ENVISAT-1 MISSION CFI SOFTWARE

MISSION CONVENTIONS DOCUMENT

PO-IS-ESA-GS-0561

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1 SCOPE

This document describes in detail the time references and formats, coordinate systems, parameters, models, and units that will be used by the Envisat-1 mission CFI software. The description sometimes goes beyond the CFI-needed information, when deemed necessary for the sake of a correct explanation.

In particular, the present issue covers the following CFI libraries:

- PPF_LIB
- PPF_ORBIT
- PPF_POINTING

Note: This document supersede the following documents:

- XD 1 Envisat-1 Reference Definitions Document for Mission Related Software,
  PO-TN-ESA-GS-00361
- XD 2 Mission Conventions Document,
  GMVSA 2070/96

2 ACRONYMS

ANX	Ascending Node Crossing
AOCS	Attitude and Orbit Control System
CAM	Coarse Acquisition Mode
CFI	Customer Furnished Item
DRS	Data Relay Satellite
ERS	European Remote Sensing Satellite
ESA	European Space Agency
ESO	European Southern Observatory
ESOC	European Space Operations Centre
ESTEC	European Space Technology and Research Centre
ET	Ephemeris Time
FAM	Fine Acquisition Mode
FK5	Fifth Fundamental Catalogue
FAM	Fine Acquisition Mode
FCM	Fine Control Mode
FPM	Fine Pointing Mode
FOS	Envisat-1 Flight Operations Segment
IAG	International Association of Geodesy
IAU	International Astronomical Union
IERS	International Earth Rotation Service
IRM	IERS Reference Meridian
IRP	IERS Reference Pole
ITRF	IERS Terrestrial Reference Frame
JD	Julian Day
J2000.0	The TDB at 1/1/2000 at 12:00:00
LOS	Line of Sight
LNP	Local Normal Pointing
MLST	Mean Local Solar Time
MJD2000	Modified Julian Day 2000
N/A	Not Applicable
NEOS	National Earth Orientation Service
OBT	On Board Time
OCM	Orbit Control Mode
PDS	Payload Data Segment

REMASE	Re-engineering of Mission Analysis Software for Envisat-1
RRM	Rate Reduction Mode
RMS	Root Mean Square
SFCM	Stellar Fine Control Mode
SR	Satellite Reference
SRR	Satellite Relative Reference
SRAR	Satellite Relative Actual Reference
SBT	Satellite Binary Time
S/C	Spacecraft
SI	International System of Units
SSP	Sub Satellite Point
SYSM	Stellar Yaw Steering Mode
TAI	International Atomic Time
TBC	To Be Confirmed
TBD	To Be Defined
TDB	Barycentric Dynamic Time
TDT	Terrestrial Dynamic Time
TLST	True Local Solar Time
URD	User Requirements Document
UT1	Universal Time UT1
UTC	Coordinated Universal Time
YSM	Yaw Steering Mode

3 APPLICABLE AND REFERENCE DOCUMENTS

3.1 Applicable Documents

AD 1 “Re-engineering of Mission Analysis Software for Envisat-1: Statement of Work”. PO-SW-ESA-GS-0344. ESA/ESTEC/NW. Issue 3.0. 19/12/1995.

AD 2 “ESA Software Engineering Standards”. ESA PSS-05-0. ESA. Issue 2. February 1991

AD 3 “OAD Standards: Time and Coordinate Systems for ESOC Flight Dynamics Operations”. Orbit Attitude Division, ESOC. Issue 1. May 1994.

AD 4 “Envisat MRD”. PO-RS-ESA-SY-125. Issue 1.1. 17/1/96

AD 5 “Envisat SRD”. PF-RS-ESA-EN-0004 (Annex 3)

AD 6 “Envisat Service Module Flight Operations Manual. Volume 1: Nominal Activities”. PPF.MA.20000.1871.MA. Matra Marconi Space. Issue 1. 9/11/95

AD 7 “Envisat CFS Specifications”. PPF.SPE.1227000.1051.MA. Matra Espace. Issue 3 Rev 0. December 1995.

AD 8 “ICD between the DRS and the Envisat-1 System”. CD/1945/mad. D/TEL/R. K. Falbe-Hansen. Issue 5. April 1996

AD 9 “Document Change Notice: ITT Appendix 1 - Annex D. PO-RS-ESA-GS-0239 Version 3.0. 18/3/96”. PO-DN-ESA-GS-0418. Issue 1, Rev 1. 18/3/96

3.2 Reference Documents

RD 1 Envisat-1 Mission CFI Software - Description and Interface Definition Document. PO-ID-ESA-SY-00412.

RD 2 Envisat-1 Mission CFI Software - PPF_LIB Software User Manual. PO-IS-GMV-GS-0557

RD 3 Envisat-1 Mission CFI Software - PPF_ORBIT Software User Manual. PO-IS-GMV-GS-0558

RD 4 Envisat-1 Mission CFI Software - PPF_POINTING Software User Manual. PO-IS-GMV-GS-0559

RD 5 “Time and Orbit State Vector handling within the Processing Chain”. PO-TN-ESA-GS-00491. C. Garrido. P. Viau. Draft 0.9B. 8/11/96

RD 6 “ERS-1 Algorithms for orbit prediction and for the determination of related static and dynamic altitude and groundtrace quantities”. ER-RP-ESA-SY-00001. H.Klinkrad (ESA/ESTEC/ORM). Issue 1. 29/5/84.

RD 7 “Semi-Analytical Theory for Precise Single Orbit Predictions of ERS-1”.
ER-RP-ESA-SY-004. H.Klinkrad (ESA/ESTEC/WMM). Issue 1, Rev 0. 28 June 1987

RD 8 “Semianalytic Theory for a Close-Earth Artificial Satellite”
Journals of Guidance and Control Vol 3, No 4. J.J.F.Liu and R.L.Alford. July-August 1980.

RD 9 “Two self-consistent Fortran subroutines for the computation of the Earth’s motion”
Astronomy & Astrophysics Supplement Series 41, 1-8. P. Stumpff. July 1980

RD 10 “Low-precision formulae for planetary positions” in the Astrophysical Journal Supplement Series: 41, p 391-411. T.C.Van Flandern, K.F.Pulkkinen. November 1979

RD 11 “Methode of Bowring”
NGT Geodesia 93-7, p 333-335. 1993

RD 12 “IERS Bulletin A”.
NEOS Earth Orientation Bulletin. 18 April 1996.

RD 13 “Explanatory Supplement to IERS Bulletins A and B”.
International Earth Rotation Service (IERS). March 1995.

RD 14 “The Astronomical Almanac for the year 1995”.

RD 15 “Explanatory Supplement to the Astronomical Almanac for the year 1992”

RD 16 “World Geodetic System 1984”.
DMA-TR-8350.2. The Defense Mapping Agency. Second Edition. 1 September 1991.

RD 17 “U.S. Standard Atmosphere 1976”.
National Oceanic and Atmosphere Administration

RD 18 “U.S Standard Atmosphere 1962”.
National Aeronautics and Space Administration

RD 19 “Orbital Motion”.
A.E. Roy. Adam Hilger Ltd, Bristol. 1982

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P.R. Escobal. John Wiley & Sons, Inc. New York 1965

RD 22 “Principle of optics: electromagnetic theory of propagation”.
Born M, Wolf E. 1 Jan 1983.

RD 23 “Satellite Geodesy: Foundations, Methods, and Applications”.
Gunter Seeber. Walter de Gruyter. Berlin 1993.

RD 24 “Physical Geodesy”.
Weikko A. Heiskanen, Helmut Moritz. Graz 1987

RD 25 “Handbook of geophysics and the space environment”.
Adolph S. Jursa. Air Force Geophysics Laboratory. 1985

4 TIME REFERENCES AND FORMATS

4.1 Time references

The following table identifies the time references that are used in the context of Envisat-1.

Time reference	Envisat usage
Universal Time 1 (UT1)	Used as time reference for all orbit state vectors (and necessary for pointing at celestial targets).
Universal Time Coordinated (UTC)	Used as time reference for all products datation.
International Atomic Time (TAI)	Only found in Doris products (not used in PDS processing).

Table 1: Envisat-1 time references

The relationships between UT1, UTC and TAI are illustrated in the following drawing:

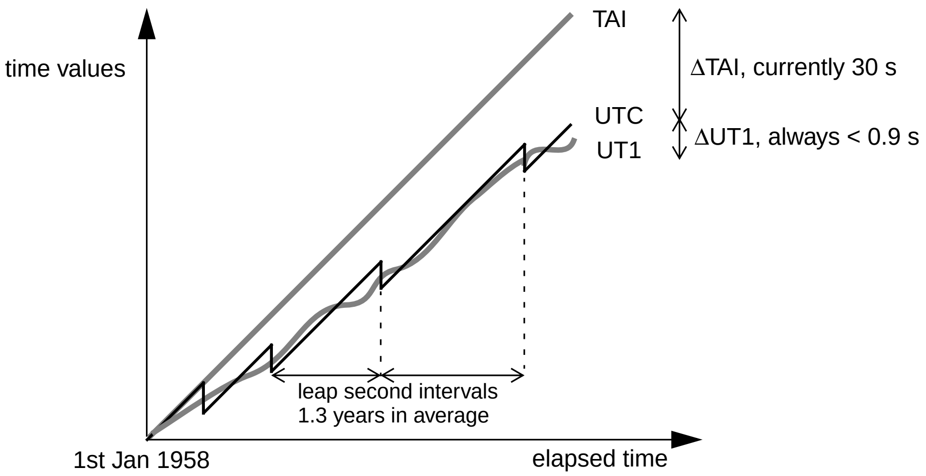

Figure 1: Relationships between UT1, UTC and TAI

*Universal Time (*UT1) is a time reference that conforms, within a close approximation, to the mean diurnal motion of the Earth. It is determined from observations of the diurnal motions of the stars, and then corrected for the shift in the longitude of the observing stations caused by the polar motion.

The time system generally used is the *Coordinated Universal Time* (UTC), previously called *Greenwich Mean Time*. The UTC is piecewise uniform and continuous, i.e. the time difference between UTC and TAI is equal to an integer number of seconds and is constant except for occasional jumps from inserted integer *leap seconds*.

The leap seconds are inserted to cause UTC to follow the rotation of the Earth, i.e. to follow UT1. This is performed in the following way:

If UT1 is predicted to lag behind UTC by more than 0.9 seconds, a leap second is inserted. The message is distributed in a *Special Bulletin C* by the International Earth Rotation Service (IERS).

The insertion of leap seconds is scheduled to occur with first preference at July 1st and January 1st at 00:00:00 UTC, and with second preference at April 1st and October 1st at 00:00:00 UTC.

ΔUT1 = UT1 - UTC is the increment to be applied to UTC to give UT1, expressed with a precision of 0.1 seconds, and which is broadcasted, and any change announced in a *Bulletin D*, by the IERS[1].

DUT1 is the predicted value of ΔUT1. Predictions of UT1 - UTC daily up to ninety days, and at monthly intervals up to a year in advance, are included in a *Bulletin A* which is published weekly by the IERS.

International Atomic Time (TAI) represents the mean of readings of several atomic clocks, and its fundamental unit is exactly one SI second at mean sea level and is, therefore, constant and continuous.

ΔTAI = TAI - UTC is the increment to be applied to UTC to give TAI.

Barycentric Dynamic Time (TDB) is the time scale applicable in the barycenter of our Solar System, with

$$TDB = TAI + 32.184 + 0.001658 \sin(g) + 0.000014 \sin(2g) \text{ [s]}$$

$$\text{with } g = 357.53 + 0.98560028 \, (t) \text{ [deg]}$$

and t = days since 2000 January 1 at Greenwich noon (12:00:00), Universal Time

J2000.0, the astronomical reference epoch, is defined as 1 January 2000 12:00:00 TDB.

4.2 Time formats

Each of these time references can be represented using several time formats.

The *Julian Day* (JD) is the interval of time in days and fraction of a day since 4713 BC January 1 at Greenwich noon (12:00:00). Julian Day is **not** used within the context of Envisat.

The *Modified Julian Day 2000* (MJD2000) is the interval of time in days and fraction of day since 2000 January 1 at 00:00:00.

$$JD = MJD2000 + 2451544.5 \text{ [decimal days]}$$

The MJD2000 format for UTC is used under the assumption that UTC is continuous between the epoch and 2000 January 1, i.e. no leap seconds are introduced.

The time format year, month, day of month, hour, minute and second follows the Gregorian calendar.

4.2.1 Envisat-1 UT1/UTC time formats

The time formats used with the UT1 and UTC time references in the context of Envisat-1 are:

1. ΔUT1 usually changes 1-2 ms per day

Time format	Time reference	Description	Envisat usage
Processing	UT1	2-elements array of 64-bits floating point numbers: • UTC, decimal days • ΔUT1, decimal seconds	Internal processing (e.g. product processing sequences)
	UTC	64-bits floating point number: • UTC, decimal days	
Transport	UT1	32-bits integer numbers: • UTC, integer days • UTC, integer seconds • UTC, integer microseconds • ΔUT1, integer microseconds	Time values exchange between computers (e.g. data file records)
	UTC	32-bits integer numbers: • UTC, integer days • UTC, integer seconds • UTC, integer microseconds	
External	UT1	Text strings: • UTC: “dd-mmm-yyyy hh:mm:ss.tttttt” • ΔUT1: “ .tttttt”	Readable output (e.g. file headers, log messages)
	UTC	Text strings: • UTC: “dd-mmm-yyyy hh:mm:ss.tttttt”	

Table 2: Envisat-1 time formats

4.3 Envisat-1 UTC time resolution

The Envisat-1 UTC time resolution is one microsecond (AD 9).

4.4 Envisat-1 on-board times

The on-board times are actually counts of on-board clock ticks, abusively called on-board times.

The on-board times used in the context of Envisat-1 are:

Time reference and format	Description	Envisat usage
Satellite Binary Time (SBT)	32-bits integer number: • count of 256 Hz clock ticks	Processing of satellite binary (No transport or external format has been defined)
On Board Time (OBT)	two 32-bits integer numbers: • obtm = most significant bits • obtl = least significant bits	Processing of instrument on-board time (No transport or external format has been defined)

Table 3:

The Satellite Binary Time (SBT) is a 32 bits counter, incremented by 1 at a frequency of about 256Hz (defined as the step-length PER_0). It varies from **00000000** (Hexadecimal) to **FFFFFFFF** (Hexadecimal), the next value being again **00000000** (Hexadecimal) and so on. This reset of the counter after **FFFFFFFF** (Hexadecimal) is called the **wrap-around**.

The On Board Time (OBT) is a generic term to represent any of the instrument counters, used to date their source packets. Most instruments use a 32 bits counter (OBT_{32}) synchronized with the SBT. Some instruments use a 40 or 43 bits counter (OBT_{40} OR OBT_{43}), where the 32 most significant bits are synchronized with the SBT (i.e. they use a more precise clock). To cope with this, OBT is represented using two 32-bits words (OBT_M and OBT_L)

Next figure shows the relationship between SBT and OBT.

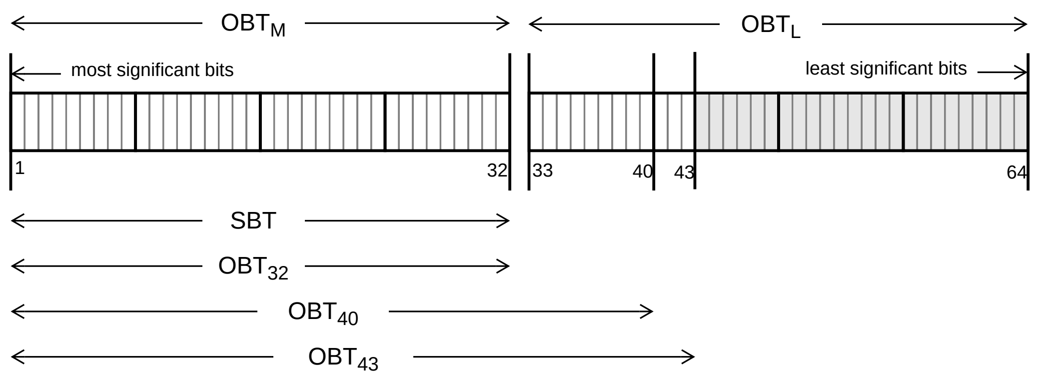

Figure 2: SBT and OBT relationship

5 COORDINATE SYSTEMS

The following coordinates systems are used in the context of Envisat-1:

Coordinate system	Envisat usage
Barycentric Mean of 2000.0	The star catalogues usually use this coordinate system to express the positions of their stars
Heliocentric Mean of 2000.0	The ephemeris of the planets are usually expressed in this coordinate system
Geocentric Mean of 2000.0	The FOS performs the internal calculations related to the predicted and restituted orbits in this coordinate system.
Mean of Date	The Mean Local Solar Time is defined in this coordinate system.
True of Date	It is the reference inertial coordinate system used for input and output in the CFI software (e.g. stars positions) It is also the reference inertial coordinate system used by the CFI software to perform its internal calculations
Earth fixed	It is the coordinate system used for input and output of the satellite state vector (i.e the orbit), and for the output for geolocation in the CFI software. It is also the coordinate system in which the predicted and restituted orbits supplied by the FOS are expressed.
Topocentric	It is the local horizontal coordinate system used to define a look direction
Satellite Reference	It is the coordinate system that constitutes the reference for the application of the selected attitude control mode.
Satellite Relative Reference	It is the coordinate system that constitutes the reference for the definition of the mispointing of the satellite
Satellite Relative Actual Reference	It is the coordinate system that constitutes the reference for the definition of a look direction relative to the satellite (e.g to express the pointing of an instrument)

Table 4: Coordinate system usage

5.1 General coordinate systems

5.1.1 Barycentric Mean of 2000.0

It is based, according to the recommendations of the International Astronomical Union (IAU), on the star catalogue FK5 for the epoch J2000.0, since the directions of its axes are defined relatively to a given number of that star catalogue positions and proper motions.

The accuracy of this reference system, realized through the FK5 star catalogue, is approximately 0.1’’.

The centre of this coordinate system is the barycenter of the Solar System. The x-y plane coincides with the predicted mean Earth equatorial plane at the epoch J2000.0, and the x-axis points towards the predicted mean *vernal equinox*, the ascending node of the ecliptic on the Earth's equator.The ecliptic is the mean plane of the Earth orbit around the Sun. The z-axis points towards north.

The word *mean* indicates that the relatively short periodic nutations of the Earth are smoothed out in the calculation of parameters for the reference epoch J2000.0.

5.1.2 Heliocentric Mean of 2000.0

It is obtained by a parallel translation of the Barycentric Mean of 2000.0 coordinate system from the barycenter of the Solar System to the centre of the Sun.

5.1.3 Geocentric Mean of 2000.0

It is obtained by a parallel translation of the Barycentric Mean of 2000.0 coordinate system from the barycenter of the Solar System to the centre of the Earth.

5.1.4 Mean of Date

The centre of this coordinate system is the centre of the Earth. The x-y plane and the x-axis are defined by the mean Earth equatorial plane and the mean vernal equinox of date. The z-axis points towards north.

The expression *mean of date* means that the system of coordinate axes are rotated with the Earth´s precession from J2000.0 to the date used as epoch and indicates that the relatively short periodic nutations of the Earth are smoothed out in the calculation of parameters for that epoch.

The precession of the Earth is the secular effect of the gravitational attraction from the Sun and the Moon on the equatorial bulge of the Earth.

5.1.5 True of Date

The centre of this coordinate system is the centre of the Earth. The x-y plane and the x-axis are defined by the true Earth equatorial plane and the true vernal equinox of date.

The expression *true of date* indicates the usage of the instantaneous Earth equatorial plane and vernal equinox. Thus the relatively short periodic nutations of the Earth are used in the calculation of parameters for that epoch.

The transformation from the Mean of Date to the True of Date is the adopted model of the nutation of the Earth.

The nutation is the short periodic effect of the gravitational attraction of the Moon and, to a lesser extent, the planets on the Earth's equatorial bulge.

5.1.6 Earth fixed

The Earth fixed coordinate system in use is the *IERS Terrestrial Reference Frame* (ITRF).

The zero longitude or IERS Reference Meridian (IRM), as well as the IERS Reference Pole (IRP), are maintained by the International Earth Rotation Service (IERS), based on a large number of observing stations, and define the IERS Terrestrial Reference Frame (ITRF).

This coordinate system rotates with the Earth, so velocities expressed in this coordinate system includes the rotation of the Earth.

5.1.7 Topocentric

Its z-axis coincides with the normal vector to the Earth's Reference Ellipsoid, positive towards zenith. The x-y plane is the plane orthogonal to the z-axis, and the x-axis and y-axis point positive, respectively, towards east and north.

5.2 CFI specific coordinate systems

5.2.1 Satellite Reference

It is a coordinate system centred on the satellite and is defined by the X_S, Y_S and Z_S axes, which are specified relatively to the reference inertial coordinate system, namely the True of Date.

The Z_S axis points along the radial satellite direction vector, positive from the centre of the coordinate system towards the satellite, the Y_S axis points along the transversal direction vector within the osculating orbital plane (i.e the plane defined by the position and velocity vectors of the satellite), orthogonal to the Z_S axis and opposed to the direction of the orbital motion of the satellite. The X_S axis points towards the out-of-plane direction vector completing the right hand coordinate system.

$$\bar{Z} = \frac{\bar{r}}{|\bar{r}|} \qquad \bar{X} = \frac{\bar{r} \wedge \bar{v}}{|\bar{r} \wedge \bar{v}|} \qquad \bar{Y} = \bar{Z} \wedge \bar{X}$$

where $\bar{X}$, $\bar{Y}$ and $\bar{Z}$ are the unitary direction vectors in the (X_S, Y_S, Z_S) axes, and $\bar{r}$ and $\bar{v}$ are the position and velocity vectors of the satellite expressed in the reference inertial coordinate system.

Next drawing depicts the Satellite Reference coordinate system:

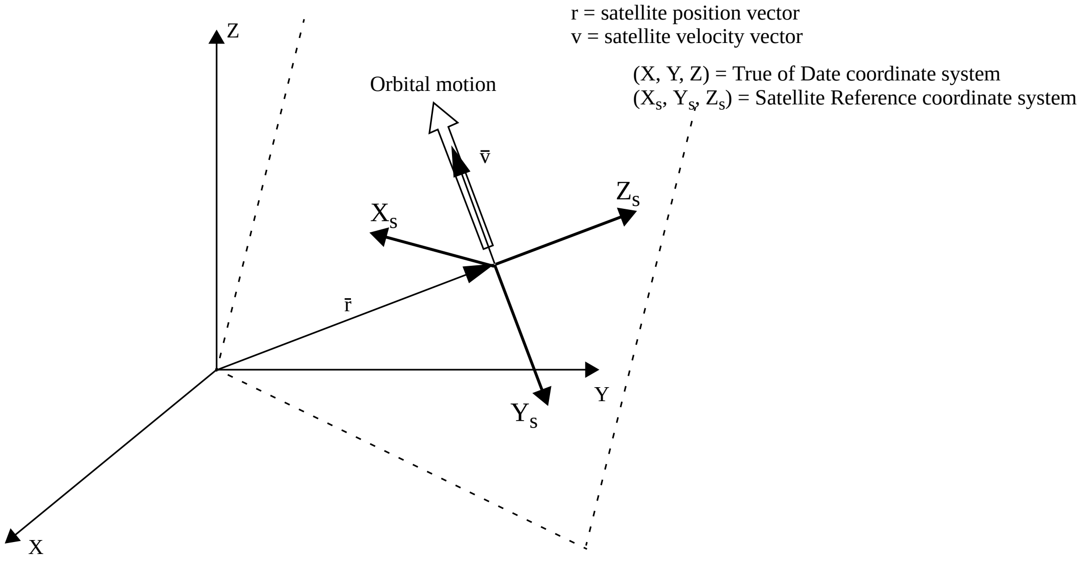

Figure 3: Satellite Reference coordinate system

5.2.2 Satellite Relative Reference

The (X'_S, Y'_S, Z'_S) is the Satellite Relative Reference coordinate system and is obtained by rotating the Satellite Reference coordinate system by three consecutive rotations: first around $-Y_S$ over a roll angle

η, then around $-X^1_s$ (i.e the rotated X_s) over a pitch angle ξ, and finally around $+Z^2_s$ (i.e the rotated Z^1_s) over a yaw angle ζ.

The roll η, pitch ξ and yaw ζ angles are function of the selected attitude control mode (see section 7.2)

Next drawing depicts the Satellite Relative Reference coordinate system:

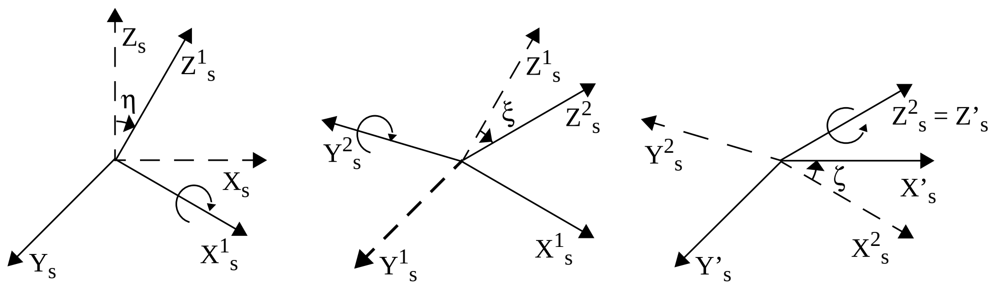

Figure 4: Satellite Relative Reference coordinate system

5.2.3 Satellite Relative Actual Reference

The (X''_s, Y''_s, Z''_s) is the Satellite Relative Actual Reference coordinate system and is obtained by rotating the Satellite Relative Reference coordinate system by three consecutive rotations: first around $-Y'_s$ over a mispointing roll angle $\Delta\eta$, then around $-X^1_s$ (i.e the rotated X'_s) over a mispointing pitch angle $\Delta\xi$, and finally around $+Z^2_s$ (i.e rotated Z^1_s) over a mispointing yaw angle $\Delta\zeta$.

The mispointing roll $\Delta\eta$, pitch $\Delta\xi$ and yaw $\Delta\zeta$ angles are defined in section 6.2.1.1

5.3 Envisat-1 specific coordinate systems

5.3.1 Local Orbital Reference Frame

The Local Orbital Reference Frame (T, R, L) is defined in AD 5.

The origin of the Local Orbital Reference Frame is the satellite in-flight centre of mass 'G'.

The unit vector L is in the direction opposite to the Earth's centre, [Geocentre].

The unit vector R is perpendicular to L and in the vertical plane containing V such that cos(V, R) > 0, where V is the **absolute velocity vector** (note that R is coplanar with L and V)

The unit vector T completes the right-handed frame (T = R x L).

This Local Orbital Reference Frame defines the absolute pointing of the satellite for the Fine Acquisition Mode 2 (FAM2), Fine Pointing Mode (FPM), and the initial and final attitude of Orbit Control Mode (OCM) with T, R, L being the pitch, roll and yaw axes respectively (see section 7.2).

[T: Tangage, R: Roulis, L: lacet]

5.3.2 Local Relative Orbital Reference Frame

The Local Relative Orbital Reference Frame ($\underline{T}_1$, $\underline{R}_1$, $\underline{L}_1$) is defined in AD 5

It has the same definition as the Local Orbital Reference Frame (T, R, L) except for the local normal pointing, i.e. the unit vector $\underline{L}_1$ is parallel to the local normal of the Earth's Reference Ellipsoid directed upward and crossing the spacecraft in-flight centre of mass 'G'.

This attitude is also known as Local Normal Pointing, please note that this is **not** a nominal Envisat-1 attitude or attitude mode.

5.3.3 Local Relative Yaw Steering Orbital Reference Frame

The Local Relative Yaw Steering Orbital Reference Frame (T', R', L') is defined in AD 5.

It has the same definition as the Local Orbital Reference Frame (T, R, L) except for the local normal pointing and for its orientation with respect to the spacecraft velocity vector:

- Local Normal Pointing L' = $\underline{L}_1$
- The unit vector R' perpendicular to L' is in the direction of V' which is the velocity vector of the subsatellite point relative to the Earth's Reference Ellipsoid (relative ground trace velocity vector)

This Local Relative Yaw Steering Orbital Reference Frame defines the absolute pointing of the satellite for the [Stellar] Yaw Steering mode ([S]YSM) with T', R', L', being the pitch, roll and yaw axes respectively (see section 7.2)

5.3.4 Envisat-1 and CFI coordinate systems relationships

The following relationships between the specific Envisat-1 and CFI coordinate systems apply:

CFI coordinate system	Envisat-1 frame	Link	Attitude control mode
Satellite Reference	Local Orbital Reference Frame	$R = -Y_S$ $T = -X_S$ $L = +Z_S$	N/A
Satellite Relative Reference	Local Orbital Reference Frame	$R = -Y'_S$ $T = -X'_S$ $L = +Z'_S$	FAM 2 / 3 FPM OCM
	Local Relative Orbital Reference Frame	$R_1 = -Y'_S$ $T_1 = -X'_S$ $L_1 = +Z'_S$	N/A
	Local Relative Yaw Steering Orbital Reference Frame	$R' = -Y'_S$ $T' = -X'_S$ $L' = +Z'_S$	[S] YSM [S] FCM
Satellite Relative Actual Reference	-	-	-

Table 5: Coordinate system relationships

5.4 Coordinate system transformations

The following picture identifies the coordinate system transformations that are relevant for Envisat-1
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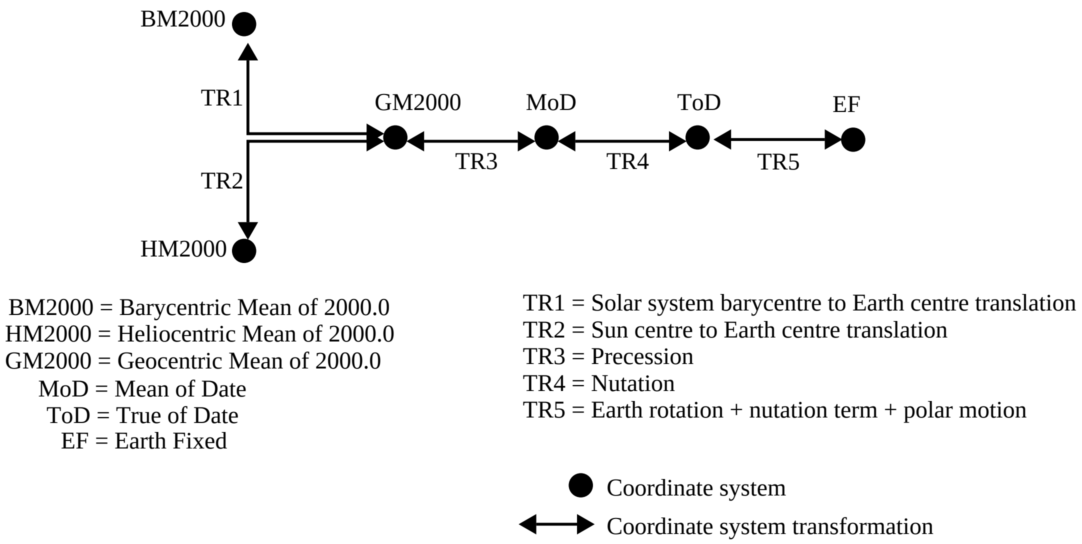

Figure 5: Coordinate system transformations

Those transformation are described in the following sections.

Note that whenever a transformation is expressed as a sequence of rotations, the following expressions apply (the angle *w* is regarded positive):

$$R_x(w) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos w & \sin w \\ 0 & -\sin w & \cos w \end{bmatrix} \quad R_y(w) = \begin{bmatrix} \cos w & 0 & -\sin w \\ 0 & 1 & 0 \\ \sin w & 0 & \cos w \end{bmatrix} \quad R_z(w) = \begin{bmatrix} \cos w & \sin w & 0 \\ -\sin w & \cos w & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

5.4.1 Barycentric Mean of 2000.0 to Geocentric Mean of 2000.0

The transformation from the Barycentric Mean of 2000.0 to the Geocentric Mean of 2000.0 coordinate system is a translation, to move the coordinate system origin from the solar system barycentre to the earth centre. It is calculated with the following expressions:

$$\vec{r}_E = \vec{r}_B - \vec{r}_{B,\,Earth}$$

$$\vec{v}_E = \vec{v}_B - \vec{v}_{B,\,Earth}$$

where $\vec{r}_E$ and $\vec{v}_E$ are the position and velocity vectors in the Geocentric Mean of 2000.0 coordinate system, $\vec{r}_B$ and $\vec{v}_B$ are the position and velocity vectors in the Barycentric Mean of 2000.0 coordinate system, and $\vec{r}_{B,\,Earth}$ and $\vec{v}_{B,\,Earth}$ are the position and velocity vectors of the Earth in the Barycentric Mean of 2000.0 coordinate system.

$\bar{r}_{B,\,Earth}$ and $\bar{v}_{B,\,Earth}$ are calculated according to RD 9.

5.4.2 Heliocentric Mean of 2000.0 to Geocentric Mean of 2000.0

The transformation from the Heliocentric Mean of 2000.0 to the Geocentric Mean of 2000.0 coordinate system is a translation, to move the coordinate system origin from the sun centre to the earth centre. It is calculated with the following expressions:

$$\bar{r}_E = \bar{r}_H - \bar{r}_{H,\,Earth}$$

$$\bar{v}_E = \bar{v}_H - \bar{v}_{H,\,Earth}$$

where $\bar{r}_E$ and $\bar{v}_E$ are the position and velocity vectors in the Geocentric Mean of 2000.0 coordinate system, $\bar{r}_H$ and $\bar{v}_H$ are the position and velocity vectors in the Heliocentric Mean of 2000.0 coordinate system, and $\bar{r}_{H,\,Earth}$ and $\bar{v}_{H,\,Earth}$ are the position and velocity vectors of the Earth in the Heliocentric Mean of 2000.0 coordinate system.

$\bar{r}_{H,\,Earth}$ and $\bar{v}_{H,\,Earth}$ are calculated according to RD 9.

5.4.3 Geocentric Mean of 2000.0 to Mean of Date

The transformation from the Geocentric Mean of 2000.0 to the Mean of Date coordinate system is a rotation, to take into account the precession of the earth rotation axis. It is performed with the following expression (see Annex A.1):

$$\bar{r}_m = R_z\left(-\frac{\pi}{2} - z\right) R_x(\theta) R_z\left(\frac{\pi}{2} - \zeta\right) \bar{r}_{J2000}$$

where $\bar{r}_m$ and $\bar{r}_{J2000}$ are the position vector in the Mean of Date and the Mean of 2000.0 coordinate system, respectively.

5.4.4 Mean of Date to True of Date

The transformation from the Mean of Date to the True of Date coordinate system is a rotation, to take into account the nutation of the earth rotation axis. It is performed with the following expression (see Annex A.2):

$$\bar{r}_t = R_z(-\delta\mu) R_x(-\delta\varepsilon) R_y(\delta\upsilon) \bar{r}_m$$

where $\bar{r}_t$ and $\bar{r}_m$ are, respectively, the position vector in the True of Date and the Mean of Date coordinate system.

5.4.5 True of Date to Earth fixed

The transformation from the True of Date to the Earth fixed coordinate system is a rotation, to take into account the earth rotation. It is performed with the following expression (see Annex A.3):

$$\bar{r}_e = R_z(H)\bar{r}_t$$

where $\bar{r}_e$ and $\bar{r}_t$ are, respectively, the position vector in the Earth fixed and in the True of Date coordinate systems.

5.4.6 Satellite Reference to Satellite Relative Reference

The transformation from the Satellite Reference to the Satellite Relative Reference coordinate system is a rotation, to take into account the satellite nominal attitude law. It is performed with the following expression:

$$\bar{r}_{srr} = R_z(\zeta)R_x(-\xi)R_y(-\eta)\bar{r}_{sr}$$

where $\bar{r}_{sr}$ and $\bar{r}_{srr}$ are, respectively, the position vector in the Satellite Reference and the Satellite Relative Reference coordinate systems, and η, ξ and ζ are the roll, pitch and yaw angles.

5.4.7 Satellite Relative Reference to Satellite Relative Actual Reference

The transformation from the Satellite Relative Reference to the Satellite Relative Actual Reference coordinate system is a rotation, to take into account a given mispointing (note that this mispointing typically depends on the instrument considered). It is performed with the following expression:

$$\bar{r}_{srar} = R_z(\Delta\zeta)R_x(-\Delta\xi)R_y(-\Delta\eta)\bar{r}_{srr}$$

where $\bar{r}_{srr}$ and $\bar{r}_{srar}$ are, respectively, the position vector in the Satellite Relative Reference and the Satellite Relative Actual Reference coordinate systems, and $\Delta\eta$, $\Delta\xi$ and $\Delta\zeta$ are the roll, pitch and yaw mispointing angles (see section 6.2.1.1)

6 PARAMETERS

6.1 Orbit parameters

6.1.1 Cartesian state vector

It comprises the cartesian components of the position $\bar{r}_{SC}$, velocity $\bar{v}_{SC}$ and acceleration $\bar{a}_{SC}$ vectors of the satellite expressed in the Earth fixed coordinate system at a given epoch.

6.1.2 Orbit radius, velocity magnitude and components

The satellite **orbit radius** is the module of the satellite position vector $\bar{r}_{SC}$:

$$R = |\bar{r}_{SC}|$$

The **velocity magnitude** is the module of the satellite velocity vector $\bar{v}_{SC}$:

$$V = |\bar{v}_{SC}|$$

The satellite **velocity** vector when is expressed in the True of Date coordinate system can be split into two **components**:

- Radial component: $\bar{v}_r = \bar{v}_{SC} \bullet \bar{Z}$
- Transversal component: $\bar{v}_t = -\overline{v_{SC}} \bullet \bar{Y}$

where $\bar{Y}$ and $\bar{Z}$ are the direction vectors of the Satellite Reference coordinate system (see section 5.2.1)

6.1.3 Osculating Kepler state vector

The osculating Kepler elements are equivalent to the cartesian state vector, at the corresponding time, expressed in the True of Date coordinate system.

The six Kepler elements, which define unambiguously the state vector, are:

- Semi-major axis (a)
- Eccentricity (e)
- Inclination (i)
- Argument of perigee (ω)
- Mean anomaly (M)
- Right ascension of the ascending node (Ω)

Other auxiliary elements, which are derived from the above six elements, are:

- Eccentric anomaly (E)
- True anomaly (υ)

- True latitude (α)
- Mean latitude (β)

The relationships between these auxiliary elements and the six Kepler elements are:

$$\tan\frac{E}{2} = \sqrt{\frac{1-e}{1+e}}\tan\frac{\upsilon}{2}$$

$$M = E - e\sin E$$ (Kepler's equation)

$$\alpha = \omega + \nu$$

$$\beta = \omega + M$$

6.1.4 Mean Kepler state vector

The osculating six Kepler elements in the True of Date coordinate system can be averaged wrt time from one ascending node to the next (or wrt mean anomaly over 2π, which is equivalent), to obtain the mean Kepler elements:

$$\bar{a}, \bar{e}, \bar{i}, \bar{\omega}, \bar{\Omega}, \bar{M}$$

6.1.5 Equinoctial state vector

The osculating Kepler elements are usually replaced by the equivalent osculating equinoctial elements for quasi-equatorial and quasi-circular orbits:

$$x_1 = a$$

$$x_2 = e_x = e\cos(\Omega+\omega)$$

$$x_3 = e_y = e\sin(\Omega+\omega)$$

$$x_4 = i_x = +2\sin(i/2)\sin(\Omega)$$

$$x_5 = i_y = -2\sin(i/2)\cos(\Omega)$$

$$x_6 = \Omega + \omega + M$$

6.1.6 Ascending node, ascending node time, nodal period, absolute orbit number

The **ascending node** of an orbit is the intersection of that orbit, when the satellite goes from the southern to the northern hemisphere, with the x-y plane of the Earth fixed coordinate system.

The **ascending node time** is the UTC time of that ascending node.

The **relative time** is the time elapsed since that ascending node till the current point in the orbit.

The **nodal period** of an orbit is the interval of time between two consecutive ascending nodes.

The Launch orbit from Kourou is regarded as **absolute orbit number** zero. From then on, each time a new ascending node is crossed the absolute orbit number is incremented by one.

6.1.7 Repeat cycle and cycle length

In the geo / helio -synchronous orbits, such as the Envisat-1 orbit, the ground track repeats precisely after a constant integer number of orbits and days. The number of days of that period is called the **repeat cycle**, whereas the corresponding number of orbits is called the **cycle length**

6.1.8 Subsatellite point, satellite nadir and ground track

The **subsatellite point (SSP)** is the normal projection of the position of the satellite in the orbit on to the surface of the Earth´s Reference Ellipsoid.

The **satellite nadir** is equivalent to the **subsatellite point**.

The trace made by the subsatellite point on the surface of the Earth's Reference Ellipsoid due to the motion of the satellite along its orbit is called the **ground track**.

6.1.9 Mean Local Solar Time and True Local Solar Time

The **Mean Local Solar Time** (MLST) is the difference between the right ascension of the selected point in the orbit, RA and the *mean longitude of the Sun*, L. This normally defines an angle, but is expressed in hours assuming 24 hours is a full circumference (2π) (see A.4).

$$MLST = (RA - L + \pi)\frac{24}{2\pi} \text{ [hours]}$$

The **True Local Solar Time** (TLST) is the difference between the right ascension of the selected point in the orbit RA and the right ascension of the Sun, RA_{Sun}, expressed in hours.

$$TLST = (RA - RA_{Sun} + \pi)\frac{24}{2\pi} \text{ [hours]}$$

The RA_{Sun} is calculated, in the Mean of Date coordinate system, according to RD 10.

6.2 Attitude coordinate systems parameters

6.2.1 Attitude coordinate systems transformation

6.2.1.1 Attitude mispointing angles

The transformation from the Satellite Reference to the Satellite Relative Reference coordinate system is accomplished by three consecutive rotations over the angles roll ξ, pitch η and yaw ζ (see 5.2.2).

The time derivative of those angles are the **roll, pitch and yaw rates**.

Both those angles and their rates are a function of the selected attitude control mode (see section 7.2).

However, those are the nominal angles and rates and, usually, there are superimposed on them a set of **mispointing angles** that makes the Satellite Relative Reference coordinate system transform to the Satellite Relative Actual Reference coordinate system.

The mispointing angles are expressed as three components, namely roll $\Delta\xi$, pitch $\Delta\eta$ and yaw $\Delta\zeta$.

The time derivative of those mispointing angles are the **mispointing rates**.

6.2.1.2 AOCS rotation amplitudes

The AOCS rotation amplitudes are the three constants C_x, C_y and C_z that define the transformation from the Satellite Reference to the Satellite Relative Reference coordinate system according to the selected attitude control mode (see section 7.2).

6.2.2 Satellite centered direction

The parameters that define a direction in the Satellite Relative Actual Reference coordinate system are the satellite related azimuth (Az) and the satellite related elevation (El):

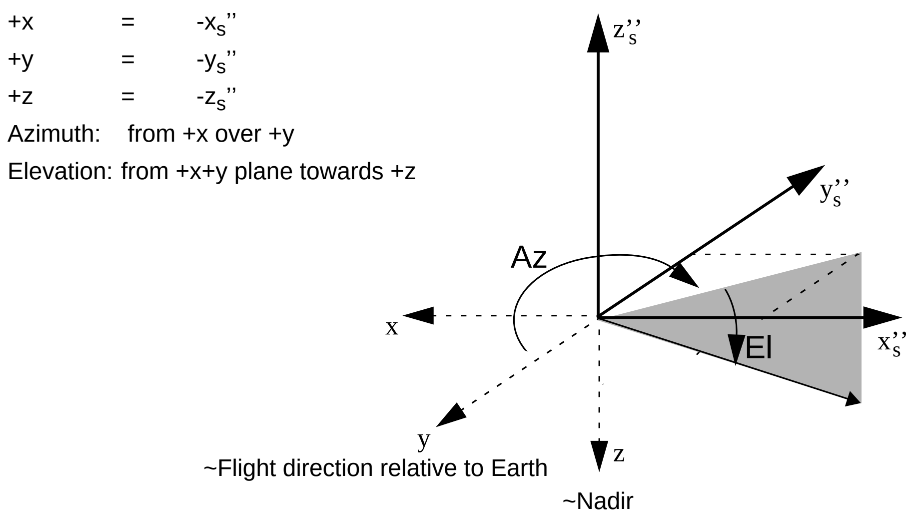

Figure 6: Satellite centred direction

Note that those are "classical" azimuth and elevation definitions in the (x, y, z) coordinate system, exactly opposite the Satellite Relative Actual Reference coordinate system (x''_s, y''_s, z''_s).

6.3 Earth related parameters

Note that altitude refers always to geodetic altitude except when the contrary is explicitly said.

6.3.1 Geodetic position

The geodetic coordinates of a point, related to the Earth´s Reference Ellipsoid, are the **geocentric longitude** λ, **geodetic latitude** φ, and **geodetic altitude** h, represented in the following drawing:

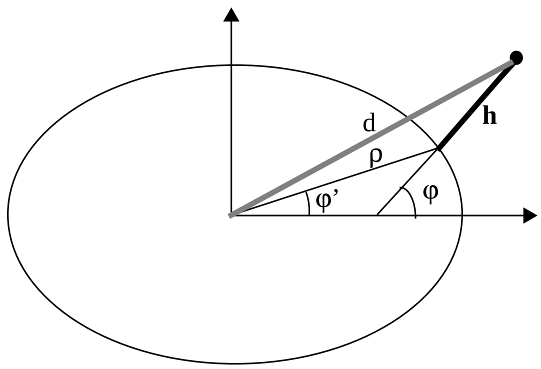

Figure 7: Geodetic position

The **geocentric latitude** φ', **geocentric radius** ρ and the **geocentric distance** d are also represented in that drawing.

The parameters **a**, **e** and **f**, i.e. the semi-major axis, the eccentricity and the flattening of the Earth's Reference Ellipsoid (see 7.4.2), define the equations that express these other parameters

The geocentric latitude φ' and the geodetic latitude φ are related by the expression:

$$\tan\varphi = \frac{1}{(1-f)^2}\tan\varphi'$$

The geocentric radius e is calculated with:

$$\rho = \frac{a\sqrt{1-e^2}}{\sqrt{1-e^2\cos^2\varphi'}}$$

The relationship between the cartesian coordinates of a point and its geodetic coordinates is:

$$x = (N+h)\cos\varphi\cos\lambda$$

$$y = (N+h)\cos\varphi\sin\lambda$$

$$z = [(1-e^2)N+h]\sin\varphi$$

where N is the **East-West radius of curvature**:

$$N = \frac{a}{\sqrt{1 - e^2 \sin^2 \varphi}}$$

The inverse transformation, from the cartesian to the geodetic coordinates, cannot be performed analytically. The iterative method that will be used will be initialized according to RD 11.

The normal projection of a point on the surface of the Earth's Reference Ellipsoid is called **Nadir**, and when that point corresponds to the position of the satellite, the projection is called subsatellite point.

Another important radius of curvature is M, the **North-South radius of curvature** (unit is m):

$$M = \frac{a(1 - e^2)}{\sqrt{(1 - e^2 \sin^2 \varphi)^3}}$$

The **radius of curvature** in any selected direction R_{Az} can be calculated with the expression (unit is m^{-1}):

$$\frac{1}{R_{Az}} = \frac{\cos^2 Az}{M} + \frac{\sin^2 Az}{N}$$

where Az is the angle of the selected direction expressed in the Topocentric coordinate system.

The **satellite centred aspect angle** $\alpha_{s/c}$ is the angle measured at the satellite between the *geometric direction*[2] from the satellite to the subsatellite point and the geometric direction from the satellite to the centre of the Earth.

The **geocentric aspect angle** α_g is the angle measured at the centre of the Earth between the geometric direction from the Earth centre to the subsatellite point and the geometric direction from the Earth centre to the satellite.

The **subsatellite point centred aspect angle** α_{ssp} is the angle measured at the subsatellite point between the geometric direction from the subsatellite point to the satellite and the geometric direction from the subsatellite point to the centre of the Earth.

The geodesic **distance** or **ground range** between two points that lay on an ellipsoid is by definition the minimum distance between those two points measured over that ellipsoid.

The **velocity** $\bar{v}_E$ **and** $\bar{a}_E$ **acceleration relative to the Earth**, i.e the Earth's Reference Ellipsoid, of a point that lays on its surface can be split into different components.

- Northward component, $\bar{v}_E \bullet \bar{N}$ or $\bar{a}_E \bullet \bar{N}$
- Eastward component, $\bar{v}_E \bullet \bar{E}$ or $\bar{a}_E \bullet \bar{E}$
- Groundtrack tangential component, $\bar{v}_E \bullet \bar{t} = v_E$ or $\bar{a}_E \bullet \bar{t}$
- Magnitude, $v_E = |\bar{v}_E|$ or $a_E = |\bar{a}_E|$
- Azimuth = the azimuth of the $\bar{v}_E$ or $\bar{a}_E$ vectors measured in the Topocentric coordinate system

where $\bar{N}$ and $\bar{E}$ are the North and East direction axes of the Topocentric coordinate system centred on that point, and $\bar{t}$ is the unitary vector tangent to the ground track at that point.

6.3.2 Ray path parameters

The path followed by the light, from its source, an observation target, to the proper instrument mounted

2. The geometric direction is defined by the straight line that connects the initial and the final point.

on Envisat-1, is bent due to the atmospheric refraction when it crosses the Earth atmosphere.

The magnitude of the bending of that path is a function of the value of the relative refraction index along the path that crosses the atmosphere.

Some parameters related to that path are the **range** S, which is the length of that path, and the signal **roundtrip time** T_r, which is the time needed by the light to travel twice that distance (unit is s):

$$T_r = 2\frac{S}{c}$$

where c = 299792458 [m s^{-1}] is the velocity of the light in a vacuum.

The **incidence angle** i of that path when it intersects an ellipsoid is the angle formed between the vector tangent at the path u, at the target from the target to Envisat-1, and the normal vector to that ellipsoid $\bar{N}$ at the incidence point.

$$\cos i = u \bullet \bar{N}$$

The **tangent altitude** h_T is the minimum distance, i.e the geodetic altitude, between that path and the Earth´s Reference Ellipsoid, and the corresponding point in the path is called the **tangent point** $\bar{r}_T$:

The **two way Doppler shift** of the signal D_s reflected by a target, is a function of the frequency of the transmitted signal f_s, and of the change in the length of the path travelled by that signal in its roundtrip, namely twice the range-rate from the corresponding Earth fixed target (see 6.6.1) to the satellite $\dot{S}$ (unit is Hz)

$$D_{2s} = -2f_s\frac{\dot{S}}{c}$$

The **one way Doppler shift** is, obviously (unit is Hz):

$$D_{1s} = -f_s\frac{\dot{S}}{c}$$

All these parameters are calculated according to the selected ray tracing model (see 7.7).

6.3.3 Earth centered direction

The parameters that define a direction from the centre of the Earth to a point in the True of Date coordinate system are the right ascension (α) and the declination (δ), shown in next figure

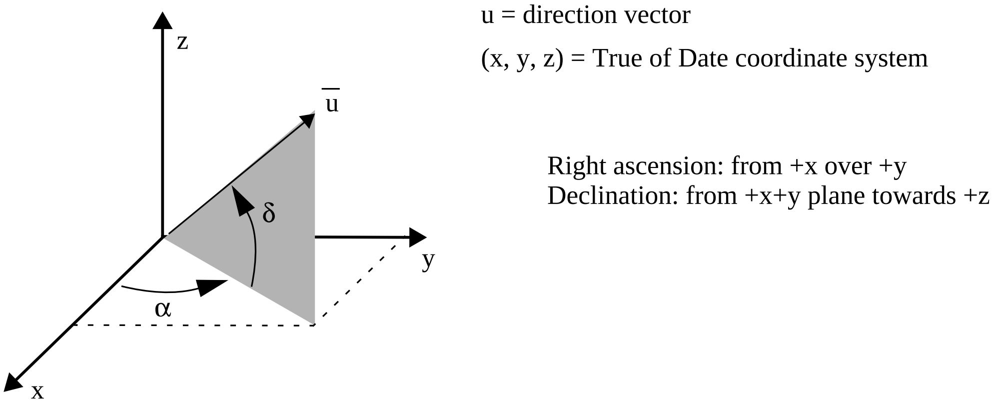

Figure 8: Earth centred direction

The same definitions can be used to define Earth centred direction in Mean of 2000 and Mean of Date coordinate systems.

6.3.4 Topocentric direction

The parameters that define a direction in the Topocentric coordinate system are the topocentric azimuth (Az) and the topocentric elevation (El), represented in the next drawing:

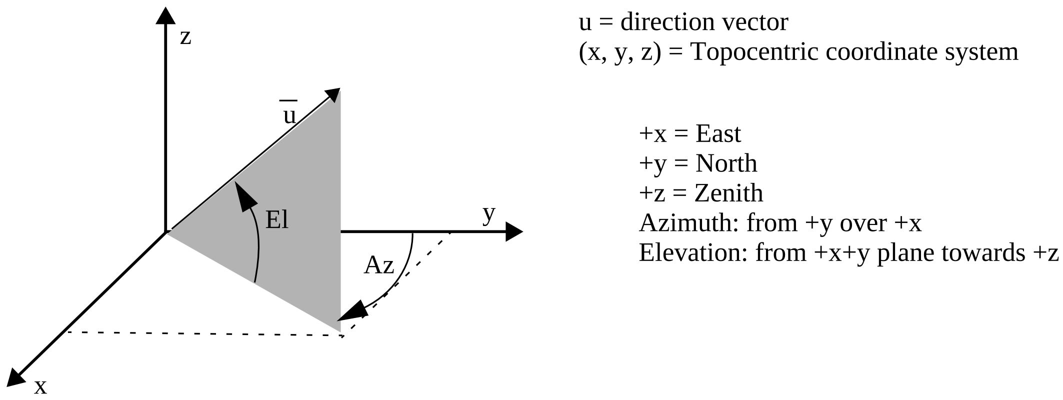

Figure 9: Topocentric direction

6.4 Ground Station parameters

6.4.1 Ground station location

The **location** of a **Ground Station** is defined by its geodetic parameters: i.e. geocentric longitude λ, geodetic latitude φ, and geodetic altitude h wrt the Earth's Reference Ellipsoid.

6.4.2 Ground station visibility

The visibility of a point from a Ground Station is limited by the **minimum link elevation** at which that point must be in order for the link between that Ground Station and that point to be established.

That minimum topocentric elevation is expressed in the Topocentric coordinate system centred at that Ground Station (see section 6.3.4), and although it is ideally a constant, in fact a real Ground Station usually has a physical mask that makes the minimum topocentric elevation be a function of the topocentric azimuth.

6.5 DRS parameters

6.5.1 DRS location

The DRS, a geo-stationary satellite, is ideally located in the equatorial plane of the Earth fixed coordinate system, and its position is therefore defined by its geocentric longitude λ expressed in that coordinate system.

6.6 Target parameters

6.6.1 Moving and Earth fixed targets

A **target** $\bar{r}_t$ is a point that is observed from the satellite and that satisfies certain conditions.

The **look direction**, or **line of sight** (LOS), $\bar{u}_0$ is the light direction, at the satellite, of the path followed by the light in its travel from the satellite to the target.

If the target moves wrt the Earth, as a result of a change in the satellite position or a change in the look direction, it is called the **moving target**.

If the target is fixed wrt the Earth, which implies that if the satellite position changes then the look direction has to change in the precise way to keep looking to that particular point fixed to the Earth, it is called the **Earth fixed target**.

In other words, the velocity of the moving target is the result of the motion of the satellite and the change in the look direction, or in the conditions that define it, with time. On the other hand, the velocity of the Earth fixed target is only a function of the position of that point wrt the Earth's Reference Ellipsoid and the rotation of the Earth fixed coordinate system.

6.6.2 Location parameters

The **location** of a **Target** is defined by its geodetic parameters: i.e. geocentric longitude λ, geodetic latitude φ, and geodetic altitude h wrt the Earth's Reference Ellipsoid, although it also can be defined by its cartesian position vector (x, y, z) expressed in the Earth fixed coordinate system.

6.7 Sun and Moon parameters

The **Sun semi-diameter** D_{Sun} is the apparent semi-diameter of the Sun, expressed in degrees, as seen from the satellite, and is calculated with the equation:

$$D_{Sun} = \frac{d_{Sun}}{R_{Sun-SC}}$$

where $d_{Sun} = 6.96 \times 10^8$ [m] is the semi-diameter of the Sun, and $R_{Sun\text{-}S/C}$ is the geometric distance between the satellite and the Sun centre.

The **Moon semi-diameter** D_{Moon} is the apparent semi-diameter of the Moon, expressed in degrees, as seen from the satellite, and is calculated with the equation:

$$D_{Moon} = \frac{d_{Moon}}{R_{Moon-SC}}$$

where d_{Moon} = 1738000 [m] is the semi-diameter of the Moon, and $R_{Moon-S/C}$ is the geometric distance between the satellite and the Moon centre.

The **area of the Moon lit by the Sun** $A_{Moon-Sun}$ is calculated with the expression:

$$A_{Moon-Sun} = \frac{1 + \cos\theta_{Sun-Moon-SC}}{2}$$

where $\theta_{Sun-Moon-S/C}$ is the angle measured at the centre of the Moon between the geometric direction from the centre of the Moon to the centre of the Sun and the geometric direction from the centre of the Moon to the satellite.

If $A_{Moon-Sun} = 0$ it is a new Moon, and if $A_{Moon-Sun} = 1$ it is a full Moon

The **satellite eclipse flag** indicates whether or not the path followed by the light from the centre of the Sun to the satellite intersects the Earth´s Reference Ellipsoid, path that is calculated according to the selected ray-tracing model (see section 7.7). It is equivalent to the **satellite to Sun visibility flag**.

The **satellite to Moon visibility flag** indicates whether or not the path followed by the light from the centre of the Moon to the satellite intersects the Earth´s Reference Ellipsoid, path that is calculated according to the selected ray-tracing model (see section 7.7).

The **target to Sun visibility flag** indicates whether or not the path followed by the light from the centre of the Sun to the target intersects the Earth´s Reference Ellipsoid, the path is calculated according to the selected ray-tracing model (see section 7.7).

7 MODELS

7.1 Envisat-1 orbit

7.1.1 Envisat-1 orbit definition

The Envisat-1 nominal orbit has the following mean elements (AD 4):

Mean element	Notation	Value
Semi-major axis	a	7159492.7 m
Eccentricity	e	0.001165
Inclination	i	98.549387 deg
Argument of perigee	ω	90.0 deg
MLST at ascending node	$MLST_{AN}$	10:00 pm
Mean altitude	$\bar{h}$	799790 m [a]

Table 6:

a. This corresponds to a minimum altitude of 786183 m and a maximum altitude of 813397 m above the Earth's Reference Ellipsoid.

The number of orbits per day is 14 + 11/35. This leads to a repeat cycle of 35 days and a cycle length of 501 orbits.

The operational orbit will be maintained within the following tolerances of the nominal mean parameters for a repeat cycle of 35 days and a cycle length of 501 orbits (AD 4):

Element	Tolerance requirement
MLST at ascending node	The MLST at the ascending node shall not deviate by more than 5 min from nominal
Ground track	The ground track shall not deviate by more than 1 Km from nominal at any point on the ground track
Mean Altitude	68 m

Table 7:

The CFI software will check the compliance of the orbit supplied on input with a set of requirements on the main osculating Kepler elements.

These requirements are less stringent than those contained in the previous table.

In fact, two categories of tolerance requirements will be checked (RD 3):

- **Tight requirements**: the orbit is very close to the nominal Envisat-1 one.

- **Loose requirements**: the orbit is far from the nominal Envisat-1 one, but still is a low eccentric, quasi polar and quasi helio-synchronous orbit, like Envisat-1.

Osculating Kepler element[a]	Tight tolerance	Loose tolerance
Semi-major axis	7118050 / 7194056 m	7000000 / 7300000 m
Eccentricity	0.000 / 0.007	0.0 / 0.1
Inclination	98.4475 / 98.6226 deg	98.0 / 99.0 deg

Table 8:

a. Derived from the mean altitude requirements

If the tight tolerance requirements are not satisfied, but the loose ones are, then a warning will be returned by the CFI software.

If even the loose tolerance requirements are not satisfied, then an error will be returned.

7.1.2 Envisat-1 orbit types

7.1.2.1 Reference orbit

The reference orbit consists of a single Envisat-1 cartesian state vector per orbit, i.e. the position and velocity vectors expressed in the Earth fixed coordinate system, corresponding to the ascending node of that orbit and its associated UTC time.

This state vector of the ascending node is calculated using the Envisat-1 longitude independent propagation mode (see 7.1.4), and imposing on it the following conditions:

- The mean argument of perigee is set to 90 degrees and the mean eccentricity to 0.001165. This orbit state results in a constant mean semi-major axis, mean eccentricity, and mean inclination, but in a secular term in the mean argument of perigee.
- The orbit is geo-synchronous, i.e. the ground track repeats precisely after a constant number of integer days (repeat cycle) and a constant number of integer orbits (cycle length)
- The orbit is helio-synchronous, i.e. the rate of the mean right ascension of the ascending node coincides with the motion of the mean Sun (see A.4):

$$\dot{\Omega} = \dot{L}_{sun}$$

which implies that the MLST of the ascending node is also constant

Typical values for the parameters of the Reference orbit are:

Parameter	Value
UT1 of the Reference orbit	Any date[a]
Repeat cycle	35 days
Cycle length	501 orbits

Table 9:

Parameter	Value
Longitude of the [Earth fixed] ascending node	0.1335 deg[b]
MLST at the ascending node	22.0 hours
Absolute orbit number of the Reference orbit	499 orbit[c]
Absolute orbit number of the requested orbit	Any orbit

Table 9:

a. Depending on Envisat-1 launch

b. Assumption: same orbit as ERS (multi-disciplinary phase)

c. Assumptions: launch orbit No 0 from Kourou + trajectory of Ariane 5 is the same as the trajectory of Ariane 4

Based on the tolerance requirements on the osculating Kepler elements (see section 7.1.1), there are only some combinations of repeat cycle[3] and cycle length that are valid (RD 3) for the Reference orbit:

Repeat Cycle (days)	Cycle length (orbits)	Repeat Cycle (days)	Cycle lengths (orbits)
3	43	7	100
8	(115)	10	143
11	(157), 158	13	186
14	201	15	(214)
16	229	17	243, 244
18	(257)	19	(271), 272, (273)
20	287	22	315
23	(328), 329, 330	24	343
25	357, 358, 359	26	(371), 373
27	(385), 386, (388)	28	401
29	(414), 415, 416	30	(431)
31	(442), 443, 444, 445	32	457, 459
33	472	34	(485), 487
35	(499), 501, 502, (503)	36	515, 517
37	(528), 529, 530, 531	38	543, 545
39	(556), 557, 560	40	(571), 573
41	(585), 586, 587, 588, 589	42	(599), 601
43	(613), (614), 615, 616, 617, (618)	44	629, 631
45	643, 644, 646, (647)	46	657, 659, (661)
47	(670), (671), 672, 673, 674, 675	48	(685), 689

Table 10:

Repeat Cycle (days)	Cycle length (orbits)	Repeat Cycle (days)	Cycle lengths (orbits)
49	(699), 701, 702, 703, (704)	50	(713), 717
51	(727), (728), 730, (733)	52	743, 745, 747
53	(756), 757, 758, 759, 760, 761, (762)	54	773, 775
55	(784), 786, 787, 788, 789	56	(799), 801, 803
57	814, 815, 818	58	(827), 829, 831, 833
59	(841), (842), 843, 844, 845, 846, 847, (848)	60	857, 859

Table 10:

All the values in this table meet the tight orbital tolerance requirements, although those in brackets are out of the Envisat-1 specifications (AD 4)

7.1.2.2 Predicted orbit

The predicted orbit consists of a single Envisat-1 cartesian state vector per orbit, i.e. the position and velocity vectors expressed in the Earth fixed coordinate system, and the UT1 time at the ascending node crossing of that orbit, or in its vicinity (RD 3)

7.1.2.3 Restituted orbit

The restituted orbit consists of a series of Envisat-1 cartesian state vectors computed for each integer minute of a day (RD 3)

7.1.3 Envisat-1 orbit propagation definition

To calculate the state vector at any point in the orbit, it is sufficient to have a state vector at one time, and then propagate that initial state vector to the required time using an orbit propagation model.

That initial state vector can come from different sources, and depending on the type of orbit, there are different requirements on the accuracy of the position and velocity vectors of that initial state[4]:

Orbit type	Supplier	Position accuracy [m] (3σ accuracy)			Velocity accuracy [m/s] (3σ accuracy)		
		Radial	Along track	Across track	Radial	Along track	Across track
Reference[a]	CFI software	68	20 sec	1250	TBD	TBD	TBD
Predicted[b]	FOS	25	900	15	0.955	0.029	0.016
Restituted[c]	FOS	25	60	15	0.040	0.027	0.014

Table 11:

a. This is the absolute accuracy, i.e. the accuracy of the initial state vector contained in the Reference Orbit Event File. The accuracy of the Reference orbit is the combination of this absolute accuracy plus the relative accuracy, which is the accuracy of the propagation model that is used to propagate the state vector (see table 13)

b. From AD 4

3. Between 3 and 60 days

c. From AD 4

7.1.4 Envisat-1 orbit propagation models

There are two orbit propagation models.

Both models have a common initialization mode, and then different propagation modes, depending on the level of accuracy required.

The initialization mode starts with an initial cartesian state vector expressed in the Earth fixed coordinate system at a certain time, supplied externally (see section 7.1.3), to calculate the time and the state vector of the true ascending node in the Earth fixed coordinate system (i.e. $z_{AN} = 0$ and $\dot{z}_{AN} > 0$)

This initialization mode implements an iterative algorithm which is based on the longitude independent propagation mode (see below).

Depending on how far the initial state vector is from the calculated true ascending node in the Earth fixed coordinate system, then the maximum propagation interval differs[5]

U_{LAT}[a]	Maximum propagation interval
$\|U_{LAT}\| \le 5$ deg	2 nodal periods wrt the time of the calculated true ascending node in the Earth fixed coordinate system
$\|U_{LAT}\| > 5$ deg	3 minutes wrt the time of the supplied initial state vector.

Table 12:

a. The osculating true latitude of the initial state vector in the True of Date coordinate system.

The two propagation modes are:

- **Longitude independent mode (reduced accuracy)**: in this case only the zonal (i.e. latitude independent) harmonics of the geoid J2, $J2^2$, J3 and J4 are used to calculate the secular perturbations of the *mean*[6] Kepler elements, and the zonal harmonic J2 is used to calculate the short periodic perturbations to transform the mean Kepler elements to the osculating Kepler elements.

This mode is based on the equations derived in RD 8.

- **Longitude dependent mode (high accuracy)**: in this case, the effect of the latitude and longitude dependent geoid anomalies as well as the effect of air-drag have been derived from observations in the form of second order correction terms to the satellite position and velocity components (radial, along track, and across track).

These correction terms are function of the longitude of the true ascending node in the Earth fixed coordinate system, and of the true latitude of the propagated state vector using the longitude independent mode, expressed in the True of Date coordinate system.

This mode is based on the equations derived in RD 7.

If $|U_{LAT}| > 5$ deg, then only the reduced accuracy propagation mode can be used.

The accuracy of these two propagation modes is, assuming a perfect initial cartesian state vector (i.e.

4. The along and across track accuracy values refer to the ground track
5. Due to the effect of the air drag, which is not considered in the initialization mode.
6. Averaged wrt the osculating mean anomaly over 2π.

a state vector exactly at ascending node), the following (RD 3):

U_{LAT}	Propagation mode	Position accuracy [m] (Maximum / RMS)			Velocity accuracy [mm/s] (Maximum / RMS)		
		Radial	Along track	Across track	Radial	Along track	Across track
$\|U_{LAT}\| \leq 5$ deg	Reduced accuracy	313 / 101	1650 / 540	229 / 80	1655 / 523	278 / 90	249 / 92
	High accuracy	60 / 5.0	49 / 15	101 / 4.5	93 / 17	118 / 7.5	60 / 5.4
$\|U_{LAT}\| > 5$ deg	Reduced accuracy	6.8 / 1.6	6.2 / 1.2	3.6 / 0.6	59 / 11	27 / 6.6	38 / 7.6

Table 13:

The following constants are used in those two propagation modes (RD 3):

Parameter	Notation	Value
Equatorial radius of the Earth	R_e	6378136 m
Earth's Gravitational constant = GM	μ	$3.98600440 \times 10^{14}$ m^3 s^{-2}
Second zonal harmonic	J2	1082.626×10^{-6}
Third zonal harmonic	J3	-2.536×10^{-6}
Fourth zonal harmonic	J4	-1.623×10^{-6}

Table 14:

7.2 Envisat-1 attitude control

There are several attitude control modes (see AD 6), whose purpose and (nominal) attitude is described in the following table:

Mode	Purpose	Attitude
Rate Reduction Mode (RRM)	Used during initial acquisition after the solar array secondary deployment, in re-acquisition case after failures, and also returning to nominal operations from safe mode. The RRM is intended to reduce the angular rates down to values lower than 0.3^{o} / sec on the 3 axes	Arbitrary attitude.

Table 15:

Mode	Purpose	Attitude
Coarse Acquisition Mode (CAM)	Allows to acquire a geocentric pointing of the pitch and roll axis with an accuracy of the order of 5 degrees, while maintaining a small angular rate on the yaw axis	Arbitrary to Geocentric, rotating
Fine Acquisition Mode 1 (FAM1)	Allows to acquire a pointing on the yaw axis lower than 2 degrees while maintaining the geocentric pointing on the pitch and roll axis.	Geocentric, rotating to FAM2 attitude.
Fine Acquisition Mode 2 (FAM2)	Stable waiting mode, ending the acquisition phase; it maintains satisfactory pointing performances while minimizing the hydrazine consumption.	The Satellite Relative Reference coordinate system is fixed to the Local Orbital Reference Frame, rotated 180 deg around Z_S.
Fine Acquisition Mode 3 (FAM3)	Transient mode between FAM2 and FPM.	Identical to FAM2 and FPM
Fine Pointing Mode (FPM)	Steady-state transition mode between YSM (not SYSM) and OCM. Triggered from the FAM2 through FAM3 to OCM or YSM.	The Satellite Relative Reference coordinate system is fixed to the Local Orbital Reference Frame, rotated 180 deg around Z_S
Orbit Control Mode (OCM)	Third operational mode, used to perform out-of-plane orbit corrections (inclination updating) and large in-plane orbit correction (eccentricity and semi-major axis updating)	During thrust phase, depending on the type of manoeuvre During in-plane correction: Identical to FPM attitude. During out-of-plane correction: Derived from FPM attitude after a rotation of 90 deg around Z_S axis

Table 15:

Mode	Purpose	Attitude
Stellar Yaw Steering Mode (SYSM)	Main of the AOCS operational modes of ENVISAT. This mode is activated either from YSM, or automatically after the SFCM. In this mode ENVISAT must ensure yaw steering pointing and local normal pointing with a pointing performance better than 0.07 deg on each axis and a rate stability below 0.003 deg/sec.	The Satellite Relative Reference coordinate system is fixed to the Local Relative Yaw Steering Reference Frame, rotated 180 deg around Z_S It moves wrt the Local Orbital Reference Frame according to the Local Normal Pointing and Yaw Steering laws.
Yaw Steering Mode (YSM)	Stable transition mode before entering in Stellar Yaw Steering Mode (SYSM),	Identical to SYSM
Stellar Fine Control Mode (SFCM)	Second operational mode of ENVISAT, dedicated to fine and short orbit corrections in the orbit plane in order to ensure a fine update of the semi-major axis and the eccentricity of the orbit.	Identical to SYSM
Fine Control Mode (FCM)	Back-up mode of the Stellar Fine Control Mode (SFCM). Same purpose as SFCM.	Identical to SYSM
Satellite Safe Mode	Ensures a survival state after a major service module anomaly	Heliocentric pointing of the $+Z_S$ face for the satellite, plus a spin motion around about the Z_S axis

Table 15:

The three rotation angles (roll, pitch, yaw) that transform the Satellite Reference to the Satellite Relative Reference coordinate system (see section 5.2.2), are calculated as follows (AD 7):

Mode	Roll	Pitch	Yaw
Rate Reduction Mode	Arbitrary attitude, not required to be simulated.		
Coarse Acquisition Mode	Arbitrary to Geocentric, rotating attitude, not required to be simulated.		
Fine Acquisition Mode 1	Geocentric, rotating to Orbital attitude, not required to be simulated.		

Table 16:

Mode	Roll	Pitch	Yaw
Fine Acquisition Modes 2 & 3 Fine Pointing Mode Orbit Control Mode[a]	0	0	0
[Stellar] Yaw Steering Mode [Stellar] Fine Control Mode	$C_Y \sin(U_{LAT})$	$C_X \sin(2U_{LAT})$	$C_Z \cos(U_{LAT})\left(1 - \frac{[C_Z \cos(U_{LAT})]^2}{3}\right)$
Satellite Save Mode	Heliocentric pointing, not required to be simulated.		

Table 16:

a. During the thrust phase of an out-of-plane correction the attitude mispointing should be increased / decreased by 90 degrees with respect to the nominal attitude mispointing.

where C_X, C_Y and C_Z are called the AOCS rotation amplitudes, in radians, and U_{LAT} is the satellite osculating true latitude in the True of Date coordinate system.

7.3 DRS-Artemis orbit

7.3.1 DRS-Artemis orbit definition

The initial DRS space segment comprises the Artemis Satellite located in the GEO orbit over Europe (16.4^o E) (AD 8).

The orbit of the DRS is known on ground to an accuracy corresponding to the following errors 20.0 Km along track, 15.0 Km across track and 15.0 Km radial. These accuracies are achieved for a 24 hour prediction and are achieved when UT is the time reference (AD 8)

The CFI software will check the compliance of the DRS orbit supplied on input with a set of requirements on the main osculating Kepler elements:

Osculating Kepler element	Tight tolerance	Loose tolerance
Semi-major axis	42000 / 43000 Km	30000 / 50000 Km
Eccentricity	0.0 / 0.1	0.0 / 0.9
Inclination	- 0.1 / + 0.1 deg	- 1.0 / + 1.0 deg

Table 17:

If the tight tolerance requirements are not satisfied, but the loose ones are, then a warning will be returned by the CFI software.

If even the loose requirements are not satisfied, then an error will be returned.

7.3.2 DRS-Artemis orbit propagation model

The 24 hour prediction of DRS will be available in equinoctial elements at a given epoch valid for certain validity period, and assuming that the user will propagate this state vector, within the validity period using the following algorithm.

$a = a_{initial}$

$e_x = e_{x\ initial}$

$e_y = e_{y\ initial}$

$i_x = i_{x\ initial}$

$i_y = i_{y\ initial}$

$\lambda = \lambda_{initial} + (t - t_{initial})\ d\lambda_{initial}\ dt$

$d\lambda_{initial}\ dt = (\mu / a^3)^{1/2}$

$\mu = 3.9860044e^{+5}\ km^3 / s^2$

($a_{initial}$, $e_{x\ initial}$, $e_{y\ initial}$, $i_{x\ initial}$, $i_{y\ initial}$, $\lambda_{initial}$) are the equinoctial elements at $t_{initial}$.

7.4 Earth

7.4.1 Earth position

The position and velocity of the Earth in the Barycentric and Heliocentric Mean of 2000.0 coordinate systems will be calculated according to RD 10

7.4.2 Earth geometry

The geometry of the Earth is modelled by a Reference Ellipsoid, namely the WGS84.

The most important parameters of the WGS84 ellipsoid are (RD 16):

Parameter	Notation	Magnitude
Semi major axis	a	6378137 m
Flattening = (a-b)/a	f	1/298.257223563
Eccentricity = ((a^2 - b^2)/a^2)$^{1/2}$	e	0.0818191908426
Semi minor axis	b	6356752.3142 m

Table 18:

The minimum distance between two points located on an ellipsoid is the length of the geodesic that crosses those two points. This geodesic distance will be calculated according to RD 24.

The surface at a certain **geodetic altitude h** over the Earth's Reference Ellipsoid is defined by:

$$x = (N + h)\cos\varphi\cos\lambda$$

$$y = (N + h)\cos\varphi\sin\lambda$$

$$z = [(1 - e^2)N + h]\sin\varphi$$

where N is the radius of curvature parallel to the meridian:

$$N = \frac{a}{\sqrt{1 - e^2\sin^2\varphi}}$$

and φ and λ are the geodetic latitude and geocentric longitude of a point on that ellipsoid.

Nevertheless, the surface at a certain geodetic altitude h over the Earth´s Reference Ellipsoid will be modelled as another ellipsoid, *concentric* with it, and with (a+h) and (b+h) as semi-major and semi-minor axis.

This simplification is quite accurate and has the advantage to allow the analytical calculation of the intersection or tangent points with such a surface.

7.4.3 Earth atmosphere

The Earth atmosphere is represented by the U.S Standard Atmosphere 1976 (RD 17), although modified by certain simplifications (see A.5):

It is also assumed that the atmosphere rotates with the same angular velocity as the Earth.

7.4.4 Refractive index

The refractive index is calculated with the Edlen's law (RD 25) although neglecting the contribution of the partial pressure of the water vapour (seeA.6)

7.5 Sun and Moon

The Sun and Moon position and velocity in the True of Date coordinate system will be calculated according to RD 10.

7.6 Stars

To calculate the look direction from the satellite to a star two consecutive steps must be performed:

- to calculate the stars coordinates in the True of Date coordinate system at the current time, taking as input a star catalogue (assumed to be expressed in the Barycentric Mean of 2000.0 coordinate system for the epoch J2000.0)
- to calculate the star coordinates in the Satellite Relative Actual Reference coordinate system at the same time

The first step must apply the following corrections (see A.7):

Correction	Description	Effect
Proper motion	Intrinsic motion of the star across the background wrt a reference epoch (e.g J2000.0) leading to a change in the apparent star position at the current epoch	Lower than 0.3 mdeg/year
Annual parallax	Apparent displacement of the position of the star caused by the difference in the position of the barycenter and the position of the Earth with the motion of the Earth around the Sun during the year	Lower than 0.3 mdeg
Light deflection	Gravitational lens effect of the Sun	Lower than 500 μdeg at the limb of the Sun and falling off rapidly with distance, e.g. to 6 μdeg at an elongation of 20 deg (so it will be <u>ignored</u>)
Annual aberration	Apparent displacement of the position of the star caused by the finite speed of light combined with the motion of the Earth around the Sun during the year	Lower than 6 mdeg
Precession	Change of the position of the star caused by the transformation from the Geocentric Mean of 2000.0 to the Mean of Date coordinate system	Lower than 6.0 mdeg/year
Nutation	Change of the position of the star caused by the transformation from the Mean of Date to the True of Date coordinate system	Lower than 13 mdeg

Table 19:

whereas the second step must apply the following ones:

Correction	Description	Effect
Satellite parallax	Apparent displacement of the position of the star caused by the difference in the position of the satellite and the position of the Earth with the motion of the satellite around the Earth during an orbit	Lower than 0.015 µdeg even for the closest stars (so it will be ignored)
Satellite aberration	Apparent displacement of the position of the star caused by the finite speed of light combined with the motion of the satellite around the Earth during an orbit	Lower than 1 mdeg for Envisat-1

Table 20:

7.7 Ray tracing

The path followed by the light when it crosses the Earth´s atmosphere is bent due to the effect of the atmospheric refraction, and therefore the direction of the light when it enters the atmosphere $\bar{u}_E$ is different to the direction of the light when it leaves the atmosphere $\bar{u}_L$.

This effect is depicted in the following drawing:

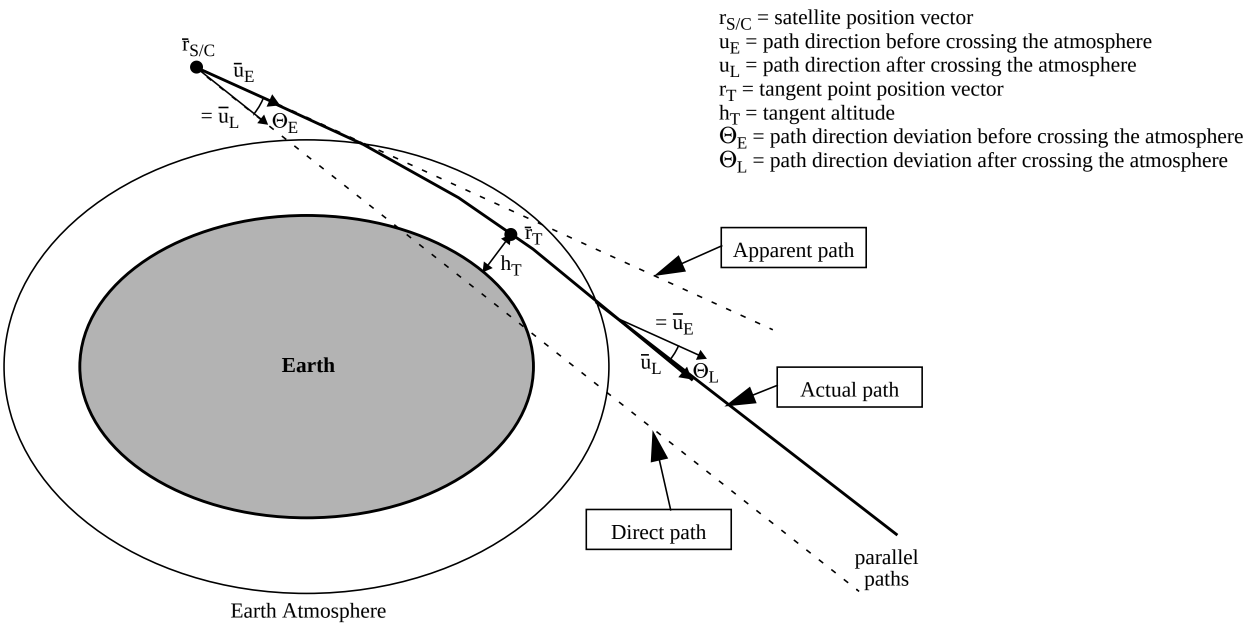

Figure 10: Ray path

Note that this drawing also represents the satellite position $\bar{r}_{SC}$, the tangent point of the light path over the Earth's Reference Ellipsoid $\bar{r}_T$ and the corresponding tangent altitude h_T, the maximum angular deviation of the light path at the entrance θ_E and at the exit θ_L of the atmosphere, where:

$$\cos\theta_E = \cos\theta_L = \bar{u}_E \bullet \bar{u}_L$$

The geometric distance $R_{T,g}$ between the satellite position and the tangent point is defined as:

$$R_{T,g} = |\bar{r}_T - \bar{r}_{SC}|$$

Finally, the range S_T is the actual distance between those two points measured along the path

A **ray tracing model** calculates the path followed by the light from the satellite through the atmosphere to the observation target.

7.7.1 No refraction model

This is the simplest model as the effect of the atmospheric refraction is not considered, therefore the path followed by the light is approximated by a straight line.

The advantage of this model is that is purely analytical.

7.7.2 Refraction models

There are two refraction ray tracing models, both based on the three following assumptions:

- The relative refractive index is a function only of the geometric altitude H above the Earth´s Reference Ellipsoid, i.e. the Earth’s atmosphere model is based on the assumption that the longitude or latitude variations of the relative refraction index are negligible
- The surface at a certain geometric altitude H will be modeled as an ellipsoid, *concentric* with the Earth’s Reference Ellipsoid, with (a+H) and (b+H) as semi-major and semi-minor axis.
- The light path lays in a plane: this plane is defined by the satellite position vector $\bar{r}_{SC}$, and by the known light direction, either $\bar{u}_E$ or $\bar{u}_L$. This assumption implies that the three dimensional effects of the light path bending are assumed to be negligible.

This simplification is quite accurate and has the advantage to allow the analytical calculation of the intersection of tangent points with such a surface

The light path is calculated by integrating the differential Eikonal´s equation in that plane:

$$\frac{d}{ds}\left(n\frac{d}{ds}\bar{r}\right) = \nabla m$$

where $\bar{r}$ is the position vector of a point in the light path, s is the arc length along that path, and m is the relative refraction index.

Note that iterative methods are usually needed to implement a refraction ray tracing model.

7.7.2.1 Standard atmosphere model

This refraction ray tracing model is based on the atmosphere model described in section 7.4.3

7.7.2.2 User’s atmosphere model

In this case the atmosphere model is based on a file supplied by the user, which defines the relative refraction index m at a discrete set of geometric altitudes H.

To have a continuous, and differentiable, relationship between the relative refraction index and the geometric altitude, the relative refraction index at any geometric altitude will be calculated by means of a cubic spline based on the two closest pair of data supplied by the user.

7.7.3 Predefined refraction corrective functions model

This model also assumes that the light path lays in a plane, i.e. the *reference* plane defined by the satellite position vector $\bar{r}_{SC}$, and by the known light direction, either $\bar{u}_E$ or $\bar{u}_L$.

It is based on the calculation of the parameters h_T, $R_{T,g}$, and S_T of the tangent point using the no refraction ray tracing model, and the calculation of a set of refraction corrective terms by means of predefined curves that depend only on the tangent altitude h_T and the wavelength of the light signal λ.

Those corrective terms are:

$$\Delta h_T = f_1(h_T, \lambda)$$

$$\Delta R_{T,g} = f_2(h_T, \lambda)$$

$$\Delta\theta_E = \Delta\theta_L = f_3(h_T, \lambda)$$

$$\Delta S_T = f_4(h_T, \lambda)$$

The tangent point, corrected for the effects of the atmosphere refraction, is calculated knowing that it lays in the *reference* plane, that the tangent altitude is $h_T + \Delta h_T$, and that the geometric distance from the satellite to the tangent point is $R_{T,g} + \Delta R_{T,g}$

The actual length of the light path is calculated as $S_T + \Delta S_T$

The light direction at the entrance or at the exit of the atmosphere, is calculated knowing that it lays in that reference plane, and that is deviated by an angle $\Delta\theta_E = \Delta\theta_L$.

The great advantage of this ray tracing model is that it is relatively quite accurate and is much faster than the *refraction* ray tracing models as it can be calculated analytically.

8 UNITS

In general, the units that will be used in all the CFI software will be the SI units, except for the angle that will use the **degree** instead of the **radian**.

Quantity	Unit	Symbol
Length	meter	m
Mass	kilogram	kg
Time	second	s
Thermodynamic temperature	kelvin	K
Amount of substance	mole	mol
Plane angle	degree	deg
Frequency	hertz	Hz
Pressure	pascal	Pa

Table 21:

ANNEX A. REFERENCE MODELS

This annex describes in detail the models mentioned throughout the document

A.1 Precession

The rotation angles of the precession model are calculated as follows (AD 3):

$$\zeta = 0.6406161T + 0.0000839T^2 + 0.0000050T^3 \text{ [deg]}$$

$$z = 0.6406161T + 0.0003041T^2 + 0.0000051T^3 \text{ [deg]}$$

$$\theta = 0.5567530T - 0.0001185T^2 - 0.0000116T^3 \text{ [deg]}$$

where T is the TDB time expressed in the Julian centuries format (1 Julian century = 36525 days).

However, the precession motion is so slow that the UTC time can be used instead of the TDB time, and therefore T can be calculated from t, the UTC time expressed in the MJD2000 format, with the following expression:

$$T = (t - 0.5)/36525 \text{ [Julian centuries]}$$

A.2 Simplified nutation

The rotation angles of the simplified nutation model are calculated with (AD 3):

$$\delta\mu = \delta\psi \cos\varepsilon$$

$$\delta\upsilon = \delta\psi \sin\varepsilon$$

where ε is the obliquity of the ecliptic at the epoch J2000.0 (unit is deg):

$$\varepsilon = 23.439291$$

$\delta\varepsilon$ and $\delta\psi$ are expressed by the *Wahr* model taking only the nine largest terms, and using UT1 instead of TDB as the time reference.

A.3 Earth rotation

The **Earth rotation angle H** is the sum of the Greenwich sidereal angle and a small term from the nutation in the longitude of the equinox.

The Greenwich sidereal angle moves with the daily rotation of the Earth and is calculated with the Newcomb's formula according to international conventions as a third order polynomial, although the third order term will be neglected in our calculations.

The nutation term is calculated with the simplified nutation model (see A.2).

$$H = G + \delta\mu$$

$$G = 99.96779469 + 360.9856473662860T + 0.29079 \times 10^{-12}T^2 \text{ [deg]}$$

where T is the UT1 time expressed in the MJD2000 format.

Note that the transformation from the Mean of Date to the Earth fixed coordinate system can be performed in one step being the $\delta\mu$ rotation term cancelled out:

$$\bar{r}_e = R_z(G)R_x(-\delta\varepsilon)R_y(\delta\upsilon)\bar{r}_{qm}$$

A.4 Mean longitude of the Sun

The mean longitude $\bar{L}$ of the Sun represents the motion of the *mean Sun* and is given, in the Mean of Date coordinate system, by (RD 10):

$$\bar{L} = 280.46592 + 0.9856473516(t - 0.5) \text{ [deg]}$$

where *t* is the UT1 time expressed in the MJD2000 format.

The motion of the mean Sun has a constant mean longitude rate, namely $\dot{\bar{L}} = 0.9856473516$ [deg/s]

A.5 Earth atmosphere

The Earth atmosphere is modelled as the U.S Standard Atmosphere 1976 but modified as follows:

- it ranges from Z = 0 Km to Z = 86 Km.
- the ratio M/M_0 decreases linearly from Z = 80 to Z = 86 Km.
- the linear relationship between T_M and H is replaced by either an arc of a circle or by a polynomial function in the vicinity of the points where the molecular-scale temperature gradient changes, in order to have a continuous and differentiable function $T_M = f(H)$

The U.S Standard Atmosphere 1976 is defined as follows (RD 17):

The air is assumed to be dry, and at altitudes sufficiently below 86 Km, the atmosphere is assumed to be homogeneously mixed with a relative-volume composition leading to a constant mean molecular weight M.

The air is treated as if it were a perfect gas, and the total pressure P, temperature T, and total density ρ at any point in the atmosphere are related by the equation of state, i.e. the perfect gas law, one form of which is:

$$P = \frac{\rho RT}{M}$$

where R = 8.31432 x 10^3 [Nm/(KmolK)] is the universal gas constant.

Besides the atmosphere is assumed to be in hydrostatic equilibrium, and to be horizontally stratified so that dP, the differential of pressure, is related to dZ, the differential of geometric altitude, by the relationship:

$$dP = -g\rho dZ$$

where g is the altitude-dependent acceleration of gravity, which can be calculated with the expression:

$$g = g_0\left(\frac{r_0}{r_0 + Z}\right)^2$$

where r_0 = 6356766 [m] and g_0 = 9.80665 [m/s^2], and that yields:

$$H = \frac{r_0 Z}{r_0 + Z}$$

where H is the geopotential altitude.

The molecular-scale temperature T_M at a point is defined as:

$$T_M = T\frac{M_0}{M}$$

where M_0 = 28.9644 [Kg/Kmol] is the sea-level value of M.

In the region from Z = 0 Km to Z = 80 Km M is constant and M = M_0, whereas between Z = 80 Km and Z = 86 Km, the ratio M/M_0 is assumed to decrease from 1.000000 to 0.9995788.

Up to altitudes up to 86 Km the function T_M versus H is expressed as a series of seven successive linear equations. The general form of these linear equations is:

$$T_M = T_{M,b} + L_{M,b}(H - H_b)$$

The value of $T_{M,b}$ for the first layer (b = 0) is 288.15 [K], identical to T_0 the sea-level value of T.

The six values of H_b and $L_{M,b}$ are:

Subscript	Geopotential altitude H_b [Km]	Molecular-scale temperature gradient LM,b [K/Km]
0	0	-6.5
1	11	0.0
2	20	1.0
3	32	2.8
4	47	0.0
5	51	-2.8
6	71	-2.0
7	84.8520 (Z = 86)	

Table 22:

Finally, the pressure can be calculated with the following expressions:

$$P = P_b\left(\frac{T_{M,b}}{T_{M,b} + L_{M,b}(H - H_b)}\right)^{\frac{g_0 M_0}{R L_{M,b}}} \quad (L_{M,b} \neq 0)$$

$$P = P_b \cdot \exp\left[\frac{-g_0 M_0 (H - H_b)}{R T_{M,b}}\right] \quad (L_{M,b} = 0)$$

The reference-level value for P_b for b = 0 is the defined sea-level value P_0 = 101325.0 N/m^2. Values of P_b for b = 1 through b ≥ 6 are obtained from the application of the appropriate equation above for the case when $H = H_{b+1}$.

A.6 Edlen’s law

The relative refraction index m at any point in the atmosphere can be calculated with the Edlen’s law:

$$m = 1 + N x 10^{-6}$$

$$N = \left[a_0 + \frac{a_1}{1-(\upsilon/b_1)^2} + \frac{a_2}{1-(\upsilon/b_2)^2}\right]\frac{P}{P_0}\frac{(T_0 + 15.0)}{T} - [c_0 - (\upsilon/c_1)^2]\frac{P_w}{P_0}$$

where P is the total air pressure in mb, T is the temperature in K, P_0 = 1013.25 mb, T_0 = 273.15 K, P_w is the partial pressure of water vapour in mb, and $\nu = 10^4/\lambda$ is the frequency in cm-1 for the wavelength λ in micrometers (RD 25)

The constants in that equation are

$a_0 = 83.42$

$a_1 = 185.08$

$a_2 = 4.11$

$b_1 = 1.140 \times 10^5$

$b_2 = 6.24 \times 10^4$

$c_0 = 43.49$

$c_1 = 1.70 \times 10^4$

The total air pressure and the temperature will be the corresponding to the atmosphere previously described, and the term in the last equation that corresponds to the partial pressure of water vapour will be neglected and therefore not calculated.

A.7 Stars position

To apply some of the necessary corrections to calculate the coordinates of a star in the Satellite Relative Actual Reference coordinate system, the following expressions shall be used (RD 14):

- Get the following variables from a star catalogue:
  - Right ascension at J2000.0 expressed in the Barycentric Mean of 2000.0: α_0 [rad]
  - Declination at J2000.0 expressed in the Barycentric Mean of 2000.0: δ_0 [rad]
  - Proper motion in the right ascension: μ_α [rad/century]
  - Proper motion in the declination: μ_δ [rad/century]
  - Radial velocity: ν [au/century]
  - Parallax: π [rad]
- Correct the star position obtained from the star catalogue (α_0, δ_0) for the proper motion and annual parallax effects using the expressions:

$$\bar{q} = (\cos\alpha_0\cos\delta_0, \sin\alpha_0\cos\delta_0, \sin\delta_0)$$

$$\bar{m} = (m_x, m_y, m_z)$$

$$m_x = -\mu_\alpha\cos\delta_0\sin\alpha_0 - \mu_\delta\sin\delta_0\cos\alpha_0 + \upsilon\pi\cos\delta_0\cos\alpha_0$$

$$m_y = \mu_\alpha \cos\delta_0 \cos\alpha_0 - \mu_\delta \sin\delta_0 \sin\alpha_0 + \upsilon\pi\cos\delta_0 \sin\alpha_0$$

$$m_z = \mu_\delta \cos\delta_0 + \upsilon\pi\sin\delta_0$$

$$\bar{P} = \bar{q} + T\bar{m} - \pi\bar{r}_{B,\,Earth}$$

where T = (t - 0.5)/36525, and t is the current TDT expressed in the MJD2000 format, and $\bar{r}_{B,\,Earth}$ is the position of the Earth in au at that TDT, expressed in the Barycentric Mean of 2000.0 coordinate system.

- Correct the star position for the annual aberration effect, using the following expressions:

$$\bar{p}_2 = \frac{\dfrac{\bar{p}_1}{\beta} + \left(1 + \dfrac{\bar{p}_1 \bullet \bar{v}}{1 + \dfrac{1}{\beta}}\right)\bar{v}}{1 + \bar{p}_1 \bullet \bar{v}}$$

$$\bar{v} = \frac{\bar{v}_{B,\,Earth}}{c} = 0.0057755\bar{v}_{B,\,Earth}$$

$$\beta = \frac{1}{\sqrt{1 - |\bar{v}|^2}}$$

where $\bar{v}_{B,\,Earth}$ is the velocity of the Earth in au/d at the current TDT expressed in the Barycentric Mean of 2000.0 coordinate system.

- The satellite aberration can be calculated with the expression (RD 13):

$$\Delta\theta = \operatorname{asin}\left[\frac{v}{c}\sin\theta - \frac{1}{4}\left(\frac{v}{c}\right)^2 \sin 2\theta\right] \text{ [rad]}$$

where $\Delta\theta$ is the change in the look direction from the satellite to the star, v is the velocity of the satellite expressed in the True of Date coordinate system, and c is the velocity of the light in a vacuum.

The following drawing sketches the satellite aberration:

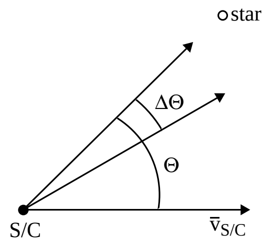

Figure 11: Satellite aberration