AIR FORCE INSTITUTE OF TECHNOLOGY

AIR UNIVERSITY
UNITED STATES AIR FORCE

SCHOOL OF ENGINEERING

WRIGHT–PATTERSON AIR FORCE BASE, OHIO

A METHOD FOR DETERMINING THE EFFECT OF DYNAMIC ERRORS OF A RADAR SATELLITE TRACKER ON ORBIT PREDICTION

THESIS

GA/EE/64-6

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A METHOD FOR DETERMINING THE EFFECT OF DYNAMIC ERRORS OF A RADAR SATELLITE TRACKER ON ORBIT PREDICTION

THESIS

Presented to the Faculty of the School of Engineering

The Institute of Technology

Air University

In Partial Fulfillment of the

Requirements for the

Master of Science Degree

in Astronautics

by

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GA 64

26 August 1964

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Foreword

This paper was generated by my desire to expend my thesis-writing effort somehow in the field of orbital mechanics and in learning something about the application of automated digital computation. This paper has served as a perfect vehicle for both of these ambitions. There have been many approaches taken to the subject on which the paper is written, but none of them seemed to use the particular tack taken herein. Evaluation of the practicality of the approach remains for some future investigator to establish by applying it to an actual satellite tracking system.

The purpose of the paper is to investigate the errors caused in orbit prediction from radar data by the system dynamic response. I recognize that determining error is one thing; correcting or compensating for it is another. Time forced me to limit myself in the preparation of this paper to the error investigation, only admitting of a possibility of a compensating scheme at the very end. This is perhaps another avenue of investigation which could lead out of this effort.

A list of terms with their definitions as used in this paper is included as Appendix F. Since many terms have more than one interpretation, my intent in preparing this glossary was to insure that the appropriate one is inferred. In general, terms are defined in the text; for convenience, however, these definitions are repeated in Appendix F.

My list of persons to whom I owe thanks for assistance in the preparation of the paper is not lengthy. At the head of the list is Major James E. McCormick, who originally proposed the problem. Although transferred to California on a permanent change of station, he nevertheless continued to be interested in the progress of the investigation. He outlined the approach to be taken in the solution of the problem in a lengthy letter several months after leaving. In addition, I am grateful for the opportunity to use the computer facility of the Systems Dynamic Analysis Division of the Foreign Technology Division, Air Force Systems Command. The personnel of the open shop in that division were extremely helpful in helping this neophyte programmer overcome the usual problems encountered in the preparation of a computer program of the scope of RADYN. I also wish to thank First Lieutenant Bryant D. Elrod, my faculty thesis advisor. His expert counselling in the area of servomechanisms made the completion of the project possible. Finally, I thank my wife, Clara Kirkwood McMullen, for her able assistance in many different ways during the preparation of this paper.

Thomas H. McMullen

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Abstract

This thesis briefly discusses the difficulty of specifying the accuracy of a satellite tracker. Equations are developed which: (1) determine azimuth and elevation angle data and fit them to fourth order time polynomials; (2) compute the orbit parameters from evaluations of these polynomials; and (3) evaluate the response of a selected tracker transfer function to the input of these time polynomials by the method of Laplace transformation. Orbit elements computed from this last evaluation are compared with the previous set to obtain the error in orbit prediction. A Fortran II program is presented which performs these computations on the IBM 7094 digital computer.

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A METHOD FOR DETERMINING THE EFFECT OF DYNAMIC ERRORS OF A RADAR SATELLITE TRACKER ON ORBIT PREDICTION

I. Introduction

The analysis of the accuracy of radar satellite trackers is an extremely broad field. To genuinely consider system accuracy one must study the validity of the data generated while the system is actually performing its task of satellite following. Under these conditions, it is very difficult to define a specification for accuracy since it varies according to the tracking environment. Accuracy is a function of such variables as target size and range, power transmitted, local target motion, such as tumbling, target position relative to the tracker, and existing atmospheric conditions, to name a few. In addition, accuracy will be different depending on whether the determination is being made on a single data point or on a set of "smoothed" data points (Ref 7:1).

The errors which cause the loss of accuracy may be divided in several different manners. One such method is to classify them as propagation errors, tracking independent errors, and tracking dependent errors. The first group is caused by the passage of the electromagnetic radiation through the various layers of the earth atmosphere. Variations in atmospheric properties make precise determination of these errors difficult. The second group, tracking independent errors, is caused principally by installation and alignment errors, wind, gravity and receiver noise. The last group, tracking dependent errors, consist of dynamic lag, glint, scintillation and compliance (Ref 7:3). This paper is concerned with a portion of this latter group; specifically, the purpose of this paper is to demonstrate the effect of dynamic errors of a radar satellite tracker in determining the orbital parameters. A computer program is herein developed whereby the dynamic errors may be determined. First, however, it is necessary to give a cursory description of the satellite tracker itself.

II. The Radar System

The simplified form of satellite tracker to be considered in this paper consists of a system capable of making range and angular measurements of the target so as to define target position with respect to the radar antenna. The range measurement depends upon measuring the time of travel of a pulse of electromagnetic energy from the tracker to the target and return. Although this measurement requires accurate timing, little mechanical motion is involved and therefore it need not be included in a study of dynamic errors. For this reason, range error will not be considered here. In subsequent calculations to investigate the effect of dynamic errors, the value of range will be considered correct.

The angle measurements consist of the azimuth and elevation angles of the satellite with respect to the tracker location. Azimuth is the angle measured from true north to the satellite projection upon the local horizontal plane. The positive direction of azimuth has been arbitrarily chosen as counter clockwise. The elevation is the angular measurement of the satellite above the horizontal plane.

The angular measurements are determined by noting the antenna position when the target is on the center line of the tracker radar beam. As is briefly discussed below, the tracker attempts to maintain the antenna position such that it is continuously directed at the target. The tracker is capable of detecting when this alignment is not achieved and in so doing generate error signals of proper sign and magnitude which are used as a control in reducing the angular error to zero. The system under consideration is a linear one so that elevation and azimuth angles of the tracker are changed by separate and independent electric servo drive systems; hence either elevation or azimuth may be altered without the one affecting the other.

The angle measurement itself depends upon the use of a highly directional antenna. Such an antenna transmits a very narrow beam and receives so as to respond most strongly to waves from one single direction. The elevation and azimuth of the tracked body is determined by the antenna orientation when it is positioned for maximum radar echo (Ref 1:7). Because of the diffraction of the electromagnetic energy transmitted by the radar system, a large antenna and short wavelength is required to produce a narrow beam (Ref 1:9). The narrowness of the beam determines the accuracy with which the radar can measure the position angles and the system angular resolution. Since the radar beam is of a conical form, the range involved in tracking earth satellites dictates a very narrow beam indeed. The width of the beam is usually specified by the beam angle between half-power points. Therefore the cross-sectional width of the beam at the target is the real determining factor of accuracy and resolution; this width varies directly with the product of range and beam width angle. As range increases, then, it is necessary to decrease the beam angle (Ref 1:10). This is achieved by both increasing the frequency of the signal used and the diameter of the antenna. This latter materially affects the dynamic response of the tracking system.

Conical scanning is often used to provide error signals for automatic tracking of targets. In this system, an antenna beam of circular cross section is moved continuously in such a way that its axis describes a cone whose apex angle is approximately equal to the beam angle, as in fig 1. If the target is directly on the cone axis, the strength of the returned signal is uniform throughout the scan; however, if the target gets off this axis, the signal strength will vary as the antenna describes the cone. If the antenna is re-oriented to cause a uniform signal, the target will be back on the cone axis and its azimuth and elevation determined (Ref 1:30). Thus the modulation envelope of the radar echos serves as the error signal. When the tracked object is on the scan axis, the error signal is zero since all pulses have the same amplitude. If the object is not on the scan axis, there is an error signal produced whose magnitude varies approximately sinusoidally and whose frequency is the same as that of the

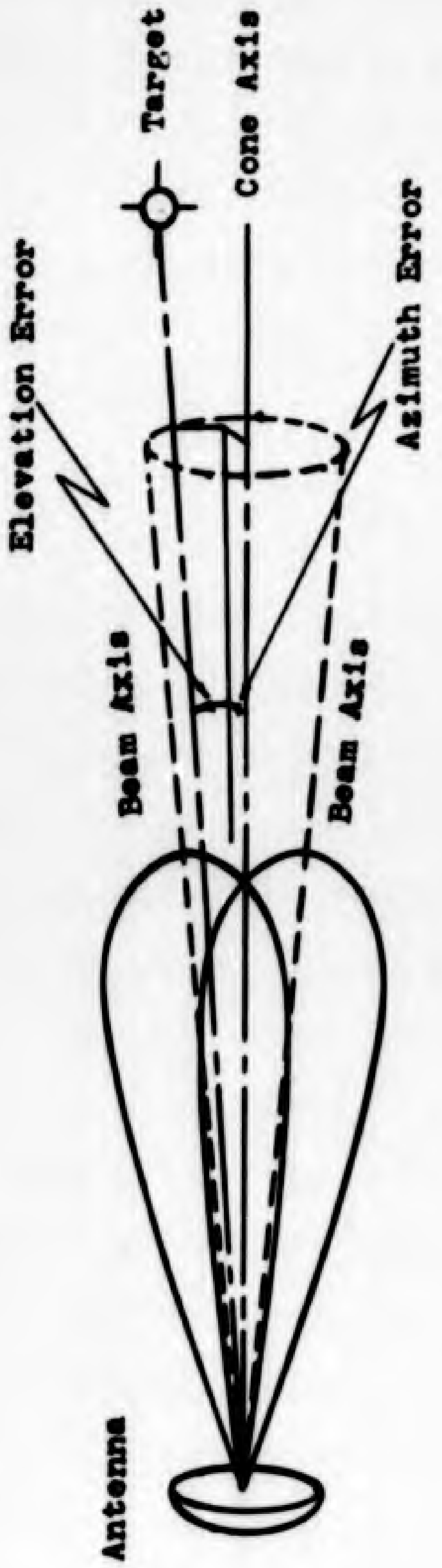

Fig 1 Geometry of Conical Scanning

(After Ref 8:29)

conical scan. Rectification of the series of returned radar pulses, then, produces an approximate sine wave of low frequency. The phase of this voltage relative to the phase of the voltage producing the conical scan determines the direction from the cone axis to the target. This signal can then be separated into two components in quadrature; these components serve as individual error signals for the azimuth and elevation servos (Ref 1:332). By appropriate signal processing these error signals are reduced to some useable scale factor in terms of volts per degree of pointing error (for instance, 7.14 volts/degree for the precision tracker of the Telstar system) (Ref 9:1332).

The fundamental element of the antenna servo system is the servo motor. The equation of motion for a DC armature controlled servo motor can be written

$$\frac{L_m J}{K_T} D^3 \phi_m + \frac{L_m B + R_m J}{K_T} D^2 \phi_m + \frac{R_m B + K_m K_T}{K_T} D \phi_m = e \quad (1)$$

where L_m = armature inductance

R_m = armature resistance

K_m = motor back emf constant

K_T = motor torque constant

T = torque of motor

i_m = armature current

J = system moment of inertia (includes antenna moment)

B = system viscous damping coefficient

D = differential operator representing a time differentiation

ϕ_m = system angular displacement

e = applied voltage

(Ref 12:38)

If the assumption is made that the armature inductance can be neglected and that the applied voltage is proportional to the angular displacement (either azimuth or elevation) of the tracked body, eq (1) assumes the form

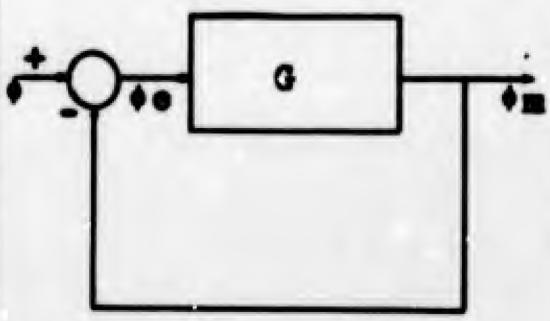

Fig 2 Antenna Servo Diagram

$$(A_1D^2 + A_2D)\phi_m = A_3\phi \qquad (2)$$

where ϕ = satellite angular displacement (azimuth or elevation)

$A_3\phi = e.$

To provide closed loop control for the system, the output is fed back and subtracted from the input to create the tracking error signal (fig 2). If G is the open loop transfer function, the system output, ϕ_m, and tracking error, ϕ_e, are related by

$$\phi_m(s) = G\phi_e(s) \qquad (3)$$

where $G = \dfrac{A_3}{A_1s^2 + A_2s}$.

Equation (3) is obtained by Laplace transforming eq (2) and replacing ϕ by ϕ_e.

The closed loop relationship is then

$$\phi_e(s) = \phi(s) - \phi_m(s) \tag{4}$$

or

$$\frac{\phi_m(s)}{\phi(s)} = \frac{G}{1 + G} = G_T \tag{5}$$

where $\phi(s)$ = the Laplace transformation of the angular displacement (azimuth or elevation) of the tracked body.

G_T = the control ratio.

It is assumed that the azimuth channel and the elevation channels are essentially similar so that eq (5) describes the response of either of them. This equation will be useful later when the system performance is to be measured. The next area to be covered, however, is the general method of attack in delineating the dynamic errors.

III. Method of Analysis

The most straightforward way to determine the errors introduced by radar tracking systems into orbit prediction is to compare the predicted orbit with the "real" orbit. Since the purpose of this paper is to consider only those errors caused by the tracker dynamic response, the assumption is made that the "real" orbits are true conics. Purturbative influences, such as earth ellipticity, atmospheric drag, and lunar gravity attraction, would serve to unnecessarily complicate the problem since their influence upon the tracker dynamic response is not detectable; moreover, inaccuracies involved in such computations would tend to disguise the problem under consideration. The orbits considered consist of ellipses with the prime focus located at the center of a spherical model of the earth. The diurnal rotation of the earth cannot be neglected; the annual precession around the sun, however, is neglected with the result that an inertial reference frame may be assumed with origin at the center of the earth.

The inputs to the satellite tracker azimuth and elevation servomechanisms are the azimuth and elevation angles described by a satellite as it crosses the "field of vision" of the tracking system. Once an orbit is established, the position of the satellite is determined for all time; hence its position with respect to a given tracking station can be determined at any value of time. This position can be described in the tracker coordinate frame of reference as a value of azimuth, elevation and range. While a closed form expression for the variation of these three variables with time is quite difficult, it is possible to compute, by a coordinate transformation, a series of these values with time as the independent variable. In this paper, the data so obtained is approximated in separate time polynomials by using a least squares fit. The functions so obtained then serve as inputs to eq (5) and the system response computed. Thus two sets of time varying vectors are obtained which describe the satellite position with respect to the tracking station. Specifically these are the following sets:

1. The input to the tracker consisting of an azimuth angle polynomial in time, an elevation angle polynomial in time, and a set of values of slant range with time (no polynomial fitting was performed on the range data since, as discussed in Chapter II, radar slant range output is considered correct).

2. The solutions to eq (5) for both the azimuth and elevation inputs, and the set of range-time data.

Both of these sets of vectors will be used independently to determine a set of parameters for the orbit. The parameters computed from set 1 are compared with the original parameter set to determine the error introduced by the least squares fit. The parameters from set 2 are compared with those of set 1 to determine the errors introduced by the system dynamic response. Since set 1 is the actual input to the tracking system, the solution to the overall problem is thereby obtained. A more detailed list of steps in the method for obtaining this solution is given below. The ensuing pages will explain in detail each of the steps.

The first step in the solution is to select the orbit upon which the analysis is to be performed. In the preparation of this paper, attention was focused on nine different orbits, the parameters of which are listed in Table I. The description of the orbits includes a set of six parameters: eccentricity, semi-major axis, right ascension of the ascending node, argument of perigee, inclination and the time at perigee. These six elements permit complete computation of the position of the satellite in space at any time. In order to reference its position with respect to the rotating earth, however, an additional parameter (which is the initial longitude of the vernal equinox) is necessary in order that the position of the orbit with respect to the earth be defined at some time. Then a knowledge of the earth rotation rate permits determination of the orbit position with respect to the earth at all times.

The orbits chosen were selected on the basis of the dynamics problem which they posed. There are, essentially, three fundamental orbits; a low, nearly circular orbit;

tracking station can be determined at any value of time. This position can be described in the tracker coordinate frame of reference as a value of azimuth, elevation and range. While a closed form expression for the variation of these three variables with time is quite difficult, it is possible to compute, by a coordinate transformation, a series of these values with time as the independent variable. In this paper, the data so obtained is approximated in separate time polynomials by using a least squares fit. The functions so obtained then serve as inputs to eq (5) and the system response computed. Thus two sets of time varying vectors are obtained which describe the satellite position with respect to the tracking station. Specifically these are the following sets:

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Table I
Orbits Studied

Parameter Set	1	2	3	4	5	6	7	8	9
Eccentricity	.005	.005	.005	.05	.05	.05	.27	.27	.27
Semimajor axis (miles)	4160	4160	4160	4560	4560	4560	5560	5560	5560
Right ascension of ascending node (deg)	-176.38	-176.38	-176.38	-176.38	-176.38	-176.38	-176.38	-176.38	-176.38
Argument of perigee (deg)	-40	-40	-40	-40	-40	-40	-40	-40	-40
Inclination (deg)	40	40	40	40	40	40	40	40	40
Time of perigee passage (sec)	0	0	0	0	0	0	0	0	0
Initial long. of ascending node	6.2	13	21.7	7.1	24	35	7.2	32.2	42.2

a slightly larger, somewhat more elliptic orbit; and finally a much larger, highly eccentric ellipse. The selection of these elements was rather arbitrarily performed, the purpose being to obtain low, medium and high altitude problems for consideration. The remainder of the six fundamental elements were held constant since it was felt that, depending upon the element considered, their variation would either have no impact on the problem or the impact could be observed in another way - ie, by variation of the initial longitude of the ascending node. This last variation was very important and was selected quite carefully, for it was used to determine the aspect that the satellite presents to the tracking station (fig 3). This last variable was selected so that in each of the fundamental orbits, the satellite would make a pass directly overhead in the first case, at an elevation angle of 40 to 50 degrees in the second case and 20 to 30 degrees in the third case. This set comprises the nine situations covered during this investigation. The tracking was similated by a single station located at Vandenberg Air Force Base (Latitude 34.75°N, Longitude 120.58°W). The ensuing discussion covers the method of analyzing a single orbit. It should be borne in mind that the same analysis was applied to each of the orbits.

The next step toward the solution is, from the given parameters, to determine a set of elevation, azimuth and

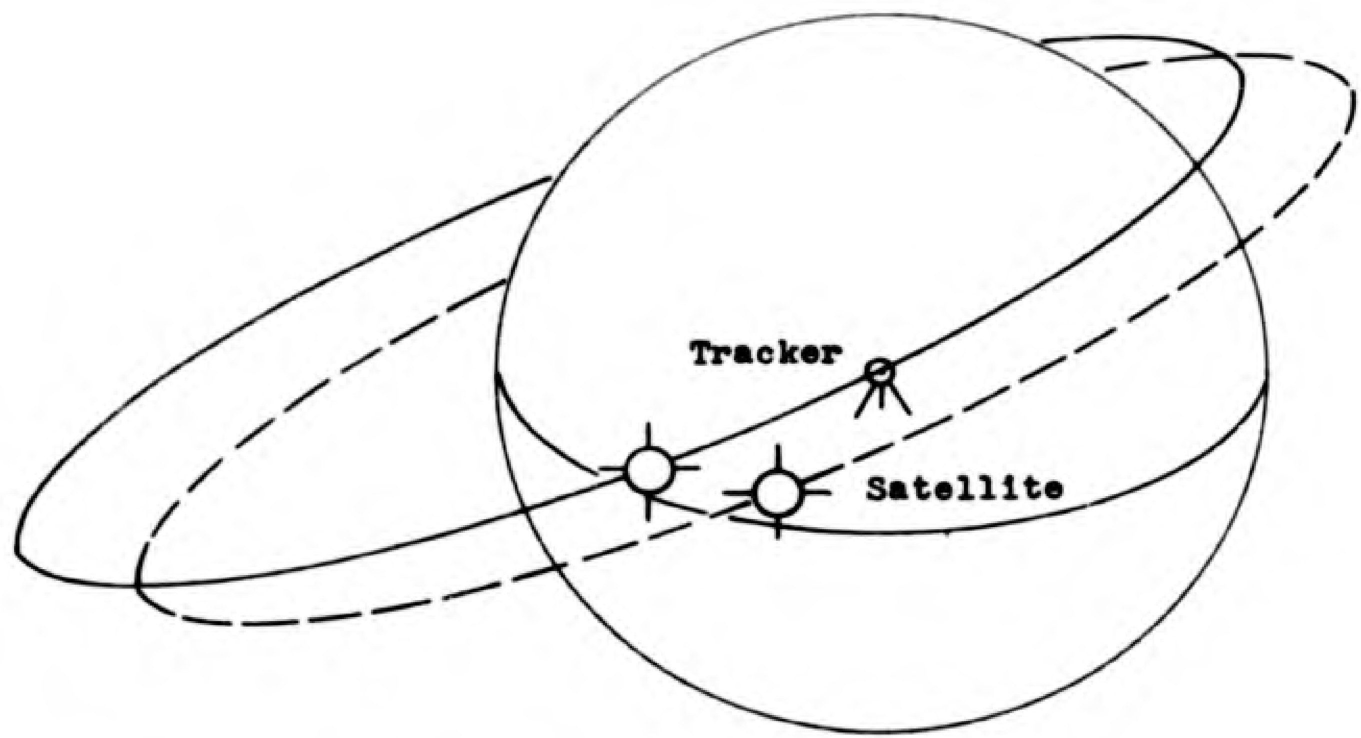


Fig 3 Effect of Change of Longitude of Vernal Equinox on Tracker-Orbit Aspect

range values corresponding to a sequence of times for one pass of the satellite across the tracking station. This is performed by applying a series of coordinate transformations to position vectors generated within the orbital plane as a function of time and the selected orbit parameters.

Next, this data is fitted to several different order polynomials and an error analysis performed in order to select the order of the polynomial required for the fit. The error analysis is performed by evaluating the polynomials at the input values of time, and then comparing the values so obtained with input values of azimuth and elevation. To prevent unnecessary complication of computation it is desirable to select a polynomial of as low order as possible. The appropriateness of the selection is finally determined by computing a set of orbit elements using values of azimuth and elevation angles from the polynomials and comparing these elements to the originally selected set.

The next link in the chain is to use the polynomials as inputs to the tracker servo system. This is performed by applying the Laplace transformation to the input time polynomials and solving eq (5). The response of the tracker servo system is obtained in the form of two functions of time proportional to the time varying azimuth and elevation angles. The orbit determined by this set is compared to the polynomial determined orbit to finally determine the dynamic errors. This comparison is performed by two methods. First, the elements themselves are compared and the absolute and relative differences determined for each element. Next, a set of fifty position vectors around the entire orbit is computed from each of the two parameter sets, with time being the basis for comparison between the two sets. The absolute maximum and root mean square value of the differences of these vectors gives an indication of the error which would exist in the prediction of the satellite position from radar data. Such an analysis is the real object of this paper. It should be observed that there is an obvious difference between the present position of the satellite as observed by the tracker and the future position predicted from tracker-observed data. The former suffers primarily from distributional errors while the other suffers from computational; however, the latter is the stiffer comparison. It is a measure, not only of the overall system dynamic error, but of the error at the individual points as well.

A program entitled RADYN, coded in the Fortran II language for the IBM 7090-94 digital computer, has been prepared by the author to perform the calculations necessary for the solution of this problem. RADYN and its subprograms are included as Appendix D. The subprograms furnished by the Systems Dynamic Analysis Division which are used in RADYN and which are not contained in the standard

7090-94 library, are included as Appendix E. The equations which the computer program is designed to solve are outlined in the next few chapters; in those cases where the development of the equations was too lengthy to include in the main body of the report, such development has been performed in Appendix C. These equations are presented in the order indicated by the preceeding paragraphs. Thus the first problem considered is the determination of elevation, azimuth and range value from a selected set of orbit parameters. A discussion of the necessary coordinate system transformations, covered in the next chapter, serves to clarify the relationship between the time-varying position vectors of the orbit and the values of azimuth, elevation, and slant range generated at the tracking system.

IV. Coordinate Transformations

The standard set of six orbital parameters used in this report serve to (1) locate the orbit plane in a non-rotating, geocentric coordinate frame; (2) describe the orbit in its plane; and (3) locate the satellite within the orbit. Therefore the parameter set completely describes the satellite kinematics. By a series of coordinate transformations, position vectors expressed in the orbital reference system may be described in the local frame of the radar tracking site. This latter is a topocentric system which has the site as origin, the local horizon as the base plane and zero azimuth oriented to true north (Ref 3:117). A comprehensive discussion of the transformations necessary to change from the inertial to the local frame will follow. First however, a brief discussion of the orbital parameters will permit clearer understanding of the rotations themselves; furthermore, it should be kept in mind that determination of these elements from the radar data is a fundamental objective in this report.

The orbit plane is described by the right ascension of the ascending node (Ω) and the inclination (i) as indicated in fig 4. The line of nodes is defined as the line of intersection of the orbit plane and the earth equatorial plane. The ascending node is the point where the line of nodes intersects the orbit as the satellite goes into the

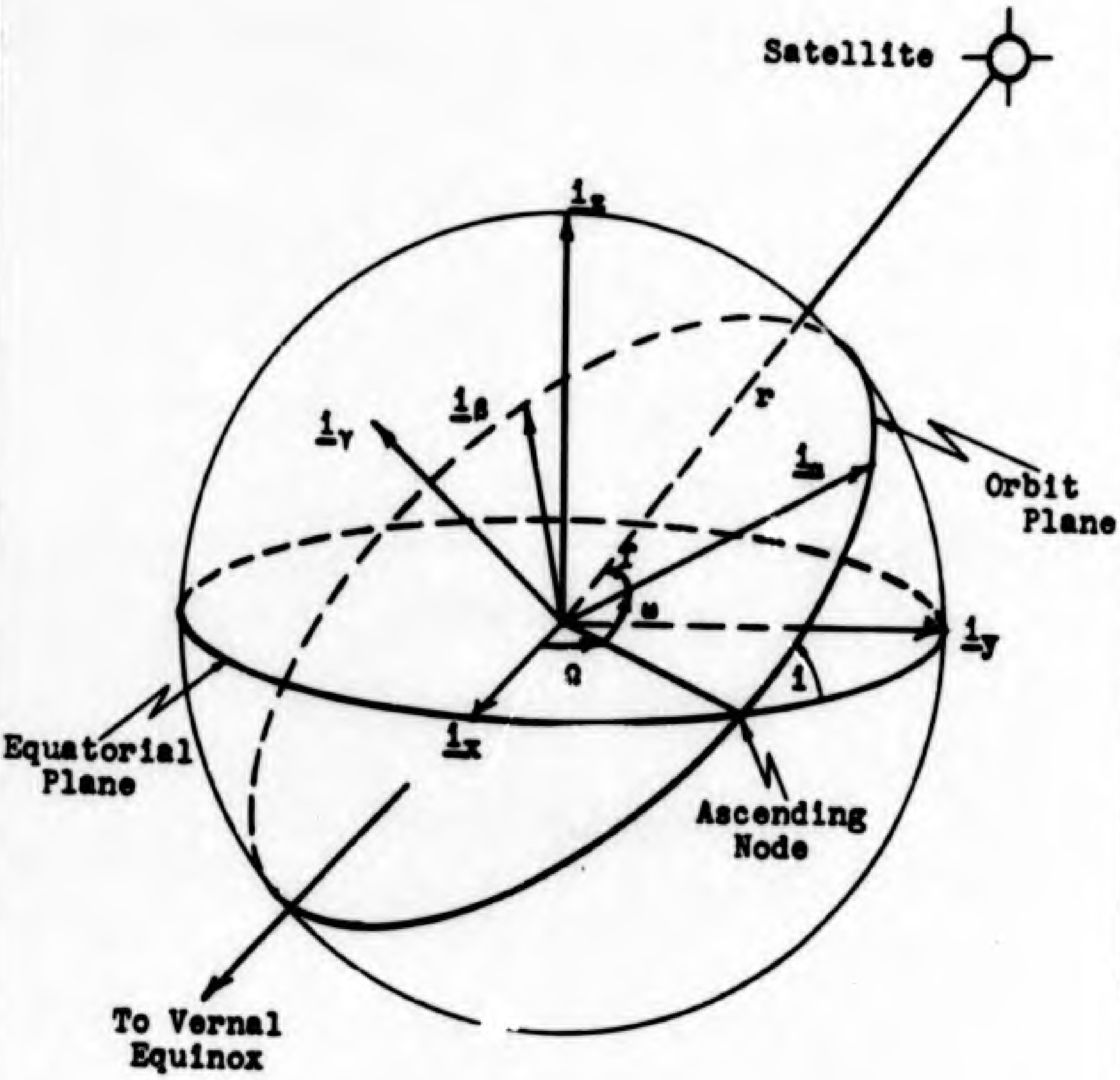


Fig 4 Orbital and Geocentric Inertial Coordinate Frames

(After Ref 4:1.7-2)

hemisphere containing the North Pole. The right ascension of the ascending node is the angle between two lines intersecting at the center of the earth, one of them directed toward the vernal equinox and the other passing through the ascending node. The orbit inclination is the angle between the equatorial plane and the orbit plane.

The in-plane description of the orbit is given by the argument of perigee (ω), the semi-major axis (a) and the eccentricity (e), which are shown in fig 5. Perigee is the point in the elliptic orbit where the orbiting body most closely approaches the prime focus (the center of the earth). The argument of perigee is the angle between two lines intersecting at the center of the earth, one of them directed to the ascending node and one of them to perigee. The semi-major axis is the distance from the center of the ellipse to perigee; in another sense, it is half the distance from apogee (furthest distance of the orbiting body from the prime focus) to perigee. The eccentricity is the ratio of the distance between the orbit center and the prime focus to the semi-major axis.

The final orbital element is the epoch which locates the body in the orbit described above. The epoch used in this report is the time of perigee passage (τ).

It is assumed that an orbit described by these six elements is given. The task to be performed in this chapter is to develop a method for describing, in the

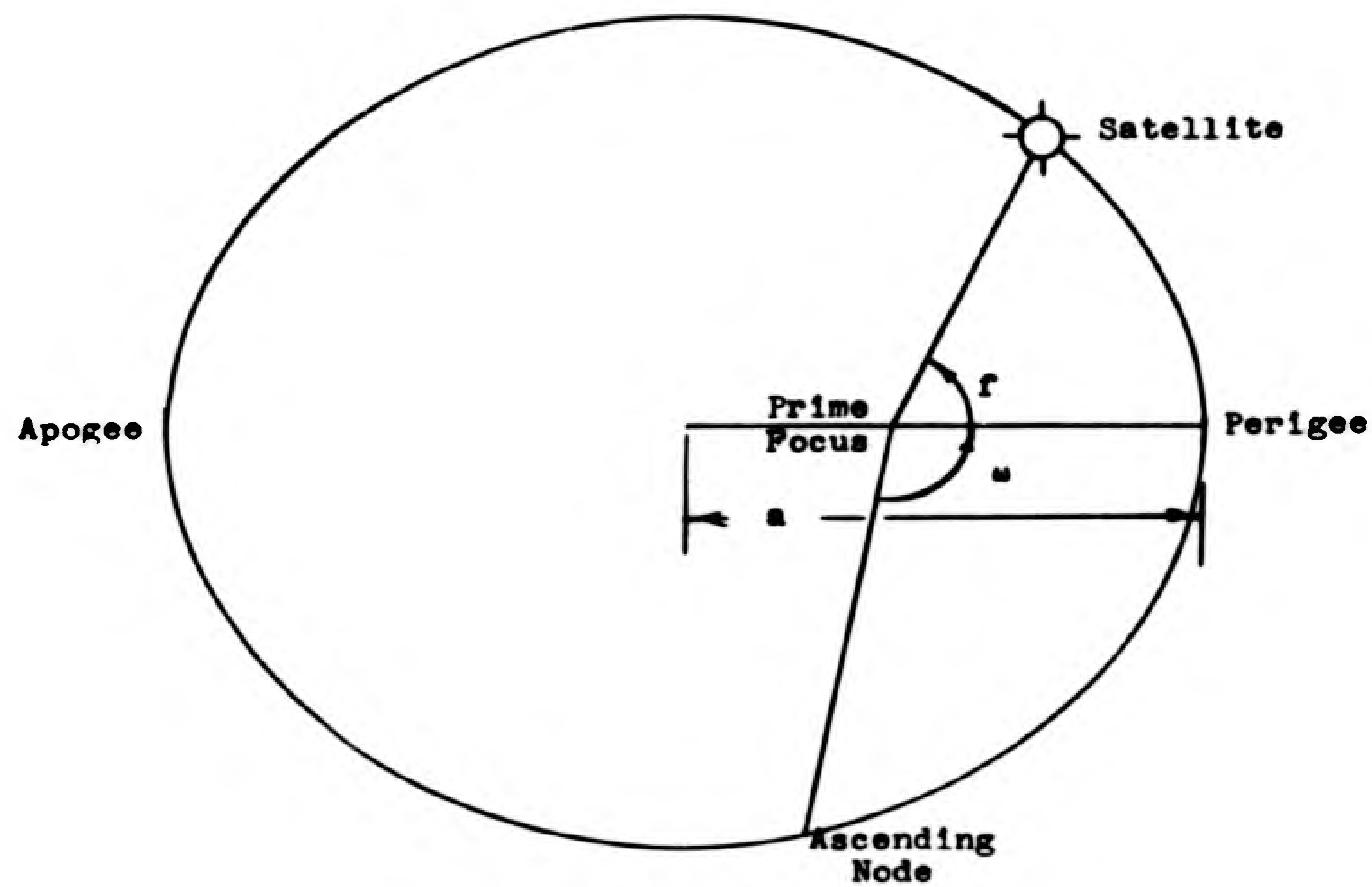


Fig 5 The Orbit

local topocentric frame, vectors already known in the orbit frame. The first step is to rotate the reference from the orbit plane to the geocentric inertial frame (fig 4). This will be done by use of the proper orthogonal rotation matrix $\underline{G}$ so that

$$\underline{x} = \underline{G}\,\underline{r} \tag{6}$$

where $\underline{r}$ is the position vector in the orbit frame

$$\underline{r} = \begin{bmatrix} r\cos f \\ r\sin f \\ 0 \end{bmatrix} \tag{7}$$

r = the magnitude of $\underline{r}$ (ie, the distance from the center of earth to the satellite)

f = the true anomaly (angle between $\underline{r}$ and perigee position vector measured in the direction of satellite rotation)

$\underline{x}$ = the position vector in the inertial frame.

$\underline{G}$ consists of an array of direction cosines resulting from forming the scalar products of all the principal unit vectors.

$$\underline{G} = \begin{bmatrix} \underline{i}_x \cdot \underline{i}_\alpha & \underline{i}_x \cdot \underline{i}_\beta & \underline{i}_x \cdot \underline{i}_\gamma \\ \underline{i}_y \cdot \underline{i}_\alpha & \underline{i}_y \cdot \underline{i}_\beta & \underline{i}_y \cdot \underline{i}_\gamma \\ \underline{i}_z \cdot \underline{i}_\alpha & \underline{i}_z \cdot \underline{i}_\beta & \underline{i}_z \cdot \underline{i}_\gamma \end{bmatrix} \tag{8}$$

$$= \begin{bmatrix} g_{11} & g_{12} & g_{13} \\ g_{21} & g_{22} & g_{23} \\ g_{31} & g_{32} & g_{33} \end{bmatrix} \tag{9}$$

From the law of cosines in sperical trigonometry applied to fig 4 it follows that

$$g_{11} = \cos \Omega \cos \omega + \sin \Omega \sin \omega \cos (180 - i)$$

$$= \cos \Omega \cos \omega - \sin \Omega \sin \omega \cos i \qquad (10)$$

But

$$\varpi \equiv \Omega + \omega$$

where ϖ is called the longitude of perigee although it is not truly a longitude since Ω and ω do not lie in the same plane.

$$\cos (\Omega + \omega) = \cos \varpi$$

$$= \cos \Omega \cos \omega - \sin \Omega \sin \omega$$

$$\cos \Omega \cos \omega = \cos \varpi + \sin \Omega \sin \omega \qquad (12)$$

$$g_{11} = \cos \varpi + \sin \Omega \sin \omega - \sin \Omega \sin \omega \cos i$$

$$= \cos \varpi + \sin \Omega \sin \omega (1 - \cos i)$$

$$= \cos \varpi + 2 \sin \Omega \sin \omega \sin^2 i/2 \qquad (13)$$

$$g_{12} = \cos \Omega \cos (90 + \omega) +$$

$$\sin \Omega \sin (90 + \omega) \cos (180 - i)$$

$$= - \cos \Omega \sin \omega - \sin \Omega \cos \omega \cos i$$

$$\sin (\Omega + \omega) = \sin \varpi = \sin \Omega \cos \omega + \cos \Omega \sin \omega$$

$$\cos \Omega \sin \omega = \sin \varpi - \sin \Omega \cos \omega \qquad (14)$$

$$g_{12} = - \sin \varpi + \sin \Omega \cos \omega - \sin \Omega \cos \omega \cos i$$

$$= - \sin \varpi + 2 \sin \Omega \cos \omega \sin^2 i/2 \qquad (15)$$

$$g_{13} = \cos \Omega \cos 90 + \sin \Omega \sin 90 \cos (90 - i)$$

$$= \sin \Omega \sin i \qquad (16)$$

$$g_{21} = \cos (90 - \Omega) \cos \omega + \sin (90 - \Omega) \sin \omega \cos i$$

$$= \sin \Omega \cos \omega + \cos \Omega \sin \omega \cos i$$

$$= \sin \varpi - \cos \Omega \sin \omega + \cos \Omega \sin \omega \cos i$$

$$= \sin \varpi - 2 \cos \Omega \sin \omega \sin^2 i/2 \qquad (17)$$

$$g_{22} = \cos (90 - \Omega) \cos (90 + \omega) +$$

$$\sin (90 - \Omega) \sin (90 + \omega) \cos i$$

$$= - \sin \Omega \sin \omega + \cos \Omega \cos \omega \cos i$$

$$= \cos \varpi - \cos \Omega \cos \omega + \cos \Omega \cos \omega \cos i$$

$$= \cos \varpi - 2 \cos \Omega \cos \omega \sin^2 i/2 \qquad (18)$$

$$g_{23} = \cos (90 - \Omega) \cos 90 +$$

$$\sin (90 - \Omega) \sin 90 \cos (90 + i)$$

$$= - \cos \Omega \sin i \qquad (19)$$

$$g_{31} = \cos \omega \cos 90 + \sin \omega \sin 90 \cos (90 - i)$$

$$= \sin \omega \sin i \qquad (20)$$

$$g_{32} = \cos (90 + \omega) \cos 90 +$$

$$\sin (90 + \omega) \sin 90 \cos (90 - i)$$

$$= \cos \omega \sin i \qquad (21)$$

$$g_{33} = \cos i \qquad (22)$$

(Ref 4:1.7-4)

The next rotation is about the z-axis to express the vector in the geocentric rotating frame. The rotation is through the angle between the x-axis and the prime meridian (fig 6). This is a time varying angle equal to the initial angle between the x-axis and the prime meridian (actually the negative of the initial longitude of the vernal equinox, λ_r) plus the angle through which the earth has rotated.

Thus **this angle may be expressed as**

$$\lambda t = \omega_e t - \lambda_r \qquad (23)$$

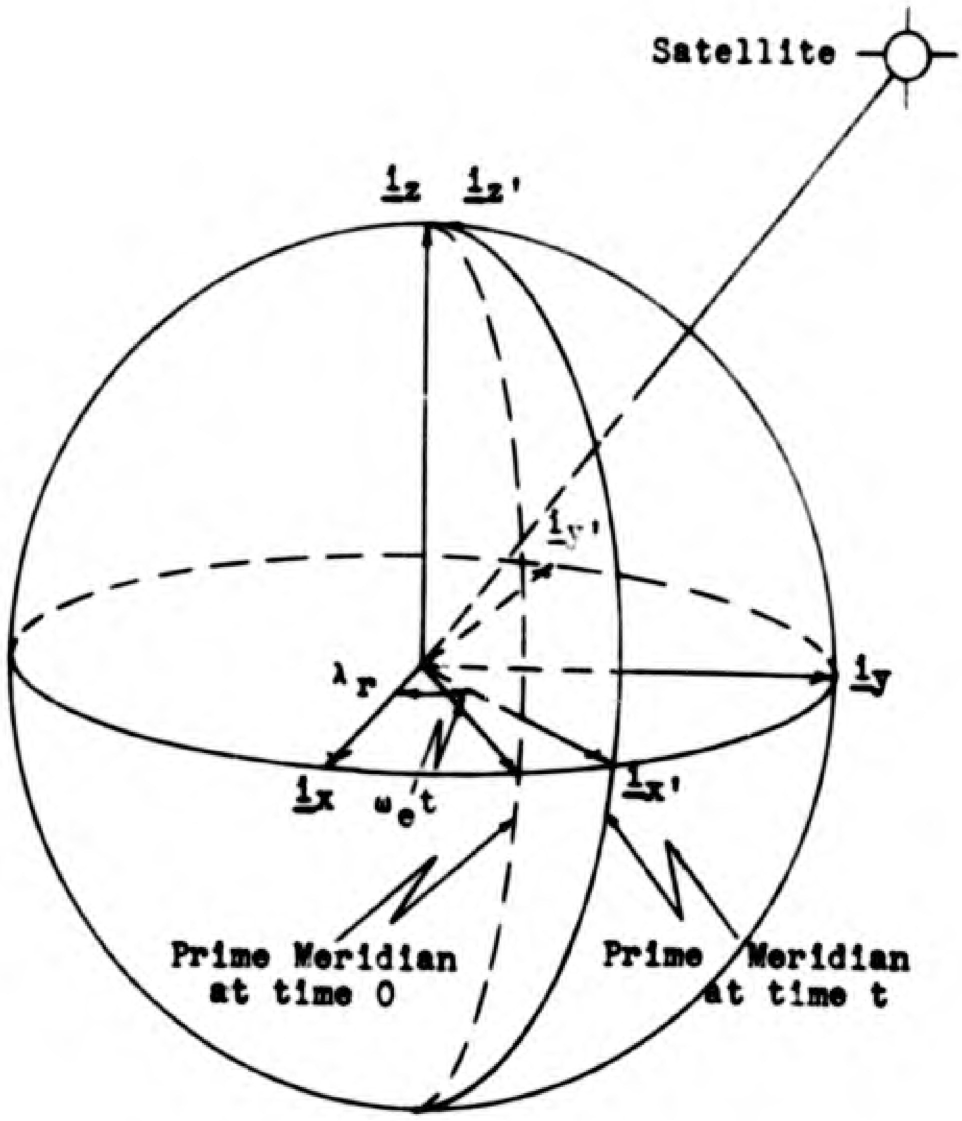


Fig 6 Rotating Geocentric System

where λ_t = the angle rotated through by time t

ω_e = earth rotation rate

= 4.1780741×10^{-3} deg/sec (Ref 5:5-25)

λ_r = initial longitude of the vernal equinox. (Ref 3 :117)

The rotation is

$$\underline{x}' = \underline{D}\, x \tag{24}$$

where $\underline{x}'$ = the vector in the rotating frame

$\underline{D}$ = the proper orthogonal rotation matrix

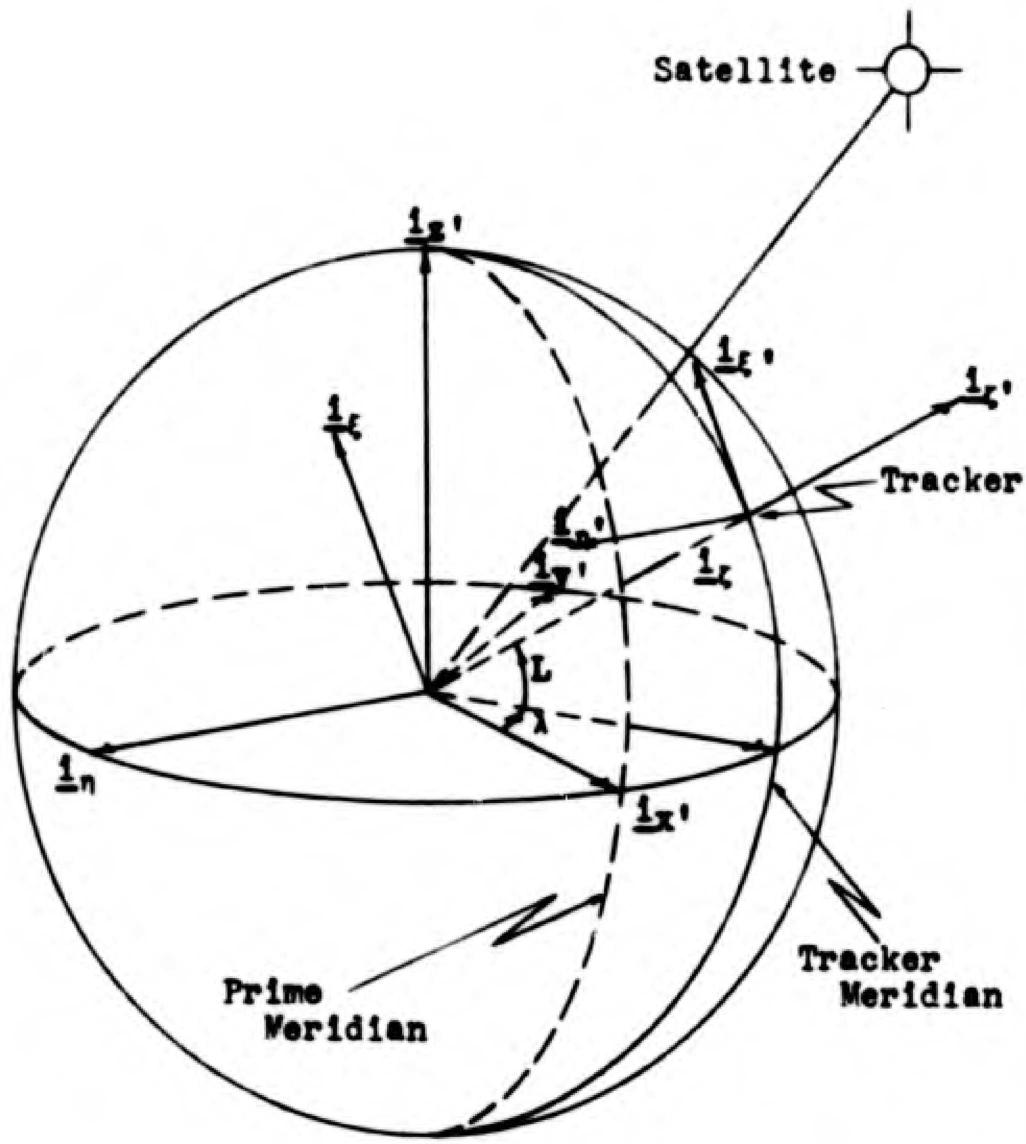
$$= \begin{bmatrix} \cos(\omega_e t - \lambda_r) & \sin(\omega_e t - \lambda_r) & 0 \\ -\sin(\omega_e t - \lambda_r) & \cos(\omega_e t - \lambda_r) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The next rotation expresses the vector in a coordinate frame which is parallel to the local topocentric frame, but whose center is located at the center of the earth (fig 7). This actually consists of two rotations, first about the z'-axis until the x"-axis is 180° past the tracking station meridian (ie, through the angle λ + 180° where λ is the tracking station longitude), then about the y"-axis until the ζ-axis lies on the station local vertical. This rotation is expressed by

$$\underline{\xi} = \underline{C}\, \underline{x}' \tag{25}$$

where $\underline{\xi}$ = the vector expressed in a geocentric rotating frame parallel to the local topocentric frame

$\underline{C}$ = the proper orthogonal rotation matrix



NOTE: The $\underline{x}''$-system is not shown here. It results from rotating the $\underline{x}'$-system about the z'-axis through the angle $\lambda + 180°$.

Fig 7 Topocentric System

$$\underline{C} = \underline{F}\,\underline{G}$$

$$\underline{F} = \begin{bmatrix} \cos(270 + L) & 0 & -\sin(270 + L) \\ 0 & 1 & 0 \\ \sin(270 + L) & 0 & \cos(270 + L) \end{bmatrix}$$

$$= \begin{bmatrix} \sin L & 0 & \cos L \\ 0 & 1 & 0 \\ -\cos L & 0 & \sin L \end{bmatrix}$$

$$\underline{G} = \begin{bmatrix} \cos(\lambda + 180) & \sin(\lambda + 180) & 0 \\ -\sin(\lambda + 180) & \cos(\lambda + 180) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -\cos\lambda & -\sin\lambda & 0 \\ \sin\lambda & -\cos\lambda & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

So that

$$\underline{C} = \begin{bmatrix} -\sin L\cos\lambda & -\sin L\sin\lambda & +\cos L \\ \sin\lambda & -\cos\lambda & 0 \\ \cos L\cos\lambda & +\cos L\sin\lambda & \sin L \end{bmatrix}$$

L = station north latitude

λ = station east longitude

The final transformation is to translate the origin of the $\underline{\xi}$ frame out the ζ-axis a distance equal to the radius of the earth. This is done in the following manner:

$$\underline{\xi} = \underline{\xi}_0 + \underline{\xi}'$$

or

$$\underline{\xi}' = \underline{\xi} - \underline{\xi}_0 \qquad (26)$$

where $$\underline{\xi}_0 = \begin{bmatrix} 0 \\ 0 \\ R_e \end{bmatrix}$$

$\underline{\xi}'$ = the vector expressed in the coordinate frame at the tracking site

R_e = earth radius

= 3960 miles

In summary, a listing is made of the various coordinate frames used.

1. $\underline{\alpha}$ - system - geocentric inertial frame with α-axis toward perigee, β-axis in the orbit plane 90° from the α-axis in the direction of rotation, γ-axis forming a right-handed system.

2. $\underline{x}$ - system - geocentric inertial frame with x-axis toward the vernal equinox, z-axis toward the North Pole, y-axis forming a right-handed system.

3. $\underline{x}'$ - system - geocentric rotating frame with x'-axis containing the prime meridian, z'-axis toward the North Pole, y'-axis forming a right-handed system.

4. $\underline{\xi}$ - system - geocentric rotating frame with ζ-axis pointing toward the tracking station, ξ-axis lying in the station meridian plane and in the northern hemisphere, the η-axis forming a right-handed system.

5. $\underline{\xi}'$ - system - topocentric rotating frame with ζ'-axis up the local vertical, ξ'-axis north, η'-axis west. The satellite position is described by radar coordinates in this frame.

Radar coordinates consist of azimuth, A, elevation, E, and range, R, as indicated in fig 8. By a standard conversion to spherical coordinates, vectors in the $\underline{\xi}'$ - frame can be expressed as functions of these three variables; specifically

$$\underline{\xi}' = \begin{bmatrix} R \cos E \cos A \\ R \cos E \sin A \\ R \sin E \end{bmatrix} \tag{27}$$

Eq (27) is combined with the previous equations of the chapter to yield

$$\begin{bmatrix} R \cos E \cos A \\ R \cos E \sin A \\ R \sin E \end{bmatrix} = \underline{\underline{C}}\ \underline{\underline{D}}\ \underline{\underline{G}}\ \underline{r} - \underline{\xi}_0 \tag{28}$$

All transformations are now defined. The next step is to determine the in-plane vectors as a function of time.

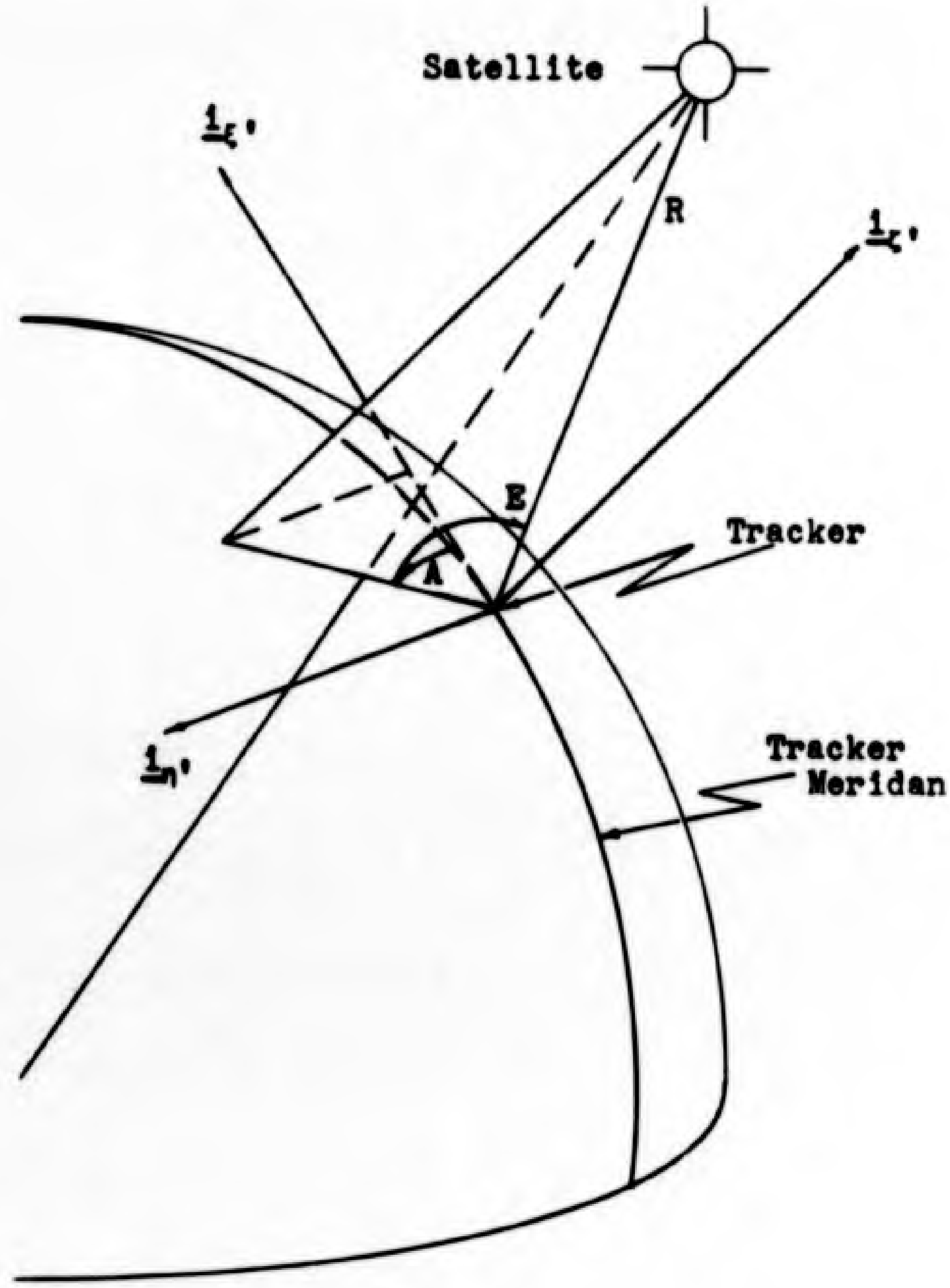


Fig 8 Radar Coordinates

V. Orbital Calculations

The development of the set of azimuth and elevation angles and radar ranges as functions of time consists of (1) determination of the satellite position in the orbital plane as a function of time and (2) performing the coordinate transformations, already described in Chapter IV, on these in-plane vectors. The in-plane coordinates consist of the true anomaly or polar angle (f) and radial distance from the prime focus (r) as indicated in fig 9. An expression of the variation of f and r with time is not easily obtained; however there is an expression relating the value of the eccentric anomaly (E) with time. If a circle is circumscribed about the orbital ellipse, the eccentric anomaly is the central angle between a radius through the perifocus and a radius cutting the circle above the satellite position. The equation relating this angle with time is commonly referred to as Kepler's equation and is

$$(t - \tau) = \frac{P}{2\pi} (E - e \sin E) \qquad (29)$$

where t = time

τ = time at which satellite passes perigee

P = orbit period.

(Ref 4:2.2-2)

This equation is developed in Appendix C.

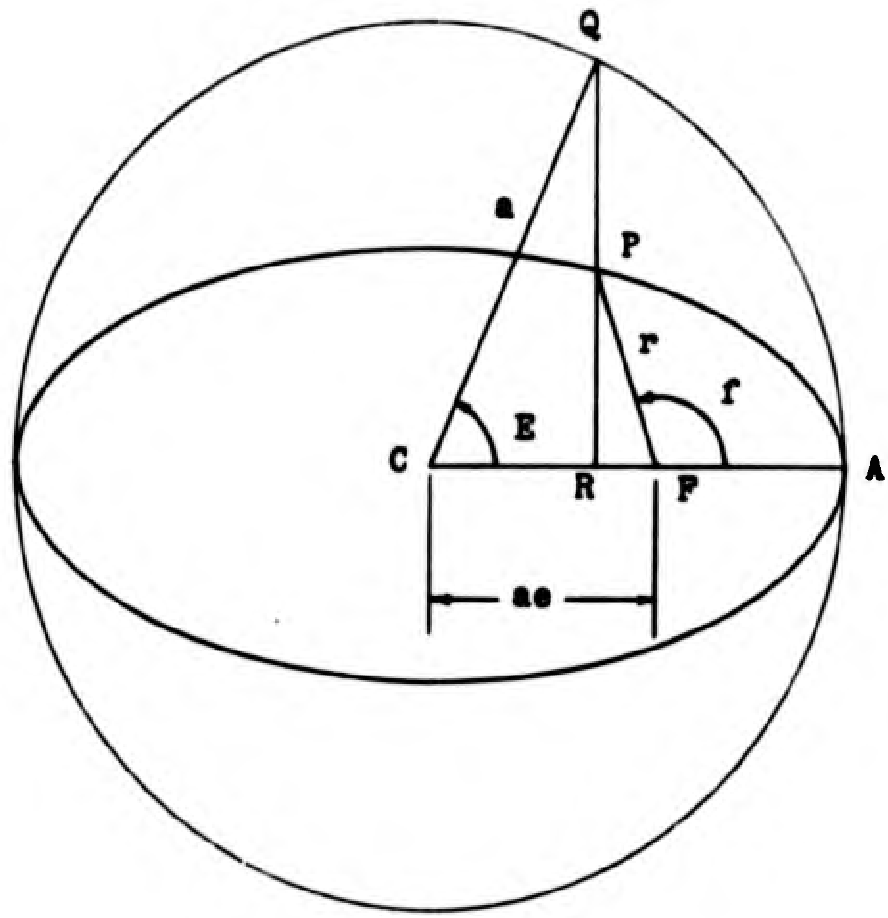


Fig 9 Orbit Geometry

(After Ref 4:2.4-1)

Equation (29) is solved directly for time, but becomes transcendental to solve for the eccentric anomaly as a function of time. An iterative solution is performed by RADYN with the equation in the form

$$E = \frac{2\pi}{P}(t - \tau) + e \sin E \qquad (30)$$

The period (P) is given by

$$P = 2\pi\sqrt{a^3/\mu} \qquad (31)$$

where μ = earth gravity constant

= universal gravity·constant times the earth mass

a = orbit semi-major axis.

(Ref 4:2.2-2)

For a given orbit, all variables of eq (30) are determined except E and t. If a value of time is selected, this equation will rapidly converge to a value for the eccentric anomaly. If time is incrementally increased through a range of values, for example from zero to P, a series of eccentric anomalies versus time are determined. These values may be converted to values of true anomaly by the equation

$$\tan \frac{f}{2} = \sqrt{\frac{1 + e}{1 - e}} \tan \frac{E}{2} \qquad (32)$$

where e = orbit eccentricity.

(Ref 4:2.4-2)

This equation is developed in Appendix C.

With the eccentric anomaly now determined as a function of time, r, the radius distance, is determined from

$$r = \frac{a\,(1 - e\,)}{1 + e\cos f} \qquad (33)$$

(Ref 4:2.4-2)

which is the equation of the ellipse in polar form. With values of f and r determined with respect to time, everything required to perform the coordinate transformations of Chapter IV are known; hence it is possible to proceed with the azimuth-elevation-range set determination by a simultaneous solution of eqs (28). It should be noted that the only values of interest are those wherein the elevation angle is positive. It is necessary to somehow determine the approximate values of the variables where the elevation angle goes from minus to plus since this starts the satellite pass over the station. In RADYN, time is incremented by a fairly large value (250 seconds) as long as elevation is negative; however, when it becomes positive, time is decreased to its previous value (the last value for which elevation was negative) and the increment is decreased to 100 seconds. When the elevation angle again becomes positive, time is again decreased to its value where last the elevation angle was negative, and the time increment is reduced to 10 seconds. This procedure is continued until the time at zero elevation is determined to within one-tenth of a second. The pass is then started and data recorded at 10 second invervals. A similar root-solving technique is used at the completion of the pass to determine the azimuth, range and time as the satellite goes below the horizon. This is also performed to an accuracy of .1 seconds.

To prepare this data as input to the tracker system it is necessary to fit it to a time polynomial by the method of least squares. Development of this theory is given in Appendix B. The LSCF2, LSCF1, LINEQ, and READT subprograms (listed in Appendix E) are used by RADYN to fit the elevation and azimuth data to time polynomials. As indicated in the Appendix B, the least squares method involves the determination of the coefficients of the selected order of polynomial such that the square of the difference between the input values of the dependent variable and the evaluation of the polynomial for the same value of the independent variable is minimized. While the theory is predicated on minimization through the standard method of equating the first derivative to zero, in practice the coefficients are determined by a system of linear equation whose number is one greater than the order of the selected polynomial since this is the number of coefficients to be determined (Ref 2:178).

An error analysis is performed by the computer program which determines the root mean square (RMS) values of the absolute and relative differences between 1st and 24th orders on the azimuth and elevation data sets. During the program development none of the curve fits gave an RMS error less than one degree. To decrease this error, three steps were taken. First, the data sets were divided into two sections, the maximum elevation angle attained being the deciding criterion for the division. In consonance with this was the second step which was to reject from the data fit points where the azimuth rate exceeded two degrees per second. The azimuth rate becomes quite high at high elevation angles; this is particularly evident when the orbit passes directly over the tracking station as is the case for which the plot in fig 10 was made (it represents the data of set 1 from Table I). The extreme azimuth rate is generally only present at one or two points. The restriction of two degrees per second is not unrealistic; in some cases the limits for extremely large antennas may be as low as .5 degrees per second (Ref 10:1253). For the orbit cases considered, this restriction seldom was used to eliminate any points; never were more than two points eliminated. When eliminations were made, the affected points occurred in the area where the data sets are divided for the fit.

The curve-fit subprograms will accept no more than 100 data points. Several times it was necessary to select every other point, or even every third point in one case

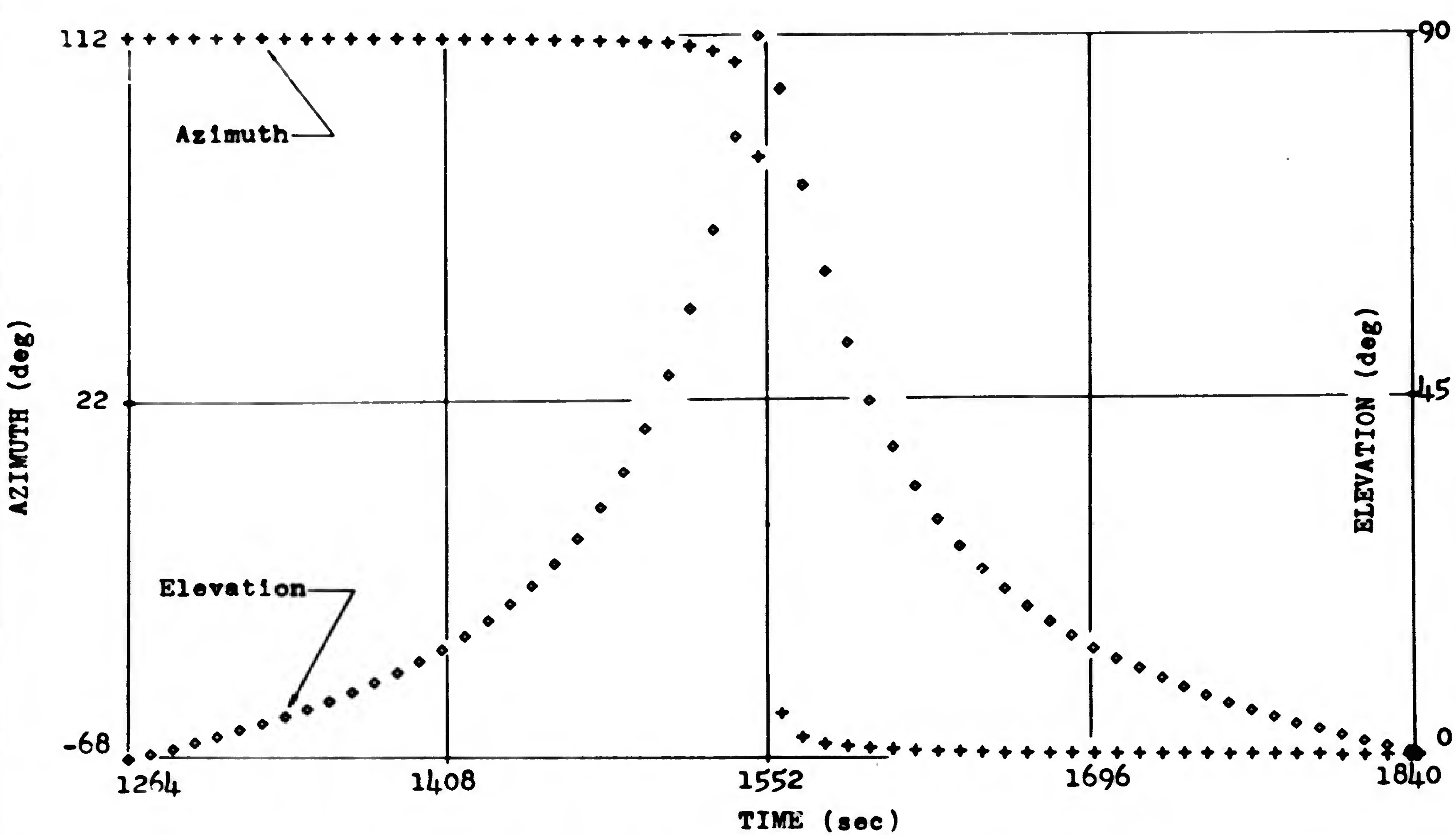


Fig 10 Azimuth and Elevation for an Overhead Pass

in order to reduce to 100 points.

The final adjustment necessary to improve the quality of the fit was to normalize the independent variable (time) so that it varied over a range from 0 to 1 for the half pass being fitted by the curve. This normalization was performed by the expression

$$t_i = \frac{T_i - T_0}{T_f - T_0} \tag{34}$$

where t_i = ith value of normalized time

T_i = ith value of real time

T_0 = real time at start of pass

T_f = real time at end.

The normalization delayed the effect of computational errors from affecting the results as higher ordered polynomials were selected. The overall effect of the process was to reduce the RMS differences to less than one degree for all the sets using polynomials of fourth order or higher. These differences decreased continuously in all cases to about 12th to 15th order at which point computational errors caused an apparent random variation of the error (although the error remained less than .1 degree). That such high ordered functions are required for accurate fitting of this type data is confirmed by Ref 11:15; however, speed and ease of computation indicated the use of a lower order polynomial. After making the three compensations listed above the fit results were so improved that is was possible to use a fourth order polynomial for this analysis and still retain RMS accuracy of the fit to within .1 degree. The polynomials are evaluated twice in between each input value and plotted (figs 11, 12, 13 and 14) to insure that they did not behave eccentrically in this region.

At this point it is considered that the input to the tracking system, namely the azimuth and elevation angles expressed as functions of time, are completely determined; prior to performing the actual solution to the response equation, however, a prediction of the orbit from the data of the evaluation of the time polynomials would well indicate the efficacy of the fit. Therefore a method will be outlined in the following pages for computing the orbit parameters from data in the form of azimuth and elevation angles and slant range.

There are two steps involved in this process; first, the transformation of these vectors into the geocentric inertial coordinate frame and second, the parameter determination from the resulting inertial vectors. The entire coordinate transformation cannot be completed from the topocentric frame to the orbit frame since it is now assumed that the orbit parameters are unknown; hence coordinate transformation, per se, cannot be carried beyond the X system.

The first step, the coordinate frame transformation, is actually the inverse of a portion of the transformation

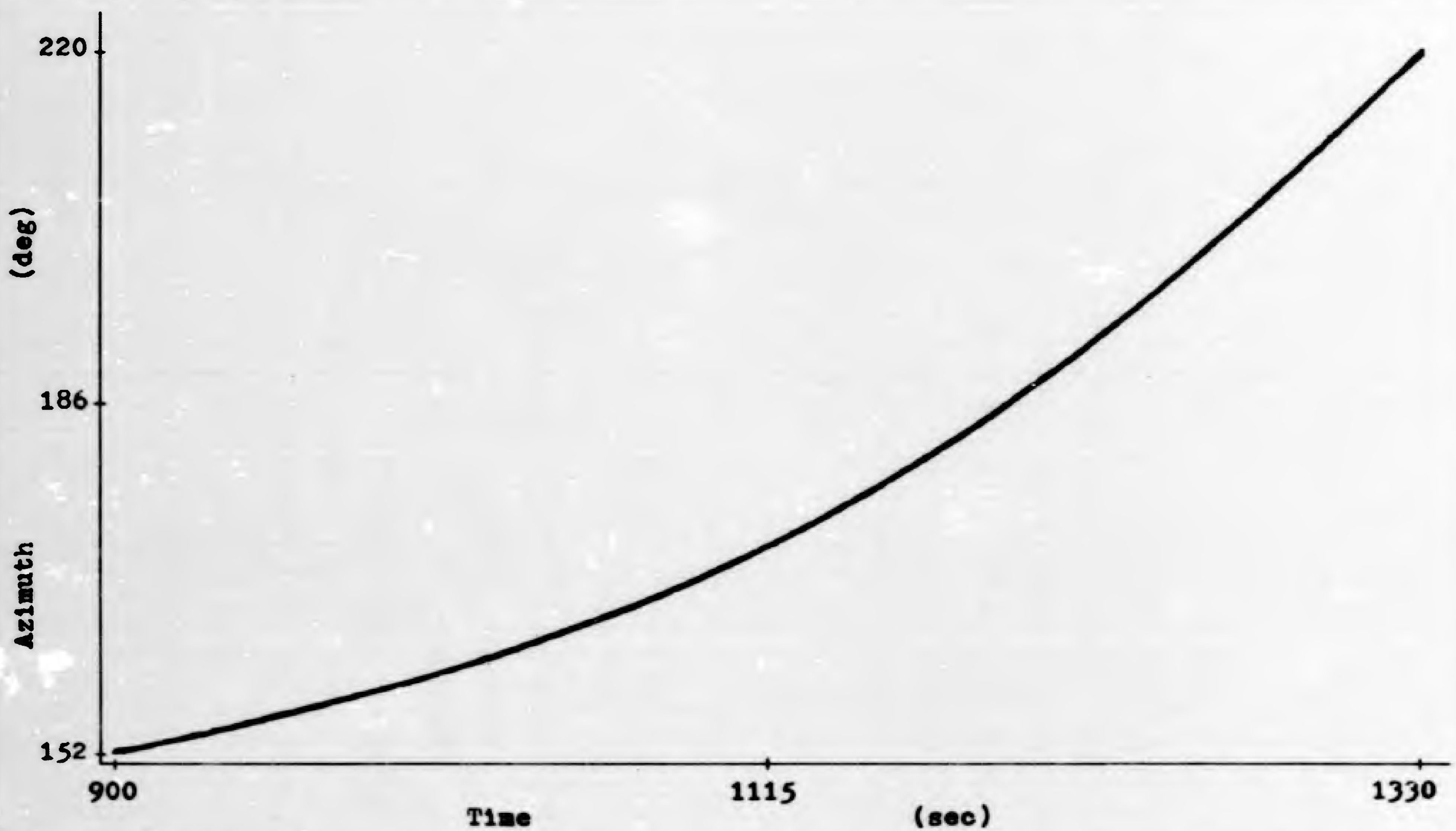


Fig 11 Azimuth Angle - Error Free and Curve Fit Plots - First Half Pass

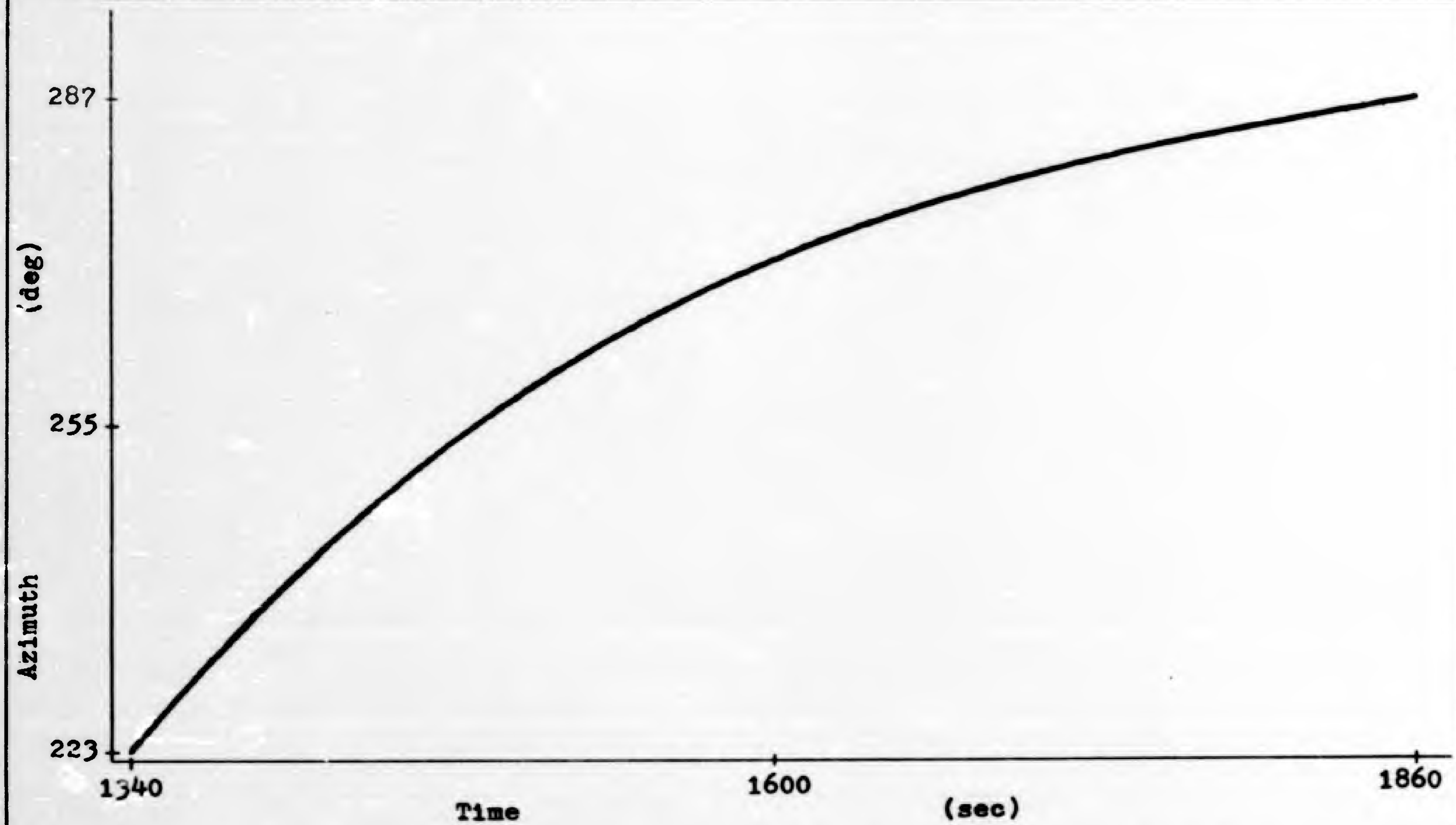


Fig 12 Azimuth Angle - Error Free and Curve Fit Plots - Last Half Pass

Fig 13 Elevation Angle - Error Free and Curve Fit Plots - First Half Pass

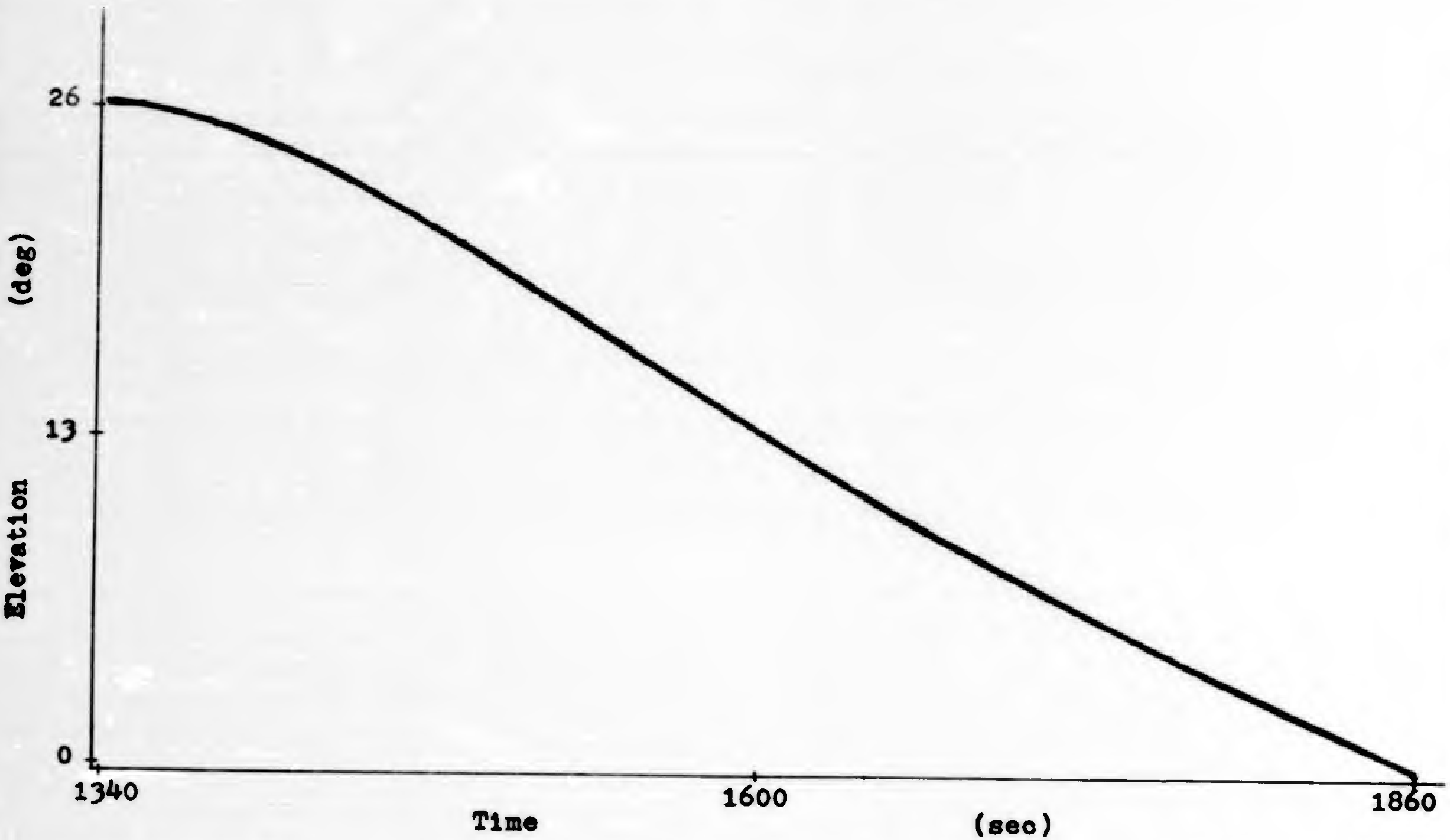


Fig 14 Elevation Angle - Error Free and Curve Fit Plots - Last Half Pass

described in Chapter IV. It was shown there that the topocentric vectors $\underline{\xi}'$ can be written

$$\underline{\xi}' = \underline{\xi}_0 + \underline{C}\,\underline{D}\,\underline{x} \tag{35}$$

where $\underline{x}$ is the vector in the geocentric inertial frame.

This can be solved for $\underline{x}$ in the form

$$\underline{x} = \underline{D}^{-1}\underline{C}^{-1}(\underline{\xi}' - \underline{\xi}_0) \tag{36}$$

As before, $\underline{\xi}_0$ is the vector describing the translation from the geocentric to the topocentric frame (ie, along the earth's radius); $\underline{C}$ is the matrix associated with the rotation to the station latitude and longitude, and $\underline{D}$ is the matrix associated with the frame rotation in going from inertial to the geocentric rotating system. Therefore these matrices are all considered known. The rotation matrices are all proper orthogonal and hence their inverse is identical to their transpose (Ref 6:12). The vector $\underline{x}$ is completely determined by the equation

$$\underline{x} = \underline{D}^T\,\underline{C}^T\,(\underline{\xi}' - \underline{\xi}_0) \tag{37}$$

The second step is to determine, utilizing these geocentric vectors, the orbital parameters. This determination is made using three of these vectors, $\underline{x}_i$, $i = 1, 2, 3$. The subscripts indicate the vector sequence in time.

The semilatus rectum of the orbit may be determined by

$$p = x_1 x_2 x_3 \left[\frac{\sin(\theta_2-\theta_3) + \sin(\theta_1-\theta_2) + \sin(\theta_3-\theta_1)}{x_3 x_2 \sin(\theta_2-\theta_3) + x_1 x_2 \sin(\theta_1-\theta_2) + x_1 x_3 \sin(\theta_3-\theta_1)} \right] \tag{38}$$

where p = the orbital semilatus rectum

θ = the angle between the line of nodes and the position vector

x = the magnitude of the position vectors.

(Ref 4:1.8-3)

The θ differences may be determined by the definition of the scalar product; for example

$$\theta_i - \theta_j = \cos^{-1}\left(\frac{\underline{x}_i \cdot \underline{x}_j}{x_i \, x_j}\right) \tag{39}$$

The principal value of the arc cosine is selected since it is not possible for the tracker to observe the satellite for more than 180 degrees of the orbit. Since the assumption has been made that the subscripts have been so ordered that they increase with time, angle $\theta_i - \theta_j$ is positive for $i > j$ and conversely negative for $i < j$.

The eccentricity may be determined from the equation

$$e^2 = \left[\frac{(p - x_1)^2}{x_1^{\,2}} + \frac{x \ (p - x_1) \cos(\theta_3 - \theta_1) - x_1 \ (p - x_3)}{x_1 x_3 \ \sin(\theta_3 - \theta_1)} \right]^2 \tag{40}$$

(Ref 4:1.8-3)

From the construction of the ellipse, the semi-major axis may be determined as

$$a = \frac{p}{1 - e} \tag{41}$$

Equations (38), (40), and (41) are developed in Appendix C.

The inclination of the orbit is equivalent to the angle between unit normals to the equatorial and orbital planes. The unit normal to equatorial plane is $\underline{i}_k$. The unit normal to the orbital plane may be expressed as the vector product of any two position vectors divided by the magnitude of the product; ie,

$$\underline{i}_n = \frac{\underline{x}_i \times \underline{x}_j}{x_i x_j \sin(\theta_j - \theta_i)} \tag{42}$$

where $\underline{i}_n$ = the desired normal.

(Ref 4:1.8-4)

The positive sense of the vector product is obtained by using a lower subscripted variable first in the product. The inclination angle is the arc cosine of the scalar product of these two unit vectors. Since the inclination angle is restricted by convention from 0 to 180 degrees, the principal value of the arc cosine is taken.

The determination of the right ascension of the ascending node necessitates the development of the equation of the orbital plane. Since the plane contains the origin, it is of the form

$$Ax + By + Cz = 0 \qquad (43)$$

$$Dx + Ey + z = 0 \qquad (44)$$

where $D = \frac{A}{C}, E = \frac{B}{C}$

By substituting the coordinates of two of the vectors into this equation, two equations are obtained which may be solved for D and E, thus yielding the plane equation. Since the line of nodes is the intersection of the orbital plane and the equatorial plane, it is the common solution of the equations

$$Dx + Ey + z = 0 \qquad (44)$$

and

$$z = 0 \qquad (45)$$

from which **is obtained**

$$\Omega = \tan^{-1} \left(\frac{Y}{x}\right)$$

$$= \tan^{-1} \left(\frac{-D}{E}\right) \qquad (46)$$

The question of which value of the arc tangent to select is resolved by noting the sign of the cosine of the angle between the y-axis and the unit normal to the orbital plane previously computed. This cosine is the scalar product of the orbital unit vector and a unit vector in the y-direction. If the cosine is negative, the angle between the two unit vectors is greater than 90° but less than 270° and the ascending node is in the front half of the equatorial plane; hence the principal value of the arc tangent is used. If the cosine is positive, the principal value plus 180° is used.

The argument of perigee is found by first determining the unit vector in the direction of perigee and then determining the angle between it and a unit vector in the direction of the ascending node. The unit vector in the direction of perigee may be expressed as a linear combination of two of the position vectors; ie,

$$\underline{i}p = A\underline{x}_1 + B\underline{x}_3 \qquad (47)$$

where $\underline{i}p$ is the desired unit vector

A and B are linear coefficients.

As developed in Appendix C, **expressions for A and B are**

$$A = \frac{1}{ex_1 \sin^2(\theta_3 - \theta_1)}[(\frac{p}{x_1} - 1) - (\frac{p}{x_3} - 1) \cos (\theta_3 - \theta_1)] \qquad (48)$$

$$B = \frac{1}{ex_3 \sin^2(\theta_3 - \theta_1)}[(\frac{p}{x_3} - 1) - (\frac{p}{x_1} - 1) \cos (\theta_3 - \theta_1)] \qquad (49)$$

(Ref 1:1.8-4)

The argument of perigee is found by taking the arc cosine of the scalar product of the perigee unit vector and the unit vector in the direction of the ascending node; from which

$$\omega = \cos^{-1} [(\underline{i}_x \cos \Omega + \underline{i}_y \sin \Omega) \cdot (A\underline{x}_1 + B\underline{x}_3)] \qquad (50)$$

The principal value of the arc cosine is used if the z-component of the perigee unit vector is positive; otherwise the negative of the principal value is used.

The final parameter to be determined is the time of perigee passage. It is computed by use of Kepler's equation (eq (29)). The angle θ_1 between the ascending node and the position vector x_1 is determined by the scalar product so that

$$\theta_1 = \cos^{-1}\left(\frac{\underline{i}_N \cdot \underline{x}_1}{x_1}\right) \tag{51}$$

where $\underline{i}_N$ is the unit vector in the direction of the node.

If the third component of $\underline{x}_1$ is positive, the principal value of the arc cosine is used; if not, the negative of the principal value is used. The true anomaly is determined as the difference of θ_1 and the argument of perigee.

$$f_1 = \theta_1 - \omega \tag{52}$$

The eccentric anomaly is obtained using eq (32)

$$\tan\frac{E_1}{2} = \sqrt{\frac{1-e}{1+e}}\tan\frac{f_1}{2} \tag{32}$$

Using Kepler's equation, then **the result becomes**

$$t - \tau = \frac{P}{2\pi}(E_1 - e\sin E_1)$$

or

$$\tau = t - \frac{P}{2\pi}(E_1 - e\sin E_1) \tag{53}$$

where τ is the time of perigee passage. All six orbital parameters are now determined.

As noted above, three vectors are needed to completely determine the orbit. In RADYN, six different sets of triple vectors are utilized to compute six sets of orbital elements from the polynomial evaluation. These sets of elements are then averaged to determine a single set which represents the curve fit. These elements are compared to the original set of elements to accurately gauge how well the curve fit approximates the initial orbit. More importantly, they serve as the standard for determining the accuracy of the set which is computed from the system response. The computation of this response is the subject of the next chapter.

VI. Dynamics Calculations

In the preceeding chapters, methods have been developed for computing sets of azimuth and elevation angles and slant ranges with varying time from a selected set of orbital parameters, and then computing the parameters from the data sets. The purpose of this chapter is to discuss the method for determining the system response as it tracks the satellite. In substance, this amounts to applying the method of the Laplacė transformation to the transfer function developed in Chapter II and the time polynomials of Chapter V.

The differential equation of either the azimuth or elevation servo system may be written from the expression of the transfer function (eq (5)). Again using ϕ as either of these angles, eq (5) may be written

$$(1 + G)\ \phi_m(s) = G\phi(s) \tag{54}$$

or

$$(s^2 + K_1 s + K_2)\ \phi_m(s) = K_2\phi(s) \tag{55}$$

The differential equation of the system is

$$(D^2 + K_1 D + K_2)\ \phi_m(t) = K_2\phi(t) \tag{56}$$

After retransforming the equation to account for initial conditions it is

$$\phi_m(s) = \frac{K_2\,\phi(s)}{s^2 + K_1 s + K_2} + \frac{\phi'_m(0) + \phi_m(0)\,(s + K_1)}{s^2 + K_1 s + K_2} \tag{57}$$

The second term on the right hand side of eq (57) is the initial conditions operator. The initial conditions assumed for this problem are that the tracking antenna has been preprogrammed to the proper elevation and azimuth angles prior to the arrival of the satellite and that it is sitting quiescent when the satellite comes above the horizon. These assumptions are valid both for the initiation of the pass and the initiation of the second half of the pass (ie, when the second set of polynomials are to be used). Therefore, $\phi'_m(0)$ is zero and $\phi_m(0) = \phi(0)$, and eq (57) is written

$$\phi m(s) = \frac{K_2\phi(s)}{s^2 + K_1 s + K_2} + \frac{\phi_m(0)\,(s + K_1)}{s^2 + K_1 s + K_2} \tag{58}$$

where K_2 is the system natural frequency squared

K_1 is twice the product of the natural frequency and the damping ratio.

The input, $\phi(t)$, to the system is a time polynomial of the form

$$\phi(t) = \phi_0 + \sum_{i=1}^{n} \phi_i t^i \tag{59}$$

As indicated in Chapter V a fourth order polynomial was selected for this analysis so that the specific form is

$$\phi(t) = \phi_0 + \phi_1 t + \phi_2 t^2 + \phi_3 t^3 + \phi_4 t^4 \qquad (60)$$

The Laplace transform of this input is

$$\phi(s) = \frac{\phi_0}{s} + \frac{\phi_1}{s^2} + \frac{2\phi_2}{s^3} + \frac{6\phi_3}{s^4} + \frac{24\phi_4}{s^5} \qquad (61)$$

The Laplace transform of the system output may now be written

$$\phi_m(s) = \left(\frac{\phi_0}{s} + \frac{\phi_1}{s^2} + \frac{2\phi_2}{s^3} + \frac{6\phi_3}{s^4} + \frac{24\phi_4}{s^5}\right)\left(\frac{K_2}{s^2 + K_1 s + K_2}\right) +$$

$$\frac{\phi_m(0)\,(s + K_1)}{s^2 + K_1 s + K_2} \qquad (62)$$

The inverse Laplace transformation will be taken of this expression term by term. The inverse of the first term is

$$\phi_{m0}(t) = \phi_0\left[1 + \frac{2\sqrt{K_2}\, e^{-\frac{1}{2}K_1 t} \sin\left(\frac{1}{2}\sqrt{4K_2 - K_1^2}\; t + \psi_a\right)}{\sqrt{4K_2 - K_1^2}}\right] \qquad (63)$$

where $\psi_a = \tan^{-1}\left(\frac{\sqrt{4K_2 - K_1^2}}{-K_1}\right)$

(Ref 12:492)

The inverse of the second term is

$$\phi_{m_1}(t) = \phi_1 [t - \frac{K_1}{K_2} + \frac{2e^{-\frac{1}{2}K_1 t} \sin (\frac{1}{2}\sqrt{4K_2 - K_1^2}\, t + \psi_b)}{\sqrt{4K_2 - K_1^2}}] \tag{64}$$

where $\psi_b = 2 \tan^{-1} (\frac{\sqrt{4K_2 - K_1^2}}{K_1})$

For the succeeding terms no inverses are available in standard listings of Laplace transformation pairs. A method for the separation of the factors into terms is given in Ref 13 and is repeated below:

$$\frac{\prod_i (s + a_i)}{(s + b_0)^m \prod_j (s + b_j)} = \sum_{\ell=1}^{m} \frac{C_\ell}{(s = b_0)^\ell} + \sum_{k=1}^{n} \frac{R_{k1}}{(s + b_k)} \tag{65}$$

for $i = 1,2,\ldots,p$

$j = 1,2,\ldots,n$

$p \quad n + 1$

where $R_{k\ell} = \left. \frac{(s + b_k) \prod_i (s + a_i)}{(s + b_0)^{m-\ell+1} \prod_j (s + b_j)} \right|_{s = -b_k}$

$$C_\ell = \delta_{(m+n-\ell)}(p) - \sum_{k=1}^{n} R_{k\ell} \quad \text{for } 1 < \ell \leq m-1$$

$$C_m = \left. \frac{\prod_i (s + a_i)}{\prod_j (s + b_j)} \right|_{s = -b_0}$$

$$\delta(m + n - \ell)(p) = \begin{cases} 1 \text{ for } m + n - \ell = p \\ 0 \text{ for } m + n - \ell \neq p \end{cases}$$

(Ref (13:5)

The inverse transformations resulting from use of this identity are listed below in order:

$$\phi_{m2}(t) = 2\phi_2 \{ \frac{K_1^{\,2} - K_2}{K_2^{\,2}} - \frac{K_1 t}{K_2} + \frac{t^2}{2} -$$

$$\frac{e^{-\frac{1}{2}K_1 t}}{K_2^{\,2}\sqrt{4K_2 - K_1^{\,2}}} [K_1 (K_1^{\,2} - 3K_2) \sin (\tfrac{1}{2}\sqrt{4K_2 - K_1^{\,2}}\, t) +$$

$$2\sqrt{4K_2 - K_1^{\,2}} (K_1^{\,2} - K_2) \cos (\tfrac{1}{2}\sqrt{4K_2 - K_1^{\,2}}\, t)] \} \qquad (66)$$

$$\phi_{m3}(t) = 6\phi_3 \{ - \frac{K_1 (K_1^{\,2} - 2K_2)}{K_2^{\,3}} + \frac{(K_1^{\,2} - K_2)t}{K_2^{\,2}} -$$

$$\frac{K_1 t^2}{2K_2} + \frac{t^3}{6} + \frac{e^{-\frac{1}{2}K_1 t}}{K_2^{\,3}\sqrt{4K_2 - K_1^{\,2}}} [\tfrac{1}{4}(\tfrac{1}{2}K_1^{\,4} - 3K_1^{\,2}(4K_2 - K_1^{\,2}) +$$

$$\tfrac{1}{2}(4K_2 - K_1^{\,2})^2) \sin (\tfrac{1}{2}\sqrt{4K_2 - K_1^{\,2}}\, t) +$$

$$K_1 \sqrt{4K_2 - K_1^{\,2}} (K_1^{\,2} - 2K_2) \cos (\tfrac{1}{2}\sqrt{4K_2 - K_1^{\,2}}\, t)] \}$$

(67)

$$\phi_{m4}(t) = 24\phi_4 \{\frac{K_1^{4} - 3K_1^{2}K_2 + K_2^{2}}{K_2^{4}} - \frac{K_1 (K_1^{2} - 2K_2)t}{K_2^{3}} +$$

$$\frac{(K_1^{2} - K_2)t^2}{2K_2^{2}} - \frac{K_1 t^3}{6K_2} + \frac{t^4}{24} - \frac{2e^{-\frac{1}{2}K_1 t}}{K_2^{4}\sqrt{4K_2 - K_1^{2}}}[\frac{K_1}{16}(\tfrac{1}{2}K_1^{4} -$$

$$5K_1^{2}(4K_2 - K_1^{2}) + 2.5(4K_2 - K_1^2)^2)\sin(\tfrac{1}{2}\sqrt{4K_2 - K_1^{2}}\,t) +$$

$$\frac{\sqrt{4K_2 - K_1^{2}}}{16}(2.5K_1^{4} - 5K_1^{2}(4K_2 - K_1^{2}) +$$

$$(4K_2 - K_1^{2})^2)\cos(\tfrac{1}{2}\sqrt{4K_2 - K_1^{2}}\,t)]\} \qquad (68)$$

The inverse of the initial condition operator is

$$\phi_{m5}(t) = \frac{2\phi_m(0)\sqrt{K_2}\, e^{-\frac{1}{2}K_1 t}}{\sqrt{4K_2 - K_1^{2}}} \sin(\tfrac{1}{2}\sqrt{4K_2 - K_1^{2}}\, t + \psi_c) \qquad (69)$$

where $\psi_c = \tan^{-1}\left(\frac{\sqrt{4K_2 - K_1^{2}}}{K_1}\right)$

The system response is

$$\phi_m(t) = \sum_{i=0}^{5} \phi_{mi}(t) \qquad (70)$$

The system response is now defined for all time. Plots of azimuth and elevation angles with and without the effects of system dynamics are shown in Figs 15 thru 18. The plots which are free of dynamic errors are from

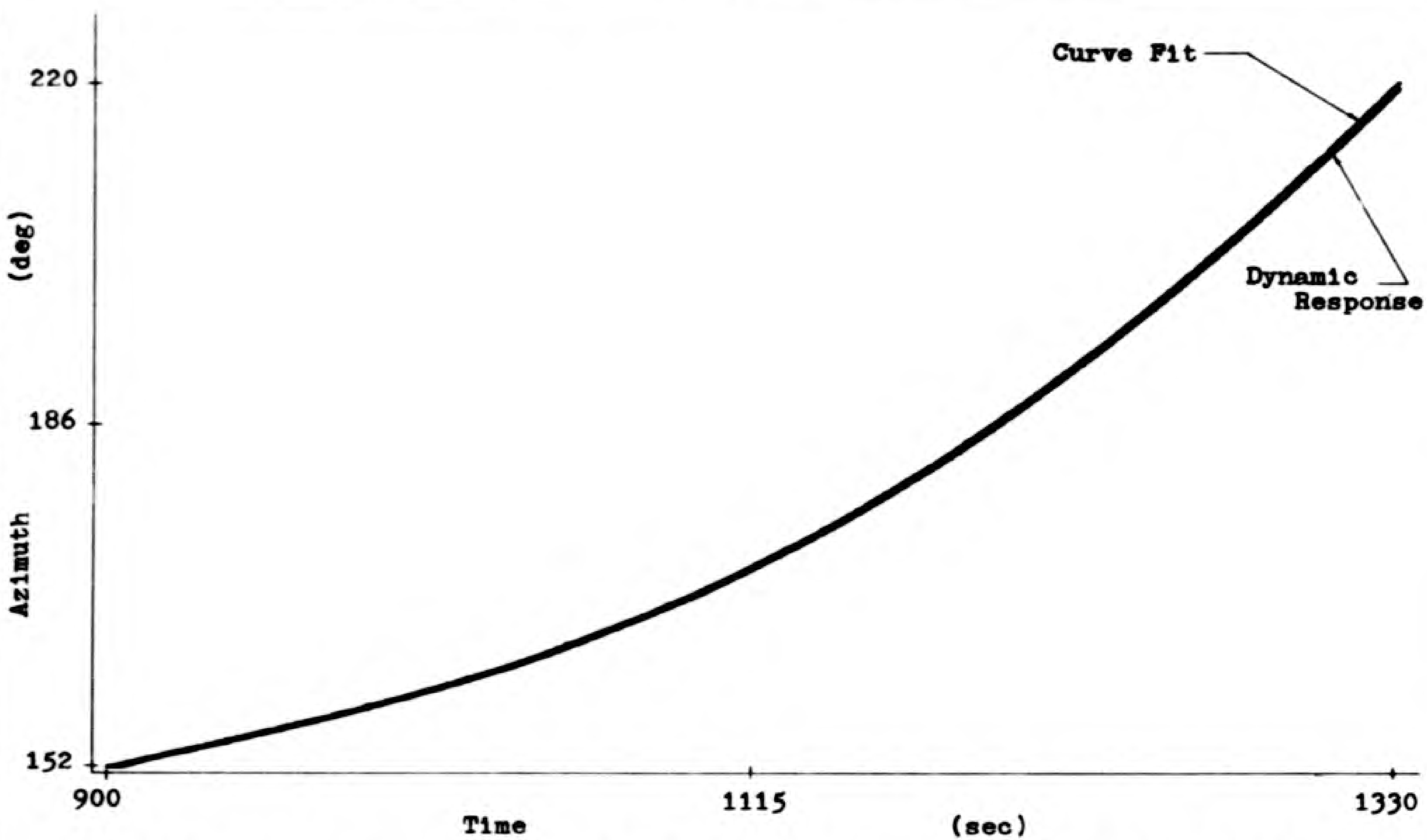


Fig 15 Azimuth Angle - Curve Fit and Dynamic Response Plots - First Half Pass

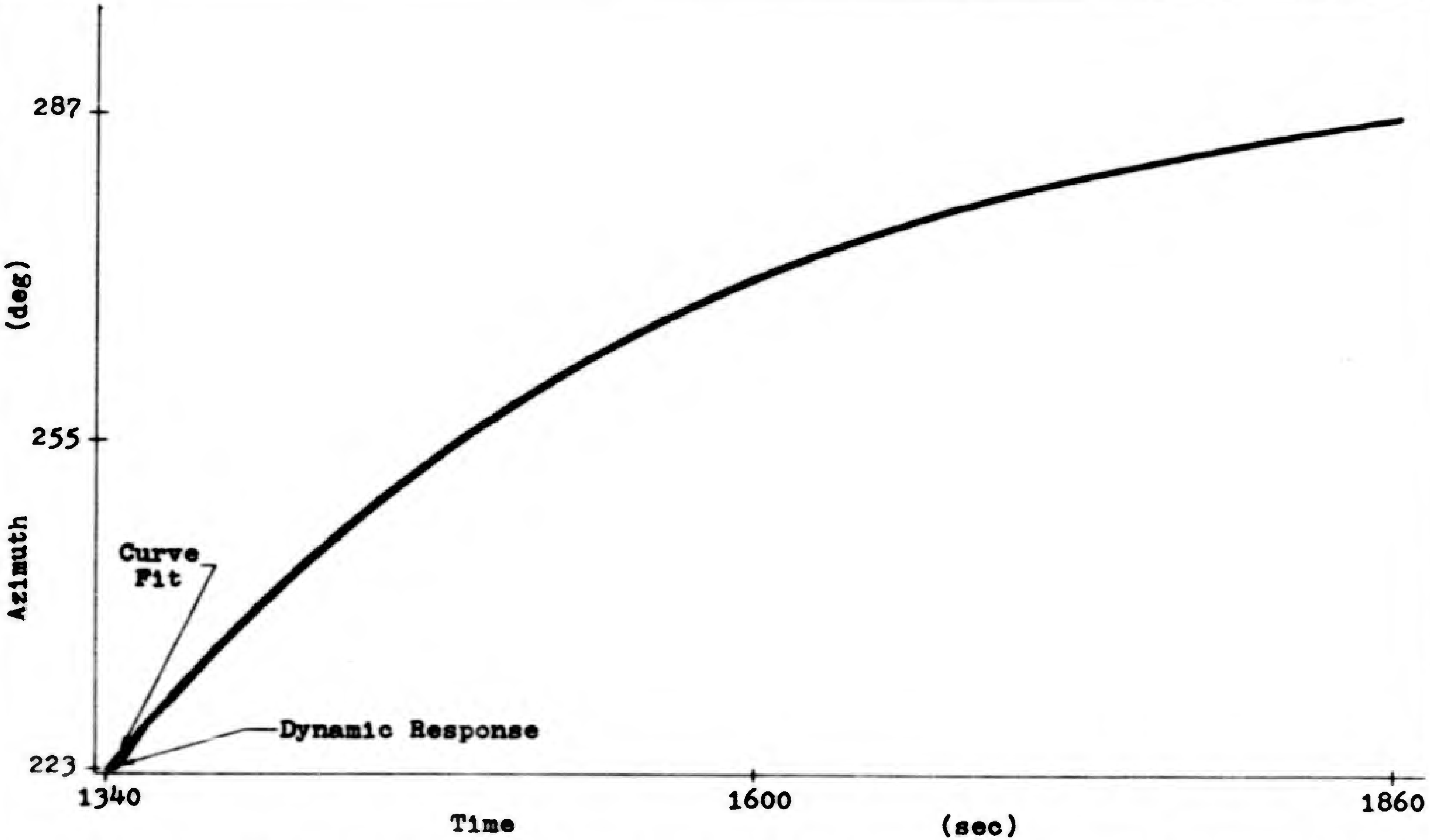


Fig 16 Azimuth Angle - Curve Fit and Dynamic Response Plots - Last Half Pass

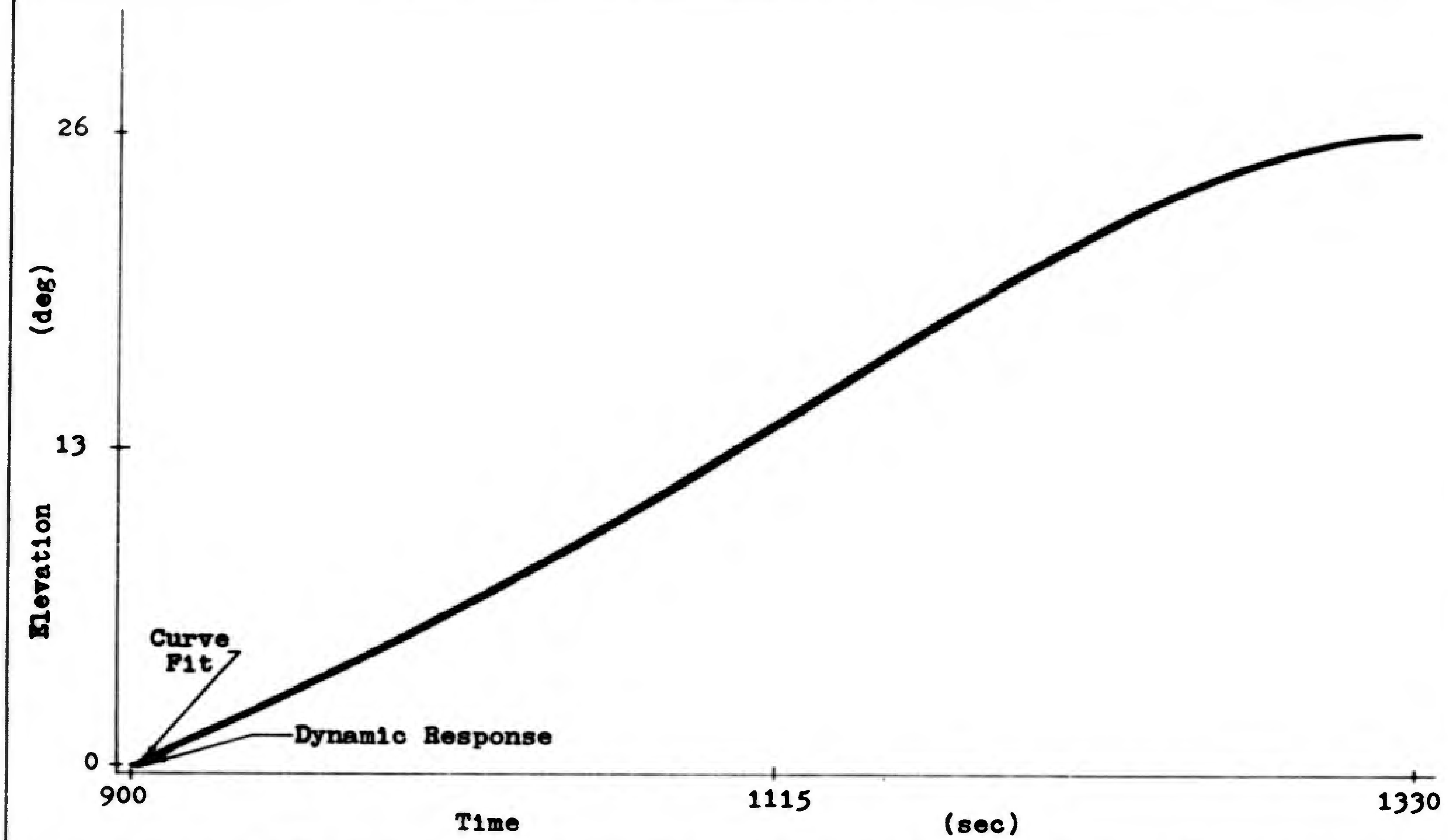


Fig 17 Elevation Angle - Curve Fit and Dynamic Response Plots - First Half Pass

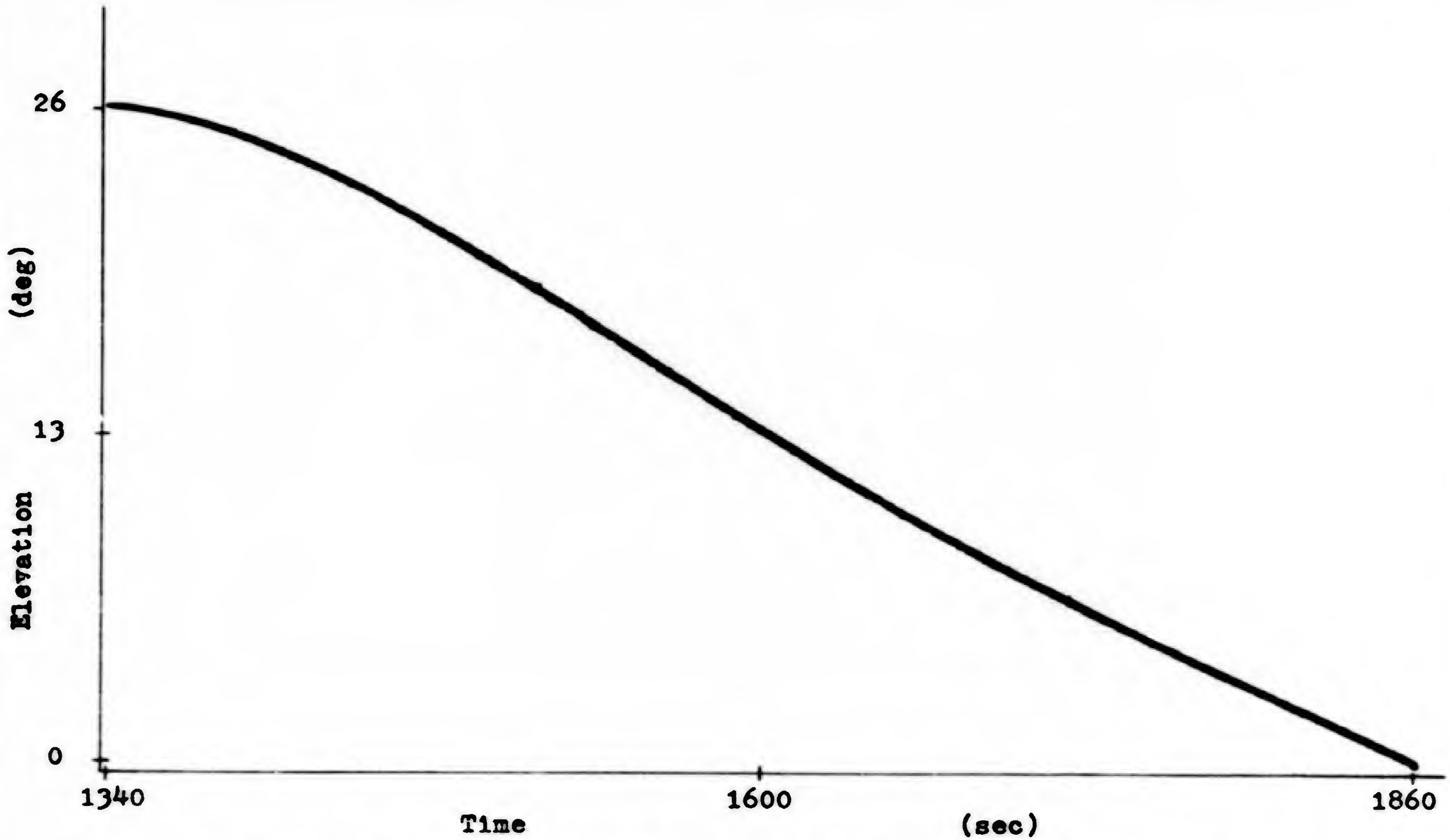


Fig 18 Elevation Angle - Curve Fit and Dynamic Response Plots - Last Half Pass

the polynomial evaluation; the response plots are from the evaluation of eq (70). The data which the plots represent are from parameter set 6 of Table I.

The remaining task is to perform the topocentric to orbit parameters computations indicated in Chapter V. A set of parameters which include system response (computed from the evaluation of eq (70)) are then compared with a set which does not (computed from the evaluation of the time polynomials); this comparison indicates the error introduced in the orbit prediction by system dynamics. In RADYN, the errors computed are formed by differencing the separate parameters of each case and by forming the relative errors. The RMS difference in position for a single orbit is also calculated. Thus the effect of the tracker dynamics upon orbit prediction is demonstrated.

VII. Results and Conclusions

This chapter briefly discusses the results obtained by performing the calculations detailed in the previous chapters upon the orbits of Table I. They will be discussed in the natural order in which they occur during the solution of a problem. A typical example is given as Appendix A. Selected for the sample was Case 6 of Table I, but it is not untypical of the results of any of the cases examined. Dimensions of the variables shown in the appendix are: (1) angles in degrees; (2) distances in miles; (3) time in seconds.

The first area of consideration lies in the area of fitting the azimuth and elevation angles to polynomials in time. The data does not approximate a low order polynomial if the time interval of the observation is very large. This fact is verified by actually performing the fits; however, it could be surmised in advance by referring to fig 10. This leads to one of three choices. The first of these is to limit the length of the period for any single fit, and perform several fits. This was partially attempted in this investigation in that the station passes were divided into two parts - that preceding, and that succeeding the maximum elevation angle of the pass. However, this was still insufficient to permit great accuracy in a low order fit. Each of the two parts into which the passes were divided might well have themselves been subdivided to provide an improvement in accuracy.

The second possibility for increasing the accuracy of the fit is to use a higher ordered polynomial. As previously noted, investigation of such a possibility indicated a monotonic decrease in the root mean square value of the error involved in the fitting process until reaching the point where calculational errors assumed sufficient importance to cause the fit error to vary randomly, although at a low level. This occurred generally between a ninth and twelth order polynomial, although for some cases it did not occur until the seventeenth order was reached. An error analysis on the orbit parameters from the evaluation of an eleventh order fit indicated the root mean square error in prediction of position during a single orbit (not considering the tracker response) to be less than one mile. This is a considerable improvement over the twenty miles or so from the fourth order fit. While all of this suggests the use of a higher order polynomial, the added complexity in the processing of the inverse transform argue strongly against it. Should higher order be considered, there is no way to determine, a priori, the degree required for a particular orbit to obtain a certain accuracy specification.

The final suggestion for improvement of the fit process is to attempt to fit the data to some function other than a polynomial. This approach was not attempted; however, it is possible that the data better approximates some other type function. This could be the subject of a future investigation. It should be borne in mind, however, that eventually the inverse transform must be taken (if the method of this paper is to be used) of whatever function is employed, and for this reason, there is a strong argument for simplicity. While an improvement in the fit could significantly decrease the error in the predicted satellite position, it would have very little effect on the form of the response of the tracking system.

The set of orbit parameters used to represent the curve fit output and the servo system output were the result of averaging six sets of orbit parameters computed from the appropriate data. The number six was arbitrarily selected as a base for the averaging; it was chosen as a number that was large enough to fairly represent the results of the fit or servo system while at the same time not requiring unnecessary computer time. Six sets, however is a rather small sample, and the averaged values may suffer a lack of accuracy from temporary system inaccuracies such as initial dynamic response. In actual practice, such computations are performed using all the data sets with correspondingly more representative results. With this in mind, a possible source of error in the comparison of the parameter set from the tracker response to the set from the curve fit is that the latter may not be representative of the data of the curve fit. Since the time polynomials only approximate the error-free data, a unique set of parameters of the data from the polynomial evaluation does not exist. The orbit computed from such data will vary depending upon the manner in which the data is used. For this reason, it is possible that the parameter set computed from evaluating the selected points on the polynomial does not represent the orbit which would be obtained by considering more of the data.

The relative errors of the predicted orbits are presented in Table II. The magnitude of the errors has little significance since they depend upon the type of tracking system selected for the investigation. The system constants and initial conditions are important in determining the response as well as the form of the transfer function. Consequently, the errors vary greatly depending upon the mathematical model utilized in the analysis. The performance of the system selected for these nine input sets may be compared by referring to Table II. The important point is that the response of any tracking system may be determined by this computer program. The arithmetic statements in subprogram DYNAM must be altered to represent the inverse transformation of the appropriate input and system dynamic description to consider any

Table II
Relative Errors after Considering Radar Dynamics

Parameter Set	1	2	3	4	5	6	7	8	9
Eccentricity	.039	.15	1.18	.043	.041	.032	.039	.0073	.065
Semi-major axis	.013	.0038	.00069	.0017	.00048	.0016	.047	.027	.075
Right ascension of ascending node	.000094	.00067	.0012	.000010	.00040	.00064	.00054	.0014	.0024
Argument of perigee	2.12	.48	.35	.048	.040	.059	.22	.14	.26
Inclination	.00079	.0021	.043	.000093	.000082	.0013	.012	.0016	.0079
Time of perigee passage	.14	.041	.0021	.0035	.0026	.0053	.014	.0095	.023
RMS predicted position	.064	.047	.026	.0045	.0046	.0050	.29	.16	.51

other system; however this is the only change to the program required in order to change the form of either the input or the system transfer function.

Table III briefly presents response errors for the tracking system using different system constants. The results in Table II were derived from a system with a damping ratio of .3 and natural frequency of .3 radians per second. Table III presents several combinations of these tracker parameters for data set 6. Although only a few different dynamic systems were checked during the development of this paper, it appears that increasing the natural frequency and decreasing the damping ratio improved the response characteristics; since an attempt at tracker optimization was not a goal of this paper, however, no extensive investigation was made in this direction.

Compensation for the effect of the system dynamic lag might be accomplished by the inverse of the procedures described in this paper. To do this, it is proposed that the actual tracker output be fitted to time polynomials and serve as inputs to the inverse of the system transfer function. The resultant solution to this problem should be devoid of response lag. The possible problems of using such a method are two; first, the mathematical model of the system must be accurately known and second, the effect of the many other errors of radar data,

Table III
Error Variation with System Constants

Damping Ratio

Natural Frequencies	.3	.5	.7	.85
.1		3.26	3.57	3.82
.3	.50		1.21	
.5			.73	

This Table shows relative root mean square error in position (times 100) for several different combinations of tracking system constants. Orbital data is from Set 6 of Table I.

as briefly mentioned in Chapter I, must be considered. Consideration of these ideas suggest two possible areas for future investigation. The first of these is to perform the analysis herein presented using the mathematical model of an existing tracking system. The resultant error could be compared with the error empirically observed in the tracker by smoothing many passes as an indication of the relative importance of the dynamic error.

The second area to be examined is the feasibility of eliminating the dynamic error by an inverse of the method developed in this paper. This would involve fitting the observed data to a time polynomial (or some other mathematical model of the input) and applying it to the inverse of the tracker transfer function. For such a process to have any meaning, the system dynamic characteristic would have to be known to a high degree of accuracy. Again, verification of results could be made against empirically observed data which has been smoothed over several passes.

The final conclusion is that the effect of tracker dynamics on orbit prediction can be determined by the method of this paper: that is,

(1) For a selected orbit, compute a series of topocentric coordinates against time to represent a pass of the satellite over the tracker;

(2) Fit the angular data so obtained to time polynomials;

(3) By the method of the Laplace transformation determine the response of the tracker to the polynomials as inputs to the transfer function of the tracking system;

(4) Compute a set of orbit parameters from data obtained from the evaluation of the functions derived in steps (3) and (4) above. Compare the parameter sets so obtained to determine the error introduced by the tracking system.

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APPENDIX A

SAMPLE PROBLEM

THIS APPENDIX CONTAINS A SAMPLE PROBLEM SOLUTION. THE INFORMATION CONTAINED ON THE FOLLOWING PAGES IS THAT WHICH WOULD BE COMPUTED DURING THE SOLUTION PROCESS. IT IS IN THE SAME FORMAT USED IN RADYN. INPUT DATA IS FROM SET 6 IN TABLE ONE.

THE GENERAL FLOW OF THE PROBLEM IS

(1) DEVELOP THE ERROR FREE DATA

(2) FIT THIS DATA TO A FOURTH ORDER POLYNOMIAL AFTER DETERMINING THE ERROR IN FITTING IT TO ALL ORDERS FROM ONE TO TWENTYFOUR.

(3) EVALUATE THE TRACKING SYSTEM RESPONSE TO THE POLYNOMIALS AS INPUTS.

(4) COMPUTE THE ORBIT PARAMETERS USING THE TWO SETS OF COMPUTED DATA (FITTED DATA AND RESPONSE DATA). THE PARAMETERS COMPUTED FROM THE FITTED DATA ARE COMPARED TO THE INITIAL INPUT. THE PARAMETERS FROM THE TRACKER RESPONSE ARE COMPARED TO THAT OF THE POLYNOMIAL. A POSITION ERROR ANALYSIS IS PERFORMED FOR ONE ORBIT.

THE SEQUENCE OF THE OUTPUT ON THE FOLLOWING PAGES IS PRESENTED BELOW

(1) THE TRACKING STATION COORDINATES AND INPUT ORBIT PARAMETERS.

(2) THE VALUES OF ELEVATION, AZIMUTH, RANGE AND TIME AT THE POINT WHERE THE DATA IS DIVIDED FOR CURVE FITTING.

(3) A PRINT-OUT OF THE DATA COMPUTED FROM THE INPUT ORBIT PARAMETERS.

(4) A CURVE FIT ERROR ANALYSIS FOR THE DATA OF THE FIRST-HALF PASS.

(5) THE COEFFICIENTS OF THE AZIMUTH AND ELEVATION TIME POLYNOMIALS COMPUTED FROM THE DATA OF THE FIRST HALF PASS (IN THIS EXAMPLE, A FOURTH ORDER POLYNOMIAL WAS USED IN FITTING THE DATA).

(6) EVALUATION OF THE POLYNOMIALS FOR THE FIRST-HALF PASS.

(7) EVALUATION OF THE TRANSFER FUNCTION FOR THE FIRST-HALF PASS.

(8) THE NEXT SEVERAL PAGES ARE THE SAME AS ABOVE EXCEPT FOR THE SECOND-HALF PASS.

(9) SIX SETS OF ORBIT PARAMETERS COMPUTED FROM AN EVALUATION OF THE TIME POLYNOMIALS. THESE SIX SETS ARE SUBSEQUENTLY AVERAGED TO FORM THE SET WHICH REPRESENT THE INPUT TO THE TRACKER. A BRIEF ERROR ANALYSIS COMPARES THE SET SO OBTAINED TO THE INPUT SET.

(10) SIX SETS OF PARAMETERS ARE PRINTED WHICH RESULT FROM THE EVALUATION OF THE TRACKER SYSTEM. THESE SETS ARE AVERAGED TO FORM THE RESPONSE SET. THIS SET IS COMPARED WITH THE PREVIOUS SET TO DETERMINE THE SYSTEM DYNAMIC ERROR.

TRACKING STATION COORDINATES.......... -120.580 DEG E LONG
.... 34.750 DEG N LAT

ORBIT PARAMETERS

ECCENTRICITY.......................... 0.0500000

SEMIMAJOR AXIS........................ 4560.0000 MILES

RIGHT ASCENSION OF ASCENDING NODE.. -176.380 DEG E OF VERNAL EQUINOX

ARGUMENT OF PERIGEE................. -40.000 DEG

LONGITUDE OF PERIGEE................ -216.380 DEG

INCLINATION.......................... 40.000 DEG

TIME OF PERIGEE PASSAGE............. 0. SEC

ORBITAL PERIOD...................... 0.62565131E 04 SEC

LONGITUDE OF VERNAL EQUINOX AT T=0... 35.000 DEG

EARTH ROTATION RATE................. 0.41780741E-02 DPS

MIDPASS PARAMETERS

TIME = 0.13804548E 04 SECONDS

ELEVATION = 0.26174C45E 02 DEGREES

AZIMUTH = 0.23390184E 03 DEGREES

RANGE = 0.10611650E 04 MILES

ERROR FREE CATA

NO.	TIME	ELEVATION	AZIMUTH	RADAR RANGE
1	9C0.0C	C.2740960E-04	0.1532413E 03	C.1977569E 04
2	900.01	C.5915274E-03	C.1532418E 03	C.1977534E 04
3	910.01	0.5720750E 00	0.1538306E 03	C.1942720E 04
4	920.01	C.1152443E C1	C.1544412E 03	C.1908125E 04
5	930.01	C.1741995E 01	C.155C747E 03	C.1873764E 04
6	940.01	C.2341011E 01	0.1557326E 03	C.1839653E 04
7	950.01	C.2949788E C1	0.1564161E 03	0.1805808E 04
8	960.01	0.3568594E 01	0.1571267E 03	C.1772246E 04
9	970.01	C.4197698E 01	C.1578659E 03	C.1738987E 04
10	980.01	0.4837346E 01	0.1586354E 03	0.1706C50E 04
11	990.01	C.5487757E 01	0.1594369E 03	C.1673457E 04
12	1000.01	C.6149136E 01	0.1602723E 03	C.1641228E 04
13	1010.01	C.6821620E 01	C.1611435E 03	C.1609390E 04
14	1020.01	C.7505322E 01	0.1620527E 03	C.1577967E 04
15	1030.01	C.8200287E 01	C.1630019E 03	C.1546985E 04
16	1040.01	0.8906484E 01	0.1639935E 03	C.1516475E 04
17	1050.01	0.9623797E 01	0.1650300E 03	C.1486465E 04
18	1060.01	0.1035200E C2	0.1661139E 03	0.1456990E 04
19	1070.01	C.1109075E 02	0.1672479E 03	C.1428C82E 04
20	1080.01	0.1183955E 02	0.1684347E 03	C.1399780E 04
21	1090.01	0.1259774E 02	C.1696773E C3	C.1372120E 04
22	1100.01	0.1336445E 02	C.1709786E 03	C.1345144E 04
23	1110.01	C.1413858E 02	C.1723416E 03	C.1318896E 04
24	1120.01	0.1491881E 02	0.1737694E 03	0.1293419E 04
25	1130.01	C.1570348E 02	C.175265CE 03	C.1268761E 04
26	1140.01	0.1649066E 02	C.1768314E 03	0.1244971E 04
27	1150.01	0.1727803E 02	0.1784715E 03	0.1222100E 04
28	1160.01	0.1806275E 02	C.1801878E 03	C.1200202E 04
29	1170.01	0.1884219E 02	C.1819829E 03	C.1179330E 04
30	1180.01	0.1961245E 02	0.1838587E 03	C.1159539E 04
31	1190.01	0.2036971E 02	0.1858167E 03	C.1140886E 04
32	1200.01	0.2110967E 02	0.1878579E 03	C.1123428E 04
33	1210.01	C.2182760E 02	C.1899825E 03	C.1107219E 04
34	1220.01	C.2251848E 02	0.1921897E 03	C.1092315E 04
35	1230.01	0.2317696E 02	0.1944777E 03	0.1078770E 04
36	1240.01	0.2379757E 02	0.1968438E 03	C.1066634E 04
37	1250.01	C.2437477E 02	0.1992837E 03	C.1055954E 04
38	1260.01	C.2490305E 02	0.2017921E 03	C.1046773E 04
39	1270.01	0.2537720E 02	0.2043622E 03	0.1039130E 04
40	1280.01	0.2579235E 02	C.2069858E 03	C.1033C57E 04
41	1290.01	C.2614424E 02	0.2096536E 03	C.1028581E 04
42	1300.01	0.2642931E 02	0.2123553E 03	C.1025719E 04
43	1310.01	0.2664483E 02	0.215C795E 03	C.1024484E 04
44	1320.01	0.2678907E 02	0.2178145E 03	C.1024878E 04
45	1330.01	0.2686130E 02	0.2205481E 03	C.1026897E 04
46	1340.01	0.2686183E 02	C.2232682E 03	C.1030530E 04
47	1350.01	0.2679199E 02	C.2259630E 03	C.1035756E 04
48	1360.01	0.2665407E 02	0.2286214E 03	C.1042549E 04

ERROR FREE CATA

NO.	TIPE	ELEVATION	AZIMUTH	RADAR RANGE
49	137C.01	C.2645118E 02	0.2312333E 03	C.1050875E 04
50	1380.01	C.2618715E 02	C.2337895E 03	C.1060694E 04
51	1390.01	0.2586635E 02	0.2362823E 03	C.1071563E 04
52	1400.01	C.2549358E 02	0.238705CE 03	C.1084633E 04
53	1410.01	C.2507383E 02	C.2410526E 03	C.1098652E 04
54	1420.01	C.2461223E 02	C.2433212E 03	C.1113965E 04
55	1430.01	0.2411385E 02	0.2455081E 03	C.1130516E 04
56	144C.01	C.2358364E 02	C.2476118E 03	C.1148249E 04
57	1450.01	0.2302633E 02	0.2496318E 03	C.1167104E 04
58	1460.01	C.2244634E 02	C.2515685E 03	C.1187C26E 04
59	1470.01	C.2184778E 02	C.2534228E 03	0.1207958E 04
60	1480.01	0.2123441E 02	0.2551964E 03	C.1229843E 04
61	1490.01	C.2060962E 02	C.2568914E 03	C.1252628E 04
62	1500.01	0.1997642E 02	0.2585102E 03	C.1276262E 04
63	1510.01	C.1933751E 02	0.2600557E 03	0.1300692E 04
64	1520.01	0.1869521E 02	0.2615306E 03	C.1325872E 04
65	1530.01	0.1805157E 02	C.262938CE 03	C.1351756E 04
66	1540.01	0.1740833E 02	C.2642808E 03	C.1378300E 04
67	1550.01	0.1676697E 02	0.2655623E 03	C.1405462E 04
68	1560.01	0.1612876E 02	C.2667852E 03	C.1433202E 04
69	157C.01	C.1549472E 02	C.2679527E 03	C.1461485E 04
70	1580.01	0.1486573E 02	0.2690675E 03	C.1490275E 04
71	1590.01	C.1424247E 02	C.2701325E 03	0.1519539E 04
72	160C.01	C.1362552E 02	0.2711502E 03	C.1549246E 04
73	1610.01	C.1301528E 02	C.2721232E 03	0.1579367E 04
74	1620.01	C.1241211E 02	0.273054CE 03	C.1609874E 04
75	1630.01	C.1181623E 02	0.2739447E 03	C.1640743E 04
76	1640.01	0.1122780E 02	0.2747976E 03	C.1671948E 04
77	1650.01	0.1064692E 02	C.2756148E 03	C.1703467E 04
78	1660.01	0.1007364E 02	C.2763981E 03	C.1735280E 04
79	1670.01	0.9507938E 01	0.2771494E 03	C.1767366E 04
80	1680.01	C.8949786E 01	0.2778704E 03	C.1799706E 04
81	1690.01	C.8399101E 01	C.2785627E 03	C.1832283E 04
82	1700.01	C.7855790E 01	C.2792278E 03	C.1865081E 04
83	1710.01	0.7319729E 01	0.2798673E 03	0.1898C83E 04
84	1720.01	0.6790778E 01	0.2804823E 03	C.1931276E 04
85	173C.01	C.6268796E 01	C.2810743E 03	C.1964646E 04
86	1740.01	C.5753625E 01	0.2816443E 03	0.1998180E 04
87	1750.01	0.5245089E 01	0.2821935E 03	C.2031866E 04
88	1760.01	0.4743028E 01	0.282723CE 03	C.2065692E 04
89	1770.01	C.4247264E 01	C.2832336E 03	C.2099649E 04
90	1780.01	C.3757621E 01	0.2837265E 03	C.2133725E 04
91	1790.01	C.3273928E 01	C.2842023E 03	C.2167911E 04
92	1800.01	0.2796012E 01	C.284662CE 03	C.2202199E 04
93	1810.01	C.2323691E 01	0.2851063E 03	C.2236579E 04
94	1820.01	C.1856807E 01	C.2855359E 03	C.2271C43E 04
95	1830.01	0.1395185E 01	0.2859516E 03	C.2305585E 04
96	1840.01	0.9386665E 00	C.286354CE 03	C.2340197E 04

ERROR FREE CATA

NO.	TIME	ELEVATION	AZIMUTH	RADAR RANGE
97	1850.01	0.4870862E-00	0.2867436E 03	C.2374871E 04
98	1860.01	C.4029682E-01	0.2871211E 03	C.2409602E 04
99	1860.11	0.3585284E-01	0.2871248E 03	0.2409949E 04
100	1860.21	C.3140942E-01	0.2871285E 03	C.2410297E 04
101	1860.31	0.2696438E-01	0.2871322E 03	C.2410645E 04
102	1860.41	0.2252425E-01	0.2871360E 03	0.2410992E 04
103	1860.51	0.1808105E-01	0.2871397E 03	C.2411340E 04
104	1860.61	0.1363913E-01	0.2871434E 03	C.2411687E 04
105	1860.71	0.9197039E-02	0.2871471E 03	0.2412035E 04
106	1860.81	0.4757679E-02	0.2871508E 03	C.2412382E 04
107	1860.91	0.3174228E-03	0.2871545E 03	0.2412730E 04

CURVE FIT ERROR ANALYSIS
45PCINTS FITTED

POLY- NOMIAL DEGREE	CIFFERENCES RMS	 MAX ABS	PELATIVE ERRCRS PMS	 MAX ABS
		AZIMUTH ANGLE (DEG)		
1	0.45E 01	0.99E 01	C.25E-01	0.48E-01
2	C.48E-C0	1.00E 00	0.29E-02	0.65E-02
3	C.18E-C0	0.63E 00	C.95E-03	0.29E-02
4	0.92E-C1	0.25E-00	C.51E-03	0.11E-02
5	0.19E-C1	0.34E-01	0.11E-03	0.19E-03
6	0.29E-02	0.86E-02	0.14E-04	0.39E-04
7	0.25E-02	0.55E-02	0.13E-04	0.25E-04
8	0.81E-03	0.18E-02	C.43E-05	0.81E-05
9	C.11E-02	0.26E-02	0.63E-05	0.17E-04
10	0.23E-02	0.57E-02	0.12E-04	0.26E-04
11	C.73E-02	0.18E-01	0.43E-04	0.12E-03
12	0.81E-02	0.15E-01	C.44E-04	0.72E-04
13	0.14E 01	0.69E 01	0.67E-02	0.32E-01
14	C.22E-C1	0.87E-01	0.11E-03	0.41E-03
15	0.12E 01	0.52E 01	0.54E-02	0.24E-01
16	0.19E 01	0.85E 01	0.88E-02	0.38E-01
17	0.22E C1	0.91E 01	0.10E-01	0.43E-01
18	C.42E 01	0.26E 02	C.19E-01	0.12E-00
19	0.14E C3	0.59E 03	0.63E 00	0.27E 01
20	0.85E 03	0.45E 04	0.39E 01	0.21E 02
21	0.12E C5	0.68E 05	0.56E 02	0.31E 03
22	0.94E 04	0.23E 01	0.44E 02	0.10E 05
23	0.59E 00	0.17E 01	0.42E 04	0.28E 05
24	C.18E-C0	0.43E-00	0.16E 04	0.10E 05

CURVE FIT ERROR ANALYSIS

45POINTS FITTED

POLY-NOMIAL DEGREE	DIFFERENCES RMS	DIFFERENCES MAX ABS	RELATIVE ERRORS RMS	RELATIVE ERRORS MAX ABS
	ELEVATION ANGLE (DEG)			
1	0.94E 04	0.23E 01	0.44E 02	0.10E 05
2	0.59E 00	0.17E 01	0.42E 04	0.28E 05
3	0.18E-00	0.43E-00	0.16E 04	0.10E 05
4	0.19E-01	0.31E-01	0.17E 03	0.11E 04
5	0.14E-01	0.41E-01	0.64E 02	0.43E 03
6	0.57E-02	0.12E-01	0.32E 02	0.21E 03
7	0.84E-03	0.16E-02	0.48E 01	0.32E 02
8	0.35E-03	0.89E-03	0.67E 00	0.45E 01
9	0.25E-03	0.66E-03	0.63E-01	0.42E-00
10	0.17E-03	0.34E-03	0.67E 00	0.45E 01
11	0.74E-03	0.13E-02	0.15E 01	0.10E 02
12	0.72E-03	0.14E-02	0.13E 01	0.84E 01
13	0.20E-00	0.87E 00	0.35E 02	0.24E 03
14	0.81E-02	0.38E-01	0.60E 00	0.40E 01
15	0.13E-01	0.73E-01	0.98E 00	0.66E 01
16	0.16E-01	0.80E-01	0.15E-00	0.10E 01
17	0.67E-01	0.29E-00	0.55E 00	0.37E 01
18	0.46E-00	0.18E 01	0.93E 00	0.62E 01
19	0.29E 02	0.14E 03	0.30E 01	0.19E 02
20	0.87E 02	0.41E 03	0.45E 01	0.21E 02
21	0.18E 04	0.11E 05	0.12E 03	0.64E 03
22	0.25E 04	0.70E 02	0.15E 03	0.41E-00
23	0.13E 03	0.18E 03	0.68E 00	0.80E 00
24	0.15E 03	0.25E 03	0.82E 00	0.11E 01

COEFFICIENTS OF TIME POLYNOMIALS

THE COEFFICIENTS OF THE AZIMUTH AND ELEVATION TIME POLYNOMIALS ARE LISTED BELOW. THESE POLYNOMIALS ARE FROM FITTING THE ERROR-FREE DATA. THE COEFFICIENTS ARE LISTED IN ASCENDING POWERS OF TIME.

AZIMUTH COEFFICIENTS	ELEVATION COEFFICIENTS
0.15312338E 03	-0.31390667E-01
0.29621314E 02	0.25987022E 02
-0.12062063E 02	-0.50789029E 01
0.79617336E 02	0.39618822E 02
-0.29501996E 02	-0.33645909E 02

POLYNOMIAL EVALUATICN

NO.	TIME	ELEVATION	REL ERR	AZIMUTH	REL ERR
1	900.00	-0.3139067E-01	0.11E 04	0.1531234E 03	0.77E-03
	900.00	-0.3118933E-01		0.1531236E 03	
	900.01	-0.3098800E-01		0.1531238E 03	
2	900.01	-0.3078667E-01	0.53E 02	0.1531241E 03	0.77E-03
	903.34	0.1703699E-00		0.1533530E 03	
	906.68	0.3710255E-00		0.1535807E 03	
3	910.01	0.5712863E 00	0.14E-02	0.1538074E 03	0.15E-03
	913.34	0.7712559E 00		0.1540333E 03	
	916.68	0.9710348E 00		0.1542586E 03	
4	920.01	0.1170720E 01	0.16E-01	0.1544835E 03	0.27E-03
	923.34	0.1370408E 01		0.1547083E 03	
	926.68	0.1570188E 01		0.1549332E 03	
5	930.01	0.1770151E 01	0.16E-01	0.1551582E 03	0.54E-03
	933.34	0.1970382E 01		0.1553838E 03	
	936.68	0.2170964E 01		0.1556100E 03	
6	940.01	0.2371977E 01	0.13E-01	0.1558370E 03	0.67E-03
	943.34	0.2573498E 01		0.1560650E 03	
	946.68	0.2775602E 01		0.1562943E 03	
7	950.01	0.2978361E 01	0.97E-02	0.1565250E 03	0.70E-03
	953.34	0.3181841E 01		0.1567573E 03	
	956.68	0.3386110E 01		0.1569914E 03	
8	960.01	0.3591229E 01	0.63E-02	0.1572275E 03	0.64E-03
	963.34	0.3797259E 01		0.1574657E 03	
	966.68	0.4004255E 01		0.1577062E 03	
9	970.01	0.4212273E 01	0.35E-02	0.1579492E 03	0.53E-03
	973.34	0.4421362E 01		0.1581948E 03	
	976.68	0.4631572E 01		0.1584433E 03	
10	980.01	0.4842946E 01	0.12E-02	0.1586948E 03	0.37E-03
	983.34	0.5055528E 01		0.1589494E 03	
	986.68	0.5269356E 01		0.1592074E 03	
11	990.01	0.5484467E 01	0.60E-03	0.1594688E 03	0.20E-03
	993.34	0.5700894E 01		0.1597338E 03	
	996.68	0.5918668E 01		0.1600027E 03	
12	1000.01	0.6137817E 01	0.18E-02	0.1602754E 03	0.19E-04
	1003.34	0.6358365E 01		0.1605523E 03	
	1006.68	0.6580333E 01		0.1608333E 03	
13	1010.01	0.6803742E 01	0.26E-02	0.1611187E 03	0.15E-03
	1013.34	0.7028606E 01		0.1614086E 03	
	1016.68	0.7254940E 01		0.1617032E 03	
14	1020.01	0.7482752E 01	0.30E-02	0.1620025E 03	0.31E-03
	1023.34	0.7712050E 01		0.1623067E 03	
	1026.68	0.7942839E 01		0.1626160E 03	
15	1030.01	0.8175120E 01	0.31E-02	0.1629304E 03	0.44E-03
	1033.34	0.8408891E 01		0.1632501E 03	
	1036.68	0.8644148E 01		0.1635752E 03	
16	1040.01	0.8880883E 01	0.29E-02	0.1639059E 03	0.53E-03
	1043.34	0.9119087E 01		0.1642422E 03	
	1046.68	0.9358746E 01		0.1645842E 03	

POLYNOMIAL EVALUATICN

NO.	TIME	ELEVATION	REL ERR	AZIMUTH	REL ERR
17	1050.01	0.9599843E 01	0.25E-02	0.1649321E 03	0.59E-03
	1053.34	0.9842361E 01		C.1652860E 03	
	1056.68	0.1008628E 02		C.1656459E 03	
18	1060.01	0.1033156E 02	0.20E-02	0.1660121E 03	0.61E-03
	1063.34	0.1057820E 02		C.1663845E 03	
	1066.68	0.1082615E 02		0.1667633E 03	
19	1070.01	0.1107538E 02	0.14E-02	0.1671486E 03	0.59E-03
	1073.34	0.1132585E 02		0.1675405E 03	
	1076.68	0.1157753E 02		0.1679390E 03	
20	1080.01	0.1183037E 02	0.78E-03	C.1683443E 03	0.54E-03
	1083.34	0.1208433E 02		0.1687564E 03	
	1086.68	0.1233936E 02		0.1691755E 03	
21	1090.01	0.1259540E 02	0.19E-03	0.1696015E 03	0.45E-03
	1093.34	0.1285241E 02		0.1700346E 03	
	1096.68	0.1311033E 02		0.1704749E 03	
22	1100.01	0.1336910E 02	0.35E-03	C.1709224E 03	0.33E-03
	1103.34	0.1362864E 02		0.1713773E 03	
	1106.68	0.1388891E 02		0.1718394E 03	
23	1110.01	0.1414983E 02	0.80E-03	0.1723090E 03	0.19E-03
	1113.34	0.1441133E 02		C.1727862E 03	
	1116.68	0.1467333E 02		0.1732708E 03	
24	1120.01	0.1493576E 02	0.11E-02	0.1737631E 03	0.37E-04
	1123.34	0.1519853E 02		0.1742630E 03	
	1126.68	0.1546157E 02		0.1747707E 03	
25	1130.01	0.1572479E 02	0.14E-02	0.1752861E 03	0.12E-03
	1133.34	0.1598809E 02		C.1758C94E 03	
	1136.68	0.1625139E 02		0.1763405E 03	
26	1140.01	0.1651459E C2	0.15E-02	0.1768795E 03	0.27E-03
	1143.34	0.1677760E 02		0.1774265E 03	
	1146.68	0.1704031E 02		0.1779814E 03	
27	1150.01	0.1730262E 02	0.14E-02	0.1785444E 03	0.41E-03
	1153.34	0.1756442E C2		0.1791154E 03	
	1156.68	0.1782561E 02		0.1796945E 03	
28	1160.01	0.1808607E 02	0.13E-02	0.1802817E 03	0.52E-03
	1163.34	0.1834568E 02		0.1808770E 03	
	1166.68	0.1860433E 02		0.1814805E 03	
29	1170.01	0.1886190E 02	0.10E-02	0.1820921E 03	0.60E-03
	1173.34	0.1911826E 02		C.1827120E 03	
	1176.68	0.1937329E 02		0.1833400E 03	
30	1180.01	0.1962685E 02	0.73E-03	0.1839763E 03	0.64E-03
	1183.34	0.1987882E 02		0.1846207E 03	
	1186.68	0.2012905E 02		0.1852734E 03	
31	1190.01	0.2037742E 02	0.38E-03	C.1859344E 03	0.63E-03
	1193.34	0.2062377E 02		0.1866035E 03	
	1196.68	0.2086796E 02		0.1872809E 03	
32	1200.01	0.2110986E 02	0.90E-05	0.1879665E 03	0.58E-03
	1203.34	0.2134929E 02		0.1886604E 03	
	1206.68	0.2158612E 02		C.1893624E 03	

POLYNOMIAL EVALUATION

NO.	TIME	ELEVATION	REL ERR	AZIMUTH	REL ERR
33	1210.01	0.2182019E 02	0.34E-03	0.1900727E 03	0.47E-03
	1213.34	0.2205133E 02		0.1907911E 03	
	1216.68	0.2227939E 02		0.1915178E 03	
34	1220.01	0.2250420E 02	0.63E-03	0.1922525E 03	0.33E-03
	1223.34	0.2272559E 02		0.1929954E 03	
	1226.68	0.2294340E 02		0.1937465E 03	
35	1230.01	0.2315744E 02	0.84E-03	0.1945055E 03	0.14E-03
	1233.34	0.2336755E 02		0.1952727E 03	
	1236.68	0.2357354E 02		0.1960478E 03	
36	1240.01	0.2377523E 02	0.94E-03	0.1968310E 03	0.65E-04
	1243.34	0.2397243E 02		0.1976221E 03	
	1246.68	0.2416496E 02		0.1984211E 03	
37	1250.01	0.2435263E 02	0.91E-03	0.1992279E 03	0.28E-03
	1253.34	0.2453524E 02		0.2000426E 03	
	1256.68	0.2471260E 02		0.2008651E 03	
38	1260.01	0.2488450E 02	0.74E-03	0.2016953E 03	0.48E-03
	1263.34	0.2505075E 02		0.2025331E 03	
	1266.68	0.2521113E 02		0.2033786E 03	
39	1270.01	0.2536544E 02	0.46E-03	0.2042316E 03	0.64E-03
	1273.34	0.2551347E 02		0.2050921E 03	
	1276.68	0.2565500E 02		0.2059601E 03	
40	1280.01	0.2578982E 02	0.98E-04	0.2068354E 03	0.73E-03
	1283.34	0.2591770E 02		0.2077181E 03	
	1286.68	0.2603842E 02		0.2086079E 03	
41	1290.01	0.2615176E 02	0.29E-03	0.2095050E 03	0.71E-03
	1293.34	0.2625749E 02		0.2104091E 03	
	1296.68	0.2635537E 02		0.2113202E 03	
42	1300.01	0.2644518E 02	0.60E-03	0.2122383E 03	0.55E-03
	1303.34	0.2652666E 02		0.2131632E 03	
	1306.68	0.2659959E 02		0.2140948E 03	
43	1310.01	0.2666372E 02	0.71E-03	0.2150332E 03	0.22E-03
	1313.34	0.2671880E 02		0.2159781E 03	
	1316.68	0.2676458E 02		0.2169295E 03	
44	1320.01	0.2680081E 02	0.44E-03	0.2178873E 03	0.33E-03
	1323.34	0.2682724E 02		0.2188513E 03	
	1326.68	0.2684360E 02		0.2198216E 03	
45	1330.01	0.2684964E 02	0.43E-03	0.2207980E 03	0.11E-02

TRANSFER FUNCTICN EVALUATICN

NO.	TIME	ELEVATION	REL ERR	AZIMUTH	REL ERR
1	900.00	-0.3177889E-01	0.12E-01	0.1531224E 03	0.61E-05
2	900.01	-0.3177853E-01	0.32E-01	0.1531224E 03	0.11E-04
3	910.01	0.3853530E-00	0.33E-00	0.1535973E 03	0.14E-02
4	920.01	0.1082982E 01	0.75E-01	0.1543853E 03	0.64E-03
5	930.01	0.1635961E 01	0.76E-01	0.1550070E 03	0.97E-03
6	940.01	0.2256669E 01	0.49E-01	0.1557065E 03	0.84E-03
7	950.01	0.2853970E 01	0.42E-01	0.1563829E 03	0.91E-03
8	960.01	0.3468087E 01	0.34E-01	0.1570848E 03	0.91E-03
9	970.01	0.4086295E 01	0.30E-01	0.1578008E 03	0.94E-03
10	980.01	0.4715077E 01	0.26E-01	0.1585412E 03	0.97E-03
11	990.01	0.5354196E 01	0.24E-01	0.1593087E 03	0.10E-02
12	1000.01	0.6005030E 01	0.22E-01	0.1601081E 03	0.10E-02
13	1010.01	0.6668349E 01	0.20E-01	0.1609434E 03	0.11E-02
14	1020.01	0.7344670E 01	0.18E-01	0.1618184E 03	0.11E-02
15	1030.01	0.8034348E 01	0.17E-01	0.1627369E 03	0.12E-02
16	1040.01	0.8737447E 01	0.16E-01	0.1637023E 03	0.12E-02
17	1050.01	0.9453827E 01	0.15E-01	0.1647179E 03	0.13E-02
18	1060.01	0.1018309E 02	0.14E-01	0.1657866E 03	0.14E-02
19	1070.01	0.1092463E 02	0.14E-01	0.1669113E 03	0.14E-02
20	1080.01	0.1167756E 02	0.13E-01	0.1680948E 03	0.15E-02
21	1090.01	0.1244081E 02	0.12E-01	0.1693393E 03	0.15E-02
22	1100.01	0.1321303E 02	0.12E-01	0.1706471E 03	0.16E-02
23	1110.01	0.1399266E 02	0.11E-01	0.1720203E 03	0.17E-02
24	1120.01	0.1477789E 02	0.11E-01	0.1734606E 03	0.17E-02
25	1130.01	0.1556668E 02	0.10E-01	0.1749696E 03	0.18E-02
26	1140.01	0.1635675E 02	0.96E-02	0.1765487E 03	0.19E-02
27	1150.01	0.1714559E 02	0.91E-02	0.1781991E 03	0.19E-02
28	1160.01	0.1793046E 02	0.86E-02	0.1799218E 03	0.20E-02
29	1170.01	0.1870836E 02	0.81E-02	0.1817175E 03	0.21E-02
30	1180.01	0.1947607E 02	0.77E-02	0.1835868E 03	0.21E-02
31	1190.01	0.2023014E 02	0.72E-02	0.1855301E 03	0.22E-02
32	1200.01	0.2096686E 02	0.68E-02	0.1875474E 03	0.22E-02
33	1210.01	0.2168232E 02	0.63E-02	0.1896388E 03	0.23E-02
34	1220.01	0.2237234E 02	0.59E-02	0.1918040E 03	0.23E-02
35	1230.01	0.2303252E 02	0.54E-02	0.1940425E 03	0.24E-02
36	1240.01	0.2365822E 02	0.49E-02	0.1963536E 03	0.24E-02
37	1250.01	0.2424457E 02	0.44E-02	0.1987364E 03	0.25E-02
38	1260.01	0.2478646E 02	0.39E-02	0.2011899E 03	0.25E-02
39	1270.01	0.2527853E 02	0.34E-02	0.2037127E 03	0.25E-02
40	1280.01	0.2571521E 02	0.29E-02	0.2063033E 03	0.26E-02
41	1290.01	0.2609066E 02	0.23E-02	0.2089600E 03	0.26E-02
42	1300.01	0.2639885E 02	0.18E-02	0.2116809E 03	0.26E-02
43	1310.01	0.2663348E 02	0.11E-02	0.2144639E 03	0.26E-02
44	1320.01	0.2678801E 02	0.48E-03	0.2173066E 03	0.27E-02
45	1330.01	0.2685568E 02	0.22E-03	0.2202064E 03	0.27E-02

PCINTS OMMITED BY FIT ARE 0

CURVE FIT ERROR ANALYSIS

62POINTS FITTED

POLY-NOMIAL DEGREE	DIFFERENCES RMS	DIFFERENCES MAX ABS	RELATIVE ERRORS RMS	RELATIVE ERRORS MAX ABS
	AZIMUTH ANGLE (DEG)			
1	0.53E 01	0.14E 02	0.21E-01	0.63E-01
2	0.10E 01	0.23E 01	0.39E-02	0.11E-01
3	0.86E-01	0.23E-00	0.33E-03	0.10E-02
4	0.80E-01	0.31E-00	0.33E-03	0.14E-02
5	0.35E-01	0.11E-00	0.14E-03	0.49E-03
6	0.84E-02	0.18E-01	0.32E-04	0.80E-04
7	0.10E-02	0.27E-02	0.40E-05	0.12E-04
8	0.27E-02	0.57E-02	0.10E-04	0.20E-04
9	0.27E-02	0.48E-02	0.10E-04	0.17E-04
10	0.12E-01	0.23E-01	0.47E-04	0.97E-04
11	0.12E-01	0.28E-01	0.47E-04	0.98E-04
12	0.33E-00	0.99E 00	0.12E-02	0.34E-02
13	0.31E-00	0.94E 00	0.11E-02	0.33E-02
14	0.93E-01	0.39E-00	0.33E-03	0.13E-02
15	0.17E-00	0.80E 00	0.61E-03	0.28E-02
16	0.14E-00	0.70E 00	0.49E-03	0.25E-02
17	0.64E 00	0.20E 01	0.22E-02	0.68E-02
18	0.65E 02	0.24E 03	0.23E-00	0.83E 00
19	0.31E 02	0.13E 03	0.11E-00	0.45E-00
20	0.58E 02	0.19E 03	0.20E-00	0.67E 00
21	0.22E 03	0.11E 04	0.78E 00	0.37E 01
22	0.67E 03	0.1[illegible] 01	0.73E 01	0.14E 04
23	0.48E-00	0.18E 01	0.11E 03	0.85E 03
24	0.22E-00	0.70E 00	0.50E 02	0.40E 03

CURVE FIT ERROR ANALYSIS

62PCINTS FITTED

POLY-NOMIAL DEGREE	DIFFERENCES RMS	DIFFERENCES MAX ABS	RELATIVE ERRCRS RMS	RELATIVE ERRCRS MAX ABS
	ELEVATION	ANGLE	(DEG)	
1	0.67E 03	0.15E 01	0.23E 01	0.14E 04
2	0.48E-00	0.18E 01	C.11E 03	0.85E 03
3	0.22E-C0	0.70E 00	0.50E 02	0.40E 03
4	0.53E-01	0.13E-C0	0.12E 02	0.92E 02
5	0.55E-C2	0.12E-01	C.94E 00	0.74E 01
6	C.52E-02	0.18E-01	C.55E 00	0.43E 01
7	0.24E-02	0.63E-02	0.30E-00	0.24E 01
8	0.57E-03	0.13E-02	0.82E-01	0.64E 00
9	C.31E-C3	0.77E-03	C.57E-01	0.44E-00
10	C.53E-03	0.13E-02	C.70E-02	0.54E-01
11	0.74E-C3	0.16E-02	0.15E-00	0.12E 01
12	C.84E-02	0.24E-01	C.65E 01	0.51E 02
13	0.66E-02	0.21E-01	C.41E 01	0.32E 02
14	0.50E-02	0.21E-01	0.38E 01	0.30E 02
15	0.30E-C1	0.15E-00	C.26E 01	0.13E 02
16	0.51E-01	0.21E-00	0.14E 02	0.10E 03
17	0.49E-01	0.19E-00	C.30E 02	0.24E 03
18	C.44E-00	0.21E 01	0.24E 03	0.18E 04
19	0.16E-C0	0.65E 00	0.57E 02	0.43E 03
20	C.75E C0	0.29E 01	0.24E 03	0.19E 04
21	0.11E C3	0.43E 03	0.98E 05	0.77E 06
22	0.62E C2	0.69E 02	0.49E 05	0.31E-00
23	0.40E C2	0.63E 02	C.15E-00	0.28E-0C
24	0.52E C2	0.74E 02	0.19E-00	0.29E-00

COEFFICIENTS OF TIME POLYNOMIALS

THE COEFFICIENTS OF THE AZIMUTH AND ELEVATION TIME POLYNOMIALS ARE LISTED BELOW. THESE POLYNOMIALS ARE FROM FITTING THE ERROR-FREE DATA. THE COEFFICIENTS ARE LISTED IN ASCENDING POWERS OF TIME.

AZIMUTH COEFFICIENTS	ELEVATION COEFFICIENTS
0.22296183E 03	0.26991532E 02
0.15357744E 03	-0.53777441E 01
-0.14158880E 03	-0.83569923E 02
0.57757708E 02	0.10110569E 03
-0.55244608E 01	-0.39178401E 02

POLYNOMIAL EVALUATION

NO.	TIME	ELEVATION	REL ERR	AZIMUTH	REL ERR
46	1340.01	0.2699153E 02	0.48E-02	0.2229618E 03	0.14E-02
	1343.34	0.2695372E 02		0.2239388E 03	
	1346.68	0.2690923E 02		0.2249043E 03	
47	1350.01	0.2685820E 02	0.25E-02	0.2258584E 03	0.46E-03
	1353.34	0.2680080E 02		0.2268011E 03	
	1356.68	0.2673718E 02		0.2277326E 03	
48	1360.01	0.2666749E 02	0.50E-03	0.2286530E 03	0.14E-03
	1363.34	0.2659188E 02		0.2295623E 03	
	1366.68	0.2651050E 02		0.2304606E 03	
49	1370.01	0.2642350E 02	0.10E-02	0.2313481E 03	0.50E-03
	1373.34	0.2633102E 02		0.2322248E 03	
	1376.68	0.2623321E 02		0.2330908E 03	
50	1380.01	0.2613020E 02	0.22E-02	0.2339461E 03	0.67E-03
	1383.34	0.2602215E 02		0.2347910E 03	
	1386.68	0.2590918E 02		0.2356254E 03	
51	1390.01	0.2579144E 02	0.29E-02	0.2364494E 03	0.71E-03
	1393.34	0.2566907E 02		0.2372632E 03	
	1396.68	0.2554219E 02		0.2380669E 03	
52	1400.01	0.2541094E 02	0.32E-02	0.2388604E 03	0.65E-03
	1403.34	0.2527545E 02		0.2396440E 03	
	1406.68	0.2513585E 02		0.2404176E 03	
53	1410.01	0.2499227E 02	0.33E-02	0.2411814E 03	0.53E-03
	1413.34	0.2484484E 02		0.2419355E 03	
	1416.68	0.2469368E 02		0.2426800E 03	
54	1420.01	0.2453892E 02	0.30E-02	0.2434148E 03	0.38E-03
	1423.34	0.2438067E 02		0.2441402E 03	
	1426.68	0.2421905E 02		0.2448562E 03	
55	1430.01	0.2405420E 02	0.25E-02	0.2455629E 03	0.22E-03
	1433.34	0.2388622E 02		0.2462603E 03	
	1436.68	0.2371522E 02		0.2469486E 03	
56	1440.01	0.2354133E 02	0.18E-02	0.2476279E 03	0.65E-04
	1443.34	0.2336465E 02		0.2482982E 03	
	1446.68	0.2318530E 02		0.2489596E 03	
57	1450.01	0.2300338E 02	1.00E-03	0.2496121E 03	0.79E-04
	1453.34	0.2281901E 02		0.2502560E 03	
	1456.68	0.2263229E 02		0.2508912E 03	
58	1460.01	0.2244332E 02	0.13E-03	0.2515179E 03	0.20E-03
	1463.34	0.2225220E 02		0.2521361E 03	
	1466.68	0.2205904E 02		0.2527460E 03	
59	1470.01	0.2186395E 02	0.74E-03	0.2533475E 03	0.30E-03
	1473.34	0.2166700E 02		0.2539408E 03	
	1476.68	0.2146832E 02		0.2545259E 03	
60	1480.01	0.2126797E 02	0.16E-02	0.2551030E 03	0.37E-03
	1483.34	0.2106607E 02		0.2556721E 03	
	1486.68	0.2086271E 02		0.2562333E 03	
61	1490.01	0.2065796E 02	0.23E-02	0.2567867E 03	0.41E-03
	1493.34	0.2045193E 02		0.2573324E 03	
	1496.68	0.2024470E 02		0.2578704E 03	

POLYNOMIAL EVALUATION

NO.	TIME	ELEVATION	REL ERR	AZIMUTH	REL ERR
62	1500.01	0.2003636E 02	0.30E-02	0.2584008E 03	0.42E-03
	1503.34	0.1982698E 02		0.2589238E 03	
	1506.68	0.1961666E 02		0.2594393E 03	
63	1510.01	0.1940547E 02	0.35E-02	0.2599475E 03	0.42E-03
	1513.34	0.1919349E 02		0.2604485E 03	
	1516.68	0.1898080E 02		0.2609422E 03	
64	1520.01	0.1876748E 02	0.39E-02	0.2614289E 03	0.39E-03
	1523.34	0.1855359E 02		0.2619086E 03	
	1526.68	0.1833923E 02		0.2623813E 03	
65	1530.01	0.1812444E 02	0.40E-02	0.2628471E 03	0.35E-03
	1533.34	0.1790932E 02		0.2633062E 03	
	1536.68	0.1769391E 02		0.2637586E 03	
66	1540.01	0.1747830E 02	0.40E-02	0.2642044E 03	0.29E-03
	1543.34	0.1726254E 02		0.2646436E 03	
	1546.68	0.1704670E 02		0.2650763E 03	
67	1550.01	0.1683084E 02	0.38E-02	0.2655026E 03	0.22E-03
	1553.34	0.1661502E 02		0.2659227E 03	
	1556.68	0.1639931E 02		0.2663364E 03	
68	1560.01	0.1618375E 02	0.34E-02	0.2667441E 03	0.15E-03
	1563.34	0.1596840E 02		0.2671456E 03	
	1566.68	0.1575332E 02		0.2675411E 03	
69	1570.01	0.1553856E 02	0.28E-02	0.2679307E 03	0.82E-04
	1573.34	0.1532417E 02		0.2683144E 03	
	1576.68	0.1511020E 02		0.2686924E 03	
70	1580.01	0.1489670E 02	0.21E-02	0.2690646E 03	0.11E-04
	1583.34	0.1468371E 02		0.2694312E 03	
	1586.68	0.1447129E 02		0.2697923E 03	
71	1590.01	0.1425946E 02	0.12E-02	0.2701478E 03	0.57E-04
	1593.34	0.1404828E 02		0.2704980E 03	
	1596.68	0.1383778E 02		0.2708428E 03	
72	1600.01	0.1362800E 02	0.18E-03	0.2711824E 03	0.12E-03
	1603.34	0.1341898E 02		0.2715167E 03	
	1606.68	0.1321075E 02		0.2718460E 03	
73	1610.01	0.1300336E 02	0.92E-03	0.2721702E 03	0.17E-03
	1613.34	0.1279682E 02		0.2724894E 03	
	1616.68	0.1259117E 02		0.2728037E 03	
74	1620.01	0.1238644E 02	0.21E-02	0.2731132E 03	0.22E-03
	1623.34	0.1218265E 02		0.2734180E 03	
	1626.68	0.1197984E 02		0.2737181E 03	
75	1630.01	0.1177802E 02	0.32E-02	0.2740135E 03	0.25E-03
	1633.34	0.1157722E 02		0.2743045E 03	
	1636.68	0.1137746E 02		0.2745909E 03	
76	1640.01	0.1117876E 02	0.44E-02	0.2748730E 03	0.27E-03
	1643.34	0.1098114E 02		0.2751508E 03	
	1646.68	0.1078460E 02		0.2754243E 03	
77	1650.01	0.1058918E 02	0.54E-02	0.2756936E 03	0.29E-03
	1653.34	0.1039488E 02		0.2759588E 03	
	1656.68	0.1020171E 02		0.2762199E 03	

POLYNOMIAL EVALUATION

NO.	TIME	ELEVATION	REL ERR	AZIMUTH	REL ERR
78	1660.01	0.1000968E 02	0.63E-02	0.2764771E 03	0.29E-03
	1663.34	0.9818799E 01		0.2767304E 03	
	1666.68	0.9629078E 01		0.2769799E 03	
79	1670.01	0.9440520E 01	0.71E-02	0.2772255E 03	0.27E-03
	1673.34	0.9253129E 01		0.2774675E 03	
	1676.68	0.9066905E 01		0.2777059E 03	
80	1680.01	0.8881851E 01	0.76E-02	0.2779407E 03	0.25E-03
	1683.34	0.8697964E 01		0.2781720E 03	
	1686.68	0.8515243E 01		0.2783999E 03	
81	1690.01	0.8333682E 01	0.78E-02	0.2786245E 03	0.22E-03
	1693.34	0.8153277E 01		0.2788457E 03	
	1696.68	0.7974019E 01		0.2790638E 03	
82	1700.01	0.7795901E 01	0.76E-02	0.2792786E 03	0.18E-03
	1703.34	0.7618911E 01		0.2794904E 03	
	1706.68	0.7443036E 01		0.2796992E 03	
83	1710.01	0.7268264E 01	0.70E-02	0.2799050E 03	0.13E-03
	1713.34	0.7094580E 01		0.2801080E 03	
	1716.68	0.6921966E 01		0.2803081E 03	
84	1720.01	0.6750404E 01	0.59E-02	0.2805055E 03	0.82E-04
	1723.34	0.6579874E 01		0.2807001E 03	
	1726.68	0.6410353E 01		0.2808922E 03	
85	1730.01	0.6241821E 01	0.43E-02	0.2810817E 03	0.26E-04
	1733.34	0.6074251E 01		0.2812687E 03	
	1736.68	0.5907617E 01		0.2814533E 03	
86	1740.01	0.5741892E 01	0.20E-02	0.2816355E 03	0.31E-04
	1743.34	0.5577045E 01		0.2818154E 03	
	1746.68	0.5413047E 01		0.2819931E 03	
87	1750.01	0.5249863E 01	0.91E-03	0.2821686E 03	0.88E-04
	1753.34	0.5087461E 01		0.2823420E 03	
	1756.68	0.4925804E 01		0.2825134E 03	
88	1760.01	0.4764855E 01	0.46E-02	0.2826827E 03	0.14E-03
	1763.34	0.4604575E 01		0.2828502E 03	
	1766.68	0.4444924E 01		0.2830158E 03	
89	1770.01	0.4285859E 01	0.91E-02	0.2831796E 03	0.19E-03
	1773.34	0.4127336E 01		0.2833417E 03	
	1776.68	0.3969312E 01		0.2835021E 03	
90	1780.01	0.3811738E 01	0.14E-01	0.2836609E 03	0.23E-03
	1783.34	0.3654568E 01		0.2838181E 03	
	1786.68	0.3497749E 01		0.2839739E 03	
91	1790.01	0.3341231E 01	0.21E-01	0.2841282E 03	0.26E-03
	1793.34	0.3184961E 01		0.2842812E 03	
	1796.68	0.3028885E 01		0.2844329E 03	
92	1800.01	0.2872946E 01	0.28E-01	0.2845833E 03	0.28E-03
	1803.34	0.2717085E 01		0.2847326E 03	
	1806.68	0.2561244E 01		0.2848807E 03	
93	1810.01	0.2405362E 01	0.35E-01	0.2850278E 03	0.28E-03
	1813.34	0.2249376E 01		0.2851738E 03	
	1816.68	0.2093222E 01		0.2853190E 03	

POLYNOMIAL EVALUATION

NO.	TIME	ELEVATION	REL ERR	AZIMUTH	REL ERR
94	1820.01	0.1936834E 01	0.43E-01	0.2854632E 03	0.25E-03
	1823.34	0.1780146E 01		0.2856066E 03	
	1826.68	0.1623087E 01		0.2857493E 03	
95	1830.01	0.1465589E 01	0.50E-01	0.2858912E 03	0.21E-03
	1833.34	0.1307577E 01		0.2860325E 03	
	1836.68	0.1148980E 01		0.2861732E 03	
96	1840.01	0.9897218E 00	0.54E-01	0.2863134E 03	0.14E-03
	1843.34	0.8297253E 00		0.2864531E 03	
	1846.68	0.6689126E 00		0.2865924E 03	
97	1850.01	0.5072045E 00	0.41E-01	0.2867313E 03	0.43E-04
	1853.34	0.3445182E-00		0.2868699E 03	
	1856.68	0.1807711E-00		0.2870083E 03	
98	1860.01	0.1587892E-01	0.61E 00	0.2871464E 03	0.88E-04
	1860.04	0.1422405E-01		0.2871478E 03	
	1860.08	0.1256895E-01		0.2871492E 03	
99	1860.11	0.1091409E-01	0.70E 00	0.2871506E 03	0.90E-04
	1860.14	0.9258986E-02		0.2871520E 03	
	1860.18	0.7603407E-02		0.2871533E 03	
100	1860.21	0.5947828E-02	0.81E 00	0.2871547E 03	0.91E-04
	1860.24	0.4292488E-02		0.2871561E 03	
	1860.28	0.2636909E-02		0.2871575E 03	
101	1860.31	0.9806156E-03	0.96E 00	0.2871589E 03	0.93E-04
	1860.34	-0.6754398E-03		0.2871602E 03	
	1860.38	-0.2331257E-02		0.2871616E 03	
102	1860.41	-0.3987551E-02	0.12E 01	0.2871630E 03	0.94E-04
	1860.44	-0.5644083E-02		0.2871644E 03	
	1860.48	-0.7300854E-02		0.2871658E 03	
103	1860.51	-0.8956909E-02	0.15E 01	0.2871672E 03	0.96E-04
	1860.54	-0.1061392E-01		0.2871685E 03	
	1860.58	-0.1227045E-01		0.2871699E 03	
104	1860.61	-0.1392794E-01	0.20E 01	0.2871713E 03	0.97E-04
	1860.64	-0.1558471E-01		0.2871727E 03	
	1860.68	-0.1724172E-01		0.2871741E 03	
105	1860.71	-0.1889896E-01	0.31E 01	0.2871754E 03	0.99E-04
	1860.74	-0.2055645E-01		0.2871768E 03	
	1860.78	-0.2221394E-01		0.2871782E 03	
106	1860.81	-0.2387166E-01	0.60E 01	0.2871796E 03	0.10E-03
	1860.84	-0.2552962E-01		0.2871810E 03	
	1860.88	-0.2718782E-01		0.2871823E 03	
107	1860.91	-0.2884603E-01	0.92E 02	0.2871837E 03	0.10E-03

TRANSFER FUNCTION EVALUATION

NO.	TIME	ELEVATION	REL ERR	AZIMUTH	REL ERR
46	1340.01	0.2698715E 02	0.16E-03	0.2229544E 03	0.33E-04
47	1350.01	0.2690922E 02	0.19E-02	0.2249762E 03	0.39E-02
48	1360.01	0.2670879E 02	0.15E-02	0.2282667E 03	0.17E-02
49	1370.01	0.2648353E 02	0.23E-02	0.2307573E 03	0.26E-02
50	1380.01	0.2619596E 02	0.25E-02	0.2334707E 03	0.20E-02
51	1390.01	0.2586691E 02	0.29E-02	0.2359543E 03	0.21E-02
52	1400.01	0.2549372E 02	0.33E-02	0.2383973E 03	0.19E-02
53	1410.01	0.2508220E 02	0.36E-02	0.2407313E 03	0.19E-02
54	1420.01	0.2463518E 02	0.39E-02	0.2429831E 03	0.18E-02
55	1430.01	0.2415617E 02	0.42E-02	0.2451476E 03	0.17E-02
56	1440.01	0.2364840E 02	0.45E-02	0.2472288E 03	0.16E-02
57	1450.01	0.2311496E 02	0.49E-02	0.2492289E 03	0.15E-02
58	1460.01	0.2255882E 02	0.51E-02	0.2511499E 03	0.15E-02
59	1470.01	0.2198284E 02	0.54E-02	0.2529944E 03	0.14E-02
60	1480.01	0.2138974E 02	0.57E-02	0.2547643E 03	0.13E-02
61	1490.01	0.2078211E 02	0.60E-02	0.2564620E 03	0.13E-02
62	1500.01	0.2016242E 02	0.63E-02	0.2580897E 03	0.12E-02
63	1510.01	0.1953300E 02	0.66E-02	0.2596495E 03	0.11E-02
64	1520.01	0.1889607E 02	0.69E-02	0.2611436E 03	0.11E-02
65	1530.01	0.1825371E 02	0.71E-02	0.2625741E 03	0.10E-02
66	1540.01	0.1760787E 02	0.74E-02	0.2639431E 03	0.99E-03
67	1550.01	0.1696038E 02	0.77E-02	0.2652528E 03	0.94E-03
68	1560.01	0.1631294E 02	0.80E-02	0.2665053E 03	0.90E-03
69	1570.01	0.1566712E 02	0.83E-02	0.2677025E 03	0.85E-03
70	1580.01	0.1502437E 02	0.86E-02	0.2688467E 03	0.81E-03
71	1590.01	0.1438599E 02	0.89E-02	0.2699397E 03	0.77E-03
72	1600.01	0.1375319E 02	0.92E-02	0.2709836E 03	0.73E-03
73	1610.01	0.1312702E 02	0.95E-02	0.2719804E 03	0.70E-03
74	1620.01	0.1250841E 02	0.98E-02	0.2729321E 03	0.66E-03
75	1630.01	0.1189818E 02	0.10E-01	0.2738406E 03	0.63E-03
76	1640.01	0.1129699E 02	0.11E-01	0.2747079E 03	0.60E-03
77	1650.01	0.1070539E 02	0.11E-01	0.2755359E 03	0.57E-03
78	1660.01	0.1012382E 02	0.11E-01	0.2763265E 03	0.54E-03
79	1670.01	0.9552561E 01	0.12E-01	0.2770817E 03	0.52E-03
80	1680.01	0.8991784E 01	0.12E-01	0.2778032E 03	0.49E-03
81	1690.01	0.8441525E 01	0.13E-01	0.2784929E 03	0.47E-03
82	1700.01	0.7901698E 01	0.14E-01	0.2791527E 03	0.45E-03
83	1710.01	0.7372087E 01	0.14E-01	0.2797843E 03	0.43E-03
84	1720.01	0.6852347E 01	0.15E-01	0.2803896E 03	0.41E-03
85	1730.01	0.6342008E 01	0.16E-01	0.2809704E 03	0.40E-03
86	1740.01	0.5840468E 01	0.17E-01	0.2815284E 03	0.38E-03
87	1750.01	0.5347001E 01	0.19E-01	0.2820653E 03	0.37E-03
88	1760.01	0.4860752E 01	0.20E-01	0.2825830E 03	0.35E-03
89	1770.01	0.4380739E 01	0.22E-01	0.2830830E 03	0.34E-03
90	1780.01	0.3905850E 01	0.25E-01	0.2835671E 03	0.33E-03
91	1790.01	0.3434849E 01	0.28E-01	0.2840369E 03	0.32E-03
92	1800.01	0.2966370E 01	0.33E-01	0.2844942E 03	0.31E-03
93	1810.01	0.2498920E 01	0.39E-01	0.2849405E 03	0.31E-03

TRANSFER FUNCTION EVALUATION

NO.	TIME	ELEVATION	REL ERR	AZIMUTH	REL ERR
94	1820.01	0.2030877E 01	0.49E-01	0.2853775E 03	0.30E-03
95	1830.01	0.1560491E 01	0.65E-01	0.2858067E 03	0.30E-03
96	1840.01	0.1085885E 01	0.97E-01	0.2862298E 03	0.29E-03
97	1850.01	0.6050601E 00	0.19E-00	0.2866483E 03	0.29E-03
98	1860.01	0.1158781E-00	0.63E 01	0.2870637E 03	0.29E-03
99	1860.11	0.1109347E-00	0.92E 01	0.2870679E 03	0.29E-03
100	1860.21	0.1059937E-00	0.17E 02	0.2870720E 03	0.29E-03
101	1860.31	0.1010513E-00	0.10E 03	0.2070762E 03	0.29E-03
102	1860.41	0.9610558E-01	0.25E 02	0.2870803E 03	0.29E-03
103	1860.51	0.9116077E-01	0.11E 02	0.2870844E 03	0.29E-03
104	1860.61	0.8621407E-01	0.72E 01	0.2870886E 03	0.29E-03
105	1860.71	0.8126640E-01	0.53E 01	0.2870927E 03	0.29E-03
106	1860.81	0.7631636E-01	0.42E 01	0.2870969E 03	0.29E-03
107	1860.91	0.7136917E-01	0.35E 01	0.2871010E 03	0.29E-03

ORBIT PARAMETERS

DETERMINED BY DIFFERENT SETS OF THREE VECTORS BEFORE CONSIDERING RADAR DYNAMICS

	SET 1	SET 2	SET 3	SET 4	SET 5	SET 6
ECCENTRICITY	0.0510541	0.0501997	0.0495584	0.0497588	0.0499754	0.0500018
SEMIMAJOR AXIS (MILES)	4575.1515	4567.3132	4556.7061	4560.6879	4565.6723	4565.4863
RIGHT ASCENSION OF THE NODE (DEG)	-176.338	-176.277	-176.282	-176.326	-176.371	-176.413
ARGUMENT OF PERIGEE (DEG)	-36.480	-38.280	-40.872	-39.685	-38.245	-38.244
LONGITUDE OF PERIGEE (DEG)	-212.819	-214.558	-217.154	-216.011	-214.617	-214.657
INCLINATION (DEG)	39.981	40.009	40.024	40.020	40.014	40.009
TIME OF PERIGEE PASSAGE (SEC)	55.3247	27.2300	-13.2417	5.0363	27.2605	27.0911
PERIOD (SEC)	6287.7218	6271.5703	6249.7352	6257.9288	6268.1907	6267.8078

COMPARISON OF INPUT PARAMETERS AND PARAMETERS FROM FIT

OF ERROR FREE DATA

NOTE. THE RELATIVE ERRORS WERE OBTAINED BY NORMALIZING THE ABSOLUTE DIFFERENCES TO THE INPUT VALUES (EXCEPT TIME OF PERIGEE PASSAGE WHICH WAS NOMALIZED TO THE PERIOD).

	INPUT	COMPUTED	ABSOLUTE DIFFERENCE	RELATIVE ERROR
ECCENTRICITY	0.0500000	0.0500913	0.9136E-04	0.1827E-02
SEMIMAJOR AXIS	4560.0000	4565.1694	0.5169E 01	0.1134E-02
RIGHT ASCENSION OF THE NODE	-176.380	-176.335	0.4536E-01	0.2572E-03
ARGUMENT OF PERIGEE	-40.000	-38.634	0.1366E 01	0.3414E-01
LONGITUDE OF PERIGEE	-216.380	-214.969	0.1411E 01	0.6521E-02
INCLINATION	40.000	40.009	0.9254E-02	0.2313E-03
TIME OF PERIGEE PASSAGE	0.	21.4502	0.2145E 02	0.3428E-02
PERIOD	6256.5132	6267.1590	0.1065E 02	0.1702E-02

THE MAXIMUM ABSOLUTE ERROR IN POSITION DUE TO THIS FIT IS 47.746 MILES. THE RMS POSITION ERROR FOR ONE ORBIT IS 20.791 MILES. THE RELATIVE VALUES ARE 0.11016633E-01 MAX-IMUM AND 0.46871948E-02 RMS FOR A SINGLE ORBIT.

ORBIT PARAMETERS

DETERMINED BY SETS OF THREE VECTORS FROM RADAR DATA

	SET 1	SET 2	SET 3	SET 4	SET 5	SET 6
ECCENTRICITY	0.0528215	0.0518396	0.0512056	0.0512828	0.0514660	0.0515531
SEMIMAJOR AXIS (MILES)	4582.2684	4567.3C79	4552.9345	4546.8868	4550.3277	4548.067C
RIGHT ASCENSION OF THE NODE (DEG)	-176.398	-176.365	-176.384	-176.459	-176.514	-176.568
ARGUMENT OF PERIGEE (DEG)	-35.141	-38.650	-42.16C	-43.67C	-42.628	-43.14C
LONGITUDE OF PERIGEE (DEG)	-211.538	-215.C16	-218.544	-220.129	-219.142	-219.709
INCLINATION (DEG)	39.933	39.959	39.974	39.969	39.962	39.956
TIME OF PERIGEE PASSAGE (SEC)	77.8412	23.4482	-30.9549	-54.4274	-38.4788	-46.4285
PERIOD (SEC)	6302.3988	6271.5594	6241.9783	6229.5447	6236.6174	6231.9703

COMPARISON OF PARAMETERS FROM FIT AND PARAMETERS

CONSIDERING TRACKER RESPONSE

NOTE. THE RELATIVE ERRORS WERE OBTAINED BY NORMALIZING THE ABSOLUTE DIFFERENCES TO THE FITTED VALUES (TIME OF PERIGEE PASSAGE WAS NORMALIZED TO THE PERIOD).

	FITTED	RESPONSE	ABSOLUTE DIFFERENCE	RELATIVE ERROR
ECCENTRICITY	0.0500913	0.0516948	0.1603E-02	0.3201E-01
SEMIMAJOR AXIS	4565.1694	4557.9654	0.7204E 01	0.1578E-02
RIGHT ASCENSION OF THE NODE	-176.335	-176.448	0.1135E-00	0.6438E-03
ARGUMENT OF PERIGEE	-38.634	-40.898	0.2264E 01	0.5860E-01
LONGITUDE OF PERIGEE	-214.969	-217.346	0.2377E 01	0.1106E-01
INCLINATION	40.009	39.959	0.5024E-01	0.1256E-02
TIME OF PERIGEE PASSAGE	21.4502	-11.5001	0.3295E 02	0.5258E-02
PERIOD	6267.1590	6252.3447	0.1481E 02	0.2364E-02

THE MAXIMUM ABSOLUTE ERROR IN POSITION INTRODUCED BY THE DYNAMIC RESPONSE OF THE SATELLITE TRACKING SYSTEM IS 51.521 MILES. THE RMS POSITION ERROR FOR ONE ORBIT IS 22.359 MILES. THE RELATIVE VALUES ARE 0.11872201E-01 MAXIMUM AND 0.49758680E-02 RMS FOR A SINGLE ORBIT.

Appendix B

Least Squares Theory

In this paper, least squares theory is used to "fit" elevation and azimuth values generated from orbit mechanics equations to time polynomials. The theory is briefly described in this appendix since in the body of the report, the theory is applied with no explanation. This discussion in general is patterned after a similar discussion in reference 2.

The overall function of the theory is to relate data in a polynomial such that the square of the difference between the value of the dependent variable computed from the polynomial and the actual value of the data at that point is a minimum. The square of the difference is minimized, rather than minimizing the difference, to avoid cancellation of differences which exist but which are of opposite sign. This latter would give an unwarranted appearance of accuracy. x_i is taken as the ith data value (either azimuth or elevation), x_i' the computed value of the dependent variable corresponding to evaluating the polynomial at t_i, the ith value of the independent variable (time) in the function to be developed. Since the function is a polynomial of order n,

$$x_i' = A_0 + A_1 t_i + \ldots + A_n t_i^n$$

$$= A_0 + \sum_{j=1}^{n} A_j t_i^j \qquad \text{(B-1)}$$

The deviation, δ_i, is

$$\delta_i = x_i - x_i'$$

$$= x_i - (A_0 + \sum_{j=1}^{n} A_j t_i^j) \qquad \text{(B-2)}$$

If there are N data points, and the error for each of these points is computed, squared, the resulting sum, E, results from

$$E = \sum_{i=1}^{N} \delta_i^{\,2} = \sum_{i=1}^{N} [x_i - (A + \sum_{j=1}^{n} A_j t_i^j)]^2 \qquad \text{(B-3)}$$

The problem is to determine A_0 and the A_j's so that E is minimized; that is, A_0 and the A_j's must be chosen so as to make the sum of the squares of the deviations E as small as possible. This is done by the usual method of calculus for minimizing functions; ie, that of equating the first partial derivative to zero. The derivatives are taken with respect to the variables to be determined, the A's.

$$\frac{\partial E}{\partial A_0} = \sum_{i=1}^{N} 2[x_i - (A_0 + \sum_{j=1}^{n} A_j t_i^j)]\,(-1) = 0 \qquad \text{(B-4)}$$

$$\frac{\partial E}{\partial A_1} = \sum_{i=1}^{N} 2[x_i - (A_0 + \sum_{j=1}^{n} A_j t_i^j)]\,(-t_i) = 0 \qquad \text{(B-5)}$$

The nth equation is

$$\frac{\partial E}{\partial A_n} = \sum_{i=1}^{N} 2\left[x_i - \left(A_0 + \sum_{j=1}^{n} A_j t_i^{\,j}\right)\right](-t_i^{\,n}) = 0 \qquad \text{(B-6)}$$

By collecting terms on the unknown coefficients the resulting equations are

$$NA_0 + \sum_{i=1}^{N} \sum_{j=1}^{n} A_j t_i^{\,j} = \sum_{i=1}^{N} x_i \qquad \text{(B-7)}$$

$$\sum_{i=1}^{N} A_0 t_i + \sum_{i=1}^{N} \sum_{j=1}^{n} A_j t_i^{\,j+1} = \sum_{i=1}^{N} x_i t_i \qquad \text{(B-8)}$$

The last equation is

$$\sum_{i=1}^{N} A_0 t_i^{\,n} + \sum_{i=1}^{N} \sum_{j=1}^{n} A_j t_i^{\,j+n} = \sum_{i=1}^{N} x_i t_i^{\,n} \qquad \text{(B-9)}$$

where A's are coefficients of the time polynomial

N is the number of data points

n is the order of the polynomial

x's and t's are known data to be fitted.

The result is n + 1 linear equations, called normal equations, in n + 1 unknowns, and therefore a unique solution for the unknowns exists and may be obtained by oridinary rules for solution of simultaneous equations.

A rule has been developed which greatly facilitates the formation of the normal equations. It is:

RULE: Form N equations by substituting the N sets of data into the desired polynomial with as yet undetermined coefficients. Next, multiply each equation by the coefficient of A_i in that equation and add the resulting equations to get the ith normal equation. Thus, the 2nd normal equation would result from the following:

$$A_o + A_1 t_1 + A_2 t_1^{\,2} + \ldots + A_n t_1^{\,n} = x_1$$

$$A_o + A_1 t_2 + A_2 t_2^{\,2} + \ldots + A_n t_2^{\,n} = x_2$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$A_o + A_1 t_N + A_2 t_N^{\,2} + \ldots + A_n t_N^{\,n} = x_N \qquad \text{(B-10)}$$

Then multiply each equation by the coefficient of A to get

$$A_o t_1 + A_1 t_1^{\,2} + A_2 t_1^{\,3} + \ldots + A_n t_1^{\,n+1} = x_1 t_1$$

$$A_o t_2 + A_1 t_2^{\,2} + A_2 t_2^{\,3} + \ldots + A_n t^{\,n+1} = x_2 t_2$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$A_o t_N + A_1 t_N^{\,2} + A_2 t_N^{\,3} + \ldots + A_n t_N^{\,n+1} = x_N t_N \qquad \text{(B-11)}$$

The sum is

$$A_o \sum_{i=1}^{N} t_i + A_1 \sum_{i=1}^{N} t_i^{\,2} + A_2 \sum_{i=1}^{N} t_i^{\,3} + \ldots +$$

$$A_n \sum_{i=1}^{N} t_i^{\,n+1} = \sum_{i=1}^{N} x_i t_i \qquad \text{(B-12)}$$

or more simply

$$\sum_{i=1}^{N} A_o t_i + \sum_{j=1}^{n} \sum_{i=1}^{N} A_j t_i^{\,j+1} = \sum_{i=1}^{N} x_i t_i \qquad \text{(B-13)}$$

This normal equation is the same as eq (B-8) (Ref 2:175).

A simple error analysis is performed in RADYN by determining the RMS error of the fit as well as the maximum error. The polynomial is evaluated at each of the values of time used in the fit, and the value so obtained subtracted from the actual value of the angle at that time so that a difference or error in the fit at that point is obtained. This is repeated for all of the values of time used in the fit, and the resulting differences are squared and added. This sum is divided by the total number of points and the square root extracted from the resulting quotient to obtain the RMS error. The relative error at each point is determined by dividing the error at that point by the correct value of the dependent variable.

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Appendix C

Equation Development

The equations used in Chapter V were in general developed in that chapter; however, where the development was long, for the sake of continuity it was delayed until this appendix. As indicated in Chapter V, in order to develop the time-position relationship of a satellite in an elliptical orbit, it is easiest to first determine the eccentric anomaly (E) at any time, and from this determine the true anomaly. Therefore, in this appendix a method for, first, determining the eccentric anomaly as a function of time (Kepler's equation), and then determining the corresponding true anomaly will be developed. Since it is assumed that the orbit parameters are known, once the true anomaly is determined, the position for that value of time is determined.

If a circle of radius a is circumscribed about the orbital ellipse and P is the satellite position on the ellipse, Q will be the point where the perpendicular to the major axis through P cuts the auxiliary circle as indicated in fig 9. The angle PFA is the true anomaly (f). The angle QCA is the eccentric anomaly (E).

Kepler's second law states that the time rate of change of area swept out by the radius vector is a constant. Therefore, one may write

$$\frac{\Delta A}{\Delta t} = \frac{\pi ab}{P} \qquad \text{(C-1)}$$

where $\frac{\Delta A}{\Delta t}$ is the constant rate of area swept out

πab is the area of the ellipse

P is the period of the ellipse.

The equation of the ellipse is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad \text{(C-2)}$$

If coordinates ($\overline{CR}$, $\overline{PR}$) are substituted into eq C-2, the result is

$$\frac{\overline{CR}^2}{a^2} + \frac{\overline{PR}^2}{b^2} = 1 \qquad \text{(C-3)}$$

From fig 9

$$\overline{CR} = a \cos E \qquad \text{(C-4)}$$

Substituting into eq C-3 yields

$$\frac{a^2 \cos^2 E}{a^2} + \frac{\overline{PR}^2}{b^2} = 1 \qquad \text{(C-5)}$$

$$\overline{PR}^2 = b^2(1 - \cos^2 E)$$

$$= b^2 \sin^2 E \qquad \text{(C-6)}$$

$$\overline{PR} = b \sin E \qquad \text{(C-7)}$$

From the figure and eq C-7 it follows that

$$\text{Area PFA} = \text{Area PRA} - \text{Area PRF} \qquad \text{(C-8)}$$

$$\text{Area PRF} = \tfrac{1}{2}(\overline{RF})\ (\overline{PR})$$

$$= \tfrac{1}{2}\ (ae - a \cos E)\ b \sin E$$

$$= \frac{ab}{2}\ (e - \cos E)\ \sin E \qquad \text{(C-9)}$$

The equation of the auxiliary circle is

$$x^2 + y^2 = a^2$$

or

$$y = \sqrt{a^2 - x^2}$$

$$= a\sqrt{1 - \frac{x^2}{a^2}} \qquad \text{(C-10)}$$

The equation of the ellipse (eq C-2) can be written

$$y = b\sqrt{1 - \frac{x^2}{a^2}} \qquad \text{(C-11)}$$

$$\text{Area PRA} = \int y\,dx$$

$$= \int b\sqrt{1 - \frac{x^2}{a^2}}\,dx \qquad \text{(C-12)}$$

But

$$\text{Area QRA} = \int y\,dx$$

$$= \int a\sqrt{1 - \frac{x^2}{a^2}}\,dx \qquad \text{(C-13)}$$

$$\text{Area PRA} = \frac{b}{a}\,\text{Area QRA} \qquad \text{(C-14)}$$

But

$$\text{Area QRA} = \text{Area QCA} - \text{Area QCR}$$

$$= \tfrac{1}{2}a^2E - \tfrac{1}{2}a^2\cos E\sin E \qquad \text{(C-15)}$$

$$\text{Area PRA} = \frac{b}{a}\left[\frac{a^2}{2}(E - \cos E\sin E)\right]$$

$$= \frac{ab}{2}(E - \cos E\sin E) \qquad \text{(C-16)}$$

$$\text{Area PFA} = \frac{ab}{2}(E - \cos E\sin E) -$$

$$\frac{ab}{2}(e\sin E - \cos E\sin E)$$

$$= \frac{ab}{2}(E - e\sin E) \qquad \text{(C-17)}$$

If time of perigee passage is τ, then time since perigee passage is

$$\Delta t = t - \tau \qquad \text{(C-18)}$$

The area swept out since perigee passage is

$$\Delta A = \text{Area PFA} \tag{C-19}$$

Combining the above yields

$$\frac{\frac{ab}{2}(E - e \sin E)}{t - \tau} = \frac{\pi ab}{P} \tag{C-20}$$

or

$$t - \tau = \frac{P}{2\pi}(E - e \sin E) \tag{30}$$

(Ref 4:2.4-2)

This is known as Kepler's equation. It cannot be solved explicity for E. An iterative solution is performed, using the equation in the form

$$E = (t - \tau)\frac{2\pi}{P} + e \sin E \tag{31}$$

This equation converges rapidly to a solution for E. The next requirement is to relate the eccentric and true anomalies in order to establish the satellite position at any time.

From fig 9 it is clear that

$$r^2 = \overline{HF}^2 + \overline{PH}^2$$

$$r^2 = (\overline{CF} - \overline{CH})^2 + \overline{PH}^2 \tag{C-21}$$

From the properties of an ellipse one obtains

$$\overline{CF} = ae \tag{C-22}$$

and

$$b = a\sqrt{1 - e^2} \tag{C-23}$$

Therefore

$$r^2 = (ae - a \cos E)^2 + b^2 \sin^2 E \tag{C-24}$$

$$r^2 = (ae - a\cos E)^2 + a^2(1 - e^2)\sin^2 E$$

$$= a^2e^2 - 2a^2 e\cos E + a^2\cos^2 E +$$

$$a^2\sin^2 E - a^2e^2\sin^2 E$$

$$r^2 = a^2(\sin^2 E + \cos^2 E) - 2a^2 e\cos E +$$

$$a^2e^2(1 - \sin^2 E)$$

$$= a^2 - 2a^2 e\cos E + a^2e^2\cos^2 E$$

$$= a^2(1 - e\cos E)^2 \qquad \text{(C-25)}$$

$$r = a(1 - e\cos E) \qquad \text{(C-26)}$$

From the polar equation of an ellipse one obtains

$$r = \frac{a(1 - e^2)}{1 + e\cos f} \qquad \text{(C-27)}$$

Equating the two values of r, yields

$$1 - e\cos E = \frac{1 - e^2}{1 + e\cos f} \qquad \text{(C-28)}$$

$$e\cos f = \frac{1 - e^2}{1 - e\cos E} - 1$$

$$= \frac{1 - e^2 - 1 + e\cos E}{1 - e\cos E}$$

$$\cos f = \frac{\cos E - e}{1 - e\cos E} \qquad \text{(C-29)}$$

From an identity of trignometry it follows that

$$\sin^2 f = 1 - \cos^2 f$$

$$= 1 - \left(\frac{\cos E - e}{1 - e\cos E}\right)^2$$

$$= \frac{1 - 2e\cos E + e^2\cos^2 E - \cos^2 E + 2e\cos E - e^2}{(1 - e\cos E)^2}$$

$$= \frac{1 - e^2 - \cos^2 E\,(1 - e^2)}{(1 - e\cos E)^2}$$

$$= \frac{(1 - e^2)\sin^2 E}{(1 - e\cos E)^2} \qquad \text{(C-30)}$$

$$\sin f = \sqrt{\frac{(1 - e^2)\sin^2 E}{(1 - e\cos E)^2}} \qquad \text{(C-31)}$$

From an identity of trigonometry it follows that

$$\tan\frac{f}{2} = \frac{1 - \cos f}{\sin f}$$

$$= \frac{1 - e\cos E - \cos E + e}{\sqrt{1 - e^2}\,\sin E}$$

$$= \frac{(1 + e) - (1 + e)\cos E}{\sqrt{1 - e^2}\,\sin E}$$

$$= \frac{(1 + e)(1 - \cos E)}{\sqrt{1 - e^2}\,\sin E}$$

$$\tan\frac{f}{2} = \sqrt{\frac{1 + e}{1 - e}}\,\tan\frac{E}{2} \qquad \text{(C-34)}$$

(Ref 4:2.4-2)

The position of the satellite is now determined for all time.

The equations necessary to determine the orbit parameters from geocentric position vectors are also used in Chapter V. Those which were not developed there are presented here with their development. The first such equation to be considered is the equation of the semilatus rectum. It can be determined by selecting three position vectors x_i, $i = 1, 2, 3$ where the subscripts indicate the vector sequence in time. The magnitude of the ith vector is

$$x_1 = \frac{p}{1 + e \cos (\theta_1 - \omega)} \tag{C-35}$$

where p = the orbital semilatus rectum

$\theta_1 - \omega$ = the true anomaly f_1

θ_1 = the angle between the line of nodes and x_1

ω = the argument of perigee.

This is the equation of an ellipse in polar form.

The purpose of the first manipulations is to develop relationships which are independent of the argument of perigee, as yet unknown. This is done through forming relations which depend on the differences of the anomalies of the position vectors which is obtainable.

$$(\theta_3 - \omega) = (\theta_3 - \theta_1) + (\theta_1 - \omega) \tag{C-36}$$

$$\begin{aligned} \cos (\theta_3 - \omega) &= \cos \left[(\theta_3 - \theta_1) + (\theta_1 - \omega)\right] \\ &= \cos (\theta_3 - \theta_1) \cos (\theta_1 - \omega) \\ &\quad - \sin (\theta_3 - \theta_1) \sin (\theta_1 - \omega) \end{aligned} \tag{C-37}$$

with $i = 3$, eq C-35 is written, after rearranging,

$$\begin{aligned} \frac{p}{x_3} &= 1 + e \cos (\theta_3 - \omega) \\ &= 1 + e\, [\cos (\theta_3 - \theta_1) \cos (\theta_1 - \omega) - \\ &\quad \sin (\theta_3 - \theta_1) \sin (\theta_1 - \omega)] \\ &= 1 + \frac{p - x_1}{x_1} \cos (\theta_3 - \theta_1) - \\ &\quad e \sin (\theta_3 - \theta_1) \sin (\theta_1 - \omega) \end{aligned} \tag{C-38}$$

Rearranging, one obtains

$$e \sin(\theta_1 - \omega) = \frac{1 - \frac{p}{x_3} + \frac{p - x_1}{x_1}\cos(\theta_3 - \theta_1)}{\sin(\theta_3 - \theta_1)}$$

$$= \frac{x_3 x_1 - p x_1 + (p x_3 - x_3 x_1)\cos(\theta_3 - \theta_1)}{x_1 x_3 \sin(\theta_3 - \theta_1)}$$

$$= \frac{x_3(p - x_1)\cos(\theta_3 - \theta_1) - x_1(p - x_3)}{x_1 x_3 \sin(\theta_3 - \theta_1)} \qquad \text{(C-39)}$$

Similarly, it is possible to replace subscript 3 by 2 to obtain

$$e \sin(\theta_1 - \omega)$$

$$= \frac{x_2(p - x_1)\cos(\theta_2 - \theta_1) - x_1(p - x_2)}{x_1 x_2 \sin(\theta_2 - \theta_1)} \qquad \text{(C-40)}$$

Equating the right hand sides of eqs C-39 and C-40 yields

$$\frac{x_3(p - x_1)\cos(\theta_3 - \theta_1) - x_1(p - x_3)}{x_1 x_3 \sin(\theta_3 - \theta_1)}$$

$$= \frac{x_2(p - x_1)\cos(\theta_2 - \theta_1) - x_1(p - x_2)}{x_1 x_2 \sin(\theta_2 - \theta_1)} \qquad \text{(C-41)}$$

$$\frac{p x_3 \cos(\theta_3 - \theta_1) - p x_1 - x_1 x_3 \cos(\theta_3 - \theta_1) + x_1 x_3)}{x_3 \sin(\theta_3 - \theta_1)}$$

$$= \frac{p x_2 \cos(\theta_2 - \theta_1) - p x_1 - x_1 x_2 \cos(\theta_2 - \theta_1) + x_1 x_2}{x_2 \sin(\theta_2 - \theta_1)}$$

$$p\left[\frac{x_3 \cos(\theta_3 - \theta_1) - x_1}{x_3 \sin(\theta_3 - \theta_1)} - \frac{x_2 \cos(\theta_2 - \theta_1) - x_1}{x_2 \sin(\theta_2 - \theta_1)}\right]$$

$$= \frac{x_1 x_3 \cos(\theta_3 - \theta_1) - x_1 x_3}{x_3 \sin(\theta_3 - \theta_1)} - \frac{x_1 x_2 \cos(\theta_2 - \theta_1) - x_1 x_2}{x_2 \sin(\theta_2 - \theta_1)}$$

(C-42)

The numerator of the left hand side becomes

$$p[x_3 x_2 \sin(\theta_2 - \theta_1)\cos(\theta_3 - \theta_1) - x_1 x_2 \sin(\theta_2 - \theta_1) -$$
$$x_2 x_3 \sin(\theta_3 - \theta_1)\cos(\theta_2 - \theta_1) + x_1 x_3 \sin(\theta_3 - \theta_1)]$$
$$= p\, x_3 x_2 [\sin(\theta_2 - \theta_1)\cos(\theta_3 - \theta_1) -$$
$$\sin(\theta_3 - \theta_1)\cos(\theta_2 - \theta_1)] -$$
$$p\, x_1 x_2 \sin(\theta_2 - \theta_1) + p\, x_1 x_3 \sin(\theta_3 - \theta_1)$$
$$= p\, x_3 x_2 \sin[(\theta_2 - \theta_1) - (\theta_3 - \theta_1)] -$$
$$p\, x_1 x_2 \sin(\theta_2 - \theta_1) + p\, x_1 x_3 \sin(\theta_3 - \theta_1)$$
$$= p\,[x_3 x_2 \sin(\theta_2 - \theta_3) - x_1 x_2 \sin(\theta_2 - \theta_1) +$$
$$x_1 x_3 \sin(\theta_3 - \theta_1)] \qquad \text{(C-43)}$$

The numerator of the right hand side of eq C-42 becomes

$$x_1 x_2 x_3 [\cos(\theta_3 - \theta_1)\sin(\theta_2 - \theta_1) - \sin(\theta_2 - \theta_1) -$$
$$\cos(\theta_2 - \theta_1)\sin(\theta_3 - \theta_1) + \sin(\theta_3 - \theta_1)]$$
$$= x_1 x_2 x_3 [\sin(\theta_2 - \theta_3) - \sin(\theta_2 - \theta_1) + \sin(\theta_3 - \theta_1)] \qquad \text{(C-44)}$$

Recombining the two sides and solving for p yields

$$p = x_1 x_2 x_3 \left[\frac{\sin(\theta_2 - \theta_3) - \sin(\theta_2 - \theta_1) + \sin(\theta_3 - \theta_1)}{x_3 x_2 \sin(\theta_2 - \theta_3) - x_1 x_2 \sin(\theta_2 - \theta_1) + x_1 x_3 \sin(\theta_3 - \theta_1)}\right]$$

$$= x_1 x_2 x_3 \left[\frac{\sin(\theta_2 - \theta_3) + \sin(\theta_1 - \theta_2) + \sin(\theta_3 - \theta_1)}{x_3 x_2 \sin(\theta_2 - \theta_3) + x_1 x_2 \sin(\theta_1 - \theta_2) + x_1 x_3 \sin(\theta_3 - \theta_1)}\right] \qquad (38)$$

(Ref 4:1.8-3)

The eccentricity is the next parameter to be considered.

By selecting the subscript i = 1, eq C-35 may be written

$$e \cos(\theta_1 - \omega) = \frac{p - x_1}{x_1} \qquad \text{(C-45)}$$

If this equation is squared and the result added to the square of C-39, there is obtained

$$e^2 \cos^2(\theta_1 - \omega) + \sin^2(\theta_1 - \omega) = e^2 \qquad \text{(C-46)}$$

$$e^2 = \frac{(p - x_1)^2}{x_1^2} + \left[\frac{x_3(p - x_1)\cos(\theta_3 - \theta_1) - x_1(p - x_3)}{x_1 x_3 \sin(\theta_3 - \theta_1)}\right]^2 \qquad \text{(40)}$$

The eccentricity e is now determined.

From eq C-35 for f = 0 one obtains

$$x_\pi = \frac{p}{1 + e} \qquad \text{(C-47)}$$

where x_π is the perigee distance.

From the construction of an ellipse in fig 9 one may obtain

$$a = ae + x_\pi$$

From eq C-49 it is evident that

$$a = ae + \frac{p}{1 + e} \qquad \text{(C-48)}$$

$$a = \frac{p}{1 - e^2} \qquad \text{(41)}$$

The final relations considered in this appendix are those which develop the constants of the unit vector in the perigee direction which appear in eq 47

$$\underline{i}p = A\,\underline{x}_1 + B\,\underline{x}_3 \qquad \text{(47)}$$

where $\underline{i}p$ = unit vector in the direction of perigee

A and B are the desired coefficients.

The scalar products of $\underline{i}_p$ with $\underline{x}_1$ and $\underline{x}_3$ are

$$\underline{i}_p \cdot \underline{x}_1 = A\, x_1^2 + B\, x_1 x_3 \cos(\theta_3 - \theta_1) \qquad \text{(C-49)}$$

$$\underline{i}_p \cdot \underline{x}_3 = A\, x_1 x_3 \cos(\theta_3 - \theta_1) + B\, x_3^{\,2} \qquad \text{(C-50)}$$

By use of fig C-1, the perigee unit vector can also be related to any position vector through the expression

$$\underline{i}_p = \underline{i}_r \cos(\theta - \omega) - \underline{i}_\theta \sin(\theta - \omega) \qquad \text{(C-51)}$$

where $\underline{i}_r$ is the unit vector in the radial direction

$\underline{i}_\theta$ is the unit vector in the transverse direction.

Therefore

$$\underline{i}_p \cdot \underline{x}_1 = x_1 \cos(\theta_1 - \omega) \qquad \text{(C-52)}$$

But eq C-35 may be written

$$x_1 \cos(\theta_1 - \omega) = \frac{p - x_1}{e} \qquad \text{(C-53)}$$

Therefore

$$\underline{i}_p \cdot \underline{x}_1 = \frac{p - x_1}{e} \qquad \text{(C-54)}$$

Similarly

$$\underline{i}_p \cdot \underline{x}_3 = \frac{p - x_3}{e} \qquad \text{(C-55)}$$

From above one obtains

$$A\, x_1^{\,2} + B\, x_1 x_3 \cos(\theta_3 - \theta_1) = \frac{p - x_1}{e} \qquad \text{(C-56)}$$

$$A\, x_1 x_3 \cos(\theta_3 - \theta_1) + B\, x_3^{\,2} = \frac{p - x_3}{e} \qquad \text{(C-57)}$$

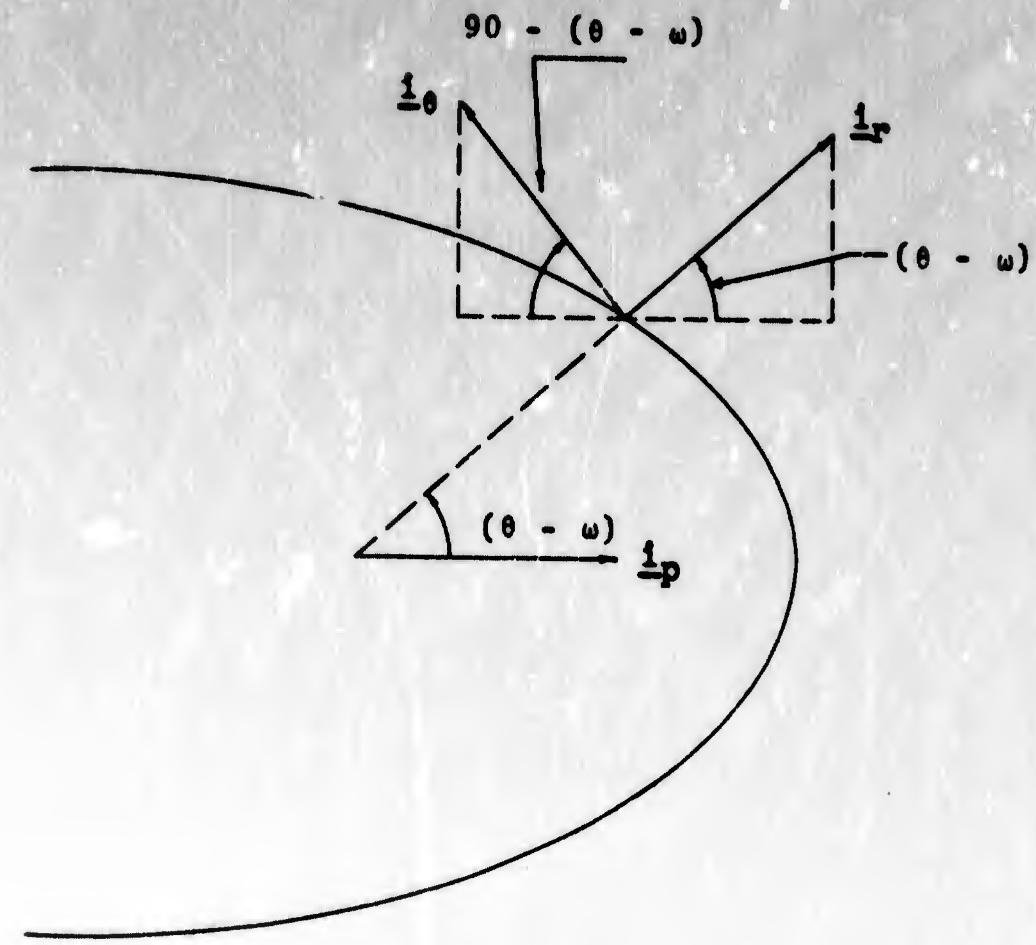

Fig C-1 Perigee Unit Vector

Solution of these two linear equations yields

$$A = \frac{1}{ex_1 \sin^2 (\theta_3 - \theta_1)} [(\frac{p}{x_1} - 1) - (\frac{p}{x_3} - 1) \cos (\theta_3 - \theta_1)] \tag{48}$$

$$B = \frac{1}{ex_3 \sin^2 (\theta_3 - \theta_1)} [(\frac{p}{x_3} - 1) - (\frac{p}{x_1} - 1) \cos (\theta_3 - \theta_1)] \tag{49}$$

(Ref 4:8.1-4)

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APPENDIX D

RADYN

THIS APPENDIX LISTS THE PROGRAM RADYN ALONG WITH THE SUBPROGRAMS DEVELOPED FOR IT. THESE PROGRAMS WERE PREPARED BY THE AUTHOR AS A PART OF THE THESIS.

RADYN CONSISTS OF THE MAIN PROGRAM AND FIVE SUBPROGRAMS. THE MAIN PROGRAM DEVELOPS THE SO-CALLED ERROR FREE DATA. THAT IS, IT DETERMINES VALUES OF AZIMUTH AND ELEVATION ANGLES AND SLANT RANGE WITH TIME FROM INPUT ORBITAL PARAMETERS. IN ADDITION, IT SERVES AS THE LOCATION FOR CONTROLLING THE ENTIRE PROBLEM SOLUTION. TO DO THIS, IT CALLS THE NECESSARY SUBPROGRAMS, SUBSEQUENTLY COLLATING THEIR OUTPUT DATA WITH THE REST OF THE COMPUTATIONAL RESULTS AS REQUIRED DURING THE SOLUTION DEVELOPMENT.

ANAL IS THE FIRST SUBPROGRAM. IT PERFORMS AN ERROR ANALYSIS OF THE CURVE FIT ROUTINE FOR POLYNOMIALS OF ORDER ONE TO TWENTY-FOUR. THE RESULTS CONSIST OF THE ROOT MEAN SQUARE AND MAXIMUM VALUES OF THE ABSOLUTE DIFFERENCE AND THE RELATIVE DIFFERENCE.

FIT IS THE SECOND SUBPROGRAM. IT IS USED TO PERFORM THE FIT TO THE SELECTED ORDER POLYNOMIAL. AS INDICATED IN THE COMMENT CARDS, THE OPTION IS AVAILABLE TO PRINT OUT ONLY THE COEFFICIENTS OF THE POLYNOMIAL, OR TO PRINT THE ENTIRE SUBPROGRAM OUTPUT, WHICH CONSISTS OF AN EVALUATION OF THE POLYNOMIAL FOR EACH VALUE OF THE INDEPENDENT VARIABLE AND A BRIEF ERROR ANALYSIS. IN ADDITION, THE OPTION IS AVAIL-

ABLE IN THIS SUBPROGRAM TO PLOT THE POLYNOMIAL INPUT DATA AND EVALUATION VERSUS TIME FOR BOTH THE AZIMUTH AND ELEVATION POLYNOMIALS.

THE NEXT SUBPROGRAM IS ORBIT. IT COMPUTES ORBIT PARAMETERS FROM SETS OF TOPOCENTRIC AZIMUTH, ELEVATION AND RANGE DATA FURNISHED IT. THE FIRST CALL FOR THIS SUBPROGRAM IS TO COMPUTE THE PARAMETERS FROM THE EVALUATION OF THE TIME POLYNOMIALS. IT COMPUTES SIX SETS WHICH ARE THEN AVERAGED BY RADYN AND COMPARED WITH THE INPUT SET. THE SECOND CALL IS TO PERFORM THE SAME FUNCTION FROM DATA OF THE TRANSFER FUNCTION EVALUATION. HERE THE COMPARISON IS WITH THE PARAMETERS BASED UPON THE POLYNOMIAL DATA.

THE NEXT SUBPROGRAM, ERROR, DETERMINES THE ROOT MEAN SQUARE DIFFERENCE IN POSITION FOR A SINGLE ORBIT BETWEEN ANY TWO SETS OF ORBITAL PARAMETERS FURNISHED IT. THIS IS PERFORMED WITH TIME BEING THE BASIS FOR COMPARING DIFFERENT POSITION VECTORS (IE, THE DIFFERENCE IN POSITION AT A GIVEN TIME IS COMPUTED).

THE FINAL SUBPROGRAM, DYNAM, EVALUATES THE SYSTEM TRANSFER FUNCTION WITH TIME. IT IS WRITTEN TO SIMILATE A SECOND ORDER SYSTEM RESPONDING TO A FOURTH ORDER INPUT.

THERE ARE FOUR INPUT STATEMENTS TO THIS PROGRAM. THE FIRST CONSISTS OF SIX FIXED POINT VARIABLES USED TO CONTROL THE OPTIONS AVAILABLE ON THE OUTPUT. THEIR USE IS EXPLAINED IN COMMENT CARDS IN THE PROGRAM ITSELF. THE SECOND READ STATEMENT OBTAINS THE TRACKING SYSTEM CONSTANTS. THE FIRST VARIABLE READ IS TWICE THE SYSTEM NATURAL FREQUENCY TIMES

THE DAMPING RATIO. THE SECOND IS THE NATURAL FREQUENCY SQUARED. THE FINAL TWO READ STATEMENTS ARE THE TRACKING STATION COORDINATES AND INPUT ORBIT PARAMETERS. ALL ANGLES ARE READ IN IN DEGREES, DISTANCES IN MILES.

```
C RADYN

      PI = 3.1415926536
      PIBY2 = PI/2.
      PIX2  = 2.*PI
      RADDEG = 180.0/PI
      WE=PIX2/86164.1
      WE1=WE*RADDEG
      DIMENSION NZ(50)
      DIMENSION DUMMY(1332),DATA(100,5)
      DIMENSION CO(32,8),CT(32,8),E(4,4,2)
      DIMENSION A (3,1,330),B (3,1),C (3,3),D (3,3),G (3,3),
     1H(3,1),X (3,1),Y (3,1),W(3,1)
      DIMENSION T(330),DEL(330),DAZ(330),RARNG(330)
      DIMENSION ACOMP(6),DLONC2(6),DARGPC(6),DLONC3(6),DINCL
     12(6),TPERIG(6),PCOMP(6),ECCOMP(6)
      DIMENSION TPOS(300),DAZPCS(300),DELPCS(300),TNORM1(300
     1),RARPCS(300)
      DIMENSION TTT(3,12),AAZ(3,12),EEL(3,12),RRANT(3,12),
     1TT(3),EL(3),AZ(3),RANT(3)
      DIMENSION TIM(3,6),CAZ(620),CEL(620),RAD(3,6,2)
      DIMENSION TA(620),TB(620),TC(200),TD(200)
      COMMON DUMMY,DATA,CO
      COMMON K20,KA2,KA3,KA4,KA6,K28
      COMMON DELTN,DELPOS,DAZPCS,TPOS
      COMMON TT,EL,AZ,RANT,B,C,RLCN4,ECCCMP,ACOMP,DLONC2,
     1DARGPC,DLONC3,DINCL2,TPERIG,PCOMP,TTT,EEL,AAZ,RRANT
      COMMON AECCOM,AACOMP,ADLCN2,ADARGP,ADLCN3,ADINCL,
     1ATPERI,APCOMP
      COMMON DELTAT,TMID1,K8,K30,NOR
      COMMON T,DEL,DAZ,RARNG
      COMMON P,TPERI,ECC,RLON2,RARGP,RINCL,SEMAX
      COMMONTIM,CAZ,CEL,RAD,CA1,CA2,CA3
      COMMON KA5,KA7
      COMMON K24,TA,TB,TC,TD
      COMMON KNTRA,BECCOM,BACOMP,BDLCN2,BDARGP,BDLCN3,
     1BDINCL,BTPERI,BPCOMP
C     KA1 IS -1 FOR ERROR ANALYSIS ONLY, 0 FOR FIT ONLY, AND
C     1 FOR BOTH
C     KA2 IS 0 TO OBTAIN POLYNOMIAL COEFFICIENTS ONLY, AND
C     3 TO GET COMPLETE CURVE FIT OUTPUT
```

```
C      KA3 IS ORDER OF POLYNOMIAL USED IN ACTUAL FIT
C      KA4 IS -1 TO EVALUATE PCINTS AT ABOUT 3.3 SECOND
C      INTERVALS AND PLCT, OTHERWISE 0.
C      KA5 MUST BE SELECTED TO MATCH THE ORDER OF THE
C      POLYNOMIAL USED IN THE PLOT BY THE FOLLOWING
C      TABLE (CAN USE 4 TO 13TH ORDER INCLUSIVE ONLY)
C        POLY      KA3       KA5
C       ORDER
C         13        13        15
C         12        12        14
C         11        11        13
C         10        10        12
C          9         9        11
C          8         8        10
C          7         7         9
C          6         6         8
C          5         5         7
C          4         4         6
C      KA7 IS -1 TO PLOT IN FIT ROUTINE, 0 TO PLOT IN DYNAM
C      ROUTINE AND +1 NOT TO PLOT
       READ INPUT TAPE 2,301,KA1,KA2,KA3,KA4,KA5,KA7
  301  FORMAT(6I10)
C      CA1 IS TWICE THE PRODUCT OF THE TRACKER NATURAL
C      FREQUENCY TIMES THE DAMPING RATIO. CA2 IS THE NATURAL
C      FREQUENCY SQUARED
       READ INPUT TAPE 2,330,CA1,CA2
  330  FORMAT(2E15.0)
   70  READ INPUT TAPE 2,15,DLAT, DLCN1, DLON2, DARGP, DINCL,
      1DLCN4
   15  FORMAT (6F10.3)
       READ INPUT TAPE 2,16, SEMAX,BGNT,BGNE,ECC
   16  FORMAT(3F10.4,F10.8)
       CLON3=CLON2+DARGP
       WRITE OUTPUT TAPE 3,17,DLCN1,DLAT
   17  FORMAT(1H1/1H0,14X,10HGA/EE/64-6/1HB,/1HA,
      114X,36HTRACKING STATION COORDINATES........
      21H.,F9.3,11H DEG E LONG/1H0,42X,9H.........,F9.3,3H DE
      37HG N LAT)
       FCTR1=SQRTF(((SEMAX * 5.28E3)**3)/(1.407639E16))
       FCTR2 = SQRTF((1.+ECC)/(1.-ECC))
       P = PIX2 *FCTR1
       AN = 1./FCTR1
       IF(DLCN3-360.)27,26,26
   26  DLCN3=CLON3-360.
       TPERI=BGNT
   27  WRITE OUTPUT TAPE 3,24,ECC,SEMAX,DLON2,DARGP,CLON3,DIN
      1CL,TPERI
   24  FORMAT(1HB14X16HCRBIT PARAMETERS/1H016X12HECCENTRICITY
      124H...................... F9.7,/1H016X12HSEMIMAJOR AX
      223HIS.....................F10.4,6H MILES/1H0,16X,3HRIG
      332HHT ASCENSION OF ASCENDING NCDE..,F9.3,9H DEG E CFIX
      44HVER-,/1H 63X11HNAL EQUINOX,/1H ,16X,13HARGUMENT CF P
      522HERIGEE.................,F9.3,4H DEG/1H0,16X,6HLONGIT
      629HUDE OF PERIGEE...............F9.3,4H DEG,/1H0,16X,
```

```
     735HINCLINATION.........................,F9.3,4H DEG,/
     81H0,16X,35HTIME CF PERIGEE PASSAGE.............,
     9F8.3,4H SEC)
      WRITE CUTPUT TAPE 3,230,P,DLON4,WE1
 230 FORMAT(1H0,14X,36HORBITAL PERICD......................
     11H.,E15.8,4H SEC,/1H0,14X,25HLCNGITUDE OF VERNAL ECUIN
     212HOX AT T=0...,F9.3,4H CEG,/1H0,14X,14HEARTH ROTATICN
     323H RATE..................,E15.8,4H CPS,/1HA)
      T(1)=BGNT
      E=EGNE
      N=1
      NN=0
      J5=0
      J4=0
      J3=0
      J2=0
      J1=-1
 105  I1=0
      I2=0
      I3=0
      I4=0
      I5=0
      I6=0
      J6=-1
  14  E1=E
      E=AN * T(N) + (ECC * SINF(E1))
      J5=J5+1
      IF(J5-10000)204,203,203
 203  WRITE OUTPUT TAPE 3,205,E,E1
 205  FORMAT(43HARUN FORCED THRU KEPLER LCCP, E ANC E1 ARE ,
    12E20.8)
      GO TC 2
 204  IF ((ABSF (E - E1)) - .CCC0001) 2,14,14
   2  J5=0
      E2 = E/2.0
      CON = COSF(E2)
      SNE = SINF(E2)
      TANF2 = FCTR2 * (SNE/CCN)
      F2 = ATANF(TANF2)
      IF (CCN) 31,32,32
  31  F = (F2 + PI)*2.
      GO TC 35
  32  IF ( SNE) 33,34,34
  33  F = (F2 +(PIX2))*2.
      GO TO 35
  34  F = F2*2.
  35  CONTINUE
      EAROT = WE*T(N)
  12  RF = ((SEMAX)*(1.00-((ECC)**2.C)))/(1.C +((ECC) * (CCS
     1F (F))))
      B(3,1)= -3960.0
      IF (RF+B(3,1)-50.)64,64,63
  64  IF (RF+B(3,1))61,61,59
  59  WRITE CUTPUT TAPE 3,60
```

```
   60 FORMAT (39H0SATELLITE IS WITHIN 50 MILES OF EARTH.)
      GO TO 63
   61 WRITE OUTPUT TAPE 3,62
   62 FORMAT (42H0SATELLITE ORBIT INTERSECTS EARTH SURFACE.)
      GO TO 70
   63 CONTINUE
      RLAT = DLAT/RADDEG
      RLON1 = DLON1/RADDEG
      RLON2 = DLON2/RADDEG
      RARGP = DARGP/RADDEG
      RINCL = DINCL/RADDEG
      RLON3 = DLON3/RADDEG
      RLON4=DLON4/RADDEG
   19 B(1,1)=0.
      B(2,1)=0.
      C(1,1)=(-SINF(RLAT))*(COSF(RLON1))
      C(1,2)=(-SINF(RLAT))*(SINF(RLON1))
      C(1,3)=COSF(RLAT)
      C(2,1)=SINF(RLON1)
      C(2,2)=-COSF(RLON1)
      C(2,3)=0.
      C(3,1)=(COSF(RLAT))*(COSF(RLON1))
      C(3,2)=(COSF(RLAT))*(SINF(RLON1))
      C(3,3)=SINF(RLAT)
      ASCEN=EAROT-RLON4
   20 D(1,1)=COSF(ASCEN)
      D(1,2)=SINF(ASCEN)
      D(1,3)=0.
      D(2,1)=-SINF(ASCEN)
      D(2,2)=COSF(ASCEN)
      D(2,3)=0.
      D(3,1)=0.
      D(3,2)=0.
      D(3,3)=1.
      RINC2=RINCL/2.0
      SRIN2=SINF(RINC2)
      RIN22=SRIN2 ** 2.
   21 G(1,1)=COSF(RLON3) +2.*(SINF(RLON2))*(SINF(RARGP))*RIN
     122
      G(1,2)=-SINF(RLON3) + 2.*SINF(RLON2)*(COSF(RARGP))*RIN
     122
      G(1,3)=(SINF(RLON2))*SINF(RINCL)
      G(2,1)=(SINF(RLON3))-(2.*(COSF(RLON2))*(SINF(RARGP))*R
     1IN22)
      G(2,2)=(COSF(RLON3))-(2.*(COSF(RLON2))*COSF(RARGP))*RI
     1N22
      G(2,3)=(-COSF(RLON2))*(SINF(RINCL))
      G(3,1)=(SINF(RARGP))*(SINF(RINCL))
      G(3,2)=(COSF(RARGP))*(SINF(RINCL))
      G(3,3)=COSF(RINCL)
      H(1,1)=RF*COSF(F)
      H(2,1)=RF*SINF(F)
      H(3,1)=0.
C     W = GH, X = DW, Y = CX, A = B + Y
```

```
      CO 3 I = 1,3
      W(I,1)=0.
      DO 3 L = 1,3
    3 W(I,1)=W(I,1) + G(I,L)*H(L,1)
      CO 5 I = 1,3
      X(I,1)=0.
      CO 5 L = 1,3
    5 X(I,1)=X(I,1) + C(I,L)*W(L,1)
      CO 7 I = 1,3
      Y(I,1)=0.
      CO 7 L = 1,3
    7 Y(I,1)=Y(I,1) + C(I,L)*X(L,1)
      CO 9 I = 1,3
    9 A(I,1,N)=B(I,1) + Y(I,1)
      RARNG(N)=SQRTF(((A(1,1,N))**2)+((A(2,1,N))**2)+((A(3
     1,1,N))**2))
      RAZ1=ATANF((A(2,1,N))/(A(1,1,N)))
      IF (A(1,1,N)) 36,37,37
   36 RAZ =RAZ1 + PI
      GO TC 40
   37 IF (A(2,1,N)) 38,39,39
   38 RAZ =RAZ1 + PIX2
      GO TC 40
   39 RAZ =RAZ1
   40 CONTINUE
      REL  = ATANF((SINF(RAZ))*(A(3,1,N))/(A(2,1,N)))
   66 CAZ(N)=RAZ*RADDEG
      CEL(N)=REL*RADDEG
   28 IF (T(N) -P) 210,210,213
  213 IF(J4)214,30,214
   30 IF(REL)260,192,123
  214 IF(REL)172,192,123
  123 J4=1
  210 CONTINUE
  207 IF(J3)135,135,200
  135 IF(I6)147,134,105
  134 IF(I5)144,133,105
  133 IF(I4)141,132,157
  132 IF(I3)128,131,155
  131 IF(I2)125,130,153
  130 IF(I1)122,120,151
  120 IF(DEL(N))136,194,137
  136 J1=0
      J2=1
  121 T(N+1)=T(N)+250.
      N=N+1
      I1=-1
      GO TO 14
  122 IF(DEL(N))121,149,124
  124 T(N)=T(N-1)+100.
      I2=-1
      GO TO 14
  125 IF(DEL(N))126,145,127
  126 T(N+1)=T(N)+100.
```

```
      N=N+1
      GO TO 14
  127 T(N)=T(N-1)+10.
      I3=-1
      GO TO 14
  128 IF(DEL(N))129,149,140
  129 T(N+1)=T(N)+10.
      N=N+1
      GO TO 14
  140 T(N)=T(N-1)+1.
      I4=-1
      GO TO 14
  141 IF(DEL(N))142,149,143
  142 T(N+1)=T(N)+1.
      N=N+1
  171 IF(J2)14,166,14
  143 T(N)=T(N-1)+.1
      I5=-1
      GO TO 14
  144 IF(DEL(N))145,149,146
  145 T(N+1)=T(N)+.1
      N=N+1
      GO TO 171
  146 T(N)=T(N-1)+.01
      I6=-1
      GO TO 14
  147 IF(DEL(N))148,149,149
  148 T(N+1)=T(N)+.01
      N=N+1
      GO TO 171
  149 T(N+1)=T(N)+.01
      N=N+1
  195 IF(J2)170,170,169
  170 GO TO 105
  169 NN=NN+1
      TPOS(NN)=T(N-1)
      CELPOS(NN)=CEL(N-1)
      CAZPOS(NN)=DAZ(N-1)
      RARPOS(NN)=RARNG(N-1)
      GO TO 105
  137 IF (J1)138,139,139
  138 J1=0
      J2=-1
      GO TO 150
  139 J2=0
  150 T(N+1)=T(N)+10.
      N=N+1
      I1=1
  165 IF(J2)14,166,14
  166 NN=NN+1
      TPOS(NN)=T(N-1)
      CELPOS(NN)=DEL(N-1)
      DAZPOS(NN)=CAZ(N-1)
      RARPOS(NN)=RARNG(N-1)
```

```
      GO TO 14
  151 IF(DEL(N))152,172,150
  152 T(N)=T(N-1)+1.
      I2=1
  167 GO TO 14
  153 IF(DEL(N))154,172,142
  154 T(N)=T(N-1)+.1
      I3=1
      GO TO 14
  155 IF(DEL(N))156,172,145
  156 T(N)=T(N-1)+.01
      I4=1
      GO TO 14
  157 IF(DEL(N))172,172,148
  172 IF(J2)105,192,105
  192 TMID=(TPOS(1)+TPOS(NN))/2.
      N1=N
      N=N+1
      T(N)=TMID
      J3=1
      GO TO 14
  194 T(N+1)=T(N)+.01
      N=N+1
      GO TO 14
  200 J3=0
      WRITE OUTPUT TAPE 3,201,TMID,DEL(N),DAZ(N),RARNG(N)
  201 FORMAT(1HB,34X,20HMIDPASS   PARAMETERS,/1H0,29X,4HTIME
     13H = E15.8,8H SECONDS,/1H0,24X,12HELEVATION = ,E15.8,
     28H DEGREES,/1H0,26X,10HAZIMUTH = ,E15.8,8H DEGREES/
     31H0,28X,8HRANGE = E15.8,6H MILES)
      K6=NN
  263 K13=K6-1
      DO 249 K11=1,K13
      IF(ABSF(DAZPOS(K11+1)-DAZPOS(K11))-179.)249,250,250
  250 IF(DAZPOS(K11+1)-DAZPOS(K11))251,249,252
  251 DO 253 K14=1,K11
  253 DAZPOS(K14)=DAZPOS(K14)-360.
      GO TO 254
  252 K15=K11+1
      DO 255 K14=K15,K6
  255 DAZPOS(K14)=DAZPOS(K14)-360.
      GO TO 254
  249 CONTINUE
  254 CONTINUE
  184 L1=0
      DO 185 L2=1,K6
      IF(L1)186,187,186
  187 WRITE OUTPUT TAPE 3,188
  188 FORMAT(1H1/1H014X10HGA/EE/64-6/1H836X14HERROR  FREE  D
     13HATA/1H014X3HNO.,2X,4HTIME,7X9HELEVATION,8X,7HAZIMUTH
     27X,11HRADAR RANGE,/1H )
  186 WRITE OUTPUT TAPE 3,189,L2,TPOS(L2),DELPOS(L2),DAZPOS(
     1L2),RARPOS(L2)
  189 FORMAT(1H ,13X,I3,F9.2,3E16.7)
```

```
      L1=L1+1
      IF(L1-48)185,197,197
  197 L1=L1-48
  185 CONTINUE
      IF(J4)262,262,260
  262 NN=0
      N=N+1
      T(N)=T(N1)+.01
      GO TO 105
  260 DO 261 K11=1,K13
      IF(DELPOS(K11+1)-DELPOS(K11))256,261,261
  261 CONTINUE
  256 AZ1=DAZPOS(K11-1)
      AZ2=DAZPOS(K11)
      AZ3=DAZPOS(K11+1)
      DIF1=ABSF(AZ1-AZ2)
      DIF2=ABSF(AZ2-AZ3)
      IF(DIF1-DIF2)282,282,281
  281 K11=K11-1
  282 CONTINUE
      K16=K11
      K7=1
      K28=1
      K20=K11
      NZ(1)=0
      NY=1
  275 IF(ABSF((DAZPOS(K20)-DAZPOS(K20-1))/
     1(TPOS(K20)-TPOS(K20-1)))-2.)277,276,276
  276 NZ(NY)=K20
      NY=NY+1
      K20=K20-1
      GO TO 275
  277 K23=1
      KA6=0
      DO 257 K8=1,2
      IF(K20-99)284,284,285
  284 K24=1
      GO TO 286
  285 A20=K20
      K26=A20/2.+.5
      IF(K26-99)297,297,298
  297 K24=2
      GO TO 286
  298 K24=3
  286 K27=K7+K20-1
      DELTAT=(TPOS(K27)-TPOS(K7))
      TMID1=TPOS(K7)
      I=0
      DO 258 II=1,K20,K24
      I=I+1
      TNORM1(K7)=(TPOS(K7)-TMID1)/DELTAT
      DATA(I,1)=TNORM1(K7)
      DATA(I,2)=DAZPOS(K7)
      DATA(I,3)=DELPOS(K7)
```

```
 258 K7=K7+K24
     K30=K20/K24
     IF(KA1)302,303,304
 302 CALL ANAL
     GO TO 305
 303 CALL FIT
     GO TO 305
 304 CALL ANAL
     CALL FIT
 305 IF(K8-1)312,312,257
 312 M2=K28+3*K24
     M4=K27-7*K24
     DO 314 M3=1,6
     RRANT(1,M3)=RARPOS(M2)
     TTT(1,M3)=TPOS(M2)
     M2=M2+2*K24
     IF(M3-3)313,313,314
 313 TTT(2,M3)=TPOS(M4)
     RRANT(2,M3)=RARPOS(M4)
     M4=M4+2*K24
 314 CONTINUE
     DO 328 KP=1,3
     KP=KP
     TIM(1,KP)=(TTT(1,KP)-TMID1)/DELTAT
     TIM(1,KP+3)=(TTT(1,KP+3)-TMID1)/DELTAT
 328 TIM(2,KP)=(TTT(2,KP)-TMID1)/DELTAT
 329 CALL DYNAM
     K20=K11+1
 278 IF(ABSF((DAZPOS(K20+1)-DAZPOS(K20))/
    1(TPOS(K20+1)-TPOS(K20)))-2.)280,279,279
 279 NZ(NY)=K20
     NY=NY+1
     K20=K20+1
     GO TO 278
 280 K7=K20
     K28=K7
 292 NY=NY-1
     NZ1=NZ(1)
     NZN=NZ(NY)
     WRITE OUTPUT TAPE 3,296,(NZ(NX),NX=1,NY)
 296 FORMAT(1HA14X26HPOINTS OMMITED BY FIT ARE ,15I3)
     IF(K24-2)294,287,299
 287 IF(K23)295,295,291
 295 WRITE OUTPUT TAPE 3,289
 289 FORMAT(43HA ONLY EVEN-NUMBERED POINTS FITTED IN ORDER
    138H TO REDUCE THEIR NUMBER TO 99 OR LESS.)
     GO TO 294
 291 WRITE OUTPUT TAPE 3,288
 288 FORMAT(42HA ONLY ODD-NUMBERED POINTS FITTED IN ORDER
    138H TO REDUCE THEIR NUMBER TO 99 OR LESS.)
     GO TO 294
 299 WRITE OUTPUT TAPE 3,300,K7
 300 FORMAT(36HAONLY EVERY 3RD POINT (STARTING WITH,I3,
    151H) FITTED IN ORDER TO REDUCE THE NUMBER TO 99 OR LES
```

```
      22HS.)
  294 K23=XMCDF(K7,2)
      IF(K8-1)307,307,257
  307 K20=K6+1-K20
      KA6=1
  257 CONTINUE
  283 DO 293 NX=1,NY
  293 NZ(NX)=0
      M2=K28+3*K24
8438  M4=K27-12*K24
      DO 318 M3=1,6
      IF(M3-3)317,317,316
  316 TTT(2,M3)=TPOS(M2)
      RRANT(2,M3)=RARPOS(M2)
      M2=M2+2*K24
  317 TTT(3,M3)=TPOS(M4)
      RRANT(3,M3)=RARPOS(M4)
  318 M4=M4+2*K24
      DO 331KP=4,6
      KP=KP
      TIM(2,KP)=(TTT(2,KP)-TMID1)/DELTAT
      TIM(3,KP-3)=(TTT(3,KP-3)-TMID1)/DELTAT
 331  TIM(3,KP)=(TTT(3,KP)-TMID1)/DELTAT
 332  CALL DYNAM
      DO 308 NOR=1,6
      NOR =NOR
      DO 322 LL=1,3
      AZ(LL)=AAZ(LL,NOR)
      EL(LL)=EEL(LL,NOR)
      TT(LL)=TTT(LL,NOR)
  322 RANT(LL)=RRANT(LL,NOR)
  308 CALL ORBIT
      WRITE OUTPUT TAPE 3,333
  333 FORMAT(1H1/1HB/1HB/1HB/1HA,44X,18HORBIT   PARAMETERS
     1/1HA,13X40HDETERMINED BY DIFFERENT SETS OF THREE VE
     239HCTORS BEFORE CONSIDERING RADAR DYNAMICS/1H0,30X
     36HSET  1,6X,6HSET  2,6X,6HSET  3,6X,6HSET  4,6X
     46HSET  5,6X,6HSET  6)
      WRITE OUTPUT TAPE 3,323,(ECCOMP(I),I=1,6),(ACOMP(I),I=
     11,6),(CLONC2(I),I=1,6),(CARGPC(I),I=1,6)
  323 FORMAT(1HA,8X,12HECCENTRICITY,F18.7,
     55F12.7,/1HA,8X,14HSEMIMAJOR AXIS,F15.4,5F12.4,/1H
     69X7H(MILES)/1H08X15HRIGHT ASCENSIONF14.3,5F12.3,/1H
     79X17HOF THE NODE (DEG)/1H08X11HARGUMENT OF,F18.3,
     85F12.3/1H 9X13HPERIGEE (DEG))
      WRITE OUTPUT TAPE3,324,(CLONC3(I),I=1,6),(DINCL2(I),I=
     11,6),(TPERIG(I),I=1,6),(PCOMP(I),I=1,6)
  324 FORMAT(1H0,8X,12HLONGITUDE OF,F17.3,5F12.3,/1H
     19X13HPERIGEE (DEG),/1H0,8X,17HINCLINATION (DEG)
     26F12.3,/1HA,8X15HTIME OF PERIGEE,F14.4,5F12.4
     3/1H 9X13HPASSAGE (SEC)/1HA,8X12HPERIOD (SEC)
     4F17.4,5F12.4)
      AECCOM=0
      AACOMP=0
```

```
      ADLON2=0
      ADARGP=0
      ADLON3=0
      ADINCL=0
      ATPERI=0
      APCOMP=0
      DO 325 I=1,6
      AECCOM=AECCOM+ECCOMP(I)/6.
      AACOMP=AACOMP+ACOMP(I)/6.
      ADLON2=ADLON2+DLONC2(I)/6.
      ADARGP=ADARGP+DARGPC(I)/6.
      ADLON3=ADLON3+DLONC3(I)/6.
      ADINCL=ADINCL+DINCL2(I)/6.
      ATPERI=ATPERI+TPERIG(I)/6.
  325 APCOMP=APCOMP+PCOMP(I)/6.
      EERR=ABSF(AECCOM-ECC)
      EREL=EERR/ECC
      AERR=ABSF(AACOMP-SEMAX)
      AREL=AERR/SEMAX
      DL2ERR=ABSF(ADLON2-DLON2)
      DL2REL=ABSF(DL2ERR/DLON2)
      DARERR=ABSF(ADARGP-DARGP)
      DARREL=ABSF(DARERR/DARGP)
      DL3ERR=ABSF(ADLON3-DLON3)
      DL3REL=ABSF(DL3ERR/DLON3)
      DINERR=ABSF(ADINCL-DINCL)
      DINREL=ABSF(DINERR/DINCL)
      TPERR=ABSF(ATPERI-TPERI)
      TPREL=ABSF(TPERR/P)
C     TIME AT PERIGEE ERROR IS NORMALIZED TO THE PERIOD
      PERR=ABSF(APCOMP-P)
      PRELL=PERR/P
      WRITE OUTPUT TAPE 3,326
  326 FORMAT(1H1/1H0,14X,10HGA/EE/64-6/1H0/1HA,17X,
     146HCOMPARISON OF INPUT PARAMETERS AND PARAMETERS
     28HFROM FIT,/1H0,35X18HOF ERROR FREE DATA/1HA,14X
     347HNOTE. THE RELATIVE ERRORS WERE OBTAINED BY NORM
     47HALIZING/1H 14X32HTHE ABSOLUTE DIFFERENCES TO THE
     528HINPUT VALUES (EXCEPT TIME OF/1H 14X,8HPERIGEE
     643HPASSAGE WHICH WAS NOMALIZED TO THE PERIOD)./1HA
     734X,5HINPUT,6X,8HCOMPUTED,2X,8HABSOLUTE,4X,8HRELATIVE
     8/1H 54X,10HDIFFERENCE,4X,5HERROR,/1H0)
      WRITE OUTPUT TAPE 3,327,ECC,AECCOM,EERR,EREL,
     1SEMAX,AACOMP,AERR,AREL,DLON2,ADLON2,DL2ERR,
     2DL2REL,DARGP,ADARGP,DARERR,DARREL,DLON3,
     3ADLON3,DL3ERR,DL3REL,DINCL,ADINCL,DINERR,
     4DINREL,TPERI,ATPERI,TPERR,TPREL,P,APCOMP,PERR,PRELL
  327 FORMAT(1H ,14X,12HECCENTRICITY,F15.7,F12.7,2E11.4,/
     11HA,14X,14HSEMIMAJOR AXIS,F13.4,F12.4,2E11.4,/1HA,14X
     215HRIGHT ASCENSION,2F12.3,2E11.4/1H 15X12HOF THE NODE
     3/1H0,14X,11HARGUMENT OF,F16.3,F12.3,2E11.4/1H 15X
     47HPERIGEE/1H0,14X,12HLONGITUDE OF,F15.3,F12.3,2E11.4,
     5/1H 15X7HPERIGEE,/1H0,14X11HINCLINATION,F16.3,F12.3,
     62E11.4,/1HA14X,15HTIME OF PERIGEE,2F12.4,2E11.4,/1H
```

```
     715X,7HPASSAGE/1HO,14X6HPERIOD,F21.4,F12.4,2E11.4)
      KNTRA=-1
      CALL ERROR
      DO 335 NOR=1,6
      NOR=NOR
      DO 334 LL=1,3
      AZ(LL)=RAD(LL,NOR,1)
      EL(LL)=RAD(LL,NOR,2)
      TT(LL)=TTT(LL,NOR)
  334 RANT(LL)=RRANT(LL,NOR)
  335 CALLORBIT
      WRITE OUTPUT TAPE 3,340
  340 FORMAT(1H1/1HB/1HB/1HB/1HA,44X,18HORBIT   PARAMETERS
     1/1HA,27X40HDETERMINED BY SETS OF THREE VECTORS FROM
     211H RADAR DATA/1HO,30X
     36HSET   1,6X,6HSET   2,6X,6HSET   3,6X,6HSET   4,6X
     46HSET   5,6X,6HSET   6)
      WRITE OUTPUT TAPE 3,323,(ECCOMP(I),I=1,6),(ACOMP(I),I=
     11,6),(DLONC2(I),I=1,6),(DARGPC(I),I=1,6)
      WRITE OUTPUT TAPE3,324,(DLONC3(I),I=1,6),(DINCL2(I),I=
     11,6),(TPERIG(I),I=1,6),(PCOMP(I),I=1,6)
      BECCOM=0
      BACOMP=0
      BDLON2=0
      BDARGP=0
      BDLON3=0
      BDINCL=0
      BTPERI=0
      BPCOMP=0
      DO 336 I=1,6
      BECCOM=BECCOM+ECCOMP(I)/6.
      BACOMP=BACOMP+ACOMP(I)/6.
      BDLON2=BDLON2+DLONC2(I)/6.
      BDARGP=BDARGP+DARGPC(I)/6.
      BDINCL=BDINCL+DINCL2(I)/6.
      BDLON3=BDLON3+DLONC3(I)/6.
      BTPERI=BTPERI+TPERIG(I)/6.
  336 BPCOMP=BPCOMP+PCOMP(I)/6.
      BEERR=ABSF(AECCOM-BECCOM)
      BEREL=BEERR/AECCOM
      BAERR=ABSF(AACOMP-BACOMP)
      BAREL=BAERR/AACOMP
      BL2ERR=ABSF(ADLON2-BDLON2)
      BL2REL=ABSF(BL2ERR/ADLON2)
      BARERR=ABSF(ADARGP-BDARGP)
      BARREL=ABSF(BARERR/ADARGP)
      BL3ERR=ABSF(ADLON3-BDLON3)
      BL3REL=ABSF(BL3ERR/ADLON3)
      BINERR=ABSF(ADINCL-BDINCL)
      BINREL=ABSF(BINERR/ADINCL)
      BTPERR=ABSF(ATPERI-BTPERI)
      BTPREL=ABSF(BTPERR/APCOMP)
      BPERR=ABSF(APCOMP-BPCOMP)
      BPRELL=BPERR/APCOMP
```

```
      WRITE CUTPUT TAPE 3,337
 337  FORMAT(1H1/1HO,14X,10HGA/EE/64-6/1HB/1HA,20X,
     148HCOMPARISON OF PARAMETERS FRCM FIT AND PARAMETERS
     2/1HO,30X,28HCONSIDERING TRACKER RESPCNSE/1HA,14X
     347HNCTE. THE RELATIVE ERRORS WERE OBTAINED BY NORM
     47HALIZING/1H 14X32HTHE ABSOLUTE DIFFERENCES TO THE
     530HFITTED VALUES (TIME OF PERIGEE/1H 14X
     638HPASSAGE WAS NORMALIZED TC THE PERIOD)./1HA
     733X,6HFITTED,6X,8HRESPONSE,2X,8HABSCLUTE,4X,8HRELATIVE
     8/1H 54X,10HDIFFERENCE,4X,5HERRCR/1HO)
      WRITE CUTPUT TAPE 3,327,AECCOM,BECCCM,BEERR,BEREL,
     1AACOMP,BACOMP,BAERR,BAREL,ADLON2,BDLCN2,BL2ERR,
     2BL2REL,ADARGP,BDARGP,BARERR,BARREL,ADLCN3,
     3BDLON3,BL3ERR,BL3REL,ADINCL,BDINCL,BINERR,
     4BINREL,ATPERI,BTPERI,BTPERR,BTPREL,APCCMP,BPCOMP,
     5BPERR,BPRELL
      KNTRA=C
      CALL ERROR
8439  GO TC 70
      ENC


      SUBRCUTINE ANAL


      DIMENSICN CUMMY(1332),CATA(100,5)
      DIMENSION CO(32,8),CT(32,8),E(4,4,2)
      DIMENSION TPOS(300),DAZPCS(300),DELPCS(300)
      DIMENSION TTT(3,12),AAZ(3,12),EEL(3,12),RRANT(3,12),
     1TT(3),EL(3),AZ(3),RANT(3)
      DIMENSION B(3,1),C(3,3)
      DIMENSICN ER(84,2,2)
      DIMENSICN T(330),DEL(330),DAZ(330),RARNG(330)
      DIMENSICN ACOMP(6),DLONC2(6),DARGPC(6),DLONC3(6),DINCL
     12(6),TPERIG(6),PCOMP(6),ECCCMP(6)
      DIMENSION TIM(3,6),CAZ(620),CEL(620),RAD(3,6,2)
      DIMENSION TA(620),TB(620),TC(200),TD(200)
      COMMCN DUMMY,DATA,CO
      COMMON K20,KA2,KA3,KA4,KA6,K28
      COMMON DELTN,DELPOS,DAZPCS,TPOS
      COMMCN TT,EL,AZ,RANT,B,C,RLCN4,ECCOMP,ACOMP,CLONC2,
     1CARGPC,DLONC3,DINCL2,TPERIG,PCCMP,TTT,EEL,AAZ,RRANT
      COMMCN AECCOM,AACOMP,ACLCN2,ADARGP,ACLCN3,ADINCL,
     1ATPERI,APCCMP
      COMMCN DELTAT,TMID1,K8,K3C,NOR
      COMMCN T,DEL,DAZ,RARNG
      COMMCN P,TPERI,ECC,RLON2,RARGP,RINCL,SEMAX
      COMMCNTIM,CAZ,CEL,RAD,CA1,CA2,CA3
      COMMCN KA5,KA7
      COMMCN K24,TA,TB,TC,TD
      COMMCN KNTRA,BECCOM,BACOMP,BCLCN2,BDARGP,BCLCN3,
     1BDINCL,BTPERI,BPCOMP
      ID =C
```

```
      CO 259 K10=1,24
      CALL LSCF2 (0,K1C,0,L,CO,CT,E,K30,2)
      CO 266 IA=1,4
      IE=IA+4*ID
      CO 266 IB=1,2
      CO 266 IC=1,2
  266 ER(IE,IB,IC)=E(IA,IB,IC)
  259 ID=ID+1
      CO 264 J=1,2
      WRITE CUTPUT TAPE 3,271,K20
  271 FORMAT(1H1,/1H ,13X,10HGA/EE/64-6,/1HB,
     129X,27HCURVE  FIT  ERRCR  ANALYSIS,/1H ,56X
     2,I3,13HPOINTS FITTED,/1HC,13X,5HPOLY-,8X,10HCIFFERENCE
     31HS,9X,15HRELATIVE ERRORS,/1H ,13X,6HNCMIAL,/1H ,13X,
     46HCEGREE,5X,3HRMS,6X,7HMAX ABS,6X,3HRMS,6X,7HMAX ABS)
      IF(J-1)268,268,269
  268 WRITE CUTPUT TAPE 3,273
  273 FORMAT(1H0,32X,23HAZIMUTH   ANGLE   (DEG))
      GO TO 274
  269 WRITE CUTPLT TAPE 3,270
  270 FORMAT(1H0,31X,25HELEVATICN   ANGLE   (DEG))
  274 IK=1
      IG=2
      IH=1
      IJ=4
  267 WRITE CUTPUT TAPE 3,265,IH,
     1((ER(I,J,K),I=IK,IJ,2),K=1,1),((ER(I,J,K),I=IG,IJ,2),K
     2=1,1)
  265 FORMAT(1H0,15X,I2,4X,E9.2,2X,E9.2,2X,E9.2,2XE9.2)
      IK=IK+4
      IG=IG+4
      IH=IH+1
      IJ=IJ+4
      IF(IH-24)267,267,264
  264 CONTINUE
      RETURN
      ENC


      SUERCUTINE FIT


      CIMENSION CUMMY(1332),CATA(100,5)
      CIMENSION CO(32,8),CT(32,8),E(4,4,2)
      CIMENSION CAZ(62C),CEL(620),X1(200),Y1(200),TA(620),
     1TB(620),TC(200),TD(200),CELPCS(300),CAZPCS(3C0),TPCS(3
     2C0)
      CIMENSION TTT(3,12),AAZ(3,12),EEL(3,12),RRANT(3,12),
     1TT(3),EL(3),AZ(3),RANT(3)
      CIMENSION B(3,1),C(3,3)
      CIMENSICN T(330),DEL(330),CAZ(330),RARAG(330)
      CIMENSION ACOMP(6),DLONC2(6),DARGPC(6),DLCNC3(6),DINCL
     12(6),TPERIG(6),PCOMP(6),ECCOMP(6)
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      CIMENSICN TIM(3,6),RAD(3,6,2)
      COMMCN DUMMY,DATA,CO
      COMMCN K20,KA2,KA3,KA4,KA6,K28
      COMMCN DELTN,DELPOS,DAZPCS,TPOS
      COMMON TT,EL,AZ,RANT,B,C,RLCN4,ECCOMP,ACOMP,CLONC2,
     1CARGPC,DLONC3,CINCL2,TPERIG,PCCMP,TTT,EEL,AAZ,RRANT
      COMMCN AECCOM,AACOMP,ACLCN2,ADARGP,ACLCN3,ADINCL,
     1ATPERI,APCOMP
      COMMCN DELTAT,TMID1,K8,K30,ACR
      COMMCN T,DEL,DAZ,RARNG
      COMMCN P,TPERI,ECC,RLON2,RARGP,RINCL,SEMAX
      COMMCNTIM,CAZ,CEL,RAD,CA1,CA2,CA3
      COMMON KA5,KA7
      COMMCN K24,TA,TB,TC,TD
      COMMCN KNTRA,BECCOM,BACCMP,BCLCN2,BDARGP,BCLCN3,
     1BDINCL,BTPERI,BPCOMP
      CALL LSCF2(O,KA3,KA2,L,CC,CT,E,K30,2)
      IF(KA2-1)1,1,3
    1 KB1=KA3+1
      WRITE CUTPUT TAPE 3,2,((CC(I,J),J=1,2),I=1,KB1)
    2 FORMAT(1H1,/1H0,14X,10HGA/EE/64-6,/1HB,26X,9HCOEFFICIE
     126HNTS  OF  TIME  POLYNOMIALS/1HA,14X,13HTHE COEFFICIE
     245HNTS OF THE AZIMUTH ANC ELEVATICN TIME POLYNO-/1H 14
     3X50HMIALS ARE LISTED BELCW. THESE POLYNOMIALS ARE FROM
     4/1H 14X44HFITTING THE ERRCR-FREE DATA. THE CCEFFICIENT
     55HS ARE/15X35HLISTED IN ASCENDING POWERS OF TIME./1HA
     618X20HAZIMUTH COEFFICIENTS9X22HELEVATICN COEFFICIENTS,
     7/1H /(21X,E15.8,15X,E15.8))
    3 IF(KA3-13)4,4,9
    4 IF(KA4)5,9,9
    5 IF(KA5-15)31,32,32
   31 CO 30 LK=KA5,14
      CO 30 LKL=1,2
   30 CO(LK,LKL)=0
   32 K29=K28+K20-1
      J=1
      JK=1
      JKL=48
      JKM=K28
      IF(K8-1)17,17,18
   17 J20=1
      GO TC 19
   18 J20=K28*3
   19 CO 20 JJ=1,K20
      CELTN=(TPOS(JKM+1)-TPOS(JKM))/(DELTAT*3.)
      CO 6 JJJ=1,3
      IF(JKL-48)22,21,21
   21 WRITE CUTPUT TAPE  3,26
   26 FORMAT(1H1/1H0,14X,10HGA/EE/64-6/1HB,32X,5HPCLYN
     117HOMIAL  EVALUATION/1HA,14X3HNO.,3X4HTIME,5X,3HELE
     26HVATICN,4X,7HREL ERR,6X,7HAZIMUTH,5X,7HREL ERR/1H )
      JKL=JKL-48
   22 JKL=JKL+1
      CNTR=JJJ-1
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```
      TA(J)=(TPOS(JKM)-TMID1)/CELTAT + CNTR*CELTN
      TB(J)=TA(J)
      CAZ(J)=((((((((((((CO(14,1)*TA(J)+CO(13,1))*TA(J)+CO(1
     12,1))*TA(J)+CO(11,1))*TA(J)+CO(1C,1))*TA(J)+CO(9,1))
     2*TA(J)+CC(8,1))*TA(J)+CO(7,1))*TA(J)+CC(6,1))*TA(J
     3)+CO(5,1))*TA(J)+CO(4,1))*TA(J)+CO(3,1))*TA(J)+CO
     4(2,1))*TA(J)+CO(1,1)
      CEL(J)=((((((((((((CO(14,2)*TA(J)+CO(13,2))*TA(J)+CC(1
     12,2))*TA(J)+CO(11,2))*TA(J)+CO(10,2))*TA(J)+CO(9,2))
     2*TA(J)+CC(8,2))*TA(J)+CO(7,2))*TA(J)+CC(6,2))*TA(J
     3)+CO(5,2))*TA(J)+CO(4,2))*TA(J)+CO(3,2))*TA(J)+CO
     4(2,2))*TA(J)+CO(1,2)
      TE=TA(J)*DELTAT+TMID1
      IF(JJJ-1)24,23,24
   23 RELEL=ABSF(CEL(J)-DELPCS(JKP))/DELPOS(JKP)
      RELAZ=ABSF(CAZ(J)-DAZPCS(JKP))/DAZPCS(JKP)
      WRITE CUTPUT TAPE 3,27,JKP,TE,CEL(J),RELEL,CAZ(J),
     1RELAZ
   27 FORMAT(1H ,13X,I3,F9.2,E15.7,E10.2,E15.7,E10.2)
      GO TC 25
   24 WRITE CUTPUT TAPE 3,28,TE,CEL(J),CAZ(J)
   28 FORMAT(1H ,17X,F8.2,E15.7,E25.7)
   25 IF(J-(3*K20-2))6,7,7
    6 J=J+1
      JKP=JKP+1
   20 JK=JK+1
    7 CONTINUE
      IF(K8-1)10,10,13
   10 LL=1
      CO 11 L=10,4C,6
      AAZ(1,LL)=CAZ(L)
      EEL(1,LL)=CEL(L)
   11 LL=LL+1
      LLL=J-21
      LLL1=LLL+12
      LL=1
      CO 12 L=LLL,LLL1,6
      AAZ(2,LL)=CAZ(L)
      EEL(2,LL)=CEL(L)
   12 LL=LL+1
 4005 GO TC 16
   13 LL=4
      CO 14 L=10,40,6
      AAZ(2,LL)=CAZ(L)
      EEL(2,LL)=CEL(L)
   14 LL=LL+1
      LLL=J-36
      LLL1=LLL+3C
      LL=1
      CO 15 L=LLL,LLL1,6
      AAZ(3,LL)=CAZ(L)
      EEL(3,LL)=CEL(L)
   15 LL=LL+1
   16 CO 8 K=1,K20
```

```
      KK=K28+K-1
      TC(K)=TPCS(KK)
      TD(K)=TC(K)
      X1(K)=CAZPCS(KK)
    8 Y1(K)=DELPCS(KK)
      IF (KA7)33,9,9
   33 CALL PLOT2(0,J,TA,CAZ,3,20.32,12.7,2,2,0,5,0.0)
      CALL PLOT2(0,K20,TC,X1,3,20.32,12.7,2,2,0,5,1.0)
      CALL PLOT2(0,J,TB,CEL,3,20.32,12.7,2,2,0,5,0.0)
      CALL PLOT2(KA6,K20,TD,Y1,3,20.32,12.7,2,2,0,5,1.0)
    9 RETURN
      END


      SUBROUTINE ORBIT


      DIMENSION DUMMY(1332),DATA(100,5),CC(32,8)
      DIMENSION TPOS(300),DAZPCS(300),DELPCS(300)
      DIMENSION A1(3),X(3,3),W1(3),V1(3),D1(3,3),RAZT(3),
     1RELT(3),B(3),C(3,3)
      DIMENSION TTT(3,12),AAZ(3,12),EEL(3,12),RRANT(3,12),
     1TT(3),EL(3),AZ(3),RANT(3)
      DIMENSION T(330),DEL(330),DAZ(330),RARNG(330)
      DIMENSION ACOMP(6),DLONC2(6),DARGPC(6),DLONC3(6),DINCL
     12(6),TPERIG(6),PCOMP(6),ECCCMP(6)
      DIMENSION TIM(3,6),CAZ(620),CEL(620),RAD(3,6,2)
      DIMENSION TA(620),TB(620),TC(200),TD(200)
      COMMON DUMMY,DATA,CO
      COMMON K20,KA2,KA3,KA4,KA6,K28
      COMMON DELTN,DELPOS,DAZPCS,TPOS
      COMMON TT,EL,AZ,RANT,B,C,RLCN4,ECCCMP,ACOMP,DLONC2,
     1DARGPC,DLONC3,DINCL2,TPERIG,PCOMP,TTT,EEL,AAZ,RRANT
      COMMON AECCOM,AACOMP,ADLCN2,ADARGP,ADLCN3,ADINCL,
     1ATPERI,APCOMP
      COMMON DELTAT,TMID1,K8,K30,NCR
      COMMON T,DEL,DAZ,RARNG
      COMMON P,TPERI,ECC,RLON2,RARGP,RINCL,SEMAX
      COMMONTIM,CAZ,CEL,RAD,CA1,CA2,CA3
      COMMON KA5,KA7
      COMMON K24,TA,TB,TC,TD
      COMMON KNTRA,BECCOM,BACCMP,BDLCN2,BDARGP,BDLCN3,
     1BDINCL,BTPERI,BPCOMP
      PI=3.1415926536
      PIX2=PI*2.
      PIBY2=PI/2.
      RADDEG=180./PI
      WE=PIX2/86164.1
      WE1=WE*RADDEG
      DLCN4=RLCN4*RADDEG
      DO 224 M=1,3
      RELT(M)=EL(M)/RADDEG
      RAZT(M)=AZ(M)/RADDEG
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  230 A1(1)=RANT(M)*COSF(RELT(M))*COSF(RAZT(M))
      A1(2)=RANT(M)*COSF(RELT(M))*SINF(RAZT(M))
      A1(3)=RANT(M)*SINF(RELT(M))
      EAROT1=TT(M)*WE
      ASCEN1=EAROT1-RLON4
  231 C1(1,1)=COSF(ASCEN1)
      C1(1,2)=SINF(ASCEN1)
      C1(1,3)=0.
      C1(2,1)=-SINF(ASCEN1)
      C1(2,2)=COSF(ASCEN1)
      C1(2,3)=0.
      C1(3,1)=0.
      C1(3,2)=0.
      C1(3,3)=1.
C     V1=A1-B, W1=(C(TRANSPOSE))V1, X1=(D1(TRANSPOSE))W1
      DO 222 I=1,3
  222 V1(I)=A1(I)-B(I)
      DO 223 I=1,3
      W1(I)=0.
      DO 223 L=1,3
  223 W1(I)=W1(I)+C(L,I)*V1(L)
      DO 224 I=1,3
      X(M,I)=0.
      DO 224 L=1,3
  224 X(M,I)=X(M,I)+D1(L,I)*W1(L)
C     XN=MAGNITUDE OF X(N)
      X1=SQRTF(X(1,1)*X(1,1)+X(1,2)*X(1,2)+X(1,3)*X(1,3))
      X2=SQRTF(X(2,1)*X(2,1)+X(2,2)*X(2,2)+X(2,3)*X(2,3))
  233 X3=SQRTF(X(3,1)*X(3,1)+X(3,2)*X(3,2)+X(3,3)*X(3,3))
C     XMDTXN=X(M) DOT X(N)
      X3DTX1=X(3,1)*X(1,1)+X(3,2)*X(1,2)+X(3,3)*X(1,3)
      X2DTX1=X(2,1)*X(1,1)+X(2,2)*X(1,2)+X(2,3)*X(1,3)
  234 X3DTX2=X(3,1)*X(2,1)+X(3,2)*X(2,2)+X(3,3)*X(2,3)
C     THETMN=THETA M MINUS THETA N
      THET31=ACOS(X3DTX1/(X3*X1))
      THET12=-ACOS(X2DTX1/(X2*X1))
  235 THET23=-ACOS(X3DTX2/(X3*X2))
C     SLRCTM=SEMILATUS RECTUM
C     SIMN=SIN(THETMN)
      SI31=SINF(THET31)
      SI12=SINF(THET12)
      SI23=SINF(THET23)
      SLRCTM=X1*X2*X3*(SI31+SI12+SI23)/(X2*X3*SI23+X1*X3*SI3
     11+X1*X2*SI12)
C     ESQUAR=ECCENTRICITY SQUARED
      TERM1=((SLRCTM-X1)/X1)*((SLRCTM-X1)/X1)
      TERM2=((X3*(SLRCTM-X1)*COSF(THET31)-X1*(SLRCTM-X3))/(X
     11*X3*SI31))**2
  237 ESQUAR=TERM1+TERM2
C     ECCOMP(NOR)=COMPUTED ECCENTRICITY
      ECCOMP(NOR)=SQRTF(ESQUAR)
C     ACCMP(NOR)=COMPUTED SEMIMAJOR AXIS
      ACCMP(NOR)=SLRCTM/(1.-ESQUAR)
C     CINCL2(NOR)=COMPUTED INCLINATION
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```
      TERM4=(X(1,1)*X(3,2)-X(3,1)*X(1,2))/(X1*X3*SI31)
      TERM4A=ABSF(TERM4)
      RINCL2=ACOS(TERM4A)
C     CHECK TO SEE IF ABSF CARC IS NECESSARY
      CINCL2(NOR)=RINCL2*RADCEG
C     EQUATION OF ORBIT PLANE IS CF FORM C1X+C2Y+Z=0
      C1=(X(1,3)*X(2,2)-X(2,3)*X(1,2))/(X(2,1)*X(1,2)-X(1,1)
     1*X(2,2))
      C2=(-X(1,3)-C1*X(1,1))/X(1,2)
C     CLONC2(NOR) IS CCMPUTEC VALLE CF RIGHT ASCENSION OF
C     THE ASCENDING NODE
      RLONC2=ATANF(-C1/C2)
      TERM5=(X(3,1)*X(1,3)-X(1,1)*X(3,3))/(X1*X3*SI31)
      IF(TERM5)240,240,239
  239 RLCNC2=RLCNC2-PI
  240 CLCNC2(NCR)=RLCNC2*RADCEG
C     UNIT VECTOR TO PERIGEE IS ALPHA TIMES VECTCR X1 PLLS
C     BETA TIMES VECTOR X3
  238 TERM6=1./(ECCOMP(NOR)*SI31*SI31)
      TERM7=(SLRCTM/X3)-1.
      TERM8=(SLRCTM/X1)-1.
      CO31=X3DTX1/(X3*X1)
      ALPHA=(TERM6/X1)*(TERM8-(TERM7*CC31))
  241 BETA=(TERM6/X3)*(TERM7-(TERM8*CO31))
C     CARGPC(NOR)=COMPLTED ARGLMENT CF PERIGEE=ACOS(DOT
C     PRODUCT OF LINE CF NODES ANC I SUB P)
      TERM9=(ALPHA*X(1,1)+BETA*X(3,1))*COSF(RLCNC2)
      TERM10=(ALPHA*X(1,2)+BETA*X(3,2))*SINF(RLCNC2)
      RARGPC=ACOS(TERM9+TERM10)
      IF(ALPHA*X(1,3)+BETA*X(3,3))4013,242,242
 4013 RARGPC=-RARGPC
  242 CARGPC(NOR)=RARGPC*RADCEG
C     PCCMP(NOR)=COMPUTED PERICC
C     CTCOMN=TIME BETWEEN PERIGEE PASSAGE ANC REACHING
C     VECTCR XN
 4014 TERM11=SQRTF(((ACOMP(NCR)*5.28E3)**3)/1.407639E16)
      TERM12=SCRTF((1.-ECCOMP(NCR))/(1.+ECCOMP(NCR)))
      PCCMP(NOR)=PIX2*TERM11
      CLCNC3(NOR)=CARGPC(NOR)+CLCNC2(NCR)
C     THETN IS THE ANGLE BETWEEN LINE OF NCCES ANC XN
      TERM13=(X(1,1)*CCSF(RLCNC2)+X(1,2)*SINF(RLCNC2))/X1
      THET1=ACCS(TERM13)
      IF(X(1,3))25C,251,251
  250 THET1=PIX2-THET1
  251 CONTINUE
C     FN IS TRUE ANOMALY CF XN
      F1=THET1-RARGPC
C     EN IS ECCENTRIC ANOMALY CF XN
      TERM15=F1/2.
      TAE1C2=TERM12*SINF(TERM15)/CCSF(TERM15)
      E1C2=ATANF(TAE1C2)
      IF(TERM15-PIBY2)253,253,252
  252 E1C2=E1C2+PI
  253 E1=2.*E1C2
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```
C     TPERIG(NOR) IS TIME OF PERIGEE PASSAGE
      TPERIG(NOR)=TT(1)-TERM11*(E1-ECCOMP(NOR)*SINF(E1))
      RETURN
      END


      SUBROUTINE ERROR


      DIMENSION T(330),DEL(330),DAZ(330),RARNG(330)
      DIMENSION B (3,1),C (3,3),G (3,3),H (3,1)
      DIMENSION TPOS(300),DAZPOS(300),DELPOS(300)
      DIMENSION DUMMY(1332),DATA(100,5)
      DIMENSION CO(32,8)
      DIMENSION W(3,2,50),DIF1(3),DIF2(50),REL1(50)
      DIMENSION ACOMP(6),DLONC2(6),DARGPC(6),DLONC3(6),DINCL
     12(6),TPERIG(6),PCOMP(6),ECCOMP(6)
      DIMENSION TTT(3,12),AAZ(3,12),EEL(3,12),RRANT(3,12),
     1TT(3),EL(3),AZ(3),RANT(3)
      DIMENSION TIM(3,6),CAZ(620),CEL(620),RAD(3,6,2)
      DIMENSION TA(620),TB(620),TC(200),TD(200)
      COMMON DUMMY,DATA,CO
      COMMON K20,KA2,KA3,KA4,KA6,K28
      COMMON DELTN,DELPOS,DAZPOS,TPOS
      COMMON TT,EL,AZ,RANT,B,C,RLON4,ECCOMP,ACOMP,DLONC2,
     1DARGPC,DLONC3,DINCL2,TPERIG,PCOMP,TTT,EEL,AAZ,RRANT
      COMMON AECCOM,AACOMP,ADLON2,ADARGP,ADLON3,ADINCL,
     1ATPERI,APCOMP
      COMMON DELTAT,TMID1,K8,K30,NOR
      COMMON T,DEL,DAZ,RARNG
      COMMON P,TPERI,ECC,RLON2,RARGP,RINCL,SEMAX
      COMMONTIM,CAZ,CEL,RAD,CA1,CA2,CA3
      COMMON KA5,KA7
      COMMON K24,TA,TB,TC,TD
      COMMON KNTRA,BECCOM,BACOMP,BDLON2,BDARGP,BDLON3,
     1BDINCL,BTPERI,BPCOMP
      PI = 3.1415926536
      PIBY2 = PI/2.
      PIX2  = 2.*PI
      RADDEG = 180.0/PI
      WE=PIX2/86164.1
      WE1=WE*RADDEG
      ARLON2=ADLON2/RADDEG
      ARINCL=ADINCL/RADDEG
      ARARGP=ADARGP/RADDEG
      BRLON2=BDLON2/RADDEG
      BRINCL=BDINCL/RADDEG
      BRARGP=BDARGP/RADDEG
      IF(KNTRA)37,36,36
   36 RLON21=ARLON2
      RARGP1=ARARGP
      RINCL1=ARINCL
      P1=APCOMP
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      TPERI1=ATPERI
      SEMAX1=AACOMP
      ECC1=AECCOM
      GO TO 9
   37 P1=P
      TPERI1=TPERI
      ECC1=ECC
      RLON21=RLON2
      RARGP1=RARGP
      RINCL1=RINCL
      SEMAX1=SEMAX
    9 DT=P/50.
      T(1)=0
      E=0
      N=1
      J5=0
      J1=-1
      I1=1
   14 E1=E
      E=(PIX2/P1)*(T(N)-TPERI1)+ECC1*SINF(E1)
      J5=J5+1
      IF(J5-10000)204,203,203
  203 WRITE OUTPUT TAPE 3,205,E,E1
  205 FORMAT(43HARUN FORCED THRU KEPLER LOOP, E AND E1 ARE ,
     12E20.8)
      GO TO 2
  204 IF ((ABSF (E - E1)) - .0000001) 2,14,14
    2 J5=0
      E2 = E/2.0
      CON = COSF(E2)
      SNE = SINF(E2)
      FCTR2=SQRTF((1.+ECC1)/(1.-ECC1))
      TANF2 = FCTR2 * (SNE/CON)
      F2 = ATANF(TANF2)
      IF (CON) 31,32,32
   31 F = (F2 + PI)*2.
      GO TO 35
   32 IF ( SNE) 33,34,34
   33 F = (F2 +(PIX2))*2.
      GO TO 35
   34 F = F2*2.
   35 CONTINUE
   12 RF = ((SEMAX1)*(1.00-((ECC1)**2)))/(1.0 +((ECC1) * (C
     1OSF(F))))
      RLON3=RLON21+RARGP1
      RINC2=RINCL1/2.0
      SRIN2=SINF(RINC2)
      RIN22=SRIN2**2
   21 G(1,1)=COSF(RLON3) +2.*(SINF(RLON21))*(SINF(RARGP1))*R
     1IN22
      G(1,2)=-SINF(RLON3) + 2.*SINF(RLON21)*(COSF(RARGP1))*R
     1IN22
      G(1,3)=(SINF(RLON21))*SINF(RINCL1)
      G(2,1)=(SINF(RLON3))-(2.*(COSF(RLON21))*(SINF(RARGP1))
```

```
     1*RIN22)
      G(2,2)=(COSF(RLON3))-(2.*(CCSF(RLCN21))*CCSF(RARGP1))*
     1RIN22
      G(2,3)=(-CCSF(RLCN21))*(SINF(RINCL1))
      G(3,1)=(SINF(RARGP1))*(SINF(RINCL1))
      G(3,2)=(COSF(RARGP1))*(SINF(RINCL1))
      G(3,3)=CCSF(RINCL1)
      H(1,1)=RF*COSF(F)
      H(2,1)=RF*SINF(F)
      H(3,1)=0.
C     W=GH
  100 CO 3 I = 1,3
      W(I,I1,N)=C.
      CO 3 L = 1,3
    3 W(I,I1,N)=W(I,I1,N) + G(I,L)*H(L,1)
      IF(N-5C)102,103,103
  102 N=N+1
      T(N)=T(N-1)+CT
      GO TC 14
  103 IF(J1)104,105,105
  104 J1=C
      I1=2
      N=1
      T(N)=0
      E=C
      IF(KNTRA)39,38,38
   38 RLCN21=BRLCN2
      RARGP1=BRARGP
      RINCL1=BRINCL
      P1=BPCCMP
      TPERI1=BTPERI
      SEMAX1=BACOMP
      ECC1=BECCOM
      GO TC 14
   39 RLCN21=ARLCN2
      RARGP1=ARARGP
      RINCL1=ARINCL
      P1=APCCMP
      TPERI1=ATPERI
      SEMAX1=AACCMP
      ECC1=AECCOM
  114 GO TC 14
  105 CO 107 I=1,50
      CO 106 J=1,3
  106 CIF1(J)=W(J,1,I)-W(J,2,I)
      CIF2(I)=SQRTF(DIF1(1)**2+DIF1(2)**2+DIF1(3)**2)
  107 REL1(I)=CIF2(I)/SQRTF(W(1,1,I)**2+W(2,1,I)**2+W(3,1,I)
     1**2)
  115 CIFMAX=DIF2(1)
      RELMAX=REL1(1)
      SQ1=CIF2(1)*CIF2(1)/50.
      SQ2=REL1(1)*REL1(1)/5C.
      JKM=1
      JKN=1
```

```
      CO 112 I=2,5C
      IF (CIF2(I)-CIFMAX)109,1C9,1C8
  108 CIFMAX=DIF2(I)
      JKM=I
  1C9 IF(REL1(I)-RELMAX)111,111,110
  11C RELMAX=REL1(I)
      JKN=I
  111 SQ1=SQ1+CIF2(I)*CIF2(I)/5C.
  112 SQ2=SQ2+REL1(I)*REL1(I)/5C.
      CIFRMS=SCRTF(SQ1)
      RELRMS=SQRTF(SQ2)
  113 CONTINUE
      IF(KNTRA)12C,118,118
  118 WRITE CUTPUT TAPE 3,119,CIFMAX,DIFRMS,RELMAX,RELRMS
  119 FORMAT(1HB14X3CHTHE MAXIMUM ABSOLUTE ERRCR IN
     126HPCSITION INTRCDUCED BY THE/1H 14X10HDYNAMIC RE
     642HSPONSE CF THE SATELLITE TRACKING SYSTEM IS/
     71H 14XF8.3,6H MILES
     241H. THE RMS PCSITICN ERRCR FOR CNE CRBIT IS/1H ,
     314XF8.3,31H MILES. THE RELATIVE VALUES AREE15.8,
     45H MAX-/1H ,14X,8HIMUM ANCE15.8,15H RMS FOR A SING
     59HLE ORBIT.)
      GO TC 121
  120 WRITE CUTPUT TAPE 3,117,CIFMAX,DIFRMS,RELMAX,RELRMS
  117 FORMAT(1HB14X30HTHE MAXIMUM ABSOLUTE ERRCR IN
     127HPCSITION CUE TO THIS FIT IS/1H 14XF8.3,6H MILES
     241H. THE RMS POSITICN ERRCR FOR CNE CRBIT IS/1H ,
     314XF8.3,31H MILES. THE RELATIVE VALUES AREE15.8,
     45H MAX-/1H ,14X,8HIMUM ANCE15.8,15H RMS FOR A SING
     59HLE ORBIT.)
  121 RETURN
      ENC


      SUBRCUTINE CYNAM


      CIMENSION CZERO(2)
      CIMENSICN T(330),DEL(33C),CAZ(330),RARNG(330)
      CIMENSION B (3,1),C (3,3),G (3,3),H (3,1)
      CIMENSION TPOS(3CO),DAZPCS(300),DELPCS(300)
      CIMENSICN DUMMY(1332),CATA(100,5)
      CIMENSICN CO(32,8)
      CIMENSION ACOMP(6),DLONC2(6),DARGPC(6),DLCNC3(6),CINCL
     12(6),TPERIG(6),PCOMP(6),ECCCMP(6)
      CIMENSICN TTT(3,12),AAZ(3,12),EEL(3,12),RRANT(3,12),
     1TT(3),EL(3),AZ(3),RANT(3)
      CIMENSICN TIM(3,6),CAZ(62C),CEL(620),RAD(3,6,2)
      CIMENSION TA(620),TB(620),TC(200),TC(2CO)
      CIMENSICN TRAFUN(2,200)
      COMMCN DUMMY,DATA,CO
      COMMON K20,KA2,KA3,KA4,KA6,K28
      COMMCN DELTN,DELPOS,DAZPCS,TPOS
```

```
      COMMCN TT,EL,AZ,RANT,B,C,RLCN4,ECCOMP,ACCMP,CLONC2,
     1CARGPC,DLCNC3,CINCL2,TFERIG,PCCMP,TTT,EEL,AAZ,RRANT
      COMMCN AECCOM,AACOMP,ACLCN2,ADARGP,ACLCN3,ADINCL,
     1ATPERI,APCCMP
      COMMCN DELTAT,TMID1,K8,K?C,NCR
      COMMCN T,DEL,DAZ,RARNG
      COMMCN P,TPERI,ECC,RLCN2,RARGP,RINCL,SEMAX
      COMMCNTIM,CAZ,CEL,RAD,CA1,CA2,CA3
      COMMCN KA5,KA7
      COMMCN K24,TA,TB,TC,TD
      COMMCN KNTRA,BECCOM,BACCPP,BCLCN2,BDARGP,BDLCN3,
     1BDINCL,BTPERI,BPCOMP
C     CONSTANTS CETERMINED HERE
      PI=3.1415926536
      PIBY2=PI/2.
      PIX2=PI*2.
      CA1=CA1*DELTAT
      CA2=CA2*(DELTAT**2)
      CA3=CA2
      CA4=CA1/2.
      CA5=SQRTF(CA2-(CA1/2.)**2)
      CB1=(CA1**2-CA2)/(CA2**3)
      CB2=CA1/(CA2**2)
      CB3=4.*CA2-CA1**2
      CB4=CA1*(CA1**2-2.*CA2)/(CA2**4)
      CB5=(CA1**4-3.*CA1**2*CA2+CA2**2)
     1/CA2**5
      CB6=SQRTF(CB3)
      PHIA=ATANF(-CA5/CA4)
      IF(-CA4)1,2,2
    1 PHIA=PHIA+PI
    2 CCNTINUE
      PHIB2=ATANF(CA5/CA4)
      IF(CA4)3,4,4
    3 PHIB=2.*(PHIB2+PI)
      GO TC 16
    4 PHIB=2.*PHIB2
   16 PHIC=ATANF(2.*CA5/CA1)
      IF(CA1)5,6,6
    5 PHIC=PHIC+PI
    6 CONTINUE
C     THESE ARE CHECKS (GOES IN 2 PAGES OVER)
      S=1.
      CKA=S**2+CA1*S+CA2-((S+CA4)**2+CA5**2)
      CKB=CA4**2+CA5**2-CA2
      KNTR=-1
   1C IF(K8-1)7,7,8
    7 LA=1
      LB=6
      LC=1
      GO TC 9
    8 LA=4
      LB=6
      LC=2
```

```
  9 CO 11 J=LA,LB
    CO 11 JJ=1,2
    CZERC(1)=CAZ(1)
    CZERC(2)=CEL(1)
    ARG=-CA4*TIM(LC,J)
    IF(ABSF(ARG)-85.)31,30,3C
 3C ENAPA=C
    GO TC 32
 31 ENAPA=EXPF(ARG)
 32 ENAPB=ENAPA
    XA=CC(1,JJ)*CA3*(1./CA2+((ENAPA)
   1*SINF(CA5*TIM(LC,J)-PHIA))/(CA5*SQRTF(CA2)))
    XB=CC(2,JJ)*(CA3/CA2)*(TIM(LC,J)-CA1/CA2+
   1ENAPA*(SINF(CA5*TIM(LC,J)+PHIB))/CA5)
    XC=2.*CO(3,JJ)*CA3*(CB1-CB2*TIM(LC,J)+TIM(LC,J)
   1**2/(2.*CA2)
   2+2.*ENAPB/(CA2**3*CB6)*(-CA1/2.*(CA1**2-3.*CA2)
   3*SINF(CB6*TIM(LC,J)/2.)-CB6*(CA1**2-CA2)
   4*CCSF(CB6*TIM(LC,J)/2.)))
    XD=6.*CO(4,JJ)*CA3*(-CB4+CB1*TIM(LC,J)-CB2*
   1TIM(LC,J)**2/2.+TIM(LC,J)**3/(6.*CA2)
   2+ENAPB/(CA2**4*CB6)*(.25*(CA1**4/2.-3.*CA1**2
   3*CB3+CB3**2/2.)*SINF(CB6*TIM(LC,J)/2.)
   4+CA1*CB6*(CA1**2-2.*CA2)*COSF(CB6*TIM(LC,J)/2.)))
    XE=24.*CC(5,JJ)*CA3*(CB5-CB4*TIM(LC,J)+CB1*TIM(LC,J)**
   12/2.-CB2*TIM(LC,J)**3/6.+TIM(LC,J)**4/(24.*CA2)
   2-2.*ENAPB/(CA2**5*CB6)*(CA1/16.*(CA1**4/2.-5.*CA1**2
   3*CB3+5./2.*CB3**2)*SINF(CB6*TIM(LC,J)/2.)
   4+CB6/16.*(5./2.*CA1**4-5.*CA1**2*CB3+
   5CB3**2)*COSF(CB6*TIM(LC,J)/2.)))
    XF=CZERO(JJ)*SQRTF(CA2)
   1/CA5*ENAPA*SINF(CA5*TIM(LC,J)+PHIC)
 18 RAC(LC,J,JJ)=XA+XB+XC+XD+XE+XF
 11 CONTINLE
 19 IF(KNTR)12,15,15
 12 IF(K8-1)13,13,14
 13 LA=1
    LB=3
    LC=2
    KNTR=0
    GO TC 9
 14 LA=1
    LB=6
    LC=3
    KNTR=0
    GO TC 9
 15 CONTINLE
    JL=K28
    JM=48
    JJJ=K28-K24
    LOT=C
    CO 24 J=1,K2C
    J=J
    IF(JM-48)22,20,2C
```

```
   20 WRITE CUT PUT TAPE 3,21
   21 FORMAT(1H1/1HO,14X,10HGA/EE/64-6/1HB,29X,7HTRANSFE
     121HR FUNCTION EVALUATICN/1HA,14X3HNO.,3X4HTIME,5X,
     29HELEVATION4X7HREL ERR6X,7HAZIMUTH,5X,7HREL ERR/1H )
      JM=JM-48
   22 JM=JM+1
      TA(J)=(TPOS(JL)-TMIC1)/CELTAT
      TB(J)=TA(J)
      TC(J)=TA(J)
      TD(J)=TA(J)
      ARG=-CA4*TA(J)
      IF(ABSF(ARG)-85.)35,34,34
   34 ENAPA=C
      GO TC 36
   35 ENAPA=EXPF(ARG)
   36 ENAPB=ENAPA
      CO 25 JJ=1,2
      XA=CC(1,JJ)*CA3*(1./CA2+((ENAPA)
     1*SINF(CA5*TA(J)-PHIA))/(CA5*SQRTF(CA2)))
      XB=CC(2,JJ)*(CA3/CA2)*(TA(J)-CA1/CA2+
     1ENAPA*SINF(CA5*TA(J)+PHIB)/CA5)
      XC=2.*CO(3,JJ)*CA3*(CB1-CB2*TA(J)+TA(J)
     1**2/(2.*CA2)
     2+2.*ENAPB/(CA2**3*CB6)*(-CA1/2.*(CA1**2-3.*CA2)
     3*SINF(CB6*TA(J)/2.)-CB6*(CA1**2-CA2)
     4*COSF(CB6*TA(J)/2.)))
      XD=6.*CO(4,JJ)*CA3*(-CB4+CB1*TA(J)-CB2*
     1TA(J)**2/2.+TA(J)**3/(6.*CA2)
     2+ENAPB/(CA2**4*CB6)*(.25*(CA1**4/2.-3.*CA1**2
     3*CB3+CB3**2/2.)*SINF(CB6*TA(J)/2.)
     4+CA1*CB6*(CA1**2-2.*CA2)*COSF(CB6*TA(J)/2.)))
      XE=24.*CO(5,JJ)*CA3*(CB5-CB4*TA(J)+CB1*TA(J)**
     12/2.-CB2*TA(J)**3/6.+TA(J)**4/(24.*CA2)
     2-2.*ENAPB/(CA2**5*CB6)*(CA1/16.*(CA1**4/2.-5.*CA1**2
     3*CB3+5./2.*CB3**2)*SINF(CB6*TA(J)/2.)
     4+CB6/16.*(5./2.*CA1**4-5.*CA1**2*CB3+
     5CB3**2)*COSF(CB6*TA(J)/2.)))
      XF=CZERO(JJ)*SQRTF(CA2)
     1/CA5*ENAPA*SINF(CA5*TA(J)+PHIC)
      TRAFUN(JJ,J)=XA+XB+XC+XD+XE+XF
   25 CONTINUE
      TE=TA(J)*DELTAT+TMID1
      JJJ=JJJ+K24
      JK=3*J-2
      RELEL=ABSF((CEL(JK)-TRAFUN(2,J))/CEL(JK))
      RELAZ=ABSF((CAZ(JK)-TRAFUN(1,J))/CAZ(JK))
      WRITE CUTPUT TAPE 3,23,JL,TE,TRAFUN(2,J),RELEL,
     1TRAFUN(1,J),RELAZ
   23 FORMAT(1H 13X,I3,F9.2,E15.7,E10.2,E15.7,E10.2)
      LOT=LOT+1
   24 JL=JL+1
      IF(KA7)27,26,27
   26 CALL PLOT2(0,LOT,TA,CAZ,3,20.,12.,2,2,C,5,0.0)
      CALL PLOT2(0,LOT,TB,TRAFUN(2),3,20.,12.,2,2,C,5,1.0)
```

```
      CALL PLOT2(0,LOT,TC,CEL,3,2C.,12.,2,2,C,5,0.C)
      CALL PLOT2(KA6,LCT,TD,TRAFUN(1),3,20.,12.,2,2,0,5,1.0)
   27 CA1=CA1/DELTAT
      CA2=CA2/DELTAT**2
      RETURN
      ENC
```

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APPENDIX E

FURNISHED SUBPROGRAMS

THE SUBPROGRAMS LISTED IN THIS APPENDIX ARE THOSE FURNISHED BY THE SYSTEMS DYNAMIC ANALYSIS DIVISION. THEY ARE, IN ORDER,

(1) ARCSINE - ARCCOSINE - A ROUTINE FOR COMPUTING THESE TWO INVERSE TRIGONOMETRIC FUNCTIONS

(2) LSCF2 - THE CURVE FIT ROUTINE AND ITS SUBPROGRAMS

(A) LSCF1 - FORMS THE NORMAL EQUATIONS

(B) LINEQ - SOLVES THE NORMAL EQUATIONS

(C) READT - READS IN DATA IN THE EVENT IT IS NOT ALREADY IN THE PROGRAM

SUBPROGRAMS LSCF1 AND LSCF2 ARE IN THE FORTRAN 2 LANGUAGE. THE OTHER TWO ARE IN FAP.

```
*       ARCSINE - ARCCOSINE ROUTINE

*       FAP
         COUNT   100
         LBL     ARCSC,X
         REM     THIS ROUTINE COMPUTES THE ARCSINE OR
         REM     ARCCOSIN WHEN CALLED BY ACOS(X)
         REM     OR ASIN(X).  IF THE ARGUMENT IS
         REM     OUT OF RANGE, AN ERROR MESSAGE WILL BE
         REM     FORTHCOMING.
         SPACE   2
         ENTRY   ACOS
         ENTRY   ASIN
         SPACE   1
ACOS     SXA     I4,4                    SAVE INDEX REDISTER 4
         CLA*    1,4
         LDQ     EC
         STQ     ES-3
         TSX     ASIN+3,4
EC       BCI     1,F ACOS
I4       AXT     0,4
```

```
        CHS
        FAD     PI
        TRA     2,4
        SPACE   2
        REM     7090 FLOATING PCINT ARCSINE SLBPROGRAM
        REM     USES SQUARE RCOT AND ARCTANGENT SUBROUTINES
        SPACE   1
ECC     BCI     1,F ASIN
ASIN    CLA*    1,4
        LCQ     ECC
        STQ     ES-3
        STO     N
        SSP
        LCQ     =1.0000001
        TLQ     ERRORS
        SXA     I1,1
        SXA     I2,2
        SUB     MIN                     TEST IF CUT CF RANGE
        TPL     T1                      IF TOO SMALL RETURN WITH
*                                       ARGUMENT
        CLA     N
        TRA     2,4
 T1     SUB     MAX                     IF TOO LARGE ERRCR RETURN
        TPL     XX
        AXT     0,3
        ACD     CONST                   TEST IF LARGER THAN SQUARE
        TPL     T2                      ROCT OF 2 CVER 2
 T3     LDQ     N
        FMP     N                       COMPUTE 1 - N SQUAREC
        STO     SQ
        CHS
        FAD     FCNE
        TXL     T4,1,0                  DIVIDE BY 1 - N SQUAREC
 T5     FDP     T,1                     OR DIVIDE 1 - N SQUAREC
        STQ     SQ                      BY N SQUARED
        PXD                             TAKE SQUARE ROOT
        LLS     8                       SEPARATE CHARCTERISTIC
        ACD     CH                      COMPUTE NEW CHARACTERISTIC
        LBT                             TEST IF EXP ODD CR EVEN
        TRA     T60
        ARS     1
 T75    ALS     27
        STO     M
        STQ     P
        LCQ     A,2                     COMPUTE LINEAR APPROX
        VLM     P,0,6
        ACD     B,2
        STO     T
        CLA     P                       APPLY NEWTCNS METHOD TWICE
        VCP     T,0,14
        LLS     21
        XCA
        ACD     T
        LRS     1
```

```
        ORA     MASK
        STO     T
        CLA     P
        DVP     T
        XCA
        ACD     T
        LRS     9                  CONVERT TO FLOATING PICNT
        ACD     M
        STO     P
        AXT     3,2                COMPUTE ARCTANGENT CONTINU
        CLA     SQ                 FRACTION
        TRA     T6
 T7     FAD     SQ
 T6     FAD     C,2
        STO     T
        CLA     MIN,2
        FDP     T
        XCA
        TIX     T7,2,1
        FAD     C
        XCA
        FMP     P
        FSB     MASK,1
        LDQ     N
        LLS                        SET PROPER SIGN
 I1     AXT     0,1
 I2     AXT     0,2
        TRA     2,4
 T2     TXI     T3,1,1
 T4     STO     T
        CLA     SQ
        TRA     T5
 T60    LRS     1
        TXI     T75,2,1
XX      XCA
        CLA     FONE
        STO     N
        XCA
        TRA     T1+2
ERRORS  CAL     =3B17
        TSX     $(STH),4
        PZE     ES
        LDQ     N
        STR
        TSX     $(FIL),4
        CALL    EXIT
        BCI     1,E16.8)
        BCI     1,    X =
        BCI     1,GE.
        BCI     1,OF RAN
        BCI     1,S OUT
        BCI     1, (X) I
        BCI     1,F ASIN
        BCI     1,MENT O
```

```
        BCI     1,**ARGU
ES      BCI     1,(50HO*
        SPACE   2
        REM     CONSTANTS
        SPACE   1
        DEC     .8125829
A       DEC     .578125829
        DEC     .30273480
B       DEC     .42187580
CH      DEC     129
        DEC     1.448631538
        DEC     3.316335425
        DEC     6.76213924
C       DEC     .1746554388
        DEC     -.2647686202
        DEC     -7.106760045
        DEC     3.709256262
MIN     OCT     140000000000
MAX     OCT     041400000001
CONST   OCT     625754150
FONE    DEC     1.0
PI      DEC     1.57079633
MASK    OCT     000007777777
        SPACE   2
        REM     VARIABLES
        SPACE   1
M       PZE
N       PZE
P       PZE
SC      PZE
T       PZE
        END
```

```
C LSCF2

C DIMENSION CARD COEFFO(32,8),COEFFT(32,8),ERRORS(4,4,2)
C CALL LSCF2(MTIN,KTH,MTOUT,LOOK,COEFFO,COEFFT,ERRORS,MPTS,
C NSETS)
C MTIN=TAPE FROM WHICH INPUT DATA WILL BE READ. WHEN ZERO
C POINTS ASSUMED IN COMMON AS A(100,5).
C KTH=DEGREE OF POLYNOMIAL FIT. MAX=24 OR (MPTS-1) WHICH
C EVER IS SMALLER
C MTOUT=TAPE ON WHICH PRINTIBLE OUTPUT IS DESIRED. WHEN ZERO
C THIS OUTPUT IS DELETED.
C LOOK=RETURNED CELL FOR DESIGNATION OF GOOD RUN(=0) OR
C NUMBER OF DATA ERRORS (NOT=0).
C COEFFO=COEFFICIENTS OF POLYNOMIAL FOR ORIGINAL SYSTEM. SEE
C SUBSCRIPT DESCRIPTION OF C1 IN LSCF1 SUBROUTINE. RETURNED
C VALUES.
C COEFFT=COEFFICIENTS OF POLYNOMIAL FOR TRANSFORMED SYSTEM.
C SUBSCRIPT DESCRIPTION SAME AS COEFFT. RETURNED VALUES.
```

```
C ERRORS=SHORT ERROR ANALYSIS CN COMPUTED DEPENDENTS. RE-
C TURNED VALUES.
C        SEE WRITE-UP FOR DESCRIPTION.
C MPTS=NUMBER OF POINTS TO BE FIT.
C NSETS=NUMBER OF SET PER POINT.
C NOTE THAT THIS SUBROUTINE USES LINEC,LSCF1,READT.
      SUBROUTINE LSCF2(MTIN,KTH,MTOUT,LOOK,CCEFFO,COEFFT,ERR
     1CRS,MPTS,NSETS)
      DIMENSION C1(32,8),C2(32,8),E(4,4,2),A(100,5),YC(100,4
     3,2),
     1XT(100),DIFFER(100,4,2),RELERR(100,4,2),DUMMY(1332),
     2ERRORS(4,4,2),COEFFO(32,8),COEFFT(32,8),CS(5C2)
      COMMCN DUMMY,A
      MT1=MTIN
      MT2=MTCUT
      K=KTH
      M=MPTS
      N=NSETS
      IF(MT1)99,101,10C
 99   MT1=2
C     READ M(NO. OF POINTS),N(NC. OF SETS) FROM FIRST CARD
 100  CALL READT(CS,MT1,L)
      IF(L)15,5,15
 5    M=CS(1)+.5
      N=CS(2)+.5
      IF(N-4)6,6,13
 6    K1=0
      LM=(M*(N+1))+2
      GO TC(1,2,3,4),N
 1    CO 7 I=3,LM,2
      K1=K1+1
      A(K1,1)=CS(I)
 7    A(K1,2)=CS(I+1)
      GO TO 102
 2    CO 8 I=3,LM,3
      K1=K1+1
      A(K1,1)=CS(I)
      A(K1,2)=CS(I+1)
 8    A(K1,3)=CS(I+2)
      GO TC 102
 3    CO 9 I=3,LM,4
      K1=K1+1
      A(K1,1)=CS(I)
      A(K1,2)=CS(I+1)
      A(K1,3)=CS(I+2)
 9    A(K1,4)=CS(I+3)
      GO TO 102
 4    CO 1C I=3,LM,5
      K1=K1+1
      A(K1,1)=CS(I)
      A(K1,2)=CS(I+1)
      A(K1,3)=CS(I+2)
      A(K1,4)=CS(I+3)
 10   A(K1,5)=CS(I+4)
```

```
      GO TO 102
  101 IF(N-4)102,102,13
C     TURNS DATA OVER TO LSCF1 FOR SOLUTION
  102 CALL LSCF1(A,M,N,K,L,C1,C2,AT,BT)
C     EXAMINE TEST CONDITION CELL FOR NORMAL RETURN
      IF(L)14,103,14
C     PERFORM ERROR ANALYSIS IN ORIGINAL AND TRANSPOSED
C SYSTEMS.
C     COMPUTE YS IN BOTH SYSTEMS
  103 FM=M
      DO 104 IS=1,2
      DO 104 NN=1,N
      E(3,NN,IS)=0
  104 E(4,NN,IS)=0
      DO 116 IS=1,2
      DO 116 NN=1,N
      NNN=NN+1
      SUMD=0
      SUMR=0
      DO 115 I=1,M
      J=K+1
      YC(I,NN,IS)=0
      GO TO (105,106),IS
  105 YC(I,NN,1)=(YC(I,NN,IS))*A(I,1)+C1(J,NN)
      J=J-1
      IF(J)108,108,105
  106 XT=AT*(A(I,1)+BT)
  107 YC(I,NN,2)=YC(I,NN,2)*XT+C2(J,NN)
      J=J-1
      IF(J)108,108,107
C     COMPUTE DIFFERENCE AND RELATIVE ERROR IN BOTH SYSTEMS
  108 DIFFER(I,NN,IS)=A(I,NNN)-YC(I,NN,IS)
      IF(ABSF(DIFFER(I,NN,IS))-E(3,NN,IS))110,109,109
  109 E(3,NN,IS)=ABSF(DIFFER(I,NN,IS))
  110 IF(A(I,NNN))111,112,111
  111 RELERR(I,NN,IS)=DIFFER(I,NN,IS)/A(I,NNN)
      GO TO 1121
  112 RELERR(I,NN,IS)=DIFFER(I,NN,IS)
 1121 IF(ABSF(RELERR(I,NN,IS))-E(4,NN,IS))114,113,113
  113 E(4,NN,IS)=ABSF(RELERR(I,NN,IS))
  114 SUMD=SUMD+((DIFFER(I,NN,IS))**2)
  115 SUMR=SUMR+((RELERR(I,NN,IS))**2)
      E(1,NN,IS)=SQRTF(SUMD/FM)
  116 E(2,NN,IS)=SQRTF(SUMR/FM)
C     PRINT OUTPUT ON MT2,IF ZERO EXIT
      IF(MT2)1161,127,117
 1161 MT2=3
  117 NN=1
      IP=0
  118 J=M
      NNN=NN+1
      IP=IP+1
C     DUMP PAGE
      WRITE OUTPUT TAPE MT2,301,IP
```

```
C     DUMP TITLE
      WRITE OUTPUT TAPE MT2,302,K,M,NN
C     DUMP HEADINGS
      WRITE OUTPUT TAPE MT2,303
      IF(J-49)119,122,122
C     DUMP J ANS WITH COEFFICIENTS
 119  IAC=K+1
      WRITE OUTPUT TAPE MT2,304,(I,A(I,1),A(I,NNN),C1(I,NN),
     2YC(I,NN,1),
     1RELERR(I,NN,1),C2(I,NN),YC(I,NN,2),RELERR(I,NN,2),I=1,
     4IAC)
C     DUMP J ANS WITHOUT COEFFICIENTS
      ID=K+2
      IF(ID-M)120,120,121
 120  WRITE OUTPUT TAPE MT2, 305,(I,A(I,1),A(I,NNN),YC(I,NN,
     21),
     1RELERR(I,NN,1),YC(I,NN,2),RELERR(I,NN,2),I=IC,M)
      IF(J-49)121,123,123
C     DUMP ERROR ANALYSIS ON SAME PAGE
 121  WRITE OUTPUT TAPE MT2,306,E(1,NN,1),E(1,NN,2),E(2,NN,1
     2),E(2,NN,2),
     1AT,E(3,NN,1),E(3,NN,2),E(4,NN,1),E(4,NN,2),BT
      NN=NN+1
      IF(NN-N)118,118,127
 122  IF(J-54)119,124,124
C     DUMP CONTINUE
 123  IP=IP+1
      WRITE OUTPUT TAPE MT2,307,NN,IP
C     DUMP NEW PAGE THEN GO TO ANS AND ERROR ANALYSIS DUMP
      WRITE OUTPUT TAPE MT2,308,NN,IP
C     DUMPS ERROR ANALYSIS ON SEPARATE PAGE
      GO TO 121
C     DUMP 53 ANS
 124  IAC=K+1
      WRITE OUTPUT TAPE MT2,304,(I,A(I,1),A(I,NNN),C1(I,NN),
     2YC(I,NN,1),
     1RELERR(I,NN,1),C2(I,NN),YC(I,NN,2),RELERR(I,NN,2),I=1,
     3IAC)
      ID=K+2
      WRITE OUTPUT TAPE MT2,305,(I,A(I,1),A(I,NNN),YC(I,NN,1
     2),
     1RELERR(I,NN,1),YC(I,NN,2),RELERR(I,NN,2),I=IC,53)
      ID=54
      J=J-53
 125  IP=IP+1
      WRITE OUTPUT TAPE MT2,307,NN,IP
      WRITE OUTPUT TAPE MT2,308,NN,IP
      WRITE OUTPUT TAPE MT2,309
      IF(J-56)120,126,126
C     DUMP 55 ANS
 126  IAC=ID+55
      WRITE OUTPUT TAPE MT2,305,(I,A(I,1),A(I,NNN),YC(I,NN,1
     2),
     1RELERR(I,NN,1),YC(I,NN,2),RELERR(I,NN,2),I=IC,IAC)
```

```
      ID=IAC+1
      J=J-55
      GO TO 125
 127  L=0
      K=K+1
      DO 128 I=1,K
      DO 128 J=1,N
      COEFFO(I,J)=C1(I,J)
 128  COEFFT(I,J)=C2(I,J)
      DO 129 I=1,4
      DO 129 NN=1,N
      DO 129 IS=1,2
 129  ERRORS(I,NN,IS)=E(I,NN,IS)
 14   LOOK=L
      RETURN
 13   L=13
 15   PRINT 200
      WRITE OUTPUT TAPE 3,200
      GO TO 14
 200  FORMAT(1X,47HDATA FOR CURVE FITTING ROUTINE HAS BEEN S
     213HSENSED AS INC
     1ORRECT.,56X,3HJBH)
C     FORMAT STATEMENTS FOR PRINT OUTPUT TAPE MT2
 301  FORMAT(1H1,110X,5H PAGE,I3)
 302  FORMAT(14X,34HLEAST SQUARES CURVE FIT OF DEGREE ,I2,9X
     2,I3,
     121H OBSERVED DATA POINTS,9X,11HSET NUMBER I2//)
 303  FORMAT(12X,1HX,15X,1HY,9X,12HCOEFFICIENTS,5X,7HY COMPU
     43HTED,4X,
     17HREL ERR,4X,12HCOEFFICIENTS,5X,10HY COMPUTED,4X,3HREL
     54H ERR/8X,
     28HOBSERVED,8X,8HOBSERVED,6X,12HORIGINAL SYS,4X,12H9RIG
     77HNAL SYS,5X,
     34HORIG,4X,15HTRANSFORMED SYS,1X,15HTRANSFORMED SYS,2X,
     65HTRANS//)
 304  FORMAT(I4,E15.7,3E16.7,E10.2,2E16.7,E10.2)
 305  FORMAT(I4,E15.7,E16.7,E32.7,E10.2,E32.7,E10.2)
 306  FORMAT(19X,11HDIFFERENCES,29X,15HRELATIVE ERRORS,22X,
     123HTRANSFORMATION Z=A(X-B)/6X,8HRMS ORG.,E9.2,6H  RMS
     55HTRANS,
     2E9.2,5X,8HRMS ORG.,E9.2,11H  RMS TRANS,E9.2,10X,4HWHER
     65HE A =,E15.7/
     36X,8HMAX ABS.,E9.2,11H  MAX ABS. ,E9.2,5X,8HMAX ABS.,E
     79.2,
     411H  MAX ABS. ,E9.2,10X,9HWHERE B =,E15.7/111X)
 307  FORMAT(40X,16HDATA OF THIS SET,I2,17H CONTINUED ON PAG
     11HE,I3)
 308  FORMAT(1H1,46X,19HCONTINUATION OF SET,I2,5H DATA,39X,
     14HPAGE,I3)
 309  FORMAT(12X,1HX,15X,1HY,26X,10HY COMPUTED,4X,7HREL ERR,
     321X,
     110HY COMPUTED,4X,7HREL ERR/8X,8HOBSERVED,8X,8HOBSERVED
     4,22X,
     212HORIGINAL SYS,5X,4HORIG,20X,15HTRANSFORMED SYS,2X,
```

```
      55HTRANS/)
       END
```

CLSCF1 LEAST SQUARES CURVE FIT (COMPUTATIONAL)

```
C DIMENSION CARD A(100,5),C1(32,8),C2(32,8)
C POINTS TO BE FITTED ASSUMED IN CORE WHEN CALL EXECUTED.
C A = POINTS TO BE FITTED. A(I,1)=INDEPENDENT VARIABLE.
C A(I,N), WHERE N = 2,3,4,5, ARE SETS OF DEPENDENT VARIABLE
C CORRESPONDING TO A(I,1).
C M = NUMBER OF POINTS. MAX = 100
C N = NUMBER OF SETS. MAX=4
C K = DEGREE OF FIT. MAX=24 OR (K-1) WHICH EVER IS SMALLER.
C L = RETURNED CHECK VALUE. L=C,RUN GOOD. L NOT=0,SYSTEM
C UNSOLVED.
C C1 = COEFFICIENTS OF POLYNOMIAL DEGREE FOR ORIGINAL
C SYSTEM. C1(1,N) IS CONSTANT TERM. C1(I,N) IS COEFFICIENT
C FOR (I-1) POWER TERM.
C      N DENOTES THE SET. RETURNED VALUES.
C C2= COEFFICIENTS OF POLYNOMIAL DEGREE FOR TRANSFORMED
C SYSTEM. SUBSCRIPTS DESIGNATED AS IN (C1I,N). RETURNED VAL-
C UES.
C AT AND BT = TRANSFORMATION CONSTANTS IN THE FORMULA
C Z=A(X-B).
      SUBROUTINE LSCF1(A,M,N,K,L,C1,C2,AT,BT)
      DIMENSION Z(32,40),S(49),D(25),A(100,5),C1(32,8),C2(32
     1,8)
C     ASSUMES THAT K LESS THAN 24
C     FORMS TRANSFORMATION CONSTANTS AT AND BT
      AT=2./(A(M,1)-A(1,1))
      BT=(A(M,1)+A(1,1))/2.
C     MAPS X INTO ZT AND FORMS MATRIX VALUES
      JJ=(2*K)+1
      DO 100 J=1,JJ
 100  S(J)=0.
      KK=K+1
      KKK=KK+N
      DO 1001 J=1,KK
      DO 1001 JJ=1,KKK
      Z(J,JJ)=0
 1001 CONTINUE
      DO 103 MM=1,M
      J=1
      KKK=2*K
      DO 103 KK=0,KKK
      ZT=AT*(A(MM,1)-BT)
      S(J)=ZT**KK+S(J)
      IF(KK-K)101,101,103
C     FORM COLUMN VECTORS OF Y
 101  IC=K+2
      IR=KK+1
```

```
      NNN=N+1
      DO 102 NN=2,NNN
      Z(IR,IC)=Z(IR,IC)+((ZT**KK)*A(MM,NN))
102   IC=IC+1
103   J=J+1
C     SETS UP MATRIX
      MC=K+1
      DO 104 IC=1,MC
      J=IC
      DO 104 IR=1,MC
      Z(IR,IC)=S(J)
104   J=J+1
C     USE LINEQ FOR SOLUTION OF SYSTEM
      CALL LINEQ(Z,C2,MC,N,L)
      IF(L)111,105,111
C     REVERTING COEFFICIENTS TO ORIGINAL SYSTEM(C2 TO C1)
105   II=K+1
      DO 110 NN=1,N
C     FORMS COEFFICIENTS FOR SYNTHETIC DIVISION
      DO 106 I=1,II
106   D(I)=(C2(I,NN))*(AT**(I-1))
C     PERFORM SYNTHETIC DIVISION
      I=1
      JJ=0
107   C1(I,NN)=0
      J=K+1
108   C1(I,NN)=D(J)+(C1(I,NN)*(-BT))
      D(J)=C1(I,NN)
      J=J-1
      IF(J-JJ)109,109,108
109   I=I+1
      JJ=JJ+1
      IF(I-K-1)107,107,110
110   CONTINUE
      L=0
111   RETURN
      END
```

CLINEQ   LINEAR EQUATIONS SOLVER

```
C     CALLING SEQUENCE...
C     DIMENSION A(32,40),X(32,8)
C     CALL LINEQ(A,X,N,K,L)
C        TO SOLVE THE LINEAR SYSTEM(S) RX=S
C        WHERE R IS THE COEFFICIENT MATRIX(N SQUARE)
C        AND S IS THE RIGHT HAND SIDE(S) (N BY K)
C        LET A(I,J)=R(I,J),I=1,N,J=1,N
C            A(I,N+J)=S(I,J),I=1,N,J=1,K
C            N=NO. OF EQUATIONS
C            K=NO. OF RIGHT HAND SIDES
C        GIVES X AS THE SOLUTION MATRIX
```

```
C          L=0 INDICATES NORMAL COMPLETION
C          L=1 INDICATES DETERMINANT=0
C          L=2 INDICATES FL. PT. OVERFLOW
C       1332 CELLS OF COMMON ARE USED
        SUBROUTINE LINEQ (A,X,N,K,L)
        DIMENSION A(32,40),X(32,8),B(32,40),C(40)
        COMMON B,C,NN,KK,NK,TAP,Z,II,IP,I1,I2,I,J,JJ
        NN=N
        KK=K
        NK=NN+KK
        DO 10 I=1,NN
        DO 10 J=1,NK
 10     B(I,J)=A(I,J)
        DO 180 I=1,NN
        II=I+1
        TAP=0.
        DO 50 J=I,NN
        Z=ABSF(B(J,I))
        IF(Z-1.)30,20,30
 20     IP=J
        GO TO 60
 30     IF(Z-TAP)50,40,40
 40     IP=J
        TAP=Z
 50     CONTINUE
 60     Z=B(IP,I)
        IF(Z)80,70,80
C       DETERMINANT=0 INDICATED BY L=1
 70     L=1
        RETURN
 80     DO 90 J=II,NK
 90     C(J)=B(IP,J)/Z
        I1=NN
        I2=NN
 93     IF(I1-IP)95,160,95
 95     Z=B(I1,I)
        IF(Z)130,100,130
 100    IF(I1-I2)110,150,110
 110    DO 120 J=II,NK
 120    B(I2,J)=B(I1,J)
        GO TO 150
 130    DO 140 J=II,NK
 140    B(I2,J)=B(I1,J)-Z*C(J)
 150    I2=I2-1
 160    I1=I1-1
        IF(I2-I)170,170,93
 170    DO 180 J=II,NK
 180    B(I,J)=C(J)
        I=NN
 185    I=I-1
        IF(I)210,210,190
 190    DO 200 JJ=1,KK
        J=NN+JJ
        I1=I+1
```

```
      CO 200 I2=I1,NN
 200  B(I,J)=B(I,J)-B(I,I2)*B(I2,J)
      GO TC 185
 210  CO 220 J=1,KK
      I1=NN+J
      CO 220 I=1,NN
 220  X(I,J)=B(I,I1)
      IF ACCUMULATOR OVERFLOW 230,24C
C     FLOATING POINT OVERFLOW INDICATED BY L=2
 230  L=2
      RETURN
C     SUCCESSFUL COMPUTATICN INDICATED BY L=C
 240  L=C
      RETURN
      END
```

C READT

```
*       FAP
        CCUNT   630
*READT
*       SINGLE PRECISION FLOATING POINT INPUT
*       SAMPLE CALLING SEQUENCES
*       IN    FORTRAN     CALL READT(A,N,L)
*       IN    FAP         CALL READT,A,N,L)
*       A     IS   THE   ARRAY NAME WHERE DATA IS TO BE STORED
*       N     IS   THE   SYMBOLIC  TAPE NUMBER
*       L     IS   THEM   ERROR  DETECTICN  CEL
*       L=0 NO ERRORS DETECTED L NCT EQUAL TO ZEROES
*                ERRORS DETECTEC
*                  N IS STCRED IN CECR.
*       A COMMA IS A FIELD TERMINATING CHARACTER.
        ENTRY   READT
 READT  SXA     SV,1
        SXA     SV+1,2
        SXA     SV+2,4
        AXT     0,1
        SXA     SC,1
        CLA     3,4
        STA     C+2
        STZ     CEXP
        CLA*    2,4
        PCC     **,2
        ARS     18
        STO     T
        CLA*    $(IOU)
        CAS     T
        TRA     *+5
        TRA     *+4
        CLS     =1
        STO     ESW
```

```
        TRA     C
        CLA     $(IOU)
        STA     *+1
        CLA     **,2
        STA     SA1+1           CHANNEL
        STA     SA5             AND TAPE NUMBER
XY      AXT     0,1
        LCI     SA5
        CNT     CA+5,1
        TXI     XX,1,1
        TRA     XX+1
XX      TXL     XY+1,1,5
Z1      SXD     *+3,1
        AXT     4,2             PICKS
        AXT     0,1             THE
        TXH     *+3,1,**        CORRECT
        TXI     *+1,2,-4        DATA
        TXI     Z1+3,1,1        CHANNEL
        LXD     Z1+3,1
        CAL     1F,2
        SLW     SA2
        CAL     2F,2
        SLW     SA2+1
        CAL     3F,2
        SLW     SA3
        CAL     4F,2
        SLW     SA3+1
        CLA     =0
        STA     B7+4
        LCI     =7
        STZ     CNT
        STZ     BCW
        STZ     EXP
        CLA     =12
        STA     CV+1
        CLA     =4
        STA     CV2+1
        STZ     ESW
        CLA     1,4             STORE FIRST
        STA     7B              DATA CELL
        TSX     SC,1
B3      TNX     *+3,4,1
        CAQ     TABLE,1,1
        TRA     TAB3,1
        TSX     SC,1
        TRA     B3
B4S     RIS     CY+1
B4      TNX     *+3,4,1
        CAQ     TABLE,1,1
        TRA     TAB4,1
        TSX     SC,1
        TRA     B4
B5S     RIS     CY
B5      TNX     *+3,4,1
```

```
        CAQ     TABLE,1,1
        TRA     TAB5,1
        TSX     SC,1
        TRA     B5
B6      TNX     *+3,4,1
        CAQ     TABLE,1,1
        TRA     TAB6,1
        TSX     SC,1
        TRA     B6
B7      SXA     77B,4
        TSX     DV-1,4
        TSX     DV1-1,4
        SXA     7B+3,1
        AXT     0,1
        CLA     BCW
        TZE     7B
        ONT     CY
        TRA     *+7
        CLA     BCW
        CAS     =32767
        TRA     B8B
        NCP
        ALS     18
        TRA     *+2
        TSX     FL,4
        ONT     CY+1
        SSM
7B      STO     **,1
        TXI     *+1,1,1
        SXA     B7+4,1
        AXT     **,1
        LCI     =7
        STZ     CEXP
        STZ     BCW
        STZ     EXP
        STZ     CNT
        CLA     =12
        STA     CV+1
        CLA     =4
        STA     CV2+1
77B     AXT     **,4
        LCQ     TQ
        TRA     B3
B8      CLA     ESW
        ACM     =1
        SLW     ESW
B8A     TNX     *+3,4,1
        CAQ     TABLE,1,1
        TRA     TAB8,1
        TSX     SC,1
        TRA     B8A
B8B     CLA     ESW
        ADM     =1
        STO     ESW
```

```
        STZ     BCW
        LXA     B7+4,1
        TRA     B7
 B8C    LXA     CV1,4
        TRA     B8
B8C     LXA     CV3,4
        TRA     B8
 B9S    RIS     CY
 B9     TNX     *+3,4,1
        CAQ     TABLE,1,1
        TRA     TAB9,1
        TSX     SC,1
        TRA     B9
 B10    TNX     *+3,4,1
        CAQ     TABLE,1,1
        TRA     TAB10,1
        TSX     SC,1
        TRA     B10
 B11    TNX     *+3,4,1
        CAQ     TABLE,1,1
        TRA     TAB11,1
        TSX     SC,1
        TRA     B11
 B12S   RIS     CY+2
        RIS     CY
 B12    TNX     *+3,4,1
        CAQ     TABLE,1,1
        TRA     TAB12,1
        TSX     SC,1
        TRA     B12
 B13S   RIS     CY
 B13    TNX     *+3,4,1
        CAQ     TABLE,1,1
        TRA     TAB13,1
        TSX     SC,1
        TRA     B13
 B14S   RIS     CY
 B14    TNX     *+3,4,1
        CAQ     TABLE,1,1
        TRA     TAB14,1
        TSX     SC,1
        TRA     B14
 B15    TNX     *+3,4,1
        CAQ     TABLE,1,1
        TRA     TAB15,1
        TSX     SC,1
        TRA     B15
 B16S   RIS     CY+2
        RIS     CY
 B16    TNX     *+3,4,1
        CAQ     TABLE,1,1
        TRA     TAB16,1
        TSX     SC,1
        TRA     B16
```

```
 B17    TNX     *+3,4,1
        CAQ     TABLE,1,1
        TRA     TAB17,1
        TSX     SC,1
        TRA     B17
 FL     SXA     AX,2
        CLA     BCW
 NM     ALS     1
        PBT
        TXI     NM,2,1
        LGR     1
        SLW     BCW
        PXA     **,2
        SSM
        ACD     =35
        STO     CEXP
 AX     AXT     **0,2
        SXA     EX-1,4
        CLA     EXP
        ONT     CY+2
        SSM
        SUB     CNT
        STO     EXP
 TZ     TZE     PK
        STO     EXP
        TMI     ZT
        CLA     N15
        STO     T
        CLA     N15+1
        STO     T+1
        TSX     MP,4
        CLA     EXP
        SUB     =15
        STO     EXP
        TZE     EX
        TRA     TZ
 ZT     LDQ     =-20
        XCA
        TLQ     EX+1
        XCA
        LLS     1
        PAX     **,2
        CLA     NP+41,2
        STO     T+1
        CLA     NP+40,2
        STO     T
        TSX     MP,4
        AXT     **,4
 EX     TRA     PK
        CLA     NP
        STO     T
        CLA     NP+1
        STO     T+1
        TSX     MP,4
```

```
        CLA    EXP
        ADD    =20
        STO    EXP
        TZE    EX
        TRA    TZ+1
MP      CLA    CEXP
        ADD    T
        STO    CEXP
        LDQ    BCW
        MPY    T+1
        SXA    SET,2
        AXT    0,2
        LLS    1
        PBT
        TXI    *-2,2,1
        LGR    1
        SLW    BCW
        TXL    SET,2,0
        CLA    CEXP
        SUB    =1
        STO    CEXP
SET     AXT    **0,2
        TRA    1,4
PK      CLA    BCW
        ADD    =0200
        PBT
        TRA    BIC
        LGR    1
        XCA
        CLA    CEXP
        ADD    =1
        STO    CEXP
        TRA    BIC+1
BIC     XCA
        CLA    CEXP
        ADD    =C200
        CAS    =0400
        TRA    ERROR
        TRA    ERROR
        TMI    ERROR
        LLS    27
        STO    BCW
        TRA    1,4
ERROR   STZ    BCW
        CLA    ESW
        ADD    =1
        STO    ESW
        LXA    B7+4,1
        TRA    7B
CV      SXA    CV1,4
        AXT    **0+12,4
        TNX    B8C,4,1
        VLM    PW+11,4,4
        LLS    4
```

```
         SSP
         ACD     BCW
         SLW     BCW
         SXA     CV+1,4
         SXA     DV,4
CV1      AXT     **,4
         TRA     1,1
         SXA     JO,2
DV       AXT     **,2
         STQ     TQ
         LDQ     BCW
         CLM
         LLS     0
         DVH     PW+11,2
         STQ     BCW
JO       AXT     **,2
         TRA     1,4
CV2      SXA     CV3,4
         AXT     **0+4,4
         TNX     B8A,4,1
         VLM     PW+11,4,4
         LLS     4
         SSP
         ACD     EXP
         SLW     EXP
         SXA     CV2+1,4
         SXA     DV1,4
CV3      AXT     **0,4
         TRA     1,1
         SXA     JO1,2
DV1      AXT     **,2
         LDQ     EXP
         CLM
         LLS     0
         DVH     PW+11,2
         STQ     EXP
JO1      AXT     **,2
         TRA     1,4
SC       AXT     0,4
         TNX     SC+3,4,1
         TRA     SCA
         TSX     SA,4
         AXT     12,4
SCA      LDQ     HOP+12,4
         SXA     SC,4
         AXT     7,4
         TRA     1,1
SA       SXA     SA4,4
SA1      AXT     5,4
         RDS     **
SA2      RCHA    SA7
         TCOA    *
SA3      TRCA    SA5
         TEFA    SA6
```

```
SA4     AXT     **,4
        TRA     1,4
SA5     BSR     **
        TIX     SA1+1,4,1
SA6     CLA     *-2
        STO     ESW
        TRA     C
SA7     IOCD    HOP,0,12
HOP     BSS     12
T       BSS     12
C       CLA     ESW
        ALS     18
        STO     **
SV      AXT     **,1
        AXT     **,2
        AXT     **,4
        TRA     4,4
CA      OCT     1000
        OCT     2000
        OCT     3000
        OCT     4000
        OCT     5000
        OCT     6000
NP      OCT     400000000102
        OCT     274712041444
        OCT     400000000077
        OCT     354074451755
        OCT     400000000073
        OCT     223445672164
        OCT     400000000071
        OCT     270357250621
        OCT     400000000065
        OCT     346453122766
        OCT     400000000061
        OCT     220072763672
        OCT     400000000056
        OCT     264111560650
        OCT     400000000053
        OCT     341134115022
        OCT     400000000047
        OCT     214571460113
        OCT     400000000044
        OCT     257727774136
        OCT     400000000041
        OCT     333715773166
        OCT     400000000035
        OCT     211340575012
        OCT     400000000032
        OCT     253630734214
        OCT     400000000027
        OCT     326577123257
        OCT     400000000023
        OCT     206157364055
        OCT     400000000020
```

```
        OCT     247613261071
        OCT     400000000015
        OCT     321556135307
        OCT     400000000011
        OCT     203044672274
        OCT     400000000006
        OCT     243656050754
        OCT     400000000003
        OCT     314631463146
PW      OCT     7346545000
        OCT     575360400
        OCT     46113200
        OCT     3641100
        OCT     303240
        OCT     23420
        OCT     1750
        OCT     144
        OCT     12
        OCT     1
N15     OCT     62
        OCT     343277244615
TC      PZE
ESW     PZE
CNT     PZE
EXP     PZE
BCW     PZE
CEXP    PZE
CY      OCT     1
        OCT     2
        OCT     4
TABLE   VFD     6/0,30/0             CODE 00 TYPE 0
        VFD     6/1,30/0             CODE 01 TYPE 0
        VFD     6/2,30/0             CODE 02 TYPE 0
        VFD     6/3,30/0             CODE 03 TYPE 0
        VFD     6/4,30/0             CODE 04 TYPE 0
        VFD     6/5,30/0             CODE 05 TYPE 0
        VFD     6/6,30/0             CODE 06 TYPE 0
        VFD     6/7,30/0             CODE 07 TYPE 0
        VFD     6/8,30/0             CODE 10 TYPE 0
        VFD     6/9,30/0             CODE 11 TYPE 0
        VFD     06/12,30/7           CODE 12 TYPE 7
        VFD     06/13,30/7           CODE 13 TYPE 7
        VFD     06/14,30/2           CODE 14 TYPE 2
        VFD     06/15,30/7           CODE 15 TYPE 7
        VFD     06/16,30/7           CODE 16 TYPE 7
        VFD     06/17,30/7           CODE 17 TYPE 7
        VFD     06/20,30/1           CODE 20 TYPE 1
        VFD     06/21,30/7           CODE 21 TYPE 7
        VFD     06/22,30/7           CODE 22 TYPE 7
        VFD     06/23,30/7           CODE 23 TYPE 7
        VFD     06/24,30/7           CODE 24 TYPE 7
        VFD     06/25,30/6           CODE 25 TYPE 6
        VFD     06/26,30/7           CODE 26 TYPE 7
        VFD     06/27,30/7           CODE 27 TYPE 7
```

```
        VFD     06/30,30/7            CODE 30 TYPE 7
        VFD     06/31,30/7            CODE 31 TYPE 7
        VFD     06/32,30/7            CODE 32 TYPE 7
        VFD     06/33,30/3            CODE 33 TYPE 3
        VFD     06/34,30/7            CODE 34 TYPE 7
        VFD     06/35,30/7            CODE 35 TYPE 7
        VFD     06/36,30/7            CODE 36 TYPE 7
        VFD     06/37,30/7            CODE 37 TYPE 7
        VFD     06/40,30/2            CODE 40 TYPE 2
        VFD     06/41,30/7            CODE 41 TYPE 7
        VFD     06/42,30/7            CODE 42 TYPE 7
        VFD     06/43,30/7            CODE 43 TYPE 7
        VFD     06/44,30/7            CODE 44 TYPE 7
        VFD     06/45,30/7            CODE 45 TYPE 7
        VFD     06/46,30/7            CODE 46 TYPE 7
        VFD     06/47,30/7            CODE 47 TYPE 7
        VFD     06/50,30/7            CODE 50 TYPE 7
        VFD     06/51,30/7            CODE 51 TYPE 7
        VFD     06/52,30/7            CODE 52 TYPE 7
        VFD     06/53,30/7            CODE 53 TYPE 7
        VFD     06/54,30/7            CODE 54 TYPE 7
        VFD     06/55,30/7            CODE 55 TYPE 7
        VFD     06/56,30/7            CODE 56 TYPE 7
        VFD     06/57,30/7            CODE 57 TYPE 7
        VFD     06/60,30/5            CODE 60 TYPE 5
        VFD     06/61,30/7            CODE 61 TYPE 7
        VFD     06/62,30/7            CODE 62 TYPE 7
        VFD     06/63,30/7            CODE 63 TYPE 7
        VFD     06/64,30/7            CODE 64 TYPE 7
        VFD     06/65,30/7            CODE 65 TYPE 7
        VFD     06/66,30/7            CODE 66 TYPE 7
        VFD     06/67,30/7            CODE 67 TYPE 7
        VFD     06/70,30/7            CODE 70 TYPE 7
        VFD     06/71,30/7            CODE 71 TYPE 7
        VFD     06/72,30/7            CODE 72 TYPE 7
        VFD     06/73,30/4            CODE 73 TYPE 4
        VFD     06/74,30/7            CODE 74 TYPE 7
        VFD     06/75,30/7            CODE 75 TYPE 7
        VFD     06/76,30/7            CODE 76 TYPE 7
        VFD     06/77,30/7            CODE 77 TYPE 7
1F      RCHF    SA7
2F      TCOF    SA2+1
3F      TRCF    SA5
4F      TEFF    SA6
1E      RCHE    SA7
2E      TCOE    SA2+1
3E      TRCE    SA5
4E      TEFE    SA6
1D      RCHD    SA7
2D      TCOD    SA2+1
3D      TRCD    SA5
4D      TEFD    SA6
1C      RCHC    SA7
2C      TCOC    SA2+1
```

```
 3C     TRCC     SA5
 4C     TEFC     SA6
 1B     RCHB     SA7
 2B     TCOB     SA2+1
 3B     TRCB     SA5
 4B     TEFB     SA6
 1A     RCHA     SA7
 2A     TCOA     SA2+1
 3A     TRCA     SA5
 4A     TEFA     SA6
        TRA      B8                 ERRORS
        TRA      C                  E
        TRA      B3                 B
        TRA      B7                 ,
        TRA      B5S                .
        TRA      B4S
        TRA      B4                 +
TAB3    TSX      CV,1               0 THRU 9
        TRA      B6
        TRA      B8
        TRA      B8
        TRA      B4
        TRA      B8
        TRA      B9S
        TRA      B8
        TRA      B8
TAB4    TSX      CV,1
        TRA      B10
        TRA      B8
        TRA      B8
        TRA      B5
        TRA      B8B
        TRA      B8
        TRA      B8
        TRA      B8
TAB5    CLA      CNT
        ADD      =1
        STO      CNT
        TSX      CV,1
        TRA      B11
        TRA      B8
        TRA      B14S
        TRA      B6
        TRA      B7
        TRA      B13S
        TRA      B12S
        TRA      B12S+1
TAB6    TSX      CV,1
        TRA      B6
        TRA      B8A
        TRA      B8A
        TRA      B8A
        TRA      B8B+3              TAB 8 FOR B8
        TRA      B8A
```

```
        TRA     B8A
        TRA     B8A
TAB8    TRA     B8A
        TRA     B8
        TRA     B8
        TRA     B9
        TRA     B8B
        TRA     B8
        TRA     B8
        TRA     B8
TAB9    CLA     CNT
        ADD     =1
        STO     CNT
        TSX     CV,1
        TRA     B15
        TRA     B8
        TRA     B14S
        TRA     B10
        TRA     B7
        TRA     B13S
        TRA     B16S
        TRA     B16S+1
TAB10   TSX     CV,1
        TRA     B10
        TRA     B8
        TRA     B14S
        TRA     B11
        TRA     B7
        TRA     B8
        TRA     B16S
        TRA     B16S
TAB11   CLA     CNT
        ADD     =1
        STO     CNT
        TSX     CV,1
        TRA     B11
        TRA     B8
        TRA     B8
        TRA     B12
        TRA     B7
        TRA     B8
        TRA     B8
        TRA     B8
TAB12   TSX     CV2,1
        TRA     B17
        TRA     B8
        TRA     B14S
        TRA     B13
        TRA     B7
        TRA     B8
        TRA     B16S
        TRA     B16
TAB13   CLA     CNT
        ADD     =1
```

```
        STO     CNT
        TSX     CV,1
        TRA     B13
        TRA     B8
        TRA     B8
        TRA     B14
        TRA     B8B
        TRA     B8
        TRA     B16S
        TRA     B16
TAB14   TSX     CV2,1
        TRA     B17
        TRA     B8
        TRA     B14S
        TRA     B15
        TRA     B7
        TRA     B8
        TRA     B16S
        TRA     B16
TAB15   CLA     CNT
        ADD     =1
        STO     CNT
        TSX     CV,1
        TRA     B15
        TRA     B8
        TRA     B8
        TRA     B16
        TRA     B8B
        TRA     B8
        TRA     B8
        TRA     B8
TAB16   TSX     CV2,1
        TRA     B17
        TRA     B8
        TRA     B8
        TRA     B17
        TRA     B7
        TRA     B8
        TRA     B8
        TRA     B8
TAB17   TSX     CV2,1
        TRA     B17
        END
```

Appendix F

Glossary of Terms

This appendix is designed to list uncommon words used in this paper. The definitions given coincide with the way the words are used in the report.

apogee - point in the elliptic orbit where the orbiting body is farthest from the center of the earth.

argument of perigee - the angle between two lines intersecting at the center of the earth, one of them directed to the ascending node and one of them to perigee.

ascending node - point where the line of nodes intersects the orbit as the satellite goes into the hemisphere containing the North Pole.

azimuth - angle measured counter clockwise from true north to satellite projection in the local horizontal plane.

diurnal rotation - rotation of earth about its axis.

eccentric anomaly - angle between radii of circle circumscribed about the orbital ellipse, one radius being through the perifocus, the other being to a point on the circle vertically above the satellite position.

eccentricity - ratio of the distance between the orbit center and the prime focus to the semi-major axis.

elevation - angular measurement of satellite above the horizontal plane.

<u>field of vision</u> (of tracking system) - space above the local horizontal plane of the tracker. It is defined by the fact that elevation angle is positive.

<u>geocentric coordinate system</u> - system with origin located at center of the earth. It may be either inertial or noninertial depending upon whether it partakes in earth diurnal rotation.

<u>inclination</u> - the angle between the equatorial plane and the orbit plane.

<u>line of nodes</u> - line of intersection of the orbit plane and the earth equatorial plane.

<u>orbit elements</u> - set of six constants which completely define an orbit. The six used in this paper are semimajor axis, eccentricity, inclination, longitude of the ascending node, argument of perigee, and time of perigee passage.

<u>orbit parameters</u> - see orbit elements.

<u>perigee</u> - point in the elliptic orbit where the orbiting body most closely approaches the center of the earth.

<u>period</u> (<u>of orbit</u>) - elapsed time between successive passage of the satellite through the same point.

<u>right ascension of the ascending node</u> - the angle between two lines intersecting at the center of the earth, one of them directed toward the vernal equinox and the other passing through the ascending node.

<u>semilatus rectum</u> - magnitude of radius vector for a true

anomaly of 90 degrees.

semi-major axis - one half the sum of the apogee distance plus the perigee distance. It is also the distance from the center of the ellipse to perigee (or apogee).

topocentric coordinate system - system with origin at the radar tracking site. It is a noninertial system.

true anomaly - angle between geocentric position vector and radius vector through perigee.

Vita

Thomas Henry McMullen was born on 4 July 1929 in Dayton, Ohio, the fourth son of Major General (then First Lieutenant) Clements McMullen and Adelaide Lewis McMullen. After graduation from Alamo Heights High School, Texas, in 1945, he attended St. Mary's University of Texas in San Antonio until 1947. At that time he entered the United States Military Academy, graduating in 1951 with a Bachelor of Science degree and a commission in the USAF. After completing pilot training in 1952, he served as a fighter pilot with the 51st Fighter Interceptor Wing during the Korean War. Since that time he has performed the job of a functional and acceptance test pilot, first at the San Antonio Air Materiel Area, Kelly Air Force Base, Texas for five and one-half years, and then at General Dynamics, Fort Worth, Texas for three and one-half years. Subsequent to this last tour, he enrolled in the Air Force Institute of Technology.

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