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16. TUPFL EMEMTARY MOTES

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Soil penetration, soil impact, seismic wave speed, soil properties, soil trajectories

A study of the performance characteristics of terradynamic impactors is reported on, including the influence of initial input velocity, impactor nose shape, and different soil condiifois in Egiln sand. The investigation also included evaluation of various sensing devices for nonintrusive weasurement of vehicle motions during the penetration process. The most useful of the experimental tools was found to be flash X-Ray radiography. Additional devices evaluated were magnetic coils, microwaves, capacitive

PREPACE
Thim report represents the reaulte of a wtudy on the sojl penetration performance of terradyamic impactors. This scudy was conducted Jan 1 , 1975 ts Sept 29, 1976 by the Engineerlng Science Department, University of Florida, Gainesville, Florida, 32611, under Contract No. F08635-75-C-0054 with the Ar Force Armament Linboratory, Egin Alr Force Sase, Florida. Mr. John Collins eerved as program manar for the Armanent Laboratcry.

This cechnical report has been reviaved and is approved for publication.

FOR THE COMMANDER
$\overbrace{\text { J. Murrar }}^{\text {(ound }}$
Chief. Wcapoa Syzeas Amalyeif Division

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cont., and other preseure transducers, atrain gagee on cost chamber walls, and breaking-arire ensors. Data analysis was performed by using classical Poncelet predictiv $n$ techaiquea, empirical Sandia penetration equations, cam-analytical Cavity Expansion Theory, and a three dimensional code for trajectory analysis developed undar this contract for use with an apsumed three dimensional force law. To obtain essential input to che aforenentioned models, a study of the acoustic wave velocity in a sand medium vas made, and results are included in this report.

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## SECTION I

## INTRODUCTION AND BACKGROUND

This report presents the results of a foint investigation conducted by the Vulnerability Assessment Branch (CLYV) of the Air Force Armament Laboratory and the University of Florida. The University gave araiyideal support so an experimental program conducted by DLYV/AFATL at Eglin Air Force Base.

The s apport included reviewing the proposed soll penetration experiments, recommencing changes, participating in some of the experiments at Eglin, making independent laboracory invostigations at the University of several types of sensors and of ultrasonic wave speeds in sand, extensive data analysis of the Eglin Experiments, study of existing terradynamic penetratior models, molifiration of the morels and application of them to tie interprptation of the Eglin experiments.

The study of the mechanice of high speed earth penetrators, includirg predictions of trafectory, depth of penetration, ravity formation, stability, and target interaction has in recent years been given the name of terradynamics. While this area of study has deen investiga:ed since the early l8th century, technological barriers have hindered experimental programs in assessing models advancef for characterizing penetrator perfcmance. The principal difficulty encountered has tsen the uriavallability of experimental toois for examining the sequential riotion of a vehscle passing through opaque loose and/or semicohesive media.

A recent review of the State of the Art of Earth Penetration Terhnology by Tr!andafilidis (Reference l) has categorized predictive penetration techniques as semi-analytical, analytical, theoretical, and empirical models. The first technique, which includes the classical penetration models based upon Newtonian mechanics, sich as roncelet (Reference 2), requires experimental data for evaluation of the inportant penetration constants. So-called analytical techniques, which include the Cavity Expansion (References 3 through 7) and Differential Force Law Models (Reference 8), rely upon knowledge of constitutive target muterial pr-perties. The theoretical models proposed (References 9 through il) are besed upon continum mechanics formulations describing the penecrator and target, and rely upon finite difference and finite element computer codes as solation techniques. Finally, empirical teciniques based upon extenaive laboratory and field testing have been Introduced with the mest extensive work in this area developed at Sandia Latoratories (References 12.13). Additional bacikround on the experimental program is presented in Section ll.

The purpose of the experimental progiram at Eglin was to ootain more complete transient records of the peneration evencs than previnus lavestigators had obtalned 1 " order to provide insight into the actual phgsical mechanisms involved, which could lead to better cerradynamic perr : ation models for predicting erajectories, penetration depths, and the tres acting on the p.ojectile. In the test program, live consecutively spact X-ray units have been used to visually record the transient position of seval penetrators. Nonspinning projectiles of stable configuration with various nos. shapes have been tested in dry and sarurated sand at three impact
velocities with nesr zesc impact obliquity. This is believed to be the most extensive use ever made of flash radiography in terradynamic research. In addition to the X-ray units, velocity coll se.s rs have been used as monitoring devices in confunction with a magne..: tape recording system.

The experimental setup at Eglin and some of the e:periments at the University on sensora are desiribed in Section Il after a short background arcount of orevious experimental studies. Data from the Eglin tests are described in Section III, with detalls tabulated in Appendix A. Analysis of the daia by classical semi-analytical penetration models and empirlcal methods is presented in Section IV. The analytical terhnique based on the spherical cavity expansion technique is discussed in Section $V$ and applied to the Eglin experiments. In Section Vi a three-dimensional terradynamic inodel is developed and applied. Sound speed measurements are reported in Section VII and a summary of the conclusions is given in Section VIII.

## SECTION II

## EXPERIMENTAL EQUIPMENT AND PROCEDURES

### 2.1 INTRODUCTION

Penetration experiments were performed by firing projectiles horizontally into sand targets contained in specially designed test chambers. After some preliminary tescs with 0.50 caliber and 20 mm standard rounds, the major part of the inveatigation used modelled 20 mm projectiles fabricated both at the afath and at the University of Florida. These projectiles were rylinders 0.02 weter in diameter by 0.22 to 0.24 meter in length. Three epecific nose shapes were inveatigated: biconic, flat ended, and step-tiar. Some of the biconic and step-tier projectiles had a hollow afterbody,but the majority of the results were obtained using solid projectiles.

Various sensing methods were investigated to determine as much as possible about the projectile's position and orientation, the shape of the cavity formed around the projectile, deformation patterns and forse distributions in the sand, and shock waves ahead of the projectile. The most successfal senting method was flash radiography. In the im mary test program five X-ray heads were fired sequentially with delay times set to record projectile position as it moved through a 1.2 -beterlong test chamber. The primary test program was planned to include firings of two projectile types (flat and step-tier projectiles) at three different velocities (approximately 210,320 and $400 \mathrm{~m} / \mathrm{sec}$ ) in dry sand and in saturated sand, with four replications of each type of ahot, and five Xray pictures taken in each shot. This program was completed successfully. Results of these tests are presented in Section III, along with a few examples of other projectile types.

Besides giving a more certain indication of projectile trajectory and attitude than any other sensing method, the $X$-rays give a good indication of the position on the projectiles where the sand separates to form a cavity, and can show also the reattachaent point on the afterbody as the projectile slows down. In the primary test program the $X$-rays showed that reattachment seldom occurred in the 1.2 meters of the trajectory observed.

The $X$-rays also revealed a detached bow wave in some cases (notably the higher-speed impacts in dry sand). The bow wave is a density discontinuity moving with the projestile, rasembling she detached shock wave ahead of a supersonic aircraft. The X-ray method was emphasized because it was the only method known that could give transient information about separation and about the shock wave shape and density gradients. Other types of sensors envisioned for use in the test program were investigated to complement the $X$-ray technique or to be used in case the $X$-ray equipment was not available.

Some of the sensing methods investigated at the University were microwaves, breaking wires, and magnetic sensors. The microwave technique was considered as alternative to the $X$-rays for continuous position monitoring, but it was not used in the experiments at Egin, since the X-ray enuipment was available. Various breaking-wire sensors and velocity screens
were used at Eglin, and the magnetic sensing mettod was used extensively both in the primary :est program and in the preliminary testing before the X-ray syatem was fuily developed. A pressure transducer in the floor of the test chamber and strain gages on the walls were also used in atrempting to build a completis data base.

The generai set-up for the primary test program at Egiln and the flash X-ray method are described in paragraph $2 . j$ after a brief review in patagraph 2.2 of some previous terradynainic experiments. Jther senscrs used in or examined for the test progran art discussed in paragraph 2.4.

### 2.2 BACKGROUND

Until fajxly recently the only experimental data availalle on ballistic penetration of soils consisted of tabulations of striking velocity $V_{0}$ versus final penetration distance $S$. Comparisons of the plots of $S$ versus $V_{0}$ wish integration of assumed force laws, e.g., of the form

$$
-d V / d t=c V^{2}+B V+Y
$$

could in principle determine the coefficients for such laws. The scatter in the data because of variations of in altu soil properties or because of tumbling or other unstable projectile behavior made conclusions from $S$ versus $V_{0}$ data difficult to draw.

In 1957 Allen, Mayfield, and Moriison (Reference 14) reported what were apparently the first laboratory investigations to record projectile transient motion. They used a photographic-electronic chroncgraph to record the successive breaking of copper grid wires located 0.1 meter apart along the trajectory and riere able to obtain better determination of force law coefficients than could be obtained from final penetration depths alone.

This brief discussion will not attempt a complete historical account of penetration experiments, but will mention a few of the more recent investigations that have obtained transient data. Some additional historical information is given in References 1 and 14 through 16 and in a 1972 survey $f$ the state of the art by McNeill (Reference 17), which also gives a bibliography. A more extensive bibliography has been prepared by Triandafilidis (Reference 1), and a 1974 annotated bibliography (Reference 18) lists Sandia Laboratories Publications related to Terradynamics.

According to McNeill, significant strides in penetrator system technology began at Sandia in 1961 with penetrators 2.4 to 3 meters in length and 0.23 to 0.46 meter 11 i diameter, with masses on the order of 450 kilograms, delivered by ground-launched rockets or by alrplanes. Sume of these testn used on-board accelerometers and telemetered data. Since that time, the accelerometer-carrying alr-dropped penetrometer has been developed into a practical tool cor rapid survey of subsurface soil properties. Wood (Reference 19) has discussed instrumentation and telemetry. Irailing wires have also been used for air gun projectiles at speeds up to $120 \mathrm{~m} / \mathrm{sec}$ (Reference 19), Murff and Coyle (Reference 20) have obtained deceleration-time records for impact at speeds up to $90 \mathrm{~m} / \mathrm{sec}$ into three soil types (compacted kaolir clay, dense ottawa sand and a mixture of kaolin clay and sand). Projectiles varied from 38 to 76 mililmeters in diameter and had masses ranging from 1.4 to 52 kilograms.

A microwave monitoring gystem was developed at the Univarsity of New Mexicu. Its use vas reported in a Ph.D diasertation in 1965 by Hakala (Reference 15). The technique showed considerable promiae, although questions of how long a path could be monitored and whether che technique could be used in moist soll were not addressed.

Successful use of flash radiogrephy in soill beginning in 1974 was reported by Cuip et al (References 21, 22). Their later woric in clay showed the existence of detached shock wave. Color enhancement techniquer of the $X$-rays revealed density variations. An automatic scannirg and iaage storing and processing technique was used. One significant resuit of the scanning technique was the discovery that the soil cavity around the projectile seemed to be larger than it had appeared in visual inspection of the radiographs. Flash radiography in soil had beer used earlier (Reference 23), but few details about it have been made public.

Although transient trajectory measurements were not wade, the share of the trajectory was revealed by post-test excayation in a 1973 Master's Thesis by Biele (Reference 24), which investigated the atability of scaled model projectiles of various nose types. Initial angles of impact were revesled by yaw cards, and plots of lateral deflection versus penetration distance were made for various initial angles. Even with quite small inicial angles ( $1-2$ degrees) lateral deflections of as much as 0.15 meter were observed in a peactration distance of $i .06$ meters.

### 2.3 SETUP OF PENETRATION EXPERIMENTS AT EGLIN

The test setup used in collecting the data base for analysis was developed in an evolutionary manner. Several sensing devices, projectile shapes, and velocity regimes were studied before the basic elements of the primary test matrix were investigated. Details of these techniques ara given elsewhere in this section and principal attention is focused upon the test assemblage as used in the March and April 1975 test program. The primary test matrix is shown below with the complete matrix described in paragraph 3.2.

TABLE 1. EXPERIMENTAL TEST MATRIX SHOT NUMHERS

| Projectile | Target | Velocity |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :---: |
| Type |  | $210 \mathrm{~m} / \mathrm{sec}$ | $320 \mathrm{~m} / \mathrm{sec}$ | $400 \mathrm{~m} / \mathrm{sec}$ |  |
| Flat Nose | Dry Sand | $15,16,17,18,19$ | $20,22,23,24$ | $14,25,26,27,29$ |  |
| Flat Nose | Wet Sand | $70,71,72,73$ | $36,37,38,74,84$ | $76,82,83,84$ |  |
| Step Tier | Dry Sand | $52,53,54,55,57$ | $56,58,59,61$ | $62,63,64,65$ |  |
| Step Tier | Wet Sand | $42,43,44,45$ | $39,40,41,49$ | $50,51,68,69$ |  |

The flat-nose projectiles used in thest experiments were solid cylinders 0.0198 meter in diameter by 0.225 meter long. For the stepcior projectiles the afterbody was a cylinder of 0.0198 mater diameter amd 0.232 meter length, with a cylindrical nose 0.0095 meter in diameter and 0.0065 meter long. The material used for the projectile; wee a high carbon concent stesi drill rod, cupplied in rod form wirh nominal dimensions of 0.02 meter in diameter and $l$ meter long. For specimens made at the University an AISI-Wl water quenched bar atock was used, while for specimens fabricated at the AFATL, AISI-01 oil quenched bar stock was used. Three of the profectiles used in the Eglin penetration experimenta are shown in Figure 1. In addition to the two projectiles described above, the photograph shows a shorter flat-nosed projectile, length 0.152 meter, used for some later tests.

In the Eglin experiments the projectiles were fired horizontally into a test chamber consisting of an open-top box of nominal dimensions 0.15 meter wide by 0.40 metar high and 1.2 meters long. The side walla and floos of the box were made of 0.0023 meter aluminum sheet framed by steel brackets and mounted on a flat wooden table platform as show. in figure 2. The ends of the test chamber were closed by fiber board that was easily penetrated by the projectiles. The test chamber was backed up by a large open-topped wooden box fitted with vertical slots to accommodate partitions. The partitions were used to fill the box with varying amounts of sand in order to contain the profectiles fnr re-use in the virious velocity regimes tested.

The buxes were filled with Eglin sand that had been sieved with a U.S. Standard Sieve Series No. 25 sieve to remove large debris, but not sieved to a controlled size range. For the dry sand tests the sand was poured slowly into the test chamber from a bucket assembly attached to an overhead crane. The wet sand tests were for the fully asturated condition. For the wet sand tests the sand was first mixed with water in a container and then shoveled into the test chamber. It was maintained in a fully saturated condition by adjusting a flow of water into the open top to compensate for leakage and maintain an essentially constant water level.

Standard triaxial tests were performed on two samples of the Eglin sand. Fcr these tests the sand was first carefully dried following procedures as described in Reference 25. Each sample was tested at three different constant values of the lateral confining pressure $\sigma_{3}$ ( 0.1962 , 0.392 , and 0.589 MPa ) with axial compressive stress $\sigma$ increasing until failure occurred (significant increase of axial strain at constant load). The two samples were a loose sand and one compacted by vibration before testing. Table 2 lists the initial density $\rho_{\rho}$ and the angle of friction $\phi$ determined for each sample by analysis of the triaxial data as well as the value $\left(\sigma_{1}-\sigma_{3}\right)_{f}$ of the stress difference at failure for each of the confining pressures.

The curve of $\sigma_{1}-\sigma_{3}$ versus axial strain $\varepsilon_{1}$ for the loose sand at the highest confining presgure will be given in Section $V$, where it is used to determine the deviatoric properties for the penetralion analysis by the spherical cavity expansiun theory method. Several confined uniaxial strain tests were also performed or dry Eglin sand. These will discussed in Sections $V$ and VII.

$$
i
$$



| Loose Sand | $\sigma_{3}$ | $\left(\overline{\left.\sigma_{1}-\sigma_{3}\right)_{f}}\right.$ |
| :---: | :--- | :--- |
| $f_{0}=1519 \mathrm{~kg} / \mathrm{m}^{3}$ | 0.1962 MPa | 0.538 MPa |
| $\phi=33.4^{\circ}$ | 0.392 | 0.983 |
| Compacted Sand | 0.589 | 1.447 |
| $\rho_{0}=1698 \mathrm{~kg} / \mathrm{m}^{3}$ |  |  |
| $\phi=39.7^{\circ}$ | 0.1962 | 0.763 |
| 0.392 | 1.423 |  |

The projectiles were fired into the test chamber with a 20 um gun. Firing velocity was controlled by varying the powder load in a primed 20 mm case. Striking velocity was measured by timing the interval between the breaking of two paper back velocity screens wich a Terminal Ballistics Data Acquisition System. The start screen was 1.22 meters fror the front of the target, and the stop screen was 0.61 meter from the target. The timing signals were also recorded on a magnetic tape system and later transferred to a pape: oscillograph record. Also recorded on the tape was the signal from a break wire on the gun muzzle 3.54 meters from the target.

To monitor the projectile filght path through the sand in the 1.2-meter-iong box, five Hewlett-Packard flash $X$-ray units were used: one 150 KV soft $X$-ray unit and four 300 KV hard $X$-ray units. They were spaced sequentially along the horizontal, with the first unit ( 150 KV ) located at 0.038 meter from the front of the box and the four 300 KV units spaced 0.38 meter between centers. Standoff distance for the 300 KV units was nominally 0.55 meter. The front ends of the i50 KV unit (small cylinder) and of one 300 KV unit (large cylinder) are visible in figure 2.

Several types of $X$-ray film were evaluated in the course of the test program, with the majority of the data recorded on Dupont Lightning Plus X-iay film. The film cassettes (not shown in Figure 2) were mounted on the outside of the box opposite from the $X$-ray units. The film plane was positioned at 0.08 meter from the centerline. In the main test program a sertes of metal letters (A throupir $Q$ ) were taped along the box, separm ated horizontally by approximately 0.07 meter along a line 0.20 meter from the top of the box, to serve as markers for locating the projectile position in the $X$-ray pictures. Some of the earlier tests, used fewer markers in the form of metal arrows. In some of the tests displacement of the soil medium was observed by suspending 0.0015 -meter steel markers that moved with the sand. Preshot and postshot $X$-ray records were made to locate the initial and final positions of the markers.

Figure 3 is a photograph of the three pilnts made from the X-ray negatives for Shet No. 26. The two pancls on the left show the nose of the fiat-ended projectile in four successive praitions at the times of the sequential firing of the $X$-rays. The fifth one in the third panel does not show up well in the reproduction but could be seell in the negative. The separation angles and the cavity around the afterbody are clearly shown in three of the positions illustrated. The aft end of the projectile is not usually visible, gince it is out of the main $X$-ray beam when the firing is correcty timed to show the nose.

A magnetic system was used to furnish supplementary velocity information after freliminary laboratory investigations at the University had estabilshed the feasibility of the method. The steel projzctiles were magnetized to a strength of about 150 gauss, as weasured at the center of the nose with a Hall-effect gaussmeter. When this magnetized projectile passed through a 0.15 -meter-diameter coil mour :ed inside the test chamber a voltage signal was generated. Four such colls , ere used in most of the Eglin tests, one on the front of the box and three inside at distances of approximately $0.49,0.80$ and 1.09 meters from the front of the box. The positiona are recorded in Appendix A for the 26 shots for which the magnetic sensor data were analyzed. The projectile can be seen passing through two of them in the X-ray picture of Figure 3. Each sensing coil as formed with 40 turns of copper wire, forming a rim about 0.004 meter thi. The voltage signals were recorded without any preamplification on the magi cic tape recording system and later transcribed to an oacillograph record. re records indicated voltage peaks of the order of 40 to 80 mv in tests witt initiai fmpact velocities around $20 \mathrm{~cm} \mathrm{~m} / \mathrm{gec}$. Some recovered projectiles showed a residual magnetic strangth at the ncse of around 20 percent of the value before firing.

Laboratory tests were made at the University on smaller diameter profectiles fired from an air gun. The time when the projectile nose arrived at the plane of the coil was precisely determined with a light beam. Comparison with the time of the peak voltage output showed that in tests with a co!l formed by two parallel wires 0.025 meter apart the peak voltage occurred precisely as the nose passed through the loop. With a 0.165 -meter-diameter coil a discrepancy was noted, indicating that the nose of the 0.215 -weter-iong by 0.0095 -meter-diameter profectile has advanced approximately 0.021 to 0.027 meter beyond the coll plane when the max!mum voltage was observed for the iow speed shots in air ( 20 to $35 \mathrm{~m} / \mathrm{sec}$ ). Since a comparable direct check could not conveniently be made with the larger flameter projectiles used in the Eglin experiments, an indirect check was made by statically mapping the radiai component of the magnetic field.

The mapping was first made for the laboratory projectiles to see if it agreed with the laboratory dynamic measu ments. At a radial distance of 0.09 meter from the projectile axis the peak radial magnetic field occurred at a distance of 0.021 meter back of the plane of the nose, in approximate agreement with the discrepancies noted above. Similar mappings of one flat-nosed and one step-tier solid projectile or the type used in the Eglin experiments showed that the maximum radial component of the magnetic field occurred at distances of 0.020 meter and 0.02 ? meter respectively, back of the nose tip planr when measured at a radial distance of 0.086 merer from the projectile axis. This indicates that the maximum response ghould occur when the projectiles have penetrated some 0.02 meter through the plane of the sensor coil,

Figure 3. Photograph of Three X-Ray Prints for Shot No. 26
assuming a circular undeformed cuil and a straight horizontal filght path through the center of che coil. Possible cources of additional error are imprecise measurement of the coil locations and especially of the time of the peak response, since with coils this large the response curve does not show a very sharp peak.

### 2.4 OTHER SENSORS IISED OR TESTED

### 2.4.1 Breaking-Wire Sensors

Both wire-grid and coated paper or plastic velocity screens are widely used to time the airborne part of a ballistic tast. They have also been used buried in soil targets or sandwiched between slabs of rock or concrete. Because it was believed that the standard wire-grid screens might disturb the deformation patterns and force flelds in the target, an attempt was made to develop wire gensors that would interfere less, by ueing finer wires in parallel arrays, less closely apaced than the screens. A developmental investigation at the University tested single wires impacted by projectiles fired from an air gun. Of particular interest was s method of verifying how much lateral motion of the wires occurred before they broke to give a signal. The 0.0095 -meter-diameter projectile was 0.15 meter long. Two pinholes in the air gun barrel near the muzzle transmitted light to a photomultiplier timing system for measuring projectile velocity. The dual-beam oscilloscofe was triggered when the aft end of the projectile passed the first pinhole (farther from the muzzle). The wire sensor was placed 0.15 meter from the second pinnole, so that the projectile nose impacted the first wire just as the aft end passed the second pinhole. The time difference between the two signals (from the second pinhole and from the oreaking wire) determined the cime delay (or advance) of the breaking-wire signal. With the known projectile velocity, the position error that would be caused by assuming that the wire brcke instancaneously in its orginal position could be determined.

Several kinds of wire were tested. The wires were stretched between supports 0.15 meter apart. The first tes:s were performed in air. Ductile wires of copper and stainless ateel stretched so much that the projectile traveled almost 0.05 meter before the wire broke in impacts at $32 \mathrm{~m} / \mathrm{sec}$. Brittle wires gave better results. A brictle 0.0001-meter-diameter tungsten wire broke after abouc 0.0025 meter of travel.

Tests were then performed with the stretched tungsten wires buried in sand. The wires broke before the projectile reached them, because of the sand pushed ahead of the flat-ended projectile. At $39 \mathrm{~m} / \mathrm{sec}$ the distance was about 0.006 meter and at $65 \mathrm{~m} / \mathrm{sec}$ about 0.009 meter from the projectile to the initial position of the wire when it broke. In all cases the breaking wire gave a good sharp step on the oscilloscope trace. The last group of testa used 0.0002 -meter-diameter steel music wire (static breaking force 89 N as compared to 17.8 N for the tungsten wire). In these last tests the wire sometmes did not break, but was deflected to one side of the projectile. A small perturbation in the voltage trace was noted at about the time the projectile reached the wire's initial position. Such a perturbation may be usable for timing purposes, although it lacks the sharp step that occuts when the wire breaks.

Two of the standard wire grid velocity screens were checked in the test apparatus at the University. With a 12.7 modiameter projectile impacting the screen in air at a speed $25.4 \mathrm{~m} / \mathrm{sec}$, the acreen bent to allow about 0.0025 meter of travel before it broke. When a ainilar test was performed in sand, the travel appeared to be about 0.0075 meter before the break.

The figures quoted for distances frow the initial wire pcsition to the projectile position when the break occurred apply, of course, only for t.ee specific projectiles, wires and configurations tested. Sensors wlll have to be calibrated in conditions similar to those they are to be used in.

The first test firings at Eglin on 6 June 1975 were planned to test breaking-wire sensors and capacitor sensors prepared at Eglin. Tungsten wires and steel music wires of the types previously tested at the University were strung between supports 0.76 meter apart, and in addition two atandard wire grid velocity screens were placed in the sand near the front end of the target. Signals were to be recorded both by counters and on magnetic tape recorders. No signals were obtained from any of the breaking-wire sensors. Post-test checks showed that the velocity screens were broken but the two other wire sensors were not broken by the 0.50 caliber projectiles. similar wire systems could be used in sand, especially with the larger ZOmm projectiles, but it wouid be necessary to check them out carefully with each projectile and test configuration. The X-ray nethod could be used as a check. Little further use of breaking-wire sensors in sand was made in the Eglin experiments because the magnetic sensors were so much better, and later the X-ray method gave ailll better resulta.

### 2.4.2 Sensors Responding to Pressure or Deformation: Capacitors, Pressure Transducers, and Strain Gages

Although the mijor effort in thr experimental progran was directed coward recording erajectory information and the bow wave formation and cav!ty formation, several types of sensors were tried that could give some additional information about arrival cimes and intensities of the stress and deformation waves in the target medium.

A capacitive transducer was developed at AFATL, consisting of two thin metal foils separated by a layer of foam rubber and encased in a flexible electrical insulating matericl. When the sensor was compressed along with the surrounding sand a voltage change occurred across the charged capacitor. This furnished timing information about the arrival of the pressure wave. With suitable calibration it could also furnish quarititative information about pressure and deformation. It also served as a good antenna for detecting and recording the actual firing times of the flash X-rays.

A pressure cell in the bottom of the test chamber 0.127 meter from the front of the box also gave information about the arrival tiae and intensity of the pressure waves and furnished a good signal.

One to five strain gage! were also mounted on the aluminum plates at the sides of the box. Good strong aignals were obtcined from the gages. The interpretation of these aignals depends on the interaction between the pressure wave in the sand and a flexural wave in the plate.

### 2.4.3 Microwaves

A microwave monitoring system was reported on in 1965 ty Hakala (Reference 15). Its oferation gives output depending on the interference between a transmitted signal and a signal reflected from the moving projectile. The transmitter and receiver were at the opposite end of the sand target from the impact point. The interference frequency is a function of the projectile velocity. Since few details about power requiraments for penetrafing various distances in dry and molst sand were avallatile, an experimental program : 0 determine some of this informatin' was undertaken at the University early in 1975.

A micrawave oscillator of maximum power 1 mw fed a variatie gain amplifier at 2 frequency of about 10 Gizz through a coupler in a microwave horn into the sand contained in a 1.2 -mettriong box. FCc these static experiments the algnal generator carrier frequency was modulated by a l kz square wave. The signal was reflected from the target. which was the end of s metal rod inserted into the opposite end of the box from the horn. Portions oi the mixed incident and reflected signals were detected by a crystal detector. The detector output (DC with amplitude varying at the modulation frequency) was fed to a Standing. Wave Ratio Meter (which contained an internal amplifier with a narrow pass bani around 1 kHz ).

When the rod was moved axially by one quarter wave length the round trip path from coupler to rod was shortened by one half wave length. The reflected and incident waves interfered and a half wave length reduciion in path was required for the detected mixed signal to go from a maximum to a minimum.

Preliminary tests shoved a atrong signal response at a distance of 0.30 meter with 3 ww power output from the microwave amplifier. At 0.60 meter the difference between the maximum and minimum response was down about 5 dB from the difference at 0.30 meter indicating a power transmission drop by about a factor of one-third, but the aignal vas atill clearly distinguishable. In fact it was still clear at 1.0 meter. Precise attenuation factors could not be obtained with the preliminary tect set-up, because of reflections from the sides of the box containing the aand.

Additional microwave studies attempted to repeat with naturally moist Eglin sand (approximately 5 percent by weight moisturecontent) the kind of measurements previously made in dry sand. With the target at a distance of only 0.15 meter howerer, the atcenuation was so great that the alternate constructive and destructive interference by the reflected ignal as the target advanced a quarter wave length was barely perceptible.

Some additional measurements with a tuned microwave horn pickup replacing the target were made verifying that detectable microwave signals were transwitted through 0.30 meter of the moist sand even with the low power signal source.

It may be possible to increase the transmitted power and to obtain a coupler that will pick up a smaller fraction of the transmittel signal to mix with the reflected signal from the target in order to enhance the interference.

For projectile velocity monitoring, the carrier wava woald not be modulated by the 1 kHz square wave. A projectile advancing at constant speod at $300 \mathrm{~m} / \mathrm{sec}$ will produce an interference frequency of approximately 20 kHz without any modslacion of the original aignal. This frequency will decrease as tise projectile slows. The amplified algal could be recorded boch on an oscilloscope and on one or two channeis of Bioaation trensient recorder. It would be recorded as a quasi-sinusoidal ignal of decresing frequency. The time between waximum and aminimur chis aignol is the time for the projectilp to advance one quarter vave length (of the order of 0.0075 me:er although precise values would have to be estabilished by celibiacion). At a projectile valocity of $300 \mathrm{~m} / \mathrm{sec}$ the time between maxtmum and minimum is about $25 \times 10^{-6} \mathrm{sec}$. increasing as the projectile slows.

The microwave system was not actually used in the Egin experiments since the $X$-ray equipment was available, but it is a poesible opiton for future use if the power reqiirements can be met.

In Section III the data collected in the Egiln experiaents will be described.

## SECTION III

## RESULTS OF EGLIN PENETRATION EXPERIMENTS

### 3.1 INTRODUCTION

During the period from 22 January to 24 May 1976 the Eglin penetration experiments included atotal of 91 shots in 17 misbions. X-ray data from two or more stations were obtained in 74 shots (No's 14 to 91 except for Shots 21, 28, 60, and 75). Appendix A lists dats obtained in Shots 14 through 91, except for the four shots for which $X$-ray data were not obtained. One page is used for each ahot, and they are listed in order. A description of the various kinda of data in Appendix $A$, both the experimentall $;$ measured data and meveral kinds of information calculated in the data analyais, is given in paragraph 3.3 after an cuerview of the primary and secondary test programs in paragraph 3.2.

### 3.2 TEST PROGRAM MATRICES

### 3.2.1 Primary Test Program Matrix

The primary test program at Eglin was planned to test two projectile configurations at three impact speeds and two target moisture conditions (dry Eglin sand and saturated Eglin sand). With four replications of each cest the plan called for 48 shots. Four extra replications brought the total to 52 shots as summarized in Table 3. The two projectile configurations were both solid cylinders of nominsl diameter 0.0198 meter, one with $n$ flat nose and the other with a step-tier nose, as described in paragraph 2.3. The wet and was fully satusated. The letters after the shot numbers indicate that special analysis was made of those shots. The letter $V$ indicates that veloc'.ty data from the $X$-rays were fitted to a Poncelet force law as described in paragraph 4.4, with information about the fitting tabulated in Appendix A. The letter $M$ indicates that the velocity results were compared with the magnetic sensor data obtained with the velocity coils, with results of the comparison listed at the end of each tabulation in Appendix A. The letter B indicates that bow wave was observed in fronc of the projectile in one or more of the X-rays (see paragraph 3.5.1).

### 3.2.2 Secondary Test Program Matrix

Shot numbers not included in the primary t:est progran are listed in Table 4.

### 3.3 DESCRIPTION OF EXPERIMENTAL DATA

For each of the 74 shota ileted in Appendix A the position and cavity separation angle information obtained from the $X$-ray records is given in the first data group. An example is shown in Table 5.
table 3. Shot numbers of experimental matrix for primary test program


TABLE 4. SHOT NUMBERS OF EXPERIMENTAL MATRIX OF SECONDARY TEST PROGRAM

| $\begin{gathered} \text { Projectile } \\ \text { Type } \\ \hline \end{gathered}$ | Target Medium | Impact Velocity Ranges Shot Numbers |  |
| :---: | :---: | :---: | :---: |
| Solid Biconic | Dry <br> Sand | $\begin{gathered} 320 \mathrm{~m} / \mathrm{sec} \\ 30 \mathrm{~V}, 31 \mathrm{~V}, 32 \mathrm{~V} \end{gathered}$ |  |
| Hollow Biconic | Dry <br> Sand | $\begin{gathered} 350 \mathrm{~m} / \mathrm{sec} \\ 35 \mathrm{~V} \end{gathered}$ | $\begin{gathered} 400 \mathrm{~m} / \mathrm{sec} \\ 3 \mathrm{~s}, 34 \end{gathered}$ |
| Solid Plat Nose 0.152 meter long | Dry <br> Sand |  | $\begin{aligned} & 1 \mathrm{sec} \\ & 9 \mathrm{VM} \end{aligned}$ |
| Hollow Step-Tier | Dry <br> Sand | $\begin{aligned} & 230 \mathrm{~m} / \mathrm{sec} \\ & 88,89 \mathrm{~V} \end{aligned}$ |  |
| $\begin{aligned} & \text { Hollow } \\ & \text { Step-Tier } \end{aligned}$ | Wet <br> Sand | $\begin{gathered} 440 \mathrm{~m} / \text { sec } \\ 90 \end{gathered}$ |  |
| Hollow Step-Tier | Water | $\begin{gathered} 230 \mathrm{~m} / \mathrm{sec} \\ 85,86 \mathrm{VM}, 87 \mathrm{VM}, 91 \mathrm{~V} \end{gathered}$ |  |
| Special <br> Model | Wet Sand | $\begin{gathered} 250 \mathrm{~m} / \mathrm{sec} \\ 46 \\ \hline \end{gathered}$ | $\begin{gathered} 550 \text { m/ sec } \\ 47,48 \\ \hline \end{gathered}$ |
|  | Dry Sand |  | 66, 67 |
| (V indicates that calculated velocities of nose and center of gravity are listed in Appendix $A$; $M$ indicates that the tabulation also includes comparison with the magnetic sensor data.) |  |  |  |

TARLE S. EXAMPLE OF FIRST DATA GROUP

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The first line below the shot number and dare givas the target medium: dry sand, wet sand or $\mathrm{H}-\mathrm{OH}$ (meaning $\mathrm{H}_{2} \mathrm{O}$ fni shots into water), the density in $\mathrm{kg} / \mathrm{m}^{3}$. the projectile's approaching velocity in meters per second as measured by the counter start and stop velocify screens as described in paragraph 2.3 and/or by the time from the break-wire on the gun muzzle to the X-iey trigger foil switch located on the froat of the box as resorded on the oscillograph strip chart. The second line gives the projectile type, mass in sg , nominal diameter and length in metera. Below each $X$-ray station number is listed the time in seconds from the foil switch trigger to the firing of that $X$-ray. The firing times were determined from the delay settings and also by noting aignals appearing on the strip chart records for various sensiors. The next two lines give the calculated center of gravity position cnordinates in meters measured from the front and bottom of the target box. Note that the first horizontal coordinate is negative, since the projectile is still outside the box.

The projectile's angle of incluation to the horizontal and the cavity separation angies at the nose, measured with respect co the projectile axis, are listed in the next two ifnes of data. A tow of ascerisks (w***) indicates missing lata. These angles were neasured on the $X$-ray negatives, as was the measurad nose width in meters. The next two lines are calculated sose position coordinates, corrected for $X$-iay bean divergence. The lase two lines give the raw data on the nose position in the X-ray picture, with the vertical position masured from the row of letter markers on the wall of the box, so that a negative coordinate indicates aistance below the letters.

For 22 shots (those not marked with a $V$ or $M$ in the experimental matrices of paragraph 3.2, no further data are tabulated in Appendix A. The other 52 shots have additional groups of calculated data to be discussed in paragraph 4.3, and the 26 marked with an $M$ in the experimental matrices of paragraph 3.2 include magnetic aensor data in the last data group, as ohown in Table 6.

TABLE 6. EXAMPLE OF LAST DATA GROUP (SHOT NO. 26)


The first line of data in this last data group (see Table 6) lists the time in seconds from the X-ray trigger coil switch time to the maximum voltage from the four magnetic sensor velocity coils. The second ine lists the times of the minimum voltage at each of the coils. These times were transcribed from the strip chart. The next two lines are computed nose positions, as will be described in paragraph 4.4. The next line lists the actual positions of the four coils as recorded in the log book, and the last two lines record the differences between the two computed nose positions and the actual positions. The significance of these differences will be discussed in paragraph 4.4.

## 34 PRELIMINARY ANALYSIS OF TABULATED DATA

### 3.4.1 Nose Positions

Nose positions as measured on the $X$-ray photos were recorded as INPUT NOSE POSITION in the first data group of the tabulations of Appendix A (see Table 5) and also the apparent nose width. This apparent nose width as compared to the known actual nose width provides a first-order correction for the divergence of the X-ray beams. A simple compurer program, based on similar triengles with apex at the X-ray source, was used to correct all apparent horizontal and vertical distances in proportion to the known correction for nose width. The corrected nose positions are tabulated imediately above the raw data input nose positions.

### 3.4.2 Center of Gravity Position

The center of gravity position was cslculated from the corrected nose position and the (uncorrected) inclination angle by using the known distance from the nose of the projectile to its center of gravity. This correction ild not account for projectile yaw. Yaw was belleved negligible because of the straightness and lateral atability of the trajectory. Further data analysis is given in Section IV.

### 3.5 DATA NOT ANALYZED

### 3.5.1 Bow Wayes

Several of the X-rays showed detached shock wave ahead of the projectile, revealed by a density discontinuity. inis occurred notably in the higher apeed impacta in dry sand. In the primary test matrix of Table 3 shot numbers marked with a B showed well defined shock waves. Thus the flat-nosed projectiles showed shock waves in the intermediate velocity range also.

The bow shock wave appeared an a roughly parabolic curve (almost a circular arc near the vertex) with vertex at a distance ahead of the projectile nose of the order of magnitude of the projectile diameter. The X-ray plctuxes have been retained for possible use in future theoretical analysis of the deformation. Figure 4 shows tracings of two of the bow waves ahead of the projectile nose in two positions in Shot 26. The distance between the two positions is not to scalr; in the figure, but the position of each bow wave is shown relative to the nose.


Figure 4. Tracing of Bow Wave for Shot No. 26

Similar shock waves were also observed in some of the preliminary shots with 0.50 caliber projectiles.

### 3.5.2 Separation Angles

In some shots the separation angles above and below appearad to be approximately symmertic with respect to the nose velocity vector, which was alightly different from the projectile heading as given by the recorded inclination angle. In almost all of the cases recorded in the primary test program only the nose was in contact with the sand. In future analyses it may be possible to relate the separation angles to the shape of the false nose of and formed in front of the flat-nose projectiles and/or to the lift forces exerted on the nose.

### 3.5.3 Marker Movements

The preshot and postshot X-rays showing movement of the small steel markers have not been analyzed. This information may be useful for evaluating future theoretical analyses of target medium deformation.

### 3.5.4 Pressure and Strain-Gage Keasurements

The pressure transducer atrip chart records msy furnish useful data to compare with the observed bow waves and/or with future theoretical analyses of stress and deformation wave propagetion in the target. The strain gage measurements on the aluminum test chamber walls were also recorded on the atrip chart. These strain pulses are also related to the pressure wave in the sand, but are strongly influenced by the response of the aluminum piate to a traveling and varying dynamic load.

### 4.1 INTRODUCTION

Trajectory plots for the 52 shots of the primary teat program are given in paragraph 4.2. Since these trajectories are very nearly straight and horizontal throughout the 1.2 -meters-long region of observation, analysis by one-dimensional penetration models is feasible. In paragraph 4.3.1 computer plots of horizontal positon versus time and of velocity versus position ace given for 21 of the shots. These were obtained by first fitting a cubic polynominal intexpolation formula to the position data and then fitting a Poncelet force law co each shot, as described in paragraph 4.3.2, which also contafns a comparison of the values obtained for the Poncelet drag cocfficients of the 41 shots of the primary marrix that have been analyzed by this method. Resulta of magnetic sensing are compared with the $X$-ray dara in paragraph 4.4. This data analysis was performed at the University.

Results of a different method of deterwing the Poncelet coefficient for each shot and also results of empirical analysis by methods similar to those developcd at Sandia Laboratories (References 12,13) are given in paragraph 4.5. Variation of the drag coefficient within a ahot is discussed in paragraph 4.6 by considering separately different segments of several trajectories. These last two data analyses were performed at the AFATL.

### 4.2 TRAJECTORIES OF PRIMARY TEST PROGRAM

Computer-plotted trajectories for the $5^{\circ}$ shots of the primary test program matrix listed in Table 3 are shown in Figures 5 through 10 based on the $x$-ray data for the positions and inclination angles. In each plot the circles mark center of gravity positions and the other end of the ine from the circle is the nose position. Shot number is shown at the left end of each trajectory. The horizontal and vertical scales are the same, but each successively numbered trajectory in a figure is plotted displaced upward one square ( 12.5 cm ) from the preceding one. The plots give a pictorial sumary of the trajectory data. Precise positions are given in the tabulations of Appendix A.

The wost remarkable feature of the trajectories is their straightness, following in most cases a nearly horizontal straight line through the 1.2 -meters-long target box. All but one of trajectories have a slight upward trend. The greatest rise, 6.2 cm , occurred for Shot 19 in Figure 5(a). Shots 16 to 19 of this group for the solid flat-nose projectile impacting dry and at about $210 \mathrm{~m} / \mathrm{sec}$ all show a continuously increasing angle of inclination, reaching 16.5 degrees in Shot 19. This was the largest inclination angle recorded. Positive Einal inclination angles were recorded for 31 shots, negative for 19 shors and zero for one, Shot 76 in Figure $7(b)$. Tn dry sand the flat-nose projectiles showed 9 positive and 4 negative final inclination augles while the atep-ifer


(b) Wet Sand

Figure 5. Trajectorien of Sulid Flat-Nosed Projectiles for Impact Velocities in the 210 m/aec Range

(a) Dry Sand

(b) Wet Sand

Figure 6. Trajectories of Solid Plat-Nosed Projectiles for Impact Velocities in the $320 \mathrm{~m} / \mathrm{sec}$ large


Figure 7. Trajectories of Solid Flat-Nosed Projectiles for Impact Velocities in the $400 \mathrm{~m} / \mathrm{sec}$ Range

(a) Dry Sand

(b) Het Sand

Figure 8. Trajectories of Solid Step-Tier Projectiles for Impact Velocities in the $210 \mathrm{~m} / \mathrm{sec}$ Range


Yigure 9. Trajectories oí Solid Step-Tier Projectilen for Impact Velocities in the $320 \mathrm{~m} / \mathrm{sec}$ Range

(a) Dry Sand

(b) Wet Sand

Pigure 10. Trajectories of Solid Stef-Tier Projectiles for Impact: Velocities in the $400 \mathrm{~m} / \mathrm{sec}$ Bange
projectiles showed 5 positive and 4 negative. In astursted sand the fiatnoses ohcred 11 positive and 2 negative final inclination angles while the step-tier noscs ahowed 7 positive and 5 negative. The iargest negative argle was -14.5 degrees in Shot 53 of Pigure $8(a)$. This was also the only trajectory that did not rise.

Some of the trajectories show a continued rise, even with a negative angle. This is most evident in Shots 27 and 29 of Yigure 7 (a) for the flet nose impacting dry sand al. $400 \mathrm{~m} / \mathrm{sec}$. The $X$-ray pictures show that during the part of the trajectory observed only the flat nose was in concact with the sand, so that no forces were acting on the afterbody urface.

Since the trajectories are so straight in the region of observation, analysis by one-dimensional penetration models is reasonable, for example the Poncelet force law to be discussed in paragraphs 4.3 and 4.5 and the Cavity Expansion Theory penetration model $1_{4}$ Section V.

## 4. 2 AHALYSIS AND DISCUSSION OF TABUTATED POSITION-TIME RESULTS BASED ON X-RAY DATA

### 4.3.1 Cubic Interpolation, X, t-plocs and V, x-plocs

- Velocity analysis has beer carried out for 52 shots for which cotaplete X-ray data ( 5 stations) were available. This inciudes 41 shots from the primary test program and 11 from the secondary program (those marked with a $V$ in Tables 3 nd 4.) In the data reduction at the University of Florida, the velocity analysis whs performed firsc by fit:ing a cubic interpolation formula to che data, and later laproved results for the $x$-component of the center of gravity velocity were obtained by fittiag the data to a formuls derived from the Poncelet force-law penetration model. The Poncelet law fittings will be discussed in paragraph 4.3.2.

The coefficienta of each cubic polynomial are liated in the tabulationg of Appendix A.

For example, for Shot 26 , the tabulated caefficiente for C.G. VELOCITY X-Comp. imply the polynomial

$$
\begin{equation*}
x=-0.1277+393.3 t-46,190 \tau^{2}+3,432,000 t^{3} \tag{1}
\end{equation*}
$$

for $\pi$ in meters and $t$ in second 3 . The soefficients were deterained by a least-squares fit. At the end of each sat uf tobulated coefficients is listed the standard deviation in waters [square root of the sum of the equares of the differences between the measured x-positions and those calculated by the cubic at the times of firing of the $X$-rays]. For Shot 26 the standard deviation is 0.0014 weter for the $x$-component, indicating a very good fit.

Velocities were calculated from the fitced cubics. For example, for Shot 26 the center of gravity $x$-component velocity is given by

$$
\begin{equation*}
V=\frac{d x}{d t}=393.3-92,380 t+10,296,000 t^{2} \mathrm{~m} / \mathrm{sec} \tag{2}
\end{equation*}
$$

Thin meuld give a good approximation to the volocity neaz the ceuter of the interval, but larger errore would be expected at the ende. Computer plote of: ( $n$ ) the calcaleted $x, t$-curve and (b) the calculated $Y_{,} x$-curva for 21 thote of the primary teme progran are ahown in Figurea 11 through
 equares. The solid curve ie the ficted crabsc, and the curve marked with vertical errokes is a cuive baned on the Poncelet force lems to bafiscucesd in paragraph 4.3.2. It if seen that the fitted cuble $x, t$-curvea agree very vell with the experimantal data, The cubic and Poacciet x,t-curves are also cloae to each othar thonugh the whole interral. Their slopes begin to differ at the ends of the intervals of observation. The $\nabla, x-$ plots by the two methods thereîora show considerable differencea at the eads. The cubic interpolation would give completely unreasomable resulte outside the interval of observation ( 0 to 1.2 meters).

### 4.3.2 Onembimasional Analysis of Velocisiea by Fitted Poncelet Force Law

The Poncelet force law (Reference 2) takea the following form, after dividing through by the mass $m$ of the projectile.

$$
\begin{equation*}
-\frac{d y}{d t}=A+g V^{2} \tag{3}
\end{equation*}
$$

where $A$ and $B$ are parameters depending on tha target matarial as well on on m. For the high velocities in the interval. of ohservation in the present program, the contribution of $A$ is negligible, and the $\nabla_{0}$-curves have been fitted by tikirg A equal to cero and detereining a beai fitt for B by a noninaear regression procedure that minimizes the standard deviation from the experimencal data of the $x, t$-curve obtained by integrating Equation (3).

Equation (3) can be integrated explictly for given initial data $\left(v_{0}, x_{0}, t_{0}\right)$ to obtain

$$
\begin{equation*}
v=\left[\left(\frac{A}{B}+v_{0}^{2}\right) e^{-2 B\left(x-x_{0}\right)}-\frac{A}{B}\right]^{1 / 2} \tag{4}
\end{equation*}
$$

or, with $A=0$

$$
\begin{equation*}
V=\nabla_{0} e^{-B\left(x-x_{0}\right)} \tag{5}
\end{equation*}
$$

With $V=d x / d t$ a second integration of Equatioa (5) gives then

$$
\begin{equation*}
x-x_{0}=\frac{1}{B} \ln \left[1+E V_{0}\left(t-t_{0}\right)\right] \tag{6}
\end{equation*}
$$

The more complicated case, with $A \notin 0$, is dimeussed in Section $V$ on the Cevity Expansion Theory.

Equation (6) was fitted to the experimental $x, t$-data. A nonlinsar regression is recuited. The procedure follmwed in this secition was to sake initial conditione $x_{0}, v_{0}$, and $t_{0}$ from the expmrimental data and the cubic





Pigure 14. Velocity Versus Position for Shot No. 19 ( $\nabla_{0}=V_{1}$ )



Figure 16. Velocity Versus Position for Shot No. $20\left(\mathrm{~V}_{\mathrm{O}} \mathrm{m}_{3}\right)$



Figure 18. Velocity Vereus Position for Shot No. $24\left(\boldsymbol{V}_{0}=V_{3}\right)$









Figure 26. Veloaity Versus Position for Shot No. $39\left(\nabla_{0}=\nabla_{1}\right)$



Figure 28. Velocity Versus Position for Shot No。 $45\left(\nabla_{0}-\nabla_{3}\right)$









 Figure 37. Position-Time Plot for Shot No. $59\left(\nabla_{0}=\nabla_{3}\right)$


Figure 38. Velocisy Versus Position for Shot No. $59\left(V_{0}=V_{3}\right)$



Figure 40. Velocity Versun Position for Shot Ho. $61\left(V_{0}=\nabla_{3}\right)$





Pigure 44. Velocity Versus Position for Shot No. $65\left(V_{0}=V_{3}\right)$



Figure 46. Velocity Versus Fosition for Shot No. $68\left(\mathrm{~V}_{0}-\mathrm{V}_{3}\right)$



Figure 48. Velocity Versus Position for Shot No. $72\left(V_{0}=V_{1}\right)$



Figure 50. Valocicy Versus Position for Snot No. $13\left(V_{0}=\nabla_{1}\right)$


interpolation results of paragraph 4.3.1. A different pcocedure will be used in paragraphs 4.5 and 4.6. Measured approach velocisies were avaylable but there was some uncertainty aiout the exact time the nose of the projentile first impacted the target. Positions at the times of the X-ray firings were more precisely known. The mafority of the aralyses ( 33 shots) were therefore made with Station 3 es initial-value point, with $V_{0}=V_{3}$ as given by the cubic interpolation, aince that point ie near the middle of the intervel where differentiation of the cubic interpolation formula shouid be most accurate. Calculared veloctites at Stations 1 and 2 were then obtained for negative values of $t-t_{0}$ and $x-x_{0}$. For 19 shots that procedure led to unreasonably large calculrted values for $V$ at $x=-0.115$ meter where $V$ should be equal to the epproach velocity. For these cases the analysis was therefore made with Station 1 selected as initial point.

In the tabulations of Appendix $A$ the value of $V_{0}$ is tabulated as vo on the line below the tabulated Poncelet drag coefficient. For example, for Shot 26 the value VJ $=267$ is ligted. Comparision with C. G.Vel.Y-Comp. values four lines above this entry shows that in this case $V_{0}{ }^{n} V_{3}$. For the 21 cases plotted in paragraph 4.3.1, the captions include either the statement $V_{0}=V_{1}$ or $V_{0}=V_{3}$, and it is also easy to tell from the $V$, $x$-pluts where chey were made to agree.

All the fitted Foncelet curves (those marked by vertical strokes) now give a reasonable agreement at $x=-11.5 \mathrm{~cm}$ with the meagured approach velocity.

The standard deviation of the x-positions calculated by Equation (6) from the experimental values is tabulated for each case in Appendix $A$ fmmediacely following the tabulated $V_{0}$.

The Poncelet drag coefficient tabulated on the sane line is related to the coefficient $B$ of Equation (1) as follows. In aerodynamics a dimensionless drag copfficient $C_{D}$ is defined such that the drag force on an object of projected area $A_{1}$ on a plane perpendicular to the velocity 1s given by

$$
\begin{equation*}
\text { Inertial Drag Force }=\rho A_{1} C_{D} v^{2} / 2 \tag{7}
\end{equation*}
$$

where $p$ is the denaity of the medium being traversed. Comparison with Equation (3), neglecting $A$, shows

$$
\begin{equation*}
B=\rho A_{1} C_{D} / 2 \mathrm{w} \quad \text { or } C_{D}=2 m B / \rho A_{1} \tag{8}
\end{equation*}
$$

The projectile masa in kilograms and the target sand density in $\mathrm{kg} / \mathrm{m}^{3}$ are tabulated for each shot in Appondix A .

Table 7 lists the Poncelet drag coefficients $C_{D}$ for each of the 41 shots of the primary test program whose velocities were analyzed. In dry sand there is little variation with lmpact velocity of the value of $C_{D}$ required to fit the $X$-ray pusition time data in the region observed. These dry sand reaults are well characterized by the Poncelet force law with a

TABLE 9 . PONCELET/DRAG COEFFICIENTS ChLCULATED FOR PRIMARY TEST PROGRAM

| Projectile Type and Sand Condition | Impact Velocity Range |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\begin{aligned} & 210 \mathrm{~m} / \mathrm{sec} \\ & \text { Shot } C_{D} \end{aligned}$ |  | $320 \mathrm{~m} / \mathrm{sec}$Shot $C_{D}$ |  | 400 m/BecShot $C_{D}$ |  |
| Solld | 17 | 1.64 | 20 | 1.77 | 25 | 1.68 |
| Fiat Noge Dry Sand | 18 | 1.62 | 22 | $2.30{ }^{\text {a }}$ | 26 | 1.65 |
|  | 19 | 1.69 | 23 | 1.72 | 27 | 1.77 |
|  |  |  |  | 1.72 | 29 | 1.71 |
|  | Avg | 1.65 | Avg | 1.74 | Avg | 1.70 |
| Solid | 70 | 1.59 | 37 | 0.95 | 76 | 0.96 |
|  | 71 | $\begin{aligned} & 1.55 \\ & 1.36 \end{aligned}$ | 38 | 0.94 | 82 | 0.73 |
| Flat Nose | 72 |  | 74 | 1.02 | 83 | 0.82 |
| Wet Sand | 73 | 1.23 | 81 | 0.96 | 84 | 0.78 |
|  | Avg | 1.43 | Avg | 0.97 | Avg. | 0.82 |
| Solid | 52 | 1.81 | 56 | 1.78 | 62 | 1.83 |
|  | 54 | 1.90 | 58 | 1.91 | 63 | 1.81 |
| Step-Xier | 55 | $\begin{aligned} & 1.89 \\ & 1.62 \end{aligned}$ | 59 | 1.67 | 64 | 1.92 |
| Dry Sand. | 57 |  | 61 | 1.82 | 65 | $0.54{ }^{\text {a }}$ |
|  | Avg | 1.80 | Avs | 1.80 | Avg | 1.85 |
| Solid | 45 | 1.24 | 39 | 0.72 | 50 | 0.79 |
| Step-Tier |  |  | 49 | 0.82 | 68 | 0.61 |
|  |  |  |  |  | 69 | 0.71 |
| Wet Sand | Avg | 1.24 | Avg | 0.77 | Avg | 0.70 |
|  | (Values marked with* exclucied from average) |  |  |  |  |  |

value of $C_{D}-1.7$ for the flat-nose projectile and $C_{D}-1.8$ for the steptier projectile. In paragraph 4.6, a different method of determining $C_{D}$ is presented, however, which shows up variations in $C_{D}$ along a trajectory.

In the wet sand (fully saturated) there is a downward trend of $C_{D}$ as the impact velocity increases, indicating that the penetration is not well characterized by a Poncelet force law with coefficients incependent of velocity. There is also more scatter in the $C_{D}$ values obtained for some of the velocity regimes in the wet sand.

The saturated sand $C_{D}$ valuea are all lower than any of the dry and values (except for Shot 65, which is belleved to be in error). Shot No. 87 into a water target at $241 \mathrm{~m} / \mathrm{sec}$ was fitted by $C_{D}=0.51$. Apparently the fully saturated sand tends to respond somewhat like a water target.

Magnetic sensor response is compared with X-ray data in the following paragraph.

### 4.4 COMPARISON OF MAGNETIC SENSOR RESULTS WITA X-RAY DATA

Por the 26 shots marked with an $M$ in Tables 3 and 4 the cubic interpolation formula described in paragraph 4.3.1 was used to comoute the nose position of the projectile at the time of maximum and minimum coil voltage from each magnetic sensing coil. These are compared to the coil position in the last data group tabulated from each of these shots in Appendix A. The differences between these two positions (tabulated in two last lines) give the appasent distance back from the nose to where the maximum outward radial compcnent of the projectile's magnetic field cut the sensing coil (or the maximim inward component for the minimum voltage case).

In Shot 26, for example, the distances were $0.024,0.023,0.035,0.042$ back to the apparent maxima. The first two of thege agree quite closely with the laboratory mapping of the magnetic field of the projectile reported in paragraph 2.3, which indicated that the maximum should occur about 0.02 meter back from the nose of the flat-nosed projectile if the magnetized projectile passed through the center of the coll. There was some discrepancy at the last two stations, but an error of 0.02 meter is quite small compared to the position coordinate of 1.076 meters at the last atation. Shot 26 had one of the straightest trajectories.

Larger discrepancles are recorded for several shots. The largest positive difference noted is for Station 2 of Shot 50 where the apparent maximum was 0.063 meter back of the nose as compared with the leborstory measurement of 0.022 meter for a solid step-tier projectile. Since this occurred at Station 2 (position 0.486 meter) it would represent about an 8 percent error in position indication.

For several stations a negative difference was noted. Tha largest was -0.027 meter for Station 3 in Shot 58 , also a solid step-tier projectile, indiccting an apparent maxirnum 0.027 meter ahead of the ncae or 0.049 meter ahead of the maximum position according to the laboratory measurement of the field, leading to a 6 percent position error at Station 3.

In the majority of the cases the apparesit arsorm were smaller than these. No labcra太ory mesaurement was made for the hollow atep-tier projectiles, but differences between computed nose positions and coil poitions for them in Shots 79,86 , and 87 were in the range of the difierences for the solid projectiles.

A possible source of the errors could be off-center projectile pathn, although both Shots 50 and 58 had quite straight trajectories according to the X-ray records. Other possible sources of error are imprecise measureant of coil positions and/or meximum time on the atrip chart and possibly diatortiona of the magnetic field of the projectile caused by the impact.

It is belleved that the X-ray data are generally more precise than the magnetic sensor data. Nevertheless the invegtigation has indicated that the magnetic sensing method can give quite good results, and it is certainily an economical method.

A different procedure for determining the Poncelet drag coefficients w11 be presented in paragraph 4.5, which also considers application of the empirical penetration formula developed at Sandia Labozatories (References 12,13 ).

### 4.5 COMPARISON OF PONCELET AND SANDIA BMPIRICAL FORMULA RESULTS

The Sandia empirical formulas (References 12,13) are rewritten here in SI uniks. Thus, according to Young (Reference 13) the total depth of penerration $D$ is given in terms of the initial impact velocity $V_{0}$ by an equation of the form
or by

$$
\begin{equation*}
D=0.0117 \operatorname{KSN}\left(W / A_{1}\right)^{1 / 2}\left(V_{0}-31.5\right) \text { for } V_{0}>61 \mathrm{~m} / \mathrm{sec} \tag{9}
\end{equation*}
$$

$$
\begin{equation*}
D=2 K S N\left(W / A_{1}\right) \operatorname{Ln}\left[1+2 V_{0}^{2}\left(10^{-4}\right)\right] \text { for } V_{0} * 61 \mathrm{~m} / \mathrm{sec} \tag{10}
\end{equation*}
$$

W is projectile weight
$A_{1}$ is cross sectional area
Since all Ampacts in the present study had $\nabla_{0}>61 \mathrm{~m} / \mathrm{sec}$, a procedure based on $a$ method used by Young (Reference 13) to modify Equation (9) for use with layered media was used to analyze the Eglin experimenta. In Equation (9), $N$ is/a nose coefficient, $S$ is a soil coefficient, and $K$ is an independencly dex́ermined parameter. Since $K, S$, and $N$ appear only as the product RSN, the procedure fcllowed was to determine the beat value of KGN to fit the experimental velocity versus position data. The projectiles for the present experiments were considerably smaller and lighter than those for which the empirical formulations and scaling laws had been shown to give good results.

According to Young (Reference 13), for $\nabla_{0}>61 \mathrm{~m} / \mathrm{sec}$ the constant $A$ in the Poncelet Equation ( 3 ) of paragraph $4.3 .2^{\circ}$ ia negligible, so that

Equarion (5) is the apprcpriate form of to position, narely (with $x_{0}=0$ )

$$
\begin{equation*}
V=V_{n} e^{-B r} \tag{11}
\end{equation*}
$$

In the procedure of this section both the Poncelet parameter $B$ and the initial velocity $V_{0}$ at $x=0$ are considered as unknowns to be determined frow the experimentel data of Appendix A as follows. The velocity $V_{1}$ at four positions $x_{i}$, each located midway between the positions of the profectile in two successive $X$-ray pictures, is calculated as $V_{1}=\Delta x_{1} / \Delta t_{1}$, where $\Delta x_{1}$ is the distance travelcd during the time between the firings of the two $x$-raya. Then the regression prosedure gives

$$
\begin{equation*}
\left.B=\left[\left(\Sigma x_{1}\right)\left(\frac{1}{4} \Sigma \ln V_{1}\right)-\sum x_{1} \ln V_{1}\right] / \sum \sum x_{i}^{2}-\frac{1}{4}\left(\sum x_{1}\right)\left(\Sigma x_{1}\right)\right] \tag{12}
\end{equation*}
$$

and

$$
\begin{equation*}
v_{0}=\operatorname{Exp}\left\{\Sigma \ln v_{i}+\frac{1}{4} B\left(\Sigma x_{1}\right)\right\} \tag{13}
\end{equation*}
$$

where $1=1$ to 4.
The Poncelet drag coefficient $C_{D}$ is then given by $C_{D}=2 m B / \rho A_{1}$ as in Equation (8).

The basis for the application of the Sandia empirical formuia to layered media (keference 13) was the assumption of constant ciecelez ion through each layer. Thus if $a_{n}$ is the acceleration magnitude nondi" sionalized with respect to $g$, the acceleration of gravity (so that $a_{n} g ;$ he dimensional deceleration), then the velocities $V_{n+1}$ at the beginning a d the end of a layer we:e related (Reference 13) by

$$
\begin{equation*}
v_{n+1}^{2}-v_{n}^{2}-2 g\left[\left(a_{n-1}+a_{n}\right) \frac{L}{2}+a_{n}\left(t_{n}-L\right)\right] \tag{14}
\end{equation*}
$$

where $t_{n}$ is the thickness of the layer and $1 t$ is assumed that $t_{n} \gg L_{\text {. . For the }}$. For flat-nosed projectile, with $L=0$, Equation (14) reduces to

$$
\begin{equation*}
v_{n+1}^{2}=v_{n}^{2}-2 a_{n} g t_{n} \tag{15}
\end{equation*}
$$

which is the basis for the following regression prucedure to determine the Sandia parameter KSN. Let

$$
\begin{equation*}
G_{i}=v_{1}^{2} /\left[0.0117\left(v_{1}-30.5\right)\left(v_{1}^{2}-v_{1}^{2}\right)(W / A)^{i / 2}\right] \tag{16}
\end{equation*}
$$

Then

$$
\begin{equation*}
K S N=\frac{\sum\left(x_{1}-x_{1}\right)\left(\frac{1}{3} \Sigma G_{1}\right)-\sum\left(x_{1}-x_{1}\right)\left(G_{i}\right)}{\left[\left(x_{1}-x_{1}\right)^{2}-\frac{1}{3} \Sigma \Sigma\left(x_{1}-x_{1}\right)\right]\left[\Sigma\left(x_{1}-x_{1}\right)\right]} \tag{17}
\end{equation*}
$$

$$
1=2 \text { to } 4
$$

The different regression analyses presented in this paragraph and in paragraph 4.3.2 gave for the classical Poncelet equations in most cases almost
the mame results for $C_{D}$. The resulte for $C_{D}$ as function of initial velucity for the solid flat-nosed projectiles in dry and and in wet sand arp sumarized in Figure 53. When the two regression results differed gignificantly the result of paragraph 4.3 .2 uss used. As shown by the solid triangles the value of $C_{D}$ is almost independent of the initial velocity in dry sand, while the values for saturated sand (marked by the solid circles) ate lower than for dry sand and show a dowaward trend with iacreasing initial velocity.

A aimilar plot of the ficted value of the Sandia parameter KSN for the same shots is shown in Figure 54. The greater acatter in the fitced values of KSN than in those of $C_{D}$ indicated that these penetration events are not well characterized by a single value of KSN for each shot. The greater discrepancies with the Sandia equation can be explained in part by the assumption of a constant deceleration wagitude in each segment, in contrast with the Poncelet prediction whin does fit the dry sand experimental data very well.

In paragraph 4.6 the m . . ible variation of the Poncelet coefficient with pueition along a $t r$. -ory is examined by fitting separace values fo: different portions the trajectory fer each shot. Some of the values $\because: \rightarrow$ : $: \rightarrow$ KSN fo: ae shots analyzed by the regression methods described In this sex...." 111 also be given in Table 8 for dry sand and Table 9 for wet sand.

### 4.6 DRAG COEFFICIENT VARIATION W:TH POSITION ALCNG A TRAJECTORY

The favorable agreement with the experimental data of the positiontime and velocity-position curves calculated by the Poncelet force luw suggeated further analysis by this model. The variation in the drag coefficient along a trajectory was examined for some of the solid flat-nose projectile rajec ories, for 9 shots in dry sand and 10 in wet sand, including examples fro.l each of the three impact velocity regimes of the primary test program. These calculations wexe made by separately evaluating the Poncelet parameter $B$ for three different segments of each trajectory by the following equation.

$$
\begin{equation*}
B=\rho C_{D} A_{1} / 2 m=\left[\ln \left(V_{n} / v_{n+1}\right)\right] /\left(x_{n+1}-x_{n}\right) \tag{18}
\end{equation*}
$$

where subscripts $n$ and $n+1$ identify values at the beginning and the end of a segment.

Tables 8 and 9, for fry sand and wet sand, respectively, exhibit in the third column the resulting drag coefficients calculated by Equatior. (18) for three segments of the trajectory for each shot. The tables also inst in the last two colums some of the values of $C_{p}$ and the Sandia parameter KSN fitted to the whole trajectory by the methods of paragraphs 4.3.2 and 4.5. When two values of $C_{p}$ were listed for a shot, the first was calculated by the mechod of paragraph 4.3.2 and the second by the method of paragraph 4.5.

As was pointed out at the end of paragraph 4.3 the $C_{0}$ values fitted to the dr:' sand shots are consistentiy higher than those for saturated sand. Moreover he wet sand values show a downard trend with increasing impact.
 Figure 53. Poncelet Drag Cnefficient $C_{D}$ versus Initial Velocity
for Flat-Nosed Projectile Tests in Dry Sand and Wet Sand


Figure 54. Sandia Penetration Coefficient KSN versus Initial Velocity for Flat-Nosed Projectile Tests in Dry Sand and Wet Sand
table 8. drag coefpicien $C^{C} C_{D}$ and kSn values for selected shots in dry sand (SOLID FLAT-NOSE PROJECTILES)

| $\left[\begin{array}{l} \text { Shot } \\ \text { No. } \end{array}\right.$ | Striking Velocity (m/sec) | $\begin{aligned} & \text { Seguent } \\ & C_{D} \end{aligned}$ | $\begin{aligned} & \text { Distance } \\ & (\mathrm{m}) \end{aligned}$ | $\begin{aligned} & \text { Average Velocity } \\ & (\mathrm{m} / \mathrm{sec}) \end{aligned}$ | $\begin{aligned} & \text { Shot } \\ & C_{D}{ }^{\star} \end{aligned}$ | Shot KSN |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 17 | 212.1 | $\begin{aligned} & 1.779 \\ & 1.590 \\ & 2.049 \end{aligned}$ | $\begin{aligned} & 0.320 \\ & 0.590 \\ & 0.901 \\ & \hline \end{aligned}$ | $\begin{aligned} & 174.1 \\ & 139.3 \\ & 108.6 \end{aligned}$ | $\begin{aligned} & 1.643 \\ & 1.779 \end{aligned}$ | 3.05 |
| 18 | 213.4 | $\begin{aligned} & 1.887 \\ & 1.673 \\ & 2.413 \end{aligned}$ | $\begin{aligned} & 0.307 \\ & 0.570 \\ & 0.826 \end{aligned}$ | $\begin{aligned} & 174.6 \\ & 139.3 \\ & 106.5 \\ & \hline \end{aligned}$ | $\begin{array}{\|l\|} \hline 1.624 \\ 2.230 \\ \hline \end{array}$ | 3.20 |
| 19 | 210.6 | 1.849 <br> 1.880 <br> 2.149 | $\begin{aligned} & 0.311 \\ & 0.573 \\ & 0.880 \\ & \hline \end{aligned}$ | $\begin{aligned} & 176.5 \\ & 139.6 \\ & 106.5 \\ & \hline \end{aligned}$ | $\begin{aligned} & 1.692 \\ & 1.973 \end{aligned}$ | 3.17 |
| 20 | 329.2 | $\begin{aligned} & 1.678 \\ & 1.910 \\ & 1.687 \\ & \hline \end{aligned}$ | $\begin{aligned} & 0.295 \\ & 0.552 \\ & 0.862 \\ & \hline \end{aligned}$ | $\begin{aligned} & 268.1 \\ & 214.8 \\ & 168.3 \\ & \hline \end{aligned}$ | $\begin{aligned} & 1.769 \\ & 1.786 \end{aligned}$ | 2.26 |
| 23 | 328.2 | $\begin{aligned} & 2.014 \\ & 1.578 \\ & 2.152 \\ & \hline \end{aligned}$ | $\begin{array}{r} 0.300 \\ 0.552 \\ 0.863 \\ \hline \end{array}$ | $\begin{array}{r} 269.5 \\ 216.0 \\ 168.7 \\ \hline \end{array}$ | $\left\lvert\, \begin{aligned} & 1.72 \% \\ & 1.867 \end{aligned}\right.$ | 1.77 |
| 24 | 327.4 | $\begin{aligned} & 1.643 \\ & 1.774 \\ & 1.814 \\ & \hline \end{aligned}$ | $\begin{aligned} & 0.269 \\ & 0.525 \\ & 0.826 \\ & \hline \end{aligned}$ | 265.9 <br> 217.8 <br> 172.0 | $\begin{array}{\|l} 1.723 \\ 1.765 \end{array}$ | 2.59 |
| 25 | 406.0 | $\begin{aligned} & 1.842 \\ & 1.400 \\ & 1.615 \\ & \hline \end{aligned}$ | $\begin{aligned} & 0.277 \\ & 0.534 \\ & 0.832 \\ & \hline \end{aligned}$ | $\begin{aligned} & 329.1 \\ & 264.2 \\ & 211.3 \\ & \hline \end{aligned}$ | $\begin{aligned} & 1.685 \\ & 1.561 \end{aligned}$ | 1.38 |
| 26 | 406.3 | $\begin{aligned} & 1.783 \\ & 1.760 \\ & 1.515 \\ & \hline \end{aligned}$ | 0.280 <br> 0.538 <br> 0.810 | 329.6 265.5 214.0 | $\begin{aligned} & 1.649 \\ & 1.670 \end{aligned}$ | 1.70 |
| 29 | 405.0 | $\begin{aligned} & 1.933 \\ & 1.761 \\ & 1.6 .65 \\ & \hline \end{aligned}$ | $\begin{array}{r} 0.281 \\ 0.522 \\ 0.839 \\ \hline \end{array}$ | 330.8 <br> 266.3 <br> 214.0 | $\begin{aligned} & 1.706 \\ & 1.735 \end{aligned}$ | 1.54 |

TABLE 9. DRAG COEFFICIENTS CD AND KSN VALUES FOR SELECTED SHOTS IN WET SAND (SOLID FLAE-NOSE PROJECTILES)

| $\begin{aligned} & \text { Shot } \\ & \text { Ho. } \end{aligned}$ | Striking Velocity (m/sec) | $\begin{gathered} \text { Segment } \\ C_{D} \end{gathered}$ | Distance (m) | Average Velocity (m/sec) | $\begin{aligned} & \text { Shot } \\ & C_{D^{*}} \end{aligned}$ | Shot KSN |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 71 | 207.9 | $\begin{aligned} & 2.167 \\ & 1.565 \\ & 1.921 \\ & \hline \end{aligned}$ | $\begin{aligned} & 0.268 \\ & 6.519 \\ & 0.826 \end{aligned}$ | $\begin{array}{r} 182.3 \\ 134.2 \\ 98.8 \\ \hline \end{array}$ | $\begin{aligned} & 1.545 \\ & 1.802 \end{aligned}$ | 1.614 |
| 72 | 214.0 | $\begin{aligned} & 1.302 \\ & 1.561 \\ & 1.673 \end{aligned}$ | $\begin{aligned} & 0.285 \\ & 0.541 \\ & 0.865 \end{aligned}$ | $\begin{aligned} & 181.1 \\ & 142.3 \\ & 104.9 \\ & \hline \end{aligned}$ | $\begin{aligned} & 1.359 \\ & 1.564 \end{aligned}$ | 3.40 |
| 73 | 212.8 | $\begin{aligned} & 1.410 \\ & 1.288 \\ & 1.697 \\ & \hline \end{aligned}$ | $\begin{aligned} & 0.287 \\ & 0.549 \\ & 0.878 \\ & \hline \end{aligned}$ | $\begin{aligned} & 180.9 \\ & 143.6 \\ & 108.3 \end{aligned}$ | $\begin{aligned} & 1.233 \\ & 1.474 \end{aligned}$ | 2.92 |
| 37 | 336.5 | $\begin{aligned} & 1.134 \\ & 0.824 \\ & 1.218 \\ & \hline \end{aligned}$ | $\begin{aligned} & 0.298 \\ & 0.55 * \\ & 0.883 \\ & \hline \end{aligned}$ | $\begin{aligned} & 279.6 \\ & 237.0 \\ & 196.6 \\ & \hline \end{aligned}$ | $\begin{aligned} & 0.947 \\ & 1.023 \end{aligned}$ | 2,214 |
| 38 | 333.1 | $\begin{aligned} & 1.032 \\ & 0.924 \\ & 0.997 \\ & \hline \end{aligned}$ | $\begin{aligned} & 0.301 \\ & 0.587 \\ & 0.917 \\ & \hline \end{aligned}$ | $\begin{array}{r} 284.0 \\ 240.2 \\ 200.1 \\ \hline \end{array}$ | $\begin{aligned} & 0.937 \\ & 0.969 \end{aligned}$ | 2.36 |
| 74 | 334.0 | $\begin{aligned} & 1.240 \\ & 0.923 \\ & 1.230 \end{aligned}$ | $\begin{aligned} & 0.309 \\ & 0.580 \\ & 0.917 \end{aligned}$ | $\begin{aligned} & 282.7 \\ & 233.4 \\ & 189.7 \end{aligned}$ | 1.016 |  |
| 81 | 333.8 | $\begin{aligned} & 1.023 \\ & 0.885 \\ & 1.308 \end{aligned}$ | $\begin{aligned} & 0.309 \\ & 0.570 \\ & 0.896 \\ & \hline \end{aligned}$ | 278.7 236.3 193.6 | 0.963 |  |
| 82 | 404.8 | $\begin{aligned} & 0.985 \\ & 0.645 \\ & 0.499 \end{aligned}$ | $\begin{array}{r} 0.308 \\ 0.562 \\ 0.889 \\ \hline \end{array}$ | 343.2 299.3 269.2 | 0.728 |  |
| 83 | 419.4 | $\begin{aligned} & 1.228 \\ & 0.697 \\ & 1.148 \\ & \hline \end{aligned}$ | $\begin{aligned} & 0.307 \\ & 0.547 \\ & 0.854 \end{aligned}$ | 347.0 298.5 258.7 | 0.819 |  |
| 84 | 405.7 | $\begin{aligned} & 0.978 \\ & 0.584 \\ & 1.322 \\ & \hline \end{aligned}$ | 0.308 <br> 0.545 <br> 0.843 | $\begin{aligned} & 333.3 \\ & 294.0 \\ & 253.0 \\ & \hline \end{aligned}$ | 0.777 |  |
| *First: listed value of Shot $C_{D}$ calculated ty method of paragraph 4.3.2 |  |  |  |  |  |  |

velocity, while for dry sand the values were essentially independent of striking velocity.

Of pa.ificular interest here is the change in $C_{D}$ along a trajectory as shown by the two or three different value listed in the third colurn for each shot. in wany cases the first segment $C_{p}$ is higher than the second and then the trend is reversed to give a third segment $C_{p}$ higher than the second. This pattern is followed in 8 nf the 10 wet sand cases anci in 4 of the 9 dry sand cames, for which three gegments were calculated.

The dry sand coefficients show less variation along the trajectory than the wet sand coeffictents, a variation of the order of 30 percent between the maximum and minimum values in dry gand and two or three times this much variation in wet gand.

This variation might imply that $C_{D}$ is velocity dependent instead of being a constant. It seems that in some cases the drag coefficient variation can be fitted to a power law in the velocity. An example of this is given in Section VI.

The apparent variation in the coefficient may, however, actually be a result of assuming an incorrect form for the force law. If the force law contains a term linearly dependent on velocity in addition to the term depending on the square of the velocity, both with constant coefficients, this would lead to an apparent variation in the $C_{D}$ determined by Equation (18), which is based on a law where the force 18 proportional to the square of the velocity.

The high drag nt the beginning of the trajectory may ilso be related to shock effects involving not only the velocity but also the .-pulse duration and the acoustic impedance of the target medium. The minimum $C_{D}$ appears to be related to momentum transfer to the target medium, where the predoninant drag is proportional to the square of the velocity. $F$ : $11 y$, as the projectile slows and the cavity collapses onto it. friction and shear resistance in the target medium become important biving rise to en increase in the apparent drag coefficisnt.

## SECTION V

## CAVITY EXPAYSION THRORY PENETRATION CALCULATION

In 1975 Maraard and Kanagud (Reference 6) pubilahed a ropori. enending the mpproximace penctration calculation method for projectilea with a heminpherical nosen besed on the theory of uxpansion of epherical cavity (CET). to profectiles with conical and ogival noses and showing how it cnult be extended co an arbitrary axially symatric projectile. The firsi i. of CET metnods for dyname penetration was by Coodier (fefererce 26; for : apherical project: s impacting an incompresaitic strain hardsaing tareat. Hanagud and Rops (Reference 3) modified the methud to accuunt approximately for target compressibility by treating the target matcilal aa locking medium. The method has been applied to penetration calculations for fletnose profectiles by Rohani (Reference 4). with the implicit asaumption that a false hemispherical nose of target material in foraed and sarifed by the projertle elong a atable ocraight path.

It should be remembéred that eev. Il quite important assumptiona are made in applying the cavity expansion metnod to penetration caiculations, so that extensive experimental verification is neressary co check on che range of approximate validity of predictions by the method. Nevartheless it has achievad some rearkable succeas in predicting penctration depths from mesarec soil properties (Reference $4,6,7$ ). Bernard and Hanagud (Reference 6) defined a dimensionless parameter $R_{s}$, which the; call the solid Reynolds number

$$
\begin{equation*}
R_{i}=\frac{\rho v^{2}}{Y} \tag{19}
\end{equation*}
$$

where $p$ is targec density, Y is carget yield strength in a unisxial strain test, and $V$ is projectile velocity. It was concluded that final penetration depth was reasonably well predicted for $R_{s}$ between aero and about 100 . They comsidered the upper bound of 100 as a conservative one, since resulta of experimengs at high values of $R_{s}$ are needed in order to establish a more realistic range. They remarked that accurate prediction of detalla of the complete deceleration history might demand a auch atronger liaitation or. $R_{\text {. }}$.

According tc the spherical cavity expansion theory for an infinite locking compressible medium the compressive nomal stress $p$ at the cavity surface is

$$
\begin{equation*}
p=p_{g}+p_{I}-p_{g}+p_{p}\left(B_{1} a k+B_{2} \dot{a}^{2}\right) \tag{20}
\end{equation*}
$$

where $P_{\text {a }}$ and $P_{I}$ are the separate contributionn of the warerial deformation (shear) and inertia, which Bernard and Hanagud (Reference 6) call the shear resistance and the dymanic pressure, respectively. In tita wivetion $\rho_{p}$ is the locked piastic denaity in the region betind the expanding spherical plastic locking shock wave, a is the inatancantous cavity reifis, a, and a are the radial veloaity and acceleration of the cavit: suriace and $P_{s}, B_{1}$, and $B_{2}$ are parameters related to properties of the arterial, The way thesu parameters are calculated will be indicated lacer in this mection.

In applying this theory to penetration by a projectile with a hemisphorical nose two very important assumptions are made (References 3,4) ir order to get a simple theory.
(1). The parts $p_{f}$ and $p_{I}$ of tha normal pressure at the tip of a hemispherical nose of radius a on a projectile traveling at speed $v$ and acceleration 0 are assumed to be equal to the values of $p_{s}$ and $P_{I}$ on a spherical cavity surface of the same instantaneous radius a, but expanding with $\dot{a}=V$ and $a=\dot{V}$.
(2). The entire hemispherical nose is assumed to be in contact with the target material, and the dynamic pressure on the projectile's hemispherical nose is assumed to vary from the stagnation point value at the nose tip to zero at the shoulder as the cosine of the polar angle measured fruin tip (culatitude), while $p_{s}$ is uniform over the nose.

If friction on the nose and all afterbody forces are neglected this leads in a straight forward manner to the following equation of motion for the projectile of mass $M$

$$
\begin{equation*}
\left(M+\frac{2}{3} \pi a^{3} \rho_{P} B_{1}\right) \frac{d V}{d t}=-\pi a^{2}\left(P_{s}+\frac{2}{3} \rho_{p} B_{2} V^{2}\right) \tag{21}
\end{equation*}
$$

The term $\frac{2}{3} \pi^{3}{ }^{3} \rho_{p} B_{1}$ is an added mass term resulting from the acceleration term $\rho_{0}{ }^{B} a^{a}$ in the expression for the dynamic pressure in Equation (20). In most clales that have been treated the added-mass term was negilgible in comparisor to elie projectile mass $M$.

In modifying the method to other exisymmetric nose shapes, Bernard and Hanagud replaced the assumption on the variation of $P_{T}$ over the nose surface by an assumption on the variation along the nose of the tangential component $V_{f}$ of the target absolute velocity. For a fully ambedded nose, the material was assumed to be 1 r . contact all along the nose surface (no separstion before the base of the nose), an assumption which they recognized was nut generally strictly correct. The normal component $V_{n}$ of target material velocity was therefore required to be equal to the normal component of the velocity of the profectile nose surface. For a conical nose of half apex angle $\phi$, it was assumed that the tanjential component of target velocity varies from $V \cos p$ at the nose tip to zero at the base of the cone, according to the law

$$
\begin{equation*}
v_{t}=\left[1-(z / L)^{2}\right]^{1 / 2} v \cos \phi \quad 0 \leq z \leq L \tag{22}
\end{equation*}
$$

where $L$ is the nose length and $z$ is the axial coordinate measured back from the tip of the nose. The velocity dependent part of the dynamic pressure $p_{I}$ was then assumed to depend on the local resultant particle velocity magnitude $v_{p}=\left[v_{n}^{2}+y_{t}^{2}\right]^{1 / 2}$ in the same way that $p_{I}$ depends on a in the spherical cavity expansion theory. Similar assumptions could be made about the dependence on the local particle acceleration on the nose surface, but because that term is usually much smaller than the velocity-dependent. term, Berrard and Hanagud (Reference 6) chose to use the nose tip acceleration $\dot{V}$, so that $P_{I}$ is given by

$$
\begin{equation*}
p_{I}=\rho_{\mu} B_{1} a \dot{V}+\rho_{p} B_{2} v_{P}^{2} \tag{23}
\end{equation*}
$$

where $V_{p}$ varles over the nose while $\dot{V}$ does not, Integration of the axdal force component over the nose then gives the following equation of motion, replacing Equation (21)

$$
\begin{equation*}
\left(M+\pi A^{3} p_{p} B_{1}\right) \frac{d V}{d t}=-\pi a^{2}\left(p_{g}+\rho_{P} B_{2}{ }^{\kappa} n^{2}\right) \tag{24}
\end{equation*}
$$

where the dimensionless nose-shape factor $f_{n}$ is given for a conical nose of length-to-diameter ratio $\mathrm{L} / \mathrm{D}$ by

$$
\begin{equation*}
f_{n}=1-\frac{2}{3} \frac{4(1 . / D)^{2}}{4(L / D)^{2}+1} \tag{25}
\end{equation*}
$$

Beaides containing the factor $f_{n}$ in place of the factor 3/2, Equation (24) differs from Equation (21) by lacking the factur $3 / 2$ in the term containing $3_{1}$.

Bemard and Hanagud (Reference 6) observed that with the assumption listed above the variation in $f_{n}$ as $L / D$ varien from zero (flat nose) to infinity (long pointed nose) prodnces a variation in predicted final penetration depth by a factor of three. For $L / D=0.5, f_{n} \mathrm{is} 3 / 2$ and the prediction is the same as for the hemispherical nose, when the contribution of the added-mass term containing $B_{1}$ is negligible.

For other fully embedded convex axisymetrical nose shapes, Bernard and Hanagud (Reference 6) gave a method for estimating $V_{n}$ and $V_{t}$ at any position on the nose by considering the circumscribed conc tangent to the nose at the point. The base of the cone was in the same plane as the actual nose base, but the tip was forward of the actusl tip. The components $\mathrm{I}_{\mathrm{n}}$ and $V_{t}$ at the point of tangency were asaumed to be equal to the values of $V_{n}$ and $V_{t}^{t}$ that would be assumed on the circumacribed cone at the point of tangency if it were an actual conical nose, calculated by the procedure described above for conical noses. This gave a continuous variation of $V_{t}$ over the nose, dropping to zero at the base of the nose. Explirit formulas for $V_{p}$ and $g_{I}$ as a function of position on the nose were given for ogives (Reference 6). For the special case where the ogive is a hemisphere, the distribution of $P_{T}$ over the nose as calculated by this procedure differs slightly from that given by the previous procedure assuming $p_{g}$ to vary as the cosine of the polar angle. But the resultant axial force is the same when the added-mass term containing $B_{1}$ is negligible.

Because of the formation of a false nose of target material, it does not seem reasonable to apply these assumptions to actual flat-nose projectiles. Neither the $X$-rey pictures nor post-test examination had shown the actual shape of the false noses formed in the Egin penetration experiments. In the following analysis of Shots 20 and 21 of the Eglin experiments two kinds of aseumptions were made for the snape of the false nose. The first assumption was a hemispherical nose, leading to Equation (21) for the equation of motion. The second kind of assum?tion was a conical nose, leading to Equation (24). The second assumption was applied for $L / D$ values of $0.5,0.4,0.2$, and 0 .

The materfal properties for the dry sand target were determined as Collows. The shear or deviatoric properties were based on a triaxial test performed in the Civil Engineering Labosatories at the University. Figure 55 shows a plot of $\sigma_{1}-\sigma_{3}$ versus $\varepsilon_{1}$, where $\sigma_{1}$ and $\varepsilon_{1}$ are axial compressive stress and strain and $\sigma_{3}$ is the constant lateral confining pressure of
0.589 pra . The dots denote the experimental curve while the two etraight liues are the bilinear fit to it. From the bilinear fit the values of

$$
\begin{equation*}
E=54 \mathrm{MPa} . \quad E_{t}=1.39 \mathrm{RPa}, \quad Y=1.4 \mathrm{MPa} \tag{26}
\end{equation*}
$$ were determined for she elautic modulus $E_{v}$ tangent modulus $E_{t}$ in the plastic regime, and yield stress $Y$ (at the intersection of the two etraight ines). The bilinear approximation, required for the aimple cavity exponsion theory, seem to be quite a reasonable one for the triaxisl test curve.

The required approximation to the compressibility properties, determined from uniaxial strain test as shown in Figure 56 is more extreme and more ariftrary. It is necessary to approximate the curve by two vertical (incompressible) lines joined by a horizontal jump representing the change in density from the locked elastic denaity at uniaxial strain $\varepsilon_{1}$, to the locked plascic deasicy at uniaxial etrain $f_{p}$ upon the pasaage of the plastic shock wave, The approximatione acsumed corfeapond to uniamial etrain values of

$$
\begin{equation*}
\text { Elastic } c_{1}=0.03 \quad \text { Plastle } c_{p}=0.094 \tag{27}
\end{equation*}
$$

The initial density was $\rho_{0}=1540 \mathrm{~kg} / \mathrm{m}^{3}$. Thea

$$
\begin{equation*}
\rho_{P}=1700 \mathrm{~kg} / \mathrm{m}^{3} \text { and } \rho_{0} / \rho_{P}=0.906 \tag{28}
\end{equation*}
$$

leading to the following numerical values for constante in the theory

$$
\begin{align*}
& B=\frac{Y}{28}-\frac{1}{3 \varepsilon_{1}}=0.00296 \quad a_{p}=1-\left(\rho_{0} / \rho_{p}\right)=0.094 \\
& 0=1-\left(\rho_{0} / \rho_{p}\right) e^{-38}=0.1020 B_{1}=1-\delta^{1 / 3}=0.533 \\
& B_{2}=1.5+\left(1+a_{p}\right) 8^{1 / 3}+0.58^{4 / 3}=1.013 \\
& P_{E}=\frac{6}{9} t\left(1-e^{-38}\right)-\frac{2}{3} t \ln \delta+\frac{2}{27} \pi^{2} E_{t}-\frac{4}{9} E_{t}\left(\sum_{n=1}^{\infty}\left(8^{n} / n^{2}\right)\right\}=3.396 \mathrm{NPa} \tag{29}
\end{align*}
$$

The formules for $B_{0}{ }^{0} p^{\prime}, B_{1}, B_{2}$, and $P_{g}$ are asiven in Ranagud and Rose (Reference 3), and differ slighty from the veraions in Bermard and Fanagud (Peference 6). The differences have to do with inclusion of various rerms that contribute litele to the actual numerical valuee obtained. With this value of $\mathrm{B}_{1}$; the added-mase term containing $\mathrm{I}_{1}$ in Equations (21)or (24) ic 0.00285 kg , which is negligible in comparison to the projectile mass m ( 0.5451 kg in Shot 70 and 0.5443 kg in Shot 25). The equation then eakes the form

$$
\begin{equation*}
\frac{d V}{d t}=-\left(A+B V^{2}\right) \tag{30}
\end{equation*}
$$

as In the Poncelet force law. Equation (30) is integrated to give

$$
\begin{equation*}
x=x_{0}+\frac{1}{B} \ln \left\{\cos \left(\sqrt{A B}\left(t-t_{0}\right)\right)+\sqrt{B / A} v_{0} \sin \left(\sqrt{A B}\left(t-t_{0}\right)\right)\right\} \tag{31}
\end{equation*}
$$

and

$$
\begin{equation*}
v=\left(\left(\frac{A}{B}+v_{0}^{2}\right) e^{-2 B\left(x-x_{0}\right)}-\frac{A}{B}\right)^{1 / 2} \tag{32}
\end{equation*}
$$



Figure 55. Bilinear Approximation to Trigxial Teat Curve for Loose Eglin Sand ( $\rho_{0}=1540 \mathrm{~kg} / \mathrm{m}^{3}$ ) with $\sigma_{3}-0.589 \mathrm{mPa}$


Figure 56. Ideal Locking Approximation to Uniaxial Strain Curve for Loose Eglin Sand (Inicial Density $1540 \mathrm{~kg} / \mathrm{m}^{3}$ )

These resilta are ploted in Figure 57 for Shot 20 and Figure 58 for Shot 25. The coefficiercs A and $B$ for the two ahota have the values ahown in Table 10.

TABLE 10. COEFFICIENTS FOR CAVITY EXPANSION THEORY PENETRATION CALCULATIONS

| Shot 20 |  |  |  | Shot 25 |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Curve | $1 . / \mathrm{D}$ | $\left.\mathrm{him} / \mathrm{s}^{2}\right)$ | $\mathrm{B}\left(\mathrm{n}^{-1}\right)$ | Curve $\mathrm{t} / \mathrm{D}$ | $\mathrm{A}\left(\mathrm{m} / \mathrm{s}^{2}\right)$ | $\mathrm{B}\left(\mathrm{m}^{-1}\right)$ |  |
| 1 | 0.5 | 1900 | 0.6614 | 1 | 0.5 | 1903 | 0.6624 |
| 2 | 0.4 | 1900 | 0.7340 | 2 | 0.4 | 1903 | 0.7351 |
| 3 | 0.2 | 1900 | 0.9009 | 3 | 0.2 | 1903 | 0.9023 |
| 4 | 0 | 1900 | 0.9922 | 4 | 0 | 1903 | 0.9936 |

Also plotted in each figure is the fitted Poncelet curve through the experimental points. The agreement is fairly good for the hemispherical nose (Curves $1, L / D=0.3$ ), overestimating the penctration by 3 to 5 percent and the final velocity by about 11 percent in Shot 20 . The xt-curve for $L / D=0.4$ (Curve 2) is essentially coincident with the experimental curve in each case. The Vx-curve is also coincident with the experimental curve in Shot 25 and overes:imates the final velocity by only 3 percent for Shot 20. It is emphasized that che parameter values used for the prediction were determined from the two static curves as shown in Figures 55 and 56. Ther ? parameters were determined before calculation of Figures 57 and 58 . No adjustments vere made to the parameter values to get a better agreement with the experimental results.

Thus, with an assumed talse conical nose vith $L / D=0.4$ the details of the deceleration history are remarkably well predicted for Shota 20 and 25 in the region of observation. The prediction with an assumed sphericei nose (or a cone with $L / D=0.5$ ) is also not bad. The solid Reynolds number for chese two shots $i s R_{s}-127$ for Shot 20 and $R_{s}=181$ for Shot 25 , both above the range of $R_{\text {a }}$ in which the cavity expansion theory was known to give good results. (See the discussion following Equation (19).)

Any attempt to apply the theory directly to the flat-nomed projectiles by using the limiting case of $L / D=0$ as in the Curves No. 4 in Figures 57 and 58 would greatly overpredict the drag and underpredict the penecration for Shots 20 and 25.

It is concluded that the cavity expansion theory method has considerable merit despite the atrong assumptions involved in its application to penetration theory. For projectiles with actual cenical or ogival noses or other nonflat axiaymetric shapea it should prove profitable to consider obilque impacts and attempt to modify the thenry to apply locally to a surface area element somewht in the manner of the aseumed differential area force lawn to be discussed in Section VI. This has not been done yet for a



Figure 58. Shot 25, Cavity Expansion Theory Predictione Compared to Experimental Data
complete trajectory, although 3ernard and Hanagud (Reference 6) discuased the eabedment process at the beginning of an oblique impact for a projectile with a conical noaz.

It ight also be possible to make similar calculation for the flat-nosed projecille, but this would aeverely test any assumed falee nose shape. It may be possible to obtain some hints about the false nose shape from atudy of the separation angles as awn in the X -rays.

## SECTION VI

## RIGID BODY MOTION IN A SOIL MEDIUM

### 6.1 EQUATIONS OF MOTION

Soil penetration prediction techniques as classified by Triandafilidis (Reference 1) are considered to fall lntc two broad categories 1.e., mathematical and experimental. Under the broad heading of mathematical further suggested subdiyisions sre semi-aralytical, analytical and chooretical. The semi-analytical cechniques listed in Reference 1 were all restricted to completely normal penetration. Even the cavity expansion models, classified as analytical, are also restricted to normal impact. Oblique impact may be analyzed using an analytical Differential Area Porce Law (Reference 8). However, a computer program based on this type of analysis has limited access because of proprietary restrictions, as well as being expensive to operate. In light of the above, study was initiated to develop a simple multidegree-of-freedom set of equations of motion for bodies of revolution in acil medium. The ground rules for this deveiopment are as follows:

1. The projectile was to be a body of revolution with zero rotajion rate about the longitudinal axis.
2. Classical six-degree-of-freedom equations of motion would be used with force and moment terms from assumed or empirical force expressions.
3. Force expressions would be selected by joins agreement between the contractor and the project engineer.
4. Reaults were to be profectile position-time tabulation using Cartesian coordinates for center of mase poaition and Euler angles for body rotations.

For the derivation a set of body fixed axte $x, y, z$ with unit vectors $\hat{i}$. $j, \hat{k}$ and an inertial frame $x^{\prime}, y^{\prime}, z^{\prime}$, with unit vertors $\hat{i}^{\prime}, \hat{j}^{\prime}, \hat{k}^{\prime}$ fixed in the soti, were selected and are shown achematically in Figure 59.

The generalizad six-degree-of-freedom equations of motion for a rigid nonsymetrical body written relative to the body axen are given as (Reference 27):
$F_{x}=m(\dot{U}+Q W-R V)$
$F_{y}=m(\dot{V}+R U-P W)$
$F_{z}=\boldsymbol{m}(\dot{W}+P V-Q U)$
$L=I_{x x} \dot{P}+I_{x y}(P R-\dot{Q})-I_{x z}(\dot{R}+P Q)+R Q\left(I_{z z}-I_{y y}\right)+I_{y z}\left(R^{2}-Q^{2}\right)$
$M=-I_{x y}(\dot{P}+R Q)+I_{y y} \dot{+}+I_{y z}(P O-\dot{R})+R P\left(I_{x x}-I_{z z}\right)+I_{x z}\left(P^{2}-R^{2}\right)$
$N=I_{x z}(Q R-\dot{P})-I_{y z}(\dot{Q}+P R)+I_{z z} \dot{R}+I_{x y}\left(Q^{2}-P^{2}\right)+Q P\left(I_{y y}-I_{x z}\right)$
where:

$$
F_{x}, F_{y}, F_{z} \text { Applied forcee in } x, y, z \text { directicns reapectively. }
$$

L,M,N Applied torques or monenta about $x, y, z$ axes respectívely.
U, $V, W$ Projectile or body translational velocitiea relative to $x, y, z$ axes respectively.
$P, Q, R \quad$ Projectile or body rotational velocities about $x, y, z$ axes respectively.

- Projectile mass.
$I_{x x}, I_{y y}$, Conventional moments of inertia and products of inertia for
$I_{z z}, I_{x y}$.
$I_{x x^{\prime}} I_{y z}$
The dot above any tem represente the time cierivative or time rate of change of that ters.

For a smatrical body the producte of inerila are zero and $I_{y y}=I_{z z}$, and Equationa (33) through (38) reduce to

$$
\begin{align*}
& F_{x}=-(\dot{U}+C W-R V)  \tag{39}\\
& F_{y}=I(\dot{V}+R U-P W)  \tag{40}\\
& F_{z}=m(\dot{H}+P V-Q U)  \tag{41}\\
& L=I_{x x} b  \tag{42}\\
& M=I_{y y} \dot{Q}+R P\left(I_{y x}-I_{y y}\right)  \tag{43}\\
& M=I_{z x} \dot{R}-Q P\left(I_{x x}-I_{y y}\right) \tag{44}
\end{align*}
$$

### 6.2 FORCE EXPRESSIONS

Thim force exerted on the projectile by the soil was sssumed to be of the form

$$
\begin{align*}
\frac{\overline{d P}}{d A} & =n_{x}\left(A_{x}+s_{x}|u|+c_{x} U^{2}\right) \hat{i} \\
& \left.+n_{y}\left(A_{y}+s_{y}|v|+c_{y} v^{2}\right)\right\}  \tag{45}\\
& +n_{z}\left(A_{z}+n_{x}|W|+C_{z} W^{2}\right) \hat{k}
\end{align*}
$$

where:
$A_{x}, B_{x}, C_{x}$, are the force coefficients to be deterwincd from A. B, C , teste performed for a certain projectile shape $y^{\prime} y^{\prime} y^{\prime} \quad$ and a given soil.
$A_{z}, B_{s}, C_{z}$
$n_{k}, n_{y}, n_{n} \quad$ are componenta of outward unit vector noresel to surface of projectile.

The forces $F_{x} F_{y}$, and $F_{z}$ may then be determined by tice projected werted oreas normal to the velocities $U, V, W$ respectively. A force is assured to exiat only when the wetted surface has an ourward velocity component sormal to the projected area. Tnis type of force diatribution applicable ouly $\mathcal{C o}$ axisymetric bodies, assume a uniformly distributed preseure over the projected wetted axea, giviag rime to a resultant force pasaing insough the geometric center of tine projected wetted aren. When the etric center of the projected wetted area coincides with the center mf the was of the projectile tnem no applied maent $L_{n} M_{,} N_{\text {, }}$ exists $\therefore$ the projectile. If the projectile is hollow or if the geometric center of the projectad area and the center of wass do not codacide then an applied moment exists and is equal to the applied forca timeo tha distance between the geometric center of the projected wetted area and the canter of mes. For a body of revolution completely mubmerged ${ }^{1} 2$ the mediun the force distribution due to lateral translational velocity $V$ is ohom in Figure 60 for the case of zero moment. In a caso where the geometric center of the area and the center of mass do not coincide the force diatribution is shown in Figure 61. The projected area used to determine the force fy for a completely submarged axisymetric projectile is the ame an for $f_{z}$. The projected area for $F_{x}$ for the complately aubeargod projectile is ofaply the cross eection of the projectile normal to the x axis.

If the projectile is only partially submerged then only a portion of the total projected grea is in contact with the soll medium and a rescitant force ami morrnt will exist as shown in Figure 62. These moments and forces are cligngiag with depth of penetration and become functions of $d \in p t h$. For this case the forces and moments are not simple expressious due to the complicated expressions required to calculace the areas. The derivation of the forces and moments required for partial peactration of a contcal nose are given in Appendix B. These equations are not included in the main body of the report as time did not permit complete computer modeling of these equations and therefore are simply included as infortation.

### 6.3 COURDINATE TRANSFORMATIONS

Tranalational and angular positions relative to an inertial frame fixed in the soil may be expressed in terma of angular and transistional velocities of the body fixed axes by use of coordinate transformations relating the two systems. The coordinate rotations required for these transformations are given in Reference 28 as:

1. Stare with body axis $x$, aligned with inertial mxis $x^{\prime}$ and rotate about body axis $z$, through an azimuthal angle Y. Tists produces a new set of body axes $X_{2}, Y_{2}, Z_{2}$.
2. Rotace abcut $Y_{2}$ through a picch angle $\theta$. This produces a new set of body axes $X_{3}, Y_{3}, Z_{3}$.
3. Finally, rotate about $X_{3}$ through a roll angle ${ }^{1}$, which brings the body into ita final bc ly axis sysiem $X, Y, Z$. (This rotation is not important for a body possessing complete symutry about the X axis.


Figure 59. Schematic of Body and Inertial Axes.
$x^{\prime}$ Positive Downward in Soil


Figure 60. Cencer of Mass and Geometric Center of Projected Area Coincident


Figure 61. Center of Mass and Geometric Center of Projected Area Not Coincident


Figure 62. Partially Submerged Projectile

The transformation equation based on these angles is given as

$$
\begin{equation*}
\bar{A}_{I}=T_{B I} \bar{A}_{B}, \tag{46}
\end{equation*}
$$

which eransforms an arbitrary vector $\bar{A}_{B}$ given in the body system to

$T_{B I}=\left[\begin{array}{lll}c \psi c \theta & (c \psi s \theta s \phi-s \psi c \phi) & (c \psi s \theta c \phi+s \psi s \phi) \\ s \psi c \theta & (s \psi s \theta s \phi+c \psi c \phi) & (s \% s \theta c \phi-c \psi s \phi) \\ -s \theta & c \theta s \phi & c \theta c \phi\end{array}\right]$
where: $c \theta=\cos \theta, s \theta=\sin \theta$, etc.
Fer orthogonal transformation auch as this the inverse of $T_{H I}$ is equal to the transpose of $T_{B I}$; therefore the inverse relation of Equation (46) is simply

$$
\begin{equation*}
\bar{A}_{B}=\left[T_{B I}\right]^{T} \bar{A}_{I} . \tag{48}
\end{equation*}
$$

By using Equation (46) the velocity vectors $U, V, W$ may be transformed to give the velocities $\dot{x}^{\prime}, \dot{y}^{\prime}, z^{\prime}$, in the inertial system. These velocities,

$$
\left\{\begin{array}{c}
\dot{x}^{\prime}  \tag{49}\\
\dot{y}^{\prime} \\
\dot{z}^{\prime}
\end{array}\right\}=\left[T_{B I}\right] \quad\left\{\begin{array}{l}
u \\
v \\
w
\end{array}\right\}
$$

may be integrated to determine the position $x^{\prime}, y^{\prime}, z^{\prime \prime}$ of the center of mass in the inertial frame.

The angular velocities $\dot{\psi}, \dot{\theta}, \dot{\phi}$ are related to the angular velocities $P, Q, R$ through a transformation matrix ROT defined by

$$
\left\{\begin{array}{l}
R  \tag{50}\\
Q \\
R
\end{array}\right\}=[R O T]\left\{\begin{array}{l}
\dot{\phi} \\
\dot{\theta} \\
\dot{\Psi}
\end{array}\right\}
$$

where:

$$
[R O T]=\left[\begin{array}{lll}
1 & 0 & -\sin \theta  \tag{51}\\
0 & \cos \phi & \sin \phi \cos \theta \\
0 & -\sin \phi & \cos \phi \cos \theta
\end{array}\right]
$$

Due to the non-orfhogonality of [ROT] the inverse of the [ROT] transformation is not [ROT]. It is given by

$$
[R O T]^{-1}=\left[\begin{array}{lll}
1 & \sin \phi \tan \theta & \cos \phi \tan \theta  \tag{52}\\
0 & \cos \phi & -\sin \phi \\
0 & \sin \phi \sec \theta & \cos \phi \sec \gamma
\end{array}\right]
$$

and


Equations (33) through (38), (49) and (53) represent the necessaxy equations. When solved simultaneously with the proper initial conditions they will yield the position and orientacion of the body as functions of time in the inertial frame.

### 6.4 GALCULATIONS

The solution of Equations (33) through (38), (49) and (53) may be obtained numerically provided expressions for the forces and moments are available. In the general case where impact is not normal the expressions are rather complicated and require consideratle computer technique and programing ability. However, a completely submerged projactile could be handled very easily for the force distribution as given by Equation (45). Also if the assumption is made that ouly the noge of the projectile is in contact with tha soil and submersion of the nose is instantaneous, a normal impact can be handled very easily.

The case of complete submersion was programmed using a MIMIC source language program. This program, available on the CDC 6600 in the Mathematical Laboratory at Egiin AFB, is essericially a fourth order Runge-Kutta numerinal method for solving simultaneous differentia: equations. A program shown as Computer Program I of Appendix C was developed for solution of Equations (33) through (38), (49) and (53), for blunt nosed cylinder, conical nosed cylinder and a hemispherical nosed cylinder. For this case the force coefficients $A_{x}$ through $C_{z}$ are assamed to be constents.

The MIMIC progrem allows for naming constants or variable parameters and for Program I, Appendix C, all the initial conditiong, ofl force coefficients, and geometric properties were given a parametel status. Only integration time and print frequency were named constints. A check ca this program was accomplished using the data for the blunt nosed cylinder listed in Figure 63. The assumption in this case is that, for normal penetration, nose submersion is instantaneous and only the nose is in contact with the soil.

For this case only the forces pardllel to the x axis of the projectile are operative; therefore all other coefficients are set to zero. It is important to note here that the coefficients used for this case were obtained irom test date of Reference 14. The results of this case are shown in Figures 64 and 65. Both plots show very


Figure 03. Data for Normal Penetration of Blunt Nosed Cylinder


Figure 64. Model Verification Usting Data of Reference 14. Projectile Velocity and Depth of Penetrition versus $T$ itme


Figure 65. Model Verification Using Data of Reference 14. Projectile Velocity versus Depth of Penetration
good agreement between model and exparimental va*ues. Buth figures show a comparison berween using constant value of $C_{x}$ for the whole velocity range and using values for $A_{x}$ and $C_{x}$ for velocities below the critical velocity of $100 \mathrm{~m} / \mathrm{sec}$. The overall effect of using - a value for $A$ is to reduce the depth of penetration at the lower velucities and brifig the projectile to rest at some finite time. The significant difference of the two casee is shown in the reduction of depth of penctiation of Figure 64.

Purther verification of the analytical model was obcained by use of experimental data of the Egin experiments. As discussed in Section II velocity measurements of aolid blunt nosed cylinders were made using an X-ray technique. The reduction in velocity during the trajectory was assumed to be attribuced to a drag term based on the expression

$$
\begin{equation*}
F_{x}=\frac{1}{2} C_{D} \mu_{s} A\left(\dot{x}^{\prime}\right)^{2} \tag{54}
\end{equation*}
$$

where $C$ is defined as the drag coefficient in the $x^{\prime}$ direction, $\rho$ is the undisturbed soil density and $A$ is the crosn sectional area of the projectile. By using this assumption and analyzing the data as in paragraph 4.6, it was found that $C_{b}$ had a variation with velocity as given in Table 11. Also, if $x$ 'and $x$ are assumed to be collinear, then $\dot{x}^{\prime}$ and $U$ are equal.

TABLE 11. VARIATION OF $\mathbf{C}_{\mathrm{D}}$ WITH VELOCITY $\dot{x}^{\prime}$.

|  | DRY SAND |
| :---: | :---: |
| $\dot{x}^{\prime} \mathrm{m} / \mathrm{sec}$ | $C_{D}$ |
| 0 | 4.0 |
| 12.7 | 3.0 |
| 25.4 | 2.2 |
| 26.2 | 2.0 |
| 304.8 | 1.9 |
| 2540.0 | 1.25 |

The data of Table 11 fit power law of the form

$$
\begin{equation*}
C_{D}=a U^{b} \tag{55}
\end{equation*}
$$



Geometric Values and Initial Conditions
mass - $5448^{m}$
${ }^{\prime} x_{x}=268 \mathrm{gm}-\mathrm{cm}^{2}$
$I_{y y}=23170 \mathrm{gm-cm}^{2}$

$$
\begin{aligned}
& I_{y y}-I_{z z} \\
& I_{x y}=I_{x z}=I_{y z}=0
\end{aligned}
$$

$U_{0}=4.06 \times 10^{4} \mathrm{~cm} / \mathrm{sec} \quad$ Three different runs

- $3.28 \times 10^{4} \quad$ with all other input
- $2.12 \times 10^{4}$ remaining constant.
$V_{0}=:_{0}=P_{0}=Q_{0}=R_{0}=\Psi_{0}-\phi_{0}=\theta_{0}=0$
$\dot{Y}_{0}^{\prime}=\dot{Z}_{o}^{\prime}-X_{o}^{\prime}=Y_{o}^{\prime}=Z_{o}^{\prime}=0$

FORCE COEFFICIENTS

$$
\begin{aligned}
& A_{x}=B_{x}-A_{y}=J_{y}=C_{y}=A_{z}=B_{z}-C_{z}-0 \\
& C_{x}=4.1\left(U_{\mathrm{cric}}\right)-0.1 \text { for all values above } U \\
& \text { prof undetermined but crit } \\
& \text { dpftoximately } 10^{4} \mathrm{~cm} / \mathrm{sec}
\end{aligned}
$$

Figure 66. Data for Sample Run of Computer Program II
where a 3.12 and $b=0.1$ when $U 1$ expremed in cm/aec. if equation (54) is compared to Equation (45) then the relation berween $C_{x}$ and $C_{D}$ le given as

$$
\begin{equation*}
c_{x}-\frac{1}{2} g_{n} c_{D} \tag{56}
\end{equation*}
$$

The density of the soil $p_{n}$ used in che experimant was $1.6 \mathrm{gm} / \mathrm{cm}^{3}$; chere$f$ re $C_{X}=0.8 C_{D}$.

Modifying the Computer Program I to include Equationa (54)to (56) resultc in the Cownuter Program II given in Appeadix C. The data used as Input for this case are given in Figure 66.

The results of these runs are given in the graphs of Figures 67 and 68. Figure 57 finows the variation of depih of penetration with variations in impact or initial velocity. Figure 68 shown correlation of model prediction with experimentally determined data.

### 6.5 CONELUSIONS

The results of the precesing paragraph show that a simple basic terradynamic approach usiug experimentaily determined force coetificients will yield reasonable results. However, it must be emphasized that this method is highly dependent on good data obtained by experiment. The lack of force coefficients for angle of attack and other than nomal impact freventa model verification for general cise.


Figure 67. Model Verification Uaing $C_{D}$ Da*a of Table 11

Figure 68. Model Verification Using $C_{D}$ Data of Figure 66

## SECTION VII

## SONIC ANE UITRASONIC WAVE SPEED GEASUREMENTS

### 7.1 INTRODUCTORY REMARKS

The preceding sections have dealt with the teating and performance characterincica of terradynamic vehiclea, inciuding trajectory, cavity ahape, separation, reattachment and stability. Predictive rechniques which allow for a quantitative asscsament of earth penetrating vehicie performance require information on certain parameters. Important among these parametera as inpute into cerradynamic modeln are the density and the acountic impedance of the target medium (Reference 29). The acoustic impedance, which is related to the wave velosity in the material, has particular importance in delineating regimes of application of penetration equations as vell es in perhaps predicting bow wave speeds observed in the $x$-ray atudies.

One technique which has been explored in the curxent test prigram for obtaining information on the above parametera is ultrasonic wave speed measurement. The use of ultrasound ss a diagnostic and measurement tool ad well documented. Poinlman (Reference 30), for example, has catalogued ultrasonic research topics in a series of volumes with frequent updating of the literature. Specific descriptions of ultrasonics in diagnostic applications have also been discissed in such references as (References 31,32), while techniques for measu:ement of meclianical parameters are contained in References 33 through 35 . :lany mechanical properiten measuremente by ultramand have been directed toward obtairing information on the elastic properties of either solid, liquid, or gaigeous media uaing the pulse-echo or through-transadseion techniques, as described for example by reskinin (Rafarence 36). Papadakia (Reference 37) and othere (Referancea 33 through 35). Some recenc properties measurements on solid heterogeneous media such as fibrous componize materielis and rock media have been reported on in References 40 through 42 . These media, while dispersive in nature, remain amenable to couventional ultrasonic testing procedurey becaune of the retention of apecimen shape during mechining. For gramiar or solid media, in which three distinct phases are present (solid, liquid and gas), and for which confined aamples are not readily produced, measurement of mechanical propertiee becomea more involved. Some data collection related to moil eedia has been reported in References 43 through 46 . Because of the phase inhomogeneity, properties measurements for soil types such as dry or eaturated ande are difficult tesk. For example, dilatacional vave apeede through dry sands by conic radiation provido reasonable proparties data, while imiler meagurements chrjugh saturated sand have proved unsatisfactory (Reference 43).

The filled porea allow rapid propagation of dilatational waves, so that meacurements of the waves tranamitted by the akeletal phase generally require the measurement of shear waves instead. These waves appear to reflect better standard measurement of the skeletal atiffness and are frequently used in testing soils properties at ultrasonic frequencies. For this reason many of the tests reported in the literaturs report data on the propagation of shear waves (Reference 44).

In obtaining such data it is important to note that considereble diaparity in the reported magnitude of acoustic waves in coila afpeara in the literature. This discrepancy is partially due to the measuring technique used, type of pulse disturbance used for generating the transmitted signal, and amplitude of resulting diaturbances. In any event it 1s important to note that current analytical models appear inadequate to predict wave velocitia, and it is neceasary to obtain quantitative measures of the wave velocities st rea: onils by experinental procedures (Reference 44).

Irs the stu:fan Eeported in thim section, wave speede in ary or molat Eglin said ( 5 to 15 percent moisture by weight) have been lavestigated with three purposes in mind: (1) to obtain input for existing penetra:ion codes requiring this information or for delineating: sunds on the usefulness of terradynamic equations (Reference 29), (2) for pote 'el relationship to observed bow shock wave speede (Reference 21), and, tor possible use as a tcol for establishing the compaction state of sand.

The major part of the investigation has been concerned with ultrasonic wave speed measurement an a function of coapaction and testing pressure. Several difficulties were encountered with the testing progran, and it atill has had only limited success. The difficulties were masiated with the dispersive nature of the sand medium, eapecially at high frequencies and low testing pressures. The ultrasonic wave speed messurement techniques and results will be degcribed in paragraph 7.2.

A brief diacussion will be given in paragraph 7.3 oi mome low frequency field measurements of sound wave apeeds.

### 7.2 EXPERIMENTAL PYOCEDURES AND RESULTS FOR ULTRASONIC WAVE SPEEDS

A Panametrics Ultrasonic Intervalometer system was available at the University that could be used either in a pulse-echo-overliap method with the same transducer used both for sending and for receiving the reflected signal or in a through-transmisaion method with separate mending and receiving transducers. Because of the dispersive uature of the medium, effort was concentrated on the through-transmisaion method. The Panameterics system can be used alone with broad band single pulses, or in conjunction with a pulsed radio-frequency (RF) oscillator it can be used with a burat of RF oscillations.

The first testing of sard samples used the aingle broad band pulse from the Uitrasonic Pulsing, Module of Panametrics system as input signal to a Panametrics Type V201 5 MHz longitudinal transducer. The sand was firsi compacted under conditions of uniaxial strain in a steel cylindur 0.05 meter in diameter under axial pressure of 4.4 to 22 MPa . After unloading, enc plates each containing one of the transducers were mounted on the cylinder and the broad band pulse appiled to one end. The broad band pulse contains all frequency components, but the received signal resembled a distorted cine wave with the first few oscillisions at a frequency 0.02 times the 5 MHz transducer resonant fiequency. An example is shown in Figure 69.

(Sweep Speed $20 \mu \mathrm{~s} / \mathrm{cm}$, vertical $50 \mathrm{mv} / \mathrm{cm}$ )
Figure 69. Received Pulse from Broadband Input Pulse

A portion of the input broadband pulse is seen at the beginning of the oscilloscope trace, driving the signal off scraen. The received pulse is amplified and mixed with the broadband pulse internally in the Panametrics unit and then displayed on an oscilloscope. Precise timiug can be accomplished in a manner similar to that for the RF bursts as will be described later in this section. More details on the operation of the Panametrics system are given in Reference 45.

The pulse displayed in Figure 69 was transmitted through a sand sample 0.0094 meter thick while under an axial pressure of about 0.2 MPa after compaction by an axial pressure of 4.4 MPa in the steel cylinder. The attenuation of the pulse by the sand was so great that this equipment and procedcre could nut te used with samples much thicker than 0.05 meter. Also the signal became erratic when the applied axial pressure during the wave speed measurement was about $0.1 \mathrm{MPa}(a p p r o x i m a t e l y ~ 15 \mathrm{psi}$ ) and disappeared altogether at some lower pressure. But the most disappointing feature of the results was that the measured wave speed appeared to depend on the thickness of the sample. This is a result of the change in shape of the pulse as it propagates through the dispersive medium. What was measured in these tests was the speed of propagation of the leading edge of the received pulse, which would be the group velocity of the transmitted wave packet of oscillations if the transmitted facket did not change its shape so much.


Figure 70. Leading-Edge Wave Speeds for a 0.01-meter-thick Dry Sand Sample


Figure 71. Leading-Edge Wave Speeds for a 0.023 -meter-thick Dry Sand Sample

Figure 70 shows the leading edge wave speed for a 0.01 -meter-thick dry sand sample versus axial testing pressure varying from about 0.05 MPa to about 0.30 MPa for specimens previously compacted at three different axial pressures. Figure 71 shows the same kind of plot for a 0.023-meter-thick dry sand sample. Figure 72 is a photograph of the steel cylinder in place in a fixture riounted in a Tinius Olsen universal testing machine to provide the ailal force during teating. The two transducer leads can be seen coning out of the fixutre. In this setup the transdurers (not visible) are inuide the end plates in direct contact with the sand.


Figure 72. Fixture for Ultrasonic Wave Speed Measurements in Sand Contained in Cylinjer Under Axial Load

Some additional tests of this type were performed on samplee with moisture conients of 5,10 , and 15 percent by weight. At the highest forming pressure the 15 percent semple wan saturated. The other samples were not saturated. The results indicated that increasing the moisture content facreases the attenuation and decreases the wave speed, but the leading-edge speed results were again imprecise because of the changing shape of the transmitted pulse.

It was proposed then to obtain an $R F$ pulser, since it was belleved that the RF bursts, contairing a single dominant RF frequency could be used to meagure group velocity of the RF burst in a manner that would not appear to depenc upon specimen thickness. It was also proposed to use the new equipment to study the effec: of varicus amounts of air and water in the three-phase sand medium. In particular it was proposed to compare results with the predictions of an equation derived by Liahov (Reference 46) and discussed by Cristescu (Reference 47)

An Arenberg Ultrasonic Laboratory oscillator, Model II (PG-65-2-C) with 400 watts peak power was obtained. It was furnished with three coils for the RF frequency ranges of 4.3 to $7.5 \mathrm{MHz}, 0.81$ to 1.1 MHz , and 0.45 to 0.62 MHz . Additional colls can extend the range to operate anywhere in the 0.13 to 190 MHz frequency range with some lose of power at the lower frequencies.

Figure 73 shows a block diagram of the Arenberg oscillator (RF pulser) connections with the Fanametrics unit. The single pulse from the Pulsing Mndule of the Panametrics unit triggers the Arenberg RF Pulser which then emits an RF frequency burst of variable length (e.g. 5 to 20 cycles at 0.5 MHz ). This input pulse travels to the sending transducer which sends a mechanical stress wave burst through the sample to the receiver and preamplifier and then to the oscilloscope (CRO). The input pulse is also attenuated and fed directly to the CRO $y$-input terminal for comparison with the received signal.

If the transmitted signal has the same wave form as the input signal, very precise timing can be obtained as follows. After the attenuated input signal and amplified output signal have been displayed on the oscilioscope (CRO) using the internal sweep of the CRO, the final precise measurement is made by switching to a sweep provided by the variable-frequency CW oscillator of the Panametrics system. The sweep frequency is adjusted so that the transmitted and received signals are made to overlap. The tine interval between the two signals is then measured by the frequency counter of the Panametrics system.

In actuality the transmitted wave pulse is modulated by the transducers and the transmission through the sand, so that the output resembles the received signal from the broadband input (see Figure 69) more than it resembles the input RF signal, which has sin essentialiy aquare envelope. But the oscillations within the pulse are at the frequency of the $R F$, and the overlapping technique can still be applied, except at low testing preesures where there is excessive distortion. For best results the RF burst should contain at least 20 cycles and the overlapping should be made to coincide at the middle of the burst, since there is some distortion at the beginning and at the end of the burst. Because the great attenuation made it necessary to use a very short specimen path, it was not possible to use such a long pulse, and usually the overlapping was performed on the second or third peak in the burst. This use of the RF technique did succeed in removing the apparent dependence of the measured wave speed on the thickness of the sample when the test was performed at axial pressures above about 0.5 MPa. Differences in wave epeed between the different specimens was within 3 percent which is consiaered very close agreement for different sand samples. The sand specimen holder was redesigned so that the transducers were not in direct contact with the sand but transmitted the signal through the steel end plates without themselves being subjected to the static axial loads. They could thus be left in place during the axiai loading to compact the sand and could measure wave speed at various times during the loading and unloading.

Figure 74 shows stress-strain curve for a uniaxial strain test cariled out in the steel cylinder. At che points merked on the curve, wave speed measurewents were performed, with the results shown in Tabie 12.


For use with broad band pulse, the $T / R$ terminal is connected directly to the sending transducer, and the receiving transducer is connected to the $R$ terminal on the Fanametrics unit for internal amplification and mixing with the input pulae.

Figure 73. Block Diagram of Interconnections Between Arenberg Oscillator (RF Pulser) and Panametrics Unit

table 12. have spedd during unlaxial strain test

| Point | Stress (MPa) | $\begin{aligned} & \text { Density Ratic } \\ & \rho / \rho_{0} \end{aligned}$ | Wave Speed (m/sec) |
| :---: | :---: | :---: | :---: |
| Loading |  |  |  |
| 1 | 3.08 | 1.065 | 910 |
| 2 | 6.14 | 1.081 | 1195 |
| 3 | 9.22 | 1.097 | . 1280 |
| 4 | 10.98 | 1.106 | 1340 |
| 5 | 13.17 | 1.114 | 1390 |
| 6 | 16.48 | 1.128 | 1470 |
| 7 | 22.17 | 1.152 | 1600 |
| Unloading |  |  |  |
| 8 | 13.61 | 1.150 | 1420 |
| 9 | 8.78 | 1.145 | 1280 |
| 10 | 6.14 | 1.142 | 1165 |
| 11 | 4.39 | 1.140 | 1070 |
| 12 | 2.19 | 1.136 | 960 |

These wave speeds are group velocities of the RP bursts. They show a very utrong dependence on the testing piessure. It is remarkable how iftcle difference was measured between the velocities duriag unlooding and those during loading, despitc the different compaction tatee and the different slopes of the loading and unloading curves. It seems clear that ultrasonic vave speed measurement will not be a good cool for detemining the compaction condition.

For the last series of tests the 5 MHz transducer was replacad by a 1 MHz Pansmetrics Type V103 transducer, and testing was again performed with bursts of 0.5 MHz . By operating nearer the transducer resonance and by using the larger 1 MHz transducers with 4 times as much frontal arisa a greater power could be transmitted. It was hoped that this would permit testing at much lower axial pressures wore like the anient pressures in the penctration experiments. A signal was in fact received at pressuras
well below the previous minimum testing pressure of 0.5 RPa. but the transmittad aignale were so badiy distorted that the pulse overlap technique could not be used. The oscillations in the transmitted pulat appeared to be at Irequency sbout 20 percent below the RF frequency of the input aignal. The reason for this is not clear, but ic la beligutd to be related to the fact that at these low pressuren the phase velocity is so luw that the wave length approaches the order of magnitude of the sand particle size. Thus the medium no longer responds as a continum. Because of the distortion, only leading-edge wave speeds could be measured at the low pressures and these with the same kind of ercors and apparent dependence of the wave speed on specimen thickness as were previously observed with the broadband pulses.at all testing pressures.

At. higher testing pressures, however, good clean RF bursts were transmitted by the 1 MHz transducers operating at 0.5 MHz , and the group velcitien could be measured quite accurately by the overlap technique.

The last serisz of tests examined the variation of the group velocity with testiag pressure for pairs of dry sand epecimens all of ubout 0.025 meter thickness with different initial densities. The two different initial compaction states at the same pressure were achieved by ahaking one of the two specimens (by tapping the side of the steel container with a hammer) to compact the a ind instead of by initially compressing it under axial load. Results for three such pairs of specimens, tested by loading to three different maximum values of axial pressure are shown in Figures 75 to 77. Each Eigure thus gives one curve for an initially loose and (solid curve) and one for an initially dense sand tested over the same range of pressures. The so-called dense sand had an initial density 4 to 7 percent greater than that for the initially loose sand. The additional density increase during the test varied fiom about 1 percent for the dense and in Figure 75 to about 5 percent for the loose sand cested to a higier pressure in Pigure 77. The arrows on each curve indicate the direction of loading or unloading. Aithough the loading and unloading curves are not identical, the wave speeds at any testing pressure do not vary a great deal from the loading curve to the unloading curve. The extreme values from the three curves are listed in Table 13. The last-point densities upon unlouding were not recorded, but they axe approximately equal to the maximum cencities.

Evidently the wave speeds are much more dependent on testing pressure than on the compaction atate, so that wave speed measurements are not a good measure of compaction state.

Because of the difficulties encountered in the program, cime did not permit further zesting of wave speeds and attenuation ac a function of moisture content. The Liahov equation (References 46,47 ) predicts significant differences in sound wave upeeds in almot aturated ands for vcry small changes in the air content. Variations in the air volume fraction from 0.005 to 0.04 would change the wave apeed by a factor of a third. It was not poasille to attempt any verification of thie theoretical prediction in the present program because of the difficuity in contriling the air content as well as the difficulty in measuring ultramonic velocities at low preseures in aand.




TABLE 13. EXIREME VALLES OF PRESSURE AND HAVE SPEED IN FIGURES 75 to 77

|  |  | $\begin{aligned} & \text { Density } \\ & \left(\mathrm{kg} / \mathrm{m}^{3}\right) \end{aligned}$ | Pressure (Mifa) | Wave Speed (m/sec) |
| :---: | :---: | :---: | :---: | :---: |
| Figure 75 |  |  |  |  |
| Loose Sand | First Point | 1638 | 1.0 | 875 |
| Teat 7 | Maximum | 1666 | 3.5 | 1140 |
|  | Last Point |  | 0.5 | 790 |
| Dense Sand | Firs $=$ Point | 1726 | 1.0 | 960 |
| Test 2 | Maximum | 1742 | 3.5 | 1170 |
|  | Last Point |  | 0.5 | 875 |
| Pigure 76 |  |  |  |  |
| Loose Sand | First Point | 1573 | 1.0 | 735 |
| Test 9 | Maximum | 1636 | 7.0 | 1230 |
|  | Last Point |  | 0.5 | 810 |
| Dense Sand | First Point | ¿677 | 1.0 | 890 |
| Test 6 | Maximum | 1723 | 7.0 | 1370 |
|  | Lest Point |  | 0.5 | 810 |
| Pigure 77 |  |  |  |  |
| Loose Sand | Firat Point | 1585 | 0.5 | 710 |
| Test 8 | Maximum | 1653 | 10.5 | 1335 |
|  | Last Point |  | 0.5 | 670 |
| Dense Sand | Pirst Point | 1644 | 0.5 | 835 |
| Test 5 | Maximun | 1723 | 10.5 | 1500 |
|  | Last Point |  | 0.5 | 810 |

The difficulty at the lower, preasures is now belleved to be caused ty the fact that the lowex wave apeeda at the lower confining presaures lead to wave lengtha of the order of magnitude of particle size. At how pressures, wave speeds of the order of $250 \mathrm{~m} / \mathrm{sec}$ have been reported (Reference 14) although not at ultrasonic frequencies. The relationship between phase velocity $c$, frequency $f$, and wave length $\lambda$ is (Reference 48)

$$
\begin{equation*}
c=\square \lambda \tag{57}
\end{equation*}
$$

At $f=1 \mathrm{MHz}$, and $c=1000 \mathrm{~m} / \mathrm{sec}$ (the order of magnitude of the gruup velocitien observed in the present program under high pressures) the wave length is $10^{-3}$ meters while the sand grain size is of the order of $10^{-4}$ to $2 \times 10^{-4}$ meters. If at low pressures a speed of $c=250 \mathrm{~m} / \mathrm{sec}$ could be expected, this would give a wave length at 1 MHz of $2.5 \times 10^{-4}$ meters, about the same as the grain size. If the same spoed $250 \mathrm{~m} / \mathrm{sec}$ prevailed at lower frequencies, it would give $\lambda=5 \times 10^{-4}$ meters at 0.5 MHz and $\lambda=10^{-3}$ meters at 0.25 MHz .
This suggests that the high frequencies are not suitable for use In the sand. A brief discussion of some lower frequency testing is given In paragraph 7.3. The discussion of Equation (57) applies to phase velocities, while the measurements reported in this section have all been group velocities. In a dispersive medium, the phase velocity of a dilatational plane wave is a function of wave length, say $c=c(\lambda)$. A wave packet, such as the RF bursts of the experiments described, contains a spectrum of phase velocities with a dominant. mean phase velocity, say $c_{c}$ at wave length $\lambda_{0}$. The packet containing wave lengths predominantly near $\lambda_{0}$ travels at a group velocity $U$, whish is reiated to $c_{o}$ by the following equation (Reference 48)

$$
\begin{equation*}
U=c_{0}-\lambda_{0}\left(\frac{d c}{d \lambda}\right)_{0} \tag{58}
\end{equation*}
$$

If the meitum is nordiapersive $d c / d \lambda=0$. Tien $U=c_{0}$ and $U$ is independent of wave length and frequency. But if the medium is dispersive, dc/d $\neq 0$, and the group velocity may differ markedly from the phase velocity. The phase velocities for the sand are not known at these frequencies, so an: evaluation of $U$ by Equation ( r 8 ) is not feasible. It was siacrved in some preliminary tests that the group velocity of the RF was somewhat frequency dependent, but time and evailable equipment did not permit a determination of the frequency dependence over a wide frequency range.
The pulsed RF measurements can be used to determine phase velocity by using a specimen of thickness equai to one wave lergth (Refersnce 39), but this is not easy to do with the short wave lengths of the ultrasound. The technique is similar to that used with continucur waves. Some preliminary rests with continious waves at lower frequencies are described in paragraph 7.3.

### 7.3 SOUND WAVE PHASE VELOCITIES

In July 1976 some preliminary field rests of low-frequency sonic phase velocities were performed at Eglin. Thn scurce for the sound waves was a 50 -pound dynamic force $M B$ vibration-testing shaker and pc r: amplifier
belonging to the University. A 0.1 -meter-diametar aluminum plate was fabricated at the University and mounted on a biaftattached to the shaker armature and extending 0.15 mbter outiside a wooder box containing the shaker. The box was partly buried in the ground.

The tryout of the equipment was conducted jointly with the Mines Branch AFATL/DIJM, which provided geophones, recording equipment, and npectrum analysis. The setup worked well. A good strong continuous wave signal was obtained at a distance of 3.7 meters from the shaker, even when the power amplifier driving the shaker was operating at very low power. Higher power sometimes led to distortion of the sine-wave signal. The technique invoived recording the signal at two stations. Frequency : was incrersed slowly until the two signals were in phase, indicating thac the two stations were one wave length $\lambda$ apart. The dilatational phase velocit, $c$ is then given by $c=f \lambda$.

The preliminary tests indicate some dependence of wave speed ou frequency, varying from about $108 \mathrm{~m} / \mathrm{sec}$ at 59 Hz to 170 to $180 \mathrm{~m} / \mathrm{sec}$ at around 100 Hz for two receiving stations 1.85 meters apart. The phase valocit at 100 Hz was compaiable to the epeed of $168 \mathrm{~m} / \mathrm{sec}$ determined from the leading edge wave speeds of pulses produced by hammer blows. Higher frequency teacs (up to about $19,000 \mathrm{~Hz}$ ) were also recorded on magnetic tape for later analysis by methods not requiring the two siguals to be in phase. Further tests of this type should be performed. It seems to be a good method for low-frequency sound-wave speed measurementa, although care must be exercised in interpreting the resulcs, which may be affected by reflections from the free surface andor from internal boundaries in stratified media, especially when the two geophones are more cisan one wave length apart.

## section VIII

## SUMMARY AND CONCLUSIONS

The results and conclusions of each phase of the investigation have been reported in previous sections. This final section aumarizes them and indicates where more detail about them may be found. Section II described the Eglin experimental program and the various types of sensors used in it or evaluated for possible use. The sequential flash X-ray technique was judged co be the most successful mathod investigated, since it not only gave more complete and precige information about the trajectory and the projectile's position and attitude at various times than did any other method but also gave information on cavity formation and meparation points and, in some cases, showed a shock wave ahead of the projectile. This Investigation is believed to be the mest extensive use ever made of flash radiography in terradynamic research. The magnetic sensors also provided good information about horizontal velocity.

The results of the experiments were described in Section III and interpreted in Section IV. The trajectory plots of paragraph 4.2 for the prinary test program showed that the flat-nosed and step-tier projectiles had followed remarkably straight and stable horizontal paths through the 1.2-meters-long teat chamber, although most of them exhibited a slight rise. Because the paths were so nearly straight, analysis by one-dimensional terradynamic models was feasitle. A cubic interpolation fomula gave a very accurate representation of the horizontal position-time data, and of the velocity near the middle of the interval. A classical Poncelet forcelaw penetration model, discussed in paragraph 4.3.2, gave an excellent account of the otserved parts of the trajectories in dry sand, with a drag coefficient essentially independent of the striking velocil in the range of velocities observed. In saturated sand, each shot could be fitted by the Poncelet model, but the dras coefficient appeared to depend on the striking velocity, which showed that the Poncelet model does not really apply.

Drag coefficient variacion along the trajectory was exhibited in paragraph 4.6. Although the velocity calculations of that section, each based on average velocity between only two stacions, tend tu magnify an error at one of the stations, they do show a trend of variation along the path, more pronounced in the wet sand cases than ini the cry. The clasaical Poncelet force law gave more consiutent results than modification of the Sandia empirical method. In all of the anslyses of Section IV, force law coefficients were determined to fit observed penetration data, anc the success of a model was judged on the basis of agreement between the coefficient values fitted to the differani siota.

The cavity-expansion penetracion model of Section $V$, on the other hand attempts to predist the penetracion behavior from statically measured soil properties. Despite the rather strong asisumptions invoived in this simple analytical model, it gave very good reauits in predicting the behavior for two flat-nosed projectiles in dry aand. It was nereusary to assume a shape for the false nose of aland carried along by the ilat-nosed profectile.

An assumed hemiapherical nose or conical nose with length-to-diameter ratio of 0.4 to 0.5 led to very close agreempat between the predicted and observed position-time an. velocity-position curves, even for shots in a velocity range higher than the range for which previous investigations had validated the method. The success of this model suggests that it whould be consiliered further, posibly for obilque impacts.

A three-dimensional trajectary analysis based on an assumed three-dimensional differential force law was presented in section VI. The procedure was carried through for a case of a straight trajectory with a drag coefficient varying with velocity according so a power law, with reasonable results. It could be applied to a trajectory with an angle of attack or an oblique impact $x f$ stitabie force coefficients could ha determined of estimated.
ection VII reported on an independent investigation of ultrasonic wave speeds as a function of sand compaction and testing pressiure. Several difficuitics were encountered in the investigation. Pulse shape changes made it impossible to establish single broadband puise propagstion speeds that weze independent of path length. This difficulty was overcoune by using RF bursts instead of single pulses. Group velocities measured for these bursts gave (with an RF frequency of 0.5 MHz ) consistent results for ambient testing pressures greater than about 0.5 MPa , but at lower prossures the signals were tou badly distorted $=0$ give consistent results. This is belleved to he a recult of the fact that the lower pressures lead to lower wave spzeds and a wave length of the order of sand grain dimensions. It is recommended thar in further studies of sound wave speeds in sand actention be concentrated on lower frequencies. The group velocities showed a greater dependence on the testing pressure than on the compaction state.

## REFERENCES

1. Triandafilidis, G.E., State of the Art of Earth Penetration Technology, TR CE-42(76)DNA-297. May 1976.
2. Poncelet, J.V., Cours de Mécanique Industrielle, First Euition, Paris, 1829.
3. Hanagud, S. and Ross, B., "Large Deformation, Deep Penetration Theory for a Compressible Strain Hardening Target Material", AIA Journal, Vol. 3, No. 5, pp. 905-911, May 1971.
4. Rohani, B., 'rifgh Velccicy Isugmeni Fineteation of Soll Tnrgata', Proc., Conference on Rapid Penetration of Terrestrial Materials, Texas A \& M University, College Station, Texas, 1972.
5. Norwood, F.R., Cylindrical Cavity Expansion in a Losking Soil, Sandia Laboratories, SLA-74-0201, Albuquerque, New Mexico, July 1974.
6. Bernard, R.S, and Hanagud, S.V., Development of a Projectile Penetration Theory, Report I Penetration Theory for Shallow to Moderate Depths, U.S. Axmy Engineers Waterways Experiment Station, rechnical Report s-75-9. Vicksburg, Mississippi, June 1975.
7. Bernard, R., Development of a Projectile Penetration Theary, Report 2, Deep Penetration Theory frr Homogeneous and Layered Targets, U.S. Army Engineers Waterways Experiment Station, Technical Keport S-75-9, Vicksburg. Missisaippi, February 1976.
8. Henderson, D. and Stephens, R.i.. "Impact and Penetration Technology" paper presented at the Fuze-Munitions Environment Characteriaation Symporium at Picatinny Arsenal, New Jersey, by AVCO Corp., November 1972.
9. Hermann, W. A Lagrangian rinite Difference Method for Two-Dimensional Motion Including Material Strength, AFtI, Technical Report WL-TR-64-107, November 1967.
10. Hageman, L. $\mathrm{J}^{\text {. }}$ and Walsh, J.M. HELP-A Muiti-Material Eulerian Program for Compressiole Fluid and Elastic-Plastic Flows in Two Space Dimensions and Time, Systems, Science and Software, 3SIR-350, Vol.I, La Jolla, California, 1970.
11. Sedgwick, R.T. Theoretical Terminal Eallistic Investigation and Studies of Impac: at Low and Very High Velocities, General Electric Co., Space Sclences Laboratory, Technical Report AFATL-TR-68-61, King of Prussia, Pennsylvania, 1968.
12. Young, C.W. The Development of Empirical Eguations for Predicting Depth of an Farth-Penetrating Profectile, Sandia Laboratories, SC-DR-67-60, May 1967.
13. Young, C.W. Empirtcal Equations fol Predicting Penetration Performance In Layered Earch Maceriais for Complex Penetrator Configurations, Sandia Laboratories, SC-DR-72-0523, December 1972.
14. Allen, W.A., Mayfield, E.B. and Korrimon, H. L. "Dynamice of a Projectile Penetratiug Sand," Jouraal of Appliad Physics, Vol. 28, Pp 370-375 and 1331-1335, 195\%.
15. Hakala, W.W. "Resistance of a Granular Medium to Normal Impact of a Rigid Projectile". Ph.D. Diaseriation, Virginia Polytechnir. Institute, Biacksburg, Virginis, June 1965.
16. Robertson, H.P., Terminal Ballistica, National Reaearch Council, Washington, D.C., 1941.
17. McNeill, R.L., "Rapid Penetration of Terreatrial Materials - The State of the Art", Proc., Conference on Rapid Penetration of Terrestrial Materials, Texas A 6 M لniversity, College Station, Texas, 1972.
18. Colp, J.L., Catidle, W.N., and Ronine, K.L., An Annotated Bibliography of Sandia Laboratories Publications Related to Terradynamics, Sandia Laboratories, SLA-73-0345, March 1974.
19. Wood, H.R., "Instrumentation of Penecration Projectiles Ueing Hard Wire * RF Datn Transmiacion", Proc. . Conference on Rapid Penetracion of Terrestrial Msterials, Texas A. \& M University, College Station, Texas. 1972.
20. Murff, J.D. and Coyle, H.M., "A Laboratory Investigation of Low-Speed Penetration," Proc., Conference on Rapid Penetration of Terrestrial Materiale, Texas A \& M University, College Station, Texas, 1972.
21. Culp, M.F., Submunition Penetration into Tactical Targeta, Lockheed Misuiles and Space Co., D434546, December 1975.
22. Culp, M.F., "Flash X-Ray Investigation of Penerator Terradynamics," paper presented at 2 'nd International Symposium on Ballistics, sponsored by American Defense Preparedness Association, Daytona Beach, Florida, March 1976.
23. Hoffman, P, R., McMath, R.R., and Migotsky, E., Profectile Penetration Studies, AFWL-TR-64-102, December 1954. (RTD-WL-TR-64-102)
24. Biele, A., An Investigation of che Terradynamic Stability of a Scaled Model Prodectile, Masters Thesis, Miesiseippi State University, August 1973.
25. Lambe, T.W. Soil Testing for Engineers, John Wiley \& Sons, New York, 1951.
26. Goodier, J.N. "On the Mechanica of Indentation and Cratering in Solid Targets of Strain Hardening Metal by Impact of Hard and Soft Spheres", Proc, ${ }^{\text {7th Symposium on Hypervelocity Impact, Vol. III, AIAA, New York, }}$ 1965, pp. 215-259.
27. Etkin, B., Dynamics of Flighi, John Wiley and Sons, N.Y., N.Y., 1959.
28. Etkir, B. Dynamics of Atmospheric Flight, John Wiiey and Sons, N.Y., N.Y., 1972.
29. Hadala, P.F. Evaluation of Emplrical snd Analytical Procedurea uned for Predicting the Rigid Body Morion of an Earth Penetrator, WES Report No. S-75-15, June, 1975.
30. Pohlman, R. Documentation in Ultrasonics (a citation series), Published by Laboratory for Ultrasonics. Aachen, Germany, 1970-1976.
31. Sharpe, R.S. Research Techniques in Nondestructive Teating, Academic Press, 1970.
32. Nondestructive Testing, NASA SP-5113, 1973.
33. RrautKramer J. and KrautRramer, H. Ultrasonic Teating of Materials, Springer-Yerlag, N.Y. Inc., 1969.
34. Green, R.E., Jr., "Ultrasonic Investigations of Mechanical Properties" in Treatise on Materiala Science and Technologz, Vol. 3. Academic Press, 1973.
35. Schreiber, E., Anderson, O.L., Soga, N. Elaetic Constants and Their Messurements. McGraw-Rill Book Co., 1973.
36. MeSkimin, H.J. "Pulse Superposition Method for Measuring Uitrasonic Wave Velocities in Solids", J.Acoustical Society of America, Vo. 33, pp. 12-16 1961.
37. Papadakis, E.P. "Absolute Accuracy of the Pulse-Rcho Overlap Method and the Pulse-Superposition Method for Ultrasonic Velocity". J.ficoustical Society of America, Vol. 52, pp. 843-949, 1972.

3B. Ross, C.A. and Sierakowski, R.L. "Elastic Wavas in Fiber-Reinforced Composites", Shock and Vibration Digest, Vol. 7, pp. 1-12, 1975.
39. Martin, A.G. "Ultrasonic Phase Velocity Measurement by Phase Comparison of Continuous or Pulsed Waves", Private Communication, 1976.
40. Friedman, M. and Bur, T.R. "Investigations of the Relations among Residual Strain, Pabric, Fracture and Ultrasonic Attenuation and Velocity in Rocks", Int.J.Rnck Mech. Min. Sci. and Geomech. Abstra. Vol. 11. Pp. 221-234, 1974.
41. Johrison, J.N., Lingle, R., Swolfs, H.S. The Determination of In Situ Anisotropic Elastic Moduli from Laboratory Ultranonic and Field Setamic Measurements, Terra Tek TR 75-28, June, 1975.
42. Barker, L.M., Lingic, R., Hendrickson, R.R., Jonason, J.N. Meaburement of Low-Pressure Static Elastic Moduli of Rock and Comparison with Ultrasonic Data, Terrs rek. TR 75-30, October, 1975.
43. Lawrence, F.V., Jr. The Responge of Soily to Dynamic Loaings; Ultrasonic Shear Wave Velocities in Sand and Clay. WES Research Report R 65-05, January, 1965.
44. Richart, I.E., Jr., Hall, J.R., Jr., and Woode, R.D. Vibrations of Solla and Foundations, Prentice-Hall Co. 1970.
45. Operations Manual for Ultrasuaic Time Intervalometer Pulsing Module 5053, Panametrics, Halthem, Mass., 1975.
46. Liahov, G.M. The Fundamentplg of the Dynamics ef Explosions in Soils and in Liquid Media [in Russian], Koscow, 1964.
47. Cristeacu, N. Dynamic Plıaticity, North-Kniland Publ. Co., 1967.
48. Lindsay, R. B., Mechanical Radiation, McGray-Hill Book Co., 1960.

## APPENDIX A

## DATA FROM EGLIN PENETRATION EXPERIMENTS

This appendix tabulates data from ii shote of the Eglin experimental program，using one page for each ahot，and listed in order by Shot Numbere． Descriptions of the various kinds of entries in the tables，both of experi－ mentally measured data and of several kinde of information calculated in the data analysis，are given in paragraph 3．Furthex explanations of mome of the calculated data groups may be found in paragrapha 3.3 and 4．4．

$$
\text { SHOT } 14 \text { ( } 10 \text { MARCH 1976. NO. } 31
$$

SAND：CRYO UENSITY：1538．KG／M由\＃3：APPFOACHING YELOCITY：420．M／S PROJECTILE：SOLIC FLAT NOSE MASS：0．545I KGO D＝0．02 ME L＝C． 225 M

| R－RAY | STATION | －－－－＊＊＊＊ | NO． 1 | NO． 2 | NO．3 | NO．4 | NO． 5 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| T\＆ME（SECOND）$\because . . . .0 .0$ ．00G＜24 CENTER OF GRAVITY HOSITION（M） $\begin{array}{lllllll}\text { HLKILONTAL } \\ \text { VEHTSCAL } & 18.0 .815 & 18.820 & 0.90 C \\ 0.167\end{array}$ |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |
| INCLINATION ANGLEGUEGIE， |  |  |  | 1－0 | 事禹串 | 中禹禹 | －6． 5 |
|  |  |  |  |  |  |  |  |
| $\begin{aligned} & \text { ABOY } \\ & \text { SELO } \end{aligned}$ |  | $\bullet \bullet$ | 坓管 | 年東 |  | －${ }^{\text {＋}}$ | ＊ ¢ $_{\text {＋}}$ |
|  |  |  |  |  |  |  | 0.0230 |
| NCSE | ZSITION | M |  | $0 \cdot 222$ | 10.535 | 18.839 | 1.013 |
| VERT | TICAL | － $0 \cdot$ | $6 \cdot 1 \angle 7$ | 0.134 | $18.12 i$ | 18．821 | 0.154 |
| INPUT NOSE POSITLUN（M） |  |  |  |  |  |  |  |
| C | IZONTAL | $\bullet \bullet \bullet \bullet \bullet \bullet$ |  |  | 禹事事事草 |  | $0.067$ |

 PROJECYILE: SOLIO FLAT MUSE MASS:C. 5452 KG . D=0.02 M. Lmu. 225 M




 PROJECTILE: SOLIO FLAT NOSE MASS:U.5شG1 KG. D=0.02 M: LEO.225 M


SANC: ORY, DENSITY:1338. KG/MF\#3: APPROACHING VELOCITY: 213. M/S PROJECTILE: SCLIU FLAT NLSE MASS:U.S4Si KG. D=0.02M. L=0.225 M

 COEFFICIENTS OF CUEIC POLYNUMIALASTANDARD DEVIATION $0.0014(\mathrm{~m})$

NUSE VEL X-COMF (M/S): 200. 17B. 78.
COEFFICIENTS OF CUEIC FOLYNOMIAL/STANOARO DEVIATION


C.G. VEL V-COMF ©M/Si: 7. B. B.

COEFFICIENTS DF CUGIC POLTNOMIAL/STANDARO DEVIATION U.1187E OU $0.6934 E 01-0.9250 E 03$ U.7028E OS/ 0.0019 (M)
 COEFFICIENTS OF CUBIC PILYNOMIALISTANDARD DEVIATION
-U.1165E UO 0.245GE 03-0.1107E OS G.3011E 06/0.0045 (M)
PONCELET ORAG COEFF• $=1.624$


SANO: ORY. DENSITY:1538. KG/M\#\#3: APMROACHING VELOCITY: 2II. M/S PHOJECTILE: SOLIU FLAI NOSE MASSiU.545y KGO D=0.02 M. L=6.225 M


SANC: CHY. DENSATY: A338. KG.M\#\#3: AFFROACHING VELOCITY: 340. M/S HKCJECIBLE: SOLIC FLAI MUSE MASS:C.545! KG. D=L.U2 M. Le0.225 M



MOSE VEL - X-COMN. (M/S): 3h3. 274 . 213. 165. 141. COEFFICIEIVTS OF CUEAC PULYNUMIALSSTANDARD DEVIATION




CUEFFICIENTS OF CUBIC PULYNOMIALISTANLARD DEVIATION

C.Ge VEL. X-COMP. (M/SI: 313. 274. 215. 165e 141.

COEFFICIENTS OF CUS:E POLYMOMIAL/STANDARO DEVIATICN

PONCELET ORAG CUEFF. $=1.769$


SANC: URY. DENSITY:15J8. KG/M象 Ji APPROACHING VELOCITY: 328. M/S



NOSE VEL Y-COMF (M/SJ: 13 - 140 . 4.


NOSE VEL• X-CDMF. (M/S): 372 (30U. 167. 137. 259.




COEFFICIENTS OF CUEIC POLYNUMLAL/STANCARU DEVIATIUN

PONCELET DRAG COEFF : $2 \cdot 496$

 PRCJECTILE: SOLIU FLAT NOSE MASS:G.54SI KG. DUG.02 M. L=0.225 M


NOSE VEL $Y$-COMP. (M/SY: 10 . 3.


NOSE VEL•X-COMP. (M/5): 309. 274. 218. 1650 131.
CCEFFICIENTS OF CUBIC POLVAUMIALISTANDARO DEVIATION


 COEFFICIENTS OF CUEIC POLYMOMINKSTAYOARN LEVIATION 0.0018 BMI

COEFFIC:ENTS OF GUBIC PULYNUMIAL/STANOARO DEVIATIDN -U.1U79E OO C.3iS4E OS CC. C72JE US U.1309E O7\% 0.0055 (M)

PONCELET OAAG CGEFF. $=1.723$

 PROJECTILE: SDI.AO FLAT NOSE MASS:0.64ASKGO O=GOO2 M. L30.225 M

 PHOJECTILE. SOLIG FLAT NUSE MASSdO.S443 KG. O=0.02 M. LEO. $22 S \mathrm{M}$




NOSE VELO X-CCMPO (M/5): 381. 335.1760



|  |
| :---: |
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CDEFFICIENTS OF CUEIC POLYNUMIALSSYANDARD DEVIATION

C.G.VEL. X-CIMS. (M/SI: 381. 335. 266. 207. 177. CIERFICIENTS JF CUERG PULYNOMIAL STANOARD DEVIATION -U.13COE OH 0.39iSE US -0.4469E OS C.3092E 07/ 0.0054 (M)

PUNCELET DRAG CUEFF. $x 1.084$


SANC: DRY. DENSITY: 1S3日. NG/MAW3: APPFOACHING VELOCITYZ $406 . ~ M / S$ PHOJECTILE: SOLIC FLAT NUSE MASS:0.5443 KG. O=0.O2 M. L=U. 225 M


SANE: LRY. OENSITY:1 S3O. KG/MFWS: APPROACHING VELOCITYI GCO. M/S HFUJECTILE: SDLAU FLAT MLSE MASS: UAE.43 KG. D=0.02 M. L=0.22S M

 COEFFICIENTS OF CUGIC FQEVNUMIALASTANOARD VEVIATION

MOSE VEL. X-COMF- (M/S): 371. 330 , 273. 189. U4. COEFFICIENTS OF CUEIC FOLYNOMIM SSTANUARD CEVIATIUN $-0.1585 E-01$ 0.3774E $63-C .3029 E G 5-0.2136 E 06 / 0.0 C 72$ (M)

C.G. VEL. Y-COMS. (M/S): 16. 13. 6.

COEFFICIENTS OF CUBIC PULYNUMIAL STAFIOAFO DE:VIATION $0.1413 E$ OO 0.17USE $02-6.3688 E$ U4 U.35i7E 06/0.0010 (M)

CDEFPICIENTS OF. CUHIC FOLYNUMIAL/STANDAHO DEVIATION
-U.1289E UU U.3781E $03-0.3429 E 05-0.2130 E 06 / 0.0073(\mathrm{MI}$
PONCELET DRAG COEFF. 1.764


SANO: DRY. OENSITV:ISJB. KIIANGHI APPRUACHING VELOCETY: AGS. M/S









 COEFFICIENTS CF CUEIC PULYNOMIALISTANDARD DEVIATION


PONCELET ORAG CCEFF. $=1.700$
$V O=265$ STAND. DEVIA. $=0.002 S$ (M)





SANG: ORY, DENSITY: 1 SSE KG/M由H3i APPROACHING VFEDCITY: $352 . ~ M / S$





COEFFICIENTS CF CUEIC POLYPMMIAL/STANDARU DEVIATIGN


 CDEFFICIENTS OF CUEIC POLYNOMIAL/STANDARD DEYIATION

 COEFFICIENTS OF CUHIC POLYNCMIAL/STANOARD OEVEATION -0.1318E OU O.363日E U3-0.2275E OS U.8028E O6/ 0.0019 (M)

PONCELET DRAG CCEFF $=0.910$




SANO: ORY: DENSITY:1SARA KG/M\#\#S: APPHOACHING VELOCITY: $352 . \mathrm{M} / \mathrm{S}$ PROJECYILE: SOLIO BICUMIC MASS:G.4964KG. D=0.02 M. LEV. 226 M











MOT 34 ( 13 MARCH. 1876, NO. 31
 PKUULGTILE: HULLUW GICUNIC MASS:O.A436 KG. Owo.62 M. L=0.226 M







| $X-R A Y$ | STATION | NO． 2 | NO．3 | NO． 4 | 5 |
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| TIME（StCOND）＊．．．．．．．．．e．－OLivISV CENTEG UF GRAVIIY FUSITIUN（M） <br>  <br>  |  | －VU0836 | －U $621^{\prime}$ | － 603981 | 006494 |
|  |  |  |  |  | 9．03） |
|  |  | 95 | U－106 | U－168 | 18 l 1 |
|  | NATIUN At | U－ 0 | 0.5 | 0． 5 | 0.0 |
| STPAFATION ANGLEIDEGREE） |  |  |  |  |  |
| $460$ | $\begin{aligned} & \text { IVE } \\ & \text { OW } \end{aligned}$ | 17.0 | 8.8 | 6 | ＋ |
|  NOSE NOSITICN GMd |  |  |  |  |  |
|  |  |  |  |  |  |
| $H C R$ $Y E$ | TICALA：． | $0.253$ | $0.107$ | $0.109$ | 18.121 |
| INPUT NOSE POSITIUN（MJ |  |  |  |  |  |
| He | CONTAL |  |  |  | あも戠中戠 |

 PROJECTILE: SOL IL FLAT NUSE MASS: 2.5449 KG. $D=0.02 \mathrm{M}$. L $=0.225 \mathrm{M}$




CCEFFICIENTS OF CUGIC PULYNUMIAL/STANOARD DEVIATIUN



C.G.VEL. X-COMN. (M/S): 311. 284. $238 . \quad 195 . \quad 157$.

COEFFICIENTS OF CUEIC PUIYNUMIAL/STANOARD DEVIATIUN

PONCELET DRAG CCEFF. $=0.948$







SAND: WET. DENSITY:Z2GSU. KG/M円\#3: APPROACHING VELOEITY: 326. M/S



SAND: WET. DANSITY: $\angle O B O$ O KG/MEFS: APPROACHING VELOCITY: E2O. M/S PRUHECTILE: SOLIL STEP-TIEF MASSiU.5645 KG. D=0.CL M. LEJ. 23 M M




| $x-\boldsymbol{N a}$ | STATION | －＊＊＊＊＊ | NO． 1 | NO． 2 | NO． 3 | NO． 4 | NO． 5 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| TSME | SECOND |  | ．000170 |  | －004836 | －008263 | ．0c7740 |
| CENTER OF GAVITY PISITIUN IM： HCRILONTML •OHOOO．O．－0．49： JERTICAL •••．．．．．．．．．．U．C52 |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |
| Samor | MATICN A | － | U－0 | ＊ | 禹我 | － 0.5 | －8．0 |
| SEPAKATIGA NIGLE（CEGHEF） |  |  |  |  |  |  |  |
| AGUV | VE－－－－ |  | 禹事禹象 | 㤟象曲菫 |  | －禹事事 | ＋ |
|  | 10TH（M | ${ }^{+} \mathrm{N}$ F | 4－0 263 |  |  | 0．0230 | 0．02＊ 0 |
| NOSE WIOTH：IN HNFILMA U．026，＊ |  |  |  |  |  |  |  |
| HCRI | $120 N T A L$ | －－ | 0.031 | 18．231 | 18．835 | 0.8381 | 1．089 |
| VERT | ICAL |  | － 092 | 18．4くi | 18．121 | 0.085 | 0.090 |
| \MPUT NUSE POSITION（M） |  |  |  |  |  |  |  |
| $\begin{aligned} & \text { HCRD } \\ & \text { VER } \end{aligned}$ | ZUNTA | －－＊－－－ | $\begin{array}{r} 0.024 \\ -0.134 \end{array}$ |  |  | $\begin{array}{r} 0.230 \\ -0.138 \end{array}$ | $\begin{aligned} & 1.081 \\ & 0.133 \end{aligned}$ |

SANC: WET. DENSITY: LIOBO. KG/MEDJI APPGCACHING VELOCITY: 210 . M/S






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OHUT 4S (LU APR.L. 1970. MO. 1 1
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SANC: WET. CHNSATY: 2 C 50 . XG/M\#\#3: APPROACHING VELOCITY: 210. MAS PROJECTIAE: SOLIU STEP-TIER MASS: 0.5662 KG. D=0.02 M. Lmo.238 M

| Ray station | NC. 2 | NO. 3 | NO. 4 | NO. 5 |
| :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |
| CENTEM OF GotiA |  |  |  |  |
|  |  |  |  |  |
| TSCLL | - 110 | 0.11 | . 1 | . 12 |
| INGLINATION | $1=0$ | 2.5 | 5 co | 6.0 |
| SEPAKATION ANGLE\UEGAEE) |  |  |  |  |
| ABUVE | 700 505 |  | 8.5 | 10.0 |
|  |  |  |  |  |
| NUSE POSITIUN (m) |  |  |  |  |
| HCFI RONTM | - 23 | 563 | 0. |  |
| VERTICAL | U.112 | 0.121 | 0.12 | 0. |
| INPUT NOSE COSITIUN (M) |  |  |  |  |
| VERTICAL -........... -6.110-0.1U8-0.098-0.089-6.082 |  |  |  |  |
|  |  |  |  |  |
| NOSE VEL. $\quad$-COMP. (M/S): 0 . 5 . 3 . <br> COEFFICIENTS OF CUOIC SOLYNUMAAL/STANOARD OEVIATION |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  <br>  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
| NOSE VEL OIRECTION(OEG) 1.6 1.6 1. 1.2 |  |  |  |  |
|  |  |  |  |  |
| AEOVE ............... 中*** |  |  |  |  |
| 8ELUE |  |  |  |  |
| COEFFICIENTS OF CUOAC PULTNUMIAL/STANUARU DEVIATION |  |  |  |  |
|  |  |  |  |  |
| U.1029E OL U.6875E OL -C.1456E U4 0.7251E 05 0.j003 (M) |  |  |  |  |
| C.G. VEL. X-COMP. (MSSJ: 198. 179. |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
| HUNCELET ORAG CLEFF $=1.241$ |  |  |  |  |
|  |  |  |  |  |
| C.G. VEG* X-COM | 183. | 145. | 117. | 95. |

SANC：WET．DENSITY：ZUSL．XG／M＊FJ：AP＠FDACMING VELOEITY：251．M／S

X－RAY STATILN ．．．．．．．．．．NO．I NO． 2 NO． 3 NO．NO．S
 CENTER OF GFAVITY HGSITBON iMA

HLIRILONTAL ．．．．．．．．．．．． 4.02
HERTLCAL ．．．．．．．．．．．．．O．．．． 114
INCLIMATION ANGLERUEES． 3.5
SEPAFAJICN ANGLE（DEGNEE）


NOSE WIOTH（M ON FILM）．C．025 NCSE POSITION（M） HCRILONTAL．．．．．．．．．．． $0.0<4$ VERTICAL＊．．．．．．．．．．．．．
ITRPUT NUSE PCSITION IMB HCRIZUNIAL ．．．．．．．．．．．． 0.021


| $\checkmark \cdot 152$ | 18.335 | 18．839 | 19 |
| :---: | :---: | :---: | :---: |
| 1 | 18.821 | 10.121 | 18.12 |
| 13．0 | ＊＊＊＊＊ | 車禹禹中 | ＊＊＊＊ |
| c0．0 | －${ }^{*}$＊${ }^{\text {\％}}$ | ＊＊＊＊ | ＋ |
| $1 \cdot 0$ | ＋4． |  |  |
| $0 \cdot 0<3 i$ |  |  | 由中种中事 |
| －． 152 | 18.535 | 18.838 | 19．144 |
| 4.118 | 10.121 | 18．121 | 10.121 |
| U． 181 | ＊＊＊＊＊＊ |  |  |
|  | ＊${ }^{\text {F }}$ |  |  |

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SMUT 4T (us ANGIL. 1970. NO. 3)
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SANC：WET．DENSITY：2USU－KG／M末末3；APPROACMINS VELOCBTY：ES2．M／S


X－RAY STATION
Nu． 1
NO． 2
NO． 3
itO．$A$
NO． 5

GENTEK OF GRAVITY FOSITIUN（MD
 VEXTICAI

INCL INATION ANGLE（UEG） SLPAKATBDN ANGLE（CEGMEE） ABDVE
 POJSE MIDTM iM ON FILMi． MCSE POSIIIGN（M）

HCHILUNYAL $\ldots . . . \bullet .$.
 INPLT NOSE FUSITIUN（M）


0.141
$\dot{8} 0$

$$
\begin{aligned}
& 0.2 n A \\
& -121
\end{aligned}
$$

$$
20.0
$$

事事事
＊戠象 0.6256

### 0.037

0.264

G．U37 0．270
$-4.116=6.640$
2400
官中 0.10235
$\checkmark 121$
.005000
－c．0．0

## 事事事

 v．u22u0.398
0.388
$y .164$
$\begin{array}{lll}0.308 & 18.339 & 19.144 \\ i=164 & 18.121 & 16.121\end{array}$

18.83

丰事事
中も韋中戠東事象象

象乎象 ＊ 4 事出事事事事
19.144
$16 \cdot 121$

审も由象

SHOT 48 ( 06 APRIL. 1976 . NO. $\rightarrow$

SAND: WEX. OENSITY:ZVSO. KG/M**3: APPAOACHING VELOCITY: 538: MCS PRUJEC:ILE: A.M.M.


SANC: WET: DENSITE:2GZGQKG/MWF3: APHROACHING VELOCITY: 327. M/S


 COEFFICIENTS OF CUGIC WOLYNOMLALASTAHOARD DEVIATION

NOSE VEL X-COMP 1 MAS I: 334. 365 . 263. 223. 192. COEFFICIENTS OF CUBIC POLYNGMIAL/STANOARD DEVIATIUN
-O.IBU9E-U1 0.34UIE 0.3-G. $2382 E$ US 0.1i63E 07/0.0016 (M)


C.G*VEL X-COMF. (M/S): 334. $305 . \quad 263 . \quad 223 . \quad 193$. COEFFICIENTS OF CUBIC POLYNOMLAL/STANDARD DEVIATION $-0.1351 E 00$ C.34UIE U3-C.23\&4E OS O.117IE OT / 0.0017 (M)

PONCELET CRAG CUEFF $=0.822$
VO = 33H. STAND. UEVIA. $=$ U.0025 (M)
C.G.VEL. X-COMP. (M/S): 334. 303. 262. 225. 194.





 COEFFICIENTS OF CUGIC PULYNOMIAL/STANUARO DEVIATION


 COEFFICIENTS OF CUBIC HOLYNGMIAL/STANDARD DEVIATION

C.G.VEL•X-COMP. (M/S): 392. 368 . 322. 263. 184. CCEFFICIENTS UF CUEIC PKLYNUMIAL/STANOAKD UEVIATION

PONCELET DRAG CUEFF• $=0.788$



SANC: WET. DENSITY: 2USU. KG/Mش 3 : APPROACHING VELOCETY: GUZ: MAS







SMUT SA 1 IN APFIL. $1976 \cdot$ NO. 31





NCSE VEL. Y-COMF. (M/SI: 7. 5. 2. 30 . NOEFFICIENTS OF CUEIC FOLYMUMIALASTANDARD TEVIATION
 COEFFICIENTS JF CUOIC POLYNUAIAL/STANDARD DEVIATION -S.7YOSE-02 O.2USOE O3-0.1〈43E 0S 0.431IE 06/0.000 (M)

C.G. VEL. Y-COMP. (M/S): 8. 5 - 2. COEFFICIENTS OF CUEIC PULYNUMIAL/STANUARO DEYOAYION

C.G. VEL. X-COMF. (M/S): 202. 178.130 . 106 .


PGNCELET DRAG CCEFF. $=14900$


RECGRDED TIME OF MAXIMUMOMINIMUM COIL VOLTAGE (SI






SMAT SO ( 14 APALL 1976. NO. $\quad 1$

SANU: DRY. DENSITY:1SJE. KG/M**3: APPRLACHING VELOCITV: 32円. M/S PMO NECTILE: SOLIU STEP-TIER MASSZU.5653 KG. DMU.02 M. L=0. 238 M



 CUEFFICIENTS OF CUUIC PGLYNUMIALSSTANLARD OEVIATION


NOSE VEL. DIMECTIUN(CEG) 1.0 103 -2.0

 COEFFICIENTS OF CNGIC FULYNOMIAL/STANOARD DEVIAJION
 COEFFICIENTS UF CUGIC POLYNTIMIAL/STAMDARC DEVIATION


PONCELET DRAG CUEFF. $: 1.760$



SAND: URY. DENSITY:ISJ8. KG/M\#E3: APPROACHING VELOCIYY: 198. M/S PRGJECTILE: SOLIU STEH-TIER MASS:C.505. KGI $O=0.92$ M. L=0.230 M



 COEFFICIENYS OF CUEIC PJLYMOMIAL/STANOARD CIEVIATIDN $-0.1329 E-61$ C.1842E 03-0.?988E U4 0.1348E $06 / 0.0094$ (M)


SANE: URY. OENSIYY: I536. KGJMAD3: APFRUACHING VELOCITY: 334: MOS PROJECTAAE: SOLIU STEMETBER MASS: U-564TKG, Da0.02 M. L=0.2.38 M

 CUSFFICIENTS OF CUBIC PULYNOMAAL/STANDARD OEVIATION $0.94 C 7 E-U 1$ J.72U7E 01 -U.1477E 4 U.17i5F 06/U.0CO7 (M)
NOSE VEL• X-COMP. (M/S): 303. 206. 212. 170. 153. COEFFICIENTS GF CUEIC PULYNGMIAL/STANOARD OEVIATIOM



C.G.VEL. V-COMF. (M/S): 6. 3 COEFFICIENTS OF CUEIC PULYNOMIAL/STANUARD DEVIATION $0.942 У E-01$ U.O2SVE CI -0.1423EC4 0.01385E O6/0.0UC4 (M)
C.GeVEL. $X$-COMP. (M/S): 303. 266. 212. 170. 153. CUEFFICIENTS OF CUEIC POLYNCMIAL/STANDARO DCVIATION

HUNCELET ORAG CGEFF. $=1.913$
VO = VELZ. STANO. OEVIA: =





 COEFFICEENTS OF CUBIC PDLVNOMIAL JTANDARO DEVIATION



COG VE: Y-COMP GMSS: -O - $\quad$ -

 COEFFICIENTS OF CU甘IC POLVNCMIALJSTANDARO DEVIATION $-0.1163 E 000.4401 E \dot{O} 0.0 .1451 E 05-6.9933 E 05 \% 0.0016$ (M)

PONCELET DRAG COEFFO 1.667





 CUEFFLCIENTS JF CUEIC POLVNOAIALSSTANOARO DEVIATION

 COEFFICIENTS OF CUEIC PUIYMUMIALISTANUARD DEVIATION





$-0.1272 E 00$ 0. $0495 E 03-0.1926 E 05$ C.4076E 06, 0.0011 (M)
PONCELET DRAG COEFF $=1.615$
VO 216. STAPM. LEVIA. C.GZ21 (H2



SANC: DRY, DENSATY: 1538 . KG/Mक ${ }^{\text {B }}$ 3 APPHOACHING VELOCITY: $1 . 冫$ ' M/S PROJECTILE: SOLIU STEKGTIER MASS:U.SO3S KG. D=0NO2 M. L=0.2in M

| xanay statiun | NO. 2 | NO. 3 | NO.4 | NO. ${ }^{\circ}$ |
| :---: | :---: | :---: | :---: | :---: |
|  CENTER OF GFAVITY POSSTION (M) <br>  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
| INC:ImATION AMGLE(NEG: SEPARATION ANGLE (VEGREE) |  |  |  |  |
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| HCRLCON |  |  |  |  |
| VEFTICAL .......... Uolvs velus veli4 U. |  |  |  |  |
| HCNICONTALVERTICAL |  |  |  |  |
|  |  |  |  |  |
|  COEFFICIENTS OF CUGIC PULYNUMIALISTANUARD DEYIATION <br>  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
| NOSE VEL. X-COMF - (M/SI: 344. 312. 258. 203. 139. COEFFICIEMTS OF CUUIC POLYNUMIAL/STANUARO DEVIATION <br>  |  |  |  |  |
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|  |  |  |  |  |
|  CUEFFICIENTS LF CUISIG. PGYNUMIAL/STANDAKD DEVIATION |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  COEFEICIENTS UF CUBIC :ULVNUMIALSSTANDAND UEVIATIOM |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
| CELET OHAG CCEFF $=1.027$ |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |

SANC: URY, LENSITY:1538. KG/M**3: APPFOACHIAG VELOCITY: 45 . M/S



CGEFFICIENTS OF CUBIC PULYNOMIAL/STANDARD DEVIATIUN


NUSE VEL X-COMP (m/S): 341. 31 . CUEEFFICIENTS UF CUEIC PELYMUMIAL/STANDARD OEVIATION


 COEFFICIENTS OF CUGIG POI.YNUMIAL/STANDAKD DEYIATION O. IU23E OC -C.41GUE OC O.GU7UE 03 C.1577E OG O.ULIA (M)
C.GeVEL. $X$-COMP. (M,S): 342. 3120 259. 196. 131. COEFFICIENTS QF. CUGIC PULYNGMIALEBTANDAND JEYIATION


PONCELET DRAG CCFFFF $=1.311$
VO $=259$. STANL. DEVIA $=0.0214(M)$
C.G. VEL. X-COMP. (M/S): 378. 327 . 259. 203. 160.
 PFOJECTIAK: SCLIO STENGTIER MASS:U.563J KG. O=U.O2 M. LEJ. 238 M


SANC: DHY, UENSITY:SJBU KG/M\&W, S: APPROACMING VELDCITY: AC7. M/S


 PROJECTBLE：A．MAH．

| x－RAY Station | ND． 1 | NU． 2 | NO． 3 | NO．＊ | NO．S |
| :---: | :---: | :---: | :---: | :---: | :---: |
| TIME（SECONO） | v0124 | ． 601158 | ．007981 | ．02c000 | －100000 |
| CENTCR OF GRAVIIV PUSIT |  |  |  | 18．839 | 19.144 |
| HCRIZUNTAL |  | － | J．055 | 18.121 | 19．121 |
|  |  | －49．0 | ＊＊＊＊＊ |  | ＊＊＊＊＊ |
| IMCLINATION ANGLE（UEG）． SEPARATION ANGLE IUEGREE | －6．6 | －49．0 | ＊ | ＊ |  |
| AEUYE ．．．．． |  |  | ＊＊中＊ |  | ＊ |
| BELOW |  | 0．0235 | $\checkmark .0240$ | ＊＊＊＊＊＊＊ | （0） |
| NUSE WIOTH IM ONFILMI． | 0.0 | 0．023 | － |  |  |
| NOSE POSITION（M） HCAIZONTAL ．．．． | 0.662 | 3．207 | 0.395 | 10.839 | 19：144 |
|  | ． 688 | 71 | ． 055 | 18.12 |  |
| I NPUT NUSE PUSITION（M） <br> HCFI $\angle O N T A L$ ．．．．．．．．．．．． | $u .069$ -0.140 | 0.274 -6.150 | 0.367 -0.177 |  <br>  | ＊事象乎 ＊中＊中 中 ${ }^{*}$ |

## 






SANE: WET. OENSITY:2USL. KG/M*F3: APPHUACHING VELOCITY: 395. MPS



 COEFFICIENTS OF CUEIC DOLYNOMIALISTANDARD OEVIATION

 CUEFFICIENTS OF CUBIC PULYNUMIAL/STANDARO DEVIATION -U.1322E OU VOHVOEE US $-0.2782 E$ US U.94S2E O6/0.0042 (M)
PONCELET ORAG COEFF. $=0.711$
$\forall O=401$. STANU. CEYIA. $=4.0062$ (M)
C.G. VEL. X-CCMP: (MSS: 401: 370. 323. 289. 249.


SANL：\＃ET：DENSITY：ZUBC．KG／M＊中3：APPFCACHING VELOCITY：2L9．M／S PFUJECTILE：SOLIL FLAT NUSE MASS：U．S4SI KG．O＝0．U2 M，i＝0．2＜5 m


NOSE YEL Y－COMF．（M／S）： 16 ．$\quad 7$－ 3.

NOSE VEL．X－COMP．（M／S）：19y．176． 132 ． 1 ． COEFFICIENTS UF CUGIC POL YNGMIAL／STANOAFD UEVIATION

| NOSE VE | OI | く日＊\ll | 0．5－1．4 |
| :---: | :---: | :---: | :---: |
| SEPARATIUN | ANGLE（UEGFEE）． | Lative ro | NOSE VEL |
| above |  |  | － 0 ＋ |
|  |  | ＊＊＊＊＊＊ |  |

C．G．VEL． Y －COMP．（M／S）：9．7．3． CUEFFICIENTS OF CUGIC POLYNUMIAL／STANOARO DEVIATION

C．G．VEL．$X$－COMH．（M／S）：199．176．132．93．6．

COEFFICIENTS OF CUGIC HOLVNUMIAL／STANDARO DEVIATION

PONCELET DRAG CUEFF $=1.507$
$V G=199$. STANC．DEVIA．$=4.0199$（M）
C．G．VEL．$X$－COMF．（M／S）：199．169．127．10C．78．


SANO: 昛T. OENSITY: ¿USG. KG/M*\#S: APPHCACHING VELOCITY: 2G8. M/S PROJECTILE: SLLIU TLAY MUSE MASS:U.S450 KG. $0=0 . C 2 \mathrm{M}$. $1=0.225 \mathrm{~m}$


SANS: WET, DENSITY: 2USU• KG/MF\#3: APJFUACHING VELOCITY: 214.M/S PEUJECTILE: SOLIO FLAT NUSE MASS:U.5448 KG. D=0.02 M. LEら. 225 m








SANO: WET. DENSIJY: $2 U 5 \cup$. KG/M事3: AMPROACHING VELOCITY: 406 - M/S PMOJECTELE: SULIC FLAT NOSE MASSSUOS448KG. DIO.OZ M. L\#U. 225 M


SAND: DRY. DENSITY:1bJE. KG/M* \#3: APMFUACHING VELDCITY: 242:M/S





 HMC JLCTILE: HOLLUE STEP=TIEMMASSiU.4218 KK:. D=C. $02 \mathrm{M} .6=0.230 \mathrm{M}$

 CLEFFICIENTS OF CUEIC WULYNUMIALASTANQAKD DEVIATION
 COEFPRGEENTS OF CUGIC PULYNOSLAL/STANOARO OEVIATION





 Huncelet ofag cueffo =1.789


RECERDED TIME OF maximumaminimum coll valtage (S)


SANU: DHYO DENSITY:1BSB. KG/MEAS: APPROACHING VFLNCITV: 234. M/S


| RAY STATION | HU. 2 | NO. 3 | NO. ${ }^{\text {d }}$ | NO. 5 |
| :---: | :---: | :---: | :---: | :---: |
|  CENTEN OF GEAVITY PLSITIUN (M) <br>  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
| INCLINATILN AN | 0.0 | -6.5 | $-10.0$ | 1 CO 5 |
| SEPARATILN ANGLLIDEGNEE, U0V 0.0 -605-1000-1C05 |  |  |  |  |
| - ¢ ¢ |  |  |  |  |
| NUSE EIDTH IM ONF1LMI. U.0¢0C 0.cz35 0.0230 0.0240 U.C350 |  |  |  |  |
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| NOSE VEL Y-COMP CUGMS : COEFFICIENTS OF CUEIR PULYANMIALASTANUAKD DEVIATIOM |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  GOEFFICIENTS DF CUBIC PULYNUMIAL/STANOARD DEVIATIIJN <br>  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
| NUSE VEL. OTRLCTIUN(UEG) 0.2 -0.4 -2.2-6.8-8.9 |  |  |  |  |
| SEPARATION ANGLE(CEGKEE). RELATIVE TU NJSE VELOCITY |  |  |  |  |
|  |  |  |  |  |
| C.G. VEL. Y-COMP. (M/SI: 6. - $\quad$ - . <br>  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
| C.Ge VEL $\quad \mathrm{X}$-COMF. (M/S): 220. 195. 140. 89. COEFFICIENTS OF. CUGIG FULYNUMIALASTANDARO DEVIATION -U.A2UAE OC J.2K40E 03-0.1743E US U.0602E 06\% C.0U42 |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
| PONCELET DPAG COEFF. 21.772 <br> $V C=220$. STAND. OEVIA. $=0.6145(M)$ <br> C.G. VEL. X-COMP. (M/bl: 22c. 188. 136. 87. 68. |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |







COEFFICIENTS OF CUGIC OOLYNUMIALISTANUAKO DEVIATIUN


 COEFFICIENTS OF CUEIC PLLYNUMIAL/STANDAFO DEVIATION

 CUEFFICIENTS OF CUEIC PLLYNLMIAL/STANDARD DEVIATIUN

- J. 1255 S OU

PCNCELET DAAG PUEFF• $\pm 0.964$



SANC: WET. OENSITY: KOSO. KGAMK\#3: APPRORCHING VELOEITY: A0SAM/5


 COLFFICIENTS OF CUBIC POLYNUAIALSSTANOARD DEVIATION

 CCEFFICIENTS OF CUBIC PGLYMUMIMLSTANDARO OEVIATIUN
(M)




 CUEFFICIENTS OF CUGIC PQLINOMIALISTANDARD DEVIATION


PONCELET DHAG COEFF. $=0.729$




 COEFFICIENTS OF CUEIG PULYNOMIAL/STANOARO DEVIATIUN




 COEFFICIENYS OF CUSIC FULYNOMIML/STANDAKO OEVIATION


PONCELET DRAG CLEFF $=0.8 \angle C$
VO =2960 STANC: UEVIA $=4.0051$ (M)


 PFOJECTILE: SULIU FLAT NUSE MASS:U.SASU KG. D=0.02 m. LEJ.225 M


SANC：M－ON，DENSITY： 1 UOU．KG／ME中3：APPRGAGHING VELOCITY：237．M／S PROJECTILE：HOLLUM STEP－TIERMASS：U•423UKGe D＝U．U2 M．L＝E． 238 m

| may stati | NO． 1 | NO． 2 | NO． 3 | NO．4 | NO． 5 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| TIME（SECONO） | $.000155$ | －60u83s | ． 00229 A | ． 004596 | ．0c8612 |
| MOKIZONTAL | －U．U6L |  |  |  |  |
| VERTICH． | － 105 | C． 113 | C． 116 | U．121 | 18.224 |
| INCLINATION ANGLEQUEG | 1．4 | U．4 | 4.0 | 1.0 | ＊＊＊＊ |
| SEPARATIDN ANGLEIUEGFEE |  |  |  |  |  |
| ABCVE | ＊${ }_{\text {事韦＊}}$ |  | ＊＊${ }_{\text {＋}}$ | ＊ | ＊＊＊＊ |
| 甘ELQW | ＊＊＊ | －＊＊＊ | －＋${ }^{\text {＋}}$ | ＊＊＊＊ | ＊＊＊ |
| NOSE MIDTH（M ON FILM）． | 0.0202 | $0 \cdot 0<30$ | U．0231 | 0.0230 |  |
| NOSE HCSITION（M） |  |  |  |  |  |
| HCRI ZUNTAL | 0.024 | 0.195 | 0.498 | 0.967 | 19．144 |
| VERTICAL | 0.107 | U．113 | 0.116 | 0.122 | 18．121 |
| INPUT NOSE POSITION（M） |  |  |  |  |  |
| HOFIZONTAL ．．．．．．．．．．．． | y－020 | 0.190 | 0.493 | 0.986 | ＊＊＊＊＊ |
| VEFTICAL－a．0．0．0．0． | －6． 116 | －0．107 | －0．104 | －0．096 | ＊＊＊＊＊＊ |

SANC:H-OH. OENSITY:IUCO. KG/m**3: APFKOAGNING VELOCITY: 237. M/S PMOJEGTILE: HOLLOE STEP-TIEHMASS: 0.4230 KG. D=0.02 M. LEN. 238 M




 COEFFICIENTS UF CUGIC! AVNUMIALSTANDARD DEVIATIGN


NOSE VEL X-CCMP (M/S : 227. 221. 2190 194. 178. COEFFICIFNTS OF CUEIC PULYNGMIALISTANDARO DENIATIUN





COEFFICIENTS OF CUEIC PULYNUMIAL/STANUARD DEVIATIGN


PONEELET DRAG CCEFF $=6.506$




SAND: ORY. DENSITY: 15JG. KG/M**3: APPRUACHING VELUCITY: 227. M/S


| - hat station | NO. 2 | NO. 3 | 10.4 | 5 |
| :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |
| CENTER OF GRAVITY PUSITICN (M) |  |  |  |  |
| HCEITZONTAL | 088 |  |  | $1.043$ |
| VEHTICAL. | $\checkmark 611$ | $\dot{\sim}$ |  | $0.084$ |
| INCLIMATION | -vos | -8.0 | -10.0 | 11.0 |
| StPAKATION ANGLE(UEGFES ) |  |  |  |  |
| a bove |  |  |  |  |
| Bildum | 5 |  |  |  |
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| NOSE POSITICN (m) |  |  |  |  |
| HCAILUNTAL | $\begin{aligned} & \text { N. } 1718 \\ & \dot{C} .118 \end{aligned}$ | $\checkmark$ | $\begin{aligned} & 0.818 \\ & 0.095 \end{aligned}$ | $\begin{aligned} & 1.14 e \\ & 4.064 \end{aligned}$ |
| VEFTICAL |  |  |  |  |
| MCHILONTAL |  |  |  |  |
| VERTICAL |  |  | - | - 163 |
| NOSE VELL Y-COMP. (M/S): <br> CUEFFICIENTS UF GUEIC POLYNUMIAL/STANDAFO DEVIATION <br>  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
| NOSE VEL. $K$-COMP. (M/S): 215. 191. 137. 77. CCEFFICIENTS OF CUEIC PULYNUMIAL/SYANDAFD DEVIATION |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
| NOSE VEL. DIMECTICN(CLG) 0.4 -0.1 -1.7 -4.7 |  |  |  |  |
|  |  |  |  |  |
| SEPARATION ANGLEIUEGREEIO HELATIVE TU NOSE VELOCITY |  |  |  |  |
| YELUE |  |  |  |  |
| C.G. YEL. Y-GCMP. IMNSI: 9. $\quad$. CCEFFICIENTS OF CULIC POLYNUMIAL/STANDARD OEVIATION <br>  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
| C.G. VEL. X-COMF. (M/Si: 215. 15i. 130. 69. 77. COEFFICIENTS OF CUUSC PGLYNCMIAL/STANUAKO DEUIATION |  |  |  |  |
|  |  |  |  |  |
| -0.1222E VO | E OS | 6SUSE U6 | / 0.0.C64 | (M) |
| PUNCELET DRAG CUEFF. $=1.734$ |  |  |  |  |
|  |  |  |  |  |
| C.G. VEL. x-CO | 185. | 135. | 97. | 69. |

SANC: VET. DEASITY: 2U53. KG/Mक 3 : APPROACHING VELOCITY: ©4S. M/S





 CUEFFICIENTS OF CUUIC FLLYNUMIALISTANOAHU DEVIATION


CUEFF:CIENTS UF CUGIC PCLYNUMIAL/STAMUAHO DEVIATIJN レ.0862E-J2 0.


 COEFFICIENTS GF CUOIC PLLYNUMIALASTINLAFUGCEVIATION O.JUCS (MJ

COEFFICIENTS OF CUOB PLLVNCMIAL/STANLAFE OEVIATIUN


PONCLAET ORAG CUEFF. $=-*$ UOC


## APPENDIX B

## DERIVATION OF FOKCES AND MOMENTS ON A RIGID BODY IN A SOIL MEDALI

The purpose of this section is to develop the expressions for the forces and moments applied to a projectile in a soll sedium. The forces are represented by $F_{x}, F_{y}$ and $F_{z}$ along a body fixed set of axes $x, y, z$ with the origin at the center of mass of the vehicle. The moments are about the $x, y$ and $z$ axes and are denoted $L, M$ and $N$ respectively and the velocity components are designated $U, V, W$ re:pectively. It is assumed that the force exerted by the goil on a differential surface area cf the body is of the form

$$
\begin{align*}
\frac{d F}{d A} \propto & \left(A_{x}+B_{x}|U|+C_{x} U^{2}\right) n_{x} \hat{i} \\
+ & \left(A_{y}+B_{y}|V|+C_{y} v i\right) n_{y} \hat{j} \\
+ & \left(A_{z}+B_{z}|w|+C_{z} W^{2}\right) n_{z} \hat{k} \tag{B-1}
\end{align*}
$$

where $n_{x}, n_{y}, n_{z}$ are components of the outward unit vector normal to the surface ${ }^{x} f^{y}$ the body. The force is assumed to exist only when the velocity is directed toward the elemental arca. The flow is assumed to be separated from the body aft of the nose.

The nrae shape that is treated is conical, with nomenclature as shown in Figure $\mathrm{b}-1$. In the cylindrical coordinate system the equation for the surface of the conical nose is given by

$$
\begin{equation*}
\rho=\frac{r}{L_{N}}(\bar{x}-x) \tag{B-2}
\end{equation*}
$$

The unit vector normal to the surface of the nose is

$$
\begin{equation*}
\hat{n}=\operatorname{Sin} \gamma \hat{i}+\operatorname{Cos} \gamma \operatorname{Cos} \beta \dot{j}+\operatorname{Cos} \gamma \operatorname{Sin} \beta \hat{k} . \tag{B-3}
\end{equation*}
$$

The area of a differential surface elemen. is givan by

$$
\begin{equation*}
d A=\frac{\rho d x d B}{\operatorname{Cos} \gamma} . \tag{B-4}
\end{equation*}
$$

The inertial reference frame $x^{\prime} y^{\prime} z^{\prime}$, a rectangular Cartesiar system, is choser with $x^{\prime}$ perpendicular to the sutface of the target and pointing inward and $y^{\prime}$ and $z^{\prime}$ chosen in any convenient direction. In order to write the equation of the target surface prior to impact, in the body fixed axis system, a coordinate system $x_{1}, y_{1}, z_{1}$ is chosen oriented parallel to the body axis system but whose origin fs coincident with that of the inertial axis system. The equation of the taiget plane in the inertial system is $x^{\prime}=0$. Using the Euler transformation matrix introducid ir Section VI the relation between the primed and sub-one system can be written aa

$$
\left\{\begin{array}{l}
x^{\prime} \\
y^{\prime} \\
z^{\prime}
\end{array}\right\}=T_{B I}\left\{\begin{array}{l}
x_{1} \\
y_{1} \\
z_{1}
\end{array}\right\}
$$

This yield

$$
\begin{equation*}
x^{\prime}=T_{11} x_{1}+T_{12} y_{1}+T_{13} z_{1}=0 \tag{B-6}
\end{equation*}
$$

where

$$
\begin{aligned}
& T_{11}=\cos \psi \cos \theta \\
& T_{12}-\operatorname{Cos} \psi \operatorname{Sin} \theta \sin -\operatorname{Sin} \psi \cos \\
& T_{13}=\operatorname{Cos} \psi \operatorname{Sin} \theta \operatorname{Cos} \phi+\operatorname{Sin} \psi \operatorname{Sin}
\end{aligned}
$$

Equation ( $B-6$ ) can be written in the body axis syscem by a simple transiation

$$
\begin{align*}
& x=x_{1}+d \\
& y=y_{1}  \tag{B-7}\\
& z=z_{1}
\end{align*}
$$

where dis the distance from the body axis origin to the inertial axis origin and is

$$
d=\sqrt{\left(x^{\prime}\right)^{2}+\left(y^{\prime}\right)^{2}+\left(z^{\prime}\right)^{2}}
$$

Substituting Equation ( $B-7$ ) into Equation ( $B-6$ ) results in

$$
\begin{equation*}
T_{11} x+T_{12} y+T_{13}-T_{11} d \tag{B-8}
\end{equation*}
$$

the equation of the target plane in the body axis system. It is convenient to write this equation in a cylindrical body axis system as shown in Figure B-1 .

$$
\begin{equation*}
T_{11} x+T_{12} \rho \operatorname{Cos} \beta+T_{13} \rho \operatorname{Sin} \beta=T_{11} d \tag{B-9}
\end{equation*}
$$

Consider the case where the nose has impacted the target and is partially immereed in the soil ss shown in Figure B-2. The locus of the intersection of the nose and the target plane will be given by the simultaneous solution of Equation ( $B-9$ ) and Equation ( $B-2$ ) which yields

$$
x=\frac{T_{1 d}-\left(r / L_{N}\right) \bar{x}\left[T_{1.2} \operatorname{Cos} \beta+T_{13} \operatorname{Sin} \beta\right]}{T_{11}-\left(r / L_{N}\right)\left[T_{12} \operatorname{Cos} \beta+T_{13} \operatorname{Sin} \beta\right]}=f_{1}\left(x^{\prime}, y^{\prime}, z^{\prime}, \varphi, \theta, \phi, \beta\right), \quad(B-10)
$$

Using Equation ( $B-1$ ) the differential force in the $x$ direction becomes

$$
d F_{x}=\left(A_{x}+B_{x}|U|+C_{x} U^{2}\right) n_{x} d A
$$

and realizing that positive value of $U$ will produce a negative force on the projectile, this force is rewritten as

$$
d F_{x}=-\frac{U}{T V T}\left(A_{x}+B_{x}|U|+C_{x} U^{2}\right) n_{x} d A .
$$

By using Equations ( $B-2$ ), ( $B-3$ ) and $(B-4)$ and $\frac{r}{L_{N}}=$ tanr the above expres-
sion may be written as

$$
d F_{x}=-\frac{U}{|U|^{(A}}\left(A_{x}+B_{x}|U|+C_{x} U^{2}\right)\left(r / L_{N}\right)^{2}(\bar{x}-x) d x d B .
$$

and upon integration, yields

$$
F_{x}=-\frac{U}{|U|^{\prime}}\left(A_{x}+B_{x}|U|+C_{x} U^{2}\right)\left(x / L_{N}\right)^{2} \iint(\bar{x}-x) d x d B
$$

where the limits for the integrais are chosen so that the integration $1 s$ uver the portion of the submerged area having a velocity component roward that surface. The integration with respect to $x$ ir performed first. The upper limit for the $x$ integral is simply $\bar{x}$. The lower limit for $x$ is a bit more complicated as shown by Figure $\mathrm{B}-3$ which shows the $\mathrm{x}-\mathrm{z}$ plane f r a case where ". $=\phi=0$. As long as the projectile has not penetrated very far the lower limit is simply siven by the equation of the intersection of the cone and the target plane, that is $f_{1}$ of Equation ( $B-10$ ). As shown in Figure B-3 when the penetration reaches the base of the nose this limit is no longer correct since integration where $x<L_{1}$ gives a contribution over the dashed part of the cone that does not exist. For this reason the lower limit becomes $f_{1}$, unless $f_{1}<L_{1}$ in which case the lower limit becomes $L_{i}$.

The limit on the $\beta$ integral will depend upon the magnitude of the angle of attack, given bilow, as compared with the cone half angle $\gamma$. If the angle of attack is less than $\gamma$ the limit will be from 0 to $2 \pi$. This is expressed as

$$
a=\cos ^{-1}\left[\frac{|U|}{\sqrt{U^{2}+V^{2}+W^{2}}}\right]<\gamma=\tan ^{-1}\left[\frac{r}{L}\right] .
$$

To summarize

$$
\begin{equation*}
F_{x}=-\frac{U}{U}\left(A_{x}+B_{x}|U|+C_{x} U^{2}\right)\left[\frac{r}{I_{N}}\right]^{2} \int_{0}^{2 \pi} \int_{1 \geq L_{1}}^{\bar{x}}(\bar{x}-x) d x d B . \tag{B-11}
\end{equation*}
$$

for $\alpha<y$ and for $\bar{x}>V_{1} \geq L_{1}$. For the case where $a>y$ one aide of the cone will be blanketed and the $\beta$ integral will be integrated only over 180 degrees. In order to determine the proper range of $B$ consider the $y-z$ coordinates ( $x$ into the plane of the paper) as shown in Figure B-4. The $V$ and $W$ components determine the projected or wetted area in contact with the soil. Let $\delta_{t}=\tan ^{-1} \frac{\mathrm{~W}}{\mathrm{~V}}$, and limits on $B$ will be $\pm \frac{\pi}{2}$ radians fron the angle $\delta_{t}$ or from $-\frac{\pi}{2}+\delta_{t}$ to $\frac{\pi}{2}+\delta_{t}$. Collecting this we write

$$
\begin{equation*}
F_{x}=-\left.\frac{U}{\mid U}\right|^{\left(A_{x}+B_{x}|U|+C_{x} u^{2}\right)}\left[\frac{r}{L_{N}}\right]^{2} \int_{-\frac{\pi}{2}+\delta_{t}}^{\frac{\pi}{2}+\delta_{t}} \int_{f_{1} \geqslant L_{1}}^{\bar{x}}(\bar{x}-x) d x d B \tag{B-12}
\end{equation*}
$$

for
$a>\cdots, \bar{x} \geq f_{1} \geq L_{1}$ and $V \geq 0$.
If $\mathrm{V}<0$ a different set of limits on $B$ ara applicable. In this cace
for $\alpha>\gamma, \quad \bar{x} \geq \sum_{1} \geq L_{2}$. and $V<0$.
Once the nose has penetcated so that it is totally submerged the force becomes

$$
\begin{equation*}
F_{x}=-\frac{U}{|U|}\left(A_{x}+B_{x}|u|+C_{x} u^{2}\right) \cdot \frac{\pi x^{2}}{n} \tag{B-14}
\end{equation*}
$$

where $n=1$ for $\alpha<y$ and $n=2$ for $a>\gamma$.
In develcping expressions for $F_{y}$ and $F_{z}$ the velocity components in the $y$ and 2 directions would be made up of a velosity term due to translation plus a tern due to rotation. Specificsily, for a point on the axis of symetry the velocity component in the $y$ direction is $V+R x$ and the velocity component in the $z$ direction is W - Qx. If these velocity terms are used, the angle $\delta_{t}$ becomes a function of $x$, namely $\delta_{t}=\tan ^{-1}[(V+R x) /(W-Q x)]$. Since $\delta_{r}$. appears in the iimits of integration with respect to $B$, theae limits now become functions of $x$, and the integration is extremely cumbersome.

One approach to this problem is to treat the two velocity terms separately and add the resulting forces. This neglects the cross product terms 2 VRx and -2 WQx in the velocity squared terms of the differential force in Equation (B-1). Another approach would be to sura the two velocity teras first and replace the longitudinal integration with respect to $x$ by integrations over several strips of finite length $\Delta x$. Then sum these gegmental integrations to obtain the resultant forces. Onily the first method is presented iere, and further consideration of this problem is left for later rescarch in which the possiblity of modifying the assumed force law should also be considered.

The expression for $F_{y}$ is written as the sum of two parts. The first is due to translation, $\left(F_{v}\right)$, and is obtained in a manner similar to the detailed procedure out infied above for $F_{x}$ and the other is a component of force due to a rotation rate $R$ radians per second about the z-axis. This component is denoted ( $F_{Y}$ ) $r^{\text {. The rotation rate will give the differentia: }}$ surface element a velocity component in the $y$ direction equal to Px. This component is treated in the same manner as the translational velocity $\because$ : $n$ order to determine the $F_{y}$ that it produces. The results are as fo:lows:

$$
\begin{equation*}
F_{y}=\left(F_{y}\right)_{T}+\left(F_{y}\right)_{r} \tag{B-15}
\end{equation*}
$$

$$
\begin{equation*}
\left(F_{y}\right)_{T}=-\frac{V}{\mid V T}\left(A_{y}+B_{y}|V|+C_{y} V^{2}\right)\left[\frac{x}{L_{N}}\right] \int_{\frac{\pi}{2}+\delta_{t}}^{\frac{\pi}{2}+\delta_{t}} \int_{f_{1} \geq L_{1}}^{-x}(\bar{x}-x) d x \cos B d B \tag{B-16}
\end{equation*}
$$

for $V>0$ and $\bar{x} \geq f_{1} \geq L_{1}$. For the case where $V<0$ the limits on the 3 incegral becoue $\frac{\pi}{2}+\delta_{t}$ to $\frac{3 \pi}{2}+\delta_{t}$.

$$
\left(F_{y}\right)_{r}=-\frac{R}{|R|}\left[\frac{r}{L_{N}}\right] \int_{\frac{\pi}{2}+\delta_{r}}^{\frac{\pi}{3}+\delta_{r}} \int_{f_{1} \geq L_{1}}^{\bar{x}}\left(A_{y}+B_{y} x|R|+C_{y} x^{2} R^{2}\right)(\bar{x}-x) d x \cos B d B
$$

for $R \geq 0, \bar{x} \geq f_{1} \geq \mathcal{L}_{1}$ and $\delta_{r}=-\tan ^{-1 Q} \frac{1}{R}$. For $R<0$ the limics on the $\beta$ integral become $\frac{\pi}{2}+\delta_{r}$ to $\frac{3 \pi}{2}+\delta_{r}$.

Once the projectile has penetrated so that the nose is totally submerged the lower limit on all of the $x$ integrals becomes $L_{1}$.

In a similar manner $F_{z}$ is composed of $\left(X_{z}\right)_{T}$ and $\left(F_{z}\right)_{r}$.

$$
\begin{equation*}
F_{z}=\left(F_{z}\right)_{T}+\left(F_{z}\right)_{r} \tag{B-18}
\end{equation*}
$$

Where

$$
\begin{equation*}
\left(F_{z}\right)_{T}=-\left|\frac{W}{W}\right|\left[\frac{r}{L_{N}}\right]\left(A_{z}+B_{z}|W| \div C_{z} W^{2}\right) \int_{\frac{\pi}{2}+\delta_{t}}^{\frac{\pi}{2}+\delta_{1}} \int_{E_{1}>L_{1}}^{\bar{x}}(\bar{x}-x) d x \sin B d \beta \tag{B-19}
\end{equation*}
$$

for $V>0$ and $\bar{x} \geq f_{1}>\mathbb{L}_{1}$. For $V<0$ the 11 mits on the $B$ integral become $\frac{\pi}{2}+\delta_{t}$ to $\frac{3 \pi}{2}+\delta_{t}$.

The component due to rotation is

$$
\begin{equation*}
\left(F_{z}\right)_{r}=\left\lvert\, \frac{Q}{Q \mid}\left[\frac{r}{L}\right] \int_{\frac{\pi}{2}}^{\frac{\pi}{2}+\delta_{r}} \int_{E_{1}>L_{1}}^{\bar{x}}\left(A_{z}+B_{z} x|Q|+C_{z} x^{2} Q^{2}\right)(\bar{x}-x) d x \sin \theta d B\right. \tag{B-20}
\end{equation*}
$$

for $R \geq 0$ and $\bar{x} \geq f_{1} \geq L_{1}$. For $R<0$ the limits on the $\beta$ integral become $\frac{\pi}{2}+\delta_{r}$ ro $\quad \frac{3 \pi}{2}+\delta_{r}$.

Once the projertile has penetrated so that the nose 13 totally submerged In the target the lower 11 mit on the $x$ integrals becomes $L_{1}$.

The momenta follow from the fact: that there is a force on the differential element of area which is located a distarce $x$ from the center of mase. See Rigure B-6. The force $\mathrm{dF}_{\mathrm{y}}$ applied to the area dA will produce a moment about the $s$-axis, denoted $N$, whose magnitude will be $x d^{p} y$.

The moment $N$ will be made up of two parts, one due to translation, $(N)$ and one due to rotation rate $(N)_{r}$.

$$
\begin{equation*}
N=(N)_{T}+(N)_{r} \tag{B-21}
\end{equation*}
$$

The results for ${ }^{(N)} \mathrm{T}_{\mathrm{T}}$ are as follows:

$$
\begin{equation*}
(N)_{T}=-\frac{V}{|V|}\left(A_{y}+B_{y}|v|+C_{y} v^{2}\right)\left[\frac{r}{L}\right] \int_{N}^{\frac{\pi}{2} R \delta_{t}} \int_{\frac{\pi}{2}+\delta_{t}}^{\bar{x}} x(\bar{x}-x) d x \operatorname{Cos} \beta d \beta \tag{B-22}
\end{equation*}
$$

for $V \geq 0$ and $\bar{x} \geq f_{1} \geq L_{1}$. For $V<0$ the 11 wits on the $\beta$ integral become $\frac{\pi}{2}+\delta_{r}$ to $\frac{3 \pi}{2}+\delta_{r}$.

The values of moment due to rotation rate are

$$
\begin{equation*}
(N)_{r}=-\frac{R}{|R|}\left[\frac{r}{L_{N}}\right] \int_{-\frac{\pi}{2}+\delta_{r}}^{\frac{\pi}{2}+\delta_{f}} \int_{f_{1} \geq L_{1}}^{\bar{S}}\left(A_{y}+B_{y} x|R|+C_{y} x^{2} R^{2}\right) x(\bar{x}-x) d x \cos \beta d B \tag{B-23}
\end{equation*}
$$

for $R \geq 0$ and $\bar{x} \geq f_{L} \geq L_{1}$. For $R<0$ the limits on the $\beta$ integral become $\frac{\pi}{2}+\delta_{r}$ to $\frac{3 \pi}{2}+\delta_{r}$.
When the complete nose is submerged in the target the lower limit on the $x$ integrals becomes $L_{1}$.

In a similar manner the moment about the $y$-axis,

$$
\begin{equation*}
M=(M)_{T}+(M)_{r} \tag{B-24}
\end{equation*}
$$

$$
\begin{equation*}
(X)_{T}=\frac{W}{|W|}\left[\frac{r}{L_{N}}\right]\left(A_{z}+B_{z}|W|+C_{z} W^{2}\right) \int_{-\frac{\pi}{2}+\delta_{t}}^{\frac{\pi}{2}+\delta_{t}} \int_{f_{1} \geq L_{1}}^{\bar{x}} x(\bar{x}-x) d x \operatorname{Sin} B d B \tag{B-25}
\end{equation*}
$$

when $V \geq 0, \bar{x} \geq f_{1} \geq L_{1}$. For $V<0$ the limits on the $B$ integral become $\frac{\pi}{2}+\delta_{t}$ to $\frac{3 \pi}{2}+\delta_{t}$.
$(M)_{r}=\frac{Q}{Q T}\left[\frac{r}{L_{V}}\right] \int_{\frac{\pi}{2}+\delta_{r}}^{\frac{\pi}{2}+\delta_{r}} \int_{f_{1} \geq L_{1}}^{\bar{x}}\left(A_{z}+B_{z} x|Q|+C_{z} x^{2} Q^{2}\right) x(\bar{x}-x) d x \sin B d B$
when $R \geq 0, \bar{x} \geq f_{1} \geq L_{1}$. For $R<0$ the limite on the 8 integral bacome $\frac{\pi}{2}+\delta_{r} \quad$ co $\frac{3 \pi}{2}+\delta_{r}$.

When the nose is completely subserged the lower limit on the $x$ integrals becomes $L_{1}$.

Under the assumptions of this force model the moment about the $x$ axis, $L$, will be equal to zero for reasonable angles of attack and obliquity.


Figure B-1. Nomenclature and Coordinate Systems


Pigure B-2. Partially Subwerged Projectile


Figure B-3. Schematic Showing Plane of Intersection of Targei and Projectile


Figure B-4. Range of Angle $B$ Shown in $y-z$ Plane


Figure E-5. Range of Angle $\beta$ Shown in $y-z$ Plane


Figure B-6. Schematic Showing Differential Force dry

1. LIST OF COMPUTER SYMBOLS

| AX | Force coefficient, $A_{x}$ |
| :---: | :---: |
| AX | Farce coefficient, $\boldsymbol{A}_{\mathbf{y}}$ |
| AZ | Force coefficient, $A_{z}$ |
| BX | Force coefficient, $B_{x}$ |
| BY | Force coefficient, ${ }^{B} y$ |
| E2 | Force coefficien:, $B_{z}$ |
| CW) | Drag cuefficient, $C_{D}$ |
| Cux | $x$ coordinate center of mass |
| Crs | $y$ coordinate center of mass |
| CM2 | 2 coordinate center of mass |
| cX | Porce coefficient, $\mathrm{C}_{\mathrm{x}}$ |
| cX | Force coefficient, $C_{y}$ |
| CZ | Force coefficient, $\mathrm{C}_{\mathrm{z}}$ |
| DT | Print time frequency |
| DTMAX | Maximum integration time |
| DTMIN | Minimum integration time |
| FIN | MIMIC finish statements |
| FX | Total force component $x$ direction, $F_{x}$ |
| FI | 'fusal force component $y$ direction, ${ }^{\prime} y$ |
| FYCO | Force component $y$ direction for cone only |
| FYCY | Force component y direction for cylinder only |
| FYHE | Force component $y$ direction for hemisphere only |
| F2 | Total force component $z$ direction, $F_{z}$ |
| F2CO | Force component $z$ direction for cone only |
| F2CY | Force component \% direction for cylinder only |
| F28E | Force component $z$ direction for hemisphere only |
| ADR | Output headinge |
| INT | MIMIC integration mabol |
| IXX | Moment of inertia. $I_{\text {x }}$ |
| IYY | Moment of incitia, $\mathrm{I}_{\text {yy }}$ |
| K1, E2, E 3 | Constmite used to select proper force cerns |
| K4, $\times 5,12$ | Constunts used to select proper mowent terns |
| L | Total applied moment about $x$ axis |

Applied moment about $x$ axis due to cone only Naturai logarithm of $C_{x}$ Appifed moment about $x$ axis due to cylinder only Applied moment about $x$ axis due to hemisphere only Length of cylinder Length of nose cone Total applied moment about $y$ axis Mass of projectile Applied moment about $y$ axis due to cone only Applied moment about $y$ axis due to cylinder only Applied moment about $y$ axis due to hemisphere oniy Total applied moment about $z$ axis Output to be listed
Applied moment about $z$ axis due to cone only Applied moment about $z$ axis due to cylinder only Rotation velocity about $x$ axis
Rotation acceleration about $x$ axis, $\dot{p}$
Universal constant $\pi=3.14159$
Euler angle $\downarrow$
Euler angular velocity, $\phi$
Initial Euler angle, $\phi_{0}$
Initall rotational veiocity, $P_{0}$
Preisure exerted by velocity $U$
Pressure exerted by velocity $V$
Pressure exerted by velocity $W$
Euler angle, $\psi$
Euler angular velocity, $\dot{\psi}$
Initial Euler angle, $\psi_{0}$
Rotational velocity about $y$ axis
Rotational acceleration about $y$ axis, $Q$
Initial angular velocity about $y$ axis, $Q_{0}$
Rotational velocity about 2 axis
Radius of cylinder, base of nose cone, temisphere Rotational acceleration about $z$ axis, $\dot{R}$

Initial angular velocity about $z$ axis, $R_{0}$
Time

| TERM | $\mathbf{1 - U / U}$ |
| :---: | :---: |
| TH | Euler angle $\theta$ |
| THD | Euler angular velocity, $\dot{\theta}$ |
| THO | Initial Euler angle, $\theta_{0}$ |
| U | Velocity relative to $x$ axis |
| UD | Acceleration relative to $x$ axis, $\dot{U}$ |
| vo | Initial velocity relative to $x$ axis, $\mathrm{U}_{0}$ |
| $v$ | Velocity relative to y axis |
| VD | Acceleration relative to y axis, V |
| vo | Initial velocity relative to $y$ axis, $V_{0}$ |
| W | Velocity relative to 2 axis |
| WD | Acceleration relative to 2 axis, $W$ |
| WO | Initial velocity relative to $z$ axis, $W_{0}$ |
| $\mathbf{X P}$ | $x^{\prime}$ axis |
| XPD | Velocity relative to $x^{\prime \prime}$ axis |
| YP | $y^{\prime}$ axis |
| YPD | Velocity relative to $y^{\prime}$ axis |
| $2 P$ | $z^{\prime}$ axis |
| 2PD | Velocity relative to $z^{\prime}$ axis |

## 2. INTRODUCTION

The necessary format and language tor programing is contained in Control Data Corp. reference manual entitled Control Data MIMIC and only the details for datainput are included here. All input data either constants (CON) or parameters (PAR) are placed at the end of the program and correspond exactly to the manner in which they are called for by the CON and PAR cardo preceding the progiam. Details of input data are shown for Computer Frogram II.

In the first case shown, Computer Program $I$, the force coefficients $A_{x}$ through $C_{2}$ are assumed to be independent of the velocity terms, therefore all these values are shown as parameters. However, in the second case, Computer ragram II, the value of $C_{x}$ is then given a variable status included in the body of the program.

Parameters K1 through K6 are used to select forces and monente depending on type of nose cone used as well as the option not to use the forces and moments on the after body. These parameters take on values of zero or unity and are defined as follows:

| Configuration | KI | K2 | K 3 | K4 | K5 | K6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Cylinder only, all moments and forces | 1 | 0 | 0 | 1 | 0 | 0 |
| Cylinder plus cone, all moments and forces | 1 | 1 | 0 | 1 | 1 | 9 |
| Cylinder plus conc, only nose moments and forces | 0 | 1 | 0 | 1 | 0 | 0 |
| Cylinder plus hemisphere, all moments and forces | 1 | 0 | 1 | 0 | 0 | 1 |
| Cylinder plus hemisphere, only nose | 0 | 0 | 1 | 0 | 0 | 1 |

3. COMPUTER PROGRAM I

CCN(E., DTMIN, OTMAX)
GAR ( $A X, B X, C X, A Y, B Y, C Y$ )
FAR (ALUZ, CZ)

FAE (MAS, IXX, IYY, $\operatorname{CinX}, \operatorname{Cr} Y, \operatorname{CML})$
FAO( GSY, LNCORAL)
FAK: (UU, B , WU, RU,QU,PO)
FAK (FMIUZYOUPSIO,XPG,YPU,ZRO)
FAR (K1,K2,K3:K4,K5,KE\}
SOLL PEAETAAT_UA -- CUAPLETELY SLEMEEGEU
FUZCF. IERMS

fyc
2: KAC $L$ LCY P PEESV
K $D O \rightarrow L M J$ HFESV
RAU RNCUPPKESH
F ${ }^{\circ}$ RAL'KAU*PKESV/2.

FZHE
$K 1+F Z C Y+K 2 * F Z C O+K 3$-F Z F
MOTE TT ERMS
LCY $\quad K \&(F Z C Y * C M Y-F Y C Y * C M Z)$
LCO. $\quad$ S* (FLCO CMY-FY UU*CMZ)

1CS - K5.FZCN* (CAX-2.4LNCO/3.)
HHE -KO F $2 H E=(C H X-5:+H A C / O$.


$6 \quad L C Y+L C O \&-H E$


```
FORCE ECUATIGNE 
    20
    @O 
```



```
    KHCDL ROU*STP(PHA)*SIN(TH)/COS(TH)
    PHID PHIUI+PHIDZ
    MHI
    PS{U OOSIN(NHIS/COS(TH)+R+COS(FHL)/COS(TH)
    NESIG FGAAPE ESUAF?SIOS
    XFS1 U*CUS(PCI)*COS(TH
    KHUZ }\quad\forall+(CUS(ESI)&SINITHO*SIN(PHI)-SI#(FSI)*COS(PHI))
```



```
    XF InT(xrS;xPO)
    YFOL U&(SIN(FSI):COS(TH))
```



```
                                H*{SIN(PSI)=SIN(TH)*CUS(PHI)-CUS{'SSII:SIN(PHID)
    YFU1&YPCZ&FPOS
    INI(YPUZYPO)
```



```
    INTIZFD,ZPOS
        HuT(i,XP),XP)
        OUT(T:XFJ;XPS
        FIN(T:AUS)
                        ENO
```

4. COMPUTER PROGRAM II



5. INPIT DATA FOR COMPUTER PROGRAM II
$D T=5.0 \times 10^{-4}$
DTMIN $1.0 \times 10^{-5}$
DTMAX $1.0 \times 10^{-3}$
AX 0.0
BX 0.0
AY 0.0
By 0.0
Cy 0.0
Az 0.0
Ez 0.0
cz 0.0
MAS $5.44 \times 10^{2}$
$\operatorname{IxX} 2.687 \times 10^{2}$
IYY $2.317 \times 10^{4}$
Cass $1.127 \times 10^{1}$
CMY 0.0
CMZ 0.0
uNCY $2.254 \times 10^{1}$
LNCO 0.0
RAD 0.992
$U_{0} \quad 4.06 \times 10^{4}$
$v_{0} \quad 0.0$
$W_{0} 0.0$
$\mathrm{P}_{0} \quad 0.0$
$Q_{0} \quad 0.0$
$R_{0} \quad 0.0$
PHIO 0.0
тнO 0.0
PSIO 0.0
XPO 0.0
YPO 0.0
2PO 0.0
$R 1 \quad 1.0$
$\mathrm{K} 2 \quad 0.0$
$\mathrm{k} 3 \quad 0.0$
$K 4 \quad 1.0$
K5 0.0
$\mathrm{K} 6 \quad 0.0$

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