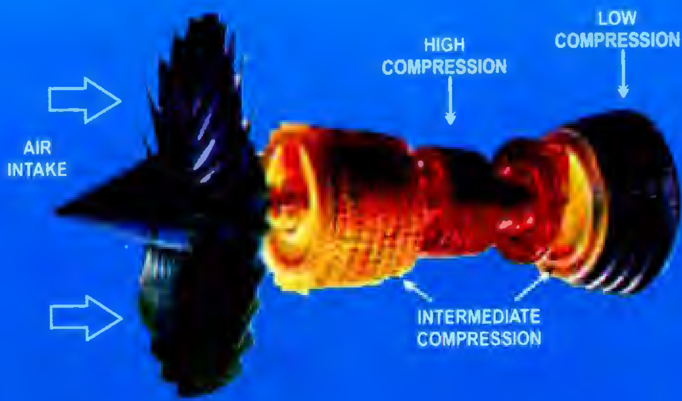
GAS TURBINES AND JET & ROCKET PROPULSION



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PREFACE

Gas turbine is relatively a new power plant which has been originated and developed in the last half a century. World War-II inducted this power plant into aircrafts as a jet propulsion engine. Today turbojet engine and its variants are the most important power plants for aircraft and missile propulsion.

Basically there are two types of propulsive devices : air breathing engines and rockets. Initially the rocket technology developed due to the requirements of military but in recent times the requirements of space travel forced a tremendous development in rocket propulsion.

In this background, Gas Turbines and Jet and Rocket Propulsion has become a very important subject for mechanical engineers and many institutions have included this subject at under-graduate and post-graduate levels.

This book has been written to serve as a text-book for the Gas Turbine and Jet and Rocket Propulsion courses taught to engineering studeuts and not for specialists in propulsion. The book is based on Gas Turbine and Jet Propulsion courses given by authors to students for several years.

The book is divided into nine chapters. Chapter 1 compares gas turbine plant with highly developed reciprocating internal combustion engine and steam turbine plants, and describes various applications of gas turbines. Chapter 2 gives the thermodynamic analysis of theoretical and actual gas turbine plants—for nuclear power generation, total energy systems and automotive field. For the study of propulsion systems a basic knowledge of gas dynamics is essential and Chapter 3 is devoted to this topic. Compressor forms a very important component of a gas turbine plant as it absorbs almost two-thirds of the power developed by the turbine. The overall efficiency of a gas turbine plant heavily depends on the performance of the compressor. As an introduction to compressors, Chapter 4 is devoted to positive displacement compressors—both reciprocating and rotary types. Chapter 5 deals with theory of dynamic compressors used in gas turbine plants, and compares the two basic type of compressors—the centrifugal and axial flow types including their design. Chapter 6 deals with gas turbine combustion chambers. The turbine design is a vast subject. Chapter 7 gives introduction to gas turbine design. Chapter 8 is devoted to jet propulsion. It deals with turbojet, turboprop and thrust argumentation devices. The athodoydsram jet and pulse jet, are also discussed. Finally, a comparison of different jet propulsion devices is given.

Chapter 9 is devoted to rocket propulsion and deals with solid and liquid propellant chemical rockets. Possible future developments in rocket technology—nuclear, electrostatic and electromagnetic rockets are also discussed.

The authors have tried to present the subject matter in a simple and clear manner so that the student can learn subject with minimum help from the teacher. The figures have been given as simple reproducible line diagrams. The theory has been illustrated with a large number of solved examples, which are graded and titled. Sufficient examples for practice have also been given. Both solved and unsolved examples have been taken from various university examinations. A large number of theory questions are given at the end of each chapter for self-testing of students.

With the rapid developments in science and technology, knowledge and information becomes out of date very soon, and each new book should justify its birth on the basis of presentation of new material. In this book an attempt has been made to draw deep into the latest literature available. It is hoped that this feature will justify this book and it will be useful to practising engineers also.

It is with great pleasure that the authors express their gratitude to many colleagues for assistance. Thanks are particularly due to Shri F.S. Mehta, Reader in mechanical engineering, University of Jodhpur, for critical comments and assistance in working out several of the illustrative examples. Thanks are due to Shri P.S. Kachhawa and S.N. Garg for preparing the drawings. Authors also thank Shri Nem Chand Jain of M/s. Standard Publishers Distributors for all help and co-operation in publishing this book.

In the first edition of ~ book some mistakes invariably creep in. The authors will be grateful if these are called to their attention. Also the authors will be happy to receive suggestions from students and teachers for improving the book in subsequent editions.

New Year Day — M.L. Mathur

1st January 1976 — R.P. Sharma

PREFACE TO THE SECOND EDITION

In the second edition of the book, the subject matter has been revised and upto date.

1st Oct. 1987 — M.L. Mathur

R.P. Sharma

CONTENTS

GLOSSARY OF SYMBOLS

A	area
a	velocity of sound
C	absolute velocity
C_d	coefficient of drag
C_L	coefficient of lift
c_p	specific heat at constant pressure
c_v	specific heat at constant volume
D, d	diameter
E	internal energy
e	specific internal energy, base of normal logarithms
F	force, thrust
f	specific thrust
g	gravitational acceleration
g_c	constant of proportionality
H	enthalpy
h	specific enthalpy, height of blade
J	Joule's equivalent
K	absolute temperature (Kelvin), work done factor
L, l	length, depth, lift
M	molecular weight, Mach number
m	mass, nuclear weight, number of blades
m	mass rate
N	rotational speed, revolution per minute
n	polytropic index
P	power
P, p	absolute pressure
Q, q	heat, rate of heat, transfer
R	gas constant, drgree of reaction
R_o	universal gas constant
r	radius, compression ratio, speed ratio
r_p	pressure ratio
S	entropy

s	specific entropy
T	absolute thermodynamic temperature
t	temperature
U	internal energy, blade speed
u	specific internal energy
V	volume, velocity relative to blade, velocity
V_e	exit velocity
V_j	jet velocity
v	specific volume
W, w	work, rate of work transfer
z	height above datum

Greek Symbols

α	angle of absolute velocity, nozzle acceleration
β	angle of relative velocity
γ	ratio of specific heat, c_p/c_v
η	efficiency
θ	temperature difference, fundamental dimension of temperature
ϕ	Flow coefficient
ρ	density
μ	Mach angle
σ	slip factor, velocity ratio, stress
ω	angular velocity
ψ	blade loading coefficient

Subscripts

a	air, atmospheric, axial velocity, actual
B	blade
b	back pressure
c	critical value, compressor
d	diffuser
e	exit
f	saturated liquid, fuel
g	saturated vapour
fg	change of phase at constant pressure
i	inlet, isentropic
m	mean
n	nozzle
o	hub

s	small stage or polytropic
t	total head or stagnation condition, turbine, throat area
p	propulsive
w	velocity of whirl
t_r	transmission
x	critical properties

Abbreviations

hp	horse power
shp	shaft horse power
thp	thrust horse power
tsfc	thrust specific fuel consumption

1

INTRODUCTION

A gas turbine is a rotary machine, similar in principle to a steam turbine. It consists of three main components—a compressor, a combustion chamber and a turbine. The air after being compressed into the compressor is heated either by directly burning fuel in it or by burning fuel externally in a heat exchanger. The heated air with or without products of combustion is expanded in a turbine resulting in work output, a substantial part, about two-thirds, of which is used to drive the compressor. Rest, about one-third, is available as useful work output.

1·1. DEVELOPMENT OF GAS TURBINE

The concept of turbine prime mover can be traced back to Hero of Alexandria who lived about 2000 years ago. John Barber, an Englishman, was first to develop an important design in 1871. This design used an impulse turbine, a reciprocating compressor, a gas producer and a combustion chamber with water injection. In 1872 Stolze in Berlin and in 1884 John Parsons in England patented their designs but both of them failed. It was Prof. Dr. R. Stodola, the world-famous teacher at the Swiss Federal Institute of Technology (from 1892 to 1929) who established the scientific and engineering basis for the steam turbine and predicted a bright future for the gas turbine at a very early date. Even the first edition of his standard book on steam turbines (1903) contained a chapter on gas turbines, including the general theory, which is still valid.

The development of gas turbines was hampered for a long time despite this general theory because of two basic reasons :—

(*i*) The lack of materials to withstand high temperatures.

(*ii*) The lack of thermodynamic and aerodynamic knowledge of flow mechanism.

Early attempts by Stolze in 1904 failed mainly because of the lack of necessary engineering know-how. Some attempts even produced negative power, *i.e.* power produced by turbine rotor was less than that required by the compressor. And the world had to wait for the development of appropriate high temperature materials and improved blade and machine efficiencies, both of which were essential for bringing the gas turbine to life. The honour of bringing the first industrial gas turbine into practice goes to Adolt Meyer of Brown Boveri when the first 4000 H P. set was demonstrated in 1919 on the occasion of Swiss National Exhibition. This was the actual birth of the gas turbine. The use of the gas turbine in military jets brought a great boost in the thermodynamic and aerodynamic knowledge during and after the First World War. So far, all the efforts were concentrated only on open cycle gas turbine and till the end of the war little was known about the closed cycle gas turbine. It was on a suggestion by Ackeret and Keller of Escher Wyss and the enterprising spirit of the same firm which made possible, after long period of development and pioneering work, the advent of closed cycle plant with the establishment of a 2000 kW closed cycle gas turbine plant in 1939. Since then this field has undergone tremendous progress and in the present state of development gas turbines have been applied to a wide range of applications which include industrial, automotive, aircraft, free piston engines and combined gas-steam cycles, and it has a bright future.

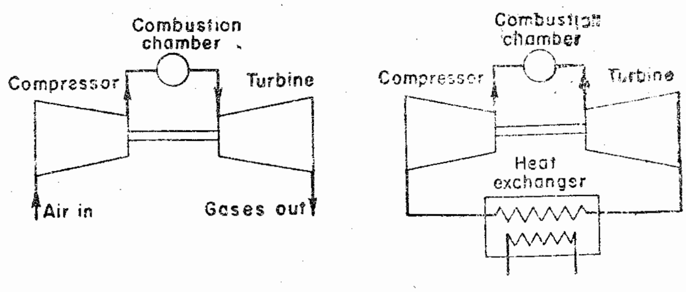
1·2. CLASSIFICATION OF GAS TURBINES

Gas turbines are classified into two main types :

1. Open cycle gas turbine.
2. Closed cycle gas turbine.

In an open cycle gas turbine [*see* Fig. 1·1 (*a*)] air is taken from the atmosphere in the compressor, and after compression its temperature is raised by burning fuel in it. The products of combustion along with the excess air are passed through the turbine, developing power and then exhausted into the atmosphere. For next cycle fresh air is taken in the compressor.

In a closed cycle gas turbine [*see* Fig. 1·1 (*b*)] the air is heated in an air heater by burning fuel externally. The working air does not



(*a*) Open cycle (*b*) Closed cycle

Fig. 1·1. Schematic diagram of open cycle and closed cycle gas turbines.

come in contact with the products of combustion. The hot air expands in the turbine and then cooled in a precooler and supplied back to the compressor. The same working fluid circulates over and again in the system.

Open cycle gas turbines can be further classified into

(*i*) Constant pressure or continuous type, and

(*ii*) Constant volume or explosion type.

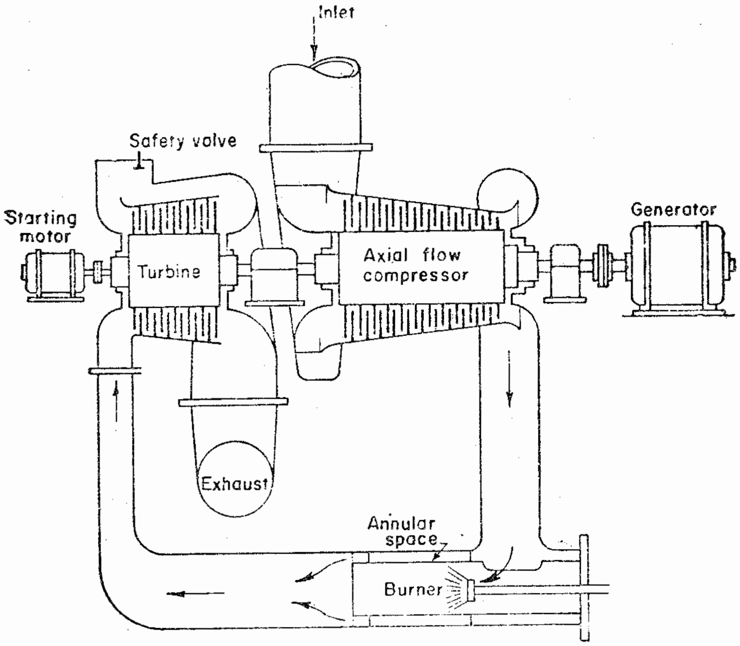
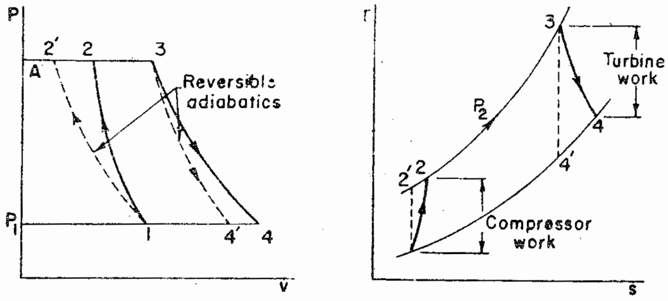


Fig. 1·2. Constant pressure type gas turbine using an axial flow compressor.

(*a*) Constant Pressure Gas Turbine

Figure 1·2 shows the diagram of a constant pressure type gas turbine using an axial flow compressor. Brayton or Joule cycle is the fundamental ideal cycle for constant pressure open cycle gas turbines. It consists of isentropic compression and expansion process and heat addition and heat rejection at constant pressure. Fig. 1·3 shows the corresponding *p-v* and *T s* diagrams.

Air is adiabatically compressed in a centrifugal or axial flow compressor to about 4 to 6 kg_f/cm^2 and is sent to a combustion



(*a*) *p-v* diagram (*b*) *T-s* diagram

Fig. 1·3. *p-v* and *T-s* diagram for a constant pressure gas turbine

chamber where a part of it (about 25 per cent), called primary air, forms a stoichiometric air-fuel mixture and is burnt at constant pressure to about 2000°C, and the rest 75 per cent of the air, called secondary and tertiary air, is fed downstream in the combustion chamber to cool the products of combustion to about 700 to 900°C by mixing with them. A temperature of about 900°C is the upper limit of temperature from metallurgical considerations. The mixture then passes to an impulse or reaction turbine where gases are expanded adiabatically. The work done by the turbine is used to drive the compressor as well as the external load. The compressor consumes about two-thirds of the total power developed by the turbine, and only about one-third is the net power available.

A gas turbine is not a self starting machine. For starting, it is first motored to some minimum speed called the *coming in* speed before the fuel is turned on. A motor of about 5 per cent of the power output of the turbine is provided for this purpose.

(*b*) Constant volume or Explosion Type Gas Turbine

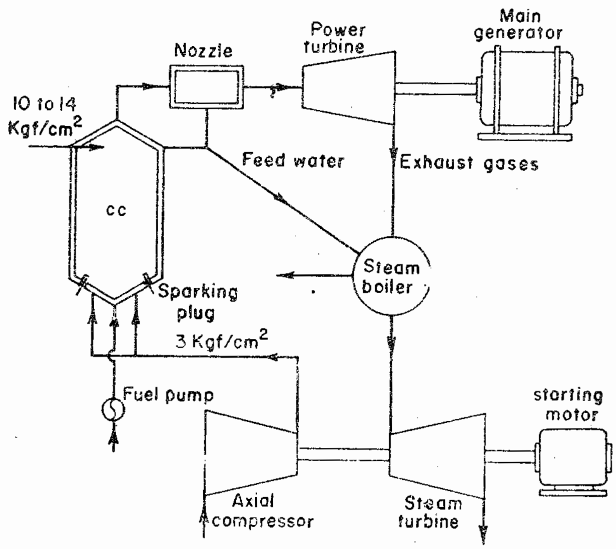


Fig. 1·4. Schematic arrangement of a constant volume Holzwarth gas turbine plant.

Figure 1·4 shows a schematic arrangement of a constant volume Holzwarth type gas turbine plant. Atkinson cycle (*see* Fig. 1·5) is the basis for this power plant. Air from the atmosphere is compressed in the axial flow compressor driven by a separate steam turbine. The compressed air at about 3 kg_f/cm^2 is sent to a combustion chamber. Fuel is supplied with the help of a fuel injector and burnt by a spark plug. The fuel explodes to develop a pressure of about 10 to 14 kg_f/cm^2 and a very high temperature. The products of combustion at a high pressure then expand in a turbine producing power. The exhaust gases, still at a high temperature, are sent to an exhaust boiler. The water used to cool the combustion chamber and nozzle is the feed water of this boiler. A steam turbine, which drives the compressor, is driven by the steam so raised.

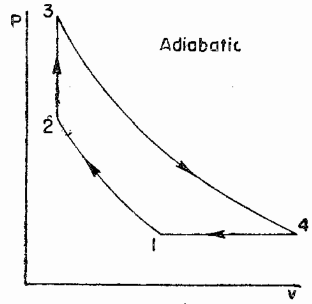


Fig. 1·5. Atkinson cycle on *P-v* diagram.

The constant volume combustion requires that the burning mixture should be isolated from compressor and the turbine ; so use of valves in the combustion chamber is necessary, resulting in an intermittent combustion which inherently impairs the smooth running of the machine. This is a big disadvantage and has been the main reason, in addition to mechanical complexity, of the disappearance of this type of gas turbine plant despite the fact that a constant volume cycle has better thermal efficiency than a constant pressure cycle.

1·3. GAS TURBINE vs. RECIPROCATING I.C. ENGINES

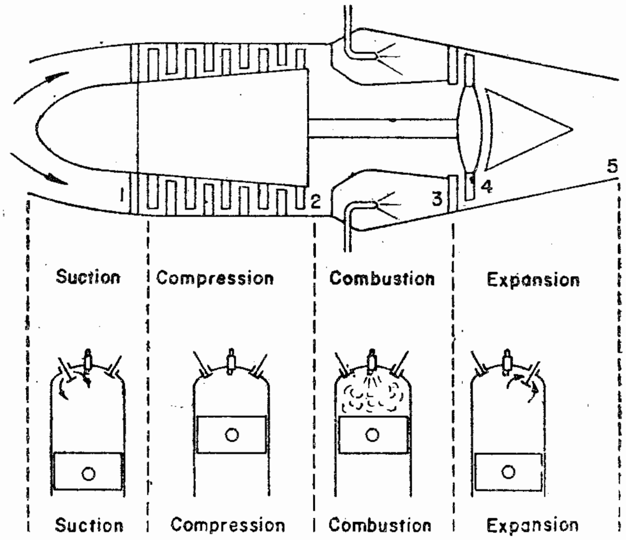
Before a complete analysis of gas turbine plant is done, it is worthwhile to discuss the comparative advantages and disadvantages of gas turbine over reciprocating I.C. Engines. Fig. 1·6 compares the gas turbine cycle with that of a reciprocating I.C. engine. It can be seen that the four basic operations of induction, compression, combustion and expansion are common to both. However, in the gas turbine the flow is continuous while in a reciprocating I.C. engine it is intermittent.

The following is a brief discussion of the advantages and disadvantages of gas turbine over the reciprocating I.C. engines.

Advantages :

1. A gas turbine power plant being a rotary machine has a simple mechanism and higher operational speed. Due to absence of reciprocating parts such as connecting rod and

piston, etc., the vibrations are virtually absent resulting in better and easy balancing and small foundations. Gas turbines, because of



1. Compressor inlet
2. Compressor outlet
3. Turbine inlet
4. Turbine outlet
5. Jet pipe exit

Fig. 1·6. Comparison of a gas turbine and reciprocating I.C. engine cycle.

less number of parts, is easier to maintain. All these characteristics give the gas turbine an exceptional reliability unmatched by reciprocating engines. The maintenance costs of a gas turbine is about 60 per cent that for a diesel engine.

2. Large amount of power developed in a gas turbine plant results in high specific output and the weight of such a plant is several times less than that for a reciprocating I.C. engine. The fact that lower pressure ratios are used in gas turbines also contributes to it by way of lighter construction used. Turbojet, turboprop and reciprocating engines have respectively a weight/power ratio of 0·13 kg/kg of thrust, 0·177 kg equivalent hp and 0·5 kg/hp or more.

3. The capacity of reciprocating engines is limited by poor volumetric efficiency at very high speeds, and detonation and

knocking, while the capacity of gas turbine seems to be unlimited. This is very important since, at present, the reciprocating I.C. engine is a highly developed product while gas turbine plant still has immense development possibilities.

4. Very little or no cooling is required for a gas turbine plant as compared to the large amount of cooling needed for reciprocating engines. This fact enhances the versatility of the gas turbine power plant. Absence of radiator is a big advantage in case of automotive use of gas turbine.

5. The mechanical efficiency of a gas turbine is more than that for reciprocating engines.

6. The lubricating oil consumption is also small. The cost of lubricating oil in the case of a gas turbine is about 0·5 to 1 per cent of the total fuel cost while for a diesel engine it amounts to about 10 to 15 per cent of the total fuel cost.

7. It is rather simpler to control the gas turbine.

8. Gas turbine, especially the closed cycle gas turbine, can burn almost any fuel ranging from kerosene to heavy oil and even peat and coal slurry. This results in a great economic advantage over the reciprocating engine in which the phenomenon of detonation dictates that the quality of fuel used should be comparatively better.

In addition to reduced fuel costs, turbine fuels are less volatile, so fire hazards are reduced. The gas turbine can be used where low quality cheap fuel is available, *e.g.*, natural gas near oil fields.

9. Gas turbines are highly suited to total energy systems where they can be used in conjunction with steam, hot air, free-piston and diesel plants.

10. Gas turbine is a comparatively rugged machine which can be left uncared for a long period.

Disadvantages :

1. The maximum efficiency of a simple gas turbine plant is lower than that of reciprocating engines. To obtain efficiency near that of reciprocating engines, gas turbines should be provided with regenerator, etc., which increases the complexity and cost of the plant.

2. The part load efficiency of gas turbine is poor. The best specific fuel consumption is near maximum output whereas for a reciprocating engine this occurs at about half the rated output. Since most of the applications do not operate at full load except for a short duration, this is a distinct disadvantage.

3. The gas turbine is not a self-starting unit. Moreover, the power required to start is also quite high, so an additional motor of high power is necessary.

4. The cost of manufacture of a gas turbine plant, at present, is higher due to the use of high heat resistant materials and special manufacturing processes used for blades. Particularly for low horse power ratings, a gas turbine becomes very expensive in initial cost as well as in running cost compared to a reciprocating engine.

5. The gas turbine is slow in its acceleration response. This is a big disadvantage for automotive use.

6. Gas turbines run at comparatively high speeds and require a reduction gear to be used for normal industrial applications.

7. The gas turbine is sensitive to the component efficiencies. Any reduction in compressor or turbine efficiency will greatly affect the overall efficiency of the plant. This requires highly matched turbine and compressor combination.

1·4. GAS TURBINE vs. STEAM TURBINE

The following is a brief comparison of gas and steam turbine plants :

1. Steam turbine is the cheapest and most easily available fuel burning plant for high power production purpose. It is in a highly developed stage after long experience and has higher efficiency than that of the gas turbine plant.

2. Steam turbine, due to higher heating surfaces required, is bulkier than a gas turbine plant. This, though, is not very important for stationary plants but is highly critical for mobile power plants.

3. Due to low pressures used in gas turbine plants specific power based on volume basis is lower than that for steam plants.

4. Little or no cooling is required in gas turbine plants whereas a large amount of cooling is required in a steam plant. This makes the gas turbine highly suitable for rail road transportation and aircraft propulsion. Closed cycle plants need about 10 to 20 per cent of the cooling water needed for steam plants.

5. Usually gas turbines run at a higher speed and the blades are also twisted as compared to steam turbine blades.

6. A gas turbine plant can be rapidly started and stopped as compared to a steam plant. Thus it is highly suitable for peak load purposes.

7. Due to the absence of components such as condenser, feed water treatment device, etc., the auxiliary equipment is small for a gas turbine. Thus a gas turbine plant requires less maintenance and is simpler to operate.

8. Manpower required to operate gas turbine plant is much less than for a steam turbine plant. The control is easier and quicker to adopt to variations in load.

1·5. *APPLICATIONS OF GAS TURBINE*

The great success of the gas turbine in the aircraft field has since long attracted many people serving a wide range of industries. The continued development towards increasing the turbine inlet temperature and improving the component efficiencies over the past decade has enlarged the sprectrum of gas turbine applications to areas such as power generation, marine and automotive propulsion, nuclear plants, utility industry, etc. The main reason of such a wide popularity being enjoyed by the gas turbine is its advantages of simplicity, high power/weight ratio, smooth running, less maintenance, multi-fuel capability, suitability for combined cycles and high reliability. The application of the aircraft gas turbine into the industrial and power generation has given a new stimulus to this widening of application spectrum of the gas turbine because of the rich experince obtained in aircraft field. Turboshaft engines are now being used for generation of electricity, auxiliary power drives, marine vehicles and ordnance prime movers, pumping of fluids, etc.

The whole gamut of the applications of the gas turbine can be divided into three broad categories :

1. Industrial
2. Power generation
3. Propulsion.

1.5.1. Industrial Gas Turbine

Since the gas turbine has many qualities which an industrial prime mover must have, its use has spread over a wide range of industrial applications ranging from petro-chemical, thermal-process-industries to general utility industries. The gas turbine itself is quite competitive in first cost and operating cost with the highly developed steam turbine. The lower manpower requirements (some gas turbine can run on unsupervised basis by remote control) and the excellent reliability of up to 99 per cent along with its basic simplicity has made the industrial gas turbine very much popular. Much of the industrial and other applications have been made possible by the adoption of aircraft gas turbine for such uses on the basis of rich experience gained in aviation field.

For the industrial gas turbine, unlike its aircraft counterpart, the frontal area and the space requirements are not critical and their selection for a particular application depends on long life, availability, thermal efficiency, pressure losses and other parameters of importance for the industry under consideration. Generally, the industrial turbine is a continuous duty unit with large power requirements and long life. The following is a brief discussion of some of the many industrial applications of the gas turbine :—

(*a*) *Thermal Process Industries*: For industries which have large mechanical and thermal energy requirements, the industrial gas turbine is ideally suited. Examples of such industries are cement, lime, and light weight aggregate manufacturing unit. The turbine can supply the power needed while the exhaust gases which contain a large amount of heat can be used to raise steam for thermal processing. Modern boilers have an efficiency of more than 90 per cent but the overall steam plant efficiency is not more than 40 per cent whereas the gas turbine with heat recovery or a total energy plant can very easily attain an efficiency figure of 75 to 80 per cent.

(*b*) *Petro-chemical Industry*. Petrochemical industries are unique in that they simultaneously need compressed air, hot gases, and mechanical power in various combinations. The gas turbine provides a heat balance for these requirements which is much superior than all other prime movers. A single gas turbine plant can produce power supply, compressed air and the hot gases or steam simultaneously. In addition to this the gas turbine can be used to drive a large number of generators, compressors and pumps, etc. The extreme reliability needs for process applications is also met by the high degree of availability and reliability inherent in a gas turbine plant.

(*c*) *Gas Compression and Processing.* Gas compression and pumping of the natural gas from gas generation site to user's requires the use of high speed centrifugal compressors. This is because of the small space requirements, less maintenance and less pipe vibrations and pulsations due to the smooth discharge of gases from a rotary compressor. The gas turbine is idealy suited for such applications because it altogether avoids the use of a costly reduction gear and its mechanical troubles. For gas processing such as liquefication, etc., the exhaust energy of the gas turbine can be used either to drive pumps or refrigeration units.

1·5·2. Combined Cycle Power Plants : Total Energy Plants.

The early use of the industrial gas turbine was mainly as a boiler auxiliary operating at variable speed to suit the combustion air needs. The turbine output available was usually only sufficient to drive the compressor, the net surplus of power, if any, was not given any importance. Then the idea of using a complete, separately fired gas turbine in front of an existing boiler to provide it with combustion air with about 17 per cent oxygen and thereby avoiding gas turbine stack losses, grew into prominence and, by 1960, the idea became a reality with the development of high temperature alloys and better component efficiency available due to advances in the state of art.

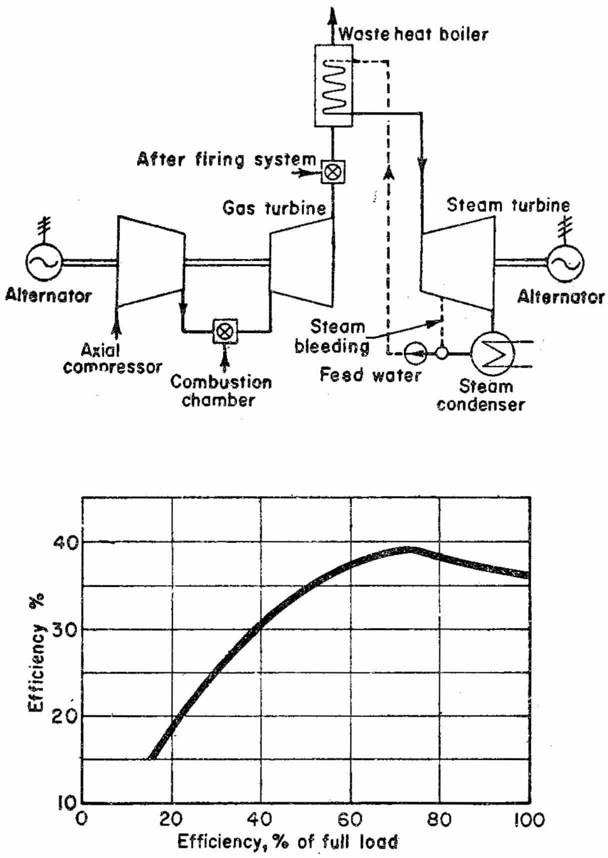


Fig. 1·7. Combined steam and gas turbine cycle.

Such combined cycles called "high-efficiency cycles" resulted in a gain of about 5 per cent in heat rate over the best steam cycle at somewhat lower capital cost. Fig. 1·7 shows the schematic diagram of a combined gas and steam turbine unit and the efficiency obtainable from it. The exhaust from the gas turbine with about 17 per cent oxygen, is used in a waste heat boiler in which additional fuel is burnt and steam is raised to be used by a steam turbine. Since the boiler is supplied with hot gases at a temperature of about 375°C to 475°C depending upon the gas turbine, the usual air heater exchanging heat with stack gases is not necessary. Rather an economiser can be used to keep stack gas temperature low. Bled steam can also be used for feedwater heating along with an economiser. Actually there can be innumerable variations of feed circuit giving minor advantages in net gain, cost or convenience depending upon the personal judgment or preference of the designer.

The reason of gain in efficiency is that by transferring some fuel to the gas turbine, additional output obtained is greater than the loss

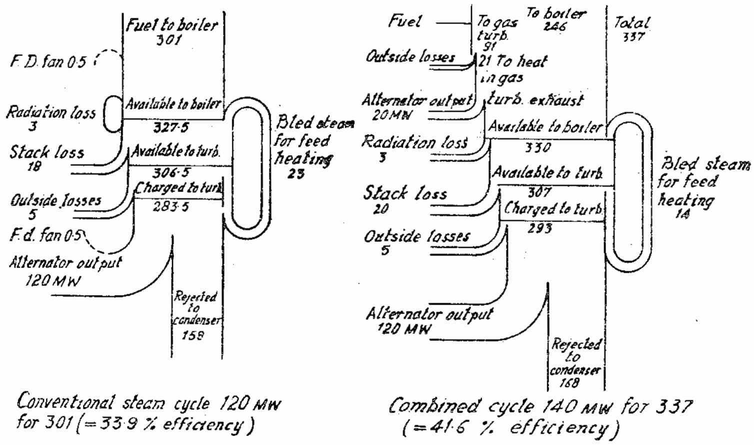


Fig. 1·8. Sankey diagram comparing the heat balance of a steam cycle with a combined cycle.

in steam turbine with a given total oxygen burning. The gas turbine used have almost cent per cent efficiency in that nearly all of its heat supply is either converted to power or reused in boiler. The magnitude of the total gain in efficiency depends upon the temperature of the gas turbine outlet and many other factors. Fig. 1·8 shows a Sankey diagram comparing the heat balance of a steam cycle with that of a combined cycle.

Ideally the gas turbine output is fixed at about 20 per cent of the steam turbine output. However, some variations occur because of the matching requirements of the flow to be passed from gas turbine to steam boiler for various applications. In 1971, a new idea was offered by Sulzer whereby the ratio of gas turbine output to steam turbine can be doubled. Fig. 1·9 shows such a scheme in which a 57 MW gas turbine is matched with a 391 MW steam turbine. A second gas turbine discharges its gases to the chimney through a duplicate economiser. The whole of the feed water heating is done by flue gases and no high pressure steam is bled for the purpose.

Another type of plant used is the recuperator plant or total energy plant shown in Fig. 1·10. In this the gas turbine is the main power source and some of its exhaust heat is recuperated in a steam cycle. The choice of the steam condition is not free (whereas it is free in case of combined cycle) but depends upon the temperatures in the recuperator. Additional fuel can be burnt in exhaust gases to increase the superheat of the steam in order to increase the steam turbine output.

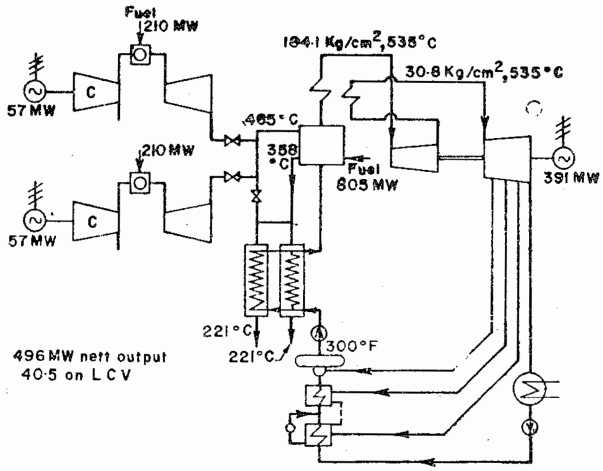


Fig. 1·9. Sulzer scheme to double gas turbine output to steam turbine output.

These plants are widely used in industries where process heat as well as power is required. Typical applications are chemical plants, textile plants and also in some electric utility plants.

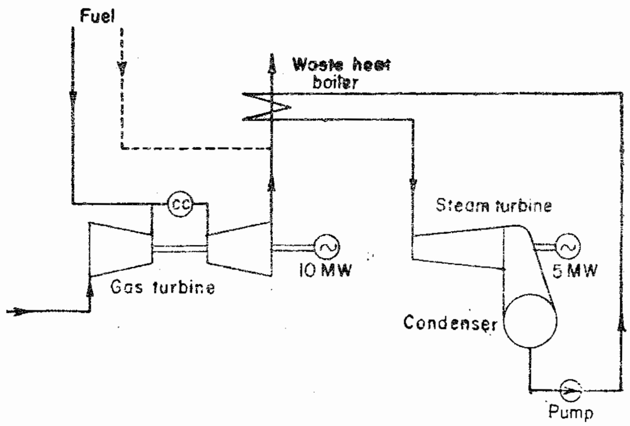


Fig. 1·10. Basic concept of recuperator cycle.

Combined diesel and gas turbine plants (CODAG) are also in use, specially in marine installations. By such a combination the part load operation of the turbine can be highly improved.

Other applications. Among other important applications are turbocharging of aircraft, diesel engines, and certain types of petrol engines, portable units for supply of compressed air, portable electricity generating sets, etc.

1·5·3. Power Generation

The rapid development in the gas turbine field and specially the use of aircraft gas turbine experience in industry has established gas turbine in power generation field. It is now widely used for base load, peaking and standby operation. Due to its ability to start from cold and carry the full load in less than two minutes, it is especially suited for the peaking and standby purposes.

The gas turbine as a part of the combined high efficiency cycle is now widely used as a base load prime mover. During the last decade the electric utility industry has experienced a tremendous and continuing load growth. The large delivery time for steam plants and non-availability of nuclear plants along with the possibility of using a cheap fuel has resulted in many "quasi base-load" gas turbine units to be established in this period. The short supply time, cheap fuel and higher availability were the main reasons for the use of gas turbines in many peaking and standby load applications.

Since the conventional steam generation methods using fossil fuels have serious environmental and long-term cost problems, the gas turbine with its characteristic advantages will be a popular choice in future baseload power plans. The combined cycle power plants would be an automatic choice due to higher efficiency, compactness, and reliability.

For arid zones, where water supply is a major problem, gas turbine electric utility plants are attractive choice.

1.5·4. Propulsion

Of the three modes of transport, namely, air, land and water, the gas turbines have enjoyed phenominal success for the first, and attracted by this success it has been applied to the other two fields also.

The use of the gas turbine for the aircraft field hardly needs any stress. It is only because of the development in the gas turbine technology that the present aviation progress has been possible. Turbojet, turboprop, by-pass engine and even the helicopters are examples of successful application of the gas turbine in the aviation field.

The great success in the aviation field attracted use of gas turbine in automobiles and now a large number of automobile models have come up in the power range of 140 to 300 h.p. The advantages of using a gas turbine in an automobile are many and are listed below.

1. Uniform torque and absence of vibrations gives smoother operation and better passenger comfort.

2. Light, compact, and less bulky.

3. Reduced air pollution due to complete combustion.

4. Cheap fuels can be used.

5 Absence of a clutch and gearbox make it easy to control and results in reduced maintenance. Also, the number of components in a gas turbine are much less than in a reciprocating engine.

6. Absence of cooling water.

7. Lubricating oil consumption is about 1/30 to 1/40th of that in reciprocating engine.

8. Since there are a few bearings cold starting is very easy even up to a temperature of $-50°C$.

But the gas turbine also has certain disadvantages for automotive applications, namely, high starting torque required necessitates high speed cranking, part load efficiency is poor, engine breaking is not easily accomplished, and above all due to large inertia of the rotating parts delay in acceleration is marked. So before it can be used in automobiles it requires certain modifications in the plant arrangements. A detailed discussion of the automotive gas turbine is included in article 2·11.

Combined diesel and gas turbine plant (CODAG) is specially suited and is widely used for marine propulsion due to its good fuel consumption characteristics, light weight, low space requirements, fast manoeuvering and high availability.

In a CODAG plant, the prime movers drive a common gear via free wheeling clutches which allows independent operation of diesel engine or the gas turbine for low speed manoeuvers. Diesels are used, with ship speed and direction controlled by adjustable pitch propellers. When cruising, the diesels are run close to their maximum speed and the propeller pitch is set to give optimum efficiency. At higher speeds the gas turbine can be started within a very short time. With a suitable free wheeling device, the gas turbine can be started without stopping the engine and at peak speed the output of the gas turbine is increased and the diesels are automatically declutched and may be run at idling speed or shut down.

The most important advantage of the marine gas turbine is the ability to change engine quickly, thus reducing dockyard work on board to be minimum. By using air craft engines in marine propulsion devices an engine can be replaced within a few hours and that is why almost all warships use aircraft type gas turbines in view of the high cost of modern weapons and the requirement of high availability. The marine gas turbine operates in a more corrosive atmosphere due to presence of sea salt in air and fuel and use of cheaper fuels.

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EXERCISES 1

1·1. How old is the concept of turbines ? Briefly trace the development of gas turbines through various stages.

1·2. What were the two main reasons for the slow development of gas turbines ?

1·3. What are the two main types of gas turbines ? What is the essential difference between them ?

1·4. Name the three major components of a gas turbine plant.

1·5. Explain with the help of a schematic diagram the working of a simple open cycle constant pressure type gas turbine. On what cycle does it work ?

1·6. Name the two types of compressors used in gas turbines.

1·7. What are the pressure and temperature ranges used in gas turbines ?

1·8. What air-fuel ratios are used in gas turbines and why ? What is the function of secondary air ? Why is it necessary ?

1·9. How much is the power consumed by the compressor of the gas turbine ?

1·10. Is gas turbine a self-starting machine ? How gas turbine is started ?

1·11. Explain with the help of a schematic diagram the working of a simple open cycle constant volume or explosion type gas turbine. On which cycle does it work ?

1·12. Why constant volume gas turbine has become obsolete ?

1·13. What are the advantages of an open cycle gas turbine as compared to reciprocating internal combustion engines ? What are the disadvantages ?

1·14. Compare the relative merits and demerits of gas and steam turbines.

1·15. Discuss the various applications of gas turbines.

1·16. Discuss the application of gas turbines in the automotive field.

2

ANALYSIS OF GAS TURBINE CYCLES

2·1. THE BASIC CYCLE

Figure 2·1 shows the Brayton or Joule cycle, the basic gas turbine cycle, on P-v and T-s diagrams.

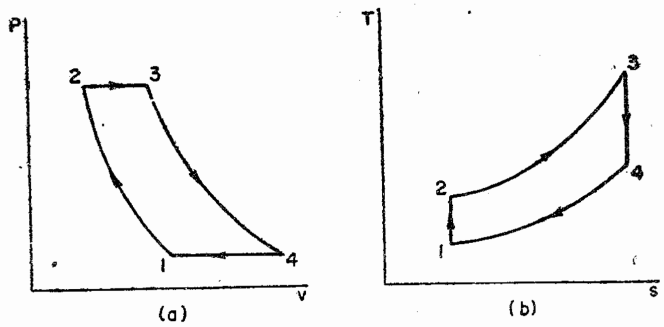


Fig. 2·1. *P-v* and *T-s* diagrams for Brayton or Joule cycle.

The air is compressed from 1 to 2 in a compressor and heat is added to this air during the process 2 to 3 which takes place in the combustion chamber. Process 3 to 4 represents the expansion of gases in a turbine. The fact that the line 3-4 is longer than the line 2-1 on T-s diagram makes it possible to extract more power by the turbine than needed to drive the compressor. In the ideal cycle processes of compression (1-2) and expansion (3-4) are assumed as isentropic and processes of heat addition (2-3) and heat rejection (4-1) are assumed as constant pressure processes.

2·2. ANALYSIS OF SIMPLE GAS TURBINE CYCLE

The actual performance of a gas turbine plant differs considerably from the Brayton cycle of Fig. 2·1 because of the deviations from the ideal cycle. These deviations include friction, shock, heat transfer and aerodynamic losses in compressor and turbine, losses in combustion chamber, piping and mechanical losses, etc With such a large number of variables affecting the performance of the gas turbine plant it will be really difficult to estimate its performance unless certain assumptions are made. To simplify the analysis the following assumptions are made :

1. There is no pressure loss in the combustion chamber and in piping, etc.

2. There is no increase in the rate of mass flow due to the addition of the fuel.

3. The specific heat remains constant at all temperatures and is same for compressor as well as turbine flow.

4. Radiation and mechanical losses are neglected.

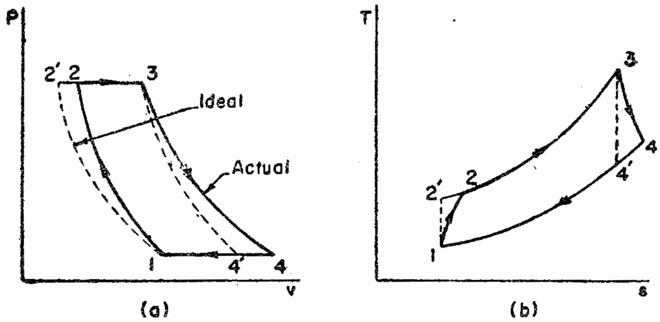


Fig. 2·2. Actual and ideal cycle on P-v and T-s diagram.

Assuming a flow of 1 kg. of air and with reference to Fig. 2·2 which shows both actual and ideal cycle, the following analysis is made :

Efficiency

Efficiency of the cycle

$$=\frac{\text{Turbine work}-\text{Compressor work}}{\text{Heat supplied}}$$

Work required for compressor

$$=h_2-h_1$$
$$=c_p(T_2-T_1)$$

Heat supplied in combustion chamber

$$=h_3-h_2$$
$$=c_p(T_3-T_2)$$

Work delivered by turbine

$$=h_3-h_4$$
$$=c_p(T_3-T_4)$$

∴ Thermal efficiency

$$=\frac{c_p(T_3-T_4)-c_p(T_2-T_1)}{c_p(T_3-T_2)}$$

$$=\frac{(T_3-T_4)-(T_2-T_1)}{T_3-T_2} \qquad (2\cdot1)$$

$$=\frac{T_3\left(1-\frac{T_4}{T_3}\right)-T_1\left(\frac{T_2}{T_1}-1\right)}{T_3-T_1 . \frac{T_2}{T_1}}$$

The isentropic efficiency of the turbine and the compressor are defined as :

Isentropic efficiency of turbine, η_t

$$=\frac{\text{Actual heat drop}}{\text{Isentropic heat drop}}$$

$$=\frac{h_3-h_4}{h_3-h_4'} \qquad (2\cdot2a)$$

or

$$\eta_t=\frac{c_p(T_3-T_4)}{c_p(T_3-T_4')}$$

$$=\frac{T_3-T_4}{T_3-T_4'} \qquad (2\cdot2b)$$

$$=\frac{\text{Actual temperature drop}}{\text{Isentropic temperature drop}}$$

Isentropic efficiency of compressor,

$$\eta_t=\frac{\text{Isentropic increase in enthalpy}}{\text{Actual increase in enthalpy}}$$

$$=\frac{h_2'-h_1}{h_2-h_1} \qquad (2\cdot3a)$$

or

$$\eta_c=\frac{c_p(T_2'-T_1)}{c_p(T_2-T_1)}$$

$$=\frac{T_2'-T_1}{T_2-T_1} \qquad (2\cdot3b)$$

$$=\frac{\text{Isentropic temperature rise}}{\text{Actual temperature rise}}$$

From equation (2·2b), we have

$$T_4=T_3-(T_3-T_4')\eta_t$$

or

$$\frac{T_4}{T_3}=1-\left\{1-\frac{T_4'}{T_3}\right\}\eta_t$$

$$=1-\eta_t\left[1-\left(\frac{P_4}{P_3}\right)^{\frac{\gamma-1}{\gamma}}\right]$$

$$\left[\because \quad \frac{T_4'}{T_3}=\left(\frac{P_4}{P_3}\right)^{\frac{\gamma-1}{\gamma}}\right]$$

Assuming equal pressure ratio for compressor and turbine, we have

$$\frac{T_4}{T_3}=1-\eta_t\left[1-\left(\frac{P_1}{P_2}\right)^{\frac{\gamma-1}{\gamma}}\right]$$

Similarly, from equation (2·3*b*), we get

$$\frac{T_2}{T_1}=1+\frac{1}{\eta_c}\left[\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}-1\right]$$

Putting the values of $\frac{T_4}{T_3}$ and $\frac{T_2}{T_1}$ in equation (2·2), we get

$$\eta=\frac{T_3\,\eta_t\left[1-\left(\frac{P_1}{P_2}\right)^{\frac{\gamma-1}{\gamma}}\right]-\frac{T_1}{\eta}\left[\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}-1\right]}{T_3-T_1\left[1+\frac{1}{\eta_c}\left\{\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}-1\right\}\right]}$$

$$=\frac{T_3\eta_t\left(1-\frac{1}{R}\right)-\frac{T_1}{\eta_c}\left(R-1\right)}{T_3-T_1\left[1+\frac{1}{\eta_c}\left(R-1\right)\right]}$$

where $$R=\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}=\frac{T_2'}{T_1}=\frac{T_3}{T_4'}$$

or $$\eta=\frac{T_3\cdot\frac{\eta_t}{R}-\frac{T_1}{\eta_c}}{\frac{(T_3-T_1)}{(R-1)}-\frac{T_1}{\eta_c}} \qquad (2·4)$$

For an ideal joule cycle,

$$\eta_c=\eta_t=1$$

$$\therefore \qquad \eta=\frac{\dfrac{T_3}{R-T_1}}{\dfrac{(T_3-T_1)}{(R-1)-T_1}}$$

$$=\frac{(T_3-T_1R)}{(T_3-T_1-T_1R+T_1)}\times\frac{R-1}{R}$$

$$=1-\frac{1}{R}$$

$$=1-\frac{1}{\left(\dfrac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}} \qquad (2{\cdot}5)$$

From equation (2·5) it is evident that the ideal air standard efficiency is independent of the turbine inlet temperature and depends only on pressure ratio and the value of the ratio of specific heats. However, in an actual cycle, with irreversibilities in the compression and expansion processes, the thermal efficiency depends both upon the pressure ratio as well as turbine inlet temperature, besides being affected by compressor and turbine efficiencies.

Work Ratio

The work done is represented by the *work ratio* which is defined as the ratio of the net work output to that of the work done by the turbine.

$$\text{Work ratio}=\frac{\text{net work output}}{\text{work done by the turbine}}$$

$$=\frac{c_p(T_3-T_4)-c_p(T_2-T_1)}{c_p(T_3-T_4)}$$

$$=1-\frac{T_2-T_1}{T_3-T_4}$$

$$=1-\frac{T_1}{T_3}\left\{\frac{T_2/T_1-1}{1-T_4/T_3}\right\}$$

$$=1-\frac{T_1}{T_3}\cdot\frac{R-1}{1-\dfrac{1}{R}}\cdot\frac{1}{\eta_c\,\eta_t}$$

or $$\text{Work ratio}=1-\frac{T_1}{T_3}R.\frac{1}{\eta_c\,\eta_t} \qquad (2{\cdot}6)$$

The net work output,

$$w=c_p\,(T_3-T_4)\eta_t-c_p(T_2-T_1)/\eta_c$$

$$=c_p\, T_3\left(1-\frac{1}{R}\right)\eta_t - c_p\, T_1(R-1)/\eta_c$$

$$=c_p\,(R-1)\left[\eta_t\,\frac{T_3}{R}-\frac{T_1}{\eta_c}\right] \tag{2·7}$$

Thus the work output of a simple cycle depends on turbine and compressor inlet temperatures and the pressure ratio.

For the ideal simple cycle gas turbine plant (Eq. 2·6),

$$\text{Work ratio}_{ideal}=1-\frac{T_1}{T_3}\,R \tag{2·8}$$

and net work output$_{ideal}$

$$=c_p(R-1)\left(\frac{T_3}{R}-T_1\right) \tag{2·9}$$

and we see that the work specific output (defined as the work done per kg of the working medium) the cycle, efficiency and air rate (defined as the air flow per unit output) are the three parameters which decide the size of a given machine. Actually, cycle efficiency includes the first parameter, namely the specific work output. Higher the cycle efficiency and higher the air rate, lower is the machine size.

2·3. EFFECT OF THERMODYNAMIC VARIABLES ON THE PERFORMANCE OF SIMPLE GAS TURBINE PLANT

From Eq. 2·4 it can be seen that under the assumptions made above there are five main thermodynamic variables affecting the performance of a single gas turbine plant. These are compressor and turbine inlet temperatures, T_1 and T_3, the pressure ratio, P_2/P_1, and compressor and turbine efficiencies, η_c and η_t. The effect of each one of these is discussed below.

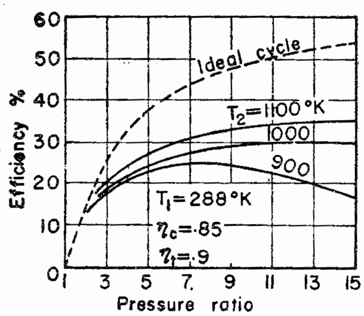


Fig. 2·3. Gas turbine cycle efficiency for different inlet temperatures.

2·3·1. Effect of turbine inlet temperature

As already discussed the ideal air standard efficiency of a simple gas turbine plant is independent of turbine inlet temperature. This is shown by the dotted line in Fig. 2·3. However, it is clear from Eq. (2·4) that turbine inlet temperature greatly affects the efficiency of an actual plant. In Fig. 2·3 the thermal efficiency have been plotted against pressure ratio for different values of turbine inlet temperatures. In drawing these curves suitable values for compressor inlet temperature, compressor and turbine isentropic efficiencies have been assumed. An increase in T_3 increases the work output from the turbine which tends to increase the thermal efficiency at a given pressure ratio but at the same time the heat supplied in the combustion chamber increases decreasing the efficiency. The rate of increase in the turbine work is greater than the rate of

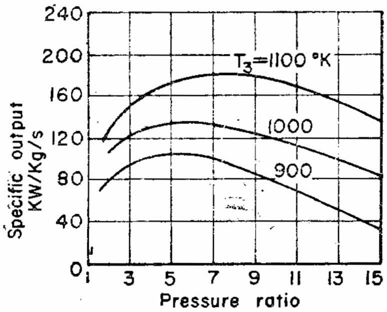


Fig. 2·4 Effect of T_3 on specific work output.

increase in heat added and, hence, for all pressure ratios increasing the turbine inlet temperature increases the cycle efficiency at a steady but decreasing rate. This is clear from the distance between constant turbine inlet temperature lines. The maximum temperature of the cycle is limited by metallurgical considerations to about 1100 K.

The effect of T_3 on specific work output is given in Fig. 2·4.

2·3·2. Effect of compressor inlet temperature

For a given peak temperature the effect of compressor inlet temperature is two fold. An increase in T_1 increases the work input to the compressor, thereby, reducing thermal efficiency, but at the same time temperature T_2 is increased and the heat supplied for obtaining a given value of T_3 reduces which tends to increase the efficiency. The work output of the turbine is not affected by T_1. Since the rate of increase of compressor work is greater than the rate of reduction in heat supply, net effect of increasing T_1 is a decrease in the efficiency of the simple gas turbine plant. The net work output decreases with an increase in T_1. A reduction in T_1 increases the shaft output as well as efficiency. But at high altitude, due to reduction in mass flow at lower pressure, the work output is decreased but a gain in thermal efficiency is obtained.

2·3·3. Effect of pressure ratio

The effect of pressure ratio on the thermal efficiency and the work output of a simple gas turbine plant is shown in Figs. 2·3 and 2·4. In Fig. 2·3 the dotted line shows that the efficiency of an ideal joule cycle will indefinitely increase with pressure ratio which is not true in practice due to irreversibilities in compression and expansion. As the pressure ratio is increased for a given value of turbine inlet temperature T_3, the efficiency increases up to a maximum value and then starts decreasing clearly indicating that there is an optimum pressure ratio for a given value of turbine inlet temperature.

Higher pressure ratio cycles are more sensitive to the efficiency of the compressors and turbines. As the pressure ratio increases the work done in turbine increases and less heat is rejected to atmosphere, so the efficiency and work both increase. But beyond a certain value of pressure ratio the efficiency of compression reduces, more compression work is required and compressor outlet temperature increases. T_2 and T_3 converge and it is not possible to add any heat in the combustion chamber, reducing work output to zero. Thus it should be noted that limitation of maximum turbine inlet temperature imposes a limit on pressure ratio which can be used.

By comparing Figs. 2·3 and 2·4 it can be seen that the pressure ratio for maximum efficiency is different from the pressure ratio for maximum specific work output. Hence the choice of pressure ratio will be a compromise. The optimum pressure ratio can be calculated for a given value of compressor and turbine efficiencies and turbine inlet temperature with the help of equation (2·9).

For an ideal cycle, with isentropic compression and expansion the work output is given by :

$$w = c_p(R-1)\left[\frac{T_3}{R} - 1\right] \tag{2·9}$$

Differentiating Eq. (2·9) with respect to R assuming T_3 and T_1 to be constant, we get

$$R = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = \sqrt{\frac{T_3}{T_1}} \tag{2·10}$$

or

$$r_p = \frac{P_2}{P_1} = \left(\frac{T_3}{T_1}\right)^{\frac{\gamma}{2(\gamma-1)}}$$

The optimum pressure ratio for maximum work output can be obtained by differentiating equation (2·7) and equating it to zero.

$$\frac{dw}{dR} = c_p(R-1)\left(-\eta_t\frac{T_3}{R^2}\right) + \left(\eta_t\frac{T_3}{R} - \frac{T_1}{\eta_c}\right)c_p = 0$$

or

$$R = \sqrt{\eta_c\eta_t T_3/T_1}$$

or

$$\frac{P_2}{P_1} = (\eta_c\eta_t T_3/T_1)^{\frac{\gamma}{2(\gamma-1)}} \tag{2·11}$$

Differentiating Eq. (2·4) with respect to R and putting it to zero, the pressure ratio for maximum overall cycle efficiency can be shown to be given by :

$$R=\frac{T_3/T_1}{1+\sqrt{(T_3/T_1-1)\left(\frac{1}{\eta_c\eta_t}-1\right)}}$$

or

$$\frac{P_2}{P_1}=\left[\frac{T_3/T_2}{1+\sqrt{(T_3/T_1-1)\left(\frac{1}{\eta_t\eta_c}-1\right)}}\right]^{\frac{\gamma}{\gamma-1}} \qquad (2·12)$$

Since $$\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}=\frac{T_2}{T_1}=\frac{T_3}{T_4}$$

We can put this in the maximum output condition of Eq. (2·10) and get,

$$\frac{T_2'}{T_1}\times\frac{T_1}{T_4'}=\frac{T_3}{T_1}$$

or

$$T_2'=T_4' \qquad (2·13)$$

i.e. for maximum work output, the temperature at the end of expansion is equal to that at the end of compression for an ideal simple gas turbine plant.

2·3·4. Effect of turbine and compressor efficiencies

From Eq. (2·4) it is evident that the cycle efficiency greatly depends on the efficiencies of the turbine and the compressor. For a given value of turbine and compressor inlet temperatures the efficiency of a simple cycle is linearly proportional to turbine efficiency as shown in Fig. 2·5. The effect of compressor efficiency is not linearly related in that it affects the heat supplied as well as the work output. A decrease in compressor efficiency decreases the heat supplied but this decrease in heat supplied is more than offset by the increase is compressor work.

A change of 1 per cent in the efficiencies of compressor and turbine can result in 3 to 5 per cent change in the cycle efficiency. Usually the turbine has a higher efficiency than the compressor and as turbine develops much more power than the compressor consumes (the net output is the difference between turbine and compressor work), a loss in turbine efficiency reduces the cycle efficiency by a larger amount than would be the case if compressor efficiency is reduced by the same amount (*see* Fig. 2·6).

Fig. 2·5 shows how improvement in turbine and compressor efficiencies affect the thermal efficiency of the cycle for a given T_1 and T_3 over a range of pressure ratios.

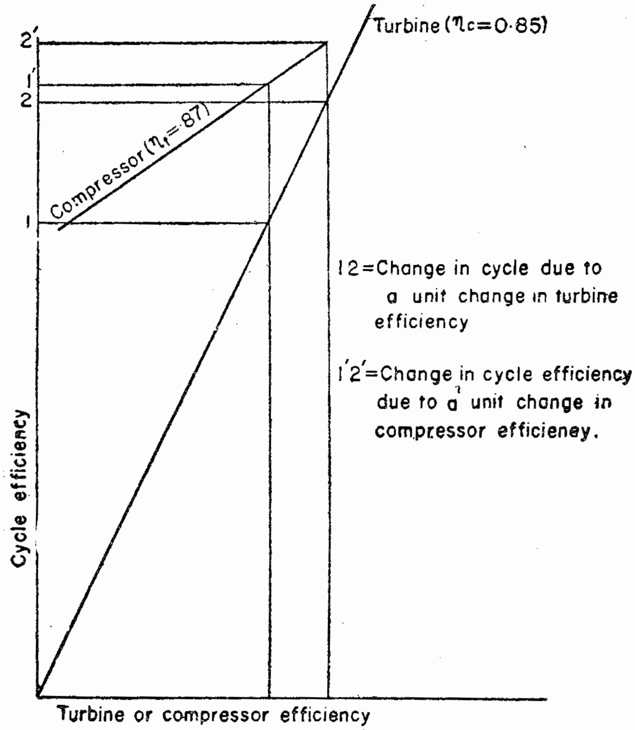


Fig. 2·5. Variation in overall efficiency with turbine or compressor efficiencies

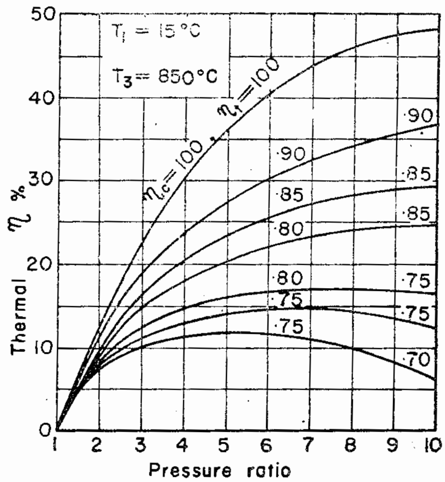


Fig. 2·6. Effect of compressor and turbine efficiency on cycle efficiency

It would be worth while, at this stage, to consider the role played by component efficiencies in the development of gas turbines. During the early development period, due to lack of thermodynamic and aerodynamic knowledge and the absence of high heat resistant materials the compressor and turbine efficiencies were about 0·67 and 0·6 respectively and the maximum turbine inlet temperatures were also low. Since the compressor consumes about 60 to 70 per cent of the total power developed by the turbine in present-day plants which have compressor and turbine efficiencies of about 0·8 to 0·9 and high turbine inlet temperature of about 900°C, it is not a surprise if those old gas turbine plants, with such low compressor and turbine efficiency figures, could not produce a net power output, (some most earliest plants even produced negative power) and the gas turbine plant had to wait for thermodynamic, aerodynamic and metallurgical developments for about 50 years to compete with other power plants. Fig. 2·6 shows the effect of compressor and turbine efficiency on cycle efficiency.

2·4. IMPROVEMENTS IN SIMPLE GAS TURBINE CYCLE

The efficiency and the specific work output of the simple gas turbine cycle is quite low inspite of increased component efficiencies. Therefore, certain modifications in the simple cycle are necessary, and for that we can look towards three main processes—compression, heat supply and expansion in the simple cycle. In what follows in the next few pages these modifications are discussed.

2·4·1. Regenerative gas turbine cycle

One of the main reasons for the low efficiency of a simple gas turbine plant is the large amount of heat which is rejected in the turbine exhaust. Due to limitations of maximum turbine inlet temperature and the pressure ratio which may be used with it, the turbine exhaust temperature is always greater than the temperature at the outlet of compressor. So, if this temperature difference is used to increase the temperature of the compressed air before entering the combustion chamber and, thereby, reducing the heat which must be supplied in the combustion chamber for a given turbine inlet temperature, an improvement in efficiency can be attained. This utilization of the heat in turbine exhaust can be affected in a heat exchanger called *re-generator*.

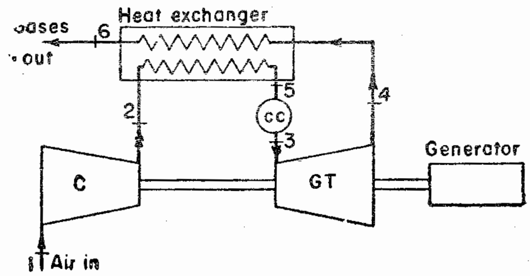


Fig. 2·7. Regenerative gas turbine cycle

Fig. 2·7 shows a schematic diagram of such an arrangement. The exhaust gases from the turbine pass through the regenerator and give their heat to the compressed air before it enters the combustion chamber, thereby, reducing the amount of heat which must be supplied in the combustion chamber to get a given turbine inlet temperature T_3. Thus regeneration improves fuel economy. The power output will be slightly reduced because of the pressure losses in regenerator and its associated pipe work.

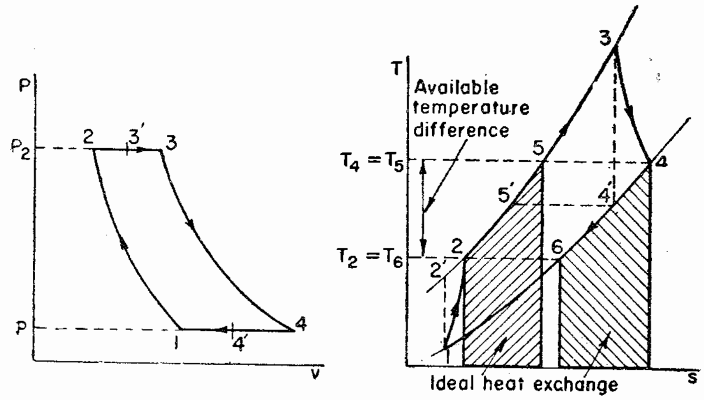


Fig. 2·8. P-v and T-s diagram for regenerative gas turbine cycle.

For a regenerative gas turbine cycle with an ideal regenerator the P-v and T-s diagrams are drawn in Fig. 2·8. It is evident that the work output is unchanged.

For complete regeneration

$$T_4 = T_3 \quad \text{and} \quad T_2 = T_6$$

The efficiency of the regenerator cycle with ideal regenerator is given by :

$$\text{Efficiency} = \frac{c_p(T_3 - T_4) - c_p(T_2 - T_1)}{c_p(T_3 - T_4)}$$

$$= \frac{(T_3 - T_4) - (T_2 - T_1)}{(T_3 - T_4)}$$

$$= 1 - \frac{T_2 - T_1}{T_3 - T_4}$$

$$= 1 - \frac{(T_2' - T_1)/\eta_c}{(T_3 - T_4')\eta_t}$$

$$= 1 - \frac{T_1}{T_3} \cdot \frac{1}{\eta_c \eta_t} \cdot (r_p)^{\frac{\gamma - 1}{\gamma}} \qquad (2·14)$$

and for isentropic compression and isentropic expansion,

$$\text{Efficiency}=1-\frac{T_1}{T_3}(r_p)^{\frac{\gamma-1}{\gamma}} \tag{2·15}$$

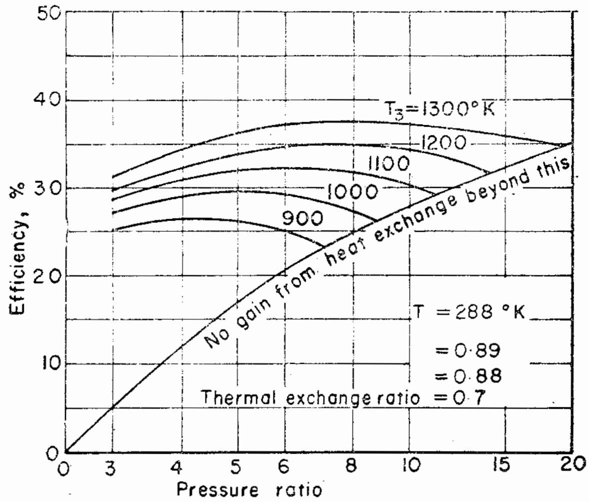


Fig. 2·9. Efficiency of regenerative gas turbine cycle.

Thus, we see that the efficiency of a regenerative cycle depends upon the turbine and compressor inlet temperatures and the pressure ratio used.

For a given inlet pressure and pressure ratio the cycle efficiency increases with increase in turbine inlet temperature (see Fig. 2·9) because more generation can be affected.

The efficiency decreases with an increase in pressure ratio with a given turbine inlet temperature, which is reverse in trend from what happens in a simple cycle (see Fig. 2·9). As the pressure ratio is increased the turbine exhaust temperature T_4 decreases, and in the limit approaches the compressor outlet temperature T_2. Also the compressor outlet temperature T_2 increases. Thus the amount of heat which can be recovered continuously decreases and in the limit, when $T_2=T_4$, no recovery can be affected. Any further increase in pressure ratio will result in heat transfer from air to exhaust gases rather than from gases to air because $T_2>T_4$. At the limit where the regenerative cycle efficiency curves meet the simple cycle curve $T_2=T_4$, and by equating the two efficiencies, we get

$$\eta=1-\frac{T_1}{T_3}(r_p)^{\frac{\gamma-1}{\gamma}}=1-\left(\frac{1}{r_p}\right)^{\frac{\gamma-1}{\gamma}}$$

or $$\frac{T_1}{T_3}=\left(\frac{1}{r_p}\right)^{\frac{2(\gamma-1)}{\gamma}}$$

or $$r_p=(T_3/T_2)^{\frac{\gamma}{2(\gamma-1)}}$$

and at this value of r_p the regenerator is superfluous. In actual operation the pressure ratio used is always less than that given by Eq. 2·18. One advantage from this low optimum pressure ratio is that the part load efficiency of the cycle is maintained almost equal to that at fuel load.

The efficiency is maximum at $r_p=1$ (see Fig. 2·9) where it is equal to the Carnot cycle efficiency, *i.e.* $\frac{T_3-T_1}{T_3}$ since the heat addition and heat rejection take place at maximum and minimum temperatures respectively. But this is of no practical significance as no work can be obtained at this pressure ratio.

Actually it is not possible to recover all the heat from the exhaust because it will require an infinite heat transfer surface area. The efficiency of regeneration is given by the thermal ratio defined as :

$$\text{Thermal ratio}=\frac{\text{actual heat transfer}}{\text{maximum possible heat transfer}}$$
$$=\frac{T_5-T_2}{T_4-T_2}$$

where T_5 is the temperature of the air after regeneration.

Efficiency of regeneration varies from 0·5 to 0·75 in practice. An increase in thermal ratio requires an increase in surface coefficient which can be obtained by increased velocity. This, however, results in increased pressure losses due to turbulence and the gain in recovery of exhaust gas heat due to increased thermal ratio is partly offset by increase in pressure drop. The pressure losses occurring in hot and cold sides of the regenerator usually amount to about 0·03 kgf/cm² and 0·04 to 0·2 kgf/cm², respectively. A pressure drop on the cold side, *i.e.* in the pressure of compressed air, increases the required pressure ratio and hence increases the compression work. Pressure drop on hot side decreases the turbine output by increasing its back pressure. Both these results in lower efficiency. Since turbine work output is substantially more than the compressor work input, pressure losses on hot side are more important than that on cold side. Generally counterflow type of regenerator gives greater regeneration efficiency than the parallel-flow type regenerator.

The work output of a regenerative cycle is substantially the same as that of a simple cycle except some loss due to pressure losses in the regenerator and the associated pipings.

Liquid metal regeneration

The poor part load efficiency of the gas turbine and hence, its specific fuel consumption can be improved by the use of regenerative turbine cycle. The minimum specific fuel consumption for a regenative turbine is obtained at a lower pressure ratio resulting in the lower cost and higher efficiency for the components like compressor, etc., because small blades can be used and losses in diffuser are also low. Another advantage of regeneration is that low specific fuel consumption occurs over a wide range of turbine inlet temperatures and is optimised at high levels of specific power.

Any reduction in weight and increase in the compactness and thermal efficiency of the turbine will be welcome because it will not only establish the current applications but will also open new vistas which hitherto have been barred due to economic considerations. Therefore, it goes without saying that a good regenerative system is of utmost importance. There are three types of regenerative systems usually employed in turbine. These are—

(*i*) Direct transfer air-to-gas exchanger.

(*ii*) Periodic flow rotary heat exchanger.

(*iii*) Liquid metal indirect transfer exchanger.

The first system has been widely used over the years. Increased knowledge of liquid metal systems and high level of proven regenerator design technology has made use of liquid metal regenerators a clear possibility in near future.

A liquid metal system consists of two independent heat exchangers coupled by a liquid metal circuit. A part of the heat energy of the turbine exhaust is passed on to a liquid metal in the turbine heat exchanger which is then pumped to the compressor heat exchanger releasing the heat to the compressor air.

Liquid metal system gives significant advantages in terms of installed weight, frontal area, engine performance and high reliability as compared with direct or rotary type regenerators. The physical separation of hot low pressure and cold high pressure exchangers minimises structural requirements, air-to-gas carryover losses and seal leakage losses are avoided, and moreover the malfunction of the regenerator does not result in idle time because turbine bypass is avoided in this system. Finned tubes are highly suitable for liquid metal-to-gas heat exchangers.

The requirements of high heat absorbing capacity, high thermal conductivity, low viscosity for high heat transfer coefficients with reduced pressure losses, low vapour pressure and the compatibility with system materials are met by Nak-78 liquid metal alloy with 78 per cent potassium.

In future the liquid metal regenerators are likely to be used widely for improving specific fuel consumption in industrial and aircraft gas turbines.

2·4·2. Intercooled gas turbine cycles

A regenerator, as discussed above, does not change the work-output of a gas turbine cycle. So rescourse to other methods is taken for increasing the work output. Two possible methods are (*i*) by reducing the work of compression and (*ii*) by increasing the work done by turbine.

Intercooling is used for decreasing the work done on the compressor. This possibility of reducing the compressor work arises from the fact that the constant pressure lines converge on *T-s* diagram as the temperature is reduced. So if the total compression is divided into a number of stages the total compressor work is smaller than that for a single stage compression for the same total pressure ratio.

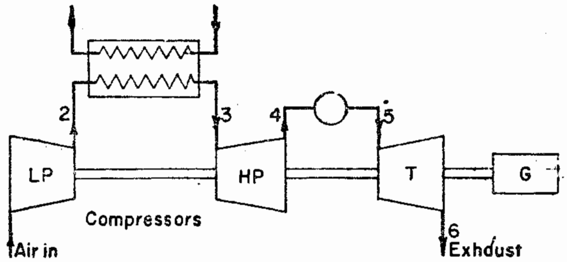


Fig. 2·10. Schematic diagram of a two-stage intercooled gas turbine.

Fig. 2·10 shows the schematic diagram of a two stage inter-cooled gas turbine cycle and Fig. 2·11 the corresponding *T-s* diagram. The performance of an intercooled cycle is shown in Fig. 2·12.

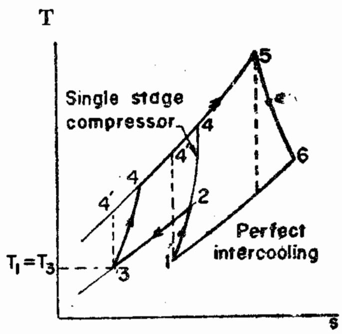


Fig. 2·11. *T-s* diagram for cycle of Fig. 2·10.

The charge from the first stage is cooled before it is led to the second stage. This process, called *intercooling*, reduces the inlet temperature of the second stage, resulting in reduction in compression work. If the charge is cooled to its initial temperature it is called *perfect intercooling*. For minimum work of compression the pressure ratio for each stage should be same with charge cooled to its initial temperature between the stages.

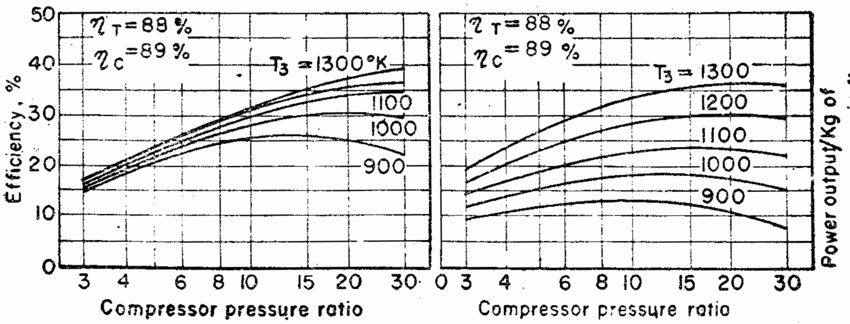


Fig. 2·12. Performance of intercooled gas turbine.

Intercooling will always increase the net workoutput of the cycle, and due to lower compressor outlet temperature there will be more scope for regeneration. However, for the same reason, the fuel supplied to obtain a given turbine inlet temperature will also increase. Therefore the thermal efficiency of the intercooled cycle is less than that for a simple cycle. One more reason of loss in efficiency is that heat is supplied at lower temperature. There is also a loss of pressure in the inter-cooler.

Intercooling is useful when the pressure ratios are high and the efficiency of the compressor is low. At low pressure ratios it is not so important and regeneration can be used to recover a substantial amount of heat from exhaust gases.

2·4·3. Gas turbine cycle with reheating

Another method of increasing the specific work output of the cycle is to use reheating to increase turbine work. The gain in work

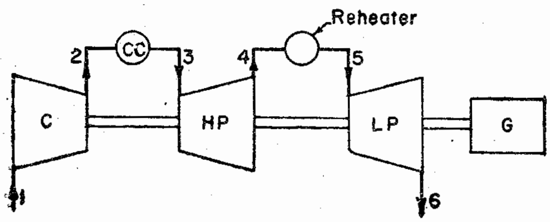


Fig. 2·13. Schematic diagram of a reheat gas turbine cycle.

output is obtained because of divergence of constant pressure lines on T-s diagram with an increase in temperature. Thus for the same expansion ratio if the exhaust from one stage is reheated in a separate combustion chamber and expanded, more output will be obtained than that obtained by expansion in a single stage. Reheating is generally done up to the upper limit of temperature T_3. Usually reheating up to two stages is done. Reheating for more than two stages is seldom done. With open cycle gas turbine the limit of reheating is the oxygen which is available for combustion.

Fig. 2·13 shows a schematic diagram of a reheat gas turbine plant and Fig. 2·14 the corresponding T-s diagram. Let the total pressure ratio r_p is divided into two stage ratios of r_{p1} and r_{p2} respectively.

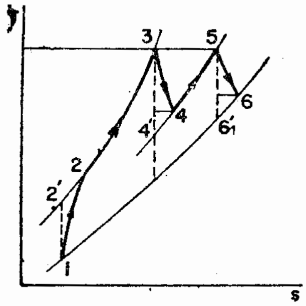


Fig. 2·14. T-s diagram of reheat cycle.

$$\because \quad r_p = r_{p1} \times r_{p2}, \qquad \therefore \quad r_{p2} = r_p / r_{p1}$$

Net work output

$$= c_p(T_3 - T_4) + c_p(T_5 - T_6) - c_p(T_2 - T_1)$$

$$= c_p(T_3 - T_4')\eta_{t1} + c_p(T_5 - T_{6'})\eta_{t2} - c_p(T_2' - T_1) \cdot \frac{1}{\eta_c}$$

where η_{t1}, η_{t2} and η_c are the efficiencies of the two turbine stages and the compressor respectively. For isentropic compression and expansion, we have

Work output W

$$= c_p[(T_3 - T_4') + (T_5 - T_6') - (T_2' - T_1)]$$

$$= c_p T_1 \left[2\frac{T_3}{T_1} - \frac{T_6'}{T_1} - \frac{T_4'}{T_1} - \frac{T_2'}{T_1} + 1 \right]$$

$$= c_p T_1 \left[2\frac{T_3}{T_1} - \frac{T_3}{T_1} \cdot \frac{T_6'}{T_1} - \frac{T_3}{T_1} \quad \frac{T_4'}{T_3} - \frac{T_2'}{T_1} + 1 \right]$$

$$= c_p T_1 \left[2\frac{T_3}{T_1} - \frac{T_3}{T_1} \cdot \frac{1}{(r_{p2})^{\frac{\gamma-1}{\gamma}}} - \frac{T_3}{T_1} \cdot \frac{1}{(r_{p1})^{\frac{\gamma-1}{\gamma}}} - (r_p)^{\frac{\gamma-1}{\gamma}} + 1 \right]$$

By differentiating Eq. (2·20) with respect to r_{p1}, keeping r_p and T_3/T_1 constant the condition for maximum work output is given by

$$r_{p1} = (r_p)^{1/2}$$

i.e., the expansion ratio must be equal in each stage. This condition can also be written as

$$(r_p)^{\frac{\gamma-1}{\gamma}} = (T_3/T_1)^{2/3}$$

and at this condition $T_2 = T_4 = T_6$. It should be noted that for any pressure ratio between 1 and r_p the work output will increase due to reheating but $r_{p1} = \sqrt{r_p}$ will give maximum work.

The efficiency of the cycle is given by

$$\text{Efficiency} = \frac{\text{Net work output}}{\text{Max. heat supplied}}$$

$$= \frac{cp[(T_3 - T_4) + (T_5 - T_6) - (T_2 - T_1)]}{cp[(T_3 - T_2) + (T_5 - T_4)]}.$$

The work output and the efficiency of a reheat cycle is shown in Fig. 2·15 and it should be noted that the pressure ratio giving maximum output is greater than that giving best performance for a simple cycle. The efficiency of the cycle is less than that for simple

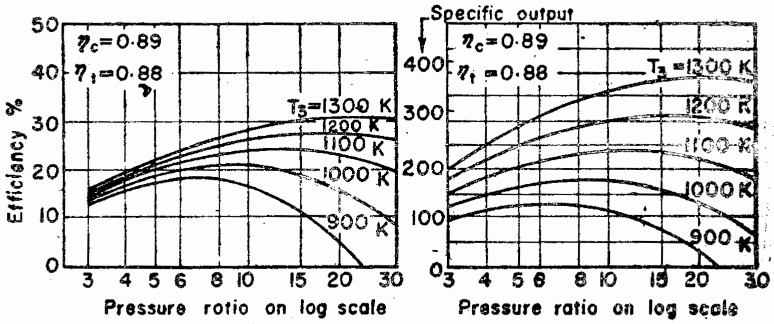


Fig. 2·15. Work output and efficiency of reheat cycle.

cycle unless regeneration is used and this reduction in efficiency increases with reduction in turbine inlet temperature. Due to complications of a second combustion chamber reheating is not used in marine plants but is extensively used in industrial plants.

The efficiency of a reheat cycle is lower than a corresponding intercooled cycle because in case of reheat, due to limitations of turbine inlet temperature, the heat added does not affect the work output, while intercooling reduces the compressor work. However, due to the fact that turbine work is greater than compressor work, the specific output of a reheat cycle is greater than that of a corresponding intercooled cycle. Efficiency of both reheat and intercooled cycles is less than that of a simple cycle while the work output is greater.

2·4·4. Gas turbine with reheating and regenerator

The fact that the turbine outlet temperature is greater than the compressor inlet temperature (*i.e.*, $T_6 > T_2$ in Fig. 2·14 for a reheat cycle gas plant) gives rise to the possibility of increasing the thermal efficiency of a reheat cycle by recovering this loss of heat in a

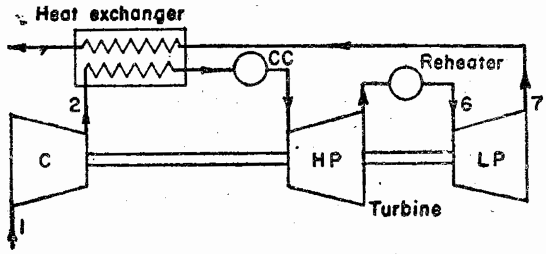


Fig. 2·16. Gas turbine cycle with reheat and regenerator

regenerator. Fig. 2·16 shows a schematic arrangement of a gas turbine plant using reheat as well as regeneration and Fig. 2·17 the corresponding T-s diagram. The heat added is reduced by the amount of the heat recovered.

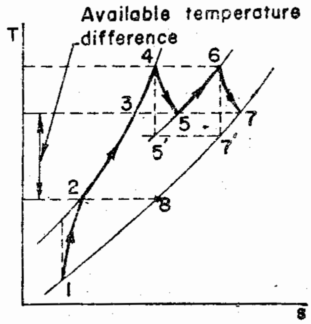


Fig. 2·17. *T-s* diagram for reheat and regenerator cycle

For a cycle with isentropic compression and expansion, ideal regeneration and the best division of pressure ratio between stages for reheat, the efficiency is given by :—

$$\text{efficiency} = \frac{c_p[2(T_4 - T_5') - (T_2' - T_1)]}{c_p[2(T_4 - T_5')]}$$

$$= 1 - \frac{T_2' - T_1}{2(T_4 - T_5')}$$

$$= 1 - \frac{1}{2}\frac{T_1}{T_4} \cdot \frac{(T_2'/T_1 - 1)}{(1 - T_4/T_5')}$$

$$=1-\frac{1}{2}\frac{T_1}{T_4}\frac{(r_p)^{\frac{\gamma-1}{\gamma}}-1}{1-\left(\frac{1}{r_p}\right)^{\frac{\gamma-1}{\gamma}}}$$

Since $\qquad r_{p1}=\sqrt{r_p}$

$$\eta=1-\frac{1}{2}\;\frac{T_1}{T_4}\;\frac{\left[(r_p)^{\frac{\gamma-1}{\gamma}}-1\right]\times(r_p)^{\frac{\gamma-1}{2\gamma}}}{(r_p)^{\frac{2\gamma-1}{2\gamma}}-1}$$

$$=1-\frac{1}{2}\cdot\frac{T_1}{T_4}\cdot(r_p)^{\frac{\gamma-1}{2\gamma}}\cdot\left[(r_p)^{\frac{\gamma-1}{2\gamma}}+1\right]$$

The efficiency curves of such a plant are similar to that of a simple regenerative cycle but the work output is similar to that of a reheat cycle. Thus both work and efficiency have been improved by a combination of reheating and regeneration.

2·4·5. Gas turbine with reheating, regeneration and intercooling.

Both the methods of increasing specific output, *i.e.*, intercooling and reheating can be used in conjunction with a generator to get high specific output and high efficiency gas turbine plant. Intercooling decreases the compressor work and because of reduced compressor outlet temperature extra heat is needed to raise the temperature to turbine inlet temperature. However, if a regenerator is used this disadvantage can be converted into an advantage in that now more regeneration is possible. Thus a combination of regenerator, intercooler and reheating can improve the performance of the plant. However, it should be noted that a high efficiency is not always sought for because sometimes it is more important to reduce plant costs than to gain in efficiency. This is particularly so where cheap fuels are being used. The extra complexity, cost and maintnance must be considered before any such plant is contemplated. The individual requirements of a particular application will have a great say in deciding whether an intercooler, a generator or reheat device should be used or not.

2·4·6. Gas turbine cycle with water injection

There are certain specific applications such as gas turbines used in naval torpedoes which require a very high specific output within the maximum temperature limit imposed by metallurgical considerations. To obtain this performance water is injected into the combustion chamber before or after the combustion. The water gets evaporated before reaching the turbine inlet. The increased amount of mass flow through the turbine results in greater specific output from the cycle. The water injected must be free from dissolved solids to avoid any loss due to deposits on turbine blades. The thermal efficiency decreases with water injection because more heat has to be supplied to vaporise the water. This latent heat of vaporisation is very difficult to recover because any condensation in the regenerator will require much greater heat transfer surface area than can be easily provided for.

2·4·7. Blade cooling

Another possible method of utilizing the benefits of using high turbine inlet temperatures to get more work output at a higher efficiency is to resort to blade cooling. Blade cooling is done to reduce the blade temperature, though the gas will still be at a high temperature. Thus, without reaching the temperature limits of the blades a higher gas temperature can be used. Blades can be cooled by passing the cool air near the root of the blades so that heat conducted to the root is carried away. Another method is to cool by passing air or liquid in blade annulus. Blade cooling is normally used for aircraft gas turbines.

2·4·8. Summary of improvements in basic gas turbine cycle with various modifications

Fig. 2·18 and Table 2·1 shows the improvements which can be obtained by various modifications of the basic gas turbine cycle.

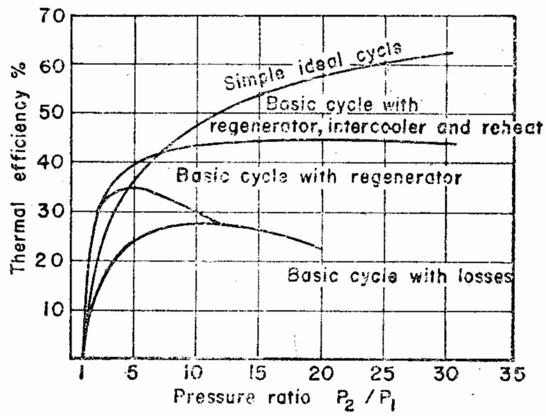


Fig. 2·18. Improvement in gas turbine cycles with various modifications.

The curves of Fig. 2·19 are drawn for compressor and turbine efficiencies of 85% and regenerator efficiency of 70%. Table 2·1 shows the general trends of effect of various modifications on work output and efficiency of the cycle. It should be noted, however, that this comparison is general in nature and not exact because of interdependence of many parameters, for example, the compressor and turbine efficiencies depend upon the pressure ratio and the losses in combustion chamber and pipe lines, etc., depend upon pressure ratio and the temperature of combustion. This is clear from Fig. 2·18 which shows that the efficiency of the basic cycle with regeneration, at first, increases but after certain pressure ratio it starts falling. And ultimately it falls to a level where regeneration is useless because at that point the temperature of air is equal to temperature of hot gases. Any further increase in pressure ratio will result in heat flow from air to hot gases.

TABLE 2·1
IMPROVEMENTS IN BASIC CYCLE BY VARIOUS MODIFICATIONS

Modification	*Effect on work output*	*Effect on efficiency*	*Remarks*
Regeration	No change	Increase	Efficiency starts
Intercooling	Increase	Decrease	falling after
Reheat	Increase	Decrease	certain pressure
Regeneration and reheat	Increase	Increase	ratio (see
Regeneration and inter-cooling	Increase	Increase	Fig. 2·18)
Intercooling and reheat	Increase	Decrease	
Regeneration, reheat and intercooling	Increase	Increase	

2·5. *ACTUAL GAS TURBINE CYCLE*

The actual gas turbine cycle differs considerably from the ideal cycle because of the following factors.

(*i*) The working medium does not follow the ideal gas law.

(*ii*) The expansion and compression processes deviate from ideal processes. This is because the processes are never instantaneous and heat transfer always occur. Moreover, some friction is always present which makes the processes irreversible. The process of diffusion in the diffuser of the compressor is always associated with some loss rendering complete diffusion impossible.

(*iii*) Complete intercooling and complete regeneration is not possible for obvious reasons.

(*iv*) There is always some pressure loss in the pipes and ducts connecting varions components of the gas turbine such as combustion chamber and the heat exchanger. All these pressure losses affect the cycle efficiency in that the available expansion ratio in the turbine is reduced by a corresponding amount.

(*v*) In actual practice the ratio of specifie heats is never constant. This is beeause of two reasons. Firstly, due to the products of combustion which expand in the turbine having a different specific heat than that of the air being compressed in the compressor. Secondly, the specific heat increases with an increases in temperature. This causes lower temperature rise for a given heat input. *i.e.*, the mean temperature at which heat is added to the cycle is lower or in other words more heat is rejected in the exhaust for a given maximum temperature and a gived expansion ratio.

Another way in which variation of specific heat affects the performance of an actual cycle is its effect on optimum pressure ratio. Usually the pressure ratio chosen corresponds to either best efficiency pressure ratio or best specific work output ratio both of which depend upon γ, the ratio of specific heats (see Equs. 2·14 and 2·16). Thus the value of optimum pressure ratio changes with the variation in specific heat.

(*vi*) The actual work output is less than the ideal work output by an amount equal to friction, windage and transmission losses which cannot be avoided.

All these factors, which tend to reduce the efficiency and work out put of the actual cycle, can be taken into account in the calculation of its performance by assigning efficiency values to each one of them. The effect of deviation of working medium from ideal gas law can be considered by taking into account the effect of change in ratio of specific heats from the γ to polytropic value n. The departure from ideal compression and losses into diffuser is taken into account by efficiency of compression η_c, and departure from adiabatic expansion and pressure losses in combustion chamber are accounted for by turbine efficiency. Efficiency of regeneration takes into account the heat losses in regenerator. Thus an accurate analysis of actual cycle is possible by the consideration of various component efficiencies.

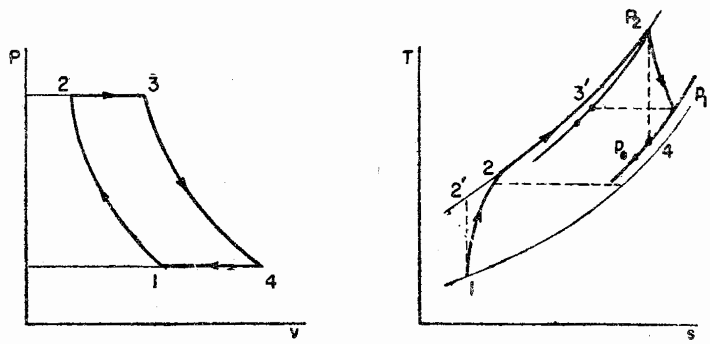


Fig. 2·19. Actual gas turbine cycle

Fig. 2·19 shows an actual cycle on P-v and T-s diagram. It can be seen that the pressure losses are apparent in lower expansion ratio which is available for the turbine. Fig. 2·20 shows the difference between cycle efficiencies for ideal and real gases for three

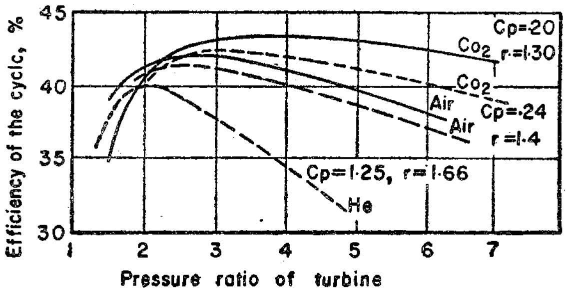


Fig. 2·20. Difference between efficiency of actual and ideal cycle

different gases. It can be seen that cycle efficiency drops with increased ratio of specific heats both for different values of γ for various gases and for changes in γ of a given gas due to influence of temperature on specific heats. For helium only one line is shown. This is because, for this gas, the effect of temperature on specific heat is negligible. From Fig. 2·20, the variation in the optimum pressure ratio for real gases from ideal gas pressure ratio can also be seen for air and carbon-dioxide. The influence of temperature on specific heat of carbon dioxide is more than that on the specific heat of air.

2·6. CLOSED CYCLE GAS TURBINE

Most of the gas turbines plants in use are open cycle type. The closed cycle gas turbine is a comparatively recent development. It was after years of pioneering work that Escher Wyss of Switzerland first introduced in 1939, a closed cycle version developing 2000 kW at a pressure ratio of 3·8 : and turbine inlet temperature of 700°C with a thermal efficiency of over 30 per cent. It took the closed cycle principle long time to establish and it was not until sixties that its use became wide-spread all over the world.

The fundamental difference between open and closed cycle gas turbines is in the method of heating the air after compression. In case of an open cycle turbine the fuel is burned in the air itself to raise it to a high temperature and then products of combustion are passed on to the turbine for expansion and which after delivering the work are finally rejected to the atmosphere. For next cycle a fresh supply of air is sucked in the compressor. In case of closed cycle turbine, on the other hand, the same air or the working fluid is circulated over and over again. The working fluid is heated by burning the fuel in a separate supply of air in a combustion chamber and transferring this heat to the working fluid, which passes through tubes fitted in this chamber. Thus the working fluid does not come into contact with the products of combustion.

Fig. 2·21 shows a schematic diagram of a closed cycle gas turbine plant. The working fluid, usually air, is compressed by the

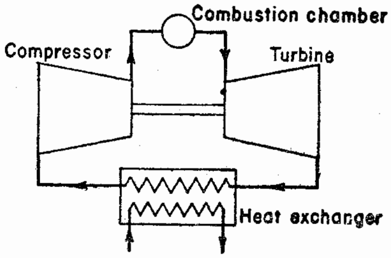


Fig. 2·21. Schematic diagram of a closed cycle gas turbine cycle.

compressor and heated in the combution chamber without coming in contact with products of combustion, and then expanded in the

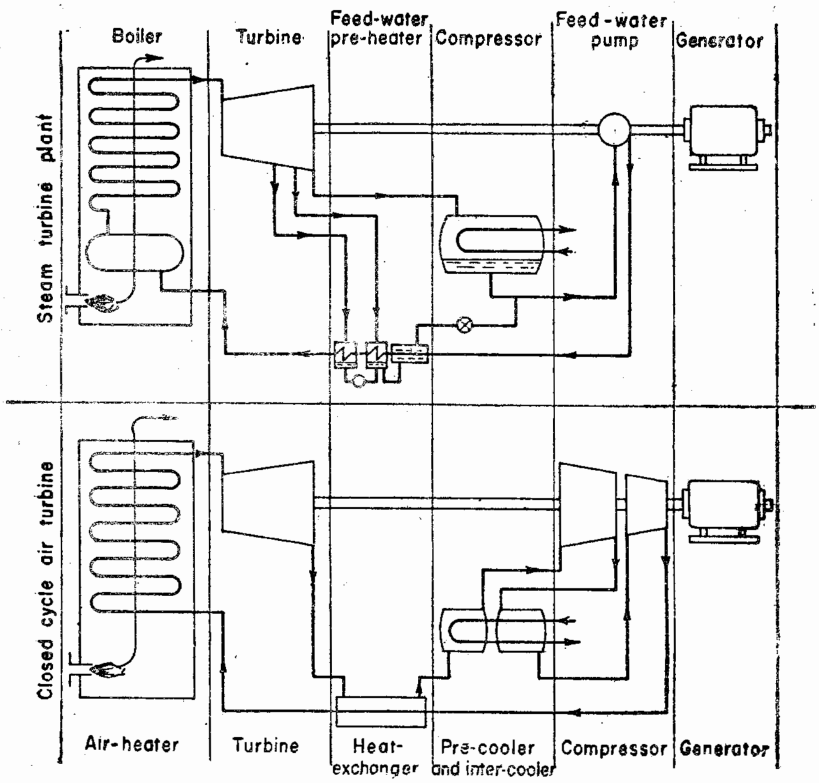


Fig. 2·22. Closed cycle gas turbine compared with steam turbine

turbine and cooled in a precooler before being supplied again to the compressor. By virtue of this indirect transfer of heat and use of a precooler, the closed cycle gas turbine is more akin to a steam turbine than to an open cycle gas turbine. Fig. 2·22 shows the striking similarity between the two types of plants. The flow area per unit output for a gas turbine plant is much less and the blade lengths do not vary much in height from inlet to outlet. In a steam plant use of high pressure steam must be resorted to for increasing the efficiency but a closed cycle gas turbine plant works on a relatively low pressure ratio of 4 to 10 (compared with those of steam turbines which is 250-500 time greater). The specific volume (m^3/kW) of a closed cycle gas turbine, at outlet, is about twenty times smaller than that of the steam turbine.

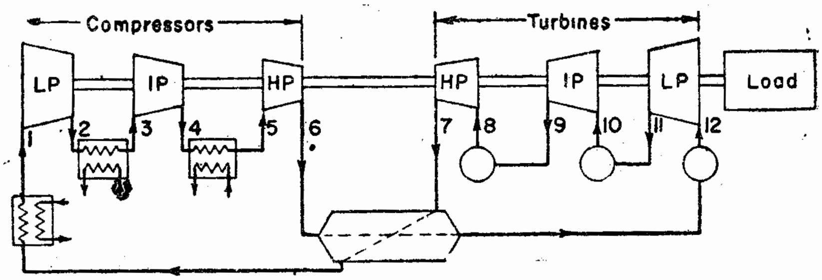


Fig. 2·23. Closed cycle gas turbine cycle with multistage intercooling and reheating.

By the use of multi-stage reheating and intercooling a closed cycle gas turbine cycle, as shown in Fig. 2·23 aud the corresponding *T-s* diagram shown in Fig. 2·24, can be approximated to the reverssible Ericsson cycle. Ericsson cycle, by virtue of the fact that in it the heat is added at highest temperature and rejected at lowest temperature, has an efficiency equal to carnot cycle efficiency.

Closed cycle gas turbine plants are highly suitable for nuclear and high output power stations, and for total energy plants. Fig. 2·29 shows the schematic diagram of a total energy plant producing

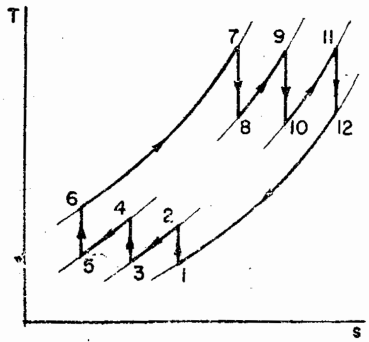


Fig. 2·24. *T-s* diagram of closed cycle gas turbine cycle with multistage intercooling and reheating.

20-30 MW power and $30\text{-}60 \times 10^6$ Kcal/h. process heat with main data given. The efficiency the plant is 34 per cent and total energy conversion is up to 85 per cent.

2·6·1. Advantages and disadvantages of closed cycle gas turbine.

Advantages :

1. Since the pressure of the working fluid is independant of atmospheric pressure, a higher pressure can be used to increase the specific output of the plant. This results in diminution of the sizes of the components both for the machines and the heat transfer apparatus used.

2. Gases other than air, which have more favourable properties can be used. Helium or helium-carbondioxide mixture gives higher efficiencies and smaller dimensions for special purposes. The properties of helium at high pressures, such as high heat transfer, low pressure drop, high sound velocity and neutrality towards radioactive materials makes it possible to build smaller heat transfer equipment and is highly suitable for nuclear plants.

3. Use of alternative working fluids such as helium, etc., gives rise to the possibility of using alternative materials as no oxidation occurs with these inert gases.

4. The power output of a closed cycle gas turbine can be controlled by changing the mass flow. The system pressure is proportional to the gas mass flow. By changing the pressure and mass flow, output changes but the temperature drop remains the same. Constant temperatures lead to constant heat drop and constant velocities in the turbine blading and hence the velocity triangles and consequently the turbine and compressor efficiencies remain constant for every power output. In case of an open cycle gas turbine the power control is affected by controlling temperature which affects the efficiency of the turbine at part load.

5. Existence of constant temperature at all loads results in low thermal stresses.

6. Due to the fact that the working fluid does not come in contact with products of combustion, even very low quality fuel can be used. All types of fuels solid, liquid, gaseous and nuclear, can be used.

7. In total energy plants direct waste heat utilization at a high temperatures for heating purposes, without affecting power cycle efficiency, can be affected because heat rejection is an isobaric process instead of isothermal change of state *i.e.* condensation

8. The turbine blades are not fouled by the products of combustion.

9. The regulation of the closed cycle gas turbine is simpler.

Disadvantages :

1. The use of high pressure requires a strong heat exchanger.

2. Since heat transfer is indirect, a part of the heat energy is lost in rediation and other losses resulting in poor combustion efficiency.

3. Due to use of air heaters and precooler the cost and bulk of a closed cycle gas turbine plant is much more than that for an open cycle gas turbine plant.

4. A coolant is needed for precooler which is a disadvantage as compared to an open cycle plant.

2·6·2. Effect of medium on performance of a closed cycle gas turbine.

In a closed cycle gas turbine the same fluid circulates round and again in the system and in the course of circulation its tempe-

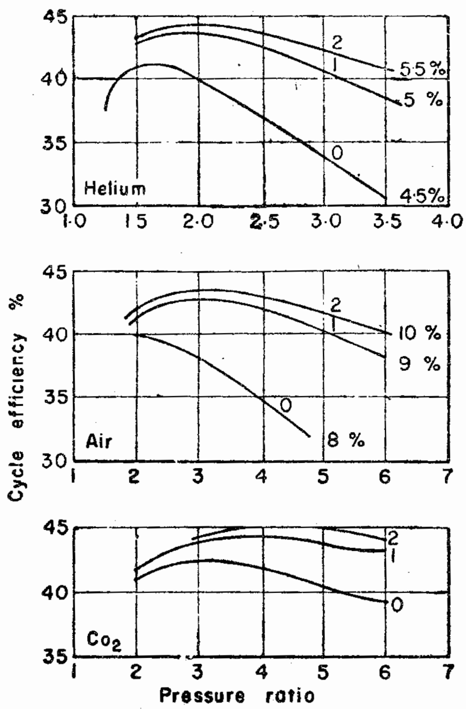


Fig. 2·25. Actual obtainable efficiencies for various gases.

rature and pressure, hence its thermodynamic properties, also change. This change in the thermodynamic property of the medium affects the performance of the closed cycle gas turbine greatly.

The ratio of specific heats, γ, influences the cycle efficiency and the optimum pressure ratio (Fig. 2·20). The cycle efficiency decreases with an increase in ratio of specific heats. Thus the ideal cycle efficiency is maximum for carbondioxide and minimum for air. However, there are other factors which affect the design and efficiency of an actual cycle. For example, due to difference in the viscocity of these gases different pressure losses occur and the cycle efficiencies obtainable are more or less same as is clear from Fig. 2·25.

However, the maximum efficiency obtainable differs due to different variation of specific heat with temperature (see Fig. 2·20).

It should be noted here, that efficiency alone is not very important in the design of a turbine plant. Other factors such as size, pressure losses and heat exchanger losses, maximum limiting speed, etc., are also quite important. Thus due consideration must be given to various design factors. One example is the optimum pressure ratio, lower the optimum pressure higher is the number of stages required for a given total pressure ratio.

In general, the optimum pressure ratios are lower for monoatomic gases (inert gases) which have a higher ratio of specific heats than that for diatomic gases such as nitrogen and air which has in turn, lower γ than that for polyatomic gases such as carbon dioxide. The lighter gases require more number of stages. Number of stages for the same circumferential velocity is given by $\frac{c_{pHe}}{c_{pair}}=\frac{1 \cdot 25}{0 \cdot 24}=5-2$ with maximum temperature remaining same for helium and air.

Once a pressure ratio corresponding either to that for maximum efficiency or maximum output is chosen all the temperatures in the cycle are fixed independent of the type of gases. With them is also fixed the temperature difference across the turbine and regenerator, etc. The heat drops are, therefore, proportional to the specific heats, or for ideal gases inversely proportional to the molecular weight. Thus we see that pressure ratio for maximum work or maximum efficiency depends upon the ratio of specific heat only while the corresponding heat drops also depends on the molecular weight.

Another important factor which affects the compressor speed and hence the maximum pressure in the gas turbine cycle, efficiency and workout is the Mach number at compressor inlet. The velocity of sound is higher in a lighter gas so that for a given tip speed, the Mach number for a lighter gas is lower than that for a heavier gas. In other words, the compressor can be run at a higher speed without exceeding the unity Mach number when a lighter gas is used. The peripherial velocities which can be used for same number of stages for helium and air are given by :—

$$\frac{U_{He}}{U_{air}}=\sqrt{\frac{c_{pHe}}{c_{pair}}}=\sqrt{5 \cdot 2}=2 \cdot 28$$

Thus a helium compressor can run at a circumferential velocity 2·28 times higher than that for air. However, such a high speed is seldom allowed by the stress limit set by the particular blade material used. The perioherial speeds of compressor rotor for nitrogen and air are limited to about 160-170 m/sec where the compressor inlet Mach number is unity. For carbon dioxide this speed is still lower because of its being heavier than air. This means that the compressor for carbon dioxide must run at a lower speed, that is number of compressor stages required will be more for carbon dioxide than for air or helium.

If the compressor is not limited in its speed by Mach number a high reaction blading can be used to attain a higher pressure coefficient.

In the range between 400°C and 800°C heat conductivity of helium is approximately 4 to 4·5 times as great as that of air. At lower temperatures it is as high as 6 times. Therefore, when helium is used the size of regenerator is reduced. However, special insulation layers, better bearing, etc., are needed with helium. New sealing problems also increase the cost.

2·7. HELIUM COOLED CLOSED GAS TURBINE FOR NUCLEAR POWER PLANTS

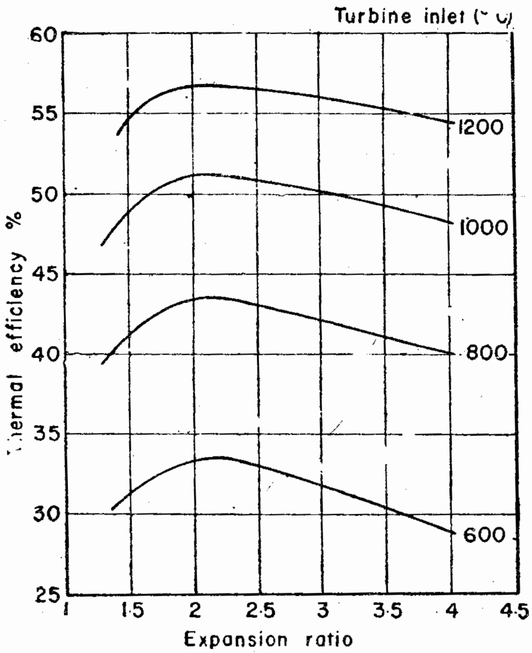


Fig. 2·26. Efficiency of helium gas turbine.

It was discussed in the previous article that helium gives high efficiency, (see fig. 2·26) and smaller dimensions for a given output of the gas turbine plant. The properties of helium at high pressures, such as high heat transfer, low pressure drop, high sound velocity, neutrality towards radioactive materials and graphite make it possible to build compact heat exchange units. Due to high heat transfer coefficient certain control processes take much less time in helium than in an air cycle of equal size.

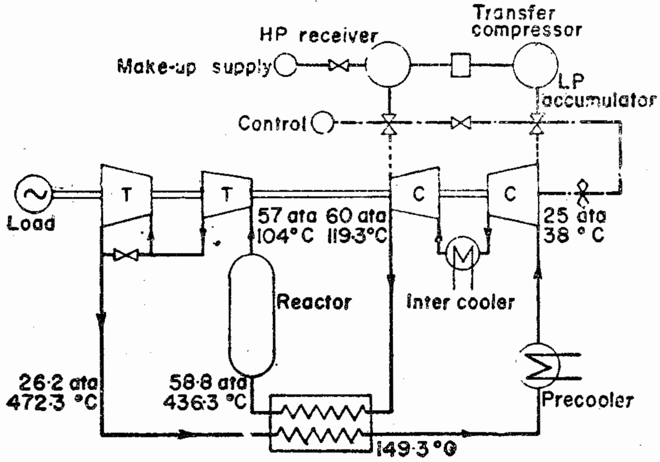


Fig. 2·27. Schematic diagram of a direct gas cycle nuclear power plant. (*Courtesy*: *Esher Wyss*).

All these properties of helium make it very attractive for nuclear power plants where previously only the steam turbines were used to convert reactor heat into mechanical work. Due to the susceptibility of water to radioactivity an intermediate heat exchanger with carbon dioxide as working medium, is generally used to transfer heat of nuclear reaction to water. Since pure helium does not become radioactive when passing through the reactor, it allows the elimination of the intermediate stage. Fig. 2·27 shows a schematic diagram of a direct gas cycle nuclear power plant. This figure also shows the main design data. The plant efficiency remains constant over a very wide range of power output, (*see* Fig. 2·28). Overall full load efficiency is about 36 to 40 per cent.

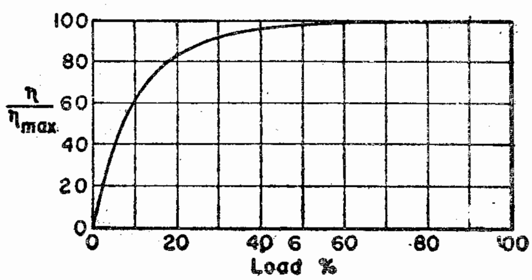
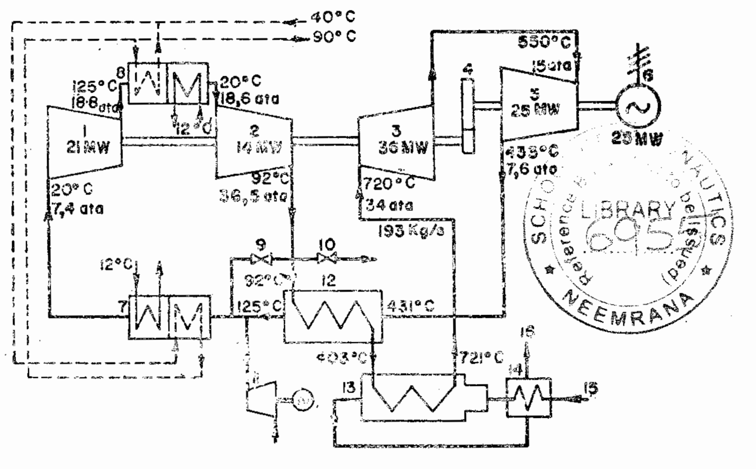


Fig. 2·28. Plant efficiency *vs* load for nuclear power plant shown in Fig. 2·27.

The advantages of direct cycle are the reduced cost, reduced bulk and reduced danger of explosion. It is easy to control and does not need special precautions, to prevent leakage of high pressure steam into the reactor. Relatively low cycle pressures are needed, thereby, making construction easier. The operation at constant temperature at all loads helps to improve safety of the operation.

2·8. TOTAL ENERGY SYSTEM INCORPORATING GAS TURBINE

The thermal efficiency of a gas turbine plant is relatively low due to high excess air of about 300 to 350 per cent and high exit temperatures of about 425 to 550°C. The efficiency lies in the range of 20 to 25 per cent for a simple gas turbine cycle, 25 to 28 per cent with regenerative heating, and up to 42 per cent for exhaust fired regenerative heating cycles. Thus about 36 to 60 per cent of the total heat supplied goes into the turbine exhaust. If this large amount of exhaust heat is utilized for producing additional power by raising steam in a boiler or for producing low pressure steam used for processing purposes, the efficiency of the gas turbine cycle can be increased to about 85 per cent. Such plants are known as total energy plants.

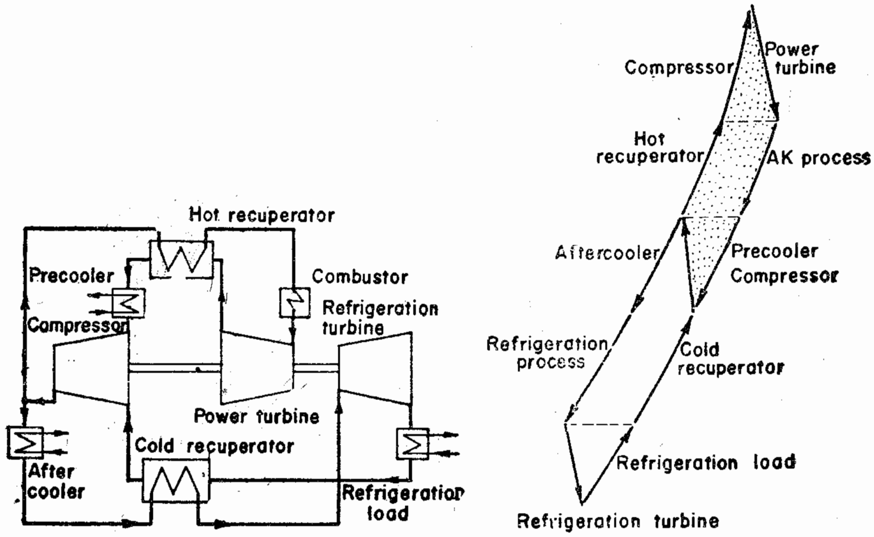


1. L.P. compressor 2. H P. compressor 3. H.P. turbine 4 Gear
5. L.P. turbine 6. Generator 7. Precooler 8. Intercooler
9 and 10. Valves 11. Charging compressor 12. Heat exchanger
13. Air heater 14, 15, 16. Compression air preheater system.

Fig. 2·29. Schematic diagram of a 25/30 MW total energy plant. Combinod power (30 MW) and heat production (20 to 60×10] kcal/h). Electrical efficiency about 34%. Total energy conversion upto 85 per cent (*Courtesy* : *Escher Wyss*).

Fig. 2·29 shows the schematic diagram of a total energy system incorporating a gas turbine. The exhaust of the gas turbine which contains about 17 to 18·5 per cent oxygen by weight is used as combustion air for the boiler. Approximately 25 per cent more exhaust gases are needed than ambient air to burn the same fuel. Almost all the heat in exhaust is utilized and the efficiency of the gas turbine is increased. The loss of efficiency in the steam cycle due to increased condenser flow is more than compensated by this. The supplementary fired boiler raises high pressure steam which is expanded in a back pressure turbine to give additional power as well as process steam. Thus efficiency of the steam cycle can be raised by about 2 to 3 per cent. A separate water heater can also be used if the process requires hot water.

The gas turbine plant in combination with a steam turbine is very flexible. Gas turbine is excellent for meeting the peak and emergency power requirement because of its low, capital cost, automatic operation, short start type and high reliability.

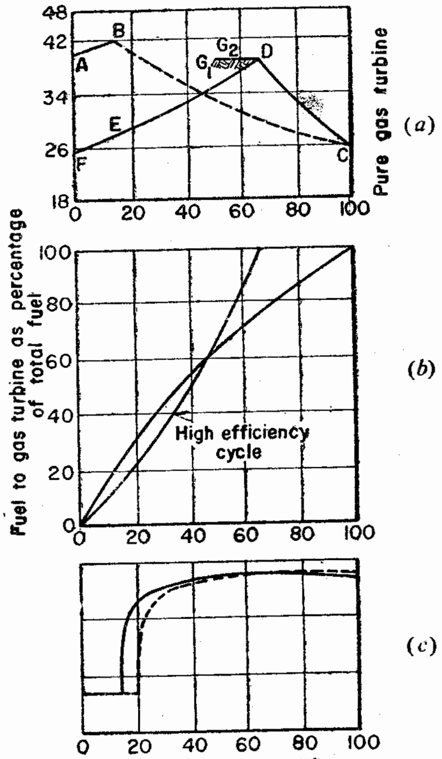


(*a*) Schematic diagram (*b*) *T-s* diagram

Fig. 2·30. Combined power and refrigeration cycle

The fact that optimum pressure ratio for turbine cycle is near the pressure ratio required by the regenerative refrigeration cycle suggests that the two cycles can be combined effectively and the same compressor can be used for both cycles reducing the overall cost. Fig. 2·30 (*a*) shows the schematic diagram of such a combined cycle and Fig. 2·30 (*b*) shows the corresponding T-s diagram. It can be seen that both the cycles are similar and optimisation of parameters for maximum efficiency is rather easy.

In article 1·5·2 of chapter 1 the combined gas and steam turbine cycle as well as the recuperator cycles were discussed. Fig. 2·31(*a*) shows the possibilities of selecting a proper design for a particular use. Point B, where the gas turbine output is zero represents the pure steam turbine with an efficiency of about 40 per cent with steam at 134 kgf/cm² and 550°C. Point B, with gas turbine to steam turbine output ratio of about 13 per cent, denotes the high efficiency combined cycle with about 42 per cent combined efficiency, *i.e.* a gain of about 5 per cent. To the right of B, the size of the gas turbine which gives a good matching gas turbine flow to boiler, becomes too large. The excess gases which must be by passed after some point to the right of B increase the stack losses. If the excess gases are not bypassed and instead an economiser is used the efficiency falls further. Point C refers to the pure gas turbine



(*a*) Gas turbine power as percentage of total power.
(*b*) Fuel burnt (*c*) Excess air

Fig. 2·31. Selecting proper design of combined cycles.

with an efficiency of 26 per cent. The dotted line BC represents the efficiencies obtainable by suitable combination of gas turbine and the steam turbine in parallel.

Point D represents the best condition for a recuperator type cycle with steam output about half that of gas turbine output, *i.e.* one-third of the total output with an efficiency of 39 per cent. The steam conditions being determined by exhaust temperature of gas turbine and other economic considerations. Points G_1 and G_2 refer to the gains which can be obtained by burning supplementary fuels.

Fig 2·31 (*b*) and (*c*) shows the fuel burnt and the excess air available with various combinations. It is interesting to note that the excess air, which is as low as 2·5 to 5 per cent from A to B, rises to about 400 per cent from C to D.

The capital cost is high along AB, and remains high upto C for steam fraction because of high steam condition but the combined cost decreases because of increase in size of the gas turbine to that of point C. For recuperator cycle, capital cost from C to D is low and almost constant per kW of output. From D to B it rises abruptly with a discontinuity.

2·9. SEMICLOSED CYCLE GAS TURBINE

Semiclosed cycle gas turbine plants are those plants in which there are two turbines, one to drive the compressor and the other is power generator turbine. The turbine which drives the compressor forms a closed circuit as shown in fig. 2·32 and is a closed cycle gas turbine. Exhaust from the closed cycle turbine is taken to a recuperator where it gives up its heat and heats up the air from the high pressure compressor before it is fed to the combus-

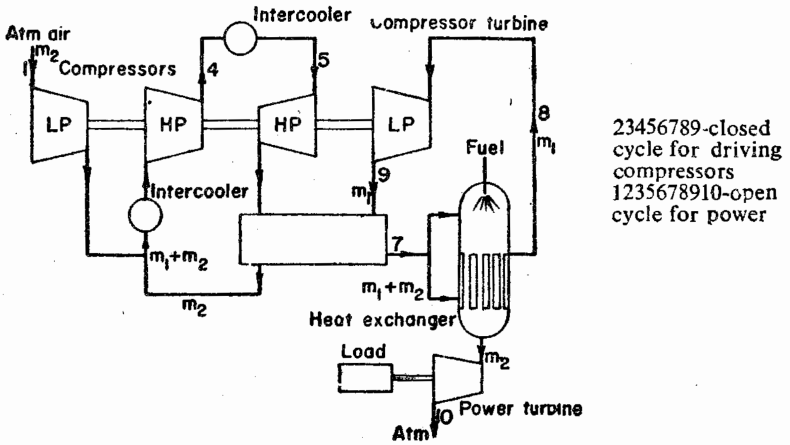


Fig. 2·32. Semiclosed gas turbine cycle.

tion chamber. The compressed air is fed to the combustion chamber through two circuits, one as combustion air in which fuel is burned and products of combustion expanded to produce power in power turbine, the other air stream passes through the combustion chamber via a heat exchanger, gets heat by taking a part of the heat of combustion and expands in the compressor turbine. Thus the heat of combustion is divided into two parts and temperature of the gases going to the turbine is reduced. Another advantage of semiclosed cycle is that the compressor turbine may be designed to match the compressor and can work at their best efficiencies while variation in load can be met by controlling the fuel supply to the combustion chamber. This results in good efficiency over almost the complete load range.

2·10. GAS TURBINE PLANT ARRANGEMENTS

Gas turbine plants have their three main components turbine, compressor and the combustion chamber arranged in different ways to meet the demands of speed and load control of the plant set by a particular application. The different arrangements affect the performance of the plant *i.e.* the efficiency and variation of the efficiency with load. The torque and speed characteristic are critical for different applications. The basic arrangements are as follows :—

2·10·1. Open cycle with single-shaft arrangement

Fig. 2·33 shows the schematic diagram of an open cycle plant with single shaft arrangement The compressor and the turbine are directly coupled by a single shaft to the load. This has an inferior flexibility characteristic. Any change in speed will change the speed of compressor and the efficiency of the compressor will decrease due to air angles being different than designed. There is also a starting difficulty. Gas turbine plants are not self-starting and must be run to about 30-40 per cent of their normal speed before they develop sufficient power to maintain the speed. This arrangement requires a large motor to accelerate the shaft. The

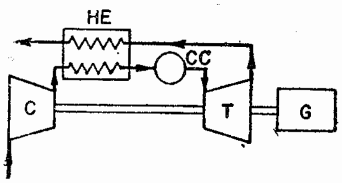


Fig 2·33. Open cycle gas turbine with single shaft arrangement.

cost of open cycle with single shaft arrangement is low. This type of arrangement is used for constant speed operation and for applications where the load is usually near to full load. At part loads the efficiency of the plant is poor.

2·10·2. Open cycle with two-Shaft arrangement

Fig. 2·34 shows the schematic arrangement of a two shaft turbine plant. It consists of two turbines and a compressor. One turbine is coupled to the compressor and the second to the load.

This arrangement is also known as free turbine arrangement. The two turbine shafts are mechanically independent and can run at

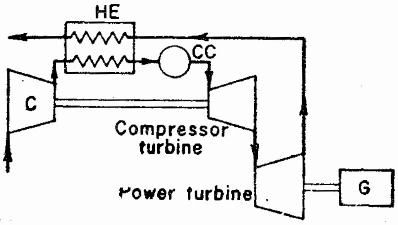


Fig. 2·34. Open cycle gas turbine with two shaft arrangement.

different speeds. Another possible arrangement is the use of two concentric shafts to couple compressor and one turbine by one shaft, and another turbine and the load by another shaft.

The two-shaft arrangement is more efficient than single shaft arrangement because one turbine coupled to the compressor can be run at a higher speed compatible to high compressor efficiency. The power supply is controlled by changing the amount of fuel supplied to the combustion chamber. However, such a control has two main disadvantages: (*i*) The amount of flow changes with the fuel supply and results in changing compressor speed. This reduces the efficiency of the compressor as air angles are now not optimum. (*ii*) The maximum temperature of the cycle is also lowered due to improper fuel-air matching, resulting in a loss of efficiency.

2·10·3. Open cycle series or parallel flow gas turbine plant

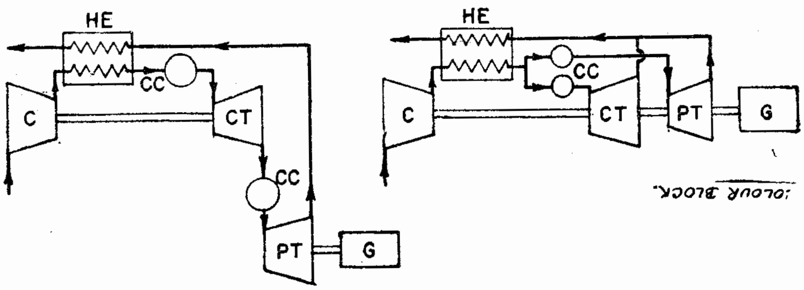


Fig. 2·35. (*a*) Open cycle series gas turbine plant (*b*) Open cycle parallel gas turbine plant.

The disadvantages discussed above can be overcome by using two combustion chambers connected in series or in parallel as shown in Fig. 2·35 (*a*) and 2·35(*b*) respectively. In series arrangement the variations in load and speed are obtained by changing the fuel supply to second combustion chamber, the compressor turbine combination still running at its optimum speed giving optimum efficiency. In parallel arrangement control is affected by changing the fuel supply to the combustion chamber of the power turbine.

With two mechanically independent shafts and the control affected by the use of two combustion chamber, it is possible to obtain a wide degree of flexibility of power and speed range. Marine propellers, heavy duty automobiles, railroad engines and wide range of industrial applications are typical examples of the use of two-shaft arrangement. The two-shaft arrangement is used to meet peak load demands because it can be quickly accelerated to meet the load requirements. If a single shaft plant is used for peaking, any sudden change in load will be met by changing the speed of the whole compressor-turbine combination resulting in long time lags and sometimes to dangerous overspeeding.

2·11. AUTOMOTIVE GAS TURBINE

The simple gas turbine with its three elements, *viz*. compressor, combustion chamber and turbine, though successful for air-crafts, is highly unsuitable for automotive applications. This is because its output torque decreases rapidly with speed and drops to zero at about 40 per cent speed. Another important reason is the poor part load performance of the simple gas turbine. Its specific fuel consumption is very high at design point due to lower pressure ratios used and, still more, it deteriorates under part load conditions. The decrease in the turbine inlet temperature at part load operation is the main reason for this poor part load efficiency. So for making the simple gas turbine suitable for automotive applications some modifications in the simple gas turbine arrangement must be done.

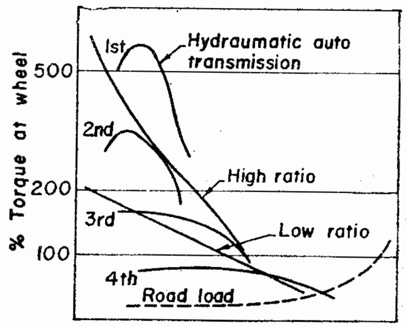


Fig. 2·36. Torque speed characteristic of a conventional engine fitted with hydromatic automatic transmission and of a gas turbine at two different pressure ratios.

Fig. 2·36 shows the torque speed characteristics of a conventional engine fitted with hydromatic automatic transmission and that of a gas turbine at two different pressure ratios. It can be seen that the torque characteristic of a turbine is very similar to a torque converter and by suitable choice of a two speed gear ratio it should give nearly the same road performance as that given by the combination of a torque converter and the conventional engine. So the first step towards modifying the simple gas turbine is that the turbine should be decoupled from the compressor and made free to make the torque-speed characteristic acceptable.

The acceleration performance of the gas turbine is lower than that of reciprocating engine, so before it can be used for automobiles the inertia of its rotating parts must be reduced.

Another point of importance is the part load fuel consumption. It is well known that almost all automotive engines, including the gas turbine, run most of the time under part load conditions and unless the part load fuel consumption is improved the use of gas turbine would be highly uneconomical. One way to do so is the use of high pressure ratio and the use of variable nozzles in the power turbine enable a high combustion temperature to be maintained at reduced loads. Another method of reducing fuel consumption is the use of heat exchanger.

Thus a free turbine, heat exchanger and variable nozzles have become more or less the standard elements of an automotive gas turbine.

2·11·1. Free power turbine

Fig. 2·37 (*a*) shows the schematic layout and Fig. 2·37 (*b*) the internal schematic of a free power turbine system. The inlet air

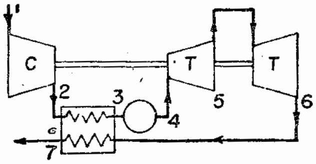


Fig. 2·37 (*a*). Free power turbine.

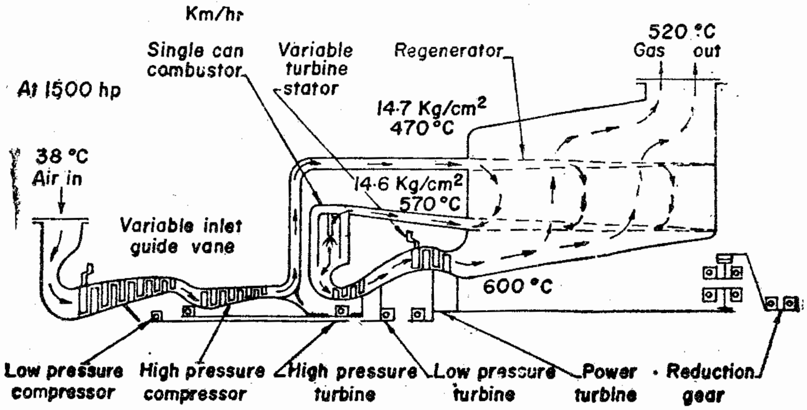


Fig. 2·37 (*b*). Typical automotive gas turbine arrangement.
(Free power turbine).

passes through the five progressively shorter height blades of the pressure compressor. The pressure ratio is increased further in the counter-rotating high pressure compressor where a centrifugal turning stage forms the transition to a radial diffuser. The pressure ratio used is 14·5 : 1.

The use of counter-rotating arrangement, the high pressure spool, *i.e.* high pressure turbine, high pressure compressor and nozzle in narrow speed range is conductive to high efficiency and increases acceleration capability.

The air from the centrifugal turning stage passes through the recuperator, gets preheated and reaches a single-can type tangential scroll combustion chamber, from where the products of combustion pass to the high pressure turbine. The first stage turbine blades and nozzle are aircooled and can withstand temperatures up to 1115°C. The uncooled second stage turbine wheel drives the low pressure compressor via coaxial shafting through the high pressure spool. A ring of variable incidence vanes direct the gases to the second stage turbine which rotates in direction opposite to that of the first stage turbine. The variable incidence vanes, *i.e.* the variable nozzles, allow the turbine to operate under optimum conditions over its entire operational range. These, thus, keep the exhaust temperature and recuperator efficiency high under part load. These also help in braking by directing, when desired, the gas flow in a direction so as to oppose the turbine flow. Thus they can protect against overspeed when the load on the turbine is suddenly lost.

Figure 2·38 shows the performance of the free power turbine. The turbine horse-power *vs* speed characteristic is significantly flat and the fuel consumption is also approaching that of best conventional engine.

The stall torque ratio, *i.e.* stall torque/design torque, is about 2·1 to 2·3 which is not sufficient for a passenger car which needs a stall torque ratio of about 4 : 1 for good acceleration. This still requires a change speed gear box. Another important thing is that for good acceleration speeds of both compressor as well as turbine should increase simultaneously but this is rarely so. The compressor accelerates faster than the turbine which is delayed due to lags in fuel control and other mechanical responses.

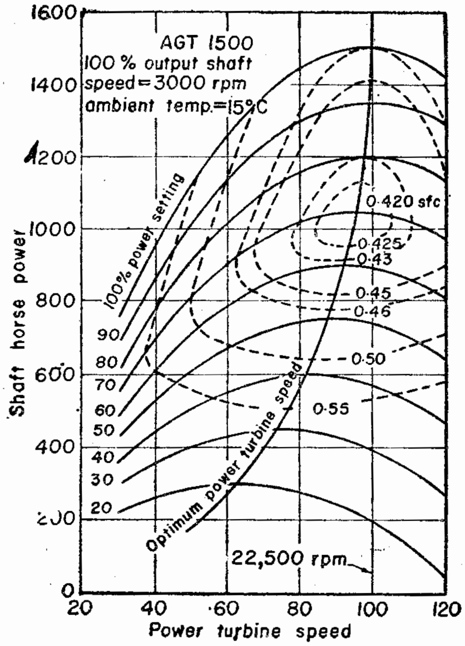


Fig. 2·38. Performance map of an automobile gas turbine.

2·11·2 Split compressor differential gas turbine (SCDGT)

Another arrangement for automotive gas turbine is the split compressor differential gas turbine (SCDGT) shown in Fig. 2·39 in which the various components are mechanically linked through a differential gear The compression is carried out in two separate compressors. The low pressure compressor is carried on one shaft and the high pressure compressor and turbine on the second shaft. These two shafts are connected through an epicyclic differential gear.

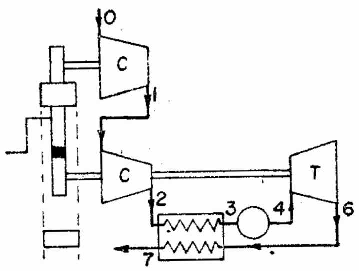


Fig. 2·39 Schematic diagram of a SCDGT power unit

The advantage of this arrangement is that its torque speed characteristic renders use of a speed changing gearbox unnecessary, except for reversing and the part load performance is much better than the free power turbine. The differential gear allows the low pressure and high pressure compressor shaft speed to vary, keeping the output shaft speed constant so that engine mass flow and power can be varied. As the high pressure compressor speed falls the output shaft speed is reduced while the low pressure compressor speed rises resulting in a higher combustion temperature. And as a consequence, the turbine torque increases since the ratio of output shaft torque to low pressure compressor torque remains constant, giving better part load efficiency.

For starting an automotive gas turbine a large amount of power is needed because the compressor speed must be quite high before it can develop a useful pressure ratio. The use of split compressor allows the cranking power to be low.

Fig. 2·40 shows the performance of a SCDGT power unit for various values of the parameter B which is analogous to "throttle" in

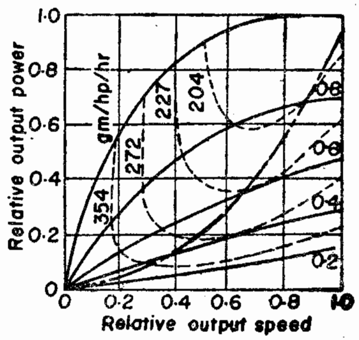


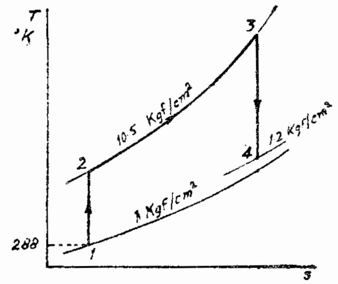
Fig. 2·40. Performance of a SCDGT power unit.

a spark-ignition engine. It shows that at high speeds the performance is quite good over a wide operational range, but performance is poor below 30 per cent output speed. The dotted line shows the optimum area of operation.

ILLUSTRATIVE EXAMPLES

2·1. Ideal gas turbine cycle : net output.

A gas turbine power unit operates at a mass flow of 30 kg/s. Air enters the compressor at a pressure of 1 kgf/cm² and temperature 15°C and is discharged from the compressor at a pressure of 10·5 kgf/cm². Combustion occurs at constant pressure and results in a temperature rise of 420°C. If the flow leaves the turbine at a pressure of 1·2 kgf/cm², determine the net power output from the unit. Take $c_p = 0{\cdot}14$ kcal/kg-K, $\gamma = 1{\cdot}4$.



F g. 2·41.

Assuming reversible adiabatic compression in the compressor, we get

$$T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$= 288 \times \left(\frac{10 \cdot 5}{1}\right)^{\frac{0 \cdot 4}{1 \cdot 4}}$$

$$= 564 \text{ K}$$

$\therefore \qquad T_2 - T_1 = 564 - 282 = 276 \text{ K}$

Temperature of the gases leaving the combustion chamber,

$$T_3 = 564 + 420 = 984 \text{ K}$$

Assuming reversible adiabatic expansion in the turbine, we get

$$T_4 = \frac{T_3}{\left(\frac{P_3}{P_4}\right)^{\frac{\gamma-1}{\gamma}}}$$

$$= \frac{984}{\left(\frac{10 \cdot 5}{1 \cdot 2}\right)^{\frac{0 \cdot 4}{1 \cdot 4}}}$$

$$= 529 \text{ K}$$

$\therefore \qquad T_3 - T_4 = 984 - 529 = 455 \text{ K}$

Net power output = turbine output − work done on compressor

$$= \text{mass rate} \times c_p\{(T_3 - T_4) - (T_2 - T_1)\}$$

$$= 30 \times 0 \cdot 24 (455 - 276) \times 5 \cdot 61$$

$$= \mathbf{7930 \text{ H.P.}}$$

Ans.

2·2. Simple gas turbine : η, work ratio, air rate, sfc, A/F ratio.

In a simple gas turbine plant air enters the compressor at 1·033 kg_f/cm^2 and 15°C and leaves at 6 kg_f/cm^2. It is then heated in *the combustion chamber to 700°C and then enters the turbine and expands to atmospheric pressure. The isentropic efficiency of compressor and turbine are 0·80 and 0·85 respectively and the combustion efficiency is 0·90. The fall in pressure through the combustion chamber is 0·1 kgf/cm². Determine (a) the thermal efficiency, (b) the work ratio, (c) the air rate in kg per shaft horse-power, (d) specific fuel consumption, and (e) the air-fuel ratio.*

Take calorific value of the fuel equal to 10,200 kcal/kg, $c_p=0·24$ and $\gamma=1·4$.

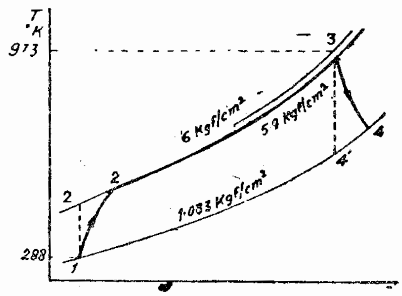


Fig. 2·42.

$$T_2'=T_1\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$=288\times\left(\frac{6}{1·033}\right)^{\frac{0·4}{1·4}}$$

$$=477\text{ K}$$

$$\eta_c=\frac{T_2'-T_1}{T_2-T_1}$$

or $$0·8=\frac{477-288}{T_2-288}$$

$\therefore$ $$T_2=524\text{ K}$$

$\therefore$ $$T_2-T_1=236$$

$$T_4'=\frac{T_3}{(P_3/P_4)^{\frac{\gamma-1}{\gamma}}}$$

$$=\frac{973}{(5·9/1·033)^{\frac{0·4}{1·4}}}$$

$$=592 \text{ K}$$

$$\eta_t = \frac{T_3 - T_4}{T_4' - T_3}$$

$$\therefore \quad 0{\cdot}83 = \frac{T_3 - T_4}{973 - 592}$$

or
$$T_3 - T_4 = 0{\cdot}83(973 - 592)$$
$$= 324 \text{ K}$$

$$\text{Net work done/kg of air} = c_p(T_3 - T_4) - c_p(T_2 - T_1)$$
$$= 0{\cdot}24 \times 324 - 0{\cdot}24 \times 236$$
$$= 21{\cdot}12 \text{ kcal/kg}$$

(a) Thermal efficiency, $\eta = \dfrac{\text{Work done}}{\text{Heat supplied}}$

$$= \frac{0{\cdot}24(324 - 236)}{0{\cdot}24(973 - 524)}$$

$$= \mathbf{17{\cdot}6\%}$$ **Ans.**

(b) Work ratio $= \dfrac{c_p\{(T_3 - T_4) - (T_2 - T_1)\}}{c_p(T_3 - T_4)}$

$$= \frac{324 - 236}{324}$$

$$= \mathbf{0{\cdot}272}$$ **Ans.**

(c) Air rate/shaft-horse-power $= \dfrac{632}{21{\cdot}12}$

$$= \mathbf{30 \text{ kg/hr}}$$ **Ans.**

(d) External heat supplied $= \dfrac{c_p(T_3 - T_2)}{\eta_{comb.}}$

$$= \frac{0{\cdot}24 \times 449}{0{\cdot}9}$$

$$= 120 \text{ kcal/kg of air}$$

Specific fuel consumption $= \dfrac{\text{Heat supplied/kg of air} \times \text{air rate}}{\text{Calorific value}}$

$$= \frac{120 \times 30}{10200}$$

$$= \mathbf{0{\cdot}353 \text{ kg/shp-hr}}$$ **Ans.**

(e) Air fuel ratio $= \dfrac{30}{0{\cdot}353}$

$$= \mathbf{85}$$ **Ans.**

2·3. Simple cycle : Comparison of ideal cycle and cycle considering component efficiencies.

An idustrial gas turbine takes in air at 1·03 kgf/cm² and 27°C and compresses it to 5·5 times the original pressure. The temperatures at the salient points are, compressure outlet, 251°C, turbine inlet, 760°C and turbine outlet, 447°C. Calculate the compressor and turbine efficiencies.

Compare for the ideal cycle and cycle considering component effieiencies—(a) thermal efficiency, (b) work ratio, (c) optimum pressure ratio for maximum output, and (d) optimum pressure ratio for maximum efficiency.

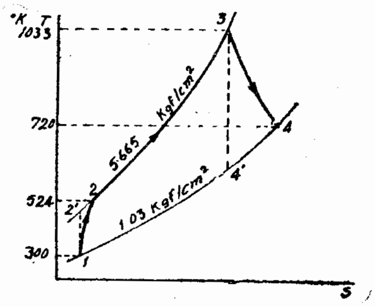


Fig. 2·43.

$$T_2'=T_1\left(\frac{P_2}{P}\right)^{\frac{\gamma-1}{\gamma}}$$

$$=300(5\cdot5)^{\frac{0\cdot4}{1\cdot4}}$$

$$=489$$

$$\eta_c=\frac{T_2'-T_1}{T_2-T_1}$$

$$=\frac{489-300}{524-300}$$

$$=0\cdot845.$$

$$T_4'=\frac{T_3}{(P_3/P_4)^{\frac{\gamma-1}{\gamma}}}$$

$$=\frac{1033}{(5\cdot5)^{\frac{0\cdot4}{1\cdot4}}}$$

$$=635$$

$$\eta_t=\frac{T_3-T_4}{T_3-T_4'}$$

$$=\frac{1033-720}{1033-635}$$

$$=0\cdot786$$

(a) Ideal cycle thermal $\eta=\dfrac{(T_3-T_4)-(T_2-T_1)}{T_3-T_2}$

$$=\frac{(1033-720)-(524-300)}{1033-524}$$

$=$**0·175** or **17·5%**. **Ans.**

Thermal efficiency $=\dfrac{T_3\,.\,\dfrac{\eta_t}{R}-T_1/\eta_c}{\dfrac{T_3-T_1}{R-1}-\dfrac{T_1}{\eta_c}}$

$$=\frac{1033\times\dfrac{0\cdot786}{1\cdot63}-\dfrac{300}{0\cdot845}}{\dfrac{1033-300}{1\cdot63-1}-\dfrac{300}{0\cdot845}}$$

$=$**0·181** or **18·1%**. **Ans.**

(b) Ideal cycle work ratio $=1-\dfrac{T_2-T_1}{T_3-T_4}$

$$=1-\frac{524-300}{1033-720}$$

$=$**0·284** **Ans.**

Work ratio $=1-\dfrac{T_1}{T_2}\,.\,R\,.\,\dfrac{1}{\eta_c\eta_t}$

$$=1-\frac{300}{1033}\,.\,(5\cdot5)^{\frac{0\cdot4}{1\cdot4}}\,.\,\frac{1}{0\cdot845\times0\cdot786}$$

$=$**0·288** **Ans.**

(c) Optimum pressure ratio for maximum output in the case of ideal cycle is given by

$$R=\sqrt{T_3/T_1}$$

$$\therefore \qquad \left(\frac{P_2}{P_1}\right)^{\frac{0\cdot4}{1\cdot4}}=\sqrt{\frac{1033}{300}}$$

or
$$\frac{P_2}{P_1}=8\cdot78 \qquad \textbf{Ans.}$$

Optimum pressure ratio for actual cycle is given by

$$R=\sqrt{\eta_c\eta_t\cdot\frac{T_3}{T_1}}$$

$$=\sqrt{0\cdot845\times0\cdot786\times\frac{1033}{300}}$$

$$=1\cdot236$$

$$\therefore \qquad P_2/P_1=(1\cdot236)^{\frac{1\cdot4}{0\cdot4}}$$

$$=2\cdot1. \qquad \textbf{Ans.}$$

(d) Optimum pressure ratio for maximum efficiency is given by

$$R=\frac{T_3/T_1}{1+\sqrt{(T_3/T_1-1)\left(\dfrac{1}{\eta_t\cdot\eta_c}-1\right)}}$$

$$=\frac{1033/300}{1+\sqrt{\left(\dfrac{1033}{300}-1\right)\left(\dfrac{1}{0\cdot845\times0\cdot786}-1\right)}}$$

$$=2\cdot83$$

$$\therefore \qquad \frac{P_2}{P_1}=(2\cdot83)^{\frac{1\cdot4}{0\cdot4}}$$

$$=9\cdot9 \qquad \textbf{Ans.}$$

In case of ideal cycle the optimum pressure ratio for maximum efficiency is given by

$$R=T_3/T_1=\frac{1033}{300}$$

$$=3\cdot5$$

$$\frac{P_2}{P_1}=12\cdot08. \qquad \textbf{Ans}$$

2·4. Condition for positive power ; η ; mass flow.

An open cycle gas turbine plant consists of a compressor, combustion chamber and turbine. The isentropic efficiencies of compressor and turbine are η_c and η_t, the maximum and minimum cycle temperatures are T_3 and T_1 respectively, and r_p is the pressure ratio for both the compression and expansion. Neglecting pressure losses and assuming that the working substance is a perfect gas, show that the necessary condition for positive power output is

$$\eta_c \eta_t T_3 > T_1 \; r_p^{\frac{n-1}{n}}.$$

Such a plant delivers 2000 hp and operates such that inlet pressure and temperature at the compressor is 1 kgf/cm² and 15°C, and that at turbine is 4 kgf/cm² and 700°C. Calculate the isentropic efficiency of the turbine and the requisite mass flow of air in kg/sec if the compressor efficiency is 85% and overall thermal efficiency is 21%.

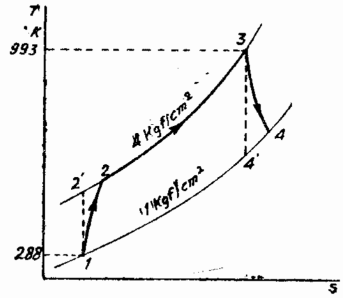


Fig. 2·44

$$T_2' = T_1 \left(P_2/P_1 \right)^{\frac{n-1}{n}}$$

$$= 288 \times 4^{\frac{0·4}{1·4}}$$

$$= 422·5 \text{ K.}$$

$$\eta_c = \frac{T_2' - T_1}{T_2 - T_1}$$

or $$0·85 = \frac{422·5 - 288}{T_2 - 288}$$

$\therefore$ $$T_2 = 446·1 \text{ K.}$$

$$T_4' = \frac{T_3}{(P_3/P_4)^{\frac{n-1}{n}}}$$

$$=\frac{973}{(4)^{\frac{0.4}{1.4}}}$$

$$=663.5 \text{ K}$$

$$\text{Overall thermal efficiency}=\frac{\text{net work output}}{\text{heat supplied}}$$

or $$0.21=\frac{c_p(973-T_4)-c_p(422.5-288)}{c_p(973-422.5)}$$

$\therefore$ $$T_4=722.8 \text{ K}$$

$$\eta_t=\frac{T_3-T_4}{T_3-T_4'}$$

$$=\frac{973-722.8}{973-663.5}=\frac{250.2}{309.5}$$

$$=\mathbf{0.81 \text{ or } 81.0\%}.$$ **Ans.**

$$\text{Net work output}=\eta\times\text{heat supplied}$$

$$=0.21\times0.24(973-422.5)$$

$$=27.75 \text{ kcal/kg}.$$

$$\text{Air mass flow rate}=\frac{2000\times75}{27.75\times427}$$

$$=\mathbf{9.57 \text{ kg/sec}}.$$ **Ans.**

2·5. Simple gas turbine ; pressure ratio for maximum output ; m ; sfc.

Deduce an expression for the specific output (kcal/kg of working fluid) of a simple constant pressure gas turbine in terms of temperatures at the beginning of compression and at the beginning of expansion, the isentropic efficiencies of the compressor and turbine, the pressure ratio and the isentropic index. The specific heat at constant pressure may be assumed constant, and the weight of the fuel added may be neglected.

Hence determine the pressure ratio at which the specific output is a maximum for the following operating conditions. Temperature at compressor inlet is 15°C, temperature at turbine inlet is 630°C, and the isentropic efficiency for compressor and turbine is 0·85 and 0·90 respectively.

Assume the working fluid to be air throughout. Determine in kg/hp-hr (a) the air flow and (b) the fuel consumption, if the calorific value of the fuel is 10,300 kcal/kg.

$$T_2'=T_1\,(r_p)^{\frac{\gamma-1}{\gamma}}$$

$$\triangle T_c'=T_2'-T_1=T_1\left(r_p^{\frac{\gamma-1}{\gamma}}-1\right)$$

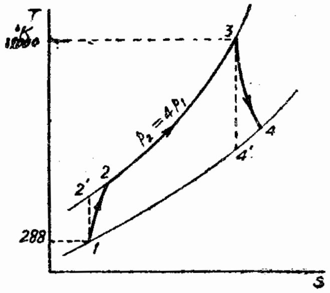


Fig. 2·45

$$\therefore \qquad \Delta T_c'=\frac{\Delta T_c}{\eta_c}=\frac{T_1}{\eta_c}\left(r_p^{\frac{\gamma-1}{\gamma}}-1\right)$$

Similarly, $\Delta T_t=\eta_t\ T_3\left(1-r_p^{\frac{1-\gamma}{\gamma}}\right)$

Now net work output $=w_t-w_c=c_p(\Delta T_t-\Delta T_c)$

or
$$w_{net}=c_p\left[\eta_t\ T_3\left(1-r_p^{\frac{1-\gamma}{\gamma}}\right)-\frac{T_1}{\eta_c}\left(r_p^{\frac{-1}{\gamma}}-1\right)\right]$$

Ans.

For optimum pressure ratio, differentiating the above relation w.r.t. r_p and equating to zero, we get

$$\frac{dw_{net}}{dr_p}=-\left(\frac{1-r}{r}\right)c_p\ \eta_t\ T_3\ r_p^{\frac{1-2\gamma}{\gamma}}-\left(\frac{r-1}{r}\right)\frac{c_p\ T_1}{\eta_c}\ r_p^{\frac{\gamma-1}{\gamma}}=0$$

$$\therefore \qquad \gamma_p=\left(\frac{T_1}{\eta_c\eta_t\ T_3}\right)^{\frac{\gamma}{2-2\gamma}}$$

Substituting the values

$$\gamma_p=\left(\frac{288}{903\times0{\cdot}85\times0{\cdot}9}\right)^{\frac{1{\cdot}4}{2-2\times1{\cdot}4}}$$

$$=\left(\frac{903\times0{\cdot}85\times0{\cdot}9}{288}\right)^{1.75}=4{\cdot}62$$

$$w_{net}=c_p\left\{\eta_t\ T_3\left(1-r_p^{\frac{1-\gamma}{\gamma}}\right)-\frac{T_1}{\eta_c}\left(r_p^{\frac{\gamma-1}{\gamma}}-1\right)\right\}$$

$$=0{\cdot}24\left\{0{\cdot}9-903\left(1-4{\cdot}62^{\frac{1-1\cdot4}{1\cdot4}}\right)-\frac{288}{0{\cdot}85}\left(4{\cdot}62^{\frac{1\cdot4-1}{1\cdot4}}-1\right)\right\}$$

$$=0{\cdot}24\left\{0{\cdot}9\times903\left(1-\frac{1}{1{\cdot}549}\right)-\frac{288}{0{\cdot}85}\left(0{\cdot}549\right)\right\}$$

$$=0{\cdot}24\ (288-186)=24{\cdot}48\ \text{kcal/kg}$$

Mass flow $=\dfrac{75\times3600}{427\times24{\cdot}48}=$ **26·9 kg/hp-hr.** **Ans.**

$$T_2=T_1\times(r_p)^{\frac{\gamma-1}{\gamma}}$$

$$=288\times(4{\cdot}62)^{\frac{0\cdot4}{1\cdot4}}$$

$$=288\times1{\cdot}55=462\ \text{K}$$

Mass of the fuel $=\dfrac{26\ 9\times0{\cdot}24(903-462)}{10{,}300}$

$=$ **0·2695 kg/hp-hr.** **Ans.**

2·6. Simple gas turbine ; efficiency and air intake taking mass of fuel into account

In a gas turbine plant, comprising a single stage compressor, combustion chamber and turbine, the compressor takes in air at 15°C and compresses it to 4 times the initial pressure with an isentropic efficiency of 85 per cent. The fuel air ratio is 0·0125 and the calorific value of the fuel is 10,000 kcal/kg. If the isentropic efficiency of the turbine is 82 per cent, find the overall thermal efficiency and the air intake for a power output of 350 hp.

Take the mass of the fuel into account. The turbine inlet temperature is 1000 K. *(I. Mech. E. April, 1959)*

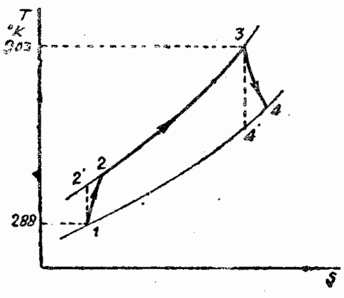


Fig. 2·46

$$T_2' = T_1 . (P_2/P_1)^{\frac{n-1}{n}}$$

$$= 288 \times (4)^{\frac{0 \cdot 4}{1 \cdot 4}}$$

$$= 425 \text{ K}$$

$$T_2 = \frac{(T_2' - T_1)}{\eta_c} + T_1$$

$$= \left(\frac{425 - 288}{0 \cdot 85} \right) + 288$$

$$= 449 \cdot 1 \text{ K}$$

Work done on compression

$$= 0 \cdot 24(449 \cdot 1 - 288) = 38 \cdot 7 \text{ kcal/kg}$$

$$T_4' = \frac{1000}{(4)^{\frac{0 \cdot 4}{1 \cdot 4}}} = 674 \text{ K}$$

$$T_4 = T_3 - \eta_t (T_3 - T_4')$$

$$= 1000 - 0 \cdot 82(1000 - 674)$$

$$= 732 \cdot 5 \text{ K}$$

Work done by turbine

$$= 0 \cdot 24(1000 - 732 \cdot 5)$$

$$= 64 \cdot 2 \text{ kcal/kg.}$$

$\therefore$ Net work output

$$= 1 \cdot 0125 \times 64 \cdot 2 - 38 \cdot 7$$

$$= 26 \cdot 3 \text{ kcal/kg of air}$$

$$= \frac{26 \cdot 3 \times 427}{75} = 15 \text{ H.P.}$$

Heat supplied $= 0 \cdot 24(1000 - 449 \cdot 1)$

$$= 132 \cdot 1 \text{ kcal/kg.}$$

$\therefore$ Overall thermal efficiency

$$= \frac{26 \cdot 3}{132 \cdot 1} = \mathbf{0 \cdot 199 \text{ or } 19 \cdot 9\%}$$ **Ans.**

Intake air required

$$= \frac{350}{15} = \mathbf{23 \cdot 38 \text{ kg./sec.}}$$ **Ans.**

2·7. Improvement in cycle with intercooling and regeneration

An ideal open cycle gas turbine plant using air operates on an overall pressure ratio of 4 and between the temperature limits of 300 K and 1000 K. Assuming constant specific heats, $c_p=0{\cdot}24$, $c_v=0{\cdot}17$, evaluate the specific work output and thermal efficiency for each of the modifications below and state the percentage change from the basic cycle. Assume optimum stage pressure ratios, perfect intercooling and perfect regeneration.

(*a*) *Basic cycle.*

(*b*) *Basic cycle with heat exchanger.*

(*c*) *Basic cycle with two stage intercooled compressor.*

(*d*) *Basic cycle with heat exchanger and intercooled compressor.*

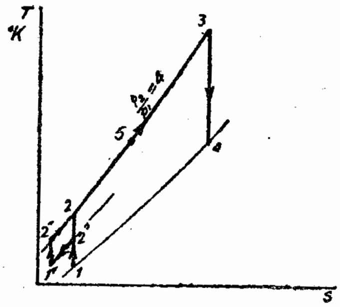


Fig. 2·47

$$c_p/c_v=\frac{0{\cdot}24}{0{\cdot}17}$$
$$=1{\cdot}4$$

(*a*) *Basic cycle.*

$$T_2'=300(4)^{\frac{0\cdot4}{1\cdot4}}$$
$$=446\text{ K}$$

$$\therefore \quad T_2'-T_1=446-300=146$$

$$T_4'=\frac{1000}{(4)^{\frac{0\cdot4}{1\cdot4}}}$$
$$=674\text{ K}$$

$$\therefore \quad T_3-T_4'=1000-674=326$$

$\therefore$ Specific work output

$$=0{\cdot}24(326-146)$$
$$=\mathbf{43{\cdot}2\ kcal/kg.}$$ **Ans.**

Thermal efficiency $=\dfrac{0.24\times180}{0.24(1000-446)}$

$=$**0·3255 or 32·55%.** **Ans.**

(*b*) *Basic cycle with heat exchanger.*

Since regeneration is perfect

$$T_4-T_2=T_5-T_2$$

or $$T_4=T_5=674 \text{ K}$$

The specific work output remains unchanged.

The thermal efficiency

$$=\frac{0.24(326-146)}{0.24(1000-674)}$$

$=$**0·552 or 55·2%.** **Ans.**

(*c*) *Basic cycle with two stage intercooled compressor.*

The pressure ratio for each stage

$$=\sqrt{4}=2$$

$\therefore$ $$T_2'=300\times2^{\frac{0.4}{1.4}}$$

$$=306 \text{ K}.$$

$\therefore$ $$T_2'-T_1=306-300=6$$

Total work done on two stages

$$=2c_p(T_2-T_1)$$

The turbine work done is not affected by intercooling, *i.e.*

$$w_t=c_p(T_3-T_4)$$

$\therefore$ Net work output

$$=c_p(T_3-T_4')-2c_p(T_2'-T_1)$$

$$=0.24\times326-2\times0.24\times6$$

$=$**75·4 kcal/kg.** **Ans.**

Thermal efficiency $=\dfrac{75.4}{0.24(1000-306)}$

$=$**0·453 or 45·3%.** **Ans.**

(*d*) *Basic cycle with heat exchanger and intercooled compressor.*

The net work output remains same as in (*c*), *i.e.* 75·4 kcal/kg

For ideal regeneration

$$T_5=T_4=674 \text{ K}.$$

$\therefore$ Thermal efficiency

$$=\frac{75.4}{0.24(1000-674)}$$

$=$**0·964 or 96·4%.** **Ans.**

2·8. Regenerative gas turbine cycle ; specific output and efficiency at full load and part load.

(a) Derive an expression for the efficiency of an ideal basic gas turbine cycle with perfect regeneration. Show the variation of efficiency with respect to cycle pressure ratio at different temperature ratios and explain how reasonably good part load efficiencies are possible with this arrangement.

(b) At full load an ideal basic regenerative cycle operates with an inlet air temperature of 20°C. The turbine inlet temperature is 850°C, and the pressure ratio for the cycle is 6 to 1. To obtain a desired part load operating point, the turbine inlet temperature is lowered to 600°C and the compression ratio to 2·5. Find the specific outputs and thermal efficiencies of the cycle at the full load and part load operating points and compare the results with those for the basic cycle operating under the same conditions.

[Aligarh, M.Sc. Engg., Jan. 1972 ; Punjab, M.E. April 1972]

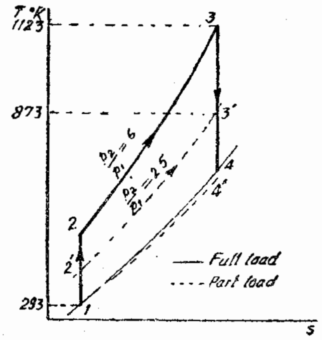


Fig. 2·48

(a) The efficiency of ideal cycle with ideal regeneration is

$$\eta=1-\frac{T_1}{T_3}(r_p)^{\frac{\gamma-1}{\gamma}},$$

where $$r_p=\frac{p_2}{p_1}$$

If $$\frac{T_2}{T_1}=\theta=\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$$

and $$\frac{p_2}{p_1}=r_p,\ \frac{T_3}{T_1}=$$

$$\eta=1-\frac{\theta}{\alpha}=1-\frac{(r_p)^{\frac{\gamma-1}{\gamma}}}{\alpha}$$

(*b*) (*i*) *Full load* $\eta = 1 - \dfrac{(r_p)^{\frac{\gamma-1}{\gamma}}}{\alpha}$

$$\alpha = \frac{1123}{293} = 3{\cdot}84$$

$$= 1 - \frac{(6)^{\frac{0{\cdot}4}{1{\cdot}4}}}{3{\cdot}84} = 1 - 0{\cdot}435 = 0{\cdot}565$$

or $= \mathbf{56{\cdot}5\%}$. **Ans.**

Specific power output

$$= c_p[(T_3 - T_4) - (T_2 - T_1)]$$

$$= c_p\, T_1 \left(\frac{T_3}{T_1} - \frac{T_4}{T_1} - \frac{T_2}{T_1} + 1 \right)$$

$$= c_p\, T_1 \left(\alpha - \frac{T_4}{T_3} \cdot \frac{T_3}{T_1} - \theta + 1 \right)$$

$$\because \frac{T_2}{T_1} = \frac{T_3}{T_4} = \theta$$

$$= c_p\, T_1 \left(\alpha - \frac{\alpha}{\theta} - \theta + 1 \right)$$

$$= 0{\cdot}24 \times 293 \left(3{\cdot}84 - \frac{3{\cdot}84}{1{\cdot}67} - 1{\cdot}67 + 1 \right)$$

$$= 0{\cdot}24 \times 293 \times 0{\cdot}87$$

$= \mathbf{61{\cdot}6\ kcal/kg}$. **Ans.**

Part load condition

$$T_3 = 600 + 273 = 873 \text{ K}$$

$$\alpha = \frac{T_3}{T_1} = \frac{873}{293} = 2{\cdot}98$$

$$\theta = \frac{T_2}{T_1} = \left(\frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}} = (2{\cdot}5)^{0{\cdot}286} = 1{\cdot}3$$

$$\eta = 1 - \frac{\theta}{\alpha} = 1 - \frac{1{\cdot}3}{2{\cdot}98}$$

$= \mathbf{0{\cdot}564\ or\ 56{\cdot}4\%}$. **Ans.**

$$w_{net} = c_p T_1 \left(\alpha - \frac{\alpha}{\theta} - \theta + 1 \right)$$

$$=0{\cdot}24\times 293\left(2{\cdot}98-\frac{2{\cdot}98}{1{\cdot}3}-1{\cdot}3+1\right)$$

$$=27{\cdot}4 \text{ kcal/kg}$$

Comparison with basic cycle without regeneration

$$\eta=1-\frac{1}{\theta}$$

(*i*) For full load $\eta=1-\dfrac{1}{1{\cdot}67}=1-0{\cdot}6$

$=$**0·4 or 40%.** **Ans.**

(*ii*) Part load $\eta=1-\dfrac{1}{1{\cdot}3}=1{\cdot}0-0{\cdot}77$

$=$**0·23 or 23%.** **Ans.**

w_{net} is not affected by regeneration and remains same.

RESULTS

	Regeneration cycle	*Simple cycle*
Full load η	56·5%	40%
w_{net}	61·6 kcal	61·6 kcal
Part load η	56·4%	23%
w_{net}	27·4 kcal	27·4 kcal

Ans.

2·9. Compound gas turbine ; $P_{L.P.\ turbine}$; $\eta_{overall}$

In a compound gas turbine the air from the compressor passes through a heat exchanger heated by the exhaust gases from the low-pressure turbine, and then into the high pressure combustion chamber. The high-pressure turbine drives the compressors only. The exhaust gases from the high pressure turbine pass through the low-pressure combustion chamber to the low pressure turbine which is coupled to an external load. The following data refer to the plant :

Pressure compression ratio in the compressor, 4 : 1

Isentropic efficiency of compressor, 0·86

Isentropic efficiency of H.P. turbine, 0·84

Isentropic efficiency of L.P. turbine, 80·0

Mechanical efficiency of drive to compressor, 0·92.

In the heat exchanger 75% of the available heat is transferred to the air.

Temperature of gases entering H.P. turbine, 660°C

Temperature of gases entering L.P. turbine, 625°C.

Atmospheric temperature and pressure are 15°C and 1·03 kgf/cm² respectively.

Assuming that the specific heat c_p of air and gas is 0·24, determine (a) the pressure of the gases entering the low pressure turbine ; (b) the overall efficiency.

[*Punjab M.E. 1969, Gwalior 1972 (S)*]

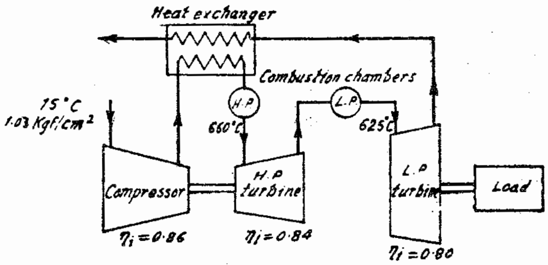


Fig. 2·49 (*a*).

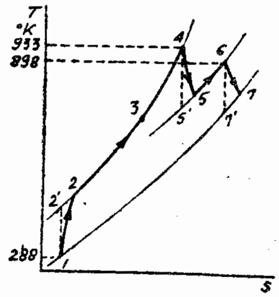


Fig. 2·49 (*b*).

$$\frac{T_2'}{T_1}=\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}=(4)^{\frac{1\cdot4-1}{1\cdot4}}=1\cdot486$$

$\therefore$ $$T_2'=288\times1\cdot486=428\text{ K}$$

$$\eta_t=\frac{T_2'-T_1}{T_2-T_1}$$

or $$0\cdot86=\frac{428-288}{T_2-288}\qquad\therefore\quad T_2=451\text{ K}$$

Now, W.D. by H.P. Turbine $\times\,\eta_{mech}$ = W.D. by compressor

$\therefore$ $$c_p(T_4-T_5)\times0\cdot92=c_p(T_2-T_1)$$

or $$(T_4-T_5)=\frac{163}{0\cdot92}=177\text{ K}$$

and $$T_5=933-177=756\text{ K}$$

For H.P. turbine η_t, $0{\cdot}84=\dfrac{T_4-T_5}{T_4-T_5'}$,

$\therefore \quad T_4-T_5'=211K,\ T_5'=722$ K

Now $\quad \dfrac{P_4}{P_5}=\left(\dfrac{T_4}{T_5'}\right)^{\frac{\gamma}{\gamma-1}}=\left(\dfrac{933}{722}\right)^{\frac{1\cdot4}{1\cdot4-1}}=2{\cdot}49$

$\therefore \quad P_5=\dfrac{4\times1{\cdot}03}{2{\cdot}49}=\mathbf{1{\cdot}655\ kgf/cm^2}$ **Ans.**

For L.P. turbine

$$\frac{T_6}{T_7'}=\left(\frac{P_6}{P_1}\right)^{\frac{\gamma-1}{\gamma}}=\left(\frac{1{\cdot}655}{1{\cdot}03}\right)^{\frac{1\cdot4-1}{1\cdot4}}=1{\cdot}143$$

$\therefore \quad T_7'=\dfrac{898}{1{\cdot}143}=785$ K

$$\eta_t=0{\cdot}8=\frac{T_6-T_7}{T_6-T_7'}$$

or $\quad T_6-T_7=(898-785)\times0{\cdot}8=90{\cdot}4$

$\therefore \quad T_7=898-90{\cdot}4=807{\cdot}6$ K

For heat exchanger

$$\eta=0{\cdot}75=\frac{c_p(T_3-T_2)}{c_p(T_7-T_2)}$$

$\therefore \quad T_3-T_2=0{\cdot}75(807{\cdot}6+451)=267{\cdot}5$ K

or $\quad T_3=451-267{\cdot}5=718{\cdot}5$ K

Overall efficiency $=\dfrac{c_p(T_6-T_7)}{c_p(T_4-T_3)+c_p(T_6-T_5)}$

$=\dfrac{90{\cdot}4}{(933-718{\cdot}5)+(898-756)}$

$=\mathbf{0{\cdot}253}$ or $\mathbf{25{\cdot}3\%}$. **Ans.**

2·10. Actual gas turbine cycle with two-stage compression and expansion ; H.P. ; sfc ; efficiency.

At design speed the following data apply to a gas turbine set employing a separate power turbine, heat exchanger, reheater and intercooler between two-stage compression.

Isentropic efficiency of compression in each stage	*80%*
Isentropic efficiency of compressor turbine	*87%*
Isentropic efficiency of power turbine	*80%*
Turbine to compressor transmission efficiency	*99%*
Pressure ratio in each stage of compression	*2 : 1*

Pressure loss in intercooler	*0·07 kgf/cm²*
Temperature after intercooling	*300 K*
Pressure loss on each side of heat exchanger	*0·1 kgf/cm²*
Thermal ratio of heat exchanger	*0·75*
Pressure loss in combustion chamber	*0·15 kgf/cm²*
Combustion efficiency of reheater	*98%*
Maximum cycle temperature	*1000 K*
Temperature after reheating	*1000 K*
Air mass flow	*25 kg/s*
Ambient air temperature	*15°C*
Ambient air pressure	*1 kgf/cm²*
Calorific value of fuel	*10,300 kcal/kg.*

For air, $c_p = 0{\cdot}24$ *and* $\gamma = 1{\cdot}4$ *during compression*

For gases, $c_p = 0{\cdot}276$ *and* $\gamma = 1{\cdot}33$ *during heating and expansion.*

Find the net power output, specific fuel consumption and overall thermal efficiency. Neglect the kinetic energy of the gases leaving the system. [*B.H.U., M.E. 1970*]

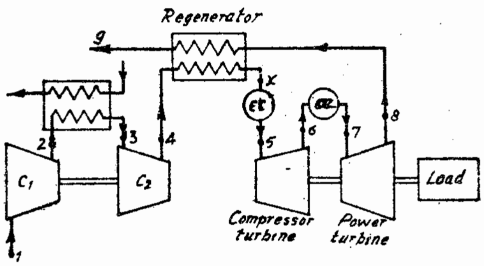


Fig. 2·50 (*a*).

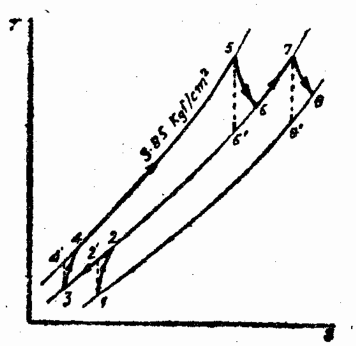


Fig. 2·50 (*b*).

$$0{\cdot}857=\frac{\gamma-1}{\gamma}\times\frac{n}{n-1}$$

or $$\frac{n-1}{n}=\frac{1{\cdot}4-1}{1{\cdot}4}\times\frac{1}{0{\cdot}857}=0{\cdot}3335$$

Work of compression in first stage is given by

$$W_{c_1}=c_p(T_2-T_1)$$
$$=c_pT_1\left\{\frac{T_2}{T_1}-1\right\}$$
$$=c_pT_1\left\{\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n_1}}\right\}$$
$$=0{\cdot}24\times280\times\left\{\begin{matrix}0{\cdot}3335\\2-1\end{matrix}\right\}$$
$$=0{\cdot}24\times280\times0{\cdot}26$$
$$=17{\cdot}5\text{ kcal/kg.}$$

Work done in the second stage,

$$W_{c_2}=0{\cdot}24\times300\times[0{\cdot}26]$$
$$=18{\cdot}75\text{ kcal}$$

Total compressor work

$$=17{\cdot}5+18{\cdot}75=36{\cdot}25\text{ kcal/kg.}$$
$$P_5=4-0{\cdot}05-0{\cdot}01$$
$$=3{\cdot}85\text{ kgf/cm}^2$$

Also $$T_4-T_3=T_3\left(\frac{T_4}{T_3}-1\right)$$
$$=T_3\left\{\left(\frac{P_4}{P_3}\right)^{\frac{n-1}{n}}-1\right\}$$
$$=300\times0{\cdot}26=78$$

$\therefore$ $$T_4=378\text{ K}$$

Now compressor work=compressor turbine work.

$\therefore$ $$36{\cdot}25=0{\cdot}276(T_5-T_6)$$

$\therefore$ $$T_5-T_6=\frac{36{\cdot}25}{0{\cdot}276}=131\text{ K.}$$

$\therefore$ $$T_6=1000-131=869\text{ K}$$

Turbine polytropic efficiency is given by

$$\eta_{p_t}=\frac{n-1}{n}\times\frac{\gamma}{\gamma-1}$$

or $$0{\cdot}882=\frac{n-1}{n}\times\frac{1{\cdot}33}{1{\cdot}33-1}$$

$\therefore$ $$\frac{n-1}{n}=\frac{0{\cdot}882\times 0{\cdot}33}{1{\cdot}33}=0{\cdot}219$$

$$T_5-T_6=T_5\left[1-\left(\frac{P_6}{P_5}\right)^{\frac{n-1}{n}}\right]$$

or $$131=1000\left\{1-\frac{1}{\gamma^{0\cdot 219}}\right\}$$

or $$\gamma^{0\cdot 219}=\frac{1000}{869}=1{\cdot}15$$

$\therefore$ $$\gamma=1{\cdot}89$$

$\therefore$ $$P_6=\frac{3{\cdot}85}{1{\cdot}89}=2{\cdot}04\text{ kgf/cm}^2$$

$$P_7=1+0{\cdot}05=1{\cdot}05\text{ kgf/cm}^2$$

$$\frac{T_6}{T_7}=\left(\frac{P_6}{P_7}\right)^{\frac{n-1}{n}}$$

$$=\left(\frac{2{\cdot}04}{1{\cdot}05}\right)^{0\cdot 219}$$

$$=1{\cdot}157$$

$$T_7=\frac{T_6}{1{\cdot}157}=\frac{869}{1{\cdot}157}$$

$$=750\text{ K}$$

Net work output/kg $=c_p'(T_6-T_7)$

$=0{\cdot}276(869-750)$

$=32{\cdot}8$ kcal/kg.

$$\text{Net shaft horse power}=\frac{\text{mass flow}\times\text{W.D./kg}\times 427}{75}$$

$$=\frac{15\times 32{\cdot}8\times 427}{75}$$

$$=\mathbf{2820}.$$ **Ans.**

Let T_x be the temperature after heat-exchanges.

Then $$0{\cdot}75=\frac{T_x-T_4}{T_7-T_4}$$

$$=\frac{T_x-378}{750-378}$$

$\therefore$ $$T_x=657\text{ K}.$$

$\therefore$ Temperature rise in the combustion chamber

$$=1000-657=343\text{ K}.$$

Overall thermal efficiency $\eta=\dfrac{T_6-T_7}{T_5-T_x}=\dfrac{119}{343}$

$=34{\cdot}8\%$ **Ans.**

Specific fuel consumption $=\dfrac{632}{10300\times0{\cdot}348}$

$=0{\cdot}176$ kg/bhp/hr **Ans.**

Without Heat Exchanger

Compression work remains same $=36{\cdot}25$ kcal/kg.

$$P_5=4-0{\cdot}1=3{\cdot}9 \text{ kgf/cm}^2$$

$$P_6=\frac{3{\cdot}9}{1{\cdot}89}=2{\cdot}06 \text{ kgf/cm}^2,\ P_7=1 \text{ kgf/cm}^2$$

$$\frac{T_6}{T_7}=(P_6/P_7)^{\frac{n-1}{n}}=(2{\cdot}06)^{0{\cdot}19}$$

$$=1{\cdot}174$$

$$\therefore\qquad T_7=\frac{869}{1{\cdot}174}=740\text{ K}$$

Hence net work output/kg $=0{\cdot}276(869-740)$

$=0{\cdot}276\times129$

$=35{\cdot}6$ kcal

Net shaft h.p. $=\dfrac{35{\cdot}6\times15\times427}{75}=$ **3050** **Ans.**

Efficiency, $\eta=\dfrac{T_6-T_7}{T_5-T_4}$

$=\dfrac{129}{622}=$ **20·8%** **Ans.**

Specific fuel consumption $=\dfrac{632}{10300\times0{\cdot}208}$

$=0{\cdot}295$ kg/bhp/hr. **Ans.**

2·11. Closed-cycle gas turbine : pressure ratio and cycle efficiency for max. work.

In a closed cycle turbine plant the working fluid at 38°C is compressed with an adiabatic efficiency of 82%. It is then heated at constant pressure to 650°C. The fluid then expands down to initial pressure in a turbine with an adiabatic efficiency of 80%. The fluid after expansion is cooled to 38°C.

The pressure ratio is such that work done per kg. is maximum. The working fluid is air which is assumed to be a perfect gas having $c_p=0{\cdot}25$ and $R=29{\cdot}2$ kg·m/kg/k.

Calculate the pressure ratio and cycle efficiency.

(Ujjain, 1971 Annual)

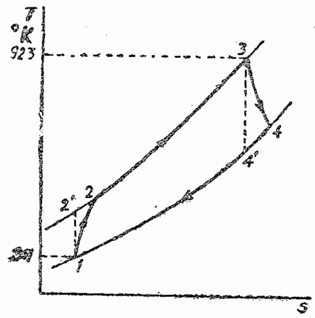


Fig. 2·51

$$c_p - c_v = \frac{R}{J}$$

$\therefore$ $$c_p = 0{\cdot}25 - \frac{29{\cdot}2}{427} = 0{\cdot}25 - 0{\cdot}0684$$

$$= 0{\cdot}1816$$

$$\gamma = \frac{c_p}{c_v} = \frac{0{\cdot}25}{0{\cdot}1816} = 1{\cdot}38$$

$$\eta_{c_1} = 0{\cdot}82 = \frac{T_2' - T_1}{T_2 - T_1}$$

$$\frac{T_2'}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = (r_p)^{\frac{\gamma-1}{\gamma}}$$

$\therefore$ $$T_2' = T_1 \ (r_p)^{\frac{\gamma-1}{\gamma}}$$

$\therefore$ $$T_2 - T_1 = 1{\cdot}22 \ T_1\left[(r_p)^{\frac{\gamma-1}{\gamma}} - 1\right]$$

$$\eta_t = 0{\cdot}80 = \frac{T_3 - T_4}{T_3 - T_4'}$$

$\therefore$ $$T_3 - T_4 = 0{\cdot}8 \ T_3\left\{1 - 1/(r_p)^{\frac{\gamma-1}{\gamma}}\right\}$$

Net work done, $W_{net} = c_p(T_3 - T_4) - c_p(T_2 - T_1)$

$$= c_p \times 0{\cdot}8 \times T_3\left\{1 - 1/(r_p)^{\frac{\gamma-1}{\gamma}}\right\} - c_p \times \ 1{\cdot}22 \ T_1\left\{(r_p)^{\frac{\gamma-1}{\gamma}} - 1\right\}$$

$$=0{\cdot}25\left[737\left\{1-1/(r_p)^{\frac{\gamma-1}{\gamma}}\right\}-380\left\{(r_p)^{\frac{\gamma-1}{\gamma}}-1\right\}\right]$$

For maximum work $\dfrac{dW}{dr}=0$ and $\dfrac{d^2W}{dr^2}=-w$

$$\therefore\quad -737\frac{d}{dr}(r_p)^{\frac{1-\gamma}{\gamma}}-380\frac{d}{dr}(r_p)^{\frac{\gamma-1}{\gamma}}=0$$

or

$$-737\,\frac{1-\gamma}{\gamma}(r_p)^{\frac{1-2\gamma}{\gamma}}-380\,\frac{\gamma-1}{\gamma}\,(r_p)^{\frac{-1}{\gamma}}=0$$

or

$$\frac{(r_p)^{\frac{1-2\gamma}{\gamma}}}{(r_p)^{-\frac{1}{\gamma}}}=\frac{380}{737}$$

or

$$(r_p)^{\frac{2(1-\gamma)}{\gamma}}=\frac{380}{737}$$

$$r_p=\left(\frac{380}{737}\right)^{\frac{\gamma}{2(1-\gamma)}}=(0{\cdot}51)^{-1{\cdot}92}$$

$$=3{\cdot}65$$

$$T_2'=311\times(3{\cdot}65)^{\frac{0{\cdot}38}{1{\cdot}38}}$$

$$=445\text{ K}$$

$$0{\cdot}82=\frac{134}{T_2-T_1}$$

$$\therefore\quad T_2-T_1=163{\cdot}5$$

$$\therefore\quad T_2=474{\cdot}5$$

$$T_4'=\frac{T_3}{(r_p)^{\frac{\gamma-1}{\gamma}}}=\frac{923}{1{\cdot}43}=645\text{ K}$$

$$0{\cdot}8=\frac{T_3-T_4}{278}$$

$$\therefore\quad T_3-T_4=222$$

$$\therefore\quad T_4=701\text{ K.}$$

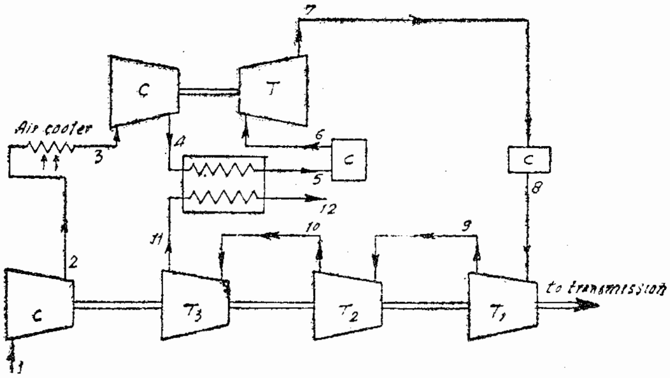
$$\eta=\frac{(T_3-T_4)-(T_2-T_1)}{T_3-T_2}=\frac{222-163{\cdot}5}{923-474{\cdot}5}$$

$$=\frac{58{\cdot}5}{448{\cdot}5}=13\%$$ **Ans.**

2·12. Automotive gas turbine

The flow diagram of an automotive gas turbine is shown in **Fig. 2·52.** *The following data refers to this turbine :*

Rated H.P.	=300
Pressure ratio	=16 : 1
Turbine inlet temperature, °C	=925
Bsfc, kg/bhp-hr.	=0·255
Exhaust temperature, °C	=316
Regenerator efficiency, %	=75
Burner efficiency, %	=96
Compressor efficiency, %	=80
Turbine efficiency, %	=85
Ambient temperature, °C	=27



(*a*)

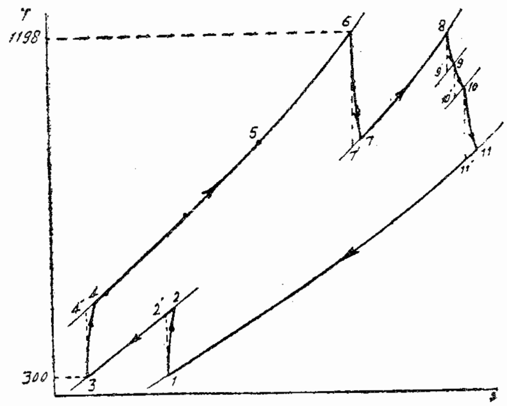


Fig. 2·52.(*b*)

Calculate the (i) thermal efficiency (ii) work ratio and (iii) temperature and pressure at salient points.

Overall pressure ratio $=16:1$

$\therefore$ Stage pressure ratio $=\sqrt{16}=4$

$$T_2'=T_1\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$=300\times\left(\frac{4}{1}\right)^{\frac{0\cdot4}{1\cdot4}}$$

$$=446\text{ K}$$

$$\eta_c=\frac{T_2-T_1}{T_2'-T_1}$$

or $$0\cdot8=\frac{T_2-300}{446-300}$$

$\therefore$ $$T_2=482\cdot5\text{ K}$$

Assuming that the air cooler cools the air to initial temperature, we have

$$T_3=T_1=300\text{ K}$$

$\therefore$ $$T_4=482\cdot5\text{ K}$$

Since the high pressure turbine drives the compressor only

$$c_p(T_4-T_3)=c_p(T_7-T_6)$$

or $$0\cdot24(482\cdot5-300)=0\cdot24(1198-T_7)$$

$\therefore$ $$T_2=1016\cdot5\text{ K}$$

$$\eta_t=\frac{T_6-T_7}{T_6-T_7'}$$

or $$0\cdot85=\frac{1198-1016\cdot5}{1198-T_7'}$$

$\therefore$ $$T_7'=983\text{ K}$$

$$\frac{T_7'}{T_6}=\frac{1}{(r_p)^{\frac{\gamma-1}{\gamma}}}$$

$\therefore$ $$r_p=\left(\frac{1198}{983}\right)^{\frac{1\cdot4}{0\cdot4}}$$

$$=2\cdot0$$

$\therefore$ $$P_7=\frac{16}{2\cdot0}=8\text{ kgf/cm}^2$$

By energy balance of heat exchanger

$$c_p(T_5-T_4)=c_p(T_{11}-T_{12})$$

$$\text{Heat exchanger effectiveness} = \frac{c_p(T_5 - T_4)}{c_p(T_{11} - T'_4)}$$

$$= \frac{c_p(T_{11} - T_{12})}{c_p(T_{11} - T'_4)}$$

or

$$0{\cdot}75 = \frac{T_{11} - 589}{T_{11} - 482{\cdot}5}$$

$\therefore$

$$= 908 \text{ K}$$

Since whole of the power output of turbine T_3 is consumed in driving the low pressure compressor, we have

$$c_p(T_2 - T_1) = c_p(T_{10} - T_{11})$$

or

$$0{\cdot}24(482{\cdot}5 - 300) = 0{\cdot}24(T_{10} - 908)$$

$$T_{10} = 1090{\cdot}5 \text{ K}$$

$$\eta_{t_3} = \frac{T_{10} - T_{11}}{T_{10} - T_{11}'}$$

or

$$0{\cdot}85 = \frac{1090{\cdot}5 - 9{\cdot}8}{1090{\cdot}5 - T_{11}'}$$

$\therefore$

$$T_{11}' = 876{\cdot}1 \text{ K}$$

$\therefore$ Pressure ratio for turbine T_3 is given by

$$r_p = \left(\frac{1090{\cdot}5}{876{\cdot}1}\right)^{\frac{1{\cdot}4}{0{\cdot}4}} = 1{\cdot}508$$

Since $p_{11} = 1 \text{ kgf/cm}^2$, $p_{10} = 1{\cdot}508 \text{ kgf/cm}^2$

$$\frac{p_8}{p_{10}} = \frac{8}{1{\cdot}508} = 5{\cdot}31$$

$\therefore$

$$\frac{p_8}{p_9} = \frac{p_9}{p_{10}} = \sqrt{5{\cdot}31} = 2{\cdot}305$$

$\therefore$

$$p_9 = \frac{8}{2{\cdot}305} = 3{\cdot}47 \text{ kgf/cm}^2$$

$$T_9' = \frac{T_8}{(r_p)^{\frac{\gamma - 1}{\gamma}}}$$

$$= \frac{1198}{(2{\cdot}305)^{\frac{0{\cdot}4}{1{\cdot}4}}} = 1070 \text{ K.}$$

$$t = \frac{T_8 - T_9}{T_8 - T_9'}$$

or

$$0{\cdot}85 = \frac{1198 - T_9}{1198 - 1170}$$

$\therefore$

$$T_9 = 1174{\cdot}2 \text{ K.}$$

2·13. Test on gas turbine plant : η_c ; m ; T_{entry}

The following data relate to a test on a simple gas turbine plant :

Ambient temperature and pressure, 10°C and 1·04 kgf/cm² respectively ; static pressure at compressor entry, 0·93 kgf/cm² ; compressor delivery total head pressure and temperature, 6 kgf/cm² and 230°C respectively ; turbine exhaust pipe temperature, 460°C, turbine horsepower 8000.

Calculate the total head isentropic efficiency of the compressor, and taking the compressor entry area as 0·10 m², calculate the air mass flow and estimate the temperature of the gases at entry to the turbine.

It may be assumed that there are no losses at compressor entry, that the entry velocity distribution is uniform and that increase of mass flow due to fuel addition is negligible.

Take c_p=0·24 and γ=1·4 for compression and c_p=0·27 for expansion.

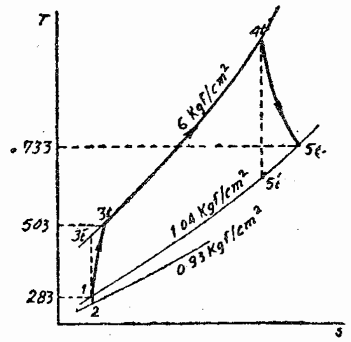


Fig. 2·53

Note. Ambient conditions are total head conditions since ambient conditions are considered to have no kinetic energy.

(a) *Compressor total head isentropic efficiency :*

Consider the total head conditions between points 1 and 3.

$$\frac{T_{3'}{}_t}{T_{1t}}=\left(\frac{P_{3t}}{P_{1t}}\right)^{\frac{\gamma-1}{\gamma}}$$

$$T_{3'}{}_t=283\times\left(\frac{6}{1\cdot04}\right)^{\frac{0\cdot4}{1\cdot4}}=466\text{ K}$$

$$\text{isentropic } \eta=\frac{T_{3}'{}_t-T_{1t}}{T_{3t}-T_{1t}}$$

$$=\frac{466-283}{503-283}=\mathbf{0{\cdot}831} \text{ or } \mathbf{83{\cdot}1\%} \qquad \textbf{Ans.}$$

(b) *Air mass flow*

$$P_{2t}=P_{1t}=1{\cdot}04 \text{ kgf/cm}^2$$
$$T_{2t}=T_{1t}=283 \text{ K}$$

$$\left(\frac{P_{2t}}{P_2}\right)=\left(\frac{T_{2t}}{T_2}\right)^{\frac{\gamma}{\gamma-1}}$$

$$\therefore \qquad \frac{1{\cdot}04}{0{\cdot}93}=\left(\frac{283}{T_2}\right)^{\frac{1{\cdot}4}{0{\cdot}4}}$$

$$\therefore \qquad T_2=283\times\left(\frac{9{\cdot}03}{1{\cdot}04}\right)^{\frac{0{\cdot}4}{1{\cdot}4}}=274 \text{ K}$$

$$T_{2t}=T_2+\frac{V_2^2}{2gJc_p}$$

$$\therefore \qquad 283=273+\frac{V_2^2}{2gJc_p}$$

$\therefore$ Velocity at entry to compressor,

$$V_2=\sqrt{2\times9{\cdot}81\times427\times0{\cdot}24(283-274)}$$
$$=134{\cdot}6 \text{ m/sec.}$$

Volume per sec. $=\text{area}\times\text{velocity}=0{\cdot}10\times134{\cdot}6$

$$=13{\cdot}46 \text{ m}^3/\text{s}=V_2$$

$$\gamma=\frac{c_p}{c_v} \qquad \therefore \quad c_v=\frac{0{\cdot}24}{1{\cdot}4}$$

$$=0{\cdot}174 \text{ kcal/kg K}$$

$$R=(c_p-c_v)J=(0{\cdot}24-0{\cdot}171)427$$
$$=29{\cdot}27 \text{ kg-m/kg K}$$

Note. Static pressure and temperature values must be used to obtain mass flow

$$P_2V_2=mRT_2$$

$$\therefore \qquad m=\frac{0{\cdot}93\times10^4\times13{\cdot}46}{29{\cdot}27\times274}=15{\cdot}6 \text{ kg/sec.}$$

Temperature at the entry to turbine

Total head temperature at entry of turbine $=T_{4t}$

Total head temperature of exhaust from turbine

$$=T_{5t}=733 \text{ K}$$

Neglecting the mass of fuel, we have

Work done by the turbine

$$=mc_p(T_{4t}-T_{5t})$$

or $$\frac{8000\times 75}{427}=15{\cdot}6\times 0{\cdot}27(T_{4t}-733)$$

$\therefore$ $$T_{4t}=\frac{8000\times 75}{427\times 15{\cdot}6\times 0{\cdot}27}+733$$

$$=1067 \text{ K.}$$ **Ans.**

EXERCISES 2

SECTION A : **Descriptive questions**

2·1. Why is it necessary to make simplifying assumptions in the analysis of gas turbine cycles ? State these assumptions.

2·2. Show the ideal Brayton or Joule cycle on P-v and T-s diagrams labelling the compressor and turbine work. Define the ideal efficiency of this cycle.

2·3. Define isentropic efficiency of (*a*) compressor, (*b*) turbine; and derive expressions for them in terms of enthalpies and temperatures. Show the effect of these efficiencies on T-s chart.

2·4. On what factors the ideal air standard efficiency of open constant pressure cycle gas turbine depend ? What are the additional factors when irreversibilities in compression and expansion are taken into account ?

2·5. Derive an expression for the efficiency of open constant pressure cycle gas turbine taking component efficiencies into account. Hence show the efficiency of ideal Joule cycle is $1-\frac{1}{(r_p)^{\gamma-1}}$ where r_p is the pressure ratio, and γ is the ratio of specific heats.

2·6. Define air rate. How does it effect the size of the gas turbine ?

2·7. Define work ratio. Which is better —a high work ratio or low work ratio? Why ?

2·8. Derive an expression for the work ratio of a simple gas turbine taking component efficiencies into account.

2·9. Show that the work output of a simple gas turbine depends on turbine and compressor inlet temperatures and the pressure ratio.

2·10. What are the three parameters on which the size of simple open cycle gas turbine depends ?

2·11. List the five main thermodynamic variables that effect the performance of a gas turbine plant. Show by diagrams how the thermal efficiency and specific work output vary with a change in these thermodynamic variables.

2·12. Draw a schematic diagram and a T-s diagram of an open cycle gas turbine with a regenerator. What is the objective of regenerative cycle ? How power output is affected by regeneration ?

2·13. Derive an expression for the efficiency of a gas turbine cycle with ideal regeneration.

2·14. What is the effect of pressure ratio on the efficiency of a regenerative cycle ? At what pressure ratio the regenerator becomes superfluous ?

2·15. Define regenerator thermal ratio or efficiency of regeneration ? What limits this effic ency ?

2·16. Draw a schematic diagram and a T-s diagram of an open cycle gas turbine with an intercooler. What is the objective of intercooling ? When intercooling is resorted to ? How does it effect the efficiency of the cycle ?

2·17. Draw a schematic diagram and a T-s diagram of an open cycle gas turbine with a reheater. Is the objective of reheating same in a steam turbine and gas turbine ?

2·18. Derive the condition for maximum output of a reheater gas turbine.

2·19. Why a reheater gas turbine is well suited for adopting regeneration ?

2·20. What are the main effects caused on the performance of an open cycle gas turbine with regeneration of the addition of an intercooler and a reheater ?

2·21. What is the effect of water injection on performance a gas turbine ?

2·22. What is the object of blade cooling in gas turbine and how it is done?

2·23. Give in tabular form the summary of improvements in basic gas turbine cycle with various modifications.

2·24. What are the factors which cause actual gas turbine cycle to differ considerably from the ideal cycle ? Discuss them.

2·25. Explain the working of a closed cycle gas turbine with the help of a schematic diagram. What are the reasons of its development in recent years.

Show that by the use of multi-stage reheating and intercooling a closed cycle gas turbine cycle approximates to reversible Ericsson cycle.

2·26. Discuss the advantages and disadvantages of a closed cycle gas turbine as compared to an open cycle gas turbine.

2·27. Why may the size of a closed cycle gas turbine be smaller than open cycle gas turbine of the same horse-power ?

2·28. What would be the advantages and disadvantages of using helium instead of air as medium in closed cycle gas turbines ?

2·29. Why helium closed cycle gas turbine is an attractive proposition for nuclear power points ?

2·30. How gas turbines can be incorporated in total energy systems ? Give two examples.

2·31. Explain the working of a semi-closed gas turbine with the help of a schematic diagram. What is its main advantage over closed cycle gas turbines ?

2·32. Describe the various gas turbine plant arrangements and their relative merits.

2·33. Give the arrangements used in automotive gas turbines.

SECTION B : **Numerical questions**

2·34. Simple gas turbine : efficiency specific output

The maximum and minimum temperatures in a simple gas turbine plant working on the Joule cycle are 1000 K and 288 K respectively. The pressure ratio is 6, and the isentropic efficiencies of the compressor and turbine are 85 and 90 per cent respectively. Calculate the efficiency and specific work output of the plant.

[T_2=515 K, T_4=639 K, η=27·6% ; output=32·1 kcal/kg]

2·35. **Effect of increase in component efficiencies on (a) thermal efficiency (b) work ratio**

In the last seventy years the gas turbine inlet temperature has increased from 500°C to 900°C, the turbine efficiencies have increased from 60 to 90 per cent, and the compressor efficiencies from 65 to 85 per cent.

For a pressure ratio of 4 calculate :

(*a*) the ideal efficiency

(*b*) the efficiency and work ratio of turbines of 70 years ago

(*c*) the efficiency and work ratio of modern turbines.

[(*a*) $\eta_{ideal}=33·6\%$; (*b*) ...... ; (*c*) ......]

2·36. Gas turbine plant : actual and ideal output and efficiency

A gas turbine operates at a pressure ratio of 7 and maximum temperature is limited to 1000 K. The isentropic efficiency of the compressor is 85 per cent and that of the turbine is 90 per cent. If the air enters the compressor at a temperature of 288 K. Calculate the specific output and efficiency of the plant and compare these values with those achieved by the ideal Joule cycle. If the unit is required to produce a power output of 750 kW, determine the necessary mass flow rate. Take c_p equal to 0·24.

[31·45 kcal/kg ; 28·58% ; 88·6 kcal/kg ; 42·7% ; 5·68 kg/s].

2·37. Comparison of ideal and actual cycle

(*a*) In a gas turbine plant the air enters the compressor at 1 kgf/cm² and 27°C. The pressure leaving the compressor is 5 kgf/cm² and the maximum temperature in the cycle is 850°C. Compare the compressor work, turbine work, work ratio and cycle efficiency for the following two cases :—

(*i*) the cycle is ideal

(*ii*) the cycle is actual with compressor efficiency of 80%, turbine efficiency of 85% and the pressure drop between compressor and turbine is 0·15 kgf/cm².

(*b*) If in the above cycle the air expands in the turbine to such a pressure that the turbine work is just equal to compressor work, and further expansion of air upto atmospheric pressure takes place in a nozzle; calculate the velocity of air leaving the nozzle.

(*c*) If an ideal regenerator is incorporated in the ideal cycle, what is the thermal efficiency of the cycle ?

[Ideal cycle $w_c=42·15$ kcal, $w_T=99$ kcal, work ratio=......, $\eta=36·55\%$, Actual cycle $w_c=52·7$ kcal, $w_T=85$ kcal, work ratio......, $\eta=22·25\%$, $c_5=692$ m/s, $\eta_{regenerator}=58·5\%$].

2·38. The layout of a gas turbine is shown in the diagram. The compressor is driven by the H.P. stage of a two-stage turbine and compresses 5 kg of air per second from 1 kgf/cm² to 5 kgf/cm² with an isentropic efficiency of 85%. The H.P. stage has an isentropic efficiency of 87% and its inlet temperature is 675°C. The L.P. stage, which is mechanically independent, has an isentropic efficiency of 82%. The expansion pressure ratios of the two turbines

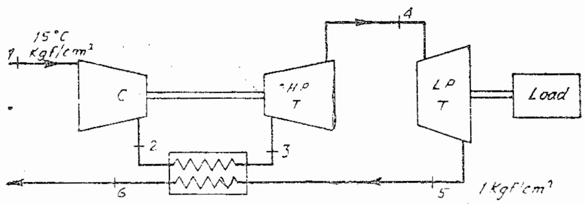


Fig. 2·54

are *not* equal. The exhaust gases from the L.P. stage pass to a heat exchanger which transfers 70% of the heat available in cooling the exhaust to raise the compressor temperature at delivery.

Assuming the working fluid to be air throughout, of constant specific heat, and neglecting pressure losses, estimate the intermediate pressure P_4 and the temperature T_4 between the two turbine stages, the horse-power output of the L.P. stage and the overall plant efficiency.

[$T_4 = 750{\cdot}5$ K, $P_4 = 1{\cdot}935$ kgf/cm^2, h.p.$=710$, $\eta = 29{\cdot}7\%$].

2·39. Gas Turbine plant : Two-stage compression separate compressor and power turbine : $m_{power\ turbine}$; η_{plant}.

In the gas turbine plant shown, each compressor operates on a pressure ratio of 3 and an isentropic efficiency of 82%. After the low pressure compressor, some of the air is extracted and passed to a combustion chamber from which the products leave at a temperature of 650°C and expand in power turbine. The remainder of the air passes through the high pressure compressor and into a combustion chamber from which it leaves at a temperature of 540°C and expands in a turbine which drives both the compressors. The isentropic efficiency of each turbine is 87%. If the temperature of the air at inlet to the low pressure compressor is 15°C, determine the percentage of the total air intake that passes to the power turbine and the thermal efficiency of the plant.

For compression assume $\gamma = 1{\cdot}4$, $c_p = 0{\cdot}24$

For heating and expansion, $\gamma = 1{\cdot}33$, $c_p = 0{\cdot}276$

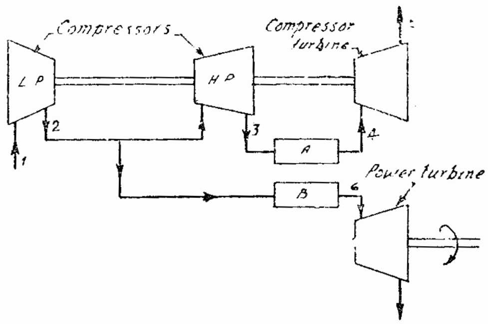


Fig. 2·55

[$T_2 = 417{\cdot}3$ K, $T_3 = 606{\cdot}1$ K, $T_4 = 813$ K, $T_5 = 516{\cdot}5$ K, $m = 14{\cdot}9\%$, $\eta = 27\%$].

2·40. Gas turbine with two-stage compression, regeneration, reheating output and efficiency

In an open cycle gas turbine plant the air is compressed in a two-stage compressor with complete inter-cooling to the initial temperature. After passing through an exhaust heat exchanger and combustion chamber, the gases are expanded in a two-stage turbine with reheating in a second combustion chamber between the stages. Both the compressor stages are driven by the high pressure turbine stage and the power output of the plant is taken from the mechanically independent low pressure turbine stage. After expansion in the low pressure turbine, the gases pass through the heat exchanger to atmosphere.

The pressure ratio of each compressor stage is 2 : 1, the air inlet pressure and temperature are 1·03 kgf/cm^2 and 15°C and the gas inlet temperature to both turbines is 700°C. If the isentropic efficiency in each compressor and turbine stage is 85% and the thermal ratio of heat exchanger is 50% determine the output per kg of air per second and the thermal efficiency of the plant, neglecting pressure losses in the heat exchanger, inter-cooler and combustion chambers and any variation in the mass flow of the working fluid due to addition of fuel.

For gases during heating and expansion take $c_p=0{\cdot}274$ and $\gamma=1{\cdot}333$.

[$T_2=363{\cdot}5$ K ; $T_4=363{\cdot}5$ K, $T_6=822$ K output=30 kcl/kg, $\eta=21{\cdot}42\%$].

2·41. Closed cycle gas turbine

The following data refer to a closed cycle gas turbine plant using helium as working fluid and incorporating two-stage compression with inter-cooling and two-stage expansion with reheating ; temperature at entry to each compression stage is 270°C ; pressure at entry to first compression stage, and at exit from the second turbine stage is 1 05 kgf/cm^2 ; first compression stage pressure ratio, 6 ; first compression stage isentropic compression efficiency, 0·85 ; temperature at inlet to each expansion stage, 1150°C ; isentropic efficiency of each expansion stage, 0·9 ; reheat pressure, 6·3 kgf/cm^2 ; for helium polytropic index n is 1·24 and R is 212 kgf-m/kg K. Calculate the cycle thermal efficiency.

[$T_2=793$ K ; $T_4=740{\cdot}5$ K, $T_6=975{\cdot}5$ K, $T_8=1049{\cdot}5$ K, $\eta_{th}=33\%$].

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3

BASIC GAS DYNAMICS

3·1. INTRODUCTION*

The majestic development of jet and rocket propulsion, high speed aircrafts, and gas turbines after the World War II has brought into prominence the field of compressible flow. A great amount of work has been done in investigating the phenomena relating to the flow of incompressible fluids in classical hydrodynamics. All these investigations have, however, excluded an important field of flow, the compressible flow in which density of the fluid varies as it flows. Classical hydrodynamic analysis does not take into account this effect, namely the variation of density, and it has been possible to describe most of the incompressible flows with the help of the law of conservation of mass and the Newton's Second Law of Motion. This approach has been quite satisfactory in describing the liquid flows and low speed gas flows in which an increase in pressure and a decrease in velocity is related in a simple manner, without any change in the density of the fluid. However, for high speed gas flows the changes in the density of the fluid are significant and are accompanied by large changes in the thermodynamic state of the fluid, *i.e.* in pressure and temperature. The above referred two laws are no longer adequate to describe the flow. The continuity equation (Law of Conservation of Mass) and the momentum equation (Newton's Second Law of Motion) become interdependent, unlike that in the incompressible flow. The relation between changes in pressure and velocity being no longer simple, it becomes imperative to take recourse to the laws of thermodynamics which take into account the thermal effects of density variation to completely describe the flow.

The variation of density along the motion of the fluid, the basic difference between compressible and incompressible fluid flow, causes significant changes in the physical nature of the compressible flow and gives rise to some typical phenomena such as shock waves which are not encountered in incompessible flow.

3·2. PROPAGATION OF SMALL DISTURBANCE—THE VELOCITY OF SOUND

A disturbance in a fluid will propagate through the fluid at a well defined velocity depending upon the elasticity of the fluid and

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*For the purpose of this chapter the velocity is denoted by V and the ratio of specific heats by k.

the magnitude of the disturbance. However, if the disturbance is small, the velocity of the propagation of the disturbance solely

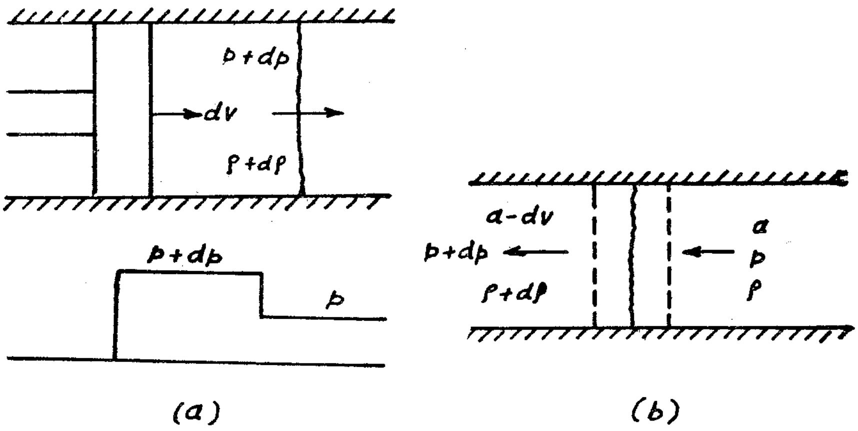


Fig. 3·1. Propagation of a small pressure disturbance.

depends upon the elasticity of the fluid. The speed with which the small disturbance is propagated through the fluid is called, by definition, the *velocity of sound* or the *acoustic velocity*. The acoustic velocity is usually taken as a property of the fluid and is very important in describing the flow of compressible fluids.

Consider a small pressure disturbance, generated with the help of a piston moving with a speed dV to the right in a long pipe of cross-sectional area A as shown in Fig. 3·1(a). Initially the fluid is assumed to be uniform and at rest. The pressure wave moves to the right with a speed a. The velocity of the fluid ahead of the pressure wave is zero and pressure is p while velocity of the fluid behind the pressure wave is dV, same as the speed of the piston. The propagation of the wave through the fluid causes a small change in the pressure and density of the fluid which are now given by $(p+dp)$ and $(\rho+d\rho)$ respectively.

For an observer travelling with the pressure wave, with the speed of sound a, the flow phenomenon is reduced to that depicted in Fig. 3·1 (b).

Writing the continuity equation for the control volume of Fig. 3·1 (b) for the case of steady flow, we get

$$(\rho+d\rho)(a-dV)A=\rho aA$$

Neglecting the higher order terms, this reduces to

$$dV=a\frac{d\rho}{\rho} \tag{3·1}$$

Again, writing the steady flow momentum equation for the control volume, and neglecting the shear forces and noting that there are no elevation changes, we have

$$pA-(p+dp)A=\rho aA(a-dV)-a$$

or

$$dp=adV \tag{3·2}$$

Substituting for dV in equation (3·2) from equation (3·1), we get,

$$a^2 = \frac{dp}{d\rho} \qquad (3\cdot3)$$

Generally this expression is written in the differential form at constant entropy as

$$a^2 = \left(\frac{\partial p}{\partial \rho}\right)_s \qquad (3\cdot4)$$

which is justified because the amplitude of pressure disturbance is small, *i.e.* the process is isentropic. Equation (3·4) is general in nature, and the acoustic velocity for a perfect gas can be found with the help of the equation of state

$$p = g\rho RT \qquad (3\cdot5)$$

For a gas following equation (3·5), the isentropic process is given by

$$\frac{p}{\rho^k} = \text{constant} \qquad (3\cdot6)$$

or, by logarithmic differentiation

$$\frac{dp}{p} = k\,\frac{d\rho}{\rho} \qquad (3\cdot7)$$

or

$$\left(\frac{\partial p}{\partial \rho}\right)_s = k\,\frac{p}{\rho} = g_o kRT = g_o \,\frac{\overline{R}T}{m} \qquad (3\cdot8)$$

where $\overline{R}$ is the universal gas constant and m the molecular weight of the gas.

Thus for a perfect gas, the velocity of sound is given by

$$a = \sqrt{g_o kRT} \qquad (3\cdot9)$$

Since for a perfect gas k, the ratio of specific heats, is a function of temperature only it can be concluded that the velocity of sound or the velocity of propagation of a small disturbance depends only on the nature of the gas, *i.e.* its molecular weight and the thermodynamic state of the gas represented by the temperature. Those gases which have smaller molecular weight experience higher velocity of sound.

By definition, the change in density of an incompressible fluid is zero. This means an infinite velocity of sound. However, the real fluids are not truly constant density fluids and always have a finite acoustic velocity.

For air, the velocity of sound is given by

$$a = \sqrt{g_o kRT}$$

$$= \sqrt{9\cdot81 \times 1\cdot4 \times 29\cdot27 \times T}$$

or

$$a = 20\cdot1\sqrt{T}\ \text{m/s}. \qquad (3\cdot10)$$

3·3. MACH NUMBER, MACH CONE, AND MACH ANGLE

A single dimensionless velocity parameter taking into account the two important parameters of compressible flow, *i.e* flow velocity and temperature, is defined as the ratio of the local flow velocity to the local acoustic velocity and is called the *Mach number* and denoted by M. Mathematically,

$$\mathrm{M}=\frac{V}{a} \tag{3·11}$$

It must be remembered that the velocity of sound is not consta (it depends upon the thermodynamic state of the fluid) and vari from point to point depending upon the local temperature an density. Thus the Mach number can vary along the flow either d to variation of flow velocity alone or of acoustic velocity alone of both, and in calculating the Mach number the local temperatu and local density must be considered.

Mach number illustrates very clearly the basic difference between various types of compressible flows. Consider a point source of disturbance initially at point O and moving with a velocity V. The point source causes infinitesimal changes in the fluid due to its motion and produces a very weak, compressive disturbance which propagates, through undisturbed fluid, in all directions with the velocity of sound, a. The propagation of this weak compressive

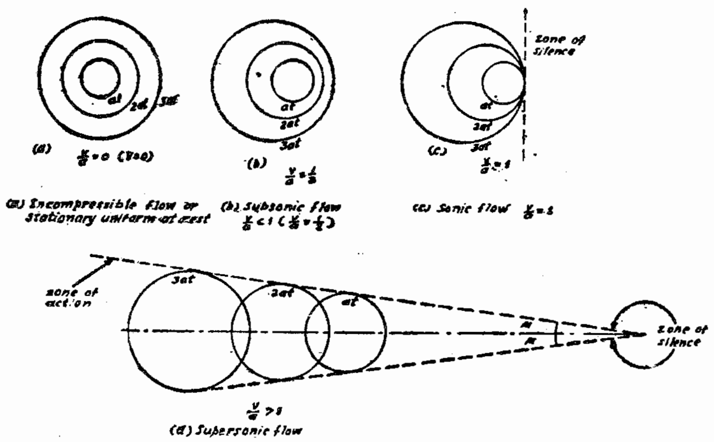


Fig. 3·2. Propagation of small disturbance through a uniform fluid at different values of V/a (Mach number).

disturbance is shown in Fig. 3·2 for various values of V/a. The location of the point disturbance moving in the fluid is denoted by numbers 1, 2, and 3 which correspond to three successive time periods, each of interval t. In case of incompressible flow when the point source is stationary ($V=0$), the pressure disturbance propagates uniformly in all directions [Fig. 3·2 (a)]. At *subsonic velocities* ($V<a$), again the disturbance surrounds the point source and propagates in all directions in fluid. However, this propagation is unsymmetrical in nature [Fig. 3·2 (b)]. Though the point source moves, it always moves in the disturbed fluid and is never able to catch up with the disturbance.

At *sonic velocity* ($V=a$), the disturbance as well as the point source travels with the same speed and form a plane front, containing the point source, which separates the undisturbed flow ahead of the disturbance from the disturbed fluid behind it. The undisturbed flow region is called *the zone of silence* and the disturbed region is called the *zone of action* [Fig. 3·2 (*c*)]. Thus for velocity range from $V=0$ to $V=a$ the point source always moves within the zone of action.

At *supersonic velocities* ($V>a$) the point source moves with a velocity greater than the velocity of propagation of the disturbance, so it is able to overtake the propagating disturbance [Fig. 3·2 (*d*)]. The disturbances are contained within a cone with the point source at its apex This cone is called the *Mach cone*, which represents the surface separating the disturbed and undisturbed flow regions. If the flow is two-dimensional, the Mach cone takes the shape of a wedge called the *Mach wedge*.

The semivertex angle of the Mach cone is called *Mach angle*, denoted by α, and by geometrical considerations of Fig. 3·2 (*d*), it can be shown that Mach angle is related to the Mach number by the relation

$$\sin \alpha = \frac{a}{V} = \frac{1}{M}$$

or

$$\alpha = \sin^{-1}\left(\frac{1}{M}\right) \tag{3·12}$$

From the above analysis it is clear that there is a fundamental difference in the nature of the subsonic and supersonic flow. In case of subsonic flow, the whole of the flow experiences the disturbance while in the supersonic flow only a portion of the fluid experiences the disturbance and the disturbance cannot reach a point ahead of the disturbing body. This is the reason that the boom of a supersonic jet is heard only after it has passed over the place where the observer is standing.

3·4. TOTAL OR STAGNATION PROPERTIES

If a flowing fluid is brought to rest isentropically the resulting state is called the *total or stagnation state* and the corresponding values of the properties describing this state are called *total or stagnation properties*.

The concept of total or stagnation condition provides us with a suitable reference condition with which the other flow conditions can be compared. This point can further be illustrated by considering a moving fluid as shown in Fig. 3·3. At the stagnation point *A*, the velocity of the fluid is zero and it experiences some compression and, thereby, an increase in pressure and temperature of the fluid at this point. The degree of this compression depends mainly upon the velocity of the fluid which varies along the pipe.

Thus any velocity cannot be taken as a reference velocity. However, the fluid as it starts flowing from the reservior converts

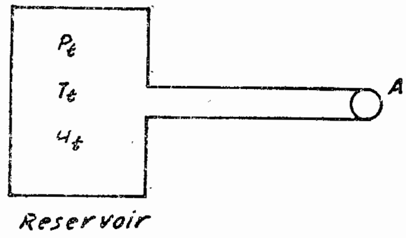


Fig. 3·3. Total or stagnation condition.

some of its energy into velocity energy during its flow through the pipe, and while reaching the stagnation point A, in the absence of heat transfer, friction, and any other type of loss, restores back this energy. The state properties at point A will, then, correspond to the reservoir properties which are constant and can, thus be taken as a convenient reference state for describing the flow of fluid while the properties in the pipe vary depending upon the local Mach number.

The conservation of energy, if written for an isentropic process, gives

$$\tfrac{1}{2}V_1^2+c_pT_1=\tfrac{1}{2}V_2^2+c_pT_2=\text{constant}$$

where subscripts 1 and 2 denote any two points in the flow. Thus the total temperature T_t, where $V=0$, is given by

$$c_pT_t=\tfrac{1}{2}V^2+c_pT$$

or

$$T_t=T+\frac{V^2}{2c_p} \tag{3·13 a}$$

or

$$\frac{T_t}{T}=1+\frac{V^2}{2c_pT}$$

which, using the relations $c_p=R\dfrac{k}{k-1}$ and the definition of Mach number, *i.e.*

$$V^2=\text{M}^2a^2=\text{M}^2kRT$$

reduces to

$$\frac{T_t}{T}=1+\frac{k-1}{2}\text{M}^2 \tag{3·13 b}$$

Equation (3·13 b) has been derived from energy equation without any reference to the reversibility or otherwise of the process and hence is also true for all adiabatic reversible and irreversible flows.

Total pressure can be obtained by combining the isentropic relation

$$\frac{P_t}{P}=\left(\frac{T_t}{T}\right)^{\frac{k}{k-1}}$$

with equation (3·13 b), as

$$\frac{P_t}{P}=\left(1+\frac{k-1}{2}\mathrm{M}^2\right)^{\frac{k}{k-1}} \tag{3·14}$$

and using the equation of state, the total or stagnation density ρ_t can be written as

$$\frac{\rho_t}{\rho}=\left(1+\frac{k-1}{2}\mathrm{M}^2\right)^{\frac{1}{k-1}} \tag{3·15}$$

These equations can be used to find properties of non-isentropic flows without error because the total or stagnation properties at a state point depend only on the local state temperature and the local Mach number and not upon the flow process.

3·5. ONE-DIMENSIONAL ADIABATIC FLOW

Flow in ducts and passages is quite often adiabatic, study of which is very important. The total enthalpy of the fluid for such a flow remains constant irrespective of whether the flow process is reversible or irreversible, *i.e.*

$$h_t=h+\frac{V^2}{2}=\text{constant} \tag{3·16 a}$$

which can be expressed in terms of Mach number, as

$$T_t=T\left(1+\frac{k-1}{2}\mathrm{M}^2\right)=\text{constant} \tag{3·16 b}$$

Equation (3·16 b), if written in the differential form becomes

$$\frac{dT}{T}=\frac{-d\left(1+\frac{k-1}{2}\mathrm{M}^2\right)}{1+\frac{k-1}{2}\mathrm{M}^2} \tag{3·16 c}$$

Because the change in temperature is independent of entropy changes, equation (3·16) is universally applicable for reversible or irreversible flows without heat transfer. For a perfect gas

$$Tdh=dh-vdp \tag{3·17}$$

$$=dh-dp/\rho$$
$$=c_p dT-dp/\rho$$
$$=\frac{kR}{k-1}dT-\frac{dp}{\rho}\times\frac{\rho RT}{p}$$
$$=\frac{kR}{k-1}dT-\frac{dp}{p}RT$$

or
$$\frac{dp}{p}=\frac{k}{k-1}\frac{dT}{T}-\frac{ds}{R} \tag{3·17 b}$$

Combining (3·17 *b*) with (3·16 *c*), we get

$$\frac{dp}{p}=-\frac{k}{k-1}\frac{d\left(1+\frac{k-1}{2}M^2\right)}{1+\frac{k-1}{2}M^2}-\frac{ds}{R} \tag{3·18}$$

which states that in an adiabatic flow the local pressure is dependent upon change in entropy. Thus for an irreversible adiabatic process there is always a loss of stagnation pressure.

Writing the equation of state in the differential form

$$\frac{dp}{p}=\frac{dT}{T}+\frac{d\rho}{\rho} \tag{3·19}$$

and using equation (3·16 *c*) and (3·18), the change in density is given as

$$\frac{d\rho}{\rho}=-\frac{1}{k-1}\frac{d\left(1+\frac{k-1}{2}M^2\right)}{1+\frac{k-1}{2}M^2}-\frac{ds}{R} \tag{3·20}$$

i.e. greater the increase in entropy (irreversibility) more will be the reduction in density of the fluid.

Similarly, using the relation $V=M\sqrt{kRT}$ and continuity equation in the logarithmically differentiated form, *i.e.*

$$\frac{dA}{A}+\frac{d\rho}{\rho}+\frac{dV}{V}=0 \tag{3·21}$$

We get

$$\frac{dV}{V}=\frac{dM}{M}-\frac{1}{2}\frac{d\left(1+\frac{k-1}{2}M^2\right)}{1+\frac{k-1}{2}M^2} \tag{3·22}$$

and
$$\frac{dA}{A}=\frac{k+1}{2(k-1)}\cdot\frac{d\left(1+\frac{k-1}{2}M^2\right)}{1+\frac{k-1}{2}M^2}+\frac{ds}{R}-\frac{dM}{M} \tag{3·23}$$

i.e. the velocity change is independent of change in entropy and that a greater area is required for a given irreversible adiabatic flow as compared to a reversible adiabatic flow.

3·6. ISENTROPIC FLOW

Most of the flows in short ducts and nozzles, which have very small time available for heat transfer and in which the frictional effects are negligible, can be exactly described by what is called a reversible adiabatic or isentropic flow. The relationship derived in the previous section are all valid for such a flow and by putting $ds=0$ in them and integrating, we get the equations describing the isentropic flow which can be written as

$$\frac{T}{T_t}=\left(1+\frac{k-1}{2}\mathrm{M}^2\right)^{-1}$$

$$\frac{P}{p_t}=\left(1+\frac{k-1}{2}\mathrm{M}^2\right)^{-\frac{k}{k-1}}$$

$$\frac{\rho}{\rho_t}=\left(1+\frac{k-1}{2}\mathrm{M}^2\right)^{-\frac{k}{k-1}} \tag{3·24}$$

$$\frac{A}{A^*}=\frac{1}{\mathrm{M}}\left(\frac{1+\frac{k-1}{2}\mathrm{M}^2}{\frac{k+1}{2}}\right)^{\frac{k+1}{2(k-1)}}$$

where A^* is the area corresponding to $\mathrm{M}=1$

Equations (3·24) completely describes the isentropic flow and shows that the local value of stagnation properties is indendent of the flow process and depends only on the local static properties and local Mach number.

3·7. ISENTROPIC FLOW IN A PASSAGE OF VARIABLE CROSS SECTION AREA

Consider reversible adiabatic or isentropic flow of a compressible fluid through a passage of variable cross-sectional area as shown in Fig. 3·4.

Fig. 3·4. Isentropic flow through a duct of variable cross-sectional area.

For one-dimensional steady isentropic flow the energy equation is

$$c_pT+\frac{V}{2}=\text{constant}$$

Using the relation $c_p=\frac{k}{k-1}R$ and the perfect gas law $p=\rho RT$, this becomes

$$\frac{V^2}{2}+\frac{k}{k-1}\frac{p}{\rho}=\text{constant}$$

which, in turn, on differentiating gives

$$VdV-\frac{k}{k-1}p\frac{d\rho}{\rho^2}+\frac{k}{k-1}\frac{dp}{\rho}=0$$

or $$VdV-\frac{k}{k-1}\rho RT\frac{d\rho}{\rho^2}+\frac{k}{k-1}\frac{dp}{d\rho}\frac{d\rho}{\rho}=0$$

or $$VdV-\frac{a^2}{k-1}\frac{d\rho}{\rho}+\frac{k}{k-1}a^2\frac{d\rho}{\rho}=0$$

or $$VdV+a^2\frac{d\rho}{\rho}\left(\frac{k}{k-1}-\frac{1}{k-1}\right)=0$$

r $$VdV+a^2\frac{d\rho}{\rho}=0$$

or $$\frac{dV}{V}-\frac{1}{M^2}\frac{d\rho}{\rho}=0 \tag{3·25}$$

Combining equations (3·25) and (3·21),we get

$$\frac{dA}{A}=-(1-M^2)\frac{dV}{V} \tag{3·26}$$

Study of this equation reveals that for subsonic flow velocities, *i.e.* $M<1$

$$\frac{dA}{A}=-\frac{dV}{V} \tag{3·27}$$

that is, in a convergent passage ($dA<0$) the velocity increases ($dV>0$) while the pressure decreases ($dp<0$). In other words, a subsonic converging passage acts as a nozzle, and a subsonic divergent passage acts as a diffuser (the pressure increases at the expense of velocity).

In case of supersonic flow velocities ($M>1$),

$$\frac{dA}{A}=\frac{dV}{V} \tag{3·27 a}$$

and in a convergent passage ($dA<0$) the velocity will also decrease, *i.e.* it acts as a diffuser, and a divergent passage will act as a nozzle. This illustrates an important feature of the compressible flow that the area change has opposite effects on subsonic and supersonic flows. A convergent flow passage acts as a nozzle for subsonic flow and as a diffuser for supersonic flow, while a divergent flow passage acts as a diffuser for subsonic flow and as a nozzle for supersonic flow. All these effects are shown in Fig. 3·5.

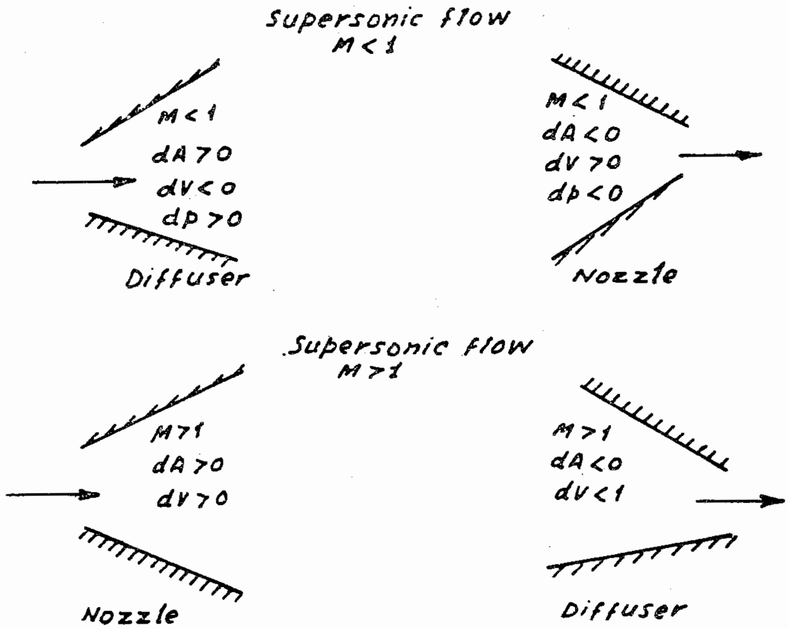


Fig. 3·5. Effect of area change for subsonic and supersonic velocities.

The relation between Mach number and area variation can be obtained by putting $ds=0$ in Eq. (3·23) and integrating it, or from equation (3·24) as

$$\frac{A_1}{A}=\frac{M}{M_1}\left[\frac{1+\frac{k-1}{2}M_1^2}{1+\frac{k-1}{2}M^2}\right]^{\frac{k+1}{2(k-1)}} \tag{3·29}$$

By logarithmic differentiation of this equation with respect to M for a given value of M_1, it can be shown that A_1/A is maximum at $M=1$. Since A_1 is fixed, the value of A is minimum at $M=1$. This area which corresponds to Mach number of unity is called *critical area* and the section at which it occurs is called the *throat*. The corresponding properties are denoted by an asterisk, *i.e.* A^*, p^*, T^*, etc.

Since the area of the throat is minimum and mass flow constant, the mass flow per unit area is maximum at the throat. It must also be noted that near $M=1$, the flow parameters are very sensitive to area variation, *i.e.* a small area change can cause substantial pressure and velocity change.

The mass flow per unit area is given as

$$\frac{W}{A}=\rho V$$

$$=\frac{p}{RT}\,M\sqrt{kRT}$$

$$=p.M\sqrt{\frac{k}{R}}\,\frac{1}{\sqrt{T}}$$

or
$$\frac{W}{A}=\sqrt{\frac{k}{R}}\,\frac{p_t}{T_t}M\,\frac{p}{p_t}\sqrt{\frac{T_t}{T}} \qquad [3\cdot30\ (a)]$$

Substituting $\frac{p}{p_t}$ and $\frac{T}{T_t}$ in terms of Mach number we get

$$\frac{W}{A}=\sqrt{\frac{k}{R}}\,\frac{p_t}{\sqrt{T_t}}M\left(1+\frac{k-1}{2}M^2\right)^{\frac{-(k+1)}{2(k+1)}} \qquad [3\cdot30\ (b)]$$

The condition for maximum mass flow rate is

$$M=1$$

and
$$\left(\frac{W}{A}\right)_{max}=\frac{W}{A^*}=\frac{\sqrt{k}}{R}\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}\frac{p_t}{\sqrt{T_t}} \qquad (3\cdot31)$$

The corresponding pressure ratio is given by

$$\frac{p}{p_t}=\frac{p^*}{p_t}=\left(\frac{2}{k+1}\right)^{\frac{k}{k-1}} \qquad [3\cdot32\ (a)]$$

For air, ($k=1\cdot4$) this reduces to

$$\frac{p}{p_t}=0\cdot528 \qquad [3\cdot32\ (b)]$$

3·8 FLOW THROUGH A CONVERGENT NOZZLE—EFFECT OF PRESSURE RATIO

Consider a convergent nozzle fitted with a back pressure reservoir whose pressure p can be varied when desired (*see* Fig. 3·6). The stagnation inlet conditions to the nozzle are denoted by p_t and T_t and are kept constant. The exit pressure is denoted by p_e.

When back pressure is equal to stagnation inlet pressure ($p_b=p_t$), there is no flow and the pressure remains constant throughout the nozzle as denoted by the line *a* in Fig. 3·5 (*b*). As p_b is reduced, flow starts, the pressure steadily decreases and flow is subsonic throughout the nozzle [line (*b*) in Fig. 3-5 (*b*)]. Any further decrease in back pressure p_b increases the flow velocity and mass flow rate. The flow velocity remains subsonic throughout till the reduction in p_b is so large that p_b/p_t is equal to p^*/p_t at which the velocity reaches a maximum (M=1) at the exit section of the nozzle.

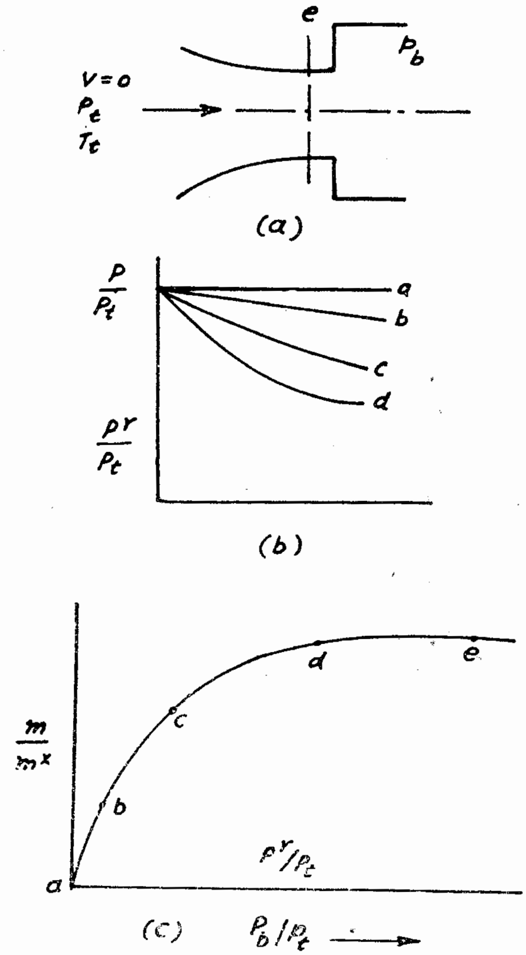
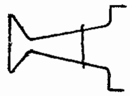


Fig. 3·6. Flow through a convergent nozzle.

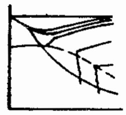
The mass flow rate is also maximum denoted by line d in Fig. 5.3 (b). Any further reduction in p_b does not cause any change in the flow pattern within the nozzle, the mass flow and the velocity at exit remaining at their maximum value. Such a flow is called *choked flow*. However, the conditions outside a nozzle exit will change, the flow will further expand to equalize its pressure with p_b and in this process a weak discontinuity (shock) will occur outside the nozzle.

3·9. FLOW THROUGH A CONVERGENT-DIVERGENT NOZZLE

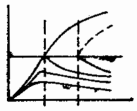
Consider a convergent-divergent nozzle shown in Fig. 3·7 (a) with a back pressure p_b which can be varied. The inlet stagnation conditions are again denoted by p_t and T_t and are kept constant.



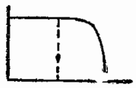
(*a*) Convergent divergent nozzle



(*b*) Pressure variation along the axis



(*c*) Velocity variation along the axis



(*d*) Mass flow *vs*. pressure ratio

Fig. 3·7 Flow through a convergent-divergent nozzle.

At $p_b = p_t$, there is no flow. At a back pressure slightly less than total pressure the flow starts, gas accelarates, and the pressure continuously decreases, but the flow is subsonic throughout the nozzle [line (*a*) in Fig. 3·7 (*b*)] ; the nozzle exit pressure is equal to p_b due to subsonic exit velocity.

At $p_b = p_1$ [Fig. 3·7 (*b*)], the flow smoothly accelerates from rest at the nozzle inlet to the velocity of sound at the throat (M=1) ; the pressure falls to p^* and after the throat the velocity starts decreasing while the pressure rises smoothly to p_b. On both sides of the throat the flow is symmetrical, equal area sections having equal velocities. Except at the throat, the velocities are subsonic everywhere. The mass flow is maximum.

At $p_b = p_2$ the flow, after smooth acceleration from rest to M=1 at the throat, continues to accelerate to a supersonic velocity after the throat [Fig. 3·7 (*b*)]. Downstream the throat, the velocity is supersonic everywhere while the pressure reduces smoothly to $p_o = p_2$.

For values of p_b between p_2 and p_1, there is no solution for the isentropic flow equation. The flow upto throat is isentropic and the velocity increases from rest to sonic velocity at the throat. After the throat the velocity further continues to increase till at section F at which a discontinuity (shock) occurs. The pressure suddenly increases as seen in Fig. 3·7 (b), while the flow is reduced to subsonic flow, and thereafter, it remains subsonic until exit where the exit pressure is equal to p_b. Mass flow remains maximum, *i.e.* it is chocking flow. Such a flow is not isentropic and the stagnation pressure before and after the discontinuity are different.

The position of the section at which the shock occurs depends upon the value of the back pressure. For a back pressure slightly less than p_1, the shock occurs near or at the throat while for a back pressure slightly more than p_2, the shock occurs near or at the exit plane of the nozzle. For a back pressure less than p_2, further expansion and hence oblique shocks will occur downstream the nozzle exit section.

ILLUSTRATIVE EXAMPLES

3·1. Dynamic temperature, dynamic pressure, kinetic pressure

Consider a uniform stream of an ideal gas having $k=7/5$ and $c_p=0{\cdot}24$ flowing at 200 m/s. What is the value of the dynamic temperature? If the total temperature is 95°C and the static pressure is 1·1 kgf/cm², state the values of the kinetic and the dynamic pressure.

The dynamic temperature

$$T_t-T_1=\frac{V_1^2}{2gJc_p}$$

$$=\frac{200^2}{2\times 9{\cdot}81\times 427\times 0{\cdot}24}=\mathbf{19{\cdot}85\ K} \qquad \textbf{Ans.}$$

$$\therefore \qquad T_1=368-19{\cdot}85=348{\cdot}15\ \text{K}$$

For isentropic process

$$\left(\frac{p_t}{p_1}\right)=\left(\frac{T_t}{T_1}\right)^{\frac{k}{k-1}}$$

$$\therefore \qquad p_t=1{\cdot}1\times\left(\frac{368}{348{\cdot}15}\right)^{\frac{1{\cdot}4}{1{\cdot}4-1}}$$

$$=1{\cdot}4\times 1{\cdot}214=1{\cdot}335\ \text{kgf/cm}^2$$

$\therefore$ Dynamic pressure $=p_t-p_1$

$$=1{\cdot}335-1{\cdot}1=0{\cdot}235\ \text{kgf/cm}^2 \qquad \textbf{Ans.}$$

Kinetic pressure $=\dfrac{\rho V^2}{2g}=\dfrac{pV^2}{RT\times 2g}$

$$=\frac{1{\cdot}1\times 200^2}{29{\cdot}27\times 348{\cdot}15\times 2\times 9{\cdot}81}$$

$$=\mathbf{2{\cdot}2\ kgf/cm^2} \qquad \textbf{Ans.}$$

3·2. Pitot tube

A pitot tube in a wind tunnel gives a static pressure reading of 40·7 kgf/cm² and a stagnation pressure reading of 98 kgf/cm². The stagnation temperature is 90°C. Calculate the acoustic velocity.

$$p_1 = 40{\cdot}7 \text{ kgf/cm}^2, \quad p_t = 98 \text{ kgf/cm}^2$$

$$T_t = 363 \text{ K}$$

$$\frac{T_t}{T} = (P_t/P)^{\frac{k}{k-1}}$$

or $$\frac{363}{T} = \left(\frac{98}{40{\cdot}7}\right)^{\frac{1{\cdot}4-1}{1{\cdot}4}}$$

$\therefore$ $\mathbf{T = 282\ K}$ **Ans.**

Acoustic velocity a is given by

$$a = 20{\cdot}1\sqrt{T}$$

$$= 20{\cdot}1\sqrt{282} = \mathbf{338\ m}$$ **Ans.**

3·3. Convergent air nozzle : p_{exit} ; A_{exit} ; W_{max}

When is it necessary to use a convergent-divergent nozzle instead of convergent nozzle only ?

A convergent nozzle is to discharge 1 kg/s of a gas, which enters the nozzle at 5 kgf/cm² and 600°C, and leaves with a speed of 500 m/s. Assuming frictionless adiabatic flow, determine the pressure at the nozzle exit, and the exit area of the nozzle. State whether the nozzle is discharging at the maximum rate for the given entry conditions, giving your reasons.

Take $c_p = 0{\cdot}27$ and $R = 29{\cdot}9$ [*Kerala, 1970 Annual*]

Considering inlet and exit of nozzle

$$h_1 + \frac{V_1^2}{2gJ} = h_2 + \frac{V_2^2}{2gJ}$$

Neglecting inlet velocity

$$h_1 - h_2 = \frac{c_2^2}{2gJ} = \frac{(500)^2}{2\times 9{\cdot}81 \times 427} = 29{\cdot}8 \text{ kcal/kg}$$

For a perfect gas the change in specific enthalpy

$$h_1 - h_2 = c_p(T_1 - T_2)$$

$$\therefore\ 29{\cdot}8 = 0{\cdot}27(873 - T_2)$$

or $$T_2 = 762{\cdot}5 \text{ K}$$

Now $c_p - c_v = \dfrac{R}{J}$ $\quad\therefore\quad c_v = 0{\cdot}27 - \dfrac{29{\cdot}9}{427} = 0{\cdot}2$

$$\therefore\quad k = \frac{c_p}{c_v} = \frac{0{\cdot}27}{0{\cdot}2} = 1{\cdot}35$$

For frictionless adiabatic flow

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} \quad \therefore\ \frac{762{\cdot}5}{873} = \left(\frac{P_2}{5}\right)^{\frac{1{\cdot}35-1}{1{\cdot}35}}$$

$\therefore$ $\mathbf{P_2 = 2{\cdot}98\ kgf/cm^2}$ **Ans.**

Now $P_2V_2=mRT_2$

$$\therefore \qquad v_2=\frac{mRT_2}{P_2}=\frac{1\times 29{\cdot}9\times 762{\cdot}5}{2{\cdot}98\times 10^4}=0{\cdot}765 \text{ m}^3/\text{s}$$

From mass flow equation

$$\dot{m}v_2=A_2C_2$$

$\therefore$ **Exit area,** $A_2=\dfrac{\dot{m}v_2}{c_2}$

$$=\frac{1\times 0{\cdot}765}{500}\times 10^4=\mathbf{15{\cdot}4\ cm^3} \qquad \textbf{Ans.}$$

For maximum discharge the exit pressure should be

$$P_t=P_1\left(\frac{2}{k+1}\right)^{\frac{k}{k-1}}$$

$$=5\left(\frac{2}{2{\cdot}35}\right)^{\frac{1{\cdot}35}{0{\cdot}35}}=2{\cdot}685 \text{ kgf/cm}^2$$

$\therefore$ Discharge is not maximum as $P_2>2{\cdot}685$ kgf/cm² **Ans.**

3·4. Convergent-divergent nozzle, given efficiency : A_t ; A_e

Show that for flow of gas through a convergent-divergent nozzle

$$\frac{P_t}{P_1}=\left(\frac{2}{k+1}\right)^{\frac{k}{k-1}}$$

where P_t = *pressure at throat*

P_1 = *pressure at inlet*

A gas expands in a convergent-divergent nozzle from 5 kgf/cm² to 1·4 kgf/cm², the initial temperature being 550°C and the nozzle efficiency is 90 per cent. All the losses take place after the throat. Find the throat and exit areas per kg of gas per second.

Take $k=1{\cdot}4$ and $R=29{\cdot}27$ kgf-m/kg/K

$$c_p=\frac{k}{k-1}R=\frac{1{\cdot}4}{1{\cdot}4-1}\times\frac{29{\cdot}27}{427}=0{\cdot}24$$

(*a*) *Throat* : $\dfrac{P_t}{P_1}=\left(\dfrac{2}{k+1}\right)^{\frac{k}{k-1}}$

Now $$\frac{T_t}{T_1}=\left(\frac{P_t}{P_1}\right)^{\frac{k-1}{k}}=\left(\frac{2}{k+1}\right)^{\frac{k}{k-1}\times\frac{k-1}{k}}$$

$$=\frac{2}{k+1}=\frac{2}{2{\cdot}4}$$

$$\therefore \qquad T_t=823\times\frac{2}{2{\cdot}4}=685 \text{ K}$$

and $$P_t=P_1\left(\frac{2}{k+1}\right)^{\frac{k}{k-1}}$$

$$=5\left(\frac{2}{2\cdot 4}\right)^{\frac{1\cdot 4}{1\cdot 4-1}}\times 10^4=2\cdot 645\times 10^4 \text{ kgf/m}^2$$

Now $P_t v_t=mRT_t$

$$\therefore \quad v_t=\frac{1\times 29\cdot 27\times 685}{2\cdot 645\times 10^4}=0\cdot 757 \text{ m}^3/\text{kg}$$

Now $V_t=\sqrt{2gJ\triangle c_p s T_t}$

$$=\sqrt{2\times 9\cdot 81\times 427\times 0\cdot 24(823-685)}=526 \text{ m/s}$$

$\therefore$ Throat area, $A_t=\dfrac{v_t}{c_t}=\dfrac{0\cdot 757\times 10^4}{526}=$ **14·4 cm²/kg/s** **Ans.**

(*b*) *Exit*

$$T_2'=T_1\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}=823\left(\frac{1\cdot 4}{5}\right)^{\frac{1\cdot 4-1}{1\cdot 4}}=578 \text{ K}$$

Efficiency of nozzle,

$$0\cdot 90=\frac{T_1-T_2}{T_1-T_2'}=\frac{823-T_2}{823-578}$$

$$\therefore \quad T_2=602\cdot 5 \text{ K}$$

$$v_2=\frac{RT_2}{P_2}=\frac{29\cdot 27\times 602\cdot 5}{1\cdot 4\times 10^4}=1\cdot 26 \text{ m}^3/\text{kg}$$

$$V_2=91\cdot 4\sqrt{0\cdot 24(823-602\cdot 5)}=665 \text{ m/s}$$

$\therefore$ Exit area, $A_2=\dfrac{v_2}{V_2}=\dfrac{1\cdot 26\times 10^4}{665}=$ **18·95 cm²/kg/s** **Ans.**

3·5. Air nozzle ; A_t ; C_{exit}, A_{exit}

A nozzle is required to pass an air flow of 1·5 kg/sec. The inlet conditions are zero velocity, pressure 3·5 kgf/cm² and temperature 425°C. The air is to be expanded to 1·4 kgf/cm². Determine the throat area required if the coefficient of discharge is assumed to be 0·98.

Also calculate the exit velocity and exit area if the nozzle efficiency is assumed to be 95 per cent.

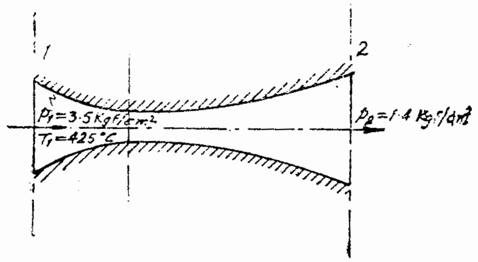


Fig. 3·8

(*i*) *Isentropic flow*

For isentropic flow the critical pressure ratio is given by

$$\frac{p^*}{p_1}=\left(\frac{2}{k+1}\right)^{\frac{k}{k-1}}$$

$$=\left(\frac{2}{1\cdot4+1}\right)^{\frac{1\cdot4}{0\cdot4}}=0\cdot528$$

$\therefore\quad p^*=0\cdot528\times3\cdot5=1\cdot848$ kgf/cm²

The temperature ratio is

$$\frac{T^*}{T_1}=\frac{2}{k+1}=\frac{2}{1\cdot4+1}=0\cdot834$$

$\therefore\quad T^*=0\cdot834\times698=581\cdot7$ K.

$$\therefore\quad v^*=\frac{RT^*}{P^*}=\frac{29\cdot27\times581\cdot7}{1\cdot848\times10^4}$$

$$=0\cdot923 \text{ m}^3/\text{kg}$$

The velocity V at the throat is given by

$$V^*=\sqrt{2gJc_p(T_1-T^*)}$$

$$=\sqrt{2\times9\cdot81\times427\times0\cdot24(698-581\cdot7)}$$

$$=483\cdot5 \text{ m/sec}$$

$$\text{Mass flow}=\frac{1\cdot5}{0\cdot98}=1\cdot53 \text{ kg/sec}$$

$$\therefore\text{ Throat area, } A=\frac{mv^*}{V^*}$$

$$=\frac{1\cdot53\times0\cdot923\times10^4}{483\cdot5}=\mathbf{29\cdot2\ cm^2}$$ **Ans.**

(*ii*) *When flow is not isentropic*

$$T_2'=T_1(p_2/p_1)^{\frac{k-1}{k}}$$

$$=698\times\left(\frac{1\cdot4}{3\cdot5}\right)^{\frac{0\cdot4}{1\cdot4}}=537 \text{ K}$$

$$\text{Nozzle efficiency, } \eta=\frac{T_1-T_2}{T_1-T_2'}$$

$$\therefore\quad 0\cdot95=\frac{698-T_2}{698-537}$$

$$T_2=545 \text{ K}$$

$$\therefore\text{ The exit velocity, } V_2=\sqrt{2gJ\,c_p(T_1-T_2)}$$

or

$$=\sqrt{2\times9\cdot81\times427\times0\cdot24(698-545)}$$

$$=555 \text{ m/sec}$$

pecific volume, $v_2 = \frac{RT_2}{P_2}$

$$= \frac{29 \cdot 27 \times 545}{1 \cdot 4 \times 10^4} = \mathbf{1 \cdot 14\ m^3/kg}$$ **Ans.**

Exit area required, $A_2 = \frac{mv_2}{V_2}$

$$= \frac{1 \cdot 5 \times 1 \cdot 14 \times 10^4}{555}$$

$$= \mathbf{30 \cdot 8\ cm^2}$$ **Ans.**

EXERCISES 3

SECTION A

3·1. What is the basic difference between compressible and incompressible fluid flow ? What difference does it cause in the physical nature of the compressible flow.

3·2. What is velocity of sound ? Derive an equation for the velocity of sound for a perfect gas.

3·3. Explain the terms Mach number, Mach cone, Mach wedge and Mach angle.

3·4. What is subsonic, supersonic, and hypersonic flows ?

3·5. Explain the terms 'zone of silence' and 'zone of action'.

3·6. Explain the concept of total or stagnation properties.

3·7. Derive expressions for stagnation temperature, stagnation pressure, stagnation density, and stagnation enthalpy.

3·8. What is one-dimensional adiabatic flow ? Derive expressions for $\frac{dp}{p}$, $\frac{d\rho}{\rho}$ and $\frac{dA}{A}$ for such a flow.

3·9. What is isentropic flow ? Deduce that in an isentropic flow convergent passage acts as a nozzle for subsonic flow and as a diffuser for supera sonic flow, while a divergent flow passage acts as a diffuser for subsonic flow- and as a nozzle for supersonic flow.

3·10. For a variable area flow passage define the terms 'critical area' and 'throat'.

3·11. Show that the condition of maximum mass flow rate for an air nozzle occurs at $\frac{p_t}{p} = 0 \cdot 528$.

3·12. Discuss the effect of pressure ratio on flow through (*a*) convergent nozzle, (*b*) convergent-divergent nozzle.

SECTION B

3·13.(*a*) Show that the change in temperature across an infinitesimal pressure pulse travelling through an ideal gas is

$$dT = \frac{T(k-1)dV}{a}$$

(*b*) Show that for an isentropic flow

$$a^* = \sqrt{\frac{2c^2 + V^2(k-1)}{k+1}}$$

(*c*) The velocity of sound of a gas is found experimentally to be 900 m/sec when the gas pressure is 1·8 kgf/cm² and the density 0·001 kg/m³. What is the value of the specific heat ratio of this gas ?

3·14. Air is supplied through a nozzle with an exit area of 10 cm². A tank supplies the air at 10 kgf/cm² and 200°C and the discharge pressure is 7·5 kgf/cm². Assuming no loss, determine the discharge temperature, the discharge velocity, the Mach number, and the mass flow.

3·15. A stream of air flowing in a duct is at a pressure of 2 kgf/cm², has a Mach number of 0·6, and flows at the rate of 0·25 kg/sec. The cross-sectional area of the duct is 1 cm².

(*a*) Compute the stagnation temperature of the stream.

(*b*) What is the maximum percentage reduction area which could be introduced without reducing the flow rate of the stream ?

(*c*) For the maximum area reduction of part (*b*), find the velocity and pressure at the minimum area, assuming no friction and no heat transfer.

3·16. Consider a supersonic nozzle constructed with a ratio of exit of the throat area of 2·0. The nozzle is supplied with air at low speed at 7 kgf/cm² and 40°C. The overall nozzle efficiency from inlet to exit is 90 per cent, but the flow is isentropic up to the throat.

Calculate the pressure, velocity and the Mach number at exit, and compare with corresponding values for isentropic flows.

3·17. Air is moving in a pipe with a velocity of 100 m/sec. The temperature and pressure at one section in the pipe is 40°C and 2kgf/cm², respectively.

(*a*) Stagnation pressure
(*b*) Stagnation temperature
(*c*) Stagnation density, and
(*d*) Stagnation enthalpy.

3·18. Prove that the velocity of sound in a Van der Waal's gas is given by

$$a = \sqrt{g\,kRT\left(\frac{1}{(1-b\rho)^2} - \frac{2a\rho}{RT}\right)}$$

3·19. The photographs of a bullet in flight show that at a great distance from the bullet the total included angle of the wave is 50·3°. The pressure and temperature of the undisturbed air are 1 kgf/cm² and 25°C, respectively. Calculate the velocity of the bullet and the Mach number of the bullet relative to undisturbed air.

3·20. Air approaches and flows around a body. At the stagnation point the pressure is 1 kgf/cm² and temperature 21°C. At this point the static pressure is 0·7 kgf/cm². Find the Mach number at this point.

3·21. Show that in a flow from reservoir, the maximum velocity that may be reached is given by

$$x = a_0\sqrt{\frac{2}{k-1}} \text{ for a perfect gas.}$$

What are the corresponding values of temperature and Mach number ? Interpret.

3·22. For a perfect gas, prove that

(i) $a^{*2}(k+1)=2a^2+V^2(k-1)$

(ii) $(p/p_1)=(\rho/\rho_1)^{k}\, e^{(k-1)(s-s_1)/R}$

3·23. A convergent-divergent nozzle is fitted into the side of a large vessel containing a gas under constant pressure and temperature. If the ratio of specific heats of the gas is 1·3 :1, calculate, from first principles, the percentage change in (a) pressure, (b) absolute temperature between the reservoir and the throat of the nozzle under the maximum flow conditions. Neglect friction and assume adiabatic expansion.

[(a) 45·6%, (b) 12%]

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4

POSITIVE DISPLACEMENT COMPRESSORS

4·1. INTRODUCTION

There is hardly any product used in our daily lives to which compressed air has not contributed in some way. Compressed air is widely used in chemical and petro chemical industries, for transmitting power, for conveying solid and powdered materials in pipe lines, in mines, and steel industry. Compressors are integral part of gas turbine plants. Compressors are used for delivering natural gas through long distance pipe lines, compressing the mixture of hydrogen and nitrogen in ammonia synthesis plant, delivering lime-kiln gas for Solvay process, circulating synthesis gases in processes for the manufacture of primary products of plastics, circulation of carbon monoxide or helium for cooling the nuclear power plants and liquefying ammonia in large refrigerating plants. In addition to steel industry, where about 10 per cent of the total capital cost of the plant comprises compressors, it is also used among instrument manufactures, and for transportation, glass, food, paper, rubber and plastic products. In a nitric acid plant about 10 per cent and in an ammonia synthesis plant about 15 to 20 per cent of total initial cost is of compressors. With regard to such a wide variety of industrial applications of compressors, it can be said that compressed air is only next best to electricity and this makes the study of various types of compressors a very important part of the education of a mechanical engineer.

The function of a compressor is to increase the pressure of the air or the gas inducted. The term fan is used to describe dynamic air compressors in which this increase in pressure is less than 0·35 kgf/cm^2, *i.e.* the density of the air does not change appreciably. Dynamic compressors, which increase the pressure of the fluid passing through them upto about 3 kgf/cm^2 are termed as *blowers*. Compressors producing vacuum are called *exhausters* or *air pumps* and those which increase the pressure of the fluid already above atmospheric pressures are called *boosters*. For the analysis of fans incompressible flow equations can be used while the compressibility effects must be considered in design and analysis of blowers and compressors. In this chapter only the positive displacement compressors are discussed. Dynamic compressor form the subject matter of the next chapter.

4·2. CLASSIFICATION OF AIR COMPRESSORS

Basically the compressors can be classified into two types, namely :—

1. Positive displacement type compressors, and
2. Dynamic compressors.

Positive displacement type of compressors are those in which successive volumes of air or gas are first confined within a closed space and then the pressure in this space is increased by decreasing its volume. Positive displacement type of compressors include reciprocating, sliding-vane rotary, liquid piston, two or three lobe rotary, and rotary screw compressors. The pressure developed by these compressors is independent of speed while the rate of flow changes with speed. Dynamic compressors are those in which the compression of air or gas is affected by the dynamic action of rotating vanes or impellers. These rotating vanes or impellers impart velocity and pressure to the flowing medium. These include centrifugal and axial flow compressors. In dynamic compressors the fluid flow is steady flow through the machine unlike discontineous flow of positive displacement type compressors. Fig. 14·1 shows the classification of compressors.

- Compressors
  - Dynamic
    - Centrifugal
    - Axial flow
    - Mixed flow
  - Positive Displacement
    - Rotary
      - Single rotor
        - Sliding vane
        - Liquid piston
      - Two rotor
        - Lobed (Roots blower)
        - Screw
    - Reciprocating

Fig. 4·1. Classification of compressors.

4·3. COMPRESSOR EFFICIENCIES

(*i*) **Isothermal efficiency**. Since for isothermal compression the work required to drive the compressor is minimum, it is considered as a standard towards which each designer will try to approach ; and the performance of the compressor is given by isothermal efficiency, which is definfed as

Isothermal efficiency

$$= \frac{\text{Isothermal work}}{\text{Actual indicated work}} \tag{4·1}$$

(*ii*) **Volumetric efficiency**. One of the effects of clearance volume is to reduce the amount of air which can be sucked in during the suction stroke The mass of air inducted is further reduced by the resistance of inlet and exhaust valves, heating of air due to compressor parts, etc. The ratio of the mass of air passing through the compressor and the mass of air which would completely fill the swept volume is defined as volumetric efficiency. The overall volumetric efficiency is given by

Overall volumetric efficiency

$$= \frac{\text{Mass of air delivered}}{\text{Mass of air corresponding to swept volume of L.P. cylinder at F.A.D. conditions}} \tag{4·2(a)}$$

where free air delivery (F.A.D) is defined as the volume of air delivered and reduced to intake pressure and temperature. The capacity of a compressor is usually given in terms of F.A.D.

Alternatively, Overall volumetric efficiency

$$= \frac{\text{Volume of free air inhaled}}{\text{Swept volume of L.P. cylinder}} \tag{4·2 (b)}$$

When the volumetric efficiency is calculated in terms of the conditions at normal pressure and temperature, abbreviated as N.T.P., it is called absolute volumetric efficiency.

Absolute volumetric efficiency

$$= \frac{\text{Mass of air delivered}}{\text{Mass of air corresponding to swept volume of L.P. cylinder at N.T.P.}} \tag{4·3}$$

Alternatively

Absolute volumetric efficiency

$$= \frac{\text{Volume of air inhaled at N.T.P.}}{\text{Swept volume of L.P. cylinder}} \tag{4·4}$$

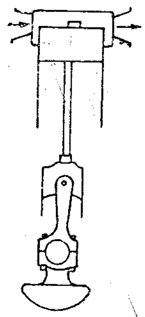
(*iii*) **Mechanical efficiency**. The mechanical efficiency of a reciprocating compressor is defined as the ratio of indicated power to the brake power of the shaft supplied to the compressor.

Mechanical efficiency

$$= \frac{\text{Indicated or air h.p.}}{\text{Shaft h.p.}} \tag{4·5}$$

4·4. RECIPROCATING COMPRESSOR

The compressing element of a reciprocating compressor is a piston in a cylinder, employing basically the same mechanical action as the intake and compression strokes of a reciprocating internal combustion engine. Fig. 4·2 shows a schematic diagram of such a compressor. During the suction stroke, when the piston travels from top dead centre to bottom dead centre, the inlet valve opens and allows the air to be sucked into the cylinder and closes as the piston reaches bottom dead centre. When the piston moves upwards the air is



·2. Schematical diagram of a reciprocating compressor.

compressed and at a rated pressure the discharge valve opens to discharge the compressed air to the system. A double acting compressor uses both ends of the cylinder for suction and discharge, thus discharging approximately twice as much air per cylinder as the single acting unit.

Fig. 4·3 shows P-v and T-s diagram of the theoretical compression cycle neglecting the clearance volume. Process P-1 represents suction, process 1-2 the compression from pressure P_1 to

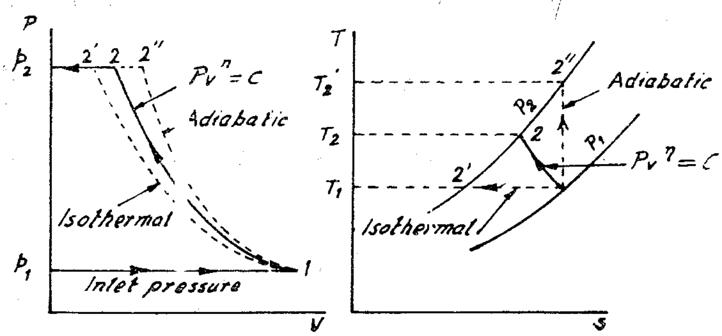


Fig. 4·3. P-v and T-s diagrams for reciprocating compressor.

pressure, P_2 and the process $2\text{-}P_2$ delivery at a constant pressure P_2. Work done in compressing the air is represented by the area of the $P\text{-}v$ diagram.

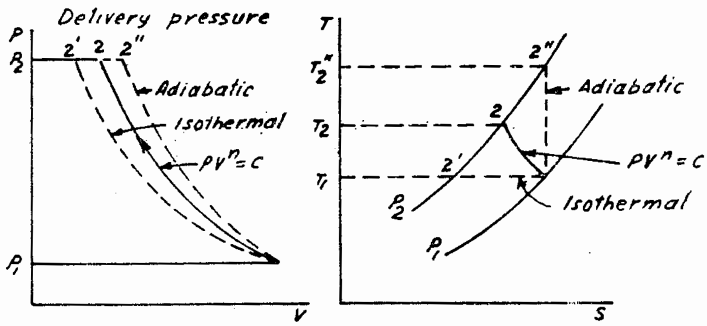


Fig. 4·4. *P-v* and *T-s* diagrams for polytropic and isentropic compression.

For isothermal compression the work done is minimum, (*see* Fig. 4·4). However, this is difficult to realize in practice and compression follows the law Pv^n=constant with value of n varying between 1·25 to 1·35.

Workdone per cycle=area of $P\text{-}v$ diagram

$$=\left[P_2V_2+\frac{P_2V_2-P_1V_1}{n-1}\right]-P_1V_1$$

$$=\frac{n}{n-1}\left[P_2V_2-P_1V_1\right]$$

$$=\frac{n}{n-1}P_1V_1\left[\frac{P_2V_2}{P_1V_1}-1\right]$$

$$=\frac{n}{n-1}\,mRT\left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}-1\right] \qquad [4\cdot6\ (a)]$$

where m is the mass of air per cycle.

The work done per cycle can also be written in terms of increase of temperature during compression by putting $PV=mRT$ in equation [4·6 (*a*)]

Work done per cycle

$$=\frac{\quad}{n-}\,mR(T_2-T_1) \qquad [4\cdot6\ (b)$$

Work done per kg of air

$$=\frac{nR}{n-}(T_2-T_1)$$

$$=c_p(T_2-T_1) \qquad [4\cdot6\ (c)]$$

It is clear from equations [4·6 (*a*)] and [4·6 (*b*)] that the work done in compressing the air depends, apart from the pressure ratio and initial air temperature, upon the index of compression, n.

For an adiabatic process (PV^{γ}=constant), work done per cycle is given by

$$W=\frac{\gamma}{\gamma-1}mRT_1\left[\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}-1\right] \qquad [4\cdot7\ (a)]$$

$$=\frac{\gamma}{\gamma-1}mR(T_2-T_1) \qquad [4\cdot7\ (b)]$$

and for an isothermal process (PV=constant) work done is given by

$$W=RT_1\log_e\left(\frac{P_2}{P_1}\right) \qquad (4\cdot8)$$

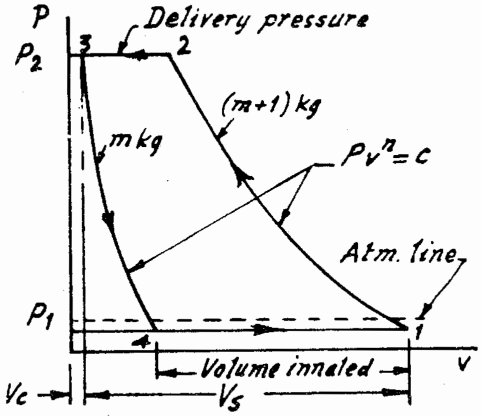


Fig. 4·5. P-v diagram of reciprocating compressor considering clearance volume.

Fig. 4·5 shows the theoretical compression cycle considering the clearance volume. At the end of the delivery the volume instead of being zero, has a value equal to V_3. During suction, first the air in the clearance volume expands to pressure P_1 and volume V_4 (process 3-4) and then suction takes place. Thus the clearance volume reduces the amount of air sucked in the suction stroke to only (V_1-V_4) and the work done per cycle, given by the area 1234, is also reduced.

Work done per cycle=area 1234

$$=\frac{n}{n-1}R(m_1-m_3)(T_2-T_1) \qquad (4\cdot9)$$

while work done per kg of air delivered remains the same as given by [4·6 (c), *i. e.*

$$\frac{n}{n-1}R(T_2-T_1)$$

The clearance volume affects only the effective stroke of the compressor during suction and reduces volumetric efficiency but the power required to drive the compressor per kg of air remains same. The decrease in volumetric efficiency is more when the compressor pressure ratio is increased.

4·4·1. Multi-stage compression

During isothermal compression there is no gain in the internal energy of air because the temperature at the end of compression is same as that at the beginning of compression. However, in actual practice the compression is always polytropic and there is a definite rise in temperature of the air. The additional work required per cycle for polytropic compression goes in increasing the temperature of the air. The deviation of the actual compression path from the isothermal path increases with higher pressure ratios (*see* Fig. 4·4) resulting in a corresponding increase in work required per cycle. To reduce the work required per cycle multi-stage compression is resorted to when the required pressure ratio exceeds about 5.

Fig. 4·6 shows the schematic diagram of a multi-stage compressor and Fig. 4·7 the corresponding P-v and T-s diagrams with

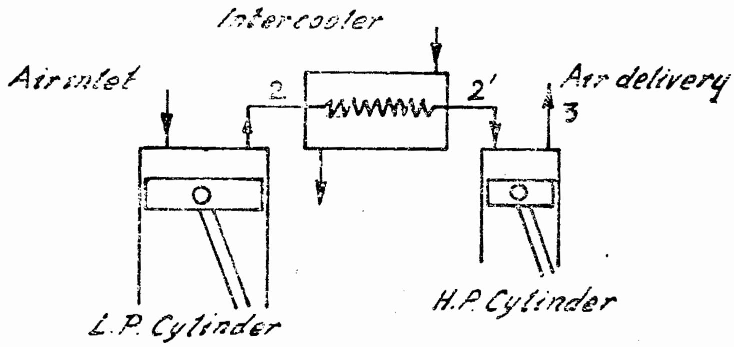


Fig. 4·6. Schematic diagram of a multi-stage compressor.

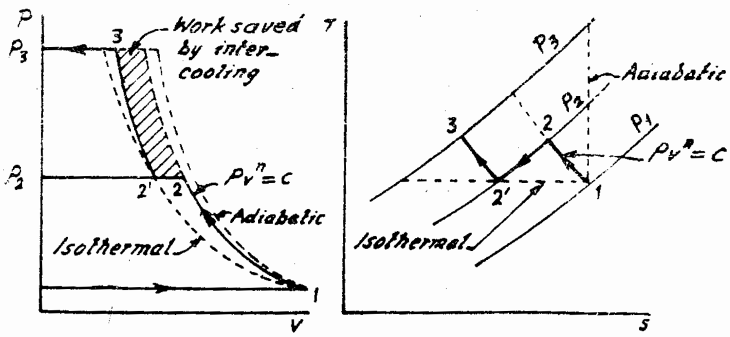


Fig 4·7. P-v and T-s diagrams with perfect intercooling.

perfect intercooling. The air after being compressed in the low pressure cylinder is cooled in an intercooler. If the air is cooled to its original temperature it is called *perfect intercooling* and if it is not cooled to its original temperature but to a higher temperature it is called *imperfect intercooling*. The work saved by intercooling is shown by the hatched area.

Assuming that (*i*) the index of compression is same in each stage, (*ii*) intercooling is done at constant pressure, and (*iii*) there is no pressure loss between stages of a multi-stage compressor, we get

Work done per cycle

=work done per cycle in (L.P. cylinder+H.P. cylinder)

$$=\frac{n}{n-1}\left[P_1V_1\left\{\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}-1\right\}+P_2V_2\left\{\left(\frac{P_3}{P_2}\right)^{\frac{n-1}{n}}-1\right\}\right] \qquad [4\cdot10\ (a)]$$

If the intercooling is complete, $P_1V_1=P_2V_2$

$$\text{and } W=\frac{n}{n-1}P_1V_1\left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}+\left(\frac{P_3}{P_2}\right)^{\frac{n-1}{n}}-2\right] \qquad [4\cdot10\ (b)]$$

$$\therefore \quad \text{H.P. required}=\frac{WN}{75\times60} \qquad (4\cdot11)$$

where N is revolutions per minute.

Differentiating equation [4·10 (*b*)] with respect to (P_2/P_1) and equating to zero the condition for minimum work in multi-stage compression can be obtained as

$$P_2=\sqrt{P_1\times P_3} \qquad (4\cdot12)$$

or
$$\frac{P_2}{P_1}=\frac{P_3}{P_2}$$

i.e. pressure ratio for each stage is same. This also gives equal work in each stage.

The total work done per cycle, then, is

$$W=2\times\frac{n}{n-1}P_1V_1\left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}-1\right] \qquad (4\cdot13)$$

Hence the conditions for minimum work are that the pressure ratio in each stage is same and the intercooling is perfect.

The above analysis is based on perfect intercooling and no pressure drop in intercooler. In practice due to imperfect cooling and pressure drop, a compressor designed with $P_2=\sqrt{P_1\times P_3}$ would have greater area of H.P. diagram so that the intermediate pressure is raised above the value of P_2.

·4·2. Performance of reciprocating compressors

Fig. 4·8 shows the actual indicator diagram of a single-stage reciprocating compressor. Due to throttling in inlet and exhaust valves the pressures at intake and delivery are different from the theoretical contant pressures. The actual work required is more due to inertia of valves, etc.

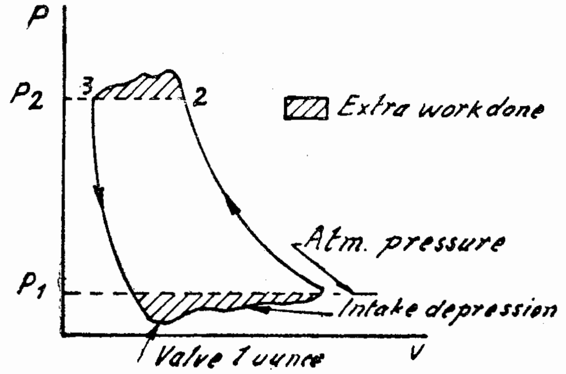


Fig. 4·8. Actual indicator diagram of a single-stage compressor.

The performance of an air compressor is usually given in terms of delivery pressure, horse power, volumetric efficiency and the capacity. Fig. 4·9 shows the performance of a typical reci-

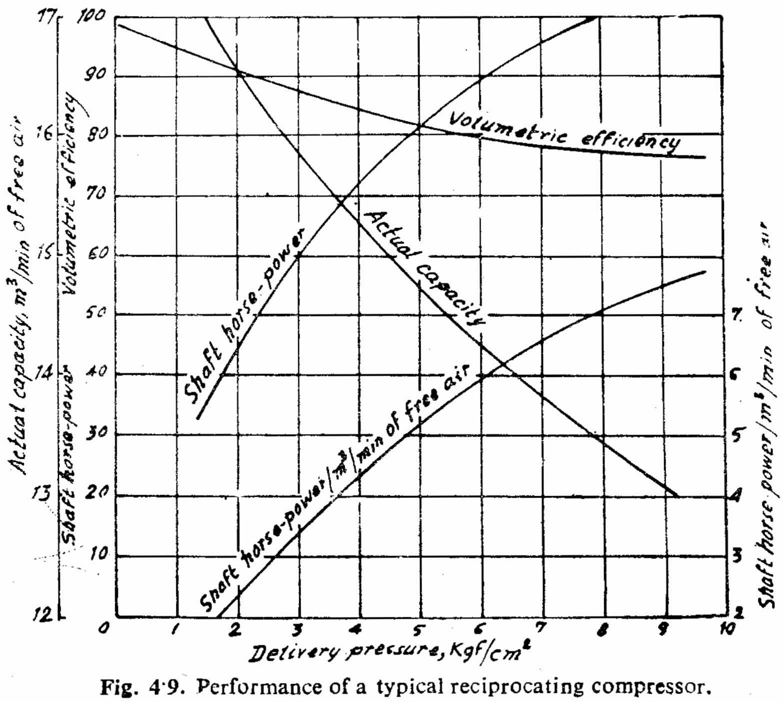


Fig. 4·9. Performance of a typical reciprocating compressor.

procating compressor. It is evident that as the pressure ratio is increased the volumetric efficiency decreases, thereby reducing the air capacity of the compressor. The power required to drive the compressor also increases with an increase in pressure ratio. One important characteristic of the reciprocating compressor is that the pressure ratio is not affected by speed and the amount of air being delivered by the compressor.

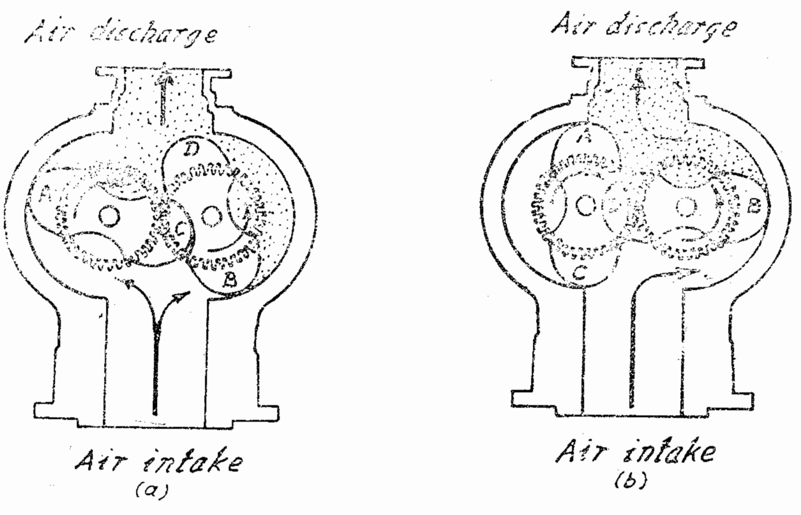
4·4·3. Applications of reciprocating compressors

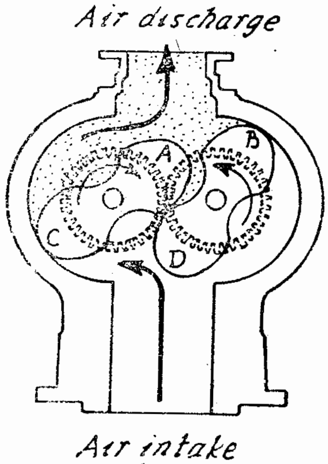
Reciprocating compressors are primarily used in industrial manufacturing plants and for on-site work. The single-stage compressor is normally used for pressures below 4 kgf/cm^2 while two stage compressor is used for pressures above 7 kgf/cm^2. Between 4 kfg/cm^2 and 7 kgf/cm^2 the single-stage unit is used for capacities below 8 to 9 m^3/min and two-stage unit for higher capacities. This type of compressor is also used in free-piston engines.

Due to high inertia of reciprocating parts and low rotational speeds this type of compressor has not been used in conjuction with machines running at very high speeds such as gas turbines. Moreover, the reciprocating compressors are suitable for comparatively low discharge volumes and high pressure. When very high discharge pressures are needed, a reciprocating compressor is the only alternative.

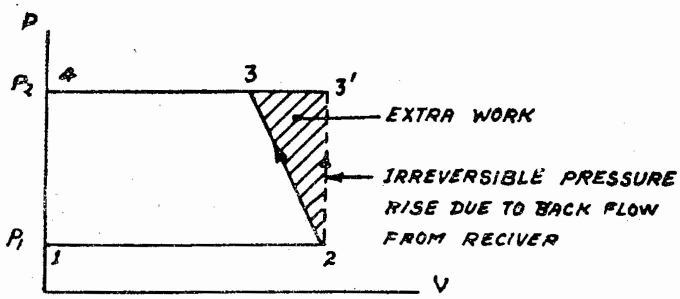
4·5. ROOTS BLOWER

Roots blower is a rotary type of positive displacement compressor. Fig. 4·9 shows a two lobe roots blower. It consists of two identi-





(c)



(*d*) *p-v* diagram of roots blower.

Fig. 4·9. Suction and discharge cycles of a two lobe roots blower.

cally formed impellers or rotors having epicycloid and hypocycloid or involute profiles. These profiles provide perfect mating. The rotors are held in position by two inter-meshed gears mounted on an external housing. However, there is no internal contact between the two rotors or between rotors and the outer housing to avoid wear, but to reduce slip and leakage this clearance is kept minimum. The rotors rotate in opposite directions.

Working of Roots blower. When the rotors rotate the air is entrapped between them and the sides of the casing at the inlet side. This volume V is positively displaced as the rotors move and its pressure remains the same as that at inlet. As soon as the rotor A is perpendicular to rotor B the discharge port is uncovered and air starts discharging. It should be noted that the air is not compressed by being entrapped between the rotors but is compressed in the discharge side of the blower. As soon as the discharge port is uncovered, *i.e.* when rotor A passes the point 2 (*see* Fig. 4·10) back flow from the air receiver occurs and the pressure is immediately increased due to back flow and discharge takes place. The amount of free air delivered to the receiver is four times that of the entrapped volume, *i.e.* $4V$ for a two lobe rotor as this happens four times in one revolution of the rotor. The air supply is intermittent even when the rotors are continuously rotating. However, by proper design and phasing of two lobes it can be ensured that when discharge from lobe A ends, lobe B starts discharging [*see* Fig. 4·10 (*c*)] and a continuous supply of air can be obtained.

The working of the Roots blower is shown in Fig. 4·10 (*d*) on a p-v diagram. The dotted line refers to a reciprocating compressor. Process 2-3 is the instantaneous pressure rise due to back flow of air from the air receiver. Thus it has an inherent irreversibility in its action and the work required is greater than a corresponding reciprocating compressor by area 23° 32′. The work done per revolution of rotor W is given by

$$W=4V(P_2-P_1) \tag{4·14}$$

This extra work done is relatively small at small pressure ratios which are usual where Roots blower is mostly used. The efficiency of the Roots blower depends upon the pressure ratio and decreases with an increase in it due to increased leakage past the lobes. At low pressure ratio the efficiency is about 80 per cent but reduces to 50-60 per cent at higher pressure ratios.

Roots blowers are built with a discharge capacity of 0·15 to approximately 1500 m³/min and run at speeds upto 7000 rev/min. A single-stage unit is sufficient for pressures up to 2 kgf/cm² while for higher pressures a two stage unit is used. The number of lobes may be 4 or 6 for the high pressure ratio compressor.

The Roots blower is used for scavenging and supercharging of two stroke internal combustion engines, low pressure bulk supply of air in steel furnaces, sewage disposal plants, low pressure gas boosters, and for blower service in general. The Roots blower is not suitable for moderate and high pressure requirements.

4·6. LYSHOLM COMPRESSOR

Lysholm compressor consists of two modified helical rotors-one grooved rotor, *i.e.* female rotor and the other a mating male rotor. These are so designed that at inlet, the air volume is gradually reduced and the pressure is increased. The nature of compression is similar to that in a reciprocating compressor in that the rotors have certain compression ratio. There is a small clearance between the lobes, hence the volumetric efficiency decreases with an increase in pressure ratio due to increased leakage. An increase in speed increases the rate of flow at a fixed pressure ratio.

The compression is smooth and continuous with a high efficiency. Lysholm compressors are built with a wide range of capacities from 3 m^3/min to 350 m^3/min at pressures upto 2 kgf/cm^2 for single stage unit and up to 7 kgf/cm^2 with two stage unit.

The volumetric efficiency of a Lysholm compressor decreases with increased pressure ratios but is better for higher speeds. The mechanical friction and fluid-friction increases with speed but on a percentage basis decreases with increasing pressure ratio.

Screw compressers are also similar in principle to Lysholm compressor. Lysholm and screw compressors are used in portable as well as general industrial plant applications. The high efficiency and high rates of flow with freedom from surging are the main characteristics of this compressor. Lysholm compressor is used in marine gas turbine power plants.

4·7. VANE TYPE COMPRESSOR

Fig. 4·10 shows the components and the basic principle of operation of a vane type compressor. It consists of a rotor with a number of vanes free to slide radially in an offset casing forming sealed sectors. The position of the inlet port is such that air is taken into the compressor cylinder when the gap between rotor and cylinder wall is increasing. The vanes are made of non-metallic fibre or carbon and are spring-loaded. The sectors vary in their volume with the position of vanes. When the vanes are fully extended it is maximum and a slight further movement isolates the filled sector from the intake port. With the rotation of sector this volume of air is reduced

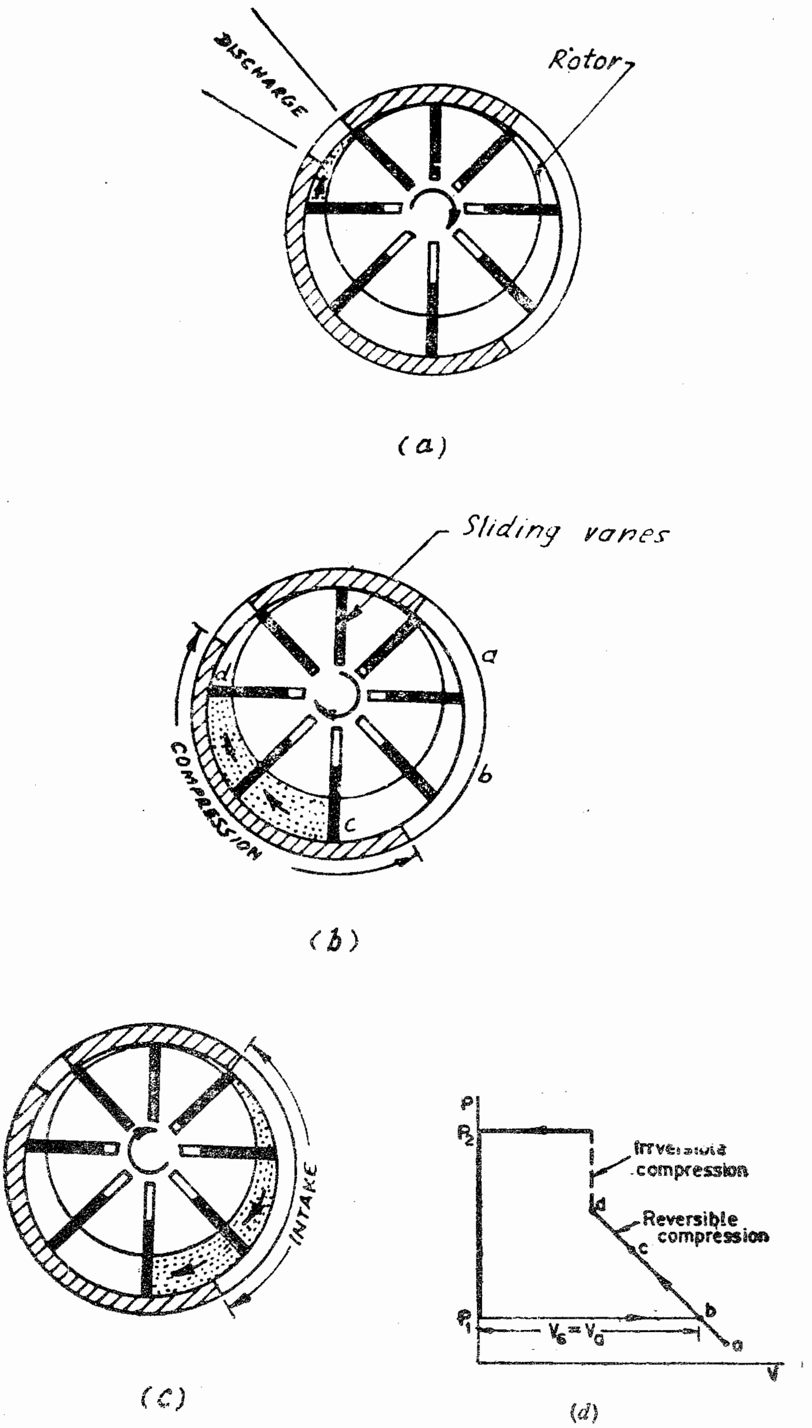


Fig. 4·10. Vane type compressor.

as the cylinder walls and the rotor body converge till the air is discharged and thus its pressure is increased. However, this is not the total compression obtained by the compressor. Further compression of the air occurs due to back flow when the delivery port is uncovered. About half of the total compression is due to vanes and this reversible compression is shown by the process *b-d* on the *p-v* diagram. The rest half of the compression is due to back flow and is irreversible.

The flow in a vane type compressor is smooth and free from pulsations. The vane type compressor can develop pressures upto 3·5 kgf/cm^2 with one stage and up to 9 kgf/cm^2 with two stages with a capacity ranging from 2·5 to 60 m^3/min. The rotational speed is limited to about 2500 r.p m. as compared to about 7000 r.p.m. of the Roots blower due to difficulties in balancing the changing acceleration of the vanes. The efficiency of a vane type compressor varies from 65 to 75 per cent and is higher than that of Roots blower if the pressure ratio is more than 1·4.

This type of compressor is used for supercharging I.C. engines, supply of air to cupola and are very popular in portable compressors used for construction purposes.

4 8. LIQUID PISTON ROTARY COMPRESSOR

The liquid piston rotary compressor (*see* Fig. 4·11) consists of a multi-blade rotor revolving in an elliptical casing partly filled with a low viscosity fluid, usually water. When the rotor turns, it carries with it the water creating a solid liquid ring turning in the casing at the same speed as that of the rotor. This liquid is forced to enter and leave the chamber alternately in a continuous cycle. This is because the rotor is round while the casing is elliptical. This action of water is just like a liquid piston and the air in the spaces between the vanes of the impeller is displaced. Vacuum can also be created by restricting the inlet air.

Liquid piston compressors have a range from 0 06 to 300 m^3/min with pressure varying from approximately 75 cm of water vacuum to over 7 kgf/cm^2. Typical applications of such compressors are in chemical processing, sewage disposal, and laboratories.

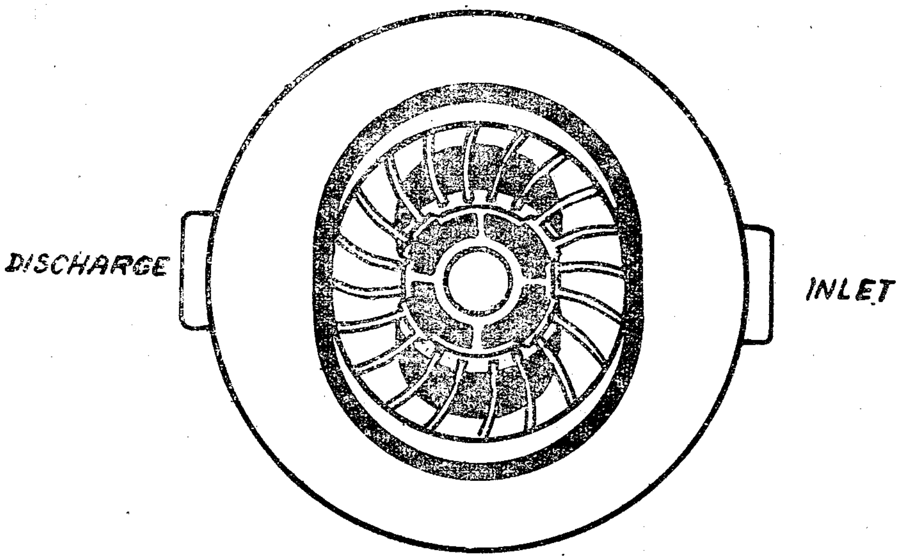


Fig. **4·11.** Liquid piston rotary compressor.

4·9. RECIPROCATING *vs.* ROTARY POSITIVE DISPLACEMENT COMPRESSORS

The reciprocating compressor due to its low speed and high inertia tends to be bulky. However, it consumes less power due to its high efficiency. It has a low initial cost. Reciprocating compressors are capable of developing very high pressure ratios and the maximum capacity is limited to about 300 m^3/minute. Reciprocating compressors are either water or air cooled while the rotary compressors are oil cooled. In rotary compressors same oil is used for lubrication as well as for cooling the air, so special detergent oil and that too in large quantities, is needed. Due to hot oil being used in main bearings leakage is a problem. This is specially so for Indian tropical conditions where oil takes relatively longer time to cool.

Rotary compressors require more power than reciprocating compressors for same mass flow as shown in Table 4·1 and this extra power manifests itself in higher outlet temperatures. Rotary compressors have outlet temperatures about 35° to 45°C higher than their reciprocating counterparts.

TABLE 4·1

POWER REQUIRED FOR VARIOUS COMPRESSORS

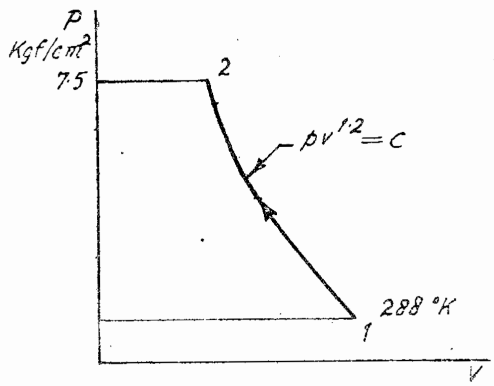
Air delivery, FAD, m^3/min	3·5	4·5/5	6	7	10·5
Positive displacement rotary	33	50	59	68	96
Reciprocating, h.p.	26	38	49	62	88

Rotary compressors are used for delivering large air quantities at relatively low pressures upto 8 kgf/cm^2. Rotary compressors are high speed machines and can be directly coupled to turbines and are smaller in size and have a uniform delivery without a large receiver between the compressors and the air main as provided in reciprocating types.

ILLUSTRATIVE EXAMPLES

4·1. Reciprocating compressor : power required

A reciprocating compressor delivers 5 kg of air per minute at a pressure of 7·5 kgf/cm². The pressure and temperature of the air before compression are 1 kgf/cm² and 15°C, respectively, and the compression process may be assumed to follow the law $pv^{1\cdot 2}=constant$. *If the rate of heat transfer from the cylinder is estimated to be 1·6 h.p., calculate the power required for the compression. Take* $c_p=0\cdot 24$.



$$T_2=T_1\times\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}$$

$$=288\times\left(\frac{7\cdot 5}{1}\right)^{\frac{0\cdot 2}{1\cdot 2}}=403\ \text{K}$$

Total power required = mass rate $\times c_p(T_2 - T_1)$ + heat transferred

$$= \frac{5 \times 0{\cdot}24(403-288)}{10{\cdot}54} + 1{\cdot}6$$

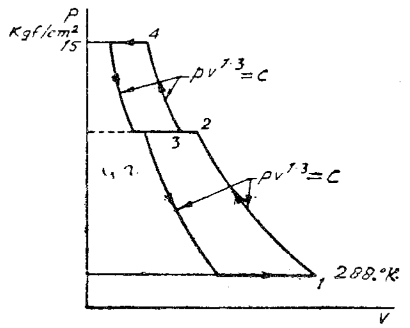
$$= 13{\cdot}1 + 1{\cdot}6 = \mathbf{14{\cdot}7\ h.p.}\quad \textbf{Ans.}$$

(Note 10·54 kcal/min = 1 h.p.)

1·2. Two-stage compressor : h.p.; η_{iso}, F.A.D. ; heat transfer

A single-acting two-stage compressor with complete intercooling delivers 5 kg/min of air at 15 kgf/cm². Assuming an intake state of 1 kgf/cm² and 15°C, and that the compression and expansion processes are reversible and polytropic with n=1·3. Calculate the power required, the isothermal efficiency and the free air delivery. Also calculate the net heat transferred in each cylinder and in the intercooler.

If the clearance ratios for the low and high pressure cylinders are 0·04 and 0·06 respectively, calculate the swept and clearance volumes for each cylinder. The speed is 420 rev/min.



Final pressure $P_3 = 15$ kg/cm² ; Initial pressure $P_1 = 1$ kg/cm²

Pressure ratio

Pressure at the end of first stage of compression,

$$P_2 = \sqrt{P_1 \times P_3}$$
$$= \sqrt{1 \times 15} = 3{\cdot}88 \text{ kgf/cm}^2$$

Work done in compression

Total work done per cycle is given by

$$W = 2m\frac{n}{n-1}RT_1\left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} - 1\right]$$

$$= \frac{2\times 5}{60} \times \frac{1{\cdot}3}{1{\cdot}3-1} \times 29{\cdot}27 \times 288[(3{\cdot}88)^{\frac{1{\cdot}3-1}{1{\cdot}3}} - 1]$$

$$= \mathbf{2240\ gf{\cdot}m/s\ or\ 29{\cdot}9\ h.p.}\quad \textbf{Ans.}$$

Isothermal work done, $W_i = mRT_1 \log \frac{P_2}{P_1}$

$$= \frac{5}{60} \times 29{\cdot}27 \times 288 \times \log_e 15$$

$$= 1908 \text{ kgf/s or } 25{\cdot}4 \text{ h.p.}$$

Isothermal efficiency, $\eta_{iso} = \frac{W_{iso}}{w}$

$$= \frac{25-4}{29{\cdot}9} = 85\% \quad \textbf{Ans.}$$

Free air delivery

Free air delivery, $V = m\frac{RT_1}{P_1}$

$$= 5 \times \frac{29\;27 \times 288}{1 \times 10^4} = \mathbf{4{\cdot}21 \; m^3/min} \quad \textbf{Ans.}$$

Heat transferred

Since there is perfect intercooling, temperature at the end of each stage is equal, *i.e.*

$$T_2 = T_3 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{0{\cdot}3}{1{\cdot}3}}$$

$$= 288\left(\frac{15}{1}\right)^{\frac{0{\cdot}3}{1{\cdot}3}} = 394 \text{ K}$$

Applying the First Law of Thermodynamics, the heat transferred in each cylinder is given by

$$Q = mc_p(T_3 - T_1) - W$$

$$= \frac{5}{60} \times 0{\cdot}24(394 - 288) - \frac{2240}{2 \times 427}$$

$$= 2{\cdot}12 - 2{\cdot}62 = \mathbf{-0{\cdot}15 \; kcal/s} \quad \textbf{Ans.}$$

Volumetric efficiency

The volumetric efficiency is given by

$$\eta_{vol} = 1 - \frac{V_d}{V_s}\left\{\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} - 1\right\}$$

For first cylinder volumetric efficiency

$$\eta_{vol}=1-\frac{V_d}{V_s}\{(P_2/P_1)^{\frac{n-1}{n}}-1\}$$

$$=1-0{\cdot}04\{(3{\cdot}88)^{\frac{0{\cdot}3}{1{\cdot}3}}-1\}=85{\cdot}2\%$$

For second cylinder

$$\eta_{vol}=1-0{\cdot}06\{(3{\cdot}88)^{\frac{0{\cdot}3}{1{\cdot}3}}-1\}=77{\cdot}92\%$$

Swept volume and clearance volume

The swept volume is given by

$$V_b-V_a=\frac{F.A.D.}{N\times\eta_{vol}}$$

First cylinder

$$V_b-V_a=\frac{F.A.D.}{N\times\eta_{vol}}=\frac{4{\cdot}21}{420\times0{\cdot}852}=\mathbf{0{\cdot}01179\ m^3}\ \textbf{Ans.}$$

Second cylinder

$$V_b-V_a=\frac{F.A.D}{P_2/P_1\times N\times\eta_{vol}}$$

$$=\frac{4{\cdot}21}{3{\cdot}88\times420\times0{\cdot}7792}=\mathbf{0{\cdot}00332\ m^3}\ \textbf{Ans.}$$

Clearance volume

Clearance volume = Clearance ratio × Swept volume

First cylinder

Clearance volume $=0{\cdot}01179\times0{\cdot}04=\mathbf{0{\cdot}000472\ m^3}$ **Ans.**

Second cylinder

Clearance volume $=0{\cdot}00332\times0{\cdot}06=\mathbf{0{\cdot}000199\ m^3}$. **Ans.**

4·3. Reciprocating compressor : unsteady flow ; time to produce a specified pressure

A reciprocating compressor delivers air to a receiver having a volume of 5 m³. The receiver is initially at a pressure of 1·5 kgf/cm² and temperature 25°C ; the pressure is to be increased to 8·5 kgf/cm², the temperature being maintained constant by means of a cooler. If the compressor is driven by a 35 h.p. motor and the rate of heat transfer from the system is 1·2 h.p., calculate the time taken to produce the specified pressure rise (Take $c_p=0{\cdot}24$ kcal/kg K, $c_v=0{\cdot}17$ kcal/kg K and atmospheric temperature $t_1=18°C$).

Initial mass of the air in the receiver,

$$m_i = \frac{P_1V_1}{RT_1} = \frac{1{\cdot}5\times5\times10^4}{29{\cdot}27\times298} = 8{\cdot}6 \text{ kg}$$

Final mass of the air in the receiver,

$$m_2 = \frac{8{\cdot}5\times5\times10^4}{29{\cdot}27\times298} = 48{\cdot}7 \text{ kg}$$

$\therefore$ Increase in mass $=48{\cdot}7-8{\cdot}6=40{\cdot}1$ kg

Writing down the energy equation, we have

$$Q+W=u_2m_2-u_im_i+h_1(m_2-m_i)$$

where Q = heat transferred

W = work done

u = internal energy

h = enthalpy

Since the temperature remains constant

$$u_i=u_2$$

$$\begin{aligned}\therefore \quad Q+W &= m_i(m_2-m_i)+h_1(m_2-m_i)\\ &= c_vT_i(m_2-m_i)+c_pT_1(m_2-m_i)\\ &= (m_2-m_1)(c_vT_i+c_pT_1)\end{aligned}$$

where T_1 is the atmospheric temperatures, $T_1=291$ K.

$$\begin{aligned}\therefore \quad Q+W &= 40{\cdot}1(0{\cdot}17\times298+0{\cdot}24\times291)\\ &= 4820 \text{ kcal}\end{aligned}$$

$(35-1{\cdot}2)\times10{\cdot}54$ kcal heat input in 60 seconds

$\therefore$ 4820 kcal input will be in

$$\frac{60\times4820}{38{\cdot}8\times10{\cdot}54} = 708 \text{ sec or } \mathbf{11{\cdot}8 \text{ min}}. \quad \textbf{Ans.}$$

EXERCISES 4

Section A

4·1. Describe the various applications of compressed air.

4·2. Define fans, blowers, exhausters and boosters.

4·3. In what way the analysis of fans differ from that of blowers and compressors ?

4·4. What are the two main types of compressors ? What are the main differences between them ?

4·5. Define isothermal efficiency. Why the efficiency of reciprocating compressors is given in terms of isothermal efficiency ?

4·6. What is free air delivery ? Why capacity of a compressor is given in terms of free air delivery ?

4·7. Define overall volumetric efficiency in terms of (*a*) mass ratio, (*b*) volume ratio. Why volumetric efficiency is less than unity ? What is absolute volumetric efficiency ?

4·8. Derive the expressions for work done in a single-stage reciprocating compressor if the compression is (*a*) adiabatic, (*b*) polytropic, (*c*) isothermal.

4·9. Define mechanical efficiency of a compressor.

4·10. What is the effect of clearance volume in a reciprocating compressor on (*a*) work done per kg of cycle, (*b*) air delivered, (*c*) volumetric efficiency ?

4·11. What are the various methods for approximating to the isothermal compression in reciprocating compressors ?

4·12. Prove that in a two-stage reciprocating air compressor, if the ntercool ing is complete, the expression for total work required, W, is given by

$$W=p_1v_1\frac{n}{n-1}\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}+\left(\frac{p_3}{p_2}\right)^{\frac{n-1}{n}}-2$$

where p_1, p_2 and p_3 are pressures at inlet to first stage, inlet to second stage and outlet from second stage respectively, and v_1 is the volume of air entering low pressure cylinder.

Hence prove that in a two-stage compressor, for maximum efficiency the intermediate pressure p_2 is the geometric mean of the initial pressure p_1 and final pressure p_3.

4·13. What are the conditions for maximum efficiency of a multi-stage reciprocating compressor ?

4·14. Show that the temperature rise of the gas in either cylinder of an ideal two-stage compressor is a minimum when the intercooler pressure $p=(p_1p_2)^{0\cdot5}$, where p_1 and p_2 are low pressure cylinder suction and high pressure cylinder delivery pressures respectively. State clearly the assumptions which are made and explain why the low pressure cylinder ratio will in practice normally exceed that in the high pressure cylinder.

4·15. Prove that the volumetric efficiency of a single-stage reciprocating air compressor having pressure ratio p_2/p_1 and clearance ratio C is given by

$$\eta_{vol}=1-C\left(\frac{p_2}{p_1}\right)^{\frac{1}{n}}-1.$$

4·16. What are the advantages of multi-stage compression ?

4·17. Discuss the typical performance curves of reciprocating air compressors.

4·18. What are the applications of reciprocating compressors ?

4·19. Describe the working of a Root's blower by the help of a neat sketch and pressure volume diagram.

4·20. What are the applications of Root's blower ?

4·21. Describe the working of a Lysholm compressor with a sketch. Where Lysholm compressor is used ?

4·22. Describe, with a sketch, the working of a vane type compressor and show its pressure-volume diagram. For what applications it is used ?

4·23. Describe with a sketch the liquid piston rotary compressor. What are its typical applications ?

4·24. Compare the relative merits and demerits of reciprocating *vs* rotary positive displacement compressors.

Section B

4·25. **Vane Compressor**

A rotary vane compressor has a free air delivery of 10 m^3/min when it compresses air from 1 kgf/cm^2 and 30°C to 2 kgf/cm^2. Estimate the power required to drive the compressor when (*a*) the ports are so placed that there is no internal compression, (*b*) the ports are so placed that there is a 30 per cent reduction of volume before back-flow occurs. Assume adiabatic compression. What is the isentropic efficiency in each instance ?

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5

DYNAMIC COMPRESSORS

5·1 DYNAMIC OR STEADY FLOW COMPRESSORS

Dynamic or steady flow compressors utilize momentum changes in the flowing air to cause the desired compression. These are of three types—centrifugal, axial, and mixed flow. In the centrifugal compressor the air is imparted a high velocity and pressure rise is by a row of moving blades, *i.e.* an impeller. The increase in kinetic energy is converted into further pressure rise by a diffuser. The flow in an axial flow compressor, as the name implies, is along the axis of the compressor while in a centrifugal compressor the flow is essentially at right angles to the axis of the compressor. The mixed flow compressor combines the advantages of both centrifugal and axial flow compressors.

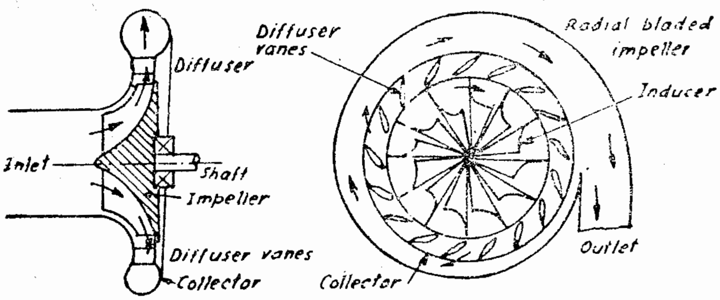


Fig. 5·1. Centrifugal compressor.

(*a*) **Centrifugal compressor**. Fig. 5·1 shows a centrifugal compressor. Air enters the eye of the impeller in an axial or radial direction and is turned through an angle of 90° in the impeller. The impeller rotating with a very high rotational speed of about 20000 to 30000 r.p.m. imparts the air a very high velocity and a small pressure rise during its radial flow in the impeller. The air then passes over a diffuser having fixed vanes which provides a gradually increasing area. The velocity of air is reduced during its passage over the diffuser and a substantial part of the kinetic energy is converted into static pressure. Single stage centrifugal compressors can develop a pressure ratio as high as 4 : 1 with capacities ranging from 15 to 300 m³/min. In multi-stage compressors pressure upto 15 kgf/cm² or more with capacities from 15 to 400 m³/min can be obtained. It can deliver practically constant pressure over a relatively wide range of capacities.

(*b*) **Axial flow compressor.** Fig. 5·2 shows an axial flow compressor. It consists of alternate rows of moving and fixed blades

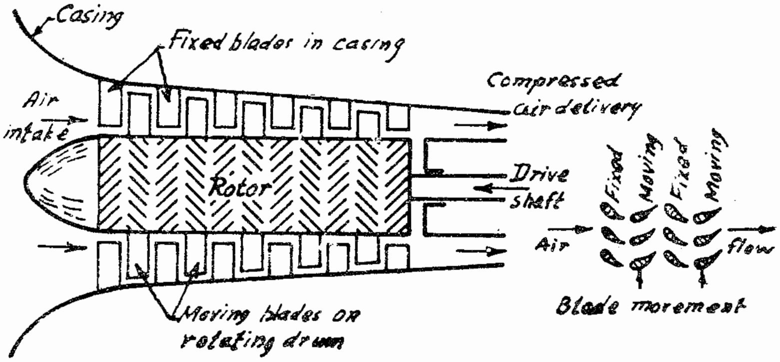


Fig. 5·2. Axial flow compressor.

—the moving blades being fixed on a central drum and the fixed blades on the outer casing. The air moves in the axial direction and a velocity is imparted to it by the moving blades, and the fixed blades convert this kinetic energy into static pressure and also guide the air to flow in the next row of moving blades. The pressure rise occurs in the moving as well as in the fixed blades. As the pressure increases in the direction of flow, the volume of air decreases and to keep the velocities same for each stage the blade height is usually decreased along the axis of the compressor (*see* Fig. 5·2).

A set of row of moving blades and fixed blades is called a stage. The pressure rise per stage is limited to about 1·2 in an axial flow compressor, so a large number of stages, usually 5 to 14, are required to give a sufficient increase in pressure. Single stage axial flow compressors can supply air in capacities from about 150 m³ to 3000 m³/min or more with pressure ratios of 1·05 : 1 to 1·2 : 1 and a multistage unit can supply upto 30000 m³/min. at pressures up to 10 kgf/cm². Axial flow compressors provide constant delivery at variable pressures and generally have a narrower stable operating range than centrifugal compressors.

(*c*) **Mixed flow compressors.** A mixed flow compressor has a hub of increasing diameters (*see* Fig. 5·3) so that the flow of air is turned through an angle less than 90° as compared to a 90° flow rotation in a centrifugal compressor having hub as a disc perpendicular to the axis of the compressor. The rotating blades give a radial and circumferential velocity and the air leaving the impeller has a three-dimensional velocity. The air is then delivered to a diffuser where pressure is increased. A mixed flow compressor is smaller than the centrifugal compressor but develops smaller

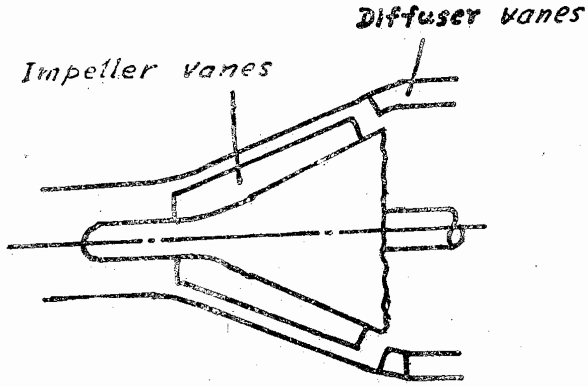


Fig. 5·3. Schematic diagram of a fixed-flow compressor.

pressure rise for the same tip speed as compared to a centrifugal compressor. The stable air flow range of mixed flow compressor is wider than the centrifugal compressor.

5·2. TOTAL HEAD OR STAGNATION PROPERTIES

Due to very high velocities which are imparted to the air in compressors it is necessary to consider total head properties of the air. The static temperature of fluid is the temperature which would be measured by a thermometer moving with the same velocity as the stream. The *total head* or *stagnation* temperature is that temperature which would be obtained if the flowing fluid is isentropically brought to rest.

By applying the energy equation to a flow through a varying area with no external heat transfer and no work, we have

$$u_1+\frac{P_1v_1}{J}+\frac{C_1^2}{2gJ}=u_2+\frac{P_2v_2}{J}+\frac{C_2^2}{2gJ}$$

or $$h_1+\frac{C_1^2}{2gJ}=h_2+\frac{C_2^2}{gJ}$$

or $$c_pT_1+\frac{C_1^2}{2gJ}=c_pT_2+\frac{C_2^2}{2gJ}$$

If T_t is the total head or stagnation temperature, then

$$c_pT_1+\frac{C_1^2}{2gJ}=c_pT_t, \quad \text{since} \quad C_2=0 \quad \text{at} \quad \text{total} \quad \text{head}$$

conditions.

$$\therefore \qquad T_t-T_1=C_1^2/2gJc_p$$

or
$$T_t-T=\frac{C^2}{2gJc_p} \tag{5·1}$$

Thus the total head or stagnation temperature considers the kinetic energy of the moving fluid. Similarly, total head or stagnation enthalpy, and total head or stagnation pressure can be defined and are given by

$$h_t-h=C^2/2gJ \tag{5·2}$$

and
$$\frac{P_t}{P}=\left(\frac{T_t}{T}\right)^{\frac{\gamma-1}{\gamma}} \tag{5·3}$$

The total head or stagnation enthalpy and total head or stagnation temperature remain constant in the absence of external heat and work transfer.

5·3. WORK DONE IN COMRESSION IN A DYNAMIC COMPRESSOR

Fig. 5·4 shows the compression cycle of a dynamic compressor on *P-v* and *T-s* diagrams. The actual compression process follows the law $PV^n=C$ and is given by the process 12. This is different from the ideal, *i.e.* isentropic compression given by the process 12′ because of friction between molecules of air, separation, eddy formation in the flow, and shock at the entry of air in the vanes. Due to these effects the work required to drive the compressor is more than in the ideal case. The extra work required is dissipated in the form of heat. In case of isentropic compression the work required is given by the area of the *P-v* diagram and on the *T-s* diagram by the area 2′ 3′ 4 5. The work done in isentropic compression is given by

$$\text{Work done per kg}=\frac{\gamma}{\gamma-1}RT_1\left[(P_{2t}/P_{1t})^{\frac{\gamma}{\gamma-1}}-1\right] \tag{5·4a}$$

$$=\frac{\gamma}{\gamma-1}R\left[T_2{'}_t-T_{1t}\right] \tag{5·4b}$$

The work done in a dynamic compressor can be found by applying steady flow energy equation at inlet and outlet of the compressor. Thus, we have

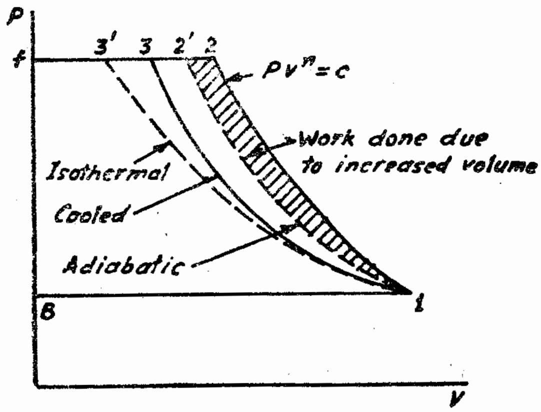
$$h_1+\frac{C_1^2}{2gJ}+w=h_2+\frac{C_2^2}{2gJ}$$

$$\therefore \qquad w=(h_2+C_2^2/2gJ)-(h_1+C_1^2/2gJ)$$

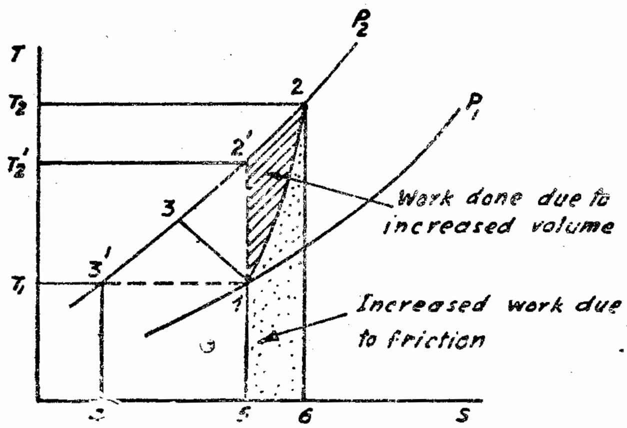
$$=h_{2t}-h_{1t} \tag{5·5 a}$$

$$=c_p(T_2{'}_t-T_{1t}) \tag{5 5 b}$$

The extra work done in actual practice is due to an increase in the volume of the air due to n being greater than γ and internal



(*a*) *P-v* diagram



(*b*) *T-s* diagram

Fig. 5·4. *P-v* and *T-s* diagrams for dynamic compressor.

frictional losses. The first one can be represented on P-v diagram (*see* Fig. 5·4), while the second cannot be represented. The internal frictional loss is represented in the T-s diagram by the area 1265. Therefore work done in an actual dynamic compressor cannot be found by the area of the P-v diagram. The area 2 3′ 4 6 represents the total actual work done in compression.

Actual work done $=c_p\times$ actual rise in temperature.

$$=c_p(T_{2t}-T_{1t}) \qquad (5\cdot6\ a)$$

$$\therefore \quad \text{Horse power} = \frac{mc_p(T_{2t}-T_{1t})J}{4500} \tag{4·6 b}$$

where m is the mass flow per minute.

5·4. EFFICIENCIES OF DYNAMIC COMPRESSORS

(a) Isentropic efficiency

The ideal compression process in dynamic compressor is isentropic, whereas in reciprocating compressors it is isothermal. It is because in reciprocating compressors due to slow speed and cylinder and interstage cooling the ideal of isothermal compression is approached. But dynamic compressors are generally uncooled and of very high speed which cause internal reheating the index of compression is always more than γ. The isentropic efficiency is defined as the ratio of isentropic work to actual work.

Isentropic efficiency (based on total head conditions)

$$= \frac{\text{Isentropic work of compression}}{\text{Actual work}}$$

$$= \frac{h_2{}'_t - h_{1t}}{h_{2t} - h_{1t}}$$

$$= \frac{c_p(T_2{}'_t - T_{1t})}{c_p(T_{2t} - T_{1t})} = \frac{(T_2{}'_t - T_{1t})}{(T_{2t} - T_{1t})} \tag{5·7 b}$$

Thus the isentropic efficiency is also equal to the ratio of the isentropic temperature rise to actual temperature rise. When the efficiency is defined on the basis of static temperature rise it is given by

Isentropic efficiency (based on static head conditions).

$$= \frac{T_2{}' - T_1}{T_2 - T_1} \tag{5·7 b}$$

For small velocities the total head isentropic efficiency and the static head isentropic efficiency are same.

(b) Polytropic or small stage efficiency

(*i*) In case of a multi-stage compressor the designer tries to obtain same efficiency for each stage. The small stage or polytropic efficiency is defined as the isentropic efficiency of an elemental (infinitesimal) stage of the compressor which remains constant throughout the whole process of compression. The fact that on a *T-s* diagram (*see* Fig. 5·5) the vertical distance increases with an increase in entropy clearly illustrates that in a multistage frictionless compression the isentropic temperature rise is more for an elemental stage at higher entropy than the temperature rise of another elemental stage at lower entropy and the sum of the isentropic temperature rises for all the elemental stages $\Sigma \triangle T_{st}'$ is greater than the single overall isentropic rise $\triangle T_t'$.

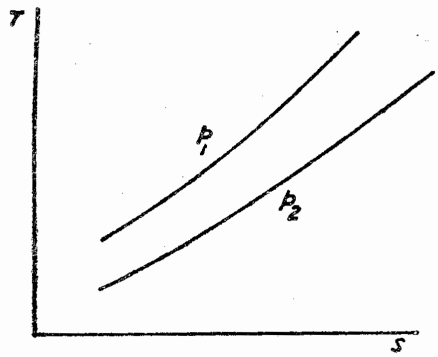


Fig. 5·5. *T-s* diagram showing that the vertical distance between two constant pressure lines increased with increase in entropy.

$$\text{Isentropic efficiency, } \eta_i = \frac{\text{Isentropic temperature rise}}{\text{Actual temperature rise}}$$

$$= \frac{T_2{}'_t - T_{1t}}{T_{2t} - T_{1t}} = \frac{\triangle T_t{}'}{\triangle T_t}$$

$$\text{Polytropic or small stage efficiency, } \eta_s = \frac{la' + ab' + bc' + cd'}{T_{2t} - T_{1t}}$$

$$\frac{\Sigma(\text{isentropic temp. rise of stages})}{\text{Actual temp. rise}} = \frac{\Sigma(\text{isentropic temp. rise of stage})}{\Sigma(\text{actual temp. rise of stages})}$$

$$= \frac{\Sigma \triangle T_{st}{}'}{\triangle T_t}$$

Since $\Sigma \triangle T_{st}{}' > \triangle T_t{}'$ the polytropic or small stage efficiency is greater than the isentropic overall efficiency. This gain in efficiency is due to preheating and will increase with increasing pressure ratios.

(*ii*) **Polytropic efficiency in terms of '*n*'**

Fig. 5·6 shows the compression process in an elemental stage. Process 1-2 is the actual compression process and the process 12′ represents the isentropic compression.

Polytropic or small stage efficiecy

$$\eta = \frac{T_2{}'_t - T_{1t}}{T_{2t} - T_{1t}} = \frac{(T_2{}'_t/T_{1t}) - 1}{(T_2{}'_t/T_{1t}) - 1}$$

$$= \frac{(P_{2t}/P_{1t})^{\frac{\gamma-1}{\gamma}} - 1}{(P_{2t}/P_{1t})^{\frac{n-1}{n}} - 1}$$

$$\left[\text{Since } \frac{T_2{}'_t}{T_{1t}} = \left(\frac{P_{2t}}{P_{1t}}\right)^{\frac{\gamma-1}{\gamma}} \text{ and } \frac{T_{2t}}{T_{1t}} = \left(\frac{P_{2t}}{P_{1t}}\right)^{\frac{n-1}{n}} \right]$$

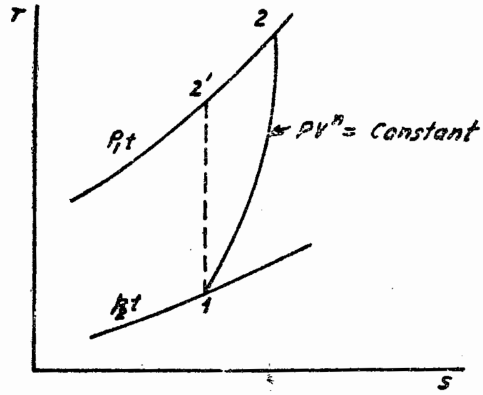


Fig. 5·6. Compression process in elemental stage.

$$=\frac{\left(\frac{P_{1t}+\triangle P_t}{P_{1t}}\right)^{\frac{\gamma-1}{\gamma}}-1}{\left(\frac{P_{1t}+\triangle P_t}{P_{1t}}\right)^{\frac{n-1}{n}}-1}$$

$$=\frac{\left(1+\frac{\triangle P_t}{P_{1t}}\right)^{\frac{\gamma-1}{\gamma}}-1}{\left(1+\frac{\triangle P_t}{P_{1t}}\right)^{\frac{n-1}{n}}-1}$$

Expanding binomially and neglecting the higher order terms, we get

$$\eta_s=\frac{\frac{\gamma-1}{\gamma}\times\frac{\triangle P_t}{P_{1t}}}{\frac{n-1}{n}\times\frac{\triangle P_t}{P_{1t}}}$$

$$=\frac{\gamma-1}{\gamma}\times\frac{n}{n-1} \qquad (5\cdot9)$$

Alternately, the polytropie efficiency can be defined as

$$=\frac{\text{Work done with polytropic compression}}{\text{Actual work done}}$$

$$=\frac{n}{n-1}\cdot\frac{R}{J}(T_{2t}-T_{1t}),\text{ for one kg of fluid}$$

$$=\frac{\frac{n}{n-1}\cdot\frac{\gamma-1}{\gamma}c_p(T_{2t}-T_{1t})}{c_p(T_{2t}-T_{1t})}$$

$$\eta_s=\frac{\gamma-1}{\gamma}\cdot\frac{n}{n-1}$$

[For expansion, as in turbine, the polytropic efficiency is defined as

$$\eta_s=\frac{\gamma}{\gamma-1}\cdot\frac{n-1}{n} \tag{5·10}$$

Since the value of n is generally unknown it is more convenient to express η_s in terms of known quantities, *i.e.* entry and delivery temperatures and pressures. We know

$$\frac{T_{2t}}{T_{1t}}=\left(\frac{P_{2t}}{P_{1t}}\right)^{\frac{n-1}{n}}$$

or
$$\ln\frac{T_{2t}}{T_{1t}}=\frac{n-1}{n}\ln\frac{P_{2t}}{P_{1t}}$$

$$\therefore\qquad \frac{n-1}{n}=\frac{\ln(P_{2t}/P_{1t})}{\ln(T_{2t}/T_{1t})}$$

Putting this value of $\frac{n}{n-1}$ in Eq. (5·9), we get the polytropic efficiency in terms of inlet and exit conditions.

$$\eta_s=\frac{\gamma-1}{\gamma}\,\frac{\ln(P_{2t}/P_{1t})}{\ln(T_{2t}/T_{1t})} \tag{5·11}$$

The polytropic efficiency defined in this way is very useful in the case of multistage compressors since it depends only on the actual temperatures and avoids the use of ideal adiabatic temperature rise.

If n is the same for all stages the overall efficiency equals that of each separate stage on the polytropic basis but not on isentropic definition. This point is further explained later.

(*iii*) **Relation between isentropic and polytropic efficiency.**

$$\eta_i=\frac{T_2{}'_t-T_{1t}}{T_{2t}-T_{1t}}$$

$$=\frac{(P_{2t}/P_{1t})^{\frac{\gamma-1}{\gamma}}-1}{(P_{2t}/P_{1t})^{\frac{n-1}{n}}-1}$$

By putting the value of $\frac{n-1}{n}$ from Eq. (5·9), we get

$$\eta_i = \frac{(P_{2t}/P_{1t})^{\frac{\gamma-1}{\gamma}} - 1}{(P_{2t}/P_{1t})^{\frac{1}{\eta_s}\frac{\gamma-1}{\gamma}} - 1} \tag{5·12}$$

The isentropic efficiency of a compressor is the ratio of the useful energy output over the total energy input. However, for a compressor useful energy is always measured in terms of pressure. Moreover, the isentropic efficiency varies with the pressure ratio used and will have different values for same pressure ratio but having different energy levels. Of two compressors having equally good internal design one which has higher pressure ratio will always have less isentropic efficiency than one having a lower pressure ratio merely because of the increasing divergence of the isentropic and polytropic processes with increasing pressure ratio. Similarly, the efficiency of a high pressure ratio turbine will be more than that of a low pressure turbine.

Thus isentropic efficiency cannot be a true measure of compressor performance. The concept of polytropic efficiency is used in the design of multistage compressors to eliminate effect of pressure ratio. If the polytropic efficiency of a stage designed by a particular method can be established it can be assumed that the similar stages will have the same polytropic efficiency and isentropic efficiency of the complete compressor can be found by using equation (5·12).

5·5. VELOCITY DIAGRAMS

The following is the nomenclature used for the analysis of centrifugal and axial flow compressors.

α_1 = exit angle from the stator or guide vanes at entrance

β_1 = inlet angle to the rotor

β_2 = outlet angle from the rotor

α_2 = inlet angle to the diffuser or the stator

U_1 = mean blade velocity at the entrance

U_2 = mean blade velocity at exit

C_1 = absolute velocity at inlet to the rotor

C_2 = absolute velocity at outlet to the rotor

V_1 = inlet relative velocity to the blade

V_2 = outlet relative velocity to the blade

C_{1w} = velocity of whirl at inlet (component of C_1 taken parallel to the plane of rotation)

C_{2w} = velocity of whirl at outlet (component of C_2 taken parallel to the plane of rotation)

C_{1a}=velocity of flow at inlet (component of C_1 taken perpendicular to the plane of rotation)

C_{2a}=velocity of flow at outlet (component of C_2 taken perpendicular to the plane of rotation)

In the case of centrifugal compressors the angles are generally measured with respect to blade direction while for axial flow compressors the angles are generally measured with respect to the direction of motion of the blade. To avoid confusion it is always desirable to mention in the problems how the angles have been measured.

In the analysis of the centrifugal and axial compressors the following assumptions are generally made :

(1) The flow phenomenon is steady.

(2) The flow is uniform throughout the cross-section of the impeller and the diffuser.

(3) There is no separation of flow.

(4) There is no shock either at the entrance or at the exit of blades.

(5) The flow through the impeller is frictionless.

(*a*) **Centrifugal compressor.**

Fig. 5·7 shows the impeller of a centrifugal compressor and the velocity diagrams at inlet and outlet. The velocity at inlet to the impeller is assumed to be axial and is equal to the velocity of flow, *i.e.*, $C_1=C_{1a}$ The velocity of whirl C_{1w} is zero at inlet to the impeller. The relative velocity of the air at inlet is V_1, which makes an angle β_1 with the direction of motion. The air flows through the impeller and in its passage is turned through an angle of 90° and ideally the outlet is in radial direction, *i.e.* the absolute velocity at outlet C_{2w} is such that its whirl component is equal to the blade tip velocity U_2. However, the large amount of the mass of air flowing through impeller has certain inertia and due to the formation of eddies the velocity of air at the tip is always less than the blade speed. This reduces the outlet whirl velocity of the air and the flow is turned through an angle less than 90° as compared to a 90° turn in ideal condition. This lagging behind of the air is taken into account by *slip factor*. Slip factor is defined as the ratio of outlet whirl velocity to the blade velocity at outlet.

$$\therefore \text{ Slip factor, } \sigma=\frac{C_{2w}}{U_2} \tag{5·13}$$

The value of the slip factor depends upon number of blades and affects the amount of power which can be supplied to the air. Its value is usually about 0·9.

Work done,

$$\begin{aligned} w_t &= \text{rate of change of momentum} \times \text{distance} \\ &= C_{2w}U_2 - C_{1w}U_1 \qquad [5{\cdot}14(a)] \\ &= \frac{C_{2w}U_2}{g} \qquad \because\ C_{1w}=0 \\ &= \frac{\sigma U_2 . U_2}{g} = \frac{\sigma U^2}{g} \qquad [5{\cdot}14\ (b)] \end{aligned}$$

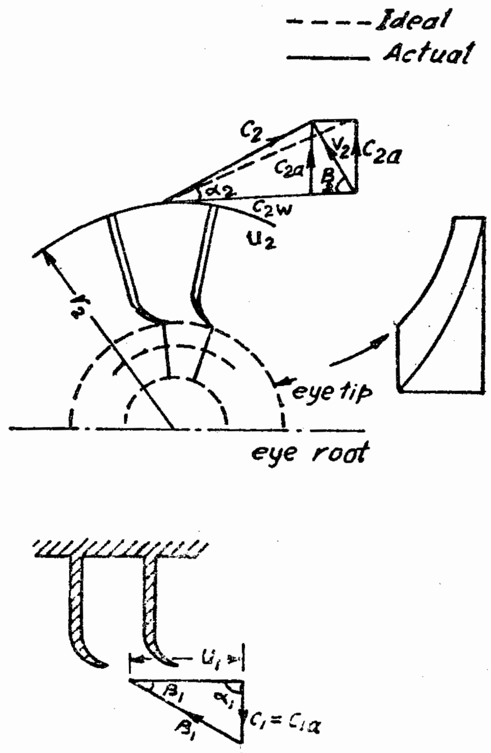


Fig. 5·7. Impeller of a centrifugal compressor and velocity diagrams at inlet and outlet. For inlet diagram actual and ideal are assumed to be same.

Eq. [(5·14 (*b*)] represents the power required by the compressor. However, the actual power required by the compressor is more than this due to the following reasons :

(*i*) The air has certain viscosity and the viscous shearing forces retard the flow near the wall and create a region of low kinetic energy than the main flow. If the energy is low the flow tends to stop and an eddying region is formed along the wall.

The formation of the eddies constitutes a loss. This is called fluid friction and windage loss.

(*ii*) In addition to the loss in eddies, the effective flow area is reduced due to the development of boundary layer near the wall and hence the velocity in the main flow increases giving non-uniform flow. This increase in velocity increases the frictional loss. More important is the fact that this increase in velocity cannot be recovered efficiently since the stagnation pressure varies across the cross-section of the flow and mixing losses occur.

(*iii*) Due to increased velocity, flow separation can take place and may also result in shock losses.

(*iv*) There is always a circulating flow within the impeller passages.

The extra work needed to drive the compressor over that given by Eq. (5·14) is taken into account by multiplying the above quantity by an experimentally determined factor called *work factor* or *power input factor*, *p*. The value of the power factor is about 1·04.

∴ Actual work required

$$w_a = \frac{p\sigma U^2}{g} \tag{5·15}$$

The energy transferred by the impeller manifests itself in rise in temperature, pressure, and velocity of the air. Usually the change in velocity is neglected and the change in temperature is given by

$$T_{2t} - T_{1t} = \frac{p\sigma\, U_2^2}{gJc_p} \qquad [(5·16\ (a)]$$

The isentropic temperature rise is given by

$$T_2'{}_t - T_{1t} = \frac{\eta_i p\sigma U_2^2}{gJc_p} \qquad [5·16\ (b)]$$

where η_i is the total head isentropic efficiency.

The pressure ratio is given by

$$\frac{P_{2t}}{P_{1t}} = \left(\frac{T_2'{}_t}{T_{1t}}\right)^{\frac{\gamma-1}{\gamma}} = \left(1 + \frac{T_{2t} - T_{1t}}{T_{1t}}\right)^{\frac{\gamma-1}{\gamma}}$$

$$= \left(1 + \frac{\eta_i p\sigma U_2^2}{gJc_pT_{1t}}\right) \tag{5·17}$$

The power input factor also takes into account the losses occuring in the diffuser. If the separation of flow is taking place near the exit of impeller it is very difficult to design a suitable diffuser configuration for efficient recovery of the kinetic energy.

Ordinarily the diffuser consists of an increasing area passage which reduces the velocity of the flow and increases the static pressure. Sometimes a guide vane diffuser is also used. Such conventional diffusers give a pressure ratio of about 3·5 : 1 and are not suitable for radial and mixed flow compressors of high performance and compact design. For such compressors the absolute velocity at exit is so high that exit Mach number is as high as 0·7 to 1·2 or more. With Mach number as high as 1·2 the diffusion ratio required is about 10 : 1 if the kinetic energy is reduced to negligible. Any attempt to do so by the conventional diffusers results in separation of flow and the dissipation losses due to mixing can no longer be balanced by extra pressure energy obtained from the diffusion process.

For this reason more than one stage of guide vanes are used in high performance centrifugal compressors. Use of more guide vane stages also reduces the mixing losses. Straight line diffusers to minimise secondary flow and resulting losses are used with swirl vanes.

Mixed flow compressors which have an exit Mach number of about 1·32 and centrifugal compressors with Mach number of about 1·24 are generally provided with a vaneless diffusing space to reduce the flow to subsonic conditions and then efficient diffusion is obtained by providing guide vanes. This also results in a large operating range to be obtained.

(*b*) **Axial flow comprssors.**

The blading arrangement and velocity diagrams at the inlet and outlet of an axial flow compressor are shown in Fig. 5·8. One stage of the compressor consists of one row of moving blade and one row of fixed blade. The air with an initial velocity C_1 enters the rotor blades and has a relative velocity equal to V_1 at an angle β_1 to the axial direction (note that reference direction is axial and not tangential as used for centrifugal compressors). The air then glides over the rotor blades and receives kinetic energy from them. A part of this kinetic energy is converted into pressure energy in the converging area between the rotor blades. Thus the absolute outlet velocity C_2 is greater than the absolute inlet velocity C_1. The air is then passed over to the stator blades where further pressure rise occurs, and in addition to this the flow is directed to enter the next stage of moving blades with an absolute velocity C_3 equal to C_1 and at an angle α_3 equal to α_1.

It is clear that the compression takes place in the moving as well as stator blades. The relative velocity is reduced from V_1 to V_2 at outlet of the rotor blades by deflecting the stream through a small angle thereby increasing the flow area and hence the pressure. Rest of the compression takes place in stator blades. The distribution of the total pressure rise into the two types of blades is indicated by a factor called *degree of reaction*. The degree of reaction is defined as the ratio of the rise in the enthalpy (hence pressure) of the air in the rotor to that of total enthalpy rise in the stage.

Degree of reaction

$$= \frac{\text{Enthalpy rise in rotor}}{\text{Total enthalpy rise in stage}}$$

$$= \frac{h_2 - h_1}{h_3 - h_1} \qquad (5\cdot18)$$

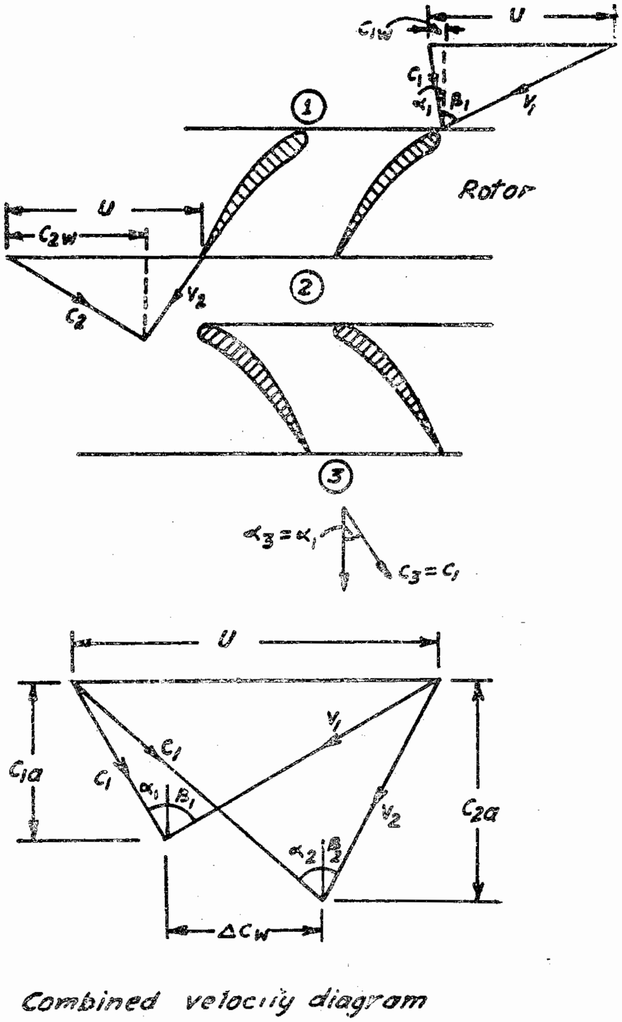


Fig. 5·8. Blading and velocity diagrams of an axial flow comppessor.

The choice of a particular degree of reaction is important in nat it affects the velocity triangles, the fluid friction and other losses. Usually a degree of reaction equal to 50 per cent is chosen. It is found that with equal inlet and outlet velocities, *i.e.* $C_1=C_2$ and constant axial velocity $C_{1a}=C_{2a}$, the 50 per cent degree of reaction gives least fluid friction and tip clearance losses, as is clear from Fig. 5·9 ; the velocity triangles become symmetrical under these conditions.

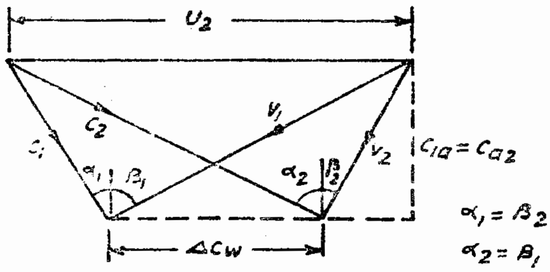


Fig. 5·9. Velocity diagram with symmetrical blading ($R=0.5$).

The axial flow compressor is designed to have a constant axial velocity. However, during first few stages of a multi-stage unit this

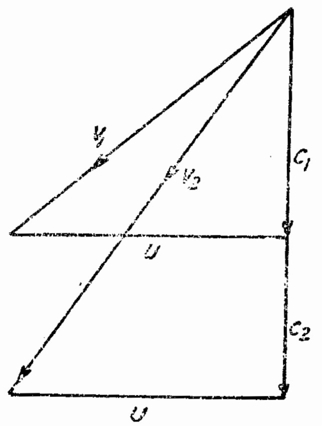


Fig. 5·10. Effect of axial velocity change on velocity diagram.

is not true and it is only after 4 or 5 stages that the axial velocity becomes constant. This affects the degree of reaction of the various stages and modifies the velocity triangles and the efficiency.

The work required per stage can be calculated from the velocity diagrams by the assumption of constant axial velocity. The work required per stage is given by

$$w=\frac{mU(C_{2w}-C_{1w})}{gJ} \qquad [5\cdot 19\ (a)]$$

$$=\frac{mU}{gJ}C_a(\tan\beta_1-\tan\beta_2)$$

$$=\frac{mU}{gJ}(U-C_a\tan\alpha_1-C_a\tan\beta_2)$$

$$=\frac{mU}{gJ}[U-C_a(\tan\alpha_1+\tan\beta_2)] \qquad [5\cdot19\ (b)]$$

Also, by steady flow energy equation the work required per stage is given by

$$w=mc_p(T_{2t}-T_{1t}) \qquad [5\cdot19\ (c)]$$

The actual work required by the compressor is not equal to that given by equation [5·19 (a)]. This expression was derived on the basis of mean values of the velocity triangles. However, this is not so and there are always deviations from the design value. If the axial velocity is above the mean value the whirl component is reduced (*see* Fig. 5·10), *i.e.* less work is transferred and if the axial velocity is less than mean value due to change in incidence angle of the blade (*see* Fig. 5·11) stalling occurs and again results in less work. So actual work is given by

$$w=\frac{mU(C_{2w}-C_{1w})\ K}{g} \qquad [5\cdot19\ (d)]$$

where K is *work done factor* and its value is about 0·86.

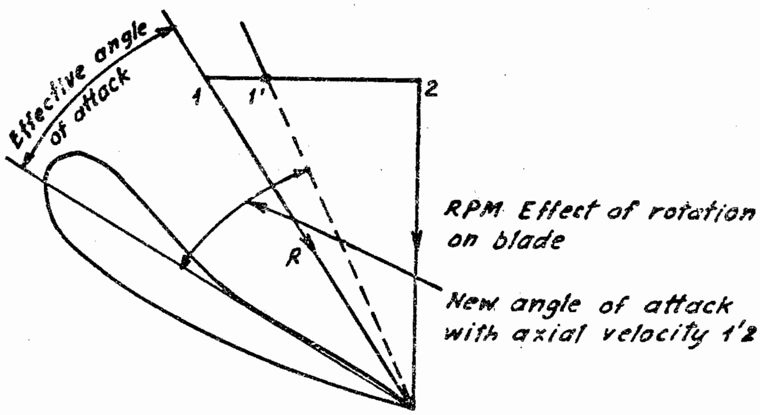


Fig. 5·11. Change in incidence angle due to change in axial velocity.

If the total head or isentropic efficiency of the stage is η_i we have,

$$\eta_i=\frac{T_2{}'_t-T_{1t}}{T_{2t}-T_{1t}}$$

or
$$\frac{T_2{}'_t}{T_{2t}}=1+\eta_i\ \frac{T_{2t}-T_{1t}}{T_{1t}}$$

Since $$\frac{P_{2t}}{P_{1t}}=\left(\frac{T_2'{}_t}{T_{1t}}\right)^{\frac{\gamma}{\gamma-1}}$$

$$\therefore \qquad \frac{P_{2t}}{P_{1t}}=\left[1+\eta_i\frac{T_{2t}-T_{1t}}{T_{1t}}\right]^{\frac{\gamma}{\gamma-1}}$$

or $$\frac{P_{2t}}{P_{1t}}=\left[1+\eta_i\frac{U(C_{2w}-C_{1w})}{gJc_pT_{1t}}\right]^{\frac{\gamma}{\gamma-1}} \tag{5·21}$$

The overall temperature ratio is given by

$$\frac{T_{2t}}{T_{1t}}=\left(\frac{P_{2t}}{P_{1t}}\right)^{\frac{\gamma-1}{\gamma\cdot\eta_i}} \tag{5·21}$$

5·6. DESIGN OF CENTRIFUGAL COMPRESSORS

Usually the pressure ratio required and the mass flow are specified to the designer. The ambient temperature and pressure range in which the machine is to operate are also known ; for example a jet engine has to operate at sea-level as well as at varying altitudes. The application to which it is going to be put decides the speed of rotation. If the compressor is coupled to an alternator or d.c. machine, its speed is exactly known. Even in other applications a rough idea of rotational speed is always available. In addition to this a rough idea of adiabatic efficiency of the compressor, the blade pressure coefficients, the inlet velocity at the impeller and the compressor outlet velocity is always available. The compressor outlet velocity is limited by the type of combustion chamber used in the gas turbine, and by the fact that if compressor is directly connected to the combustion chamber or through a heat exchanger. In the first case low compressor outlet velocities are required to avoid blowout at low loads and/or at high altitude, while in the second case a high velocity is preferred as it results in greater heat transfer. However, this also increases the pressure losses in the heat exchanger; thus a compromise is necessary.

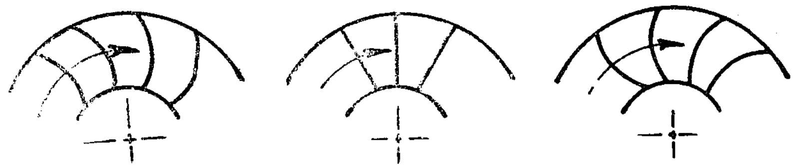
The impeller tip speed and outlet diameters are fixed from considerations of stress limits, allowable bearing loads, rotational speed, and space limitations, apart from the theoretical considerations.

Usually a new design is an improvement of the part designs, so dynamic similarily rules can always be applied to get approximate values of tip speed, rotational speed, various efficiencies, etc. However, in case of new designs and special type of machines such past experience is not available. Thus, an actual design is always based on fundamental theoretical analysis plus on past experience available in terms of empirical rules and formulae, and a good insight and experience is always a valuable asset for the designer.

In what follows is a brief discussion of design methods used to calculate the various dimension of a centrifugal compressor.

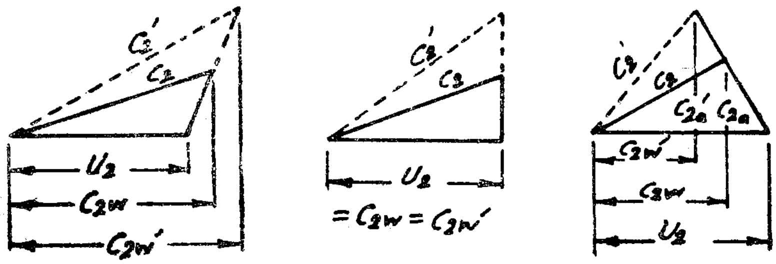
5·6·1. Types of impellers.

There are three basic types of impellers, namely, the forward curved vanes, radial vanes, and backward curved vanes. These are shown in Fig. 5·12. Fig. 5·13 shows the corresponding velocity triangles for same tip diameter and rotational speed.



(*a*) Forward curved vane (*b*) Radial vane (*c*) Backward curved vane

Fig. 5·12. The three types of impellers.



(*a*) Forward (*b*) Radia (*c*) Backward

Fig. 5·13. Exit velocity diagrams for different types of impellers. (Dotted lines show the effect of increased volume flow at constant speed).

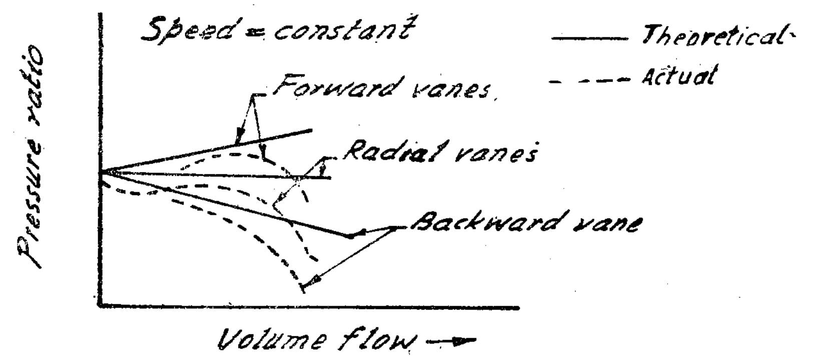


Fig. 5·14. Pressure *vs* volume flow at constant rotational speed·

In an impeller the energy is transferred in the form of kinetic energy which is converted to potential energy (pressure) in diffuser.

It can be seen that for a given tip speed the forward curved vane impeller transfers maximum energy and then comes the radial vane impeller while the backward curved vane impeller transfers least amount of energy. Thus the use of forward vanes results in a smaller diameter and lesser weight for the machine. However, it should be noted that ultimately this kinetic energy is converted into pressure in a diffuser and diffuser is the least efficient of all the parts of the compressor. So higher the impeller exit velocity more are the pressure losses. If efficiency is not to be seriously affected the diffuser length will have to be made quite large.

The pressure ratio obtainable from a given compressor depends upon the rotational speed, the impeller diameter, and the volume flow. Fig. 5·13 shows the effect of change in volume flow on the exit velocity diagrams of three types of impellers. Direction of velocity C_2 depends upon the tip speed and its magnitude upon volume flow. An increase in flow increases the energy transfer for a forward curved vane impeller and decreases for a backward curved vane impeller and remains same for radial vane impeller. This results in a rising pressure ratio *vs* volume flow characteristic for forward curved vane impeller, drooping characteristic for backward curved impeller, and constant pressure characterstic for radial vane impeller as shown in figure 5·14.

One important performance characteristic of a compressor is its stable operational range. The rising characterstic of forward curved vane impeller gives stable operation over a very narrow range. This is because first with increasing volume flow the pressure ratio continues to increase but after certain flow level the losses in the diffuser increase so rapidly that obtainable pressure ratio falls off rapidly as shown by dotted line in Fig. 5·14. Such a curve has a small stable operational range near its peak which is characterised by high losses so that efficiency in the stable range is poor. The characteristic for radial vane impeller remains constant and has reasonable efficiency.

Another important point is the ease of manufacture and stress levels. Usually impeller vanes are forged. Thus both the forward and backward curved vane impellers are difficult to manufacture and are subject to higher stress levels because of their curvature. They are subjected to bending stresses in addition to centrifugal stresses.

Thus, though the forward curved vane impeller gives high energy transfer, its operational range is very limited and that too with a lower overall efficiency. The increased difficulty in manufacture and high stress levels are further disadvantages. Considering that about two-third of power developed by gas turbine is used in driving the compressor, this poor efficiency results in a great loss of power.

Radial vane impellers are most popular because of ease of operation and good stable operational range with reasonably good efficiency. Almost all aircraft gas turbine engines and most other engines use this type of vanes.

Backward curved vaned impellers are used only in industrial turbines where usually lower pressure ratios are needed. This also gives good efficiency because the exit velocity (and hence loss in diffuser) is small.

5·6·2. Design of impeller.

There are various theories of impeller design. In one of the theories a suitable value of the pressure difference across the blade, called the blade loading, is assumed and inviscid flow equations are written and integrated to find the density and pressure distribution and various dimensions of the impeller are found by assuming a suitable variation of radial velocity. However, it is difficult to obtain a suitable value of blade loading without extenstive development work and the procedure for calculation is cumbersome and time consuming. Another theory considers the fact that the flow losses occur due to sudden change in flow path such as bends, etc. So the impeller passage must vary smoothly and must be kept straight as far as possible. This requires for a given mass flow and rotational speed, a hyperboloid surface as the inner wall of the impeller. The theoretical blade shape found under such assumption is rather difficult to manufacture. In the third theory, which will be followed in this book, the impeller is designed to have purely axial inlet and purely radial exit. The product of passage height and passage width gives the effective passage area between two impeller blades. This area is perpendicular to the flow direction. The variation of passage area from inlet to exit is determined by assuming that it forms a circular cone of very small cone angle.

Impeller shape

Fig. 5·15 shows the typical impeller shape along with its main dimensions. These are, D the tip diameter, D_0 the hub diameter, D_1 the impeller inlet diameter, U_2 the tip velocity, etc. The main object of this section is to give a method of determining these dimensions based on theoretical considerations and practical experience.

(*i*) *Hub diameter.* The hub-diameter D_0 is usually obtained by strength and vibration considerations. The size of impeller shaft and the thickness of the metal required determines the hub diameter which must invariably be checked from vibration point of view. The manufacturing method used for milling the blades, etc., of the impeller can also require some modification in the calculated value of hub diameter. The use of a hollow impeller shaft may need enlarged hub diameter. The axial width l of the hub is also determined from strength considerations.

(*ii*) *Impeller inlet diameter*: The annular or eye diameter, i.e. the impeller inlet diameter D_1 is determined from the consideration of pressure losses. The pressure losses are proportional to the relative velocity at inlet for a given mass flow and rotational speed. The eye diameter D_1 is calculated on the assumption of minimum

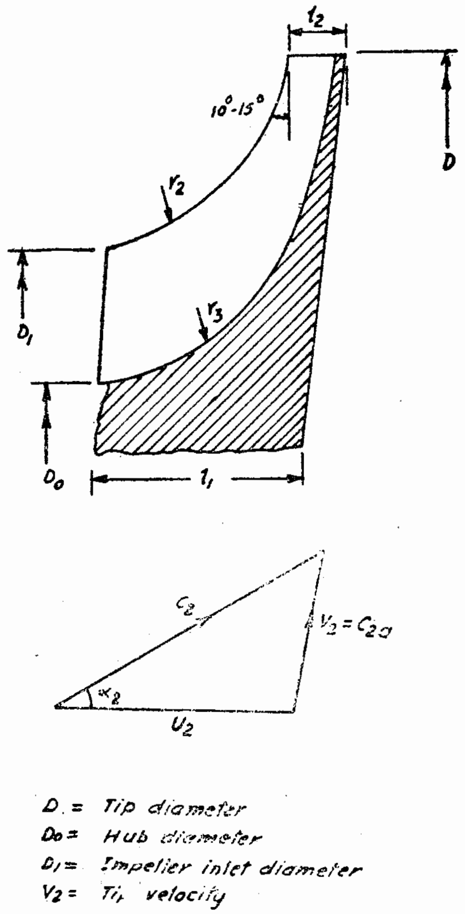


Fig. 5·15. Impeller shape and velocity diagram at exit.

Mach number at inlet. For such a calculation it is assumed that the absolute inlet velocity is purely axial and the tip velocity U_1 is assumed to be the mean value U_m. The mean value is determined on the assumption that the mean diameter D_m divides the total annular area into two equal part, (*see* Fig. 5·16), *i.e.*

$$D_m = \sqrt{\frac{D_1^2 + D_o^2}{2}} \tag{5-22}$$

thus

$$U_1 = U_m = \pi D_m N$$

$$= \pi N \sqrt{(D_o^2 + D_1^2)/2} \tag{5-23}$$

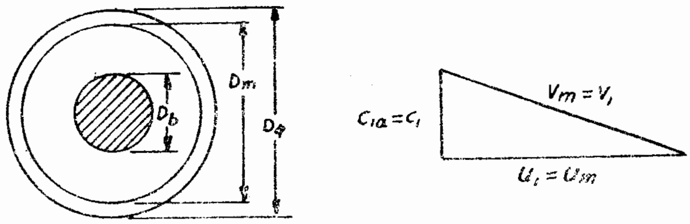


Fig. 5·16. Mean diagram.

The axial velocity is assumed to be normal and is given by

$$C_{1a}=\frac{Q_1}{\frac{\pi}{4}(D_1^2-D_0^2)}$$

where Q_1 is the volume flow.

The relative velocity at inlet is given by

$$V_1^2=U_m^2+C_{1a}^2$$

$$=\pi^2N^2\frac{(D_1^2+D_0^2)}{2}+\left[\frac{4Q_1}{\pi(D_1^2-D_0^2)}\right]^2 \qquad (5·24)$$

If a_1 is the sonic velocity at inlet, then Mach number at inlet is given by

$$M^2=V_1^2/a_1^2$$

$$=\frac{\pi^2N^2(D_1^2+D_0^2)}{2a_1^2}+\left[\frac{4Q_1}{\pi a_1(D_1^2-D_0^2)}\right]^2 \qquad (5·25)$$

Since the hub diameter is designed from strength and vibration considerations it will not be a variable in equation (5·25) and can be assumed constant. Then for a given volume flow Q_1 and rotational speed N, the condition for minimum Mach number at impeller inlet can be found by differentiating equation (5·25) with respect to D_1 and equating it to zero. This gives

$$-\left(\frac{4Q_1}{\pi a_1}\right)^2\times\frac{4(D_1^2-D_0^2)D_1}{(D_1^2-D_0^2)^4}+\frac{\pi^2N^2 2D_1}{2a_1^2}=0 \qquad (5·26)$$

or
$$\frac{64Q_1^2}{\pi^2}\times\frac{1}{(D_1^2-D_0^2)^3}=\pi^2N^2$$

or
$$(D_1^2-D_0^2)^3=\frac{64}{\pi^4}\left(\frac{Q_1}{N}\right)^2$$

or
$$D_1^2-D_0^2=\left(\frac{64}{\pi^4}\right)^{1/3}\left(\frac{Q_1}{N}\right)^{2/3}=0·87\left(\frac{Q_1}{N}\right)^{2/3}$$

or
$$D_1=\sqrt{0·87\left(\frac{Q_1}{N}\right)^{2/3}+D_0^2} \qquad (5·27)$$

Diameter D_1 is the diameter which corresponds to the minimum Mach number. However, usually a limiting Mach number of 0·8 or even 0·9 is generally used to determine the impeller diameter for a given volume flow and rotational speed. It should be assured by suitably increasing the diameter D_1 that the axial velocities are not very high otherwise losses would be too high. The ratio of eye diameter to tip diameter varies from 0·45 to 0·70. This is to ensure smooth variation of impeller passage from eye to the tip of the impeller.

(*iii*) *Impeller outlet diameter.* The impeller outlet diameter is usually determined by stress limitations, the rotational speed, and space limitations. Usually the outlet diameter is given along with the rotational speed. If not given, a suitable pressure ratio can be assumed. With values of power input factor based on past experience, the tip velocity U_2 can be found from equation (5·17). Once the tip velocity is found the outlet diameter can be found from the relation

$$U = \pi DN \tag{5·28}$$

Usually the speed N is fixed by the driven machine and so value of D is determined by

$$D = \frac{U}{\pi N}$$

However, where there is a choice of selecting the rotational speed, the stress, and stress limits, etc., both the diameter D and speed N to be usedshould be decided. The diameter thus obtained must be checked against the empirical rule that eye diameter to impeller outlet diameter ratio should be between 0·45 to 0·70. Too large a ratio will result in incomplete accelaration of the air while too small a value will not allow a smooth passage variation from eye to outlet diameter. Lower ratios must be used where efficiency is very important.

(*iv*) *Impeller outlet width.* Out of the total energy transferred by the rotor to the air about half appears as static pressure head and rest half as kinetic energy. So the pressure at impeller outlet p_2 is assumed to be equal to

$$p_2 = p_1 + \frac{p_2 - p_1}{2} \tag{5·29}$$

and the velocity head is neglected because for determining the volume flow at outlet only pressure and temperature are required.

Thus the value of static pressure p_2 at impeller outlet can be found. This must be checked against the empirical rule that this must be about one-fourth of the ideal pressure rise $(p_{2i} - p_1)$ where the ideal pressure rise is given by

$$\frac{p_{2i}}{p_1} = \left(1 + \frac{U_2^2}{gJc_pT_1}\right)^{\frac{\gamma}{\gamma-1}} \tag{5·30}$$

wo values differ a mean value can be taken.

Similarly, the temperature rise from eye to outlet may be assumed about half of the total temperature rise, and again neglecting the outlet velocity, we have

$$T_2 - T_1 = \frac{U_2^2}{2gJc_p} \tag{5·31}$$

or

$$\frac{T_2}{T_1} = 1 + \frac{U_2^2}{2gJc_pT_1}$$

Since

$$\frac{Q_2}{Q_1} = \left(\frac{T_2}{T_1}\right) \Big/ \left(\frac{p_2}{p_1}\right) \tag{5·32}$$

With the help of equations (5·29), (5·30) and (5·32) we can find the outlet volume flow Q_2. Once the outlet volume flow Q_2 is known the outlet velocity C_2, the outlet area A_2, and axial width of impeller at outlet l_2, can be found from the equation

$$C_2 = U_2 \tan \alpha_2 \tag{5·33}$$

where a suitable value of outlet blade angle α_2 is assumed. Usually with vaned diffusers it varies from 5° to 15° while for vaneless diffusers its value should vary from 12° to 18°. The lower values are used for smaller tip speeds. These values of outlet angle give a reasonable stable operational range to the compressor.

Then outlet area is given by

$$A_2 = \frac{Q_2}{C_2} \tag{5·34 a}$$

The area A_2 can also be given by

$$A_2 = (\pi DQ - nt)l_2 \tag{5·34 b}$$

where n is the number of blades in the impeller and t its thickness. The number of blades depend on the size of the impeller. About 18 to 22 blades are used in small compressors and 28 to 32 in larger ones. Usually an odd number is chosen in spite of the fact that an even number facilitates manufacturing. This is because an odd number of blades does not allow vibration impulses to grow, The number of blades, if high will cause deterioration of compressor efficiency as the reduced outlet area results in higher outlet velocities and hence increased pressure losses in the diffuser. Too small a number of blades will result in increased slip factor and more turbulence. The thickness of blades again depends upon the size of the impeller. The thickness does not greatly affect the performance but thinner blades with a taper of 4° to 6° included angle are preferred. Taper provides a stiff tip to withstand vibration of the impeller Thicker blades are used for large diameter impellers and they require a greater taper than that used for smaller diameter impellers.

The axial depth l_2 of impeller at outlet can be found from the relation.

$$l_2 = A_2/\pi D \tag{5·35 a}$$

or from
$$l_2 = \frac{A_2}{(\pi D - nt)} \tag{5·35 b}$$

if blade thickness is considered.

(v) *Entrance buckets.* Entrance buckets designates the first part of flow passage from eye diameter to outlet diameter as shown in Fig. 5·17. At this section the entrance velocity is assumed to be purely axial and the tip velocity corresponding to mean diameter D_m is used to calculate the passage height h_m.

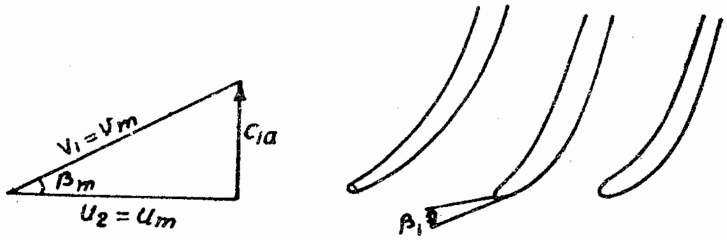


Fig. 5·17. Entrance bucket section and corresponding velocity diagrams.

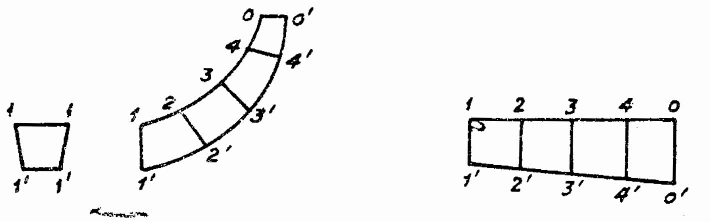
At the mean diameter D_m, the theoretical passage height is given by

$$h_m = \frac{\pi D_m \sin \beta_m}{n}$$

where n is the number of blades and $\tan \beta_m = \dfrac{C_{1a}}{U_m}$. The value of U_m can be found from equation (5·23). The value of the theoretical passage width must be increased by a factor of 1·3 for best results when angle β_m is less than 45°. For higher angles a greater factor is usually used to ensure a sufficient area corresponding to this angle.

Once the passage height is found the inlet Mach number should again be checked at the mean diameter D_m to ensure that sufficient passage area has been provided.

(vi) *Impeller passage.* The impeller passage from eye to tip is designed such that it provides an increasing area at a rate matching with the diffusion rate. The radius r_1 (*see* Fig. 5·18) is chosen such that the passage smoothly joins the eye and outlet areas, while the radius r_2 is fixed from the considerations of manufacturing process, strength and weight. One usual method of determining the passage is the use of an equivalent circular cone as shown in Figure 5·18. The cone angle must be very small for good efficiency.



(*a*) Impeller passage (*b*) Equivalent circular cone

Fig. 5·18. Impeller passage and the equivalent circular cone.

5·6·3. Centrifugal compressor diffuser

Normally the diffuser of a centrifugal compressor is an annular space with or without vanes in which the air is slowed down and thus the kinetic energy is converted into static pressure (*see* Figs. 5·19).

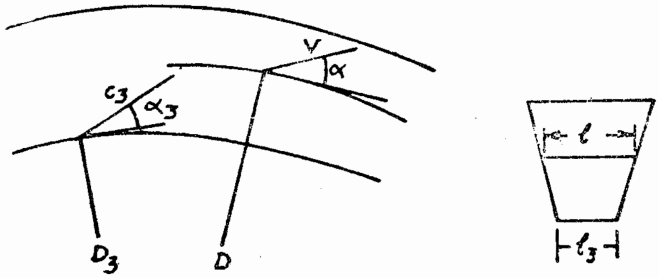


Fig. 5·19(*a*). Vaneless diffuser.

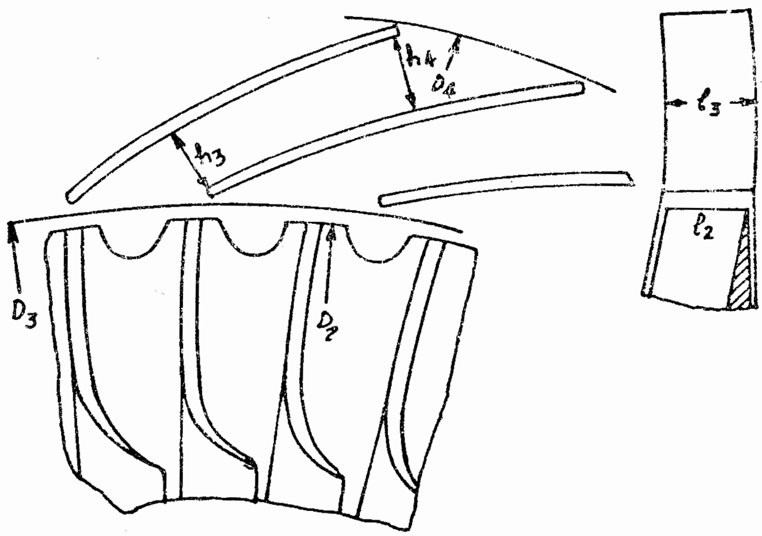


Fig. 5·19(*b*). Vaned diffuser.

When the air comes out of the impeller and enters the diffuser it behaves like a free vortex if there are no obstructions present. Such a vortex is given by

$$r^2\omega = \text{constant} \qquad [5\cdot37\ (a)]$$

or

$$r \times r\omega = r \times C \cos\alpha = \text{constant} \qquad [5\cdot37\ (b)]$$

$$\therefore \quad rC \cos\alpha = r_3 C_3 \cos\alpha_3 \qquad (5\cdot38)$$

Writing down the continuity equation in the radial direction, we have

$$\rho C \sin\alpha\, A = \rho_3 C_3 \sin\alpha_3 A_3$$

$$\rho C \sin\alpha . \pi D l = \rho_3 C_3 \sin\alpha_3 . \pi D_3 l_3$$

From equation (5·38), we get

$$l\rho \tan\alpha = l_3 \rho_3 \tan\alpha_3$$

or

$$\tan\alpha_3 = \frac{\rho l}{\rho_3 l_3} \qquad (5\cdot39)$$

Writing down the energy equation between the impeller inlet and outlet, we have

$$T_1 + \frac{C_1^2}{2gJc_p} + \frac{T_{t1}\left(R_t^{\frac{\gamma-1}{\gamma}} - 1\right)}{\sigma} = T_2 + \frac{}{2gJc_p} \qquad (5\cdot40)$$

and energy equation between impeller outlet and diffuser inlet and at some section of diffuser, we have

$$T_2 + \frac{C_2^2}{2gJc_p} = T_3 + \frac{C_3^2}{2gJc_p} = T + \frac{C^2}{2gJc_p} \qquad (5\cdot41)$$

From the above equations T can be found and put in

$$\rho/\rho_3 = (T/T_3)^{\frac{1}{n-1}} \qquad (5\cdot42)$$

where the value of n can be found from the equation

$$n = \frac{\eta_D \gamma}{1 - \gamma(1 - \eta_D)} \qquad (5\cdot43)$$

where η_D is the diffuser efficiency which varies from 82 to 92 per cent.

The diffuser inlet axial length l_3 is made equal to the total tip width l_2 plus some clearance on rear side of impeller so that the radial velocity as well as the angle α_3 can be found from.

$$\tan\alpha_3 = \frac{(\pi D - nt) l_2}{\pi D l_3} \cdot \frac{C_2}{m U_2} \qquad (5\cdot44)$$

where m is a constant whose value is less than unity. The exact value depends upon the number of blades.

Thus from equation (5·44), $\tan\alpha_3$ can be obtained, and from Eq. (5·42) ρ/ρ_3 can be obtained, or ρ/ρ_3 can be taken to vary between 1·08 to 1·18. When both these values are put in equation (5·39), the value of $l \tan \alpha$ can be found, *i.e.* the diffuser outlet angle and outlet axial width can be estimated based on past experience. The diffuser shape obtained is approximately a logrithmic spiral.

The diffuser inlet diameter is decided by the clearance between impeller outlet and diffuser inlet. The ratio of diameters is usually

$$\frac{D_3}{D_2}=1{\cdot}15 \text{ to } 1{\cdot}40 \tag{5·45}$$

the larger values being preferred where efficiency is more important.

The outer diameter D_4 is again decided by empirical rules. For a vaneless diffuser this should be

$$\frac{D_4}{D_2}=2{\cdot}2 \tag{5·46}$$

while for a diffuser with vanes it should be

$$\frac{D_4}{D_2}=1{\cdot}8 \text{ to } 2{\cdot}0 \tag{5·47}$$

For good efficiency a large value is used but in some applications such as in aircrafts where space is more important than efficiency, diameter ratio as small as 1·5 is used. This requires a large number of vanes for good diffusion, which are undesirable as they are conducive to vibrations.

The passage height h_3 is given by

$$A_3=nl_3h_3 \tag{5·48}$$

where the area A_3 is selected to be about 1·6 to 2·1 times the effective exit area of the impeller. The effective impeller outlet area is defined as the area required for the absolute velocity C_2, *i.e.* effective impeller outlet area is given by

$$A_2'=\pi Dl_2 \sin \alpha_2 \tag{5·49}$$

or

$$A_2'=(\pi D-nt)\, l_2 \sin \alpha_2 \tag{5·49 (a)}$$

when number of vanes and their thickness is considered. The ratio A_3/A_2 allows sufficient area for air flow in the diffuser. This ratio is intimately connected with the stable operation of the compressor. A very high ratio of about 2·3 or so may result in a very narrow stable operational range. As regards the number of vanes, at least six of them are required for good efficiency and the vane thickness should normally not exceed 3 to 4 mm. The ratio of diffuser vane outlet area to inlet area should be

$$\frac{l_4.h_4}{l_3.h_3}=1{\cdot}5 \text{ to } 2{\cdot}0 \tag{5·50}$$

Usually a value of 1·6 is preferred. In deciding the outlet area of the diffuser thought should be given to the area of the passage into which it is to discharge and the total diffusion required. For example, some applications, such as a combustion chamber, require that exit velocity should not be high lest blow out occurs while heat exchanger requires a higher outlet velocity for efficient heat transfer. The rate of expansion from diffuser inlet to diffuser outlet is made such that the passage height increases by a unit length for every 7 to 14 units of passage length.

DESIGN OF AXIAL FLOW COMPRESSORS

5·7. DEGREE OF REACTION

Since the degree of reaction, which has already been defined as the ratio of the increase in enthaply in the rotor to that of the total increase in enthalpy in the stage, has important effect upon the velocity diagrams and the efficiency of the compressor it is worth while to derive an expression for it. Usually the inlet and exit velocity of the compressor is neglected in such a derivation.

The total increase in the enthalpy in a stage is equal to the work done in the stage and is given by

$$h=w=c_p\triangle T=\frac{UC_a}{gJ}(\tan\beta_1-\tan\beta_2) \tag{5·51}$$

This work is utilized in increasing the enthalpy and the kinetic energy of the rotor such that

Increase in enthalpy in the rotor

=Workdone in the stage − Change in kinetic energy of the rotor

$$=w-\frac{1}{2gJ}(V_1^2-V_2^2) \tag{5·52}$$

But $V_1^2=C_a^2+(C_a\tan\beta_1)^2$

and $V_2^2=C_a^2+(C_a\tan\beta_2)^2$

By putting these values and Eq. (5·52) in Eq. (5·51), we get

Increase in enthalpy in the rotor

$$=\frac{UC_a}{gJ}(\tan\beta_1-\tan\beta_2)-\frac{C_a^2}{2gJ}(\tan^2\beta_1-\tan^2\beta_2)$$

∴ Degree of reaction

$$=\frac{\dfrac{UC_a}{gJ}(\tan\beta_1-\tan\beta_2)-\dfrac{C_a^2}{2gJ}(\tan^2\beta_1-\tan^2\beta_2)}{UC_a(\tan\beta_1-\tan\beta_2)/gJ}$$

$$=1-\tfrac{1}{2}\frac{C_a}{U}(\tan\beta_1+\tan\beta_2) \tag{5·53}$$

Also, from the velocity diagrams, we have

$$\frac{U}{C_a}=\tan\alpha_1+\tan\beta_1 \tag{5·54}$$

and $$\frac{U}{C_a}=\tan\beta_2+\tan\alpha_2 \tag{5·55}$$

or $$\frac{2U}{C_a}=(\tan\alpha_1+\tan\alpha_2)+(\tan\beta_1+\tan\beta_2)$$

or $$\tan\beta_1+\tan\beta_2=\frac{2U}{C_a}-(\tan\alpha_1+\tan\alpha_2)$$

Putting this into Eq. (5·53), we get

Degree of reaction

$$R=1-\tfrac{1}{2}\frac{C_a}{U}\left\{\frac{2U}{C_a}-(\tan\alpha_1+\tan\alpha_2)\right\}$$

or $$R=\tfrac{1}{2}\;\frac{C_a}{U}(\tan\alpha_1+\tan\alpha_2) \tag{5·56}$$

Thus the degree of reaction can be controlled by the air angles and by the axial velocity C_a of the air which is kept constant.

For a 50 per cent reaction it follows from Eq. (5·56), that

$$\tan\alpha_1+\tan\alpha_2=U/C_a \tag{5·57}$$

Comparing it with equations (5·54) and (5·55), we get

$$\alpha_1=\beta_2 \qquad \text{and} \qquad \alpha_2=\beta_1 \tag{5·58}$$

i.e., the velocity diagram is symmetrical. With other values of R, the velocity diagram is unsymmetrical and the difference $(C_2^2-C_1^2)$ is less than $(V_1^2-V_2^2)$, *i.e.* the pressure rise in the fixed

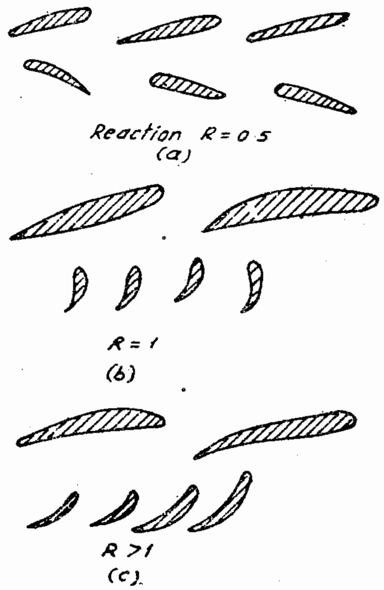


Fig. 5·20. Blading for different degree of reaction.

blades is a small percentage of the total pressure rise. The non-symmetrical type of blading is unsuitable for very high rotational speeds due to Mach number limitations. The stages required for a given pressure ratio is more in case of non-symmetrical blading than with symmetrical blading. Fig. 5·20 shows the blading for different degrees of reaction.

5·8. VORTEX THEORY

In the previous section we have discussed symmetrical and non-symmetrical type of blading arrangements. Another type of blading called free vortex blading is also used which is designed to produce axially directed velocity leaving the rotating blades. In the earlier blade arrangements no consideration was given to the fact that velocity triangles change with the radius and these were calculated at a particular radius only. The change in peripheral velocity with radius will modify the velocity triangles. The vortex design is an attempt to take into account the effect of change in radius of the blades.

Whenever the fluid has an angular velocity as well as a velocity in the direction parallel to the axis of rotation it is said to have 'vorticity'. The flow through an axial flow compressor is vortex flow in nature. The rotating fluid is subjected to a centrifugal force and to balance this force a radial pressure gradient is necessary.

Consider a small element of fluid at radius r with pressures and velocities as shown in Fig. 5·21. The centrifugal force acting

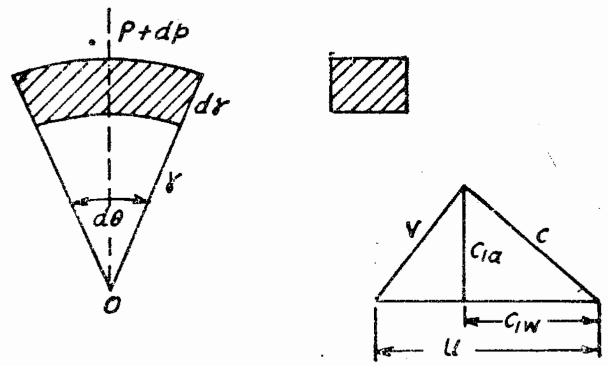


Fig. 5·21. Small element at radius r and condition for radial equilibrium.

on this element is $\rho r\, dr\, d\theta.\ C_w^2/gr$. By resolving the forces along the radial direction, the condition for radial equilibrium is obtained as follows :

$$(P+dP)(r+dr)d\theta-P.r.d\theta-P.dr.2\ \frac{d\theta}{2}=\rho\ dr\ r\ d\theta\ \frac{C_w^2}{gr}$$

or
$$\frac{dp}{dr}=\frac{\rho C_w^2}{gr} \tag{5·59}$$

The total enthalpy h at any radius r for the flowing fluid, which is assumed to be perfect, is given by

$$h=c_pT+C^2/2gJ$$

Since
$$C^2=C_a^2+C_w^2$$

and
$$c_pT=\frac{\gamma}{\gamma-1}\cdot\frac{R}{J}\ .T=\frac{\gamma}{\gamma-1}\ \frac{P}{\rho J}$$

We have

$$h=\frac{\gamma}{\gamma-1}\ \frac{P}{\rho J}+\frac{C_a^2}{2gJ}+\frac{C_w^2}{2gJ}$$

Differentiating the above equation with respect to r, we get

$$\frac{dh}{dr}=\frac{C_a dC_a}{drgJ}+\frac{C_w dC_w}{drgJ}+\frac{\gamma}{\gamma-1}\ \frac{1}{gJ}\left[\frac{1}{\rho}\cdot\frac{dP}{dr}-\frac{P}{\rho^2}\ \frac{d\rho}{dr}\right] \tag{5·60}$$

Assuming that for a small change in pressure dP across the annulus wall the flow is isentropic, *i.e.* it obeys the law $P/\rho^\gamma=$constant, or in differential form

$$\frac{P}{dr}=\frac{\rho}{rP}\cdot\frac{dP}{dr}$$

From Eq. (5·59) and (5·60), we get

$$gJ\frac{dh}{dr}=C_a\frac{dC_a}{dr}+C_w\ \frac{dC_w}{dr}+\frac{g\gamma}{\gamma-1}\left[\frac{1}{\rho}\ \frac{dP}{dr}-\frac{1}{\rho\gamma}\ \frac{dP}{dr}\right]$$
$$=C_a\frac{dC_a}{dr}+C_w\ \frac{dC_w}{dr}+\frac{g\gamma}{\gamma-1}\ \frac{1}{\rho}\cdot\ \frac{\gamma-1}{\gamma}\cdot\frac{dP}{dr}$$

By putting the value of dP/dr, we get

$$gJ\frac{dh}{dr}=C_a\frac{dC_a}{dr}+C_w\ \frac{dC_w}{dr}+\frac{C_w^2}{r} \tag{5·61}$$

This equation is the basic equation for all vortex flow. In the design of axial flow compressors it is assumed that the flow is free vortex flow in which the whirl velocity varies inversely with radius.

We assume that work input at all radii is equal to the total head temperature and hence the enthalpy will remain constant for all radii. The axial velocity is also assumed to be constant, *i.e.*

$$dh/dr=0$$

and
$$dC_a/dr=0$$

By putting these conditions in equation (5·61), we get

$$dC_w/C_w = -C_w/r$$

which on integration gives

$$C_w \times r = \text{constant}$$

i.e. the whirl velocity varies inversely as the radius, or in other words the condition of free vortex blading satisfies the radial equilibrium requirements.

The air angles needed can be calculated. In Fig. 5·22 are shown the blade angles as they vary from root to tip for a 50 per

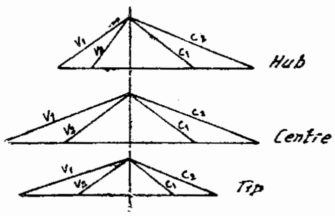


Fig. 5·22. Free vortex velocity diagram at three section.

cent reaction blading and a free vortex blading. It can be seen that free vortex blading has a considerable twist. So the Mach number will also change considerably during the passage of flow through the stage, and if the inlet Mach number is high shocks will occur. For this reason the blade and axial speeds are limited, resulting in bigger dimensions. Also, since the vortex blade is designed to produce an axially directed velocity leaving the row. for good efficiency, the air is actually accelerated in the fixed blades and the pressure reduces instead of increasing in the fixed blades. This gives less overall pressure rise per stage and the number of stages required is more than that for other types of blading.

The avoidance of vorticity was earlier thought to be essential for attaining high efficiencies. However, the marked twists in the rotor blades of a free-vortex design and the large frontal areas are the main disadvantages of this type of design. Moreover, with this design the best efficiency condition, *i.e.* 50 per cent reaction, can only be realized at one radius and not over the entire span of blades. So now it is thought that free vortex design is not essential for good efficiency and stable flow ; and the constant reaction blading is used to maintain constant degree of reaction from root to tip of the blade for best efficiency and the condition of constant axial velocity is relaxed.

5·9. AEROFOIL THEORY

In order to achieve high pressure ratio with compressors with high efficiency it is necessary to minimise, as completely as possible, the losses occurring in the flow. Minimization of the losses require a detailed knowledge of the flow process and for that the aerodynamic design of the flow passage is necessary. One important aspect of the aerodynamic design is the study of blade profiles and the effect of the presence of other blades on the flow phenomena, *i.e.* cascading of the blades in a compressor.

Aerofoil geometry

Fig. 5·23 shows the geometry of an aerofoil. The *mean camber line* is the line representing the locus of all points midway

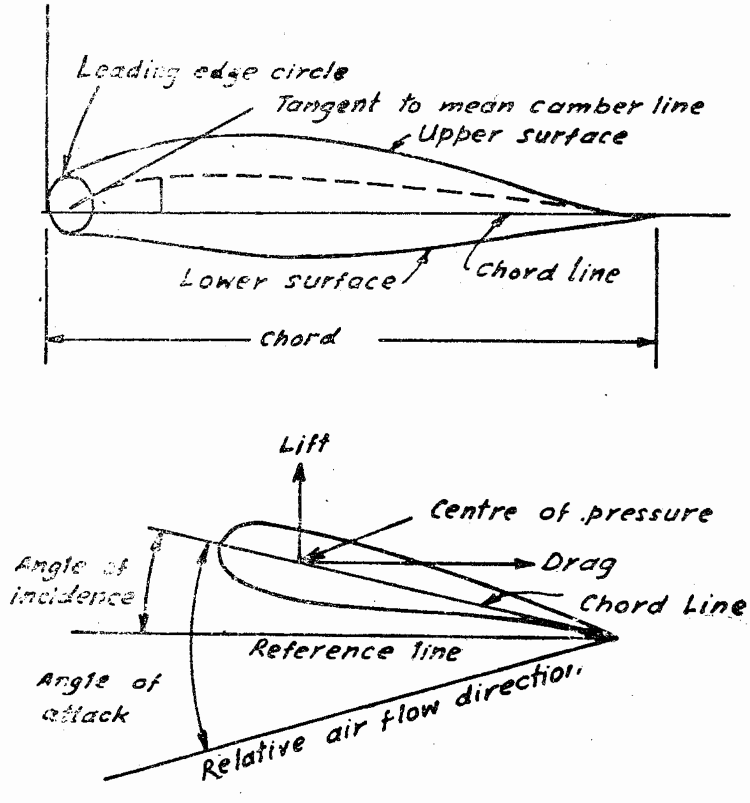


Fig. 5·23. Geometry of an aerofoil.

between the upper and lower surfaces of an aerofoil as measured perpendicular to the mean line. *Chord line* is the line joining the ends of the mean camber line. *Camber* is the maximum rise of the mean line from the chord line. *Leading edge radius* is the radius of a circle tangent to the upper and lower surfaces, with its centre located on the tangent to the mean camber line through the leading edge of this line. The *angle of attack* is the angle between the relative air flow direction and the reference line. The aerofoil is assumed to be stationary and the relative air flow direction is equal and opposite to the velocity of the aerofoil. *Angle of incidence* is the angle between the chord line and the reference measurement whereas the angle of attack decides the desposition of the chord line of the aerofoil with respect to the air flow. Another important aerofoil dimension is the *aspect ratio, i.e.* the ratio of blade height to blade chord.

Lift and Drag coefficient of an aerofoil

There are two types of forces acting on an aerofoil moving through the fluid. First are the shearing forces due to fluid friction of the air and the surface of the aerofoil and the second are the pressure forces acting on the aerofoil. Since static pressure cannot produce a resultant force on a moving aerofoil, the only reason for the development of the pressure forces is the dynamic pressure given by $\frac{1}{2}\rho V^2$. The maximum force which may be developed is given by

$$F=\tfrac{1}{2}\rho V^2 \times A \tag{5·63}$$

where A is the area of flow over which dynamic pressure is acting. However, the actual force developed by the aerofoil is much greater than that given by Eq. (5·63). Let us consider a vane as shown in Fig. 5·25 deflecting a flow. The force developed on the vane is given by :—

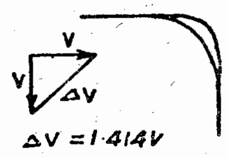


Fig. 5·24. Deflection of flow by an aerofoil.

$$F = \text{rate of change of momentum}$$

$$= m\frac{dV}{dt} = \rho AV \times \triangle V$$

$$= 1·414\ \rho\ AV^2$$

which is 2·828 times greater than given by simple energy equation.

The aerofoil utilizes this aerodynamic mechanical advantage obtained by deflecting a flow and is capable of producing forces greater than those given by Eq. (5·63). The forces developed by the aerofoil, *i.e.* the ability of the aerofoil to deflect a stream of air depends on its angle of attack, the curvature of the aerofoil (*i.e.* camber), the thickness of the aerofoil, and the velocity of air.

The resultant force acting on the aerofoil can be resolved into two forces-one drag force D in the direction of motion representing the frictional forces tending to retard the motion, and the other lift force L perpendicular to direction of motion. Lift is the basic force causing the aeroplane to maintain its flight.

The drag force is a resultant of the pressure distribution over the aerofoil and the skin friction and represents a loss, and the ratio of lift to drag L/D is considered as an indication of the efficiency of the aerofoil.

The lift and drag force can be given as

$$D=C_D \frac{1}{2g} \rho V^2 A \tag{5·65}$$

and

$$L=C_L \frac{1}{2g} \rho V^2 A \tag{5·66}$$

where C_D and C_L are the drag and lift coefficients respectively and A is the projected area of the aerofoil. These coefficients depend upon the shape of the profile.

The variation of lift and drag coefficients with angle of incidence is shown in Fig. 5·25. If the angle of incidence is too much the flow breaks away from the profile.

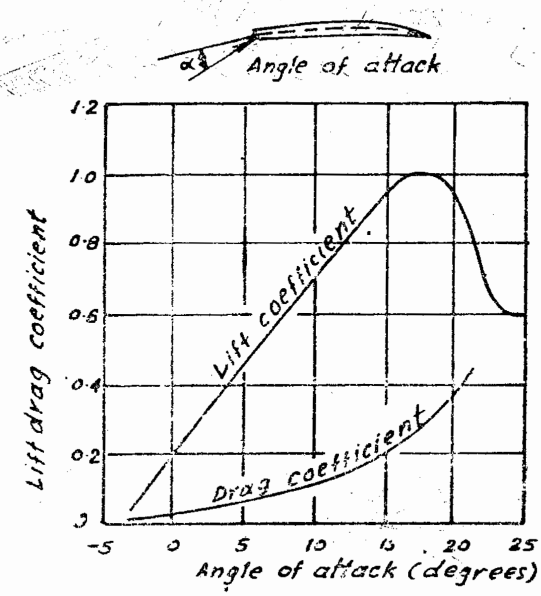


Fig. 5·25. Variation of drag and lift coefficients with angle of attack.

5·10. CASCADE THEORY

The work done on a rotating blade can be expressed in terms of lift and drag forces and hence the lift and drag forces can be expressed in terms of blade angles at inlet and exit.

Let us consider a blade moving with a velocity U, the air entering with a relative velocity V_1 at an angle β_1 with the axial direction and leaving the blade with a relative velocity V_2 at an angle β_2. V_1 and V_2 have a mean velocity V_m at an angle β_m. (*See* Fig, 5·26).

The resultant, of drag and lift forces can be resolved into two components L_w' and L_a', in rotational and axial directions respectively. The rotational component L_w' represents the force required to overcome the resistance of motion and L_a' represents the pressure force developed. The useful lift per unit length of blade in the direction of rotation is given by

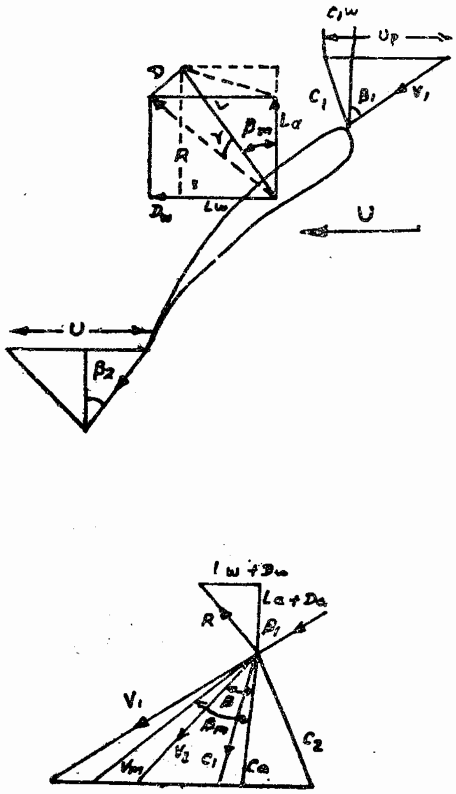


Fig. 5·26. Blade inlet and outlet velocity diagrams.

$$L_w'=L_w+D_w=L\cos\beta_m+D\sin\beta_m$$

$$=C_L\frac{1}{2g}\rho V_m^2 C\cos\beta_m+C_D\frac{1}{2g}\rho V_m^2 C\sin\beta_m$$

$$=\frac{1}{2g}\rho V_m^2 C\cos\beta_m(C_L+C_D\tan\beta_m) \tag{5·67}$$

where area $A=C\times 1$, C being the chord.

Since $\quad C_a=V_m\cos\beta_m$

$$L_w'=\frac{\rho C_a^2}{2g}C\left(\frac{C_L+C_D\tan\beta_m}{\cos\beta_m}\right)$$

Work done $\quad =L_w'\times U$

$$=\frac{\rho C_a^2 U_C}{2g\cos\beta_m}(C_L+C_D\tan\beta_m) \tag{5·68}$$

From Eq. (5·68) the total work done per unit mass flow is

$$W=\frac{U C_a}{\rho}(\tan\beta_1-\tan\beta_2) \tag{5·69}$$

$$\therefore \quad W'=mW=\rho C_a sW$$

$$=\rho\frac{C_a^2}{g}U s(\tan\beta_1-\tan\beta_2) \tag{5·70}$$

where s is the distance between two blades.

Equating Eqs (5·68) and (5·70), we get

$$\frac{\rho C_a^2 U_C}{2g\cos\beta_m}(C_L+C_D\tan\beta_m)=\frac{\rho C_a^2 Us}{g}(\tan\beta_1-\tan\beta_2)$$

or

$$C_L=2\frac{s}{C}(\tan\beta_1-\tan\beta_2)\cos\beta_m-C_D\tan\beta_m \tag{5·71}$$

Neglecting C_D in Eq. (5·71), which is very small as compared to C_L, we have

$$C_L=2\frac{s}{C}(\tan\beta_1-\tan\beta_2)\cos\beta_m \tag{5·72}$$

If dP is the loss of pressure in a cascade, this must be equal to the drag force in the direction of mean velocity. Writing these forces for unit blade length and equating, we get

$$\frac{1}{2g}\rho V_m^2 C_D C=s\cos\beta_m.d_p \tag{5·73}$$

Putting this into equation (5·72), we get

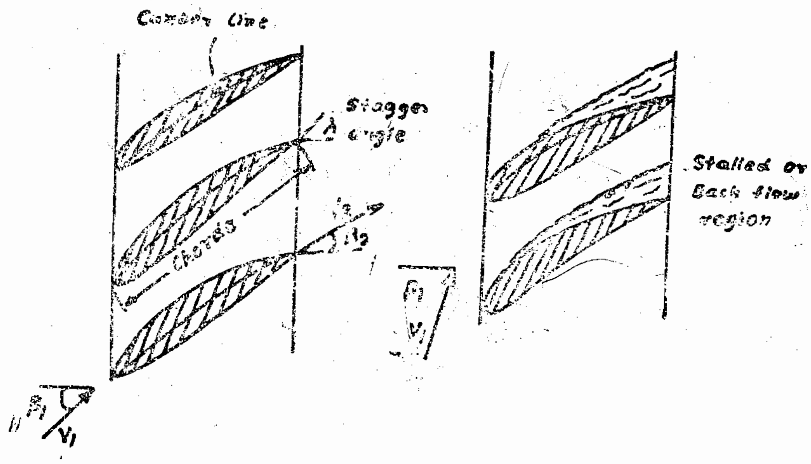
$$C_D=\frac{s}{C}\cdot\frac{d_p}{\frac{1}{2g}\rho V_1^2}\cdot\frac{\cos^3\beta_m}{\cos\beta_1} \tag{5·74}$$

Fig. 5·25 shows the lift and drag coefficients of a fixed geometrical form for different angles of incidence. Such data can easily be obtained from wind tunnel tests and a wide range of geometrical forms of the cascade can be investigated. From these coefficients the blade angles and hence the efficiency of the aerofoil can be found. The efficiency of a cascade is defined as the ratio of actual to theoretical pressure rise in the cascade. Thus the aerodynamic investigations can give us a great insight into the design of the blades conducive to high efficiencies at higher pressure ratios.

5·11. SURGING IN COMPRESSORS AND ITS CONTROL

All dynamic compressors have certain characteristic instability of flow over their operational range. This instability is associated with separation of flow from the blade surface and formation of eddies and can be seen to be of two types—*stall* and *surge*. Both these types of instabilities are associated with the positive slope of the pressure mass flow curve of the compressor, *i.e.* when the pressure rise increases with an increase in mass flow. Stall, in general, is characterised by reverse flow near the blade tip which upsets the velocity distribution and hence adversely affects the performance of the succeeding stages. Surge is a violent overall instability of flow in which flow fluctuations of large amplitudes occur throughout the compressor which can result in destructively severe vibrations and noise ; in comparison stall results in relatively mild local flow fluctuations. Both surge and stall impair the performance and limit the range of operation of the compressors.

The mechanism of the instability of flow can be understood as follows : Consider a typical *cascade* shown in Fig. 5·27. It is well known that a cascade blade at certain angles of attack with the moving air develops a lift force. As the angle of attack increases this lift force increases. However, further increase in the angle of attack, more than a critical value, results in a drastic decrease in the lift coefficient along with a rapid increase in drag coefficient. (*see* Fig. 5·25). This is because an increase in angle β_1 in Fig. 5·27 does not increase β_2 much but raises the pressure coefficient. When this increase is critical a separation of flow occurs as the air finds it difficult to follow the convex surface of the blade. With still more increase in β_1, the angle β_1 changes drastically and flow losses due to separation are high, thereby, reducing the lift coeffieient and increasing the drag coefficient. With reference to Fig. 5·25, it can be seen that this will reduce the component (L_w+D_w) which is responsible for pressure rise in the cascade. This phenomenon is called "*stalling*". Due to stalling the pressure rise will be reduced. The loss of pressure causes a recession of delivery, and intermittent back flow occurs. A similar thing happens when angle β_1 is reduced below a certain value. Stall due to too high angle of attack is called "***positive stall***" and that due to too low angle of attack is called "***negative stall***".



(a) Normal operation. (b) Stalled ooeration.

Fig. 5·27. Stalling in a cascade.

The stall does not occur on all blades at one time, not even on all blades of a single stage, but usually occurs on a set of blades. This stall moves in a direction opposite to that of blade rotation with a velocity equal to half the rotational velocity and is called *rotating stall*. Due to this rotation, which dissipates a very large amount of energy the flow, though having instable flow tendency, is affected only in the way that the pressure flow curve has a positiveslope and reduces the performance of the compressor. An increase in camber and stagger angles increase the tendency of the blade to stall. The cascade spacing and the thickness of the blades also affect the tendency to stall. A decrease in axial velocity increases the angle of incidence and stalling tends to occur when the mass flow is reduced, *i.e.* at starting, closing, or at part load.

Surging is also associated with separation of flow or stalling but the main cause of it is the dynamic instability of the system as a whole. Propagating stall may cause severe body vibrations of the whole mass of air in the compressor. One important characteristic of propagating stall is that the exited forces developed by it have the same frequency as the natural frequency of the blades. This is very dangerous as it may set severe vibrations and damage the blades.

The phenomena of surging can best be explained with reference to the characteristic curve of Fig. 5·28. For a constant speed operation of the compressor, starting from low flows the pressure rise increases with an increase in flow up to point B, where it reaches the maximum value. This region has a positive slope and is unstable.

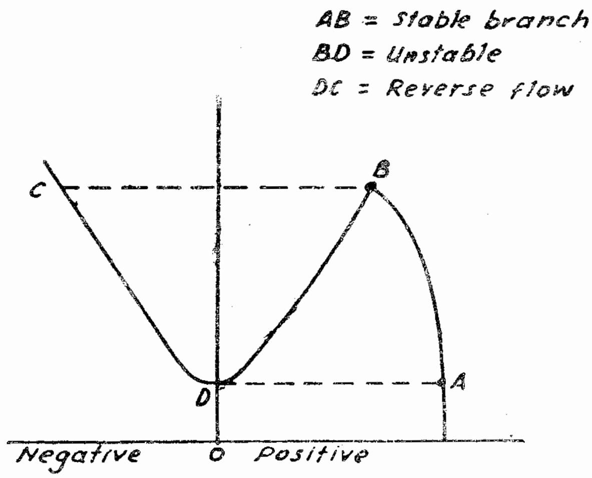


Fig. 5·28. Pressure volume flow characteristics at an axial compressor.

Beyond point A the compressor operates in normally stable region. During operation in unstable region, say at point D, if the flow is reduced there will be an increase in work and pressure ratio but the specific volume will decrease in the first stage. In the second stage this flow has slower axial velocity due to the reduced mass flow as well as specific volume. This results in decrease in inlet angle β_1 and an increase in the angle of attack and hence stalling will occur in the rest of the compressor stages. The effect of reducing flow is cumulative and the pressure falls with reduced mass flow. But if the pressure in the receiver or pipe down stream does not fall so rapidly, flow will have a tendency to flow back to the compressor. After some time the pressure in the downstream pipe reduces and forward flow takes place. This forward and backward flow of the air results in severe pulsations and the phenomenon is called *surging*. It should be noted that while stall produces local pulsations the surge causes pulsations in the whole system and the characteristics of the connecting system greatly affect the onset of surge.

For operation in stable region an increase in mass flow reduces the pressure rise and ultimately at point C the mass flow is maximum but pressure ratio is unity, *i.e.* the efficiency is zero. Such flow is called *chocking flow*. Surge and chocking decides the lower and upper limits of flow.

There is no absolute remedy for surge. Usually the pressure ratio in a single unit is limited to about 7 to 9 and by two stage compression pressure ratios as high as 12 to 16 can be obtained. Interstage bleeds, whereby a portion of the compressor air flow is bled off, is used to help to reduce instability at low speed

5·12. CHARACTERISTIC CURVES OF DYNAMIC COMPRESSORS

Figs. 5·29 and 5·30 show the characteristic curves of centrifugal and axial flow compressors respectively. Pressure ratio has been plotted for various mass flow rates at different speeds. Iso-efficiency curves have also been drawn. The whole of the compressor characteristics can be divided into stable and unstable regions separated by the surge line. The surge line shows the limit for the stable operation of the compressor—to its right is the stable region and to its left is unstable region. It is a single parameter line, *i.e.* it is a fixed mass flow rate which produces surge at a given pressure ratio.

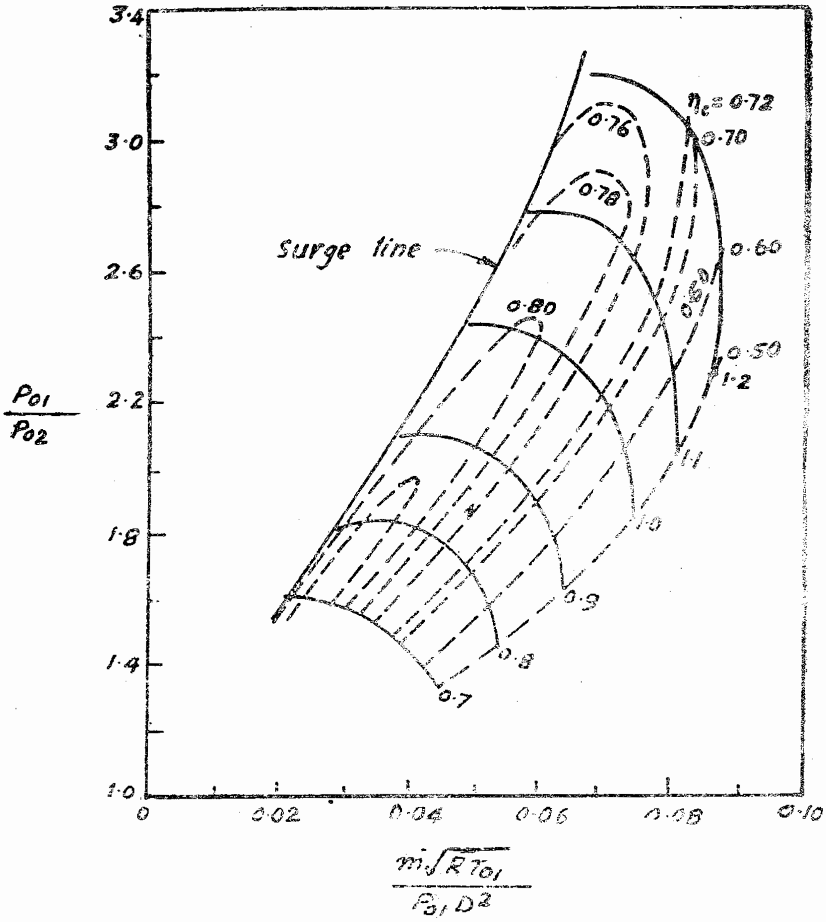


Fig. 5·29. Typical cenrifugal compressor characteristics.

The efficiency varies with the mass flow at a given speed and the maximum value is nearly same for all speeds. A curve representing the locus of points of maximum efficiency has been drawn in

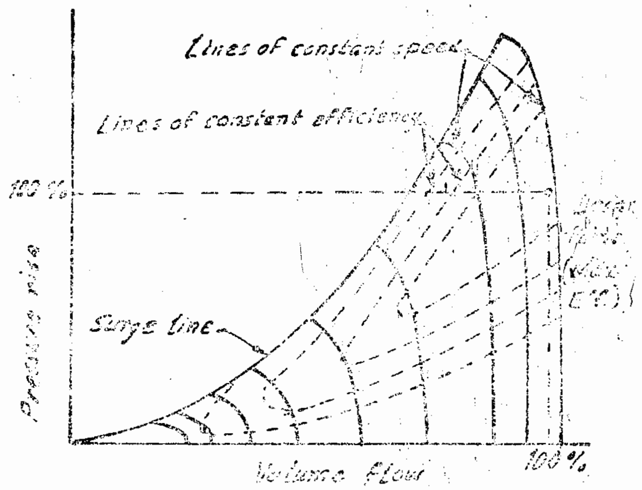


Fig. 5·30. Characteristic curves for an axial flow compressor

Fig. 5·29. It can be seen that this most efficient stable operation line is roughly parallel to the surge line. Surge points are reached in case of axial flow compressors before the pressure ratio mass flow curve reaches a maximum and hence the stable operation of the axial flow compressor is limited to a narrower range.

The possible pressure ratio and the mass flow, both increase with speed. For axial flow compressor the *pressure ratio-mass flow* curve is very steep the and pressure ratio rapidly decreases with an increase in mass flow.

5.13. CENTRIFUGAL *vs* AXIAL FLOW COMPRESSORS

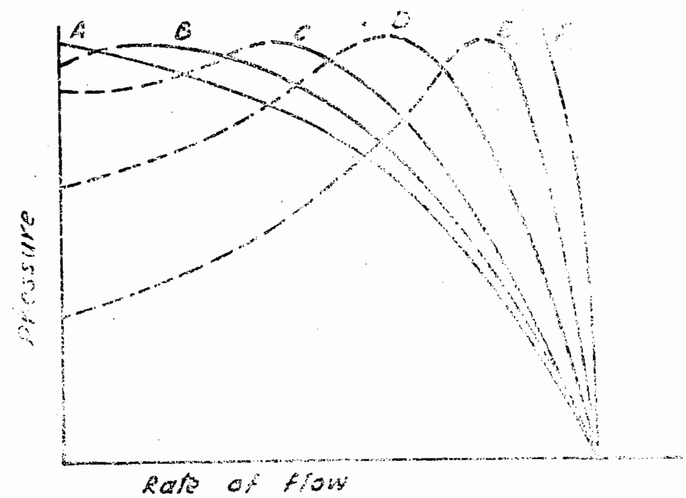
The centrifugal compressor is relatively simple in construction, cheap and has a high pressure ratio of about 4 : 1 per stage as compared to about 1·2 : 1 for axial flow compressors. The efficiency of a centrifugal compressor varies from about 70 to 82 percent over a wide range of speed This is lower than that of about 85 to 90 percent obtainable in an axial flow compressor. The centrifugal compressor has a large frontal area for a given mass flow and is not suitable for multi-staging due to losses between stages. Since centrifugal compressors can develop higher pressure ratios per stage, they are used for supercharging I.C. Engines, in gas turbine plants, for fertilizer, nitric acid and air separation plants. Relatively small size of the impeller eye limits the capacity of centrifugal compressor to values which are generally lower than that for axial flow compressors of the same size.

The axial flow compressor has relatively large inlet area and hence high flow capacity per unit of cross sectional area. However, since the centrifugal component of work input is absent to compensate, an axial flow compressor needs relatively higher inlet velocities.

Thus the maximum pressure ratio developed is relatively low because the inlet Mach number becomes more than unity (supersonic) rather earlier as compared to a centrifugal compressor.

The axial flow compressor has a large flow capacity with relatively low pressure ratios (*see* Fig. 5·30), is compact and has a good efficiency. However, the operational range is very narrow and for this reason it is not used for normal industrial applications. Axial flow compressor is widely used in steel industry due to its high capacity and compactness. Due to low frontal area they are also widely used in jet engines.

Figs. 5·31 and 5·32 show the pressure, mass flow and efficiency speed curves for different types of compressors. In Fig. 5·31 the dotted lines represent the unstable operation. It can be seen that centrifugal pumps have a falling pressure flow curve with no unstability. With centrifugal fans, which are nothing but low pressure compressors, the characteristic curve depends upon type of blading. Centrifugal compressors also have a surge limit but its stable range is relatively broader than the axial flow compressors. In positive



A = Centrifugal pump
B = Centrifugal fan (backward curved vanes)
C = Centrifugal fan (Forward curved vanes)
D = Centrifugal compressor
E = Axial flow compressor
F = Positive displacement compressor

Fig. 5·31. Pressore-mass flow curves for different compressor.

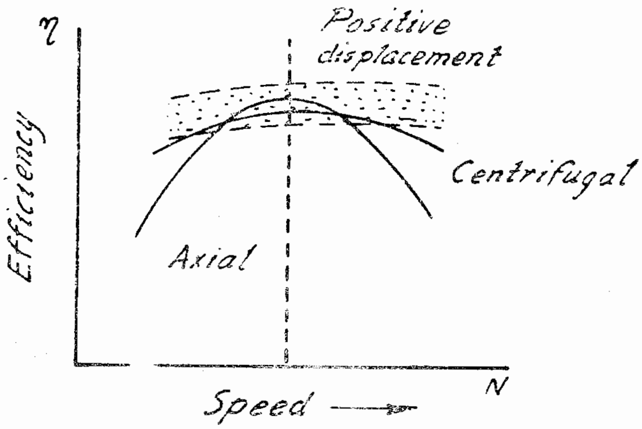
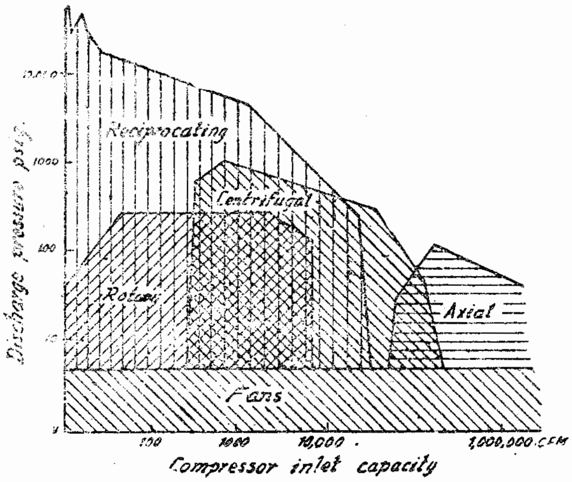


Fig. 5·32. Efficiencies of positive displacement, centrifugal and axial flow compressors.

displacement compressors there is a slight decrease in volumetric efficiency with decrease in pressure, but no back-flow can occur and there is no unstable region. The efficiency of positive displacement compressors is higher than that of dynamic compressors (*see* Fig. 5·32) over full range of operation. The axial compressor has a very narrow high efficiency operarional range.

Fig 5·33 shows the normal pressures and flow capacities over which various types of compressors are used and it can be seen that



5·35. Range of operation of various type of compressors, reciprocating compressor are used where very high pressures are required and axial flow compressors where high mass flows are required. Table 5·1 shows the advantages of the main types of compressors.

TABLE 5·1

ADVANTAGES OF BASIC TYPES OF COMPRESSORS

1	Reciprocating	—	Highest efficiency, both at full and part load
2.	Rotary (Positive displacement)	—	Compact, high speed, can be free from lubrication contamination and usually has higher efficiency than the centrifugal (not as high as reciprocating).
3.	Centrifugal	—	Freedom from pulsation in the discharge line and from lubrication contamination, low maintenance.
4.	Axial flow	—	Highest capacity. Freedom from lubrication contamination low maintenance.
5.	Fan	—	Low pressure, freedom from lubrication con-contamination, pulsation. High speed, low maintenance.

ILLUSTRATIVE EXAMPLES

5·1 Rotary Compressor : t_{outlet} ; hp

Air flows at the rate of 5 kg/s through a rotary compressor. The air, initially at a pressure of 1 kgf/cm² and temperature 15°C, enters the compressor with negligible velocity and leaves at a pressure of 30 kgf/cm² through a 300 mm diameter pipe. If the compressor is reversible and adiabatic and the mechanical efficiency of the compressor is 80%, calculate the temperature of the air leaving the compressor and the power required to drive the compressor.

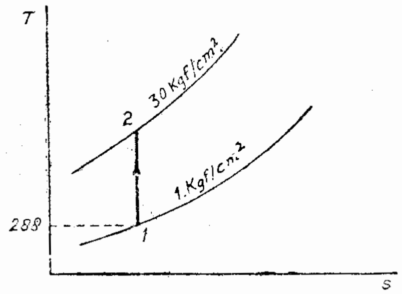


Fig. 5·34

Temperature after compression is given by

$$T_2 = T_1 \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$= 288 \left(\frac{30}{1}\right)^{\frac{0 \cdot 4}{1 \cdot 4}} = 762 \cdot 5 \text{ K}$$

$$= \mathbf{489 \cdot 5°C}. \qquad \textbf{Ans.}$$

Power required to drive the compressor

$$= \frac{mc_p(T_2 - T_1)}{\eta_m}$$

$$= \frac{5 \times 0 \cdot 24(762 \cdot 5 - 288) \times 5 \cdot 61}{0 \cdot 80}$$

$$= 3990 \text{ hp}. \qquad \textbf{Ans.}$$

5 2. Centrifugal compressor ; η_{isen} ; hp

Show on a temperature-entropy diagram a frictionless adiabatic compression. Shade the area which represents the work done on the gas during compression. On the same diagram show an adiabatic compression with friction and shade and label the area which represents the extra work required due to friction.

The pressure of air being compressed in a centrifugal compressor is doubled. The inlet temperature is 27°C and final temperature 105°C. Calculate the isentropic efficiency of the compressor and horse power required to drive it, if 30 kg of air are compressed per minute. $c_p = 0 \cdot 239$, $c_v = 0 \cdot 171$.

$$\gamma = \frac{c_p}{c_v} = \frac{0 \cdot 239}{0 \cdot 171} = 1 \cdot 4$$

Now $$\frac{T_2'}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = 2^{\frac{1 \cdot 4 - 1}{1 \cdot 4}}$$

$$\therefore \quad T_2' = 300 \cdot 2^{\frac{1 \cdot 4 - 1}{1 \cdot 4}} = 366 \text{ K}$$

$$\eta_{isentropic} = \frac{T_1 - T_2'}{T_2 - T_1}$$

$$= \frac{366 - 300}{378 - 300} = \mathbf{0 \cdot 845 \text{ or } 84 \cdot 5\%} \qquad \textbf{Ans.}$$

$$\text{hp} = \frac{Jmc_p(T_2 - T_1)}{4500}$$

$$= \frac{427 \times 30 \times 0 \cdot 239 \, (378 - 300)}{4500} = \mathbf{53 \cdot 1} \qquad \textbf{Ans.}$$

5·3. Rotary compressor : tip dia ; annulus area ; hp

A single-sided straight vaned rotary compressor is required to deliver 10 kg of air per second with a total pressure ratio of 4·5 : 1, and operates at a speed of 16000 r.p.m. The air inlet pressure and temperature are 1·03 kgf/cm² and 27°C. Find :

(a) rise in total temperature,

(b) tip speed of the impeller,

(c) tip diameter,

(d) inlet eye annulus area,

(e) theoretical horse power to drive the compressor.

The slip factor is 0·94 and the compressor isentropic efficiency is 78%. The air enters the inlet eye axially with a velocity of 150 m/sec. Given $c_p=0{\cdot}24$, $\gamma=1{\cdot}4$.

Inlet stagnation temperature

$$T_{t1}=T_1+\frac{C^2}{2gJc_p}$$

$$=300+\frac{150^2}{2\times 9{\cdot}81\times 427\times 0{\cdot}24}$$

$$=311{\cdot}2\text{K}$$

For isentropic compression

$$T_{t2}'=T_{t1}\times\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$=311{\cdot}2\times(4{\cdot}5)^{\frac{0\cdot4}{1\cdot4}}$$

$$=443\text{K}$$

Compressor isentropic efficiency

$$\eta_c=\frac{T_{t2}'-T_{t1}}{T_{t2}-T_{t1}}$$

$$0{\cdot}78=\frac{443-311{\cdot}2}{T_{t2}-T_{t1}}$$

Total temperature rise

$$=T_{t2}-T_{t1}$$

$$=\frac{443-311 \cdot 2}{0 \cdot 78}=\mathbf{168 \cdot 9°C} \qquad \textbf{Ans.}$$

$\therefore$ Workdone on compressor

$$=c_p(T_{t2}-T_{t1})$$

$$=0 \cdot 24 \times 168 \cdot 9=40 \cdot 55 \text{ kcal/kg}$$

$$\text{Work input}=\frac{\text{slip factor} \times (\text{blade velocity})^2}{g}$$

or

$$40 \cdot 55 \times 427=\frac{0 \cdot 94 \times U_2^2}{9 \cdot 81}$$

$\therefore$

$$U_2=\sqrt{\frac{40 \cdot 55 \times 427 \times 9 \cdot 81}{0 \cdot 94}}$$

$$=\mathbf{425 \text{ m/s.}} \qquad \textbf{Ans.}$$

$$\text{Blade tip diameter}=\frac{60 \times 425}{17 \times 1600}=0 \cdot 5075 \text{ m}$$

Air specific volume at entry

$$=\frac{29 \times 300}{1 \times 100}=7 \cdot 0 \text{ m}^3/\text{kg}$$

$\therefore$ Annular area at eye inlet

$$=\frac{10 \times 87}{150}=5 \cdot 8 \text{ m}^2$$

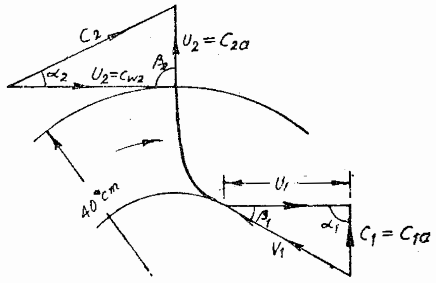
Theoretical horse power to drive the compressor

$$=\frac{40 \cdot 55 \times 10 \times 427}{75}=\mathbf{2310 \text{ hp}} \qquad \text{Ans.}$$

5·4. Centrifugal compressor : velocity diagram ; T_{exit} ; P_{exit}

Describe the functions of the impeller and diffuser in a centrifugal compressor.

Air from a practically quiescent atmosphere, pressure 1·03 kgf/cm² and temperature 15°C, enters axielly a centrifugal compressor fitted with radial blades, and the air leaves the diffuser with negligible velocity. The tip diameter of the impeller is 40 cm and the speed of rotation 20000 rev/min. Neglecting all losses, find the temperature and pressure of the air as it leaves the compressor. $\gamma=1 \cdot 4$. *[Agra, 1972 annual]*



1=Inlet to impeller
2=Outlet from impeller
3=Outlet from diffuser.

Fig. 5·36

The function of the impeller is to transfer the energy of drive (work input) to the air causing a rise in both static pressure and temperature and a rise in velocity. The diffuser converts the kinetic energy into pressure energy, *i e.* the air velocity is reduced causing a rise in both static pressure and temperature.

Peripheral velocity of impeller tip

$$U_2=\frac{\pi DN}{60}=\frac{\pi\times 0{\cdot}4\times 20000}{60}=419 \text{ m/s}$$

$$\text{Work input}=\frac{1}{gJ}(U_2C_{W_2}-U_1C_{W_1}) \text{ kcal/kg}$$

As entry is axial $C_{w1}=0$ and since there are no losses the relative velocity at the impeller tip is radial making $C_{W_2}=U_2$

$$\therefore \qquad W=\frac{U^2}{gJ}=\frac{419^2}{9{\cdot}81\times 427}=41{\cdot}9 \text{ kcal/kg}$$

Now $\qquad W=c_p.\triangle T_t$

$\therefore$ The rise in total temperature

$$\triangle T_t=\frac{W}{c_p}=\frac{41{\cdot}9}{0{\cdot}24}=175°\text{C}$$

Since the exit velocity is negligible the rise in static temperature across the machine is same as rise in total temperature

$$=175°\text{C}.$$

$\therefore$ Exit temperature from diffuser

$=T_1+175=288+175=$ **463 K.** **Ans.**

For isentropic compression

$$\frac{p_3}{p_1}=\left(\frac{T_3}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$$

$$\therefore \qquad p_3=1{\cdot}03\left(\frac{463}{288}\right)^{\frac{1\cdot 4}{1\cdot 4-1}}$$

$=$ **5·43 kgf/cm².** **Ans.**

5·5. Centrifugal supercharger ; impeller dia. $A_{annulur}$; hp

A single stage c ntrifugal supercharger is to be designed to maintain a total head pressure of 1·5 kgf/cm² in the manifold of an internal combustion engine when the ambient temperature and pressure are −7°C and 0·67 kgf/cm². The mixture flow is 1 kg/s and for the mixture R=28 and γ=1·33.

If the isentropic efficiency of 75 per cent is expected, an entry velocity of 110 m/s is acceptable and a rotational speed of 20000 rev/min is convenient, calculate :

(*a*) *a suitable impeller diameter* ;

(*b*) *the annulus area required for the entry* ;

(*c*) *the theoretical power required to drive the impeller* ;

(*d*) *the density of the mixture at the impeller tip.*

Assume no prewhirl and radial discharge.

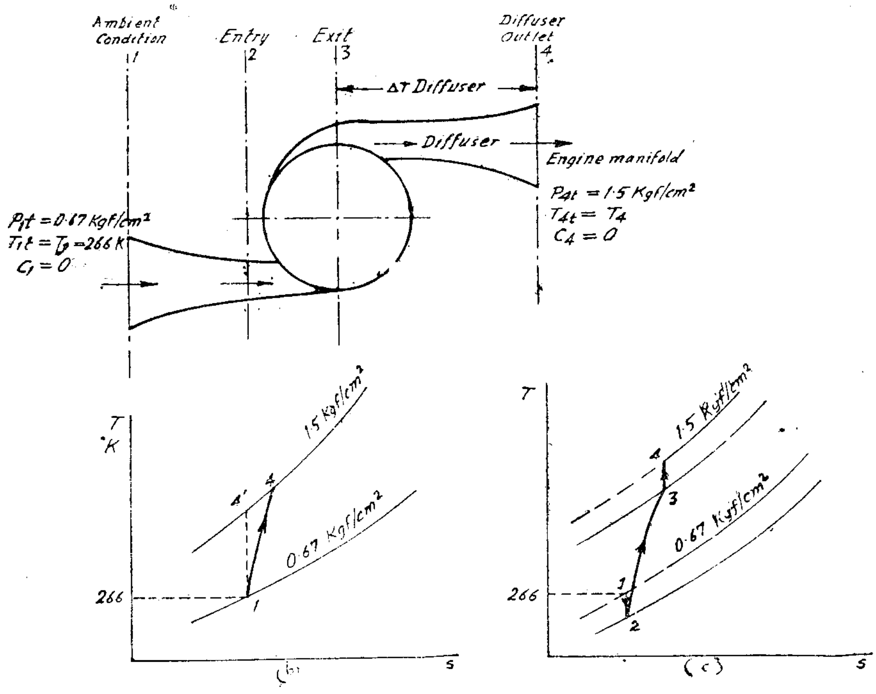


Fig. 5·37

$$c_v = \frac{R}{J(\gamma - 1)}$$

$$= \frac{28}{427(1{\cdot}33 - 1)} = 0{\cdot}198 \text{ kcal/kg K}$$

$$c_p = \gamma \times c_v = 1{\cdot}33 \times 0{\cdot}198 = 0{\cdot}264 \text{ kcal/kg K}$$

Let 1 refer to ambient condition

2 refer to entry to impeller

3 refer to exit from-impeller

4 refer to exit from diffuser or entry to engine mainifold.

Neglecting ambient velocity and delivery velocity in engine manifold (*i.e.* $C_1=C_4=0$), the conditions at 1 and 4 are total head conditions. (*see* Fig. 5·37).

Considering isentropic compression from 1 to 4 and writing suffix t for total head conditions, we get

$$\frac{T_4{}'_t}{T_{1t}}=\left(\frac{P_{4t}}{P_{1t}}\right)^{\frac{\gamma-1}{\gamma}}=\left(\frac{1\cdot5}{0\cdot67}\right)^{\frac{1\cdot33-1}{1\cdot33}}=1\cdot222$$

$$\therefore \quad T_4{}'_t=266\times1\cdot222=326 \text{ K}$$

$$\eta_{isentropic}=\frac{T_4{}'_t-T_{1t}}{T_{4t}-T_{1t}}$$

or
$$0\cdot75=\frac{326-266}{T_{4t}-266}$$

$$\therefore \quad T_{4t}=346 \text{ K}=T_4$$

(*a*) Applying the law of conservation of energy to section 1 and 4.

$$h_1+\frac{C_1^2}{2gJ}+w=h_4+\frac{C_4^2}{2gJ}$$

As C_1 and C_4 are zero

$$W=h_4-h_1=c_p(T_4-T_1)$$
$$=0\cdot264(346-266)=21\cdot12 \text{ kcal}$$

With no prewhirl, $W=\dfrac{C_w.U}{gJ}$

As discharge is radial velocity of whirl at exit=tangential velocity of impller at exit U.

$$\therefore \quad W=\frac{U^2}{gJ}=21\cdot12 \text{ kcal}$$

or
$$U=\sqrt{9\cdot81\times427\times21\cdot12}=296 \text{ m/s}$$

Now
$$U=\frac{\pi DN}{60}$$

$\therefore$ **Impeller diameter,**

$$D=\frac{296\times60}{\pi\times20000}=0\cdot282 \text{ m or } \mathbf{28\cdot2 \text{ cm}} \qquad \textbf{Ans.}$$

(*b*) Assuming frictionless adiabatic flow from ambient (section 1) to entry (section 2), by law of conservation of energy, we get

$$h_1+\frac{C_1^2}{2gJ}=h_2+\frac{C_2^2}{2gJ}$$

or $$h_1 - h_2 = \frac{C_2^2}{2gJ} - 0 = c_p(T_1 - T_2)$$

or $$0 \cdot 264(266 - T_2) = \frac{(110)^2}{2 \times 9 \cdot 81 \times 427}$$

$\therefore$ $$T_2 = 260 \cdot 5 \text{ K}$$

Let P_2 be the static pressure at entry.

Then $$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma - 1}{\gamma}}$$

$\therefore$ $$P_2 = 0 \cdot 67 \left(\frac{260 \cdot 5}{266}\right)^{\frac{1 \cdot 33}{1 \cdot 33 - 1}} = 0 \cdot 618 \text{ kgf/cm}^2$$

Note carefully that static conditions must be used to obtain mass or specific volume.

$$P_2 v_2 = mRT_2$$

or $$0 \cdot 618 \times 10^4 \times v_2 = 1 \times 28 \times 260 \cdot 5$$

$\therefore$ $$v_2 = 1 \cdot 18 \text{ m}^3/\text{kg}$$

Now Volume = area of cross-section × velocity

$\therefore$ $$\text{Area of entry} = \frac{v_2}{C_2} = \frac{1 \cdot 18}{110}$$

$$= 0 \cdot 0107 \text{ m}^2 \text{ or } \mathbf{107 \text{ cm}^2}$$ **Ans.**

(c) $$hp = \frac{J \times m \times c_p(T_{4t} - T_{1t})}{75}$$

$$= \frac{427 \times 1 \times 0 \cdot 264(346 - 266)}{75} = \mathbf{120}$$ **Ans.**

(d) To find the density at impeller tip 3, calculate static temperature and pressure at 3. In diffuser 3-4 kinetic energy is converted to heat. Assuming no loss in the diffuser.

$$\frac{C_3^2}{2gJ} - \frac{C_4^2}{2gJ} = c_p(T_3 - T_4)$$

or $$\frac{296^2}{2 \times 9 \cdot 81 \times 427 \times \cdot 24} = T_3 - 346 \qquad [c_4 = 0]$$

or $$T_3 = 346 + 44 = 390 \text{ K}$$

$$\frac{p_3}{p_4} = \left(\frac{T_3}{T_4}\right)^{\frac{\gamma}{\gamma - 1}}$$

$$= \left(\frac{390}{346}\right)^{\frac{1 \cdot 33}{1 \cdot 33 - 1}} = 1 \cdot 665$$

$\therefore$ $$p_3 = 1 \cdot 5 \times 1 \cdot 665 = 2 \cdot 5$$

$\therefore$ $$\rho_3 = \frac{2 \cdot 5}{29 \cdot 27 \times 390} = 3 \cdot 32 \times 10^{-4} \text{ kg/m}^3$$ **Ans.**

5·6. Centrifugal compressor : vane angles at root and tip, Mach number.

Discuss the relative merits of axial flow and radial flow compressors for internal combustion turbine plants.

A double-sided centrifugal compressor has impeller eye root and tip diameters of 18 cm and 30 cm and is to deliver 16 kg of air per second at 16000 rev./min. The design ambient conditions are 15°C and 1 kgf/cm² and the compressor is to be a part of a stationary power plant. Find suitable values for the impeller vane angles at root and tip of the eye if air is given 20° of pre-whirl at all radii. The axial component of inlet velocity is constant over the eye and is 150 m/s.

Find also the maximum Mach number at the eye. What is the disadvantage of reducing the Mach number by introducing pre-whirl and how may it be mitigated ? Take $c_p = 0{\cdot}24$.

[*Banaras, M.Sc., (Engg.) 1970*]

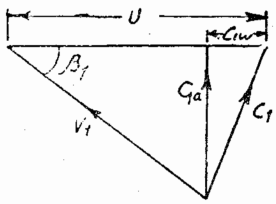


Fig. 5·38.

Annulus area of the impeller

$$= \frac{\pi}{4}(d_2^2 - d_1^2) = \frac{\pi}{4}[(0{\cdot}3)^2 - (0{\cdot}18)^2] = 0{\cdot}0452 \text{ m}^2$$

$$U_{tip} = \frac{\pi DN}{60} = \frac{\pi \times 0{\cdot}3 \times 16000}{60} = 25{\cdot}2 \text{ m/s}$$

$$U_{root} = \frac{\pi \times 0{\cdot}18 \times 1600}{60} = 151 \text{ m/s}$$

$$C_{1f} = 150 \text{ m/s}$$

$$\therefore \quad C_1 = \frac{150}{\cos 20^\circ} = \frac{150}{0{\cdot}9397} = 160 \text{ m/s}$$

Temperature equivalent of C_1

$$= \frac{C_1^2}{2gJc_p} = \frac{160^2}{2 \times 9{\cdot}81 \times 427 \times 0{\cdot}24} = 12{\cdot}7°\text{C}$$

$$T_1 = 288 - 12{\cdot}7 = 275{\cdot}3°\text{C}$$

Now $\dfrac{T_{1t}}{T_1} = \left(\dfrac{P_{1t}}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$ or $\dfrac{288}{275{\cdot}3} = \left(\dfrac{1}{P_1}\right)^{\frac{1{\cdot}4-1}{1{\cdot}4}}$

or $$P_1=0{\cdot}843 \text{ kgf/cm}^2$$

$$\rho_1=\frac{P_1}{RT_1}=\frac{0{\cdot}843\times10^4}{29{\cdot}27\times275{\cdot}3}=1{\cdot}047 \text{ m}^3/\text{kg}$$

Now $$\tan 20°=\frac{C_{1w}}{C_{1a}}$$

or $$C_{1w}=150\times0{\cdot}364=55{\cdot}6 \text{ m/s}$$

$$\tan \beta_{1tip}=\frac{C_1a}{U_{tip}-C_{1w}}=\frac{150}{252-55{\cdot}6}$$

$\therefore$ $\beta_{1tip}=$**37·4°**. **Ans.**

$$\tan \beta_{1root}=\frac{150}{151-55{\cdot}6}$$

$\therefore$ $\beta_{1root}=$**57·5°** **Ans.**

Maximum relative velocity at inlet

$$=\sqrt{150^2+(252-55{\cdot}6)^2}=247 \text{ m/s}$$

$$\text{Maximum inlet Mach number}=\frac{247}{\sqrt{g\gamma RT}}$$

$$=\frac{247}{\sqrt{9{\cdot}81\times1{\cdot}4\times29{\cdot}27\times275{\cdot}3}}=\mathbf{0{\cdot}755}$$ **Ans.**

It is possible to reduce the relative velocity at inlet and hence the Mach number by introducing prewhirl, by allowing the air to be prawn over the curved inlet guide vanes. The disadvantage is that the work capacity of the compressor is reduced. The Mach number is high at the tip only. It is thus preferable to reduce the prewhirl gradually from a maximum at the eye tip to zero at the eye root by suitably twisting the inlet guide vanes.

5·7. Axial air compressor: velocity diagram, isentropic efficiency.

An axial compressor is fitted with half reaction blading, the blade inlet and outlet angles being 50° and 15° when measured from the axial direction. The mean diameter of a certain blade pair is 85 cm and the ns at 5500 rev/min. Calculate the necessary isentropic efficiency of the stage if the pressure ratio of compression is to be 1·4 when the air inlet temperature is 25°C.

$$\text{Mean blade speed}=\frac{\pi DN}{60}=\frac{\pi\times0{\cdot}85\times5500}{60}=244{\cdot}5 \text{ m/s}$$

Draw the velocity diagram to scale and read

$$\Delta C_w=155 \text{ m/s}$$

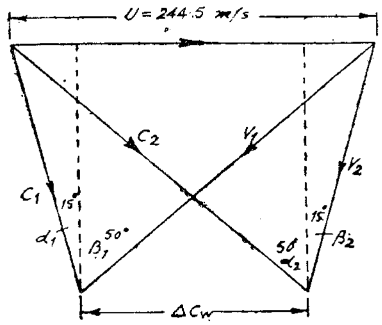


Fig 5·39.

$$\text{Work input/kg} = \frac{U \times \Delta C_w}{g} = \frac{244 \cdot 5 \times 155}{9 \cdot 81} = 3860 \text{ kgf/m}$$

$$\text{Isentropic work} = Jc_p(T_2 - T_1)$$

$$= Jc_p \left[\left(\frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\gamma}} T_1 - T_1 \right]$$

$$= Jc_p T_1 \left[\left(\frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right]$$

$$= 427 \times 0 \cdot 24 \times 298 \left[(1 \cdot 4)^{\frac{1 \cdot 4 - 1}{1 \cdot 4}} - 1 \right]$$

$$= 3363 \text{ kgf/m}$$

$$\text{Ssage efficiency} = \frac{\text{Isentropic work}}{\text{Actual work}} \times 100$$

$$= \frac{3363}{3860} \times 100 = \mathbf{87 \cdot 3\%}$$ **Ans.**

5·8. Axial flow compressor : η_c ; η_p ; $C_{delivery\ pipe}$

Derive an expression, in terms of entry and delivery pressures and temperatures and the ratio of the specific heats, for the polytropic or small stage efficiency of a rotary compressor. Explain briefly why it is sometimes desirable to express compressor efficiency in this form.

The following data refer to a test on an axial flow compressor : Atmospheric temperature and pressure at inlet, 20°C and 1·03 kgf/cm² respectively ; total head temperature and pressure in the delivery pipe, 160°C and 3 5 kgf/cm² respectively ; static pressure in delivery pipe, 3 kgf/cm².

Calculate (a) the total head isentropic and polytropic efficiencies of the compressor, and (b) the air velocity in the delivery pipe.

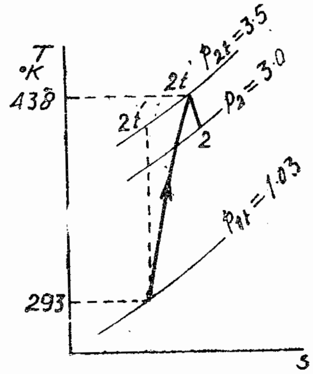


Fig. 5·40.

$$\text{Polytropic, efficiency } \eta_s = \frac{\log_e \left(\frac{p_{2t}}{p_{1t}}\right)^{\frac{\gamma-1}{\gamma}}}{\log_e \left(\frac{T_{2t}}{T_{1t}}\right)} \qquad (1)$$

where t refers to total or stagnation condition.

The other expression for polytropic or small stage efficiency is

$$\eta_s = \frac{\gamma-1}{\gamma} \cdot \frac{n}{n-1} \qquad (2)$$

However, since the value of n is generally unknown the form of Eq. (2) is not convenient and hence it is desirable to express polytropic efficiency in terms of entry and delivery temperatures and pressures as given by Eq. (1). The use of polytropic efficiency is in the design of multi-stage machines where a common stage efficiency may be adopted. The polytropic efficiency is greater than isentropic efficiency in compressors and less in turbines.

$$\frac{T_2'{}_t}{T_{1t}} = \left(\frac{p_{2t}}{p_{1t}}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{3\cdot5}{1\cdot03}\right)^{\frac{1\cdot4-1}{1\cdot4}} = 1\cdot419$$

$\therefore$ $$T_2'{}_t = 293 \times 1\cdot419 = 415 \text{ K}$$

Isentropic η, $$\eta_t = \frac{T_2'{}_t - T_{1t}}{T_{2t} - T_{1t}} = \frac{415-293}{433-293} = \mathbf{0\cdot871} \quad \textbf{Ans.}$$

Polytropic η, $$\eta_s = \frac{\log_e \left(\frac{p_{2t}}{p_{1t}}\right)^{\frac{\gamma-1}{\gamma}}}{\log_e \left(\frac{T_{2t}}{T_{1t}}\right)} = \frac{\log_e \left(\frac{3\cdot5}{1\cdot03}\right)^{\frac{1\cdot4-1}{1\cdot4}}}{\log_e \left(\frac{433}{293}\right)}$$

$$= \mathbf{0\cdot895} \qquad \textbf{Ans.}$$

To calculate the air velocity in the delivery pipe we need to know the static temperature. Consider isentropic process 2t-2

$$\frac{T_{2t}}{T_2}=\left(\frac{p_{2t}}{p_2}\right)^{\frac{\gamma-1}{\gamma}}=\left(\frac{3\cdot5}{3}\right)^{\frac{1\cdot4-1}{1\cdot4}}=1\cdot0454$$

$\therefore$ $$T_2=\frac{433}{1\cdot0454}=414\text{ K}$$

Now $$T_{2t}=T_2+\frac{C_2^2}{2gJc_p}$$

$\therefore$ $$C=\sqrt{2gJc_p(T_{2t}-T_2)}$$
$$=\sqrt{2\times9\cdot81\times427\times0\cdot24(433-414)}$$
$$=\mathbf{195\cdot5\ m/s}$$ **Ans.**

5·9. Velocity diagram ; no of stages ; hp

The mean value of the velocity of the blades in the second stage of an uncooled axial flow compressor is 230 m/s, and the axial velocity of the air is 150 m/s. The shape of the fixed and the moving blades are the same and are such that the difference between the tangents of the angles at outlet and inlet is 0·56. If the adiabatic efficiency of the stage is 85 per cent and the air enters the stage with a temperature of 290 K, find the similar stages required to raise the pressure of the air from 1 to 3·8 kgf/cm² and the power required to compress a mass flow of 1000 kg per minute. Assume c_p remains 0·24. The mechanical efficiency of the compressor is 99 per cent, and that the difference in the kinetic energy of the air entering successive stages is negligible.

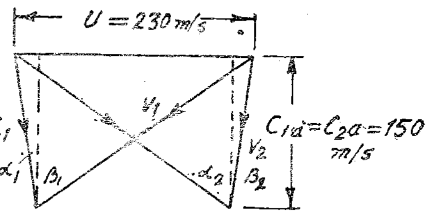


Fig. 5·41.

Work done/kg

$$=\frac{U}{gJ}(T_2-T_1)$$

$$=\frac{U(C_{2w}-C_{1w})}{Jg}$$

$$=\frac{U}{Jg}\ C_a(\tan\beta_1-\tan\beta_2)$$

$$=\frac{230\times150}{9{\cdot}81\times427}\times(-0{\cdot}56)=-4{\cdot}61 \text{ kcal}$$

Now $\quad$ W.D.$=c_p\triangle T$

$\therefore$ Rise in temperature in second stage

$$\triangle T=\frac{4{\cdot}61}{0{\cdot}24}=19{\cdot}2^\circ\text{C}.$$

Pressure ratio for the second stage

$$\frac{\quad}{p_{1t}}=\left[1+\eta\frac{T_{2t}-T_{1t}}{T_{1t}}\right]^{\frac{\gamma}{\gamma-1}}$$

$$=\left[1+0{\cdot}85\times\frac{19{\cdot}2}{290}\right]^{\frac{1\cdot4}{1\cdot4-1}}=1{\cdot}211$$

Total pressure ratio$=3{\cdot}8$

Assuming x no. of stages

$$(1{\cdot}211)^x=3{\cdot}8$$

$\therefore$ $\quad x=6{\cdot}96$ or say **7** **Ans.**

$$\text{hp}=\frac{mC_p\triangle T\times\text{no. of stages}\times427}{\eta_{mech}\times4500}$$

$$=\frac{1000\times0{\cdot}24\times19{\cdot}2\times7\times427}{0{\cdot}99\times4500}$$

$$=3100$$ **Ans.**

5·10. Axial-flow Compressor : h.p., blade angles

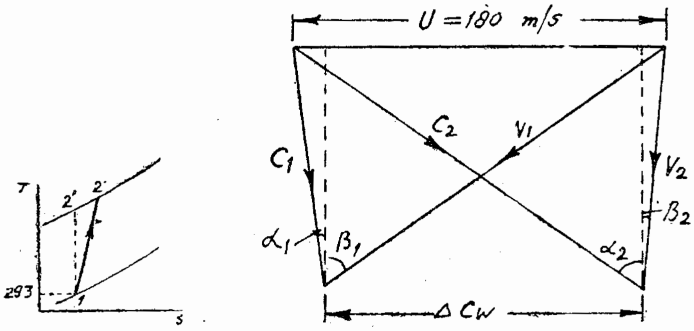
An eight-stage axial flow compressor takes in air at a temperature of 20°C at the rate of 3 kg/s. The pressure ratio is 6 and the isentropic efficiency is 0·89 ; the compression process is adiabatic. The stages of the compressor are similar and operate with 50% reaction ; in each stage the mean blade speed is 180 m/s and the uniform axial velocity of flow of the air is 110 m/s.

Determine the power to the air and the direction of the air at entry to and exit from the rotor and the stator blades.

Assume air to be a perfect gas for which $c_p=0{\cdot}24$ kcal/kg K and γ is 1·4.

It is assumed that the temperature change is same in each stage. Then the power may be obtained by considering the overall conditions.

$$T_2'=T_1\times\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$



(a) (b)

Fig. 5·42.

$$=293\times 6^{\frac{0\cdot 4}{1\cdot 4}}=488\text{ K}$$

$$\eta=\frac{T_2'-T_1}{T_2-T_1}$$

$\therefore$ $$0\cdot 89=\frac{488-293}{T_2-293}$$

$\therefore$ $$T_2=512\text{ K}$$

$\therefore$ Power to the air $$=\frac{mc_p\triangle TJ}{75}$$

$$=\frac{3\times 0\cdot 24\times(512-293)427}{75}$$

$$=\mathbf{900\ hp}$$ **Ans.**

Temperature change per stage

$$=\triangle T_s=\frac{\triangle T}{8}=\frac{512-293}{8}=27\cdot 38$$

Work done/kg of air per second

$$=\frac{U\times\triangle C_w}{g}$$

or $$\frac{180\times\triangle C_w}{9\cdot 81}=\frac{0\cdot 24\times 219}{8}\times 427$$

or $$\triangle C_w=\frac{0\cdot 24\times 219\times 427\times 9\cdot 81}{180\times 8}$$

$$=153\text{ m/sec}$$

$$\triangle C_w=C_a(\tan\beta_1-\tan\beta_2)$$

or $$153=110(\tan\beta_1-\tan\beta_2) \qquad (i)$$

and $$R=\frac{1}{2}\frac{C_a}{U}(\tan\beta_1+\tan\beta_2)$$

$$\therefore \quad 0.5=\frac{1}{2}\times\frac{110}{180}(\tan\beta_1+\tan\beta_2) \qquad (ii)$$

$$\therefore \quad \tan\beta_1+\tan\beta_2=\frac{153}{110}$$

and $$\tan\beta_1-\tan\beta_2=\frac{180}{110}$$

$$\therefore \quad \tan\beta_1=\frac{153+180}{110\times 2}=\frac{333}{110\times 2}$$

$$\therefore \quad \beta_1=\mathbf{56°\ 40'}$$ **Ans**

We also have,

$$\tan\beta_2=\frac{180-153}{110\times 2}=\frac{27}{220}$$

$$\therefore \quad \beta_2=7°\ 6'$$

$$\left.\begin{aligned}\alpha_1=\beta_2=7°\ 6'\\ \alpha_2=\beta_1=56°\ 40'\end{aligned}\right\}$$ **Ans.**

5·11. Axial flow compressor : reaction at blade tip.

A stage of an axial flow compressor is designed to have (a) radial equilibrium and uniform axial velocity over the annulus at inlet to and at outlet from the rotor, (b) the same work done per unit mass flow at all radii. Show that over any cross-section between the blade rows :

$$C_w . r=\text{constant}$$

in which C_w is the tangential velocity at a radius r from the compressor axis and the flow is reversible. Assume that over the annulus at inlet to the rotor the stagnation enthalpy and entropy are uniform.

If the stage reaction R at the blade root is 30%, estimate the reaction at the blade tip where the radius is 30% greater than at the root. Assume that at a given radius the absolute velocities in and out of the stage are the same both in magnitude and direction, and that the stage reaction is equal to the actual enthalpy rise in the rotor divided by the actual enthalpy rise in the stage. [*Panjab, M.E., 1972 April*]

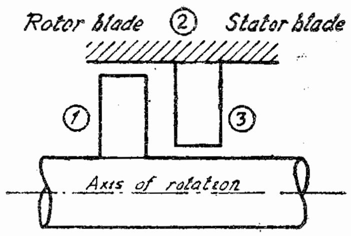


Fig. 5·43.

$$C_1=C_3\ ;\ C_{1f}=C_{2f},\ R=\frac{h_1-h_2}{h_1-h_3}$$

By steady flow energy equation for the rotor (1 to 2), we get

$$h_1+\frac{C_1^2}{2g}=h_2+\frac{C_2^2}{2g}+W_{rotor}$$

$$\therefore \qquad h_1-h_2=W_{rotor}+\frac{C_2^2-C_1^2}{2g}$$

$$=W_{rotor}+\frac{1}{2g}[(C_{2w}^2+C_{2f}^2)+(C_{1w}^2+C_{1f}^2)]$$

Similarly, for the whole stage (1 to 3)

$$h_1-h_3=W_{stage}=W_{rotor}$$

(since $C_1=C_3$ and $W_{stator}=0$)

$$\therefore \qquad R=\frac{h_1-h_2}{h_1-h_3}=\frac{W_{rotor}+\frac{1}{2g}(C_{2w}^2-C_{1w}^2)}{W_{rotor}}$$

$$=1+\frac{\frac{1}{2g}(C_{2w}^2-C_{1w}^2)}{U\,\frac{(C_{1w}-C_{2w})}{g}}$$

$$=1-\frac{1}{2U}(C_{1w}+C_{2w})$$

$$\therefore \qquad R_{rooi}=1-\left(\frac{C_{1w}+C_{2w}}{2U}\right)_{root}$$

$$\therefore \qquad \left(\frac{C_{1w}+C_{2w}}{2U}\right)_{root}=1-R_{root}=1-0{\cdot}3=0{\cdot}7$$

Now $\qquad C_w.r=\text{constant}$

$$\therefore \qquad C_{1w(tip)}.r_{tip}=C_{1w(root)}.r_{root}$$

$$\therefore \qquad C_{1w(tip)}=C_{1w(root)}\,\frac{r_{root}}{r_{tip}}$$

Similarly $\qquad C_{2w(tip)}=C_{2w(root)}\times\frac{r_{root}}{r_{tip}}$

$$\therefore \quad C_{1w(tip)}+C_{2w(tip)}=(C_{1w}+C_{2w})_{root}\times\frac{r_{root}}{r_{tip}}$$

and $\qquad \frac{U_{tip}}{\varphi_{tip}}=\frac{U_{root}}{r_{root}}$

$$\therefore \qquad U_{tip}=U_{root})\times\frac{r_{tip}}{r_{root}}$$

$$\therefore \qquad \left[\frac{C_{1w}+C_{2w}}{2U}\right]_{tip}=(C_{1w}+C_{2w})_{root}\times\frac{r_{root}}{r_{tip}}\times\frac{1}{2U_{root}}\times\frac{r_{root}}{r_{tip}}$$

$$=\left(\frac{C_{1w}+C_{2w}}{2U}\right)_{root}\times\left[\frac{r_{root}}{r_{tip}}\right]^2$$

$$=0.7\times\left(\frac{1}{1.3}\right)^2=0.414$$

$$\therefore \qquad R_{tip}=1-\left(\frac{C_{1w}+C_{2w}}{2U}\right)_{tip}$$

$$=1-0.414=0.586 \qquad \textbf{Ans.}$$

The degree of reaction increases from 0·3 at root to 0·586 at tip (almost 95·5% increase).

5·12. Axial flow compressor : U_w ; static pressure

In a stage of an axial-flow air compressor there is radial equilibrium over the cross-section of the annulus at exit from the rotor blades. Assuming that over this cross-section there is constant stagnation enthalpy and constant entropy, show that

$$r^2.\frac{\partial}{\partial r}(u_f)^2+\frac{\partial}{\partial r}(ru_w)^2=0$$

in which u_f is the axial component of the air velocity, u_w is the tangential component and r the distance from the compressor axis. If the hub-tip ratio is 0·8, the axial velocity is proportional to r, and at the hub,

$u_f=150$ m/s, $\qquad u_w=215$ m/s

Static pressure=1·4 kgf/cm²

Static temperature=20°C.

Show that at the tip, u_w will be approximately 138 m/s and determine the static pressure there. **(Punjab, M.E., 1971-72)**

$$r^2\frac{\partial}{\partial r}(u_f)^2+\frac{\partial}{\partial r}(ru_w)^2=0$$

Differentiating $r^2\,2u_f.\dfrac{\partial u_f}{\partial r}+2r^2.u_w\dfrac{\partial u_w}{\partial r}+2.u_w^2.r=0$

or $$u_f\frac{\partial u_f}{\partial r}+u_w+\frac{\partial u_w}{\partial r}+\frac{u_w^2}{r}=0$$

[which is the vortex equation derived when h_0 is a constant)

Given $\qquad u_f=kr \qquad$ and $u_f^2=k^2r^2$

Substituting the values :

$$2k^3r^3+\frac{\partial}{\partial r}+\frac{\partial}{\partial r}(ru_w)^2=0$$

$$\therefore \qquad \int_{r_R u_{w,R}}^{r_R u_{w,T}}\partial(ru_w)^2=-\int_{r_R}^{r_T}2k^2r^3\,dr$$

or $$\left(r^2u_w^2\right)_{r_\mathrm{R}u_{w,\mathrm{R}}}^{r_\mathrm{T}u_{w,\mathrm{T}}} = -\left(\frac{k^2.r^4}{2}\right)_{r_\mathrm{R}}^{r}$$

$$\therefore \quad r_\mathrm{T}^2u^2_{w,\mathrm{T}} - r_\mathrm{R}^2u^2_{w,\mathrm{R}} = \frac{k^2}{2}\left(r_\mathrm{R}^4 - r_T^4\right)$$

$$= \frac{u_{f,\mathrm{R}}^2 r_\mathrm{R}^4}{2r_\mathrm{R}^2} - \frac{u_{f,\mathrm{T}}^2 r_\mathrm{T}^4}{2r_\mathrm{T}^2}$$

$$\therefore \quad r_\mathrm{T}^2u^2_{w\mathrm{T}} - r_\mathrm{R}^2u^2_{w,\mathrm{R}} = \tfrac{1}{2}u^2_{f\mathrm{R}} - \tfrac{1}{2}u_{f\mathrm{T}}^2r_\mathrm{T}^2$$

$$\therefore \quad u^2_{w\mathrm{T}} - \left(\frac{r_\mathrm{R}}{r_\mathrm{T}}\right)^2 u^2_{w\mathrm{R}} = \frac{u_{f\mathrm{R}}^2}{2}\left(\frac{r_\mathrm{R}}{r_\mathrm{T}}\right)^2 - \tfrac{1}{2}u^2_{f,\mathrm{T}}$$

$$u^2_{w\mathrm{T}} = u^2_{w\mathrm{R}}\left(\frac{r_\mathrm{R}}{r_\mathrm{T}}\right)^2 + \frac{u^2_{f\mathrm{R}}}{2}\left(\frac{r_\mathrm{R}}{r_\mathrm{T}}\right)^2 - \tfrac{1}{2}u^2_{f\cdot\mathrm{R}}\left(\frac{r_\mathrm{T}}{r_\mathrm{R}}\right)^2$$

$$\left[\text{as } u_{f\mathrm{T}} = u_{f\mathrm{R}}\cdot\frac{r_\mathrm{T}}{r_\mathrm{R}}\right]$$

$$\therefore \quad u^2_{w\mathrm{T}} = 0.8^2(215)^2 + \frac{0.8^2}{2}(150)^2 - \tfrac{1}{2}\left(\frac{1}{0.8}\right)^2(150)^2$$

$$\therefore \quad u_{w\mathrm{T}} = \mathbf{138.5\ m/s}$$ **Ans.**

$$h_{o,\mathrm{R}} = c_pT_\mathrm{R} + \frac{u_\mathrm{R}^2}{2gJ} = 0.24\times293 + \frac{215^2+150^2}{2\times9.81\times427} = 78.6$$

$$u^2_\mathrm{T} = u^2_{f\mathrm{T}} = u^2_{w\mathrm{T}} = u^2_{f\mathrm{R}}\left(\frac{r_\mathrm{T}}{r_\mathrm{R}}\right)^2 + u^2_{w,T}$$

$$= 150^2\times\frac{1}{0.8^2} + 138.5^2 = 544\times10^2$$

and $$\frac{u_T^2}{2gJ} = \frac{544\times10^2}{2\times9.81\times427} = 6.49$$

$$\therefore \quad c_pT_\mathrm{T} = 78.6 - 6.49 = 72.11$$

and Tip temperature, $T_\mathrm{T} = \dfrac{72.11}{0.24} = 300$ K or 27°C

Now $$\frac{p_\mathrm{T}}{p_\mathrm{R}} = \left(\frac{T_\mathrm{T}}{T_\mathrm{R}}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{300}{293}\right)^{\frac{1.4}{1.4-1}}$$

$\therefore$ Tip static pressure, $p_\mathrm{T} = 1.4\left(\dfrac{300}{293}\right)^{\overline{1.4}} = \mathbf{1.53\ kgf/cm^2}$

Ans.

5.13. Aerofoil blading : pressure rise ; hp ; number of stages

An axial flow compressor takes in air at a pressure of 0.7 kgf/cm² and a temperature of 5°C. The blading is of aerofoil type, the values of C_L and C_d being 0.6 and 0.05 respectively, for the angles of incidence used, which is zero. The chord area of the aerofoil blades is 20 cm² and the blade length 6.5 cm ; the mean diameter of blade ring is 60 cm and its speed is 5000 rev/min. There are 50 blades on each *blade ring and the blades occupy one-tenth of the axial area of flow. The quantity of free air compressed is 850 m^3/min.*

Assuming adiabatic compression, calculate the pressure rise per blade ring and the theoretical horse-power required per stage. If pressure per stage is same for all stages, how many stages are required to produce a final pressure of 3·5 kgf/cm^2 ? What would then be the theoretical horse-power required to drive the compressor ? $R=29{\cdot}27$. Assume there is no prewhirl.

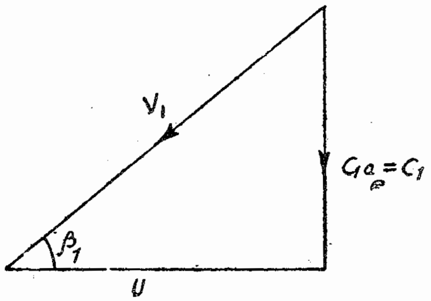


Fig. 5·44.

Mass/unit volume, $p_1v_1=mRT_1$

$$m=\frac{0{\cdot}7\times10^4\times1}{29{\cdot}27\times278}=0{\cdot}86 \text{ kg/m}^3$$

$$\rho=\frac{m}{g}=\frac{0{\cdot}86}{9{\cdot}81}=0{\cdot}0875$$

$$Q=k.\pi.d.b.C_{1a}.$$

Blades occupy 1/10th of flow area.

$$\therefore \qquad \frac{850}{60}=0{\cdot}9\times\pi\times0{\cdot}6\times0{\cdot}065\times C_{1a}$$

$$\therefore \qquad C_{1a}=\frac{850}{60\times0{\cdot}9\times\pi\times0{\cdot}6\times0{\cdot}065}=128{\cdot}5 \text{ m/s}$$

$$\text{Blade velocity, } U=\frac{\pi DN}{60}=\frac{\pi\times0{\cdot}6\times5000}{60}=157 \text{ m/s}$$

Assuming inlet velocity triangle to be a right angled triangle

$$\tan\beta_1=\frac{128{\cdot}5}{157}=0{\cdot}819$$

$$\therefore \qquad B_1=39^\circ\ 19'$$

$$\sin\beta_1=0{\cdot}6336, \quad \cos\beta_1=0{\cdot}7736$$

$$V_1=\frac{128{\cdot}5}{\sin\beta_1}=\frac{128{\cdot}5}{0{\cdot}6336}=203 \text{ m/s}$$

$$\text{Lift, } L=\frac{C_L \rho A V^2}{2}$$

$$=0{\cdot}6\times0{\cdot}0875\times\frac{20}{10^4}\times\frac{(203)^2}{2}=2{\cdot}162 \text{ kg}$$

$$\text{Drag, } D=\frac{C_D \rho A V^2}{2}=\frac{0{\cdot}05}{6}\times2{\cdot}162=0{\cdot}1801 \text{ kg}$$

$$\text{hp}=\frac{(L\sin\beta_1+D\cos\beta_1)n\pi dN}{4500}$$

$$=\frac{(2{\cdot}162\times0{\cdot}6336+0{\cdot}1801\times0{\cdot}7736)\times50\times\pi\times0{\cdot}6\times5000}{4500}$$

$$=\mathbf{157{\cdot}9}$$ **Ans.**

Mass of air compressed/sec

$$=\frac{850\times0{\cdot}86}{60}=12{\cdot}19 \text{ kg}$$

Energy imparted to 1 kg of air

$$=\frac{157{\cdot}9\times75}{12{\cdot}19}=971 \text{ kgf-m}$$

$$\therefore \qquad 971=RT_1\frac{\gamma}{\gamma-1}\left[\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}-1\right]\times\frac{1}{\eta_{adiabatic}}$$

$$=29{\cdot}27\times278\times\frac{1{\cdot}4}{0{\cdot}4}\left[\left(\frac{P_2}{P_1}\right)^{\frac{0{\cdot}4}{1{\cdot}4}}-1\right]\times1$$

$$\therefore \qquad \frac{P_2}{P_1}=\mathbf{1{\cdot}129}$$ **Ans.**

Rise of pressure required

$$=3{\cdot}5-0{\cdot}7=2{\cdot}8 \text{ kgf/cm}^2$$

$$\frac{P_2}{P_1}=\frac{3{\cdot}5}{0{\cdot}5}=5=(1{\cdot}129)^n$$

$$\therefore \qquad n=13{\cdot}35, \text{ say } 14 \text{ stages}$$

$$\therefore \qquad \text{Total hp}=157{\cdot}9\times4$$

$$=\mathbf{2210}$$ **Ans.**

EXERCISES 5

SECTION A

5·1. How dynamic compressors are classified ?

5·2. Explain the concept of total head or stagnation properties. Derive expressions for stagnation temperature and stagnation pressure.

5·3. Show the work done in a dynamic compressor on p-v and T-s diagrams. What is the increase in the work done due to non-isentropic compression ?

Derive an expression for the work done considering the actual process.

5·4. What is the difference between the isentropic efficiency based on total head condition and static condition ?

5·5. Explain the concept of polytropic efficiency. What is the need of this concept ?

5·6. Derive the expression for polytropic efficiency of an axial flow compressor in terms of

(a) n and γ ;

(b) inlet and exit total head temperatures and pressures ;

5·7. Derive a relation between isentropic and polytropic efficiency.

5·8. Sketch the velocity diagrams for a centrifugal compressor and derive the expression

$$\frac{p_{2t}}{p_{1t}}=\left(1+\frac{\eta_t\, p\sigma U_2^2}{gJc_pT_{1t}.}\right)$$

5·9. Explain the terms 'slip factor' and 'power input factor'.

5·10. Sketch the velocity diagrams for an axial flow compressor and derive the relation

$$\frac{p_{2t}}{p_{1t}}=\left[1+\eta_i\frac{U(C_{2w}-C_{1w})}{gJc_p-T_{1t}}\right]^{\frac{\gamma}{\gamma-1}}$$

5·11. Explain the term 'degree of reaction'. Why the degree of reaction is generally kept 50 per cent ?

5·12. What are the three main types of centrifugalcompressor impellers ? Draw the exit velocity diagrams for these three types. Discuss the characteristics of the three types and give their fields of application.

5·13. Briefly explain how a centrifugal compressor impeller is designed ?

5·14. Briefly explain how a centrifugal compressor diffuser is designed ?

5·15. Derive an expression for degree of reaction and show that for 50% reaction the blades are symmatrical, *i.e.* $\alpha_1=\beta_2$ and $\alpha_2=\beta_1$.

5·16. What is vortex theory ? Derive an expression for vortex flow.

5·17. What is an aerofoil ? Define, with a sketch, the various terms used in aerofoil geometry.

5·18. What is lift and drag coefficient of an aerofoil ? Write an express on for lift and drag. Show by a graph how the lift and drag coefficients vary with angle of attack.

5·19. What is meant by cascade theory ?

5·20. Explain the phenomena of stalling, surging, and choking in rotary compressors. How these can be controlled ?

5·21. Compare the characteristic curves of centrifugal and axial flow compressors.

5·22. Compare centrifugal and axial flow compressors and comment on their fields of applications.

5·23. Discuss the advantages and disadvantages and range of operation of various types of compressors (positive displacement and dynamic).

SECTION B

5·24. Rotary compressor : power input.

A rotary compressor, having a pressure ratio 10 : 1, takes in 25 kg of air per second at 15°C. Calculate the required power input in kW.

Take $c_p=0{\cdot}24$ and $\gamma=1{\cdot}4$.

[$T_2=557$ K, power 8260 kW]

5·25. Supercharger : hp absorbed.

A supercharger deals with a fuel-air mixture of 1 to 15·3, the volume of mixture produced by 1 kg fuel occupying 12 m³ at a temperature of 0°C and a pressure of 1·03 kgf/cm². The supercharger compresses the mixture from a pressure of 0·55 kgf/cm² to 1·03 kgf/cm², the initial temperature being −5°C. If γ for the mixture is 1·39 and the isentropic efficiency of compression is 85 percent. Find the horse power absorbed in driving the supercharger when the engine is using 4·7 kg of fuel per minute.

[$T_2=329{\cdot}2$ K, hp=103]

5·26. Centrifugal compressor ; η_i ; hp ; Δs

A centrifugal compressor delivers 100 kg of air per minute at a pressure of 2 kgf/cm² when compressing from 1 kgf/cm² and 15°C. If the temperature of the air delivered is 97°C and no heat is added to the air from external sources during compression, determine the efficiency of the compressor relative to ideal adiabatic compression and estimate the horse power absorbed. Take $\gamma=1{\cdot}4$.

Calculate also the change in the entropy of the air during compression and sketch the compression process on a temperature-entropy diagram.

[$T_2=351$ K ; $\eta_i=76{\cdot}9\%$; hp=187 ; $\Delta s=0{\cdot}01265$]

5·27. Axial flow compressor ; no. of stages.

An axial flow compressor provides a pressure ratio of 5 : 1 with an overall head isentropic efficiency of 87 per cent when the total head inlet temperature is 15°C. The compressor is designed for 50% reaction with inlet and outlet air angles from the rotor blades of 44° and 13° respectively. The mean blade speed and axial velocity are constant throughout the compressor. Assuming a blade velocity of 170 m/s and work input factor of 0·85, find the number of stages required.

[$T_2-T_1=193$ K ; $C_a=142{\cdot}1$ m/s ; $\Delta Cw=104{\cdot}2$ m/s, W.D.=3·6 kcal, Temperature rise=15°C ; No. of stages=13]

5·28. Total head pressure and temperature.

Show from first principles that the total head temperature rise of the air passing through a centrifugal compressor is directly proportional to the square of the impeller speed. Develop an expression for the corresponding total head pressure ratio in terms of this rotational speed, explaining how the former is influenced by a change in the intake total head temperature.

The overall impeller diameter of such a compressor is 50 cm. Air is inspired at a total head temperature of 15°C and the isentropic efficiency of compression is 80%. Neglecting the effect of slip and other losses, determine the compressor total head pressure ratio at a speed of 16,000 r.p.m. Find also the increase in total head temperature.

5·29. Axial compressor ; r ; h, vane angles.

An axial flow air compressor is required to deliver 20 kg of air per second at a speed of 9000 r.p.m. Static temperature and pressure at inlet to first stage may be taken as one atmosphere and 300 K. Take stage temperature rise to be 17°C and the peripheral velocity at mean radius as 180 m/s, axial velocity 150 m/s.

Find the mean radius, height of blades and air angles for 50 per cent reaction machine.

5·30. Centrifugal compressor : p_2/p_1 ; hp ; vane angles

In a double-sided centrifugal compressor the following data is given :

Overall diameter of impeller	=50 cm
Eye tip diameter	=30 cm
Eye root diameter	=15 cm
R.P.M.	=15000
Total mass flow	=18 kg/s
Inlet total head temperature	=295 K
Total head isentropic efficiency	=78%
Power input factor	=1·04
Slip factor	=0·9

Find (*a*) the total head pressure ratio ;

(*b*) the hp required to drive the compressor ;

(*c*) the inlet angles of the vanes at the root and tip impeller eye.

Assume that the velocity of air at inlet is 150 m/s and is axial, and remains constant across the eye annulus. [*Chandigarh, M.Sc. (Engg.), 1969*]

[U=393 m/s ; P_{2t}/P_{1t}=**3·08** ; **hp=3530**, β_{1root}=**57·8°**, β_{1tip}=**32·4°**]

5·31. Aerofoil blading ; hp ; p_2.

Air at 1·03 kgf/cm² and temperature 15°C is compressed by an axial flow compressor fitted with aerofoil blading. The values of C_L for the aerofoil blade at the angle used is 0·35 and C_D is 0·06. The cord area is 16·5 cm², the mean diameter of the blade ring is 46 cm, length of the blade is 5 cm and speed of the impeller is 8000 r.p.m. The blades occupy 20 per cent of the axial area of the flow and there are 40 blades per impeller. The quantity of free air compressed per minute is 565 m³. The outlet angle of the fixed blade is 90°.

Calculate the theoretical horse power required per stage and the final theoretical pressure leaving the stage.

Assume adiabatic efficiency 0·85, γ=1·4, and the best angle of incidence=0. R=29·27.

[C_{a1}=163 m/s ; U=192·6 m/s, β_1,=40·2°, L=2·3 kg, D=0·394 kg, **hp=183**, m=11·5 kg, W=1192 kgf-m ; p_2/p_1=1·126 ; **p_2=1·152 kgf/cm²**]

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6

GAS TURBINE COMBUSTION CHAMBER

6·1. INTRODUCTION

The main purpose of a gas turbine combustion chamber is to introduce heat energy into the mass of air compressed by the compressor, by burning fuel in it so that the products of combustion can be expanded to get useful work output. Combustion in a gas turbine is a continuous process and peak combustion temperatures exist continuously in the combustion chamber. This is in contrast to reciprocating engines where the peak cycle temperature is encountered only for a very short duration in the cycle. Moreover, due to space limitations and of energy and momentum requirements the volume flow rate as well as rate of heat release is very high in a gas turbine combustion chamber and the residence time of fuel is very small, of the order of a few milli-seconds. Thus continuously high combustion temperatures, large continuous flow, and high heat energy release make the design and development of a gas turbine combustion chamber rather difficult. Most combustion chamber designs are empirical in nature and are result of years of hard development work. There is still a great deal to learn about combustion phenomena in a gas turbine combustion chamber. In what follows the salient features of gas turbine combustion chamber design and its performance are discussed in its present state of art.

6·2. REQUIREMENTS

Very high rates of fuel consumption in gas turbine plants require that the design of the combustion chamber, where all the energy is developed, must be given a close consideration if such a plant is to compete with other forms of power plants. The following are, in brief, the main requirements of a gas turbine combustion chamber :

1. High combustion efficiency.
2. Minimum pressure losses.
3. Stable and smooth combustion over the full operational range.
4. Uniform exit temperature.
5. Low combustion chamber wall temperature compatible with long chamber life.

6. Easy starting.
7. Minimum size and weight.
8. Freedom from carbon deposits and smoke.
9. High degree of reliability.
10. Easy to dismantle and service.

Many of the above requirements are mutually incompatible ; for example, a high degree of turbulence necessary for good combustion efficiency results in higher pressure losses ; and again, it is very easy to get a good combustion efficiency and uniform temperature distribution if there is no limit on length of the chamber. Moreover, the above requirements vary widely in their importance over the type of engine and its application. For aircraft purposes small space, low weight, and high temperature but with a small life of a few hundred hours is thought to be sufficient. For industrial applications a much longer working life (about 100000 hours) is important but size is not very important. In addition to this, a particular application may impose further requirements. Hence the design of a combustion chamber is essentially an exercise in compromise of many mutually incompatible requirements.

6·3. COMBUSTION PROCESS IN GAS TURBINES

For good combustion efficiency the fuel must be atomised into small droplets and must be thoroughly mixed with the air. The inlet temperature and the turbulence required for mixing should be sufficient to reduce ignition delay so that while passing through the combustion chamber with a moderately low velocity the fuel-air mixture gets sufficient time for complete burning and uniform temperature distribution before it reaches the exit.

Fig 6·1 (*a*) shows a combustion chamber in its simplest form, *i.e.* a long duct. Such a combustion chamber will have a very high degree of pressure loss during combustion which may reach a value up to about 25 percent of the total compressor pressure. The pressure loss in a combustion chamber consists of two parts : (*i*) Burning losses which include the losses due to turbulence and change in momentum of gases. (*ii*) loss due to the skin friction between the gases and the wall. To reduce the pressure losses a diffuser as shown in Fig. 6·1 (*b*) is fitted which reduces the velocity to about one-fifth or one-sixth of the original velocity, *i.e.* from about 75 m/s at the entrance of combustion chamber to about 12 to 15 m/s in the combustion chamber. But since the flow is continuous, even at this low velocity it is not possible to have stable combustion and to achieve this the air should be made almost stagnant or a reversal of flow is sometimes necessary. This is done by providing a baffle as shown in Fig. 6·1 (*c*) or by swirlers.

However, one more shortcoming still remains. It is the fact that in a gas turbine plant the overall air-fuel ratio used is about

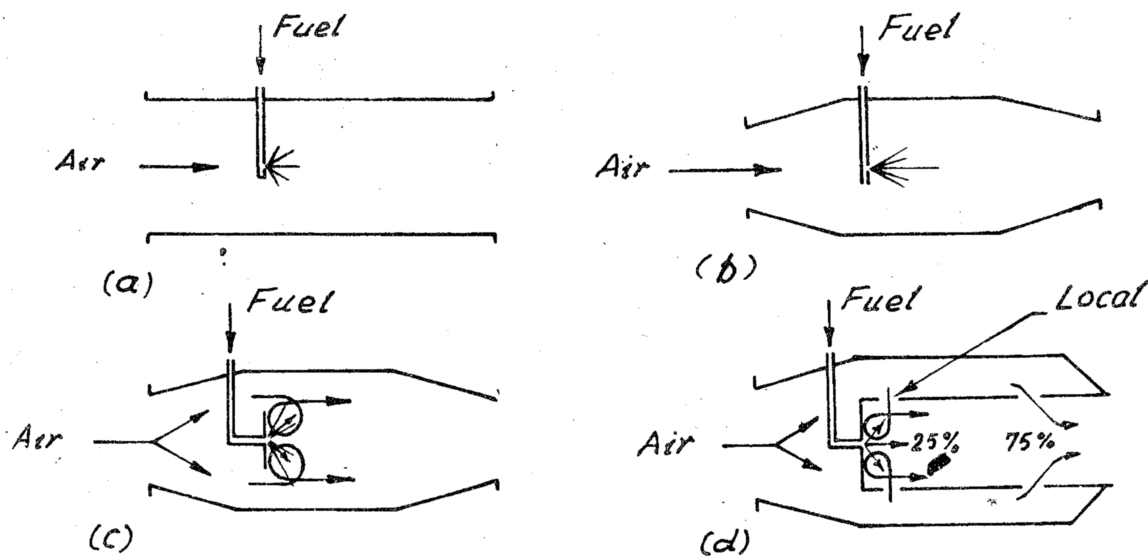


Fig. 6·1. Development of a gas turbine combustion chamber.

60 : 1 to 70 : 1, which is well outside the inflammability limits. So a device must be provided by which a local air-fuel ratio of about 15 : 1 to 13 : 1 can be produced for efficient burning of the fuel. This is done by providing, instead of a baffle, an inner chamber as shown in Fig. 6·1(*d*). This chamber has a number of holes through which the air necessary to produce a near stoichiometric air-fuel ratio is provided. This air is called *primary air*. The products of combustion in this inner chamber are at very high temperature (about 2000°C) which cannot be used in turbine, so a secondary supply of air is provided downstream the inner chamber through similar holes to reduce the temperatures to an acceptable level. The *secondary air* is supplied in such a manner that a thorough mixing of air and products of combustion takes place to give uniform temperature at the exit of the chamber. The secondary air is also used to cool the chamber. The primary air amounts to about 25 percent of the total air supply.

Thus a combustion chamber essentially consists of four important parts : fuel injector, diffuser, inner chamber, and outer chamber supplying the secondary air. Each combustion chamber will provide for separate combustion and mixing zones, efficient atomisation of fuel for good combustion and some method of recirculation of a part of products of combustion for maintaining combustion stability.

6·4. TYPES OF COMBUSTION CHAMBER

A wide number of configurations are used for combustion chamber depending upon the gas turbine engine and its applications to meet the specific requirements of a given use. However, all of them can be classified into three main types :

1. Tubular or can type.
2. Annular type.
3. Tubo-annular or can-annular type.

1. Tubular or can type combustion chamber

As shown in Fig. 6·2, a tubular or can type combustion chamber consists of an inner chamber which admits the primary air through swirl vanes near the fuel jets of the injector nozzle. Some air from outer chamber is also fed through other holes near the primary

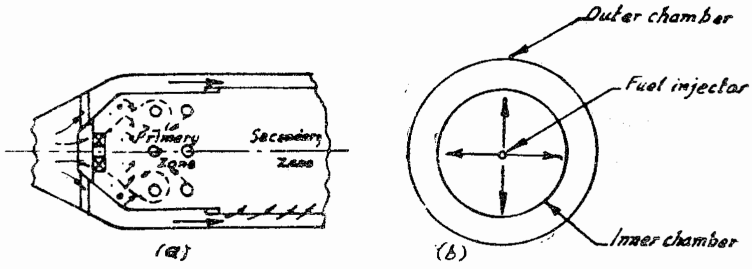


Fig. 6·2. Schematic diagram of a tubular or can combustion chamber of a turbojet engine using swirlers.

zone. Other perforations downstream the chamber admit secondary air. Fig. 6·3 shows another form of can type combustion chamber.

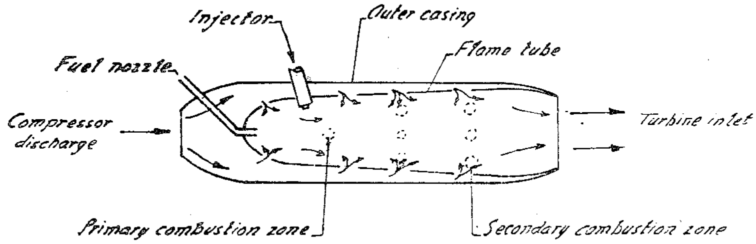


Fig. 6·3. Schematic diagram of a typical 'can type' combustor.

An inner chamber is surrounded by a larger outer chamber through which compressed air flows. A part of this air is fed to the primary zone and the rest of the air travels in axial direction and is continuously fed to the inner chamber till near the dilution zone where all the secondary air is introduced in the inner chamber.

The air flowing through the outer chamber keeps the inner chamber cool and also does not allow the heat to go to the outer wall of holes. A part of the primary air which is introduced through holes gets preheated in this way resulting in better vaporisation and improved combustion.

In a combustion chamber the number of cans varies from 6 to 16 and are arranged around the axis of the gas turbine. Each forms a complete burner in itself. To ensure simultaneous combustion all the cans are interconnected.

Can type of combustion chambers are widely used for industrial applications where bulk or space are not of critical importance. These are not suitable for aircraft engines, due to large frontal area and weight. This type of combustion chamber is most adaptable with the centrifugal type of compressor since diffuser divides the compressed air into channels. This gives good control of combustion. However, it does not utilize space efficiently and large areas of metal are required to enclose a given gas flow thus, making the combustion chamber heavier. With the replacement of centrifugal compressor by axial compressor, the popularity of can type combustion chambers is reducing. Large curvature of chamber surfaces gives it high strength and resistance to warping.

2. Annular combustion chamber

Annular combustion chamber is a concentric chamber surrounding the axis of the rotor thus forming an annular shape between the rotor and the outer casing. The fuel is injected into an inner annular chamber where primary air is also supplied. This is surrounded by another annular chamber, exactly like that in can type chamber, which supplies the secondary air and also the cooling air.

Fig. 6·4 shows a double annular combustion chamber which has two concentric annular chambers.

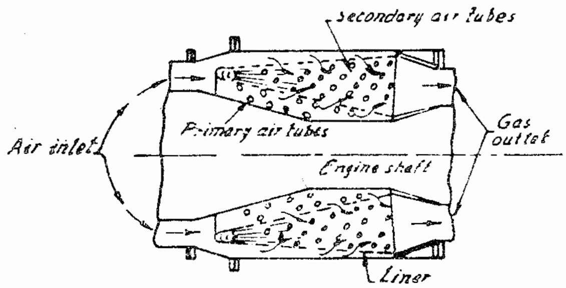


Fig. 6·4. Annular type combustion chamber.

Annular combustion chambers have least pressure drop due to larger volume per unit surface area and are more efficient than can type chambers. It requires about half the diameter of can type chamber for the same mass flow. However, any change in the flow velocity will result in significant change in the temperature distribution, and distortion of inner annular chamber is critical. This is because of lower degree of curvature of the chamber surfaces.

3. Tubo-annular combustion chamber

A tuboannular or can-annular type of combustion chamber, which is a combination of tubular and annular, chamber, consists of a number of cylindrical chambers arranged inside a single annular casing all around its circumference as shown in Fig. 6·5. Each burner is a small annular chamber in that there is an inner chamber through which the air flows to cool the inside of the chamber. Annular combustion chambers have characteristics of both can and annular type of chambers. The large curvature of linear surface makes it more resistance to warping and higher strength while the annular arrangement gives efficient space utilization resulting in

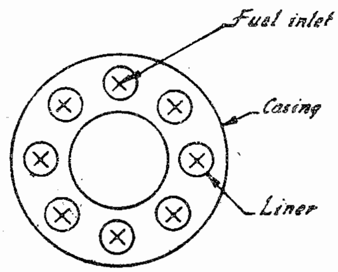


Fig. 6·5. Can-annular combustion chamber.

reduced diameter and weight. This type of combustion chamber is robust and has low pressure losses and is used for engines with higher pressure rates. The table 6·1 compares the three types of combustion chambers.

TABLE 6.1. COMPARISON OF COMBUSTION CHAMBERS

S.N.	*Chamber Type*	*Advantages*	*Disadvantages*
1.	Tubular or can type	(*i*) Mechanically robust. (*ii*) Fuel and air flow patterns easily matched (*iii*) Cheap and easy to develop.	(*i*) High pressure loss. (*ii*) Requires more space and is heavy. (*iii*) Larger frontal area.
2.	Annular	(*i*) Minimum pressure loss. (*ii*) Minimum length and weight. (*iii*) Minimum engine frontal area.	(*i*) Sensitive to change in flow velocity. (*ii*) Difficult to develop. (*iii*) Uniform temperature distribution difficult to obtain. (*iv*) Serious buckling problem of outer annular chamber.
3.	Tubo-annular or can-annular	(*i*) Mechanically robust. (*ii*) Lower pressure loss. (*iii*) Shorter and lighter than tubular chamber.	(*i*) Requires interconnectors. (*ii*) Less compact than annular.

6·5. FLOW PATTERN IN A COMBUSTION CHAMBER

A gas turbine combustion chamber has two main zones. One is the primary zone of recirculation to stabilize the burning over the widely varying fuel-air ratios covering fuel operational range of the gas turbine. This recirculation is obtained by supplying a small portion of the total air with the help of a swirler or some other device. The recirculation or the back flow of hot gases causes them to mix with the incoming fuel-air mixture and ensures good combustion stability over a wide range of fuel-air ratios.

Fig. 6 6 shows a typical industrial gas turbine combustion chamber and its flow pattern. The corresponding pressure, tem-

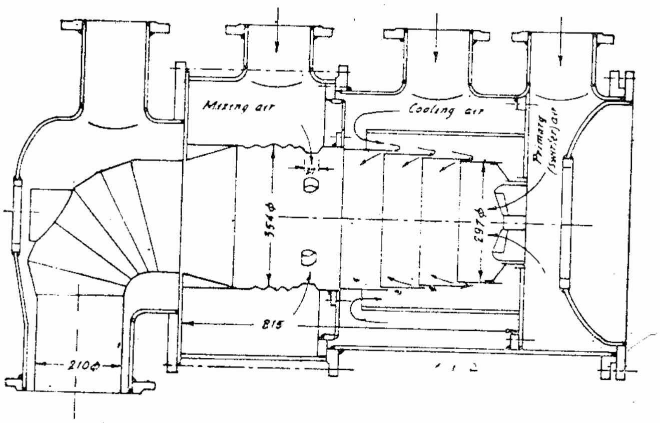
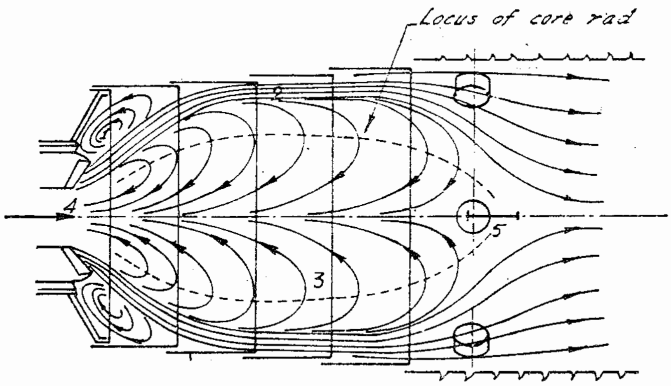


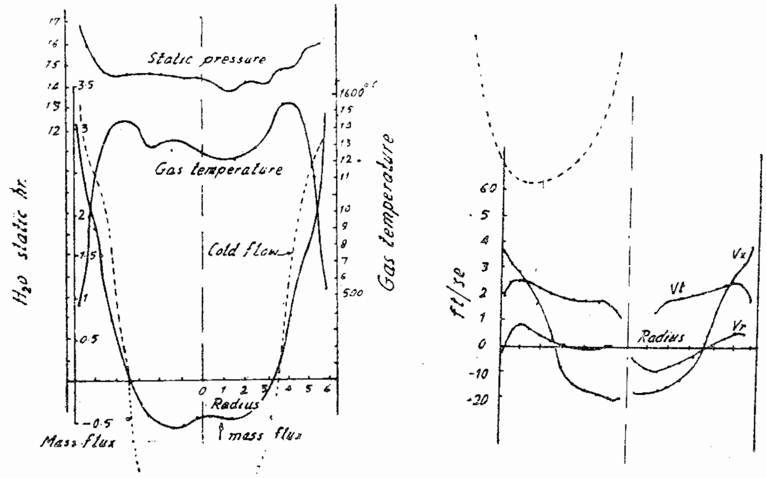
Fig. 6·6 (*a*). Section showing constructional features of an industrial gas turbine combustion chamber (Dimensions in mm).



1. Strong torodial movement near swirler end of chamber.
2. Major part of flow adjacent to wall where velocity has negligible radial component.
3. Curved stream lines leading to flow reversal.
4. Inner stagnation zone.
5. Outer stagnatian zone.

Fig. 6·6 (*b*). Flow pattern in the industrial gas turbine combustion chamber shown above.

perature and velocity profiles are shown in Fig. 6·7. The swirler imparts angular momentum to the primary air resulting in strong torodial movement near swirler end of the chamber due to appreciable radial pressure gradients at this section. The axial velocity is maximum near the chamber wall (where the radial component of the velocity is negligible) outside the boundary layer. Then it decreases gradually towards the axis of the chamber and reaches a zero velocity at the core radius. Inside the central core the axial velocity is negative, *i.e.* recirculation occurs (*see* Figs. 6·6 and 6·7)



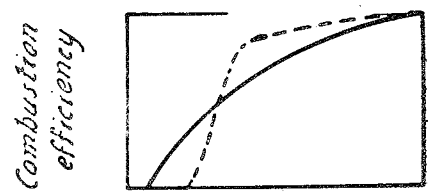
(*a*) Pressure and temperature profiles. (*b*) Velocity profiles.

Fig. 6·7. Pressure, temperature and velocity profiles in gas turbine combustion chamber.

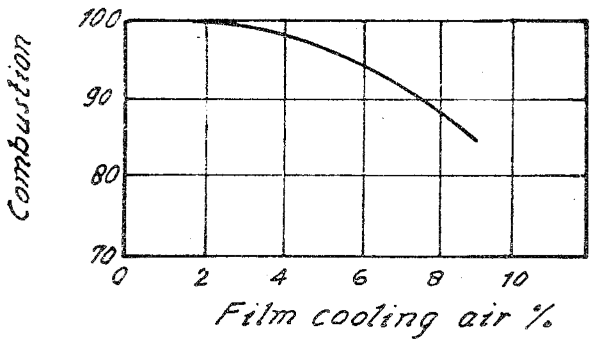
which is necessary for stable operation. Recirculation increases the residence time of the fuel-air mixture allowing more time for combustion, thus resulting in efficient combustion. The tangential velocity is zero at centre indicating fixed body rotation.

In the second combustion zone secondary air is introduced gradually to complete the combustion and reduce the temperature of the burnt gases before reaching the turbine inlet. As the cooling air is added progressively at various sections downstream the chamber the specific angular momentum decreases resulting in a decrease in the tangential velocity as the flow travels downstream. This is because the cooling air has momentum in axial direction only. The wall friction losses, in addition to the effect of cooling air, cause the static pressure to decrease at outer radius of flow and increase at centre of the flow (*see* Fig. 6·8). Thus the axial pressure gradients near the chamber wall and around centre have opposite directions giving rise to reversal of flow.

Due to combustion the local stagnation temperature increase is not uniform which results in further non-uniformity due to different combustion rates. This reduces the radial pressure gradient in central core of the combustion chamber. The different burning rates are evident from the temperature profiles in Fig. 6·7.



(a)



(b) Combustion efficiency *vs* film cooling air.

Fig 6·8. Effect of type of mixing and amount of film cooling.

The flow pattern can be greatly modified by the degree of swirl, size and location of primary and secondary zone air holes, and the fuel spray pattern.

6·6. PERFORMANCE AND OPERATING CHARACTERISTICS OF COMBUSTION CHAMBERS

The following is a brief and general discussion of the main performance and operating characteristics of a gas turbine combustion system. The characteristics which are of importance for a given chamber greatly depend upon its application and are decided accordingly. Therefore, here no attempt has been made to discuss the individual application requirements.

6·6·1. Combustion efficiency

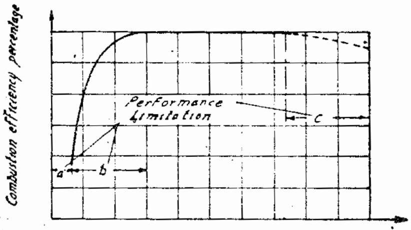
Combustion efficiency is defined as the ratio of actual heat release to the theoretical heat release in a combustion chamber and is given by :

Combustion efficiency

$$= \frac{\text{Actual total head temperature rise in combustion chamber}}{\text{Theoretical total head temperature rise}} \qquad (6\cdot1)$$

The heat released in a given combustion system depends upon a great number of factors, the important being inlet pressure and inlet temperature. Any increase in pressure and temperature at combustor inlet reduces the ignition lag. Noting the fact that the residence time of the fuel-air mixture in a gas turbine is very short, of the order of a few milli-seconds, this effect becomes very important. Higher the inlet temperature lesser is the ignition lag, hence better is the combustion. The inlet pressure for a given combustor also affects the inlet velocity and in turn the turbulence and the quenching effect of air near the wall and the pressure losses in the chamber [*see* Fig. 6·8 (*a*)]. The type of mixing used to cool the products of combustion in secondary zone, and the amount of film cooling also affect combustion efficiency as shown in Fig. 6·8 (*b*).

Fig. 6·9 shows the relationship between combustion efficiency and combustion pressure. The figure is divided into four main parts. Performance limit "*a*" refers to the impossibility of combustion due to poor atomisation and heat losses, zone "*b*" is the regime in which the reaction rates determine the combustion efficiency and the atomisation is relatively unimportant. This regime ranges from



1. a=Atomi·ation and heat losses
2. b=Chemical reaction rates
3. c=Evaporation and/or mixing.

Fig.6·9. Combustion efficiency as related to combustion pressure.

a pressure of 0·15 to 2 kgf/cm². The third zone refers to normal combustion chamber operation. It can be seen that very high combustion efficiencies, very near to 100 per cent are obtainable in this range and as the pressure increases the combustion efficiency becomes independent of reaction rates. However, after a certain pressure the vaporisation and mixing process in the combustor is adversely affected and combustion efficiency starts falling. This is because of the reduced spray angle and reduced penetration at high combustion pressures both of which result in over rich local fuel-air ratios resulting in an increase in smoke. The inadequate mixing at higher pressures due to poor atomisation results in reduced combus-

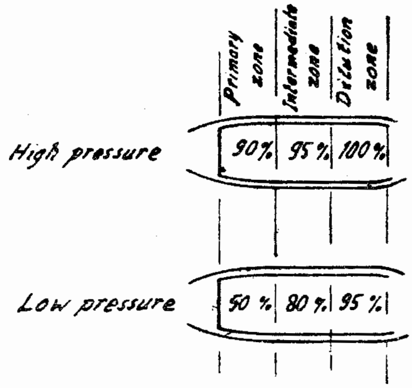


Fig. 6.10, Influence of pressure on combustion efficiency of different combustion zones.

tion efficiencies as denoted by zone C. The combustion efficiencies obtainable in various zones of a combustor at higher and lower pressures are shown in Fig. 6·10.

The combustion efficiency of jet-engines usually decreases with increase in altitude. The reduced heat release rate allows a higher inlet velocity in the combustor (combustion in a combustor can be approximated by Rayleigh flow), *i,e.* the residence time is reduced. This reduction in residence time further reduces the combustion efficiency because now there is less time available during which the process of atomisation, vaporisation, mixing and combustion must be completed and the effect of altitude is cumulative in nature. Fig. 6·11 shows variation in combustion efficiency with altitude for three different types of combustion chambers.

Usually it is possible to obtain a combustion efficiency higher than 95 percent under normal operating conditions. However, performance parameters also warrant close consideration for different applications. For example in an industrial combustor a very good combustion efficiency of nearly 100 percent can be obtained because it is possible to have high residence time (as high as 6 milisec), while due to high flow requirements for an aircraft combustor the residence time is very low and good combustion efficiency over the operational range is rather difficult to achieve.

2. Combustion Intensity

One measure of the heat release in a given combustion chamber is *combustion intensity* which is defined as :

$$\text{Combustion intensity} = \frac{\text{heat released in kcal}}{\text{hr} \times \text{m}^3 \times \text{atm}}$$

From the above relation it is clear that combustion intensity is an indication of the heat release rate in a given space and with a certain pressure, thus takes into account the inlet conditions. High combustion intensity means a smaller space. Table 6·2 shows typical combustion intensities for various combustion systems The aircraft combustors, due to severe space limitations, require very high combustion intensity while industrial gas turbine combustors require a rather low of combustion intensity.

TABLE 6·2. TYPICAL COMBUSTION INTENSITY

S.N.	*Process*	*Operating condition*	*Combustion intensity kcal/m³-hr-atm*
1.	Bunsen flame (town gas).	Low pressure,	$1·3\times10^6$
2.	Fuel-oil flames from commercial atomisers.	Burning in open.	0 09 to $0·2\times10^6$
3.	Oil fired boilers.		0·036 to $0·18\times10^6$
4.	Industrial gas turbine combustor.	Normal condition.	$0·2\times10^6$
5.	Aircraft engine (4 stroke).	2400 rpm, 110 octane fuel.	about 4×10^6
6.	V-1 engine.	Normal runing.	about $1·8\times10^6$
7,	V-2 rocket.	Maximum thrust.	11×10^6
8.	Aero turbine engine.	Design conditions.	1·5 to 4×10^4
9.	Marine Boiler.		0·1 to 10^6
10.	Domestic coal boiler.		$0·006\times10^6$

3. Pressure losses

For any flow process there must be a pressure loss. However, since occurrence of a pressure loss in a Brayton cycle means reduction in the work output and cycle efficiency, the pressure losses in a combustor must be reduced to a minimum. Pressure losses also result in lower mass flow and, hence, in reduced thrust in case of aircraft engines. At a pressure ratio of 4 : 1 an increase in combustor pressure loss by 3 percent is equivalent to 2 percent reduction in compressor efficiency. Since about 70 percent of the power developed by the turbine is used in driving the compressor, this loss amounts to a big drop in the net output of the turbine. Combustor pressure losses can be divided into two main parts :

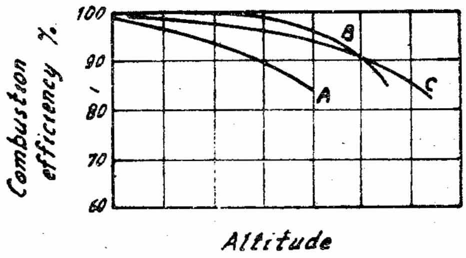


Fig. 6·11. Variation of combustion efficiency with altitude for three different combustors.

1. Pressure losses due to change in momentum as a result of addition of heat.

2. Skin and fluid friction pressure losses.

The friction pressure losses and momentum pressure losses can be calculated by assuming one-dimensional adiabatic flow in the combustion zone. However, the analysis is cumbersome and beyond the scope of this book.

The turbulence and skin friction losses depend upon the type of mixing of secondary air with products of combustion. Higher the turbulence greater is the pressure loss. Hence the requirement of low pressure loss and uniform temperature distribution at combustor outlet cannot be satisfied simultaneously and a compromise is always used by altering the relative amounts of primary and secondary air by suitable modifications in the manner in which the air is introduced in the combustion chamber.

Another important fact which governs the acceptable pressure loss in a combustor is its length. Fig. 6·12 shows how the length of a combustor can be reduced at the expense of pressure losses. This also shows the reduction in thrust due to increased pressure losses. The combustor length can be reduced by providing more mixing so that combustion is complete over a shorter length. This is done by allowing a higher air velocity at the exit of the compressor diffuser and allowing it to enter at steep angle, thus promoting deep penetration of incoming air into hot gases. The consequent rapid mixing results in efficient combustion. One advantage of this method is that the increased pressure losses are partially compensated due to lower level of parasite losses in the compressor diffuser because now relatively less diffusion is required and higher exit velocities can be obtained.

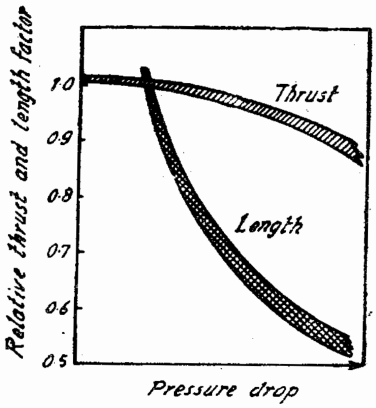


Fig. 6·12. Effect of pressure drop on combustor length and engine output.

4. Combustion chamber length

The combustion chamber length is important because reduced length means reduced bulk and weight both of which are of prime importance in aircraft turbines. For industrial turbines, only when space is limited this factor might be of importance but normally length is not important for such uses.

The reduction in length as obtainable by accepting a higher pressure loss in the combustion chamber has been already described. Anothr method of reducing turbine engine length at the expense of a higher pressure loss is the integration of turbine stators into the burning zone of combustion chamber as shown in Fig. 6·13. Most of the secondary or diluent combustion air passes through the hollow turbine stators located within the combustor to cool them and then goes out through the holes in the surface of an aerofoil to mix with the primary airflow to complete the combustion process. Thus the

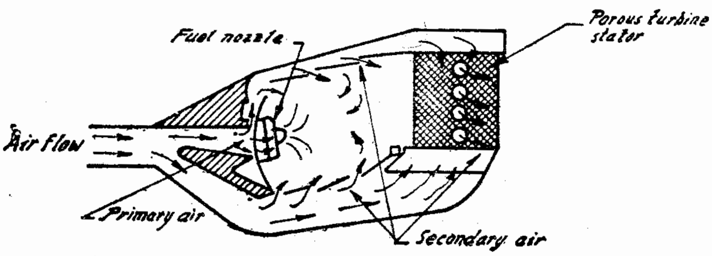


Fig. 6·13. Combustor with integral turbine stators.

combustion dilution and turbine inlet turning process are accomplished togather and w thin the same length. Careful selection of size and location of the passages in the stator can provide a very good temperature distribution at turbine inlet.

Another method of reducing the overall length is double banking of flame-tubes as shown Fig. 6·14. This, however, results in lower combustion efficiency because the intermediate zone of combustion (*see* Fig. 6·10), whose function is to recover the dissociation loss, is removed. This zone, at low pressure, is very similar

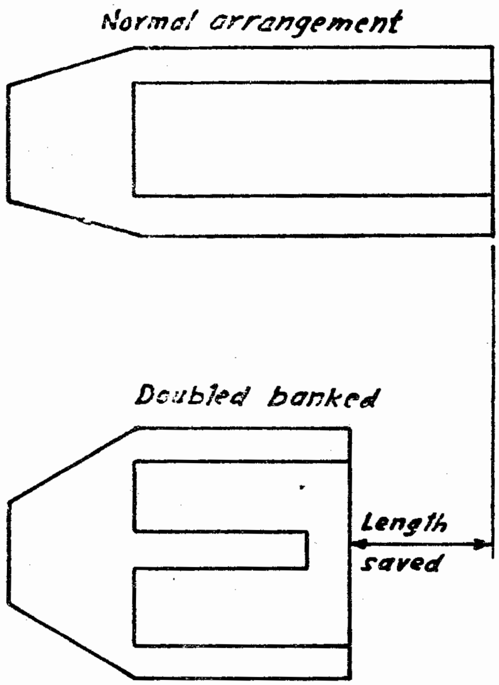


Fig. 6·14. Saving in overall length by double banking.

to primary zone and the combustion efficiency falls by a few per-cent. This loss of combustion efficiency results in higher specific fuel consumption and must be weighed against reduced weight and size for a particular application.

5. Temperature distribution

The turbine blades are the most stressed part of the gas turbine. The high rotational velocities and the high operating temperatures (which reduce the material strength) cause them to operate very near their strength limits. The temperature at the turbine inlet, therefore, should be uniform otherwise the turbine blades would expand unevenly, and highly localized stress levels would set up. This might result in warpage and even cracking of the blades. Thus very serious stress conditions are created by non-uniform temperature distribution at turbine inlet. Theoretically the temperature near the walls should be lower than that at centre because the secondary air is blown almost longitudinally near the liner walls to keep them cool. The temperature is higher in the central core till the opposite boundary layer is reached. This temperature distribution is made uniform by providing more mixing length and higher turbulence in the secondary combustion zone. This, however, results in increased pressure losses. Even in the central core the temperature is uneven because of different burning rates across the combustor cross-section. This is because the uneven local fuel-air ratio results in lower flame velocities near the outer surface of this central core and higher velocities in the central core. In addition to this the recirculation necessary for stabilization of combustion in the primary zone also causes uneven temperature distribution (*see* Figs. 6·7 and 6·15).

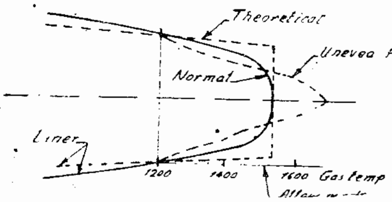


Fig 6·15. Temperature distribution as affected by fuel-air ratio distribution.

Other possible reasons of uneven temperature distribution are operation outside the stability limits, carbonization, inefficient combustion and uneven fuel-air ratio distribution due to injection system used. The control of temperature distribution at combustion chamber exhaust is very important for avoiding unduly high stresses on turbine blades. Unfortunately no theoretical treatment is possible and development work is the only approach used to control this temperature distribution.

6. Blowout

If the flame propagation velocity is more than the axial velocity of the fuel-air mixture, the flame will travel out of combustion chamber exit section and if much less than mixture velocity it can travel upstream ; in both cases the result is blowout. The flame propagation velocity is maximum at near stoichiometric fuel-air ratio and decreases for lean or reach mixtures (Fig. 6·16). Thus

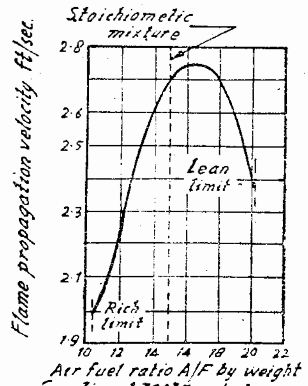


Fig. 6·16. Flame velocity *vrs.* fuel-air ratio.

the blowout occurs when too rich or, too lean air-fuel mixtures are encountered. This may happen during sudden acceleration or deceleration from high speed. Due to inertia of the rotating and other parts during acceleration, too much fuel is injected resulting in very rich mixtures and during deceleration the fuel flow drops much more rapidly than air flow causing lean mixture blowout. At high altitude operation the temperatures and pressures are low. Low pressure causes poor atomisation and lean fuel-air mixture; and low temperature and low pressure both causes ignition lag to increase to levels where no burning can take place because it is more than the residence time of the mixture. Incomplete burning and flame quenching may also result in blowout. Another possible reason of blowout may be a comparatively lean mixture which is extinguished by high turbulence of the secondary air flow. Any disturbance in temperature, pressure, and velocity profile which may cause surging in the compressor can also result in blowout. Thus careful matching of the various parts of the combustion chamber as well as those of gas turbine components is essential to avoid possible blowout tendencies within the operational range of the gas turbine.

7. Stability

Earlier it was mentioned that when the flame propagation velocity is either less or more than air velocity blowout occurs. However, it is neither necessary nor possible that the flame velocity be exactly equal to air velocity because the flame front is never normal to the air flow direction and there is always a range of flame velocity for a given air velocity which will result in stable flame. Since flame velocity depends upon fuel-air ratio, there is a rich as well as a lean fuel-air ratio limit, for a given air velocity as shown in Fig. 6 17 within which a stable operation is possible. As the air velocity increases the range of fuel-air ratio for stable

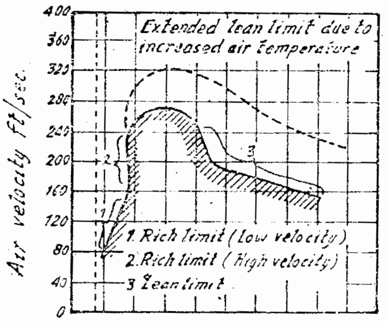


Fig. 6·17. Flame stability limits at different air speeds and fuel-air ratios.

operation narrows and after a certain air velocity no combustion can take place.

Any factor which affects the ignition delay, flame velocity and residence time is likely to affect the stability loop. The effect of increased air temperature in enlarging the stability loop is shown in Fig. 6·17 and the effect of atomisation is shown in Fig. 6·18. This figure also shows the stability limits for different types of combustion chambers.

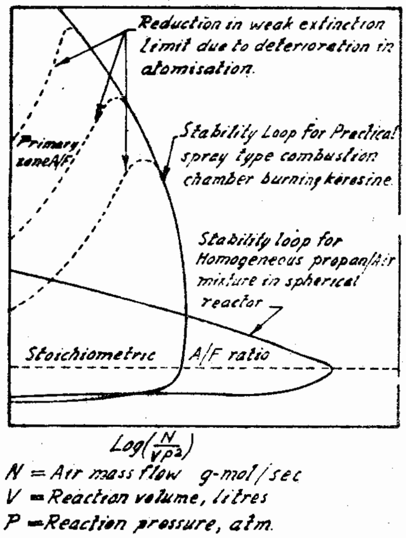


Fig.6·18. Stability characteristics of various combustion chambers and effect of atomisation on stability.

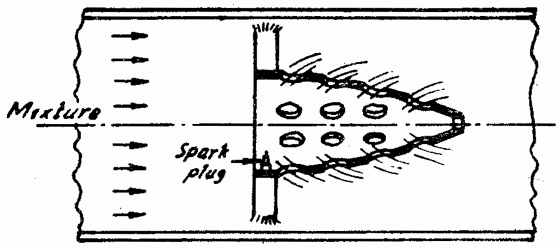


Fig 6·19. (*a*) T·ansverse jet flame holder.

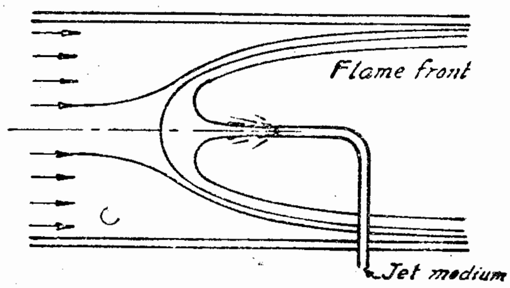


Fig. 6'19 (*b*) Reverse jet flame holder.

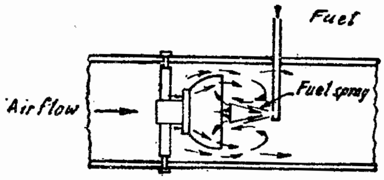


Fig 6·19. (*c*) Baffie for stabilizing flame.

Usually the performance range of a combustion chamber is increased by production of a combustible air-fuel mixture under all conditions of operation by means of swirlers, baffles, and flame shields. All these devices divert the primary air into a low velocity region where suitable fuel-air ratio is generated and burned. This protected region may be produced by a transverse jet flame holder in which hot gases pass through a wall having a number of holes [Fig. 6·19 (*a*)] or a reverse jet flame holder [Fig. 6·19 (*b*)], or a baffle plate [Fig. 6·19 (*c*)], which acts as an obstacle for the main air flow and generates low velocity region uprstream the baffle. The swirler (*see* Fig. 6·6) also stabilizes the flow by producing recirculation of hot products of combustion.

Good stability range is obtained by good atomisation along with optimum fuel-air mixing even at the expense of higher pressure loss in the combustors of aircraft engines which have to operate over a very wide performance range.

8. Starting and initiation of combustion

A spark plug is used to initiate the combustion. Usually the normal mixture ratios obtained in gas turbine combustion chambers give good starting characteristics. The spark plug must be designed to ensure re-starting at high altitude where pressure, temperature, and densities are low.

For ramjet combustors a considerable forward speed is necessary before ignition can occur, therefore, such engines are launched by some other propulsion device before starting of combustion.

9. Carbon deposits and smoke

In the primary combustion zone there is a region in which of hot gases due to flow reversal meet the incoming fuel spray in such a manner that the oxygen availability is very low whereas, the temperature is very high. This results in craking of the fuel and production of carbon in the primary zone. A part of this carbon gets deposited on injection nozzle and rest goes into the secondary or dilution zone. If in the later part of the primary zone or in the dilution zone oxidation of carbon occurs, a smoke-free exhaust is obtained. Thus carbon deposits on burner and smoke are not related and both depend upon design factors.

The vane angle of swirler used for reversal of flow is very important. There is always an optimum vane angle (which corresponds to optimum ratio for well stirred flow to plug flow in combustion chamber) on both sides of which more carbonisation occurs. The effect of vane angle is important as it affect the length and richness of the reversal zone.

Both carbon formation and smoke are results of over-rich fuel-air mixture. Under some operating conditions, such as takeoff, this is enevitable and smoke always results. Blue smoke is the result of chilling of flame while black smoke is produced due to lack of available air.

Another important design factor affecting the carbonisation is the cone angle of the injection nozzle. Reduction in cone angle below a certain value always results in increased carbon formation.

Thus we see that swirler, injector, fuel-air ratio, and fuel characteristics greatly affect the production of carbon which can be oxidized to avoid smoke at the exhaust of the combustor by proper matching of dilution and primary air flows. Hence smoke and carbon-free operation is greatly a matter of design.

It must be noted that occurrence of smoke need not be associated with loss of combustion efficiency. Smoke can be produced even at a combustion efficiency of 99·5 percent. However, it does result in other losses and causes atmospheric pollution.

10. Combustion chamber life

Combustion chamber working life is very important in view of very high combustion intensity associated with it. Adequate cooling of chamber walls and use of high heat resistant material are the two solutions to the problems of metal deterioration which may result in warping and cracking of liner material. Good combustion chamber life is again a matter of design.

6·7. FUEL INJECTION IN COMBUSTION CHAMBER

Good atomisation of fuel over the full working range is essential for stable, efficient, and smoke-free operation of a combustion chamber. The fuel droplets, smaller than about 10 microns, evaporate too rapidly and are unable to maintain stable combustion while droplets of size greater than about 200 to 250 microns remain unburned or partially burnt due to short residence time and pass on to exhaust in the form of smoke. The method of fuel injection, thus, has a pronounced effect on the performance of a combustor.

There are four basic methods of introducing fuel into the combustion chamber of a gas turbine. These are :

1. Spray injection system.
2. Vaporiser system.
3. Air blast system.
4. Air spray system.

Spray injection system, in its simplest form, consists of a single hole fixed orifice which is designed to give maximum fuel flow at a reasonable injection pressure. Noting the fact that at low speed and high altitude the fuel flow rate is only about 5 to 10 percent of that at sea level and full speed and that at least a pressure of 2 kgf/cm^2 is required for good atomisation, a very wide pressure range is required to meet the operational range of the combustion chamber. This is because the pressure required varies in proportion to the square of velocity for a given nozzle area. In the simplex or Lub-bock nozzle [Fig. 6·20 (*a*)] the orifice area is varied by means of spring-loaded piston responsive to fuel pressure so that a wide range of a fuel flows can be obtained with a relatively small injection pressure range

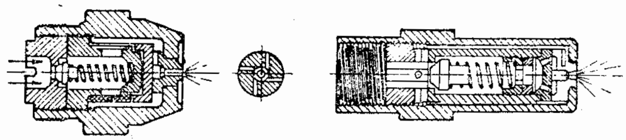


Fig. 6·20. (*a*) Simplex fuel nozzle. (*b*) Single unit duplex nozzle.

Another type of nozzle used to reduce the injection pressure range for good atomisation is the duplex nozzle [Fig. 6·20 (*b*)]. This consists of two fuel passages leading to a small vortex chamber. At low fuel rates flow passes through only one passage, the other remaining closed by a spring-loaded relief valve to maintain sufficient pressure for good atomisation. At high fuel flows both the passages supply fuel to the vortex chamber.

Spray injection system gives relatively low maximum heat release but wide burning limits, especially a very good weak extinction limit. A typical value is 1000 air-fuel ratio compared with a value of 400 for the vaporiser system. The injection nozzle is susceptible to blockage by deposition of carbon.

In the vaporiser system the fuel and air are mixed within a vaporising device before being injected into the primary zone. The fuel-air mixing is better than spray injection and maximum heat release is obtained with uniform mixture distribution at stoichiometric value. Vapour combustion system gives almost a constant combustion efficiency over a wide range of fuel pressures and mixture ratios. The burning rate is, however, limited by evaporation rate, gaseous diffusion, and chemical reaction rates, resulting in a rather inferior stability loop for vaporiser system. This system also requires an auxiliary jet for starting.

TABLE 6·3. COMPARISON OF VARIOUS TYPES OF FUEL INJECTORS

S. No.	Injector type	Advantages	Disadvantages
1.	pray atomiser	(i) Wide stability limits. (ii) Easy to modify during chamber development. (iii) Mechanically robust. (iv) Good low extinction limit.	(i) Fuel distribution and hence outlet temperature distribution varies with amount of fuel flow supplied. (ii) At high pressures produces exhaust smoke. (iii) Needs high fuel pump pressure.
2.	Vaporising system.	(1) Requires relatively low fuel pressures. (2) Outlet temperature distribution almost negligibly affected by amount of fuel supplied. (3) Burns with a blue flame and hence emits little luminous radiation and produces little exhaust smoke.	(1) Needs auxiliary fuel jet for starting. (2) Difficult to design and develop. (3) Slow response to fuel changes. (4) Mechanically suspect, specially at high pressures. (5) Fairly narrow stability limits.
3.	Air blast system	(1) Outlet temperature distribution fairly insensitive to fuel flow rates. (2) Little exhaust smoke. (3) Mechanically robust. (4) At low fuel pressures operation is satisfactory.	(1) Very narrow stability limits. (2) Poor combustion performance at low chamber velocities such as during start up.
4.	Air spray system	(1) Efficient combustion and good start up. (2) Good low extinction limits and wide stability limits. (3) Exhaust temperature distribution insensitive to fuel flow.	(1) Needs high degree of matching for good operation. (2) Cost and weight both increase. (3) This difficulty might be overcome in future as this system is in its infancy.

In the air blast system fuel is caused to flow over a plate located in a high velocity stream. Fuel as it passes over the plate, gets atomised by high velocity air and passes on to the combustion zone. The fuel air mixture is greatly dependent on the flow pattern because the fuel drops are air borne. This is somewhat similar to spill type burner in which the fuel is given an atomising vortex and only a small quantity of fuel is injected while additional fuel needed spills back to the combustion zone with the recirculation of the fuel vortex. Air blast system gives a very good exhaust temperature distribution, smokeless operation, and is cheap and reliable. However, its low extinction limit is very poor.

In the air spray system, both spray and air-blast atomisation is used. The spray supplies fuel at low fuel flow rates to give it a good low extinction limit, efficient combustion and, good startup ,and low load running operation. Under normal operating conditions the supply of fuel from the two sources are so distributed that at maximum fuel flow rates only the air blast system supplies the fuel. Thus the disadvantages of spray system such as smoke, sensitively to fuel flow of the exhaust temperature distribution is avoided.

Table 6·3 compares the various types of fuel injection systems.

6·8. CONCLUSION

Because of a wide range of operations and their conflicting requirements gas turbine combustion chamber design is essentially an excercise in compromise. Wide range of air and fuel flow, and the resulting fuel-air ratio distribution, rapid acceleration, deceleration, take-off, starting and high altitude operation (where inlet air temperature, pressure and densities are low) are some of the factors which must be taken into consideration while designing the combustion chamber. The method of fuel injection affects almost all the factors influencing combustion. Not only a compromise and matching of various combustion chamber parts is necessary but also matching with other parts of the gas turbine such as compressor and turbine is essential for stable, efficient and reliable operation of a combustion chamber. Thus, though the combustion chamber seems to be the simplest part of a gas turbine system, it requires much more insight and experience in fields like thermodynamics, combustion, aerodynamics, and material science. It is really a difficult part to design. And to most designers the path of trial and error based on past experience is the only alternative.

EXERCISES 6

6·1. What is the main purpose of a gas turbine combustion chamber ? Why the design of combustion chamber is rather difficult?

6·2. What are the main requirements of a gas turbine combustion chamber ? Are these requirements mutually compatible ?

6·3. What are the reason for pressure losses in a combustion chamber? How they can be reduced ?

6·4. What is the normal range of air-fuel ratios used in gas turbine? How combustion is obtained at this air-fuel ratio?

6·5. What is primary and secondary air in gas turbine combustion chamber? What are their functions?

6·6. What are the main types of gas turbine combustion chamber? Discuss their relative merits and demerits?

6·7. Sketch a tubular or can type combustion chamber and explain its construction. Where such chambers are used?

6·8. Sketch an annular type of combustion chamber. What are its advantages and disadvantages?

6·9. Sketch and explain how a tubo-annular combustion chamber combines the advantages of both tubular and annular type of combustion chambers.

6·10. How stabilisation of flame is obtained in a typical gas turbine combustion chamber? Show by a sketch the flow patterns in a typical combustion chamber.

6·11. Discuss in brief the main performance and operating characteristics of a gas turbine combustion system. Illustrate your answer by suitable performance curves.

6·12. How combustion efficiency of a gas turbine combustion chamber is defined? What are the main factors on which it depends?

6·13. What is the importance of combustion intensity in a combustion chamber? Is the combustion intensity same for an aircraft gas turbine and industrial gas turbine?

6·14. What is the importance of combustion chamber length?

6·15. What are the reasons for uneven temperature distributions in gas turbines? What harm can it cause?

6·16. What is blowout in gas turbines? When does it occur?

6·17. Define flame velocity, ignition delay, residence time, and stability with reference to gas turbine combustion chamber. How flame stabilisation can be achieved?

6·18. What factors affect the formation of carbon deposits and smoke in gas turbine combustion chamber?

6·19. List the methods of introducing fuel into the combustion chamber of a gas turbine and discuss their advantages and disadvantages.

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7

ELEMENTARY TURBINE DESIGN

7·1. INTRODUCTION

The analysis of turbines is very similar to that of dynamic compressors discussed in chapter 5. Fig. 7·1 shows the two types of

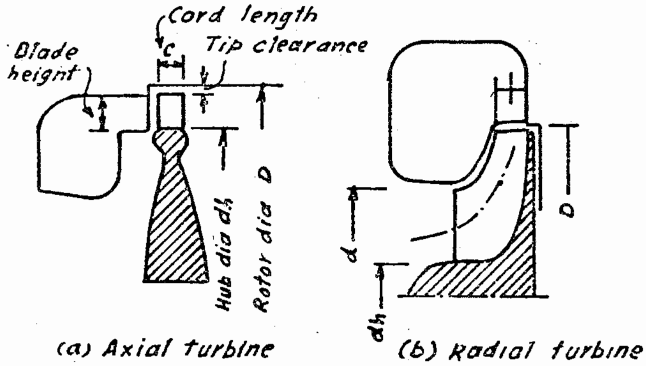


Fig. 7·1. Turbine geometry.

turbines, namely, the axial flow turbine and the radial flow turbine. With the exception of a few, in almost all applications the axial flow type of the turbine is employed and, therefore, the treatment of the radial flow type has been avoided in this chapter. This chapter discusses the various theories of turbine design. The limiting factors in the turbine design, the materials used, and various cooling methods are also discussed.

7·2. IMPULSE AND REACTION TURBINE

The turbines can be classified into two types :

(*i*) Impulse turbine

(*ii*) Reaction turbine.

In the impulse turbine, the gases expand to a high velocity in the nozzle and pass over to the moving blades. The moving blades simply divert the gas flow to the next stage or to exit, and in the process convert the kinetic energy into useful work. There is no expansion or pressure drop over the moving blades.

The reaction turbine is characterised by the fact that the pressure drop or expansion takes place both in the nozzles (or stator blades) as well as in the moving blades.

In case of an impulse turbine the blade passage area remains constant while for reaction turbine the passage area varies continuously to allow for the continued expansion of the gas stream over the moving blades.

7·3. COMPOUNDING OF TURBINES

Highest efficiency is obtained when the blade speed is half that of the gas stream velocity. Many times, the nozzle outlet velocity and hence, the blade tip speed for the given pressure drop is so high that the stresses developed may go beyond the allowable stress limits. Therefore, it is common to divide the total pressure drop into a number of stages or to convert the kinetic energy into useful work output by using more than one stage.

7·3 1. Velocity-compounded impulse turbine

Fig. 7 2 shows a velocity-compounded or Curtis turbine stage. In this type of compounding the whole of the pressure drop occurs in a single nozzle and the resultant velocity is used over a number of stages to keep the blade speed low. The pressure and the velocity as they vary over the turbine section are shown in Fig. 7·2.

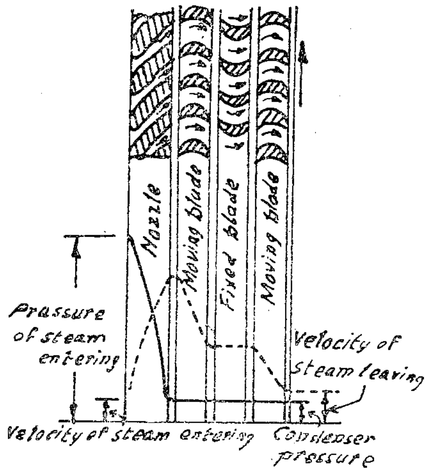


Fig. 7·2. Velocity-compounded impulse turbine.

7·3·2. Pressure-compounded impulse turbine

In pressure-compounded impulse turbine the total pressure drop, rather than the kinetic energy, is divided into a number of pressure drops over the stages (Fig. 7·3). In each stage, which consists of a nozzle and a moving blade, the gases are expanded and the kinetic energy is used in moving the rotor, and useful work is obtained. Such a stage is sometimes referred as Reteau stage. The corresponding pressure and velocity diagrams are given in Fig. 7·3.

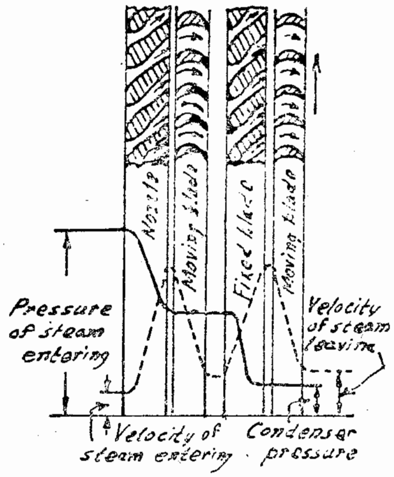


Fig. 7·3. Pressure-compounded impulse turbine.

7·4. EFFICIENCY OF A TURBINE

The efficiency of a turbine can be defined in a number of ways depending upon the use to which it is put. The expansion in the turbine can be regarded as adiabatic and so the ideal turbine is that turbine in which the expansion is isentropic. The turbine work is given by the drop in the stagnation enthalpy, *i.e.*

$$W_{T_{ideal}} = \Delta h_0 = c_p \Delta T_0 \tag{7·1}$$

For turbines in which the exhaust energy is not utilized, the efficiency is defined as total-to-static turbine efficiency. η_{ts}, (Fig. 7·4)

$$\eta_{ts} = \frac{T_{01} - T_3}{T_{01} - T_3''} = \frac{h_{01} - h_3}{h_1 - h_3''}$$

This is the ratio of the actual turbine work done to the ideal work corresponding to total inlet conditions and static exit conditions. Single-stage turbines and turboprops for which the exhaust velocity is incidental fall in this category.

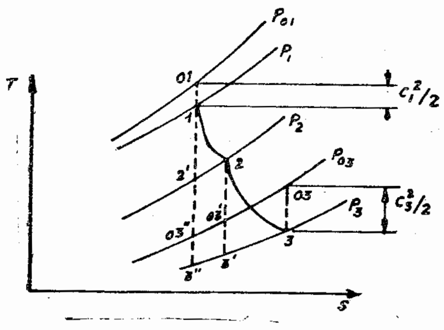


Fig 7·4. *T-s* diagram illustrating efficiency of turbine.

For turbojet, etc., where the exhaust energy is not a loss because the gases are accelarated to a high velocity for propulsive thrust generation, total-to total efficiency is used. This is defined as

$$\eta_{tt}=\frac{T_{01}-T_{03}}{T_{01}-T_{03}''}=\frac{h_{01}-h_{03}}{h_{1}-h_{03}''} \tag{7·2}$$

which is the ratio of the actual work done to the ideal work done corresponding to total inlet and total exit conditions. This definition also applies to multi-stage turbines because the exhaust energy from a stage does not go waste.

Another term which is sometimes used is the "work ratio". This is defined as the ratio of actual total head temperature drop to the isentropic temperature drop from total head inlet to static outlet pressure, and is given by

$$\text{Work ratio}=\frac{T_{0_1}-T_{0_3}}{T_{0_1}-T_{0_3}''} \tag{7·3}$$

ELEMENTARY DESIGN OF A TURBINE

(*i*) TWO DIMENSIONAL DESIGN

7·5 VELOCITY DIAGRAMS, WORK AND EFFICIENCY

Fig. 7·5 shows the typical velocity diagram for an axial flow turbine. The gas enters the nozzle or stator with a velocity C_1 and gets expanded to pressure p_2 and temperature T_2, and leaves the nozzle with an absolute velocity C_2 at angle α_2 from the axial direction. (Angles measured in the direction of rotation are taken as positive). The gas, then, enters the moving blade at angle β_2 with a relative velocity V_2 and gets deflected. The gas leaves the blade with a relative velocity V_3 at an angle β_3, C_3 being its absolute velocity at an angle α_3 which is also called swirl angle.

In an impulse stage, V_3 will be either slightly less than V_2 due to frictional loss or equal to V_2 but in a reaction stage V_3 will always

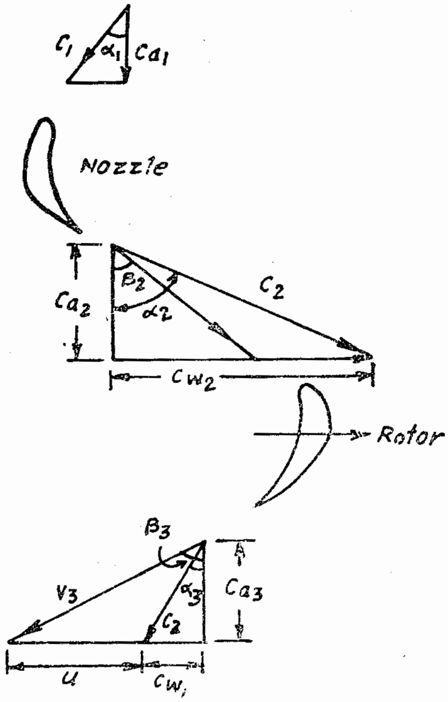


Fig. 7·5. Velocity diagrams for a turbine stage.

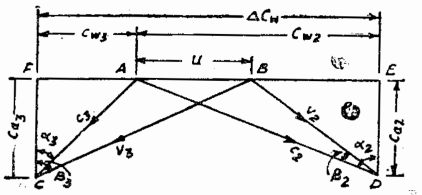


Fig. 7·6. Combined inlet and outlet velocity diagram for the moving blade.

be greater than V_2 because, then, a part of the pressure drop will occur in the moving blade which will increase the gas velocity. *Since the blade speed U increases, for a given rpm with increase in the radius, these diagrams are for a particular radius only.* The blade diagrams will be different for root, tip, and other radii points. However, due to the high hub to tip ratio used in axial flow turbines, the radial component of the velocity can be neglected. In such a case this two-dimensional analysis becomes quite representative and we can draw the above diagrams for the mean radius and assume that they are valid for other sections also.

If the moving blade inlet and outlet diagrams are superimposed as shown in Fig. 7·6, the change in whirl velocity, $\triangle C_w$, is given by the distance EF. Due to this change in whirl velocity, *i.e.* change in angular momentum, a torque is produced on the rotor. The work done per kg of flow rate is given by

$$W=\frac{AB\times EF}{g}=\frac{U\times \triangle C_w}{g}$$

or
$$W=\frac{(C_{w2}+C_{w3})U}{g} \tag{7·4}$$

W, the work done per kg of flow rate is also called specific work.

The efficiency of the rotor blade is given by

$$\text{Blading efficiency, } \eta_B=\frac{\text{Work done on blades}}{\text{Energy supplied to the blade}}$$

$$=\frac{AB\times EF/g}{\frac{1}{2}V_g^2/2g}=\frac{g(C_{w2}+C_{w3})U}{V_g^2}$$

Since
$$C_{w2}=V_2 \sin \beta_2$$
$$C_{w3}=V_3 \sin \beta_3$$

$$\therefore \qquad \eta_B=\frac{gU(V_2 \sin \beta_2+V_3 \sin \beta_3)}{C_2^2}$$

$$=\frac{2U}{C_2^2}V_2 \sin \beta_2\left(1+\frac{V_3}{V_2}\frac{\sin \beta_3}{\sin \beta_2}\right)$$

For an impulse turbine $\frac{V_3}{V_2}$ is the blade velocity coefficient K, and if $C=\sin \beta_3/\sin \beta_2$, then

$$\eta_B=\frac{2U}{C_2^2}V_2 \sin \beta_2 (1+KC)$$

From geometry

$$V_2 \sin B_2=C_2 \sin \alpha_2-U$$

$$\eta_B=\frac{2U}{C_2^2}\cdot (C_2 \sin \alpha_2-U)(1+KC)$$

$$=2\sigma (\sin \alpha_2-\sigma)(1+KC)$$

where $\sigma=\frac{U}{C_2}$ is blade to gas speed ratio.

or
$$\eta_B=2(1+KC)(\sigma \sin \alpha_2-\sigma^2) \tag{7·5}$$

Putting $\frac{d\eta_B}{d\sigma}$ equal to zero for best efficiency

$$\frac{d\eta_B}{d\sigma}=2(1+KC)(\sin\alpha_2-2\sigma)=0$$

or
$$\sigma=\frac{\sin\alpha_2}{2} \tag{7·6}$$

And the maximum blade efficiency is

$$\eta_{Bmax}=(1+KC)\frac{\sin^2\alpha_2}{2} \tag{7·7}$$

If there is a change in the axial velocity, axial thrust is produced, which is given by

Axial thrust = Mass × Change in axial velocity

$$=\frac{1}{g}(C_{a_3}-C_{a_2}) \tag{7·8}$$

Energy lost per unit mass flow due to friction over the moving blade is given by

$$\frac{V_2^2-V_3^2}{2g}$$

and the energy lost in exist per unit mass flow is $\frac{C_3^2}{2g}$.

7·6. DEGREE OF REACTION

The degree of reaction is defined as the ratio of the enthalpy drop in the moving blade to the total enthalpy drop over the stage. This is given by

$$R=\frac{h_2-h_3}{h_{0_1}-h_0} \tag{7·9}$$

If the flow is adiabatic

$$h_{0_1}=h_{0_2}$$

and
$$R=\frac{h_3-h_2}{h_{0_2}-h_0}$$

From inlet and outlet velocity diagrams,

$$V_2^2=C_{a_2}^2+C_{w_2}^2=C_{a_2}^2+C_{a_2}^2\tan^2\beta_2=C_{a_2}^2(1+\tan^2\beta_2)$$

$$V_3^2=C_{a_3}^2+C_{w_3}^2=C_{a_3}^2+C_{a_3}^2\tan^2\beta_3=C_{a_3}^2(1+\tan^2\beta_3)$$

Heat drop across the ideal blade is equal to the change in relative velocity, *i.e.*

$$h_2-h_3=\frac{1}{2g}(V_3^2-V_2^2)$$

$$=\frac{1}{2g}C_{a_3}^2(1+\tan^2\beta_3)-\frac{1}{2g}C_{a_2}^2(1+\tan^2\beta_2)$$

If the axial velocity is constant over the stage, *i.e.*

$$C_{a1}=C_{a_2}=C_{a_3}=C_a$$

then

$$h_2-h_3=\frac{1}{2g}C_a\ (\tan^2\beta_3-\tan^2\beta_2)$$

or

$$h_2-h_3=\frac{1}{2g}C_a\,(\tan\beta_3+\tan\beta_2)\ (\tan\beta_3-\tan\beta_2)$$

Once again from the velocity diagrams,

$$C_{a_2}\tan\alpha_2=U+C_{a_2}\tan\beta_2$$

and

$$C_{a_2}\tan\alpha_3=U+C_{a_2}\tan\beta_3$$

Since $C_a=C_{a_2}=C_{a_3}$

We get

$$\tan\alpha_2=\frac{U}{C_a}+\tan\beta_2$$

$$\tan\alpha_3=\frac{U}{C_a}+\tan\beta_3 \qquad (7\cdot10)$$

or

$$\tan\alpha_2+\tan\alpha_3=\tan\beta_2+\tan\beta_3 \qquad (7\cdot11)$$

and

$$h_2-h_3=\frac{1}{2g}C_a\,(\tan\beta_3-\tan\beta_2)\ (\tan\alpha_2+\tan\alpha_3)$$

The change in the stagnation enthalpy is equal to the specific work output, W.

$$h_{01}-h_{03}=h_{02}-h_{03}$$

$$=\frac{U(C_{w2}+C_{w3})}{g}$$

$$=\frac{U(C_{a2}\tan\alpha_2+C_{a3}\tan\alpha_3)}{g}$$

$$=\frac{U}{g}C_a\,(\tan\alpha_2+\tan\alpha_3)$$

Degree of reaction, $R=\dfrac{h_2-h_3}{h_{02}-h_{03}}=\dfrac{1}{2}\ \dfrac{C_a}{U}\ (\tan\beta_3-\tan\beta_2)$ (7·12)

If β_m is the mean flow angle through the moving blade the degree of reaction can be approximated by

$$R=\frac{1}{2}\ \frac{C_a}{U}\ \tan\beta_m \qquad (7\cdot13)$$

7·7 FLOW AND BLADE LOADING COEFFICIENTS

The flow coefficient, ϕ, is defined as the ratio of the inlet velocity C_a to the blade speed U.

$$\phi=\frac{C_a}{U} \tag{7·14}$$

The blade loading coefficient, ψ, is defined as the ratio of the specific work of the stage to the square of the blade velocity, *i.e.*

$$\psi=\frac{W}{U^2}=\frac{U(C_{w1}+C_{w1})}{U^2} \tag{7·15}$$

$$=\frac{C_{w1}}{U}+\frac{C_{w2}}{U}$$

If $C_{a_2}=C_{a_3}=C_a$, the blade loading coefficient is given by

$$\psi=\frac{C_a}{U}\ (\tan\alpha_2+\tan\alpha_3)$$

$$=\frac{C_a}{U}\ (\tan\beta_2+\tan\beta_3) \tag{7·16}$$

These two parameters, ϕ and ψ, are non-dimensional parameters and are often used to plot the design charts.

7·8. VELOCITY RATIO

Another important non-dimensional parameter is the ratio of the blade speed to the velocity which would be obtained by isentropic expansion through the stage, *i.e.*

$$\sigma_i=\frac{U}{C_1}=\frac{U}{\sqrt{2(h_{01}-h_{03}''}} \tag{7·17}$$

Sometimes the velocity ratio is also defined as the ratio of the blade speed to the nozzle outlet velocity, *i.e.*

$$\sigma=\frac{U}{C_2} \tag{7·18}$$

Since $$\eta_{tt}=\frac{h_{01}-h_{02}}{h_{01}-h_{03}''},$$

$$h_{01}-h_{03}=C_i^2\ \sigma_i\eta_{tt}$$

We can express the blade loading coefficient ψ in terms of total-to-total efficiency and the velocity ratio

$$\psi=\frac{W}{U^2}=\frac{h_{01}-h_{03}}{U^2}$$

$$=\frac{C_i^2\ \eta_{tt}}{U^2}$$

$$=\frac{\eta_{tt}}{\sigma_i^2} \tag{7·19}$$

This is an important relation applicable to all stages.

7·9 DESIGNS FOR VARIOUS DEGREES OF REACTION

(*i*) **Zero degree of reaction.** The degree of reaction is given by

$$R=\tfrac{1}{2}\frac{C_a}{U}(\tan\beta_3-\tan\beta_2) \tag{7·12}$$

For zero degree of reaction

$$\beta_3=\beta_2 \tag{6·20}$$

The specific work output

$$W=\frac{U}{g}(C_{w2}+C_{w3})$$

$$=\frac{U}{g}C_a(\tan\beta_3+\tan\beta_2)$$

$$=2\,\frac{C_aU}{g}\tan\beta_2 \tag{7·21}$$

The blade loading coefficient

$$\psi=\frac{W}{U^2}=\frac{2C_a\tan\beta_3}{U} \tag{7·22}$$

The deflection by the rotor

$$=\beta_2+\beta_3=2\beta_3$$

$$=2\,\frac{C_a}{U}\,2\tan\,\frac{(\beta_2+\beta_3)}{2}$$

or

$$\psi=2\,\frac{C_a}{U}\tan\,\frac{\delta}{2} \tag{7·23}$$

where $\delta=2\beta_2=2\beta_3$ is the rotor deflection angle.

Fig. 7·7 shows the performance for such a stage on a ψ-ϕ chart with the help of constant α_2 and β lines.

If the exit swirl angle α_3 is zero, then

$$\tan\beta_3=\frac{U}{C_a}$$

and

$$\psi=\frac{2C_a\tan\beta_3}{U}=\frac{2C_a}{U}\cdot\frac{U}{C_a}=2 \tag{7·24}$$

i.e. the change in tangential velocity is twice the blade speed for such a blading.

(*ii*) **Fifty per cent reaction**

$$R=\frac{C_a}{2U}(\tan\beta_3-\tan\beta_3)=0·5$$

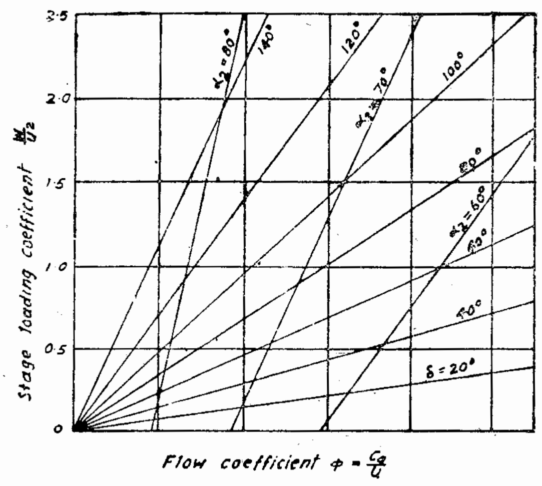


Fig 7·7. ψ-ϕ chart for zero reaction stage.

$$\tan \beta_3 - \tan \beta_2 = \frac{U}{C_a}$$

Since from velocity triangles

$$\tan \alpha_2 = \frac{U}{C_a} + \tan \beta_2 \tag{7·10}$$

We get, $\tan \beta_3 = \tan \alpha_2$

or $\beta_3 = \alpha_2$

Similarly, from the relation

$$\tan \alpha_3 = \frac{U}{C_a} + \tan \beta_3$$

We get, $\tan \beta_2 = \tan \alpha_3$

or $\beta_2 = \alpha_3$

i.e. for fifty per cent reaction stage the blade velocity diagram is symmetrical as shown in Fig. 7·8.

The specific work output is given by

$$W = \frac{U}{g}(C_{w2} + C_{w3})$$

$$= \frac{U}{g}(C_2 \sin \alpha_2 - U + C_2 \sin \alpha_2)$$

$$= \frac{U}{g}(2C_2 \sin \alpha_2 - U)$$

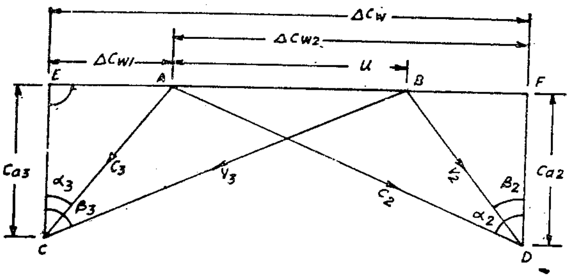


Fig. 7·8. 50 per cent reaction stage velocity diagram.

$$=\frac{C_2^2}{g}\left(2U\ \frac{C_2}{C_2^2}\sin\alpha_2-\frac{U^2}{C_2^2}\right)$$

$$=\frac{C_2^2}{g}\left(2\sigma\sin\alpha_2-\sigma^2\right)$$

where σ is the velocity ratio U/C_2

Total heat drop$=\dfrac{2(C_2^2-C_3^2)}{2g}$ for a 50 per cent reaction turbine because $\dfrac{C_2^2-C_3^2}{2g}$ is the energy equivalent to heat drop in moving blades.

$\therefore$ Total energy supplied

$$=\frac{2(C_2^2-C_3^2)}{2g}+\frac{C_3^2}{2g}$$

$$=\frac{2C_2^2-C_3^2}{2g}$$

Since from the velocity diagram of Fig. 7·8

$$C_3=V_2$$

$$C_3^2=V_2^2=C_2^2+U^2-2C_2U\sin\alpha_2$$

Total energy supplied

$$=\frac{2C_2^2-C_2^2-U^2+2C_2U\sin\alpha_2}{2g}$$

$$=\frac{1}{2g}(C_2^2-U^2+2C_2\,U\sin\alpha_2)$$

$$=\frac{C_2^2}{2g}(1-\sigma^2+2\sigma\sin\alpha_2)$$

$$\text{Efficiency}=\frac{\dfrac{C_2^2}{g}(2\sigma\ \text{sid}\ \alpha_2-\sigma^2)}{\dfrac{1}{2g}C_2^2(1-\sigma^2+2\sigma\sin\alpha_2)}$$

$$=2-\frac{2}{1-\sigma^2+2\sigma\sin\alpha_2} \tag{7·25}$$

For maximum efficiency

$$\frac{d}{d\sigma}(1-\sigma^2+2\sigma \sin \alpha_2)=0$$

or
$$-2\sigma+2\sigma \sin \alpha_2=0$$

or
$$\sigma=\sin \alpha_2 \tag{7·26}$$

and the maximum efficiency is given by

$$\eta_{B_{max}}=\frac{2(2 \sin^2 \alpha_2-\sin^2 \alpha_2)}{(1-\sin^2 \alpha_2+2 \sin^2 \alpha_2)}=\frac{2 \sin^2 \alpha_2}{1+\sin^2 \alpha_2} \tag{7·27}$$

The blade loading coefficient ψ is given by

$$\psi=\frac{W}{U^2}=U(C_{w2}+C_{w2})$$

$$=\frac{UC_a(\tan \beta_2+\tan \beta_3)}{U^2}$$

$$=\frac{UC_a(2 \tan \beta_3-U/C_a)}{U^2}$$

or
$$=2\frac{C_a}{U} \tan \alpha_2-1 \qquad (\because \ \beta_3=\alpha_2)$$

If the outlet swirl angle α_3 is zero, we get

$$\tan \beta_3=\tan \alpha_2=\frac{U}{C_a}$$

and
$$\psi=2\frac{C_a}{U}\frac{U}{C_a}-1=1$$

Then the change in the tangential velocity across the rotor is equal to the blade speed.

ψ-ϕ charts similar to that of Fig. 7·7 can be plotted for design purposes.

7·10 DESIGN WITH AXIAL OUTLET VELOCITY

The situation where the exit swirl angle α_3 is zero has already been discussed in the two previous articles. Here, however, some general considerations are made.

Fig. 7·9 shows the velocity diagram for such a case. The degree of reaction R is given by

$$R=\frac{C_a}{2U}(\tan \beta_3-\tan \beta_2)$$

$$=\frac{C_a}{2U}\left(\frac{U}{C_a}-\tan \beta_2\right)$$

$$=\frac{1}{2}-\frac{1}{2}\frac{C_a}{U} \tan \beta_2$$

The specific work output is given by

$$W = U\,C_a(\tan\beta_2 + \tan\beta_3)$$
$$= U\,C_a\,(\tan\alpha_2 + \tan\alpha_3)$$
$$= U\,C_a\tan\alpha_2$$

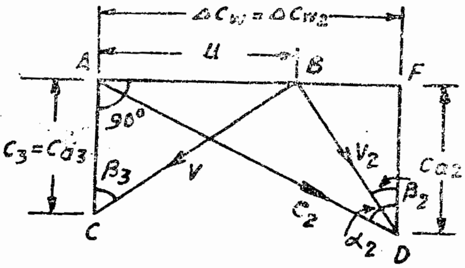


Fig. 7·9 Velocity diagram with axial outlet velocity

Blade loading coefficient

$$\psi = \frac{W}{U^2} = \frac{U\,C_a\tan\alpha_2}{U^2}$$

We have, $\tan\alpha_3 = 0$, and $\tan\beta_3 = U/C_a$

Again, $\tan\alpha_3 = \tan\beta_2 + \tan\beta_3$

$$= (\tan\beta_2 - \tan\beta_3) + 2\tan\beta_3$$

$$= -R\,\frac{2U}{C_a} + \frac{2U}{C_a} \qquad \text{(by Eq. 7·10)}$$

$$= \frac{2U}{C_a}(1-R)$$

$$\therefore \qquad \psi = \frac{U\,C_a}{U^2}\,\frac{2U}{C_a}(1-R) = 2(1-R) \qquad (7{\cdot}28)$$

The above equation gives

$\psi = 1$ for 50 per cent reaction

and $\psi = 2$ for zero reaction. These situations have already been discussed above.

7·11 OFF-DESIGN PERFORMANCE

The knowledge of the off-design performance of a turbine is essential for proper matching of the compressor and the turbine. Also, for overall assessment of the gas turbine efficiency for various applications such as aircraft engine, automobiles, etc., the operation under conditions different from design conditions is of importance.

Under the assumption of constant axial velocity

$$C_{a2}=C_{a3}=C_a, \text{ we have}$$

$$\psi=\frac{C_a}{U}(\tan\beta_2+\tan\beta_3)$$

By equation (7·11)

$$\psi=\frac{C_a}{U}\left(\tan\alpha_2+\tan\beta_3-\frac{U}{C_a}\right) \tag{7·29}$$

or

$$\psi=\phi(\tan\alpha_2+\tan\alpha_3)-1$$

Similarly, the reaction R can be written with the help of equation (7·11) as

$$R=\tfrac{1}{2}+\tfrac{1}{2}\phi(\tan\beta_3-\tan\alpha_2) \tag{7·30}$$

Experimental evidence suggests that the angles α_2 and β_3 remain substantially constant under off design operating conditions. These angles are sometimes referred as "must" angles.

Thus $\tan\alpha_2+\tan\beta_3=$constant, is the condition determining the off-design performance. Lines of constant ($\tan\alpha_2+\tan\beta_3$) on ψ-ϕ chart passing through the design point will represent the off-design performance of the turbine and since ($\tan\beta_3-\tan\alpha_2$) is also substantially constant, reaction at any off-design point will be the function of the flow coefficient ϕ.

For a 50 per cent reaction turbine

$$\tan\beta_3=\tan\alpha_2$$

Putting this into (Eq. 7·30) the degree of reaction R, is $R=\frac{1}{2}=$ constant, *i.e.* the degree of reaction remains constant under off-design conditions also. This is because of the fact that if α_2 and β_3 are constant the shape of the triangles do not change and they remain symmetrical.

(*ii*) THREE DIMNSIONAL DESIGN

7·12. RADIAL EQUILIBRIUM THEORY

Radial equilibrium theory has been one of the conventional methods of designing an axial flow impeller considering the three-dimensional flow. This theory assumes that the radial velocity is absent in the flow, *i.e.* radial equilibrium always exists. The stream lines lie on circular, cylindrical surfaces which are axisymmetric. The shifting of the streamlines takes place completely within the blades and downstream the blades there is no radial velocity component.

As already discussed in the chapter on compressor (*see* section 5·8), the momentum equation under such assumptions becomes

$$\frac{dp}{dr}=\frac{\rho C_w^2}{gr} \tag{5·59}$$

which under the usual assumptions of constant enthalpy and entropy takes the form

$$gJ\,\frac{dh}{dr}=C_a\,\frac{dC_a}{dr}+C_w\,\frac{dC_w}{dr}+\frac{C_w^{\,2}}{r} \qquad (5\cdot61)$$

This equation is the basic equation for all vortex flows.

7·12·1. Free vortex design

If now it is assumed that the work input at all radii is equal to the total head temperature and hence, the enthalpy will remain constant for all radii and further that the axial velocity is also constant, *i.e.*

$$\frac{dh}{dr}=0 \quad \text{and} \quad \frac{dC_a}{dr}=0$$

equation (5·61) reduces to

$$\frac{dC_w}{C_w}=\frac{-C_w}{r}$$

$$C_w\times r=\text{constant} \qquad (5\cdot62)$$

Such a flow which follows this equation, *i.e.* in which the whirl or tangential velocity varies inversely with the radius, is called a "free vortex flow."

Free vortex flow is a particular case of the radial equilibrium flow which is a reversible, constant stagnation enthalpy flow having a tangential component of velocity which varies inversely as the radius and the axial component is uniform throughout the flow. The constancy of stagnation enthalpy of flow, *i.e* the work done on any moving blade is invariant, implies that circulation is constant along the blade length and no vorticity is produced by the blade in the flow. The flow remains irrotational. Because of this reason it was thought that any design based on this theory would be highly efficient and the vortex theory became very popular.

From the velocity triangles, we have

$$\tan\alpha=\frac{C_w}{C_a}=\frac{\text{constant}}{r\,C_a}$$

Since C_a is constant

$$\tan\alpha\propto\frac{1}{r}$$

The work output per unit flow at a radius r is

$$h_{0_2}-h_{0_3}=U(C_{w1}+C_{w2})$$

$$=Wr\left(\frac{C_2}{r}+\frac{C_3}{r}\right)$$

where $r\,C_{w1}=C_1$ and $r\,C_{w2}=C_2$ corresponds to two stations before and after the rotor and C_2 and C_3 are constants.

$$h_{0_2}-h_{0_3}=W(C_2+C_3)$$
$$=\text{constant}$$

This proves that the work done on moving blade is constant as discussed above, The variation of air angles from root to tip is shown in Fig. 7·10 in the free vortex design which are given by

$$\tan\beta_2=\frac{C_2}{rC_{a_2}}-\frac{Wr}{C_{a_2}}$$

$$\tan\beta_3=\frac{Wr}{C_{a_3}}+\frac{C_3}{C_{a_3}} \qquad (7{\cdot}31)$$

and $$\tan\alpha_2=\frac{C_2}{r\,C_{a_2}}$$

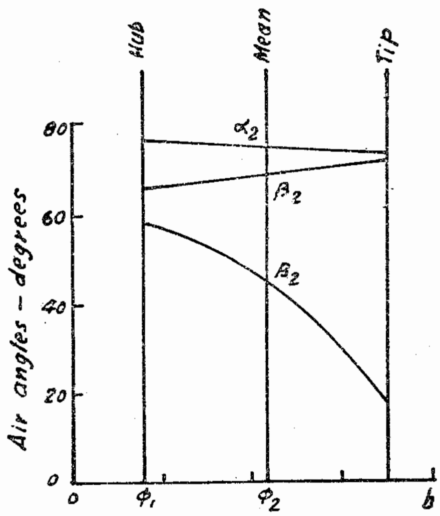


Fig. 7·10. Variation of blade angles along the blade height.

The degree of reaction R is given by

$$R=\frac{1}{2}\frac{C_a}{U}(\tan\beta_3-\tan\beta_2) \qquad (7{\cdot}12)$$

which, with the help of equation (7·31) becomes

$$R=\frac{C_3-C_2}{2U_r}+1$$

or $$R=\frac{C_3-C_2}{2Wr^2}+1 \qquad (7{\cdot}32)$$

If we assume that the reaction at the root r_n is zero, then

$$0=\frac{C_3-C_2}{2W_{rn}^2}+1$$

which when combined with (7 12) gives

$$R=1-\frac{r_h^2}{r^2} \tag{7·33}$$

describing the variation of the degree of reaction from root to tip, the degree of reaction increasing toward the tip.

Usually the reaction at the hub is kept zero or little more than zero so that the reaction may not be too much at the tip. Too much reaction at tip results in leakage loss. Thus we see that the hub to tip radius ratio is very important in that it affects the degree of reaction along the blade. Smaller the hup-tip ratio higher will be the reaction at the tip for impulse condition at the root and more wil be the leakage losses.

Though the free vortex theory has been a very popular basis of axial flow impeller designs, it nevertheless has certain disadvantages. One of them is that it leads to a twisted rotor blade along its span and on which the incident velocity, and hence the Mach number, varies considerably from hub to tip. Some designes, in order to have straighter blades, depart from free vortex design. In most of these designs the condition of radial equilibrium is satisfied by balancing the radial pressure gradient with the centripetal acceleration due to the tangential component of the whirl velocity. This renders the radial velocity negligible, though not totally absent and surprisingly the efficiency of these designs is not poorer than the free vortex design.

Another factor which must be considered is that in free vortex design, we assume entropy to be constant, *i.e.* the viscosity effects are neglected whilst it plays an important part in building up low energy fluid adjacent to the annular walls. This results from wall boundary layers and their interaction with flows around the blades in region adjacent to the wall. It also affects the blade wakes. In compressor design this is accounted by the "work done factor" which may reduce the average work by 15 to 20 per cent for multistage turbines.

Another, disadvantage of the free vortex design is that the root and tip sections are subject to adverse flow conditions of low reaction and adverse pressure gradients. Both these are not conducive to good efficiency. In the free vortex design the velocity ratio is given by

$$V_{rim}=\sqrt{\frac{U_R2}{2gJ\triangle\ H}} \tag{7·34}$$

where U_r is rim wheal speed, which is related to efficiency as given in Fig. 7·11. This shows that the efficiency decreases as the velocity ratio is decreased. So most early designs used a high velocity ratio, *i.e.*, high speeds (U_R) or low stage work ($\triangle H$). From the detailed studies of the spanwise efficiency characteristics of the blade for different pressure ratio, velocity ratio, and variable area turbines, it became clear that the main reason for decrease in efficiency with a decrease in rim velocity ratio is the poor root efficiencies. High root losses always occur with low velocity ratio, highly loaded stages in free vortex designs. In such designs large loss occurs in the end wall regions, particularly at the low reaction root. This poor performance is perhaps due to separation of the boundary layer fluid in this region which results in large local losses and blockage of the flow forcing the mainstrean flow towards the midspan.

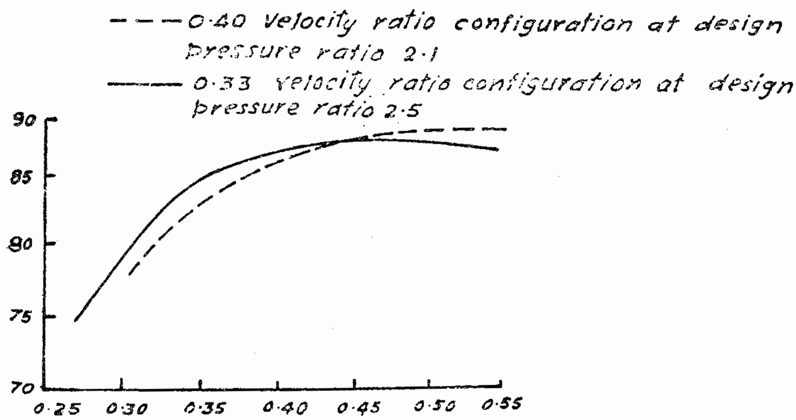


Fig. 7·11. Measured efficiency *vs* rim velocity ratio for free vortex design.

7·12 2. Controlled-vortex method

Since the free vortex design results in poor root efficiency due to low root reaction, the obvious expedient to improve the efficiency of the axial flow impeller is to increase the root reaction. One method is to reduce the gas angle or provide more swirl leaving the blade as compared to the conventional design but this imposes additional swirl on the next stage or exit guide vane. Seeing the fact that some swirl is already present in conventional free vortex design, it may become quite severe for high flow stages.

Another alternative is to adjust the aerofoil designs at all radial stations so that the adverse flow conditions at root and tip are avoided. This is the basis of the controlled-vortex method in which the designer can alter and optimise each aerofoil section by varying the mainstrean flow pattern. Two approaches, which increase the root reaction are :

(*i*) Constant spanwise work distribution with the stator angle icrea ed athe root.

(*ii*) Varying the spanwise work distribution.

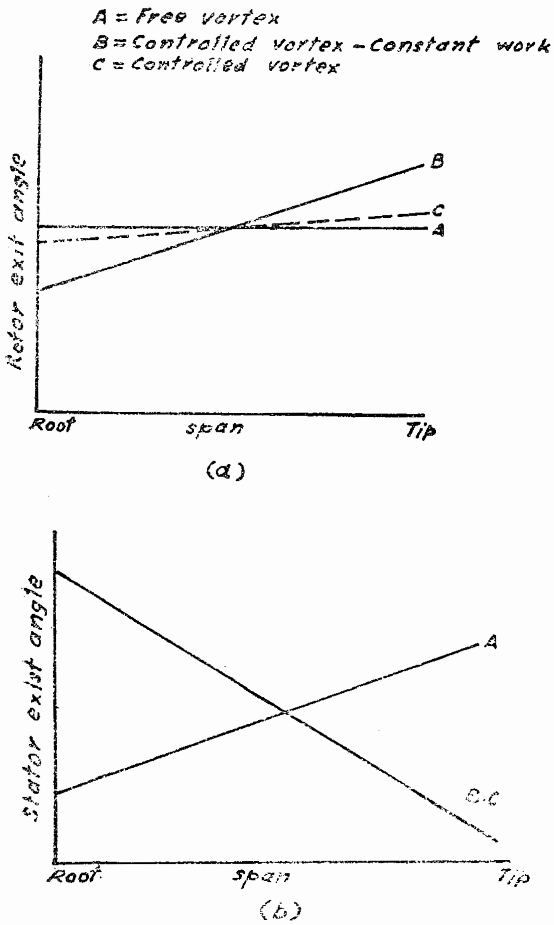


Fig. 7·12. Variation of blade and vane exit angles for various flow pattern.

In the first case, the rotor exit angle at the root is reduced [*see* Fig. 7·12(*a*)] for constant work. This requires a decrease in the stator exit angle at the tip to keep the stator area at a given value (*see* Fig. 7·12 (*b*)]. However, this results in a very much skewed rotor swirl pattern in that there is a very high swirl at the root and very little at the tip as clear from curve *B* of Fig. 7·13 (*a*). This would require a highly twisted stator blade to accept the large swirl gradient. And because of this reason this approach becomes unacceptable, though it has significantly raised the reaction at the root. (*see* fig. 7·14).

A=Free vortex
B=Controlled vortex-constant work
C=Controlled vortex

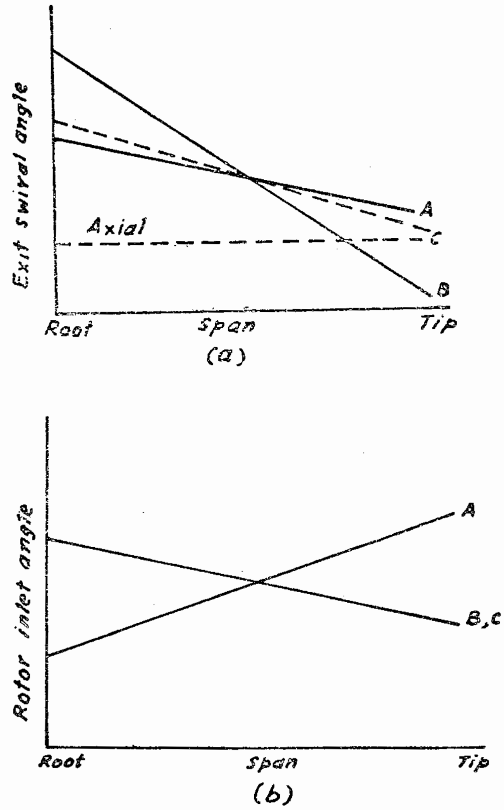


Fig. 7·13. Variation of blade inlet and exit swirl angle for various flow pattern.

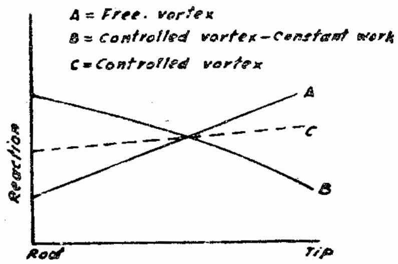
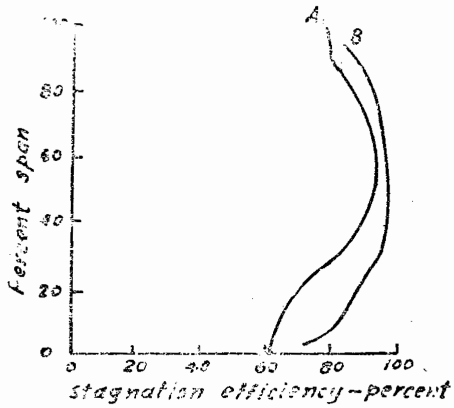


Fig. 7·14. Variation of reaction for various flow pattern.

Varying the spanwise work distribution (*see* fig. 7·16) is a better approach. If work at the root is reduced, less turning would be needed and the large swirl at the blade root can be reduced. The opposite situation holds true for the tip where additional work can be done without incurring problems.

The curves marked C in Figs. 7·12 to 7·14 show the behaviour of the non-uniform work controlled-vortex stage. Variation in work distribution results in increased root reaction and significant gain over the conventional free-vortex stage.



A=Free vertex design at design pressure ratio and 0·33 velocity ratio
B=Controlled vertex turbine at design pressure ratio and 0·285 volocity ratio

Fig. 7·15. Comparison of free vortex and controlled vortex turbine spanwise measured efficiency.

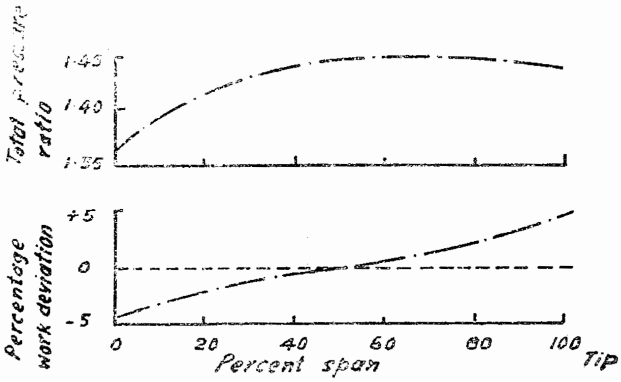


Fig. 7·16. Controlled vortex total pressure ratio and work variation spanwise.

Fig. 7·15 shows that there is a marked improvement in efficiency particularly in the root region. As is clear from Fig. 7·14 and 7·17 only a small change in the work profile and exit angles is required to achieve the desired result. Fig. 7·18 shows the free-vortex blading against the controlled-vortex design.

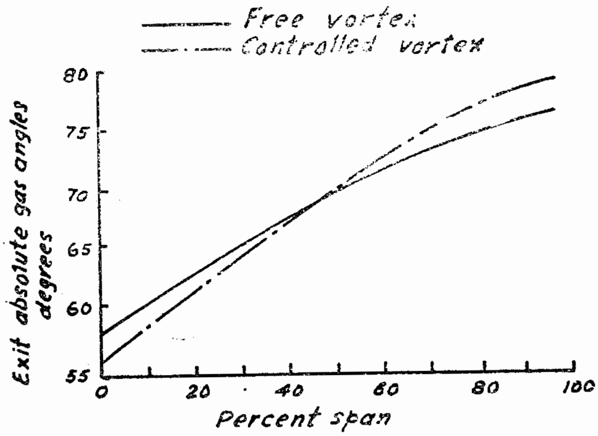


Fig. 7·17. Predicted exit gas angle comparison between free vortex and controlled vortex.

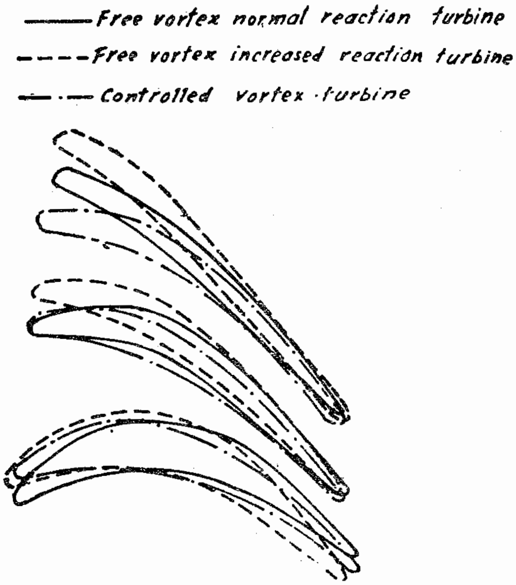


Fig. 7·18. Blade sections for free and controlled vortex.

The controlled vortex stage has a higher efficiency than the free-vortex stage for nearly the entire span (*see* fig. 7·19). The high reaction free-vortex design, obviously, shows better efficiency than the low reaction free-vortex stage. At the tip, however, the normal reaction stage has better efficiency than the high reaction stage due to lower tip leakage losses.

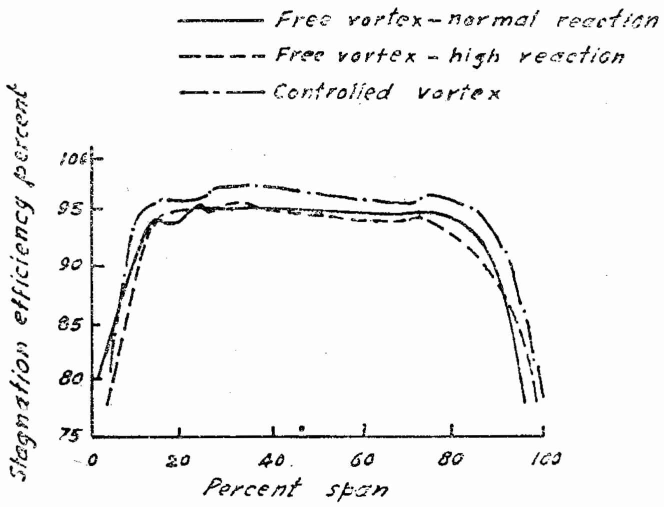


Fig. 7·19. Measured free vortex *versus* controlled vortex spanwise transverse efficiency for the blading of Fig. 7·18

Due to relatively small change in each stage, relative to the free vortex design, the stacking up of non-uniform work profiles does not cause any trouble in multi-stage controlled-vortex turbine. The controlled-vortex turbine, rather has a less tendency to force the flow away from the walls giving a flow per unit area more uniform than the free vortex stages. Improved efficiency near the end walls also results in an improved temperature distribution.

In the radial equilibrium theory it is assumed that there is no radial component of the fluid velocity. This implies that the circulation is constant along the blade length and no vorticity is

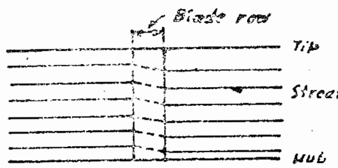


Fig. 7·20. Streamlines under the assumption that all the radial flow takes place in the blade passages.

generated by the blade. The flow remains irrotational. If at all any radial flow takes place it is within the blade passages. The streamlines under such an assumption would be like those shown in Fig. 7·20. However, this assumption that all the radial motion takes place within the blade row and that between the blade rows the streamlines are circular cylindrical surfaces is not true. It has been found that the radial equilibrium theory is not adequate because there are appreciable radial velocities outside the blade row as is clear from Fig. 7·21 which shows the distribution of the axial component of the velocity at various axial distances for a single row of stationary inlet guide vanes. This confirms that the radial flow regime is not fully established within the blade row and radial flow does exist outside the blade row.

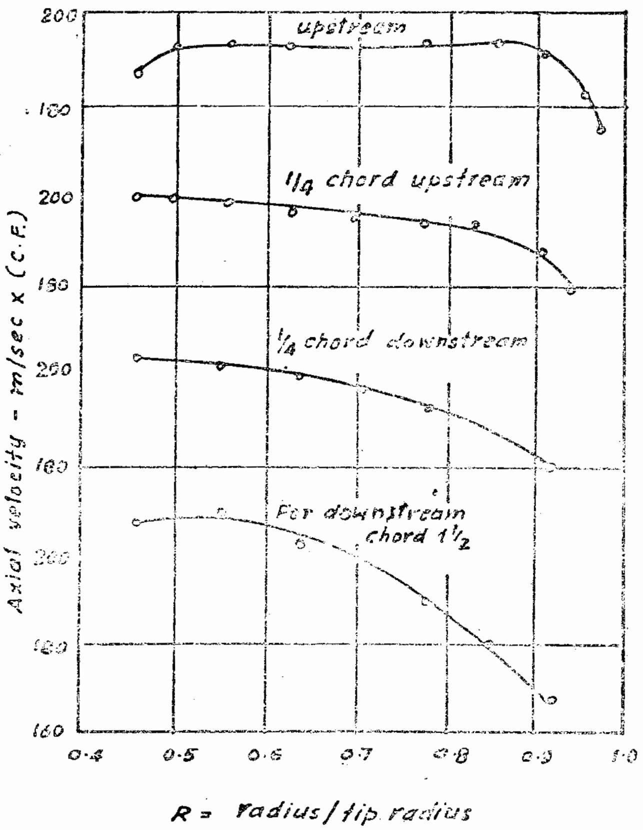


Fig. 7·21. Axial velocity profiles upstream and downstream of an isolated row of guide vanes.

With the increase in speed and the pressure ratios the compressibility effects have become important. Due to these effects radial density gradient and hence radial velocity distributions occur. Thus the radial equilibrium theory tacitly neglects the compressibility effects. This also assumes that the effect of a discrete blade is not transmitted to the flow which again is not true.

7·13. ACTUATOR DISC THEORY

The fact that radial equilibrium is not achieved at any station because of the radial flow effects and mutual interference of the blade rows has rendered the radial equilibrium theory inadequate. Actuator disc theory is an attempt to a refinement over this. This concept o f actuating disc theory has been borrowed from airscrew theory or t heory of propellers.

The basis of the actuating disc theory is that each blade row in a turbomachine can be represented by a plain of discontinuity such that the streamlines, *i.e.*, the mass flux is continuous across the disc and the radial equilibrium is established at a large distance from the disc. Fig. 7·22 shows the streamlines for such a case.

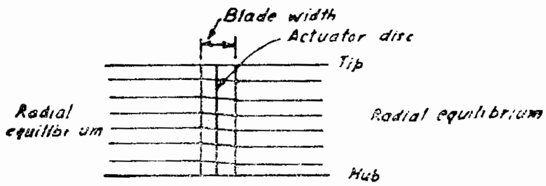


Fig. 7·22. Streamlines under the assumption of the actuator disc.

This model by assuming that radial velocities exist outside the blade rows gives a better picture of the flow and is also simpler for the designers than other methods.

Hawthorne and Horlock (1) have analysed the flow under the assumptions of incompressible flow, parallel annulli, small radial velocities, small axial vorticity and by neglecting the viscosity effects. Later Hawthorne and Pingrose (10) modified the theory to account for the compressibility effects. The detailed analysis of the theory is out of scope of this book and those interested are referred to the work of Hawthorne, Horlock and Pingrose.

With the easy availability of digital computing facilities the application of the actuating disc theory is increasing. Other methods such as streamline curvature method have come into vogue, but still the understanding of the flow phenomena is far from complete and the designers well have to make use of some degree of empericism.

7·14. FACTORS LIMITING THE TURBINE DESIGN

The aim of every designer is to get the largest possible thermo, dynamic performance commensurate with the desired reliability, and ease of operation at minimum of the cost. In the process he has to decide a large number of design parameters determining the performance of the aerodynamic (blading), hydrodynamic (bearings), mechanical (blades, disc, shafts) and combustion elements of the gas turbine plant. Though he has a large number of the various combinations of these design parameters, he has to face the limits imposed by the requirements of the particular application for which the machine is being designed, the availability of materials and their mechanical and other properties, and the research and development effort necessary to realize the projected design.

The work output and the speed of the turbine are usually fixed by the application considerations, and so also the turbine inlet pressure and inlet temperature. Then the designer has recourse to the various geometrical parameters like rotor diameter to blade height ratio, nozzle angle, blade angles, ratio of rotor clearances to blade height, chord to blade height ratio and the outlet swirl angle. Almost all these geometric parameters affect the various losses in a turbine stage and some of them are optimum parameters, *i.e.* they give highest efficiencies under given conditions.

The following is a brief discussion of the limiting factors in the turbine design.

(*i*) **Stress considerations**

The specific work output is given by

$$W=\frac{U\triangle C_w}{g}=\frac{U(C_{w2}+C_{w3})}{g} \tag{7·35}$$

In the specific output is high then the number of stages, and hence the total losses, required for a given output are reduced. Either the blade tip speed or the change in the tangential velocity can be increased to get this effect.

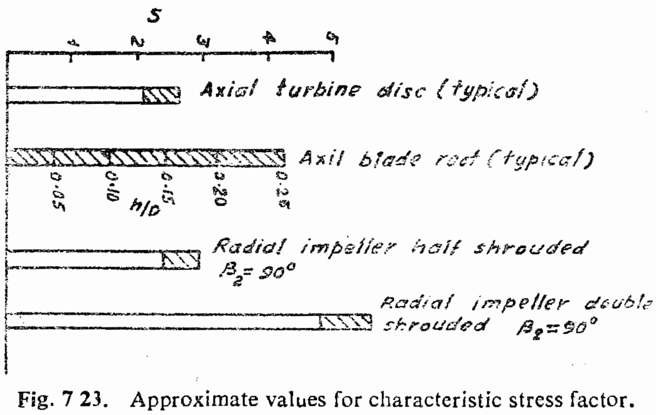


Fig. 7 23. Approximate values for characteristic stress factor.

The maximum rim speed is limited by the allowable stresses of the material used for the turbine discs. These stresses are a function of the wheel-tip speed, and the specific weight of the rotor material and is given by

$$\text{stress } \sigma = SU^2 \frac{\rho}{g} \tag{7·36}$$

where the factor S is a characteristic value which depends mainly on the wheel type as is clear from Fig. 7·23. These mean values are valid for wheels in which small thermal stresses exist and for axial wheels the blade length is not more than 20 per cent of the disc diameter, *i.e.* $h/D<0·2$. Lower S factor values, *i.e.* higher tip speed, can be obtained by providing suitable disc and blade taper ratios.

(*ii*) **Operating temperatures.** The maximum allowable stress σ_{max} depends upon the material composition and on the disc temperature. As is clear from Fig. 7·24, after a temperature of about

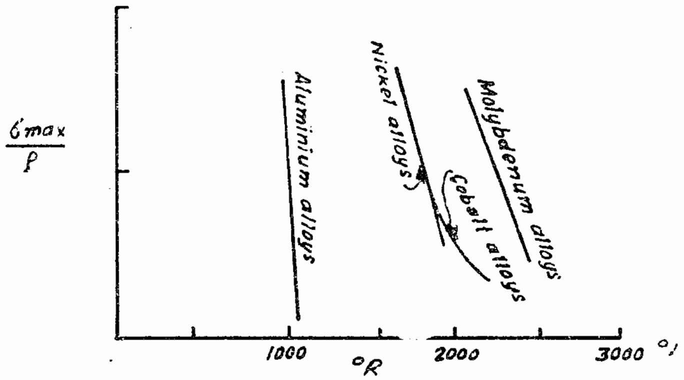


Fig. 7·24. Maximum allowable stresses for different materials for 1000-hr rupture life.

500°C aluminium alloys cannot be used and molybdenum base alloys have the highest strength under elevated temperatures. Thus there exists a maximum turbine velocity ratio which must not be exceeded for safe operation.

The inter-relation between the turbine disc trmperature and the main stream gas temperature is, thus, also important. This brings into play the vital role of the cooling system used and its effectiveness. The blade themperature can approximately be taken to utilize about 85 per cent of dynamic gas temperature and the turbine disc temperature can be taken equal to the blade temperature. However, these temperatures would mainly depend upon the blade cooling method used and the thermal conductivity of the material of the blade and disc.

Fig. 7·25 shows the allowable turbine velocity ratio and gas temperature for different spouting velocities and materials. It may be noted that allowable gas temperatures increase with increasing spouting velocity. The influence of the turbine velocity ratio depends upon the slope of the $\frac{\sigma}{\rho}=f(T_b)$ curve for the material. For

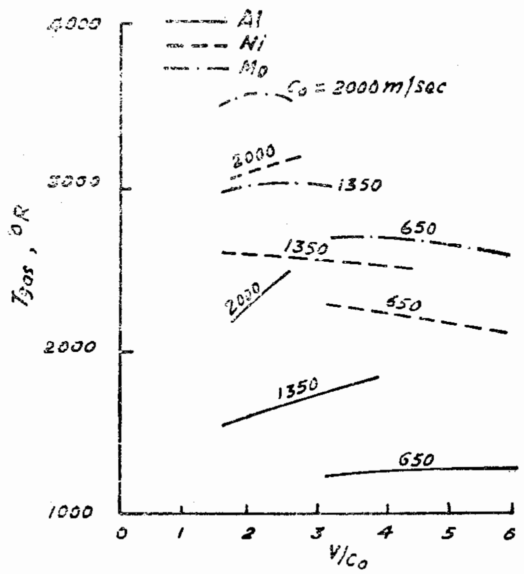


Fig. 7·25. Allowable turbine velocity ratio and gas temperatures for different spouting velocities and materials 1000-hour rupture life.

materials with a steep slope, the admissible gas temperature increases with increasing turbine-velocity ratios upto the point of maximum stress. For materials having gradual slope, like nickel and molybdenum alloys, it may increase, decrease or be almost independent of the turbine velocity ratio. This is important since the efficiency of the turbine is dependent upon the velocity ratio and higher the velocity ratio more is the efficiency (*see* Fig. 7·11).

(*iii*) **Blade fixing.** Fixing of blades is a problem of great importance in gas turbine construction and affects the maximum rim

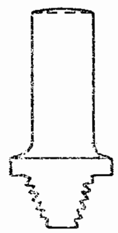


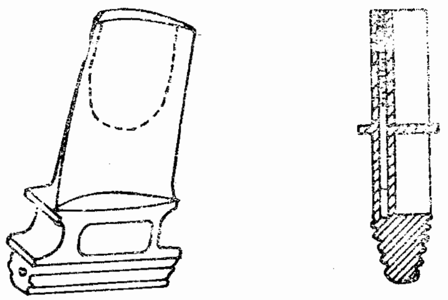
Fig. 7·26. Fir tree type of attachment.

speed which can be used. The critical area is the disc/blade attachment which transfers the centrifugal load on the blade into the disc rim. The most common type of the attachment used is the "fir tree" design shown in Fig. 7·26 in which the forces are transmitted by the individual teeth, as in the screw thread. This results in stress concentration at the bases of the teeth and creep takes place in a very small zone of pronounced shear stress, and after some time a crack develops leading to fracture due to shearing off of the teeth.

The pitch/chord ratio is important in that it affects the area of the stress concentration zone.

Since the centrifugal and gas loads induce bending and shear stresses in the teeth of the fir-tree the local stresses are generally higher than the mean blade centrifugal stresses. Therefore the temperature at the blade fixing must be lower than that in the blade profile to maintain the same creep life because the creep life decreases with increase in temperature.

Cooling [*see* Fig. 7·27 (*b*)] of the root is one way of keeping the disc temperature low. Another solution is the use of extended root design and the space between the blade roots is sealed to prevent the working fluid bypassing the blades. Such a design is called "long-shank" design. The use of long shank bucket also results in more uniform temperature distribution.



(*a*) Precision-cast hollow bucket (*b*) Double cooled bucket

Fig. 7·27. Types of blades.

Mechanical considerations favour a large taper from the vane tip to root whereas the aerodynamic considerations limit this taper. The solution is the use of a hollow bucket [*see* Fig. 7·27 (*a*)] which reduces the vane stresses at bucket and decreases the slope of the allowable temperature variation *versus* height. The hollow tip also acts as a two tooth labyrinth seal and tends to reduce leakage through the bucket tip clearance.

Unfortunately the embrittlement and sensitivity to notch is increased with an improvement in creep strength of most materials at high temperatures. So an element of suitable structural design, especially the attachment of blade roots to discs, plays an important role in the overall design and its life.

(*iv*) **Degree of reaction**. Since the blade temperature decreases with a decrease in the degree of reaction, an impulse turbine is obviously more suitable for operation at high gas temperature. Another important effect is the variation of the degree along the blade. It increases from root to the tip. Usually the design is made on the basis of mean radius value. The reaction at hub is kept zero to avoid tip losses due to excessive reaction at the tip. This limits the hub to tip radius ratio of the turbine.

(*v*) **Mach number**. The possibility of high losses due to occurrence of shock waves requires that the Mach number along the turbine blade should be low. The maximum Mach number occurs at the root and directly depends upon the mean axial velocity used in the design. The blade shape is also quite sensitive to Mach number along the blade. Fig. 7·28 shows a typical example of supersonic turbine blades as affected by Mach numbers on upper

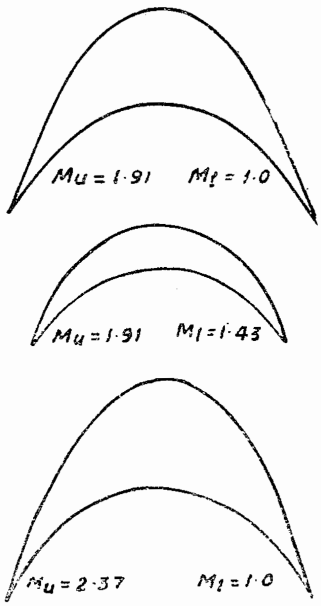


Fig. 7·28

and lower surfaces of the blade. It can be seen that an increase in lower-surface (concave) Mach number M_l is more acceptable at high upper-surface mach number M_u since the decrease in thickness/chord ratio is less than the case with low values of upper-surface mach number M_u, *i.e.* at low values of inlet Mach number. It is also interesting to note that blade shape is more sensitive to changes in M_l than in M_u.

Higher the pitch/chord ratio less is the profile losses. The blade loading is also dependent upon the surface Mach numbers. The maximum blade loading occurs where there is minimum M_l on concave surface and maximum M_u on convex surface.

(*vi*) **Outlet flow**. Impulse turbine volume flow at outlet determines the rotor exit velocity triangle as well as the rotor inlet velocity triangle since there is no density change between inlet and outlet in an idle impulse turbine. Even for reaction turbines, it is more singnificant because it controls the outlet velocity triangle while inlet flow even does not control inlet velocity triangle except for the unusual case of 100 per cent reaction. This also limits the outlet M_u. If outlet M_u is decreased then the pressure on the outlet convex surface increases with the possibility of flow separation and the resultant losses and divergence of the angle.

(*vii*) **Vibrations**. The blade shape is determined by the aerodynamic considerations so that resonance between natural frequency of a blade and any of the forcing frequency, if present, would cause fatigue fracture due to vibrations. Moreover, each blade leaves a slight disturbance in the medium flowing away from it and this 'wake' effect acts upon the rotor blades on next stage. The pulses so caused recur with a frequency equal to multiplication of the rotor speed and the number of blades.

The vibrations are avoided by keeping the natural frequency lower by employing a large pitch and small number of blades because slender and narrow blades designed by mechanical and aerodynamic considerations, have several modes of vibrations, especially, bending and torsional modes, which have quite high natural frequencies.

7·15. MATERIALS FOR GAS TURBINES

The materials used in gas turbine are often stressed to their very limits under the action of high temperatures and pressures. The materials used, therefore, must be tested for temperatures that are expected to be encountered, creep strength, yield strength and ductility, thermal and mechanical shock resistance, oxidation and corrosion resistance, embrittlement, adaptability for the manufacturing and the working processes, the expected life of the various components, etc. In addition to the above, the cost and the availability of the material plays an important role in its selection for use in gas turbines.

The detailed metallurgical discussion of the various properties and manufacturing processes is out of scope of this book. However, in what follows a brief idea is given of the important properties of the alloys and their relevance to the gas turbine.

(*i*) **Creep**. Most engineering materials, when stressed to the elastic limit for very long durations, undergo a permanent deformation and ultimately fail under a stress much below the maximum short-run strength of the material. This phenomenon of permanent deformation under prolonged load is called creep and the ultimate failure is termed as rupture. Creep occurs because of the fact that individual crystals slip along the crystallographic planes and some gain boundary movement also occurs under such stresses. The net effect is that the material behaves like a plastic material.

High creep strength is very important for the gas turbine materials because if permanent deformations take place the clearances at many closely-fit parts as well as the flow passages would be reduced resulting in sharp declines in the efficiency or complete failure of the plant.

Fig. 7·29 shows a typical creep strain *vs* time plot. This curve can be divided into three important stages. The rate of creep

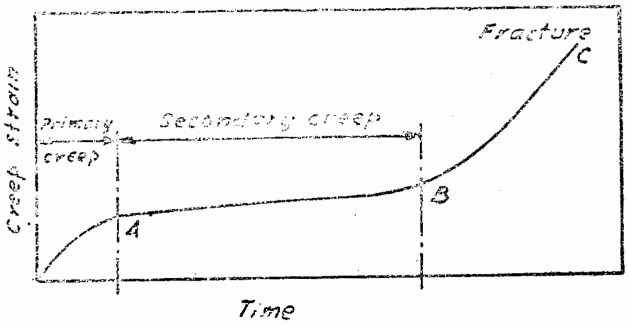


Fig. 7·29. Typical creep strain *versus* time plot.

in the first stage, called primary creep, decreases rapidly with time and is followed by the second stage, the secondary creep zone, in which the rate of creep remains constant or reduces slowly. The secondary creep zone is relevant for gas turbines because it covers the effective life of the components. The rate of creep again increases during the third stage and is ultimately followed by rupture.

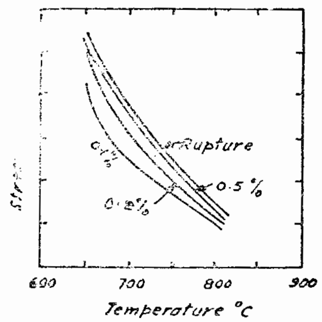
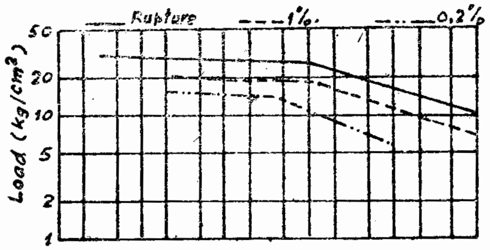
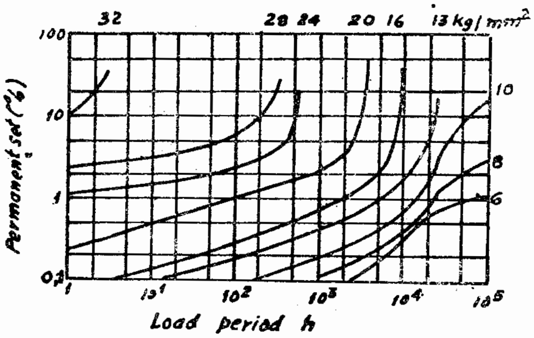


Fig. 7·30. Creep strength of Nimonic 80 A alloy for different strain in 1000 hrs.

The creep strength of a material depends upon the temperature and the amount and the type of stresses occurring in the particular component. It rapidly decreases with increase in the temperaure. Fig. 7·30 shows the creep strength of a Nimonic 80*A* alloy for strains in 1000 hours. For design purposes not the rupture stress but the stress producing 0·1 or 0·2 per cent creep strain for about 100000 hours of stressing is used. Therefore, a large number of tests covering very long periods are necessary to detect some processes which appear only after very long operating periods. Fig. 7·31 shows the creep curves of an austenitic nickel-chromium steel with additions



(*a*) Creep curves



(*b*) Creep limit and rupture curves

Fig. 7·31. Creep curves

of molybdenum, niobium and tungsten, the upper diagram showing curves for 0·2% and 1% elongation limits and long-time rupture curve. It can be seen that some of the test pieces have failed while some others are still under test after 110000 hours. The use of 100000 hours creep strength is the standard practice today for design calculations. Some manufactures even test specimen taken from the same lot from which turbine parts were made and keep ahead of the actual turbine, so that a failure can be predicted before it actually occurs.

The alloy group which is of interest to present and future high temperature gas turbines are molybdenum, cobalt, nickel and

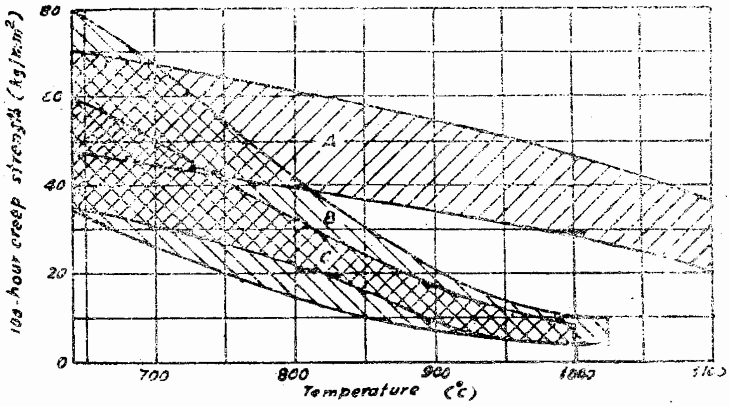


Fig. 7·32. (a) 100 hour creep strength of highly heat resistant nickel-cobalt base and molybdenum base alloy at various temperatures
A Molybdenum alloys ; *B* Nickel alloys ; *C* Cobalt alloys ;
D Columbium alloys.

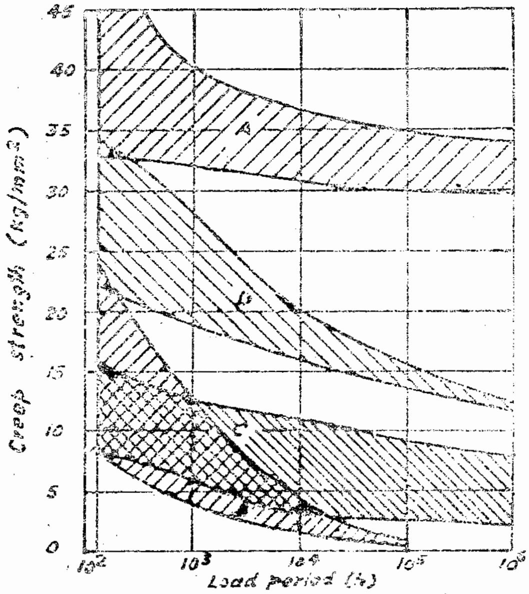


Fig. 7·32. (b) Creep strength (extraporated after Larson-Miller) for four different heat resistant alloys at temperatures 900°C upto 1,000,000 hours.

columbium (niobium) alloys. Fig. 7·32(*a*) shows 100 hour creep strength of highly heat resistant nickel-cobalt base and molybdenum base alloys at various temperature. Fig. 7·32 (*b*) shows the creep strength of these alloys up to 1,00,000 hours at 900°C extrapolated by Larson-Miller method.

It can be seen that creep strength for molybdenum, cobalt, and nickel alloys overlap in temperature range of 650°C. The nickel alloys have the highest creep strength, molybdenum alloys the next, and cobalt alloys the least in this temperature range. However, the pattern changes rapidly with rising temperature—nickel and cobalt alloys falling quickly in strength while molybdenum alloys show a much slower drop. Even at temperature below 700°C they show best creep strength.

From Fig. 32(*b*) which also includes data for columbium alloys, it is clear that under very-long periods and high temperature of 900°C the molybdenum alloys gives best performance. Columbium alloys have creep strengths more or less equal to that of molybdenum alloys for short-test periods but fall short of them for longer periods. In addition, columbium alloys which are much more costly than molybdenum alloys show much lower heat resistance. Cobalt alloys show the poorest performance at long-test periods.

Tantalum alloys are also good high heat resisting materials but are much more costly than niobium alloys. Tungston alloys grow brittle above 400°C and cromium-base alloys also show brittleness at about 500°C.

Molybdenum alloys have so far the best creep strength for short and long period tests and are of major interest. These alloys are also noted for their very good behaviour with respect to helium to which they have excellent resistance at temperatures up to 2500°C. Thus for closed cycle gas turbines using helium these alloys are an automatic choice.

Now, due to developments in manufacturing and machining methods, molybdenum alloys with the addition of titanium and circonium can be cast into blocks of one tone or more from which turbine parts can be forged. The fact that these alloys can also be welded gives them added advantage in the manufacturing programme over other materials.

(*ii*) **Surface corrosion.** With the use of elevated pressure and temperature the surface corrosion has become one of the barriers which must be crossed before reliable gas turbines can be produced.

The industrial gas turbine is the biggest sufferer in this matter. For such turbines use of inexpensive fuel such as crude oil is a must. Even with a small gas turbine the saving so obtained amounts to several thousand rupees per annum. These fuels have an ash content of about 0·01 to 0·10 per cent, which also contains a high percentage of vanadium. The combustion products are highly corrosive.

Another factor which makes the industrial gas turbine more susceptable to corrosion is that unlike its aviation counter part it cannot fly away from the dirt in the inlet air to which it is exposed. The fact that air-fuel ratio for a gas turbine is 50 or more, makes even a small degree of contamination of air much more disastrous than even a much large degree of fuel contamination. Moreover, the life required of an industrial turbine is very high typical example is a Westinghouse turbine manufactured for Mississippi River Transmission Corporation which has completed 131,000 hours of operation before shut-down for inspection. The turbine was 17 years old when it started its endurance run.

The high temperature corrosion is mainly due to presence of vanadium, sulphur and some sodium compounds in the fuel. The non-volatile vanadium occurring as an organic compound accumulates into the residue during distillation. This, when burned, gets converted into vanadium oxide (V_2O_5). The V_2O_5 content of such ashes varies from 20 to 40 per cent and can be as high as 90 per cent in some cases.

All known heat-resisting steels are severely attacked by ash containing vanadium at temperatures higher than the ash melting point. Even a few grams of vanadium cause catastropic damage in a few days. Moreover, the attack becomes intense at elevated

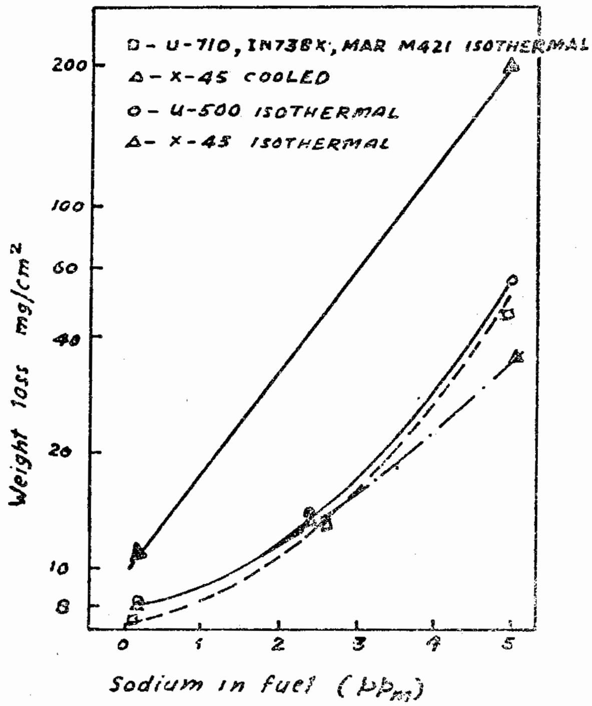


Fig. 7·33. Effect of sodium on various alloys (2 ppm V, 900°C and 150 hour test).

temperatures. But corrosion can begin at temperatures below the melting point of V_2O_5, which is 675°C, because due to invariable presence of sulphur and sodium, complex sodium vanadates are formed which are potentially more corrosive at lower temperatures than V_2O_5 itself. Thus the corrosion attack can start at temperatures of about 600°C. In addition to above, the V_2O_5 fluxes the protective oxide film from the surface and, thus, permits further oxidation of the exposed base metal.

Fig. 7·33 shows that even a small amount of sodium, in the presence of vanadium is quite corrosive. These data are taken in the pressurised passage at 800°F metal temperature and 3 atm pressuse for 150 hours. Fig. 7·34 gives the corrosion rate of several alloys when burning 3-GT fuel compared to normal oxidation after 150 hours. This fuel contained 5 ppm sodium and 2 ppm vanadium. Very high corrosion rates are apparent.

Removal of vanadium from the fuel is very costly ; therefore, two approaches are used to solve the problem. These are :

(1) Dilute the ash so that its vanadium content is harmlessly low.

(2) Increase the melting point of the ash above that of metal temperature likely to be encountered.

The second approach is more promising and additives like dolomite, magnesium oxide, calcium oxide, silicic acid, alumina, kaolin etc., may be added.

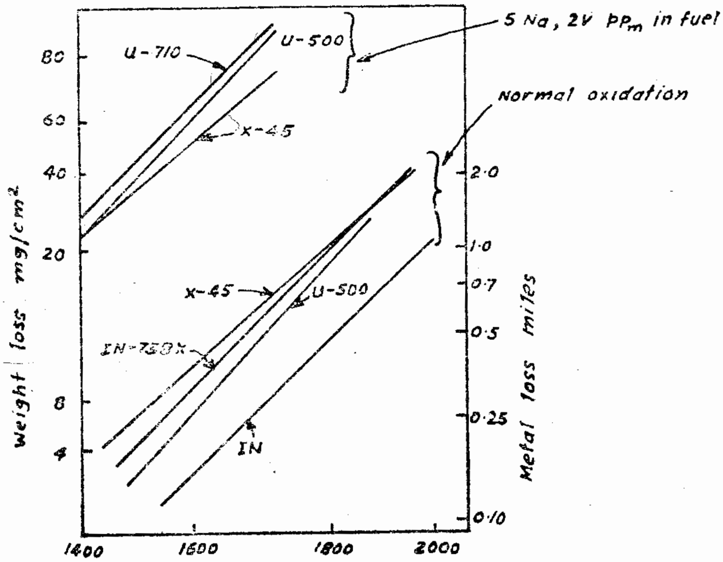


Fig. 7·34. Corrosion rate of several alloys when burning 3-GT fuel compared to normal oxidation.

(*iii*) **Typica alloys for gas turbines.** A large number of alloys, coatings, and surface treatments, have been investigated for gas turbine use. Early machines used stainless steels, stellite (a cobalt base alloy) and inconel (a nickel base alloy). For the temperature characteristics of these early gas turbines, these alloys had adequate mechanical properties but exhibiting a tendency towards corrosion.

Table 7·1 shows the super alloys and their compositions which are likely to be used in present and future gas turbines. Nickel-base alloys have more resistance to vanadium attack and also have good mechanical properties, but unfortunately they have poor resistance to sulfidation. In general, higher chromium alloys have better (or at least the attack is less severe) corrosion resistance than those predominantly nickel. Presence of aluminium reduces resistance to scaling. The present trend in the development of nickel base alloys of increasing aluminum and decreasing chrominm is, thus, resulting in poor corrosion resistance. This trend is likely to be reversed for super alloys.

Inconel X, so far widely recommended, has good resistance to V_2O_5 but poor resistance to attack by sulphur, INCO 713 C has better mechanical properties and better oxidation resistance. Udimet 500 with high cromium content is quite resistant to surface corrosion and is among the nickel-base alloys being widely recommended. Other alloys under test are INCO 738 X, with a better strength at elevated temperatures and X-45, N155 and MU21. Type 310 stainless steel is also of interest.

Some other very high strength nickel-base alloys are TAZ-8, MAR-M 200, IN-100 and Hastelloy-X. TAZ-8 which has, by weight 8% tantalium, 6% cromium, 6% alminium, 4% molybdenum 4% tungsten, 2·5% vanadium, 1% zirconium, 0·125% carbon and rest nickel, is basically a cast alloy in which vanadium is the main binding material. This alloy has high heat resistance and good mechanical strength. However, it is highly oxidized at 900°C because the melting point of V_2O_5 is only 675°C. So a modified version of TAZ-8, called TAZ-8A has been produced by removing vanadium and adding 2·5% columbium which forms a relatively high melting point intermediate compound Ni_3 Cb, and also 0·004% boron which improves stress rupture life and hot workability of some nickel-base alloys. Another alloy of interest which has a high oxidation resistance is Hastelloy-X. But it is still to be investigated for commercial application.

Above of temperature of 900 or 1000°C and more, most of the alloy which possess high heat-resistance are no longer sufficiently scale resistant. This is especially true for the molybdenum alloys, which are otherwise very interesting due to their high creep strength at high temperatures. The closed cycle gas turbine is better placed than its open cycle counterpart because of the fact that the working medium need not necessarily be air or some oxidizing gas. Instead

TABLE : 7·1
CHEMICAL COMPOSITION (IN MASS %) OF SOME TYPICAL HIGH TEMPERATURE SUPER ALLOYS

Alloy	*Cr*	*Ni*	*Co*	*Al*	*Ti*	*Fc*	*W*	*M8*	*Columbion*
Inconel X	15	BAL*	Nil	0·7	2·5	7	Nil	Nil	1
Inco 713C	12·5	BAL	Nil	6	Nil	1	Nil	4	2
Inco 738X	16	BAL	8·5	3·4	3·4	Nil	2·6	1·8	Nil
INCO 700	15	BAL	28	3	2·2	0·7	Nil	3·7	Nil
Udimet 500	18	BAL	18	3	3	Nil	Nil	4	Nil
Udimet 520	15	BAL	12	2	3	Nil	1	6	Nil
Udimet 700	19	BAL	18	4·3	3·5	Nil	Nil	5	Nil
Udimet 710	18	BAL	15	2·5	5	Nil	1·5	3	Nil
X-45	25	10	BAL	Nil	Nil	2	7	Nil	N 1
N 155	21	20	20	—	—	BAL	2·5	3	1
M 421	15	BAL	10	4·3	1·8	Nil	3·5	1·8	Nil
310	25	20	2%Si			BAL			
Hastelloy H	21·65	BAL	1·65	0 80Si	0·13C	17·0	0·85	9·46	
Rene 41	19·25	BAL	11·07	1·60	3·19	0·30	·0017 Boron	9·81	0 05Si ·01C
GMR-235	15·29	BAL	Nil	2·54	2·13	10·16		5·16	·042B ·11C ·5Si

BAL means base alloy

a neutral gas like helium can be used without any allowance for scale formation.

The temperatures and pressures used are increasing continuously over the past years and the demands on the material are increasing so steadly that very soon even the austentic steels will not remain satisfactory. The trend is now towards non-ferrous alloys of high heat resistance. Ceramics and ceramals are also under active consideration for use in turbine blades.

It is felt that ultimately some combination of blade coating additive treatment and blade cooling may be proper answer to the corrosion problem in gas turbine. Blade cooling is an instrument in reducing both oxidation and a sulphidation type of attack. Studies have shown that the amount of corrosion attack exhibited by super alloys is dependent upon metal surface rather than the gas-stream temperature and is an exponential function of the metal temperature. This fact makes cooling even more important.

7·16. COOLING OF TURBINE BLADES

The advantages of employing high turbine inlet gas temperatures are well known. Use of higher turbine inlet gas temperature results in higher thermal efficiency and lower specific fuel consumption. As is clear from Fig. 7·35 the output increases approximately by 12 percent and efficiency by 3 percent for a non-regenerative cycle gas turbine or 8 per cent for a regenerative cycle unit for each 55°C increase in the inlet temperature. Output and efficiency increase approximately by 2·5 per cent for each 1 per cent increase in turbine efficiency and 1·5 per cent for each 1 per cent increase in compressor efficiency. This would require a proper matching of the turbine efficiency and the inlet temperature to be used. Also, in an aircraft gas turbine the minimum specific fuel consumption conditions require a high turbine inlet temperature. If weight is more important, then the optimum turbine inlet temperature for maximum specific thrust is still higher. Thus, the present trend for using high compression ratio and high turbine inlet temperatures mean that there will be a large thermal stress on the blades in addition to the usual centrifugal stresses. This problem can be met in two ways :

(*i*) Development of high-heat resistant materials.

(*ii*) Development of better turbine blade cooling methods so that a higher turbine inlet temperature can be used without increasing the blade metal temperature.

It is true that the best approach is to use the first method because the use of second method involves a partial loss of simplicity for which the gas turbine is famous, and also increased maintenance. However, the development of a new high-heat resistant material involves arduous development work and may take a couple of years before it can be commercially used, and thus, it is onys because of the developments in the field o f blade cooling that it hla been possible to use high temperature operating conditions which otherwise would have remained metallurgically impossible for many

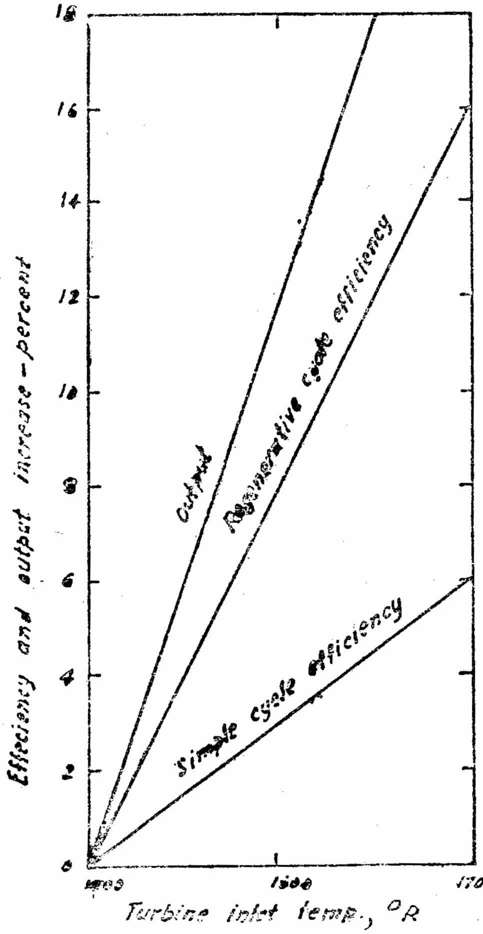


Fig. 7·35. Effect of turbine inlet temperature on gas turbine output and efficiency.

years to come. Currently a maximum cycle temperature as high as 1500 K has been used. By employing blade cooling one can not only use the present-day materials to sustain higher temperatures and pressures, but also cheaper materials can be used for some parts. Thus the cost can be reduced drastically by resorting to cooling of blades, which seems to be the only promising line of development towards the use of higher turbine temperatures.

The blade cooling systems may be broadly divided into two main categories :

(1) Air cooling

(2) Liquid cooling.

These can further be divided into

(*i*) Internal air cooling

(*a*) Hollow blade, with or without inserts

(*b*) Solid blade with radial holes, with or without inserts.

(*ii*) External air cooling

(*a*) Film cooling

(*d*) Effusion cooling using porous blades or transpiration cooling

(*e*) Root cooling

(*iii*) Internal liquid cooling

(*f*) Forced connection cooling

(*g*) Free convection cooling in the open thermosyphon

(*h*) Free convection cooling in closed thermosyphon

(*i*) Evaporative cooling in closed thermosyphon.

(*iv*) External liquid cooling

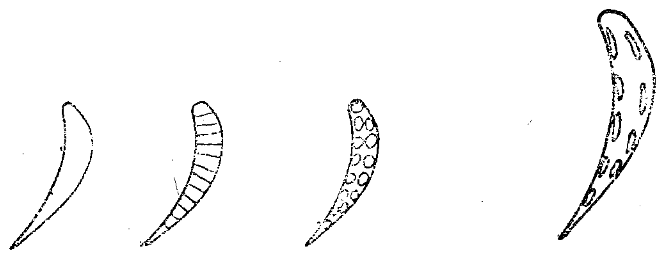
(*j*) Sweet cooling using porous blades

(*k*) Spray cooling.

The principle underlining most of the above system is to allow the heat to flow into the blade and thence remove it by the coolant. The main exception is film cooling. In this type of cooling an insulating layer of air is provided around the blade which reduces the heat flow into the blade. Some manufactures also use a ceramic coating over the blades to reduce heat flow into it.

7·16·1. Internal air cooling

Internal air cooling through a completely hollow blade or through a number of radial holes in an otherwise solid blade is



(*a*) Hollow lade

(*b*) Partition included to give more heat surface

(*c*) Hollow blades with tubes to increase heat transfer surface

(*d*) Cooling with elliptical holes

Fig. 7·36. Blade cooling arrangements.

relatively simple, and after 1960 has come into wide use for industrial, aircraft, and marine gas turbines. Air is bled from some point in the high pressure compressor circuit and fed to passages formed within the nozzle guide vanes and the rotor blades. The air removes the heat and joins the main gas stream of the turbine. About 2 per cent of the total turbine flow per set of rotor and stator is required to keep the blade temperatures within limits. The heat transfer co-efficient of the hollow blades can be increased by partitioning it or by packing tubes inside the hollow blade (*see* Fig. 7·36).

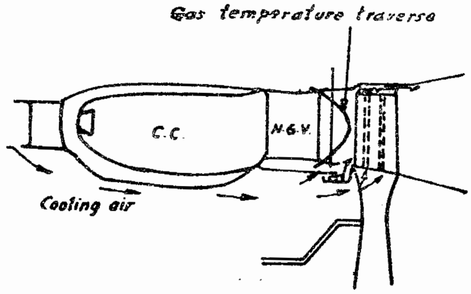
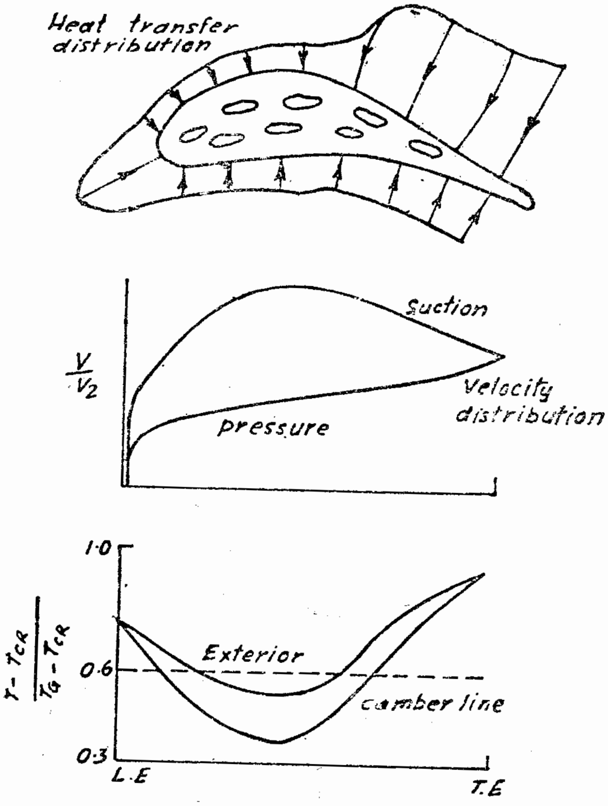


Fig. 7·37. Typical internal air cooling system

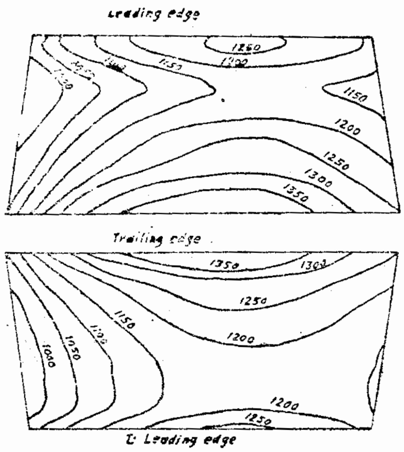
Fig. 7·37 shows a typical internal air cooling system in which cooling air is supplied into passages starting from within the blade root and running spanwise to blade tip, where it is discharged. In order to ensure that overtip leakage of main gas stream is reduced and that all cooling passages discharge against more or less the same static pressure a shroud and a sealing system is used over the blade tip. The static pressure difference between the coolant and the main gas stream depends upon the detailed seal design.

Fig. 7·38 shows the heat transfer distribution along the blade. The cooling effectiveness of the air is reduced towards the blade tip as it gets heated. Therefore, the general level of the temperature tends to rise from root to tip. If the radial temperature distribution of the gas relative to blade is very non-uniform, it may reach its highest value away from the tip as is clear from Fig. 7·39. This is very important because it reduces the maximum stress occurring at the blade tip. The local temperature distribution also affects the oxidation and corrosion resistance of the blade.

Another important factor in internal air cooling system is the acceleration of the coolant flow to the rim speed of the turbine rotor. The temperature of the coolant rises by an amount $U_{rim}^2/2\ c_p$, which can be as high as 50° to 60°C. That is, the air gets preheated even before it had actually started the cooling process. This becomes very undesirable where high pressure ratio compressors are used. To avoid this, an alternative feed system is used which allows the air to expand through a number of prewhirl nozzles. By matching the tangential speed of the air entering the turbine wheel with that of



L.E. = Leading edge T.E. = Trailing edge
Fig. 7·38 Heat transfer distribution along the blade.

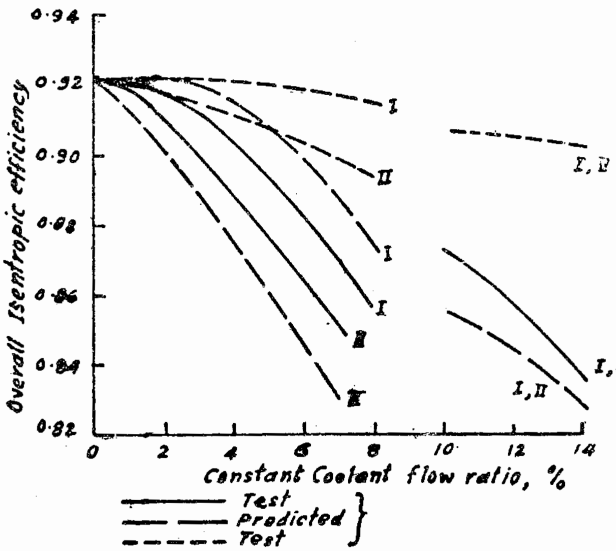


Operating conditions
Reynolds no. 3×10^5 ;
Gas temperature,
mean 1427 K,
Peak 1507 K ; Cooling
flow ratio 1·35%

Fig. 7·39. Blade surface isotherms.

the rim speed, a reduction of 50° to 60°C in the air temperature is obtained. This allows a reduction in the necessary amount of coolant flow, though at a loss of certain pressure head. However, this is sometimes an advantage in that the static pressure ratio (coolant inlet/outlet pressure) required to be sustained by the tip seal is reduced. In a typical turbine with prewhirls the coolant flow required was reduced by 4 per cent and the static pressure ratio from 1·8 to 1·3.

Since air is taken from the compressor, some direct loss of turbine power occurs. If the coolant air joins back the main gas stream after its cooling function some of this loss is recovered. However, work has to be done in pumping the cooling air, and losses occur at various take-off and joining places, and most important of all when the coolant air mixes with the main gas stream, some mixing losses occur and the main flow is disturbed. This mixing of cold air into hot main stream also results in some heat losses. The combined effect of all is to reduce the power and efficiency by some degree. Fig. 7·40 shows that the efficiency falls very sharply with the increase in the coolant air flow.



Efficiency based on inlet mass flow alone.

I Ist. stage rotor cooled.

II 2nd stage rotor cooled.

I, II Both rotors cooled (2nd stage coolant ratio constant at 7%)

Fig. 7·40. Effect of cooling air discharge on the overall efficiency of a two stage turbine.

7·16·2. External air cooling

In case of film cooling, a layer of coolant air is formed on the surface of the blade by injecting air streams at various places. A cold boundary layer is formed which acts as an effective insulator for the blade. This method, thus, prevents the heat flow to the blade unlike other methods in which heat flow is allowed into the blade from where it can be removed.

The coolant has a high velocity parallel to the cooled wall and removes the heat flux by convection.

In transpiration or effusion cooling the blade is made of a porous material so that by discharging the coolant through these semi-infinite number of small pores, a cool boundary layer is produced. This method is seen to be the ultimate in air cooling because it combines the conventional convective cooling (as the air flows along the cool side, *i.e.* within the turbine blade) with the film cooling on the other (hot) side where it emerges from the pores. This has been possible because of the development of sintered material with the help of the powder metallurgy. The sintering process ensures the maximum contact between the laminae and also the required physical strength. Thus, in addition to convective and film cooling, the heat is also transferred by the highly effective interstitial heat transfer process. This is like packed-bed type of convective heat transfer.

The cooling obtained by transpiration depends upon the material used for the blades. They range from electron-beam drilled sheet, *e.g.* M_6 and M_8, woven gauges of varying fineness *e.g.* Rigimesh 12×64 and 50×250, where the figures indicate the number of warn and welf wires per inch, to the closely controlled permeability porous materials.

The fine gauges, made from wire diameters of the order of 0·076 mm behave like the true porous material. The drilled sheet also behaves like a porous material but performs a little less well than a fine gauze. Porous materials used are bronzes or stainless steels formed by powder metallurgy, *e.g*, "Porosint" bonze of porosity grades *A*, *B*, *C*, *D* and porous stainless steel of porosity grades *H* and *E*. Tests show that about 3 to 3·5% of the mainstream flow per row of blades is required to keep blade temperature below 1200 K with cooling air at 300 K. The exact value would naturally depend upon the convective heat transfer coefficient between the coolant and the inner surface of the porous materials.

The future of the transpiration cooling will depend upon how closely the permeability along and across the span can be controlled for optimum heat transfer distribution and the long-life service of these materials. The porous materials are likely to be blocked by presence of any foreign particles in coolant and mainstream flow. The resistance to corrosion and oxidation of these materials is still under investigation. The losses due to mixing and other aeorodynamic effects of this type of cooling are very small and the fall in efficiency in the turbine is less than 2 per cent. Developments in material technology suggest that very soon this method of cooling will become a standard practice in future high temperature turbines. Temperatures at least as high as 1500 K are feasible with transpiration or effusion cooling.

7·16·3. Internal liquid cooling systems

(*i*) *Forced convection cooling.* In this system water is pumped radially outwards through holes drilled in blade section and returned through two parallel holes. These holes have chordwise cross-connections at the tips. This system has been very little used because of the difficulty in providing a continuous flow system within the narrow confines of the blade interior. Furthermore, it is quite difficult to match the performance of free-convection system presently in use, even after using organic liquids of higher boiling point.

(*ii*) *Open ended free convection system.* The open-ended free-convection system consists of a heated tube, closed at one end, and opening at the other into a large reservoir of Fluid (*see* Fig. 7·41). Due to the centrifugal accelaration the heated fluid flows over the walls towards the reservoir by natural convection. Simultaneously a core of cool fluid from reservoir penetrates in the tube and fills it. The pressure is high enough to prevent formation of steam in the blade cavities. If any steam is formed it is formed at the inner free surface of the water. This steam can be lead away for condensation and recirculation or can be used to drive an auxiliary steam turbine, or in a steam jet refrigerating plant to increase cycle efficiency by pre-cooling inlet air.

Blades with cavities of circular cross-section give better results than rectangular or other shapes because it has better heat transfer characteristics. It is better to use a larger hole than a few small holes because the pronounced mixing of hot and cold steam can be avoided. The orifice shape is also important. The sharp-edged type gives better overall heat transfer than the rounded type.

The main difficulties with the open-ended free-convection system are—

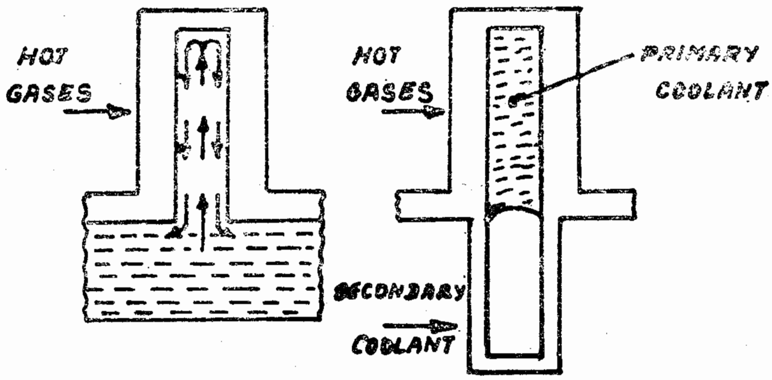
(1) The blades are more prone to corrosion because water is the only practical coolant.

(2) There is more likelihood of foreign material accumulation in the blind end of the system. This may result in catastrophic failure of turbine blades.

(3) Ebullition takes place in the tube. This results in uneven and out-of-balance forces on the turbine blades causing severe vibrations. The control of coolant flow is also difficult due to ebullition.

(4) Coriolis acceleration, due to rotation acting on the fluid density distribution, distorts the flow and sometimes causes orifice instability,

The mixing problem due to orifice instability can be mitigated by maximising the cross-sectional area and by inducing a separation of hot and cold flow by suitable tube inclination,



(*a*) Open ended system (*b*) Closed ended system

Fig. 7·41. Internal liquid cooling systems.

(*iii*) *Closed ended free convection cooling.*

Fig. 7·41 (*b*) shows the closed-ended free-convection cooling system and Fig. 7·42 its various arrangements. In this system the

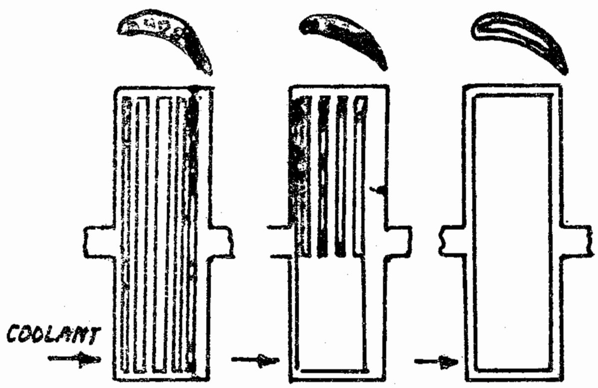


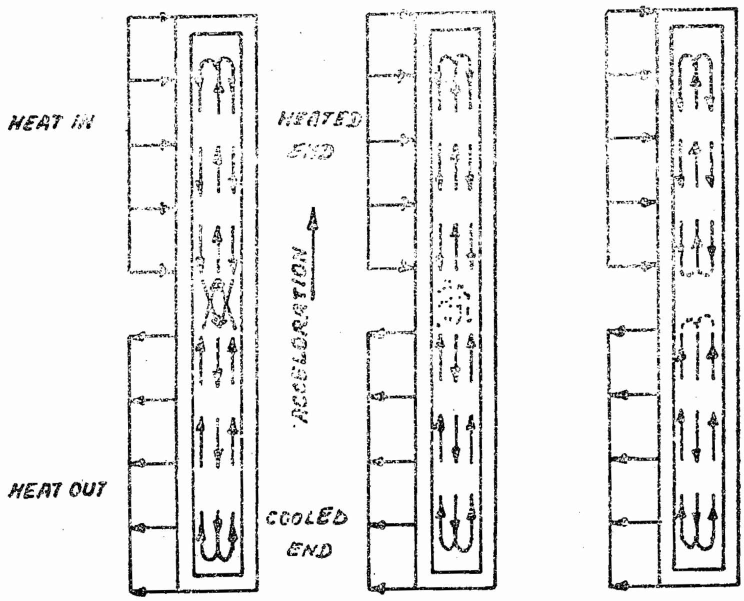
Fig. 7·42. Various internal arrangements of blades in closed ended system.

primary coolant is completely enclosed by extending the blade root and cooling passage into the rotor drum and sealing it at both ends. This avoids the possibility of failure due to accumulation of foreign material. The heat is transferred from blades to the closed thermo-syphon and subsequently transferred to a secondary fluid.

The circulation of the coolant at the heated and cooled ends of the blade passage is in opposite directions as seen in Fig. 7·41.

The motion of the primary fluid in the closed-end thermo-syphon is a combination of the three coupling modes shown in Fig. 7·43. These modes co-exist in a complicated manner depending upon the Prandtl number and strongly affect the heat transfer co-efficients.

A radial focusing jet is generated by the approaching cold and hot coolant annuli. This jet is highly stable at very high Prandtl numbers associated with the laminar flow. When the jet is highly stable the heat transfer across the coupling is only because of axial conduction as seen in Fig. 7·43 (*c*). This higbly stable jet breaks down immediately after formation if a large temperature



(*a*) Mixing (*b*) Convective (*c*) Conduction

Fig. 7·43. Coupling modes in closed-end free-connection system.

difference is present. Then the hot and cold streams burst open into each other at high Prandtl number and a convective mode of coupling is produced resulting in maximum heat transfer. This is shown in Fig. 7·43 (*b*). For still higher temperature difference and low Prandtl number the jet becomes turbulent [*see* Fig. 7·43 (*a*)]. Free exchange of momentum and heat transfer takes place. The overall heat transfer is intermediate between that of the maximum by convective mode and the minimum by conduction mode. The effective cooling can also be increased by employing inclined tubes as in open-ended free-convection system.

One advantage of closed thermosyphon system is that it allows a wider choice of primary coolant. Substance which would have unacceptable properties like toxicity, chemical unstability, cost or sheer weight for an open thermosyphon system, would now be enclosed in permanently sealed tubes and water or air can be used as the secondary coolant. One important example of such a choice is the possibility of the use of liquid metals for cooling purposes. Liquid metals have high critical temperatures and thus allow a higher temperature, very near to the maximum with present materials without the use of excessive pressures on blade. Sodium-potassium eutectide mixture (Na K) with water as secondary coolant allows a mean blade temperature of 570°C with a gas temperature of 1320°C. The maximum metal temperature at the trailing edge of the blade does not exceed 760°C.

Another advantage is that the primary coolant can be used up to its critical point where the heat transfer coefficient tends to be very large.

(*iv*) *Evaporative cooling.* In this system of cooling the blade cavity is only partially filled with the primary coolant and the advantage of high heat transfer due to boiling in the heated length and condensation in the cooling length is taken. The use of reduced amount of fluid also results in reduced mechanical difficulties by ebullition induced vibrations and hydrostatic pressures.

For gas turbine blade the use of water has the great disadvantage of having a low critical temperature for evaporative cooling. But at the same time water is highly superior to other liquids due to its very high thermal conductivity and latent heat. Some liquid metals, especially mercury, is quite suitable for this type of cooling if a condensing mercury which would wet a metallic wall can be prepared.

7·16·4. External liquid cooling

External liquid cooling systems include spray cooling and sweat cooling. In spray cooling water is injected over the blades in such a manner that complete blade is covered while sweat cooling utilizes a porous blade material for passing the liquid to the blade surface.

Spray cooling is more prone to corrosion and erosion of blades. It also renders the gas relatively colder for use in the later stages of a multistage turbine in addition to having adverse aerodynamic effects.

Use of sweat cooling largely depends upon the development of materials with closely controlled porosity for ensuring the optimum distribution of the coolant around the blade surface.

Liquid cooling systems have greater thermodynamic advantages over its air counterpart. However, these systems are not suitable for aircraft turbines, which needs maximum cooling, because of the necessity of carrying large quantities of water. The difficulty of designing a very reliable liquid-cooling system at very high speeds used in marine and industrial turbines is also a big disadvantage. This is coupled to the fact that yet much is to be known about the heat-transfer characteristics of liquid cooling systems with various geometries, liquids and other parameter over the working temperature range.

ILLUSTRATIVE EXAMPLES

7·1. Multi-stage impulse gas turbine : No. of stages

A multi-stage gas turbine is to be designed with impulse stages, and is to operate with an inlet pressure and temperature of 6 kgf/cm² and 900 K, and an outlet pressure of 1 kgf/cm². The isentropic efficiency of the turbine is likely to be 85 per cent. All the stages are to have a nozzle outlet angle of 15°, equal inlet and outlet blade angles, a mean blade speed of 250 m/s and equal inlet and outlet gas velocities. Estimate the number of stages required. Assume $c_p = 0{\cdot}276$ *and* $\gamma = 1{\cdot}333$.

For isentropic expansion

$$T_2' = \frac{T_1}{\left(\dfrac{P_2}{P_1}\right)^{\frac{n-1}{n}}}$$

$$= \frac{900}{\left(\dfrac{6}{1}\right)^{\frac{0{\cdot}333}{1{\cdot}333}}} = 575 \text{ K}$$

The isentropic efficiency is given by

$$\eta_t = \frac{T_1 - T_2}{T_1 - T_2'}$$

$$\therefore \qquad 0{\cdot}85 = \frac{T_1 - T_2}{900 - 575}$$

$\therefore$ The total temperature drop

$$T_1 - T_2 = 0{\cdot}85(900 - 575)$$

$$= 276{\cdot}3 \text{ K}.$$

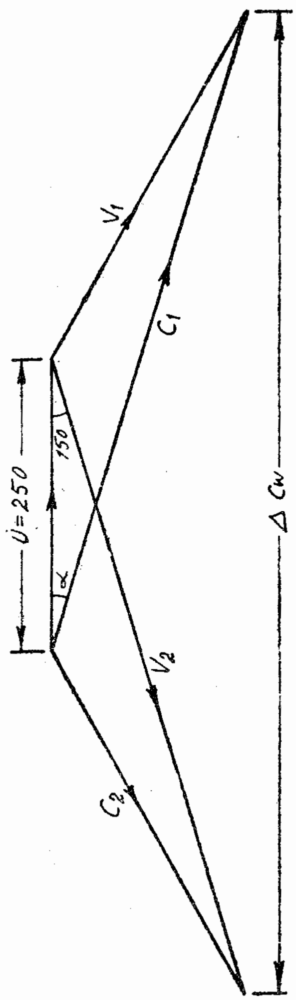


Fig. 7·44

For optimum blade speed ratio, the absolute velocity C_1 is given by

$$C_1=\frac{2U}{\cos\alpha_1}=\frac{2\times 250}{\cos 15}=517{\cdot}6 \text{ m/s}$$

Given $\beta_2=\beta_1$

and $C_0=C_2=C_1 \sin\alpha_1=517{\cdot}6 \sin 15=134{\cdot}1$ m/s

The temperature drop manifests itself in increased velocity of flow. Therefore, by writing down the energy conservation for the nozzle, we get

$$c_p(T_0-T_1)=\tfrac{1}{2}(C_1^2-C_0^2)$$

∴ The stage temperature drop

$$T_0-T_1=\frac{C_1^2-C_0^2}{2c_p gJ}=\frac{517{\cdot}6^2-134{\cdot}1^2}{2\times 0{\cdot}276\times 9{\cdot}81\times 427}=108{\cdot}7 \text{ K}$$

∴ Total number of stages required

$$=\frac{276{\cdot}3}{108{\cdot}7}=2{\cdot}54$$

i.e., 3 stages **Ans.**

7·2. Axial Turbine : Impulse wheel

In an axial-flow gas turbine using 30 kg of gas per sec the gas at entry to the nozzle ring has a pressure of 4·5 kgf/cm², the temperature of 925°C and a velocity 150 m/s. The gas expands with an adiabatic efficiency of 95 per cent to a pressure of 1·75 kgf/cm². Calculate :

(*a*) *the gas speed leaving the nozzle taking* $c_p=0{\cdot}276$ *and* $\gamma=1{\cdot}34$

(*b*) *the nozzle outlet angle required to give a whirl velocity of 670 m/s.*

The turbine rotor has a mean blade speed of 320 m/s, and the blade angles at entry and exit are equal. Find the blade angle required and, assuming an impulse wheel with an efficiency of 90 per cent, the horse-power developed.

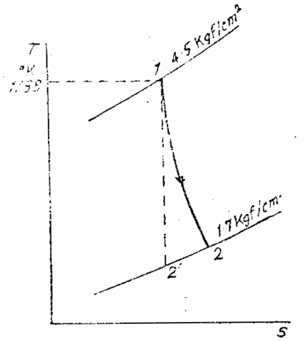


Fig.7·45

$$T_2' = \frac{T_1}{\left(\frac{P_1}{P_2}\right)^{\frac{\gamma-1}{\gamma}}}$$

$$= \frac{1198}{\left(\frac{4\cdot5}{1\cdot75}\right)^{0\cdot286}} = 913 \text{ K}$$

$$0\cdot95 = \frac{T_1 - T_2}{T_1 - T_2'}$$

$\therefore$ $T_1 - T_2 = 0\cdot95 \times (1198 - 913)$

$= 271$

Now $\dfrac{C_2^2 - C_1^2}{2gJ} = c_p(T_1 - T_2)$

$\therefore \dfrac{C_2^2 - (150^2)}{2 \times 9\cdot81 \times 427} = 0\cdot276 \times 271$

$\therefore$ $C_2 = 803$ m/s **Ans.**

Using angles from the axial direction

$$\alpha_0 = \sin^{-1}\frac{670}{C_2} = \sin^{-1}\frac{670}{803}$$

$\therefore$ $\alpha_0 = 56\cdot6°$. **Ans.**

$C_{a1} = U_2 \cos \alpha_0 = 803 \cos 56\cdot6$

$= 456$

$$\alpha_1 = \tan^{-1}\frac{350}{C_{a1}} = \tan^{-1}\frac{350}{456}$$

$\therefore$ $\alpha_1 = 37\cdot4°$

$\beta = 90 - \alpha_1 = 90 - 37\cdot4 = 52\cdot6°$ **Ans.**

Blade efficiency $= \dfrac{\text{W.D.}}{\text{K.E. supplie}}$

$$= \frac{U(\Delta V_w)}{g} \times \frac{2g}{U_2^2}$$

$= 0\cdot9$

$\therefore$ $U(\Delta C_w) = \dfrac{0\cdot9}{2} \times (803)^2 = 291000$

$\therefore$ Horse power developed

$$= \frac{MU(\Delta U_w)}{75 \times g}$$

$$= \frac{30 \times 291000}{75 \times 9\cdot81}$$

$= 11880$ H.P. **Ans.**

7·3. Reaction gas turbine : velocity diagram ; W.D. ; c_{exit} ; t_t ; and t_{2f} ; blade angles

The final stage of a gas turbine rotor with part reaction blading is supplied with gas at 1·5 kgf/cm² and a static temperature of 680°C. At mean section where the blade speed is 200 m/s the gas velocity at entry to the moving blades has a periphural component 430 m/s and an axial component 330 m/s. The gas expands through the stage to 1·04 kgf/cm² with an actual drop of temperature of 67°C. $c_p=0·276$.

(*a*) *Calculate the work done per kg of gas and the peripheral gas velocity at exit.*

(*b*) *Calculate the initial and final total temperatures of the gas.*

(*c*) *Assuming the annular area of the cross-section through the rotor to be constant find the axial gas velocity at exit and the required blade angle.*

Work done is proportional to the drop in total temperature

$$\text{W.D.}=Jc_p\triangle T=427\times0{·}276\times67$$

$$=7900 \text{ kcal/kg.} \qquad \textbf{Ans.}$$

If the blade peripheral velocity is U.

Then $$\text{W.D.}=\frac{U}{g}(C_{w1}-C_{w2})=Jc_p\triangle T$$

$$\therefore \quad C_{w1}-C_{w2}=\frac{7900\times9{·}81}{200}=388$$

$$\because \quad C_{w1}=430 \text{ m/s}$$

$$\therefore \quad C_{w2}=430-388=42 \qquad \textbf{Ans.}$$

$$C_1=\sqrt{430^2+330^2}=541 \text{ m/sec}$$

Temperature equivalent of this velocity

$$T_{c1}=\frac{C_1^2}{2gJc_p}=\frac{541^2}{2\times9{·}81\times427\times0{·}276}=126{·}8°\text{C}$$

$\therefore$ Total temperature at inlet to blade

$$T_t=T_1+T_{c_1}$$

$$=680+126{·}8=806{·}8°\text{C.} \qquad \textbf{Ans.}$$

Total temperature at exit

$$=T_1-\triangle T$$

$$=806{·}8-67$$

$$=739{·}8°\text{C} \qquad \textbf{Ans.}$$

Now $\quad$ Mass $\times$ Specific volume$=v_a\times A$

Mass and A being constant, we get

$$\frac{C_a}{\text{Specific volume}}=\text{constant}$$

$$\text{Specific volume}=\frac{RT}{p}$$

$$\therefore \qquad \frac{p\times C_a}{T}=\text{constant}$$

or $$\frac{p_1 C_{a_1}}{T_1}=\frac{p_2 C_{a_2}}{T_2}$$

$$T_2=T_t-T_{c_2}$$

$$=(739\cdot8+273)-\frac{C_{a_2}}{2gJc_p}$$

$$T_2=T_{2t}-T_{c_2}\;=(739\cdot8+273)-\frac{C_{a_2}}{2gJc_p}$$

or $$\frac{1\cdot5\times330}{953}=\frac{1\cdot04C_{a_2}{}^2}{1012\cdot8-\dfrac{C_{a_2}{}^2}{2\times9\cdot81\times427\times\cdot24}}$$

or $$1012\cdot8-\frac{C_{a_2}{}^2}{2\times9\cdot81\times427\times0\cdot24}=\frac{953\times1\cdot04}{1\cdot5\times330}C_{a_2}{}^2$$

or $$1012\cdot8-0\cdot000496\;C_{a_2}{}^2=2C_{a_2}{}^2$$

$$\therefore \qquad C_{a_2}{}^2=\frac{1012\cdot8}{2\cdot000496}$$

$$\therefore \qquad C_{a_2}=22\cdot4 \text{ m/s}$$

$$\tan\alpha_2=\frac{AB}{C_{a_2}}=\frac{200}{22\cdot4}=8\cdot94$$

$$\therefore \qquad \alpha_2=83\cdot6^\circ.$$ **Ans.**

7·4. Turbine design on free vortex theory

In a single-stage turbine designed on free vortex theory the following particulars are given :

Mass flow through turbine	$=18$ *kg/s*
Inlet total head temperature	$=1000$ *K*
Inlet total head pressure	$=4$ *kgf/cm*^{2}
Axial velocity at nozzle exit	$=260$ *m/s*
Blade speed at mean diameter	$=305$ *m/s*
Nozzle angle at mean diameter	$=25°$
Ratio of tip to root radius	$=1\cdot4$

The gas leaves the stage in an axial direction. Assuming that all the nozzle loss amounts to 4 per cent of the isentropic heat drop across the nozzle, find :

(a) *the total throat area of the nozzle ;*
(b) *the nozzle efflux angle at root and tip ;*
(c) *the static pressure after nozzle at root and tip ;*
(d) *the gas inlet angles at root and tip of the blades ;*
(e) *the rate of work done on the turbine blades in hp*

Assume a mean specific heat at constant pressure of 0·274 throughout the cycle. *(B.H.U., M.E., 1970)*

For no loss up to throat

$$\frac{P^*}{P_t} = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{2}{2\cdot333}\right)^{\frac{1\cdot333}{1\cdot333-1}} = 0\cdot542$$

$\therefore$ $$P^* = 4\times0\cdot542 = 2\cdot168 \text{ kgf/cm}^2$$

Also $$\frac{T^*}{T_t} = \left(\frac{P^*}{P_t}\right)^{\frac{\gamma-1}{\gamma}} = \frac{2}{\gamma+1} = \frac{2}{2\cdot333}$$

$\therefore$ $$T^* = 1000\times\frac{2}{2\cdot333} = 858 \text{ K}$$

$$T_t = T^* + \frac{C^2}{2gJc_p}$$

$\therefore$ $$C = \sqrt{2gJc_p(T_t - T^*)}$$

$$= \sqrt{2\times9\cdot81\times427\times0\cdot274(1000-858)}$$

$$= 572 \text{ m/s}$$

$$\rho^* = \frac{P^*}{RT^*} = \frac{2\cdot168\times10^4}{29\cdot27\times858} = 0\cdot863 \text{ m}^3\text{/kg}$$

(a) Throat area

$$A = \frac{G}{\rho U^*} = \frac{18}{0\cdot863\times572} = 364 \text{ cm}^2$$ **Ans.**

(b) Angle α_1 at any radius r and α_{1m} at the design radius r_m are related by the equation

$$\tan\alpha_1 = \frac{r}{r_m}\tan\alpha_{1m}$$

Given $$\frac{\text{Tip radius}}{\text{Root radius}} = 1\cdot4$$

$$\therefore \quad \frac{\text{Mean radius}}{\text{Root radius}}=1{\cdot}2$$

$$\alpha_{1m}=25^\circ$$

$$\tan \alpha_{1\,root}=\frac{r_{root}}{r_{mean}}\times \tan \alpha_{1m}$$

$$=\frac{1}{1{\cdot}2}\tan 25^\circ$$

$$=0{\cdot}389$$

$$\therefore \quad \alpha_{1\,root}=21^\circ\ 15'$$ **Ans.**

$$\tan \alpha_{1\,tip}=\frac{r_{tip}}{r_{root}}\times \tan \alpha_{1\,root}$$

$$=1{\cdot}4\times 0{\cdot}389=0{\cdot}5446$$

$$\therefore \quad \alpha_{1\,tip}=28^\circ\ 34'.$$ **Ans.**

The gas inlet angle at root and tip (β_1)

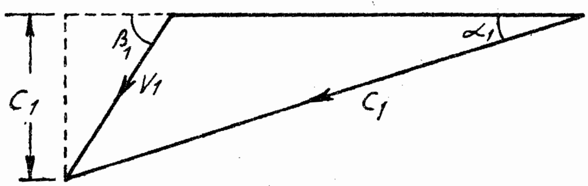


Fig. 7·45

$$U_r=\frac{r_m}{r_r}\times U_m$$

$$=\frac{305}{1{\cdot}2}=254{\cdot}2 \text{ m/sec}$$

$$V_{w1}=\frac{r_r}{r_m}\times V_{w1m}=\frac{r_r}{r_m}\times\frac{V_{a1}}{\tan \alpha_{1m}}$$

$$=1{\cdot}2\times\frac{260}{\tan 25^\circ}$$

$$=\frac{1{\cdot}2\times 260}{0{\cdot}4663}=669 \text{ m/s}$$

$$\tan \beta_{1r}=\frac{V_{a1}}{V_{w1r}-U_r}=\frac{260}{699-254{\cdot}2}=0{\cdot}629$$

$$\beta_{1r}=32^\circ\ 6'.$$ **Ans.**

$$\cot \beta_1 = \frac{r}{r_m} \cot \beta_{1m} - \left(\frac{r}{r_m} - \frac{r_m}{r} \right) \cot \alpha_{1m}$$

Replacing m by r

$$\cot \beta_{1T} = \frac{r_T}{r_r} \cot \beta_{1\gamma} - \left(\frac{r_T}{r_r} - \frac{r_r}{r_T} \right) \cot \alpha_{1\gamma}$$

$$= \frac{1 \cdot 4}{0 \cdot 627} - \left(1 \cdot 4 - \frac{1}{1 \cdot 4} \right) \times \frac{1}{0 \cdot 389} = 0 \cdot 522$$

$\therefore$ $\beta_{1T} = 62° \, 26'$ **Ans.**

(c) Since there is no swirl, $V_w = V_{w1}$. The same work will be done per kg of gas at all radii

$\therefore$ Work done/sec.

$$= \frac{Q \times U \times V_w}{g}$$

$$= \frac{18 \times 254 \cdot 2 \times 669}{9 \cdot 81 \times 75}$$

$$= \mathbf{4160 \ H.P.}$$ **Ans.**

(*f*) Static pressure after the nozzle at root and tip.

First consider the root

$$C_{1r} = \frac{V_{a1}}{\sin \alpha_{1r}}$$

$$= \frac{260}{\sin 21° 15'} = 717 \text{ m/s}$$

$$T_1 = T_t - \frac{C_{1r}^2}{2gJc_p} = 1000 - \frac{(717)^2}{2 \times 9 \cdot 81 \times 427 \times 0 \cdot 274}$$

$\therefore$ $T_1 = 776 \cdot 5$ K.

Since the losses are $0 \cdot 04$ of the isentropic heat drop,

$$T_1 - T_1' = 0 \cdot 04 (T_t - T_1'')$$

or $$776 \cdot 5 - T_1' = 0 \cdot 04 \times 1000 - 0 \cdot 04 \ T_1'$$

$\therefore$ $T_1' = 767$ K

$$P_{1r} = P_t \times \left(\frac{T_1'}{T_2} \right)^{\frac{\gamma}{\gamma - 1}} = 4 \left(\frac{767}{1000} \right)^{\frac{1 \cdot 333}{0 \cdot 333}}$$

$$= 1 \cdot 384 \text{ kgf/cm}^2$$

For tip

$$C_{1T} = \frac{V_{a1}}{\sin \alpha_{1T}} = \frac{260}{\sin 28° \, 34'} = 544$$

$$T_1-T_t-\frac{C_{1t}^2}{2gJc_p}=1000-\frac{(544)^2}{2\times9{\cdot}81\times427\times0{\cdot}274}$$

$$=871{\cdot}7$$

$$T_1-T_1'=0{\cdot}04(T_t-T_1')$$

$$\therefore \quad 871{\cdot}7-T_1'=0{\cdot}04(1000-T_1')$$

$$\therefore \quad T_1'=\frac{831{\cdot}7}{0\ 96}=866\text{ K}$$

$$\text{Pressure at tip}=P_t\times\left(\frac{T_1'}{T_t}\right)^{\frac{\gamma}{\gamma-1}}$$

$$=4\times\left(\frac{866}{1000}\right)^4$$

$$=2{\cdot}249\text{ kgf/cm}^2$$ **Ans.**

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8

JET PROPULSION

8·1. INTRODUCTION

All the propulsive devices are based on two fundamental laws, namely, Newton's second and third laws of motion. All these devices cause a change in momentum of a mass of fluid, called *propellant*, by imparting it energy, and the *reaction* of this accelerating mass of fluid produces, in a direction opposite to that of the mass flow, a *propulsive force* called *thrust*. This thrust propels the engine.

The propulsive devices can be classified into two main catagories :

1. Air breathing engines.
2. Rocket engines.

As the name implies. air breathing engines depend on the atmosphere for their supply of oxident (air) while rocket engines carry their own supply of oxident. In the first type of engine the air is taken in, accelerated either by a propeller or by expanding high pressure turbine exhaust in a nozzle or by a combination of both. This accelerated mass of air is ejected at the rear of the engine, the resultant change in momentum of the fluid produces a thrust to prope the engine. In a rocket engine the fuel is burned in the combustion chamber, the source of oxygen being within the rocket, and the products of combustion expanded in a nozzle imparting them a high velocity which produces the propulsive force. The performance of air-breathing engines is, therefore, affected by the ambient conditions and the forward speed of the aircraft while rocket performance is not affected by the forward speed and very little affected by ambient condition. In this chapter only the air-breathing engines have been discussed. Rocket engines form the subject matter of next chapter.

Air-breathing engines can further be classified as follows :

1. Reciprocating Engines.
2. Gas Turbine Engines.
   - (*i*) Turbojet.
   - (*ii*) Turboprop.

Note : In this chapter absolute velocity has been taken as V and not C. V here is not relative velocity.

3. Ram Jet.
4. Pulse Jet.
5. Ram Rocket

The reciprocating engine develops its propulsive force or thrust by accelarating the air with the help of a propeller driven by it, the exhaust of engine imparting almost negligible amount of thrust to that developed by the propeller.

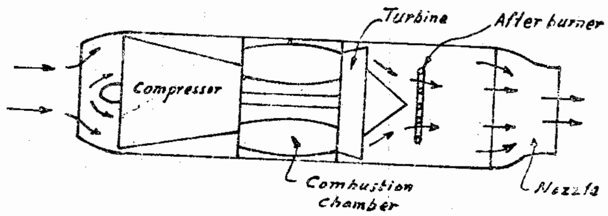


Fig. 8·1. Schematic diagram of a turbojet engine.

A turbojet engine, as shown in Fig. 8·1, consists of a diffuser, compressor, combustion chamber, turbine and a nozzle. The air is inducted in the diffuser which reduces its velocity, thereby, raising its pressure. The air is then compressed in a compressor of either axial or centrifugal type. The compressed air then passes to a combustion chamber where fuel is added and burnt. The products of combustion are fed to a turbine which produces power just sufficient to drive the compressor and other auxiliaries such as fuel pump, lubricating pump, etc. This amounts approximately to about two-thirds of the available energy in the exhaust. Exhaust from the turbine, which is at a higher pressure than ambient air pressure, is further expanded in a nozzle to a very high velocity. The reaction of this high velocity jet produces the propulsive force. About one-third of the available exhaust energy is used to produce thrust.

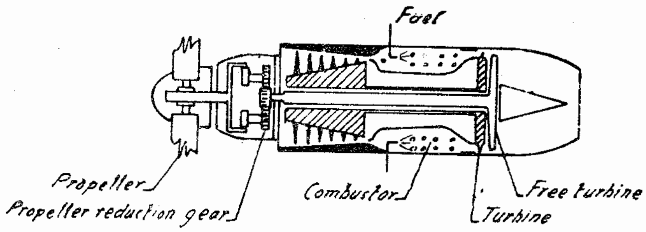


Fig. 8·2. Schematic diagram of a turboprop engine.

The turboprop engine (*see* Fig. 8·2) differs from the turbo jet in that it uses a propeller to increase the mass flow of air. In order to turn both the compressor and the propeller the turbine extracts almost all the available energy in the exhaust leaving very little energy for the acceleration of exhaust gases in the nozzle. This small energy which produces thrust by acceleration of exhaust gases in the nozzle amounts to about 10 percent of the total available energy in the exhaust. The air flow produced by the propeller bypasses the main engine and is ducted around the engine. The total thrust produced is the sum of the thrusts produced by the propeller and the nozzle respectively. The compressor used in a turboprop is generally of axial flow type. The turbine used may be coupled to both the compressor and propeller through suitable reduction gear or a two stage turbine may separately drive them.

In a ramjet, simplest of all the propulsive devices, there is neither a compressor nor a turbine. It consists of only three main components : diffuser, combustion chamber and the exhaust nozzle. The pressure of the incoming air is increased by the ram effect in the diffuser to a level sufficient for combustion. Fuel is injected in the combustion chamber and the products of combustion are expanded in a nozzle to produce thrust for the propulsion of the craft. In a turbojet engine the turbine is needed only to drive the compressor. As the compressor is eliminated in the ramjet due to compression obtained by ram effect, turbine gets eliminated automatically and the device has no moving parts. The ramjet will not operate statically unless a continuous stream of air is passed into the diffuser to produce sufficient pressure rise for combustion. For this reason it must be launched by some other power plant before it starts working. This results in very high specific fuel consumption for the ramjet.

The pulse-jet resembles a ram jet except that it has shutter valves which make the process of induction intermittent. Air pressure is raised in the diffuser which opens the shutter valves and enters the combustion chamber. The fuel is burnt in the combustion chamber and the consequent rise in pressure closes the valves and combustion takes place at constant volume. In this way pulse-jet cycle resembles the Otto cycle. The high pressure and high temperature gases, then, expand in the nozzle producing high exhaust velocities. Due to this high degree of acceleration a fall in the pressure of combustion chamber occurs and the pressure differential between the air in the diffuser and combustion chamber forces open the shutter valves, supplying fresh air for burning. Thus the cycle is repeated at a frequency of about 40 to 50 cycles per second depending upon the natural frequency of the duct, and a buzzing noise is produced.

8·2. THRUST

Before we undertake the detailed discussion of various propelling devices, their thermodynamic cycles and performances, it is worthwhile to discuss some of the basic laws of thrust production and the factors which affect the performance of the engine.

Let us consider the control volume of a schematic propulsive device shown in Fig. 8·3. A mass m_a of air enters the control volume

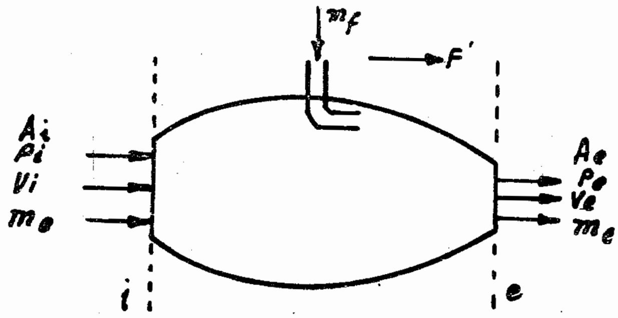


Fig. 8·3. Schematic diagram of a propulsive device.

with a velocity V_i and pressure p_i at the point i and the products of combustion of mass m_e leaves the control volume with a velocity V_e and pressure p_e at the point e. The flow is assumed to be steady and reversible outside the control volume, the pressure and velocity being constant over the entire control volume except that at the exhaust area A_e. Force F' is the force necessary to balance the thrust produced due to change in momentum of the fluid as it passes through the control volume.

If p_a is the atmospheric pressure, then writing the momentum equation, we get

$$m_eV_e-m_aV_a=F+(p_i-p_a)A_i-(p_e-p_a)A_e \tag{8·1}$$

or thrust $$F=(m_eV_e-m_aV_a)+(p_e-p_a)A_e-(p_i-p_a)A_i$$

We have, by mass balance

$$m_e=m_a+m_f \tag{8·2}$$

where m_e, m_a, and m_f are the mass flow rates of exhaust gases, air, and fuel respectively. If

$$f=\text{Fuel air ratio}=\frac{m_f}{m_a}$$

$$m_e=m_a(1+f) \tag{8·3}$$

$$\text{Thrust}=\underbrace{m_a[(1+f)V_e-V_a]}_{\text{Momentum thrust}}+\underbrace{(p_e-p_a)A_e-(p_i-p_a)A_i}_{\text{Pressure thrust}} \tag{8.4}$$

From equation (8·4) it is clear that the net thrust produced is made up two parts—momentum thrust and the pressure thrust. If the exhaust velocity V_e from the control volume is subsonic, then $p_e \approx p_a$ and also $p_i \approx p_a$, so that the pressure thrust is zero. Similar is the case for propeller engines. For supersonic exhaust velocity the pressure p_e may differ from p_a. However, the pressure thrust developed is so small as compared to the momentum thrust that it can safely be neglected for simple calculations and the net thrust is given by

$$\text{Thrust,} \quad F = m_a[(1+f)V_e - V_a] \tag{8·5}$$

The thrust, given by equation (8·5), can be increased by increasing the mass flow or increasing the velocity of the exhaust jet for a given V_a. Equation (8·5) has been derived for a simple control volume shown in Fig. 8·3, but is equally applicable to a plane flying at a forward speed V_i. In the latter case the velocities are considered relative to the aircraft. The air has a velocity V_i, equal to aircraft forward speed, relative to the engine. Thus, we see that a large amount of thrust can be obtained either by propelling a large mass of air and increasing its velocity by a small amount, as in propeller engines or by increasing the velocity of a small mass of air to a high value as in turbojet engine. In the case of turbojet the fuel-air ratio f is very small (about 0 01 to 0·02) and hence the mass of fuel can be neglected safely without causing much error in the performance calculations.

The thrust is, then, given by

$$F = m_a(V_e - V_a) \tag{8·6}$$

However, in the case of ramjet the fuel air ratio is near stoichiometric value.

Equations (8·4) to (8·6) clearly show an important point in that the thrust developed by air-breathing engines greatly depends upon the forward speed of the aircraft, which is represented by V_a here. In the case of a rocket engine V_a is zero and the thrust is independent of the rocket forward speed. The dependence of thrust on forward speed of the aircraft has led to the development of different types of jet propulsion devices for use in various types of aircrafts.

The thrust developed by the engine overcomes the drag of the aircraft and in doing so does some work, called the thrust power, given by

$$\text{Thrust power} = F.V_0$$
$$= m_a[1+f)V_e - V_a]\, V_0$$

and neglecting the amount of fuel flow

$$\text{Thrust power} = m_a(V_e - V_a)V_0$$

It may be pointed out here that an engine driven propeller is rated in horse power while turbojet, ramjet, etc. are rated on the basis of thrust developed.

Thrust per unit of mass flow is referred as specific thrust and neglecting the pressure thrust and fuel flow is given by,

$$\text{Specific thrust} = (V_e - V_0) \tag{8·7}$$

The specific fuel consumption is also defined on the basis of thrust.

Thrust specific fuel consumption (*tsfc*)

$$=\frac{\text{Mass flow per hour}}{\text{Thrust produced}}$$

$$tsfc=\frac{m_f}{T}$$

or
$$tsfc=\frac{m_f}{m_a[(1+f)V_e-V_0]}\ \text{kg/hr-kgt} \tag{8·8}$$

For turbojet and turboprop engines the thrust specific fuel consumption is based on take-off conditions, *i.e.* static thrust and fuel consumption while for ramjet and pulse jet, because of the operating principle, cruising speed is taken as reference. For piston engines the specific fuel consumption is given in terms of kg/h.p.-hr.

Propeller-engines, and turbo-prop and turbofan engines, which accelerate a large mass of fluid to a relatively lower velocity for each unit of fuel burned, will have according to equation (8·8) lower specific fuel consumption than turbojet engines accelerating a small mass of fluid to a higher velocity since the value of f more or less remains constant. The value of f is constant because the maximum allowable temperature for all turbines is almost the same which is obtained at a given fuel-air ratio. Thrust per unit mass of the fuel, rather than fuel economy, is a rough indication of the size of the engine for a given thrust.

8·3. THRUST *VS.* THRUST HORSE POWER

No direct comparison is possible between reciprocating engine and jet engine because the former is rated in horsepower while the latter is rated in terms of thrust. One method is to convert the horse power developed by a reciprocating engine into thrust developed by the propeller. Alternatively, the thrust of a jet engine can be converted into thrust horsepower (*thp*). However, th ere seems to be no direct comparison between thrust and thrust horse-power because the latter depends upon the aircraft speed according to the relation

$$\text{Thrust horsepower } (thp)=F\times V \tag{8·9}$$

where F=Thrust in kg/hr

V=Aircraft velocity, km/hr

Because of this variation of thrust horsepower with speed it is not used to rate the jet engines; instead thrust per kg of mass flowing or fuel burnt is used to rate such engines.

Take-off thrust. Take-off thrust is the thrust available while starting a propulsive device.

From equation (8·6) the take-off thrust ($V_0=0$) is given by

$$F_{static}=m_a-V_e,\ i.e.$$

the static thrust per unit mass flow of air is directly proportional to the exhaust jet velocity. While the thermal efficieney at $V_0=0$ is given by

$$\eta_{th}=\frac{(1+\quad \frac{1}{\;})}{f\times C.V.}$$

Combining both the above equations, we get

$$F_{static}=\frac{2.\eta_{th}\times C.V.\times m_f}{V_e} \tag{8·10}$$

Equation (8·10) shows that for a given fuel rate and thermal efficiency the take-off thrust is inversely proportional to exhaust velocity. So if a large mass of air is accelerated to a smaller exhaust velocity the static thrust would be more than when for the same energy conversion a small air mass is accelerated to high velocity. Equation (8·10) clearly illustrates the takeoff characteristics of various propulsive devices. The static thrust is highest for propeller engines (see Fig. 8·4) which accelerate a large mass flow to a relatively smaller velocity, while it is less for a turbojet engine; turboprop engine falls between the two. The ram jet does not have any take-off thrust because unless it moves with a very high speed no compression will occur and no power can be taken from it.

In Fig. 8·4 is plotted the thrust developed by an adjustable pitch propeller and a jet engine at various aircraft speeds. It can be seen that a jet engine develops almost the same thrust over the

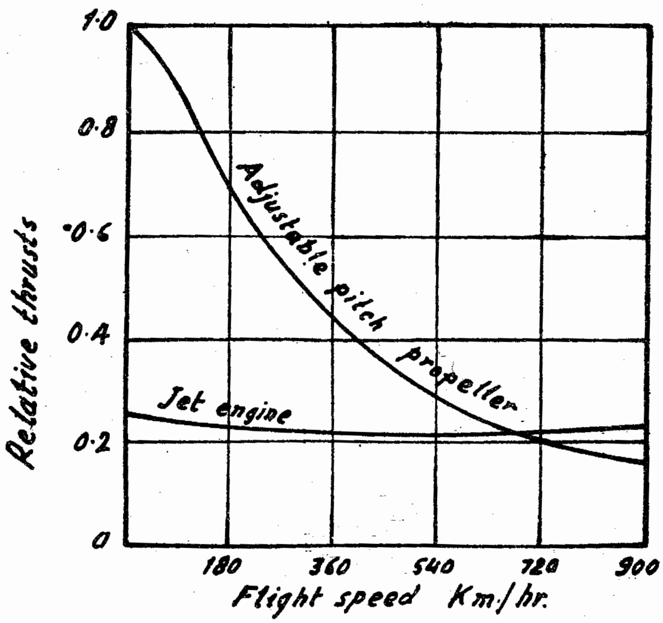


Fig. 8·4. Thrust *vs* speed for propeller driven and turbojet engine.

whole speed range while a propeller engine has very high thrust at low speed which rapidly decreases with the aircraft speed. This gives the propeller engine very good take-off and initial climb characteristics. Howevtr, the rapid decay in thrust with flight speed limits the flight range of the propeller engine. Turboprop engines have better take-off characterstics than the turbojet engine.

8·4. EFFICIENCIES

For the evaluation of the performance of a jet propulsion plant the efficiencies of its components must be found along with other performance parameters. Evalution of the efficiencies will give an indication of the state of development of a particular component and improvements can be effected to increase the overall efficiency of the plant.

(*i*) **Ram efficiency.** Almost all the jet propulsion plants, except the propeller engine and the rockets, utilize a diffuser to increase the pressure of the incoming air by converting its kinetic energy into pressure energy in it. Some devices such as pulse jet and ram jet which do not have a compressor rely exclusively on the diffuser to raise the air pressure. Hence the efficiency of diffuser is important in that it greatly affects the power output and the thrust. The compression obtained in a diffuser is called the *ram compression* (fig. 8·5) and its efficiency is called as *ram efficiency.*

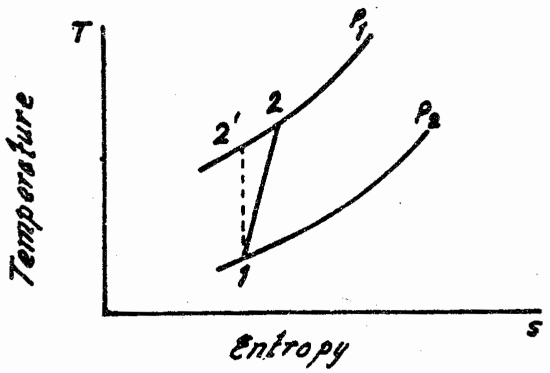


Fig. 8·5. Ram compression.

In the calculation of ram efficiency total temperature and pressure conditions are used to consider the effect of forward speed of the propulsive device. If V_o is the forward speed of the engine, T_t and T_a the total and static temperature, then

$$T_t = T_a + \frac{V_o^2}{2gJc_p} \tag{8·11}$$

In an ideal diffuser the pressure will rise to the isentropic pressure given by

$$\frac{p_t'}{p_a}=\left(\frac{T_t'}{T_a}\right)^{\frac{\gamma}{\gamma-1}}=\left[1+\frac{V_o^2}{2gJc_pT_a}\right]^{\frac{\gamma}{\gamma-1}}$$

or
$$p_t'=p_a\left[1+\frac{V_o^2}{2gJc_pT_a}\right]^{\frac{\gamma}{\gamma-1}} \tag{8·12}$$

However, due to losses in friction and shock, etc., the actual pressure will not rise to the value given by equation (8·12). If η_d is the efficiency of the diffuser, the actual pressure rise is equal to isentropic pressure rise multiplied by the efficiency.

or
$$p_{t2}-p_a=\left[p_a\left(1+\frac{V_o^2}{2gJc_pT_a}\right)^{\frac{\gamma}{\gamma-1}}-p_a\right]\eta_d$$

where p_2 is the final pressure in the diffuser.

$$\therefore \quad p_{t2}=p_a+\eta_d\cdot p_a\left[\left(1+\frac{V_o^2}{2gJc_pT_a}\right)^{\frac{\gamma}{\gamma-1}}-1\right] \tag{8·13}$$

If the velocity is written in terms of Mach number, we get

$$\frac{p_{t2}}{P_a}=\left[1+\eta_d\frac{\gamma-1}{2}M^2\right]^{\frac{\gamma}{\gamma-1}} \tag{8·14}$$

Alternately, the efficiency of a diffuser is defined as a pressure recovery factor such that if multiplied by the isentropic pressure it will give the actual final temperature, *i.e.*

$$\text{Final pressure,}\quad p_2=\eta_r\times p_a\left[1+\frac{V_o^2}{2gJc_pT_a}\right]^{\frac{\gamma}{\gamma-1}} \tag{8·15}$$

where η_r is called the ram efficiency.

It can be seen from these expressions that the amount of ram compression which can be obtained in a diffuser depends greatly upon the forward speed of the aircraft. The efficiency of the diffuser varies from 0·7 to 0·9 depending upon flight speed.

(*ii*) **Propulsive efficiency**. Propulsive efficiency is the measure of the effectiveness with which the kinetic energy imparted to the fluid is transferred into useful work. The useful work done by the system is the product of the thrust and the flight velocity, *i.e.* $F\times V_0$ and is called *thrust power*. The kinetic energy imparted to the fluid is the difference between the kinetic energy at exit and kinetic energy at inlet and is called *propulsive power*. The difference between propulsive power and the thrust power is called the leaving losses.

Kinetic energy of air at inlet

$$=m_a \frac{V_o^2}{2}$$

Kinetic energy of gases at outlet

$$=m_a(1+f)\ \frac{V_e^2}{2}$$

$$\therefore \text{ Propulsive power}=m_a(1+f)\ \frac{V_e^2}{2}-m_a\ \frac{V_o^2}{2}$$

$$=m_a\left[(1+f)\ \frac{V_e^2}{2}-\frac{V_o^2}{2}\right] \tag{8·16}$$

$$\text{Thrust power}=F\times V_o=m_a\ [(1+f)\ V_e-V_o]\ V_o \tag{8·17}$$

$\therefore$ Propulsion efficiency

$$=\frac{\text{Thrust power}}{\text{Propulsive power}}$$

$$=\frac{m_a[(1+f)\ V_e-V_o]\ V_o}{m_a\left[(1+f)\ \frac{V_e^2}{2}-\frac{V_o^2}{2}\right]}$$

or

$$\eta_p=\frac{(1+f)\ [V_e-V_o]\ V_o}{\left[(1+f)\ \frac{V_e^2}{2}-\frac{V_o^2}{2}\right]} \tag{8·18}$$

For turbojet engines the mass of fuel is very small, about 1 or 2 per cent of the mass of air. Neglecting the mass of fuel, we have

$$\eta_p=\frac{2(V_e-V_o)\ V_o}{V_e^2-V_o^2}$$

$$=\frac{2V_o}{V_e+V_o}=\frac{2r}{1+r} \tag{8·19}$$

where r is the speed ratio $\frac{V_o}{V_e}$.

Differentiating equation (18·19) with respect to the speed ratio r and putting equal to zero, the condition for best propuslive efficiency is obtained which is $r=1$, *i.e* the velocity of the gas jet is equal to the velocity of the aircraft and at this point the value of propulsive efficiency is 100 percent. However, from equation (8 6), it is clear that at the condition of maximum propulsive efficiency the thrust developed is zero and no useful work output can be obtained from the power plant. So it is not practical to maximise propulsive efficiency of a jet engine. Other parameters such as thrust, etc., must be considered for performance evaluation.

$$\text{Thrust power} = [m_a(1+f)\,V_e - V_o]V_o$$
$$= m_a.V_e^2\left[(1+f) - \frac{V_o}{V_e}\right]\frac{V_o}{V_e}$$
$$= m_a.V_e^2[(1+f)-r]r \tag{8·20}$$

Neglecting the mass of fuel

$$\text{Thrust power} = m_a.V_e^2(1-r)r \tag{8·21}$$

By differentiating with respect to r and equating to zero, we see that maximum thrust occurs at $r=0·5$, *i.e.* when the speed of the aircraft is half the speed of the gas exhaust jet, and the corresponding value of the propulsive efficiency is given by

$$\eta_p = \frac{2r}{1+r} = \frac{2\times 0·5}{1+0·5} = 0·667 \tag{8·22}$$

Thus it is evident from the above analysis that for a turbojet, ramjet, and pulse jet, the points of maximum efficiency and maximum thrust are different and some compromise must be done to get reasonable thrust with a good propulsive efficiency. Turboprop engines are essentially two-fluid stream engines and this analysis cannot be applied to them. For each stream a separate equation must be written.

In the case of propeller-engine aircrafts the propulsive efficiency is based on the brake horse power of the engine, *i.e.*

$$\eta_p = \frac{\text{Thrust power}}{\text{Brake horse-power}}$$

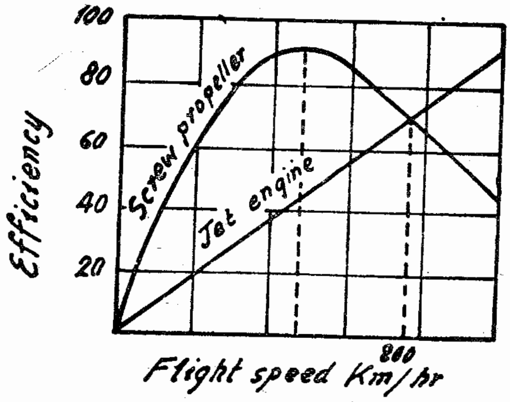


Fig. 8·6. Propulsive efficiencies of air screw and turbojet engine.

Fig. 8·6 shows the populsive efficiencies of a screw-propeller and a jet engine. The effieiency of a jet engine increases with the aircraft velocity. For the propeller engine the efficiency at first increases rapidly but after a certain speed it starts falling because of the air reaching supersonic velocities which results in shock and flow separation losses. For velocities greater than about 800 km/hr, the jet engine is more efficient than the propeller engine.

In the case of rockets, the mass of the air entering the power plant is zero, *i.e.* $f=\frac{m_f}{m_a}$ tends to infinity. The propulsive efficiency can also be written as :

$$\eta_p=\frac{\text{Thrust power}}{\text{Thrust power}+\text{Energy lost at exit}}$$

$$=\frac{m_a[(1+f)\,V_e-V_o]V_o}{m_a[(1+f)\,V_e-V_o]\,V_o+m_a(1+f)\,\frac{V_e^2}{2}}$$

$$=\frac{1}{1+\frac{(1+f)\,V_e^2}{2V_o\{(1+f)\,V_e-V_o\}}}$$

$$=\frac{1}{1+\frac{(1+f)}{2r[(1+f)-r]}}=\frac{1}{1+\frac{1}{2r\left(1-\frac{r}{1+f}\right)}} \qquad (8\cdot23a)$$

Putting $f=\infty$,

$$\eta_p=\frac{1}{1+\frac{1}{2r}} \qquad (8\cdot23b)$$

i.e. **the rocket propulsion efficiency would increase with r and would be maximum at $r=\infty$.**

***(iii)* Thermal Efficiency**. Thermal efficiency of a propulsive device is an indication of the degree of utilization of energy in fuel in accelerating the fluid flow and is defined as the increase in the kinetic energy of the fluid divided by total energy in fuel.

$$\text{Thermal efficiency}=\frac{\text{Propulsive power}}{\text{Fuel flow rate}\times\text{Calorific value of fuel}}$$

$$=\frac{m_a\left[(1+f)\,\frac{V_e^2}{2}-\frac{V_o^2}{2}\right]}{m_f\times C.V.}$$

or

$$\eta_{th}=\frac{(1+f)\,\frac{V_e^2}{2}-\frac{V_o^2}{2}}{\frac{m_f}{m_a}\times C.V.}=\frac{(1+f)\,\frac{V_e^2}{2}-\frac{V_o^2}{2}}{f\times C.V.} \qquad (8\cdot24)$$

It should be noted that this expression applies only to turbojet, ramjet, and pulse jet engines and does not apply to propeller engines and rocket engines.

For propeller engines thermal efficiency is defined as

$$\eta_{th}=\frac{\text{brake horse power}}{\text{Fuel rate}\times C.V.\text{ of fuel}} \tag{8·25}$$

In the case of turbo-prop engine shaft power as well as thrust power both are used for propulsion. However, the shaft power is considerably larger than the thrust power and it is usual to define thermal efficiency on the equivalent shaft power basis such that the thrust power due to exhaust jet is also included. For calculating thrust power some suitable velocity for the aircraft is selected.

(*iv*) **Propeller efficiency**. The propeller produces thrust power by accelerating the air. The propeller itself is driven by the engine. The efficiency of the propeller is defined as the ratio of the thrust power to the shaft power.

$$\text{Propeller efficiency}=\frac{\text{Thrust power}}{\text{Shaft power}}$$

$$=\frac{F\times V_0}{shp} \tag{8·26}$$

In the case of turboprop engine the thrust power developed by the exhaust is also considered.

(*v*) **Transmission efficiency**. In many cases the engine or the turbine output cannot be directly applied to the propeller; some form of transmission is involved between the engine and the propeller in the form of a reduction gear. The main reason for providing reduction gear in the case of a turbo-prop engine is high rotational speed of the turbine at which the propeller cannot be rotated efficiently. In addition to this some layout problems always occur. Due to friction and other losses the output from the transmission system is always less than input to it and the transmission efficiency is defined as :

$$\text{Efficiency of transmission}=\frac{\text{Output of the transmission}}{\text{Input to the transmission}} \tag{8·27}$$

(*vi*) **Overall efficiency**. The overall efficiency of a propulsive device is the ratio of the useful workdone to the chemical energy supplied in the form of fuel.

$$\text{Overall efficiency}=\frac{\text{Useful propulsive work}}{\text{Chemical energy supplied}}$$

Overall efficiency, η_0

$$=\frac{\text{Engine output}}{\text{Engine input}}$$

$$\times\frac{\text{Transmission output}}{\begin{array}{c}\text{Transmission input}\\ \text{(engine output)}\end{array}}\times\frac{\text{Useful propulsive work}}{\begin{array}{c}\text{Input to the propulsive}\\ \text{device (Transmission}\\ \text{output)}\end{array}}$$

$$= \text{Thermal efficiency} \times \text{Transmission efficiency} \times \text{Propulsive efficiency}$$

or

$$\eta_0 = \eta_{th} \times \eta_{tr} \times \eta_p \qquad (8\cdot23)$$

It follows from the above that the overall effect of the propulsive device is made of a power plant converting some percentage of the chemical energy supplied to it into a form useful for propulsion (shaft horse power in case of propeller devices and acceleration of mass flow for jet devices). This includes the losses in the transmission, if used, (the transmission converting the energy in a more useful form, *i.e.* in acceleration of the fluid) ; the final propulsive device converting this energy into useful work output. This is shown in Fig. 8·7.

Propulsive Devices

In what follows in the next few pages, the various propulsive devices, their principal parts, thermodynamic cycles and performances are discussed in detail and compared.

8.5 AIR SCREW

In an aircrew, the source of power is a reciprocating internal combustion engine which drives a propeller connected to it. The propeller displaces rearwards, a large mass of air accelerating it in the process. Due to this acceleration of the fluid a propulsive force is produced which drives the aircraft.

Nearly all the early aircrafts used reciprocating engines as the source of energy to drive the propeller. Early use of steam engine failed miserably due to very high weight and bulk of such engines The extensive use of aircrafts for military purposes led to a very rapid development of reciprocating internal combustion engines during the two world wars and it is now a highly developed piece of equipment as compared to its industrial counterpart. Further development resulted in the use of highly supercharged and turbocharged engines. All these engines were gasoline engines, the diesel engine in spite of its good fuel economy and reliability was not used due to higher weight.

Howerever, the use of reciprocating engines is continuously on the decline because its development has reached a stage of near saturation as far as maximum power developed is concerned, while the demand of present day aircrafts, in terms of high flight speeds, long-distance travels and high load carrying capacities, is soaring to new heights. A power output more than about 5000 HP is difficult to obtain without modifications in the present reciprocating engine plant. The output can be increased by increasing the cylinder sizes, installing large number of cylinders or by running the engine at higher speeds. Unfortunately all these methods of raising the output of the engine increase the engine size, frontal area of the aircraft, complexity and cost of the plant. The drag of the plane will also increase to critical values with increase in engine size.

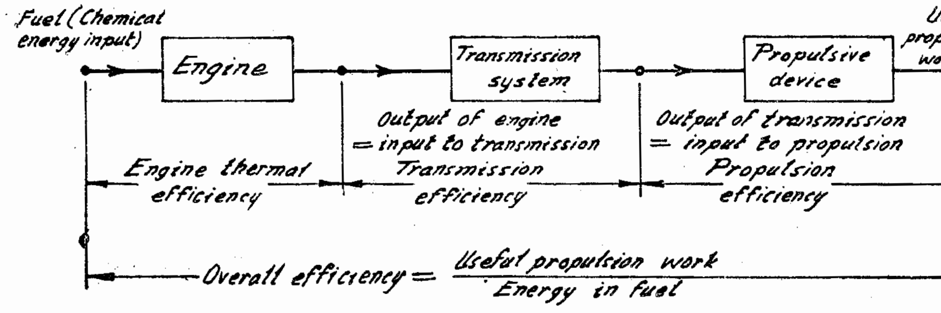


Fig. 8·7. Block diagram for aircraft en·ine efficiencies.

Thus the speed of airscrew is limited to a range of about 700 km/hr because of the power requirement and the propeller. The propeller loses its effectiveness at higher speeds due to separation of flow and shock waves as the air velocity approaches the sonic velocity. At lower speeds the propulsive efficiency of the propeller is about 95 percent.

For small aircrafts flying at velocities less than about 500 to 650 km/hr reciprocating engine is in an enviable position due to its excellent fuel economy and good take-off characteristics. However, due to comparatively large drop in power with altitude operation and the need of using high octane fuels, along with the difficult cooling and lubrication problems, high weight/power ratio, and larger frontal area of such engines these are being replaced by turbojets in higher speed ranges.

Rapid developments in design of turbojets and turboprop engines are very soon likely to explode the best fuel economy myth of the reciprocating engines as they are nearing the specific fuel consumption value of such engines. Still the reciprocating engine is likely to be used for small aircrafts needing only a few hundred horse-power both because of good take-off characteristics and due to difficulties in the development of smaller gas turbine engines giving reasonable fuel economy and cost.

8·6. TURBOJET

The very rapid fall in the propeller efficiency at higher speeds led to the use of turbojet in the high speed flight range in place of recipro-

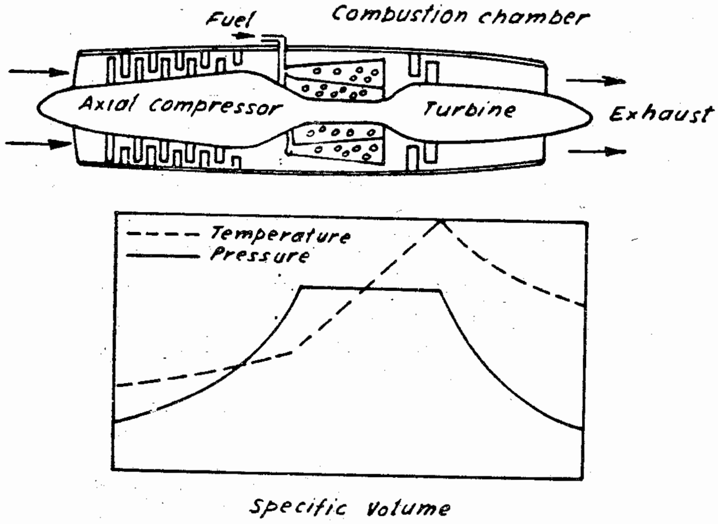


Fig. 8·8. Internal arrangement of a turbojet using axial flow compressor.

cating engines. Turbojet engines, with all their modifications to improve their performance, are now widely used.

(*a*) **Components.** As already discussed a turbojet consists of a diffuser at the intake, a compressor to raise air pressure, a combustion chamber to burn the fuel and a turbine developing power just sufficient to drive the compressor and auxiliaries, and an exhaust nozzle as a propulsive device. Of the total compression of air, a part is obtained by the ram compression in the diffuser and rest in the compressor. Similarly, a part of the total expansion ratio available is used in turbine to drive the compressor and rest used to produce thrust with the help of the exhaust nozzle.

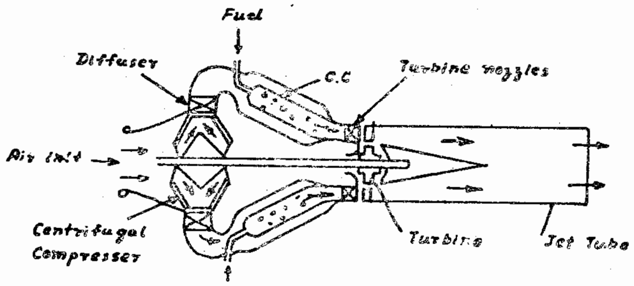


Fig. 8·9. Turbojet using centrifugal compressor.

A schematic diagram of a turbojet engine and its internal arrangement is shown in the Fig. 8·9. The associated pressure, temperature and velocity are also shown in Fig. 8.8.

The compressor used in a turbojet can be either centrifugal type or axial flow type. The use of a particular type of compressor gives the turbojet typical characteristics. The centrifugal compressor produces a high pressure-ratio of about 4 : 1 to 5 : 1 in a single stage and usually a double-sided rotor is used to reduce the engine diameter. The turbojet using a centrifugal compressor has a short and sturdy appearance (*see* Fig. 8·9). The advantages of centrifugal compressor are high durability, ease of manufacture and low cost, and good operation under adverse circumstances such as icing and when sand and small foreign articles are inhailed in inlet duct.

The axial-flow compressor is more efficient than the centrifugal type and gives the turbojet a long, slim, streamlined appearance (*see* Fig. 8·8). The engine diameter is reduced which results in low aircraft drag. A multi-stage axial flow compressor can develop a pressure-ratio as high as 6 : 1 or more. The air handled by it is more than that handled by a centrifugal compressor of the same diameter.

The demand for increased power has led to the use of split or two-spool axial-flow compressor. Fig. 8·10 shows a turbojet using such a compressor. A very high pressure ratio of about 9 : 1 to 13 : 1 is obtained by using a high pressure and a low pressure rotor driven by separate shafts. The use of high pressure ratio gives very good specific fuel consumption (0·75 kg/kg thrust per hr) and use of two rotors allows greater efficiency because firstly, the high pressure rotor can be governed for speed and secondly, the low pressure rotor can be allowed to run at a speed giving maximum efficiency.

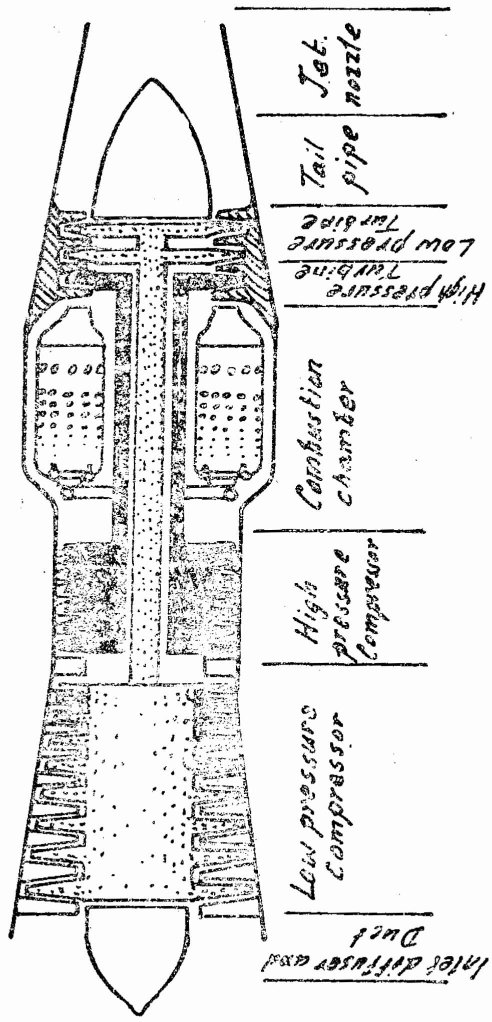


Fig. 8·10. Turbojet using a split or two-spool axial-flow compressor.

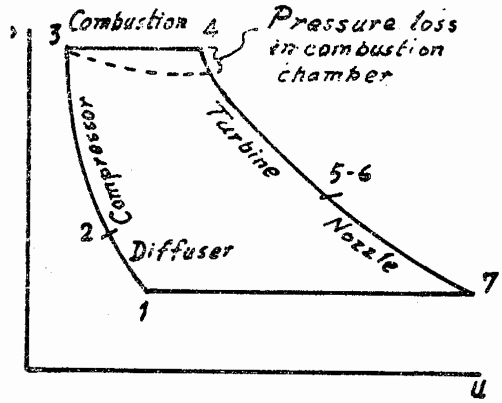
The turbojets having centrifugal compressor have about 20 per cent weight advantage over the axial flow turbojets. Thrust per unit length is more for the first type while thrust per unit diameter is more for the second type. The axial flow turbojets have about 6 to 8 per cent less specific fuel consumption.

(*b*) **Thermodynamic cycle**. Fig. 8·11 shows the basic thermodynamic cycle of a turbojet engine on p-v and T-s diagrams. This is Joule or Brayton cycle. In the analysis of the turbojet cycle following assumptions are made.

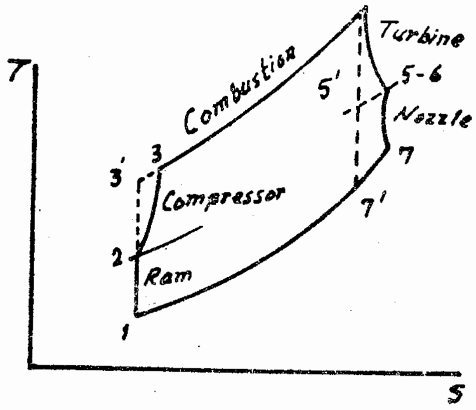
(*i*) There is no loss of pressure in the combustion chamber.

(*ii*) The specific heat is constant.

(*iii*) Power developed by the turbine is just sufficient to drive the compressor.



(*a*) *p-v* diagram



(*b*) *T-s* diagram

Fig. 8·11. Thermodynamic cycle of a turbojet engine on *p-v* and *T-s* diagrams.

At the inlet to the diffuser air enters with a velocity equal to the forward velocity of the aircraft. In diffuser air velocity is decreased and pressure increased. In the ideal case the pressure will rise such that the velocity at the exit of the diffuser is zero. However, in actual practice the air will have a velocity of about 60·90 m/s at diffuser exit. If η_d is the efficiency of the diffuser then the total pressure at the end of diffusion process is given by :

$$\frac{p_{t2}}{p_1}=\left(1+\eta_d\frac{\gamma-1}{2}M^2\right)^{\frac{\gamma}{\gamma-1}}$$

From the diffuser air goes into a compressor. If η_c is the compressor efficiency and p_{t3}/p_{t2} the pressure ratio, we get

$$\eta_c=\frac{h_{t3}'-h_{t2}}{h_{t3}-h_{t2}}$$

or
$$h_{t3}-h_{t2}=\frac{1}{\eta_c}(h_{t3}'-h_{t2})=\frac{c_p}{\eta_c}(T_{t3}'-T_{t2})$$

$$=\frac{c_p T_{t2}}{\eta_c}\left(\frac{T_{t3}'}{T_{t2}}-1\right)$$

$$=\frac{c_p T_{t2}}{\eta_c}\left[\left(\frac{p_{t3}}{p_{t2}}\right)^{\frac{\gamma-1}{\gamma}}-1\right]$$

The heat supplied to the combustion chamber is given by

$$q=(h_{t4}-h_{t3})$$

We assume that there is no pressure loss in the combustion chamber so that the pressure remains constant and full pressure is available for expansion in the turbine and nozzle.

Since we have assumed that turbine and compressor work are same

$$h_{t3}-h_{t2}=h_{t4}-h_{t5}$$

If η_t is the turbine efficiency and (p_{t5}/p_{t4}) turbine pressure ratio,

$$h_{t4}-h_{t5}=\eta_t\, c_p\, T_{t4}\left[1-\left(\frac{p_{t5}}{p_{t4}}\right)^{\frac{\gamma-1}{\gamma}}\right]$$

By equating the turbine and compressor work, we get

$$c_p\frac{T_{t2}}{\eta_c}\left[\left(\frac{p_{t3}}{p_{t2}}\right)^{\frac{\gamma-1}{\gamma}}-1\right]=\eta_t\, c_p\, T_{t4}\left[1-\left(\frac{p_{t5}}{p_{t4}}\right)^{\frac{\gamma-1}{\gamma}}\right] \qquad (8·30)$$

From the equation (8·30), knowing the ram compression and the compressor pressure ratio, the required turbine ratio to produce a power equal to that absorbed by the compressor can be obtained and from the turbine pressure ratio the nozzle pressure can be obtained.

If η_n is the efficiency of the nozzle we have

$$\eta_n = \frac{h_{t6} - h_7}{ht_6 - h_7'} \tag{8·31}$$

Points 5 and 6 are the same assuming that there is no loss in passing the gas from turbine exhaust to the nozzle. It should be noted that in equation (8·31) h_7 is used instead of total enthalpy h_{t7} because the exhaust nozzle efficiency is an indication of the percentage of total energy converted into velocity energy.

$$\therefore \quad h_{t6} - h_7 = \eta_n \; c_p \; T_{t6} \left[1 - \left(\frac{p_7}{p_{t6}} \right)^{\frac{\gamma-1}{\gamma}} \right]$$

The exhaust velocity of the nozzle can be obtained by writing energy balance equation.

$$h_{t6} - h_7 = \frac{V_e^2}{2}$$

or
$$\frac{V_e^2}{2} = \eta_n c_p T_{t6} \left[1 - \left(\frac{p_7}{p_{t6}} \right)^{\frac{\gamma-1}{\gamma}} \right]$$

or
$$V_e = \sqrt{2 c_p \eta_n T_{t6} \left[1 - \left(\frac{p_7}{p_{t5}} \right)^{\frac{\gamma-1}{\gamma}} \right]}$$

Thrust developed $= m_a[(1+f) \, V_e - V_o]$

$$\left\{ = m_a(1+f) \sqrt{2 c_p \eta_n T_{t5} \left[1 - \left(\frac{p_7}{T_{t6}} \right)^{\frac{\gamma-1}{\gamma}} \right]} - V_o \right\} \tag{8·32}$$

From the above, thrust specific fuel consumption and also the thermal efficiency can be calculated.

(*c*) **Performance of a turbojet engine.** With the help of the above analysis it is possible to estimate the performance of a turbojet engine as affected by component efficiencies and other parameters. Fig. 8·12 shows the thrust specific fuel consumption for various compression pressure ratios at two different Mach numbers. It is evident that for a given Mach number there is only one compressor pressure ratio which gives best fuel economy for given values of component efficiencies and maximum allowable temperature. As the pressure ratio increases with a given maximum temperature fuel consumption decreases to a minimum. After that further increase in pressure ratio will not improve the fuel economy until maximum temperature is not raised. For a given pressure ratio higher maximum temperature will result in more thrust. Maximum thrust produced per unit mass of fuel is at a lower compression ratio than which produces minimum specific fuel consumption at the same turbine inlet temperature.

In addition to above parameters, three more variables—flight speed, altitude (inlet temperature and pressure), and fuel flow rate greatly affect the performance of a jet engine.

The turbojet is almost constant thrust engine. The static thrust of such engines is very low as compared to propeller engine

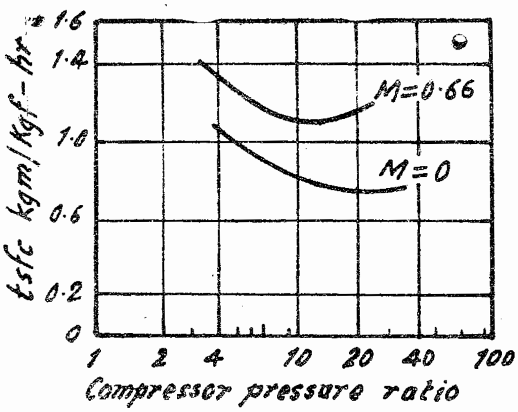


Fig. 8·12. Thrust specific fuel consumption *vs* compressor pressure ratio for a turbojet engine.

aircrafts for which cruise thrust is about 60 per cent of the take-off thrust. The thrust at first decreases with increase in speed because the velocity V_0 in the thrust equation increases. After a

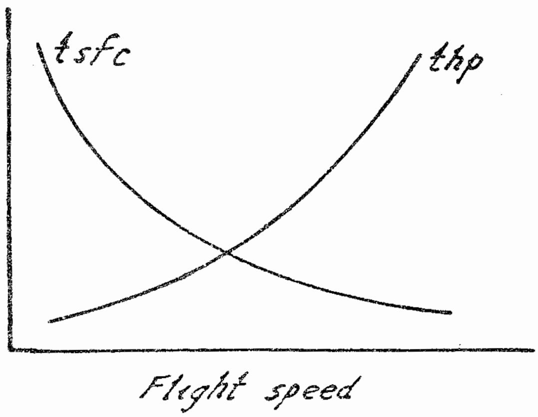


Fig. 8·13. Thrust specific fuel consumption and thrust horse power *vs.* flight speed.

minimum value, the thrust starts increasing (*see* Figs. 8·4 and 8·14) due to increased ram compression at higher speeds. The specific fuel consumption based on thrust horse power reduces because with almost constant thrust the thrust horse power increases as shown in Fig. 8·13. Therefore, the maximum speed limit is the most efficient operational point for the turbojet.

As the altitude increases, the thrust decreases due to decrease in density, pressure, and temperature of the air (*see* Fig. 8·14).

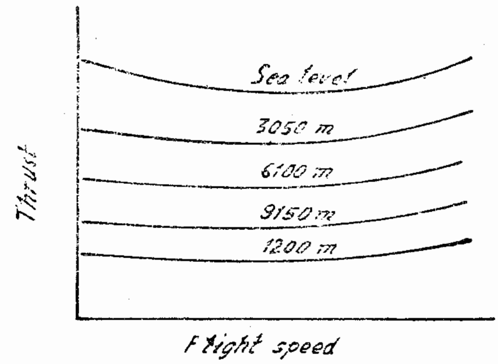


Fig. 8·14. Effect of altitude on thrust at maximum rpm.

However, the rate of decrease of thrust is less than the rate of decrease of density with altitude because some loss due to reduced density is compensated by lesser drag. The thrust is maximum at sea level. But, due to considerable reduction in drag (at an altitude of 8000 m the drag is reduced to less than 25 per cent of sea level drag), the turu bojet is most efficient when flown at high altitudes and at relatively high speeds.

The fuel consumption on fuel mass per km of travel increases with speed as power developed is increased.

The operational range of turbojet engine is about 800 to 1100 km/hr and the specific fuel consumption is about 0·9 to 1·5 kgf-hr/kg at cruising speeds and are still greater at lower speeds. The altitude limit is about 7500 m.

(*d*) **Turbojet vs. engine-propeller.** Figs. 8·15 and 8·4 compare the thrust and the power for the two types of propulsive

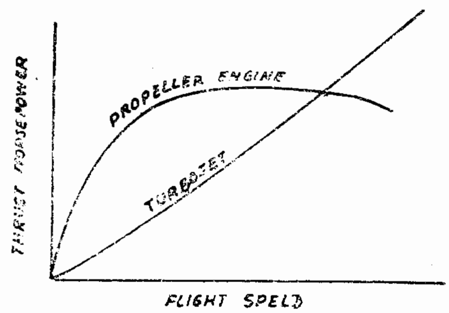


Fig. 8·15. Power *vs* speed characteristics of turbojet and propeller engine.

devices. It is seen that the engine propeller has very good take-off thrust as compared to a turbojet engine. The maximum speed of turbojet engine is more than the engine-propeller. However, in the lower speed range the power available for engine-propeller is very high and the two are equal until when the turbojet has reached a minimum speed of about 700 km/hr. So below this speed, engine propeller is better. For turbojet the fuel economy is also poor at lower speeds.

(e) **Advantages and disadvantages of turbojet**

Advantages of turbojet

1. The power weight ratio of a turbojet is about 4 times that of a reciprocating engine propeller system.

2. It is simple, easy to maintain, and requires lower lubricating oil consumption. Furthermore, complete absence of liquid cooling results in reduced frontal area.

3. There is no limit to the power output which can be obtained from a turbojet while the piston engines have reached almost their peak power and further increase will be at the cost of comlexity and greater engine weight and frontal area of the aircraft.

4. The speed of a turbojet is not limited by the propeller and it can attain higher flight speeds than engine propeller aircrafts.

Disadvantages

1. The fuel economy at low operational speeds is extremely poor.

2. It has low take-off thrust, and hence poor starting characteristics.

18·6. THRUST AUGUMENTATION

The poor take-off characteristic of the turbojet engine can be improved by augmenting the thrust. The thrust from a turbojet is given by

$$F = m_a[(1+f)\ V_e - V_o]$$

in which the exhaust V_e is the function of the maximum temperature in the cycle. Higher the maximum temperature higher is the value of V_e. Another method of increasing thrust is to increase the mass flow rate. Improved thrust results in shorter take-off distances, high climb rate and good manoeuvrability at high altitudes. The thrust augmentation can be affected by the following methods :

(*i*) **Reheat or afterburner**. The exhaust of the turbojet turbine has sufficient oxygen to burn as there is always excess air

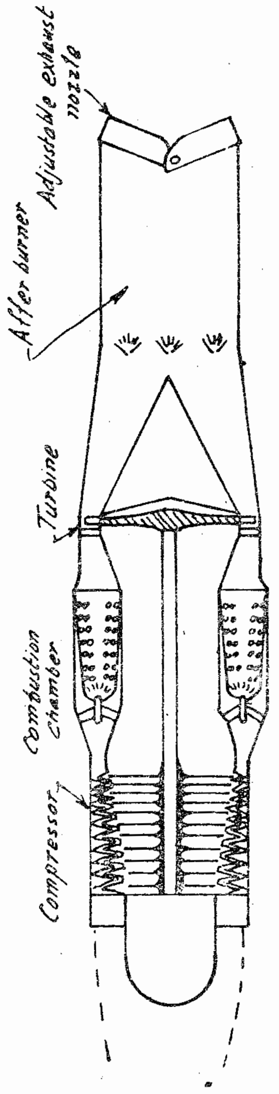


Fig. 8·16. Turbojet with an afterburner.

available. This allows the burning of additional fuel in the exhaust of the turbine before it reaches the nozzle. It is called *after-burning*. This increases the mass flow and the velocity of the exhaust jet, thereby, boosting the thrust. A boost of about 30 per cent can be obtained in this manner. However, the fuel consumption increases rapidly. For about 20 per cent thrust increase by use of reheat the overall fuel consumption may be increased by more than 100 per cent and this additional mass of fuel has to be carried by the turbojet. Therefore, it is used only for take-off or for high climbing rates and for a very short duration. Use of reheat for thrust augmentation requires that the exhaust jet area should be variable to meet the demands of increased flow rate, *i.e.* to avoid chocking flow (otherwise some loss of thrust will always occur during unaugmented operation). Fig. 8·16 shows a turbojet with an afterburner.

(*ii*) **Water-methanol injection.** To improve thrust at take-off or at high altitudes a mixture of water and methanol is injected in the combustion chamber (Fig. 8·17). Water evaporates and there is an increase in mass flow giving greater thrust. The loss in evaporating water is compensated by the energy of methanol and the working temperatures are kept same.

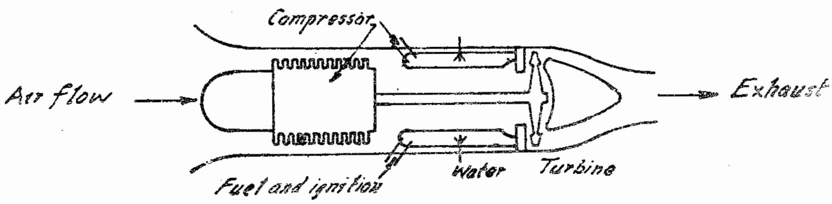


Fig. 8·17. Water injection in combustion chamber.

Actually due to the fixed pressure ratio of turbine, water injection in the combustion chamber tends to reduce air flow by an amount corresponding to the amount of water injected. However, a decrease in airflow results in compressor shifting to a higher pressure. This is because the turbine keeps the compressor speed same and if air flow is reduced higher pressure will be developed by the compressor. So water injection causes a lower mass flow and higher pressure ratio of compressor and a higher total mass flow through the turbine, increasing thrust.

Water can also be injected in the compressor (*see* Fig. 8·18). By injecting water into compressor inlet the inlet air is cooled. This results in increased compressor pressure ratio in addition to increased mass flow. Increased compressor pressure ratio allows a greater expansion ratio for the exhaust nozzle so that thrust is increased both by increase in the mass flow as well as in the exhaust velocity. Amount of water injection at compressor inlet can correspond to inlet air saturation or to compressor exit air saturation. In the latter case extra water is evaporated during the mechanical compression process.

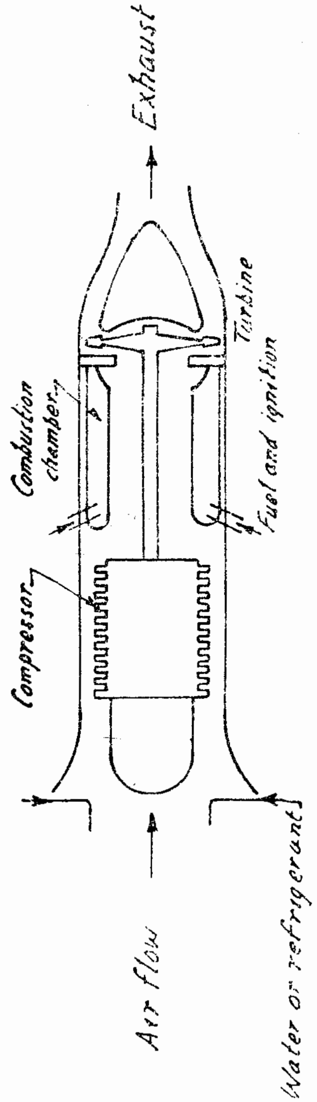


Fig. **8·18.** Water injection in compressor inlet.

The amount of extra thrust at a given flight speed is almost the same with water injection in combustion chamber and water injection for compressor exit saturation. Thrust augmentation of about 30 per cent is possible by water injection. The specific liquid consumption is very high.

(*iii*) **Bleed-burn cycle.** Since in a turbine excess air is also present, a small percentage of high pressure air from the compressor is bled to an auxiliary combustion chamber by by-passing the turbine (*see* Fig. 8·19). In auxiliary combustion chamber the

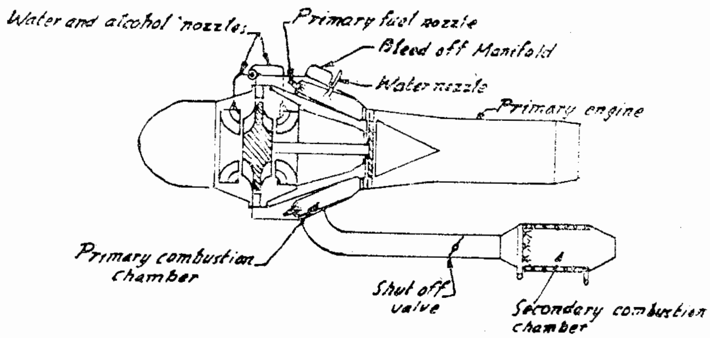


Fig. 8·19. Air bleed system.

bled air is heated by an additional fuel supply to a higher temperature than would be permissible in the main engine on account of the limiting temperature at the turbine blades. The hot gases are then discharged forming an additional jet. A shut off valve is used to bring the engine to normal position. Water is injected into main combustion chamber to replace the mass of the extracted air, thus maintaining the discharge of main jet at the same level. This method is usually used for take-offs only due to high rate of liquid consumption which cannot be carried with engine while in flight. The augmented thrust ratio is highest for this method among the three methods.

Afterburning seems to be the only practical method for thrust augmentation during flight. The air bleed-off system gives maximum thrust augmentation but at the expense of large fuel consumption. This is used only when a large take-off thrust is needed. For smaller thrust augmentation ratio water injection is used because of simplicity and light weight. Afterburner is used for medium thrust augmentation ratio. Afterburner combined with water injection can also be used.

8·8. TURBOPROP ENGINE

A little thought over the thrust and thrust specific consumption equations will reveal that a higher thrust per unit mass flow of fuel can be obtained by increasing the mass flow of air and that this also results in better fuel economy. This fact is utilized in a turboprop engine which is an intermediate between a pure jet engine and a propeller engine. Turboprop engine attempts to increase the air flow by using a propeller driven by the turbine in addition to the thrust produced by the exhaust nozzle.

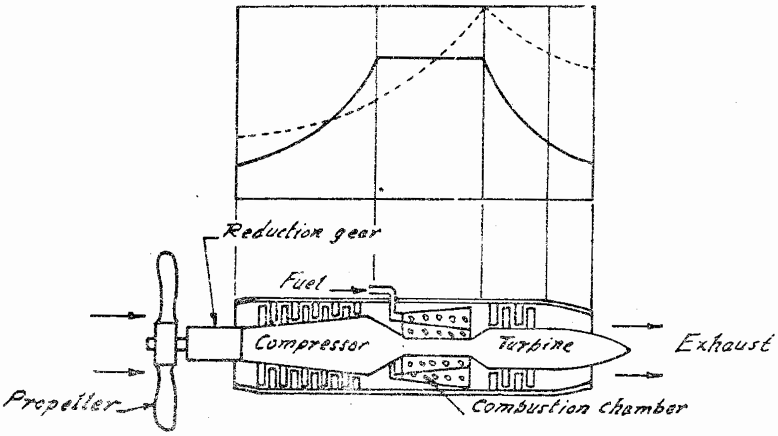


Fig. 8·20. Schematic diagram of a turboprop engine.

Fig. 8·20 shows a schematic diagram of a turboprop engine The air enters the diffuser as in a turbojet and compressed in a compressor before passing to the combustion chamber. The compressor in the turboprop is essentially an axial flow compressor. The products of combustion expand in a two-stage or multi-stage turbine. One stage of the turbine drives the compressor and the other drives the propeller. Thus the turbine expansion is used to drive both compressor as well as propeller and less energy is available for expansion in the nozzle. Due to lower speeds of propeller a reduction gear is necessary between turbine and the propeller. About 80 to 90 per cent of the available energy in exhaust is extracted by the turbine while rest, about 10 to 20 per cent, contributes the thrust by increasing the exhaust jet velocity.

(*a*) **Thermodynamic cycle.** The thermodynamic cycle of turboprop is similar to that of a pure jet engine except that now more energy is used in the turbine as seen from the *T-s* diagram of Fig. 8·21. About 80 to 90 per cent of the total energy is used in the turbine and only about 10 to 20 per cent is used in exhaust nozzle. The propeller produces its own thrust and thus the turboprop engine is essentially a two fluid stream engine.

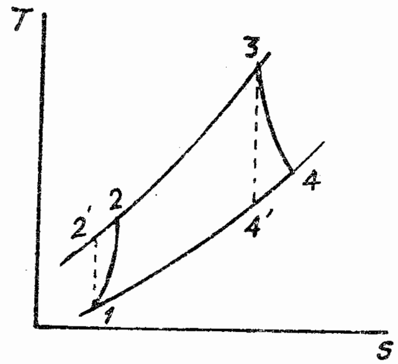


Fig. 8·21. *T-s* diagram for a turboprop engine.

The optimum power ratio for turboprop engine can be determined as follows :

Let the total expansion be divided into parts such that :

h_1=enthalpy drop in the nozzle.

h_2=enthalpy drop in the turbine.

η_{tr}=transmission efficiency of the propeller and gears.

η_t= turbine efficiency.

η_n=nozzle efficiency.

m_a=mass rate of flow in turbine.

Total thrust=Nozzle thrust+Propeller thrust.

$$=m_a[V_e-V_o]+FV_o$$

$$=m_a[\sqrt{2gh_1\,\eta_n}-V_o]+m_a\eta_t\,h_2\,\eta_{tr}$$

$$=m_a[\sqrt{2gh_1\,\eta_n}-V_o+\eta_t\,\eta_{tr}\,h_2] \qquad (8\cdot33)$$

By assigning suitable values to η_t, η_n and η_{tr} this equation can be optimised for maximum thrust.

(*b*) **Performance**. Turboprop engines combine in them the high take-off thrust and good propeller efficiency of the propeller engines at speeds lower than 800 km/hr and the small weight, lower frontal area, and reduced vibration and noise of the pure jet engine.

Its operational range is between that of propeller engines and turbojets though it can operate in any speed upto 800 km/hr.

The power developed by the turboprop remains almost same at high altitudes and high speeds as that under sea-level and take-off conditions because as speed increases ram effect also increases. The specific fuel consumption increases with increase in speed and altitude. The thrust developed is high at take-off and reduces at increased speed.

(c) Advantages and Disadvantages

Advantages

1. Turboprop engines have a higher thrust at take-off and better fuel economy.

2. The frontal area is less than propeller engines so that the drag is reduced

3. The turboprop can operate economically over a wide range of speeds, ranging from low speeds where pure jet engine is uneconomical to high speeds of about 800 km/hr where the propeller engine efficiency is low.

4. It is easy to maintain and has lower vibrations and noise.

5. The power output is not limited as in the case of propeller engines.

6. The multi-shaft arrangement allows a great flexibility of operation over a wide range of speeds.

Disadvantages

1. The main disadvantage is that at high speeds, due to shocks and flow separation, the propeller efficiency decreases rapidly, thereby, putting up a maximum speed limit on the engine.

2. It requires a reduction gear which increases the cost and also consumes certain energy developed by the turbine in addition to requiring more space.

(d) Applications

The turboprop engine is widely used in commercial and military aircrafts due to it's high flexibility of operation and good fuel economy. It is likely to eliminate propeller engines from moderate power and speed aircrafts.

8·9. BYPASS AND DUCTED FAN ENGINES

Bypass and ducted fan engines are very similar to turboprop engines except that instead of a propeller these engines use a low pressure compressor or a fan for accelerating the secondary stream of air flow.

In bypass engines (*see* Fig. 8·22) the secondary air flow completely bypasses the turbine, combustion chamber and the nozzle such that the thrust obtained is the sum of that produced by the low-pressure fan and that produced by exhaust nozzle.

In ducted fan engines (*see* Fig. 8·23) the secondary air flow, though bypasses the turbine and combustion chamber, is mixed with the main flow in the exhaust nozzle and expands in it to produce thrust.

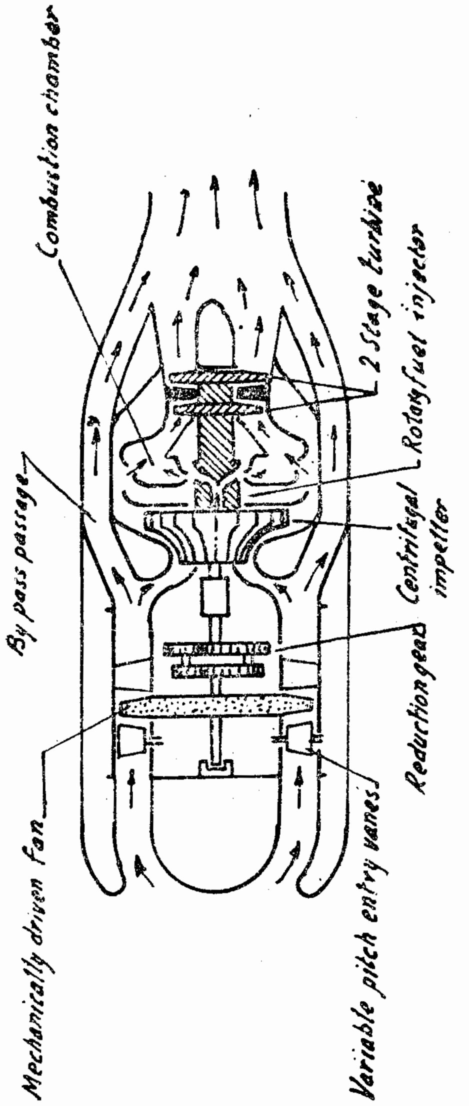


Fig. 8·22. Bypass engine.

Both these types use two-stage compressors—one low pressure compressor for secondary air flow and another main compressor

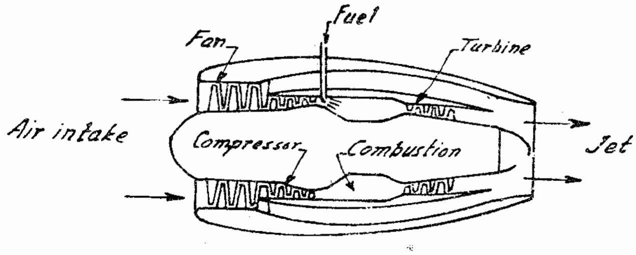


Fig. 8·23. Ducted fan engine.

driven by a two-stage turbine. Sometimes the two compressor stages can be made integral on the same shaft driven by a single turbine.

By a simple analysis it can be shown that the thrust and fuel economy can be improved by increasing the secondary air flow. A typical example is shown in Fig. 8 24. It may be noted that as

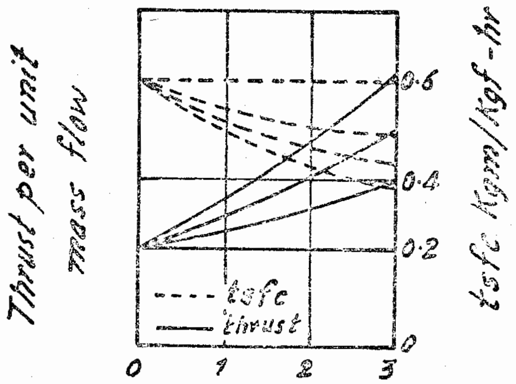


Fig. 8·24. Effect of by-pass ratio on performance of a turbofan engine.

the bypass ratio is increased the drag also increases so the best bypass ratio is that which gives maximum increase in the thrust. A value of about 1·2-1·4 is usual.

The ducted fan engine results in lower exhaust velocities which means lesser noise.

8·10. REGENERATIVE DUCTED FAN ENGINE

Fig. 8·25 shows a ducted fan engine utilizing the exhaust heat of the turbine for preheating the air by ducting it to turbine exhaust and back to the combustion chamber.

Use of regeneration results in exceptional fuel economy, very good endurance and long range capabilities. The part load operation

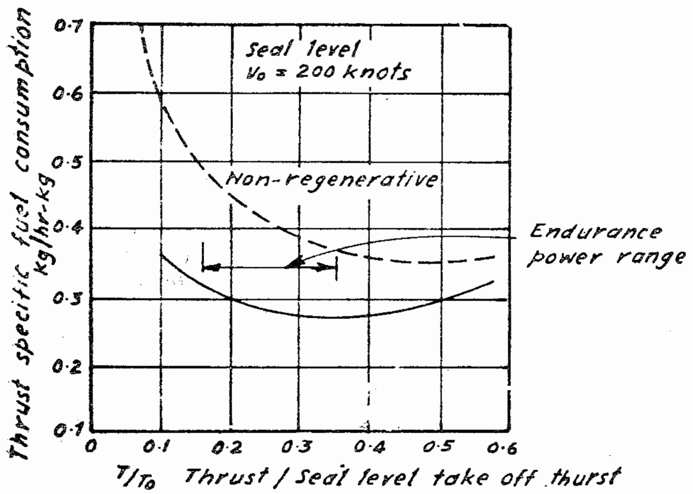


Fig. 8·25. Specific fuel concemption for regenerative and non-regenerative engine.

of the aircraft also becomes economical. The fuel consumption can be decreased by 25 to 30 per cent as seen in Fig. 18·25.

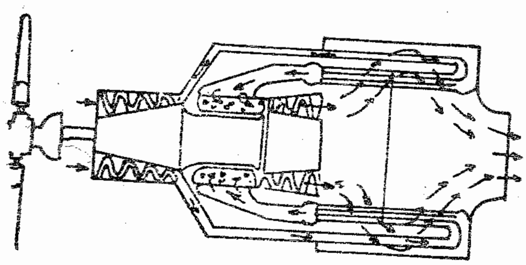


Fig. 8·26. Direct gas-to-air regenerator engine.

Now-a-days regeneration is widely used for long range aircrafts on patrol, observation and search missions. The weight of the aircraft is also reduced since has to carry less fuel and this amounts to about 30-50% longer range for present day non-regenerative ducted-fan engines.

8·11. TURBOSHAFT ENGINE

Turboshaft engine is that which uses whole of the available expansion in a turbine to drive a propeller coupled to it through a reduction gear. In this arrangement the exhaust gas does not contribute to the production of thrust. Use of a turbine results in smaller and light weight engine, with less noise and vibration as compared to a reciprocating engine power plant. This type of arrangement is used in helicopters.

8·12. RAM JET

The fact of obtaining very high pressure ratios of about 8 to 10 by ram compression has made it possible to design a jet engine without a mechanical compressor. A deceleration of the air from Mach number 3 at diffuser inlet to Mach number 0·3 in combustion chamber would cause pressure rise ratio of more than 30. Due to shock and other losses inevitable at such velocities all of this pressure rise is not available ; still whatever we get is more than sufficient for raising the air pressure to combustion pressure.

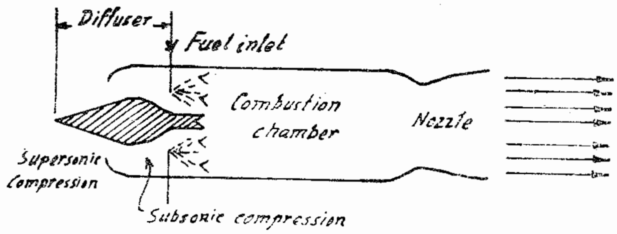


Fig. 8·27. Schematic diagram of a ramjet engine.

Fig. 8·27 shows a schematic diagram of a ram jet. Ram jet is simplest of all the air-breathing engines and consists of only three main parts—a diffuser, combustion chamber with fuel-injector, and an exhaust nozzle. There is no moving part in the ram jet.

Air enters the diffuser at a very high velocity and gets compressed. Since usually the ram jet operates at supersonic speeds, the total compression obtained is partially due to shock waves at the inlet of diffuser and partially in the diverging section of diffuser. The air then enters the combustion chamber, fuel is injected and mixed with the air and burnt. Flame holder or stabiliser is used to reduce the velocity of air mixture in some part of the combustion chamber, so that combustion is stable in an otherwise high velocity flow. Combustion is initiated with the help of a spark plug. By the time burning takes place and the temperature of the gases increased, the fresh supply of air to the diffuser builds up pressure at the end of the diffuser so that these gases cannot expand towards the diffuser, but instead, are obliged to expand in combustion chamber towards the tail pipe and further expand in the nozzle to produce a very high velocity jet. The thrust provided by this high acceleration of exhaust gases is applied to the ram jet structure through pressure at the end of diffuser and shear forces all over the surface of the engine.

(*a*) **Thermodynamic cycle.** The ram jet thermodynamic cycle is exactly same as that of turbojet engine, *i.e.* Brayton cycle. Fig. 8·28 shows this cycle for a ram jet on *T-s* diagram. In an ideal

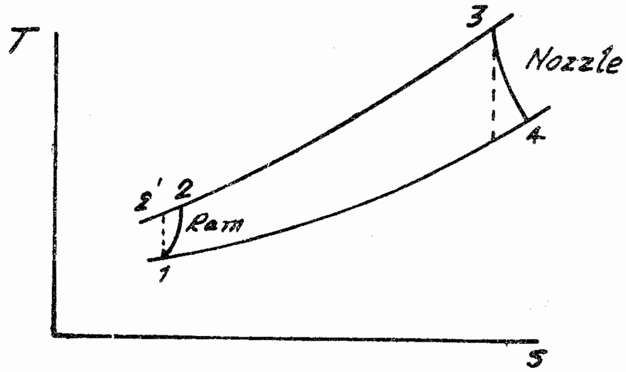


Fig. 8·28. Ram jet cycle on *T-s* diagram.

cycle process 1-2 is isentropic ram compression and process 3-4′ is the isentropic expansion in the nozzle. In actual practice there will be losses due to shock, friction and mixing at the diffuser and losses in the nozzle. The actual compression and expansion is shown by the processes 1-2 and 3-4 respectively. Combustion is represented by the process 2-3.

Since the expansion and compression is assumed isentropic, in an ideal ram jet cycle the stagnation pressure must remain constant and the pressure ratio of ram compression and nozzle expansion ratio must be same.

$$\frac{pt_2}{p_1}=\left[1+\frac{\gamma-1}{2}M_1^2\right]^{\frac{\gamma}{\gamma-1}}$$

and
$$\frac{p_{t3}}{p_4}=\left[1+\frac{\gamma-1}{2}M_e^2\right]^{\frac{\gamma}{\gamma-1}}$$

where M_1 and M_2 are inlet and exhaust Mach numbe

$$\therefore \quad \frac{p_{t1}}{p_1}=\frac{p_{t3}}{p_4}$$

which gives $M_1 = M_e$

or $$\frac{V_0}{\sqrt{\gamma R T_1}} = \frac{V_e}{\sqrt{\gamma R T_4}}$$

or $$V_e = V_0 \sqrt{\frac{T_4}{T_1}} \tag{8·34}$$

By putting this into thrust equation (8·33) thrust and thrust specific consumption can be found,

(*b*) **Performance**. Fig. 8·29 shows the thrust specific fuel consumption and net thrust per unit weight of a ram jet engine. It can be seen that at low flight speeds the thrust fuel consumption is very high while at high speeds it reduces considerably. This is because at low speeds the compression is poor.

As is clear from the Fig. 8·29, the thrust is maximum at sea level and decreases as the altitude increases, dueto reduction in

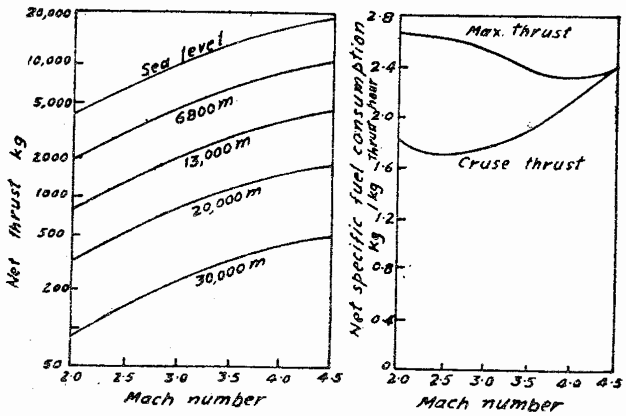


Fig. 8·29. Performance of a ram jet engine.

density with altitude. However, the velocity of the ram jet increases due to lower resistance at high altitude, thereby, improving specific fuel consumption, if the increase in thrust by increased velocity is more than decrease in thrust due to reduced density ; otherwise the specific fuel consumption will be independent of altitude.

The thrust, at a given altitude, rises rapidly with an increase in the flight speed due to increased pressure ratio but at too high Mach numbers the losses in diffuser reduce the thrust. Ram jets have highest thrust per unit weight of the plant amongst air breathing engines and is only next to rockets in this respect.

Though a ram jet can operate at subsonic velocities just below the sonic velocity, it is most efficient at high velocities of about 2400-6000 km/hr and at very high altitudes.

(c) Advantages and disadvantages

Advantages of ram jet

1. Ram jet is very simple and does not have any moving part. It is very cheap and requires almost no maintenance.

2. Due to the fact that a turbine is not used to drive the mechanical compressor, the maximum temperature which can be allowed in ram jet is very high, about 2000°C as compared to about 900°C in turbojets. This allows a greater thrust to be obtained by burning fuel at air-fuel ratio of about 15 : 1, which gives higher temperatures.

3. The specific fuel consumption is better than turbojet engines at high speed and high altitudes.

4. There seems to be no upper limit to the flight speed of the ram jet.

Disadvantages of ram jet

1. Since the compression of air is obtained by virtue of its speed relative to the engine, the take-off thrust is zero and it is not possible to start a ram jet without an external launching device.

2. The engine heavily relies on the diffuser and it is very difficult to design a diffuser which will give good pressure recovery over a wide range of speeds.

3. Due to high air speed, the combustion chamber requires flame-holder to stabilize the combustion.

4. At very high temperatures of about 2000°C dissociation of products of combustion occurs which will reduce the efficiency of the plant if not recovered in nozzle during expansion.

(*d*) **Applications**. Due to its high thrust at high operational speed, it is widely used in high-speed aircrafts and missiles. Subsonic ram jets are used in target weapons, in conjunction with turbojets or rockets for getting the starting torque.

8·12. PULSE JET

Pulse jet is very similar to ram jet in construction except that in addition to the diffuser at intake, combustion chamber and exhaust nozzle, it has mechanically operated flapper valves which can allow or stop air flow in the combustion chamber. Thus pulse jet is an intermittent flow, compressorless type of device with minimum number of moving parts. Pulse jet was the power plant of German *V*-1 bomb popularly known as 'Buzz bomb' first used in World war II in 1944. Fig. 8·30 (*a*) shows the schematic diagram of the pulse jet and Figs 8·30 (*b*) and (*c*) respectively show its theoretical and actual *p-v* diagrams.

The operation of the pulse jet is as follows: During starting compressed air is forced into the inlet which opens the spring loaded

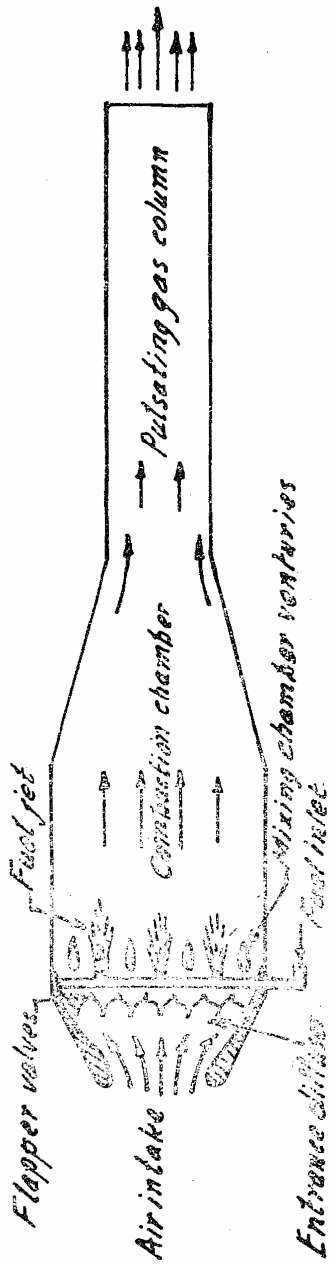


Fig. 8·30 (a) Schematic diagram of a pulse jet engine.

flapper valves ; the air enters combustion chamber into which fuel is injected and burnt with the help of a spark plug. Combustion occurs with a sudden explosion [process 2-3 in Fig. 8·30(*b*)], *i.e.* the combustion is constant volume combustion instead of constant pressure

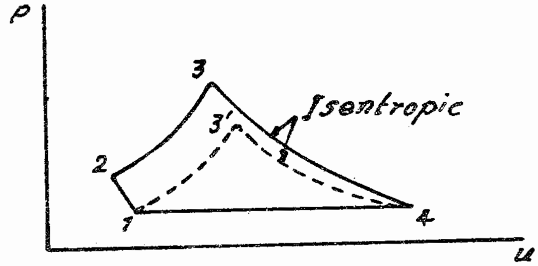


Fig. 8·30 (*b*). Theoretical pulse jet cycle on *p-v* diagram.

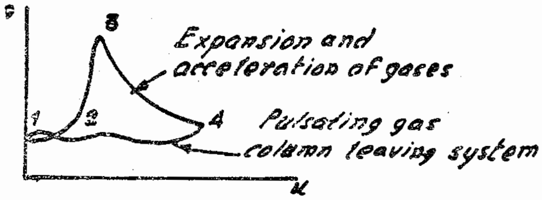


Fig. 8·30 (*c*) Actual pulse jet cycle on *p-v* diagram.

combustion as in other propulsive devices. The pulse jet cycle is more near to Otto cycle. Ram action can also be used to increase the pressure of the cycle (*see* Fig. 8·30(*b*)). Due to increase in pressure in the combustion chamber, the flapper valves close stopping the air supply and at the same time the gases are forced out of the nozzle with a very high velocity. Because of this high velocity, the pressure continues to fall since vacuum is created in the combustion chamber. This allows the flapper valves to open and fresh air enters the combustion chamber. Some air also reenters from the tail pipe because pressure in the tail pipe is also very low, and the next cycle starts. This increase in pressure due to back flow helps in better combustion as it will be hot by the time it reaches combustion chamber and also increases the thrust due to additional filling. The frequency of cycles depends upon the duct shape and working temperature. In V-1 rocket it was about 40 cycles per second which corresponds to about 2400 r.p.m. of a two stroke reciprocating engine.

Once the cycle is started neither the initial flow of compressed air nor the spark plug is necessary as the inlet diffusion will produce sufficient compression and any residual flame or even a hot part in the combustion chamber will ignite the fuel. Though the air flow is intermittent, the fuel is continuously supplied.

The pulse jet has low thermal efficiency and limited speed range. In early designs the efficiency obtained was about 2 to 3 per cent with a total flight life of 30 to 60 minutes. The maximum operating speed of the pulse jet is seriously limited by two factors : (*i*) It is not possible to design a good diffuser at high speeds. (*ii*) The flapper valves, the only mechanical part in the pulse jet, also have certain natural frequency and if resonance with the cycle frequency occurs then the valve may remain open and no compression will take place. Also, as the speed increases it is difficult for the air to flow back. This reduces the total compression pressure as well as the mass flow of air which results in inefficient combustion and lower thrust. The reduction in thrust and efficiency is quite sharp as the speed increases. At subsonic speeds it might not operate as the speed is not sufficient to raise the air pressure to combustion pressure.

Advantages and Disadvantages

Advantages of pulse jet

1. This is very simple device only next to ram jet and is light in weight. It requires very small and occasional maintenance.

2. Unlike ram jet, it has static thrust because of the compressed air starting ; thus it does not need a device for initial propulsion. The static thrust is even more than the cruise thrust.

3. It can run on almost any types of liquid fuels without much effect on the performance. It can also operate on gaseous fuel with a little modifications.

4. Pulse jet is relatively cheap.

Disadvantages of pulse jet

1. The biggest disadvantage is very short life of flapper valves and high rates of fuel consumption. The specific fuel consumption is as high as that of ram jet.

2. The speed of the pulse jet is limited within a very narrow range of about 650-800 km/hr because of the limitations in the aerodynamic design of an efficient diffuser suitable for a wide speed range.

3. The operational range of the pulse jet is also limited in altitude range.

4. The high degree of vibrations due to intermittent nature of the cycle and the buzzing noise has made it suitable for pilotless crafts only.

5. It has lower propulsive efficiency than turbojet engines.

Applications

Pulse jet is highly suited for bombers like the German *V*-1 ; it has also been used in some helicopters, target aircrafts missiles, etc.

8·13. RAM ROCKET

Ram rocket (*see* Fig. 8·31) is an attempt to combine the initial thrust of a rocket with the lower fuel combustion of ram jet. Afterburner can also be used in a ram-jet for thrust agumentation. However, use of a rocket, as shown in the schematic diagram, allows it to start itself from rest and accelerate to its efficient operating speed range. The rocket is shut down at high speed. Due to high specific fuel consumption of rocket, the ram rocket unit has very high specific fuel consumption at lower speeds.

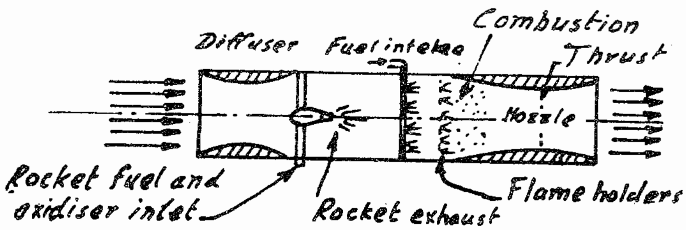


Fig. 8·31. Schematic diagram of a ram rocket.

9·14. COMPARISON OF VARIOUS PROPULSION DEVICES

In comparing various types of propulsion devices for a particular application, the weight of the power plant with its associated control equipment is most important. This includes the weight of the fuel which must be carried with the engine to meet its particular requirements. For short range vehicles fuel consumption is not so important a factor as in case of long range vehicles. The weight of the fuel is directly related to the fuel consumption of the vehicle. So depending upon the long or short range requirements and the fuel consumption, each type of power plant is suitable only for a particular speed range, altitude range and for military or commercial or experimental applications. A judicious choice of a particular plant for a given application requires a detailed study of the characteristics of the various plants suitable to the needs to be met and their economy. Such a comparison cannot be generalized. Still, however, in what follows, an attempt has been made to compare various propulsion devices.

The most important performance parameters which must be evaluated are thrust to weight ratio, specific fuel consumption, and the variation of thrust with altitude. Fig. 8·32 and 8·33 show such a comparison for various types of propulsion devices. It can very well be seen that though rocket and ram jet have very high thrust but their specific fuel consumption is nearly prohibitive. Ram jets are efficient only at very high speeds. Rockets improve their thrust with an increase in altitude while for all other devices the thrust reduces with increase in altitude Table 8·1 shows the components and fuel used for different propulsion devices.

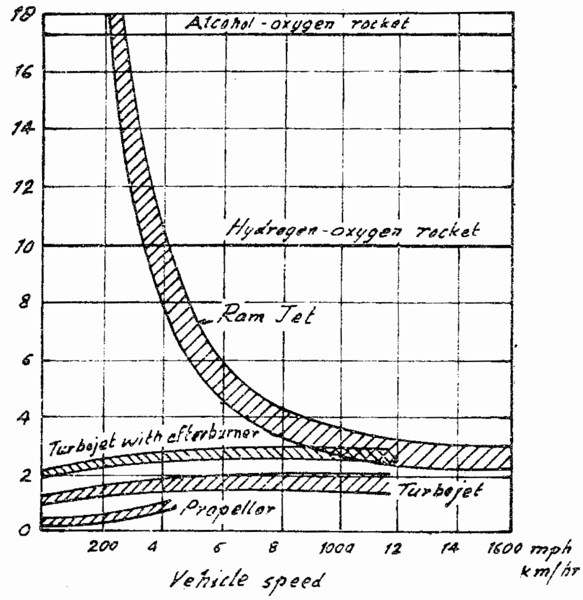


Fig. 8·32. Propellant or fuel consumption *vs* flight speed for propulsion devices.

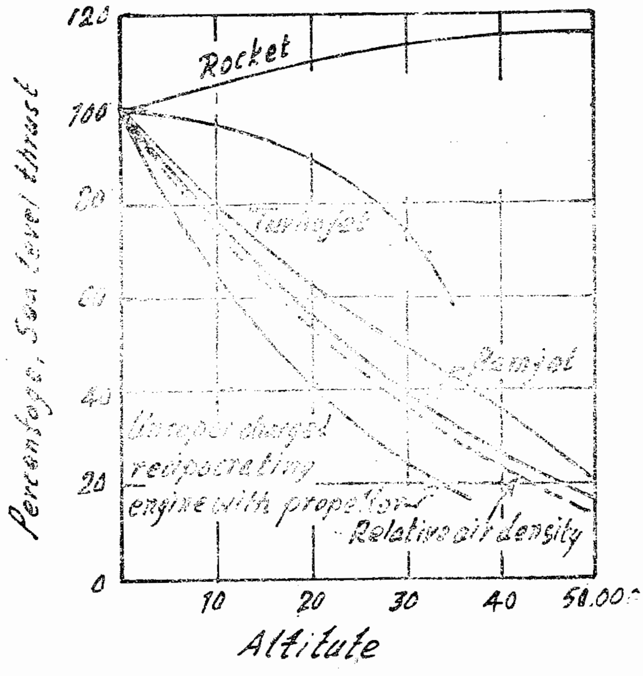


Fig. 8·33. Variation of thrust with altitude for different propulsion devices.

TABLE 8·1

COMPONENT AND FUELS USED FOR VARIOUS PROPULSIVE DEVICES

S.No.	*Type of en ine*	*Components*	*Compressing device*	*Source of oxygen for combustion*	*Propulsion device*	*Fuel used*
1.	Propeller Engine	Piston engine propeller	Engine	Atmospheric air	Propeller	High octane
2.	Turbojet	Diffuser, combustion, chamber, exhaust nozzle, compressor and turbine	Diffuser, compressor.	—do—	Exhaust nozzle	Kerosene, av gasoline
3.	Turboprop	Diffuser, compressor, combustion chamber, turbine, propeller, reduction gear, exhaust nozzle.	Diffuser, compressor.	—do—	Propeller, exhaust nozzle	Kerosene, tion fuel.
4.	Turbofan (ducted fan or by-pass engine)	Diffuser, fan, compressor, combustion chamber, turbine.	Diffuser. compressor.	—do—	Fan nozzle or nozzle alone	Kerosene, av fuels.
5.	Ram jet	Diffuser, combustion chamber, exhaust nozzle.	Diffuser	—do—	Exhaust nozzle	Wide rang liquid fuels
6.	Pulse jet	Diffuser, flapper valves, combustion chamber, exhaust nozzle.	Diffuser, back flow in tail-pipe	—do—	Exhaust nozzle	—do—
7.	Rocket	Combustion chamber, exhaust nozzle, fuel and oxident storage.	———	Carries its own oxident supply.	Exhaust nozzle	Solid, liquid pellants.

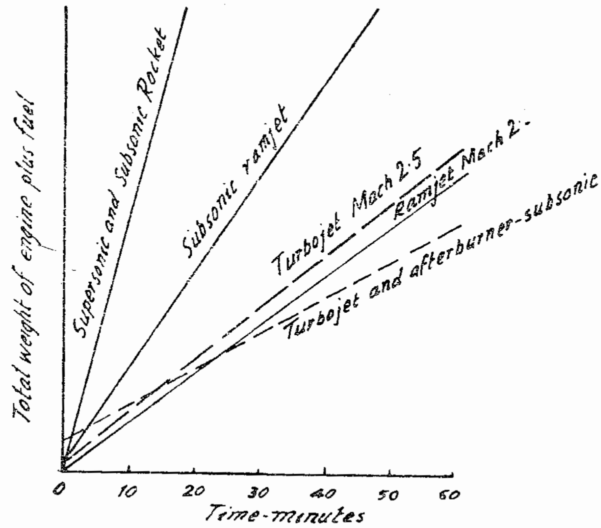


Fig. 8·34. Total weight *vs* flight time for different propulsion systems.

ILLUSTRATIVE EXAMPLES—8

8·1. Ideal gas turbine : h.p. : exhaust thrust

A gas turbine power unit consumes 30 kg of air per second. The air enters the compressor at a pressure of 1 kgf/cm² and temperature 15°C and leaves at a pressure of 35 kgf/cm². The combustion process occurs at a constant pressure and produces a temperature rise of 480°C. The air then expands through the turbine to a pressure of 1·3 kgf/cm². Calculate the net power output from the unit assuming that the flow through the compressor and turbine is reversible and adiabatic.

If the flow from the turbine is discharged to atmosphere at 1·01 kgf/cm² through a circular nozzle 300 mm in diameter, calculate the thrust produced by the unit.

For reversible and adiabatic compression

$$T_2 = T_1 \left(\frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\gamma}}$$

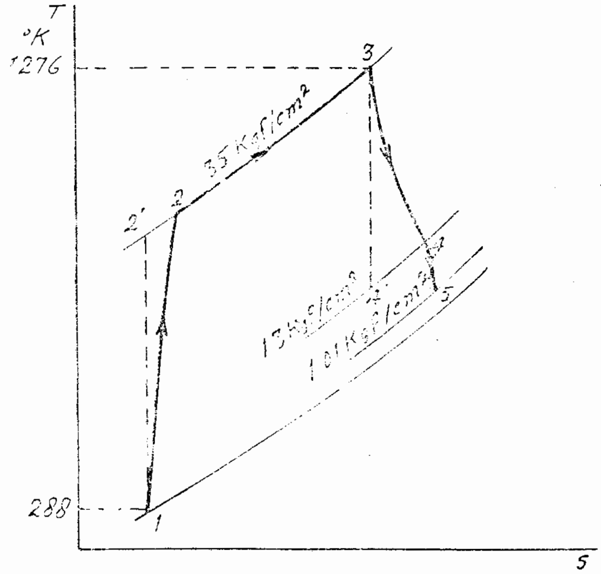


Fig. 8·35

$$=288\times\left(\frac{35}{1}\right)^{\frac{0\cdot4}{1\cdot4}}=796\text{ K}$$

$$T_2-T_1=796-288=508$$

$$T_3=796+480=1276\text{ K}$$

$$\text{Turbine expansion ratio}=\frac{35}{1\cdot3}$$

For reversible and adiabatic expansion

$$T_4=\frac{T_3}{\left(\frac{p_3}{p_4}\right)^{\frac{\gamma-1}{\gamma}}}=\frac{1276}{\left(\frac{35}{1\cdot3}\right)^{\frac{0\cdot4}{4\cdot1}}}=498\text{ K}$$

$$T_3-T_4=1276-498=778$$

$$\text{Net work output}=m\,c_p[(T_3-T_4)-(T_2-T_1)]$$

$$=30\times0\cdot24(778-508)\times5\cdot612$$

$$=11000\text{ hp.}$$

Thrust from the nozzle

Assuming that the expansion in the nozzle is reversible and adiabatic we have

$$T_5 = \frac{T_4}{\left(\frac{p_4}{p_5}\right)^{\frac{\gamma-1}{\gamma}}} = \frac{498}{\left(\frac{1\cdot3}{1\cdot01}\right)^{\frac{0\cdot4}{1\cdot4}}} = 464 \text{ K}$$

$\therefore$ Change in enthalpy $= c_p(T_4 - T_5) = 0\cdot24 \times (498 - 464)$

$$= 0\cdot24 \times 34$$

$$= 8\cdot16 \text{ kcal/kg}$$

Change in enthalpy = change in K.E.

$$c_p(T_5 - T_6) = \frac{C^2}{2gJ}$$

$$\therefore \quad C = \sqrt{2gJ\, c_p(T_4 - T_5)}$$

$$\sqrt{2 \times 9\cdot81 \times 427 \times 8\cdot16}$$

$$\doteq 262 \text{ m/sec.}$$

$$\text{Thrust} = \frac{\text{mass rate} \times \text{change in velocity}}{g}$$

$$= \frac{30 \times 262}{9\cdot81}$$

$$= \mathbf{800\ kgf}$$

Ans.

8·2. Jet propulsion

A jet-propelled plane consuming air at the rate of 20 kg/s, is to fly at a Mach number 0·6 at an altitute of 5000 m where the pressure is 0·55 kgf/cm² and temperature is −20°C. The diffuser which has a pressure coefficient of 0·9 decelerates the flow to a negligible velocity. The compressor pressure ratio is 5 and the maximum temperature in the combustion chamber is 1000°C. After expanding in the turbine, the gases continue to expand in the nozzle to a pressure of 0·7 kgf/cm². The isentropic efficiencies of the compressor, turbine, and nozzle are 0·81, 0·85 and 0·92 respectively. The calorific value of the fuel is 10000 kcal/kg. Assuming that the products of combustion have the same properties as air, find

(a) *the power input to the compressor ;*

(b) *the power output of the turbine ;*

(c) *the fuel-air ratio ;*

(d) *the exit Mach number ;*

(e) *the thrust provided by the engine ; and*

(f) *the thrust power developed.*

Solution

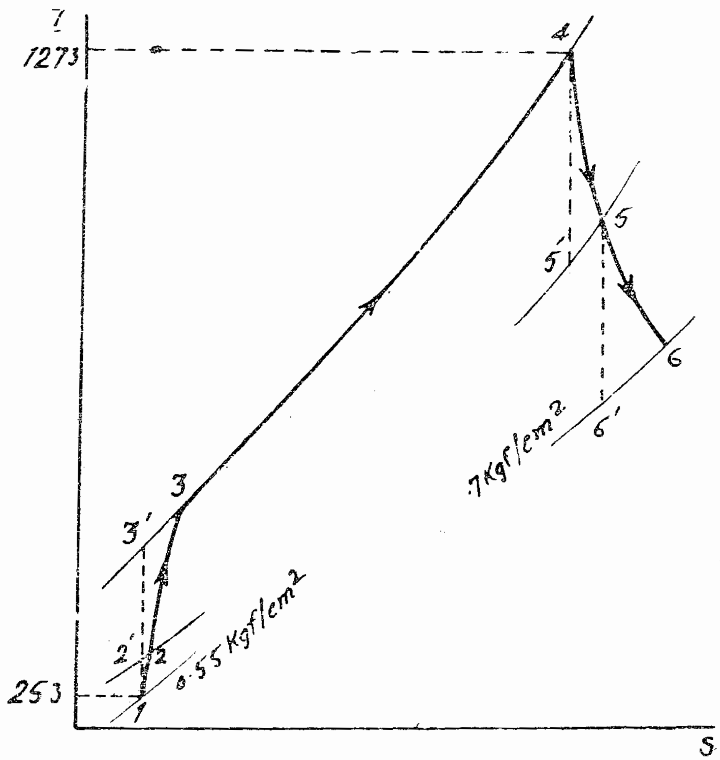


Fig. 8·36

For air, the velocity of sound is given by

$$a = 20{\cdot}1\sqrt{T_1}$$
$$= 20{\cdot}1\sqrt{253} = 320{\cdot}0 \text{ m/sec}$$

∴ The plane velocity

$$V = Ma = 0{\cdot}6 \times 320 = 192 \text{ m/sec.}$$

By energy equation, we have

$$c_pT_1 + \frac{V_1^2}{2g} = c_pT_2 + \frac{V_2^2}{2g}$$

Since V_2 is negligible, we have

$$T_2 = T_1 + \frac{V_1^2}{2gJc_p} = 253 + \frac{192^2}{2\times 9{\cdot}81\times 0{\cdot}24\times 427}$$
$$= 271{\cdot}4 \text{ K}$$

If the flow in the diffuser is isentropic, then

$$p_2' = p_1\left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}} = 0{\cdot}55\times\left(\frac{271{\cdot}4}{253}\right)^{\frac{1\cdot4}{0\cdot4}}$$
$$= 0{\cdot}7 \text{ kgf/cm}^2$$

Diffuser efficiency is given by

$$\eta_{diff.}=\frac{p_2-p_1}{p_2'-p_1}$$

$$\therefore \quad 0{\cdot}9=\frac{p_2-0{\cdot}55}{0{\cdot}7-0{\cdot}55}$$

$$\therefore \quad p_2=0{\cdot}55+0{\cdot}9(0{\cdot}7-0{\cdot}55)$$

$$=0{\cdot}685 \text{ kgf/cm}^2$$

$$\therefore \quad p_3=5\times0{\cdot}685=3{\cdot}43 \text{ kgf/cm}^2$$

$$T_3'=T_2\times\left(\frac{p_3}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$=271{\cdot}4(5)^{0{\cdot}286}=430 \text{ K.}$$

The compressor efficiency is

$$\eta_c=\frac{T_3'-T_2}{T_3-T_2}$$

$$\therefore \quad 0{\cdot}81=\frac{430-271{\cdot}4}{T_3-271{\cdot}4}$$

$$\therefore \quad T_3=467{\cdot}4 \text{ K}$$

Compressor work per/kg $=c_p(T_3-T_2)$

$$=-0{\cdot}24(467{\cdot}4-271{\cdot}4)=47 \text{ kcal}$$

Power input to compressor

$$=\frac{20\times{\cdot}47\times3600}{427\times75}=\mathbf{5350\ hp}$$ **Ans.**

(*b*) The power output of the turbine is equal to the power input to the compressor. **Ans.**

(*c*) $T_4=273+1000=1273$ K

Let m be the air-fuel ratio.

$\therefore$ Amount of gases in turbine

$$=(m+1) \text{ kg/kg of air}$$

Using conservation of energy

Enthalpy of air + Calorific value of fuel = Enthalpy of gases

$$\therefore \quad m.c_pT_3+C.V=(m+1)c_pT_4$$

or $$m\times0{\cdot}24\times467{\cdot}5+10000=(m+1)\times0{\cdot}24\times1273$$

$$\therefore \quad m=50{\cdot}5.$$

or Air-Fuel ratio $=50{\cdot}5$ **Ans.**

(*d*) $W_{compressor}=W_{turbine}$

$$\therefore \quad c_pm(T_3-T_2)=c_p(m+1)(T_4-T_5)$$

or $$T_4-T_5=\frac{m}{m+1}(T_3-T_2)$$

$$=\frac{50{\cdot}5}{51{\cdot}5}\times196=192{\cdot}2$$

$$\therefore \quad T_5=1080{\cdot}8 \text{ K}$$

Turbine efficiency is

$$\eta_t = \frac{T_4 - T_5}{T_4 - T_5'}$$

$$\therefore \quad T_5' = T_4 - \frac{(T_4 - T_5)}{\eta_t}$$

$$= 1273 - \frac{1273 - 1080{\cdot}8}{0{\cdot}85}$$

$$= 1047 \text{ K}$$

and

$$p_5 = \frac{p_4}{\left(\frac{T_4}{T_5'}\right)^{\frac{\gamma}{\gamma-1}}} = \frac{3{\cdot}43}{\left(\frac{1273}{1047}\right)^{3{\cdot}5}} = 1{\cdot}71$$

$\therefore$ Nozzle expansion ratio

$$= \frac{p_5}{p_6} = \frac{1{\cdot}71}{0{\cdot}7} = 2{\cdot}44$$

$$\therefore \quad T_6' = \frac{T_5}{\left(\frac{P_5}{P_6}\right)^{\frac{k-1}{k}}} = \frac{1080{\cdot}8}{(2{\cdot}44))^{0{\cdot}286}} = 839 \text{ K}$$

The isentropic efficiency of nozzle is

$$0{\cdot}92 = \frac{T_5 - T_6}{T_5 - T_6'}$$

$$= \frac{1080{\cdot}8 - T_6}{1080{\cdot}8 - 839}$$

$$\therefore \quad T_6 = 858{\cdot}8$$

Nozzle exit velocity

$$V_6 = \sqrt{2gJc_p(T_5 - T_6)}$$

$$= \sqrt{2 \times 9{\cdot}81 \times 427 \times 0{\cdot}24 \times 222}$$

$$= 670 \text{ m/sec}$$

The sonic velocity corresponding to T_6 is

$$a_6 = 20{\cdot}1\sqrt{T_6}$$

$$= 20{\cdot}1\sqrt{858{\cdot}8}$$

$$= 589 \text{ m/sec.}$$

$\therefore$ The Mach number at exit

$$M_6 = \frac{V_6}{a_6} = \frac{670}{589} = \mathbf{1{\cdot}14}$$ **Ans.**

(e) Thrust force $F = \frac{m}{g}\left[\left(\frac{1+m}{m}\right)V_6 - V_1\right]$

$$= \frac{20}{9{\cdot}81}\left[\frac{51{\cdot}0}{50} \times 670 - 192\right]$$

$$= \mathbf{1001 \text{ kgf}}.$$ **Ans.**

(f) Thrust power, $P = \frac{F \times V_1}{75} = \frac{1001 \times 192}{75} = \mathbf{2570}$ **Ans.**

8·3. Turbojet : nozzle diameter ; thrust ; s.f.c.

A simple turbojet unit operates with a turbine inlet temperature of 1000 K, a total head pressure ratio 4 allowing for combustion chamber pressure loss, and a mass flow of 25 kg/s under design conditions. The following component efficiencies may be assumed.

Total head isentropic efficiency of compressor	$=80\%$	
Total head isentropic efficiency of turbine	$=85\%$	
Combustion efficiency	$=98\%$	
Nozzle efficiency	$=90\%$	
Mechanical transmission efficiency	$=99\%$	
Calorific value of fuel	$=10300$ *kcal/kg*	
For air,	$c_p=0{\cdot}24,$	$\gamma=1{\cdot}4$
For gases,	$c_p=0{\cdot}276$	$\gamma=1{\cdot}33$

Calculate the propelling nozzle diameter, design thrust, and specific fuel consumption when the unit is stationary and at sea level where the ambient conditions may be taken as 1 kgf/cm² and 300 K.

The relation for the fuel between the ideal combustion temperature rise and fuel/air ratio at 300 K is given by

Combustion temperature rise (°C)	100	208	405	592	770
Fuel/air ratio	0·00225	0·005	0·010	0·015	0·02

(*Banaras Hindu Univercity, M.E.* 1970)

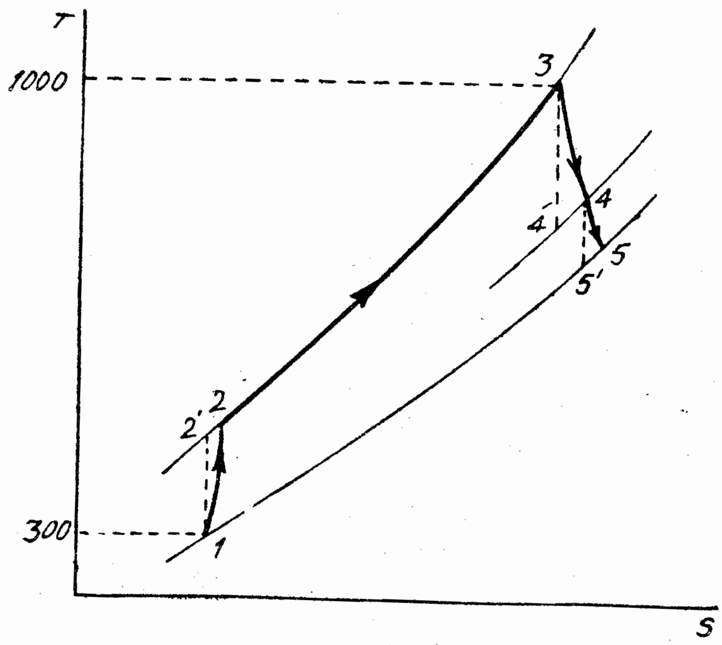


Fig. 8·37 (*a*)

$$\frac{T_2'{}_t}{T_1}=\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}=4^{\frac{1\cdot4-1}{1\cdot4}}=1\cdot486$$

$\therefore$ $T_2'{}_t=300\times1\cdot486=445$

Compressor efficiency $\eta_c=0\cdot8=\dfrac{T_2'{}_t-T_{1t}}{T_{2t}-T_{1t}}=\dfrac{145}{T_{2t}-T_{1t}}$

$\therefore$ $T_{2t}-T_{1t}=181$ K

$\therefore$ $T_{2t}=481\cdot2$ K

Now, Work output of turbine = Work of compressor

$\therefore$ $c_{pg}\times0\cdot99(T_{3t}-T_{4t})=c_{pa}(T_{2t}-T_{1t})$

or $0\cdot276\times0\cdot99(1000-T_{4t})=0\cdot24(181\cdot2)$

$\therefore$ $T_{3t}=840\cdot5$ K

Now $\dfrac{T_{3t}-T_{4t}}{T_{3t}-T_4'{}_t}=0\cdot85$

$\therefore$ $T_4'{}_t=812\cdot5$ K

And $\dfrac{T_{3t}}{T_4'{}_t}=\left(\dfrac{p_{3t}}{p_{4t}}\right)^{\frac{\gamma-1}{\gamma}}$ $\therefore$ $\dfrac{p_3}{p_4}=2\cdot31$

$\therefore$ $p_{4t}=\dfrac{4}{2\cdot31}=1\cdot73$ kgf/cm³

Nozzle : $\dfrac{T_{4t}}{T_5'{}_t}=\left(\dfrac{p_{4t}}{p_5}\right)^{\frac{\gamma-1}{\gamma}}$

or $\dfrac{840\cdot5}{T_5'}=\left(\dfrac{1\cdot73}{1}\right)^{\frac{1\cdot33-1}{1\cdot33}}=1\cdot145$

$\therefore$ $T_5'=732\cdot5$ K

To find velocity C_5, find $T_{5t}-T_5$

Efficiency of nozzle

$0\cdot9=\dfrac{T_{4t}-T_5}{T_{4t}-T_5'}$ (Note $T_{4t}=T_{5t}$)

or $0\cdot9=\dfrac{840\cdot5-T_5}{840\cdot5-732\cdot5}$

$\therefore$ $T_5=743\cdot2$; $T_{4t}-T_5=96\cdot3$ K

Now $\dfrac{C_5^2}{2gJc_p}=T_{5t}-T_5$

$=T_{4t}-T_5=96\cdot3$

$$C_5 = \sqrt{2gJc_p(T_{5t} - T_5)}$$
$$= \sqrt{2 \times 9{\cdot}81 \times 427 \times 0{\cdot}276 \times 96{\cdot}3}$$
$$= 472{\cdot}5 \text{ m/s}$$

$$\text{Thrust/kg/s} = \frac{mC_5}{g} = \frac{1 \times 472{\cdot}5}{9{\cdot}81} = 48{\cdot}1$$

Total Thrust $= 48{\cdot}1 \times 25$

$= \mathbf{1202{\cdot}5 \text{ kg/s}}$ **Ans.**

At this stage the operating pressure ratio $\frac{p_5}{p_{4t}}$ should be checked against the critical pressure ratio $\frac{p_c}{p_{4t}}$. If $\frac{p_5}{p_{4t}} > \frac{p_c}{p_{4t}}$ then a simple convergent nozzle would be sufficient for the expansion. If $\frac{p_5}{p_{4t}} < \frac{p_c}{p_{4t}}$ a convergent-divergent nozzle is needed. In actual practice only convergent nozzles are used and the expansion takes place upto p_c (if p_c is higher than p_1). Hence in addition to the momentum thrust there will be a pressure thrust given by $(p_c - p_1)$

$$\frac{p_1}{p_{4t}} = \frac{p_5}{p_{4t}} = \frac{1}{1{\cdot}73} = 0{\cdot}58$$

$$\frac{p_c}{p_{4t}} = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{2}{2{\cdot}33}\right)^{\frac{1{\cdot}33}{1{\cdot}33-1}} = 0{\cdot}54$$

Here $$\frac{p_5}{p_{4t}} > \frac{p_c}{p_{4t}}$$

Hence there is no question of contribution from pressure thrust.

Specific volume at outlet

$$v_5 = \frac{RT_5}{P_5} = \frac{29{\cdot}27 \times 743\ 2}{1 \times 10^4}$$
$$= 2{\cdot}174 \text{ m}^3\text{/kg}$$

Now area of nozzle $\times$ velocity $=$ volume,

$$A \times C_5 = \dot{m}v$$

or $$A \times 472{\cdot}5 = 25 \times 2{\cdot}174$$

$\therefore$ $$A = 0{\cdot}115$$

or **d = 38 25 cms.** **Ans.**

Actual diameter should be slightly greater than this value because of boundary layer effects.

Temperature rise during combustion

$$= T_{3t} - T_{2t}$$
$$= 1000 - 481{\cdot}2 = 518{\cdot}8 \text{ K}$$

From the given data, plot the graph between fuel-air ratio and combustion temperature rise and find the fuel-air ratio for 518·8°C temperature rise. It comes to 0·013.

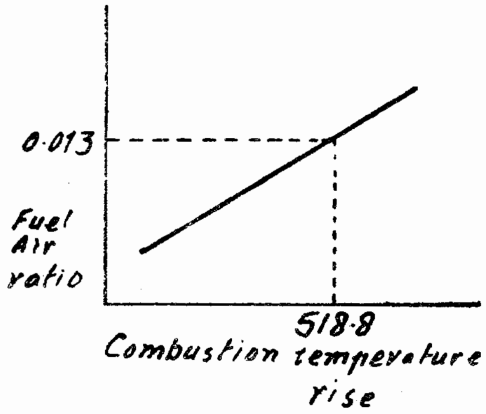


Fig. 8·37 (*b*)

Now, combustion efficiency is 98%

$\therefore$ Fuel air ratio $= \dfrac{0 \cdot 013}{0 \cdot 98} = 0 \cdot 01326$ kg. of fuel/kg air

$\therefore$ Specific fuel consumption/kg thrust/hr

$$= \frac{0 \cdot 01326}{48 \cdot 1} \times 3600$$

$= 0 \cdot 992$ **kg/thrust/hr.** **Ans.**

8·4. Turbojet ; thrust ; s.f.c.

A turbojet aircraft is flying at 800 km/h at an altitude of 10,000 m where the pressure and temperature of the atmosphere are 0·25 kgf/cm² and −45°C respectively. The compressor pressure ratio is 10 : 1 and the maximum cycle temperature is 800°C. Calculate the thrust developed and the specific fuel consumption in kg/kg thrust/hr. Assume the following data :

Intake duct efficiency	$=90\%$
Isentropic efficiency of compressor	$=90\%$
Total head pressure loss in the combustion chamber	$=0 \cdot 14$ kgf/cm^2
Calorific value of fuel	$=10300$ *kcal/kg*
Combustion efficiency	$=98\%$
Isentropic efficiency of turbine	$=92\%$
Mechanical efficiency of drive	$=98\%$
Jet pipe efficiency	$=92\%$
Nozzle outlet area	$=0 \cdot 08$ m^2

c_p, for compression gas$=0{\cdot}24$, for combustion gas$=0{\cdot}275$

γ, for compression process$=1{\cdot}4$, for combustion and expansion process$=1{\cdot}333$.

The nozzle in the turbojet is convergent.

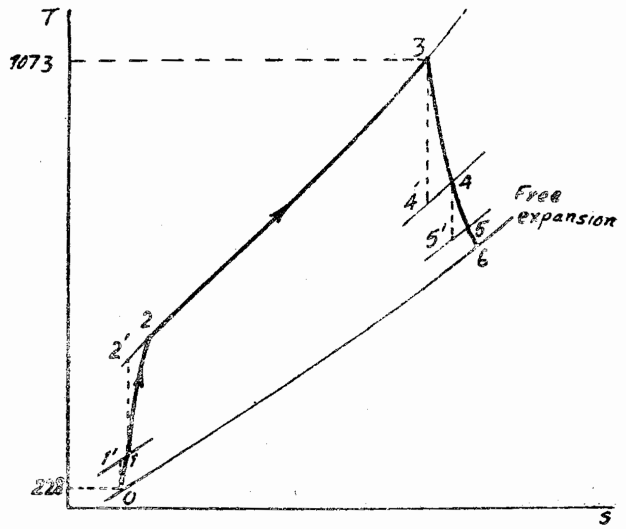


Fig. 8·38

Kinetic energy of air at inlet

$$=\left(\frac{800\times10^3}{3600}\right)^2=\frac{1}{2\times9{\cdot}81\times427}$$

$$=5{\cdot}9 \text{ kcal/kg}$$

$$T_{t1}-T_0=\frac{5{\cdot}9}{c_p}=\frac{5{\cdot}9}{0{\cdot}24}=24{\cdot}6 \text{ K}$$

$\therefore$ $$T_{t1}=(-45+273)+24{\cdot}6=252{\cdot}6 \text{ K}$$

Now intake-efficiency

$$0{\cdot}9=\frac{T_{t1}'-T_0}{T_{t1}-T_0}$$

$\therefore$ $$T_{t1}'-T_0=0{\cdot}9\times24{\cdot}6=22{\cdot}14$$

$\therefore$ $$T_{t1}'=(-45+273)+22{\cdot}14=250{\cdot}14 \text{ K}$$

As 0-1$'$ is isentropic compression

$$\frac{p_{t1}'}{p_0}=\left(\frac{T_{t1}'}{T_0}\right)^{\frac{\gamma}{\gamma-1}}=\left(\frac{250{\cdot}14}{228}\right)^{\frac{1{\cdot}4}{0{\cdot}4}}=1{\cdot}1^{3{\cdot}5}=1{\cdot}396$$

$$\therefore \qquad p_{t_1}'=p_{t1}=1{\cdot}396\times0{\cdot}25=0{\cdot}349 \text{ kgf/cm}$$

Compressor

$$\frac{T_{t_2}'}{T_{t1}}=\left(\frac{p_{t2}}{p_{t1}}\right)^{\frac{\gamma-1}{\gamma}}=10^{\frac{0\cdot4}{1\cdot4}}=1{\cdot}932$$

$$\therefore \qquad T_{t_2}'=1{\cdot}932\times252{\cdot}6=489 \text{ K}$$

$$\eta_c=0{\cdot}9=\frac{T_{t_2}'-T_{t_1}}{T_{t_2}-T_{t_1}}$$

$$\therefore \qquad T_{t_2}-T_{t_1}=\frac{489-252\,6}{0{\cdot}9}=\frac{236{\cdot}4}{0}=263 \text{ K}$$

$$\therefore \qquad T_{t_2}=263+252{\cdot}6=515{\cdot}6 \text{ K}$$

Also $\quad p_{t2}=10\times p_{t1}=10\times0{\cdot}349$

$$=3{\cdot}49 \text{ kgf/cm}^2$$

Hence $\quad p_{t_3}=p_{t_3}-$ loss of total pressure in combustion chamber

$$=3{\cdot}49-0{\cdot}14=3{\cdot}35 \text{ kgf/cm}^2$$

Turbine

The power developed by the turbine is equal to that required by the compressor and to overcome friction

$$\therefore \quad c_{pg}(T_{t_3}-T_{t_4})=\frac{c_{pa}(T_{t_2}-T_{t_1})}{0{\cdot}98}$$

or

$$T_{t_3}-T_{t_4}=\frac{0{\cdot}24}{0{\cdot}275}\times\frac{263}{0{\cdot}98}=234$$

$$\therefore \qquad T_{t_4}=T_{t_3}-234=1073-234=839 \text{ K}$$

$$\eta_T=0{\cdot}92=\frac{T_{t_3}-T_{t_4}}{T_{t_3}-T_{t_4}'}$$

$$\therefore \qquad T_{t_3}-T_{t_4}'=\frac{234}{0{\cdot}92}=254{\cdot}2$$

$$\therefore \qquad T_{t_4}'=1073-254{\cdot}2-818{\cdot}8 \text{ K}$$

Then, $$\frac{p_{t3}}{p_{t4}'}=\left(\frac{1073}{818{\cdot}8}\right)^{\frac{1\cdot333}{0\cdot333}}=(1{\cdot}311)^{4\cdot01}=2{\cdot}96$$

$$p_{t4}'=\frac{2{\cdot}35}{2{\cdot}96}=1{\cdot}131 \text{ kgf/cm}^2$$

For choked flow in the nozzle

$$\frac{p^*}{p_{t4}'}=\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}=\left(\frac{2}{2{\cdot}333}\right)^{\frac{1{\cdot}333}{0{\cdot}333}}=0{\cdot}54$$

$$\therefore \quad p^*=0{\cdot}54\times1{\cdot}131$$
$$=0{\cdot}611\ \text{kgf/cm}^2=p_5$$

Therefore in this nozzle the flow is choked and an unresisted expansion from 0·611 kgf/cm² to atmospheric pressure of 0·25 kgf/cm² takes place outside the nozzle.

Temperature at throat

$$\frac{T^*}{T_{t4}}=\frac{2}{\gamma+1}=\frac{2}{2{\cdot}333}$$

$$\therefore \quad T^*=\frac{839\times2}{2{\cdot}333}=720\ \text{K}=T_5'$$

Now jet pipe efficiency

$$0{\cdot}92=\frac{T_{t4}-T_5}{T_{t4}-T_5'}$$

$$\therefore \quad T_{t4}=T_5=0{\cdot}92(839-720)=109{\cdot}5\ \text{K}$$

$$\therefore \quad T_5=839-109{\cdot}5=729{\cdot}5\ \text{K}.$$

Now $$R=\frac{c_{p}J(\gamma-1)}{\gamma}$$

$$=\frac{0{\cdot}275\times427\times0{\cdot}333}{1{\cdot}333}=29{\cdot}35\ \text{kgf-m/kg- K}$$

Hence $$v_5=\frac{RT_5}{p_5}=\frac{29{\cdot}35\times729{\cdot}5}{0{\cdot}611\times10^4}=3{\cdot}5\ \text{m}^3/\text{kg}$$

Also for the nozzle, from the flow equation

$$c_{pg}\,T_4+\frac{c_4^2}{2g_0J}=cp_v\,T_5+\frac{C_5^2}{2g_0J}$$

C_5=jet velocity leaving the aircraft.

$$\therefore \quad \frac{C_5^2}{2g_0J}=c_{pg}(T_{t4}-T_5)$$

$$=0{\cdot}275\times109{\cdot}5$$

$$=30{\cdot}2\ \text{kcal/kg}$$

$$\therefore \quad C_5=504\ \text{m/s}$$

Then

Mass flow $$=\frac{A_5C_5}{v_5}=\frac{0{\cdot}08\times504}{3{\cdot}5}$$

$$=11{\cdot}5\ \text{kg/sec}$$

Now, momentum thrust

$$=\frac{m(C_5-C_{aircraft})}{g_0}$$

$$=\frac{11\cdot5(504-222)}{9\cdot81}$$

$$=330\cdot5 \text{ kgf}$$

Pressure thrust $=(p_5-p_{atm})A$

$$=(0\cdot611-0\cdot250)10^4\times0\cdot08$$

$$=288\cdot8 \text{ kgf}$$

Total thrust $=330\cdot5+288\cdot8$

$$=\mathbf{619\cdot3 \text{ kgf}}$$ **Ans.**

Now, heat supplied

$$=mc_{pg}(T_{3t}-T_{2t})$$

$$=11\cdot5\times0\cdot275(1073-515\cdot6)$$

$$=1761 \text{ kcal/sec}$$

$$\therefore \qquad m_f.C.V.=\frac{1761}{0\cdot98}$$

$$\therefore \qquad m_f=\frac{1761}{0\cdot98}\times\frac{1}{10{,}300}=0\cdot1748 \text{ kg/s}$$

Hence, specific fuel consumption

$$=\frac{0\cdot1748\times3600}{619\cdot3}$$

$$=\mathbf{1\,018 \text{ kg/kgf-hr}}$$ **Ans.**

EXERCISES 8

SECTION A :

8·1. What are the two main propulsive devices and what is the basic difference between them ?

8·2. Classify the air-breathing engines and list the main components of each engines. Which are the suitable fuels for each one of them ?

8·3. Draw the schematic diagrams of the various propulsion engines using turbine and show the basic difference in the mode of production of thrust in propeller, by-pass, and turbojet engine.

8·4. Why a ram jet engine does not require a compressor and a turbine ?

8·5. Draw a schematic diagram of a turbojet engine and explain its working by the help of a *T-s* diagram.

8·6. What is ram effect ?

8·7. What is the total thrust comprised of ? Derive a general equation for the thrust developed in a jet engine. Hence, show that the thrust developed greatly depends upon the forward speed of aircraft.

8·8. Define thrust specific fuel consumption and show that the turbojet engine will have higher thrust specific fuel consumption than other air-breathing engines.

8·9. Define ram efficiency and derive an expression for it.

8·10. Why jet propulsion engines are not rated in terms of horse-power ?

8·11. Define propulsion efficiency and derive an expression for it. Discuss its optimum values.

8·12. How overall efficiency of a jet engine is related to its thermal, transmission, and propulsion efficiencies ?

8·13. Show by suitable graphs the variation of thrust and thrust horse-power with flight velocity for a turbojet engine and compare these with that for a reciprocating engine.

8·14. Compare the specific fuel consumption, thrust, and thrust horse-power of a reciprocating engine and a turbojet engine at varying altitudes.

8·15. Which compressor , axial flow or centrifugal, is more suitable for a turbojet engine ? Why ?

8·16. Briefly state advantages and disadvantages of a turbojet engine.

8·17. When thrust augmentation is necessary in an aircraft engine ? Briefly describe the various methods of thrust augmentation in a jet engine. For which applications these various methods are suitable ?

8·18. What is the difference between a turboprop engine and a bypass engine ? When a bypass engine would be preferred over a turboprop or a turbojet engine ?

8·19. Discuss the statement "A turboprop engine combines in itself the desirable characterist ics of a turbojet engine and a reciprocating engine for specific applications."

8·20. Explain why a reciprocating engine having the same available power output at 200 km/hr as a turboprop engine, has a lesser available thrust horse-power at 800 km/hr ?

8·21. Draw a schematic diagram of a ram jet engine and describe its operation. What are its advantages and disadvantages ?

8·22. How an air-craft having a ram jet engine takes off ?

8·23. Draw a schematic diagram of a pulsejet engine and describe its operation. What are the advantages and disadvantages of pulse jet engine ?

8·24. How ram rocket combines the desirable characteristics of a rocket and a ram jet ?

SECTION B

8·1. Jet Propulsion : Propulsion, thermal and overall efficiencies.

The leaving velocity is 1750 km/hr from a jet and the inlet velocity is 880 km/hr. The specific fuel consumption is 12 kg per hr for each kg of thrust. Gasoline of 10200 kcal/kg lower heating value is used as fuel. For 2000 kg thrust compute the air flow in kg/sec. Compute the probable propulsion and thermal efficiences, and finally estimate the overall efficiency of this jet-propell. ed unit.

(Aligarh, M.Sc. Engg., 1972)

8·2. **Turbojet unit : net thrust; given nozzle efficiency**

In a turbojet unit flying at a speed of 1000 km/hr at an altitute where the ambient temperature is $-43°C$ the maximum temperature in the cycle was $700°C$. The pressure ratio was 10. Assuming that the static pressure at compressor inlet and nozzle outlet are equal to the local atmospheric pressure and that the gases leave the turbine with a velocity of 90 m/s, calculate the net thrust of the engine for a mass flow rate of 35 kg/s. Given the isentropic efficiency of the nozzle 0·95.

[$W_{turbine}=51·45$ kcal, enthalpy drop nozzle $=58·2$ kcal $C_5=705$ m/s, $C_1=278$ m/s. Thrust $=14900$ kg-m/sec].

8·3. **Jet Propulsion : Power required ; exit speed ; thrust.**

Air enters a jet propulsion engine at the rate of 2000 kg/min at $15°C$ and $1·03$ kg/cm^2 and is compressed adiabatically to $182°$ C and four times the entry pressure. Products of combustion enter the turbine at $815°C$ and leave it at $650°C$ to enter the tail nozzle.

Find the isentropic efficiency of the compressor, the power required to drive the compressor, the exit speed of the gases and the thrust created when flying at 800 km/hr. Assume isentropic efficiency of turbine same as that of the compressor and the nozzle efficiency 90 per cent.

For air $\gamma=1·4$ and $c_p=0·238$ kcal/kg- K.

[$\eta_c=84\%$, h p.$=7560$, $p_4=2·055$ kgf/cm^2, $T_5=775·4$ K, $C_{exit}=552$ m/s Thrust $=1120$ kg.]

8·4. **Turbojet engine : fuel consumption.**

In a turbojet with a forward-facing ram intake, the jet velocity relative to the propelling nozzle at exit is twice the flight velocity. Determine the rate of fuel comsumption in kg/s when developing a thrust of 2500 kgf under the following conditions.

Ambient pressure and temperature	$0·7$ kgf/cm^2 and $0°C$ respectively
Compressor stagnation pressure ratio	5 : 1
Pressure at nozzle exit	$0·7$ kgf/cm^2
Flight Speed	800 km/hr
Enthalpy of combustion of fuel	10500 kcal/kg
Efficiencies :	
Ram	100%
Stagnation, compressor	85%
Stagnation. turbine	90%
Nozzle	95%
Combustion	98%

Assume that the mass flow of fuel is small compared with the mass flow of air and that the working fluid throughout has the properties of air at low temperature. Neglect extraneous pressure drops.

8·5. **Jet propsion : power to drive compressor, air fuel ratio, thrust.**

In a jet propulsion unit, air is compressed by means of an uncooled rotary compressor, the pressure at delivery being 4 times that at the entrance, and the temperature rise during compression is 1·2 times that for frictionless adiabatic compression. The air is then led to a combustion chamber where the fuel is burned under constant pressure conditions. The products of combustion at $800°C$ pass through a turbine which drives the compressor. The exhaust gases from the turbine are expanded in a nozzle down to atmospheric pressure. The atmospheric pressure is $1·03$ kgf/cm^2, and the temperature is $10°C$.

Assuming that the values of R and γ after combustion remain same as that for air, estimate (*a*) the power required to drive the compressor per kg of air per second, (*b*) the air-fuel ratio, if the calorific value of the fuel is 10400 kcal/kg, and (*c*) the thrust developed per kg/sec. The velocity of approach may be neglected. The gases are expanded isentropically in both the turbine and the nozzle.

8·6. **Jet propulsion : given all efficiencies ; thrust, sfc**

A gas turbine plant for jet propulsion has a centrifugal compressor, a combustion chamber and a turbine, the power output of the turbine being just sufficient to drive the compressor. The gases leaving the turbine expand to atmospheric pressure in the nozzle. The aircraft is flying at 250 metres per second at 12,000 metre altitude. The ambient conditions at this altitude are pressure 0·20 kgf/cm² and temperature 217 K. Calculate the thrust and specific fuel consumption assuming the following data :

Isentropic compressor efficiency	=80%
Isentropic turbine efficiency	=85%
Pressure ratio (allowing for combustion chamber pressure loss)	=5 : 1
Combustion efficiency	=95%
Nozzle diameter	=30 cms
Nozzle efficiency	=90%
Ram efficiency	=90%
Mechanical transmission efficiency	=98·5%
Maximum cycle temperature	=950 K
Theoretical Fuel air ratio	=0·0136

For air, $c_p=0{\cdot}24$, $\gamma=1{\cdot}4$

For gases, $c_p=0{\cdot}276$, $\gamma=1{\cdot}33$

[*B.H.U., M.E., 1971*]

[$p_{ram}=0{\cdot}306$ kg/cm², $T_{2t}=429{\cdot}1$ K, $T_{4t}=790$ K, $p_{4t}=0{\cdot}6275$ kgf/cm², $p_5=0{\cdot}339$ kgf/cm², $T_5=691$ K, $C_5=478$ m/s, m=5·66 kg/s, momentum thrust =131·5 kg, pressure thrust=98·4 kg. **total thrust=229·9 kg, sfc=1·26 kg/kg thrust hr.**]

8·7. **Ram jet : Specific thrust and sfc**

(*a*) For turbopro engine it is usual to employ propulsion efficiencies close to unity whereas the practice for turbojet is to employ propulsion efficiencies between 0·55 to 0·7 only. Explain why ?

(*b*) Derive an expression for overall efficiency of turbojet device and show that for a given jet velocity it is maximum when the flight speed is approximately half that of the jet velocity.

(*c*) A Ramjet is flying at Mach number 2 at an altitute of 9000 m where the pressure and temperature are 0·3 kgf/cm² and 35°C. If the maximum combustion temperature is limited to 1050°C and the adiabatic efficiencies of the diffuser and nozzle are 100%, using Air Tables, find out the specific thrust developed and the specific fuel consumption.

Assume suitable values for the gas velocity through the ramjet Take the calorific value of the fuel as 10,300 kcal/kg.

(*Aligarh, M. Sc. Engg., 1972*)

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9

ROCKET PROPULSION

9.1 INTRODUCTION

A rocket propulsion device, though working on the same principle as that of a jet engine *i.e.* of obtaining a propulsive force as a reaction to the acceleration of a mass of fluid, characterises itself in that it carries, with it, its own supply of oxidant. The fact that it does not depend upon atmosphere for the supply of the oxidant allows it to operate at very high altitudes and even in vacuum.

The history of rocket propulsion can be traced back to eighth century when the Greeks knew about flying fire. Though the actual period when the use of rocket principle started is quite uncertain, the Chinese were, perhaps, the first to use this principle in fire arrows as early as in 1235. In spite of the fact that a large number of investigators were working on rockets, it was only during the two World Wars that rocket was given its due importance for warfare and other uses. Noted amongst these early workers were Alexander Rynin and Tsiolkovskiy (1903) in Russia, H. Oberth, Fritz Von Opel, Ziolkovsky and E. Sanger in Germany and Dr. R.H. Goddard in U.S.A. It was in 1919 that Dr. Goddard published what is called as the first report on rockets. After that, with the advent of German V-1 and V-2 rockets, the rockets were fully established. Werner Von Brown pioneered these V-1 and V-2 rockets during 1924-42. First came the V-1 rocket working on constant volume explosion type combusion; it was actually a pulse jet. Then in 1944 came the V-2 rocket—a liquid propulsion device using liquid oxygen and ethyal alcohol in water as main propellants. The main propellants were burned in the combustion chamber and the products expanded through a nozzle to obtain thrust. To drive the centrifugal pumps used to inject the main propellants into the combustion chamber, as second propellant, hydrogen peroxide and calcium permanganate were used. These, when combined, produced superheated steam which was used to drive a separate turbine to drive the auxiliaries, such as the injector pump, etc. During and after the war Americans studied the captured V-2 rockets and by the second World War the rockets established themselves for war purposes and now they have been used for war as well as other purposes such as inter-planetary travel, assisting in take off of heavily loaded jet crafts, starting ram jets, etc. All the post war efforts culminated into the magnificent moon landing by Niel Armstrong in 1969 ; and the success of the Russian Lunar vehicle "Lunokhod" has opened new horizons for the exploration of inter-steller systems and a great boost to the insatiable explorative instinct of the man.

9·2 CLASSIFICATION OF ROCKETS

The necessary energy and the momentum which must be imparted to a propellant as it is expelled from the engine to produce a thrust can be given in many ways. Chemical, nuclear, or solar energy can be used and the momentum can be imparted by expanding from a pressure or by electrostatic or electromagnetic force. Alternatively, the propellant can be heated with the help of electric power and expelled in a nozzle. Fig. 10 1 gives the classification of rockets on the basis of the methods used for accelerating the propellant.

Chemical rockets, so far the most widely used rockets, depend upon the burning of the fuel inside the combustion chamber and expanding it through a nozzle to obtain thrust. The important characterstic of chemical rockets is that the propulsive energy is obtained from within the fuel and the oxidant. The propellant may be solid or liquid. In the free radical propellant system, the propellant have certain free redicals in them so that when they recombine enormous amount of energy is released which can be used for getting propulsion.

The vast store of the atomic energy is utilized in case of nuclear propulsion. Fission or fusion both can be used to increase the energy of the propellant.

In case of electrodynamic propulsion devices a separate energy source is used and the propellant is accelarated by electrostatic or electromagnetic forces. The propellant is either discrete charged-particles which are accelerated by electrostatic forces or a stream of electrically conducting material so that it can be accelerated by electromagnetic forces.

The first type is called ion rocket, and the second type is called plasma jet. Photon propulsion is the ultimate in propulsion efficiency, the propellant being expelled in the form of photons travelling with the velocity of light. Photon propulsion does not seem to be technically feasible at present.

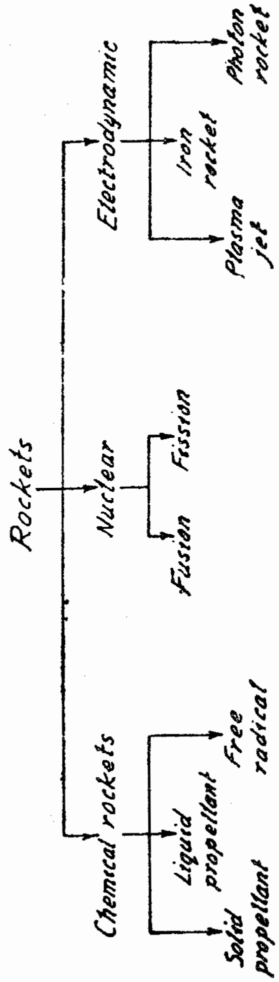


Fig. 9·1 Classification of rockets

9·3 PRINCIPLE OF ROCKET PROPULSION

The rocket engine in its simplest form consists of a combustion chamber and an expanding nozzle. The fuel and the oxidant, which after burning constitute the exhaust from the nozzle, are called propellants, as it is these only which produce the propulsive forces. Propellant gases are generated into the combustion chamber by burning propellants and are then expanded in a nozzle to a supersonic velocity, thereby, requiring a convergent-divergant nozzle. These high velocity gases going out of the nozzle produce the thrust and propel the rocket.

If $\dot{m}$ = mass rate of propellant

V_e = exhaust velocity from the nozzle

F = thrust

P = power required to give an exhaust velocity of V_e

L = accelaration of the rocket

m = mass of the rocket

then $$P = \dot{m}\frac{V_e^2}{2}$$

and $$F = \dot{m}\, V_e = m\alpha$$

Combining these, we get,

$$\frac{P}{m} = \frac{\alpha V_e}{2} \tag{9·1}$$

From the above equation it is clear that the velocity and the thrust which can be obtained from a given type of rocket propulsion system greatly depend upon the power which is available for imparting kinetic energy, and the mass of the rocket vehicle.

In the case of chemical rockets this energy is extracted from the propellant itself and depends greatly upon the characteristics of the propellant. Such rockets are energy-limited in that their performance is limited by the low exhaust velocity obtained by the use of present-day propellants. Same is the case for nuclear rockets.

In the case of ion, plasma, and photon propulsion, this energy is supplied from a separate power source which is usually an electric power obtained by nuclear or solar source. Such rockets are power limited as it is very difficult to produce such a huge amount of power within the small mass of the rocket.

In order to obtain high exhaust velocity, *i.e.* high specific impulse for a given thrust, very large amount of power is required.

In the case of nuclear rockets this is limited by the temperature by the rocket walls. In other methods of electrodynamic propulsion huge equipment is needed to convert such large amount of power into kinetic energy which increases the mass of the vehicle beyond economic and practical limits. For low exhaust velocity the mass flow required will be high. Since this mass of propellants has to be carried with in the rocket, a compromise is always made in terms of best economy obtained. For a given power, very high velocities may not be of any use as the thrust is so low that it will not take any payload. It must be noted that the useful output of all propulsive devices is thrust, the pull which motivates the craft. Fig. 9·2 shows the range of the accelarations and the exhaust velocities obtainable from different types of rocket propulsion systems. For going out of the earth's gravitational field

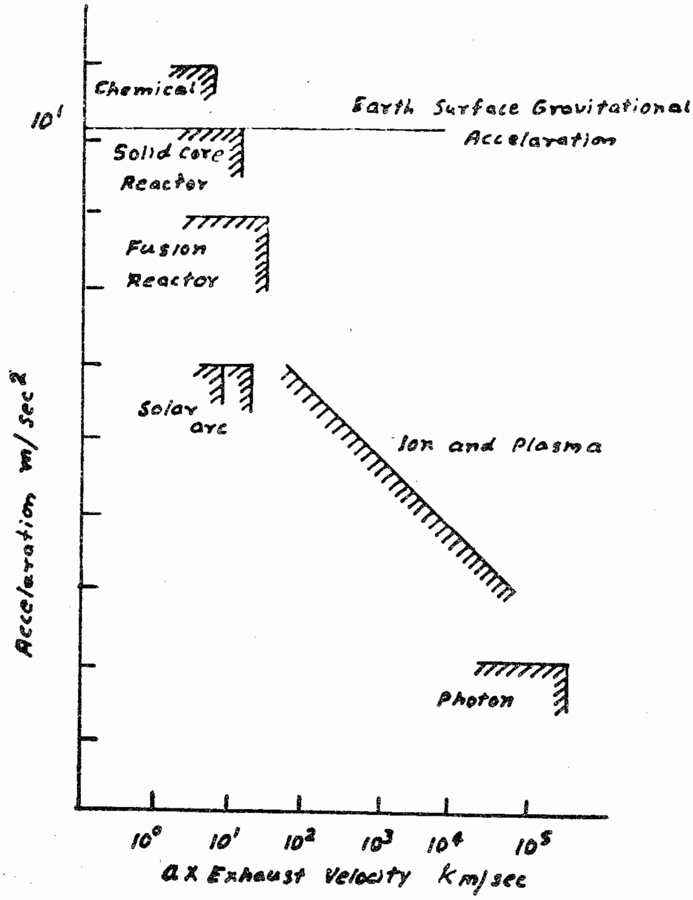
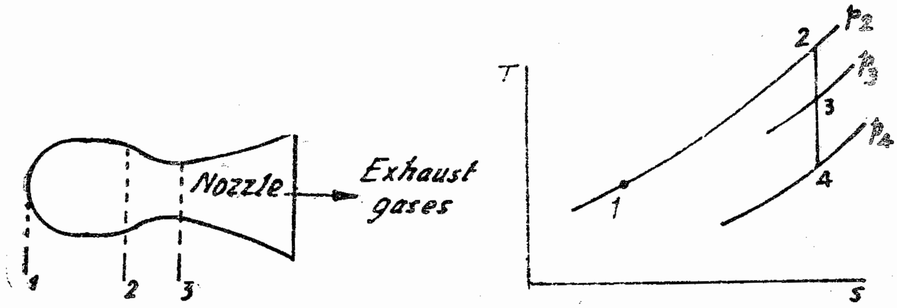


Fig. 9·2. Range of accelerations and exhaust velocities obtainable by different rocket propulsion systems.

high acceleration is needed so chemical and nuclear systems must be used. Once the gravity limit is crossed the low acceleration available by ion or plasma devices is sufficient for inter-planetary travels.

9·4. ANALYSIS OF AN IDEAL CHEMICAL ROCKET

Fig. 9·3 (*a*) shows the schematic diagram of an ideal chemical rocket and its corresponding *T*-*s* diagram.



9·3. (*a*) Schematic diagram of a chemical rocket and its corresponding *T*-*s* diagrams.

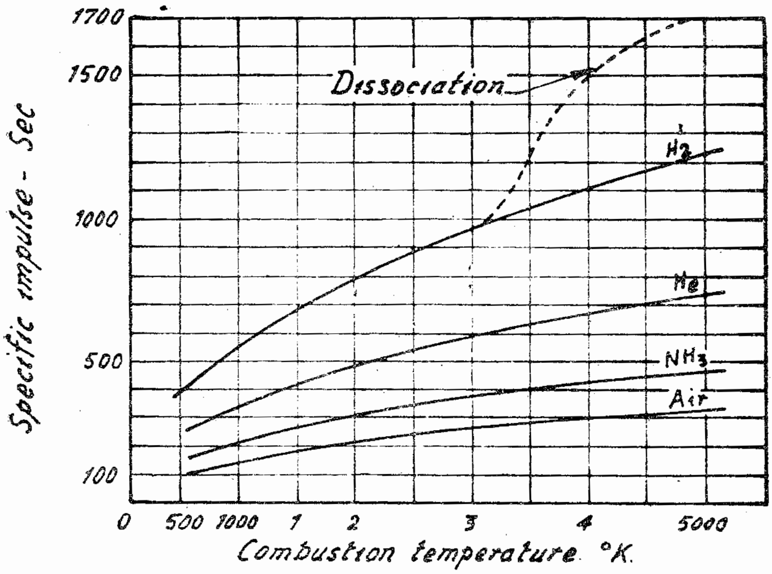


Fig. 9·3. (*b*) Specific impulse as a function of temperature for several different fluids.

The propellant is stored in the combustion chamber at pressure p_1 and we assume that the burning occurs at constant pressure. The products of combution enter the convergent-divergent nozzle at point 2 (point 3 corresponds to nozzle throat) and get expanded isentropically in the nozzle, leaving it with an exhaust velocity V_e. We also assume that the gases behave as perfect gas.

Since the expansion is isentropic, we get

$$V_e=\sqrt{2\,c_p T_2\left[1-\left(\frac{p_4}{p_2}\right)^{\frac{\gamma-1}{\gamma}}\right]} \tag{9·2}$$

Putting c_p in terms of universal gas constant R_o, molecular weight M, and ratio of specific heats γ, we get

$$V_e=\sqrt{\frac{2\gamma R_o}{(\gamma-1)M}T_2\left[1-\left(\frac{p_4}{p_2}\right)^{\frac{\gamma-1}{\gamma}}\right]} \tag{9·3}$$

If Q is the heat supplied in the form of chemical energy per unit mass of propellant, we get

$$Q=c_p(T_2-T_1)$$

Then, from equation (9·3), it follows that

$$V_e=\sqrt{\frac{2\gamma R_o}{(\gamma-1)M}\left(T_1+\frac{Q}{c_p}\right)\left[1-\left(\frac{p_4}{p_2}\right)^{\frac{\gamma-1}{\gamma}}\right]} \tag{9·4}$$

If A_t is the area of the throat of the nozzle and p_t the pressure at throat it can be proved that the mass flow rate is given by

$$m=\frac{A_t\,p_2}{\sqrt{\frac{R_e}{M}T_2}}\sqrt{\gamma\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}} \tag{9·5}$$

The thrust produced is given by

$$F=mV_e+(p_4-p_a)A_4 \tag{9·6}$$

where p_a is the atmospheric pressure. It should be noted that the term $(p_4-p_a)A_4$ has not been neglected because the exhaust velocity V_e is supersonic.

By putting equations (9·4) and (9·5) into equation (9·6) the thrust produced by a chemical rocket can be written as

$$F=\left[\frac{A_t\,p_2}{\sqrt{\frac{R_o}{M}T_2}}\sqrt{\gamma\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}}\right]$$

$$\times\left\{\frac{2\gamma R_o}{(\gamma-1)M}\left(T_1+\frac{Q}{c_p}\right)\left\{1-\left(\frac{p_4}{p_2}\right)^{\frac{\gamma-1}{\gamma}}\right\}\right\}+(p_4-p_a)A_4 \tag{9·7}$$

As is clear from the above equation, the thrust, depends upon the pressure in the combustion chamber, the properties of the propellant and the geometrical shape of the rocket.

To obtain high thrust the molecular weight of the propellants must be as low as possible (*see* Equ. 9·7). Most of the propellants, presently in use, produce CO, H_2O, CO_2, N_2 and H_2, and the average molecular weight of such species is about 25 and seldom less than 20 or so. It means that a limit on the performance of a chemical rocket is imposed by the fact that the molecular weight of the products of combustion cannot be reduced below certain limit. Alternative fuels such as hydrogen and fluorine (molecular weight 8·9) or hydrogen and oxygen (molecular weight 9) or even some light metals like lithium with a molecular weight of 7 can be used. Even with the use of such light materials this limit cannot be lowered indefinitely because the use of lower molecular weight reactants do not necessarily give lighter products.

Another factor which limits the thrust obtainable is the maximum allowable temperature as well as the maximum temperature which can be produced by chemical reactions. At very high temperatures, dissociation does not allow the whole of the heat-energy to be converted into the kinetic energy and the maximum obtainable temperatures are limited. The pressurisation of combustion chamber, to a certain extent, curbs the dissociation but above a pressure of about 20 kgf/cm² this curbing tendency almost vanishes. Thus the chemical rocket is energy limited in the sense that in the first instance a large amount of heat release from the propellant is difficult to obtain, and if obtained it is still more difficult to utilize it due to dissociation. The effect of variation in the ratio of specific heats is not very much due to the presence of dissociation. The value of γ varies between 1·2 to 1·3 for almost all rocket propulsion fuels. Table 9·1 and Fig. 9·3(*b*) gives theoretical heats which can be obtained in chemical rockets and the specific impulse obtainable with different fluids.

TABLE 9·1. THEORETICAL HEATS WHICH CAN BE OBTAINED IN CHEMICAL ROCKETS

	Highest temperature	*Lowest molecular weight*	*Lowest ratio of specific heats*
System	Cyanogen and ozone	Hydrogen and flugrine	Hydrogen peroxide and gasoline
Specific impulse, sec.	270	373	248
	Flame temperature 5525°C	Molecular weight 8·9	Ratio of specific heats 1·20

Apart from the effect of the characteristics of the propellant the area-ratio of the nozzle and its shape also play an important part in dictating the performance of the chemical rocket as it affects the velocities obtainable as well as the drag on the rocket. A detailed analysis of this is beyond the scope of this book.

9·5 OPTIMUM EXPANSION RATIO FOR ROCKET

The thrust developed by a rocket is given by

$$F=mV_e+(p_e-p_a)A_e \tag{9·6}$$

where m=mass flow rate

If the divergent portion of the convergent-divergent nozzle of the rocket is increased the area A_e, velocity V_e, and static pressure p_e at nozzle exit will vary. By differentiating equation (9·6) with respect to p_a and equating to zero, the condition for maximum thrust is obtained as

$$\frac{dF}{dp_a}=m\frac{dV_e}{dp_e}+A_e+(p_e-p_a)\frac{dA_e}{dp_e}=0 \tag{9 7}$$

Euler's equation can be written as

$$\frac{dp_e}{dv_e}=-\rho_e V_e \tag{9·8}$$

By continuity equation $m=\rho_e V_e A_e$, we have

$$dp_e/dV_e=-m/A_e \tag{9·9}$$

From equations (9·7) and (9·9), we have

$$-m\times\frac{A_e}{m}+A_e+(p_e-p_a)\frac{dA_e}{dp_e}=0$$

or
$$(p_e-p_a)\frac{dA_e}{db_e}=0 \tag{9·10}$$

Since $\frac{dA_e}{dp_e}\neq 0$, the condition for maximum thrust is given by

$$p_e=p_a \tag{9·11}$$

i.e. the optimum expansion condition is that when the static pressure at the exit of the rocket nozzle is the same as the ambient pressure. This conculsion can also be arrived at by simple physical reasoning. Suppose that the length of the diverging nozzle passage is increased over that corresponding to the optimum expansion ratio $(p_a=p_e)$; this will result in further expansion in the nozzle and pressure at exit of the nozzle will be less than ambient pressure p_a and the second term in the equation (9·6) will become negative and thrust will decrease. On the other hand, any length of the nozzle which is less than that corresponding to $p_e=p_a$ will result in lesser expansion and hence in reduced exhaust velocity and thrust will again be reduced. So the condition $p_e=p_a$ gives the best expansion condition for a rocket nozzle with a given threat diameter.

9 6 THE CHEMICAL ROCKET

Chemical rockets are classified as follows on the basis of the type of propellant used :

1. Solid propellant rockets.
2. Liquid propellant rockets.
3. Free radical rockets.

The nature of the propellant to be used greatly affects the overall shape and size of the rocket and major differences in details occur. One typical example is that in the case of solid propellant rockets no fuel pump and injector is required as in is required liquid-propellant rockets.

9·6·1. (*a*) Solid propellant rocket

Fig. 9·4 shows a schematic diagram of a solid propellant rocket. It consists of a combustion chamber, an expansion nozzle, and an igniter. The peculiar characteristic of this type of rocket is that whole of the fuel is contained in the combustion system which has its oxidant within the fuel itself.

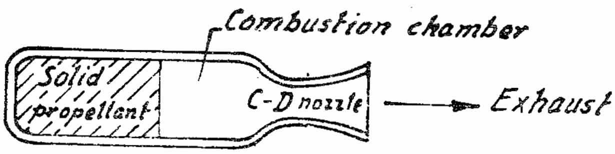


Fig. 9·4 Schematic diagram of solid a propellant rocket

An igniter, which is a detonator or a fuse containing highly reactive primary explosive material like lead oxide, is used to start burning of the propellant. The burning occurs with rapid decomposition and heat release. This heat is conducted back to the propellant and thus the process is self-sustaining. Such a process is called *burning by deflagration*. The burning rate depends upon the pressure of the chamber and increases with an increase in pressure. Burning may be controlled or uncontrolled. To obtain maximum thrust whole of the combustion space is filled with the propellant and it is allowed to burn like a cigarette. However, this results in very small operational time and difficulty in the design of nozzle, because in solid propellant rockets only cooling available is due to external environment. Therefore, the burning rate is usually controlled by pressure as well as by giving different shapes and sizes to the propellant. This propellant shape is called *grain.* The various types of grains are shown in Fig. 9·5. The pressures developed in a solid propellant rocket are about 80 to 200 kgf/cm^2. The thrust and duration of operation depends upon the shape, size and the material of the propellant. Usually the fuel is used in cast or extruded form.

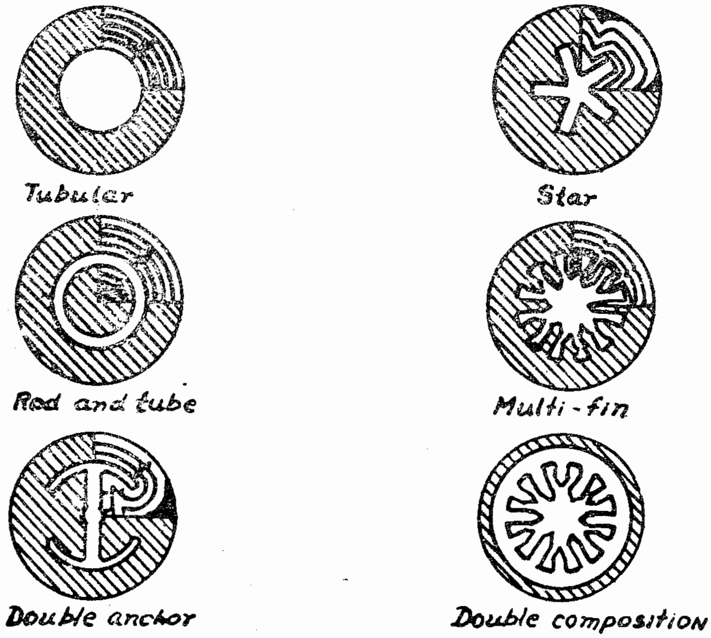


Fig. 9·5. Internal-burning charge designs.

Solid propellant rockets are used in assisted-takeoff missiles, projectiles and a number of other applications. For small rocket missiles almost invariably solid propellent rocket is used as it is quite adequate for the range in question and is much more easy to handle than the liquid propellant rocket of same size.

Solid propellant rockets have limited duration of burning and it is difficult to control the burning of the propellant as well as cooling of the combustion chamber.

9·6·1. (*b*) Solid propellants

In general, the solid propellants can be classfied into two main categories :

1. Composite propellants.
2. Double-base propellants.

Composite propellants consist of an oxidiser in a ground crystalline shape mixed in the fuel of plastic nature acting as a binder to hold the mixture and to keep a uniform composition. Typical oxidisers are ground crystals of potassium, lithium or ammonium perchlorate or nitrate. The binder may be rubber, asphalt or elastomers. Composite propellants are difficult to cast and have a high oxidiser content which results in high density of the exhaust. The mechanical properties of the composite propellants depend upon the nature of the binder used, for example, use of polystyrene as a binder results in hard structure of the grain. The burning rate of the propellant can be controlled by controlling the ratio of the mass of the fuel to the oxidiser, by addition of catalyst or by the shape and size of the grain. Double-base propellants or the homogeneous propellants are characterised by nitrocellulose containing $[C_6H_{72}(ONo_3)_3]$ compounds which act as a plastic and gives the fuel a colloidal characteristic. These are true mono-propellants in that each molecule of the propellant has, in it, the necessary amount of fuel and oxidiser. Double-base propellants, combinations of nitroglycerin $[C_3H_5(No_3)_3]$ and nitrocellulose with small quantities of additives are very commonly used. The additives are provided to impart stability of combustion and freedom from ageing which are necessary for achieving high performance.

Such propellants are plastic in nature, have a very high viscosity and their appearance is smooth, almost waxy. These are made in required shape by casting or extrusion.

Now-a-days this classification of solid propellant has become arbitrary in nature because of the development of such nitrocelluloses nitroglycerin based propellants which also have oxidesers like potassium perchlorate.

In table 9·2 are given the typical solid propellants, their chemical formula and properties.

TABLE 9·2 SOLID PROPELLANTS

Solid propellant system	*Flame temperature K.*	*Molecular Weight*	*Specific impulse, seconds*
$KClO_4$ and C_2H_4O	1800-3030	25-35	165-210
NH_4ClO_4 and C_2H_4O	1800-2150	22-25	175-240
Nitrocellulose and nitroglycerine	2360-3140	22-28	105-195
Asphalt and perchlorate	2360-2640	——	180-195

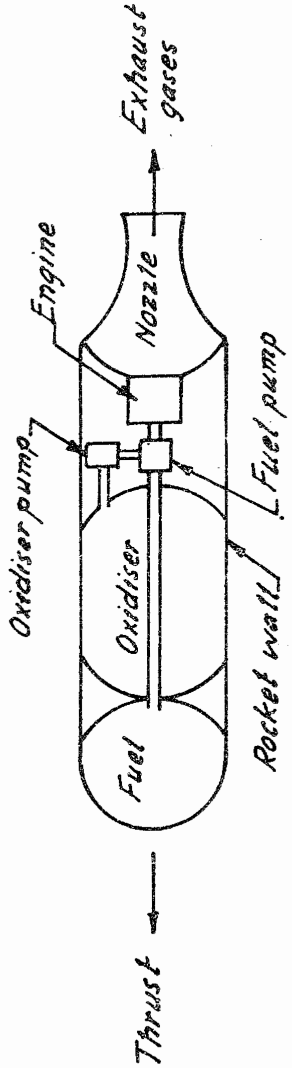


Fig. 9·6. Schematic diagram of a liquid propellant rocket.

9·6·2. (*a*) Liquid propellant rocket

Fig. 9·6 shows a schematic diagram of a liquid propellant rocket consisting of fuel and oxidiser tanks, pump and injecting device, combustion chamber and an expanding nozzle. The propellant is stored in the rocket and with the help of pumps injected into the main combustion chamber, the fuel also cooling the combustion chamber in the process. Fuel is burned into the combustion chamber and the products expanded in the nozzle to produce thrust.

Some device must be used to increase the pressure of the liquid propellant to inject them into the combustion chamber. This device may be any one of the following :

(*i*) Pressure-feed system.

(*ii*) Pump feed system.

(*iii*) Bleed system.

In pressure-feed system a pressurised gas cylinder containing an inert gas, usually nitrogen or helium, is used to force the fuel and the oxidiser into the combustion chamber. This is shown in Fig. 9·7. Such a device is simple, cheap, and reliable.

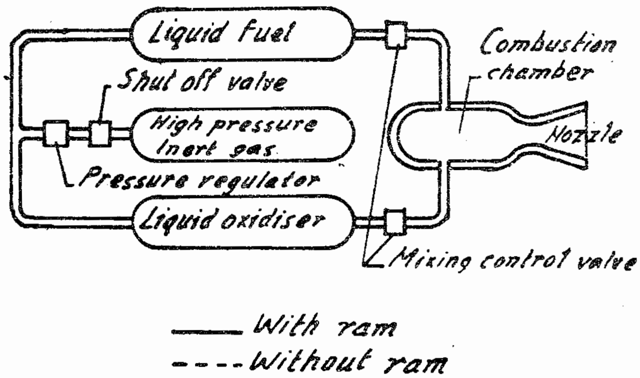


Fig. 9·7. Inert gas pressure feed liquid propellant rocket.

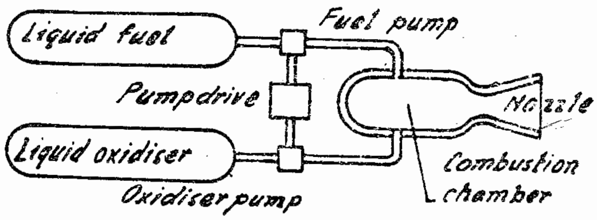


Fig. 9·8. Pump feed liquid propellant rocket.

However, it suffers from a big disadvantage that if long operational period is required the weight of the inert gas tank will be high. Therefore, usually such a system is used for low operational period requirements. The pressure of the inert gas is quite high about 120 to 250 kgf/cm^2, *i.e.* it will require bulky containers.

Another alternative to the use of an inert gas tank is a gas generator, in which fuel and oxidiser can be fed from main tanks and a constant pressure is produced which will work just as the inert pressurised tank. The product temperature in the gas generator will be less than that produced in main combustion chamber.

Still, another alternative is the use of hypergolic fuels. Hypergolic fuels are those which are self-igniting, *i.e.* as soon as the fuel comes in contact with the oxidiser it will ignite. Use of a hypergolic fuel in a tank can also be used to generate pressure necessary for the injection in main chamber.

In the pump-feed system a separate set of pumps is used to drive the fuel into the combustion chamber. The pump is driven by a turbine which is run either on a mono-propellant fuel or the same fuel as in main chamber. A schematic diagram of such a system is shown in Fig. 9·8. The source of turbine power, if it is a separate chamber, will result in increased weight and reduced efficiency. However, a part of it can be compensated by the thrust agumentation obtained by the exhaust of this turbine which is expanded in another nozzle. This extra jet can also be used for yaw- and roll-control of missiles. Such a system does not limit the duration of the flight and is used for high power and long duration flights.

For the ignition of the fuel, it must be injected in an atomised form, mixed properly and vapourised for efficient burning. The mechanism of burning will, however, depend upon the type of the propellant used.

If the propellant is monopropellant, *i.e.* one which does not need an oxidiser to release its chemical energy, the process of burning is exactly like that in an internal combustion engine except that no air is introduced with the fuel. The fuel is injected, vaporised, ignited and completely decomposed before entering the nozzle.

If the propellant is bipropellant, *i.e.* which has a fuel and an oxidiser, both of which are combined in the combustion chamber, first of all a very through mixing of the two must take place. The ratio of the fuel and the oxidiser decides the temperature of the reaction products.

If the fuel is hypergolic it does not require much of vaporising or mixing and it will instantaneously start burning as soon as it comes into contact with the oxidiser.

Except for hypergolic fuels which do not require any source of ignition, all other propellants must be ignited with the help of an igniter, which may be a spark plug, an ignited glow plug or any other suitable type of igniter. Once the ignition starts all propellants have self-sustaining characteristic.

The pressures in the combustion chamber of a liquid propellant rocket are about 20 kgf/cm² to 45 kgf/cm² depending upon the nature of the propellant, feed system, and design of the injector and cooling system used. Cooling is necessary to reduce the temperature of the rocket wall to about 3000°C. This is done either by regenerative cooling or by film cooling. In case of regenerative cooling the propellant is circulated around the nozzle and the combustion chamber before it is injected so that it takes away heat from rocket walls and gets preheated. In film cooling the fuel is injected through a series of holes or through porous sections in such a way that a thin film of the propellant is formed on the inner surface of the combustion chamber such that the wall is cooled and is not subjected to direct heat from the products of combustion.

9·6·2. (*b*) **Liquid Propellants.** As already discussed there are two basic types of liquid propellants, namely :

(*i*) Monopropellant, and

(*ii*) Bipropellant.

Monopropellants are those which do not require an oxidiser and can produce thermal energy by decomposition either by a catalyst or by ignition. Typical examples are nitro-methane, trinitrotoluene, nitroglycerine and hydrazine. Monopropellants are usually hazardous and are not preferred. They are used for limited purposes such as for a source of power to the turbine in a pump-feed system, etc.

Bipropellant, as the name implies, is a combination of a fuel and an oxidiser, which when combines produces thermal energy. Typical examples of such combinations are hydrogen peroxide and alcohol, liquid oxygen and liquid hydrogen, etc. Some other such combinations are given in table 9·3.

The above combinations may or may not be hypergolic. Now-a-days, a combination of metallic fuels like lithium or beryllium with either oxygen or flourine and liquid hydrogen, what is called as a tripropellant, is also being used to improve the performance of chemical rockets.

Table 9·3 shows the common liquid propellants used along with the approximate obtainable thrust them from. Fluorine is the most power fue oxidiser, next is the chlorine trifluoride which is so active that even the glass-fibre cotton, a fireproof material burns in it. Hydrogen peroxide and ozone are also good oxidisers. Liquid hydrogen has maximum heat value but is very explosive. Kerosene and gasoline were used as fuels for *V*-2 rocket by Germans. Kerosene is also used with some powdered metals as rocket fuel. Hydrazine requires comparatively smaller quantities of oxidiser for burning.

TABLE 9·3 COMMON BIPROPELLANTS USED IN LIQUID PROPELLANT ROCKETS

Oxidiser	*Fuel*	*Temperature, °K*	*Molecular weight*	*Specific impulse, seconds*
Hydrogen peroxide (H_2O_2)	Gasoline	2930	21	248
Red fuming nitric acid (HNO_3)	Gasoline	3120	25	240
Liquid oxygen	Alcohol	3340	22	259
Liquid oxygen	Liquid ammonia	—	—	—
Liquid oxygen	Hydrazine (N_2H_4)	3240	18	280
Liquid oxygen	Liquid hydrogen	2760	9	364
Liquid fluorine	Liquid hydrogen	3090	8·9	373
Liquid fluorine	Ammonia	4260	19	306
Nitric acid	Aniline (COH_5NH_2)	—	—	—
Nitrogen tetroxide	Hydrazine	—	—	—
Ozone	Cyanogen	523	—	270
Hydrogen peroxide	Ethyl diamene ($C_2H_4N_2H_4$)	—	—	—

9·6·2. (*c*) **Requirements of a liquid propellant.** Any liquid or a combination of liquids, if capable of producing mainly gaseous products by an exothermic chemical reaction, can be regarded as a liquid propellant. However, to be ideally suitable for liquid propellant rocket it must have certain specific characteristics which are given below.

1. The calorific value of the propellant must be high.

2. In order to minimise the space requirements its density should be high.

3. It should be easy to store and handle.

4. It must have low corrosion characteristics.

5. Its toxicity must be low.

6. Its performance must not be greatly affected by ambient temp erature.

7. Its ignition must be smooth and reliable.

8. It must be chemically stable and must not deteriorate over reasonable storage periods.

9. The change in viscosity with temperature should be low so that at low temperatures the pumping work is not high or at high temperatures it does not simply come out of the tanks.

As far as known none of the presently available propellants meets all these requirements and still lot of development work is needed in this field even after so much successful launchings of rockets and satellites.

9·7. ADVANTAGES OF LIQUID PROPELLANT ROCKETS OVER SOLID PROPELLANT ROCKETS

The following are some of the advantages of liquid-propellant rockets over the solid-propellant rockets :—

1. Duration of operation can be increased.

2. Use of cooling allows the metal walls to keep their strength.

3. Use of lower pressures in conjunction with cooling makes it possible to use less expensive or non-critical materials.

4. Size of the combustion chamber is smaller as whole of the fuel need not be stored in it.

5. It is very easy to control such a rocket by simply controlling the propellant flow.

6. Heat losses to the wall are reduced to preheating of fuel during cooling process.

Liquid propellant rockets are extensively used with aircraft, for assisted take-off, ballistic missiles, thrust agumentation, and in space flight.

9·8 FREE RADICAL PROPULSION

If a stable chemical material is supplied with sufficient energy to break the energy bonds some unstable free radicals will be produced. If these free radicals are used in a rocket so that they recombine and release a vast amount of energy, the limitations of a chemical rocket can be partially overcome.

Molecular hydrogen with certain concentration of unstable free radicals in it seems to be the best propellant due to its lower molecular weight. However, it is long time before such a system can be used in practice due to difficulties in generating free radicals, preserving them for sufficient time and controlling the recombination rates.

9·9 NUCLEAR PROPULSION

The maximum exhaust velocity which can be obtained in a chemical rocket is limited to about 4·5 km/sec because of the lower energy release from the propellant and other limitations such as attainment of a high temperature in the combustion chamber. It is well known that in a nuclear reaction vast amounts of energy, 10^6 to 10^8 times that of a chemical reaction, can be obtained. This high rate of energy release will result in very low mass of the propelling device and a wider lattitude in selection of propellant material. Detailed study of these reactions and related equipment is beyond the scope of this book and only a simple discription is given below.

There are two types of nuclear reactions which can be used to produce energy, namely

1. Fission.
2. Fusion.

In fission a heavy molecules is broken into fragments by the bombardment of neutron on its nucleus. When the neutron is absorbed into the nucleus, the heavy molecule splits up into number of fragments and energy is released in the process. This energy is in the form of heat energy. In addition to this, the reaction also produces about 2·5 neutrons which makes the reaction self-sustaining.

Fig. 9·9 shows a schematic diagram of a nuclear propulsion unit. It consists of a nuclear core having ${}_{92}U^{235}$ as the nuclear

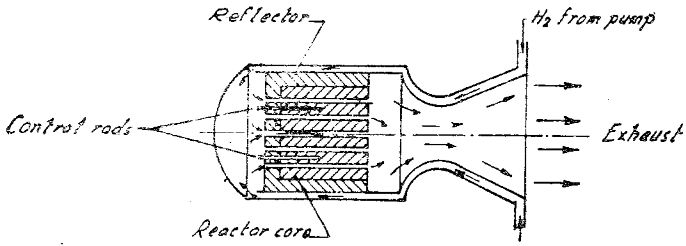


Fig. 9·9. Schematic diagram of solid core nuclear-heated hydrogen rocket.

fuel. The reaction is controlled by the control roads. A reflector is provided to avoid leakage of neutrons. Hydrogen is used as a propellant which takes up heat from the reactor and expands in the nozzle.

In the second type of reaction, *i.e.* thermo-nuclear reaction or fusion, charged nuclei of light elements are fused into a heavy nucleus and in the process vast amount of energy is released. To start fusion reaction, coulomb repulsion forces between the nuclei must be overcome which presents a formidable problem as it requires temperatures of the order of 10^{60} K. Nevertheless, nuclear fusion might be achieved in near future and the future propulsion devices are very likely to be nuclear.

Nuclear propulsion has been successfully used in submarines, ships, and some experimental rockets.

ELECTRODYNAMIC PROPULSION

9·10. ION PROPULSION

Fig. 9·10 shows the schematic diagram of an ion rocket. In this type of rocket, the propellant consists of charged particles which

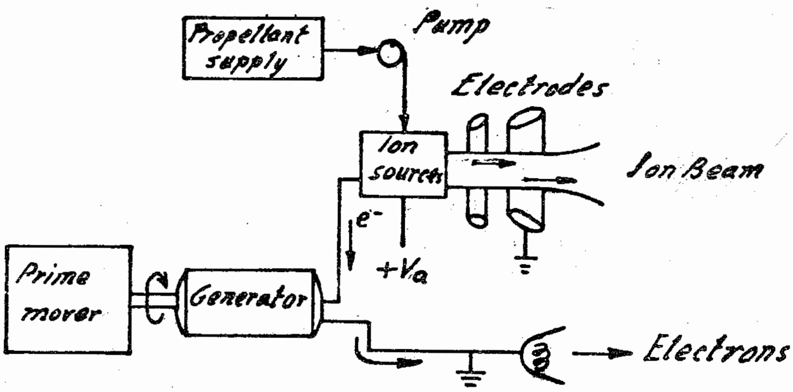


Fig. 9·10. Schematic diagram of ion rocket.

are accelerated by electrostatic forces, just as in a cathode-ray tube. The neutral propellant is supplied to an ion chamber where it is ionised. The ions are extracted from this chamber by the electrodes at very high potential. These electrodes also accelerate them and give a definite beam shape. The electrons produced in the ion source, which is always kept at high positive voltage are taken out with the help of a generator and dissipated in a hot filament. A nuclear power plant may be used to provide high voltage across the electrodes and the propellant is ionised from the heat developed by this power plant.

Ion propulsion can only be carried out in vacuum. However, the outer space environment which is near vacuum can also be used. The specific impulse, *i.e.* the exhaust velocities are very high but the thrust is so low that such a system is suitable only for outer space travel where gravitational forces are not to be overcome by the rocket :

9·11. PLASMA PROPULSION

Plasma propulsion differs from ion propulsion in that the electrical forces are not directly used to accelerate the propellant ;

Instead the propellant is first heated to a very high temperature and then expanded through a nozzle. It is a hybrid thermal-electrical propulsion system in that it relies on expansion of a hot plasma for bulk of its thrust but its energy is supplied electrically rather than by chemical combination or by nuclear fusion or fission.

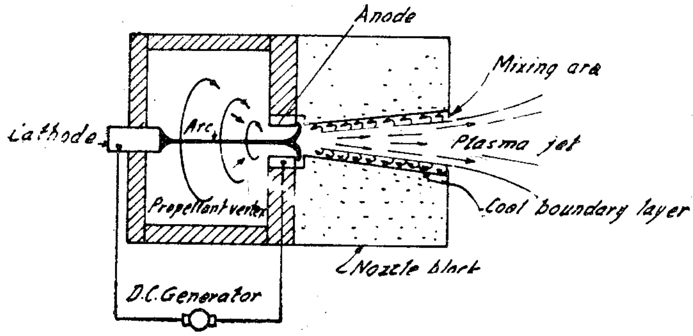


Fig. 9·11. Schematic diagram of a plasma jet.

A high voltage source, which again may be a nuclear reactor, is used to produce an electric arc which heats up the propellant to a high temperature. The propellant is either hydrogen or helium as both have low molecular weight and high maximum stagnation temperature before they dissociate. Helium is preferable as it has a higher stable temperature. Once the propellant is heated to a high temperature it is expanded in a conventional nozzle to get propulsive force.

For starting, the electrodes are brought closer than the normal position to produce a spark. The chamber core becomes a good conductor in the presence of positive ions and after the arc is established, the electrodes are taken apart to their normal position. Energy leaves the arc in electromagnetic radiation and is consumed in increasing the potential energy of ionisation and dissociation. The temperature of the material in the region of arc is increased. The propellant is fed in the form of vortex to concentrate the arc into a narrow region. This allows a very high temperature to be obtained in the central core—much more than the wall materials can withstand since the propellant vortex also cools the wall. A temperature as high as 50,000 K has been used. This high temperature plasma escapes through the nozzle and a propulsive force is created.

Since ionisation of hydrogen takes place only at about 10,000 K a large amount of energy supplied to the arc is not available for producing thrust. Unless this energy is used back by proper recombination, the efficiency will be very low.

Production of an efficient plasma arc, the difficulty of cooling the rocket wall, and the need of very high power for arcing are the major limitations of plasma-jet propulsion system. Hot plasma erodes the nozzle, and the life of electrodes is limited to a few minutes. Not much information is available about it but it certainly has a great future potential.

9·12. PHOTON PROPULSION

Photons are electromagnetic quanta of energy which do not carry an intrinsic mass but carry a momentum equal to hf/c where h is Plank's constant, f the frequency of radiation, and c is the velocity of light. When photons are produced some mass, according to Einstein's equation $m=E/c^2$, disappears, *i.e.* in other words if photons are emitted some mass is transferred. Fig. 9·12 shows a schematic diagram of a photon propulsion unit in which a hot nuclear power-source generates a flux of photons which is shaped by a photon absorber and then escapes through the nozzle producing a thrust equal to

$$F=\frac{d\left(\frac{hf}{c}\right)}{dt}=\frac{1000P_e}{c}$$

where P_e is the power in jet in kilowatts.

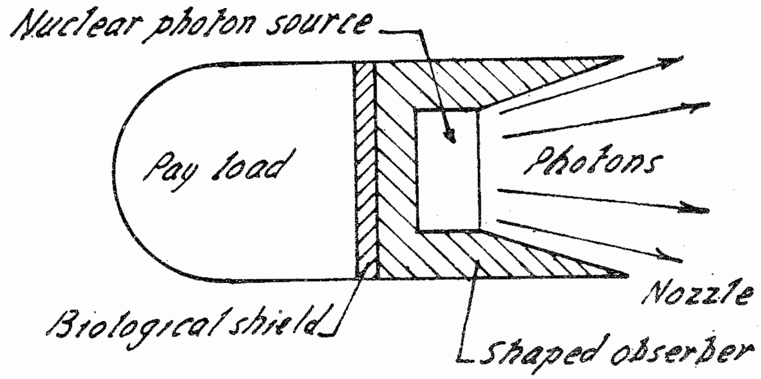


Fig. 9·12. Schematic diagram of a photon propulsion unit.

EXERCISES 9

Section A

9·1. What is the basic difference between rocket propulsion and jet propulsion? Can rockets work in vacuum ?

9·2. How rockets can be classified ? What is the stage of development of each type ?

9·3. Derive a general expression for the thrust produced by a chemical rocket and hence discuss the importance of the molecular weight of the propellants.

9·4. What are the factors which limit the thrust obtainable from chemical rockets ?

9·5. What is the condition for the maximum thrust in a chemical rocket ? Derive an expression for the same.

9·6. Draw a schematic diagram of a solid propellant rocket and explain its working. What are the applications of this type of rocket ?

9·7. Briefly describe the two types of solid propellant rockets.

9·8. Draw a schematic diagram of a liquid propellant rocket. What are the different sytems of injecting liquid propellants into the combustion chamber ?

9·9. Discuss the mechanism of burning of various types of liquid propellants.

9·10. What is meant by monopropellant and bipropellant fuels for rockets ? Give important examples of each type. What is the difference in the application of a monopropellant and a bipropellant rocket engine ?

9·11. What are the desirable requirements of a liquid propellant for rockets ?

9·12. Compare the advantages and disadvantages of solid and liquid propellants ?

9·13. Explain the free radical propulsion?

9·14. Describe how nuclear energy can be used for propulsion of rockets. Has it been used so far ?

9·15. Discuss, with suitable sketches, the three systems of electrodynamic rocket propulsion.

9·16. What factors are important in the comparison of propulsive devices ? State the optimum operational range, specific fuel consumption and relative weights for various propulsion devices.

9·17. What are the advantages and disadvantages of rocket engines ?

9·18. What are the various types of oxidisers in common use in rockets ?

9·19. Describe the use of hydrogen peroxide as a monopropellant. What is the function of hydrogen peroxide in a bipropellant rocket engine?

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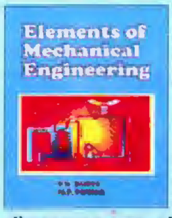

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