EXPERIMENT AND NUMERICAL EVALUATION OF ACOUSTIC SCATTERING AT A ROUGH SURFACE

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Abstract

The Helmholtz integral is often used as the basis for theories on scattering of acoustic signals from rough surfaces, but several approximations must be made to obtain analytical solutions. Many of the mathematical difficulties in analytical computations can be avoided by numerically evaluating the Helmholtz integral. A numerical technique is described which utilizes measured values of the acoustic pressure of the incident beam, including the side lobes, thereby removing the uncertainty which normally results from approximating the form of the incident beam. A comparison of numerical, analytical, and experimental results is made for the case of a vertically incident beam and a moderately rough surface. The numerical computations are shown to yield values of the scattered pressure which agree well with experimental data for both rough and smooth surfaces.

These initial results indicate that the pressure reflected or scattered from a smooth, or slightly rough surface is sensitive to the phase of the side lobes of the incident beam. The rms pressure scattered from a very rough surface is apparently less sensitive to the form of the approximation used to specify the incident pressure.

The geometrical slope factor, $f(\theta_1,\theta_2)$, is often used in the literature to account for the effect of finite surface slopes. Its validity is verified numerically for moderate scattering angles and a sinusoidal surface, and the limitations on its validity are discussed. A further study of the effect of surface slopes, and a study of the general boundary value problem, including the effects of shadowing and the limitations of the Kirchhoff approximation, are suggested as useful extensions of this work.

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Acknowledgements

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I. Introduction

A. Background.

In the broadest sense, this report deals with the scattering of acoustic and electromagnetic waves from rough surfaces. In this sense the topic involves numerous and diverse areas of research. Without attempting to provide an exhaustive list of these areas, it may be useful to mention a few in order to illustrate the point, and to provide a background for this work.

The first major efforts in studying the effects of rough surfaces on propagating acoustic and electromagnetic waves were in connection with the reverberation or clutter which occurs in sonar and radar returns.[1-3] The problem of reverberation is a major one in target detection. A specific example of this kind of a problem is the detection and classification of fish in shallow water or at shallow depths in deep water. Similarly, interference and distortion are produced in communications signals which may impinge on and be affected by the surface of the sea, the sea floor, the surface of the earth, layers in the atmosphere, and so on. In problems such as these, effort is directed mainly toward calculating, eliminating, or compensating for the corruption of information-carrying signals which impinge on a scattering surface. This is the so-called 'direct' problem.

In the 'inverse' problem, the emphasis is on using scattering phenomena to obtain information about the scattering surface. An example of this approach is the attempt to determine by acoustic means the roughness and composition of the sea floor and, ideally, some of the properties of the sub-bottom layers. These are the objectives that have been emphasized in this laboratory. There is considerable interest also in scattering from the surface of the sea, with a view toward understanding the growth and propagation of ocean waves and the interaction of the wind and the surface of the water.[4] Another example is found in the field of meteorology. Both acoustic and radar beams are scattered from layers in the atmosphere in order to study its structure and dynamics.[5] And finally, radar signals back-scattered from the moon and the planets are analyzed for the purpose of determining some of the statistical roughness parameters of the surfaces of those bodies.[6]

The extent to which either the direct or the inverse problem can be solved depends largely on whether a tractable and complete theoretical description of the scattering process can be derived. One of the more formidable aspects of the theoretical problem is obtaining solutions that are applicable to the case of very rough surfaces (large surface slopes). Another is the inclusion of realistic transmitted beams.

In the following paragraphs a very brief historical view of the development of acoustic theory is given. For a more extensive review of scattering see References 7-9.

The first theoretical treatment of acoustic scattering was apparently by Rayleigh[10] in 1895. He considered the case of a plane wave normally incident on a corrugated surface. He assumed that the scattered field can be expressed as a superposition of plane waves propagating away from the surface. After some controversy in the literature, this assumption has been shown to be invalid at points close to the scattering surface.[11] More recently, Uretsky[12] has formulated an 'exact' solution for a sinusoidal surface, but his solution is limited to the case of an incident plane wave.

Several papers were published in the early 1950's which laid the foundation for most of the recent work in scattering theory. Several of these papers were published in Russian by Brekhovskikh[13] and Isakovich.[14] Of special importance in western literature was a paper published in 1953 by Eckart.[15] He adapted the Helmholtz integral[16,17] -- a classical method of physical optics -- to the problem of acoustic scattering from a random surface. He introduced the statistics of the surface into the theory in a simple manner, and obtained the first and second moments of the scattered signal. His basic procedure was shown by LaCasce and Tamarkin[18] to be useful also for corrugated surfaces, and it has subsequently formed the basis for much of the theoretical work found in the literature.

The basic validity of the Helmholtz integral approach has been well established. Its primary virtue is perhaps the fact that it is amenable to approximation techniques which lead to closed-form results. At the same time, its primary shortcoming is that mathematical difficulties <u>necessitate</u> numerous approximations. The degree of success that is ultimately realized in solving the direct and inverse problems with this approach will depend largely on the validity of those approximations.

B. Approximations and difficulties in acoustic scattering theory.

For the purposes of this discussion, six factors can be enumerated as sources of difficulty in the application of the Helmholtz integral technique to the scattering of acoustic waves from rough surfaces. (The same, or similar, factors arise in the scattering of electromagnetic waves, but the specific concern of this report is with the acoustic case.) Some of these factors are not explicitly labled as approximations, but in each of these cases it will be clear that there is a direct correspondance with some approximation in the theory. The six are:

1) The Kirchhoff approximation.
2) The incident pressure.
3) Phase approximations.
4) The zero-slope approximation.
5) Description of the surface.
6) Shadowing.

These are all further discussed below, but first, it should be pointed out that they are not all independent. For example, 'The Kirchhoff approximation' and 'Shadowing' could be logically combined under the heading 'Boundary values'. Also 'The incident pressure' and 'Phase approximations' could be combined under some more general title. Nevertheless, the present arrangement is convenient, and is also consistent with current treatments of these topics in the literature.

The Kirchhoff approximation -- Although the Helmholtz integral is formally exact, a fundamental limitation on its power is imposed by the need to know and to specify boundary values; specifically, the acoustic pressure of the scattered signal on the surface, and its derivative with respect to the local surface normal. In the Kirchhoff approximation, these quantities are replaced at each point by the values that they would have if the incident wave were reflected from a plane tangent to the surface at that point.[15, 16] Eckart[15] was among the first to apply this approximation to acoustic scattering -- its use was a basic step in his formulation. This approximation is considered to be valid if the minimum radius of curvature of the surface is much larger than the acoustic wavelength.[6,19,20] This condition is satisfied in the present case and the Kirchhoff approximation is used in both the numerical and analytical computations.

The incident pressure -- The standard procedure in the literature is to represent the incident pressure as a product of an amplitude term and a phase term. The amplitude term includes a directivity function for the incident beam, and for spherical waves, it includes a geometrical spreading factor. The directivity function usually approximates the main lobe of the incident beam by some simple analytical expression such as a Gaussian -- side lobes are ignored. Often, the amplitude of the incident wave is assumed to be constant, or uniform, within some finite ensonified area. Similarly, the phase distribution in the incident beam is approximated by simple expressions which correspond either to plane waves or to spherical waves. Two comments can be made regarding the use of these approximations. First, if plane waves or a uniform amplitude is assumed, or if the side lobes are ignored, one would expect the resulting theory to apply mainly to cases in which the scattering surface is of finite size and is completely ensonified by the main lobe of the beam. There are several examples of laboratory experiments using artificial surfaces in which this arrangement has been used.[18, 21, 22] This is probably not a useful procedure to pursue in applying theory to field measurements. Second, for a directional source, the amplitude and phase of the beam are complicated functions of position in the region close to the source. A spherical wave (point-source) approximation is therefore not valid except at large distances from the source. Its validity must be verified for a given application, particularly in laboratory experiments where it is often desirable to use large, highly directional sources.

Recently, in this laboratory, acoustic scattering from a wind-blown water surface was studied as a function of the width of the acoustic beam.[23] An analytical theory, derived from the formulation of Eckart[15] was used to calculate the mean square pressure of the signal scattered from the rough surface as well as the pressure reflected from the smooth surface. The Fresnel phase approximation was used in the theory in the manner of Gulin[24] and Horton and Melton.[25] The surface was randomly rough and the spatial correlation functions of the surface are shown in Figure 1b. Computations using these correlation functions gave the solid line shown in Figure 1a. The experimental data are represented by the dashed curve, and the shaded area shows the scatter in the data. These curves and the results

Figure 1. (a) Normalized mean square pressure versus beamwidth for a wind-blown water surface. p is the scattered acoustic pressure, p_o is the pressure reflected from the smooth surface, and * means complex conjugate. The beamwidth is the angle between the half-power points of the incident beam. The scattering angles were $\theta_1 = \theta_2 = 0^o$ (see Figure 2, p.15) and the incident beams were symmetrical about the z axis. The rms roughness of the surface was 0.14 cm, the acoustic frequency was 200 kHz, and the data were measured at depths of 50 and 100 cm. (b) Spatial correlation functions, $\Psi(\xi)$ and $\Psi(\eta)$, in the downwind and crosswind directions, respectively. ξ and η are the corresponding correlation parameters. The correlation functions are defined as follows:

$$\Psi(\xi) = \left\langle \zeta(x,y)\ \zeta(x+\xi,y) \right\rangle_x / \left\langle \zeta^2(x,y) \right\rangle ,$$

$$\Psi(\eta) = \left\langle \zeta(x,y)\ \zeta(x,y+\eta) \right\rangle_y / \left\langle \zeta^2(x,y) \right\rangle ,$$

where $\zeta(x,y)$ is the surface displacement, and $\left\langle\ \right\rangle_x$ denotes an ensemble average taken over the x coordinate; and similarly for y.

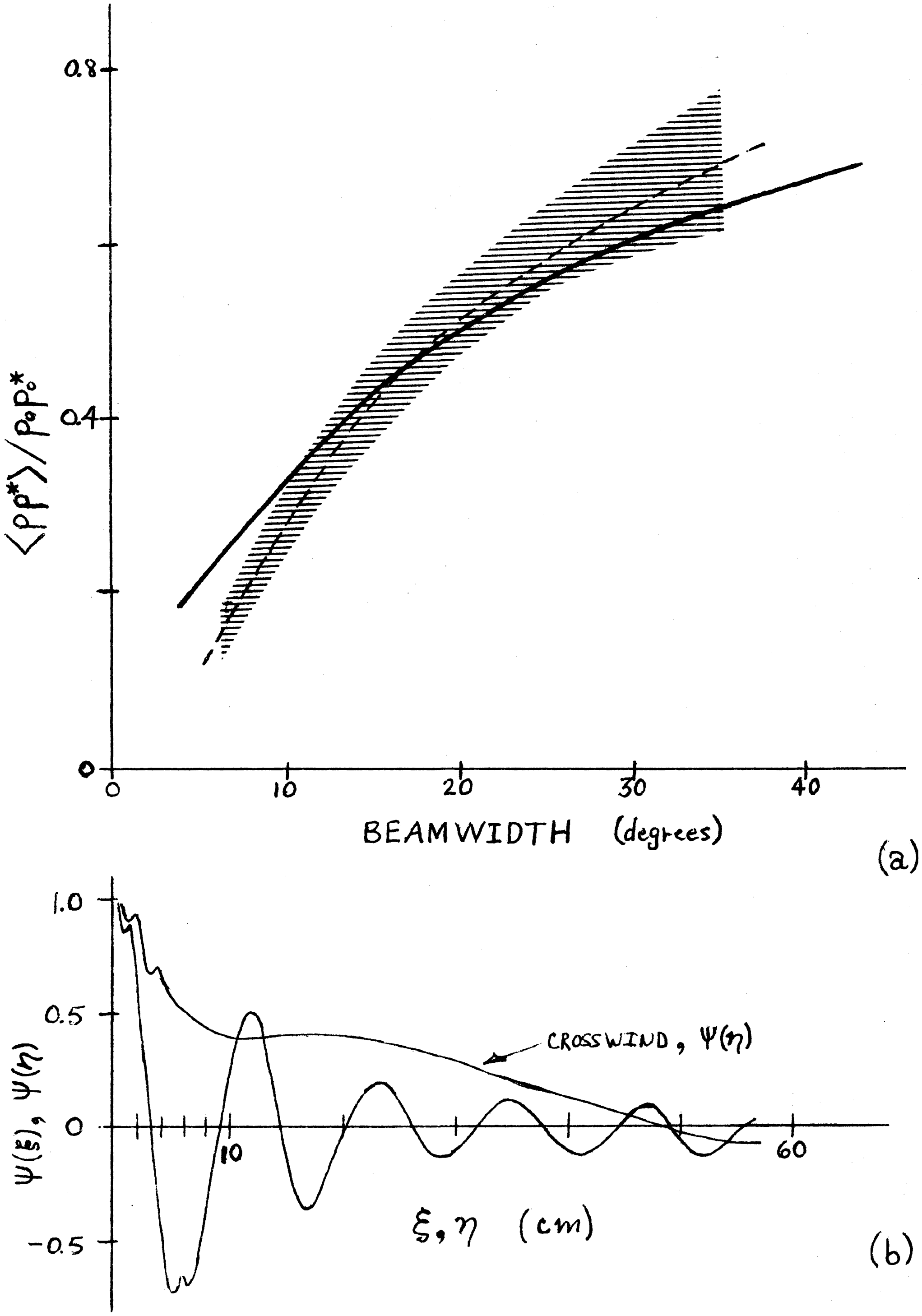

Figure 1. Sound scattered at a randomly rough windblown surface. Vertical incidence.

(a) Normalized mean square pressure versus beamwidth. Hashed - experiment; solid - theory.

(b) Spatial correlation functions.

$\sigma = 0.14$ cm $\quad\quad$ $f = 200$ kHz of subsequent measurements show a persistent and apparently systematic discrepancy between theory and experiment which worsens as the beam-width is reduced. The determination of the source of this discrepancy was part of the motivation for doing a numerical integration of the Helmholtz integral. It appeared to be largely attributable to the use of an approximation for the incident pressure. But it was not clear how the errors were apportioned between the calculated pressures for the smooth and rough surfaces. This matter is studied numerically in the present work. Measured values of the incident pressure are inserted directly into the computation in order to avoid the problem of specifying its form mathematically. The normal derivative of the incident pressure is obtained from simple approximations (described in Section II. B.). The accuracy of this procedure is checked by comparing computed and measured pressures of signals reflected from the smooth water surface.

Phase approximations -- A well-known problem in physical optics is the approximation of the phases of the incident and scattered waves in ways which lead to tractable integrals. More specifically, the problem involves approximating the path lengths of the incident and scattered rays. Thus, there is at least an operational distinction between this process and the specification of the phase distribution in the incident beam. Discussions of the applicability and effects of the Fraunhofer and Fresnel approximations in acoustic scattering have been given by Melton and Horton,[26] Horton and Melton,[25] and by MacDonald and Spindel.[27] In this work, the Fresnel approximation is used in the derivation of the analytical theory (Section III). The primary emphasis is on phase approximations made in the specification of the incident pressure.

The zero-slope approximation -- The applicability of most scattering theories is limited to surfaces of moderate roughness.[8] In Eckart's original formulation, and in most of the subsequent variations on that theory, this limitation is due, in part, to the approximation that the surface slope is zero everywhere. A technique for including surface slopes in this kind of a theory has been described for the case of the Fraunhofer approximation by Beckmann and Spizzichino [6] and by Tolstoy and Clay.[28] They used an approximate integration by parts to evaluate the terms involving the surface slope. The result is a simple geometrical 'slope factor' which multiplies the remaining integral. It is often used for both periodic and random surfaces, but the conditions of its validity are not clear from its derivation. A slope correction curve obtained by using this factor is compared in Section V. B. with a curve obtained from numerical computations. Other theoretical formulations which take account of surface slopes are found in References 29-32.

Description of the surface -- In the Eckart theory,[15] it is necessary to obtain the bivariate probability density function (PDF) of the surface displacement. If the surface has a Gaussian PDF, the rms scattered pressure can be expressed in terms of a spatial correlation function. In the literature it has been argued that exponential or linear correlation functions are improper due to the non-zero values of the first derivatives at the origin.[33, 34] In practice, however, it is often observed that these functions yield results that closely approximate measured values. The important point seems to be simply that the scattered pressure may depend strongly on the correlation function near its origin. This is demonstrated in Reference 23 (see Figure 1) where a numerical integration utilizing measured values of the correlation function yielded good results, whereas a quadratic approximation failed badly. Clay[35] has shown, using the same data, that a linear approximation yields equally good results. Also, Clay, Medwin, and Wright[36] have shown that the coherent pressure (the ensemble average, $\langle p \rangle$) is strongly affected by the form of the PDF of a rough water surface. These problems are perhaps not well suited to a numerical analysis. On the other hand, the exact shape of the scattering surface (to the degree that it can be measured) can be utilized in the numerical calculation and its effect can be studied. This is done to some extent in the present work, where the sinusoidal surface used in the analytical theory is an approximation of the real surface.

<u>Shadowing</u> -- At large angles of incidence, portions of a rough surface are not ensonified by the direct transmitted beam. Geometrical shadowing factors have been derived to account for this effect,[37-41] but examples of their use are not numerous. A recent example is a paper by Gardner,[42] who considered the effect of shadowing on backscattering from the sea surface at incident angles greater than 80°. He found that shadowing can account for effects previously ascribed

to scattering from bubbles or biological scatterers. Clay, et al[36] have used Wagner's[39] shadowing factor in a study of coherent scattering from a wind-blown water surface. They comment that it seems to work although, in this instance, it is not clear why. It is possible that their result can be explained by the diffraction of sound into the geometrical shadows, and by multiple scattering at the surface. These phenomena have apparently not yet been dealt with and no attempt is made to do so here. A study of shadowing is also beyond the scope of the present work, and shadowing does not occur for the surface and scattering angles used here.

C. Objectives.

The effects of none of the factors just discussed have yet been fully evaluated; and in fact, substantial difficulties are encountered in attempts to do so.

Scattering theories are often tested in the laboratory, where the scattering parameters can be controlled and measured more easily and more accurately than is possible in the field. Yet, even under those circumstances, it is difficult to satisfy assumptions made in the derivation of the theory and to isolate, either analytically or experimentally, the effects of any one of the factors that are sources of difficulty. For this reason it is worthwhile to consider a numerical evaluation of the Helmholtz integral. Most of the mathematical difficulties encountered in an analytical solution do not exist in this case. Although attention must be paid to the errors inherent in numerical computations (truncation and roundoff errors), these errors are shown in a later section to be negligible in the computations reported here.

There are two main limitations on the usefulness of the numerical technique. First, the displacement and slope of the scattering surface must be known at each sample point. This suggests that an artificially produced surface be used. Also, restricting the roughness of the surface to a function of only one variable greatly simplifies the problem without altering the basic physics of the scattering process. Bourianoff and Horton[43] implemented this approach by computer simulation of the sea surface in a numerical study of the statistics of acoustic backscattering. Second, because the integrand is an oscillatory function of surface position, the required number of computations (sample points) may be prohibitively large. That number is roughly proportional to the number of ensonified Fresnel zones on the surface, and can be minimized for a given geometry and signal wavelength by using highly directional transmitted beams. In laboratory experiments those parameters often have values such that numerical computations are feasible. Comparisons of analytical, numerical, and experimental results are thus possible. Because a linear scaling law applies to acoustic experiments, conclusions concerning the effects of the factors enumerated earlier can be expected to be valid for scattering in the sea as well as for scattering in the laboratory.

In this work, acoustic scattering from a periodic surface is studied in order to obtain initial results and to evaluate the practicality and usefulness of extending the work. Measurements of the pressure of the scattered acoustic signals were made in a laboratory water tank, with the transmitter and receiver restricted to the x-z (y = 0) plane. Periodic, long-crested waves were produced on the water surface by a mechanical wave generator. The geometry of the experiment is shown in Figure 2. A cross-section of the water waves very closely approximated a sinusoid, so the numerical and experimental results are compared with values obtained from an analytical expression derived for a sinusoidal surface. The numerical computations are limited to the case of vertical incidence ($\theta_1 = 0^o$), but measurements of the backscattered signal were also made and are compared with the analytical results. The emphasis in this work is on the effects of phase approximations, particularly in the incident pressure, and on the effect of neglecting surface slopes.

The chart shown in Figure 3 shows the computations that have been made. The approximations and conditions used in each computation are shown in blocks and are arranged vertically in four columns. Each of the first three columns on the left corresponds to a distinct representation of the incident pressure. The fourth column corresponds to the analytical solution.

Very briefly, the three representations of the incident pressure were chosen for the following reasons: 1) In order to evaluate the numerical procedure, it is important to avoid as many approximations as possible. A set of directly measured values of both phase and amplitude is the best available representation of the incident pressure. 2) The most reasonable analytical representation of the phase of the incident pressure is that of a point

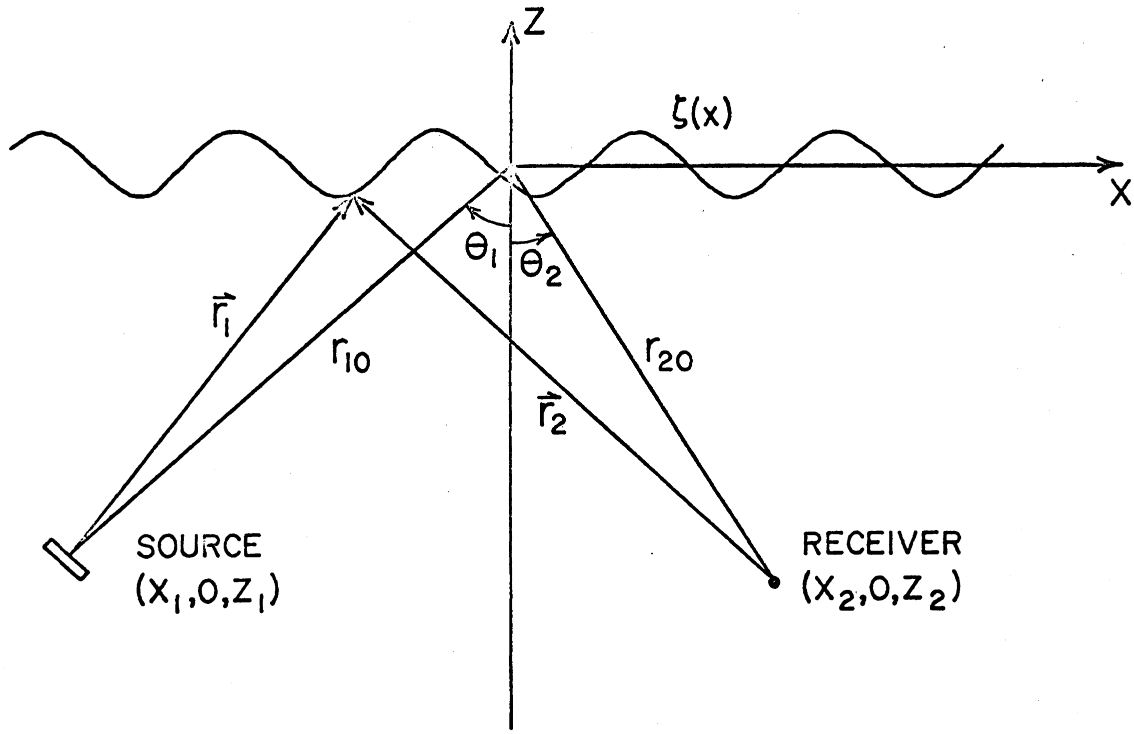

Figure 2. The scattering geometry, shown, for simplicity, in two dimensions. The transmitter and receiver are restricted to the x-z plane, but the surface corrugations effectively extend to $\pm\infty$ in the y direction. The waves propagate in the x direction and are assumed to be periodic.

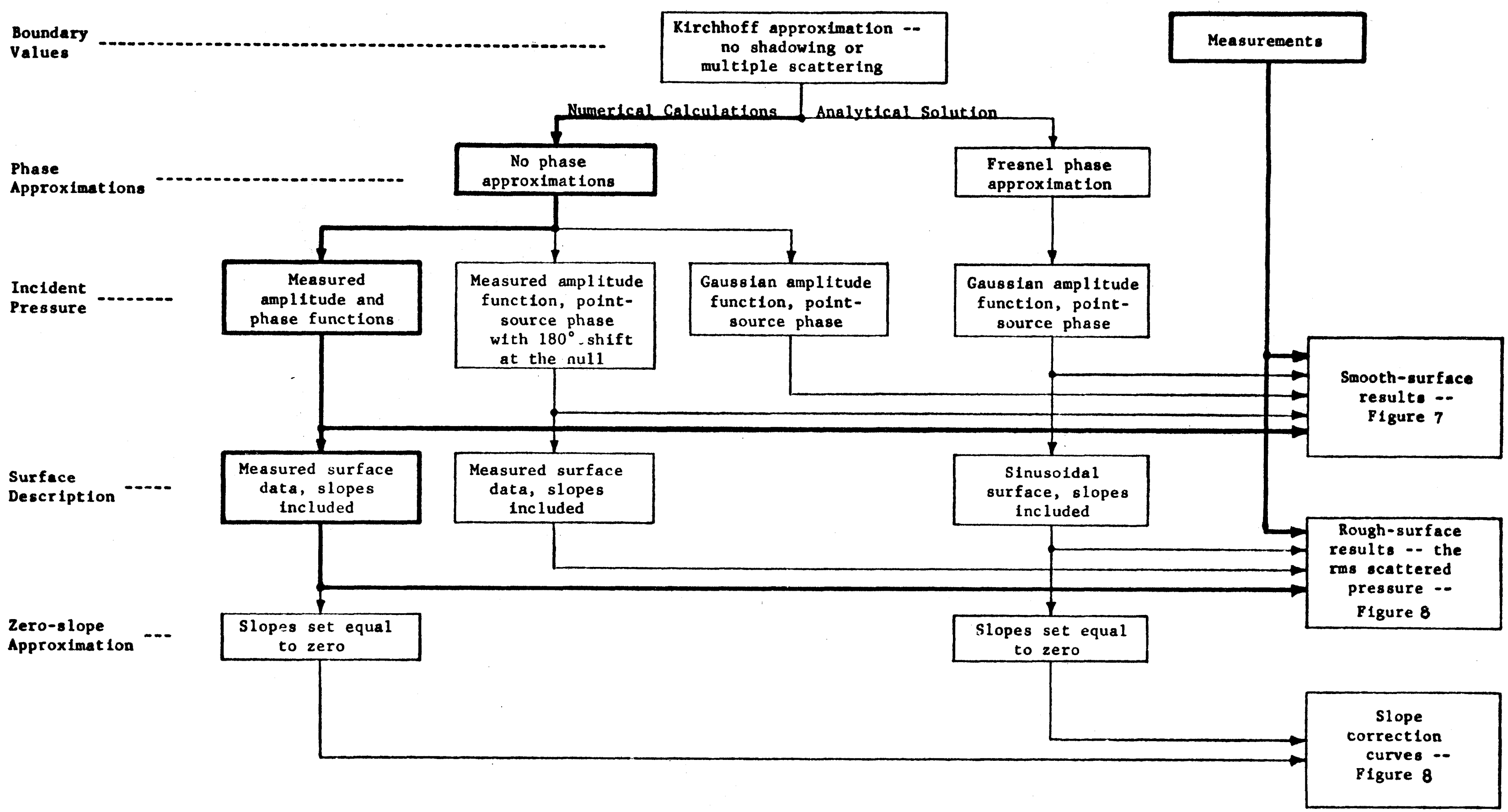

Figure 3. A chart outlining the computations and showing the approximations and conditions used. The relationship of the measured data and the organization of the results are also shown.

source. But in order to include the side lobes of the beam, it is necessary to add 180^{o} phase shifts at points corresponding to the amplitude nulls between the lobes of the beam. This kind of a function deviates slightly from the measured values (see Figure 5, p. 24). The effect of this deviation is evaluated for both the smooth and rough surfaces. 3) A Gaussian amplitude function and a point-source phase term are often used to specify the incident pressure. The effect of neglecting the side lobes is of particular interest here.

The blocks on the far right represent the graphs shown in Section V. The numerical computation using the measured values of both the amplitude and phase of the incident pressure is of primary interest. For the purpose of reference, this will be referred to as the 'exact' computation, and the parts of the chart which pertain to it are drawn with heavy lines. The results of all of the computations are compared with measured data -- also shown by heavy lines. Further explanations of the computations, and of the notations within the blocks, are given in following sections of the text. Results of analytical computations and measurements for the case of backscattering are also shown in Section V but are not included in the chart.

II. Numerical evaluation of the Helmholtz integral.

A. Theory

For the case of scattering of an acoustic wave from a pressure-release surface, the monochromatic Helmholtz integral, in the Kirchhoff approximation, is[6,15,16]

$$P = -(4\pi)^{-1} \int_S \underset{\sim}{n} \cdot \nabla (p'G) dS \qquad (1)$$

where p is the scattered acoustic pressure at a point receiver,

p' is the incident pressure at the scattering surface,

$$G = \exp(ikr_2)/r_2 \quad , \qquad (2)$$

r_2 is the length of a vector from the observation point, $(x_2, 0, z_2)$, to the scattering point,

∇ is the gradient operator,

$\underset{\sim}{n}$ is a unit vector normal to the surface at the scattering point,

S is the surface,

the time factor $\exp(-i\omega t)$ is ignored.

The geometry is shown in Figure 2.

Let the displacement of the water surface be $\zeta(x,y) = \zeta(x)$, and let its x derivative be $\zeta_x(x)$. Then the normal unit vector and the differential surface area are

$$\underset{\sim}{n} = (\zeta_x \underset{\sim}{x} - \underset{\sim}{z})/(1 + \zeta_x^{\,2})^{\frac{1}{2}} \qquad (3)$$

and $dS = (1 + \zeta_x^{\,2})^{\frac{1}{2}}\, dsdy \quad , \qquad (4)$

where $\underset{\sim}{x}$ and $\underset{\sim}{z}$ are unit vectors.

Substituting into Equation 1 and performing the indicated operations yields

$$p = (4\pi)^{-1} \iint_{-\infty}^{\infty} G(p'U + V)dxdy \quad , \tag{5}$$

where

$$U = r_2^{-1}(ik - r_2^{-1})[\zeta - z_2 - \zeta_x(x-x_2)] \quad , \tag{6}$$

$$V = \partial p'/\partial z - \zeta_x \partial p'/\partial x \quad , \tag{7}$$

k is the acoustic wavenumber = $2\pi/\lambda$.

The scattered pressure is computed for many equally-spaced positions of the surface. (For the computations reported here, twenty-three positions were used.) Then the rms scattered pressure is obtained from

$$p_{rms} = (\lambda_s^{-1} \int_0^{\lambda_s} p\, p^*\, dx)^{\frac{1}{2}} \quad , \tag{8}$$

where λ_s is the wavelength of the surface. Simpson's rule is used to evaluate the integrals in both Equation 5 and Equation 8. A discussion and listing of a computer program to perform this computation are presented in Appendix A (program SCATTER).

The zero-slope approximation consists in setting $\zeta_x(x) = 0$ (i.e., $\underset{\sim}{n} = -\underset{\sim}{z}$). The error resulting from the use of this approximation can be easily evaluated numerically. The computation of p_{rms} is performed twice; once using values of $\zeta_x(x)$ computed from measured values of $\zeta(x)$, and again, with $\zeta_x = 0$. The error is the difference of the two results. In the remainder of this report, it will be referred to in terms of a 'slope correction' to be applied to values of the scattered pressure calculated in the zero-slope approximation.

Finally, the pressure reflected from a smooth surface is obtained by setting $\zeta = \zeta_x = 0$.

A monochromatic incident signal has been assumed in this derivation. This assumption is permissible in a pulsed-mode experiment if the pulse (or ping) length is sufficiently long that the surface is completely ensonified for a period greater than the duration of the receiver gate aperture. In this experiment, a pulse length of 400 μsec was used. This pulse length was long enough to satisfy this criterion and yet so short that the wave surface could be regarded as stationary for the period of one measurement. (Frequency shifted components of the scattered signal are not of interest here.)

B. Computational procedure.

Two sets of input data are read into the computer: first, the surface displacement, $\zeta(x)$, and its x derivative, $\zeta_x(x)$; and second, the incident pressure and its x and z derivatives on the mean water surface.

Surface data -- To obtain a representative set of data for use in the computation, the surface displacement was measured in the center of the ensonified area and recorded on analog tape for a period of 5 minutes. That record was then digitized and analyzed to find an 'overall' rms value (the rms displacement for the entire 5 minute record). An 'average' block of 600 data (values of ζ), corresponding to a wave train approximately 60 cm long (1.5 sec., $\sim 7\lambda$s), was then selected using the criterion that its rms value be within 0.5% of the overall rms displacement.

The assumption was made that the quantity measured, $\zeta(t,x = 0)$, is insignificantly different from the desired quantity, $\zeta(t = 0,x)$, which is much more difficult to measure. This is justifiable because dispersion and attenuation do not appreciably alter the waveform as it propagates across the ensonified area.

The wave spectrum is sharply peaked at $\sim$ 4.5 Hz and the waveform closely approximates a sine wave with a slight flattening of the troughs and sharpening of the peaks. This is the waveform that one would expect for gravity waves with a very narrow frequency spectrum. In Figure 4, surface displacement probability density functions obtained from the 'average' block and from the entire 5 minute record are compared with the PDF of a cosine wave. The computer program used to compute the PDF's of the surface data is presented in Appendix C.

Each digitized value of the surface displacement corresponds to a scattering point in the numerical integration. The sampling rate was chosen so that the truncation error in a numerical integration of the scattering function in the x direction would be acceptably small. Using the usual estimate of error for Simpson's rule,[44] with point spacings $\Delta x = 0.0998$ cm and $\Delta y = 0.2$ cm, the upper bound on truncation errors in the integration is about 1%. The computation is normally performed with single precision arithmetic -- which corresponds to nine decimal digits in the computer used, a Univac 1108. Test computations in double precision arithmetic (eighteen decimal

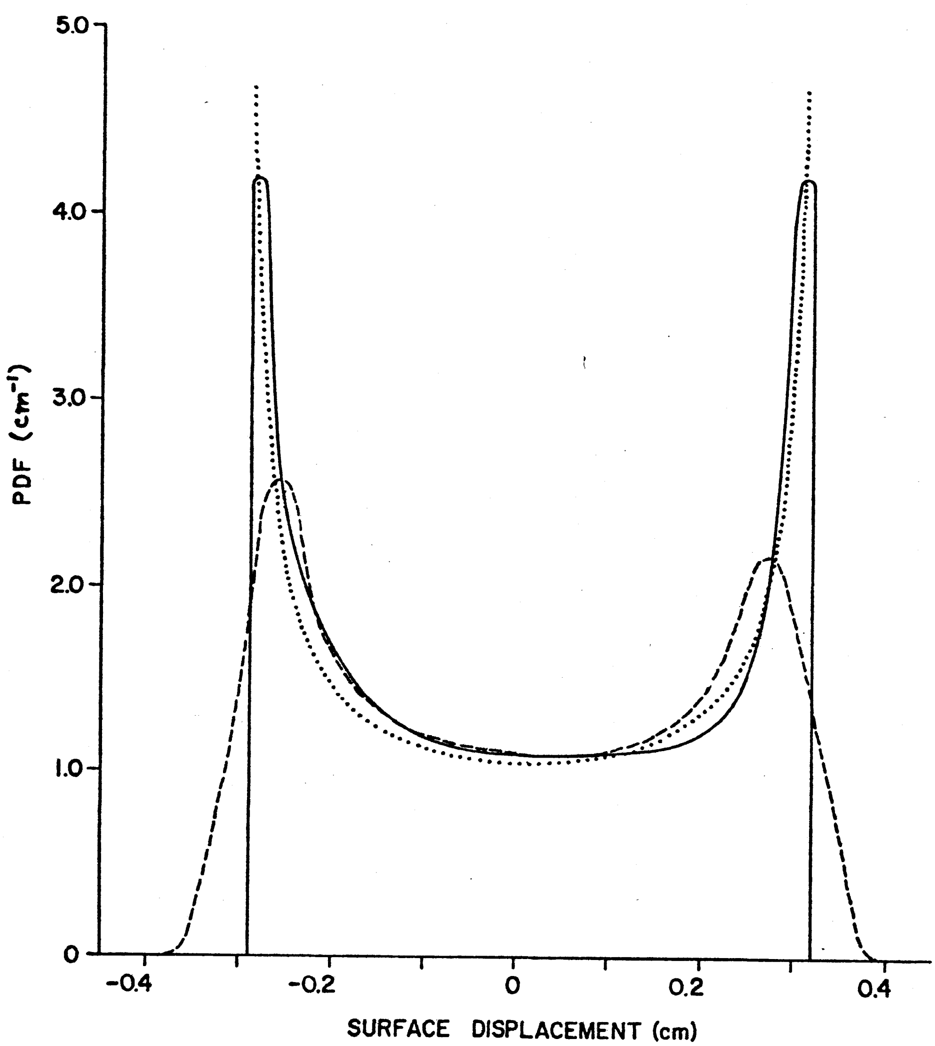

Figure 4. Normalized probability density functions (PDF's) for surface displacement. The solid curve is derived from the 600 work 'average' block; the dashed curve, from the entire 5 minute record; and the dotted curve, from a cosine function having an amplitude of 0.305 cm. The cosine PDF has been shifted on the abscissa to facilitate a comparison with the other curves.

digits) have shown that roundoff errors arise only in the fourth significant digit of the computed scattered pressure.

A value of ζ_x was computed for each of the 600 ζ's in the 'average' block. The surface was sampled 88 times per surface wavelength, so the simplest approximation formula was adequate. That is,

$$\zeta_{xi} \simeq \frac{\zeta_{i+1} - \zeta_{i-1}}{2\Delta x} \quad . \tag{9}$$

For further details of the analysis of the surface data, and of the computation of the x derivatives, see Appendix A, program TRANSLATE.

<u>Incident pressure data</u> -- It was mentioned earlier that for this calculation the transmitted beam is required to be both vertically incident and symmetrical about the z axis. In addition, the depth of the transducer was fixed at 50.1 cm. Consequently, when the water surface is smooth, the incident pressure distribution at the surface is a function only of ρ, where ρ is in the surface and is the radial distance from the beam axis. The incident pressure was measured by placing the transducer at a depth of 100.0 cm and traversing the field 50.1 cm above it with a small hydrophone. Measured values of the amplitude and phase of the pressure are shown in Figure 5.

The measured pressure can be written as

$$p'(\rho,z) = |p'| \ \exp(i\varphi) \quad . \tag{10}$$

At each point on the radius the partial derivatives with respect to ρ and z are

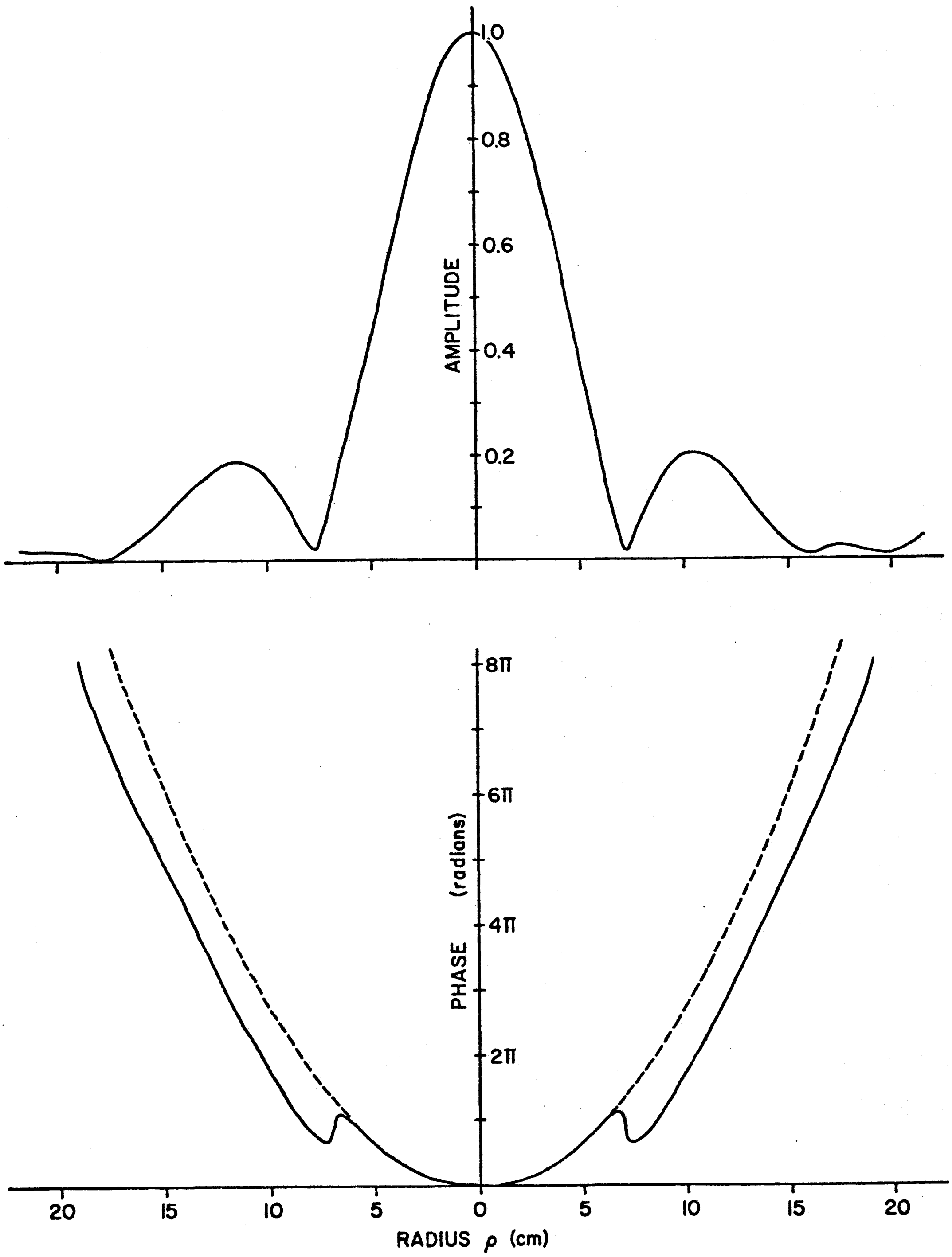

Figure 5. Normalized values of the amplitude and phase of the incident pressure measured in a plane parallel to the surface of the transducer at a distance of 50.1 cm. The dashed line represents the phase of a point source.

$$\frac{\partial p'}{\partial \rho} = \left[\left(\frac{\partial |p'|}{\partial \rho}\right)^2 + |p'|^2 \left(\frac{\partial \varphi}{\partial \rho}\right)^2\right]^{\frac{1}{2}} \exp[i(\varphi + \tan^{-1} w)], \qquad (11)$$

$$\frac{\partial p'}{\partial z} \simeq \frac{-kr_{10}\,|p'|}{r_1} \exp\,[i(\varphi + \pi/2)] \quad , \qquad (12)$$

$$\text{where} \quad w = |p'| \left(\frac{\partial |p'|}{\partial \rho}\right)^{-1} \frac{\partial \varphi}{\partial \rho} \quad . \qquad (13)$$

At each scattering point the x derivative of p' is obtained from the identity $\partial p'/\partial x = (\partial p'/\partial \rho)(\partial \rho/\partial x)$, where $\partial \rho/\partial x = x/\rho$. The partials with respect to ρ of $|p'|$ and φ are obtained from approximation formulas equivalent to Equation 9 (see Appendix A, program PGEN2). The data, shown in Figure 5, were measured at increments of $\Delta\rho = 0.25$ cm. Test computations have shown that this is adequate even for points near the null in the amplitude profile; the scattered and reflected pressures are insensitive to the values of the partials at the null.

At points on the rough surface, the incident pressure and its derivatives are obtained by perturbing the phases of the nearest smooth-surface values. In the neighborhood of each point, the phase shifts are approximately equal to $k\Delta r_1$. This was checked by numerically computing the incident pressure and its derivatives by a method similar to that used by Zemanek[45] to compute the pressure in the near-field of a piston source. On this basis, phase errors resulting from the perturbation procedure are estimated to be typically an order of magnitude smaller than the uncertainty ($\pm$ 0.2 radian) in the measurement of φ. Amplitude corrections were ignored because they are approximately proportional to $\Delta r_1/r_1$, and are an order of magnitude less than the uncertainty ($\pm$5%) in the measured values of $|p'|$.

It is clearly necessary to limit a numerical integration to a finite surface area. For this computation, the area of integration was chosen to include the main lobe and the first side lobe of the incident beam; the maximum radius was set at 17.0 cm. Errors resulting from this choice are likely to be most easily seen in a comparison of computed and measured pressures reflected from a smooth surface. Such a comparison is made in Section V. A. and the error is shown to be small.

III. Analytical solution - sinusoidal surface.

Three approximations are made here in addition to the Kirchhoff approximation already used in the derivation of Equation 5. First, the Fresnel approximation is used in computing the phases of the incident and scattered rays. Second, the geometrical spreading factor, r_2^{-1}, is set equal to r_{20}^{-1} and is removed from the integral. Third, the incident pressure is approximated by the product of a Gaussian directivity function and a point source phase function as follows:

$$p' = \exp[-\left(\frac{x \cos\theta_1}{L_x}\right)^2 - \left(\frac{y}{L_y}\right)^2] \exp(ikr_1) \ . \qquad (14)$$

L_x and L_y are the linear beamwidths in the x and y directions, measured, as before, in a plane parallel to the face of the transducer and separated from it by a distance r_{10}. As in the numerical calculation, the time factor, $\exp(-i\omega t)$, is not explicitly included.

In the Fresnel approximation the total path length of a scattered ray is

$$r_1 + r_2 = r_{10} + r_{20} + A_1 x^2 + A_2 x + A_3 y^2 + A_4 \zeta \tag{15}$$

where
$$A_1 \equiv \frac{\cos^2\theta_1}{2r_{10}} + \frac{\cos^2\theta_2}{2r_{20}} \quad , \tag{16}$$

$$A_2 \equiv \sin\theta_1 - \sin\theta_2 \quad , \tag{17}$$

$$A_3 \equiv \frac{1}{2r_{10}} + \frac{1}{2r_{20}} \quad , \tag{18}$$

$$A_4 \equiv \cos\theta_1 + \cos\theta_2 \tag{19}$$

Substitution of Equations 14 and 15 into Equation 5 yields

$$p = \frac{\exp[ik(r_{10} + r_{20})]}{4\pi r_{20}} \iint_{-\infty}^{\infty} [ikA_4 + (2a_x^2 x - ikA_2)\zeta_x] \exp[-(a_x^2 x^2 - ikA_2 x + a_y^2 y^2 - ikA_4 \zeta)]dxdy \quad , \tag{20}$$

where

$$a_x \equiv A_x \exp(-i\varphi_x/2) \quad , \tag{21}$$

$$a_y \equiv A_y \exp(-i\varphi_y/2) \quad , \tag{22}$$

$$A_x \equiv (L_x^{-4} \cos^4\theta_1 + k^2 A_1^2)^{1/4} \quad , \tag{23}$$

$$A_y \equiv (L_y^{-4} + k^2 A_3^2)^{1/4} \quad , \tag{24}$$

$$\varphi_x \equiv \tan^{-1}(kA_1 L_x^2/\cos^2\theta_1) \quad , \tag{25}$$

$$\varphi_y \equiv \tan^{-1}(kA_3 L_y^2) \quad . \tag{26}$$

It is convenient at this point to also define the quantity

$$\nu^2 \equiv 4A_x^{\ 2}/(k^2 \cos\varphi_x) \quad . \tag{27}$$

The scattered pressure can be written as the sum

$$p = p_a + \delta p \quad , \tag{28}$$

where p_a is the approximate scattered pressure corresponding to the term ikA_4 in the square brackets of Equation 20, and δp is the slope correction obtained from the remaining terms in the brackets.

The scattering surface is

$$\zeta_m(x) = \zeta_o \cos K\,(x-x_m) \quad , \tag{29}$$

where ζ_o is the amplitude, and $K = 2\pi/\lambda_s$. The term x_m is provided as a means of translating the surface, and the subscript m is used below to refer to the mth position of the surface. Alternatively, a continuously propagating surface can be obtained by replacing Kx_m by $\omega_s t$, where ω_s is the angular frequency of the surface.

We now use the Bessel function expansion[46],

$$\exp[ia \cos K(x-x_m)] = \sum_{n=-\infty}^{\infty} J_n(a) \exp[inK(x-x_m)] \quad , \tag{30}$$

where

$$a \equiv kA_4\zeta_o \quad . \tag{31}$$

Substitution of Equations 29 and 30 into Equation 20 yields the scattered pressure in the small slopes approximation,

$$p_{am} = \frac{kA_4 \exp(i\varphi_o)}{4\pi r_{20}} \sum_{n=-\infty}^{\infty} J_n(a) \exp(-inKx_m) \iint_{-\infty}^{\infty} \exp - [a_x^2 x^2$$

$$- i(kA_2 + nK)x + a_y^2 y^2] \, dxdy \quad , \qquad (32)$$

and the slope correction term,

$$\delta p_m = \frac{\exp i(\varphi_o - \pi/2)}{4\pi r_{20}} \sum_{n=-\infty}^{\infty} J_n(a) \exp(-inKx_m) \iint_{-\infty}^{\infty} (2a_x^2 x - ikA_2)\zeta_x$$

$$\exp - [a_x^2 x^2 - i(kA_2 + nK)x + a_y^2 y^2] \, dxdy \qquad (33)$$

where $\varphi_o \equiv k(r_{10} + r_{20}) + \pi/2$. (34)

Integration of Equations 32 and 33 yields

$$p_m = p_{am} + \delta p_m$$

$$= \frac{kA_4}{4r_{20}A_xA_y} \sum_{n=-\infty}^{\infty} (1 + \epsilon_n) J_n(a) \exp[-\alpha^2(n)] \exp[i\varphi_m(n)], \qquad (35)$$

where $\alpha^2(n) \equiv (A_2 + nK/k)^2/\nu^2$, (36)

$$\varphi_m(n) \equiv \varphi_o + (\varphi_x + \varphi_y)/2 - \alpha^2(n) \tan \varphi_x - nKx_m + n\pi/2 \quad . \qquad (37)$$

The term ϵ_n is the fractional slope correction associated with the amplitude of the nth order diffraction lobe. Its form is

$$\epsilon_n = \left(\frac{nK}{kA_4}\right)^2 \quad . \qquad (38)$$

The rms pressure, obtained from Equations 8 and 35, is

$$p_{rms} = \frac{kA_4}{4r_{20}A_xA_y} \left\{ \sum_{n=-\infty}^{\infty} (1 + \epsilon_n)^2 J_n^{\,2}(a) \exp\left[-2\alpha^2(n)\right] \right\}^{\frac{1}{2}} \quad . \qquad (39)$$

Computer programs for evaluating Equations 35 and 39 are presented in Appendix B.

As a function of receiver angle, p_{rms} has the form of a diffraction pattern. The directions of the maxima are given by the familiar diffraction grating formula,

$$\sin\theta_2 = \sin\theta_1 \pm nK/k \quad ; \qquad (40)$$

and the amplitudes of the diffraction peaks are proportional to the Bessel functions, $J_n(a)$. The zeroth order terms in Equations 35 and 39 are the specular components of the pressure and the mean square pressure, respectively, and do not include slope corrections. The angular width of the diffraction peaks is dependent on the surface wavelength and, via the quantity ν, on the beam width, the geometry and the acoustic wavelength. Much of the width is due to the curvature of the acoustic waves. This factor is accounted for in the Fresnel approximation by the term $(kA_1)^2$ in ν (see Equations 23 and 27). Further, ν is roughly proportional to the angular beamwidth $(\sim L_x/r_{10})$. Analagous to the case of a coarse diffraction grating in optics, broad diffraction peaks are obtained if the diameter of the ensonified area is not much larger than the surface wavelength. Further, for small values of the ratio K/k, the orders overlap and may not be resolved in a measurement of the rms pressure.

For directions corresponding to diffraction maxima, ϵ_n can be rewritten in the geometrical form

$$\epsilon(\theta_1, \theta_2) = \left(\frac{\sin\theta_2 - \sin\theta_1}{\cos\theta_2 + \cos\theta_1}\right)^2 \quad . \tag{41}$$

Tolstoy and Clay[28] define the term $f(\theta_1, \theta_2)$. This is the geometrical slope factor mentioned in the Introduction. Comparing their expression for the scattered pressure with Equation 35, we find that

$$f(\theta_1, \theta_2) = A_4[1 + \epsilon(\theta_1, \theta_2)]/2 \quad . \tag{42}$$

Beckman and Spizzichino[18] define a similar term which includes an additional factor of sec θ_1. The differences are due to definitions and are of no consequence.

In those previous derivations, no distinction was made between periodic and random surfaces, although a geometrical form is perhaps not what one would expect for the slope factor in the latter case. However, this result is consistent with the concept that the pressure scattered in a specific direction from a rough surface is the resultant pressure diffracted from those wavelength components of the surface roughness which satisfy Equation 40.[1, 47] The value of the fractional slope correction is the same for all of those components since they are all characterized by a constant value of the term nK. But a question of validity arises for the following reason. For the sinusoidal surface the slope correction at a given angle is a resultant value determined by the contributions of overlapping diffraction peaks. This is accounted for by the inclusion of the quantity $1 + \epsilon_n$ within the summations in Equations 35 and 39; and only for very narrow beamwidths is it justifiable to remove that term from the summations.

In the references referred to above, the term $f(\theta_1, \theta_2)$ is used (for a random surface) as a coefficient external to the scattering integral. Apparently, implicit in this useage is the assumption that the beamwidth is small and that the diffraction peaks associated with each component of the surface spectrum are very narrow. For the beamwidths commonly used, it is not clear that this assumption can be made.

IV. Experiment

The measurements were made in a water tank 1.80 m in diameter and 1.50 m deep. Waves were produced at one end of the tank by vertically oscillating a horizontal, partially submerged, wooden rod 5.0 cm in diameter and 125 cm in length. The driving mechanism was a Slo-Syn stepping motor connected to the rod by a rack and pinion. A mechanical switch connected to the motor shaft was used to reverse the motor. The stepping rate of the motor was constant in the periods between reversals.

Measurement of the wave heights was by resistance probes, a standard method described in the literature.[23, 48] To calibrate the wave probes, a micrometer head was mounted above the water surface. It was connected electrically in series with a battery, a resistor, and the water in the tank. Touching the water with a probe wire at the end of the micrometer completed the circuit and produced a voltage drop across the resistor. This was observed on an oscilloscope. Wave crests just touching the end of the wire produced a string of pulses on the oscilloscope. The wave height was obtained directly from the difference in micrometer readings for the rough and smooth surfaces.

Small variations in the shape and amplitude of the waves occurred because it was not possible to totally eliminate reflections from the tank walls, or to completely stabilize the wave generation process. The mean amplitude of the waves, measured at different points within the ensonified area, did not vary more than the uncertainty in the measurement of the mean value, approximately $\pm$ 5%.

The source transducer was a 5.1 cm diameter Clevite G7 ceramic disk with a thickness mode resonance at 205.6 kHz. The edge and bottom face of the disk were covered with 0.5 cm of styrofoam to eliminate transmissions in unwanted directions. For the measurements at vertical incidence the source transducer was mounted on a horizontal arm and positioned in the center of the tank at a depth of 50.1 cm.

The receiving hydrophones were (approximately) square pieces of ceramic about 1.5 mm on a side. These were cut from a thin disk having a thickness mode resonance at about 1200 kHz. A hydrophone was mounted on a horizontal arm suspended from pivots on the y axis at the water surface. The hydrophone was thus free to move along an arc in the x-z plane (see Figure 2). For these measurements the radius, r_{20}, of the arc was fixed at 49.2 cm.

For backscattering measurements, both the source transducer and a hydrophone were mounted on the pivoted arm, with the hydrophone offset 6° in the x-z plane.

A small piece of styrofoam was attached to the hydrophone on the side nearest the source. This effectively shielded the hydrophone from spurious signals scattered from the source, yet had no measureable effect on signals received directly from the water surface.

The electronics are straightforward and are described in detail elsewhere.[49] Figure 6 is a block diagram which shows the basic features of the apparatus for the pulse generation and for the measurement of the mean square signal. Briefly, the signal from the hydrophone is amplified, squared, gated, and integrated. The gate is set to pass only a part of the signal scattered from the water surface, and is controlled by a digital time-delay generator. The output of the integrator is read on a digital voltmeter after a specified number of pulses have been transmitted.

Throughout the experiment, a system calibration was maintained by repeated measurements, at a fixed angle, of the pressure reflected from the smooth water surface. A disadvantage of this procedure is that in order to compare the measurements with theoretical results, a theoretical calculation must be made for the reflected pressure as well as for the scattered pressure.

V. Results

A. Smooth surface

The reflected pressure as a function of receiver angle was calculated numerically using three different representations of the incident pressure. The results are shown in Figure 7.

The solid curve is the 'exact' result obtained using the measured values of both the amplitude and phase of the incident pressure. The accuracy of this calculation is confirmed by the measured values of the reflected amplitude which have been fit by least squares to the calculated curve. These measurements were limited by the size of

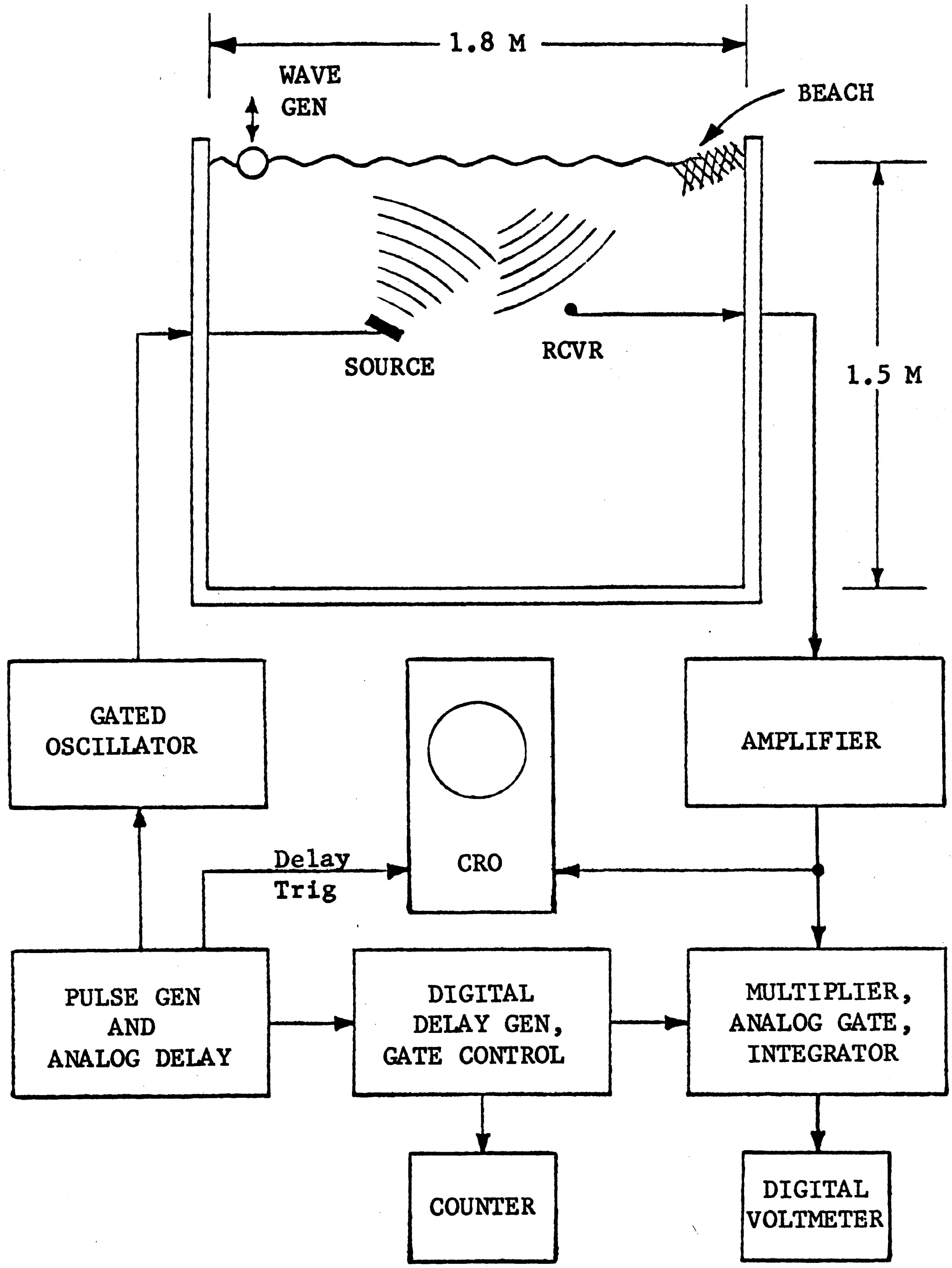

Figure 6. Apparatus for laboratory scattering measurements.

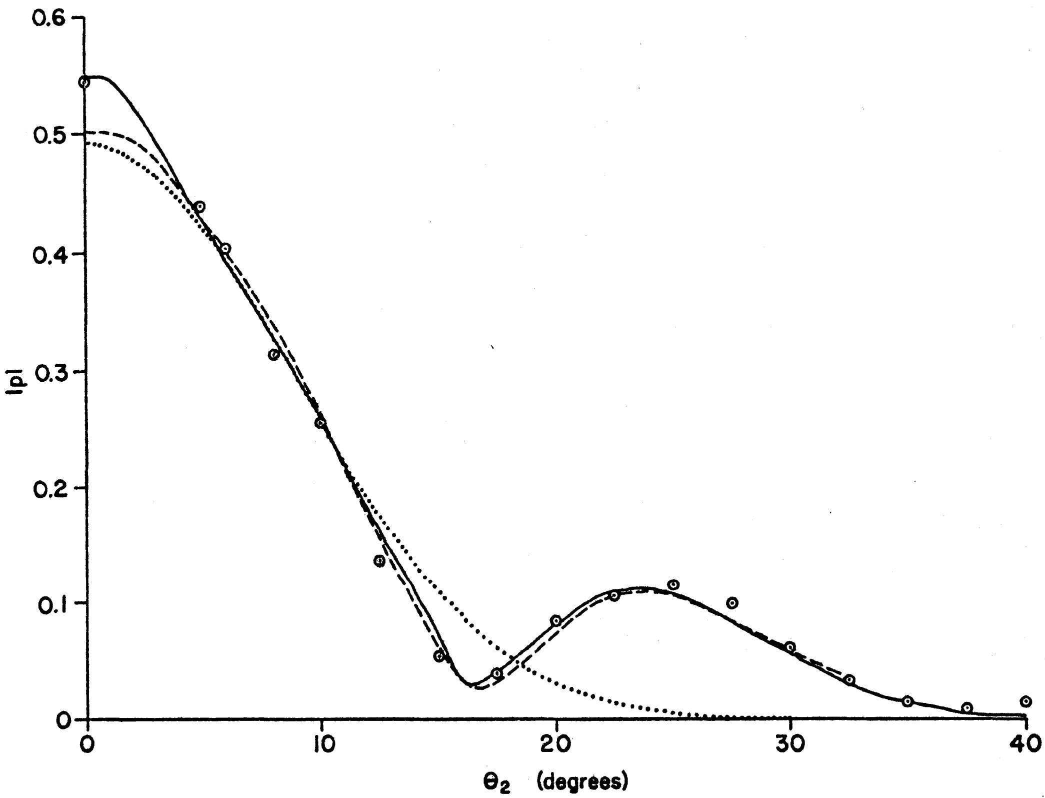

Figure 7. The amplitude of the pressure reflected from the smooth surface. The computed curves were obtained by numerical integration using the measured values of the incident pressure (solid curve), the measured amplitude and a point-source phase with a 180° shift (dashed curve), and a Gaussian amplitude function with a point-source phase (dotted curve). The circles are measured values.

the source transducer to receiver angles greater than about 6°. Therefore, measurements were also made of the direct transmitted pressure using a geometrical configuration consistent with the concept of an image source. These data are indistinguishable from the reflected data, as expected, and the data shown at 0° and 5° were taken from these measurements.

In the second calculation (dashed curve), values of the measured incident amplitude were used, but the phase was approximated by the function kr_1 with a 180° phase shift added at the radius corresponding to the null in the amplitude profile. It is apparent in Figure 5 that approximating the phase in this way is accurate to within 0.5 radian out to a radius of 15 cm, where the amplitude is 95% down from the peak value. This phase error produces an error of approximately 8.5% in the computed amplitude at $\theta_2 = 0°$. The phase errors in this approximation are not too different from the errors in the Fresnel approximation, where, at a radius of 15 cm, the phase error for $\theta_2 = 0°$ is about 0.9 radian.

In the third calculation (dotted curve), the incident pressure was approximated by Equation 14, a Gaussian amplitude function and a point-source phase function. The width of the Gaussian was obtained by fitting it to the half-amplitude points of the measured amplitude profile of Figure 5. The effect of this approximation is to ignore the side lobes of the incident beam, and the resulting error in the reflected amplitude at $\theta_2 = 0°$ is about 10%. One might expect larger errors if fewer Fresnel zones were ensonified by the incident beam. The analytical solution, Equation 35, was also evaluated for this case.

The essential difference between this calculation and the numerical integration is that the latter does not involve the Fresnel approximation. The difference in the results is very small, however, because the Fresnel phase errors are less than 0.2 radian within the area ensonified by the significant part of the Gaussian beam.

B. Rough Surface

The effective roughness of a scattering surface is often specified by the quantity[50]

$$g = k^2\sigma^2(\cos\theta_1 + \cos\theta_2)^2 \quad , \tag{43}$$

where σ is the rms displacement of an ensemble of surface points about their mean. A surface is normally regarded as 'rough' if $g > 1$. For these measurements, $\sigma = 0.21$ cm, $k = 8.73$ cm^{-1}, and θ_1, $\theta_2 < 45°$; so the range of g is approximately 6.8 - 13.6.

Scattering at vertical incidence -- Calculated and measured values of the rms scattered pressure are shown in Figure 8. Calculated values of the slope correction are also shown. These curves were computed using the same representations of the incident pressure that were used in computing the curves of Figure 7. However, here, the dashed curves are the analytical results, obtained from Equation 39; and the dotted curve is the numerical result obtained with the measured incident amplitude and the modified (180° shifts) point-source phase function. The solid curves again refer to the 'exact' numerical computation with the qualification that the solid slope correction curve is the difference between the 'exact' result and the equivalent result with the slopes set to zero.

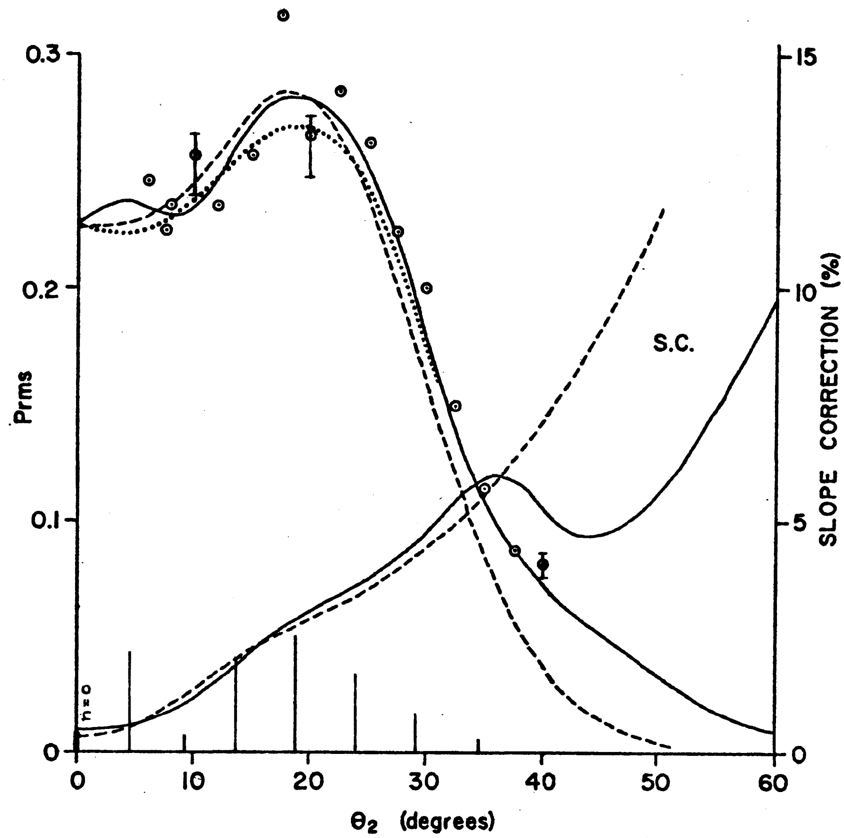

Figure 8. The rms scattered pressure and the slope correction (the curves labled s.c.) at vertical incidence. The solid and dotted curves were computed numerically; the dashed curves are from the analytical solution, Equation 39; and the circles are measured values. Calculated relative amplitudes of the diffraction peaks are shown at the base of the graph.

The agreement of the pressure curves at small values of θ_2 is somewhat better than that of the corresponding curves for the smooth surface. The approximations of the incident beam apparently become increasingly valid as the roughness of the surface is increased. This may be due to a relative decrease in the coherence of the point scatterers illuminated by the side lobes. In a more detailed view, we note that the small peak evident in the numerical curve at $\theta_2 \simeq 4°$ is not resolved in the other two curves. It is clear from Equation 40 that this peak is a diffraction maximum of order n = 1. Failure to resolve it is a consequence of approximating the incident pressure. It was pointed out in Section III that the non-planar nature of the incident acoustic waves has a strong broadening effect on the diffraction peaks of the scattered signal. Comparison of the solid and dotted curves shows that small phase errors, particularly in the representation of the side lobes, can have a significant effect on the computed widths of the diffraction peaks. In the examples shown here, the effect of phase approximations is to reduce the resolution of the individual diffraction peaks, but an erroneous enhancement of the resolution is also possible. For example, it is easy to show that in a Fraunhofer or plane wave approximation the peaks are dramatically narrowed.

Each of the data in Figure 8 is an average of 20,000 - 30,000 returns accumulated over a period of 2-3 minutes. The scatter in the data is due to longer-period fluctuations in the amplitude and shape of the water waves, and to possible effects of air bubbles near the surface of the water and on the transducers. The vertical bars on the data points at 10°, 20°, and 40° show the scatter resulting from five measurements at each of those angles. It was desirable to forgo longer averaging times in order to minimize problems which tended to arise in long-term measurements -- namely, problems with bubbles and with instability in the rather unreliable wave generator. Nevertheless, on the average, the measured data deviate from the computed values (solid curve) by less than 4%.

The data were multiplied by the same constant used to fit the smooth-surface data to the 'exact' curve in Figure 7. All three curves fit the data reasonably well when $\theta_2 < 20°$. For larger values of θ_2, the data increasingly diverge from the analytical curve, whereas a good fit is maintained with the numerical curves. This is partially attributed to the effect of the side lobe of the incident beam. In the geometry of this measurement, a portion of the side lobe is forward-scattered in the direction of the receiver with an angle of incidence of approximately 12°. With the aid of Equations 31 and 40 and a table of Bessel functions, it is easy to show that the diffracted orders associated with the side lobe have their maximum amplitudes in the neighborhood of $\theta_2 = 40°$, with orders of lesser amplitude occurring at both smaller and larger values of θ_2. Small adjustments in the parameters used to calculate the analytical curve can improve its fit with the data in part of the angular range, but not without degrading the fit elsewhere. The amplitude used for the surface roughness was $\zeta_o = 0.305$ cm, and was taken from Figure 4.

Backscattering -- The numerical integration was restricted to the case of a vertically incident beam. Therefore, only analytical curves and experimental data are shown in Figure 9. The data have been matched to the numerical curve (computed for vertical incidence) at the point $\theta_1 = 0°$.

If is evident upon comparison of Figures 8 and 9 that the same kinds of comments that were made with regard to the scattering at vertical incidence are also appropriate in this case. First, the agreement between the computed curve and the data is reasonably good, at least at small values of θ_1. Second, the discrepancy between the computed curve and the datum at $\theta_1 = 2.5°$ suggests again that the analytical solution yields a lesser degree of resolution than that which is physically and numerically realizable. The accuracy of that datum is supported by other data that are not shown here. Finally, side lobe contributions are clearly evident at values of θ_1 greater than 20°.

Slope correction -- Slope correction curves, given in terms of percentage contribution, are plotted in Figures 8 and 9. In Figure 8 it is possible to compare the analytical result with that obtained by numerical integration. The agreement is excellent at receiver angles less than 36°. At larger angles the curves diverge with the numerical curve clearly showing the reduced slope dependence appropriate to the forward-scattered side lobe.

This follows from Equations 38 and 39 where we note that each diffraction order has a slope correction term associated with it. Further, the correction terms increase rapidly with increasing order.

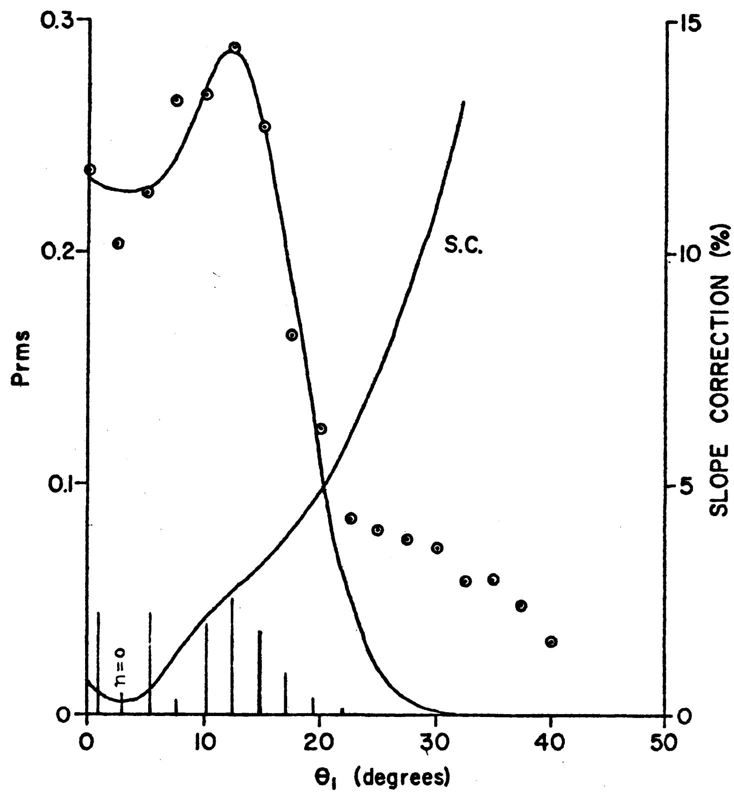

Figure 9. The rms scattered pressure and the slope correction for backscattering. The curves are from the analytical solution, Equation 39, and the circles are measured values. Calculated relative amplitudes of the diffraction peaks are shown at the base of the graph.

The prominent side lobe contributions apparent in Figures 8 and 9 consist of low-order components, but occur at the same angles as higher-order main lobe components. Thus, in this angular range, the percentage slope correction will reflect the dominance of the lower-order side lobe components. This is in agreement with an intuitive argument; namely that in forward scattering, the average surface slope tends to zero and results in very small values of the slope correction. Maximum values of the slope correction occur in backscattering where the average surface slope is increasingly biased as the angle of incidence is increased.

According to Equation 41, the fractional slope correction in backscattering increases rapidly at angles of incidence greater than those of the present measurement. At large angles, it may be an important factor in backscattering from the sea surface and sea floor. For example, at $\theta_1 = -\theta_2 = 60°$, a moderate angle, the slope correction is $\epsilon = \tan^2\theta_1 = 3$. It would be useful to extend the numerical part of this work to backscattering from random surfaces and to a study of the slope correction at large angles of incidence.

VI. Conclusion

A comparison of numerical, analytical, and experimental results has been made for the case of a vertically incident beam and a moderately rough surface. The numerical computations have been shown to yield values of the scattered pressure which agree well with experimental data. Simple approximation formulas, applied to values of the incident pressure on the mean scattering surface are apparently satisfactory (within the limits of the Kirchhoff approximation) for computing the scattered pressure and its normal derivative on the rough surface. The use of measured values of both the phase and amplitude of the incident pressure makes possible the accurate calculation of the scattered acoustic pressure for both smooth and rough surfaces. This approach appears to be an effective means for minimizing some of the difficulties inherent in studies of acoustic scattering; particularly, difficulties associated with the specification of the incident pressure and with phase approximations in both the incident and scattered waves. Side lobes of the incident beam are included in the numerical calculations and are not a source of uncertainty in a comparison between theoretical and experimental results.

The present computations indicate that the pressure reflected or scattered from a smooth, or slightly rough surface is sensitive to the phase of the side lobes of the incident beam. Phase approximations result in errors in the calculated pressure that are difficult to avoid in analytical scattering theories. The rms pressure scattered from a very rough surface is apparently less sensitive to the form of the approximation used to specify the incident pressure. This is a useful conclusion in terms of both analytical computations and further applications of the numerical technique, and its limitations should be determined.

The validity of the geometrical slope factor, $f(\theta_1,\theta_2)$(Equation 42), has been verified numerically for moderate scattering angles and a sinusoidal surface. But there are at least two difficulties associated with its use. First, the side lobes of the incident beam are not included in the derivations leading to $f(\theta_1,\theta_2)$ -- so it is not valid at scattering angles where the side lobes contribute significantly to the scattered pressure. Second, its validity is uncertain for the case of a random surface because the effect of the widths of the overlapping diffraction peaks is not accounted for. (This effect is accounted for in the case of a sinusoidal surface.) It is important to extend the study of the effect of surface slopes to backscattering and to non-sinusoidal surfaces.

Other extensions of this work are feasible. A study of the general boundary value problem, which includes the effects of shadowing and the limitations of the Kirchhoff approximation, would probably be most useful. For that purpose, surfaces of greater roughness should be used and the computation should be extended to large angles of incidence. In that case, it may be useful to utilize the point-source phase approximation. Errors associated with its use are not large in the present case, and they may be reduced by moderately increasing (in comparison with the present value) the distance of the source from the center of the scattering surface. The use of this approximation would simplify the computation, reduce its cost, and reduce the labor required to obtain initial values of the incident pressure.

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Appendix

Computer Programs

The purpose of this appendix is to document the computer programs used in the work described in the text. For the most part, these programs have been written for a specific purpose so are not expected to be useful in a general sense. However, they may be helpful in an extension of the present work or in making similar kinds of computations. A brief description of each program is given in terms of the purpose of the program and the method used. Definitions of terms and details of the programs are given mainly by numerous comment cards within the programs. The programs were written in Fortram V for a Univac 1108 computer.

A. Programs used in the numerical integration of the Helmholtz integral.

The numerical integration is performed by program SCATTER. In addition, two auxilliary programs provide input data: program TRANSLATE provides values of the surface displacement, ζ, and its x derivative, ζ_x; and program PGEN2 provides values of the incident acoustic pressure, p', and its partial derivatives on the scattering surface. The relationships of these auxilliary programs to the measured input data and to program SCATTER are shown in Figure A1. Modifications of the computational procedure will be necessary in order to apply the method of numerical analysis to other geometrical configurations. But it is hoped that the techniques used here will

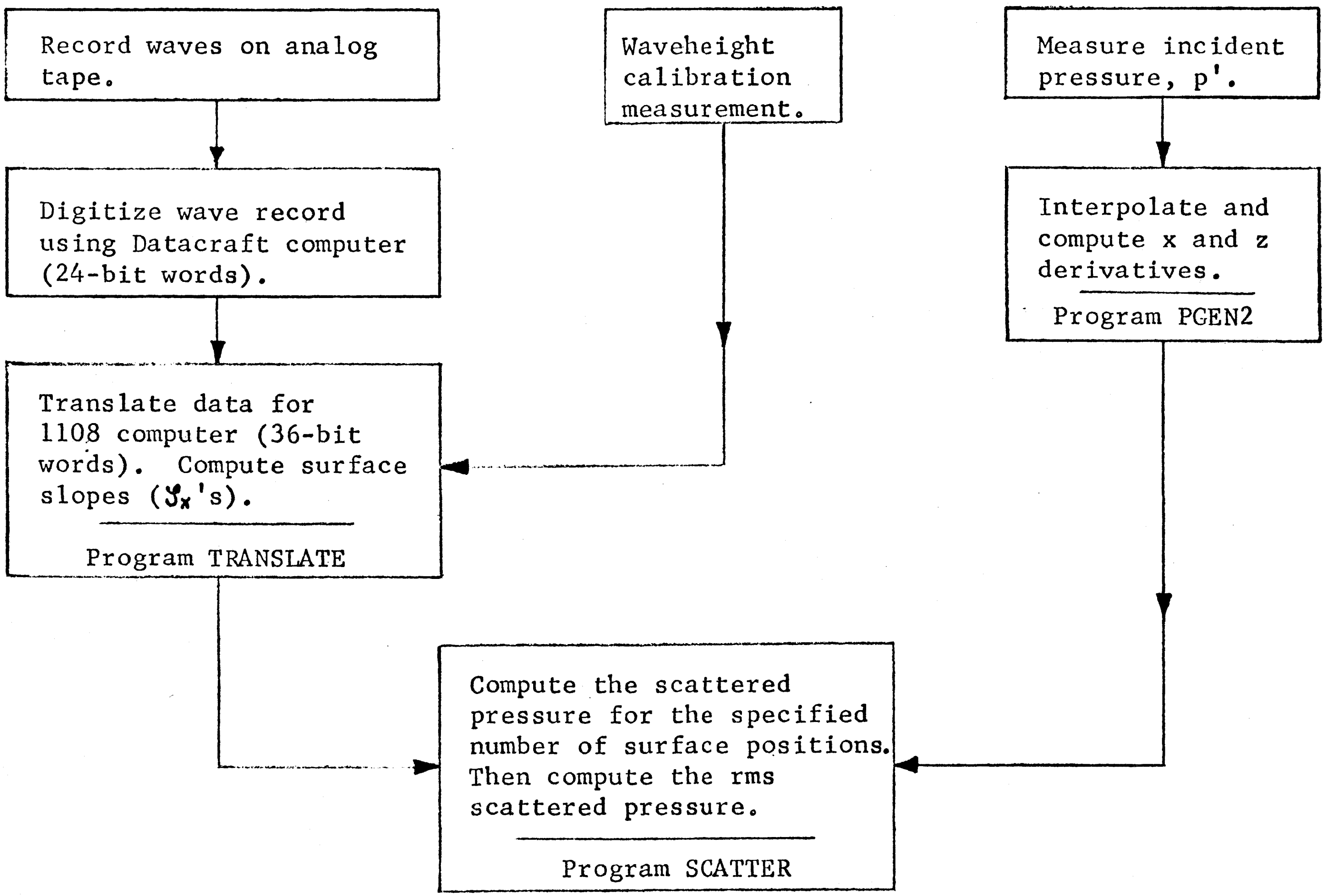

Figure A1. Data, procedures, and programs used in the numerical evaluation of the Helmholtz integral.

be both useful and instructive in extensions of this work.

Program SCATTER -- The real and imaginary parts of Equation 5 in the text are computed separately, but concurrently. Using Equations 6 and 7, the real part of the scattered pressure is

$$p_{re} = \frac{1}{2\pi}\int_{-X_{max}}^{X_{max}}\int_{0}^{Y(x)}\left[T_1\cos(kr_2)-T_2\sin(kr_2)\right] r_2^{-1}dydx \quad ; \qquad \text{(A1)}$$

and the imaginary part is

$$p_{im} = \frac{1}{2\pi}\int_{-X_{max}}^{X_{max}}\int_{0}^{Y(x)}\left[T_1\sin(kr_2)-T_2\cos(kr_2)\right] r_2^{-1}dydx \quad ; \qquad \text{(A2)}$$

where
$$T_1 = \left(\frac{kp'_{im}}{r_2} + \frac{p'_{re}}{r_2^2}\right)\left[(x-x_2)\zeta_x - (\zeta-z_2)\right] - \left[\zeta_x\left(\frac{\partial p'}{\partial x}\right)_{re} - \left(\frac{\partial p'}{\partial z}\right)_{re}\right] , \qquad \text{(A3)}$$

$$T_2 = \left(\frac{kp'_{re}}{r_2} - \frac{p'_{im}}{r_2^2}\right)\left[(x-x_2)\zeta_x - (\zeta-z_2)\right] + \left[\zeta_x\left(\frac{\partial p'}{\partial x}\right)_{im} - \left(\frac{\partial p'}{\partial z}\right)_{im}\right] . \qquad \text{(A4)}$$

The limits of integration are determined by the (arbitrarily defined) dimensions of the ensonified area on the surface. The length of the ensonified area in the x direction is $2X_{max}$. Because of the symmetry which exists when both the source and the receiver are in the x-z (y=0) plane, the integration on y is performed only over the half-plane $y > 0$.

The notation used for position coordinates in SCATTER differs slightly from that used in the text and in Equations A1-A4; The source coordinates are $(0, 0, z_o)$; the receiver coordinates are (x, o, z); and the surface coordinates are (x', y', ζ). Also, the distance from the scattering point to the receiver is R, rather than r_2.

The computational procedure described here was chosen because it is direct and because of the symmetry in the scattering geometry. Many variations are possible, and some are probably necessary in order to make similar calculations for other geometrical configurations. Throughout the present program, parallel computations are made for terms corresponding to positive and negative values of the surface coordinate, x'. This may not be a generally useful procedure. Equations A1-A4 can be simplified along with the entire computational procedure if the phase of the incident pressure can be approximated by the function kr_1. It was decided to read the input data from punched cards exclusively because the number of input data required in the present computations was not large (402 and 602 cards were supplied by PGEN2 and TRANSLATE, respectively). Data access from tape, drum, or disk files is more efficient than cards for other scattering geometries and for random surfaces.

MAIN

```
C
C THIS IS PROGRAM SCATTER.
C IT COMPUTES THE SCATTERED ACOUSTIC PRESSURE AND THE RMS SCATTERED
C PRESSURE BY NUMERICALLY INTEGRATING THE HELMHOLTZ INTEGRAL.  THE
C COMPUTATION IS LIMITED TO THE SPECIAL CASE OF SYMMETRY ABOUT THE X
C AXIS.  ALSO, THE OBSERVATION POINT IS RESTRICTED TO THE X-Z PLANE.
C THE PRESSURE DISTRIBUTION ON THE MEAN SURFACE IS REQUIRED TO BE
C SYMMETRICAL ABOUT THE Z AXIS.  VALUES OF THE INCIDENT PRESSURE AND ITS
C GRADIENT ARE SPECIFIED AT POINTS ALONG A RADIUS, RHO, AND ARE
C SUPPLIED ON CARDS BY PROGRAM PGEN2.  A DIMENSION STATEMENT LIMITS THE
C NUMBER OF CARDS TO 500.
C THE SCATTERING SURFACE IS ASSUMED TO BE PERIODIC.
C VALUES OF THE SURFACE DISPLACEMENT AND ITS X DERIVATIVE ARE
C SPECIFIED AT POINTS ON THE X AXIS, AND ARE SUPPLIED ON CARDS BY
C PROGRAM TRANSLATE.
C THE SCATTERING POINTS ARE ARRANGED IN ROWS PARALLEL TO THE Y AXIS.
C THE COMPUTATIONAL PROCEDURE IS TO FIND THE CONTRIBUTION OF POINTS
C ON A ROW, THEN THE ROW, THEN ALL ROWS.  INTEGRATION IS BY MEANS OF
C SIMPSON'S RULE.
C RESULTS ARE COMPUTED FOR SEVERAL OBSERVATION ANGLES, THETA.  FOR EACH
C ANGLE, THE SCATTERED SIGNAL IS CALCULATED FOR MANY POSITIONS OF
C THE SURFACE.
C THE RMS SCATTERED PRESSURE IS OBTAINED AFTER THE SCATTERED PRESSURE
C HAS BEEN CALCULATED FOR THE SPECIFIED NUMBER OF SURFACE POSITIONS.
C THE SQUARED PRESSURE MAGNITUDES ARE AVERAGED BY ANOTHER APPLICATION
C OF SIMPSON'S RULE.
C THE ARGUMENTS OF SIN AND COS ARE APPROXIMATED IN THE INNER DO LOOPS
C TO GAIN SPEED.  ARRAYS SSIN AND CCOS ARE COMPUTED AND STORED... VALUES
C ARE AT INCREMENTS OF 1. DEGREE.
C
C ALL NAMES PREFIXED BY A 'Q' REFER TO VALUES FOR NEGATIVE XP.
C XP, YP, AND ZETA ARE THE SURFACE COORDINATES.
C THE CARTESIAN COORDINATES OF THE OBSERVATION POINT ARE X,O.,Z.
C ZO IS THE DEPTH OF THE TRANSDUCER.
C TRE AND TIM ARE THE REAL AND IMAGINARY CONTRIBUTIONS TO THE
C SCATTERING INTEGRAL FROM A SINGLE POINT, XP,YP,ZETA.
C RE AND RIM ARE THE REAL AND IMAGINARY CONTRIBUTIONS OF ONE ROW (AFTER
C INTEGRATING WRT YP).
C PRE AND PIM ARE THE REAL AND IMAGINARY COMPONENTS OF THE SCATTERED
C PRESSURE FOR ONE POSITION OF THE SURFACE.
C
C
C PARAMETER MW MUST BE .GE. THE NUMBER OF SURFACE DATA CARDS.
C PARAMETER MAXI MUST BE .GE. THE NUMBER OF XP'S USED IN THE
C CALCULATION.
C PARAMETER MAXJ MUST BE .GE. THE NUMBER OF SCATTERING POINTS USED IN
C THE Y DIRECTION.
C PARAMETER MNT IS THE MAXIMUM NUMBER OF OBSERVATION ANGLES.
C PARAMETER MS IS THE MAXIMUM NUMBER OF SURFACE POSITIONS.
C THESE PARAMETERS MUST BE THE SAME IN ALL SUBROUTINES.
C
      PARAMETER MW=600
      PARAMETER MAXI=250,MAXJ=100
      PARAMETER MNT=14,MS=25
C
```

```
      C
            COMMON/INTEG1/NYP,II,NXP,DXP,DYP,TWOPI
            COMMON/INTEG2/TRE(MNT,MAXJ),TIM(MNT,MAXJ),QTRE(MNT,MAXJ)
            COMMON/INTEG3/QTIM(MNT,MAXJ),RE(MNT,MAXI),RIM(MNT,MAXI),QRE(MNT,
           1MAXI),QRIM(MNT,MAXI)
            COMMON/TERMS/PRE(MNT,MS),PIM(MNT,MS)
            COMMON/PERTB/SSIN(360),CCOS(360)
            COMMON NTHETA,NS
            DIMENSION XP(MW),ZETA(MW),ZETAX(MW)
            DIMENSION TERM1(MNT),QTERM1(MNT)
            DIMENSION ZETHZ(MNT,MW),XPMX(MNT),QXPMX(MNT)
            DIMENSION X(MNT),Z(MNT),NY(MW)
            DIMENSION XP2(MW),YP2(MAXJ),THET(MNT)
            DIMENSION RHO(500),PMAG(500),PPHAZ(500),DXMAG(500),DXPHAZ(500)
            DIMENSION DZMAG(500),DZPHAZ(500)
            DIMENSION PP(MS)
            DATA TWOPI/6.2831853/
      C
      C
      C   NUMB IS THE NUMBER OF POSITIONS OF THE SCATTERING SURFACE FOR WHICH
      C   THE H-K INTEGRAL IS EVALUATED.  IT MUST BE ODD.
      C   KSHIFT IS THE NUMBER OF INCREMENTS (DXP) THAT THE SURFACE IS SHIFTED.
      C   SET KSHIFT AND NUMB SO THAT KSHIFT*(NUMB-1)*DXP IS AS CLOSE AS
      C   POSSIBLE TO WLNGTH.
      C   SET IROUGH = 0 IF THE CALCULATION IS TO BE MADE FOR A SMOOTH SURFACE.
      C   THE DATA CARDS FROM TRANSLATE ARE READ EVEN IF IROUGH = 0.
      C   XPMAX IS THE MAXIMUM VALUE OF XP AND YP ACTUALLY USED IN THE
      C   CALCULATION.
      C   DYP IS THE SPACING OF SCATTERING POINTS IN THE Y DIRECTION.
      C   THETA IS THE OBSERVATION ANGLE WRT VERTICAL.
      C   NTHETA IS THE NUMBER OF THETAS.  IT MUST BE .LE. MNT.
      C   RADIUS IS THE DISTANCE FROM THE CENTER OF THE COORDINATE SYSTEM TO
      C   THE OBSERVATION POINT.
      C   NW IS THE NUMBER OF WAVE DATA READ IN.
      C   DXP IS THE SPACING OF THE WAVE DATA.
      C   WLNGTH IS THE WAVELENGTH OF THE WATER WAVES.
      C   ZETA IS THE VERTICAL DISPLACEMENT OF THE SCATTERING SURFACE.
      C   ZETAX IS THE X DERIVATIVE OF ZETA.
      C   SEE PROGRAM PGEN2 FOR DEFINITIONS OF RHO, PMAG, PPHAZ, DZMAG, ETC.
      C
      C
      C   INPUT CARDS.          ******************************
      C
            READ 1000, NUMB,KSHIFT,IROUGH
       1000 FORMAT(3I10)
            READ 1020, XPMAX,DYP
       1020 FORMAT(2F10.0)
            READ 1030, NTHETA,RADIUS
            READ 1025, (THET(NT), NT=1,NTHETA)
       1025 FORMAT(5F10.0)
      C
      C   DATA FROM PROGRAM TRANSLATE.
      C
            READ 1030, NW,DXP,WLNGTH
       1030 FORMAT(I10,3F10.0)
            DO 20 I=1,NW
            READ 1040, KARD,ZETA(I),ZETAX(I)
```

```
 1040 FORMAT(I10,2E12.5)
      IF(KARD .NE. 1) GO TO 510
   20 CONTINUE
C
      IF(IROUGH .NE. 0) GO TO 80
      DO 40 I=1,NW
      ZETA(I)=0.
   40 ZETAX(I)=0.
C
C  DATA FROM PROGRAM PGEN2.
C
   80 READ 1050, NRHO,DRHO,WL,Z0
 1050 FORMAT(I5,3F10.4)
      DO 100 I=1,NRHO
      READ 1060, KARD,RHO(I),PMAG(I),PPHAZ(I),DXMAG(I),DXPHAZ(I),DZMAG(I
     1),DZPHAZ(I)
 1060 FORMAT(I5,F10.0,E13.0,F8.0,E13.0,F8.0,E13.0,F8.0)
      IF(KARD .NE. I) GO TO 520
  100 CONTINUE
C
C                             ******************************
C
      REAL K
      K=TWOPI/WL
C
      PRINT 1070, XPMAX
 1070 FORMAT('1THE RADIUS OF THE SCATTERING AREA IS NOMINALLY XPMAX =',
     1F5.1,' CM.')
      PRINT 1080, RADIUS
 1080 FORMAT(//' THE OBSERVATION POINTS LIE ON AN ARC OF RADIUS',F6.2,'
     1CM.')
      PRINT 1090, NW,DXP,WLNGTH
 1090 FORMAT(//' THE NUMBER OF SURFACE DATA READ IN IS NW =',I5,'.  THEI
     1R SPACING IS DXP =',F6.4,' CM.'/' THE WAVELENGTH OF THE WAVES IS W
     2LNGTH =',F6.3,' CM.')
      PRINT 1100
 1100 FORMAT(//' THE FIRST AND LAST SURFACE DATA CARDS ARE'/7X,'CARD ',4
     1X,'ZETA ',6X,'ZETAX '/)
      IINC=NW-1
      PRINT 1110, (I,ZETA(I),ZETAX(I), I=1,NW,IINC)
 1110 FORMAT(1H ,I10,2E12.5)
      PRINT 1120, WL,Z0
 1120 FORMAT(//' THE ACOUSTIC WAVELENGTH IS WL =',F6.4,' CM.'/' THE TRAN
     1SDUCER IS',F6.2,' CM BELOW THE MEAN WATER SURFACE.')
      PRINT 1130, NRHO,DRHO
 1130 FORMAT(//' THE NUMBER OF PRESSURE DATA READ IN IS NRHO =',I5,'.  T
     1HEIR SPACING IS DRHO =',F7.5,' CM.')
      PRINT 1140
 1140 FORMAT(//' THE FIRST AND LAST PRESSURE DATA CARDS ARE'/7X,'KARD ',
     16X,'RHO ',7X,'PMAG ',6X,'PPHAZ ',6X,'DXMAG ',6X,'DXPHAZ',6X,'DZM
     2AG ',6X,'DZPHAZ'/)
      IINC=NRHO-1
      PRINT 1150, (I,RHO(I),PMAG(I),PPHAZ(I),DXMAG(I),DXPHAZ(I),DZMAG(I)
     1,DZPHAZ(I), I=1,NRHO,IINC)
 1150 FORMAT(1H ,I10,F10.5,E13.6,F12.5,E13.6,F12.5,E13.6,F12.5)
C
C  THE SIGN OF Z0 IS CHANGED BECAUSE THE COORDINATE SYSTEM NOW BEING
```

```
C   USED IS DIFFERENT FROM THAT USED IN PGEN2.
C
       ZO=-ZO
C
       PRINT 1160, DXP,DYP
  1160 FORMAT(//' THE SPACING OF THE SCATTERING POINTS IS DXP =',F6.4,' C
      1M IN THE X DIRECTION, AND DYP =',F6.4,' CM IN THE Y DIRECTION.')
C
C   CONSTRUCT 360-ELEMENT ARRAYS FOR THE SIN AND COS APPROXIMATIONS.
C
       DANGL=TWOPI/360.
       ANGL=-DANGL/2.
       DO 120 J=1,360
       ANGL=ANGL+DANGL
       SSIN(J)=SIN(ANGL)
   120 CCOS(J)=COS(ANGL)
C
C   MXP AND KSHIFT ARE PARAMETERS USED IN MANIPULATING THE SURFACE DATA.
C   NXP IS THE NUMBER OF XP'S IN XPMAX.
C
       NXP=XPMAX/DXP+1.5
       IF(NXP.EQ.(NXP/2)*2) NXP=NXP+1
       MXP=NXP
       MXPO=MXP
       XPMAX=(MXP-1)*DXP
C
   140 PRINT 1190, MXP,NUMB,KSHIFT
  1190 FORMAT(//' MXP =',I4,' SURFACE POINTS ARE INITIALLY ASSIGNED TO TH
      1E +XP DATA SET.'/' THE H-K INTEGRAL WILL BE EVALUATED FOR NUMB =',
      2I3,' SURFACE POSITIONS.'/' FOR EACH SHIFT OF THE SURFACE, MXP WILL
      3 BE INCREASED BY KSHIFT =',I3,'.')
C
C   COMPUTE AND STORE THE X COORDINATES OF THE SCATTERING POINTS,
C   THEIR SQUARES, AND THE SQUARES OF THE Y COORDINATES.
C   ALSO COMPUTE, STORE, AND PRINT THE NY'S, WHERE NY(I) IS THE NUMBER OF
C   SCATTERING POINTS IN THE ITH ROW.  THEY MUST ALL BE ODD.
C
       DO 150 J=1,MAXJ
   150 YP2(J)=((J-1)*DYP)**2
       DO 155 I=1,MXP
       XP(I)=(I-1)*DXP
       XP2(I)=XP(I)**2
       MY=SQRT(XPMAX**2-XP2(I))/DYP+1.5
       IF(MY .EQ. 1) MY=3
       IF(MY .EQ. (MY/2)*2) MY=MY+1
       IF(XP2(I)+((MY-1)*DYP)**2 .GT. RHO(NRHO)**2) GO TO 500
   155 NY(I)=MY
       PRINT 1198
  1198 FORMAT(//' THIS LIST IS THE ARRAY NY(I).'/)
       PRINT 1199, (NY(I), I=1,MXP)
  1199 FORMAT(1H ,20I6)
C
C   COMPUTE THE COORDINATES OF THE OBSERVATION POINTS, AND THE ZETMZ'S.
C   ZETMZ MEANS 'ZETA MINUS Z'.
C
   156 DO 158 NT=1,NTHETA
       THETA=THET(NT)/57.295
```

```
      X(NT)=RADIUS*SIN(THETA)
      Z(NT)=-RADIUS*COS(THETA)
      DO 158 I=1,NW
  158 ZETMZ(NT,I)=ZETA(I)-Z(NT)
      XP(1)=.00001
C
C  A LOOP ON SURFACE POSITION BEGINS HERE.
C  THE SURFACE IS SHIFTED BY RETURNING TO STATEMENT 160.  THE INCREMENT
C  KSHIFT IS ADDED TO MXP, AND THEN THE INDICES OF ZETA AND ZETAX ARE
C  CHANGED BY THAT AMOUNT.
C
      NS=1
      GO TO 170
  160 NS=NS+1
      MXP=MXPO+(NS-1)*KSHIFT
C
C  ESTABLISH THE SURFACE COORDINATE XP.  THAT IS, BEGIN A LOOP ON THE
C  ROWS OF SCATTERING POINTS.  ALSO, COMPUTE SOME TERMS THAT ARE
C  INDEPENDENT OF THE COORDINATE YP.
C  II IS THE INDEX OF A ROW.
C  NNXP IS THE +XP SURFACE DATA INDEX FOR THAT ROW.
C  NNXQ IS THE CORRESPONDING -XP INDEX.
C
  170 DO 290 II=1,NXP
      NYP=NY(II)
      NNXP=MXP+1-II
      NNXQ=MXP-1+II
      DO 220 NT=1,NTHETA
      XPMX(NT)=XP(II)-X(NT)
      QXPMX(NT)=-XP(II)-X(NT)
      TERM1(NT)=ZETAX(NNXP)*XPMX(NT)-ZETMZ(NT,NNXP)
  220 QTERM1(NT)=ZETAX(NNXQ)*QXPMX(NT)-ZETMZ(NT,NNXQ)
C
C  LOOP ON THE YP'S IN THE ROW.
C  JJ IS THE INDEX OF A POINT IN THE ROW.
C
      DO 280 JJ=1,NYP
C
C  THE RADIAL DISTANCE FROM THE ORIGIN TO THE PROJECTION OF THE
C  SCATTERING POINT ONTO THE Z=0 PLANE IS RH.  THE INCIDENT PRESSURE IS
C  SPECIFIED AT A SET OF RADII, RHO(I).
C  FIND THE RHO CLOSEST TO THE CURRENT SCATTERING POINT.
C  LET I BE ITS INDEX.
C
      RH2=XP2(II)+YP2(JJ)
      RH=SQRT(RH2)+.00001
      R1=SQRT(RH2+(ZETA(NNXP)-ZO)**2)
      QR1=SQRT(RH2+(ZETA(NNXQ)-ZO)**2)
      I=RH/DRHO+1.5
      PGENR=SQRT(RHO(I)**2+ZO**2)
      DF=K*(R1-PGENR)
      QDF=K*(QR1-PGENR)
C
C  OBTAIN THE INCIDENT PRESSURE AT THE SCATTERING POINTS BY PERTURBING
C  THE PHASES OF THE SMOOTH-SURFACE VALUES SUPPLIED BY PGEN2.
C  SUBROUTINE PRTB PERFORMS THIS FUNCTION.
C
```

```
      P=PMAG(I)
      PZ=DZMAG(I)
      PX=DXMAG(I)
      F=PPHAZ(I)
      FZ=DZPHAZ(I)
      FX=DXPHAZ(I)
      CALL PRTB(P,F,DF,PREP,PIMP)
      CALL PRTB(P,F,QDF,QPREP,QPIMP)
      CALL PRTB(PZ,FZ,DF,DZREP,DZIMP)
      CALL PRTB(PZ,FZ,QDF,QDZREP,QDZIMP)
      CALL PRTB(PX,FX,DF,DXREP,DXIMP)
      CALL PRTB(PX,FX,QDF,QDXREP,QDXIMP)
C
C  TERM IS THE SINE OF THE ANGLE BETWEEN RH AND THE Y AXIS. IT IS
C  NEEDED TO COMPUTE THE PARTIAL OF P WRT X.
C
      TERM=XP(II)/RH
C
      DXREP=TERM*DXREP
      QDXREP=-TERM*QDXREP
      DXIMP=TERM*DXIMP
      QDXIMP=-TERM*QDXIMP
C
C  COMPUTE INTEGRAND TERMS WHICH DON'T INVOLVE THETA.
C
      TERM4=ZETAX(NNXP)*DXREP-DZREP
      QTERM4=ZETAX(NNXQ)*QDXREP-QDZREP
      TERM5=ZETAX(NNXP)*DXIMP-DZIMP
      QTERM5=ZETAX(NNXQ)*QDXIMP-QDZIMP
C
C  LOOP ON THE VALUES OF THETA (OBSERVATION POINTS).
C
      DO 280 NT=1,NTHETA
C  R2 MEANS R-SQUARED.
      R2=XPMX(NT)**2+YP2(JJ)+ZETMZ(NT,NNXP)**2
      QR2=QXPMX(NT)**2+YP2(JJ)+ZETMZ(NT,NNXQ)**2
      R=SQRT(R2)
      QR=SQRT(QR2)
      TERM2=PREP/R2+K*PIMP/R
      QTERM2=QPREP/QR2+K*QPIMP/QR
      TERM3=K*PREP/R-PIMP/R2
      QTERM3=K*QPREP/QR-QPIMP/QR2
      T1=TERM1(NT)*TERM2-TERM4
      QT1=QTERM1(NT)*QTERM2-QTERM4
      T2=TERM1(NT)*TERM3+TERM5
      QT2=QTERM1(NT)*QTERM3+QTERM5
C
C  FIND THE PHASES OF THE SCATTERED RAYS.
C  REMOVE INTEGER MULTIPLES OF TWOPI FROM K*R, THEN APPROXIMATE SIN AND
C  COS BY SSIN AND CCOS.
C  THE FUNCTION AINT REMOVES THE FRACTIONAL PART OF G.
C
      G=R/WL
      QG=QR/WL
      NG=(G-AINT(G))*360.+1.
      NQG=(QG-AINT(QG))*360.+1.
      S=SSIN(NG)
```

```
      QS=SSIN(NQG)
      C=CCOS(NG)
      QC=CCOS(NQG)
C
C  THE FOLLOWING TERMS ARE VALUES OF THE INTEGRAND AT THE SCATTERING
C  POINTS.
C
      TRE(NT,JJ)=(T1*C+T2*S)/R
      QTRE(NT,JJ)=(QT1*QC+QT2*QS)/QR
      TIM(NT,JJ)=(T1*S-T2*C)/R
  280 QTIM(NT,JJ)=(QT1*QS-QT2*QC)/QR
C
C  INTEGRATE WRT YP.  THAT IS , FIND THE CONTRIBUTION OF THE ROWS AT
C  + AND - XP.
C
  290 CALL INTGY2
C
C  INTEGRATE WRT XP.  THAT IS, INTEGRATE OVER ALL THE ROWS TO OBTAIN THE
C  SCATTERED PRESSURE FOR THE CURRENT SURFACE POSITION.
C
      CALL INTGX2
C
C  ALL TERMS HAVE BEEN COMPUTED FOR THIS SURFACE POSITION.
C  EITHER SHIFT THE SURFACE AND REPEAT THE CALCULATION, OR STOP.
C
      IF(NS .EQ. NUMB) GO TO 400
      GO TO 160
C
C  FOR EACH RECEIVER ANGLE FIND THE MAGNITUDE OF THE SCATTERED PRESSURE,
C  THE PHASE, AND THE MAGNITUDE-SQUARED, P2.  PRINT THEM.  THEN AVERAGE
C  THE P2'S AND TAKE THE SQUARE ROOT TO GET PRMS.
C
  400 DO 460 NT=1,NTHETA
      PRINT 1200, THET(NT)
 1200 FORMAT('1**********************************'/'      THETA =',F5.1,'
     1DEGREES,'/'**********************************')
      PRINT 1290
 1290 FORMAT(/25X,'   NS',9X,'PRE   ',9X,'PIM   ',8X,'PMAGN ',5X,'PHASE',
     18X,'PP'/)
      DO 440 NS=1,NUMB
      A=PRE(NT,NS)
      B=PIM(NT,NS)
      P2=A*A+B*B
      PMAGN=SQRT(P2)
      PP(NS)=P2
      CALL ANGLE(A,B,PHASE)
  440 PRINT 1300, NS,A,B,PMAGN,PHASE,P2
 1300 FORMAT(27X,I3,3F14.9,F10.6,F14.9)
      IF(NUMB .LT. 3) GO TO 460
C
C  FIND PRMS AND PRINT IT.
C
      H=1./(NUMB-1)
      CALL INTEG1(PP,NUMB,H,PMS )
      PRINT 1310, PRMS                  <-- PRMS = SQRT(PMS)
 1310 FORMAT(82X,F14.9,' = PRMS')
  460 CONTINUE
```

```
      STOP
C
  500 PRINT 1320
 1320 FORMAT(///' INSUFFICIENT DATA FROM PGEN2.  QUIT.  REDUCE XPMAX.')
      STOP
  510 PRINT 1330
 1330 FORMAT(///' SURFACE DATA CARDS NOT IN ORDER.  QUIT.')
      STOP
  520 PRINT 1340
 1340 FORMAT(///' PRESSURE DATA CARDS NOT IN ORDER.  QUIT.')
      STOP
      END
C
END OF COMPILATION:        NO  DIAGNOSTICS.
```

SUB1

```
C
      SUBROUTINE  INTG2
C
C  THIS SUBROUTINE USES SIMPSON'S RULE TO INTEGRATE THE SCATTERING
C  TERMS CORRESPONDING TO ROWS (ENTRY INTGY2) AND STRIPS (ENTRY INTGX2).
C
      PARAMETER MNT=14,MS=25
      PARAMETER MAXI=250,MAXJ=100
      COMMON/INTEG1/NYP,II,NXP,DXP,DYP,TWOPI
      COMMON/INTEG2/TRE(MNT,MAXJ),TIM(MNT,MAXJ),QTRE(MNT,MAXJ)
      COMMON/INTEG3/QTIM(MNT,MAXJ),RE(MNT,MAXI),RIM(MNT,MAXI),QRE(MNT,
     1MAXI),QRIM(MNT,MAXI)
      COMMON/TERMS/PRE(MNT,MS),PIM(MNT,MS)
      COMMON NTHETA,NS
C
      ENTRY INTGY2
      IMAX=NYP-3
      W=DYP/3.
C
      DO 200 NT=1,NTHETA
      F1=TRE(NT,1)+TRE(NT,NYP)
      G1=TIM(NT,1)+TIM(NT,NYP)
      QF1=QTRE(NT,1)+QTRE(NT,NYP)
      QG1=QTIM(NT,1)+QTIM(NT,NYP)
      F2=TRE(NT,NYP-1)
      G2=TIM(NT,NYP-1)
      QF2=QTRE(NT,NYP-1)
      QG2=QTIM(NT,NYP-1)
      F3=0.
      G3=0.
      QF3=0.
      QG3=0.
C
      DO 100 I=2,IMAX,2
      F2=F2+TRE(NT,I)
      G2=G2+TIM(NT,I)
      QF2=QF2+QTRE(NT,I)
      QG2=QG2+QTIM(NT,I)
      F3=F3+TRE(NT,I+1)
      G3=G3+TIM(NT,I+1)
      QF3=QF3+QTRE(NT,I+1)
  100 QG3=QG3+QTIM(NT,I+1)
       RE(NT,II)=W*(F1+4.*F2+2.*F3)
      RIM(NT,II)=W*(G1+4.*G2+2.*G3)
      QRE(NT,II)=W*(QF1+4.*QF2+2.*QF3)
  200 QRIM(NT,II)=W*(QG1+4.*QG2+2.*QG3)
      RETURN
C
      ENTRY INTGX2
      IMAX=NXP-3
      W=DXP/3.
C
      DO 400 NT=1,NTHETA
      F1=RE(NT,1)+RE(NT,NXP)
      G1=RIM(NT,1)+RIM(NT,NXP)
```

```
            QF1=QRE(NT,1)+QRE(NT,NXP)
            QG1=QRIM(NT,1)+QRIM(NT,NXP)
            F2=RE(NT,NXP-1)
            G2=RIM(NT,NXP-1)
            QF2=QRE(NT,NXP-1)
            QG2=QRIM(NT,NXP-1)
            F3=0.
            G3=0.
            QF3=0.
            QG3=0.
      C
            DO 300 I=2,IMAX,2
            F2=F2+RE(NT,I)
            G2=G2+RIM(NT,I)
            QF2=QF2+QRE(NT,I)
            QG2=QG2+QRIM(NT,I)
            F3=F3+RE(NT,I+1)
            G3=G3+RIM(NT,I+1)
            QF3=QF3+QRE(NT,I+1)
        300 QG3=QG3+QRIM(NT,I+1)
            PPRE=F1+4.*F2+2.*F3
            PPIM=G1+4.*G2+2.*G3
      C
      C  IN THE NEXT TWO STATEMENTS, THE Q PRESSURE COMPONENTS ARE ADDED TO
      C  THE +XP TERMS.
      C
            PRE(NT,NS)=(PPRE+QF1+4.*QF2+2.*QF3)*(W/TWOPI)
        400 PIM(NT,NS)=(PPIM+QG1+4.*QG2+2.*QG3)*(W/TWOPI)
            RETURN
            END
      C
END OF COMPILATION:        NO  DIAGNOSTICS.
```

SUB2

```
C
      SUBROUTINE ANGLE(X,Y,A)
C
C  THIS IS A SUBROUTINE TO COMPUTE THE PHASE ANGLE OF A COMPLEX NUMBER.
C
      PI=3.14159265
      IF(ABS(X).GT.1.E-10 .AND. ABS(Y).GT.1.E-10) GO TO 5
C  IF BOTH X AND Y ARE ZERO, A WILL BE SET = 0.
      IF(ABS(X).LT.1.E-10 .AND. ABS(Y).LT.1.E-10) GO TO 130
      IF(ABS(X).LT.1.E-10 .AND. Y.LT.0.) GO TO 100
      IF(ABS(X).LT.1.E-10 .AND. Y.GT.0.) GO TO 110
      IF(ABS(Y).LT.1.E-10 .AND. X.LT.0.) GO TO 120
      IF(ABS(Y).LT.1.E-10 .AND. X.GT.0.) GO TO 130
    5 A=ATAN(Y/X)
      IF(A) 10,10,50
   10 IF(X) 20,20,30
   20 A=A+PI
      GO TO 200
   30 A=A+2.*PI
      GO TO 200
   50 IF(X) 20,20,200
  100 A=1.5*PI
      GO TO 200
  110 A=.5*PI
      GO TO 200
  120 A=PI
      GO TO 200
  130 A=0.
  200 RETURN
      END
C
```

END OF COMPILATION: NO DIAGNOSTICS.

SUB3

```
C
      SUBROUTINE INTEGI(F,N,D,V)
C
C THIS IS A SUBROUTINE TO INTEGRATE A FUNCTION, F, VIA SIMPSON'S RULE.
C
      DIMENSION F(N)
      F1=F(1)+F(N)
      F2=F(N-1)
      F3=0.
      IMAX=N-3
      DO 10 I=2,IMAX,2
      F2=F2+F(I)
   10 F3=F3+F(I+1)
      V=(D/3.)*(F1+4.*F2+2.*F3)
      RETURN
      END
C
```

END OF COMPILATION: NO DIAGNOSTICS.

SUB4

```
C
      SUBROUTINE PRTB(P,F,DF,PREAL,PIMAG)
C
C THIS IS A SUBROUTINE TO PERTURB THE PHASES OF THE INCIDENT PRESSURE.
C
      COMMON/PERTB/SSIN(360),CCOS(360)
      DATA TWOPI/6.2831853/
C
      G=(F+DF)/TWOPI+10.
C
C THE SUM F+DF CAN BE NEGATIVE.  THEREFORE, 10 WAVELENGTHS ARE ADDED TO
C ENSURE A POSITIVE VALUE FOR THE ARRAY INDEX, NG.
C
      NG=(G-AINT(G))*360.+1.
      PREAL=P*CCOS(NG)
      PIMAG=P*SSIN(NG)
      RETURN
      END
C
```

END OF COMPILATION: NO DIAGNOSTICS.

```
@XQT
MAP 017K-05/26-20:52

ADDRESS LIMITS     001000 014447      040000 154401
STARTING ADDRESS   012735

WORDS DECIMAL          5928 IBANK        39170 DBANK

                 SEGMENT MAIN             001000 014447     040000 154401

 NTABS/FORIO                                            2   040000 040044
 UWERRS/NAGFORFUN02              1   001000 001610      2   040045 040504
 NFFTIS/FORIO                    1   001611 002667      2   040505 040736
 NFFTOS/FORIO                    1   002670 003256      2   040737 041000
 ERUS
 SQRTS/NAGFORFUN10               1   003257 003326      2   041001 041012
 SINCOSS/NAGFORFUN10             1   003327 003474      2   041013 041042
 FORIOS2/FORIO                   1   003475 006167      2   041043 043547
 NISYMS/FORIO                    1   006170 006267      2   043550 043554
 ATANS/NAGFORFUN10               1   006270 006530      2   043555 043614
 FORIOS1/FORIO                   1   006531 011730      2   043615 044703
 TERMS (COMMON BLOCK)                                       044704 046177
 INTEG3 (COMMON BLOCK)                                      046200 104247
 INTEG2 (COMMON BLOCK)                                      104250 114417
 INTEG1 (COMMON BLOCK)                                      114420 114425
 BLANK$COMMON (COMMON BLOCK)                                114426 114427
 PERTB (COMMON BLOCK)                                       114430 115747
 SUB4                            1   011731 011772      0   115750 115764
                                                        2   PERTB
 SUB3                            1   011773 012072      0   115765 116012
 SUB2                            1   012073 012330      0   116013 116030
 SUB1                            1   012331 012734      0   116031 116107
                                 3   INTEG1             2   BLANK$COMMON
                                 5   INTEG3             4   INTEG2
                                                        6   TERMS
 MAIN                            1   012735 014447      0   116110 154401
                                 3   INTEG1             2   BLANK$COMMON
                                 5   INTEG3             4   INTEG2
                                 7   PERTB              6   TERMS

 SYS$*RLIB$. LEVEL 36
 END OF COLLECTION - TIME 1.175 SECONDS
```

SAMPLE OUTPUT

```
THE RADIUS OF THE SCATTERING AREA IS NOMINALLY XPMAX = 17.0 CM.

THE OBSERVATION POINTS LIE ON AN ARC OF RADIUS 49.20 CM.

THE NUMBER OF SURFACE DATA READ IN IS NW =  600.  THEIR SPACING IS DXP = .0977 CM.
THE WAVELENGTH OF THE WAVES IS WLNGTH = 8.800 CM.

THE FIRST AND LAST SURFACE DATA CARDS ARE
     CARD      ZETA         ZETAX

        1    .21983+00   -.14639+00
      600   -.28540+00   -.33782-01

THE ACOUSTIC WAVELENGTH IS WL = .7198 CM.
THE TRANSDUCER IS 50.10 CM BELOW THE MEAN WATER SURFACE.

THE NUMBER OF PRESSURE DATA READ IN IS NRHO =  401.  THEIR SPACING IS DRHO = .05000 CM.

THE FIRST AND LAST PRESSURE DATA CARDS ARE
     KARD      RHO         PHAG         PPHAZ       DXMAG          DXPHAZ     DZMAG         DZPHAZ

        1     .00000    .100000+01    3.78667    .000000           .00000    .872907+01    5.35746
      401   20.00000    .170000-01    2.78814    .550442-01       4.28620    .137818+00    4.35893

THE SPACING OF THE SCATTERING POINTS IS DXP = .0977 CM IN THE X DIRECTION, AND DYP = .2000 CM IN THE Y DIRECTION.

MXP = 175 SURFACE POINTS ARE INITIALLY ASSIGNED TO THE +XP DATA SET.
THE H-K INTEGRAL WILL BE EVALUATED FOR NUMB = 23 SURFACE POSITIONS.
FOR EACH SHIFT OF THE SURFACE, MXP WILL BE INCREASED BY KSHIFT =  4.

THIS LIST IS THE ARRAY NY(I).

   87   87   87   87   87   87   87   87   87   87   87   87   87   87   87   87   87   87   87   87
   85   85   85   85   85   85   85   85   85   85   85   85   85   85   85   85   85   85   85   85
   85   85   85   83   83   83   83   83   83   83   83   83   83   83   83   83   83   81   81   81
   81   81   81   81   81   81   81   79   79   79   79   79   79   79   79   79   79   77   77   77
   77   77   77   77   75   75   75   75   75   75   75   73   73   73   73   73   73   73   71   71
   71   71   71   71   69   69   69   69   69   67   67   67   67   67   65   65   65   65   63   63
   63   63   63   61   61   61   61   59   59   59   59   57   57   57   55   55   55   53   53   53
   51   51   51   49   49   49   47   47   47   45   45   43   43   43   41   41   39   39   37   37
   35   33   33   31   29   29   27   25   23   21   19   17   15   11    3
```

•••••••••••••••••••••••••••••••••

THETA = .0 DEGREES.

•••••••••••••••••••••••••••••••••

NS	PRE	PIM	PMAGN	PHASE	PP*
1	.075735444	.074120705	.105970451	.774623	.011229736
2	.105919360	.052027809	.118007642	.456585	.013925804
3	.159945803	.088086168	.182597460	.503400	.033341832
4	.275549386	.230057718	.358962417	.695665	.128854016
5	.368657842	.309901230	.481609151	.699023	.231947375
6	.322326325	.214764187	.387321461	.587745	.150017913
7	.168334100	.062726230	.198505197	.321503	.039404313
8	.109896071	.010424079	.110389346	.094571	.012185808
9	.112101615	.028317080	.115622789	.247426	.013368629
10	.109484127	.000476201	.109485161	.004349	.011987001
11	.055204276	-.066643352	.086538132	5.404181	.007488848
12	-.031997426	-.078430170	.084706120	4.325028	.007175127
13	-.120596207	-.048163686	.129858328	3.521564	.016863185
14	-.184807168	-.079950394	.201359764	3.549896	.040545755
15	-.202438163	-.190323496	.277856514	3.896156	.077204242
16	-.182291122	-.277034733	.331629757	4.130405	.109978296
17	-.167170521	-.270591632	.318065736	4.158987	.101165812
18	-.171099192	-.192812670	.257782191	3.986587	.066451658
19	-.145663049	-.114758985	.185438260	3.808874	.034387348
20	-.066103105	-.108449982	.127007948	4.164994	.016131019
21	.008301147	-.128079873	.128348598	4.777111	.016473363
22	.039959343	-.060497470	.072503055	5.296119	.005256693
23	.065415917	.047663454	.080938539	.629678	.006551047
					.227339 = PRMS

@FIN

NUMB = 23
NTHETA = 7

RUNID: 6C0643 PROJECT: USER:

ITEM	AMOUNT	COST(DOLLARS)
CPU TIME	00:14:44.323	$35.37
I/O REQUESTS	396	$0.28
I/O-WORDS TRANSFERRED	367728	$0.13
CORE USAGE	11.515	$3.60
CARDS IN	1579	$0.47
PAGES PRINTED	16	$0.33
JOB CHARGE	1	$0.12
TOTAL COST		$40.50

THE ABOVE DOLLAR AMOUNTS ARE APPROXIMATE AND ARE BASED ON RATES FOR CONVENIENCE RUNS

INITIATION TIME: 20:51:48-MAY 26,1973

TERMINATION TIME: 21:20:55-MAY 26,1973

Program TRANSLATE -- The essential function of this program is to read a set of digitized values of the surface displacement from magnetic tape, compute approximate surface slopes for a selected subset of the data, and punch cards containing the displacements and slopes for that subset. In the work reported in the text, the surface data were digitized and recorded on tape with the aid of a small Datacraft computer. This machine uses 24-bit words, whereas a Univac 1108 computer, for which this program was written, is a 36-bit machine. Thus, much of the program is concerned with simply reading the tape and translating the data into the proper format. Subroutine RREC is the primary program unit for performing this task.

The digitized data on the tape are positive binary integers with 10 significant bits in the present case. Numbers proportional to the displacement about the mean surface level are obtained by subtracting the number $2^{10}/2$ from each datum. A proportionality factor is then used to convert these numbers to surface displacements in centimeters (cards 234-254).

The 'selected subset' of data referred to above is an N-word block of data which is considered to be representative of the entire data set; that is, it is an 'average' block. It is selected using the criterion that its rms displacement be very close (normally within .5%) to the rms displacement of the entire data set (cards 176-194).

The surface slope, ζ_x, (the x derivative of the displacement,

γ) is computed for each value of γ by means of Equation 9 in the text. Subroutine XDERIV performs this function.

Definitions of terms, including the quantities required on input cards, are given by comment cards within the program.

Subroutine PLDATA was supplied by B. Sternberg of this laboratory.

```
@ASG,TH 177/TAPE PASSWORD.,8C9,REEL ID
@FOR,ASI MAIN
C
C  THIS IS PROGRAM TRANSLATE.  IT READS A MAGNETIC TAPE CONTAINING
C  BLOCKS OF NUMBERS RELATED TO THE DISPLACEMENT OF THE WATER SURFACE.
C  THE NUMBERS ARE CONVERTED TO WAVEHEIGHTS IN CENTIMETERS AND
C  X-DERIVATIVES ARE CALCULATED.  THE DATA IN A SELECTED BLOCK ARE
C  PUNCHED ON CARDS FOR USE IN PROGRAM SCATTER.
C  THIS PROGRAM HAS BEEN WRITTEN FOR THE PARTICULAR CASE WHERE THE DATA
C  ARE RECORDED ON 9-TRACK TAPE, IN 24-BIT WORDS, AND IN N-WORD BLOCKS.
C  THE DATA WORDS ARE CONVERTED FROM 24 BITS TO 36 BITS.
C  THE MEAN IS FOUND AND IS SUBTRACTED FROM EACH DATUM.
C  AN RMS VALUE IS COMPUTED FOR EACH BLOCK.
C  THE MEAN OF THE RMS VALUES IS CALCULATED, AND AN 'AVERAGE' BLOCK
C  HAVING AN RMS VALUE CLOSE TO THE MEAN IS FOUND.  ONLY THE DATA IN
C  THAT BLOCK ARE USED FURTHER.
C  THE NUMBERS IN THAT AVERAGE BLOCK ARE CONVERTED TO CM.
C  X DERIVATIVES ARE CALCULATED.
C  CARDS ARE PUNCHED FOR USE BY PROGRAM SCATTER.
C
C  N IS THE NUMBER OF WORDS IN A DATA BLOCK.
C  FOR THIS PROGRAM, N MUST BE AN INTEGER MULTIPLE OF 3.
C  N CAN BE CHANGED BY CHANGING THE PARAMETER STATEMENTS.
C  NFILES IS THE NUMBER OF TAPE FILES TO BE SKIPPED.
C  NB IS A NUMBER .GE. THE NUMBER OF BLOCKS (OR RECORDS) IN ANY FILE TO
C  BE SKIPPED.
C  SRATE IS THE EFFECTIVE DIGITIZER SAMPLING RATE.
C  EFFECTIVE MEANS (DIGITIZER SAMPLING RATE)*(RECORDER RECORD SPEED/
C  RECORDER PLAYBACK SPEED).
C  SIGN IS SET .LT. 0. IF THE DATA ARE INVERTED ON THE TAPE.
C  PKHT IS THE ASSUMED PEAK HEIGHT OF THE WATER WAVES.
C  SPEED IS THE SPEED OF THE WATER WAVES.
C  WLNGTH IS THE WAVELENGTH OF THE WAVES.
C  NB IS THE NUMBER OF N-WORD BLOCKS IN THE FILE.
C  IB IS THE INDEX OF THE AVERAGE BLOCK.  IT CAN HAVE ANY VALUE FROM
C  1 TO NB.
C  EPS IS THE FRACTIONAL DIFFERENCE BETWEEN THE AVERAGE RMS AND THE RMS
C  OF THE AVERAGE BLOCK.
C  RMSHT IS THE RMS SURFACE DISPLACEMENT IN CM FOR THE DATA IN THE
C  AVERAGE BLOCK.
C  DXP IS THE HORIZONTAL SEPARATION IN CM OF THE SURFACE DATA.
C  ZETA IS THE SURFACE DISPLACEMENT IN CM.
C  ZETAX IS THE APPROXIMATE X DERIVATIVE OF ZETA.
C
C
      PARAMETER N=600
      COMMON/REC/IY(N)
      COMMON/DERIV/DXP,ZETA(N),ZETAX(N)
      DIMENSION LABEL(7),RMS(1000)
      DIMENSION TEMP(N),DUM(N)
      DATA DUM/N*0./
      REAL MEAN(1000)
      NN=N
C
```

```
C  INPUT CARDS.                                   ********************
C
      READ 10, NFILES,MB
   10 FORMAT(2I10)
      READ 20, SRATE
      READ 20, SIGN
      READ 20,PKHT,SPEED,WLNGTH
   20 FORMAT(3F10.0)
C
C                                                 ********************
C
      PRINT 30, NFILES
   30 FORMAT(1H1,' NFILES =',I2,' TAPE FILES HAVE BEEN SKIPPED.')
      PRINT 40, SRATE SIGN
   40 FORMAT(1H0,'THE EFFECTIVE SAMPLING RATE IS',F6.1,' SAMPLES PER SEC
     1OND.'/' SIGN =',F4.0//)
      PRINT 50, PKHT
   50 FORMAT(' THE PEAK HT OF THE AVERAGE WAVE IS',F6.3,' CM.')
      PRINT 60, SPEED
   60 FORMAT(' THE SPEED OF THE WAVES IS',F6.2,' CM PER SECOND.')
      PRINT 70, WLNGTH
   70 FORMAT(' THE WAVELENGTH OF THE WATER WAVES IS',F6.3,' CM.')
C
C  SKIP FILES THAT ARE NOT OF INTEREST.  LABELS ARE FILES.
C
      IF(NFILES .NE. 0) CALL SKIPF(NFILES,MB)
C
C  READ AND PRINT THE TAPE LABEL.
C  IOTPIN IS AN MACC 1108 LIBRARY SUBROUTINE TO READ A DATA BLOCK FROM
C  MAGNETIC TAPE.  SEE THE REFERENCE MANUAL FOR THE 1108 COMPUTER.
C  IN THIS CASE THE LABEL IS STRICTLY NUMERIC AND IS RECORDED AS 24-BIT
C  WORDS.  FLD MANIPULATES THE BITS TO READ THE WORDS IN A 36-BIT (1108)
C  FORMAT.
C
      CALL IOTPIN(17,1,IY,N,LNZ,$80)
   80 LABEL(1)=FLD(0,24,IY(1))
      LABEL(2)=FLD(0,12,IY(2))
      LABEL(3)=FLD(12,24,IY(2))
      LABEL(4)=FLD(0,24,IY(3))
      LABEL(5)=FLD(0,12,IY(4))
      LABEL(6)=FLD(12,24,IY(4))
      LABEL(7)=FLD(0,24,IY(5))
      PRINT 90,(LABEL(J), J=1,7)
   90 FORMAT(///' TAPE NUMBER',I6,10X,'DATA TAKEN ON ',I2,'/',I2,'/',I2/
     128X,'DATA DIGITIZED ON ',I2,'/',I2,'/',I2)
C
C  AN EOF SHOULD FOLLOW THE LABEL.
C  FIND THE LABEL EOF.
C  SUBROUTINE IOTPSP SPACES LOGICAL UNIT 17 (HERE) PAST THE NEXT DATA
C  BLOCK.  IT TRANSFERS CONTROL TO STATEMENT 100 IF AN EOF IS FOUND.
C
      CALL IOTPSP(17,$100)
      PRINT 95
   95 FORMAT(//' ***** LABEL EOF NOT FOUND. *****')
```

```
C
C  BEGIN THE ANALYSIS OF THE DATA.
C  READ THE DATA BLOCK BY BLOCK.  FOR EACH BLOCK COMPUTE THE RMS AND THE
C  MEAN.  ALSO COMPUTE THE AVERAGE RMS AND THE AVERAGE MEAN.  FINALLY,
C  COMPUTE THE STANDARD DEVIATIONS OF THE RMS'S AND MEANS.
C  BECAUSE THE WAVES ARE APPROX PERIODIC, USE ONLY THAT NUMBER OF DATA
C  (NS) WHICH CORRESPOND TO AN INTEGER NUMBER OF WAVELENGTHS.
C  SUBROUTINE RREC CONVERTS 24-BIT WORDS TO 36-BIT WORDS.  CONTROL GOES
C  TO STATEMENT 180 IF AN EOF IS FOUND, AND TO 160 IF A TAPE ERROR IS
C  FOUND.
C
      DXP=SPEED/SRATE
      SPWL=WLNGTH/DXP
      T=N-1
      WLPBL=AINT(T*DXP/WLNGTH)
      NS=SPWL*WLPBL+1.
      J=0
      K=0
      SUM2=0.
      SUM3=0.
      SUM4=0.
      SUM5=0.
  100 SUM=0.
      SUM1=0.
      J=J+1
      CALL RREC(NW,$180,$160)
      K=K+1
      DO 120 I=1,NS
      SUM=SUM+FLOAT(IY(I))
  120 SUM1=SUM1+FLOAT(IY(I))**2
      F=NS
      MEAN(J)=SUM/F
      RMS(J)=SQRT(SUM1/F-MEAN(J)**2)
      SUM2=SUM2+MEAN(J)
      SUM3=SUM3+MEAN(J)**2
      SUM4=SUM4+RMS(J)
      SUM5=SUM5+RMS(J)**2
      GO TO 100
  160 PRINT 170, J
  170 FORMAT(//' ***** TAPE ERROR IN BLOCK NO.',I4,' *****'//)
      MEAN(J)=0.
      RMS(J)=0.
      GO TO 100
  180 FK=K
      NB=J-1
      AVGMN=SUM2/FK
      SDMEAN=SQRT((SUM3-FK*AVGMN**2)/(FK-1.))
      AVGRMS=SUM4/FK
      SDRMS=SQRT((SUM5-FK*AVGRMS**2)/(FK-1.))
      PRINT 190, NS,WLPBL
  190 FORMAT(///' THE FIRST',I4,' WORDS IN EACH BLOCK ARE USED TO COMPUT
     1E THE MEAN AND THE RMS.  THESE WORDS CORRESPOND TO',F5.1,' WAVELEN
     2GTHS.')
      PRINT 200, K,AVGMN,SDMEAN
```

```
      200 FORMAT(///' THE AVERAGE OF THE MEANS OF',I4,' ERROR-FREE BLOCKS IS
         1',F10.5/' THE STANDARD DEVIATION OF THE MEANS IS',F10.5)
          PRINT 210, AVGRMS,SDRMS
      210 FORMAT('0THE AVERAGE OF THE RMSES IS',F10.5/' THE STANDARD DEVIATI
         1ON OF THE RMSES IS',F10.5)
          PRINT 220
      220 FORMAT('1THIS IS ARRAY MEAN.'//)
          PRINT 225,(MEAN(J), J=1,NB)
      225 FORMAT(1H ,10F12.5)
          PRINT 240
      240 FORMAT('1THIS IS ARRAY RMS.'//)
          PRINT 225,(RMS(J), J=1,NB)
C
C  FIND A BLOCK CONTAINING DATA WITH AN RMS CLOSE TO AVGRMS.
C
          EPS0=.005
          EPS=EPS0
      260 J=0
      270 J=J+1
          IF(ABS(RMS(J)/AVGRMS-1.) .LT. EPS) GO TO 300
          IF(J-NB) 270,280,280
      280 EPS=EPS+EPS0
          IF(EPS-10.*EPS0) 260,260,330
      300 PRCNT=100.*EPS
          IB=J
          PRINT 320, IB,RMS(IB),PRCNT
      320 FORMAT(//' BLOCK NUMBER',I4,' HAS AN RMS VALUE OF',F7.2,'.  THIS I
         1S WITHIN',F4.2,' PERCENT OF THE MEAN RMS.')
          GO TO 340
      330 PRINT 335
      335 FORMAT(//' ***** CANT FIND AN RMS(J) CLOSE TO AVGRMS, QUIT. ****')
          CALL EXIT
C
C  REWIND THE TAPE AND RETURN TO THE BEGINNING OF THE WAVE DATA.
C
      340 CALL IOTPRW(17)
          NFP1=NFILES+1
          CALL SKIPF(NFP1,MB)
C
C  GO TO BLOCK NUMBER IB, READ IT, AND COMPUTE THE MEAN AND RMS VALUES
C  AGAIN, AS A CHECK.
C
      360 IMAX=IB-1
          DO 370 I=1,IMAX
      370 CALL IOTPSP(17)
          CALL RREC(NW,$480,$440)
          DO 380 I=1,N
      380 TEMP(I)=FLOAT(IY(I))-MEAN(IB)
          SUM=0.
          SUM1=0.
          DO 390 I=1,NS
          SUM=SUM+FLOAT(IY(I))
      390 SUM1=SUM1+FLOAT(IY(I))**2
          F=NS
```

```
            CKMEAN=SUM/F
            CKRMS=SQRT(SUM1/F-CKMEAN**2)
            PRINT 400, CKMEAN,CKRMS
        400 FORMAT(//' CHECK THE MEAN AND RMS OF BLOCK IB.'/10X,'MEAN =',F10.5
           1,10X,'RMS =',F10.5)
            PRINT 420, NW
        420 FORMAT(//' CHECK IF NW=N.  THE NUMBER OF DATA READ INTO ARRAY TEMP
           1 IS',I5,'.')
            GO TO 500
        440 PRINT 445, IB
        445 FORMAT(//' ***** TAPE ERROR IN BLOCK IB =',I4,'.  TRY ANOTHER BLOC
           1K. *****')
            GO TO 270
        480 PRINT 490
        490 FORMAT(//' ***** EOF FOUND BY RREC. QUIT. *****')
            CALL EXIT
      C
      C  FIND THE LARGEST POSITIVE AND NEGATIVE VALUES IN THE AVERAGE BLOCK.
      C  URSRCH IS AN MACC LIBRARY SUBROUTINE TO FIND THE LARGEST OR SMALLEST
      C  ELEMENT IN AN ARRAY.
      C
        500 CALL URSRCH(0,N,TEMP,LP,TMPMAX,0,DUM)
            CALL URSRCH(1,N,TEMP,LN,TMPMIN,0,DUM)
            PRINT 520, LP,TMPMAX
        520 FORMAT('0DATUM NUMBER',I4,' IS THE LARGEST POSITIVE DATUM IN THE A
           1VERAGE BLOCK.  IT HAS THE VALUE',F10.4)
            PRINT 530, LN,TMPMIN
        530 FORMAT('0DATUM NUMBER',I4,' IS THE LARGEST NEGATIVE DATUM IN THE A
           1VERAGE BLOCK.  IT HAS THE VALUE',F10.4)
      C
      C  THE FACTOR CF WILL CONVERT THE NUMBERS IN ARRAY TEMP TO CM.
      C
            TMP=TMPMIN
            IF(SIGN .GT. 0.) TMP=TMPMAX
            CF=PKHT/TMP
            RMSHT=ABS(CF)*RMS(IB)
            DO 550 I=1,N
        550 ZETA(I)=CF*TEMP(I)
      C
            PRINT 580, CF
        580 FORMAT('0THE WAVEHT CONVERSION FACTOR IS CF =',E10.4)
            PRINT 590, RMSHT
        590 FORMAT(' THE RMS WAVEHT OF THE AVERAGE BLOCK IS RMSHT =',F5.3,' CM
           1.')
            PRINT 600, DXP
        600 FORMAT(' THE HORIZONTAL SEPARATION OF DATA ON THE SURFACE IS DXP =
           1',F6.4,' CM.')
            PRINT 620
        620 FORMAT('1THIS IS ARRAY ZETA.'//)
            PRINT 630,(ZETA(I), I=1,N)
        630 FORMAT(1H ,10F12.5)
      C
      C  COMPUTE X DERIVATIVES AND CREATE OUTPUT.
      C
```

```
              CALL XDERIV
              PRINT 650
          650 FORMAT(1H1,8X,'I ',7X,'ZETA ',7X,'ZETAX')
              PRINT 670, (I,ZETA(I),ZETAX(I), I=1,50)
          670 FORMAT(1H ,I10,2F12.5)
              CALL PLDATA(N,DUM,TEMP,1)
              PUNCH 680,NN,DXP,WLNGTH
          680 FORMAT(I10,2F10.5)
              PUNCH 700, (I,ZETA(I),ZETAX(I), I=1,NN)
          700 FORMAT(I10,2E12.5)
              END
        C
        &FOR,SIA SUB1
        C
              SUBROUTINE XDERIV
        C
        C  THIS SUBROUTINE COMPUTES THE APPROXIMATE X DERIVATIVE OF FUNCTION
        C  ZETA.
        C
        C
              PARAMETER N=600
              COMMON/DERIV/DXP,ZETA(N),ZETAX(N)
        C
              IMAX=N-1
              D=-2.*DXP
              ZETAX(1)=2.*(ZETA(2)-ZETA(1))/D
              ZETAX(N)=2.*(ZETA(N)-ZETA(N-1))/D
              DO 20 I=2,IMAX
           20 ZETAX(I)=(ZETA(I+1)-ZETA(I-1))/D
              RETURN
              END
        C
        &FOR,SIA SUB2
        C
              SUBROUTINE RREC(NWORDS,$,$)
        C
        C  THIS IS A SUBROUTINE TO READ 1 RECORD OF TAPE FROM UNIT NO. 17 AND
        C  CONVERT 9-TRACK, 24 BIT WORDS  TO 36 BIT WORDS.
        C
              PARAMETER N=600
              DIMENSION IX(N)
              COMMON/REC/IY(N)
        C
              M=2*N/3
              CALL IOTPIN(17,1,IX,M,LNZ,$20,$30)
        C
        C  GO TO 20 IF AN EOF IS FOUND.
        C  GO TO 30 IF A TAPE ERROR IS FOUND.
        C
              NWORDS=LNZ/3
              IMAX=NWORDS-2
              N84=8**4
              DO 10 I=1,IMAX,3
              J=2*(I+2)/3-1
```

```
      IY(I)=FLD(0,24,IX(J))
      IY(I+1)=FLD(24,12,IX(J))*N84+FLD(0,12,IX(J+1))
   10 IY(I+2)=FLD(12,24,IX(J+1))
      RETURN
   20 RETURN 2
   30 RETURN 3
      END
C
@FOR,SIA SUB3
C
      SUBROUTINE PLDATA(LENT,A,B,NPLINE)
C
C  THIS IS A SUBROUTINE TO PLOT (VIA PRINTER) THE DATA IN THE AVERAGE
C  BLOCK.
C
      PARAMETER NC=18
      INTEGER M(NC)
      IMPLICIT INTEGER (A-Z)
      REAL A(2),B(2)
C
      CALL URPRTM(0,0)
      PRINT 4000
 4000 FORMAT(25X,' THE AVERAGE BLOCK.       UNITS = DIGITIZER BITS'/
     -T6,'-1000',T16,'-800',T26,'-600',T36,'-400',T46,'-200',T58,'0',
     -T67,'200',T77,'400',T87,'600',T97,'800',T107,'1000',T113/
     -5X,10('..+.......'),'..+..')
C
C  CLEAR PRINTER BUFFER.
C
      DO 21 I=1,NC
   21 M(I)=6H
      DO 4 I=1,LENT,NPLINE
      K=0
      CALL COL(0.0,1H.)
      DO 3 J=1,NPLINE
      IJ=I+J-1
      IF(IJ.GT.LENT) GO TO 3
      CALL COL(B(IJ),1H*)
      CALL COL(A(IJ),1H.)
      IB=B(IJ)
      IA=A(IJ)
    3 CONTINUE
      PRINT 4001, I,M,IB,IA
 4001 FORMAT(1X,I4,1X,17A6,A2,2I10)
C
C  CLEAR PLOT BUFFER OF SYMBOLS.
C
      DO 31 K=1,NC
   31 M(K)=6H
    4 CONTINUE
      CALL URPRTM(6,3)
      RETURN
C
      SUBROUTINE COL(VAL,SYM)
```

```
      REAL VAL
      INTEGER SYM,COLV
      COLV=(VAL+1049.)/20.
      COLV=MAX(1,COLV)
      COLV=MIN(104,COLV)
      CALL CHSTOR(SYM,COLV,M)
      RETURN
C
      SUBROUTINE CHSTOR(CHAR,POS,ARRAY)
      DIMENSION ARRAY(2)
      IMPLICIT INTEGER(A-Z)
      WORD=(POS+5)/6
      BIT=ABS(MOD((POS-1)*6,36))
      FLD(BIT,6,ARRAY(WORD))=FLD(0,6,CHAR)
      RETURN
      END
C
@FOR,SIA SUB4
C
      SUBROUTINE SKIPF(NFILES,MB)
C
C  THIS IS A SUBROUTINE TO SKIP TAPE DATA FILES THAT ARE NOT OF
C  CURRENT INTEREST.
C
      DO 60 I=1,NFILES
   20 CALL IOTPSP(17,$60)
      DO 20 J=1,MB
   40 FORMAT(///' NO EOF FOUND BY IOTPSP.  QUIT.')
      PRINT 40
      CALL EXIT
   60 CONTINUE
      RETURN
      END
C
C
@XQT
          1         250             }
   400.                             } DATA CARDS (INPUT)
   -1.                              }
   .32        39.1        8.80      }
@FIN
```

Program PGEN2 -- This program was written specifically for a vertically incident beam with cylindrical symmetry about the beam axis. Its function has already been described. Very briefly, it reads input data, expands the number of data by linear interpolation, computes partial derivatives, and punches cards. The input data are values of the amplitude and phase of the transmitted pressure measured along a radius in a plane at a specified distance from the source. Referring to Figure 5, we see that these quantities are smoothly varying functions of the radius, ρ, except for relatively rapid changes associated with the nulls of the amplitude profile. So the sampling interval is determined primarily by the need to adequately determine the shape of those functions near the nulls.

The first part of the program (cards 58-69) is a linear interpolation procedure applied to both the magnitude and phase data. Uncertainties in the measurement of these quantities is likely to be greater than interpolation errors.

Partial derivatives are computed from Equations 11-13 in the text.

```
@FOR,SIA MAIN
C
C  THIS IS PROGRAM PGEN2.
C  ITS PURPOSE IS TO PROVIDE VALUES OF THE INCIDENT PRESSURE AND ITS
C  DERIVATIVES FOR USE BY PROGRAM SCATTER.  IT REQUIRES MEASURED VALUES
C  OF THE MAGNITUDE AND PHASE OF THE INCIDENT PRESSURE ON A RADIUS IN
C  THE MEAN SCATTERING SURFACE.
C  IN THIS PROGRAM THE RADIAL DISTANCE IS CALLED 'X' FOR BREVITY, BUT
C  IT SHOULD BE UNDERSTOOD THAT, IN TERMS OF PROGRAM SCATTER, IT IS
C  REALLY THE QUANTITY 'RHO'.
C  PGEN2 INTERPOLATES TO INCREASE THE NUMBER OF VALUES, AND IT COMPUTES
C  THE X (RHO) AND Z DERIVATIVES AT EACH VALUE OF X.
C  THE INTERPOLATED MAGNITUDES AND PHASES OF THE INCIDENT PRESSURE AND
C  ITS DERIVATIVES ARE PUNCHED ON CARDS.
C                          **********
C  Z IS THE VERTICAL DISTANCE FROM THE TRANSDUCER SURFACE TO THE PLANE
C  IN WHICH THE MEASUREMENTS WERE MADE.
C  WL IS THE ACOUSTIC WAVELENGTH.
C  NCARDS IS THE NUMBER OF POINTS FOR WHICH  MAGN AND PHASE ARE READ IN.
C  AFTER INTERPOLATION, VALUES FOR NX POINTS ARE PRINTED AND PUNCHED.
C  NINT IS THE  NUMBER OF INTERPOLATION POINTS DESIRED BETWEEN THE INPUT
C  POINTS.
C  DXO IS THE HORIZONTAL SPACING BETWEEN POINTS CORRESPONDING TO THE INPUT
C  DATA.  AFTER INTERPOLATION, THE SPACING IS DX.
C  PO IS THE PRESSURE MAGNITUDE.
C  PHI IS THE PHASE OF THE MEASURED PRESSURE IN MULTIPLES OF TWOPI.
C  THE OUTPUT (PUNCHED AND PRINTED) PHASES ARE IN RADIANS.
C  DXMAG IS THE MAGN OF THE DERIVATIVE OF P WRT X.
C  DXPHAZ IS THE PHASE OF THE DERIVATIVE.
C  SIMILARLY FOR Z.
C
      DIMENSION PO(500),PHI(500),X(500)
      DIMENSION DPODX(500),DPHIDX(500)
      REAL K
C
C  INPUT CARDS.          ******************************
C
      READ 20, Z,DXO,WL
   20 FORMAT(3F10.0)
      READ 25, NCARDS,NINT
   25 FORMAT(2I10)
C
      NX=(NCARDS-1)*NINT+1
C
      READ 30,(PO(I),PHI(I), I=1,NX,NINT)
   30 FORMAT(2F10.0)
C
C                        ******************************
C
      PRINT 35
   35 FORMAT(1H1,'THE INPUT ARRAYS ARE',22X,4HPO  ,8X,3HPHI//)
      PRINT 40,(I,PO(I),PHI(I), I=1,NX,NINT)
   40 FORMAT(30X,I5,2F12.4)
C
```

```
      TWOPI=6.283185
      K=TWOPI/WL
C
C  FILL UP THE PO AND PHI ARRAYS BY INTERPOLATING.
C
      FINT=NINT
      IFIN=NINT-1
      JFIN=NX-NINT
      DO 50 J=1,JFIN,NINT
      DP=(PO(J+NINT)-PO(J))/FINT
      DPHI=(PHI(J+NINT)-PHI(J))/FINT
      DO 50 I=1,IFIN
      FI=I
      PO(J+I)=PO(J)+FI*DP
   50 PHI(J+I)=PHI(J)+FI*DPHI
C
      DX=DXO/FINT
C
      PRINT 52, NX,DX,WL,Z
   52 FORMAT(1H1,'NX =',I5,5X,'DX =',F8.4,5X,'WAVELENGTH =',F8.4,5X,'Z =
     1',F8.4///)
      PUNCH 54, NX,DX,WL,Z
   54 FORMAT(I5,3F10.4)
      PRINT 58
   58 FORMAT(11X,4HX    ,7X,6HPMAG  ,3X,5HPHAZ ,7X,6HDXMAG ,8H DXPHAZ ,7X
     1,6HDZMAG ,8H DZPHAZ //)
C
C  ATTACH VALUES OF X TO THE PRESSURE DATA.
C
      DO 60 I=1,NX
      FI=I
   60 X(I)=(FI-1.)*DX
C
C  COMPUTE PARTIAL DERIVATIVES AND CONVERT PHASES TO RADIANS.
C  THE FUNCTION AINT IS AN 1108 FORTRAN INTRINSIC FUNCTION TO REMOVE
C  THE FRACTIONAL PART OF A REAL NUMBER.
C
      DPHIDX(1)=0.
      DPODX(1)=0.
      IMAX=NX-1
      DO 80 I=2,IMAX
      DPODX(I)=(PO(I+1)-PO(I-1))/(2.*DX)
   80 DPHIDX(I)=(PHI(I+1)-PHI(I-1))*TWOPI/(2.*DX)
      DPODX(NX)=DPODX(IMAX)
      DPHIDX(NX)=DPHIDX(IMAX)
C
      DO 100 I=1,NX
      Q=PHI(I)
      AA=TWOPI*(Q-AINT(Q))
      S=SIN(AA)
      C=COS(AA)
      DPXRE=-PO(I)*DPHIDX(I)*S+DPODX(I)*C
      DPXIM=PO(I)*DPHIDX(I)*C+DPODX(I)*S
      R=SQRT(X(I)**2+Z*Z)
```

```
              DZMAG=K*PO(I)*Z/R
              QD=Q+.25
              DZPHAZ=TWOPI*(QD-AINT(QD))
      C
      C  CONVERT REAL AND IMAG TERMS TO MAGN AND PHASE.
      C
              DXMAG=SQRT(DPXRE**2+DPXIM**2)
              CALL ANGLE(DPXRE,DPXIM,DXPHAZ)
      C
              PRINT 110,I,X(I),PO(I),AA,DXMAG,DXPHAZ,DZMAG,DZPHAZ
          100 PUNCH 110, I,X(I),PO(I),AA,DXMAG,DXPHAZ,DZMAG,DZPHAZ
          110 FORMAT(I5,F10.5,3(E13.6,F8.5))
              END
      C
      @FOR,SIA SUB1
              SUBROUTINE ANGLE(X,Y,A)
              PI=3.14159265
              IF(ABS(X).GT.1.E-10 .AND. ABS(Y).GT.1.E-10) GO TO 5
              IF(ABS(X).LT.1.E-10 .AND. ABS(Y).LT.1.E-10) GO TO 130
              IF(ABS(X).LT.1.E-10 .AND. Y.LT.0.) GO TO 100
              IF(ABS(X).LT.1.E-10 .AND. Y.GT.0.) GO TO 110
              IF(ABS(Y).LT.1.E-10 .AND. X.LT.0.) GO TO 120
              IF(ABS(Y).LT.1.E-10 .AND. X.GT.0.) GO TO 130
            5 A=ATAN(Y/X)
              IF(A) 10,10,50
           10 IF(X) 20,20,30
           20 A=A+PI
              GO TO 200
           30 A=A+2.*PI
              GO TO 200
           50 IF(X) 20,20,200
          100 A=1.5*PI
              GO TO 200
          110 A=.5*PI
              GO TO 200
          120 A=PI
              GO TO 200
          130 A=0.
          200 RETURN
              END
      C
      @XQT
         50.1      .25       .7198
               81        5
         1.        0.
         .997      .001
         .992      .003
         .982      .005
         .965      .007
         .948      .013
         .927      .02
         .904      .024
         .875      .036
         .853      .052
```

00163	.82	.063
00164	.784	.08
00165	.744	.1
00166	.704	.12
00167	.665	.14
00168	.62	.166
00169	.581	.195
00170	.537	.224
00171	.493	.258
00172	.442	.294
00173	.398	.33
00174	.347	.374
00175	.3	.407
00176	.254	.45
00177	.206	.49
00178	.161	.518
00179	.121	.548
00180	.083	.556
00181	.04	.44
00182	.015	.33
00183	.025	.345
00184	.053	.367
00185	.077	.4
00186	.102	.445
00187	.123	.495
00188	.141	.554
00189	.156	.614
00190	.17	.675
00191	.183	.74
00192	.192	.805
00193	.198	.872
00194	.2	.942
00195	.201	1.007
00196	.2	1.082
00197	.199	1.157
00198	.196	1.23
00199	.19	1.3
00200	.185	1.375
00201	.175	1.445
00202	.167	1.525
00203	.157	1.595
00204	.143	1.688
00205	.125	1.78
00206	.112	1.88
00207	.098	1.97
00208	.084	2.055
00209	.075	2.125
00210	.067	2.205
00211	.054	2.295
00212	.044	2.38
00213	.035	2.46
00214	.027	2.554
00215	.02	2.632
00216	.015	2.71
00217	.012	2.775
00218	.011	2.86
00219	.013	2.947
00220	.018	3.06
00221	.02	3.165
00222	.022	3.27
00223	.025	3.35
00224	.026	3.445
00225	.025	3.555
00226	.023	3.64
00227	.021	3.71
00228	.02	3.81
00229	.019	4.
00230	.017	4.24
00231	.017	4.36
00232	.016	4.53
00233	.017	4.72
00234	⊘FIN	

B. Programs based on analytical expressions for acoustic scattering from a sinusoidal surface.

Two programs have been written for the purpose of evaluating Equations 35 and 39 in the text.

Program PSIN1 -- The scattered pressure, p_m, is evaluated for a specified number of positions of the scattering surface. The surface is translated stepwise a total distance of one surface wavelength. Several quantities related to the scattered pressure are computed and printed for each position of the surface (see card 211, statement 200), and all of these quantities, except phases, are averaged over the surface wavelength (cards 215-223). One of the quantities obtained is the rms scattered pressure, p_{rms}. Another is the coherent component of the scattered pressure, $\langle p \rangle$. Values of the percentage slope correction are also computed.

The equations and definitions of terms used in this program are taken directly from Equations 15-38 in the text, and the computational procedure is clearly outlined by comment cards in the program.

Both of the subroutines in this program are useful in a variety of applications. Subroutine INTEG1, used here to compute the averages of the pressure terms, is a simple routine for the numerical integration of a function of one variable.

```
@ASG,MT 11.,T,MATHPK
@COPY,G 11.,TPFS.
@FREE 11.
@FOR,SIA MAIN
C
C  THIS IS PROGRAM PSIN1.
C  IT COMPUTES THE PRESSURE AND THE RMS PRESSURE SCATTERED IN THE X-Z
C  PLANE FROM A CORRUGATED SURFACE.  A SLOPE CORRECTION IS INCLUDED.
C  THE CORRUGATIONS ARE COSINUSOIDAL AND ARE PARALLEL TO THE Y AXIS.
C  A HIGHLY DIRECTIVE TRANSMITTED ACOUSTIC BEAM AND A POINT RECEIVER
C  ARE ASSUMED.
C  THE AMPLITUDE PROFILES OF THE INCIDENT PRESSURE ARE ASSUMED TO BE
C  GAUSSIAN.  THEY ARE MEASURED ON A HORIZONTAL PLANE AT A DISTANCE R1
C  FROM THE SOURCE TRANSDUCER.
C  BWX IS THE BEAMWIDTH = FWHM/1.665 MEASURED IN THE X-Z PLANE.
C  BWY IS THE BEAMWIDTH MEASURED IN THE Y-Z PLANE.
C  THE PHASE OF THE INCIDENT PRESSURE IS THAT OF A POINT SOURCE.
C  MODE SPECIFIES THE SCATTERING GEOMETRY.
C       MODE=1 ... SPECULAR SCATTERING.
C       MODE=-1 ... BACKSCATTERING.
C       MODE=0 ... TRANSMITTER ANGLE FIXED AT ZERO.
C  MODE0 IS THE INITIAL VALUE OF MODE.
C  DMODE IS THE INCREMENT.
C  NMODE IS THE NUMBER OF VALUES OF MODE.  NORMALLY 1, 2, OR 3.
C  THETA0 IS THE INITIAL VALUE OF THE RECEIVER ANGLE WRT VERTICAL.
C  DTHETA IS THE INCREMENT.
C  NTHETA IS THE NUMBER OF VALUES USED.
C  OFFSET IS THE ANGULAR DISPLACEMENT OF THE TRANSDUCER WRT ITS NORMAL
C  POSITION FOR THE SCATTERING MODE SPECIFIED.  IT IS POSITIVE IF THE
C  DISPLACEMENT IS IN THE + THETA1 DIRECTION.
C  THETA1 IS + 'UPWIND'.
C  THETA2 IS + 'DOWNWIND'.
C  ZETA IS THE AMPLITUDE OF THE CORRUGATIONS.  IT MUST BE .GT. ZERO.
C  ZETA0 IS THE INITIAL VALUE OF ZETA.
C  DZETA IS THE INCREMENT BY WHICH ZETA IS INCREASED.
C  NZETA IS THE NUMBER OF VALUES OF ZETA USED.
C  WL IS THE ACOUSTIC WAVELENGTH.
C  SWL IS THE SURFACE WAVELENGTH.
C  K IS THE ACOUSTIC WAVENUMBER.
C  SK IS THE SURFACE WAVENUMBER.
C  R1 IS THE RADIUS OF THE ARC ON WHICH THE TRANSMITTER MOVES.
C  R2 IS THE RADIUS OF THE ARC ON WHICH THE RECEIVER MOVES.
C  M IS THE INDEX FOR HORIZONTAL DISPLACEMENT OF THE SURFACE.
C  XM IS THE MTH HORIZONTAL DISPLACEMENT OF THE SURFACE.
C  MMAX IS THE NUMBER OF VALUES OF XM.  IT MUST BE ODD FOR SIMPSON'S
C  RULE AVERAGING.
C  BESSEL FUNCTIONS ARE USED IN THE FORM J(N), WHERE N IS THE ORDER AND
C  NMAX=16 IS THE MAXIMUM ORDER.  IT CAN BE INCREASED IF NECESSARY.
C  A IS THE ARGUMENT OF THE BESSEL FUNCTIONS.
C
C
      REAL J(100),J0
      DIMENSION PPRE(100),PPIM(100)
      DIMENSION PRE(100),PIM(100),PP2(100),P2(100)
```

```
C
      REAL MODEO,MODE
      READ 40, MODEO,DMODE,NMODE
      READ 40, BWX,BWY
      READ 40, THETAO,DTHETA,NTHETA
      READ 40, OFFSET
      READ 40, ZETAO,DZETA,NZETA
      READ 40, R1,R2
      READ 40, WL,SWL,MMAX
   40 FORMAT(2F10.0,I10)
C
      PRINT 45
   45 FORMAT('1THE INPUT CARDS ARE')
      PRINT 50, MODEO,DMODE,NMODE
      PRINT 50, BWX,BWY
      PRINT 50, THETAO,DTHETA,NTHETA
      PRINT 50, OFFSET
      PRINT 50, ZETAO,DZETA,NZETA
      PRINT 50, R1,R2
      PRINT 50, WL,SWL,MMAX
   50 FORMAT(24X,2F10.4,I10)
C
      DATA NMAX/16/
      DATA TWOPI/6.2832/
      REAL K
      K=TWOPI/WL
      SK=TWOPI/SWL
      A3=(1./R1+1./R2)/2.
      BWXO=BWX
C
C  COMPUTE THE MAGNITUDE OF THE PRESSURE REFLECTED VERTICALLY FROM A
C  SMOOTH SURFACE.
C
      AXO=SQRT(SQRT(BWX**-4+(K*A3)**2))
      AYO=SQRT(SQRT(BWY**-4+(K*A3)**2))
      PO=K/(2.*R2*AXO*AYO)
      PRINT 60, PO
   60 FORMAT('OTHE PRESSURE REFLECTED FROM A SMOOTH SURFACE IS PO =',
     1E10.5)
C
C  COMPUTE DXM.
C
      D=1./(MMAX-1)
      DXM=SWL*D
      PRINT 62, DXM
   62 FORMAT('OTHE SURFACE SHIFT INCREMENT IS DXM =',F8.4)
C
C  ESTABLISH THE GEOMETRY.
C
      MODE=MODEO-DMODE
      DO 260 MD=1,NMODE
      MODE=MODE+DMODE
C
C  ESTABLISH ZETA.
```

```
C
      ZETA=ZETA0-DZETA
      DO 260 IZ=1,NZETA
      ZETA=ZETA+DZETA
      PRINT 65, ZETA
   65 FORMAT('1***************'/'    ZETA =',F5.3/' ***************')
C
C  ESTABLISH THE TRANSMITTER AND RECEIVER ANGLES, THETA1 AND THETA2.
C
      THETA2=THETA0-DTHETA
      DO 260 NT=1,NTHETA
      THETA2=THETA2+DTHETA
      ANG2=THETA2/57.295
      CA2=COS(ANG2)
      THETA1=MODE*THETA2+OFFSET
      ANG1=THETA1/57.295
      CA1=COS(ANG1)
C
      PRINT 70, THETA1,THETA2
   70 FORMAT(' THETA1 =',F6.2,' DEGREES.'/' THETA2 =',F6.2,' DEGREES.')
      PRINT 75
   75 FORMAT(12X,'PPRE ',7X,'PPIM ',6X,'PPMAG ',6X,'PPPHAZ',7X,'PRE  ',
     17X,'PIM  ',7X,'PMAG  ',7X,'PPHAZ',7X,'PP2  ',7X,'P2'/)
C
C  COMPUTE SOME TERMS.
C
      A1=(CA1**2/R1+CA2**2/R2)/2.
      A2=SIN(ANG1)-SIN(ANG2)
      A4=CA1+CA2
      A=K*ZETA*A4
      BWX=BWX0/CA1
      AX2=SQRT(BWX**-4+(K*A1)**2)
      AX=SQRT(AX2)
      AY2=SQRT(BWY**-4+(K*A3)**2)
      AY=SQRT(AY2)
      PHIX=ATAN(K*A1*BWX**2)
      TX=TAN(PHIX)
      CX=COS(PHIX)
      PHIY=ATAN((K*A3)*BWY**2)
      XY=(PHIX+PHIY)/2.
      GNU2=4.*AX2/(K*K*CX)
      CP=K*A4/(4.*R2*AX*AY)
C
C  BESJ IS A UNIVAC MATH-PAC ROUTINE TO COMPUTE BESSEL FUNCTIONS.
C
      CALL BESJ(A,0.,NMAX,J)
      J0=J(1)
      DO 80 NN=1,NMAX
   80 J(NN)=J(NN+1)
C
C  ESTABLISH XM.
C
      XM=-DXM
      DO 200 M=1,MMAX
```

```
          XM=XM+DXM
C
C  COMPUTE THE APPROXIMATE SCATTERED PRESSURE, PP(M), AND THE CORRECTED
C  PRESSURE, P(M), FOR MMAX SURFACE POSITIONS.
C  THE REAL AND IMAGINARY PARTS ARE COMPUTED SEPARATELY.
C
C  IN COMPUTING PHASES, THE CONSTANT PHIO=K*(R1+R2) IS IGNORED.
C
          Q=XY-A2*A2*TX/GNU2
          C=COS(Q)
          S=SIN(Q)
          PPREM=J0*EXP(-A2*A2/GNU2)*C
          PPIMM=PPREM*S/C
          PREM=PPREM
          PIMM=PPIMM
C
          REAL N
          DO 100 NN=1,NMAX
          N=NN
          ALFP2=(A2+N*SK/K)**2/GNU2
          ALFM2=(A2-N*SK/K)**2/GNU2
          EPS=(N*SK/(K*A4))**2
          QP=XY-ALFP2*TX-N*SK*XM+.25*N*TWOPI
          QM=XY-ALFM2*TX+N*SK*XM-.25*N*TWOPI
          C=COS(QP)
          S=SIN(QP)
          CM=COS(QM)
          SM=SIN(QM)
          EC=C*EXP(-ALFP2)+CM*EXP(-ALFM2)*((-1.)**N)
          ES=S*EXP(-ALFP2)+SM*EXP(-ALFM2)*((-1.)**N)
          PPREM=PPREM+J(NN)*EC
          PPIMM=PPIMM+J(NN)*ES
          PREM=PREM+(1.+EPS)*J(NN)*EC
      100 PIMM=PIMM+(1.+EPS)*J(NN)*ES
          PPRE(M)=CP*PPREM
          PPIM(M)=CP*PPIMM
          PRE(M)=CP*PREM
          PIM(M)=CP*PIMM
          PP2(M)=(PPREM**2+PPIMM**2)*CP*CP
          PPMAG=SQRT(PP2(M))
          P2(M)=(PREM**2+PIMM**2)*CP*CP
          PMAG=SQRT(P2(M))
C
C  COMPUTE THE PHASES OF PP(M) AND P(M).
C
          CALL ANGLE(PPRE(M),PPIM(M),PPPHAZ)
          CALL ANGLE(PRE(M),PIM(M),PPHAZ)
C
      200 PRINT 220, PPRE(M),PPIM(M),PPMAG,PPPHAZ,PRE(M),PIM(M),PMAG,PPHAZ,
         1PP2(M),P2(M)
      220 FORMAT(5X,2(3E12.4,F12.3),2E12.4)
C
C  AVERAGE THE REAL AND IMAGINARY PARTS OF THE PRESSURE TERMS AND FIND
C  THE MAGNITUDES AND PHASES OF THE AVERAGES.
```

```
C
      CALL INTEG1(PPRE,MMAX,D,APPRE)
      CALL INTEG1(PPIM,MMAX,D,APPIM)
      CALL INTEG1(PRE,MMAX,D,APRE)
      CALL INTEG1(PIM,MMAX,D,APIM)
      CALL INTEG1(PP2,MMAX,D,APP2)
      CALL INTEG1(P2,MMAX,D,AP2)
      CALL ANGLE(APPRE,APPIM,APPPHZ)
      CALL ANGLE(APRE,APIM,APPHZ)
      APPMAG=SQRT(APPRE**2+APPIM**2)
      APMAG=SQRT(APRE**2+APIM**2)
C
      PRINT 230
  230 FORMAT(56X,'*** AVERAGES ***')
      PRINT 220, APPRE,APPIM,APPMAG,APPPHZ,APRE,APIM,APMAG,APPHZ,APP2,
     1AP2
C
      PPRMS=SQRT(APP2)
      PRMS=SQRT(AP2)
      PPRMSN=PPRMS/PO
      PRMSN=PRMS/PO
      SLPCOR=(1.-PPRMS/PRMS)*100.
C
      PRINT 250, PPRMS,PRMS
  250 FORMAT(//9X,'THE RMS SCATTERED PRESSURE W/O SLOPE CORRECTION IS PP
     1RMS =',E13.5/  9X,'THE RMS SCATTERED PRESSURE WITH SLOPE CORRECTIO
     2N IS PRMS =',E13.5)
  260 PRINT 265, PPRMSN,PRMSN,SLPCOR
  265 FORMAT(9X,'THE SAME QUANTITIES DIVIDED BY PO ARE PPRMSN =',E13.5
     1/48X,'PRMSN =',E13.5/9X,'THE CONTRIBUTION OF THE SLOPE CORRECTION
     2TO PRMSN IS SLPCOR =',F7.2,' PERCENT.')
      END
C
@FOR,SIA SUB1
C
      SUBROUTINE ANGLE(X,Y,A)
C
C  THIS IS A SUBROUTINE TO COMPUTE THE PHASE ANGLE OF A COMPLEX NUMBER.
C
      PI=3.14159265
      IF(ABS(X).GT.1.E-10 .AND. ABS(Y).GT.1.E-10) GO TO 5
C  IF BOTH X AND Y ARE ZERO, A WILL BE SET = 0.
      IF(ABS(X).LT.1.E-10 .AND. ABS(Y).LT.1.E-10) GO TO 130
      IF(ABS(X).LT.1.E-10 .AND. Y.LT.0.) GO TO 100
      IF(ABS(X).LT.1.E-10 .AND. Y.GT.0.) GO TO 110
      IF(ABS(Y).LT.1.E-10 .AND. X.LT.0.) GO TO 120
      IF(ABS(Y).LT.1.E-10 .AND. X.GT.0.) GO TO 130
    5 A=ATAN(Y/X)
      IF(A) 10,10,50
   10 IF(X) 20,20,30
   20 A=A+PI
      GO TO 200
   30 A=A+2.*PI
      GO TO 200
```

```
   50 IF(X) 20,20,200
  100 A=1.5*PI
      GO TO 200
  110 A=.5*PI
      GO TO 200
  120 A=PI
      GO TO 200
  130 A=0.
  200 RETURN
      END
C
@FOR,SIA SUB2
C
      SUBROUTINE INTEG1(F,N,D,V)
C
C  THIS IS A SUBROUTINE TO INTEGRATE A FUNCTION, F, VIA SIMPSON'S RULE.
C
      DIMENSION F(N)
      F1=F(1)+F(N)
      F2=F(N-1)
      F3=0.
      IMAX=N-3
      DO 10 I=2,IMAX,2
      F2=F2+F(I)
   10 F3=F3+F(I+1)
      V=(D/3.)*(F1+4.*F2+2.*F3)
      RETURN
      END
C
@XQT
   0.         0.              1
   5.35       5.35
   0.         40.             2
   0.
   .305       0.              1
   50.1       49.2
   .7198      8.8             49
@FIN
```

SAMPLE OUTPUT

....

ZETA = .305

....

THETA1 = .00 DEGREES.
THETA2 = .00 DEGREES.

PPRE	PPIM	PPMAG	PPPHAZ	PRE	PIM	PMAG	PPHAZ	PP2	P2
.2257+00	-.3111+00	.3843+00	5.340	.2260+00	-.3120+00	.3852+00	5.339	.1477+00	.1484+00
.2169+00	-.2937+00	.3651+00	5.349	.2176+00	-.2943+00	.3660+00	5.349	.1333+00	.1340+00
.1814+00	-.2504+00	.3091+00	5.339	.1823+00	-.2506+00	.3099+00	5.341	.9557-01	.9606-01
.1092+00	-.1996+00	.2231+00	5.224	.1097+00	-.1948+00	.2236+00	5.225	.4978-01	.4999-01
.1903-01	-.1283+00	.1277+00	4.860	.1837-01	-.1286+00	.1299+00	4.854	.1682-01	.1687-01
-.4517-01	-.5188-01	.6879-01	3.996	-.4667-01	-.5176-01	.6970-01	3.979	.4732-02	.4858-02
-.5221-01	.1882-01	.5549-01	2.796	-.5331-01	.1952-01	.5677-01	2.791	.3080-02	.3223-02
-.1778-01	.6276-01	.6523-01	1.847	-.1742-01	.6345-01	.6579-01	1.839	.4259-02	.4329-02
.2436-02	.8097-01	.8101-01	1.541	.4118-02	.8096-01	.8106-01	1.520	.6562-02	.6571-02
-.4144-01	.9913-01	.1074+00	1.967	-.3978-01	.9857-01	.1063+00	1.954	.1154-01	.1130-01
-.1481+00	.1370+00	.2017+00	2.395	-.1479+00	.1367+00	.2014+00	2.395	.4068-01	.4056-01
-.2606+00	.1824+00	.3161+00	2.531	-.2623+00	.1830+00	.3199+00	2.532	.1012+00	.1023+00
-.3086+00	.2032+00	.3695+00	2.559	-.3111+00	.2043+00	.3722+00	2.560	.1366+00	.1385+00
-.2606+00	.1824+00	.3181+00	2.531	-.2623+00	.1830+00	.3199+00	2.532	.1012+00	.1023+00
-.1481+00	.1370+00	.2017+00	2.395	-.1479+00	.1367+00	.2014+00	2.395	.4068-01	.4056-01
-.4144-01	.9913-01	.1074+00	1.967	-.3978-01	.9857-01	.1063+00	1.954	.1154-01	.1130-01
.2437-02	.8097-01	.8101-01	1.541	.4118-02	.8096-01	.8106-01	1.520	.6562-02	.6571-02
-.1778-01	.6276-01	.6523-01	1.847	-.1742-01	.6345-01	.6579-01	1.839	.4254-02	.4329-02
-.5221-01	.1881-01	.5549-01	2.796	-.5331-01	.1952-01	.5677-01	2.791	.3080-02	.3223-02
-.4517-01	-.5189-01	.6879-01	3.996	-.4667-01	-.5176-01	.6970-01	3.979	.4732-02	.4858-02
.1903-01	-.1283+00	.1297+00	4.860	.1837-01	-.1286+00	.1299+00	4.854	.1682-01	.1687-01
.1092+00	-.1946+00	.2231+00	5.224	.1097+00	-.1948+00	.2236+00	5.225	.4978-01	.4999-01
.1814+00	-.2504+00	.3091+00	5.339	.1823+00	-.2506+00	.3099+00	5.341	.9557-01	.9606-01
.2169+00	-.2737+00	.3651+00	5.349	.2176+00	-.2943+00	.3660+00	5.349	.1333+00	.1340+00
.2257+00	-.3111+00	.3843+00	5.340	.2260+00	-.3120+00	.3852+00	5.339	.1477+00	.1484+00
				*** AVERAGES ***					
-.6484-02	-.3264-01	.3327-01	4.516	-.6484-02	-.3264-01	.3327-01	4.516	.5080-01	.5112-01

THE RMS SCATTERED PRESSURE W/O SLOPE CORRECTION IS PPRMS = .22540+00
THE RMS SCATTERED PRESSURE WITH SLOPE CORRECTION IS PRMS = .22611+00
THE SAME QUANTITIES DIVIDED BY PO ARE PPRMSN = .45548+00
PRMSN = .45691+00
THE CONTRIBUTION OF THE SLOPE CORRECTION TO PRMSN IS SLPCOR = .31 PERCENT.

THETA1 = .00 DEGREES.
THETA2 = 20.00 DEGREES.

PPRE	PPIM	PPMAG	PPPHAZ	PRE	PIM	PMAG	PPHAZ	PP2	P2
-.1730-01	.1442+00	.1452+00	1.690	-.2033-01	.1537+00	.1550+00	1.702	.2108-01	.2404-01
.2162+00	.2322-01	.2174+00	.107	.2254+00	.2861-01	.2272+00	.126	.4726-01	.5161-01
.1410+00	-.2735+00	.3078+00	5.188	.1492+00	-.2814+00	.3185+00	5.200	.9471-01	.1015+00
-.2085+00	-.3037+00	.3684+00	4.111	-.2134+00	-.3145+00	.3801+00	4.116	.1357+00	.1445+00
-.3903+00	.3331-01	.3918+00	3.056	-.4024+00	.3376-01	.4038+00	3.058	.1535+00	.1631+00
-.1577+00	.3556+00	.3890+00	1.988	-.1622+00	.3668+00	.4010+00	1.987	.1513+00	.1608+00
.2347+00	.2944+00	.3765+00	.898	.2425+00	.3031+00	.3882+00	.896	.1417+00	.1507+00
.3590+00	-.7144-01	.3661+00	6.087	.3697+00	-.7432-01	.3771+00	6.085	.1340+00	.1422+00
.1114+00	-.3434+00	.3610+00	5.026	.1137+00	-.3531+00	.3709+00	5.024	.1303+00	.1376+00
-.2246+00	-.2782+00	.3575+00	4.033	-.2307+00	-.2842+00	.3661+00	4.030	.1278+00	.1340+00
-.3504+00	.8747-02	.3505+00	3.117	-.3577+00	.9879-02	.3578+00	3.114	.1229+00	.1280+00
-.2115+00	.2610+00	.3359+00	2.252	-.2150+00	.2668+00	.3426+00	2.249	.1128+00	.1174+00

.5415-01	.3030+00	.3078+00	1.394	.5671-01	.3092+00	.3143+00	1.389	.9473-01	.9881-01
.2277+00	.1265+00	.2604+00	.507	.2341+00	.1272+00	.2664+00	.498	.6782-01	.7099-01
.1779+00	-.7858-01	.1963+00	5.871	.1826+00	-.8345-01	.2008+00	5.855	.3853-01	.4031-01
.3362-01	-.1239+00	.1284+00	4.977	.3081-01	-.1266+00	.1303+00	4.951	.1649-01	.1699-01
-.3061-01	-.7159-01	.7787-01	4.308	-.3177-01	-.7011-01	.7697-01	4.287	.6063-02	.5924-02
-.5402-01	-.5126-01	.7447-01	3.901	-.5252-01	-.5230-01	.7412-01	3.925	.5546-02	.5493-02
-.1087+00	.6602-02	.1087+00	3.081	-.1116+00	.3817-02	.1116+00	3.107	.1186-01	.1246-01
-.6106-01	.1411+00	.1538+00	1.979	-.6585-01	.1448+00	.1591+00	1.998	.2364-01	.2530-01
.1393+00	.1448+00	.1975+00	.823	.1376+00	.1516+00	.2048+00	.834	.3901-01	.4194-01
.2113+00	-.7015-01	.2226+00	5.963	.2196+00	-.7230-01	.2312+00	5.965	.4957-01	.5345-01
.1781-01	-.2091+00	.2098+00	4.797	.1727-01	-.2182+00	.2189+00	4.791	.4403-01	.4791-01
-.1545+00	-.4993-01	.1624+00	3.454	-.1640+00	-.5109-01	.1718+00	3.444	.2638-01	.2950-01
-.1729-01	.1442+00	.1452+00	1.690	-.2032-01	.1537+00	.1550+00	1.702	.2108-01	.2404-01
				*** AVERAGES ***					
-.2010-02	-.3433-02	.3978-02	4.183	-.2010-02	-.3433-02	.3978-02	4.183	.7487-01	.7935-01

THE RMS SCATTERED PRESSURE W/O SLOPE CORRECTION IS PPRMS = .27362+00
THE RMS SCATTERED PRESSURE WITH SLOPE CORRECTION IS PRMS = .28169+00
THE SAME QUANTITIES DIVIDED BY PO ARE PPRMSN = .55293+00
PRMSN = .56924+00
THE CONTRIBUTION OF THE SLOPE CORRECTION TO PRMSN IS SLPCOR = 2.87 PERCENT.
THETA1 = .00 DEGREES.
THETA2 = 40.00 DEGREES.

PPRE	PPIM	PPMAG	PPPHAZ	PRE	PIM	PMAG	PPHAZ	PP2	P2
.5876-02	.3910-01	.3954-01	1.422	.6804-02	.4170-01	.4225-01	1.409	.1564-02	.1785-02
.3271-01	.3178-03	.3291-01	.010	.3486-01	-.4780-03	.3486-01	6.269	.1083-02	.1215-02
.8019-02	-.2374-01	.2506-01	5.038	.7600-02	-.2480-01	.2594-01	5.010	.6279-03	.6728-03
-.1304-01	-.1670-01	.2119-01	4.049	-.1318-01	-.1694-01	.2146-01	4.051	.4491-03	.4605-03
-.2475-01	.2718-02	.2509-01	3.033	-.2601-01	.2126-02	.2610-01	3.060	.6297-03	.6812-03
-.5310-02	.3091-01	.3137-01	1.741	-.6261-02	.3280-01	.3339-01	1.759	.9838-03	.1115-02
.3308-01	.9478-02	.3442-01	.280	.3559-01	.1038-01	.3707-01	.284	.1184-02	.1374-02
.9017-02	-.3085-01	.3214-01	4.997	.9434-02	-.3364-01	.3494-01	4.986	.1033-02	.1221-02
-.2478-01	-.3964-02	.2510-01	3.300	-.2747-01	-.3639-02	.2771-01	3.273	.6300-03	.7677-03
.4604-02	.1633-01	.1697-01	1.296	.5793-02	.1857-01	.1945-01	1.268	.2880-03	.3784-03
.7351-02	-.1488-01	.1659-01	5.171	.8707-02	-.1689-01	.1909-01	5.198	.2753-03	.3646-03
-.2477-01	.5545-03	.2477-01	3.119	-.2744-01	-.2307-03	.2744-01	3.150	.6136-03	.7528-03
.6562-02	.3250-01	.3316-01	1.372	.6509-02	.3561-01	.3620-01	1.390	.1100-02	.1310-02
.3713-01	-.1085-01	.3868-01	5.999	.4046-01	-.1142-01	.4204-01	6.008	.1496-02	.1767-02
-.1429-01	-.3861-01	.4117-01	4.358	-.1538-01	-.4200-01	.4473-01	4.361	.1695-02	.2001-02
-.3771-01	.1776-01	.4168-01	2.701	-.4101-01	.1929-01	.4532-01	2.702	.1737-02	.2054-02
.2146-01	.3556-01	.4153-01	1.028	.2337-01	.3866-01	.4517-01	1.027	.1725-02	.2041-02
.3341-01	-.2475-01	.4158-01	5.646	.3623-01	-.2688-01	.4511-01	5.645	.1729-02	.2035-02
-.2655-01	-.3234-01	.4184-01	4.025	-.2870-01	-.3485-01	.4515-01	4.024	.1750-02	.2038-02
-.3296-01	.2615-01	.4207-01	2.471	-.3530-01	.2808-01	.4511-01	2.470	.1770-02	.2035-02
.2363-01	.3526-01	.4245-01	.980	.2521-01	.3763-01	.4529-01	.980	.1802-02	.2051-02
.3844-01	-.1977-01	.4323-01	5.808	.4101-01	-.2100-01	.4608-01	5.810	.1869-02	.2123-02
-.1542-01	-.4111-01	.4391-01	4.354	-.1643-01	-.4392-01	.4689-01	4.354	.1928-02	.2199-02
-.4172-01	.1090-01	.4312-01	2.886	-.4459-01	.1185-01	.4614-01	2.882	.1860-02	.2128-02
.5879-02	.3910-01	.3954-01	1.422	.6807-02	.4170-01	.4225-01	1.409	.1564-02	.1785-02
				*** AVERAGES ***					
-.8220-07	.4199-06	.4230-06	1.766	-.1109-06	.4573-06	.4705-06	1.809	.1243-02	.1440-02

THE RMS SCATTERED PRESSURE W/O SLOPE CORRECTION IS PPRMS = .35251-01
THE RMS SCATTERED PRESSURE WITH SLOPE CORRECTION IS PRMS = .37953-01

THE SAME QUANTITIES DIVIDED BY PO ARE PPRMSN = .71234-01
PRMSN = .76696-01
THE CONTRIBUTION OF THE SLOPE CORRECTION TO PRMSN IS SLPCOR = 7.12 PERCENT.

@FIN

RUNID: B09606 PROJECT: USER:

LOAD MATHPK B/3 B09606

ITEM	AMOUNT	COST(DOLLARS)
CPU TIME	00:00:09.778	$0.65
I/O REQUESTS	530	$0.63
I/O-WORDS TRANSFERRED	373057	$0.22
CORE USAGE	0.397	$0.22
CARDS IN	310	$0.15
PAGES PRINTED	12	$0.39
TAPE UNITS USED	1	$0.40
JOB CHARGE	1	$0.20
TOTAL COST		$2.86

THE ABOVE DOLLAR AMOUNTS ARE APPROXIMATE AND ARE BASED ON RATES FOR STANDARD RUNS

INITIATION TIME: 03:20:00-MAY 16,1973

TERMINATION TIME: 03:21:18-MAY 16,1973

Program PSIN2 -- The rms scattered pressure is evaluated directly from Equation 39. A considerable simplification is obvious in comparing this program with PSIN1, but the programs are basically similar and additional comments do not seem to be necessary. One note, however; the present output format is not a particularly efficient one and should probably be changed.

```
@ASG,MT 11.,T,MATHPK
@COPY,G 11.,TPFS.
@FREE 11.
@FOR,SIA MAIN
C
C  THIS IS PROGRAM PSIN2.
C
C  IT COMPUTES THE RMS PRESSURE SCATTERED IN THE X-Z PLANE FROM A
C  CORRUGATED SURFACE.  A SLOPE CORRECTION IS INCLUDED.
C  THE CORRUGATIONS ARE COSINUSOIDAL AND ARE PARALLEL TO THE Y AXIS.
C  A HIGHLY DIRECTIVE TRANSMITTED ACOUSTIC BEAM AND A POINT RECEIVER
C  ARE ASSUMED.
C  THE AMPLITUDE PROFILES OF THE INCIDENT PRESSURE ARE ASSUMED TO BE
C  GAUSSIAN.  THEY ARE MEASURED ON A HORIZONTAL PLANE AT A DISTANCE R1
C  FROM THE SOURCE TRANSDUCER.
C  BWX IS THE BEAMWIDTH = FWHM/1.665 MEASURED IN THE X-Z PLANE.
C  BWY IS THE BEAMWIDTH MEASURED IN THE Y-Z PLANE.
C  THE PHASE OF THE INCIDENT PRESSURE IS THAT OF A POINT SOURCE.
C  MODE SPECIFIES THE SCATTERING GEOMETRY.
C        MODE=1 ... SPECULAR SCATTERING.
C        MODE=-1 ... BACKSCATTERING.
C        MODE=0 ... TRANSMITTER ANGLE FIXED AT ZERO.
C  MODE0 IS THE INITIAL VALUE OF MODE.
C  DMODE IS THE INCREMENT.
C  NMODE IS THE NUMBER OF VALUES OF MODE.  NORMALLY 1, 2, OR 3.
C  THETA0 IS THE INITIAL VALUE OF THE RECEIVER ANGLE WRT VERTICAL.
C  DTHETA IS THE INCREMENT.
C  NTHETA IS THE NUMBER OF VALUES USED.
C  OFFSET IS THE ANGULAR DISPLACEMENT OF THE TRANSDUCER WRT ITS NORMAL
C  POSITION FOR THE SCATTERING MODE SPECIFIED.  IT IS POSITIVE IF THE
C  DISPLACEMENT IS IN THE + THETA1 DIRECTION.
C  THETA1 IS + 'UPWIND'.
C  THETA2 IS + 'DOWNWIND'.
C  ZETA IS THE AMPLITUDE OF THE CORRUGATIONS.  IT MUST BE .GT. ZERO.
C  ZETA0 IS THE INITIAL VALUE OF ZETA.
C  DZETA IS THE INCREMENT BY WHICH ZETA IS INCREASED.
C  NZETA IS THE NUMBER OF VALUES OF ZETA USED.
C  WL IS THE ACOUSTIC WAVELENGTH.
C  SWL IS THE SURFACE WAVELENGTH.
C  K IS THE ACOUSTIC WAVENUMBER.
C  SK IS THE SURFACE WAVENUMBER.
C  R1 IS THE RADIUS OF THE ARC ON WHICH THE TRANSMITTER MOVES.
C  R2 IS THE RADIUS OF THE ARC ON WHICH THE RECEIVER MOVES.
C  BESSEL FUNCTIONS ARE USED IN THE FORM J(N), WHERE N IS THE ORDER AND
C  NMAX=16 IS THE MAXIMUM ORDER.
C  A IS THE ARGUMENT OF THE BESSEL FUNCTIONS.
C
C
      REAL J(100),J0
C
      REAL MODE0,MODE
      READ 40, MODE0,DMODE,NMODE
      READ 40, BWX,BWY
      READ 40, THETA0,DTHETA,NTHETA
```

```
      READ 40, OFFSET
      READ 40, ZETAO,DZETA,NZETA
      READ 40, R1,R2
      READ 40, WL,SWL
   40 FORMAT(2F10.0,I10)
C
      PRINT 45
   45 FORMAT('1THE INPUT CARDS ARE')
      PRINT 50, MODEO,DMODE,NMODE
      PRINT 50, BWX,BWY
      PRINT 50, THETAO,DTHETA,NTHETA
      PRINT 50, OFFSET
      PRINT 50, ZETAO,DZETA,NZETA
      PRINT 50, R1,R2
      PRINT 50, WL,SWL
   50 FORMAT(24X,2F10.4,I10)
C
      DATA NMAX/16/
      DATA TWOPI/6.2832/
      REAL K
      K=TWOPI/WL
      SK=TWOPI/SWL
      A3=(1./R1+1./R2)/2.
      BWXO=BWX
C
C  COMPUTE THE MAGNITUDE OF THE PRESSURE REFLECTED VERTICALLY FROM A
C  SMOOTH SURFACE.
C
      AXO=SQRT(SQRT(BWX**-4+(K*A3)**2))
      AYO=SQRT(SQRT(BWY**-4+(K*A3)**2))
      PO=K/(2.*R2*AXO*AYO)
      PRINT 60, PO
   60 FORMAT('0THE PRESSURE REFLECTED FROM A SMOOTH SURFACE IS PO =',
     1E10.5)
C
C  ESTABLISH THE GEOMETRY.
C
      MODE=MODEO-DMODE
      DO 260 MD=1,NMODE
      MODE=MODE+DMODE
C
C  ESTABLISH ZETA.
C
      ZETA=ZETAO-DZETA
      DO 260 IZ=1,NZETA
      ZETA=ZETA+DZETA
      PRINT 65, ZETA
   65 FORMAT('1***************'/'    ZETA =',F5.3/' ***************')
C
C  ESTABLISH THE TRANSMITTER AND RECEIVER ANGLES, THETA1 AND THETA2.
C
      THETA2=THETAO-DTHETA
      DO 260 NT=1,NTHETA
      THETA2=THETA2+DTHETA
```

```
      ANG2=THETA2/57.295
      CA2=COS(ANG2)
      THETA1=MODE*THETA2+OFFSET
      ANG1=THETA1/57.295
      CA1=COS(ANG1)
C
      PRINT 70, THETA1,THETA2
   70 FORMAT(' THETA1 =',F6.2,' DEGREES.'/' THETA2 =',F6.2,' DEGREES.')
C
C COMPUTE SOME TERMS.
C
      A1=(CA1**2/R1+CA2**2/R2)/2.
      A2=SIN(ANG1)-SIN(ANG2)
      A4=CA1+CA2
      A=K*ZETA*A4
      BWX=BWXO/CA1
      AX2=SQRT(BWX**-4+(K*A1)**2)
      AX=SQRT(AX2)
      AY2=SQRT(BWY**-4+(K*A3)**2)
      AY=SQRT(AY2)
      PHIX=ATAN(K*A1*BWX**2)
      CX=COS(PHIX)
      GNU2=4.*AX2/(K*K*CX)
      CP=K*A4/(4.*R2*AX*AY)
C
C BESJ IS A UNIVAC MATH-PAC ROUTINE TO COMPUTE BESSEL FUNCTIONS.
C
      CALL BESJ(A,0.,NMAX,J)
      J0=J(1)
      DO 80 NN=1,NMAX
   80 J(NN)=J(NN+1)
C
C COMPUTE THE RMS PRESSURE.
C
      PPSUM=J0*J0*EXP(-2.*A2*A2/GNU2)
      PSUM=PPSUM
C
      REAL N
      DO 200 NN=1,NMAX
      N=NN
C
      ALFP2=(A2+N*SK/K)**2/GNU2
      ALFM2=(A2-N*SK/K)**2/GNU2
      E=EXP(-2.*ALFP2)+EXP(-2.*ALFM2)
      EPS=(N*SK/(K*A4))**2
      PPSUM=PPSUM+(J(NN)**2)*E
  200 PSUM=PSUM+(((1.+EPS)*J(NN))**2)*E
C
C PPRMS IS THE SMALL SLOPES APPROXIMATION OF PRMS.
C EPS IS THE SLOPE CORRECTION TERM.
C
      PPRMS=CP*SQRT(PPSUM)
      PRMS=CP*SQRT(PSUM)
C
```

```
C  SLPCOR IS THE PERCENTAGE CONTRIBUTION OF THE SLOPE CORRECTION.
C
      SLPCOR=(1.-PPRMS/PRMS)*100.
      PPRMSN=PPRMS/PO
      PRMSN=PRMS/PO
C
      PRINT 250, PPRMS,PRMS
  250 FORMAT(/ 9X,'THE RMS SCATTERED PRESSURE W/O SLOPE CORRECTION IS PP
     1RMS =',E13.5/  9X,'THE RMS SCATTERED PRESSURE WITH SLOPE CORRECTIO
     2N IS PRMS =',E13.5)
  260 PRINT 265, PPRMSN,PRMSN,SLPCOR
  265 FORMAT(9X,'THE SAME QUANTITIES DIVIDED BY PO ARE PPRMSN =',E13.5
     1/48X,'PRMSN =',E13.5/9X,'THE CONTRIBUTION OF THE SLOPE CORRECTION
     2TO PRMSN IS SLPCOR =',F7.2,' PERCENT.'/)
      END
C
@XQT
   -1.        0.              1
   6.154      6.154
   0.         2.              10
   0.
   .305       0.              1
   81.5       81.5
   .7198      8.8
@FIN
```

C. A program for the computation of probability density functions for surface displacement: program PDF.

Much of this program is nearly identical to program TRANSLATE, and most of the discussion of that program is pertinent here also. The reason for this is that both programs were intended to utilize the same input (surface displacement) data. Here, a PDF for the surface displacement is computed both for the entire data set (file), as read from the digital tape, and for the 'average' data block previously described.

The basic procedure for computing the PDF of the entire file is as follows: An N-word data block is read and converted to 36-bit words by subroutine RREC. The data are placed in array IY which is common to the main program and to subroutines RREC and DSTFUN. At entry DSTF1, subroutine DSTFUN sorts the data into histogram bins. Another data block is then read and sorted. When all blocks have been read, the histogram is normalized by a call to DSTFUN at entry DSTF2. The PDF of the 'average' block is obtained by one call to each of the entry points of DSTFUN.

Much of the program between cards 124 and 246 is concerned with computing mean and rms values, and with selecting an average block. These operations are unnecessary in terms of simply obtaining a PDF for a given data set. From that point of view, the essential, and most useful, part of the program is subroutine DSTFUN.

```
@ASG,TH 17/PASSWORD.,8C9,REEL ID
@FOR,ASI MAIN
C
C  THIS IS PROGRAM PDF.
C  IT HAS BEEN WRITTEN TO ANALYZE WAVEHEIGHT DATA THAT HAS BEEN
C  RECORDED ON 9-TRACK MAGNETIC TAPE, IN 24-BIT WORDS, AND IN N-WORD
C  BLOCKS.
C  THE DATA WORDS ARE CONVERTED FROM 24 BITS TO 36 BITS.
C  THE MEAN IS FOUND AND IS SUBTRACTED FROM EACH DATUM.
C  AN RMS VALUE IS COMPUTED FOR EACH BLOCK.
C  THE MEAN OF THE RMS VALUES IS CALCULATED, AND AN 'AVERAGE' BLOCK
C  HAVING AN RMS VALUE CLOSE TO THE MEAN RMS IS FOUND.
C  NORMALIZED SURFACE DISPLACEMENT PROBABILITY DENSITY FUNCTIONS (PDF'S)
C  ARE OBTAINED FOR THE WHOLE DATA SET AND FOR THE AVERAGE BLOCK.
C
C  N IS THE NUMBER OF WORDS IN A DATA BLOCK.
C  FOR THIS PROGRAM, N MUST BE AN INTEGER MULTIPLE OF 3.
C  N CAN BE CHANGED BY CHANGING THE PARAMETER STATEMENTS.
C  NFILES IS THE NUMBER OF TAPE FILES TO BE SKIPPED.
C  MB IS A NUMBER .GE. THE NUMBER OF BLOCKS (OR RECORDS) IN ANY FILE TO
C  BE SKIPPED.
C  IYMAX IS THE MAXIMUM VALUE OF THE DATA (WHICH IS ASSUMED TO BE IN
C  POSITIVE INTEGER FORM).  IYMAX=1024 FOR A 10-BIT DIGITIZER.
C  NSIZE IS THE WIDTH OF THE HISTOGRAM BINS.
C  SIGN IS SET .LT. 0. IF THE DATA ARE INVERTED ON THE TAPE.
C  SRATE IS THE EFFECTIVE DIGITIZER SAMPLING RATE.
C  EFFECTIVE MEANS (DIGITIZER SAMPLING RATE)*(RECORDER RECORD SPEED/
C  RECORDER PLAYBACK SPEED).
C  PKHT IS THE ASSUMED PEAK HEIGHT OF THE WATER WAVES.
C  SPEED IS THE SPEED OF THE WATER WAVES.
C  WLNGTH IS THE WAVELENGTH OF THE WAVES.
C  NB IS THE NUMBER OF N-WORD BLOCKS IN THE FILE.
C  IB IS THE INDEX OF THE AVERAGE BLOCK.  IT CAN HAVE ANY VALUE FROM
C  1 TO NB.
C  EPS IS THE FRACTIONAL DIFFERENCE BETWEEN THE AVERAGE RMS AND THE RMS
C  OF THE AVERAGE BLOCK.
C  RMSHT IS THE RMS SURFACE DISPLACEMENT IN CM FOR THE DATA IN THE
C  AVERAGE BLOCK.
C  DXP IS THE HORIZONTAL SEPARATION IN CM OF THE SURFACE DATA.
C
C
      PARAMETER N=600,IYMAX=1024
      COMMON IY(N)
      COMMON/DSF/DF(IYMAX),NSIZE,NIY
      DIMENSION LABEL(7),RMS(1000)
      DIMENSION TEMP(N),X(IYMAX)
      DATA DF/IYMAX*0./
      REAL MEAN(1000)
      NN=N
      IYM=IYMAX
C
C  INPUT CARDS.                                ********************
C
      READ 10, NFILES,MB
```

```
   10 FORMAT(2I10)
      READ 10, NSIZE
      READ 20, SIGN
      READ 20, SRATE
      READ 20,PKHT,SPEED,WLNGTH
   20 FORMAT(3F10.0)
C
C                                                      *********************
C
      PRINT 30, NFILES
   30 FORMAT(1H1,' NFILES =',I2,' TAPE FILES HAVE BEEN SKIPPED.')
      PRINT 35, IYM,NSIZE,SIGN
   40 FORMAT(1H0,'THE EFFECTIVE SAMPLING RATE IS',F6.1,' SAMPLES PER SEC
      PRINT 40, SRATE
   35 FORMAT(' IYMAX =',I5/' NSIZE =',I3/' SIGN =',F4.0)
     1ND.'//)
      PRINT 50, PKHT
   50 FORMAT(' THE PEAK HT OF THE AVERAGE WAVE IS',F6.3,' CM.')
      PRINT 60, SPEED
   60 FORMAT(' THE SPEED OF THE WAVES IS',F6.2,' CM PER SECOND.')
      PRINT 70, WLNGTH
   70 FORMAT(' THE WAVELENGTH OF THE WATER WAVES IS',F6.3,' CM.')
C
C SKIP FILES THAT ARE NOT OF INTEREST.  LABELS ARE FILES.
C
      IF(NFILES .NE. 0) CALL SKIPF(NFILES,MB)
C
C READ AND PRINT THE TAPE LABEL.
C IOTPIN IS AN MACC 1108 LIBRARY SUBROUTINE TO READ A DATA BLOCK FROM
C MAGNETIC TAPE.  SEE THE REFERENCE MANUAL FOR THE 1108 COMPUTER.
C IN THIS CASE THE LABEL IS STRICTLY NUMERIC AND IS RECORDED AS 24-BIT
C WORDS.  FLD MANIPULATES THE BITS TO READ THE WORDS IN A 36-BIT (1108)
C FORMAT.
C
      CALL IOTPIN(17,1,IY,N,LNZ,$80)
   80 LABEL(1)=FLD(0,24,IY(1))
      LABEL(2)=FLD(0,12,IY(2))
      LABEL(3)=FLD(12,24,IY(2))
      LABEL(4)=FLD(0,24,IY(3))
      LABEL(5)=FLD(0,12,IY(4))
      LABEL(6)=FLD(12,24,IY(4))
      LABEL(7)=FLD(0,24,IY(5))
      PRINT 90,(LABEL(J), J=1,7)
   90 FORMAT(///' TAPE NUMBER',I6,10X,'DATA TAKEN ON ',I2,'/',I2,'/',I2/
     128X,'DATA DIGITIZED ON ',I2,'/',I2,'/',I2)
C
C AN EOF SHOULD FOLLOW THE LABEL.
C FIND THE LABEL EOF.
C SUBROUTINE IOTPSP SPACES LOGICAL UNIT 17 (HERE) PAST THE NEXT DATA
C BLOCK.  IT TRANSFERS CONTROL TO STATEMENT 100 IF AN EOF IS FOUND.
C
      CALL IOTPSP(17,$100)
      PRINT 95
   95 FORMAT(//' ***** LABEL EOF NOT FOUND. *****')
```

```
C
C  BEGIN THE ANALYSIS OF THE DATA.
C  READ THE DATA BLOCK BY BLOCK.  FOR EACH BLOCK COMPUTE THE RMS AND THE
C  MEAN.  ALSO COMPUTE THE AVERAGE RMS, THE AVERAGE MEAN, AND THE
C  STANDARD DEVIATIONS OF THE RMS'S AND MEANS.
C  BECAUSE THE WAVES ARE APPROX PERIODIC, USE ONLY THAT NUMBER OF DATA
C  (NS) WHICH CORRESPOND TO AN INTEGER NUMBER OF WAVELENGTHS.
C  FINALLY, USE SUBROUTINE DSTFUN TO COMPUTE THE NORMALIZED
C  PDF OF ALL THE DATA.
C  SUBROUTINE RREC CONVERTS 24-BIT WORDS TO 36-BIT WORDS.  CONTROL GOES
C  TO STATEMENT 180 IF AN EOF IS FOUND, AND TO 160 IF A TAPE ERROR IS
C  FOUND.
C  AN N-WORD BLOCK IS READ BY RREC, THEN THE DATA ARE SORTED INTO
C  HISTOGRAM BINS BY SUBROUTINE DSTF1.
C
      NIY=IYMAX/NSIZE +1
      DXP=SPEED/SRATE
      SPWL=WLNGTH/DXP
      T=N-1
      WLPBL=AINT(T*DXP/WLNGTH)
      NS=SPWL*WLPBL+1.
      J=0
      K=0
      SUM2=0.
      SUM3=0.
      SUM4=0.
      SUM5=0.
  100 SUM=0.
      SUM1=0.
      J=J+1
      CALL RREC(NW,$180,$160)
      IF(NW .EQ. N) GO TO 110
      PRINT 105
  105 FORMAT(//' ***** NW DOES NOT EQUAL N *****')
      GO TO 160
  110 CALL DSTF1
      K=K+1
      DO 120 I=1,NS
      SUM=SUM+FLOAT(IY(I))
  120 SUM1=SUM1+FLOAT(IY(I))**2
      F=NS
      MEAN(J)=SUM/F
      RMS(J)=SQRT(SUM1/F-MEAN(J)**2)
      SUM2=SUM2+MEAN(J)
      SUM3=SUM3+MEAN(J)**2
      SUM4=SUM4+RMS(J)
      SUM5=SUM5+RMS(J)**2
      GO TO 100
  160 PRINT 170, J
  170 FORMAT(//' ***** TAPE ERROR IN BLOCK NO.',I4,' *****'//)
      MEAN(J)=0.
      RMS(J)=0.
      GO TO 100
  180 FK=K
```

```
      NB=J-1
      AVGMN=SUM2/FK
      SDMEAN=SQRT((SUM3-FK*AVGMN**2)/(FK-1.))
      AVGRMS=SUM4/FK
      SDRMS=SQRT((SUM5-FK*AVGRMS**2)/(FK-1.))
C
C  SUBROUTINE DSTF2 NORMALIZES THE HISTOGRAM.
C
      CALL DSTF2
      PRINT 190, NS,WLPBL
  190 FORMAT(///' THE FIRST',I4,' WORDS IN EACH BLOCK ARE USED TO COMPUT
     1E THE MEAN AND THE RMS.  THESE WORDS CORRESPOND TO',F5.1,' WAVELEN
     2GTHS.')
      PRINT 200, K,AVGMN,SDMEAN
  200 FORMAT(///' THE AVERAGE OF THE MEANS OF',I4,' ERROR-FREE BLOCKS IS
     1',F10.5/' THE STANDARD DEVIATION OF THE MEANS IS',F10.5)
      PRINT 210, AVGRMS,SDRMS
  210 FORMAT('0THE AVERAGE OF THE RMSES IS',F10.5/' THE STANDARD DEVIATI
     1ON OF THE RMSES IS',F10.5)
      PRINT 220
  220 FORMAT('1THIS IS ARRAY MEAN.'//)
      PRINT 225, (MEAN(J), J=1,NB)
  225 FORMAT(1H ,10F10.5)
      PRINT 240
  240 FORMAT('1THIS IS ARRAY RMS.'//)
      PRINT 225, (RMS(J), J=1,NB)
C
C  FIND A BLOCK CONTAINING DATA WITH AN RMS CLOSE TO AVGRMS.
C
      EPS0=.005
      EPS=EPS0
  260 J=0
  270 J=J+1
      IF(ABS(RMS(J)/AVGRMS-1.) .LT. EPS) GO TO 300
      IF(J-NB) 270,280,280
  280 EPS=EPS+EPS0
      IF(EPS-10.*EPS0) 260,260,330
  300 PRCNT=100.*EPS
      IB=J
      PRINT 320, IB,RMS(IB),PRCNT
  320 FORMAT(//' BLOCK NUMBER',I4,' HAS AN RMS VALUE OF',F7.2,'.  THIS I
     1S WITHIN',F4.2,' PERCENT OF THE MEAN RMS.')
      GO TO 340
  330 PRINT 335
  335 FORMAT(//' ***** CANT FIND AN RMS(J) CLOSE TO AVGRMS. QUIT. *****')
      CALL EXIT
C
C  REWIND THE TAPE AND RETURN TO THE BEGINNING OF THE WAVE DATA.
C
  340 CALL IOTPRW(17)
      NFP1=NFILES+1
      CALL SKIPF(NFP1,MB)
C
C  GO TO BLOCK NUMBER IB, READ IT, AND COMPUTE THE MEAN AND RMS VALUES
```

```
      C  AGAIN, AS A CHECK.
      C
        360 IMAX=IB-1
            DO 370 I=1,IMAX
        370 CALL IOTPSP(17)
            CALL RREC(NW,$480,$440)
            DO 380 I=1,N
        380 TEMP(I)=FLOAT(IY(I))-MEAN(IB)
            SUM=0.
            SUM1=0.
            DO 390 I=1,NS
            SUM=SUM+FLOAT(IY(I))
        390 SUM1=SUM1+FLOAT(IY(I))**2
            F=NS
            CKMEAN=SUM/F
            CKRMS=SQRT(SUM1/F-CKMEAN**2)
            PRINT 400, CKMEAN,CKRMS
        400 FORMAT(//' CHECK THE MEAN AND RMS OF BLOCK IB.'/10X,'MEAN =',F10.5
           1,10X,'RMS =',F10.5)
            PRINT 420, NW
        420 FORMAT(//' CHECK IF NW=N.  THE NUMBER OF DATA READ INTO ARRAY TEMP
           1 IS',I5,'.')
            GO TO 500
        440 PRINT 445, IB
        445 FORMAT(//' ***** TAPE ERROR IN BLOCK IB =',I4,'.  TRY ANOTHER BLOC
           1K. *****')
            GO TO 270
        480 PRINT 490
        490 FORMAT(//' ***** EOF FOUND BY RREC. QUIT. *****')
            CALL EXIT
      C
      C  FIND THE LARGEST POSITIVE AND NEGATIVE VALUES IN THE AVERAGE BLOCK.
      C  URSRCH IS AN MACC LIBRARY SUBROUTINE TO FIND THE LARGEST OR SMALLEST
      C  ELEMENT IN AN ARRAY.
      C
        500 CALL URSRCH(0,N,TEMP,LP,TMPMAX,0,DUM)
            CALL URSRCH(1,N,TEMP,LN,TMPMIN,0,DUM)
            PRINT 520, LP,TMPMAX
        520 FORMAT('0DATUM NUMBER',I4,' IS THE LARGEST POSITIVE DATUM IN THE A
           1VERAGE BLOCK.  IT HAS THE VALUE',F10.4)
            PRINT 530, LN,TMPMIN
        530 FORMAT('0DATUM NUMBER',I4,' IS THE LARGEST NEGATIVE DATUM IN THE A
           1VERAGE BLOCK.  IT HAS THE VALUE',F10.4)
      C
      C  THE CONVERSION FACTOR CF WILL ALLOW THE COMPUTATION OF ARRAY X IN
      C  CENTIMETERS.
      C
            TMP=TMPMIN
            IF(SIGN .GT. 0.) TMP=TMPMAX
            CF=PKHT/TMP
            RMSHT=ABS(CF)*RMS(IB)
            PRINT 580, CF
        580 FORMAT('0THE WAVEHT CONVERSION FACTOR IS CF =',E10.4)
            PRINT 590, RMSHT
```

```
  590 FORMAT(' THE RMS WAVEHT OF THE AVERAGE BLOCK IS RMSHT =',F5.3,' CM
     1.')
      PRINT 600, DXP
  600 FORMAT(' THE HORIZONTAL SEPARATION OF DATA ON THE SURFACE IS DXP =
     1',F6.4,' CM.')
C
C  COMPUTE VALUES FOR THE ABSCISSA OF THE HISTOGRAM.
C
      X0=FLOAT(NSIZE-IYMAX)/2.
      DO 650 I=1,NIY
  650 X(I)=(X0+FLOAT((I-1)*NSIZE))*CF
C
C  PRINT THE X ARRAY AND THE PDF ARRAY FOR THE WHOLE DATA SET.
C
      PRINT 670
  670 FORMAT('1ARRAYS X AND DF.'//)
      PRINT 680, (X(I),DF(I), I=1,NIY)
  680 FORMAT(1H F12.4,F14.6)
C
C  COMPUTE AND PRINT THE X AND PDF ARRAYS FOR THE AVERAGE BLOCK.
C
      DO 690 I=1,NIY
  690 DF(I)=0.
      CALL DSTF1
      CALL DSTF2
C
      PRINT 700
  700 FORMAT('1ARRAYS X AND DF FOR THE AVERAGE BLOCK.'//)
      PRINT 680, (X(I),DF(I), I=1,NIY)
      END
C
@FOR,SIA SUB1
C
      SUBROUTINE DSTFUN
C
C  THIS SUBROUTINE SORTS INTEGER DATA INTO BINS TO PRODUCE A
C  NORMALIZED HISTOGRAM.  DSTF1 SORTS, DSTF2 NORMALIZES.
C
      PARAMETER N=600,IYMAX=1024
      COMMON IY(N)
      COMMON/DSF/DF(IYMAX),NSIZE,NIY
C
      ENTRY DSTF1
C
      DO 20 J=1,N
      K=IY(J)/NSIZE+1
   20 DF(K)=DF(K)+1.
      RETURN
C
      ENTRY DSTF2
C
      SUMDF=0.
      DO 40 I=1,NIY
   40 SUMDF=SUMDF+DF(I)
```

```
            DFNORM=1./(SUMDF*NSIZE)
            DO 60 I=1,NIY
         60 DF(I)=DF(I)*DFNORM
            RETURN
            END
      C
      @FOR,SIA SUB2
      C
            SUBROUTINE RREC(NWORDS,$,$)
      C
      C  THIS IS A SUBROUTINE TO READ 1 RECORD OF TAPE FROM UNIT NO. 17 AND
      C  CONVERT 9-TRACK, 24 BIT WORDS  TO 36 BIT WORDS.
      C
            PARAMETER N=600
            DIMENSION IX(N)
            COMMON IY(N)
      C
            M=2*N/3
            CALL IOTPIN(17,1,IX,M,LNZ,$20,$30)
      C  GO TO 20 IF AN EOF IS FOUND.
      C  GO TO 30 IF A TAPE ERROR IS FOUND.
            NWORDS=LNZ/3
            IMAX=NWORDS-2
            N24=8**4
            DO 10 I=1,IMAX,3
            J=2*(I+2)/3-1
            IY(I)=FLD(0,24,IX(J))
            IY(I+1)=FLD(24,12,IX(J))*N24+FLD(0,12,IX(J+1))
         10 IY(I+2)=FLD(12,24,IX(J+1))
            RETURN
         20 RETURN 2
         30 RETURN 3
            END
      C
      @FOR,SIA SUB3
      C
            SUBROUTINE SKIPF(NFILES,MB)
      C
      C  THIS IS A SUBROUTINE TO SKIP TAPE DATA FILES THAT ARE NOT OF
      C  CURRENT INTEREST.
      C
            DO 60 I=1,NFILES
            DO 20 J=1,MB
         20 CALL IOTPSP(17,$60)
            PRINT 40
         40 FORMAT(///' NO EOF FOUND BY IOTPSP.  QUIT.')
            CALL EXIT
         60 CONTINUE
            RETURN
            END
      C
      @XQT
               1        250
              15
          -1.
          400.
          .32       39.1       8.80
      @FIN
```