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COMPUTER PROGRAM FOR STEADY TRANSONIC FLOW OVER THIN AIRFOILS BY FINITE ELEMENTS

HUNTSVILLE RESEARCH & ENGINEERING CENTER
LOCKHEED MISSILES & SPACE COMPANY, INC.

OCTOBER 1975

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AIR FORCE FLIGHT DYNAMICS LABORATORY
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NOTICE

This technical report has been reviewed and is approved for publication.

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A finite element program is described for the analysis of subsonic and transonic flows over thin airfoils by solving the nonlinear transonic potential equation based on small disturbance theory. The present numerical algorithm is developed using the concept of finite elements in conjunction with the least square method of weighted residuals. Since the governing equation is of the elliptic-hyperbolic type, a one-sided assembly technique was devised and adapted in the supersonic region to restore the directional property of the flow, which was removed by the exclusion of entropy from the transonic potential equation. The finite element discretization results in a system of banded nonlinear algebraic equations, which is solved by direct iterations. The elements presently used include triangles and quadrilaterals with the perturbed potential function and velocity components as nodal unknowns. Boundary conditions of both Dirichlet and Neumann types are therefore imposed conveniently. Also, secondary unknowns, such as local Mach number and pressure coefficient, etc., are computed directly without resorting to numerical differentiation. The present computer program is separated into two parts: the first part (designated as STRANL-I) generates, from a limited number of input cards, the necessary mesh information and, if desired, produces a mesh plot and optimal nodal numbering as well; the second part (STRANL-II) carries out the analysis and displays the pressure coefficients along the chord line on printer plots. Two sample cases of flow over a NACA 64 A006 and a 6% thick circular arc are given to demonstrate the applicability and usage of the program. The solution procedures are found to be very accurate and quite efficient, permitting the aerodynamic forces to be calculated to engineering accuracy in tens of seconds on a CDC 6600 computer.

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FOREWORD

This report was prepared by personnel in the Computational Mechanics Section of the Lockheed Missiles & Space Company, Inc., Huntsville Research & Engineering Center, Huntsville, Alabama for the Air Force Flight Dynamics Laboratory, Wright-Patterson Air Force Base, Ohio. The computer programs were developed under Project 1370, "Dynamic Problems in Flight Vehicles," Task 137004, "Design Analysis," Contract F33615-75-C-3125. Capt. Gerald Van Keuren, AFFDL/FYS, is the Air Force Project Engineer.

S.T.K. Chan was the principal investigator for the study including computer program development under the supervision of M.R. Brashears. The authors are grateful for the assistance of the following Lockheed-Huntsville personnel: V.Y.C. Young, for the invaluable technical discussions and A.J. Jordan for aid in computer program conversion.

The authors submitted this report in July 1975 as an AFFDL technical report to cover research performed from April through June 1975.

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LIST OF SYMBOLS

Symbol

a	local speed of sound
c	chord length of airfoil
C_p	pressure coefficient
$g(x)$	thickness distribution of the airfoil
L_i	right-hand side vector of the resulting system of equations
$M = V/a$	local Mach number
M_∞	freestream Mach number
N_i	shape functions
p	local static pressure
p_o	stagnation pressure
R	residual resulting from any approximate solution
S_{ij}	system coefficient matrix
u, v	perturbed velocity components in the x- and y-directions, respectively
U_∞	freestream speed
$x = x'/c$, $y = y'/c$	nondimensional x and y coordinates

Greek

γ	ratio of specifc heats (γ = 1.4 for air)
ϵ	small number used as convergence criterion
θ	under-relaxation factor
$\phi = \phi'/(U_\infty \cdot c)$	nondimensional perturbation velocity potential
ϕ_i	undetermined parameters at nodal points
χ	integral expression for the squared errors over the entire flow domain
∇	gradient of a scalar function

In addition, a repeated index implies summation unless specified otherwise. Also, subscripts after a comma indicate partial differentiations.

1. INTRODUCTION

The aim of the present study was to develop an efficient and accurate numerical algorithm for the analysis of steady transonic flow over nonlifting thin airfoils. With this objective, the present formulation is therefore based on the small disturbance but nonlinear transonic potential equation for inviscid, compressible fluid.

It is well known that two major difficulties exist in solving the transonic flow problem: (1) the governing partial differential equation is nonlinear, and (2) the equation is of mixed elliptic-hyperbolic type. Because of these difficulties, numerical procedures must therefore be used, with provisions made to properly account for the above transonic effects. Among the numerical approaches, the finite difference relaxation technique is the most well developed and widely used in recent transonic calculations. On the other hand, the finite element technique, in spite of its tremendous success in the field of solid and structural mechanics, has never been attempted until the present investigation, obviously because conventional finite element technique is invalid in solving mixed-type equations. This difficulty, nevertheless, was successfully resolved by a special assembling technique devised for the supersonic region, while the major advantages of the conventional finite element method are retained. These include the complete flexibility in element arrangement, its effectiveness to cope with complex geometric shapes and boundary conditions, and the ease with using efficient higher order approximations. For these reasons, the present approach appears to be superior in solution accuracy and computational efficiency compared to other existing numerical techniques.

The present numerical algorithm is developed using the concept of finite elements in conjunction with the least squares method of weighted residuals

The basic element presently used has a cubic expansion for the perturbed velocity potential inside the element, with nodal unknowns representing the function itself and the two perturbed velocity components. The resulting system of nonlinear algebraic equations is banded, although nonsymmetric. This system of algebraic equations is solved by direct iterative procedures.

The theory and numerical procedures, together with the usage of the computer programs, are discussed in the following sections. The theory including the small disturbance potential equation with its corresponding boundary conditions and the related secondary unknowns are summarized in Section 2. The numerical procedures employed in the present approach are discussed in Section 3. These include the finite element formulation based on least squares, elements used in the computation, special considerations for the supersonic region, treatment of boundary conditions and a description of the iterative procedures. In Section 4, the two parts of the computer code, namely, STRANL-I and STRANL-II are described separately, in the aspects of scope and flow chart, description of variables, subroutines used, and finally input and output. Two sample cases are given in Section 5 to demonstrate how to use these computer programs, which are listed in the Appendixes.

2. THEORY — ASSUMPTIONS AND BASIC EQUATIONS

The objective of the present study was to develop an efficient and accurate numerical algorithm for the analysis of steady transonic flow over nonlifting thin airfoils. With this objective in mind, the formulation was therefore based on the small disturbance but nonlinear transonic potential equation for inviscid, compressible fluid. The embedded shock is assumed to be weak and boundary layer effects are neglected. These assumptions render the small perturbation theory possible and lead to the following mathematical problem to be solved.

Differential Equation

$$\left[1 - M_\infty^2 - M_\infty^2 (1 + \gamma) \phi_{,x}\right] \phi_{,xx} + \phi_{,yy} = 0 \qquad (1)$$

Boundary Conditions

$$(1 + u) \frac{dg}{dx} - v = 0 \quad \text{on the airfoil} \qquad (2)$$

$$\nabla \phi = 0 \quad \text{at infinity} \qquad (3)$$

$$v = 0 \quad \text{on line of symmetry} \qquad (4)$$

where ϕ = perturbed velocity potential function, u,v = perturbed velocity components in the x- and y-directions, respectively, M_∞ = freestream Mach number, γ = ratio of specific heats, taken to be 1.4 for air, and g = function of x defining the geometry of the airfoil. Equation (1) is in dimensionless form and the x-axis is aligned with the undisturbed flow direction. The dimensional (with primes) and nondimensional quantities are related by

$$x = \frac{x'}{c}, \quad y = \frac{y'}{c} \quad \text{and} \quad \phi = \frac{\phi'}{U_\infty c}$$

where c and U_∞ are the characteristic length and speed, which are currently taken as the chord length and the freestream speed, respectively.

Figure 1 shows the flow field, together with the steady transonic governing equation and corresponding boundary conditions. Because only nonlifting thin airfoils are being considered, the airfoils are necessarily symmetric and hence only half of the flow field is depicted and analyzed. As the figure shows, the flow tangency condition on the airfoil is retained in its nonlinear form, as defined by Eq. (2), and is to be applied along the actual airfoil surface. Another approach is to use the linearized boundary condition and apply it along the airfoil chordline, as commonly adapted in most existing finite difference codes. Apparently, use of the nonlinear form is more accurate when the magnitude of u becomes non-negligible. On the other hand, use of the linearized boundary condition could enhance solution convergence somewhat as the nonlinearity now exists in the governing equation only. Extensive studies and comparisons are certainly required in order to assess the two approaches. In the present computer code, the nonlinear form is used unless otherwise specified through an input control key, in which case the linear form will be used. On the line of symmetry, as is well known, the Neumann condition of no normal flow is to be imposed. For the far field, because only a finite domain can be treated in the analysis, the condition of no disturbance at infinity is assumed to be valid on the outer boundary which is usually taken as a few chord lengths away from the airfoil. Numerical experimentations indicate that a flow region with $H \geq 2c$ and $L \geq 4c$ is generally required to yield solution with adequate accuracy. For flow with higher freestream Mach number, the flow region should be extended somewhat to account for effects from the enlarging supersonic pocket.

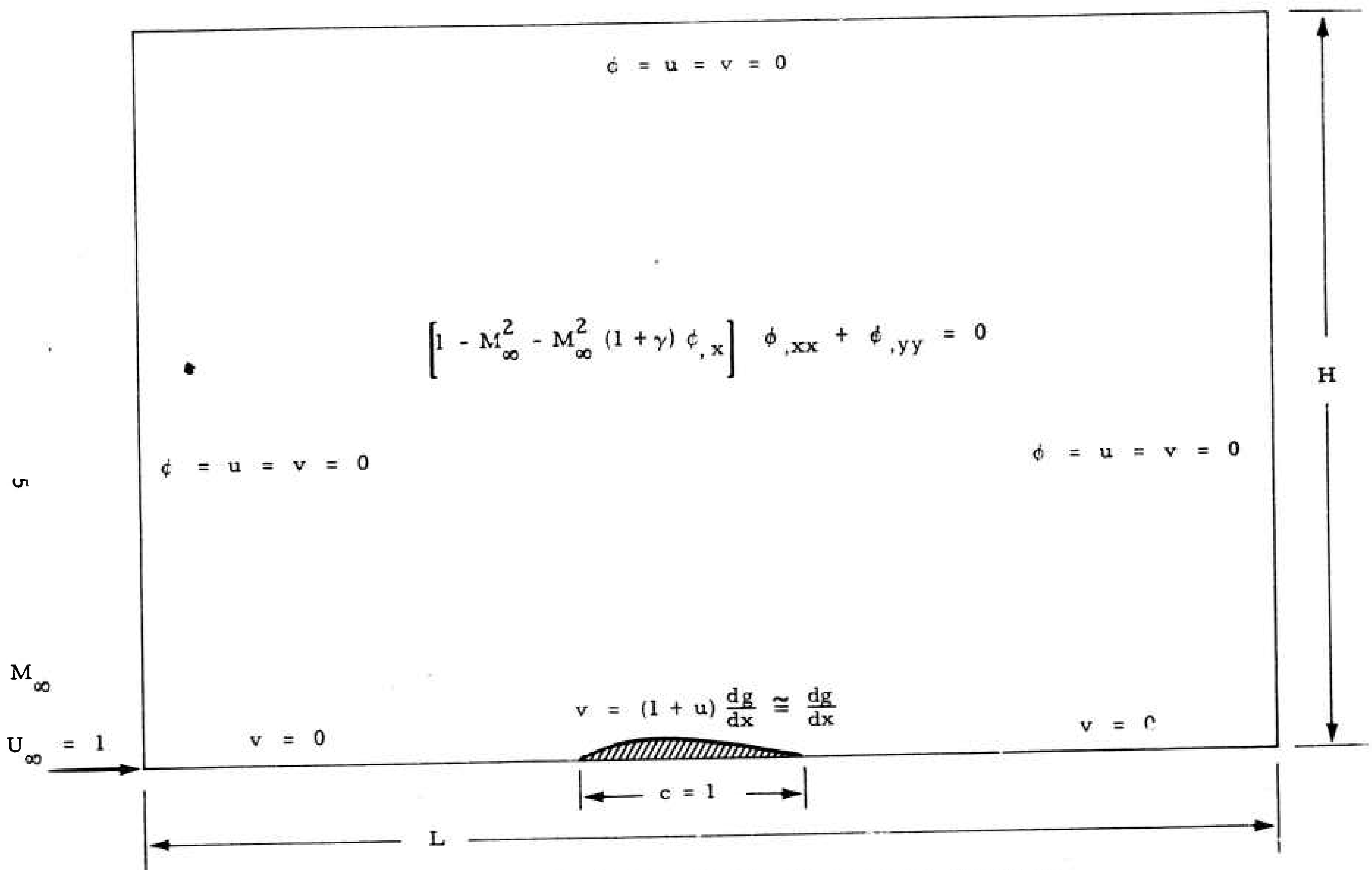


Figure 1 - Schematic of Flow Field and Governing Equations

For the problem of lifting airfoils, the entire flow domain must be considered. In this case, an asymptotic solution including at least the circulation as unknown should be applied on the finite outer boundary. This is part of a current study conducted for AFFDL and will be implemented later.

Once the flowfield solution in terms of the perturbed velocity potential has been obtained, all secondary unknowns can be calculated subsequently. These include

$$a = \left[\frac{\gamma - 1}{2} (U_\infty^2 - V^2) + \left(\frac{U_\infty}{M_\infty}\right)^2\right]^{1/2} \tag{5}$$

$$M = \frac{V}{a} \tag{6}$$

$$\frac{p}{p_o} = \frac{1}{\left[1 + \frac{\gamma - 1}{2} M^2\right]^{\frac{\gamma}{\gamma - 1}}} \tag{7}$$

$$C_p = -\left[\frac{2u}{U_\infty} + (1 - M_\infty^2)\frac{u^2}{U_\infty^2} + \frac{v^2}{U_\infty^2}\right] \cong -\frac{2u}{U_\infty} \tag{8}$$

In the above, $U_\infty = 1$, the normalized freestream speed, a = local speed of sound, M = local Mach number, V = the total velocity, p = local static pressure, p_o = stagnation pressure, and C_p = pressure coefficient.

3. NUMERICAL SOLUTION PROCEDURES

The concept of finite elements in conjunction with the method of weighted residuals (MWR) is the basis of the present numerical procedures in solving the small perturbation transonic flow equation. Of all the MWR methods, the Galerkin and the least squares approaches were investigated (Chan et al., Ref. 1), of which only the latter approach was found applicable to mixed flow analyses. The success with the least squares formulation is attributable to the resulting matrix being positive definite and well conditioned. This approach was therefore incorporated in the present computer program, with numerical procedures described in the following subsections. These include the finite element formulation, element description, special considerations in the supersonic region, the imposition of boundary conditions and iterative procedures.

3.1 Finite Element Formulation

The finite element method, in conjunction with the least squares method of weighted residuals, is used herein to solve numerically the small perturbation transonic equation. In this approach, a set of locally defined trial functions with undetermined parameters is assumed as the approximate solution, and the integral expression for the square of errors committed by the approximate solution is formulated. Then the integral of square errors in the entire domain is minimized with respect to the undetermined parameters to yield a system of algebraic equations. In actual computations, the minimization process is performed at element level and then an assembling process is invoked to obtain the system of algebraic equations. It is these processes which enable us to take care of the supersonic pocket with convenience and success. This special assembly technique is described in Section 3.3.

Written in the usual manner with repeated indices implying summation, the approximate solution has the following form

$$\hat{\phi} = N_i \phi_i \tag{9}$$

in which N_i's represent the shape functions and ϕ_i's are the undetermined parameters at nodal points. The residual then becomes

$$R = \left\{ \left[1 - M_\infty^2 - M_\infty^2 \; (1+\gamma) \, N_{k,x} \, \phi_k \right] N_{j,xx} + N_{j,yy} \right\} \phi_j \tag{10}$$

from which an integral expression for the square errors is obtained as

$$\chi = \iint R^2 dA \tag{11}$$

Upon the minimization of χ with respect to the undetermined parameters, a system of algebraic equations in the following form is obtained

$$S_{ij} \, \phi_j = 0 \tag{12}$$

Herein the banded system matrix S_{ij} is defined as

$$S_{ij} = \iint P_i \, Q_j \, dA \tag{13}$$

with P_i and Q_j determined by the following expressions

$$Q_j = \left[1 - M_\infty^2 - M_\infty^2 \; (1+\gamma) \, N_{k,x} \, \phi_k \right] N_{j,xx} + N_{j,yy} \tag{14}$$

and

$$P_i = Q_i - M_\infty^2 \; (1+\gamma) N_{k,xx} \, \phi_k \, N_{i,x} \tag{15}$$

As stated earlier, the system matrix is obtained by combining appropriate contributions from all the elements. The element matrices, in turn, are evaluated effectively by numerical integration to avoid the tedious and error prone algebraic manipulations. The one presently used is a 7 point Gaussian quadrature, which can integrate exactly a quintic polynomial in the element. It is to be noted that although the system of equations, Equation (12), is homogeneous; it does possess a non-trivial solution once the boundary condition on the airfoil is imposed.

3.2 Element Description

The basic element used in the present program is the nonconforming cubic triangular element developed by Bazeley et al. (Ref. 2). Also used in the program are quadrilateral and trapezoidal elements constructed from these triangular elements. These three types of elements can be mixed and used freely in the entire flow region except that only trapezoids should be used to cover adequately the supersonic region in order to enact the special treatment discussed later.

The basic triangular element is shown in Fig. 2, which at each vertex has the function itself and its two first derivatives (velocity components) as undetermined parameters. This type of element was adapted mainly because boundary conditions of both Dirichlet and Neumann types can be imposed with

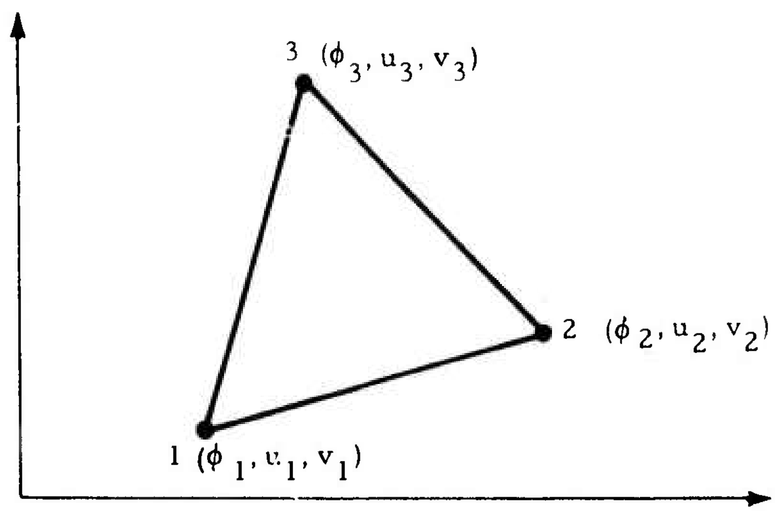


Figure 2 - Triangular Element with Undetermined Parameters

equal convenience. In addition, because velocities at nodes are treated as primary unknowns, secondary unknowns such as local Mach number, and pressure coefficients, etc. can be computed directly without resorting to numerical differentiation and thus assuring higher accuracy. Furthermore, the use of higher order elements can usually improve computational efficiency as evidenced in most finite element analyses.

In the element, the approximate solution is assumed as

$$\phi^e = N_i \phi_i \quad (i = 1 \text{ to } 9)$$

in which ϕ_i's are the nine undetermined parameters of ϕ and its first derivatives at the nodal points as shown in the figure and N_i's are the corresponding shape functions, which are expressed in terms of area coordinates.

Defined in the following are these shape functions and their first and second derivatives.

Letting

$$a_i = x_j y_k - x_k y_j$$

$$b_i = y_j - y_k$$

$$c_i = x_k - x_j$$

$$\Delta = \text{area of triangle 1-2-3} = (b_j c_k - b_k c_j)/2$$

$$\alpha = 0.5\,(c_k - c_j)$$

$$\beta = 0.5\,(b_j - b_k)$$

$$H = \zeta_i \zeta_j \zeta_k$$

$$H_x = b_i \zeta_j \zeta_k + b_j \zeta_k \zeta_i + b_k \zeta_i \zeta_j$$

$$H_y = c_i \zeta_j \zeta_k + c_j \zeta_k \zeta_i + c_k \zeta_i \zeta_j$$

$$H_{xx} = 2(\zeta_i b_j b_k + \zeta_j b_k b_i + \zeta_k b_i b_j)$$

$$H_{yy} = 2(\zeta_i c_j c_k + \zeta_j c_k c_i + \zeta_k c_i c_j)$$

with i=(1,2,3), j=(2,3,1), k=(3,1,2), then one has for ℓ=(1,4,7), i=(1,2,3),

$$N_\ell = \zeta_i^2 (3-2\zeta_i) + 2H$$

$$N_{\ell,x} = \frac{1}{2\Delta} \left[6b_i\zeta_i(1-\zeta_i) + 2H_x \right]$$

$$N_{\ell,y} = \frac{1}{2\Delta} \left[6c_i\zeta_i(1-\zeta_i) + 2H_y \right]$$

$$N_{\ell,xx} = \frac{1}{(2\Delta)^2} \left[6b_i^2(1-2\zeta_i) + 2H_{xx} \right]$$

$$N_{\ell,yy} = \frac{1}{(2\Delta)^2} \left[6c_i^2(1-2\zeta_i) + 2H_{yy} \right]$$

for ℓ=(2,5,8), i=(1,2,3)

$$N_\ell = \zeta_i^2(c_k\zeta_j - c_j\zeta_k) + \alpha H$$

$$N_{\ell,x} = \frac{1}{2\Delta} \left[2b_i\zeta_i(c_k\zeta_j - c_j\zeta_k) + 2\Delta\zeta_i^2 + \alpha H_x \right]$$

$$N_{\ell,y} = \frac{1}{2\Delta} \left[2c_i\zeta_i(c_k\zeta_j - c_j\zeta_k) + \alpha H_y \right]$$

$$N_{\ell,xx} = \frac{1}{(2\Delta)^2} \left[2b_i^2 (c_k\zeta_j - c_j\zeta_k) + 4b_i(2\Delta)\zeta_i + \alpha H_{xx} \right]$$

$$N_{\ell,yy} = \frac{1}{(2\Delta)^2} \left[2c_i^2(c_k\zeta_j - c_j\zeta_k) + \alpha H_{yy} \right]$$

for ℓ=(3,6,9), i=(1,2,3)

$$N_{\ell} = \zeta_i^2(b_j\zeta_k - b_k\zeta_j) + \beta H$$

$$N_{\ell,x} = \frac{1}{2\Delta}\left[2b_i\zeta_i(b_j\zeta_k - b_k\zeta_j) + \beta H_x\right]$$

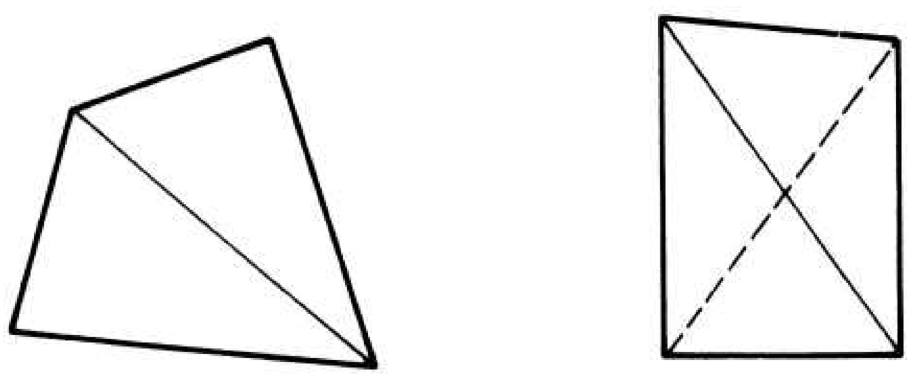
$$N_{\ell,y} = \frac{1}{2\Delta}\left[2c_i\zeta_i(b_j\zeta_k - b_k\zeta_j) + 2\Delta\zeta_i^2 + \beta H_y\right]$$

$$N_{\ell,xx} = \frac{1}{(2\Delta)^2}\left[2b_i^2(b_j\zeta_k - b_k\zeta_j) + \beta H_{xx}\right]$$

$$N_{\ell,yy} = \frac{1}{(2\Delta)^2}\left[2c_i^2(b_j\zeta_k - b_k\zeta_j) + 4c_i(2\Delta)\zeta_i + \beta H_{yy}\right]$$

The undetermined parameters are arranged in the order of ϕ_1, u_1, v_1, ϕ_2, u_2, v_2, ϕ_3, u_3, and v_3.

Quadrilateral and trapezoidal elements as shown in Fig. 3 are also used in the present program, the former in purely subsonic flow region to save data input and the latter in the mixed and supersonic flow region. The element matrix for a quadrilateral is obtained by combining appropriately the matrices for two triangles, while matrix for a trapezoidal element is



a. Quadrilateral Element b. Trapezoidal Element

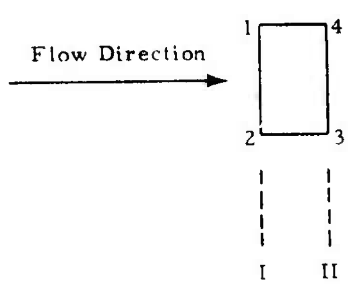
Figure 3 - Elements Constructed from Basic Triangles

obtained by combining and averaging contributions from four triangles, two left-running and two right-running triangles. The averaging process is designed to remove the bias effects inherent in the quadrilaterals presently used. In any event, the present program is coded in such a way that the computer is given the jurisdiction over which type of element to use, according to the flow behavior in the element. Of course, it does no harm to use trapezoids where the usage of quadrilaterals is considered adequate, except that computation time will increase slightly.

3.3 Special Considerations for Supersonic Region

As is well known, the finite element method is a powerful tool for solving problems governed by elliptic equations. However, for problems governed by either parabolic or hyperbolic equations, the conventional finite element assembly technique must be modified so that the proper influence of solution in time (or time-like direction) is considered. For the present problem, the x-axis is a time-like direction which implies that the solution at the upwind station will determine the solution at its downwind station but not the contrary. This consideration leads to the well known upwind influence finite difference operator (backward finite difference). An equivalent technique in finite element analysis has been devised and applied to solve the transonic flow equation.

Consider the rectangular element sketched below with upwind station I and downwind station II, each having two nodal points. With the element type chosen, the element matrices can be constructed in the usual manner. However, before assembling the element matrices into the system matrix, the coefficient of the



first term in the transonic equation should be evaluated

$$C = 1 - M_\infty^2 - M_\infty^2 (1 + \gamma) u \tag{16}$$

The sign of the coefficient C being positive, zero, or negative will define the equation as elliptic, parabolic or hyperbolic. If C is non-positive for all nodes in the element, the rows representing the improper downwind influence on solution at an upwind station are ignored during assembly. This feature is taken care of automatically in the program, requiring only a little care with the nodal arrangement in the element. In the anticipated supersonic region, element node points should be arranged in the order as indicated in the above sketch, starting with the upper left corner node and proceeding in counter-clockwise direction. On the other hand, if the sign of C is positive at any of the four nodes, no special assembling is invoked. As stated earlier, only trapezoidal elements are to be used in the anticipated supersonic and mixed flow region, while triangular elements and quadrilaterals consisting of only two triangles are considered unsuitable because of their bias nature. However, these two latter types of elements can be used effectively in the subsonic flow region.

3.4 Boundary Conditions and Iterative Procedures

As stated earlier, the imposition of boundary conditions for the present problem can be carried out conveniently because the elements presently used have function value and two first derivatives as primary unknowns. The associated boundary conditions thus can be treated as the essential type, i.e., having prescribed values. Standard finite element methodology is therefore followed by assembling first an unconstrained problem and then modifying the matrix equations accordingly. More specifically, on the outer boundary the flow is assumed to be undisturbed and hence the corresponding unknown parameters are set equal to zero. On the line of symmetry, the velocity component v is also set equal to zero. On the airfoil, the nonlinear form of flow tangency on the airfoil is imposed by replacing the algebraic equation for v by

$$v_i - \frac{dg}{dx} u_i = \frac{dg}{dx} \tag{17}$$

in which u_i and v_i are unknown parameters at node "i" on the airfoil. When the linear form is desired, the corresponding equation becomes

$$v_i = \frac{dg}{dx} \tag{17a}$$

and is applied along the chordline. The selection between these two approaches is made through a control key in the input.

In the present program, no special efforts have been devoted to treat the leading edge and trailing edge singularities. Any rigorous approach should consider also the invalidity of small perturbation theory in these regions, which is certainly beyond the scope of the present study. Nevertheless, these regions are relatively small and the flow is usually subsonic in nature, which implies any error committed is generally localized and becomes less important elsewhere. For non-blunt airfoil such as a 6% thick circular arc, using half of the actual airfoil slope at each respective place in the computation seems to work well. However, for blunt airfoil such as NACA 64 A006, a finite slope near the leading edge, say 1% behind the airfoil leading edge, could be used in computing the vertical velocity component at that point. A still better approach is to use very fine elements in that region and treat the leading edge as a singular point. In practical computations, however, an element with its area approaching zero might create a new numerical problem because all shape function derivatives involve element area as divisor. Therefore, caution must be taken in using extremely small elements.

With the equations properly assembled and boundary conditions imposed, the system of nonlinear algebraic equations is finally solved by iterative procedures in the form

$$S_{ij} (\overline{\phi}) \phi_j^{(n)} = L_i \tag{18}$$

to solve for the solution $\phi^{(n)}$ at the n^{th} iteration. The function $\bar{\phi}$ is, in turn, defined as

$$\bar{\phi} = \theta \phi^{(n-1)} + (1-\theta) \bar{\phi}^{(n-1)} \tag{19}$$

in which θ is a relaxation factor in the range $0 < \theta \leq 1$. For subsonic flows, θ is usually taken as unity; for flow in the transonic regime, however, it has been found that the use of an under relaxation factor is required. The value of θ should decrease from unity for barely critical flow to 0.4 or even 0.3 for supercritical flow. The optimum under-relaxation factor generally depends on the freestream Mach number and the mesh being used. At the present, no systematic investigations have been done in this regard, and therefore its value should be selected by numerical experimentations. Generally speaking, a smaller under-relaxation factor will make the solution more stable but at the same time tends to slow down the rate of convergence.

Equation (18) is to be solved subject to certain prescribed convergence criterion. The one presently used is that the relative change of local Mach number between two consecutive iterations should be less than a prescribed small number for all the nodes in the flow field, that is

$$\left| \frac{M^{(n)} - M^{(n-1)}}{M^{(n)}} \right| < \epsilon \tag{20}$$

Numerical experimentations indicate that a solution with adequate accuracy is generally obtainable with ϵ in the range $0.01 \leq \epsilon \leq 0.001$.

4. PROGRAM DESCRIPTIONS AND USAGE

As stated earlier, the present computer program is capable of analyzing steady transonic flow over nonlifting thin airfoils by solving the small disturbance but nonlinear transonic potential equation. The numerical algorithm is based on utilizing the finite element technique in conjunction with the least squares method of weighted residuals. To solve a particular transonic flow problem, an appropriate finite element mesh must first be set up, followed by using the numerical procedures summarized in Section 3. Thus the present programs are separated into two parts – the first part which generates the necessary mesh information from a limited number of input cards and the second part which carries out the analysis with element mesh generated in the first part. By doing so, the generated mesh can be fully inspected for its correctness prior to the analysis, and storage required in the second program can be accordingly determined to suit each particular problem. These two programs are designated STRANL-I and STRANL-II (Steady Transonic Flow by Lockheed, parts I and II, respectively) and are described in more detail in the following.

4.1 STRANL-I

4.1.1 Scope and Flow Chart

This program reads in a limited number of input cards and generates mesh information such as element nodes, nodal coordinates, boundary nodes and airfoil slope, etc., to be used in the second program. Additionally, the present program has two options – one for renumbering the nodes to yield a smaller bandwidth and the other to plot the generated mesh for quick inspection. A schematic flow chart of the program is shown in Fig. 4.

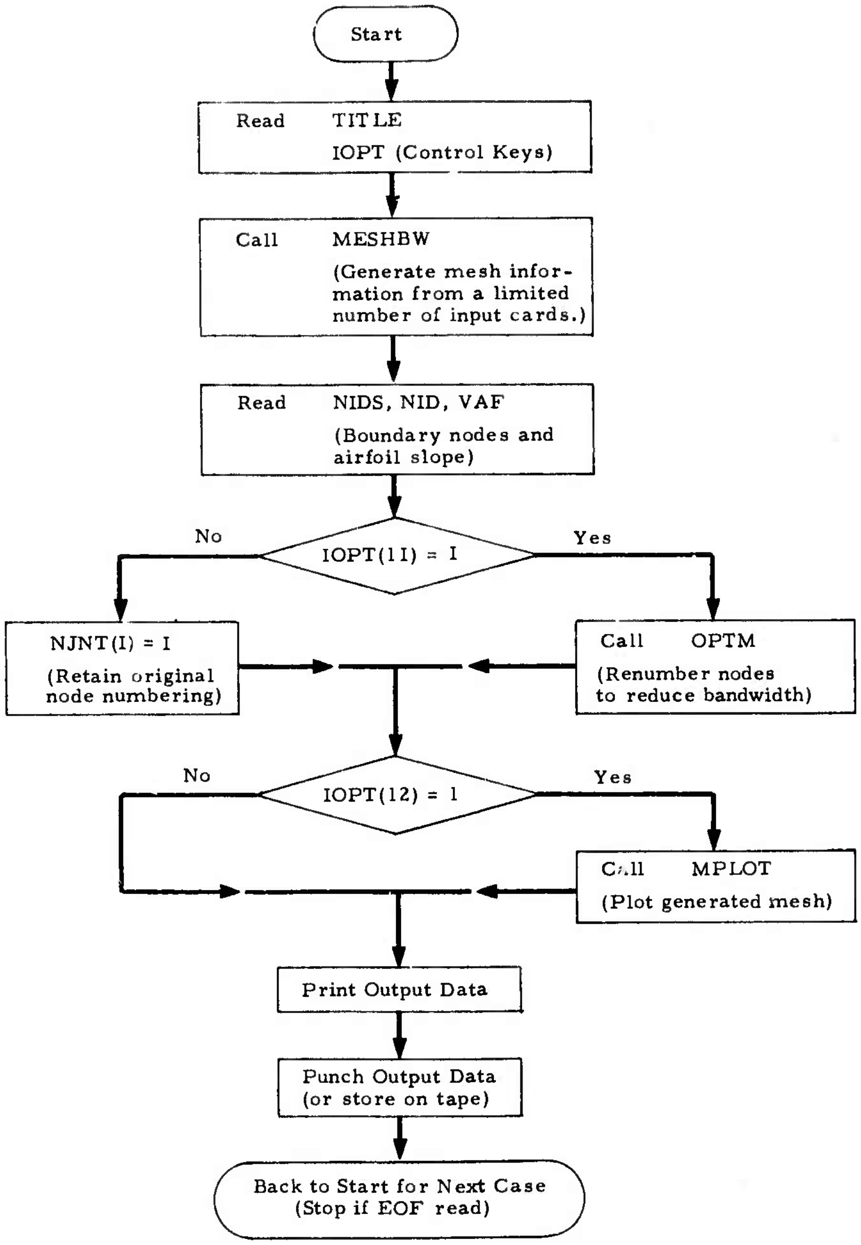


Figure 4 - Flow Chart of STRANL-I

The program as presently dimensioned requires approximately 42_8 words to run and can accommodate the following maxima:

- 200 elements
- 180 nodal points
- 50 nodes for each type of boundary conditions
- 10 nodes connected to any node in the mesh

4.1.2 Description of Variables

TITLE	Array used to describe the problem under consideration.
IOPT	Array containing the option keys which are activated when read in as "1." Presently there are two keys in the program, IOPT (11) for node renumbering option and IOPT (12) for mesh plot option.
NDEL	Array containing element node points.
X	Array containing the x-coordinate of nodal points.
Y	Array containing the y-coordinate of nodal points.
NJNT	Array containing the new nodal numbers if node renumbering option is executed; otherwise it retains the original nodal numbering.
MEMJT NEWJT JOINT JMEM	Arrays to provide adjustable dimensions used in subroutine OPTM for node renumbering.
NIDS	Array containing the actual number of boundary nodes for far field, on the line of symmetry, and along the airfoil surface.
NID	Array containing the nodal numbers for each type of boundary nodes mentioned above.
VAF	Array containing the airfoil slope for nodes on the airfoil surface.
NEM	Maximum number of elements allowed.
NPM	Maximum number of nodal points allowed.

NCN	Maximum number of nodes connected to any node in the mesh.
NEMN	Product of NPM with NCN.
NELS	Total actual number of elements.
NPS	Total actual number of nodal points.
MAXN	The maximum difference in nodal numbers between two connected nodes.
NBW	Full bandwidth of the resulting system matrix.

4.1.3 Subroutines

MESHBW: This subroutine is called to generate mesh information including the following: array of element nodes, total number of elements, x- and y-coordinates of nodal points, and the maximum difference in nodal numbers between two connected nodes. In generating this information, a small amount of input cards is required, which will be described under Section 4.1.4.

OPTM: This subroutine is called, when desired, to renumber the nodal points so as to yield as small a bandwidth as possible. The algorithm first establishes a table containing the nodal numbers connected to each of the nodal points, and then considers each node in turn as the origin of the new numbering system to search for one resulting in a smallest bandwidth. In consequence this subroutine generates an array of pointers relating the original numbering scheme to the new numbering scheme. This array is to be used in constructing the matrix equation, and is used again to refer back to the original numbering at output time. Using this method, one need not worry about the bandwidth problem and is unaware that any renumbering has taken place.

MPLOT: This routine plots the mesh generated together with the original node numbering, using the CALCOMP plotter. Correctness of the mesh can thus be checked quickly prior to running the second program.

4.1.4 Input and Output

All input and output are referred to the original numbering scheme, with input in the form of punch cards, and output in the form of printout and punch cards. If subroutine MPLOT is called, the mesh is also plotted for the convenience of inspection.

Input

Input cards to this program should be prepared and provided in the order described below.

A. TITLE CARD (7A10, A2)

Col. 1-72 Description of the problem under study

B. OPTIONS CARD (40I2)

Col. 22 Punch "1" if node renumbering is desired
Col. 24 Punch "1" if mesh plot is desired

C. ELEMENT CARDS (16I5)

These are cards supplied to generate element nodal numbers in groups. One card is needed for each group of elements whose nodes are related in a regular pattern. The number of elements in a group can vary from one to any positive integer number, depending on how the nodes are numbered originally. There can be as many cards as required to generate the element nodes, and a blank card is added at the end to terminate the process.

Col.	Description
1-5	Number of nodes per element. Punch 3 for triangular elements, and punch 4 for quadrilateral and trapezoidal elements.
6-10	Number of elements in one direction (called first direction).
11-15	Number of elements in the other direction (called second direction).
16-20	Increment of nodal numbers in the first direction.
21-25	Increment of nodal numbers in the second direction.

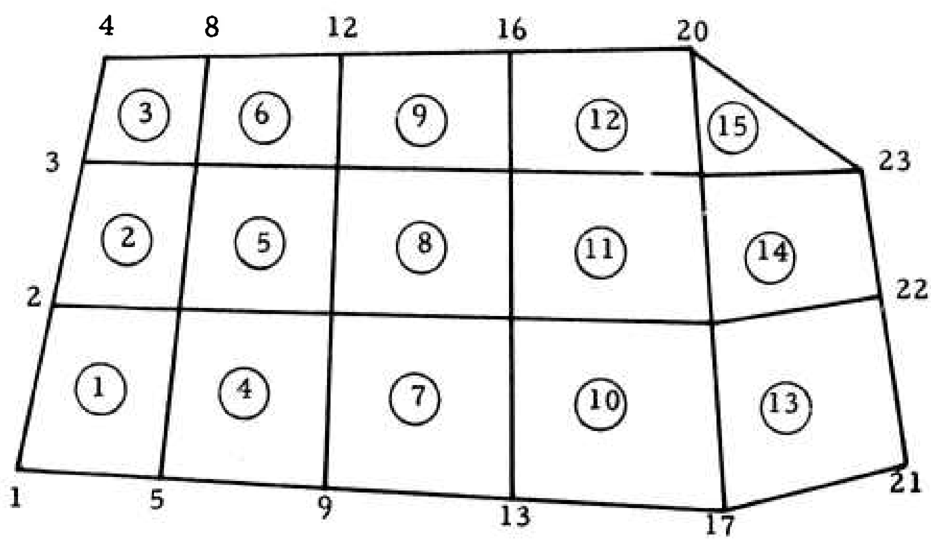
26-30	First nodal number in the element
31-35	Second nodal number in the element
36-40	Third nodal number in the element
41-45	Fourth nodal number in the element ("0" for triangles)

As stated before, element nodes are presently ordered in the counterclockwise direction, and the upper left corner node should be taken as the first nodal point in an element located in the anticipated supersonic region. This is required to enact the assembling process correctly in the supersonic pocket.

For example, to generate element nodes for a mesh shown below, three input cards are required: one card for the first 12 elements, second card for the next two elements, and the third card for the triangular element. The three cards are:

4	4	3	4	1	2	1	5	6
4	1	2	0	1	18	17	21	22
3	1	1	0	0	20	19	23	0

(Ended by a blank card when all elements are covered)



D. CARD FOR TOTAL NO. OF NODES (I5)

Col. 1-5 Total number of nodes for the entire flow region.

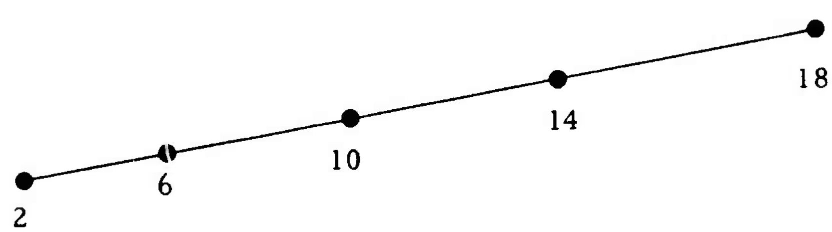
E. NODE COORDINATE CARDS (3I5, 5F10.0)

The x- and y-coordinates of nodes are also generated in connected groups. A connected group of nodes are those falling on a straight line with a constant ratio of nodal distances and constant increment in nodal numbers as well. The intermediate nodal points can thus be generated by linear interpolation. Again, the process is terminated by encountering a blank card.

<u>Col.</u>	<u>Description</u>
1-5	Node number of the beginning nodal point.
6-10	Total number of nodes on the line.
11-15	Increment of nodal numbers between two consecutive nodes.
16-25	Ratio of distances between three consecutive nodes.
26-35	x-coordinate of the beginning node.
36-45	y-coordinate of the beginning node.
46-55	x-coordinate of the end node.
56-65	y-coordinate of the end node.

For example, to generate the coordinates for the nodes depicted below with a constant distance ratio of 1.2, the input card should read as

2 5 4 1.2 x_2 y_2 x_{18} y_{18}



F. CARD FOR NO. OF NODES ON BOUNDARIES (16I5)

Col.	Description
1-5	Number of nodes on the outer boundary (called NFARF)
6-10	Number of nodes on line of symmetry (called NWAKE)
11-15	Number of nodes on the airfoil surface (called NBODY)

G. CARDS FOR BOUNDARY NODES (16I5)

(a) Nodes on the outer boundary (NFARF entries)

(b) Nodes on line of symmetry (NWAKE entries)

(c) Nodes on the airfoil surface (NBODY entries)

H. CARDS FOR SLOPE ON AIRFOIL (8F10.0)

Airfoil slope for nodes on the airfoil surface (NBODY entries)

Output

The output from this program is in the form of printout, punch cards, and also a plot if the mesh plot option is invoked.

The following items are printed in the order as described:

- Title of the problem under study
- Control keys specified in the options card
- Total number of elements, number of nodal points, and the full bandwidth
- Element numbers and element node points
- Nodal numbers and their corresponding x- and y-coordinates
- Nodes on each type of boundaries
- Slope for nodes on the airfoil surface

The above items except the first two, together with the NJNT array relating the new node numbering to the original node numbering, are then punched and saved as input to the second program (STRANL-II).

4.2 STRANL-II

4.2.1 Scope and Flow Chart

This program carries out the analysis following the numerical procedures described in Section 3, with any mesh information generated and supplied through the first program. In summary this program solves the small perturbation transonic potential equation via the least squares finite element technique. The embedded supersonic pocket is taken care of by a proper assembling scheme and the resulting system of nonlinear algebraic equations is solved by direct iterative scheme. The present program has three options, one for inputting nonzero initial guess of the solution, one for computing in the same run solutions for higher Mach number(s) using results computed for lower Mach number case(s), and the last one for selecting the form of boundary condition on the airfoil. A schematic flow chart of the program is shown in Fig. 5.

The program as presently dimensioned requires approximately 160_8 K words for the program itself and can accommodate the following maxima:

- 200 elements
- 180 nodal points
- 50 nodes for each type of boundary conditions
- 84 in full bandwidth of the resulting matrix equations

4.2.2 Description of Variables

TITLE	Array used to describe the problem under consideration
IOPT	Array containing the option keys which are activated when read in as "1." Presently there are three options: IOPT(1) for continuation for high Mach number cases while using existing results computed for lower Mach number, IOPT(2) for inputting non-zero initial guess (referred to the new numbering system), and IOPT(3) for selecting the linearized boundary condition applied along the airfoil chordline.

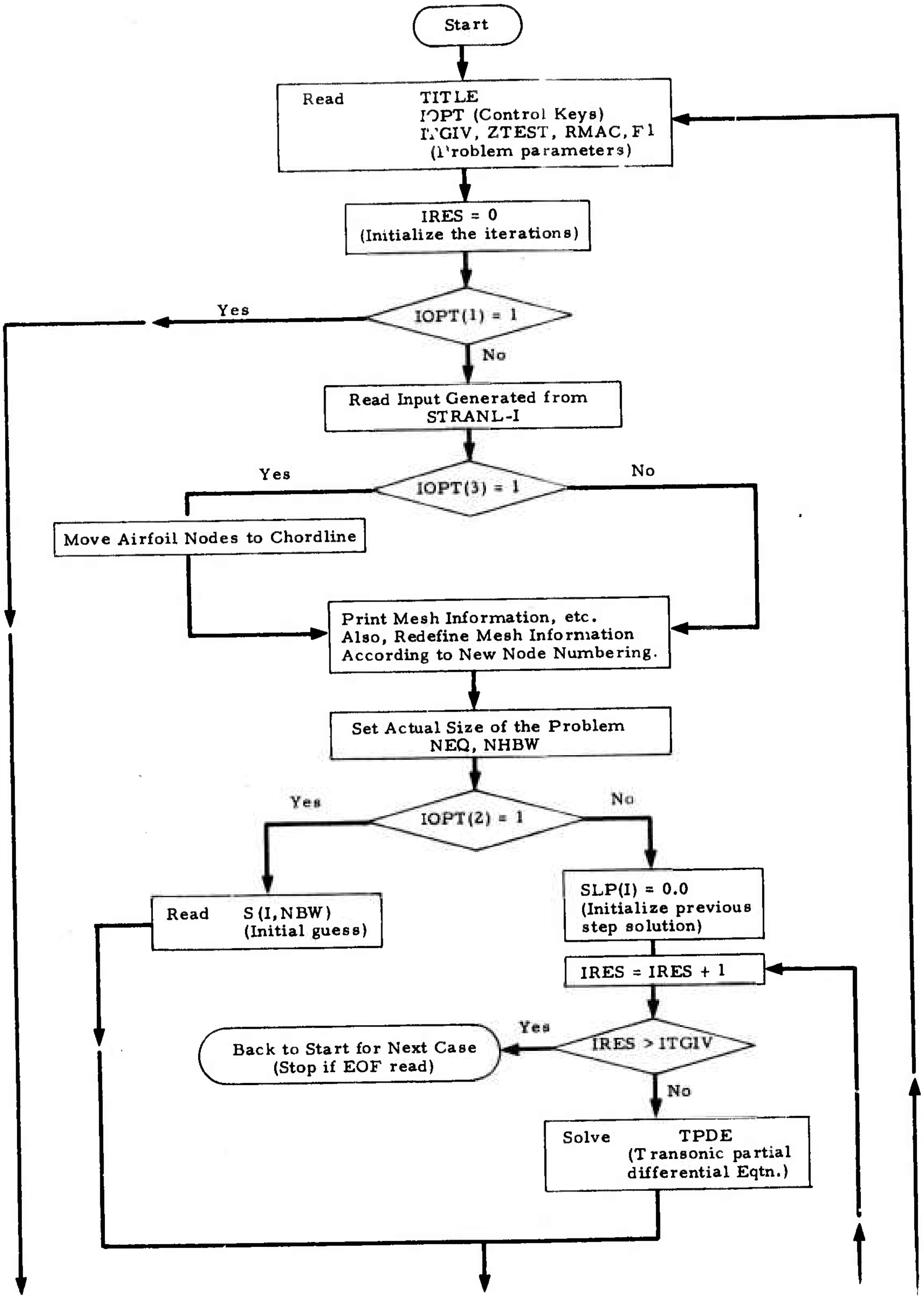


Figure 5 - Flow Chart of STRANL-II

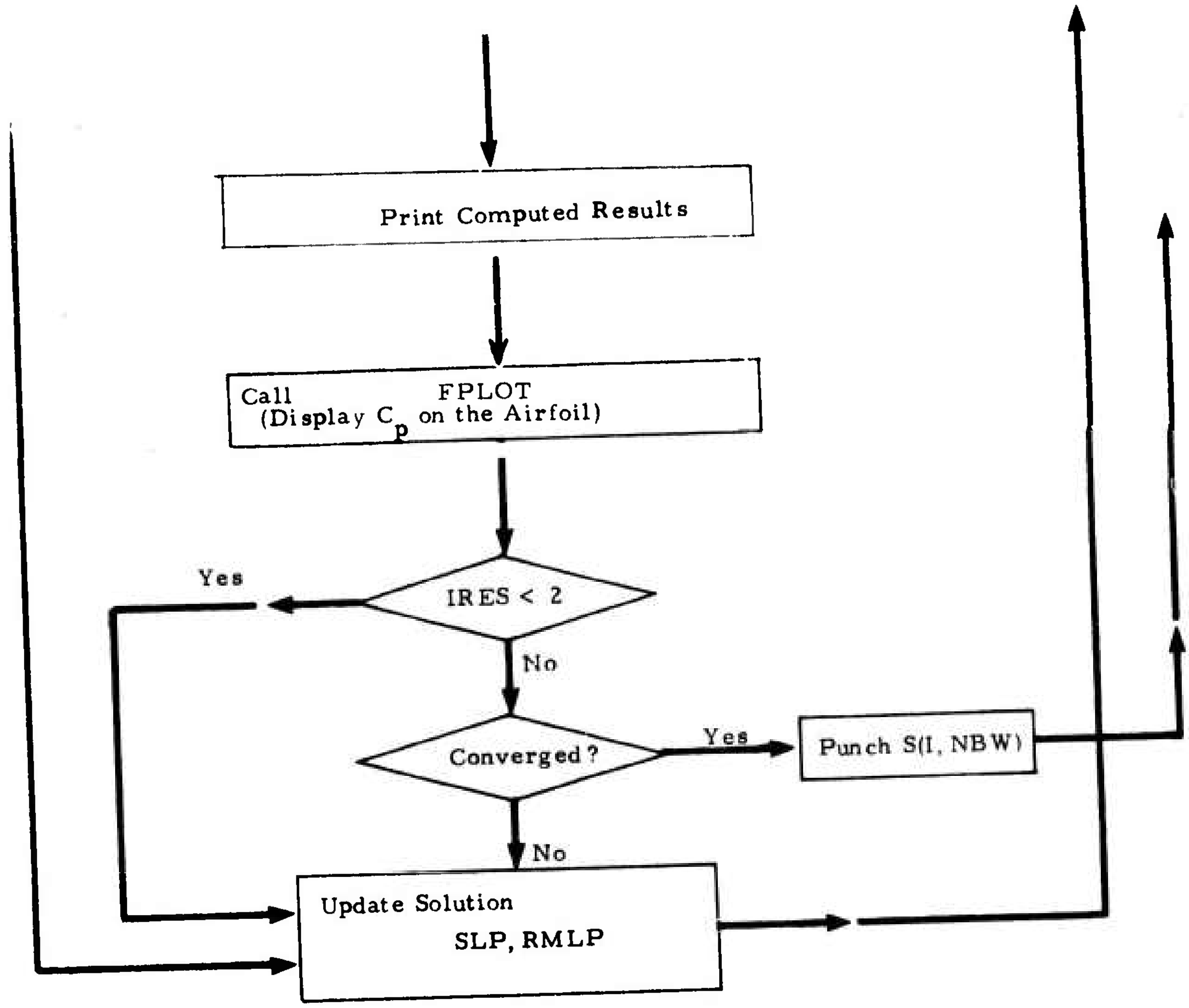


Figure 5 -(Concluded)

NOD	Array containing element node points
S	The resulting system matrix with main diagonal terms stored in the column numbered NHBW. After solving, the column numbered NBW contains the solution.
SLP	Array containing the solution obtained from rexalation procedures. This solution is used in generating the system matrix to solve for the next iterative solution.
X	Array containing the x-coordinate of nodal points.
Y	Array containing the y-coordinate of nodal points.
RML	Array containing values of local Mach number at nodal points for the current iteration.
RMLP	Array containing values of local Mach number at nodal points for the previous iteration.
COF	Array containing values of $M_\infty^2 + M_\infty^2 (1 + \gamma)u$ evaluated at nodal points.
NJNT	Array relating the new node numbering to the original node numbering schemes.
NIDS	Array containing the actual number of boundary nodes for far field, on the line of symmetry, and along the airfoil, respectively.
NID	Array containing the nodal numbers for each type of boundary nodes mentioned above.
VAF	Array containing the airfoil slope for nodes on the airfoil surface.
AR	Array to store values for use in subroutine FPLOT.
LR	Another array to be used in the subroutine FPLOT.
NEM	Maximum number of elements allowed.
NPM	Maximum number of nodal points allowed.

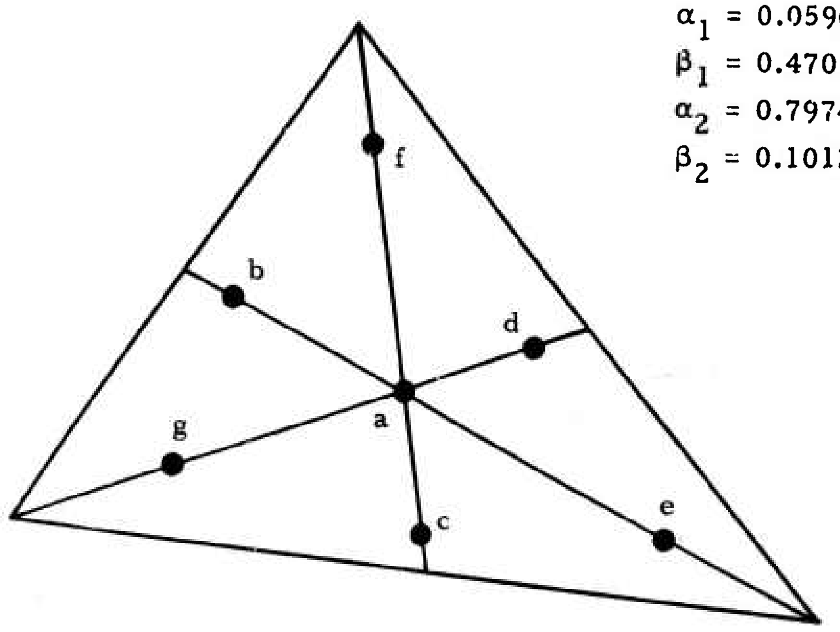
NCM	Maximum full bandwidth allowed in the resulting system matrix.
NA	Maximum number of nodes allowed on the airfoil surface.
NRM	Maximum number of equations allowed.
NELS	Total actual number of elements.
NPS	Total actual number of nodal points.
NBW	Full bandwidth of the resulting system matrix.
NHBW	Half bandwidth of the resulting system matrix.
NEQ	Number of equations in the resulting system of equations.
ITGIV	Cut-off number of iterations.
ZTEST	Small number used to check convergence based on the relative change of local Mach number between two consecutive iterations.
RMAC	Freestream Mach number.
SQMAC	Square of the freestream Mach number.
F1	Under-relaxation factor with $0 < F1 \leq 1.0$.
IRES	Number of iterations.
POT	Perturbed velocity potential.
UPT	Perturbed velocity component in the x-direction.
V	Perturbed velocity component in the y-direction.
U	Total velocity component in the x-direction.
PRATIO	The ratio of local static pressure to stagnation pressure.
CP	Pressure coefficient.
DELM	Change of local Mach number between two consecutive iterations.

4.2.3 Subroutines

NEWK: This subroutine is called in the main program to generate system matrix for the entire flow field by assembling appropriately the element matrices. The elements are, in general, a combination of triangles, quadrilaterals and trapezoids. For a 4-node element, this subroutine also determines either to treat it as a quadrilateral or as a trapezoid, depending on the local behavior of the governing equation being elliptic or otherwise. Subroutine EMQT is called to generate the element matrices.

EMQT: This subroutine generates element matrix for elements in the form of triangles, quadrilaterals and trapezoids. Element matrix for the later two types is obtained by combining appropriately (and averaging for the last type) contributions from triangular elements which are in turn evaluated in subroutine EMTC.

EMTC: This subroutine evaluates the element matrix for a triangular element by numerical integration. The one presently used is a 7-point Gaussian quadrature, which can integrate exactly a quintic polynomial in the element. Shown below are the locations of these Gaussian points with their corresponding weights shown on page 31.



$\alpha_1 = 0.05961587$
$\beta_1 = 0.47014206$
$\alpha_2 = 0.79742699$
$\beta_2 = 0.10128651$

Points	Area Coordinates			Weights
a	1/3	1/3	1/3	0.225
b	α_1	β_1	β_1	
c	β_1	α_1	β_1	0.13239415
d	β_1	β_1	α_1	
e	α_2	β_2	β_2	
f	β_2	α_2	β_2	0.12593918
g	β_2	β_2	α_2	

DERV: This is a subroutine called by EMTC to evaluate the first and second derivatives of the shape functions at a Gaussian point, based on the expressions listed in Section 3.2.

BNDEQ: This is an equation solver for banded nonsymmetric system of algebraic equations, utilizing the direct Gaussian elimination scheme. In this solver the system matrix is arranged in a twisted form such that main diagonal terms are stored in the column numbered NHBW and the right-hand side vector is in the column NBW. After solving, the column numbered NBW contains the solution vector.

FPLOT: FPLOT collects and stores data as it becomes available, and upon signal, produces a printer plot in practically any orientation and size. It should be regarded as a general purpose output routine for displaying output data in graphical form.

Description of Parameters

M1	Size of the main storage array, and it should never be larger than 800, which corresponds to 400 points to be plotted.
IPNT	Counter initialized — usually IPNT = 0 - in the calling program. . It is incremented by 2 each time a new data point is entered in AR.

AR	Main storage array. It should be in a dimension statement in the calling program. For example, DIMENSION AR(800).
LR	An array of bytes used to hold the curve number. It should be dimensioned for M1/2. The type declaration LOGICAL*1 LR(400) should be in the calling program.
ISTOP	Flag used to signal that all data has been entered. ISTOP = 0 causes data to be stored. If ISTOP = -1 and NC = NCMAX the program immediately branches to the plotting section. If ISTOP = 1 and NC = NCMAX a data point is stored and then the plotting section is entered.
NC	Curve number for which data is being entered. It must be a positive integer less than 21.
NCMAX	The number of curves to appear on the graph. This is the largest value NC will have.
V1	The horizontal coordinate of the data point to be plotted.
V2	The vertical coordinate of the data point to be plotted.

4.2.4 Input and Output

Input

The bulk of input to this program is provided through STRANL-I in the form of punch cards. To run several cases of various freestream Mach number but using the same mesh and boundary conditions, three additional cards should be furnished for each case to be computed. These cards should be provided in the following order

A. TITLE CARD (7A10, A2)

Col. 1-72 Description of the problem under study

B. OPTIONS CARD (40I2)

Col. 2 Punch "1" if it is a continuation in computation from one Mach number to another.

Col. 4	Punch "1" if cards for nonzero initial guess is furnished in the input data. If so, insert them following those generated from STRANL-I.
Col. 6	Punch "1" if linearized boundary condition along the chordline is desired.

NOTE: As seen in the flow chart, when IOPT(1) = 1, the last two options become inoperative.

C. PROBLEM PARAMETERS CARD (I5, 3F10.0)

Col. 1-5	Cut-off number of iterations.
Col. 6-15	Small number used to check convergence based on the relative change of local Mach number between two consecutive iterations, usually taken in the range from 0.01 to 0.001.
Col. 16-25	Freestream Mach number
Col. 26-35	Under relaxation factor with the value of 1.0 for subsonic cases, and decreasing for supercritical flows.

For the first case of the run, punch cards from STRANL-I are placed after the above cards. For subsequent cases using the same mesh, three additional cards as described above are required for each case.

<u>Output</u>

The output from this program include printout, referred to the original numbering system, and punch cards as well for the cases whose solutions are convergent.

The printouts are in the following order:

- Title of the problem under study
- Convergent limit

- Control keys specified in the options card
- Total number of element, number of nodal points, and the full bandwidth of the resulting matrix equations
- Element numbers and element node points
- Nodal numbers and their corresponding x- and y-coordinates
- Boundary nodes for each type of boundaries
- Slope for nodes on the airfoil surface
- For each iteration, the computed results at all nodal points are printed. These include the perturbed velocity potential, total velocity components in the x- and y-directions, the value of M_∞^2 (1 + 2.4 u), local Mach number, the ratio of local static pressure to stagnation pressure coefficient, and finally the change of local Mach number between two consecutive iterations.
- For each iteration, the value of C_p along the airfoil is displayed graphically.

5. SAMPLE CASES

Two sample cases are presented herein to demonstrate the usage of the computer programs described in the previous section regarding input and typical output from the programs. The two problems are flow over a NACA 64 A006 airfoil analyzed with a coarser mesh and flow over a 6% thick circular arc computed with a finer mesh where the nodes are renumbered to reduce the bandwidth.

In computing a transonic flow field for a given airfoil shape with the present programs, the following procedures are generally followed:

- A desired mesh is sketched with each node assigned a number.
- Based on the mesh sketched, an appropriate set of input cards is prepared and supplied to STRANL-I to generate the required mesh information in the form of punch cards.
- The above punch cards, with three additional cards for each case and cards for non-zero initial guess if available, are used as input to STRANL-II to compute the flow field.
- During the execution of STRANL-II, results for each iteration are printed, and for solution satisfying convergent criterion the solution is saved in the form of punch cards for possible later use.

The following paragraphs describe in more detail the input and output for the two sample problems.

5.1 Flow Over a 6% Thick Circular Arc Airfoil

This problem was analyzed using the mesh shown in Sketch 1 which consists of 154 elements together with 170 nodes. Note that a relatively regular mesh is adapted in the anticipated supersonic region while the

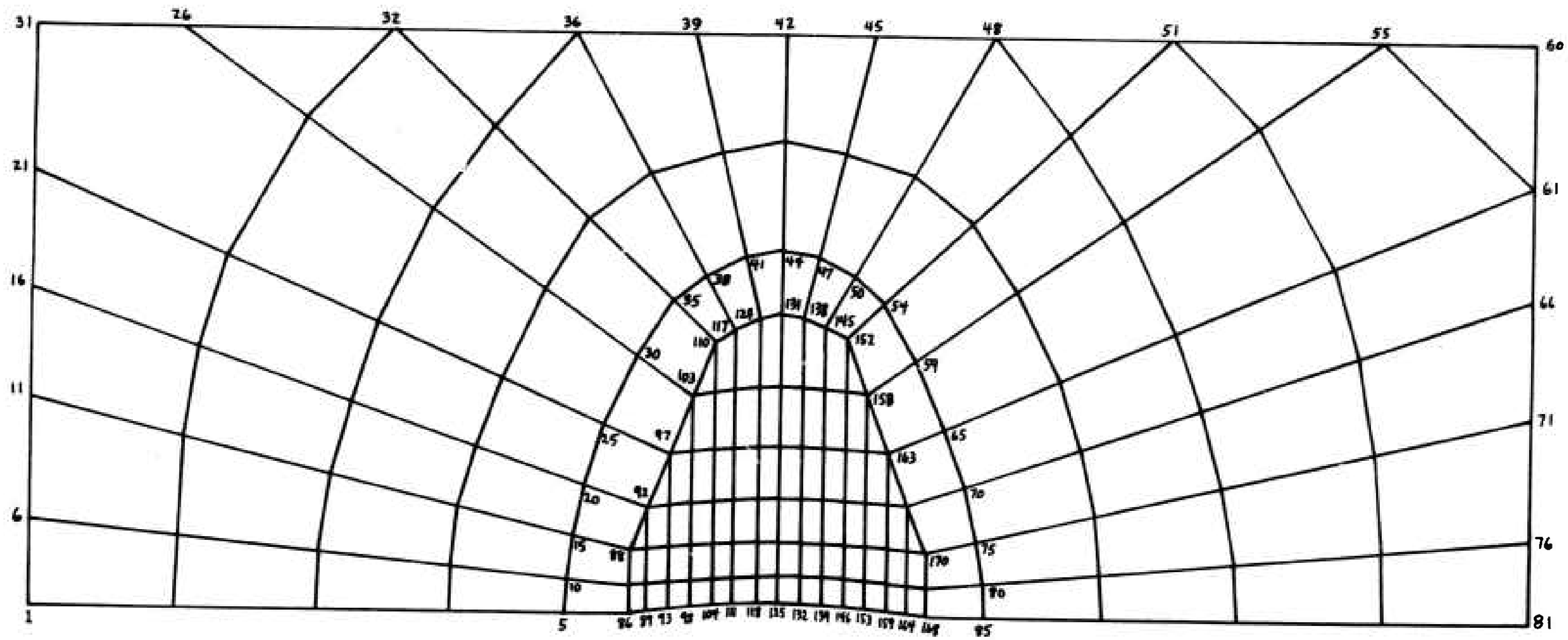


Figure 6 - Freehand Sketch of Finite Element Mesh for 6% Thick Circular Arc Airfoil

remaining elements are arranged rather arbitrarily. Also, the nodal points need only be numbered according to convenience, as the node renumbering option will be invoked to renumber them to yield a smallest bandwidth. With the present mesh, although a little more effort is required in preparing input cards to the STRANL-I program, the elements are considered to have been used more effectively than a regular mesh. Summarized below are the input and output to the STRANL-I and STRANL-II programs for the present problem.

5.1.1 STRANL-I

Input

The necessary set of input cards is listed on the next three pages. Note that both options for plotting mesh and node renumbering are to be enacted, and the final mesh will have nodes on the actual airfoil surface rather than on the chordline.

INPUT CARDS TO STRANL-I FOR CIRCULAR ARC AIRFOIL

STEADY TRANSONIC FLOW--MESH 6--154 ELEMENTS,170 NODES

1 1

3	1	1	0	0	31	21	26	
4	5	4	5	1	6	1	2	7
3	1	1	0	0	26	27	32	
4	3	1	1	0	27	28	33	32
3	1	1	0	0	32	33	36	
4	2	5	1	3	33	34	37	36
4	2	1	1	0	48	49	53	52
3	1	1	0	0	48	52	51	
3	1	1	0	0	51	56	55	
4	3	1	1	0	51	52	57	56
3	1	1	0	0	55	61	60	
4	4	1	1	0	55	56	62	61
4	4	4	1	5	61	62	67	66
4	1	1	0	0	10	5	86	87
4	1	1	0	0	15	10	87	88
4	1	1	0	0	20	15	88	92
4	1	1	0	0	25	20	92	97
4	1	1	0	0	30	25	97	103
4	1	1	0	0	35	30	103	110
4	1	1	0	0	38	35	110	117
4	1	1	0	0	41	38	117	124
4	1	1	0	0	44	41	124	131
4	1	1	0	0	44	131	138	47
4	1	1	0	0	47	138	145	50
4	1	1	0	0	50	145	152	54
4	1	1	0	0	152	158	59	54
4	1	1	0	0	158	163	65	59
4	1	1	0	0	163	167	70	65
4	1	1	0	0	167	170	75	70
4	1	1	0	0	170	169	80	75
4	1	1	0	0	169	168	85	80
4	2	1	1	0	87	86	89	90
3	1	1	0	0	88	91	92	
4	3	1	1	0	90	89	93	94
3	1	1	0	0	92	96	97	
4	4	1	1	0	94	93	98	99
3	1	1	0	0	97	102	103	

4	5	1	1	0	99	98	104	105
3	1	1	0	0	103	109	110	
4	6	6	1	7	105	104	111	112
4	5	1	1	0	147	146	153	154
3	1	1	0	0	152	151	158	
4	4	1	1	0	154	153	159	160
3	1	1	0	0	158	157	163	
4	3	1	1	0	160	159	164	165
3	1	1	0	0	163	162	167	
4	2	1	1	0	165	164	168	169
3	1	1	0	0	167	166	170	

(blank card)

170

1	5	1	0.910	0.00	0.00	1.80	0.00
6	5	1	0.910	0.00	0.30	1.81	0.10
11	5	1	0.910	0.00	0.70	1.82	0.26
16	5	1	0.870	0.00	1.10	1.86	0.43
21	5	1	0.834	0.00	1.50	1.92	0.66
31	1	0	1.00	0.00	2.00		
26	5	1	0.910	0.50	2.00	2.02	0.86
32	4	1	0.910	1.20	2.00	2.12	1.08
36	3	1	0.870	1.80	2.00	2.25	1.15
39	3	1	0.870	2.20	2.00	2.38	1.22
42	3	1	0.910	2.50	2.00	2.50	1.25
45	3	1	0.870	2.80	2.00	2.62	1.22
48	3	1	0.870	3.20	2.00	2.75	1.15
51	4	1	0.910	3.80	2.00	2.88	1.06
55	5	1	0.910	4.50	2.00	2.98	0.86
60	1	0	1.00	5.00	2.00		
61	5	1	0.834	5.00	1.50	3.08	0.66
66	5	1	0.870	5.00	1.10	3.12	0.46
71	5	1	0.910	5.00	0.70	3.15	0.26
76	5	1	0.910	5.00	0.30	3.20	0.10
81	5	1	0.910	5.00	0.00	3.20	0.00
86	3	1	1.20	2.00	0.00	2.00	0.22
89	4	1	1.20	2.05	0.00572	2.05	0.35
93	5	1	1.20	2.125	0.01315	2.125	0.55
98	6	1	1.20	2.20	0.01922	2.20	0.75
104	7	1	1.20	2.275	0.02394	2.275	0.92
111	7	1	1.20	2.35	0.02731	2.35	0.96

```
118   7   1   1.20   2.425   0.02933   2.425   1.00
125   7   1   1.20   2.50    0.03000   2.50    1.02
132   7   1   1.20   2.575   0.02933   2.575   1.00
139   7   1   1.20   2.65    0.02731   2.65    0.96
146   7   1   1.20   2.725   0.02394   2.725   0.92
153   6   1   1.20   2.80    0.01922   2.80    0.75
159   5   1   1.20   2.875   0.01315   2.875   0.55
164   4   1   1.20   2.95    0.00572   2.95    0.35
168   3   1   1.20   3.00    0.00      3.00    0.22
 (blank card)
 21    8   15
  1    6   11   16   21   26   31   32   36   39   42   45   48   51   55   60
 61   66   71   76   81
  2    3    4    5   82   83   84   85
 86   89   93   98  104  111  118  125  132  139  146  153  159  164  168
  .06022  +0.10820  +0.09004  +0.07193  +0.05389  +0.03589  +0.01794  +0.00000
  .01794  -0.03589  -0.05388  -0.07193  -0.09004  -0.10820  -0.06022
```

Output

As stated before, output of this program includes printouts, punch cards and a plot for the generated mesh.

Printouts and punch cards for the present mesh are listed on the next 12 pages.

PRINT OUTS FROM STRAIN-I FOR CIRCULAR ARC AIRFOIL

STEADY TRANSONIC FLOW--MESH 6--154 ELEMENTS,170 NODES

0-0-0-0-0-0-0-0-0-0 1-0-0-0-0-0-0-0-0-0

NO. OF ELEMENTS= 154 NO. OF NODES= 170   FULL BANDWIDTH= 34

ELE. NO. AND ELEMENT NODES

1	31	21	26	0
2	6	1	2	7
3	11	6	7	12
4	16	11	12	17
5	21	16	17	22
6	26	21	22	27
7	7	2	3	8
8	12	7	8	13
9	17	12	13	18
10	22	17	18	23
11	27	22	23	28
12	8	3	4	9
13	13	8	9	14
14	18	13	14	19
15	23	18	19	24
16	28	23	24	29
17	9	4	5	10
18	14	9	10	15
19	19	14	15	20
20	24	19	20	25
21	29	24	25	30
22	26	27	32	0
23	27	28	33	32
24	28	29	34	33
25	29	30	35	34
26	32	33	36	0

27	33	34	37	36
28	34	35	38	37
29	36	37	40	39
30	37	38	41	40
31	39	40	43	42
32	40	41	44	43
33	42	43	46	45
34	43	44	47	46
35	45	46	49	48
36	46	47	50	49
37	48	49	53	52
38	49	50	54	53
39	48	52	51	0
40	51	56	55	0
41	51	52	57	56
42	52	53	58	57
43	53	54	59	58
44	55	61	60	0
45	55	56	62	61
46	56	57	63	62
47	57	58	64	63
48	58	59	65	64
49	61	62	67	66
50	62	63	68	67
51	63	64	69	68
52	64	65	70	69
53	66	67	72	71
54	67	68	73	72
55	68	69	74	73
56	69	70	75	74
57	71	72	77	76
58	72	73	78	77
59	73	74	79	78
60	74	75	80	79
61	76	77	82	81
62	77	78	83	82
63	78	79	84	83
64	79	80	85	84
65	1	5	86	87
66	15	10	87	88
67	20	15	88	92
68	25	20	92	97
69	30	25	97	103
70	35	30	103	110
71	38	35	110	117
72	41	38	117	124
73	44	41	124	131
74	44	131	138	47
75	47	138	145	50

76	50	145	152	54
77	152	158	59	54
78	158	163	65	59
79	163	167	70	65
80	167	170	75	70
81	170	169	80	75
82	169	168	85	80
83	87	86	89	90
84	88	87	90	91
85	88	91	92	0
86	90	89	93	94
87	91	90	94	95
88	92	91	95	96
89	92	96	97	0
90	94	93	98	99
91	95	94	99	100
92	96	95	100	101
93	97	96	101	102
94	97	102	103	0
95	99	98	104	105
96	100	99	105	106
97	101	100	106	107
98	102	101	107	108
99	103	102	108	109
100	103	109	110	0
101	105	104	111	112
102	106	105	112	113
103	107	106	113	114
104	108	107	114	115
105	109	108	115	116
106	110	109	116	117
107	112	111	118	119
108	113	112	119	120
109	114	113	120	121
110	115	114	121	122
111	116	115	122	123
112	117	116	123	124
113	119	118	125	126
114	120	119	126	127
115	121	120	127	128
116	122	121	128	129
117	123	122	129	130
118	124	123	130	131
119	126	125	132	133
120	127	126	133	134
121	128	127	134	135
122	129	128	135	136
123	130	129	136	137
124	131	130	137	138
125	133	132	139	140

126	134	133	140	141
127	135	134	141	142
128	136	135	142	143
129	137	136	143	144
130	138	137	144	145
131	140	139	146	147
132	141	140	147	148
133	142	141	148	149
134	143	142	149	150
135	144	143	150	151
136	145	144	151	152
137	147	146	153	154
138	148	147	154	155
139	149	148	155	156
140	150	149	156	157
141	151	150	157	158
142	152	151	158	0
143	154	153	159	160
144	155	154	160	161
145	156	155	161	162
146	157	156	162	163
147	158	157	163	0
148	160	159	164	165
149	161	160	165	166
150	162	161	166	167
151	163	162	167	0
152	165	164	168	169
153	166	165	169	170
154	167	166	170	0

OLD NODE	NEW NODE	X(I)	Y(I)
1	167	0.	0.
2	168	.51551E+00	0.
3	169	.98453E+00	0.
4	170	.14115E+01	0.
5	166	.18000E+01	0.
6	161	0.	.30000E+00
7	162	.51838E+00	.24272E+00
8	163	.99016E+00	.19060E+00
9	164	.14194E+01	.14316E+00
10	165	.18100E+01	.10000E+00
11	153	0.	.70000E+00
12	154	.52124E+00	.57399E+00
13	155	.99557E+00	.45931E+00
14	156	.14272E+01	.35496E+00
15	157	.18200E+01	.26000E+00
16	144	0.	.11000E+01
17	145	.56614E+00	.89607E+00
18	146	.10537E+01	.71865E+00
19	147	.14872E+01	.56429E+00
20	148	.18600E+01	.43000E+00
21	134	0.	.15000E+01
22	135	.61743E+00	.12299E+01
23	136	.11324E+01	.10046E+01
24	137	.15618E+01	.81670E+00
25	138	.19200E+01	.66000E+00
26	122	.50000E+00	.20000E+01
27	123	.93532E+00	.16735E+01
28	124	.13315E+01	.13764E+01
29	125	.15920E+01	.11060E+01
30	126	.20200E+01	.86000E+00
31	133	0.	.20000E+01
32	111	.12000E+01	.20000E+01
33	112	.15350E+01	.16640E+01
34	113	.18418E+01	.13582E+01
35	114	.21200E+01	.10800E+01
36	101	.18000E+01	.20000E+01
37	102	.20406E+01	.15455E+01
38	103	.22500E+01	.11500E+01
39	91	.22000E+01	.20000E+01
40	92	.22963E+01	.15829E+01

41	93	.23800E+01	.12200E+01
42	81	.25000E+01	.20000E+01
43	82	.25000E+01	.16073E+01
44	83	.25000E+01	.12500E+01
45	71	.28000E+01	.20000E+01
46	72	.27037E+01	.15829E+01
47	73	.26200E+01	.12200E+01
48	61	.32000E+01	.20000E+01
49	62	.29534E+01	.15455E+01
50	63	.27500E+01	.11500E+01
51	50	.38000E+01	.20000E+01
52	51	.34640E+01	.16567E+01
53	52	.31582E+01	.13443E+01
54	53	.28800E+01	.10600E+01
55	38	.45000E+01	.20000E+01
56	40	.40647E+01	.16735E+01
57	41	.36685E+01	.13764E+01
58	42	.33080E+01	.11060E+01
59	43	.29800E+01	.86000E+00
60	39	.50000E+01	.20000E+01
61	28	.50000E+01	.15000E+01
62	29	.43826E+01	.12299E+01
63	30	.38676E+01	.10046E+01
64	31	.34392E+01	.81670E+00
65	32	.30800E+01	.66000E+00
66	16	.50000E+01	.11000E+01
67	17	.44278E+01	.90520E+00
68	19	.39299E+01	.73572E+00
69	21	.34958E+01	.58828E+00
70	23	.31200E+01	.46000E+00
71	18	.50000E+01	.70000E+00
72	7	.44782E+01	.57399E+00
73	8	.39880E+01	.45931E+00
74	10	.35493E+01	.35496E+00
75	12	.31500E+01	.26000E+00
76	20	.50000E+01	.30000E+00
77	9	.44845E+01	.24272E+00
78	2	.40154E+01	.19060E+00
79	3	.35885E+01	.14316E+00
80	5	.32000E+01	.10000E+00
81	22	.50000E+01	0.
82	11	.44845E+01	0.
83	4	.40154E+01	0.

84	1	.35885E+01	0.
85	6	.32000E+01	0.
86	160	.20000E+01	0.
87	159	.20000E+01	.10000E+00
88	158	.20000E+01	.22000E+00
89	152	.20500E+01	.57200E-02
90	151	.20500E+01	.10030E+00
91	150	.20500E+01	.21380E+00
92	149	.20500E+01	.35000E+00
93	143	.21250E+01	.13150E-01
94	142	.21250E+01	.11316E+00
95	141	.21250E+01	.23317E+00
96	140	.21250E+01	.37718E+00
97	139	.21250E+01	.55000E+00
98	132	.22000E+01	.19220E-01
99	131	.22000E+01	.11742E+00
100	130	.22000E+01	.23526E+00
101	129	.22000E+01	.37668E+00
102	128	.22000E+01	.54637E+00
103	127	.22000E+01	.75000E+00
104	121	.22750E+01	.23940E-01
105	120	.22750E+01	.11418E+00
106	119	.22750E+01	.22246E+00
107	118	.22750E+01	.35241E+00
108	117	.22750E+01	.50834E+00
109	116	.22750E+01	.69546E+00
110	115	.22750E+01	.92000E+00
111	110	.23500E+01	.27310E-01
112	109	.23500E+01	.12124E+00
113	108	.23500E+01	.23395E+00
114	107	.23500E+01	.36921E+00
115	106	.23500E+01	.53151E+00
116	105	.23500E+01	.72628E+00
117	104	.23500E+01	.96000E+00
118	100	.24250E+01	.29330E-01
119	99	.24250E+01	.12708E+00
120	98	.24250E+01	.24438E+00
121	97	.24250E+01	.38515E+00
122	96	.24250E+01	.55406E+00
123	95	.24250E+01	.75676E+00
124	94	.24250E+01	.10000E+01
125	90	.25000E+01	.30000E-01
126	89	.25000E+01	.12970E+00

127	88	.25000E+01	.24934E+00
128	87	.25000E+01	.39290E+00
129	86	.25000E+01	.56518E+00
130	85	.25000E+01	.77192E+00
131	84	.25000E+01	.10200E+01
132	80	.25750E+01	.29330E-01
133	79	.25750E+01	.12708E+00
134	78	.25750E+01	.24438E+00
135	77	.25750E+01	.38515E+00
136	76	.25750E+01	.55406E+00
137	75	.25750E+01	.75676E+00
138	74	.25750E+01	.10000E+01
139	70	.26500E+01	.27310E-01
140	69	.26500E+01	.12124E+00
141	68	.26500E+01	.23395E+00
142	67	.26500E+01	.36921E+00
143	66	.26500E+01	.53151E+00
144	65	.26500E+01	.72628E+00
145	64	.26500E+01	.96000E+00
146	60	.27250E+01	.23940E-01
147	59	.27250E+01	.11418E+00
148	58	.27250E+01	.22246E+00
149	57	.27250E+01	.35241E+00
150	56	.27250E+01	.50834E+00
151	55	.27250E+01	.69546E+00
152	54	.27250E+01	.92000E+00
153	49	.28000E+01	.19220E-01
154	48	.28000E+01	.11742E+00
155	47	.28000E+01	.23526E+00
156	46	.28000E+01	.37668E+00
157	45	.28000E+01	.54637E+00
158	44	.28000E+01	.75000E+00
159	37	.28750E+01	.13150E-01
160	36	.28750E+01	.11316E+00
161	35	.28750E+01	.23317E+00
162	34	.28750E+01	.37718E+00
163	33	.28750E+01	.55000E+00
164	27	.29500E+01	.57200E-02
165	26	.29500E+01	.10030E+00
166	25	.29500E+01	.21380E+00
167	24	.29500E+01	.35000E+00
168	15	.30000E+01	0.
169	14	.30000E+01	.10000E+00
170	13	.30000E+01	.22000E+00

```
NODES AT FARFIELD
   1    6   11   16   21   26   31   32   36   39   42   45   48   51   55   60   61   65   71   76
  81

NODES ON LINE OF SYMMETRY
   2    3    4    5   82   83   84   85

NODES ON THE AIRFOIL
  86   89   93   98  104  111  118  125  132  139  146  153  159  164  168

SLOPE ALONG NODES ON AIRFOIL
     .60220E-01     .10820E+00     .90040E-01     .71930E-01     .53890E-01     .35890E-01     .17940E-01    0.
    -.17940E-01    -.35890E-01    -.53880E-01    -.71930E-01    -.90040E-01    -.10820E+03    -.60220E-01
```

PUNCH CARDS FROM STRANL-I FOR CIRCULAR ARC AIRFOIL

154	170	84	21	8	15										
31	21	26	0	6	1	2	7	11	6	7	12	16	11	12	17
21	16	17	22	26	21	22	27	7	2	3	8	12	7	8	13
17	12	13	18	22	17	18	23	27	22	23	28	8	3	4	9
13	8	9	14	18	13	14	19	23	18	19	24	28	23	24	29
9	4	5	10	14	9	10	15	19	14	15	20	24	19	20	25
29	24	25	30	26	27	32	0	27	28	33	32	28	29	34	33
29	30	35	34	32	33	36	0	33	34	37	36	34	35	38	37
36	37	40	39	37	38	41	40	39	40	43	42	40	41	44	43
42	43	46	45	43	44	47	46	45	46	49	48	46	47	50	49
48	49	53	52	49	50	54	53	48	52	51	0	51	56	55	0
51	52	57	56	52	53	58	57	53	54	59	58	55	61	60	0
55	56	62	61	56	57	63	62	57	58	64	63	58	59	65	64
61	62	67	66	62	63	68	67	63	64	69	68	64	65	70	69
66	67	72	71	67	68	73	72	68	69	74	73	69	70	75	74
71	72	77	76	72	73	78	77	73	74	79	78	74	75	80	79
76	77	82	81	77	78	83	82	78	79	84	83	79	80	85	84
10	5	86	87	15	10	87	88	20	15	88	92	25	20	92	97
30	25	97	103	35	30	103	110	38	35	110	117	41	38	117	124
44	41	124	131	44	131	138	47	47	138	145	50	50	145	152	54
152	158	59	54	158	163	65	59	163	167	70	65	167	170	75	70
170	169	80	75	169	168	85	80	87	86	89	90	88	87	90	91
88	91	92	0	90	89	93	94	91	90	94	95	92	91	95	96
92	96	97	0	94	93	98	99	95	94	99	100	96	95	100	101
97	96	101	102	97	102	103	0	99	98	104	105	100	99	105	106
101	100	106	107	102	101	107	108	103	102	108	109	103	109	110	0
105	104	111	112	106	105	112	113	107	106	113	114	108	107	114	115
109	108	115	116	110	109	116	117	112	111	118	119	113	112	119	120
114	113	120	121	115	114	121	122	116	115	122	123	117	116	123	124
119	118	125	126	120	119	126	127	121	120	127	128	122	121	128	129
123	122	129	130	124	123	130	131	126	125	132	133	127	126	133	134
128	127	134	135	129	128	135	136	130	129	136	137	131	130	137	138
133	132	139	140	134	133	140	141	135	134	141	142	136	135	142	143
137	136	143	144	138	137	144	145	140	139	146	147	141	140	147	148
142	141	148	149	143	142	149	150	144	143	150	151	145	144	151	152
147	146	153	154	148	147	154	155	149	148	155	156	150	149	156	157
151	150	157	158	152	151	158	0	154	153	159	160	155	154	160	161
156	155	161	162	157	156	162	163	158	157	163	0	160	159	164	165
161	160	165	166	162	161	166	167	163	162	167	0	165	164	168	169
166	165	169	170	167	166	170	0								

.	0.	5.155E-01	0.	9.846E-01	0.	1.412E+00	0.
1.800E+00	0.	0.	3.000E-01	5.184E-01	2.427E-01	9.901E-01	1.906E-01
1.419E+00	1.432E-01	1.810E+00	1.000E-01	0.	7.000E-01	5.212E-01	5.740E-01
9.956E-01	4.593E-01	1.427E+00	3.550E-01	1.820E+00	2.600E-01	0.	1.100E+00
5.661E-01	8.961E-01	1.059E+00	7.186E-01	1.487E+00	5.643E-01	1.860E+00	4.300E-01
.	1.500E+00	6.174E-01	1.230E+00	1.132E+00	1.005E+00	1.562E+00	8.167E-01
1.920E+00	6.600E-01	5.000E-01	2.000E+00	9.353E-01	1.674E+00	1.331E+00	1.376E+00
1.692E+00	1.106E+00	2.020E+00	8.600E-01	0.	2.000E+00	1.200E+00	2.000E+00
1.536E+00	1.664E+00	1.842E+00	1.358E+00	2.120E+00	1.080E+00	1.800E+00	2.000E+00
2.041E+00	1.545E+00	2.250E+00	1.150E+00	2.200E+00	2.000E+00	2.296E+00	1.583E+00
2.380E+00	1.220E+00	2.500E+00	2.000E+00	2.500E+00	1.607E+00	2.500E+00	1.250E+00
2.800E+00	2.000E+00	2.704E+00	1.583E+00	2.620E+00	1.220E+00	3.200E+00	2.000E+00
2.959E+00	1.545E+00	2.750E+00	1.150E+00	3.800E+00	2.000E+00	3.464E+00	1.657E+00
3.158E+00	1.344E+00	2.880E+00	1.060E+00	4.500E+00	2.000E+00	4.065E+00	1.674E+00
3.669E+00	1.376E+00	3.308E+00	1.106E+00	2.980E+00	8.600E-01	5.000E+00	2.000E+00
5.000E+00	1.500E+00	4.383E+00	1.230E+00	3.868E+00	1.005E+00	3.438E+00	8.167E-01
3.080E+00	6.600E-01	5.000E+00	1.100E+00	4.428E+00	9.052E-01	3.930E+00	7.357E-01
3.497E+00	5.883E-01	3.120E+00	4.600E-01	5.000E+00	7.000E-01	4.470E+00	5.740E-01
3.988E+00	4.593E-01	3.549E+00	3.550E-01	3.150E+00	2.600E-01	5.000E+00	3.000E-01
4.484E+00	2.427E-01	4.015E+00	1.906E-01	3.588E+00	1.432E-01	3.200E+00	1.000E-01
5.000E+00	0.	4.484E+00	0.	4.015E+00	0.	3.588E+00	0.
3.200E+00	0.	2.000E+00	0.	2.000E+00	1.000E-01	2.000E+00	2.200E-01
2.050E+00	5.720E-03	2.050E+00	1.003E-01	2.050E+00	2.138E-01	2.050E+00	3.500E-01
2.125E+00	1.315E-02	2.125E+00	1.132E-01	2.125E+00	2.332E-01	2.125E+00	3.772E-01
2.125E+00	5.500E-01	2.200E+00	1.922E-02	2.200E+00	1.174E-01	2.200E+00	2.353E-01
2.200E+00	3.767E-01	2.200E+00	5.464E-01	2.200E+00	7.500E-01	2.275E+00	2.394E-02
2.275E+00	1.142E-01	2.275E+00	2.225E-01	2.275E+00	3.524E-01	2.275E+00	5.083E-01
2.275E+00	6.955E-01	2.275E+00	9.200E-01	2.350E+00	2.731E-02	2.350E+00	1.212E-01
2.350E+00	2.339E-01	2.350E+00	3.692E-01	2.350E+00	5.315E-01	2.350E+00	7.263E-01
2.350E+00	9.600E-01	2.425E+00	2.933E-02	2.425E+00	1.271E-01	2.425E+00	2.444E-01
2.425E+00	3.851E-01	2.425E+00	5.541E-01	2.425E+00	7.568E-01	2.425E+00	1.000E+00
2.500E+00	3.000E-02	2.500E+00	1.297E-01	2.500E+00	2.493E-01	2.500E+00	3.929E-01
2.500E+00	5.652E-01	2.500E+00	7.719E-01	2.500E+00	1.020E+00	2.575E+00	2.933E-02
2.575E+00	1.271E-01	2.575E+00	2.444E-01	2.575E+00	3.851E-01	2.575E+00	5.541E-01
2.575E+00	7.568E-01	2.575E+00	1.000E+00	2.650E+00	2.731E-02	2.650E+00	1.212E-01
2.650E+00	2.339E-01	2.650E+00	3.692E-01	2.650E+00	5.315E-01	2.650E+00	7.263E-01
2.650E+00	9.600E-01	2.725E+00	2.394E-02	2.725E+00	1.142E-01	2.725E+00	2.225E-01
2.725E+00	3.524E-01	2.725E+00	5.083E-01	2.725E+00	6.955E-01	2.725E+00	9.200E-01
2.800E+00	1.922E-02	2.800E+00	1.174E-01	2.800E+00	2.353E-01	2.800E+00	3.767E-01
2.800E+00	5.464E-01	2.800E+00	7.500E-01	2.875E+00	1.315E-02	2.875E+00	1.132E-01
2.875E+00	2.332E-01	2.875E+00	3.772E-01	2.875E+00	5.500E-01	2.950E+00	5.720E-03
2.950E+00	1.003E-01	2.950E+00	2.138E-01	2.950E+00	3.500E-01	3.000E+00	0.
3.000E+00	1.000E-01	3.000E+00	2.200E-01				

```
   1    6   11   16   21   26   31   32   36   39   42   45   48   51   55   60
  61   66   71   76   81
   2    3    4    5   82   83   84   85
  86   89   93   98  104  111  118  125  132  139  146  153  159  164  168
6.022E-02 1.082E-01 9.004E-02 7.193E-02 5.389E-02 3.589E-02 1.794E-02 0.
1.794E-02-3.589E-02-5.388E-02-7.193E-02-9.004E-02-1.082E-01-6.022E-02
 167  168  169  170  166  161  162  163  164  165  153  154  155  156  157  144
 145  146  147  148  134  135  136  137  138  122  123  124  125  126  133  111
 112  113  114  101  102  103   91   92   93   81   82   83   71   72   73   61
  62   63   50   51   52   53   38   40   41   42   43   39   28   29   30   31
  32   16   17   19   21   23   18    7    8   10   12   20    9    2    3    5
  22   11    4    1    6  160  159  158  152  151  150  149  143  142  141  140
 139  132  131  130  129  128  127  121  120  119  118  117  116  115  110  109
 108  107  106  105  104  100   99   98   97   96   95   94   90   89   88   87
  86   85   84   80   79   78   77   76   75   74   70   69   68   67   66   65
  64   60   59   58   57   56   55   54   49   48   47   46   45   44   37   36
  35   34   33   27   26   25   24   15   14   13
```

5.1.2 STRANL-II Program

With the present mesh, three cases of freestream Mach numbers are to be computed. These are

$$M_\infty = 0.806 \quad \text{(Subcritical)}$$
$$M_\infty = 0.861 \quad \text{(Barely Critical)}$$
$$M_\infty = 0.909 \quad \text{(Supercritical)}$$

Input

Listed below are typical input cards to the STRANL-II program to compute flow field for the above cases.

```
STEADY STATE SOLUTION -- CIRCULAR ARC -- 170 NODES -- M=0.806
 0 0 0
    6  0.005        0.806        1.000
   (Insert cards generated in STRANL-I)
STEADY STATE SOLUTION -- CIRCULAR ARC -- 170 NODES -- M=0.861
 1
    8   0.01        0.861        1.000
STEADY STATE SOLUTION -- CIRCULAR ARC -- 170 NODES -- M=0.909
 1
   12   0.01         0.909        0.500
```

The above cards indicate there is no non-zero initial guess furnished in order to compute the case $M_\infty = 0.806$, while cases with higher Mach number will use results computed for lower Mach number as initial guess. In all cases, the nonlinear boundary condition on the actual airfoil surface is to be used, as suggested by IOPT (3) = 0 in the options card.

Output

The output from this program includes printouts and punch cards for the convergent solution(s).

The printouts are in the following order:

- Title of the problem under study
- Convergent limit
- Control keys specified in the options card
- Total number of elements, number of nodal points, and the full bandwidth of the resulting matrix equations
- Element numbers and element node points
- Nodal numbers and their corresponding x- and y-coordinates
- Boundary nodes for each type of boundary
- Slope for nodes on the airfoil surface
- For each iteration, the computed results at all nodal points are printed, together with a printer plot displaying the pressure coefficient on the airfoil surface.

The above items except problem parameters and computed results have also been printed in the STRANL-I program, but are repeated here for completeness. For brevity, only results for a typical iteration are listed on the next five pages for reference.

5.2 Flow Over a NACA 64 A006 Airfoil

This problem was analyzed using the mesh shown in Sketch 2, which consists of 120 quadrilaterals with 150 nodes. For this particular mesh, it is apparent that nodal numbering along the shorter (vertical) direction yields the smallest bandwidth and, therefore, no nodal renumbering is required. Since the present mesh has a rather regular pattern, less input cards are required. The procedures described for the circular arc airfoil problem also applied here. For brevity, only the input cards to both programs and some typical printouts from STRANL-II are listed here for reference. Again, the nonlinear boundary condition on the actual airfoil surface is to be used in computations for all the cases.

MACH NUMBER= .909   RELAX. FACTOR = .5000

NO. OF ITERATION= 10

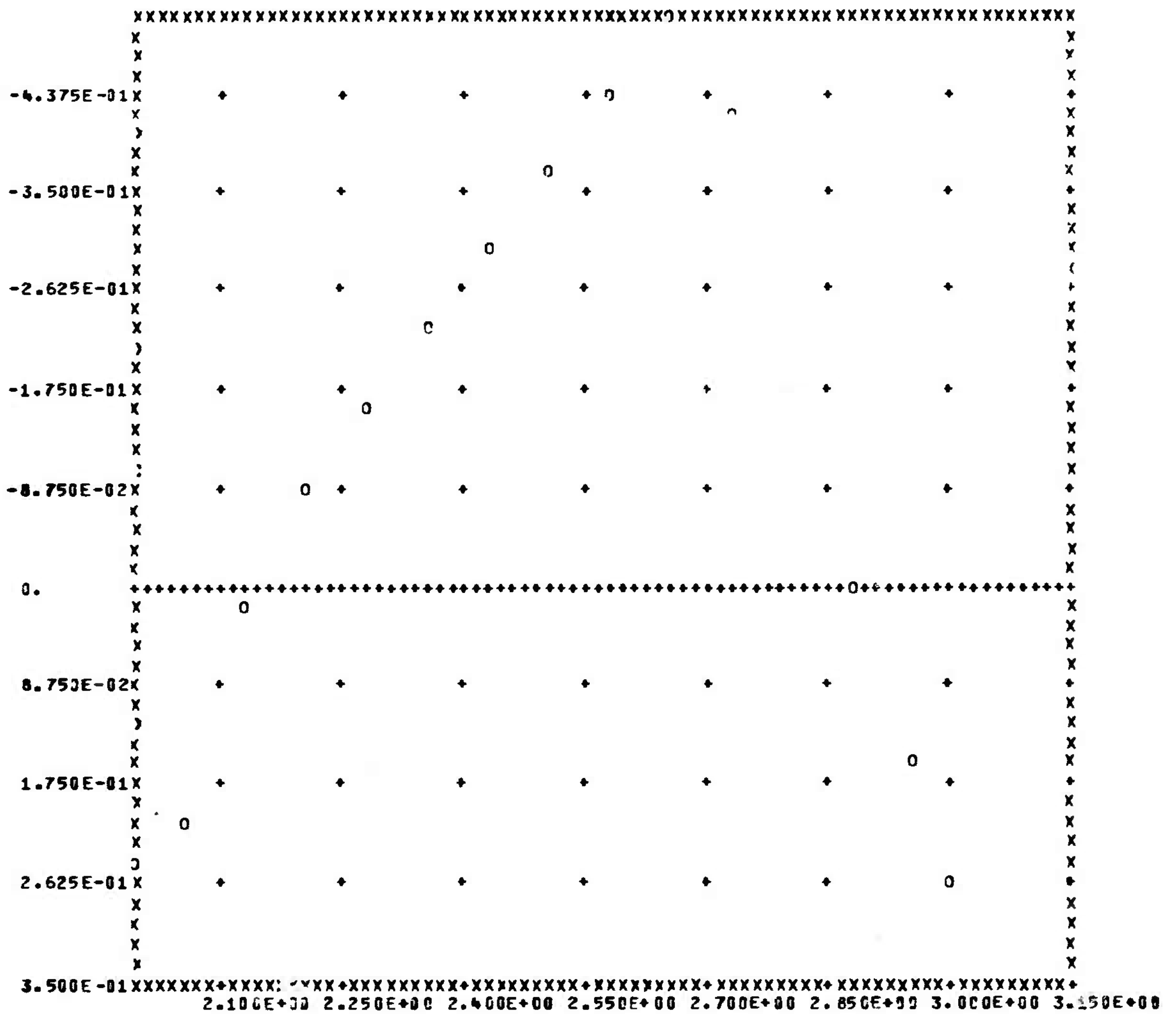
NODE	PHIT	UCOM	VCOM	COF	LMAC	P/PO	[illegible]	DELM
1	0.	1.0000E+00	0.	8.2628E-01	9.0900E-01	5.8550E-01	-0.	0.
2	3.8695E-04	1.0016E+00	0.	8.2939E-01	9.1066E-01	5.8444E-01	-3.1311E-03	3.2793E-06
3	-4.9242E-04	9.9516E-01	0.	8.1669E-01	9.0388E-01	5.8877E-01	9.6736E-03	2.5074E-06
4	-4.7991E-03	9.8493E-01	0.	7.9639E-01	8.9309E-01	5.9569E-01	3.0110E-02	-1.0479E-05
5	-2.2363E-02	9.2713E-01	0.	6.8178E-01	8.3315E-01	6.3445E-01	1.4481E-01	-3.5696E-05
6	0.	1.0000E+00	0.	8.2628E-01	9.0900E-01	5.8550E-01	-0.	0.
7	3.4937E-04	1.0010E+00	-2.6900E-04	8.2825E-01	9.1005E-01	5.8482E-01	-1.9908E-03	2.9373E-06
8	-5.0832E-04	9.9545E-01	5.9023E-05	8.1726E-01	9.0419E-01	5.8858E-01	9.0918E-03	1.2897E-06
9	-4.9457E-03	9.8247E-01	-3.1468E-04	7.9152E-01	8.9051E-01	5.9735E-01	3.5092E-02	-1.1724E-05
10	-2.2565E-02	9.3087E-01	9.7688E-03	6.8918E-01	8.3703E-01	6.3193E-01	1.3734E-01	-3.8060E-05
11	0.	1.0000E+00	0.	8.2628E-01	9.0900E-01	5.8550E-01	-0.	0.
12	2.6680E-04	1.0009E+00	-2.5953E-04	8.2812E-01	9.0998E-01	5.8487E-01	-1.8552E-03	2.7157E-06
13	-4.1593E-04	9.9695E-01	7.4553E-04	8.2024E-01	9.0578E-01	5.8756E-01	6.0885E-03	1.8695E-06
14	-4.9100E-03	9.8345E-01	1.7844E-03	7.9345E-01	8.9154E-01	5.9669E-01	3.3057E-02	-9.5256E-06
15	-2.0862E-02	9.4555E-01	1.8065E-02	7.1830E-01	8.5226E-01	6.2205E-01	1.0806E-01	-3.8684E-05
16	0.	1.0000E+00	0.	8.2628E-01	9.0900E-01	5.8550E-01	-0.	0.
17	2.0168E-04	1.0004E+00	-3.2747E-04	8.2711E-01	9.0944E-01	5.8521E-01	-8.3709E-04	1.6406E-06
18	-4.6034E-04	9.9674E-01	4.9715E-04	8.1982E-01	9.0555E-01	5.8770E-01	6.5165E-03	4.0436E-07
19	-5.4069E-03	9.8050E-01	4.1753E-03	7.8762E-01	8.8846E-01	5.9867E-01	3.8911E-02	-1.0347E-05
20	-1.9186E-02	9.6044E-01	2.2839E-02	7.4784E-01	8.6778E-01	6.1200E-01	7.8322E-02	-4.0782E-05
21	0.	1.0000E+00	0.	8.2628E-01	9.0900E-01	5.8550E-01	-0.	0.
22	1.3464E-04	1.0004E+00	-2.1069E-04	8.2712E-01	9.0945E-01	5.8521E-01	-8.4712E-04	1.4736E-06
23	-4.3765E-04	9.9818E-01	6.7750E-04	8.2267E-01	9.0707E-01	5.8673E-01	3.6456E-03	2.3270E-07
24	-5.3162E-03	9.8298E-01	7.1685E-03	7.9253E-01	8.9107E-01	5.9699E-01	3.3939E-02	-1.0485E-05
25	-1.5548E-02	9.7876E-01	2.4170E-02	7.8416E-01	8.8693E-01	5.9965E-01	4.1821E-02	-4.0712E-05
26	0.	1.0000E+00	0.	8.2628E-01	9.0900E-01	5.8550E-01	-0.	0.
27	3.6239E-05	9.9996E-01	-4.0177E-04	8.2621E-01	9.0896E-01	5.8552E-01	7.0386E-05	4.4929E-07
28	-5.3314E-04	9.9702E-01	1.6148E-03	8.2037E-01	9.0585E-01	5.8751E-01	5.9556E-03	-7.4935E-07
29	-4.5399E-03	9.8702E-01	1.0133E-02	8.0053E-01	8.9535E-01	5.9425E-01	2.5837E-02	-1.0267E-05
30	-1.2301E-02	9.8739E-01	2.3377E-02	8.0127E-01	8.9598E-01	5.9384E-01	2.4648E-02	-4.4201E-05
31	0.	1.0000E+00	0.	8.2628E-01	9.0900E-01	5.8550E-01	-0.	0.
32	0.	1.0000E+00	0.	8.2628E-01	9.0900E-01	5.8550E-01	-0.	0.
33	-4.1138E-04	9.9818E-01	2.1401E-03	8.2267E-01	9.0707E-01	5.8673E-01	3.6405E-03	-7.8275E-07
34	-3.3095E-03	9.9409E-01	1.0188E-02	8.1456E-01	9.0280E-01	5.8946E-01	1.1715E-02	-1.0243E-05
35	-8.1910E-03	9.9906E-01	1.9706E-02	8.2442E-01	9.0821E-01	5.8600E-01	1.4905E-03	-3.3120E-05
36	0.	1.0000E+00	0.	8.2628E-01	9.0900E-01	5.8550E-01	-0.	0.
37	-1.9747E-03	9.9957E-01	9.4790E-03	8.2543E-01	9.0859E-01	5.8576E-01	7.6502E-04	-5.1349E-06
38	-6.7616E-03	1.0021E+00	1.7448E-02	8.3051E-01	9.1142E-01	5.8395E-01	-4.5735E-03	-3.9993E-05
39	0.	1.0000E+00	0.	8.2628E-01	9.0900E-01	5.8550E-01	-0.	0.
40	-1.6041E-03	9.9974E-01	8.0433E-03	8.2577E-01	9.0876E-01	5.8565E-01	4.4695E-04	-2.1641E-05
41	-5.0520E-03	1.0071E+00	1.3243E-02	8.4045E-01	9.1667E-01	5.8060E-01	-1.4474E-02	-6.5961E-05
42	0.	1.0000E+00	0.	8.2628E-01	9.0900E-01	5.8550E-01	-0.	0.
43	-1.0950E-03	1.0039E+00	5.2960E-03	8.3382E-01	9.1305E-01	5.8291E-01	-7.6382E-03	-3.1134E-05
44	-3.4062E-03	1.0150E+00	7.9555E-03	8.5602E-01	9.2498E-01	5.7531E-01	-3.0098E-02	-5.7364E-05
45	0.	1.0000E+00	0.	8.2628E-01	9.0900E-01	5.8550E-01	-0.	0.
46	-3.2705E-04	1.0058E+00	5.7650E-04	8.3771E-01	9.1511E-01	5.8159E-01	-1.1533E-02	-8.3370E-06
47	-1.5873E-03	1.0175E+00	1.6916E-03	8.6107E-01	9.2766E-01	5.7360E-01	-3.5140E-02	-6.5835E-05

48	0.	1.0000E+00	0.	8.2628E-01	9.0900E-01	5.8550E-01	-0.	0.
49	1.0310E-03	1.0026E+00	-5.0687E-03	8.3141E-01	9.1175E-01	5.8374E-01	-5.1995E-03	4.9079E-05
50	8.2268E-04	1.0169E+00	-5.5606E-03	8.5973E-01	9.2696E-01	5.7405E-01	-3.3819E-02	-3.6762E-05
51	0.	1.0000E+00	0.	8.2628E-01	9.0900E-01	5.8550E-01	-0.	0.
52	3.6270E-04	9.9822E-01	-2.6220E-03	8.2276E-01	9.0712E-01	5.8670E-01	3.5447E-03	6.2856E-06
53	2.6511E-03	9.9621E-01	-8.4278E-03	8.1875E-01	9.0503E-01	5.8804E-01	7.5072E-03	5.5844E-05
54	3.9623E-03	1.0177E+00	-1.2062E-02	8.6141E-01	9.2792E-01	5.7343E-01	-3.5626E-02	6.6903E-05
55	0.	1.0000E+00	0.	8.2628E-01	9.0900E-01	5.8550E-01	-0.	0.
56	2.4331E-05	9.9976E-01	-1.5707E-04	8.2580E-01	9.0874E-01	5.8566E-01	4.8985E-04	-5.0026E-07
57	5.7991E-04	9.9639E-01	-1.7267E-03	8.1911E-01	9.0518E-01	5.8794E-01	7.2229E-03	5.7358E-06
58	4.0033E-03	9.9378E-01	-8.3914E-03	8.1395E-01	9.0246E-01	5.8968E-01	1.2359E-02	4.5936E-05
59	8.4627E-03	1.0083E+00	-1.8917E-02	8.4283E-01	9.1805E-01	5.7972E-01	-1.7063E-02	1.7086E-04
60	0.	1.0000E+00	0.	8.2628E-01	9.0900E-01	5.8550E-01	-0.	0.
61	0.	1.0000E+00	0.	8.2628E-01	9.0900E-01	5.8550E-01	-0.	0.
62	4.5659E-05	9.9981E-01	3.7383E-05	8.2590E-01	9.0880E-01	5.8563E-01	3.8529E-04	-2.7257E-06
63	5.3027E-04	9.9918E-01	-9.0767E-04	8.2466E-01	9.0813E-01	5.8605E-01	1.6351E-03	-1.3014E-07
64	4.9490E-03	9.8631E-01	-6.0436E-03	7.9913E-01	8.9457E-01	5.9475E-01	2.7311E-02	4.3221E-05
65	1.2316E-02	9.8584E-01	-2.1100E-02	7.9820E-01	8.9429E-01	5.9492E-01	2.7840E-02	1.5860E-04
66	0.	1.0000E+00	0.	8.2628E-01	9.0900E-01	5.8550E-01	-0.	0.
67	8.1673E-07	9.9981E-01	1.9883E-04	8.2591E-01	9.0880E-01	5.8562E-01	3.7510E-04	-4.2541E-06
68	6.5559E-04	9.9796E-01	-5.9666E-04	8.2223E-01	9.0684E-01	5.8688E-01	4.0923E-03	-1.4084E-06
69	5.0795E-03	9.8068E-01	-3.0546E-03	7.8797E-01	8.8864E-01	5.9855E-01	3.8561E-02	3.2954E-05
70	1.5946E-02	9.7350E-01	-2.3008E-02	7.7373E-01	8.8140E-01	6.0322E-01	5.2351E-02	1.2677E-04
71	0.	1.0000E+00	0.	8.2628E-01	9.0900E-01	5.8550E-01	-0.	0.
72	-1.0952E-04	1.0003E+00	4.0547E-04	8.2681E-01	9.0928E-01	5.8532E-01	-5.3137E-04	-6.5350E-06
73	7.4483E-04	9.9698E-01	-9.5870E-04	8.2028E-01	9.0580E-01	5.8755E-01	5.0458E-03	-3.7966E-06
74	4.6593E-03	9.8525E-01	-1.3871E-03	7.9702E-01	8.9343E-01	5.9548E-01	2.9467E-02	2.9262E-05
75	1.9426E-02	9.5794E-01	-2.1473E-02	7.4288E-01	8.6516E-01	6.1370E-01	8.3342E-02	1.2839E-04
76	0.	1.0000E+00	0.	8.2628E-01	9.0900E-01	5.8550E-01	-0.	0.
77	-2.4236E-04	1.0006E+00	3.9981E-04	8.2739E-01	9.0959E-01	5.8512E-01	-1.1196E-03	-6.3261E-06
78	8.6454E-04	9.9518E-01	-6.0301E-04	8.1672E-01	9.0399E-01	5.8876E-01	9.6385E-03	-5.5397E-06
79	4.1335E-03	9.8786E-01	1.1098E-03	8.0220E-01	8.9618E-01	5.9371E-01	2.4256E-02	2.8330E-05
80	1.9262E-02	9.3684E-01	-7.7255E-03	7.0102E-01	8.4314E-01	6.2796E-01	1.2558E-01	9.7568E-05
81	0.	1.0000E+00	0.	8.2628E-01	9.0900E-01	5.8550E-01	-0.	0.
82	-2.9877E-04	1.0012E+00	0.	8.2864E-01	9.1026E-01	5.8469E-01	-2.3786E-03	-6.2440E-06
83	9.2877E-04	9.9419E-01	0.	8.1476E-01	9.0286E-01	5.8943E-01	1.1614E-02	-1.0079E-05
84	4.0472E-03	9.9075E-01	0.	8.0793E-01	8.9922E-01	5.9176E-01	1.8489E-02	2.8015E-05
85	1.9666E-02	9.3220E-01	0.	6.9182E-01	8.3834E-01	6.3108E-01	1.3481E-01	8.7118E-05
86	-4.3498E-02	8.7258E-01	5.2547E-02	5.7360E-01	7.7957E-01	6.6933E-01	2.4926E-01	-6.4602E-05
87	-3.8062E-02	9.1175E-01	5.1871E-02	6.5127E-01	8.1896E-01	6.4368E-01	1.7246E-01	-6.8185E-05
88	-3.2317E-02	9.4428E-01	4.3064E-02	7.1579E-01	8.5178E-01	6.2236E-01	1.0904E-01	-6.7060E-05
89	-4.9289E-02	8.8929E-01	9.6221E-02	6.0673E-01	7.9997E-01	6.5604E-01	2.1004E-01	-7.5702E-05
90	-4.1824E-02	9.3558E-01	6.4258E-02	6.9853E-01	8.4408E-01	6.2735E-01	1.2399E-01	-7.8001E-05
91	-3.5193E-02	9.5280E-01	5.2329E-02	7.3267E-01	8.6106E-01	6.1635E-01	9.1281E-02	-7.5712E-05
92	-2.8842E-02	9.6836E-01	4.2244E-02	7.6355E-01	8.7670E-01	6.0624E-01	6.1327E-02	-7.1520E-05
93	-5.2524E-02	9.8730E-01	8.8897E-02	8.0110E-01	8.9980E-01	5.9139E-01	1.7461E-02	-8.3345E-05
94	-4.4365E-02	9.7152E-01	7.3789E-02	7.6981E-01	8.8197E-01	6.0285E-01	5.1367E-02	-9.2748E-05
95	-3.6635E-02	9.7878E-01	5.6503E-02	7.8421E-01	8.8835E-01	5.9874E-01	3.9162E-02	-8.9754E-05
96	-2.9352E-02	9.8524E-01	4.5467E-02	7.9700E-01	8.9452E-01	5.9478E-01	2.7422E-02	-8.4593E-05
97	-2.2360E-02	9.9195E-01	3.6061E-02	8.1032E-01	9.0118E-01	5.9050E-01	1.4789E-02	-7.1666E-05
98	-5.1190E-02	1.0392E+00	7.4751E-02	9.0405E-01	9.5385E-01	5.5703E-01	-8.4286E-02	-1.0239E-04
99	-4.4272E-02	1.0204E+00	6.6148E-02	8.6669E-01	9.3299E-01	5.7022E-01	-4.5199E-02	-1.0605E-04
100	-3.6988E-02	1.0079E+00	5.7071E-02	8.4187E-01	9.1906E-01	5.7908E-01	-1.8986E-02	-1.0541E-04
101	-2.9713E-02	1.0050E+00	4.6142E-02	8.3613E-01	9.1539E-01	5.8142E-01	-1.2062E-02	-9.9956E-05

102	-2.2652E-02	1.0035E+00	3.7322E-02	8.3323E-01	9.1345E-01	5.8265E-01	-8.4058E-03	-8.9571E-05
103	-1.5989E-02	1.0027E+00	2.8856E-02	8.3171E-01	9.1234E-01	5.8336E-01	-6.3141E-03	-6.9611E-05
104	-4.6564E-02	1.0800E+00	5.8199E-02	9.8483E-01	9.9718E-01	5.3002E-01	-1.6440E-01	-1.2154E-04
105	-4.1423E-02	1.0610E+00	5.5536E-02	9.4719E-01	9.7618E-01	5.4305E-01	-1.2567E-01	-1.2043E-04
106	-3.5674E-02	1.0441E+00	5.0497E-02	9.1379E-01	9.5759E-01	5.5468E-01	-9.1147E-02	-1.1545E-04
107	-2.9494E-02	1.0291E+00	4.4442E-02	8.8394E-01	9.4106E-01	5.6511E-01	-6.8277E-02	-1.1930E-04
108	-2.3153E-02	1.0204E+00	3.6834E-02	8.6666E-01	9.3139E-01	5.7123E-01	-4.2155E-02	-1.1099E-04
109	-1.6935E-02	1.0141E+00	2.9659E-02	8.5426E-01	9.2446E-01	5.7564E-01	-2.9132E-02	-9.4989E-05
110	-1.1169E-02	1.0084E+00	2.2422E-02	8.4295E-01	9.1819E-01	5.7963E-01	-1.7336E-02	-7.2532E-05
111	-3.9191E-02	1.1132E+00	3.9953E-02	1.0508E+00	1.0333E+00	5.0793E-01	-2.3022E-01	-1.7175E-04
112	-3.5392E-02	1.0922E+00	4.0630E-02	1.0091E+00	1.0099E+00	5.2221E-01	-1.8755E-01	-1.6846E-04
113	-3.0847E-02	1.0728E+00	3.9761E-02	9.7074E-01	9.8843E-01	5.3543E-01	-1.4820E-01	-1.5876E-04
114	-2.5675E-02	1.0546E+00	3.6473E-02	9.3455E-01	9.6833E-01	5.4795E-01	-1.1104E-01	-1.3849E-04
115	-2.0150E-02	1.0380E+00	3.1483E-02	9.0164E-01	9.5015E-01	5.5936E-01	-7.7242E-02	-1.2564E-04
116	-1.4622E-02	1.0253E+00	2.5234E-02	8.7638E-01	9.3626E-01	5.6814E-01	-5.1276E-02	-1.0260E-04
117	-9.4835E-03	1.0148E+00	1.8938E-02	8.5559E-01	9.2490E-01	5.7536E-01	-2.9954E-02	-7.7762E-05
118	-2.9436E-02	1.1465E+00	2.0568E-02	1.1168E+00	1.0706E+00	4.8562E-01	-2.9716E-01	-2.2033E-04
119	-2.7193E-02	1.1223E+00	2.4908E-02	1.0688E+00	1.0431E+00	5.0201E-01	-2.4786E-01	-2.1015E-04
120	-2.4125E-02	1.0985E+00	2.6942E-02	1.0215E+00	1.0164E+00	5.1824E-01	-1.9931E-01	-1.9664E-04
121	-2.0330E-02	1.0763E+00	2.6644E-02	9.7768E-01	9.9184E-01	5.3332E-01	-1.5442E-01	-1.6546E-04
122	-1.6032E-02	1.0553E+00	2.3978E-02	9.3600E-01	9.6873E-01	5.4770E-01	-1.1176E-01	-1.2560E-04
123	-1.1608E-02	1.0367E+00	1.9594E-02	8.9915E-01	9.4848E-01	5.6042E-01	-7.4106E-02	-9.7588E-05
124	-7.4795E-03	1.0212E+00	1.4473E-02	8.6839E-01	9.3172E-01	5.7102E-01	-4.2756E-02	-7.5891E-05
125	-1.7194E-02	1.1803E+00	0.	1.1839E+00	1.1096E+00	4.6280E-01	-3.6635E-01	-1.9991E-04
126	-1.6827E-02	1.1538E+00	6.9217E-03	1.1313E+00	1.0788E+00	4.8075E-01	-3.1183E-01	-1.8151E-04
127	-1.5651E-02	1.1263E+00	1.2217E-02	1.0767E+00	1.0473E+00	4.9947E-01	-2.5545E-01	-1.4796E-04
128	-1.3663E-02	1.0988E+00	1.4947E-02	1.0222E+00	1.0165E+00	5.1817E-01	-1.9948E-01	-1.1853E-04
129	-1.1032E-02	1.0737E+00	1.5079E-02	9.7244E-01	9.8868E-01	5.3528E-01	-1.4858E-01	-7.8693E-05
130	-8.1423E-03	1.0492E+00	1.2644E-02	9.2390E-01	9.6188E-01	5.5199E-01	-9.9037E-02	-7.0594E-05
131	-5.4242E-03	1.0271E+00	9.5853E-03	8.7994E-01	9.3790E-01	5.6710E-01	-5.4332E-02	-7.4979E-05
132	-2.4448E-03	1.2133E+00	-2.1767E-02	1.2494E+00	1.1489E+00	4.4045E-01	-4.3507E-01	-1.1959E-04
133	-4.0955E-03	1.1860E+00	-1.2491E-02	1.1952E+00	1.1163E+00	4.5893E-01	-3.7820E-01	-1.0811E-04
134	-5.0475E-03	1.1581E+00	-4.3002E-03	1.1398E+00	1.0837E+00	4.7787E-01	-3.2054E-01	-5.4269E-05
135	-5.1725E-03	1.1291E+00	1.6698E-03	1.0823E+00	1.0505E+00	4.9757E-01	-2.6114E-01	2.0692E-05
136	-4.6113E-03	1.1035E+00	3.9567E-03	1.0315E+00	1.0216E+00	5.1503E-01	-2.0886E-01	8.3615E-05
137	-3.9674E-03	1.0641E+00	2.8656E-03	9.5337E-01	9.7801E-01	5.4191E-01	-1.2889E-01	-2.5333E-05
138	-3.2514E-03	1.0341E+00	3.5502E-03	8.9388E-01	9.4543E-01	5.6235E-01	-6.8393E-02	-8.7853E-05
139	1.4833E-02	1.2458E+00	-4.4713E-02	1.3138E+00	1.1889E+00	4.1835E-01	-5.0419E-01	-1.3438E-05
140	1.1159E-02	1.2171E+00	-3.4075E-02	1.2568E+00	1.1537E+00	4.3775E-01	-4.4353E-01	-1.0540E-05
141	7.9215E-03	1.1884E+00	-2.3234E-02	1.1999E+00	1.1193E+00	4.5721E-01	-3.8350E-01	3.5196E-05
142	5.4588E-03	1.1485E+00	[illegible].3119E-02	1.1207E+00	1.0727E+00	4.8437E-01	-3.0090E-01	-5.1012E-05
143	3.0309E-03	1.0942E+00	-1.[illegible]828E-02	1.0130E+00	1.0113E+00	5.2133E-01	-1.9007E-01	-3.4453E-04
144	7.2562E-04	1.0612E+00	-8.[illegible]602E-03	9.4773E-01	9.7494E-01	5.4382E-01	-1.2322E-01	-2.0742E-04
145	-6.8367E-04	1.0358E+00	-3.4706E-03	8.9728E-01	9.4727E-01	5.6118E-01	-7.1838E-02	-1.1666E-04
146	3.2045E-02	1.2008E+00	-6.4700E-02	1.2245E+00	1.1358E+00	4.4782E-01	-4.1283E-01	1.2783E-04
147	2.6497E-02	1.1805E+00	-5.8387E-02	1.1842E+00	1.1115E+00	4.6172E-01	-3.7007E-01	5.5112E-05
148	2.0741E-02	1.1448E+00	-5.0041E-02	1.1134E+00	1.0696E+00	4.8617E-01	-2.9571E-01	-1.7911E-04
149	1.4916E-02	1.1051E+00	-4.1342E-02	1.0347E+00	1.0243E+00	5.1341E-01	-2.1381E-01	-3.[illegible]27E-04
150	9.5489E-03	1.0733E+00	-2.8778E-02	9.7163E-01	9.8854E-01	5.3537E-01	-1.4836E-01	-2.7875E-04
151	5.2425E-03	1.0508E+00	-1.8267E-02	9.2697E-01	9.6365E-01	5.5088E-01	-1.0233E-01	-1.2616E-04
152	1.9950E-03	1.0276E+00	-1.0672E-02	8.8098E-01	9.3848E-01	5.6674E-01	-5.5416E-02	-4.0700E-05
153	4.3111E-02	1.0871E+00	-7.8195E-02	9.9899E-01	1.0065E+00	5.2430E-01	-1.816[illegible]E-01	1.0152E-03
154	3.5599E-02	1.0649E+00	-7.3574E-02	9.5501E-01	9.8169E-01	5.3961E-01	-1.3598E-01	2.4661E-04
155	2.7545E-02	1.0573E+00	-6.3334E-02	9.3988E-01	9.7264E-01	5.4526E-01	-1.1915E-01	2.4818E-04

156	1.9714E-02	1.0523E+00	-4.8574E-02	9.3005E-01	9.6639E-01	5.4917E-01	-1.0749E-01	2.1684E-04
157	1.2761E-02	1.0411E+00	-3.4494E-02	9.8780E-01	9.5361E-01	5.5718E-01	-8.3698E-02	1.4265E-04
158	7.0212E-03	1.0300E+00	-2.2982E-02	8.8583E-01	9.4132E-01	5.6494E-01	-6.0744E-02	5.4792E-05
159	4.6137E-02	9.9230E-01	-8.9347E-02	8.1102E-01	9.0510E-01	5.8799E-01	7.4024E-03	-3.6964E-04
160	3.7880E-02	9.9731E-01	-7.5309E-02	8.2095E-01	9.0916E-01	5.8540E-01	-2.9384E-04	4.2474E-05
161	2.9688E-02	9.9961E-01	-6.0994E-02	8.2551E-01	9.1056E-01	5.8459E-01	-2.9389E-03	1.9739E-04
162	2.1879E-02	1.0095E+00	-4.7664E-02	8.4519E-01	9.2032E-01	5.7827E-01	-2.1355E-02	2.3865E-04
163	1.4715E-02	1.0172E+00	-3.6052E-02	8.6047E-01	9.2802E-01	5.7337E-01	-3.5835E-02	2.9570E-04
164	4.3242E-02	9.1429E-01	-9.8926E-02	6.5632E-01	8.2549E-01	6.3943E-01	1.6035E-01	2.9171E-04
165	3.5875E-02	9.3544E-01	-6.5133E-02	6.9826E-01	8.4400E-01	6.2740E-01	1.2414E-01	1.5628E-04
166	2.9225E-02	9.6292E-01	-5.2963E-02	7.5275E-01	8.7159E-01	6.0954E-01	7.1116E-02	1.7088E-04
167	2.2735E-02	9.8321E-01	-4.3285E-02	7.9298E-01	8.9229E-01	5.9621E-01	3.1666E-02	2.1222E-04
168	3.7777E-02	8.6833E-01	-5.2291E-02	5.6516E-01	7.7531E-01	6.7211E-01	2.5760E-01	1.8345E-04
169	3.2463E-02	9.2276E-01	-5.1771E-02	6.7310E-01	8.3016E-01	6.3639E-01	1.5077E-01	1.8692E-04
170	2.6823E-02	9.5518E-01	-4.3141E-02	7.3739E-01	8.6305E-01	6.1506E-01	8.7436E-02	1.6841E-04

C_p for 6% Thick Circular Arc (M_∞ = 0.909)



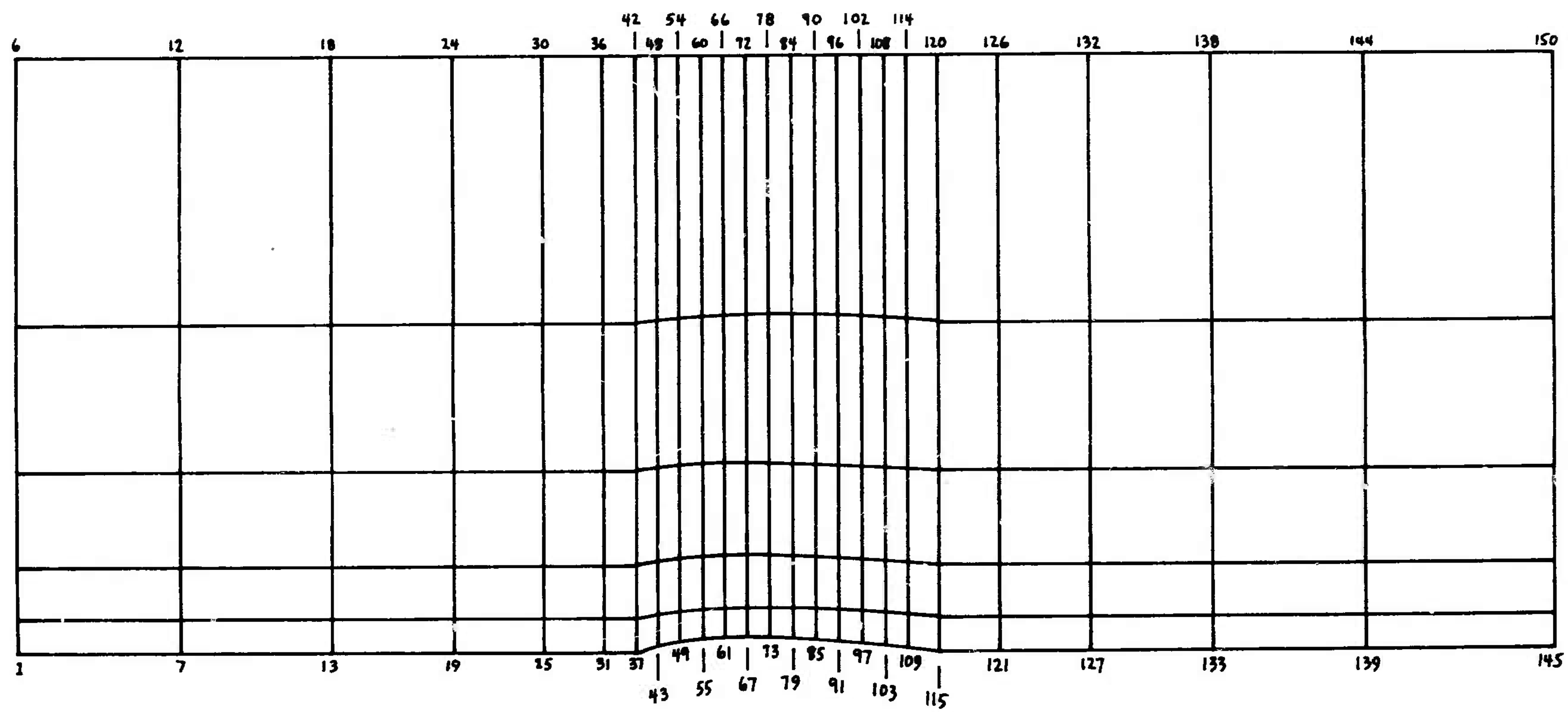


Figure 7 - Freehand Sketch of Finite Element Mesh for NACA 64 A006 Airfoil

The cases to be computed are:

$M_\infty = 0.825$	(Subcritical)
$M_\infty = 0.875$	(Barely Critical)
$M_\infty = 0.900$	(Supercritical)
$M_\infty = 0.940$	(Definite Supercritical)

INPUT CARDS TO STRANL-I FOR NACA 64 A006

```
STEADY TRANSONIC FLOW--NACA64A006--MESH 5   150 NODES
                               1
   4     5    24     1     6     2     1     7     8
  (Blank Card)
 150
   1     6     1   1.70       0.000      0.000      0.000      2.000
   7     6     1   1.70       0.500      0.000      0.500      2.000
  13     6     1   1.70       1.000      0.000      1.000      2.000
  19     6     1   1.70       1.400      0.000      1.400      2.000
  25     6     1   1.70       1.700      0.000      1.700      2.000
  31     6     1   1.70       1.900      0.000      1.900      2.000
  37     6     1   1.70       2.000      0.000      2.000      2.000
  43     6     1   1.70       2.075      .01684     2.075      2.000
  49     6     1   1.70       2.150      .02283     2.150      2.000
  55     6     1   1.70       2.225      .02657     2.225      2.000
  61     6     1   1.70       2.300      .02896     2.300      2.000
  67     6     1   1.70       2.375      .02988     2.375      2.000
  73     6     1   1.70       2.450      .02945     2.450      2.000
  79     6     1   1.70       2.525      .02739     2.525      2.000
  85     6     1   1.70       2.600      .02438     2.600      2.000
  91     6     1   1.70       2.675      .02048     2.675      2.000
  97     6     1   1.70       2.750      .01602     2.750      2.000
 103     6     1   1.70       2.825      .01126     2.825      2.000
 109     6     1   1.70       2.900      .00065     2.900      2.000
 115     6     1   1.70       3.000      0.000      3.000      2.000
 121     6     1   1.70       3.200      0.000      3.200      2.000
 127     6     1   1.70       3.500      0.000      3.500      2.000
 133     6     1   1.70       3.900      0.000      3.900      2.000
 139     6     1   1.70       4.400      0.000      4.400      2.000
 145     6     1   1.70       5.000      0.000      5.000      2.000
  (Blank Card)
  35     9    14
   1     2     3     4     5     6    12    18    24    30    36    42    48    54    60    66
  72    78    84    90    96   102   108   114   120   126   132   138   144   145   146   147
 148   149   150
   7    13    19    25    31   121   127   133   139
  37    43    49    55    61    67    73    79    85    91    97   103   109   115
      0      0.1040     0.0638     0.0400     0.0220     0.0044    -0.0174    -0.0344
  .0465    -0.0562    -0.0622    -0.0636    -0.0636    -0.0636
```

INPUT CARDS TO STRANL-II FOR NACA 64 A006

```
STEADY TRANSONIC FLOW--NACA64A006--MESH 5 --M=0.825
        (blank card)
     6  0.005        0.825       1.000
          (Insert cards generated in STRANL-I)
STEADY TRANSONIC FLOW--NACA64A006--MESH 5--M=0.875
 1
    12  0.01         0.875       0.500
STEADY TRANSONIC FLOW--NACA64A006--MESH 5--M=0.900
 1
    12  0.01         0.90        0.500
STEADY TRANSONIC FLOW--NACA64A006--MESH 5--M=0.940
 1
    16  0.01         0.940       0.400
```

MACH NUMBER= .940   RELAX. FACTOR = .4000

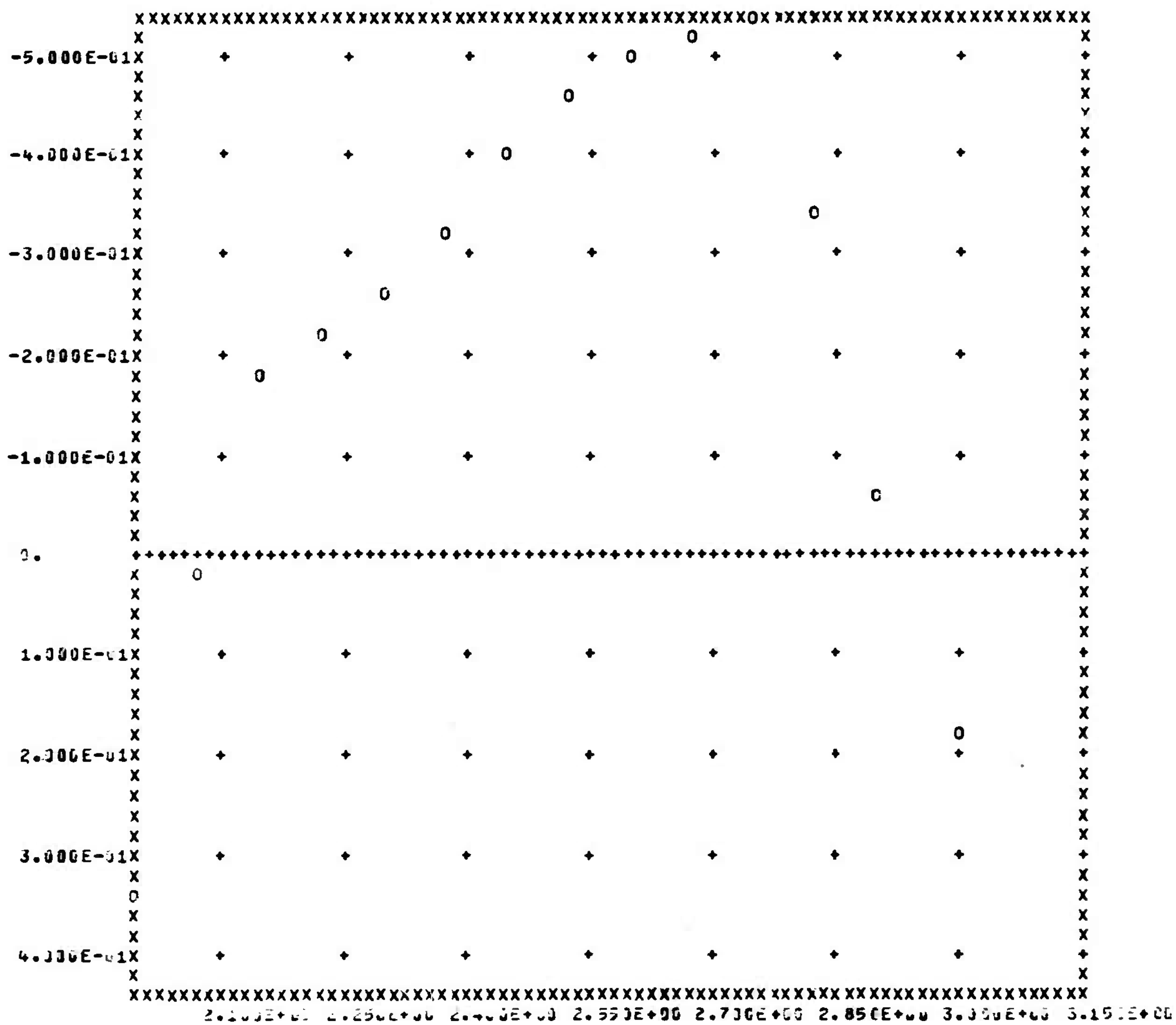
NO. OF ITERATION= 15

NODE	PHIT	UCOM	VCOM	COF	LMAC	P/PO	CP	DELM
1	0.	1.0000E+00	0.	8.8360E-01	9.4000E-01	5.6578E-01	-0.	0.
2	0.	1.0000E+00	0.	8.8360E-01	9.4000E-01	5.6578E-01	-0.	0.
3	0.	1.0000E+00	0.	8.8360E-01	9.4000E-01	5.6578E-01	-0.	0.
4	0.	1.0000E+00	0.	8.8360E-01	9.4000E-01	5.6578E-01	-0.	0.
5	0.	1.0000E+00	0.	8.8360E-01	9.4000E-01	5.6578E-01	-0.	0.
6	0.	1.0000E+00	0.	8.8360E-01	9.4000E-01	5.6578E-01	-0.	0.
7	1.7541E-04	1.0007E+00	0.	8.8549E-01	9.4078E-01	5.6528E-01	-1.4338E-03	1.7269E-05
8	1.3097E-04	1.0007E+00	9.6939E-05	8.8517E-01	9.4082E-01	5.6526E-01	-1.4843E-03	1.7012E-05
9	2.1243E-04	1.0010E+00	2.6298E-04	8.8565E-01	9.4197E-01	5.6510E-01	-1.9349E-03	1.6444E-05
10	2.5149E-04	1.0010E+00	1.0966E-05	8.8572E-01	9.4111E-01	5.6507E-01	-2.0019E-03	1.1701E-05
11	1.3296E-04	1.0007E+00	-1.9688E-04	8.8515E-01	9.4082E-01	5.6526E-01	-1.4742E-03	6.4003E-06
12	0.	1.0000E+00	0.	8.8360E-01	9.4000E-01	5.6578E-01	-0.	0.
13	-2.1681E-04	9.9771E-01	0.	8.7875E-01	9.3747E-01	5.6738E-01	4.5780E-03	4.3387E-05
14	-2.4437E-04	9.9730E-01	-4.9249E-04	8.7789E-01	9.3702E-01	5.6766E-01	5.3976E-03	4.0992E-05
15	-3.3361E-04	9.9693E-01	-1.2790E-04	8.7769E-01	9.3661E-01	5.6792E-01	6.1398E-03	4.1556E-05
16	-2.3749E-04	9.9690E-01	2.4012E-04	8.7782E-01	9.3657E-01	5.6795E-01	6.2077E-03	3.2268E-05
17	-9.5335E-05	9.9859E-01	3.1009E-04	8.8062E-01	9.3845E-01	5.6676E-01	2.8103E-03	2.0649E-05
18	0.	1.0000E+00	0.	8.8360E-01	9.4000E-01	5.6578E-01	-0.	0.
19	-5.3761E-03	9.7452E-01	0.	8.2956E-01	9.1200E-01	5.8358E-01	5.0964E-02	-3.0065E-06
20	-5.7810E-03	9.7557E-01	1.7945E-03	8.3189E-01	9.1316E-01	5.8284E-01	4.8851E-02	-8.2753E-06
21	-5.3280E-03	9.7951E-01	2.9508E-03	8.4015E-01	9.1746E-01	5.8009E-01	4.0983E-02	-2.5783E-06
22	-4.1490E-03	9.8580E-01	4.3614E-03	8.5349E-01	9.2437E-01	5.7570E-01	2.8394E-02	-4.3199E-06
23	-1.9769E-03	9.9232E-01	4.0965E-03	8.6731E-01	9.3153E-01	5.7114E-01	1.5368E-02	-1.3051E-06
24	0.	1.0000E+00	0.	8.8360E-01	9.4000E-01	5.6578E-01	-0.	0.
25	-1.5727E-02	9.5255E-01	0.	7.8297E-01	8.8815E-01	5.9887E-01	9.4909E-02	-1.7574E-04
26	-1.5611E-02	9.5212E-01	2.1767E-03	7.8267E-01	8.8770E-01	5.9916E-01	9.5751E-02	-1.8559E-04
27	-1.5561E-02	9.5263E-01	9.4676E-03	7.8325E-01	8.8834E-01	5.9875E-01	9.4646E-02	-1.6523E-04
28	-1.1860E-02	9.6503E-01	1.3653E-02	8.2945E-01	9.0178E-01	5.9012E-01	6.9935E-02	-1.3499E-04
29	-5.1927E-03	9.8612E-01	1.0568E-02	8.5417E-01	9.2477E-01	5.7544E-01	2.7760E-02	-7.4434E-05
30	0.	1.0000E+00	0.	8.8360E-01	9.4000E-01	5.6578E-01	-0.	0.
31	-3.6382E-02	8.5723E-01	0.	5.3083E-01	7.8755E-01	6.6413E-01	2.8554E-01	-3.4993E-04
32	-3.4402E-02	8.8266E-01	3.0396E-02	6.3477E-01	8.1451E-01	6.4657E-01	2.3467E-01	-3.6600E-04
33	-2.3160E-02	9.3252E-01	3.3760E-02	7.4049E-01	8.6729E-01	6.1232E-01	1.3496E-01	-3.6204E-04
34	-1.8639E-02	9.6808E-01	2.6238E-02	8.1590E-01	9.0537E-01	5.8782E-01	6.3849E-02	-2.9493E-04
35	-7.4151E-03	9.9046E-01	1.6133E-02	8.6336E-01	9.2962E-01	5.7236E-01	1.9087E-02	-1.4693E-04
36	0.	1.0000E+00	0.	8.8360E-01	9.4000E-01	5.6578E-01	-0.	0.
37	-5.3719E-02	8.2723E-01	1.2739E-01	5.1722E-01	7.6674E-01	6.7769E-01	3.4554E-01	-4.7527E-04
38	-4.3859E-02	9.1973E-01	6.7440E-02	7.1338E-01	8.5563E-01	6.1986E-01	1.6054E-01	-5.1661E-04
39	-3.3596E-02	9.5445E-01	4.6519E-02	7.8760E-01	8.9143E-01	5.9676E-01	9.1108E-02	-4.7993E-04
40	-2.1374E-02	9.7828E-01	3.1719E-02	8.3753E-01	9.1667E-01	5.8060E-01	4.3445E-02	-3.9595E-04
41	-8.1959E-03	9.9399E-01	1.8178E-02	8.7086E-01	9.3355E-01	5.6986E-01	1.2013E-02	-1.9477E-04
42	0.	1.0000E+00	0.	8.8360E-01	9.4000E-01	5.6578E-01	-0.	0.
43	-5.7163E-02	9.9435E-01	1.0341E-01	8.7163E-01	9.3969E-01	5.6597E-01	1.1290E-02	-6.0815E-04
44	-4.7247E-02	9.5829E-01	8.3698E-02	7.9515E-01	8.9831E-01	5.9234E-01	8.3423E-02	-6.2698E-04
45	-3.5329E-02	9.7879E-01	5.2375E-02	8.3860E-01	9.1819E-01	5.7963E-01	4.2443E-02	-6.0904E-04
46	-2.2184E-02	9.8916E-01	3.3945E-02	8.6062E-01	9.2869E-01	5.7295E-01	2.1670E-02	-4.7843E-04
47	-6.3629E-03	[illegible]	1.3355E-02	[illegible]	9.3693E-01	5.6772E-01	5.9157E-03	-2.2201E-04

48	0.	1.0000E+00	0.	8.8360E-01	9.4000E-01	5.6578E-01	-0.	0.
49	-5.3948E-02	1.0852E+00	6.9237E-02	1.0643E+00	1.0391E+00	5.0444E-01	-1.7045E-01	-7.2321E-04
50	-4.6715E-02	1.0450E+00	6.7012E-02	9.7897E-01	9.9279E-01	5.3273E-01	-8.9946E-02	-6.8696E-04
51	-3.5743E-02	1.0044E+00	5.4659E-02	8.9285E-01	9.4648E-01	5.6168E-01	-8.7206E-03	-7.6572E-04
52	-2.2350E-02	1.0027E+00	3.4838E-02	8.8924E-01	9.4361E-01	5.6349E-01	-5.3201E-03	-6.1558E-04
53	-8.4247E-03	1.0008E+00	1.9110E-02	8.8539E-01	9.4113E-01	5.6506E-01	-1.6844E-03	-2.4207E-04
54	0.	1.0000E+00	0.	8.8360E-01	9.4000E-01	5.6578E-01	-0.	0.
55	-4.5690E-02	1.1055E+00	4.4220E-02	1.1073E+00	1.0612E+00	4.9117E-01	-2.1098E-01	-1.2345E-03
56	-4.1933E-02	1.0782E+00	4.5909E-02	1.0495E+00	1.0295E+00	5.1023E-01	-1.5542E-01	-1.1967E-03
57	-3.3731E-02	1.0480E+00	4.4830E-02	9.8535E-01	9.9486E-01	5.3146E-01	-9.5958E-02	-9.0979E-04
58	-2.1638E-02	1.0157E+00	3.4116E-02	9.1683E-01	9.5804E-01	5.5440E-01	-3.1336E-02	-7.6358E-04
59	-8.1372E-03	1.0062E+00	1.8713E-02	8.9672E-01	9.4705E-01	5.6132E-01	-1.2378E-02	-2.6844E-04
60	0.	1.0000E+00	0.	8.8360E-01	9.4000E-01	5.6578E-01	-0.	0.
61	-3.7917E-02	1.1284E+00	2.4825E-02	1.1559E+00	1.0876E+00	4.7557E-01	-2.5684E-01	-1.2763E-03
62	-3.5038E-02	1.1031E+00	2.9706E-02	1.1023E+00	1.0579E+00	4.9316E-01	-2.0626E-01	-1.2343E-03
63	-2.9335E-02	1.0722E+00	3.3106E-02	1.0368E+00	1.0221E+00	5.1474E-01	-1.4449E-01	-1.3873E-03
64	-1.9654E-02	1.0375E+00	2.9527E-02	9.6306E-01	9.8234E-01	5.3921E-01	-7.4936E-02	-6.7309E-04
65	-7.4775E-03	1.0116E+00	1.7438E-02	9.0830E-01	9.5309E-01	5.5751E-01	-2.3293E-02	-3.9221E-04
66	0.	1.0000E+00	0.	8.8360E-01	9.4000E-01	5.6578E-01	-0.	0.
67	-2.7243E-02	1.1570E+00	5.0906E-03	1.2165E+00	1.1216E+00	4.5589E-01	-3.1393E-01	-1.2049E-03
68	-2.6273E-02	1.1307E+00	1.2981E-02	1.1608E+00	1.0901E+00	4.7410E-01	-2.6146E-01	-1.0057E-03
69	-2.3194E-02	1.0926E+00	2.0590E-02	1.0809E+00	1.0453E+00	5.0068E-01	-1.8520E-01	-1.4730E-04
70	-1.6233E-02	1.0556E+00	2.3197E-02	1.0015E+00	1.0027E+00	5.2661E-01	-1.1119E-01	-3.4322E-04
71	-6.2867E-03	1.0208E+00	1.4968E-02	9.2780E-01	9.6331E-01	5.5109E-01	-4.1689E-02	-7.3001E-04
72	0.	1.0000E+00	0.	8.8360E-01	9.4000E-01	5.6578E-01	-0.	0.
73	-1.4035E-02	1.1969E+00	-2.0827E-02	1.3012E+00	1.1710E+00	4.2817E-01	-3.9388E-01	-3.8125E-04
74	-1.5397E-02	1.1613E+00	-6.2252E-03	1.2256E+00	1.1268E+00	4.5291E-01	-3.2251E-01	-2.4358E-04
75	-1.5154E-02	1.1206E+00	7.3099E-03	1.1394E+00	1.0781E+00	4.8118E-01	-2.4128E-01	-5.4612E-04
76	-1.1397E-02	1.0726E+00	1.4910E-02	1.0375E+00	1.0220E+00	5.1480E-01	-1.4512E-01	-1.5234E-03
77	-4.3715E-03	1.0313E+00	1.1025E-02	9.5003E-01	9.7501E-01	5.4378E-01	-6.2649E-02	-1.1281E-03
78	0.	1.0000E+00	0.	8.8360E-01	9.4000E-01	5.6578E-01	-0.	0.
79	1.3356E-03	1.2253E+00	-4.2151E-02	1.3614E+00	1.2074E+00	4.0845E-01	-4.5365E-01	7.2579E-04
80	-1.3111E-03	1.1991E+00	-2.9364E-02	1.3058E+00	1.1739E+00	4.2655E-01	-3.9822E-01	1.1016E-04
81	-4.3752E-03	1.1494E+00	-8.3164E-03	1.2004E+00	1.1125E+00	4.6115E-01	-2.9874E-01	-1.5431E-03
82	-5.0703E-03	1.0933E+00	5.5973E-03	1.0815E+00	1.0460E+00	5.0030E-01	-1.8666E-01	-2.3214E-03
83	-1.7177E-03	1.0406E+00	5.4887E-03	9.6965E-01	9.8539E-01	5.3732E-01	-8.1152E-02	-1.1625E-03
84	0.	1.0000E+00	0.	8.8360E-01	9.4000E-01	5.6578E-01	-0.	0.
85	1.3817E-02	1.2467E+00	-5.7973E-02	1.4068E+00	1.2357E+00	3.9349E-01	-4.9344E-01	1.2901E-04
86	1.4312E-02	1.2261E+00	-4.7254E-02	1.3631E+00	1.2086E+00	4.0775E-01	-4.5218E-01	-1.7966E-04
87	7.5380E-03	1.1816E+00	-2.7951E-02	1.2688E+00	1.1522E+00	4.3859E-01	-3.6327E-01	-1.5938E-03
88	2.7363E-03	1.1120E+00	-5.5344E-03	1.1210E+00	1.0678E+00	4.8725E-01	-2.2392E-01	-6.4178E-04
89	1.4116E-03	1.0417E+00	-1.3123E-03	9.7202E-01	9.8664E-01	5.3654E-01	-8.3389E-02	4.5324E-04
90	0.	1.0000E+00	0.	8.8360E-01	9.4000E-01	5.6578E-01	-0.	0.
91	3.3225E-02	1.2630E+00	-7.0982E-02	1.4414E+00	1.2577E+00	3.8212E-01	-5.2603E-01	-1.5718E-03
92	3.2187E-02	1.2443E+00	-6.3060E-02	1.4016E+00	1.2328E+00	3.9498E-01	-4.8853E-01	-1.2351E-03
93	2.2335E-02	1.2086E+00	-4.6829E-02	1.3259E+00	1.1865E+00	4.1970E-01	-4.1715E-01	-3.8337E-04
94	1.1882E-02	1.1295E+00	-2.2155E-02	1.1582E+00	1.0888E+00	4.7488E-01	-2.5893E-01	1.2964E-03
95	4.3307E-03	1.0322E+00	-3.2343E-03	9.5180E-01	9.7592E-01	5.4321E-01	-6.4321E-02	4.0231E-03
96	0.	1.0000E+00	0.	8.8360E-01	9.4000E-01	5.6578E-01	-0.	0.
97	5.3554E-02	1.2686E+00	-7.8908E-02	1.4532E+00	1.2656E+00	3.7807E-01	-5.3723E-01	-2.8128E-03
98	5.1396E-02	1.2590E+00	-7.5276E-02	1.4329E+00	1.2528E+00	3.8463E-01	-5.1303E-01	-2.1227E-03
99	3.8783E-02	1.2250E+00	-6.6772E-02	1.3608E+00	1.2085E+00	4.0785E-01	-4.5008E-01	8.0451E-04
100	2.2048E-02	1.1382E+00	-4.2064E-02	1.1767E+00	1.0999E+00	4.6839E-01	-2.7644E-01	2.6189E-03
101	6.3069E-03	1.0313E+00	-1.5347E-02	[illegible]	9.7499E-01	[illegible]	-5.2514E-02	[illegible]

102	0.	1.0000E+00	0.	8.8360E-01	9.4000E-01	5.6578E-01	-0.	0.
103	7.6473E-02	1.1710E+00	-7.4478E-02	1.2463E+00	1.1417E+00	4.4450E-01	-3.4209E-01	-1.0885E-03
104	6.8452E-02	1.1833E+00	-7.7495E-02	1.2723E+00	1.1570E+00	4.3595E-01	-3.6655E-01	-1.2033E-03
105	5.4252E-02	1.1758E+00	-7.7898E-02	1.2565E+00	1.1478E+00	4.4105E-01	-3.5165E-01	-3.6283E-04
106	3.2076E-02	1.1344E+00	-6.4822E-02	1.1687E+00	1.0967E+00	4.7029E-01	-2.6884E-01	9.8719E-04
107	1.0212E-02	1.0570E+00	-2.3832E-02	1.0045E+00	1.0044E+00	5.2560E-01	-1.1402E-01	-3.4800E-03
108	0.	1.0000E+00	0.	8.8360E-01	9.4000E-01	5.6578E-01	-0.	0.
109	8.4248E-02	1.0323E+00	-6.5656E-02	9.5216E-01	9.7842E-01	5.4165E-01	-6.4658E-02	4.1564E-03
110	7.5988E-02	1.0117E+00	-7.0450E-02	9.0842E-01	9.5572E-01	5.3586E-01	-2.3412E-02	2.5705E-03
111	6.3940E-02	1.0447E+00	-7.2401E-02	9.7850E-01	9.9294E-01	5.3264E-01	-9.9498E-02	3.6934E-03
112	4.2243E-02	1.1197E+00	-6.5902E-02	1.1374E+00	1.0792E+00	4.8051E-01	-2.3936E-01	7.1036E-04
113	1.5256E-02	1.0623E+00	-3.3500E-02	1.0156E+00	1.0106E+00	5.2174E-01	-1.2451E-01	2.4760E-03
114	0.	1.0000E+00	0.	8.8360E-01	9.4000E-01	5.6578E-01	-0.	0.
115	8.0157E-02	9.1432E-01	-5.8183E-02	7.0297E-01	8.4976E-01	6.2367E-01	1.7036E-01	5.8495E-04
116	7.4077E-02	9.2907E-01	-5.7728E-02	7.3318E-01	8.6487E-01	6.1389E-01	1.4186E-01	1.0883E-03
117	6.4048E-02	9.6496E-01	-5.5633E-02	8.0930E-01	9.0334E-01	5.8912E-01	7.0073E-02	8.3486E-04
118	4.5802E-02	9.9785E-01	-5.5673E-02	8.7904E-01	9.3934E-01	5.6620E-01	4.3045E-03	3.7530E-04
119	1.9827E-02	1.0220E+00	-4.3377E-02	9.3019E-01	9.6547E-01	5.4974E-01	-4.3929E-02	1.0389E-03
120	0.	1.0000E+00	0.	8.8360E-01	9.4000E-01	5.6578E-01	-0.	0.
121	5.3261E-02	8.8620E-01	0.	6.4227E-01	8.1766E-01	6.4452E-01	2.2760E-01	9.1341E-04
122	5.3144E-02	9.0084E-01	-1.9245E-02	6.7331E-01	8.3324E-01	6.3439E-01	1.9832E-01	7.7058E-04
123	5.3388E-02	9.2414E-01	-3.2326E-02	7.2274E-01	8.5831E-01	6.1813E-01	1.5171E-01	6.2144E-04
124	4.1951E-02	9.5412E-01	-4.1481E-02	7.8631E-01	8.9083E-01	5.9715E-01	9.1757E-02	2.5837E-04
125	2.0294E-02	9.8783E-01	-3.9680E-02	8.5779E-01	9.2746E-01	5.7373E-01	2.4343E-02	-2.4095E-05
126	0.	1.0000E+00	0.	8.8360E-01	9.4000E-01	5.6578E-01	-0.	0.
127	3.5095E-02	9.4857E-01	0.	7.7454E-01	8.8387E-01	6.0163E-01	1.0285E-01	3.3871E-04
128	3.4919E-02	9.3877E-01	-4.0379E-03	7.5376E-01	8.7334E-01	6.0841E-01	1.2246E-01	3.9521E-04
129	3.3501E-02	9.3864E-01	-1.1741E-02	7.5347E-01	8.7327E-01	6.0846E-01	1.2273E-01	3.8947E-04
130	2.3086E-02	9.4847E-01	-2.2282E-02	7.7432E-01	8.8403E-01	6.0152E-01	1.0307E-01	3.1810E-04
131	1.4672E-02	9.7291E-01	-2.6567E-02	8.2615E-01	9.1064E-01	5.8445E-01	5.4177E-02	1.2038E-04
132	0.	1.0000E+00	0.	8.8360E-01	9.4000E-01	5.6578E-01	-0.	0.
133	1.4360E-02	9.5410E-01	0.	7.8626E-01	8.8983E-01	5.9779E-01	9.1004E-02	2.0572E-04
134	1.4139E-02	9.5843E-01	-3.5110E-03	7.9545E-01	8.9452E-01	5.9477E-01	8.3133E-02	1.7405E-04
135	1.3332E-02	9.6114E-01	-5.4497E-03	8.0120E-01	8.9747E-01	5.9288E-01	7.7711E-02	1.6527E-04
136	1.1260E-02	9.6692E-01	-8.1187E-03	8.1345E-01	9.0376E-01	5.8885E-01	6.6162E-02	1.4386E-04
137	6.1092E-03	9.7872E-01	-1.0527E-02	8.3846E-01	9.1665E-01	5.8061E-01	4.2568E-02	1.0455E-04
138	0.	1.0000E+00	0.	8.8360E-01	9.4000E-01	5.6578E-01	-0.	0.
139	1.9507E-03	9.9410E-01	0.	8.7110E-01	9.3349E-01	5.6990E-01	1.1791E-02	-1.2401E-04
140	2.0340E-03	9.9303E-01	1.0828E-03	8.5881E-01	9.3230E-01	5.7365E-01	1.3945E-02	-1.1456E-04
141	2.1114E-03	9.9316E-01	-7.4428E-05	8.5910E-01	9.3245E-01	5.7056E-01	1.3671E-02	-1.1345E-04
142	1.3567E-03	9.9385E-01	-1.3052E-03	8.7057E-01	9.3321E-01	5.7007E-01	1.2292E-02	-9.2988E-05
143	9.7879E-04	9.9505E-01	-1.9524E-03	8.7311E-01	9.3454E-01	5.6923E-01	9.8928E-03	-5.3348E-05
144	0.	1.0000E+00	0.	8.8360E-01	9.4000E-01	5.6578E-01	-0.	0.
145	0.	1.0000E+00	0.	8.8360E-01	9.4000E-01	5.6578E-01	-0.	0.
146	0.	1.0000E+00	0.	8.8360E-01	9.4000E-01	5.6578E-01	-0.	0.
147	0.	1.0000E+00	0.	8.8360E-01	9.4000E-01	5.6578E-01	-0.	0.
148	0.	1.0000E+00	0.	8.8360E-01	9.4000E-01	5.6578E-01	-0.	0.
149	0.	1.0000E+00	0.	8.8360E-01	9.4000E-01	5.6578E-01	-0.	0.
150	0.	1.0000E+00	0.	8.8360E-01	9.4000E-01	5.6578E-01	-0.	0.

C_p for NACA 64 A006 (M_∞ = 0.940)



Appendix A
FORTRAN LISTING OF STRANL-I

Appendix A

A FORTRAN listing of the source deck for the STRANL-I program is presented in the following pages. The program, as presently configured, required 42_8K words to execute.

```
      PROGRAM MAIN(INPUT=65,OUTPUT=131,PUNCH=65,TAPE5=INPUT,
     1      TAPE6=OUTPUT)
C
C     GENERATE 2-D MESH CONSISTING OF TRIANGLES AND QUADRILATERALS
C     THE PRESENT PROGRAM HAS TWO OPTIONS
C     IOPT(11)=1  RENUMBER NODES TO YIELD SMALLER BANDWIDTH
C     IOPT(12)=1   CALL MPLOT TO PLOT GENERATED MESH
C     THE PROGRAM AS PRESENTLY DIMENSIONED ALLOWS THE FOLLOWING MAXIMA
C     200 ELEMENTS,180 NODES,10 CONNECTING NODES TO ANY NODE,
C     50 NODES FOR EACH TYPE OF BOUNDARY CONDITIONS
C     DEVELOPED AND CODED BY STEVENS CHAN OF LOCKHEED-HUNTSVILLE,ALA.
C     EXCEPT SUBROUTINE MPLOT WHICH IS CODED AND SUPPLIED BY
C     CAPT. GERRY VANKEUREN OF AFFDL, WRIGHT-PATTERSON AIR FORCE BASE
C
      DIMENSION TITLE(8),IOPT(20),NDEL(200,4),MEMJT(1800)
      DIMENSION X(180),Y(180),NJNT(180),NEWJT(180),JOINT(180),JMEM(180)
      DIMENSION NIDS(3),NID(3,50),VAF(50)
      EQUIVALENCE (NIDS(1),NFARF),(NIDS(2),NWAKE),(NIDS(3),NBODY)
      DATA NEM,NPM,NCN,NEMN/200,180,10,1800/
C
C     READ AND GENERATE MESH INFORMATION ,ETC.
C
   10 READ(5,805)(TITLE(I),I=1,8)
      IF(EOF(5))2000,50
   50 CONTINUE
      READ(5,820)(IOPT(I),I=1,20)
      CALL MESHBW(NEM,NPM,NDEL,X,Y,NELS,NPS,MAXN)
C     READ BOUNDARY NODES AND AIRFOIL SLOPE
      READ 825, NIDS
      DO 120 I=1,3
      NS=NIDS(I)
  120 READ 825, (NID(I,J),J=1,NS)
      READ 830, (VAF(I),I=1,NBODY)
C
C     EXECUTE NODE RENUMBERING AND MESH PLOT OPTIONS
C
      IF (IOPT(11) .EQ. 1)  GO TO 140
      DO 130 I=1,NPS
  130 NJNT(I)=I
      GO TO 150
  140 CALL OPTM(NPM,NEM,NPS,NELS,4,NCN,NEMN,MEMJT,NEWJT,JOINT,
     1            JMEM,NDEL,NJNT,MAXN)
  150 CONTINUE
      IF (IOPT(12).EQ.1)  CALL MPLOT(X,Y,NDEL,NPM,NEM,NELS,NPS)
C
C     COMPUTE FULL BANDWIDTH AND PRINT OUTPUT DATA
C
      NBW=6*(MAXN+1)
      PRINT 910, (TITLE(I),I=1,8)
      PRINT 820, (IOPT(I),I=1,20)
      PRINT 920, NELS,NPS,NBW
      PRINT 930
      DO 220 N=1,NELS
  220 PRINT 825, N,(NDEL(N,J),J=1,4)
      PRINT 935
      DO 230 I=1,NPS
  230 PRINT 940, I,NJNT(I),X(I),Y(I)
      PRINT 951, (NID(1,I),I=1,NFARF)
```

```
      PRINT 952, (NID(2,I),I=1,NWAKE)
      PRINT 953, (NID(3,I),I=1,NBODY)
      PRINT 955, (VAF(I),I=1,NBODY)
C
C     PUNCH OUTPUT DATA TO BE USED AS INPUT TO STRANL--II
C
      PUNCH 825, NELS,NPS,NBW,(NIDS(I),I=1,3)
      PUNCH 825, ((NDEL(I,J),J=1,4),I=1,NELS)
      PUNCH 840, (X(I),Y(I),I=1,NPS)
      DO 350 I=1,3
      NS=NIDS(I)
  350 PUNCH 825, (NID(I,J),J=1,NS)
      PUNCH 840, (VAF(I),I=1,NBODY)
      PUNCH 825, (NJNT(I),I=1,NPS)
      GO TO 10
  805 FORMAT(7A10,A2)
  820 FORMAT(40I2)
  825 FORMAT (16I5)
  830 FORMAT (8F10.0)
  840 FORMAT (1P8E10.3)
  910 FORMAT(1H1,2X,7A10,A2//)
  920 FORMAT (1H0,-NO. OF ELEMENTS=',I4,'  NO. OF NODES=',I4,
     1         '    FULL BANDWIDTH=',I4//)
  930 FORMAT ('0ELE. NO. AND ELEMENT NODES'/)
  935 FORMAT ('0OLD NODE','  NEW NODE', 6X,'X(I)',12X,'Y(I)'/)
  940 FORMAT (I6,I10,2E15.5)
  951 FORMAT ('0NODES AT FARFIELD'/(20I5))
  952 FORMAT ('0NODES ON LINE OF SYMMETRY'/(20I5))
  953 FORMAT ('0NODES ON THE AIRFOIL'/(20I5))
  955 FORMAT ('0SLOPE ALONG NODES ON AIRFOIL'/(8E15.5))
 2000 STOP
      END
```

```
      SUBROUTINE MESHBW(NEM,NPM,NDEL,X,Y,NELS,NPS,MAXN)
C
C     GENERATE MESH INFORMATION INCLUDING ELEMENT NODE ARRAY,
C     NODAL COORDINATES, AND MAX. DIFFERENCE IN NODES
C
      DIMENSION NDEL(NEM,4),X(NPM),Y(NPM)
      DIMENSION IDUMP(4),DUMP(4)
 1020 FORMAT(16I5)
 1150 FORMAT (3I5,5F10.0)
C
C     GENERATE ELEMENT NODE NUMBERS AND THEIR MAXIMUM DIFFERENCE
C
      DO 100 N=1,NEM
      DO 100 J=1,4
  100 NDEL(N,J)=0
      NELS = 0
      MAXN=0
  120 READ 1020, NPE,KEI,KEJ,KDI,KDJ,(IDUMP(I),I=1,4)
      IF (NPE .EQ. 0)  GO TO 129
      J1=-KDJ
      DO 124 J=1,KEJ
      J1 = J1 + KDJ
      J2 = J1 - KDI
      DO 124 I=1,KEI
      NELS = NELS + 1
      J2 = J2 + KDI
      DO 124 K=1,NPE
  124 NDEL(NELS,K) = IDUMP(K) + J2
      DO 126 I=1,NPE
      DO 126 J=I,NPE
      NDIFF=NDEL(NELS,I)-NDEL(NELS,J)
  126 IF (IABS(NDIFF) .GT. MAXN)  MAXN=IABS(NDIFF)
      GO TO 120
  129 IF (NELS .GT. NEM)  GO TO 300
C
C     GENERATE NODAL COORDINATES
C
      READ 1020, NPS
  130 READ 1150, ND,NNI,NDI,RATIO,(DUMP(I),I=1,4)
      IF (ND .EQ. 0)  GO TO 140
      TEMP2 = 1.0
      IF (NNI .LT. 3)  GO TO 135
      J1 = NNI - 1
      TEMP1 = 1.0
      TEMP2 = 0.0
      DO 134 I=1,J1
      TEMP2 = TEMP2 + TEMP1
  134 TEMP1 = RATIO*TEMP1
  135 TEMP1 = (DUMP(3)-DUMP(1))/TEMP2
      TEMP2 = (DUMP(4)-DUMP(2))/TEMP2
      J1 = ND
      TEMP3 = 0.0
      TEMP4 = 0.0
      DO 137 I=1,NNI
      X(J1) = DUMP(1) + TEMP3
      Y(J1) = DUMP(2) + TEMP4
      J1 = J1 + NDI
      TEMP3 = TEMP3 + TEMP1
```

```
      TEMP4 = TEMP4 + TEMP2
      TEMP1 = RATIO*TEMP1
  117 TEMP2 = RATIO*TEMP2
      GO TO 130
  140 IF (NPS .GT. NPM)  GO TO 400
      RETURN
C     PRINT ERROR MESSAGE IF DIMENSIONS FOR ELEMENTS OR NODES EXCEEDED
  300 PRINT 310, NELS,NEM
  310 FORMAT ('0TOO MANY ELEMENTS',5X,'NELS=',I4,5X,'NEM=',I4)
      CALL EXIT
  400 PRINT 410, NPS,NPM
  410 FORMAT ('0TOO MANY NODES', 5X,'NPS=',I4,5X,'NPM=',I4)
      CALL EXIT
      END
```

```
      SUBROUTINE OPTM(NPM,NEM,NPS,NELS,NPE,NCN,NEMN,MEMJT,NEWJT,
     1     JOINT,JMEM,NDEL,NJNT,MAXN)
C
C     RENUMBER THE NODES TO REDUCE BANDWIDTH, WITH NJNT CONTAINING THE
C     NEW NODAL NUMBERS (LOCATION IN ARRAY REPRESENTS THE OLD NODAL NO.)
C     NPM=MAX. NO. OF NODES.    NPS=ACTUAL NO. OF NODES
C     NEM=MAX. NO. OF ELEMENTS.    NELS=ACTUAL NO. OF ELEMENTS
C     NPE=MAX. NO. OF NODES AMONG ALL THE ELEMENTS
C     NCN=ESTIMATED MAX. NO. OF NODES CONNECTED TO ANY NODE
C     NDEL=ELEMENT NODE POINTS
C     MAXN=MAX. DIFF. BETWEEN NODAL POINTS IN ANY ELEMENT
C     MEMJT,NEWJT,JOINT,JMEM ARE INTERMEDIATE WORKING SPACE
C
      DIMENSION NDEL(NEM,NPE),NJNT(NPM),MEMJT(NEMN)
      DIMENSION NEWJT(NPM),JOINT(NPM),JMEM(NPM)
C
C     ESTABLISH NODES CONNECTED TO EACH OF THE NODAL POINT
C
      DO 10 J=1,NPS
   10 JMEM(J)=0
      DO 60 J=1,NELS
      DO 50 I=1,NPE
      JNTI=NDEL(J,I)
      IF (JNTI .EQ. 0)  GO TO 60
      JSUB=(JNTI-1)*NCN
      DO 40 II=1,NPE
      IF (II .EQ. I)  GO TO 40
      JJT=NDEL(J,II)
      IF (JJT .EQ. 0)  GO TO 50
      MEMI=JMEM(JNTI)
      IF (MEMI .EQ. 0)  GO TO 30
      DO 20 III=1,MEMI
      JP=JSUB+III
      IF (MEMJT(JP) .EQ. JJT)  GO TO 40
   20 CONTINUE
   30 JMEM(JNTI)=JMEM(JNTI)+1
      NPOS=JSUB+JMEM(JNTI)
      MEMJT(NPOS)=JJT
   40 CONTINUE
   50 CONTINUE
   60 CONTINUE
      DO 100 I=1,NPS
      IF (JMEM(I) .LE. NCN)  GO TO 100
      PRINT 90, I,NCN
   90 FORMAT ('0ERROR....NODE',I5,'  HAS MORE THAN',I5,'  CONN. NODES')
      CALL EXIT
  100 CONTINUE
C
C     THE ORIGIN OF THE NEW NUMBERING SYSTEM IS LOCATED AT EACH NODE
C     IN TURN TO SEARCH FOR THE ONE WITH SMALLEST BANDWIDTH
C
      DO 160 IK=1,NPS
      DO 120 J=1,NPS
      JOINT(J)=0
  120 NEWJT(J)=0
      MAX=0
      I=1
      NEWJT(1)=IK
```

```
      JOINT(IK)=1
      K=1
  130 NEW=NEWJT(I)
      K4=JMEM(NEW)
      IF (K4 .EQ. 0)  GO TO 145
      JSUB=(NEWJT(I)-1)*NCN
      DO 140 JJ=1,K4
      JP=JSUB+JJ
      K5=MEMJT(JP)
      IF (JOINT(K5) .GT. 0)  GO TO 140
      K=K+1
      NEWJT(K)=K5
      JOINT(K5)=K
      NDIFF=IABS(I-K)
      IF (NDIFF .GE. MAXN)  GO TO 160
      IF (NDIFF .GT. MAX)  MAX=NDIFF
  140 CONTINUE
      IF (K .EQ. NPS)  GO TO 150
  145 I=I+1
      GO TO 130
  150 MAXN=MAX
      DO 155 J=1,NPS
  155 NJNT(J)=JOINT(J)
  160 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE MPLOT(XC,YC,NODE,NN,NE,NELS,NPS)
C
C     USE CALCOMP PLOTTER TO PLOT GENERATED MESH
C     CODED BY CAPT. GERRY VANKEUREN OF AFFDL, WRIGHT-PATTERSON AFB
C
      DIMENSION XC(NN),YC(NN),NODE(NE,4),XP(7),YP(7)
C     DEFINE PLOT SIZE AND PLACE PEN IN INITIAL POSITION
      DO 5 L=1,NPS
      XC(L)=XC(L)*8.
    5 YC(L)=YC(L)*8.
      CALL PLOT(0.0,-3.0,-3)
      CALL PLOT(1.0,3.0,-3)
C     DRAW EACH ELEMENT IN TURN
      DO 10 I=1,NELS
      NPE=5
      IF (NODE(I,4).EQ.0)  NPE=4
      NPP=NPE-1
      DO 9 J=1,NPP
      XP(J)=XC(NODE(I,J)))
    9 YP(J)=YC(NODE(I,J))
      XP(NPE)=XP(1)
      YP(NPE)=YP(1)
      XP(NPE+1)=YP(NPE+1)=0.0
      XP(NPE+2)=YP(NPE+2)=1.0
   10 CALL LINE(XP,YP,NPE,1,0,0)
C     WRITE NODAL NUMBERS AND FINALLY END THE PLOT
      DO 20 L=1,NPS
      PL=FLOAT(L)
   20 CALL NUMBER(XC(L),YC(L),.14,PL,0,-1)
      CALL PLOTE
      RETURN
      END
```

Appendix B
FORTRAN LISTING OF STRANL-II

Appendix B

A FORTRAN listing of the source deck for the STRANL-II program is presented in the following pages. The program, as presently configured, requires 177_8K words to execute.

```
      PROGRAM MAIN(INPUT=65,OUTPUT=131,PUNCH=65,TAPE5=INPUT,
     1     TAPE6=OUTPUT)
C
C     STEADY TRANSONIC FLOW ANALYSIS BY FINITE ELEMENT METHOD
C     USING LEAST SQUARES WITH TRIANGULAR AND QUADRILATERAL ELEMENTS
C     IOPT(1)=1, USE RESULTS OF PREVIOUS CASE AS STARTING SOLUTION
C     WHILE THE OTHER OPTION IS IGNORED
C     IOPT(2)=1, READ IN NON-ZERO INITIAL GUESS
C     IOPT(3)=1, APPLY LINEARIZED BOUNDARY CONDITION ON CHORDLINE
C     THE PROGRAM AS PRESENTLY DIMENSIONED ALLOWS THESE MAXIMA
C     200 ELEMENTS, 180 NODES, 50 NODES FOR EACH TYPE OF BOUNDARY COND.
C     MAX. FULL BANDWIDTH = 84
C     DEVELOPED AND CODED BY STEVENS CHAN OF LOCKHEED-HUNTSVILLE,ALA.
C
      DIMENSION TITLE(8),IOPT(20),NOD(200,4),S(540,84),SLP(540)
      DIMENSION X(180),Y(180),RML(180),RMLP(180),COF(180),NONT(180)
      DIMENSION NIDS(3),NID(3,50),VAF(50),AR(100)
      LOGICAL LR(50)
      EQUIVALENCE (NIDS(1),NFARF),(NIDS(2),NWAKE),(NIDS(3),NBODY)
      DATA NEM,NPM,NCM,NA/200,180,84,50/
      DATA PI/3.1415926/,GAMMA/1.40/
      NRM=3*NPM
      IA=2*NA
      CONST=0.5*(GAMMA-1.0)
      EXP=-GAMMA/(GAMMA-1.0)
C
C     READ TITLE, CONTROL KEYS, AND PROBLEM PARAMETERS
C
  100 READ(5,805)(TITLE(I),I=1,8)
      IF(EOF(5))2000,101
  101 CONTINUE
      READ(5,820)(IOPT(I),I=1,20)
      READ 830, ITGIV,ZTEST,RMAC,FI
      PRINT 910, (TITLE(I),I=1,8),ZTEST
      PRINT 820, (IOPT(I),I=1,20)
      IRES=0
      RFT=1.0
      SQMAC=RMAC**2
      IF (IOPT(1) .EQ. 1)  GO TO 382
C
C     READ AND PRINT MESH DATA, BOUNDARY NODES, AND AIRFOIL SLOPE
C
      READ 825, NELS,NPS,NBW,(NIDS(I),I=1,3)
      READ 825, ((NOD(I,J),J=1,4),I=1,NELS)
      READ 840, (X(I),Y(I),I=1,NPS)
      DO 110 I=1,3
      NS=NIDS(I)
  110 READ 825, (NID(I,J),J=1,NS)
      READ 840, (VAF(I),I=1,NBODY)
```

```
  120 READ 825, (NJNT(I),I=1,NPS)
C
C     IF IOPT(3)=1, APPLY LINEARIZED BOUNDARY CONDITION ON CHORDLINE
C     OTHERWISE APPLY NONLINEAR BOUNDARY CONDITION ON AIRFOIL SURFACE
C
      IF (IOPT(3) .NE. 1)  GO TO 116
      DO 115 J=1,NBODY
      I=NID(3,J)
  115 Y(I)=0.0
  116 CONTINUE
  200 PRINT 920, NELS,NPS,NBW
      PRINT 930
      DO 220 N=1,NELS
  220 PRINT 825, N,(NOD(N,J),J=1,4)
      PRINT 935
      DO 230 I=1,NPS
  230 PRINT 940, I,NJNT(I),X(I),Y(I)
      PRINT 951, (NID(1,I),I=1,NFARF)
      PRINT 952, (NID(2,I),I=1,NWAKE)
      PRINT 953, (NID(3,I),I=1,NBODY)
      PRINT 955, (VAF(I),I=1,NBODY)
C
C     REDEFINE MESH DATA, ETC. USING NEW NODAL NUMBERING SYSTEM
C
      DO 238 N=1,NELS
      DO 238 I=1,4
      IF (NOD(N,I) .EQ. 0)  GO TO 238
      KK=NOD(N,I)
      NOD(N,I)=NJNT(KK)
  238 CONTINUE
      DO 239 I=1,3
      IS=NIDS(I)
      DO 239 J=1,IS
      KK=NID(I,J)
  239 NID(I,J)=NJNT(KK)
      DO 243 I=1,NPS
      RMLP(I)=X(I)
  243 RML(I)=Y(I)
      DO 244 II=1,NPS
      I=NJNT(II)
      X(I)=RMLP(II)
  244 Y(I)=RML(II)
C
C     HALF BANDWIDTH AND NUMBER OF EQUATIONS
C
      NHBW=NBW/2
      NEQ=3*NPS
C
C     READ NONZERO INITIAL GUESS OR PROCEED WITH ZERO SOLUTION
C
      IF (IOPT(2) .NE. 1)  GO TO 250
      READ 840, (S(I,NBW),I=1,NEQ)
      GO TO 295
  250 DO 260 I=1,NEQ
  260 SLP(I)=0.0
C
C     ITERATIONS START HERE AND CHECKED IF ITGIV IS EXCEEDED. IF SO,
C     PRINT FAIL TO CONVERGE AND PROCEED TO NEXT CASE. OTHERWISE,
```

```
C     CONTINUE TO ITERATE
C
  265 IRES=IRES+1
      IF (IRES .GT. ITGIV)  GO TO 600
C     FORMULATE SYSTEM OF ALGEBRAIC EQUATIONS
      DO 266 I=1,NEQ
      DO 266 J=1,NBW
  266 S(I,J)=0.0
      CALL NEWK(SQMAC,NRM,NCM,NEQ,NBW,NEM,NELS,NOD,SLP,S,COF,NPM,X,Y)
C     IMPOSE B. C. FOR FARFIELD, LINE OF SYMMETRY, AND ON AIRFOIL
      DO 274 I=1,NFARF
      IE=3*NID(1,I)-3
      DO 272 II=1,3
      IE=IE+1
      DO 270 K=1,NBW
  270 S(IE,K)=0.0
  272 S(IE,NHBW)=1.0
  274 CONTINUE
      DO 280 I=1,NWAKE
      IE=3*NID(2,I)
      DO 278 K=1,NBW
  278 S(IE,K)=0.0
  280 S(IE,NHBW)=1.0
      DO 285 J=1,NBODY
      IE=3*NID(3,J)
      DO 282 K=1,NBW
  282 S(IE,K)=0.0
      S(IE,NHBW)=1.0
      IF (IOPT(3) .NE. 1)  S(IE,NHBW-1)=-VAF(J)
  285 S(IE,NBW)=VAF(J)
C     CALL BANDED SOLVER TO SOLVE THE SYSTEM OF EQUATIONS
      CALL BNDEQ(S,NRM,NCM,NEQ,NHBW)
C
C     PRINT COMPUTED RESULTS
C
  295 PRINT 970, RMAC,RFT
      PRINT 975, IRES
      DO 305 I=1,NPS
      J=NJNT(I)
      II=3*NJNT(I)
      POT=S(II-2,NBW)
      UPT=S(II-1,NBW)
      V=S(II,NBW)
      U=UPT+1.0
      QSQ=U*U+V*V
      ASQ=CONST*(1.0-QSQ)+1.0/SQMAC
      RML(J)=SQRT(QSQ/ASQ)
      PRATIO=(1.0+CONST*RML(J)**2)**EXP
      CP=-2.*UPT
      COF(J)=SQMAC*(1.0+2.4*UPT)
      DELM=RML(J)-RMLP(J)
  305 PRINT 978, I,POT,U,V,COF(J),RML(J),PRATIO,CP,DELM
C
C     DISPLAY PRESSURE COEFFICIENT CP
C
      PRINT 985
      ISTOP=0
      IPNT=0
```

```
      DO 320 J=1,NBODY
      IF (J .EQ. NBODY)  ISTOP=1
      I=NID(3,J)
      UPT=S(3*I-1,NBW)
      V=S(3*I,NBW)
      CP=-2.*UPT
  320 CALL FPLOT(IA,IPNT,AR,LR,ISTOP,1,1,X(I),CP)
      IF (IRES .LT. 2)  GO TO 382
C
C     CHECK CONVERGENCE. IF SO, PUNCH CONVERGED SOLUTION AND
C     PROCEED TO NEXT CASE. OTHERWISE, UPDATE SOLUTION AND CONTINUE
C     TO ITERATE
C
      DO 340 I=1,NPS
      PCTE=1.0-RMLP(I)/RML(I)
      IF (ABS(PCTE) .LT. ZTEST)  GO TO 340
      RFT=F1
      GO TO 382
  340 CONTINUE
      PUNCH 840, (S(I,NBW), I=1,NEQ)
      GO TO 100
  382 DO 385 I=1,NPS
  385 RMLP(I)=RML(I)
      RFTC=1.0-RFT
      DO 390 I=1,NEQ
  390 SLP(I)=RFT*S(I,NBW)+RFTC*SLP(I)
      GO TO 265
  600 PRINT 980
      GO TO 100
  805 FORMAT(7A10,A2)
  820 FORMAT (40I2)
  825 FORMAT (16I5)
  830 FORMAT (I5,3F10.0)
  840 FORMAT (1P8E10.3)
  910 FORMAT(1H1,2X,7A10,A2//'  CONVERGENCE LIMIT =',F6.4//)
  920 FORMAT (1H0,-NO. OF ELEMENTS=',I4,'  NO. OF NODES-',I4,
     1          '   FULL BANDWIDTH=',I4//)
  930 FORMAT ('OELE. NO. AND ELEMENT NODES'/)
  935 FORMAT ('OOLD NODE',' NEW NODE', 6X,'X(I)',12X,'Y(I)'/)
  940 FORMAT (I6,I10,2E15.5)
  951 FORMAT ('ONODES AT FARFIELD'/(20I5))
  952 FORMAT ('ONODES ON LINE OF SYMMETRY'/(20I5))
  953 FORMAT ('ONODES ON THE AIRFOIL'/(20I5))
  955 FORMAT ('OSLOPE ALONG NODES ON AIRFOIL'/(8E15.5))
  970 FORMAT (1H1, -MACH NUMBER=',F6.3,5X,'RELAX. FACTOR =',F6.4/)
  975 FORMAT (1H0,4X,-NO. OF ITERATION=',I4/
     1         1H0,7X,-NODE',8X,'PHIT',11X,'UCOM',11X,'VCOM',
     2        12X,'COF-,11X,'LMAC',11X,'P/PO',13X,'CP',11X,'DELM'/)
  978 FORMAT (I10,1P8E15.4)
  980 FORMAT ('OFAIL TO CONVERGE IN SPECIFIED NO. OF ITERATIONS')
  985 FORMAT (1H1)
 2000 STOP
      END
```

```
      SUBROUTINE NEWK(SQMAC,NRM,NCM,NEQ,NBW,NEM,NELS,NOD,SLP,S,
     1     COEF,NPM,X,Y)
C
C     GENERATE SYSTEM MATRIX BY ASSEMBLLING CONTRIBUTIONS FROM
C     ALL THE ELEMENTS
C     SUBROUTINE EMQT CALLED TO GENERATE ELEMENT MATRIX
C
      DIMENSION COEF(NPM),X(NPM),Y(NPM),XQ(4),YQ(4),PM(12),BB(12,12)
      DIMENSION NOD(NEM,4),S(NRM,NCM),SLP(NRM)
      NHBW=NBW/2
      DO 480 N=1,NELS
      I1=1
      IF (NOD(N,4)) 402,402,404
  402 NPEL=3
      NTRS=1
      GO TO 410
  404 NPEL=4
      NTRS=4
      I2=0
      DO 408 I=1,4
      NI=NOD(N,I)
  408 IF (COEF(NI) .GT. 1.00)   I2=I2+1
      IF (I2 .EQ. 0)  NTRS=2
      IF (I2 .EQ. 4)  I1=3
  410 DO 425 I=1,NPEL
      NI=NOD(N,I)
      XQ(I)=X(NI)
      YQ(I)=Y(NI)
      DO 425 J=1,3
      IS=3*(NI-1)+J
      IE=3*(I-1)+J
  425 PM(IE)=SLP(IS)
      CALL EMQT(XQ,YQ,PM,SQMAC,BB,NTRS)
      DO 450 I=I1,NPEL
      NR=3*(NOD(N,I)-1)
      IE=3*(I-1)
      DO 450 II=1,3
      NR=NR+1
      IE=IE+1
      DO 450 J=1,NPEL
      NC=3*(NOD(N,J)-1)-NR+NHBW
      JE=3*(J-1)
      DO 450 JJ=1,3
      NC=NC+1
      JE=JE+1
  450 S(NR,NC)=S(NR,NC)+BB(IE,JE)
  480 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE EMQT(XQ,YQ,PMQ,SQMAC,EQ,NTRS)
C
C     GENERATE MATRIX FOR A QUADRILATERAL OR TRIANGLE
C     SUBROUTINE EMTC CALLED TO GENERATE MATRIX FOR A BASIC TRIANGLE
C
      DIMENSION EQ(12,12),ET(9,9),XQ(4),YQ(4),XT(3),YT(3),MP(3,4)
      DIMENSION PMQ(12),PMT(9)
      DATA MP/1,2,3,3,4,1,2,3,4,4,1,2/
      FTOR=1.0
      IF (NTRS .EQ. 4)  FTOR=0.5
      DO 100 I=1,12
      DO 100 J=1,12
  100 EQ(I,J)=0.0
      DO 150 II=1,NTRS
      DO 105 I=1,3
      N1=MP(I,II)
      IT=3*(I-1)
      IQ=3*(N1-1)
      DO 102 J=1,3
      IT=IT+1
      IQ=IQ+1
  102 PMT(IT)=PMQ(IQ)
      XT(I)=XQ(N1)
  105 YT(I)=YQ(N1)
      CALL EMTC(ET,XT,YT,PMT,SQMAC)
      DO 130 K=1,3
      NR=3*(MP(K,II)-1)
      IE=3*(K-1)
      DO 130 KK=1,3
      NR=NR+1
      IE=IE+1
      DO 130 L=1,3
      NC=3*(MP(L,II)-1)
      JE=3*(L-1)
      DO 130 LL=1,3
      NC=NC+1
      JE=JE+1
  130 EQ(NR,NC)=EQ(NR,NC)+ET(IE,JE)*FTOR
  150 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE EMTC(A,XL,YL,PEL,SQMAC)
C
C     EVALUATE ELEMENT MATRIX FOR A TRIANGLE BY GAUSSIAN QUADRATURE
C     SUBROUTINE DERV CALLED TO EVALUATE SHAPE FUNCTION DERIVATIVES
C     AT THE GAUSSIAN POINTS
C
      DIMENSION A(9,9),P(9),Q(9),NP(5),B(3),C(3),XL(3),YL(3),S(3),
     1          DNX(9),DNXX(9),DNYY(9),PEL(9)
      DIMENSION EINT(3,7),WT(7)
      DATA LMAX/7/,WT/0.225,3*0.13239415,3*0.12593918/
      DATA EINT/3*0.33333333,0.05961587,3*0.47014206,0.05961587,
     1    3*0.47014206,0.05961587,0.79742699,3*0.10128651,
     2    0.79742699,3*0.10128651,0.79742699/
      DATA NP/1,2,3,1,2/,GAMMA/1.40/
      DO 2 I=1,9
      DO 2 J=1,9
    2 A(I,J)=0.
      DO 4 I=1,3
      J=NP(I+1)
      K=NP(I+2)
      B(I)=YL(J)-YL(K)
    4 C(I)=XL(K)-XL(J)
      AREA=0.5*(B(2)*C(3)-B(3)*C(2))
      CST1=1.0-SQMAC
      CST2=SQMAC*(1.0+GAMMA)
      DO 100 L=1,LMAX
      DO 10 I=1,3
   10 S(I)=EINT(I,L)
      CALL DERV(AREA,B,C,S,DNX,DNXX,DNYY)
      U=0.
      UX=0.
      DO 30 I=1,9
      U=U+DNX(I)*PEL(I)
   30 UX=UX+DNXX(I)*PEL(I)
      ALPHA=CST1-CST2*U
      DO 40 I=1,9
      P(I)=ALPHA*DNXX(I)+DNYY(I)
   40 Q(I)=P(I)-CST2*UX*DNX(I)
      WEIGHT=WT(L)*AREA
      DO 60 I=1,9
      CST=WEIGHT*Q(I)
      DO 60 J=1,9
   60 A(I,J)=A(I,J)+CST*P(J)
  100 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE DERV(AREA,B,C,S,DNX,DNXX,DNYY)
C
C     EVALUATE SHAPE FUNCTION DERIVATIVES AT GAUSSIAN POINT
C
      DIMENSION B(3),C(3),S(3),DNX(9),DNXX(9),DNYY(9),NP(5)
      DATA NP/1,2,3,1,2/
      TWOA=2.*AREA
      TWOASQ=TWOA**2
      DO 200 I=1,3
      J=NP(I+1)
      K=NP(I+2)
      SI=S(I)
      SJ=S(J)
      SK=S(K)
      BI=B(I)
      BJ=B(J)
      BK=B(K)
      CI=C(I)
      CJ=C(J)
      CK=C(K)
      SISQ=SI*SI
      BISQ=BI*BI
      CISQ=CI*CI
      ALFA=0.5*(CK-CJ)
      BETA=0.5*(BJ-BK)
      HX=BI*SJ*SK+BJ*SK*SI+BK*SI*SJ
      HXX=2.*(SI*BJ*BK+SJ*BK*BI+SK*BI*BJ)
      HYY=2.*(SI*CJ*CK+SJ*CK*CI+SK*CI*CJ)
      CSS=6.*SI*(1.-SI)
      CS=6.*(1.-2.*SI)
      L=3*I-2
      DNX(L)=BI*CSS+2.*HX
      DNXX(L)=BISQ*CS+2.*HXX
      DNYY(L)=CISQ*CS+2.*HYY
      CS=CK*SJ-CJ*SK
      L=L+1
      DNX(L)=2.*BI*SI*CS+TWOA*SISQ+ALFA*HX
      DNXX(L)=2.*BISQ*CS+4.*BI*TWOA*SI+ALFA*HXX
      DNYY(L)=2.*CISQ*CS+ALFA*HYY
      BS=BJ*SK-BK*SJ
      L=L+1
      DNX(L)=2.*BI*SI*BS+BETA*HX
      DNXX(L)=2.*BISQ*BS+BETA*HXX
  200 DNYY(L)=2.*CISQ*BS+4.*CI*TWOA*SI+BETA*HYY
      DO 300 I=1,9
      DNX(I)=DNX(I)/TWOA
      DNXX(I)=DNXX(I)/TWOASQ
  300 DNYY(I)=DNYY(I)/TWOASQ
      RETURN
      END
```

```
      SUBROUTINE BNDEQ(A,NRMAX,NCMAX,N,ITERM)
C
C     EQUATION SOLVER FOR BANDED NON-SYMMETRIC SYSTEM OF EQUATIONS
C     SOLUTION STORED IN THE LAST COLUMN A(I,2*ITERM)
C
      DIMENSION A(NRMAX,NCMAX)
      CERO=1.E-6
      PARE = CERO**2
      NBND=2*ITERM
      NBM = NBND - 1
C     BEGINS ELIMINATION OF THE LOWER LEFT
      DO 1000 I=1, N
      IF ( ABS(A(I,ITERM))  .LT.  CERO)  GO TO 410
      GO TO 430
  410 IF ( ABS(A(I,ITERM))  .LT.  PARE)   GO TO 1600
      PRINT 420, A(I,ITERM), I
  420 FORMAT (' WARNING. ILL-CONDITIONED A-MATRIX. A=',E16.6,' I=',I4)
  430 JLAST = MIN0(I+ITERM-1, N)
      L = ITERM + 1
      DO 500 J=1, JLAST
      L = L - 1
      IF ( ABS(A(J,L))  .LT.  PARE)  GO TO 500
      B = A(J,L)
      DO 450 K=L, NBND
  450 A(J,K) = A(J,K) / B
      IF (I  .EQ.  N)  GO TO 1200
  500 CONTINUE
      L=0
      JFIRST = I + 1
      IF (JLAST  .LE.  I)  GO TO 1000
      DO 900 J= JFIRST, JLAST
      L=L+1
      IF ( ABS(A(J,ITERM-L))  .LT.  PARE)  GO TO 900
      DO 600 K=ITERM, NBM
  600 A(J,K-L) = A(J-L,K) - A(J,K-L)
      A(J,NBND) = A(J-L,NBND) - A(J,NBND)
      IF (I  .GE.  N-ITERM+1)  GO TO 900
      DO 800 K=1, L
  800 A(J,NBND-K) = -A(J,NBND-K)
  900 CONTINUE
 1000 CONTINUE
C     BACK-SUBSTITUTION
 1200 L = ITERM - 1
      DO 1500 I=2, N
      DO 1500 J=1, L
      IF (N+1-I+J  .GT.  N)  GO TO 1500
      A(N+1-I,NBND) = A(N+1-I,NBND) - A(N+1-I+J,NBND)*A(N+1-I,ITERM+J)
 1500 CONTINUE
      RETURN
C     PRINT THE ENTIRE MATRIX IF ZERO ON MAIN DIAGONAL
 1600 PRINT 1601
 1601 FORMAT (' COMPUTATION STOPPED IN BNDEQ BECAUSE ZERO APPEARED ON
     1MAIN DIAGONAL.   THE MATRIX FOLLOWS.')
      DO 1602 I=1, N
 1602 PRINT 1603, (A(I,J), J=1, NBND)
 1603 FORMAT (10E12.4)
      STOP
      END
```

```
      SUBROUTINE FPLOT(M1,IPNT,AR,LR,ISTOP,NC,NCMAX,V1,V2)
C
C     PRINTER PLOT FOR DISPLAYING COMPUTED RESULTS
C
      LOGICAL   LM(2),LN(120),LP(4),LX(4),LR(1),LC(20)
      INTEGER RI(2),ST(2),SG(2)
      DIMENSION AR(M1),N(120),RHO(30),Z(15),IOFF(2),ISP(2),SF1(2),SF2(2)
     1,ID(2),ISG(2),II(4),IP(4),IM(4),IX(4),IR(802),IA(238)
      EQUIVALENCE (IP(1),LP(1)),(IM(1),LM(1)),(LN(1),N(1),RHO(1)),
     1(IX(1),LX(1)),(N120,RI(1)),(N58,RI(2))
      DATA (Z(I),I=1,15)/1.,1.25,1.5,1.75,2.,2.5,3.,3.5,4.,4.5,5.,6.,
     17.,8.,9./
      DATA (ISP(I),I=1,2),(IX(I),I=1,4),(IP(I),I=1,4),(IM(I),I=1,4)/
     110,5,4*1HX,4*1H+,4*0/
      DATA ID(1),ID(2),ISG(1),ISG(2)/120,58,1,-1/
      DATA LN(1),II,LC(1)/1,80,50,0,1,1H0/
      IF(ISTOP)173,172,172
  172 J=IPNT+2
      IF(J.GT.M1)GO TO 173
      IPNT=J
      AR(J-1)=V1
      AR(J)=V2
      DO 10 K=1,4
   10 IM(K) = NC
      J=J/2
      LR(J)=LM(2)
  173 IF(NC.LT.NCMAX)RETURN
      IF(ISTOP.EQ.0)RETURN
      DO 171 I=1,2
      RI(I)=ID(I)
      SG(I)=ISG(I)
      IF(II(I).NE.0)RI(I)=II(I)
      IF(II(I+2).EQ.1)SG(I)=-SG(I)
  171 ST(I)=(RI(I)+1-SG(I)*(RI(I)-1))/2
      IF(RI(1).GT.120)RI(1)=120
      IF(RI(2).GT.238)RI(2)=238
      N20 = N120/6
      N120 = 6* N20
      DO 3 I=1,2
      AMIN=AR(I)
      AMAX=AR(I)
      DO 7 J=1,IPNT,2
      AA1=AR(J)
      IF(AMIN.GT.AA1)AMIN=AA1
    7 IF(AMAX.LT.AA1)AMAX=AA1
      IF (ABS(AMIN) .LT. .000001)  AMIN = 0.0
      IF (ABS(AMAX) .LT. .000001)  AMAX = 0.0
      R=AMAX-AMIN
      IF (ABS(R).LT.1.E-9.AND.ABS(AMAX).LT.1.E-9) R=1.0
      IF (ABS(R).LT.1.E-9.AND.AMAX.LT.0.)         R=-AMAX
      IF (ABS(R).LT.1.E-9.AND.AMAX.GT.0.)         R= AMAX
      DO 22 J=1,15
      B=ALOG10(R/(RI(I)-2)/Z(J))
      M=B
      IF(B.LT.0)M=M-1
      C=Z(J)*10.**(M+1)
      B=AMIN/Z(J)/10.**(M+1)
      I1=B
```

```
      IF(B.LT.0)I1=I1-1
      IF((RI(I)-2+I1)*C-AMIN)18,19,19
   18 C=10.*C
   19 IF(J.EQ.1)SMIN=C
   22 IF(C.LT.SMIN)SMIN=C
      SF1(I)=(1./SMIN)*SG(I)
      B=AMIN/SMIN
      M=B
      IF(B.LT.0)M=M-1
      SF2(I)=ST(I)-M*SG(I)
      RHO(I)=SF2(I)+0.5
      M=SF2(I)
      DO 25 J=1,10
      IF(M-J-((M-J)/ISP(I))*ISP(I))25,3,25
   25 CONTINUE
    3 IOFF(I)=J
      DO 101 I=1,N58
  101 IA(I)=0
       DO 102 J=1,IPNT,2
      IR(J)=SF1(1)*AR(J)+RHO(1)
      IT=SF1(2)*AR(J+1)+RHO(2)
      IF(J.NE.1)GO TO 109
      IR(IPNT+2)=2
      IR(J+1)=0
  108 I3=IT
      GO TO 102
  109 IF(IT-I3)104,105,105
  104 IR(J+1)=IR(IPNT+2)
      IR(IPNT+2)=J+1
      GO TO 108
  105 I=IT+1
  106 I=I-1
      I1=IA(I)
      IF(I1)106,106,107
  107 IR(J+1)=IR(I1)
      IR(I1)=J+1
  102 IA(IT)=J+1
      LAST=IPNT+2
      JJ=IOFF(2)
      LZH=SF2(1)
      LZV=SF2(2)
      DO 100 I=1,N58
      DO 40 J=1,N120
   40 N(J)= 1H
      IF((I-1)*(I-N58))140,151,140
  151 DO 141 J=1,N120
  141 N(J) = IX(1)
  140 LN(1)=LX(1)
      LN(N120)=LX(1)
      IF(LZH.LE.0.OR.LZH.GT.N120)GO TO 131
      LN(LZH)=LP(4)
  131 NB=1
      IF(I.NE.JJ)GO TO 35
      NB=2
      JJ=JJ+5
      I3=IOFF(1)
      DO 32 J=I3,N120,10
   32 LN(J)=LP(4)
```

```
      IF(I.NE.LZV)GO TO 35
      DO 135 J=1,N120
  135 N(J) = IP(1)
   35 I3=IA(I)
      IF(I3.EQ.0)GO TO 121
  120 LAST=IR(LAST)
      I2=IR(LAST-1)
      I1=LAST/2
      LM(2)=LR(I1)
      IMM = IM(2)
      LN(I2)=LC(IMM)
      IF(LAST.NE.I3)GO TO 120
  121 GO TO (38,41),NB
   38 WRITE(6,39)(N(J),J=1,N120)
   39 FORMAT(1IH           ,120A1)
      GO TO 100
   41 AA1=I
      VALUE=(AA1-SF2(2))/SF1(2)
      WRITE(6,42)VALUE,(N(J),J=1,N120)
   42 FORMAT(1H ,1PE10.3,120A1)
  100 CONTINUE
      I3=IOFF(1)
      J=0
      DO 49 I=I3,N120,10
      J=J+1
      AA=I
   49 RHO(J)=(AA-SF2(1))/SF1(1)
      IF(IOFF(1)-5)62,62,63
   62 WRITE(6,50)(RHO(I),I=1,J'
   50 FORMAT(10X,12(1PE10.3))
      RETURN
   63 IF(J.GE.12)J=11
      WRITE(6,64)(RHO(I),I=1,J)
   64 FORMAT(16X,11(1PE10.3))
      RETURN
      END
```

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