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Technical Report: NAVTRADEVCEN 1205-6

SIMULATION OF HELICOPTER AND V/STOL AIRCRAFT, VOLUME VI, XC-142 ANALOG COMPUTER PROGRAM STUDY: XC-142A SIMULATION EQUATIONS MECHANIZATION

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ABSTRACT

This report presents the analysis and simplification procedures that are required to define and program the mathematical model for the XC-142A aircraft in a form which is suitable for mechanization and solution on a general purpose analog computer. This program will enable USNTDC to perform dynamic simulation studies for a V/STOL tilt-wing aircraft.

Section II contains the complete mathematical model of the XC-142 with accompanying denotation and validation.

In Section III, three sets of simulation equations are presented. These sets represent the complete six degrees of freedom equations, longitudinal mode equations, and lateral-directional mode equations.

Section IV contains the mechanization functional block diagrams along with the patching and operating instructions required for their utilization. Section IV also specifies the analog computer installation which is required to solve the mechanizations.

The subsequent sections contain: a discussion of program limitations, conclusions, and recommendations.

FOREWORD

NAVTRADEVCEN Technical Report 1205-6 presents the VTOL analysis required for derivation of simulation equations for the XC-142 tilt wing VTOL. An XC-142 math model is defined and presented in a form suitable for mechanization and solution on a general purpose analog computer.

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SECTION I

INTRODUCTION

The study described in this report is an extension of the tilt-wing aircraft investigation originally reported in NAVTRADEVCEN 1205-2. The purpose of this additional effort is to define a mathematical model for the XC-142A aircraft in a form amenable to mechanization and solution on a general purpose analog computer. This program can then be used to conduct dynamic simulation investigation for a V/STOL tilt-wing aircraft as exemplified by the XC-142A.

In the following sections are descriptions of the mathematical model, the simulation equations, and the mechanisation in functional flow charts along with patching and operating instructions.

Since flight characteristics are of prime importance, no attempt has been made to simulate systems or unusual environmental characteristics in this report.

SECTION II

MATHEMATICAL MODEL

NAVTRADEVCEN Reports 1205-2 and 1205-3, which were prepared by Melpar, Inc., presented an unabridged mathematical model for the XC-142A aircraft which reflected data that was available at the time. Since the publication of those reports, however, Ling-Temco-Vought (manufacturer of the XC-142A) further defined the aerodynamic coefficient data and incorporated flexibility and Mach number effects into their basic equations. As a result of these developments, it was necessary to modify the math model of the XC-142A. This modification effort involved data acquisition, data reduction, and data analysis and interpretation. Appendix A defines the symbols used in the following pages.

A. Data Collection

The unabridged mathematical model for the XC-142A is based on the new data obtained from Ling-Temco-Vought, which is presented as Appendix B. It represents the latest data available on the aerodynamic characteristics of the XC-142A. Figure 1 is a three-view arrangement of the XC-142A and Table 1 lists some of the dimensional data of the aircraft.

B. Data Reduction

After compiling the manufacturers data (Appendix B), it was necessary to convert it to a form which could readily be used by a general purpose analog computer. The majority of the aerodynamic variables were presented in polynomial form including some 5th degree terms. The conversion process entailed generating digital computer (SDS-920) programs which would solve L-T-V's mathematical expressions, defining the excursion limits of the particular variable involved, plotting the digital computer outputs, fitting the resulting curves with straight line segments, and finally rewriting the equations as they would be expressed for analog computer simulation. Appendix C presents the individual functions both in graphic and tabulated form. Appendix D contains several typical digital computer (SDS-920) programs which were used to convert Ling-Temco-Vought data to the functions as presented in Appendix C

C. Data Analyses and Interpretation

The following paragraphs are devoted to a term by term analysis of the various aerodynamic parameters and equations of motion. Included is a comparison of the equations as they appeared in 1205-2, in the new polynomial form and as they are converted to analog form.

The equations of motion presented provide a continuous solution of the aerodynamic characteristics for:

1. Aftward and lateral airspeeds to 100 ft/sec and forward airspeeds to 700 ft/sec.

2. Altitude density variations from sea level to 25,000 ft. for standard day conditions.

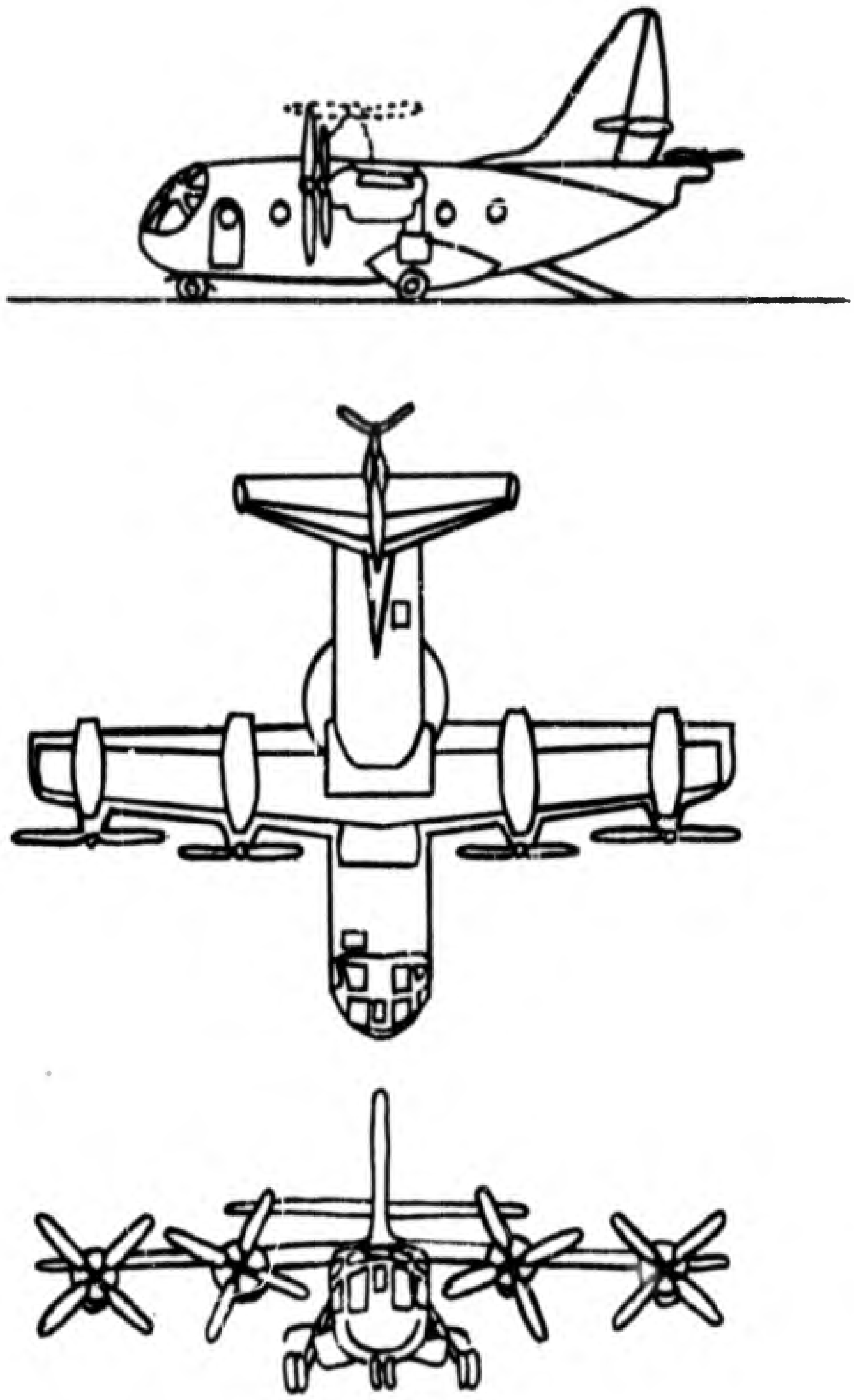


Figure 1. Three-View Arrangement

CHARACTERISTIC	VALUE
Normal Gross Weight	37,474 pounds
Center of Gravity (aft of the leading edge of the mean aerodynamic chord (MAC))	
Max Forward	10% MAC
Max Aft	28% MAC
Wing	
Total Area	534.37 ft^2
Span	67.5 ft
Aspect Ratio	8.53
Dihedral Angle	-2.12°
Airfoil Section	NACA 63-318 (Mod)
Mean Aerodynamic Chord	8.072 ft
Trail Edge Flaps - Double slotted	
Maximum Deflection	60°
Deflection for take off (STOL)	40°
Deflection for landing (STOL)	60°
Leading Edge Flaps	
Deflection	87°
Ailerons - Plain	
Maximum Deflection (wing up)	$\pm 50^\circ$
Maximum Deflection (wing down)	$\pm 20^\circ$
Horizontal Stabilizer - All moving	
Area	163.5 ft^2
Span	31.14 ft
Aspect Ratio	5.08

Table 1. XC-142A Physical Characteristics

CHARACTERISTIC	VALUE
Horizontal Stabilizer - All moving (Cont'd)	
Dihedral Angle	0°
Airfoil Section (root)	NACA 0015
Airfoil Section (tip)	NACA 0012
Maximum Deflection (leading edge up)	40°
Maximum Deflection (leading edge down)	-40°
Hinge Line, % of tail mean geometric chord	13%
Vertical Tail	
Area: Fin to rudder hinge	95 ft^2
Area: Rudder aft of hinge	21.6 ft^2
Aspect Ratio	1.87
Airfoil Section (root)	NACA 0018
Airfoil Section (tip)	NACA 0012
Fuselage	
Length	50 ft
Length (including tail rotor)	58.12 ft
Outside Height	10.72 ft
Outside Width	9.25 ft
Maximum cross-sectional area	90 ft^2
Propellers	
Diameter	15.625 ft
Number of Blades	4
Tail Rotor	
Diameter	8 ft
Number of Blades	3

Table 1. XC-142A Physical Characteristics (Cont'd)

3. Gross weight and C.G. conditions with corresponding variation due to wing incidence.

Included are:

1. Mach effects

2. Airframe flexibility effects

3. Control surface back-off due to aerodynamic loading and simulated P/C hinge moment limits.

1. <u>Axis Systems</u>

In addition to the conventional axis systems (inertial, body, stability and wind) required to define the cumulative aerodynamic characteristics of the XC-142A tilt wing aircraft, two additional types of axis systems must be employed. The first type arises because the relative wind acting upon the wing may differ from the relative wind acting upon the fuselage. This phenomenon is caused by propeller inflow velocities. Therefore the need exists to define a wing stability axis system which will be used to compute wing angle of attack and velocity. These parameters will be subsequently utilized to determine the aerodynamic forces created by the wing. Figure 2 shows the wing tilt angle (i_w) and the variable distances ($x_{a.c.}$ and $z_{a.c.}$) that track the aerodynamic center (a.c.) of the wing as the wing is tilted through the angle i_w. The aerodynamic center of the wing as located in the x-z body axis plane is the origin of the wing stability axes. The wing stability axis system is analogous to the aircraft axis system but the x axes of the two systems are in general not parallel to each other because the wing relative wind differs significantly from the fuselage relative wind. This is caused by the fact that the wing experiences an additional velocity caused by propeller inflow which does not affect the fuselage. The wing stability axis system has its origin translated by $x_{a.c.}$ and $z_{a.c.}$ from the aircraft stability axis origin (which is located at aircraft c.g. nominally) as in Figure 2.

The second type of axis system, the propeller axis system, is shown in Figure 3. The axis system is repeated for each main propeller, so that there are actually four main propeller axis systems. The propeller axis system will enable the development of the propeller thrust (T_n), propeller torque (Q_n) in terms of propeller power, a normal force (N_n^*) perpendicular to T_n along the propeller blade, and propeller moments (M_n) and (Y_n).

Further development of these propeller forces and moments will be considered in the discussion of aerodynamic effects.

Before continuing, we will develop the aircraft body axes to inertial axes transformation. The matrices are:

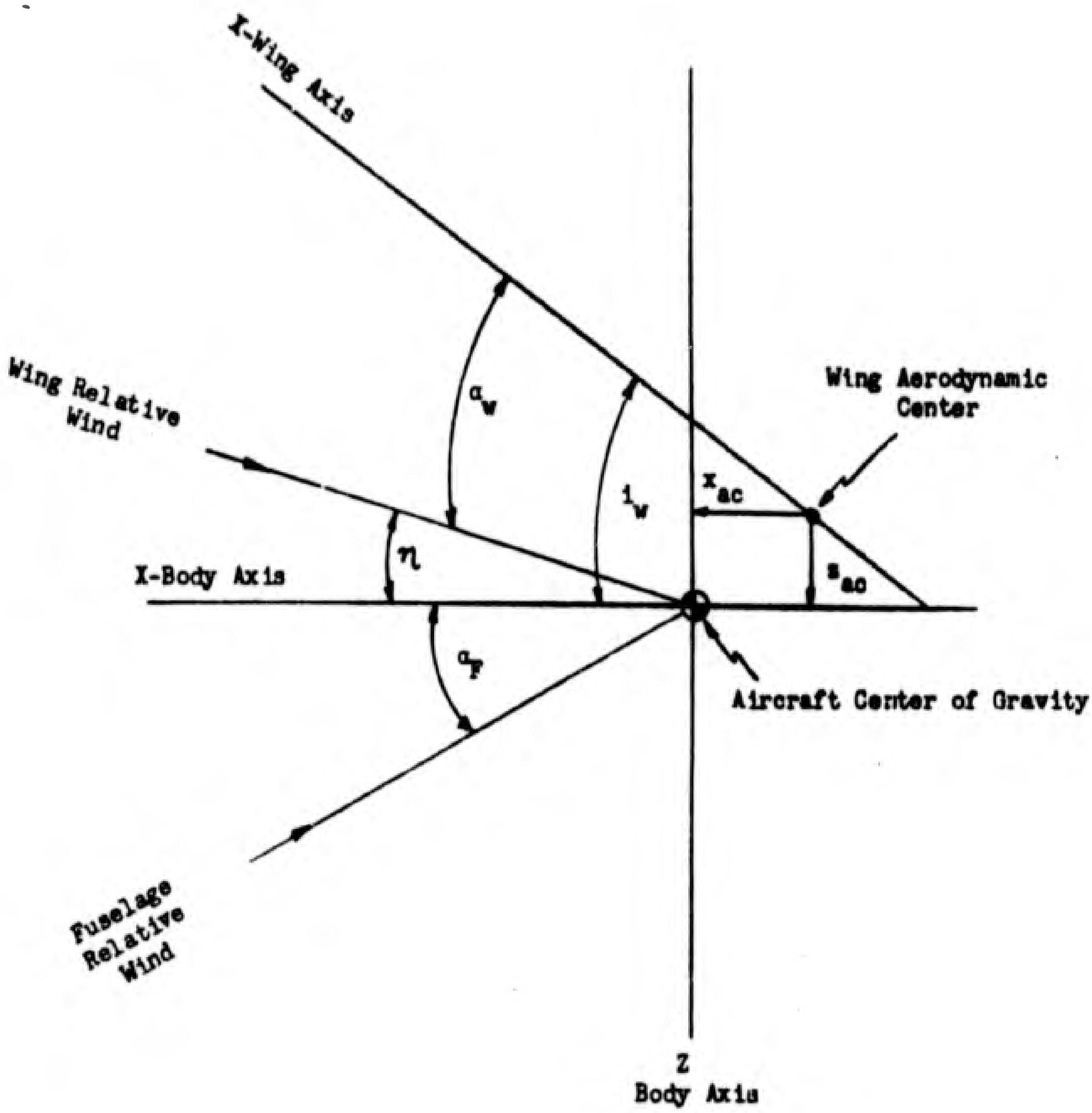


Figure 2. Wing Stability Axis System

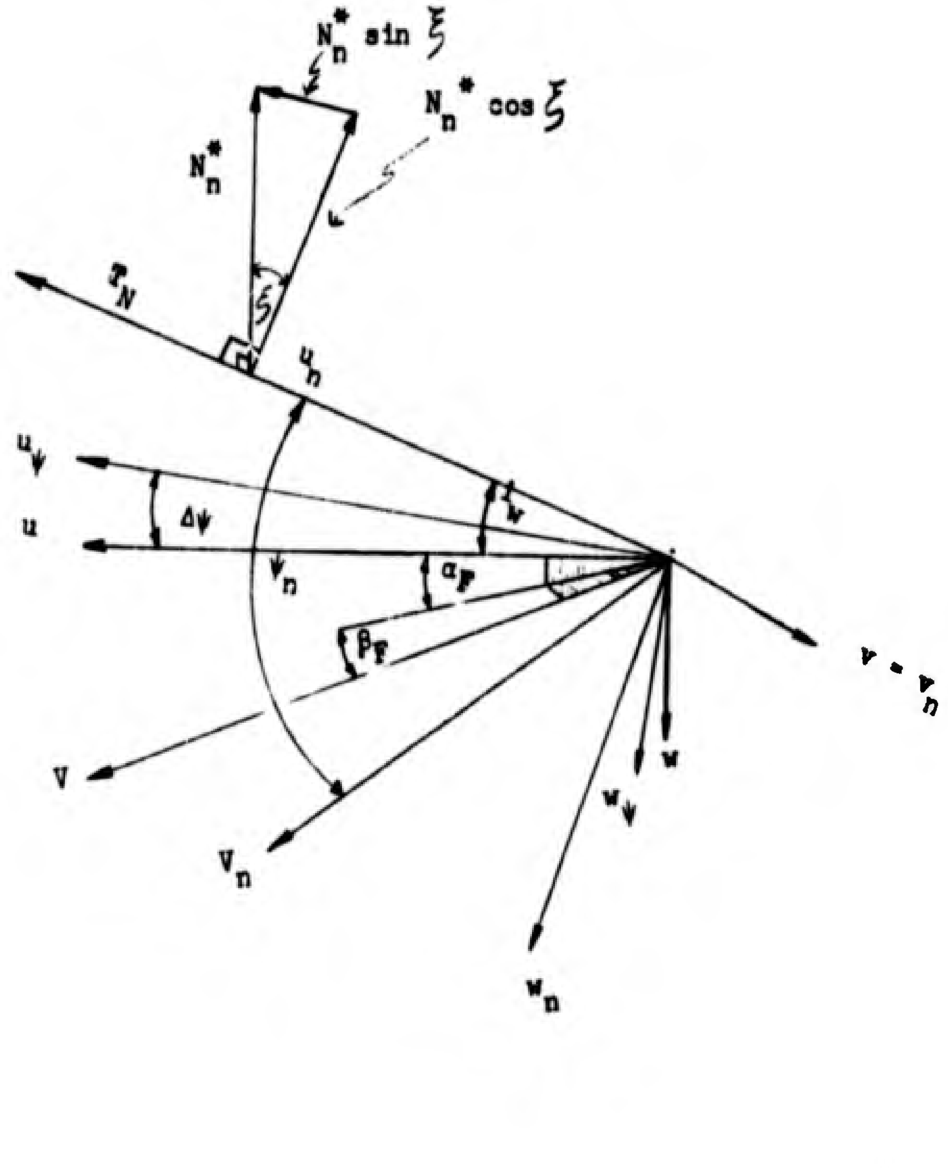


Figure 3. Propeller and Fuselage Angles

$$
\begin{bmatrix} N1 \\ N2 \\ N3 \end{bmatrix} = \begin{bmatrix} \cos\psi\cos\theta & \cos\psi\sin\theta\sin\Phi - \sin\psi\cos\Phi & \cos\psi\sin\theta\cos\Phi + \sin\psi\sin\Phi \\ \sin\psi\cos\theta & \sin\psi\sin\theta\sin\Phi + \cos\psi\cos\Phi & \sin\psi\sin\theta\cos\Phi - \cos\psi\sin\Phi \\ -\sin\theta & \cos\theta\sin\Phi & \cos\theta\cos\Phi \end{bmatrix} \times \begin{bmatrix} X1 \\ X2 \\ X3 \end{bmatrix}
$$

In these matrices N1 is north, N2 is east and N3 down for the inertial axes, and X1 is x, X2 is y and X3 is z for the aircraft body axes.

The inertial to body axis rates are:

$$p = -\dot\psi\sin\theta + \dot\Phi \qquad (1.1)$$

$$q_1 = \dot\psi\cos\theta\sin\Phi + \dot\theta\cos\Phi \qquad (1.2)$$

$$r = \dot\psi\cos\theta\cos\Phi - \dot\theta\sin\Phi \qquad (1.3)$$

And conversely:

$$\dot\Phi = p + q_1\tan\theta\sin\Phi + r\tan\theta\cos\Phi \qquad (1.4)$$

$$\dot\theta = q_1\cos\Phi - r\sin\Phi \qquad (1.5)$$

$$\dot\psi = r\frac{\cos\Phi}{\cos\theta} + q_1\frac{\sin\Phi}{\cos\theta} \qquad (1.6)$$

2. Aerodynamic Forces and Moments

In order to develop expressions for X_a, Y_a, Z_a, $\mathcal{L}_a$, M_a and N_a, individual contributions from the major airframe components of the XC-142A aircraft will be considered. The major components to be considered are the wing, the main propellers, the vertical tail and rudder, the horizontal stabilizer, the tail rotor and the fuselage. After the aerodynamic force and moment expressions for each of these major components are developed, they will be summed to get the total aerodynamic force and moment expressions.

In the equations of motion of the basic aircraft, the internal moments (the right side of the moment equations) include gyroscopic effects due to the rotating mass of an engine. The gyroscopic effects are due to the main engines and the tail rotor. For the main engines we have the following terms representing gyroscopic effects:

$$+(I_E\dot\Omega_E)\cos i_w - q_1(I_E\Omega_E)\sin i_w \qquad \text{for } \mathcal{L}_a \text{ term.}$$

$$+ p(I_E\Omega_E) \sin i_W + r\,(I_E\Omega_E) \cos i_W$$ for M_a term.

$$- (I_E\dot{\Omega}_E) \sin i_W - q_1\,(I_E\Omega_E) \cos i_W$$ for N_a term.

For the tail rotor we have:

$$+ q_1\, I_{TR}\Omega_{TR}$$ for $\daleth_a$ term.

$$0$$ for M_a term.

$$- pI_{TR}\Omega_{TR}$$ for N_a term.

Data received from L-T-V shows that I_E equals 1.352 slug-ft^2 for four engines, and the inertia of main and tail props and gear boxes was 3.287 slug-ft^2. Since these gyroscopic terms are small in comparison with other terms, they have been deleted from the abridged mathematical model.

The equations of motion without expansion of the aerodynamic terms are:

$$X_a = m(\dot{U} + Wq_1 - Vr) + mg \sin \theta \tag{2.1}$$

$$Y_a = m(\dot{V} + Ur - Wp) - mg \cos \theta \sin \phi \tag{2.2}$$

$$Z_a = m(\dot{W} + Vp - Uq_1) - mg \cos \theta \cos \phi \tag{2.3}$$

$$\daleth_a = I_{xx}\dot{p} - I_{xz}\,(\dot{r} + pq_1) + (I_{zz} - I_{yy})\, q_1 r \tag{2.4}$$

$$M_a = I_{yy}\dot{q}_1 - I_{xz}\,(r^2 - p^2) + (I_{xx} - I_{zz})\, pr \tag{2.5}$$

$$N_a = I_{zz}\dot{r} - I_{xz}\,(\dot{p} - q_1 r) + (I_{yy} - I_{xx})\, pq_1 \tag{2.6}$$

We are now ready to develop the aerodynamic forces and moments for each major component of the aircraft. It should be noted that the expressions developed are applicable for hovering, transition and conventional flight.

a. Air Flow Variables

The following air flow variables are required to evaluate the equation for standard day conditions:

$$MN = \frac{V_T}{a} = \text{Mach Number} \tag{2.7}$$

where:

$$a = 1117.0 - 4[\frac{h}{1000}] \text{ ft/sec} = \text{speed of sound} \qquad (2.8)$$

or

$$a = 1117.0 - .0004h \text{ ft/sec}$$

The air density (ρ) may be expressed as:

$$\rho = .00238 - 6.783 \times 10^{-8} h + 6.188 \times 10^{-13} h^2 \qquad (2.9)$$

Freestream dynamic pressure is defined as:

$$q = \frac{1}{2} \rho V_T^2 \qquad (2.10)$$

where:

$$V_T^2 = u^2 + v^2 + w^2$$

The slipstream dynamic pressure (q_s) is developed from the momentum equation as follows:(1)

From the momentum equation we may write

$$T = m_p \, \Delta V_{\alpha=0} = \rho \frac{\pi}{2} D^2 \left(V + \frac{\Delta V_{\alpha=0}}{2}\right) \Delta V_{\alpha=0}$$

where:

m_p = mass flow through propeller

$\Delta V_{\alpha=0}$ is the increment of slipstream velocity due to T_n at $\alpha=0$.

Rearrangement gives:

$$\frac{(\Delta V_{\alpha=0})^2}{2} + V(\Delta V_{\alpha=0}) - \frac{T}{\rho \frac{\pi}{4} D^2} = 0$$

solving the quadratic equation yields

$$\Delta V_{\alpha=0} = -V \pm \sqrt{V^2 + \frac{2T}{\rho \frac{\pi}{4} D^2}}$$

(1) NACA Rpt. TN-3307

$$(\Delta V_{\alpha=0} + V)^2 = V^2 + \frac{2T}{\rho \frac{\pi}{4} D^2}$$

This may be expressed in terms of dynamic pressure as:

$$q_s = q_{\alpha=0} = q_F + \frac{T}{\frac{\pi}{4} D^2} = q_F + \frac{T}{767} \qquad (2.11)$$

where: $$T = \sum_{n=1}^{4} T_n = T_1 + T_2 + T_3 + T_4 \qquad (2.12)$$

The above relationships have been derived for the condition of $\alpha=0$ of the model. The slipstream dynamic pressure (q_s) would be expected to be a function of angle of attack; however to include these effects would needlessly complicate the presentation, since LTV has not included them in their math model.

The slipstream mass ratio ($\frac{m}{m''}$) is an indication of the increased flow over the wing surface due to propeller wash. The slipstream ratio is presented in Appendix B as the following polynomial:

$$\frac{m}{m''} = [1 - K_1 C_{T,S} - K_2 C_{T,S}^{\;2} - K_3 C_{T,S}^{\;5}]$$

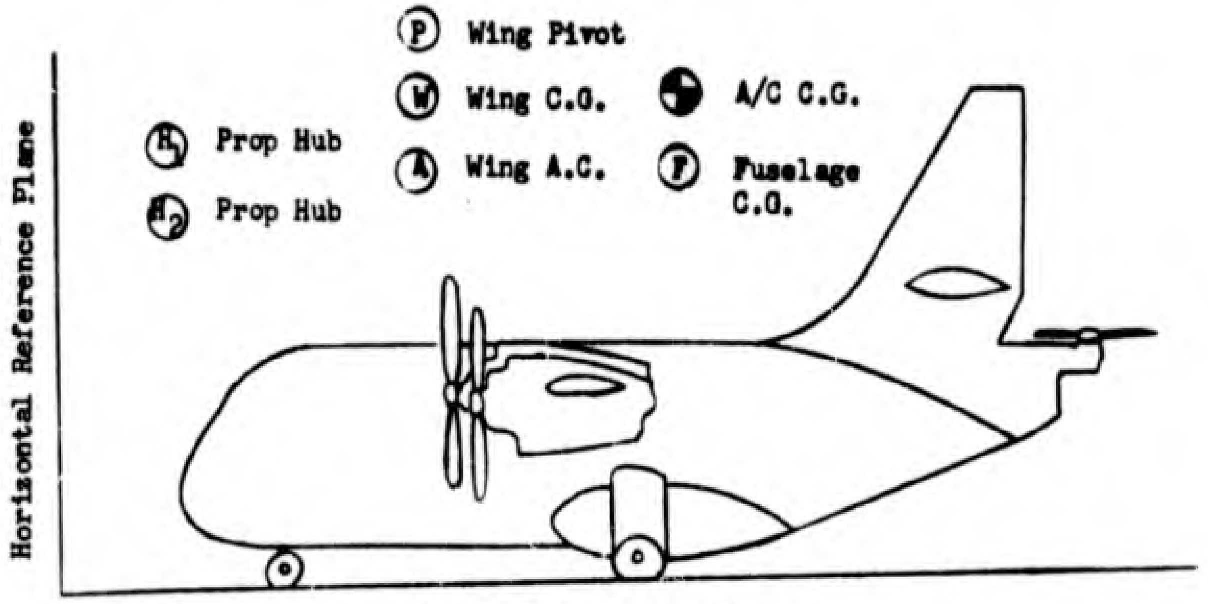
where: $K_1 = .15$ $K_2 = .25$ $K_3 = .20$

The slipstream mass ratio may be written for analog simulation as follows:

$$\frac{m}{m_w} = f(\frac{C_{T,S}}{G-1}) \qquad (2.13)$$

b. Weight and Balance and Moment Arms

The following are equations presented by Ling-Temco-Vought denoting the variables needed to find the c.g. positions and moment arms. LTV has established the equations by considering the wing and fuselage as separate components. Figure 4 shows the relationship of the moment arms. It should be noted that the reference planes are denoted horizontal and vertical to depict horizontal and vertical distances from them.



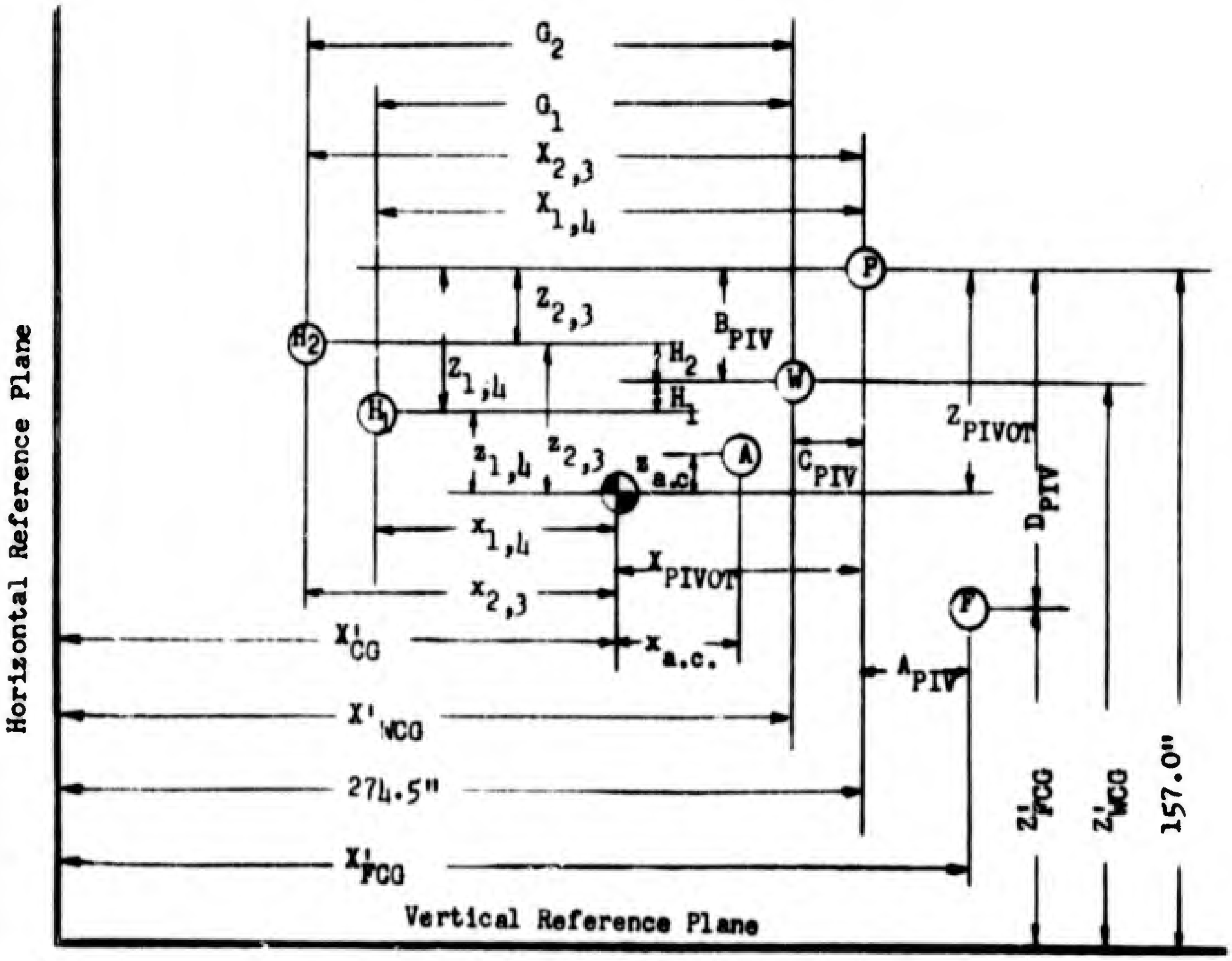


Figure 4. Relationship of Moment Arm Coefficients

Input from Operator:

1) W_T, airplane gross weight

2) X_{CG}, longitudinal C.G. location, wing down in percent mean geometric chord

3) Z'_{CG}, vertical C.G. location, wing down, use W.L. in inches

Constants:

1) Wing weight (W_W) = 11,729 lbs.

2) Wing vertical C.G. location (Z'_{WCG}) = 141.39 in.

3) Wing Longitudinal C.G. location (X'_{WCG}) = 265.28 in.

4) Longitudinal distance to wing pivot point, 274.50 in.

5) Vertical distance to wing pivot point, 157.0 in.

6) Distance from wing pivot to propeller hub for engines 1 and 4

$X = 50.33$ in.
$Z = 19.50$ in.

7) Distance from wing pivot to propeller hub for engines 2 and 3

$X = 68.02$ in.
$Z = 13.10$ in.

Calculate:

1) $X'_{CG} = 245.45 + 96.86 \left(\frac{X_{CG}}{100}\right) \sim$ in.

X'_{CG} = longitudinal C.G. location based on a reference of station zero

2) $W_F = W_T - W_W \sim$ lb.

$m = \frac{W_T}{32.2} \sim$ slugs

W_F = fuselage gross weight

W_W = wing gross weight

3) $X'_{FCG} = \frac{W_T}{W_F} [X'_{CG} - X'_{WCG} (1 - \frac{W_F}{W_T})] \sim$ in.

X'_{FCG} = longitudinal C.G. of fuselage based on a reference of station zero

4) $Z'_{FCG} = \frac{W_T}{W_F} [Z'_{CG} - Z'_{WCG} (1 - \frac{W_F}{W_T})] \sim$ in.

Z'_{FCG} = vertical C.G. of fuselage based on a reference of station zero

5) $A_{PIV} = - \frac{W_F}{W_T} [274.50 - X'_{FCG}] (\frac{1}{12}) \sim$ ft.

A_{PIV} = longitudinal, fuselage C.G. location based upon a reference at the wing pivot point

6) $B_{PIV} = - \frac{W_W}{W_T} [157.00 - Z'_{WCG}] (\frac{1}{12}) \sim$ ft

B_{PIV} = vertical, wing C.G. location based upon a reference at the wing pivot point

7) $C_{PIV} = - \frac{W_W}{W_T} [274.50 - X'_{WCG}] (\frac{1}{12}) \sim$ ft.

C_{PIV} = longitudinal, wing C.G. location based upon a reference at the wing pivot point

8) $D_{PIV} = - \frac{W_F}{W_T} [157.00 - Z'_{FCG}] (\frac{1}{12}) \sim$ ft.

D_{PIV} = vertical, fuselage C.G. location based upon a reference at the wing pivot point

9) $E_W = B_{PIV} + \frac{3.56}{12} \sim$ ft.

10) $F_W = C_{PIV} + \frac{4.835}{12} \sim$ ft.

E_W and F_W locate the wing A.C. at 25 percent MAC (on the chord).

11) $G_1 = C_{PIV} + \frac{56.33}{12} \sim$ ft.

G_1 is the longitudinal distance from the wing C.G. to the propeller hub for engines one and four.

12) $G_2 = C_{PIV} + \frac{68.02}{12} \sim ft.$

G_2 has the same definition as G_1 but is for engines two and three.

13) $H_1 = B_{PIV} + \frac{19.50}{12} \sim ft.$

H_1 is the vertical distance from the wing C.G. to the propeller hub for engines one and four.

14) $H_2 = B_{PIV} + \frac{13.10}{12} \sim ft.$

H_2 has the same definition as H_1 but is for engines two and three.

X_{PIVOT} and Z_{PIVOT} are the distances from the center of gravity to the wing pivot along the X and Z body axis respectively.

$$X_{PIVOT} = A_{PIV} + B_{PIV} \sin i_w + C_{PIV} \cos i_w$$

$$Z_{PIVOT} = D_{PIV} + B_{PIV} \cos i_w - C_{PIV} \sin i_w$$

The equations for the moment arms from the c.g. of the aircraft in the body axes to the aerodynamic center (ac) of the wing in the x-z plane

$$x_{a.c.} = A_{PIV} + E_W \sin i_w + (F_W - K_{ac_1} \delta F - K_{ac_2} MN) \cos i_w$$

$$z_{a.c.} = D_{PIV} + E_W \cos i_w - (F_w - K_{ac_1} \delta F) \sin i_w$$

$$K_{ac_1} = .77$$

$$K_{ac_2} = 0$$

The equations for the moment arms from the c.g. to the propeller hub, x_n, y_n, and z_n are:

$$x_1 = x_4 = A_{PIV} + G_1 \cos i_w + H_1 \sin i_w$$

$$x_2 = x_3 = A_{PIV} + G_2 \cos i_w + H_2 \sin i_w$$

$$z_1 = z_4 = D_{PIV} + H_1 \cos i_w - G_1 \sin i_w$$

$$z_2 = z_3 = D_{PIV} + H_2 \cos i_w - G_2 \sin i_w .$$

The above equations have been programmed and solved over the entire center of gravity and wing incidence angle excursion range. By plotting computer outputs the equations have been rewritten into analog form as:

C.G. to Wing Pivot

$$X_{PIV} = f(\frac{c.g.}{H-1}) \qquad (2.14)$$

$$Z_{PIV} = -3.10 + .007\ i_w^{(o)} \qquad (2.15)$$

C.G. to Wing A.C.

$$x_{a.c.} = -[.8 + .1719\ i_w] + f(\frac{c.g.}{H-5}) + 2.865\ \delta F\ f(\frac{i_w}{H-2}) \qquad (2.16)$$

$$z_{a.c.} = -2.86 + f(\frac{\delta F}{H-3})\ i_w \qquad (2.17)$$

C.G. to Propeller Hub

$$x_1 = x_4 = f(\frac{i_w}{H-4}) + f(\frac{c.g.}{H-5}) \qquad (2.18)$$

$$x_2 = x_3 = f(\frac{i_w}{H-6}) + f(\frac{c.g.}{H-5}) \qquad (2.19)$$

$$z_1 = z_4 = f(\frac{i_w}{H-7}) \qquad (2.20)$$

$$z_2 = z_3 = f(\frac{i_w}{H-8}) \qquad (2.21)$$

$$y_1 = -y_4 = -27.75 \qquad (2.22)$$

$$y_2 = -y_3 = -13.33 \qquad (2.23)$$

c. Main Propeller

There are four mutually similar systems of axes used to describe forces and moments generated by the main propellers during hover and low aircraft velocities. An analysis which was developed by Ling-Temco-Vought (Appendix B) for the XC-142A is adopted herein to describe main propeller forces and moments. The subscript n (n = 1, 2, 3, 4) denotes the particular propeller. They are numbered left to right looking from the top--1 and 2 are port propellers; 3 and 4 are starboard propellers. The approach used to develop the required propeller equations is to consider first the propeller geometry, next state the aerodynamic coefficients and finally write expressions for the force and moment contributions of the main propellers.

(1) Main Propeller Geometry. From Figure 3 the wind vector with respect to each propeller is formed as $V_n^2 = u_n^2 + v_n^2 + w_n^2$.

For each propeller we define the inflow angle, ψ_n, which is the angle between the u_n and V_n velocities.

$$\cos \psi_n = \frac{u_n}{V_n}$$

$$\sin \psi_n = \frac{(w_n^2 + v_n^2)^{1/2}}{V_n} \qquad (2.24)$$

The location of the projected inflow vector in the disk plane ξ_n is defined as the angle between the velocities $V_n \sin \psi_n$ and w_n.

$$\sin \xi_n = \frac{v_n}{V_n \sin \psi_n}$$

$$\cos \xi_n = \frac{w_n}{V_n \sin \psi_n} \qquad (2.25)$$

The velocity expressions u_n, v_n and w_n will now be developed. The origins of each propeller axis are located along a line parallel to the y-body axis at approximately the center of mass of each nacelle. In each propeller the axis system origin is located by x_n, y_n and z_n body axis components which are multiplied by the appropriate body axis angular velocity in order to give tangential velocity components of u_n, v_n and w_n.

$$u_n = u_\psi \cos i_w - w_\psi \sin i_w - y_n [p \sin i_w + r \cos i_w] + q_1[x_n \sin i_w + z_n \cos i_w] \qquad (2.26)$$

$$v_n = v + x_n r - z_n p \qquad (2.27)$$

$$w_n = w_\psi \cos i_w + u_\psi \sin i_w + y_n [p \cos i_w - r \sin i_w] - q_1[x_n \cos i_w - z_n \sin i_w] \qquad (2.28)$$

where:

$$u_\psi = u \cos \Delta\psi - w \sin \Delta\psi$$

$$w_\psi = w \cos \Delta\psi + u \sin \Delta\psi$$

$$\Delta\psi = [\frac{\psi}{C_L}]\, C_L'' , \quad [\frac{\psi}{C_L}] = .202$$

where $\Delta\psi$ is the angular change in propeller relative wind due to lift forces created by the wing.

(2) Main Propeller Aerodynamic Coefficients. The aerodynamic coefficients presented for the main propellers follow those presented in Appendix B. First let us define the advance ratio (J_n) for each main propeller and the advance ratio normal to the propeller disk (J_n').

$$J_n = \frac{60V_n}{N_n D} \quad \text{and} \quad J_n' = J_n \cos \psi_n \qquad (2.29)$$

or:

$$J_n = \frac{3.84\, V_n}{N_n} \quad \text{and} \quad J_n' = \frac{3.84\, u_n}{N_n}$$

The symbol N_n is the particular propeller RPM, the number 60 changes RPM to RPS, D is the diameter of the propeller and B_n (Melpar notation) the blade pitch angle of the particular propeller. The aerodynamic coefficients are then developed in terms of advance ratio and blade pitch.

$$C_{T_n} = C_{T_o} + \frac{\partial C_T}{\partial J'} J_n' + \frac{\partial^2 C_T}{\partial J'^2} (J_n')^2 + \frac{\partial C_T}{\partial B} B_n \qquad (1205\text{-}2)$$

This is expanded by LTV in Appendix B to:

$$C_{T_n} = C_{T_o} + C_{T_\beta} \beta_n + C_{T_{J'}} J_n' + C_{T_{J'^2}} J_n'^2 +$$

$$C_{T_{J^2\beta}} \beta_n (J_n')^2 + C_{T_{J^2\beta^2}} (\beta_n)^2 (J_n')^2 +$$

$$C_{T_{J^2\beta^3}} (\beta_n)^3 (J_n')^2$$

Since the data presents the coefficients as constants, C_{T_n} may be written for analog simulation as follows:

$$C_{T_n} = [f(\frac{J_n'}{P-1}) B_n + f(\frac{J_n'}{P-2})] K_1 + [f(\frac{J_n'}{P-3}) B_n - f(\frac{J_n'}{P-4})](1 - K_1) \qquad (2.30)$$

where: $K_1 = 1$ when $B_n \leq .5235$ Radians

$K_1 = 0$ when $B_n > .5235$ Radians

$$C_{P_n} = C_{P_o} + \frac{\partial C_P}{\partial J'} J'_n + \frac{\partial C_P}{\partial B} B_n + \frac{\partial^2 C_P}{\partial B \partial J'} \cdot B_n J'_n + \frac{\partial C_P}{\partial J'^2} (J'_n)^2 \qquad (1205\text{-}2)$$

This is expanded by LTV in Appendix B to:

$$C_{P_n} = C_{P_o} + C_{P_\beta} \beta_n + C_{P_{\beta^2}} \beta_n^2 + C_{P_{J^2}} (J'_n)^2 + C_{P_{J^3}} (J'_n)^3 + C_{P_{J'^2}} (J'_n)^2 \beta_n + C_{P_{J'^3\beta}} (J'_n)^3 \beta_n$$

Since the data gives the coefficients as constants, C_{P_n} may be written for analog simulation as follows:

$$C_{P_n} = f\left(\frac{B_n}{P\text{-}11}\right) - f\left(\frac{B_n}{P\text{-}12}\right) f\left(\frac{J'_n}{P\text{-}13}\right) \qquad (2.31)$$

$$C_{N_n} = \frac{\partial}{\partial B} \left[\frac{\partial (C_N \cot \psi)}{\partial J'}\right] B_n J_n \sin \psi_n \qquad (1205\text{-}2)$$

This is expanded by LTV in Appendix B to:

$$C_{N_n} = [C_{N_{J\beta}} + C_{N_{J'\beta}} (J'_n)] J_n \beta_n \sin \psi'_n$$

And since the data presents the coefficients as constants, C_{N_n} may be written for analog simulation as follows:

$$C_{N_n} = [f\left(\frac{J_n}{P\text{-}5}\right) \cdot f\left(\frac{B_n}{P\text{-}6}\right) \cdot f\left(\frac{\psi_n}{P\text{-}7}\right)] \qquad (2.32)$$

$$C_{Y_n} = \frac{\partial}{\partial B} \left[\frac{\partial (C_Y \cot \psi)}{\partial J'}\right] B_n J_n \sin \psi_n + \frac{\partial}{\partial J'} \cdot \left[\frac{\partial (C_Y \cot)}{\partial J'}\right] J'_n J_n \sin \psi_n \qquad (1205\text{-}2)$$

This is expanded by LTV in Appendix B to:

$$C_{Y_n} = [C_{Y_{J\beta}} + C_{Y_{J\beta^2}} \beta_n] J_n \beta_n \sin \psi_n$$

And since the data presents the coefficients as constants, C_{Y_n} may be written for analog simulation as follows:

$$C_{Y_n} = [f(\frac{J_n}{P-8}) \cdot f(\frac{B_n}{P-9}) \cdot f(\frac{\psi_n}{P-10})] K_2 \qquad (2.33)$$

where: $K_2 = 1.0$ when $n = 1$ and 2

$K_2 = -1.0$ when $n = 3$ and 4

$$C_{M_n} = \frac{\partial}{\partial J} (\frac{\partial C_M}{\partial \psi_n}) \psi_n J_n \qquad (1205-2)$$

This is expanded by LTV in Appendix B to:

$$C_{M_n} = C_{M_\psi} \psi_n$$

where:

$$J_n \leq 0.5 \rightarrow C_{M_\psi} = .0593 J_n - .0573 J_n^2 - .01642 (\beta_n - .2094)$$

$$0.5 < J_n \leq 1.0 \rightarrow C_{M_\psi} = .01535 + .003625 (J_n - .05) - .01642 (\beta_n - .2094)$$

$$J_n > 1.0 \rightarrow C_{M_\psi} = .0171925 - .01642 (\beta_n - .2094)$$

And since the data presents the coefficients as constants, C_{M_n} may be written for analog simulation as follows:

$$C_{M_n} = [f(\frac{B_n}{P-14}) + f(\frac{J_n}{P-15})] \psi_n \qquad (2.34)$$

C_{T_n} is the coefficient of thrust (T_n), C_{P_n} is the coefficient of power used to express torque (Q_n), C_{N_n} is the coefficient of normal thrust (N_n^*)-- the thrust component perpendicular to T_n. C_{Y_n} and C_{M_n} are the lateral and longitudinal hub moment coefficients that appear during wing tilt.

(3) Main Propeller Force and Moment Expressions. Before expressing the force and moment contribution in body axes due to the propellers, the individual propeller forces and moments developed in propeller axes are stated in terms of the coefficients.

$$T_n = D^4 \left(\frac{N_n}{N_o}\right)^2 \left(\frac{\rho}{\rho_o}\right) C_{T_n} \rho_o N_o^2$$

$$N_n^* = D^4 \left(\frac{N_n}{N_o}\right)^2 \left(\frac{\rho}{\rho_o}\right) C_{N_n} \rho_o N_o^2$$

$$Y_n = D^5 \left(\frac{N_n}{N_o}\right)^2 \left(\frac{\rho}{\rho_o}\right) C_{Y_n} \rho_o N_o^2$$

$$M_n = D^5 \left(\frac{N_n}{N_o}\right)^2 \left(\frac{\rho}{\rho_o}\right) C_{M_n} \rho_o N_o^2$$

$$Q_n = \frac{D^5}{2} \left(\frac{N_n}{N_o}\right)^2 \left(\frac{\rho}{\rho_o}\right) C_{P_n} \rho_o N_o^2$$

In these equations, N_o is the maximum RPM of the propellers and ρ_o is the air density at sea level on a standard day. Using D = 15.625 ft. and N_o = 1232 RPM the equations may be written as follows:

$$T_n = 2.513 \times 10^7 \rho \left[\frac{N_n}{1232}\right]^2 C_{T_n} \qquad (2.35)$$

$$N_n^* = 2.513 \times 10^7 \rho \left[\frac{N_n}{1232}\right]^2 C_{N_n} \qquad (2.36)$$

$$Y_n = 3.9266 \times 10^8 \rho \left[\frac{N_n}{1232}\right]^2 C_{Y_n} \qquad (2.37)$$

$$M_n = 3.9266 \times 10^8 \rho \left[\frac{N_n}{1232}\right]^2 C_{M_n} \qquad (2.38)$$

$$Q_n = 6.249 \times 10^7 \rho \left[\frac{N_n}{1232}\right]^2 C_{P_n} \qquad (2.39)$$

These equations then enable us to write the propeller force and moment contributions in aircraft body axes. Observe that each equation is subscripted by n so that each propeller individually influences the forces and moments.

$$(\Delta X_a)_p = \sum_{n=1}^{4} (T_n \cos i_w - N_n^* \cos \xi_n \sin i_w) \qquad (2.40)$$

$$(\Delta Y_a)_p = \sum_{n=1}^{4} (-N_n^* \sin \xi_n) \qquad (2.41)$$

$$(\Delta Z_a)_p = \sum_{n=1}^{4} (-T_n \sin i_w - N_n^* \cos \xi_n \cos i_w) \qquad (2.42)$$

$$\begin{aligned}(\Delta \mathcal{L}_a)_p = &+ [(\Delta Z_a)_{p_1} - (\Delta Z_a)_{p_4}] y_1 + [(\Delta Z_a)_{p_2} - (\Delta Z_a)_{p_3}] y_2 \\ &- [(\Delta Y_a)_{p_1} + (\Delta Y_a)_{p_4}] z_1 - [(\Delta Y_a)_{p_2} + (\Delta Y_a)_{p_3}] z_2 \\ &- \sum_{n=1}^{4} (Y_n \cos \xi_n) \sin i_w - \sum_{n=1}^{4} (M_n \sin \xi_n) \sin i_w\end{aligned} \qquad (2.43)$$

$$\begin{aligned}(\Delta M_a)_p = &M_{T_{PIVOT}} + \sum_{n=1}^{4} T_n(\cos i_w) Z_{PIVOT} + \sum_{n=1}^{4} T_n(\sin i_w) X_{PIVOT} \\ &- (N_1^* \cos \xi_1 \sin i_w + N_4^* \cos \xi_4 \sin i_w) s_1 \\ &- (N_2^* \cos \xi_2 \sin i_w + N_3^* \cos \xi_3 \sin i_w) s_2 \\ &+ (N_1^* \cos \xi_1 \cos i_w + N_4^* \cos \xi_4 \cos i_w) x_1 \\ &+ (N_2^* \cos \xi_2 \cos i_w + N_3^* \cos \xi_3 \cos i_w) x_2 \\ &- \sum_{n=1}^{4} (Y_n \sin \xi_n) + \sum_{n=1}^{4} (M_n \cos \xi_n)\end{aligned} \qquad (2.44)$$

where

$$M_{T_{PIVOT}} = 1.625 (T_1 + T_4) + 1.092(T_2 + T_3)$$

$$(\Delta N_a)_p = -[(\Delta X_a)_{p_1} - (\Delta X_a)_{p_4}]\, y_1 - [(\Delta X_a)_{p_2} - (\Delta X_a)_{p_3}]\, y_2$$

$$+ [(\Delta Y_a)_{p_1} + (\Delta Y_a)_{p_4}]\, x_1 + [(\Delta Y_a)_{p_2} + (\Delta Y_a)_{p_3}]\, x_2$$

$$- \sum_{n=1}^{4} (Y_n \cos \xi_n) \cos i_w - \sum_{n=1}^{4} (M_n \sin \xi_n) \cos i_w$$

(2.45)

In equations (2.40) through (2.45) the letter p denotes the effects of the main propellers and p subscripted p_n where n = 1, 2, 3, 4 is a particular propeller. For example, in $(\Delta \mathcal{L}_a)_p$ the term $(\Delta Z_a)_{p_1}$ equals $(-T_1 \sin i_w - N_1 \cos \xi_1 \cos i_w)$. In order to better appreciate these equations let us consider the aircraft in normal forward flight where the propeller wind vector is parallel to the x-z plane ($\xi_n = 0$) and there is no tilt of the wing ($i_w = 0$). The equations (2.40) through (2.45) then become:

$$(\Delta X_a)_p = \sum_{n=1}^{4} T_n \qquad (2.46)$$

$$(\Delta Y_a)_p = 0 \qquad (2.47)$$

$$(\Delta Z_a)_p = \sum_{n=1}^{4} (-N_n^*) \qquad (2.48)$$

$$(\Delta \mathcal{L}_a)_p = + (-N_1^* + N_4^*)\, y_1 + (-N_2^* + N_3^*)\, y_2 \qquad (2.49)$$

$$(\Delta M_a)_p = M_{T_{PIVOT}} + \sum_{n=1}^{4} T_n\, z_{PIVOT} \qquad (2.50)$$

$$+ (N_1^* + N_4^*)\, x_1 + (N_2^* + N_3^*)\, x_2 + \sum_{n=1}^{4} M_n$$

$$(\Delta N_a)_p = -(T_1 - T_4)y_1 - (T_2 - T_3)y_2 - \sum_{n=1}^{4} Y_n \qquad (2.51)$$

Equation (2.46) is the total thrust and (2.48) is the total normal force due to the propellers. Equation (2.49) is the rolling moment contribution which will be zero if outboard (n=1 and 4) and inboard (n=2 and 3) normal propeller forces are balanced; (2.50) is the pitching moment contribution; and (2.51) is the turning moment contribution which will be negligible if the outboard (n=1 and 4) and inboard (n=2 and 3) thrusts are balanced and $Y_n = 0$. The incremental propeller forces and moments equations (2.40) through (2.45) will be included in the total aerodynamic forces and moments.

d. Wing

The calculation of wing aerodynamics forces and moments is complicated by the wing tilt during vertical and transition flight. These forces and moments are developed in wing stability axes by first considering the wing geometry and then defining wing aerodynamic coefficients in accord with Appendix B.

(1) Wing Geometry. In wing stability axes there occurs an induced velocity (ΔV) due to the propeller wash across the wing. This gives the effect of increased lift. In order to describe the effect, a coefficient of thrust of the wing ($C_{T,S}$) is defined as a function to total aircraft velocity (V_T).

$$C_{T,S} = \frac{T}{q_s S_p} \qquad (2.52)$$

where S_p, is the total disk area of the four propellers and q_s is the slipstream dynamic pressure.

$$q_s = (q_F + \frac{T}{S_p}), \text{ where } q_F = 1/2\, \rho V_T^2 \qquad (2.53)$$

At low forward speed during transition and in hover, the effect of $C_{T,S}$ is at a maximum and is dependent upon wing tilt and wing flap angle.

From the geometry of Figure 5, the u_w component of wing velocity is the sum of the two velocity vectors u_p and ΔV. The velocity u_p is the average of the individual propeller velocities, u_1, u_2, u_3 and u_4. The rigorous equation for u_p is:

$$u_p = [\sum_{N=1}^{4} u_N]/4$$

which when expanded and simplified is:

$$u_p = u_\psi \cos i_w - w_\psi \sin i_w + \frac{1}{2} q_1[(x_1 + x_2)\cos i_w - (z_1 + z_2)\sin i_w]$$

This expression has been simplified for the purposes of simulation to:

$$u_p = u \cos i_w - w \sin i_w \qquad (2.54)$$

for the following reasons:

1. u_p represents an intermediate calculation in resolving u_w ($u_w = u_p + \Delta V$), thus removing to some degree the accuracy criteria for u_p.

2. LTV data, see page 110 of this report, indicates that the simplification is allowable.

3. From the rigorous expression for u_p, the quantity containing the rate term q_1 can be shown to be negligible in comparison to the fuselage velocity terms u_ψ and w_ψ. Assuming a maximum value for q_1 which will generally occur at moderate speeds and for zero wing incidence, the maximum value for x_1 and x_2 are 4.2 and 5.6 respectively. Thus the total contribution of the rate term is approximately 4.9 ft/sec as opposed to characteristic values of 300 and 50 for u_ψ and w_ψ respectively.

4. Normally $\Delta\psi$ will attain its largest values at low airspeeds, approximately .25 radians, and its smallest values at relatively high speeds, approximately .020 radians. Consequently, only small errors result in the assumption that $u_\psi \approx u$ and $w_\psi \approx w$ and then only near the flight stall region. In addition, in those flight regimes where $\Delta\psi$ is large, propeller thrust is also very large, which yield substantial values of ΔV, approximately 150 ft/sec for 90% thrust. Therefore, the error arising in u_w from the assumption that $u_\psi \approx u$ and $w_\psi \approx w$ is masked by the fact that u_p represents a fraction of the total wing velocity.

The lateral wing velocity v_w is equal to the lateral body velocity, v, since we are concerned with wing stability axes. The vertical velocity of the wing as defined in the wing stability axis system is w_w and is described by the fuselage velocities u and w rotated through the angle i_w.

$$w_w = u \sin i_w + w \cos i_w$$

The total velocity (V_w) in the wing axis is then:

$$V_w = [w_w^2 + (u_p + \Delta V)^2 + v_w]^{\frac{1}{2}} \qquad (2.55)$$

where $v_w = v$

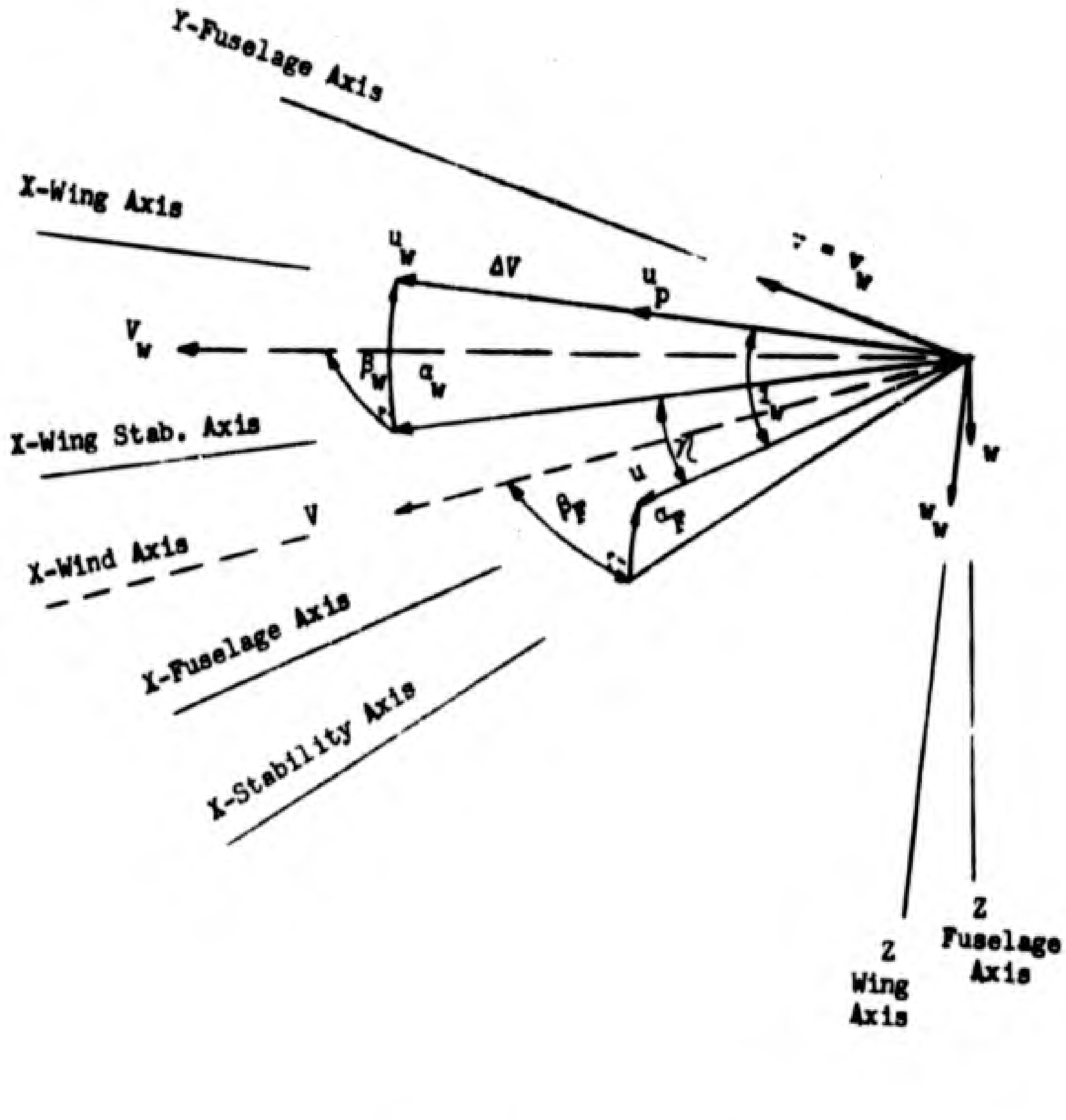


Figure 5. Wing Axes

The expression for induced velocity (ΔV) is defined from momentum theory.[2] ΔV is similar to the mean inflow velocity ($W_{i_{MEAN}}$) developed for helicopter inflow analysis.[3]

$$\Delta V = -u_p + \left(u_p^2 + \frac{2T}{\rho S_p}\right)^{1/2}$$

Rearranging we have:

$$(u_p + \Delta V) = u_w = \left(u_p^2 + \frac{2T}{\rho S_p}\right)^{1/2}$$

Now substituting equation (2.54) for u_p

$$u_w = \left[\frac{2T}{\rho S_p} + (u \cos i_w - w \sin i_w)^2\right]^{1/2} \qquad (2.56)$$

From the foregoing the wing angle of attack (α_w), and the wing sideslip angle (β_w) can be stated as:

$$\alpha_w = \tan^{-1}\left(\frac{w_w}{u_p + \Delta V}\right) = \sin^{-1}\frac{w_w}{(u_w^2 + w_w^2)^{1/2}} \qquad (2.57)$$

$$= \cos^{-1}\frac{u_w}{(u_w^2 + w_w^2)^{1/2}}$$

with the sign of α_w as positive down from plus x_w.

$$\beta_w = \tan^{-1}\frac{v_w}{\sqrt{u_w^2 + w_w^2}} = \sin^{-1}\frac{v_w}{V_w} = \cos^{-1}\frac{\sqrt{u_w^2 + w_w^2}}{V_w}$$

(2.58)

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2. For example, see Airplane Aerodynamics, Dommasch et.al.
3. NAVTRADEVCEN 1205-1, Section 3.

(2) Wing Aerodynamic Coefficients. In order to develop the wing forces and moments five aerodynamic coefficients will be defined as in Appendix B. Since the development is in wing axes, rolling (p) and turning (r) rates necessary to define these coefficients are transformed from body axes. The pitching rate (q_1) is the same in wing axes since q_1 lies in the x-z plane and the x_w - z_w plane. The transformation of the angular rates (p and r) is accomplished by a rotation, η. From Figure 4 η is equal to (i_w - α_w). We can then write for the wing rolling rate (p_w) and the wing turning rate (r_w) the following equations.

$$p_w = p \cos \eta - r \sin \eta \qquad (2.59)$$

$$q_w = q_1 \qquad (2.60)$$

$$r_w = p \sin \eta + r \cos \eta \qquad (2.61)$$

The aerodynamic coefficients of the wing are C_D, C_L, (C_{ℓ_w}), (C_{m_w}) and (C_{n_w}). They were defined as follows in accordance with their development in 1205-2.

$$C_D = C_{D_o} + \frac{C_L^2}{\pi ARe} + \frac{\partial C_D}{\partial \delta F} \cdot \delta F + \frac{\partial^2 C_D}{\partial \delta^2 F} \cdot \delta^2 F$$

The strong flap dependence ($\delta^2 F$) in the C_D expression above, is due to the importance of flap during transition (low speed domain).

$$C_L = [C_{L_o} + C_{L_{\delta F}} \cdot \delta F + C_{L_{\alpha_F}} \cdot \alpha_w + \frac{\partial C_{L_{\alpha_F}}}{\partial \delta F} \cdot \delta F \cdot \alpha_w$$

$$C_{L_w} = C_L \frac{m}{m_w} \quad \text{Lift due to mass flow increase}$$

$$(C_{\ell_w}) = C_{\ell_{\beta_F}} \cdot \beta_w + C_{\ell_{\delta A}} \cdot \delta A + \frac{b}{2V_w} C_{\ell_p} \cdot p_w + \frac{b}{2V_w} C_{\ell_r} \cdot r_w$$

$$(C_{m_w}) = C_{m_o} + \frac{\partial C_m}{\partial \delta F} \cdot \delta F + \frac{c}{2V_w} C_{m_{q_1}} \cdot q_1$$

$$(C_{n_w}) = C_{n_{\beta_F}} \cdot \beta_w + C_{n_{\delta A}} \cdot \delta A + \frac{b}{2V_w} C_{n_p} \cdot p_w + \frac{b}{2V_w} C_{n_r} \cdot r_w$$

Appendix B expands the above equations to:

$$C_L = [C_{L_o} (1 - 2.25\, C_{T,S} + 1.25\, C_{T,S}^{\;2}) +$$

$$(C_{L_\alpha} + C_{L_{\alpha_{\delta F}}} \cdot \delta F)\, \alpha'' + (C_{L_{\delta F}} + C_{L_{\delta F^2}}\ \delta F$$

$$+ C_{L_{\delta F^3}}\ \delta F^2)\ \delta F]\ [F]_{\substack{WING\\FLEX}}\ [F]_{\substack{WING\\MACH}}$$

$$C_L'' = C_L \frac{m}{m''}$$

$$C_D = C_{D_o} + \frac{C_L^2}{\pi ARe} + C_{D_{\delta F}}\ \delta F + C_{D_{\delta F^2}}\ \delta F^2$$

$$C_{m_W} = C_{m_o} + C_{m_{\delta F}}\ \delta F + C_{m_q} \frac{\bar{c}\ q''}{2V''}$$

$$C_{\ell_W} = [C_{\ell_{\beta_o}} + C_{\ell_{\beta_{C_L}}} (C_L'') + C_{\ell_{\beta_{i_W}}} (\frac{\pi}{2} - i_W)]\ \beta''$$

$$+ [C_{\ell_p} \frac{b}{2} \frac{p''}{V''}]\ [F]_{FLEX-P} + [C_{\ell_{r_{C_L}}}\ C_L'' \frac{b\ r''}{2V''}]$$

$$+ [\Delta C_\ell]_{\delta A} \cdot [F]_{\delta A\ FLEX}\ [F]_{\delta A\ B/O} + [\Delta C_\ell]_{\Delta T}$$

$$C_{n_W} = C_{n_{\beta_{C_L}2}} (C_L'')^2\ \beta'' + C_{n_{p_{C_L}}} (C_L'') \frac{b\ p''}{2V''} + C_{n_{r_{C_L}2}} \cdot (C_L'')^2 \cdot$$

$$\frac{b\ r''}{2V''} + [\Delta C_n]_{\Delta T} + [\Delta C_n]_{\delta A}\ [F]_{\delta A\ FLEX} \cdot [F]_{\delta A\ B/O}$$

where:

$$[F]_{\substack{FLEX\\PROP}} = 1 + .000177\ q_s$$

$$[F]_{\substack{WING\\FLEX}} = 1 + .000312\, q_s$$

$$[F]_{\substack{WING\\MACH}} = 1 - .1246MN + .7544\, MN^2$$

$$[F]_{\delta A\ FLEX} = 1 - .00109\, q_s$$

$$[F]_{\delta A\ B/O} = 1 - .0014\, q_s + .00000080\, q_s^{\,2}$$

$$[\Delta C\ell]_{\Delta T} = [.04\, C_{T,S} + .08\, C_{T,S}^{\,2} + .12\, C_{T,S}^{\,3}]\, C_L'' \frac{\Delta T}{\Sigma T}$$

$$[\Delta C_n]_{\Delta T} = [-.060\, C_{T,S}^{\,2} - .140\, C_{T,S}^{\,3}]\, C_L'' \frac{\Delta T}{\Sigma T}$$

$$\frac{\Delta T}{\Sigma T} = \left[\frac{(T_1 + T_2) - (T_3 + T_4)}{\Sigma T_n}\right]$$

ΔT indicates that these terms are included to account for the difference between wind tunnel test results and calculated results.

$$[\Delta C\ell]_{\delta A} = \left\{ C_{\ell_{\delta A}}\, \delta A_{LT} + C_{\ell_{\delta A^2}} |\delta A_{LT}| \delta A_{LT} - C_{\ell_{\delta A}C_L}\, \delta A_{LT}\, C_L'' + C_{\ell_{\delta A^2 C_L}} (\delta A_{LT})^2\, C_L'' \right\}$$

$$- \left\{ C_{\ell_{\delta A}}\, \delta A_{RT} + C_{\ell_{\delta A^2}} |\delta A_{RT}| \delta A_{RT} - C_{\ell_{\delta A}C_L}\, \delta A_{RT}\, C_L'' + C_{\ell_{\delta A^2 C_L}} (\delta A_{RT})^2\, C_L'' \right\}$$

$$[\Delta C_n]_{\delta A} = \Big\{ [C_{n_{\delta A}} + C_{n_{\delta A}\, C_L} \; C_L'' + C_{n_{\delta A}\, C_1\, C_{TS}} \; C_L'' \; C_{T,S}$$

$$+ C_{n_{\delta A}\, C_L^2\, C_{T,S}} \; (C_L'')^2 \; C_{T,S}] \; \delta A_{LT} \Big\}$$

$$- \Big\{ [C_{n_{\delta A}} + C_{n_{\delta A}\, C_L} \; C_L''$$

$$+ C_{n_{\delta A}\, C_L\, C_{T,S}} \; C_L'' \; C_{T,S} + C_{n_{\delta A}\, C_L^2\, C_{T,S}} \; (C_L'')^2 \; C_{T,S}] \delta A_{RT}$$

The wing coefficients may be written for analog simulation as follows:

$$C_L = [f(\frac{C_{T,S}}{W-1}) + f(\frac{\delta F}{W-2}) \; \alpha_w + f(\frac{\delta F}{W-3})] \; f(\frac{q_s}{W-6}) \; f(\frac{MN}{W-7}) \quad (2.62)$$

$$C_{L_w} = C_L \; f(\frac{C_{T,S}}{U-1}) \quad (2.63)$$

$$C_D = .04975 \; C_L^2 + f(\frac{\delta F}{W-4}) \quad (2.64)$$

$$C_{m_w} = -.06 - .5157 \; \delta F - 4.613 \; \frac{q_w}{V_w} \quad (2.65)$$

$$C_{l_w} = [.0367 - .0573 \; C_{L_w} - f(\frac{i_w}{W-14})] \; \beta_w - [15.1875 \; \frac{p_w}{V_w}] \; f(\frac{q_s}{W-5})$$

$$+ [8.4375 \; C_{L_w} \; \frac{r_w}{V_w}] + [\Delta C_l]_{\delta A} \; f(\frac{q_s}{W-8}) + [\Delta C_l]_{\Delta T} \quad (2.66)$$

$$C_{n_w} = [.029 \; (C_{L_w})^2 \; \beta_w - 2.26125 \; C_{L_w} \; \frac{p_w}{V_w} - .5906 \; C_{L_w} \; \frac{r_w}{V_w}$$

$$+ [\Delta C_n]_{\Delta T} + [\Delta C_n]_{\delta A} \; \cdot \; f(\frac{q_s}{W-8})] \quad (2.67)$$

where:

$$[\Delta C_\ell]_{\Delta T} = f(\frac{C_{T_s}}{W-9})\ C_{L_w}\ [\frac{(T_1 + T_2) - (T_3 + T_4)}{\Sigma T}] \qquad (2.68)$$

$$[\Delta C_n]_{\Delta T} = f(\frac{C_{T_s}}{W-10})\ C_{L_w}\ [\frac{(T_1 + T_2) - (T_3 + T_4)}{\Sigma T}] \qquad (2.69)$$

$$[\Delta C_\ell]_{\delta A} = .126\ [\delta A_{LT} - \delta A_{RT}] \qquad (2.70)$$

$$[\Delta C_n]_{\delta A} = [.007736 - .01518\ C_{L_w}][\delta A_{LT} - \delta A_{RT}] \qquad (2.71)$$

(3) Wing Force and Moment Expressions. Before writing the force and moment expressions for the wing, equations (2.63) through (2.67) will be transformed to the aircraft body axes through the angle η.

The body axes wing coefficients are then:

$$(C_x)_w = -C_D \cos\eta - C_L \sin\eta \qquad (2.72)$$

$$(C_y)_w = 0 \qquad (2.73)$$

$$(C_z)_w = C_D \sin\eta - C_L \cos\eta \qquad (2.74)$$

$$(C_\ell)_w = (C_{\ell_w}) \cos\eta + (C_{n_w}) \sin\eta \qquad (2.75)$$

$$(C_m)_w = (C_{m_w}) + \frac{z_{a.c.}}{c} (C_x)_w - \frac{x_{a.c.}}{c} (C_z)_w \qquad (2.76)$$

$$(C_n)_w = -(C_{\ell_w}) \sin\eta + (C_{n_w}) \cos\eta \qquad (2.77)$$

x_{ac} and z_{ac} are the respective distances from the c.g. of the aircraft in body axes to the aerodynamic center (ac) of the wing in the x-z plane. c is the mean aerodynamic chord.

The wing force and moment contributions to the total force and moment equations for analog simulation are as follows:

$$(\Delta X_a)_w = (C_x)_w\ S[q_s]\ f(\frac{C_{T_s}}{G-1}) = 534.37\ (C_x)_w\ f(\frac{C_{T_s}}{G-1})\ q_s \qquad (2.78)$$

$$(\Delta Z_a)_w = (C_z)_w \, S[q_s] \, f(\frac{C_{T,S}}{G-I}) = 534.37 \, (C_z)_w \, f(\frac{C_{T,S}}{G-I}) \, q_s \tag{2.79}$$

$$(\Delta \mathcal{L}_a)_w = (C_{\mathcal{L}})_w \, bS[q_s] \, f(\frac{C_{T,S}}{G-I}) = 36069.975 \, (C_{\mathcal{L}})_w \, f(\frac{C_{T,S}}{G-I}) \, q_s \tag{2.80}$$

$$(\Delta M_a)_w = (C_m)_w \, cS[q_s] \, f(\frac{C_{T,S}}{G-I}) = 4313.435 \, (C_m)_w \, f(\frac{C_{T,S}}{G-I}) \, q_s \tag{2.81}$$

$$(\Delta N_a)_w = (C_n)_w \, bS[q_s] \, f(\frac{C_{T,S}}{G-I}) = 36069.975 \, (C_n)_w \, f(\frac{C_{T,S}}{G-I}) \, q_s \tag{2.82}$$

Equations (2.78) through (2.82) are the wing force and moment contributions. $[f(C_{T,S}) \, q_s]$ is the dynamic pressure term incorporating the effects of having thrust produced by the propellers and having wash across the wing. Consider what happens if the aircraft is flying and the engines are turned off (T = 0). Then $q_s = q_F$ and $f(C_{T,S}) \approx 1$. Thus $[f(C_{T,S}) \, q_s] \approx q_F$ with the engines off and equations (2.78) through (2.82) are wing force and moment equations that would be expected to occur in unpowered flight.

e. Vertical Stabilizer and Rudder. Forces and moments for vertical tail (vt) and rudder arise from the relative wind pushing against the vertical tail surfaces thereby causing a turning moment. This produces a side force, as well as rolling and turning moments.

Dynamic pressure acting against the vertical tail and rudder yields a side force $(Y_a)_{vt}$ and rolling and turning moments $(\Delta \mathcal{L}_a)_{vt}$, $(\Delta N_a)_{vt}$ respectively which are nondimensionalized in terms of aerodynamic coefficients as C_y, C_{ℓ}, and C_n.

$$C_y = C_{y_{\beta_F}} \cdot \beta_F + C_{y_{\delta R}} \cdot \delta R \tag{1205-2}$$

$$C_{\ell} = C_{\ell_{\beta_F}} \cdot \beta_F + C_{\ell_{\delta R}} \cdot \delta R + \frac{b}{2V_B} [C_{\ell_r} \cdot r + C_{\ell_p} \cdot p] \tag{1205-2}$$

$$C_n = C_{n_{\beta_F}} \cdot \beta_F + C_{n_{\delta R}} \cdot \delta R + \frac{b}{2V_B} [C_{n_p} \cdot p + C_{n_r} \cdot r] \tag{1205-2}$$

Appendix B includes Mach number and flexibility effects by use of the following terms:

$[F]_{\beta VT \atop FLEX}$ is the flexibility effect of the vertical tail,

$[F]_{\delta R \atop FLEX}$ is the flexibility effect of the rudder,

$[F]_{VT \atop MACH}$ term corrects for velocity effects, and

$[F]_{pVT \atop FLEX}$ is the flexibility term of the vertical tail due to rolling rate

$$[F]_{VT \atop MACH} = (1 + .28\ MN^2)$$

$$[F]_{\beta VT \atop FLEX} = (1 - .000406\ q_F)$$

$$[F]_{\delta R \atop FLEX} = (1 - .000516\ q_F)$$

$$[F]_{pVT \atop FLEX} = (1 - .000475\ q_F)$$

Appendix B expands the C_y term for vertical tail and rudder in the following form.

$$\Delta C_y = \left\{ C_{y_\beta}\ \beta_v\ [F]_{\beta VT \atop FLEX} + (C_{y_{\delta R}} + C_{y_{\delta R_\beta}}\ \beta_v)\ \delta R_{OUT}\ [F]_{\delta R \atop FLEX} + C_{y_r} \cdot \frac{br}{2V}\ [F]_{\beta VT \atop FLEX} \right\} [F]_{VT \atop MACH}$$

Rewriting the C_y equations for analog simulation

$$C_y = \left\{ -.745\ \beta_F\ [F]_{\beta VT \atop FLEX} + .235\ \delta R_{OUT}\ [F]_{\delta R \atop FLEX} \right\} [F]_{VT \atop MACH} \qquad (2.83)$$

Where $\beta_v = \beta_F$, since LTV has assumed that the sidewash angle is negligible.

Appendix B gives the C_{ℓ} term due to vertical tail and rudder as:

$$\Delta C_{\ell} = \Big\{ C_{\ell_\beta} \beta_v [F]_{\substack{\beta VT \\ FLEX}} + (C_{\ell_{\delta R}} + C_{\ell_{\delta R_{\beta_v}}} \beta_v) \delta R_{OUT} [F]_{\substack{\delta R \\ FLEX}}$$

$$+ C_{\ell_r} \frac{b}{2} \frac{r}{V} [F]_{\substack{\beta VT \\ FLEX}} \Big\} [F]_{\substack{VT \\ MACH}}$$

Rewriting this equation for analog simulation:

$$C_{\ell} = \Big\{ - .0946\, \beta_F [F]_{\substack{\beta VT \\ FLEX}} + 1.431 \frac{r}{V_B} [F]_{\substack{\beta VT \\ FLEX}}$$

$$+ .039\, \delta R_{OUT} [F]_{\delta R\ FLEX} \Big\} [F]_{\substack{VT \\ MACH}} \qquad (2.84)$$

Assuming $\beta_v = \beta_F$

Appendix B expands the C_n term due to vertical tail and rudder effects as:

$$\Delta C_n = \Big\{ C_{n_\beta} \beta_v [F]_{\substack{\beta VT \\ FLEX}} + (C_{n_{\delta R}} + C_{n_{\delta R_\beta}} \beta_v) \delta R_{OUT} [F]_{\delta R\ FLEX}$$

$$+ C_{n_r} \frac{b}{2} \frac{r}{V} [F]_{\substack{\beta VT \\ FLEX}} + C_{n_p} \frac{b}{2} \frac{p}{V} [F]_{\substack{PVT \\ FLEX}} \Big\} [F]_{\substack{VT \\ MACH}}$$

where: $\delta R_{OUT} = [\delta R_{IN} + \frac{H.M.}{K_{B/O}}]$

Note: Maximum rudder pedal deflection is limited in accordance with the following equation.

or:

$$\delta R_{IN}\Big|_{LIMIT} = \left[\frac{K_{B/O} [H.M. - C_{H\beta} \beta_v q_F SR \bar{c}_R] - H.M. C_{H_{\delta R}} q_F SR \bar{c}_R}{K_{B/O} C_{H_{\delta R}} q_F SR \bar{c}_R} \right]$$

$SR = 22.81\ ft^2$

$\bar{c}_R = 2.68\ ft$

$K_{B/O} = 29,350\ \frac{ft\text{-}lb}{Rad}$

$$H.M. = \left[\frac{K_{B/O} \, [C_{H_{\delta R}} \, \delta R_{IN} + C_{H_\beta} \, \beta_v] \, q_F \, \delta R \, \bar{c}_R}{K_{B/O} - C_{H_{\delta R}} \, q_F \, \delta R \, \bar{c}_R} \right]$$

Limited to $\pm$ 1200 ft-lbs.

where:

$$C_{H_{\delta R}} = [-.573] \qquad MN \leq .15$$

$$= [-.573 - .535 \, (MN - .15)] \text{ for } MN > .15$$

per radian

$$C_{H_\beta} = .124 \, [1 + .926 \, MN^3] \sim \text{per radian}$$

Rewriting the C_n equation for analog simulation:

$$C_n = [(.243 \, \beta_F - 4.6575 \, \frac{r}{V_B}) \, [F]_{\substack{\beta VT \\ FLEX}} - .0831 \, \delta R_{OUT} \, [F]_{\substack{\delta R \\ FLEX}}$$

$$+ 33.75 \, \frac{P}{V} \, f(\frac{M}{VT-7}) \, [F]_{\substack{PVT \\ FLEX}} \,] \cdot [F]_{\substack{VT \\ MACH}} \qquad (2.85)$$

where:

$$[F]_{\substack{\beta VT \\ FLEX}} = f(\frac{q_F}{V_T-3})$$

$$[F]_{\substack{\delta R \\ FLEX}} = f(\frac{q_F}{V_T-4})$$

$$[F]_{\substack{PVT \\ FLEX}} = f(\frac{q_F}{V_T-5})$$

$$[F]_{\substack{VT \\ MACH}} = f(\frac{MN}{V_T-6})$$

$$\delta R_{OUT} = [\delta R_{IN} + \frac{H.M.}{29,350}]$$

$$H.M. = \left[\frac{29350 \, q_F (C_{H_{\delta R}} \cdot \delta R_{IN} + C_{H_\beta} \cdot \beta)}{480.118 - q_F \, C_{H_{\delta R}}} \right]$$

Limited to $\pm$ 1200 ft-lbs.

$$C_{H_{\delta R}} = f(\frac{MN}{V_T - 2})$$

$$C_{H_\beta} = f(\frac{MN}{V_T - 1})$$

$$\delta R_{IN}\Big|_{LIMIT} = \left[\frac{29350\,[C_{H_{\delta R}}\,\delta R_{IN} + C_{H_\beta}\,\beta_F\,]}{480.118 - C_{H_{\delta R}}\,q_F}\right]$$

The forces and moments for the vertical tail can then be expressed in the following equations:

$$(\Delta Y_a)_{vt} = C_y\,Sq\,(\frac{q_{vt}}{q})$$

$$(\Delta \mathcal{L}_a)_{vt} = C_{\mathcal{L}}\,bSq\,(\frac{q_{vt}}{q})$$

$$(\Delta N_a)_{vt} = C_n\,bSq\,(\frac{q_{vt}}{q})$$

Here q is the dynamic pressure and q_{vt} is the vertical tail dynamic pressure.

In Appendix B $\frac{q_{vt}}{q}$ is written as n_v and is assumed equal to 1.0, the equations may then be written as follows:

$$(\Delta Y_a)_{vt} = 534.37\,q\,C_y \qquad (2.86)$$

$$(\Delta \mathcal{L}_a)_{vt} = 36069.975\,q\,C_{\mathcal{L}} \qquad (2.87)$$

$$(\Delta N_a)_{vt} = 36069.975\,q\,C_n \qquad (2.88)$$

This force and these moments will be included in the total aerodynamic forces and moments.

f. Horizontal Stabilizer. The equations for forces and moments for the horizontal stabilizer (hs) as presented in Appendix B differ so widely from those of 1205-2 that only the new equations are presented.

We define

$$\alpha_{t_{RIGID}} = i_{t_{RIGID}} + \alpha_F - \epsilon + \ell_{HT} \frac{q}{V} + \ell_{HT} \frac{\partial \epsilon}{\partial \alpha_F} \frac{\dot{w}}{V^2}$$

where: $\ell_{HT} = 24.3 - X_{P.V}$

$$\frac{\partial \epsilon}{\partial \alpha_F} = \left\{ [(f_1 C_{L_{\alpha_{WING}}} [F]_{\substack{WING \\ FLEX}} [F]_{\substack{WING \\ MACH}} \cdot \frac{m}{m''} - 1) \sqrt{1 - C_{T,S}} + 1] \right.$$

$$\left. + f(C_{T,S}) [C_{T,S} \sqrt{1 - C_{T,S}} \cos i_w + \frac{2}{q_s S_p} \frac{\partial}{\partial \alpha_F} (N_2^* + N_3^*)] \right\}$$

where:

$$\frac{\partial}{\partial \alpha_F} (N_2^* + N_3^*) = \boxed{B} [1 + (\frac{\Delta\psi}{C_L}) \frac{m}{m''} C_{L_{\alpha_{WING}}} \sqrt{1 - C_{T,S}}] \cos i_w$$

$$\boxed{B} = \left\{ [C_{N_{J\beta}} + C_{N_{J^2\beta}} (J_2') J_2 \beta_2 (16.557) \rho N_2^2 + [C_{N_{J\beta}} + C_{N_{J^2\beta}} (J_3') J_3 \beta_3 \right.$$

$$\left. (16.557) \rho N_3^2 \right\}$$

Here α_t is the angle of attack of the tail, α_F is the angle of attack of the fuselage, i_t is the angle of incidence of the tail and ϵ is the downwash angle.

In accord with Appendix B we have

$$\epsilon = f_1 C_L'' + i_w + \alpha_F - \alpha'' + \epsilon_0 + [\Delta\epsilon]_{PROPS}$$

where:

$$[\Delta\epsilon]_{PROPS} = f(C_{T,S}) [C_{T,S} \alpha'' + \frac{2(N_2^* + N_3^*)}{q_s S_p}]$$

$$f(C_{T,S}) = \left\{ \frac{1}{(2-C_{T,S})(1 + \sqrt{1 - C_{T,S}})} \right\}$$

for the XC-142A.

In order to account for flexibility effects in $a_{t_{RIGID}}$, Appendix B gives the following equation for $a_{t_{FLEX}}$.

$$a_{t_{FLEX}} = \left[\frac{a_{t_{RIGID}} - [\frac{\Delta a_t}{n_z}] \frac{(\Sigma F_z)_1}{W_t} + [\frac{\Delta a_t}{\dot{q}}] \dot{q}}{1 - [\frac{\Delta a_t}{HM}] H_{a_t} + Z_{a_t} (\frac{1}{W_t} [\frac{\Delta a_t}{n_z}] - [\frac{\Delta a_t}{Z_t}])} \right]$$

where: W_t = airplane gross weight

$$H_{a_t} = C_{h_{a_t}} \bar{c}_h q_F S_h [F]_{UHT\ MACH}$$

$$\bar{c}_h = 5.44 \text{ ft}, \; S_h = 163.5 \text{ ft}^2, \; C_{h_{a_t}} = -.481/\text{rad}$$

$$Z_{a_t} = - C_{L_{a_t}} S q_F [F]_{UHT\ MACH}$$

$$[F]_{UHT\ MACH} = [1 - .0706 \text{ MN} + .5233 \text{ MN}^2]$$

$$(\Sigma F_z)_1 = [(\Delta F_z)_{WING} + (\Delta F_z)_{PROPS} + (\Delta F_z)_{FUS} + (\Delta F_z)_{TR}]$$

$$[\frac{\Delta a_t}{n_z}] = .00288 \frac{\text{Rad}}{G}, \quad [\frac{\Delta a_t}{HM}] = 2.234 \times 10^{-6} \frac{\text{Rad}}{\text{Ft-lb}}$$

$$[\frac{\Delta a_t}{\dot{q}_1}] = -.00212 \text{ sec}^2, \quad [\frac{\Delta a_t}{Z_t}] = 11.5 \times 10^{-7} \frac{\text{Rad}}{\text{lb.}}$$

The lift (C_{L_t}) and drag (C_{D_t}) coefficients of the tail can then be expressed in the following relations:

$$C_{L_t} = C_{L_{a_t}} \cdot a_{t_{FLEX}} [F]_{UHT\ MACH}$$

$$C_{D_t} = C_{D_{o_t}} + (C_{L_t})^2 K_t$$

where: K_t = .299

These equations are rewritten for analog simulation as:

$$\alpha_{t_{RIGID}} = i_{t_{RIGID}} + \alpha_F - \epsilon + (24.3 - X_{PIV})\left(\frac{q_1}{V_T} + \frac{\partial\epsilon}{\partial\alpha_F}\frac{\dot{w}}{V_T^2}\right) \qquad (2.89)$$

$$\epsilon = .079\, C_{L_w} + i_w + \alpha_F - \alpha_w + [\Delta\epsilon]_{PROPS} + .0594 \qquad (2.90)$$

where:

$$[\Delta\epsilon]_{PROPS} = f\left(\frac{C_{T,S}}{UHT-1}\right)\left[C_{T,S}\,\alpha_w + \frac{N_2^* + N_3^*}{383.5\, q_s}\right] \qquad (2.91)$$

$$\alpha_{t_{FLEX}} = \left[1 - f\left(\frac{MN}{UHT-5}\right) f\left(\frac{h_p}{UHT-6}\right)\right]\left[\alpha_{t_{RIGID}} - \frac{.00288}{W_T}(\Sigma F_z)_1 - .00212\,\dot{q}_1\right] \qquad (2.92)$$

$$\Sigma F_{z_1} = (\Delta Z_a)_p + (\Delta Z_a)_w + (\Delta Z_a)_f$$

$$\frac{\partial\epsilon}{\partial\alpha_f} = \left[\left(.3397\, f\left(\frac{q_s}{W-6}\right) \cdot f\left(\frac{MN}{W-7}\right) \cdot f\left[\frac{C_{T,S}}{G-1}\right] - 1\right) f\left(\frac{C_{T,S}}{UHT-3}\right) + 1\right]$$

$$+ f\left(\frac{C_{T,S}}{UHT-1}\right)\left[f\left(\frac{C_{T,S}}{UHT-4}\right)\cos i_w + \frac{.0026}{q_s} \cdot \frac{\partial(N_2^* + N_3^*)}{\partial\alpha_F}\right] \qquad (2.93)$$

where:

$$\frac{\partial(N_2^* + N_3^*)}{\partial\alpha_F} = \boxed{B}\; f\left(\frac{C_{T,S}}{UHT-2}\right)\cos i_w \qquad (2.94)$$

$$\boxed{B} = \frac{N_2^*}{\sin\psi_2} + \frac{N_3^*}{\sin\psi_3} \qquad (2.95)$$

$$C_{L_t} = 1.146\,\alpha_{t_{FLEX}}(1 - .0706\, MN + .5235\, MN^2) \qquad (2.96)$$

$$C_{D_t} = .00244 + .299\,(C_{L_t})^2 \qquad (2.97)$$

The horizontal stabilizer (hs) can contribute forces in the x and z directions, and a pitching moment. The equations are as follows:

$$(\Delta X_a)_{hs} = -[C_{D_t} \cos(i_{t_{RIGID}} - \alpha_{t_{RIGID}}) + C_{L_t} \sin(i_{t_{RIGID}} - \alpha_{t_{RIGID}})] \, Sq\left(\frac{q_{hs}}{q}\right) \quad (2.98)$$

$$(\Delta Z_a)_{hs} = -[-C_{D_t} \sin(i_{t_{RIGID}} - \alpha_{t_{RIGID}}) + C_{L_t} \cos(i_{t_{RIGID}} - \alpha_{t_{RIGID}})] \, Sq\left(\frac{q_{hs}}{q}\right) \quad (2.99)$$

$$(\Delta M_a)_{hs} = -(\Delta X_a)_{hs} \cdot h_{hs} + (\Delta Z_a)_{hs} \cdot \ell_{hs} \quad (2.100)$$

$$\frac{q_{hs}}{q} = 1.0$$

$$h_{hs} = 7.5 \text{ ft.}$$

$$\ell_{hs} = 24.3 - X_{PIV}$$

ℓ_{hs} is distance from aircraft c.g. to the aerodynamic center (a.c.) of the horizontal stabilizer and h_{hs} is the height of a.c. above the c.g. Both ℓ_{hs} and h_{hs} are measured in the x-z plane of the aircraft body axes. S is the wing area, ρ is the air density, c is the mean aerodynamic chord and the angle $(i_{t_{RIGID}} - \alpha_{t_{RIGID}})$ is used to transform C_{L_t} and C_{D_t} to body axes.

$(\Delta X_a)_{hs}$, $(\Delta Z_a)_{hs}$ and $(\Delta M_a)_{hs}$ will be included in the total aerodynamic forces and moments.

g. Tail Rotor. The Z force $(\Delta Z_a)_{TR}$ and the pitching moment $(\Delta M_a)_{TR}$ developed at the tail will be obtained by finding a tail rotor advance ratio (J_{TR}). From J_{TR} the tail rotor thrust coefficient $(C_{T_{TR}})$ and tail rotor power coefficient $(C_{P_{TR}})$ will be found. In turn, the tail rotor thrust (T_{TR}) and torque (Q_{TR}) is obtained and consequently $(\Delta Z_a)_{TR}$ and $(\Delta M_a)_{TR}$. T_{TR} is positive in the -z direction.

We define the total tail rotor velocity (V_{TR}) as:

$$V_{TR} = [(u_{TR})^2 + (v_{TR})^2 + (w_{TR})^2]^{1/2}$$

Here u_{TR}, v_{TR}, and w_{TR} are defined as:

$$u_{TR} = u \cos \epsilon + w \sin \epsilon$$

$$v_{TR} = v - \ell_{TR}\, r$$

$$w_{TR} = - \ell_{TR} q_1 + u \sin\epsilon - w \cos \epsilon \qquad (2.101)$$

ℓ_{TR} is the distance from the center of the tail rotor hub to the aircraft c.g. and $(\psi)_{TR}$ locates the tail rotor with respect to the aircraft body axes.

$$(\psi)_{TR} = \cos^{-1} \frac{w_{TR}}{V_{TR}}$$

Here ϵ is again the downwash angle and is defined as for the horizontal stabilizer.

The advance ratio for the tail rotor is

$$J_{TR} = \frac{60\ V_{TR}}{N_{TR}\ D_{TR}} \quad \text{or} \quad J'_{TR} = \frac{60(-w_{TR})}{N_{TR}\ D_{TR}} \quad \text{or} \quad J'_{TR} = \frac{-7.5\ w_{TR}}{N_{TR}} \qquad (2.102)$$

where N_{TR} is the RPM and D_{TR} the diameter of the tail rotor.

We now define $(C_{T_{TR}})$ and $(C_{P_{TR}})$ as is done in 1205-2.

$$C_{T_{TR}} = C_{T_{TR}}(B_{TR}) + \frac{\partial C_{T_{TR}}}{\partial J'_{TR}}(J'_{TR})$$

$$C_{P_{TR}} = \frac{\partial^2 C_p}{\partial B_{TR}^2}(B_{TR})^2$$

Here B_{TR} is the collective pitch of the tail rotor blades.

Appendix B expands these equations to:

$$C_{T_{TR}} = .18622\ \beta_{TR} + 2.692\ |\beta_{TR}|\beta_{TR} - 2.822\ \beta_{TR}^3$$

$$- .3773\ |\beta_{TR}^3|\ \beta_{TR} - .10\ J'_{TR}$$

$$C_{P_{TR}} = 1.084\,(\beta_{TR})^2$$

These equations may be written for analog simulation as,

$$C_{T_{TR}} = f\left(\frac{B_{TR}}{TR-1}\right) - 0.1\, J'_{TR} \qquad (2.103)$$

$$C_{P_{TR}} = f\left(\frac{B_{TR}}{TR-2}\right) \qquad (2.104)$$

The thrust (T_{TR}) and torque (Q_{TR}) of the tail rotor is:

$$T_{TR} = D^4_{TR} \left(\frac{\rho}{\rho_o}\right) \left(\frac{N_{TR}}{(N_o)_{TR}}\right)^2 C_{T_{TR}} \qquad (1205-2)$$

$$Q_{TR} = D^5_{TR} \left(\frac{\rho}{\rho_o}\right) \left(\frac{N_{TR}}{(N_o)_{TR}}\right)^2 C_{P_{TR}} \qquad (1205-2)$$

These equations are presented in Appendix B as,

$$T_{TR} = 6.448 \times 10^6\, \rho \left[\frac{N_{TP}}{2380}\right]^2 C_{T_{TP}}$$

$$Q_{TR} = 8.206 \times 10^6\, \rho \left[\frac{N_{TP}}{2380}\right]^2 C_{P_{TP}}$$

This may be written for analog simulation as:

$$T_{TR} = 1.1378\, \rho\, f\left(\frac{N_{TR}}{TR-3}\right) C_{T_{TR}} \times 10^6 \qquad (2.105)$$

$$Q_{TR} = 1.4486\, \rho \left(\frac{N_{TR}}{TR-3}\right) C_{P_{TR}} \times 10^6 \qquad (2.106)$$

Consequently the force and moment terms can be written directly.

$$(\Delta Z_a)_{TR} = -T_{TR} \qquad (2.107)$$

$$(\Delta M_a)_{TR} = -T_{TR}\, \ell_{TR}, \text{ where } \ell_{TR} = (32.08 - X_{PIV}) \qquad (2.108)$$

$$(\Delta N_a)_{TR} = Q_{TR} \qquad (2.109)$$

h. Fuselage. In a very direct manner we can write the effects of the fuselage (F) on the total aerodynamic forces and moments.

We have for the forces

$$(\Delta X_a)_F = -\frac{1}{2}\rho V_T^2 \, S \, C_{D_o}$$

C_{D_o} is the equilibrium drag coefficient.

$$(\Delta Y_a)_F = +\frac{1}{2}\rho V_T^2 \, S \, C_{y_{\beta_f}} \cdot \beta \cdot \frac{d\beta_f}{d\beta}$$

$$\beta_f = \sin^{-1}\frac{v}{V_B} = \cos^{-1}\frac{\sqrt{u^2 + w^2}}{V_B}$$

$$\frac{d\beta_f}{d\beta} = [1 + K_1 C_{T,S} + K_2 C_{T,S}^{\;2} + K_3 C_{T,S}^{\;3}]$$

$K_1 = 1.6$

$K_2 = -1.4$

$K_3 = 0$

$C_{y_{\beta_f}}$ is the change in side force with respect to a changing sideslip angle.

$$(\Delta Z_a)_F = -\frac{1}{2}\rho V_T^2 \, S \, C_{L_{\alpha_F}} \cdot \alpha_F$$

$$\alpha_F = \sin^{-1}\frac{w}{(u^2 + w^2)^{1/2}}$$

$C_{L_{\alpha_F}}$ is the change in lift coefficient with varying angle of attack. This is also known as the lift curve slope.

We have for the moments:

$$(\Delta \mathcal{L}_a)_F = 0$$

$$(\Delta M_a)_F = \frac{1}{2}\rho V_T^2 S c (C_{m_o} + C_{m_{\alpha_F}} \cdot \alpha_F)$$

C_{m_o} is the aerodynamic pitching moment coefficient in equilibrium flight and $C_{m_{\alpha_F}}$ is the longitudinal static stability derivative.

$$(\Delta N_a)_F = \frac{1}{2}\rho V_T^2 S b C_{n_{\beta_f}} \cdot \beta_f \cdot \frac{d\beta_f}{d\beta}$$

$C_{n_{\beta_f}}$ is the static directional or "weathercock" derivative.

The fuselage forces and moments are written for analog simulation as follows:

$$(\Delta X_a)_f = -(11.008 + 25.115 K_1) q$$

$$(\Delta Y_a)_f = -306.2 f(\frac{C_{T,S}}{F-1}) \beta_f q$$

$$(\Delta Z_a)_f = -183.823 \alpha_f q$$

$$(\Delta M_a)_f = (17.254 + 3364.48 \alpha_f) q$$

$$(\Delta N_a)_f = -4761.237 f(\frac{C_{T,S}}{F-1}) \beta_f q$$

K_1 = 1.0 if landing gear down

= 0 if landing gear up

In the above expressions $q = \frac{1}{2}\rho V_T^2$ which is the free stream dynamic pressure (q), b is the wing span, c is the mean aerodynamic chord and S is the wing area.

3. Term Analysis

During the process of reducing L-T-V data, a careful analysis was made of each term. Aside from the equation simplification derived from converting the aerodynamic coefficient equation to function-type expressions, only the gyroscopic terms of the engine and tail rotor were deleted from the rigorous-equations.

In pursuing the problem further, an attempt was made to justify the elimination of the term $\Delta\psi$ (propeller parameter). On the surface, it seemed a reasonable assumption. To determine the effect of this term, a digital program was developed for the SDS 920 computer. The program was designed to determine level flight aircraft trim requirements over a fairly broad spectrum of speed and altitude. Initially, the programs used the rigorous equation to generate trim requirements. Next the rigorous equations were modified such that $\Delta\psi$ was defined as zero. Comparison of the computer output for rigorous and modified equation shows conclusively that the term, $\Delta\psi$, is not negligible. It has a significant influence on propeller pitching moment. In tests near the stall regime tail incidence angle differed by as much as two (2) degrees from the rigorous results. In addition, wing angle of attack differed by as much as one-half (0.5) degree.

The advantage of eliminating these variable ($\Delta\psi$) is the reduction of computer hardware by one position servo and at least two resolvers. Since the term could not be neglected completely, it was decided to test the validity of making a small angle assumption of $\Delta\psi$. The rigorous equations were modified by defining $\cos \Delta\psi \equiv 1.0$ and $\sin \Delta\psi \equiv .202\, C_{L_{A.C.}}$. Comparison of the results produced by the rigorous equations with those wherein the small angle assumption of $\Delta\psi$ was made demonstrated that, even under exaggerated flight conditions, there was less than a 0.28 degree difference in tail incidence angle and less than 0.16 degree difference in wing angle of attack. Figures 5 through 10 illustrate these test results.

Although $\Delta\psi$ could not be eliminated completely, the advantages derived from the small angle assumption justification do result in a significant savings in terms of analog mechanization.

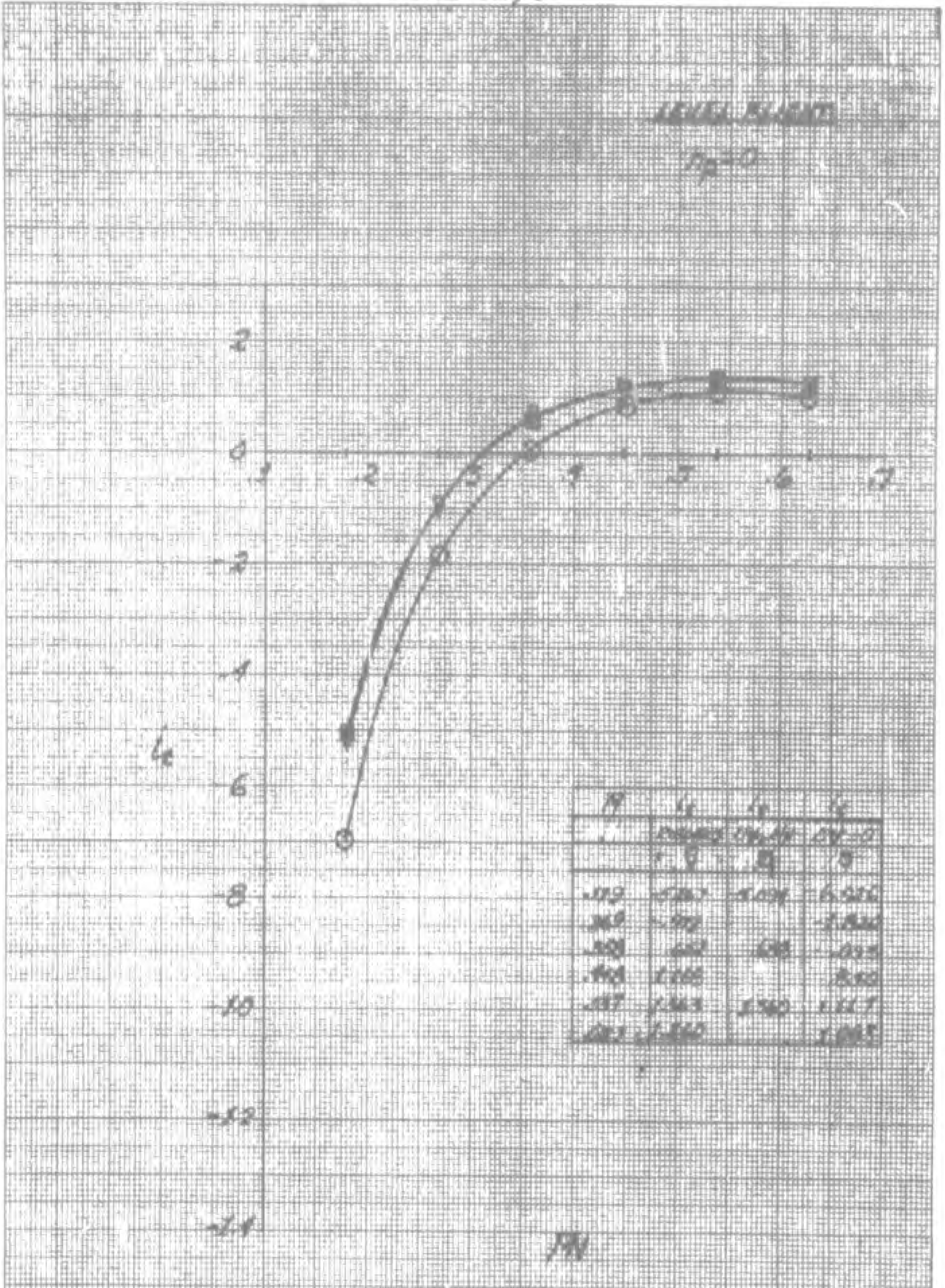


Figure 6. Investigation of [illegible] Simplification

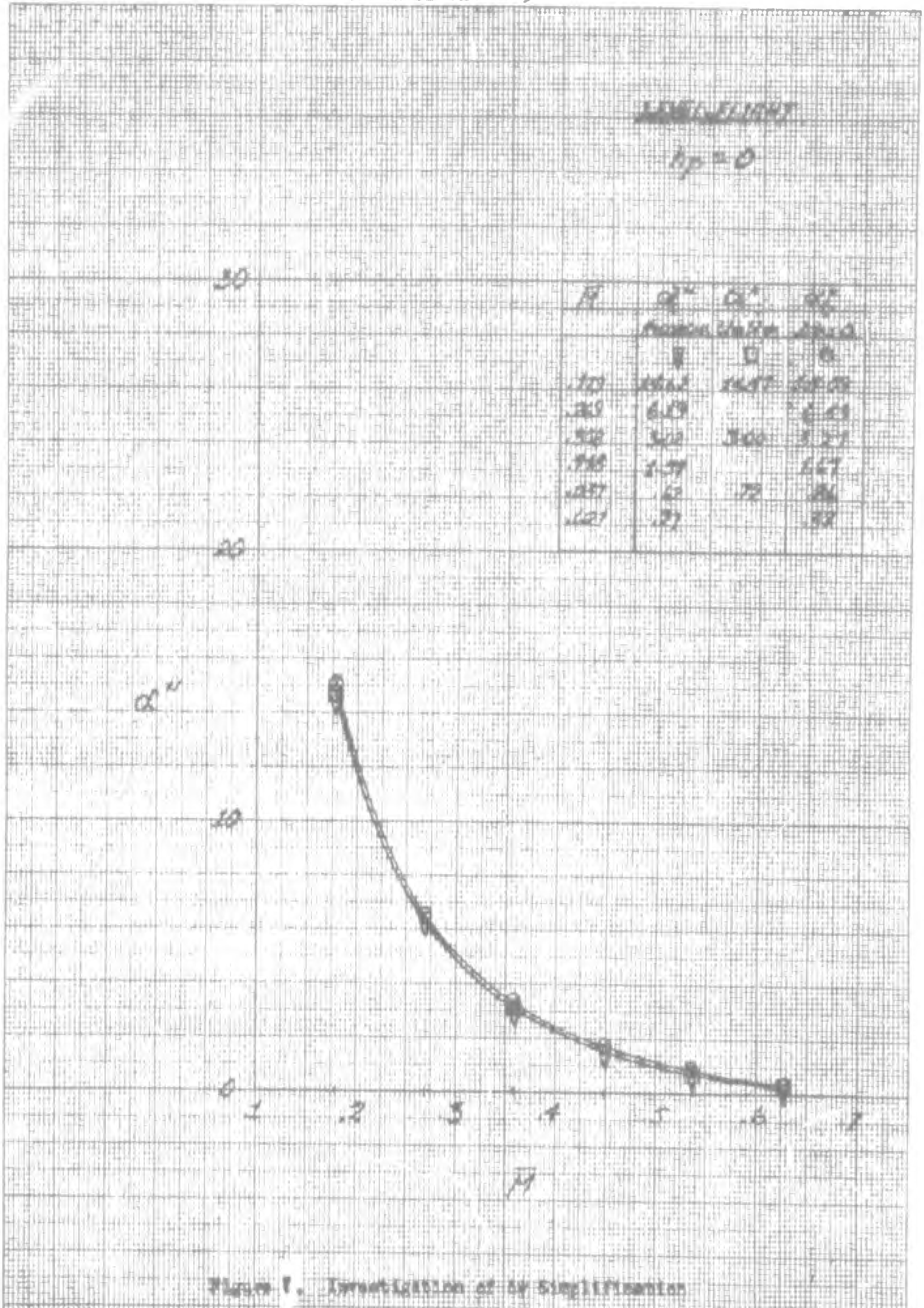


Figure 7. Investigation of [illegible] Simplification

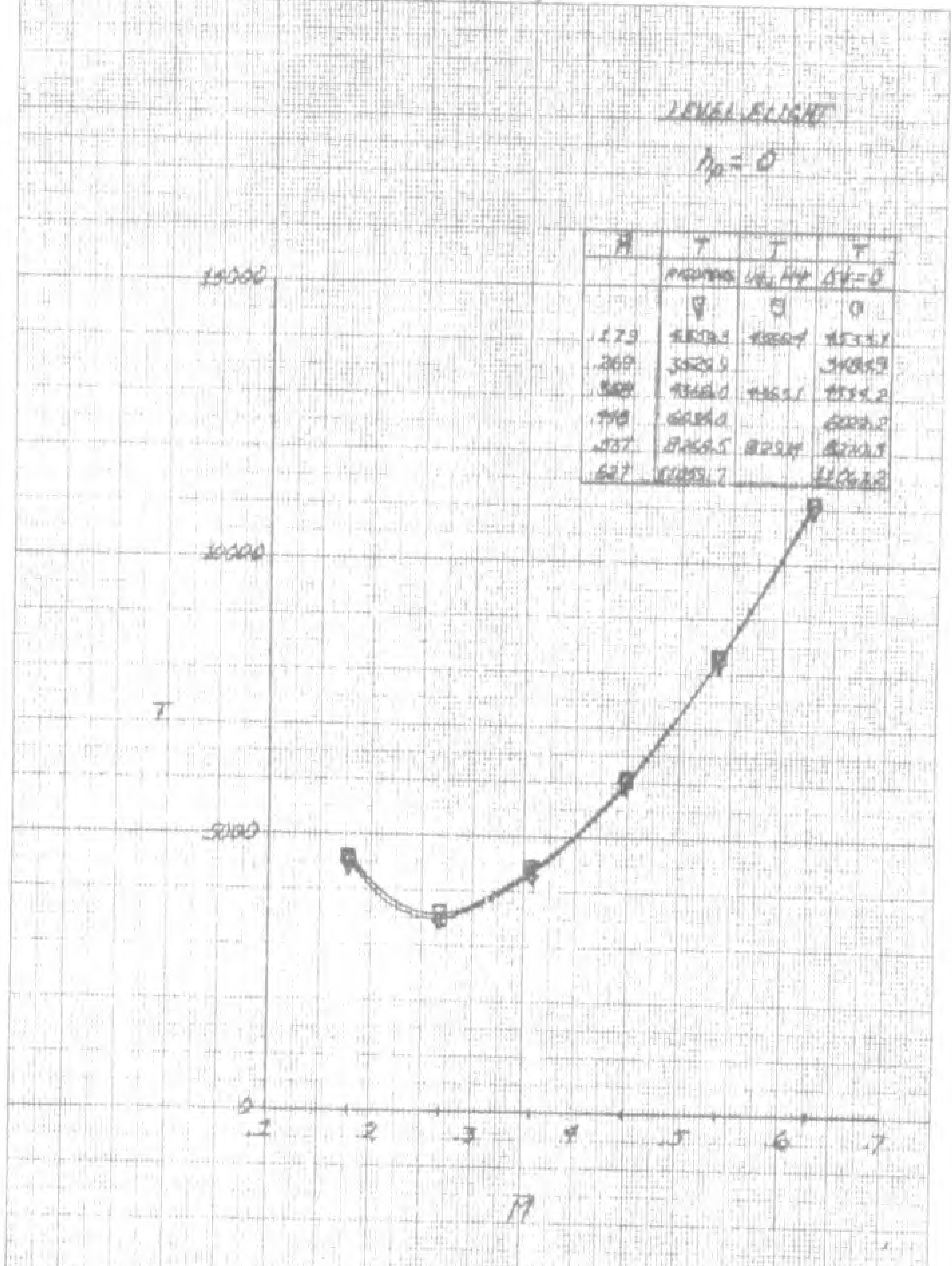


Figure 8. Investigation of Δγ Simplification

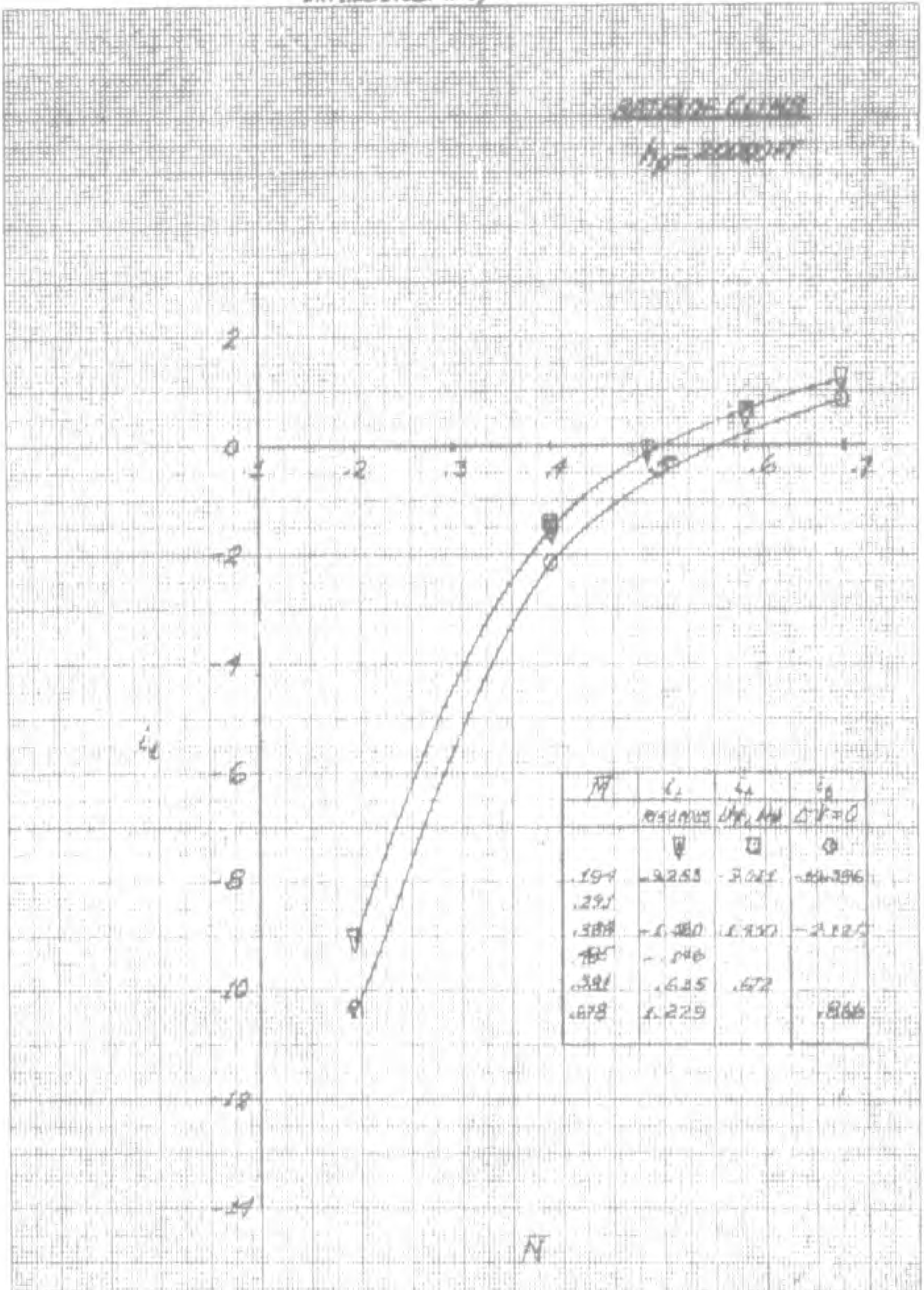


Figure 9. Investigation of ΔV Simplification

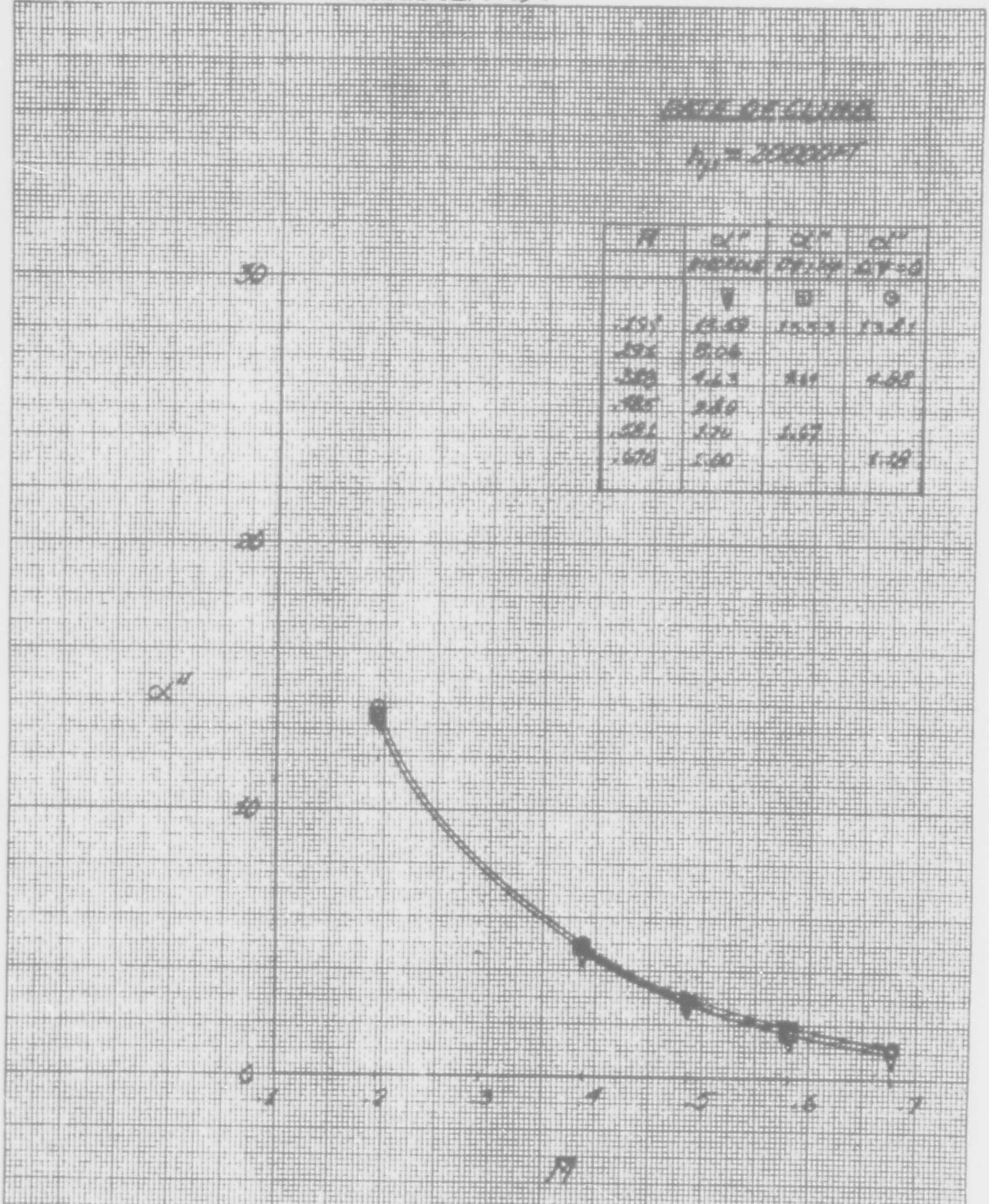


Figure 30. Investigation of Δw Simplification

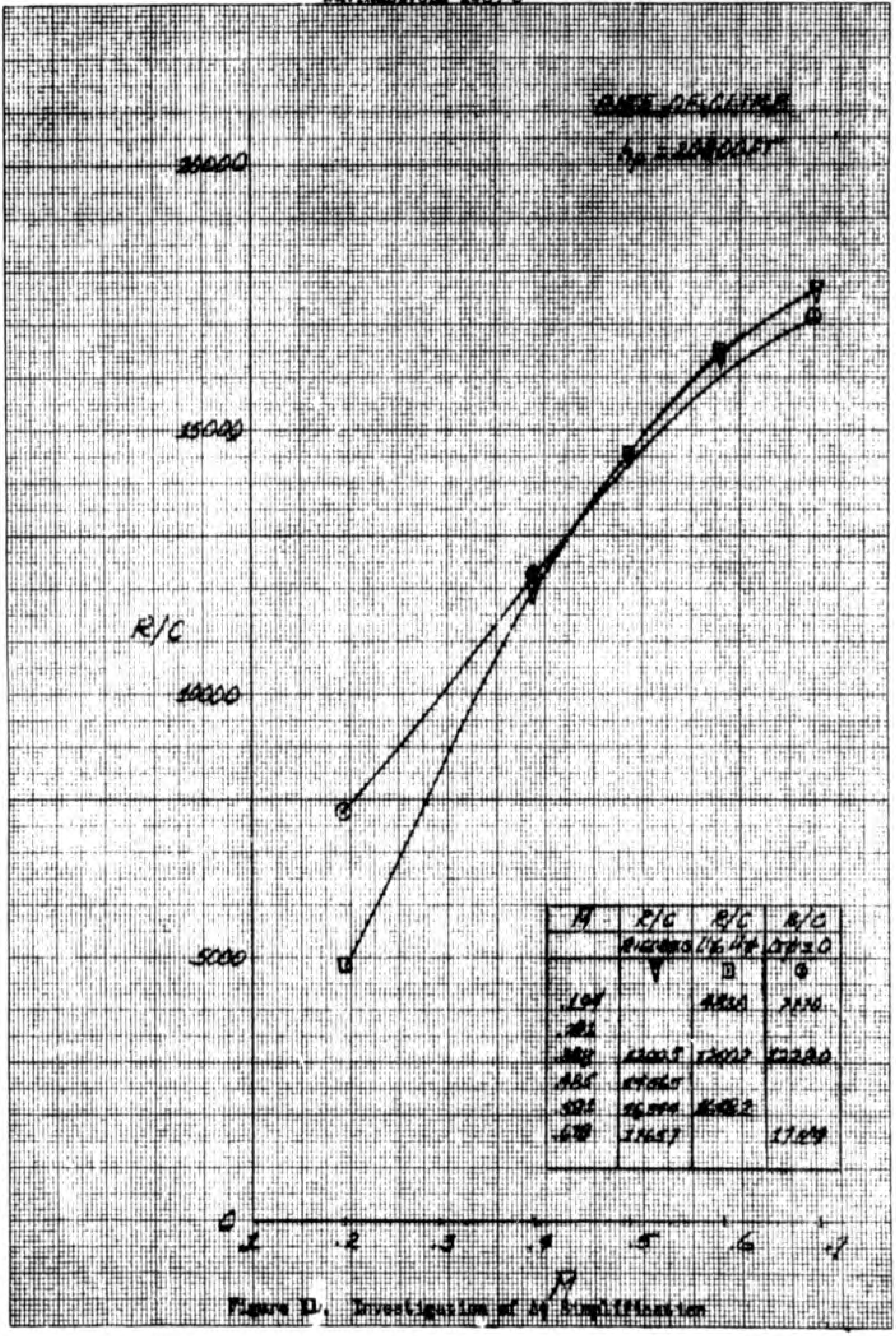


Figure D. Investigation of Δq Simplification

SECTION III

SIMULATION EQUATIONS

Having reduced Ling-Temco-Vought data to a form amenable to analog computer mechanization, the next task to be performed was to specify the requirements of an analog computer which would allow static and dynamic testing of individual mathematical model terms. A computer arrangement which would permit the testing of all six degrees of freedom was the goal to be achieved. Unfortunately, the complexity of this type aircraft demands a component arrangement which represents a very large general purpose computer installation. Consequently, the next step became one of deciding how the mathematical model could be partitioned to reduce component requirements without jeopardizing the validity of results established in a term-by-term analysis of the mathematical model.

Melpar decided (based on the purpose of the computer) that the XC-142A mathematical model could be broken down into two secondary mathematical models. One which would rigorously define the longitudinal characteristics of the aircraft, while the other would demonstrate the lateral-directional properties of the aircraft under predetermined longitudinal constraints.

It should be noted here, that any breakdown of the rigorous mathematical model into an arrangement representing less than six degrees of freedom is made at the sacrifice of certain static and dynamic properties (depending on the type of breakdown selected). The breakdown described in the previous paragraph represents a best choice, insofar as a minimum sacrifice of static and dynamic interplay is involved.

Another basis by which the mathematical model could be simplified is that of using only those elements required to perform specific tests such as: Level flight tests, Rate of climb tests, Maximum acceleration and deceleration tests, Rudder effectiveness tests, Aileron effectiveness test, Static lateral tests, etc. A breakdown of this nature, however, would be incompatible with the purpose for which the simulation is to be designed. It is evident that few dynamic properties of the mathematical model could be tested in such a configuration, no interplay between the longitudinal and lateral terms would be accounted for, and terms which might appear to be negligible during one test might have a significant effect in another type of test.

The following table presents the three mathematical models that were flow diagrammed in functional form.

DESCRIPTION	SIMULATION EQUATIONS: SIX DEGREES OF FREEDOM	SIMULATION EQUATIONS: LONGITUDINAL MODE	SIMULATION EQUATIONS: LATERAL-DIRECTIONAL MODE
EQUATIONS OF MOTION	$\dot{u} = \frac{X_a}{m} + vr - wq_1 - g \sin \theta$	$\dot{u} = \frac{X_a}{m} - wq_1 - g \sin \theta$	
	$\dot{v} = \frac{Y_a}{m} + wp - ur + g \cos \theta \sin \phi$		$\dot{v} = \frac{Y_a}{m} + w \cdot p - ur + g \sin \phi$
	$\dot{w} = \frac{Z_a}{m} + uq_1 - vp + g \cos \theta \cos \phi$	$\dot{w} = \frac{Z_a}{m} + uq_1 + g \cos \theta$	
	$\dot{p} = \frac{1}{I_{xx}} [\mathcal{L}_a + I_{xz} (\dot{r} + pq_1) - (I_{zz} - I_{yy}) rq_1]$		$\dot{p} = \frac{\mathcal{L}_a}{I_{xx}} + \frac{I_{xz}}{I_{xx}} \dot{r}$
	$\dot{q}_1 = \frac{1}{I_{yy}} [M_a + I_{xz} (r^2 - p^2) + (I_{zz} - I_{yy}) pr]$	$\dot{q}_1 = \frac{M_a}{I_{yy}}$	
	$\dot{r} = \frac{1}{I_{zz}} [N_a + I_{xz} (\dot{p} - rq_1) - (I_{yy} - I_{xx}) pq_1]$		$\dot{r} = \frac{N_a}{I_{zz}} + \frac{I_{xz}}{I_{zz}} \dot{p}$
FORCES AND MOMENTS	$X_a^{*} = (\Delta X_a)_p + (\Delta X_a)_w + (\Delta X_a)_{ho} + (\Delta X_a)_f$	$X_a = X_a^{*}$	
	$Y_a^{**} = (\Delta Y_a)_p + (\Delta Y_a)_{vt} + (\Delta Y_a)_f$		$Y_a = Y_a^{**}$
	$Z_a^{*} = (\Delta Z_a)_p + (\Delta Z_a)_w + (\Delta Z_a)_{ho} + (\Delta Z_a)_{TR} + (\Delta Z_a)_f$	$Z_a = Z_a^{*}$	
	$\mathcal{L}_a^{**} = (\Delta \mathcal{L}_a)_p + (\Delta \mathcal{L}_a)_w + (\Delta \mathcal{L}_a)_{vt}$		$\mathcal{L}_a = \mathcal{L}_a^{**}$
	$M_a^{*} = (\Delta M_a)_p + (\Delta M_a)_w + (\Delta M_a)_{ho} + (\Delta M_a)_{TR} + (\Delta M_a)_f$	$M_a = M_a^{*}$	
	$N_a^{**} = (\Delta N_a)_p + (\Delta N_a)_w + (\Delta N_a)_{vt} + (\Delta N_a)_f + (\Delta N_a)_{TR}$		$N_a = N_a^{**}$

NOTE: For clarity and brevity, if the "six degree of freedom term" is applicable directly to the "Longitudinal Mode", it is superscripted with a *; if to the "Lateral-Directional Mode", with a **.

Table 2. Simulation Equations

DESCRIPTION	SIMULATION EQUATIONS: SIX DEGREES OF FREEDOM	SIMULATION EQUATIONS: LONGITUDINAL MODE	SIMULATION EQUATIONS: LATERAL-DIRECTIONAL MODE
FUSELAGE TERMS	$(\Delta X_a)_f^* = -(11.008 + 25.115 K_1)q$ $K_1 = 1$ when landing gear down $= 0$ otherwise	$(\Delta X_a)_f = (\Delta X_a)_f^*$	
	$q^* = f(\frac{h}{[illegible]}) V^2$	$q = q^*$	$q = q^*$
	$V^{2**} = u^2 + v^2 + w^2$	$V^2 = u^2 + w^2$	$V^2 = V^{2**}$
	$(\Delta Y_a)_f^{**} = f(\frac{C_{Y,\beta}}{[illegible]}) \beta_f q$		$(\Delta Y_a)_f = (\Delta Y_a)_f^{**}$
	$\beta_f^{**} = \sin^{-1} \frac{v}{V}$		$\beta_f = \beta_f^{**}$
	$(\Delta Z_a)_f^* = -183.823\, \alpha_f q$	$(\Delta Z_a)_f = (\Delta Z_a)_f^*$	
	$\alpha_f^* = \sin^{-1} (\frac{w}{(u^2 + w^2)^{1/2}})$	$\alpha_f = \alpha_f^*$	
	$(\Delta M_a)_f = (17.254 + 3364.42\, \alpha_f)q$	$(\Delta M_a)_f = (\Delta M_a)_f^*$	
	$(\Delta N_a)_f^{**} = f(\frac{C_{Y,\beta}}{[illegible]}) \beta_f q$		$(\Delta N_a)_f = (\Delta N_a)_f^{**}$
WING TERMS	$C_{Y,\beta}^* = \frac{\Sigma Y}{767\, q_w}$	$C_{Y,\beta} = C_{Y,\beta}^*$	$C_{Y,\beta} = C_{Y,\beta}^*$
	$q_w^* = q - \frac{T}{767}$	$q_w = q_w^*$	$q_w = q_w^*$
	$(\Delta X_a)_w^* = 536.37\, C_x\, f(\frac{C_{Y,\beta}}{[illegible]})\, q_w$	$(\Delta X_a)_w = (\Delta X_a)_w^*$	
	$C_x^* = -C_D \cos\eta - C_L \sin\eta$	$C_x = C_x^*$	
	$\eta^* = i_w - \alpha_w$	$\eta = \eta^*$	$\eta = \eta^*$
	$C_L^* = [f(\frac{C_{Y,\beta}}{[illegible]}) + f(\frac{[illegible]}{[illegible]})\, \alpha_w + f(\frac{[illegible]}{[illegible]})]$	$C_L = C_L^*$	$C_L = C_L^*$
	$f(\frac{[illegible]}{[illegible]})\, f(\frac{[illegible]}{[illegible]})$	$MW = MW^*$	$MW = MW^*$

Table 2. Simulation Equations (Cont'd)

DESCRIPTION	SIMULATION EQUATIONS: SIX DEGREE OF FREEDOM	SIMULATION EQUATIONS: LONGITUDINAL MODE	SIMULATION EQUATIONS: LATERAL-DIRECTIONAL MODE
WING TRANS (CONT'D)	$M^* = f(\frac{h}{\bar{c}_T}) V$		
	$C_{L_w}^* = C_L f(\frac{C_T S}{\bar{c}_T})$	$C_{L_w} = C_{L_w}^*$	$C_{L_w} = C_{L_w}^*$
	$\alpha_w^* = \sin^{-1} \frac{w_w}{\sqrt{u_w^2 + w_w^2}}$	$\alpha_w = \alpha_w^*$	$\alpha_w = \alpha_w^*$
	$\beta_w^{**} = \sin^{-1} \frac{v_w}{V_w}$		$\beta_w = \beta_w^{**}$
	$u_w^* = \sqrt{\frac{2T}{761\rho} \pm (u \cos i_w - w \sin i_w)^2}$	$u_w = u_w^*$	$u_w = u_w^*$
	$w_w^* = w \cos i_w + u \sin i_w$	$w_w = w_w^*$	$w_w = w_w^*$
	$V_w^{2**} = u_w^2 + v^2 + w_w^2$	$V_w^2 = u_w^2 + w_w^2$	$V_w^2 = V_w^{2**}$
	$C_D^* = .04975\, C_L^2 + f(\frac{\delta F}{\bar{c}_T})$	$C_D = C_D^*$	
	$(AZ_a)_w^* = 534.37\, C_z\, f(\frac{C_T S}{\bar{c}_T}) q_w$	$(AZ_a)_w = (AZ_a)_w^*$	
	$C_z^* = -C_L \cos\eta + C_D \sin\eta$	$C_z = C_z^*$	
	$(AM_a)_w^* = 4313.435\, C_m\, f(\frac{C_T S}{\bar{c}_T})\, q_w$	$(AM_a)_w = (AM_a)_w^*$	
	$C_m^* = C_{m_w} + \frac{X_{ac}}{8.072} C_z - \frac{Z_{ac}}{8.072} C_x$	$C_m = C_m^*$	
	$C_{m_w}^* = -.06 - .5157\, \delta F - 4.613 \frac{q_1}{V_w}$	$C_{m_w} = C_{m_w}^*$	

Table 2. Simulation Equations (Cont'd)

DESCRIPTION	SIMULATION EQUATIONS		
	SIX DEGREES OF FREEDOM	LONGITUDINAL MODE	LATERAL-DIRECTIONAL MODE
WIND TERMS (CONT'D)	$(\Delta L_a)_w^{**} = 36069.975\, C_\ell\, f(\frac{C_T \delta}{Q-1})\, q_s$		$(\Delta L_a)_w = (\Delta L_a)_w^{**}$
	$C_\ell^{**} = C_{\ell_w} \cos\eta + C_{n_w} \sin\eta$		$C_\ell = C_\ell^{**}$
	$C_{\ell_w}^{**} = [.0367 - .0573\, C_{L_w} - f(\frac{t_w}{Q-11})]\, \beta_w$		$C_{\ell_w} = C_{\ell_w}^{**}$
	$- 15.1875 \frac{p_w}{V_w} f(\frac{q_s}{Q-5}) + 8.4375\, C_{L_w} \cdot$		
	$\frac{rw}{V_w} + .126\, (\delta a_{LT} - \delta a_{RT})\, f(\frac{q_s}{Q-8})$		
	$+ f(\frac{C_T \delta}{Q-9})\, C_{L_w} [\frac{T_1 + T_2 - T_3 - T_4}{\Sigma T}]$		
	$p_w^{**} = p \cos\eta - r \sin\eta$		$p_w = p_w^{**}$
	$r_w^{**} = r \cos\eta + p \sin\eta$		$r_w = r_w^{**}$
	$(\Delta N_a)_w^{**} = 36069.975\, C_n\, f(\frac{C_T \delta}{Q-1})\, q_s$		$(\Delta N_a)_w = (\Delta N_a)_w^{**}$
	$C_n^{**} = C_{n_w} \cos\eta - C_{\ell_w} \sin\eta$		$C_n = C_n^{**}$
	$C_{n_w}^{**} = .029\, C_{L_w}^2\, \beta_w - 2.26125\, C_{L_w} \frac{p_w}{V_w}$		$C_{n_w} = C_{n_w}^{**}$
	$- .5906\, C_{L_w} \frac{r_w}{V_w} + f(\frac{C_T \delta}{Q-10})\, C_{L_w} \cdot$		
	$[\frac{T_1 - T_2 - T_3 - T_4}{\Sigma T}] + f(\frac{q_s}{Q-8}) \cdot$		
	$(.007736 - .01518\, C_{L_w})\, (\delta a_{LT} - \delta a_{RT})$		

Table 2. Simulation Equations (Cont'd)

DESCRIPTION	SIMULATION EQUATIONS		
	SIX DEGREES OF FREEDOM	LONGITUDINAL MODE	LATERAL-DIRECTIONAL MODE
PROP THRUS	$(\Delta X_a)_p^{**} = \sum_{n=1}^{4} (T_n \cos i_w - H_n^* \cos \xi_n \sin i_w)$ $T_n^* = 2.513 \times 10^7 \rho \left(\frac{N_n}{1732}\right)^2 C_{T_n}$ $C_{T_n}^* = [f(\frac{J'_n}{P-1}) B_n + f(\frac{J'_n}{P-2})] K_2$ $+ [f(\frac{J'_n}{P-3}) B_n - f(\frac{J'_n}{P-4})] (1 - K_2)$ $K_2 = 1$ when $B_n \leq .5235$ Rad. $= 0$ when $B_n > .5235$ Rad. $\dot{J}'_n = \frac{3.84\, u_n}{N_n}$ $u_n = u_\varphi \cos i_w - w_\varphi \sin i_w$ $- y_n [p \sin i_w + r \cos i_w]$ $+ q_1 [x_n \sin i_w + z_n \cos i_w]$ $v_n^* = v + x_n r - z_n p$ $J_n^* = \frac{3.84\, V_n}{N_n}$ $w_n = w_\varphi \cos i_w + u_\varphi \sin i_w$ $+ y_n [p \cos i_w - r \sin i_w]$ $- q_1 [x_n \cos i_w - z_n \sin i_w]$	$(\Delta X_a)_p = \sum_{n=1}^{4} (T_n \cos i_w - H_n^* \sin i_w)$ $T_n = T_n^*$ $C_{T_n} = C_{T_n}^*$ $J'_n = J_n'^*$ $u_n = u_\varphi \cos i_w - w_\varphi \sin i_w$ $+ q_1 [x_n \sin i_w + z_n \cos i_w]$ $J_n = J_n^*$ $w_n = w_\varphi \cos i_w + u_\varphi \sin i_w$ $- q_1 (x_n \cos i_w - z_n \sin i_w)$	$(\Delta X_a)_p = (\Delta X_a)_p^{**}$ $T_n = T_n^*$ $C_{T_n} = C_{T_n}^*$ $J'_n = J_n'^*$ $u_n = u_\varphi \cos i_w - w_\varphi \sin i_w$ $- y_n [p \sin i_w + r \cos i_w]$ $v_n = v_n^*$ $J_n = J_n^*$ $w_n = w_\varphi \cos i_w + u_\varphi \sin i_w$ $+ y_n [p \cos i_w - r \sin i_w]$

Table 2. Simulation Equations (Cont'd)

DESCRIPTION	SIMULATION EQUATIONS: SIX DEGREES OF FREEDOM	SIMULATION EQUATIONS: LONGITUDINAL MODE	SIMULATION EQUATIONS: LATERAL-DIRECTIONAL MODE
PROP THRUST (CONT'D)	$V_n^{**} = \sqrt{u_n^2 + v_n^2 + w_n^2}$	$V_n = \sqrt{u_n^2 + w_n^2}$	$V_n = V_n^{**}$
	$u_\psi^* = u \cos \Delta\psi - v \sin \Delta\psi$	$u_\psi = u_\psi^*$	$u_\psi = u_\psi^*$
	$v_\psi^* = v \cos \Delta\psi + u \sin \Delta\psi$	$v_\psi = v_\psi^*$	$v_\psi = v_\psi^*$
	$\Delta\psi^* = .202\, C_{L_\psi}$	$\Delta\psi = \Delta\psi^*$	$\Delta\psi = \Delta\psi^*$
	$\phi_n^* = \cos^{-1} \frac{u_n}{V_n} = \sin^{-1} \frac{\sqrt{w_n^2 + v_n^2}}{V_n}$	$\phi_n = \phi_n^*$	$\phi_n = \phi_n^*$
	$\xi_n^{**} = \sin^{-1} \frac{v_n}{\sqrt{v_n^2 + w_n^2}}$	$\xi_n = 0$	$\xi_n = \xi_n^{**}$
	$(T_n^*)^* = 2.513 \times 10^7 \rho \left(\frac{N_n}{1232}\right)^2 C_{T_n}$	$T_n^* = (T_n^*)^*$	$T_n^* = (T_n^*)^*$
	$C_{T_n}^* = f\left(\frac{J_n}{P-5}\right) f\left(\frac{B_n}{P-6}\right) f\left(\frac{\phi_n}{P-7}\right)$	$C_{T_n} = C_{T_n}^*$	$C_{T_n} = C_{T_n}^*$
	$(\Delta Y_a)_p^{**} = \sum_{n=1}^{4} (-T_n^* \sin \xi_n)$		$(\Delta Y_a)_p = (\Delta Y_a)_p^{**}$
	$(\Delta Z_a)_p^{**} = \sum_{n=1}^{4} (-T_n \sin i_w - T_n^* \cos \xi_n \cos i_w)$	$(\Delta Z_a)_p = \sum_{n=1}^{4} (-T_n \sin i_w - T_n^* \cos i_w)$	$(\Delta Z_a)_p = (\Delta Z_a)_p^{**}$
	$(\Delta L_a)_p^{**} = (\Delta Z_{a_1} - \Delta Z_{a_4}) y_1 + (\Delta Z_{a_2} - \Delta Z_{a_3}) y_2 - (\Delta Y_{a_1} + \Delta Y_{a_4}) z_1 - (\Delta Y_{a_2} + \Delta Y_{a_3}) z_2 - \sum_{n=1}^{4} [(T_n \cos \xi_n + N_n \sin \xi_n) \sin i_w]$		$(\Delta L_a)_p = (\Delta L_a)_p^{**}$

Table 2. Simulation Equations (Cont'd)

DESCRIPTION	SIMULATION EQUATIONS: SIX DEGREES OF FREEDOM	SIMULATION EQUATIONS: LONGITUDINAL MODE	SIMULATION EQUATIONS: LATERAL-DIRECTIONAL MODE
PROP THRUS (CONT'D)	$Y_n^{**} = 3.9266 \times 10^8 \rho \left[\frac{N_n}{1232}\right]^2 C_{Y_n}$		$Y_n = Y_n^{**}$
	$C_{Y_n}^{**} = [f(\frac{J_n}{F-8})\ f(\frac{B_n}{F-9})\ f(\frac{\psi_n}{F-10})] K_3$		$C_{Y_n} = C_{Y_n}^{**}$
	$K_3 = 1$ when $n = 1$ and 2 $= -1$ when $n = 3$ and 4		
	$N_n^{*} = 3.9266 \times 10^8 \rho \left[\frac{N_n}{1232}\right] C_{N_n}$	$N_n = N_n^{*}$	$N_n = N_n^{*}$
	$C_{N_n}^{*} = [f(\frac{B_n}{F-14}) + f(\frac{J_n}{F-15})]\ \psi_n$	$C_{N_n} = C_{N_n}^{*}$	$C_{N_n} = C_{N_n}^{*}$
	$Q_n^{*} = 6.2494 \times 10^7 \rho \left[\frac{N_n}{1232}\right]^2 C_{P_n}$	$Q_n = Q_n^{*}$	$Q_n = Q_n^{*}$
	$C_{P_n}^{*} = f(\frac{B_n}{F-11}) - f(\frac{B_n}{F-12})\ f(\frac{J'_n}{F-13})$	$C_{P_n} = C_{P_n}^{*}$	$C_{P_n} = C_{P_n}^{*}$
	$(\Delta M_a)_p = 1.625\ (T_1 + T_4) + 1.092\ (T_2 + T_3)$ $+ (N_1^{*} \cos \xi_1 + N_4^{*} \cos \xi_4)(\cos i_w x_1$ $- \sin i_w z_1) + (N_2^{*} \cos \xi_2 + N_3^{*} \cos \xi_3) \cdot$ $(\cos i_w x_2 - \sin i_w z_2)$ $+ \sum_{n=1}^{4} [T_n(\cos i_w Z_{PIV} + \sin i_w X_{PIV})$ $- Y_n \sin \xi_n + N_n \cos \xi_n]$	$(\Delta M_a)_p = 1.625\ (T_1 + T_4) + 1.092(T_2 + T_3)$ $+ (N_1^{*} + N_4^{*})(\cos i_w x_1 - \sin i_w z_1)$ $+ (N_2^{*} + N_3^{*})(\cos i_w x_2 - \sin i_w z_2)$ $+ \sum_{n=1}^{4} [T_n(\cos i_w Z_{PIV} + \sin i_w X_{PIV})$ $+ N_n]$	
	$(\Delta N_a)_p^{**} = -(\Delta X_{a_1} - \Delta X_{a_4}) y_1 - (\Delta X_{a_2} - \Delta X_{a_3}) y_2$ $+ (\Delta Y_{a_1} + \Delta Y_{a_4}) x_1 + (\Delta Y_{a_2} + \Delta Y_{a_3}) x_2$ $- \sum_{n=1}^{4} [Y_n \cos \xi_n + N_n \sin \xi_n] \cos i_w$		$(\Delta N_a)_p = (\Delta N_a)_p^{**}$

Table 2. Simulation Equations (Cont'd)

DESCRIPTION	SIMULATION EQUATIONS: SIX DEGREES OF FREEDOM	SIMULATION EQUATIONS: LONGITUDINAL MODE	SIMULATION EQUATIONS: LATERAL-DIRECTIONAL MODE
VERTICAL TAIL TERMS	$(\Delta \mathcal{L}_a)^{**}_{vt} = 36069.975\ q\ (\Delta C_{\mathcal{L}})$		$(\Delta \mathcal{L}_a)_{vt} = (\Delta \mathcal{L}_a)^{**}_{vt}$
	$(\Delta C_{\mathcal{L}})^{**} = [(-.0946\ \beta_f + 1.431\ \frac{r}{V})\ f(\frac{q}{VT\text{-}3}) + .039\ \delta R_{OUT}\ f(\frac{q}{VT\text{-}4})]\ f(\frac{MN}{VT\text{-}6})$		$(\Delta C_{\mathcal{L}}) = (\Delta C_{\mathcal{L}})^{**}$
	$\delta R^{**}_{OUT} = [\delta R_{IN} + \left\{\frac{q}{480.118 - q\ f(\frac{MN}{VT\text{-}2})}\right\} \left\{f(\frac{MN}{VT\text{-}2})\ \delta R_{IN} + f(\frac{MN}{VT\text{-}1})\ \beta_f\right\}]$		$\delta R_{OUT} = \delta R^{**}_{OUT}$
	$(\Delta N_a)^{**}_{vt} = 36069.975\ q\ (\Delta C_n)$		$(\Delta N_a)_{vt} = (\Delta N_a)^{**}_{vt}$
	$(\Delta C_n)^{**} = [(.243\ \beta_f - 4.6575\ \frac{r}{V})\ f(\frac{q}{VT\text{-}3}) - .0831\ \delta R_{OUT}\ f(\frac{q}{VT\text{-}4}) + 33.75\ \frac{p}{V}\ f(\frac{MN}{VT\text{-}7})\ f(\frac{q}{VT\text{-}5})]\ f(\frac{MN}{VT\text{-}6})$		$(\Delta C_n) = (\Delta C_n)^{**}$
	$(\Delta Y_a)^{**}_{vt} = 534.37\ q\ (\Delta C_y)$		$(\Delta Y_a)_{vt} = (\Delta Y_a)^{**}_{vt}$
	$(\Delta C_y)^{**} = [-.745\ \beta_f\ f(\frac{q}{VT\text{-}3}) + .235\ \delta R_{OUT} \cdot f(\frac{q}{VT\text{-}4})]\ f(\frac{MN}{VT\text{-}6})$		$(\Delta C_y) = (\Delta C_y)^{**}$
UNIT HORIZONTAL TAIL	$(\Delta X_a)^{*}_{hs} = -534.37\ q[C_{D_t}\ \cos\ (i_{t_{RIGID}} - \alpha_{t_{RIGID}}) + C_{L_t}\ \sin\ (i_{t_{RIGID}} - \alpha_{t_{RIGID}})]$	$(\Delta X_a)_{hs} = (\Delta X_a)^{*}_{hs}$	

Table 2. Simulation Equations (Cont'd)

DESCRIPTION	SIMULATION EQUATIONS: SIX DEGREES OF FREEDOM	SIMULATION EQUATIONS: LONGITUDINAL MODE	SIMULATION EQUATIONS: LATERAL-DIRECTIONAL MODE
UNIT HORIZONTAL TAIL (CONT'D)	$(\Delta M_a)^*_{hs} = -7.5\,(\Delta Z_a)_{hs} + (24.3 - X_{PIV})(\Delta Z_a)_{hs}$	$(\Delta M_a)_{hs} = (\Delta M_a)^*_{hs}$	
	$C_{D_t}^* = .00244 + .299\,C_{L_t}^2$	$C_{D_t} = C_{D_t}^{**}$	
	$C_{L_t}^* = 1.146\,\alpha_{t_{FLEX}}\,f(\frac{M}{MS-7})$	$C_{L_t} = C_{L_t}^*$	
	$\alpha^*_{t_{RIGID}} = i_{t_{RIGID}} + \alpha_f - \epsilon + (24.3 - X_{PIVOT})\cdot(\frac{q_1}{V_T} + \frac{\partial\epsilon}{\partial\alpha_f}\frac{\dot{w}}{V_T^2})$	$\alpha_{t_{RIGID}} = \alpha^*_{t_{RIGID}}$	
	$\epsilon^* = .079\,C_{L_w} + i_w + \alpha_f - \alpha_w + [\Delta\epsilon]_{PROPS} + .0594$	$\epsilon = \epsilon^*$	
	$[\Delta\epsilon]^*_{PROPS} = f(\frac{C_{T,S}}{MS-1})\,[C_{T,S}\,\alpha_w + \frac{N_2^* + N_3^*}{303.5\,q_w}]$	$[\Delta\epsilon]_{PROPS} = [\Delta\epsilon]^*_{PROPS}$	
	$\frac{\partial\epsilon}{\partial\alpha_f}^* = (.3397\,f(\frac{q_s}{MS-5})\,f(\frac{M}{MS-7})\,f(\frac{C_{T,S}}{MS-1}) - 1)\,f(\frac{C_{T,S}}{MS-3}) + 1 + f(\frac{C_{T,S}}{MS-1})\cdot[f(\frac{C_{T,S}}{MS-4})\cos i_w + \frac{.0026}{q_w}\frac{\partial(N_2^* + N_3^*)}{\partial\alpha_f}]$	$\frac{\partial\epsilon}{\partial\alpha_f} = \frac{\partial\epsilon}{\partial\alpha_f}^*$	
	$\frac{\partial(N_2^* + N_3^*)}{\partial\alpha_f} = f(\frac{C_{T,S}}{MS-2})\cos i_w\,[\frac{N_2^*}{\sin\psi_2} + \frac{N_3^*}{\sin\psi_3}]$		
	$\alpha^*_{t_{FLEX}} = [1 - f(\frac{M}{MS-7})\,f(\frac{h_p}{MS-8})]\,[\alpha_{t_{RIGID}} - \frac{.00288}{V_t}(\Sigma F_z)_1 - .00212\,\dot{q}]$	$\alpha_{t_{FLEX}} = \alpha^*_{t_{FLEX}}$	

Table 2. Simulation Equations (Cont'd)

DESCRIPTION	SIMULATION EQUATIONS: SIX DEGREES OF FREEDOM	SIMULATION EQUATIONS: LONGITUDINAL MODE	SIMULATION EQUATIONS: LATERAL-DIRECTIONAL MODE
UNIT HORIZONTAL TAIL (CONT'D)	$(\sum F_z)_1 = (\Delta Z_a)_w + (\Delta Z_a)_p + (\Delta Z_a)_f$		
	$(\Delta Z_a)^*_{hs} = 536.37\, q\, [-C_{L_t} \cos (i_{t_{RIGID}} - \alpha_{t_{RIGID}}) + C_{D_t} \sin (i_{t_{RIGID}} - \alpha_{t_{RIGID}})]$	$(\Delta Z_a)_{hs} = (\Delta Z_a)^*_{hs}$	
TAIL ROTOR THRUST	$(\Delta Z_a)^*_{TR} = - T_{TR}$	$(\Delta Z_a)_{TR} = (\Delta Z_a)^*_{TR}$	
	$T^*_{TR} = 1.1378\, \rho\, f(\frac{N_{TR}}{TR-3})\, C_{T_{TR}} \times 10^6$	$T_{TR} = T^*_{TR}$	
	$C^*_{T_{TR}} = f(\frac{B_{TR}}{TR-1}) \quad - .1\, J'_{TR}$	$C_{T_{TR}} = C^*_{T_{TR}}$	
	$J'^*_{TR} = \frac{-7.5\, w_{TR}}{N_{TR}}$	$J'_{TR} = J'^*_{TR}$	
	$w^*_{TR} = u \sin \epsilon - w \cos \epsilon - q_1 (32.08 - X_{PIVOT})$	$w_{TR} = w^*_{TR}$	
	$(\Delta M_a)^*_{TR} = - T_{TR} (32.08 - X_{PIVOT})$	$(\Delta M_a)_{TR} = (\Delta M_a)^*_{TR}$	
	$Q^*_{TR} = 1.4486\, \rho\, f(\frac{N_{TR}}{TR-3})\, C_{P_{TR}} \times 10^6$	$Q_{TR} = Q^*_{TR}$	$Q_{TR} = Q^*_{TR}$
	$C^*_{P_{TR}} = f(\frac{B_{TR}}{TR-2})$	$C_{P_{TR}} = C^*_{P_{TR}}$	$C_{P_{TR}} = C^*_{P_{TR}}$
	$(\Delta N_a)^{**}_{TR} = - Q_{TR}$		$(\Delta N_a)_{TR} = (\Delta N_a)^{**}_{TR}$

Table 2. Simulation Equations (Cont'd)

SECTION IV

COMPUTER

A. Introduction

One of the principle purposes of the study was the specification of the requirements which a general purpose analog computer must meet for programming a mathematical model of the XC-142A aircraft. The computer needed to mechanize the mathematical model as presented by LTV would require a very large general purpose analog computer. To simulate the flight control system, LTV's computer complex consisted of approximately 500 amplifiers in conjunction with a digital computer. Consequently, LTV's mathematical model based on polynomial approximations was converted to a more practical form, wherein the equations are presented in terms of non-linear straight line functions.

In order to form a basis for producing the recommended flow charts to mechanize the abridged mathematical models presented in this report, the following rules were established:

1. The computer will be a d-c analog computer (for high resolution and accuracy).
2. Position servo-mechanisms will be available.
3. It will be possible to connect at least two (2) potentiometers in series without loss of computational accuracy.
4. Each summing amplifier will be capable of handling:

   3 unity gain inputs
   2 ten gain inputs
   2 five gain inputs

5. Diode function generators will be available.
6. Components capable of generating the sine and cosine of angles will be available.

In the event that Rule 3 cannot be met by the computer installation, an additional flow chart is presented which inserts isolation amplifiers in place of Rule 3.

Three functional schematics or flow charts are offered to describe the recommended mechanization of the XC-142A mathematical model. Functional A shows the computer requirements which will provide experimentation in the longitudinal mode of flight. Functional B shows the computer requirements which will provide experimentation in the lateral-directional mode of flight. Functional C shows the computer requirements which will provide experimentation in all six degrees of freedom.

In addition, Functional D is presented in the event that two potentiometers in series cannot be mechanized. This functional is for six degrees of freedom.

Assumptions:

Functional A has been generated on the basis of the following assumptions:

1. aircraft velocities: $v = p = r = 0$

2. aircraft accelerations: $\dot{v} = \dot{p} = \dot{r} = 0$

3. aircraft attitude: $\phi = 0$

Functional B has been generated on the basis of the following assumptions:

1. aircraft velocities: w = constant; u = constant (not necessarily zero)

2. aircraft accelerations: $\dot{u} = \dot{w} = \dot{q}_1 = 0$

3. aircraft attitude: θ = constant (not necessarily zero)

Functionals C and D make no assumptions with regard to aircraft velocities, accelerations, or attitude.

In addition to the component and capability assumptions, the scale factors, as they appear on the functionals, are based on the constraints given in Table 3. The parameters delineated in Table 3 are invariant for any specific test or run. They may, however, be changed by the operator prior to a given test.

B. <u>Functional Presentation</u>

In the interests of clarity and simplicity, component symbology was held to a minimum. The classical amplifier symbol was used for summers, inverters, servos and integrators. Summers are those amplifiers which have more than one input. Inverters are those amplifiers which have only one input. Servos are those amplifiers which have more than one input and whose outputs feed a motor generator assembly shown in block form. Servos were used only when three or more function generations of a variable were required or when two or more multiplications of a variable were required. Multipliers and sine-cosine generators were presented by an indicative block, since a variety of component arrangements may be employed to perform these operations.

In many instances, summer amplifiers are shown with multiplier circuits forming a feedback loop to perform a division operation. The feedback circuit shown on the functional must be the only feedback for this type circuit.

TABLE 3

Parameter	Original Value
m_1	1163.8 slugs
I_{xx}	173,000 slug-ft^2
I_{yy}	122,000 slug-ft^2
I_{zz}	267,000 slug-ft^2
I_{xz}	8,750 slug-ft^2
ρ	.00238 slug/ft^3
h_p	0 ft
N_n	1,232 RPM
N_{TR}	2,360 RPM
c.g.	20% MAC

Adjustable Constants

The method for presenting equations or component parts of equations (whichever applies) at any given point on the functionals is to indicate the mathematical quantity above the line connecting any two components. The scale factor which applies to a mathematical quantity is presented below the line connecting any two components. For example:

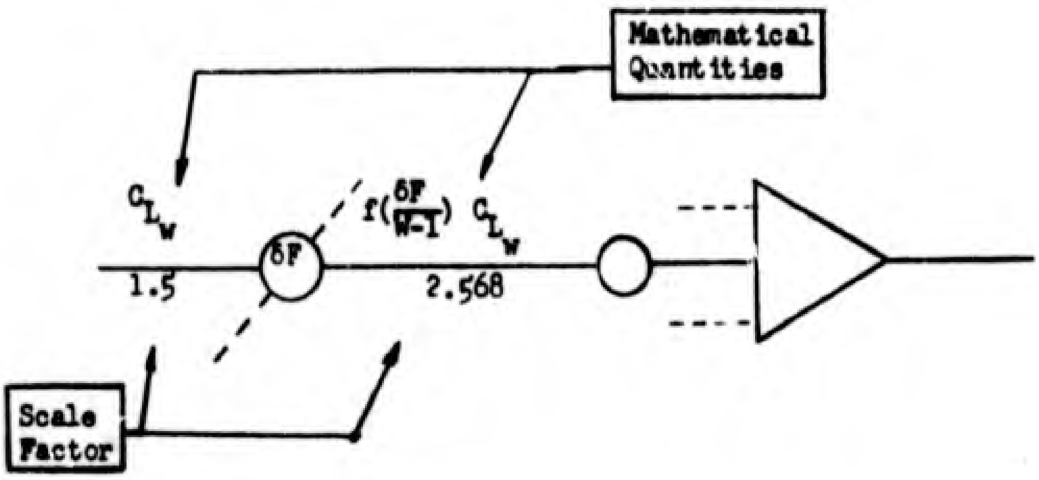


Table 4 defines and depicts the component symbols presented in the functional.

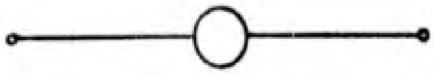
In general, the scale factor indicated for a given mathematical expression represents the maximum numerical value which can be achieved by the expression. In some cases, the scale factor chosen represents an estimate of the maximum value which can be realistically achieved by a particular aircraft parameter. Categorically, force and moment terms are based on estimated scale factors while non-linear functions and aerodynamic coefficient representations are based on calculated scale factors. In either case, the meaning of the scale factor remains unchanged. That is to say, a scale factor (at a given point) when multiplied by the maximum voltage ratio $\left(\frac{\text{maximum voltage at a given point}}{\text{voltage reference}}\right)$ is equal to the maximum value of the mathematical expression.

Maximum Function Value = (scale factor) (maximum voltage ratio)

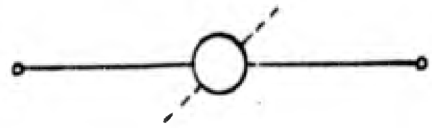
The expression above is independent of the reference voltage. Therefore it applies to any reference voltage.

TABLE 4

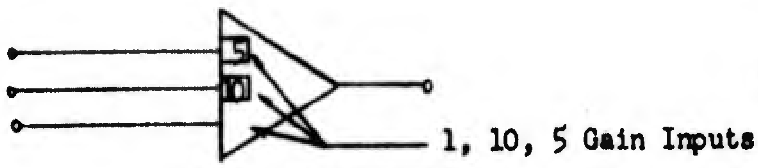
FUNCTIONAL COMPONENT SYMBOLOGY



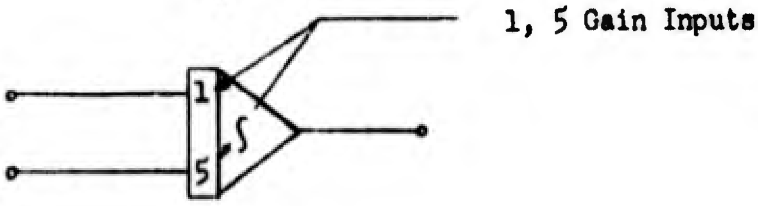
Manual Potentiometer



Servo Driven Potentiometer

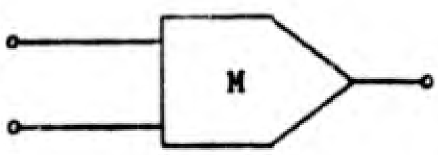


Summer or Inverter

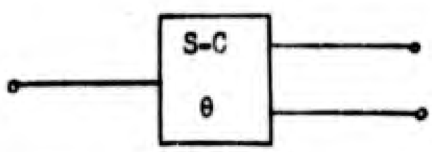


Integrator

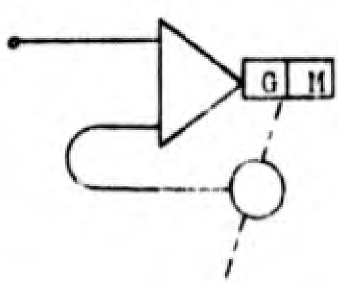
Diode Function Generator



Multiplier



Sine-Cosine Potentiometer or Resolver



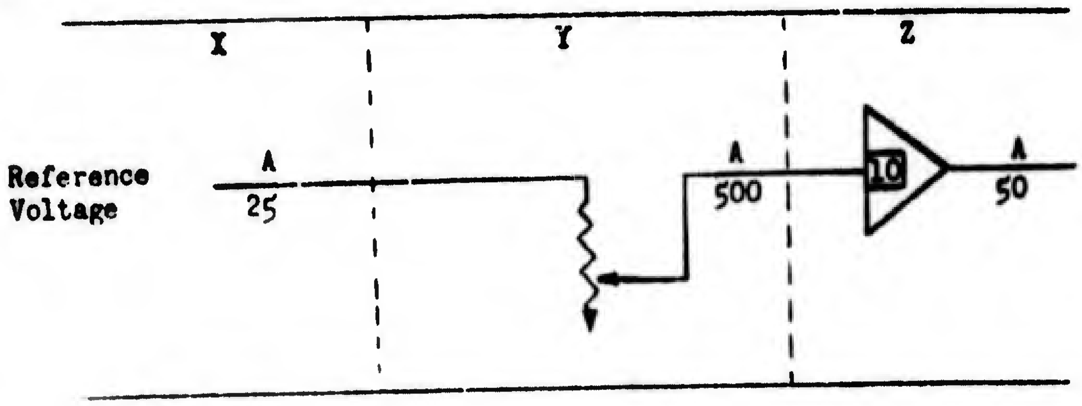
Servomechanism

Up to this point no reference to voltage gain devices has been made. If the expression above were to be used to determine the value of a scale factor, it would be necessary to modify the expression to take into account the voltage gain properties of electronic components.

In order to assist the reader in the understanding of scale factors as given in this report, let us present here several sample computations.

Example No. 1

Suppose hypothetical conditions were such that the term A had to be developed as shown below and that the quantity A represented some constant of 25 units.



Reference voltage (VR = 1.0) would exist in block X and this is arbitrarily assigned the quantity 25 units. Next we have a potentiometer whose wiper is fixed at such a point as to attenuate the source signal by a factor of 20. Therefore the maximum voltage ratio measurable in this block would be one-twentieth of the voltage reference $(XR)_{MAX} = (\frac{1}{20})$. From the previous equation

$$F\,V_{MAX} = (S.F.)\ (V.R.)_{MAX}$$

or $$S.F. = (F.V.)_{MAX}/(V.R.)_{MAX}$$

by substitution

$$S.F. = \frac{A_{(MAX)}}{\frac{1}{20}} = (25)(20) = 500$$

Therefore if we are to interpret A correctly in block Y, the scale factor must be 500. Block Z shows the signal representative of A as going through a ten gain amplifier, therefore the maximum voltage ratio measurable in this block would be 10 times the maximum voltage measurable in block Y $(10)\ (\frac{1}{20}) = 0.5$. Once again reverting to the basic equation

$$S.F. = (F.X.)_{MAX}/(V.R.)_{MAX}$$

substitution yields

$$S.F. = \frac{A_{(MAX)}}{.5} = \frac{25}{.5} = 50$$

Therefore if we are to interpret A correctly in Block Z the scale factor must be 50. Furthermore, from this example we may observe that

1. Attenuating devices increase the size of scale factors

2. Gain device decrease the size of scale factors

Therefore a more rigorous form for the expression maximum function value = scale factor (maximum voltage ratio) or

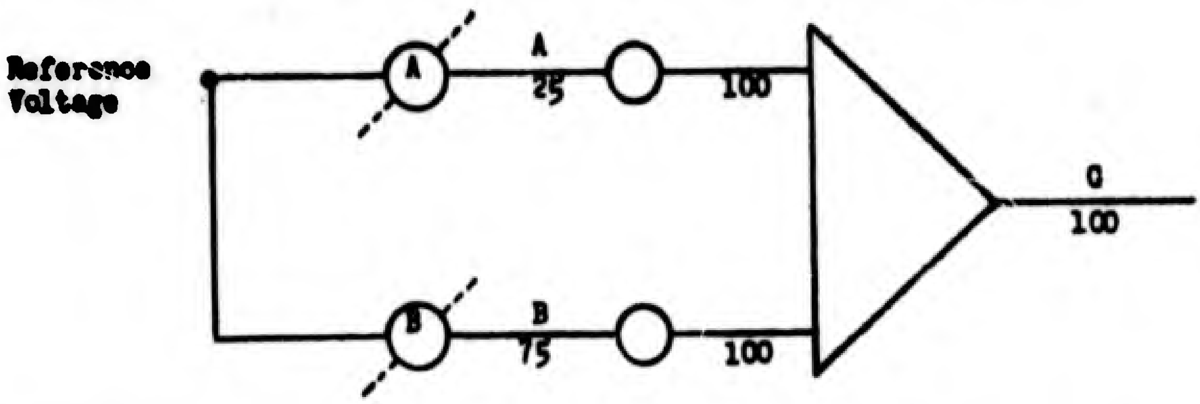
$$FV_{MAX} = (SF)\ (VR)_{MAX}$$

would be

$$FV_{MAX} = (SF)\ (VR_{MAX})\ (Gain) \text{ or } SF = \frac{FV_{MAX}}{(VR)_{MAX}\ (Gain)}$$

Example No. 2

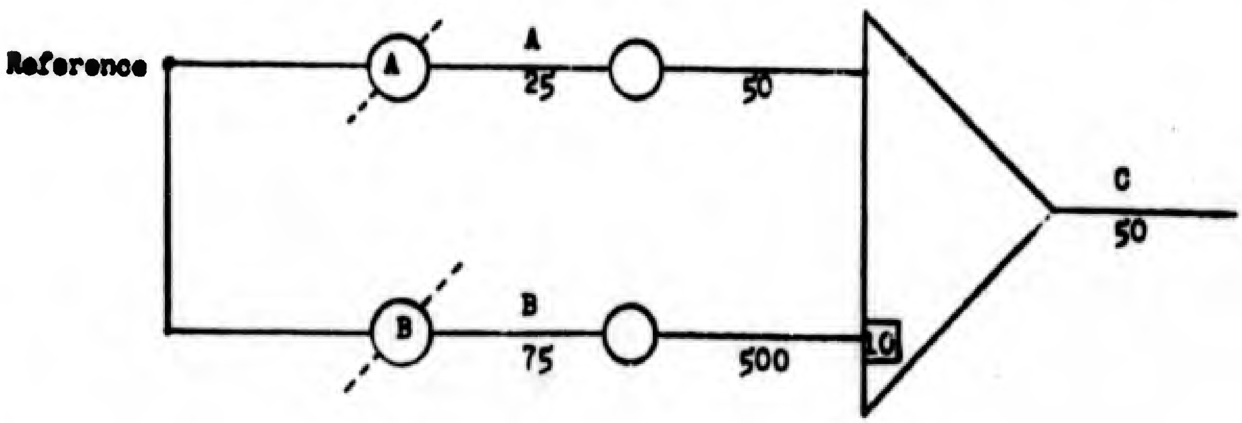
Given a circuit which will solve the expression A + B = C, where the maximum value of A is 25 and the maximum value of B is 75. If it is certain that both the quantities A and B can achieve their maximum value simultaneously we can proceed to mechanize the circuitry as follows:



The potentiometers directly preceding the amplifier may be described as weighting components. For example, if maximum voltage were present on line A, we would interpret the conditions as 25 units. Similarly, if maximum voltage were present on the B line, we would interpret the conditions as 75 units. In other words, the same voltage on both lines represents different quantities. Remembering that maximum voltage (which is the reference voltage) at line C must represent 100 units, we observe that for maximum output the input from line A as seen by the amplifier must be 25/100 of the reference voltage and line B must be 75/100 of the reference voltage (assuming a unity gain amplifier).

Example No. 3

Now let us take the case where it is not known that both A and B can be achieved simultaneously and further it has been established that C would probably never exceed 50 units. Mechanization of the circuit would be as follows:



The B line input into the amplifier is now of 10 gain input rather than unity gain input. Referring to the principal of the first example given, we can establish that since the potentiometer in line B must increase the scale factor it could be increased to 500 units. Subsequently, if a gain of 10 were selected the scale factor of this signal on the output side of the amplifier should accurately be interpreted as 50, thus satisfying all the previously stated conditions.

In this configuration saturation of the amplifier is possible but the initial stipulations indicate that unusual conditions must exist for this situation to occur which implies that saturation conditions have exceeded the scope of the problem.

In order to transform the general flow charts of this report to a specific general purpose analog computer, the following steps should be taken:

1. Expand the multiplier and sine-cosine blocks to represent the available computer. Sine-cosine generation may be mechanized by employing servo driven high resolution resolvers, diode function generators, multitapped potentiometers, or specially wound precision potentiometers, depending upon the configuration of the computer.

2. Based on the servo capability of the machine, decide how best to mechanize the non-linear functions of the mathematical model. One approach to this problem may be to use available diode function generators where only one or two functions of a particular parameter are required. Continue this process until the function generator availability has been exhausted. In all cases where deviations from the general flow charts have been employed make appropriate adjustments to the flow charts.

3. Review the flow chart scaling. If desired, it is possible to reduce the number of manual potentiometers required by judiciously adjusting the scale factor on the output side of summing or inverting amplifiers.

4. Having established the component requirement and configuration of a specific computer, assign identifying numbers to each specific component on the flow charts. This procedure will enable the operator to rapidly locate signals to be monitored during specific tests and aid in patching and debugging the system.

C. Computer Specification

In order to efficiently mechanize the XC-142A equations for making dynamic studies, it is necessary to have available a general purpose analog computer which meets the following minimum requirements:

1. It must have 84 amplifiers which can be used as summers plus additional requirements as stated below.

2. It must have 42 servos capable of driving up to 5 tapped potentiometers of varying basic resistance in order to allow cascading of at least two potentiometers*, or sufficient servos to drive 120 tapped potentiometers of like basic resistance plus 72 additional isolation amplifiers, or function generators plus attendant multipliers and amplifiers; or a combination of these configurations. The servo motor amplifiers must be capable of summing or 14 additional summing amplifiers must be provided.

3. It must have 17 multipliers plus those amplifiers required by the multipliers.

4. It must have 55 inverter amplifiers.

5. It must have 4 integrating amplifiers.

6. It must have 47 sine-cosine devices which when fed both polarities of the representation of a parameter yields as output the sine and cosine of that parameter in any of the four possible sign combinations.

* Table 5 shows a comparison of equipment count between a computer having the capability of cascading at least two potentiometers and one not having the capability.

COMPONENT	(Functional C) WITH CASCADED POTENTIOMETERS	(Functional D) WITHOUT CASCADED POTENTIOMETERS
Summer	145	145
Inverter	82	188
Integrator	8	8
Multiplier	39	39
Sine-Cosine Generator	77	77
Servo	52	95
Diode Function Generator	6	6
Tapped Potentiometers	116	116
Linear Potentiometers	52	52
Manual Potentiometer	431	431

TABLE 5. COMPONENT COMPARISON

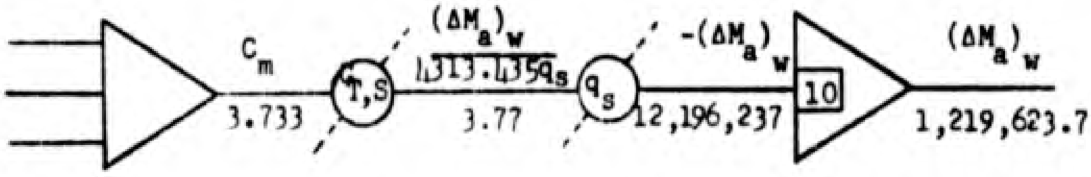
D. Patching Instructions

Example 1

Given the expression

$$(\Delta M_a)_w = 4313.435\ C_m\ f(\frac{C_{T,S}}{G-1})\ q_s$$

the expression would appear on the functional block diagram as:



Assuming inputs necessary to compute C_m are available, the following procedure would be used to mechanize the equation for $(\Delta M_a)_w$.

1. Connect the output of the summing amplifier generating C_m to the terminal of the servo driven potentiometer, $f(\frac{C_{T,S}}{G-1})$, which represents maximum position $C_{T,S}$.

2. Connect the other end of this function generator to ground reference.

   Note: If the function $f(\frac{C_{T,S}}{G-1})$ were bi-polar, it would be necessary to produce $-C_m$ with an inverter amplifier in which case step 2 would be changed to say; connect the other end of this function generator to the output of the inverter amplifier outputing $-C_m$. A positive ground reference should be provided on function generators which are bi-polar.

3. Connect the wiper arm terminal of the functional generator producing $f(\frac{C_{T,S}}{G-1})$ to the terminal of the q_s function generator which is representative of maximum q_s.

4. Connect the other end of the function generator to ground reference.

5. Connect the wiper arm terminal of the q_s function generator to a ten gain input of a selected inverter amplifier.

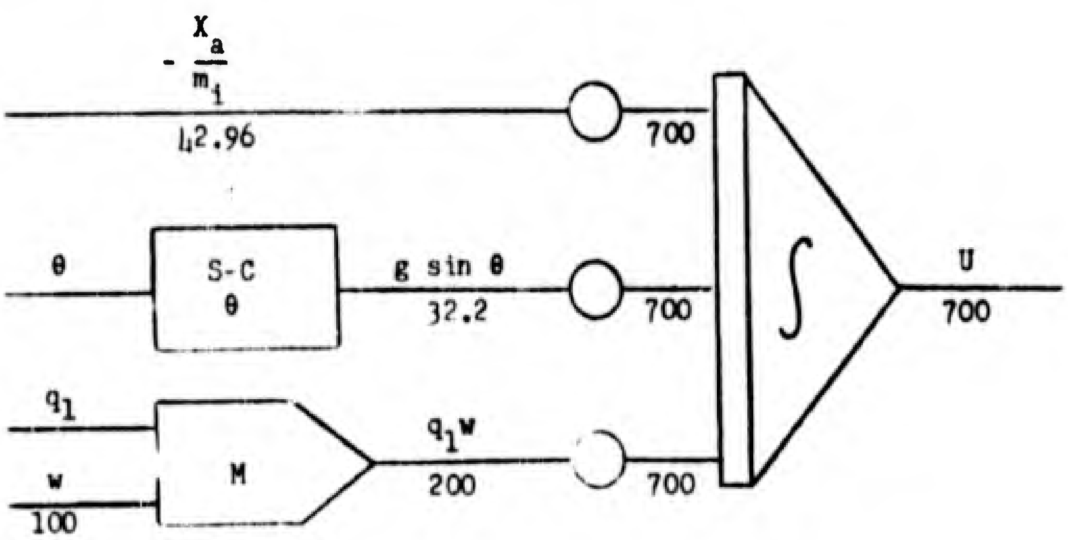
   Note: Voltage gains are usually employed in this mechanization for the purpose of operating the computer at reasonable voltage levels, while operating the simulator within the flight envelope of the XC-142 aircraft.

Example 2

Given the expression

$$U = \int \left(\frac{X_a}{m_i} - q_1 w - g \sin \theta\right) dt$$

the depicted mechanization



Given a typical general purpose computer and assuming the parameters X_a/m_i, θ, q_1 and w were available, the patch operation would be as follows:

1. Select the integrator amplifie to be used for outputting the parameter U.

2. Select three manual potentiometers.

3. Connect the wiper terminal of each potentiometer to one of the inputs of the integrator amplifier (all inputs must be unity gain inputs).

4. Connect one end of each potentiometer to the ground reference (if necessary).

5. Connect the terminal whose output is X_a/m_i to the open terminal of one of the manual potentiometers.

6. Adjust this potentiometer to the setting which satisfies the following equation

$$\left(\text{Pot setting} = \frac{\text{scale factor in}}{\text{scale factor out}}\right)$$

$$\text{Pot setting} = \frac{42.96}{700} = .0614$$

7. Assuming a diode function generator is available which will output the sine function of the input, connect the terminal whose output is θ to the input terminal of this diode function generator.

8. Connect the output of the diode function generator to the open terminal of one of the unused manual potentiometers.

9. Using the formula given in step 6, adjust this manual potentiometer to the setting $\frac{32.2}{700} = .0805$. Patching the multiplier circuit which produces $q_1 w$ depends upon the particular computer used and the method by which multiplication is achieved.

Note: A common technique used in general purpose analog computers for performing the multiplication operation is called Quarter-Square.

Mathematically if the product XY is desired, one method of achieving this is:

$$\frac{1}{4}(X - Y)^2 = \frac{1}{4}X^2 - \frac{1}{2}XY + \frac{1}{4}Y^2$$

$$-\frac{1}{4}(X + Y)^2 = -\frac{1}{4}X^2 - \frac{1}{2}XY - \frac{1}{4}Y^2$$

Summing the two expressions yields -XY. This method is employed in the TR-48 analog computer. Component requirements for producing the multiplication operation are: (1) Both negative and positive polarity of each quantity must be available, (2) a multiplier module ([illegible] squaring cards) must be available, and (3) one inverter amplifier in addition to the multiplier module is required. Assuming that at least one polarity of each quantity to be multiplied is available, 3 inverter amplifiers and one multiplication module will be necessary to perform the multiplication operation.

10. Connect the terminal whose output is q_1 to the +X input terminal of the multiplier module. Connect the terminal whose output is $-q_1$ to the -X input terminal of the multiplier module.

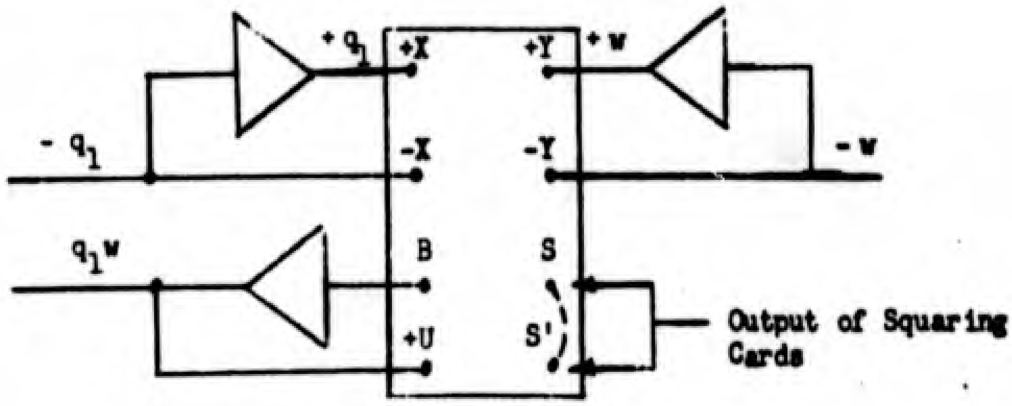
11. Connect the terminal whose output is w to the +Y input terminal of the multiplier module. Connect the terminal whose output is -w to the -Y input terminal of the multiplier module.

12. Connect the output of the squaring cards to each other.

13. Connect the output of the squaring cards to the input of a unity gain inverter amplifier.

14. Connect the output of the inverter amplifier to the feedback circuit of the multiplier module.

    Note: If output polarity reversal is desired, interchange the connections as given either in step 10 or step 11.



TR-48 Multiplier Patching Configuration

E. Sample Calculation

The process utilized to transform manufacturers data to the flow charts presented in this report is demonstrated by the following example:

(a) LTV data

$$(\Delta F_Z)_W = C_2 \, S \, \frac{m}{m''} \, q_s$$

where

$$\frac{m}{m''} = \{ 1 - R_1 \, C_{T,S} \text{ [illegible] } - R_5 \, (C_{T,S})^5 \}$$

$$C_Z = -C_L \cos\eta + C_D \sin\eta$$

$$\eta = i_w - \alpha''$$

$$C_L = [C^{*}_{L_0} + (C_{L_\alpha} + C_{L_{\alpha_{\delta F}}}\ \delta F)\ \alpha'' + C^{**}_{L_{\delta F}}\ \delta F]\ [F]_{\substack{WING\\FLEX}}\ [F]_{\substack{WING\\MACH}}$$

$$C^{*}_{L_0} = C_{L_0}\ [1 - 2.25\ C_{T,S} + 1.25\ (C_{T,S})^2\ ]$$

$$C_{T,S} = \frac{\Sigma T}{q_s\ S_p}$$

$$C^{**}_{L_{\delta F}} = [C_{L_{\delta F}} + C_{L_{\delta F^2}}\ (\delta F) + C_{L_{\delta F^3}}\ (\delta F)^2\ ]$$

$$[F]_{\substack{WING\\FLEX}} = [1 + .000312\ q_s\ ]$$

$$[F]_{\substack{WING\\MACH}} = [1 - .1246\ \bar{M} + .7544\ (\bar{M})^2\ ]$$

$$C_D = [C_{D_0} + \frac{C_L^2}{\pi\ ARe} + C_{D_{\delta F}}(\delta F) + C_{D_{\delta F^2}}\ (\delta F)^2]$$

$\pi AR = 26.8$	$C_{L_0} = .08$	$C_{L_{\delta F^2}} = 4.104$
$e = .75$	$C_{L_\alpha} = 4.30$	$C_{L_{\delta F^3}} = -2.703$
$S = 534.37$	$C_{L_{\alpha_{\delta F}}} = 1.642$	$C_{D_0} = .013$
$S_p = 767$	$C_{L_{\delta F}} = .444$	$C_{D_{\delta F}} = -.0306$
$R_1 = .15$		$C_{D_{\delta F^2}} = .2955$
$R_2 = .25$		
$R_5 = .20$		

(b) Melpar's analog simulation expression

$$(\Delta Z_a)_w = 534.37\ f\left(\frac{C_{T,S}}{G-1}\right) q_s (C_z)_w$$

where:

$$(C_z)_w = -\ C_L \cos\eta + C_D \sin\eta$$

$$C_L = \left[f\left(\frac{C_{T,S}}{W-1}\right) + f\left(\frac{\delta_F}{W-2}\right) \alpha_w + f\left(\frac{\delta_F}{W-3}\right)\right] f\left(\frac{q_s}{W-6}\right) \cdot f\left(\frac{MN}{W-7}\right)$$

$$C_D = .04975\ C_L^2 + f\left(\frac{\delta_F}{W-4}\right)$$

Appendix B contains a presentation of the above functions equating Melpar's expression to LTV data.

$$f\left(\frac{C_{T,S}}{G-1}\right) = \frac{m}{m''}$$

$$f\left(\frac{C_{T,S}}{W-1}\right) = C^*_{L_0}$$

$$f\left(\frac{\delta_F}{W-2}\right) = C_{L_\alpha} + C_{L_{\alpha_{\delta F}}} (\delta F)$$

$$f\left(\frac{\delta_F}{W-3}\right) = C^{**}_{L_{\delta F}} (\delta F)$$

$$f\left(\frac{q_s}{W-6}\right) = [F]_{\substack{WING\\FLEX}}$$

$$f\left(\frac{MN}{W-7}\right) = [F]_{\substack{WING\\MACH}}$$

$$.04975 (C_L)^2 = C_L^2/\pi \cdot AR \cdot e$$

$$f\left(\frac{\delta_F}{W-4}\right) = C_{D_0} + C_{D_{\delta F}} (\delta F) + C_{D_{\delta F^2}} (\delta F)^2$$

(c) Mechanization discussion

The basic rules to following in mechanizing a given expression are:

1. Expand the expression into its fundamental elements.

2. Gather terms which are common to all elements of the equation (constant and variable terms).

Proceeding with these rules in mind, let us generate the $(\Delta Z_a)_w$ expression:

Given the expression

$$(\Delta Z_a)_w = 534.37\ f\left(\frac{C_{T,S}}{G-1}\right) q_s\ (C_Z)_w$$

we expand the expression and gather common terms

$$(\Delta Z_a)_w = 534.37\ f\left(\frac{C_{T,S}}{G-1}\right) q_s\ [-C_L \cos\eta + C_D \sin\eta]$$

which when expanded further yield

$$(\Delta Z_a)_w = 534.37\ f\left(\frac{C_{T,S}}{G-1}\right) q_s \left\{ -(C_L)(\cos\eta) + [.04975\ C_L^2 + f\left(\frac{\delta E}{W-4}\right)]\sin\eta \right\}$$

where

$$C_L = [f\left(\frac{C_{T,S}}{W-1}\right) + f\left(\frac{\delta_F}{W-2}\right)\alpha_w + f\left(\frac{\delta_F}{W-3}\right)]\ f\left(\frac{q_s}{W-6}\right) \cdot f\left(\frac{MN}{W-7}\right)$$

Working from the above expression, we see that one summing amplifier, six servo driver function generators, and three manual potentiometers are required to generate C_L (see Figure 11). Continuing, one multiplier is required to generate the term (C_L^2). The addition of $f(\frac{\delta_F}{W-4})$ to (C_L^2) requires a summing amplifier and attendant manual potentiometer. Generating the quantities $C_L \cos\eta$ and $[.04975(C_L^2) + f(\frac{\delta_F}{W-4})]\sin\eta$ requires one servo driven cosine function generator, and one servo driven sine function generator. The addition of resulting quantities requires another summing amplifier and associated manual potentiometers. Having generated the quantities contained within the braces, we must then feed the output of this summary amplifier through two servo driven potentiometers connected in a series circuit to affect the multiplication of the quantity within braces by $f(\frac{C_{T,S}}{G-1})$ and q_s. The constant 534.37 is taken

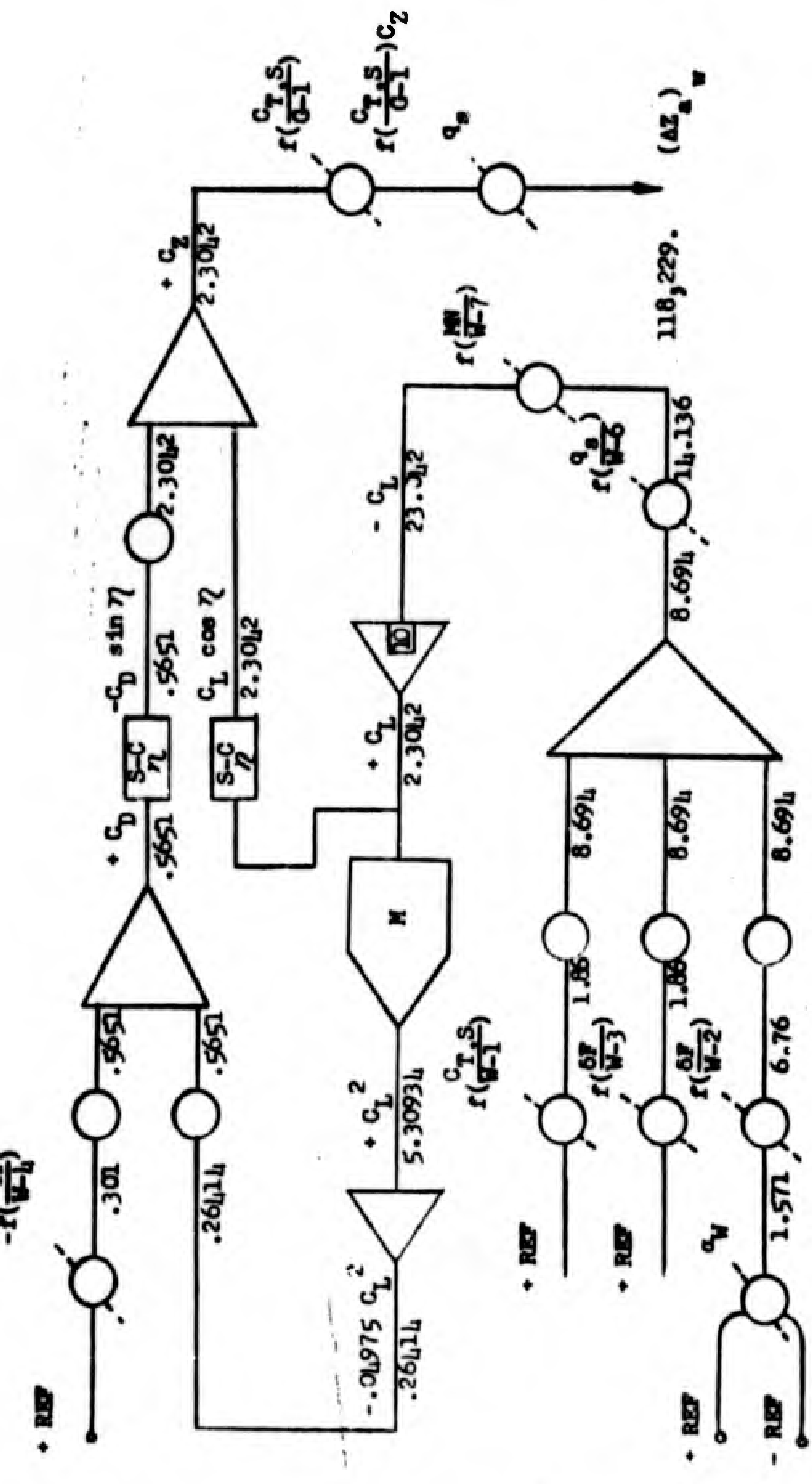


Figure 12. Sample $(\Delta Z_a)_w$ Mechanization

into consideration at the output of the q_s potentiometer by incorporating it into the scale factor under the mathematical expression (ΔZ_a). This scale factor $(\Delta Z_a)_w$ has direct bearing on the weight that this term will have in subsequent computer operations.

F. Computer Operation

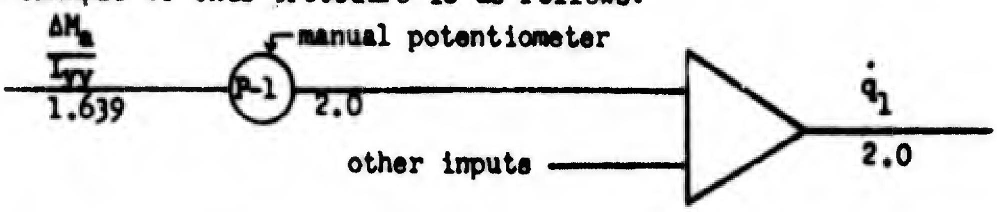
Before performing a test of either the longitudinal, lateral or six-degrees-of-freedom computers, manual potentiometers controlling the following terms should be adjusted (as required) to establish initial conditions:

1. N wing propeller speeds
2. N_{TR} tail propeller speed
3. ρ air density (equivalent to setting pressure altitude)
4. m_1 aircraft mass (equivalent to setting gross weight)
5. I_{xx} moment of inertia
6. I_{yy} moment of inertia
7. I_{zz} moment of inertia
8. I_{xz} cross product of inertia
9. c.g. aircraft center of gravity

If values other than those given in table 3 are desired, the following procedure should be followed:

1. Establish the new desired value of the parameter to be altered.
2. List all expressions which contain the term to be altered.
3. List the manual potentiometers which can be adjusted to reflect the desired change.
4. Compute the new manual potentiometer settings.
5. Adjust the affected manual potentiometers.
6. Begin new-test.

An example of this procedure is as follows:



where (ΔM_a) is scaled to 200,000 ft-lbs.

I_{yy} is set at 122,000 slug-ft^2

$$\therefore \quad \dot{q}_1 \text{ is scaled to } \frac{\Delta M_a}{I_{yy}} = \frac{200,000}{122,000} = 1.639$$

from the formula:

$$\text{Pot setting} = \frac{\text{scale factor into potentiometer}}{\text{scale factor out of potentiometer}}$$

The original pot setting for P-1 would be $\frac{1.639}{2.0} = .8195$.

Now then, if a new value of I_{yy} was desired, for example, 170,000 it would be necessary to change the pot setting of P-1 by the formula shown above. Thus we have:

$$\text{P-1} = \frac{\frac{200,000}{170,000}}{2.0} = \frac{1.1765}{2.0} = .5883$$

From the above it can be seen that given a constant pitching moment M, the new and higher amount of inertia I_{yy} results in a greater attenuation (compare initial pot setting with the new pot setting) of the moment signal thereby lowering the signal representing pitching acceleration ($\dot{q}_1$). This correctly is equivalent to lowering the dynamic response of the aircraft. Since parameters such as those given in Table 3 may appear in many terms of the mathematical model, care must be exercised to verify that all terms containing the variable, to be altered, be adjusted. Table 6 presents a list of the scale factors used to generate the flow charts. These values represent the estimated maximum values which would be achieved by specific aircraft parameter within a reasonable flight envelope.

In addition to the initial condition procedure mentioned above, it will be necessary to provide operator control of the many XC-142A moveable surfaces.

TABLE 6. SCALE FACTORS

Parameter	Scale Factor
θ	1.571 rad.
Φ	1.571 rad.
p	2.0 rad/sec
q_1	1.0 rad/sec
r	1.0 rad/sec
M_a	200,000 ft-lbs
N_a	200,000 ft-lbs
7_a	400,000 ft-lbs
X_a	50,000 lbs
Y_a	20,000 lbs
Z_a	70,000 lbs
u	700 f/s
v	100 f/s
w	100 f/s
u_n	700 f/s
v_n	100 f/s
w_n	100 f/s
u_w	750 f/s
v_w	100 f/s
w_w	100 f/s
V	714 f/s
V_w	762 f/s
q	700 lbs/ft^2

TABLE 6. SCALE FACTORS
(Cont'd)

Parameter	Scale Factor
q_s	750 lbs/ft^2
MN	.7
N_n^*	18284 lbs
T_n	10466 lbs
i_w	1.745 rad
$i_{t_{RIGID}}$	.698 rad
B_{TR}	.349 rad
B_n	1.047 rad
J_n	2.1818
J_n'	2.1818
ψ_n	1.571 rad
ξ_n	1.571 rad
$\Delta\psi$	.47 rad
$\alpha_{t_{RIGID}} - i_{t_{RIGID}}$	3.14 rad
η	3.14 rad
ϵ	3.14 rad
α_f	3.14 rad
β_f	1.571 rad
α_w	1.571 rad
β_w	1.571 rad
δF	1.047 rad
δA_{LT}	.8726 rad
δA_{RT}	.8726 rad
δR_{IN}	.5236 rad

A list of the input requirements for controlling the simulated XC-142A aircraft are:

a) Input requirements for testing the longitudinal mode computer are:

1) i_w wing incidence angle

2) δF flaps position

3) B_n wing propeller pitch angles

4) B_{TR} tail propeller pitch angle

5) i_t tail incidence angle

b) Input requirements for testing the lateral mode computer are:

1) i_w wing incidence angle

2) δF flaps position

3) B_n wing propeller pitch angles

4) δA aileron surface deflection

5) δR rudder surface deflection

In addition, it will be necessary to provide constant inputs representative of the aircraft velocities u and w. This can be accomplished through the use of manual potentiometers.

c) Input requirements for testing the six-degree-of-freedom computer are:

1) i_w wing incidence angle

2) δF flaps position

3) B_n wing propeller pitch angle

4) B_{TR} tail propeller pitch angle

5) i_t tail incidence angle

6) δA aileron surface deflection

7) δR rudder surface deflection

If smooth continuous control is desired for a particular input, manual potentiometers should be used. In cases when both a positive and negative value is possible, the potentiometer must have position reference voltage at one end of the potentiometer and negative reference at the other end of the potentiometer.

If pulse or step movement of the input parameters are desired, pulse generators or voltage level changes should be used; such as one-shot multivibrators and Schmitt triggers, respectively.

G. Functional Flow Charts

The functional flow charts are contained in the attached pocket at the end of this report.

SECTION V

DISCUSSION

The purpose of this report is to define a mathematical model for the XC-142A aircraft in a form which can be mechanised on a general purpose analog computer. The computer is to be used to enable USNTDC to perform dynamic simulation studies for this class of aircraft. Since we are most vitally concerned with flight characteristics, no attempt has been made in this report to deal with systems or unusual environmental characteristics.

The equations and mechanization require that the operator establish pressure altitude, aircraft mass distribution, aircraft surface deflections and propeller pitch angles and speeds. The implication of the above constraints in no way impairs the purpose for which the simulation is to be designed. They merely hold constant certain variables which, in actual flight, change slowly with respect to real time. For any given problem, aircraft gross weight, moments of inertia, propeller speeds, and altitude will be constant. It will be possible to deflect the aircraft movable surfaces by using manual potentiometers, pulse generators, and/or voltage level changers. In all probability, manual potentiometers will be used to provide continuous control of wing incidence angle, flap angle, main propeller pitch angles, and tail propeller pitch angle. Pulse generators and voltage level changers will be used to simulate rapid movement of the movable surfaces which control the longitudinal and lateral characteristics of the aircraft. Manual potentiometers will allow the operator to simulate smooth continuous control of the movable aircraft surfaces.

As was stated previously, any mathematical model representing less than six degrees of freedom sacrifices static and dynamic fidelity. This is plainly to be noted in the Euler rate equations which in turn affect the aircraft force and moment summation by means of the gravity terms.

The longitudinal mode mechanization will provide accurate static and dynamic results in the pitch channel only. It will be possible to test aircraft dynamic response to pulse and step type movements of the unit horizontal tail and tail propeller pitch angle. It will be possible to perform acceleration and deceleration tests, rate of climb tests, longitudinal trim tests, hovering tests and hover-to-transition-to-cruise tests.

If the assumption that all four propellers are operating identically is made, mechanization requirements for the longitudinal mode computer will be reduced considerably.

The lateral mode mechanization will provide (under certain conditions) accurate static and dynamic results in the roll and turn channels. The most significant constraints in the lateral mode computer are those which apply to the speed of the aircraft along its longitudinal and vertical axes (u and w). These variables will be controlled by the operator by means of manual potentiometers. Therefore, it will be possible to test terms which control the lateral characteristics of the aircraft under given longitudinal conditions.

Selections of u and w can be made such that the aircraft is in a level flight configuration, rate of climb configuration, or hovering condition. In this computer configuration, it will be possible to test rudder and aileron effectiveness, roll and turning characteristics to surface deflections, the effects of differential propeller pitch angle scheduling; in short all terms which demonstrate significant influence on the lateral characteristics of the XC-142A aircraft.

SECTION VI

CONCLUSIONS

The most accurate and effective method for performing a term by term analysis of the XC-142A aircraft is one in which all six degrees of freedom are incorporated into the mathematical model.

The abridged mathematical models presented in this report represent the simplest and at the same time most useful form possible for term by term analyses of this type aircraft. The mathematical model breakdowns, longitudinal and lateral, can be used effectively in a term-by-term analysis of this type aircraft.

The longitudinal mode mechanization may be further simplified by a significant degree if it is assumed that the propellers are being operated identically and therefore that the force and moment outputs of one propeller when multiplied by four represent the total propeller outputs.

The wing parameter, $\Delta\psi$, is amenable to the small angle assumption but may not be neglected. Tests have also indicated that it may be possible to replace the unit horizontal tail parameter $\frac{\partial \epsilon}{\partial q}$ by a constant or functions of u and w pending the results of dynamic studies of the longitudinal mode. From the abridged equation it is quite apparent that significant amounts of computer components may be saved by verifying the validity of this indication.

SECTION VII

RECOMMENDATIONS

It is recommended that USNTDC mechanize the full six-degree-of-freedom abridged mathematical model presented in this report. Further, it is recommended that in the event of an expansion of the testing scope (to include capabilities of cockpit flying of tests) that flight control mixing expressions be added to the simulation.

If the available computer facilities are such that they cannot encompass the six degree of freedom simulation, only slightly degraded performance will be obtained by mechanizing the longitudinal and lateral mode computers for which flow diagrams have been provided.

APPENDIX A

SYMBOLOGY

The symbology used in this report is defined in the following pages. Because of the nature of the XC-142A aircraft, a number of new parameters appear when presenting the mathematical model. In many cases Melpar ascribed to the nomenclature used by LTV; however to avoid repeated use of several symbols, Melpar has deviated from the manufacturer's symbology.

One notable exception is the use of the symbol, ד, (resh) in place of the more common symbol ℓ, when referring to rolling coefficients or moments. This exception has been made primarily to avoid confusion between the symbol, L, which alludes to aircraft terms representing lift and the symbol, ℓ, which normally denotes aircraft terms representing rolling characteristics.

The following list notes the differences existing in symbology between Melpar and Ling-Temco-Vought.

Melpar	LTV	Melpar	LTV
ד	L	w_n	w_n'
$C_ד$	C_ℓ	ψ_n	ψ_n'
m/m_w	m/m''	C_{y_n}	$C_{y_n}^*$
$C_{T,S}$	C_{T_S}	C_{m_n}	$C_{m_n}^*$
q	q_F	y_n	y_n^*
V_w	V''	M_n	M_n^*
α_w	α''	f(fusel)	F(Fuselage)
β_w	β''	vt	VT
p_w	p''	B_n	β_n
r_w	r''	MN	M
q_w	q''	hs	UHT
u_n	u_n'	TR	TP
v_n	v_n'		

SYMBOLOGY

Symbol	Definition	Units	Pos. Sense
a	Speed of sound	ft/sec.	
AR	Aspect Ratio		
a.c.	Aerodynamic Center	% MAC	
b	Span (defined by subscript)	ft.	
B	Blade Pitch Angle (defined by subscript)	rad	always pos.
c	Chord (define by subscript)	ft.	
$\bar{c}$	Mean Aerodynamic Chord	ft.	
c.g.	Center of Gravity		
C_D	Coefficient of Drag (defined by subscript) - in stability axes		
C_H	Hinge Moment Coefficient (defined by subscript)		
C_{y_n}	Propeller Lateral Hub Moment Coefficient		
C_L	Coefficient of Lift (defined by subscript)		
C_l	Rolling Moment Coefficient (defined by subscript)		
$C_{L_{af}}$	Coefficient of Lift due to propeller inflow on Wing		
C_m	Pitching Moment Coefficient (defined by subscript)		
C_{M_n}	Propeller Longitudinal Hub Moment Coefficient		
C_n	Yawing Moment Coefficient (defined by subscript)		
C_{N_n}	Propeller Yawing Moment Coefficient (defined by subscript)		
C_P	Power Coefficient (defined by subscript)		

SYMBOLOGY

Symbol	Definition	Units	Pos. Sense
C_T	Thrust Coefficient (defined by subscript)		
C_{T_S}	Coefficient of Thrust at wing due to inflow velocity		
C_X	Drag Force Coefficients in Body Axes		
C_Y	Side Force Coefficients in Body Axes		
C_Z	Lift Force Coefficients in Body Axes		
D	Drag	lbs.	Pos. Aft.
D	Diameter (defined by subscript)	ft.	
e	Span Efficiency Factor		
f	Indicates, Function of		
[F]	Flexibility term		
g	Acceleration of Gravity	ft/sec^2	downward
h	Reference height (defined by subscript)	ft.	upward
h	Altitude (defined by subscript)	ft.	upward
H.M.	Hinge Moment		
I	Moment of Inertia (defined by subscript)	$slug\text{-}ft^2$	
i	Incidence Angle (defined by subscript)	rad.	
i_w	Wing Incidence	rad.	clockwise
J	Propeller Advance Ratio (defined by subscript)		
J'	'Normal' Component of J (defined by subscript)		
K	Constant		

SYMBOLOGY

Symbol	Definition	Units	Pos. Sense
L	Lift Force (defined by subscript)	lbs.	upward
⌉	Rolling Moment (defined by subscript)	ft-lbs.	roll right
ℓ	Reference length (defined by subscript)	ft.	
m	Mass (defined by subscript)	slugs	
m/m_v	Slipstream Mass Flow Ratio		
M	Pitching Moment (defined by subscript)	ft-lbs	nose up
MAC	Mean Aerodynamic Chord	ft.	
MOC	Mean Geometric Chord	ft.	
MN	Mach Number	ft.	
N	Yawing Moment (defined by subscript)	ft-lbs.	nose right
N	Rotational Velocity	R.P.M.	
N*	Normal Force	lbs.	
p	Rolling Rate	rad/sec	roll right
q_1	Pitching Rate	rad/sec	nose up
q	Dynamic Pressure (free stream)	lbs/ft^2	
q_s	Dynamic Pressure (slipstream)	lbs/ft^2	
Q	Torque (defined by subscript)	ft-lbs.	
r	Yawing or Turning Rate	rad/sec	nose right
RPM	Revolutions per Minute		
RPS	Revolutions per Second		
S	Area (defined by subscript)	ft^2	
S_p	Total Propeller Disk Area	ft^2	

SYMBOLOGY

Symbol	Definition	Units	Pos. Sense
t	Tail		
T	Thrust (defined by subscript)	lbs.	forward
U	X-axis velocity in the airmass	ft/sec.	forward
u	Longitudinal Velocity (defined by subscript)	ft/sec.	forward
v	Side velocity (defined by subscript)	ft/sec.	to right
V	Y-axis Velocity in the air mass	ft/sec.	to right
V	Velocity (defined by subscript)	ft/sec.	always
V_T	Relative Velocity	ft/sec.	
w	Vertical Velocity (defined by subscript)	ft/sec.	downward
W	Z-axis Velocity in the air mass	ft/sec.	downward
W	Weight (defined by subscript)	lbs.	
X	Longitudinal Axial Force, in Body Axes	lbs.	forward
Y	Lateral Axial Force, in Body Axes	lbs.	to right
Z	Vertical (Normal) Axial Force, in Body Axes	lbs.	downward

SUBSCRIPTS

Symbol	Definition
a	Body Axes
a.c.	Aerodynamic Center
A	Aileron
B	Blade
c.g.	Center of Gravity

SUBSCRIPTS

Symbol	Definition
D	Drag
E	Engine
f	Fuselage
F	Flap
$F_{c.g.}$	Fuselage Center of Gravity
GE	Ground Effect
hs	Horizontal Stabilizer
LT	Left
n	Identifies component (Engine, Propeller, etc.)
o	Initial, or Steady State Condition, or Sea Level
p	Propeller or Propeller Axes
PIV, PIVOT	Point of Rotation of Wing
P	Power
q_1	Pitching Rate
Q	Torque
R	Rudder
r	Turning or Yawing Rate
RT	Right
s	Slipstream
t	Tail
T	Thrust
TR	Tail Rotor
vt	Vertical tail
v	Side velocity

SUBSCRIPTS

Symbol	Definition
w	Wing, Wind Axes
$W_{c.g.}$	Wing Center of Gravity
X	Longitudinal Force (Body Axes)
x	Distance along x axis from c.g. to component axis system origin or center or to a.c.
Y	Lateral Force (Body Axes)
y	Distance along y axis from c.g. to component axis system origin or center or to a.c.
Z	Normal Force (Body Axes)
z	Distance along z axis from c.g. to component axis system origin to center or to a.c.
xx	About x body axis
yy	About y body axis
zz	About z body axis
xz	Product of Inertia (xz plane)

GREEK SYMBOLS

Symbol	Definition	Units	Pos. Sense
α	Angle of Attack (ref. A/C Body Axes) (defined by subscript for component axis reference)	rad.	relative wind below ref. axis.
β	Angle of Sideslip (ref. A/C Body Axes) (defined by subscript for component axis reference)	rad.	relative wind from right
Δ	Increment		
δA	Aileron deflection angle	rad.	left down right up
δF	Flap deflection angle	rad.	always pos.

GREEK SYMBOLS

Symbol	Definition	Units	Pos. Sense
ϵ	Downwash angle (+) upwash (-)	rad.	
η	(with axis subscript) = Normal g's force		
θ	Pitch Angle	rad.	above horizon
ρ	Air Density	slug/ft^3	
ϕ	Angle of roll (defined by subscript if other than body axes)	rad.	rt. wing down
ψ	Angle of Yaw (defined by subscript if other than body axes)	rad.	yaw right
Ω	Angular Velocity	rad/sec	clockwise
ξ_n	Angle between the component of the relative wind vector in the propeller disk plane and the propeller Z axis.	rad.	rel. wind from right
ψ_n	Angle between the u_n and V_n velocities	rad.	rel. wind from right
η	$(i_w - \alpha_w)$		
δR_{IN}	Rudder deflection angle	rad.	T.E. to left
δR_{OUT}	Rudder deflection angle	rad.	T.E. to left
$\Delta\psi$	Angular change in propeller relative wind	rad.	

APPENDIX B

MANUFACTURER'S DATA

DESIGN INFORMATION RELEASE

MODEL IN MIS EFF.		DIR. NO.		REV.
XC-142 FLIGHT CONTROL SYSTEM SIMULATOR		2-53310/210-207(S&C-77)		A
EQUATIONS FOR THE COMPUTER SIMULATION		9/18/63	PAGE 1	
	SYSTEM	REF. ERA		

Fill in block below for Information Request		Fill in block below for Information Release	
TO	GROUP	IN REPLY TO DIR. NUMBER	
REQ. BY	GROUP	REL. TO C. C. Calvin	GROUP 0-55200
REASON		PREPARED BY G. C. Booth	DATE 9/18
		CHECKED BY H. Shields	DATE 9-20-63
CV ONLY ☒ DIR ☐ BUWEPS ☐ ☐		GROUP APP.	DATE 9-23-63
		PROJECT APP.	DATE 9-2[illegible]

J. T. Curtis (2), J. T. Hooks, A. W. Shaw, H. F. Stahl, S. J. Craig, H. F. Shields, G. T. Upton

DESIGN INFORMATION

The enclosed information defines the aerodynamic representation of the XC-142 airplane to be used in the Flight Control System Simulator.

XC-142A FLIGHT CONTROL SYSTEM SIMULATOR - EQUATIONS FOR THE COMPUTER REPRESENTATION OF FLIGHT CHARACTERISTICS

1. EXTENT OF THE COMPUTER SIMULATION

The computer representation of the airframe will provide for the continuous solution of the aerodynamic characteristics and the equations of motion for:

1) Aftward and lateral airspeeds to 50 knots and forward airspeeds to 400 knots.
2) Altitude density variations from sea level to 25,000 feet for either hot, standard, or cold day conditions.
3) Any initial weight and C.G. condition with the corresponding variation due to wing incidence.

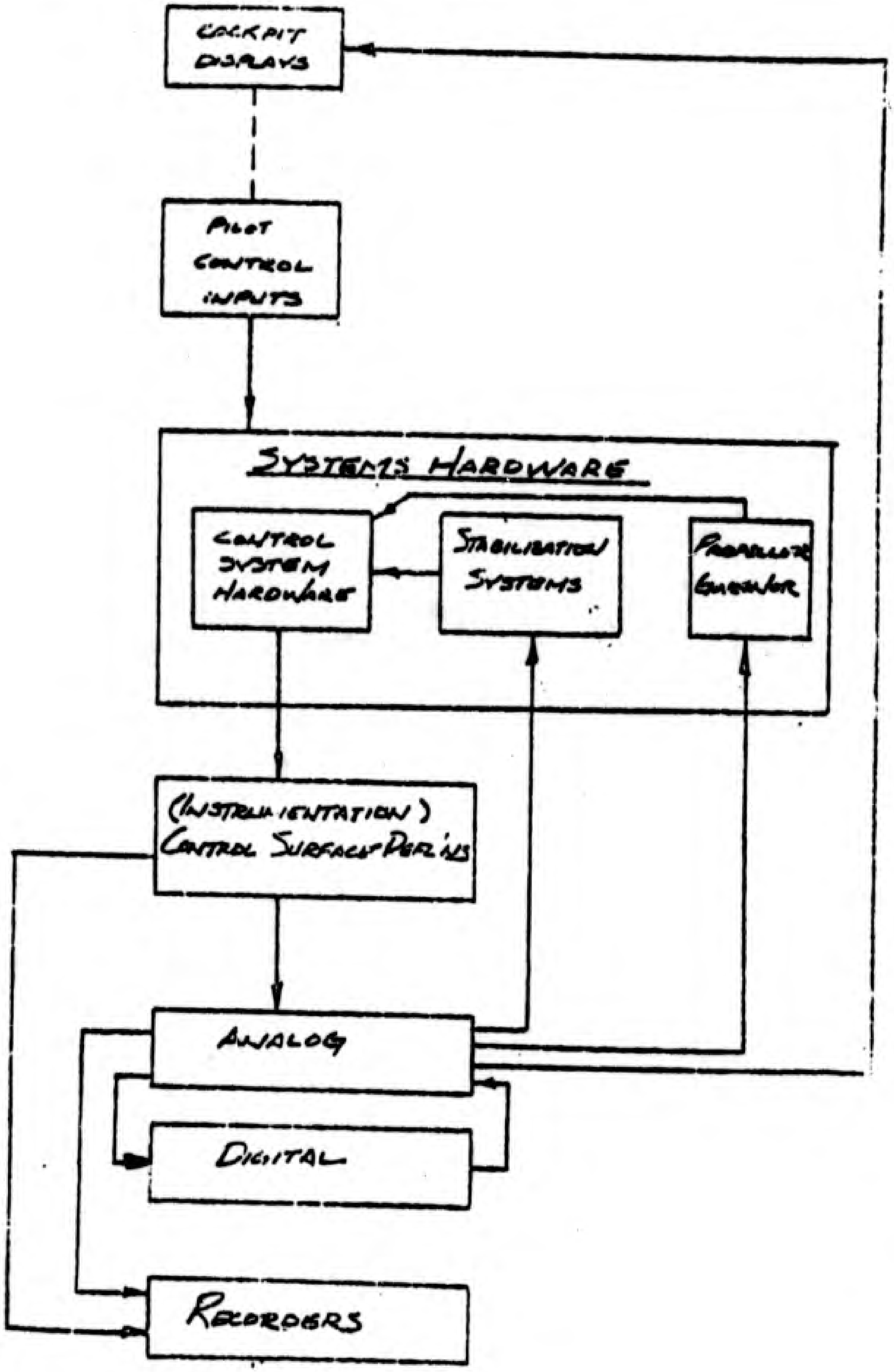
Included are:

1) Mach effects.
2) Airframe flexibility effects.
3) Control surface back-off due to aerodynamic loading and simulated P/C hinge moment limits.

A representation of the engine power characteristics, the generation of propeller governor signals and gyro and accelerometer sensor signals will be provided also.

It is most desirable that the computer setup be mechanized to facilitate minimum time loss when switching from the evaluation of discreet flight conditions to continuous flights (from ground take-off) or vice versa.

2. General Signal Flow Diagram



3. EQUATIONS OF MOTION

A BODY AXIS SYSTEM IS USED WITH THE ORIGIN LOCATED AT THE C.G. AND THE X-AXIS FORWARD IN THE W.L. PLANE OF THE C.G. ALONG B.L. 0. THE BODY AXES ARE LOCATED RELATIVE TO THE INITIAL SPACE REFERENCE BY THE DISPLACEMENTS $\psi, \theta, \phi, X_e, Y_e,$ & Z_e.

$$\psi = \int\left(\frac{r\cos\phi + q\sin\phi}{\cos\theta}\right)dt$$

$$\theta = \int(q\cos\phi - r\sin\phi)dt$$

$$\phi = \int(p + \dot{\psi}\tan\theta\cos\phi)dt$$

$$X_e = \int\dot{X}_e\,dt \quad , \quad Y_e = \int\dot{Y}_e\,dt \quad , \quad Z_e = \int\dot{Z}_e\,dt$$

WHERE

$$\begin{bmatrix}\dot{X}_e \\ \dot{Y}_e \\ \dot{Z}_e\end{bmatrix} = \begin{bmatrix}\cos\theta\cos\psi & \sin\phi\sin\theta\cos\psi - \cos\phi\sin\psi & \cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi \\ \cos\theta\sin\psi & \cos\phi\cos\psi + \sin\phi\sin\theta\sin\psi & \cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi \\ -\sin\theta & \sin\phi\cos\theta & \cos\phi\cos\theta\end{bmatrix}\begin{bmatrix}u \\ v \\ w\end{bmatrix}$$

$$\Sigma F_x = m[\dot{u} - vr + wq] + mg \sin\theta$$

$$\Sigma F_Y = m[\dot{v} - wp + ur] - mg \cos\theta \sin\phi$$

$$\Sigma F_Z = m[\dot{w} - uq + vp] - mg \cos\theta \cos\phi$$

$$\Sigma L = I_x[\dot{p} + J_x(\dot{r} + pq) + Erq]$$

$$\Sigma M = I_Y[\dot{q} + J_Y(r^2 - p^2) - Gpr]$$

$$\Sigma N = I_Z[\dot{r} + J_Z(\dot{p} - rq) + Fpq]$$

WHERE

$$E = \frac{I_Z - I_Y}{I_X} \qquad F = \frac{I_Y - I_X}{I_Z} \qquad G = \frac{I_Z - I_X}{I_Y}$$

$$J_x = \frac{-I_{XZ}}{I_X} \qquad J_Y = \frac{-I_{XZ}}{I_Y} \qquad J_Z = \frac{-I_{XZ}}{I_Z}$$

THE FORCE AND MOMENT SUMMATION TERMS REPRESENT THE TOTAL OF THE COMPONENT CONTRIBUTIONS, i.e.,

$$\Sigma F_X = \left\{(\Delta F_X)_{WING} + (\Delta F_X)_{PROPS} + (\Delta F_X)_{FUS} + \ldots\right\}$$

<u>LOAD FACTORS @ C.G.:</u>

$$n_{X_{CG}} = \frac{\Sigma F_X}{W} \qquad n_{Y_{CG}} = \frac{\Sigma F_Y}{W} \qquad n_{Z_{CG}} = \frac{-\Sigma F_Z}{W}$$

4. Weight, Inertia, C.G., Moment Arms

Weight and inertia will be constants defined for the various loading conditions to be investigated. (The variation of inertias with wing incidence is sufficiently small that it is neglected.) The variation of the airplane C.G. with wing incidence is accounted for in the following moment arm equations. Changes of the reference wing down C.G. location are made by modifying the constants A_{PIV} and D_{PIV}.

Moment Arms

C.G. to wing pivot

$$\bar{x}_{PIV} = A'_{PIV} + B'_{PIV} \sin i_W + C'_{PIV} \cos i_W$$

$$\bar{z}_{PIV} = D_{PIV} + B_{PIV} \cos i_W - C_{PIV} \sin i_W$$

C.G. to wing a.c.

$$\bar{x}_W = A'_{PN} + E_W \sin i_W + (F'_W - K'_{ac_1}\delta_F - K_{ac_2}\bar{M}) \cos i_W$$

$$\bar{z}_W = D_{PN} + E'_W \cos i_W - (F_W - K_{ac_1}\delta_F) \sin i_W$$

C.G. to prop. hub

$$\bar{x}_1 = \bar{x}_4 = A_{PN} + G_1 \cos i_W + H_1 \sin i_W$$

$$\bar{x}_2 = \bar{x}_3 = A_{PN} + G_2 \cos i_W + H_2 \sin i_W$$

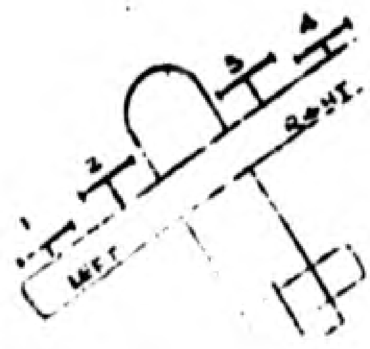
$$\bar{z}_1 = \bar{z}_4 = D_{PN} + H_1 \cos i_W - G_1 \sin i_W$$

$$\bar{z}_2 = \bar{z}_3 = D_{PN} + H_2 \cos i_W - G_2 \sin i_W$$

$\bar{y}_1 = -27.75$ FT $\quad \bar{y}_4 = 27.75$ FT

$\bar{y}_2 = -13.33$ FT $\quad \bar{y}_3 = 13.33$ FT

x, y, z subscripts ref: prop numbering bel…



5. AERODYNAMIC FORCES AND MOMENTS

5.1 AIR FLOW VARIABLES

MACH NUMBER, $\bar{M}$

$\bar{M} = \dfrac{V}{a}$ WHERE a IS SPEED OF SOUND

(1) STANDARD DAY: $a = 1117 - 4\left(\dfrac{h}{1000}\right) \sim$ FT/SEC

(2) HOT DAY: $a = 1162 - 4.15\left(\dfrac{h}{1000}\right) \sim$ FT/S

(3) COLD DAY:

$0 \le h \le 3300$ FT ; $a = 980 + 16.06\left(\dfrac{h}{1000}\right) \sim$ FT/S

$3300' \le h \le 10800$ FT ; $a = 1033 \sim$ "

$10800' \le h \le 25000$ FT ; $a = 1033 - 4.022\left(\dfrac{h - 10800}{1000}\right) \sim$

AIR MASS DENSITY, ρ

[illegible] $0 \le h \le 25000$ FT:

(1) STANDARD DAY:

$$\rho = .00238 - .00006783\left(\frac{h}{1000}\right) + .0000006188\left(\frac{h}{1000}\right)^2$$

(2) HOT DAY:

$$\rho = .00219 - .0000624\left(\frac{h}{1000}\right) + .0000005694\left(\frac{h}{1000}\right)^2$$

[illegible] $0 \le h \le 10000$ FT:

(3) COLD DAY:

$$\rho = .00309 - .0002088\left(\frac{h}{1000}\right) + .00000908\left(\frac{h}{1000}\right)^2$$

DYNAMIC PRESSURE

FREESTREAM, $q_F = \frac{1}{2}\rho V^2$; $V^2 = (u^2 + v^2 + w^2)$

+ SLIPSTREAM, $q_S = q_F + \frac{\Sigma T}{S_P}$; $S_P = 767\ FT^2$

SLIPSTREAM MASS FLOW RATIO,

$$\frac{m}{m''} = \left[1 - R_1 C_{T_S} - R_2 C_{T_S}^2 \quad , - R_5 C_{T_S}^5\right.$$

WHERE $C_{T_S} = \frac{\Sigma T}{q_S S_P}$

$$\Sigma T = \sum_{n=1}^{4} T_n = (T_1 + T_2 + T_3 + T_4)$$

$R_1 = .15$
$R_2 = .25$
$R_3 = R_4 = 0$
$R_5 = .20$

f'Cr.

5.2 Force and Moment Components

5.2.1 Wing

5.2.1.1 Wing Variables

$$u_W = +\sqrt{\frac{2\Sigma T}{\rho S_p} \pm \left(u\cos\dot{i}_W - w\sin\dot{i}_W\right)^2} \quad \text{(see note)}$$

$$w_W = w\cos\dot{i}_W + u\sin\dot{i}_W$$

$$v_W = v$$

$$V'' = \sqrt{(u_W)^2 + (v_W)^2 + (w_W)^2}$$

$$\alpha'' = \sin^{-1}\frac{w_W}{\sqrt{u_W^2 + w_W^2}} = \cos^{-1}\frac{u_W}{\sqrt{u_W^2 + w_W^2}}$$

$$\beta'' = \sin^{-1}\frac{v_W}{V''} = \cos^{-1}\frac{\sqrt{u_W^2 + w_W^2}}{V''}$$

$$\left.\begin{aligned} p'' &= p\cos\eta - r\sin\eta \\ r'' &= r\cos\eta + p\sin\eta \end{aligned}\right\} \text{ where } \eta = (i_W - \alpha'')$$

$$q'' = q$$

NOTE: THE + OR - SIGN DETERMINED BY THE SIGN OF THE QUANTITY $(u\cos i_W - w\sin i_W)$.

5.2.1.2 Wing "Power-Off" Coefficients

$$C_L = \left[C_{L_0}^{*\prime} + (C_{L_\alpha} + C_{L_{\alpha\delta_F}}\delta_F)\alpha'' + C_{L_{\delta_F}}^{**}\delta_F\right]\left[F\right]_{FLEX}\left[F\right]_{\dots}$$

$$C_L'' = C_L \frac{m}{m''}$$

$$C_D = C_{D_0}' + \frac{C_L^2}{\pi A R e} + C_{D_{\delta_F}}'\delta_F + C_{D_{\delta_F^2}}'\delta_F^2$$

$$C_{m_W} = C_{m_0}' + C_{m_{\delta_F}}\delta_F + C_{m_q}'\frac{\bar{c}}{2}\frac{q''}{V''}$$

$$C_{l_W} = \left[C_{l_{\beta_0}} + C_{l_{\beta C_L}}'(C_L'') + C_{l_{\beta i_W}}(\tfrac{\alpha}{2} - i_W)\right]\beta''$$

$$+ \left[C_{l_p}\frac{b}{2}\frac{p''}{V''}\right]\cdot\left[F\right]_{FLEX\cdot P} + \left[C_{l_{r C_L}}'\cdot C_L''\cdot\frac{b}{2}\cdot\frac{r''}{V''}\right]$$

$$+ \left[\Delta C_l\right]_{\delta_A}\cdot\left[F\right]_{\delta_A FLEX}\cdot\left[F\right]_{\delta_A B/O}$$

$$+ \left[\Delta C_l\right]_{\frac{\delta_F}{2}\Delta T}'$$

$$C_{n_W} = C_{n_{\beta C_L^2}}(C_L'')^2\beta'' + C_{n_{p C_L}}'(C_L'')\frac{b}{2}\frac{p''}{V''}$$

$$+ C_{n_{r C_L^2}}'(C_L'')^2\frac{b}{2}\frac{r''}{V''} + \left[\Delta C_n\right]_{\frac{\delta_F}{2}\Delta T}$$

$$+ \left[\Delta C_n\right]_{\delta_A}\cdot\left[F\right]_{\delta_A FLEX}'\cdot\left[F\right]_{\delta_A B/O}$$

$$C_{L_0}^{*} = C_{L_0}'\left[1 - 2.25C_{T_S} + 1.25C_{T_S}^2\right]$$

$$C_{L_{\delta_F}}^{**} = \left[C_{L_{\delta_F}}' + C_{L_{\delta_F^2}}'\delta_F + C_{L_{\delta_F^3}}'\delta_F^2\right]$$

$$LF]_{FLEX \cdot P} = (1 + .000177\, q_s)$$

$$LF]_{\substack{WING \\ FLEX}} = (1 + .000512\, q_s)$$

$$LF]_{\substack{WING \\ MACH}} = \left[\frac{1}{\sqrt{1 - .8\bar{M}}} \right] \text{ OR } \left[1 - .1246\bar{M} + .7644\, \bar{M}^2 \right]$$

$$LF]_{\delta_A FLEX} = [1 - .00109\, q_s]$$

$$LF]_{\delta_A B/O} = [1 - .0014\, q_s + .0000008\, q_s^2]$$

$$[\Delta C_L]_{\Delta T} = [a_1 C_{T_S} + a_2 C_{T_S}^2 + a_3 C_{T_S}^3]\, C_L'' \frac{\Delta T}{\Sigma T}$$

$$[\Delta C_n]_{\Delta T} = [a_4 C_{T_S}^2 + a_5 C_{T_S}^3]\, C_L'' \frac{\Delta T}{\Sigma T}$$

where $\frac{\Delta T}{\Sigma T} = \left[\frac{(T_1 + T_2) - (T_3 + T_4)}{\Sigma T} \right]$

ALTERNATE AILERON EFFECTIVENESS C/C

$$[\Delta C_L]_{\delta_A} = C_{\ell_{\delta_A}}^{*} (\delta_{A_{LT}} - \delta_{A_{RT}})$$

$$[\Delta C_n]_{\delta_A} = [C_{n_{\delta_A}}^{*} + C'_{n_{\delta_A C_L}} \cdot C_L''] (\delta_{A_{LT}} - \delta_{A_{RT}})$$

OCT 24, 1963

Aileron Effectiveness Modification: The first two equations on page 10, $[\Delta C_l]_{\delta_A}$ & $[\Delta C_n]_{\delta_A}$, must be rewritten in terms of the separate aileron angles, δ_{ALT} & δ_{ART}, rather than the combination $(\delta_{ALT} - \delta_{ART})$. Revise as follows:

$$[\Delta C_l]_{\delta_A} = \left\{ C'_{l_{\delta_A}} \delta_{ALT} + C'_{l_{\delta_A^2}} |\delta_{ALT}| \delta_{ALT} - C'_{l_{\delta_A C_L}} \delta_{ALT} C_L'' + C'_{l_{\delta_A^2 C_L}} (\delta_{ALT})^2 C_L'' \right\} - \left\{ C'_{l_{\delta_A}} \delta_{ART} + C_{l_{\delta_A^2}} |\delta_{ART}| \delta_{ART} - C'_{l_{\delta_A C_L}} \delta_{ART} C_L'' + C'_{l_{\delta_A^2 C_L}} (\delta_{ART})^2 C_L'' \right\}$$

$$[\Delta C_n]_{\delta_A} = \left\{ \left[C_{n_{\delta_A}} + C_{n_{\delta_A C_L}} C_L'' + C'_{n_{\delta_A C_L C_{T_s}}} C_L'' C_{T_s} + C_{n_{\delta_A C_L^2 C_{T_s}}} (C_L'')^2 C_{T_s} \right] \delta_{ALT} \right\} - \left\{ \left[C'_{n_{\delta_A}} + C'_{n_{\delta_A C_L}} C_L'' + C'_{n_{\delta_A C_L C_{T_s}}} C_L'' C_{T_s} + C'_{n_{\delta_A C_L^2 C_{T_s}}} (C_L'')^2 C_{T_s} \right] \delta_{ART} \right\}$$

Note:

1) All coefficients same as previously given.
2) Note change in use of absolute value of δ_A.
3) Both δ_{ALT} & δ_{ART} are positive trailing edge down.

5.2.1.3 WING COEFFICIENTS IN BODY AXES

$$C_X = -C_D \cos\eta - C_L \sin\eta$$

$$C_Z = -C_L \cos\eta + C_D \sin\eta$$

$$C_l = C_{l_W} \cos\eta + C_{n_W} \sin\eta$$

$$C_n = C_{n_W} \cos\eta - C_{l_W} \sin\eta$$

$$C_m = C_{m_W} + C_X \frac{\bar{z}_W'}{\bar{c}} - C_Z \frac{\bar{x}_W'}{\bar{c}}$$

5.2.1.4 WING FORCE & MOMENT CONTRIBUTIONS

$$(\Delta F_X)_W = C_X S \frac{m}{m''} q_s$$

$$(\Delta F_Z)_W = C_Z S \frac{m}{m''} q_s$$

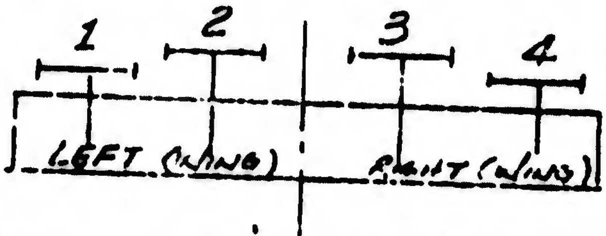
$$(\Delta L)_W = C_l' b S \frac{m}{m''} q_s$$

$$(\Delta M)_W = C_m \bar{c} S \frac{m}{m''} q_s$$

$$(\Delta N)_W = C_n b S \frac{m}{m''} q_s$$

5.2.2 MAIN PROPELLERS

APPLY THE FOLLOWING GENERAL EQUATIONS TO A PARTICULAR PROP BY SETTING $n = 1, 2, 3,$ OR 4 PER THE FOLLOWING PROP. NUMBERING SYSTEM.



GENERATION OF THE ~~PROP~~ CHARACTERISTICS OF THE 4 INDIVIDUAL PROPS IS REQUIRED.

5.2.2.1 VELOCITY COMPONENTS IN PROP. AXES

$$\Delta\phi = \left[\frac{\phi'}{C_L}\right] C_L''$$

$$u_\phi = u\cos\Delta\phi - w\sin\Delta\phi$$

$$w_\phi = w\cos\Delta\phi + u\sin\Delta\phi$$

$$u_n' = u_\phi \cos i_W - w_\phi \sin i_W - \bar{y}_n\left[p\sin i_W + r\cos i_W\right] + q\left[\bar{x}_n \sin i_W + \bar{z}_n \cos i_W\right]$$

$$v_n' = v + \bar{x}_n r - \bar{z}_n p$$

$$w_n' = w_\phi \cos i_W + u_\phi \sin i_W + \bar{y}_n\left[p\cos i_W - r\sin i_W\right] - q\left[\bar{x}_n \cos i_W - \bar{z}_n \sin i_W\right]$$

5.2.2.2 INFLOW ANGLES

$$\phi_n' = \cos^{-1}\frac{u_n'}{V_n'} = \sin^{-1}\frac{\sqrt{(w_n')^2 + (v_n')^2}}{V_n'}$$

$$V_n' = \sqrt{(u_n')^2 + (v_n')^2 + (w_n')^2}$$

5.2.2.3 LOCATION OF PROJECTED INFLOW VECTOR IN DISK PLANE

$$\xi_n = \sin^{-1} \frac{v_n'}{\sqrt{(v_n')^2 + (w_n')^2}} = \cos^{-1} \frac{w_n'}{\sqrt{(v_n')^2 + (w_n')^2}}$$

5.2.2.4 ADVANCE RATIOS

$$J_n = \frac{60\, V_n'}{\bar{N}_n\, D} \qquad \text{WHERE } \bar{N}_n \text{ IS PROP RPM}$$

$$J_n' = \frac{60\, V_n' \cos \psi_n'}{\bar{N}_n\, D} = \frac{60\, u_n'}{\bar{N}_n\, D}$$

5.2.2.5 PROPELLER COEFFICIENTS

$$C_{T_n} = \Big[C_{T_0}' + C_{T_\beta}' \beta_n + C_{T_J}' J_n' + C_{T_{J^2}}' (J_n')^2 + C_{T_{J^2\beta}}' \beta_n (J_n')^2 + C_{T_{J^2\beta^2}}' (\beta_n)^2 (J_n')^2 \Big] + C_{T_{J^3\beta}} (\beta_n)(J_n')^3$$

$$C_{N_n}^* = \Big[C_{N_{J\beta}}' + C_{N_{J^2\beta}}' (J_n') \Big] J_n \beta_n \sin \psi_n'$$

$$C_{Y_n}^* = \Big[C_{Y_{J\beta}}' + C_{Y_{J\beta^2}}' (\beta_n) \Big] J_n \beta_n \sin \psi_n'$$

$$C_{P_n} = \Big[C_{P_0} + C_{P_\beta}' \beta_n + C_{P_{\beta^2}} \beta_n^2 + C_{P_{J^2}} (J_n')^2 + C_{P_{J^3}} (J_n')^3 + C_{P_{J^2\beta}}' (J_n')^2 \beta_n + C_{P_{J^3\beta}}' (J_n')^3 \beta_n \Big]$$

$$C_{M_n}^* = C_{M_\psi} \psi_n'$$

WHERE

$$[J_n \le 0.5] \rightarrow C_{M_\psi} = [.0593\, J_n - .0573\, J_n^2 - .01642(\beta_n - .2094)]$$

$$[0.5 < J_n \le 1.0] \rightarrow C_{M_\psi} = [.01535 + .003685(J_n - 0.5) - .01642(\beta_n - .2094)]$$

$$[J_n > 1.0] \rightarrow C_{M_\psi} = [.0171925 - .01642(\beta_n - .2094)]$$

5.2.2.6 PROPELLER FORCES & MOMENTS

$$T_n = 16.557\,\rho\,\bar{N}_n^2\,C_{T_n} = 2.5130\times10^{7}\rho\left[\frac{\bar{N}_n}{1232}\right]^2 C_{T_n}$$

$$N_n^* = 16.557\,\rho\,\bar{N}_n^2\,C_{N_n}^* = 2.5130\times10^{7}\rho\left[\frac{\bar{N}_n}{1232}\right]^2 C_{N_n}^*$$

$$Y_n^* = 258.70\,\rho\,\bar{N}_n^2\,C_{Y_n}^* = 3.9266\times10^{8}\rho\left[\frac{\bar{N}_n}{1232}\right]^2 C_{Y_n}^*$$

$$M_n^* = 258.70\,\rho\,\bar{N}_n^2\,C_{M_n}^* = 3.9266\times10^{8}\rho\left[\frac{\bar{N}_n}{1232}\right]^2 C_{M_n}^*$$

$$Q_n = 41.173\,\rho\,\bar{N}_n^2\,C_{P_n} = 6.2494\times10^{7}\rho\left[\frac{\bar{N}_n}{1232}\right]^2 C_{P_n}$$

NOTE: ASTERISKS ARE USED TO DISTINGUISH THE DISCREPANCY OF "CONVENTIONAL" PROPELLER NOTATION FROM CONVENTIONAL AIRPLANE NOTATION.

5.2.2.7 Force & Moment Components in Body Axes

$$(\Delta F_x)_p = \sum_{n=1}^{4} (T_n \cos i_W - N_n^* \cos \xi_n \sin i_W)$$

$$(\Delta F_Y)_p = \sum_{n=1}^{4} (-N_n^* \sin \xi_n)$$

$$(\Delta F_Z)_p = \sum_{n=1}^{4} (-T_n \sin i_W - N_n^* \cos \xi_n \cos i_W)$$

$$\begin{aligned}(\Delta L)_p = & (\Delta F_{Z_1} - \Delta F_{Z_4})\bar{y}_1 + (\Delta F_{Z_2} - \Delta F_{Z_3})\bar{y}_2 \\ & - (\Delta F_{Y_1} + \Delta F_{Y_4})\bar{z}_1 - (\Delta F_{Y_2} + \Delta F_{Y_3})\bar{z}_2 \\ & - \sum_{n=1}^{4} (Y_n^* \cos \xi_n) \sin i_W - \sum_{n=1}^{4} (M_n^* \sin \xi_n) \sin i_W\end{aligned}$$

$$\begin{aligned}(\Delta M)_p = & 1.625 (T_1 + T_4) + 1.092 (T_2 + T_3) \\ & - (N_1^* \cos \xi_1 \sin i_W + N_4^* \cos \xi_4 \sin i_W)\bar{z}_1 \\ & - (N_2^* \cos \xi_2 \sin i_W + N_3^* \cos \xi_3 \sin i_W)\bar{z}_2 \\ & + (N_1^* \cos \xi_1 \cos i_W + N_4^* \cos \xi_4 \cos i_W)\bar{x}_1 \\ & + (N_2^* \cos \xi_2 \cos i_W + N_3^* \cos \xi_3 \cos i_W)\bar{x}_2 \\ & + \sum_{n=1}^{4} (T_n \cos i_W)\bar{z}_{PIV} + \sum_{n=1}^{4} (T_n \sin i_W)\bar{x}_{PIV} \\ & - \sum_{n=1}^{4} (Y_n^* \sin \xi_n) + \sum_{n=1}^{4} (M_n^* \cos \xi_n)\end{aligned}$$

$$\begin{aligned}(\Delta N)_p = & -(\Delta F_{X_1} - \Delta F_{X_4})\bar{y}_1 - (\Delta F_{X_2} - \Delta F_{X_3})\bar{y}_2 \\ & + (\Delta F_{Y_1} + \Delta F_{Y_4})\bar{x}_1 + (\Delta F_{Y_2} + \Delta F_{Y_3})\bar{x}_2 \\ & - \sum_{n=1}^{4} (Y_n^* \cos \xi_n) \cos i_W - \sum_{n=1}^{4} (M_n^* \sin \xi_n) \cos i_W\end{aligned}$$

5.2.3 FUSELAGE

$$(\Delta F_X)_F = -C_{D_{0_F}}' S q_F$$

$$(\Delta F_Y)_F = C_{Y_{\beta_F}} \beta \frac{d\beta_F}{d\beta} S q_F$$

$$(\Delta F_Z)_F = -C_{L_{\alpha_F}} \alpha_F S q_F$$

$$(\Delta M)_F = (C_{m_{0_F}} + C_{m_{\alpha_F}}' \alpha_F) \bar{c} S q_F$$

$$(\Delta N)_F = (C_{n_{\beta_F}} \beta \frac{d\beta_F}{d\beta}) b S q_F$$

WHERE $\beta = \sin^{-1} \frac{v}{V} = \cos^{-1} \frac{\sqrt{u^2 + w^2}}{V}$

$$q_F = \frac{1}{2} \rho V^2$$

$$\frac{d\beta_F}{d\beta} = \left[1 + k_1 C_{T_S} + k_2 C_{T_S}^2 \right]$$

$k_1 \approx +1.6$

$k_2 \approx -1.4$

$k_3 \approx 0$

5.2.4 VERTICAL TAIL & RUDDER

5.2.4.1 COEFFICIENTS

$$\Delta C_l = \left[C'_{l_\beta} \beta_v \cdot [F]_{\beta V.T. FLEX} + (C'_{l_{\delta_R}} \quad) \delta_{R_{OUT}} \cdot [F]_{\delta_R FLEX} + C'_{l_r} \frac{b}{2} \frac{r}{V} [F]_{\beta V.T. FLEX} \right] \cdot [F]_{V.T. MACH}$$

$$\Delta C_n = \left[C'_{n_\beta} \beta_v \cdot [F]_{\beta V.T. FLEX} + (C'_{n_{\delta_R}} \quad) \delta_{R_{OUT}} \cdot [F]_{\delta_R FLEX} + C'_{n_r} \frac{b}{2} \frac{r}{V} \cdot [F]_{\beta V.T. FLEX} + C_{n_p} \frac{b}{2} \frac{p}{V} \cdot [F]_{p V.T. FLEX} \right] \cdot [F]_{V.T. MACH}$$

$$\Delta C_Y = \left[C'_{Y_\beta} \beta_v [F]_{\beta V.T. FLEX} + (C'_{Y_{\delta_R}} \quad) \delta_{R_{OUT}} \cdot [F]_{\delta_R FLEX} - \frac{r}{V} [F]_{\beta V.T. FLEX} \right] \cdot [F]_{V.T. MACH}$$

$$[F]_{\beta V.T. FLEX} = \overset{F_6}{(1 - .000406 q_F)}$$

$$[F]_{\delta_R FLEX} = \overset{F_7}{(1 - .000516 q_F)}$$

$$[F]_{p V.T. FLEX} = \overset{F_8}{(1 - .000495 q_F)}$$

$$[F]_{V.T. MACH} = \overset{F_9}{(1 + .28 \bar{M}^2)}$$

$$\beta_v \doteq \left[\text{ASSUME } \beta_v \doteq \beta \right]$$

5.2.4.2 (Air Loaded) Rudder Deflection, $\delta_{R_{OUT}}$

$$C_{H_\beta} = .124\,(1 + .926\,\bar{M}^3) \sim \text{PER RAD}$$

$$C_{H_{\delta_R}} = [-.573] \text{ FOR } \bar{M} \leq 0.15$$

$$= [-.573 - .585(\bar{M} - .15)] \text{ FOR } \bar{M} > 0.15$$

~ PER RAD

$$H.M. = \left[\frac{K'_{B/O}\,[C'_{H_{\delta_R}}\,\delta_{R_{IN}} + C'_{H_\beta}\beta'_V]\,q_F\,S_R\,\bar{c}_R}{K'_{B/O} - C'_{H_{\delta_R}}\,q_F\,S_R\,\bar{c}_R}\right]$$

LIMITED @ ± 1200 FT-LBS

$$\delta_{R_{OUT}} = \left[\delta_{R_{IN}} + \frac{H.M.}{K_{B/O}}\right]$$

WHERE $\delta_{R_{IN}}$ IS LIMITED @ THE PARTICULAR VALUE CORRESPONDING TO THE H.M. LIMIT. EXPLICITLY,

$$\delta_{R_{IN}}\Big|_{LIMIT} = \left[\frac{K_{B/O}\{H.M. - C_{H_\beta}\beta_V\,q_F\,S_R\,\bar{c}_R\} - H.M.\,C_{H_{\delta_R}}\,q_F\,S_R\,\bar{c}_R}{K_{B/O}\,C_{H_{\delta_R}}\,q_F\,S_R\,\bar{c}_R}\right]$$

$$S_R = 22.81 \text{ FT}^2 \qquad \bar{c}_R = 2.48 \text{ FT}$$

$$K_{B/O} = 28{,}350 \ \frac{\text{FT-LB}}{\text{RAD}} \sim (\text{PRELIMINARY})$$

5.2.4.3 Force & Moment Contributions

$$(\Delta L)_{V.T.} = \Delta C_l\, b S q_F\, \eta_V$$

$$(\Delta N)_{V.T.} = \Delta C_n\, b S q_F\, \eta_V$$

$$(\Delta F_Y)_{V.T.} = \Delta C_Y\, S q_F\, \eta_V$$

$$\eta_V = [\text{ASSUME } \eta_V = 1]$$

SEE REVISION NOTE P. 15

UNIT HORIZONTAL TAIL

$$\varepsilon = f_1 C_L'' + i_W + \alpha_F' - \alpha'' + \varepsilon_0 + [\Delta\varepsilon]_{PROPS}$$

$$[\Delta\varepsilon]_{PROPS} = f(C_{T_S})\left[C_{T_S}\alpha'' + \frac{2(N_2^* + N_3^*)}{q_S S_P}\right]$$

$$\text{\&} \quad f(C_{T_S}) = \left\{\frac{1}{(2 - C_{T_S})(1 + \sqrt{1 - C_{T_S}})}\right\}$$

$$\alpha_{t_{RIGID}} = \left[i_{t_{RIGID}} + \alpha_F - \varepsilon + l_{HT}\frac{q}{V} + l_{HT}\frac{\dot{w}}{V^2}\right]$$

WHERE $l_{HT} = 24.3 - \bar{x}_{PV}$

$$\frac{\partial\varepsilon}{\partial\alpha_F} = \left\{\left[\left(f_1 \cdot C_{L\alpha_{WING}} \cdot [F]_{\substack{WING\\FLEX}} \cdot [F]_{\substack{WING\\MACH}} \frac{m}{m''} - 1\right)\sqrt{1 - C_{T_S}} + 1\right]\right.$$

$$\left. + f(C_{T_S}) \cdot \left[C_{T_S}\sqrt{1 - C_{T_S}}\cos i_W + \frac{2}{q_S S_P}\frac{\partial}{\partial\alpha_F}(N_2^* + N_3^*)\right]\right\}$$

$$\frac{\partial}{\partial\alpha_F}(N_2^* + N_3^*) = \boxed{B} \cdot \left[1 + \left(\frac{\psi}{C_L}\right)\frac{m}{m''} C_{L\alpha_{WING}}\sqrt{1 - C_{T_S}}\right]\cos i_W$$

$$\boxed{B} = \left\{\left[C_{N_{J\beta}} + C_{N_{J^2\beta}}(J_2')\right] J_2\beta_2 (16.557)\rho \bar{N}_2^2 + \left[C_{N_{J\beta}} + C_{N_{J^2\beta}}(J_3')\right] J_3\beta_3 (16.557)\rho \bar{N}_3^2\right\}$$

REV A

$$H_{\alpha_t} = C_{h_{\alpha_t}} \bar{c}_h S_h q_F [F]_{\substack{UHT\\MACH}}$$

$$Z_{\alpha_t} = -C_{L_{\alpha_t}} S q_F [F]_{\substack{UHT\\MACH}}$$

$$\left\{ \begin{array}{l} \bar{c}_h = 5.44 \text{ FT} \\ S_h = 163.5 \text{ FT}^2 \\ C_{h_{\alpha_t}} = -.481/\text{RAD} \end{array} \right.$$

$$(\Sigma F_z)_1 = \left[(\Delta F_z)_{WING} + (\Delta F_z)_{PROPS} + (\Delta F_z)_{FUS} + (\Delta F_z)_{TP}\right]$$

DELETE IF DONE IN SIMULATOR

$$[F]_{\substack{UHT\\MACH}} = \left[1 - .0706\bar{M} + .5233\bar{M}^2\right]$$

$$\alpha_{t_{FLEX}} = \left[\frac{\alpha_{t_{RIGID}} - \left[\frac{\Delta\alpha_t}{n_z}\right]\frac{(\Sigma F_z)_1}{W_t} + \left[\frac{\Delta\alpha_t}{\dot{q}}\right]\dot{q}}{1 - \left[\frac{\Delta\alpha_t}{HM}\right]H_{\alpha_t} + Z_{\alpha_t}\left(\frac{1}{W_t}\left[\frac{\Delta\alpha_t}{n_z}\right] - \left[\frac{\Delta\alpha_t}{Z_t}\right]\right)}\right]$$

WHERE W_t = AIRPLANE GROSS WEIGHT (SEE LOADING CONDITIONS)

$$C_{L_t} = C_{L_{\alpha_t}} \alpha_{t_{FLEX}} [F]_{\substack{UHT\\MACH}}$$

$$C_{D_t} = C_{D_{0_t}} + k_t (C_{L_t})^2$$

$$(\Delta F_x)_{UHT} = -\left[C_{D_t}\cos(i_{t_{RIGID}} - \alpha_{t_{RIGID}}) + C_{L_t}\sin(i_{t_{RIGID}} - \alpha_{t_{RIGID}})\right] S q_F$$

$$(\Delta F_z)_{UHT} = -\left[C_{L_t}\cos(i_{t_{RIGID}} - \alpha_{t_{RIGID}}) - C_{D_t}\sin(i_{t_{RIGID}} - \alpha_{t_{RIGID}})\right] S q_F$$

$$(\Delta M)_{UHT} = -(\Delta F_x)_{UHT}\bar{h}_{HT} + (\Delta F_z)_{UHT} l_{HT}$$

$$\bar{h}_{HT} = 7.5 \text{ FT}$$

$$\left[\frac{\Delta\alpha_t}{n_z}\right] = .00218 \frac{\text{RAD}}{g} \qquad \left[\frac{\Delta\alpha_t}{HM}\right] = 2.234\times10^{-6} \frac{\text{RAD}}{\text{FT LB}}$$

$$\left[\frac{\Delta\alpha_t}{\dot{q}}\right] = -.00212 \text{ SEC}^2 \qquad \left[\frac{\Delta\alpha_t}{Z_t}\right] = 11.5\times10^{-7} \frac{\text{RAD}}{\text{LB}}$$

← {BASIC AIRPLANE FEB '64}

5.2.6 TAIL PROPELLER

$$u_E = u \cos \epsilon + w \sin \epsilon$$

$$w_E = u \sin \epsilon - w \cos \epsilon$$

$$u_{TP} = u_E$$

$$v_{TP} = v - l_{TP}\, r$$

$$w_{TP} = w_E - l'_{TP}\, q$$

$$V_{TP} = \sqrt{(u_{TP})^2 + (v_{TP})^2 + (w_{TP})^2}$$

$l_{TP} = 32.08$ [illegible] $\bar{x}_{PN}$

$$\psi_{TP} = \sin^{-1} \frac{\sqrt{(u_{TP})^2 + (v_{TP})^2}}{V_{TP}} = \cos^{-1} \frac{w_{TP}}{V_{TP}}$$

$$J'_{TP} = \frac{60\, w_{TP}}{N_{TP} D_{TP}}$$

$$C_{T_{TP}} = \Big[.18622\, \beta_{TP} + 2.692 \left|\beta_{TP}\right| \beta_{TP} - 2.822 (\beta_{TP})^3 - 3.773 \left|\beta_{TP}^3\right| \beta_{TP} - .10\, J'_{TP} \Big]$$

$$C_{P_{TP}} = 1.084\, (\beta_{TP})^2$$

$$T_{TP} = 1.1378\, \rho\, \Omega_{TP}^2\, C_{T_{TP}} = 6.4448 \times 10^6 \rho \left[\frac{N_{TP}}{2380}\right]^2 C_{T_{TP}}$$

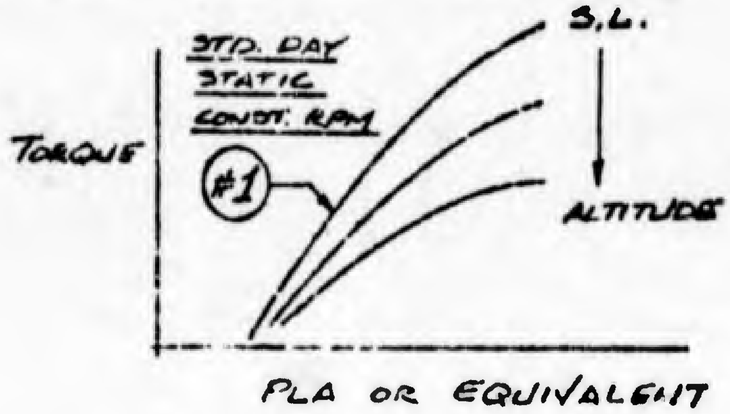
$$Q_{TP} = 1.4486\, \rho\, \Omega_{TP}^2\, C_{P_{TP}} = 8.206 \times 10^6 \rho \left[\frac{N_{TP}}{2380}\right]^2 C_{P_{TP}}$$

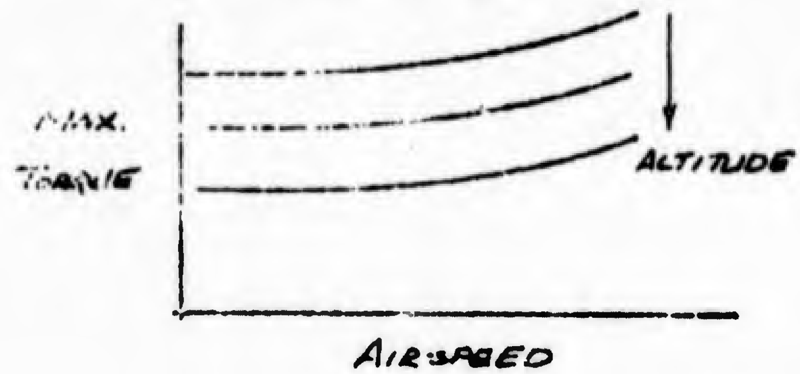
$$(\Delta F_z)_{TP} = -T_{TP}$$

$$(\Delta M)_{TP} = -T_{TP}\, l_{TP}$$

6.0 ENGINE SIMULATION

GENERAL CHARACTERISTICS OF ENGINE TORQUE OR SHP ARE AS FOLLOWS:





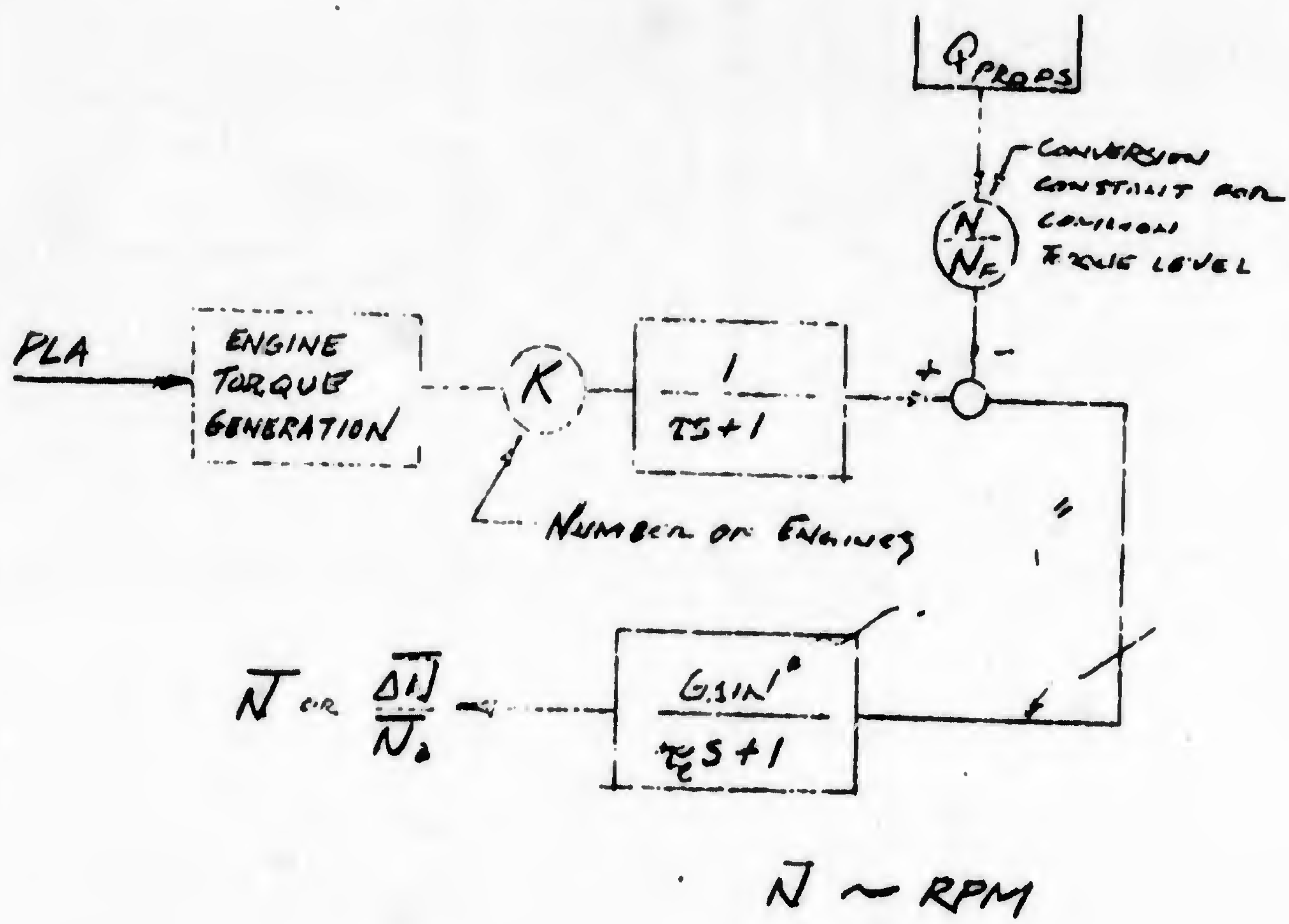
THESE CHARACTERISTICS CAN BE SIMULATED IN EITHER OF TWO WAYS:

(1) GENERAL (CURVE FIT) EQUATIONS IN DIGITAL TO PROVIDE TORQUE AVAILABLE AT ANY & ALL FLIGHT CONDITIONS. THIS WILL REQUIRE AN A-D CHANNEL FOR PLA (POWER LEVER ANGLE).

(2) GENERATE STATIC TORQUE VS PLA @ S.L. (CURVE #1) IN ANALOG TO BE USED FOR TRANSITION WORK. PROVIDE PARALLEL ANALOG CIRCUITRY FOR "HIGH SPEED" FLIGHT FOR EQUATION OF FORM

$$Q_{ENGINE} = \left[Q_o\Big|_{A.P.} + \frac{\partial Q}{\partial V}\Delta V + \frac{\partial Q}{\partial PLA}(PLA) + \frac{\partial Q}{\partial RPM}(RPM) \right.$$

BLOCK DIAGRAM ENGINE SIMULATION



1. GOVERNOR DRIVE SIGNAL

THE PRESENT PLAN IS TO DRIVE THE PROPELLER GOVERNOR WITH AN ELECTRIC MOTOR. THE MOTOR DRIVE SIGNAL WILL BE PROPORTIONAL TO RPM GENERATED ABOVE.

8. STABILIZATION SYSTEM SIGNALS

SINGLE CHANNEL SIGNALS ARE REQUIRED AS FOLLOWS:

θ, ϕ } 3-WIRE

p, q, r, $n_{z_{acc}}$ } 7-WIRE

NO GYRO DYNAMICS WILL BE SIMULATED.

$$n_{z_{acc}} = n_{z_{cg}} + \frac{x_{acc}}{g}\dot{q}$$

(circled term $\frac{x_{acc}}{g}\dot{q}$:) THIS TERM APPEARS TO BE NEGLIGIBLE AT PRESENT

9. COCKPIT DISPLAYS DRIVE SIGNALS

INSTRUMENT — SIGNAL

1) ARTIFICIAL HORIZON* ..... θ & ϕ

2) DIRECTIONAL GYRO & COURSE INDICATOR** .... ψ

3) AIRSPEED TAPE (TRUE).... $u_{KNOTS} = .5921 \times u_{FT/SEC}$ ~ KNOTS

4) RATE OF CLIMB .... $\dot{h} = -\dot{Z}_e \times 60$ ~ FT/MIN

5) WING STALL WARNING..... α'' (SET MIN SAFE SPEED LINE @ $\alpha' = 14.4°$)

6) GROSS ALTIMETER .... $h = -Z_e$ ~ FT

7) SENSITIVE ALTIMETER .... $h = -Z_e$ ~ FT

8) NORMAL LOAD FACTOR .... $n_{Z_{COCKPIT}} = n_{Z_{C.G.}} + \frac{\bar{x}}{g}\dot{q}$

(NOMINAL $\frac{\bar{x}}{g} = .555$)

9) WING INCIDENCE .... i_W

10) FLAP POSITION .... δ_F

11) SIDE "G" .... n_Y

COMMAND INSTRUMENTS

*ATTITUDE COMMAND BAR IN HOVER:

$$\theta_c = K_{A_\theta}\ddot{X}_e + K_{R_\theta}\dot{X}_e + K_{D_\theta}X_e \sim \pm 10° \text{ LIMIT}$$

$$\phi_c = K_{A_\phi}\ddot{Y}_e + K_{R_\phi}\dot{Y}_e + K_{D_\phi}Y_e \sim \pm 10° \text{ LIMIT}$$

(ASSUME $\ddot{X}_e = \dot{u}$ & $\ddot{Y}_e = \dot{v}$ IF NECESSARY)

POSITIVE θ_c COMMANDS DRIVE BAR UP FROM REF.

POSITIVE ϕ_c " " " LEFT WING DOWN.

**SITUATION DISPLAY

HOVER: X_c & Y_c — X_c POSITIVE NORTH; Y_c " EAST

ILS APPROACH: h_e & Y_c

$$h_e = [(h - h_{REF}) - m(X_c - X_{REF})]$$

WHERE h_{REF} & X_{REF} WILL BE GIVEN BIAS VALUES.

m WILL BE GIVEN AS A POSITIVE NUMBER.

COCKPIT COMPUTER CONTROL SWITCHES

(1) OPERATE/RESET/HOLD

(2) INSTRUMENT SCALE SWITCH: (x1, x10, x100)

SWITCH SCALE ON FOLLOWING –

	SWITCH	SCALE
a) SENSITIVE ALTIMETER	x1	800 FT. MAX.
	x10	8000 FT. MAX.
	x100	80,000 FT. MAX
b) COMMAND BAR (HOVER)	x1	PER GIVEN GAINS
	x10	$\frac{1}{10}$ " "
	x100	N/A
c) X-Y SITUATION (HOVER)	x1	200 FT PER MARK
	x10	2000 FT " "

10 WIND/GUST SIMULATION

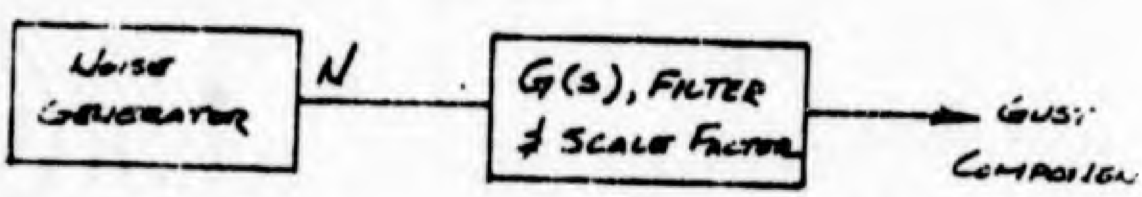
GUSTY WIND WILL BE SIMULATED BY SUPERIMPOSING RANDOM (NOISE) GUSTS ONTO CONSTANT MEAN WIND LEVELS. THE RANDOM GUSTS CONTRIBUTIONS CAN BE ASSUMED TO BE IN THE AIRPLANE BODY AXES SYSTEM AS GENERATED. THE MEAN WIND (GIVEN IN EARTH SYSTEM COMPONENTS u_e & v_e) SHOULD BE CONVERTED FROM THE EARTH SYSTEM INTO THE BODY SYSTEM BY THE FOLLOWING APPROXIMATE TRANSFERS:

$$u_W = u_e \cos\psi + v_e \left[\qquad + \sin\psi \right]$$

$$v_W = -u_e \sin\psi + v_e \left[\cos\psi - \qquad \right]$$

$$W_W = u_e \theta - v_e \phi$$

THE RANDOM GUST COMPONENTS ARE GENERATED FROM THREE NOISE SOURCES.



THE GUST COMPONENTS ARE DEFINED

$$u_G = \frac{K_u}{\sqrt{N_u}}\left[\frac{1+\tau_1 s}{(1+\tau_2 s)^2}\right] \times \text{GENERATOR OUTPUT}$$

$$v_G = \frac{K_v}{\sqrt{N_v}}\left[\frac{1+\tau_1 s}{(1+\tau_2 s)^2}\right] \times \text{GENERATOR OUTPUT}$$

$$w_G = \frac{K_w}{\sqrt{N_w}}\left[\frac{1+\tau_1 s}{(1+\tau_2 s)^2}\right] \times \text{GENERATOR OUTPUT}$$

N_u, N_v, & N_w ARE THE POWER SPECTRAL DENSITY OUTPUTS OF THE THREE SOURCES IN $\frac{(FT/SEC)^2}{RAD/SEC}$ UNITS.

TABULATION OF τ & K CONSTANTS FOR 3 WIND LEVELS

MEAN WIND ~ FT/SEC →	16.9	33.8	50.7
K_u	2.54	3.60	4.40
K_v	2.54	3.60	4.40
K_w	.889	1.255	1.535
τ_1 ~ SEC	3.08	1.535	1.03
τ_2 ~ SEC	1.775	.895	.596

FOR REFERENCE, THE ABOVE INFORMATION SHOULD PROVIDE STANDARD DEVIATIONS PER THE TABLE ON NEXT PAGE.

STANDARD DEVIATIONS

MEAN WIND ~ FT/SEC	16.9	33.8	50.7
σ_u ~ FT/SEC	3.34	6.68	10.02
σ_v ~ FT/SEC	3.34	6.68	10.02
σ_w ~ FT/SEC	1.18	2.36	3.54

II. DEFINITION OF CONSTANTS

AIRPLANE DIMENSIONAL CONSTANTS

S	534.37 FT²	D	15.625 FT
S_P	767.0 FT²	$\bar{C}_R$	2.68 FT
b	67.5 FT	S_R	22.81 FT²
$\bar{c}$	8.072 FT	D_{TP}	8.0 FT

WEIGHT AND LOADING CONDITIONS WITH ASSOC. CONSTANTS

Case No	①	②	③	④	⑤
Weight lb	37474	→	→	26560	41513
m SLUGS	1163.8	→	→	824.84	1289.2
REF C.G. % E	15.0	20	28	24.64	19.25
I_X SLUG-FT	173,000	→	→	172,000	178,000
I_Y "	122,000	→	→	93,700	120,000
I_Z "	267,000	→	→	243,000	267,000
I_{XZ} "	8750	7000	4500	7610	8290
$(X_L)_{cg}$	119.64	→	→	128.07	123.4
A_{PN} FT	-.968	-.564	+.081	-.091	-.648
B_{PN}	-.407	-.407	-.407	-.574	-.368
C_{PN}	-.240	-.240	-.240	-.339	-.217
D_{PN}	-2.706	-2.706	-2.706	-1.836	-2.432
E_W	-.11	-.11	-.11	.278	-.071
F_W	.161	.161	.161	.062	.134
G_1	3.953	3.953	3.953	3.855	3.977
G_2	5.427	5.427	5.427	5.329	5.451
H_1	1.217	1.217	1.217	1.051	1.257
H_2	.684	.684	.684	.518	.724
$K_{\alpha c_3}$	.77	—	—	—	—
$K_{\alpha c_1}$	0				

AERODYNAMIC COEFFICIENTS

WING:

C_{L_0}	.08	$C_{l_r C_L}$	.25
C_{L_α}	4.30	$C_{n_\beta C_L^2}$	.029
$C_{L_\alpha \delta_F}$	1.642	$C_{n_p C_L}$	−.067
$C_{L_{\delta_F}}$	.444	$C_{n_r C_L^2}$	−.0175
C_{D_0}	.013	$C_{l_{\delta_A}}$	.14325
πAR	26.8	$C_{l_{\delta_A^2}}$	−.02758
e	.75	$C_{l_{\delta_A} C_L}$	.0573
$C_{D_{\delta_F}}$	−.0306	$C_{l_{\delta_A^2} C_L}$	−.04925
$C_{D_{\delta_F^2}}$	.2955	$C_{n_{\delta_A}}$	.0287
C_{m_0}	−.06	$C_{n_{\delta_A} C_L}$	−.0258
$C_{m_{\delta_F}}$	−.5157	$C_{n_{\delta_A} C_L C_{TS}}$	−.0954
C_{m_q}	−1.143	$C_{n_{\delta_A} C_L^2 C_{TS}}$	.0287
$C_{l_{\beta_0}}$	.0367		
$C_{l_\beta C_L}$	−.0573	a_1	.04
$C_{l_\beta i_W}$	−.0292	a_2	.08
C_{l_p}	−.45	a_3	.12
$C_{L_{\delta_F^2}}$	4.104	a_4	−.06
$C_{L_{\delta_F^3}}$	−2.703	a_5	.14

MAIN PROPELLERS:

ϕ_{CL}	.202	$C_{Y_{J\beta}}$	.1051 for M 1 & 2 −.1051 for M 3 & 4
C_{T_0}	.028	$C_{Y_{J\beta^2}}$	−.05644 for M 1 & 2 +.05644 for M 3 & 4
C_{T_β}	.5845	C_{P_0}	.013
C_{T_J}	−.1	C_{P_β}	.04011
$C_{T_{J^2}}$	−.234	$C_{P_{\beta^2}}$	.8208
$C_{T_{J^2\beta}}$	.3724	$C_{P_{J^2}}$	−.1
$C_{T_{J^2\beta^2}}$	−.1313	$C_{P_{J^3}}$	−.080
$C_{N_{J\beta}}$	.0534	$C_{P_{J^2\beta}}$	.1432
$C_{N_{J^2\beta}}$	.1028	$C_{P_{J^3\beta}}$	.06309
		$C_{T_{J^2\beta^3}}$	−.01656

FUSELAGE:

$C_{D_{0F}}$	.0206	$C_{m_{0F}}$	.004
$C_{Y_{\beta F}}$	−.573	$C_{m_{\alpha F}}$	.780
$C_{L_{\alpha F}}$	.~~344~~	$C_{m_{\beta F}}$	−.132

VERTICAL TAIL ...

$C_{l\beta}$	-	-.0946	$C_{l\delta_R}$	-	.0390
$C_{m\beta}$	-	.243	$C_{m\delta_R}$	-	-.0851
$C_{Y\beta}$	-	-.745	$C_{Y\delta_R}$	-	.235
C_{lr}	-	.0424	$C_{l\delta_R\beta}$	-	0
C_{mr}	-	-.138	$C_{m\delta_R\beta}$	-	0
C_{Yr}	-	0	$C_{Y\delta_R\beta}$	-	0
C_{mp}	-	.061 @ M = 0			
		.064 @ M = .1			
		.0675 @ M = .6			
		.0705 @ M = .7			

UNIT HORIZONTAL TAIL:

$C_{L_{\alpha t}}$	-	1.146	f_1	-	.079
$C_{D_{0t}}$	-	.00244	f_2	-	0
K_t	-	.299	f_3	-	0
ε_0	-	.0594	f_4	-	0

TAIL PROPELLER:

$C_{T\beta_{TP}}$	-	.1862	$C_{T\beta^4_{TP}}$	-	-3.773
$C_{T\beta^2_{TP}}$	-	2.692	C_{TJ}	-	-.1
$C_{T\beta^3_{TP}}$	-	-2.822	$C_{P\beta^2_{TP}}$	-	1.084

12. RANGE OF VARIABLES

VARIABLE	RANGE		
	HOVER	TRANSITION	CRUISE
X_e			N/A
Y_e	0 TO -800 FT		N/A
Z_e	0 TO -8000 FT		0 TO -25000 FT
u	±84.5 FPS	TO 254 FPS	TO 676 FPS
v	±84.5 "	→	→
w	±84.5 "	→	→
$\dot{u}$			
$\dot{v}$			
$\dot{w}$			
ΣF_x	.6 weight		
ΣF_y	.2 weight		
ΣF_z	1.5 weight		
ψ	CYCLIC	→	→
θ	±.5 RAD	→	→
ϕ	±.5 RAD		±90 DEG
p	±1 RAD/SEC	→	±2 RAD/SEC
q	±.5 "	→	→
r	±.5 "	→	→
$\dot{p}$	±1 RAD/SEC²	→	±2 RAD/SEC²
$\dot{q}$	±1 RAD/SEC²	→	→
$\dot{r}$	±1 RAD/SEC²	→	→
ΣL	175,000 FT-LB	→	350,000 FT-LB
ΣM	150,000 FT-LB	→	→
ΣN	150,000 "	→	→

VARIABLE	RANGE		
	HOVER	TRANSITION	CRUISE
i_W	100 DEG	→	—
δ_F		60 DEG	→
δ_{ALT} or δ_{ART}	±50 DEG	±50 DEG	±20 DEG
δ_R	±30 DEG	→	→
i_t	30 ±10 DEG	→	±10 DEG
β_n			±7 DEG
β_{TP}	±20 DEG	→	N/A

13 VARIABLES TO RECORD

THIS IS A PRELIMINARY LIST TO ESTABLISH MINIMUM RECORDER REQUIREMENTS.

	UPSET	DOWN SET
1	i_W	M (MACH NUMBER)
2	δ_F	α_F
3	$R/C = -\dot{Z}_E$	R/C
4	$h = -Z_E$	h
5	θ	θ
6	u	V
7	LOSF (LONG. STICK FORCE)	LOSF
8	LOSP (" " POS'N)	LOSP
9	$\dot{\delta}_{TP}$	n_z @ C.G.
10	i_t	i_t
11	α_t	α_t'
12	α''	α''
13	$\dot{q}$	$\dot{q}$
14	q	q
15	θ	β_1
16	PSA1 (PITCH STAB. ACT., CH 1)	n_z @ COCKPIT
17	PSA2 (" " " " 2)	PILOT'S PITCH TRIM INPUT
18	PITCH TRIM ACT. POS'N	PITCH TRIM ACT. POS'N
19	PILOT'S PITCH TRIM INPUT	
20	n_z @ C.G.	
21	ADA (ALT. DAMP. ACT. POS'N)	
22	β_1	
23	w	LONG. FEEL PACKAGE INPUT
24	$n_{x_{c.g.}}$ OR COLLECT. FEEL ISOL. ACT.	TAIL PROP AUTO TRIM ACT. POS'N
		UHT AUTO TRIM ACT. POS'N

	UP SET	DOWN SET
25	CSP (COLLECT. STK POS'N)	THROTTLE POS'N
26	PLA (PWR. LEVER ∠)	PLA
27	GOVERNOR ACT. OUT. POS'N	GOVERNOR ACT. OUT. POS'N
28	ΣT (TOTAL THRUST)	ΣT
29	ΣQ_{ENG} (TOTAL ENGINE TORQUE)	ΣQ_{ENG}
30	RPM	RPM
31	$\dot{p}$	$\dot{p}$
32	p	p
33	ϕ	ϕ
34	LASP (LAT. STK POS'N)	LASP
35	LASF (" " FORCE)	LASF
36	v	β_F
37	RSA1 (ROLL STAB. ACT. CH 1)	δ_{RIN}
38	RSA2 (" " " 2)	δ_{ROUT}
39	$(\beta_1 - \beta_2)$	ΣRSA
40	δ_{ALT}	δ_{ALT}
41	δ_{ART}	δ_{ART}
42	δ_{RIN}	ΣYSA
43	r	r
44	ϕ	ψ
45	RPP (RUD PEDAL POS'N)	RPP
46	RPF (" " FORCE)	RPF
47	YSA1 (YAW STAB ACT CH1)	n_y @ C.G.
48	YSA2 (" " " 2)	$\dot{r}$
49	X_e	
50	Y_e	
51	Z_e	
52	ψ OR $(u+u_G+u_W)$	
53	θ OR $(v+v_G+v_W)$	
54	ϕ OR $(w+w_G+w_W)$	

	UP SET	DOWN SET
55	LAT. INPUT, AILERON INTEGRATOR	
56	DIR. " , " "	
57	OUTPUT OF " "	
58	LAT. INPUT, DIFF. BLADE INTEGRATOR	
59	DIR. " " " "	
60	OUTPUT OF " " "	

X-Y PLOTTERS: MINIMUM OF TWO.

14. CHECKOUT

14.1 STATIC - AERODYNAMICS WILL PROVIDE COMPLETE STATIC CHECK NUMBERS FOR SEVERAL CONDITIONS TO COMPLEMENT CHECKS OF THE COMPUTER GROUP.

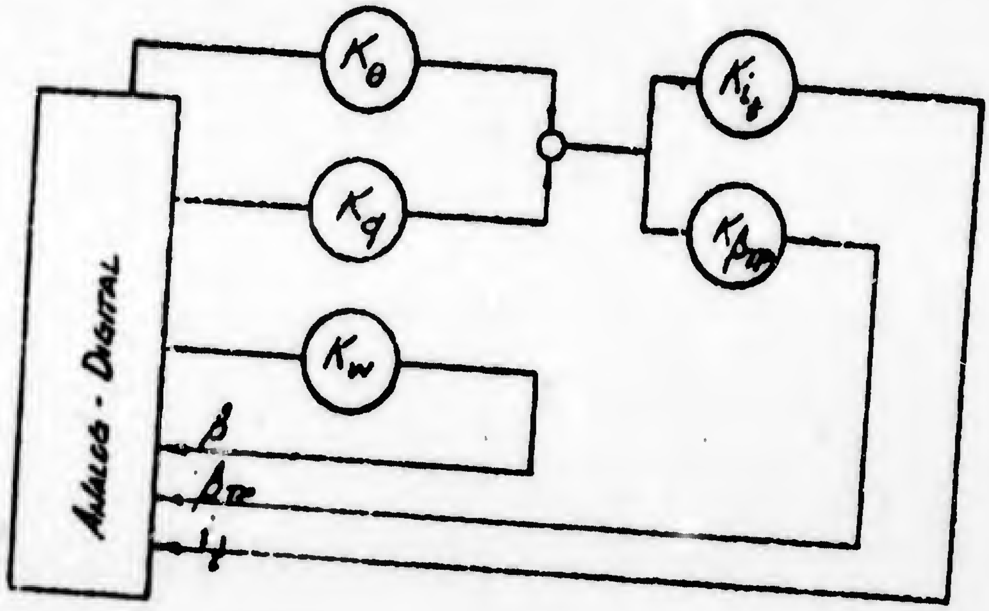
14.2 DYNAMIC

OPEN LOOP - AERO. WILL PROVIDE AIRFRAME RESPONSE CHECKS FOR SEVERAL CONDITIONS COVERING THE FLIGHT ENVELOPE OF THE AIRPLANE IN THE 'CRUISE' CONFIGURATION.

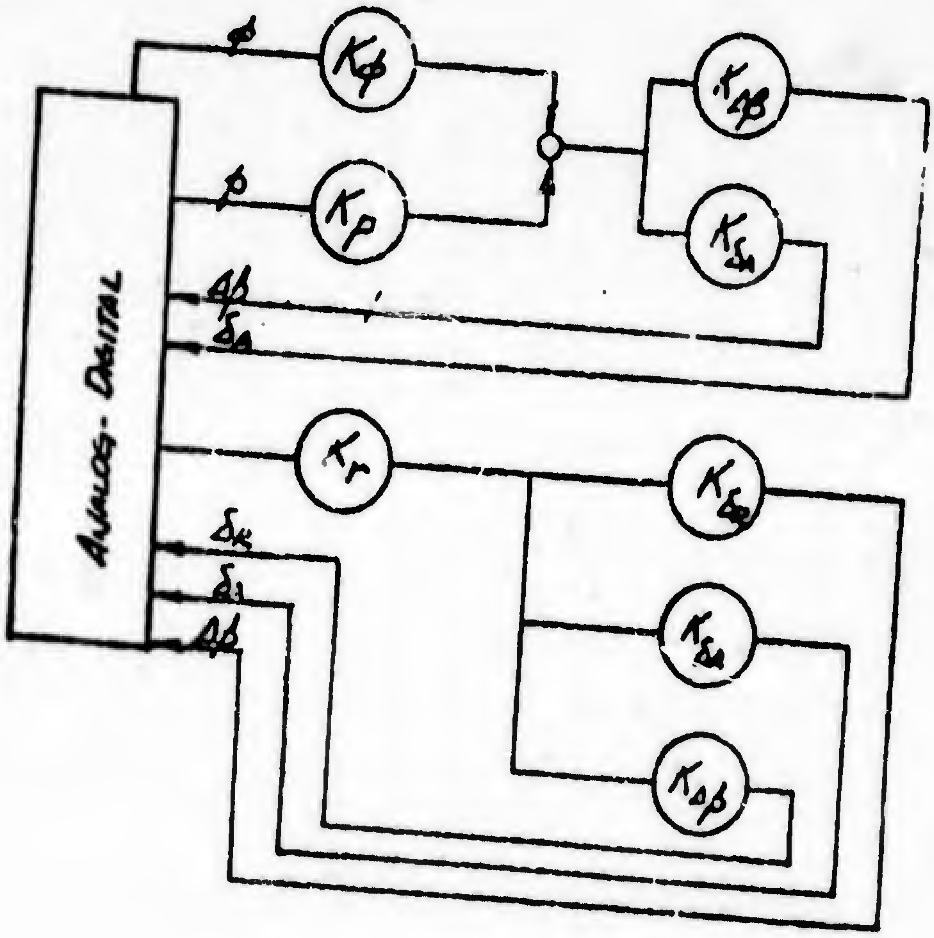
CLOSED LOOP - ACCURATE OPEN LOOP CHECKS MAY BE DIFFICULT THROUGH THE HOVER-TRANSITION FLIGHT REGIME DUE TO THE INSTABILITY OF THE BASIC AIRFRAME. IT IS NECESSARY THAT SIMPLE ANALOG FEEDBACKS BE PROVIDED FOR MAKING CLOSED LOOP COMPUTER CHECKS. THESE FEEDBACKS MAY BE USED TO PROVIDE "IDEAL" STABILIZATION SYSTEM RESPONSES AS REQUIRED. THE BLOCK DIAGRAMS FOLLOWING DEFINE THESE CLOSED LOOP REQUIREMENTS.

14. CHECKOUT

LONGITUDINAL:

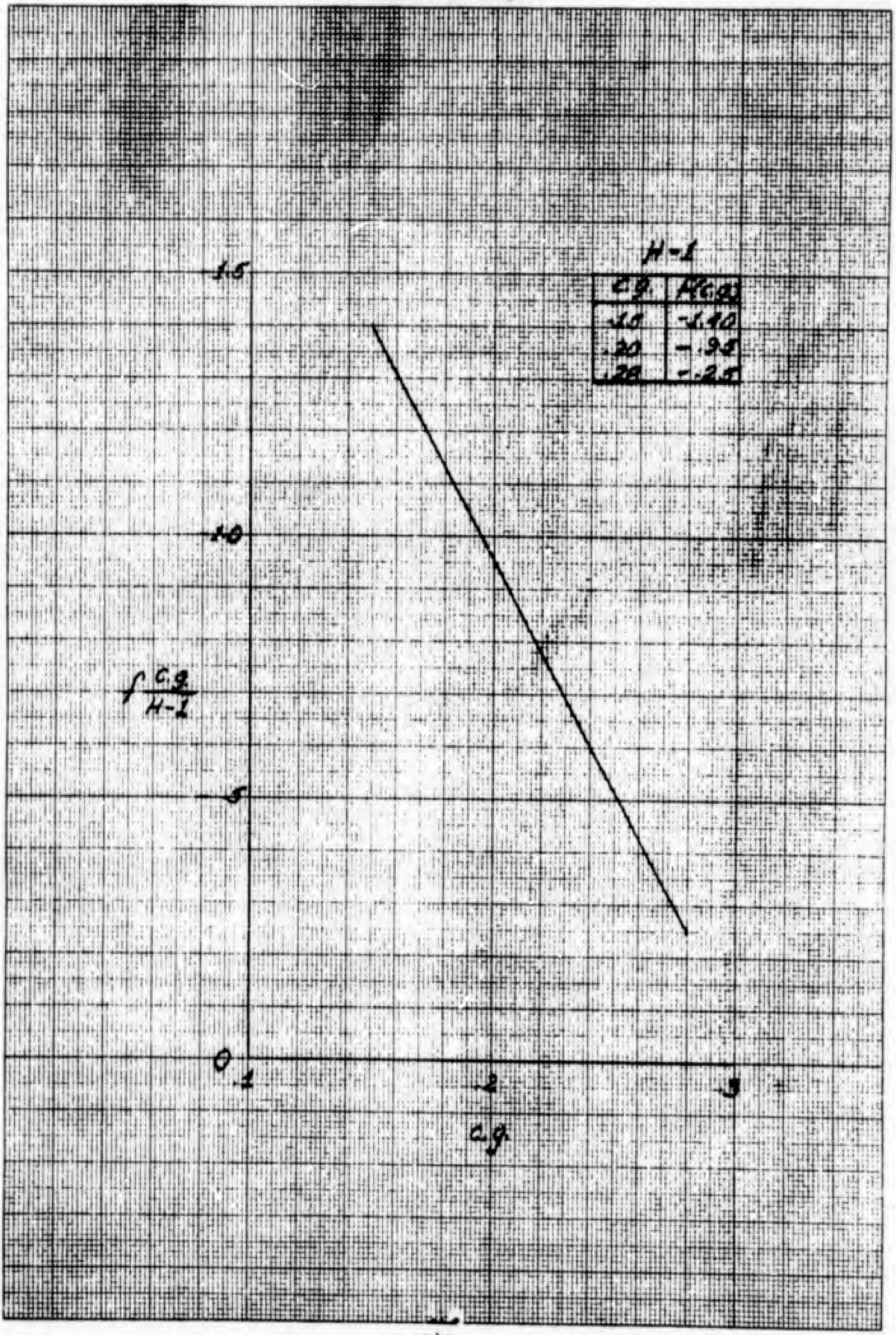


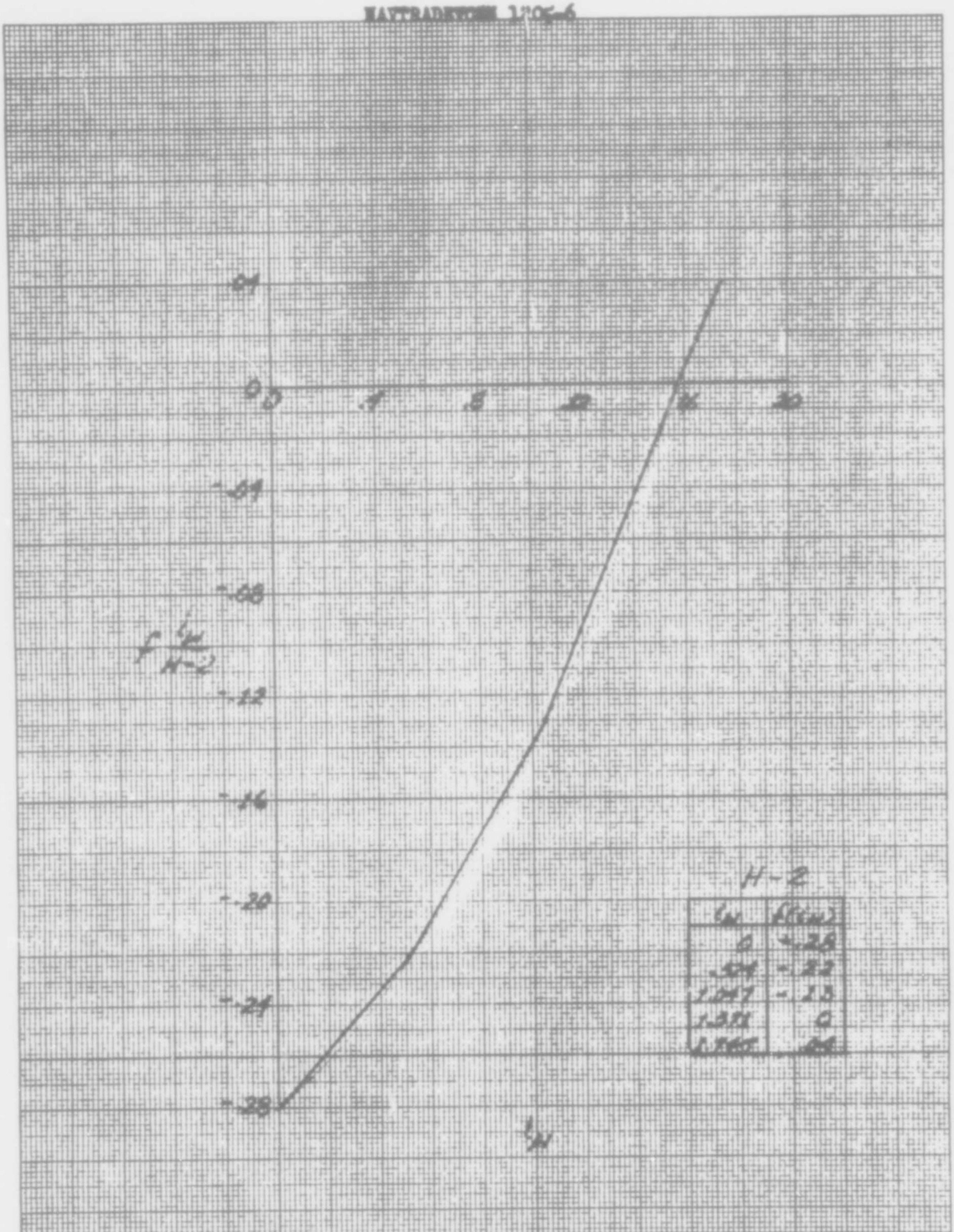
LATERAL - DIRECTIONAL:

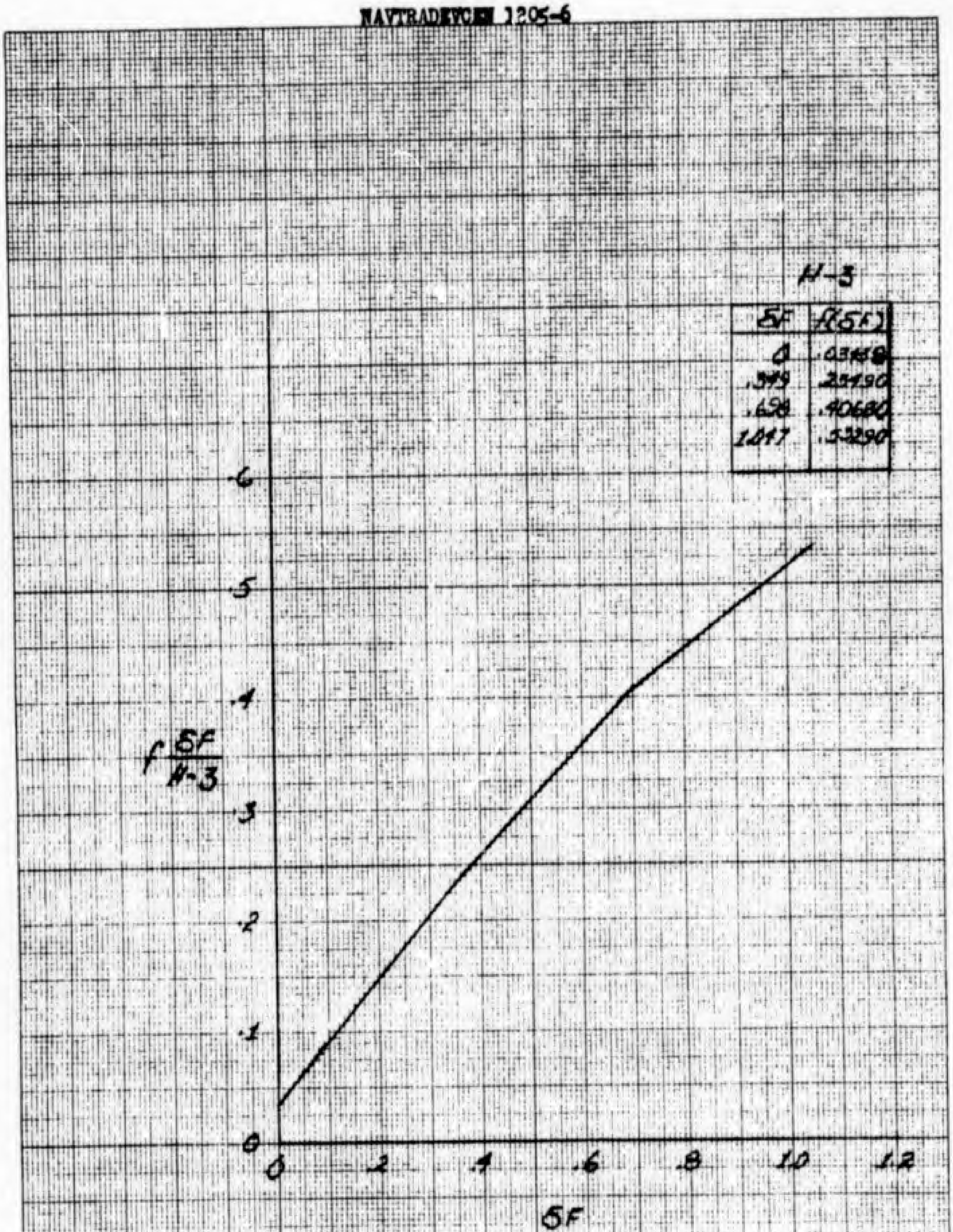


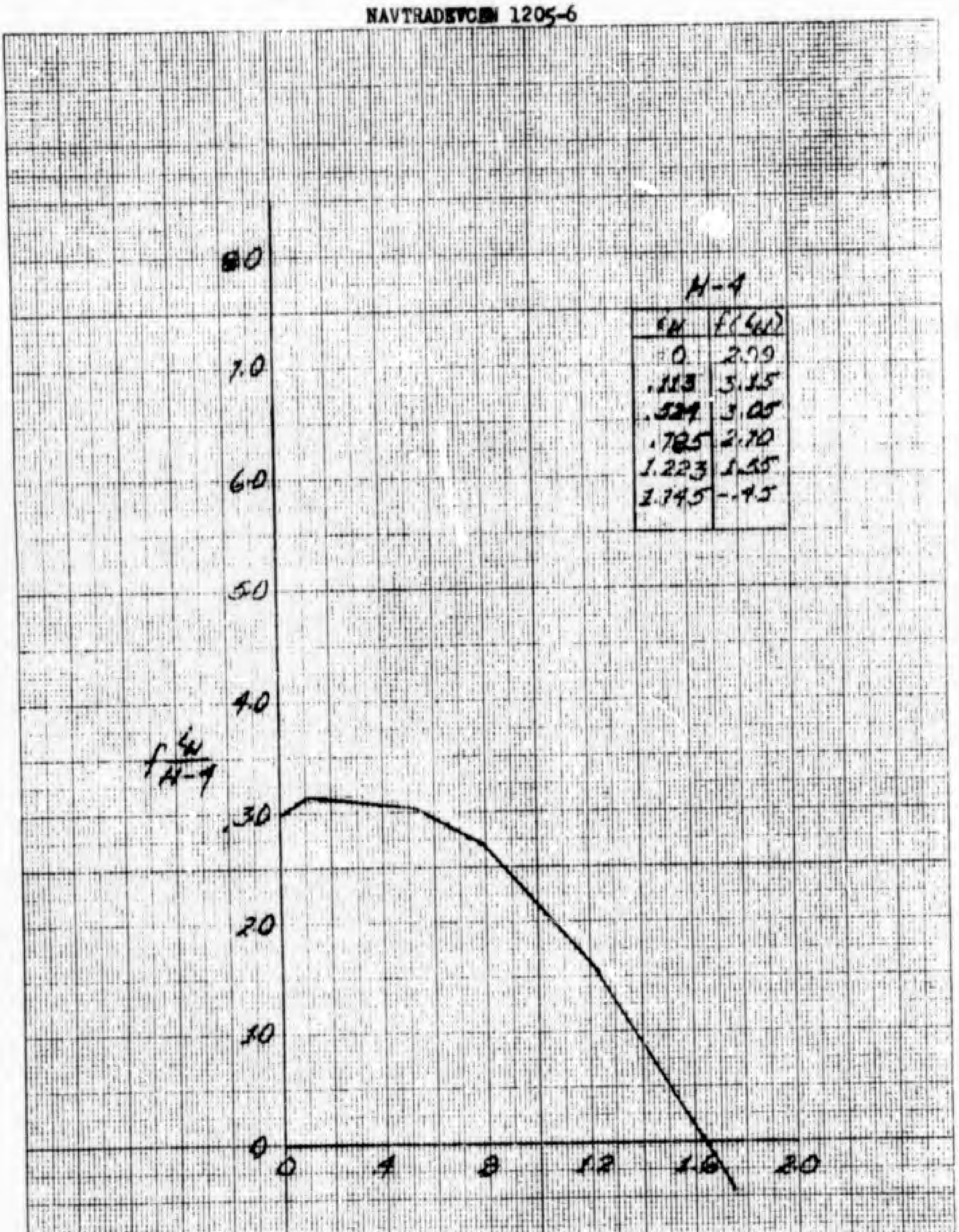
APPENDIX C

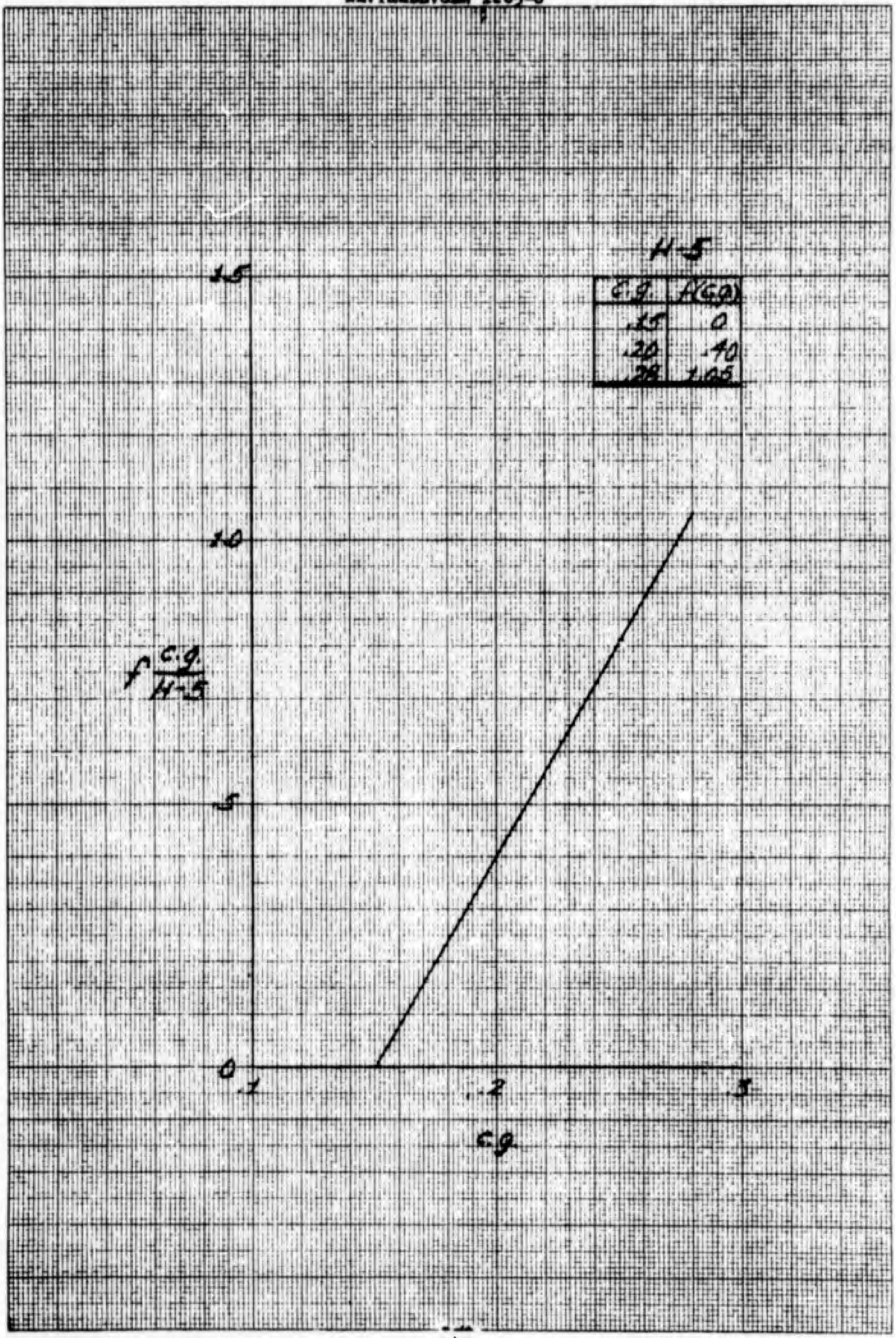
GRAPHICAL AND TABULAR FUNCTIONS

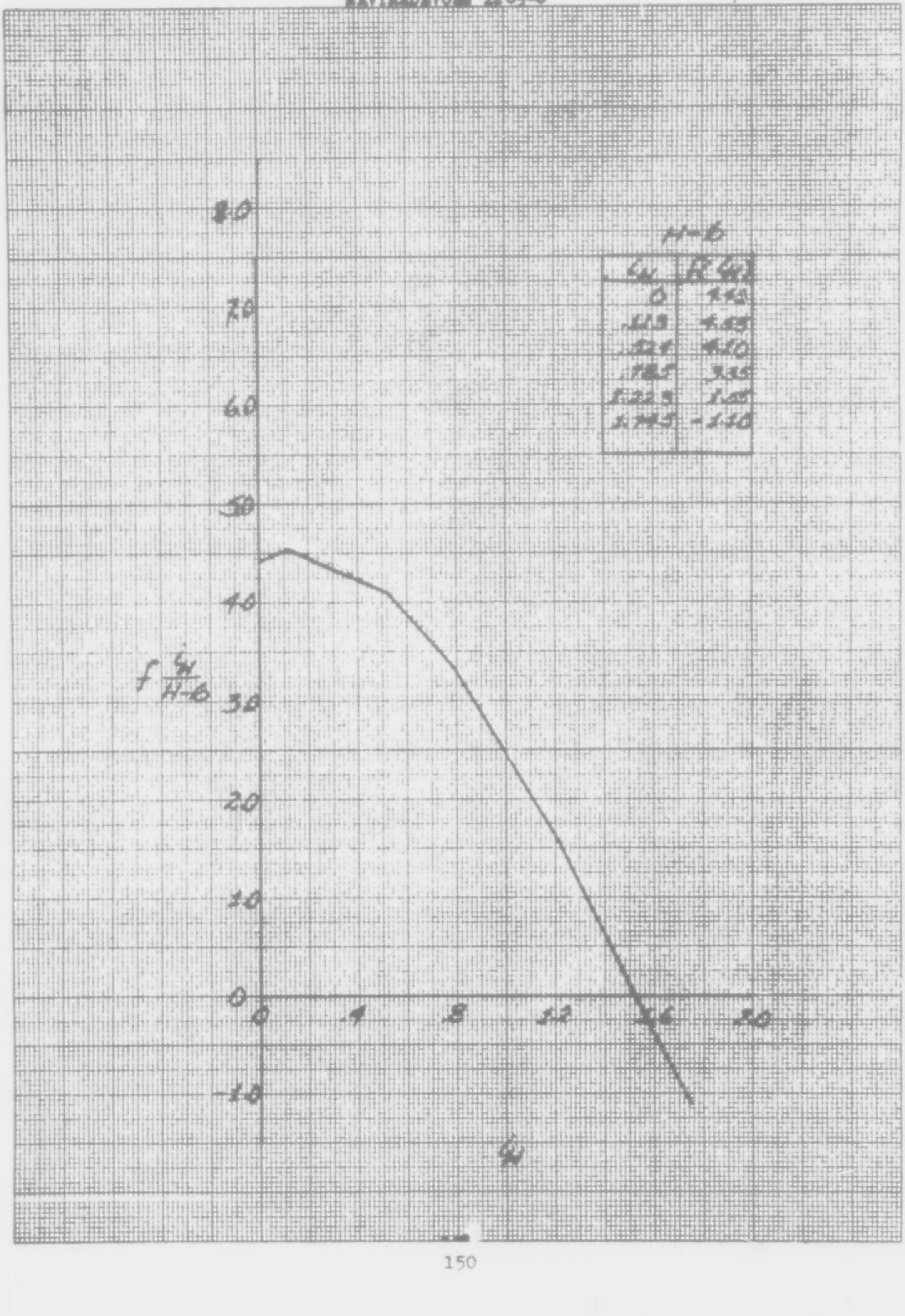

H-1
c.g. | f(c.g.)
.10 | -1.40
.20 | -.95
.28 | -.25
1.5
1.0
.5
0
f c.g. / H-1
.1
.2
.3
c.g.

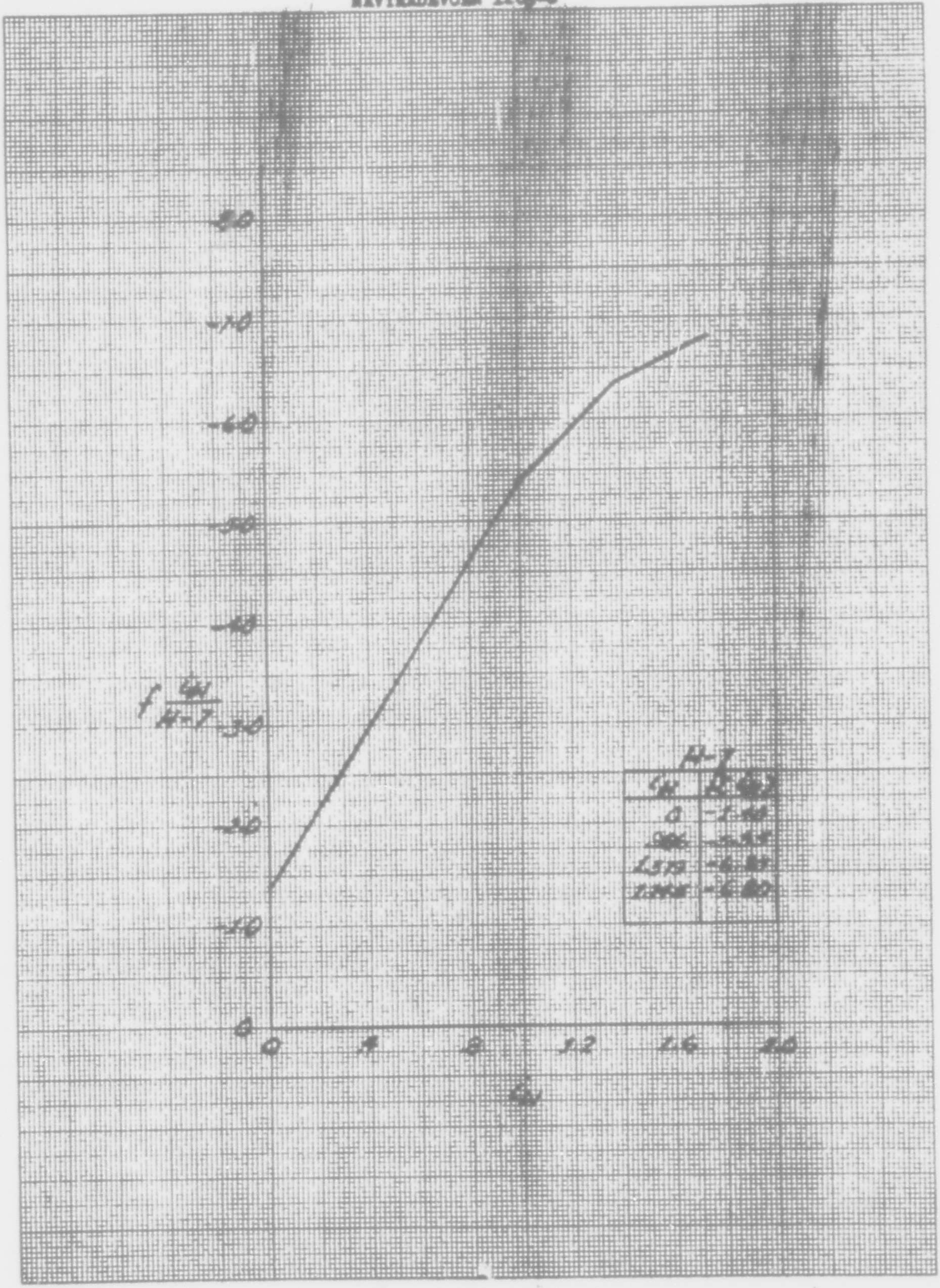

H-2
0 -.28
.524 -.22
1.047 -.13
1.371 0
1.565 .04

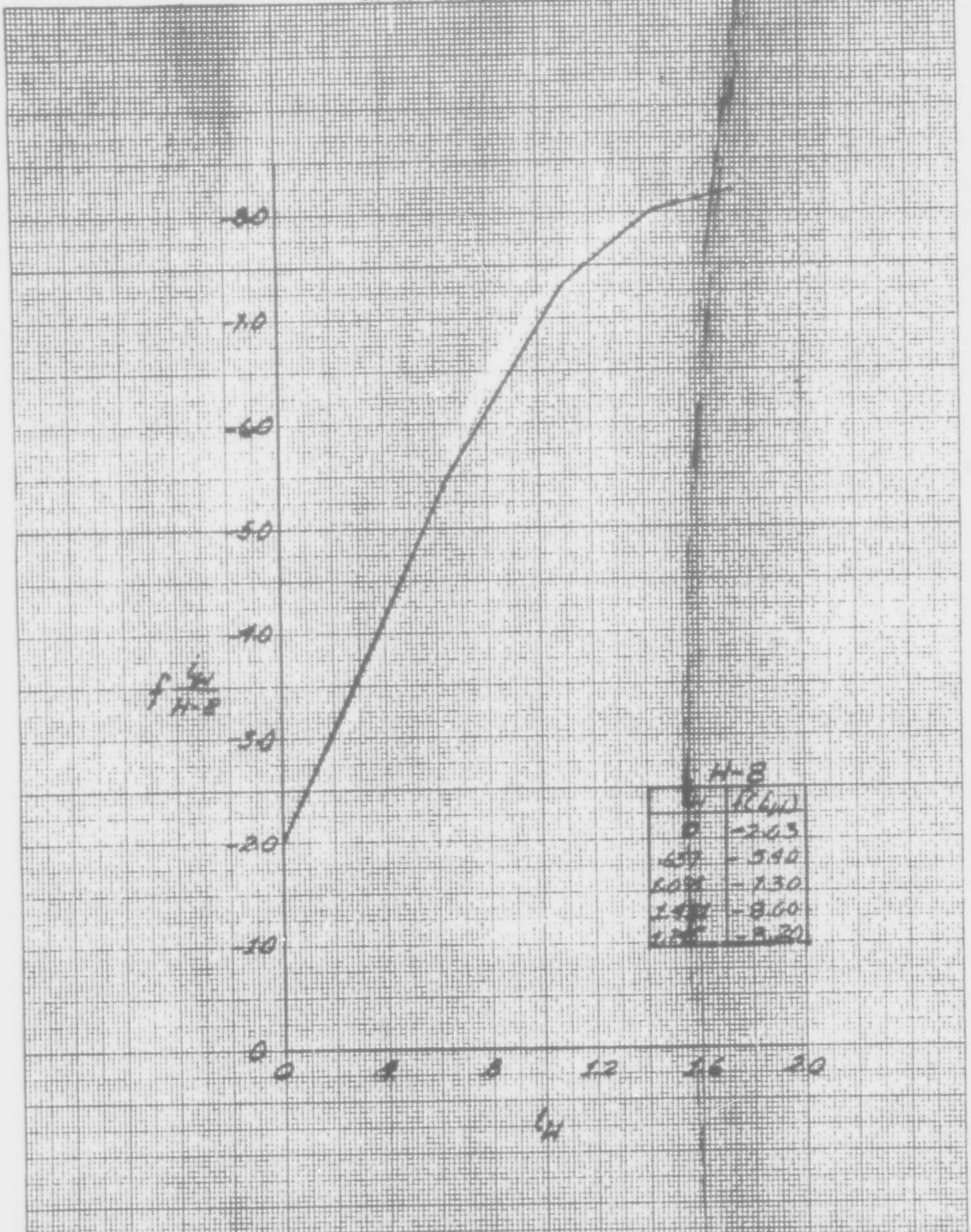

H-3
δF
f(δF)
0
.03158
.349
.23490
.698
.40680
1.047
.53290
.6
.5
.4
.3
.2
.1
0
f δF / H-3
0
.2
.4
.6
.8
1.0
1.2
δF

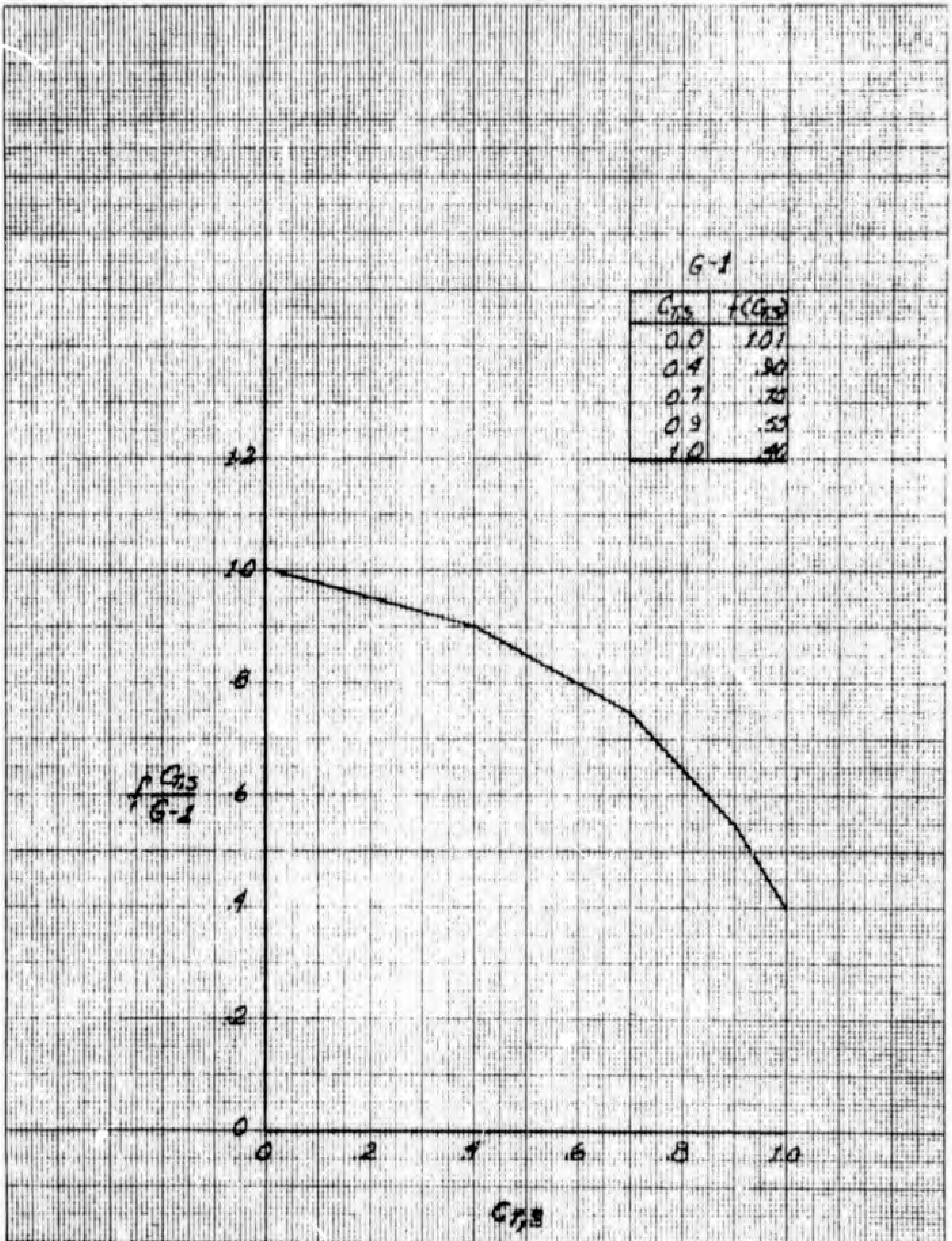

H-4
ξw | f(ξw)
0 | 2.99
.113 | 3.15
.524 | 3.05
.785 | 2.70
1.223 | 1.55
1.745 | -.45
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0
-1.0
f ξw / H-4
0
.4
.8
1.2
1.6
2.0
ξw

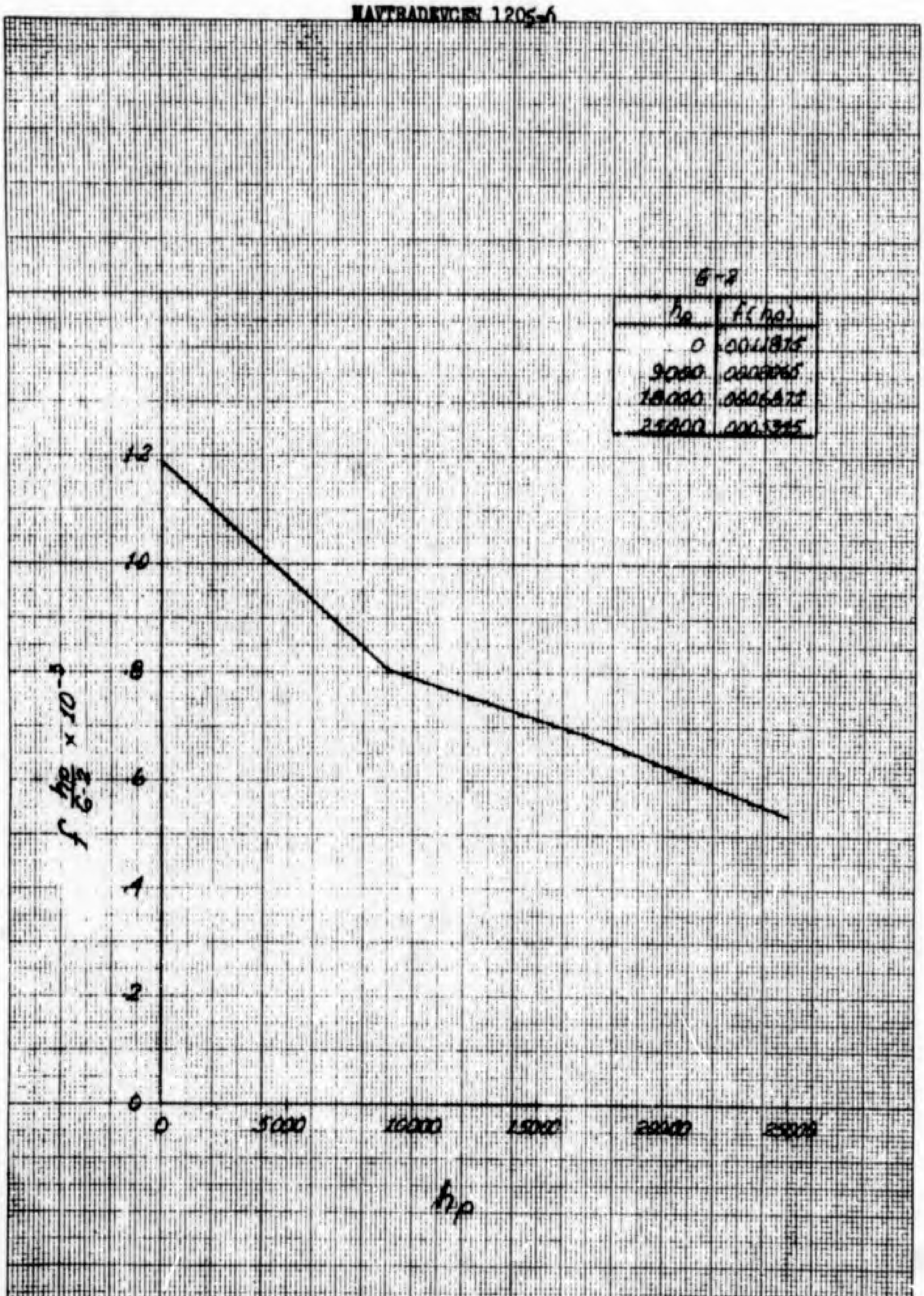

H-5
c.g.
f(c.g.)
.15
0
.20
.40
.28
1.05
1.5
1.0
.5
0
.1
.2
.3
c.g.
f c.g./H-5

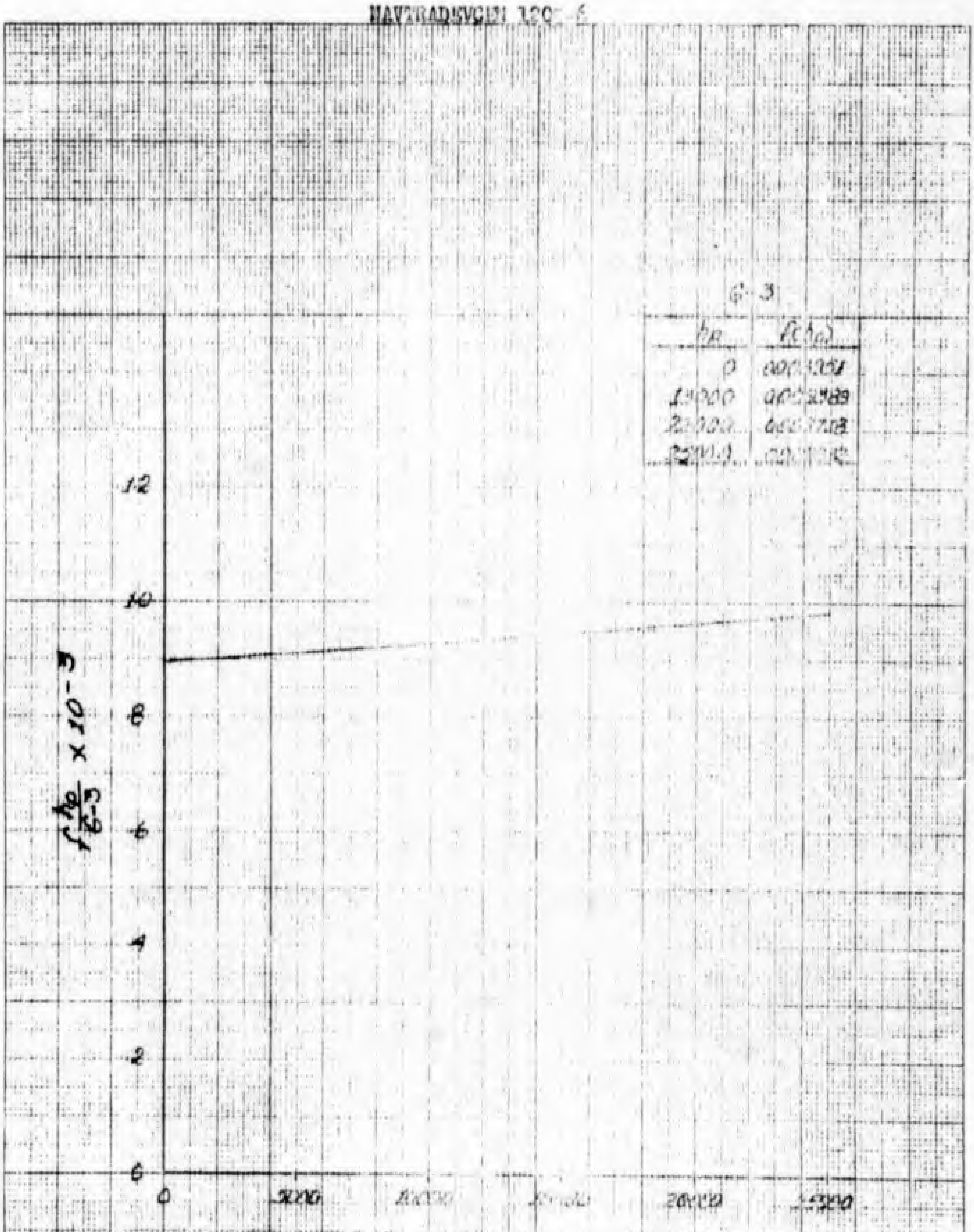

8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0
-1.0
f ω/H-6
0
.4
.8
1.2
1.6
2.0
ω
H-6
ω | F(ω)
0 | 4.45
.115 | 4.55
.524 | 4.10
.785 | 3.35
1.225 | 1.05
1.745 | -1.10

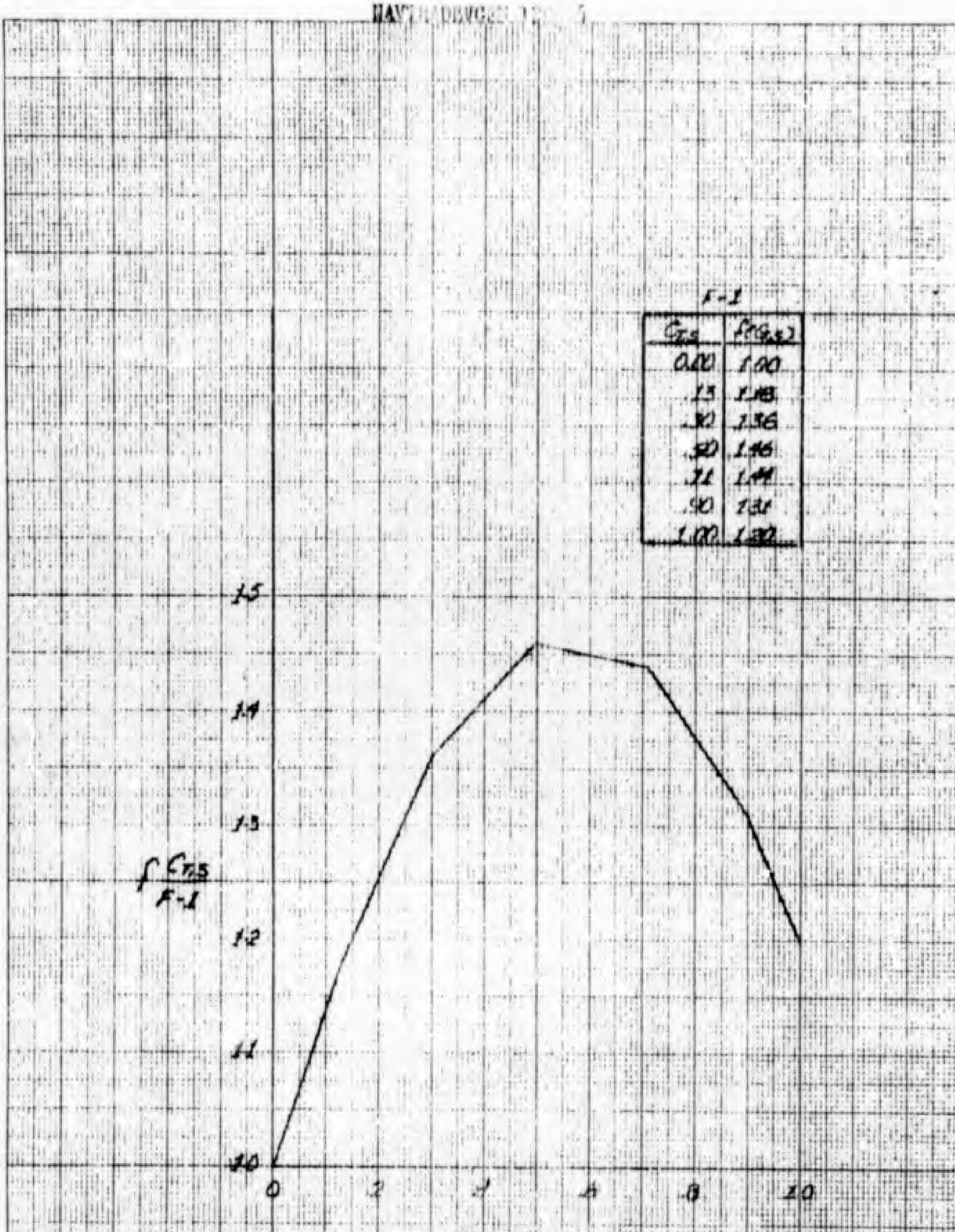

-8.0
-7.0
-6.0
-5.0
-4.0
-3.0
-2.0
-1.0
0
0
.4
.8
1.2
1.6
2.0
H-7
0
-1.40
1.570

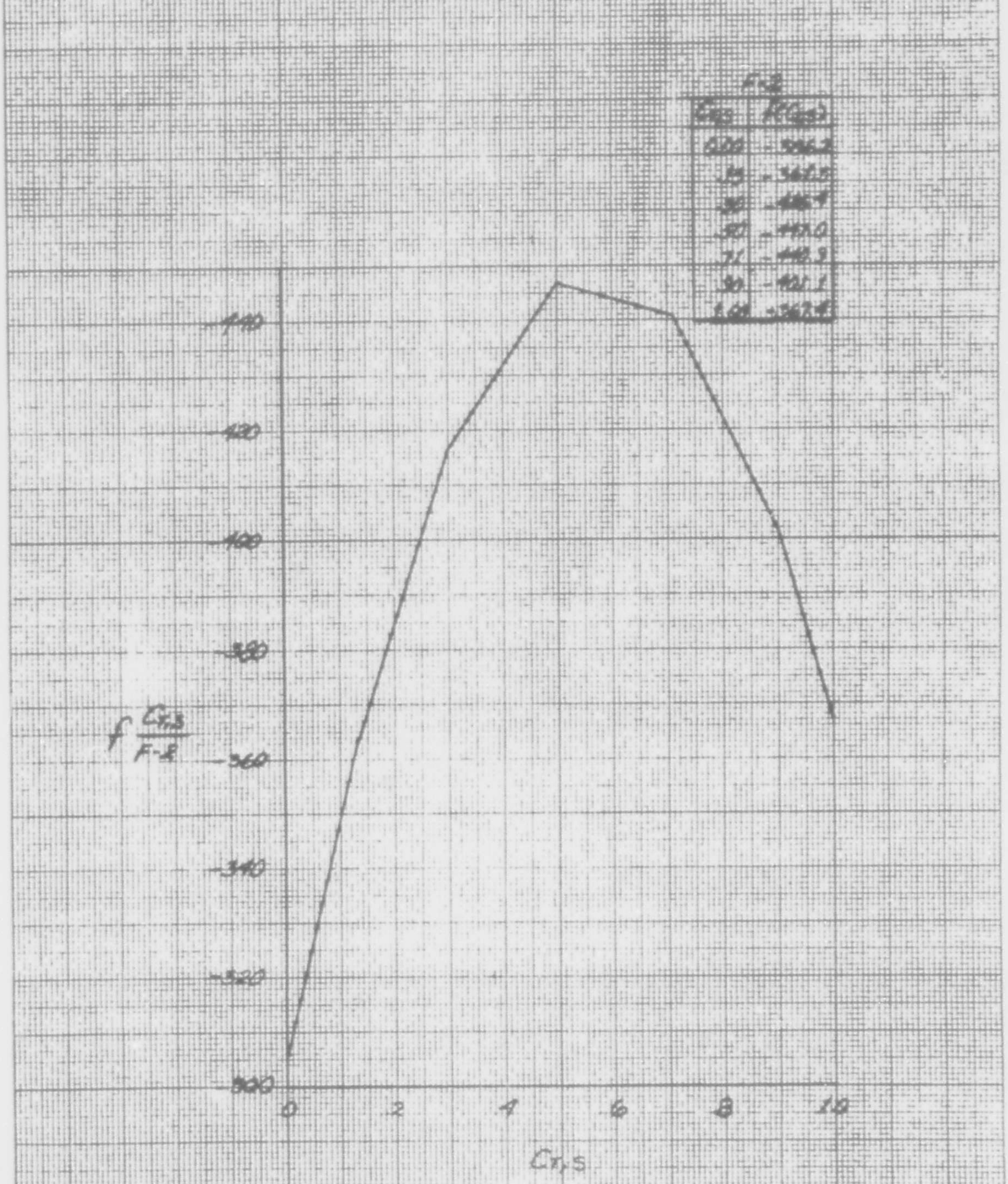

H-8
-2.03
-5.40
-7.30
-8.00
-8.20
-8.0
-7.0
-6.0
-5.0
-4.0
-3.0
-2.0
-1.0
0
.4
.8
1.2
1.6
2.0

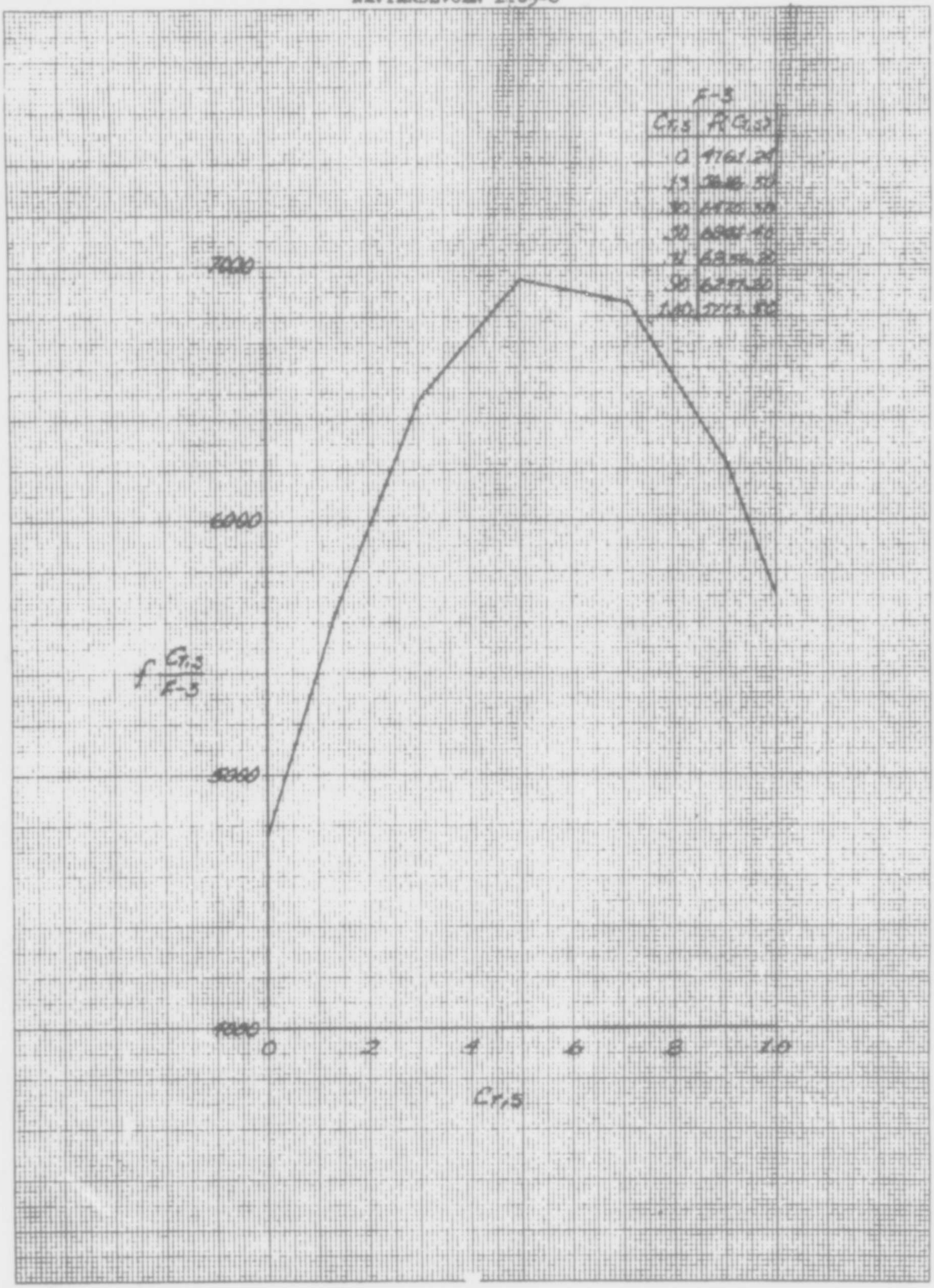

G-1
$C_{T,S}$ | $f(C_{T,S})$
0.0 | 1.0
0.4 | .90
0.7 | .75
0.9 | .55
1.0 | .40
1.2
1.0
.8
.6
.4
.2
0
$f\frac{C_{T,S}}{G-1}$
0
.2
.4
.6
.8
1.0
$C_{T,S}$

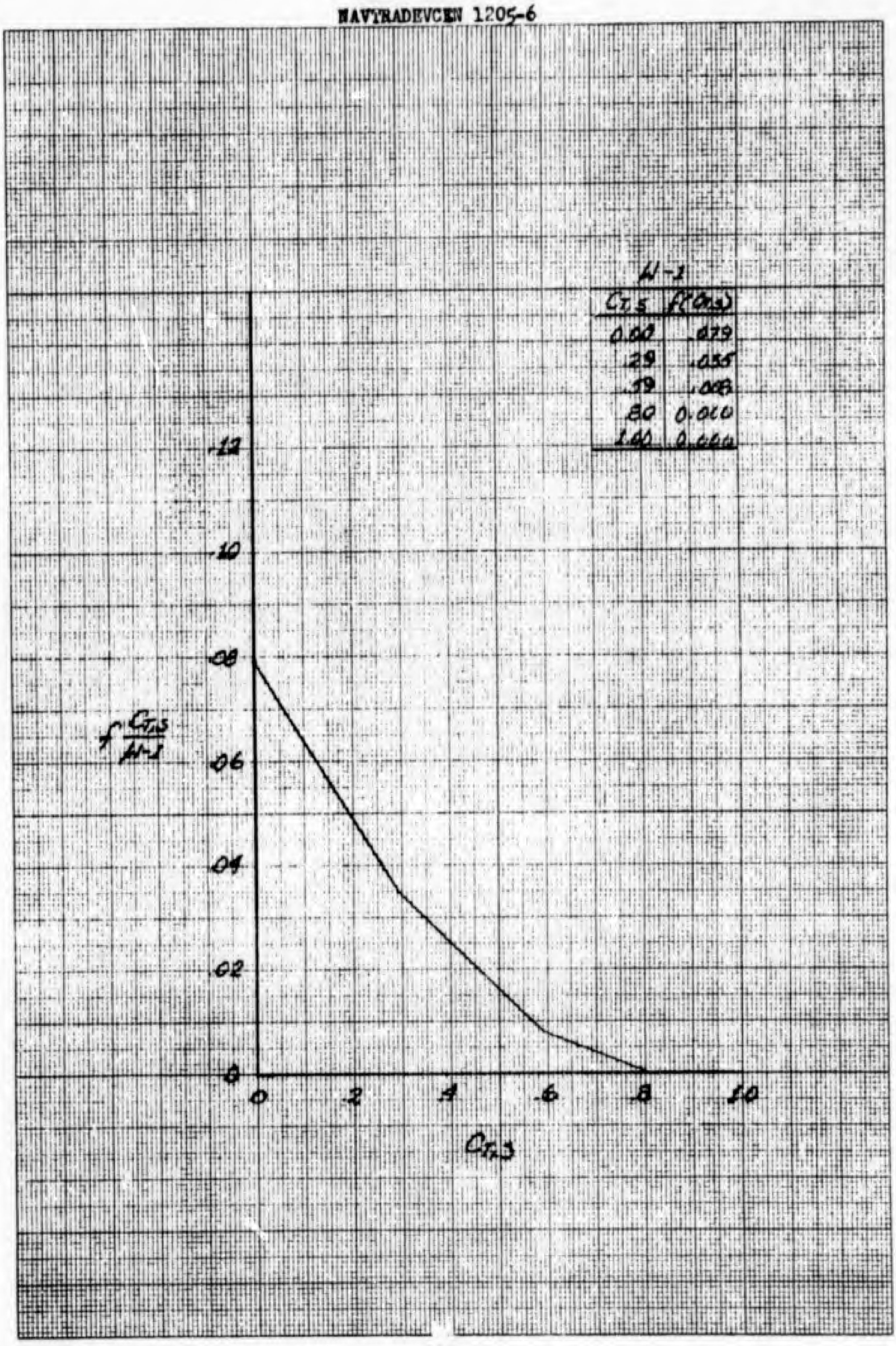

E-2
h_p | $f(h_p)$
0 | .0011875
9000 | .0008065
18000 | .0006475
25000 | .0005345
12
10
8
6
4
2
0
0
5000
10000
15000
20000
25000
h_p


G-3
hp
f(hp)
0
12
10
8
6
4
2
0
5000
10000
20000
25000

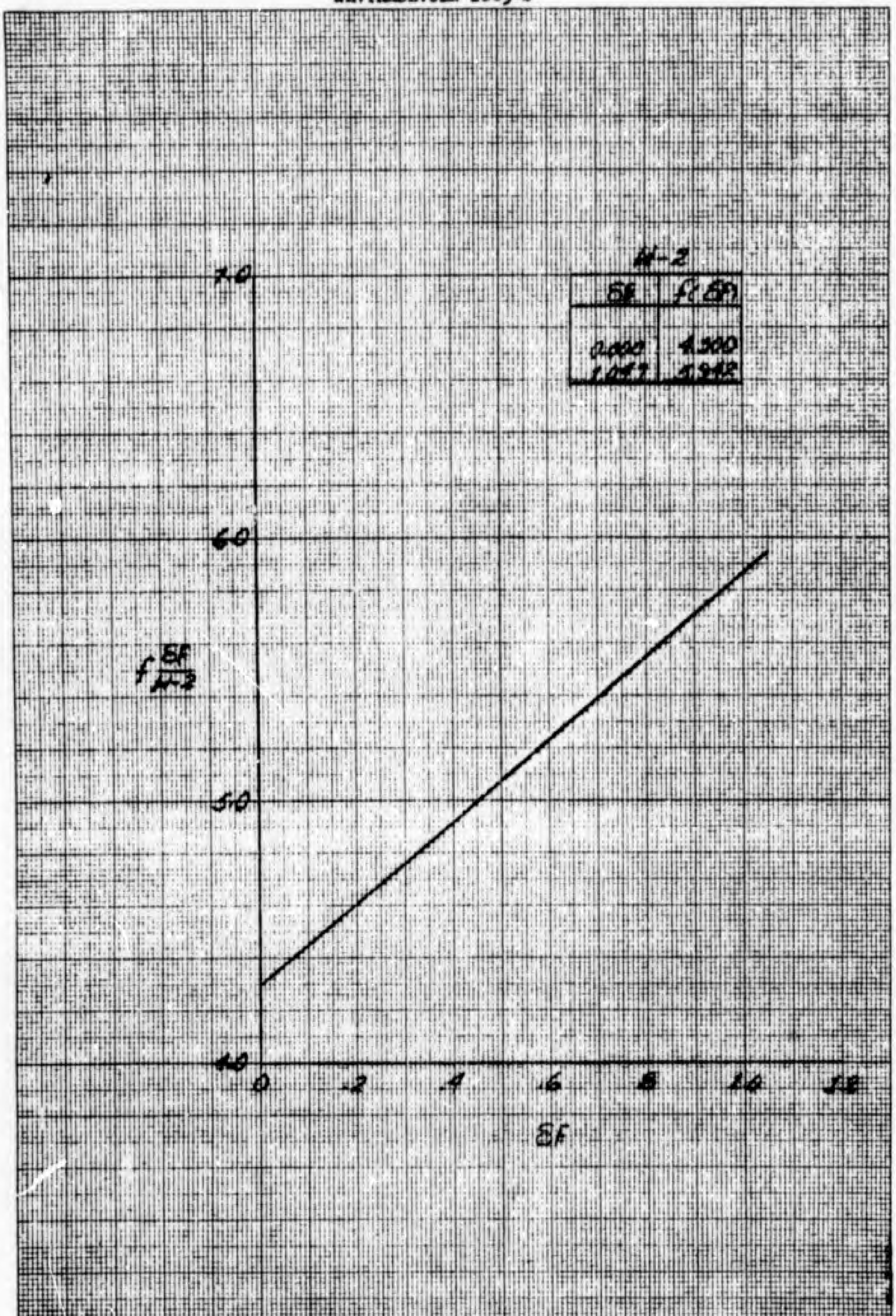

F-1
$C_{T,S}$ | $f(C_{T,S})$
0.00 | 1.00
.13 | 1.18
.30 | 1.36
.50 | 1.46
.71 | 1.44
.90 | 1.31
1.00 | 1.20
1.5
1.4
1.3
1.2
1.1
1.0
0
.2
.4
.6
.8
1.0
$f\frac{C_{T,S}}{F-1}$

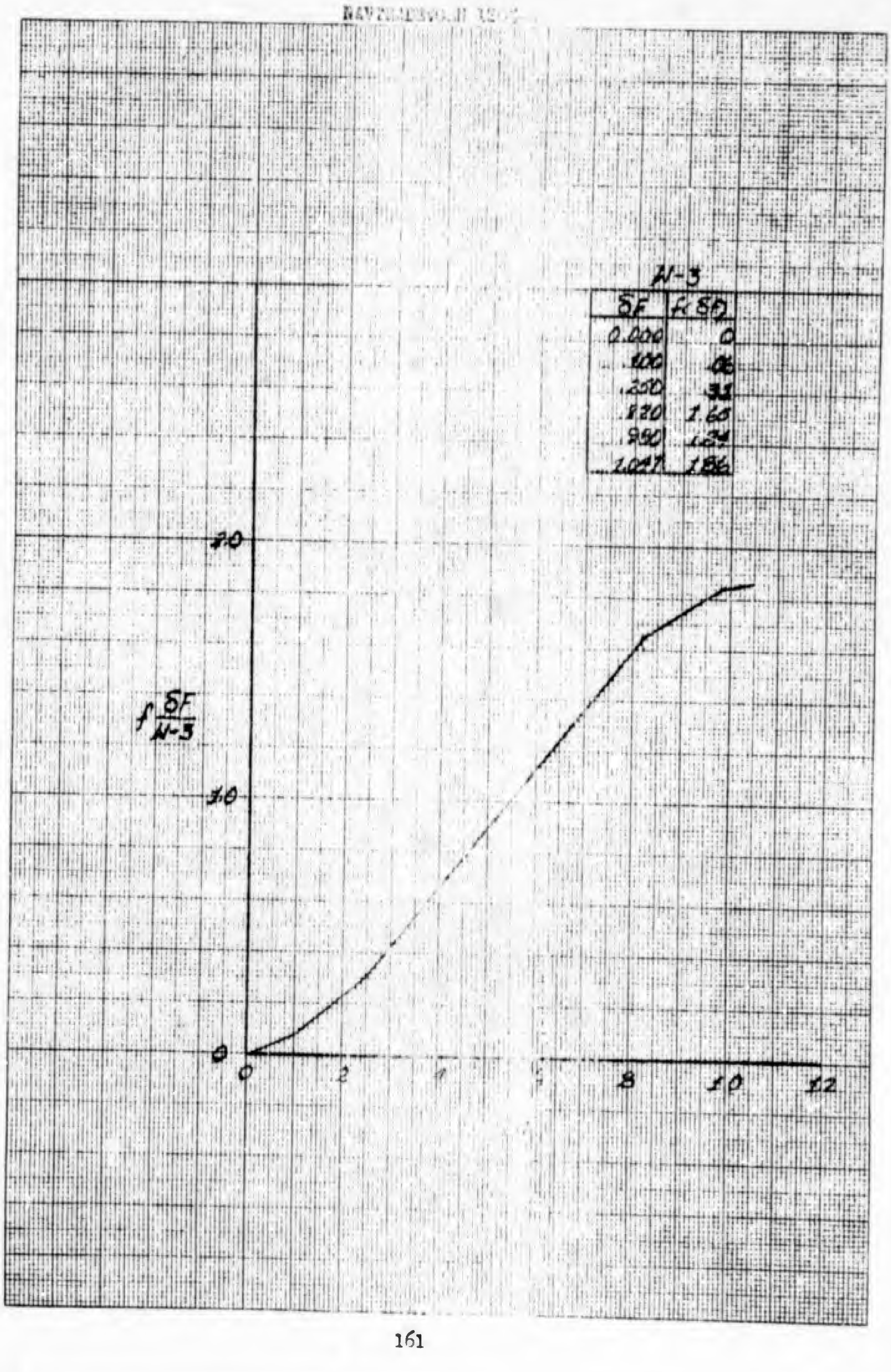

F-2
.00 -306.2
.15 -360.5
.30 -416.4
.50 -447.0
.71 -440.9
.90 -401.1
1.00 -367.4
-440
-420
-400
-380
-360
-340
-320
-300
0
.2
.4
.6
.8
1.0
F-2

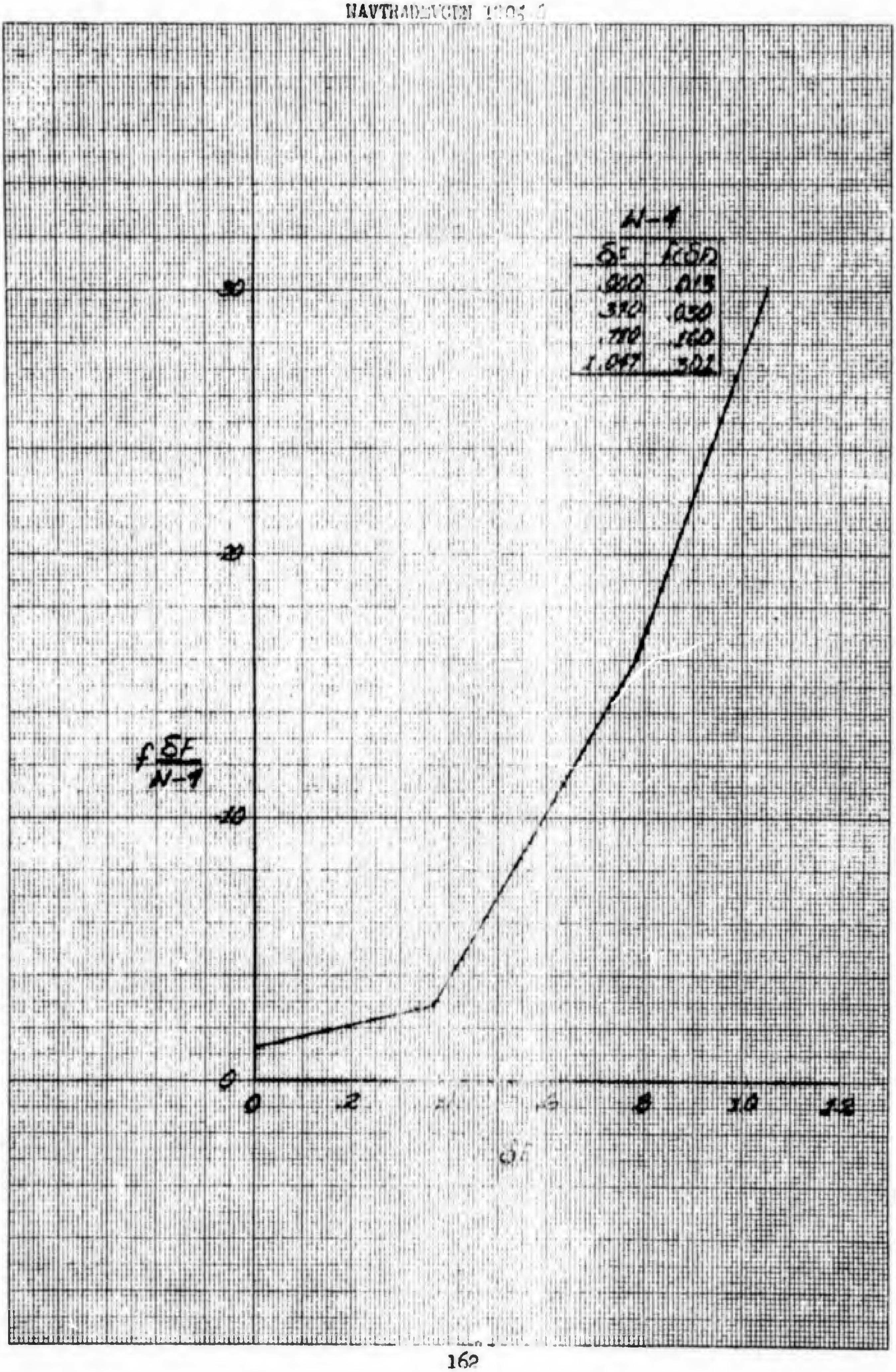

F-3
7000
6000
5000
4000
0
.2
.4
.6
.8
1.0
Cr,5

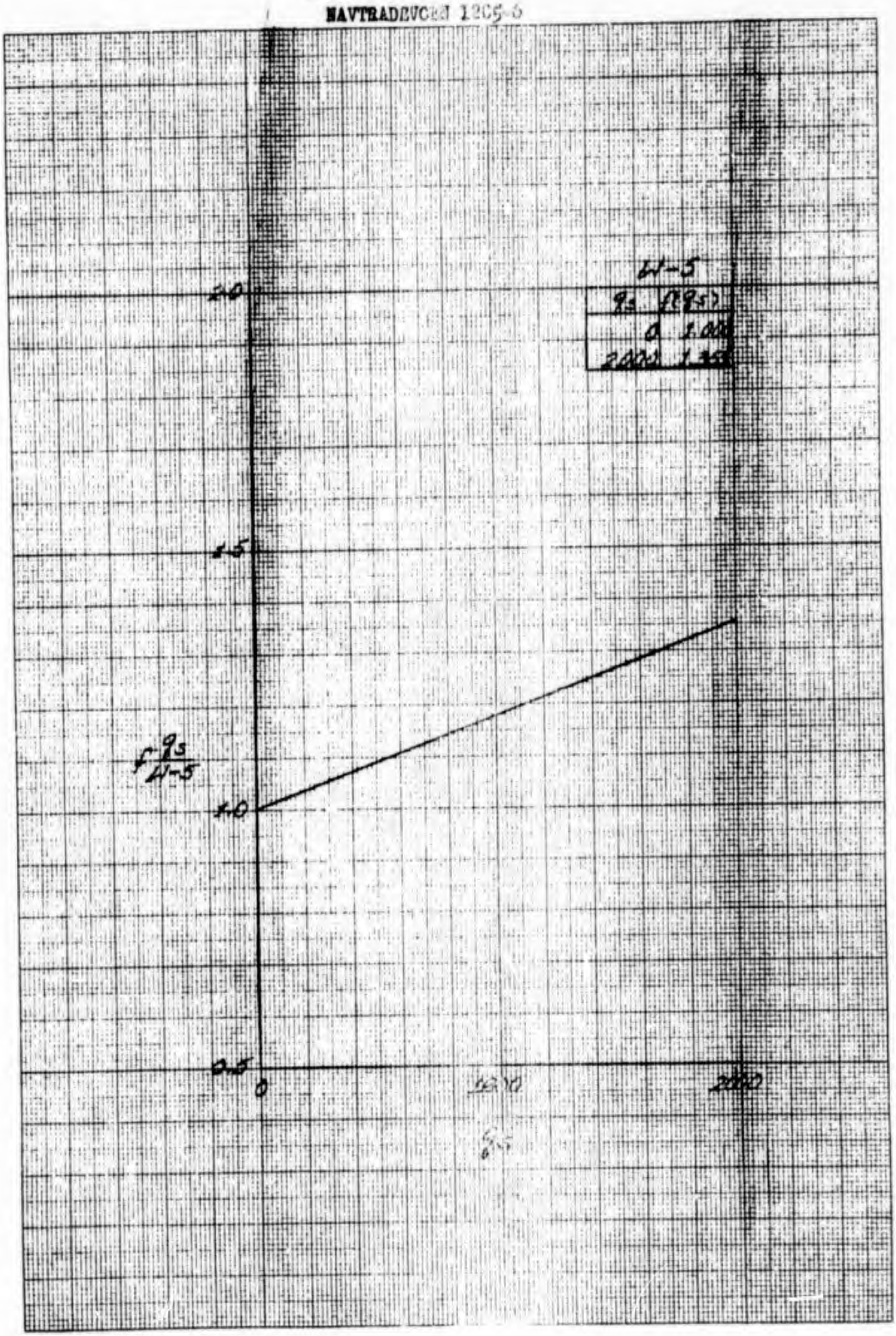


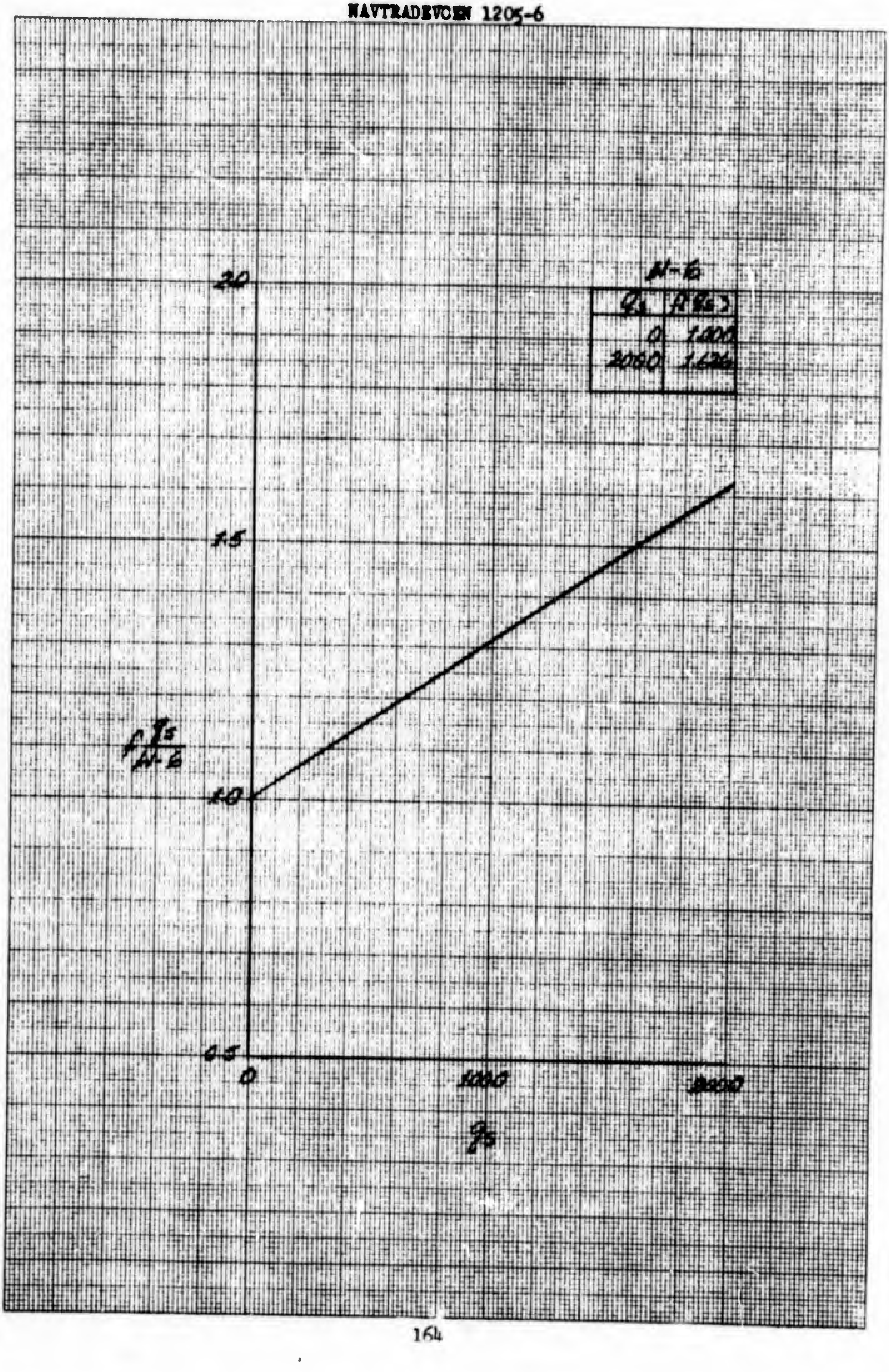
H-1
$C_{T,S}$ | $f(C_{T,S})$
0.00 | .079
.29 | .035
.59 | .008
.80 | 0.000
1.00 | 0.000
$f\frac{C_{T,S}}{H-1}$
.12
.10
.08
.06
.04
.02
0
0
.2
.4
.6
.8
1.0
$C_{T,S}$

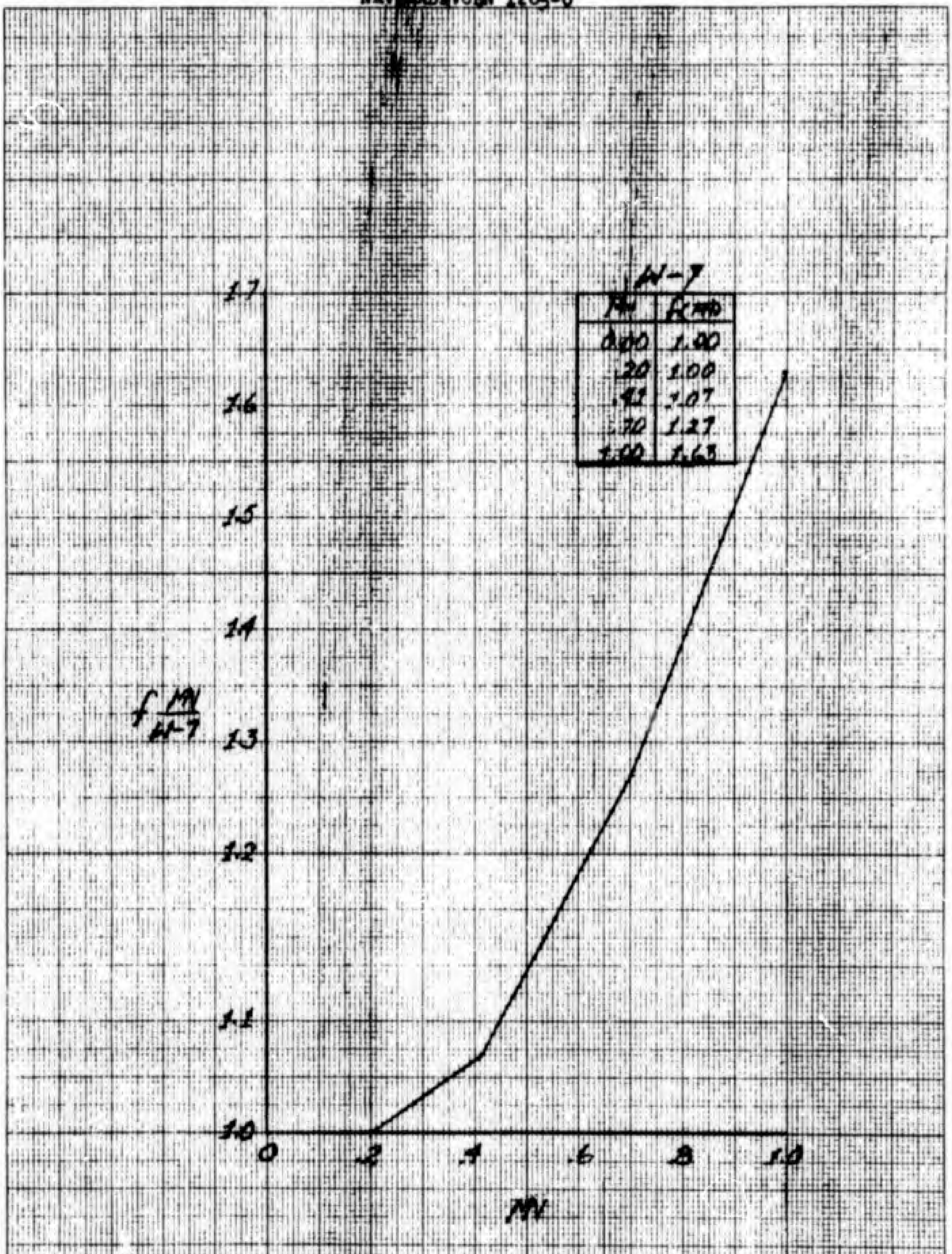

M-2
δf
f(δf)
0.000
1.047
4.300
5.942
7.0
6.0
5.0
4.0
f δf/M-2
0
.2
.4
.6
.8
1.0
1.2
δf

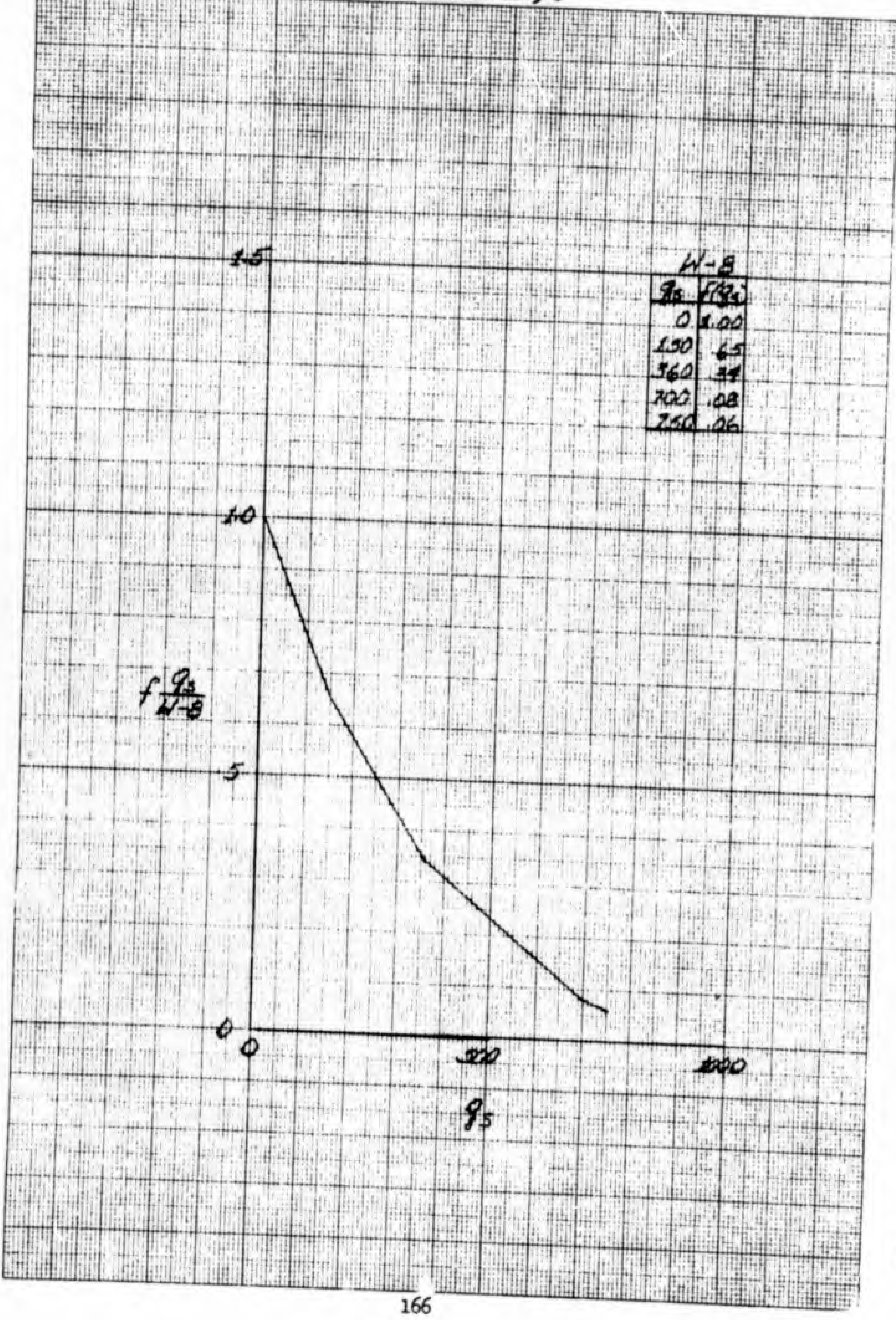

N-3
δF f(δF)
0.000 0
.100 .06
.250 .33
.820 1.60
.950 1.84
1.047 1.86
2.0
1.0
0
f δF/N-3
0
2
4
6
8
10
12

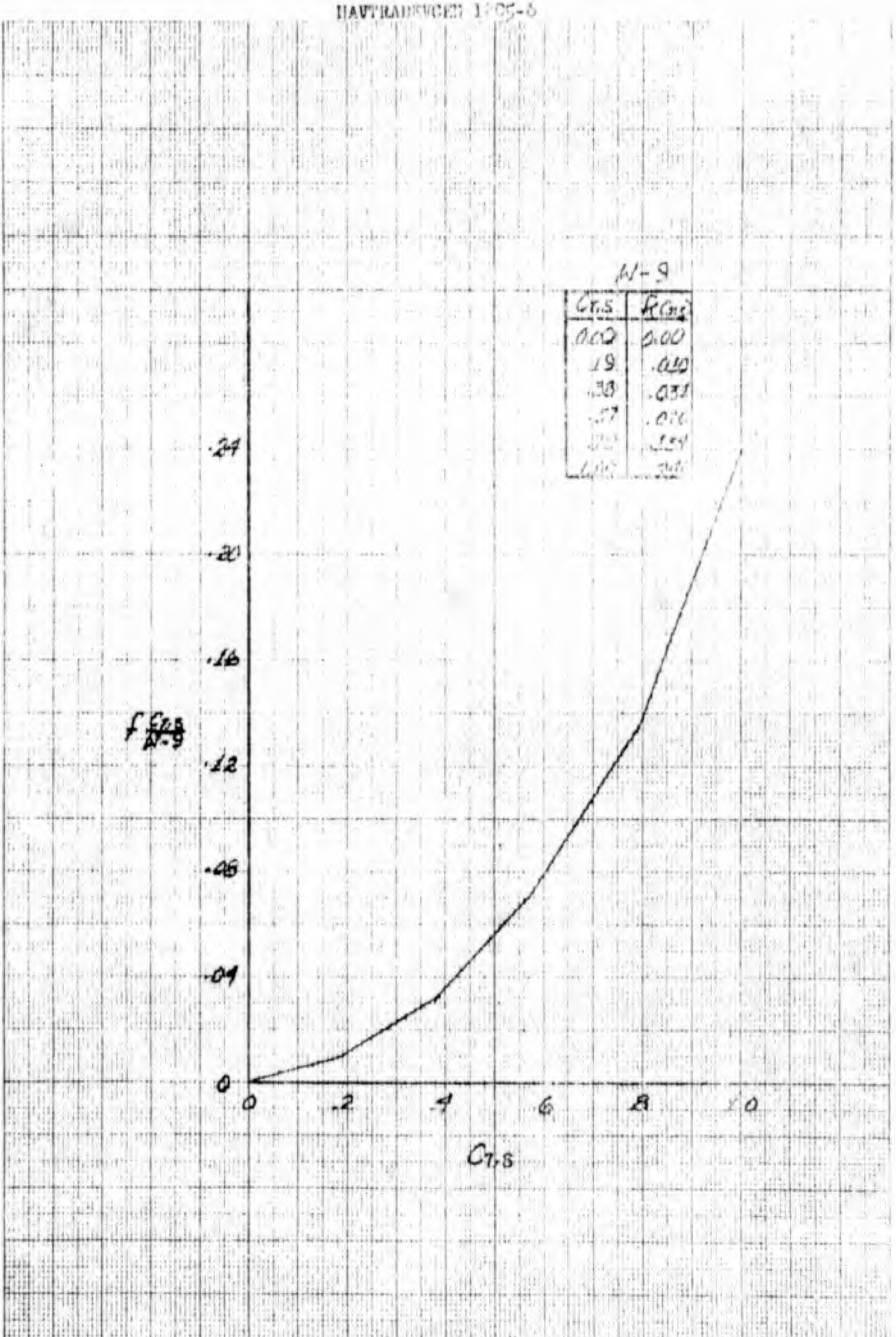

N-4
δF | f(δF)
.000 | .015
.370 | .030
.770 | .160
1.047 | .302
30
20
10
0
f δF / N-4
0
.2
.4
.6
.8
1.0
1.2
δF

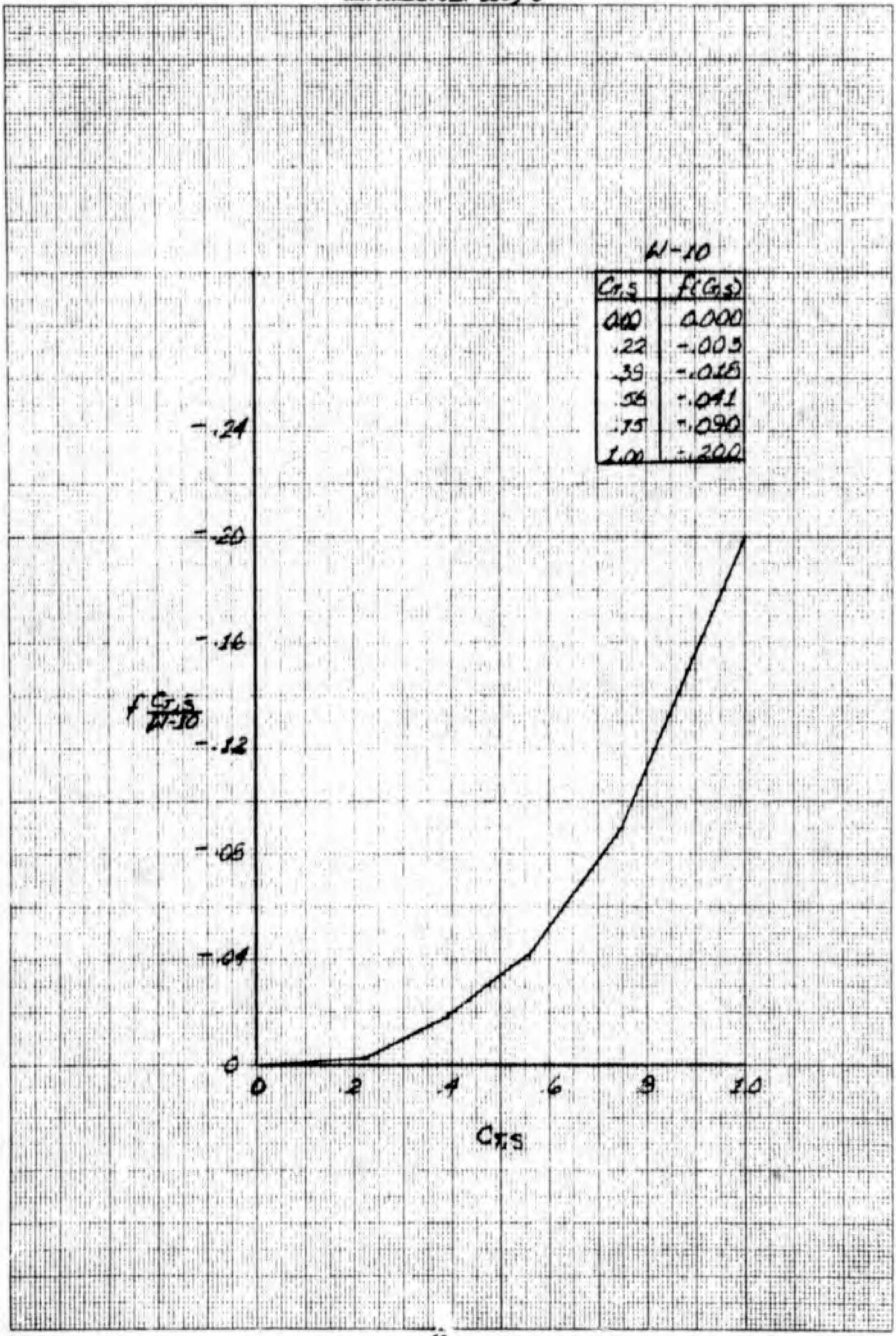

W-5
q_s f(q_s)
0 1.000
2000 1.35
2.0
1.5
1.0
0.5
$f\frac{q_s}{W-5}$
0
1000
2000
q_s

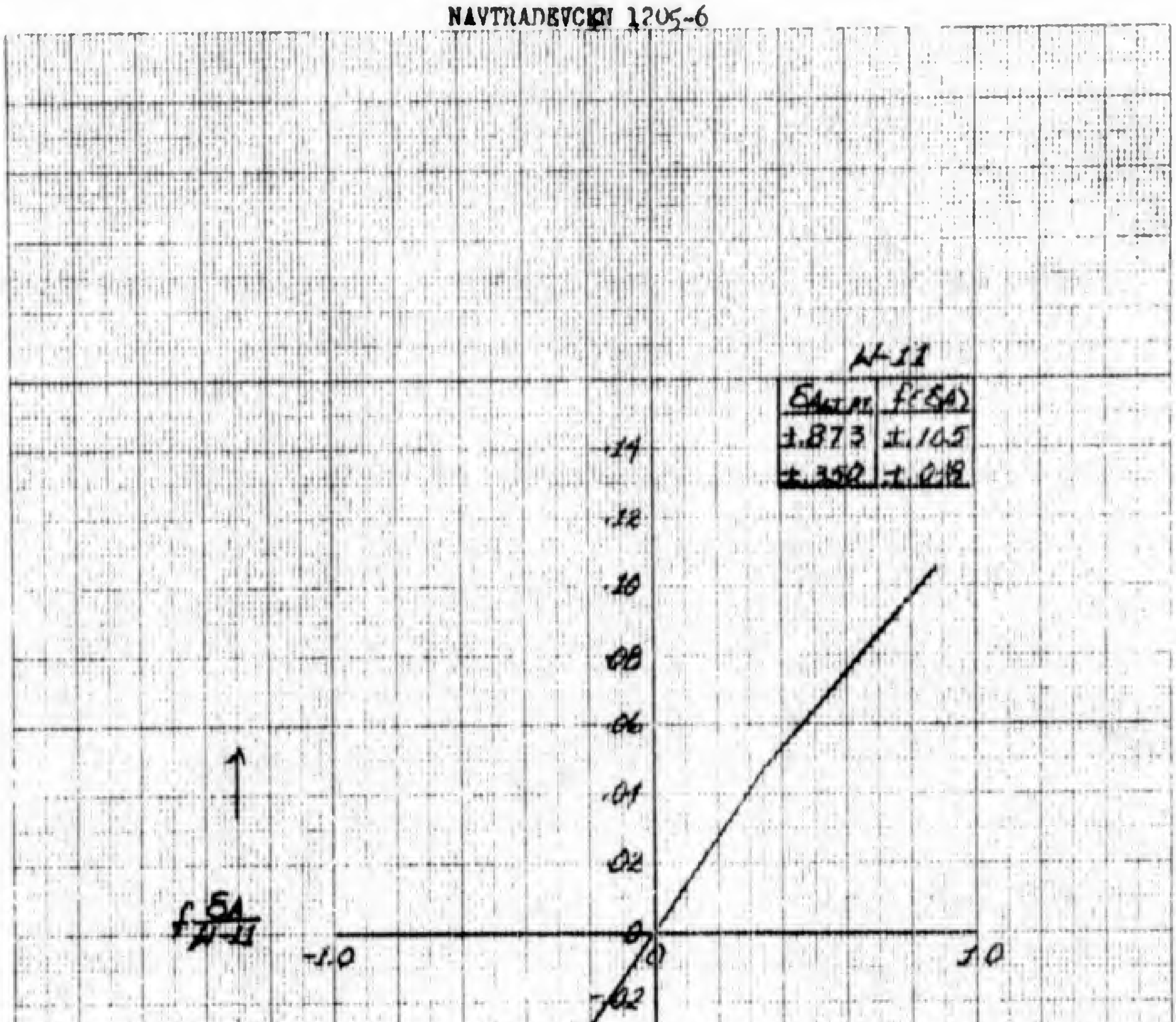

N-6
q_s
$f(q_s)$
0
1.000
2060
1.626
2.0
1.5
1.0
0.5
0
1000
2000
$f\frac{q_s}{N-6}$
q_s

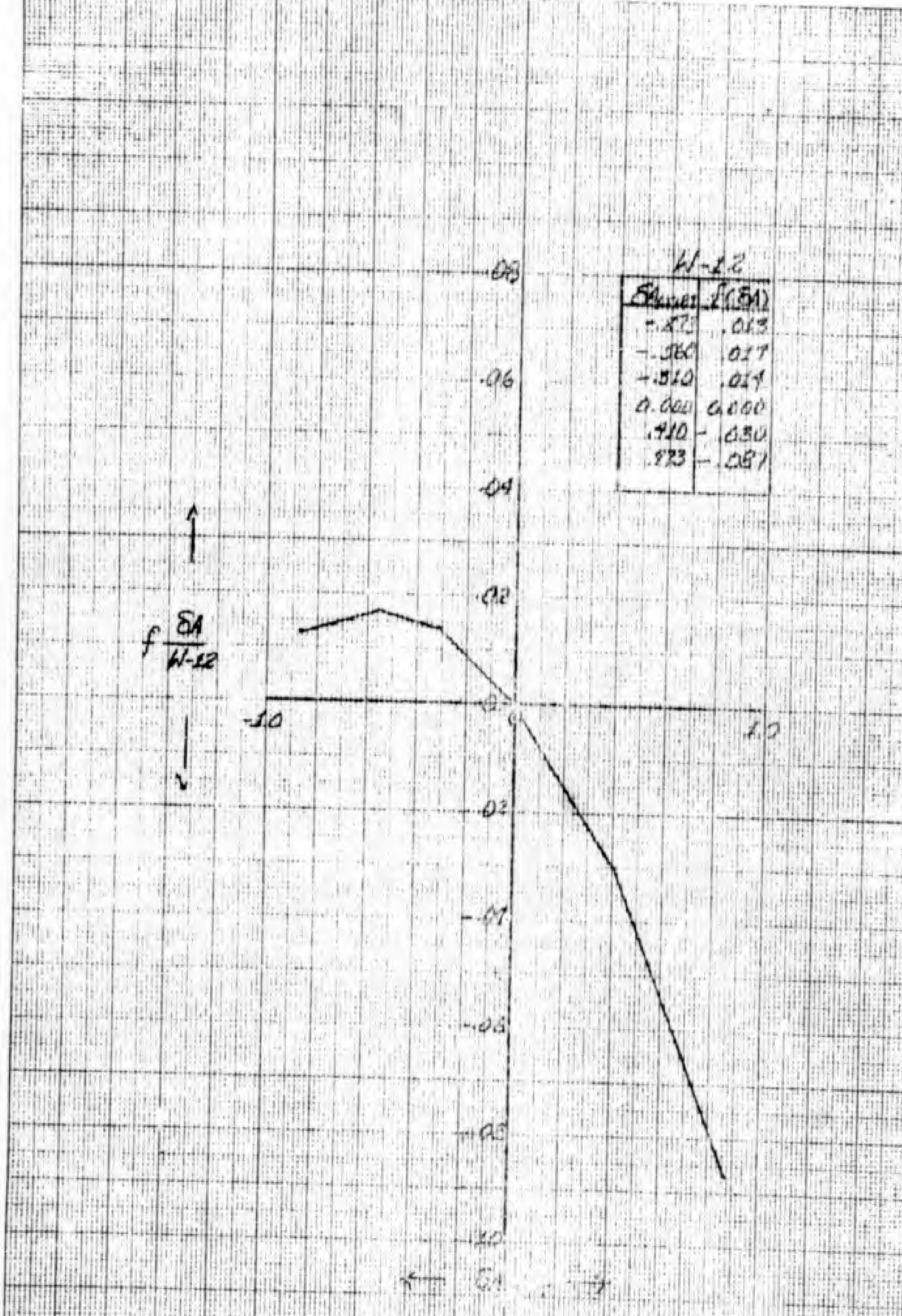

W-7
0.00 1.00
.20 1.00
.41 1.07
.70 1.27
1.00 1.63
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1.0
0
.2
.4
.6
.8
1.0

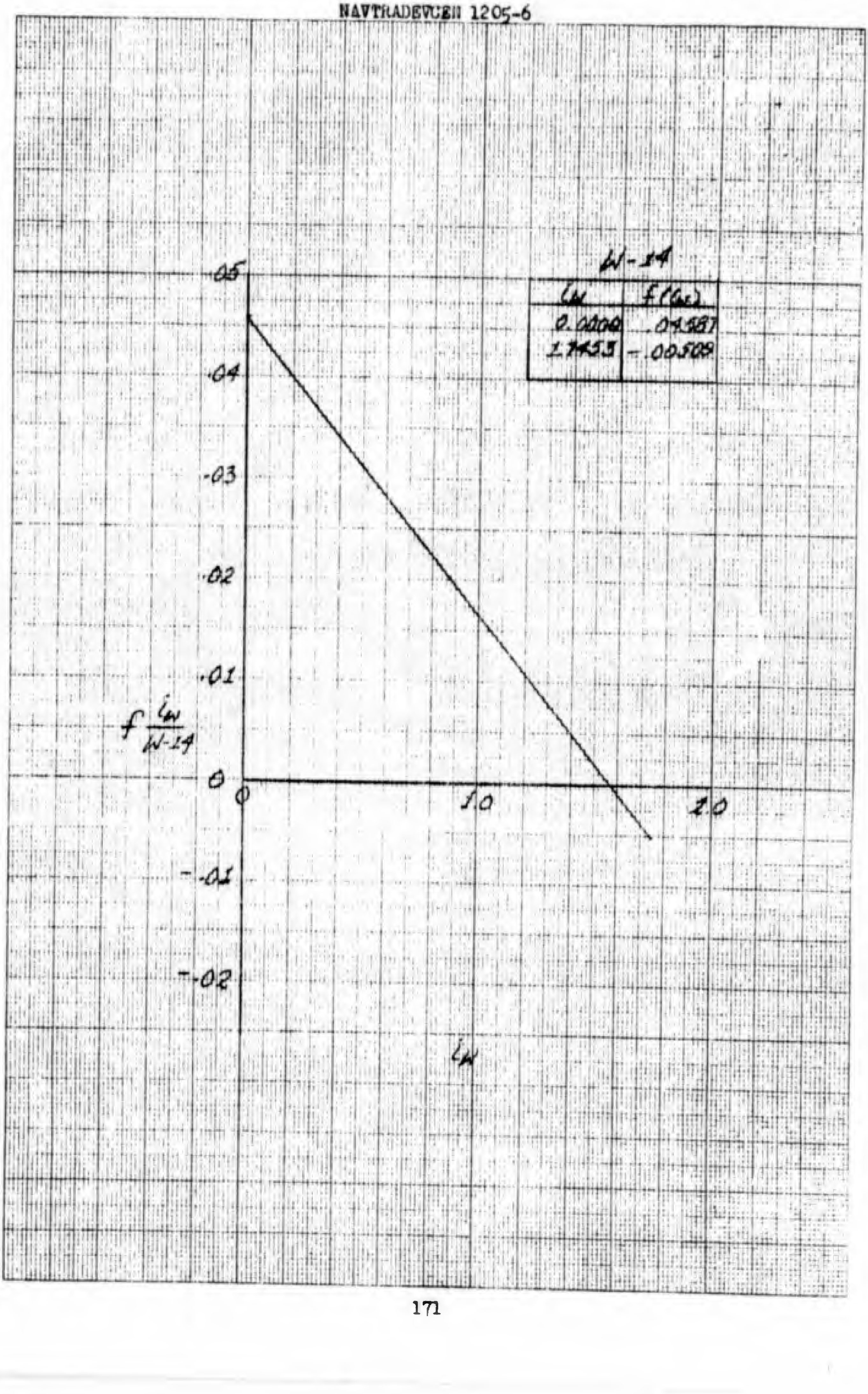

W-8
q5 f(q5)
0 1.00
150 .65
360 .34
700 .08
750 .06
1.5
1.0
.5
0
0
500
1000
q5
f q5/W-8

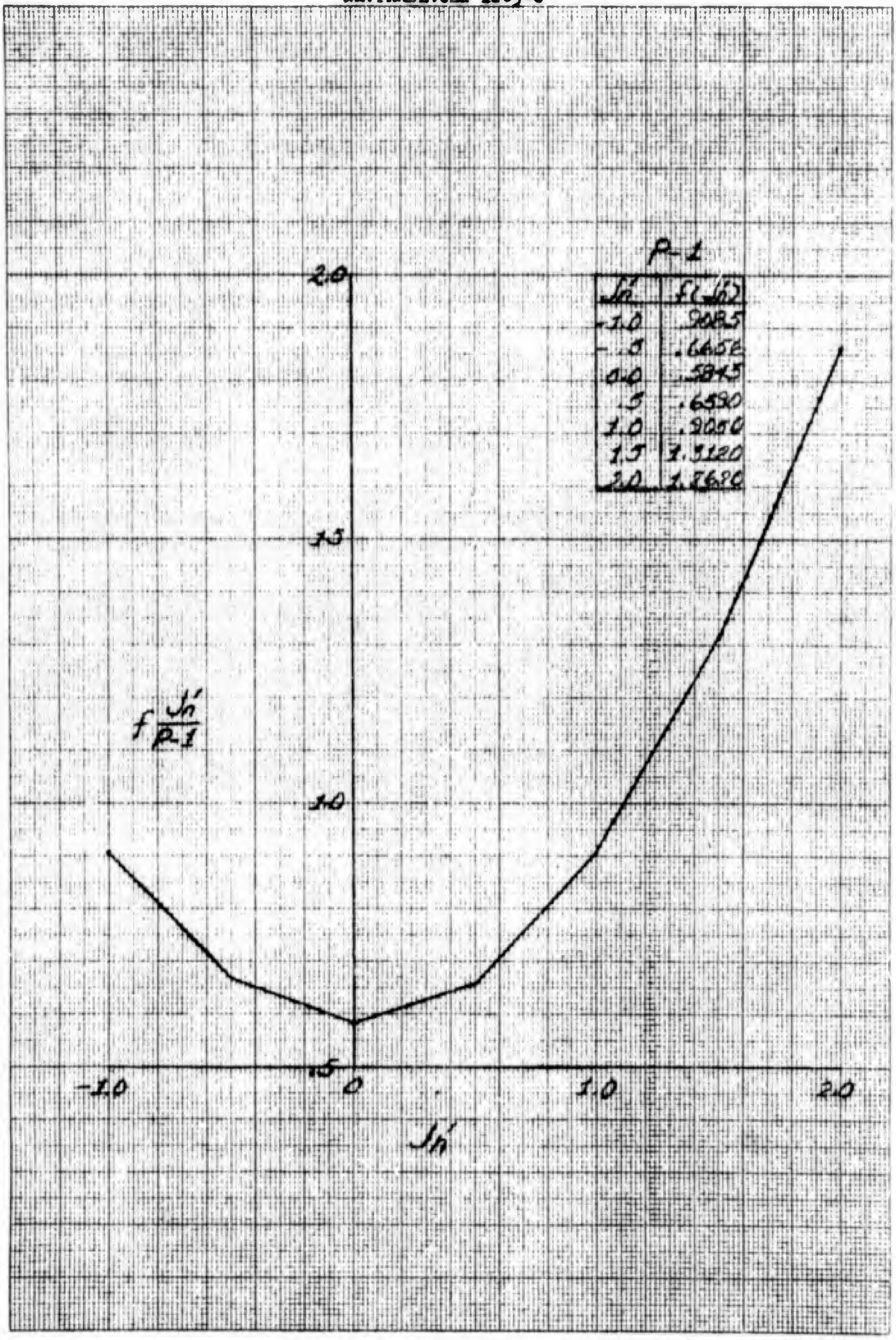

N-9
$C_{T,S}$
0.00
.19
.010
.38
.031
.57
.24
.20
.16
.12
.08
.04
0
0
.2
.4
.6
.8
1.0
$C_{T,S}$


W-10
C_{T,S} | f(C_{T,S})
0.00 | 0.000
.22 | -.003
.39 | -.018
.56 | -.041
.75 | -.090
1.00 | -.200
-.24
-.20
-.16
-.12
-.08
-.04
0
f C_{T,S} / W-10
0
.2
.4
.6
.8
1.0
C_{T,S}

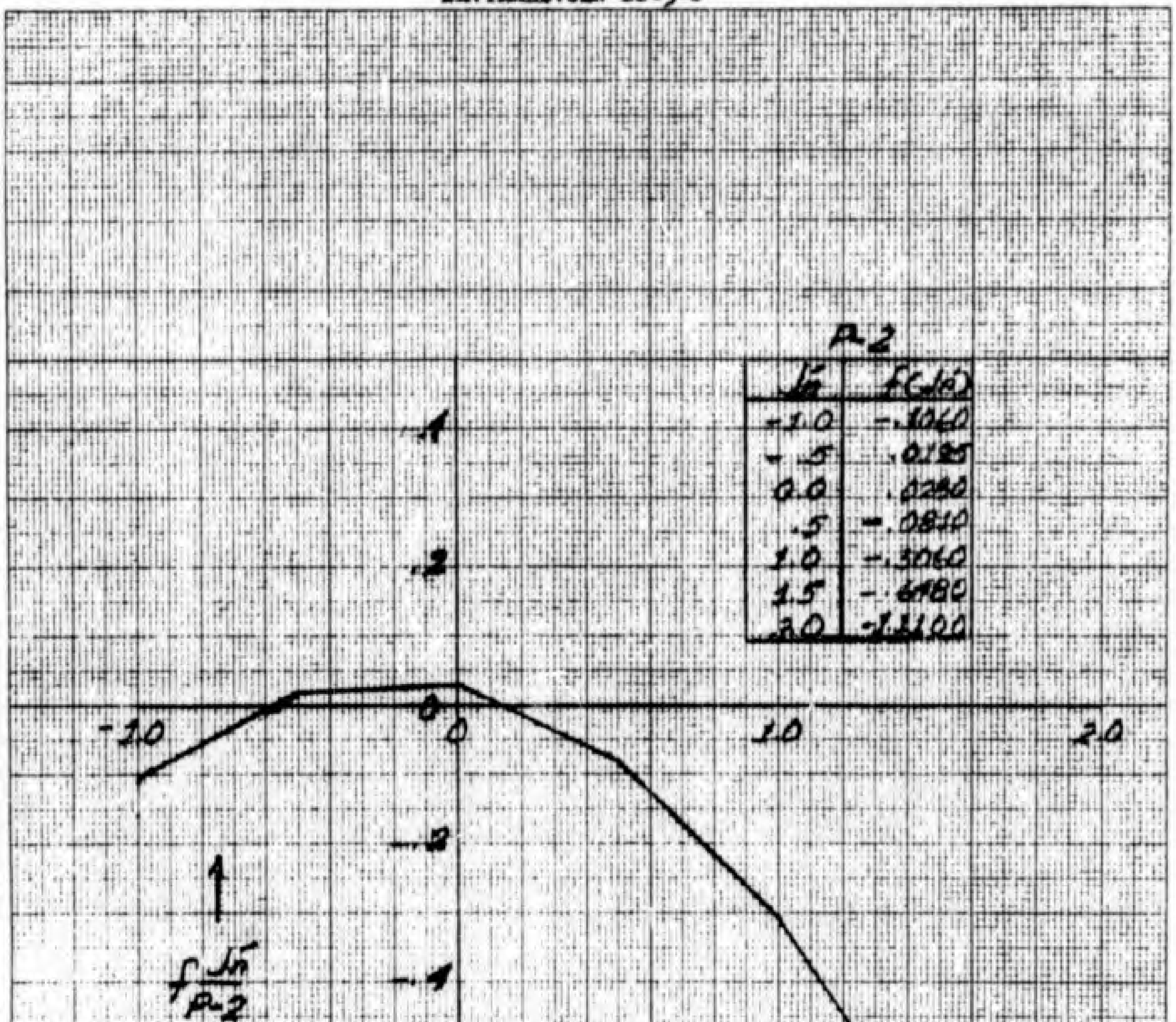

W-11
δA LT,RT | f(δA)
±.873 | ±.105
±.350 | ±.048
.14
.12
.10
.08
.06
.04
.02
0
-.02
-.04
-.06
-.08
-.10
-.12
-.14
-1.0
0
1.0
f δA W-11
δA LT,RT

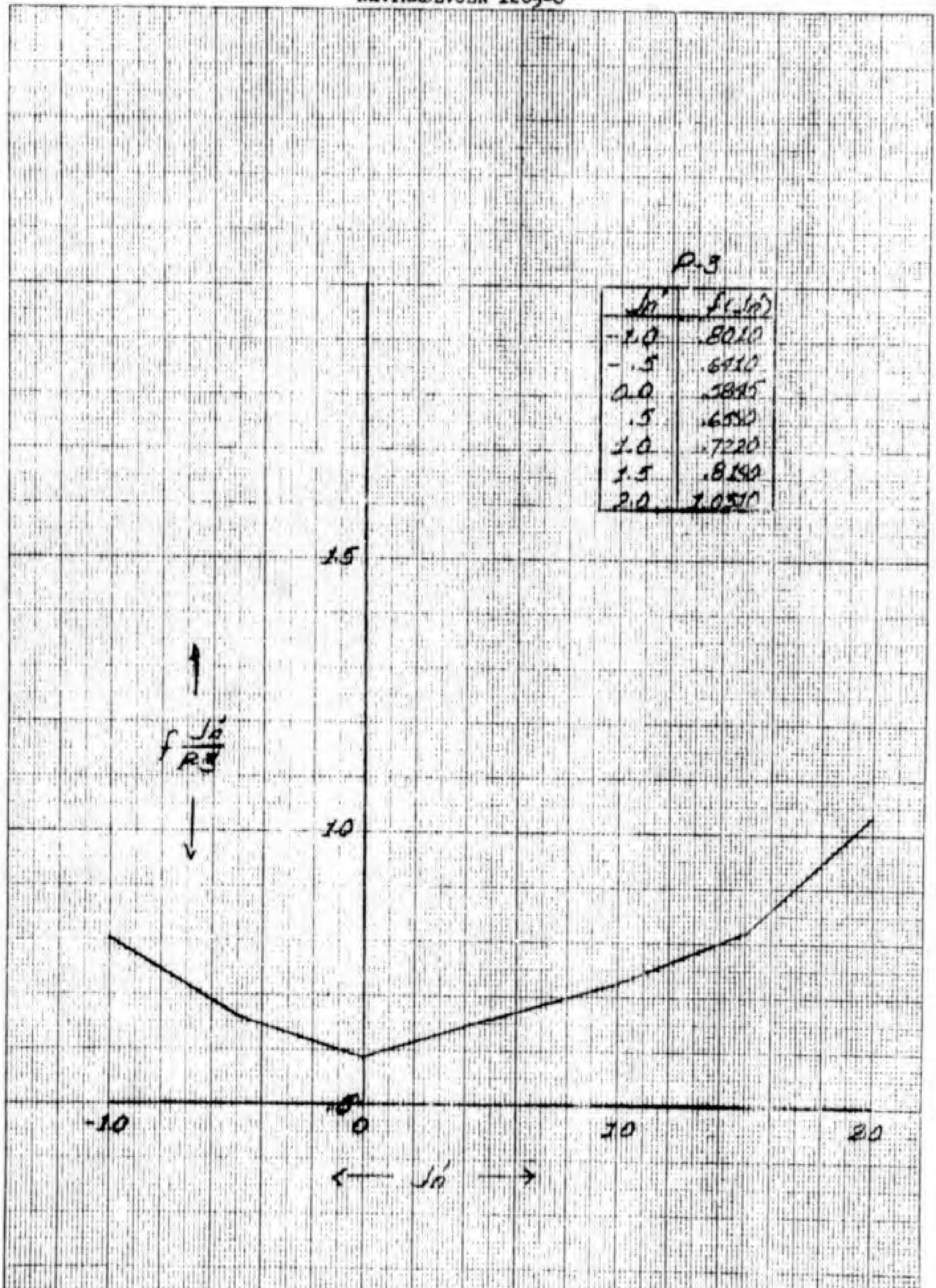

W-12
f(δA)
.013
.017
.014
0.000
.410
.873
- .030
- .087
.08
.06
.04
.02
-1.0
1.0
f δA / W-12
δA


W-14
i_W | $f(i_W)$
0.0000 | .04587
1.7453 | -.00509
.05
.04
.03
.02
.01
0
-.01
-.02
$f\frac{i_W}{W-14}$
0
1.0
2.0
i_W



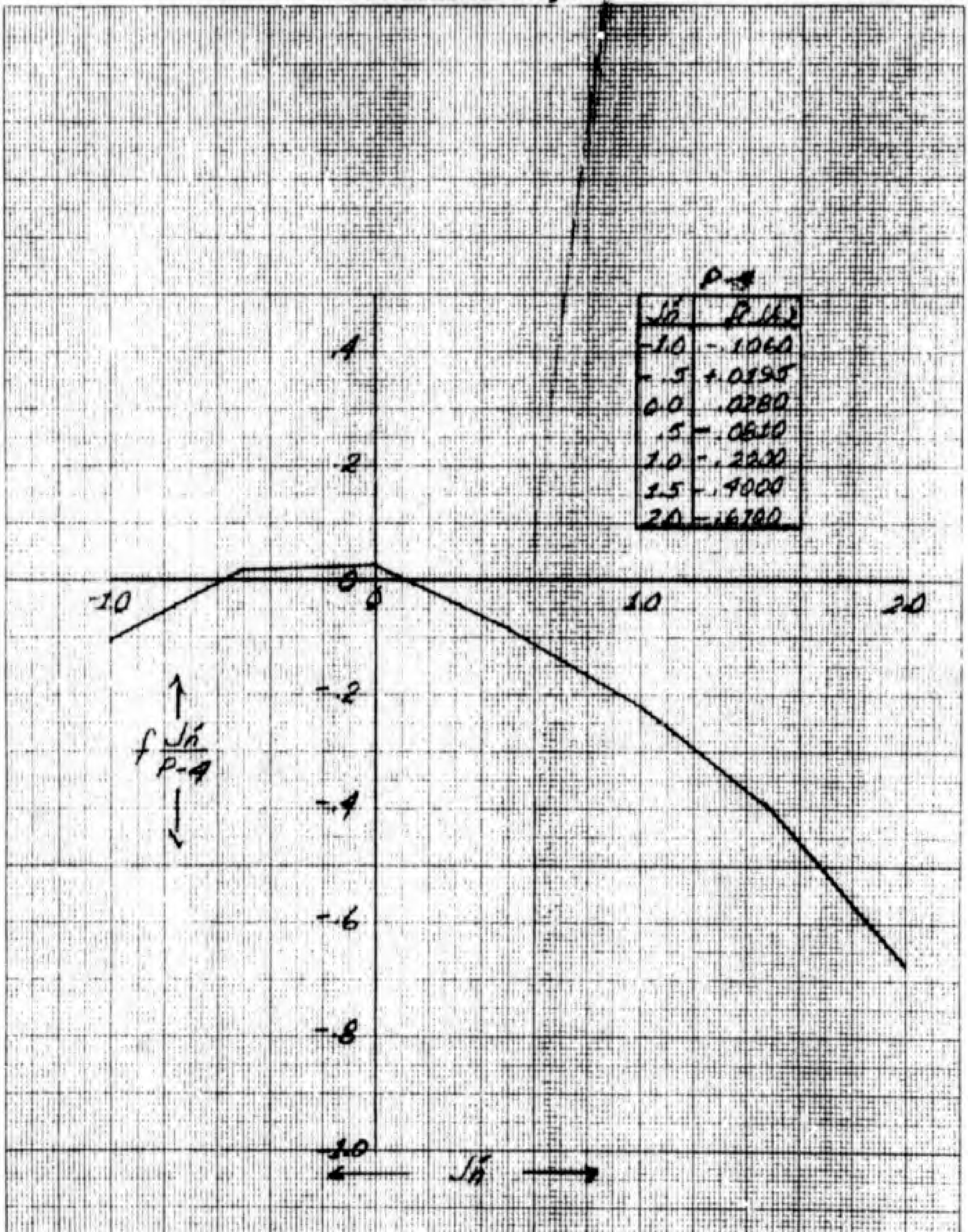
Jh'	f(Jh')
-1.0	.9085
-.5	.6656
0.0	.5845
.5	.6590
1.0	.9050
1.5	1.3120
2.0	1.8680


p=2
√h | f(√h)
-1.0 | -.1060
-.5 | .0185
0.0 | .0280
.5 | -.0810
1.0 | -.3060
1.5 | -.6480
2.0 | -1.1100
.4
.2
0
-1.0
0
1.0
2.0
-.2
-.4
-.6
-.8
-1.0
-1.2
f √h / p=2
√h



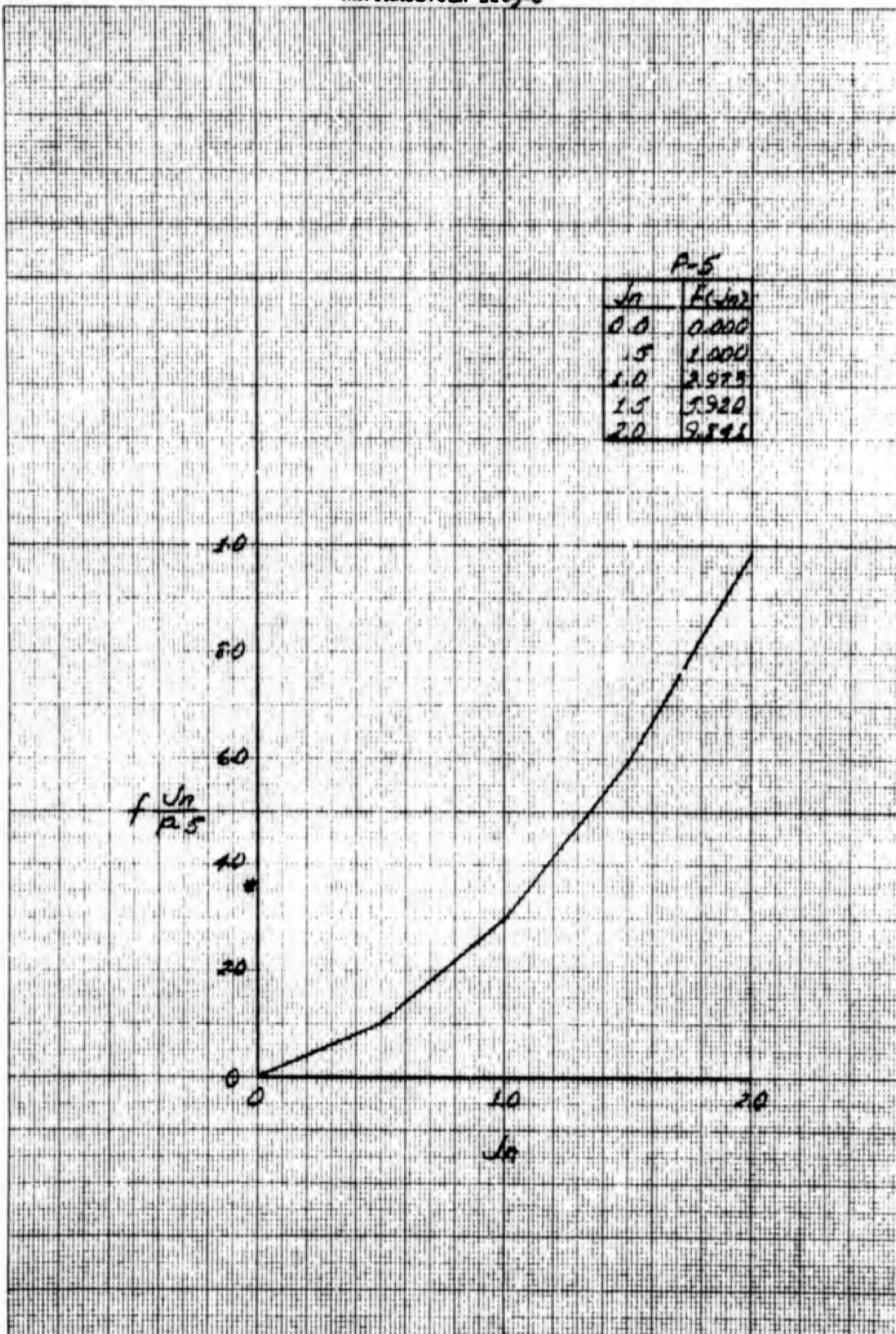
P-3

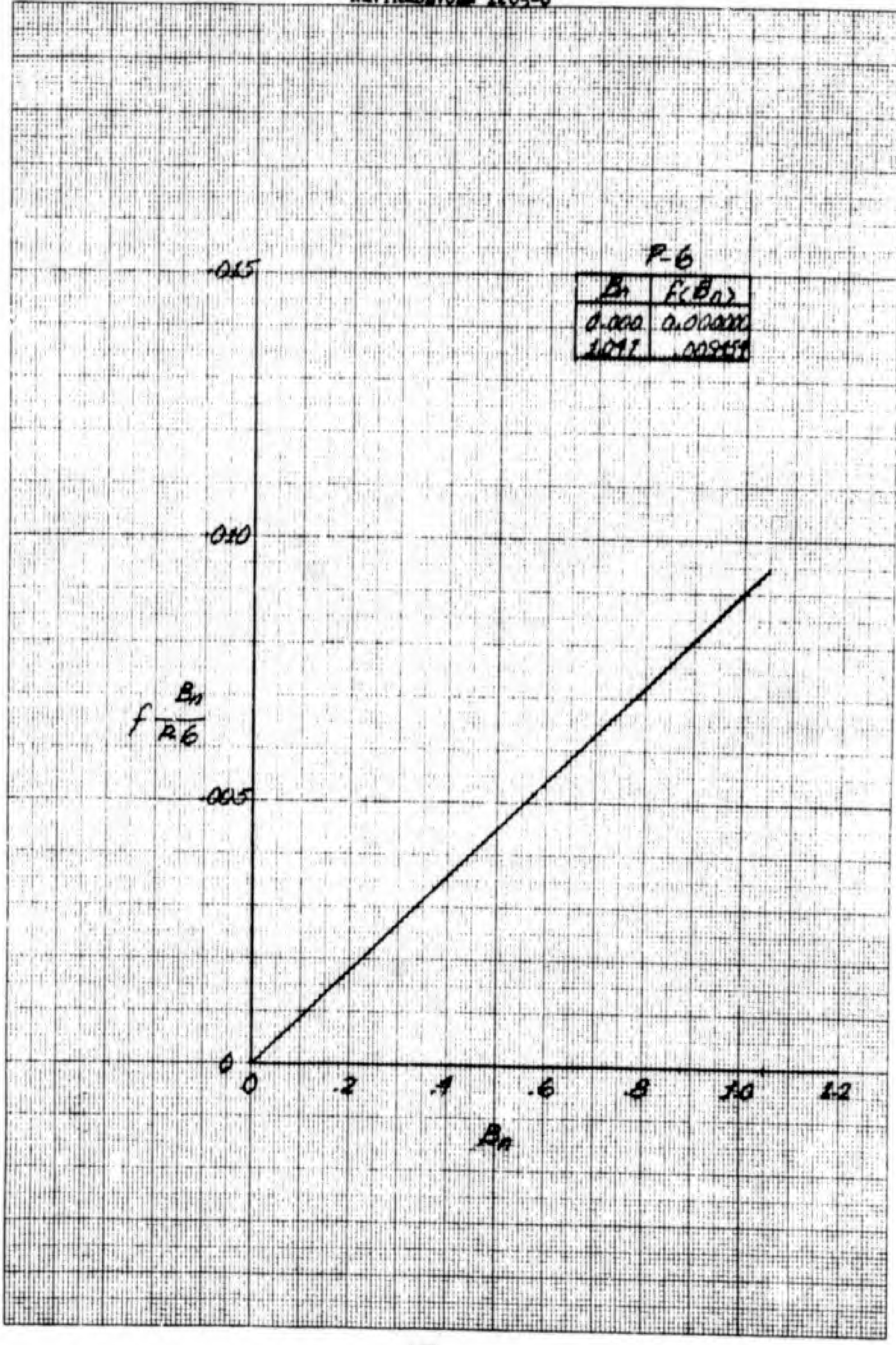
Jo'	f(Jo')
-1.0	.8010
-.5	.6410
0.0	.5845
.5	.6550
1.0	.7220
1.5	.8180
2.0	1.0510

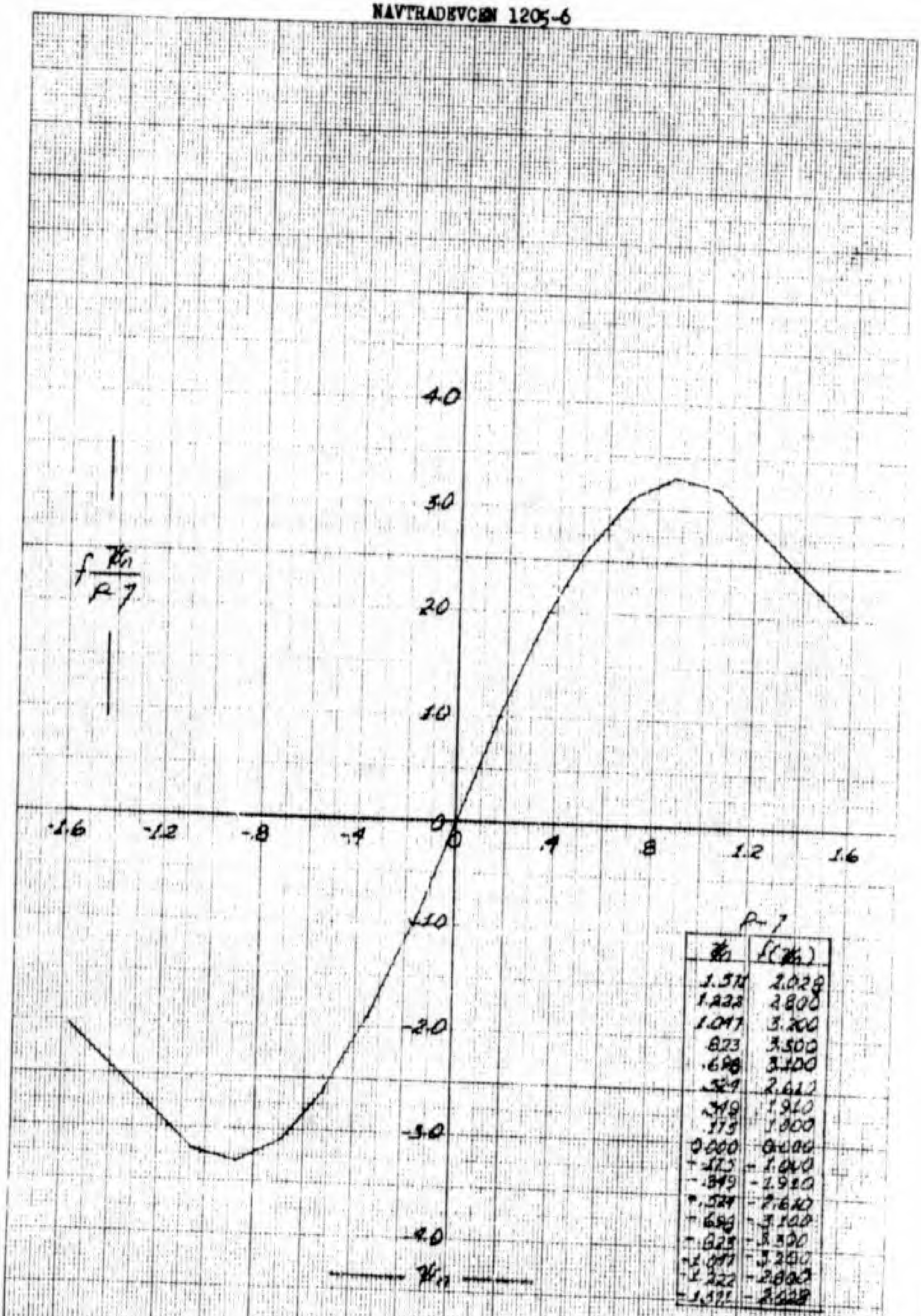


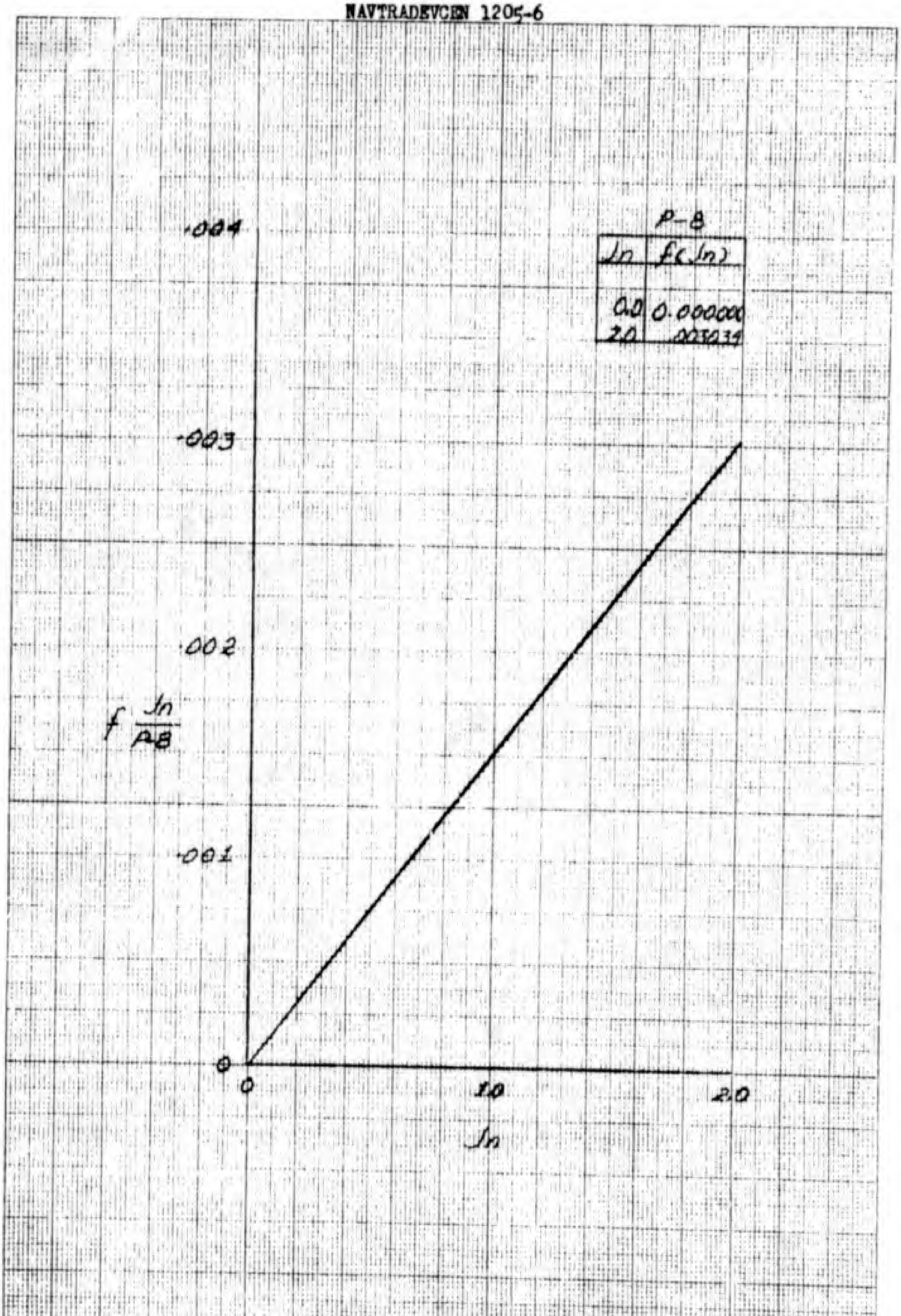
P-4

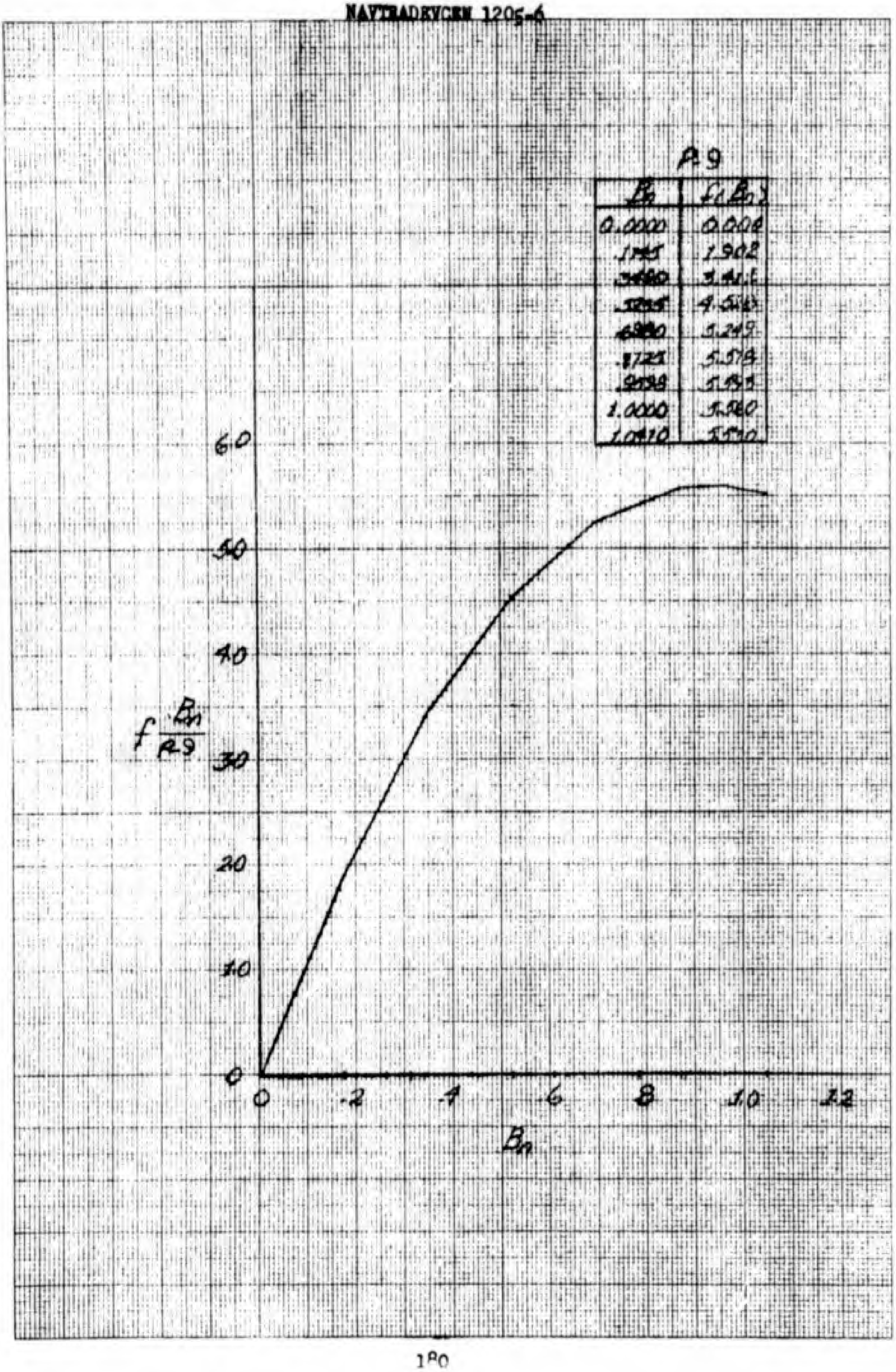
Jn	f(Jn)
-1.0	-.1060
-.5	+.0195
0.0	.0280
.5	-.0610
1.0	-.2200
1.5	-.4000
2.0	-.6700


P-5
Jn | F(Jn)
0.0 | 0.000
.5 | 1.000
1.0 | 2.975
1.5 | 5.920
2.0 | 9.841
10
80
60
40
20
0
f Jn/P-5
0
1.0
2.0
Jn


P-6
B_n
$f(B_n)$
0.000
0.00000
1.041
.009459
.015
.010
.005
0
$f\frac{B_n}{P6}$
0
.2
.4
.6
.8
1.0
1.2
B_n

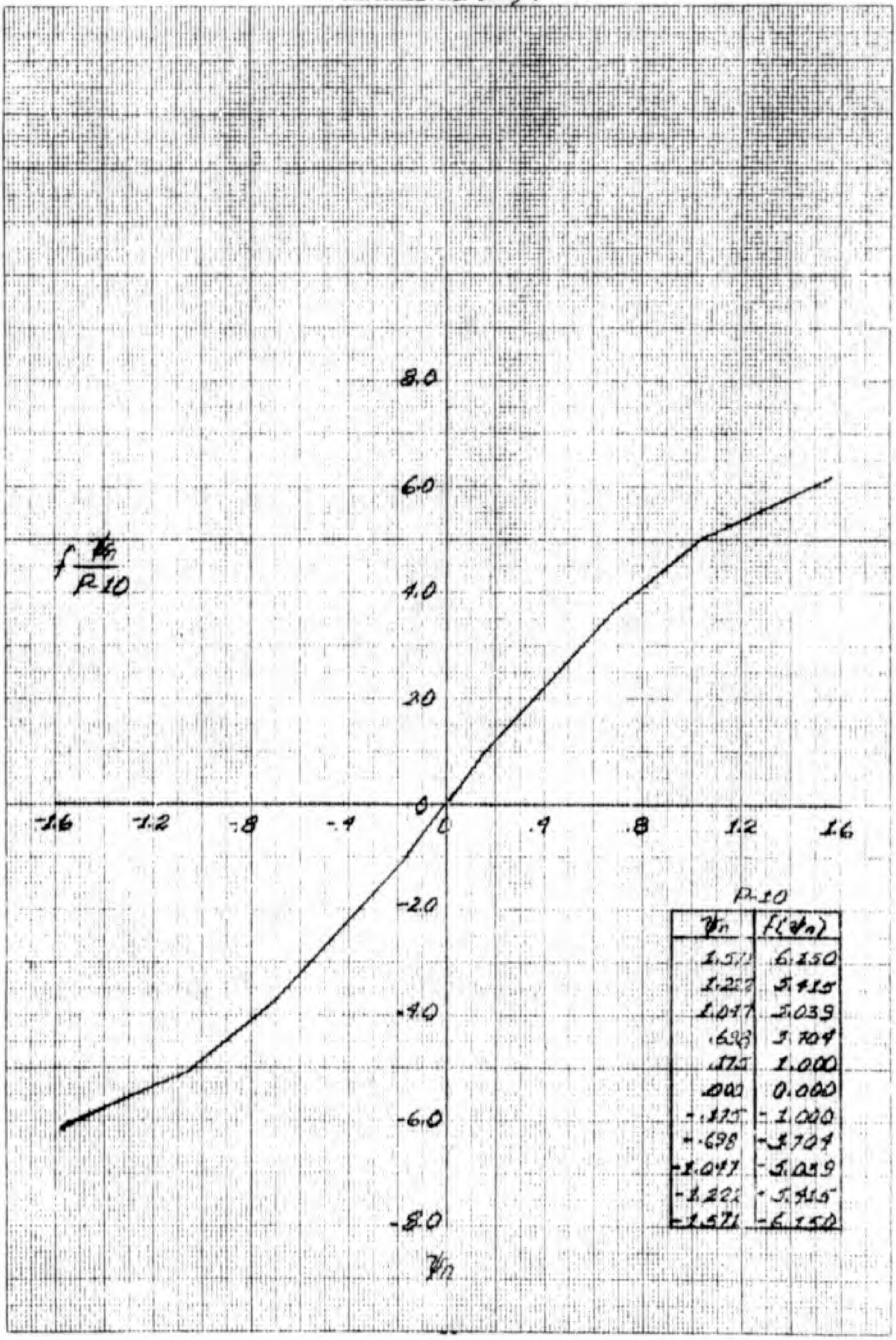

4.0
3.0
2.0
1.0
-1.0
-2.0
-3.0
-4.0
-1.6
-1.2
-.8
-.4
0
.4
.8
1.2
1.6
P-7
1.222 2.800
1.047 3.200
.823 3.500
.698 3.100
[illegible] 2.610
.349 1.910
.175 1.000
0.000 0.000
.175 -1.040
-.349 -1.910
.524 -2.610
.698 -3.100
.823 -3.500
-1.047 -3.200
-1.222 -2.800
[illegible] -2.028


P-8
Jn | f(Jn)
0.0 0.000000
2.0 .003039
.004
.003
.002
.001
0
f Jn/P8
0
1.0
2.0
Jn



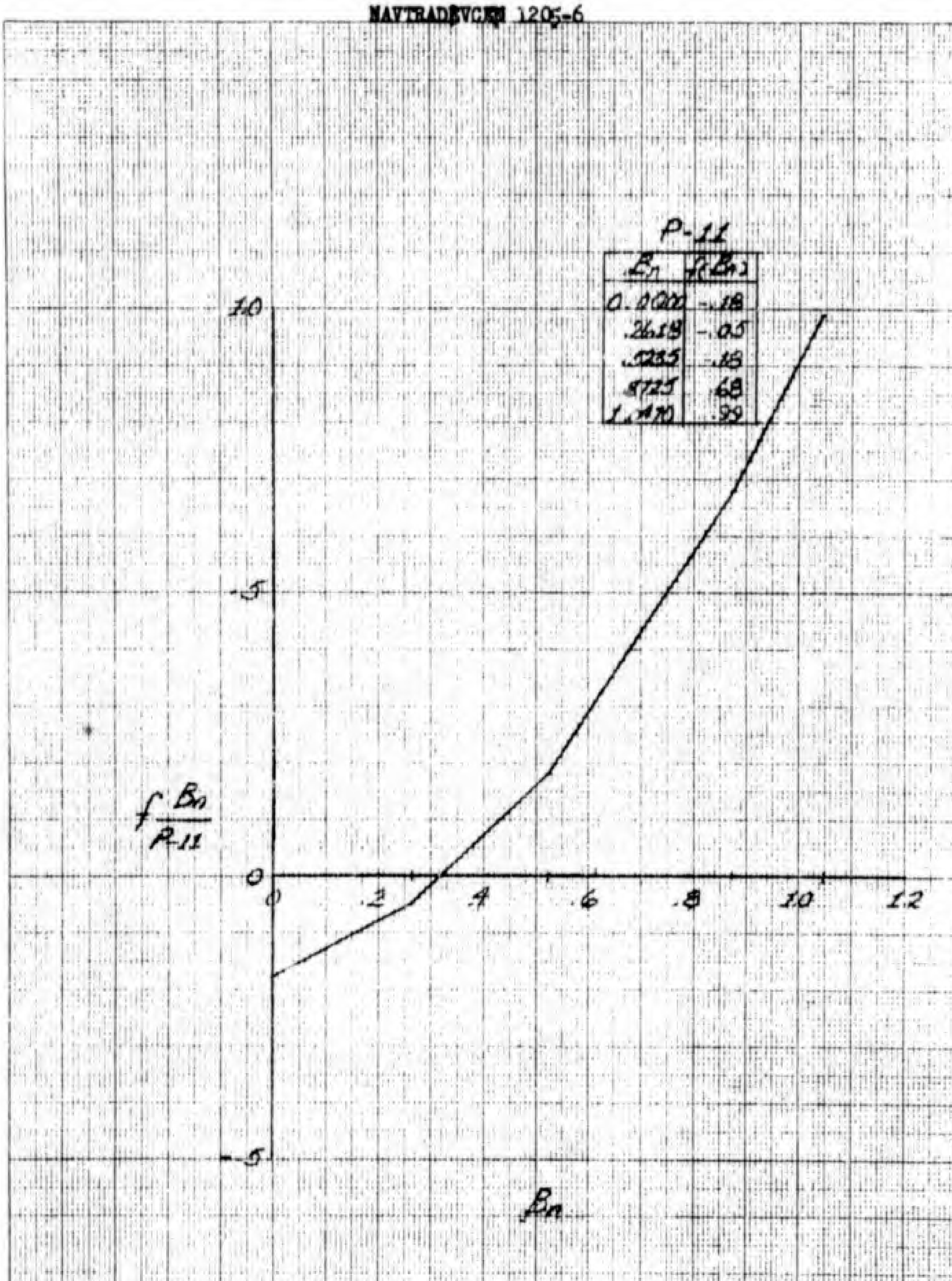
R9

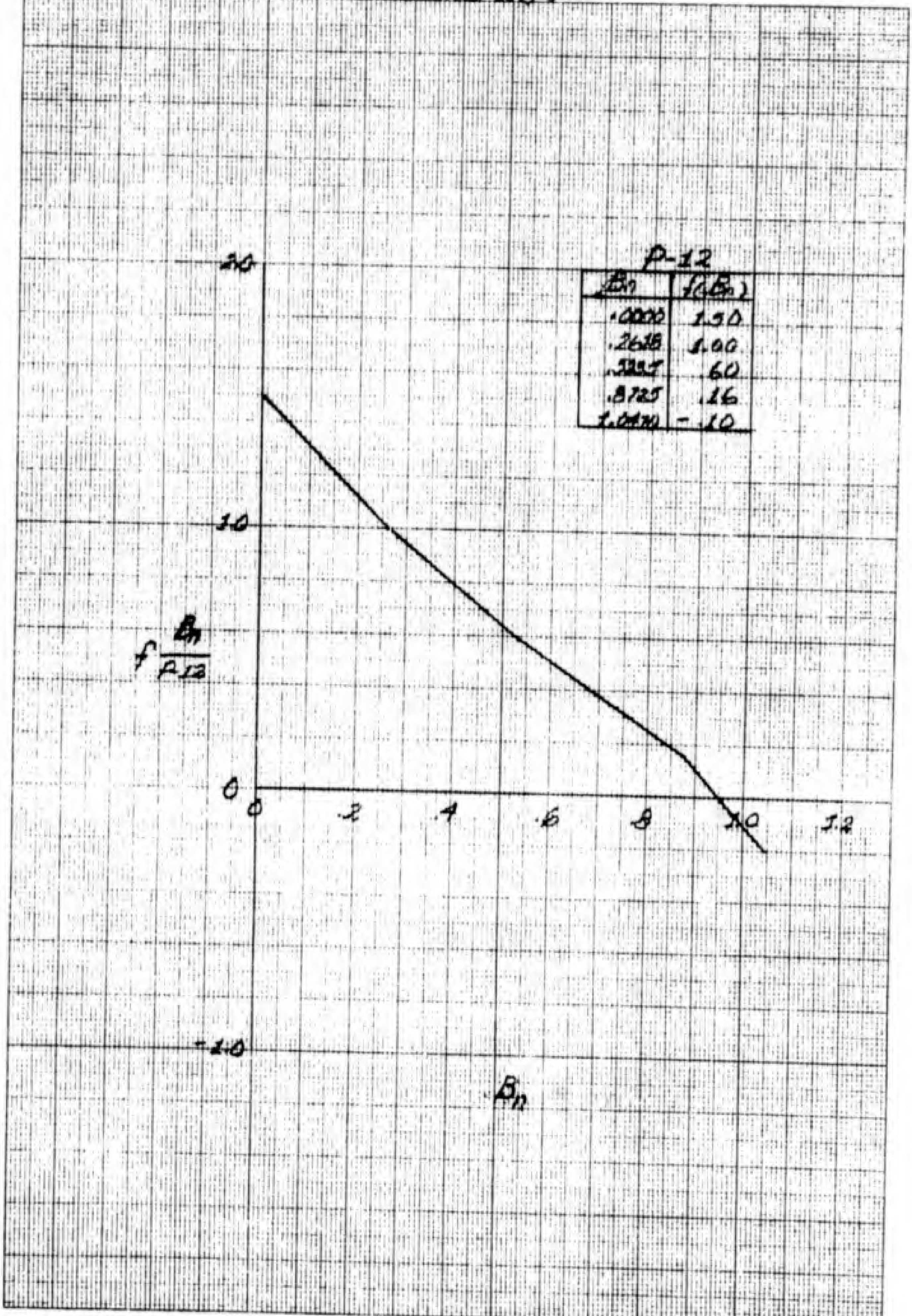
B_n	$f(B_n)$
0.0000	0.000
.1745	1.902
.3490	[illegible]
.5235	[illegible]
.6980	5.249
.8725	5.578
[illegible]	[illegible]
1.0000	5.560
1.0470	5.530

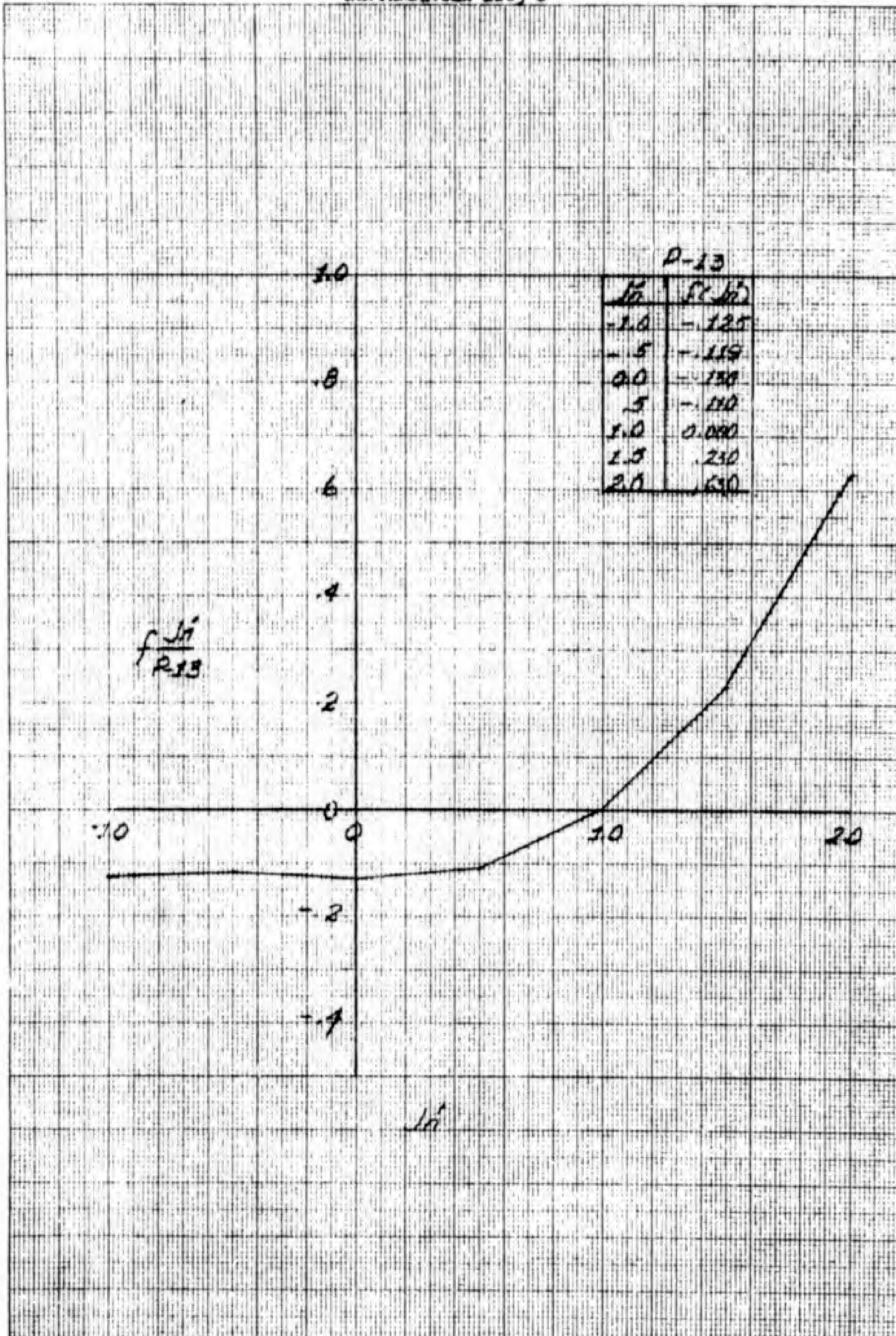


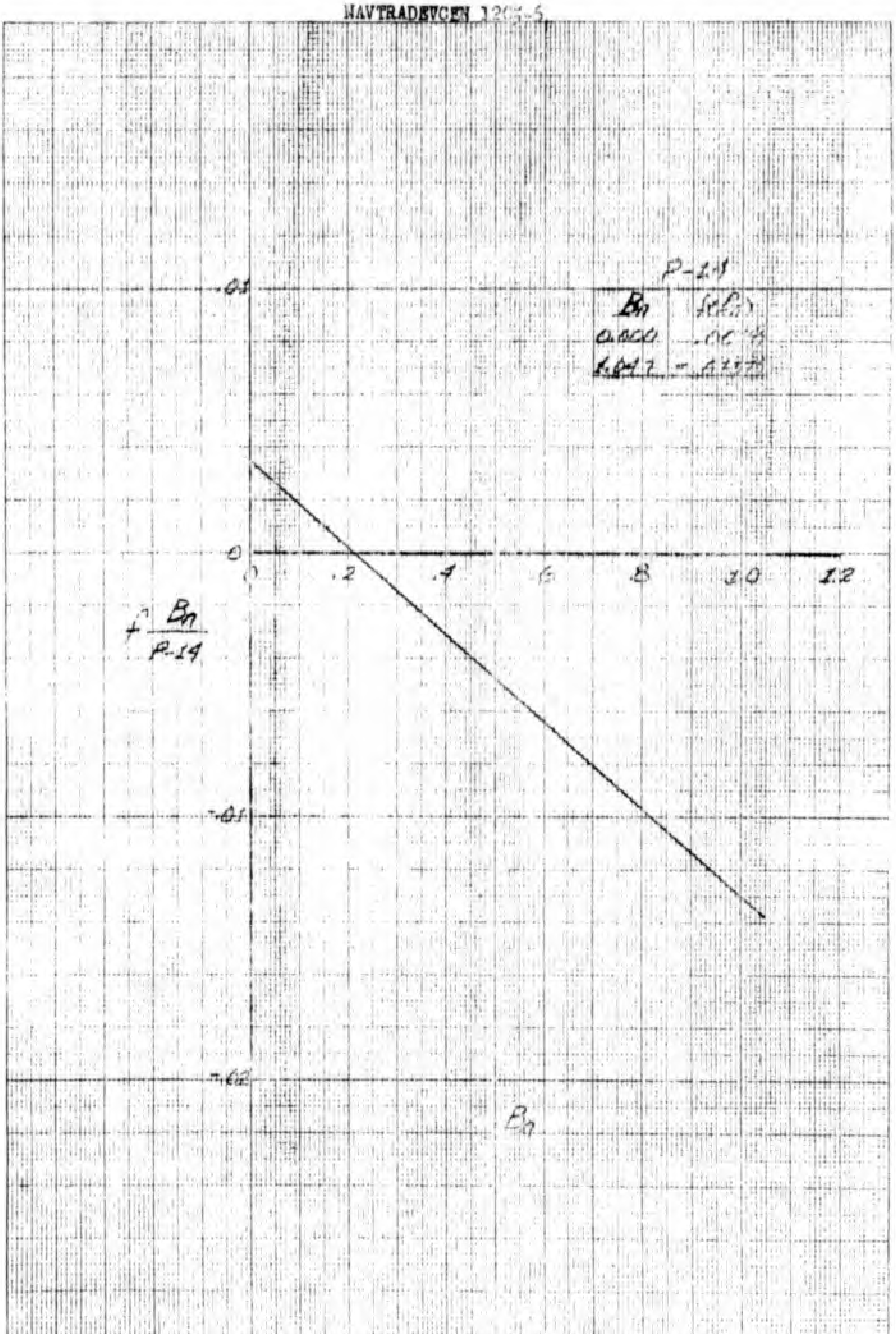
P-10

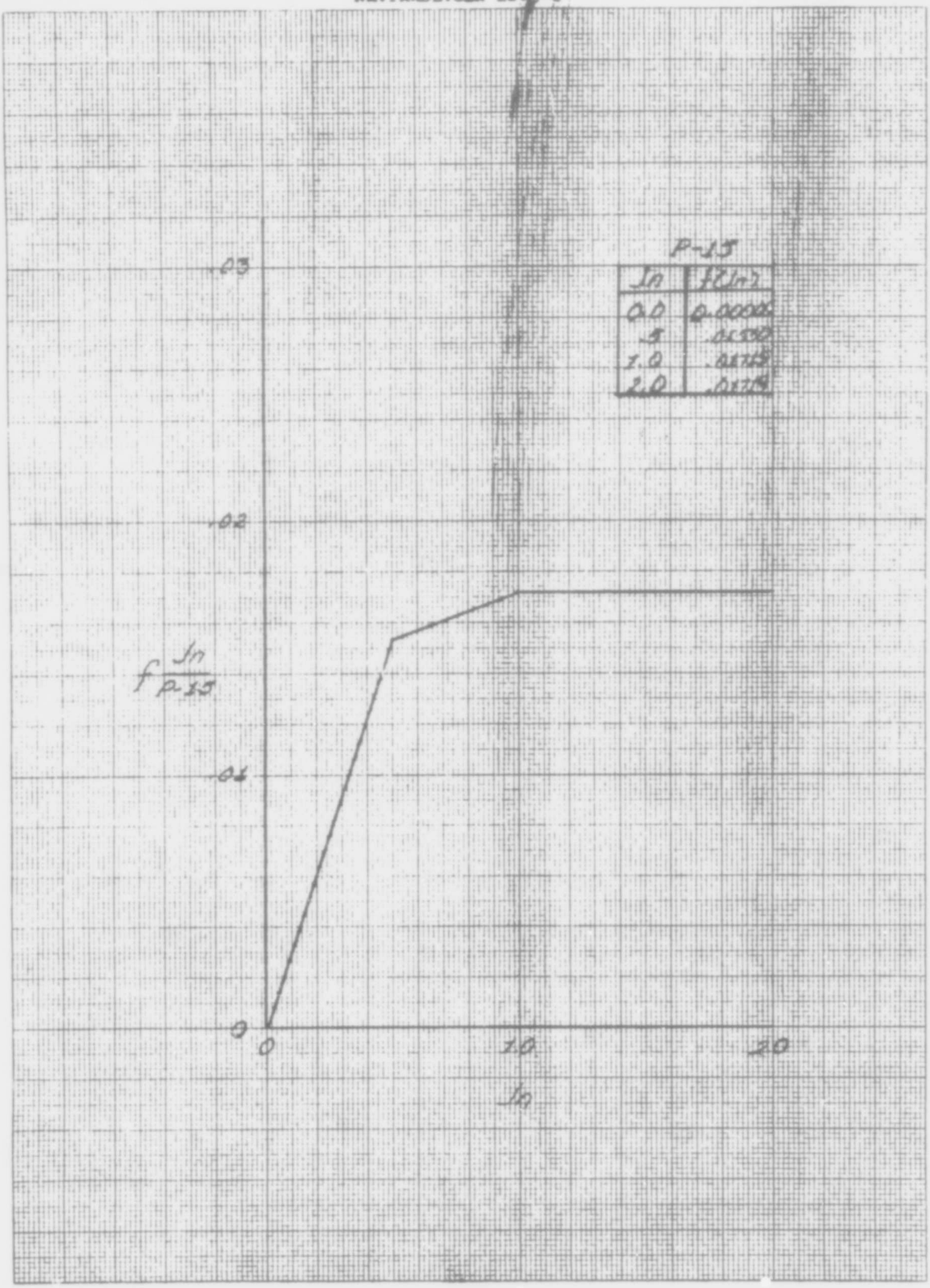
ψ_n	$f(\psi_n)$
1.571	6.150
1.222	5.415
1.047	5.039
.698	3.704
.175	1.000
.000	0.000
-.175	-1.000
-.698	-3.704
-1.047	-5.039
-1.222	-5.415
-1.571	-6.150

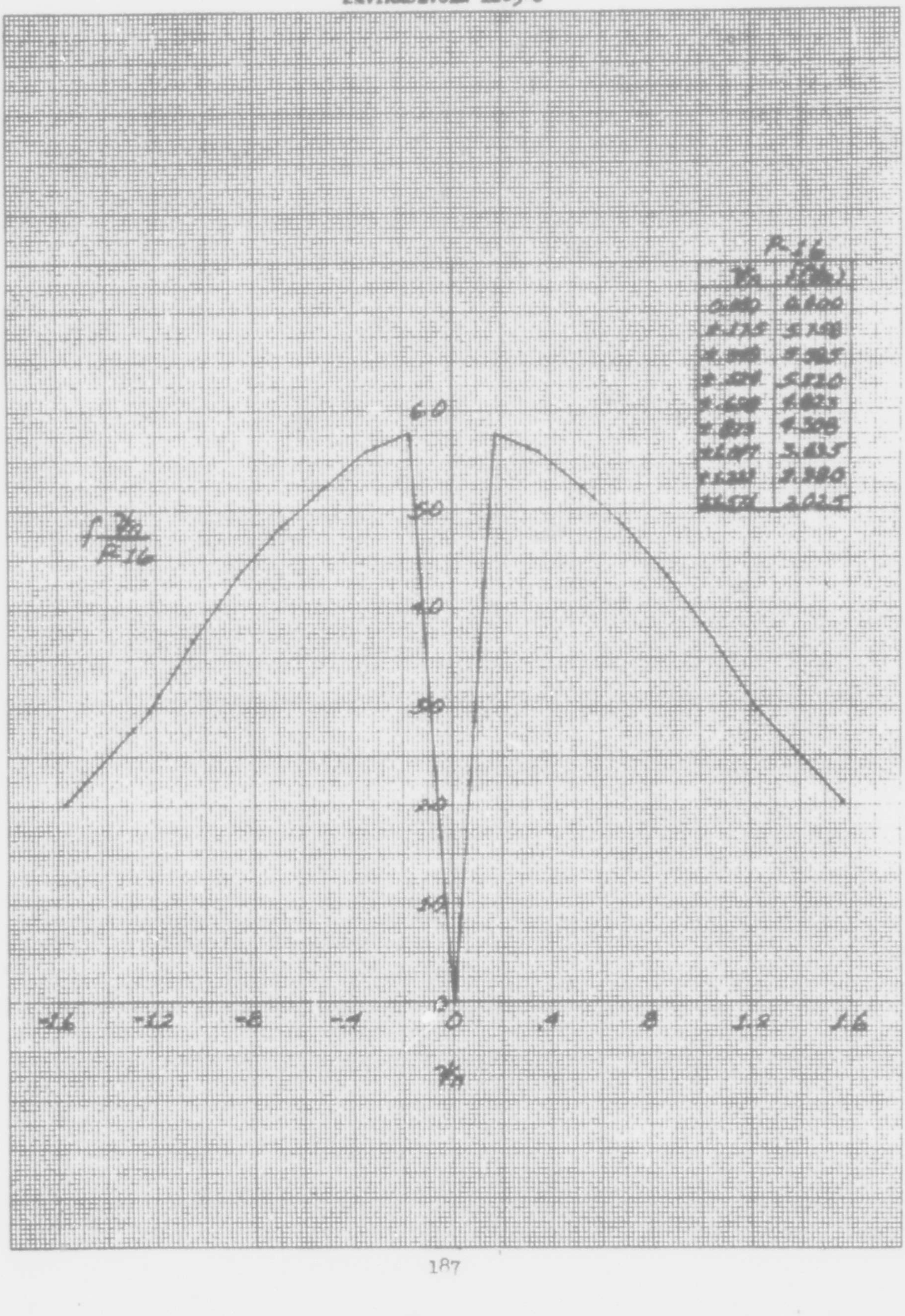

P-11
B_n
$f(B_n)$
0.0000
-.18
.2618
-.05
.5235
.18
.8725
.68
1.1470
.99
1.0
.5
0
-.5
0
.2
.4
.6
.8
1.0
1.2
$f\frac{B_n}{P\text{-}11}$
B_n

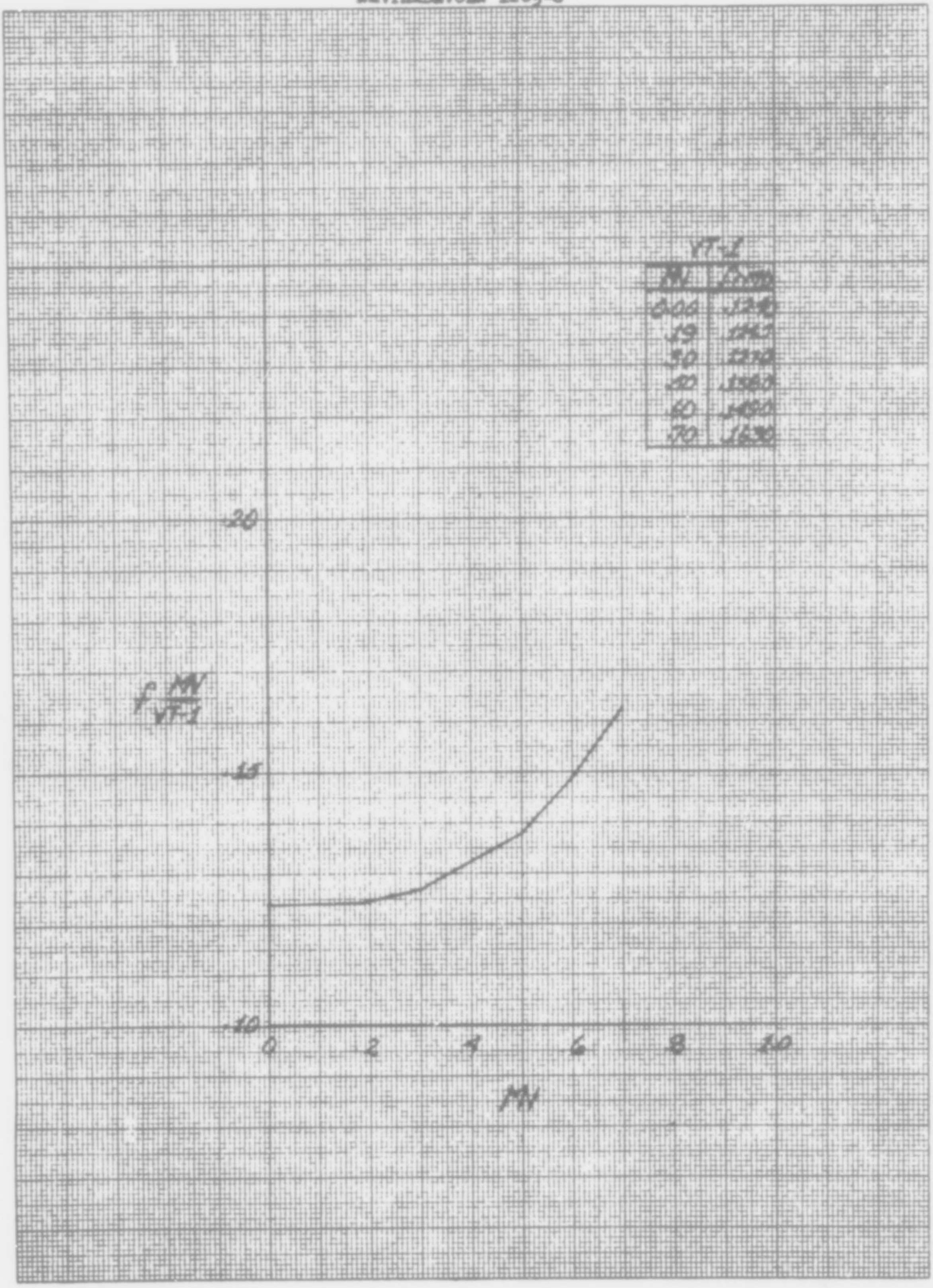

P-12
Bn | f(Bn)
.0000 | 1.50
.2618 | 1.00
.5235 | .60
.8725 | .16
1.0470 | -.10
2.0
1.0
0
-1.0
0
.2
.4
.6
.8
1.0
1.2
f Bn/P12
Bn

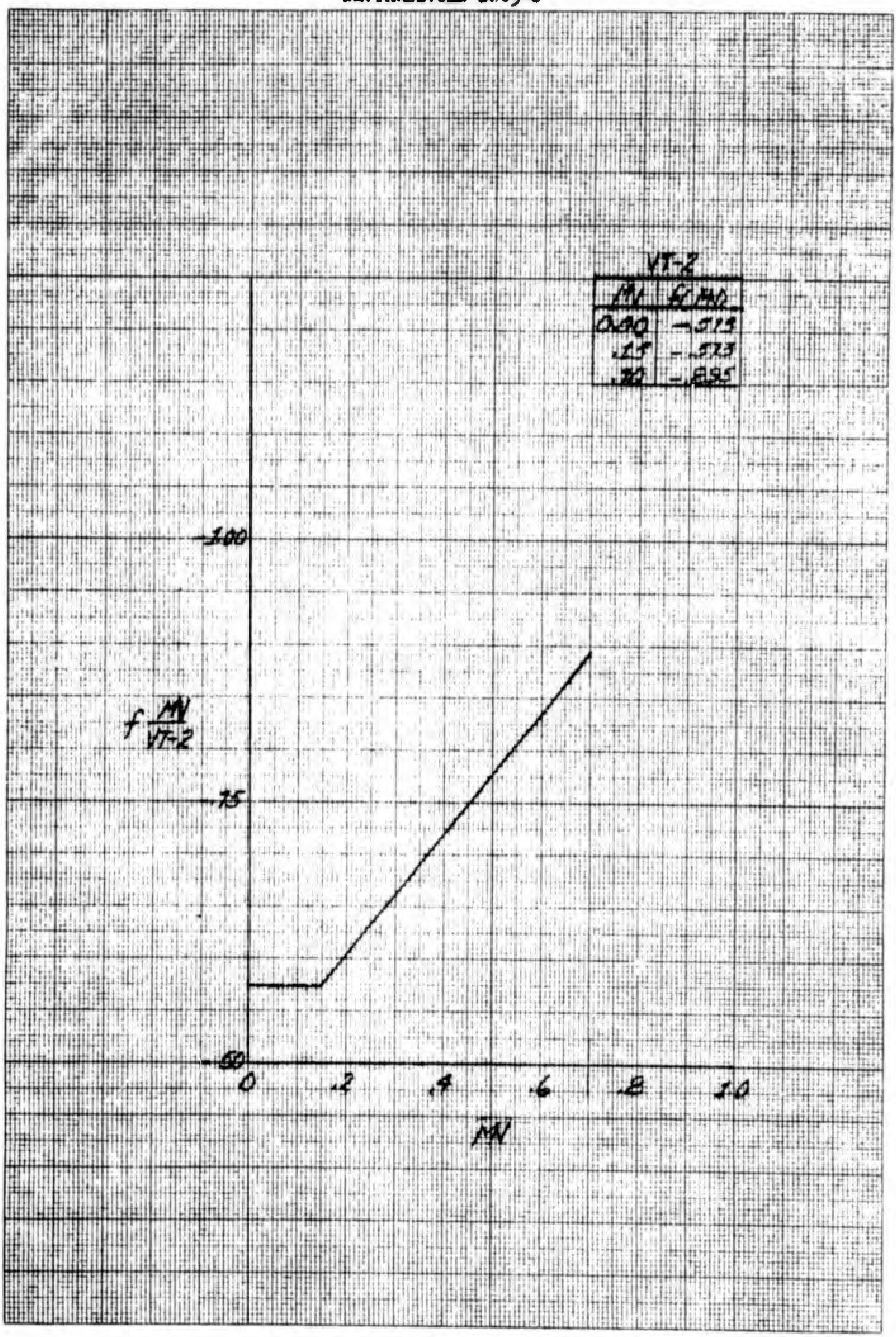

D-13
-1.0 -.125
-.5 -.119
0.0 -.130
.5 -.110
1.0 0.000
1.5 .230
2.0 .630
1.0
.8
.6
.4
.2
0
-.2
-.4
-1.0
0
1.0
2.0
R-13

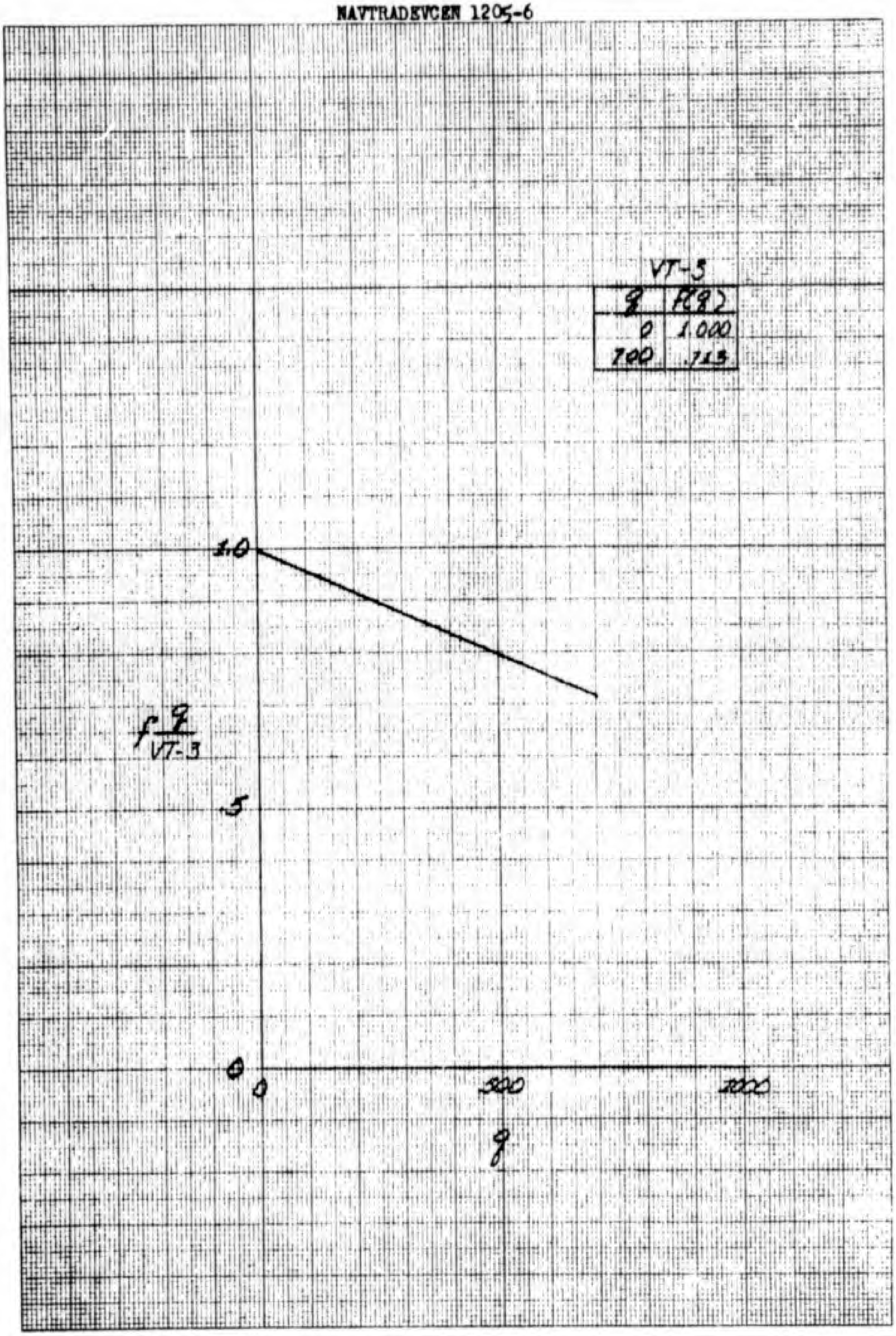

Bn
0.000
1.047
0
.2
.4
.6
.8
1.0
1.2
Bn

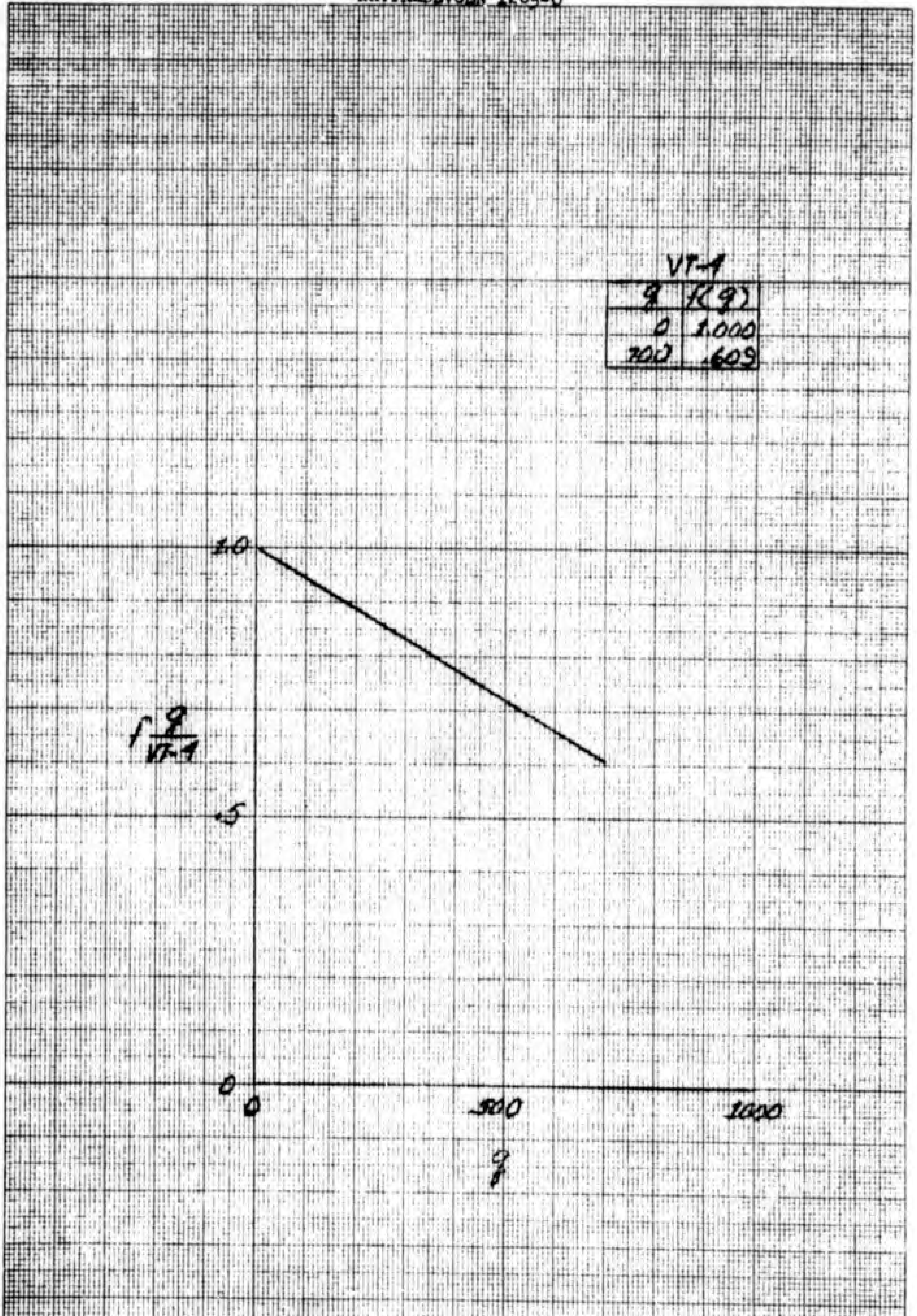

P-15
In | f(In)
0.0 | 0.0000
.5 | .0150
1.0 | .0172
2.0 | .0172
.03
.02
.01
0
f In/P-15
0
1.0
2.0
In

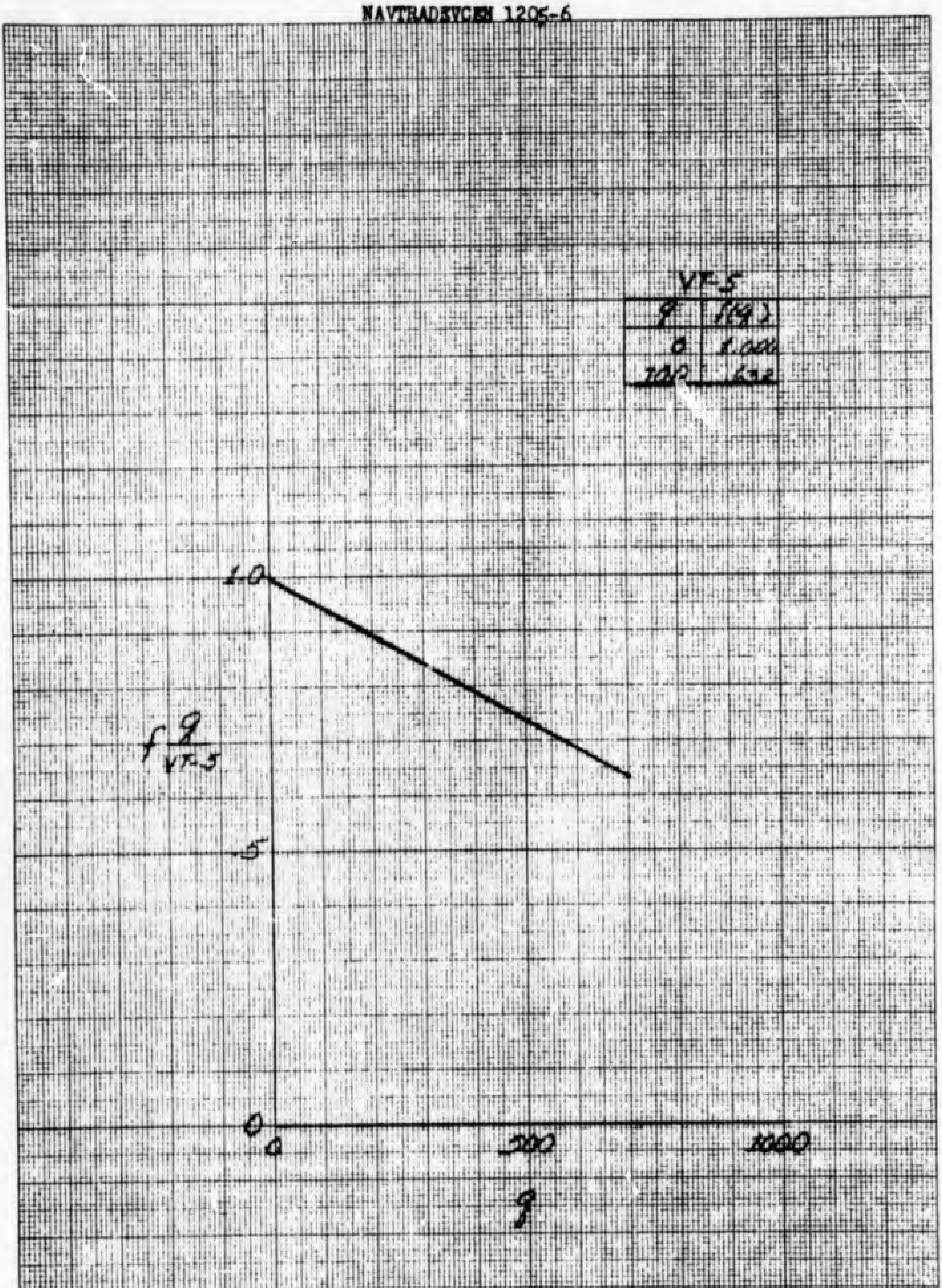

6.0
-1.6
-1.2
-.8
-.4
0
.4
.8
1.2
1.6

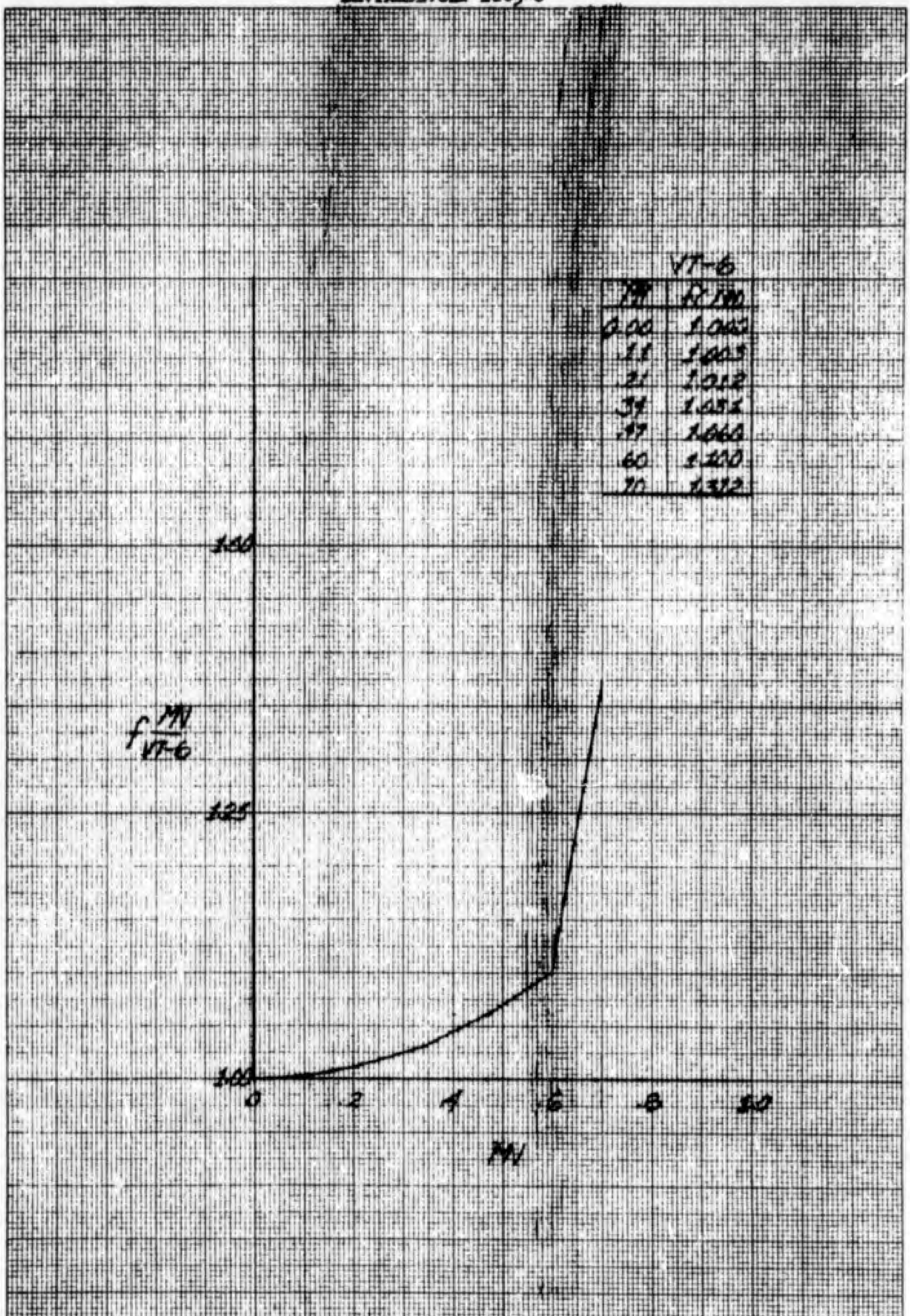

YT-1
MN | f(MN)
0.06 | .1240
.19 | .1245
.30 | .1270
.50 | .1380
.60 | .1490
.70 | .1630
.20
.15
.10
0
.2
.4
.6
.8
1.0
f MN/YT-1
MN


VT-2
MN
0.00 -.573
.15 -.573
.30 -.855
-1.00
-.75
-.50
0
.2
.4
.6
.8
1.0
f MN/VT-2
MN


VT-3
q
f(q)
0
1.000
700
.723
1.0
.5
0
$f\frac{q}{VT-3}$
0
500
1000
q

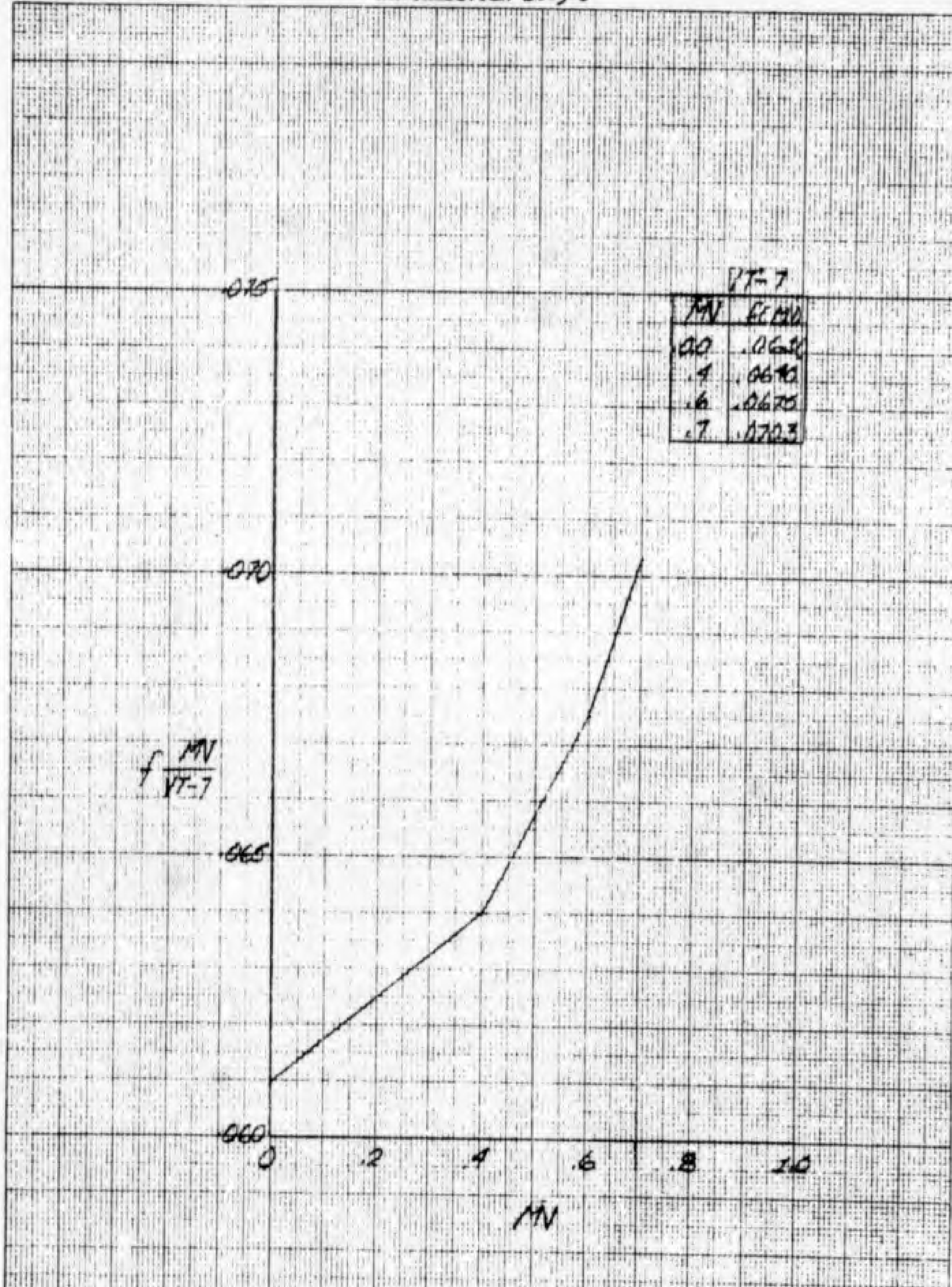

VT-4
q | f(q)
0 | 1.000
700 | .609
1.0
.5
0
0
500
1000
q
f q/VT-4

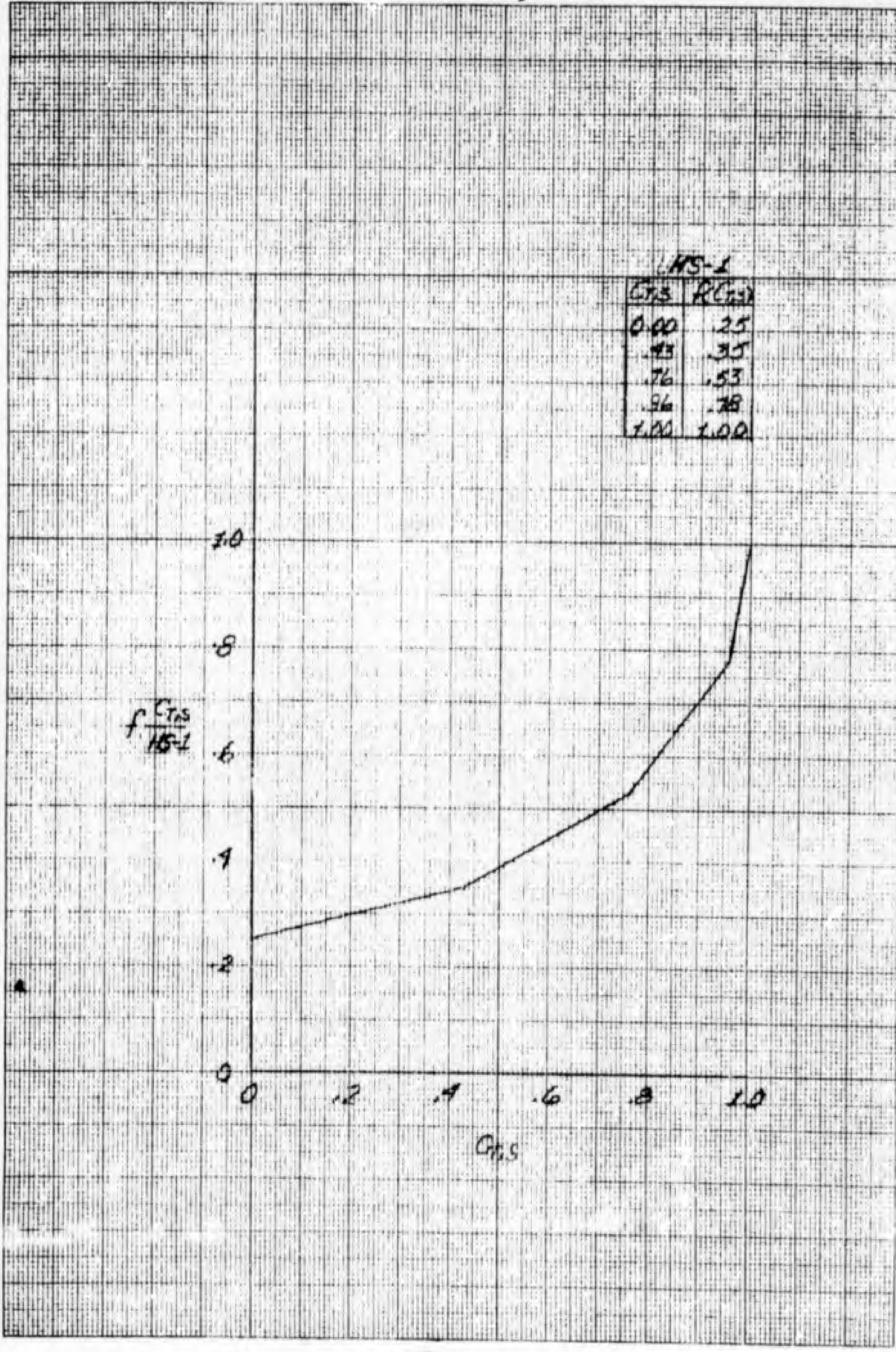

VF-5
q | f(q)
0 | 1.000
700 | .632
1.0
.5
0
0
500
1000
q

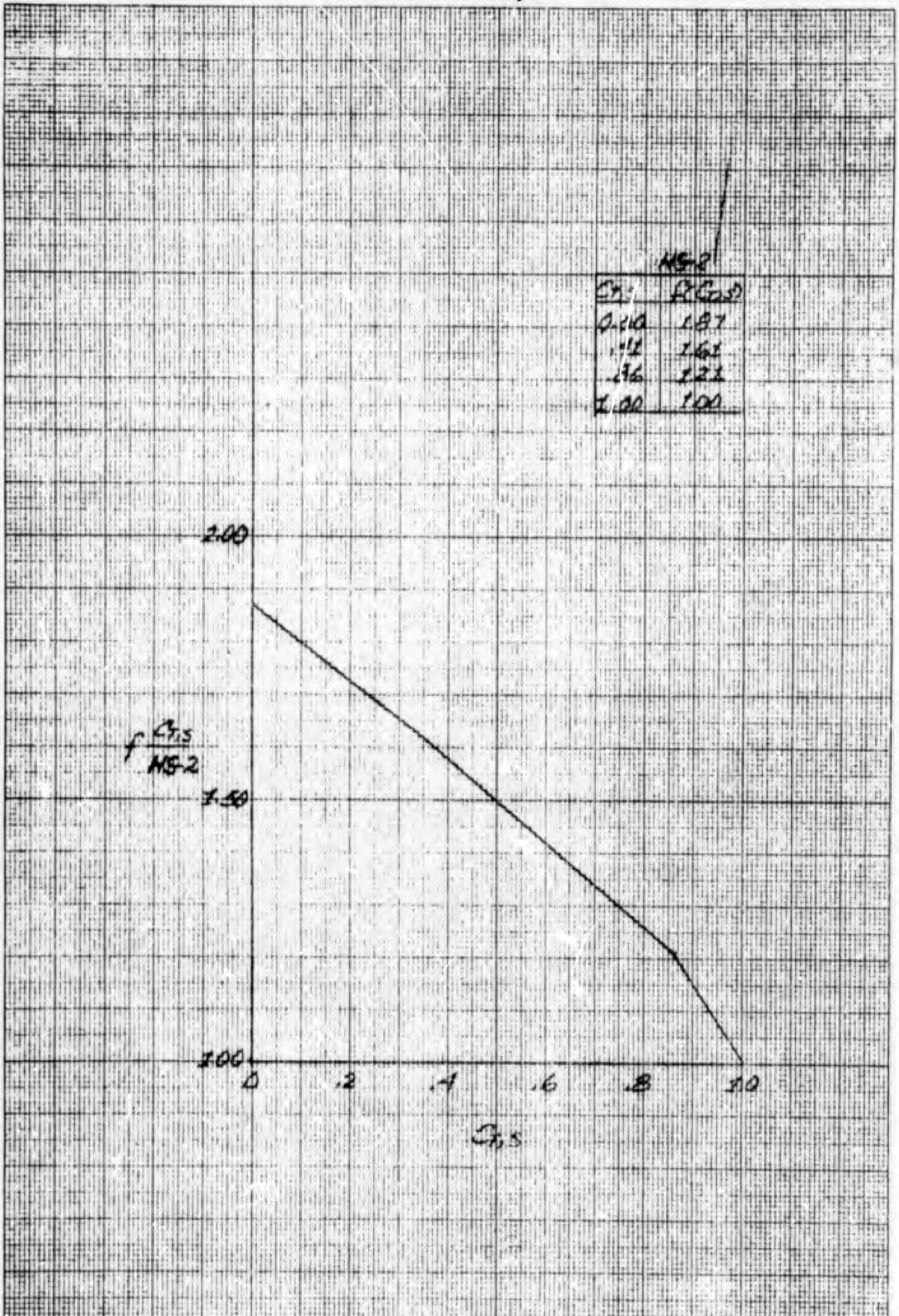


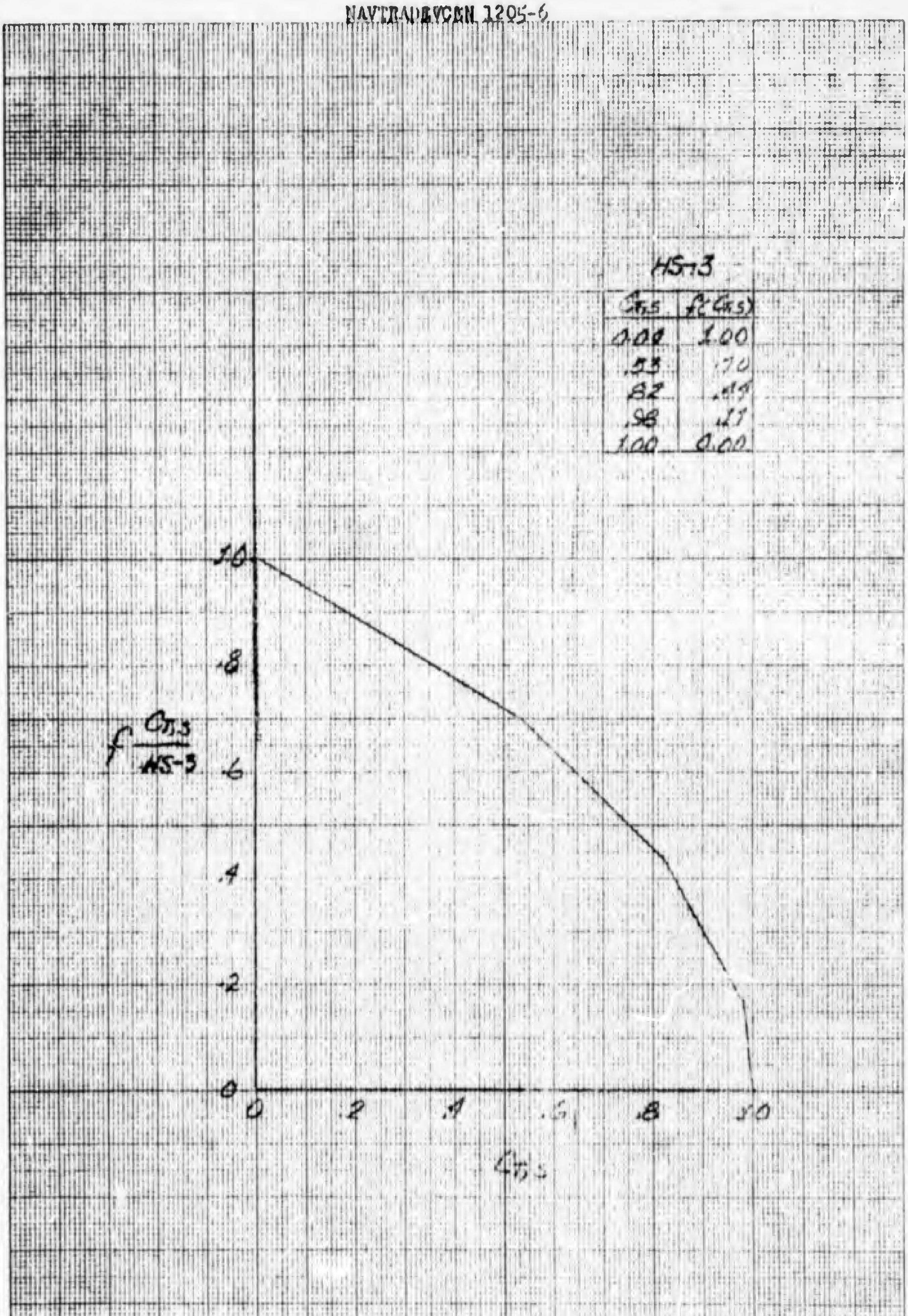
VT-6

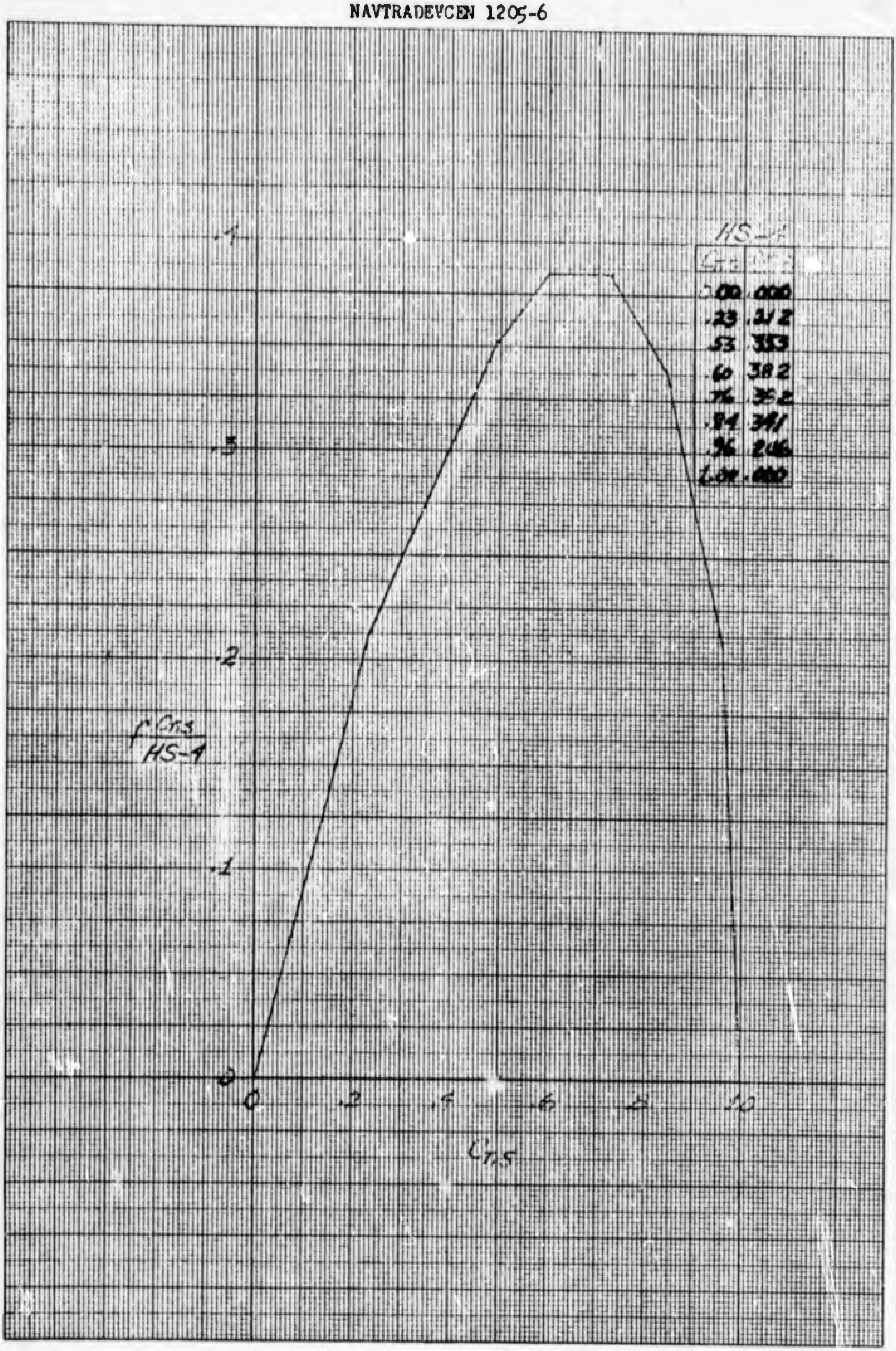
M	f(M)
0.00	1.002
.11	1.005
.21	1.012
.34	1.031
.47	1.060
.60	1.100
.70	1.372

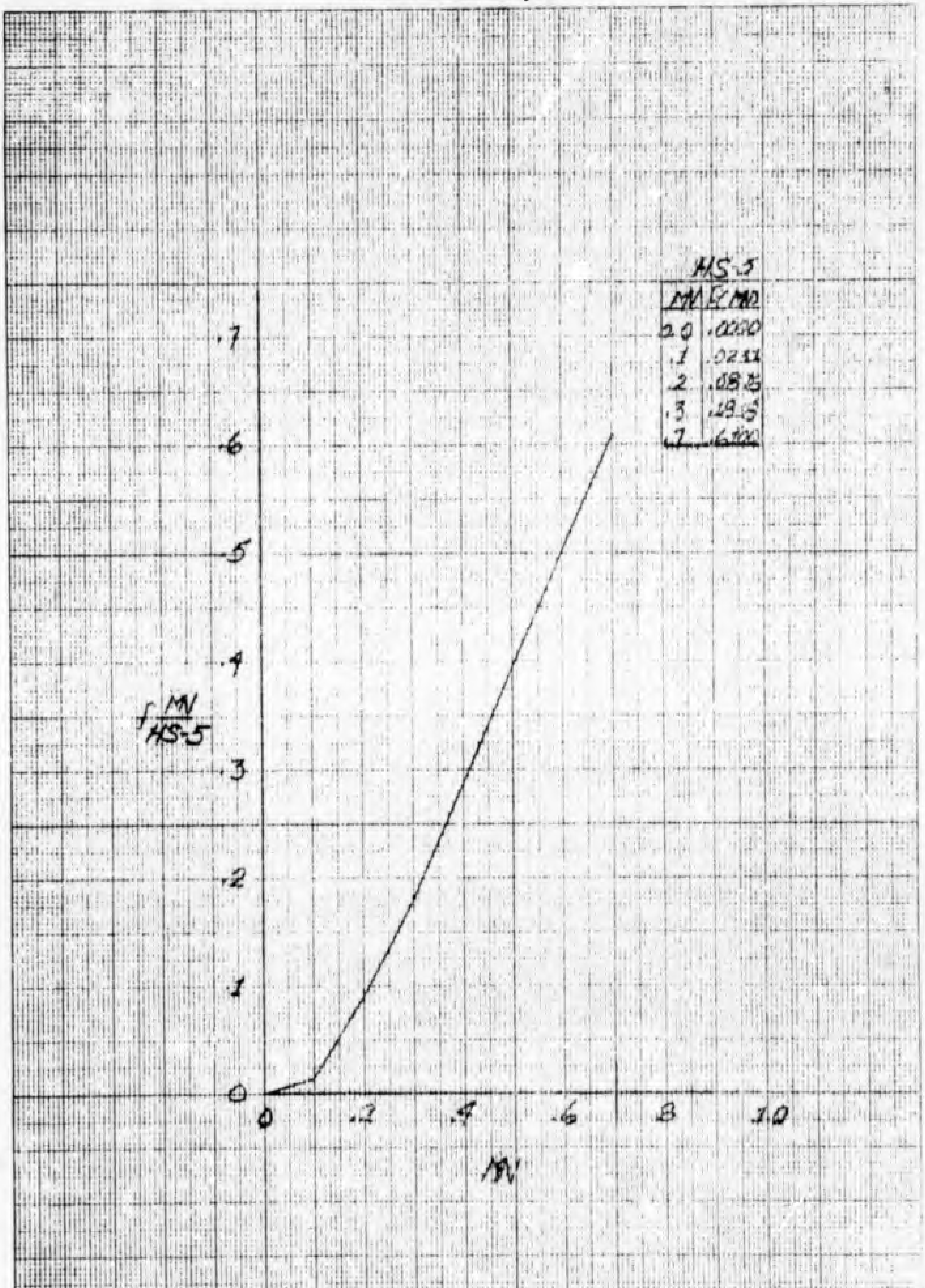

.075
.070
.065
.060
$f\frac{MN}{VT-7}$
0
.2
.4
.6
.8
1.0
MN
VT-7
MN
Ec MN
0.0
.0620
.4
.0640
.6
.0675
.7
.0703

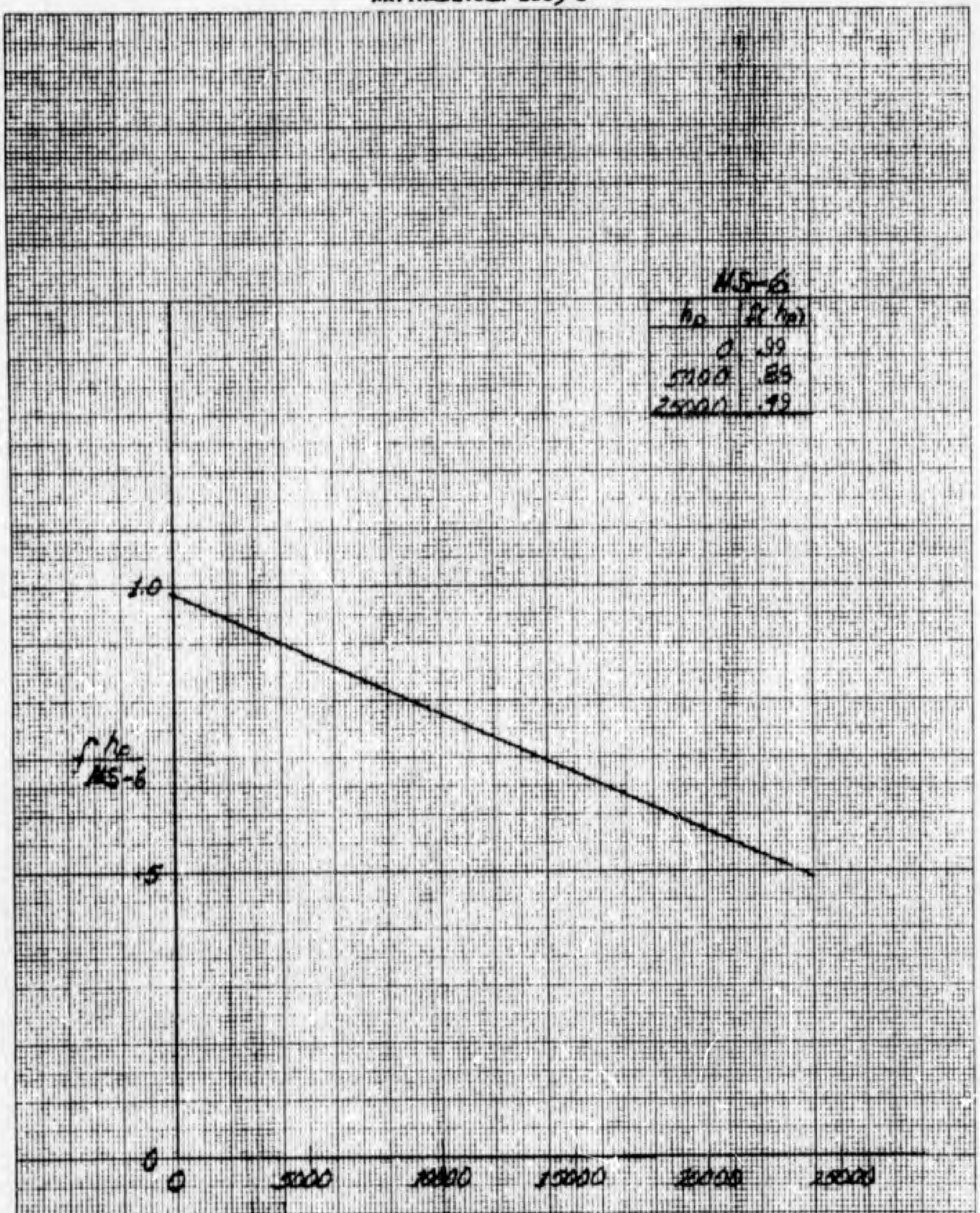

HS-1
C_{TS} f(C_{TS})
0.00 .25
.43 .35
.76 .53
.96 .78
1.00 1.00
1.0
.8
.6
.4
.2
0
f $\frac{C_{TS}}{HS-1}$
0
.2
.4
.6
.8
1.0
C_{TS}

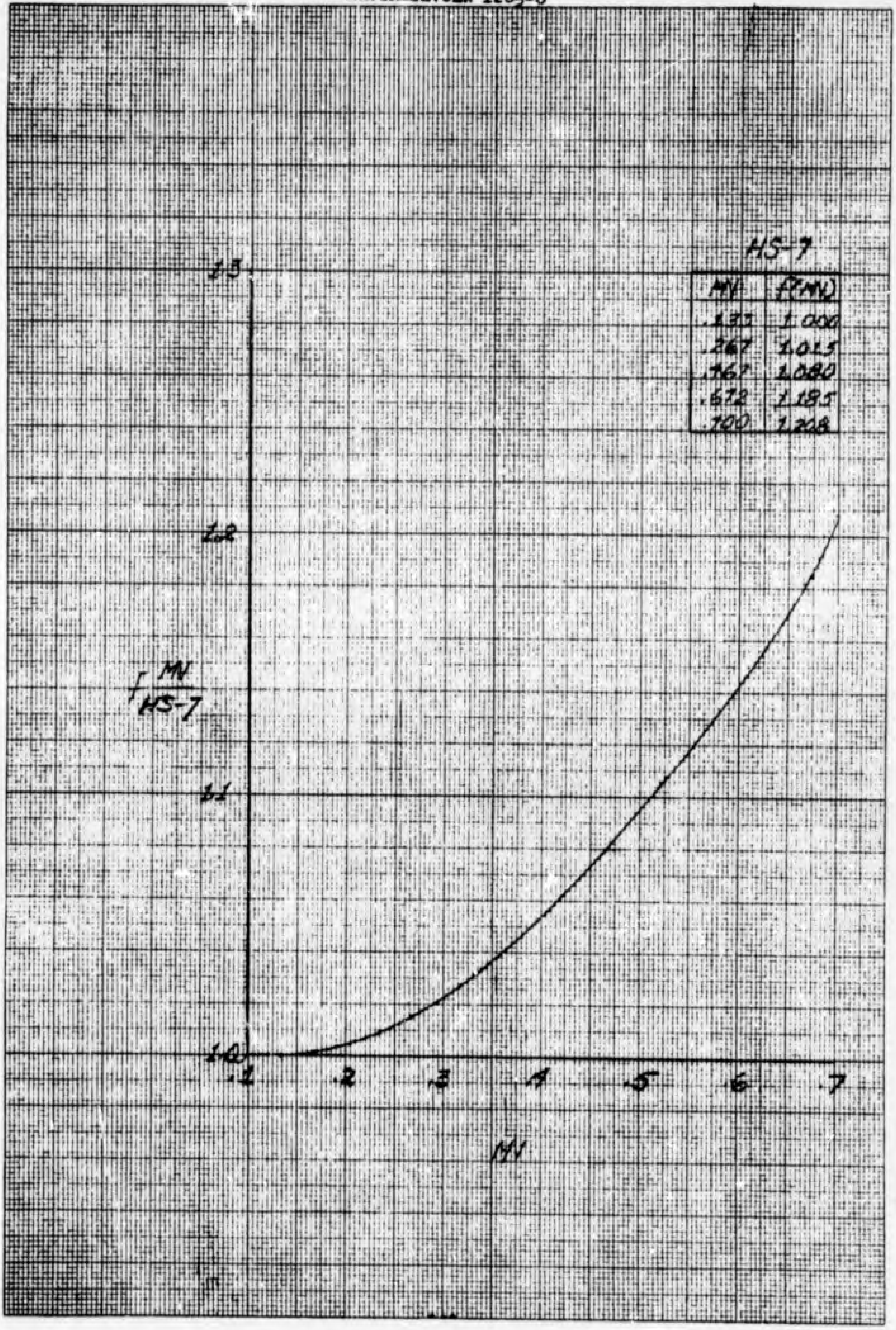

HS-2
$C_{T,S}$
$f(C_{T,S})$
0.10 1.87
.41 1.61
.86 1.21
1.00 1.00
2.00
1.50
1.00
0
.2
.4
.6
.8
1.0
$f\frac{C_{T,S}}{HS\text{-}2}$
$C_{T,S}$

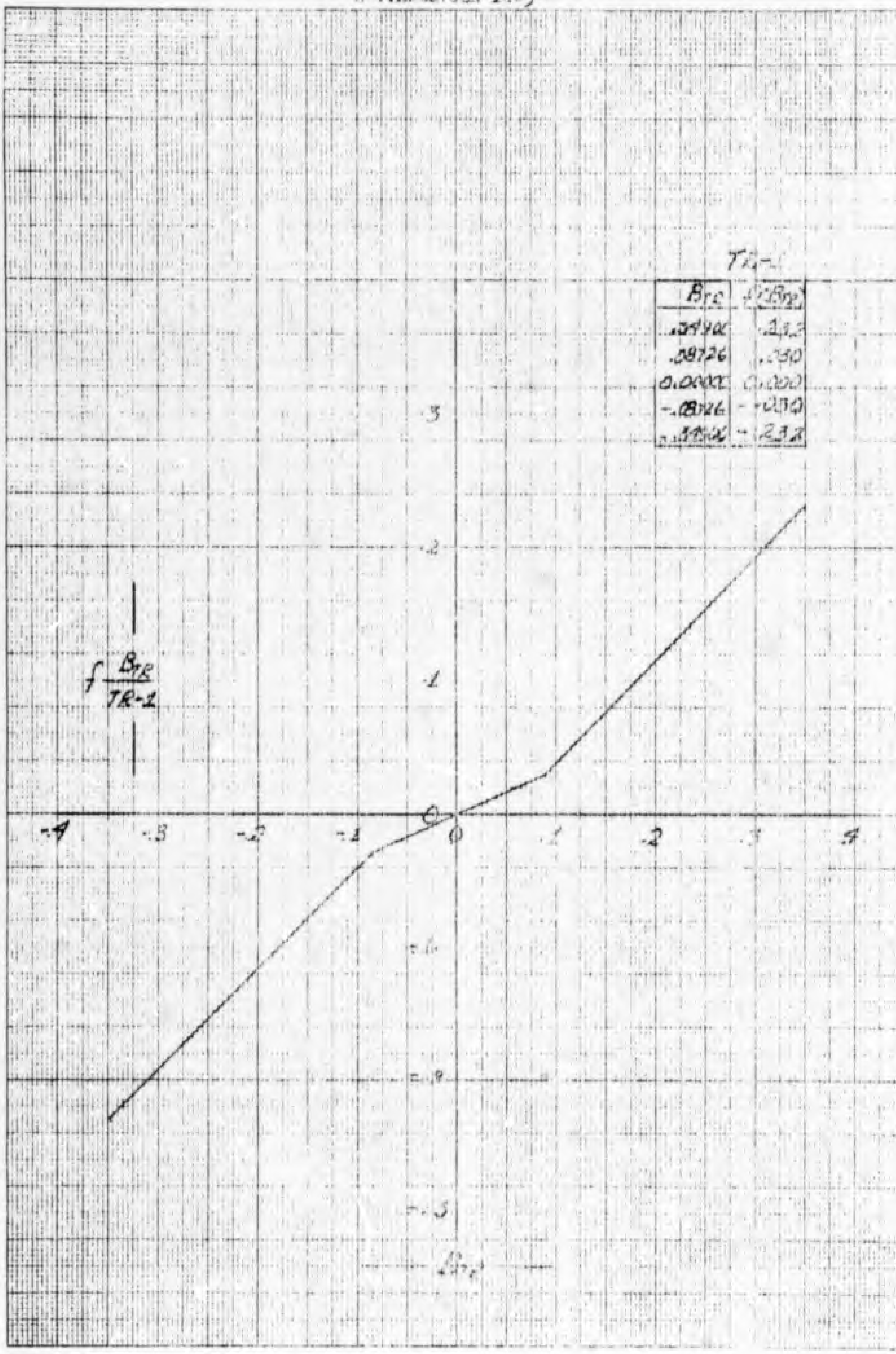

HS-3
$C_{T,S}$
$f(C_{T,S})$
0.00
1.00
.55
.70
.82
.45
.96
.17
1.00
0.00
$f\frac{C_{T,S}}{HS-3}$
1.0
.8
.6
.4
.2
0
.2
.4
.6
.8
1.0
$C_{T,S}$

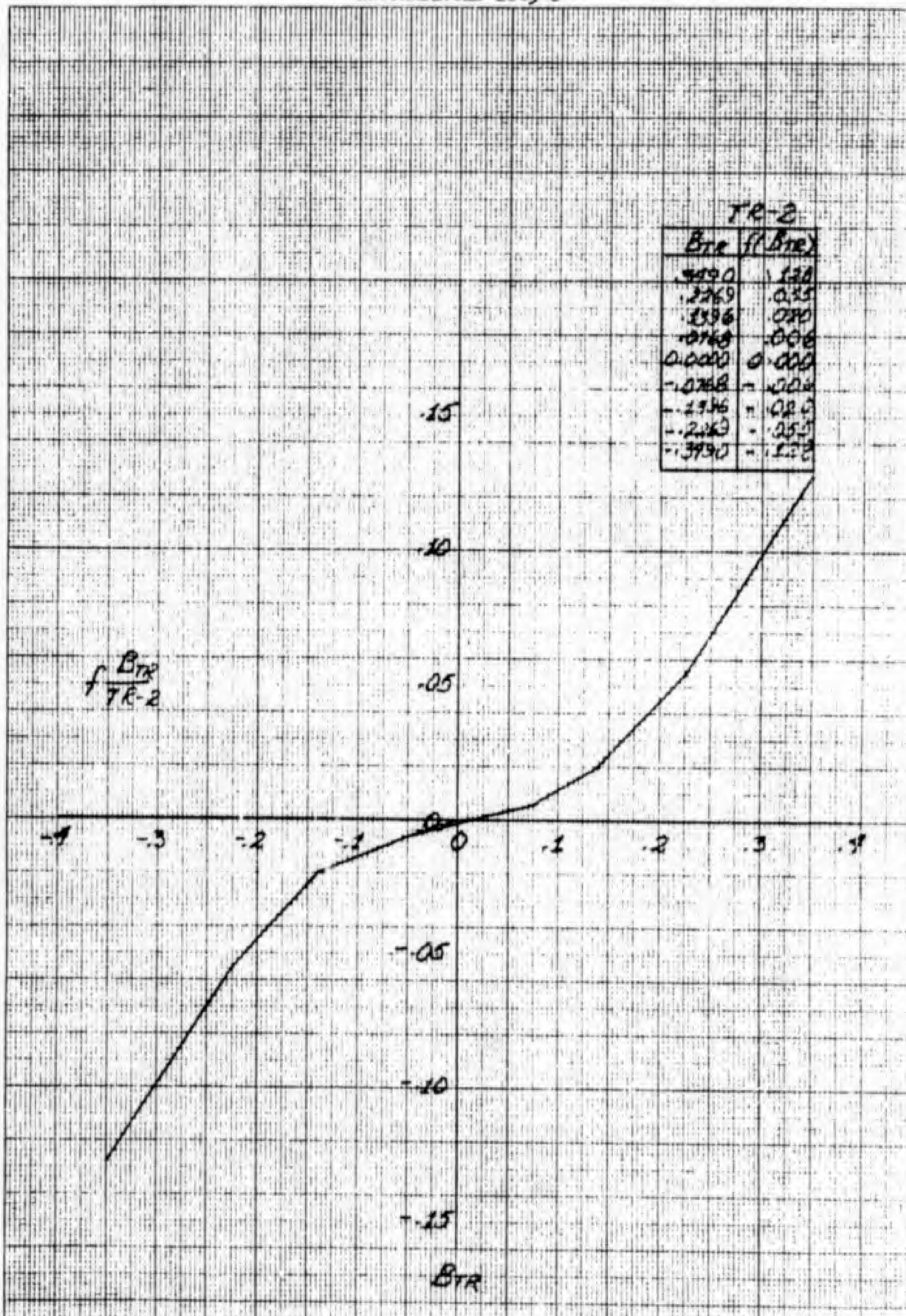

HS-4
0.00 .000
.23 .212
.53 .353
.60 .382
.76 .352
.84 .341
.96 .206
1.04 .000
.4
.3
.2
.1
0
f Cts / HS-4
0
.2
.4
.6
.8
1.0
$C_{T,S}$

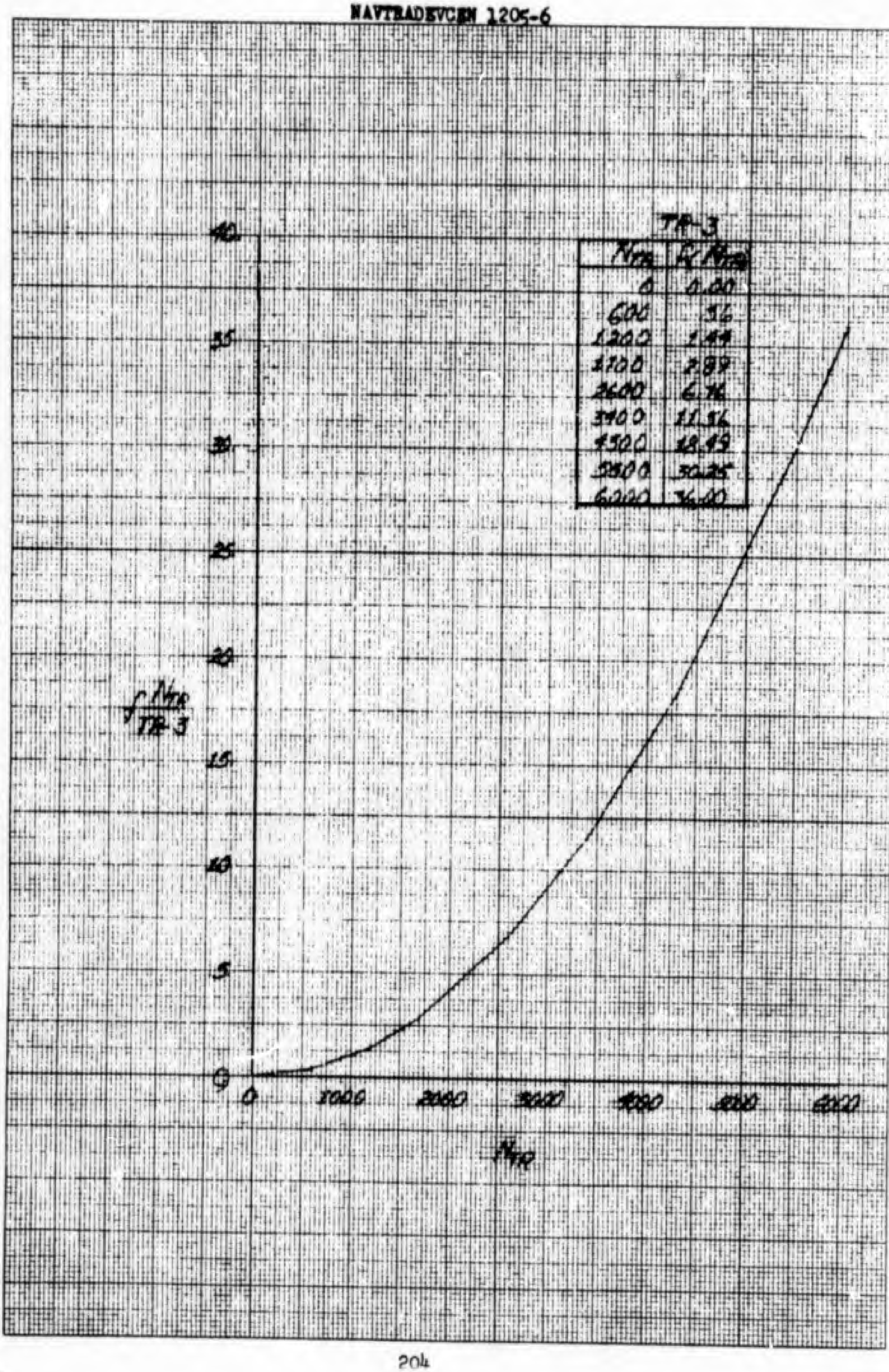

HS-5
MN
0.0 .0000
.1
.2
.3
.7
.7
.6
.5
.4
.3
.2
.1
0
$f\frac{MN}{HS-5}$
0
.2
.4
.6
.8
1.0
MN


M5-6
.99
.88
25000
.49
1.0
.5
0
0
5000
10000
15000
20000
25000
M5-6


HS-7
MN | f(MN)
.133 | 1.000
.267 | 1.015
.467 | 1.080
.672 | 1.185
.700 | 1.208
1.3
1.2
1.1
1.0
f MN/HS-7
.1
.2
.3
.4
.5
.6
.7
MN


TR-1
B_{TR} | $f(B_{TR})$
.3490 | .232
.08726 | .050
0.0000 | 0.000
-.08726 | -.050
-.3490 | -.232
$f\frac{B_{TR}}{TR\text{-}1}$
B_{TR}
-.4 -.3 -.2 -.1 0 .1 .2 .3 .4
.3 .2 .1 -.1 -.2 -.3


TR-2
B_TR f(B_TR)
.2269
.1396
0.0000 0 .000
-.0768
-.3490
.15
.10
.05
-.05
-.10
-.15
-.4
-.3
-.2
-.1
0
.1
.2
.3
.4
f B_TR / TR-2
B_TR



TR-3

N_{TR}	$f\,N_{TR}$
0	0.00
600	.36
1200	1.44
1700	2.89
2600	6.76
3400	11.56
4300	18.49
5500	30.25
6000	36.00

APPENDIX D

COMPUTER PROGRAMS

The following two SDS-920 digital computer programs show how the polynomial expressions presented by L-T-V were converted to a form amenable to analog simulation.

```
*       FUNCTION GENERATOR FOR XC-142A
C       FUNCTION OF CTS

        DO 500 CTS=0.,1.0,.1
        DELT=.04*CTS+.08*CTS**2+.12*CTS**3
        DELN=-.06*CTS**2-.14*CTS**3

        TYPE 501,CTS,DELT,DELN

   501  FORMAT(10X,F5.1,2F20.5)
        CONTINUE

C       FUNCTION OF DELTA-F

        DO 600 DELF=0.,1.5,.1
        WCLDF=.444*DELF+4.102*DELF**2-2.703*DELF**3

        TYPE 601,DELF,WCLDF
   601  FORMAT(10X,F5.1,F15.4)
        CONTINUE

        STOP
*       END
```

OUTPUTS

DELT	DELF
DELN	WCLDF

FORTRAN NAME	VARIABLE
CTS	$C_{T,S}$
DELF	$C_{L_{\delta F}}$
DELN	$\dfrac{[\Delta C_n]\ \Delta T}{C_L{}''\ \dfrac{\Delta T}{\Sigma T}}$
DELT	$\dfrac{[\Delta C_\ell]\ \Delta T}{C_L{}''\ \dfrac{\Delta T}{\Sigma T}}$
WCLDF	$C_{L_{\delta F}}^{**}$

```
*       PROPELLER COEFFICIENTS (CN AND CY)
C
        TYPE 15
   15   FORMAT($CYN1-CYN FOR N=1,2        CYN2-CYN FOR N=3,4$)
C
        DO 50 JRATIO=0.0,2.0,0.5

        TYPE 5, JRATIO
    5   FORMAT($  J=$,F4.1)

        DO 50 BETA=0.0,60.0,5.0

        TYPE 10, BETA
   10   FORMAT($  BETA=$,F5.1)

        DO 50 PSI=0.0,70.0,10.0
C
        BETAR=BETA*.01745
        PSIR=PSI*.01745
        JPRIME=JRATIO*COSF(PSIR)
        TEMP1=JRATIO*BETAR*SINF(PSIR)
C
        CNN=(.0534+.1028*JPRIME)*TEMP1
        CYN1=(.1051-.0564*BETAR)*TEMP1
        CYN2=(-.1051+.0564*BETAR)*TEMP1

        TYPE 20, PSI,CNN,CYN1,CYN2
   20   FORMAT(F10.1,3F15.6)

   50   CONTINUE

        STOP
*       END

        OUTPUTS
          CYN1       JPRIME      PSI
          CYN2       PSIR        CNN
```

SUBPROGRAMS REQUIRED

COSF SINF

THE END

FORTRAN NAME	VARIABLE	FORTRAN NAME	VARIABLE
BETA	$\beta_{(deg)}$	CYN2	$C_{Y_n}^*$ for n = 3,4
BETAR	$\beta_{(rad)}$	JRATIO	J_n
CNN	$C_{N_n}^*$	JPRIME	J'_n
CYN1	$C_{Y_n}^*$ for n = 1,2	PSI	ψ_n (deg)
		PSIR	ψ_n (rad.)

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13. ABSTRACT

This report presents the analysis and simplification procedures that are required to define and program the mathematical model for the XC-142A aircraft in a form which is suitable for mechanization and solution on a general purpose analog computer. This program will enable the Naval Training Device Center to perform dynamic simulation studies for a V/Stol tilt-wing aircraft.

Section II contains the complete mathematical model of the XC-142 with accompanying denotation and validation.

In Section III, three sets of simulation equations are presented. These sets represent the complete six degrees of freedom equations, longitudinal mode equations, and lateral-directional mode equations.

Section IV contains the mechanization functional block diagrams along with the patching and operating instructions required for their utilization. Section IV also specifies the analog computer installation which is required to solve the mechanizations.

The subsequent sections contain: a discussion of program limitations, conclusions, and recommendations.

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VTOL ANALOG PROGRAM						

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(PAGE 2)

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