

# Pressure Buildup Analysis: A Simplified Approach

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## Summary

Pressure transients are modeled by the logarithmic approximation of the exponential integral ( $Ei$ ) function during the infinite-acting period. Use of the logarithmic approximation has been made for studying pressure buildup, drawdown, and falloff behavior. The conventional methods of transient analyses have been applied successfully over the past 30 years for calculating permeability-thickness product and skin. Unfortunately, estimation of average reservoir pressure ( $\bar{p}$ ) from pressure buildup tests for closed systems has lacked the desired level of accuracy because of the uncertainties associated with the definition of drainage shape in field applications.

A three-constant equation has been developed from the logarithmic approximation of the  $Ei$  function to describe the transient pressure behavior. The equation developed traces a rectangular hyperbola, which is unique to the well at the time of testing. Because of the very nature of the equation, it is possible to extrapolate a buildup curve beyond the infinite-acting period to obtain  $\bar{p}$  directly regardless of the drainage shape and boundary conditions. Consequently, we always obtain a superior  $\bar{p}$  estimate compared with the conventional methods, whose applications are often uncertain in actual cases. The proposed method also enables one to calculate the permeability-thickness product and skin with accuracy comparable to the conventional methods. The theoretical validity and applicability of the method have been demonstrated by examples.

## Introduction

The conventional methods<sup>1-3</sup> of pressure buildup analysis are well known. These methods have been

discussed in great detail by Ramey and Cobb<sup>4</sup> for closed or no-flow boundary systems, and by Kumar and Ramey<sup>5</sup> and Ramey *et al.*<sup>6</sup> for constant-pressure boundary systems. A comprehensive review of these works can be found in Ref. 7.

Horner's method<sup>2</sup> of buildup analysis is by far the most popular in the petroleum industry. Ramey and Cobb<sup>4</sup> and Cobb and Smith<sup>8</sup> concluded that the Horner graph is superior to both the Miller-Dyes-Hutchinson<sup>3</sup> (MDH) and Muskat<sup>1</sup> graphs regardless of the producing time in closed systems. Even though determination of permeability-thickness product and skin are relatively straightforward, the estimation of static reservoir pressure remains somewhat difficult for a well that produces at pseudosteady state before shut-in. Horner's method requires correction of the extrapolated false pressure,  $p^*$ , to obtain the static reservoir pressure,  $\bar{p}$ , for closed reservoir boundaries. To correct  $p^*$  for various reservoir drainage shapes, Mathews, Brons, and Hazebroek<sup>9</sup> (MBH) generated dimensionless pressures as a function of dimensionless producing time. These MBH pressure function curves have been used extensively in the industry and are presented in Refs. 7, 10, and 11.

Ramey and Cobb<sup>4</sup> also suggest a method for extrapolating a Horner straight line to average reservoir pressure for a well producing at pseudosteady state in a known reservoir drainage boundary. Odeh and Al-Hussainy<sup>12</sup> proposed a technique that correlates  $p^*$  to  $\bar{p}$  as a function of drainage shape. Likewise, the Dietz<sup>13</sup> method of estimating  $\bar{p}$  from an MDH plot also requires the knowledge of drainage shape. Once the pseudosteady-state flow is achieved, application of Slider's<sup>14</sup> technique appears to be superior to the other methods just mentioned. The main advantage of Slider's desuperposition method is that the reservoir shape definition no longer is required.

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The problem with the application of the MBH pressure function and its variations<sup>12,13</sup> lies in defining the true drainage shape in the field. Lack of enough well control and the complexity of reservoir geology largely contribute to the poor drainage shape definition. More important, the location of the well within a reservoir shape has a greater impact on the estimation of  $\bar{p}$  than does the shape itself.

Taylor<sup>15</sup> and Taylor and Caudle<sup>16</sup> recently demonstrated that the MBH pressure functions cannot be used successfully in multiwell reservoirs even for theoretical examples with known flow streamlines. This problem stems from the very sensitive nature of the MBH function on well location in a given drainage shape. Taylor developed a computer model using Lin's<sup>17</sup> bounding technique to generate a specific MBH pressure function for any reservoir system. Although this method appears to be an improvement on the existing MBH pressure functions, both lack of an analytical solution and the requirement that the well be produced at pseudosteady state before shut-in impose severe limitations on the use of this method. Rigorously, the methods of MBH and Taylor are valid only for closed systems—i.e., no flow at the outer boundary. In many cases oil reservoirs are under some form of enhanced recovery, and fluid injection would eliminate the use of MBH pressure functions in such reservoirs.

Prediction of true average reservoir pressure is crucial for any type of reservoir study; yet there seems to be no easy analytical tool available to practicing engineers for estimating  $\bar{p}$ . Ramey *et al.*<sup>6</sup> presented the dimensionless pressure functions for constant-pressure boundary systems. A subsequent publication by Kumar<sup>18</sup> gives a more elegant analysis of pressure-transient behavior under various degrees of pressure maintenance by water injection. Unfortunately, only one reservoir shape, a well in the center of a square, was analyzed rigorously. Lack of similar pressure functions for other reservoir geometries makes Kumar's work somewhat limited in practical application.

Mead<sup>19</sup> recently proposed an empirical method for determining  $\bar{p}$  from the asymptote of a rectangular hyperbola. He observed that a rectangular hyperbola characterizes the buildup behavior after the wellbore storage effect dissipates. Mead's analysis did not present a mathematical basis to confirm his observation but, nevertheless, the results were indisputable. It was unclear from Mead's work whether the region beyond the infinite-acting period can be modeled by the same rectangular hyperbola that characterized the infinite-acting period.

This paper expands upon the work of Mead and presents the mathematical basis for a simplified pressure buildup analysis procedure. The technique enables one to determine  $\bar{p}$  directly from the field data without prior knowledge of the drainage shape and to obtain good estimates of  $kh$  and  $s$ . This method thus provides a powerful tool to the practicing engineer because of its simplicity and accuracy in any type of reservoir drainage system: infinite-acting, closed, or pressure-maintained.

## Theory

The Horner working equation for a well shut in after producing at a constant rate in an infinite-acting reservoir is

given by<sup>2,7,10</sup>

$$p_{ws} = p_i - \frac{m}{2.303} \ln \left( \frac{t_p + \Delta t}{\Delta t} \right) \quad \dots \dots \dots (1)$$

The logarithmic term on the right side of Eq. 1 may be written as

$$\begin{aligned} \ln \left( \frac{t_p + \Delta t}{\Delta t} \right) &= \ln \left( 1 + \frac{t_p}{\Delta t} \right) = \ln \alpha + \frac{t_p - (\alpha - 1)\Delta t}{\Delta t} \\ &= \ln(\alpha + x), \quad \dots \dots \dots (2) \end{aligned}$$

where  $\alpha$  is a constant and

$$x = \frac{t_p - (\alpha - 1)\Delta t}{\Delta t} \quad \dots \dots \dots (3)$$

The right side of Eq. 2 may be expanded as follows.<sup>20</sup>

$$\begin{aligned} \ln(\alpha + x) &= \ln \alpha + 2 \left[ \left( \frac{x}{2\alpha + x} \right) + \frac{1}{3} \left( \frac{x}{2\alpha + x} \right)^3 \right. \\ &\quad \left. + \frac{1}{5} \left( \frac{x}{2\alpha + x} \right)^5 + \dots \right] \\ &= \ln \alpha + 2 \sum_{n=1}^{\infty} \frac{1}{2n-1} \left( \frac{x}{2\alpha + x} \right)^{2n-1} \quad \dots \dots \dots (4) \end{aligned}$$

The term  $x/(2\alpha + x)$  under the summation sign of Eq. 4 may be written, after substituting for  $x$  from Eq. 3, as

$$\frac{x}{2\alpha + x} = \frac{[t_p - (\alpha - 1)\Delta t]/\Delta t}{2\alpha + [t_p - (\alpha - 1)\Delta t]/\Delta t}$$

or

$$\frac{x}{2\alpha + x} = \frac{2(t_p + \Delta t)}{t_p + (\alpha + 1)\Delta t} - 1 \quad \dots \dots \dots (5)$$

Note that in Eq. 4,  $\alpha$  must be a positive constant and  $x$  must lie between  $-\alpha$  and plus infinity. Now if  $x/(2\alpha + x)$  is a small term as given by Eq. 5, only the first two terms on the right side of Eq. 4 need to be considered. We demonstrate validity of this assumption in the Field Example.

Therefore, we can rewrite Eq. 2 as

$$\ln \left( 1 + \frac{t_p}{\Delta t} \right) = \ln(\alpha + x) = \ln \alpha + 2 \left( \frac{x}{2\alpha + x} \right)$$

or

$$\ln \left( 1 + \frac{t_p}{\Delta t} \right) = \ln \alpha - 2 + \frac{4(t_p + \Delta t)}{t_p + (\alpha + 1)\Delta t} \quad \dots \dots \dots (6)$$

Combining Eqs. 1 and 6, we obtain

$$p_{ws} = p_i - \frac{m}{2.303} (\ln \alpha - 2) - \frac{4m(t_p + \Delta t)}{2.303[t_p + (\alpha + 1)\Delta t]}$$

Assuming that  $t_p + \Delta t \approx t_p$  and rearranging, we have

$$(p_{ws} - a)(b + \Delta t) = c, \quad \dots \dots \dots (7)$$

where

$$a = p_i - \frac{m}{2.303} (\ln \alpha - 2), \quad \dots \dots \dots (8)$$

$$b = \frac{t_p}{\alpha + 1}, \quad \dots \dots \dots (9)$$

and

$$-c = \frac{4mt_p}{2.303(\alpha + 1)} \quad \dots \dots \dots (10)$$

Eq. 7 represents a *rectangular hyperbola* with an asymptote equal to  $a$ . Because Eq. 7 has been derived for an infinite-acting reservoir, any change in the boundary condition does not alter its form. For instance, in a reservoir of closed outer boundaries,  $p_i$  should be replaced by  $p^*$  in both Eqs. 1 and 8. Thus, constant  $a$  in Eq. 7 implicitly accounts for the various boundary conditions of infinite-acting, no-flow, and different degrees of pressure maintenance.

Let us briefly examine the implications of the assumption:  $t_p + \Delta t \approx t_p$ . That is,  $\Delta t \ll t_p$ . This condition, however, does not mean that the producing time has to be long enough to reach pseudosteady state,  $t_{pss}$ . Thus, a buildup behavior following a short flow period (infinite-acting) can be analyzed with Eq. 7 to obtain meaningful reservoir properties. The condition  $\Delta t \ll t_p$  also implies that the MDH method of semilog analysis is applicable. In a conventional semilog analysis, the MDH graph straightens buildup data to much shorter shut-in times compared with a Horner plot. Such an apparent limitation is not a serious drawback in the proposed method when  $t_p$  is small, because few data points in the early shut-in period are needed to generate a unique hyperbola.

### Reservoir Static Pressure, $\bar{p}$

Horner's method gives  $p_i$  or  $\bar{p}$  directly from a plot of  $p_{ws}$  vs. the logarithm of the time ratio,  $(t_p + \Delta t)/\Delta t$ , for an infinite reservoir. For a developed reservoir, the extrapolated Horner false pressure,  $p^*$ , at a time ratio of unity has to be corrected to obtain  $\bar{p}$ .

Interestingly enough, the use of Eq. 7 always should yield  $\bar{p}$ , because at infinite shut-in time the wellbore shut-in pressure will build up to the static pressure,  $\bar{p}$ , provided that the adjacent wells do not interfere. The preceding statement is also valid for a well located in a pressure-maintained reservoir. Thus, the asymptote of

Eq. 7 is the static pressure,  $\bar{p}$ .

$$\bar{p} = a. \quad \dots \dots \dots (11)$$

### Constant $\alpha$

The constant  $\alpha$ , which appears in Eqs. 7, 8, and 9, can be determined from either of these equations for a given buildup test.

For an infinite-acting reservoir,

$$p_i = \bar{p}. \quad \dots \dots \dots (12)$$

Combining Eqs. 8, 11, and 12, we obtain

$$\ln \alpha = 2$$

or

$$\alpha = e^2 = 7.289. \quad \dots \dots \dots (13)$$

Thus, Eq. 13 is valid for  $r_{inv} < r_e$ . For a finite reservoir,  $\bar{p} < p_i$ ; therefore,  $\alpha > 7.389$ . Because the value of  $\alpha$  is +7.389 or greater, it satisfies one of the conditions for Eq. 4 to be valid.

### Permeability-Thickness Product, $kh$

An expression for the semilog slope,  $m$ , can be obtained by combining Eqs. 9 and 10.

$$m = 0.5758 \frac{(-c)}{b} \quad \dots \dots \dots (14)$$

The semilog slope of Eq. 1 is given by

$$m = \frac{162.6q\mu B}{kh} \quad \dots \dots \dots (15)$$

Eqs. 14 and 15 are combined to obtain an equation for  $kh$  in terms of the constants of the rectangular hyperbola,

$$kh = \frac{282.39q\mu Bb}{-c} \quad \dots \dots \dots (16)$$

Mead<sup>19</sup> empirically determined that the maximum slope on the MDH type of plot ( $p_{ws}$  vs.  $\log \Delta t$ ) on a Cartesian graph yields the semilog slope,  $m$ . In Appendix A we give theoretical reasoning to Mead's intuitive finding.

### Skin Factor, $s$

The van Everdingen<sup>21</sup> skin factor can be calculated from the following equation, once constants  $a$  and  $b$  are evaluated from the field data.

$$s = \frac{1}{2} \left[ \frac{2.303}{m} (a - p_{wf|\Delta t=0}) + \ln \alpha + 5.4316 - \ln \frac{k}{\phi \mu c_t r_w^2} - \ln t_p \right] \quad \dots \dots \dots (17)$$

The derivation of Eq. 17 is given in Appendix B. The pressure drop resulting from the skin is given by<sup>7,10,11</sup>

$$\Delta p_s = 0.87ms \dots (18)$$

### Solution of the Rectangular Hyperbola Equation

The ideal solution of Eq. 7 is to perform a nonlinear regression analysis to estimate the three constants *a*, *b*, and *c*. Eq. 7 is rearranged in the form:

$$p_{ws} = a + \frac{c}{b + \Delta t} \dots (19)$$

A linear regression can be performed for the variables *p<sub>ws</sub>* and *1/(b + Δt)* to obtain optimal values of *a*, *b*, and *c*. Because Eq. 19 is a three-constant equation, a trial-and-error procedure has to be employed by assuming values of *b* until a value of the regression coefficient close to unity is obtained. A programmable calculator can be used to perform this trial-and-error calculation.\*

An alternative solution to the regression analysis lies in the approach of Mead.<sup>19</sup> Mead proposed that because Eq. 7 contains three constants, any three sets of pressure-time data may be used to describe the unique rectangular hyperbola for the well. He gave the following solutions for the constants, where *t* = Δ*t* and *p* = *p<sub>ws</sub>*.

$$a = \frac{(p_3 - p_2)(p_2 t_2 - p_1 t_1) - (p_2 - p_1)(p_3 t_3 - p_2 t_2)}{(p_3 p_2)(t_2 - t_1) - (p_2 p_1)(t_3 - t_2)} \dots (20)$$

$$b = \frac{p_3 t_3 - p_2 t_2 - a(t_3 t_2)}{p_3 - p_2} \dots (21)$$

$$c = (b - t_1)(a - p_1) \dots (22)$$

Mead gave a program listing for an HP-67/97 calculator to solve Eqs. 20, 21, and 22. Although Mead's approach provides satisfactory results, the method does not provide unique answers. Because data scatter is inherent in any field test, the values of *a*, *b*, and *c* vary depending on the choice of data sets. Also, since Eq. 7 is not an exact equivalent of Eq. 1, the constants *b* and *c* will be somewhat sensitive to points chosen to describe the rectangular hyperbola.

The regression analysis procedure described previously provides the most satisfactory results. However, one may wish to get a good estimate for the value of *b* following Mead's approach and then perform the trial-and-error regression analysis either graphically or numerically.

\*A program listing is available upon request from either author.

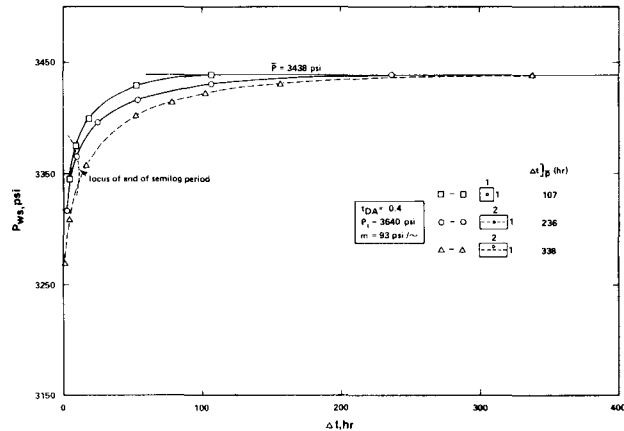


Fig. 1—Buildup behavior in various drainage areas.

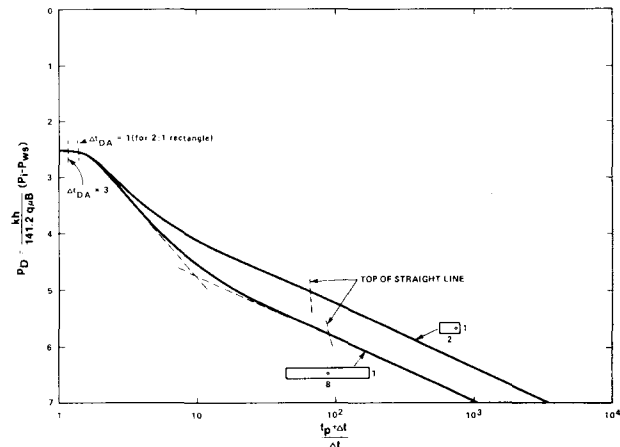


Fig. 2—Theoretical Horner graphs at *t<sub>DA</sub>* = 0.4 (adapted from Ref. 20).

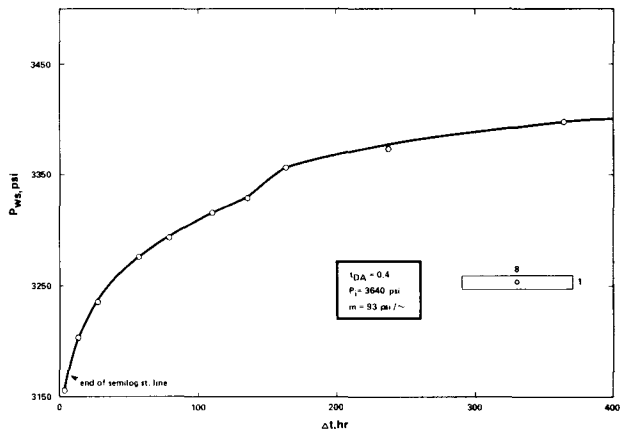


Fig. 3—Buildup behavior in an 8:1 rectangular drainage boundary.

**TABLE 1—PRESSURE BUILDUP DATA FOR A WELL IN THE CENTER OF A CLOSED SQUARE<sup>22</sup>**

$p_{ws}$ (psi)	$\Delta t$ (hours)		
3,191*	0.10*	Known Reservoir Data	
3,222	0.20	$p_i$ , psi	3,640
3,251	0.41	$q_o$ , STB/D	450
3,273	0.71	$t_p$ , hours	473
3,297	1.30	$B_o$ , RB/STB	1.20
3,315	2.01	$\mu_o$ , cp	0.8
3,323	2.49	$c_i$ , psi <sup>-1</sup>	$14 \times 10^{-6}$
3,345	4.38		
3,359	6.14	Calculated Data	
3,375	9.46	$m$ , psi/~	93
3,399	18.92	$t_{DA}$	0.40
3,428*	52.55**	$\phi hA$ , cu ft	$21 \times 10^6$
3,438*	107*		

\*Beginning of semilog straight line.

\*\*Note that the last two data sets are calculated from the theoretical Horner graph.

**TABLE 2—PRESSURE BUILDUP DATA FOR A WELL IN THE CENTER OF A SQUARE WITH CONSTANT-PRESSURE BOUNDARIES<sup>5,6,7</sup>**

Shut-in Time $\Delta t$ (hours)	Pressure $p_{ws}$ (psi)		
0	3,561	Known Reservoir Data	
0.333	3,851	$t_p$ , hours	4,320
0.500	3,960	$q$ , STB/D	350
0.667	4,045	$\mu$ , cp	0.80
0.883	4,104	$c_i$ , psi <sup>-1</sup>	$17 \times 10^{-6}$
1	4,155	$A$ , acres	7.72
2*	4,271*	$B$ , RB/STB	1.136
3	4,306	$h$ , ft	49
4	4,324	$r_w$ , ft	0.29
5	4,340	$\phi$	0.23
6	4,352		
7	4,363		
8	4,371		
9	4,380		
10	4,387		
20	4,432		

\*Beginning of semilog straight line.

**TABLE 3—COMPARISON OF REGRESSION AND SEMILOG ANALYSES (DATA OF TABLE 2)**

	Regression Analysis of the Hyperbola, $\Delta t$ (hours)			Semilog Analysis
	2-20*	6-20	2-7	
$a = \bar{p}$ , psig	4,491.4	4,516.65	4,436.79	4,458
$m$ , psi/~	172.59	159	183.07	152
$b$ , hours	5.2	8.84	2.16	—
$-c$ , psi-hr	1,558.76	2,441.93	686.81	—
$r^2$	0.99669	0.99982	0.99823	—

\*2-20 implies that the pressure-time data for  $2 < \Delta t < 20$  were used for the regression analysis.

## Application of the Rectangular Hyperbola Equation

### Theoretical Examples

**No-Flow Boundary.** Denson *et al.*<sup>22</sup> presented pressure buildup data for a well in the center of a closed square as given in Table 1. We use these data to predict  $\bar{p}$  by Eq. 7. We also examine whether the same  $\bar{p}$  can be obtained for other reservoir geometries, given the same reservoir  $kh$ ,  $t_p$ , and  $\bar{p}_i$ . The pressure-time data for other reservoir geometries were obtained from the theoretical Horner graphs.<sup>23</sup>

Fig. 1 is a Cartesian plot of shut-in pressure vs. shut-in time. The graph compares the buildup profiles obtained for three different locations of a well in two reservoir geometries. We observe distinctly different buildup characteristics asymptotically reaching the same  $\bar{p}$  in each case with less than 0.5% error. This observation suggests that a rectangular hyperbola can describe *uniquely* a well's buildup behavior in closed systems.

A very interesting feature of Fig. 1 is that the infinite-acting period, where the semilog analysis applies, ended in less than 10 hours in all cases. Only the late-time data helped to characterize the different pressure profiles to attain the same  $\bar{p}$ . Thus the very nature of the alternate form of the logarithmic approximation of the  $Ei$  function allows us to obtain  $\bar{p}$  directly. In this respect, the proposed method has a distinct advantage over the conventional semilog analysis of Horner and MDH.

We have found that analysis of only the late-time data, dominated by the boundary effects, also gives the correct value of  $\bar{p}$ , as one might expect intuitively. However, the proposed method requires analysis of only the infinite-acting data for calculating the correct  $kh$  and  $s$  from Eqs. 16 and 17, respectively.

We also investigated the applicability of Eq. 7 in describing the buildup behavior in other reservoir boundaries, such as an asymmetric well location in a rectangle and a well located in the center of a long rectangle. The unusual feature about these reservoir shapes is that the MBH pressure function,  $p_{DMBH}$ , can become negative for certain producing times. Fig. 2 displays theoretical Horner graphs at  $t_{DA}$  equals 0.4 for a well one-eighth of the length away from the side in a 2:1 rectangle and for a well in the center of an 8:1 rectangle. Both buildup behaviors exhibit an increase in the rate of buildup following the end of the semilog period. A transition period, indicated between the dotted lines for an 8:1 rectangle, can be observed before the change in the buildup rate occurs.

To study this unusual shape effect, we obtained a buildup graph using the Denson *et al.*<sup>22</sup> data for a well in an 8:1 rectangle. The theoretical Horner graph<sup>23</sup> was employed to generate the graph displayed in Fig. 3. A break in the rather smooth buildup behavior at 136 hours is observed, followed by a rapid pressure increase over a short time interval and then the final buildup period. The buildup to the static pressure ultimately is achieved after 3,638 hours of shut-in. This unusual buildup behavior can be appreciated by inspecting Fig. 2, the semilog graph.

Note that the final buildup curve in Fig. 3 spans the majority of total buildup time,  $163 \text{ hours} < \Delta t < 3,638 \text{ hours}$ ; thus one has to use the data beyond 163 hours to

obtain the correct  $\bar{p}$ . Although such a long shut-in period may be impractical from the standpoint of testing, this unusual buildup behavior can be avoided once a  $(t_{DA})_{pss}$  of 3.0 is reached. The  $(t_{DA})_{pss}$  will be approached with decreasing length of the semilog line on a Horner graph. For instance, the end of the semilog straight line occurs after just 5 hours of shut-in when  $t_{DA}$  equals 0.4. Further increase in  $t_{DA}$  would result in a duration of the semilog period of less than 5 hours.

**Constant-Pressure Boundary.** Simulated pressure buildup data for a well in the center of a square drainage area with constant-pressure boundaries are provided in Table 2.<sup>5-7</sup> Other relevant data also are given in the table.

The linear regression analysis performed on the data beyond 2 hours yields the following results. For the intercept,  $a = \bar{p} = 4,491.40$  psi (30 968.20 kPa) [4,458 psi (30 737.91 kPa) by semilog analysis]. For the slope,  $-c = 1,558.757$  psi-hr (10 747.63 kPa·h). The regression coefficient,  $r^2$ , equals 0.99669. The semilog slope,  $m$ , is calculated as follows.

$$m = \frac{2.303}{4} \frac{-c}{b} = 172.59 \text{ psi/} \sim (1190.01 \text{ kPa/} \sim)$$

[152 psi/  $\sim$  (1048.04 kPa/  $\sim$ ) by semilog analysis].

We observe that the proposed method predicts  $\bar{p}$  to be 0.75% higher than the semilog method. The  $m$  value is predicted even higher, at 13.55%; consequently, the permeability-thickness product,  $kh$ , will be underestimated by the same margin. The constant  $a$  (equal to  $\bar{p}$ ) is relatively insensitive to changes in the other two constants  $b$  and  $c$  of Eq. 7. Therefore, the regression analysis can be optimized to give a better value of  $m$ , comparable to the semilog analysis.

The error involved in neglecting the higher-order terms in Eq. 4 is about 3.32% at  $\Delta t = 10$  hours. This error is incurred because of the translation of the Horner equation to its equivalent rectangular hyperbola equation, Eq. 7. However, the translational error can be reduced significantly if only the late infinite-acting data are analyzed for estimating  $kh$  and  $s_D$  and the early-time data for  $\bar{p}$ . Table 3 shows the results of the analyses.

The problem examined here assumes that all four boundaries of the reservoir are fully pressure-maintained. Kumar<sup>18</sup> demonstrated that the degree of pressure maintenance has rather significant influence on the  $P_{DMBH}$  at a given value of  $t_{DA}$ . Consequently, the correction for  $p^*$  to  $\bar{p}$  is dependent on the correct estimation of the degree of pressure maintenance in field applications. One also should bear in mind that incorrectly guessing the shape of the drainage area or the well location in a given shape has very serious consequences on the  $\bar{p}$  estimate.

Thus the simplicity and ease of *directly* calculating the  $\bar{p}$  with less than 1% error makes the proposed technique superior to conventional techniques.

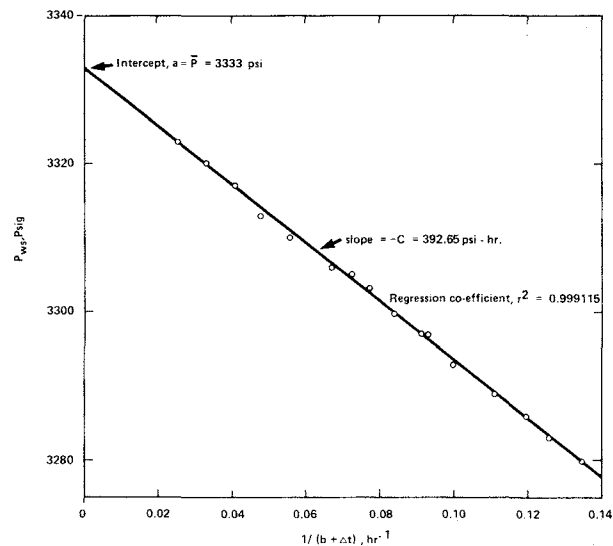
### Field Example

Table 4 provides buildup and other pertinent test data reported by Earlougher.<sup>7</sup> A regression analysis was made for the data for  $2.51 < \Delta t < 37.54$  hours as shown in Fig. 4. The following results were obtained from Fig.

**TABLE 4—PRESSURE BUILDUP TEST DATA FOR THE FIELD EXAMPLE<sup>7</sup>**

$\Delta t$ (hours)	$p_{ws}$ (psig)	Known Reservoir Data	
0.0	2,761	$q_o$ , STB/D	4,900
0.94*	3,266*	$t_p$ , hours	310
1.05	3,267	$r_w$ , ft	4.25/12
1.15	3,268	$C_t$ , psi <sup>-1</sup>	$22.6 \times 10^{-6}$
1.36	3,271	$h$ , ft	482
1.68	3,274	$\mu_o$ , cp	0.20
1.99	3,276	$\phi$	0.09
2.51	3,280	$B_o$ , RB/STB	1.55
3.04	3,283	Depth, ft	10,476
3.46	3,286	A, acres	502.65
4.08	3,289	$r_e$ , ft	2,640
5.03	3,293		
5.97	3,297		
6.07	3,297		
7.01	3,300		
8.06	3,303		
9.00	3,305		
10.05	3,306		
13.09	3,310		
16.02	3,313		
20.00	3,317		
26.07	3,320		
31.03	3,322		
34.98	3,323		
37.54	3,323		

\*Beginning of semilog straight line.



**Fig. 4—Regression analysis of the field example data.**

4. For the intercept,  $a = \bar{p} = 3,333$  psig (22 981.04 kPa) [3,342 psig (23 043.09 kPa) by semilog analysis]. For the slope,  $-c = 392.65$  psi-hr (2907.32 kPa·h), and  $b = 4.898321$  hours. The regression coefficient,  $r^2$ , equals 0.999115.

The permeability-thickness product is calculated from Eq. 16 as follows.

$$kh = \frac{282.39q\mu Bb}{-c} = \frac{(282.39)(4,900)(0.21)(4.8983)}{392.65}$$

$$= 5,351.15 \text{ md}\cdot\text{ft} \text{ (1.61 md}\cdot\text{m)}$$

or

$$k = 11.10 \text{ md} \text{ (12.8 md by semilog analysis).}$$

The semilog slope is

$$m = \frac{2.303(-c)}{4b} = \frac{2.303(392.65)}{(4)(4.8983)}$$

$$= 46.15 \text{ psi/}\sim \text{ (318.20 kPa/}\sim \text{) [40 psi/}\sim \text{ (275.80 kPa/}\sim \text{) by semilog analysis].}$$

The skin factor can be estimated with Eq. 17:

$$s = \frac{1}{2} \left[ \frac{2.303}{m} (a - p_{wf}|_{\Delta t=0}) + \ln \alpha + 5.4316 - \ln t_p \right. \\ \left. - \ln \frac{k}{\phi \mu c_t r_w^2} \right] = \frac{1}{2} \left\{ \frac{2.303}{46.152} (3,333 - 2,761) \right. \\ \left. + \ln 62.287 + 5.4316 - \ln 310 \right. \\ \left. - \ln \left[ \frac{11.10}{(0.09)(0.2)(22.6 \times 10^{-6})(4.25/12)^2} \right] \right\}$$

$$= 6.6 \text{ (8.6 by semilog analysis).}$$

The results obtained by the proposed method are in good agreement with those of semilog analysis. We observe that the difference between values of  $m$  and  $s_D$  calculated from both the methods is rather large; however, if we use  $m = 40$  psi/  $\sim$  (275.80 kPa/  $\sim$ ) in Eq. 17, we obtain a skin value of 8.67.

These differences are expected to occur because the rectangular hyperbola is not an *exact* equivalent of the Horner equation. For instance, at  $\Delta t = 10$  hours, the term  $x/(2\alpha + x)$  of Eqs. 4 and 5 is equal to  $-0.3424$ . Therefore, the value of the largest term neglected is equal to  $-0.01338$ , which is 3.9% of the first term of the series. Given the degree of error involved in any graphical procedure, we believe that the proposed technique is an adequate approximation of the conventional graphical methods of Horner and MDH. The main advantage of the new method is that it always would yield a superior static reservoir pressure estimate in field applications where the drainage shape is not known accurately.

Note that  $\alpha$  of  $+64.287$  calculated from Eq. 8 is positive, a necessary condition for Eq. 4 to be valid. Also,  $x = [t_p - (\alpha - 1)\Delta t]/\Delta t$  having a minimum value of  $-(\alpha - 1)$  when  $\Delta t \rightarrow \infty$  satisfies the second necessary condition:  $-\alpha < x < +\infty$  for Eq. 4 to be valid.

## Discussion

We should recognize that the ideal geometrical reservoir shapes for which mathematical solutions are available do not exist in reality. Also, in multiwell reservoirs, the true no-flow boundaries are the physical boundaries. Although effective no-flow boundaries between wells might exist when all wells are producing, these boundaries move depending on relative producing rates. Thus one ought to estimate approximate reservoir drainage shape even though the same well may be tested every year because of the changing production and/or injection scenarios. Furthermore, whenever a fluid is injected into a reservoir, the *degree* of pressure maintenance has a profound effect on the  $p_{DMBH}$  function.

The facts of the preceding paragraph present serious complications for routine buildup analysis. Taylor<sup>15</sup> and Taylor and Caudle<sup>16</sup> highlighted this complication by using computer-generated flow streamlines for a homogeneous multiwell reservoir with no-flow boundaries. Examination of streamlines dictated the use of a 4:1 rectangle reservoir shape for the drainage boundary. However, Taylor's rigorous solution to the specific problem indicated a higher value for the MBH pressure function. Therefore, the existing methods do not easily afford correct  $\bar{p}$  solution to the actual field problems.

This observation, however, does not imply that the MBH pressure functions are no longer valuable. On the contrary, they can serve a very useful purpose. One can estimate  $p^*$  from a Horner plot and  $\bar{p}$  by the proposed method to calculate the  $p_{DMBH}$ . Corresponding to the  $t_{DA}$  and known  $p_{DMBH}$  for a buildup test, an estimate of the *drainage shape* can be made from the MBH graphs by trial and error. This independent method of estimating a drainage shape from well test data is a significant finding of this study.

The method presented here provides a simple analytical tool for estimating accurate  $\bar{p}$  within a well's drainage volume. All the assumptions<sup>10,11</sup> made while the *Ei* solution to the diffusivity equation is derived are implicit in the rectangular hyperbola equation.

The assumptions for a line-source well are: an ideal, isotropic, and homogeneous formation of constant permeability, porosity, and thickness; a single-phase noncompressible flowing fluid of constant viscosity; small pressure gradients everywhere in the system; and small total system effective isothermal compressibility. One assumption deserves special comment, however. When we make the logarithmic approximation of the *Ei* function, we assume that  $t_D/r_D^2 > 100$ —i.e., the line-source well. Recently, Morrison<sup>24</sup> demonstrated that for short-time drillstem tests in tight formations, Edwardson *et al.*<sup>25</sup> solutions of the  $p_D$  function for a finite-radius wellbore are applicable. Use of the line-source solution causes significant errors when  $kh$  and  $\bar{p}$  are estimated by the Horner method. The proposed method is subject to the condition that  $t_D/r_D^2 > 100$  is true for the reservoir system being investigated.

The rectangular hyperbola predicts transient buildup pressure behavior in the infinite-acting regime and beyond. The early-time data affected by wellbore storage and skin must be eliminated from the analysis. Thus identification of the beginning of the semilog straight line or the end of the storage effect by the well-known log-log type-curve analysis<sup>26-28</sup> is a prerequisite for the use of the proposed method.

The proposed method can be very valuable in a situation where the early-time data are dominated by the wellbore storage phenomenon, while the late-time data are affected by the boundary effects; consequently, eliminating the semilog period in the process. Wells located in a dense spacing and/or near a boundary could exhibit such a characteristic behavior. We still would be able to use the boundary-affected data to obtain a valid  $\bar{p}$  solution. The early-time data could be analyzed by the conventional type-curve analysis for estimating  $kh$  and  $s$ . To our knowledge, no valid  $\bar{p}$  estimation is possible for a situation just described using a conventional technique.

Although the method as presented here has been developed for liquid reservoirs, the application of this technique to gas reservoirs can be made easily. The use of pseudopressure<sup>29</sup> and real time provides an accurate estimate of reservoir properties in most gas-well situations. Strictly speaking,  $p_{DMBH}$  functions are not directly applicable to gas reservoirs for determining  $\bar{p}$ .  $p_{DMBH}$  functions were developed originally for liquid systems where the product of  $\mu c_t$  remains essentially constant for the duration of a well test. However, for a gas reservoir, the pseudopressure transformation does not linearize the diffusivity equation completely because of the nonconstant  $\mu c_t$  and consequently may cause a significant error in the  $\bar{p}$  estimate. Kazemi<sup>30</sup> suggested an iterative scheme to circumvent the problem that stems from the partially linearized diffusivity equation by evaluating the  $\mu c_t$  product at the prevailing average reservoir pressure. More recently, Ziauddin<sup>31</sup> proposed a correction term for the  $p_{DMBH}$  function when determining  $\bar{p}$  in gas reservoirs. This correction term was obtained from Kale and Mattar's<sup>32</sup> semianalytical solution of the diffusivity equation for the gas flow. Interestingly enough, our proposed method is independent of the preceding problem because the rectangular hyperbola directly yields  $\bar{p}$ , unlike an indirect method involving the  $p_{DMBH}$  function.

Extension of the proposed method to drawdown, specifically to reservoir limit testing, has been addressed recently in Ref. 33. For the sake of brevity, we defer discussion on further application of the proposed model to fractured wells, injection wells, and the gas wells for deliverability testing.

## Conclusions

The following conclusions can be made from this study.

1. Pressure buildup behavior of a well can be defined uniquely by a rectangular hyperbola regardless of the boundary conditions—i.e., infinite-acting, no-flow, or various degrees of pressure maintenance at the outer boundary in a homogeneous reservoir.

2. The rectangular hyperbola equation and the logarithmic approximation of the  $Ei$  function that is used to generate the buildup equation of Horner and MDH are

equivalent. Unlike the Horner and MDH methods, the hyperbola can predict pressure behavior beyond the infinite-acting period in most instances. The exceptions are cases where wells are located in either asymmetrical drainage shapes or long narrow rectangles that are characterized by negative  $p_{DMBH}$  values before the  $(t_{DA})_{pss}$  is reached.

3. The asymptote of the three-constant rectangular hyperbolic equation directly yields  $\bar{p}$ , the static reservoir pressure. This technique is therefore superior to the conventional methods of Horner and MDH where  $\bar{p}$  is estimated by indirect methods for closed systems.

4. The proposed method offers a distinct advantage over the conventional methods, because knowledge of neither the well/reservoir configuration nor the boundary condition is required for a routine buildup analysis.

5. The permeability-thickness product and skin also can be calculated from the constants of the hyperbolic equation. However, the conventional methods, when correctly used, would provide superior results.

6. Reservoir drainage shape can be estimated from the MBH pressure functions when the proposed method is used in conjunction with Horner's  $p^*$ .

## Nomenclature

- $a$  = asymptote of rectangular hyperbola, psi (kPa)
- $A$  = drainage area, sq ft ( $m^2$ )
- $b$  = constant of rectangular hyperbola, hours
- $B$  = formation volume factor, RB/STB (res  $m^3$ /stock-tank  $m^3$ )
- $c$  = constant of rectangular hyperbola, psi-hr (kPa·hr)
- $c_t$  = total system compressibility,  $psi^{-1}$  ( $kPa^{-1}$ )
- $h$  = formation thickness, ft (m)
- $k$  = formation permeability, md
- $m$  = slope of linear portion of semilog plot of pressure buildup curve,  $psi/\sim$  ( $kPa/\sim$ )
- $n$  = exponent of logarithmic expansion series, Eq. 4
- $p_{DMBH}$  = MBH pressure, dimensionless
- $p_i$  = initial reservoir pressure, psi (kPa)
- $p_s$  = pressure drop resulting from skin, psi (kPa)
- $p_{wf}$  = flowing bottomhole pressure, psi (kPa)
- $p_{ws}$  = bottomhole shut-in pressure, psi (kPa)
- $\bar{p}$  = volumetric average pressure in drainage area, psi (kPa)
- $\bar{p}_i$  = volumetric average pressure before the last production period in the drainage area of the test well, psi (kPa)
- $p^*$  = pressure obtained from extrapolation of the Horner straight line to a time ratio of unity, psi (kPa)
- $q$  = volumetric producing rate, STB/D (stock-tank  $m^3/d$ )
- $r_D$  = radial distance, dimensionless
- $r_e$  = external radius of drainage boundary, ft (m)



- $r_{inv}$  = radius of investigation, ft(m)
- $r_w$  = wellbore radius, ft (m)
- $s$  = skin factor, dimensionless
- $S$  = semilog slope as defined by Eq. A-2,  
psi/~ (kPa/~)
- $t_D$  = dimensionless producing time based on  
 $r_w$
- $t_{DA}$  = dimensionless producing time based on  $A$
- $(t_{DA})_{pss}$  = dimensionless producing time at the  
beginning of pseudosteady-state flow
- $t_p$  = producing time, hours
- $t_{pss}$  = producing time at the beginning of  
pseudosteady-state flow, hours
- $\Delta t$  = shut-in time, hours
- $(\Delta t)_p$  = shut-in time required to reach  $p$ , hours
- $x$  = dimensionless variable defined by Eq. 3
- $\alpha$  = dimensionless constant introduced in  
Eq. 2
- $\phi$  = porosity, fraction
- $\mu$  = viscosity, cp (mPa·s)

### Subscripts

- max = maximum
- o = oil
- 1,2,3 = separate sets of data

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### APPENDIX A

#### The Maximum Slope of the MDH Graph

Mead<sup>19</sup> had determined empirically that the maximum slope of the MDH graph plotted on Cartesian paper gave the semilog slope,  $m$ . The correctness of Mead's finding is given in the following analysis.

Eq. 19 is reproduced here:

$$p_{ws} = a + \frac{c}{b + \Delta t} \dots \dots \dots (A-1)$$

The semilog slope,  $S$ , for a Cartesian MDH graph is given by

$$S = \frac{dp_{ws}}{d \ln \Delta t} = \frac{dp_{ws}}{d(\Delta t)} \cdot \frac{d(\Delta t)}{d \ln \Delta t} = \Delta t \frac{dp_{ws}}{d(\Delta t)} \quad \dots \dots \dots (A-2)$$

Differentiating Eq. A-1 with respect to  $\Delta t$ , we obtain

$$\frac{dp_{ws}}{d(\Delta t)} = \frac{-c}{(b + \Delta t)^2} \quad \dots \dots \dots (A-3)$$

Combining Eqs. A-2 and A-3, we have

$$S = (-c) \frac{\Delta t}{(b + \Delta t)^2} \quad \dots \dots \dots (A-4)$$

Note that  $S$  is always positive since  $c$  is a negative quantity, while  $b$  is positive. To obtain the shut-in time,  $\Delta t$ , at which  $S$  reaches a maximum, we differentiate Eq. A-4 with respect to  $\Delta t$  and set the equation to zero.

$$\frac{ds}{d(\Delta t)} = (-c) \left[ \frac{1}{(b + \Delta t)^2} - \frac{2\Delta t}{(b + \Delta t)^3} \right] = 0$$

or

$$\Delta t = b \quad \dots \dots \dots (A-5)$$

We now show that  $m$  is a maximum at  $\Delta t = b$  by obtaining the negative value of the differential,  $d^2 S / d(\Delta t)^2$ .

$$\frac{d^2 S}{d(\Delta t)^2} = (-c) \left[ \frac{-2}{(b + \Delta t)^3} - \frac{2}{(b + \Delta t)^3} + \frac{6\Delta t}{(b + \Delta t)^4} \right]$$

At  $\Delta t = b$ , we have

$$\frac{d^2 S}{d(\Delta t)^2} = (-c) \left( \frac{1}{2.6667b^3} - \frac{1}{2b^3} \right) = -ive \quad \dots \dots \dots (A-6)$$

Eq. A-6 is always negative because  $b$  is positive and  $c$  is negative. Therefore, the value of  $S_{max}$  is given by Eq. A-4.

$$S_{max/\Delta t=b} = (-c) \frac{\Delta t}{(b + \Delta t)^2} = \frac{-c}{4b} \quad \dots \dots \dots (A-7)$$

Eq. 14 of the text indicates that the semilog slope,  $m$ , is given by

$$\frac{m}{2.303} = \frac{-c}{4b}$$

Hence the maximum slope on the Cartesian graph of  $p_{ws}$  vs.  $\Delta t$  equals the semilog slope of Horner or MDH.

## APPENDIX B

### Derivation of the Equation for Skin

The flowing bottomhole pressure,  $p_{wf}$ , for a well in an infinite-acting reservoir where the logarithmic approximation of the line-source solution applies, is given by<sup>7,10,11</sup>

$$p_{wf} = p_i - \frac{m}{2.303} \left( \ln t_p + \ln \frac{k}{\phi \mu c_t r_w^2} - 7.4316 + 2s \right) \quad \dots \dots \dots (B-1)$$

The Horner buildup equation is reproduced here from the text:

$$p_{ws} = p_i - \frac{m}{2.303} \ln \left( \frac{t_p + \Delta t}{\Delta t} \right) \quad \dots \dots \dots (B-2)$$

$$p_{wf} - p_{ws} = p_i - \frac{m}{2.303} \left[ \ln t_p - \ln \left( \frac{t_p + \Delta t}{\Delta t} \right) + \ln \frac{k}{\phi \mu c_t r_w^2} - 7.4316 + 2s \right] \quad \dots \dots \dots (B-3)$$

Recalling Eq. 6 of the text, and assuming that  $t_p + \Delta t \approx t_p$ , we have

$$\ln \left( \frac{t_p + \Delta t}{\Delta t} \right) = \ln \alpha - 2 + \frac{4t_p}{t_p + (\alpha + 1)\Delta t} \quad \dots \dots \dots (B-4)$$

Combining Eqs. B-3 and B-4, we obtain

$$p_{ws} - p_{wf} = \frac{m}{2.303} \left[ \ln t_p - \ln \alpha + 2 - \frac{4t_p}{t_p + (\alpha + 1)\Delta t} + \ln \frac{k}{\phi \mu c_t r_w^2} - 7.4316 + 2s \right]$$

or

$$p_{ws} - p_{wf} = \frac{m}{2.303} \left( \ln t_p - \ln \alpha - 5.4316 + \ln \frac{k}{\phi \mu c_t r_w^2} + 2s \right) = \frac{4mt_p}{2.303[t_p + (\alpha + 1)\Delta t]}$$

Rearranging, we have

$$(p_{ws} - a') \left( \frac{t_p}{\alpha + 1} + \Delta t \right) = \frac{-4mt_p}{2.303(\alpha + 1)}$$

or

$$(p_{ws} - a')(b' + \Delta t) = c, \dots\dots\dots (B-5)$$

where

$$a' = p_{wf} + \frac{m}{2.303} \left( \ln t_p - \ln \alpha - 5.4316 + \ln \frac{k}{\phi \mu c_t r_w^2} + 2s \right), \dots\dots (B-6)$$

$$b' = \frac{t_p}{\alpha + 1}, \dots\dots\dots (B-7)$$

and

$$c' = \frac{-4mt_p}{2.303(\alpha + 1)}, \dots\dots\dots (B-8)$$

Note that constants  $b'$  and  $c'$  of Eq. B-5 are identical to the constants  $b$  and  $c$ , respectively, of Eq. 7. Therefore,

$$a' = a. \dots\dots\dots (B-9)$$

Eq. B-6 can be rearranged to obtain an expression for skin,  $s$ ,

$$s = \frac{1}{2} \left[ (a - p_{wf}) \frac{2.303}{m} - \ln t_p + \ln \alpha + 5.4316 - \ln \frac{k}{\phi \mu c_t r_w^2} \right], \dots\dots\dots (B-10)$$

### SI Metric Conversion Factors

acre	× 4.046 856	E+03	= m <sup>2</sup>
bbl	× 1.589 873	E-01	= m <sup>3</sup>
cp	× 1.0*	E-03	= Pa·s
cu ft	× 2.831 685	E-02	= m <sup>3</sup>
ft	× 3.048*	E-01	= m
psi	× 6.894 757	E+00	= kPa
psi <sup>-1</sup>	× 1.450 377	E-01	= kPa <sup>-1</sup>

\*Conversion factor is exact.

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## Comments from J.E. Warren, JPT Executive Editor

### Pressure Buildup Analysis: A Simplified

**Approach** develops a three-constant equation (a rectangular hyperbola) to approximate the exponential integral ( $Ei$ ) function that describes transient pressure behavior. This equation is unique to the well at the time of testing and makes it possible to obtain the volumetric average pressure ( $\bar{p}$ ) directly, regardless of the boundary conditions and the shape of the drainage area in most instances. This paper extends the work of Mead and provides a powerful tool for the practicing engineer because of its simplicity and accuracy in any type of reservoir drainage system. The authors conclude that, in addition to  $\bar{p}$ , the constants of the rectangular hyperbolic equation permit the permeability-thickness product and the skin resistance to be estimated directly and the reservoir drainage shape to be estimated indirectly.

Obviously, the proposed method can be applied to a number of cases involving transient pressure behavior whenever an approximation to an approximation is adequate. (Unfortunately, this probably will generate a new series of papers treating all previously published topics via a lower-order approximation.)

SPE 11867

## Discussion of Pressure Buildup Analysis: A Simplified Approach

Neil V. Humphreys, SPE, Mobil R&D Corp.

The following addresses a paper (Jan. 1983, *J. Pet. Tech.*, Pages 178-88) by A.R. Hasan and C.S. Kabir.

The author's approach to a long-standing problem in well-test analysis is most innovative, and appears to offer a very welcome solution to a persistent problem. Their analysis of the problem is most interesting, and for the examples quoted appears to offer a superior solution technique. However, one of the assumptions implicit in their analysis gives cause for concern.

A fundamental assumption in this paper is that higher-order terms of the expansion

$$\ln(\alpha+x) = \ln\alpha + 2 \left[ \left( \frac{x}{2\alpha+x} \right) + \frac{1}{3} \left( \frac{x}{2\alpha+x} \right)^3 + \frac{1}{5} \left( \frac{x}{2\alpha+x} \right)^5 + \dots \right]$$

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may be ignored. In fact, the authors propose consideration of the first two terms of the expansion only, stating that justification for this will be shown in a field example. I show that this assumption is not always valid.

Using the authors' nomenclature,

$$\frac{x}{2\alpha+x} = \frac{2(t_p + \Delta t)}{t_p + (\alpha+1)\Delta t} - 1,$$

which may be rearranged as

$$\frac{x}{2\alpha+x} = \frac{2[(t_p/\Delta t) + 1]}{(t_p/\Delta t) + \alpha + 1} - 1 = F.$$

Defining this as  $F$ , the given expansion of  $\ln(\alpha+x)$  may be rewritten as

$$\ln(\alpha+x) = \ln\alpha + 2F + \frac{2F^3}{3} + \frac{2F^5}{5} + \dots$$

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$$\ln(\alpha + x) = \ln\alpha + 2 \left[ \left( \frac{x}{2\alpha+x} \right) + \frac{1}{3} \left( \frac{x}{2\alpha+x} \right)^3 + \dots \right]$$

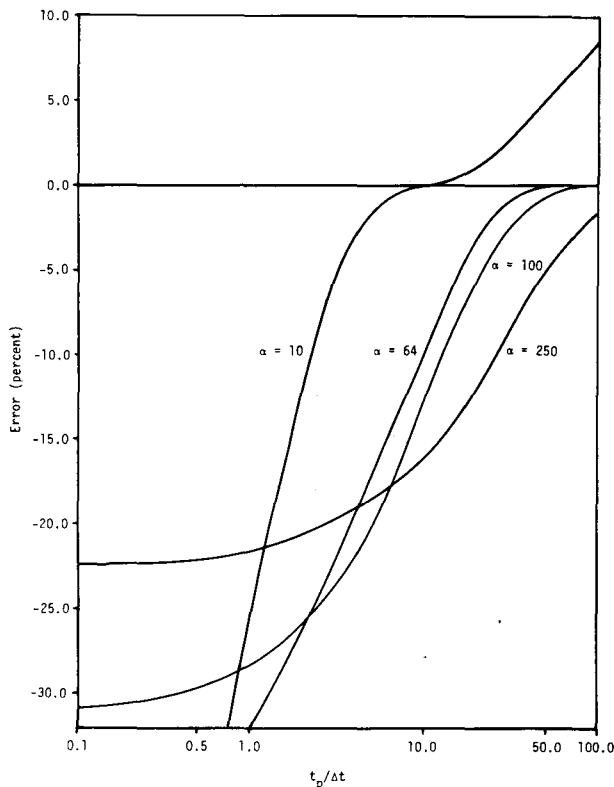


Fig. D-1—Error implied by consideration of only the first two terms of the expansion as opposed to the first three terms.

$$\ln(\alpha + x) = \ln\alpha + 2 \left[ \left( \frac{x}{2\alpha+x} \right) + \frac{1}{3} \left( \frac{x}{2\alpha+x} \right)^3 + \dots \right]$$

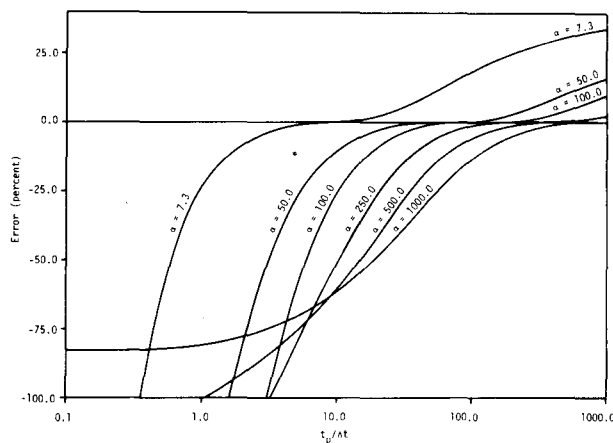


Fig. D-2—Error implied by consideration of only the first 2 terms of the expansion as opposed to the first 11 terms.

TABLE D-1—ERROR (%) FOR VARIOUS  $\alpha$  IMPLIED BY CONSIDERATION OF ONLY THE FIRST TWO TERMS OF THE EXPANSION\* AS OPPOSED TO CONSIDERING THE FIRST THREE TERMS OF THE EXPANSION\*

$t_p/\Delta t$	$\alpha = 7.389$	10.00	50.00	62.287	100.00	250.00	500.00	1,000.00
0.10	-109.6396	-96.7097	-41.3190	-36.9307	-30.8250	-22.4695	-18.4527	-15.5836
0.50	-40.2914	-48.4931	-37.8346	-34.6005	-29.6488	-22.1586	-18.3318	-15.5345
1.00	-17.3637	-25.5962	-34.0153	-31.9548	-28.2590	-21.7775	-18.1820	-15.4736
10.00	0.2105	0.0030	-7.0849	-9.4123	-12.9291	-16.1067	-15.7195	-14.4224
31.00	4.7657	2.7804	-0.2035	-0.7249	-2.6162	-8.3980	-11.3424	-12.2761
50.00	7.3651	5.2589	0.0000	-0.0260	-0.5792	-4.8084	-8.5400	-10.6438
100.00	10.3318	8.5238	0.5568	0.1583	0.0000	-1.1035	-4.1579	-7.3890
250.00	12.5711	11.2288	3.6451	2.5237	0.9618	0.0000	-0.4396	-2.5701
500.00	13.4050	12.2809	6.1810	5.1097	3.2264	0.4005	0.0000	-0.3939
1,000.00	13.8394	12.8377	7.9462	7.1094	5.5297	2.1005	0.3589	0.0000

$$*\ln(\alpha + x) + \ln\alpha + 2 \left[ \left( \frac{x}{2\alpha+x} \right) + \frac{1}{3} \left( \frac{x}{2\alpha+x} \right)^3 + \dots \right]$$

TABLE D-2—ERROR (%) FOR VARIOUS  $\alpha$  IMPLIED BY CONSIDERATION OF ONLY THE FIRST 2 TERMS OF THE EXPANSION\* AS OPPOSED TO CONSIDERING THE FIRST 11 TERMS OF THE EXPANSION\*

$t_p/\Delta t$	$\alpha = 7.389$	10.00	50.00	62.287	100.00	250.00	500.00	1,000.00
0.10	-441.8062	-616.3900	-448.4360	-364.7826	-260.5470	-148.6116	-107.6193	-82.9639
0.50	-66.4600	-103.1356	-259.9822	-246.4687	-210.0029	-139.3406	-104.7172	-81.9666
1.00	-22.9251	-39.8270	-165.1508	-171.7534	-167.4648	-129.0813	-101.2706	-80.7475
10.00	0.2155	0.0030	-9.8180	-14.5857	-25.0572	-49.6313	-61.2616	-63.0202
31.00	6.2353	3.3335	-0.2096	-0.7785	-3.1489	-14.6732	-27.7098	-39.5742
50.00	11.1308	7.2471	0.0000	-0.0262	-0.6190	-6.7996	-16.5096	-28.2575
100.00	18.9886	14.5087	0.5981	0.1632	0.0000	-1.2424	-5.8865	-14.2049
250.00	27.5297	23.7068	5.0285	3.2159	1.0838	0.0000	-0.4713	-3.3456
500.00	31.4854	28.4683	10.6849	8.1602	4.4552	0.4295	0.0000	-0.4224
1,000.00	33.7300	31.3094	16.5929	14.0026	9.6044	2.7017	0.3849	0.0000

$$*\ln(\alpha + x) + \ln\alpha + 2 \left[ \left( \frac{x}{2\alpha+x} \right) + \frac{1}{3} \left( \frac{x}{2\alpha+x} \right)^3 + \dots \right]$$

Thus, the percentage error incurred by truncating the series at the second term as opposed to the third term of the expansion may be evaluated from

$$\text{error} = \frac{(2F^3/3)}{\ln\alpha + 2F + (2F^3/3)} \times 100\%.$$

This has been evaluated for various values of  $\alpha$  and  $t_p/\Delta t$ , and is shown in Table D-1 and Fig. D-1. It is apparent, because of the additive nature of the series, that disregard of higher-order terms will result in some additional, significant, error. This is shown in Table D-2 and Fig. D-2, where the first 11 terms of the expansion are considered. It is observed from the tables that, for a given value of  $\alpha$ , the error incurred by ignoring higher-order terms of the expansion is highly significant when the value of  $t_p/\Delta t$  is not of the same order of magnitude as the value of  $\alpha$ . This is particularly apparent for an infinite acting system ( $\alpha=7.389$ ), where, at small values of  $t_p/\Delta t$ , the error is greater than 100%.

Note from Table D-2 that when  $\alpha=t_p/\Delta t$ , the error implied by the authors' assumptions becomes negligible.

This would imply that for the authors' method to be exactly applicable,  $\alpha=t_p/\Delta t$ . Unfortunately, this makes the authors' Eq. 6 nonlinear in  $\Delta t$ , and thus precludes the representation of this equation by a rectangular hyperbola, which is fundamental to the authors' approach.

It is interesting to note that for examples in the paper where  $\alpha$  may be calculated, the associated  $t_p/\Delta t$  values are such that the error induced by the authors' assumption is small. This may be fortuitous, or it may imply some physical dependence of  $\alpha$  on  $t_p$ , which makes the authors' assumption valid. Such a relationship is not addressed in the paper, nor is one immediately apparent from cursory examination of the authors' equations.

It is possible that such a relationship may be demonstrated, either theoretically or empirically. Until this dependence has been demonstrated, I believe that users of the approach presented in the paper should exercise considerable caution, and it is imperative that they calculate the error implied by the authors' assumption over the entire range of  $t_p/\Delta t$  values they intend to analyze. This appears particularly necessary for systems which approximate to infinite-acting (i.e., low values of  $\alpha$ ) and for low or high values of  $t_p/\Delta t$ .

## SPE 11926

# Authors' Reply to Discussion of Pressure Buildup Analysis: A Simplified Approach

A.R. Hasan, SPE, U. of North Dakota  
C.S. Kabir,\* SPE, Dome Petroleum Ltd.

We appreciate this opportunity to clarify certain points presented in our paper. Of particular significance and interest is the assumption concerning truncation error, made while deriving the rectangular hyperbola equation. Mr. Humphreys' Discussion in this regard is very illuminating.

As he points out, neglecting the higher-order terms in the infinite series expansion for  $\ln(\alpha+x)$  is fundamental to the derivation of the hyperbola equation. This assumption is valid whenever the value of  $\alpha$  is close to  $t_p/\Delta t$ . The dramatically large truncation errors presented in Tables D-1 and D-2 by Humphreys are true for very low  $t_p/\Delta t$  values because of the inherent assumption in our derivation,  $t_p + \Delta t = t_p$ . However, we point out that  $t_p/\Delta t < 1.0$  implies shut-in time greater than the producing time. As Earlougher and Kazemi<sup>1</sup> note, whenever  $\Delta t > t_p$ , no useful *additional* data are obtained for the buildup segment of the test because the radius of investigation is governed by the producing time.

Additionally, the constants  $a$ ,  $b$  (and hence  $\alpha = t_p/b - 1$ ), and  $c$  of the hyperbola equation are obtained by optimization. The optimization procedure leads to a value of  $\alpha$  that does not deviate greatly from  $t_p/\Delta t$ , indicating a minimum truncation error. Thus, we have shown in the boxed portion of Humphreys' Table D-2 the error (%) that would typically apply to *field data*

when analyzed by our method.

Examination of Humphreys' Fig. D-2 reveals that the error band decreases substantially with an increase in  $\alpha$ , because the length of an  $\alpha$  curve on the zero-percent-error line increases dramatically as  $t_p/\Delta t$  is increased. This observation implies that, with an increase in a well's producing life, reliability of the rectangular hyperbola predictions is enhanced.

Thus, the excellent results obtained by the use of a rectangular hyperbola are *not* fortuitous, but rather are inevitable. Mead<sup>2</sup> and much of our own in-house analysis substantiate this point.

With regards to the value of  $\alpha = e^2 = 7.389$  for an infinite-acting reservoir, we wish to point out that this value was calculated on a theoretical basis. The underlying thought was to show that for a finite reservoir,  $\alpha > 7.389$ . Our recent observation indicates that  $\alpha > 7.389$  is true even for an infinite-acting reservoir. Thus,  $\alpha = 7.389$  should be construed as a minimum value for any set of test data. All the parameters in the hyperbola equation, including  $\alpha$ , *must* be determined from the field data as outlined in the second paragraph of Page 181 of our paper.

## References

1. Earlougher, R.C. Jr. and Kazemi, H.: "Practicalities of Detecting Faults from Buildup Testing," *J. Pet. Tech.* (Jan. 1980) 18-20.
2. Mead, H.N.: "A Practical Approach to Transient Pressure Behavior," paper SPE 9901 presented at the 1981 SPE California Regional Meeting, Bakersfield, March 25-26.

\*Now with ARCO Oil & Gas Co.

# Discussion of Pressure Buildup Analysis: A Simplified Approach

Dennis L. Bowles, SPE, Cities Service Co.  
Christopher White, Cities Service Co.

In their paper (*J. Pet. Tech.*, Jan. 1983, Pages 178–88), authors A.R. Hasan and C.S. Kabir ambitiously undertook the task of clarifying the often complex field of pressure-transient analysis. They developed a three-constant hyperbolic equation approximating the logarithmic approximation of the exponential integral (line-source solution). They then stated that the hyperbolic constants may be used to estimate  $kh$ ,  $s$ , and  $\bar{p}$ .

A careful analysis of their work has led us to suspect that application of the proposed method will often yield results significantly different from those obtained by conventional analysis.

## Theory

We examined the applicability of the Hasan-Kabir method during three time ranges: (1) wellbore dominated, (2) infinite-acting, and (3) late transient or boundary-affected.

1. The Hasan-Kabir method considers pressure change to be inversely related to time. Thus, it cannot fit fractured well data for which  $\Delta p$  is proportional to  $\sqrt{\Delta t}$  or storage-dominated behavior for which  $\Delta p$  is linearly proportional to  $\Delta t$ . Analysis of these regimes by the proposed method will result in errors similar to those incurred by misapplication of conventional techniques. Hasan and Kabir do not claim to be able to analyze linear flow or wellbore storage data.

2. Serious difficulties arise when the Hasan-Kabir method is applied to infinite-acting data. They use the common assumption that  $t_p \gg \Delta t$  or  $t_p + \Delta t \approx t_p$ . The implications of that statement with regard to the hyperbolic approximation must be assessed very carefully.

For small  $\Delta t$  ( $t_p/\Delta t \geq 100$ ), we find that significant error results from truncation of the infinite series<sup>1</sup> (Eq. 1) after the second term.

$$\ln(\alpha + x) = \ln \alpha + 2 \sum_{n=1}^{\infty} \frac{1}{2n-1} \left( \frac{x}{2\alpha+x} \right)^{2n-1}, \dots (1)$$

where

$$x = \frac{t_p - (\alpha - 1)\Delta t}{\Delta t}, \dots (2)$$

For an infinite system, Hasan and Kabir show that

$$\alpha = e^2 \approx 7.389, \dots (3)$$

The truncation error for an infinite system is tabulated vs.  $t_p/\Delta t$  in Table D-1. Note that the error may exceed

20%, indicating that the Hasan-Kabir method does not approximate infinite-acting behavior even when  $t_p \gg \Delta t$ .

For large  $\Delta t$ ,  $t_p$  is not much larger than  $\Delta t$  and  $t_p + \Delta t \neq t_p$ . Thus, the Hasan-Kabir method improperly superposes the pressure transient induced by the production preceding shut-in for the buildup test.

In summary, the Hasan-Kabir method cannot accurately model infinite-acting behavior for large or small  $\Delta t$ . We provide a quantitative demonstration of this statement later in this Discussion.

3. The form of the rectangular hyperbola proposed by Hasan and Kabir for analysis of pressure transients is based on the line-source solution in an infinite reservoir. The hyperbola does not bear a close mathematical resemblance to the bounded reservoir solution, which is in terms of Bessel's functions rather than the exponential integral. Therefore, there is no theoretical basis for using the Hasan-Kabir method to analyze data during the late-time (boundary-affected) portion of the test.

All of these factors caused us to have profound reservations regarding the basis and accuracy of the Hasan-Kabir method. We therefore proceeded to formulate comparisons between the Hasan-Kabir method and classical techniques of analysis. We accomplished this by the well-established method of solving the diffusivity equation with appropriate boundary conditions in Laplace space<sup>2,3</sup> and numerically inverting using the Stehfest algorithm.<sup>4</sup>

**Effects of Truncation Error.** The parameters shown in Table D-2 were used to generate an infinite reservoir data set. Two intervals—one meeting all criteria given by Hasan and Kabir, the other including some wellbore-dominated data—were analyzed using the same type of iterative linear regression that they used. Fig. D-1 and Table D-2 clearly indicate that erroneous results are obtained for  $\bar{p}$ ,  $k$ , and  $s$ ; the errors are even more acute when storage-dominated data are included in the analysis. Since no boundary-affected data were analyzed, a correct model should extrapolate to  $\bar{p} = p_i$ . The hyperbolas indisputably and falsely indicate a boundary where conventional methods suggest no such thing. These analyses underestimate both  $\bar{p}$  and drainage area. Thus, use of these parameters in volumetrics will result in a substantial underestimation of reserves. The net result could be failure to develop a potentially profitable discovery.

**Uniqueness of the Solution.** In their Summary and Conclusions, Hasan and Kabir state that the pressure behavior of a well can be defined uniquely by a rectangular hyperbola regardless of boundary conditions. They do, however, note that the constants  $b$  and  $c$  will be somewhat sensitive to the time interval chosen to



**TABLE D-1—TRUNCATION  
ERROR FOR INFINITE SYSTEM**

$t_p/\Delta t$	Truncation Error (%)
1	22.9
6.389	0.0
10	0.2
100	19.2
1,000	42.5
10,000	56.6

**TABLE D-2—PARAMETERS AND RESULTS FOR EXAMPLE 1 (FIG. D-1)**

Parameters		Results of Hasan-Kabir Analysis		
$k$ , md	10		Including Storage Data	Infinite- Acting Data Only
$h$ , ft	500			
$\mu$ , cp	0.20			
$B_o$ , RB/STB	1.2	$k$ , md	4.54	8.55
$t_p$ , hours	3,792	$\bar{p}$ , psig	2,970.4	2,979.1
$t_{pD}$	$10^8$	$s$	-1.3	3.2
$s$	5	$m$ , psig/cycle	85.88	45.65
$C_D$	250	$b$ , hours	16.89	115.85
$\rho_i$ , psig ( $=\bar{p}$ )	3,000	$-c$ , psi-hr	2,519.7	9,185.1
$\phi$	0.10	apparent $r_{eD}$	$\sim 15,000$	
$c_t$ , psi $^{-1}$	$20 \times 10^{-6}$	apparent $r_e$ , ft	$\sim 7,500$	
$r_w$ , ft	0.50			
$\rho_{wf}$ , psig	2504.7			

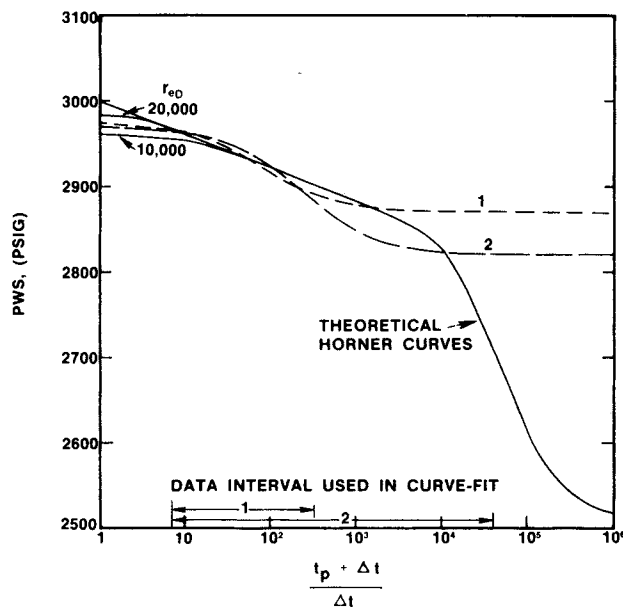
**TABLE D-3—PARAMETERS AND RESULTS FOR  
EXAMPLE 2 (FIG. D-2) (field example from Ref. 5)**

Parameters		Results of Hasan-Kabir Analysis*		
$q_o$ , STB/D	4,900	$k$ , md	11.10	(12.66)
$t_p$ , hours	310	$\rho$ , psig	3,333	(3,331.3)
$t_{pD}$	$2.028 \times 10^7$	$s$	6.6	(7.95)
$r_w$ , /12 ft	4.25	$m$ , psig/cycle	46.15	(40.92)
$c_t$ , /psi $^{-1}$	$22.6 \times 10^{-6}$	apparent $r_{eD}$	5,000	
$h$ , ft	482	$r_e$ , ft	1,771	
$\mu$ , cp	0.20	$b$ , hours	4.898	
$\phi$	0.09	$-c$ , psi-hr	392.65	
$B_o$ , RB/STB	1.55			
$A$ , acres	502.65			
$r_e$ , ft	2,640			
$r_{eD}$	7,454			

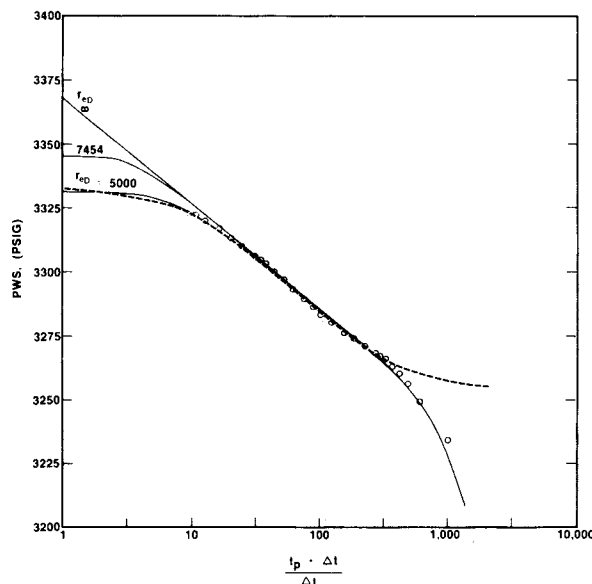
\*Our analysis results appear in parentheses.

**TABLE D-4—PARAMETERS AND RESULTS  
FOR EXAMPLE 3 (FIG D-3)**

Parameters (same as Example 1 except)						
	$t_p$ , hours	379,200				
	$t_{pD}$	$10^{10}$				
	$s$	0				
	$C_D$	0				
	$\rho_{wf}$ , psig	2,596.1				
Results of Analysis by Hasan-Kabir Method						
Data Set	$k$ (md)	$m$ (psig/cycle)	$\bar{p}$ (psig)	$s$	$b$ (hours)	$-c$ (psi-hr)
1	5.35	72.94	2927.9	-6.4	190.2	24,080.0
2	12.87	30.32	2856.5	-2.2	6.392	557.6
3	13.05	29.90	2895.4	-0.53	6.374	5,494
4	22.62	17.25	2934.2	+10.2	617.2	5,556.4



**Fig. D-1**—Horner plot of two hyperbolic fits to infinite reservoir data, showing that the Hasan-Kabir method falsely indicates no-flow boundaries. The hyperbolas (Curves 1 and 2), infinite solution, and two-bounded solutions are displayed.



**Fig. D-2**—Horner plot of the field example from Hasan and Kabir, originally from Ref. 5. The hyperbola (dashed line) and several conventional solutions are shown.

describe the rectangular hyperbola. They attempt to demonstrate the robustness of their method with a series of examples. It was difficult to investigate the validity of their examples because of numerous typographical errors throughout their paper.

Their field example is included as our Example 2.<sup>5</sup> Note that  $t_p$  is not much greater than  $\Delta t$  for all of the test and that this is a bounded reservoir for which their approximation (of the infinite solution) is not appropriate. There is therefore no reason to expect that their method will yield accurate results for this example. However, the data displayed in Table D-3 and Fig. D-2 indicate that in this case the Hasan-Kabir method gives a reasonable estimate of  $\bar{p}$  although their estimates of  $k$  and  $s$  exhibit more error than one would desire. It is also important to note that an engineer using the Hasan-Kabir method would enter material-balance calculations with a PV 120% too high, since our Horner plot indicates that  $r_{eD} = 5,000$  rather than 7,454 that Hasan and Kabir and Earlougher<sup>5</sup> use as a parameter. In some cases, such an error may lead to a decision to develop a field that is actually uneconomic due to insufficient reserves. One should also note that the goodness of fit which Hasan and Kabir obtain in this example is partly because the data analyzed lie in the time interval in which truncation error is relatively small (see Table D-1). Also, boundary effects are apparent during the last 21 hours of the test, and they extrapolate less than one cycle in Horner time. Therefore it seems that their agreement with classical analysis results from fortuitous selection of data rather than the reliability of their method.

It is prudent to examine the sensitivity of the Hasan-Kabir method to the time interval selected for analysis. We generated a purely infinite-acting data set and selected 17 points spanning 4 decades of magnitude. We

subdivided these data into four sets as noted by the legend in Fig. D-3. The data sets were analyzed by the Hasan-Kabir method. The results of these analyses are shown in Table D-4. After comparing the parameters and results, we noted the following.

1. None of the hyperbolas model all the infinite-acting data.
2. One would hope that including more infinite-acting data would improve the curve fit and provide better estimates of  $\bar{p}$ ,  $k$ , and  $s$ . We find that this is not the case.
3. None of the fits yield the correct  $\bar{p}$ . The hyperbolas should all extrapolate to  $\bar{p} = p_i$ .
4. The analyst may choose almost any value of  $\bar{p}$  by choosing the appropriate data set.
5. The error in estimates of permeability ( $k$ ) may exceed 120%. Errors in estimates of skin factor ( $s$ ) are even more severe.
6. All properties are apparent functions of time.

## Conclusions

For clarity, we address the conclusions made by Hasan and Kabir point-by-point.

1. The rectangular hyperbola does not uniquely define the pressure behavior of a well. It is insensitive to boundary conditions, and use of the Hasan-Kabir method can lead to erroneous conclusions regarding  $\bar{p}$  and drainage area.

2. The hyperbolic equation is a truncated series approximating the logarithmic approximation of the exponential integral function and is not equivalent to the buildup equation of Horner or Miller-Dyes-Hutchinson. Even if it were equivalent, there is no reason to believe that the hyperbola can predict pressure behavior beyond the infinite-acting period. It will certainly be in error in highly asymmetric drainage shapes as noted by Hasan

and Kabir.

3. The method of Hasan and Kabir is an empirical curve-fitting technique and does not yield  $\bar{p}$  directly. Conventional methods are direct, although the use of published corrections may cause them to appear to be indirect. We do not find the Hasan-Kabir method superior to conventional analysis.

4. The proposed method is insensitive to boundary conditions. As our examples show, substantial errors in volumetric calculations can result.

5. Conventional methods will always yield superior estimates of  $kh$  and skin. Hasan and Kabir concede this point in their paper.

6. In light of the factors we have discussed, use of the Hasan-Kabir method to estimate drainage shape indirectly will necessarily result in erroneous conclusions.

## Nomenclature

- $a$  = intercept in hyperbolic equation, psi (kPa)  
 $b$  = constant in hyperbolic equation, hours  
 $B$  = formation volume factor, RB/STB (res m<sup>3</sup>/stock-tank m<sup>3</sup>)  
 $c$  = constant in hyperbolic equation, psi-hr (kPa·h)  
 $c_t$  = total reservoir compressibility, psi<sup>-1</sup> (kPa<sup>-1</sup>)  
 $C_D$  = dimensionless wellbore storage coefficient  
 $h$  = formation thickness, ft (m)  
 $k$  = permeability, md  
 $\bar{p}$  = average pressure, psi (kPa)  
 $p_D$  = dimensionless pressure  
 $p_i$  = initial pressure, psi (kPa)  
 $p_{wf}$  = flowing well pressure at time of shut-in, psi (kPa)  
 $p_{ws}$  = static well pressure, psi (kPa)  
 $q$  = flow rate, B/D (m<sup>3</sup>/d)  
 $r_e$  = radius of reservoir, ft (m)  
 $r_{eD}$  = dimensionless radius of reservoir,  $r_e/r_w$   
 $r_w$  = radius of wellbore, ft (m)  
 $s$  = skin factor  
 $t_D$  = dimensionless time  
 $t_p$  = production time, hours  
 $t_{pD}$  = dimensionless production time  
 $\Delta t$  = shut-in time, hours  
 $\Delta t_D$  = dimensionless shut-in time  
 $x$  = dimensionless constant introduced by Hasan and Kabir, defined by Eq. 2  
 $\alpha$  = dimensionless constant introduced by

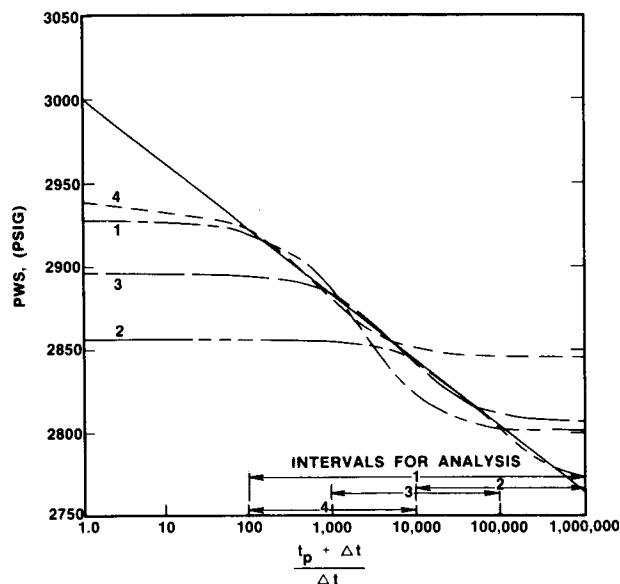


Fig. D-3—Series of hyperbolic fits to a single semilog straight line, demonstrating that the Hasan-Kabir method does not generate unique solutions.

Hasan and Kabir

$\mu$  = viscosity, cp (Pa·s)

$\phi$  = porosity, fraction

## Subscripts

1, 2, 3, 4 = different data sets

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## SI Metric Conversion Factors

bbbl	× 1.589 873	E-01	= m <sup>3</sup>
cp	× 1.0*	E-03	= Pa·s
ft	× 3.048*	E-01	= m <sup>3</sup>
psi	× 6.894 757	E+00	= kPa

\*Conversion factor is exact.

JPT

# Author's Reply to Discussion of Pressure Buildup Analysis: A Simplified Approach

SPE 11925

A.R. Hasan, SPE, U. of North Dakota  
C.S. Kabir,\* SPE, Dome Petroleum Ltd.

Bowles and White make a very *determined* effort to disprove the conclusions we reached in our paper. We must confess to being somewhat taken aback by their vehement dislike of our proposed analysis method.

We discuss their concerns in the same order as they address various flow regimes: (1) wellbore-dominated or early-time behavior, (2) infinite-acting reservoir ( $t_p < t_{pss}$ ) behavior, and (3) infinite-acting period in a bounded system ( $t_p > t_{pss}$ ), during a buildup test.

## Wellbore-Dominated ( $\Delta t < \Delta t_{bst}$ ) Period\*\*

We clearly state in our paper that the wellbore-dominated data *must* be excluded for a valid analysis. This exclusion is also *required* for the conventional Horner and MDH analyses. Same is true when the early-time data are influenced by the fracture linear flow.

We cannot comprehend why Bowles and White bring up this point despite our clear statement on the applicability of the hyperbola. In their Example 1 (Table D-2) they even use the early-time data to demonstrate the inadequacy of the hyperbola!

## Infinite-Acting Reservoir ( $t_p < t_{pss}$ ) Period

They attempt to demonstrate the limitations of the hyperbola, for an infinite-acting reservoir, by showing the so-called truncation error in their Table D-1. We address the question of truncation error in our Reply to Mr. Humphreys.

We point out that for an infinite-acting reservoir, Horner's method always provides superior results because  $\bar{p}$  is obtained by *direct* extrapolation of the semilog line to the Horner time ratio,  $(t_p + \Delta t)/\Delta t$ , of unity. Thus, there is no advantage of using our suggested approach although the results obtained are quite acceptable.

Here, Table R-1 compares the results obtained by Bowles and White's own conventional analysis with those of rectangular hyperbola. To a practicing engineer, the results obtained by the two methods could be considered to be very good even though the assumption of  $t_p + \Delta t \approx t_p$  was made for the development of the rectangular hyperbola equation. Interestingly enough, the hyperbola predicts a  $\bar{p}$  which is 99.303% of the true value—an accuracy considered unacceptable by Bowles and White!

We offer the following on the discrepancy of  $k$  and  $s$  values as shown in Table R-1. One of the most important uses of  $k$  value, obtained from well testing, is in reservoir simulation studies. A reservoir engineer could appreciate that a reservoir having 8.55-md permeability does not perform very differently for most problems

when a  $k$  of 10 md is used or vice versa.

We note that the skin calculated from the hyperbola is lower than the true skin of 5.0. However, from a practical point of view, any decision on a stimulation job is very unlikely to be affected by the difference in the skin pressure drop that we calculate in Table R-1.

Thus, when a rectangular hyperbola is used to describe the pressure behavior in an *infinite-acting reservoir* ( $t_p < t_{pss}$ ), the results obtained are acceptable for all practical purposes, as shown by the results (Table R-1). However, we recommend use of Horner graph for simplicity and accuracy.

**Uniqueness of the Solution.** To prove nonuniqueness of the rectangular hyperbola solutions, Bowles and White chose an *extreme, theoretical* example to illustrate their point of view (Example 3). We cannot conceive of a situation where a well produces for 43.29 years ( $t_p = 379,200$  hours) without  $r_{inv}$  reaching the reservoir boundaries. This fact implies that  $r_e$  is at least  $10^6$  ft! One other impracticality is readily apparent from this example. They analyze the data over two to four log cycles. A practicing engineer rarely gets an opportunity to analyze data over a  $1\frac{1}{2}$  log cycle without the storage and/or skin effects. For the example cited (Example 3), a  $(t_p + \Delta t)/\Delta t = 100$  translates to a shut-in time of 3,830 hours or 160 days. We are unaware of such long shut-in times being used for reservoirs having transmissivity in the order of 25,000 md-ft/cp.

Even in this extreme case, for Data Set 2, they show a maximum error of 4.8% in the  $\bar{p}$  estimate. Data Set 2 reflects a realistic shut-in time of 38 hours. Permeability of 12.87 md compares favorably with the true value of 10 md. Skin, however, is more optimistic than desired.

Because of the assumption of  $t_p + \Delta t = t_p$ , the hyperbola is not expected to provide unique solutions as Bowles and White had hoped to obtain. We restate that the Horner method is obviously superior whenever  $t_p < t_{pss}$ . The primary application of hyperbola lies in bounded and/or pressure-maintained systems, where  $\bar{p}$  is difficult to determine by the conventional methods.

The use of the word "unique" in our paper is admittedly rather confusing. We meant to say that the constants of the hyperbola are defined by the data from infinite-acting period no matter what the *shape* of the reservoir is, hence the unfortunate choice of the word unique. However, we clearly indicated that the constants  $a$ ,  $b$ , and  $c$  are somewhat sensitive to the choice of the data set (see Table 3 of our paper, Page 182). Our statement concerning the equivalence of Horner and hyperbola equations (our second conclusion) is also somewhat ambiguous. We intended to say that the mathematical derivation we presented shows the equivalence between these two equations for the infinite-acting data under the constraints we have already discussed.

\*Now with ARCO Oil & Gas Co.

\*\* $\Delta t_{bst}$  = shut-in time prior to the beginning of the semilog straight line.

### Bounded Systems ( $t_p \geq t_{pss}$ )

Bowles and White state that the hyperbola is *not* capable of predicting pressures in bounded systems, because the hyperbola does not bear a close mathematical resemblance to the bounded reservoir solution. We refer them to Pages 2-92 of Ref. 1, where Eq. 1 is presented.

$$p_D = \frac{p^* - p_{wf}}{p_i q_D} = \frac{1}{2} (\ln t_D + 0.809). \dots\dots\dots (1)$$

We quote Ref. 1: "Eq. 1 [Eq. 2-128 of Ref. 1] represents the pressure at a well in a reservoir of *any shape*, may be obtained by replacing  $p_i$  by  $p^*$  in Eq. 2 [Eq. 2-75 of Ref. 1], which is the equation applicable to an infinite reservoir."

$$p_D = \frac{p_i - p_{wf}}{p_i q_D} = \frac{1}{2} (\ln t_D + 0.809). \dots\dots\dots (2)$$

No further discussion of this point is necessary.

One of the most remarkable and contradictory features about their Discussion is that they do *not* believe in the hyperbola's ability to predict pressures in bounded systems, yet they use a field example where  $t_p > t_{pss}$  (310 hours > 264 hours) to illustrate their point of view, which is not clearly understood.

While citing Example 2 (field example in our paper, also Earlougher<sup>2</sup>) they state that the agreement between their analysis and ours is perhaps coincidental and fortuitous. Could Bowles and White explain why we obtain a  $\bar{p}$  anywhere near their value if the hyperbola does not trace beyond the infinite-acting period?

We prepared Table R-2 to compare the various results. One readily observes that Earlougher obtains a slightly higher  $\bar{p}$  because of lower  $p_{DMBH}$  value for a correspondingly higher drainage area. Earlougher<sup>2</sup> clearly states that the drainage radius of 2,640 ft is only an *estimate*. For all practical purposes, it matters very little whether the true  $p$  is 3,342 or 3,331.3 psig.

We note with interest that our analysis yields a  $\bar{p}$  of 3,333 psig (for the data for  $2.51 < \Delta t < 37.54$  hours) without having to enter the drainage radius as a variable. This observation indicates the superiority of our approach in field applications.

Bowles and White express undue concern regarding PV calculations. If a material-balance approach is used, one could easily obtain  $\bar{p}$  as a function of producing time for a known reservoir drainage area from Eq. 3.<sup>3</sup>

$$\frac{kh}{141.3qB\mu} (p_i - \bar{p}) = 2\pi t_{DA} \dots\dots\dots (3)$$

Because hyperbola could predict  $\bar{p}$  accurately without the knowledge of the drainage area,  $A$ , use of Eq. 3 conceivably could give an estimate of  $A$ .

Bowles and White also state in their Example 2 that the last 21 hours of the test are *affected* by the boundary. Interestingly enough, Earlougher<sup>2</sup> could draw a semilog straight line through the points in question. We concur with Earlougher's analysis, and we fail to understand what Bowles and White are trying to say.

**TABLE R-1—COMPARISON OF RESULTS PRESENTED BY BOWLES AND WHITE—EXAMPLE 1 ( $t_p < t_{pss}$ )**

Parameter	Bowles and White	Rectangular Hyperbola
$k_o$ , md	10.0	8.55
$p_i (= \bar{p})$ , psig	3,000.0	2,979.1
$s$	5.0	3.2
$\Delta p_{skin}$ , psi	169.78	127.09

**TABLE R-2—COMPARISON OF VARIOUS RESULTS—EXAMPLE 2 ( $t_p > t_{pss}$ )**

Parameter	Bowles and White	Rectangular Hyperbola	Earlougher <sup>2</sup>
$\bar{p}$ , psig	3,331.3	3,333.0	3,342.0
$k_o$ , md	12.66	11.10	12.80
$s$	7.95	6.60	8.60

### Concluding Remarks

1. We do not understand what the *confused* and *contradictory* arguments of Bowles and White accomplish. All the conclusions reached in our paper are *valid*.

2. The use of a hyperbola is clearly meant for bounded and/or pressure-maintained reservoirs where  $\bar{p}$  is difficult to determine. For an infinite-acting reservoir ( $t_p < t_{pss}$ ), Horner's approach is certainly desirable. We may not have stated this point clearly in our paper, but it was certainly implied because all the examples presented satisfy the condition  $t_p > t_{pss}$ .

3. Agreement between our results and those of conventional analyses is certainly *not* fortuitous, but has theoretical basis. Mead<sup>4</sup> has proved the validity of the solutions for numerous field examples. Our in-house analysis of a large number of wells indicates the same.

4. Whenever a well/reservoir configuration is possible to determine for field applications, conventional methods (Horner in conjunction with MBH) would provide superior results. However, as explained in our paper, such geometries for which MBH solutions are available are very difficult to define in the real world.

5. Conventional methods provide superior estimates of skin and permeability. These values could also be obtained from the rectangular hyperbola with accuracy comparable to the conventional methods.

### References

1. *Gas Well Testing: Theory and Practice*, fourth edition, Alberta Energy Resources Conservation Board, Calgary, Alta., Canada (1979).
2. Earlougher, R.C. Jr.: *Advances in Well Test Analysis*, Monograph Series, SPE, Dallas (1977) 5.
3. Ramey, H.J. Jr. and Cobb, W.M.: "A General Pressure Buildup Theory for a Well in a Closed Drainage Area," *J. Pet. Tech.* (Dec. 1971) 1493-1505; *Trans., AIME*, 251.
4. Mead, H.N.: "A Practical Approach to Transient Pressure Behavior," paper SPE 9901 presented at the 1981 SPE California Regional Meeting, Bakersfield, March 25-26.

### SI Metric Conversion Factors

cp	× 1.0*	E-03	= Pa·s
ft	× 3.048*	E-01	= m
psi	× 6.894 757	E+00	= kPa

\*Conversion factor is exact.

# Discussion of Pressure Buildup Analysis: A Simplified Approach

SPE 13676

Liu Ruei Jin, SPE, Inst. of Chang-Qing Oil Field

This Discussion addresses "Pressure Buildup Analysis: A Simplified Approach" by Hasan and Kabir (Jan. 1983 *JPT*, Pages 178-80) and the relevant Discussions and Replies (May 1983 *JPT*, Pages 905-17). I agree with some of the authors' work in estimation of static reservoir pressure,  $\bar{p}$ , but I think that this approach may be simplified and made more rigorous by the following method.

From the authors' Eqs. 7 and 19, respectively, and because of the unique slope in a line, in a set of any three points of Fig. 4 we have

$$\frac{\frac{p_2 - p_1}{1} - \frac{p_3 - p_2}{1}}{b + t_1 - b + t_2} = 0,$$

where

- $p$  = reservoir pressure, psi [kPa],
- $b$  = constant of rectangular hyperbola, hours,
- and
- $t$  = time, hours.

When  $t_2 - t_1 = t_3 - t_2$ , and  $p_1$ ,  $p_2$ , and  $p_3$  are the corresponding pressures, respectively, this equation is true also. After simplifying and rearranging,

$$b = \frac{t_1 p_1 - t_2 p_2 + t_3 p_3 - t_3 p_2}{2p_2 - p_1 - p_3} \dots \dots \dots (1)$$

(SPE 13676)

Again, as a result of the unique slope of the line and the authors' Eq. 7, we have

$$(p_1 - a)(b + t_1) = (p_2 - a)(b + t_2) = (p_3 - a)(b + t_3),$$

where  $a$  is the asymptote of rectangular hyperbola, psi [kPa]. Then,

$$a = \frac{bp_2 - bp_1 + t_2 p_2 - t_1 p_1}{t_2 - t_1} = \bar{p} \dots \dots \dots (2)$$

From the authors' Table 4, after a regression analysis for  $2.51 < \Delta t < 37.54$  hours, the Miller-Dyes-Hutchinson (MDH) linear equation may be obtained as follows.

$$t_p = 3267.023 + 37.423 \ln t \dots \dots \dots (3)$$

and we can estimate permeability,  $k$ , and skin factor,  $s$ , with Eq. 3 and other data directly. If Eq. 3 is extrapolated to  $t_1 = 10$  hours,  $t_2 = 20$  hours, and  $t_3 = 30$  hours. Therefore,  $p_1 = 3,304.45$  psig [22 780 kPa],  $p_2 = 3,316$  psig [22 863 kPa], and  $p_3 = 3,322$  psig [22 904 kPa], respectively. Substituting for them to my Eqs. 1 and 2, we may obtain  $b = 11.62$  and  $a = 3,341$  psig [23 035 kPa] =  $\bar{p}$ .

This is in better agreement with field data and the 3,342 psig [23 042 kPa] obtained by the semilog analysis than Hasan and Kabir's 3,333 psig [22 980 kPa]. Perhaps this modified procedure is more simplified in practice and rigorous in mathematics.

SOLVING THE RECTANGULAR HYPERBOLA  
EQUATION FOR PRESSURE BUILDUP ANALYSIS

by

J. P. MEYER, A. R. HASAN, AND C. S. KABIR

12353

Application of the rectangular hyperbola equation to pressure buildup analysis has been recently discussed.<sup>1</sup> We suggested an iterative linear regression scheme for estimating the values of the parameters  $a$ ,  $b$ , and  $c$  in the hyperbolic equation that leads to a direct estimation of the average reservoir pressure  $\bar{p}$ . The purpose of this note is to discuss a direct regression scheme as well as an improved iterative regression scheme for estimating the three parameters in the hyperbolic equation.

Direct Regression Approach

The rectangular hyperbolic equation

$$(p-a)(b+\Delta t) = c \quad (1)$$

can be rearranged in the following form:

$$p = a' \Delta t + b' (p \Delta t) + c' \quad (2)$$

where

$$a' = a/b \quad (3)$$

$$b' = -1/b \quad (4)$$

$$c' = (ab+c)/b \quad (5)$$

Eq. 2 is linear in variables  $\Delta t$  and  $p\Delta t$  with parameters  $a'$ ,  $b'$ , and  $c'$ .

Setting

$$x_1 = \Delta t \quad (6)$$

$$x_2 = p\Delta t \quad (7)$$

we have

$$p = a' x_1 + b' x_2 + c' \quad (8)$$

To determine coefficients  $a'$ ,  $b'$ , and  $c'$ , linear least squares regression may be used.<sup>2</sup> This is done by minimizing the squared residual error between the observed pressures and those calculated by Eq. 8,

$$\begin{aligned} \epsilon &= \sum (p_0 - p)^2 \\ &= \sum (p_0 - a' x_1 - b' x_2 - c')^2 \end{aligned} \quad (9)$$

By differentiating Eq. 9 with respect to each parameter and setting the result to zero, we obtain the following set of normalized equations:

$$a' \sum x_1^2 + b' \sum x_1 x_2 + c' \sum x_1 = \sum P x_1 \quad (10)$$

$$a' \sum x_1 x_2 + b' \sum x_2^2 + c' \sum x_2 = \sum P x_2 \quad (11)$$

$$a' \sum x_1 + b' \sum x_2 + c' N = \sum P \quad (12)$$

where  $N$  is the total number of data points. Eq. 10, 11, and 12 can be expressed in matrix form

$$\begin{matrix} A & \cdot & C & = & D \\ = & - & - & & \end{matrix} \quad (13)$$

or

$$\begin{bmatrix} \sum x_1^2 & \sum x_1 x_2 & \sum x_1 \\ \sum x_1 x_2 & \sum x_2^2 & \sum x_2 \\ \sum x_1 & \sum x_2 & N \end{bmatrix} \begin{bmatrix} a' \\ b' \\ c' \end{bmatrix} = \begin{bmatrix} \sum P x_1 \\ \sum P x_2 \\ \sum P \end{bmatrix} \quad (14)$$

The solution of this linear set for the column vector  $C$ , whose elements are  $a'$ ,  $b'$ , and  $c'$  is of the general form

$$\begin{matrix} C & = & A^{-1} \cdot D \\ = & & = \end{matrix} \quad (15)$$

Packaged programs are available for TI 59, HP67, and other calculators for solving the 3 x 3 matrix of Eq. 14.



After the least square coefficients have been determined, the original parameters a, b, and c of the hyperbola (and hence average reservoir pressure,  $\bar{p} = a$ , and semilog slope  $m = -0.5758 c/b$ ) can be computed from Eqs. 3, 4, and 5. We have developed a program for a TI 59 calculator which gives  $\bar{p}$ , b, and m directly from pressure-time data input and is available upon request.

### Iterative Regression Approach

Eq. 1 is written in the following form:

$$p = a + \frac{c}{b + \Delta t} \quad (16)$$

If the value of b is known, the parameters a and c can be obtained by linear regression with  $1/(b + \Delta t)$  as the independent variable. Since the value of b is unknown, an iterative solution is needed to search for the optimum value of b for which the correlation coefficient is maximum. Any such search will be aided by a good starting value for b. There are two methods of estimating an initial value of b.

Method 1: For Eq. 1 to be exactly the same as the Horner equation, it is necessary that the expansion of  $\ln(\alpha + x)$  satisfy the condition<sup>1</sup>

$$\frac{x}{2\alpha + x} = \frac{2(t + \Delta t)}{t + (\alpha + 1)\Delta t} - 1 = 0 \quad (17)$$

or

$$\Delta t = t / (\alpha - 1) \quad (18)$$

Because  $\alpha \gg 1$

$$b = t_p / (\alpha + 1) \approx t_p / \alpha$$

and

$$\Delta t = t_p / (\alpha - 1) \approx t_p / \alpha \quad (19)$$

or

$$b = \Delta t$$

Since  $\Delta t$  is variable, an average shut-in time should be used to approximate  $b$ . Experience indicates that the geometric mean of the first and last shut-in time data (that would be used for the analysis) is an excellent starting value for  $b$ . Thus

$$b = \sqrt{(\Delta t)_1 (\Delta t)_n} \quad (20)$$

Method 2: Another way to estimate a starting value for  $b$  would be to use three data points and three equations to solve for the three parameters in Eq. 1 or 16. Algebraic manipulations lead to the following expression for  $b$ :

$$b = \frac{(\Delta t_2 - \delta \Delta t_1)}{(\delta - 1)} \quad (21)$$

where

$$\delta = \left( \frac{P_1 - P_3}{P_2 - P_3} \right) \left( \frac{\Delta t_3 - \Delta t_2}{\Delta t_3 - \Delta t_1} \right) \quad (22)$$

The three sets of data should include the early, middle, and late time data belonging to the semilog period.

### Field Example

Table 1 compares the two approaches using the field example cited in the paper (Table 4, Ref. 1). Ten data points between  $2.51 \leq \Delta t \leq 10.05$  (excluding  $\Delta t = 5.97$ ) were used. The entries in Table 1 show that the two methods yield virtually identical values of  $\bar{p}$ ,  $m$ ,  $k$ , and  $s$ , although the direct regression approach takes less computation time on a TI59 calculator than the iterative method.

Table 1: Comparison of Solution Methods

	<u><math>\bar{p}</math></u> (psig)	<u><math>m</math></u> (psi/cycle)	<u><math>k</math></u> (md)	<u><math>s</math></u>	<u>Computation</u> <u>Time (min)</u>	<u>Standard</u> <u>Deviation</u> ( $\sigma$ psi)
Direct Regression	3334	46.12	11.66	6.58	1:05	0.365
Iterative Regression	3333	46.51	11.57	6.45	7:00	0.358

where

$$\sigma = \frac{\sum (p_0 - p)^2}{N - 1}$$

### References:

<sup>1</sup>Hasan, A. R., and Kabir, C. S., "Pressure Buildup Analysis: A Simplified Approach," J. Pet. Tech. (Jan. 1983) 178-188.

<sup>2</sup>Guttman, I., and Wilkes, S., Introductory Engineering Statistics, John Wiley and Sons, New York, N. Y., pp. 241-259, 1965.