELLIP
      IF(IFUNC.EQ.2.AND.ILRUZ.EQ.1) FACT = - FACT
      DATA(1,II) = DATA(1,II) + FACT*(XR*CF - XI*SF)
      DATA(2,II) = DATA(2,II) + FACT *(XR*SF + XI*CF)
  270 CONTINUE
  200 CONTINUE
      RETURN
      END

      SUBROUTINE PNCH(X,NPY,DT,XMAX,PHI,R,T0,INSTM)
      DIMENSION X(1),IX(1025),IY(1025)
      DEG = R/111,195
      BACKAZ = 360. - PHI
      TMAX =(NPY-1)*DT
      CPM = 60./DT
      CPMM=500./XMAX
      WRITE(43,6) INSTM
      WRITE(42,6) INSTM
    6 FORMAT(I5,4X,1H1,4X,1H1,3X,2H-1,4X,1H2,4X,1H2)
      WRITE(43,5)
      WRITE(42,5)
    5 FORMAT(1H )
      WRITE(43,3) R,DEG,BACKAZ,T0,DT,CPMM
      WRITE(42,3) R,DEG,BACKAZ,T0,DT,CPMM
      WRITE(43,4)TMAX,CPM
      WRITE(42,4)TMAX,CPM
    3 FORMAT(10X,5F10,2,1PE10,3,6H 3000,)
    4 FORMAT(25X,F10.2,F10.3)
```

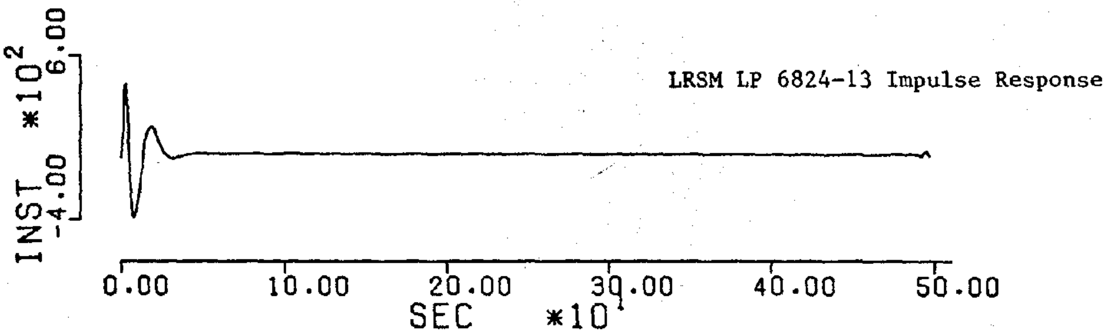
```
      DO 100 I = 1,NPY
      IX(I) = I-1
      IY(I) = 5000*X(I)/XMAX
  100 CONTINUE
      NPY1 = NPY+1
      IX(NPY1)=-9999
      IY(NPY1)=-9999
      WRITE(43,2)((IX(I),IY(I)),I=1,NPY1)
    2 FORMAT(16I5)
      WRITE(42,2)((IX(I),IY(I)),I=1,NPY1)
      RETURN
      END
```

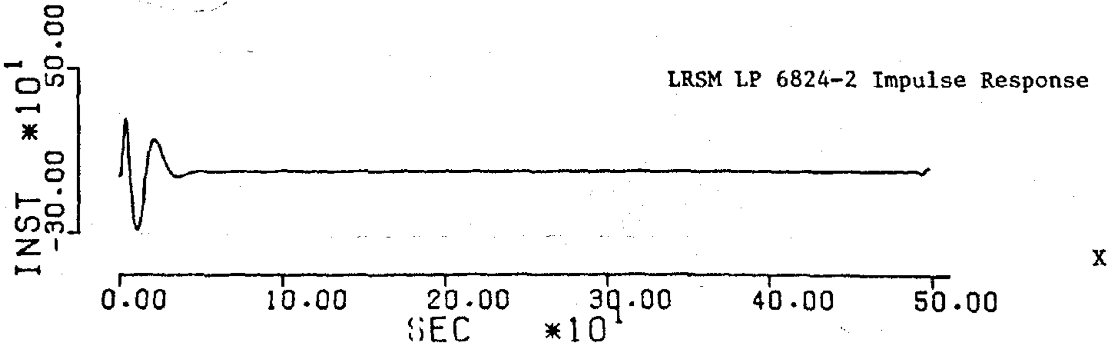
```
 DATA    PAGE  1

    1    1 0.488
 85.       145.      350.
 7.5
 0.250E 23
 1.5       0.01

 2.0       0.01

 2.5       0.005

 5.0       0.0007

 10.       0.0002

 20.       0.0001

 22.       0.00008

 30.       0.00003

 35.       0.00006

 100.      0.00006


 352.3      DBQ
 854.3      0.0         2                                  1  480  5.0         0

 346.7      SLM
 244.9      0.0         2                                  1  480  5.0         0

 345.4      FLO
 265.2      0.0         2                                  1  480  5.0         0

 76.4       BLWV
 749.6      0.0         2                                  1  480  5.0         0

 262.9      HBOK
 854.3      0.0         3                                  1  480  5.0         0

-500.
 -1.
```

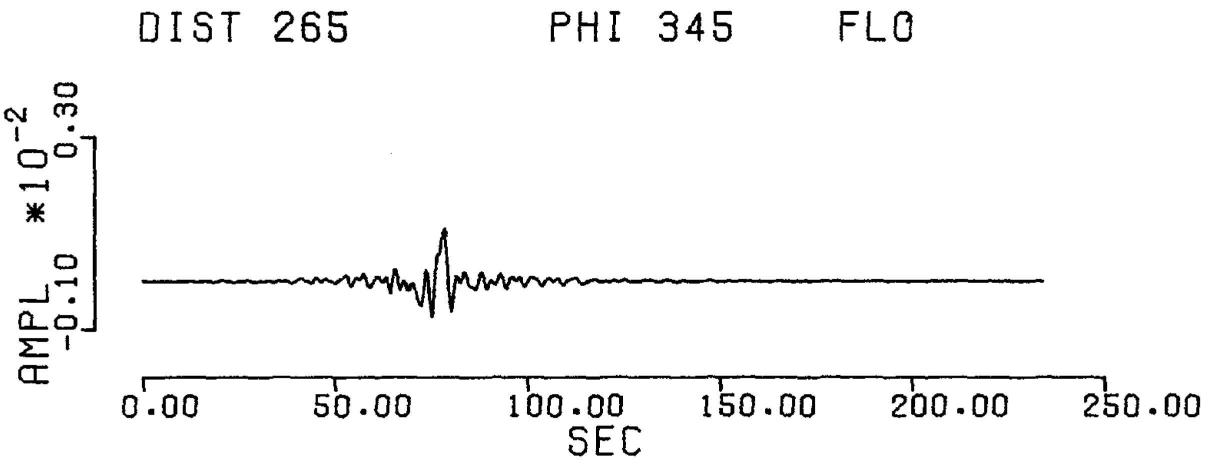
LOVE H = 7.5

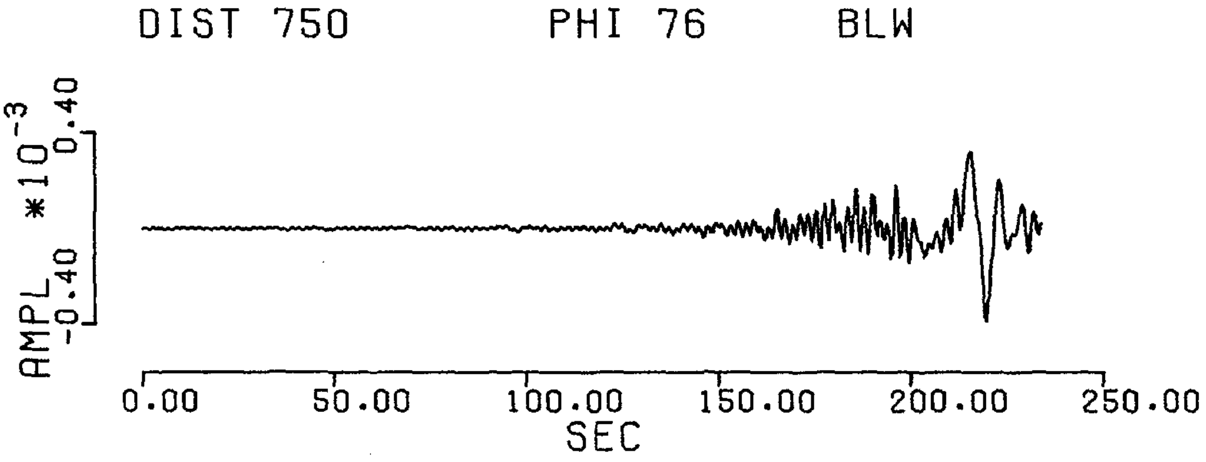

15-100 WWSSN Impulse Response


30-100 WWSSN Imoulse Response


LRSM LP 6824-13 Impulse Response


LRSM LP 6824-2 Impulse Response


DIST 265
PHI 345
FLO
AMPL *10^-2
0.30
-0.10
0.00
50.00
100.00
150.00
200.00
250.00
SEC


DIST 750
PHI 76
BLW
AMPL *10^-3
0.40
-0.40
0.00
50.00
100.00
150.00
200.00
250.00
SEC

```
EQ         DT =      0.488

      DIP       SLIP    STRIKE
   85.000   145.000   350.000

DEPTH =      7.500
LOVE

    PERIOD     S(REAL)              S(IMAG)
     2.00 0.                    -0.7958E 22
     2.50 0.                    -0.9947E 22
     3.00 0.                    -0.1194E 23
     3.50 0.                    -0.1393E 23
     4.00 0.                    -0.1592E 23
     5.00 0.                    -0.1989E 23
     6.00 0.                    -0.2387E 23
     7.00 0.                    -0.2785E 23
     8.00 0.                    -0.3183E 23
     9.00 0.                    -0.3581E 23
    10.00 0.                    -0.3979E 23
    11.00 0.                    -0.4377E 23
    12.00 0.                    -0.4775E 23
    13.00 0.                    -0.5173E 23
    14.00 0.                    -0.5570E 23
    15.00 0.                    -0.5968E 23
    16.00 0.                    -0.6366E 23
    17.00 0.                    -0.6764E 23
    18.00 0.                    -0.7162E 23
    19.00 0.                    -0.7560E 23
    20.00 0.                    -0.7958E 23
    22.00 0.                    -0.8754E 23
    24.00 0.                    -0.9549E 23
    26.00 0.                    -0.1035E 24
    28.00 0.                    -0.1114E 24
    30.00 0.                    -0.1194E 24
    35.00 0.                    -0.1393E 24
    40.00 0.                    -0.1592E 24
    45.00 0.                    -0.1790E 24
    50.00 0.                    -0.1989E 24
    60.00 0.                    -0.2387E 24
    70.00 0.                    -0.2785E 24
    80.00 0.                    -0.3183E 24
    90.00 0.                    -0.3581E 24
   100.00 0.                    -0.3979E 24
   150.00 0.                    -0.5968E 24
   200.00 0.                    -0.7958E 24

   PERIOD     GAMMA     GAMMA     GAMMA     GAMMA     GAMMA     GAMMA     GAMMA     GAMMA     GAMMA     GAMMA
     2.00   0.01000   0.        0.        0.        0.        0.        0.        0.        0.        0.
     2.50   0.00500   0.        0.        0.        0.        0.        0.        0.        0.        0.
     3.00   0.00414   0.        0.        0.        0.        0.        0.        0.        0.        0.
     3.50   0.00328   0.        0.        0.        0.        0.        0.        0.        0.        0.
     4.00   0.00242   0.        0.        0.        0.        0.        0.        0.        0.        0.
     5.00   0.00070   0.        0.        0.        0.        0.        0.        0.        0.        0.
     6.00   0.00060   0.        0.        0.        0.        0.        0.        0.        0.        0.
     7.00   0.00050   0.        0.        0.        0.        0.        0.        0.        0.        0.
     8.00   0.00040   0.        0.        0.        0.        0.        0.        0.        0.        0.
     9.00   0.00030   0.        0.        0.        0.        0.        0.        0.        0.        0.
    10.00   0.00020   0.        0.        0.        0.        0.        0.        0.        0.        0.
    11.00   0.00019   0.        0.        0.        0.        0.        0.        0.        0.        0.
    12.00   0.00018   0.        0.        0.        0.        0.        0.        0.        0.        0.
    13.00   0.00017   0.        0.        0.        0.        0.        0.        0.        0.        0.
    14.00   0.00016   0.        0.        0.        0.        0.        0.        0.        0.        0.
    15.00   0.00015   0.        0.        0.        0.        0.        0.        0.        0.        0.
```

```
   16.00  0.00014  0.       0.       0.       0.       0.       0.       0.       0.       0.
   17.00  0.00013  0.       0.       0.       0.       0.       0.       0.       0.       0.
   18.00  0.00012  0.       0.       0.       0.       0.       0.       0.       0.       0.
   19.00  0.00011  0.       0.       0.       0.       0.       0.       0.       0.       0.
   20.00  0.00010  0.       0.       0.       0.       0.       0.       0.       0.       0.
   22.00  0.00008  0.       0.       0.       0.       0.       0.       0.       0.       0.
   24.00  0.00008  0.       0.       0.       0.       0.       0.       0.       0.       0.
   26.00  0.00008  0.       0.       0.       0.       0.       0.       0.       0.       0.
   28.00  0.00008  0.       0.       0.       0.       0.       0.       0.       0.       0.
   30.00  0.00008  0.       0.       0.       0.       0.       0.       0.       0.       0.
   35.00  0.00006  0.       0.       0.       0.       0.       0.       0.       0.       0.
   40.00  0.00006  0.       0.       0.       0.       0.       0.       0.       0.       0.
   45.00  0.00006  0.       0.       0.       0.       0.       0.       0.       0.       0.
   50.00  0.00006  0.       0.       0.       0.       0.       0.       0.       0.       0.
   60.00  0.00006  0.       0.       0.       0.       0.       0.       0.       0.       0.
   70.00  0.00006  0.       0.       0.       0.       0.       0.       0.       0.       0.
   80.00  0.00006  0.       0.       0.       0.       0.       0.       0.       0.       0.
   90.00  0.00006  0.       0.       0.       0.       0.       0.       0.       0.       0.
  100.00  0.00006  0.       0.       0.       0.       0.       0.       0.       0.       0.
  150.00  0.00006  0.       0.       0.       0.       0.       0.       0.       0.       0.
  200.00  0.00006  0.       0.       0.       0.       0.       0.       0.       0.       0.
PHI =    352.300  DBQ
  674.600     0.       2    0    0    0    0    0    1  480     5.000    0    0
          MAX AMPL = 0.3871E-03

PHI =    346.700  SLM
  244.900     0.       2    0    0    0    0    0    1  480     5.000    0    0
          MAX AMPL = 0.1110E-02

PHI =    345.400  FLO
  265.200     0.       2    0    0    0    0    0    1  480     5.000    0    0
          MAX AMPL = 0.1097E-02

PHI =     76.400  BLW
  749.600     0.       2    0    0    0    0    0    1  480     5.000    0    0
          MAX AMPL = 0.3897E-03

PHI =    262.900  HBO
  854.300     0.       3    0    0    0    0    0    1  480     5.000    0    0

DEPTH =    -1.000
```

Appendix A.

A. L. Levshin and Z. A. Yanson
Surface waves in vertically and radially inhomogeneous media.
Algorithms for the interpretation of seismic data. Computational
Seismology Series, Volume 5, pp 147-177, Moskva, 1971.

Translated by R. B. Herrmann, Saint Louis University, 1972.

INTRODUCTION

In recent years, the data from surface waves generated by earthquakes and explosions have been used for a detailed investigation of the properties of the earth and of the source of the disturbance (for example, to discern zones of reduced velocity or large velocity gradients, to estimate the distribution of absorption with depth, to determine the mechanism and time function of sources, etc.). In order to solve this relatively difficult mathematical problem, simplifications are usually made in the model of the medium (layered homogeneous medium) and of the source (point source) (5,12,16-20); these simplifications can be shown to be insufficient. In our work, based on results obtained in (2,4,15), we will expound the basic elements of a more general theory of surface waves, valid for very minimal restrictions on the model and source.

We will consider vertically (radially) inhomogeneous media with arbitrary rules for the variation of the elastic constants and density with depth (radius); to find the solution of the equation of motion in such a medium, the spectral theory of operators will be used (6,9). The seismic sources will be treated as fields of volume forces, localized spatially and temporally; the only restrictions imposed on the properties of these fields are those of physical existence. An exact solution is constructed for such a source; asymptotic expansions are made at large distances from the source where the field of the disturbance will separate into propagating Love and Rayleigh waves. Then formulae are obtained for some elementary force fields.

(Some results of Saito's work (22) are used here, but that work is marked by approaches to the final solution which seem quick to us. Hence, a different treatment of the seismic source and the asymptotic expansion is given).

1. Displacement field in an elastic halfspace

Statement of the problem. We will consider an elastic halfspace with coordinates z, r, ϕ ($0\leq z<\infty$, $0\leq r<\infty$, $0\leq\phi<2\pi$). The equations of motion are (7):

$$\frac{\partial\widehat{rz}}{\partial r}+\frac{1}{r}\frac{\partial\widehat{\varphi z}}{\partial\varphi}+\frac{\partial\widehat{zz}}{\partial z}+\frac{\widehat{rz}}{r}=\rho\frac{\partial^2 u_z}{\partial t^2}-F_z,$$

$$\frac{\partial\widehat{rr}}{\partial r}+\frac{1}{r}\frac{\partial\widehat{r\varphi}}{\partial\varphi}+\frac{\partial\widehat{rz}}{\partial z}+\frac{\widehat{rr}-\widehat{\varphi\varphi}}{r}=\rho\frac{\partial^2 u_r}{\partial t^2}-F_r, \qquad (1.1)$$

$$\frac{\partial\widehat{r\varphi}}{\partial r}+\frac{1}{r}\frac{\partial\widehat{\varphi\varphi}}{\partial\varphi}+\frac{\partial\widehat{\varphi z}}{\partial z}+\frac{2\widehat{r\varphi}}{r}=\rho\frac{\partial^2 u_\varphi}{\partial t^2}-F_\varphi.$$

Here $\widehat{rz}$, $\widehat{rr}$, $\widehat{r\phi}$, $\widehat{\phi\phi}$, $\widehat{\phi\phi}$, $\widehat{zz}$ are components of the stress tensor; u_z u_r, u_ϕ are components of the displacement vector u(t, z, r, ϕ) in the directions a_z, a_r, a respectively; F_z, F_r, F_ϕ are components of the vector volume force F(t,z,r,ϕ) acting at the source along the same directions; t is the time.

For the components of stress we have the following relations:

$$\widehat{rz} = \mu\left(\frac{\partial u_z}{\partial r} + \frac{\partial u_r}{\partial z}\right), \qquad \widehat{\varphi z} = \mu\left(\frac{1}{r}\frac{\partial u_z}{\partial \varphi} + \frac{\partial u_\varphi}{\partial z}\right), \tag{1.2}$$

$$\widehat{rr} = \lambda\Delta + 2\mu\frac{\partial u_r}{\partial r}, \qquad \widehat{\varphi\varphi} = \lambda\Delta + 2\frac{\mu}{r}\left(\frac{\partial u_\varphi}{\partial \varphi} + u_r\right),$$

$$\widehat{r\varphi} = \mu\left(\frac{\partial u_\varphi}{\partial r} - \frac{u_\varphi}{r} + \frac{1}{r}\frac{\partial u_r}{\partial \varphi}\right), \qquad \widehat{zz} = \lambda\Delta + 2\mu\frac{\partial u_z}{\partial z}.$$

where Δ - dilation.

$$\Delta = \frac{\partial u_z}{\partial z} + \frac{1}{r}\frac{\partial u_\varphi}{\partial \varphi} + \frac{\partial u_r}{\partial r} + \frac{u_r}{r}. \tag{1.3}$$

The Lamé constants λ and μ and the density ρ are piecewise continuous positive functions of a single coordinate z; for $z>Z,\lambda,\mu,\rho$ are constant, and the velocity of transverse waves $b = \sqrt{\mu/\rho}$ and of compressional waves $a = \sqrt{(\lambda+2\mu)/\rho}$ are maximal (this is necessary since Z can be as large as desired):

$$b(Z+0) = \max b(z), \quad a(Z+0) = \max a(z)^{1}.$$

The components of displacement and stress are continuous and bounded everywhere in the region $0 \leq z<\infty$; the surface z = 0 of stress, i.e.

$$\widehat{rz} = \widehat{\varphi z} = \widehat{zz} = 0 \quad \text{при} \quad z = 0. \tag{1.4}$$

Initial conditions:

$$\mathrm{u} = \frac{\partial \mathrm{u}}{\partial t} = 0 \quad \text{при} \quad t < 0. \tag{1.4a}$$

Source. The force field F(t,z, r, ϕ) is described as a real source localized in space and time. The following conditions are imposed on F:

1) F(t,z,r,ϕ) = 0 for t<0)
2) F(t,z,r,ϕ) is absolutely integrable and satisfies the Dirichlet conditions for all arguments. Then we are permitted the following representations:

$$\mathbf{F}(t, z, r, \varphi) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{ipt} \int_{0}^{\infty} \Big[\sum_{m=-\infty}^{\infty} \sum_{i=1}^{3} f_m^{(i)} \mathbf{A}_m^{(i)} \Big] \xi \, d\xi \, dp, \qquad (1.5)$$

where

$$\mathbf{A}_m^{(1)} = \mathbf{a}_z Y_m, \quad \mathbf{A}_m^{(2)} = \mathbf{a}_r \frac{\partial Y_m}{\partial r} \frac{1}{\xi} + \mathbf{a}_\varphi \frac{\partial Y_m}{\partial \varphi} \frac{1}{\xi r}, \qquad (1.6)$$

$$\mathbf{A}_m^{(3)} = \mathbf{a}_r \frac{\partial Y_m}{\partial \varphi} \frac{1}{\xi r} - \mathbf{a}_\varphi \frac{\partial Y_m}{\partial r} \frac{1}{\xi}, \quad Y_m = e^{im\varphi} J_m(\xi r).$$

Here J_m is a Bessel function of the first kind of order m. The system of vector functions $A_m^{(i)}$ are fields and mutually orthogonal. The coefficients $f_m^{(i)}$ (z,ξ, p) are found using the orthogonality relation

$$\int_0^\infty \int_0^{2\pi} [\mathbf{A}_m^{(i)}(\lambda r), \overline{\mathbf{A}}_l^{(j)}(\gamma r)] \, r \, d\varphi \, dr = 2\pi \delta_{ij} \delta_{ml} \frac{\delta(\gamma - \lambda)}{\sqrt{\gamma \lambda}}$$

(δ_{ij} is the Kronecker delta, $\delta(\gamma - \lambda)$ is the Dirac delta function[1]) and equal

$$f_m^{(i)} = \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{-ipt} \int_0^\infty \int_0^{2\pi} (\mathbf{F}, \overline{\mathbf{A}}_m^{(i)}) \, r \, d\varphi \, dr \, dt.$$

Specifically for i = 1,2,3 we have:

$$f_m^{(1)} = \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{-ipt} \int_0^\infty \int_0^{2\pi} [F_z \overline{Y}_m(\xi r)] \, r \, d\varphi \, dr \, dt,$$

$$f_m^{(2)} = \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{-ipt} \int_0^\infty \int_0^{2\pi} \Big[\Big(F_r \frac{\partial \overline{Y}_m}{\partial r} + F_\varphi \frac{\partial \overline{Y}_m}{\partial \varphi} \frac{1}{r} \Big) \Big] \frac{r}{\xi} \, d\varphi \, dr \, dt, \qquad (1.7)$$

$$f_m^{(3)} = \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{-ipt} \int_0^\infty \int_0^{2\pi} \Big[\Big(F_r \frac{\partial \overline{Y}_m}{\partial \varphi} \frac{1}{r} - F_\varphi \frac{\partial \overline{Y}_m}{\partial r} \Big) \Big] \frac{r}{\xi} \, d\varphi \, dr \, dt,$$

$$\overline{Y}_m = e^{-im\varphi} J_m(\xi r).$$

For the force components of F_z, F_r, F_ϕ we have been using (1.5), (1.6):

$$F_z = \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{ipt} \int_0^\infty \Big[\sum_{m=-\infty}^{\infty} f_m^{(1)} Y_m \Big] \xi \, d\xi \, dp,$$

$$F_r = \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{ipt} \int_0^\infty \Big[\sum_{m=-\infty}^{\infty} \Big(f_m^{(2)} \frac{\partial Y_m}{\partial r} + f_m^{(3)} \frac{\partial Y_m}{\partial \varphi} \frac{1}{r} \Big) \Big] d\xi \, dp, \qquad (1.8)$$

$$F_\varphi = \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{ipt} \int_0^\infty \Big[\sum_{m=-\infty}^{\infty} \Big(f_m^{(2)} \frac{\partial Y_m}{\partial \varphi} \frac{1}{r} - f_m^{(3)} \frac{\partial Y_m}{\partial r} \Big) \Big] d\xi \, dp.$$

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[1] $\overline{A}_1^{(j)}$ is complex conjugate of $A_1^{(j)}$

Formulae for displacement. The solutions arising in non-stationary problems of the theory of elasticity have the form

$$\mathbf{u}(t, z, r, \varphi) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{ipt} \mathbf{U}\, dp, \qquad (1.9)$$

where

$$\mathbf{U}(p, z, r, \varphi) = \text{v.} \left[\int_0^{\infty} \sum_{-\infty}^{+\infty} \sum_{i=1}^{3} V_m^{(i)}(z, \xi, p)\, A_m^{(i)} \xi\, d\xi \right]$$

with a solution similar to the stationary problem of elasticity theory, which is derived in the same way with conditions that $V_m,(z,\xi,p)$ be square integrable on the interval $z \in (0,\infty)$. Here and later on, V. indicates that the integration contour goes along the real axis, and around the poles of the integrand with small semicircles above the pole (for integration with ξ) and below the pole (with integration with p). Hence for the components of displacement along the directions a_z, a_r, a_ϕ we obtain:

$$u_z = \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{ipt}\, \overline{\text{v.}} \int_0^{\infty} \left[\sum_{m=-\infty}^{\infty} V_m^{(1)} Y_m \right] \xi\, d\xi\, dp,$$

$$u_r = \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{ipt}\, \overline{\text{v.}} \int_0^{\infty} \left[\sum_{m=-\infty}^{\infty} \left(V_m^{(2)} \frac{\partial Y_m}{\partial r} + \frac{V_m^{(3)}}{r} \frac{\partial Y_m}{\partial \varphi} \right) \right] d\xi\, dp, \qquad (1.10)$$

$$u_\varphi = \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{ipt}\, \overline{\text{v.}} \int_0^{\infty} \left[\sum_{m=-\infty}^{\infty} \left(\frac{V_m^{(2)}}{r} \frac{\partial Y_m}{\partial \varphi} - V_m^{(3)} \frac{\partial Y_m}{\partial r} \right) \right] d\xi\, dp.$$

Placing (1.8), (1.10) in equation (1.1) and in the boundary condition (1.4), and accepting the permissibility of moving the double differentation under the integral sign, we obtain the following equations:

1. For $V_m^{(1)}, V_m^{(2)}$:

$$l_1(V_m^{(1)}, V_m^{(2)}) \equiv \frac{d}{dz}\left[(\lambda + 2\mu) \frac{dV_m^{(1)}}{dz} - \xi\lambda V_m^{(2)} \right] - \\ - \xi\mu \frac{dV_m^{(2)}}{dz} + V_m^{(1)}(p^2\rho - \xi^2\mu) = -f_m^{(1)}, \qquad (1.11)$$

$$l_2(V_m^{(1)}, V_m^{(2)}) \equiv \frac{d}{dz}\left[\mu \frac{dV_m^{(2)}}{dz} + \xi\mu V_m^{(1)} \right] + \\ + \xi\lambda \frac{dV_m^{(1)}}{dz} + V_m^{(2)}(p^2\rho - \xi^2\lambda - 2\xi^2\mu) = -f_m^{(2)},$$

with boundary conditions

$$\sigma_{zz} \equiv (\lambda + 2\mu) \frac{dV_m^{(1)}}{dz} - \xi\lambda V_m^{(2)} = 0, \quad \tau_{rz} \equiv \mu \frac{dV_m^{(2)}}{dz} + \xi\mu V_m^{(1)} = 0 \\ \text{при} \quad z = 0. \qquad (1.12)$$

The functions $V_m^{(1)}$, $V_m^{(2)}$ σ_{zz}, and τ_{rz} are continuous and bounded for all z.

2. For $V_m^{(3)}$

$$l_3(V_m^{(3)}) \equiv \frac{d}{dz}\left(\mu \frac{dV_m^{(3)}}{dz}\right) + V_m^{(3)}(p^2\rho - \xi^2\mu) = -f_m^{(3)}, \quad (1.13)$$

with boundary condition

$$\tau_{\varphi z} \equiv \mu \frac{dV_m^{(3)}}{dz} = 0 \quad \text{при} \quad z = 0. \quad (1.14)$$

$V_m^{(3)}$ and $\tau_{\phi z}$ are continuous and bounded for all z. The left side of equation (1.11) and the boundary condition (1.12) define a self-adjoint operator L in the region of integration with the square integrable vector function $\begin{vmatrix} V_m^{(1)} \\ V_m^{(2)} \end{vmatrix}$ The left side of (1.13) and the boundary condition (1.14) defines a self-adjoint operator L_3 in the region of integration with the function $V_m^{(3)}$. If the vector function $\begin{vmatrix} f_m^{(1)} \\ f_m^{(2)} \end{vmatrix}$ and the function $f_m^{(3)}$ are also square integrable on the interval $z\varepsilon(o,\infty)$, which follows from the conditions imposed on the function $F(t,z,r,\phi)$, then the following relations between the functions $V_m^{(1)}$, $V_m^{(2)}$, $V_m^{(3)}$ and the eigenfunctions of the above described operators are valid (2,6,10).

<u>Expression of V_m in terms of eigenfunctions.</u> $V_m^{(i)}$ (i = 1,2) can be expressed in the following manner:

$$V_m^{(i)} = \sum_{k=1}^{k_R(\xi)} c_{km}^R \tilde{V}_k^{(i)} + \int_{p_0^2}^{\infty} c_m^R(\beta)\,\tilde{V}^{(i)}(\beta, z)\,d\beta. \quad (1.15)$$

Here, for the coefficients c_{km}^R, c_m^R we have:

$$c_{km}^R = \frac{1}{p_{kR}^2(\xi) - p^2}\,\frac{D_{km}^R}{I_{kR}}, \qquad c_m^R = \frac{D_m^R}{\beta(\xi) - p^2}, \quad (1.16)$$

$$D_{km}^R = \int_0^\infty (f_m^{(1)}\tilde{V}_k^{(1)} + f_m^{(2)}\tilde{V}_k^{(2)})\,dz, \qquad D_m^R = \int_0^\infty (f_m^{(1)}\tilde{\tilde{V}}^{(1)} + f_m^{(2)}\tilde{\tilde{V}}^{(2)})\,dz,$$

$$I_{kR} = \int_0^\infty \rho\,[(\tilde{V}_k^{(1)})^2 + (\tilde{V}_k^{(2)})^2]\,dz, \qquad p_0^2 = \xi^2 b^2 (Z + 0).$$

Here $\tilde{V}_k^{(1)}$ and $\tilde{V}_k^{(2)}$, $\tilde{V}^{(1)}$ and $\tilde{V}^{(2)}$ are the eigenfunctions of the operators constituting the left-hand sides of (1.11) with the boundary conditions (1.12).

The first part of the function corresponds to the discrete spectra of eigenvalues p_{kR}^2 $(k = 1, 2, \ldots k_r(\xi))$; $\xi^2 \nu_R^2 < p_{kR}^2 < p_0^2$, where ν_R is the minimal velocity of Rayleigh waves in a halfspace with constants equal a(z), b(z), ρ(z) for some value of depth (1,2). The second part corresponds to the continuous spectra of eigenvalues $\beta(p_0^2 \leq \beta < \infty)$. Here the wave number ξ plays the role of a free parameter. Formula (1.16) is obtained using the orthogonality relations:

$$\int_0^\infty \rho\,[\tilde{V}_i^{(1)}\tilde{V}_j^{(1)} + \tilde{V}_i^{(2)}\tilde{V}_j^{(2)}]\,dz = 0 \quad \text{при } i \neq j,$$

$$\int_0^\infty \rho\,[\tilde{V}^{(1)}(\beta)\overline{\tilde{V}}(p^2) + \tilde{V}^2(\beta)\,\tilde{V}^{(2)}(p^2)]\,dz = \delta(p^2 - \beta),$$

where $\overline{\tilde{V}}^{(i)}$ is the complex conjugate of $\tilde{V}^{(i)}$.

Similarly $V_m^{(3)}$ can be expressed as

$$V_m^{(3)} = \sum_{k=1}^{k_L(\xi)} c_{km}^L \tilde{V}_k^{(3)} + \int_{p_0^2}^\infty c_m^L(\beta)\,\tilde{V}^{(3)}(z,\beta)\,d\beta, \qquad (1.17)$$

where

$$c_{km}^L = \frac{1}{p_{kL}^2(\xi) - p^2}\,\frac{D_{km}^L}{I_{kL}}, \qquad c_m^L = \frac{D_m^L}{\beta(\xi) - p^2};$$

$$D_{km}^L = \int_0^\infty f_m^{(3)}\tilde{V}_k^{(3)}dz, \qquad D_m^L = \int_0^\infty f_m^{(3)}\overline{\tilde{V}}^{(3)}dz, \qquad (1.18)$$

$$I_{kL} = \int_0^\infty \rho\,(\tilde{V}_k^{(3)})^2\,dz.$$

For this derivation the following orthogonality conditions were used:

$$\int_0^\infty \rho\tilde{V}_i^{(3)}\tilde{V}_j^{(3)}dz = 0 \quad \text{при } i \neq j,$$

$$\int_0^\infty \rho\tilde{V}^{(3)}(p^2)\,\overline{\tilde{V}}^{(3)}(\beta)\,dz = \delta(p^2 - \beta).$$

Placing these expressions for V_m into (1.10), we obtain the complete precise formulae for displacements.

$$u_z = \frac{1}{2\pi}\int_{-\infty}^{+\infty} e^{ipt}\left\{ \text{v.}\int_0^{\infty}\left[\sum_{m=-\infty}^{\infty}\left(\sum_{k=1}^{K_R(\xi)} c_{km}^R \bar{V}_k^{(1)} + \int_{p_0^2}^{\infty} c_m^R(\beta)\,\bar{V}^{(1)}d\beta\right)Y_m\right]\xi d\xi\right\} dp,$$

$$u_r = \frac{1}{2\pi}\int_{-\infty}^{+\infty} e^{ipt}\left\{ \text{v.}\int_0^{\infty}\left[\sum_{m=-\infty}^{\infty}\left(\left(\sum_{k=1}^{K_R(\xi)} c_{km}^R \bar{V}_k^{(2)} + \int_{p_0^2}^{\infty} c_m^R(\beta)\,\bar{V}^{(2)}d\beta\right)\frac{\partial Y_m}{\partial r} + \right.\right.\right.$$
$$\left.\left.\left. + \frac{1}{r}\left(\sum_{k=1}^{K_L(\xi)} c_{km}^L \bar{V}_k^{(3)} + \int_{p_0^2}^{\infty} c_m^L(\beta)\,\bar{V}^{(3)}d\beta\right)\frac{\partial Y_m}{\partial \varphi}\right)\right] d\xi\right\} dp, \quad (1.19)$$

$$u_\varphi = \frac{1}{2\pi}\int_{-\infty}^{+\infty} e^{ipt}\left\{ \text{v.}\int_0^{\infty}\left[\sum_{m=-\infty}^{\infty}\left(\frac{1}{r}\left(\sum_{k=1}^{K_R(\xi)} c_{km}^R \bar{V}_k^{(2)} + \int_{p_0^2}^{\infty} c_m^R(\beta)\bar{V}^{(2)}d\beta\right) \times \right.\right.\right.$$
$$\left.\left.\left. \times \frac{\partial Y_m}{\partial \varphi} - \left(\sum_{k=1}^{K_L(\xi)} c_{km}^L \bar{V}_k^{(3)} + \int_{p_0^2}^{\infty} c_m^L(\beta)\,\bar{V}^{(3)}d\beta\right)\frac{\partial Y_m}{\partial r}\right)\right] d\xi\right\} dp.$$

It can be shown that the derived solution for non-stationary problem satisfies the null initial conditions (1.4a). Thus for fixed ξ the functions of p in (1.19), multiplied by exp(ipt), are only the coefficients c_{km}^Q, c_m^Q which are analytic everywhere except at a finite number of points for Im $p \geq 0$, decreasing as $p \to \infty$ not slower than $O(1 / p^{1+|\varepsilon|})$. Changing the order of integration among p and ξ taking the integration contour with respect to p in the lower halfspace, we obtain the null condition on u(t) and du/dt for t<0.

Asymptotic expressions for large r. At large distances r, which are not commensurable with the length dimensions of the seismic source, the main part of the disturbance given by formula (1.19) becomes the Rayleigh and Love surface waves. Their contribution equals the sum of residues of the poles in the integrand, contained in the sum $\left[\sum_{k=1}^{K_Q(\xi)}\right]$.

Keeping only parts, decreasing not faster than r^{-1}, we obtain the following asymptotic formula for displacements at large r (5):

$$u_z(t,z,r,\varphi)=\frac{1}{\sqrt{2\pi r}}\,\mathrm{Re}\int_{\bar p}^{\infty}\frac{\exp i\left(pt-\frac{\pi}{4}\right)}{p}\times$$
$$\times\left[\sum_{k=1}^{K_R(p)}U_{kR}(p,\varphi)\,\tilde V_k^{(1)}(p,z)\,\frac{\sqrt{\xi_{kR}}}{C_{kR}I_{kR}}\exp(-i\xi_{kR}r)\right]dp,$$
$$u_r(t,z,r,\varphi)=\frac{1}{\sqrt{2\pi r}}\,\mathrm{Re}\int_{\bar p}^{\infty}\frac{\exp i\left(pt-\frac{3\pi}{4}\right)}{p}\times$$
$$\times\left[\sum_{k=1}^{K_R(p)}U_{kR}(p,\varphi)\,\tilde V_k^{(2)}(p,z)\,\frac{\sqrt{\xi_{kR}}}{C_{kR}I_{kR}}\exp(-i\xi_{kR}r)\right]dp, \qquad (1.20)$$
$$u_\varphi(t,z,r,\varphi)=\frac{1}{\sqrt{2\pi r}}\,\mathrm{Re}\int_{\bar p}^{\infty}\frac{\exp i\left(pt+\frac{\pi}{4}\right)}{p}\times$$
$$\times\left[\sum_{k=1}^{K_L(p)}U_{kL}(p,\varphi)\,\tilde V_k^{(3)}(p,z)\,\frac{\sqrt{\xi_{kL}}}{C_{kL}I_{kL}}\exp(-i\xi_{kL}r)\right]dp.$$

Here for Q = R, L $K_Q(p)$ is the maximum number of harmonics of Rayleigh (R) and Love (L) waves which exist for a given p.

$$U_{kQ}(p,\varphi)=\sum_{m=-\infty}^{\infty}D_{km}^{Q}\exp im\left(\varphi+\frac{\pi}{2}\right), \qquad (1.21)$$

ξ_{kQ} is the wave number, a root of the equation $p_{kQ}^2(\xi) - p^2 = 0$. The phase and group velocities of the k'th harmonic v_{kQ} and C_{kQ} are related to ξ_{kQ} by

$$v_{kQ}=\frac{p}{\xi_{kQ}}, \qquad C_{kQ}=\frac{dp_{kQ}}{d\xi}(\xi=\xi_{kQ}). \qquad (1.22)$$

$\bar p$ is the limiting frequency; oscillations with frequencies less than $\bar p$ make up the quasi-static part of the disturbance and are not of interest to us.

For the derivation of formula (1.20) the following asymptotic relations were used:

$$\mathrm{v.}\int_0^\infty \frac{\Phi(\xi)\, J_m(\xi r)\, \xi\, d\xi}{p_{kQ}^2(\xi) - p^2} \approx \sqrt{\frac{\pi \xi_{kQ}}{2r}}\, \frac{\Phi(\xi_{kQ})}{p C_{kQ}} \times$$

$$\times \exp\left\{-i\left[\xi_{kQ} r - \left(m - \frac{1}{2}\right)\frac{\pi}{2}\right]\right\},$$

$$\mathrm{v.}\int_0^\infty \frac{\Phi(\xi)\, \dfrac{dJ_m(\xi r)}{dr}\, d\xi}{p_{kQ}^2(\xi) - p^2} \approx \sqrt{\frac{\pi \xi_{kQ}}{2r}}\, \frac{\Phi(\xi_{kQ})}{p C_{kQ}} \times$$

$$\times \exp\left\{-i\left[\xi_{kQ} r - \left(m - \frac{3}{2}\right)\frac{\pi}{2}\right]\right\}.$$

The vertical u_z and radial u_r components of displacement make up the Rayleigh wave; the tangential component u_ϕ makes up the Love wave.

Expressions for p_{kQ}, C_{kQ} by integrals of eigenfunctions. The calculus of variation can be used to obtain formulae for p_{kQ} and C_{kQ} (2, 15, 23):

$$p_{kQ}^2 = (\xi^2 G_{1k}^Q + 2\xi G_{2k}^Q + G_{3k}^Q)/I_{kQ}, \qquad C_{kQ} = (\xi G_{1k}^Q + G_{2k}^Q)/p_{kQ} I_{kQ}, \tag{1.23}$$

where G_{jk}^Q are the following integrals:

$$G_{1k}^R = \int_0^\infty [(\lambda + 2\mu)(\tilde{V}_k^{(2)})^2 + \mu(\tilde{V}_k^{(1)})^2]\, dz,$$

$$G_{2k}^R = \int_0^\infty \left[\mu \frac{d\tilde{V}_k^{(2)}}{dz} \tilde{V}_k^{(1)} - \lambda \frac{d\tilde{V}_k^{(1)}}{dz} \tilde{V}_k^{(2)}\right] dz,$$

$$G_{3k}^R = \int_0^\infty \left[(\lambda + 2\mu)\left(\frac{d\tilde{V}_k^{(1)}}{dz}\right)^2 + \mu\left(\frac{d\tilde{V}_k^{(2)}}{dz}\right)^2\right] dz, \tag{1.24}$$

$$G_{1k}^L = \int_0^\infty \mu (\tilde{V}_k^{(3)})^2 dz,$$

$$G_{2k}^L \equiv 0,$$

$$G_{3k}^L = \int_0^\infty \mu \left(\frac{d\tilde{V}_k^{(3)}}{dz}\right)^2 dz.$$

Methods for calculating p_{kR}, $\tilde{V}_k^{(1)}$ and $\tilde{V}_k^{(2)}$ are described in (11,21), and the methods for $p_k L$, $\tilde{V}_k^{(3)}$ in (2,15).

SURFACE WAVES FROM ELEMENTARY SOURCES. We will consider what form equation (1.20) takes for some elementary sources: axially symmetric vertical and radial impacts, torsional impact, field of horizontal forces, dipoles, center of compression. The fields of many more complex sources can be obtained by adding the fields of these elementary sources with respective constants of proportionality. Because the function U_{kQ} in equation (1.20) is the only term which is affected by the source, it is only necessary to consider the expressions for U_{kQ} for various impacts

1. Vertical axially symmetric impact. Let

$$\mathbf{F} = F_z(t, z, r)\, \mathbf{a}_z.$$

In this case we get from (1.7):

$$f_0^{(1)}(z, \xi, p) = \int_{-\infty}^{+\infty} e^{-ipt} \int_0^{\infty} F_z J_0(\xi r)\, r dr dt;$$

$$f_m^{(1)} = 0 \quad \text{при } m \neq 0; \; f_m^{(2)} \equiv f_m^{(3)} \equiv 0;$$

$$D_{k0}^R = \int_0^{\infty} f_0^{(1)}(z, \xi_{kR}, p)\, \tilde{V}_k^{(1)}(z, \xi_{kR})\, dz;$$

$$D_{km}^R = 0 \quad \text{при } m \neq 0; \; D_{km}^L \equiv 0.$$

In summary,

$$U_{kR} = \int_0^{\infty} f_0^{(1)} \tilde{V}_k^{(1)} dz, \quad U_{kL} = 0. \qquad (1.25)$$

In particular, for an ideally concentrated vertical force at the point z = h, r = 0, $\mathbf{F} = \delta(z-h) \frac{\delta(r)}{r} \varphi(t) \mathbf{a}_z$ *

$$U_{kR} = \tilde{V}_k^{(1)}(h, p)\, S(p). \qquad (1.26$$

Here and later on $S(p) = \int_{-\infty}^{\infty} \Phi(t) \exp(-ipt) dt$ – the source time spectrum.

II. Radial axially symmetric impact. Let

$$\mathbf{F} = F_r(t, z, r)\, \mathbf{a}_r.$$

In this case we obtain from (1.7):

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* Translator's note: these concentrated forces are not normalized by the factor $1/2\pi$.

$$f_0^{(2)} = -\int_{-\infty}^{+\infty} e^{-ipt} \int_0^{\infty} F_r J_1(\xi r)\, r\, dr\, dt; \qquad f_m^{(2)} = 0 \quad \text{при} \quad m \neq 0;$$

$$f_m^{(1)} \equiv f_m^{(3)} \equiv 0;$$

$$D_{k0}^{R} = -\int_0^{\infty} f_0^{(2)} V_k^{(2)} dz; \qquad D_{km}^{R} = 0 \quad \text{при} \quad m \neq 0; D_{km}^{L} \equiv 0.$$

Hence,

$$U_{kR} = -\int_0^{\infty} f_0^{(2)} V_k^{(2)} dz, \qquad U_{kL} = 0. \tag{1.27}$$

In the ideal case of a radial source* concentrated at the point z = h, r = 0, $\mathbf{F} = 2\delta(z-h)\frac{\delta(r)}{r^2}\varphi(t)\,\mathbf{a}_r$

$$U_{kR} = -\xi_{kR} V_k^{(2)}(h, p)\, S(p). \tag{1.28}$$

III. Rotational impact. Let

$$\mathbf{F} = F_{\varphi}(t, z, r)\,\mathbf{a}_{\varphi}.$$

From (1.7) we get

$$f_0^{(3)} = \int_{-\infty}^{+\infty} e^{-ipt} \int_0^{\infty} F_{\varphi} J_1(\xi r)\, r\, dr\, dt; \qquad f_m^{(3)} = 0 \quad \text{при} \quad m \neq 0;$$

$$f_m^{(1)} \equiv f_m^{(2)} \equiv 0; \qquad D_{km}^{R} \equiv 0; \qquad D_{k0}^{L} = \int_0^{\infty} f_0^{(3)} V_k^{(3)} dz;$$

$$D_{km}^{L} = 0 \quad \text{при} \quad m \neq 0.$$

The final result is

$$U_{kR} = 0, \qquad U_{kL} = \int_0^{\infty} f_0^{(3)} V_k^{(3)} dz. \tag{1.29}$$

In the case of a concentrated torque at z = h, r = 0 and

$$\mathbf{F} = 2\delta(z-h)\frac{\delta(r)}{r^2}\varphi(t)\,\mathbf{a}_{\varphi}$$

$$U_{kL} = \xi_{kL} V_k^{(3)}(h, p)\, S(p). \tag{1.30}$$

IV. Field of a horizontal force with fixed orientation. We will consider only the particular case of this force field

$$\mathbf{F} = F_T(t, z, r)\,\mathbf{a}_T,$$

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* Translator's note: these concentrated forces are not normalized by the factor 1/2π

where a_T is a horizontal unit vector of fixed orientation:

$$(\mathbf{a}_T, \mathbf{a}_z) = 0, \ (\mathbf{a}_T, \mathbf{a}_r) = \cos(\delta - \varphi),$$

Then,

$$(\mathbf{a}_T, \mathbf{a}_\varphi) = \sin(\delta - \varphi).$$

$$\mathbf{F} = F_T [\cos(\delta - \varphi)\, \mathbf{a}_r + \sin(\delta - \varphi)\, \mathbf{a}_\varphi]$$

and from (1.7) we have

$$f_m^{(1)} \equiv 0; \quad f_m^{(2)} = f_m^{(3)} = 0 \quad \text{при} \ m \neq \pm 1;$$

$$f_1^{(2)} = \frac{e^{-i\delta}}{2} f_T; \quad f_{-1}^{(2)} = -\frac{e^{-i\delta}}{2} f_T;$$

$$f_1^{(3)} = \frac{e^{-i\delta}}{2i} f_T; \quad f_{-1}^{(3)} = \frac{e^{i\delta}}{2i} f_T;$$

$$f_T = \int_{-\infty}^{+\infty} e^{-ipt} \int_0^\infty F_T J_0(\xi r)\, r\, dr\, dt.$$

Summing over m, we obtain the following relation:

$$U_{kR} = i \cos(\delta - \varphi) \int_0^\infty f_T V_k^{(2)} dz, \tag{1.31}$$

$$U_{kL} = -i \sin(\delta - \varphi) \int_0^\infty f_T V_k^{(3)} dz.$$

In the case of an ideal concentrated force acting at the point r = 0, z = h,

$$\mathbf{F} = \delta(z - h) \frac{\delta(r)}{r} \varphi(t)\, \mathbf{a}_T;$$

$$U_{kR} = i \cos(\delta - \varphi) V_k^{(2)}(h, p) S(p), \tag{1.32}$$

$$U_{kL} = -i \sin(\delta - \varphi) V_k^{(3)}(h, p) S(p).$$

V. An arbitrarily oriented concentrated force. Let

$$\mathbf{F} = \delta(z - h) \frac{\delta(r)}{r} [\mathbf{a}_z \cos\beta + \mathbf{a}_T \sin\beta]\, \varphi(t).$$

Combining (1.25) and (1.32), we get

$$U_{kR} = [\cos\beta V_k^{(1)}(h, p) + i \sin\beta \cos(\delta - \varphi) V_k^{(2)}(h, p)] S(p), \tag{1.33}$$

$$U_{kL} = -i \sin\beta \sin(\delta - \varphi) V_k^{(3)}(h, p) S(p).$$

VI. Dipole without moment. The field of a dipole is made up of a pair of forces without moment

$$\mathbf{F}_\pm = \pm \delta(z - h) \frac{\delta(r)}{r} \varphi(t) [\mathbf{a}_z \cos\beta + \mathbf{a}_T \sin\beta],$$

we obtain the result by using the following operator on U_{kQ} IN (1.33) (keeping only those parts decreasing slower than r^{-1}):

$$\left[\cos\beta \frac{d}{dh} + i\xi_{kQ} \cos(\delta - \varphi) \sin\beta \right];$$

We obtain the results:

$$U_{kR} = \left[\cos^2\beta \, \frac{d\tilde{V}_k^{(1)}}{dh}(h, p) - \xi_{kR} \sin^2\beta \cos^2(\delta - \varphi) V_k^{(2)}(h, p) + \right. \tag{1.34}$$

$$\left. + \frac{i}{2} \sin 2\beta \left(\cos(\delta - \varphi) \xi_{kR} V_k^{(1)}(h, p) + \frac{d\tilde{V}_k^{(2)}}{dh}(h, p)\right)\right] S(p),$$

$$U_{kL} = -i \sin\beta \sin (\delta - \varphi) \left[\cos\beta \frac{d\tilde{V}_k^{(3)}}{dh}(h, p) + \right.$$

$$\left. + i\xi_{kL} \cos(\delta - \varphi) \sin\beta V_k^{(3)}(h, p)\right] S(p).$$

VII. Dipole with moment. Let a pair of forces act in the same direction as in VI, but allow the force to have a moment; the axis of the dipole, i.e., the line associated with the points where the force acts, is given by the vector a_n, where

$$(\mathbf{a}_n, \mathbf{a}_z) = \cos\gamma, \quad (\mathbf{a}_n, \mathbf{a}_r) = \sin\gamma \cos(\alpha - \varphi);$$

$$(\mathbf{a}_n, \mathbf{a}_\varphi) = \sin\gamma \sin(\alpha - \varphi).$$

The angles γ, δ, β, α are not independent, and are related by the relation $\mathrm{ctg}\gamma \mathrm{ctg}\beta = -\cos(\delta - \alpha)$. The displacement field of this dipole valid for large distances r is obtained by using the following operator on U_{kQ} in (1.33):

$$\left[\cos\gamma \frac{d}{dh} + i\xi_{kQ} \cos(\alpha - \varphi) \sin\gamma\right].$$

$$U_{kR} = \left[\cos\gamma \cos\beta \frac{d\tilde{V}_k^{(1)}}{dh}(h, p) - \xi_{kR} \sin\beta \sin\gamma \cos(\delta - \varphi) \cos(\alpha - \varphi) \times \right.$$

$$\times V_k^{(2)}(h, p) + i\xi_{kR} \sin\gamma \cos\beta \cos(\alpha - \varphi) V_k^{(1)}(h, p) +$$

$$\left. + i \cos\gamma \sin\beta \cos(\delta - \varphi) \frac{d\tilde{V}_k^{(2)}}{dh}(h, p)\right] \times S(p),$$

$$U_{kL} = -i \sin\beta \sin(\delta - \varphi) \left[\cos\gamma \frac{d\tilde{V}_k^{(3)}}{dh}(h, p) + i\xi_{kL} \sin\gamma \cos(\alpha - \varphi) \times \right.$$

$$\left. \times V_k^{(3)}(h, p)\right] \times S(p).$$

VIII. Center of expansion. The center of expansion can be considered to be equivalent to a source composed of three orthogonal dipoles without moment. We will take one dipole to be vertical ($\beta = 0$), and the other two horizontal ($\beta = \pi/2$, $\delta = 0$ and $\pi/2$). The resultant field is

$$U_{kR} = \left[\frac{d\tilde{V}_k^{(1)}}{dh}(h, p) - \xi_{kR} V_k^{(2)}\right] \times S(p), \quad U_{kL} = 0. \tag{1.35}$$

2. Displacement fields in an elastic sphere

Statement of the problem. We will consider an elastic sphere with

coordinates R, θ, φ $(0 \leqslant R \leqslant R_0,\ 0 \leqslant \theta \leqslant \pi,\ 0 \leqslant \varphi < 2\pi)$.
motion in this coordinate system are (7):

$$\frac{1}{R}\frac{d\widehat{\theta R}}{\partial\theta} + \frac{1}{R\sin\theta}\frac{\partial\widehat{\varphi R}}{\partial\varphi} + \frac{\partial\widehat{RR}}{\partial R} + \frac{1}{R}(2\widehat{RR} - \widehat{\theta\theta} - \widehat{\varphi\varphi} + \widehat{\theta R}\times$$
$$\times\operatorname{ctg}\theta) = \rho\frac{\partial^2 u_R}{\partial t^2} - F_R,$$

$$\frac{1}{R}\frac{\partial\widehat{\theta\theta}}{\partial\theta} + \frac{1}{R\sin\theta}\frac{\partial\widehat{\theta\varphi}}{\partial\varphi} + \frac{\partial\widehat{\theta R}}{\partial R} + \frac{1}{R}[3\widehat{\theta R} + (\widehat{\theta\theta} - \widehat{\varphi\varphi})\times$$
$$\times\operatorname{ctg}\theta] = \rho\frac{\partial^2 u_\theta}{\partial t^2} - F_\theta, \quad (2.1)$$

$$\frac{1}{R}\frac{\partial\widehat{\theta\varphi}}{\partial\theta} + \frac{1}{R\sin\theta}\frac{\partial\widehat{\varphi\varphi}}{\partial\varphi} + \frac{\partial\widehat{\varphi R}}{\partial R} +$$
$$+\frac{1}{R}[3\widehat{\varphi R} + 2\widehat{\theta\varphi}\operatorname{ctg}\theta] = \rho\frac{\partial^2 u_\varphi}{\partial t^2} - F_\varphi.$$

Here θR, $\theta\theta$, ϕR, $\phi\phi$, RR are components of the stress tensor; u_R, u_θ, u_ϕ are components of the displacement vector u along the directions a_R, a_θ , a_ϕ ; F_R, F_θ, F_ϕ are components of the vector volume force acting at the source along the same directions.

For the components of stress we have the following relations

$$\begin{aligned}
\widehat{\theta R} &= \mu\left(\frac{\partial u_\theta}{\partial R} - \frac{u_\theta}{R} + \frac{1}{R}\frac{\partial u_R}{\partial\theta}\right),\\
\widehat{\theta\theta} &= \lambda\Delta + 2\frac{\mu}{R}\left(\frac{\partial u_\theta}{\partial\theta} + u_R\right),\\
\widehat{\theta\varphi} &= \frac{\mu}{R}\left(\frac{\partial u_\varphi}{\partial\theta} - u_\varphi\operatorname{ctg}\theta + \frac{1}{\sin\theta}\frac{\partial u_\theta}{\partial\varphi}\right),\\
\widehat{\varphi R} &= \mu\left(\frac{\partial u_\varphi}{\partial R} - \frac{u_\varphi}{R} + \frac{1}{R\sin\theta}\frac{\partial u_R}{\partial\varphi}\right),\\
\widehat{\varphi\varphi} &= \lambda\Delta + 2\frac{\mu}{R}\left(u_R + u_\theta\operatorname{ctg}\theta + \frac{1}{\sin\theta}\frac{\partial u_\varphi}{\partial\varphi}\right),\\
\widehat{RR} &= \lambda\Delta + 2\mu\frac{\partial u_R}{\partial R}.
\end{aligned} \quad (2.2)$$

Here the dilatation has the form

$$\Delta = \frac{\partial u_R}{\partial R} + \frac{2u_R}{R} + \frac{1}{R\sin\theta}\frac{\partial u_\varphi}{\partial\varphi} + \frac{1}{R}\frac{\partial u_\theta}{\partial\theta} + \frac{u_\theta}{R}\operatorname{ctg}\theta. \quad (2.3)$$

The Lamé constants λ, μ and the density ρ are piecewise continuous positive functions of a single coordinate R; the components of displacement and stress are continuous and bounded for all points in 0, R_0 . The surface of the sphere is free from stress, i.e.

$$\widehat{\theta R} = \widehat{\varphi R} = \widehat{RR} = 0 \ \text{при}\ R = R_0. \quad (2.4)$$

The initial conditions are

$$u = \frac{\partial u}{\partial t} = 0 \quad \text{при} \quad t < 0. \qquad (2.4a)$$

The force field $F(t,R,\theta,\phi)$ is described as a real source localized in space and time. The following limitations are laid upon F:

1) $\mathbf{F}(t, R, \theta, \varphi) = 0$ при $t < 0$;
2) $\mathbf{F}(t, R, \theta, \varphi)$

is absolutely integrable with respect to t and satisfies the Dirichlet conditions for all arguments. Then we are permitted the following representation:

$$\mathbf{F} = \frac{1}{2\pi} \int\limits_{-\infty}^{+\infty} e^{ipt} \left[\sum_{n=0}^{\infty} \sum_{m=-n}^{n} \sum_{i=1}^{3} f_{mn}^{(i)}(R, p)\, \mathbf{A}_{mn} \right] dp, \qquad (2.5)$$

where

$$\begin{aligned}
&\mathbf{A}_{mn}^{(1)} = \mathbf{a}_R Y_{mn},\\
&\mathbf{A}_{mn}^{(2)} = \left(\mathbf{a}_\theta \frac{\partial Y_{mn}}{\partial \theta} + \mathbf{a}_\varphi \frac{1}{\sin\theta} \frac{\partial Y_{mn}}{\partial \varphi} \right) \frac{1}{N},\\
&\mathbf{A}_{mn}^{(3)} = \left(\mathbf{a}_\theta \frac{\partial Y_{mn}}{\partial \varphi} \frac{1}{\sin\theta} - \mathbf{a}_\varphi \frac{\partial Y_{mn}}{\partial \theta} \right) \frac{1}{N}, \qquad (2.6^{1})\\
&\quad Y_{mn}(\theta, \varphi) = e^{im\varphi} P_n^m(\cos\theta),\\
&\quad N = \sqrt{n(n+1)}.
\end{aligned}$$

$p_n^m(\cos\theta)$ is the associated Legendre polynomial, defined by the following formulae (8):

$$P_n^m(x) = \frac{(1-x^2)^{m/2}}{2^n \cdot n!} \frac{d^{n+m}(x^2-1)^n}{dx^{n+m}} \quad \text{при} \quad m \geqslant 0,$$

$$P_n^m(x) = (-1)^m \frac{(n-|m|)!}{(n+|m|)!} P_n^{|m|}(x) \quad \text{при} \quad m < 0.$$

The system of spherical vector functions $A_{mn}^{(i)}$ are a field of a system of vectors satisfying the following orthogonality conditions on a unit sphere:

$$\int\limits_0^{2\pi}\int\limits_0^{\pi} (\mathbf{A}_{mn}^{(i)}, \overline{\mathbf{A}}_{lq}^{(j)}) \sin\theta \, d\theta \, d\varphi = 4\pi \delta_{ij} \delta_{ml} \delta_{nq} \frac{(n+m)!}{(n-m)!\,(2n+1)}.$$

---

[1] For n = 0 N = 0, Y_{mn} = const and (2.6) loses meaning, we will consider that $A_{00}^{(2)} = A_{00}^{(3)} = 0$.

Another system of vector functions was suggested for the solution of a similar problem in elastic wave theory in the work of G. I. Petrashin (11), which are linear combinations of the system $A_{mn}^{(i)}$. In distinction with (11) where the wave field in the sphere is given in the form of a summation of potentials and solenoidal fields, the analysis of the wave fields according to the system $A_{mn}^{(i)}$ permits a separate study of the spheroidal and torsional oscillations of a sphere (22). All proofs on the orthogonality and completeness of the system, studied in (11) are easily applied to the system $A_{mn}^{(i)}$.

The coefficients $f_{mn}^{(i)}$ making up the impact F in the system $A_{mn}^{(i)}$ are

$$f_{mn}^{(i)}(R,\, p) = \frac{2n+1}{4\pi}\frac{(n-m)!}{(n+m)!}\int_{-\infty}^{+\infty} e^{-ipt}\int_0^{2\pi}\int_0^{\pi}(\mathbf{F},\, \overline{\mathbf{A}}_{mn}^{(i)})\sin\theta\, d\theta\, d\varphi\, dt.$$

Specifically for i = 1, 2, 3 we have:

$$\begin{aligned}
f_{mn}^{(1)} &= \frac{(n-m)!}{(n+m)!}\frac{2n+1}{4\pi}\int_{-\infty}^{+\infty} e^{-ipt}\int_0^{2\pi}\int_0^{\pi} F_R\overline{Y}_{mn}\sin\theta\, d\theta\, d\varphi\, dt,\\
f_{mn}^{(2)} &= \frac{(n-m)!}{(n+m)!}\frac{2n+1}{4\pi N}\int_{-\infty}^{+\infty} e^{-ipt}\int_0^{2\pi}\int_0^{\pi}\left[F_\theta\frac{\partial\overline{Y}_{mn}}{\partial\theta} + F_\varphi\frac{1}{\sin\theta}\frac{\partial\overline{Y}_{mn}}{\partial\varphi}\right]\times\\
&\qquad\times\sin\theta\, d\theta\, d\varphi\, dt,\\
f_{mn}^{(3)} &= \frac{(n-m)!}{(n+m)!}\frac{2n+1}{4\pi N}\int_{-\infty}^{+\infty} e^{-ipt}\int_0^{2\pi}\int_0^{\pi}\left[F_\theta\frac{1}{\sin\theta}\frac{\partial\overline{Y}_{mn}}{\partial\varphi} - F_\varphi\frac{\partial\overline{Y}_{mn}}{\partial\theta}\right]\times\\
&\qquad\times\sin\theta\, d\theta\, d\varphi\, dt,\\
\overline{Y}_{mn}(\theta,\, \varphi) &= e^{-im\varphi}p_n^m(\cos\theta).
\end{aligned}\tag{2.7}$$

For the force components F_R, F_θ, F_ϕ we obtain from (2.5), (2.6):

$$\begin{aligned}
F_R &= \frac{1}{2\pi}\int_{-\infty}^{+\infty} e^{ipt}\left[\sum_{n=0}^{\infty}\sum_{m=-n}^{n} f_{mn}^{(1)}Y_{mn}\right]dp,\\
F_\theta &= \frac{1}{2\pi}\int_{-\infty}^{+\infty} e^{ipt}\left[\sum_{n=1}^{\infty}\frac{1}{N}\sum_{m=-n}^{n}\left(f_{mn}^{(2)}\frac{\partial Y_{mn}}{\partial\theta} + f_{mn}^{(3)}\frac{1}{\sin\theta}\frac{\partial Y_{mn}}{\partial\varphi}\right)\right]dp,\\
F_\varphi &= \frac{1}{2\pi}\int_{-\infty}^{+\infty} e^{ipt}\left[\sum_{n=1}^{\infty}\frac{1}{N}\sum_{m=-n}^{n}\left(f_{mn}^{(2)}\frac{1}{\sin\theta}\frac{\partial Y_{mn}}{\partial\varphi} - f_{mn}^{(3)}\frac{\partial Y_{mn}}{\partial\theta}\right)\right]dp.
\end{aligned}\tag{2.8}$$

<u>Formulae for displacements</u>. We will seek displacements in the form

$$\mathbf{u}(t,\, R,\, \theta,\, \varphi) = \frac{1}{2\pi}\mathbf{v}.\int_{-\infty}^{+\infty} e^{ipt}\left[\sum_{n=0}^{\infty}\sum_{m=-n}^{n}\sum_{i=1}^{3} V_{mn}^{(i)}\mathbf{A}_{mn}^{(i)}\right]dp, \tag{2.9}$$

where $V_{mn}^{(i)} = V_{mn}^{(i)}(R,p)$. Hence for the projection of displacements along the directions a_R, a_θ, a_ϕ we obtain:

$$u_R = \frac{1}{2\pi} \text{v.} \int_{-\infty}^{+\infty} e^{ipt} \left[\sum_{n=0}^{\infty} \sum_{m=-n}^{n} V_{mn}^{(1)} Y_{mn} \right] dp,$$

$$u_\theta = \frac{1}{2\pi} \text{v.} \int_{-\infty}^{+\infty} e^{ipt} \left[\sum_{n=1}^{\infty} \frac{1}{N} \sum_{m=-n}^{n} \left(V_{mn}^{(2)} \frac{\partial Y_{mn}}{\partial \theta} + V_{mn}^{(3)} \frac{1}{\sin\theta} \frac{\partial Y_{mn}}{\partial \varphi} \right) \right] dp, \tag{2.10}$$

$$u_\varphi = \frac{1}{2\pi} \text{v.} \int_{-\infty}^{+\infty} e^{ipt} \left[\sum_{n=1}^{\infty} \frac{1}{N} \sum_{m=-n}^{n} \left(V_{mn}^{(2)} \frac{1}{\sin\theta} \frac{\partial Y_{mn}}{\partial \varphi} - V_{mn}^{(3)} \frac{\partial Y_{mn}}{\partial \theta} \right) \right] dp.$$

Placing (2.8), (2.10) into equation (2.1) and the boundary conditions, we arrive at the following equation for $V_{mn}^{(i)}$.

1. For $V_{mn}^{(1)}$, $V_{mn}^{(2)}$:

$$\begin{aligned} l_1(V_{mn}^{(1)}, V_{mn}^{(2)}) \equiv \frac{d}{dR} \left[(\lambda + 2\mu) \frac{dV_{mn}^{(1)}}{dR} + \frac{2\lambda}{R} V_{mn}^{(1)} - \frac{\lambda N}{R} V_{mn}^{(2)} \right] + \\ + \frac{\mu}{R^2} \left[4 \frac{dV_{mn}^{(1)}}{dR} R - 4V_{mn}^{(1)} + \right. \\ \left. + N \left(3V_{mn}^{(2)} - R \frac{dV_{mn}^{(2)}}{dR} - NV_{mn}^{(1)} \right) \right] + p^2 \rho V_{mn}^{(1)} = - f_{mn}^{(1)}, \\ l_2(V_{mn}^{(1)}, V_{mn}^{(2)}) \equiv \frac{d}{dR} \left[\mu \left(\frac{dV_{mn}^{(2)}}{dR} - \frac{V_{mn}^{(2)}}{R} + \frac{NV_{mn}^{(1)}}{R} \right) \right] + \\ + \frac{\lambda N}{R} \left(\frac{dV_{mn}^{(1)}}{dR} + \frac{2}{R} V_{mn}^{(1)} - \frac{N}{R} V_{mn}^{(2)} \right) + \\ + \frac{\mu}{R^2} \left(5NV_{mn}^{(1)} + 3R \frac{dV_{mn}^{(2)}}{dR} - V_{mn}^{(2)} - 2N^2 V_{mn}^{(2)} \right) + p^2 \rho V_{mn}^{(2)} = - f_{mn}^{(2)} \end{aligned} \tag{2.11}$$

with the boundary conditions:

$$\begin{aligned} \sigma_{RR} \equiv (\lambda + 2\mu) \frac{dV_{mn}^{(1)}}{dR} + \frac{2\lambda}{R} V_{mn}^{(1)} - \frac{\lambda N}{R} V_{mn}^{(2)} = 0, \\ \tau_{\theta R} \equiv \mu \left(\frac{dV_{mn}^{(2)}}{dR} - \frac{V_{mn}^{(2)}}{R} + \frac{NV_{mn}^{(1)}}{R} \right) = 0 \quad \text{при} \quad R = R_0, \\ V_{mn}^{(1)} = V_{mn}^{(2)} = 0 \quad \text{при} \quad R = 0. \end{aligned} \tag{2.12}$$

The functions $V_{mn}^{(1)}$, $V_{mn}^{(2)}$, σ_{RR} and $\tau_{\theta R}$ are continuous and bounded everywhere on 0, R_o.

2. For $V_{mn}^{(3)}$ we have

$$\begin{aligned} l_3(V_{mn}^{(3)}) \equiv \frac{d}{dR} \left[\mu \left(\frac{dV_{mn}^{(3)}}{dR} - \frac{V_{mn}^{(3)}}{R} \right) \right] + \frac{3\mu}{R} \frac{dV_{mn}^{(3)}}{dR} - \\ - \frac{\mu}{R^2} (N^2 + 1) V_{mn}^{(3)} + p^2 \rho V_{mn}^{(3)} = - f_{mn}^{(3)} \end{aligned} \tag{2.13}$$

and the boundary condition:

$$\tau_{\varphi R} \equiv \mu\left(\frac{dV^{(3)}_{mn}}{dR} - \frac{V^{(3)}_{mn}}{R}\right) = 0 \quad \text{при} \quad R = R_0,$$
$$V^{(3)}_{mn} = 0 \quad \text{при} \quad R = 0. \tag{2.14}$$

The functions $V^{(3)}_{mn}$, $\tau_{\phi R}$ continuous and bounded everywhere on $\left[0,\ R_o\right]$.

Expressions for V_{mn} in terms of eigenfunctions. $V^{(i)}_{mn}$ (i = 1, 2) can be expressed in the following manner:

$$V^{(i)}_{mn} = \frac{1}{R_0^2}\sum_{k=1}^{\infty} c^{S}_{kmn}\widetilde{V}^{(i)}_{kn}, \tag{2.15}$$

where for the coefficients c^S_{kmn} we have:

$$c^{S}_{kmn} = \frac{1}{p^2_{knS} - p^2}\,\frac{D^{S}_{kmn}}{I_{knS}}, \tag{2.16}$$
$$D^{S}_{kmn} = \int_0^{R_0} \left(f^{(1)}_{mn}\widetilde{V}^{(1)}_{kn} + f^{(2)}_{mn}\widetilde{V}^{(2)}_{kn}\right) R^2\, dR,$$
$$I_{knS} = \frac{1}{R_0^2}\int_0^{R_0} \rho R^2 \left[(\widetilde{V}^{(1)}_{kn})^2 + (\widetilde{V}^{(2)}_{kn})^2\right] dR.$$

Here $\widetilde{V}^{(i)}_{kn}(R)$, i = 1, 2 is the eigenfunction, p^2_{knS} is the eigenvalue of the operator consisting of the left hand side of (2.11) together with the boundary conditions (2.12), for a given integer value of the parameter n.

Similarly

$$V^{(3)}_{mn} = \frac{1}{R_0^2}\sum_{k=1}^{\infty} c^{T}_{kmn}\widetilde{V}^{(3)}_{kn}, \tag{2.17}$$

where

$$c^{T}_{kmn} = \frac{1}{p^2_{knT} - p^2}\,\frac{D^{T}_{kmn}}{I_{knT}},$$
$$D^{T}_{kmn} = \int_0^{R_0} f^{(3)}_{mn}\widetilde{V}^{(3)}_{kn} R^2\, dR, \tag{2.18}$$
$$I_{knT} = \frac{1}{R_0^2}\int_0^{R_0} \rho R^2 (\widetilde{V}^{(3)}_{kn})^2\, dR.$$

where $\tilde{V}^{(3)}_{kn}$ is the eigenfunction, a p^2_{knT} is the eigenvalue of the operator consisting of the left-hand side of (2.13) and the boundary conditions (2.14).

To derive (2.15) - (2.18) these orthogonality conditions were used:

$$\int_0^{R_0} \rho R^2 \left(\tilde{V}^{(1)}_{kn}\tilde{V}^{(1)}_{ln} + \tilde{V}^{(2)}_{kn}\tilde{V}^{(2)}_{ln}\right) dR = 0,$$

$$\int_0^{R_0} \rho R^2 \left(\tilde{V}^{(3)}_{kn}\tilde{V}^{(3)}_{ln}\right) dR = 0 \quad \text{при} \quad k \neq l.$$

Placing the expressions developed for $V^{(i)}_{mn}$ (R,p) into (2.10) the following formulae for displacements are obtained:

$$u_R = \frac{1}{2\pi R_0^2} \text{v.} \int_{-\infty}^{+\infty} e^{ipt} \left[\sum_{n=0}^{\infty} \sum_{m=-n}^{n} \sum_{k=1}^{\infty} c^S_{kmn}(p)\, \tilde{V}^{(1)}_{kn}(R)\, Y_{mn}\right] dp,$$

$$u_\theta = \frac{1}{2\pi R_0^2} \text{v.} \int_{-\infty}^{+\infty} e^{ipt} \left[\sum_{n=1}^{\infty} \frac{1}{N} \sum_{m=-n}^{n} \sum_{k=1}^{\infty} \left(c^S_{kmn}(p)\, \tilde{V}^{(2)}_{kn}(R) \frac{\partial Y_{mn}}{\partial\theta} + \right.\right.$$

$$\left.\left. + c^T_{kmn}(p)\, \tilde{V}^{(3)}_{kn}(R) \frac{1}{\sin\theta} \frac{\partial Y_{mn}}{\partial\varphi}\right)\right] dp, \qquad (2.19)$$

$$u_\varphi = \frac{1}{2\pi R_0^2} \text{v.} \int_{-\infty}^{+\infty} e^{ipt} \left[\sum_{n=1}^{\infty} \frac{1}{N} \sum_{m=-n}^{n} \sum_{k=1}^{\infty} \left(c^S_{kmn}(p)\, \tilde{V}^{(2)}_{kn}(R) \frac{1}{\sin\theta} \frac{\partial Y_{mn}}{\partial\varphi} - \right.\right.$$

$$\left.\left. - c^T_{kmn}(p)\, \tilde{V}^{(3)}_{kn}(R) \frac{\partial Y_{mn}}{\partial\theta}\right)\right] dp.$$

The proof that this constructed solution (2.19) satisfies the initial conditions (2.4a) proceeds analogously with that of the plane case for each term in the series $\sum\limits_{m,\,n}$.

Expression of displacements in the form of a sum of proper oscillations of a sphere. In formula (2.19) only products of the type c^Q_{mn} exp(ipt) depend on p. Thus calculation of integrals with respect to p reduces to evaluating a number of integrals of the type

$$\text{v.} \int_{-\infty}^{+\infty} \frac{e^{ipt}\psi(p)}{\alpha^2 - p^2}\, dp.$$

By introducing the conditions on the source, $\psi(p)$ can have singularities for Im $p \geq 0$ and must be analytic in the lower halfspace. As $p \to \infty$, $\psi(p) \to 0$. Besides this $\psi(-p) = \overline{\psi(p)}$. Hence it is not difficult to obtain

$$\text{v.}\int_{-\infty}^{+\infty} e^{ipt}\frac{\psi(p)}{\alpha^2-p^2}\,dp = 2\,\mathrm{Re}\left[\text{v.}\int_{0}^{\infty} e^{ipt}\frac{\psi(p)}{\alpha^2-p^2}\,dp\right] =$$

$$= \frac{2\pi}{\alpha}\mathrm{Re}\left[\psi(\alpha)\,e^{i(\alpha t-\pi/2)}\right] + a(t).$$

The function $\alpha(t)$ is associated with the particular source function $f_{mn}^{(i)}$ in the upper halfplane for $p = 0$. Its form, either exponential decay with time or static impact, is not of interest to us.

Placing this evaluation into (2.19) and ignoring $\alpha(t)$ we get

$$u_R = \frac{1}{R_0^2}\mathrm{Re}\sum_{n=0}^{\infty}\sum_{m=-n}^{n}\sum_{k=1}^{\infty}\exp\left[i\left(p_{knS}t-\frac{\pi}{2}\right)\right]D_{kmn}^{S}\frac{\widetilde{V}_{kn}^{(1)}(R)}{p_{knS}}\frac{Y_{mn}}{I_{knS}},$$

$$u_\theta = \frac{1}{R_0^2}\mathrm{Re}\sum_{n=1}^{\infty}\frac{1}{N}\sum_{m=-n}^{n}\sum_{k=1}^{\infty}\left\{\exp\left[i\left(p_{knS}t-\frac{\pi}{2}\right)\right]D_{kmn}^{S}\frac{\widetilde{V}_{kn}^{(2)}(R)}{p_{knS}I_{knS}}\frac{\partial Y_{mn}}{\partial\theta} + \right.$$

$$\left. + \exp\left[i\left(p_{knT}t-\frac{\pi}{2}\right)\right]D_{kmn}^{T}\frac{\widetilde{V}_{kn}^{(3)}(R)}{p_{knT}I_{knT}\sin\theta}\frac{\partial Y_{mn}}{\partial\varphi}\right\}, \tag{2.19a}$$

$$u_\varphi = \frac{1}{R_0^2}\mathrm{Re}\sum_{n=1}^{\infty}\frac{1}{N}\sum_{m=-n}^{n}\sum_{k=1}^{\infty}\left\{\exp\left[i\left(p_{knS}t-\frac{\pi}{2}\right)D_{kmn}^{S}\times\right.\right.$$

$$\left.\times\frac{\widetilde{V}_{kn}^{(2)}(R)}{I_{knS}p_{knS}\sin\theta}\frac{\partial Y_{mn}}{\partial\varphi}\right] - \exp\left[i\left(p_{knT}t-\frac{\pi}{2}\right)\right]D_{kmn}^{T}\frac{\widetilde{V}_{kn}^{(3)}(R)}{I_{knT}p_{knT}}\left.\frac{\partial Y_{mn}}{\partial\theta}\right\}.$$

Thus we have expressed the displacements in the form of proper S spheroidal and toroidal T oscillations of a sphere with discrete frequencies p_{knS} and p_{knT}. The radial component u_R is composed only of spheroidal oscillations; the components u_θ and u_ϕ are made up of both spheroidal and torsional components.

Asymptotic values of displacement for $n \sin\theta >> |m|$. We will follow (13) for separating the propagating surface waves from the displacements described in (2.19a). We will change the order of summation

(i.e., change the sum $\sum_{n=o}^{\infty}\sum_{m=-n}^{\infty}\sum_{k=1}^{\infty}$ to $\sum_{k=1}^{\infty}\sum_{m=-\infty}^{\infty}\sum_{n=o}^{\infty}$) and

we will change the summation with respect to n to an integration along a contour L, enclosing positive part of the real axis in the plane of the complex variable ν, where n = Integer (Re ν):

$$\sum_{n=0}^{\infty} f_n P_n^m(\cos\theta) = -\frac{1}{2i}\oint_L \frac{f(\nu)\,P_\nu^m(\cos(\pi-\theta))\,(-1)^m\,d\nu}{\cos\pi\left(\nu+\frac{1}{2}\right)}.$$

Expressing $(\cos\pi(\nu+1/2))^{-1}$ for $\mathrm{Im}\,\nu \geq 0$ by the series

$$2\sum_{l=0}^{\infty}(-1)^l\exp\left[i(2l+1)(\nu+{}^1\!/_2)\pi\right],$$

and for $\mathrm{Im}\,\nu < 0$ the series

$$2\sum_{l=0}^{\infty}(-1)^l\exp\left[-i(2l+1)(\nu+{}^1\!/_2)\pi\right],$$ fixing the contour of integration to the real axis and introducing a new variable $p = p_{kQ}(\nu)$ we have

$$\sum_{n=0}^{\infty} f_n P_n^m(\cos\theta) \approx 2\sum_{l=0}^{\infty}(-1)^{l+|m|}\int_0^{\infty} f(p)\,P_\nu^m(\cos(\pi-\theta)) \times \\ \times \sin\left[(2l+1)\left(\nu+\frac{1}{2}\right)\pi\right]\frac{d\nu}{dp}\,dp.$$

Excluding from consideration the low frequency part of the field with $p < \bar{p}$ and applying the asymptotic expansion of the associated Legendre polynomial for large values $n \sin\theta \gg |m|$, we arrive at

$$\sum_{\mathrm{Ent}[\nu(\bar{p})]}^{\infty} f_n P_n^m(\cos\theta) \approx \frac{2\sqrt{2}}{\sqrt{\pi\sin\theta}}\sum_{l=0}^{\infty}(-1)^l\int_{\bar{p}}^{\infty}\nu^{m-1/2} f(p)\frac{d\nu}{dp} \times \\ \times \cos\left[\left(\nu+\frac{1}{2}\right)(\pi-\theta)-\frac{\pi}{4}+\frac{m\pi}{2}\right]\sin\left[(2l+1)\left(\nu+\frac{1}{2}\right)\pi\right]dp.$$

$$\sum_{\mathrm{Ent}[\nu(\bar{p})]}^{\infty} f_n \frac{dP_n^m(\cos\theta)}{d\theta} \approx \frac{2\sqrt{2}}{\sqrt{\pi\sin\theta}}\sum_{l=0}^{\infty}(-1)^l\int_{\bar{p}}^{\infty}\nu^{m+1/2} f(p)\frac{d\nu}{dp} \times \\ \times \cos\left[\left(\nu+\frac{1}{2}\right)(\pi-\theta)-\frac{3\pi}{4}+\frac{m\pi}{2}\right]\sin\left[(2l+1)\left(\nu+\frac{1}{2}\right)\pi\right]dp.$$

Using these formulas for transforming (2.19a), we can obtain the following asymptotic expression for the displacements in surface waves, going around the sphere l times and passing through the pole $\theta = 0$ and the antipodes $\theta = \pi$, g times:

$$u_R(t, R, \theta, \varphi) = \frac{1}{\sqrt{2\pi R_0 \sin\theta}}\,\dot{\mathrm{R}}\mathrm{e}\int_{\bar{p}}^{\infty}\frac{1}{p}\exp\left[i\left(pt+\frac{\pi}{4}(2g-1)\right)\right] \times \\ \times\left\{\sum_{k=1}^{K_S(p)}\frac{U_{kS}(p,\varphi)}{I_{k\nu S}C_{kS}}\sqrt{\frac{\nu_{kS}}{R_0}}\,\widetilde{V}_{k\nu}^{(1)}\exp\left[-i\left(\nu_{kS}+\frac{1}{2}\right)\widetilde{\theta}\right]\right\}dp,$$

$$(2.20)$$

$$u_\theta(t, R, \theta, \varphi) = \frac{1}{\sqrt{2\pi R_0 \sin\theta}}\,\mathrm{Re}\int_{\bar{p}}^{\infty}\frac{1}{p}\exp\left[i\left(pt-\frac{\pi}{4}(2g+3)\right)\right] \times \\ \times\left\{\sum_{k=1}^{K_S(p)}\frac{U_{kS}(p,\varphi)}{I_{k\nu S}C_{kS}}\sqrt{\frac{\nu_{kS}}{R_0}}\,\widetilde{V}_{k\nu}^{(2)}\exp\left[-i\left(\nu_{kS}+\frac{1}{2}\right)\widetilde{\theta}\right]\right\}dp,$$

$$u_\varphi(t, R, \theta, \varphi) = \frac{1}{\sqrt{2\pi R_0 \sin\theta}}\,\mathrm{Re}\int_{\bar{p}}^{\infty}\frac{1}{p}\exp\left[i\left(pt-\frac{\pi}{4}(2g-1)\right)\right] \times \\ \times\left\{\sum_{k=1}^{K_T(p)}\frac{U_{KT}(p,\varphi)}{I_{k\nu T}C_{kT}}\widetilde{V}_{k\nu}^{(3)}\exp\left[-i\left(\nu_{kT}+\frac{1}{2}\right)\widetilde{\theta}\right]\right\}dp.$$

Here for Q = S, T, $K_Q(p)$ is the maximum number of harmonics in Rayleigh waves (Q = S) and Love waves (Q = T) for a given p;

$$U_{kQ} = \sum_{m=-\infty}^{\infty} \nu_{kQ}^{m-1} D_{kmn}^{Q} \exp\left[im\left(\varphi + (-1)^{g}\pi/2\right)\right], \qquad (2.21)$$

where ν_{kQ} is the analog of the wave number, a root of the equation $P_{kQ}^2(\nu) - p^2 = 0$.

The phase and group velocities of the k-th harmonic v_{kQ} and C_{kQ} along the surface of the sphere are expressed through ν_{kQ}:

$$v_{kQ} = \frac{pR_0}{\nu_{kQ} + {}^1/_2}, \qquad C_{kQ} = R_0 \frac{dp_{kQ}}{d\nu}\,(\nu = \nu_{kQ}). \qquad (2.22)$$

The multiple passages of the waves through the poles $\widetilde{\theta}\,(0 \leq \widetilde{\theta} < \infty)$ are related to the coordinate of the observation point $\theta\,(0 \leq \theta < \pi)$

$$\widetilde{\theta} = (-1)^{g}\theta + 2\pi(g - l).$$

The components u_F and u_θ form the Rayleigh wave; u_ϕ, the Love wave.

<u>Expressions for P_{kQ}, C_{kQ} in terms of integrals of the eigenfunctions</u>.

The method of variational analysis can be used (25) for Q = S, T to obtain

$$p_{k\nu Q}^2 = \frac{N^2 G_{1k}^{Q} + 2NG_{2k}^{Q} + G_{3k}^{Q}}{R_0^2 I_{k\nu Q}}, \qquad C_{kQ} = \frac{NG_{1k}^{Q} + G_{2k}^{Q}}{R_0 p_{kQ} I_{k\nu Q}}. \qquad (2.23)$$

Here the integrals G_{1k}^Q-G_{3k}^Q have the form

$$G_{1k}^{S} = \int_0^{R_0} \left[(\lambda + 2\mu)(\widetilde{V}_{k\nu}^{(2)})^2 + \mu(\widetilde{V}_{k\nu}^{(1)})^2\right] dR,$$

$$G_{2k}^{S} = \int_0^{R_0} \left[\left(\mu\widetilde{V}_{k\nu}^{(1)} \frac{d\widetilde{V}_{k\nu}^{(2)}}{dR} - \lambda\widetilde{V}_{k\nu}^{(2)} \frac{d\widetilde{V}_{k\nu}^{(1)}}{dR}\right) R - (2\lambda + 3\mu)\,\widetilde{V}_{k\nu}^{(1)}\widetilde{V}_{k\nu}^{(2)}\right] dR,$$

$$G_{3k}^{S} = \int_0^{R_0} \left\{\left[(\lambda + 2\mu)\left(\frac{d\widetilde{V}_{k\nu}^{(1)}}{dR}\right)^2 + \mu\left(\frac{d\widetilde{V}_{k\nu}^{(2)}}{dR}\right)^2\right] R^2 + \right.$$

$$\left. + 2\left(2\lambda\widetilde{V}_{k\nu}^{(1)} \frac{d\widetilde{V}_{k\nu}^{(2)}}{dR} - \mu\widetilde{V}_{k\nu}^{(2)} \frac{d\widetilde{V}_{k\nu}^{(1)}}{dR}\right) R + 4(\lambda + \mu)(\widetilde{V}_{k\nu}^{(1)})^2 - \mu(\widetilde{V}_{k\nu}^{(2)})^2\right\} dR,$$

$$G_{1k}^{T} = \int_0^{R_0} \mu(\widetilde{V}_{k\nu}^{(3)})^2\, dR, \qquad (2.24)$$

$$G_{2k}^{T} \equiv 0,$$

$$G_{3k}^{T} = \int_0^{R_0} \mu\left[\left(\frac{d\widetilde{V}_{k\nu}^{(3)}}{dR}\right)^2 R^2 - 2\widetilde{V}_{k\nu}^{(3)} \frac{d\widetilde{V}_{k\nu}^{(3)}}{dR} R - (\widetilde{V}_{k\nu}^{(3)})^2\right] dR,$$

$$N = \sqrt{\nu(\nu + 1)}.$$

Methods for calculation of $\tilde{V}^{(i)}_{k\nu}$-$P^{i}_{k\nu Q}$ are given in (3, 4, 14).

SURFACE WAVES FROM ELEMENTARY SOURCES. The source type influences only the terms U_{kQ} in (2.20). We will consider expressions for U_{kQ} for several elementary impacts, noting that $\nu \gg |m|$.

I. <u>Radial axially symmetric force.</u> Let

$$\mathbf{F} = F_R(t, R, \theta)\,\mathbf{a}_R.$$

From (2.7) we obtain:

$$\begin{aligned} f^{(1)}_{0\nu} &= (\nu_{kS} + {}^1/_2) \int_{-\infty}^{+\infty} e^{-ipt} \int_0^{\pi} F_R P_\nu(\cos\theta)\sin\theta\, d\theta\, dt, \\ f^{(1)}_{m\nu} &= 0 \quad \text{при} \quad m \neq 0, \quad f^{(2)}_{m\nu} \equiv f^{(3)}_{m\nu} \equiv 0, \\ U_{kS} &= \frac{1}{\nu_{kS}} \int_0^{R_0} f^{(1)}_{0\nu} \widetilde{V}^{(1)}_{k\nu}(R)\, R^2\, dR, \\ U_{kT} &= 0. \end{aligned} \tag{2.25}$$

In particular, for an ideally concentrated radial force at the point R = H, θ = 0

$$\mathbf{F} = \delta(R - H)\frac{\delta(\theta)}{H^2 \sin\theta}\varphi(t)\,\mathbf{a}_R$$

$$U_{kS} = \widetilde{V}^{(1)}_{k\nu}(H) \times S(p), \tag{2.26}$$

where $S(p) = \int_{-\infty}^{+\infty} e^{-ipt}\varphi(t)\, dt$ - time spectra of the source.

II. <u>Axially symmetric force directed along a meridian.</u> Let

$$\mathbf{F} = F_\theta(t, R, \theta)\,\mathbf{a}_\theta.$$

In this case we have from (2.7):

$$\begin{aligned} & f^{(1)}_{m\nu} \equiv f^{(3)}_{m\nu} \equiv 0; \quad f^{(2)}_{m\nu} = 0 \quad \text{при} \quad m \neq 0; \\ & f^{(2)}_{0\nu} = \int_{-\infty}^{+\infty} e^{-ipt} \int_0^{\pi} F_\theta \frac{dP_\nu(\cos\theta)}{d\theta} \sin\theta\, d\theta\, dt; \\ & U_{kS} = \frac{1}{\nu_{kS}} \int_0^{\infty} f^{(2)}_{0\nu} \widetilde{V}^{(2)}_{k\nu}\, R^2\, dR; \\ & U_{kT} = 0. \end{aligned} \tag{2.27}$$

For an ideally concentrated meridianal source acting at R = H, θ = 0

$$\mathbf{F} = \frac{2\delta(R-H)\,\delta(\theta)\,\varphi(t)}{H^3\sin^2\theta}\,\mathbf{a}_\theta, \quad U_{kS} = -\frac{\nu_{kS}}{H}V^{(2)}_{k\nu}(H)\,S(p). \quad (2.28)$$

III. *Rotational impact*. Let

$$\mathbf{F} = F_\varphi(t, R, \theta)\,\mathbf{a}_\varphi.$$

From (2.7) we obtain

$$f^{(1)}_{m\nu} \equiv f^{(2)}_{m\nu} \equiv 0; \quad f^{(3)}_{m\nu} = 0 \quad \text{при} \quad m \neq 0;$$

$$f^{(3)}_{0\nu} = -\int_{-\infty}^{+\infty} e^{-ipt}\int_0^\pi F_\varphi\,\frac{dP_\nu(\cos\theta)}{d\theta}\sin\theta\,d\theta\,dt;$$

$$U_{kS} = 0; \quad U_{kT} = -\frac{1}{\nu_{kT}}\int_0^{R_0} f^{(3)}_{0\nu}V^{(3)}_{k\nu}R^2\,dR. \qquad (2.29)$$

In the special case of an ideally concentrated rotational impact acting at the point R = H, θ = 0,

$$\begin{aligned} \mathbf{F} &= 2\delta(R-H)\,\frac{\delta(\theta)\,\varphi(t)}{H^3\sin^2\theta}\,\mathbf{a}_\varphi, \\ U_{kT} &= \frac{\nu_{kT}}{H}V^{(3)}_{k\nu}(H)\,S(p). \end{aligned} \qquad (2.30)$$

IV. *Tangential force field with fixed azimuth*. We will consider only a particular case of this force field:

$$\mathbf{F} = F_T(t, R, \theta)\,\mathbf{a}_T,$$

where $\vec{a}_T$ is a unit horizontal vector of fixed azimuth:

$$(\mathbf{a}_T, \mathbf{a}_R) = 0, \quad (\mathbf{a}_T, \mathbf{a}_\theta) = \cos(\delta - \varphi),$$

$$(\mathbf{a}_T, \mathbf{a}_\varphi) = \sin(\delta - \varphi).$$

Then,

$$\mathbf{F} = F_T\,[\mathbf{a}_\theta\cos(\delta - \varphi) + \mathbf{a}_\varphi\sin(\delta - \varphi)]$$

and we obtain from (2.7):

$$f^{(1)}_{m\nu} \equiv 0; \quad f^{(2)}_{m\nu} = f^{(3)}_{m\nu} = 0 \quad \text{при} \quad m \neq \pm 1;$$

$$f^{(2)}_{1\nu} = \frac{e^{-i\delta} f_{T\nu}}{2\nu(\nu+1)}; \quad f^{(2)}_{-1\nu} = \frac{-e^{-i\delta} f_{T\nu}}{2};$$

$$f^{(3)}_{1\nu} = \frac{e^{-i\delta} f_{T\nu}}{2i\nu(\nu+1)}; \quad f^{(3)}_{-1\nu} = \frac{e^{i\delta} f_{T\nu}}{2i};$$

$$f_{T\nu} = \int\limits_{-\infty}^{+\infty} e^{-ipt} \int\limits_{0}^{\pi} F_T \left(\frac{dP^1_\nu}{d\theta} + \frac{P^1_\nu}{\sin\theta} \right) \sin\theta \, d\theta \, dt.$$

summing with respect to m,

$$U_{kS} = \frac{(-1)^g}{\nu^2_{kS}} i \cos(\delta - \varphi) \int\limits_0^{R_0} f_{T\nu} \tilde{V}^{(2)}_{k\nu} R^2 \, dR,$$

$$U_{kT} = -\frac{(-1)^g}{\nu^2_{kT}} i \sin(\delta - \varphi) \int\limits_0^{R_0} f_{T\nu} \tilde{V}^{(3)}_{k\nu} R^2 \, dR. \tag{2.31}$$

In the case of a concentrated force at R = H, θ = 0,

$$\mathbf{F} = \frac{\delta(R-H)\,\delta(\theta)\,\varphi(t)\,\mathbf{a}_T}{H^2 \sin\theta}$$

$$U_{kS} = (-1)^g i \cos(\delta - \varphi) \tilde{V}^{(2)}_{k\nu}(H) S(p),$$

$$U_{kT} = -(-1)^g i \sin(\delta - \varphi) \tilde{V}^{(3)}_{k\nu}(H) S(p). \tag{2.32}$$

V - VIII. Other elementary sources. The formulas for the remaining elementary sources considered in Section 1 can be obtained from equations (1.33)-(1.36) upon replacing U_{kR} by U_{kS}, U_{kL} by U_{kT}, ξ_{kR} by ν_{kS}/H, ξ_{kT} by ν_{kT}/H, $\tilde{V}^{(1)}_k$ by $\tilde{V}^{(1)}_{k\nu}$, $\tilde{V}^{(2)}_k$ by $(-1)^g \cdot \tilde{V}^{(2)}_{k\nu}$, and $\tilde{V}^{(3)}_k$ by $(-1)^g \tilde{V}^{(3)}_{k\nu}$.

Asymptotic formulae for (2.19) - (2.32) as $pR_o/b(R_o) \to \infty$. Let $pR_o/b(R_o) \to \infty$, and let the quantity $p(R_o - R)/b(R_o) = pz/b(R_o)$ under these conditions. Then it is not difficult to show that equations (2.19) - (2.32) tend toward the corresponding formulas (1.19) - (1.32), if it is noted that as $pR_o/b(R_o) \to \infty$ and

$$g = 0, \quad l = 0, \quad H = R_0 - h; \quad R_0\theta \to r;$$

$$U_{kR} \to -U_{kz}; \quad U_{k0} \to U_{kr}; \quad \frac{\nu_{kQ}}{R} \approx \frac{\nu_{kQ}}{H} \to \xi_{kQ};$$

$$\tilde{V}^{(i)}_{k\nu}(R_1) \tilde{V}^{(i)}_{k\nu}(R_2) \to \tilde{V}^{(i)}_k(z_1) \tilde{V}^{(i)}_k(z_2), \tag{2.33}$$

where $z_1 = R_0 - R_1, \quad z_2 = R_0 - R_2, \quad i = 1, 2, 3;$

$$\tilde{V}^{(1)}_{k\nu}(R_1) \tilde{V}^{(2)}_{k\nu}(R_2) \to -\tilde{V}^{(1)}_k(z_1) \tilde{V}^{(2)}_k(z_2).$$

3. Some properties of the derived solutions

We will consider some properties of the derived equations (1.20) and (2.20), which are essential for understanding the excitation and propagation of surface waves. Let us consider that the force field F, describing the seismic source, is localized in some zone, situated in the halfspace near the initial coordinates $z = 0$, $r = 0$, and for the sphere near the pole $R = R_0$, $\theta = 0$. We will locate a non-disturbing receiver at the point z, r, ϕ (R, θ, ϕ), which registers the q'th component of displacement at this point ($q = z, r, \phi$ in a halfspace and $q = R, \theta, \phi$ for a sphere). The quantity r (or θ) has the meaning of epicentral distance, ϕ - azimuth at the epicenter to the station, z (or R) - depth of receiver, measured from the free surface (or center of sphere).

The theoretical seismogram $u_q(t)$ of the surface wave, recorded by such a receiver, is described by equations (1.20) or (2.20) for sufficiently large r (or θ). The Rayleigh surface wave (index Q = R or S) will be recorded only for $q = z, r$, or R, θ: it is polarized in a vertical plane (section of great circle), passing through the epicenter and receiver. The Love surface wave (wave index Q = L or T) will be recorded only for $q = \phi$: it is linearly polarized, the displacement vector normal to the polarization plane of Rayleigh waves.

Each seismogram can be considered as a superposition of an infinite number of harmonics (in other terminology - normal waves, overtones, or modes): the index k ($k = 1, 2, \ldots,) \infty$) designates the number of the harmonic. Let $u_{kq}(t)$ be the contribution of the k'th harmonic to the q'th component of the seismogram. Then

$$u_k(t) = \sum_{k=1}^{\infty} u_{kq}(t). \qquad (3.1)$$

We will consider further not the seismogram itself, $u_q(t)$ or $u_{kq}(t)$ but their spectra, i.e., the Fourier transform with respect to time of the form $\int_0^\infty \Phi(p)e^{ipt}dt$; we designate these spectra as $\Phi_q(p)$ and $\Phi_{kq}(p)$ respectively. It is evident that

$$\Phi_q = \sum_{k=1}^{\infty} \Phi_{kq}, \qquad (3.2)$$

$$u_{kq}(t) = \frac{1}{\pi} \operatorname{Re} \int_0^{\infty} \Phi_{kq}(p) e^{ipt} dp. \qquad (3.3)$$

Surface wave spectra. The spectral density Φ_{kq} differs from zero only for $p > P_k^Q$, where $\overline{p}_k^Q$ is the limiting frequency in the spectra of the k'th harmonic of the kQ'th wave. The boundary frequencies are related as

$$\overline{p}_{k+1,Q} > \overline{p}_{kQ} > \overline{p}_{k-1,Q} > \cdots > \overline{p}_{1Q}.$$

In a halfspace $p_1L = p_1R = 0$, in a sphere $p_1S > 0$, $p_1T > 0$. However since we are not interested in the low frequency part of the spectra of the disturbance with $p < \overline{p}$, which is impossible to express in the form of a propagating wave, it will be accepted that for surface waves $\Phi_{kq} = 0$ for $p < \max(\overline{p}, \overline{p}_{kQ})$.

The spectral density can be expressed in the form of a product

$$\Phi_{kq} = \prod_{i=0}^{4} B_{kq}^{(i)}. \tag{3.4}$$

For the terms $B_{kq}^{(i)}$ we have the following formulae:

a) in a halfspace (where Q = R for q = z,r; Q = L for q = ϕ):

$$B_{kq}^{(0)} = B^{(0)} = \sqrt{\frac{\pi}{2}}\, e^{-i\frac{\pi}{4}}, \tag{3.5}$$

$$B_{kq}^{(1)} = [v_{kQ}(p)\, C_{kQ}(p)\, I_{kQ}(p)]^{-1}, \tag{3.6}$$

$$B_{kq}^{(2)} = \frac{\exp[-ir\xi_{kQ}(p)]}{\sqrt{r\xi_{kQ}(p)}}, \tag{3.7}$$

$$B_{kq}^{(3)} = U_{kQ}(p, \varphi), \tag{3.8}$$

$$B_{kq}^{(4)} = \alpha_q \widetilde{V}_k^{(j_q)}(p), \quad j_z = 1, \quad j_r = 2, \quad j_\varphi = 3, \tag{3.9}$$

$$\alpha_z = 1, \quad \alpha_r = -i, \quad \alpha_\varphi = i;$$

b) in a sphere (where Q = S for q = R, θ, Q = T for q = ϕ):

$$B_{kq}^{(0)} = B^{(0)} = \sqrt{\frac{\pi}{2}}\, e^{-i\frac{\pi}{4}}, \tag{3.5a}$$

$$B_{kq}^{(1)} = [v_{kQ}(p)\, C_{kQ}(p)\, I_{k\nu Q}]^{-1}, \tag{3.6a}$$

$$B_{kq}^{(2)} = \frac{\exp\left\{-i\left[\left(\nu_{kQ}(p) + \frac{1}{2}\right)\widetilde{\theta} - \frac{\pi}{2}\, g\right]\right\}}{\sqrt{\nu_{kQ}(p)\sin\theta}}, \tag{3.7a}$$

$$B_{kq}^{(3)} = U_{kQ}(p, \varphi), \tag{3.8a}$$

$$B_{kq}^{(4)} = \alpha_q \widetilde{V}_{k\nu}^{(j_q)}(R), \quad j_R = 1, \quad j_\theta = 2, j_\varphi = 3,$$

$$\alpha_R = 1, \quad \alpha_\theta = i(-1)^{g+1}, \quad \alpha_\varphi = i(-1)^{g}. \tag{3.9a}$$

g is the number of passages through the epicenter and anti-epicenter.

The multiplier $B_{kq}^{(1)}$ depends only the properties of the medium (variation of velocity and density with depth or radius) and the type of waves recorded.

The multiplier $B_{kq}^{(2)}$ describes the propagation effect of the waves: the numerator defines the phase delay of the oscillations for a given frequency, the denominator--the decrease of amplitude due to geometric spreading along the path r (or θ). The additional phase contribution of $\pi g/2$ in a sphere arises on account of g-passages through the epicenter and anti-epicenter.

The multiplier $B_{kq}^{(3)}$ for a given wave depends only on the source mechanism, and also on the azimuth of the recording station with respect to the source. For an axially symmetric source $B_{kq}^{(3)}$ does not depend on ϕ.

The multiplier $B_{kq}^{(4)}$ depends on the depth of burial of the receiver and its orientation (i.e., for which component of displacement it is set up to measure).

<u>Polarization of Rayleigh waves</u>. The ratio of spectral densities Φ_{kr}/Φ_{kz} or $\phi_{k\theta}/\phi_{kR}$ defines the polarization of Rayleigh waves. Since $B_{kr}^{(j)} = qB_{kz}^{(j)}$ and $B_{kR}^{(j)} = B_{k\theta}^{(j)}$ for $j < 4$, then

$$\frac{\Phi_{kr}}{\Phi_{kz}} = \frac{B_{kr}^{(4)}}{B_{kz}^{(4)}} = -i\left[\frac{\tilde{V}_k^{(2)}(p, z)}{\tilde{V}_k^{(1)}(p, z)}\right], \tag{3.10}$$

$$\frac{\Phi_{k\theta}}{\Phi_{kR}} = \frac{B_{k\theta}^{(4)}}{B_{kR}^{(4)}} = i(-1)^{g+1}\left[\frac{\tilde{V}_{k\nu}^{(2)}(R)}{\tilde{V}_{k\nu}^{(1)}(R)}\right]. \tag{3.10a}$$

Thus the eigenfunctions $\tilde{V}_k^{(i)}$, $\tilde{V}_k^{(i)}$ define the Rayleigh wave polarization ellipse; the direction of rotation and ratio of the ellipse axes depends on the depth of burial of the receiver; The apparent change in the direction of rotation for a sphere for changes in g is associated with the fixed choice of direction in designating u_θ (+ for motion away from the epicenter).

<u>Reciprocity principle</u>. For simple sources of the concentrated force type, it is easy to establish the principle of reciprocity--the invariance of spectral density ϕ_{kz} for change in the placement of the source and receiver with preservation of the source orientation with respect to the receiver and vice versa. In fact for a simple vertical force at depth we have according to (1.26):

$$
\begin{aligned}
B_{kq}^{(3)} &= \widetilde{V}_k^{(1)}(p,h)\,S(p),\\
\Phi_{kz}^{\downarrow}(h,z) &= \Big[\prod_{i=0}^{2} B_{kz}^{(i)}\Big] S(p)\,\widetilde{V}_k^{(1)}(p,h)\,\widetilde{V}_k^{(1)}(p,z),\\
\Phi_{kr}^{\downarrow}(h,z) &= \Big[\prod_{i=0}^{2} B_{kz}^{(i)}\Big] S(p)\,\widetilde{V}_k^{(1)}(p,h)\,(-i)\,\widetilde{V}_k^{(2)}(p,z).
\end{aligned}
\tag{3.11}
$$

For a simple horizontal force at depth h, oriented toward the receiver according to (2.32) we have

$$
\begin{aligned}
B_{kq}^{(3)} &= i\widetilde{V}_k^{(2)}(p,h)\,S(p),\\
\Phi_{kz}^{\rightarrow}(h,z) &= \Big[\prod_{i=0}^{2} B_{kz}^{(i)}\Big] S(p)\,i\widetilde{V}_k^{(2)}(p,h)\,\widetilde{V}_k^{(1)}(p,z),\\
\Phi_{kr}^{\rightarrow}(h,z) &= \Big[\prod_{i=0}^{2} B_{kz}^{(i)}\Big] S(p)\,\widetilde{V}_k^{(2)}(p,h)\,\widetilde{V}_k^{(2)}(p,z).
\end{aligned}
\tag{3.12}
$$

For a simple horizontal force at depth h perpendicular to the epicenter-receiver direction, from (1.32)

$$
\begin{aligned}
B_{kq}^{(3)} &= -i\widetilde{V}_k^{(3)}(p,h)\,S(p),\\
\Phi_{k\varphi}^{\rightarrow}(h,z) &= \Big[\prod_{i=0}^{2} B_{kq}^{(i)}\Big] S(p)\,\widetilde{V}_k^{(3)}(p,h)\,\widetilde{V}_k^{(3)}(p,z).
\end{aligned}
\tag{3.13}
$$

From this the reciprocity relations follow:

$$
\begin{aligned}
\Phi_{kz}^{\downarrow}(h,z) &= \Phi_{kz}^{\downarrow}(z,h),\\
\Phi_{kr}^{\rightarrow}(h,z) &= \Phi_{kr}^{\rightarrow}(z,h),\\
\Phi_{k\varphi}^{\rightarrow}(h,z) &= \Phi_{k\varphi}^{\rightarrow}(z,h),\\
\Phi_{kr}^{\downarrow}(h,z) &= -\Phi_{kz}^{\rightarrow}(z,h),\\
\Phi_{kz}^{\rightarrow}(h,z) &= -\Phi_{kr}^{\downarrow}(z,h).
\end{aligned}
\tag{3.14}
$$

Similar relations for a sphere are not difficult to obtain.

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* English translations available.

Additional force systems. These forces have been normalized to represent unit forces of couples acting at the source. They are suitable for direct substitution in to (1.20). The result (1.20) agrees with the theoretical equations of Saito and Tsai and Aki (1970).

(a) Arbitrarily directed force. (δ, β). The force is represented as

$$\vec{F} = \frac{\delta(z-h)\delta(r)}{2\pi r}\left[\cos(\delta-\phi)\sin\beta\vec{a}_r + \sin(\delta-\phi)\sin\beta\vec{a}_\phi + \cos\beta\vec{a}_z\right]s(t)$$

where δ is the aximuth of the force from north and β is the angle that the force makes with the positive z axis, which is taken to be downward. Note that $0 \leq \delta \leq 2\pi$ and $0 \leq \beta \leq \pi$. The $\vec{a}_q$ are the unit vectors in the q'th direction. s(t) is the source time function. The U_{kQ} become

$$U_{kR} = \left[\cos\beta V_k^{(1)}(h,p) - i\sin\beta\cos(\delta-\phi)V_k^{(2)}(h,p)\right] \cdot S(p)/2\pi$$

$$U_{kL} = -i\sin\beta\sin(\delta-\phi)\,V_k^{(3)}(h,p) \cdot S(p)/2\pi$$

where $S(p) = \int_{-\infty}^{\infty} \exp(-ipt)=(t)\,dt;$

(b) Dipole without moment. (δ,β).

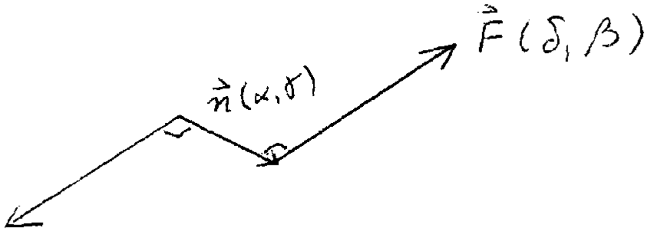
$$U_{kR} = \Big[\cos^2\beta\frac{dV_k^{(1)}}{dh}(h,p) - k_R\sin^2\beta\cos^2(\delta-\phi)V_k^{(2)}(h,p)$$
$$+\frac{i}{2}\sin 2\beta\cos(\delta-\phi)\Big(k_R V_k^{(1)}(h,p) + \frac{dV_k^{(2)}}{dh}(h,p)\Big]\cdot S(p)/2\pi$$

$$U_{kL} = i\sin\beta\sin(\delta-\phi)\left[\cos\beta\frac{dV_k^{(3)}}{dh}(h,p) + i\,k_L\cos(\delta-\phi)\sin\beta V_k^{(3)}(h,p)\right]\cdot S(p)/2\pi$$

(c) Couple (δ,β), (α,γ). The positive force in the direction (δ,β) is converted into a couple by the application of an operator to (a):

$$\left[\cos\gamma\frac{d}{dh} - ik_Q\cos(\alpha-\phi)\sin\alpha\right], \quad Q = L \text{ or } R\,;$$

graphically, the couple appears as



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[1] This differs from section V by factor $(2\pi)^{-1}$ which is needed for normalization to a unit force.

The u_{kQ} become

$$U_{kR} = \{\cos\gamma\cos\beta\frac{dV_k^{(1)}}{dh}(h,p) - k_R\sin\beta\sin\gamma\cos(\delta-\phi)\cos(\alpha-\phi)V_k^{(2)}(h,p)$$

$$+ ik_R\sin\gamma\cos\beta\cos(\alpha-\phi)V_k^{(1)}(h,p)$$

$$+ ik_R\cos\gamma\sin\beta\cos(\delta-\phi)\frac{dV_k^{(2)}}{dh}(h,p)\}.\ \frac{S(p)}{2\pi}$$

$$U_{kL} = -i\sin\beta\sin(\delta-\phi)\left[\cos\gamma\frac{dV_k^{(3)}}{dh}(h,p)\right.$$

$$\left. + ik_L\sin\gamma\cos(\alpha-\phi)V_k^{(3)}(h,p)\right].\ S(p)/2\pi$$

where (δ,β) and (α,γ) are related by the condition of perpendicularity

$$\text{ctg}\gamma\text{ctg} = -\cos(\delta-\alpha).$$

(d) Two perpendicular dipoles without moment of equal magnitude and opposite sign (δ,β), (α,γ). For this set of dipoles, the angles define the directions of the tensional T and compressional P axes by

$$(\delta,\beta) = T \qquad (\alpha,\gamma) = P.$$

$$U_{kR} = \{(\cos^2\beta - \cos^2\gamma)\frac{dV_k^{(1)}}{dh}(h,p)$$

$$-k_R\left[\sin^2\beta\cos^2(\delta-\phi) - \sin^2\gamma\cos^2(\alpha-\phi)\right] V_k^{(2)}(h,p$$

$$+ \frac{i}{2}.\left[\sin 2\beta\cos(\delta-\phi) - \sin 2\gamma\cos(\alpha-\phi)\right].$$

$$.\left[k_R V_k^{(1)}(h,p) - \frac{dV_k^{(2)}}{dh}(h,p)\right]\}\ \frac{S(p)}{2\pi}$$

$$U_{kL} = \{-\frac{i}{2}\left[\sin 2\beta\sin(\delta-\phi) - \sin 2\gamma\sin(\alpha-\phi)\right]\frac{dV_k^{(3)}}{dh}(h,p)$$

$$+ \frac{1}{2}k_L V_k^{(3)}(h,p)\left[\sin^2\beta\sin(2\delta-2\phi) - \sin^2\gamma\sin(2\alpha-2\phi)\right]\}\ \frac{S(p)}{2\pi}$$

with the necessary perpendicularity relation

$$\text{ctg}\gamma\text{ctg}\beta = -\cos(\delta-\alpha).$$

(e) Double couple without moment (δ,β), (α,γ). The angles are defined as in (c).

$$\begin{aligned} U_{kR} = \{ \; & 2 \cos\gamma\cos\beta \frac{dV_k^{(1)}}{dh}(h,p) - 2k_R V_k^{(2)} \sin\beta\sin\gamma\cos(\delta-\phi)\cos(\alpha-\phi) \\ & + i \left[k_R V_k^{(1)}(h,p) + \frac{dV_k^{(2)}}{dh} \right] (h,p) \quad . \\ & \quad . \left[\sin\gamma\cos\beta\cos(\alpha-\phi) + \sin\beta\cos\gamma\cos(\delta-\phi) \right] \} . \; S(p)/2\pi \end{aligned}$$

$$\begin{aligned} U_{kL} = \{ \; & -i \left[\sin\beta\sin(\delta-\phi)\cos\gamma + \sin\gamma\cos\beta\sin(\alpha-\phi) \right] \frac{dV_k^{(3)}}{dh}(h,p) \\ & -k_L V_K^{(1)}(h,p)\sin\beta\sin\gamma\sin(\alpha+\delta - 2\phi) \} \quad S(p) 2\pi \end{aligned}$$

and

$$\mathrm{ctg}\gamma\,\mathrm{ctg}\beta = - \cos(\delta-\alpha)$$

(f) Explosive source.

$$U_{kR} = \left[\frac{dV_k^{(1)}}{dh}(h,p) - k_R V_k^{(2)}(h,p) \right] S(p)/2\pi$$

$$U_{kL} = 0.$$

(g) Double couple mechanism in terms of strike, dip, and slip of motion on the fault plane. The orthogonality conditions on the couples or dipoles of (d) or (e) imply that only three quantities are required to define motion on the fault plane. These quantities are the fault plane dip $\underline{d}$, the strike $\underline{\theta}$, and the slip $\underline{s}$. These parameters are shown in Figure A.1. The dip is measured from the horizontal in a downward direction and varies between 0° and 90°. The strike is measured clockwise from north and varies from 0° through 360°. The slip angle gives the direction of motion of the hanging wall with respect to the foot wall; the slip angle varies from 0° to 360° and is measured counterclockwise from the strike.

With this convention, the U_{kQ} become

$$\begin{aligned}
U_{kR} = \frac{S(p)}{2\pi} \Big\{ & \sin s \sin 2d \left[\frac{dV_k^{(1)}}{dh} (h,p) + \frac{1}{2} \xi_{kR} V_k^{(2)} (h,p) \right] \\
& + \xi_{kR} V_k^{(1)} (h,p) \Big[- \cos s \sin d \sin 2(\phi-\theta) \\
& \qquad - \frac{1}{2} \sin s \sin 2d \cos 2(\phi-\theta) \Big] \\
& \qquad + i \left[\xi_{kR} V_k^{(1)} (h,p) + \frac{dV^{(2)}}{dh} (h,p) \right] \cdot \\
& \qquad \Big[- \cos d \cos s \cos(\phi-\theta) \\
& \qquad + \cos 2d \sin s \sin(\phi-\theta) \Big] \Big\}
\end{aligned}$$

$$\begin{aligned}
U_{kL} = \frac{S(p)}{2\pi} \Big\{ & - i \Big[\sin s \cos 2d \cos(\phi-\theta) \\
& \qquad + \cos s \cos d \sin(\phi-\theta) \Big] \frac{dV_k^{(3)}}{dh} (h,p) \\
& + \xi_{kL} V_k^{(3)} (h,p) \Big[\cos s \sin d \cos 2(\phi-\theta) \\
& \qquad - \frac{1}{2} \sin s \sin 2d \sin 2(\phi-\theta) \Big] \Big\}
\end{aligned}$$

A reverse fault striking north and dipping 45° to the east is given by $\theta = 0^\circ$, $d = 45^\circ$, and $s = 90^\circ$. A right lateral vertical strike slip fault which strikes north is given by $\theta = 0^\circ$, $d = 90^\circ$, and $s = 180^\circ$.

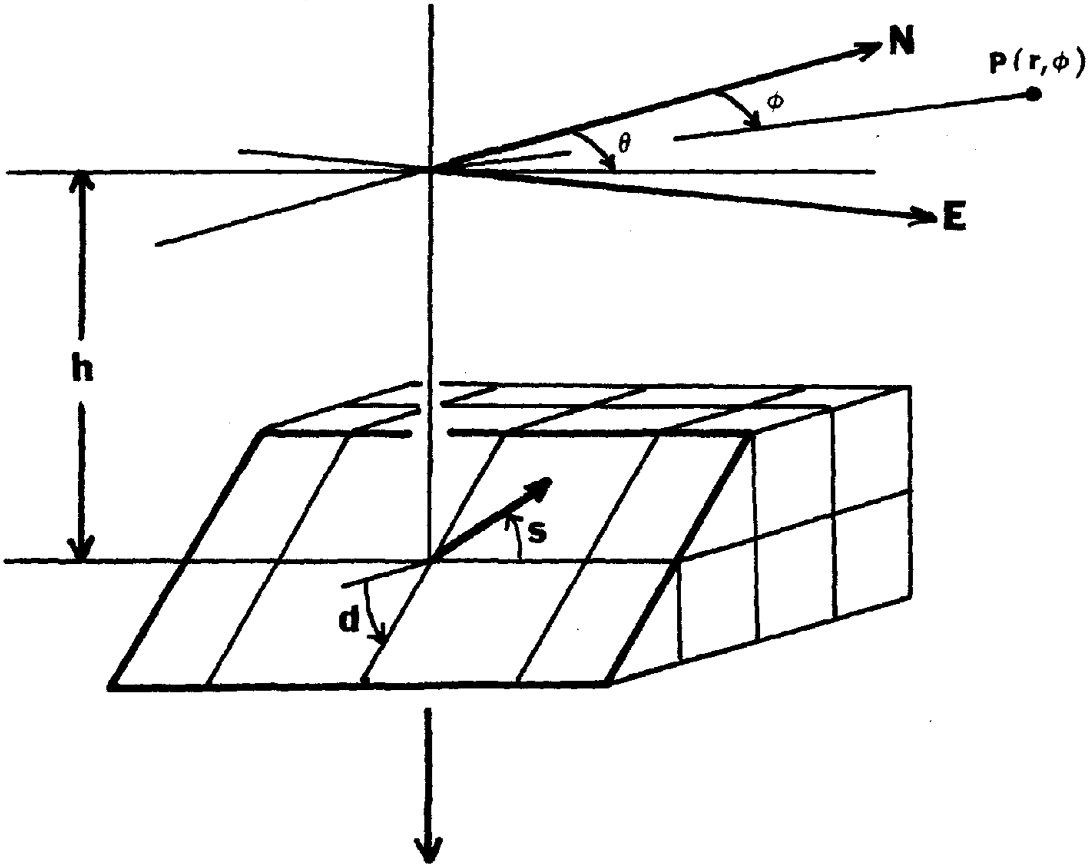

Fig. A. 1. Coordinate and fault plane geometry

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