
of UNSTEADY SEPARATED FLOW $\infty$
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> UNITED STATES AIR FORCE ACADEMY AUGUST $10-11,1983$


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TITLE:
Supermaneuverability.
Wing Rock Flow Phenomena.
Potential Applications of Forced Unsteady Flows.
Unsteady Stall Penetration of an Oscillating Swept Wing.
Simultancous Flow Visualization and Unsteady Lift Measurement on an Oscillating Lifting Surface.
A Visual Study of a Delta Wing in Steady and Unsteady Motion.
Comparative Visualization of Accelerating Flow around Various Bodies, Starting from Rest.
Prediction of Dynamic Stall Characteristics Using Advanced Nonlinear Panel Methods.
Numerical Solution of the Navier-Stokes Equations for Unsteady Separated Flows.
Unsteady Aerodynamic Loading of an Airfoil due to Vortices Released Intermittently from Its Upper Surface.
A Navier-Stokes Calculation of the Airfoil Dynamic Stall Process.
Some Structural Features of Unsteady Separating Turbulent Shear Flows.
Can the Singlarity Be Removed in Time-Dependent Flows?
On the Shedding of vorticity at Separation.
Unsteady Separated Fluws. Forced and common Verticity about Oscillating Airfoils.
Unsteady Separated Flows. Generation and UBe by Insects.
Theoretical Study of Non-linear linsteady Actodynamics of a Non-Rigid Lifting Body.


## COMPONENT PART NOTICE (CON't)

## AD\#\#:

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## TILLE:

Theoretical Investigation of Dynamic Stall Using a Momentum Integral Method.
Preliminary Results from the Unsteady Airfoil Model USTAR2. Experiments on Controlled, Unsteady, Separated Turbulent Boundary Layers. Genesis of Unsteady Separation. Flow Separation Induced by Periodic Aerodynamic Interference. Leading Edge Separation Criterion for an Oscillating Airfoil. Natural Unsteadiness of a Separation Bubble behind a Backward-Facing Step.
AFOSR/FJSRL/L. COLORADO
WORKSHOP ON L'NSTEADY SEPARATED FLOWS

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WORNSHOP ON L'.NSTEADY SEPARATED FLUW'S

- PROCRAM -
hednesday, 10 dugust 1983
8:00 AM Welcome
$\quad$ Introductory Remarks

| 8:15 AM | "Supermaneuverability," h' Herbet (Invited Presentation) |
| :--- | :--- |
| $9: 15$ | "Wing Rock Flow Phenomena," L. Eriesson |
| $9: 45$ | Coffee Break |
| $10: 00$ | "Aerodynamic Agility Resulting from Energetic Separated Flows," | D. $\therefore$. Kennedy

10:30 "Potential Applications of Forced lnsteady Flows," fi. Viets, G. M. Palmer, R. J. Bechke

| 11:00 | "Ensteady stall Penetration of an uscillating Swept "ing," <br> F. Carta |
| :---: | :---: |
| 11:30 | lunch ( SCOClub - buses leave at front of buildirg) |
| 1:00 PM | "Correlation of lift and Boundary-layer Activity on an Oscillating Lifting Surface," K. Bass, J. Johnsun, and J |

1:30

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SESSLUS:
$4: 45$
$5: 15$ Oscillating Lifting Surface," K. Bass, J. Johnsun, and J. Unruh
"A lisual Study of a Delta \%ing; in Steady and !nsteduy sotion,"

"Comparative Visualization of Acceleratine Flows around Various Bodies, Starting from Rest," P. fremath, M. Balaes, adad in. Bank

Cortee Sreak

"Prediction wi byamic Stall characteristices lising iownacd

 Separated tlums," in. !. :!.s :ney

 mad (., -s. Chisu
 Shsier-Stches Equat iuns," S. A. Shatroth

 shear Fluws." a. Sizpsor



| $5:+5$ | "On the Sheding oi Yorticity at separation," D. Teliunis D. Marhioulakis, and M. S. Camer |
| :---: | :---: |
| $0: 15$ | Return to \#otels (buses leave iront of building) |
| Thursday, 11 dugust 1983 |  |
| $3: 30 \mathrm{~A}$ | "Lnstedady separated flow: Forced and Comon Vorticity about Uscillating Airfoils," M. Robinson and M. W. Luttges |
| $0: 00$ | "Passive and Active Device-Controlled linsteady Separated Flowitelds," $\because$. Nugib, D. Noga, and $i^{\prime}$. Reisenthel |
| $4: 30$ | "Unsteady separated flows: (ieneration and lise by Insects," <br> Y. ㅇ. luttjes, C. Sumps, M. kliss, and M . Robinsun |
| 10:100 | Cutite 3resin |
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| . 000 |  <br>  , b! : ! Cornc |
| .40 | Le:口 りiscussion <br>  |

## Abstract:

Supermaneuverability is defined as the capability of a righter aircraft to execute tactical maneuvers with controlled side slipping and at angles or attack beyond aaximum $11 f t$. This paper deals particularlywith post stall maneuverability at zero side slipping since this element of supermaneuverability is relatively unknown. The analysis is based on optimum control calcuiation or simplifled maneuver eleaents and on extensive manoed and computerized close alr combat simulation This arialysis explains the tactical adrantage observed during combat simulations and leals to the derfaltion of a typlcal maneuver duty cycle which $s$ consistant with conventicnal alr combat manouvers with all aspect veapons. Reference is made to earlier studies about meceuvers with thrust vectoring and thrust reversal. Finaliy, requiremerts are given for the necessary leve: of thrust-to-weight ratios and control power :nc:uding trdications of technica: solutions.

## i. Introduction

There are three dirrerent concepts (rig. I) or : eproving =aneuverability by weans or tiliting er.e: :e thrust:
a) Inrlight ehrust reversal

Thrust vectoring
Post Stal: maneuvering (PST)
a) has teen considered as a teceleration sevice (1) which permits to slow the alreraft Jcun rapidiy in:o a speed regime of better turn performance, however, it does not directiy contr:buie :o ganeuver performance in tarms of :=provire a charge of the direction ur rlighe.
(b) has been discussed \{2\} in conjunction wi:n concigurations, such as the Bhe Harrier. i?.ru=t vectorire offers an additional degree of reecon lo establish a saneuver state. Hovever, : i requires engine oxti momentum to poiat at the a:rcrar: e.e. over the veciorire range, which is ratr:y ifecompatio: with arterburfor :ryia:lat:on. a notsceable :eproverent of alscebal sapabl:ity has teen fesonstra:ed.
(e) $:=$ ine subject or inis paper. Ine erfe:ne :s rixec lo :he a!rerar: fuse:age and :tus exd: mosenture $=\mathrm{a}$ :xay: in :ine with the c.e. The on! y diferesce :o corventiona: atrcrati :y the requireacrit for arge anc:os of a:iack in excesy of maxizum itift ancie of a:tack. Post sta: fl:eti cond!tions have a:ready teen ceeoryirated, hewerer, tact:ca: p:E-aareuvers require a level or contro: :ab: : : : y far beyond that of contexporary alrcrar:. Aiso, : te :aci:cal advantace was unkfoun

There :s s:z: iar:iy iciveer :te concep:s ( 3 ) atit (c). For 20:n corsep:= ine Eatr. :t. zuzia:rec :urn iertoranncy :s :ea:! anc : :si:ed io ex:-rze riten: cord:i:cr:. T..o iac::ca:


Fig. 1 Schemes of using thrust def:ection for saneuver erhancement.
advartage is based on stiort ierm and highiy tnstantaneous mantrers anc the acti:eremen: of sman:l radif of turn (b) coulc even inc:ude (c) if the alrcrart were alicwec :o exceed the stal!
 adrantage of thrust veciorine ly marefinaland eay not justify ine overajl fesien penaliles.

The tact:cad atvan:age of FST-Eareuvering in ghort rarige atr comiat defends on the weafons used. There : a craree : r combat Eaneurar chararter!yidcy (3) caused by ines a: myfec:
 sivuat:ons rarticu:ar:y insianeatecuy qurn performance coetired wi:n sas: : ir: rasit has been found dec:sive. Ste rev veafors -


 =es.s or fur:ter dmproved enerey -


 ccetal effect:rene:ン。

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o 5 sec iverage duration of a PST mareuver
$0 \quad 108$ of total engagement time in PST
o lower i－level by aboat 118
o ：ower iverage maneuvering speed by about $0 . \hat{1}$ \＆$M$
and tactical advantages：
o quickrr intc firing position anneu；ering
－longer a firing position maintaining
o more first shoot ojportunities
－less counter hits
－easier and quicker to switch alternate targets
o dictating tactics throughout entire speed regime
－Eure missiles deployable
As an overa：i resu：t it was round：
－exanarga ratio in dual combat with equa：neapons against an opporent of equa：ccnventional maneuver Ferformance is about ？：？
－excharge ratio ir muitipie comtat remains io te iargeiy dependert on the number：ratio，however，can be sienificantiy imprcved．For exampie $\therefore$ ：［4？is single PST－capable fighter ＋ay ：：e to newtraiize two sonven：：cna：cpponents．Computer ：imuiatiory［7］involving iarger numetr：of opponents are showirg an ：ncruase of the re：ative adyartage of ps：－maneuvering（：IE．2）．

A ：aree acourt of tactical data has beer Edinezed ir．more thar． 3000 simulated engagements c：z fisferer．z？nned combat yimulators rlown by is operationa：pi：ois cf inree airforese． s：i：i， 1 i remntred somewtat dirricust to frecise：y understand the advantage．Sowetimes tre adiar．tage is attributed to ruselage pointing ove：：the ：arger s－rarze．ictualiy，oriy a re：a：iveiy sma：i number of gun shots have been
 ：Ir：re ：imitalion to $1=30^{\circ}$ dit no：iead io any＝ienificani degraiation of tactical success ？

Eto eatority of rifisg opportunities occurms

 ：ご：apasi：：：y，therefore，must de inierpreted a a mar．euverif．g scheme rather chan a simbaiar．s ter：ce．
：n Eenerai，ithe fay－crf is based on i trado 0：：oss of ererey versuy posic：ona：anj tiee a！vantage．$A$ profer fet－maneuver freceeding areag－on ongagerant piovises a cecisive liee atvantage at acmentary expense of enersy．in
 ie acmicramiot iy a ro：i－rate acvaniage，
 ＂：ét rar．c ：urn to a ：e：hard iurn deazrst an a：©errs：e daree：a ccrven：ionad aircract vou：t Aave ：o uritcac，ro：：arit re－loac．witr fre
 ye：cc：iy vecict at corisiar：－very nien－ar．fie


．．．urned simu：ations islicing maneuvers）．
is a resuit，the alreraft with PSr－capability was able to jictate the tactical course of the ergagement．Miss：：e capabijity aid not allow the conventional opponent（s）to disengage． Overail speed and ；oadractor was observed to be ：ower than in conventional air combat．In particular，there was iess time at limit g－ ：cadings．

## 3．Manepyer anadisis

For any flighi condition iasrcraft atitude， velocity vector，fower setting，aititude）ite manouver staie（！orgitudinal and latera： acceieration）is a Cuncifnn of the sum of a：： rorces，aerrdynacic rorcez，engine in：et and ex：：eceericum．The anajys：sha：le inalted io
 F：8．（3）and（：8．id！are rosu：ty or mass po：rit ca．zu：at：onz for a pariscular adrerali JraE fo：ar ai a pariscu：ar a：t：ince ard écr maxis：ue eng：re power．F：E．（3）fortaine 0 no：izona： rikht cons：itcns．

Fig．（4）descrites two cond：ticns of a vortica． Eaneuver．tne rifenest and ：cwest po：nt with ine ve：oc：iy vecior po：risme nor：zonia：iy．Trere：s a $E E F=$ itne disi：rgutstr：r．g Deiveer．acceieraied ars de．e：eraiect watelver ataies a：id itere are
 coectration or turr raies ard＝acs ruever corresponde to a iari：csiar tastus or iurn． $\because$ ت．（z）ant＊：aresfec：a：こazes ot mary＝uct： ：－aeraes for an inesn：ie nueter of ri：ent －ons：：ons．$\lambda$ a a：rerar：earcuver wob：d be a seguetice or＝arebver $=: a t e s$ at con：：fuousiy
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- that a turn rate advantage beyond maximum lift angie of attack as compared to "corner speed" can be expected at very : ow speeds
- that a significant turn radius advantage can be achieved at angles of attack vell beyond maximum ifft
- that the hygh angle or at ack part cf an advantageous PST-maneuver wou:d tave to penetrate a relativedy : ow speed regime ( $0.05>\mathrm{M}>0.2$ )
- Lhat no significant advantage could bo exjected frow PST-maneuvers 1 Imited to mover ste angles of attack (: $550^{\circ}$ ).

Performance of a PST-maneuver is an optizum control problemwitt angie of at:ack and tank argles as independz..t variabies (inrust vas Cound ic be maximut etroughicit the maneuver for best resulis). Pay-off functica in real cocdat woudd be an earlier firing opporiuniliy and io屯eny counterfire. In order to set betier probiem irangparency, Enimun tiee saneuvers uith defined staridng and end conditions have Deen calculated in [9!. There was ar advantage of exceed!ng maxdmum: ift angie of atiack and Instantaneols genetration in:0 ine PST regiec whenever certajn geowetrdca! consiratnts tad io se zatisf:od.

Che of the mosi afp:icabie anajyitcad zaneuvers. Sor exami - o, is inat of $\geqslant 180^{\circ}$ charge of headine



the point of depuriture at indtial speed and : ititude (fig. (5)). The same aircraft, hovaver bemi:ed : o maxtmum diftant: of attack woudd roidov a differer: oftimum riight path and need sore isee. Fig. (S) shous the assuciated itme n:stories for angle of ateack and spect. Typ:caldy, the PE:-Eaneuver dy characterdzed by a rap:d pi:ch uf io PET-iriniderces witharast
recovery into the conventional flight regime with only a rew seconds of persistence at PSTconditions.

Fig. (l) is a time history of the same analytical minimum iime maneuver as shoan in rig. (5) an (6), plotted in terms of rate of velocity vector turn is. speed. It shous the type of PST-maneuver cycle which also has been round in actual combat maneuvers. In comparison with the $a-1$ imited optimum maneuver the PSTmareuver reatures a lower average rate of turn ( $18^{\circ} / s$ against $22^{\circ} / s$ ) but less total cnange of headings and a significantly smadier average radius of turn. Fig. (3) shows the energy management of both the conventiona: and the PST maneuver.
of course, there is a large variety of maneuver types for which PST-capability can provide an advantage. Mass point trajectory 31 culation are useful but 1 imited. Actual PSTaneuver performance will depend upon availa: e sontrol power and dynamic response. Fig. 9 is a suevary of simulator trials with actual serodynamic and mass data or a righter aircrart sesign, for which a suitable control syster was designed ancorporating low spaed contro: entancement by mears of $\pm$ !nc $\quad$ iffiux defiection ir pitch and yaw.

For certain refeatable maneuver tyfes and Cor a E:ven ! dmitation in argie of aitack (abscissa in fig. 9) fisote were trying io

 limited fighter with ail aspect weapons. Result of computer simuiations.
filght regime. During the PST-maneuver the weapon was even pointing away from the target.

Fig. (13) is another representation of the






Angle of Allack :':






Fig． 13 Typical air combat engagement．Re－ lative positions of opponents and their aspects and firing opportunities Iime intervals in sec．Result of computer simulations．
－ame ergagement．It shows how the pilots in aircrart 2 （lert hand side of rig．（13））and in aircrart 1 （right hand side of rig．（13））would viow their opponents as they look outside their cockpit windous．Aircrart symbols in fig．（13） represent the actual aspect relative to the opponent at the indinated time intervals．The rigure，therefore，also shows convergence into a riring position and the ime in riring position for a given weapon cone．

Aircraft 2 performs the PST－section of the maneuver（second 8 －15）far outside aircraft 1 weapon range in its backward look angle sector and is getting rirst riring opportunity in tise 23．second being in a sare beam position relative to the target．There is active riring time without counterfire for several seconds unt！d the opponent would gass each nther almost head on．The pllot of aircrart 2 is never ：ooking into his opponent firing cone until his cirst missile hits the target．He recovered the anergy lost during the PST－maneuver right arter ithe attack against aircrare 1 （fig．14）． Average speed of the PST－alrcrart observed throughout aany simulated dual and ajdifiole combat encagements 1 s ondy 5 － 108 lower as compared to the conventional opponents． Obviously，the aircrart is vulnerable during the high angle of attack phase or a PST－maneuver， however，the pliot has always the option to regirain dr the momentary loss of speed constitutes a tactical disadvantage（provided conventional performance does not surfer too much froe its incorporation into the overal：
（esign）．
PST－maneuvers are consistent with the general dynamic characteristics of air combat with all aspect weapons［10］．There is the same Energy




ごニーー4：－05：．
overall left hand maneuver cycle of the turn rate vs. speed time history with the addition of a short term right hand cycle excursion into the PST-regime.

## 4. PST regime and requicements

Limits are given by controleability, engine power and structural constraints (fig. 15). Actual usaga, however, is dictated by tactical advantagas. For high performance aircraft (thrust-to-weight $>1.0$ ) there is a 10 w speed regime of possible sustained maneuver. In combat, however: these are no sustained maneuvers (constant maneuver states); excess thrust is always used to re-accelerate or to gain altitude. As a general rule PST-capability requires

- surficieat control power in pitch, roll and yaw at mach numbers as low as 0.1 and incidence up to $70^{\circ}$.
- Aigh angle of attack compatibility up to $70^{\circ}$ at machnumbers as high as 0.6 ( $4000 \mathrm{maltitude)} \mathrm{for} \mathrm{example} \mathrm{with}$ regard to aircraft stability or air intake rlow.
- Thrust/weight ration > 1.0 rig. (16) shows the time advantage of the optimized PST-maneuver (fig. 5) at different thrust-to-weight ratios. With decreasing engine power the observed maximue values or angle to attack are becoming smaller. At thrust-to-weight rations of less than 0.6 there is no tactical advantage in exceeding maximum lift even if the capability is avaiiable.
- Surficient control pover, particularly in pitch and yaw.


Fig. is the PST f:Asht regime (atrcraf: deperdent:


Fig. 16 Thrust dependance of the iime advantage in a minimum time mareuver with limited PST capability. Results of trajectory optimieations.

PST-maneuvers are characterized by high pitch rates and rotation in yaw and roll at the same time. Coordinated flight with zero sidesilp requires rotation around the velocity vector. Since the pilot does not recognize the velocity vector the control system has to be sechanizad accordingly. A lateral stick inpu: would have to produce more yaw and less roll at increasing angle of attack. This caused sowe conrusion with piluts during simulated combat, because a pilot tends to use body axis as reference.

As a result of manned combat simuiations a requireaent for velocity vector roli acceleralion was developed. It is plotted in rig. (17) as a runction of anglo of attack for a speed of $M=0.2$ at 6000 altitude. A eranslation in body axis wotion shows that for a conventional aircraft the demand for roll could mareinally be satisfied with aerodynamic controls, however, for pitch and yav a control augmentation vould be required. For example, a $10^{8}$ conical deflection of the jet exhaust would surfice if the nozzle actuation meets certain dynami= requirements. Such nozzle would have to be integrated in the aircraft control system. It was round that nozzie control is an enhancement of handing characteristics and thus combat erfectiveness even in the conventional rlight regime heyond $10^{\circ}$ angie of attack.

## 5. Sumary

The capability of exceeding maximus ilrt angle of attack - Post Stall (PST) maneuvering can iaprove ruture ciose air coabat erfectiveness to degree unachievabie by conventional perforaance. The tactical advantage is attributed to combination of fairiy high turn rates and sall turn radid at PST rlight conditions in all aspect veapon environment. PST maneuvers are short pertod and nighiy instantareous and constitute a irade of short :erz : oss of energy against posit:onad

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# WING ROCK FLOW PHENOMENA 

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## ABSTRACT

Flow mechanisms that can generate wing-rock type oscillations are described. It is shown that the slender wing roci phenomenon, the limit cycle oscillation in roll observed for very slender delta wings, is caused by asymmetric leading edge vortices and that vortex breakdown can never be the cause of it as it has a damping efiect. For that reason slender wing rock is only realized for delta wings with more than 74 leading edge sweep for which asymmetric vortex shedding occurs before vortex breakdown. For straight or moderately swept wings the flow mechanism causing wing rock is two-dimensional in nature, closely felated to the dynamic stall phenomenon. Pointed forebodies provide a ehird flow mechanism, asymaetric vortex shedding sensitive to body motion, which can generate a rocking motion of a slender vehicle unless it is completely axisymmetric.

## INTRODUCTICN

The steadily increasing demands on performance expose present day aerospace vehicles co unsteady flow flelds which generate highily nonlinear aerodynamics that exhioit significant coupling between longitudinal and lateral degrees of freedomp. The conplex vehicle dynamics are caused by separatei flow effec:s of various types, which have laryely eluded theoretical descrifiion. Consequently, the designer is to a darge extent dependent upon ex.sting experiapntal capabilities for dynamic testing, where dynamic support interferencej adds complexity to the already complicated separated flou characteristics. Thus, it can te rather
difficult to obtain a true description of non-linear pitch-yaw-roll coupling phenomena such as wing rock and nose slice. In the present paper existing experimental results are examined to obtain a description of the underlying Eluid mechanics.

## DISCUSSION



It can be seen that the main effect is the induced sidesilip. Thus, one can compare the $C_{i}(i)$-characteristacs for a siender delta wing ${ }^{9}$ (fig. 2a) with the $\mathrm{C}_{\mathrm{m}}(\mathrm{a})$-characteriselcs for cylinder-tlare body (F19. 2b). In both cases the characteristics change in a discontinuous isasion, and are likely to be associated with nysteresis, coth phenomena sypical for the effects of separated flow. the serodyramac stifiness. $C_{i}$. in Fig. 2a and C.nid in Fig. 2t. increases draratacally wher the discontinuty is encountered. (with or
without associated hysteresis). Both discontinuous changes of the aerodynamic characteristics are associated with convective time lag effects ${ }^{\text {i.S. } 10 \text {. This }}$ causes the statically stabilizing effects to become dynamically destabilizing, as is illustrated in Fig. 3 for the cylirder-flare body. The separated shear layer impacting on the flare at time $t$, when $u(t)=0$, was generated by the nose at a time increment -t earlier, when the angle of attack was $\alpha(t-\lambda t)>0$. Thus, a residual tlare force exists at $\alpha(t)=0$, which drives the motion, and consequently, is undamping.

Thus, the statically stabi.izing separation-induced flare force is dynamicably destabilizing. It is shown in Refs. 7 and 8 how the unsteady aerodynamics measured for large amplitude oscillations $(\dot{\sim})$ around - = 0 can ve predicted from static aerodynamic characteristics when accounting for the convective time lag effect (Fig. 1).


Two types ot separation-induced discontinusties occur for the slender deita wing. Cne is caused ky the breakdown of the leading edge vortices. It is the three-dimensional equivaleat to airfoil stal:. For very slender Jetea wings another discontinuous change of the derodynamics can occur before vortex breakdoun due to asymmetric leading edge vortices. The asymmetric rortex phenomenon has reen studied extensively in che case of slender tocies of revolutionll it and has ceen goserved also on slender delta vings ${ }^{\text {fi }}$ (Fig. 6). Vortex asymetry occurs before vortex breakdown oniy for very siender delta wifgs. \#A: $16^{\circ}$ according to experiments (Fig. 7). In orcer to use fig. 7 to explore the effects $c$ sidesiff. , and roll angle. $\because$ an erfective afex hali-angle is formulated as toldows for srall angles. $A$ 15\% - $15 \%$.

$$
\begin{aligned}
& \bar{y}_{\boldsymbol{A}} \text { in } 30 \\
& \text { 10A }=\text { inna s:no } 20 \\
& j \theta_{A}=3 \operatorname{cosa} 3 c
\end{aligned}
$$

The $C$ evaluated from stati measurements for a $70^{\prime}$ delta wing shows the effects of both vortex burst and vortex asymmetry (Fig. 8). The first break in the $C_{2,3}$-characteristics, at $a-20^{\circ}$, is caused by vortex burst on the windward wing half (Pig. 9). Eqs. (3a) and (3b) give $\vec{A} A=24.3^{\circ}$ for $\alpha=20^{\circ}$, $3=4^{\circ}$ (C) evaluated for $\beta= \pm 4^{\circ}$ ). The loss of lift due to vortex burst on the rigkt wing-half causes the observed decrease of the derivative magnitude $\mid C_{2 i-1}$ (Fig. 9). At $\alpha-35^{\circ}$ one obtains ${ }^{2} A=15.1^{\circ}$ for $j=-4^{\prime}$, giving vortex lift-off on the leeward wing half (Fig. 9). Th:s causes a loss of lift which results in an increase of the magnitude of the rolling moment derivative, $\mid C i j l$, in agreement with the experimental cesults (Fig. 8). Even more erratic $C_{l_{\sim}}$-characteristics have been measured on an $86.5^{\circ}$ swept delta wing ${ }^{9}$ (Fig. 10a). The expected behavior, following that for the $82.5^{\circ}$ swept delta wing, ( ${ }^{\mathrm{H} A}=7.5^{\circ}$ ) 13 indicated by a solid line. The zig-zag behavior of the dotted line connecting the experimental resuits can be understood if one studies the Cl(.)-characteristics (Fig. lOb). It appears that Detween $i=0$ and $3=5^{\circ}$ the vortex asymetry switched several times, with vortex lift-off alternating between the two wing halves. A static hysteresis of $j= \pm 5^{\circ}$ is indiccted.

Experimental results ${ }^{6}$ demonstrate that wing rock starts before vortex breakdown (fig. 11), and that wing rock is associated with a loss of the time-average lift 6.15 (Figs. 11 and 12 ). It is, of sourfe, to te expected that the "lift-off" of one of the leading edge vortices (fig. 6) will cause loss of dift. Thus, wing rock is caused by the vortex-asymmetry and not be the vortex breakdown. Fiqures 13 and 14 dllustrate the fluid mecranical reascns for chis. At an $\dot{-i} A$ vortex asymmetry occurs, the wing half with the lifted-off vortex loses lift and "dips down", rotating around the roll axis (fig. 13). As desult or the ancreasing robl angle the effective apex angle "it is increased. Eqs. (3a) and (3b!, and the jnrtex attaches açain. This produces a restoring roliling moment, the positive aerodynamic spring needed tor the sigid tody oscillation in roll (sig. 1). Due to the convective time lag effect discussed earlier tre wing is dynaricaliy unstatle in roll until the amplitude has reached the diadt cycle ragnitude, at which the damping on both sides of the discontinuity suffices so talance the undainping induced sy it, as is lllustrated by the cylinder-flare results lu Fiq. 4. According co Fiq. is wing rock should start occuring for an
 which is in exceident agreement with experdmental resules".

Thus, the discontinuity introduced by the vortex asymmetry has all the characteristics needed for the limit cycle oscillation in roll. Figure 14 demonstrates that vortex breakdown is lacking these characteristics. If for some reason, due to external disturbances for example, the vortex burst becomes asymmetric, as is sketched in Fig. 14, the resulting net loss of lift on one wing half will cause it to "dip down". This increases 0 and thereby ${ }^{\prime}$, , Eqs. (3a) and (3b), causing this wing half to penetrate further into the vortex burst region, and no switch to a restoring moment occurs. The opposite wing half gets out of the vortex burst region, generating increased lift that adds to the statically destabilizing rolling moment. Thus, no restoring moment, no positive aerodynamic spring, is generated and no rigid body o-oscillation is possible. If che positive spring is provided by the structure, as an the case of elastic vehicle dynamics, the dynamic effect of the vortex breakdown would be dynamically stabilizing, damping, as the vorsex burst is also associated with time lag effects. Thus, vortex breakdown has aerodynamic characteristics completely opposite to those needed to cause slender wing rock.

Although vortex burst cannot cause wing rock, it is involved in the cases of wing rock observed for very high. angles of attack ${ }^{\text {b }} \mathrm{l}$, : 35 (Fig. 9). Figure 15 illustrates the fluid rectanics for $42^{\circ}$ and $\quad A=10^{\circ}$, tor wrich a bimit cycle amplitude of

- 32 has been measured". At $v=0$,
$1=A=10$, asymmetric vortex burst exista. The wing hal: with the largest 1:15t loss dips sown, increasing s and i. when "A 0 for $0 .-11^{\prime}$ and the vortex-induced lift on the cosite dramaticaliy or lost completely, a testoring robling moment is generated. gecause of the integrated darping eftects discussed earlier in connection vith Eig. 13 the ampilitude $\therefore$ has to exceed - -11 substantially before net zero damping is reached and the $1: 1$ int cycle oscillation called ving rock is established.
whereas Nquyen et al' measured no vang rock for their 80 delta wing below - $27^{\circ}$ Levin and katz measured wing rock asteady at $\quad 20^{\circ}$ tot the same ieading edqe sweep (fig. 16i. This eariy ving rock occurance is frotatily. as che authors suggest e caused ty the centertocy used on their model isee :nset 10 Fig. 16!. The smalle: $1: \pi \mathrm{m}$ cycle arflitude. $\cdots$ : : $^{\circ}$ (F:g. 1B) confared to . v 34. which was tie :art: cycie arglitude reasured by ima
 to the iesser $\because 0$ riex-irduced duads ex:s:ing at the love: angie of attack'.

Nouyen et al ${ }^{6}$ showed that the oscillations in roll damped down to zero amplitude if the $80^{\circ}$ delta wing ( it = $10^{\circ}$ ) was yawed to $;=10^{\circ}$ at $6=27^{\circ}$ (Eig. 17). This is, of course, to De expected as the windward wing half has $A>15{ }^{\circ}$. Eqs. (2), (3a) and ( 3 c ), leaving it outside of the boundary for asymmetric vortex shedding (Fig. 13), whereas the leeward wing half with i. ${ }^{\circ}<5^{\circ}$ remains inside the tagion for vortex-asymmetry. Thus, neither wing half crosses the boundary, and the wing-rock-inducing discontinuity is never encountered. Correspondingly, oscillations in yaw around $=0$ are darped for li: = 10 according to the experimental results ${ }^{6}$ (Fig. 18).

It was noted by the authors in Ret. 18 that the normal force measured during wing rock was below that measured in static tests. Thus, at $u=20^{\circ}$ the mean or time aver:ge normal force is $C_{V O R}=0.64$ for -ULIM - $14^{\circ}$ (Fiq. 16) whereas the static data showed $C_{\text {NOH }}$ to vary 5 rom CNOK 0.80 to CNOR 0.65 when $u$ increased from 0 to $15 \%$ As the static data show no rollifig moment at
$=0=0$ for $a<32^{\circ}$, it is covious that the vortices stayed symmetric in the static case, whereas in the dynamic test vortex-asymmetry must have been present to cause the wing rock. The likely reason for this anomaly is the lerge centerbody. Whereas a thin splitter plate of similar height has been found to trigge: early verex asymmetry also in static tests by forcing asymmetric stacnstion :low conditions on the topside ${ }^{0}$, the dateral extent of the center body in Ref. is appacently allowed symmetric vortex formation in the static test. As a matrer of fact the ving rock motior was not self-induced at $=20^{\circ}$ but had to te started at a righer angle of aftack, in which case it vould fersist when the angle of attack was reduced to " $20^{\circ}$. Even at $u=35^{\circ}$ the vortices remained symmetric for 15 seconds (fig. 12). This cannot, toweve: explain the big difference observed at " $30^{\circ}$, viere in the dynamic test with CLill = $30^{\circ}$ CJOR varied betweer. $C_{\text {NOR }}=0.86$ and $C$ vorn 0.5 whereas the static test gave $C_{\text {vir }}=1.2 \mathrm{~g}$ and $C_{\text {NOR }}=0.8$ for $s=0$ and $=30^{\circ}$ sespectively. in this case it is the early vortey. burst otserved in the dynamic test that is the likely reason for the addíional ilft loss.
in :egard to the usage of the resuits in F:g. P . vhach are ctea:ned Ec: symere:c fiow cordit:ons.
 conditions a:scussed in figa. v, 13. 14. and 15, the :0!:owing needs :o te sald. wheress vu:tex zurst is :elatively wraftected ty the presence or atsence o: ine vortex an ine opposite unag-tiait. the asymetis: vortex sheddinq is very
dependent upon the "crowding" of the companion vortex. It is the strength of the vortex, represented by $\alpha$ in Fig. 7 , and the closeness of the opposite vortex, represented by "A in fig. 7, which together determine whether or not -1ift-off" of the vortex will occur. In a first approximation the effect of side slip on the "closeness parameter" can be neglected. That 15

$$
\begin{equation*}
\left(A_{A}\right)_{E F F}=\left[\left(A_{A}\right)_{L}+\left(A_{A}\right)_{R}\right] / 2 \tag{4}
\end{equation*}
$$

It is shown in Ref. 19 that the vortex strength and associated aerodynamic loads are determined by the parameter "/"A rather than by $u$ alone. Consequently, the indicated changes of IA in Fig. 13 should be substituted by cranges of $\left(\alpha /{ }^{\circ} A\right)_{\mathrm{EFF}}$. That is, the changes would occur in the vertical rather than in the horizontal plane. The conclusions would, hovever, be the same in regard to the effects of roll sante 0 .

In Ref. 21 simple analytic method : 3 presentes, which can predict the Hamt cycle amplitude for the wing rock oscillations measured by Nguyen et $1: 0^{\circ}$.

## WING ROCK OF NON-SLENDER WINGS

A completely different flow mechanism is the cause of wing rock of straight ox moderately swept wings. It is closely related to dynanic stall. The experimental resulta in Fig. it illustrate that plunging oscillations of an airfoil can be undaeped in the gtall region. It is shown in Ref. 23 that this will be the case if the stall is associated with a significant loss of lift. It is the "leading-edge jet". the soving wallwall jet analogy diecuseed in Ref. 24 (Fig. 15), that protuces the negative aerodynamic damping in plunge 22 shown in Fig. 14.
is an alrcraft it perturbed when thying close to etall, the down-rolling Eing halt will experience the upstreas soving wall effect lllustrated for the down-stroke :n Fig. 15. As it prosotes separation, the loss of lift is ancreased beyond the static lift logs. wore the higher the plunging rate ! $\dot{z}$ 1a. This generates a rolling moment that drives the motion. l.e.. it is undamping. the delayed giall due to domaresean soving wall effecis on the opposite wing (upatroke in Fig. 15) will add to the undasping rolling monent.

Thus. the induced eifecte of the tocsi pluriging velocity $z$ wi:i di:ve the wing in roil. what stops the wing rollisg sotion to protuce uing cock? Eq. (b) and Fig. it give the anser.

When the roll angle 0 has been increased enough to cause ${ }^{\text {EFF }}$ to decrease below $\alpha_{\text {STALL }}$ on the downgoing wing half, the flow will reattach to generate the lift needed to produce a restoring rolling moment; the aeridynamic apring needed for wing rock, as was described carlier. The flow reattachmenf ${ }_{5}$ is associated with time lag effects ${ }^{25}$, creating negative a\&rodynanic damping to be added to the "leading edge jet" effect discussed earlier. Thus, the condition for wing rock exists also for a conventional wing.

Associated with wing-rock is the oscillation in yaw called nose slice. The down-rolling wing half will move back due to the stall-induced drag increase. The increased leading edge sweep angle will promote flow reattachment, thus reinforcing the aerodynamic spring of the wing rock. For a straight or moderately swept wing, the side slip due to nose slice will dominate over the roll-induced side slip, ${ }^{i}$ EFF in Eq. (2), as $\alpha$ is small. The opposite is true for a slender delta wing where the drag increase due to nose slice in small and ${ }^{3}$ EFF is the dominant ide slip component $(\alpha$ is large in Eq. (2)). Thus, one expects the coupling between wing rock and nose slice to be weak for highly swept wings and strong for straight or moderately swept winge.

## BODY ROCX

That a pointed forebody can provide a third mechanise for wing rock. or body-rock was demonstrated recently ${ }^{26}$. Wings and cail surfaces could be removed tros the model of an advanced aircraft without stopping the rocking sotion. Obviously, it must be the vortices shed from the pointed torebody that supplied the iriving mechanian for chis body rock motion. It has been established that the formation of asyametric body vortices can be dominated by the body motion 27 , and that the vortex that ia not liftod of: noves inbourd to remain very close to the surtace near the centerlino of the body26. 29 (Fig. lo). placing ine cockpit in the inset sketch of fig. 16 and considering ine data by fidiesio (Fig. 17), one stares to wee what this chird flow zechanian de.
it is shown an Ref. is that at a critica! keynatide numer negative ragnus lite o: large magnaluie will be generated at very majest rotapion rate on a circular cyidnder. it de deacribed in Rel. 27 how inas llow phenuzenon, which ls caused by soving wal! fisecen on boundary layer eransilion, ean explati the cesulia in Fig. 1 . Tha: 1s. :he if:eci:on is: even $\rightarrow$ very s:om fois:ion to:er=inex the ifrection ju
the vortex asymmetry．Based upon these results，one obtains the piciure sketched in Fig． 18 for the vortex－ induced effects on the cockpit．
 The experimentally observed bcdy rock： 6 was obtained on a model that oniy had the roll degree－of－freedom （DOF）．FOr an alrcraft in free flight the asymetry will generate the largest effect in the yaw DOF．That is．the motion will be nose－slice－dominated with relatively weak feedback from the roll DOF il．ustrated in fig．18．How－ ever，ins，vortex inteiaction with an af：fir ${ }^{\prime \prime}$ this third flow rechanism may become zuch more stgnificant in regary ： 0 ：is ：mpact on pitcti－yaw－rol： soupl：：：9．
$\therefore$ should be noted that the inst： ：iow sechan：sz ex：s：s orily ：n t：mb：ed sange $ر$ ：a and Re．$O$ ：course，the Er：ileal Reynolds nueber sange ay be reiaisuely wile，as che isamsilion ：nduced separarion ayymerery Euves comarse ithe nose itp un a porated oq：ve ur ce：se as the Reynolis number 13 1ncresses（F：9．19）．

## concics：ons

An analysis J：wing＝ock pheno＝ena tias s：own ehe ！o：：um：ng：

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& 0 \text { Sierter wisg tock :s catsel by } \\
& \text { asy=ret: } \text { :ent:rg elge } \\
& \text { vor::ces. }
\end{aligned}
$$

－Vortex breakdown has a damping effect on the roll oscillations and can never cause wing rock．

0 Thus，slender wing rock will only occur for delta wings with more than $74^{\circ}$ leading edge swet？．
－Wing rock of a conventional wing can occur at stall if the stall causes an abrupt lift loss．
－Eody rock can be generated on body alone by asymmetric vortex shedding from a slender nose if the body is not completely axisymmetric．

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3．Ericsson，L．E．＂Sepa：azed Fion E：tects on tite Static and Evoar：Statd：．ty ot g：int losed Cy：rde：F：ase Rodies＊ ：iasa Cg－759：9，Vecerte：：96s．
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Fia. 6 a-". Boundaries for Vortex Asummetry and iurtex Burst (Ref. 16)

itia. 9 Development of Wina Rock for an $80^{\circ}$ Del:a Ning (Ref. 18)


Fin. 7 Effect of Wing Rock on Lift (Ref. 6)


Fia. ? Wing Dock Caused by Asymetric Vortices.


Fia. io Effect of Vortex. Surst on Lateral Stabllity.




Fig. 11 Fime -istory of $\operatorname{Wing}$ Rock, $1=30^{\circ}$ and $u=20^{\circ}$ (Ref. 18)


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& 13 \text { Effec: of Sides!id on Roll }
\end{aligned}
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Fiq. 14 Airfoil Damping in Plunqina Oscillations (Ref. 22)


- Downstaone


Fin. 15 "Leading Edue Jet" Effect

Fiq. 16 Effect of Asymmetric Vortex Shedding
on Side force and iormal Force




Fin. 17 Effect of Spinning Nose Tib on VortexInduced Side Force (Ref. 30)

[^0]

Fiq. 18 Vortex-Induced Load on Aircraft
Fiq. 18 Vortex-induced Load on Aircraft
Cockpit
 Cockpit

# POTENTIAL APPLICATIONS OF FORCED LNSTEADY FLOWS* 

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## Abstract

Furced unsteady flows are examined from the puint of vicw of potential appilcation of new devices which are made possible by the improved u:derstanding of these tlows. In particular, iluws are exami:ned winich require tu external driver but wheilin their enterg irom the freestream धelocity. "ther iluws require no moving parts at al! to seaerate the unsteadaness.
incroduction
Öntedfy separitud thows mas appear is a consequesec ot the mution wi the surlace st the body , 1 ds d result of the unstcoainess in the flow over that sortide. In manv isses, when contronted by Su, it iln's, the wbiwtive is to eliminate or at least antrol the separation.

The urrenc rese.irin prosram inss taken a somewitht ii:icren: perspective. The sbjective of the
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flow visualization results of Figure 1) which have un important eifect on the flowfield. Aithough these nozzles can be employed to control boundary lavers, most of the effort has been applied to free jets. The jet is controlled by the iluidic feedback sivscem wrapped around the body of the jet nozzle in Figure 2. An alternate version employing a difierent fluidic ieedback has been found to be capable of producing an unsteady jet even if the phase of the jet flow is different from that of the surroundings.


Figure 1. Smoke flow visualization of a fluidically oscillating ict.
nozzle plenum feedeack loop




dnother embodiment of a fluidically controlled jet nozzle is shown in Figure 3 . In this case the dynamic orientation of the jet is determined by a pair of rotating valves located on either side of the nozzle exit. Since the jet is alwas inclined toward the closed valve, this nozzle has the advantage that the phase position tan be controlled in addition to the frequency, A disadvantage is that the control valves must be externally driven


Pisure 4. Schematic of the rotor vortes generatar.

Fie second aevice is tram shaped rutor shown
 : :ce: ream veloiity is greater than the tip speed if Ehe rotur, the appearance of the rotor tip will resul: in the production of a vortex behind the rutor. The flowiield thereby produced has been prive! and found to have application to controlliag
 It wement in roarwiord :aina: ramps (Ficure jb).





leads to an improved mixing rate between the tuel and uxidizer. The presence of the unsteady flow in the diffuser (Figure 50 ) has resulted in cases where the diffusiun is more elfective, especially at larger angles.


Figure jc. Schematic of the vortex generator applied to a dump combustor geometry.


Figure Si. Sibematis wi the vortex senerator appliud to di:iuser sewnetry.

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The Jesismembodied be the unsteadi li: : enser evpertaent (Fisure ju) (an be ddapted to the roter
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Figure 7．kotor jet flowfield seen in a moving coordinate system．

Another problem of current interest is the application of the rocos vortex ，senerator to reduce the Jrag vi a bluit body vehicle．An example is a van in which the objective is to reduce the drag but to affect the volume of the van in a minimal way．Thus，simply stream－ ！ining the van has severe limitations．The method proposed is to employ the rotor to enable the llow to turn through a larger angle and thus only require a small loss of van wolume due to eliminating the rear corner is shown in Figure 8．In this figure，the transturmation to a courdinate isame structure dominates the thow and should leat to I Jrap reduction since it substantially reduces the size of the wake behind the といるいに。





















through an analog to digital convertar．The probe positioning has been automated in one direction and is driven by the microcomputer．

The velocity of the air at any given point in the flowfield is continuously measured by the hot－ wire probe．Since the structure of the flowfield generated by the rotor of Figure 4 is coherent and synchronously varying with respect to the position of the rotor upstream，the velocity is sampled with respect to the position of the rotor．Thus a com－ plete＂time picture＂of the structure is obtained as it passes the probe．In addition，the coherency of the flow structures allows the signals from several successive rotations to be averaged to improve the signal to noise ratio and reduce the effects of turbulence on the measurements．The signals from the hot wire are processed and recorded by the microcomputer through an analog to digital converter．

The shaft of the rotur in Figure 4 has been fitted with a light chopper disk oi two tracks． One track has a single hole used to indicate the start of the rotior orbit and the second track has thirtr－six equally spaced lobes（ever： $10^{\circ}$ ）used to indicate the position of the rotor in its orbit． These two tracks are＂read＂by lisht emitting diodes （LED）plroto transistor inickups．The resulting signals are passed through an intertace which pro－ vides sicnal conditioning to prepare the sianals tor the mi rocomputer．

screw．The position accuracy of the system has been found to be within $\pm 1 / 2$ turn of the lead screw or about 0.020 inches．The bottom indicator has a reproductibility of better than 0.010 inches．

A computer program to record，process，and sture the daca from the hot－wire probe has been aritten and tested．The program takes 100 samples of eath of the two wires in the probe for each of the 36 positions of the rotur．This data is averaged and reflected through the calibration curves tor the hot wires to yield 36 two component velucities for two wires．The data is displayed on the computer console and stored as a disk file for further analysis．The procedure abce is then repeated for each prose position．In this manner， the complete time protile of the convected structure is shtained alons the probe path．The probe step size in the riaht direction is programable．

The remainder of this paper will concern itself with the fact that an external Iriver or power source is not needed to turn the rotor shown in Figure 4．This result improves the utility of the rotor and additional potential applications are discussed．

## The Self－Powered Rotur

The results and putential applicaticns of the rotur device cited in the sections concerned with past and current efforts were based on a rotor requiring an external source of power to turn and generate the vorticity．With proper design，how－ ever，the rotor motion can be self induced with the required power being drawn directly from the free－ strean velocity．Perhaps more interesting than the fact that the motion is self induced is the direction of rotation．As shewn in Figure i，the self induced direction of rotation is countercluck－ wise．Therefore，an increase in the rotational speed will also increase the strength of the resulting vortices．

The basic mechanism which drives the rotor in a direction which opposes the freestream velucity is shown sthematicallv in Figure tha．The relative pressure distribution is shown on the exposed portion of the rotur for the diven conif buration． Athoush there is a stamation pressure eftect near the upstream side wi the rotor，the remainder ot the upper surfiace is dominated a low pressure distributiun．Une misht sas that the rotor＂flies＂ in the treestream．From the distribution，it is chear that an integration vielding the mument thout the wis of the toror will result in a counter lucir wi い ：moment．


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The startup operation of the unpowered rotor device is as follows: Even if the rotor is weil balanced and relatively free to turn (i.e. low trictional resistance), at low freestream velocities the rotor remains in a statio urientation. As the freestream velocity is increased, the rotor assumes the position shown in Figure llb with the cusp shape directly above the shaft. A turther increase in the freestream velocity results in an osciliation or rocking of the rotor about this position. The amplitude of the escillations increase with freestream velocity until the rotor betins to turn in che counterclockwise direction. The rotor was never observed to turn in the clockwise direction.


Fi ure lat. Se!t driven rotor speed as a tumetion ui ireestream velucity.
the particular rotur discussed in Re:ereate 10 has beon ematam in det.it to jetermine the rebthonshi; betwera the treestrean velucity and $\because$ thet ins: speed it produces. The results, shown in Fisure lat, we well represented by a lincar dopetadenc. Plas result is cemphosized by the nonti"enstumat resules at Vifote litb were the rutit


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It should be emphasized that the results of Figares $12 a$ and $b$ are singular results for this particular rotor shape and for the frictional resistance of this bearing configuration. A change in the rotor shape or the frictional resistance would change the value of the curve but is not expected to affect the functional dependence.

From the point of view of potential application, the fact that the rotor can be employed without the need for an external driver is significant. It will allow more widespread use of the concept behind the device, namely the potential use of unsteady flows to achieve results superior to thuse found with numinally steady tlows. All of the rotor based devices discussed in the preceding sections will be positivelv affected, from the point of view of application, by the possisility of self induced rotation.

## Uther Putential Applications

A wise range of potential applications exists or unstedy iluw devices. Since an external driver is unnecessary to power the rotor vortex generator, the potential applications for this particalar techaique have improved. The iollowing examples are ali related to flow control near a solid wall and are nut meant to be at all inclusive. Many wher embodimeats of these concepts will appear it the : uture.







 - ! : "




From the point of vew of the influence on the ! low, the concept suggested in Figure 13a is likely to be superior. However, supporting the required struciure within the seometrical constraints of the nozzle sy become difificult. From that poilt it view, the susgestion of Figure 130 mav be more sisile shieved.


Fiatue la. shewatic of a rotary valve seotmetry applied to +wall ject.

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cease. The flow will reattach to the wall and the process will repeat itselt. The net result will be an intermittent fluid "flap" without the need for moving parts.


Figure in. Sohematic at a sladile device to produce in intermitent :hapina.

Analyses



























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k_{c}=\frac{\frac{\pi!}{4!}}{4}
$$



$$
C_{p}(x, t)=-p(x, t) / q \cos 2 x
$$


$1=0^{0} \alpha=\alpha_{\mu}=9^{\circ}=10 \mathrm{~Hz}$ $\because=0.30 x=.125$





13. 2 effect of sueep a:Cle on overmli wave speed











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\begin{equation*}
a=a_{M}+\bar{a} \sin \omega t=a_{M}+\bar{a} \sin 2 \pi r_{0}=a_{v} \tag{5}
\end{equation*}
$$







$a_{v}=a_{v o_{0}}+k_{c}$
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 $M_{c}=1 .$.


$\therefore$ O. 5 TYPICAL MOTION TIME HISTORIES FOR THREE MEAN ANGLES Of ATTACK

derivative of Eq. (5),

$$
\dot{\alpha}=\omega \bar{\alpha} \cos \omega x=\omega \bar{\alpha} \sqrt{1-\sin ^{2} \omega t}
$$

A conbination of Eqs. (5) and (7) yiel.1s

$$
\begin{equation*}
\dot{a}=\omega \sqrt{a^{2}-\left(\sigma-a_{M}\right)^{2}} \tag{3}
\end{equation*}
$$

waich can be converted wi . . rate, 4, .............a (cf, Ref. b)

$$
\begin{equation*}
A=\frac{c \cdot \dot{a}}{2 a-j_{c}}=k_{c} \quad \sqrt{a^{2}-\left(a-\sigma_{H}\right)^{2}} \tag{3}
\end{equation*}
$$

The values of $i$ for the intersections of the $\alpha_{v}=$ i: Se: line rith the three notion curves in Fig. $j$ are toted on the Eicure and are sem to vaty Eron

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\begin{equation*}
\varepsilon=q_{v}-q_{v_{0}}-a k_{c} \tag{10}
\end{equation*}
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fig. s correlation of relative vortex inception angle kith a

















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$\left(1=\mathrm{BHz}, H_{c}=0.40\right)$, जido $\bar{a}=3$ DEG
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F1G. II HARMONIC AINALYSIS OF THE S:EEP EFFECT UN THE LITT RESPGNSE AT..$=15$ 2EG, ${ }_{6}=0.075$ $\left(t=8+2, V_{c}=0.40\right)$, N1. $\overline{\boldsymbol{\alpha}}=3$ 2EG











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o. Ca:za, F. D., G. L. Comerford, R. G. Carlson and R. 4. Blackwell: Investigation of Airfoil Dynanic Stall and Its Influence on delicopter Control Loads. USAASRDL Technical Report T2-51, September, 1972.
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B. Jormont, A . E. $A$ tathematical : Iodel of "nsteady derodynanics and Radial Elow for dpplication to tlelicopter Rotors. "SAlliLRDL


Robert L. Bass*<br>James E. Johnson**, and<br>James F. Unruh ${ }^{\dagger}$

## Abstract

Boundary layer and trailing edge flow activities were recorded using hydrogen bubble flow visualization techniques on an oscillating lifting surface in a two-dimensional water tunnel. Simultaneous with flow documentation, unsteady lift was measured over a range of reduced frequencies from 0.5 tc 10. Unsteady loads using classical, inviscid theories were predicted for the experimental conditions investigated. Reduced frequency bands exhibiting poor agreement between experiment and theory were identified and a correlation to observed flow phenomena was accomplished. The results support thesutilization of a separate viscous model near the trailing edge coupled with an inviscid flow field model to predict unsteady loads. The results further show that for certain reduced frequency bands, classical inviscid solutions may be applicable and adequate.

## Nomenclature

| $b$ | 1/2 chord length |
| :---: | :---: |
| $c$ | Chord length |
| $C(k)$ | Theodorsen function |
| $C_{L i}$ | Oscillatory lift coefficient |
| $C_{1}{ }^{\text {o }}$ | Two-dimensional steady state lift curve slope |
| ; | Oscillation frequency |
| k | Reduced irequency ( $\omega \mathrm{b} / \mathrm{V}$ ) |
| t | Tine |
| $\because$ | Free stream velocity |
| < | Distance along chord from leading edje |
| ‘ | Instantaneous angle of attack. |
| $\because$ | uscillatory angle of attack amplitude |
| ${ }^{1}$ | Mean angle of attack: |
|  | Soundary layer thickness |
| - | Boundary layer displacement thichness |
| $\therefore$ | Width of lifting surface |
| $\because$ | Fluid kinematic viscosity |
| - | Circular frequency |
| . | Fluid density |
| '" | Phase shif: in Theodorsen's function |

introduc:ion
The innalites $0^{\circ}$ the douncary isyer (viscous *eses) nave an irportant jearing on liting sur-- the ansiead, zero and hydrodynamic bemavior. inascid :neories are not adequate in many practical d.Wlications in their prediction of uns:eady loads or -ister incedtion.
:-mpoved :heories of lifing suriace dinamic :eromance which account for real fluid efiect: we regalies to acivance the seate of unsteddy aerom: "yrodgnamics. this redirztion nas led nurerJus researchersi-9 :0 reeevaluate sssumptions used on invescis uns:ead sprosinamic theories. Bots





analytical and experimental efforts have been undertaken. Most studies have evaluated the applicability of the Kutta condition in unsteady flows. Significant experimental actisity has been undertaken to study trailing-edge loading and flow patterns on oscillating lifting surfaces. Theoretical efforts have also been initiated to include viscous corrections to inviscid solutions to allow better prediction of oscillatory loads. These research, efforts have been accomplished primarily in 1974-1980.

In the composite, these efforts, which are discussed in Reference 10, represent both analytical and experimental research investigating the effects of viscosity on unsteady loads on oscillating lifting surfaces. The experimental efforts show that for many cases, viscous (real fluid) effects drastically alter the trailing-edge conditions on oscillating lifting surfaces and, thus, the classical Kutta condition is not maintained. The experimental work represents measurements of trailing-edge pressures and flowfield patterns under various oscillatory conditions. In some cases low reduced 'requencies have been utilized and in other cases high reduced frequencies. Theoretical work has replaced the kutta condition with auxiliary conditions which appear appropriate relative to the viscous flow phenomena at the trailing edge and which alter the associated circulation.

In recent years, unsteady fluid dynamics has received considerabie attention as it relates to unsteady separated flows, and there is a wealth of litermiure in high Reynolds number dyna: ic stall and unsteady viscous-inviscid interaction on usclilatory airfoils. $11-13$ Current interest in high anqle of actack susermaneuverability dircraft has emphasized the need to advance the state of knowledge in unsteady separated fiows. Also recent interest in low speed (Reynolds nurber) drone serodynamics 19-21 has required a re-evaluation of unsteady separated flow activit, di conditions wrern viscous effects on separation onenorema ant associated lodes can resul: in simiticantioneront lifing surface response than would be encountere: at the Reynolds numbers associated with conventional or hioh speed arcraft. Residrders ou :he application, the frotected operation of advance: lifitn: suffaces with ative load controls in
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o evaluating the applicability of the Kutta condition over a wide range of reduced frequencies, and

- establishing flow re imes where poor agreement exists between :lassical unsteady loads theory and experiment.

Experiments were undertaken in a two-dimensional water tunnel since higher Reynolds numbers could be achieved than with models in a wind tunnel, and hydrogen bubble techniques could be utilized to visualize the unsteady boundary layer activity. This work provides data whereby total oscillatory lift was recorded simultanecusly with the recording of unsteady boundary layer activity.

## Experimental fpparatus

## Flow Tunnel and Instrumentation

Figure 1 shows the water tunnel contraction, test section and diffuser used in this study.

?i.i. 1 Test section with balance and hydrofoll
A rectanguidr two-almensional test section of
6.1 , 30.5 cm provided a lest section aspect ratio
$\therefore$ The test saction length is 61 cm with a
$\because 5.7$ • 30.5 cm high viewing ared. A 1500 watt
un: : lamp placed over a blexiglass window 1.2 cm
whe oy 45.7 ctong locnied on the :op surfite
orgoded liont at 30 to the viewing ancle and pro-

- ised lliumina! ©on for thom visuatiatton. The
wier cunnel test section shown in finure 1 was
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3.08 pipe was used for :he returt linn i $\therefore \therefore$
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## Flow Visualization Equipment

Hydrogen bubbles were utilized to provide flow visualization for this study. A grid of platinum wire was placed upstream of the test model to provide visualization of the stream lines. In addition, hydrogen bubble generators were imbedded at different locations on the model surface to allow direct injection of tracers into the oscillating foil boundary layer. Photography of the hydrogen bubble tracers was successfully accomplished with a 16 mm high-speed motion picture camera with zoom lens. The water tunnel test section viewing window was marked with a vertical line for a tracer reference point. The developed film was analyzed frame-by-frame on a Vanguard Motion Analyzer where flight trajectories of hudrogen bubbles were easily determined. The film analyzer was configured to computerize the coordinates of motion of the flow tracers. The computerized space-time histories of individual bubbles were used to establish flow separation and reattachment locations and reverse flow activity during a foil oscillation cycle. These data were used for recording qualitative information or boundary layer activity and were not used for Inad determination

## Dynamometer Design

A dynamometer, based upon the work of Epperson and Pengelley ${ }^{22}$, was used for measuring the dynamic lift coefficient on the oscillating model. The dynamoneter, shown in Figure 1 is classified as an external dynamic balance which can resolve two forces (vertical and herizontal) and one monent about a given axis perpendicular to the plane containing the two forces. Electric resistance strain gages attached to the structural elements of the A-frame flexures mounted back-to-bdck were incorporated in a Wheatstone bridge circuit in such a manner that the bridge output was proportional to the applied external moment.

The ai, foil section was supporied by a shaft connected to an external dynamoneter unit lucated on each side of the tunnel tect section. The dymamometer units were attaches io a clevis that was. dilowed to rotaie upon shaft bearinas mounted on the tunnel side walls. Dscillatary eotion was inparted by : hyaraulic eervo cylinder located on the lower structure of the test section. A unique pressure balanced sesl was fabricated mish absores no load the inhibits the flow of liguio from the test section to the surroundinas.
Mode: Design
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Table I Model lest conditions

| Run | Freesiream velocily， $V$ ．im＇s | Oscillation frequency． $f, \mathrm{~Hz}$ | Reduced frequ acy． $k=2 . / b / 1$ | Reduced veloctly． $1 k$ | Angle of attack．deg $\alpha(1)=\alpha_{0}+\dot{\alpha} \sin 2 \pi / f$ | $R \mathrm{r}=2 \mathrm{~V} \cdot \mathrm{biv}$ | No ．＂ $R e^{\bullet}=1^{\prime} v$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1）Velo atty bariation |  |  |  |  |  |  |  |
| HVI | 982 | 0132 | 0.648 | 54 | $589 \cdot 0.95 \sin 083 /$ | 16．10s | 219 |
| HV？ | 39 | 0147 | 180 | i 59 | $5825+1025 \sin 09251$ | 6，390 | 118 |
| HV3 | 1623 | 0151 | 0 ＋45 | こ．29 | $5775+1025$ un 09471 | 26.620 | 28. |
| HV4 | 1： 91 | 0144 | 0.592 | ＇ 81 | $580+10 \sin 0 \times 0 \times 1$ | 20.650 | 247 |
| 2）Frequency variation |  |  |  |  |  |  |  |
| HW．I | ＋33 | 0.387 | 4.28 | 0.14 | $59+09 \sin 2431$ | 7.105 | 146 |
| HW：？ | 418 | 0329 | 3.76 | 0 206 | $605+0.95 \sin 2071$ | 6，865 | 143 |
| Hu． 3 | 407 | 0836 | 98.4 | 0 102 | $6.05+0.95 \sin 5.201$ | 6.70 | 141 |
| HW． 4 | 369 | 0.241 | 3.13 | （1）319 | $505+0.95 \sin 1.511$ | 6.060 | 135 |
| 3）Mean angle－of－allack variatun |  |  |  |  |  |  |  |
| $\mathrm{H} \mathrm{x}_{0} .1$ | 399 | 0436 | 5.24 | （） 191 | 1．95＋ $1.95 \sin 2741$ | 6.545 | 140 |
| Has ${ }^{\text {a }}$ | 418 | 0602 | 689 | 0149 | $560+1$ shr 3.781 | 6.860 | 143 |
| $\mathrm{Ha} 0_{0}{ }^{\text {？}}$ | 456 | 0 0）9 | 6.04 | 0166 | 7 $65+1.15$ wi 3611 | 7.485 | 150 |
| $\mathrm{Ham}_{0}$－4 | 434 | 0633 | 6.98 | 0143 | －155＋105 m1：3．981 | $\bigcirc .125$ | 146 |
| $\mathrm{H} \mathrm{a}_{0}{ }^{5}$ | 450 | 0.649 | $6 \%$ | 0.145 | $-5.29+1.05 \sin +081$ | $\cdots .389$ | 144 |
| d）Pitichamplitudesartation |  |  |  |  |  |  |  |
| Holit | 584 | 0503 | $\pm 09$ | 02.44 | $425+159 \sin 3161$ | 9.655 | 170 |
| Hir－${ }^{\text {d }}$ | ， 76 | $04^{7}$ | 391 | $0: 56$ | $425+255 \sin 2.951$ | 9.450 | 168 |
| His 31 | 478 | 0） 4 | 369 | 0.271 | $420+48 \mathrm{sm} 280 \mathrm{l}$ | 9．475 | 168 |

28＊－haved on theoretial alue for an cyamalent fal ntate ot hore in

Table：Model boundary－layef observations

| How regime ident | Run | A | 11 | 1． 6 m ： | f． Hz | Observed boundary－layer actusts |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\wedge$ | HV？ | 045 | ：？ | 16？ | 015 | Bl veparallor at $x \quad 2 b \geq 075$ <br>  <br>  |
|  | HVA | 059 | $1 *$ | 12： | 1） 14 | 12 separation at $12 n=01^{\text {a }}$ <br> Reatlached BLar $\mathbf{Y}$ 2bғ0 $\downarrow$ <br>  <br> Separation zone acturls sumilat blH： |
| 4 | 11） | 00.5 | $1 \times$ | 9： | 013 |  separation point du：ing foil pithire whe <br> Reallached Bt at 1 ： A a 0 th <br> TE separation al 0 0 $3 \leq \leq 1$ ： $0 \leq 0^{\cdots}$ <br> ITF separation zune bsillatei dutise fat whe |
| 1 | Hu id | 13＊1 | 1： | ＊ | 1）1， |  14 eparatoonalo e （ll separallow zone uscillate adting ionl witel |
| 1） | 川！ | i． | $0 \times 1$ | （1） | 119 |  <br>  |
| 1 | H64 | 11 | 912 | 1＇ | （1） 21 |  |
|  | HW： | 33 | 0 Ot | 41 | 013 | Identualtobl |
|  | HW | 4 i | （1） | $+1$ | 0 17 | Tommastor－ |
|  | 吅1 | 45 | 010 | 10 | 034 |  |
| 1 | ． | 10. | $0(\mathrm{~m})$ | $4 y$ | 110 | 1．0nmshedsfagat ll at \％ 10 |

[^1]
(3) $k=10$

unsteady lift measurements were made. Figure 5 shows a comparison of theory and experiment for osciliatory lift coefficients $C_{L \bar{x}}$ os versus reduced velocity ( $1 / k$ ). Also shown on this figure are the various flow regimes (see Table 2) where different distinct boundary layer activity was observed during the flow visualization.

\[

$$
\begin{equation*}
C(k)^{\prime}=C(k) e^{i \in} \tag{3}
\end{equation*}
$$

\]

A phase shift between the virtual mass and oscillatory lift contributions to $C_{L \bar{x}_{O S}}$ was introduced to synthesize changes in the Kutta condition.

From the data shown in Figure 5, it can be seen that agrement vetween theory and experiment is good in those regions where boundary layer activity was well behaved. For $1 / k<0.75$ ( $k: 1.33$ ) good agreement between theory and experiment is noted. Also, for $1 / k>2(k<0.5)$, reasonable agreement is noted. However, at the knee of the theoretical curves ( $0.75<1 / k<2.0$ ), significant deviation between measured and predicted oscillatory lift coefficient is noter. This flow regime represents conditions where the gredtest degree of boundary layer activity is recorded wherein significant separation and reattachment of the boundary layer was observed. Also, the flow near the trailing edge was separated with flow alterrating around the irailing edge region as the foil osclllated trinough its maximum and minimum angle of incidence. since such trailing edge flow conditions will significantly affec: the circulation around the osEllating foil in comparison to that predicted from lisssicai inviscis theory. lift predictions were alic carried out for various values of ?. As noted in Figure 5. 3 30 phase lay in $C(k)$ provides im:roved agreement between theory and experiment.

## Conclusions and Recommendations

ne presented s;multaneous unsteady loads eassements and boundary layer flow visualization :mble acditiond insight into the effects of vis. . sit: or unstesty lods on ostillatina lifting jurecos, and dudilunal insight into the flow con: :? ons inich oceur in different reduced frequenc: eanis. The results in this paper snow that:

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    3 FMe vulta condition, based on Tudil:a:`ve
        , iow visudl:adion, was violated ?or wany
        - :ne utserged osclllatmg fut: tes: con-
        iltions.
        joserved boundary layer pmenomena supporis
        :"e ",pothesis that insteddy boundary layer
        * :%e. is responsible for boor surement
        bevween oderent theore:ical predic:ions and
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quency bands classical inviscid solutions may be applicable and adequate.

## Acknowledgements

The authors of this paper wish to express their appreciation to Dr. H. Norman Abramson who first suagested this work. Also, we want to express our gratitude to Mr. C. M. Wood for his dedicated work in conducting the experiments. Mr. Victoriano Hernandez for his skillful art work on the figures. and Mrs. Adeline K. Raeke for typing the text.

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## Abstract

Two delta wings with a leading edge sweep of $45^{\circ}$ and $60^{\circ}$ were studied in a towing tank at chord Reynolds number up to $3.5 \times 10^{5}$; The wings were pitched about the quarter chord point through typiEal angles of attack of $15^{\circ} \pm 15^{\circ}$, with a reduced frequency in the range of. 0 to 3 . In the steady state flow, dye visualizations revealed the existence of a shear layer near the leading edge that rolls up and Forns discrete vortices parallel to the leading edge. These vortices were observed to pair at least once as thev were convected downstream. Similar phenomena were ubserved in the unsteady sase, except that the vortices shed from the leading $t^{2 d}$ ee were modulated and altered hy the unsteady motion, which was an order of magnitude lower in frequency. In general, the unsteadiness delaved separation and promoted hysteresis similar to results obtained in unsteady two-dimensional Alricils.

## Nomenclature

```
IR ispect ratio
    root chor:!
    pitching frequency
    reduced irequencv, -fc/lom
N
` ving semispan
t time (sec)
\becauser towing speed
y(: angle of attack
    mpex angle
    Ingle hetween vortex cores
    perturhation wavelenpth
    klnematic viscosity
```


## 1. Introduction

 If: ni : experiences stall it large angle of attack. "? ee ieparition on the unper surfine redtuces the
 foerelses. The stall angle for most airiolls is iromm :2. Tho aerodynamic characteristics of a f.l:, fing are considerahly tifferent. The leading adge iuction perks iredicted by potential theos? do
 rmonth auction peaks inbound ni the leading edgers ore desected. The lift is mininly enntributed b: - lieste two peaks which are produced is the flow ..natates in the leeward stde of the wing ind forms 1 pult at statinntry leadlafe edge vortices.





the separated vortical structures rather than by the attached flow near a convex surface. These vortices exist to angles of attack as large as $30^{\circ}$ or more. The lift keeps increasing with $\alpha$ until the vortex breaks down. Hence, a delta wing is a good means to obtain high lift at large angle of attack.

Sy using air bubbles for visualization in a water channel, Elle (1958) showed that the separation vortices have very concentrated cores. Furthermore, the angle between the two cores, $\gamma$, is not a sensitive function of the angle of attack. The ratio between $\gamma$ and the spex angle, $B$, is alwavs between 0.6 and 0.7 , but the vortex cores lift away from the wing surface with increasing $\alpha$. Fink and Taylor (1967) investigated the pressure distribution on a wing with $B=20^{\circ}$. The suction peaks of the spanwise pressure distribution at several chordwise locations always occurred at about $60 \%$ of the semispan from the center for all tested angles of attack, $5^{\circ}<\alpha<30^{\circ}$. In other words, the suction peaks were located under the vortex cores. A more detalled visualization (Fink, 1967) showed that there is a counter rotating vortex associated with edch primary separation vortex. The existence of the counter rotating vortex sould also be inferred from the total head survey.

Under many practical situations, e.g. fast maruevering of an aircraft, the flow is not steady. The unsteady aerodynamic properties are significantly different from those in steady flow. For a two-dimensional airfoil, the lift, drag and moments expertence large hysteresis during the execution of one rycle (ScAlister and Carr, $1978 \&$ McCroske?, !982). A large separation vortex develops near the leading edge and convects aleng the chord. High level surface pressure fluctuations are produced. on the unsteisdy delta wit.g the information ahout the time evolving vortices is very limited itamourne pt al., 196?). This is one of the main reasons for thls work.

## $\therefore$ Experiment.al Approsch

## 2.! Models and Test Condfions

Wo delta whas with a leading edge sweep of $45^{\circ}$ and no $0^{\circ}$ werr used for the prement investigation. The root chor! a! hoth whens was 25 cm , and the
 (t) $3.5 \times 10^{\prime}$. Fis. 1 is stetch of :he as de!ta



 surt 1 e with , shorn lentine wite.


Fig. 1 Schematic of the $45^{\circ}$ Deita Wing.

The four-bar mechanism shown in Fig. 2 was used to sting-mount and to pitch the delta wing fround the desired position along the ch.ord. In the experiments reported herein the wing was pitched around the $1 / 4$ chord position. The mean angie of attack could be set from $0^{\circ}$ to $45^{\circ}$. A Boston Ratiotrol motor derived the four-bar linkages to produce approximately sinusoidal cscillations of amplitude $\pm 5^{\circ}, \pm 10^{\circ}$ and $\pm 15^{\circ}$ about a given mean angle of attack. The reduced frequency, $K \equiv \pi \mathrm{fc} / \mathrm{U}_{\infty}$, was varied in the range of 0 to 3 , and a digital readout displayed the inctantaneous angle of attack wit the wing.


Fig. 2 Photograph of the Pitching Mechanism

## $\therefore$.こ. Fing Tank S:stem

The wings used in the present investigation were towed at speeds in the range of 10 to [01 cm isec ehrough the water thannel described he fis-rl-H.k et 11. (1981). The towing tank is 18 m long, 1.2 m wide, and 0.9 m deep. The pitching nechinlsm was rigidly mounted on a carriage thit rhes an two tracks monted on tup of the towing switem. During towing, the carriage was supportod $\because$ in n!l film which Insured $\because$ ibrationless tow,
 bense 10.1 perant.

### 2.3 Flow Visualization

Food color and fluorescent dyes were used in the present investigation. The food color dyes were Alluainated with conventional flood lights. The fluorescent dyes were excited with sheets of laser projected in the desired plane. To produce a sheet of light, a 5 watt argon laser (Spectra Physics, Model 164) was used with a mirror mounted on an optical scanner having a 720 Hz natural frequency (General Scanning, Inc.). A sine-wave signal generator, set at a frequency equal to the inverse of the camera shutter speed, derived the optical scanner to produce light sheets approximately 1 mm thick.

Side views of the flow fleld were obtained using a vertical sheet of laser in the $x-y$ plane at $z=30 \mathrm{~cm}(40 \%$ of the root chord) and a stationary camera outside the tank as shown in Fig. 3. End views were obtained using a vertical sheet of laser in the $y^{-z}$ plane at $x=20 \mathrm{ca}(80 \%$ of the root chord). The camera was stationary and by necessity outside the towing channel; thus the view was from a $45^{\circ}$ oblique angle and the horizontal scale was contracted by about 30 percent.


Fig. 3 Definition Sketch

Dye sheets or tre lines iever seeped into the boundary laver elarougit a svetem of slots and holes sut the suction slde of the withe. The slots were 0.2 min wide and were milled it $a$ is angle to minimite the thow dfsturbance The holes were o. omm dymeter and wert ifaced it 1 im cortiot duenter.



thickness, due to the inhibition of vertical motion caused by introducing a weak saline stratification in the tank. The dye layers remained quiescent until disturbed by the flow field on and around the wing. Thus, the boundary layer flow as well as the potential flow could be observed silice the dye layers existed in both flow regions.

## 3. Experimental Results

### 3.1 Steady State Flow

Dye visualization techniques were used to observe the flow on a delta wing at fixed angle of artack in the range of $0^{\circ}$ to $45^{\circ}$. The trailing edge separation could be seen even at zero angle of attack. As $\alpha$ increased above $5^{\circ}$, two stationary vortices occurred near the leading edge as found by other investigators. At the interface between these large primary vortices and the external potential flow, a thin shear layer was formed by the velocity difference between the free stream and the separated region. The shear layer was unstable and small secondary vortices were generated. In the present operating conditions about five small vortices rolled around the primary vortex/potential flow interface. At angles of attack above $10^{\circ}$, the separated region was sufficiently thick that the smaller secondary vortices were at least one diameter removed from the wing. Under these conditions, the secondary vortices merged in a pairing process as they were convected around the edge of the primary vortex, as shown in rig. 4 . In this -ide view using a vertical sheet of laser, the $45^{\circ}$ delta wing was at angle of attack $\alpha=10^{\circ}$, the chord Reynolds number was $R_{C}=2.5 \times 10^{4}$, and the flow was from left to right. The pairing process appeared to be quite similar to the one observed in plane mixing lavers (Winant and Browand, 1974). The physical mechanism for generating the small vortices in the delta wing case is believed to be the same as that In a free shear layer (Browr. and Roshko, i 774 ). In the delta wing case, ruwever, the flow field is more complicated be sause of the complex neometry and the non-planai velocity fleld.


Fig. 4 Side View with Wing Fixed at $a=10^{\circ}$

The ddscrete vortlces were also observed on - - ho sheep, sharp leading edpe delta wing is shown in the lop ifow in Fig. 5. Here, $1=10^{\circ}$.



apex. The pairing process was observed on cine films of this run.


Fig. 5. Top View with the $60^{\circ}$ Deita Wing Fixed at $\alpha=10^{\circ}$.

At smaller angles of attack, the vortex merging seemed to be inhibited by the presence of the wing surface. At larger attack angles, intense mixing made it more difficult to observe the pairing process. It is not clear from the present experiments whether the primary vortex only caused the secondary vortices by setting up the inftial shear layer, or if it was possibly the result of several mergings of the secondary vortices.

Similar vortex formation and pairing were observed in the pitching case. However, the process was modulated by the lower frequency oscillation of the wing as shown in the next section.

### 3.2 Lnsteady Flow

The flow visualization results of the $44^{\circ}$ sweef delta wing undergoing a sinusoldal pitching motion are presented in here and in Flow Research Film No. 55 (avallable on request). Nn the suction side of the pitching delta wing, the leading edge separation vortices executed a grow-decay cycle durlag one period. Fig. h is a side view of the $45^{\circ}$ wing undergoing the pitching wotion $(t)^{\circ}=15+5 \sin (0.4 t)$. we chord Revnolds number $R_{c}$ - $2.5 \times 10^{4}$ and reduced irequency $K=0.5$. Both the upward and downard motions are shown side by side for the angles of attack of $10^{\circ}, 12^{\circ}, 14^{\circ}, 16^{\circ}, 18^{\circ}$ and $20^{\circ}$. At a particular litack angle, the flow patterns were very difierent during the upward and downard guplons. The bysteresis loop elearly extsted. Th.


 - ! ! ! 1:!


Fig 6 Side View of the Pitching Delta Wing


Fig 7 Top View of the Pitching Delta Wing

Fig． 7 shows top view of the wing undergoing the pitching motion $\alpha(t)^{\circ}=15+15 \sin (0.8 t)$ ，at chord Reynolds number $R_{c}=2.5 \times 10^{4}$ and reduced fre－ quency $K=1,0$ ．Both the upward and downward motions are shown side by side for the attack angles $\alpha=0^{\circ}, 5^{\circ}, 10^{\circ}, 15^{\circ}, 20^{\circ}, 25^{\circ}$ and $30^{\circ}$ ．The three dye slots on the left side of the wing are closer to the leading edge as compared to the ones on the right side．During the up stroke，the separation first started across the whole trailing edge at $\alpha=2^{\circ}$ ．As the angle of attack increased， the separation propagated upstream from the two corners at the tralling edge toward the aper．The propagation speed along the leading edge was approximately equal to the flow speed（ $10 \mathrm{~cm} / \mathrm{sec}$ ）． At $:=30^{\circ}$ ，the separation front reached the apex ind the separation vortices were fully developed． During the down stroke，the vortices became smaller and diminished in size at about $\alpha=10^{\circ}$ ，but the flns was still separated near the trailing edge．

Fig． 8 shows an end view of the pitching wing bhtined using a vertical sheet of light in the $y-z$ plane at $x=0.8 \mathrm{c}$ ．The view is from $=45^{\circ}$ oblique angle and the flow is out from the plane of the photograph．In this run the wing underwent the itching motion $\alpha(t)^{\circ}=15+10 \sin (0.2 t)$ ，at chord Revnolds number $R_{c}=2.5 \times 10^{4}$ and reduced frequency $K=0.25$ ．The up stroke at the angles of attack $q=5^{\circ}, 10^{\circ}, 15^{\circ}$ and $20^{\circ}$ is shown in the figure． The dye was released only from the slots on the left side of the wing．Hence，only one of the loading edge separation vortices is marked．The vortex grows as the angle of attack is increased． In the cine film from which the frames stown in fig． 8 were obtained，several discrete vortices rolled around the primary vortex and merged in a pairing process as mentioned above．

The dye layers technique was used to visualize the boundary layer flow as well as the potential fow around the wing．The horizontal dye layers were excited with vertical sheets of laser in the $\because-\because$ and $\because-:$ ilanes．When the wing passed through the quiescent dye layers，the local motion of the lult could he inferred from the deformation of the lavers．In the end views，the dye layers near the anter portion of the span moved downard during the up stroke is the separation vortex grew，indicating． t strong fownash in that region．Duriag the down stroke a second blob of dye appeared next to the ipparation vortex and nearer to the center portion n！the span，As the angle of attack decreasd，the －wirtetnn vortex ditaintshed while the second bloh ：He srow in size．The nature of this motion ！ ＊ut clect it present，but it could be a separat！on bebhl，wear the trailing edge similar to the one hearved by hinkelmann and Barlow（1980）on a sma！！ apoct rat io rectangular wing at constant angle ot －！がな。

## －U！：－！：！© R R duce：Frequenc：







a Increasing
Fig． 8 End View of the Pitching Delta Wing．

In general，the unsteadiness delayed separa－ tion and promoted hysteresis similar to results obtained in unsteady two－dimensional airfoils．As the attack angle decreased from its maximum value at least one and of ten two large separated regions appeared on the wing as a result of the unsteadi－ ness．As the oscillation frequency increased，the formation of the separated region appeared further downstream．At $K=1$ ，the streamlines remained parallel to the wing surface until high attack ingles．Significant changes occurred when the reduced frequency was above 3 ．No large scale separation was then ohserved on the wing；however． a strong vortex was shed at the trailing edge as a passed through its maximum value．Fig． 9 is a sidc view of the pitching wing using the dye layers technique．The wing underwent the pitching，motion $\alpha(t)^{\circ}=15+15 \sin (2.6 t)$ ，at chord Reynolds number $R_{C}=2.5 \times 10^{4}$ and reduced frequency $K=3.3$ ．The photograph shows the wing at angle of attack $\alpha=25^{\prime \prime}$ during the up stroke．The dye layers in the outer flow regions indicates that the potential flow is following the motion of the wing，and that the separation region is restricted to be very close to the wing surface．


Fig． 9 Side View at High Reduced Frequency

Fig. 10 is an end view at the same run conditions depicted in Fig. 9, but at an attack angle $\alpha=20^{\circ}$ during the down stroke. Both the separation vortex and the second blob of dye mentioned in the previous section appear in the photograph.


Fig. 10 End Viow at High Reduced Frequency.

A tentative explanation for the distinct change at $K \geq 3$ is as follows. The reduced frequency car be viewed as the ratio of the chord to a "perturbation" wavelength: $K=\pi f c / U_{\infty}=\pi c / \lambda$. In other words, the perturbation wavelength is about equal to the chord length at $K=3$. As a matter of fact, the reattachment point can be seer near the trailing edge in Fig. 9. After the flow is reattached, a thin shear layer with intense vortices formed and a strong vortex counter rotating with respect to the attached wing circulation was shed from the trailing edge. The induced velocity of this vortex kept the potential flow moving downward with the wing. If the perturbation wavelength was longer than the chord, i.e. $K<\pi$, the wake remalned thick. The diffused vortex could not enforce a thin separation region on the wing. Based on these arguments, it appears that a parameter based on the chord length and the perturbation wavelength is more appropriate to describe the flow than the corventional reduced frequency.

## 4. Conclusions

Twe delta wings with a leading edge sweep of $45^{\circ}$ and $60^{\circ}$ were studied in a towing tank at chord Revnolits number up to $3.5 \times 10^{5}$. The wings were pitched about the quarter chord point with sinusoldal ascillations of amplitude $\pm 5^{\circ}, \pm 10^{\circ}$ and $\pm 15^{\circ}$ about a given mean attack angle that varied in the range $0^{\circ}$ to $45^{\circ}$. Food color and fluorescent dees were used to visuallze the flow fleld on and around the wings. Sheets of laser excited the etunrescent dye in yield detalled flow information in the desired plane.

The steady state flow fleld was stud!ed at angles of attack of $0^{\circ}$ to $45^{\circ}$. Separation was , hser:ed near the tralling edge on the centerline at small attack angles. As a increased above $5^{\circ}$, leading edge separation occurred forming two large scale srationay vortices approximately parallel in the leading edge as found by previous inverstigators. In atdition, the dyed shear laver near the leadiat - !ee was nbserved to roll up and form alserete قurtlees along ipproxtmately straight Ines emanatthe from the wing pex, sinllar to the noes comonly ubserved in tree she ar lavers. These portices rallo! wound the pr!mar: zartwe potenctial :lw



Similar phenomena were observed in the unsteady case, except that the vortices shed from the leading edge were modulated and altered by the unsteady motion which had an order of magnitude lower frequency. The leading edge separation vortices executed a grow-decay cycle during one pitching period. As the angle of attack increased, the first observable separation was along the trailing edge of the wing. As $\alpha$ increased further, the separated region increased by moving from the wing tips along the leading edge toward the spex. At large angle of attack, the potential flow field had a downward component near the center of the span indicating strong downwash in this zegion. As a decreased from its maximum value, at least one and of ten two large separated regions appeared on the wing as a result of the unsteadiness. In general, the unsteadiness delayed separation and promoted hysteresis similar to results obtained on unsteady two-dimensional adrfoils. As the oscillation frequency increased, the formation of the separated region appeared further downstream. At a reduced frequency $K=1$, the stream! ines remained parallel to the wing surface until high attack angles. At $K=3$, no large scale separation was observed on the wing; however, a scrong vortex was shed at the tralling edge as $\alpha$ passed through its maximum value.

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## Experimental Setup

A large, low turbulence, open return wind tunnel was converted to unsteady operation as described by Freymuth, Palmer and Bank (1983). The tunnel started from rest, then maintained a nearly constant acceleration of $2.44 \mathrm{~m} / \mathrm{sec}^{2}$ ( $25 \%$ of gravity) for 5 seconds prior to power shutof:. The tunnel had a tesi section 0.9 m $X$ O.g ï across and was 20 m Iong with Plexiglas front and top walls allowing for photigraphy.

We investigated a cylinder, a sphere, a flat plate, a round plate and an MACA 0015 airfoil. These bodies had the same diameter or chordlength $c=15.2 \mathrm{~cm}$; the Reynolds number was $R=5200$.

A strip of liquid titanium tetrachloride $\left(\mathrm{TiCL}_{4}\right)$ was painted in flow direciion on the body surface, releasing dense white fumes. The method is described in detail by Freymuth, Bank and Palmer (1983). Since smoke is released in the retions of vorticity production we assume that smoke fatterns reveal vorticity patterns near the body and in the near wake behina the body, where differences in vorticity diffusion and smoke diffusion do not represent a problem a froblem may exist for low Reynolds number fiow, far downstream, and when vortices of oppoiite sion are adjaceat to each other (smoke dens:ty does not have a distinction in sign).

Smoke patterns were recorded ior 3 sec dith a 16 mm Bolex movie cagera at 54 frames pe: Eec and with a shutter sfeed of $1 / 500$ sec. Decause of space limitations, oniy a restricted number of frames of an acceleration seouence can de show: in the Figures (additiona: rrames can te obtatred upon request from the authors). In the fiou:e caftions we denote the ime between frames thown as $\therefore$ it and the time from acceceration start io it.e firsi frame shown as $t .$.

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there is a weak tendency of the main vortex to decay into smaller ones prior to turbulent decay. For the sphere the secondary vortex is visible as a vortex ring on the sphere surface.

Upon close examination quantitative differences exist between the two cases. Separatior for the sphere occurs later (larger $t_{1}$ ) than for the cylinder but transition to turbulence proceeds faster, resulting in a smailer primary vortex. In the late stages of turbulent decay a turbulent vortex street Gevelops behind the cylinder (first observed by Prandtl 1905) whereas the sphere wake remains rather symmetric and diffuse.

Figure 2 compares accelerated flow around a fiat plate (left columit, a round plate (middle column) and an : iACA 0015 airfoll (right column), ail placed perpendicuiar to the flow. For the round plate oniy the lower edge is visualized. thele vortex deve iopment is quiltatively similar for the 3 cases the montrast to Figure 1 is considerat:e. The separation tongue which jreviously devisoped into the main vortex now Biartis formation of a vortex group consisting of A to da, 00 sma: vortices, eome of them in a tabe of pairing. The 2 vortex grcupy venind the :of and bo:ion edses o: the body then interact mh.-: trey becume turbulen:. Secondary flow is :o: established in these cases except near the :ourd upper edge of the airfoii. Decay into -intuierce ta slower than in Figure 1 and :s $\therefore$ owest for the round airfoil edge.

Facure 3 comfates the eymetrisu :ACA 0015

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 "erond ryaze, iect solumri. Also a second Joutir! rotatios vor"ex :y lnevilence :tear the

 e?,. : se zecontary vortex. The for=ation of そin zure e:acorate pationns is frever:ed by

















and vortex development creeps up from the rear of the airfoil, the sharp edged reverse alrfoil and flat plate retain vortex formation close to the leading edge, with vortices follewing each other in rapid succession. These vortices get quickly turbulent, especialiy for the fiat fiate.

For the flat plate, the movie was taken at a Wider angle of view to show some of the wake downstream of the trailing edge. Details of the wa!:e for the airfoil are reported by Freymuth, Palmer and Bank (1983), showing an unstable shear layer and a subsequent vortex street.

## Conclusiens

ite have compared accelerating flow around a :imited number of bodies under similar experimental conditions. Obviously a more general lnvestigation is possible by considering additional body shapes and by changing some experimental conditions simultanecusly for all bodies, in particular the Reynolds number.

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## Abstract

This paper presents preliminary results from a program of work in which a surface singularity panel method is being extended for modelling the dynamic interaction between a separated wake and a surface unciergoing an unsteady motion. The method combines the capabilities of an unsteady time-stepping code and a technique for modelling extensive separation using free vortex sheets. Routines are being developed for treating the dynamic interaction between the separated wake and the solid buundary in an environnent where the separation point is moving with time. The behavior of these routines is being examined in a parallel effort using a ewo-dimensional pilot version of the three-dimensional code. This allows refinements in the procedures to be quickly developed and tested prior to installation into the main code.

The extonded code is being coupled with an unsteady integral boundary layer method to examine the prediction of dynamic stall characteristics. The boundary layer code is accessed during the timestef cycle and provides the separation locations as well as the boundary layer displacement effect:- the latter is thudelled in the potential flow code using the source transpiration technique.

The preliminary results presented here inciude basic unsteady test cases for both the potentia? -low and boundary layer routines. Some exploratory sevarated flow calculations are included from a series of numerical studies on the stability of the calculation procedure. Correlations with experinental dynami. stall results have yet to be performed.

## Introduction

Flow separation on the lifting surfaces of a vehicit at hign angle of attach is always complicated by a ceriain degree of unsteadiness. but, when the vehicle itself is undergoing unsteady motion or defurmation, or if it enters a different flow field rapid!/, then the cumplexity of the separated flow is evr gredter, and culminates in the phenomenon of dyant stall. If the angle of attack oscillates arount the 5 atic stall anyle, the flutd dynamic forces and miments usually exhmit large anounts of nos:eresie and a cundition of negalive derodynamic dawing ufien devilups during part of the cyile. This can lead to the cundition of flutter in 1 single degree uf ereedum uscillation rigid body moilun. (Nurmally, in attachoc ilow, flutter only uccurs when the bod. not., e includes riultiple degrees of freedum. e.g. , wnoled bending and tursiun

- Haver ?usented at the AFOSR/FJSRL norkshop on Or icuay Separded Fluws, USir a ademi. Culuratw Sorings. Co, Augus: 10-11, :983.
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of an aircraft wing.) During a rapid increase in angle of attack, the stauic stall angle can be greatly exceeded, resulting in excursions in the dynamic force and moment values that are far greater than their static counterparts. The consequences of dynamic stall are far-reaching and lead to such problems as wing drop, yaw (sometimes leading to spin eritry), wing rocking and buffeting as well as stall flutter.

Although a great deal has been learned about dynamic stall characteristics--mainly through experimental observation--there is not at this time a completely satisfactory theoretical method (1), í) for predicting the dynamic stall characteristics for new untested shapes even for the two-dimensional case. Moreover, quantitative comparisons of experimental test oata on new geometries can be obscured by the effects of three-dimensional wind-tunnel interactions, wall interference and experimental uncertainties (3). In the present work a possible theoretical approach is examined for predicting dynamic stall characteristics. The approach combines an unsteady time-stepping method (4) with a steady inviscid/viscid iterative code (5) that includes an extensive separation model. The latter has proven very successful in the steady case. Both codes are applicable to general three-dimensional shapes and have been developed from the same basic panel method (6).

Extensive investigations of the dynamic stall characteristics of airfoiis oscillating in pitch have been reviewed by McCroskey (1), (2). In practical aerudynamic environments, the unsteadyness can be a combination of several motions. Unsteady motions other than fitching have been investigated by, among othors, Liiva et al. (7), Lang (8), Lang and Francis (9), Maresca et al. (10), and Saxena et al. (11). In these references the phenomena of fiunging, flap, spoller and rectilinear oscillations were examined. There are very few theoretical approaches, nowever, and in a recent review. McCroskey (2) concludes that at the present time the engineer who needs answers should turn to one of the empirical currelation techniques, even though these are not completely satisfactory and only supply broad details cf furce and monent characteristics. The method of Ericsun and Reding is perhaps une of the mosi comprehensive of these methods. Their idtes: paper (12) incurporstes a number of findings from the systematic experiments of Carr et al. (13).

Early theuretica! approaches is dynamic stall dduressed the deep siall aspect which is duminates by the passage uf vortices shed from the vicinity of the leading edge. Han (14) used just a thin plane nodel of the airfull. Later work by Eauduei al. (15) extended the basic model to thith sec:ions using a parel method. The rain arawback with the apnooch is :rat: irucial assurptions regarsina the location and ilem uf vortex shedding nave to be made in uraer iw iewfur the calculditons. Also. the resul:s from $(15$ ) are seesl:ive to (a) tie antle at with the ores ;at leates :he suroce, (s) the
time at which vortex emissions terminate so as to start the reat+achment process arid (c) the viscous diffusion of the free vortices.

Calculations of the characteristics of the thin boundary layer in the attached flow regions of an oscillating airfoil using unsteadj methods (16), (17), have demonstrated good qualitative agreement with experimental observations. However, one feature at least needs further examination in regard to improved modelling: it is refurted (18), (19), that when incidence is increasing beyond the static stall angle, the location of zero skin friction in the turbulent boundary layer and the catastrophic separation can occur at different stations, Figure 1. Apparently, a long thin tongue of reversed flow precedes the main separation zone. This is not observed under quasi-steady conditions.


Fig. 1. Model of Sears and Telionis (19) for Up-stream-Moving Separation of an Unsteady Boundary Layer.

Crimi and Reeves (20) combined a potential flow method with an unsteady boundary layer analys is in a viscous/inviscid interaction approach. The potential flow nodel was based on chordline singularities and so excluded modelling of the thick wake. Also, detail of the stagnation poirt location in relation to the curved leading edge was missing. Emphasis was placed on the details of leading-edge bubble Dursting and application to trailing-edge separation was rot attempted. Shamroth and Kreskovsky (21) developed a similar technique but with improvec treatment of the separated flow region, transition phenomena and potenital flow region. However, the procedure failed to predict the flow field about the stalled airfoil. They concluded that the effect on the uuter inviscid solution due to the finite wake displacement must be modelled.

In spite of the shortcomings of the above approaches, the general technique of matching various viscous and inviscid regions remains an attractive alternative to the full Navier-Stokes treatment.
$\therefore$ 'though, in principle, the latter can overcome linltations of the potential flow/boundary layer iterative approach, such treatment is limited at ihis time to laminar flows at Reynulds numbers much lower than realistic for most practical dpplisations (2). Prugress twoards higher Reynolds numbers is being made, but applications to general probiens is still d lung way tway (22).

The present method goes beyond the capabilities of the farlier theoretical approaches in that both trailing-edge and leading-edge stall with vortex passage can be included in principle. The method, developed for the three-dimensional case, is applicable to arbitrary configurations and to general motions; i.e., not just pirch oscillation. In addition, because the basis of the method is a surface singularity panel code, a more reliable and direct coupling between the inviscid and viscous analysis is assured. Moreover, modelling of the separated zone in the trailing-edge region and more detailed treatment of the vortex/surface interaction should make the approach more viable for applications to dynamic separation problems.

## Potential Flow Methods

General
Although remarkable advances are being made in flow field calculations using finite-difference and also finite-element methods, the surface integral approach using panel methods coupled with special routines for nonlinear effects still offers distinct advantages for many real flow problems. In particular, panei methods offer greater versatility for practical application to complicated configurations and are consiaerably more efficient in terms of computing effort. However, the concept of zonal model-ling--in which a local Navier-Stokes analysis is coupled with a panel method--should not be overlooked. Ultimately, such a coupling should lead to an improved modelling of vortices (e.g., vortex cores, vortex dissipation and breakdown), thick viscous regions, local separation bubbles, and shock wave/ boundary interactions.

Over the past decade, panel methods have seen a trend towards higher-order formulations (23), (25) and (26) At the outset it was argued that compared with the earlier low-order methods the more continuous representation of the surface singularity distribution should allow a reduction in parel density for a given solution accuracy and, hence, should lead to luwer computing costs. No such benefits have appeared su far for the general three-dimensional case. In fact, preliminary investigations (27) have indicated that the prediction accuracy for problems with complicated interactions, such as vortex/surface or high curvature situations, depends more on the density of control points where the boundary conditions are enforced; the order of the singularity distribution plays a minor role. In the meantime, further developments of piecewise constant singulaiity panel methods, e.g., Morino (28) and AMI's Program VSAERO (6), are giving cumuarable accuracy to the higher-urder methods at much lower computing costs.

The low computing cost of Program VSAERO makes it practical to apply the method to nonlinear probleas requiring iterative solutions, e.g., wake re laxation fo: high-lift configurations, multiplecomponent problems and rotor cases; and viscous/inviscid calculations with coupled boundary layer analyses, including the case with extensive separation (5), and also time-stepping calculations for three-dimensional unsteady problems ( 4 ) that are beyond the scope of a rarmonic analysis. This method, therefore, offers an attractive oasis for a
practical tool aimed at predicting the aerodynamic characteristics of dynamic =tall problems. At the outset, this code was being developed in two different directions; viz, one was for extensive setaration modelling under "steady" conditions, while the other was for unsteady time-stepping calculations. These two capabilities, described in the section below, have now been brought together in one code.

## Separated Flow Model

Under essentially steady conditions, the pressure distribution in a trailing-edge separation region is usually characterized by a constant pressure region extending back to the trailing edge followed by a short recompression region (e.g., (29)). A simplification of this characteristic is modelled in the two-dimensional CLMAX program (30), (31) using a pair of constant strength vortex sheets to enclose the separated region, figure 2. The length of the sheets required a semi-empirical apbroach and the condition that the sheets be forcefree is satisfied in an interative cycle in which segments of the sheet are aligned with calculated local flow directions. The method combines boundary layer and potential flow codes in an outer inviscid/ viscid iteration cycle. The potential flow pressure distribution--which includes the influence of the free vortex sheets--is passed to the boundary layer routine which then supplies the separation points and the boundary layer displacement thickness distribution for tile next iteration. The boundary layer displacement effect in the attached flow region is modelled by transpiration (i.e., source distribution) rather than by a displacement surface. The infin advantage of the transpiration approach is that the matrix of influence coefficients in the panel method remains essentially the same from one iteradion to the next; only the wake condition changes.

The thin vortex sheet model of the upper searated shear layer was demonstrated by Young and hoad (32) to be a reasonable representation of the flow as far back as the trailing edge. Fur example, a comparison from (32) of a laser-velocimeter flow survey, and a CLMAX program calculation is shown here in Figure 3. Downstream of the trailing edge the vortex sheet model becomes less representative of the flow; however, a later evaluation of a graded vorticity mode! over the recumpression zone showed little effect on the airfoil solution. Mure detailed modelling of the recompression zone (such as, for example, the approaches used by Gross (33) or Zunlwalt (34) would be desirable in cases where the wake interacts closely with a downstream component.

A particular feature of the vertex sheet model enclosing the region of low energy is that pressures can be calculated directly in the separated zone (30). This is an additional advantage over the displacement surface approach of Henderson (35) and over the source outflow model of Jacob (36). The CL MAX method generally gives very close agreement with experimental pressure distributions (30). (31). An initial extension of this method to the unsteady case is reported in (31) where quasi-steady sulutons coupled with a phase shift model were used. Extension of the riodel to the three-amensional case is reported in (37) for a stripwise model and in (5) for a mure general treatment. The separation model has a! so been successfully installed in a transonic finite -difference cote (38).

The same basic model is also applicable for the unsteady case; here, however, the assumption of constant vorticity is no longer valid. In fact, a dynamic wake model is essential and will be discussed below after tile description of the unsteady formulation.


Fig. 2. Mathematical Flow Model (Steady).


Fig. 3. Location of Experimental Free Shear Layer and CLMAX Calculated Vortex Sheet Centerlines for Low Mach Number Case (from (32)).

## Unsteady Method

Formulation. Consider the whole of space divided into two regions by the surface of the configuration and assume the existence of Laplacian velocity potential distributions in the two regions. i.e., : in the flow field and : in the blade interior. If we now apply Green's third identity to the two regions, then the total potenital at d point, $P$, on the inside surface of the boundary can be written:

$$
\begin{aligned}
& :=-\iint(-i) n \cdot\binom{\vdots}{\vdots} \text { is }-(i \cdot) \\
& \text {-I! (.) ra) } \\
& \text { • } \\
& \iint \cdots(-): s
\end{aligned}
$$

Here, $r$ is the length of the vector from the surface element to the point, $P$, and S-P signifies that the point, $P$, is excluded from the surface integration. Equation (1) includes the contribution fron the wake surface, $W$.

The Dirichlet boundary condition is now applied in the interior region to render a unique distribution. In principle, any potential flow can be applied. However, the flow, $i_{i}=i_{u}$, implied by Morino (28) and used by Johnson and Rubbert (25), has proven to be very reliable in practice. With this flow, Eq. (1) becomes

$$
\begin{align*}
J= & \int_{S-p}: n \cdot\left(\frac{1}{n}\right) d S-2=2 p \\
& +\iint\left(: U-\sum_{U}\right) n \cdot\left(\frac{1}{r}\right) d W \\
& -\iint_{5}: n \cdot \therefore \tag{2}
\end{align*}
$$

where : the periturbation potential in the flow field, has been substituted for: : : .

The first two terms in Eq. (2) give the perturDation potentid due to a distribution of normal duublets if strength, : , un the configuration surace. Similarly, the third term represents a doublet distribution of strength, :U - : L, or the wake and the fourth temil represents a source distribution of strength, $\| \cdot \cdot i$, on the configuration surface.

Equation (2) is Dasically the same as the fur"ulation given by Murino (28). who used a direct appication of treen's theurem in the fluw field. The present appruach to the problem is a scocide case of a multi-domain fomulation which has lead to the more general three-dimensiunal methud in which large regiuns of separated flow are mudelled in a binilar way to that in the CLMAx program (5). (30) snd (31)

The suurce termi In Eu. 汶) can be evaluated srecil, from the cundition of mu fluw benetration at the surface. The fluw velucity relative so the dody-fi.e. frame is at any ins:ant of the

$$
\begin{equation*}
\gamma=v \cdot v-n \quad R . \tag{3}
\end{equation*}
$$

 s. ff pisilun. ant re ine instantaneous anset fluw send anguiar veluci: $\because$, respecilvely, and a is be relative busitiun ve: :ur besemen a puthe $n$ Ge ruta:tun atis dole a bunt in the suldace.

Pur zeru encers:iun. $\delta \cdot n=u$. dence.
and Eq. (2) becomes

$$
\begin{align*}
0 & =\iint_{S-p} \dot{n} \cdot \nabla\left(\frac{1}{r}\right) d S-2 \cdots D_{p} \\
& +\iint_{W}\left(: U \cdot L_{L}\right) \underline{n} \cdot\left(\frac{1}{r}\right) d W \\
& -\iint_{S} \frac{1}{r}(\underline{n} \cdot \underline{V}-i n \cdot \underline{n} \cdot \underline{R}) d S
\end{align*}
$$

This is the basic equation of the method. It is solved for the unknown surface perturbation potential, $\uparrow$, or surface doublet distribution, $\dot{\text {, }}$, at a number of time steps as the configuration proceeds through the motion. The wake surface is transported at the end of each step using calculated velocities of points on the wake surface. The doublet distribution, $2 U-i l$, on each wake surface is known from solutions at earlier time steps. The unsteady Kutta condition

$$
\begin{equation*}
\frac{i v}{i t}+v \frac{h}{i s}=0 \tag{5}
\end{equation*}
$$

is satisfied at points alung each wake separation line at each time sted.

At each time step the flow solution is determined with reference to the body-fixed frame. The incompressitile pressure coefficient is, therefore, given by

$$
\begin{equation*}
C_{p}=\left(v_{5}^{2}-v^{2}+2 \frac{\pi}{t}\right), v . \tag{6}
\end{equation*}
$$

where $\underline{V}_{S}=\underline{h} \quad \underline{R}-\underline{V}$ is the instantaneous velucity of a puint $\ln ^{-}$the surface relative tu a stationary reference frame, and $V$ is alven by Eq. (3).

Jumerical Procedure. The general arrangement of the cunfiguration is shown in Figure 4 . The $x$, $y .:$ coordinate system with unit vecurs. I. ., is


fixed relative to the configuration. For symmetrical applications, the $z-x$ plane is regarded as the plane of symmetry.

A numerical procedure has been assembled in a time-stepping mode to obtain the unsteady pressure distribution and forces and moments. The surface of the winn is represented by planar quadrilateral panels over each of which the doublet and source distributions are assumed constant. With this assumption, the surface integrals in Eq. (4) can be performed in the closed-form for each panel.

Equation (4) is then satisfied simultaneously at a point at the center of each panel. If there are $N$ panels representing the configuration surface, Eq. (4) becomes:

$$
\sum_{\substack{k=1 \\ K}}^{N}\left\{_{K} \mu_{K} C_{J K}\right\}_{1}-2 T_{H_{J}}+E_{J}=0 ; \quad J=1, N
$$

where ${ }^{\prime} K$ is the unknown doublet value on panel $K$. (Note: ${ }^{2} K={ }^{2} / 4 \pi$.)

$$
E_{J}=\sum_{K=1}^{N_{W}}:-W_{K} C_{J K}: \Delta_{R_{J}}-V_{2} \cdot \dot{-N}_{J}
$$

where in the number of panels in the wake, varies with time and and $V$.. take their instantaneus values at each time $\bar{s}$ tep.

$$
r_{R_{j}}=\sum_{k=1}^{N} i \underline{n} \quad R_{k} \cdot n_{k} B_{j k}: / 4
$$

is the source distribution due to rotation about the dxis, $h$, and

$$
i_{j}=\sum_{k=1}^{N} i_{i} n_{k} B_{j k} ; 4
$$

wre itie cumbunents of a inree-part source distribution Jue to the relative translation of the cunfiguration and the onset flow. (ivete: in a sym"etrical case the $y$-component is zero.)

The suantities, $B$ jo and $C_{\text {ap }}$ are the velocity sutentid influence coefficients for the cunstant suarce and loablet distridutions. reseectively, unt barel baciong on the control punt un panel J. -hese macluse contritutions frum the mace anel in the symetrical iase. Expressions for thase ine iaence coefficients have been aven oy Mumo in . al

different expressions are installed in the VSAERO code based on planar panels.

Equation (7) is solved by a direct method for $N \leq 320$ and by an iterative method for $N>320$.

The surface pressure distribution is calculated using Eq. (6). The surface gradient of $\perp$ is evaluated on each panel by differentiating a two-way parabolic fit through the doublet values on the panel and its four immediate neighbors At the separation lines a simple differencing is applied for the gradients approaching the separation line.

The gradient of $\phi$ with respect to time is evaluated by central differencing over two time steps; i.e.,

$$
\frac{j t^{t-j t}}{\partial t}=\left(t^{t}-p^{t-2 \Delta t}\right) / 2 \Delta t
$$

For harmonic motions the real and imaginary pressures are obtained by Fourier analys is for the first harmonic based on solutions over a complete cycle. The calculations start with inciderice $x_{0}$ and a regular (i.e., steady) wake. Two iterations are performed to render the wake force free. An uscillatory doublet component based on a linearized solution is then superimposed along each wake line before starting the time-step model. Time-step calculations proceed over a half cycle before applying the Fourier analysis.

At each time step a new panel is formed at the head of each columin of wake panels and all the existing wake panel corner points are convected downstream at the local velocity. Each wake panel keeps the doublet value it received at the time it was formed. This doublet value is based on the conditions at the separation line and satisfies Eq. (5). It is assumed that the shedding vccurs at cunstant vorticity over the time interval, t. In this way the doublet strength, $t+i t$, on the new wake pane $i$ is related to the strength, $\mathrm{N}_{\mathrm{N}}^{\mathrm{t}}$, of the previous wake panel at the separation line by

$$
\omega_{N}^{++\therefore t}=2 T^{t}-\cdots{ }^{t} .
$$

where .- is the resultant touble: dalue at :te separation line.

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Weatment of the tree-shear idyer mutel than was
usea for Pre secauv cose. Reluc:ares are billl cal-
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the calculated velocity vectors for a small time interval, at. In this way, as time progresses, a dynamic wake model is generated. At each step a new piece of free sheet is shed from the calculated separation point; the strength and size of this new segment is determined by the local upstream velocity condition. The location of the separation--calculated using an unsteady boundary layer code, see the next section--can now move with time.

It is convenient to regard the local vorticity (i.e., doublet gradient) on the free vortex sheets in two components; a streanwise component and a cross-flow component. The streamwise component is already force free and is related to the spanwise rate $0^{i}$ shedding of circulation from the wing. The cross-flow vorticity component is associated with direct dumping of bound circulation from the separation line ard must be transported with the local flow velocity in order to be force free. This cross-flow yorticity component --which was assumed constant with streanmise distance in the steady case-- now varies along each streamline on the free sheets for two $r$ asons. first, the vorticity value being convected unto the free sheet at each separation point is varying in time because of varying onset flow conditions and because of the changing separation locations, secondly, stretching by the eritire configuration of solid surfaces and free wake sheets. This stretching of the doublet disbribution carried by the free sheets yields varying vorticity values when the doublet gradient is evaluated. In this way the free sheets can become highly distorted and centers of vortex roll-up may form. Special treatment of the sheets is therefore essential if numerical stability is to be maintained. Two routines are being evaluated in this work but they have not been fully implemented at this time.
(i) Vortex Arralgamation

To cater for vortex roll-up in a reasonable manner it is essential to include a vortex core model in which integrated vorticity is accumblated rather than to follow a detailed calculation of multiple turns of a vortex spiral. An amalganation scheme similar to that of Moure (39) is being used. when the angle betwean neighburing samments resresenting a sheet exceeds a specified anyle, the segment end points are merged $t_{i}$ a new lucation at the centroid of their combined circulation. A number of such cores are allowed in the new routine to deal with complex motions. A viscous core expression can be applied to each vurtex core when com;luting the field velucities.

## (ii) Redistribution

Having perfurted the vurtex-core allalgamatiun calculation hiong each free sheet the points defininy the intermediate free sheets are redistributed with equal spacing in a manner similar to Fink and Suh (40) and Sorikaya and Schuaff (11). Purtiuns : The sheet between dinalyandect cures are iceated independently mers, Figure 5. Thi ereatment. which is applied to both the sheet geuneiry and its fublet distribution, uses a biquadratic interpoie. tlue: scheme based on surface distance alung the neet.

This routine should nel: stablize the numer. ual culcalations. especially in :he initial part uc each sheet when ene sebaration loca: an is orana won tree in ere enree-simensiund ase the moti,


Fig. 5. Illustration of Multiple Vortex Core Amalgamation and Redistribution Scheme.
tribution scheme is being arranged along the calculated mean streamlines in the wake sheets. This causes some difficulty if amalgamation is not proceeding uniformiy along all lines on a sheet.

## Calculations

As the routines are being developed, preliminary calculations are being performed to check the basic operation of the code. Figure 6 shows the growth of indicial lift and circulation for a NACA 0012 impulsively started from rest at an angle of attack of .1 rad. The curves are compared with Wagner's function for indicial lift and R.T. Jones indicial circulation for a flat plate. These initial calculations, which used 31 panels around the section, are in good agreement and indicate a slightly higher trend which is consistent with a higher steady state circulation for the thickness case.


Fig. 6. Indicial lift and Circulation for an :mpulsively Started Two-Dimenstural Flat Blate.

Sume recent refinements developed in the :w almantunal ollit code have sian ficantly reduced the computimg requrbent of these the-stepaing calculations. For examble. flare : shows the ef-
 seas in the a somer truther and semstistrates drath: cunvergence.


Fig. 7. Computed Indicial Lift for an Impulsively Started NACA 0012 Section--E: fect of Numiver of Tifie Steps.

The procedure has been tested also for the harmonic oscillation case. Earlier calculations required 80 time steps per cycle for a NACA 0012 oscillating in pitch about the quarter chord. These culloared favorably with the Theodorsen flat plate function over a range of reduced frequency, Figure 8. The lew calculations are also in good agreement but were performed with only 16 time steps per cycle.

 function of Reduced frequency. $\sin :$.

Figure 9 shows the computed results, $C_{L}$ versus time using only 4 time steps per cycle. This is in remarkably guod agreement with the 16 and also 32 time-step 'cycle solutions, demonstrating an extremely good convergence characteristic.


Fig. 9. Computed C for a NACA 0012 Section Oscillating in Pitch about the QuarterChord.

Time-stepping calculations have also been performed for cases with prescribed extensive separations. The purpose of these calculations was to check the basic unsteady circulation shedding model in the potential flow code. For the first set of tests, the wake panels were simply transported at the onset flow velocity after the initial growth as determined from the surface conditions at sedaration. Severai triangular shapes were considered, each starting impulsively from rest and proceeding forwarcis over 10 time steps for a total time of $\therefore=: U . / h=3.0$, where $h$ is the triangle base height. Separation was prescribed at the corners. Fiqure 10 (a) shows the computed history of the drag coefficient from pressure integration for a 60 ir: angle with blunt face forward. A tutal of 40 panels was used to represent the triangle surface. The calculation was repeated in the presence of wind tunnel walls (also panelled) with a 10 biuckage ratiu. The indicated bluckage correction is sumewhat lower than that given by standard techniques. Figure $10(b)$ compares the computed pressure distributions for this triangle in and out of the tunnel. This "base " pressure has unly a sthall variation and is quite close to experimental measurements. Figure 11 shows a summary of computed drag cuefficient versus triangle semi-apex angle. The calculaced values are slightly high in relation tu the experimental data collected frum several suurces by Heerner in Aerudymamic: Drag.

(a) History of Drag.

(b) Calculated Pressure Distribution at r $=3.0$.

Fiq. 10. Calculations on a Triangular Section Started Impulsively from Rest.

One further case was run for the 60 apex-forward tilangle in free air with the full wake velucity calculation routine turned on but without the amalqamation and redistribution schemes at this stage. The calculated $C_{0}$ for this case falls beluw the experimental value. Figure 11. A series of contouted wake shapes is snown in figure 12 . These are samples from a total of 40 time-step calculations The tutal computing time for this case was 195 cecunds un a PRIME 550 minicumputer--this is equivalent tu less than : seconds of CRAY time. The sulutluri chould benefit fromi the nullericial daminc proviled by the dinalyamation and redistribution schenes lescribed earlier.

Finally, a iest calculation was performed for a NACA 0012 section in a state of pitch from $10^{\circ}$ to $30^{\circ}$ with ac $/ 2 \mathrm{U}_{\infty}=.175$. The calculation used 30 paneis and 10 time steps. Separation points were prescribed and the motion was started impulsively


Fig. 11. Calculated Drag Cuefficient of Two-Dimensional wedges as a Function of Apex Angle.
from rest
Fiqure $13(a)$ shows a sample of the computed a. ke shapes and demonstrates a redsonathe numerical behavior. Sample pressure distributions are shown in Figure $13(b)$. The : assaue of the leading-edge: vortex is clearl, shown. This is assucided will d lucal requon uf reversed fluw. These dre prellan-
 cal behavior of the calculdton ;roredure and puten. tial fluw madel. Future cosses will anclude the coupled thuidary lager catiolation - or ored elta the seraratiun mont. At and trop the calculated results will be condared wion wervental daco.



$$
\begin{equation*}
V_{T}=\frac{l}{k} \sqrt{\frac{\omega^{W}}{U}} \operatorname{sgn}\left(i_{W}\right) \tag{20}
\end{equation*}
$$

and $k=0.41$ is the von Karman constant.
The similarity solutions have shown that the entrainment coefficient, $C_{E}$, can be expressed as

$$
\begin{equation*}
C_{E}=C_{E_{S}}-\frac{1}{U_{e}} \frac{\partial \delta}{\partial t} \tag{21}
\end{equation*}
$$

where $C_{E_{S}}$ is the entrainuent coefficient for the sieady case as giver by:

$$
\begin{equation*}
C_{E_{S}}=\sqrt{\frac{C_{f}}{2}}(0.074 G-1.0957 / G) \tag{22}
\end{equation*}
$$

Substi:ution of Eq. (21) intu Eqs. (14) and :5) and the intruduction of $h^{*}=\ldots, / /$ give

$$
\frac{1}{U_{e}} \frac{s}{t}+\frac{f}{x}=R_{t}
$$

$$
H * \frac{\because *}{x}+\frac{l+H^{*}}{\sqrt{e}_{e}} \frac{\cdot *}{\cdot t}
$$

 $\therefore$

$$
\begin{aligned}
& 3_{2}=i_{E_{S}} \cdot \frac{v_{e}}{v_{e}}
\end{aligned}
$$

of the secund degree for the parameter, $=\frac{d x}{U_{e} d t}$ :

Equations (23) and (24) can be solved in various ways. Initial conditions at $t=t c$ and boundary conditions at the stagnation point ( $t \cdot 0$ ) are sufficient to determine a solution in the region where the flow is attached, i.e., ? 2 . In the present paper, the time derivatives are treated as forcing terms and the integration is performed in the $x$-direction using a Runge-Kutte methud.

The buundary layer procedure has been tested dyainst expermments and the calculations of other investigaturs. The results are shown in Fiqures 14 and 15. Figure $14\langle a\}$, (b) and (c) shows the mean quanitites (momenturn thickness, skin friction and shape factor) for the experiment cunducted by Cousteix (45) un a flat plate. The main free strean velucity is $22 \mathrm{~m} / \mathrm{sec}$ and the mutiun is har:lunic with respect tu time with :he freunency of 32 hz . The oresent calculation (sulid ine) agrees very moll with the calculation by luusteix and the curre ation with experiment is aisu very guva.

(c) Mean Shape Factur.

Fig. 14. Cunciuded.

Figure 15 shows the comparisun with the calculation of Nash et al. (46) for a munotonically time-varying fluw on a flat plate. The present calculation predicts the separation at the end of the plate when ist $=0.682$ as in the Nash et al. calculatiun. The uverall results are in good ayreement with their calculation.






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Finally, an experimental data case from (3) was run and the computed lift variation with compared with the measured data in Figure 18. The airfuil is a NACA 0012 and is oscillating in pitch about the quarter-chord line with $=8.1+4.9$
 Reynolds number was $4 \times 10^{6}$. This reduced frequency condition is very close to the changeover from a lead to a lay situation and so there is urily a simall difference between the upswing and duwnswing curves.
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## Introduction

Adrances in computer hardware during the last decade have permitted the numerical solution of the Xaver-itoke equations. weiginal efforts comensrase on steaty flu problems, however, mure reently anstady fiows have been aderessed. the $\because$ the best uses of the Navier-stahes numericas probram in : solve separated thos due to the anibour amacter at the problem. (:or prohiems in



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## arovicu：un











＇hen movinc ilong the airfoil surface，tris train of yortines not orive makes the flow unsteady but also may sause signitioant znanges
 ane rot yet availabie tor zomparisor，we ara presentine here t ineoretionl annlysis for computine the unsiegdj int and drae of iro airfuil when vurtises are roleasej inturmstten：iy ：ron ts upper surtace．

Eespite the tuct that vortices observed in the Iaboritory in most Ate？：iormed ：rom rolitis $1 p$ of Fortex sher se resilitne from bountary layer separi：lun at the sinarpitpo： the spoider or rotor is sketano．in FiE．$\because$ ，they
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is shown in Fin. 1 , the transfornation

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\begin{equation*}
\left.z=\frac{1}{2} ;+\frac{1}{5}\right) \tag{1}
\end{equation*}
$$

aups a olvc!e u. Madius a (>1/ in the $\zeta$ (=弓 $+i \eta$ ) plane, centered at $\boldsymbol{\zeta}=$ ? - a, into a symmetrio Joukowski iirfoil in the physical z ( $=x+i y) p l a n e$, whose chord is sligntiy ionger than 2 . A irse-stream velocity of magnitude ot one nalf and at an angle of attack $\alpha$ is maffed without rhanging its orientation :nto a uniform stream of unit speed in the physical plane. For the convenience of cormulation a $\zeta^{\prime}\left(=\xi^{\prime}+i \eta^{\prime}\right)$ plane is introduced in $\overline{\mathrm{s}} \mathrm{i} .{ }^{\circ}$, whose origin coincides wth the center of the circle. The relation

$$
\begin{equation*}
=\therefore \div 1-1 \tag{i2}
\end{equation*}
$$

:10:is io: coorinate transfomation between $\zeta$ tni と' planes.
\& liscretu vortex oi circulation $k: a t$ g fosition zi outsite the airfoil is transtomed i worlane tu Et. (1) intu a fortex of the same *: $\cdot$ ungth at h' outside the cirche. Tu satisfy the fomiary bondition that the flow be tangent $\because$ - acirate ini of fultll: the requremen: -!: re tota! verasition be conservei, an


 : Ev:-5-r rortiees in the physicai pline, be it -ru artex poneratel ty i spoller or a vortex sion in t!e witu, t set o! three yortinns is
 :थ.alion ras: luso:nted. appose the minill















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$$
\frac{d w}{d r^{\prime}}=\frac{1}{2} e^{-i \cdot x}\left(1-\frac{a}{3} e^{i \cdot:}\right)+\frac{i \cdot}{2 \cdot-1}
$$

$$
\begin{equation*}
\frac{d z}{d \zeta}=\frac{\ddot{i}-1}{\ddot{y}} \tag{b}
\end{equation*}
$$

This expression ho: is at any puint in the physical flow except at the trailing edge of the airfoil and the center of a free vortex.

In the unsteady flow field, a waxe vortex is shed at any instant of time. Let this be the mth vortex whose position is द̆m in the transformed piane. Its circulation $\boldsymbol{r}_{\mathrm{m}}$. determired by satisfying the Kutta condition that $\zeta^{\prime}=a$ be a stugration point, hes the expression


$$
\frac{3-1}{1,1-11}
$$



















$\ddot{u}_{n}$, leri"ed afte: taring a proper limit.

$$
\begin{aligned}
& =\frac{n}{n-1}\left[e^{-i n}-\left(\frac{a}{n}\right)^{n} e^{i x}+\frac{i}{n}\right. \\
& \left.+\underset{j \neq n}{m} \frac{i k_{i}}{-1} \frac{1}{m-j}-\underset{j=1}{m} \frac{i k_{j}}{i-a,-1}\right] \\
& \text { _ } \\
& -\left({ }_{i}-1\right)
\end{aligned}
$$

$$
\begin{align*}
& \text { ario: ore vomputat based on the Zlasius } \\
& \text { tourm tur instenty :Ion. } \\
& D+i:=i:-i . e^{-i z} \sum_{j=1}^{m} \dot{k}_{j}(u+i v) \frac{\ddots}{{ }_{j}} \frac{i}{j_{j}-1}
\end{align*}
$$

sheet rolls into a spiral, which becomes tighter and tighter as time progresses. dhen distances among vortices become too small, inaccurate results are expected which appear in the form of scattered vortices in the waxe. To eliminate this chaotic motion of the wake vortices, we adopt Aoore's method to use a concentrated tip vortex to represen: the tightly rolled jortion of a vortex sheet. More specifically, when the total number of wake vortices exceeds 20 in the computation, the first two vortices at the tip are combined into one situated at thear centroij posifion, whose circulation is the sum of the two individual strengths.

Resuits
ror the $70_{0}^{\circ}$ thick symetric siffoi:, computations have been made by varying angie of s:tuck, chordwise position of the rejetised vortices, and the geriod at whish they yre released.

At zero ang-e oftther and wan vortines of Circulation 0.5 are release with a period ? $=5$ a: a height of 3.1 ato\% the 1 pper surface at the midchord position, the behavionof list voefficient at the par:y stuee is photewi $i: \%$ aig. 4. "he sudden relpase of g fortex musos a nesative itift on the airfoil. Lift increasos wath tima in an oscillatory manner; the moan
 oscilation, xaluaty appoachos an asymptuti

 ?us i shape that is thoot ifention? to that at tay other sucie, wxenet thit the witirn var. -s















neighboring released vortices and stronger interactions between these vortices and the wake vortex sheet. It is interesting to note that a: 1 wake vortices have counterclockwise circulations. Within that period, lift climbs to a maximum and then drops to a minimum, both occurring when a released vortex leaves the trailing edge.

The variation of mean lift coefficient with time is plotted in Fig. B for different values of T. Solid lines represent results for the arrangement that vortices are released at the midchord location. The plot reveals that for a shorter $T$ when vortices are released at a higher frequency, a higher average lift can be generated on the airfoil. Each of the higher ? curves approaches an asymptotic value, which increases with increasing frequency. The curves for lower values of ? seem to have the same behavior, but a tremendous amount of computer time is needed to find their asymptotic lift values. For example to reach $t=200$ for the " = 2 curve, the CPU time is 2700 seconds on a Eyber $170-720$ computer. No attempt has been made to extend the com ritation beyond that time. OONever, it should be pointed out that for T aquala ? or the mean lift coefficient can be:one greyte: than 0.5, which is the strtic lif: soefficien if the circulation of $y$ releasei vortex is applied around the airfoil. For a period equal to or greater than 4 , the noan :itt cannot exceel this value.

The dashed and the dash-doted lines : :n Fis. tre usel to represent results obtained sibn the vo:tox-generating mechanism 13 it the : i-itort ind $z^{+-\bullet h o r d ~ p o s i t i o n s, ~ r e s p e c t i v e l ~} \ddot{y}$. $\because$ these two cases fortices are still released it tre sama ini:ial height of u. 1 above the人 ou: Iirtoll surface. The influenct $v$ : Gine tig whodwise position on lift is not tou stong is reves: it in the tifure. Genera!:y





















generated on a wing utilizing a properly designed vortex-triggering device installed on its upper surface. Despite the negligibly smeil drag shown in the result, it can be expected to be substantial in reality when vortices are generated by moving a spoiler into and cut of the airfoil surface.

The strength of the released vortices is arbitrarily assigned in the present analysis. It actually should be determined more reallstically from empirical data or from viscous flow computations. The Iatter approach is being taken by us in an attempt to solve the problem of an unsteady viscous flow past an oscillating spoiler.

## hcknowiedgment

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 Fiows Abost a donkowak Artol: in the Presurner
 $\therefore$ AnAtry:


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E1g. 1 r.ow past a NALA vノly airtio土 Nitha spoiler at the quarter-chord position on the apper surface. Photograph was taken from the left rear gide of the N1:ng. Streti lines were truced out by smoxe invected at different heifnts :rom a vertisa! v!amn instal: led fu: 1pstrosim.

$\because \because r$
$\because: \cdots$


Pig. \& initiqi variation ui litt coefficion for $\alpha=0$ and $?=5$. Circulation $u$ : vortioes roleased it the in:tornurd








Scientific Research Associates, Inc. Glastonbury, Connecticut 06033

## Abstract

A time-dependent Navier-Stokes calculation procedure has been applied to the problem of an NiACA VOl2 airfoil oscillating in pitch in a low Kach aumber, high Reynolds number environment. The calculated results show many of the known physical features, including sudden suction surface separation, vortices shed at the leading and trailing edges and the return to attached flow at low inciderces. Both the lift and moment coefficient curves show the expected features and the calculated wall pressure cuefficients show strong correspondence to meisured datal.

## intruduction

The unsteady flow field about in isolated airfuil is an importan:, roblen commonly encountered in i number oi practiad : lluw situations. These include sirfuil flutter, vibration, buffut and gust response, ss bell th the problem of dirfoll dunamie stall. ilas latter problen, airfuit dynamic stall, is the subject of tike present paper. Airioil denamic seall arurs ia a variets of situations. Une important OWh wilid blas initiated the present study is the adilupter retur prohlem. As the tedicupter blade tranels througs the rotor disc, the blade cxperi-
 don the bhate :ill te unstalled (i.e., the tho .ath tot watan any large separaced regions ieading - A decrease in thade itit or a generacion of large diade momene wet:leients); however, fur example, ia foreurd : tight uter the retreationg purtion of the
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flow fields. A recent discussion of much of this work is given by Shamroth and Cibeling ${ }^{8}$. Although most airfoil Navier-Stokes analyses have focused upon either steady airfoils or impulsively started airfolls, some attention has been given to the more complex problem of airfoils oscillating in pitch. For example, Mehta ${ }^{9}$ investigated an NaCA 0012 airfoil oscilsating in pitch in an incompressible laminar environment and Wu, Sampath and Sankar ${ }^{10}$ considered a $9 \%$ thick Joukowski airfoil, again oscillating in an incompressible, laminar flow environment. In later works Shamroth ${ }^{11}$ investigated an NaCa 0012 airfoil oscillating in pitch between $0^{\circ}$ and $10.5^{\circ}$ in a curbulent environment ard Tassa and Sankar ${ }^{2} 2$ examined an airfoil uscillating between 1$)^{\circ}$ and $20^{\circ}$ in turbulent fluw showing d $\because$ namic stall lonps but nu comparison witi data was performed. The present paper iocuses upon cal.eulations of a high Reynolds, turbuleat, 0.3 Math number tlow about an AACi vol2 airfoil oscillating in pitch and compares ialculated orface pressures with experimental measurements.

## Anal:>0

## The ciordinute senem

The presence of founding surians it a comput.at
 Limes presents significont dicibulbes for numerteal techniques which sulve : tee sivier-乌t kees equathens. If a boundith surtace (sucti is the air:oil surtue) dien not andacide with a innolinate libe serious numertab errors maly arise tat the apiliotthen of buandur. ond!tions for etae problem and considerable éfirt mav he requited : radace : bex


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The form of the equations expressed in the more common courdinate systems can be found in staudard fluid dÿnamic texts. One possible mppranco for solving the equations in general nonorthogonal fora is the strong conservation approwth such its that used by Steger ${ }^{55}$. A second pussible approsth solves 3 set of equations in shich the metric coefficients du not appear witlija derivatives. This thas been termed the quasi-lathear torm by Shamroth and Gubli:ng and the haln rule conservation form by Handman ${ }^{\text {G }}$. Altwagh securate results bave been whtalled with buth turms of the equations, in cases whir the satobian $u$ transformation is independent ui time the latter form is less sensitive tu the precise manner in which the matrices are evaluIted ${ }^{8}, 15,10$. Therefore, the preseat effort nililizes the quasi-linear furme Disiousiion of cuordinate sostems having time-dependent Jokonij:is nits been giont :y Thomas and Lombard ${ }^{17}$.

If ble spatial variables are transiurmed irom the liartexian ourdinates ( $x, \because$ ) 10 a atem sot of心rdinates (., ) where

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\begin{equation*}
=(x, 1, t) \quad=(x, \because, t) \quad:=1 \tag{1}
\end{equation*}
$$


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governing equations are replaced by an implicit time difference approximation, uptionallv a backward difference or Crank-Nicolsun scheme. Terns involving nonlinearities th the inplicit time levei are linearized br Tavlur expansion in time about the solution It the known time level, and spatial difference approximations are introduced. The result is a system of multidimensional cuupled (but linear) differene .quations fur the dependent varables at the urknown or implicit time level. To swive titese difesence equativas, the Douglas-Gunn pronedure for suberateng Al:ern.ating-direct.zon implicit (ivl) sohemes as perturbations of fundamental implicit aifferemce schemes is introduced. This technique leius : 0 iقsicms of coupled linear difference equatiuns having narrow biock-handed matrix structures whioh dan ve sulved efticiently bu itandard blookclimi:ation methods.

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\begin{align*}
& \text { (2) (\%) (\%) }  \tag{3}\\
& \text { 4(1) } 1-1
\end{align*}
$$

t.tional domain into the problem than car be removed throush specificiation of physical boundary conditions. These additional variables van be removed in a varjety of wavs: for example, by wie of one-sided differencing, bv use of equations applied in a anesided manner at the boundaries or through so-cialled "extraneous boundary conditions". If extraneous ionditions are utilized, thev should be chosen to be as reasomable as posisibie irom a physical point of view. The hopethesis is alopted that if a sulution - un be obtained with the extrameous boundary conditions arplied, then any detective intluence of these extrameous conditions will result only irom the , ippoximations inherent in them, and the solutions should be assessed in this manner.

 : Luns fur the airtsil flow ifeld problem. f̈ile suter oumdir* has divided into :"ur segments. These were all uipotream ixundar:, a downstream boundary and twa






























the bulndary laver thickness : i.e.,
$=\cdot \vee D$
$=.109^{\circ}$
(3)
where 2 is the van-itriest damping factor, is the von-turman constant and $:$ is the boundary layer thinkness. In the wake regio. downstream of the airfoil trailing edge a is taken as $9.2 h$ where $h, i s$ the wake thickness. It is recugnized that an acoum rate representation of tine rurbulent viscosity for c!is ver! comple: flow with multiple stied vortices and large separated zunes will require a more general turbulence model. Honever, in tiese initial donnmic stall studies the main forus is upon asses-
 the drammi, stail prut ebs, and identifying :ide sensitive parametors of tar rrubiem. Seudies iitit atwo cquation model are iurrent $\because$ in pregress.

## Art:fincu] jissipation

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\begin{equation*}
R e_{i x}=|a-\partial b / \partial x| \Delta x / \Delta e \tag{7}
\end{equation*}
$$

where ${ }^{2}$ is the cotal or effective viscosity including both laminar and turbulent contributions, and $\therefore x$ is the grid spacing. The dissipation coefficient $d_{x}$ is non negative and is chosen as the larger of zero and the local quantity $d_{e}\left(\sigma_{x} \operatorname{Red}_{x}-1\right)$. The dissiyation parameter $\sigma_{x}$ is a specified constant and represents the inverse of the cell Reynolds rumber below which no artificial dissipation is added. The dissipation coefficient $d_{y}$ is evaluated in .a analogous manner and is based on the local cell keynolds number $\mathrm{Re}_{j y}$ and grid spacing $\Delta y$ for the $y$-direction and the specified parameter s.. It should be noted that recently calculaiions have been run with artificial dissipation added in the conservative form $\therefore\left(i^{n-1} d_{x} \quad 3: / 3 x\right) / 3 x$ and no significant difference between the ferms was noted.

The question arises as to the values of $\sigma_{x}$ and $\because$ which should be chosen. Based upon the results of Ref. 2f, as well as several other investigations for a variety of viscous subsonic and transonic flows it was concluded that setting between 0.1 and 0.025 suppressed non-physical oscillations, gave solutions which were insensitive to the precise value of $:$ in this range and gave good agreement with data. Therefure, ecund urder damping in the consirvative form wats used in the present calculations with heing set in the range between 0.1 and 0.05 .

## Result:

The analysis described previously was applied to the flow about an NaCa 0nl2 airfoil oscillating sinusoidally in pitch. The airfoll was immersed in a stream of Revnolds numbers equal to $2.08 \times 10^{6}$ and Yic: numer equal to 0.30. The airfoil mean inciSence was set $1512^{\circ}$ and uscillated with an ampli:ade of $8^{\circ}$ at a reduced irequency bersed upon semihard of 0.125. These conditions correspond to Run if. 415 of the data of $5 t$. Hilaire and Carta ${ }^{2}$ ? The raliulation was run for slightly more than one ひrle usiag 3 hignly stretched grid with the first , wint away trom the airfoll being $0.1: \times 10^{-5}$ chords (rum the alriuil. approximately 950 time steps wers required to proce through a iocle fus this reduced :requesty. Higher -edticed frequencies required feber :ine steps. Fice flculation was inithated from a ratverged veady sulution it " In.ilence. The ialwhation wis begun whth the artificial disstpation matacter, being set to 0.05 . During the nigh fa idene downstrake portion of the ealculation sumertat probicas sppeares it $s=19.7^{\circ}$, whith

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 at:tes are believed related to mexh rexulut? atid the :cmorary focrease fo artilichat dissipat tor, was Abopted gather thati adding gore grid puinto simp: tue ed cionsmbe necensit:












Fig. 1 - Calculated and measured pressure coefficient, $s=12^{\circ}$.





Fin. 3-Ciculated and measured pressure distrihution, $t=14.7^{\circ}$.
1.i: Lo: indicated stali at $19.5^{\circ}$. Merefore, at incidene vilues greater than $17.5^{\circ}$ the excelient quan: Deative agreement shown in Figs. $:$ and 2 no longer
 rumained. Fir ex.mple, the data at $19.5^{\circ}$ is presented wle: ste catculatiun at $1=19.9^{\circ}$, 0 in Fis. 3. d: sinugh these are at difierent values of © the represent pressure distributions at approxi--atc: : the same beremental time atcer stall is inithated: the distributions are remarkably similar, Fig. 3, is wel! is tigures not presented hore due : , phe ifimitations, indleate that although the - Aloulction nredbets stal: tu occur atter the meas1.red intidence, once stall occurs, when referenced : the :lac of it.all, the calculated and measured bressure distrbutions becume quite simblar. The E.bjor discrean: in :le caliulated and messured Yifues apeat tu lie in the prediction $B$ voriex taf: intiot. Elas, in curn, is like! to be dejendoen: vet :urjuiente and :ransbibn adeling. Cum-






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Cumpardsons be:west measured and daiculated dles
 $\therefore: \because$ bwing :hese fibures if stould be averd that lift and anment weitiotent are lategrated quantities. Re.atlye! sanill difiere:tes in pressure distribum

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 actiliteat ln f!s. ©. As tat se sen, the results




Fig. : - Calculated and measured pressure distribution, $A=12.5^{\circ}$.


Fig. 5 - Calculatod atd metsured pressure cistribut ton, $=6.5^{\circ}$.


whereas the calculated lift continues to increase until $20^{\circ}$. Both measurement and calculation show a precipitous drop near the maximum incidence. The major disagreement in the measured and calculated lift coefficient is over the low incidence half of the downstroke. The measured coefficient continued to decrease until $s=12^{\circ}$, whereas the calculated coefficient plateaued at $1 \approx 16^{\circ}$, indicating a somewhat more rapid recovery from stall. This agreemen: is regarded as reasonable with the major difference oceurring at low incidence during vtal! recoverv. A cumparison between calculated and measured moment coefficient is shown in Fig. 7. As can be seen in Fig. 7, the current comparison shows good qualitative agreement, the major difference being the larger negative moment cuefficient alculated than measured near 'max. Over'll the agreement is quite good.

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$0=95^{\circ}, \quad 0<0$


$$
a=165^{\circ}, \quad a<0
$$


$3=95^{\circ}, \quad a<0$

the appearance of the trailing edge counterclockwise vorticity appears to arise from two sources. The first source is the pressure surface boundary layer being convected around the trailing edge and up the suction surface by the large clockwise leading edge separation zone. The second source of this negative vorticity is a secondary boundary layer arising as the large clockwise separated zone is brought to a no-slip condition at the airfoil surface. By $a=18.3^{\circ}$, $\dot{x}<0$ the leading edge vortex has clearly broken away and the trailing eage vortex of opposite sign has increased in size and is moving somewhat upstrean. This is demonstrated more clearly in the vorticity contour plot. By $x=16.5^{\circ}, \&<0$ the vortices are tending to interact and begin to move downstream. This pracess also has been discussed by Robinson and Luttges? $5^{5}$ Finally by $x=9.5^{\circ}, i$ the flow has filly recovered.

## Concluding Remarks

A Navier-Stokes calculation procedure has been applied to the problem of an isolated airfoil oscillating sinusoidaily in pitch. The calculation procedure has solved the governing equations by an implicit technique with turbulence represented via a mixing length model. A highly stretched grid was used to resolve the turbulent boundary layer. Although further effort remains in regard to the turbulence model, boundary conditions, etc., the resulting calculation showed many of the observed flow field features, includirg the vortex shedding and separation processes. In addition, comparisons between predicted and measured surface pressure distribution showed strong qualitative agreement.

## diknowledgeent

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## ABSTRACT

Some physical features of unsteady separating turbulent boundary layers are presented for practical Reynolds numbers and reduced frecuencies for helicopter and turbomachinery ilcws. Upstream of detachment in moderate amplitude flows, the flow is quasi-steady, i.e., the fhase-averaged flow is described by the steacy free-stream flow structure. Results presented here show that cscillation waveform ana amplitude strongly intluence the detachec flon benavior.

## i. IITAFCLCTIOR

insitady :urbulen: tundary lajers are of corsiceratile interest because of unsteady dercc:rmic phermena asscciated with olades in rometessors wo surbines are with helicopter ro:ors in :ransid:ing motion. They are par:icblarl, :ripor:an: auring high litt or loading corcelions wher separation may be presen: during a por:upi ct the uscillation cycle. Inder suct coclitons there is significant mieraction Detmee. the thich torbulent shedr iayer and the iriscic licn. Trus. infomation on the siruc:ure $=$ stwarating ureteady iurtulent shear arees is recessary for the uncerstancirg anc * Fere calciation c: :liese oracitical flows.
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detached flow behivior. This is an important result since some practical unsteady flows have non-sinusoidal waveforms.

In spite of its importance, relatively littie fundamental work has been done to cescribe the structure of unsteady separating turbulent shear layers. To this writer's know?edge, measurements of :urbulent Reynoids stresses in an unsteady separating turbulent shear flow have been made only at Sriu and by Cousteix et al. ${ }^{7}$ dt O.N.E.R.A. in France. Pariin et al. ${ }^{3}$ and Jayaraman et al. g at Stanford examined some cases with smali amourts of near-wall reversed flow during part of a cycle, but with ro flow detachment. The arplitude to rean tree-stream velocity ratio was about 0.3 , 0.15, ard 0.3 in these investigations, respectively. Direct measuierients of the uscillating Reynolds shearing and nurrial stresses, surface shearing stress, and fraction of time backflow was present were made at practical Reynolas numbers and reduced frequencies ty the sMU group.

As background for this paper, a brief summaty is given in section $!1$ or the nature a sieady freestream separating turbulent boundary layer as determined oy the work of simpsen et al.2.? and shilon et al.:0 at sp. The resuits trom the sinusoldally unsteady senaratil: turbuler: shear is,er of smpser eedi.: 5 are discissed if sec:un $: \therefore$ : seciler $:$ : presents the resculis orote recert worn trat -ndicate that the co.".sition wateome anc arollituce strencly terowine the senasice $u^{*}$ a to:aching iurtuien: setar layer.

mittent transitory detachment (ITO) occurs with backflow 20 ; of the time; transitory detachment (TD) occurs with backflow 50, of the time; and detachment ( 0 ) occurs where the time-average wall shear stress is zero.

Figure 1 shows a oualitative sketch of the second steady-freestrear bottom wail turbulent separater flow stuales at SMU and the locations ut iD, ITO, and I when determined 1 nim from the wall. The mean flow upstream of :0 obeys the "iaw uf the wal!" anc the "law of the wake" as iong as the maximul snearing stress -ouv fiax is less than $: .5$ iw, where $i_{w}$ is the local wall shearing stress. when $-i \frac{\overline{u v}}{\text { max }}$, 1.5 ; $w$, the Dury and schefiela'? mean velocity profile cirrelaion and the lis of ine wall apply wic:ream at :TD. LD to one-third of the turbuierce energ; production ir the ater tecict: is Wur : remal seresses eptec:e. which modities : Me reictions between dissipe:iun rate, furbu:erce ene:qy. and :urbulen: shear stress :nat - Moscrved pareher upsereat.
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scanning laser anemometer reveal a relatively coherent structure of the large eddios. 14 The backflow does not come from far downstream. The frequency of passage $n$ of these large-sciile structures varies is $U_{e} / \mathrm{c}$ and is about an order of magnitude smaller than the frequency far upsiream of detachment. Reynolds shearing stresses in the backflow must be modeled by reiating them to the turbulence structure and not to local mean velocity gradients. The mean velocity profiles in the backflow are a result of : ime-averaging of the large fluctuations and are not related to the cause of the turbulence.

Commonily used ilurbuience mocels for attached flows do not work well for separated flows. Edton ${ }^{15}$ diccussed computations of the Simpsori et al. ${ }^{3}$ flow that vere made for the :980-1981 AFOSR-HTTM-Stanford Conferences on complex furmulent flows. The integral, eddy viscosity, and bixing length me'hous triat were used were essentially attached boundar: layer methnes with some dc hoc change that woulc make :ne setacned flow mean velocity profiles look ghod. Urforturatoply, :he calculated 2eynolds shearimy seresses were muct: iower inall the cata. One $\mathfrak{y}$ equation model was used shat betier predic:ed the shearirg stress, but overoredicted the jrow or rase of ere srear idyer. Collins ara Sinasor it also showed ind wher s:tached flow mirneg lengen ndede: are imposed on detachate flows, low ming lengths and iow snear siresses -ust be bees thorce to get good rear velocl:y frolies.













given phase wt ot a cycle.; Figure 3 shows the sinusoidal waveform $\dot{U}_{e} \bar{U}_{e}$. Upstream of where intermittent backflow begins $\left(\gamma_{p u}<1\right)$, the flow and turbulence structure behave in a quasisteady manner.

After the beginning of detachnent, large amplitude ard phase variations develop through each flow and the structure is not quasi-steady. Unsteady effects procuce hysteresis in relationships between flow paraneters. As the frees:ream velocity during a cycle begins :o increase, the fraction of tine that the flow Poves downsiream ruu at a given phase of the cocie increases as backflow fluid is wasted Nownstrean. As the free-siream velocity nears :ne maximur value in a bycle, the increasingly wcierse pressure gradient causes progressively gredter near wall backflow at downstream loca: ors while you renains high at the upsiream fart of the cetacher 'luw. After the freecorne itioci:g begins or cecelerate, the icasiar where piuw reversal begins moves ats:!tat". This gycte is repestes as the free-

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averaged backfiow is greatest when ipu is low and when $\hat{u}^{2}$ and $\hat{v}^{2}$ are near maximum values. In other words $\hat{U}$ in the backflow is nearily in phase with $\hat{u^{2}}$ and $\hat{v^{k}}$. This is consistent with the general observation from the steady flow that $\vec{u}^{2}$ and $v^{2}$ are greater when there is more miean backflow.

The stead; flow resul:s show that $-\overline{u v} /$ decreases with decreasing $\bar{\gamma}_{\text {Du }}$. In the steady flow -uv is greater with less ensembleaveraged backflow or greater ${ }^{\text {y }}$ fu. In other words, $\dot{b}$ is lower and $\dot{U}$. is nearly ir phase witt -uv.r. is in the steady iree-strear flow, -üv/ $\sqrt{u^{2} v^{2}}$ decreases with decreasing 'fu' dithough there is some hysteresis and phase lag for the unsteady ficw.

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tne experiments discussed above，an axiai－ compressor－type waveform，and a large amplituae usciliation，＇yhich were proauced in the jxill wirid ：unnel．The parameter $R=\left(U_{e}\right.$ rax $\left.-U_{e} \min \right) / 2 \mathrm{C}_{e}$ is 0．3．0．238，and 0.752 for these cases． respectively．in each case exallured the mean tree－stream velocity if was the same rear the test－section throd：（1．6čri）．

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## ACKNOWLEDGMENTS

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## AD-P004 165

## Abstract

The evolution of unsteady roundary lavers on jsci:lating airfoits is studied. T,re computational fifficulties associated with the movement of the ctagnation noint as a function of space and time are so'ved by using a novei rumerical schene. ralculations are performed for pressure cistributions tyoica! of those founc near the leadinn edge Df airfills. Gesults are mresented for two cases. in :he first, solutions are obtainet for fiow with seraration and with resescriten pressure distrthution: thev infer that a sinnu?arity develons and is of the same type as that ntserved on a circu'ar cy'imer startec inpulsive?y from rest. In the secorn, results are ohtained for the sare flow in. the viscous flow so? in icns are interacted with -he externa! fiow thy using an inversp houndarv'uner rethor. The imteraction seems to remove the sinnu'ariv, hovever, "hese results are rrelimimar" and need in he checte and improved yoon.

## 1. 0 :n+rnciuctinn

r recen: 'ears some manortant tevelopments nave iccurred in the thpory of tve-difersinnal iar*mar montarv-laver ficws. the erisia' disccuer., the " . : "cracelen and cher", is bhat the solu-- igm ga eme moumeary-layer encuaciems wien moundary enctiones corresponcira to a circlitar cylinder :ar:ed irpulsively erom rest tevelons a simgularity. This, with th semeting the kelceity of the cyomser and a iis rasius. occurs at a tire u... a a A. ant at an arouar tis:ance a illo fron the ograst stancation peime. it this eire the position ? :erostimiricion is $: \quad$ ex close !o, tun mo coimeitem: wion, the singularity. Tmis tiscover: sias mace ty snivimq the cover:inn houndary-
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surface at some stage in the cycle and causes stall to cccur shortiv afterwards. The occurrence of the vortex is probably associated with a breaknown of the unsteadv boundary layer.

The purposes of the present research are to determine the relationship between unsteady separation and singularities in the solution, and :o explore the nossibilities of removing this singularity by interaction of the viscous and inviscid equations. In this naper we outline the current status of the work, report our progress and describe the nroblems shich remain to be solven!. So far we have exarined tre evolution of the boundary laver near tre nose $n^{*}$ an oscil'ating airfoil and found that, wher, the reduced frequency is of the same orter as in the experinen's on dyamic stall. -he unsteady houncary iaver ceases to tehave in a smonth manner just downstream of sedaration ant hefore one cycle has reen combleted; as with the impulsively started circular cylinder, this irregdar behavior sionals the onset of a singularity in the solution of the doundary-layer equations. The equations and solution procedures used in the present study are tescriber in eections 2 and 3 . respectively, and the results are presentes and tiscussed in Section :.

A: aresent we are in tre process of exarining tre lint between this singularity and the externa? fow. Previcus stucies' cf steaty in:eractive soundary 'ayers have been reported for the 'eadingedre region of thin airfoi's of the type consiaered here. :. is the angie of attack. it was observe:: tha: the boundary layer near the nose is well bemaver anc unsepars:ec if : ig 1.15 and ai:hcụ̣" there is significant adverse oressure gracient. i: - igher values of $:$, mewever. separation occurre:" with an assncidted sincularity which recuires :"e use of an intaracitye ehecry. :itith sush a :heary. Ca!cu'aicon or flows with smil separation aresented po fifficu!:ies, tu: at higher values of the solu:ions 'ailed to corverge, orsvicing mo sieady-s:aie sclution. in ipporiam: opiec:ive $\mathrm{s}^{\text {a }}$ ar siutv $0^{\circ}$ iniersecive unsieaty finms is ic ce:erminc me:ter a similar treakdown will accur.
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Usually the boundary-layer calculations for the above equations are performed for prescr ibed boundary conditions jiven by

$$
\begin{equation*}
u(s, 0, t)=0, v(s, n, t)=0, u\left(s, n_{e}, t\right)=u_{e}(s, t) \tag{3}
\end{equation*}
$$

and we shall refer to this as the standard prohlem. in the interactive problem we determine $u_{p}(s, t)$ partly from inviscid theory and partly from the pressure distrioution resulting from the blowing velocity $d / d s\left(u_{e}{ }^{5 \star}\right)$ induced hy the houndary layer. Tius we write

$$
\begin{equation*}
u_{e}(s, t)=u_{e}^{0}(s, t)+u_{c}(s, t) \tag{4}
\end{equation*}
$$

Where $u f(s, t)$ is the slip velocity at the airfoil according to inviscid theory and $u_{c}(s, t)$ is related to the blowing velocity by a varistion of the Hilbert integra?

$$
\begin{equation*}
u_{c}(s, t)=\frac{1}{\pi} \int_{-0}^{\infty} \frac{d}{d s}\left(u_{e}(*) \frac{d t}{s-T}\right. \tag{5}
\end{equation*}
$$

Strictly, Eq. (5) is valid only for straiginc walls but it can be generalized for any airfoil shape as discussed by reiect and Clark ${ }^{6}$. For unseparater boundary layers the effect of $u_{c}(s, t)$ is generally weak but onie sepa-ation occurs, its effect is enhanced sitnificantly in the neighborhood of separatiun. Thet this must he so mav he seen by noting that otherwise the integrand in Ea. (5) would develop a strong singularity at separation and cause the sulutions to break down further downstrean. As discussed in Refs. 7 and 8, it is sufficient to rediare Ea. (5) hy

$$
\begin{equation*}
u_{c}(s, t)=\frac{1}{-} \vdots_{s_{a}}^{s_{n}} \frac{\left(u^{i} e^{i *}\right)}{s-} d v \tag{6}
\end{equation*}
$$

where the prine denotes differentiation with respect to s .

In the standary problem, the solution of poundary-layer equations requires that the external velocity distribution be specified. Since the oresent efiort is uirected toward iolutions near the leading edge of the airfoll. P loral model for the potential flow has theen chosen in the place of a full-notential-fl'a code. He consider an eilipse with major axis $2 a$ and minor $a ;$ is $2 a:(\because \cdot \cdot 1)$ at an angle of attack ilt!. The surface of the hody is defined hy

$$
x=-a \cos { }^{\circ}, \quad v=a \sin i \quad-\infty \quad i+\cdots
$$

san with these definitions and io first annroximation. the external velocity around the ellinse can be teduced fron inviscid $r$ ? ow theory to the

$$
\begin{equation*}
\bar{u}_{\mathrm{e}}^{0}(s, t)=\frac{:{ }_{0}}{\sqrt{1+?_{i}^{2}}} \tag{171}
\end{equation*}
$$

uere $\vec{u}(s, t)$ denotes a dinensionless velocity, is is.-), the parareter tenotes a dimen: sfonless cistance retaied to the $z$ - and $y$-coordinates of the ellipse by $v=1 /=a, y=a-?$ red-
sured from the nose, and $5(\equiv \alpha / \tau)$ represents a dimensionless angle of attack. The parameter is also related to the surface distarce $s$ by

$$
\begin{equation*}
s=a \tau^{2} \int_{0}^{5}\left(1+E^{2}\right)^{1 / 2} d r \tag{8}
\end{equation*}
$$

We next define a dimensionless distance 7 by

$$
\begin{equation*}
\eta=\left[\frac{R(1+\tau)}{2 \tau^{2}\left(1+\xi^{2}\right)}\right]^{1 / 2} \frac{n}{2} \tag{9a}
\end{equation*}
$$

with $R=2 a u_{0} / v$, and a dimensionless stream function $f$ by

$$
\begin{equation*}
f(r, n, t)=\left[(1+\tau) a u_{\infty} v \tau^{2}\right]^{-1 / 2} 2_{i j}(s, n, t) \tag{9b}
\end{equation*}
$$

Introducing these relations together with Eq. (8)
into Eqs. (1) and (2), it can be shown that the continuity and momentum equations can be written as

$$
\begin{aligned}
& f{ }^{\prime}+\frac{?}{1+\tau^{2}} f^{2}+\left(1+r^{2} \bar{u}_{e} \frac{\stackrel{\rightharpoonup}{u}_{e}}{?+}\right.
\end{aligned}
$$

$$
\begin{align*}
& +\left(1+:^{2}\right) \frac{: f^{\prime}}{\frac{i t}{t}} \tag{10}
\end{align*}
$$

Here primes denote differentiation with respect to $n$ and $t 1=11+i u v i / a:^{2}$.

$$
\text { The boundary conditions for } f \text { and } f^{\prime} \text { become }
$$

$f=f^{\prime}=0$ at $A=0$,

$$
\begin{equation*}
f^{\prime} \cdot\left(1+:^{2}\right)^{1 / 2} \bar{u}_{e}\left(r, t_{1}\right) \text { as } \cdots \tag{11}
\end{equation*}
$$

The definition $e f \bar{u}_{e}\left(5, t_{t}\right)$ is given by

$$
\begin{equation*}
\bar{u}_{e}(r, t)=\frac{\left.u_{e} e^{r}, t\right)}{u_{m p}(r+t}=\frac{r+e_{0}(t,)}{\sqrt{i+t^{2}}}+\frac{u_{c}}{u_{0}} \tag{12}
\end{equation*}
$$

where with a prime denoting differentiation with respect to?

$$
\begin{equation*}
\frac{u_{c}}{u_{2}}=\frac{2^{1 / 2}}{--R^{1 / 2}} \frac{\left(1+:^{1 / 2}\right.}{\left(1+r^{2}\right)^{1 / 2}}: \frac{\therefore^{\prime}\left(r \cdot t_{1}\right)}{r-?} d! \tag{13}
\end{equation*}
$$

and the dirensionless sispiacement thickness is given by


Substituting Ea. (18) intc Eo. (13), and using Ea. (12), the eicge boundary conctition in Eq. (11) can

where

$$
\begin{equation*}
\varepsilon=\frac{1}{7 \tau}\left[\frac{2(1+t)}{R}\right]^{1 / 2} \tag{16}
\end{equation*}
$$

To complete the formulation of the problem, initial conditions must be specified in the ( $t, n$ ) plane at some $s=s_{0}$, either on the lower or upper surface of the airfoil as well as initial conditions in the plane on both surfaces of the airfoil. In the latter case, if we assume that steady-flow conditions prevail at $t=0$, then the initial conditions in the ( $s, n$ )-plane can easily be generated for both surfaces by solving the governing equations for steady flow, which in this case, are given by Eq. (1) and by

$$
\begin{equation*}
u \frac{i u}{i s}+v \frac{\vdots u}{i n}=u_{e} \frac{d u_{e}}{d s}+v \frac{i^{2} u}{i n^{2}} \tag{17}
\end{equation*}
$$

There is no problem with the initial conditions for Eqs. (1) and (17) since the calculations start at the stagnation point.

The generation of the initial conditions in the ( $t, n$ )-plane are not so straightforward to obtain as is discussed in section 3.1.

### 3.0 Solution Procedure

The solution procedure for the set of equations and boundary and initial conditions given in Section 2 can 0 behieved in two parts conce:ned, respectively, with the leading edge and downstream region. These two parts are considered in the following two subsactions. In both cases the solution procedure makes usc of Keller's box method, which is a two-point finite-difference scheme extensively used for the solution of parabolic partial-differential equations, as discussed by Bradshaw et al.'.

### 3.1 Leading-Edge Region

The generition of the initial conditions in the ( $t, n$ )-plant at $s=s_{0}$ requires a special numerical procedure. Given, as we are, the complete velocity profile distribution on the previous time line, there is, in principle, no difficulty in computing values on the next time line by an explicit nethod, but if we wish to avoid the stability oroblems associated with such a method by using an inplicit method, we are imediately faced with the problem of generating a starting profile on the new tine line.
in order to explain the problem further, it is instructive to see what happens to the stagnation point as a function of time. For this purpose
 of the form $\mathrm{o}\left(1+A \sin ^{-1} 1\right)$, and let $\bar{u}$ us'u.11. : ), then Eq. (7) becomes

$$
\begin{equation*}
\vec{u}_{e}\left(\cdot t_{1}\right)=\frac{i_{0}\left(1+A \sin ^{-} t_{1}\right)}{1+4} \tag{18}
\end{equation*}
$$

where $\bar{\omega}$ is related to the dimensional frequency by

$$
\bar{\omega}=\frac{a \tau^{2}}{(T+\tau) u_{c s}} \omega
$$

Here 5.0 and $A$ denote parameters that need to be specified. Since by definition $\bar{u} \ell=0$ at the stagnation point, its Incation, ${ }_{5}$, is given by

$$
\begin{equation*}
r_{s}=-r_{0}\left(1+A \sin \bar{\omega} t_{p}\right) \tag{19}
\end{equation*}
$$

and so the upper and lower surfaces of the airfoil as functions of time are defined, in particular, by $r_{5}>5_{s}$ and $5<5_{s}$. For example, let us take $A=1$, $w=\pi / 4$ and piot $F_{, ~} /{ }^{5}, 0$ in the ( $t, \xi$ )-plane, as shown in Figure 1 for one cycle $(0 \leq t 1 \leq 8)$.


Fig. 1. Variation of stagnation point with time for one cycle according to Eq. (19), with $\bar{\omega}=7 / 4$, $A=1$.

When $t_{1}=2$, the stagnation point ${ }^{5}$ is at $-2_{0}$, when $t \mid=6$ is at 0 , etc. If 's were fixed, we could assume that $u=0$ at $:=0$ for all time and all $\eta$, but this is not the case. it is also possible to assume that the stagnation point is coincident with zero u-velocity for a prescribed time. However, we should note that the stagnation point given by Eq. (19) is based or the vanishing of the external velocity. For a time-dependent flow, this does not necessarily imply that the u-velocity is zero across the layer for a given "-location and specified time. This point is substantiated by the results shown in Figure 2 taken from Ref. 10 and obtained with a novel numerical procedure called the characteristic box scheme. it is also evident from Figure ? that flow reversals do occur due to the movement of the locus of zero u-velocity across the layer. This causes numerical instabilities which can be avoided by using either the zig-zag box or the characteristic box finite-difference scheme. The details of these numerical schemes have been reported in Ref. 9 and, with special reference to oscillating airfoils, in Ref. 10.

### 3.2 Nownstream Region

A solution to the leading-edge region, cbtained by the procedure of Section 3.1, may be used us initial conditions for the solution of the system of equations given by Eqs. (11) and (15) both with eitner standard or inverse procedures. in


Fig. 2. Velocity profiles in the immediate neighborhood of the stagnation line at different times for $\bar{J}=-1 \dot{A}$ and $A=1$. The dashed lines indicate the locus of zero $u$-velocity across the layer.
practice a standard procedure is used up to a specified :-location after which the calculations may proceed by either standard or inverse procedures. For example, to study the evolution of the boundary layer on an oscillating airfoil with prescribed pressure distribution, we use the standard procedure and where the inviscid and viscous flow equations are solved interactively, the inverse procedure is used after a short distance from the leading-edge region.

To solve the equations for both standard and inverse problems we use modified forms of Keller's box scheme. In the latter case we use the Mechul function formulation ${ }^{9}$ which treats the external velocity as an unknown.

The box scheme reduces Eq. (10) to a firstorder system. With $\bar{u}_{e}, f^{\prime}$ and $f^{\prime \prime}$ represented by $w, r$ and $v$, respectively, we write

$$
\begin{align*}
& f^{\prime}=r  \tag{20a}\\
& r^{\prime}=v  \tag{20~b}\\
& w^{\prime}=0 \tag{20c}
\end{align*}
$$

and obtain

$$
\begin{align*}
& v^{\prime}+\frac{-}{1+r^{2}} r^{?}+\left(1+r^{2}\right) w \frac{i w}{?}+\left(1+r^{2}\right)^{3 / 2} \frac{i w}{i t_{1}} \\
& \quad=r \frac{r}{?}-v \frac{f}{?}+\left(1+r^{2}\right) \frac{r}{t_{1}} \tag{204}
\end{align*}
$$

With this notation, the boundary conditions given by Eq. (11) can be written as

$$
\begin{gather*}
f=r=0 \quad \text { at } \quad=0  \tag{2la}\\
r_{e}=(1+-2)^{l / 2} w_{e} \text { at }=-e_{e} \tag{2lb}
\end{gather*}
$$

For the standard problen $w_{e}$ is $k k_{i} w n$ and is given by Eq. (18). A fourth boundary condition is repuired for the inverse problem and is obtained from

Eq. (15). Introducing a discrete approximation to Eq. (15), it can be written in the form

$$
\begin{equation*}
r_{e}\left(\varepsilon_{i}\right)-c_{i}{ }^{\varepsilon A\left(r_{i}\right)}=g_{i} \tag{22}
\end{equation*}
$$

where $c_{i j}$ is the matrix of interaction coefficients defining the relationship between the displacement thickness and external flow and the parameter $\mathrm{g}_{\mathrm{i}}$ represents terms whose values are assumed to be known. It is gi en by
$g_{i}=\bar{i}_{i}+r_{0}+\varepsilon \sum_{k=1}^{i=1} c_{i k} s\left(r_{k}\right)+\varepsilon \sum_{k=1+1}^{i} c_{i k} s\left(r_{k}\right)$

The system of Eqs. (20) to (22) has been solved by the numer:cal procedure of ref. 9 for the standard and inverse formulations.

### 4.0 Nature of the Singularity for an

One phase of the calculations for the oscillating airfoil was carried out by choosing ${ }^{5} 0=1$, $A=-1 / 2$ and $=0.1$. With these choices the maximum value of xeff, defined by

$$
\begin{equation*}
r_{e f f}=(1+A \sin -1) \tag{24}
\end{equation*}
$$

is sufficient to provoke separation with a strong singularity if the houndary layer were steady. At present $: 0, A$, Tare being varied to examine their effect on the nature of singularity.

The unsteady flow calculations displayed in Fig. 3 show that the boundary layer eventually separates, the flow remaining smooth. However, just downstream of separation, it is evident that 3 sin . gularity develops in the solution in the neighborhood of : $=2.12$ and $\mathrm{T}_{1}=308.75^{\circ}$ and that it is not possible to continue the solution beyond this time without conceptual changes in the mathematical and physical formulation of the problem. While this is a satisfying conclusion, and nay be interpreted as giving theoretical support to experinental observations of dymamic stall, it should be


Fig. 3. Computed results for the oscillating airfoil, $A=-1 / 2, \bar{\omega}=0.1$. (a) Displacement thickress *. (b) Displacement velocity, $d / d r$. ( $\bar{u}_{e^{i *}}$ ). (c) Wall shear parameter, $f_{W}^{\prime \prime}$.
treated with some caution. Boundary-layer singuTarities have been the subject of much controversy in recent years and it is clearly important to make sure that any irregularities in a computed solution are not creatures of the numerical method used. :!c, however, feel confident that the calculations reported here are accurate and that the singularity is real.

Figure 3a shows that the variation of the displacement thickness

$$
\begin{equation*}
\because=\frac{i *}{a}\left(\frac{1+i}{\square}\right) \frac{1}{5} \tag{25}
\end{equation*}
$$

is generally smooth except in the neighborhood of $=2.12$ and for $-\mathrm{T}_{1}=308.75^{\circ}$. The first sign of irregularity is the steepening of the slope of : when ${ }^{-} t=300^{\circ}$. A local maximum of $*$ occurs at " $=2.12$ when ${ }^{-} t_{1}=308.75^{\circ}$. When the same results are plotted for a displacement velocity, (d/dr) ( $\mathrm{u}_{\mathrm{e}} \cdot *$ ), (Fig. 3b), we observe that the steepening of the displacement velocity near: $=2.12$ is dramatic. For exanple, for $\mathrm{t}_{1}=300^{\circ}$, the peak is at : = 2.125, for $t_{1}=305^{\circ}$, it is at $=2.105$, for It $=307.5^{\circ}$, it is at $=2.09$ and finally for $\mathrm{T}_{1}=308.75^{\circ}$, the peak moves to $=2.08$. it should be noted that the maximum value of displacenent velocity moves towards the separation point with increasing ${ }^{-} t$; the same behavior will be shown to occur for the circular cylinder discussed below.

As shown in Fig. 3c, the wall shear parameter $f_{w}^{*}$ shows no signs of irregularity for $\overline{-t} 1-308.75^{\circ}$ but a deep ninimun in $f_{w}^{*}$ occurs near $=2.15$, i.e. near the peak of ${ }^{*}$.
:t is interesting and useful to compare the results presented in Fig. 3 for an oscillating airfoil with those obiained for a circular cylinder started inpulsively fron rest. This comparison
lends support to the accuracy of the present caiculation method and at the same time enables us to compare the characteristics of two distinctly different unsteady flows near the singularity location. The circular cylinder problem has been extensively studied as reported in Refs. 11 and 12 and the present results shown below are in close agreement: with those of previous authors, but with subtle differences which may have important implications.

Figure 4 shows the results obtained by Cebeci $i^{2}$ for the circular cylinder problem. As in the case of the oscillating airfoil, the flow separates and remains smooth up to the separation point. However, just downstream of separation with increasing time, a singularity develops in the neighborhood of $n=112^{\circ}$ and $t=3.0$ and it was not possible to continue the boundary-layer calculations beyond this time and angular location. From Fig. 4a we see that while the variation of displacement thickness is smooth for values of a less than $108^{\circ}$, it begins to steepen dramatically thereafter. The same results are plotted in Fig. 4 b to demonstrate that, as in Fig. 3b, the displacement velocity exhibits a maximum which increases rapidly with $t$ ime. Aga in the maximum shift towards the location of separation with increasing time.

The results of local skin-friction cuefficient calculations in Fig. 4 c follow similar trends to those obtained for the oscillating airfoil. In buth cases, the distributions pass through zero with no signs of irregularity and do noi exh ibit any breakdown before the time corresponding to the singularity.

The very careful calculations of van Domelen and Shen are reproduced on Figures 5 and 6 for displacement thickness and velocity profiles, respectively. The corresponding displacenent thickness results of Cebec $i$ together with the new calculations of the velocity profiles are reproduced for


Fig. 4. Computed results of Cebeci ${ }^{2}$ for the circular cylinder. (a) Displacement thickness $\{* / L$. (b) Displacement velocity, $\mathrm{d} / \mathrm{d}^{\circ}\left(\mathrm{u}_{\mathrm{e}}{ }^{〔 *)}\right.$. (c) Wall shear parameter, $\mathrm{f}_{\mathrm{w}}^{\prime \prime}$.


Fig. 5. Comparisor, between the displacement thickness values obtained by van Commelen and Shen (circles) and by ceheci ${ }^{2}$ (solid line) for the circular cyinder. (a) $t=2.0,(b) t=2.5,(c) t=2.75$. ( $x$ is in radians.)

(a)


Fin. 5. Comorison between the velocity profiles obtained by van iomelen and shen. (solid lines) and by Cebect (symbols). (a) ?.75, (b) $t=9.984375$ (van Dormelen and Shen) and $t=9.0875$ (present calculations).
comparison purposes. As can be seen from Fig. 5, the agreement between the sets of calculations for three values of $t=2,2.5$ and 2.75 is excellent. The velocity profiles of Fig. 6a, which correspond to a time $t=2.75$ as in Fig. 5, are also in excellent agreement for various angular locations. In contrast, the calculated velocity profiles of van Dormelen and Shen ${ }^{13}$ at $t=2.984375$ show differences from the present results obtained at $t=2.9875$. The figure confirms the expected close agreement of the two sets of results at the two smallest angular locations, but significant differences at the two highest values. The trend is different in that the present results show that the location and the magnitude of the maximum negative velocity increases with angular location. Also the tendency for flattening of the velocity profiles in the vicinity of the singularity is not confirmed by the present results.

Figure 7 shows the velocity profiles obtained by Cebec $i^{2}$ for two values of $t$ as a function of angular location. It is clear that the magnitude of negative velocity increases with angular location and suggests that as the singularity is approached, the magnitude of the negative velocity will tend to infinity.

Figure 8 allows comparison of the displacement velocities obtained by Cowley ${ }^{3}$ and by the present method for four values of time. Ye would expect, from the previous comparisons that the two sets of results would be in close accord at least for times

(a)


Fig. 7. Compuced velocity profiles for the circislar cyinder according to the calculations of Cebec $i^{i}$. (a) $:=2.5$, ( $t$ ) $:=2.75$. ( $x$ is in radians.)
up to 2.75. The figure shows the expected close agreement until the naximum value is approached. The discrepancies apparent at higher values of $\theta$ cannot readily be erplained, and it should be noted that the location and time of singularity occur at different values of $\theta$; the results of Cowley and van Dommelen and Shen appear to agree in this respect. The reasons for these discrepancies are presently under investigation.

### 4.0 Effect of Interaction on the Singularity

The interaction procedure discussed in Section 3 has been applied to the flow problem previously examined in Section 4 with the standard method. The results are shown in Fig. 9 and are discussed below. In contrast to the standard problem which makes the implicit assumption of infinite Reynolds number, the interaction requires the specification of a finite Reynolds number. In addition, a thickness ratio t has to be specified and, since the definition of $\varepsilon$ involves $R$ and $\tau$, the calculations are performed for a specific value of $\varepsilon_{,}(\equiv 4.5 \times$ $10^{-3}$ ) for the present results. Other values of $\varepsilon$ are being examined to determine the effect of combinations of Reynolds number and thickness ratio.

The present calculations were performed in the following way. For all values of time with $\bar{w}$ t ranging from 0 to $360^{\circ}$, the standard method with the leading-edge region procedure of Section 3 was used to generate the initial conditions at a short distance from the leading edge, $5=0.5$. With these initial conditions and for each value of $\overline{\mathrm{u}} \mathrm{t}$, the inverse method was used to calculate the unsteady flow from : $=0.5$ to $5=5.5$, for the specified value of $\varepsilon$. Since the system of equations is now elliptic, several sweeps in the --direction were necessary to aci,ieve a converged solution. Where flow reversal was encountered, as happens for values of $\overline{\mathrm{J}} \mathrm{t} \boldsymbol{\rho}>270^{\circ}$ and $\mathrm{r}>2$, up to three sweeps are required; where separation did not exist, a single sweep was sufficient. It is to be expected that the value of c will influence the number of sweeps and, since it is linked to physical parameters, will affect the singularity and the size of the bubble.

Figure 9a shows the variation of displacement thickness "* and Fig. 96 the wall shear parameter $f_{b}^{\prime \prime}$ as a function of nondimensional distance - and time. It is evident that for values of © < 2.5 , the solutions are well thaved. As expected, the fisplacement thickness $r$ increases with for all values of time and reaches a maximum around $t_{1}=300^{\circ}$ as a consequence of the change in the angle of attack. in the same range of t, the shear stress parameter decreases for all values of ${ }^{-} t$ and reaches a ninimum corresponding to the maximum in displacement thickness.

For values of - 2.5 , the solut ions rema in well behaved until around $\mathrm{T}_{1}=290^{\circ}$. The general trends are in accord with the expectations and there is negligible difference between the results obtained with the standard and interactive nethods for values of ${ }^{-1} t$ up to the maximum for which the standard method alloked solutions. The wiggles apparent in the solutions for high values of ${ }^{-1}$ rema in to te explained. in particular, the influsence of numerical parameters such as $:$ and ' $t 1$ spacing together with the assigned value of, need to be systematically explored. Yevertheless, it is inportant to note that the singularity no longer


Fig. 8. Comparison between the displacement velocity values obtained by Cowley (solid lines) and by the present method (dashed lines)for the circular cylinder. (a) $t=2.5$, (b) $t=2.6$, (c) $t=2.7$, (d) $\mathrm{t}=$ ? .8 .


Fig. ${ }^{a}$. Effect of interaction on the variation of (a) displacement thickness :*. (b) wall shear parameter for an oscillating airfoll with. $=4.5 \times 10^{-3}$. Solid lines in the insert represent the resuits obtained by the standard method and dashed lines those by the inverse method.
exisis and. in contrast to the standard method which allowed the solutions to be performed up to - 1 . $308.35^{\circ}$. the calculation: with the inverse method were performed for complete cycie. it is also inportant to note that the chosen value of, inplies a high Reynolds number (. $10^{\mathrm{f}}$ ) for a thict. ness ratio of : 0.l. Since typical leadingedge bubbles are associated with lower values of ?eynoids number, we expect that interactive calculations can be performed over a wide range of angle of attack without pronieas associated with the singularity.

## 6. 0 Acknowledgment

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## Abstract

Steady and unsteady velocity components over a backward-facing circular arc are neasured by LaserDoppler Velocinetry. A periodic disturbance is added to the mean flow and the response of unsteady separation is investigated. Special attention is given to the distribution and the flux of vorticity.

## 1. Introduction

derodynamicists are interested in the phenonenon of separation because it controls the behavior of separated flows and therefore the loading Jf airfoils and phenoinena like steady or unsteady stall. The study of unsteady separation has generated sone controversies, nostly because of the widelf varied tools of investigation and quite aften because of confusing terininiolagy. The significance of the phenomenon lies in its interac:ion with the entire flow fielh. Its liosal properties may explain the detailed struciure of the 'low but these should only be viewed as a step :owards the understanding and eventually predic. :ion of the global effects. In this paper we disziss our initial experinental efforts in this fires:tion. We study now the vorticity generated in the boundary layer is convected and shed in the free strean. iforebver, we are interested in the interaction of the separated free-shear layer with the dead water and the rigid wall.

Experinental vori on unsteady separation has been initited in the past faw decadys followigy :7e plonearing work of Sears ${ }^{2}$ Moore and zott ${ }^{2}$. ins experinental work of didal and Ludwicy per:ilis onlfg to stady flow, and the wark of Jespilard and Miller is confined to hign frequency oscilla:ury flow. nur anumladye at the beginning of the 1970's 101s: inis complex phenomenun was therefore reiaradil, adrrum and sapiously in need of nore ine ensive experimental investigation. Some projres; vas acnieved by nethois of flow visualiza:1 an and by hot-wira aneworeter technifues. Sush nethods 7 age alreddy been enployed oy Senesus et si . derlé, Rulter. Vagis and Feter and Menruse ey to sidy ansteddy viscous (low phenotent. Dut :he specific ases consliered athathe seat: n! :ne mokelis aere desifined ior a stidy of :he ansle "um fiely. in ali these sidiles the Dinsary iders aer: s: :nth ende it was essem.




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aspects of the flow. This was accomplished by flow visualization methods and water-ylycerin inixtures to achieve thich. boundary layers with not so small velocities. Steady flows aver moving surfaces were first examined in an open channel. Unsteady effects, transient and oscilatory were later conducted in a closed water tunnel designed and constructed for this purpose, with two different systems of pressure disturbance generators.
: Hore recently, :Aezaris and Telionis ${ }^{14}$ ootained detailed LDV measurenents over a rearward facing circular arc. In the work of Ref. it, unsteadiness is introduced by a flap puisating over the circular are. The nodel enployed in the present study is the same with the model of Mezaris and Telionis and will be described in the indin body of the paper.
it is weil snown that the diplitude ratio is salled for a flat plate by the irequency parameter $k=$ ase where w, $x$ and if are the erequency, the distance along the wall and fipe free strean velo.. city respeciively ( (ightnili ${ }^{5}$ ). dith increasing $x$, or equivalently .., the anplitude profiles have been proveif analytically and experimentelly (Telionis ${ }^{\text {IF }}$ ) to nave smaller oversnouts wion mpproach the wall. The oscillatory gart of the flom tends to becure pluz ilow ant :he stobes layer is conflied cluser and closer to ine wall. This is not the case if the adearse pressare jramtent increas.es with y (fef. (if). in fact, the fitd of tezaris and relfonis in indicate overonouts abou: in times lerger than the apllide of : ex outer fion.
in the present paper ate interguce the alstarbance in the uncoming si-edn rather :nan :heogizn 1 pulsating fian. de :hen easilne wore carefsily the fiow in ine dicintly )! seiarlitum ly a mure puwerfal atat acquist:iom sfsean anl: wernies Trefuetcy doasin ansiysts is ael is Fospier serles exadasimes li sil dave ojr is. Morejoren.

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i3. I The jp: water tunnel showing the recent nodifications: (1) i rotating vatit friven oy a variable speed notur and (b) a by-pass system with d control valve.

## 2. Facilities and the Mode:

To conduct the research reported in this , mper, sote modifications and additions were ferassary. The bastc tasix was to convert the tunnel tu in unsteddy ater tunnei. Disturbances aer: intruduced upstreain of the setiling chamber by a rotating rane (fig. 1). for the past 5 yesris. our experimental wori on unsteddy serodyndanics as perforsed with stesty stream. Uni: =. 11 ll ness was introduced by dynanic notion of the oted 3 p part of it. Modifications were proposed ant carried out :i) conver: the tannel to an oscil1::inj :unnel. ints, of course, required extes oftiorts to reduce the ispoulence in the tunnel and retisliarat: the faclitity. ©o contro! the rean fisw. I rotsting vane was ins:alled imediateij dowre the : est section ss shown in fig. 1 . inis 1:na was couplej tu a HELLEK DC wotor win vaplshle soces contro!. The unit controls utana:l:11: the syeri : 0 -inin: 3.5 : of :me set salue. : : is itso ejupped alith an optlesl eacoder writen CH se inter!sced atrecily at: the insorstary -0ryser.
: very slmodcant eactur 11 stadies of





 tebles : cuntrji ingesendent ly buth the ampll.








Any perijaicity externally added to tannel generates free-streain turbulence. iany existing unsteaday flow facilitios uperate with turbulence levels of the order of ! \%. Careful stautes rejuire nuch lower iurbulence levels. To iutet the fiow in our tunnel we, $\mathrm{a}^{\text {thend }}$ died the literature on honeycombs and screens 18 -22 and cogtacted iersonally an exper: in the fleld (Nagion'). is a result of our investigations. we instalied in the settiling inamber a secund set of finer honeycombs and isets of fine screens.

Typical -esulis are snown in Tables 1 and 2. obtsined with an LDV tracker and counter respectively. in this table, the tinnel speed and the by-dass valve are controlled independently. $i:$ is surartsing that the by-pass system infliences greatly the tursulence level, even thouyn $i$ : is far spstreat of the lest sectlon. de hate sisg performes an exhausitive stady of :argulence :-n. quency specirs ensiples of anich are inclades in AC! . it.
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 rout man square af the stanal and if is the turbalence level. The by-pass opening is also indiked in




 -. $93 \%$

[^2]Convergence, the boundary layer develupes like, $q$ flat plate 3lasius layer (Mezaris and Tellorits ${ }^{14}$ ) Furtier downstrean, the top surface diverges to generate a region of adverse pressure jradient. However, the botton plate is continued further downtrean. In this way the separated region is not affected by mirror linage separation. It is recalled that flows about symetric bodies, as for exanple a circu!ar cylinder are controlled by the fiteraction of two shear layers with opposite sitins of vorticity, whicn eventually results into periodic snedding of large scale vortices. The situation is fifferent in the case of an dirfoil tt an angle of attack, whereby the separating flow over ine suction side develops with little or no influence of the trailing edge vorticity. This s exactly the situation which is simulated by our rij. Yedsurenents are conducted on the diverging section which has the shape of a circular arc as shown schematically in Fig. ?.

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profiles of the u-component of the velocity, mean and amplitudes respectively.


Fiz. 9 Selected velucity wavefor is $1: x=50 \mathrm{~mm}$. Front to hack iurresonit to: $=6,5.2,4.8,4.2$, $\therefore .3$ and 2.8 mm .

=13. 10 Reduced medn relucli:y profiles at stations $=$ i) Enrougd gonm.
instantaneous jrofiles st adeh station das fur any of the phase points fir which data are stored nave deen plotied and sisdied. de selecied :o display nere in Fiz. 12 sixicen profilees ajuispaced wi:nin a period di the flrs: pos:redin siation. $x$ dsme. It wilich the point fi eero sain friction vanispres. at inis station. :he skin frizion is diways positive and its mini un value is zeru. Another station of sijntflcance for the varlation a! zero skin fricilun is the location at Which the ixin friction is slwajs negative Dut its naxinun is equal to zern. Enis uicurs approxinately at the sistion, $x=65$ an (Fig. 13). Incidentally, ficiursing ts the definition of Despara and alliers. :nis is the ixation of separation. Howevar. inis is villid for nigh opouyn frepuencies © ar whin ine point of separation dues not respond $\therefore$ ine periodic flustustions of the velocity.
*) deteratine : te nor atal :ompunent of the vel 厄ity. ine Eypunen: in a sirec:ton intifnes oy


?onent $v$ can be calculated by a simple aigebraic formula.


Fig. 12 Velacity profiles at $x=45 \mathrm{~mm}$ for 16 values of the perind.


Fis. 13 Velocity profiles at $x=65 i m$ for 15 values of the period.


F1., It ine wayeforas of displacement thicaness. The syinbuls 0, © , $\because, x, \$, 4, x$ corresponi to stations 9 tinrubinn $;$ in this jruer. The period of ascillition is 5 sec.
stream. This is the well known property of boundary layers whereby outer flow decelerations result in thickening of the boundary layer. However, further downstream a peculiar behavior is observed. The minimum of the displacement thickness snifts upstream to about the quarter point of the period until $x=45 \mathrm{~mm}$. Downstreain of tnis station the minimuin and therefore the entire waveforin of the alimost sinusuidal signal shifts ayain towards the midpoint in the phase. This suggests that the station $x=45$ inn is the origin of a wavelike phenomenon which propagates periodically in joth directions, upstreain and downstreain. Incidentally, this station is the station of the apper:nost position of zero skin friction. Tnis phenomenon is more clearly illustrated in Figs. 15 and 1 o where the displacement thickness is plotted versas :he akitl distance, with the period appearing as a arameter.

5. The Snedding of Vorticity

A classical relationship between circulation, r. and vorticity, 2 , dictates that

$$
\begin{equation*}
\Gamma=\iiint_{d} A \tag{2}
\end{equation*}
$$

where $A$ is the ared contained by the contour of integration of the circulation integral. Howarth (see discussion in Ref. 24) has demonstrated that if the area is chosen appropriately, the rate of shedding of vortiticy at separation beconles

$$
\begin{equation*}
\frac{d \Gamma}{d t}=\int_{0}^{\delta} \Omega u d y \tag{3}
\end{equation*}
$$

which within the ooundary layer approximation yields

$$
\begin{equation*}
\frac{d \Gamma}{d t}=\frac{1}{2} v_{e}^{2} \tag{4}
\end{equation*}
$$

where $U$ is the edge velocity at separation. This simple formula has been used extensively in the literature of discrete vortex dynamics. The validity of this formula and its possible improvement and extension to unsteady flow is the topic of an on-going investigation at $\mathrm{VFi}: \mathrm{SU}^{\circ}$.

In all the attempts to calculate the strengtn If the discrete vortex via Eq. (4), the position and initial velocity of the vortex are arbitrarily assigned. The present group is workiny on an interactive approach wion will allow the calculetion of this necessary infurmation. In the presant paper ae have made the first steps towards neasuring the flux of vorticity.

Jur data have been employed so far to calculate the fuantity va/oy inion is equal to vorticlity within the boundary-layer approximation. instantaneous profiles of this quantity have been calculated and are plotted in Fig. 17 for 4 phases of the flow. A lot nore phase values have been plutted but are omitted here due to space limitations. A carefal inspection of Fig. 17 indicates the periodic lift-off of the shear layer. However, it is known that the second tern of vorticliy, the quantity $\quad v / \mathrm{x}$ may not be negligible in ene neighbornood of separation.

In the continuation of our research we plan $\therefore$ calculate the combined effect of the terns is'y and ev/ix. : Moreover we intend to calculate the instantaneous moments of vorticity, nainely the center of gravity of the vorticity and the vorticley flux which will generate information on the posi:ion, $y_{c}$, a agnitude, $\therefore$, and convective velo: lty $U_{c}, V_{c}$ of a discrete vortex equivalent to the tatal vorticity shed by the boundary layer. imese pantities can be calculated by the formulas

$$
\begin{align*}
& \because=0 d y  \tag{5}\\
& f_{c}=\int_{0}^{y d y}  \tag{0}\\
& y_{i}=i \text { udr } \\
& \text { vjy } \tag{3}
\end{align*}
$$



Fi.f. 17 Vorticity profiles for 4 phases of the periodic oscillation. Top to su:tum, $/ / 10,5 \mathrm{~F} / 10$, $91 / 10$ and $13 T / 16$.

I: is proposed here that the nascent vorticies and their initisi ation in discrete vortex dindinics ; tould be deternined in this way to eliminate the arbitrary issumption commonly employed in this theory. The present authors intend to calculate the instan:aneous values of these quantities and compare wi:n interacting boundary-layer calcula:Ions.

## 5. Conclusions

The data obtained su far in this investiga:ion and presected nere display sone inportant trends in the developing of unsteddy separating il uws. Most interes:ing is the fact that the anplitude of oscillations !ncreases farther along the separating free shear layer. Vurticity is therefore juided aday fron the wal! jut it is pul. sa:ing with a vach nituner amplitude than aitnin
the attached boundary layer. Work on this problen is currently deing continued. Sore data are obtained, but the data already stored and presentec nere in raw forn can be used to generste useful infornation. In an expanded version of this paper we intend to present data on the flux of vorticity and circulation and its ultimate fate once separated fran the solid boundary.

## Acin now ledgement

Inis work is supported by 4 OOSK irant Vo. 32:)223 and is monitored by Captain ilichael $S$. francis.

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# Unsteady Separated Flow: Forced and Common Vorticity About Oscillating Airfoils 

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## AD-P004 167

ABSTRACT

Flow perturbations induced through dynamic sinusoidal oscillations of an NACA 0015, NACA. 0012, and flat plate were examined across a wide range of test conditions. Phase-locked multiple exposure flow visualization in conjunction with corroborative hotwire anemometry documented the development of temporally and spatially synchronus leading and trailing edge vortices induced through unsteady flow separation. Airfoil oscillation dynamics directly influenced vortex initiation, development and traversing velocities. The results suggest the existence of optimal combinations of variables for maximizing both vortex strength and residence time over the a土rfoil.
:ITRODUCTION
Much of the new impetus for research into unsteady separated flows has resulted from the fotential use of large scale vortices for improving aerodynamic performance (McCroskey 1982, et 31). The possible exploitation or large scale vortices as an additional energy source has already been demonstrated. Current research activities (Carret ad. 1977. McCroskey el al. 1975) have shown that the unsteady 1 ift and nocent coerficients produced via dynamic airfoil csciliations can be cour to rive times greater than steady state counterparts.

However, before a realistic utilization of zuch unsteady vortical structures is possibie, many fundamental questions regarding the physics of vortex jevelopment in a temporally dependent : : curdeid must be answerod. The dejendence of vor:ex initiation, development, strength and inceraction with the alrfoil are required as a cunction of airfol: zeovetry and osciblation dymames in order to Jetermine both tho physics of deveiopment as well as derin!ng the control and repeatabilizy of vortex production. The gresent study focuses on the reiations between ino rorcine variades (airroil geonetiy and osci:dation dynamics) and the resuicant vortex sevelopmenta: behav!or.

## METHODS

Exicr: $e$ erts dere conJucted on an NACA 0012,


 v:=ua: :=a:ton :n contunction with hoidire aresometry.

A solid aluminum NACA 0012 with $10^{\prime \prime}$ chord, a hollow-core aluminum NACA 0015 with six inch chord, and a 6-inch and $12-$ inch chord by 0.25 inch thick solid aluminum flat plate were used for the experi..ent. Stiffeners were placed along the span of the $12^{\prime \prime}$ flat plate to minimize flexing of the plate during rapid oscillation. The airoils were driven with a $1 / 3 \mathrm{H} . \mathrm{P}$. D.C. motor using a 6 to 1 gear reduction connected to a variable displacement scotch yoke. Sinusoidal oscillation rate could be held constant while the magnitude of the uscillation angle was changed by a radial bearing adjustment on the rly wheel of the D.C. motor. A rotating potentiometer on the Cly wheel was used to determine the oscillation period and specific airfoil attitude during any portion of the rotation cycle.

Flow visualization was obtained using a smoke rake constructed of a NACA 0015 airfoil with $1 / 8$-inch diameter tubes inserted in the trailing edge. The smoke rake was located in the settling chamber of the $2^{\prime} \times 2^{\prime}$ wind tunnel in order to minimize disturbances to the test section. The rake could be moved vertically to optimize the position of the smokelines. Dense smoke generated from heated Rosco rog juice was stored in a 55-gallon drum and delivered to the rake at modest pressure. The pressure and ensuing flow rates were adjusted to prevent smoke from being emitted as a turbulent jot.

Synemronization of data acquisition wath the arifoil angle during cec: - ia:ion was aecomplished using a 0 :0 5 vel: :isep outiui from the potentioneter on the $f$ fy wheel which corresponded :o the e io 2-oscidlation of the airfol:. Voltage discrimination leve!s from 0 to 5 יolts could be preset on the electronic trigeer. When the selected voltage :eve! was reached, a pu:ze was generated to trigger both the stroboscofic Lamp for f:ow visuadization and LSI 11/2̉3 microprocessor for velocity measurements. All angle-dependent daia co: :eciton could be zynchronized with the proper phase angle of the airfo!l. Synchronization sienals were checked regulariy by atroboscopic examination of a!rfin: pesition reiative $: 0$ s itxed escrometer scale.

S:ng:e and =ul:ip:e ptase :ocied siroooscof:c (7a sec dura:ion, po:r: sourcal
 = SLP casera and hish too Jf:-X : : : = deveiojed a: ín 300. The inase-:ccies au: : fie expozure gro:ografne hien:: gries : ice re;e:j: veness of
flow rield disturbances.

Velocity measurements were made using a conventional constant temperature $2-n e e d l e$ hotwire probe constructed of 0.000i-inch Wollaston wire. Using various linearizing circuits referenced elsewhere (Francis, et al. 1978), a 0 to 5 volt, full scale output was preset for the velocity of the mean flow. The hotwire probe was mounted on an orthogonally driven traverse mechanism in the wind tunnel test section.

Hotwire data acquisition and subsequent reduction were accomplished with an LSI $11 / 23$ microprocessor. Upon receipt of the phase-locked trigger signal, the analog signal was digitized and stored at a sampling frequency of 1000 Hz . Data were collected over two complete oscillation cycles for each data sample and ten successive data samples runs were averaged.

## RESULTS

The unsteady flow rield produced by an airfoil driven through sinusoidal pitch osc:ilations was extrewely complex due to the temporal and spatial interdependence on osciliation dynamics. Changes in any independent parameters altered the flow field. Such complexity did not prohibit highly reproducible :'Jow structure synchronized with the airfoil osciliation. The most prominent structures were the (1) initiation and growth of a leading edge vortex, (2) the interaction between the leading edge vortex and the upper airfoll surface, and (3) the development of a trailing edge vortex. These structures were stable enough to permit repeatable visualizations using multiple exposure photographs and to permit averaged velocity histories using hot-wire anemometry when both were synchronized with speciric phase angles of the oscillation cycle. The initiation, jeve.opment, and shedding characteristics of these flow structures were studied in detail.

The general patterns of vortex formation remained guite constant over the extensive range of test conditions and airfoil geometries examined. Figure 1 depicts the vortex formation over an oscillating rlat plate. As the rlat piate approached the maximum angle of attack, a sea:! strucinre was visualized about the rlat piate deading edge at approximately 0.2 chord (Fig. lA). Further into the oscillation cycie, :his smal: structure rapidiy grew in $91 z 0$ and was ead.iy !dentiried as a leading edge vortex. -uper:mposed smokedines rrom successive phase:ocked, mu:ispie exposure photographs attested to the reproducible appearance and growth of this :eading edge vortex. The presence of the leading edge vurtex cver the alrfuld surface reattached what otherwise would have been separated flow under static test condilions. As the clockwise roisting vortex was about to sned into the wake (F:E. 1:), a vortex extibiling counierciocku:se circulaiton was indilated around the iradiang edee trom deneath io above the airfod.. whereas the presence of the besdine edge vortex had -ea:iacted sefaraied riow, the indtiation and :apid growth of the tra:ilng edge vortex rosuited : n complete 'ratsciysesc' riou separation froe the a:rfo:i ieading edec (fig. li). in the waike, the iest:ng and irat:ing edge vor:ices cceotrec


Fig. 1 - Vortex development over an oscillating 12" flat plate; Re 87,300; a = $15^{\circ} \pm 5^{\circ} \cos (\mathrm{T})$; osc pt. 0.75 c ; $K=1.0$; $A-j$ correspond to $1 / 10$ increments of the oscillation cycle
to rorm a tandem structure which remained coherent for many chord diameters downstream. Though fig. 1 depicts vortex development for a single set of conditions, similar development sequences have been observed across a wide range of flow conditions and airfoil geometries. Criticai dirferences in vortex initiation, development, repeatability and convecting velocity did emerge. Abterations in these characterdstics were examined in detall ard are presented later.

The use of short duration, multiple exposure photographs permitted documentation of vortex reproducibidity. Whereas superimposed smokelines from successive phase-locked exposures (Fig. 1) dentify repeatabie voriex character:stics, spat!aliy disparate or difruse smoked:nes indicate osciliation-independent riows. The datter structures are evident in the difruse smoke!ines about the circumference of the leading edge vortex in the wake where viscous diffusion has sta-ted the breakdown into turbulence. Similarly, difruse patterns also were ovident near the alrfoi: beadirg edge where turbuient sefaration had occurrec.

## VOhtEX structure

F:ow V:zua::=a::on ar.d noi-wire anemoce: :y prov:Aed :nこtent tato the sirueture of tie :ea::ng edee vuriex. Mu: : ; : e-exposure v:rua.:zat:ons or a stie:e yo:iex :atiated over



Fis. 2 - Velocities induced by a traversing vortex over an oscillating flat plat; $\operatorname{Re} 38,000 ; 1=15^{\circ} \pm 5^{\circ} ; \mathrm{K} 1.5$;0sc pt. $0.75 c$; vortex located over $0.2 c$; $A-E$ correspond to probe positions of 3.01 , j.j1, 3.77, 4.20 and 5.58 inches above the airfoil surface.
induced velocity


rotation of outlying streamlines down and around a turbulent vortex circumference. The absence of smoke from the vortex core created a striking contrast between the turbulent vortex circumference and the laminar rotation induced in the potential field. The visualizations suggested the existence of a strong shear layer between the inner core and the vortex cirsumference. In addition, growth of the vortex from a separation tongue (see Freymuth, these Froceedings) emanating from the boundary layer would have little smoke. Diffusion at the turbulent circumference of the vortex into the potential field suggests that a decrease in the induced velocity would correspond to an increase in radius away from the vortex core.

Anemometric measurements which give a timedependent profile of the velocitics of the vortex substantiated visualizations. A hot-wire probe positioned at various chord locations above and normal to the airfoil surface yielded velocity profiles at each location over successive oscillation cycles. Two representative cycles are indicated in the data of Fig. 2. The velocity perturbations increased as the probe was immersed first in the potential flow field (Fig. 2A) and later in the vortex. A peak velocity maximum was obtained with the probe located tangent to the vortex circumference (Fig. 2B). Further movement toward the vortex center produced a velocity minimum. The probe recorded the passage of the high velocity region of the vortex cimcumference and entered the slow moving inner core. The two maxiza on either side of the velocity minima indicated the passage of the vortex as the probe entered and exited the inner core. Though only two oscillation cycles are shown, profiles at any location were quite reproducible when phase-locked to airfoil oscillation.

Multiple exposure photographs in Fig. 2 indicate the relative positions of the hot-wire probe and vortex which produced velocity maxima and minima peaks. In the rar rield, outside the vortex core (Fig. 2A), the velocity maxima occurred with the probe tangent to the vortex diameter. Similarly, inside the vortex core (Fig. 2E), the minima between the two velocity maxima was observed with the probe again tangent to the diameter and on a perpendicular bi:jctor through the vortex center. A plot of these instantaneous peak velocities along this bisector line (Fig. 2a) shows the instantaneous velocity rleld induced by the passing vortex. Inside the core (Fig. 2a, 0.0 to 0.15 c ), the vortex behaves as a solid body rotation. Outside the viscous core, the velocity induced in the potential field diminishes as $1 /$ औ with $R$ being the effective vortex radius. This result agrees with the ciassic solutions derived by Oseen (1911) and Hamel (1916) as reviewed by Schlichting (:979) for the velocities induced by a vortex dilament as a function of radial distance.

Repeatec gea:renents across most test condition, ditu ajr:"01! Geometries indicated yimbar structu: : $\because$ any leading edge vortex. Inte similardty permitied a characterization to De aade of t!. -- vorisces dased upon redatdve disacin. .... circuiation velocily. The ctrcumference diaseter anc the peak velocity obiainet bhere were eedected as meazures :o be HECt acrose va::ous iest conditlons. Frese

$\because$ _- 2 - Mean angle of attack effects on vortex developwent; 5" NACA 3015 airfoil; Fe 50,000 ; $15^{\circ}$; osc. pe. 0.25 c ; $\because$ 0.5; F-i corresfond to mean angles of $0,5,10,20$ and 30 degrees respectively.
characteristic values as well as vortex initlation point in the oscillation cycle, and the average convecting velocity of the vortex center over the airfoil surface provides for a rather complete comparizon of unsteady flow itructures elicited in the rollowing tests.

MEA: ANGLE OF ittack
To create a leading edge vortex, it was necessary to oscillate adrfoils or plates such that the critical stall angle was exceeded during some portion of the oscillation cycle. Thus, the static stall angle provided a good reference angle for determining whether or not a lead, ng edge vortex would be produced. Figure 3 shows the dependence of vortex development on mean angle or attack. For ap up to $5^{\circ}$ (Fig. 3 if), the multiple exposure photographs ind!cated no icadine edze vortices had been generated. A further increaze in mean angle (Fig. $3 \mathrm{H}, \mathrm{m} \mathrm{m}^{\circ}$ ) produced small deaditis and tralling edge vortices most evident in the wake. Leading edge vortex diazeter increased 258 as the wesn angle of attack was increased rrom $10^{\circ}$ to $15^{\circ}$. For ${ }^{\circ}$ m increments between $15^{\circ}$ to $30^{\circ}$, vortex size remained uncranged.

Vortex characterisidcs became qu:te sensitive to other tezt parameters at nigher mean angles $(\mathrm{m}) 20^{\circ}$ ). With an oscijiation angle or $\pm 5^{\circ}$ ar a reduced frequency of $x=0.25$, the -eading edge voriex separated froe the alrfoll at approxieaiely 308 chord. fre deiacheent was not icialiy depencent upon the eean angle of attack. Oup:ication of the saze geozetr:c conc:i:ons bu:

vortex which remained attached shedding into the wake only at the airfoil trailing edge.

## OSCILLATION ANGLE

The magnitude of the oscillation angle provided another reference condition for the production of synchronous vortices. Small oscillation angles $\left(a_{\omega} \leq 0.5\right)$ at rapid oscillation rates ( $K \geq 2.0$ ) were shown to effectively attach modestly separated flow. Instantaneous ( $<7 \mu \sec d a r a t i o n$ ) single exposure photographs documented the presence of vortex development similar to that shown in Fig. 1. Multiple exposure phase-locked photographs, however, showed no spatially and temporally synchronous vortices. Oscillation angles up to $\alpha_{\omega} \pm 3$ at K values < 0.25 similarly produced little evidence of synchronus flow structures. However, further increases in either the reduced frequency parameter or oscillation amplitude produced repeatable disturbances. Hence, a threshold condition existed requiring a combination of dynamic parameters of sufficient strength to elicit repeatable flow structure.

REDUCED FREQUENCY
The reduced frequency parameter, oscillation angles, and oscillation axes directly arfected the size, velocity profile and repeatability of vortex formation of these dynamic parameters, the reduced frequency ( $K$ ) provided the most predictable alteration of the flow fields. From a series of multiple exposure photographs (l2/cycle), the location of the vortex center was tracked as a runction of time and rraction or the oscillation cycle. Across the Reynolds numbers ( $60,000-140,000$ ) and oscillation angles ( $3-5^{\circ}$ ) examined (Fig. 4), K provided an accurate index of the vortex convection velocity. The straight line drawn through the data pointsestablished the average traversing speed across the alrfoll at 30 to $40 \$$ of the free stream velocity.

The individual vortex positions (Re $=$ $60,000, K=0.5$, and $K=0.75$ ) reveal instances of momentary delays. These delays were most prominent as the alrroil reversed direction from the maximue angle of attack and pitched downward. The characteristic circulation veiocity of the vortex circumference showed tha', maxlmim delays were obtained rrom vortices exhibiting the

POSMON OF VORTEX CENTER


largest reference profile velocity maxima. As the airfoil approached the lowest positions, a rapid acceleration in traversing velocity ensued. Though the reduced frequency was a good measure of the average vortex position, it did not adequately predict either vortex delay or acceleration characteristics. All dynamic parameters as well as airfoil geometry were observed to affect vortex motion.

The flow field structures related to $K$ changes ranged from weab: poorly synchronized vortices at low $K$ values (Fig. 5A $K=0.25$ ), to a succession of compact, intense, and closely spaced leading edge vortices well synchronized to airfoil osciliation at high $K$ values (Fig. $5 E$ $k=1.75)$.


Fié. : - Resuced rrequency errects on yoriex tevrioplone; NACA 0012; $7 \mathrm{c} 75.200 ;$ a $15^{\circ} \pm 5^{\circ}$; osc. it. 0.25 c ; A-E vortesfond to \% = 0.25, 0.5, 1.0. :.5 ara l.is respecidve:y.
it K va:ues greater: han 1.5, s.enificant a : ieralions in the deveiopmental patiern of :rald:ng edge vorilcity occurre: The presence of mu:tiple teading edge voritces over the a:rfodi surface displaced the formation of iradding odee vorisces inio the vake. The irai:ime edge voriex unich had procuced complete : Aow separaiton cyer ine airyoll al fover $X$
 =cfaration at nigh $K$ va:ues (E:E. 5 ( E). :itus, attached E:ov over the atre:d resulied :Arowghout ine onitre osc:l:afion eycie a: c.evates values o! E. 21.5 acroze the citer





Fig. 6 - Vortex magnitude, diameter and relative circulation based upor the peak velocity obtained at various chord locations; NACA 0012 ; a $15^{\circ} \pm 5^{\circ}$; osc. pt. 0.25 .
peak circulation. velocity increate of 168 as the reduced frequency uas changed from 0.25 to 0.5 . At these low $X$ values, the vortex diaseter changed slightly across cest conditions with larger diameters occurring at a Reynolds number of 140,000. Bolh the characteristic circulation velocity and diameter were combined to obtain a realtive circulation index r. Though the characteristic maximum vortex velocity decreased in almost linear fashion across the airfoil chord, vortex diameter increased rapidly from 0.25 chord to the trailing edge. The net erfect was a continual increase in relative circulation as the vortex passed rrom the leading to the trailing edge.

## OSCILLATION AXIS

Similar to the reduced frequency parametor, the oscillation axis location directly affected the velocity magnitudes and repeatability of vortex development. In order to minimize the effects of airfoil eecmetry, a rlat platews oscillated about two different äis locations; 0.25 and 0.75 C . Figure 7 contrasts the vortex development between the two osciliation positions for cenerwise duplicate test conditions. The difruse smokelines in Fig. 7, plaies A - E do not exhibit tho highly regeatable characterisetcs observed wieh oscillations about 0.75 c . Both plates $A$ and $F$ were taken at the same position in the osciliation cycle, دT z 2-or ine maxdmus angle of atiack $20^{\circ}$. The ieading edge voriox in plate \& (oscil:aidon ax:s 0.25C) apfears much :arger and furthe down the chord inan dn g:.ite f (oscidiazion axis 0.75C). A beteer satch beiweon condizions based upon vortex diameter and : dacement over the adrrod: occurs between $A$ anc $G$ at 208 furiher into the osci:dation cycde. This 208 phase shift produced a vortex sise and docalion astch plate for plate inroughou: ine eni:re oyciliabion cycio. Changing ine oscdidation axis from 0.25C so 0.75C vould appear :c nare selayed ine derpiopaent of ine deadine edge roriex by 208 of :to ose:laa:ion per:od. $\alpha$ cela: © ed ana:yzis of roriex in:itaz:on ans traversine ve:oci:y az a functionc! int tyramic ;asace:ers :s presented inter.

$$
\because 0: 0 c:: y=c a z u r e=c t:=\{5: \epsilon, \text { हj shovec }
$$


$\therefore \quad 7-$－$\quad \therefore$ ci：ation axes errect on vortex Levelopaent； $12^{n}$ rlat plate； Se 70,$000 ; 15^{\circ} \pm 5^{\circ} \operatorname{sos}(-T) ; K 2.5$ ； i－E and F－J correspond to $1 / 5$ a：ctements of the uscil：ation cycie
velocities ard vortex diameters with changes in osci：iation axes and $K$ ．An average circulation veiocity increase of 158 was obtained for reduced frequencies of $1.0,1.5$ ，and 2.5 by relocaling tie osciliation axis from 0.25 to 0.75 C ．The vortex diameter decreased an average value of 10 ：o 158．Cireuiation veloc：ties increased 30 ： vetween $K$ values of 1.0 and 2.5 tor a fixed osciliation position．Also，average voriex tiafeters decreased to 20508 with elevated va：ues of $K$ ．Whereas the magnitude of the e：resialion velocities were directiy proporticnal ：o ：nereases in ：he oscillation axis and $K$ ，the ：：： 0 of i．．e voriex diameter was inversely re：ated．

A A：rec：currebation exteted beiveen the －agn：iude or the circuistion veiocity ans the ：－foitab：lity of vertex genoratiun．F！oy
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vortex development over a flat plate


Fig．$\quad$－Yortex magnitude，Jiameter gnd reiative $\therefore: c u!z t i o n$ based upon ine feak ve：ociey obtained at 0．1 こnさrs ：：cremen： $12^{\circ}$ riat p：aie； Pe 0,$000 ;+15^{\circ} \pm 5^{\circ}$ ；osc．pi．0．？5 and 0.75 c．
indicated．The decrease was followed by a rapid acceleration of the circulation velocity at 0.5 C with a corresponding decrease in vortex diameter． Telocity decay commenced again from 0.7 C to the tralilng edge．The velocity reduction rollowed by the rapid acce！eration of the vortex over midchord was comensurate with the formation of a second vortex over the ：eadingedec．Thus，it appears that strong interaction between successively generated vortices at tith $K$ valwes a！tered the normal vortex ve：ocily decays．

## AidiFO：L GEOMETPY

 toin ：eading and iral：ine esfe yor：ises．Under stalic conditions，the fiat piate extititet fiou eeparalion al sidghi：y ：over sean aftita of atiscik $4, ~ 10^{\circ}$ ；ian ine NACA OU12 aris 00：5


 Exar ataties of atiack thar．ihe two cententiona：


 ：¢D：ing tice roflex．





Seast part of the key to this difference appeared to be the sharp leading edge of the osoillating flat plate. When either the NACA 0012 or 0015 airfoils were rotaied $i 80^{\circ}$ and oscillated about 0.75 C with the sharp trailing edge forward, the leading edge vortices produced were qualitatively the same as those generated rrom a flat plate oscillated about 0.75C. The most intense, cohesive, and repeatable vortex structures were observed from the rlat plate and reversed NACA 0012 and $6 C 15$ airfoils oscillated about 0.75 C at the maximum liniting frequency of the oscillatory mechanism (-15 Hz).

Significant difrerences were cbserved, however, in trailing edge vortex development between the rlat plate and reversed NaCA airfoils. Passage of the leading edge vortex cver the rounded trailing edge of the Naca 0012 a:rfo:l always triggered a talling edge vortex independer. of reduced 1 requency. In cuntrast, the sharp trailing edge or both the flat plate and a conventionally oriented NACA OC12 airfoil procuced trailing edge vortices at difforent fositions in the oscillation cycle, relatively irdependent or the leading edge vortex location. For edther the normal or reversed geometry, deve: opeent of the trailing edge vortex at K < 1.25 produced fiow sefaration whereas $X$ values in excess of 1.5 resulied ill attached flow through :he eniare osci:lation cyc:e.

## MOREX :MTMATION

The icen::rdcation of leading edge vortex ::: : : atiton us:ne flow visualization and hot-wire ,reeosetry presents a unique probien. Though ": vel visualization aay not necessarily undicate the initial cuidd-up or voriseity, it :s crucial in :dentifying a deveioped vertex structure. ald tes: conditions wheh produced synchron:zed iraling edge vor:ices :eveajed fully develofed vor:-ces by 0.1 to 0.2 chord. For analysis furposes, therecore, the 0. en chord :ocation or $^{\text {en }}$ :he ieading ecge vortex was selected as a spatial :ererence for vortex inlifation. The composite Fiexte 9 cites the sontribution of oscillation ane: ( $\mathrm{s}_{\mathrm{w}}$, Eear angle ( m ), and the reduced trequency parameier ( $K$ ) to the initiation or a :rajing odge vortex re:ative do tra oscijlation jeie. Ai: data points were obla ied froa fiow Yisuadizalion and hot-wife anemometry using a 12 : f. C I:at plese anc reversed MACA OC15 atrfo: - tht 3 in. C. Oyci:iation of votr deviceswaz seou: 0.75 Ca a $\mathrm{x}=\mathrm{x}$
 *as.es of a voriex roreed during ine upistroic t: : :e oscil:a:inn cye:e and increastre va:ues 3 is Ae!ajes tre appearamce of a vortex unil! or =uct goyonc 2.0\%. I: :s no:ab:e that in:y -c.a:ton vaz asyeptot!c, fur:ner excursions
 LS ver:ex :ns:iation

Aisn : : f:gure ヲ, :ecreazing iés vero arre:atec atr. ite increastre:y eariy afpearance

 tcy:cov were we:: :n:0 it.e Covns:-oke por::0n c: :t.e oyci:ia:ory sy:ie. is a aiftoacted jop :..e yor:ice mpiearos tur: fie :he up: roxe por::or. itste, :te effocts c! by and tare


## NTUTON OF LEAONG EDGE VORTEX



Fig. 9 - Vcrtex iniriation over a $12^{n}$ ilat plate and a reversed :AC: 0015 atrfoil; osc. pt. 0.75 c .
asymptotic limits. Sowewhat surprisinedy, im seems a somewhat iess efrective variable than a across the values teited.

Varialions in reduced frequency parameter also produced the large alterations in vortex initiation. The index of initiation shows that increases in $K$ delayed the appearance of leading edge vortices in almost :inear rashion. The felay increased through $\%$ values of 1.25 with more modest delays occurring at higher $K$ values.
reaversing or convecting velocity
It was noted earlier (Fig. 4) that momentary delays occurred in the traversing motion of the leading edge vortex passing from the leading to the trailing edie. Hot-wire measurements at 0.1 $C$ increments over an oscillating rlat plate for two dirferent oicillat:ne axes ( 0.25 and C.i5C) and three different values of K ( $1.0,1.5$, and 2.5) snowed simblar restits (5is. 10). The leadirg edge vortex did not traverse the asrfoti surface with a unifora velocity after initiation. Vortices acce:erated over the airfoil surface at d:fferent rates dependent upon the test conditions.

Piotting the vortex fosit:on as a rurction or the oscil!ation cycle may distort ine depencence of vortex developent on airco: oscilia:ion. When the saze position data were pioitedin roal tise (Fig. II), the dynazics of vortex initiation and traversing ve:ocity were odserved indefencer: of ose:liation dyiaz:cs. The tise required :o corfiete one osciliation cycle for each or the inree X values :s :nctaite above the time scale in fie. 11. Thougt ine oscidlatior rate lierered by a ractor or z.5, vortox andtiation ans positien as runction of real t:=e remained quite sim: dar across conditions. In contrast, Fte. 10 3xess the position dala to ose:ilation eycie prases. The actuad deponcence or fortex insitation and position on rea! idee seez to dra:cate a charactertstic inteize:on ary sevinotsent :tee that is somextat dnajencen: of fme afro:d oze:d:ation dyna=ies.

Oiter characier:se:cz of voriex tm:tafion ares posit:on. nowever, were oterves io se sefencent upon ir.e atrio:: osci:iation syear:cs.
 :rereases $!n$ : te reduces requercy iaraea:et :csu:tot is. yor:ex iravergirg yo:oc:iy

VORTEXPOSITION VS OSCILLATION CYCLE


Es. 19 - V:-tex fosition as a runction of oscidiation cycle; $12^{n}$ Clat piate; fe $? 3,000$; $15^{\circ} \pm 5^{\circ}$; osc. pt. 0.25 and J.is こ.

ircrearnts. This offect vas previousiy noted in the fiow visuailiation study (Fig. 4). At reauced requencies where the ieaoing edge vortex reatined orer the alrfoll surface through one coafiete esci:jation cycle ( $\mathrm{X} \leq 0.75$ ), the rortax exfersenced conveciton acceleration immedidieiy arler instiation (Fig. $11 \mathrm{~T}=0-40$ asec).

For all the conditions lested, ine cenvection morements readined surficiently linear ec approx:mate "arerafo" traversing yelocitioy Eviveen 0.2 and 0.9 C (Fig. 12). F:ow \% sualizat:on of the :eacife edge vortox cen.eerod over erese polnts perestled a rapid assessaent of irayeraime ve:ocizy across a wide parfacter rance. The average iraveraling velocity obtalsed with inis technique agreed vith aneaceetric teats Lo within 25. Alihough a eirect corfelalion ex:ate Detween the recuect frequency and
 $\therefore$ :Opo of iths zorredation occurz al approilmaiely
 =atac:yam!c f:ow aejaration tie is the trabilne edec roriox ceasea. Mesn anfie of atlack Ga stous ar. inverge corredalion to lraversing : 0 :oc:iy such that tich areles of altack resuli :: a siover variex convec:ion Sizijat results were otlaind for coin ite revtraet lidh sols ant
 Eecse:ry and chor: iere:t


Fig. 12 - Vortex traversing velocity over a $12^{\text {n }}$ flat piate and a reversed NACA 0015 airfoil; csc. pt. 0.75 c .

## DISCUSSION

The observations made in the previousiy dsscribed tests may be organized into two general categories: (i) initiation, development and convection velocities or voritces and (2) inherert vortex characteristics. As will be noted later, vortex characteristics appear to relate to the convection velocities.

INITIATION:
Assuming that the static stall angle ds oxceeded at some time in the sinusoidal oseillation cycle, safncrements are related directly to earlier vortex occurrence during upward pitching of the lifting surface: the larger the a, the earlier in the cycle a vortex is initiated. Both ${ }^{\text {d }}$ and $X$ value increments delay the appoarance of a vortex to iater portions of the oscillation cycie. When the oscillation axis is mered back rrom 0.25 to 0.75 chord locations, vortex initiation is siallarly delayed. Thus, earlier vortex initiation der:ves frow $\mathrm{m}_{\mathrm{m}}^{-\mathrm{a}}$ stall while dater vortex initiation derives frow any test condition which increases 1 valuns. Across the tests done in the present series of stud!.s, vorter appearance occurred as oardy as midway through the upward oltching and as late as miduay througt the dowrward pitchirg of the oscillation cycle.

PhaYEREZMG OR CONVECTBG VELOCITY:
The traversing velocity (V/V.) of the lessing edee rortex vas affected $: n$ a anner ana: agous to the initiation increaenty in both the ruduced fequency faraee:er fin and oscislatior anf: i: use increasect be vortex iravursine re:ocisy. Over the condsions iosied, sfereased K restided in ilinar chasees, but oscidialion ampl:tuce produced re:oede:es thich tpproachet an azyppiot:c itsi: of 0.4 Y. tar 4 $26^{\circ}$. \& sigilar asympiozie conci:sot probabiy exdsis a: Aister reduces frequenc:ey as we: b; for ll de doublful ital ite trayersiac relocily vou:t excoed the local rroe gitean reiocily vaiue. Tre traver=:nc robocisy yag bsyeraciy zeiator to incruanisa $:=$ ite aean ase: of atiack. A decreaze of 338 :n fraversing volocily reatiled from lincreasirg tae acan anf:e


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blockage of the free stream flow（greater y ）for loager periods of time（reducej $K$ values） resulted in sicuer traversing velocities． Vortices developing in the shadowed wake of the airfoil remained ror longer periods over the airfoil．

## RELATIVE CIPCULATION：

The size，strength and repeatability of leading edge vortices were directly altered by changes in the oscillation axes and varying the oscillation rate．Higher peak circulation velocities with simultaneous decreases in vortex diameters were raalized with elther increased reduced rrequency parameter or rearward oovements or the oscillation axes．Through wistiple exposure flow visuailzation and ensemble averaged hotwire signals，the concise，energetil？and extremeiy coherent vortices（axis 0．75C，K＞1．0） were repeatibie to within 28 variation of incuced velocity over 30 consecutive osciliation sycies．

When the peak veiocity was combined with the corresponding vortex diameter，the relative circiuation estitate ：remalned relatively unchanged across conditions．The initiad Cormation（vortex over 0.2 chord）value of the c：reviation appeared constant across $K$ values and oscillation positions for the fiat ilate．As the vortex traversed the airco：l surface rrem beading ：0 irali：sg edge，tre relative circuiation ：sereased by a factor of 2．5．Hence，the vortic：ty appearad su tui：d with time over the a：re：：rather tran decay with isme alter ：n：：isai：on．The magnitude of the relative c：rcu：ation was inverseiy reiated to the cse：baticn rate．though the poak veiocity tncreased with $k$ ，the decrease ist the vortex 1：azeter resuited in an overai：decrease in the ：c：aitve circuidtion a：tigbier \％values．Fcr Ine ：iat piaie escillated about 0．75C．
decteases approximateiy 258 ay tre resuced teequency was ：nc：eased trom 1.0 ：0 2.5. $\dot{A}:$ houer esen vo：－iex possessed a recuced circuiation at figrer $K$ valwes，the sota overaj： yor：ex fre：woree abou：：he a！rro：：way hither ssene mudtip：e vorisces res：ded over the a：mol：

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frequencies permitted a greater period of time for the jortex to grow in size and remain over the airfoil surface in the wake region of the blocked flow．Since the relative alrculation indices were quite constant，higher reduced rrequencies resulted in vortices with higher circulation velocities and smalier diameters． Less time was avallable for the development of the vortex in the shadowed wake；hence，the vortices reazined concentrated．

## CONCLUSION

Harmonic oscillation of airfoils at angles of attack in excess of static stall were observed to produce complex interactive rlow rield structures．The most predominate of these were the cormation or both a leadirg and trailing vortex which were temporally and spatialiy dependent upon the alrcill osciliation dynamics． These structures were surricientiy raproducible to purmi：multipie exposure flow visualizations and ensemble averaged hotwire anemometer frofiles to be aade phase locked to the atrfoil uscillation．The presence or the teading edge vortex was observed io reattach otherwise separated 510 w at angles of attack which produced complete flow separaiion under non－osciliating cond：t：ons．

The dyram：cs or tre afro：：osciliation directly inriuences voriex init：ation， deve：opment and traversing ve：ocity．the usci：！ation rate（reduced requency），mean angle c！attack，oscillation ang！s，oscillation axes ar wel：as the airro：seometry were shown io cirectiy abier the vortex circu：ation velocity and size，yst， bert the relative circuia：ion index quite constant．upti＝a：seiection or the a：rfo：：seometry and osci：latcry parameters snou： 4 permit gaxima：！：f：entancemen：：hrough increasing vortex residence t：＝e and cireulation ve：ocity over the a：ryo：surface．Much adtit：ona：wurk remains ： 0 be done ！n two sreas： （1）＝easuring the icial yor：ic：iy rie：t dur：rg a！：soi：osc：：：ation ant（2）examinine the in：eraction or zu：：ifie vortices over the a：rfol： at ebevated values or K ．Bo：n inese approaches nave reati：y apparent exf：anatory and tocnoica： exfio：iat：on vailues．

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## Abstrac:

The novel iff generation mechanism postulated iy Weis-Fogh (1973) anc evaluated by Lighthill (1973) in regard to ho. oring irsects provided graphic evidence ior the possible utility of unsteady flows. The present report summarizes flight mechanisms in dragonflies that appear to exploit unsteady flows to achieve :3ther remarkabie aerodynamics. We envision that such studies may provide a means of identifying enucial combinations of unsteady variables that are effective in both generatine and using unsteady flows.

Dragonfiy flight was studied both in unrestrained, normal specimens and in tethered, aboratory-tested specimens. In both instances, hioh speed photography permitted the sharacteri:ation of wing motions inc. uding stroke -ength, stroke angles, stroke frequency, angies of attack ard phase angies for the tandem pair of tizngs. In the laboratory such observations were cuf:ed with force balance masures such that instantareous correlations of wing motion and :Et were ootained. Fiow visualization also was sotained for tetnered insects furing eliaited : Isht episodes. Simp!e osciilating plate models we:e used in a zero fiow test to simulate at : cast some aspects of observed dragonfiy ierodynamios.

The $\therefore$ isht modes of a dragonf:y inc:ude (1) :: er-speed forward and Lpward maneuvers, (2) $\therefore$ :Us:e or soarine maneuver:, and (3) hovering or $\therefore$ Aw mareuvers in any direstion. Combined modes tis :"ult ran3zticns from one mode to ancther we observed often without mator charges in body $\therefore$ :itude or ding synamies. The wing geoge:rtes ace simpie and static except for pasilve defornations produced by interdctions with rearby eneresact fuit. hing dynamirs include wing beat :re weredes of $25-35 \mathrm{~Hz}$, wing atroke anedes of :5 Cordard and $20^{\circ}$ ber:ind the root attactment i.: re: : as anges or $40^{\circ}$ beneath and $53^{\circ}$ atove A. ie hortzontal, anezes o: attacin of 1 to $50^{\circ}$ and indse argles for che dandet whes of near ? :o 220

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staticnary vortices surrounding the upper wing stroke quadrant.
is a working mode? of dragonfly flight mechanisms, summarizes the observed wing positions, angles of attack, flow fields and lift peaks we have been able to characterize. In essence, it appears that inigh angles of attack of the wings generate a local circulation exploitable to provide relatively high lift under hovering conditions. Angies of attack appear smaller if thrust is to be provided and if net fiow is to be achieved.

In view of these observations, verticaliyoriented flat piates of varying thicknesses were driven back and forth in a zero flow test condition to evaluate the effects of stroke ieneth and stroke frequency. The results of these studies were quite ciear. For very thin piates, stroke jength dictated vortex size and stroke frequency had littie effect other than a modest enhancement of fiow structure cohesiveness. The addition of angularity to the : at plates resuited ir net flow dominated by vorilces. These observations are consistent with the notion that Cl ow structure may be initiated by a wirs and can be expected to persevere. This freserved local flow structure co:id be expioited by another wing or by ti:e same ning on a late: st:"ove.

Ove: sid, these experimenis 2 ridicase that uncteady flow: may be ased to support quite sophteticated insect filght maneuvers. ?o :ig:n:tiant change in wing jeometry is needed to achirve such rlicht and oniy modest aiterations in fynaric wing sercine variab:es are requited. The objervaticns raste here injtcate inat dragonrites use mechani:ms quite dirterent f:om thote durd by ihe Chaloid dusi, sis deacrated by hede- hoch $^{2}$. Other meanj of exi: oit:ng arsteady ce;u:uted riows may exist dico within the insect wor:d. Many of these fossibilitices remain to e e examined and io be and:y:rs !n :resary io Erne:cor, zuntrol and uer.

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 Ghe Cha:cid was as we: as ather bisec:s.








configurations are often more simple in insects and muscle－elicited changes in configuration during various flight modes are more limited． Unlike birds，for example，many insects achieve excellent aerodynamics and maneuverability in the absence of a variety of deployable wing devices． Aiso，insects are able to accomplish a wide range of flight behaviors without the aid of an elaborate set of neurai controls．Thus，attempts to understand biological Elight are best focused upon these accomplished，but simple，flisht practitioners．

Work focused upon the Chalcid wasp is inited by several considerations．First，the wasp is extremely small（－ 0.5 cm ）and exhibits i＇：izht in a very low Reynold＇s number range．To dc so，wing beat frequencies approach 300 Hz ． A $i s o$ ，this wasp has not received much attention in regard to either fiight musciiature or neural f：isht control mechanisms．

ヨecause of these limitations with the Thacoid wasp，we have elected tc study flight in the dragcnfly．Weis－Fogh（1973）cited this iruect as ore which most probabiy nad to utilize rovei lift generation mechanisms．More recently， Sorberg（1975）and Savage et al．（1979）have prov．ded evidence that these insects produce i．iner amounts of lift than can be anticipated ．：！：usual stexdy state aerodynamic principles． anrik on anderijing neural cuntrol mechanisms （Paingie，1958；Neviile，1960）indicates that bo．．e inseats use central pattern exenerators to frive winc covements．These movements are subjed to only minor ！isexander，1982；01berg． 1333：Eetdbaux modirications．Thus，the daccre：apfears to acnieve excelient aerodyna－
$\therefore$ ：tut usire jtandard zteady state mechanisms the nc：defenting ufon exaborate wing control． Ti．e wine mo：icne exhibited by dragonflies reveal i fini：y～tereotyped kineaailcs and liuld－wing inieractions．So dedicated are the wing motions， －Abi ausculature has but two cardinal ＂：ons：eievation and depression．Fiexion and rxienstur，etaracteriutics of insecta which ：etra：t ir ru：t the wing：when not in use，are ra：ted from both the control and muscuiature of －：M fragnf：y（こ：arx，1940）． iu an dside，it fe notabie that rossid ©otse indleate dracollles have survived sance ：．．•ime ur $: 170$ saurs，essentially unchanged

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predictable．These insects inevitably depart rapldly at a $45^{\circ}$ angle from the horizon and in the direction of body orientation．The insect net was simply moved to an intercept position． Within an hour of capture，the insects were in the laboratory being subjected to cooling ffor next day testing）or to chloroform anesthetization（for handling purposes）．Each insect was weighed，fully measured and then tethered to a smail wood beam using cyanoacrylate．A variety of tests ensued as described below．

To relate laboratory observations with naturai flight behavior in the dragonfly， unrestrained flight of the dragonfly was observed in the habitat from which they were collected． Flight modes were recorded and simple aerodynamic values were estimated．In addition，the wing kinematics were recorded using telescopic photography（ 35 mm camera； $1 / 1000$ shutter speed； 250 ma lens； 1000 ASA color film）．Most photographs were taken as the dragonflies hovered near favorite reed－iop perches．

Lift measurements and simultaneous wing motion analyses were achieved with tethered dragonflies mounted to a one－cimensional strain－ gauge force balance．The amplified force balance output was displayed on one channel of a CRT oscilloscope and a photodiode output was displayed on the second channe2．As the stroboscopic（－ 0.5 msec ）illumination was triggered for photographing wing motion，the diode signal on the oscilloscope marked related instantaneous lift values．ilternativeiy， coniinuous Strobotach（Gen．Radio）output provided diode marking of lift vadues associated with videotaped，phase－related wing motions． From a basge number of elicited flight episodes visualized for 21 dragonflies，it was possible to describe both the wing kineaatics and the associated，phase－related ift vaiues．In many instances，it was necessary to place a mark on the rear wings of the dragonilizes io assure subsequert discrimination of the front from the rear wines．In some instances，the cyanoacrylic cement used for tethering spread to legs of the insecte or to other thoracic structures．These specirens were not uned since the spread migtt have altered filght fatterns and ：ift teneration． In general，the insects sould be tosted over numerous rijeht episodes for periods of at ：east two hours．iltered wirg motions and decreased lift generation were wyed as indicators of deter：oratine rateht behavior．

F：ow visuaileation way achieved juring Aragonfly flight of：zodes．heated kerosene zane was delivered（＜ 10 em sec－）rros a 1 ：m dameter tube aff：cximately 10 ca in f：ori or t：e test gifecizens．The iaminar smoke st：eam was positioned io triersept the Eetian wing fosition approximate：y nid－spar．i：：tes：ino a！ic photegraply was tone in a $50 \times 50 \times 30$ eazero riou dox corst：ructed of こ．ear cast aery．！ （F：exte：as）．in atout the rid：te or a $:=1$ et：
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wing interactions observed during dragcafly testing. Finally, a novel means of testing dragonflies was developed to permit more detailed flow-wing visualizations. This novel procedure is described tozether with a projection of its potential in model studies of insect flight.

## Results

Before turning our attention to the laboratory studiez, a few of our observations on natural flight will be summarized (Table 1). After spending much of its development as a prodatory water nymph, the dragonfly emerges as the flying adult form from a metamorphosis that takes place on a twig or reed that grows out of the water habitat of the nymph. As the wings are unfoldirg and drying, the adult, itself, is subject to predation by birds. Thereafter, jrasonflies are not subject to heavy predation becalize both color and manedverability provide Jice:lent survival probability.

Filght in the adult dragonfly serves several ciolceical functions and roles. During feeding, $\therefore$ :i dragonfiy alternates between gliding, powered $\because:$ teht and aerobatics. During territory patrols, a prelude to mating, the dragonfly glices with an occasional powered wingbeat sequence to maintain i:titude. In instances of startle or predation theat, episodes of escape behavior consist of powered flight upward at $45^{\circ}$ angles mixet with d:'tnatic aerobatics. Mating consists of mase and remait draconflies flying in tandem with cretarit, albeit poor:y coordinated, fing motion m!ibited by one or both members of the union. :is 11: of the above circumstances, periods of uvering occur. Hovering also occurs as a fr: : de to a dragonfiy returning to a favored : :rah atop a reed.

Feduced to rlicht modes, the dragonfly -st: oits escape, elidine art hovering. Escare curisist of short durstion ( $1-5$ secs) episodes of ic: an:Itude, high frequency (- 30 Hz ) wingbeat $\because$ Ote that ropel the insect at speede estimated
 : te: rareiy interrupted by wing motion. This mule sintirues for feriods or $i 0$ or more seconds. $\therefore$ :e:ativo: moderate speed of $1 \mathrm{~m} \mathrm{sec}^{-1}$ !s
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Flight Characteristics of Unrestrained Dragonflies

|  | Modes | Duration | Speed | Maneuyering |
| :---: | :---: | :---: | :---: | :---: |
| 1) | Escape | $<5 \mathrm{sec}$ | >50 $\mathrm{sec}^{-1}$ | modest; <br> upward, <br> forward e $45^{\circ}$ <br> angle from body axis |
| 2) | Gliding | $>10 \mathrm{sec}$ | - $1 \mathrm{~m} \mathrm{sec}^{-1}$ | ```little; low negative angle glide path; occasional wing use to regain altitude``` |
| 3) | hovering | $>10 \mathrm{sec}$ | -** | excellent <br> but slow <br> movement in <br> virtualiy <br> any <br> direction in any plane |

i) feeding on the wing consists of brief bursts of directions, sharp turns and rapid reversals of direction; similar aerobatic displays observed in dragonfly disputes over territory
ii) roll instabilities seem prominent
iii) aerobatic displays appear to be comprised of rapid (<1 10 msec) ction from one llight mode to a:other

Table 9 . Cbserved natura: : 1eht : haviors of dragonflies. Based up, uvserved major fight characteristics exhibited durino feeding, patroling, territory displays, ating and predation avoidance, these eneral categories summarize the most reliably ubserved rlight modes.
:he wing hifemattes or te?tered sragonrizes to those of Jragonflies exhib:ting naturad rizht. a few of these jhotograshs are slown in Fis. 1. The wing poilitons are quite ste:lar :o thoye : hotographed ireviously ina: :or, i 775 ; :icrbere.
 characierietice are eeen diu Uur iaboratory iesta. Ttis comarison 1 f faportant sirce the ist Eeneration mea; ured in iethered specimens (whether usiree noreal wif.g kinegat!es or not) was yuito tien. untoubtediy, the wirg-illuid Interactions supporting such hieh : :f: are of prime dmpuriance Du:, is atctiton, is seoms b:c:og:ca: iy unldaciy ina: Kichiy dieturted wire ao:tone hou: te capabie of titot :1ft seneration. In any evert, the photograpts of natura: ristet c:ear:y ztow tha: itec teineres tragonr:tes






E- 1 Examples of wing positions photcgrapued fus :overing fragon:": tes. The photographs note diken with 1 msec stut:er speed "rie 1 exerhoto :ens and natural light. - of 122 wuch phucg:aphe were used to du:un..:s the aimiarities in wine


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Eici Simultaneous photographs of dragonfly wing motions and photodiode marked, lift generation. These frontal views of the dragonfly revealed wing stroke angles while side views (not shown) revealed cauda:-rostral angles as well as the ctometric angle of attack of the wings. In these photographs, the rear wings were ma:ked to allow easier differentiation of front and rear wing angles.
box both wine cotions and osciliographic records vouid se photosrafhed simuitaneously. it typical serius of wine positions and associated ift trones are provided in Fig. 3. Using these photographs and Strobotach illuminated viceo tapes, it was possible to construct the summary o: wing kinematics as shown In Fig. 4. As may be seen, the wing tips trace out an oval juring each cye:e. Beginning at the tof of this oval, the m:ne tips move jownward and rosward, then upon reachirg the bottom of the oval they iwist upward i.f: :e zovine backward to return to the sop of the $\therefore$ a. Zotr the front and rear wing of the tandem [7.: Ahow 3imilar motions, although the rear wing Ba: : ead the rront one by as much as $150^{\circ}$ in some ©...tances, the wings are ciear:y $190^{\circ}$ out or :ta.e with each other, so it is dirficuie io tevermine which wing ieade etrough a typica: w:etecat cycie. Tre frequency or itrgbeat eycie t. GFFroximately $25-35$ !iz and di any instant is - te came cor both wines. iverage wirg itp :e:ont:tes were about to: lureer Jurine ufserones


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Eige 4 Summary of two dimensions of the wing motions. Using video tafes and 35 mm photographs, the typical wirg angles are indicated across a ru: wingbeat cycie. Both front and rear wing gotions are indicated such that phase angles between them can be determined. These motions can be summarized as a wing tip motion which traces out an ovoid path slanted downward and forward then upward and backward through a complete wingbeat.
angics are derscted on the upsirote :han the downstroke, in these sumeariesi, no atiomf has been aade en represent wine iwisitne a:one ibe spar; however, a modest ancunt or iwistine propagates rrom the root to the wing itf eact time the wine changes dtrection fron upstrokes to downetrokes and vice versa.

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A）

B）


Ene E Sumary of beometric angles of attack shroughout a winbeat．These angles are inticative of wine tip characteristics and to no：ind！cate a smal！amount of spanw：se twisting．Hypothesized riow and thrust vectors are also indicated．The angle representations are indicated over a ij；ica：to asec wireteat eycie duration at ajprox！ateiy 5 meec ：ntervals．


 －．te anrosynafic sa：vis：ions．To compare
 い二athot ：tr：was otiathou trest the force A．Atce＝eatures over conficio witebeat eycies．




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Eige 6 Summary of lift generation during a typical wingbeat cycle．All wingbeat cycles have been normalized to the indicated time scale to correci for variations in wingueat rrequency．As indicated，lift peaks of approximately 7 If are generated once during each wingbeat cycie，Such peaks give rise to inordinately high apparent $C_{2}$ values．
wing maximum lift was recorded in all test episodes．Sometimes the lirt peak even exhibited two components which we sfeculate might reflect the peak lift contribution of the two wines moving through the maximum ilft production portion of the wingbeat cyele at eitehtiy A：frerent times．Also，throughowt a typica！test episode variations in peak lift amplitudes occurred．Such variations afpeared to redate to wingbeat stroke amplitudes but these relationships remain to be more carefuliy measured．The erfect of wingbeat rrequency was feeiligible in iorms of instantaneous lirt peanis Lut ilf，of course，have a direct impact on iotal amount or lirt generated per unit tiee．
inxious to determire how these wise andematics produce such ：．igh lift，we arranged a tiow visuailizsiten ies：for tettered Aragonif：les． Tl．bća kerosene zmoke de：dvered a shor：distance ancad of the tracont：les in the zero r：ow



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 ：unt waso show tatter bomogeroows stoke









Ene 2 Stroboscopic fiow visualizations of wake struetures atout a dragonfly durlng a iethered llight episode. Smoke was deitvered imeediaiedy ahead of the fraeonrly wine at a jow velocity.

Detween the source of the wake vortex structures ard possitie ding tip vortices. The latter moved f.sto the dower turbulent wake and were quite :ators: cumpared :o the tormer. Thus, we :ere athe co socument at ieast the :esldua: evidence of a nove: riuid-wane dnteraction.

Ensobraect by thoee rintings, we sought a way :o betier uraersiand the rlow-wing atiteracilons mpioyed by the sragontiy to achieve tubt: :f: Fwe approacies aere trios: (1) the facctir:y was exilolied to become an automation =ode: or tis ri!gnt kinerai!ce and (2) an uac: : :ailne Elat plate mode: waz used for :Aactic : Iow visuadization. Bott approaches are :r. ihe:: !nfancy but Lot!: appear io have good fcien:1s:.

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 wnodes: a=ribtude and foeiucney-de;uraur:






ELg. \& Multiple exposure flow visualizations about an automaton model of the tethered dragonfly. All but one forward wing have been removed. The vortex formation is cleariy documented for a variety of wingllow interactions throughout a portion of the wing stroke cycie. The driven wing kinematics are quite similar to those of the uurestrainad and tethered (nondriven) dragontiles.
override the edectrical stimuli and initlate rlight kinematics of varying rrequericies. The dragonclies tested to date rarely elect to dis:upt our automaton procedures in this way. The result is a reproducible visualization or wise-fiuld interactions at any phase, in wha: appears io be, a ratieer normal wisebeat eycie. The preidminary results of this app:osch are encourasing. As may be sect in Fie. 3, the single front wing (others removed to provide fiou tamble!ty) edstis reproducitie vortices durtre cartaln portions or the triven wiretes: cycie. Since the visuationtions :eitesent t-5 stroboscof: © exfoewres iaker al : i.e sa=0 ::=e Geday boiveti a silmu:us ful:e and the oniot of I:Iumination, soth the autoeaton characte:!Et: es of the wine sotion and the reprofucto:e siructures of the $t: \frac{0}{}$ are ovident. we bave $x$ : analysed tho wing-fiow brioraci:ons fa:iy bu: we can fefinitedy etie the prezence of wisieaty enereszos r:iws grosucos sy dereonf:y wite
 secr. :: the cuafiote tour utre abtocaton zote: as we:l. Ti.e stae, shape and iosi:ion or irese








Eize 2 Flow visualization for an oscillating flat plate. In these photographs the vortex structures elicited by a thin ( -0.5 mm ) plate are shown for a variety of stroke iengths and oscillation frequencies.

In our second model approach employing an oscillating rlat plate, we simply sought a d:dactic indication of flow-plate interactions using a variety of thin plates. In these tests the effecte of cscillation ampilitude, oscillation rate and flate thickness were visualized. inomometry data were coilected in a single timension using an overheated winiature theralisier. The experimental parameters vere quichly reduced to those which produced cohesive vortex structures. dithin this parameter range :ortex cohesivencss was rated from the f!ow Uisuailiations and then piotted in regard to the sipropriate irotuction parameters. The resu: is were strikingly consistent (Fig. 9). Very conesive structures occurred with extremely inin piaies (- 0.5 mm$)$. hs fiaie thickness inereased, bo:h osc:liation rrequency and oscillation af: litude improved r : ow cohesiveness. As plate intckeses decreased, uscillation rrequency had a :arge effoct on fiou cohesiveness but stroka :ese:h had lilt:e offect (Fie. 10). The addit:on ot sea: : amourts or aneuiarity to the oscil:ating fitie dic no: ctariee itis reia:lon. 3ut in tratances of angu:arily tnat exceeded $20-25^{\circ}$, a ferin!te net thow was induced in the liourte:c.

Overa: :, these obsarvations stbstant:ste an :aroran: b:as obisited fros fiou visualisat!ons abcut dragonf:y wing . Fire:, the flow struciure ly exceeding:y cohestre and, secons:y, winebea: a=f: !tuce has lit:lo efroc: upon suc! structures. Fre use of piate anetes Incicates that tho

 vortex ;rosuciton ard zoteationess. : aifears








OSCILLATION FREQUENCY (HZ)

Elacll Sumary of stroke length and osciliation frequency effec:s on vortex formation by an oscillating rlat plate. Approximateiy inn test conditions were evaluated for Cow struciure cohestveness prior :o a cuantification (orcinal scailng) of the relations depicted here.
and of theme: ves.

## RAchesion

The re:at!chantgz Detween tragonr: $\quad$ : : C: gone:ration ent atne k!neatics tave stour the presence of tish itr: once gurine each winejos: cycic. d:so, the :otal amount of :lft eterat:on eeasured vas qul:e :arge. These observat:ons are trconalstont with s:oafy-siato aerotynamica and indteate that the sragonf:y auzt ec;ioy unsteacy or energizod separates fiows :o ach:eve : :f:

 no: been achlovec for ciher hover:se b:secks.

The wite at:eeat:es of :ho sraconf:y Stefor





l:overirg, escape and gliding. These racts suggest that the dragonfly must utilize unsteady rlows and related ift generation mechanisms which differ substantially from those of the Chalcid wasp. The flow visualization studies corroborate such a difference. Visualized vortex-dominated flows are immediately adjacent to the wing's mid-span. They persist in this spatial relation to the wing over an appreciable amount of the wingbeat cycle. And, these flows are lateralized such that each side or the insect interacts with a local flowfield that is somewhat independent of the rlowfield on the other side. It is tempting to speculate that the lateralization of clowfields underlies the roll instabilities seen in the natural flight of tragonflies.

Cne major conciusion may be drawn from the above observations: dragonflies use unsteady mechanisms that differ in many ways from those used by the Cha:cid wasp. Further, it now appears :ikely that other biological organisms coulj utilize jet other unsteady flow characteriztics to achieve lift and rlight behaviors. This is not a deterrent to attempting to understand and, perhaps, emulate the use of such riows. Rather, it is encouraging that many exploitation possibilities may exist: each optinized for a range of different aerodynamic needs.

For us, our work has just begun. Given the data at tanc, we must determine how the dragonfly produces the unsteady separated rlows we have v:sus:ized. de eust determire how these rlows ate sontro: Icd. ins, wa must deteraine how such $\therefore$ :ums inierag: with the dragonfiy wings to roduce the renariabie :If: values we have tosuernted.

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## Abstract

A recently developed general theory of aerodynamics is utilizet in an investigation of non-linear unsteady flow problems involving a non-rigid lifting body. It is shown that this theory, developed on the basis of viscous llow equations, permits the important interactive !!uid dynamic elements dominating the aerodynamics of non-linear unsteady flows to be identified and their contributions evaluated individually. Ciosed form expressions for the lift, the drag, and the power expenditure of the Weis-Fogh problem, considerec as a special case of llexible lifting bodies, are presented and discussed.

## 1. Introduction

The subject of unsteady aerodynarnics has been, for more than hall a century, an active field of fluid dynamics research. Previous theoretical and exferimental efforts have demonstrated that, under cersan restrictive circumstances, unsteady llows can be approximated by small departures from steady or uniform behavior. The addition of unsteady phenomena is steady ones reasonably describe such flows. The resulang linearized equations describing the flows are ufien amenable to mathematical treatinent. A large beaty of valuable literature has been developed over the years bealing with various aspects of lunear unsteady thows. Most of the fundamenta! concepts of unsteady thous that are adequately described by the dinear theory are tow well-inderstood. In contrast, in the domain of non-linear unsteady !lows, where strong unsteady eflects andilidate the lineat simplifications, the mathematical and experimental ditficulties attendant to a rigorous freatine.lt of unsteady serodynamic proolems are amenense. Vant of the essental and unque features of the pilinear unstedy flows are not well-understood sorsyy.

In recen: bears, :here has been a napid growen of eesearch se invity in non-lifeat unsteady arpodynamics. blus: ut the curce:te cesearin sopa's in non-linear ansteady xerodyratnics are motwated by applatations in :urso-rimhtues. inarte propellers, helicop:er ru:ors, e: ., where stions !low ninteachoms is an in:ransk part of :'re overali beriavige. The mimmization or alioviation of larare adverse effects isused by fluw uisteasiness is of fithar: oftern in tiex woftistions. A munber ot
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The present paper describes some of the recent efforts of the present authors in establishing an adequate understanding of the various inviscid and viscous mechanisms of generating unsteady aerodynamic forces. The work described here is a part of a long range research program, underway since early 1970s, in computational aerodynamics and in theoretical aerodynarnics. This research program has progressed through several stages. During its initial stages, computational and theoretical approaches designed specifically for non-linear unsteady flow problems were conceived, developed, calibrated, and thoroughly tested through analyses and numerical illustrations. Most recently, these approaches have been utilized in studies of several infortant problems of non-linear unsteady aerodynamics. These studies demonstrated that a genera! theory for aerodynamic forces and moments in viscous flows, developed previously by the first author of the present paper, is ideally-suited for non-linear unsteady aerodynamic problegis.

In previous papers ${ }^{2,3}$, detailed and rigorous derivations of the general serodynamic theory and preliminary results of a theoretical study of the vortex/airfoil interaction problem have been presented. In the present paper, important concepts related to the application of this general theory are reviewed. In particular, it is shown that, with this general theory, the total air load on a solid body can be divided into severa! components, each representing the contribution of 1 distinct physical process. This distinguishing feature of the general theory can be utilized in the establishment of a reasonable understanding of the detailed mechanisin of production of aerodynamic sorces. It is anticipated that such in understanding will in time provide a rationa! basis $f x$ the alleviation, control, or utilization of large non-linear aerodyinamic forces in various applications.

The contents of the prescnt paper are centered upon the theoretical treatment of the Weis-Fog' problem. Exte:isive computational efforts, hewever, are bein' earried out concurrently with theoreticai siudies. In fact, computational resilts have provided in the past and are continual!y providimg inportant physical insights :o risteady aerodynamic problems. Numerical procedures utilized in the present rexearch progratn emphasize integral-feprexentation formulations of the viscous tlow equations. This ix: aulation perithts the whution tield to be confmed to the vortical region af the tiow. Howerer, once the virtarity tistribution is compuied, then the evaluation o! the co-!lowinh posential low surrourding the voricical region is strainhtiorward. The resulting mumertial procedure is pariw wiarly well-su:ted for dubputing lugh feytolds rumber external thats. For inexr tlows, the wortical regive. which tiay encompass bexndapy labers. iecirculatory flut ext wanes, *omprises whty a sona!
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parametric studies of two-dimensional flows can be performed economically using widely avaidable computers such as the CDC-6600 computer. Threedimensional computations have been carried out for several flows involving simple boundary geonetries. The amounts of computation required for these flows are not unreasonable. Computed results for the unstea'y flow problems discussed in this paper and in Reference 3 will be presented in future articles. In this connection, it is worthy of note tha: the general viscous theory of aerodynamics used in the present study relates the aerodynamic forces and momenss acting on lifting bodies to their vortica! environinents. The general theory and the integral-representation approach are therefore ideally suited for one another in a combined theoretical and computational research prggram.

Previous applications, 2 of the general viscous theory of aerodynamics are concerned with flows past rigid lifting bodies. The Weis-Fogh motion considered in this paper, however, involves two wings aitached and yet moving relative to one another. The present study is, in this context, a precursor to a more comprehensive study of the unsteady derodynarnics of flexible (non-rigid) lifting bodies. It has beer recognized for a number of yeirs that unsteady aerodynamics of tlexible lifting bodies is intherent to acuatic propulsion and thight of animals. Obviuusly, a reasonable understanding of the physical mechanisms oi generation of unsteady aerodvnamic torces accompanying large amplizude motions of flexible witing surfaces is, beyond its biological significances, of decisive inpori to nuvel designs $u$ ! airborne vehicles utilizing large unsteady forces.

## 2. Vorticity Dynumics

Linsteady incompressible motions of viscous tlund are governed by the iaw of mass conservation and Newton's laws of motion. The mathematical statements o! these laws are faniliarly expressed, in terms of the veluctity vector $v$ and the pressure $p$, as the continuity and the Navier-Stokes equations. It is however, advan'ageous to introduce the concept of voritaty vecice a celined by

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\begin{equation*}
7 \times \dot{v}=\dot{j} \tag{1}
\end{equation*}
$$

and to consider the vericity iranspor: equi :ton

$$
\begin{equation*}
\frac{3 \dot{j}}{3 g}=(\dot{w} \cdot \eta) \dot{v}-(\dot{v} \cdot \eta) \dot{w} \cdot v \nabla^{2} \dot{w} \tag{2}
\end{equation*}
$$

There are everal mapor advantages in the use of tie corcep: of voricity. In the tirst place, the re:narhable success of the well-known circulation theory in predicting the litt force anplies that the vorticity of tie tow, which ultinately stould be resparisible for the circulation. is secountate for forces exeried by the tlutd on azordyratnac surfaces. Secondly, it is well hnown itat? viscous eltec:s are present wily in tie vortical part of :ise tlow. This tac: suggesis that it is pussible, in siugyint tluws about solid buty, to contite the solution to the viscous pegion of the !low througt the use of the wetictiy soncept. Thirdly, the comept of vorticity pertm:s the overall tlow problem so be deriomposed into a infenatis aspect and a ancice asperi. This tevinpustion tactitstes the wentituation ot imporisns phonceizl processes assuctated with varivas types of ! tows. Pirese :napor athantates stfeted oy the ase s! the - oxficity coriepts have been eiriployed by the preatl
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utilized previously in studies of various steady and timedependent flow problems. This advantage is briefly reviewed below.

The kinematic aspect of the viscous flow problem is described by Eq. (I) and the continuity equation. This aspec.t expresses the instantaneous relationship between the velocity field and the vorticity field. The kinetic aspect of the problem is described by Eq. (2). This asper.t is concerned with the redistribution of vorticity in the fluid through various kinetic processes. In studies of unsteady flows, it is convenient to follow the kinetic development of the vorticity field in the fluid. A knowledge of the velocity field is needed in the solution or the vorticity transport equation. This velocity field is kinemaitally a function of the vorticity field. The vorticity transport equation is non-linear and its mathematical arialysis presents great difficulties. It is, however, possible to obtain a significant amount of understanding about the kinetic processes involved in unsteady flows without detailed mathematical analyses.

Consider a finite solid body immersed in an infinite incompressible fluid with uniform viscosity. The solid body is initially at rest in the fluid which is also at rest. Subsequent prescribed motion of the solid body induces a correspunding unsteady motion of the fluid. It has been shown that vorticity is neither created nor destroyed in the interior of the fluid domain . Vorticity, however, is continually being generated at the solid boundary in contact with the fluid folluwing the initiation of the solid motion. This vorticity spreads into the interior of the fluid by the process of viscous diffusion and, once there, is transported away from the solid surface by both cenvection and diffusion. Since the transport oi vorticity by convection is 3 finite rate process and that by diffusion is effectively finite rate, the vortical region of the llow is of finite extent at any finite time level after the initiation of the solid motion. Outside the vorisal region, the flow is irrotational and therefore inviscid. If the flow Reynolds number is not small, then the effective rate of viscous diffusiot, is much smaller than that of convection. Therefore, a large region of the !luid, ahead and to the side of the solid, is free of vorticity and is inviscid.

The general pattern of unsteady llow development can be briefly described as follows. Is a consequence of the sulid motion relative to the !luid, voritity is generated continually at the flund/solid intertace. Once generated, the vorticity moves along the solid surface as long as the flow reinains attached. That is, succe the effective rate of viscous dilfusion is much smaller than That of convection, the vorticity, generated on the sulud surtace, cannot perietrate far into the interior of the fluid domain before being carried downstreain by the fluid motion. A thin layer of voricity :"acent to the solid timundary is therefore present. This layer is simply the well-known boundary layer. The vorticity within the boundary layer continually moves downstream with the tlusd and, at the sarne time, is continually being replenished through the generation of vortacity on the solid surface. This process of replenishment is present in botit steady and unsteady fluws. In steady llows, the replenishinent process and the vorticity transpixt process balace one another and the vorticity distribution th the boundart laver is independent of tithe in a reference trane attached to tire wht. In unsteady flows, the replenishimen: and t:anspor: processes do not balance one arotier and voricity sistribution in the boundary laver is fitme-deperstent. In buth steady and unsteady liols. onalare tie boundary tavers are thin, it as often conventent is eepresplit the torthity th the layers by vorier simeis.

The reperesentation of a boundary laver br a vories
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approximates the location of the vorticity aiross the boundary layer by a given point adjacent to the solid surface. The strength of the concentrated vortex sheet is simply the integrated vorticity across the boundary lay er. The vortex sheet moves along the solid surface. The distinction between the inviscid assumption and the present approximation is not merely a matter of semantics. While the two concepts often lead to the same conveniences in analyses and computation, the presen: approximation is based on a viscous flow viewpoint arid experiences no conceptual difficulties associated with previous inviscid theories.

The vorticity in the boundary layer eventually leaves the vicinity of the solid surface through several possible avenues. If no massive separation of the flow occurs on the solid surface, then the vorticity in the boundary layer eventually feeds into a wake layer. This occurs, for example, in the case of a thin airfoil at a stnal! angle of attack. The two boundary layers at the two sides of the airfoil in this case merge at the trailing edge, with both layers feeding vorticity into the wake layer. In steady flows, the total flux of vorticity entering the wake is zero. In unsteady flows, a net flux of vort:city enters the wake. Since the boundary layers are thin, the wake layer, which is a continuation of the joundary layers, is also thin initially. As the vorticity laver inoves away from the solid through the convective process, viscous dilfusion produces only a slow growith in the thickness of the wake layer. In consequence, it is reasonable in many applications to represent the witke layer also by a vortex sheet. The wake layer is usually unsiable. The velocity field associated with the vorticity in the wake layer causes the wake layer to "roll-up". If the roll-up process keurs at a large distance from the solid, then it is reasonable to represent the rolled-up vorticity by a sungle vortex filament in analyzing the t!ow ne ar the solid. If the roll-up process occurs near the solid, however, then detailed structure of the rolledup borticity may be recessarv. In any event, the total strength of the rolled-up vorticity needs to be known in order to determine correcily the aerodynamic forces actunt: on the solit. For three-dimensional llows, the vort:city in the boundary laver leaves the vicinty of the solid surfaces also through the formation of tip vortices which usually roll up.
la applications where flow separation is an ampurtant feature, the representation of the vortacity in :the boundary laye? part of the flow by a vortex sheet is siall permissible. Quansitatively accurate solution to the tluw problein in this case requires a knowledge of the cetaded vorticity distribuition in the separsied (recirculating) part of the flow. This disiributed vor!acity is mot accurately reppexented by concenirated vovere sheets or vortex filamenis. lt is well hinown. however, that in unsteavy tion yoner assemblies uiten move inore ar less as at mility. In cunsequence. considerable physica! ansigh: can be ganced througtl a vortex stiee:/flament reppesentation even in cases of !lums con:aining trassive xeparated regions.

## 3. Aeroduratime Theoiv for Viscous Flows

Aerodynamic torces and mumetis siang on whad bodies imenersed and thoving in viscuss !ludes ian se teterimined. in permiple, through a guantitative ainuledxe of :ine ofiatiec tluid motion around the bodies. The xiqusition ot Le:a.ted infumation abul: the peat ilowt!eld assuciates with hiting surtaces, however, pexen:s inmense, viten insupmuntasle, mathemation side erperinmial difticul:tes. Historsiallo, the:c:xe.
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with the details of the fluid motion. In particular, the circulation theory is known to predict the lift force accurately for certain types of lifting surfaces, e.g., thin auloils with sharp trailing edges, under certain flow envirorments, e.g., small angles of attack.

The circulation theory is today, as it was fifty years ago, the foundation of accepted theories of aerodynamics. Considerable uncertainties and conceptual difficulties, however, exist regarding the application of the circulation theory in cases where the lifting surface does not possess a sharp trailing edge or where more than one trailing edge is present, where massive flow separation occurs, and where ti.e lifting surface is three-dimensional and its motion is timedependent. These uncertainties arise mainly because of the perfect-fluid assumption in the mathematical development of the theory. The viscous origin of circulation has been long recognized and several yellknown werks, e.g., by Von Karman and Millikan by Howarth , and oy Sears, nave dealt with certain aspects of viscous phencmena that produce circulation. A thorough understar:ding of the viscous mechanisms of generation of steady and time-dependent aerodynamic forces, however, is not available and it is usually difficult to interpret the application of the circulation theory as an approximation of the viscous flow phenomena.

A general theory ${ }^{2}$ for aerodynamic forces and moments in viscous flows was rigorously established recent!y on the basis of the Navier-Stokes and continuity equations. No simplifying assumptions, other than those contained in the Navier-Stokes equations, were introduced in the derivation of this theory.

The general theory comprises the following three mathernatical statements:

$$
\begin{align*}
& \int_{\mathbf{R}} \omega d \mathbf{L}=0  \tag{3}\\
& F=-\frac{D}{d-1} \frac{d}{d t} \int_{R_{m}} \vec{r} \times \omega d R \cdot D \frac{d}{d t} \int_{R_{s}} \approx d R(4)  \tag{4}\\
& \vec{u}=\frac{D}{2} \frac{d}{d t} \int_{R_{m}} r^{2} \dot{d} d R \cdot \rho \frac{d}{d t} \int_{R_{s}} \dot{i} \times \dot{v} d R(5) \tag{5}
\end{align*}
$$

where $R$ is the infinite unlimited region jointly occupied by the Iluid and the solid bodies; $F$ is the aerodynamic force acting on the solid bodies; $d$ is the dimensionality of the problem, i.e., $d=2$ for iwo firmensional flows and $d=3$ for three-dimensional fluws, $r$ is a position veçtor; $R$ is the region occupied by the sulid sodies, and II is the moment o! aerodynamic tox:e acting on the solid bodies.

Equations (3), (f) and (5) are valid for :he incumpressible motion of in infinite fiusd with unitorm viscosity and with une or more sutiu hod:en !!nmersed in the tluid. The thotions are considered in start trom rest and tife generally itme-dependent. Steady llows, athen they exist, afe considered to be spproached asimptotically at large tune levels after the onset of the -notion. Equation (3) states dhat the combined toial ruxictily of the lluid and the wolid bodies is zeru. The voricity in the solsd bodies, as aefined by Eq. (1), is winply ixice the angular velceity of the witut maties. In cases where tive region occupied by the soled bodies is negligibly sinall, $x$ where the solud borties undergo unly oranslational thotions, the wortectivin the witc vanisics and the iotal voricity in the lluid is zeri. for these :̈ases, existing inviscid serviynamic ineories corfectly requife the sutal cifculation of stie wiole sysiceri, inklud:if :he buund roriex, :he siar:ing voriex and :tir - aine vortses, :o be eepu. The r!tecis n! fotation u! :he

by Eqs. (3) are generally not included in inviscid analyses.
Equation (4) states that the aerodynamic force acting on the solid bodies is composed of two contributions. The first term on the right side of Eq. (4) gives the contribution of the time variation of the first moment of the vorticity. The second term gives the contribution of the inertia force of the fluid displaced by the solid bodies. Equation (5) states that the moment of aerodynamic force is composed of two contributions, a contribution of the total second momert of the vorticity fieid and a contribution of the momen. of inertia. The inertia terms in Eqs. (6) and (7) of course vanish in cases where the solid region is negligibly small or where the : ilid bodies experience no acceleration.

As discussed earlier, it is convenient to divide the overall unsteady flow problem into its kinetic and ninematic aspects. The general theory described here relates the insteady aerodynamic forces and moments acting on the solid bodies to the kinetic development of the vorticity field. The task of analyzing the kinetics and the kinematics of the flow remains.

It is clear from the general theorv, that all the information about aerodynamic forces and moments are contained in the time-dependent vorticity environment of the lifting body. No information about the potential field surrounding the vortical region is needed in the theorv. Under certain restrictive circumstances, it is porsible to sperify the vorticity field approximately without actually solving the vorticity transport equation. The general theory described above then permits the uns:eady serodvnamic forces and moments to be - .termined in a straightforward manner.

It reeds to be emphasized, even at the risk of appearing repetitive, that the general theory described above is exact in that it is an exact consequence of the viscous flow equations. This iact, however, does not inmbit the introduction of approximations to the equations given in this section. As discussed in Section 2 of this paper, the word "upprovimation" is used here to inaliate that the prec:: - distribution of the vorticaty in the tluid is compromised in exchange for convenience in the evaluation of unsteady aerudynamic forces and noments. Thiough this approximation, the general theort sfiers an opportunity of establishing important physical ins: pht $^{\text {pito }}$ the mechanisins of generation of large unsieady serodynarnic forces. This opportunity is aridasle even witn relatively imprecise approximations of :he vorticit distribution. I inder circumstances where the vorticity distributions can be accurately spproximated, the general theory leads to accurate preduethons o! unstedty derodynamue forces and moments in both the linear and the non-linear domains.

The approximations to the general theory. are conceptallv di!ferent from the inviscia fluid assumption whien is the $t$ asis of classical theories. Since a truly inviscid :luic boes not exist in nature and since the limit of vanishingly sinall viscosity is distinct from a zero viscosity, the sukcess of the inviscid theories must depend uon the auspicious circumstance that inviscid conclusions coincide with certain approximations of viscou conclusions. The general viscous theory has been shownㄹ. inceed, to yield, at various levels of spproximsion. well-known conclusions of classicial inviscid theories.

For high Revnolds number exiernal flows contatint in appreciable regions of separation, as that sescribed i. Section 2, the voriciey distribution in the tluid is accurately represented by vortex shects and vortex illaments. The integrals in Eqs. 13). (it), and (5) over the region $R$, tien reduces io integrals over surfaces in Enesedimensional flows and over lines in : w u-umensiuna! !iows. Under these circumsianies, the analises beciome considerably simpler. To deter mine the serodym:mic torce, the following spproximation of

Eq. (4) may be used:

$$
\vec{F}=-\frac{\rho}{d-1} \frac{d}{d t} \int_{S_{+}+W} \vec{r} \times \vec{\gamma} d B+\rho \frac{d}{d t} \int_{R_{c}} \hat{v} d R(\sigma)
$$

where $W$ is the wake surfaces (or lines) including tip vortex sheets and starting yortex where they exist, and $S^{+}$is a surface enveloping the solid surface $S$ and at infinitesimal distance from S. The distinction between $S$ and $S^{+}$is conceptually important. With the approximation discussed, the vortex sheet represents the boundary layer vorticity which is in the fluid domain. The velocity of the vortex sheet is different from the solid surface velocity.

In Eq. (6), the vortex sheet on $S$ approximates the boundary layer adjacent to the solid surface. The vortex strength $\gamma$ on $S$ is therefore the integrated vorticity across the thickness of the boundary layer'. To the accuracy of the boundary layer approximation, the vorticity is the negative of the normal derivative of the velocity component in the direction tangent to the solid surface. One therefore obtains, upon integrating the vorticity along the normal direction,

$$
\begin{equation*}
\gamma(s)=-v_{s}(s, \delta)+v_{s}(s, 0) \tag{7}
\end{equation*}
$$

where $s$ in the boundary layer coordinate tangential to the solid surface $v_{s}$ is the tangential velocity component, $\delta$ is the normal coordinate at the edge of the boundary layer and 0 is the normal coordinate on the solid surface. If $v_{s}(s, 0)=0$, then $\gamma$ is smply the negative of the boundary layer edge velocity.

Equation (7) is easily generalized to threedimensional applications. According to Eq. (7), the vortex sheet on the solid surface represents a discontinuity in tangential velocity between the solid and the inviscid flow surrounding the boundary layer. This discontinuity is consistent with the approximation of the boundary layer, whici, possesses a finite albeit small thickness, by a sheet. This approximation of course is not suitable for the computation of the kinetic transport of vorticity within the boundary layer. The approximation, nevertheless, is well-suited for the computation of aerodynamic forces. In many applications, distribution of the vortex stiength on the solid can be computed without actually performing boundary layer calculations. For example, if the vorticliy distribution in the detached part of the flow is known, then the strength of the vortex sheet on $S$ is uniquely determined, as discussed in Rei. S, without computing the detalled flow within the boundary layer. It is mentioned in passing that the concept described ir. Ref. 5 is similar to that used in the panel/vortex lattice nethods currently receiving a great deal of attention vithin the aerodynamics communtty. In many existing panel codes, fictitious source-sink distributions over $\$$ are used. It is not difficult to show however, that these source-sink distributions are equivalent to vortex distributions over $S$. The vortex distributions over $S$, as discussed earlier, are approximations of real vorticity in the boundary laver. Computationally, the use of vortex distributions over $S$ is as convenient as the use of sourcesink distributions ${ }^{10}$. Also, in vortex lattice methods, the vortices in the interior of the fluid domain are usually aliowed to convect but not to diffuse. This restriction, towever, is ngt necessary and can be removed in viscous computations.

Once the vortex distributions over $S$ and $W$ are computed, Eq. (6) immediately gives the aerodynamic force F. The use of Eq. (6) clearly offers distinctive ajvantages over the prevailing surface pressure-shear stress integration nethod since, with Eq. (6), both the unsteady dag and the lift can be evaluaied in a straig':forward inanner directly froin the vortex
distributions.
In general, the vortical region in the fluid is composed of a vortical system near the solid bodies and a vortical system trailing the solid bodies. The near vortical system in general contains attached boundary layers and detached recirculating flows. The vorticity in the trailing vortical system represents the vorticity shed from the near vortical system at previous time levels. Shortly after the initiation of the motion of a solid body, the vorticity region is confined to thin layers near the solid body. In the case of a lifting body, a concentrated dose of vortici:y, i.e., a starting vortex, leave; the vicinity of the body shcitly after the motion's onset. The average velocity of this starting vortex is initially one half of the freestream velocity. This fact is consistent with the well-known Wagner's effect and can be shown by analyzirg the vorticity distribution in the boundary layer as it leaves the solid body's trailing edge. With increasing time, the starting vortex moves in the general downstream direction, becomes diffused, and approaches the velocity of the freestream. Between the starting vortex and the near vortical s) stem is stretched the remainder of the trailing vortical system which, for convenience, is called the vortical wake. The line of demarcation between the near vortical system and the wake need not be delineated precisely. The division of the overall vortical $s$ 'stem into its several components is exteenely useful since, with the general viscous serodynamic theory, the contributions of each of these components to the aerodynamic force and moment can be considered individualı; For example, with Eq. (4), the integral over $R_{f}$ can be written as the sum of four integrals over, respectively, ihe unsteady boundary layers, the recirculating regions, the vorti:al wake, and the starting vortex. By separating; the overall aerodynamic force into contributions by the several flow components, considerable physical insights can be developed.

In the case of a high Reynolds number flow containing no appreciable recirculating regions, Eq. (6) shows that the overall aerodynamic force is composed of lcur contributions. The first contribution, represented by the vortex moment integral over $S^{+}$, is due to the developinent of the unsteady boundary layer. The second and third contribitions are due to, respectively, the movernents of the wake and the starting vortex, and are represented bi the vortex mornent integral suer $W$. The fourth contribution is due to the solid body acceleration.

The preceding discussion concerning the acrodynamic force is applicable also to the mornent .oi aerodynamic force. In the case of a high Reynus nuinber flow containing no appreciable recarculating regions, Eq. ( 5 ) yields an equation expressing timen terms of vortex shert strengths over $S^{*}$ and $W$ and the effects of solid bexiy acceleration.

## 4. Weis-Fogh Mechanism

It is well-inown that, according to inviscid theories, steady lift force acting on an airfoll is proportional to the circulation around the airfoil. For an airfoil initially at rest and is set into motion impulsively, the circulation is developed through the shedding of a starting vortex. That is, because of the need to conserve total vorticity, the acquisition of a circulation around an airforl is accompanied by the release of a starting vortex. The circulation around the starting vortex is equal in inapnitude and upposite in sense to the circulation acquired by the arfoul. The process of vortex shedding is not difficult io understand in the context of viscous flow. Indeed. the starting vortex is a direct consequence of the unsteady boundary laver activities around the arfoll. There evist, however, conceptual ditfroulties in understanding the process of voriex shedding in the
context of an inviscid fluid, i.e., a fluid with a zero viscosity rather than a vanishingly small viscosity.

Weis-Fogh ${ }^{\text {II }}$ observed that certain types of insects, e.g., Encarsia Formosa, with a pair of wings pivoted together at their trailing edge, rotate their wings individually in opposite directions. The rotation generates a circulation about each wing. If the two wings are identical in shape and their rotational speed is identical, then the circulation magnitude of each wing is equal to that of the other wing. The senses of the circulations of the two wings are opposite to one another. Because of symmetry, no trailing edge shedding of vortices occurs. In fact, if vortices were shed from the trailing edges of the two wings, they would annihilate one another because they are of opposite senses.

Weis-Fogh described the above mentioned mechanism of generating circulation as the "fling" phase of a fling-clap cycle. At the beginning of the cycle, the two wings are close to each other. The wings' leading edges separate from one another as the wings fling apart, i.e., rotate about their trailing edges and open up into a $V$ shape. After reaching a certain opening angl', the two wings break apart and move in opposite directins arounc the body of the insect. The wings eventually flip and return to their initial position through a clap motion. In this paper, the fling, or opening, phase of the motion is examined. The analyses presented, however, is obviously also applicable directly to the clap phase of the motion.

The two-dimensional definition of the Weis-Fogh motion is given in detail by Lighthill ${ }^{12}$. The fling phase of the motion is shown schematically in Figure 1, where the wing pair is modeled by a pair of flat plates of chord c. The points $A_{1}$ and $A_{2}$ are stationary and are the junctures of the two wings, with $A_{2}$ on the upper surfaces and $A_{1}$ on the lower surtace's of the wings. During the fling phase, the wings rotate about the points $A_{1}$ and $A_{2}$ with a angular velocity

$$
\begin{equation*}
\Omega=\dot{\theta} \tag{8}
\end{equation*}
$$

where $a$ is the half angle of the opening of the wing pair. The motion is symmetric about the line EF. The fluid infinitely far from the wing pair is stationary. Each of the two wings acquires a circulation during the fling phase. Since the two circulations of the two wings are ellual in magnitude and opposite in sense, the total cilculation about the two wings is zero. At the moment of breaking apart of the wings, each wing possesses a circulation suitable for generating lift during its subsequent motion. It is easy to see thiai if the range $0<x<\frac{\pi}{2}$ corresponds to the fling phase, then the range $\frac{\pi}{2}<1<\frac{\pi}{\pi}$ corresponds to the clap phase of the Weis-Fogh motion.

Lighthill ${ }^{2}$ emphasized the absence of the trailir gedge shedding of vortices in the Weis-Fogh motion. He indicated that it is remarkable that the Weis-Fogh mechanism works "for a !luid uf zeru viscosity; not simply in the limit of vanishing viscosity". He presented a two-dimensional inviscid analyses which showed that the circulation around each wing during the tling phase is proportional to the wings' angular velocity $\Omega$ and to a function of the opening angle $a$ of the wings. He expressed this function in the form of an integral and presented cqumputed results for this function. Edwards and Cheng recently extended Lighthill's work and presented a closed form expression for the wing's circulation during the fling phase.

Although vortices are not expected to be shed at the frailing edges of the two wings during the fling phase, the rotation of the wing $\left\{\begin{array}{l}\text { may cause leading edge }\end{array}\right.$ shedding of vortices. Lighthill ${ }^{2}$ examıned this leading edge separation phenomenon and concluded that its effect in the wing's circulation is weak. Subsequent
experiments by Maxworthy ${ }^{14}$, however, showed a substantial effect of the leading-edge separration. Results of a recent study by Edwards and Cheng ${ }_{14}^{3}$ are in general agreement with Maxworthy's observation ${ }^{14}$.

Weis-Fogh suggested that the opening of the wing pair causes the necessary circulation to be generated irnmediately and thus avoiding any delays in the build-up of the maximum lift required by the well-known Wagner's effect. Also, in the case of the conventional wing, substantiai work must be done by the wing on the fluid to provide the kinetic energy associated with the starting vortex. In consequence, the wing experiences a large unsteady drag immediately after the start of its motion. Since the starting vortex is absent, the Weis-Fogh wing is not expected to experience a large drag immediately after the breakup of the wing pair. To the present authors' knowledge, although the problem of generation of circulation on the wing pair has received the attention of previous inves:igators, the questions of unsteady lift force, unsteady dag force, and power expenditure experienced by the wings during the fling phase of the Weis-Fogh motion have not been resolved. Answers to these questions are important in establishing a comprehensive understanding of non-linear unsteady aerodynamics and in future utilization of large unsteady aerodynamic forces.

For the problem under consideration, the processes of generation and transport of vorticity described in Section 2 of this paper lead to an unsteady boundary layer wherever the flow near the solid surface is essentially tangential to the surface. The thickness of this boundary layer is comparable to the diffusion leng th $(v \tau)^{2}$, where $\tau$ is the time duration of the fling phase of the Weis-Fogh motion. If this diffusion length is much smaller than the chord of the wings, then the boundary layer vorticity is accurately approximated by a vortex sheet. For the present problem, flow near the lower surface of the wing is expected to remain attached throughout the lling phase. The flow is, however, expected to separate from the leading edge. resulting in leading-edge vortex shedding.

Following Lighthill ${ }^{2}$, the problem is reformulated in a conforinally mapped plane, $\zeta=\xi+i n$, through the use of a Schwarz-Christoffel transformation. A closed form expression for the transfigmation function presented by Edwards and Cheng is used. The transformation function is

$$
\begin{equation*}
z=1 k(1 \cdot \zeta)^{1-\alpha / \pi}(\zeta-1)^{\alpha / \pi} \tag{9}
\end{equation*}
$$

where $z=x+i y$ is the physical plane described in Figure 1 , and $k$ is $A$ :onstant given by

$$
\begin{equation*}
k=\frac{1}{2} \subset\left(\frac{\alpha}{\pi}\right)^{-\alpha / \pi}\left(1-\frac{x}{\pi}\right)^{-1 \cdot \frac{x}{\pi}} \tag{10}
\end{equation*}
$$

The transformation (9) maps the wing pair onto the straight line seginent lying on the $\mathrm{j}-\mathrm{axis}$ in the range $-1<5$ is shown in Fig. 2, with the two wings, $A_{1} B A_{2}$ and $A_{1} C A_{2}$, mapped respectively onto the lower and upper surfaces of the segment. The $y$-axis EF is inapped onto the reinaining parts of the $\xi$-axis, as shown in Fig. 2.

During the flang phase, the line EF retnans to be a line of symmetry. It is therefore necessary only to consider the left half complex 2 -plane, which is mapped unto the upper hal! complex s-plane. The normal velucity of the wing $A, B A$, during the fling phase is . $12 \mid$. The :mpermeabld boundary condition on the wing $A_{1} B A_{2}$ therefore gives the following boundars condition fur the stream function $b$.

$$
\begin{equation*}
\because-\frac{1}{2} \because:, 2 \tag{!!}
\end{equation*}
$$

The stream function vanishes on the line of symmetry $E A_{1}-A_{2} F$ and also infinitely far from the wing.

It ${ }^{2}$ is simple to show that, with the potential function $\phi_{1}$, the complex potential, $W_{1}=\phi_{1}+i \psi_{I}$, expressed in terms of $\zeta$ below satisfies these conditions for the stream function:

$$
\begin{equation*}
W_{1}=-\frac{1 k^{2}}{2 \sin 2 \alpha}\left[(\zeta+1)^{2-2 \alpha / \pi}(\zeta-1)^{2 \alpha / \pi}-\zeta^{2}-2\left(1-\frac{2 \alpha}{\pi}\right) \zeta\right] \tag{12}
\end{equation*}
$$

In Eqs. (12) and the following equations, the subscript 1 is used to inoicate the omission of the leading-edge vortex shedding phenomena. The complex velocity in the $\zeta^{-}$ plane, $v_{\zeta}=d W / d \zeta$, is given by

$$
\begin{align*}
v_{\zeta 1}=-\frac{\Omega k^{2}}{\sin 2 a} & {\left[\left(\zeta-1+\frac{2 \alpha}{\pi}<\zeta+1\right)^{1-2 \alpha / \pi}(\zeta-1)^{-1+2 \alpha / \pi}\right.} \\
& \left.-\left(\zeta+1-\frac{2 a}{\pi}\right)\right] \tag{43}
\end{align*}
$$

The complex velocity in the z -plane is given by the right side of Eq. (13) divided by the derivative $\mathrm{dz} / \mathrm{d} \zeta$. It is, however, more convenient to use a transformed version of Eq. (6) in studies of aerodynamic forces acting on the wings.

In the present problem, the wing's cross-sectional area is negligibly small. The last integral in Eq. (6) therefore vanishes. The contributions of the unsteady boundary layer and the wake can be treated individually. One has therefore, with the wake, i.e., the shed vortices, omitted,

$$
\begin{equation*}
F_{l x}+i F_{l y}=i \rho \frac{d}{d t} \int_{s^{+}} \gamma_{1} d s \tag{14}
\end{equation*}
$$

where $F_{1 x}$ and $F_{1 y}$ are respectively the horizontal and the vertical components of aerodynamic force acting on the wing pair due to the motion of the wing pair.

Because of symmetry, the vortex strength on the right wing is equal in magnitude and opposite in sense to the vortex strength on the left wing. The lift force $L$, acting on each wing is mereiore one half of the total vertical force $F_{y}$ and is siven by the time variation of the vortex momeht on a s: isie wing. One therefore has

$$
\begin{equation*}
L_{1}=0 \frac{d}{d t} \int_{A_{1} B A_{2}} x y_{1} d s \tag{is}
\end{equation*}
$$

where the integration is over both the upper and the lower surfaces of the left wing.

Since the wing has no tangential velocity on its surface, the last term in Eq. (7) vanishes. The vortex strength $y$ is therefore simply the negative of the tangential velocity at the fluid side of the vortex sheet. Since $d z=d ;(d z / d ;)$, Eq. (20) can be rewritten as

$$
\begin{equation*}
L_{1}=\nu \frac{d}{d t} \int_{-1}^{1} x_{y} y_{v} d t \tag{16}
\end{equation*}
$$

where $Y_{\text {: }}$ is the negative of the :-component of velocity, given by Eq. (13), on the 5 -axis.

On tie $\xi-a \times 15$, for the interval $-1<5<1$, one obtains from Eqs. (9) and (13) the following expressions

$$
\begin{equation*}
x=-\sin a k(1 \cdot 5)^{1-2 / \pi}(1-5)^{1 / 7} \tag{17}
\end{equation*}
$$

and

$$
\begin{gather*}
y_{: 1}=-\frac{2 h^{2}}{\sin 2 z}\left[\left(5-1 \cdot \frac{2 \pi}{7}\right)(1 \cdot 5)^{1-28 / 7}(1-5)^{-1} \cdot 21 / 7 \cos 2 x\right. \\
\cdot\left(5 \cdot 1-\frac{2 x}{7}\right) \tag{13}
\end{gather*}
$$

Plaring Eqs. (17) and (1s):ntu Eq. (6), one abrains.
after performing the integration,

$$
\begin{equation*}
L_{1}=\rho c^{3} \frac{d}{d t}\left[\Omega f_{1}(\alpha)\right] \tag{19}
\end{equation*}
$$

where

$$
\begin{equation*}
f_{1}(\alpha)=\frac{\pi}{3}_{\frac{(\alpha)}{\pi}}{ }^{1-3 \alpha / \pi}\left(1-\frac{\alpha}{\pi}\right)^{-2}+3 \alpha / \pi\left(1-\frac{2 \alpha}{\pi}\right) \csc 2 \alpha \tag{20}
\end{equation*}
$$

Equation (21) can be rewritten as

$$
\begin{equation*}
\frac{L_{1}}{\rho c^{3}}=\Omega f_{1}+\Omega^{2} \frac{d f_{1}}{d \alpha} \tag{2I}
\end{equation*}
$$

where $\dot{\Omega}$ is the angular acceleration of the wing.
Equation (21) states that the unsteady lift component $L$ has two contributions. One contributor is the angular acceleration of the wing, its effect being directly proportional to the angular acceleration. The other coritributor is the angular velocity of the wing, its effect being proportional to the square of the angular velocity. Both contributions are functions of the opening angle, $\alpha$, of the wing pair, as given by $f_{1}(\alpha)$ and its derivative.

For convenience, the horizontal force icting on the wing $A_{1} B A_{2}$ is designated as an unsteady drag. This drag cannot be determined fromi the vorticity moment on a single wing alorie. The total horizontal force acting on the wing pair is zero because of symmetry. The two wings are hinged together at their trailing edges. In Fig. 3 is shown a free body diagram for the wing, with N , and $S_{\text {, }}$ representing respectively the normal and tangehtial components of the unsteady aerodynamic force acting on the wing. $H_{l}$ is the reaction at the hinge, i.e., the force exerted by the wing $A_{1} C A_{2}$ on the wing $A_{1} B A_{2}$. Because of symmerry, this reaction is directed horizontally. The free body diagrar. shows that the unsteady lift $L$ is the sum of the vertical components of S , and N ? The unsteady drag is the sum of the horizontal components of $S_{1}$ and $N_{1}$ and is the negative of $H_{1}$

The unsteady normal force $N_{1}$ acting on each wing is given by

$$
\begin{equation*}
v_{1}=\int_{A_{1}}^{B} p_{1} d s-\int_{B}^{A_{2}} p_{1} d s \tag{22}
\end{equation*}
$$

where $p_{1}$ is the pressure acing $\infty$ the wing. Since the ouundary layer cannot support a significant pressure difference ucross the layer, it is permissible to let this pressure $p$ be the pressure at the boundary layer's outer edge, where the vorticity is negligibly small and the !!ow is pistential. From the inviscid momentum equation, one obtairis

$$
\begin{equation*}
\left.7 i o \frac{3}{3 t} \cdot p \cdot p \frac{v^{4}}{2}\right)=0 \tag{23}
\end{equation*}
$$

At the edge of the bourdary laver, the norinal velucity of the t!ow is neg!igibl small compared to the ingential velocity. The :angental velocity inagnitude is equal to the strength $y$ ut the vortex sheet representing the boundary layer. Using this information, one obtains tron Equ. (22) arid (23),

$$
\begin{align*}
N_{1} & =-0 \frac{d}{d t} \int_{-1}^{1} 2 \frac{d r}{d r} d! \\
& =\frac{2}{2} \int_{-1}^{1} r_{5}^{2} \frac{d r}{d!} d \zeta \tag{26}
\end{align*}
$$


I s:mp !qs. (12) unc (18), (! wil be stown that

$$
\begin{equation*}
\frac{N_{1}}{\rho c^{3}}=\frac{d}{d t}\left[\Omega f_{1}(\alpha)\right] \csc \alpha-\Omega^{2} g_{1}(\alpha) \operatorname{ctn} \alpha \tag{25}
\end{equation*}
$$

where

$$
\begin{equation*}
g_{1}(\alpha)=\frac{\pi}{2} \csc ^{2}(2 \alpha)\left(\frac{\alpha}{\pi}\right)^{1-4 \alpha / \pi}\left(1-\frac{\alpha}{\pi}\right)^{-3+4 \alpha / \pi}\left(1-\frac{2 \alpha}{\pi}\right)^{2} \tag{26}
\end{equation*}
$$

Equations (21) and (25) gives the following expressions for the unsteady tangential force and the unsteady drag

$$
\begin{equation*}
\frac{S_{1}}{\rho c^{3}}=\Omega^{2} g_{l}(\alpha) \tag{27}
\end{equation*}
$$

and

$$
\begin{equation*}
\frac{D_{1}}{\rho c^{3}}=\frac{d}{d t}\left[\Omega f_{1}(\alpha)\right] \operatorname{cts} \alpha-\Omega^{2} g_{1}(\alpha) \csc \alpha \tag{28}
\end{equation*}
$$

The normal forces can be expressed in the form

$$
\begin{equation*}
\frac{N_{l}}{\rho c^{3}}=\dot{\Omega} f_{n l}(\alpha)+\Omega^{2} g_{n l}(\alpha) \tag{29}
\end{equation*}
$$

where the subscript $n$ indicates the fact that the functions $f_{n l}(\alpha)$ and $g_{n l}(\alpha)$ are related to contributions to N .

The forces $L_{1}$ and $D_{1}$ are easily expressed in forms similar to Eq. (29). The tahgential force $S_{\mathcal{Y}}$ is independent of the angular acceleration and is proportional to the angular velocity, as is shown in Eq. (27). The physical significance of this rather remarkable teature of the tangential force is not yet explained. It is clear, however, that this tangential force is due to the wellknown leading edge suction effect which is also present in steady flows past airfoils at non-zero angles of attack. In the case of a thin airfoil represented by a infinitesimally thin flat plate, this suction force can be determined through a limiting process. The procedure just described for the Weis-Fogh problem when applied to the flat plate, at an angle of attack and a free stream velocity $u_{\infty}$, gives a suction force value of $-\rho u_{\infty} r \sin \alpha,{ }^{\infty}$ where $\Gamma$ is the circulation (boundary layer ${ }^{\infty}$ vorticity) around the plate. This suction force corresponds to a zero drag as is expected for steady inviscid flows.

The functions $f, g_{\text {, }} f_{d \prime}$, etc. are represented graphically in Figures $44^{\prime}, 5,8$ and 7 . It is worthy of note that all these functions are easily expressible in terms of the two func:ions $f_{1}$ and $g_{1}$ and the derivatives of these two functions.

It is of interest to determine the power expenditure needed to maintain a prescribed motion of the wing. The power requirement $P_{f}$ for the case of no leading-eage vortex shedding is given by

$$
\begin{equation*}
p_{1}=\int_{A_{1}}^{B} p_{1} v_{n} d s-\int_{B}^{A_{2}} p_{1} v_{n} d s \tag{30}
\end{equation*}
$$

Since, on the wing, $v=. ? r$, where $r$ is the distance iro:n the origin, one has, in the $\zeta$-plane,

$$
P_{1}=-2!\frac{d}{d t}\left[\int_{-1}^{1} 91^{r} \frac{d r}{d!} d!-\frac{2!!}{2} \int_{-1}^{1} r^{2} \leq 1^{r /( } \frac{d r}{d!}\right) d t
$$

Placing Eqs. (9), (!2), and (13) :nto Eq. (31), one ubtans, after considerable :nampulations,

$$
\begin{equation*}
\frac{P_{1}}{O c^{6}}=\frac{d}{d t}\left[:^{2} t,(2)\right] \tag{32}
\end{equation*}
$$

ahere

$$
\mathrm{f}_{\mathrm{pl}}(\alpha)=\frac{\pi}{8}\left(\frac{\alpha}{\pi}\right)^{1-4 \alpha / \pi}\left(1-\frac{\alpha}{\pi}\right)^{-3+4 \alpha / \pi}\left(1-\frac{2 \alpha}{\pi}\right)^{2} \csc ^{2} 2 \alpha
$$

Equation (33) can be expressed as

$$
\begin{equation*}
\frac{P_{1}}{\rho c^{4}}=2 \dot{\Omega} \Omega f_{p l}(\alpha)+\Omega^{3} g_{p l}(\alpha) \tag{34}
\end{equation*}
$$

where $g_{p l}$ is the derivative of $f_{p l}$ with respect to $\alpha$.
The function: of $\mathrm{f}_{\mathrm{pl}}$ and $\mathrm{g}_{\mathrm{pl}}$ are shown graphically in Figure 8. The function $f_{p l}$ ls positive, as expected. The function $\mathrm{g}_{\mathrm{p}}$ is always negative. This suggests that, during the fling phase of the Weis-Fogh motion, if the wing is moving at a constant velocity, then the fluid performs work on the wing and not the other way around! It is likely that this apparent paradox is a consequence of the omission of the leading-edge vortex shedding phenomena.

## 5. Leading-Edge Vortex Shedding

The analyses of the preceding section omits the presence of vortices shed from the wing's leading edge. Using the present general viscous theory of aerodynamics, the effects of the shed vortices on the aerodynamic forces, moments, and power requirement can all be expressed as functions of the distribution (location and streng th) of shed vortices.

To demonstrate the use of the general theory, consider a distribution of shed vorticity $\omega_{\text {. }}(\zeta)$ in the $\zeta$ plane. Because of symmetry, one has $\omega_{-}(\zeta \zeta) /=-\omega_{r}(\zeta)$. A complex porential $W_{2}$ associated with this vorlicity distribution exists in the region free of shed vorticity. This complex potential is

$$
\begin{equation*}
w_{2}=-\frac{i}{2 \pi} \int_{R_{w}} \ell r_{1}\left(\frac{\zeta-\zeta_{0}}{\zeta-\bar{\zeta}_{0}}\right) \omega_{\zeta}\left(\zeta_{0}\right) d R_{\zeta_{0}} \tag{35}
\end{equation*}
$$

where $R_{w}$ is the region occupied by the shed vorticity in the upper complex $\zeta$-plane.

It is easy to show that the irnaginary part of $W_{2}$ vanishes on the $\xi$-axis. The complex potentia? $W=W_{l} \cdot W_{2}$ therefore satisfies the boundary conditions for $\psi$ stated in Section 4. This complex potential $W$ therefore is the correct complex potential fo: the fling ohase of the Wers-Fogh motion, with vorticity shed from the leading edge of wings taken into sccount. The complex porential is invariant under a conformal iransformation. The distribution of the vorticity lield in the z-plane corresponding to $\omega_{r}$ is obtainable through the transformation relation, Eq. (9).

Equation (35) states that the effecis of the shed vurtices can be considered in a piecewise maniner. That is, the region of integration $R_{w}$ can be divided intu seberetits and the effects of vorticity within each reg:nent studed individually and the results summed. To Allastrate this proc ss, consider the effects of vorticity in a stiall region, say $5 R_{\text {w }}$, around the point : = iv. The :otal verticity in this smill region is designated or Consider this total vorticity to be a vortex located at $\therefore:$ : By reason of symmetry, there exists a vortex of sirength . Ir at $\zeta$ - $\bar{j}$ as shown in Fig. 2. The complex potential it $^{4}$ for this Yoriex pair is

$$
\begin{equation*}
x=-\frac{1}{2} \div \operatorname{zn}\left(\frac{i-i}{1-3}\right) \tag{36}
\end{equation*}
$$

1: is east io show that the sireng:h ot : 1 en orrespacing jatr of ivemes in the $\therefore$-plane are diso
 pair th the e-plane. as sienthec on Fin. i, are express:sie
in terms of $\zeta_{0}$ and $\bar{\zeta}_{0}$ through Eq. (9).
The vortex strength $\delta \gamma_{\zeta}$ on the wing in the $\zeta$ plane is simply the negative ${ }^{\zeta}$ of the $\xi$ component of velocity on the 5 -axis for $-1<\xi<1$. This velocity component is easily obtained from Eq. (36). The increment lift $\delta L$ due to the vortex pair $\delta \Gamma$ and $-\delta \Gamma$ is related to $\delta \gamma_{\zeta}$ through

$$
\begin{equation*}
\delta L=\rho \frac{d}{d t} \int_{-1}^{1} \times \delta \gamma_{\zeta} d \xi \tag{37}
\end{equation*}
$$

It can be shown that

$$
\begin{equation*}
\int_{-1}^{1} x \delta Y_{\zeta} \delta \zeta=k \eta_{0} \delta \Gamma \tag{38}
\end{equation*}
$$

where $\eta_{0}$ is the $n$ coordinate of the vortex $\delta \Gamma$ and $k$ is a function of a defined by Eq. (10). It is clear from Eqs. (37) and (38) that the increment lift $\delta \mathrm{L}_{2}$ is dependent upon the vortex strength $\delta \Gamma$ the movement of $\delta \Gamma$ in the $\eta$ direction, and the opening angle of the wing pair through $k$. If all the shed vorticity in the flow is reasonably represented by a single pair of vortices with known strength and location, then Eqs. (37) and (38) permit the lift due to shed vorticity to be evaluated simply. In general, it is possible to represent the shed vorticity by a series of pairs of vortices. The lift increment due to each pair of vortices can be determined using Eqs. (37) and (38). The sum of all the lift increments and $L_{p}$, given by Eq. (19) is the total lift acting on a weis-Fogh wing during the fling phase. Indeed, with a distribution of shed vorticity, the total lift $L$ is obviously the sum of $L_{1}$ and $L_{2}$, the contribution of the shed vorticity to the lift is given by

$$
\begin{equation*}
L_{2}=\rho \frac{d}{d t}\left[k \int_{R_{w}} n_{0}\left(\zeta_{0}\right) d R{ }_{o}\right] \tag{39}
\end{equation*}
$$

The total normal force $N$ and the total power requirements $P$ are expressible in terms of the potential function and vortex strength in forms identical to Eqs. (24) and (31), with the subscripts "l" removed. In these equations, the potential function $\phi$ is the sum of $p$ and $\Phi_{2}$, the real part of $W_{2}$. The form of Eqs. (24) and (31) states that the contributions of $\$$ and $\$ 2$ to the normal force and the power can be indididually evaluared and added together to give the contribution of $\$$. The total vortex sheet strength $\gamma$ is the sum of $\gamma_{1}$ and $\gamma_{2}$, the vortex sheet strength due to $W_{2}$. The vortex sheet strength, however, appears in Eqs. (24) and (31) as squared terms. In consequence, the contribution of the vortex sheet $y$, which represents the unsteddy bouncary layer on the wing, is not simply the sum of the contributions of $\%_{1}$ and $r_{2}$. If one let, for example,

$$
v_{2}=-\nu \frac{d}{d t} \int_{-1}^{1} \nu 2 \frac{d r}{d \xi} d \xi-\frac{\rho}{2} \int_{-1}^{1} r_{2} 2 \frac{d r}{d \xi} d \xi(40)
$$

then

$$
\begin{equation*}
\left.v=v_{1} \cdot v_{2}-0 \int_{-1}^{1} r_{01}\right\rangle_{52} \frac{d r}{d \tau} d \tau \tag{4}
\end{equation*}
$$

Similarly, one has

$$
\begin{equation*}
P=P_{1} \cdot P_{2}-\cdot!!\int_{-1}^{1} \quad r_{i} \quad r_{2} r\left(\frac{d t}{d!}\right) d! \tag{i+2}
\end{equation*}
$$

Closed forin expressions for $V$ and $p$ have been ubtaned by the present authors. Expresstions for $S$ and $D$ are easaly sbitanable from those for $L$ and $V$. Arcause of the leng:th limitation of the present paper, discuasions of these eesults are pos"poned tor the future.

## 6. Concluding Remarks

The present work ias yielded useful results for the lift and the drag acting on a wing undergoing the fling and the clasp phases of the Weis-Fogh motion. C.loser form expressions have been obtained for the unsteady lift, the drag, and the power expenditure of the WeisFog'l wing. These results have led to the identification of inajor contributors to unsteady aerodynamic forces acting on the wing and the power requirement to sustain prescribed wing motions.

It should be pointed out that, compared with aerodynamics of rigid lifting bodies, the Weis-Fogh problem is substantially more difficult to treat. For the rigid body problem, the use of the general viscous theory, as stated as Eqs. (3), (4) and (5) in this paper, is particularly convenient since the overall vortical system can be divided into components and the contribution of each component to airloads can be studied individually, Even though the unsteady flow may be in the non-linear domain, the linear addition of the individual contributions gives the correct total aerodynamic force and moment acting on the rigid body. The line of action of the aerodynamic force and consequently the power expenditure are easily obtainable from the total force and moment. For the Weis-Fogh motion, the totai force and moment acting on the wing pair can be determined also in a manner similar to that for the rigid body problem. The force and moment acting on each individual wing, however, cannot be evaluated using Eqs. ('4) and (5), excepting that the lift on each wing, because of symmetry, happens to be one half of the total lift on the wing pair. In the present paper, the unsteady total normal force acting on the wing and the power expenditure of the wing are determined from the pressure distribution on the wing. The drag force and the leading-edge suction force are determined from the lift and the normal force. Since the pressure field is not a linear function of the voticity field, the total unsteady force and the power expenditure for each wing is not a simple sum of individual contributions of the several vortical fluw components. Eqs. (41), for example, shows that the total normal force is composed of three terms. The terms $\mathrm{N}_{1}$ and $\mathrm{N}_{2}$ are respectively the individual contributions of the wing motion and the shed vorticity (wahe). In addition to these two contributions, there is 1 third contribution due to the "interaction" of the first two contributors. This third contribution is expressed in the form of an integral in Eq. (42). The integrand of this integra! contains the product of $\gamma_{1}$ and $\gamma_{2} . \gamma_{1}$ and $\gamma_{2}$ are the vortex sheets representing the vorticity in unsteady boundary layers resulting from, respectively, the wing mution and the shed vorticity. From Eqs. (42), it is clear that the wing motion and the shed vorticity both contribute to the power expenditure of the wing. In addition, there is a third contribution to the power expenciture due to the interaction of the first itu cont:butors.

The Weis-rogh problem is a special case of the problem of iwn-linear ansteady aerodynamies of nonfigid lifing bodies. The present study is, in this contert, 3 precursor to a inore comprehensive study of unsieady aerodynamics of flexible lifting bodies. It is antripated that the developinent of a ruutine capability for preducting snsteady serodynamic behavior in the nonlinear domain will requare extensive and persistent efforts wer a number of years. The work described in this article represents only a few initial steps in search of this capability. In that contert, the results of the present study are miouraging in that the recently developed general iscous theorv of apeodvatmis is shown :0 be well suted for the :heoretulal sisu of unciesd jerodvasente probleons, inctudeng :lose assuitated al:i) !lexible l:! ang budes.

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Figure 1. Weis-Fogh Motion

$\dot{C}_{0}^{6}$
Figure 2. Wings in Transform Plane $\zeta$


Figure 3. Free Bady Datgran

$$
\frac{v}{c^{3}}=\therefore t_{n 1} \cdot \theta_{e_{n 1}}^{2}
$$



Figure 7. Weis-Fogh Leading-Edge Suction Functions

$$
\frac{S}{\rho c^{3}}=\Omega^{2} g_{s l}
$$



Figure 3. Weis-Fogh Power Functions

$$
\frac{p_{1}}{\nu c^{4}}=2 \cdot \Omega \Omega{ }_{p l}(\alpha)+\Omega^{3} g_{p l}(\alpha)
$$

# THEORETICAL INVESTIGATION OF DYNAMIC STALI. 

 USING A MOMENTUM INTEGRAL METHOD
## E.J. Jumper <br> and

J.E. Hitchcock

Air Force Institute of Techaology Wright-Patterson AFB, Ohio

## Abstract

An analytical study into the gust response of an airfoil is presented. The momentum-integral equation for steady flow is extended into the unsteddy flow regime to predict the behavior of an airioil that experiences a constant-rate-ut-change of angle-oi-artack gust. The von Karman-Pohlhausen mechod of integration is successfully modified to incurporate the additional transient flow terms; the equation vi clusure necessay to do this is also presented. Finally, compuation of the flow about a Joukowsixi airfoil using the new equat iore is performed and the results are presented and discussed. It will be showil that these results are in axteenent with existing experimental data.

## 1. Intruduction

In 1932, Sax von Kiramer published che results of wind-tunne $i$ experiments simulating d wing vacounterian t constant-rateot-change of angle-otsttack, $t$, wist. His results demonstrated chat the wink encountered stall at hiplier ankles of attack t!at: inctee statie-stall case and the maximum
 portionate amunt to the indereased stall angle ut attsex'. Sitace that time, a number of empirical studies have been undertaken in the veneral ared of ? Pready : low how reterted to as dynamic itall. 'rew, buwever, have concenterated on the cu.sstant-1 rase. Uae is these tew is the work of Deckeas and Buebler", which was a siow visualization stady and wore receatly by baley whath was a combinat iun llow UEsbilizathon-pressute messurement study'. These
 As atu:ction ot the llow ind the cunstant i. A summary © the resules of eriereace ? and 3 is





 tmpurtate to ate chat blese two experiments ditart














 -: : •••


Fie. 1 - Sumary u: data for ingriste o: stall vis reduced angular rate, tane:! :stall retereace 3)

Because of their simplicety, ataterat methatis have bee: helptul i:s unders.dading the interp! y : the phystall phenomens leadiak iu stall. fat:
 unsteady flow because the clusare rquat on iecear sary to perturm the igtrigations was but ava:ifitre. this paper presents an extensb: ot E!e interarat method to unstosuy :live Aos





## 






$$
\begin{equation*}
\delta_{2}=\int_{0}^{j}\left(1-\frac{u}{v_{e}}\right) \frac{u}{u_{e}} d y \tag{3}
\end{equation*}
$$

The introduction of the transient terms into the momentum－integral equation requires an additional equation which，until now，has not been kncan． This closure equation，however，may be arrived at as follows for the case in which transients are not too severe．First the observation is made that the displacement thickness is releated to the boundary layer thickness by

$$
\begin{equation*}
\Sigma_{1}=C_{1} S \tag{4}
\end{equation*}
$$

dssuming $C_{1}$ is slowly varying in time，

$$
\begin{equation*}
\frac{\therefore 1}{\therefore t}=C_{1} \frac{j}{3 t} \tag{iij}
\end{equation*}
$$

Further，fur all laminat tlows the foundary layer thiekness is related te the pusental tlo b：

$$
=氏 \frac{\bar{v}}{\sqrt{v_{x}}}
$$

い tilat，



$$
-\frac{\pi}{\because}=\cdots
$$



```
\[
\cdot \cdot \omega: \dot{=} \frac{1}{x} \cdot \frac{1}{1} \frac{1}{x} \cdot \ddot{x}-\frac{\vdots}{x}
\]
```










such that the velocity profile is
$\frac{u}{r}=\left(2 r-2 r^{3}+r^{4}\right)+\frac{\therefore}{6}\left(n-3 r^{2} \cdot 3 r^{2}-r^{*}\right) \quad(13)$
Eq（13）may now be used to evaluate fidnd：irom Eqs（2！and（3）

$$
\begin{equation*}
\frac{5}{5}=\frac{3}{10}-\frac{\vdots}{120} \tag{114}
\end{equation*}
$$

and

$$
\begin{equation*}
\therefore=\frac{37}{315}-\frac{1}{945}-\frac{\therefore}{9072} \tag{15}
\end{equation*}
$$

Theste equatiuns，Eq，（ $A \rightarrow$ ）and（ib）may be cumbined ：cytoid

 thand side of riq is
$\therefore \because+12+\frac{17}{3!}-\frac{3}{4+3}-\frac{1}{1!\square}=\therefore$
 t＂rw paramere，$\because$ ，ehaft is an expl：it tuncz．．．a


a： 1





$$
-8-
$$

$$
\therefore+\cdots:[\cdot+\cdots
$$

$$
\begin{equation*}
K=2\left(\frac{U_{e}}{j x}+\frac{1}{U_{e}} \frac{j U_{e}}{i t}\right) \tag{24}
\end{equation*}
$$

Eqs. (12), (18), (19), (20), (22), (23), and (24) now constitute a modified set of equations similar to the steady state equations which may be stepwise numerically integrated in a manner equivalent tw the von Karman-Pohlhausen method ${ }^{6}$.

## 111. Application to Constant-i Gust

The mettrod outlined sbove was applied to the case of an $15 \%$ thick symmetrical Joukowski airfoil a ergung constant-a gust. The unsteady pocencial llow fieid was sulved tor the Joukowsí: alrtoit (u) poovide the flow parameters $\mathrm{C}_{0}$, , C . $: x$, and $k_{e} /$, $t$ nerded tor the momentum-incegralmethod solution of the boundary layer. The transtent potentisl field was approximated as psectosteady by neglectiag the starting voltices, an effec: which we suspect is small but which is the subject of a separate investisation to be published at a tuture dater.
l:a order to determine the siall angle, separaton at the quarter chord was derined as stall for t!e purposes in thas investigation. The angie ot Attatik at stall was determiand ior a number ut llow andtans and worstant is, and the irsuits plated




$\therefore \therefore$
18:
w.. ? 1 ! : ...ar. .


Alchough the Kramer experiment used a different airfoil, the similarity of the leading eage geometries leads one co suspect that the resulis shou'd be generally comparable. The offect shown in Figure 2 was compared to the results of reference I as follows. Kramer's result may be written as

$$
\begin{equation*}
C_{\text {max dyn }}=C_{\max t}+0.36 \frac{C_{\dot{4}}^{0}}{U_{\infty}} \tag{25}
\end{equation*}
$$

where is in cadians per secund. The results of Figure 2 may be wricten as

$$
\begin{equation*}
A_{\text {stall } d \ddot{n}} \neq \ddots_{s t a l l} \mathrm{st}+0.096 \frac{\frac{1}{c i}}{U_{i}} \tag{27}
\end{equation*}
$$

i: written for $\dot{j}$ in radians per seconds. Since these resules are not in terms of $C_{\text {t max }}$ they are not directly comparable to fq (26): Max mever, if we Jssume that the eltect of increasing estal: is tu simpty disolace the $C_{1}$ vs acurve by staliparable amount isee Figare 3), then $C_{\text {: }}$, nay be arrived at by simp:y mutiplying by the stope of the unstalled $C$ : $v s$ i rurve. This assumption is similar to thit wi the proudo-stedy-state assumption in that the eftect of tie starting vortices is assumed negligi-
 per tadan), Eq (27) predicts a C:max re'ationship

$$
c_{1 \max \text { Jyn }}=C_{\text {max st }}+0.101 \frac{i}{U} \quad \text { ant }
$$

The comparison to kramer's result. Fq (.6) is ㄷ.rorkstic.


In regard to the first, tnere is still work to be done in establishing the linits to the applicability of the assumptions leading to Eq (8), but the tools for such a study are available in the solution irself, and work in this direction is already underway ${ }^{\text {a }}$. Sensitivity studies carried out to date have shown that the worst case errors incurred from these assumptions change the slope of the curve of figure 2 by loss than $10 \%$. In regard to the second, we now have available in the unsteady momentum-integral method a tool to help understand the pieces of physics which are at play in delaying the stall, and although not detailed here, it is clear from plotting the various modified Puhlhausen parameters fron the solution that the transient term in the ext:rnal flow is balancing the unfavorable pressure jradient to delay separation. Much more is to be learned from further exploitation of ti:e unsteady method because of the fact that our solution is not at all limited to constant-1 cases.

Finally, it should be mentioned that we have also begun work on trying to adapt the method to a :un-Newtonian boundary'. It is too early to say whether this work will be successtul, but the intention is to address the discrepancies in the experaments so graphicaliy demonstated by a comparision of figures 1 and 2.

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p:

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## Abstract

Preliminary resulte from an UnSTeady Airfoll analysis in 2 dimensions have been obtained from a computer code (USTAR2) developed by the present authors. This computer code is based upon an analysis which utilizes a doublet panel method to model the afrfoll surface, an integral unsterfy buundary layer scheme to model the viscous attached flow, and discrete vartices to model the detached boundary layers which form the alrfoll wake region. This model has been insed to successfully predict steady lift and drag coefflcients as well ds pressure distribuctons for several alrfolls with both attached and detached boundary layers. In addition, calculations have been made for a llolted number of cases for both attacted and detached unsteady flow situations. These ealculations are compared in a cursury way with experimental data to polnt out some of the strengths and weaknesses of the present formulation.

## Introduction

Unsteady aerodynatices is in important phenomenom which has been studied more intensively in recent vears. These studies have been rade in conmection with applications pertalning to hellcopters, axial flow turblies and coapressors with inlet distortions, vertical axis wind turbines, and misalles and faxed-wing alreraft undergoing ripts mineuvers. Lifting surfaces subjected to the-dependent freesiresm velocity or the-dependent body zotions aiay, it suwe ios: $=$, $:!=2$ heye stonftcaiat stall reglons on their surtaces.

A number of approaches have been taken vith regard to the prediction of unsteady stalled alriolls. Yost of ehese approaches have been revieved by yecresionyi. ${ }^{\text {e. }}$ In general, these apsroaches ranhe from empletcal modelsi,0, to zodels bayed on the Navter-Stukes equations ${ }^{7}{ }^{3}$. The eapirical models are generally applicable to seall sinusoldal prich oscillativis about nome relat!vely tou angle of attack. The Nayter-stokes swiutions tend th consume iarge amounts of conputer thace and are usually llalted to low Reynolds aamber soiutions. Unfurtunately, theer are fex podely which can be consldered as representint something in bervera the extrese eaxes of almost total eantricisa and the lengthy gore exart solu: tons of the Xavier-Siokes equations. There have been seversl boundary laver codes beveloped whtich

[^3]
iraduate Research listet., Jept. Yech. Fingr.
can be used to predict some of the behavior assoclated with unsteady stall $9-11$. There have also been models of the potential flow behavior related to the shedding of leading edge vorticity as typified by the work of Ham ${ }^{12}$. More recently, Katzil simulated the unsteady separated flow over a thin cambered airfoll. The models of both Has ard Katz required empirical information regarding tine appearance and position of the separation point.

## GSTAR2 Analysis

Recataty, the present authors formulated and began development on an analytical modalit which is potentially capable of predicting dynamic effects for stalled and unstalled airiolls undergelng arbitrary alrfoll sotions. This model does not require input of alrfoll section data and may thus be used to examine arbitrary alrfoil shapes. An UnSTeady AiRfoil model in ? dimensions based on this analysis has been taplemented via a computer code (USTAR2). Execution times for this code are short when cmared to Vavier-Stokes solutions and little empiricism is required. In order to valldate this analysis, however, comparison between USTAR? predictions and experimental data aust be made for a number of cases. This paper presents some of the prellalnary comparisony.

The USTAR2 model is a two-diaeneional Incompressibie formulation which is based on state-of-the-ar! sethods with sowe extensions. The potential flow regluns near the atrioll are modeled usinn the doublet panel analvsls of Ashleyts uhich consists of a Creen's function representation of the potential $t:$ iti $\quad$ resuiti.... fros the cotion of the alrfoll and the presence ot associated tralling vakes. Thls potential llow ts based upon the isplace equation tor the velocity potentlal:(t, ()

$$
\begin{equation*}
\because:-0 \tag{!}
\end{equation*}
$$

Whteh ls valld tor both steady and unstradr ilow. By ircen's theorem, s soation to (!) ay be representel by inergeals over the boundaptes of the flow where thone boundarles are replaced br suriaces acrows with potentia! jumpe occur. These surtacey, an depleted in flgure $:$ are represented by the alrioll sur:acex and the wake sherts whith epring trow the tralling etipe and any separat!on polnt. ilsh Green's theorez, the Asturbance potential at air :teld point $\dot{r}$, dur to the slrtul! and vaike surtares. ay te urtiten as

$$
\begin{aligned}
& (i)-\int 5 \cdots 1 \begin{array}{c}
i-k \\
1
\end{array} d s \\
& \text { - } \int \because \because \quad!\quad!d s
\end{aligned}
$$

where $\checkmark$ and 3 are doublet distribucions on the alrfoll and wake surfaces $S$ and $W$ respectively, is the surface normal at the source point, and $R$ is the distance separating the field point and source point. Boundary conditions include a kinematic surfare tangency condition given by

$$
\begin{equation*}
\frac{i 2}{i n}+\left(\dot{u}_{0}-\dot{u}_{s}\right) \cdot \dot{n}=0 \quad \text { on } S \tag{3}
\end{equation*}
$$

where $\dot{\psi}_{\text {and }} \dot{\Delta}_{s}$ are the freestream and airfoil surface velocities respectively and in is the outward normal to the airfoll surface. An additional boundary condition is the trailing edge flow condition " which, in the present model, requires that the fiow direction at the trailing edge be along the crailing edge bisector. Equations (2) and (3) are solved for 3 and 0 by first ellminating; to form a single equation in . The airforl and wake surfaces are then discretized to form a set of linear algebraic equationa in terms of the unknown 0 . The potential, is then obtained from (2). The pressure distribution around the alrfoll is obtalned from the unsteady Bernoulil equation. Detalls of this analysis are presented $\mathrm{in}^{2}$ referencelt.


Yigure : Schemathe of boundary (Vortex) Sheers tor Unyteady Pltching Yotion with Separation

The mplasy function of the bound. f laver analyals is to predict the presence and veation of any bundary separation polne on the atrituld surtace. The presture aradient and edge velucley distributions, which are used in boundary layer iatculationm, are obtalned from the potential flow sude: which say include sheets of voritetiy shed irse bhe boundary laver separstion polat. Theresore, a strong coupling the veen the boundary iaver analysts and potential llow analysla extats for sepataied flow sliualtons. the turbulent boundary layer analyals used in the present work ls essentlaliy that due to helo. Fersiger, and Xilnein. This uniteady Integral pechntque stres exceltent result: tor the steady llows of ?ll:ann. lierring. and Vorbury: Siratiord. Same! and loubert (iee Coles and llisit. reterence :l): Kia!': Stapsun and sericklindis: and wiekharte (see tita, reterence is). Yure taporiantly. this acehod gredtcen the unstesty toundey taver tata o! aratison: and houderilie. et a:.:! and coapares

while being an order of magnitude faster. The formulation for the laminar portion of the boundary layer is based upon the author's unsteady extension of Thwaltes' method (see Cebeccl and Bradshaw ${ }^{24}$. Transition is assimed to occur either due to laminar boundary layer separation or according to a natural transition criteria due to Cebect and Salth ${ }^{25}$.

## Experimental Data

An abundance of data for sceady flow over airfoils exists and can be used to check the ability of the USTAR2 model to predict such flows. One such case is shown in Figure 2 where the data of Sheldahl and Klimas 26 are compared with USTAR2. Unsteady data are, on the other hand, relativeiy scarce. In the present paper the unsteady experimental data consists of some recentiy obtained data for an alrfoil undergoing a plich up motion froa zero angle of attack. In each case the pitching rate, i, remains constant. These data consist of airfoll surface pressure data, fiow visualization dat. and surface hot-vire data. All data vere ubtair.ed in the USAF Academy lowspeed 2 ft $y$ ft wind tunnel. The experimental arrangements will be briefly described.


Figure : Life and Drag Coetstetents for a vaca mals atrioll with re - oss.0iv

Arfoll sur!ace pressure data vere obtatned by francls, beexce. and Retelleit. Detalls of the tent selup can be tound in that reforencr. A computer conerolled plech uscillator vas used 80 tapart constant $i$ pleching to o-lnch chors NaCA OOI: alrfoll. Prossure tapa vere located at 19 postitions along the surtace of the atrioll. Approxtatciy ? 5 repetiftuns of each case sere run so 4 : 0 wtialn ensezbie avetages of the wurtace presture cuetitetenti at the pressure ports. These data were then used to obsa!a tlit and trag coreftetents for the alrioll as a iuncetion of :tse.

Ylow visualteation data, were sbiathed br



tunnel upstream of the pitching airfoil. The wire was placed in a plane normal to the axis of rotation of the pitching airfoil. A smoke producing 011 (Roscoline) was coated on a 0.005 inch diameter tungsten wire which was in turn teated electrically to produce a large number of smoke streaks in the flow. These streaks have a rather uniform spacing due to the regular spacing of smoke material droplets which are formed when the wire is coated. The smoke was iiluminated by a high intensity strobe light placed downstream of the aitfoll. The proper sequencing of airfoil. pitch comanas with strobe light and smoke wire triggering was accompiished by competer control. A 35 mm camera looking along the pitch axis was used to record the visual data.

In the work by Walker, Helin, and Strickland ${ }^{28}$, a NACA 0015 airfoil with a 6 -inch chord was instrumented with an array of hot-wires. As indicated in figure 3, seven hot-wires were mounted on the upper surface of the alrfoll (suction side). The hot-wire sensing elements (TSI-10 hot films) were soldered to pairs of number 9 sewing needles which protruded above the airfoll surface approximarely 0.20 inches. The needle supports in turn were mounted in electrically trisulated plugs which were machined flush


Figure 3. Surface Hot-Wire Conflguration
With the alrfoll surface. A TSI model 1050 hot-vire anemometer system, along with an inharse linestizer, vere used to obtain velocity signals. Approxinately 25 repetitions of each case were run to cbeain ensemble averdiges of the velocity sinnal from each probe.

## Discussion of Results

Lift and drag data are shown in figure 4 tor a MaCA 0012 sirfoil pitching up from a sero angle of attark at a non-diaensional pltching rate $K$ of 0 v89. The non-zero lift coefficient at zero angle of attack is due to the so-called "plech circulation." The slope o! the lift curve in this substall region can be seen to be considerably less than for the steady case due to the dounvash on the alrfoll produced by the vortex sheet springing !roo the rat:ing edge. The results from a staple analysis given in Reference 29 are seen to agree with the experimental lift and drag verv well up to and s litele beyond the stall angle which is approxitately 10 degrees


Figure i. Lift and Drag Coefficient for a NACA 0012 Airfoll Pitching Up at Ccnstant $\dot{a}$ about the 31.7 Percent Chord Eor $\operatorname{Re}=77,700$
for the present case. The USTAR2 analysis is seen to predict beth lift and drag coefficients reasonebly well below an angle of attack equal to about 25-30 degrees. The small, abrupt jumps observed in the USTAR2 resisles can be attributed to the tendency for the boundary layer separation point to "lock in" on surface panel edges. This problea can be corrected in subsequent versions of USTAR2 by making ainor changes in the procefures for introducing "edge velocities" into the boundary layer sub=outine. The lack of agreement at hiph angles of attack is thought to be partially due to discrete vake vortex core growths which are excessively large in the USTAR2 analysis. This reduces tive effect of the large scale vortex which grows on the suction side of the airfoil. The core growth parameter was arbitrarily selected in the past and shouid be reduced to represent more realistle grouth rates. Examination of pressure coefficient data bears out the fact that the influence of the vortex moving over the alrfoll is too reak in the USTAR? analysis. The effect on lift for two different pitching rites is shown in figure 5 . Drag coefflelents for the cases in figure 5 are similar to chose measured and predicted in Figure 4.


Figure 5. Comparison of Lift Coefficients for a NACA 0012 Atrfoil Pitching Up at Constant $\dot{\alpha}$ about the 31.7 Percent Chord for $\operatorname{Re}=77,700\left(K=\frac{\alpha c}{2 u_{\infty}}\right)$

The wake geometry obtalned from the USTAR2 analysis is compared in Figure 6 to flow visualization data. The vortex sheets obtained in the USTAR2 analysis basically represent "streaklines" in that most of the fluid particles which make up these sheets were either injected ints the flow at the leading or trailing edge. While exact comparisons between visual and calculated results are difficult to isake, it does apear that the predicted large scale vortex is not as tightly rolled up as that indicated by flow visualization. This again indicates that the discrete wake vortex cores are growing at excessive rates in the USTAR2 analysis.

Velocities obtalned from three of the seven surface mounted hotwire probes are shown for a particular case in figure 7. Near the nose of the atrioil (7\% chord) the velocity measured by the probe rises until the probe becomes immersed in the separated boundary layer which occurs at an angle of attack of about $19^{\circ}$. The probe is located above the alrfoll surface about $3 \%$ of a chord length. The calculated edge velocity drops much sooner since the separated vortex sheet passes over the $7 \%$ chord position almost immediately after the boundary layer separates from the nose. Therefore, agreement between experiment and analysis is at least qualitatively correct at the $7 \%$ chord position. For this particular flow situation, the experimental work indicates that a significant region of reversed flow first appears at an angle of attack of about 25 to 30 degrees somewhat downstream of the nose (20\% chord). The magnicude of the reversed flow reaches a maximum at about the $60 \%$ chord and his a value of about 1.5 times the freestream velocity. Further downstream (87\% chord),



Figure 6. Wake Geometries Obtalned from Flow Visualization and USTAR2 $\left(a=45^{\circ}\right.$, $K=0.109$, $\mathrm{Re}=62,500$, Pivot Point $=$ 0.25 Chord, NACA 0015)
the reverse flow due to the initial large scale vortex passage is reduced and occurs at a higher angle of attack. As can be seen froa Figure 7 , results obtained from the USTAR2 analysis at $60 \%$ chord are in poor agrement with the experimental daca above an angle of attack of 25 degrees. The prediction of reversed $f$ low due to the vortex passage lags tlat of the measured data by a considerable iength of time. The immediate reasons for this lag are not clear, but tay also be associated with an incorrect discrete vortex core growth rate.


Figure 7. Velocities Obtained from Surface Mounted Hot-Wire Anemometers ( $\mathrm{K}=0.089$, $\operatorname{Re}=62,500$, Pivot Polnt $=.317$ Chord, naca 0015)

## Conclusions

Preliminary results from the USTAR2 anal-
vsis are encouraging, but indicate that
additional requirements are necessary for
satisfactory fredictions at large angles
of attack for the cases studied. in-depth
correlations between the surface pressure
data, flow visualization data, and surface
velocity data should be made in order to more
completaly understand the constant it flows
used in this paper to test the validity of
che lSTAR2 analysis.

## Acknowledgement

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## AD-P004 172

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#### Abstract

Experiments on turbulent boundary layers subjected to controlled unsteadiness have beefiperformed in a special water channel. The flow is steady in the development section upstream of the unsteady test section, where the boundary layer is subjected to an oscillating adverse free-stream velocity gradient sufficient to induce flow reversal near the wall. Measurements of the mean, oscillatory, and turbulence components of the streamwise velocity in the boundary layer indicate that the mean velocity and mean turbulent stresses are unaffected by the oscillation, whereas the periodic components of these quantities are strongly dependent upon frequency. At low frequenEies the boundary layer behaves quasistaticly; at intermediate frequencies the boundary layer behavlor correlates with the Strouhal number based on the length of the unsteady region; and at high frequencles the outer region of the boundary layer noves as a slug while the sublayer behaves as a Stokes layer described by laminar equations. Reverse fiows occur in this Stokes layer, but the boundary layer remains thin and hence attached to the surtiace. Although the boundary layer would separate from the surface at zero frequency, separatic. is prevented by rather slow oscilla:luns, and hence unsteadiness can be used as a means for separation control.


## Nomenclature

| ${ }_{k}^{4}$, a | amplitude of in $=\left(. / L_{0}^{2}\right)\left(U_{E} / d x\right)$, scceleration parameter |
| :---: | :---: |
| L | length of unsteady region, 0.61 ta |
| $i_{s}$ | = $\sqrt{2 / T}$, Stokes layer thickress, m |
| $P_{1,1}$ | phase of us |
| K. | $=U_{0} \because / \mathrm{l}$, momentum thickness Reynolds Number |
| St ${ }_{\text {r }}$ |  |
| St. | - 儿。 |
| 4 | streanwise velority, ta/s |
| ù | periodic component of $u$ |
| $\square$ | $=U$, inean componeat of a |
| (u) | $\bar{j}+$ in phase average of $u$ |
| ${ }^{\prime}$ | local frec-strean velucity, m/s |
| $i_{0}$ | tree-itream velocity it itart of the unsteady region, ra/s |
| $\times$ | stresmwise coordinate, m |
| * | $x$ it stift of the unsteady resion, it |
| i' | $=\left(x-x_{0}\right) / \mathrm{L}$ |
| $\gamma$ | dishance frum surface, m |
|  | * Do, boandary layer thlekness ( ontes), a <inematis viscosity, gís, |
|  | irepuency, rad/s |

## Introduction and Objectives

Unsteady turbulent boundary layers have many important applications. Prediction of the behavior of such boundary layers requires models, which need physical. Insight for their development and sound data for their evaluation. The Stanford unsteady boundzry layer research program is designed to meet this need.

The objective of this program is the development of understanding of unsteady turbulent boundary lavers and the collection of definitive data for the development and evaluation of predictive models. This paper summarizes results pertinent to this workshop; for full detalls see $\operatorname{Re}^{c}$. (1).

## Experimental System

The experiments are performed in a special water channel shown in Fig. 1. Water was chosen as the workiag fluid for ease of flow visualization, laser anemometry, and control of unsteadiness over a wide range of pertinent frequencies.

A unfque feature of our apparatus is the maintenance of steady flow in the region upstream of the unsteady flow test section. This makes the boundary layer entering the test section a standard, two-dimensional, flat-plate turbulent boundary layer, and gives the modeler a very well known inflow condition. The unsteady flow takes the form of oscillation (or step change) in the free-stream velocity gradient between zero (flat plate) and an adverse condition.

In order to maintain the desired steady flow upstrean of the unsteady test section, the channel flow rate must be held constant. This is acconplished by presenting d coastant resistance to the flow and by fixing the latet and discharge pressures. The fixed resistance is provided by slots it: the dlicharge plate cosc. plate in Fig. 1), and a cunstant-head taik and open iump ifx -he pressures.

The ansteady behavior in the test iecelon is ereated by contrisl ot the way !n which the llaid leaves the tost section. A bleed plate on $E$ lie channel wall opposite the test surfice and 1 sinilar phate at the end of the recovery section can be used to remove all or part of the flow. Be aselllating the slutted thow-control plate back and forth, the relative mounts of ilow extracted through these two bleed flites is ascillated, while mintulaing the total flow fixed, and tats provides the controlled :instedinecis. Gith d unltorm exit bleed fron the wall opposile the test houndary liyer, the free-stren velucitv secol by the teit boindiry haver decresses lineirly fin the downtread l!extton, end with no opposite-wa!! hleed the


by the test boundary layer is as shown in Fig. 2. The actual variation deviates slightly from this design target, as we shall discuss shortly.

The inlet section was carefully developed to produce a very two-dimensional, quiet inlet flow. Boundary layer suction removes the nozzle boundary layers, and suction from the wall opposite the test surface is used to maintain a uniform free-stream velocity over the test surface in the developing region. The side-wall boundary layers are removed at the entrance to the test section, the intent being to maintain two-dimensionality of the flow in the unsteady test section and downstream recovery region (see Ref. 1 for documentation).

Measurements of the streamwise velocity component have been made using a single-component DISA Laser Doppler Anemometer with tracker. The LDA signals are prefiltered to remove blas noise and then examined in real tine with a dedicated MINC inicrocomputer. The mean, turbulence, and periodic components of the signal are extracted, and then time-averages and phase-averages of these quantities are formed. A shaft position encoder on the oscillating plate provides the tining signals required for phase averaging at 512 points in each cycle.

Great care has been taken to make this a quality experiment. For example, the water temperature is :nafintained to within $0.5{ }^{\circ} \mathrm{F}$ of a set temperature by continuous refrigeration, an impotant feature that enables stable long-term operation. The water is filtered to 5 aicrons to control the particle sise (important for LDA). Accelerometer measureaeats and analysis have assured that there are no vibration problems. All aeasuring equipment has been carefully calibrated and these calibrations have been periudically checked. Degradation of the ilow due to algat growth can be detected by appropriate velucity measurements, and has been eliminated by a program of preventative cleaning and water refreshing.

The entire experiment operates under the control it the MINC computer, which sets the frequeacy . 3 uscillation, posittons the LDA, and acquires and frocesses the data. This enables us to program cont inaous rans, which is necessary because of the long time of data acquisition, especially at low :requenistes. Continuous production oferation for a serix or $(\mathrm{m}$ ) is now common.
'Jur first mator report (Ref. 1) describes the extensive quallficition expertments that were made © , establish the two-diaensionallty of the flow, the iteadiness of the upstredin region, and the rifllability of the unsteady data. These wlll not be repeited here.

## Experiments

To date three sroups if experiments have been G:A!t:ted, $t$ w) with frequency oscllitions and one Whth step changey. The luw-anplitude osclllation ind tepexpertacats do not produce flow reversal. The histh-saplltude itepexpertaent loses show flow riverial, is alll a ingh-amplleude step experlisent tuw in iroseres. In this paper we shall repurt
 lation experbaent af relevance to this aorkshop and suge vonelasions that are independent of aphltidn.



The experiments were run at a number of frequencies over the range $0.1-2.0 \mathrm{~Hz}$. The boundary layer parameters at the entrance to the unsteady region for these experiments were:

Free-stream velocity $U_{0} \quad 0.732 \mathrm{~m} / \mathrm{s}$
Coles' boundary layer thyckness $\delta$
Displacement thickness $\delta^{*}$
Momentum thickness $\cap$
Shape factor $H$
Momentum thickness Reynolds No. $R_{9}$ Friction coefficient $C_{f}$
Kinematic viscosity $v$
3.55 cm
0.59 cm
0.42 cm
1.42

2790
0.00322
$1.09 \times 10^{-6} \mathrm{~m}^{2} / \mathrm{s}$
Measurements were made at four streamuise locations designated by $X$ '. The data cover the Strouhal number ranges

$$
\begin{aligned}
0.039 & <S t_{\delta}<1.54 \\
0.22 & <S t_{x}<15.7 .
\end{aligned}
$$

The oscillating plate mechanis: produces a free-stream velocity oscillation that is very nearly sinusoidal, and the resulting boundary layer response is essentially sinusoidal, so for purposes of this discussion we shall treat them as such. We decompose the velocity into three components.

$$
u=\bar{u}+\tilde{u}+u
$$

where $i$ is the time-average, it is the oscillating component, and $u^{\prime}$ is the turbulence. The timeaverage is extracted by deraging over a large integer number of cycles, and the oscillating component is extracted by averaging over a large number of cycles at the same phase angle. See Ref. 1 for procedural details.

Ref. 1 and its assoclated microfiche contain approximately 1000 graphs and tables of data from these expertments. A digital tape for use by modelers is also avallable.

## Free-Stream Dynamics

Figure 3 shows the thac-average free-strean velocity in the test reston over the range ot fre quenctes. Vote that the frew-stream velocity gradient is linear as intended and essentially Independent of frequency. The acceleration parameter cortesponding to this gradleat is

$$
k=\frac{v}{\frac{d U}{2}} \frac{\mathrm{E}}{2 \mathrm{a}}=-1.95 \times 11^{-7 n}
$$

This represents 1 itrong suerse-pressure krationt for which, in iteddy flow, ane woulld oxpret itho development of apparatlon.

In 1 mporeant observation is thit the bomatar: layer remalaed attached darlag ill int theare unstesdy expertanent. !lowever, when we siet the frequency to sers) (at the me in free-strestas sasle te show in Fig. 3) a separition bullds up over a period of 1 flnuto or so, and the beoundary leyer





 ber:.

Figure 4 shows the amplitude of the unsteady component in the free stream. Note that the amplitude grows linearly from the start of the test section, as per the design target (Fig. 2). However, unlike the design target the phase of this variation is not constant in the streamise direction. Figure 5 shows the phase lag; note that the lag decreases downstream in the test section, and over the second half of the unsteady region is very stall at all frequencies. These phase lags are probably not iaportant in discussing the qualitative aspects of the boundary layer behavior, but probably should be considered by modelers comparing a prediction to the data.

## Time-Averaged Behavior

Figure 6 shows the time-average velocity proiiles in the boundary layer near the end of the unsteady section. The scales are staggered to prevent confusion; when these data are laid on top uE one another, very little difference can be seen, indicating that the idean velocity proflle is essentially unaftected by the oscillation frequency.

Pigure 7 shows the time-average of the streamwise component of Reynolds stress, norialized on the local free-stream velocity at the outer edge of the boundary layer, near the end of the unsteady sectlon. Again we see that the time-average turbulence protiles are essentially all the same, indi‘ating no significant effect of the oscillation on the time-averaged turbulence.

The observation that the the-average flow is a.te ingnficantly affected by the oscillation is Ousistent with other recent observations in nasteady boundary layers. It also explains why , rediction nethuts based on iteady flow mode!s utcen do ver: well in predleting transtory stall phensmena.

## Unsteady Component

Fifure $y$ shows the amplitude of the unsteady Component it the end of the test section, normalleit on the iree-stream amplitude. Here a sifilfiallt virlition with frequency is seen. Vote that it low frequencles the amplatude is greater in the boundary layer than in the tree itream, by over a : Actar of 2 it 1 ). lz . At high frequencies the aplltule is andorm over most of the layer, indi-
 re:bon th low irequen:les, sugestive of cotalmed Itw-)t-the-wall behavior ita this range.
"t sure 7 shows the phate of che instedy comprone te fote partlealarly fhe hifitirequency ses, there the suter reiton of the boundary laver anver in phase dith the free streata, and there is at ri,fd ehmbe varlition in phatic near the surtace. The phase lead near the surface appotaches if ${ }^{\circ}$ as the Irequency is facreased; te fa lateresting that thin is the theoretteal phase lead iore latione it 能es layer.









Figure 10 shows the phase-average profiles at 2 Hz over a portion of the cycle. Note that the lowest three curves display reversed flow near the wall. At this Erequency and location the flow reversal occurs from about $130^{\circ}$ to $210^{\circ}$, and is greatest at about $165^{\circ}$, consistent with the phase lead shown in Fig. 9.

## Flow Regimes

Ailalysis of the governing equations and the data suggests that there are three regimes of flow in unsteady boundary layers. These can be exprersed in terms of the time scale for the oscillation,
$T_{0}=1 / \omega$
the fime scale for the unsteady region,

$$
T_{u}=\left(x-x_{0}\right) / u_{0}
$$

and the time scale for the large eddies

$$
T_{e}=\Sigma / U_{0} .
$$

The tatios of these time scales define the two pertinent Strouhal numbers,

$$
S t_{s}=T_{u} / T_{0} \quad S t_{x}=T_{u} / T_{0}
$$

When $T_{o}$ is long compared to $T_{u}$ and $T_{e}$, both the overall flow and the eddies have the to respond to the change, and the flow behaves quasistaticly. Sluce the zero-trequency flow here is a separated flow, and the oscillating fluws are not, it would seem that this regime was not reached in the experiments, and hence that the quasistatic regime recilies Strouhal numbers at least as low as

$$
\mathrm{St}_{5}<0.04 \quad \mathrm{St} \mathrm{x}_{\mathrm{x}}<0.2 .
$$

When $T_{0}$ is comparable with $T_{u}$ but long compared to $T_{e}$, the eddies have $t i n e ~ t o r e n d ~ a n d$ the entire boundary layer should reflect the influence of the uscillation. is a resilt, integral quantities such as the displacement thickness vary throughout the cycle. The equations (Ref. l) suggest that the behavior should correlate on $S t x$. Figure 11 shows the displacement thickness is a function of $S t_{x}$, normalized by the amplitude of the freestream fluctuation, for the full set of hifhamplltude and low-amplitude osclllations. Note that, for $0.5<5 t_{x}<3$ the correlation on St is excellent, indicitary that $S t x$ is Indeed the correct correlating parameter for this range of flows (a sindlar plot $u$ st, does not collapse the data). Vote also that it higher frequencles the displacement thlekars varlition becozes independent of St,
when $f_{\text {, }}$ is thort compared to i.., l.e., it, is sutfleter: ly ugh, the edjles do not haw the to respond to the acelliation. Dur fiat sumerest that

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analytical solution of the Stokes equation. The same holds for the phase (Fig. 13).

## Acknowledgements

Dr. Pradip Parikh did the detailed design of the experimental facility and conducted important preliminary experiments, and has provided much helpful advice throughout this program. The work is supported by the Army Research Office and by the Aeromechanics Laboratory, RTL (AVRADCOM); companion work contributing to the facility has been supported by NASA-Ames Research Center and by the National Science Foundation.

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figure 2.
Wesinintariet tor frem-strest velocity ssilllitun




Phure 7. Phase of the periudtc component of velixelty in the inomdary layer


Figure 11. Amplitude of the periodic component of the displacement thickness, normalized on the amplitude of the free-stream velocity oscillation, for all oscillatory experiments



Figure 12. Amplitude of the periodtc component in the Stokes layer, nornallaed on the amplitude of the frec-atream velocity oselllation, for all high-frequelley oscillatury expertisents


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Ho, Chih-Ming<br>University of Southern California<br>Los Angeles, California 90089-1454

## Abstract

Unsteady separation is a problem of areat technical importance but with little basic understanding. A very limited amount of experimental data is available because of the difficulties involved in measuring the temporally evolving separated flows. In this presentation we first examine the flow field of a downstream moving separation in detail and then we explore the possibility of applying the learned physical mechanism to the upstream noving separation problems and to the unsteady sepazation on a lifting surface. .

## Introduction

The unsteady separation was recognized to be intrinsically different from steady separation in the mid 1950's. Rott Sears ${ }^{2}$ and Moure ${ }^{3}$ pointed out that the vanishing of the wall shear stress could not be the criterion for unsteady separaticn. Instead separation should occur at the zero shear stress location in a coordinate system convected with the separation speed. This has been called the MRS criterion since then. The obvious problem in this criterion is that the separation speed is not known a priori. Large amounts 0 : theoretical effort have been spent on this challenging topic. The most complete summar: is the menouraph by Teleionis. ${ }^{4}$ i.:.tensive experimental works are available in stodying the effects produced by unsteady separacion on liftina :urfaces. ${ }^{5}$ Exper:mental investigations on the proc.. asses in oolved in unsteady separation howcue: tre very :ew. The main problem is that the veloc:ty field needs to be well tocumen a but severe limitations exist in mosurin: the time evolving separated ッobotry tiel. sepatat:on uscally inそl:es two or even three spatial धartables. in che unsteaty case, ime is an aflat:o:a: Ur-1able. تherefore the measurnments rotare the ase of any probes as well as tarse and :ast dara seorage sustoms. Fu:Germon, :eversu : ! owers in mos: separac:ons an! :re we:l tore!op! to:
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the surface pressure fluctuations were surveyed in detail. The data clarified several important issues. The detachment point in this flow moves downstream. We then attempted to apply the mechanisms learned from the downstream moving detachment point to a case of upstream moving detachment point, e.g. separation on the cylinder or the airfoil. The results are promising.

Unsteady Separation With Downstream Moving Detachment Point

Moore ${ }^{3}$ pointed sut that the flow reversal will not occur in a situation with the downstream moving detachment point. (Fig. 1).


Ei:. L. Separation with townstream mourna detachment :oint (Mnore 1953).

This adrantageous characteristic allows us to investanate the velocity in reat detall bÿ usinu hot-abe proves. at laterestin: unsteat? separation occurted is the 1menn:in: jet case. ihen she voherene


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## !stead: veparation Detachmest ionnt

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process and signisicantly modify the at－\(\sigma\) dyramic properties of the unsteady air－ foils．

\section*{Acknowledgment}

The author would like to thank \(N\) ． Didden and S．H．Chen Eor their great help duriag the course of this research．

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