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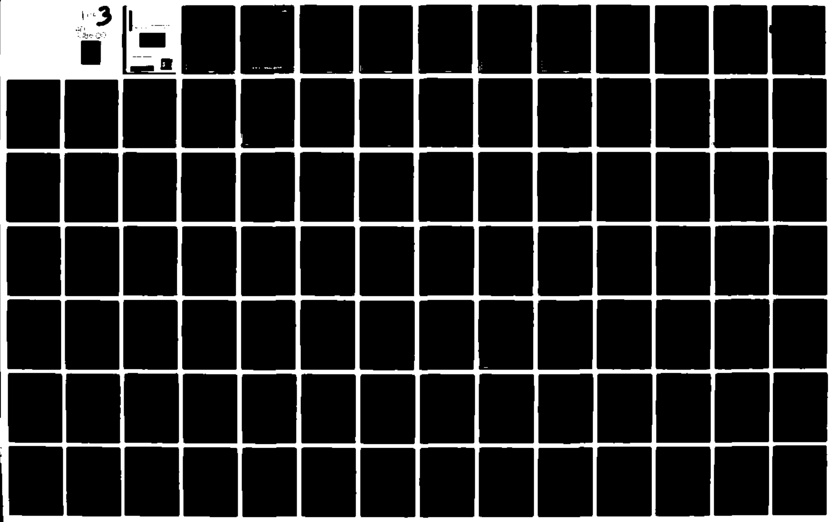

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RESPONSE OF THICK CYLINDRICAL SHELLS TO TRANSIENT INTERNAL LOADINGS

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The case of wave propagation in a thick-walled elastic cylindrical shells subject to dynamically applied internal pressure is examines for various shell geometries and modes of application of the internal loading. Shells of both infinite as well as semi-infinite length are treated. In both cases the loading is considered to be axisymmetric. The investigation culminates in the determination of dynamic behavior of the thick shell subject to a band of constant intensity pressure moving at constant velocity along the inner surface of the shell. Displacement and strain components at any point in the shell may be evaluated in terms of dimensionless variables from the computer program presented.

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ABSTRACT

The case of wave propagation in a thick-walled elastic cylindrical shell subject to dynamically applied internal pressure is examined for various shell geometries and modes of application of the internal loading. Shells of both infinite as well as semi-infinite length are treated. In both cases the loading is considered to be axisymmetric. The investigation culminates in the determination of dynamic behavior of the thick shell subject to a band of constant intensity pressure moving at constant velocity along the inner surface of the shell. Displacement and strain components at any point in the shell may be evaluated in terms of dimensionless variables from the computer program presented.

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NOMENCLATURE

$r, z,$	Cylindrical coordinates
σ_{rr}	Radial stress component
σ_{zz}	Vertical stress component
$\sigma_{\theta\theta}$	Tangential stress component
σ_{rz}	Vertical shear stress component
U_r	Radial particle velocity component
U_z	Vertical particle velocity component
λ, μ	Lame's constants
t	Time dimension
ρ	Material density
n_r, n_z	Cylindrical components of the unit normal vecter to the wave surface
G_L	Longitudinal wave velocity
G_S	Shear wave velocity
R	Internal radius
R_o	External radius
α	Dimensionless ratio = $\frac{\lambda}{\lambda+2\mu}$
β	Dimensionless ratio = $\frac{G_S}{G_L}$
τ	Dimensionless time = $\frac{G_L t}{R}$
A_i	Known quantity at a point i
B_i	Dimensionless time = $\tau - \tau_i$
[ ]	Bracket notation = [The value at the rear of the wave front] - [The value at the front of the wave front]
∂	Partial differentiation

CHAPTER I

INTRODUCTION

Background

Transient motion in solids has been of interest principally in mechanical, mining, and geophysical engineering. In recent years, however, stress wave propagation problems are becoming increasingly important in aeronautical and nuclear engineering in problems associated with structures under impulsive loading. In the current study a thick cylindrical shell subjected to impulsive loads applied at the interior is considered by the application of the method of characteristics.

It is known that when an elastic body is disturbed by a load which is abruptly or gradually applied on a portion of it, elastic waves propagate through the body with two different velocities which depend upon elastic properties. Waves involving dilatation (or voluminal change) are usually called dilatation or longitudinal waves and those involving rotation (or no voluminal change) are called transverse or shear waves.

In such a boundary value problem as considered in this study nonplanar elastic waves, like spherically and cylindrically expanding waves, are propagating through the medium. These nonplanar waves, in contrast to plane waves, change the shape of the wave front as they propagate, thus varying the distribution of stresses and velocities of the disturbed particle in the elastic body.

Various kinds of wave propagation problems which can be represented in terms of a set of partial differential equations have been studied previously using different approaches such as the integral transform method, the finite element method, and the method of characteristics which is relevant to this study. When the theory of characteristics is applied to elastic wave propagation problems it involves the solution of a system of first-order partial differential equations which governs the deformation in the dynamic field. It is more advantageous to apply the method of characteristics rather than the integral transform method to solve complex boundary value problems, since those first-order partial differential equations involve stresses and particles as dependent variables.

In solving a problem expressed in terms of a system of partial differential equations which involves more than two independent variables certain integral or solution surfaces may exist in the solution space. The integral surfaces consist of what are called the characteristics (or the characteristic curves or waves), and the relations governing the dependent variables along these characteristics will be called characteristic equations.

If only discontinuities in the first partial derivatives of continuous dependent variables on the wave fronts are assumed, the characteristic equations can be derived by the employment of Hadamard's kinematical discontinuity relations(1949)(so called "weak" discontinuity relations). In their work on cylindrical and spherical wave propagation by the method of characteristics[6], Chou and Koenig determined the distribution of stresses behind the wave fronts of the one-dimensional case where the load is abruptly applied. Their computational procedure is that they first derive the characteristic equations compatible along the leading wave front by the employment of the weak discontinuity relations and then evaluate the solution domain behind the wave front by numerical stepwise integration of the characteristic equations. Their method, however, is restricted to one dimensional problems.

Hadamard's work was successfully extended by T.Y. Thomas[2] to kinematical discontinuity relations across a wave front where the dependent variables themselves are discontinuous (so called "strong" discontinuity relations). Subsequently these strong discontinuity relations provided the capability for solving impulsive loading problems of two-dimensional cases by the method of characteristics. Recently in a series of his works[3][4][5][6], M. Ziv successfully applied the theory of characteristics to two-dimensional wave propagation problems where the load is applied abruptly to the boundary. In this case the discontinuity of the dependent variables will occur on the wave front. A computational method with a computer code was presented in his work[6] for the transient motion of a half-space subject to an impulsive load applied radially and uniformly at the boundary of a cylindrical cavity, and his work is extended in the present study to the case of stress propagation in a thick cylindrical shell due to internal axisymmetric impulsive loading.

The computational procedure for solving an impulsive loading problem of the two-dimensional case by the method of caracteristics can be roughly divided into three stages as follows:

1. Characteristic Formulation

   Combining Newton's laws of motion with the elastic relations obtained from Hooke's law gives us a system of first-order partial differential equations involving the variables of stresses and particle velocities, and after certain mathematical procedures involving the application of weak discontinuity relations we obtain a system of characteristic equations along the characteristics.

2. Application of Strong Discontinuity relations

   Strong discontinuity relations are superimposed on the characteristic equations obtained in the previous stage. Then, we can compute the decay of the stresses along the wave front.

3. Numerical Integration

   The solution domain behind the wave front is divided into a grid system by the characteristics along which the characteristic equations expressed in finite difference form will be integrated in a stepwise manner.

An incident longitudinal wave generated by an abruptly applied load will emanate from the inner surface of the tube through the medium and after some time elapses it strikes the outer free surface. Thus we must consider a reflection of waves from the outer free surface as well as from the free top surface (of all four cases to be considered only in CASE 1 does a free top surface exists). Since all action is linear elastic we can apply the principle of superposition to the interacted region of incident and reflected waves.

If an incident longitudinal wave strikes a free surface normally, it is totally reflected with a 180° change in phase, namely a compression wave will be reflected as a tensile wave and vice versa. It should be noted here that near a free surface where an incident compression wave was reflected and transformed into a tensile wave spectacular fracture caused by so called "spalling"[7](p203) may occur. Spalling occurs when a high intensity compressive wave reflects at a free surface and will play a main role in fracture of structures such as thick cylindrical concrete shells under impulsive loading. Furthermore it is a fact that after reflection at the outer free surface incident spherical and cylindrical waves change the nature of divergent waves into that of convergent waves, namely the intensity of the reflected tensile wave will increase as it propagates towards the inner surface. Therefore it is likely that spalling might occur not only near the outer free surface but also in the region far from the reflected surface.

Present Boundary Value Problems

Before presenting the characteristic formulation the boundary value problems to be considered are discussed here in detail. Two basic structures are shown in Figure 1 and Figure 2 while we consider four cases of loading and geometry as follows:

1. CASE 1

   A semi-infinitely long thick cylindrical shell subjected to a uniformly and radially applied impulsive load on the entire inner surface of the shell.

2. CASE 2

   An infinitely long thick cylindrical shell subjected to a uniformly and radially applied impulsive load along the semi-infinite length of the interior (from $z=0$ to $z=+\infty$).

3. CASE 3

   The same shell as CASE 2 subjected to a uniformly and radially applied impulsive load along the finite length of the interior (from $z=-a$ to $z=+a$).

4. CASE 4

   The same shell with the same intial conditions as CASE 3, however, the load of finite length is travelling with a constant speed in the direction of the z-axis.

Loading in CASE 1 and CASE 2 is axisymmetric and is given in terms of a step radial-stress input which is an abruptly applied permanent constant load. We also will consider the case of a rectangular input which is equivalent to a case involving a loading and unloading process. The rectangular input case which is essential to calculate the travelling-load case (CASE 4) can be evaluated by means of the superposition of two step-input cases.

CHAPTER II

CHARACTERISTIC FORMULATION

Basic Dynamic Field Equations

The basic dynamic field equations governing the deformation in the solid stem from combining the equations of motion with the equations of the stress-strain relationships. The equations of motion for linear elastic, isotropic, homogeneous material in cylindrical coordinates are written as:

$$\frac{\partial\sigma_{rr}}{\partial r}+\frac{\partial\sigma_{rz}}{\partial z}+\frac{1}{r}(\sigma_{rr}-\sigma_{\theta\theta})=\rho\frac{\partial U_r}{\partial t}$$

$$\frac{\partial\sigma_{rz}}{\partial r}+\frac{\partial\sigma_{zz}}{\partial z}+\frac{\sigma_{rz}}{r}=\rho\frac{\partial U_z}{\partial t},$$

and the equations of the stress-strain relationships after differentiation with respect to time are given as:

$$\frac{\partial\sigma_{rr}}{\partial t}=(\lambda+2\mu)\frac{\partial U_r}{\partial r}+\lambda\left(\frac{U_r}{r}+\frac{\partial U_z}{\partial z}\right)$$

$$\frac{\partial\sigma_{zz}}{\partial t}=(\lambda+2\mu)\frac{\partial U_z}{\partial z}+\lambda\left(\frac{U_r}{r}+\frac{\partial U_r}{\partial r}\right)$$

$$\frac{\partial\sigma_{\theta\theta}}{\partial t}=(\lambda+2\mu)\frac{U_r}{r}+\lambda\left(\frac{\partial U_r}{\partial r}+\frac{\partial U_z}{\partial z}\right)$$

$$\frac{\partial\sigma_{rz}}{\partial t}=\mu\left(\frac{\partial U_z}{\partial r}+\frac{\partial U_r}{\partial z}\right).$$

These symbols are defined in the nomenclature for the geometry shown in Figure 1 and Figure 2.

Characteristic Equations

The derivation of the characteristic equations from the above dynamic field equations was presented in detail by Ziv in references[3][5]. The general orthogonal scheme for the characteristic formulation[6], however, is repeated here for convenience.

When stress waves propagate in the r,z,t space, they form cones as shown in Figure 3 and the cones consist of, or are characterized by, the infinite number of the characteristics whose origin is the apex of the cone. In our scheme, however, we consider only four characteristic curves which are formed by the intersection of the surface of the cone with two planes passing through the t-axis. These planes intersect orthogonally with each other at the t-axis and each plane has two intersection lines with the cone to form two characteristic curves (or lines). We define these orthogonal characteristic curves as bicharacteristic curves or the bicharacteristics.

The characteristic equations governing the variation of stresses and particle velocities along the bicharacteristic curves are reduced by the orthogonal scheme, in which the bicharacteristic curves are separated by 90° from each other. These characteristic equations along the integration paths shown in Figure 4 are written as follows according to reference[6]:

$$d\sigma_{rr}-\rho G_L dU_r=(\lambda(\frac{\partial U_z}{\partial z}+\frac{U_r}{r})-G_L(\frac{\partial\sigma_{rz}}{\partial z}-\frac{\sigma_{\theta\theta}-\sigma_{rr}}{r}))dt$$

for the bicharacteristic curve $(n_r=1, n_z=0)$ $\theta=0^\circ$ along $\frac{dr}{dt}=G_L$, $\frac{dz}{dt}=0$

$$d\sigma_{rr}+\rho G_L dU_r=(\lambda(\frac{\partial U_z}{\partial z}+\frac{U_r}{r})+G_L(\frac{\partial\sigma_{rz}}{\partial z}-\frac{\sigma_{\theta\theta}-\sigma_{rr}}{r}))dt$$

for the bicharacteristic curve $(n_r=-1, n_z=0)$ $\theta=0^\circ$ along $\frac{dr}{dt}=-G_L$, $\frac{dz}{dt}=0$,

$$d\sigma_{rz}-\rho G_S dU_z=(\mu\frac{\partial U_r}{\partial z}-G_S(\frac{\partial\sigma_{zz}}{\partial z}+\frac{\sigma_{rz}}{r}))dt$$

for the bicharacteristic curve $(n_r=1, n_z=0)$ $\theta=0^\circ$ along $\frac{dr}{dt}=G_S$, $\frac{dz}{dt}=0$,

$$d\sigma_{rz}+\rho G_S dU_z=(\mu\frac{\partial U_r}{\partial z}+G_S(\frac{\partial\sigma_{zz}}{\partial z}+\frac{\sigma_{rz}}{r}))dt$$

for the bicharacteristic curve $(n_r=-1, n_z=0)$ $\theta=0^\circ$ along $\frac{dr}{dt}=-G_S$, $\frac{dz}{dt}=0$,

$$d\sigma_{zz}-\rho G_L dU_z=(\lambda(\frac{\partial U_r}{\partial r}+\frac{U_r}{r})-G_L(\frac{\partial\sigma_{rz}}{\partial r}+\frac{\sigma_{rz}}{r}))dt$$

for the bicharacteristic curve $(n_r=0, n_z=1)$ $\theta=90^\circ$ along $\frac{dr}{dt}=0$, $\frac{dz}{dt}=G_L$,

$$d\sigma_{zz}+\rho G_L dU_z=(\lambda(\frac{\partial U_r}{\partial r}+\frac{U_r}{r})+G_L(\frac{\partial\sigma_{rz}}{\partial r}+\frac{\sigma_{rz}}{r}))dt$$

for the bicharacteristic curve $(n_r=0, n_z=-1)$ $\theta=90^\circ$ along $\frac{dr}{dt}=0$, $\frac{dz}{dt}=-G_L$,

$$d\sigma_{rz}-\rho G_S dU_r=(\mu\frac{\partial U_z}{\partial r}-G_S(\frac{\partial\sigma_{rr}}{\partial r}-\frac{\sigma_{\theta\theta}-\sigma_{rr}}{r}))dt$$

for the bicharacteristic curve $(n_r=0, n_z=1)$ $\theta=90^\circ$ along $\frac{dr}{dt}=0$, $\frac{dz}{dt}=G_S$,

$$d\sigma_{rz}+\rho G_S dU_r=(\mu\frac{\partial U_z}{\partial r}+G_S(\frac{\partial\sigma_{rr}}{\partial r}-\frac{\sigma_{\theta\theta}-\sigma_{rr}}{r}))dt$$

for the bicharacteristic curve $(n_r=0, n_z=-1)$ $\theta=90^\circ$ along $\frac{dr}{dt}=0$, $\frac{dz}{dt}=-G_S$,

$$d\sigma_{rr}=((\lambda+2\mu)\frac{\partial U_r}{\partial r}+\lambda(\frac{U_r}{r}+\frac{\partial U_z}{\partial z}))dt$$

$$d\sigma_{zz}=((\lambda+2\mu)\frac{\partial U_z}{\partial z}+\lambda(\frac{U_r}{r}+\frac{\partial U_r}{\partial r}))dt$$

$$d\sigma_{rz}=\mu(\frac{\partial U_z}{\partial r}+\frac{\partial U_r}{\partial z})dt$$

$$d\sigma_{\theta\theta}=((\lambda+2\mu)\frac{U_r}{r}+\lambda(\frac{\partial U_r}{\partial r}+\frac{\partial U_z}{\partial z}))dt$$

$$dU_r=\frac{1}{\rho}(\frac{\partial\sigma_{rr}}{\partial r}+\frac{\partial\sigma_{rz}}{\partial z}-\frac{\sigma_{\theta\theta}-\sigma_{rr}}{r})dt$$

$$dU_z=\frac{1}{\rho}(\frac{\partial\sigma_{rz}}{\partial r}+\frac{\partial\sigma_{zz}}{\partial z}+\frac{\sigma_{rz}}{r})dt.$$

The last six equations are for the static line $dr=dz=0$. These equations apparently form a system of fourteen simultaneous equations with the fourteen unknowns σ_{rr}, $\sigma_{\theta\theta}$, σ_{zz}, σ_{rz}, $\frac{\partial\sigma_{rr}}{\partial r}$, $\frac{\partial\sigma_{zz}}{\partial z}$, $\frac{\partial\sigma_{rz}}{\partial r}$, $\frac{\partial\sigma_{rz}}{\partial z}$, U_r, U_z, $\frac{\partial U_r}{\partial r}$, $\frac{\partial U_r}{\partial z}$, $\frac{\partial U_z}{\partial r}$, and $\frac{\partial U_z}{\partial z}$.

For convenience, dimensionless variables are now introduced as follows:

$$\bar{U}_r=\frac{U_r}{G_L},\qquad \bar{\sigma}_{rr}=\frac{\sigma_{rr}}{\lambda+2\mu},$$

$$\bar{U}_z=\frac{U_z}{G_L},\qquad \bar{\sigma}_{\theta\theta}=\frac{\sigma_{\theta\theta}}{\lambda+2\mu},$$

$$\tau=\frac{G_L t}{R},\qquad \bar{\sigma}_{zz}=\frac{\sigma_{zz}}{\lambda+2\mu},$$

$$\bar{r}=\frac{r}{R},\qquad \bar{\sigma}_{rz}=\frac{\sigma_{rz}}{\lambda+2\mu},$$

$$\bar{z}=\frac{z}{R},$$

also, let $\alpha=\frac{\lambda}{\lambda+2\mu}$ and $\frac{G_S}{G_L}=\beta=(\frac{\mu}{\lambda+2\mu})^{\frac{1}{2}}$.

Rewriting the fourteen characteristic equations in terms of the dimensionless variables after the bars have been removed

$$d\sigma_{rr}-dU_r=\left(\alpha\left(\frac{\partial U_z}{\partial z}+\frac{U_r}{r}\right)-\frac{\partial\sigma_{rz}}{\partial z}+\frac{\sigma_{\theta\theta}-\sigma_{rr}}{r}\right)d\tau \qquad (1)$$

along $dr=d\tau$,

$$d\sigma_{rr}+dU_r=\left(\alpha\left(\frac{\partial U_z}{\partial z}+\frac{U_r}{r}\right)+\frac{\partial\sigma_{rz}}{\partial z}-\frac{\sigma_{\theta\theta}-\sigma_{rr}}{r}\right)d\tau \qquad (2)$$

along $dr=-d\tau$,

$$d\sigma_{rz}-\beta dU_z=\beta\left(\beta\frac{\partial U_r}{\partial z}-\frac{\partial\sigma_{zz}}{\partial z}-\frac{\sigma_{rz}}{r}\right)d\tau \qquad (3)$$

along $dr=\beta d\tau$,

$$d\sigma_{rz}+\beta dU_z=\beta\left(\beta\frac{\partial U_r}{\partial z}+\frac{\partial\sigma_{zz}}{\partial z}+\frac{\sigma_{rz}}{r}\right)d\tau \qquad (4)$$

along $dr=-\beta d\tau$,

$$d\sigma_{zz}-dU_z=\left(\alpha\left(\frac{\partial U_r}{\partial r}+\frac{U_r}{r}\right)-\frac{\partial\sigma_{rz}}{\partial r}-\frac{\sigma_{rz}}{r}\right)d\tau \qquad (5)$$

along $dz=d\tau$,

$$d\sigma_{zz}+dU_z=\left(\alpha\left(\frac{\partial U_r}{\partial r}+\frac{U_r}{r}\right)+\frac{\partial\sigma_{rz}}{\partial r}+\frac{\sigma_{rz}}{r}\right)d\tau \qquad (6)$$

along $dz=-d\tau$,

$$d\sigma_{rz}-\beta dU_r=\beta\left(\beta\frac{\partial U_z}{\partial r}-\frac{\partial\sigma_{rr}}{\partial r}+\frac{\sigma_{\theta\theta}-\sigma_{rr}}{r}\right)d\tau \qquad (7)$$

along $dz=\beta d\tau$,

$$d\sigma_{rz}+\beta dU_r=\beta\left(\beta\frac{\partial U_z}{\partial r}+\frac{\partial\sigma_{rr}}{\partial r}-\frac{\sigma_{\theta\theta}-\sigma_{rr}}{r}\right)d\tau \qquad (8)$$

along $dz=-\beta d\tau$,

$$d\sigma_{rr}=\left(\frac{\partial U_r}{\partial r}+\alpha\left(\frac{U_r}{r}+\frac{\partial U_z}{\partial z}\right)\right)d\tau \qquad (9)$$

along $dr=dz=0$,

$$d\sigma_{zz}=\left(\frac{\partial U_z}{\partial z}+\alpha\left(\frac{U_r}{r}+\frac{\partial U_r}{\partial r}\right)\right)d\tau \qquad (10)$$

along $dr=dz=0$,

$$d\sigma_{rz}=\beta^2\left(\frac{\partial U_z}{\partial r}+\frac{\partial U_r}{\partial z}\right)d\tau \qquad (11)$$

along $dr=dz=0$,

$$d\sigma_{\theta\theta}=\left(\frac{U_r}{r}+\alpha\left(\frac{\partial U_r}{\partial r}+\frac{\partial U_z}{\partial z}\right)\right)d\tau \tag{12}$$

along $dr=dz=0$,

$$dU_r=\left(\frac{\partial\sigma_{rr}}{\partial r}+\frac{\partial\sigma_{rz}}{\partial z}-\frac{\sigma_{\theta\theta}-\sigma_{rr}}{r}\right)d\tau \tag{13}$$

along $dr=dz=0$,

$$dU_z=\left(\frac{\partial\sigma_{rz}}{\partial r}+\frac{\partial\sigma_{zz}}{\partial z}+\frac{\sigma_{rz}}{r}\right)d\tau \tag{14}$$

along $dr=dz=0$.

In order to investigate the behavior of the shell at a point(r,z,τ) by the finite difference technique these characteristic equations are expressed in the finite difference form. Letting f be any variable of the equation, df can be written as $df=f-f_i$ where f_i denotes the known variable at the point i and f denotes the unknown variable at the point(r,z,τ). If we take the first characteristic equation along $dr=d\tau$ we obtain two equations as follows:

$$\sigma_{rr}-\sigma_{rr1}-U_r+U_{r1}=\left(\alpha\left(\frac{\partial U_z}{\partial z}+\frac{U_r}{r}\right)-\frac{\partial\sigma_{rz}}{\partial z}+\frac{\sigma_{\theta\theta}-\sigma_{rr}}{r}\right)(\tau-\tau_1)$$

$$\sigma_{rr}-\sigma_{rr1}-U_r+U_{r1}=\left(\alpha\left(\frac{\partial U_z}{\partial z_1}+\frac{U_{r1}}{r}\right)-\frac{\partial\sigma_{rz}}{\partial z_1}+\frac{\sigma_{\theta\theta 1}-\sigma_{rr1}}{r}\right)(\tau-\tau_1).$$

After adding these two equations, dividing by 2 , and gathering the unknown terms on the left hand side we finally come up with

$$\left(1+\frac{B_1}{2r}\right)\sigma_{rr}-\left(1+\alpha\frac{B_1}{2r}\right)U_r-\frac{B_1}{2r}\sigma_{\theta\theta}-\alpha\frac{B_1}{2}\frac{\partial U_z}{\partial z}+\frac{B_1}{2}\frac{\partial\sigma_{rz}}{\partial z}=A_1$$

where

$$A_1=(\sigma_{rr}(1-\frac{B}{2r})-U_r(1-\frac{\alpha B}{2r})+(\alpha\frac{\partial U_z}{\partial z}-\frac{\partial\sigma_{rz}}{\partial z}+\frac{\sigma_{\theta\theta}}{r})\frac{B}{2})_1,$$

$$B_1=\tau-\tau_1.$$

Likewise the characteristic equations (2) through (14) are transformed and written in accordance with reference[6]:

$$
\begin{bmatrix}
1+\frac{B_1}{2r} & -\frac{B_1}{2r} & 0 & 0 & 0 & 0 & 0 & \frac{B_1}{2} & -1-\alpha\frac{B_1}{2r} & 0 & 0 & 0 & 0 & -\alpha\frac{B_1}{2} \\
1-\frac{B_2}{2r} & \frac{B_2}{2r} & 0 & 0 & 0 & 0 & 0 & -\frac{B_2}{2} & 1-\alpha\frac{B_2}{2r} & 0 & 0 & 0 & 0 & -\alpha\frac{B_2}{2} \\
0 & 0 & 0 & 1+\beta\frac{B_3}{2r} & 0 & \beta\frac{B_3}{2} & 0 & 0 & 0 & -\beta & 0 & -\beta^2\frac{B_3}{2} & 0 & 0 \\
0 & 0 & 0 & 1-\beta\frac{B_4}{2r} & 0 & -\beta\frac{B_4}{2} & 0 & 0 & 0 & \beta & 0 & -\beta^2\frac{B_4}{2} & 0 & 0 \\
0 & 0 & 1 & \frac{B_5}{2r} & 0 & 0 & \frac{B_5}{2} & 0 & -\alpha\frac{B_5}{2r} & -1 & -\alpha\frac{B_5}{2} & 0 & 0 & 0 \\
0 & 0 & 1 & -\frac{B_6}{2r} & 0 & 0 & \frac{B_6}{2} & 0 & -\alpha\frac{B_6}{2r} & 1 & -\alpha\frac{B_6}{2} & 0 & 0 & 0 \\
\beta\frac{B_7}{2r} & -\beta\frac{B_7}{2r} & 0 & 1 & \beta\frac{B_7}{2} & 0 & 0 & 0 & -\beta & 0 & 0 & 0 & -\beta^2\frac{B_7}{2} & 0 \\
-\beta\frac{B_8}{2r} & \beta\frac{B_8}{2r} & 0 & 1 & -\beta\frac{B_8}{2} & 0 & 0 & 0 & \beta & 0 & 0 & 0 & -\beta^2\frac{B_8}{2} & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\alpha\frac{B_9}{2r} & 0 & -\frac{B_9}{2} & 0 & 0 & -\alpha\frac{B_9}{2} \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -\alpha\frac{B_9}{2r} & 0 & -\alpha\frac{B_9}{2} & 0 & 0 & -\frac{B_9}{2} \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\beta^2\frac{B_9}{2} & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{B_9}{2r} & 0 & -\alpha\frac{B_9}{2} & 0 & 0 & -\alpha\frac{B_9}{2} \\
-\frac{B_9}{2r} & \frac{B_9}{2r} & 0 & 0 & -\frac{B_9}{2} & 0 & 0 & -\frac{B_9}{2} & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -\frac{B_9}{2r} & 0 & -\frac{B_9}{2} & -\frac{B_9}{2} & 0 & 0 & 1 & 0 & 0 & 0 & 0
\end{bmatrix}
\bullet
\begin{bmatrix}
\sigma_{rr} \\ \sigma_{\theta\theta} \\ \sigma_{zz} \\ \sigma_{rz} \\ \frac{\partial\sigma_{rr}}{\partial r} \\ \frac{\partial\sigma_{zz}}{\partial z} \\ \frac{\partial\sigma_{rz}}{\partial r} \\ \frac{\partial\sigma_{rz}}{\partial z} \\ U_r \\ U_z \\ \frac{\partial U_r}{\partial r} \\ \frac{\partial U_r}{\partial z} \\ \frac{\partial U_z}{\partial r} \\ \frac{\partial U_z}{\partial z}
\end{bmatrix}
=
\begin{bmatrix}
A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \\ A_6 \\ A_7 \\ A_8 \\ A_9 \\ A_{10} \\ A_{11} \\ A_{12} \\ A_{13} \\ A_{14}
\end{bmatrix}
$$

Matrix 1

where

$$A_1=(\sigma_{rr}(1-\frac{B}{2r})-U_r(1-\frac{B}{2r})+(\alpha\,\frac{\partial U_z}{\partial z}-\frac{\partial\sigma_{rz}}{\partial z}+\frac{\sigma_{\theta\theta}}{r})\,\frac{B}{2})_1$$

$$A_2=(\sigma_{rr}(1+\frac{B}{2r})+U_r(1+\frac{B}{2r})+(\alpha\,\frac{\partial U_z}{\partial z}+\frac{\partial\sigma_{rz}}{\partial z}-\frac{\sigma_{\theta\theta}}{r})\,\frac{B}{2})_2$$

$$A_3=(\sigma_{rz}(1-\frac{B}{2r})-\beta U_z+\beta(\beta\,\frac{\partial U_r}{\partial z}-\frac{\partial\sigma_{zz}}{\partial z})\,\frac{B}{2})_3$$

$$A_4=(\sigma_{rz}(1+\frac{B}{2r})+\beta U_z+\beta(\beta\,\frac{\partial U_r}{\partial z}+\frac{\partial\sigma_{zz}}{\partial z})\,\frac{B}{2})_4$$

$$A_5=(\sigma_{zz}-U_z+(\alpha(\frac{\partial U_r}{\partial r}+\frac{U_r}{r})-\frac{\partial\sigma_{rz}}{\partial r}-\frac{\sigma_{rz}}{r})\,\frac{B}{2})_5$$

$$A_6=(\sigma_{zz}+U_z+(\alpha(\frac{\partial U_r}{\partial r}+\frac{U_r}{r})+\frac{\partial\sigma_{rz}}{\partial r}+\frac{\sigma_{rz}}{r})\,\frac{B}{2})_6$$

$$A_7=(\sigma_{rz}-\beta U_r+\beta(\beta\,\frac{\partial U_z}{\partial r}-\frac{\partial\sigma_{rr}}{\partial r}+\frac{\sigma_{\theta\theta}-\sigma_{rr}}{r})\,\frac{B}{2})_7$$

$$A_8=(\sigma_{rz}+\beta U_r+\beta(\beta\,\frac{\partial U_z}{\partial r}+\frac{\partial\sigma_{rr}}{\partial r}-\frac{\sigma_{\theta\theta}-\sigma_{rr}}{r})\,\frac{B}{2})_8$$

$$A_9=(\sigma_{rr}+(\frac{\partial U_r}{\partial r}+\alpha(\frac{U_r}{r}+\frac{\partial U_z}{\partial z}))\,\frac{B}{2})_9$$

$$A_{10}=(\sigma_{zz}+(\frac{\partial U_z}{\partial z}+\alpha(\frac{U_r}{r}+\frac{\partial U_r}{\partial r}))\,\frac{B}{2})_{10}$$

$$A_{11}=(\sigma_{rz}+\beta^2(\frac{\partial U_z}{\partial r}+\frac{\partial U_r}{\partial z})\,\frac{B}{2})_{11}$$

$$A_{12}=(\sigma_{\theta\theta}+(\frac{U_r}{r}+\alpha(\frac{\partial U_r}{\partial r}+\frac{\partial U_z}{\partial z}))\,\frac{B}{2})_{12}$$

$$A_{13}=(U_r+(\frac{\partial\sigma_{rr}}{\partial r}+\frac{\partial\sigma_{rz}}{\partial z}-\frac{\sigma_{\theta\theta}-\sigma_{rr}}{r})\,\frac{B}{2})_{13}$$

$$A_{14}=(U_z+(\frac{\partial\sigma_{rz}}{\partial r}+\frac{\partial\sigma_{zz}}{\partial z}+\frac{\sigma_{rz}}{r})\,\frac{B}{2})_{14}.$$

and $B_1=\tau-\tau_1$, $B_2=\tau-\tau_2$, $B_3=\tau-\tau_3$, $B_4=\tau-\tau_4$, $B_5=\tau-\tau_5$, $B_6=\tau-\tau_6$, $B_7=\tau-\tau_7$, $B_8=\tau-\tau_8$, $B_9=\tau-\tau_9$. The subscripts 1 through 9 denote the points where the quantities are already known.

At this stage the applicability of the above characteristic equations is confined to the case where the dependent variables σ_{rr}, $\sigma_{\theta\theta}$, σ_{zz}, σ_{rz}, U_r, and U_z are continuous while their first partial derivatives may be discontinuous across the wave front. In the present boundary value problems, however, the load is abruptly applied at the boundary. Therefore discontinuities in the dependent variables themselves exist across the bicharacteristic curves. Furthermore the discontinuities may occur not only due to the impulsive load but also due to the reflections from the boundaries. This means that strong discontinuities must be considered for the reflected region even if the load is applied gradually. In the next section the strong discontinuity relations will be applied to the characteristic equations to be compatible with the strong motion of the present boundary value problems.

Strong Discontinuity Relations

If strong discontinuities which mean the discontinuities in the dependent variables themselves exist along the bicharacteristic curves, strong discontinuity relations must be imposed on the characteristic equations along the discontinuities. Observing the characteristic equations (1) through (14) it is seen that every equation apparently involves partial derivatives of the dependent variables which are not differentiable along the bicharacteristic curves carrying discontinuities.

Now we assume that all characteristic equations except the static equations (9) through (14) would be along the bicharacteristic curves which are carrying discontinuities. Accordingly in order to superimpose strong discontinuity relations the characteristic equations (1) (8) should be first rewitten using the well known bracket notation as follows:

$$d[\sigma_{rr}]-d[U_r]=(\alpha([U_{z;z}]+\frac{[U_r]}{r})-[\sigma_{rz;z}]+\frac{[\sigma_{\theta\theta}]-[\sigma_{rr}]}{r})d\tau \tag{1a}$$

along $dr=d\tau$,

$$d[\sigma_{rr}]+d[U_r]=(\alpha([U_{z;z}]+\frac{[U_r]}{r})+[\sigma_{rz;z}]-\frac{[\sigma_{\theta\theta}]-[\sigma_{rr}]}{r})d\tau \tag{2a}$$

along $dr=-d\tau$,

$$d[\sigma_{rz}]-\beta d[U_z]=\beta(\beta[U_{r;z}]-[\sigma_{zz;z}]-\frac{[\sigma_{rz}]}{r})d\tau \tag{3a}$$

along $dr=\beta d\tau$,

$$d[\sigma_{rz}]+\beta d[U_z]=\beta(\beta[U_{r;z}]+[\sigma_{zz;z}]+\frac{[\sigma_{rz}]}{r})d\tau \tag{4a}$$

along $dr=-\beta d\tau$,

$$d[\sigma_{zz}]-d[U_z]=(\alpha([U_{r;r}]+\frac{[U_r]}{r})-[\sigma_{rz;r}]-\frac{[\sigma_{rz}]}{r})d\tau \tag{5a}$$

along $dz=d\tau$,

$$d[\sigma_{zz}]+d[U_z]=(\alpha([U_{r;r}]+\frac{[U_r]}{r})+[\sigma_{rz;r}]+\frac{[\sigma_{rz}]}{r})d\tau \tag{6a}$$

along $dz=-d\tau$,

$$d[\sigma_{rz}]-\beta d[U_r]=\beta(\beta[U_{z;r}]-[\sigma_{rr;r}]+\frac{[\sigma_{\theta\theta}]-[\sigma_{rr}]}{r})d\tau \tag{7a}$$

along $dz=\beta d\tau$,

$$d[\sigma_{rz}]+\beta d[U_r]=\beta(\beta[U_{z;r}]+[\sigma_{rr;r}]-\frac{[\sigma_{\theta\theta}]-[\sigma_{rr}]}{r})d\tau \tag{8a}$$

along $dz=-\beta d\tau$,

where

$$[f]=f\Big|_{\substack{\text{The value of } f\\ \text{at the rear of}\\ \text{the wave front}}} - f\Big|_{\substack{\text{The value of } f\\ \text{at the front of}\\ \text{the wave front}}}$$

; = Partial differentiation.

The strong discontinuity relations to the relevant case are derived by Ziv by means of the kinematical compatibility relations given in reference[2] by Thomas. Those strong discontinuity relations are presented in reference[6] as follows:

$$[U_{z;z}]_{1a}=[U_{z;z}]_{2a}=\frac{[U_r]}{r} \qquad (15)$$

$$[\sigma_{rz;z}]_{1a}=[\sigma_{rz;z}]_{2a}=\frac{[\sigma_{rr}]-[\sigma_{zz}]}{r} \qquad (16)$$

$$[U_{r;z}]_{3a}=[U_{r;z}]_{4a}=-\frac{[U_z]}{r} \qquad (17)$$

$$[\sigma_{zz;z}]_{3a}=[\sigma_{zz;z}]_{4a}=\frac{2[\sigma_{rz}]}{r} \qquad (18)$$

$$[U_{r;r}]_{5a}=\frac{[U_z]}{R+z} \qquad (19)$$

$$[\sigma_{rz;r}]_{5a}=\frac{[\sigma_{zz}]-[\sigma_{rr}]}{R+z} \qquad (20)$$

$$[U_{r;r}]_{6a}=[\sigma_{rz;r}]_{6a}=0 \qquad (21)$$

$$[U_{z;r}]_{7a}=-\frac{[U_r]}{R+z} \qquad (22)$$

$$[\sigma_{rr;r}]_{7a}=\frac{2[\sigma_{rz}]}{R+z} \qquad (23)$$

$$[U_{z;r}]_{8a}=[\sigma_{rr;r}]_{8a}=0, \qquad (24)$$

where the subscripts refer to the corresponding characteristic equations.

Substituting these relations (15) (24) into the corresponding equations (1a) (8a) gives us

$$d[\sigma_{rr}]-d[U_r]=(2\alpha[U_r]-2[\sigma_{rr}]+[\sigma_{\theta\theta}]+[\sigma_{zz}])\frac{dr}{r} \quad (1b)$$

$$d[\sigma_{rr}]+d[U_r]=(2\alpha[U_r]+2[\sigma_{rr}]-[\sigma_{\theta\theta}]-[\sigma_{zz}])\frac{dr}{r} \quad (2b)$$

$$d[\sigma_{rz}]-\beta d[U_z]=-\beta(\beta[U_z]+3[\sigma_{rz}])\frac{dr}{r} \quad (3b)$$

$$d[\sigma_{rz}]+\beta d[U_z]=\beta(-\beta[U_z]+3[\sigma_{rz}])\frac{dr}{r} \quad (4b)$$

$$d[\sigma_{zz}]-d[U_z]=(\alpha(\frac{[U_z]}{R+z}+\frac{[U_r]}{r})-\frac{[\sigma_{zz}]-[\sigma_{rr}]}{R+z}-\frac{[\sigma_{rz}]}{r})dr \quad (5b)$$

$$d[\sigma_{zz}]+d[U_z]=(\alpha[U_r]+[\sigma_{rz}])\frac{dr}{r} \quad (6b)$$

$$d[\sigma_{rz}]-\beta d[U_r]=\beta(-\beta\frac{[U_r]}{R+z}-\frac{2[\sigma_{rz}]}{R+z}+\frac{[\sigma_{\theta\theta}]-[\sigma_{rr}]}{r})dr \quad (7b)$$

$$d[\sigma_{rz}]+\beta d[U_r]=\beta([\sigma_{rr}]-[\sigma_{\theta\theta}])\frac{dr}{r} \quad (8b)$$

or in the finite difference form

$$(1+\frac{B_1}{r})[\sigma_{rr}]-\frac{B_1}{2r}[\sigma_{\theta\theta}]-\frac{B_1}{2r}[\sigma_{zz}]-(1+\alpha\frac{B_1}{r})[U_r]=[A_1] \quad (1c)$$

$$(1-\frac{B_2}{r})[\sigma_{rr}]+\frac{B_2}{2r}[\sigma_{\theta\theta}]+\frac{B_2}{2r}[\sigma_{zz}]+(1-\alpha\frac{B_2}{r})[U_r]=[A_2] \quad (2c)$$

$$(1+3\beta\frac{B_3}{2r})[\sigma_{rz}]-\beta(1-\beta\frac{B_3}{2r})[U_z]=[A_3] \quad (3c)$$

$$(1-3\beta\frac{B_4}{2r})[\sigma_{rz}]+\beta(1+\beta\frac{B_4}{2r})[U_z]=[A_4] \quad (4c)$$

$$-\frac{B_5}{2(R+z)}[\sigma_{rr}]+(1+\frac{B_5}{2(R+z)})[\sigma_{zz}]-\alpha\frac{B_5}{2r}[U_r]-(1+\frac{\alpha B_5}{2(R+z)})[U_z]+\frac{B_5}{2r}[\sigma_{rz}]=[A_5] \quad (5c)$$

$$[\sigma_{zz}]-\alpha\frac{B_6}{2r}[U_r]+[U_z]-\frac{B_6}{2r}[\sigma_{rz}]=[A_6] \quad (6c)$$

$$(1+\frac{\beta B_7}{R+z})[\sigma_{rz}]-\beta(1-\frac{\beta B_7}{2(R+z)})[U_r]-\beta\frac{B_7}{2r}[\sigma_{\theta\theta}]+\beta\frac{B_7}{2r}[\sigma_{rr}]=[A_7] \quad (7c)$$

$$[\sigma_{rz}]+\beta[U_r]-\beta\frac{B_8}{2r}[\sigma_{rr}]+\beta\frac{B_8}{2r}[\sigma_{\theta\theta}]=[A_8] \quad (8c)$$

where

$$[A_1]=((1-\frac{B}{r})[\sigma_{rr}]-(1-\alpha\frac{B}{r})[U_r]+(\frac{[\sigma_{zz}]+[\sigma_{\theta\theta}]}{r})\frac{B}{2})_1$$

$$[A_2]=((1+\frac{B}{r})[\sigma_{rr}]+(1+\alpha\frac{B}{r})[U_r]-(\frac{[\sigma_{zz}]+[\sigma_{\theta\theta}]}{r})\frac{B}{2})_2$$

$$[A_3]=((1-\frac{3\beta B}{2r})[\sigma_{rz}]-\beta(1+\frac{\beta B}{2r})[U_z])_3$$

$$[A_4]=((1+\frac{3\beta B}{2r})[\sigma_{rz}]+\beta(1-\frac{\beta B}{2r})[U_z])_4$$

$$[A_5]=((1-\frac{B}{2(R+z)})[\sigma_{zz}]-(1-\frac{\alpha B}{2(R+z)})[U_z]+\frac{\alpha B}{2r}[U_r]+\frac{B}{2(R+z)}[\sigma_{rr}]-\frac{B}{2r}[\sigma_{rz}])_5$$

$$[A_6]=([\sigma_{zz}]+[U_z]+\frac{\alpha B}{2r}[U_r]+\frac{B}{2r}[\sigma_{rz}])_6$$

$$[A_7]=((1-\frac{\beta B}{R+z})[\sigma_{rz}]-\beta(1+\frac{\beta B}{2(R+z)})[U_r]+\frac{\beta B}{2r}[\sigma_{\theta\theta}]-\frac{\beta B}{2r}[\sigma_{rr}])_7$$

$$[A_8]=([\sigma_{rz}]+\beta[U_r]+\frac{\beta B}{2r}[\sigma_{rr}]-\frac{\beta B}{2r}[\sigma_{\theta\theta}])_8.$$

CHAPTER III

COMPUTATIONAL METHOD FOR CASE 1

Characteristic Equations along Strong Discontinuity

The discussion in reference[6] on the behavior of waves and their discontinuities must hold for CASE 1 as long as the incident wave front does not reach the outer free surface of the shell, since the initial and boundary conditions in the present case are exactly same as those in [6]. Therefore the characteristic equations (1b),(2b),(3b),(5b), and (6b) should be employed for the two dimensional region in which every grid point of integration mesh has strong nature of discontinuity in dependent variables due to the impulsive load and the reflected waves from the top free surface. In other words those five bicharacteristic curves which construct the orthogonal integration mesh in the two dimensional region carry strong discontinuities so that we should employ the characteristic equations (1b),(2b),(3b),(5b), and (6b) instead of (1),(2),(3),(5), and (6). The characteristic equations (1c) (6c) and static equations (9),(10), and (12) are rewritten in matrix form as follows after the bracket notation has been removed:

$$
\begin{bmatrix}
1+\frac{B_1}{r} & -\frac{B_1}{2r} & -\frac{B_1}{2r} & 0 & -1-\alpha\frac{B_1}{r} & 0 & 0 & 0 \\
1-\frac{B_2}{r} & \frac{B_2}{2r} & \frac{B_2}{2r} & 0 & 1-\alpha\frac{B_2}{r} & 0 & 0 & 0 \\
0 & 0 & 0 & 1+3\beta\frac{B_3}{2r} & 0 & -\beta+\beta^2\frac{B_3}{2r} & 0 & 0 \\
-\frac{B_5}{2(R+z)} & 0 & 1+\frac{B_5}{2(R+z)} & \frac{B_5}{2r} & -\alpha\frac{B_5}{2r} & -1-\frac{\alpha B_5}{2(R+z)} & 0 & 0 \\
0 & 0 & 1 & -\frac{B_6}{2r} & -\alpha\frac{B_6}{2r} & 1 & 0 & 0 \\
1 & 0 & 0 & 0 & -\alpha\frac{B_9}{2r} & 0 & -\frac{B_9}{2} & -\alpha\frac{B_9}{2} \\
0 & 0 & 1 & 0 & -\alpha\frac{B_9}{2r} & 0 & -\alpha\frac{B_9}{2} & -\frac{B_9}{2} \\
0 & 1 & 0 & 0 & -\frac{B_9}{2r} & 0 & -\alpha\frac{B_9}{2} & -\alpha\frac{B_9}{2}
\end{bmatrix}
\bullet
\begin{bmatrix}
\sigma_{rr} \\ \sigma_{\theta\theta} \\ \sigma_{zz} \\ \sigma_{rz} \\ U_r \\ U_z \\ \frac{\partial U_r}{\partial r} \\ \frac{\partial U_z}{\partial z}
\end{bmatrix}
=
\begin{bmatrix}
[A_1] \\ [A_2] \\ [A_3] \\ [A_5] \\ [A_6] \\ A_9 \\ A_{10} \\ A_{12}
\end{bmatrix}
$$

Matrix 2

where

$$[A_1]=\{(1-\frac{B}{r})\sigma_{rr}-(1-\alpha\frac{B}{r})U_r+(\frac{\sigma_{zz}+\sigma_{\theta\theta}}{r})\frac{B}{2}\}_1$$

$$[A_2]=\{(1+\frac{B}{r})\sigma_{rr}+(1+\alpha\frac{B}{r})U_r-(\frac{\sigma_{zz}+\sigma_{\theta\theta}}{r})\frac{B}{2}\}_2$$

$$[A_3]=\{(1-\frac{3\beta B}{2r})\sigma_{rz}-\beta(1+\frac{\beta B}{2r})U_z\}_3$$

$$[A_4]=\{(1-\frac{B}{2(R+z)})\sigma_{zz}-(1-\frac{\alpha B}{2(R+z)})U_z+(\alpha\frac{U_r}{r}+\frac{\sigma_{rr}}{R+z}-\frac{\sigma_{rz}}{r})\frac{B}{2}\}_4$$

$$[A_6]=(\sigma_{zz}+U_z+(\alpha U_r+\sigma_{rz})\frac{B}{2r})_6$$

$$A_9=(\sigma_{rr}+(\frac{\partial U_r}{\partial r}+\alpha(\frac{U_r}{r}+\frac{\partial U_z}{\partial z}))\frac{B}{2})_9$$

$$A_{10}=(\sigma_{zz}+(\frac{\partial U_z}{\partial z}+\alpha(\frac{U_r}{r}+\frac{\partial U_r}{\partial r}))\frac{B}{2})_{10}$$

$$A_{12}=(\sigma_{\theta\theta}+(\frac{U_r}{r}+\alpha(\frac{\partial U_r}{\partial r}+\frac{\partial U_z}{\partial z}))\frac{B}{2})_{12}.$$

It is worthwhile to note here that the bracketed value of the dependent variables in (1b),(2b),(3b),(5b), and (6b) represent "Jumps" across the corresponding bicharacteristic curves carrying strong discontinuities so that the jumps are equivalent to the values of the dependent variables which are discontinuous across the bicharacteristics. However there are no jumps of the dependent variables in the static equation (9), (10), and (12) since the dependent variables are continuous across the bicharacteristics $dr=dz=0$. It can be noted that the static equations (9) through (12) are merely a restatement of the constitutive equations in the basic field equations.

It is known that in reflections of spherical longitudinal waves at the boundary both longitudinal and shear waves are generated as reflected waves[6](P158). Therefore in CASE 1 after the reflection at the outer free surface we must consider not only reflected longitudinal waves but also reflected shear waves which emanate from the outer surface into the two-dimentional region. These reflected shear waves are characterized by the bicharacteristic curve $dr=-\beta d\tau$. Accordingly we should employ the characteristic equation (4c) and superimpose it on Matrix 2. The characteristic equation (4c) is superimposed on (3c) to obtain

$$(2+\frac{3\beta}{2r}(B_3-B_4))[\sigma_{rz}]+\frac{\beta^2}{2r}(B_3+B_4)[U_z]=[A_{34}] \qquad (3c')$$

where

$$[A_{34}]=((1-\frac{3\beta B}{2r})[\sigma_{rz}]-\beta(1+\frac{\beta B}{2r})[U_z])_3+((1+\frac{3\beta B}{2r})[\sigma_{rz}]+\beta(1-\frac{\beta B}{2r})[U_z])_4.$$

Regions of Influence and Typical Reccurring Points

A region in the solution domain where all points are under the same type of motion is defined here as a region of influence. In other words all points of each region of influence are affected by the same boundary and initial conditions which induce the same type of particle motion in the region. All points in each region of influence are categorized into three groups which are defined as follows:

1. Leading Points

   Points that are located on a leading wave front and do not comply with the orthogonal scheme.

2. Intermediary Points

   Points that have an immediate contact with one or more leading points via their bicharacteristic curves and require a special orthogonal mesh configulation.

3. <u>Regular Points</u>

Points that comply with the perfect symmetric mesh configulation.

Four regions of influence and twenty four recurring points in the entire solution domain are shown in Figure 5. Before the leading wave reflects at the outer surface the solution domain is divided into two regions of influence in accordance with the case of reference[6]. One is a one-dimensional region r,τ, influenced by the loaded boundary and the other is a two-dimensional region r,z,τ, influenced by the free top surface as well as by the loaded boundary. After the reflection of the leading wave from the outer surface, however, the entire domain is divided into four regions of influence. In addition to the two previous regions of influence we must add two reflected regions which are influenced by the outer boundary. One is a reflected one-dimensional region r,τ, influenced not only by the loaded boundary but also by the outer boundary. The other is a reflected two-dimensional region r,z,τ, influenced by the outer boundary as well as the loaded boundary and the top surface.

Accordingly all points in each region of influence are categorized into three groups previously defined with respect to their computational schemes. Therefore twenty four recurring points a through p in the solution domain can be defined as shown in Figure 5. All recurring points are categorized into the three groups in the following way: Points a, a', b, c, c', d, e, e', f, n and p belong to regular points, points g, g', i, i', j, k, k', and m belong to the leading points and points h, h', ℓ, ℓ', and o belong to the intermediary points.

In the next section the boundary and initial conditions will be prescribed and the computational scheme of integration for each recurring point will be discussed in addition to the derivation of the decays of the leading waves in terms of the closed-form solutions.

Computational Scheme for Each Recurring Point

The recurring points were defined with respect to the region of influence, the group of points, and the boundary conditions. Subsequently we must detect twenty four recurring points to determine the dependent variables in the entire solution domain. Once the neighborhood of the leading wave which implies the intermediary points as well as the leading points is determined, we can apply the perfect symmetric scheme to the rest of the solution domain, since all points remaining are regular points which comply with the perfect symmetric mesh.

The following dimensionless values of the boundary conditions are prescribed when both the intensity of σ_{rr}-input and the inner radius of the shell are considered to be unity.

The boundary conditions for the loaded boundary $(r, z>0, t)$ are given as $\sigma_{rr}=1$ and $\sigma_{rz}=0$. The initial conditions for the loaded boundary are given as $U_r=-1$ due to $[U_r]=-[\sigma_{rr}]$ the dynamical conditions of discontinuities across a wave front[8](P140), $U_z=\partial U_z/\partial z=0$, $\sigma_{\theta\theta}=\sigma_{zz}=a\sigma_{rr}$ due to static equations (9),(10),(12), and $\partial U_r/\partial r=0.5$ due to the known decay expression for a one-dimensional cylindrical leading wave.

The boundary conditions for point $(R,0,\tau)$ are given as $\sigma_{rr}=1$ and $\sigma_{zz}=\sigma_{rz}=0$. The initial conditions for the corner point are given as $U_r=-1$, $\sigma_{\theta\theta}=a\sigma_{rr}$, $\partial U_z/\partial z=0$ and $\partial U_r/\partial r=0.5$.

The boundary conditions for points $(R<r<R_0,0,\tau)$ on the free top surface are given as $\sigma_{zz}=\sigma_{rz}=0$.

The boundary conditions for points $(R_0,z>0,\tau)$ on the outer free surface are given as $\sigma_{rr}=\sigma_{rz}=0$.

All particles in the shell except for those on the loaded boundary are considered to be at rest at $\tau=0$.

Stresses and particle velocities at the typical recurring points a through p are evaluated in the following way:

a. <u>Regular Inner Two-Dimensional Points a(r,z,τ)</u>

These points comply with the perfect symmetric mesh configulation referring to Matrix 2.

a'. Reflected Regular Inner Two-Dimensional Points a'(r,z,t)

After replacing (3c) in Matrix 2 with (3c') due to reflected shear waves from the outer surface, Matrix 2 becomes

$1+\frac{B_1}{r}$	$-\frac{B_1}{2r}$	$-\frac{B_1}{2r}$	0	$-1-\alpha\frac{B_1}{r}$	0	0	0		σ_{rr}		$[A_1]$
$1-\frac{B_2}{r}$	$\frac{B_2}{2r}$	$\frac{B_2}{2r}$	0	$1-\alpha\frac{B_2}{r}$	0	0	0		$\sigma_{\theta\theta}$		$[A_2]$
0	0	0	$2+\frac{3\beta}{2r}(B_3-B_4)$		$\frac{\beta^2}{2r}(B_3+B_4)$	0	0		σ_{zz}		$[A_{34}]$
$-\frac{B_5}{2(R+z)}$	0	$1+\frac{B_5}{2(R+z)}$	$\frac{B_5}{2r}$	$-\alpha\frac{B_5}{2r}$	$-1-\frac{\alpha B_5}{2(R+z)}$	0	0	●	σ_{rz}	=	$[A_5]$
0	0	1	$-\frac{B_6}{2r}$	$-\alpha\frac{B_6}{2r}$	1	0	0		U_r		$[A_6]$
1	0	0	0	$-\alpha\frac{B_9}{2r}$	0	$-\frac{B_9}{2}$	$-\alpha\frac{B_9}{2}$		U_z		A_9
0	0	1	0	$-\alpha\frac{B_9}{2r}$	0	$-\alpha\frac{B_9}{2}$	$-\frac{B_9}{2}$		$\frac{\partial U_r}{\partial r}$		A_{10}
0	1	0	0	$-\frac{B_9}{2r}$	0	$-\alpha\frac{B_9}{2}$	$-\alpha\frac{B_9}{2}$		$\frac{\partial U_z}{\partial z}$		A_{12}

Matrix 3

b. <u>Regular Loaded Boundary Two-Dimensional Points $b(R, z>0, t)$</u>

In order to meet the boundary conditions $\sigma_{rr}=1$ and $\sigma_{rz}=0$ Matrix 2 becomes

$$\begin{bmatrix} \frac{B_2}{2r} & \frac{B_2}{2r} & 1-\alpha\frac{B_2}{r} & 0 & 0 & 0 \\ 0 & 1+\frac{B_5}{2(R+z)} & -\alpha\frac{B_5}{2r} & -1-\frac{\alpha B_5}{2(R+z)} & 0 & 0 \\ 0 & 1 & -\alpha\frac{B_6}{2r} & 1 & 0 & 0 \\ 0 & 0 & -\alpha\frac{B_9}{2r} & 0 & -\frac{B_9}{2} & -\alpha\frac{B_9}{2} \\ 0 & 1 & -\alpha\frac{B_9}{2r} & 0 & -\alpha\frac{B_9}{2} & -\frac{B_9}{2} \\ 1 & 0 & -\frac{B_9}{2r} & 0 & -\alpha\frac{B_9}{2} & -\alpha\frac{B_9}{2} \end{bmatrix} \bullet \begin{bmatrix} \sigma_{\theta\theta} \\ \sigma_{zz} \\ U_r \\ U_z \\ \frac{\partial U_r}{\partial r} \\ \frac{\partial U_z}{\partial z} \end{bmatrix} = \begin{bmatrix} [A_2]-(1-\frac{B_2}{r}) \\ [A_5]+\frac{B_5}{2(R+z)} \\ [A_6] \\ A_9+1 \\ A_{10} \\ A_{12} \end{bmatrix}$$

Matrix 4

c. <u>Regular Free Top Boundary Points c(R<r<Ro,0,t)</u>

After applying the boundary conditions $\sigma_{zz} = \sigma_{rz} = 0$ Matrix 2 becomes

$$
\begin{bmatrix}
1+\frac{B_1}{r} & -\frac{B_1}{2r} & -1-\alpha\frac{B_1}{r} & 0 & 0 & 0 \\
1-\frac{B_2}{r} & \frac{B_2}{2r} & 1-\alpha\frac{B_2}{r} & 0 & 0 & 0 \\
0 & 0 & -\alpha\frac{B_6}{2r} & 1 & 0 & 0 \\
1 & 0 & -\alpha\frac{B_9}{2r} & 0 & -\frac{B_9}{2} & -\alpha\frac{B_9}{2} \\
0 & 0 & -\alpha\frac{B_9}{2r} & 0 & -\alpha\frac{B_9}{2} & -\frac{B_9}{2} \\
0 & 1 & -\frac{B_9}{2r} & 0 & -\alpha\frac{B_9}{2} & -\alpha\frac{B_9}{2}
\end{bmatrix}
\bullet
\begin{bmatrix}
\sigma_{rr} \\ \sigma_{\theta\theta} \\ U_r \\ U_z \\ \frac{\partial U_r}{\partial r} \\ \frac{\partial U_z}{\partial z}
\end{bmatrix}
=
\begin{bmatrix}
[A_1] \\ [A_2] \\ [A_6] \\ A_9 \\ A_{10} \\ A_{12}
\end{bmatrix}
$$

Matrix 5

c'. <u>Reflected Regular Free Top Boundary Points c'($R<r<R_o,0,t$)</u>

We apply the same boundary conditions as the points c to Matrix 3 to obtain

$$
\begin{bmatrix}
1+\frac{B_1}{r} & -\frac{B_1}{2r} & -1-\alpha\frac{B_1}{r} & 0 & 0 & 0 \\
1-\frac{B_2}{r} & \frac{B_2}{2r} & 1-\alpha\frac{B_2}{r} & 0 & 0 & 0 \\
0 & 0 & -\alpha\frac{B_6}{2r} & 1 & 0 & 0 \\
1 & 0 & -\alpha\frac{B_9}{2r} & 0 & -\frac{B_9}{2} & -\alpha\frac{B_9}{2} \\
0 & 0 & -\alpha\frac{B_9}{2r} & 0 & -\alpha\frac{B_9}{2} & -\frac{B_9}{2} \\
0 & 1 & -\frac{B_9}{2r} & 0 & -\alpha\frac{B_9}{2} & -\alpha\frac{B_9}{2}
\end{bmatrix}
\bullet
\begin{bmatrix}
\sigma_{rr} \\ \sigma_{\theta\theta} \\ U_r \\ U_z \\ \frac{\partial U_r}{\partial r} \\ \frac{\partial U_z}{\partial z}
\end{bmatrix}
=
\begin{bmatrix}
[A_1] \\ [A_2] \\ [A_6] \\ A_9 \\ A_{10} \\ A_{12}
\end{bmatrix}
$$

Matrix 6

d. <u>Regular Inner Edge Point d(R,0,z)</u>

When $\sigma_{rr}=1$, $\sigma_{zz}=\sigma_{rz}=0$, Matrix 2 becomes

$$
\begin{bmatrix}
\frac{B_2}{2r} & 1-\alpha\frac{B_2}{r} & 0 & 0 & 0 \\
0 & -\alpha\frac{B_6}{2r} & 1 & 0 & 0 \\
0 & -\alpha\frac{B_9}{2r} & 0 & -\frac{B_9}{2} & -\alpha\frac{B_9}{2} \\
0 & -\alpha\frac{B_9}{2r} & 0 & -\alpha\frac{B_9}{2} & -\frac{B_9}{2} \\
1 & -\frac{B_9}{2r} & 0 & -\alpha\frac{B_9}{2} & -\alpha\frac{B_9}{2}
\end{bmatrix}
\bullet
\begin{bmatrix}
\sigma_{\theta\theta} \\
U_r \\
U_z \\
\frac{\partial U_r}{\partial r} \\
\frac{\partial U_z}{\partial z}
\end{bmatrix}
=
\begin{bmatrix}
[A_2] - (1-\frac{B_2}{r}) \\
[A_6] \\
A_9 - 1 \\
A_{10} \\
A_{12}
\end{bmatrix}
$$

Matrix 7

e. <u>Regular Inner One-Dimensional Points $e(r,\tau)$</u>

Since these points are located in the region of weak one-dimensional motion, the matrix for these points is obtained from Matrix 1 as follows:

$$
\begin{bmatrix}
1+\frac{B_1}{2r} & -\frac{B_1}{2r} & 0 & -1-\alpha\frac{B_1}{2r} & 0 \\
1-\frac{B_2}{2r} & \frac{B_2}{2r} & 0 & 1-\alpha\frac{B_2}{2r} & 0 \\
1 & 0 & 0 & -\alpha\frac{B_9}{2r} & -\frac{B_9}{2} \\
0 & 0 & 1 & -\alpha\frac{B_9}{2r} & -\alpha\frac{B_9}{2} \\
0 & 0 & 0 & -\frac{B_9}{2r} & -\alpha\frac{B_9}{2}
\end{bmatrix}
\bullet
\begin{bmatrix}
\sigma_{rr} \\ \sigma_{\theta\theta} \\ \sigma_{zz} \\ U_r \\ \frac{\partial U_r}{\partial r}
\end{bmatrix}
=
\begin{bmatrix}
A_1 \\ A_2 \\ A_9 \\ A_{10} \\ A_{12}
\end{bmatrix}
$$

Matrix 8

f. Regular Loaded Boundary One-Dimensional Points $f(R,\tau)$

To be compatible with the boundary conditions $\sigma_{rr}=1$, $\sigma_{rz}=0$ Matrix 8 becomes

$$\begin{bmatrix} \frac{B_2}{2r} & 0 & 1-\alpha\frac{B_2}{2r} & 0 \\ 0 & 0 & -\alpha\frac{B_9}{2r} & -\frac{B_9}{2} \\ 0 & 1 & -\alpha\frac{B_9}{2r} & -\alpha\frac{B_9}{2} \\ 1 & 0 & -\frac{B_9}{2r} & -\alpha\frac{B_9}{2} \end{bmatrix} \bullet \begin{bmatrix} \sigma_{\theta\theta} \\ \sigma_{zz} \\ U_r \\ \frac{\partial U_r}{\partial r} \end{bmatrix} = \begin{bmatrix} A_2-(1-\frac{B_2}{2r}) \\ A_9-1 \\ A_{10} \\ A_{11} \end{bmatrix}$$

Matrix 9

g. Leading Inner One-Dimensional Points $g(R+\tau,\tau)$

The geometric decay of the incident leading wave in the medium can be obtained in a closed-form solution. It is a cylindrical longitudinal wave front carrying strong discontinuities. The wave front appears as a line in the r-z plane, this implies that the decay is a function of the independent variable r only. Therefore the characteristic equation(1a) along $dr=d\tau$ is reduced to

$$d[\sigma_{rr}]-d[U_r]=(\alpha[U_r]+[\sigma_{\theta\theta}]-[\sigma_{rr}])\frac{dr}{r}.$$

From the equations (2a) as $d\tau=0$ or from the dynamical conditions for discontinuities across a wave surface[8](p141) we obtain

$$[U_r]=-[\sigma_{rr}]$$

and also the static equations (9), (10), and (12) give us the following expression:

$$[\sigma_{\theta\theta}]=[\sigma_{zz}]=\alpha[\sigma_{rr}].$$

Substituting these relations into the former equation gives us (with $d\tau=dr$) the well-known expression

$$\frac{d[\sigma_{rr}]}{[\sigma_{rr}]}=-\frac{1}{2}\frac{dr}{r} \text{ or } [\sigma_{rr}]=Kr^{-\frac{1}{2}}.$$

When the initial condition $\sigma_{rr}=1$ is applied K becomes unity. Finally we can summarize the quantities of leading one-dimensional points $g(R+\tau,\tau)$ as follows:

$$\sigma_{rr}=r^{-\frac{1}{2}},\ \sigma_{\theta\theta}=\sigma_{rr},\ \sigma_{zz}=\alpha\sigma_{rr},\ \sigma_{rz}=0,$$
$$U_r=-\sigma_{rr},\ U_z=0,\ \frac{\partial U_r}{\partial r}=0.5r^{-\frac{3}{2}} \text{ and } \frac{\partial U_z}{\partial z}=0.$$

g'. <u>Reflected Leading Inner One-Dimensional Points $g'(R_o<R+\tau,\tau)$</u>

This is a cylindrical longitudinal tensile wave which propagates via the bicharacteristic curve $dr=-d\tau$. Therefore we employ the characteristic equation (2a). Since the stresses are irrespective of the z-coordinate the equation (2a) becomes

$$d[\sigma_{rr}]+d[U_r]=(\alpha[U_r]-[\sigma_{\theta\theta}]+[\sigma_{rr}])\frac{d\tau}{r},$$

as $d\tau=0$ in (1a) we obtain

$$[\sigma_{rr}]=[U_r]$$

and from the static equations (9), (10), and (12)

$$[\sigma_{\theta\theta}]=[\sigma_{zz}]=\alpha[\sigma_{rr}].$$

Using these relations (with $d\tau=-dr$) we finally come up with the reverse decay expression as follow:

$$\frac{d[\sigma_{rr}]}{[\sigma_{rr}]}=-\frac{1}{2}\frac{dr}{r} \text{ or } [\sigma_{rr}]=Kr^{-\frac{1}{2}}.$$

This expression is same as that of the incident wave, but we should notice that the value of r decreaces as the reflected wave propagates towards the inner surface. Theis implies that after the reflection at the outer surface the incident divergent wave turns to the reflected convergent wave.

h. and h' Intermediary One-Dimensional Points h.(Ro>R+τ-Δτ,τ) and h'(Ro<R+τ-Δτ,τ)

Although Matrix 8 is valid for points h and h', B_2 should be replaced by Δτ/2 for h and B should be replaced by Δτ/2 for h'. B_1 for both hand h' is replaced by Δτ.

i. Leading Free Top Surface Points i(R+τ,0,τ)

These leading points are sheared by the incident wave and all longitudinal spherical waves reflected from the free top surface. Those waves on the points i are characterized by the bicharacteristic curve dr=dτ. Therefore we use the characteristic equation (1b) and by equations (2b), (9), and (12) as dτ=0, we have $[U_r]=-[\sigma_{rr}]$ and $[\sigma_{\theta\theta}]=\alpha[\sigma_{rr}]$, and (1b) becomes $2d[\sigma_{rr}]=-((2+\alpha)[\sigma_{rr}]-[\sigma_{zz}])\frac{dr}{r}$. But the boundary condition is $[\sigma_{zz}]=0$, thus the decay is given by

$$\frac{d[\sigma_{rr}]}{[\sigma_{rr}]}=-(1+\frac{\alpha}{2}) \quad \text{or} \quad [\sigma_{rr}]=r^{-(1+\frac{\alpha}{2})},$$

and in summary

$$\sigma_{rr}=r^{-(1+\frac{\alpha}{2})}, \quad \sigma_{\theta\theta}=\alpha\sigma_{rr}, \quad \sigma_{rz}=0, \quad U_r=-\sigma_{rr} \quad U_z=\alpha r^{-\frac{1}{2}},$$

$$\frac{\partial U_r}{\partial r}=(1+\frac{\alpha}{2})r^{-(2+\frac{\alpha}{2})}, \text{ and } \frac{\partial U_z}{\partial z}=0.$$

The particles at points i are subject to the compressive stress component σ_{zz} which refers to the inner leading one-dimensional points g. Since $[\sigma_{zz}]=-[U_z]$ (from (6b) as $d\tau=0$), U_z at points i is equal to $\sigma_{zzg}=\sigma_{rrg}=\alpha r^{-\frac{1}{2}}$.

i'. Reflected Leading Free Top Surface Points i'($R_0<R+\tau,\tau$)

The reflected leading wave front is propagating towards the inner surface via the bicharacteristic curve $dr=-d\tau$. We substitute $[U_r]=[\sigma_{rr}]$ and $[\sigma_{\theta\theta}]=\alpha[\sigma_{rr}]$ that come from (1b), (9), and (12) as $d\tau=0$ into the characteristic equation (2b) to obtain $2d[\sigma_{rr}]=((2+\alpha)[\sigma_{rr}]-[\sigma_{zz}])\frac{dr}{r}$. But the free suface condition leads us to (with $d\tau=-dr$)

$$\frac{d[\sigma_{rr}]}{[\sigma_{rr}]}=-(1+\frac{\alpha}{2})\frac{dr}{r} \quad \text{or} \quad [\sigma_{rr}]=r^{-(1+\frac{\alpha}{2})},$$

and in summary

$$\sigma_{rr}=r^{-(1+\frac{\alpha}{2})}, \quad \sigma_{\theta\theta}=\alpha\sigma_{rr}, \quad \sigma_{zz}=0, \quad U_r=\sigma_{rr}, \quad U_z=\alpha r^{-\frac{1}{2}},$$
$$\frac{\partial U_r}{\partial r}=-(1+\frac{\alpha}{2})r^{-(2+\frac{\alpha}{2})}, \text{ and } \frac{\partial U_z}{\partial z}=0.5\alpha r^{-\frac{3}{2}}.$$

The particles at points i' are subjected to the tensile stress component σ_{zz} which refers to the reflected inner leading one-dimensional points g'. Since $[\sigma_{zz}]=-[U_z]$ (from (6b) as $d\tau=0$), U at points i' becomes $\sigma_{zzg'}=\sigma_{rrg'}=\alpha r^{-\frac{1}{2}}$.

j. Leading Two-Dimensional Loaded Boundary Points j(R,τ,τ)

We can refer to Reference[6] for the calculation of these points. The dependent variables are integrated by the following matrix:

$$
\begin{bmatrix}
0 & 1+\frac{B_5}{2(R+z)} & -1-\frac{\alpha B_5}{2(R+z)} & 0 & 0 \\
0 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & -\frac{B_9}{2} & -\alpha\frac{B_9}{2} \\
0 & 1 & 0 & -\alpha\frac{B_9}{2} & -\frac{B_9}{2} \\
1 & 0 & 0 & -\alpha\frac{B_9}{2} & -\alpha\frac{B_9}{2}
\end{bmatrix}
\bullet
\begin{bmatrix}
\sigma_{\theta\theta} \\
\sigma_{zz} \\
U_z \\
\frac{\partial U_r}{\partial r} \\
\frac{\partial U_z}{\partial z}
\end{bmatrix}
=
\begin{bmatrix}
[A_5] + \frac{B_5}{2}\left(\frac{\sigma_{rr}}{R+z}+\alpha\frac{U_r}{r}\right)_j \\
[A_6] + \frac{\alpha B_6}{2r}U_{rj} \\
A_9 - \sigma_{rr} + \frac{\alpha B_9}{2r}U_{rj} \\
A_{10} + \frac{\alpha B_9}{2r}U_{rj} \\
A_{12} + \frac{B_9}{2r}U_{rj}
\end{bmatrix}
$$

Matrix 10

k. <u>Leading Inner Two-Dimensional Points $k(r, z > \tau + R - r, \tau)$</u>

These points are not compatible with the orthogonal scheme for the characteristic equations. Points k, however, lie on the reflected spherical wave which passes through two known points so that we can apply a linear interpolation between these two points which are known from previous calculations. One of the known points is a point i and the other is possibly a point ℓ. If the latter point does not coincide with a point ℓ and appears between two points, another interpolation between the two ℓ points is necessary. All calculations of these interpolations are carried out in the same manner as in reference[6].

k'. <u>Reflected Leading Inner Two-Dimensional Points $k'(r, z > R + \tau - 2R_0 + r, \tau)$</u>

According to Fiure 6 we can apply a similar interpolation scheme to evaluate stresses and particle velocities at these points k'. Two known points from which a point k' is calculated by interpolation exist in the τ-constant plane. One is a point i' and the other is possibly a point ℓ'. If the latter point does not coincide with a point ℓ' which is a known grid point of integration, it is calculated by another interpolation that can refer to [6].

ℓ. Intermediary Two-Dimensional Points $\ell(r,R+\tau-r,\tau)$

These points are located on the reflected logitudinal cones and directly on the integration grid points. But these points do not comply with the orthogonal mesh configulation, since the bicharacteristic curves drawn from one of these points must terminate on the preceding reflected longitudinal cone $(r-\Delta\tau,0,r-R+\Delta\tau)$. It should be noted that every reflected longitudinal wave generated by the incident wave on its free top surface carries new strong discontinuities. Accordingly the integration paths must be drawn from the preceding reflected cone other than ordinary grid points. The quantities at points are evaluated by Matrix 2. The details of the calculations for B_i and $[A_i]$ are presented in reference[6].

ℓ'. Reflected Intermediary Two-Dimensional Points $\ell'(r,R+\tau-2R+r,\tau)$

Although the similar calculation scheme for B_i and $[A_i]$ can be applied to these reflected points, the governing matrix is Matrix 3 due to the reflected shear wave.

m. and o. <u>Outer boundary Two-Dimensional Points m(R ,z>R+τ-Ro,τ), o(R ,R+τ-Ro,τ)</u>

After applying the boundary conditions $\sigma_{rr} = \sigma_{rz} = 0$ Matrix 2 becomes

$$
\begin{bmatrix}
-\frac{B_1}{2r} & 0 & -1-\alpha\frac{B_1}{r} & 0 & 0 & 0 \\
0 & \frac{B_5}{2r} & -\alpha\frac{B_5}{2r} & -1-\frac{\alpha B_5}{2(R+z)} & 0 & 0 \\
0 & -\frac{B_6}{2r} & -\alpha\frac{B_6}{2r} & 1 & 0 & 0 \\
0 & 0 & -\alpha\frac{B_9}{2r} & 0 & -\frac{B_9}{2} & -\alpha\frac{B_9}{2} \\
0 & 0 & -\alpha\frac{B_9}{2r} & 0 & -\alpha\frac{B_9}{2} & -\frac{B_9}{2} \\
1 & 0 & -\frac{B_9}{2r} & 0 & -\alpha\frac{B_9}{2} & -\alpha\frac{B_9}{2}
\end{bmatrix}
\bullet
\begin{bmatrix}
\sigma_{\theta\theta} \\ \sigma_{zz} \\ U_r \\ U_z \\ \frac{\partial U_r}{\partial r} \\ \frac{\partial U_z}{\partial z}
\end{bmatrix}
=
\begin{bmatrix}
[A_1] \\ [A_5] \\ [A_6] \\ A_9 \\ A_{10} \\ A_{12}
\end{bmatrix}
$$

Matrix 11

The evaluations of B_i and $[A_i]$ at these points m and o can be done by the same methods as those for the points k and ℓ respectively.

n. <u>Regular Outer Corner Point n(Ro,0,τ)</u>

When $\sigma_{rr} = \sigma_{rz} = 0$ Matrix 2 becomes

$$
\begin{bmatrix}
-\frac{B_1}{2r} & -1-\alpha\frac{B_1}{r} & 0 & 0 & 0 \\
0 & -\alpha\frac{B_6}{2r} & 1 & 0 & 0 \\
0 & -\alpha\frac{B_9}{2r} & 0 & -\frac{B_9}{2} & -\alpha\frac{B_9}{2} \\
0 & -\alpha\frac{B_9}{2r} & 0 & -\alpha\frac{B_9}{2} & -\frac{B_9}{2} \\
1 & -\frac{B_9}{2r} & 0 & -\alpha\frac{B_9}{2} & -\alpha\frac{B_9}{2}
\end{bmatrix}
\bullet
\begin{bmatrix}
\sigma_{\theta\theta} \\ U_r \\ U_z \\ \frac{\partial U_r}{\partial r} \\ \frac{\partial U_z}{\partial z}
\end{bmatrix}
=
\begin{bmatrix}
[A_1] \\ [A_6] \\ A_9 \\ A_{10} \\ A_{12}
\end{bmatrix}
$$

Matrix 12

p. <u>Regular One-Dimensional outer Surface Points p(Ro,t)</u>

When $\sigma_{rr} = \sigma_{rz} = 0$ Matrix 8 becomes

$$
\begin{bmatrix}
-\frac{B_1}{2r} & 0 & -1-\alpha\frac{B_1}{r} & 0 \\
0 & 0 & -\alpha\frac{B_9}{2r} & -\frac{B_9}{2} \\
0 & 1 & -\alpha\frac{B_9}{2r} & -\alpha\frac{B_9}{2} \\
1 & 0 & -\frac{B_9}{2r} & -\alpha\frac{B_9}{2}
\end{bmatrix}
\bullet
\begin{bmatrix}
\sigma_{\theta\theta} \\ \sigma_{zz} \\ U_r \\ \frac{\partial U_r}{\partial r}
\end{bmatrix}
=
\begin{bmatrix}
[A_1] \\ A_9 \\ A_{10} \\ A_{12}
\end{bmatrix}
$$

Matrix 13

CHAPTER IV

COMPUTATIONAL METHODS FOR CASE 2, CASE 3 AND CASE4

Boundary and Initial Conditions in case 2

The geometry and the loading for CASE 2 are shown in Figure 1 and discussed in detail in Chapter I. The shell considered here is infinitely long and then it is obvious that the shell does not have a free top surface at z=0 which in CASE 1 generates reflected two-dimensional longitudinal waves. However all discussion in CASE 1 on the characteristic formuration and the computational method except on the application of the boundary conditions must hold for CASE 2. In other words we can assume that spherical longitudinal waves reflect from the plane z=0, accordingly the characteristic formuration and all computational procedures in CASE 1 are valid for CASE 2 with the exception of the application of the following conditions at the plane z=0.

The following boundary conditions are prescribed when both the intensity of σ_{rr}-input and the inner radius of the shell are considered to be unity.

The boundary conditions for point (R,0,τ) are given as $\sigma_{rr}=1$ and $\sigma_{rz}=0$. The initial conditions for point (R,0,0) are given as $U_r=-1$, $\sigma_{\theta\theta}=\sigma_{zz}=a\sigma_{rr}$, and $\partial U_z/\partial z=-\frac{\alpha(1+a)}{2}$, $\partial U_r/\partial r=1$ as will be shown later.

The boundary conditions for points $(R, z<0, z)$ are $\sigma_{rr}=\sigma_{rz}=0$. The initial conditions for those points are $\sigma_{\theta\theta}=\sigma_{zz}=U_r=U_z=0$.

The boundary and initial conditions for loaded boundary points $(R, z>0)$ and outer surface points $(R_o, -\infty<z<+\infty)$ are the same as those in CASE 1.

Recurring Points in CASE 2

Figure 6 shows the four regions of influence and twenty five recurring points in the r-z plane. The quantities at all recurring points except points c, d, i, i', n, and q can be evaluated by the same procedure as in CASE 1. The quantities at the recurring points where we can not apply the same computational method as in CASE 1 are computed in the following way:

c. Regular Two-Dimensional Outer surface Points $C(R-\tau+R_o<z<R+\tau-R_o, \tau)$

When $\sigma_{rr}=\sigma_{rz}=0$ Matrix 2 becomes

$$\begin{bmatrix} -\frac{B_1}{2r} & -\frac{B_1}{2r} & -1-\alpha\frac{B_1}{r} & 0 & 0 & 0 \\ 0 & 1+\frac{B_5}{2(R+z)} & -\alpha\frac{B_5}{2r} & -1-\frac{\alpha B_5}{2(R+z)} & 0 & 0 \\ 0 & 1 & -\alpha\frac{B_6}{2r} & 1 & 0 & 0 \\ 0 & 0 & -\alpha\frac{B_9}{2r} & 0 & -\frac{B_9}{2} & -\alpha\frac{B_9}{2} \\ 0 & 1 & -\alpha\frac{B_9}{2r} & 0 & -\alpha\frac{B_9}{2} & -\frac{B_9}{2} \\ 1 & 0 & -\frac{B_9}{2r} & 0 & -\alpha\frac{B_9}{2} & -\alpha\frac{B_9}{2} \end{bmatrix} \bullet \begin{bmatrix} \sigma_{\theta\theta} \\ \sigma_{zz} \\ U_r \\ U_z \\ \frac{\partial U_r}{\partial r} \\ \frac{\partial U_z}{\partial z} \end{bmatrix} = \begin{bmatrix} [A_1] \\ [A_5] \\ [A_6] \\ A_9 \\ A_{10} \\ A_{12} \end{bmatrix}$$

Matrix 14

d. Regular Two-Dimensional Edge Point d(R,O,z)

When $\sigma_{rr}=1$, $\sigma_{rz}=0$ Matrix 2 becomes

$$
\begin{bmatrix}
\frac{B_2}{2r} & \frac{B_2}{2r} & 1-\alpha\frac{B_2}{r} & 0 & 0 & 0 \\
0 & 1+\frac{B_5}{2(R+z)} & -\alpha\frac{B_5}{2r} & -1-\frac{\alpha B_5}{2(R+z)} & 0 & 0 \\
0 & 1 & -\alpha\frac{B_6}{2r} & 1 & 0 & 0 \\
0 & 0 & -\alpha\frac{B_9}{2r} & 0 & -\frac{B_9}{2} & -\alpha\frac{B_9}{2} \\
0 & 1 & -\alpha\frac{B_9}{2r} & 0 & -\alpha\frac{B_9}{2} & -\frac{B_9}{2} \\
1 & 0 & -\frac{B_9}{2r} & 0 & -\alpha\frac{B_9}{2} & -\alpha\frac{B_9}{2}
\end{bmatrix}
\bullet
\begin{bmatrix}
\sigma_{\theta\theta} \\ \sigma_{zz} \\ U_r \\ U_z \\ \frac{\partial U_r}{\partial r} \\ \frac{\partial U_z}{\partial z}
\end{bmatrix}
=
\begin{bmatrix}
[A_2]-(1-\frac{B_2}{r}) \\ [A_5]+\frac{B_5}{2(R+z)} \\ [A_6] \\ A_9-1 \\ A_{10} \\ A_{12}
\end{bmatrix}
$$

Matrix 15

i. Leading Two-Dimensional Incident Points $i(R+\tau,0,\tau)$

The leading wave front at points i carries strong discontinuities via the bicharacteristic curve $dr=d\tau$. Therefore we employ the characteristic equation (1b)

$$d[\sigma_{rr}]-d[U_r]=(2\alpha[U_r]-2[\sigma_{rr}]+[\sigma_{\theta\theta}]+[\sigma_{zz}])\frac{d\tau}{r},$$

substituting the dynamical condition for wave surface $[\sigma_{rr}]=-[U_r]$ (or from (2b) as $d\tau=0$) to obtain

$$d[\sigma_{rr}]=-(\alpha+1)[\sigma_{rr}]+\frac{1}{2}([\sigma_{\theta\theta}]+[\sigma_{zz}])\frac{d\tau}{r},$$

and from the static equations (9), (10), and (12), $[\sigma_{\theta\theta}]=[\sigma_{zz}]=\alpha[\sigma_{rr}]$. We finally come up with (as $d\tau=dr$)

$$\frac{d[\sigma_{rr}]}{[\sigma_{rr}]}=-\frac{dr}{r} \quad \text{or} \quad [\sigma_{rr}]=Kr^{-1}.$$

Since $\sigma_{rr}=1$ at (R,0,0) K is taken to be unity. This is the well-known expression for the geometric decay of the spherical incident leading wave. All quantities for points i are obtained as the following closed-form solution:

$$\sigma_{rr}=r^{-1},\ \sigma_{\theta\theta}=\alpha\sigma_{rr},\ \sigma_{zz}=\alpha\sigma_{rr},$$
$$\sigma_{rz}=0,\ U_r=-\sigma_{rr},\ U_z=-\sigma_{zz},\ \frac{\partial U_r}{\partial r}=r^{-2},\ \text{and}\ \frac{\partial U_z}{\partial z}=0.$$

i'. Reflected Leading Two-Dimensional Points i'(R<r<R+τ,0,τ)

The reflected leading wave propagates along the bicharacteristic curve dr=-dτ. Therefore we take the characteristic equation (2b) to determine the decay of the reflected spherical wave,

$$d[\sigma_{rr}]+d[U_r]=(2\alpha[U_r]+2[\sigma_{rr}]-[\sigma_{\theta\theta}]-[\sigma_{zz}])\frac{d\tau}{r}.$$

From (1b) as dτ=0 we have $[\sigma_{rr}]=[U_r]$ and from the static equations $\sigma_{\theta\theta}=\sigma_{zz}=\alpha\sigma_{rr}$. We substitute these relations into (2b) to obtain (as dτ=-dr)

$$\frac{d[\sigma_{rr}]}{[\sigma_{rr}]}=-\frac{dr}{r}\ \text{or}\ [\sigma_{rr}]=Kr^{-1}.$$

This reverse decay expression implies the convergent spherical wave. In summary the closed-form solutions are as follows:

$$\sigma_{rr}=r,\ \sigma_{\theta\theta}=\alpha\sigma_{rr},\ \sigma_{zz}=\alpha\sigma_{rr},$$
$$\sigma_{rz}=0,\ U_r=\sigma_{rr},\ U_z=\sigma_{zz},\ \frac{\partial U_r}{\partial r}=-r^{-2},\ \text{and}\ \frac{\partial U_z}{\partial z}=0.$$

n. Regular Inner Surface Points n(0,-τ<z<0,τ)

When $\sigma_{rr}=\sigma_{rz}=0$ Matrix 2 becomes

$$
\begin{bmatrix}
\frac{B_2}{2r} & \frac{B_2}{2r} & 1-\alpha\frac{B_2}{r} & 0 & 0 & 0 \\
0 & 1+\frac{B_5}{2(R+z)} & -\alpha\frac{B_5}{2r} & -1-\frac{\alpha B_5}{2(R+z)} & 0 & 0 \\
0 & 1 & -\alpha\frac{B_6}{2r} & 1 & 0 & 0 \\
0 & 0 & -\alpha\frac{B_9}{2r} & 0 & -\frac{B_9}{2} & -\alpha\frac{B_9}{2} \\
0 & 1 & -\alpha\frac{B_9}{2r} & 0 & -\alpha\frac{B_9}{2} & -\frac{B_9}{2} \\
1 & 0 & -\frac{B_9}{2r} & 0 & -\alpha\frac{B_9}{2} & -\alpha\frac{B_9}{2}
\end{bmatrix}
\bullet
\begin{bmatrix}
\sigma_{\theta\theta} \\ \sigma_{zz} \\ U_r \\ U_z \\ \frac{\partial U_r}{\partial r} \\ \frac{\partial U_z}{\partial z}
\end{bmatrix}
=
\begin{bmatrix}
[A_2] \\ [A_5] \\ [A_6] \\ A_9 \\ A_{10} \\ A_{12}
\end{bmatrix}
$$

Matrix 16

q. <u>Leading Inner-Surface Points $q(R,-\tau,\tau)$</u>

If the boundary conditions for points j are set to the same value as those for points q, the closed-form solution of σ_{zz} for points j is equal to that for points q. It follows that in order to calculate the decay of σ_{zz} at points q we can employ the characteristic equation (5b) along the bicharacteristic curve dz=d which characterizes the leading spherical wave front at points j under the boundary conditions $\sigma_{rr}=\sigma_{rz}=0$. The characteristic equation (5b) with $\sigma_{rr}=\sigma_{rz}=0$ is written as

$$d[\sigma_{zz}]-d[U_z]=(\alpha[U_z]-[\sigma_{zz}])\frac{d\tau}{R+z}.$$

From the dynamical condition or (6b) as $d\tau=0$ we have $[\sigma_{zz}]=-[U_z]$, then the equation becomes

$$2d[\sigma_{zz}]=-(1+\alpha)[\sigma_{zz}]\frac{d\tau}{R+z},$$

as $dr=dz$ we obtain

$$\frac{d[\sigma_{zz}]}{[\sigma_{zz}]}=-\frac{1+\alpha}{2}\frac{dz}{(R+z)} \quad \text{or} \quad [\sigma_{zz}]=K(R+z)^{-\frac{1+\alpha}{2}}.$$

Since $\sigma_{zz}=\alpha$ at (R,0,0) and if R=1, K becomes α. It follows that

$$[\sigma_{zz}]=\alpha(1+z)^{-\frac{1+\alpha}{2}}.$$

Finally the quantities for points q are obtained as the following closed-form solution:

$$\sigma_{rr}=0,\ \sigma_{zz}=\alpha(1+|z|)^{-\frac{1+\alpha}{2}},\ \sigma_{\theta\theta}=\sigma_{zz},$$

$$\sigma_{rz}=0,\ U_r=0,\ U_z=-\sigma_{zz},\ \frac{\partial U_r}{\partial r}=0,\ \text{and}\ \frac{\partial U_z}{\partial z}=\frac{\alpha(1+\alpha)}{2}(1+z)^{-\frac{3+\alpha}{2}}.$$

Superposition of Shifted Loads for CASE 3

Our aim here is to obtain the transient stress distribution in an infinitely long thick cylindrical shell subject to an impulsive constant load of finite length. It is obtained by applying the principle of superposition since all action is linear elastic.

We now consider two cases of loading that can be readily obtaind from the previous calculation for CASE 2. One of those loading cases is, as shown in Figure 6, that its semi-infinite load along the inner surface shifts upwards(in the negative direction of z) by a half of the load width. The other is that the semi-infinite load shifts downwards also by a half of the load width and the load is applied in the reverse direction, namely the input stress is changed to tensile stress while the stress intensity remains unity ($\sigma_{rr}=-1$). After the superposition we obtain the stress distribution for a case of a finite-width load whose center line lies on the r-axis. In the practical computation, however, we have to shift the load of the latter case one grid point lower than a half of the load width as shown in Figure 6.

Consecutive superposition Scheme for CASE 4

Up to here we considered only step constant input shown in Figure 8(a) for CASE 1, CASE 2, and CASE 3. However, in order to obtain a stress distribution of the travelling load case(CASE 4) rectangular inputs which have various duration of loading time must be considered. A rectangular σ_{rr} input can be obtained by superimposing a negative unit step input whose striking time is shifted to τ on a unit step input as shown in Figure 8. Once a stress distribution of case 3 due to a rectangular input load which has certain duration of loading is obtained, we can compute the stress distribution of the travelling load case by means of the consecutive superposition of the previous result. The speed of the travelling load can be controlled by the duration of loading time in CASE 3.

In view of Figure 9 it is seen that a load of finite length is applied at $\tau=0$ as shown in (a) and then at $\tau=\tau_a$ the load is removed, however at the same time another load which is shifted downwards by one grid point distance is applied and again at $\tau=\tau_b$ the load is removed when another shifted load is applied. This process is repeated until the incident wave due to the first loading reaches the inner surface after reflecting at the outer surface. The stress distribution at $\tau=\tau_a$

is obtained by superimposing the stresses due to the first loading at $\tau=\tau_a$ on the stresses due to the second load applied at $\tau=\tau_a$. Thus what we obtained by superposition is an accumulated stress distribution due to many rectangular input loads that account for a travelling load of finite length.

CHAPTER V

RESULTANT MOTION IN THICK SHELL

Computer programs

The transient motion in a thick shell was obtained with aid of CDC CYBER computer. Five separate programs are utilized to obtain the stress distribution of the present boundary value problems. The first and second programs, TRES1 and TRES2 are an extension of the coputer code CHAR2DZ developed by M. Ziv[6]. Next three programs, RECTINP, TRES3, and TRES4 were developed during the couse of this work to culminate the computation of the transient motion in a thick shell subject to an impulsive travelling load of finite length on the interior. Also another program PLOTALL is utilized to plot the time history of the stresses and the particle velocities which are computed by the above five programs. These programs are written in FORTRAN IV language.

The program TRES1 determines the stress distribution of CASE 1 in which the load is applied as a step input, so the duration of loading is permanent. The output of TRES1 stored on TAPE1 can be plotted on the CALCOMP plotting machine by using the program PLOTALL.

The second program TRES2 is used to determine the stress distribution of CASE 2 in which the duration of loading is permanent. The output is stored on TAPE5 which will be retrieved separately by the programs, RECTINP, TRES3, and PLOTALL for their computation and plotting purposes. RECTIMP is a program to compute various cases of a rectangular input load for CASE 2. This program retrieves TAPE5 and stores its output on TAPE6.

The program TRES3 carry out the calculation of the superposition scheme for CASE 3. This program retrieves TAPE5 obtained by TRES2 or RECTIMP. Since the program RECTIMP stores its output on TAPE6, before attempting to run TRES3 of a rectangular input loading case TAPE6 should be renamed as TAPE5 which is compatible with the format in TRES3.

The last program TRES4 determines the stress distribution of a travelling load case (CASE 4). TAPE10 obtained by TRES3 is retrieved by TRES4 which will store the output on TAPE15.

All program listings are presented in Appendix A-F.

Recorded Plots

Nine groups of plots were obtaind as shown in Figures 10-63 which represent the time history of the stresses and particle velocities for all cases. The following data common to all cases are prescribed during the computation: inner radius of the shell R=1.00, outer radius R_o=1.30, Poisson's ratio ν=0.15, intensity of input σ_{rr}=1.0, step size of integration $\Delta\tau$=0.02, and number of time steps is 30.

The first group of plots shows the time history of the stresses and the particle velocities of CASE 1 at six detected points. Observing Figure 14 one can see that three jumps occur during the entire time of record. Those jumps are due to the arrivals of the incident longitudinal wave, the first refrected longitudinal wave from (1.0,0,0), and the refrected wave from the outer surface in the order of arrival time.

The next three groups of plots Figure 16 through 33 show the time history of the stresses and the particle velocities of CASE 2. Three cases of the duration of loading are computed for CASE 2, that are 0.04, 0.08, and permanent cases. It should be noticed that in Figure 23 or 29 of the rectangular input cases a high tensile stress wave which is due to the reflected logitudinal wave from the outer free surface passes through the points located around the half way of width of the shell. If the duration of loading is permanent a reflected tensile stress wave will be cancelled by a compression wave issued from the loaded boundary.

The next three groups of plots are recorded for CASE 3 and the last two groups of plots are obtained for CASE 4 in which speeds of the travelling load are 0.5 and 1.0. Observing Figure 52 through 63 one can see that in those plots many jumps in the stresses and the particle velocities occur during the entire time of record. This is because that many discontinuous wave fronts due to the loading and unloading process exist in the solution domain. For example in Figure 63 the first jump occurs at τ=0.08 and after that a jump is observed step by step until the last time step, since, in this case, the speed of the travelling load is 1.0 which implies that the travelling load is moving step by step.

Thus, displacement and strain components at any point in the shell may be evaluated in terms of dimensionless valuables from the computer program presented.

REFERENCES

[1] Chou, P.C. and Koenig, H.A.
"A Unified Approach to Cylindrical and Spherical Elastic Waves by the Method of Characteristics", Journal of Applied Mechanics, ASME Transactions, Vol. 33 NO 1 p159-p168, 1966

[2] Thomas, T.Y.
"The General Theory of Compatiblity Conditions", International Journal of Engineering Science, Vol. 4 p207-p233, 1966

[3] Ziv, M.
"Two Spatial Dimensional Elastic Wave Propagation by the Theory of Characteristics", International Journal of Solids and Structures, Vol. 5 p1135-p1151, 1969

[4] Ziv, M.
"The Decay of Leading Elastic Waves by the Theory of Characteristics", International Journal of Engineering Science, Vol. 8 p483-p497, 1970

[5] Ziv, M.
"Multi-Dimensional Wave Propagation in Solid due to Impact Loading by the Method of Characteristics", T.A.E Report NO 178, Technion - Israel Institute of Technology, 1975

[6] Ziv, M.

"General Transient Solution Method for the Entire Sudden Deformation Domain of Two-Dimensional Solids", T.A.E. Report NO 204, Technion - Israel Institute of Technology, 1974

"Transient Response of an Elastic Half Space to an Enbedded Cylindrical Load by a Two-Spatial Characteristic Computer Code", T.A.E. Report NO 250, Technion - Israel Institute of Technology, 1975

[7] Rinehart, J.S.

"Stress Transients in Solids", HyperDynamics, 1975

[8] Achenbach, J.D.

"Wave Propagation in Elastic Solids", North-Holland, 1973

APPENDIX A

FIGURES IN CHAPTERS I-IV

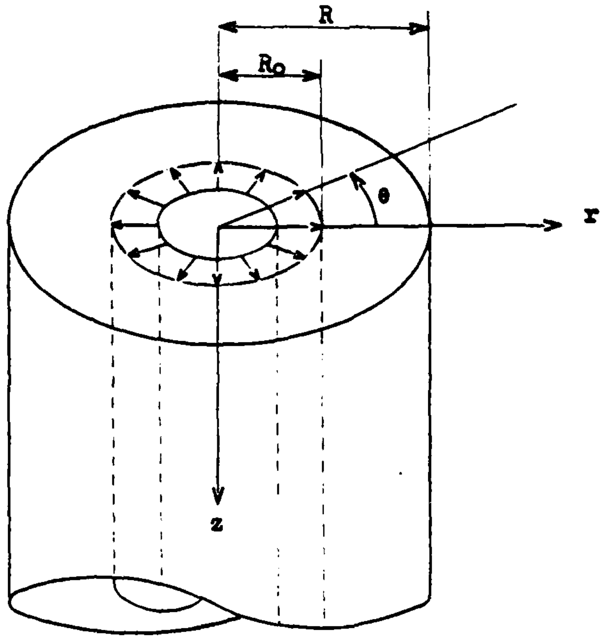


Figure 1. Geometry, loading, and coordinate system for CASE 1.

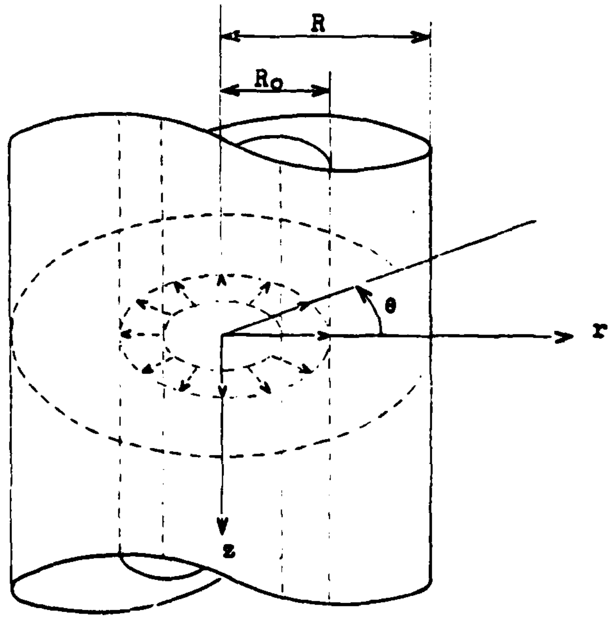


Figure 2. Geometry, loading, and coordinate system for CASE 2.

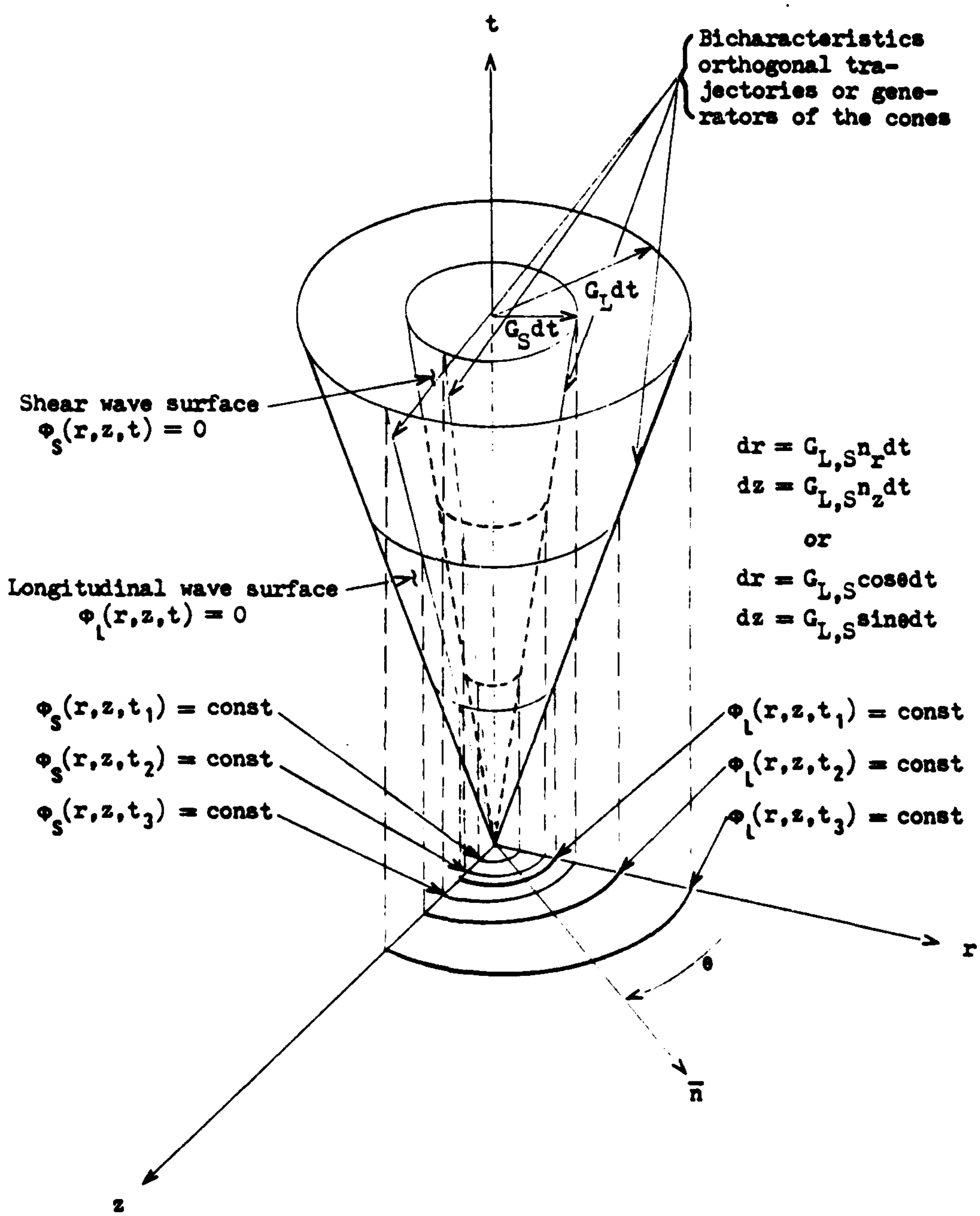


Figure 3. Longitudinal and shear wave cones and the bicharacteristic curves.

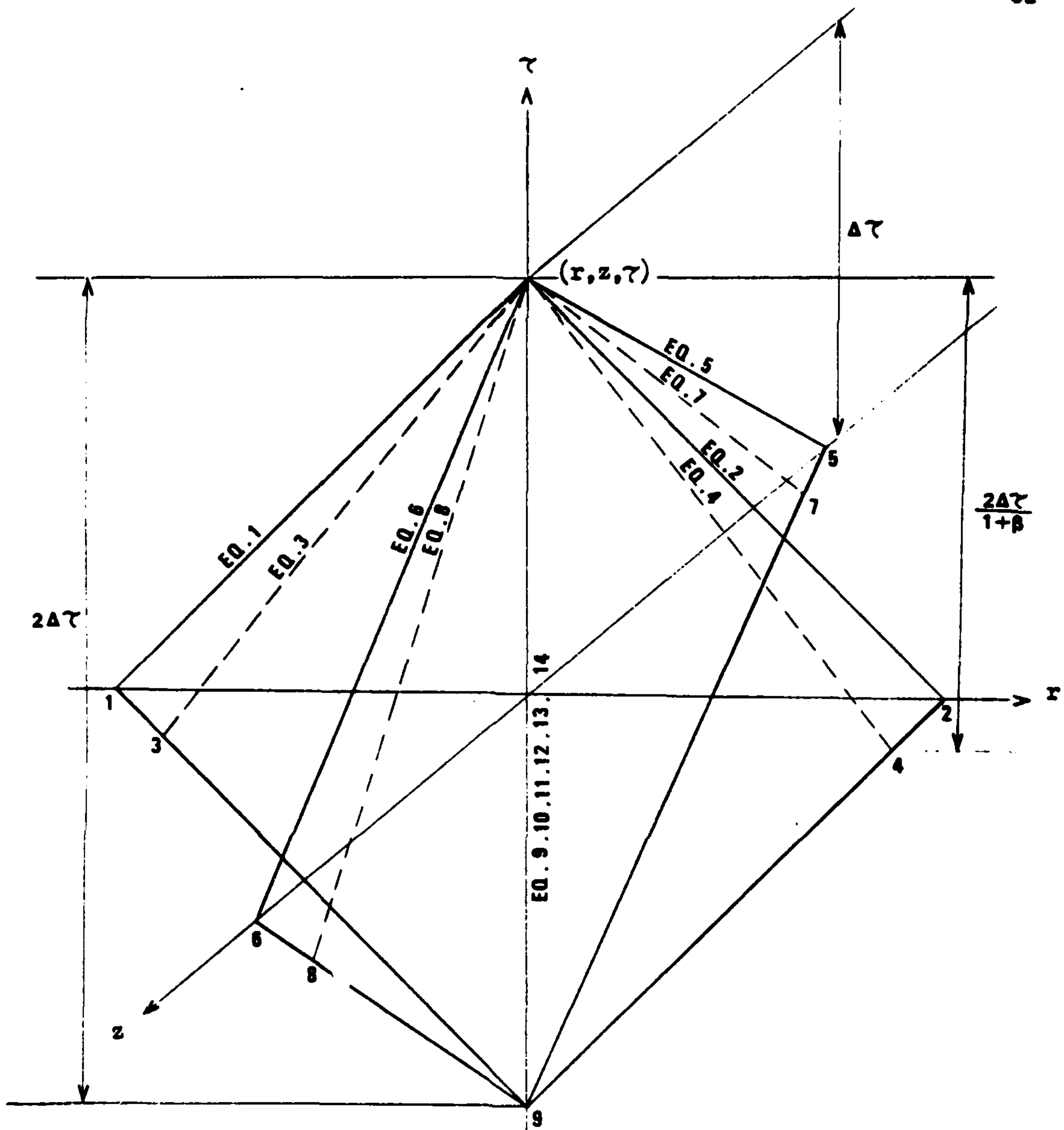


Figure 4. Orthogonal symmetric mesh element in the r,z,τ space.

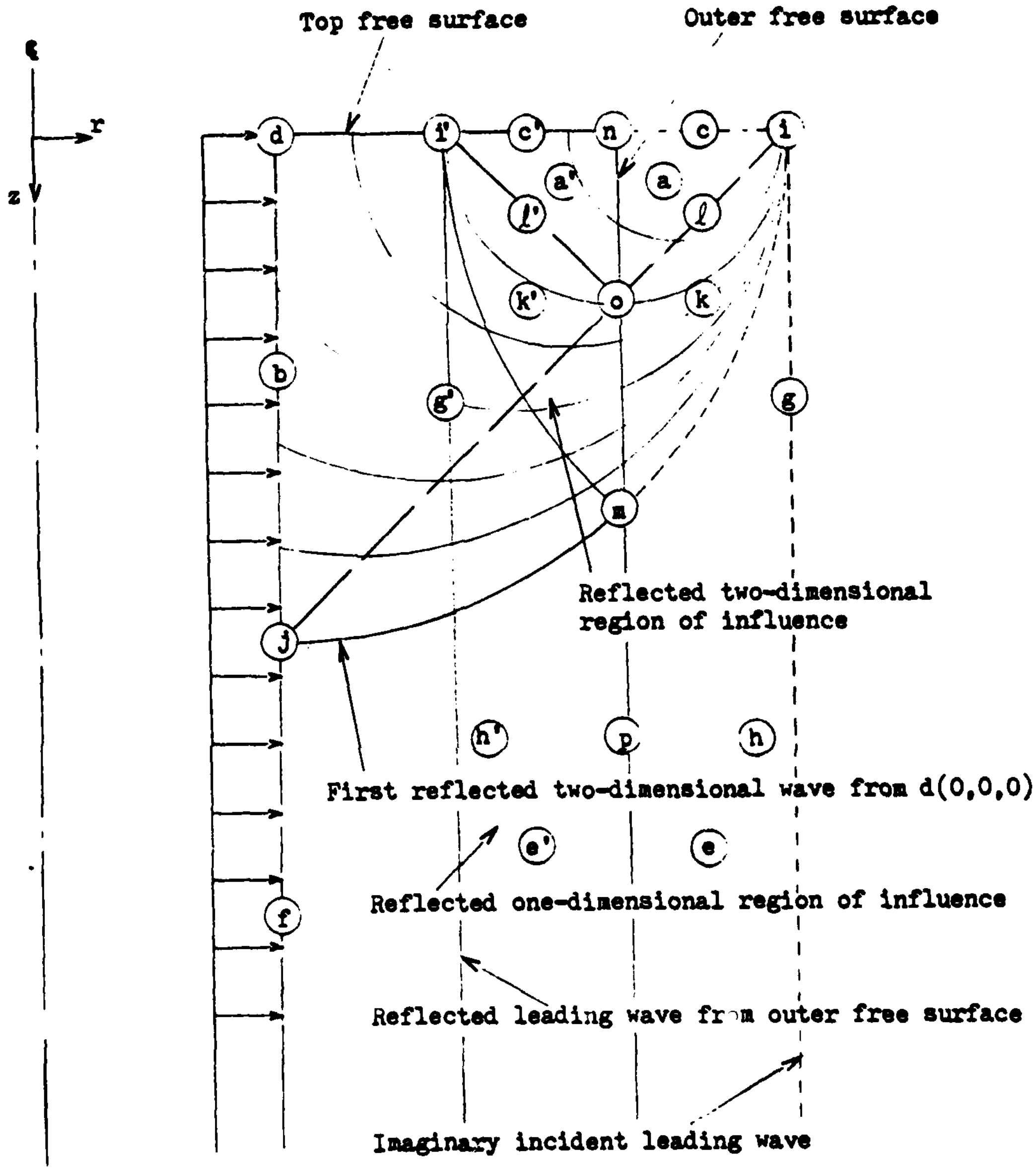


Figure 5. Influence regions and twenty four recurring points in CASE 1.

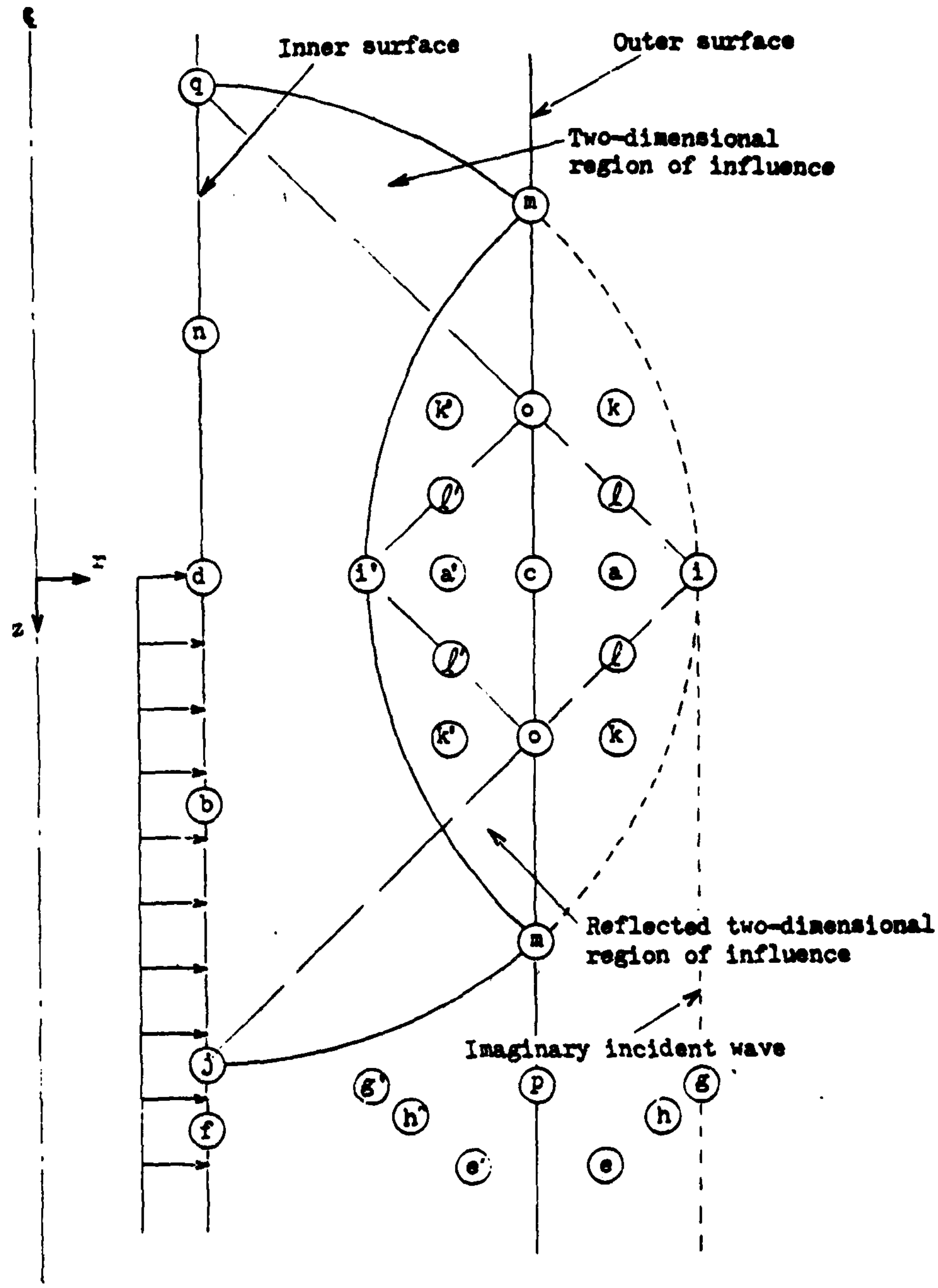


Figure 6. Influence regions and twenty five recurring points in CASE 2.

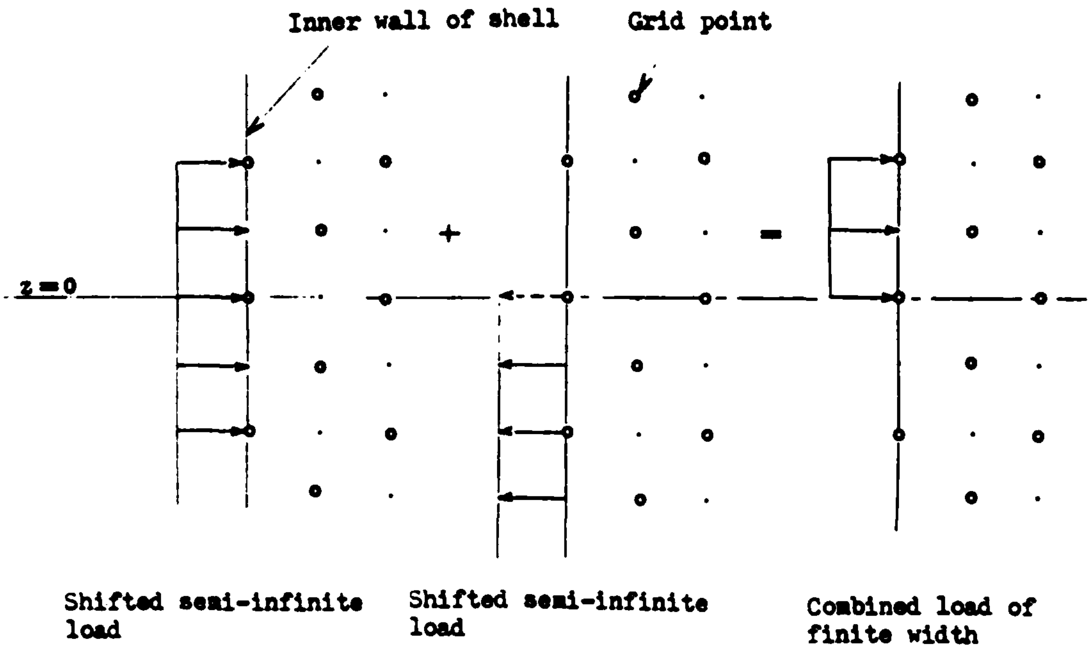


Figure 7. Superposition scheme of shifted loads for CASE 3.

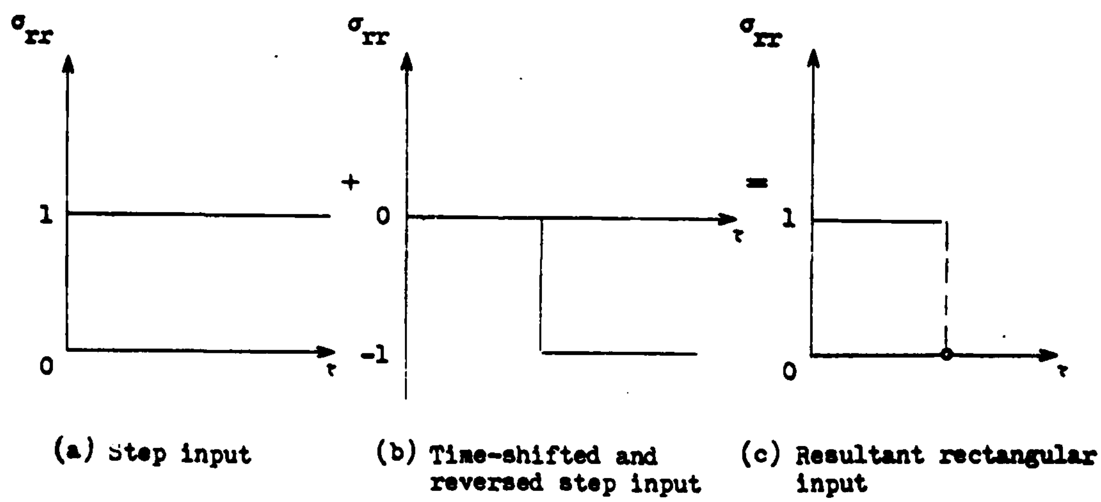
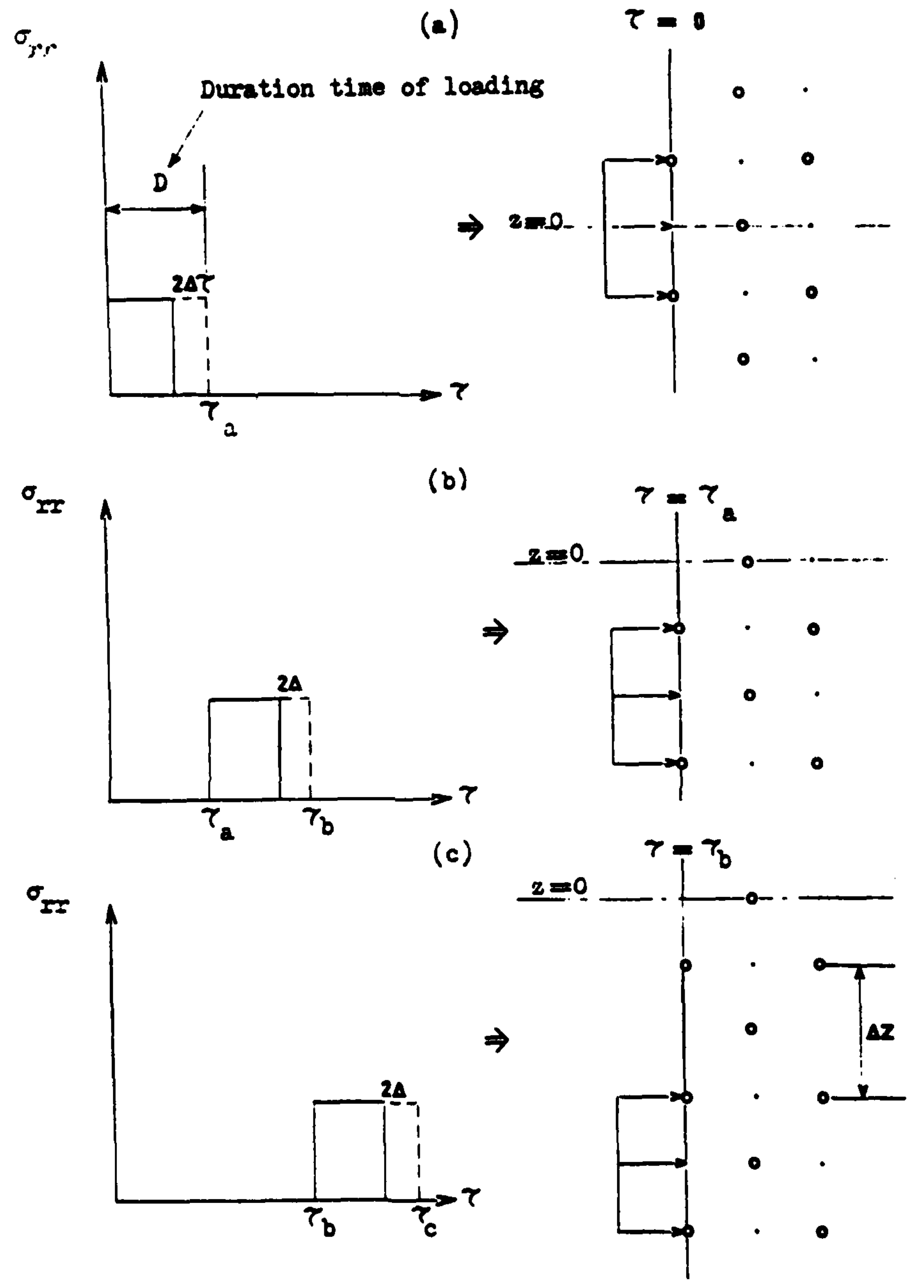


Figure 8. Impulsive rectangular-input loading.



Speed of travelling load $= \frac{\Delta z}{D}$

Figure 9. Consecutive superposition scheme for CASE 4.

APPENDIX B

PROGRAM LISTING OF TRES1

```
      PROGRAM TRES1(OUTPUT,TAPE1,TAPE2)
C     *****************************************************************
C   CASE 2 : TRANSIENT RESPONSE OF A SEMI-INFINITELY LONG TUBE
C            SUBJECTED TO LOAD APPLIED FROM THE INTERIOR AND ALONG
C            THE LENGTH OF THE TUBE
C
C  ----------------------------------------------------------------------
C   CO-ORDINATE SYSTEM FOR THE ARRAYS USED IN THE PROGRAM:
C
C   J - COORDINATE FOR Z-AXIS. J=1 AT Z=0.0.
C   K - COORDINATE FOR R-AXIS. K=1 AT R=DRO.
C   L - COORDINATE FOR VARIABLES. 1 - SITT       5 - UZ
C                                 2 - UR         6 - DUZDZ
C                                 3 - DURDR      7 - SIRR
C                                 4 - SIZZ       8 - SIRZ
C  NT - TIME COORDINATE. 1 - TOW, 2 - TOW-DT, 3 - TOW-2*DT
C
C  ----------------------------------------------------------------------
C
C  INPUT DATA
C
C     N     ---    NUMBER OF DIVISIONS ACROSS THICKNESS OF CYLINDER
C     INDEX ---    TOTAL NUMBER OF INTEGRATION(MUST BE 2*N)(MAXIMUM 30)
C     DRO   ---    INTERNAL RADIUS
C     DR1   ---    EXTERNAL RADIUS
C     RNE   ---    POISSON'S RATIO
C     FF    ---    DIMENSIONLESS LOADING INTENSITY
C
C     THE FORM OF OUTPUT IS SELECTED BY SPECIFYING IPRINT AND INPUT
C     THE INFORMATION REQUIRED FOR THE OUTPUT
C
C     IPRINT             FORM OF OUTPUT
C     ------             --------------
C       1          PRINT FOR A SPECIFIED TIME
C       2          PRINT FOR A SPECIFIED POINT
C       3          PRINT FOR BOTH SPECIFIED TIME AND POINTS
C     ---------------------------------------------------------
C
C     IFROM  ---   STARTING TIME FOR PRINTING
C     ITILL  ---   TIME FOR TERMINATION OF PRINTING
C     IWRITE ---   TIME INTERVAL OF PRINTING
C     NPRINT ---   NUMBER OF POINTS SPECIFIED (MAXIMUM  9)
C     JPRINT ---   J - COORDINATE OF SPECIFIED POINTS
C     KPRINT ---   K - COORDINATE OF SPECIFIED POINTS
C
C**********************************************************************
C
      COMMON/FO/FF,FB,FR
      COMMON/ALL/I,TOW,R,Z,K,J,DT5,ALFA,BETA,BETA2,G2,BG,CBA,CABA,PIE5,N
```

```
      COMMON/RE/IFROM,ITILL,IWRITE,IJ1,IPRINT,NPRINT
      COMMON/PRINT/JPRINT(9),KPRINT(9)
      DIMENSION T3(31,8),TT(34,8)
      DIMENSION Q3(31,8),QJ(2,31,8)
      DIMENSION ZA(41)
      COMMON/TN/T(34,31,8),T1(34,31,8),T2(34,31,8),TOTAL(34,31,8)
      COMMON/QN/Q(34,31,8),Q1(34,31,8),Q2(34,31,8)
C
C     INPUT TO THE PROGRAM
C
      DATA INDEX,N,DRO,DR1,RNE,FF/30,15,1.,1.30,.15,1./
      DATA(JPRINT(M),M=1,2)/2,2/
      DATA(KPRINT(M),M=1,2)/1,2/
      DATA IPRINT,IFROM,ITILL,IWRITE,NPRINT/1,1,30,1,2/
      N1=N+1
      INDX1=INDEX+1
      INDX4=INDEX+4
      CALL GALIM(RNE,DRO,INDEX,INDX1,INDX4,T3,TT,
     +DR1,Q3,QJ,ZA,N1)
      STOP
      END
C
C
      SUBROUTINE GALIM(RNE,DRO,INDEX,INDX1,INDX4,T3,TT,
     +DR1,Q3,QJ,ZA,N1)
C
C***********************************************************************
C     PURPOSE: TO FIX THE SCHEME OF INTEGRATION                        *
C***********************************************************************
C
      COMMON/ALL/I,TOW,R,Z,K,J,DT5,ALFA,BETA,BETA2,G2,BG,CBA,CABA,PIE5,N
      COMMON/PRINT/JPRINT(9),KPRINT(9)
      COMMON/RE/IFROM,ITILL,IWRITE,IJ1,IPRINT,NPRINT
      COMMON/FO/FF,FB,FR
      COMMON/AA/B1,B2,B3,B5,B6,B9
      DIMENSION ZA(N1)
      COMMON/TN/T(34,31,8),T1(34,31,8),T2(34,31,8),
     +TOTAL(34,31,8)
      COMMON/QN/Q(34,31,8),Q1(34,31,8),Q2(34,31,8)
      DIMENSION T3(INDX1,8),T4(3,8),TT(INDX4,8)
      DIMENSION Q3(INDX1,8),Q4(3,8),QJ(2,INDX1,8)
C
C     CALCULATION OF THE CONSTANTS
C
      FR=-FF
      FRZ=0.
      DT=(DR1-DRO)/FLOAT(N)
      DT5=DT/2.0
      ALFA=RNE/(1.0-RNE)
      BETA2=(1.0-ALFA)/2.0
      BETA=SQRT(BETA2)
      G2=DT/(1.0+BETA)
```

```
      BG=2.0*BETA*G2
      CBA=(1.0-BETA)/(1.0+BETA)
      CABA=2.0*BETA/(1.0+BETA)
      B1=DT5
      B2=DT5
      B3=G2
      B5=DT5
      B6=DT5
      B9=DT
      IJ=1
      PIE5=2.0*ATAN(1.0)
C
C     SET ALL ARRAYS EQUAL TO ZERO
C
      DO 50 L=1,8
      DO 49 NT=1,3
      T4(NT,L)=0.0
      Q4(NT,L)=0.0
   49 CONTINUE
      DO 50 K=1,INDX1
      T3(K,L)=0.0
      Q3(K,L)=0.0
      QJ(1,K,L)=0.0
      QJ(2,K,L)=0.0
      DO 50 J=1,INDX4
      T(J,K,L)=0.0
      T1(J,K,L)=0.0
      T2(J,K,L)=0.0
      TOTAL(J,K,L)=0.0
      Q(J,K,L)=0.0
      Q1(J,K,L)=0.0
      Q2(J,K,L)=0.0
   50 CONTINUE
C
C     INITIAL CONDITIONS
C
      T1(1,1,7)=FF/DRO**(1.0+ALFA/2.0)
      T1(1,1,1)=ALFA*T1(1,1,7)
      T1(1,1,2)=-T1(1,1,7)
      T1(1,1,3)=FF*(1.0+ALFA/2.0)/DRO**(2.0+ALFA/2.0)
      T1(1,1,5)=ALFA*FF/SQRT(DRO)
      DO 55 J=2,INDX4
      T1(J,1,7)=FF/SQRT(DRO)
      T1(J,1,1)=ALFA*T1(J,1,7)
      T1(J,1,4)=T1(J,1,1)
      T1(J,1,2)=-T1(J,1,7)
      T1(J,1,3)=0.5*FF/DRO**1.5
   55 CONTINUE
      DO 60 L=1,8
      T4(1,L)=T1(1,1,L)
      DO 60 J=1,INDX4
      TT(J,L)=T1(J,1,L)
```

```
   60 CONTINUE
C
C     THE FRAME OF INTEGRATION
C
      DO 2000 I=1,INDEX
      SIGN=1.0
      TOW=I*DT
      IA=I+1
      IA1=IA
      IB=I+2
      IC=I+3
      KK=3
      DO 1000 JM=2,IC
      KK=5-KK
      J=I+4-JM
      Z=FLOAT(J)*DT-DT
      IF(JM .GT. 3) GO TO 200
      IF(JM .EQ. 3) GO TO 300
C
  100 K=1
      R=DRO
      CALL LOADED(DRO,INDX4,INDX1,DT,T,T1,T2,SIGN,DR1)
      IF(I .EQ. 1) GO TO 105
      DO 110 K=2,I
      R=FLOAT(K)*DT-DT+DRO
      CALL GENER(DRO,INDX4,INDX1,DT,T,T1,T2,SIGN,DR1)
  110 CONTINUE
  105 K=I+1
      R=FLOAT(I)*DT+DRO
      CALL WAVE(DRO,INDX4,INDX1,T,SIGN,FF)
      DO 115 K=1,IA
      DO 115 JJ=IC,INDX4
      DO 115 L=1,8
      T(JJ,K,L)=T(IB,K,L)
  115 CONTINUE
      DO 120 JJ=2,IA
      DO 120 L=1,8
      T(JJ,IA,L)=T(IB,IA,L)
  120 CONTINUE
      GO TO 1000
C
  300 K=1
      R=DRO
      CALL LEAD(DRO,INDX4,INDX1,DT,T,T1,T2,SIGN,DR1)
      DO 305 L=1,8
      T3(1,L)=T(J,1,L)
      T4(3,L)=T4(2,L)
      T4(2,L)=T4(1,L)
  305 CONTINUE
      T4(1,7)=FF/(DRO+TOW)**(1.0+0.5*ALFA)
      T4(1,1)=ALFA*T4(1,7)
      T4(1,2)=-T4(1,7)
```

```
      T4(1,3)=FF*(1.0+0.5*ALFA)/(DRO+TOW)**(2.0+0.5*ALFA)
      T4(1,5)=FF*ALFA/SQRT(DRO+TOW)
      T4(1,4)=0.0
      T4(1,6)=0.0
      T4(1,8)=0.0
      IF(I .LE. 2) GO TO 1000
      DO 310 K=3,I,2
      DO 310 L=1,8
      T(J,K,L)=T(IB,K,L)
  310 CONTINUE
  320 GO TO 1000
C
  200 IF(J.EQ. 1) GO TO 500
C
  400 IF(KK .EQ. 2) GO TO 405
      K=1
      R=DRO
      CALL LOAD1(DRO,INDX4,INDX1,DT,T,T1,T2,SIGN,DR1,FRZ)
  405 DO 445 K=KK,I,2
      R=FLOAT(K)*DT-DT+DRO
      LM=(J-1)*(J-1)-I*I+(K-1)*(K-1)
      IF(LM .GT. 0) GO TO 430
      ML=K+J-I-2
      IF(ML) 410,415,425
  410 CALL GENER1(DRO,INDX4,INDX1,DT,T,T1,T2,SIGN,DR1)
      GO TO 445
  415 CALL DIAG(DRO,INDX4,INDX1,DT,T,T1,T2,T4,SIGN,DR1)
      DO 420 L=1,8
      T3(K,L)=T(J,K,L)
  420 CONTINUE
      GO TO 445
  425 CALL GINTER(DRO,INDX4,INDX1,DT,T3,T4,T,SIGN,DR1,N1)
      GO TO 445
  430 DO 440 L=1,8
      T(J,K,L)=T(IB,K,L)
  440 CONTINUE
  445 CONTINUE
      GO TO 1000
C
  500 IF(KK .EQ. 2) GO TO 505
      K=1
      R=DRO
      CALL BOAX1(DRO,INDX4,INDX1,DT,T,T1,T2,SIGN,DR1)
  505 IF(I .LE. 2) GO TO 515
      IAN=I-1
      DO 510 K=KK,IAN,2
      R=FLOAT(K)*DT-DT+DRO
      CALL FREE(DRO,INDX4,INDX1,DT,T,T1,T2,SIGN,DR1)
  510 CONTINUE
  515 K=I+1
      DO 520 L=1,8
      T(J,K,L)=T4(1,L)
```

```
  520 CONTINUE
 1000 CONTINUE
      SIGN=-1.0
      IF(I-N)1985,599,604
C     INITIAL REFLECTED CONDITIONS
  599 Q1(1,N1,7)=FR/DR1**(1.0+ALFA/2.0)
      Q1(1,N1,1)=ALFA*Q1(1,N1,7)
      Q1(1,N1,2)=-SIGN*Q1(1,N1,7)
      Q1(1,N1,3)=SIGN*FR*(1.0+ALFA/2.0)/DR1**(2.0+ALFA/2.0)
      Q1(1,N1,5)=ALFA*FR/SQRT(DR1)
      DO 600 J=2,INDX4
      Q1(J,N1,7)=FR/SQRT(DR1)
      Q1(J,N1,1)=ALFA*Q1(J,N1,7)
      Q1(J,N1,4)=ALFA*Q1(J,N1,7)
      Q1(J,N1,2)=-SIGN*Q1(J,N1,7)
      Q1(J,N1,3)=0.5*SIGN*FR/DR1**1.5
  600 CONTINUE
      DO 602 L=1,8
      Q4(1,L)=Q1(1,N1,L)
      DO 602 M=1,N1
      QJ(1,M,L)=Q4(1,L)
  602 CONTINUE
      DO 603 J=1,INDX4
      DO 603 K=1,INDX1
      DO 603 L=1,8
  603 Q(J,K,L)=Q1(J,K,L)
      GO TO 4000
  604 IR=I-N
      KW=N1-IR
      DO 606 L=1,8
      Q4(3,L)=Q4(2,L)
      Q4(2,L)=Q4(1,L)
  606 CONTINUE
      R=FLOAT(2*N-I)*DT+DRO
      Q4(1,7)=FR/R**(1.0+ALFA*0.5)
      Q4(1,1)=ALFA*Q4(1,7)
      Q4(1,2)=-SIGN*Q4(1,7)
      Q4(1,3)=FR*SIGN*(1.0+ALFA*0.5)/R**(2.0+0.5*ALFA)
      Q4(1,5)=FR*ALFA/R**0.5
      Q4(1,4)=0.
      Q4(1,6)=0.
      Q4(1,8)=0.
      DO 3000 JM=2,IC
      J=I+4-JM
      IM=MOD(JM+N,2)+2
      Z=FLOAT(J-1)*DT
      J1=I-N+1
      J2=2+SQRT(FLOAT(I*I-N*N))
      IF(J-J1)640,630,610
  610 IF(J.LT.J2)GO TO 620
      K=N1
      R=DR1
```

```
      FB=-T(J,N1,7)
      CALL LOADED(DRO,INDX4,INDX1,DT,Q,Q1,Q2,SIGN,DR1)
      IF(IR.EQ.1)GO TO 614
      DO 612 KR=2,IR
      K=N-KR+2
      R=FLOAT(K)*DT-DT+DRO
      CALL GENER(DRO,INDX4,INDX1,DT,Q,Q1,Q2,SIGN,DR1)
  612 CONTINUE
  614 K=KW
      R=FLOAT(K-1)*DT+DRO
      CALL WAVE(DRO,INDX4,INDX1,Q,SIGN,FR)
      DO 616 K=KW,N1
      DO 616 JJ=J2,INDX4
      DO 616 L=1,8
      Q(JJ,K,L)=Q(IB,K,L)
  616 CONTINUE
      DO 618 L=1,8
      DO 617 M=1,N1
  617 QJ(2,M,L)=QJ(1,M,L)
      QJ(1,1,L)=Q(J2,N1,L)
  618 CONTINUE
      DO 619 JJ=2,J2
      DO 619 LL=1,8
  619 Q(JJ,KW,LL)=Q(IB,KW,LL)
      CALL RBOUND(DRO,INDX4,INDX1,DT,Q,Q1,Q2,Q4,QJ,J1,SIGN,T,ZA,N1,DR1)
      GO TO 3000
  620 IF(IM.EQ.2)GO TO 622
      K=N1
      R=DR1
      AJM=1.0+FLOAT(I*I-N*N-(J-1)*(J-1))/FLOAT(2*(I-N))
      M=AJM
      DO 624 L=1,8
  624 Q(J,K,L)=(AJM-FLOAT(M) )*QJ(1,M+1,L)+(FLOAT(M+1)-AJM)*QJ(1,M,L)
  622 DO 629 KR=IM,IR,2
      K=N-KR+2
      R=FLOAT(K-1)*DT+DRO
      LM=(J-1)**2-I*I+(2*N-K+1)**2
      IF(LM.GT.0)GO TO 626
      CALL RINTER(Q,QJ,Q4,INDX4,INDX1,ZA,N1)
      GO TO 629
  626 DO 625 L=1,8
      Q(J,K,L)=Q(IB,K,L)
  625 CONTINUE
  629 CONTINUE
      GO TO 3000
  630 K=N1
      R=DR1
      DO 633 L=1,8
      Q(J,K,L)=QJ(1,N1,L)
      Q3(N1,L)=Q(J,K,L)
  633 CONTINUE
      IF(IR.LE.2)GO TO 3000
```

```
      DO 636 KR=3,IR,2
      K=N-KR+2
      LM=(J-1)**2-I*I+(2*N-K+1)**2
      IF(LM.GT.0)GO TO 634
      CALL RINTER(Q,QJ,Q4,INDX4,INDX1,ZA,N1)
      GO TO 636
  634 DO 635 L=1,8
      Q(J,K,L)=Q(IB,K,L)
  635 CONTINUE
  636 CONTINUE
      GO TO 3000
  640 IF(J.EQ.1)GO TO 700
      IF(IM.EQ.2)GO TO 641
      K=N1
      R=DR1
      FB=-T(J,N1,7)
      FRZ=-T(J,N1,8)
      CALL LOAD1(DRO,INDX4,INDX1,DT,Q,Q1,Q2,SIGN,DR1,FRZ)
      IF(IR.LE.2)GO TO 3000
  641 DO 649 KR=IM,IR,2
      K=N-KR+2
      R=FLOAT(K-1)*DT+DRO
      LM=(J-1)**2-I*I+(2*N-K+1)**2
      LA=(J-1)**2-(I-N)**2+(N-K+1)**2
      ML=J+2*N-K-I
      IF(LM.GT.0)GO TO 647
      IF(LA.GT.0)GO TO 646
      IF(ML)642,643,645
  642 CALL GENER1(DRO,INDX4,INDX1,DT,Q,Q1,Q2,SIGN,DR1)
      GO TO 649
  643 CALL DIAG(DRO,INDX4,INDX1,DT,Q,Q1,Q2,Q4,SIGN,DR1)
      DO 644 L=1,8
      Q3(K,L)=Q(J,K,L)
  644 CONTINUE
      GO TO 649
  645 CALL GINTER(DRO,INDX4,INDX1,DT,Q3,Q4,Q,SIGN,DR1,N1)
      GO TO 649
  646 CALL RINTER(Q,QJ,Q4,INDX4,INDX1,ZA,N1)
      GO TO 649
  647 DO 648 L=1,8
      Q(J,K,L)=Q(IB,K,L)
  648 CONTINUE
  649 CONTINUE
      GO TO 3000
  700 IF(IM.EQ.2)GO TO 705
      K=N1
      R=DR1
      FB=-T1(J,N1,7)
      CALL BOAX1(DRO,INDX4,INDX1,DT,Q,Q1,Q2,SIGN,DR1)
  705 IF(IR.LE.2)GO TO 715
      DO 710 KR=IM,IR,2
      K=N-KR+2
```

```
      R=FLOAT(K-1)*DT+DRO
      CALL FREE (DRO,INDX4,INDX1,DT,Q,Q1,Q2,SIGN,DR1)
  710 CONTINUE
  715 K=KW
      DO 720 L=1,8
      Q(J,K,L)=Q4(1,L)
  720 CONTINUE
 3000 CONTINUE
 4000 CONTINUE
 1985  CONTINUE
      DO 1975 J=1,INDX4
      DO 1975 L=1,8
      DO 1970 K=1,IA1
 1970 TOTAL(J,K,L)=Q(J,K,L)+T(J,K,L)
      IF(I.NE.2*N)GO TO 1975
      TOTAL(J,1,L)=TOTAL(J,1,L)+TT(J,L)
 1975 CONTINUE
      CALL RESULT(RNE,DRO,DR1,DT,I,FF,INDX4,INDX1,TOTAL,T1,IJ,N1)
      DO 1980 J=1,INDX4
      DO 1980 L=1,8
      DO 198C K=1,IA1
      T2(J,K,L)=T1(J,K,L)
      T1(J,K,L)=T(J,K,L)
      Q2(J,K,L)=Q1(J,K,L)
      Q1(J,K,L)=Q(J,K,L)
 1980 CONTINUE
2000  CONTINUE
      RETURN
      END
C
C
      SUBROUTINE AMAT(DRO,INDX4,INDX1,DT,T,T1,T2,A,SIGN,DR1)
C
C************************************************************************
C     PURPOSE: TO CALCULATE THE MATRIX AND VECTOR BASED ON MATRIX 2     *
C                                                                       *
C     COLUMNS 1 TO 8 OF A(II,JJ) REPRESENT THE MATRIX WHILE A(II,9) IS *
C     THE COLUMN VECTOR SUCH THAT:                                      *
C           A(1,9)= [A2]                  A(5,9)= [A6]                  *
C           A(2,9)=  A9                   A(6,9)= [A1]                  *
C           A(3,9)=  A10                  A(7,9)= [A5]                  *
C           A(4,9`=  A12                  A(8,9)= [A3]                  *
C
      COMMON/ALL/I,TOW,R,Z,K,J,DT5,ALFA,BETA,BETA2,G2,BG,CBA,CABA,PIE5,N
      COMMON/P.A/B1,B2,B3,B5,B6,B9
      COMMON/FO/FF,FB,FR
      DIMENSION T(INDX4,INDX1,8),T1(INDX4,INDX1,8),T2(INDX4,INDX1,8)
      DIMENSION A(8,9)
      F=FF
      IF(SIGN.EQ.-1.0)F=FR
      DO 600 II=1,8
      DO 600 JJ=1,9
```

```
      A(II,JJ)=0.0
  600 CONTINUE
C
C     CALCULATING THE MATRIX
C
      A(1,1)=B2/R
      A(1,2)=1.-2.*ALFA*B2/R
      A(1,4)=B2/R
      A(1,7)=1.-2.*B2/R
C
      ALFA9=B9*ALFA
      ALFA9R=ALFA9/R
      A(2,2)=-ALFA9R
      A(2,3)=-B9
      A(2,6)=-ALFA9
      A(2,7)=1.
C
      A(3,2)=-ALFA9R
      A(3,3)=-ALFA9
      A(3,4)=1.
      A(3,6)=-B9
C
      A(4,1)=1.
      A(4,2)=-B9/R
      A(4,3)=-ALFA9
      A(4,6)=-ALFA9
C
      A(5,2)=-ALFA*B6/R
      A(5,4)=1.0
      A(5,5)=1.0
      A(5,8)=-B6/R
C
      A(6,1)=-B1/R
      A(6,2)=-1.-2.*ALFA*B1/R
      A(6,4)=-B1/R
      A(6,7)=1.+2.*B1/R
C
      A(7,2)=-ALFA*B5/R
      A(7,4)=1.0+B5/(Z+DRO)
      A(7,5)=-1.0-ALFA*B5/(Z+DRO)
      A(7,7)=-B5/(Z+DRO)
      A(7,8)=B5/R
C
      A(8,5)=-SIGN*BETA+BETA2*B3/R
      A(8,8)=1.0+3.0*SIGN*BETA*B3/R
C
C     CALCULATING THE VECTOR
C
      ZIR=Z*Z-TOW*TOW+(R-DRO)*(R-DRO)
      IF(SIGN.EQ.-1.0)ZIR=Z*Z-TOW*TOW+(2.0*DR1-DRO-R)**2
      KNEG=K-1
      KPOS=K+1
```

```
      JNEG=J-1
      JPOS=J+1
C
      IF(KNEG .EQ. 0) GO TO 620
      A(6,9)=(T1(J,KNEG,1)+ALFA*T1(J,KNEG,2)-T1(J,KNEG,7))
     +*B1/(R-DT)+T1(J,KNEG,7)-T1(J,KNEG,2)
      IF(ZIR .GT. 0.0) GO TO 610
      A(6,9)=A(6,9)+(T1(J,KNEG,4)+ALFA*T1(J,KNEG,2)-
     +T1(J,KNEG,7))*B1/(R-DT)
C
  610 CONTINUE
      SRZ=CABA*T1(J,KNEG,8)+CBA*T2(J,K,8)
      UZ=CABA*T1(J,KNEG,5)+CBA*T2(J,K,5)
      A(8,9)=SRZ*(1.0-3.0*SIGN*BETA*B3/(R-BG))+UZ*(-SIGN*BETA-BETA2*B3/
     +(R-BG))
C
  620 CONTINUE
      A(1,9)=(T1(J,KPOS,7)+ALFA*T1(J,KPOS,2)-T1(J,KPOS,1))
     +*B2/(R+DT)+T1(J,KPOS,7)+T1(J,KPOS,2)
      IF(ZIR .GT. 0.0) GO TO 640
      A(1,9)=A(1,9)+B2*(T1(J,KPOS,7)+ALFA*T1(J,KPOS,2)-
     +T1(J,KPOS,4))/(R+DT)
C
  640 CONTINUE
      IF(JNEG .EQ. 0) GO TO 650
      A(7,9)=T1(JNEG,K,4)-T1(JNEG,K,5)+B5*ALFA*T1(JNEG,K,2)/R
     ++B5*(T1(JNEG,K,7)-T1(JNEG,K,4)+ALFA*T1(JNEG,K,5))/(DRO+Z-DT)
     +-B5*T1(JNEG,K,8)/R
C
  650 CONTINUE
      A(5,9)=T1(JPOS,K,4)+T1(JPOS,K,5)+B6*ALFA*T1(JPOS,K,2)/R
     ++B6*T1(JPOS,K,8)/R
C
      IF(SIGN .EQ. -1 .AND. K .EQ. 2*N-I+2) GO TO 670
      IF(K .EQ. I .AND. SIGN .EQ. 1) GO TO 660
      A(2,9)=T2(J,K,7)+B9*T2(J,K,3)+ALFA9*T2(J,K,2)/R+ALFA9*T2(J,K,6)
      A(3,9)=T2(J,K,4)+B9*T2(J,K,6)+ALFA9*T2(J,K,2)/R+ALFA9*T2(J,K,3)
      A(4,9)=T2(J,K,1)+B9*T2(J,K,2)/R+ALFA9*T2(J,K,3)+ALFA9*T2(J,K,6)
      RETURN
C
  660 B=B2
      GO TO 680
  670 B=B1
  680 CON=F/R**1.5
      R2=R+SIGN*DT/2.0
      AXE=SIGN*F*B*(1.0-2.0*ALFA)/R2**1.5
      A(2,9)=CON*R+SIGN*0.5*B9*CON-SIGN*ALFA*B9*CON
      A(3,9)=ALFA*CON*R+0.5*SIGN*B9*ALFA*CON
      A(4,9)=ALFA*CON*R+0.5*SIGN*B9*ALFA*CON-CON*SIGN*B9
      IF(SIGN .EQ. 1) A(1,9)=AXE
      IF(SIGN .EQ. -1) A(6,9)=AXE
      RETURN
```

```
      END
C
C***********************************************************************
C       SUBROUTINES RINT AND CINT ARE USED TO ADJUST MATRIX 2 FOR EACH
C       CATEGORY OF POINTS
C***********************************************************************
C
      SUBROUTINE RINT(A,N,M,LA,LB)
C
C INTERCHANGE ROWS LA AND LB
C
      DIMENSION A(N,M)
      DO 10 II=1,M
      SAVE=A(LA,II)
      A(LA,II)=A(LB,II)
   10 A(LB,II)=SAVE
      RETURN
      END
C
      SUBROUTINE CINT(A,N,M,LA,LB)
C
C INTERCHANGE COLUMNS LA AND LB
C
      DIMENSION A(N,M)
      DO 10 II=1,N
      SAVE=A(II,LA)
      A(II,LA)=A(II,LB)
   10 A(II,LB)=SAVE
      RETURN
      END
C
C
      SUBROUTINE WAVE(DRO,INDX4,INDX1,T,SIGN,F)
C
C***********************************************************************
C        INCIDENT LEADING WAVE POINTS -- G--
C***********************************************************************
C
      COMMON/ALL/I,TOW,R,Z,K,J,DT5,ALFA,BETA,BETA2,G2,BG,CBA,CABA,PIE5,N
      COMMON/PO/FP,FB,FR
      DIMENSION T(INDX4,INDX1,8)
      T(J,K,7)=F/R**0.5
      T(J,K,1)=ALFA*T(J,K,7)
      T(J,K,2)=-SIGN*T(J,K,7)
      T(J,K,3)=F*SIGN*0.5/R**1.5
      T(J,K,4)=ALFA*T(J,K,7)
      RETURN
      END
C
C
      SUBROUTINE BOAX1(DRO,INDX4,INDX1,DT,T,T1,T2,SIGN,DR1)
C
```

```
C***********************************************************************
C     REGULAR CORNER POINT -- D --
C***********************************************************************
C
      COMMON/ALL/I,TOW,R,Z,K,J,DT5,ALFA,BETA,BETA2,G2,BG,CBA,CABA,PIE5,N
      COMMON/FO/FF,FB,FR
      COMMON/AA/B1,B2,B3,B5,B6,B9
      DIMENSION T(INDX4,INDX1,8),T1(INDX4,INDX1,8),T2(INDX4,INDX1,8)
      DIMENSION A(8,9),AB(5,5),COE(5)
      F=FF
      IF(SIGN.EQ.-1.0)F=FB
    5 CALL AMAT(DRO,INDX4,INDX1,DT,T,T1,T2,A,SIGN,DR1)
      CALL CINT(A,8,9,4,6)
      IF(SIGN.EQ.-1.0)CALL RINT(A,8,9,1,6)
      DO 15 II=1,5
      DO 10 JJ=1,5
   10 AB(II,JJ)=A(II,JJ)
   15 COE(II)=A(II,9)
      T(J,K,7)=F
      T(J,K,4)=0.0
      T(J,K,8)=0.0
      COE(1)=COE(1)-(1.0-2.0*B2/R)*T(J,K,7)
      IF(SIGN.EQ.-1.0)COE(1)=A(1,9)-(1.0+2.0*B1/R)*T(J,K,7)
      COE(2)=COE(2)-T(J,K,7)
      CALL MATINV(AB,COE,5,5,1,DET,KS)
      IF(KS .EQ. 1) GO TO 20
      T(J,K,1)=COE(1)
      T(J,K,2)=COE(2)
      T(J,K,3)=COE(3)
      T(J,K,5)=COE(5)
      T(J,K,6)=COE(4)
      RETURN
   20 WRITE (1,25)
   25 FORMAT(*SINGULAR AT REGULAR CORNER POINT -- D --*)
      STOP
      END
C
C
      SUBROUTINE FREE(DRO,INDX4,INDX1,DT,T,T1,T2,SIGN,DR1)
C
C***********************************************************************
C      REGULAR FREE SURFACE POINT -- C --
C***********************************************************************
C
      COMMON/ALL/I,TOW,R,Z,K,J,DT5,ALFA,BETA,BETA2,G2,BG,CBA,CABA,PIE5,N
      COMMON/AA/B1,B2,B3,B5,B6,B9
      DIMENSION T(INDX4,INDX1,8),T1(INDX4,INDX1,8),T2(INDX4,INDX1,8)
      DIMENSION A(8,9),AB(6,6),COE(6)
   10 CALL AMAT(DRO,INDX4,INDX1,DT,T,T1,T2,A,SIGN,DR1)
      CALL CINT(A,8,9,4,7)
      DO 7 II=1,6
      DO 6 JJ=1,6
```

```
    6 AB(II,JJ)=A(II,JJ)
    7 COE(II)=A(II,9)
      T(J,K,4)=0.0
      T(J,K,8)=0.0
      CALL MATINV(AB,COE,6,6,1,DET,KS)
      IF(KS.EQ.1) GO TO 4
      T(J,K,1)=COE(1)
      T(J,K,2)=COE(2)
      T(J,K,3)=COE(3)
      T(J,K,7)=COE(4)
      T(J,K,5)=COE(5)
      T(J,K,6)=COE(6)
      RETURN
    4 WRITE (1,5)
    5 FORMAT(*SINGULAR AT REGULAR FREE SURFACE POINT -- C -- *)
      STOP
      END
C
C
      SUBROUTINE LEAD(DRO,INDX4,INDX1,DT,T,T1,T2,SIGN,DR1)
C
C***********************************************************************
C       LEADING LOADED BOUNDARY POINT (TWO DIMENSIONAL) -- J --
C***********************************************************************
C
      COMMON/ALL/I,TOW,R,Z,K,J,DT5,ALFA,BETA,BETA2,G2,BG,CBA,CABA,PIES,N
      COMMON/AA/B1,B2,B3,B5,B6,B9
      COMMON/FO/FF,FB,FR
      DIMENSION T(INDX4,INDX1,8),T1(INDX4,INDX1,8),T2(INDX4,INDX1,8)
      DIMENSION A(8,9),AB(5,5),COE(5)
      IF(K.EQ.I) B9=DT5
      CALL AMAT(DRO,INDX4,INDX1,DT,T,T1,T2,A,SIGN,DR1)
      CALL CINT(A,8,9,2,6)
      CALL RINT(A,8,9,7,1)
      DO 7 II=1,5
      DO 6 JJ=1,5
    6 AB(II,JJ)=A(II,JJ)
    7 COE(II)=A(II,9)
      T(J,K,2)=T(I+2,1,2)
      T(J,K,7)=T(I+2,1,7)
      T(J,K,8)=0.0
      COE(1)=COE(1)+B5*T(J,K,7)/(Z+DRO)+B5*ALFA*T(J,K,2)/R
      COE(2)=COE(2)-T(J,K,7)+B9*ALFA*T(J,K,2)/R
      COE(3)=COE(3)+B9*ALFA*T(J,K,2)/R
      COE(4)=COE(4)+B9*T(J,K,2)/R
      COE(5)=COE(5)+B6*ALFA*T(J,K,2)/R
      CALL MATINV(AB,COE,5,5,1,DET,KS)
      IF(KS.EQ.1) GO TO 3
      T(J,K,1)=COE(1)
      T(J,K,3)=COE(3)
      T(J,K,4)=COE(4)
      T(J,K,5)=COE(5)
```

```
      T(J,K,6)=COE(2)
      B9=DT
      RETURN
    3 WRITE (1,5)
    5 FORMAT( *SINGULAR AT LEADING LOADED BOUNDARY 2-D POINT  -J- *)
      STOP
      END
C
C
      SUBROUTINE LOADED(DRO,INDX4,INDX1,DT,T,T1,T2,SIGN,DR1)
C
C**********************************************************************
C     REGULAR LOADED BOUNDARY POINT (ONE DIMENSIONAL) -- F --
C**********************************************************************
C
      COMMON/ALL/I,TOW,R,Z,K,J,DT5,ALFA,BETA,BETA2,G2,BG,CBA,CABA,PIE5,N
      COMMON/AA/B1,B2,B3,B5,B6,B9
      COMMON/FO/FF,FB,FR
      DIMENSION T(INDX4,INDX1,8),T1(INDX4,INDX1,8),T2(INDX4,INDX1,8)
      DIMENSION A(8,9),AB(4,4),COE(4)
      F=FF
      IF(SIGN.EQ.-1.0)F=FB
      IF(K.EQ.I .AND. SIGN .EQ. 1) GO TO 2
      IF(SIGN .EQ. -1 .AND. K .EQ. 2*N-I+2) GO TO 8
      GO TO 5
    2 B2=B1/2.
      B9=B1
      GO TO 5
    8 B9=B1
      B1=B1/2.0
    5 CALL AMAT(DRO,INDX4,INDX1,DT,T,T1,T2,A,SIGN,DR1)
      IF(SIGN.EQ.-1.0)CALL RINT(A,8,9,1,6)
      DO 7 II=1,4
      DO 6 JJ=1,4
    6 AB(II,JJ)=A(II,JJ)
    7 COE(II)=A(II,9)
      T(J,K,5)=0.0
      T(J,K,6)=0.0
      T(J,K,7)=F
      T(J,K,8)=0.0
      AB(1,2)=AB(1,2)+ALFA*B2/R
      AB(1,4)=0.
      COE(1)=COE(1)-(1.0-2.0*B2/R)*T(J,K,7)
      IF(SIGN.EQ.-1.0)COE(1)=A(1,9)-(1.0+2.0*B1/R)*T(J,K,7)
      COE(2)=COE(2)-T(J,K,7)
      CALL MATINV(AB,COE,4,4,1,DET,KS)
      IF(KS.EQ.1) GO TO 3
      DO 9 L=1,4
    9 T(J,K,L)=COE(L)
      B1=DT5
      B2=B1
      B9=2.*B1
```

```
      RETURN
    3 WRITE (1,4)
    4 FORMAT( *SINGULAR AT REGULAR LOADED BOUNDARY 1-D POINT  - F -*)
      STOP
      END
C
C
      SUBROUTINE LOAD1(DRO,INDX4,INDX1,DT,T,T1,T2,SIGN,DR1,FRZ)
C
C************************************************************************
C     REGULAR LOADED BOUNDARY POINT (TWO DIMENSIONAL) -- B --
C************************************************************************
C
      COMMON/ALL/I,TOW,R,Z,K,J,DT5,ALFA,BETA,BETA2,G2,BG,CBA,CABA,PIE5,N
      COMMON/AA/B1,B2,B3,B5,B6,B9
      COMMON/FO/FF,FB,FR
      DIMENSION T(INDX4,INDX1,8),T1(INDX4,INDX1,8),T2(INDX4,INDX1,8)
      DIMENSION A(8,9),AB(6,6),COE(6)
      F=FF
      IF(SIGN.EQ.-1.0)F=FB
      CALL AMAT(DRO,INDX4,INDX1,DT,T,T1,T2,A,SIGN,DR1)
      IF(SIGN.EQ.1.0)CALL RINT(A,8,9,6,7)
      IF(SIGN.EQ.-1.0)CALL RINT(A,8,9,1,7)
      DO 7 II=1,6
      DO 6 JJ=1,6
    6 AB(II,JJ)=A(II,JJ)
    7 COE(II)=A(II,9)
      T(J,K,7)=F
      T(J,K,8)=0.5*(1.0-SIGN)*FRZ
      COE(1)=COE(1)-(1.0-2.0*B2/R)*T(J,K,7)
      IF(SIGN.EQ.-1.0)COE(1)=A(1,9)+B5/(DRO+Z)*T(J,K,7)
     +  -B5/R*T(J,K,8)
      COE(2)=COE(2)-T(J,K,7)
      COE(6)=COE(6)+B5*T(J,K,7)/(Z+DRO)
      IF(SIGN.EQ.-1.0)COE(6)=A(6,9)-(1.0+2.0*B1)*T(J,K,7)
      IF(SIGN.EQ.-1.0)COE(5)=COE(5)+B6/R*T(J,K,8)
      CALL MATINV(AB,COE,6,6,1,DET,KS)
      IF(KS.EQ.1) GO TO 3
      DO 9 L=1,6
    9 T(J,K,L)=COE(L)
      RETURN
    3 WRITE (1,4)
    4 FORMAT( *SINGULAR AT REGULAR LOADED BOUNDARY 2-D POINT  - B - *)
      STOP
      END
C
C
      SUBROUTINE GENER(DRO,INDX4,INDX1,DT,T,T1,T2,SIGN,DR1)
C
C************************************************************************
C     REGULAR INNER ONE DIMENSIONAL POINTS -- E --
C************************************************************************
```

```
C
      COMMON/ALL/I,TOW,R,Z,K,J,DT5,ALFA,BETA,BETA2,G2,BG,CBA,CABA,PIE5,N
      COMMON/AA/B1,B2,B3,B5,B6,B9
      DIMENSION T(INDX4,INDX1,8),T1(INDX4,INDX1,8),T2(INDX4,INDX1,8)
      DIMENSION A(8,9),AB(5,5),COE(5)
      IF(SIGN.EQ.1.0.AND.K.EQ.I)GO TO 2
      IF(SIGN.EQ.-1.0.AND.K.EQ.2*N-I+2)GO TO 8
      GO TO 5
    2 B2=B1/2.0
      B9=B1
      GO TO 5
    8 B9=B1
      B1=B1/2.0
    5 CALL AMAT(DRO,INDX4,INDX1,DT,T,T1,T2,A,SIGN,DR1)
      CALL CINT(A,8,9,5,7)
      CALL RINT(A,8,9,5,6)
      DO 7 II=1,5
      DO 6 JJ=1,5
    6 AB(II,JJ)=A(II,JJ)
    7 COE(II)=A(II,9)
      AB(1,2)=AB(1,2)+ALFA*B2/R
      AB(1,4)=0.
      AB(1,5)=AB(1,5)+B2/R
      AB(5,2)=AB(5,2)+ALFA*B1/R
      AB(5,4)=0.
      AB(5,5)=AB(5,5)-B1/R
      CALL MATINV(AB,COE,5,5,1,DET,KS)
      IF(KS.EQ.1) GO TO 3
      DO 9 L=1,4
    9 T(J,K,L)=COE(L)
      T(J,K,5)=0.0
      T(J,K,6)=0.0
      T(J,K,7)=COE(5)
      T(J,K,8)=0.0
      B1=DT5
      B2=B1
      B9=2.0*B1
      RETURN
    3 WRITE (1,4)
    4 FORMAT(*SINGULAR AT REGULAR INNER ONE DIMENSIONAL POINT  - E -*)
      STOP
      END
C
C
      SUBROUTINE GENER1(DRO,INDX4,INDX1,DT,T,T1,T2,SIGN,DR1)
C
C*************************************************************************
C     REGULAR INNER TWO DIMENSIONAL POINTS — A —
C     SOLVE MATRIX 2 WITHOUT ALTERATION
C*************************************************************************
C
      COMMON/ALL/I,TOW,R,Z,K,J,DT5,ALFA,BETA,BETA2,G2,BG,CBA,CABA,PIE5,N
```

```
      COMMON/AA/B1,B2,B3,B5,B6,B9
      DIMENSION T(INDX4,INDX1,8),T1(INDX4,INDX1,8),T2(INDX4,INDX1,8)
      DIMENSION A(8,9),AB(8,8),COE(8)
      CALL AMAT(DRO,INDX4,INDX1,DT,T,T1,T2,A,SIGN,DR1)
      DO 7 II=1,8
      DO 6 JJ=1,8
    6 AB(II,JJ)=A(II,JJ)
    7 COE(II)=A(II,9)
      CALL MATINV(AB,COE,8,8,1,DET,KS)
      IF(KS.EQ.1) GO TO 3
      DO 9 L=1,8
    9 T(J,K,L)=COE(L)
      RETURN
    3 WRITE (1,4)
    4 FORMAT(*SINGULAR AT REGULAR INNER TWO DIMENSIONAL POINT  - A -*)
      STOP
      END
C
C
      SUBROUTINE GINTER(DRO,INDX4,INDX1,DT,T3,T4,T,SIGN,DR1,N1)
C
C*********************************************************************
C     LEADING INNER TWO DIMENSIONAL POINT -- K --
C     CALCULATED BY INTERPOLATION ALONG THE REFLECTED LONGITUDINAL WAVE
C*********************************************************************
C
      COMMON/ALL/I,TOW,R,Z,K,J,DT5,ALFA,BETA,BETA2,G2,BG,CBA,CABA,PIE5,N
      DIMENSION T(INDX4,INDX1,8),T3(INDX1,8),T4(3,8)
      Y=TOW-(R-DRO)
      IF(SIGN.EQ.-1.0)Y=TOW-2.0*DR1+DRO+R
      X=(Z*Z/Y+Y)/2.
      RK=(X-Y)/Z
      TETA1=ATAN(RK)/PIE5
      TETA=1.-TETA1
      RK=(TOW-X)/DT+1.
      IF(SIGN.EQ.-1.0)RK=2*N-I+1+X/DT
      KA=IFIX(RK)
      KB=KA+1
      C2=RK-FLOAT(KA)
      C1=1.-C2
      DO 154 L=1,8
  154 T(J,K,L)=TETA*(C1*T3(KA,L)+C2*T3(KB,L))+TETA1*T4(1,L)
      RETURN
      END
C
C
      SUBROUTINE DIAG(DRO,INDX4,INDX1,DT,T,T1,T2,T4,SIGN,DR1)
C
C*********************************************************************
C     INTERMEDIARY TWO DIMENSIONAL POINT -- L --
C*********************************************************************
C
```

```
      COMMON/ALL/I,TOW,R,Z,K,J,DT5,ALFA,BETA,BETA2,G2,BG,CBA,CABA,PIE5,N
      COMMON/AA/B1,B2,B3,B5,B6,B9
      DIMENSION T(INDX4,INDX1,8),T1(INDX4,INDX1,8),T2(INDX4,INDX1,8)
      DIMENSION [illegible] 9),AB(8,8),COE(8),E(8),T4(3,8)
      B1=DT5
      B2=Z*DT/(2.*(Z+2.*DT))
      IF(SIGN.EQ.1.0)GO TO 5
      BT=B1
      B1=B2
      B2=BT
    5 B3=(Z+(1.-BETA)*DT-SQRT((DT-BETA*(Z+DT))**2+(1.-BETA2)*Z**2))
     +/(2.*(1.-BETA2))
      B5=DT5
      B6=Z*DT/(2.*(2.*Z+DT))
      B9=(Z+DT-SQRT(Z*Z+DT*DT))/2.
      CALL AMAT(DRO,INDX4,INDX1,DT,T,T1,T2,A,SIGN,DR1)
      DO 7 II=1,8
      DO 6 JJ=1,8
    6 AB(II,JJ)=A(II,JJ)
    7 COE(II)=A(II,9)
      R2=R+2.0*B2
      CALL ENV(INDX4,INDX1,DT,Z,R2,E,T,T1,T4,B2)
      COE(1)=E(7)*(1.+B2/R2)+E(2)*(1.+ALFA*B2/R2)-B2*E(1)/R2
     ++B2*(ALFA*E(2)+E(7)-E(4))/R2
      Z6=Z+2.*B6
      CALL ENV(INDX4,INDX1,DT,Z6,R,E,T,T1,T4,B6)
      COE(5)=E(4)+E(5)+B6*ALFA*E(2)/R+B6*E(8)/R
      CALL ENV(INDX4,INDX1,DT,Z,R,E,T,T1,T4,B9)
      COE(2)=E(7)+B9*(E(3)+ALFA*(E(2)/R+E(6)))
      COE(3)=E(4)+B9*(E(6)+ALFA*(E(2)/R+E(3)))
      COE(4)=E(1)+B9*(E(2)/R+ALFA*(E(3)+E(6)))
      R3=R-2.0*B3*BETA
      CALL ENV(INDX4,INDX1,DT,Z,R3,E,T,T1,T4,B3)
      COE(8)=E(8)*(1.0-SIGN*3.0*BETA*B3/R3)-E(5)*(SIGN*BETA+B3*BETA2/R3)
      CALL MATINV(AB,COE,8,8,1,DET,KS)
      IF(KS .EQ. 1) GO TO 3
      DO 209 L=1,8
  209 T(J,K,L)=COE(L)
      B1=DT5
      B2=DT5
      B6=DT5
      B3=G2
      B9=DT
      RETURN
    3 WRITE (1,4)
    4 FORMAT(* SINGULAR AT INTERMEDIARY TWO DIMENSIONAL POINT  - L -*)
      STOP
      END
C
C
      SUBROUTINE ENV(INDX4,INDX1,DT,ZL,RL,E,T,T1,T4,BL)
C
```

```
C***********************************************************************
C     VALUES OF VARIABLES L ARE CALCULATED AT THE INTERSECTION OF THE
C     BICHARACTERISTIC CURVES FROM A POINT ON THE DIAGONAL AND THE
C***********************************************************************
C
      COMMON/ALL/I,TOW,R,Z,K,J,DT5,ALFA,BETA,BETA2,G2,BG,CBA,CABA,PIE5,N
      DIMENSION T(INDX4,INDX1,8),T1(INDX4,INDX1,8),T4(3,8),E(8)
      C1=2.0*BL/DT
      C2=1.0-C1
      THETA=ATAN(ZL/(ABS(RL-R)+DT))/PIE5
      THETA1=1.0-THETA
      DO 10 L=1,8
      E(L)=C1*(THETA*T1(J,K-1,L)+THETA1*T4(2,L))+
     +C2*(THETA*T(J+1,K-1,L)+THETA1*T4(1,L))
   10 CONTINUE
      RETURN
      END
C
      SUBROUTINE RBOUND(DRO,INDX4,INDX1,DT,Q,Q1,Q2,Q4,QJ,J1,SIGN,T,ZA,
     +  N1,DR1)
C     INTERMEDIATE TWO-DIMENSIONAL POINTS -- L--
      COMMON/ALL/I,TOW,R,Z,K,J,DT5,ALFA,BETA,BETA2,G2,BG,CBA,CABA,PIE5,N
      COMMON/AA/B1,B2,B3,B5,B6,B9
      COMMON/FO/FF,FB,FR
      DIMENSION ZA(N1)
      DIMENSION T(INDX4,INDX1,8),QJ(2,INDX1,8)
      DIMENSION Q(INDX4,INDX1,8),Q1(INDX4,INDX1,8),Q2(INDX4,INDX1,8)
      DIMENSION A(8,9),AB(6,6),COE(6),E(8),Q4(3,8)
      R=FLOAT(N)*DT+DRO
      DO 200 M=1,N1
      ZN=SQRT(      FLOAT(  (I+N-2*M+2)*(I-N)   )  )
      Z=ZN*DT
      ZA(M)=ZN
      NZ=ZN
      IF(M.EQ.1)GO TO 200
      QJ(1,M,7)=-((ZN-FLOAT(NZ))*T(NZ+2,N1,7)
     ++(FLOAT(NZ+1)-ZN)*T(NZ+1,N1,7)  )
      QJ(1,M,8)=-((ZN-FLOAT(NZ))*T(NZ+2,N1,8)
     +        +(FLOAT(NZ+1)-ZN)*T(NZ+1,N1,8)  )
      B1=0.25*(TOW-R+DRO-Z**2/(TOW+R-2.0*(M-2)*DT-DRO)    )
      B9=0.5*(TOW-(M-2)*DT-SQRT(Z**2+(R-(M-2)*DT-DRO)**2)    )
      B5=0.25*(TOW+Z-(M-2)*DT+(R-(M-2)*DT-DRO)**2/(Z-TOW+(M-2)*DT))
      IF(B5 .GT. DT5) B5=DT5
      B6=0.25*(TOW-Z-(M-2)*DT-(R-(M-2)*DT-DRO)**2/(TOW+Z-(M-2)*DT))
      CALL AMAT(DRO,INDX4,INDX1,DT,Q,Q1,Q2,A,SIGN,DR1)
      CALL RINT(A,8,9,1,7)
      DO 7 II=1,8
      DO 6 JJ=1,8
    6 AB(II,JJ)=A(II,JJ)
    7 COE(II)=A(II,9)
      RA=TOW-(M-2)*DT
      RB=TOW-(M-1)*DT
```

```
      TA=ASIN(  DT*SQRT(FLOAT( (I+N-2*(M-2))*(I-N)  )  )/RA   )
      IF(I.EQ.N1)GO TO 8
      TB=ASIN(  DT*SQRT(FLOAT( (I-1+N-2*(M-2))*(I-1-N)  )  )/RB   )
    8 CONTINUE
      R1=R+2.0*B1
      T2=ATAN(Z/(R1 - (M-2)*DT - DRO)  )
      CALL ENV1(R1,Z,T2,TA,TB,B1,E,QJ,Q4,DT,M,INDX1,N1)
      COE(6)=E(7)*(1.-B1/R1)-E(2)*(1.-ALFA*B1/R1)+B1*E(1)/R1
     +  +B1*(ALFA*E(2) - E(7) + E(4)  )/R1
      COE(6)=COE(6)-(1.+2.*B1/R)*QJ(1,M,7)
      Z6=Z+2.0*B6
      T6=ATAN(Z6/(R-(M-2)*DT-DRO) )
      CALL ENV1(R,Z6,T6,TA,TB,B6,E,QJ,Q4,DT,M,INDX1,N1)
      COE(5)=E(4)+E(5)+B6*ALFA*E(2)/R+B6*E(8)/R
      COE(5)=COE(5)+B6/R*QJ(1,M,8)
      T9=ATAN(Z/(R-(M-2)*DT-DRO )    )
      CALL ENV1(R,Z,T9,TA,TB,B9,E,QJ,Q4,DT,M,INDX1,N1)
      COE(2)=E(7)+B9*( E(3)+ALFA*(E(2)/R+E(6) )    )
      COE(2)=COE(2)-QJ(1,M,7)
      COE(3)=E(4)+B9*( E(6)+ALFA*(E(2)/R+E(3) )    )
      COE(4)=E(1)+B9*( E(2)/R+ALFA*(E(3)+E(6) )    )
      IF(B5.EQ.DT5)GO TO 9
      Z5=Z-2.0*B5
      T5=ATAN(Z5/(R-(M-2)*DT-DRO)    )
      CALL ENV1(R,Z5,T5,TA,TB,B5,E,QJ,Q4,DT,M,INDX1,N1)
      GO TO 11
    9  DO 10 L=1,8
   10 E(L)=(ZN-FLOAT(NZ))*Q1(NZ+1,N1,L)+(FLOAT(NZ+1)-ZN)*Q1(NZ,N1,L)
   11 CONTINUE
      COE(1)=E(4)-E(5)+B5*ALFA*E(2)/R-B5*E(8)/R
      COE(1)=COE(1)+B5/(Z+DRO)*QJ(1,M,7)
     +  -B5/R*QJ(1,M,8)
      IF(M.EQ.N1  )COE(1)=A(1,9)+B5/(Z+DRO)*QJ(1,M,7)
     +     -B5/R*QJ(1,M,8)
      CALL MATINV(AB,COE,6,6,1,DET,KS)
      DO 209 L=1,6
      QJ(1,M,L)=COE(L)
  209 CONTINUE
  200 CONTINUE
      B1=DT5
      B5=DT5
      B6=DT5
      B9=DT
      RETURN
      END
C
C
      SUBROUTINE ENV1(RL,ZL,TETA,TA,TB,BL,E,QJ,Q4,DT,M,INDX1,N1)
C***********************************************************************
C     CALCULATION OF QUANTITIES AT TERMINAL POINTS BICHARACTERISTIC
C     CURVES OF POINTS -- M--
C***********************************************************************
```

```
      COMMON/ALL/I,TOW,R,Z,K,J,DT5,ALFA,BETA,BETA2,G2,BG,CBA,CABA,PIE5,N
      DIMENSION QJ(2,INDX1,8),Q4(3,8),E(8)
      TTA=TETA/TA
      TTA1=1.0-TTA
      IF(I.EQ.N1)GO TO 8
      TTB=TETA/TB
      GO TO 9
 8    TTB=0.0
 9    CONTINUE
      TTB1=1.0-TTB
      C2=2.0*BL/DT
      C1=1.0-C2
      DO 10 L=1,8
      E(L)=C1*(TTB*QJ(2,M-1,L)+TTB1*Q4(2,L) )
     + +C2*(TTA*QJ(1,M-1,L)+TTA1*Q4(1,L) )
   10 CONTINUE
      RETURN
      END
C
C
      SUBROUTINE RINTER(Q,QJ,Q4,INDX4,INDX1,ZA,N1)
C***********************************************************************
C     INTERMEDIATE TWO-DIMENSIONAL POINTS -- N --
C***********************************************************************
      COMMON/ALL/I,TOW,R,Z,K,J,DT5,ALFA,BETA,BETA2,G2,BG,CBA,CABA,PIE5,N
      DIMENSION ZA(N1)
      DIMENSION QJ(2,INDX1,8),Q4(3,8),Q(INDX4,INDX1,8)
      Y=I-2*N-1+K
      X=0.5*Y+0.5*FLOAT((J-1)*(J-1))/Y
      A=X-FLOAT(I-N)
      AJAX=X*X-A*A
      AJ=SQRT(AJAX)
      AJM=(FLOAT( I*I-N*N)-AJ*AJ)/FLOAT( 2*(I-N))+1.0
      TJK=ASIN(FLOAT(J-1)/X  )
      TAJ=ASIN(AJ/X)
      TETA=TJK/TAJ
      M=AJM
      C1=(ZA(M)-AJ)/(ZA(M)-ZA(M+1)  )
      DO 10 L=1,8
   10 Q(J,K,L)=TETA*(C1*QJ(1,M+1,L)+(1.-C1)*QJ(1,M,L))+(1.-TETA)*Q4(1,L)
      RETURN
      END
C
      SUBROUTINE MATINV(A,B,N,N1,MSUB,DET,KS)
      DIMENSION A(1),B(1)
      TOL=0.0
      KS=0
      JJ=-N
      DO 65 J=1,N
      JY=J+1
      JJ=JJ+N+1
      BIGA=0.0
```

AD-A118 600 MASSACHUSETTS UNIV AMHERST DEPT OF CIVIL ENGINEERING F/G 20/14
RESPONSE OF THICK CYLINDRICAL SHELLS TO TRANSIENT INTERNAL LOAD--ETC(U)
AUG 82 T HAN-URA, W A NASH DAAG29-77-G-0095
UNCLASSIFIED ARO-14700.2-EG NL

2 OF 3
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A118600

```
      IT=JJ-J
      DO 30 I=J,N
      IJ=IT+I
      IF(ABS(BIGA)-ABS(A(IJ)))20,30,30
   20 BIGA=A(IJ)
      IMAX=I
   30 CONTINUE
      IF(ABS(BIGA)-TOL)35,35,40
   35 KS=1
      RETURN
   40 I1=J+N*(J-2)
      IT=IMAX-J
      DO 50 K=J,N
      I1=I1+N
      I2=I1+IT
      SAVE=A(I1)
      A(I1)=A(I2)
      A(I2)=SAVE
   50 A(I1)=A(I1)/BIGA
      SAVE=B(IMAX)
      B(IMAX)=B(J)
      B(J)=SAVE/BIGA
      IF(J-N)55,70,55
   55 IQS=N*(J-1)
      DO 65 IX=JY,N
      IXJ=IQS+IX
      IT=J-IX
      DO 60 JX=JY,N
      IXJX=N*(JX-1)+IX
      JJX=IXJX+IT
   60 A(IXJX)=A(IXJX)-(A(IXJ)*A(JJX))
   65 B(IX)=B(IX)-(B(J)*A(IXJ))
   70 NY=N-1
      IT=N*N
      DO 80 J=1,NY
      IA=IT-J
      IB=N-J
      IC=N
      DO 80 K=1,J
      B(IB)=B(IB)-A(IA)*B(IC)
      IA=IA-N
   80 IC=IC-1
      RETURN
      END
C
C
      SUBROUTINE RESULT(RME,DRO,DR1,DT,I,FF,INDX4,INDX1,T,T1,IJ,N1)
      COMMON/RE/IFROM,ITILL,IWRITE,IJ1,IPRINT,NPRINT
      COMMON/PRINT/JPRINT(9),KPRINT(9)
      DIMENSION T(INDX4,INDX1,8),T1(INDX4,INDX1,8)
      IF(IJ .EQ. 2) GO TO 90
      IJ1=1
```

```
      IF(IPRINT .EQ. 1) GO TO 75
   75 CALL OUTP(RNE,DRO,DR1,DT,I,FF,INDX4,INDX1,T1)
      IJ=2
   90 IF(I .GT. ITILL) RETURN
      GO TO(100,200,300) IPRINT
  100 IF(I .NE. IFROM ) RETURN
      IFROM=IFROM+IWRITE
      CALL OUTP1(DRO,DT,I,INDX4,INDX1,T,N1)
      RETURN
  200 CALL OUTP2(DRO,DT,I,INDX4,INDX1,T,T1)
      RETURN
  300 IF(I .NE. IFROM) GO TO 310
      IFROM=IFROM+IWRITE
      CALL OUTP1(DRO,DT,I,INDX4,INDX1,T,N1)
  310 CALL OUTP2(DRO,DT,I,INDX4,INDX1,T,T1)
      RETURN
      END
C
C
      SUBROUTINE OUTP(RNE,DRO,DR1,DT,I,FF,INDX4,INDX1,T1)
C
C*********************************************************************
C     PRINTS THE VALUES OF THE INPUT CONSTANTS AND THE INITIAL       *
C     CONDITIONS                                                     *
C*********************************************************************
C
      DIMENSION T1(INDX4,INDX1,8)
      INDEX=INDX1-1
      CALL DATE(NDATE)
      WRITE(1,9) NDATE
      WRITE (1,10) RNE,DRO,DR1,FF,DT,INDEX
    9 FORMAT(A12/* TRANSIENT RESPONSE OF SEMI-INFINITELY LONG*
     +* TUBE SUBJECT TO ABRUPTLY APPLIED LOAD(CASE 1)*//
     +* DURATION TIME OF LOAD = PERMANENT*/* WIDTH OF LOAD*
     +* = SEMI-INFINITE*/)
   10 FORMAT(* THE INPUT CONSTANTS */21(1H-)//*POISSON'S RATIO =*,
     +F5.3,/*INNER RADIUS =*,F4.2/*OUTER RADIUS =*,F4.2/*NON-*
     +*DIMENSOINAL SIRR =*,F4.2/*STEP SIZE FOR INTEGRATION =*,F6.4/
     +*NUMBER OF TIME STEPS =*,I3///* THE INITIAL CONDITIONS *
     +*(TIME  T=0.0)*/38(1H-)/)
      WRITE (1,20) (T1(1,1,L),T1(2,1,L),L=1,8)
   20 FORMAT(*POINT - R=1.0  Z=0.0*,27X,*POINTS - R=1.0  Z>0.0*/
     +20(1H-),27X,21(1H-)//2(* SITT =*,F7.4,32X)/2(*   UR =*,F7.4,32X)/
     +2(*DURDR =*,F7.4,32X)/2(* SIZZ =*,F7.4,32X)/2(*   UZ =*,F7.4,32X)
     +/2(*DUZDZ =*,F7.4,32X)/2(* SIRR =*,F7.4,32X)/2(* SIRZ =*,F7.4,
     +32X)/72(1H-)/)
      RETURN
      END
C
C
      SUBROUTINE OUTP1(DRO,DT,I,INDX4,INDX1,T,N1)
C
```

```
C***********************************************************************
C     PRINTS VALUES OF VARIABLES L AT SPECIFIED TIME FOR POINTS (J,K)
C     FOR J=1,INDEX+4 AND K=1,INDEX+1
C***********************************************************************
C
      DIMENSION T(INDX4,INDX1,8)
   30 TOW=FLOAT(I)*DT
      WRITE (1,40) TOW
   40 FORMAT(//* AT TIME T=*,F7.4/19(1H-)//5X,*Z*,8X,*R*,7X,*SIRR*,5X,
     +*SITT*,5X,*SIZZ*,5X,*SIRZ*,6X,*UR*,7X,*UZ*/72(1H-))
      IA=I+1
      IC=I+3
      KK=1
      IF(MOD(I,2) .EQ. 0) KK=2
      DO 60 J=1,IC
      KK=3-KK
      Z=DT*FLOAT(J-1)
      WRITE (1,50)
   50 FORMAT(* *)
      DO 60 K=KK,N1,2
      R=FLOAT(K-1)*DT+DRO
      WRITE (1,70) Z,R,T(J,K,7),T(J,K,1),T(J,K,4),T(J,K,8),
     +              T(J,K,2),T(J,K,5)
   60 CONTINUE
   70 FORMAT(8(2X,F7.4))
      WRITE (1,80)
   80 FORMAT(72(1H-))
      RETURN
      END
C
      SUBROUTINE OUTP2(DRO,DT,I,INDX4,INDX1,T,T1)
C
C***********************************************************************
C     PRINTS VALUES OF VARIABLES L FOR SPECIFIED POINTS (JPRINT,KPRINT)
C***********************************************************************
C
      COMMON/RE/IFROM,ITILL,IWRITE,IJ1,IPRINT,NPRINT
      COMMON/PRINT/JPRINT(9),KPRINT(9)
      DIMENSION T(INDX4,INDX1,8),T1(INDX4,INDX1,8)
      DIMENSION R(9),Z(9),TOW(51),VAR(9,51,8)
      IF(IJ1 .NE. 1) GO TO 100
      IJ1=2
      DO 10 JJ=1,NPRINT
      R(JJ)=FLOAT(KPRINT(JJ))*DT-DT+DRO
      Z(JJ)=FLOAT(JPRINT(JJ))*DT-DT
   10 CONTINUE
      DO 20 L=1,8
      DO 20 LL=1,NPRINT
      J10=JPRINT(LL)
      K10=KPRINT(LL)
      TOW(1)=0.0
      VAR(LL,1,L)=T1(J10,K10,L)
```

```
   20 CONTINUE
  100 IF(MOD(I,2) .EQ. 1) RETURN
      IP=I/2+1
      TOW(IP)=FLOAT(I)*DT
      DO 120 JJ=1,NPRINT
      J10=JPRINT(JJ)
      K10=KPRINT(JJ)
      DO 110 L=1,8
      VAR(JJ,IP,L)=T(J10,K10,L)
  110 CONTINUE
  120 CONTINUE
  300 IF(I .LT. ITILL) RETURN
      DO  340 JJ=1,NPRINT
      WRITE (2,310) JJ,R(JJ),Z(JJ)
C 310 FORMAT(///5X,I2,5X*THE POINT:  R =*,F7.4,2X*Z =*,F7.4/3X,45(1H-)
C    + //7X*TOW*,6X*SIRR*,6X*SITT*,6X*SIZZ*,6X*SIRZ*,7X*UR*,8X*UZ*/
C    +72(1H-))
  310 FORMAT(I2,2F7.4)
      DO 330 KK=1,IP
      WRITE (2,320) TOW(KK),VAR(JJ,KK,7),VAR(JJ,KK,1),VAR(JJ,KK,4),
     +VAR(JJ,KK,8),VAR(JJ,KK,2),VAR(JJ,KK,5)
  320 FORMAT(7(3X,F7.4))
  330 CONTINUE
C     WRITE (2,335)
  335 FORMAT(72(1H-)/)
  340 CONTINUE
      RETURN
      END
```

APPENDIX C

PROGRAM LISTING OF TRES2

```
      PROGRAM TRES2(OUTPUT,TAPE5,TAPE6)
C     ****************************************************************
C     CASE 2:  TO OBTAIN THE TRANSIENT RESPONSE OF AN INFINITELY LONG,
C              THICK CYLINDRICAL SHELL TO AN EMBEDDED CYLINDRICAL LOAD
C              BY TWO-DIMENSIONAL METHOD OF CHARACTERISTICS.  THE LOAD
C              WHICH IS APPLIED SUDDENLY AT T=0, IS UNIFORM ALONG THE
C              LENGTH OF THE SHELL FROM Z=0 TO Z=INFINITY.
C
C    ------------------------------------------------------------------
C     COORDINATE SYSTEM FOR THE ARRAYS USED IN THE PROGRAM:
C
C     J - COORDINATE FOR Z-AXIS. J=1 AT Z=0.0.
C     K - COORDINATE FOR R-AXIS. K=1 AT R=DRO.
C     L - COORDINATE FOR VARIABLES. 1 - SITT       5 - UZ
C                                   2 - UR         6 - DUZDZ
C                                   3 - DURDR      7 - SIZR
C                                   4 - SIZZ       8 - SIRZ
C    NT - TIME COORDINATE. 1 - TOW, 2 - TOW-DT, 3 - TOW-2*DT
C
C    ------------------------------------------------------------------
C
C    INPUT DATA
C
C       N     ---    NUMBER OF DIVISIONS ACROSS THICKNESS OF CYLINDER
C       INDEX ---    TOTAL NUMBER OF INTEGRATION(MUST BE 2*N)(MAXIMUM 30)
C       DRO   ---    INTERNAL RADIUS
C       DR1   ---    EXTERNAL RADIUS
C       RNU.  ---    POISSON'S RATIO
C       FP    ---    DIMENSIONLESS LOADING INTENSITY
C
C       THE FORM OF OUTPUT IS SELECTED BY SPECIFYING IPRINT AND INPUT
C       THE INFORMATION REQUIRED FOR THE OUTPUT
C
C       IPRINT              FORM OF OUTPUT
C       ------              --------------
C         1          PRINT FOR A SPECIFIED TIME
C         2          PRINT FOR A SPECIFIED POINT
C         3          PRINT FOR BOTH SPECIFIED TIME AND POINTS
C       ---------------------------------------------------------
C
C       IFROM  ---   STARTING TIME FOR PRINTING
C       ITILL  ---   TIME FOR TERMINATION OF PRINTING
C       IWRITE ---   TIME INTERVAL OF PRINTING
C       NPRINT ---   NUMBER OF POINTS SPECIFIED (MAXIMUM 9)
C       JPRINT ---   J - COORDINATE OF SPECIFIED POINTS
C       KPRINT ---   K - COORDINATE OF SPECIFIED POINTS
C
C*********************************************************************
      COMMON/FO/FF,FB,FR,FM
```

```
      COMMON/ALL/I,TOW,R,Z,K,J,DTS,ALFA,BETA,BETA2,G2,BG,CBA,CABA,FIXS,N
      COMMON/RE/IFROM,ITILL,IWRITE,IJ1,IPRINT,NPRINT
      COMMON/PRINT/JPRINT(9),KPRINT(9)
      DIMENSION T3(31,8),TT(34,8)
      DIMENSION Q3(31,8),QJ(2,31,8)
      DIMENSION ZA(31)
      COMMON/TN/T(34,31,8),T1(34,31,8),T2(34,31,8),TOTAL(34,31,8)
      COMMON/QN/Q(34,31,8),Q1(34,31,8),Q2(34,31,8)
C
C     INPUT TO THE PROGRAM
C
      DATA INDEX,N,DRO,DR1,RNE,FF/30,15,1.,1.30,.15,1.0/
      DATA(JPRINT(M),M=1,2)/2,2/
      DATA(KPRINT(M),M=1,2)/1,2/
      DATA IPRINT,IFROM,ITILL,IWRITE,NPRINT/1,1,30,1,2/
      N1=N+1
      INDX1=INDEX+1
      INDX4=INDEX+4
      CALL GALIM(RNE,DRO,INDEX,INDX1,INDX4,T3,TT,
     +DR1,Q3,QJ,ZA,N1)
      STOP
      END
C
C
      SUBROUTINE GALIM(RNE,DRO,INDEX,INDX1,INDX4,T3,TT,
     +DR1,Q3,QJ,ZA,N1)
C
C*********************************************************************
C     PURPOSE: TO FIX THE SCHEME OF INTEGRATION                      *
C*********************************************************************
C
      COMMON/ALL/I,TOW,R,Z,K,J,DTS,ALFA,BETA,BETA2,G2,BG,CBA,CABA,FIXS,N
      COMMON/PRINT/JPRINT(9),KPRINT(9)
      COMMON/RE/IFROM,ITILL,IWRITE,IJ1,IPRINT,NPRINT
      COMMON/FO/FF,FB,FR,FM
      COMMON/AA/B1,B2,B3,B5,B6,B9
      COMMON/LINK/BD(8)
      DIMENSION ZA(N1)
      COMMON/TN/T(34,31,8),T1(34,31,8),T2(34,31,8),
     +TOTAL(34,31,8)
      COMMON/QN/Q(34,31,8),Q1(34,31,8),Q2(34,31,8)
      DIMENSION T3(INDX1,8),T4(3,8),TT(INDX4,8)
      DIMENSION Q3(INDX1,8),Q4(3,8),QJ(2,INDX1,8)
C
C     CALCULATION OF THE CONSTANTS
C
      FR=-FF
      FRE=0.
      DT=(DR1-DRO)/FLOAT(N)
      DTS=DT/2.0
      ALFA=RNE/(1.0-RNE)
      BETA2=(1.0-ALFA)/2.0
```

```
      BETA=SQRT(BETA2)
      G2=DT/(1.0+BETA)
      BG=2.0*BETA*G2
      CBA=(1.0-BETA)/(1.0+BETA)
      CABA=2.0*BETA/(1.0+BETA)
      B1=DT5
      B2=DT5
      B3=G2
      B5=DT5
      B6=DT5
      B9=DT
      IJ=1
      PIES=2.0*ATAN(1.0)
C
C     SET ALL ARRAYS EQUAL TO ZERO
C
      DO 50 L=1,8
      BD(L)=0.
      DO 49 NT=1,3
      T4(NT,L)=0.0
      Q4(NT,L)=0.0
   49 CONTINUE
      DO 50 K=1,INDX1
      T3(K,L)=0.0
      Q3(K,L)=0.0
      QJ(1,K,L)=0.0
      QJ(2,K,L)=0.0
      DO 50 J=1,INDX4
      T(J,K,L)=0.0
      T1(J,K,L)=0.0
      T2(J,K,L)=0.0
      TOTAL(J,K,L)=0.0
      Q(J,K,L)=0.0
      Q1(J,K,L)=0.0
      Q2(J,K,L)=0.0
   50 CONTINUE
C
C     INITIAL CONDITIONS
C
      P1=ALFA
      P2=1.0
      T1(1,1,7)=FF/DRO**P2
      T1(1,1,1)=ALFA*T1(1,1,7)
      T1(1,1,2)=-T1(1,1,7)
      T1(1,1,3)=P2*FF/DRO**(P2+1.)
      T1(1,1,4)=P1*T1(1,1,7)
      T1(1,1,5)=T1(1,1,4)
      T1(1,1,8)=0.
      T1(1,1,6)=-FF*ALFA*(1.0+ALFA)/2.0
      DO 55 J=2,INDX4
      T1(J,1,7)=FF/SQRT(DRO)
      T1(J,1,1)=ALFA*T1(J,1,7)
```

```
      T1(J,1,4)=T1(J,1,1)
      T1(J,1,2)=-T1(J,1,7)
      T1(J,1,3)=0.5*FF/DRO**1.5
   55 CONTINUE
      DO 60 L=1,8
      T4(1,L)=T1(1,1,L)
      DO 60 J=1,INDX4
      TT(J,L)=T1(J,1,L)
   60 CONTINUE
C
C     THE FRAME OF INTEGRATION
C
      DO 2000 I=1,INDEX
      SIGN=1.0
      TOW=I*DT
      IA=I+1
      IA1=IA
      IB=I+2
      IC=I+3
      KK=3
      DO 1000 JM=2,IC
      KK=5-KK
      J=I+4-JM
      Z=FLOAT(J)*DT-DT
      IF(JM .GT. 3) GO TO 200
      IF(JM .EQ. 3) GO TO 300
C
  100 K=1
      R=DRO
      CALL LOADED(DRO,INDX4,INDX1,DT,T,T1,T2,SIGN,DR1)
      IF(I .EQ. 1) GO TO 105
      DO 110 K=2,I
      R=FLOAT(K)*DT-DT+DRO
      CALL GENER(DRO,INDX4,INDX1,DT,T,T1,T2,SIGN,DR1)
  110 CONTINUE
  105 K=I+1
      R=FLOAT(I)*DT+DRO
      CALL WAVE(DRO,INDX4,INDX1,T,SIGN,FF)
      DO 115 K=1,IA
      DO 115 JJ=IC,INDX4
      DO 115 L=1,8
      T(JJ,K,L)=T(IB,K,L)
  115 CONTINUE
      DO 120 JJ=2,IA
      DO 120 L=1,8
      T(JJ,IA,L)=T(IB,IA,L)
  120 CONTINUE
      GO TO 1000
C
  300 K=1
      R=DRO
      CALL LEADM(P1,DRO,INDX4,INDX1,T,FF)
```

```
      DO 305 L=1,8
      T3(1,L)=T(J,1,L)
      T4(3,L)=T4(2,L)
      T4(2,L)=T4(1,L)
  305 CONTINUE
      R=DRO+TOW
      T4(1,7)=FF/R**P2
      T4(1,1)=ALFA*T4(1,7)
      T4(1,2)=-T4(1,7)
      T4(1,3)=P2*FF/R**(P2+1.)
      T4(1,4)=P1*T4(1,7)
      T4(1,5)=-T4(1,4)
      T4(1,6)=T4(1,2)/R
      T4(1,8)=0.
      IF(I .LE. 2) GO TO 1000
      DO 310 K=3,I,2
      DO 310 L=1,8
      T(J,K,L)=T(IB,K,L)
  310 CONTINUE
  320 GO TO 1000
C
  200 IF(J.EQ. 1) GO TO 500
C
  400 IF(KK .EQ. 2) GO TO 405
      K=1
      R=DRO
      FM=FF
      CALL LOAD1(DRO,INDX4,INDX1,DT,T,T1,T2,SIGN,DR1,FRZ)
  405 DO 445 K=KK,I,2
      R=FLOAT(K)*DT-DT+DRO
      LM=(J-1)*(J-1)-I*I+(K-1)*(K-1)
      IF(LM .GT. 0) GO TO 430
      ML=K+J-I-2
      IF(ML) 410,415,425
  410 CALL GENER1(DRO,INDX4,INDX1,DT,T,T1,T2,SIGN,DR1)
      GO TO 445
  415 CALL DIAG(DRO,INDX4,INDX1,DT,T,T1,T2,T4,SIGN,DR1)
      DO 420 L=1,8
      T3(K,L)=T(J,K,L)
  420 CONTINUE
      GO TO 445
  425 CALL GINTER(DRO,INDX4,INDX1,DT,T3,T4,T,SIGN,DR1,M1)
      GO TO 445
  430 DO 440 L=1,8
      T(J,K,L)=T(IB,K,L)
  440 CONTINUE
  445 CONTINUE
      GO TO 1000
C
  500 IF(KK .EQ. 2) GO TO 505
      K=1
      R=DRO
```

```
      DO 501 L=1,8
  501 BD(L)=T1(I+2,1,L)-T1(2,1,L)
      BD(5)=-BD(5)
      FM=FF
      CALL LOAD1(DRO,INDX4,INDX1,DT,T,T1,T2,SIGN,DR1,FRE)
  505 IF(I .LE. 2) GO TO 515
      IAN=I-1
      DO 510 K=KK,IAN,2
      R=FLOAT(K)*DT-DT+DRO
      DO 506 L=1,8
  506 BD(L)=T1(I+2,K,L)-T1(2,K,L)
      BD(5)=-BD(5)
      CALL GENER1(DRO,INDX4,INDX1,DT,T,T1,T2,SIGN,DR1)
  510 CONTINUE
  515 K=I+1
      DO 520 L=1,8
      T(J,K,L)=T4(1,L)
  520 CONTINUE
 1000 CONTINUE
      SIGN=-1.0
      IF(I-N)1985,598,604
C     INITIAL REFLECTED CONDITIONS
  598 CONTINUE
      Q1(1,N1,7)=FR/DR1**P2
      Q1(1,N1,1)=ALFA*Q1(1,N1,7)
      Q1(1,N1,2)=-SIGN*Q1(1,N1,7)
      Q1(1,N1,3)=SIGN*P2*FR/DR1**(P2+1.)
      Q1(1,N1,4)=P1*Q1(1,N1,7)
      Q1(1,N1,5)=-Q1(1,N1,4)
      Q1(1,N1,6)=Q1(1,N1,2)/DR1
      Q1(1,N1,8)=.0
      DO 600 J=2,INDX4
      Q1(J,N1,7)=FR/SQRT(DR1)
      Q1(J,N1,1)=ALFA*Q1(J,N1,7)
      Q1(J,N1,4)=ALFA*Q1(J,N1,7)
      Q1(J,N1,2)=-SIGN*Q1(J,N1,7)
      Q1(J,N1,3)=0.5*SIGN*FR/DR1**1.5
  600 CONTINUE
      DO 602 L=1,8
      Q4(1,L)=Q1(1,N1,L)
      DO 602 M=1,N1
      QJ(1,M,L)=Q4(1,L)
  602 CONTINUE
      DO 603 J=1,INDX4
      DO 603 K=1,INDX1
      DO 603 L=1,8
      Q(J,K,L)=Q1(J,K,L)
  603 CONTINUE
      GO TO 4000
  604 IR=I-N
      KN=N1-IR
      DO 606 L=1,8
```

```
      Q4(3,L)=Q4(2,L)
      Q4(2,L)=Q4(1,L)
  606 CONTINUE
      R=FLOAT(2*N-I)*DT+DRO
      Q4(1,7)=FR/R**P2
      Q4(1,1)=ALFA*Q4(1,7)
      Q4(1,2)=-SIGN*Q4(1,7)
      Q4(1,3)=SIGN*P2*FR/R**(P2+1.0)
      Q4(1,4)=P1*Q4(1,7)
      Q4(1,5)=-Q4(1,4)
      Q4(1,6)=Q4(1,2)/R
      Q4(1,8)=0.
      DO 3000 JM=2,IC
      J=I+4-JM
      IM=MOD(JM+N,2)+2
      Z=FLOAT(J-1)*DT
      J1=I-N+1
      J2=2+SQRT(FLOAT(I*I-N*N))
      IF(J-J1)640,630,610
  610 IF(J.LT.J2)GO TO 620
      K=N1
      R=DR1
      FB=-T(J,N1,7)
      CALL LOADED(DRO,INDX4,INDX1,DT,Q,Q1,Q2,SIGN,DR1)
      IF(IR.EQ.1)GO TO 614
      DO 612 KR=2,IR
      K=N-KR+2
      R=FLOAT(K)*DT-DT+DRO
      CALL GENER(DRO,INDX4,INDX1,DT,Q,Q1,Q2,SIGN,DR1)
  612 CONTINUE
  614 K=KW
      R=FLOAT(K-1)*DT+DRO
      CALL WAVE(DRO,INDX4,INDX1,Q,SIGN,FR)
      DO 616 K=KW,N1
      DO 616 JJ=J2,INDX4
      DO 616 L=1,8
      Q(JJ,K,L)=Q(IB,K,L)
  616 CONTINUE
      DO 618 L=1,8
      DO 617 M=1,N1
      QJ(2,M,L)=QJ(1,M,L)
  617 CONTINUE
      QJ(1,1,L)=Q(J2,N1,L)
  618 CONTINUE
      DO 619 JJ=2,J2
      DO 619 LL=1,8
  619 Q(JJ,KW,LL)=Q(IB,KW,LL)
      CALL RBOUND(DRO,INDX4,INDX1,DT,Q,Q1,Q2,Q4,QJ,J1,SIGN,T,ZA,N1,DR1)
      GO TO 3000
  620 IF(IM.EQ.2)GO TO 622
      K=N1
      R=DR1
```

```
      AJM=1.0+FLOAT(I*I-N*N-(J-1)*(J-1))/FLOAT(2*(I-N))
      M=AJM
      DO 624 L=1,8
      Q(J,K,L)=(AJM-FLOAT(M) )*QJ(1,M+1,L)+(FLOAT(M+1)-AJM)*QJ(1,M,L)
  624 CONTINUE
      Q(J,K,7)=-T(J,K,7)
      Q(J,K,8)=-T(J,K,8)
  622 DO 629 KR=IM,IR,2
      K=N-KR+2
      R=FLOAT(K-1)*DT+DRO
      LM=(J-1)**2-I*I+(2*N-K+1)**2
      IF(LM.GT.0)GO TO 626
      CALL RINTER(Q,QJ,Q4,INDX4,INDX1,ZA,N1)
      GO TO 629
  626 DO 625 L=1,8
      Q(J,K,L)=Q(IB,K,L)
  625 CONTINUE
  629 CONTINUE
      GO TO 3000
  630 K=N1
      R=DR1
      DO 633 L=1,8
      Q(J,K,L)=QJ(1,N1,L)
      Q3(N1,L)=Q(J,K,L)
  633 CONTINUE
      IF(IR.LE.2)GO TO 3000
      DO 636 KR=3,IR,2
      K=N-KR+2
      LM=(J-1)**2-I*I+(2*N-K+1)**2
      IF(LM.GT.0)GO TO 634
      CALL RINTER(Q,QJ,Q4,INDX4,INDX1,ZA,N1)
      GO TO 636
  634 DO 635 L=1,8
      Q(J,K,L)=Q(IB,K,L)
  635 CONTINUE
  636 CONTINUE
      GO TO 3000
  640 IF(J.EQ.1)GO TO 700
      IF(IM.EQ.2)GO TO 641
      K=N1
      R=DR1
      FB=-T(J,N1,7)
      FRZ=-T(J,N1,8)
      CALL LOAD1(DRO,INDX4,INDX1,DT,Q,Q1,Q2,SIGN,DR1,FRZ)
      IF(IR.LE.2)GO TO 3000
  641 DO 649 KR=IM,IR,2
      K=N-KR+2
      R=FLOAT(K-1)*DT+DRO
      LM=(J-1)**2-I*I+(2*N-K+1)**2
      LA=(J-1)**2-(I-N)**2+(N-K+1)**2
      ML=J+2*N-K-I
      IF(LM.GT.0)GO TO 647
```

```
      IF(LA.GT.0)GO TO 646
      IF(ML)642,643,645
  642 CALL GENER1(DRO,INDX4,INDX1,DT,Q,Q1,Q2,SIGN,DR1)
      GO TO 649
  643 CALL DIAG(DRO,INDX4,INDX1,DT,Q,Q1,Q2,Q4,SIGN,DR1)
      DO 644 L=1,8
      Q3(K,L)=Q(J,K,L)
  644 CONTINUE
      GO TO 649
  645 CALL GINTER(DRO,INDX4,INDX1,DT,Q3,Q4,Q,SIGN,DR1,N1)
      GO TO 649
  646 CALL RINTER(Q,QJ,Q4,INDX4,INDX1,ZA,N1)
      GO TO 649
  647 DO 648 L=1,8
      Q(J,K,L)=Q(IB,K,L)
  648 CONTINUE
  649 CONTINUE
      GO TO 3000
  700 IF(IM.EQ.2)GO TO 705
      K=N1
      R=DR1
      FB=-T(J,N1,7)
      DO 701 L=1,8
 701  BD(L)=Q1(I+2,N1,L)-Q1(2,N1,L)
      BD(5)=-BD(5)
      CALL LOAD1(DRO,INDX4,INDX1,DT,Q,Q1,Q2,SIGN,DR1,FRZ)
  705 IF(IR.LE.2)GO TO 715
      DO 710 KR=IM,IR,2
      K=N-KR+2
      R=FLOAT(K-1)*DT+DRO
      DO 706 L=1,8
 706  BD(L)=Q1(I+2,K,L)-Q1(2,K,L)
      BD(5)=-BD(5)
      CALL GENER1(DRO,INDX4,INDX1,DT,Q,Q1,Q2,SIGN,DR1)
  710 CONTINUE
  715 K=KW
      DO 720 L=1,8
      Q(J,K,L)=Q4(1,L)
  720 CONTINUE
 3000 CONTINUE
 4000 CONTINUE
 1985  CONTINUE
      DO 1975 J=1,INDX4
      DO 1975 L=1,8
      DO 1970 K=1,IA1
       TOTAL(J,K,L)=Q(J,K,L)+T(J,K,L)
 1970 CONTINUE
      IF(I.NE.2*N)GO TO 1975
      TOTAL(J,1,L)=TOTAL(J,1,L)+TT(J,L)
 1975 CONTINUE
      CALL RESULT(RNE,DRO,DT,I,FF,INDX4,INDX1,TOTAL,T1,IJ,N1,
     +TT,DR1,N)
```

```
      DO 1980 J=1,INDX4
      DO 1980 L=1,8
      DO 1980 K=1,IA1
      T2(J,K,L)=T1(J,K,L)
      T1(J,K,L)=T(J,K,L)
      Q2(J,K,L)=Q1(J,K,L)
      Q1(J,K,L)=Q(J,K,L)
 1980 CONTINUE
2000  CONTINUE
      RETURN
      END
C
C
      SUBROUTINE AMAT(DRO,INDX4,INDX1,DT,T,T1,T2,A,SIGN,DR1)
C
C*****************************************************************
C     PURPOSE: TO CALCULATE THE MATRIX AND VECTOR BASED ON MATRIX 2   *
C                                                                     *
C     COLUMNS 1 TO 8 OF A(II,JJ) REPRESENT THE MATRIX WHILE A(II,9) IS *
C     THE COLUMN VECTOR SUCH THAT:                                    *
C           A(1,9)= [A2]                A(5,9)= [A6]                  *
C           A(2,9)=  A9                 A(6,9)= [A1]                  *
C           A(3,9)=  A10                A(7,9)= [A5]                  *
C           A(4,9)=  A12                A(8,9)= [A3]                  *
C*****************************************************************
C
      COMMON/ALL/I,TOW,R,Z,K,J,DTS,ALFA,BETA,BETA2,G2,BG,CBA,CABA,PIES,N
      COMMON/AA/B1,B2,B3,B5,B6,B9
      COMMON/FO/FF,FB,FR,FM
      COMMON/LINK/BD(8)
      DIMENSION T(INDX4,INDX1,8),T1(INDX4,INDX1,8),T2(INDX4,INDX1,8)
      DIMENSION A(8,9)
      F=FF
      IF(SIGN.EQ.-1.0)F=FR
      DO 600 II=1,8
      DO 600 JJ=1,9
      A(II,JJ)=0.0
  600 CONTINUE
C
C     CALCULATING THE MATRIX
C
      A(1,1)=B2/R
      A(1,2)=1.-2.*ALFA*B2/R
      A(1,4)=B2/R
      A(1,7)=1.-2.*B2/R
C
      ALFA9=B9*ALFA
      ALFA9R=ALFA9/R
      A(2,2)=-ALFA9R
      A(2,3)=-B9
      A(2,6)=-ALFA9
      A(2,7)=1.
```

```
C
      A(3,2)=-ALFA9R
      A(3,3)=-ALFA9
      A(3,4)=1.
      A(3,6)=-B9
C
      A(4,1)=1.
      A(4,2)=-B9/R
      A(4,3)=-ALFA9
      A(4,6)=-ALFA9
C
      A(5,2)=-ALFA*B6/R
      A(5,4)=1.0
      A(5,5)=1.0
      A(5,8)=-B6/R
C
      A(6,1)=-B1/R
      A(6,2)=-1.-2.*ALFA*B1/R
      A(6,4)=-B1/R
      A(6,7)=1.+2.*B1/R
C
      A(7,2)=-ALFA*B5/R
      A(7,4)=1.0+B5/(Z+DRO)
      A(7,5)=-1.0-ALFA*B5/(Z+DRO)
      A(7,7)=-B5/(Z+DRO)
      A(7,8)=B5/R
C
      A(8,5)=-SIGN*BETA+BETA2*B3/R
      A(8,8)=1.0+3.0*SIGN*BETA*B3/R
C
C     CALCULATING THE VECTOR
C
      ZIR=Z*Z-TOW*TOW+(R-DRO)*(R-DRO)
      IF(SIGN.EQ.-1.0)ZIR=Z*Z-TOW*TOW+(2.0*DR1-DRO-R)**2
      KNEG=K-1
      KPOS=K+1
      JNEG=J-1
      JPOS=J+1
C
      IF(KNEG .EQ. 0) GO TO 620
      A(6,9)=(T1(J,KNEG,1)+ALFA*T1(J,KNEG,2)-T1(J,KNEG,7))
     +*B1/(R-DT)+T1(J,KNEG,7)-T1(J,KNEG,2)
      IF(ZIR .GT. 0.0) GO TO 610
      A(6,9)=A(6,9)+(T1(J,KNEG,4)+ALFA*T1(J,KNEG,2)-
     +T1(J,KNEG,7))*B1/(R-DT)
C
  610 CONTINUE
      SRZ=CABA*T1(J,KNEG,8)+CBA*T2(J,K,8)
      UZ=CABA*T1(J,KNEG,5)+CBA*T2(J,K,5)
      A(8,9)=SRZ*(1.0-3.0*SIGN*BETA*B3/(R-BG))+UZ*(-SIGN*BETA-BETA2*B3/
     +(R-BG))
C
```

```
  620 CONTINUE
      A(1,9)=(T1(J,KPOS,7)+ALFA*T1(J,KPOS,2)-T1(J,KPOS,1))
     +*B2/(R+DT)+T1(J,KPOS,7)+T1(J,KPOS,2)
      IF(ZIR .GT. 0.0) GO TO 640
      A(1,9)=A(1,9)+B2*(T1(J,KPOS,7)+ALFA*T1(J,KPOS,2)-
     +T1(J,KPOS,4))/(R+DT)
C
  640 CONTINUE
      IF(JNEG .EQ. 0) GO TO 645
      A(7,9)=T1(JNEG,K,4)-T1(JNEG,K,5)+B5*ALFA*T1(JNEG,K,2)/R
     ++B5*(T1(JNEG,K,7)-T1(JNEG,K,4)+ALFA*T1(JNEG,K,5))/(DRO+Z-DT)
     +-B5*T1(JNEG,K,8)/R
      GO TO 650
  645 A(7,9)=BD(4)-BD(5)+B5*ALFA*BD(2)/R+B5*(BD(7)-BD(4)+ALFA*
     +BD(5))/(DRO+Z-DT)-B5*BD(8)/R
C
  650 CONTINUE
      A(5,9)=T1(JPOS,K,4)+T1(JPOS,K,5)+B6*ALFA*T1(JPOS,K,2)/R
     ++B6*T1(JPOS,K,8)/R
C
      IF(SIGN .EQ. -1 .AND. K .EQ. 2*N-I+2) GO TO 670
      IF(K .EQ. I .AND. SIGN .EQ. 1) GO TO 660
      A(2,9)=T2(J,K,7)+B9*T2(J,K,3)+ALFA9*T2(J,K,2)/R+ALFA9*T2(J,K,6)
      A(3,9)=T2(J,K,4)+B9*T2(J,K,6)+ALFA9*T2(J,K,2)/R+ALFA9*T2(J,K,3)
      A(4,9)=T2(J,K,1)+B9*T2(J,K,2)/R+ALFA9*T2(J,K,3)+ALFA9*T2(J,K,6)
      RETURN
C
  660 B=B2
      GO TO 680
  670 B=B1
  680 CON=F/R**1.5
      R2=R+SIGN*DT/2.0
      AXE=SIGN*F*B*(1.0-2.0*ALFA)/R2**1.5
      A(2,9)=CON*R+SIGN*0.5*B9*CON-SIGN*ALFA*B9*CON
      A(3,9)=ALFA*CON*R-0.5*SIGN*B9*ALFA*CON
      A(4,9)=ALFA*CON*R+0.5*SIGN*B9*ALFA*CON-CON*SIGN*B9
      IF(SIGN .EQ. 1) A(1,9)=AXE
      IF(SIGN .EQ. -1) A(6,9)=AXE
      RETURN
      END
C
C*************************************************************************
C      SUBROUTINES RINT AND CINT ARE USED TO ADJUST MATRIX 2 FOR EACH
C      CATEGORY OF POINTS
C*************************************************************************
C
      SUBROUTINE RINT(A,N,M,LA,LB)
C
C INTERCHANGE ROWS LA AND LB
C
      DIMENSION A(N,M)
      DO 10 II=1,M
```

```
      SAVE=A(LA,II)
      A(LA,II)=A(LB,II)
   10 A(LB,II)=SAVE
      RETURN
      END
C
      SUBROUTINE CINT(A,N,M,LA,LB)
C
C INTERCHANGE COLUMNS LA AND LB
C
      DIMENSION A(N,M)
      DO 10 II=1,N
      SAVE=A(II,LA)
      A(II,LA)=A(II,LB)
   10 A(II,LB)=SAVE
      RETURN
      END
C
C
      SUBROUTINE WAVE(DRO,INDX4,INDX1,T,SIGN,F)
C
C***********************************************************************
C       INCIDENT LEADING WAVE POINTS — G—
C***********************************************************************
C
      COMMON/ALL/I,TOW,R,Z,K,J,DTS,ALFA,BETA,BETA2,G2,BG,CBA,CABA,PIES,N
      COMMON/FO/FF,FB,FR,FM
      DIMENSION T(INDX4,INDX1,8)
      T(J,K,7)=F/R**0.5
      T(J,K,1)=ALFA*T(J,K,7)
      T(J,K,2)=-SIGN*T(J,K,7)
      T(J,K,3)=F*SIGN*0.5/R**1.5
      T(J,K,4)=ALFA*T(J,K,7)
      RETURN
      END
C
C
C
      SUBROUTINE LEADM(P1,DRO,INDX4,INDX1,T,FF)
C
C***********************************************************************
C      LEADING INNER-SURFACE POINT — Q —
C***********************************************************************
C
      COMMON/ALL/I,TOW,R,Z,K,J,DTS,ALFA,BETA,BETA2,G2,BG,CBA,CABA,PIES,N
      DIMENSION T(INDX4,INDX1,8)
      T(J,K,7)=0.
      T(J,K,8)=0.
      T(J,K,3)=0.
      DECAY=(1.+ALFA)/2.0
      T(J,K,4)=P1*FF/ABS(Z+DRO)**DECAY
      T(J,K,5)=T(J,K,4)
```

```
      T(J,K,6)=DECAY*P1*FF/ABS(Z+DRO)**(DECAY+1.)
      T(J,K,2)=0.0
      T(J,K,1)=T(J,K,4)
      DO 1 L=1,8
 1    T(J,K,L)=T(I+2,1,L)-T(J,K,L)
      RETURN
      END
C
      SUBROUTINE LOADED(DRO,INDX4,INDX1,DT,T,T1,T2,SIGN,DR1)
C
C*********************************************************************
C     REGULAR LOADED BOUNDARY POINT (ONE DIMENSIONAL) --- F ---
C*********************************************************************
C
      COMMON/ALL/I,TOW,R,Z,K,J,DTS,ALFA,BETA,BETA2,G2,BG,CBA,CABA,PIES,N
      COMMON/AA/B1,B2,B3,B5,B6,B9
      COMMON/FO/FF,FB,FR,FM
      DIMENSION T(INDX4,INDX1,8),T1(INDX4,INDX1,8),T2(INDX4,INDX1,8)
      DIMENSION A(8,9),AB(4,4),COE(4)
      F=FF
      IF(SIGN.EQ.-1.0)F=FB
      IF(K.EQ.I .AND. SIGN .EQ. 1) GO TO 2
      IF(SIGN .EQ. -1 .AND. K .EQ. 2*N-I+2) GO TO 8
      GO TO 5
    2 B2=B1/2.
      B9=B1
      GO TO 5
    8 B9=B1
      B1=B1/2.0
    5 CALL AMAT(DRO,INDX4,INDX1,DT,T,T1,T2,A,SIGN,DR1)
      IF(SIGN.EQ.-1.0)CALL RINT(A,8,9,1,6)
      DO 7 II=1,4
      DO 6 JJ=1,4
    6 AB(II,JJ)=A(II,JJ)
    7 COE(II)=A(II,9)
      T(J,K,5)=0.0
      T(J,K,6)=0.0
      T(J,K,7)=F
      T(J,K,8)=0.0
      AB(1,2)=AB(1,2)+ALFA*B2/R
      AB(1,4)=0.
      COE(1)=COE(1)-(1.0-2.0*B2/R)*T(J,K,7)
      IF(SIGN.EQ.-1.0)COE(1)=A(1,9)-(1.0+2.0*B1/R)*T(J,K,7)
      COE(2)=COE(2)-T(J,K,7)
      CALL MATINV(AB,COE,4,4,1,DET,KS)
      IF(KS.EQ.1) GO TO 3
      DO 9 L=1,4
    9 T(J,K,L)=COE(L)
      B1=DTS
      B2=B1
      B9=2.*B1
      RETURN
```

```
    3 WRITE (5,4)
    4 FORMAT(*SINGULAR AT REGULAR LOADED BOUNDARY 1-D POINT  - F -*)
      STOP
      END
C
C
      SUBROUTINE LOAD1(DRO,INDX4,INDX1,DT,T,T1,T2,SIGN,DR1,FRZ)
C
C***********************************************************************
C     REGULAR LOADED BOUNDARY POINT (TWO DIMENSIONAL) -- B --
C***********************************************************************
C
      COMMON/ALL/I,TOW,R,Z,K,J,DTS,ALFA,BETA,BETA2,G2,BG,CBA,CABA,PIES,N
      COMMON/AA/B1,B2,B3,B5,B6,B9
      COMMON/FO/FF,FB,FR,FM
      COMMON/LINK/BD(8)
      DIMENSION T(INDX4,INDX1,8),T1(INDX4,INDX1,8),T2(INDX4,INDX1,8)
      DIMENSION A(8,9),AB(6,6),COE(6)
      F=FM
      IF(SIGN.EQ.-1.0)F=FB
      CALL AMAT(DRO,INDX4,INDX1,DT,T,T1,T2,A,SIGN,DR1)
      IF(SIGN.EQ.1.0)CALL RINT(A,8,9,6,7)
      IF(SIGN.EQ.-1.0)CALL RINT(A,8,9,1,7)
      DO 7 II=1,6
      DO 6 JJ=1,6
    6 AB(II,JJ)=A(II,JJ)
    7 COE(II)=A(II,9)
      T(J,K,7)=F
      T(J,K,8)=0.5*(1.0-SIGN)*FRZ
      COE(1)=COE(1)-(1.0-2.0*B2/R)*T(J,K,7)
      IF(SIGN.EQ.-1.0)COE(1)=A(1,9)+B5/(DRO+Z)*T(J,K,7)
     +  -B5/R*T(J,K,8)
      COE(2)=COE(2)-T(J,K,7)
      COE(6)=COE(6)+B5*T(J,K,7)/(Z+DRO)
      IF(SIGN.EQ.-1.0)COE(6)=A(6,9)-(1.0+2.0*B1)*T(J,K,7)
      IF(SIGN.EQ.-1.0)COE(5)=COE(5)+B6/R*T(J,K,8)
      CALL MATINV(AB,COE,6,6,1,DET,KS)
      IF(KS.EQ.1) GO TO 3
      DO 9 L=1,6
    9 T(J,K,L)=COE(L)
      RETURN
    3 WRITE (5,4)
    4 FORMAT(*SINGULAR AT REGULAR LOADED BOUNDARY 2-D POINT  - B - *)
      STOP
      END
C
C
      SUBROUTINE GENER(DRO,INDX4,INDX1,DT,T,T1,T2,SIGN,DR1)
C
C***********************************************************************
C     REGULAR INNER ONE DIMENSIONAL POINTS -- E --
C***********************************************************************
```

```
C
      COMMON/ALL/I,TOW,R,Z,K,J,DTS,ALFA,BETA,BETA2,G2,BG,CBA,CABA,PIES,N
      COMMON/AA/B1,B2,B3,B5,B6,B9
      DIMENSION T(INDX4,INDX1,8),T1(INDX4,INDX1,8),T2(INDX4,INDX1,8)
      DIMENSION A(8,9),AB(5,5),COE(5)
      IF(SIGN.EQ.1.0.AND.K.EQ.I)GO TO 2
      IF(SIGN.EQ.-1.0.AND.K.EQ.2*N-I+2)GO TO 8
      GO TO 5
    2 B2=B1/2.0
      B9=B1
      GO TO 5
    8 B9=B1
      B1=B1/2.0
    5 CALL AMAT(DRO,INDX4,INDX1,DT,T,T1,T2,A,SIGN,DR1)
      CALL CINT(A,8,9,5,7)
      CALL RINT(A,8,9,5,6)
      DO 7 II=1,5
      DO 6 JJ=1,5
    6 AB(II,JJ)=A(II,JJ)
    7 COE(II)=A(II,9)
      AB(1,2)=AB(1,2)+ALFA*B2/R
      AB(1,4)=0.
      AB(1,5)=AB(1,5)+B2/R
      AB(5,2)=AB(5,2)+ALFA*B1/R
      AB(5,4)=0.
      AB(5,5)=AB(5,5)-B1/R
      CALL MATINV(AB,COE,5,5,1,DET,KS)
      IF(KS.EQ.1) GO TO 3
      DO 9 L=1,4
    9 T(J,K,L)=COE(L)
      T(J,K,5)=0.0
      T(J,K,6)=0.0
      T(J,K,7)=COE(5)
      T(J,K,8)=0.0
      B1=DTS
      B2=B1
      B9=2.0*B1
      RETURN
    3 WRITE (5,4)
    4 FORMAT('SINGULAR AT REGULAR INNER ONE DIMENSIONAL POINT  - E -')
      STOP
      END
C
C
      SUBROUTINE GENER1(DRO,INDX4,INDX1,DT,T,T1,T2,SIGN,DR1)
C
C*************************************************************************
C     REGULAR INNER TWO DIMENSIONAL POINTS -- A --
C     SOLVE MATRIX 2 WITHOUT ALTERATION
C*************************************************************************
C
      COMMON/ALL/I,TOW,R,Z,K,J,DTS,ALFA,BETA,BETA2,G2,BG,CBA,CABA,PIES,N
```

```
      COMMON/AA/B1,B2,B3,B5,B6,B9
      COMMON/LINK/BD(8)
      DIMENSION T(INDX4,INDX1,8),T1(INDX4,INDX1,8),T2(INDX4,INDX1,8)
      DIMENSION A(8,9),AB(8,8),COE(8)
      CALL AMAT(DRO,INDX4,INDX1,DT,T,T1,T2,A,SIGN,DR1)
      DO 7 II=1,8
      DO 6 JJ=1,8
    6 AB(II,JJ)=A(II,JJ)
    7 COE(II)=A(II,9)
      CALL MATINV(AB,COE,8,8,1,DET,KS)
      IF(KS.EQ.1) GO TO 3
      DO 9 L=1,8
    9 T(J,K,L)=COE(L)
      RETURN
    3 WRITE (5,4)
    4 FORMAT(*SINGULAR AT REGULAR INNER TWO DIMENSIONAL POINT  - A -*)
      STOP
      END
C
C
      SUBROUTINE GINTER(DRO,INDX4,INDX1,DT,T3,T4,T,SIGN,DR1,N1)
C
C***********************************************************************
C     INTERMEDIATE INNER TWO DIMENSIONAL POINT -- X --
C     CALCULATED BY INTERPOLATION ALONG THE REFLECTED LONGITUDINAL WAVE
C***********************************************************************
C
      COMMON/ALL/I,TOW,R,Z,K,J,DTS,ALFA,BETA,BETA2,G2,BG,CBA,CABA,PIES,N
      DIMENSION T(INDX4,INDX1,8),T3(INDX1,8),T4(3,8)
      Y=TOW-(R-DRO)
      IF(SIGN.EQ.-1.0)Y=TOW-2.0*DR1+DRO+R
      X=(Z*Z/Y+Y)/2.
      RK=(X-Y)/Z
      TETA1=ATAN(RK)/PIES
      TETA=1.-TETA1
      RK=(TOW-X)/DT+1.
      IF(SIGN.EQ.-1.0)RK=2*N-I+1+X/DT
      KA=IFIX(RK)
      KB=KA+1
      C2=RK-FLOAT(KA)
      C1=1.-C2
      DO 154 L=1,8
  154 T(J,K,L)=TETA*(C1*T3(KA,L)+C2*T3(KB,L))+TETA1*T4(1,L)
      RETURN
      END
C
C
      SUBROUTINE DIAG(DRO,INDX4,INDX1,DT,T,T1,T2,T4,SIGN,DR1)
C
C***********************************************************************
C     INTERMEDIATE TWO DIMENSIONAL POINT -- L --
C***********************************************************************
```

```
C
      COMMON/ALL/I,TOW,R,Z,K,J,DT5,ALFA,BETA,BETA2,G2,BG,CBA,CABA,PIE5,NO
      COMMON/AA/B1,B2,B3,B5,B6,B9
      DIMENSION T(INDX4,INDX1,8),T1(INDX4,INDX1,8),T2(INDX4,INDX1,8)
      DIMENSION  A(8,9),AB(8,8),COE(8),E(8),T4(3,8)
      B1=DT5
      B2=Z*DT/(2.*(Z+2.*DT))
      IF(SIGN.EQ.1.0)GO TO 5
      BT=B1
      B1=B2
      B2=BT
    5 B3=(Z+(1.-BETA)*DT-SQRT((DT-BETA*(Z+DT))**2+(1.-BETA2)*Z**2))
     +/(2.*(1.-BETA2))
      B5=DT5
      B6=Z*DT/(2.*(2.*Z+DT))
      B9=(Z+DT-SQRT(Z*Z+DT*DT))/2.
      CALL AMAT(DRO,INDX4,INDX1,DT,T,T1,T2,A,SIGN,DR1)
      DO 7 II=1,8
      DO 6 JJ=1,8
    6 AB(II,JJ)=A(II,JJ)
    7 COE(II)=A(II,9)
      R2=R+2.0*B2
      CALL ENV(INDX4,INDX1,DT,Z,R2,E,T,T1,T4,B2)
      COE(1)=E(7)*(1.+B2/R2)+E(2)*(1.+ALFA*B2/R2)-B2*E(1)/R2
     ++B2*(ALFA*E(2)+E(7)-E(4))/R2
      Z6=Z+2.*B6
      CALL ENV(INDX4,INDX1,DT,Z6,R,E,T,T1,T4,B6)
      COE(5)=E(4)+E(5)+B6*ALFA*E(2)/R+B6*E(8)/R
      CALL ENV(INDX4,INDX1,DT,Z,R,E,T,T1,T4,B9)
      COE(2)=E(7)+B9*(E(3)+ALFA*(E(2)/R+E(6)))
      COE(3)=E(4)+B9*(E(6)+ALFA*(E(2)/R+E(3)))
      COE(4)=E(1)+B9*(E(2)/R+ALFA*(E(3)+E(6)))
      R3=R-2.0*B3*BETA
      CALL ENV(INDX4,INDX1,DT,Z,R3,E,T,T1,T4,B3)
      COE(8)=E(8)*(1.0-SIGN*3.0*BETA*B3/R3)-E(5)*(SIGN*BETA+B3*BETA2/R3)
      CALL MATINV(AB,COE,8,8,1,DET,KS)
      IF(KS .EQ. 1) GO TO 3
      DO 209 L=1,8
  209 T(J,K,L)=COE(L)
      B1=DT5
      B2=DT5
      B6=DT5
      B3=G2
      B9=DT
      RETURN
    3 WRITE (5,4)
    4 FORMAT(* SINGULAR AT INTERMEDIARY TWO DIMENSIONAL POINT  - L -*)
      STOP
      END
C
C
      SUBROUTINE ENV(INDX4,INDX1,DT,ZL,RL,E,T,T1,T4,BL)
```

```
C
C*********************************************************************
C     CALCULATION OF QUANTITIES AT TERMINAL POINTS OF
C     BICHARACTERISTIC CURVES OF POINTS L
C*********************************************************************
C
      COMMON/ALL/I,TOW,R,Z,K,J,DT5,ALFA,BETA,BETA2,G2,BG,CBA,CABA,PIES,N
      DIMENSION T(INDX4,INDX1,8),T1(INDX4,INDX1,8),T4(3,8),E(8)
      C1=2.0*EL/DT
      C2=1.0-C1
      THETA=ATAN(EL/(ABS(RL-R)+DT))/PIES
      THETA1=1.0-THETA
      DO 10 L=1,8
      E(L)=C1*(THETA*T1(J,K-1,L)+THETA1*T4(2,L))+
     +C2*(THETA*T(J+1,K-1,L)+THETA1*T4(1,L))
   10 CONTINUE
      RETURN
      END
C
      SUBROUTINE RBOUND(DRO,INDX4,INDX1,DT,Q,Q1,Q2,Q4,QJ,J1,SIGN,T,ZA,
     +  N1,DR1)
C
C*********************************************************************
C     INTERMEDIATE TWO-DIMENSIONAL POINTS -- M --
C*********************************************************************
      COMMON/ALL/I,TOW,R,Z,K,J,DT5,ALFA,BETA,BETA2,G2,BG,CBA,CABA,PIES,N
      COMMON/AA/B1,B2,B3,B5,B6,B9
      COMMON/FO/FF,FB,FR,FM
      DIMENSION ZA(N1)
      DIMENSION T(INDX4,INDX1,8),QJ(2,INDX1,8)
      DIMENSION Q(INDX4,INDX1,8),Q1(INDX4,INDX1,8),Q2(INDX4,INDX1,8)
      DIMENSION A(8,9),AB(6,6),COE(6),E(8),Q4(3,8)
      R=FLOAT(N)*DT+DRO
      DO 200 M=1,N1
      ZN=SQRT(      FLOAT(  (I+N-2*M+2)*(I-N)  )  )
      Z=ZN*DT
      ZA(M)=ZN
      NZ=ZN
      DO 12 L=1,8
   12 QJ(1,M,L)=0.0
      IF(M.EQ.1)GO TO 200
      QJ(1,M,7)=-((ZN-FLOAT(NZ))*T(NZ+2,N1,7)
     ++(FLOAT(NZ+1)-ZN)*T(NZ+1,N1,7)  )
      QJ(1,M,8)=-((ZN-FLOAT(NZ))*T(NZ+2,N1,8)
     +       +(FLOAT(NZ+1)-ZN)*T(NZ+1,N1,8)  )
      B1=0.25*(TOW-R+DRO-Z**2/(TOW+R-2.0*(M-2)*DT-DRO)   )
      B9=0.5*(TOW-(M-2)*DT-SQRT(Z**2+(R-(M-2)*DT-DRO)**2)   )
      B5=0.25*(TOW+Z-(M-2)*DT+(R-(M-2)*DT-DRO)**2/(Z-TOW+(M-2)*DT))
      IF(B5 .GT. DT5) B5=DT5
      B6=0.25*(TOW-Z-(M-2)*DT-(R-(M-2)*DT-DRO)**2/(TOW+Z-(M-2)*DT))
      CALL AMAT(DRO,INDX4,INDX1,DT,Q,Q1,Q2,A,SIGN,DR1)
      CALL RINT(A,8,9,1,7)
```

```
      DO 7 II=1,8
      DO 6 JJ=1,8
    6 AB(II,JJ)=A(II,JJ)
    7 COE(II)=A(II,9)
      RA=TOW-(M-2)*DT
      RB=TOW-(M-1)*DT
      TA=ASIN(   DT*SQRT(FLOAT( (I+N-2*(M-2))*(I-N)  )  )/RA    )
      IF(I.EQ.N1)GO TO 8
      TB=ASIN(   DT*SQRT(FLOAT( (I-1+N-2*(M-2))*(I-1-N)  )  )/RB    )
    8 CONTINUE
      R1=R-2.0*B1
      T1=ATAN(Z/(R+2.*B1 - (M-2)*DT - DRO)  )
      CALL ENV1(R1,Z,T1,TA,TB,B1,E,QJ,Q4,DT,M,INDX1,N1)
      COE(6)=E(7)*(1.-B1/R1)-E(2)*(1.-ALFA*B1/R1)+B1*E(1)/R1
     +  +B1*(ALFA*E(2) - E(7) + E(4)  )/R1
      COE(6)=COE(6)-(1.+2.*B1/R)*QJ(1,M,7)
      Z6=Z+2.0*B6
      T6=ATAN(Z6/(R-(M-2)*DT-DRO) )
      CALL ENV1(R,Z6,T6,TA,TB,B6,E,QJ,Q4,DT,M,INDX1,N1)
      COE(5)=E(4)+E(5)+B6*ALFA*E(2)/R+B6*E(8)/R
      COE(5)=COE(5)+B6/R*QJ(1,M,8)
      T9=ATAN(Z/(R-(M-2)*DT-DRO )    )
      CALL ENV1(R,Z,T9,TA,TB,B9,E,QJ,Q4,DT,M,INDX1,N1)
      COE(2)=E(7)+B9*( E(3)+ALFA*(E(2)/R+E(6) )    )
      COE(2)=COE(2)-QJ(1,M,7)
      COE(3)=E(4)+B9*( E(6)+ALFA*(E(2)/R+E(3) )    )
      COE(4)=E(1)+B9*( E(2)/R+ALFA*(E(3)+E(6) )    )
      IF(B5.EQ.DT5)GO TO 9
      Z5=Z-2.0*B5
      T5=ATAN(Z5/(R-(M-2)*DT-DRO)    )
      CALL ENV1(R,Z5,T5,TA,TB,B5,E,QJ,Q4,DT,M,INDX1,N1)
      GO TO 11
    9 DO 10 L=1,8
   10 E(L)=(ZN-FLOAT(NZ))*Q1(NZ+1,N1,L)+(FLOAT(NZ+1)-ZN)*Q1(NZ,N1,L)
   11 CONTINUE
      COE(1)=E(4)-E(5)+B5*ALFA*E(2)/R-B5*E(8)/R
      COE(1)=COE(1)+B5/(Z+DRO)*QJ(1,M,7)
     +  -B5/R*QJ(1,M,8)
      IF(M.EQ.N1  )COE(1)=A(1,9)+B5/(Z+DRO)*QJ(1,M,7)
     +     -B5/R*QJ(1,M,8)
      CALL MATINV(AB,COE,6,6,1,DET,KS)
      DO 209 L=1,6
      QJ(1,M,L)=COE(L)
  209 CONTINUE
  200 CONTINUE
      B1=DT5
      B5=DT5

      B6=DT5
      B9=DT
      RETURN
      END
```

```
C
C
      SUBROUTINE ENV1(RL,ZL,TETA,TA,TB,BL,E,QJ,Q4,DT,M,INDX1,N1)
C
C*************************************************************************
C     CALCULATION OF QUANTITIES AT TERMINAL POINTS OF
C     BICHARACTERISTIC CURVES OF POINTS M
C*************************************************************************
C
      COMMON/ALL/I,TOW,R,Z,K,J,DT5,ALFA,BETA,BETA2,G2,BG,CBA,CABA,PIES,N
      DIMENSION QJ(2,INDX1,8),Q4(3,8),E(8)
      TTA=TETA/TA
      TTA1=1.0-TTA
      IF(I.EQ.N1)GO TO 8
      TTB=TETA/TB
      GO TO 9
  8   TTB=0.0
  9   CONTINUE
      TTB1=1.0-TTB
      C2=2.0*BL/DT
      C1=1.0-C2
      DO 10 L=1,8
      E(L)=C1*(TTB*QJ(2,M-1,L)+TTB1*Q4(2,L) )
     + +C2*(TTA*QJ(1,M-1,L)+TTA1*Q4(1,L) )
   10 CONTINUE
      RETURN
      END
C
C
      SUBROUTINE RINTER(Q,QJ,Q4,INDX4,INDX1,ZA,N1)
C************************************************************************
C     INTERMEDIATE TWO-DIMENSIONAL POINTS -- N --
C*************************************************************************
C
      COMMON/ALL/I,TOW,R,Z,K,J,DT5,ALFA,BETA,BETA2,G2,BG,CBA,CABA,PIES,N
      DIMENSION ZA(N1)
      DIMENSION QJ(2,INDX1,8),Q4(3,8),Q(INDX4,INDX1,8)
      Y=I-2*N-1+K
      X=0.5*Y+0.5*FLOAT((J-1)*(J-1))/Y
      A=X-FLOAT(I-N)
      AJAX=X*X-A*A
      AJ=SQRT(AJAX)
      AJM=(FLOAT( I*I-N*N)-AJ*AJ)/FLOAT( 2*(I-N))+1.0
      TJK=ASIN(FLOAT(J-1)/X   )
      TAJ=ASIN(AJ/X)
      TETA=TJK/TAJ
      M=AJM
      C1=(ZA(M)-AJ)/(ZA(M)-ZA(M+1)   )
      DO 10 L=1,8
   10 Q(J,K,L)=TETA*(C1*QJ(1,M+1,L)+(1.-C1)*QJ(1,M,L))+(1.-TETA)*Q4(1,L)
      RETURN
      END
```

```
C
      SUBROUTINE MATINV(A,B,N,N1,MSUB,DET,KS)
      DIMENSION A(1),B(1)
      TOL=0.0
      KS=0
      JJ=-N
      DO 65 J=1,N
      JY=J+1
      JJ=JJ+N+1
      BIGA=0.0
      IT=JJ-J
      DO 30 I=J,N
      IJ=IT+I
      IF(ABS(BIGA)-ABS(A(IJ)))20,30,30
   20 BIGA=A(IJ)
      IMAX=I
   30 CONTINUE
      IF(ABS(BIGA)-TOL)35,35,40
   35 KS=1
      RETURN
   40 I1=J+N*(J-2)
      IT=IMAX-J
      DO 50 K=J,N
      I1=I1+N
      I2=I1+IT
      SAVE=A(I1)
      A(I1)=A(I2)
      A(I2)=SAVE
   50 A(I1)=A(I1)/BIGA
      SAVE=B(IMAX)
      B(IMAX)=B(J)
      B(J)=SAVE/BIGA
      IF(J-N)55,70,55
   55 IQS=N*(J-1)
      DO 65 IX=JY,N
      IXJ=IQS+IX
      IT=J-IX
      DO 60 JX=JY,N
      IXJX=N*(JX-1)+IX
      JJX=IXJX+IT
   60 A(IXJX)=A(IXJX)-(A(IXJ)*A(JJX))
   65 B(IX)=B(IX)-(B(J)*A(IXJ))
   70 NY=N-1
      IT=N*N
      DO 80 J=1,NY
      IA=IT-J
      IB=N-J
      IC=N
      DO 80 K=1,J
      B(IB)=B(IB)-A(IA)*B(IC)
      IA=IA-N
   80 IC=IC-1
```

```
      RETURN
      END
C
C
      SUBROUTINE RESULT(RNE,DRO,DT,I,FF,INDX4,INDX1,T,T1,IJ,N1,
     +TT,DR1,N)
C
      COMMON/RE/IFROM,ITILL,IWRITE,IJ1,IPRINT,NPRINT
      COMMON/PRINT/JPRINT(9),KPRINT(9)
      DIMENSION T(INDX4,INDX1,8),T1(INDX4,INDX1,8)
      DIMENSION TT(INDX4,8)
      IF(IJ .EQ. 2) GO TO 90
      IJ1=1
      IF(IPRINT .EQ. 1) GO TO 75
   75 CALL OUTP(RNE,DRO,DT,I,FF,INDX4,INDX1,TT,DR1,N)
      IJ=2
   90 IF(I .GT. ITILL) RETURN
      GO TO(100,200,300) IPRINT
  100 IF(I .NE. IFROM ) RETURN
      IFROM=IFROM+IWRITE
      IF(MOD(I,2).EQ.0) CALL OUTP1(DRO,DT,I,INDX4,INDX1,T,N1)
      RETURN
  200 CALL OUTP2(DRO,DT,I,INDX4,INDX1,T,T1)
      RETURN
  300 IF(I .NE. IFROM) GO TO 310
      IFROM=IFROM+IWRITE
      IF(MOD(I,2).EQ.0) CALL OUTP1(DRO,DT,I,INDX4,INDX1,T,N1)
  310 CALL OUTP2(DRO,DT,I,INDX4,INDX1,T,T1)
      RETURN
      END
C
C
      SUBROUTINE OUTP(RNE,DRO,DT,I,FF,INDX4,INDX1,TT,DR1,N)
C
C*********************************************************************
C     PRINTS THE VALUES OF THE INPUT CONSTANTS AND THE INITIAL       *
C     CONDITIONS                                                     *
C*********************************************************************
C
      COMMON/LINK/BD(8)
      DIMENSION TT(INDX4,8),TA(8),TB(8),BDI(8)
      INDEX=INDX1-1
      CALL DATE(NDATE)
      WRITE(5,7) NDATE
      WRITE(5,8)
    7 FORMAT(A12/* TRANSIENT RESPONSE OF INFINITELY LONG TUBE*
     +* SUBJECT TO ABRUPTLY APPLIED LOAD(CASE 2)*/)
    8 FORMAT(* DURATION TIME OF LOAD = PERMANENT*/* WIDTH OF*
     +* LOAD = SEMI-INFINITE*/)
      WRITE(5,10) RNE,DRO,DR1,FF,DT,INDEX
   10 FORMAT(* THE INPUT CONSTANTS */21(1H-)//*POISSONS RATIO =*,F5.3
     +/*INNER RADIUS =*,F4.2/*OUTER RADIUS =*,F4.2/*NON-DIMENSIONAL *
```

```
     +*SIRR =*,F4.2/*STEP SIZE FOR INTEGRATION =*,F6.4/*NUMBER OF *
     +*TIME STEPS =*,I3//* THE INITIAL CONDITIONS (TIME  T=0.0)*
     +/38(1H-)/)
      DO 15 L=1,8
   15 BDI(L)=0.
      WRITE(5,20) (BDI(L),TT(1,L),TT(2,L),L=1,8)
   20 FORMAT(*POINTS- R=1.0,Z<0.0*,6X,*POINT - R=1.0,Z=0.0*,6X,
     +*POINTS- R=1.0,Z>0.0*/
     +3(19(1H-),6X)//3(* SITT =*,F7.4,12X)/3(*   UR =*,F7.4,12X)/
     +3(*DURDR =*,F7.4,12X)/3(* SIZZ =*,F7.4,12X)/3(*   UZ =*,F7.4,12X)
     +/3(*DUZDZ =*,F7.4,12X)/3(* SIRR =*,F7.4,12X)/3(* SIRZ =*,F7.4,
     +12X)/72(1H-)/)
      RETURN
      END
C
C
      SUBROUTINE OUTP1(DRO,DT,I,INDX4,INDX1,T,N1)
C
C***********************************************************************
C     PRINTS VALUES OF VARIABLES L AT SPECIFIED TIME FOR POINTS (J,K)
C     FOR J=1,INDEX+4 AND K=1,INDEX+1
C***********************************************************************
C
      DIMENSION T(INDX4,INDX1,8),Q(8)
      TOW=FLOAT(I)*DT
      WRITE (5,40) TOW
   40 FORMAT(//* AT TIME T=*,F7.4/19(1H-)//5X,*Z*,8X,*R*,7X,*SIRR*,5X,
     +*SITT*,5X,*SIZZ*,5X,*SIRZ*,6X,*UR*,7X,*UZ*/72(1H-))
      IA=I+1
      IB=I+2
      IC=I+3
      KK=2
      DO 45 JJ=1,IB
      KK=3-KK
      J=IC-JJ+1
      Z=-DT*FLOAT(J-1)
      WRITE(5,50)
      DO 45 K=KK,N1,2
      R=FLOAT(K-1)*DT+DRO
      DO 42 L=1,8
   42 Q(L)=T(I+2,K,L)-T(J,K,L)
      WRITE(5,70)Z,R,Q(7),Q(1),Q(4),Q(8),Q(2),Q(5)
   45 CONTINUE
      KK=1
      IF(MOD(I,2).EQ.0) KK=2
      DO 60 J=1,IC
      KK=3-KK
      Z=DT*FLOAT(J-1)
      WRITE (5,50)
   50 FORMAT(* *)
      DO 60 K=KK,N1,2
      R=FLOAT(K-1)*DT+DRO
```

```
      TM=-T(J,K,5)
      WRITE (5,70) Z,R,T(J,K,7),T(J,K,1),T(J,K,4),
     +T(J,K,8),T(J,K,2),TM
   60 CONTINUE
   70 FORMAT(8(2X,F7.4))
      WRITE (5,80)
   80 FORMAT(72(1H-))
      RETURN
      END
C
C
      SUBROUTINE OUTP2(DRO,DT,I,INDX4,INDX1,T,T1)
C
C***********************************************************************
C     PRINTS VALUES OF VARIABLES L FOR SPECIFIED POINTS (JPRINT,KPRINT)
C***********************************************************************
C
      COMMON/RE/IFROM,ITILL,IWRITE,IJ1,IPRINT,NPRINT
      COMMON/PRINT/JPRINT(9),KPRINT(9)
      DIMENSION T(INDX4,INDX1,8),T1(INDX4,INDX1,8)
      DIMENSION R(9),Z(9),TOW(51),VAR(9,51,8)
      IF(IJ1 .NE. 1) GO TO 100
      IJ1=2
      DO 10 JJ=1,NPRINT
      R(JJ)=FLOAT(KPRINT(JJ))*DT-DT+DRO
      Z(JJ)=FLOAT(JPRINT(JJ))*DT-DT
      IF(JPRINT(JJ).LT.0)Z(JJ)=Z(JJ)+2.*DT
   10 CONTINUE
      DO 20 L=1,8
      DO 20 LL=1,NPRINT
      TEMP=JPRINT(LL)
      J10=ABS(TEMP)
      K10=KPRINT(LL)
      TOW(1)=0.0
      IF(JPRINT(LL).LT.0)GO TO 15
      VAR(LL,1,L)=T1(I+1,K10,L)-T1(J10,K10,L)
      GO TO 20
 15   VAR(LL,1,L)=VAR(J10,K10,L)
   20 CONTINUE
  100 IF(MOD(I,2) .EQ. 1) RETURN
      IP=I/2+1
      TOW(IP)=FLOAT(I)*DT
      DO 120 JJ=1,NPRINT
      TEMP=JPRINT(JJ)
      J10=ABS(TEMP)
      K10=KPRINT(JJ)
      DO 110 L=1,8
      IF(JPRINT(JJ).LT.0)GO TO 105
      VAR(JJ,IP,L)=T(J10,K10,L)
      GO TO 110
 105  VAR(JJ,IP,L)=T(I+2,K10,L)-T(J10,K10,L)
  110 CONTINUE
```

```
  120 CONTINUE
  300 IF(I .LT. ITILL) RETURN
      DO  340 JJ=1,NPRINT
      WRITE (6,310) JJ,R(JJ),Z(JJ)
C 310 FORMAT(///5X,I2,5X*THE POINT:  R =*,F7.4,2X*Z =*,F7.4/5X,46(1H-)
C    + //7X*TOW*,6X*SIRR*,6X*SITT*,6X*SIZZ*,6X*SIRZ*,7X*UR*,8X*UZ*/
C    +72(1H-))
  310 FORMAT(I2,2F7.4)
      DO 330 KK=1,IP
      WRITE (6,320) TOW(KK),VAR(JJ,KK,7),VAR(JJ,KK,1),VAR(JJ,KK,4),
     +VAR(JJ,KK,8),VAR(JJ,KK,2),VAR(JJ,KK,5)
  320 FORMAT(7(3X,F7.4))
  330 CONTINUE
C     WRITE (6,335)
  335 FORMAT(72(1H-)/)
  340 CONTINUE
      RETURN
      END
```

APPENDIX D

PROGRAM LISTING OF TRES3

```
      PROGRAM TRES3(OUTPUT,TAPE10,TAPE5)
C     ******************************************************************
C
C     CASE3 : TRANSIENT RESPONSE OF AN INFINITELY LONG TUBE SUBJECTED
C             TO A LOAD OF FINITE WIDTH FROM THE INTERIOR. THE DATA OF
C             CASE2 IS RETRIEVED FROM TAPE5 FOR SUPERPOSITION.
C
C             ARRAYS; TOTAL(J,K) - STRESSES AFTER SUPERPOSITION
C                     TU(J,K)    - DATA FROM TAPE5 AT EACH TIME STEP
C                                  LOAD IS SHIFTED HALF WIDTH OF LOAD
C                                  UPWARD.
C                     TL(J,K)    - DATA FROM TAPE5 FOR EACH TIME STEP
C                                  LOAD IS SHIFTED DOWNWARD(Z+DIRECTION)
C
C     ******************************************************************
      DIMENSION TOTAL(600,8),TU(600,8),TL(600,8),ROW(20),L(8)
C
C     INPUT DATA
C     WIDTH STANDS FOR LOAD WIDTH
C     SET NLOAD=1 IF DURATION OF LOAD IS PERMANENT
C         NLOAD=2 IF DURATION OF LOAD IS FINITE
C
      DATA WIDTH,NLOAD/0.04,1/
C
      CALL READ(WIDTH,NW,NLOAD,NWIDTH,NWH,JOE,DT,INDX,INDX1,RI,
     +          SIRR,IDT)
      DO 1000 I=3,INDX,2
      DO 10 J1=1,8
      READ(5,100)(ROW(J),J=1,20)
   10 WRITE(10,100)(ROW(J),J=1,20)
  100 FORMAT(20A4)
      NP=(INDX/2+3-JOE)/2
      NP1=NP+1
      NP2=NP+2
      IF(JOE.EQ.1)LCI=NP1*(2*(I-1)+5)
      IF(JOE.EQ.2)LCI=(NP1+NP2)*(2*(I-1)+4)/2+NP2
      LC=LCI-(2*(I-1)+5)
      LN=2*NP+JOE-1
      CALL RED(I,LCI,LC,LCA,NP,NP1,JOE,NWH,LN,TU,TL)
      KK=2
      NC=2*(I-1)+5+NWIDTH
      KT=0
      DO 12 J=1,NC
      NN=J-(NC+1)/2
      Z=DT*FLOAT(NN)
      KK=3-KK
      DO 12 K=KK,INDX1,2
      KT=KT+1
      TOTAL(KT,2)=FLOAT(K-1)*DT+RI
```

```
      TOTAL(KT,1)=Z
      DO 12 M=3,8
      TOTAL(KT,M)=TU(KT,M)-TL(KT,M)
      IF(Z.EQ.HW)TOTAL(KT,M)=TOTAL(KT-LN*NWH,M)
      IF(Z.EQ.HW.A.M.EQ.8)TOTAL(KT,8)=-TOTAL(KT,8)
   12 CONTINUE
      WRITE(10,200)
      JS=NP1
      JE=LCA-NP
      IF(JOE.EQ.1) GO TO 13
      JS=JS+MOD(NWH,2)
      JE=JE-MOD(NWH,2)
   13 DO 14 J1=JS,JE
      WRITE(10,201) (TOTAL(J1,M),M=1,8)
      IF(J1.EQ.JE) GO TO 14
      IF(TOTAL(J1,1).NE.TOTAL(J1+1,1)) WRITE(10,200)
   14 CONTINUE
      IF(I.EQ.INDX-1) WRITE(10,202)
  200 FORMAT( )
  201 FORMAT(8F9.4)
  202 FORMAT(72(1H-))
 1000 CONTINUE
      STOP
      END
C
      SUBROUTINE READ(WIDTH,HW,NLOAD,NWIDTH,NWH,JOE,DT,INDX,INDX1,RI,
     +                SIRR,IDT)
C-----------------------------------------------------------------------
C     PURPOSE; TO READ AND PRINT THE INITIAL CONDITION
C-----------------------------------------------------------------------
      DIMENSION ROW(20),CI(5,8)
      REWIND 5
      IF(NLOAD.EQ.1) READ(5,100) RI,RO,SIRR,DT,INDX
      IF(NLOAD.EQ.2) READ(5,101) DTL,RI,RO,SIRR,DT,INDX
  100 FORMAT(//////////14X,F4.2/14X,F4.2/22X,F4.2/27X,F6.4/22X,I3)
  101 FORMAT(///24X,F5.2///////14X,F4.2/14X,F4.2/22X,F4.2/27X,F6.4/22X,
     +I3)
      NWIDTH=WIDTH/DT
      IF(MOD(NWIDTH,2).EQ.1)WRITE(10,199)
  199 FORMAT(* WIDTH OF LOAD IS UNSUITABLE *)
      NWH=NWIDTH/2
      WIDTH=2.0*DT*NWH
      HW=WIDTH/2.0
      NWIDTH=WIDTH/DT
      CALL DATE(NDATE)
      WRITE(10,200) NDATE
      IF(NLOAD.EQ.1) WRITE(10,201) WIDTH
      IF(NLOAD.EQ.2) WRITE(10,202) DTL,WIDTH
  200 FORMAT(A12/* TRANSIENT RESPONSE OF INFINITELY LONG *
     +*TUBE SUBJECT TO ABRUPTLY APPLIED LOAD(CASE 3)*/)
  201 FORMAT(* DURATION TIME OF LOAD = PERMANENT*/
     +* WIDTH OF LOAD =*,F4.2)
```

```
  202 FORMAT(* DURATION TIME OF LOAD =*,F5.2/
     +* WIDTH OF LOAD =*,F4.2)
      REWIND 5
      READ(5,102)
  102 FORMAT(////)
      DO 10 J1=1,14
      READ(5,203)(ROW(J),J=1,20)
   10 WRITE(10,203)(ROW(J),J=1,20)
  203 FORMAT(20A4)
      WRITE(10,204) RI,HW,RI,HW,RI,HW,HW,RI,HW,RI,HW
  204 FORMAT(*R=*,F3.1,*,Z<-*,F4.2,4X,*R=*,F3.1,*,Z=-*,F4.2,2X,
     +*R=*,F3.1,*,-*,F4.2,*<Z>*,F4.2,2X,*R=*,F3.1,*,Z=*,F4.2,4X,
     +* R=*,F3.1,*,Z>*,F4.2/13(1H-),4X,13(1H-),2X,18(1H-),2X,
     +12(1H-),5X,12(1H-)/)
      READ(5,103)
  103 FORMAT(//)
      READ(5,104)((CI(J,K),J=1,3),K=1,8)
  104 FORMAT(3(7X,F7.4,12X))
      READ(5,105)
  105 FORMAT( )
      DO 11 K=1,8
      CI(4,K)=CI(2,K)
   11 CI(5,K)=CI(1,K)
      CI(4,5)=-CI(4,5)
      CI(4,6)=-CI(4,6)
      CI(4,8)=-CI(4,8)
      WRITE(10,205)((CI(J,K),J=1,5),K=1,8)
  205 FORMAT(5(* SITT =*,F7.4,3X)/5(*   UR =*,F7.4,3X)/
     +       5(*DURDR =*,F7.4,3X)/5(* SIZZ =*,F7.4,3X)/
     +       5(*   UZ =*,F7.4,3X)/5(*DUZDZ =*,F7.4,3X)/
     +       5(* SIRR =*,F7.4,3X)/5(* SIRZ =*,F7.4,3X)/82(1H-))
      IDT=DTL/DT
      INDX1=INDX/2+1
      IND=INDX1-1
      IF(MOD(IND,2).EQ.0) JOE=2
      IF(MOD(IND,2).EQ.1) JOE=1
      RETURN
      END
C
      SUBROUTINE RED(I,LCI,LC,LCA,NP,NP1,JOE,NWH,LN,TU,TL)
C-----------------------------------------------------------------------
C     PURPOSE; TO READ THE VARIABLES FROM TAPES AND STORE THEM
C              IN THE ARRAY T(J,K)
C-----------------------------------------------------------------------
      DIMENSION TU(600,8),TL(600,8),TT(8)
      NHH=NWH/2
      IF(JOE.EQ.1)LCA=LC+2*NP*NWH
      IF(JOE.EQ.2)LCA=LC+(NWH*NP+NHH)*2+MOD(NWH,2)
      KP=LN*NWH+1
      K1=KP
   10 READ(5,100)(TT(J),J=1,8)
  100 FORMAT(8F9.4)
```

```
      IF(TT(2).EQ.0.0) K1=K1-1
      IF(TT(2).EQ.0.0) GO TO 11
      DO 12 L=1,8
      TL(K1,L)=TT(L)
      TU(K1-KF+1,L)=TT(L)
   12 IF(K1.GT.LCA-LN*NNH) TU(K1,L)=TT(L)
   11 K1=K1+1
      LCI=LCI-1
      IF(LCI.GT.0) GO TO 10
      RETURN
      END
```

APPENDIX E

PROGRAM LISTING OF TRES4

```
      PROGRAM TRES4(OUTPUT,TAPE15,TAPE10)
C    ******************************************************************
C
C     CASE 4 : TRANSIENT RESPONSE OF AN INFINITELY LONG TUBE SUBJECTED
C              TO A MOVING LOAD OF FINITE WIDTH.  THE DATA OF CASE 2
C              IS RETRIEVED FROM TAPE10 FOR SUPERPOSITION.
C
C    ******************************************************************
      COMMON CI(5,6),T(600,8,5)
C
C     INPUT DATA
C     SPEED = SPEED OF TRAVELLING LOAD
C     SPEED IS GIVEN BY SPEED=2*DT/DTL
C     (WHERE DT IS STEP SIZE OF INTEGRATION, DTL IS DURATION OF LOAD)
C
C     IN CASE 4 SPEED OF TRAVELLING LOAD IS GIVEN BY SPEED=1/NS
C
      DATA SPEED/0.5/
C
      CALL READ(SPEED,DTL,WIDTH,RI,RO,DT,INDX)
      INDX1=INDX/2+1
      NSP1=2.005/SPEED+1
      IND=INDX1-1
      IF(MOD(IND,2).EQ.0) JOE=2
      IF(MOD(IND,2).EQ.1) JOE=1
      NP=(INDX/2+3-JOE)/2
      NP1=NP+1
      CALL SUPO(NP,NSP1,JOE,WIDTH,RI,DT,INDX)
      STOP
      END
C
      SUBROUTINE READ(SPEED,DTL,WIDTH,RI,RO,DT,INDX)
C-----------------------------------------------------------------------
C     PURPOSE; TO READ THE INITIAL CONDITIONS FROM TAPE10 AND PRINT
C              THEM ON TAPE15
C-----------------------------------------------------------------------
      COMMON CI(5,6)
      DIMENSION ROW(20)
      REWIND 10
      REWIND 15
      READ(10,100) DTL,WIDTH,RI,RO,DT,INDX
  100 FORMAT(///24X,F5.2/16X,F4.2//////14X,F4.2/14X,F4.2//
     +27X,F6.4/22X,I3)
      READ(10,101)(CI(I,2),I=1,5),(CI(I,5),I=1,5),(CI(I,5),I=1,5),
     +            (CI(I,6),I=1,5),(CI(I,1),I=1,5),(CI(I,4),I=1,5)
  101 FORMAT(///////7X,F7.4,4(10X,F7.4)/7X,F7.4,4(10X,F7.4)//
     +7X,F7.4,4(10X,F7.4)/7X,F7.4,4(10X,F7.4)//7X,F7.4,4(10X,F7.4)/
     +7X,F7.4,4(10X,F7.4))
      DF=SPEED-2.0*DT/DTL
```

```
      IF(DF.GT.1.0E-5) WRITE(15,500)
  500 FORMAT(//* DATA IN TAPE10 IS NOT SUITABLE TO THIS PROBLEM*//)
      CALL DATE(NDATE)
      WRITE(15,200) NDATE,SPEED,WIDTH
  200 FORMAT(A12/* TRANSIENT RESPONSE OF INFINITELY LONG TUBE SUBJECT*
     +* TO IMPULSIVE TRAVELLING LOAD (CASE 4)*//
     +* SPEED OF TRAVELLING LOAD =*,F4.2/* WIDTH OF LOAD =*,F4.2)
      REWIND 10
      READ(10,103)
  103 FORMAT(////)
      DO 1 I=1,26
      READ(10,104)(ROW(J),J=1,20)
    1 WRITE(15,104)(ROW(J),J=1,20)
  104 FORMAT(20A4)
      RETURN
      END
C
      SUBROUTINE SUPO(NP,NSP1,JOE,WIDTH,RI,DT,INDX)
C-----------------------------------------------------------------------
C     PURPOSE; TO FIX THE SCHEME OF SUPERPOSITION
C-----------------------------------------------------------------------
      COMMON CI(5,6),T(600,8,5)
      DIMENSION ROW(20)
      NSP=NSP1-1
      NP1=NP+1
      NP2=NP+2
      NWIDTH=WIDTH/DT
      DO 1000 I=3,INDX,2
      NT=(FLOAT(I)-1.999)/FLOAT(NSP)
      NI=(I-NSP*NT)/2
      NSM=(NSP1+1)/2
      DO 10 L=1,8
      READ(10,100)(ROW(J),J=1,20)
   10 WRITE(15,100)(ROW(J),J=1,20)
  100 FORMAT(20A4)
      NWH=NWIDTH/2
      IF(JOE.LE.1)LCI=NP1*(2*(I-1)+3+NWIDTH)
      IF(JOE.EQ.2)LCI=(NP1+NP2)*(2*(I-1)+2+NWIDTH)/2+NP2
      LC=LCI-(2*(I-1)+3+NWIDTH)
      CALL RED(I,NI,LCI,LC,NSP1,NSM)
      IF(I.LT.NSP1) GO TO 1000
      IF(I.GT.NSP1) GO TO 14
      CALL INT(RI,DT,NI,NSP1,LC,WIDTH)
      GO TO 1000
   14 IF(JOE.LE.1)LCB=LC-2*NP*NSP
      IF(JOE.EQ.2)LCB=LC-(NP+NP1)*NSP
      DO 23 J1=1,LCB
   23 T(J1,1,NI)=T(J1,1,NI)+2.0*DT
      DO 24 JA=1,LC
      DO 24 JB=1,LCB
      IF(T(JB,1,NI).NE.T(JA,1,NSM).OR.T(JB,2,NI).NE.T(JA,2,NSM))
     +GO TO 24
```

```
      DO 25 L=3,8
   25 T(JA,L,NSM)=T(JA,L,NSM)+T(JB,L,NI)
   24 CONTINUE
      WRITE(15,200)
      DO 26 JD=1,LC
      DO 27 L=1,8
   27 T(JD,L,NI)=T(JD,L,NSM)
      WRITE(15,101)(T(JD,M,NI),M=1,8)
  101 FORMAT(8F9.4)
      IF(JD.EQ.LC) GO TO 26
      IF(T(JD,1,NSM).NE.T(JD+1,1,NSM)) WRITE(15,200)
  200 FORMAT( )
   26 CONTINUE
      IF(I.EQ.INDX-1) WRITE(15,201)
  201 FORMAT(72(1H-))
 1000 CONTINUE
      RETURN
      END
C
      SUBROUTINE RED(I,NI,LCI,LC,NSP1,NSM)
C-----------------------------------------------------------------------
C     PURPOSE; TO READ DATA FROM TAPE10
C-----------------------------------------------------------------------
      COMMON CI(5,6),T(600,8,5)
      DIMENSION TT(8)
      K1=1
   13 READ(10,101)(TT(J),J=1,8)
  101 FORMAT(8F9.4)
      IF(TT(2).EQ.0.0) K1=K1-1
      IF(TT(2).EQ.0.0) GO TO 11
      DO 12 L=1,8
      IF(I.LE.NSP1) T(K1,L,NI)=TT(L)
   12 IF(I.GT.NSP1) T(K1,L,NSM)=TT(L)
   11 K1=K1+1
      LCI=LCI-1
      IF(I.LT.NSP1.AND.TT(2).NE.0.0) WRITE(15,101)(TT(J),J=1,8)
      IF(I.LT.NSP1.AND.TT(2).EQ.0.0) WRITE(15,200)
  200 FORMAT( )
      IF(LCI.GT.0) GO TO 13
      RETURN
      END
C
      SUBROUTINE INT(RI,DT,NI,NSP1,LC,WIDTH)
C-----------------------------------------------------------------------
C     PURPOSE; TO SUPERPOSE THE INITIAL CONDITIONS
C-----------------------------------------------------------------------
      COMMON CI(5,6),T(600,8,5)
      WL=WIDTH/2.0+2.0*DT
      WU=-WIDTH/2.0+2.0*DT
      DO 10 J1=1,LC
      IF(T(J1,2,NI).NE.RI) GO TO 10
      IF(T(J1,1,NI).GE.WU) GO TO 11
```

```
      DO 12 L=3,8
   12 T(J1,L,NI)=T(J1,L,NI)+CI(1,L-2)
      GO TO 10
   11 IF(T(J1,1,NI).GT.WU) GO TO 13
      DO 14 L=3,8
   14 T(J1,L,NI)=T(J1,L,NI)+CI(2,L-2)
      GO TO 10
   13 IF(T(J1,1,NI).GE.WL) GO TO 15
      DO 16 L=3,8
   16 T(J1,L,NI)=T(J1,L,NI)+CI(3,L-2)
      GO TO 10
   15 IF(T(J1,1,NI).GT.WL) GO TO 17
      DO 18 L=3,8
   18 T(J1,L,NI)=T(J1,L,NI)+CI(4,L-2)
      GO TO 10
   17 DO 19 L=3,8
   19 T(J1,L,NI)=T(J1,L,NI)+CI(5,L-2)
   10 CONTINUE
      WRITE(15,200)
      DO 20 J1=1,LC
      WRITE(15,101)(T(J1,J,NI),J=1,8)
  101 FORMAT(8F9.4)
      IF(J1.EQ.LC) GO TO 20
      IF(T(J1,1,NI).NE.T(J1+1,1,NI)) WRITE(15,200)
   20 CONTINUE
  200 FORMAT( )
      RETURN
      END
```

APPENDIX F

PROGRAM LISTING OF RECTINP

```
      PROGRAM RECTINP(OUTPUT,TAPE5,TAPE6)
C   *****************************************************************
C
C     CASE 2 : TRANSIENT RESPONSE OF AN INFINITELY LONG TUBE SUBJECTED
C              TO A RECTANGULAR INPUT LOAD.        THE DATA OF CASE 2
C              IS RETRIEVED FROM TAPE5 FOR SUPERPOSITION.
C
C   *****************************************************************
      COMMON CI(3,6),T(600,8,5)
C
C     INPUT DATA
C     NTS = NUMBER OF TIME STEPS WHEN THE CONSTANT LOAD IS REMOVED
C     DURATION OF LOADING IS GIVEN BY DTL=2*DT*NTS
C     (WHERE DT IS STEP SIZE OF INTEGRATION)
C
C     IN CASE 4 SPEED OF TRAVELLING LOAD IS GIVEN BY SPEED=1/NTS
C
      DATA NTS/2/
C
      NS=2*NTS-1
      CALL READ(NS,RI,RO,DT,INDX)
      INDX1=INDX/2+1
      NSP1=NS+2
      IND=INDX1-1
      LN=IND+1
      IF(MOD(IND,2).EQ.0) JOE=2
      IF(MOD(IND,2).EQ.1) JOE=1
      NP=(INDX/2+3-JOE)/2
      NP1=NP+1
      CALL SUPO(NP,NSP1,JOE,RI,DT,INDX,LN,NTS)
      STOP
      END
C
      SUBROUTINE READ(NS,RI,RO,DT,INDX)
C-------------------------------------------------------------------
C     PURPOSE; TO READ THE INITIAL CONDITIONS FROM TAPE5 AND PRINT
C              THEM ON TAPE6
C-------------------------------------------------------------------
      COMMON CI(3,6)
      DIMENSION ROW(20)
      REWIND 5
      REWIND 6
      READ(5,100)RI,RO,DT,INDX
  100 FORMAT(//////////14X,F4.2/14X,F4.2//27X,F6.4/22X,I3)
      READ(5,101)(CI(I,2),I=1,3),(CI(I,5),I=1,3),(CI(I,3),I=1,3),
     +            (CI(I,6),I=1,3),(CI(I,1),I=1,3),(CI(I,4),I=1,3)
  101 FORMAT(///////3(7X,F7.4,12X)/3(7X,F7.4,12X)//
     +3(7X,F7.4,12X)/3(7X,F7.4,12X)//3(7X,F7.4,12X)/3(7X,F7.4,12X))
      DTL=DT*(NS+1)
```

```
      CALL DATE(NDATE)
      WRITE(6,200) NDATE,DTL
  200 FORMAT(A12/* TRANSIENT RESPONSE OF INFINITELY LONG TUBE SUBJECT*
     +* TO RECTANGULAR INPUT LOAD (CASE 2)*//
     +* DURATION TIME OF LOAD =*,F5.2/
     +* WIDTH OF LOAD = SEMI-INFINITE*)
      REWIND 5
      READ(5,103)
  103 FORMAT(////)
      DO 1 I=1,26
      READ(5,104)(ROW(J),J=1,20)
    1 WRITE(6,104)(ROW(J),J=1,20)
  104 FORMAT(20A4)
      RETURN
      END
C
      SUBROUTINE SUPO(NP,NSP1,JOE,RI,DT,INDX,LN,NTS)
C-----------------------------------------------------------------------
C     PURPOSE; TO FIX THE SCHEME OF SUPERPOSITION
C-----------------------------------------------------------------------
      COMMON CI(3,6),T(600,8,5)
      DIMENSION ROW(20),TEM(600,8)
      NSP=NSP1-1
      NP1=NP+1
      NP2=NP+2
      DO 1000 I=3,INDX,2
      NT=(FLOAT(I)-1.999)/FLOAT(NSP)
      NI=(I-NSP*NT)/2
      NSM=(NSP1+1)/2
      DO 10 L=1,8
      READ(5,100)(ROW(J),J=1,20)
   10 WRITE(6,100)(ROW(J),J=1,20)
  100 FORMAT(20A4)
      IF(JOE.LE.1)LCI=NP1*(2*(I-1)+5)
      IF(JOE.EQ.2)LCI=(NP1+NP2)*(2*(I-1)+4)/2+NP2
      LC=LCI-(2*(I-1)+5)
      CALL RED(I,NI,LCI,LC,NSP1,NSM,LN,DT,NTS)
      IF(I.LT.NSP1) GO TO 1000
      IF(I.GT.NSP1) GO TO 14
      CALL INT(RI,DT,NI,NSP1,LC,NSM)
      GO TO 1000
   14 IF(JOE.LE.1)LCB=LC-2*NP*NSP
      IF(JOE.EQ.2)LCB=LC-(NP+NP1)*NSP
      DO 23 L1=1,LC
      DO 23 L=1,8
   23 TEM(L1,L)=T(L1,L,NSM)
      LCB=LCB+NTS*LN
      DO 24 JA=1,LC
      DO 24 JB=1,LCB
      IF(T(JB,1,NI).NE.T(JA,1,NSM).OR.T(JB,2,NI).NE.T(JA,2,NSM))
     +GO TO 24
      DO 25 JC=3,8
```

```
   25 T(JA,JC,NSM)=T(JA,JC,NSM)-T(JB,JC,NI)
   24 CONTINUE
      WRITE(6,200)
      DO 26 JD=1,LC
      DO 27 L=1,8
   27 T(JD,L,NI)=TEM(JD,L)
      IF(JD.LE.LC-LN) GO TO 28
      DO 29 M=1,NTS
      T(JD+M*LN,1,NI)=TEM(JD,1)+2*M*DT
      DO 29 L=2,8
   29 T(JD+M*LN,L,NI)=TEM(JD,L)
   28 WRITE(6,101)(T(JD,M,NSM),M=1,8)
  101 FORMAT(8F9.4)
      IF(JD.EQ.LC) GO TO 26
      IF(T(JD,1,NSM).NE.T(JD+1,1,NSM)) WRITE(6,200)
  200 FORMAT( )
   26 CONTINUE
      IF(I.EQ.INDX-1) WRITE(6,201)
  201 FORMAT(72(1H-))
 1000 CONTINUE
      RETURN
      END
C
      SUBROUTINE RED(I,NI,LCI,LC,NSP1,NSM,LN,DT,NTS)
C-----------------------------------------------------------------------
C     PURPOSE; TO READ DATA FROM TAPE 5
C-----------------------------------------------------------------------
      COMMON CI(3,6),T(600,8,5)
      DIMENSION TT(8)
      K1=1
   13 READ(5,101)(TT(J),J=1,8)
  101 FORMAT(8F9.4)
      IF(TT(2).EQ.0.0) K1=K1-1
      IF(TT(2).EQ.0.0) GO TO 11
      DO 12 L=1,8
      IF(I.LE.NSP1) T(K1,L,NI)=TT(L)
   12 IF(I.GT.NSP1) T(K1,L,NSM)=TT(L)
      IF(K1.LE.LC-LN) GO TO 11
      DO 14 M=1,NTS
      T(K1+M*LN,1,NI)=TT(1)+M*2*DT
      DO 14 L=2,8
   14 T(K1+M*LN,L,NI)=TT(L)
   11 K1=K1+1
      LCI=LCI-1
      IF(I.LT.NSP1.AND.TT(2).NE.0.0) WRITE(6,101)(TT(J),J=1,8)
      IF(I.LT.NSP1.AND.TT(2).EQ.0.0) WRITE(6,200)
  200 FORMAT( )
   15 IF(LCI.GT.0) GO TO 13
      RETURN
      END
C
      SUBROUTINE INT(RI,DT,NI,NSP1,LC,NSM)
```

```
C-----------------------------------------------------------------------
C     PURPOSE; TO SUPERPOSE THE INITIAL CONDITIONS
C-----------------------------------------------------------------------
      COMMON CI(3,6),T(600,8,5)
      DO 10 J1=1,LC
      DO 17 L=1,8
   17 T(J1,L,NSM)=T(J1,L,NI)
      IF(T(J1,2,NI).NE.RI) GO TO 10
      IF(T(J1,1,NI).GE.0.) GO TO 11
      DO 12 L=3,8
   12 T(J1,L,NSM)=T(J1,L,NI)-CI(1,L-2)
      GO TO 10
   11 IF(T(J1,1,NI).GT.0.) GO TO 13
      DO 14 L=3,8
   14 T(J1,L,NSM)=T(J1,L,NI)-CI(2,L-2)
      GO TO 10
   13 DO 16 L=3,8
   16 T(J1,L,NSM)=T(J1,L,NI)-CI(3,L-2)
   10 CONTINUE
      WRITE(6,200)
      DO 20 J1=1,LC
      WRITE(6,101)(T(J1,J,NSM),J=1,8)
  101 FORMAT(8F9.4)
      IF(J1.EQ.LC) GO TO 20
      IF(T(J1,1,NSM).NE.T(J1+1,1,NSM)) WRITE(6,200)
   20 CONTINUE
  200 FORMAT( )
      RETURN
      END
      END
```

APPENDIX G

PROGRAM LISTING OF PLOTALL

```
      PROGRAM PLOTALL(OUTPUT,TAPE1,TAPE3,TAPE5,TAPE7,TAPE10,TAPE12,
     +                TAPE15,TAPE17,TAPE20)
C**********************************************************************
C
C     PURPOSE; TO PLOT THE VARIABLES AT SELECTED POINTS ACCORDING TO
C              TIME ON THE CALCOMP PLOTTER AND TO PRINT THE VARIABLES
C              AT THOSE POINTS ACCORDING TO TIME ON TAPE NTO
C
C              THE DATA OF THIS PROGRAM IS RETRIEVED FROM TAPE NTI
C
C**********************************************************************
      DIMENSION Z(6),R(6),T(6,6,35),XA(35),YA(35)
C
C     INPUT DATA
C     SET NCASE=1 IF NUMBER OF CASE IS 1
C         NCASE=2 FOR CASE2
C         NCASE=3 FOR CASE3
C         NCASE=4 FOR CASE4
C
C     SET NOPTION=1 IF ONLY PLOTTING IS NEEDED
C         NOPTION=2 IF ONLY PRINTING IS NEEDED
C         NOPTION=3 IF BOTH OF PLOTTING AND PRINTING ARE NEEDED
C
C
C     SET NLOAD=1 IF DURATION OF LOAD IS PERMANENT
C         NLOAD=2 IF DURATION OF LOAD IS FINITE
C
C     Z(K) STANDS FOR Z-COORDINATE OF THE SELECTED POINTS
C     R(K) STANDS FOR R-COORDINATE OF THE  SELECTED POINTS
C     K; THE MAXIMUM NUMBER OF THE POINTS IS  6
C
      DATA NCASE,NOPTION,NLOAD/2,3,2/
      DATA (Z(K),K=1,6)/0.00,0.00,0.00,0.08,0.08,0.08/
      DATA (R(K),K=1,6)/1.02,1.14,1.30,1.02,1.14,1.30/
C
      IF(NCASE.EQ.1) NTI=1
      IF(NCASE.EQ.1) NTO=3
      IF(NCASE.EQ.2) NTI=5
      IF(NCASE.EQ.2) NTO=7
      IF(NCASE.EQ.3) NTI=10
      IF(NCASE.EQ.3) NTO=12
      IF(NCASE.EQ.4) NTI=15
      IF(NCASE.EQ.4) NTO=17
      REWIND NTI
      IF(NCASE.NE.4) GO TO 1
      READ(15,100) DTL,WIDTH
  100 FORMAT(///27X,F4.2/16X,F4.2)
      GO TO 2
    1 IF(NCASE.NE.2) GO TO 3
```

```
      IF(NLOAD.EQ.1) READ(5,101)
  101 FORMAT(////)
      IF(NLOAD.EQ.2) READ(5,102) DTL
  102 FORMAT(///25X,F4.2/)
      GO TO 2
    3 IF(NCASE.NE.3) GO TO 4
      IF(NLOAD.EQ.1) READ(10,103) WIDTH
  103 FORMAT(////16X,F4.2)
      IF(NLOAD.EQ.2) READ(10,104) DTL,WIDTH
  104 FORMAT(///25X,F4.2/16X,F4.2)
    4 IF(NCASE.EQ.1) READ(1,101)
    2 READ(NTI,105) RI,RO,DT,INDX
  105 FORMAT(/////14X,F4.2/14X,F4.2//27X,F6.2/22X,I3)
      INDX1=INDX/2+1
      INDX2=INDX1+1
      IND=INDX1-1
      IF(MOD(IND,2).EQ.0) JOE=2
      IF(MOD(IND,2).EQ.1) JOE=1
      CALL READ(DT,INDX,INDX1,IND,JOE,WIDTH,Z,R,T,XA,
     +          NCASE,NTI,NTO)
      IF(NOPTION.NE.1) CALL WRIT(NLOAD,DTL,WIDTH,INDX1,IND,Z,R,T,XA,
     +                           NCASE,NTI,NTO)
      IF(NOPTION.NE.2) CALL SBPL(NLOAD,DTL,WIDTH,DT,INDX1,IND,INDX2,
     +                           Z,R,T,XA,YA,NCASE,NTI,NTO)
      STOP
      END
C
      SUBROUTINE READ(DT,INDX,INDX1,IND,JOE,WIDTH,Z,R,T,XA,
     +                NCASE,NTI,NTO)
C-----------------------------------------------------------------------
C     PURPOSE; TO READ DATA FROM TAPE10
C-----------------------------------------------------------------------
      DIMENSION Z(6),R(6),T(6,6,IND),XA(IND),TT(8),CI(5,6)
C
C     READ INTIAL CONDITIONS
C
      IF(NCASE.GT.1) GO TO 20
      READ(1,100)(CI(K,2),K=2,3),(CI(K,5),K=2,3),(CI(K,3),K=2,3),
     +           (CI(K,6),K=2,3),(CI(K,1),K=2,3),(CI(K,4),K=2,3)
  100 FORMAT(///////7X,F7.4,39X,F7.4/7X,F7.4,39X,F7.4//7X,
     +       F7.4,39X,F7.4/7X,F7.4,39X,F7.4//7X,F7.4,39X,F7.4/
     +       7X,F7.4,39X,F7.4/)
      GO TO 21
   20 IF(NCASE.GT.2) GO TO 22
      READ(5,101)(CI(K,2),K=1,3),(CI(K,5),K=1,3),(CI(K,3),K=1,3),
     +           (CI(K,6),K=1,3),(CI(K,1),K=1,3),(CI(K,4),K=1,3)
  101 FORMAT(///////7X,F7.4,2(19X,F7.4)/7X,F7.4,2(19X,F7.4)//7X,F7.4,
     +       2(19X,F7.4)/7X,F7.4,2(19X,F7.4)//7X,F7.4,2(19X,F7.4)/7X,
     +       F7.4,2(19X,F7.4)/)
      GO TO 21
   22 READ(NTI,102)(CI(K,2),K=1,5),(CI(K,5),K=1,5),(CI(K,3),K=1,5),
     +              (CI(K,6),K=1,5),(CI(K,1),K=1,5),(CI(K,4),K=1,5)
```

```
102 FORMAT(///////7X,F7.4,4(10X,F7.4)/7X,F7.4,4(10X,F7.4)//
   +7X,F7.4,4(10X,F7.4)/7X,F7.4,4(10X,F7.4)//7X,F7.4,4(10X,F7.4)/
   +7X,F7.4,4(10X,F7.4)/)
 21 DO 1 K=1,6
    IF(R(K).NE.1.0) GO TO 1
    WL=WIDTH/2.0
    WU=-WL
    IF(NCASE.LE.2) WU=0.0
    IF(Z(K).GE.WU) GO TO 2
    DO 3 L=1,6
  3 T(K,L,1)=CI(1,L)
    GO TO 1
  2 IF(Z(K).GT.WU) GO TO 4
    DO 5 L=1,6
  5 T(K,L,1)=CI(2,L)
    GO TO 1
  4 IF(NCASE.LE.2) GO TO 23
    IF(Z(K).GE.WL) GO TO 6
 23 DO 7 L=1,6
  7 T(K,L,1)=CI(3,L)
    GO TO 1
  6 IF(Z(K).GT.WL) GO TO 8
    DO 9 L=1,6
  9 T(K,L,1)=CI(4,L)
    GO TO 1
  8 DO 10 L=1,6
 10 T(K,L,1)=CI(5,L)
  1 CONTINUE
    NWIDTH=WIDTH/DT
    NP=(INDX1+2-JOE)/2
    NP1=NP+1
    NP2=NP+2
    DO 11 I=2,INDX
    IF(NCASE.EQ.1) GO TO 40
    IF(MOD(I,2).EQ.0) GO TO 11
 40 IF(NCASE.GT.1) GO TO 24
    IF(JOE.EQ.1) LCI=NP1*(I+2)
    RM=(I-2)*0.5+2.0
    IM=RM
    DM=RM-IM
    IF(DM.EQ.0.0.A.JOE.EQ.2) LCI=(NP1+NP2)*IM
    IF(DM.GT.0.0.A.JOE.EQ.2) LCI=(NP1+NP2)*IM+NP2
    GO TO 25
 24 IF(NCASE.GT.2) GO TO 26
    IF(JOE.EQ.1) LCI=NP1*(3+2*I)
    IF(JOE.EQ.2) LCI=(NP1+NP2)*(I+1)+NP2
    GO TO 25
 26 IF(NCASE.NE.3) GO TO 27
    IF(JOE.EQ.1) LCI=NP1*(2*(I-1)+3+NWIDTH)
    IF(JOE.EQ.2) LCI=(NP1+NP2)*(2*(I-1)+2+NWIDTH)/2+NP1
    GO TO 25
 27 IF(JOE.EQ.1) LCI=NP1*(2*(I-1)+3+NWIDTH)
```

```
      IF(JOE.EQ.2) LCI=(NP1+NP2)*(2*(I-1)+2+NWIDTH)/2+NP1
C
C     READ STRESSES AT SPECIFIED POINTS
C
   25 READ(NTI,103)
  103 FORMAT(///////)
   12 READ(NTI,104)(TT(J),J=1,8)
  104 FORMAT(8F9.4)
      IH=I/2+1
      DO 13 K=1,6
      IF(Z(K).NE.TT(1).OR.R(K).NE.TT(2)) GO TO 13
      XA(IH)=DT*(I-1)
      DO 14 L=3,8
   14 T(K,L-2,IH)=TT(L)
   13 CONTINUE
      LCI=LCI-1
      IF(LCI.GT.0) GO TO 12
   11 CONTINUE
      RETURN
      END
C
      SUBROUTINE WRIT(NLOAD,DTL,WIDTH,INDX1,IND,Z,R,T,XA,NCASE,NTI,NTO)
C-----------------------------------------------------------------------
C     PURPOSE; TO PRINT THE VARIABLES AT THE SPECIFIED POINTS
C              ACCORDING TO TIME
C-----------------------------------------------------------------------
      DIMENSION Z(6),R(6),T(6,6,IND),XA(IND)
      REWIND NTO
      IF(NCASE.GT.1) GO TO 10
      WRITE(3,200) NCASE
  200 FORMAT(5X,*CASE*,I2/5X,*DURATION TIME OF LOAD = PERMANENT*/5X,
     +       *WIDTH OF LOAD = SEMI-INFINITE*)
      GO TO 11
   10 IF(NCASE.GT.2) GO TO 12
      IF(NLOAD.EQ.1) WRITE(7,200) NCASE
      IF(NLOAD.EQ.2) WRITE(7,201) NCASE,DTL
  201 FORMAT(5X,*CASE*,I2/5X,*DURATION TIME OF LOAD = *,F6.3/5X,
     +       *WIDTH OF LOAD = SEMI-INFINITE*)
      GO TO 11
   12 IF(NCASE.NE.3) GO TO 13
      IF(NLOAD.EQ.1) WRITE(12,202) NCASE,WIDTH
  202 FORMAT(5X,*CASE*,I2/5X,*DURATION TIME OF LOAD = PERMANENT*/5X,
     +       *WIDTH OF LOAD =*,F6.3)
      IF(NLOAD.EQ.2) WRITE(12,203) NCASE,DTL,WIDTH
  203 FORMAT(5X,*CASE*,I2/5X,*DURATION TIME OF LOAD =*,F6.3/5X,
     +       *WIDTH OF LOAD =*,F6.3)
      GO TO 11
   13 WRITE(17,204) NCASE,DTL,WIDTH
  204 FORMAT(5X,*CASE*,I2/5X,*SPEED OF TRAVELLING LOAD =*,F6.3/5X,
     +       *WIDTH OF LOAD =*,F6.3)
   11 DO 1 K=1,6
      WRITE(NTO,205) R(K),Z(K)
```

```
  205 FORMAT(/////* THE POINT  R=*,F6.3,* Z=*,F7.3/32(1H-)//5X,*TOW*
     +,5X,*SIRR*,5X,*SITT*,5X,*SIZZ*,5X,*SIRZ*,6X,*UR*,7X,*UZ*/63(1H-))
      DO 2 I=1,IND
    2 WRITE(NTO,206) XA(I),(T(K,J,I),J=1,6)
  206 FORMAT(7F9.4)
    1 CONTINUE
      RETURN
      END
C
      SUBROUTINE SEPL(NLOAD,DTL,WIDTH,DT,INDX1,IND,INDX2,Z,R,T,XA,YA,
     +                NCASE,NTI,NTO)
C-----------------------------------------------------------------------
C     PURPOSE; TO PLOT THE VARIABLES AT THE SELECTED POINTS
C              ACCORDING TO TIME
C-----------------------------------------------------------------------
      DIMENSION Z(6),R(6),T(6,6,IND),XA(INDX2),YA(INDX2)
      CALL PLOTS(20)
      CALL OPTION(2)
      DO 1 K=1,6
      IF(K.EQ.1) CALL PLOT(0.5,1.5,-3)
      IF(K.GT.1) CALL PLOT(15.5,0.0,-3)
      CALL FACTOR(0.55)
      YA(INDX1)=-3.0
      YA(INDX2)=0.5
      XA(INDX1)=0.0
      XA(INDX2)=DT*FLOAT(IND)/5.0
C
C     DRAW X,Y-AXIS
C
      CALL AXIS(0.,0.,9HTIME-AXIS,-9,10.0,0.,XA(INDX1),XA(INDX2))
      CALL AXIS(0.,0.,13HVARIABLE-AXIS,13,11.0,90.,YA(INDX1),YA(INDX2))
      DO 2 I=1,6
      DO 3 J=1,IND
    3 YA(J)=T(K,I,J)
      Y1=12-I*0.5
C
C     PLOT STRESSES
C
      IF(I.LE.2) CALL NEWPEN(3)
      IF(I.GT.2.AND.I.LT.5) CALL NEWPEN(4)
      IF(I.GE.5) CALL NEWPEN(2)
      CALL LINE(XA,YA,IND,1,1,I)
      CALL SYMBOL(8.78,Y1,.21,I,0.,-1)
      IF(I.EQ.1) CALL SYMBOL(999.,999.,.21,5H-SIRR,0.,5)
      IF(I.EQ.2) CALL SYMBOL(999.,999.,.21,5H-SITT,0.,5)
      IF(I.EQ.3) CALL SYMBOL(999.,999.,.21,5H-SIZZ,0.,5)
      IF(I.EQ.4) CALL SYMBOL(999.,999.,.21,5H-SIRZ,0.,5)
      IF(I.EQ.5) CALL SYMBOL(999.,999.,.21,3H-UR,0.,3)
      IF(I.EQ.6) CALL SYMBOL(999.,999.,.21,3H-UZ,0.,3)
    2 CONTINUE
      FCASE=NCASE
      CALL NEWPEN(1)
```

APPENDIX H

PLOTS OF DEPENDENT VARIABLES

CASE 1
DURATION TIME OF LOAD = PERMANENT
WIDTH OF LOAD = SEMI-INFINITE

THE POINT R= 1.000 Z= 0.000

TOW	SIRR	SITT	SIZZ	SIRZ	UR	UZ
0.0000	1.0000	.1765	0.0000	0.0000	-1.0000	.1765
.0400	1.0000	.1391	0.0000	0.0000	-.9707	.1695
.0800	1.0000	.1029	0.0000	0.0000	-.9417	.1634
.1200	1.0000	.0678	0.0000	0.0000	-.9130	.1574
.1600	1.0000	.0337	0.0000	0.0000	-.8847	.1516
.2000	1.0000	.0008	0.0000	0.0000	-.8568	.1460
.2400	1.0000	-.0312	0.0000	0.0000	-.8293	.1405
.2800	1.0000	-.0621	0.0000	0.0000	-.8021	.1353
.3200	1.0000	-.0920	0.0000	0.0000	-.7755	.1302
.3600	1.0000	-.1208	0.0000	0.0000	-.7492	.1252
.4000	1.0000	-.1487	0.0000	0.0000	-.7235	.1205
.4400	1.0000	-.1757	0.0000	0.0000	-.6982	.1159
.4800	1.0000	-.2017	0.0000	0.0000	-.6734	.1115
.5200	1.0000	-.2267	0.0000	0.0000	-.6491	.1072
.5600	1.0000	-.2508	0.0000	0.0000	-.6253	.1031

THE POINT R= 1.120 Z= 0.000

TOW	SIRR	SITT	SIZZ	SIRZ	UR	UZ
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
.0400	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
.0800	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
.1200	.8840	.1560	0.0000	0.0000	-.8840	.1667
.1600	.8798	.1259	0.0000	0.0000	-.8567	.1597
.2000	.8756	.0968	0.0000	0.0000	-.8298	.1534
.2400	.8715	.0685	0.0000	0.0000	-.8032	.1474
.2800	.8674	.0412	0.0000	0.0000	-.7771	.1418
.3200	.8634	.0148	0.0000	0.0000	-.7514	.1364
.3600	.8594	-.0108	0.0000	0.0000	-.7261	.1313
.4000	.8556	-.0355	0.0000	0.0000	-.7012	.1264
.4400	.8517	-.0594	0.0000	0.0000	-.6769	.1218
.4800	-.0360	-.2385	0.0000	0.0000	-1.5369	-.0494
.5200	-.0349	-.2902	0.0000	0.0000	-1.5319	-.0247
.5600	-.0246	-.3402	0.0000	0.0000	-1.5181	-.0665

THE POINT R= 1.300 Z= .020

TOW	SIRR	SITT	SIZZ	SIRZ	UR	UZ
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
.0400	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
.0800	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
.1200	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
.1600	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
.2000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
.2400	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
.2800	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
.3200	0.0000	-.0299	.0038	0.0000	-1.4927	-.0088
.3600	0.0000	-.0743	-.0038	0.0000	-1.4704	-.0088
.4000	0.0000	-.1179	-.0110	0.0000	-1.4529	-.0236
.4400	0.0000	-.1588	-.0028	0.0000	-1.4373	-.0373
.4800	0.0000	-.2006	-.0040	0.0000	-1.4225	-.0439
.5200	0.0000	-.2419	-.0041	0.0000	-1.4085	-.0518
.5600	0.0000	-.2828	-.0041	0.0000	-1.3951	-.0597

THE POINT R= 1.000 Z= .080

TOW	SIRR	SITT	SIZZ	SIRZ	UR	UZ
0.0000	1.0000	.1765	.1765	0.0000	-1.0000	0.0000
.0400	1.0000	.1382	.1707	0.0000	-.9768	0.0000
.0800	1.0000	.0797	.0261	0.0000	-.9633	.1383
.1200	1.0000	.0442	.0268	0.0000	-.9200	.1316
.1600	1.0000	.0101	.0271	0.0000	-.8800	.1254
.2000	1.0000	-.0225	.0274	0.0000	-.8429	.1194
.2400	1.0000	-.0537	.0277	0.0000	-.8081	.1135
.2800	1.0000	-.0836	.0280	0.0000	-.7751	.1079
.3200	1.0000	-.1124	.0283	0.0000	-.7439	.1024
.3600	1.0000	-.1399	.0287	0.0000	-.7141	.0972
.4000	1.0000	-.1664	.0290	0.0000	-.6856	.0921
.4400	1.0000	-.1918	.0293	0.0000	-.6582	.0872
.4800	1.0000	-.2162	.0296	0.0000	-.6319	.0824
.5200	1.0000	-.2396	.0299	0.0000	-.6066	.0779
.5600	1.0000	-.2621	.0302	0.0000	-.5822	.0735

THE POINT R= 1.120 Z= .080

TOW	SIRR	SITT	SIZZ	SIRZ	UR	UZ
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
.0400	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
.0800	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
.1200	.9449	.1667	.1667	0.0000	-.9449	0.0000
.1600	.9107	.0909	.0181	-.0096	-.8660	.1408
.2000	.8985	.0623	.0231	-.0208	-.8244	.1299
.2400	.8895	.0338	.0237	-.0208	-.7892	.1235
.2800	.8816	.0065	.0240	-.0192	-.7562	.1177
.3200	.8746	-.0195	.0243	-.0176	-.7250	.1122
.3600	.8682	-.0445	.0245	-.0163	-.6955	.1070
.4000	.8623	-.0684	.0247	-.0153	-.6674	.1020
.4400	.8569	-.0913	.0248	-.0143	-.6406	.0972
.4800	-.0931	-.2800	-.1417	-.0135	-1.5597	.0926
.5200	-.0492	-.3358	-.0050	-.0153	-1.5203	-.0532
.5600	-.0841	-.3709	-.0180	-.0268	-1.5425	-.0482

THE POINT R= 1.300 Z= .060

TOW	SIRR	SITT	SIZZ	SIRZ	UR	UZ
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
.0400	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
.0800	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
.1200	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
.1600	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
.2000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
.2400	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
.2800	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
.3200	0.0000	-.0422	.0105	0.0000	-1.5093	-.0164
.3600	0.0000	-.2978	-.5008	0.0000	-1.5329	.4742
.4000	0.0000	-.2694	-.0174	0.0000	-1.4902	-.0148
.4400	0.0000	-.3123	-.0175	0.0000	-1.4547	-.0227
.4800	0.0000	-.3533	-.0107	0.0000	-1.4312	-.0374
.5200	0.0000	-.3949	-.0120	0.0000	-1.4102	-.0442
.5600	0.0000	-.4357	-.0120	0.0000	-1.3921	-.0521

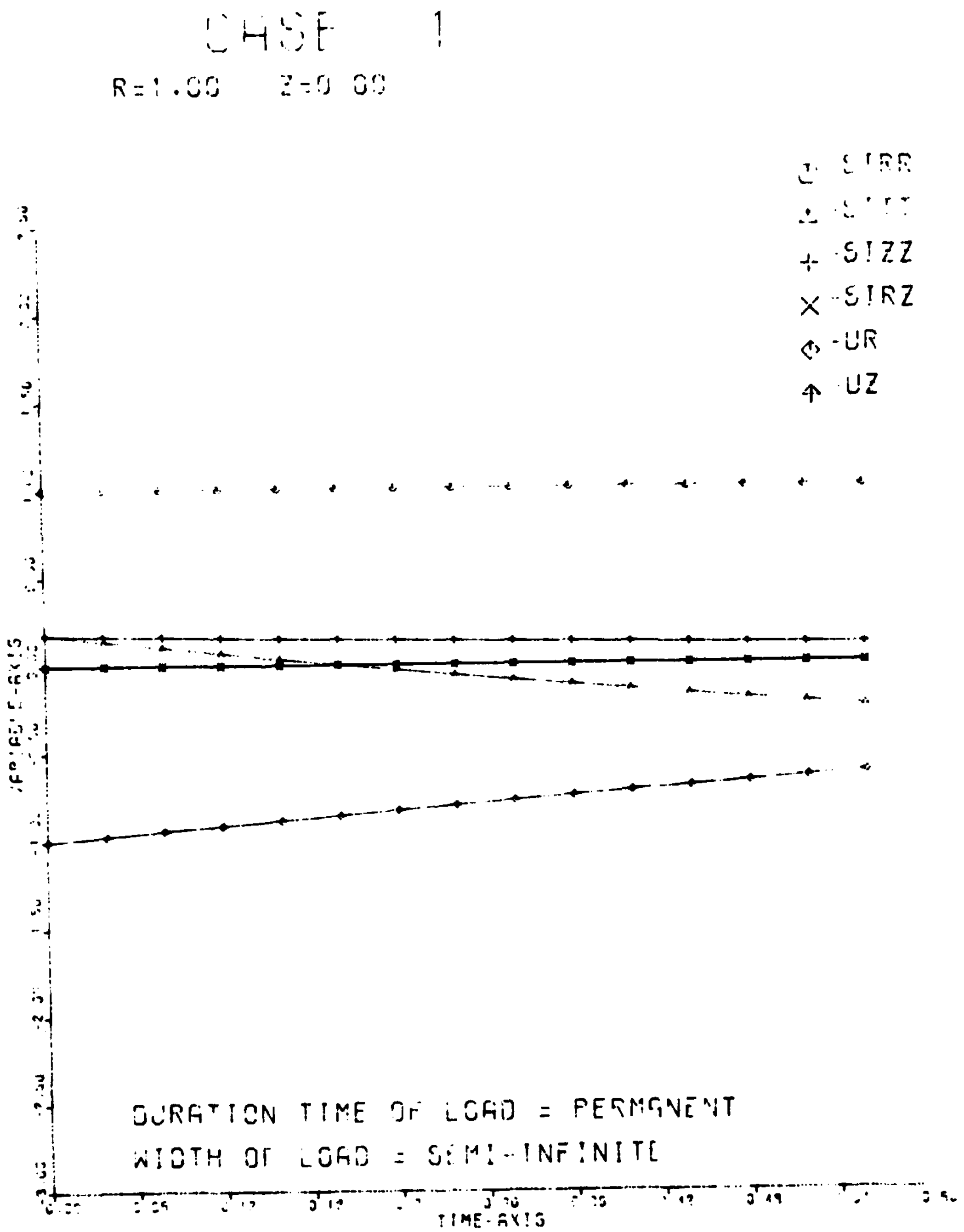
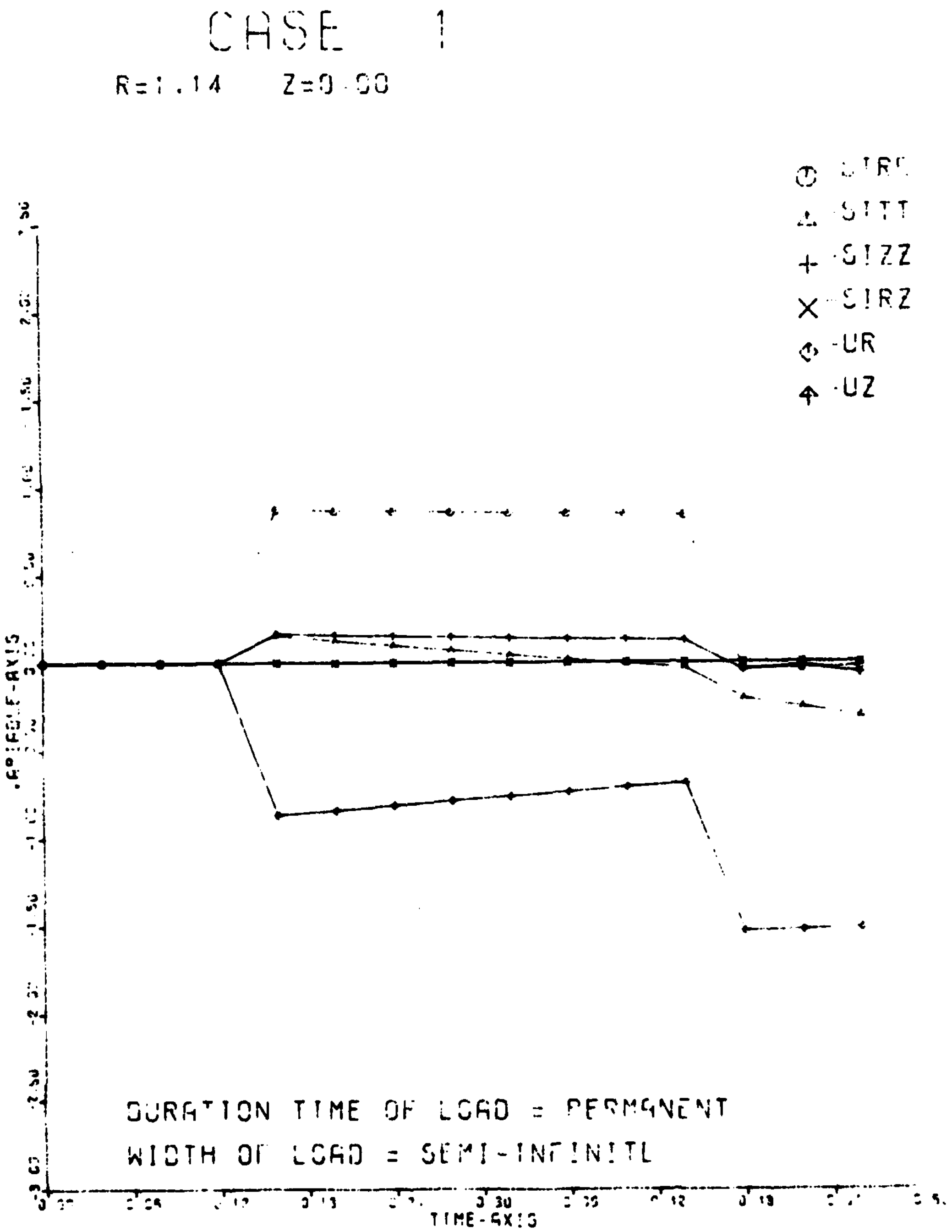


Figure 10



CASE 1
R=1.14 Z=0.00
SIRR
SITT
SIZZ
SIRZ
UR
UZ
TIME-AXIS
DURATION TIME OF LOAD = PERMANENT
WIDTH OF LOAD = SEMI-INFINITE

Figure 11

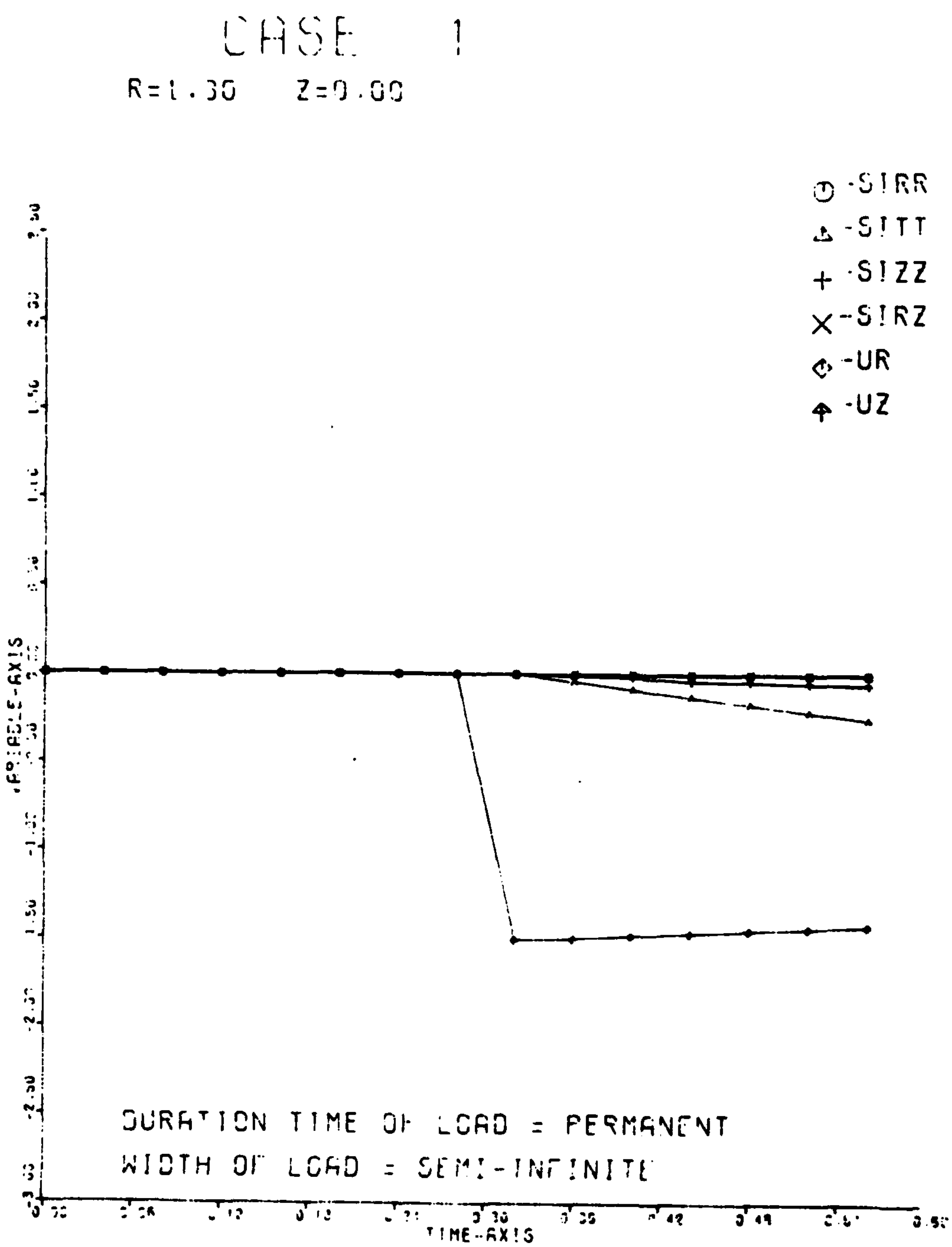

CASE 1
R=1.30 Z=0.00
-SIRR
-SITT
-SIZZ
-SIRZ
-UR
-UZ
DURATION TIME OF LOAD = PERMANENT
WIDTH OF LOAD = SEMI-INFINITE
TIME-AXIS

Figure 12

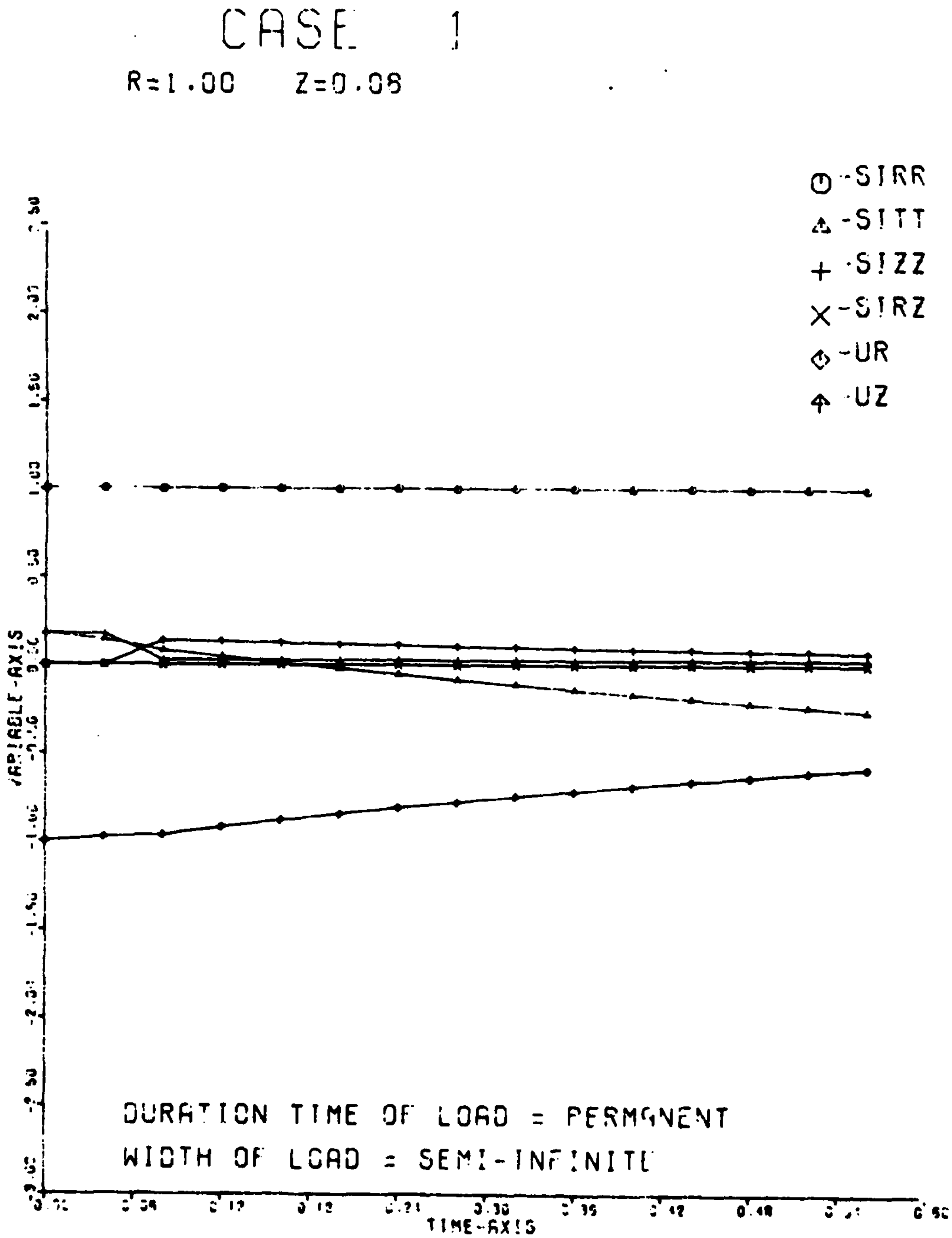

CASE 1
R=1.00 Z=0.06
O -SIRR
△ -SITT
+ -SIZZ
X -SIRZ
◇ -UR
↑ -UZ
VARIABLE-AXIS
TIME-AXIS
DURATION TIME OF LOAD = PERMANENT
WIDTH OF LOAD = SEMI-INFINITE

Figure 13

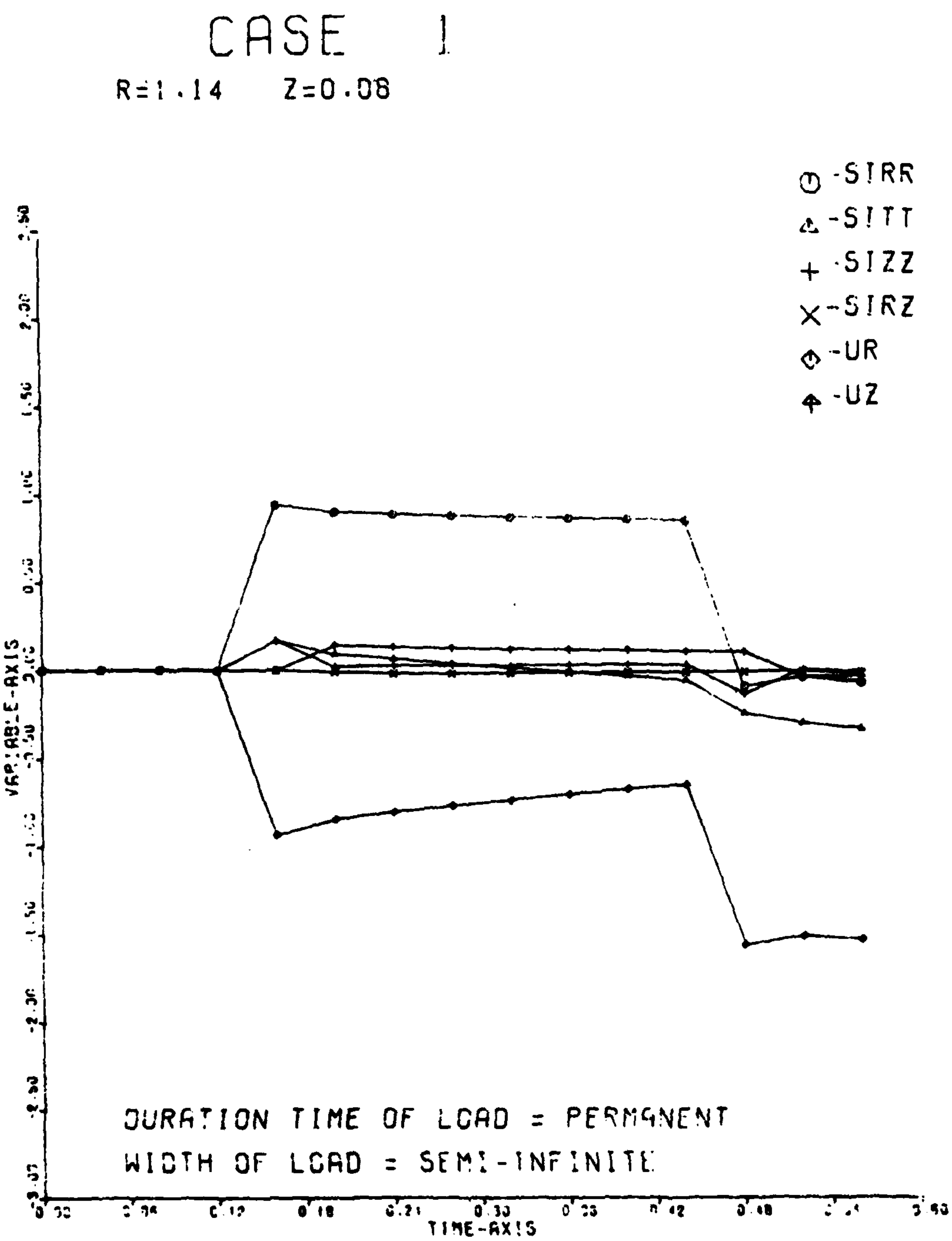

CASE 1
R=1.14 Z=0.08
-SIRR
-SITT
-SIZZ
-SIRZ
-UR
-UZ
VARIABLE-AXIS
TIME-AXIS
DURATION TIME OF LOAD = PERMANENT
WIDTH OF LOAD = SEMI-INFINITE

Figure 14

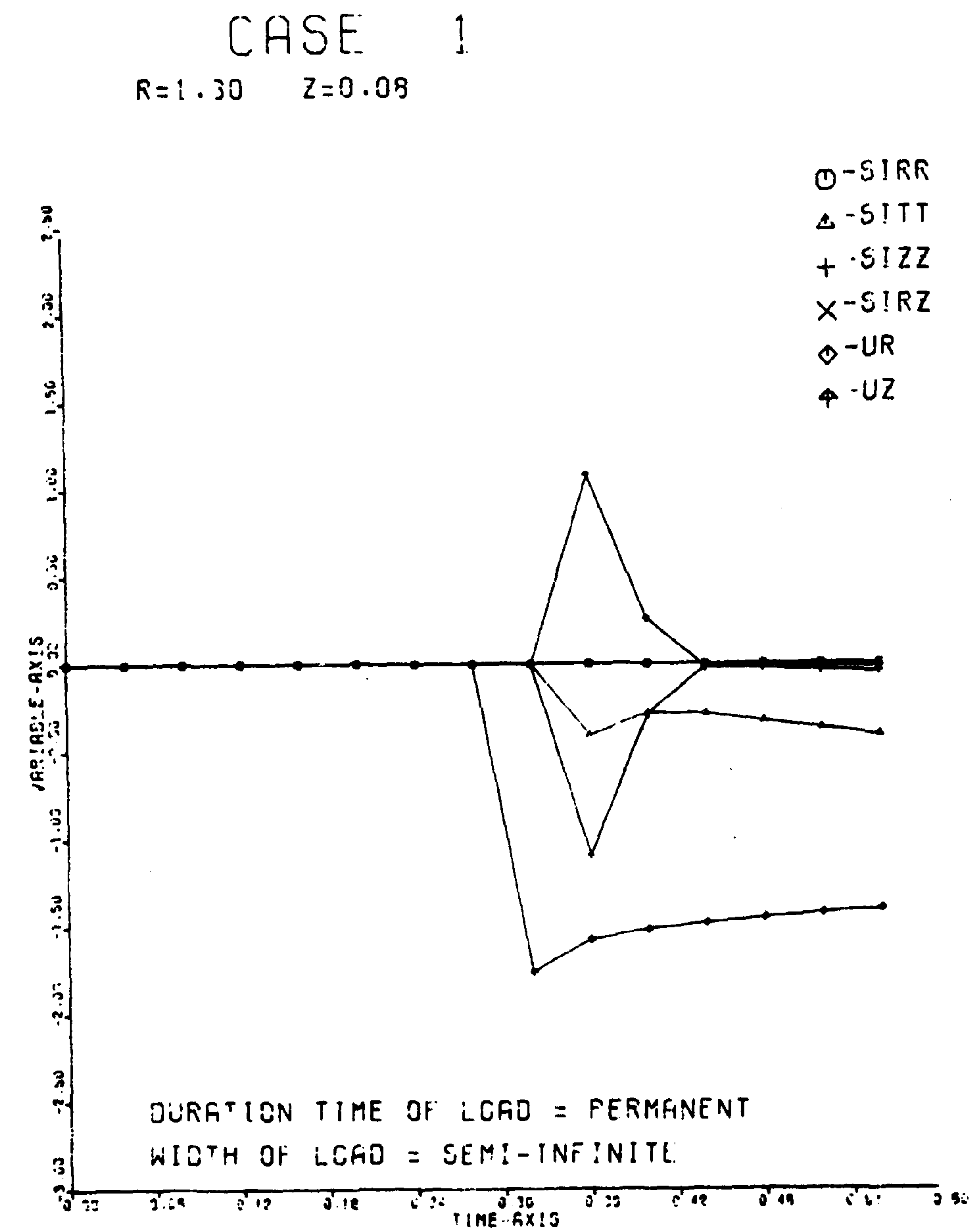

CASE 1
R=1.30 Z=0.08
-SIRR
-SITT
-SIZZ
-SIRZ
-UR
-UZ
VARIABLE-AXIS
TIME-AXIS
DURATION TIME OF LOAD = PERMANENT
WIDTH OF LOAD = SEMI-INFINITE

Figure 15

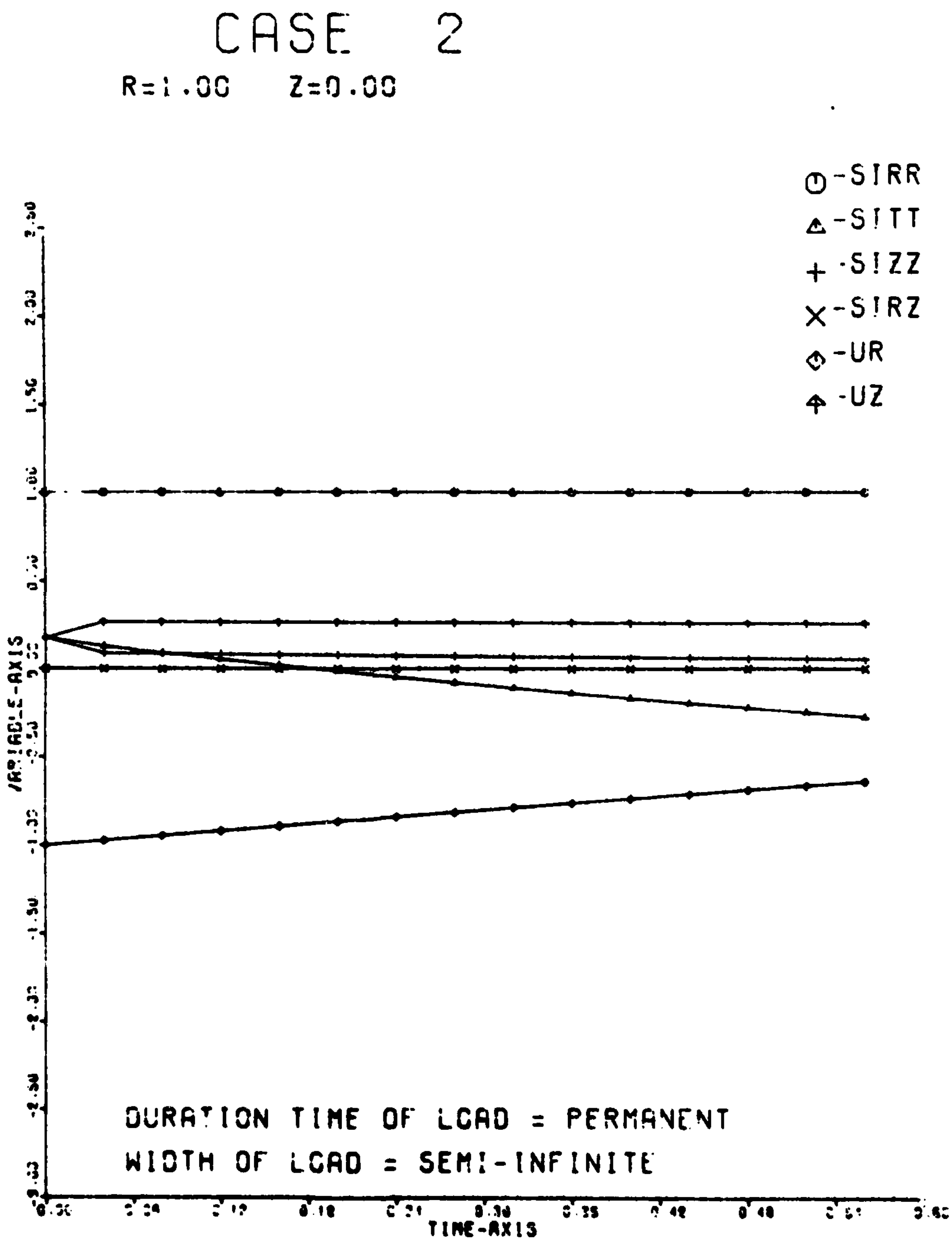

CASE 2
R=1.00 Z=0.00
-SIRR
-SITT
-SIZZ
-SIRZ
-UR
-UZ
VARIABLE-AXIS
TIME-AXIS
DURATION TIME OF LOAD = PERMANENT
WIDTH OF LOAD = SEMI-INFINITE

Figure 16

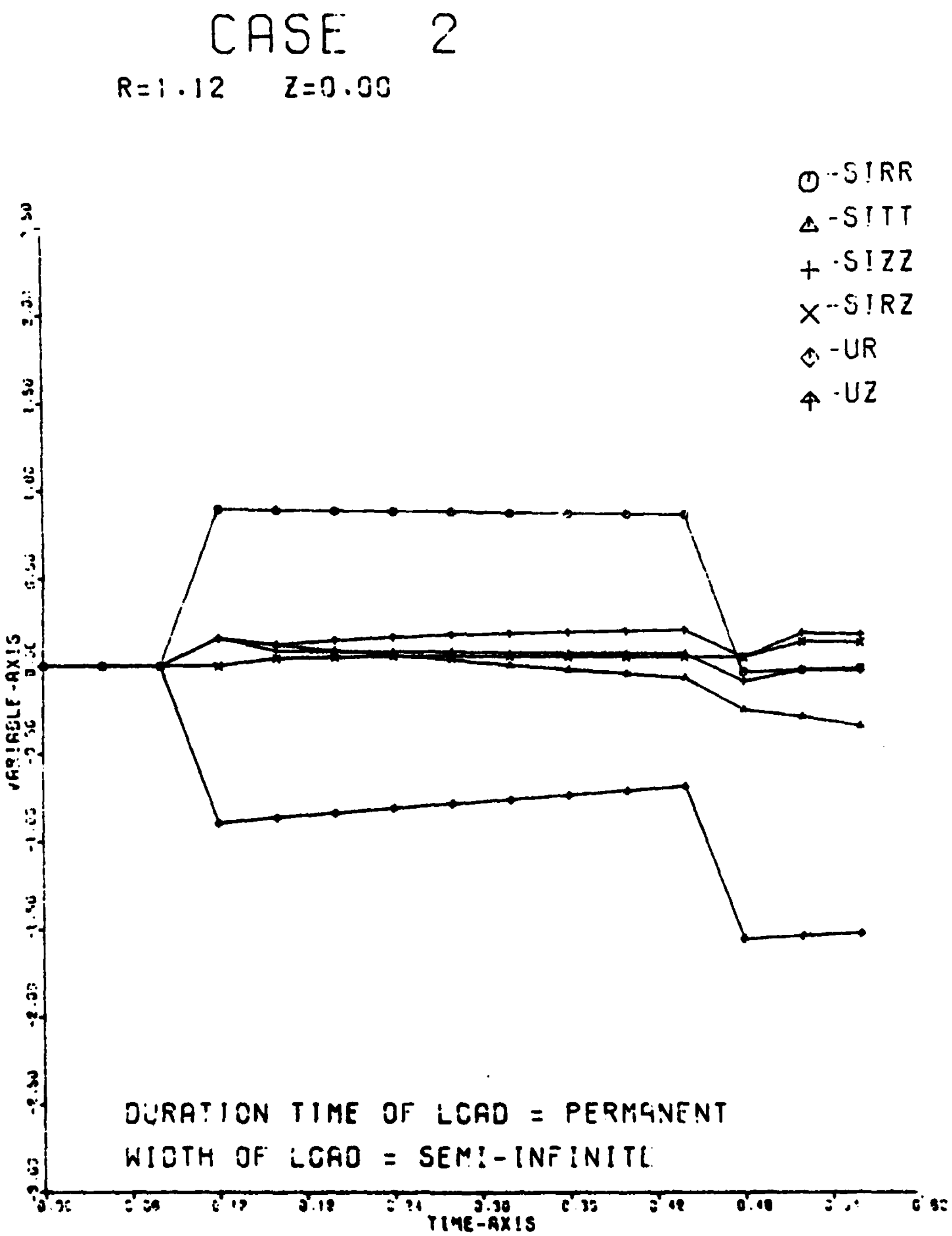

CASE 2
R=1.12 Z=0.00
-SIRR
-SITT
-SIZZ
-SIRZ
-UR
-UZ
VARIABLE-AXIS
TIME-AXIS
DURATION TIME OF LOAD = PERMANENT
WIDTH OF LOAD = SEMI-INFINITE

Figure 17

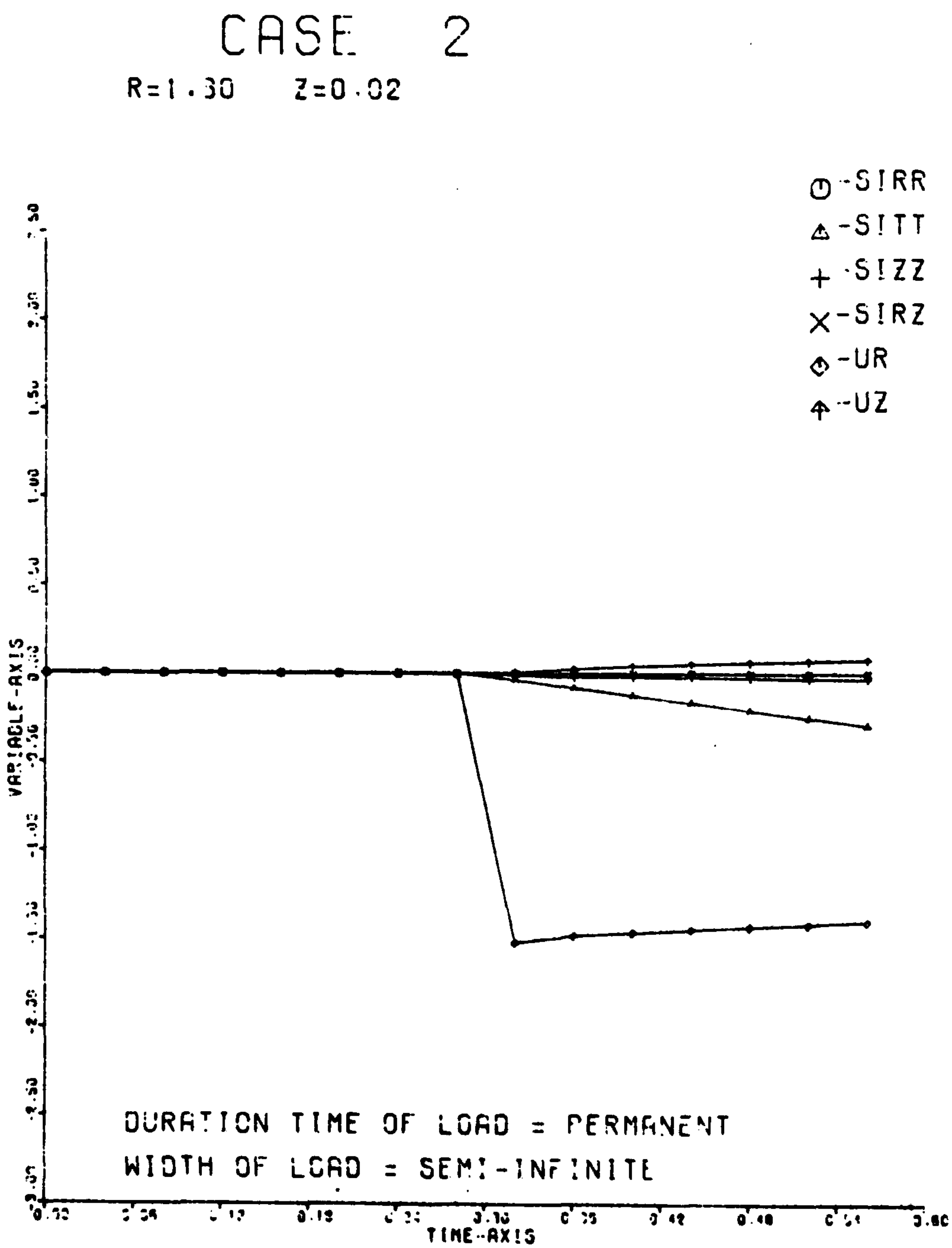

CASE 2
R=1.30 Z=0.02
-SIRR
-SITT
-SIZZ
-SIRZ
-UR
-UZ
VARIABLE-AXIS
DURATION TIME OF LOAD = PERMANENT
WIDTH OF LOAD = SEMI-INFINITE
TIME-AXIS

Figure 18

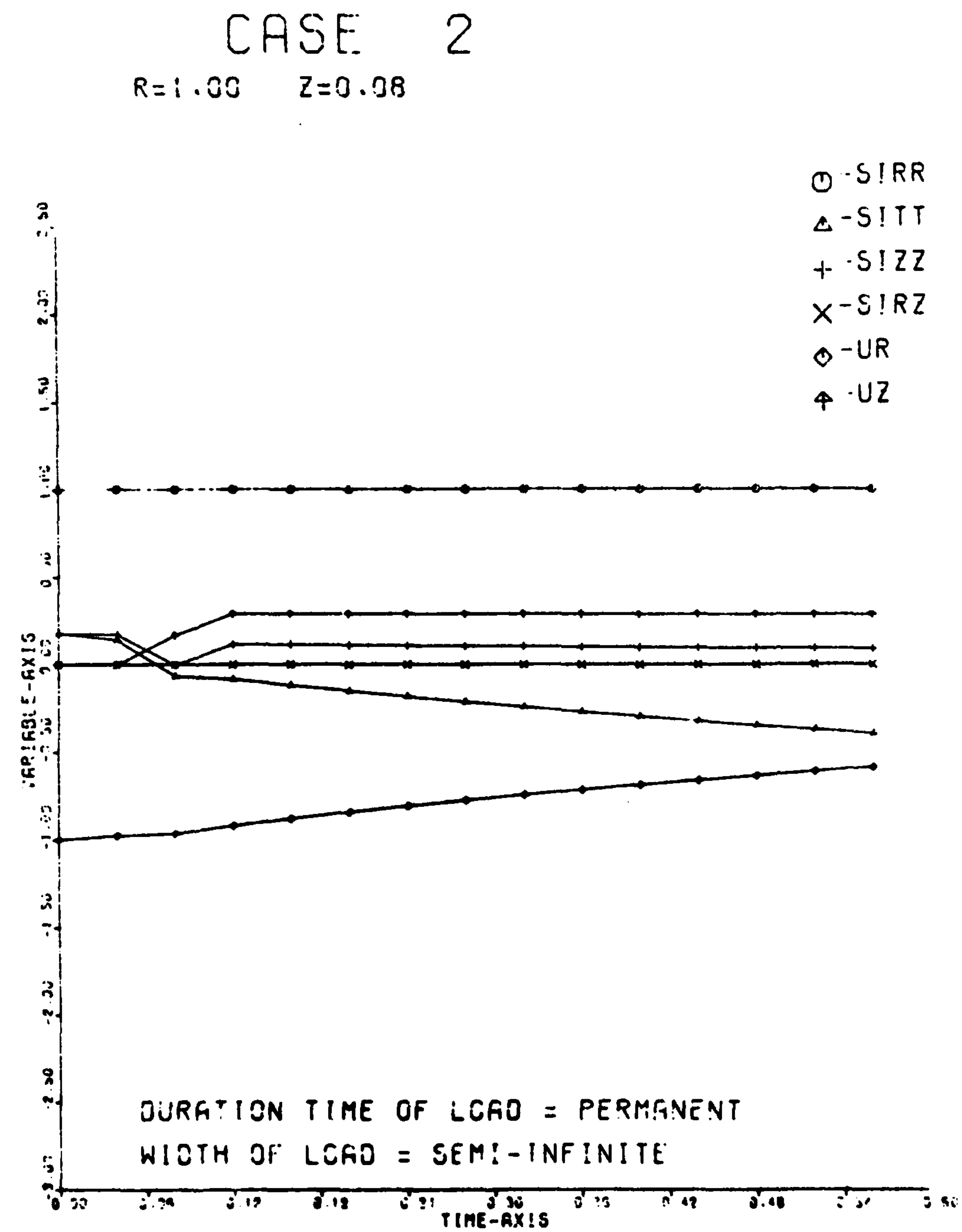

CASE 2
R=1.00 Z=0.08
-SIRR
-SITT
-SIZZ
-SIRZ
-UR
-UZ
VARIABLE-AXIS
TIME-AXIS
DURATION TIME OF LOAD = PERMANENT
WIDTH OF LOAD = SEMI-INFINITE

Figure 19

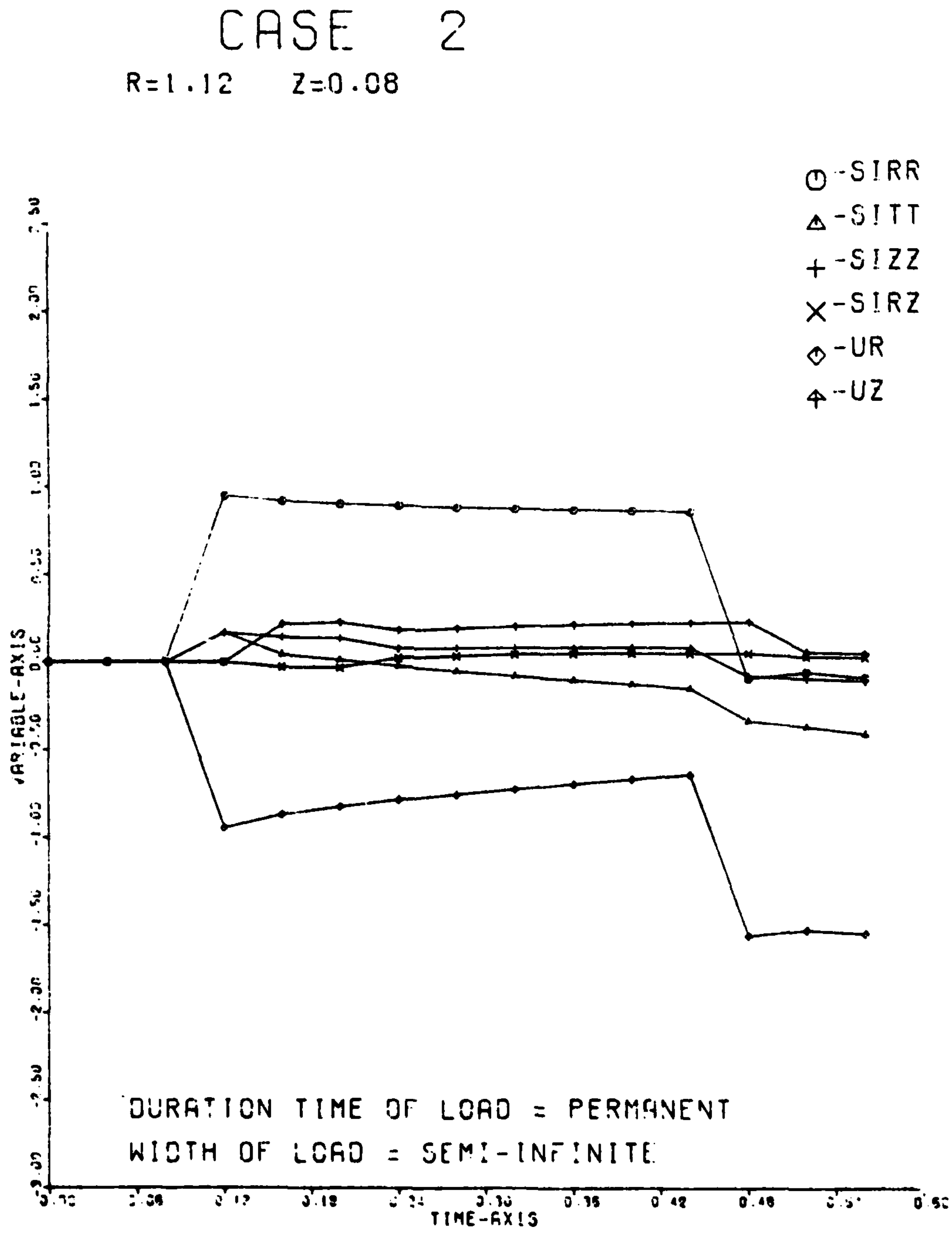

CASE 2
R=1.12 Z=0.08
-SIRR
-SITT
-SIZZ
-SIRZ
-UR
-UZ
VARIABLE-AXIS
TIME-AXIS
DURATION TIME OF LOAD = PERMANENT
WIDTH OF LOAD = SEMI-INFINITE

Figure 20

CASE 2

R=1.30 Z=0.06

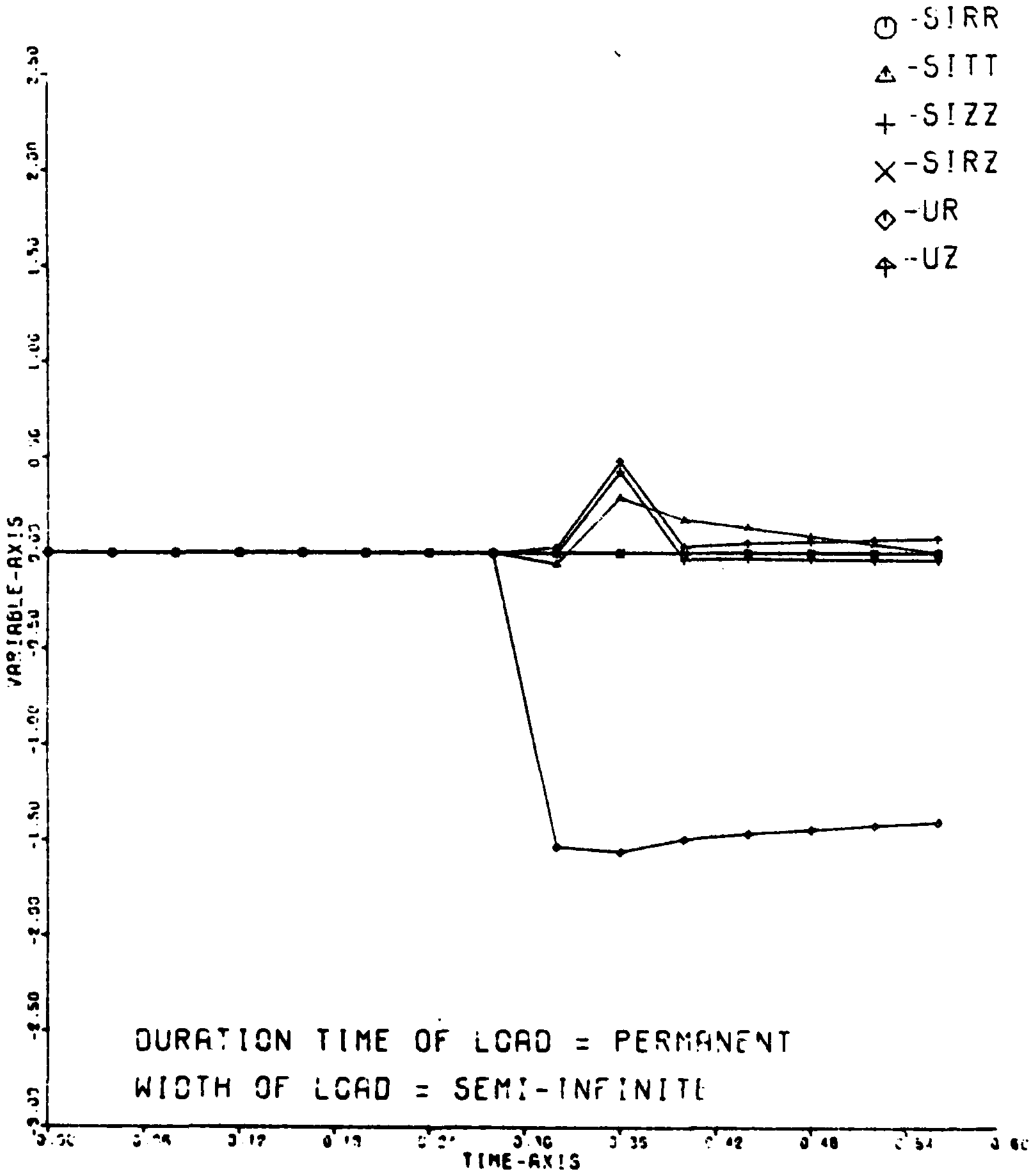


Figure 21

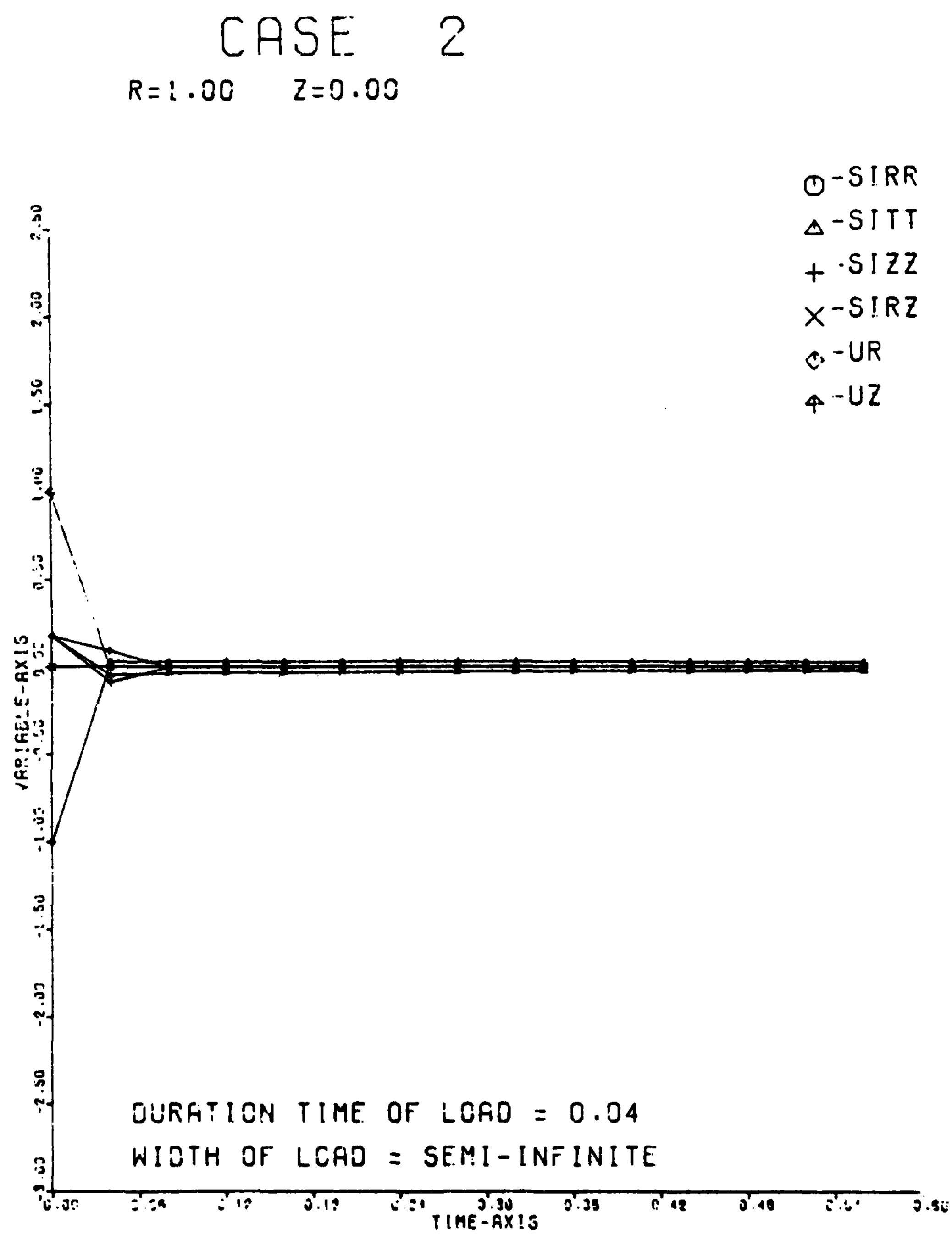

CASE 2
R=1.00 Z=0.00
-SIRR
-SITT
-SIZZ
-SIRZ
-UR
-UZ
VARIABLE-AXIS
TIME-AXIS
DURATION TIME OF LOAD = 0.04
WIDTH OF LOAD = SEMI-INFINITE

Figure 22

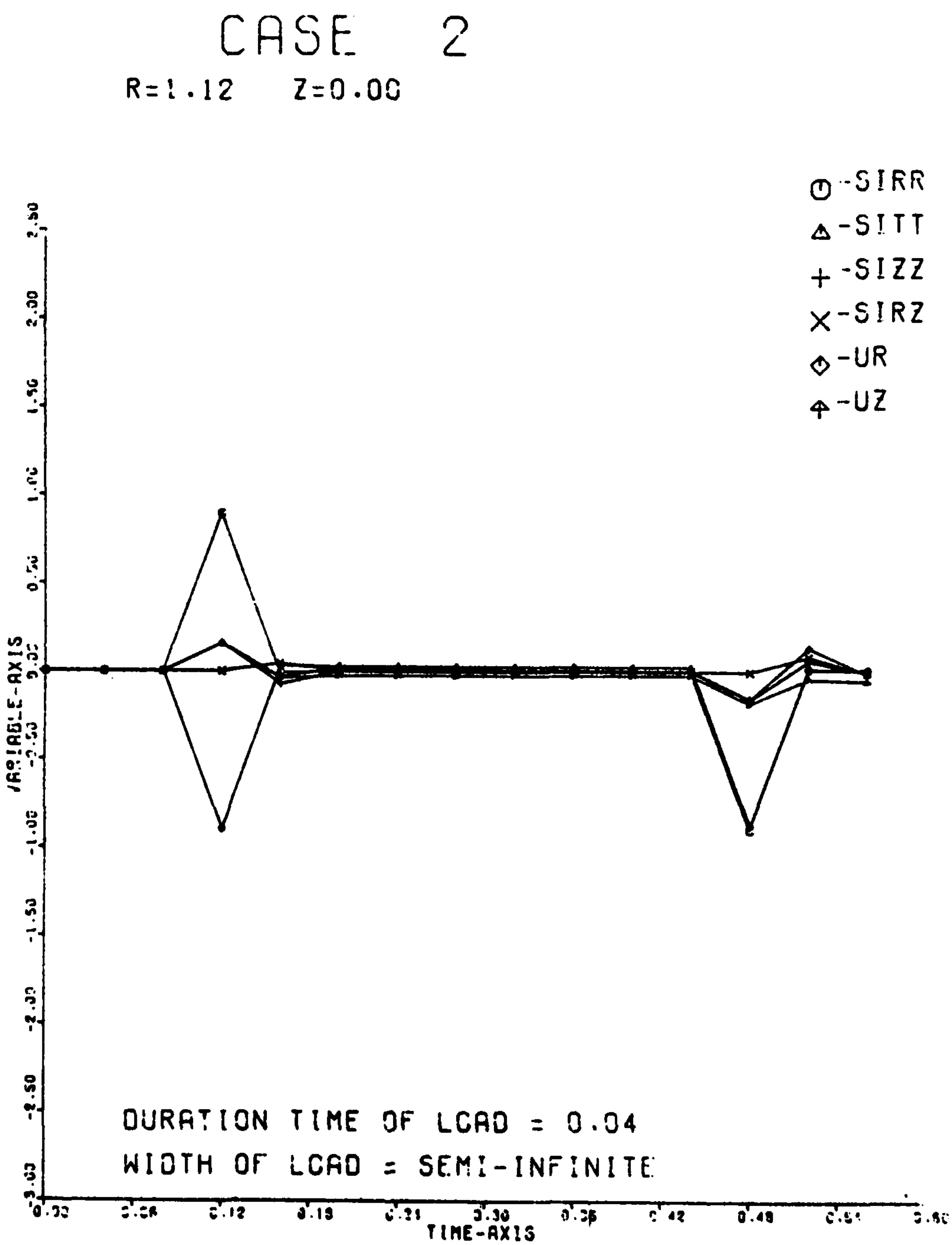

CASE 2
R=1.12 Z=0.00
O-SIRR
△-SITT
+-SIZZ
×-SIRZ
◇-UR
♁-UZ
VARIABLE-AXIS
TIME-AXIS
DURATION TIME OF LOAD = 0.04
WIDTH OF LOAD = SEMI-INFINITE

Figure 23

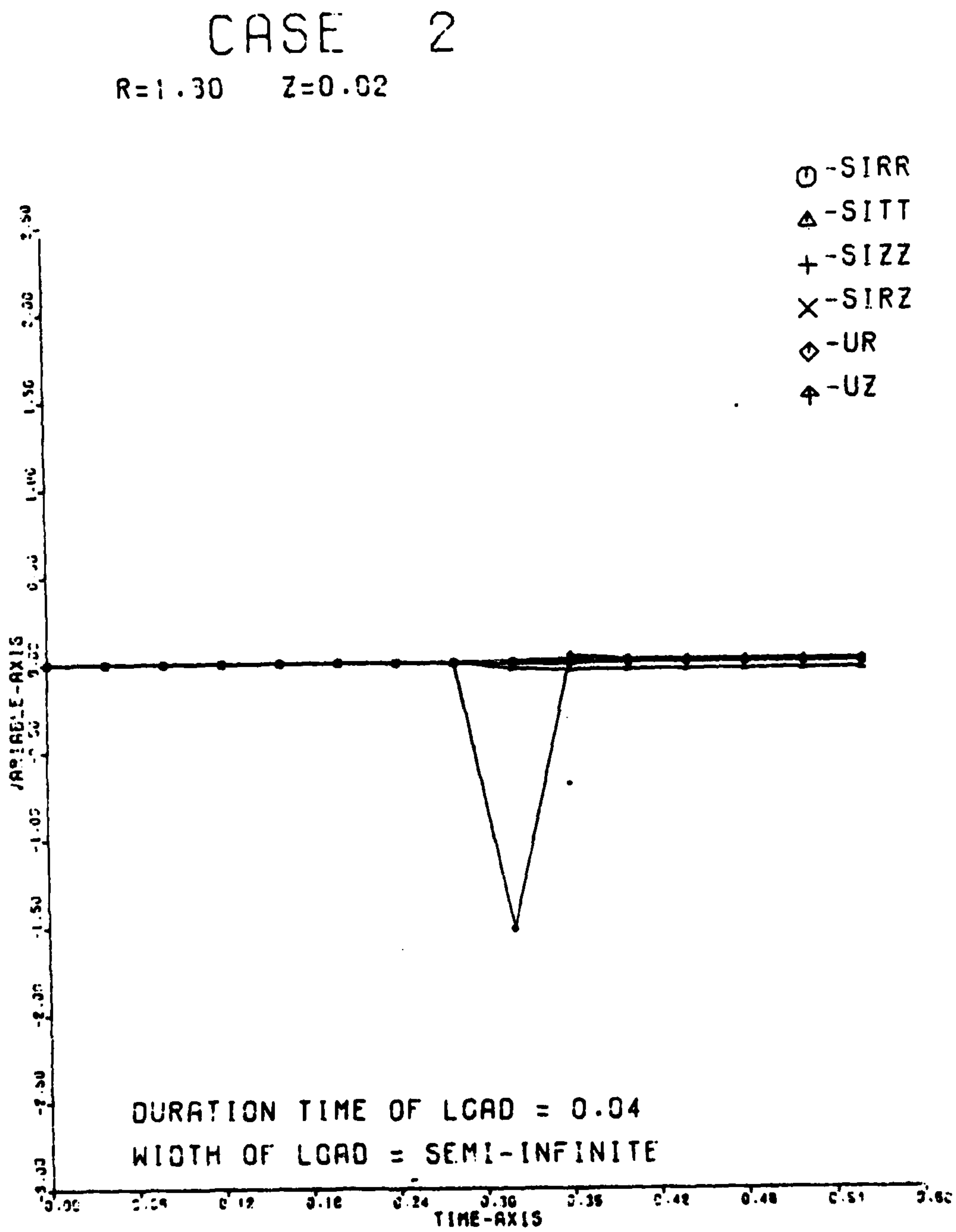

CASE 2
R=1.30 Z=0.02
-SIRR
-SITT
-SIZZ
-SIRZ
-UR
-UZ
VARIABLE-AXIS
TIME-AXIS
DURATION TIME OF LOAD = 0.04
WIDTH OF LOAD = SEMI-INFINITE

Figure 24

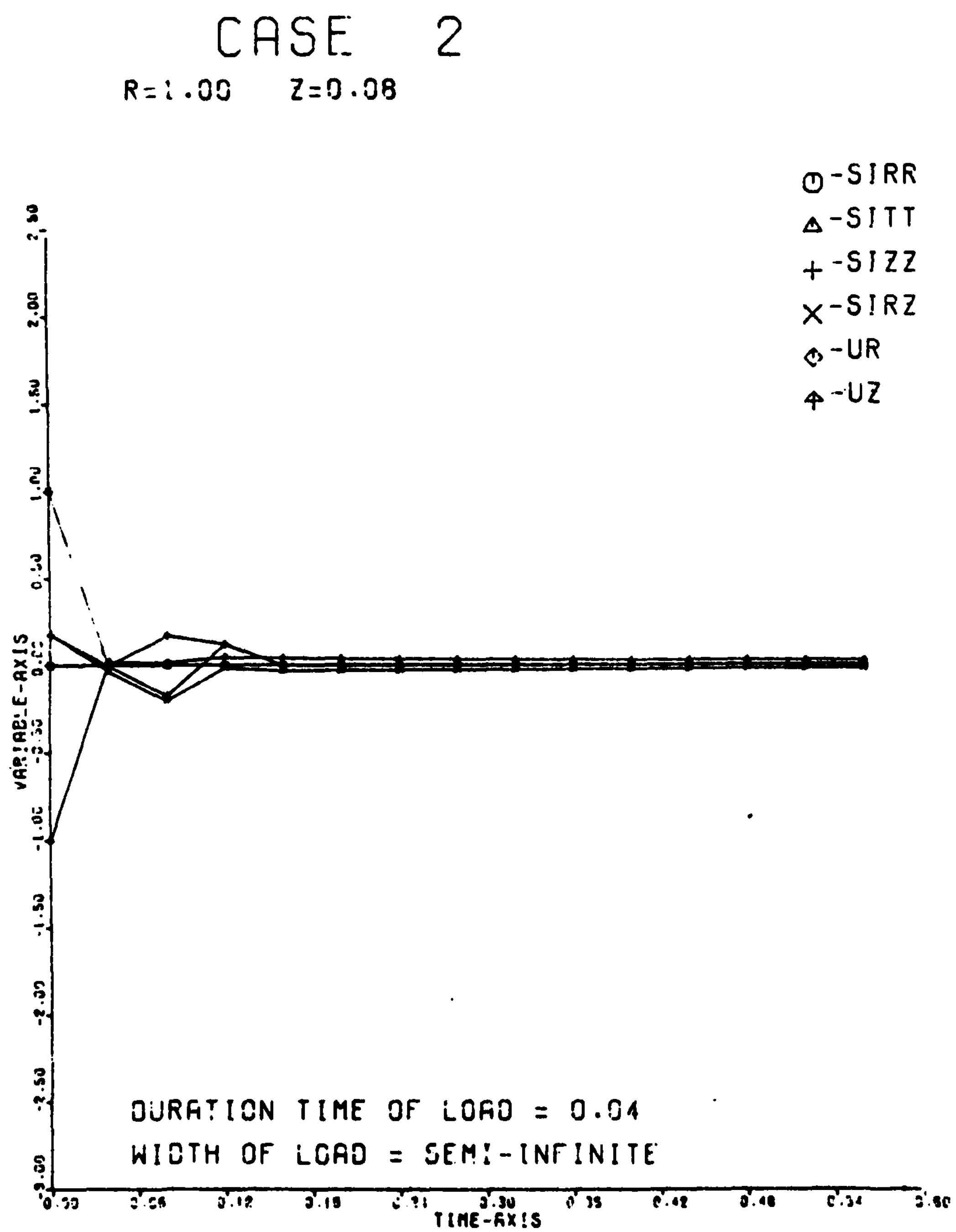

CASE 2
R=1.00 Z=0.08
-SIRR
-SITT
-SIZZ
-SIRZ
-UR
-UZ
VARIABLE-AXIS
TIME-AXIS
DURATION TIME OF LOAD = 0.04
WIDTH OF LOAD = SEMI-INFINITE

Figure 25

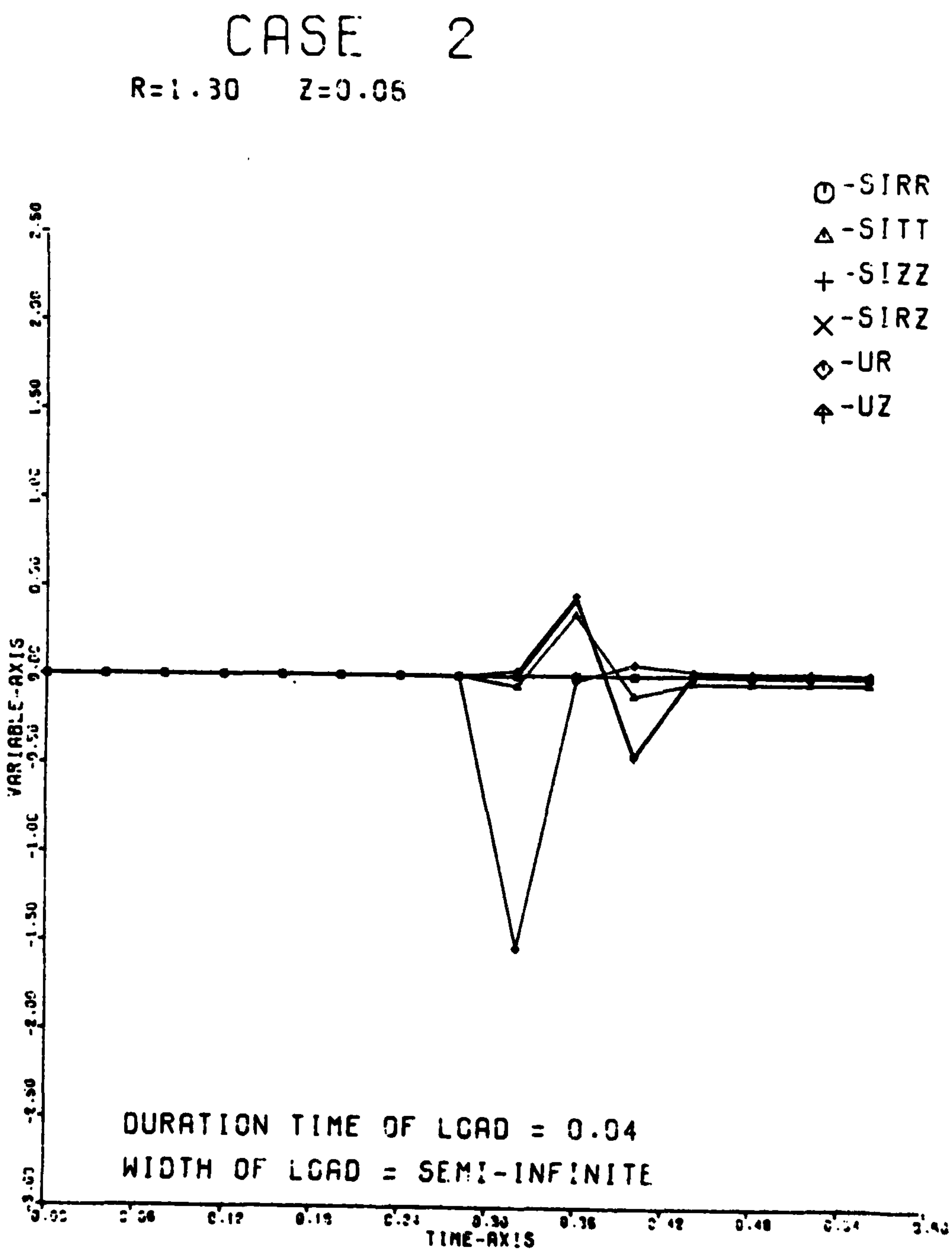

CASE 2
R=1.30 Z=0.06
O-SIRR
Δ-SITT
+-SIZZ
X-SIRZ
◇-UR
↑-UZ
VARIABLE-AXIS
TIME-AXIS
DURATION TIME OF LOAD = 0.04
WIDTH OF LOAD = SEMI-INFINITE

Figure 26

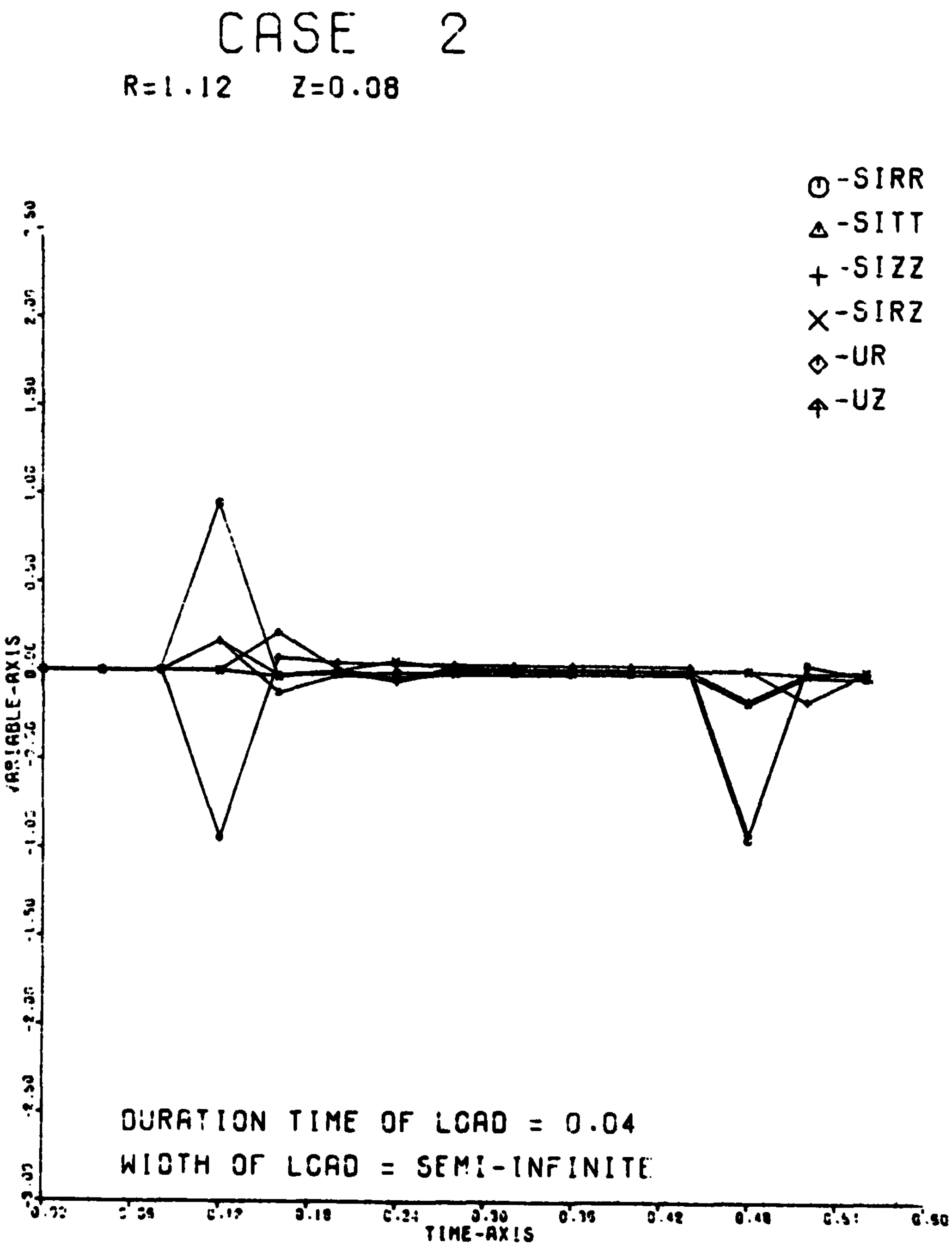

CASE 2
R=1.12 Z=0.08
-SIRR
-SITT
-SIZZ
-SIRZ
-UR
-UZ
VARIABLE-AXIS
TIME-AXIS
DURATION TIME OF LOAD = 0.04
WIDTH OF LOAD = SEMI-INFINITE

Figure 27

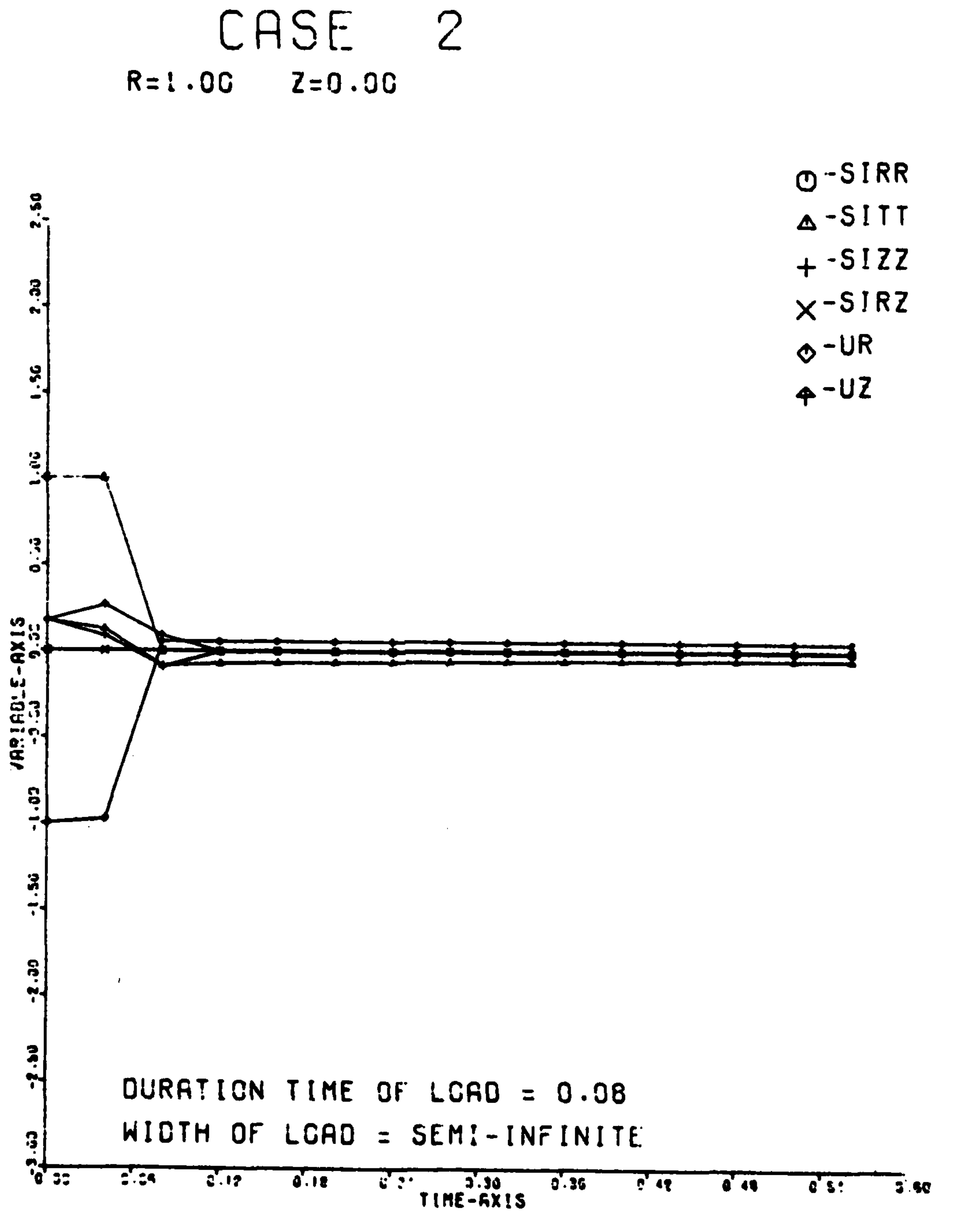

CASE 2
R=1.00 Z=0.00
-SIRR
-SITT
-SIZZ
-SIRZ
-UR
-UZ
VARIABLE-AXIS
TIME-AXIS
DURATION TIME OF LOAD = 0.08
WIDTH OF LOAD = SEMI-INFINITE

Figure 28

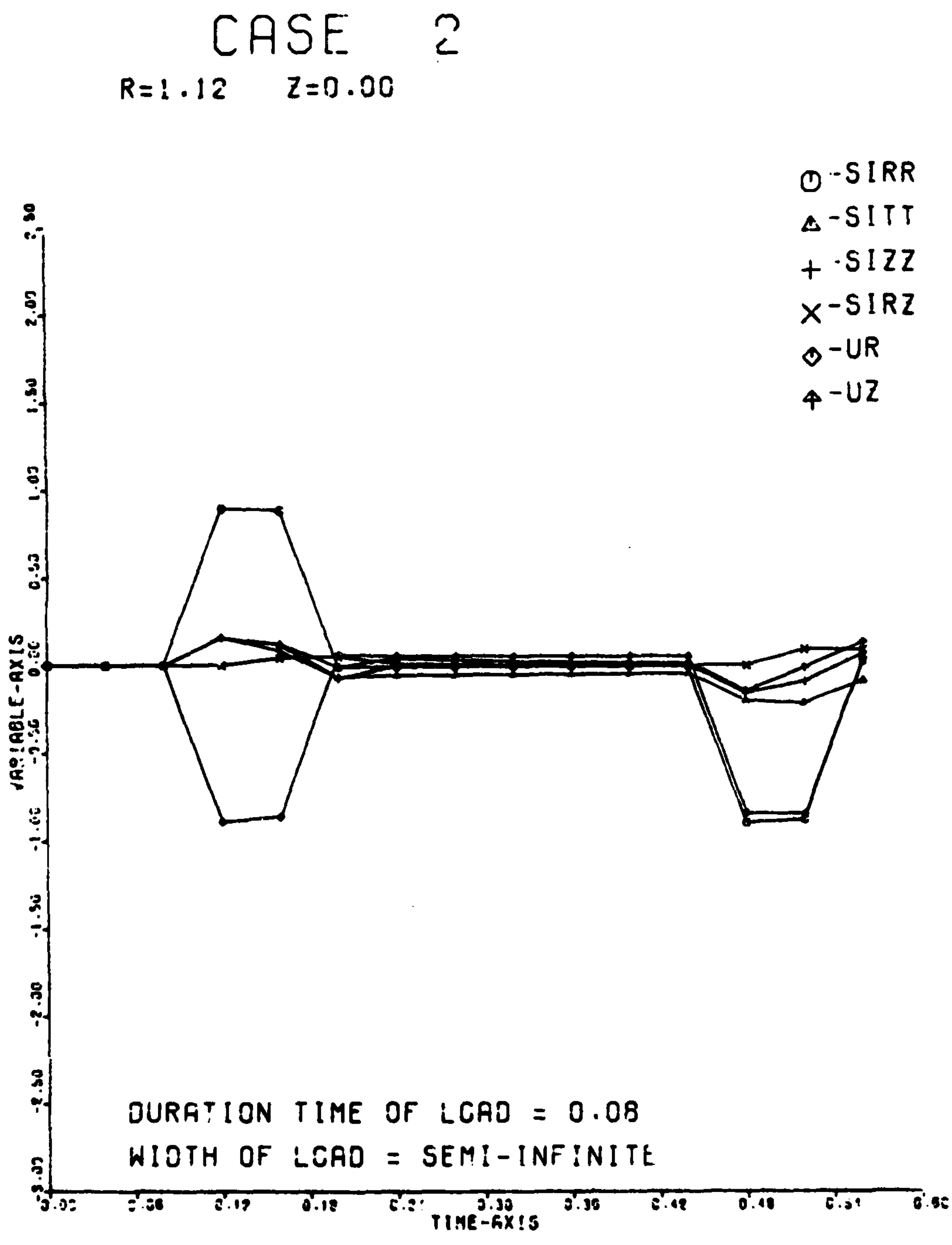

CASE 2
R=1.12 Z=0.00
-SIRR
-SITT
-SIZZ
-SIRZ
-UR
-UZ
VARIABLE-AXIS
TIME-AXIS
DURATION TIME OF LOAD = 0.08
WIDTH OF LOAD = SEMI-INFINITE

Figure 29

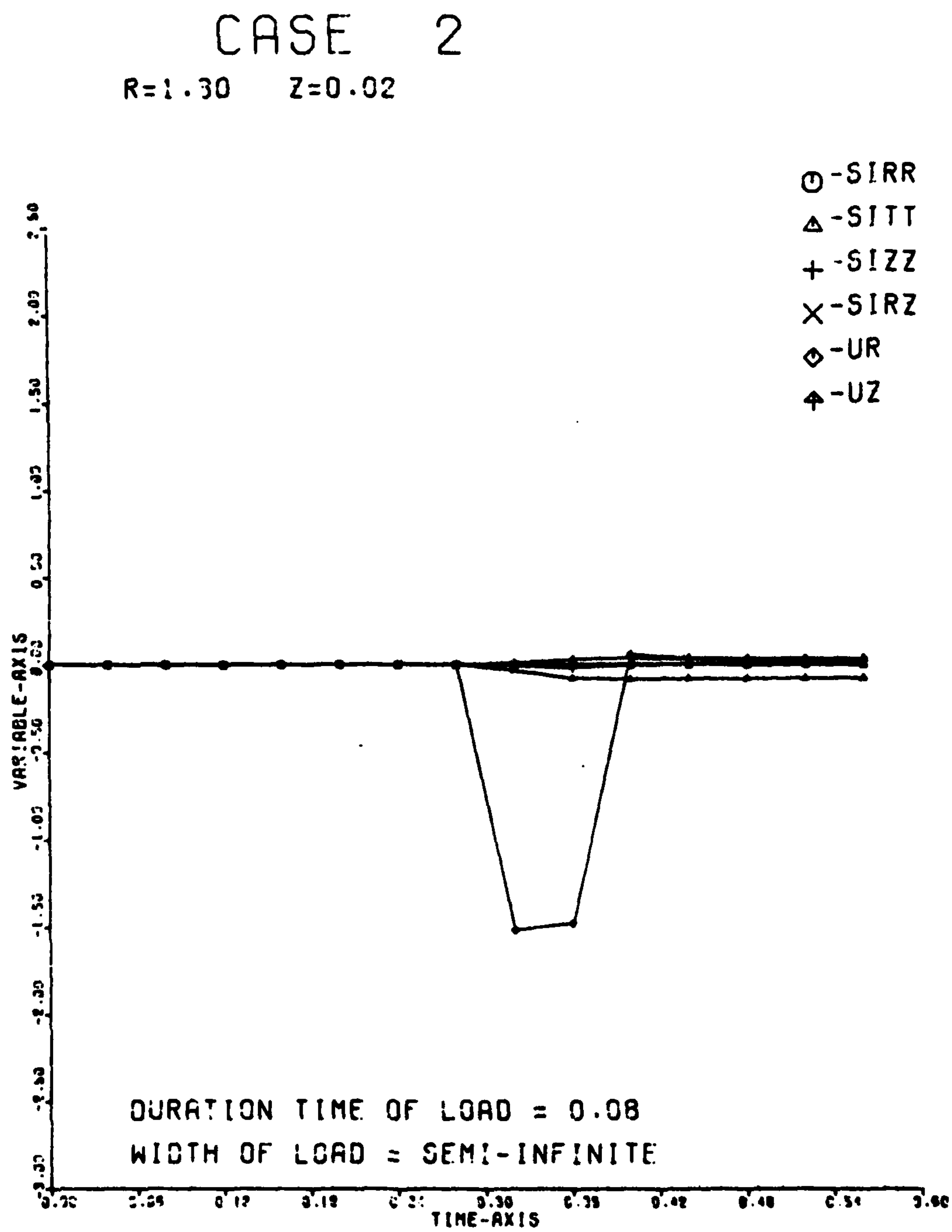

CASE 2
R=1.30 Z=0.02
-SIRR
-SITT
-SIZZ
-SIRZ
-UR
-UZ
VARIABLE-AXIS
TIME-AXIS
DURATION TIME OF LOAD = 0.08
WIDTH OF LOAD = SEMI-INFINITE

Figure 30

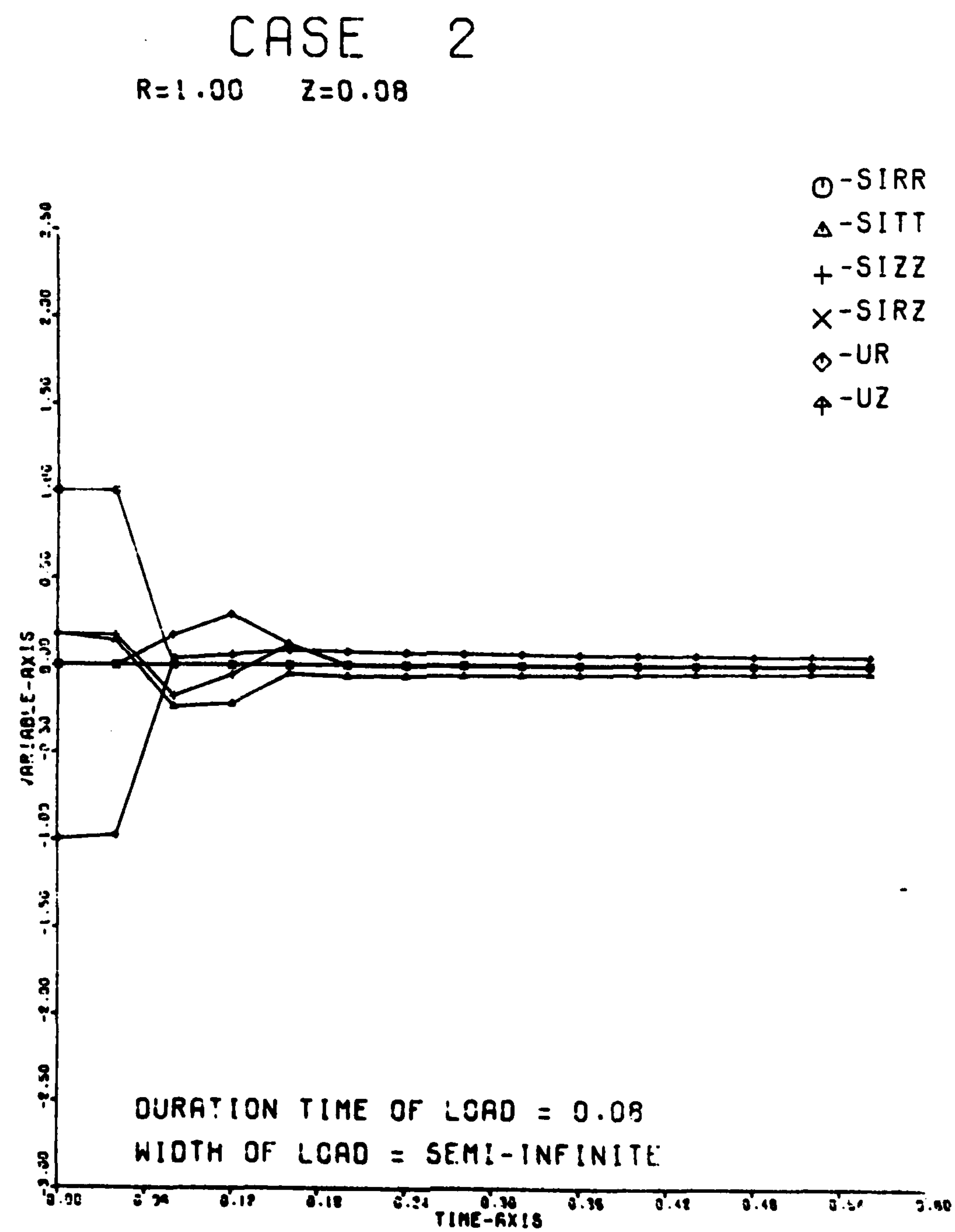

CASE 2
R=1.00 Z=0.08
O-SIRR
△-SITT
+-SIZZ
X-SIRZ
◇-UR
↑-UZ
VARIABLE-AXIS
TIME-AXIS
DURATION TIME OF LOAD = 0.08
WIDTH OF LOAD = SEMI-INFINITE

Figure 31

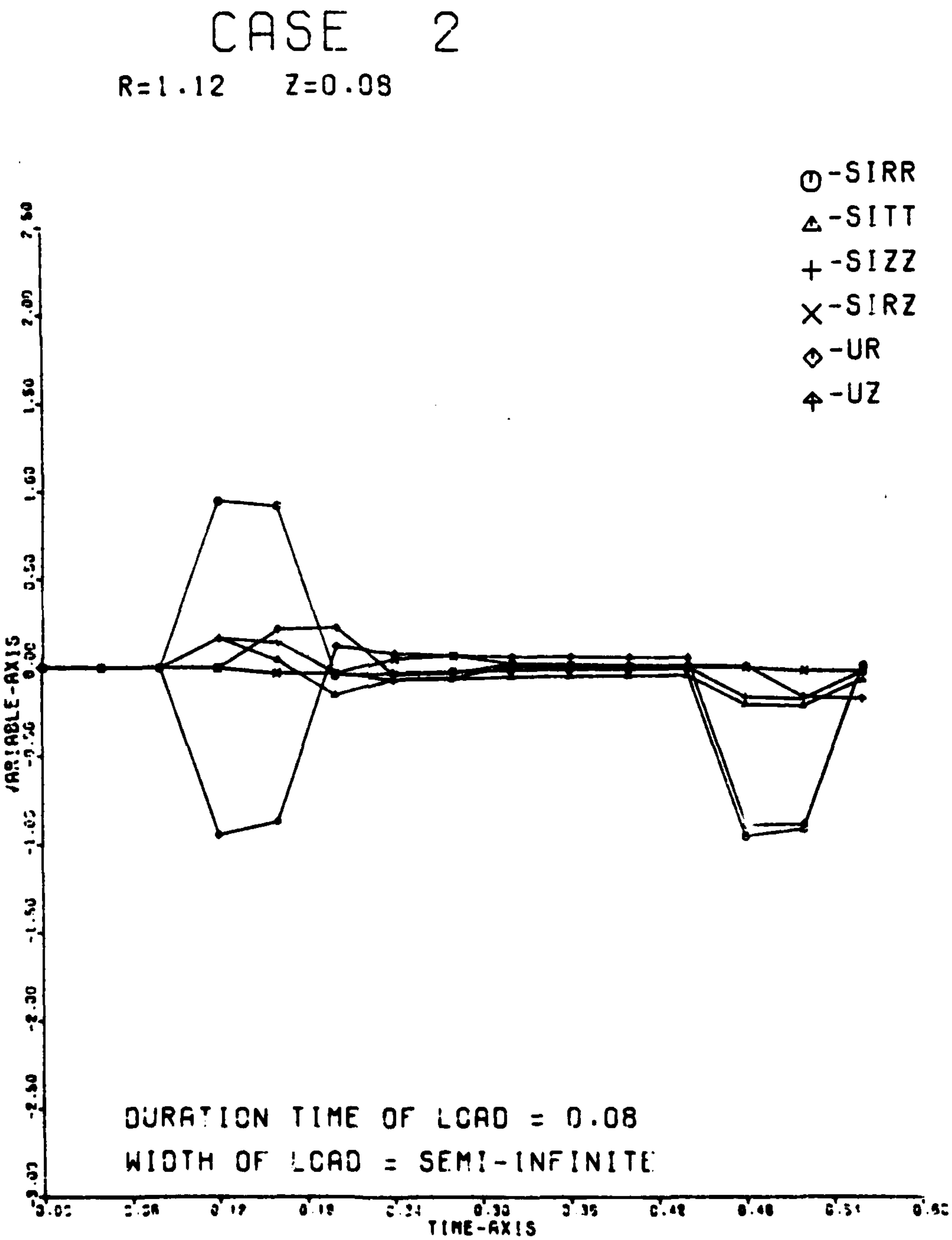

CASE 2
R=1.12 Z=0.08
-SIRR
-SITT
-SIZZ
-SIRZ
-UR
-UZ
VARIABLE-AXIS
TIME-AXIS
DURATION TIME OF LOAD = 0.08
WIDTH OF LOAD = SEMI-INFINITE

Figure 32

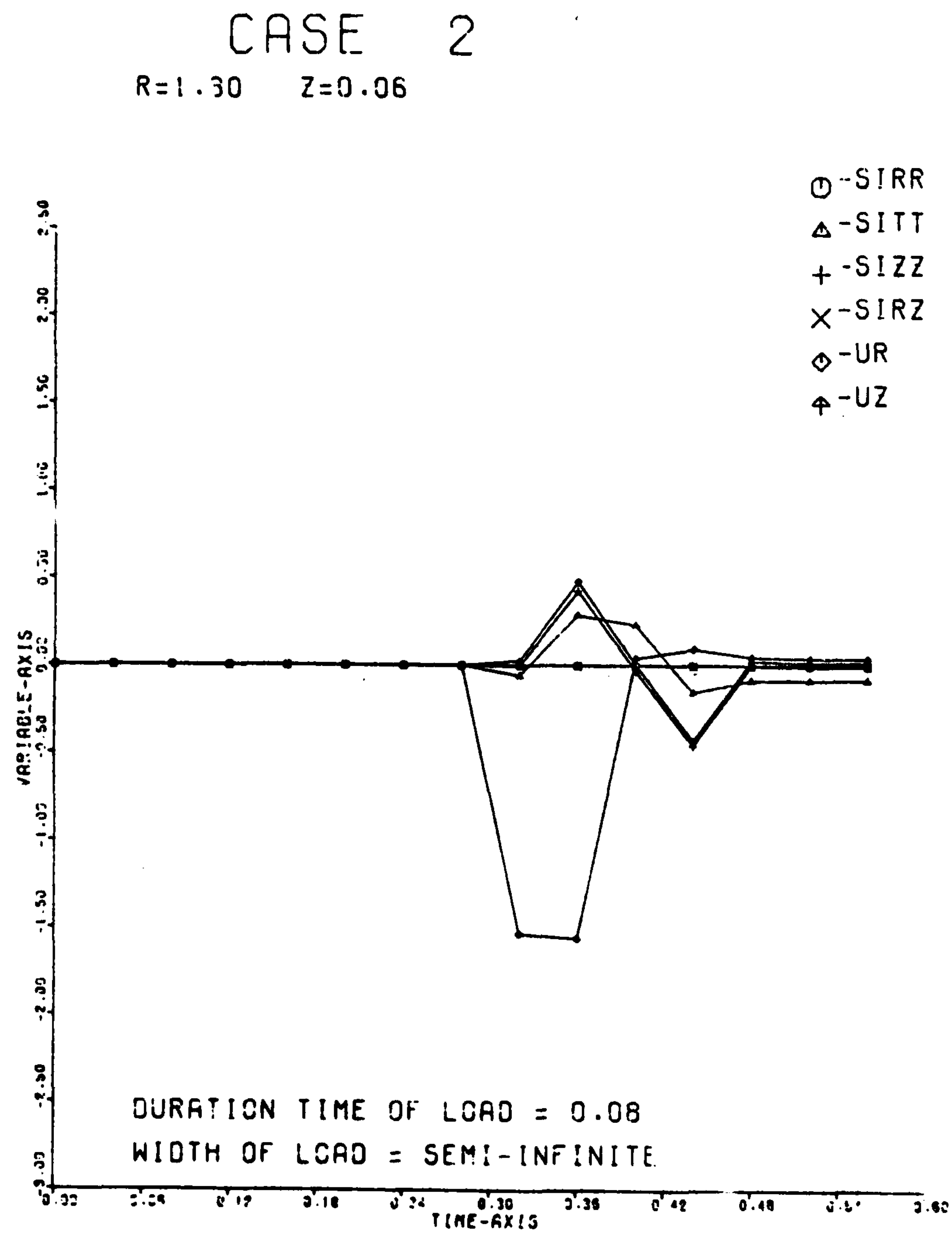

CASE 2
R=1.30 Z=0.06
-SIRR
-SITT
-SIZZ
-SIRZ
-UR
-UZ
VARIABLE-AXIS
TIME-AXIS
DURATION TIME OF LOAD = 0.08
WIDTH OF LOAD = SEMI-INFINITE

Figure 33

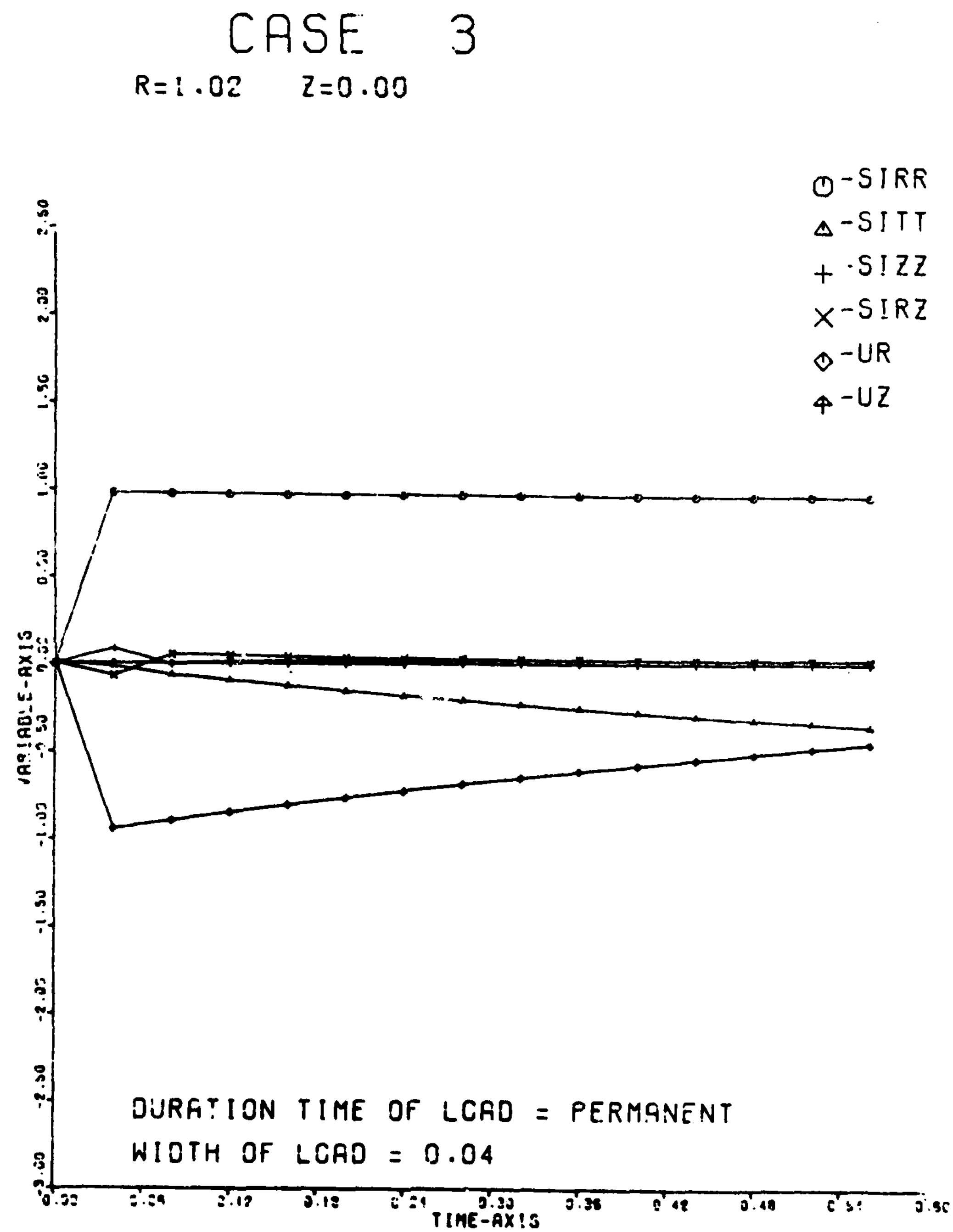

CASE 3
R=1.02 Z=0.00
-SIRR
-SITT
-SIZZ
-SIRZ
-UR
-UZ
VARIABLE-AXIS
TIME-AXIS
DURATION TIME OF LOAD = PERMANENT
WIDTH OF LOAD = 0.04

Figure 34

CASE 3

R=1.14 Z=0.00

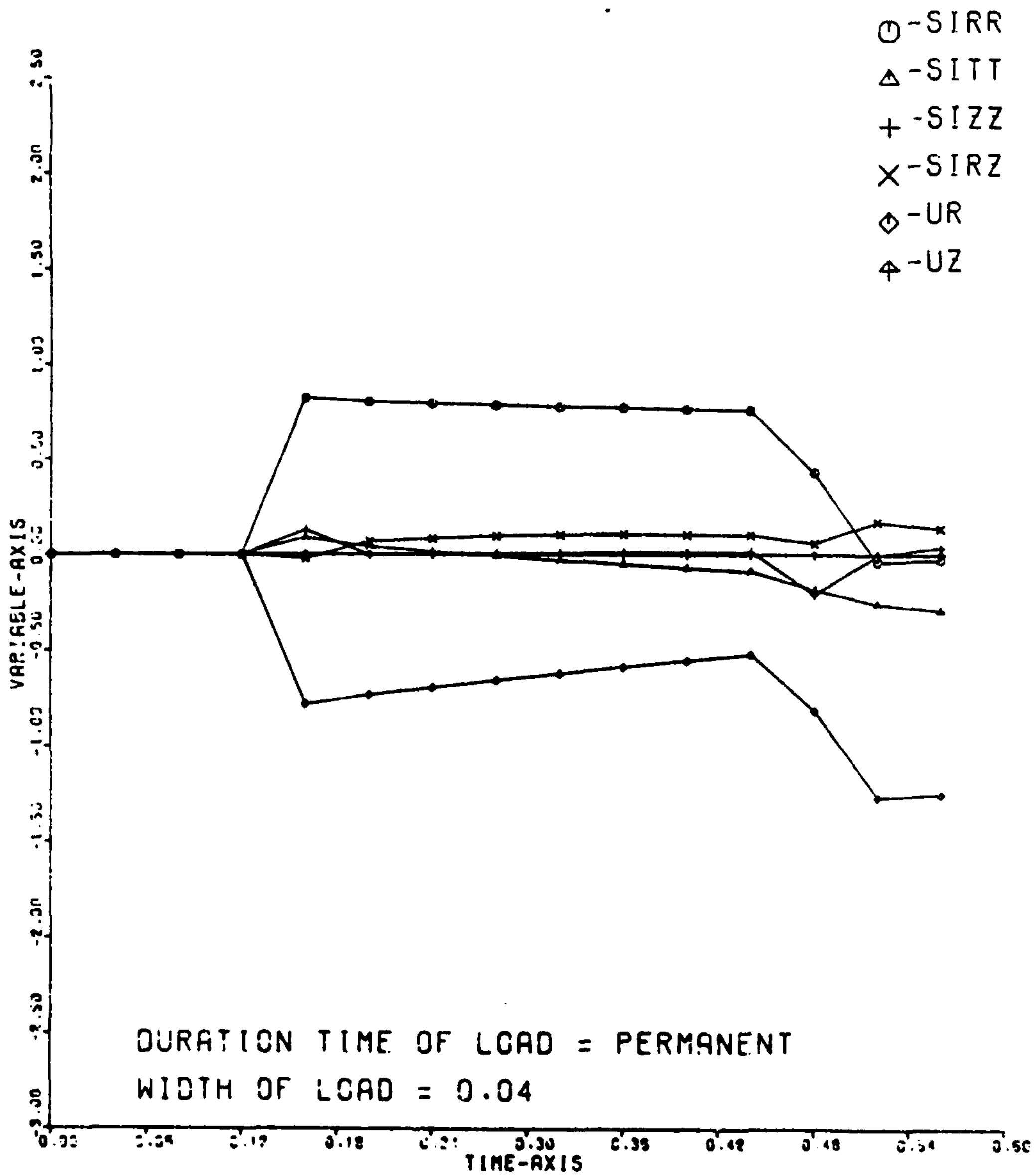


Figure 35

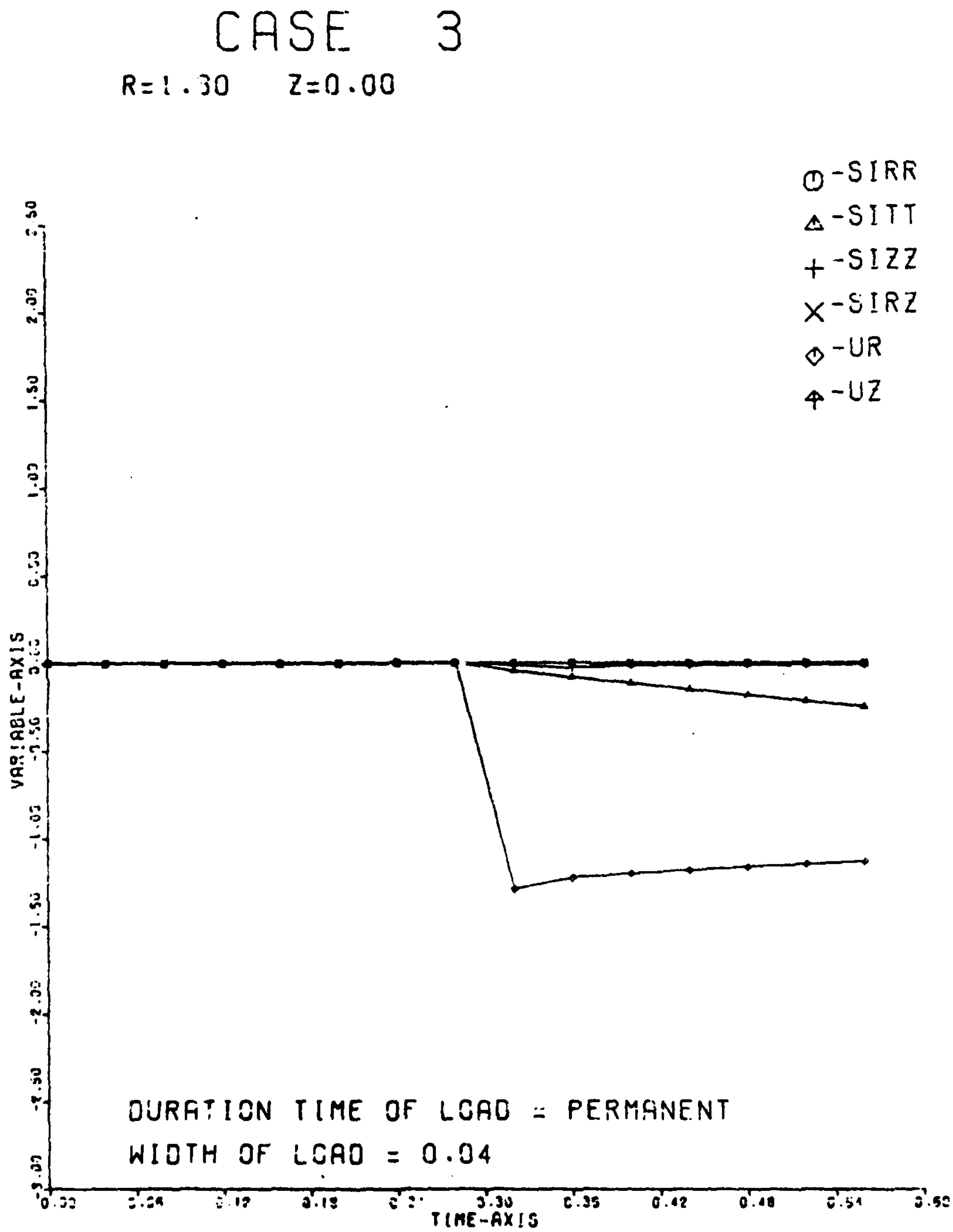

CASE 3
R=1.30 Z=0.00
-SIRR
-SITT
-SIZZ
-SIRZ
-UR
-UZ
VARIABLE-AXIS
TIME-AXIS
DURATION TIME OF LOAD = PERMANENT
WIDTH OF LOAD = 0.04

Figure 36

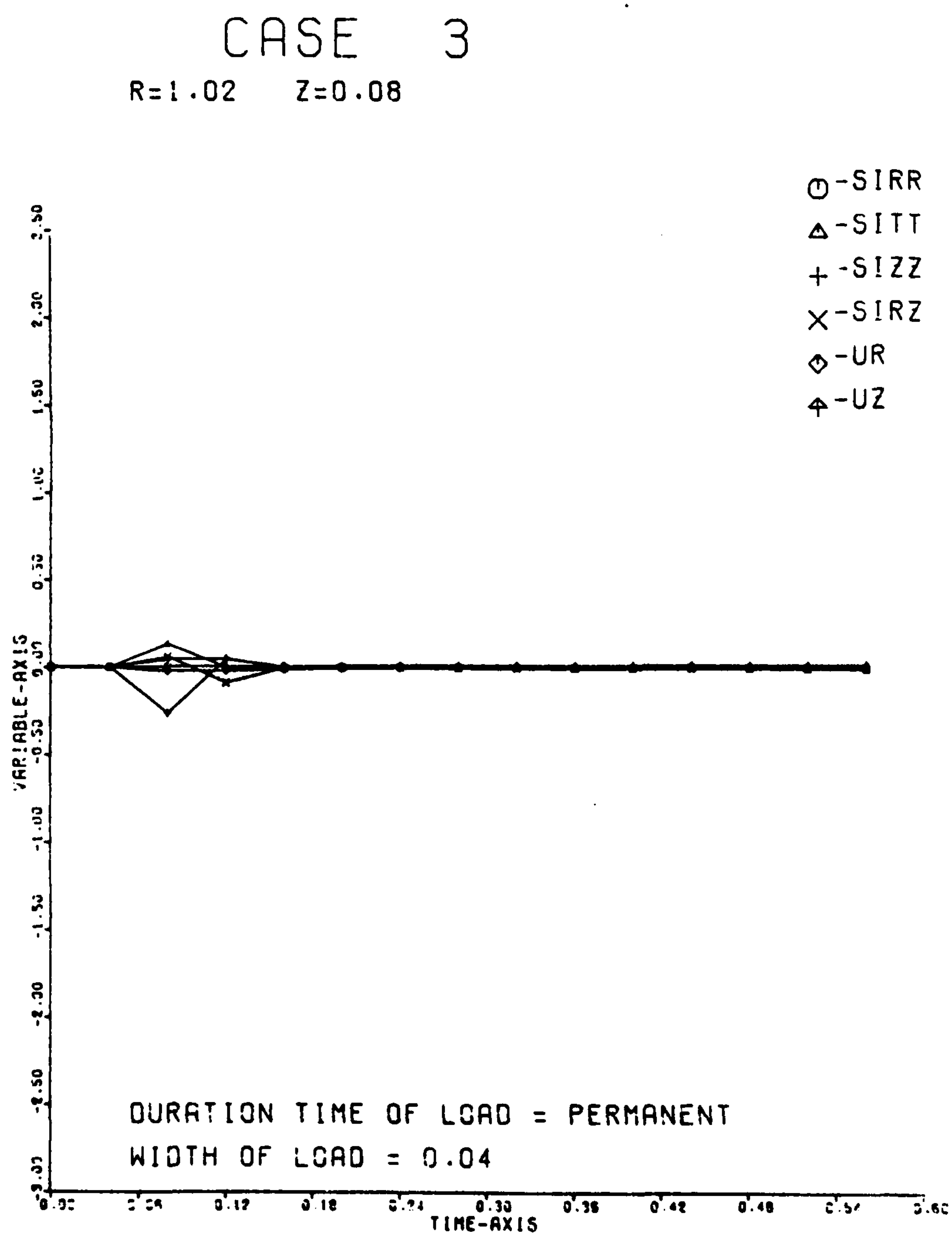

CASE 3
R=1.02 Z=0.08
-SIRR
-SITT
-SIZZ
-SIRZ
-UR
-UZ
VARIABLE-AXIS
DURATION TIME OF LOAD = PERMANENT
WIDTH OF LOAD = 0.04
0.12
0.18
0.30
0.36
0.42
0.48
TIME-AXIS

Figure 37

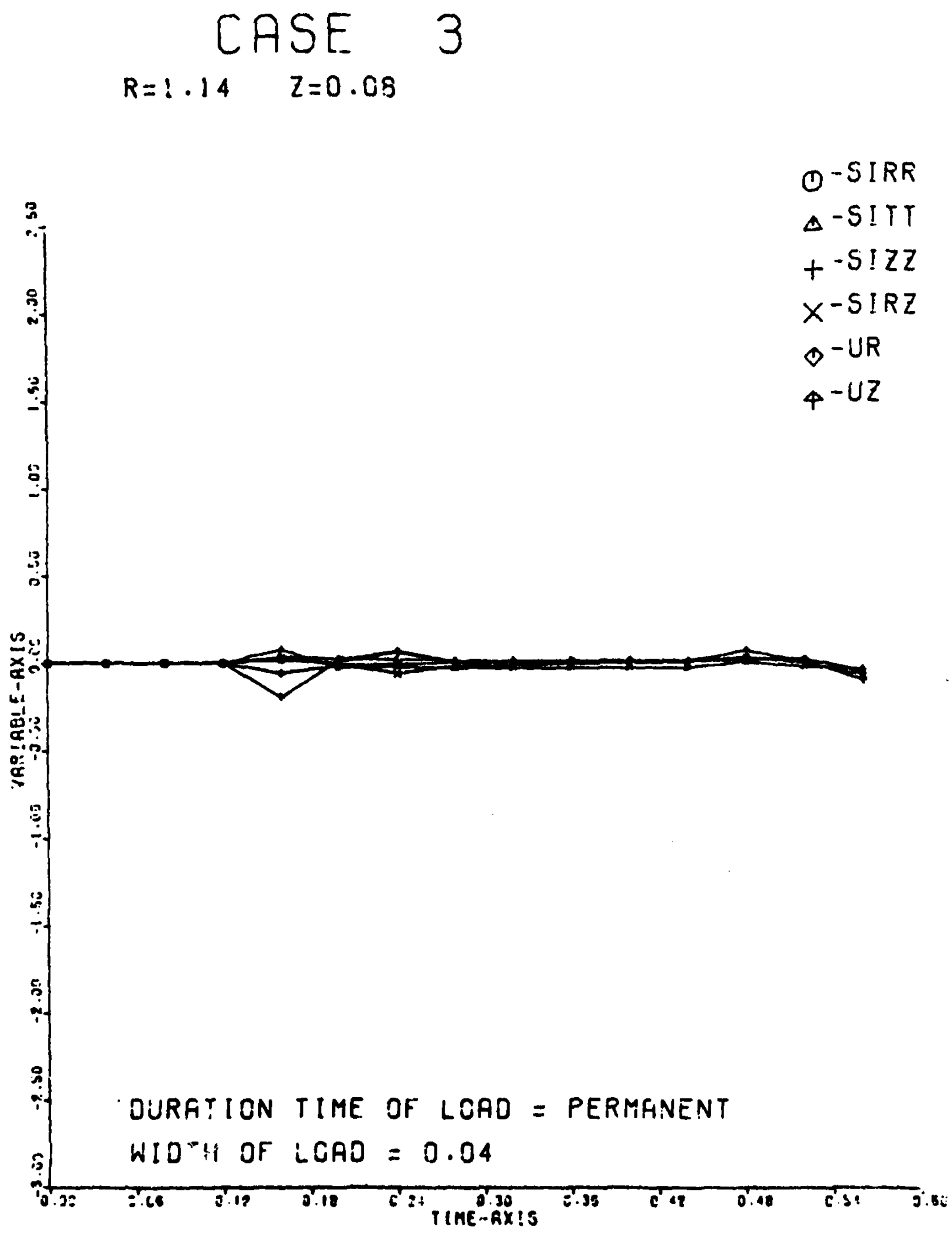

CASE 3
R=1.14 Z=0.08
-SIRR
-SITT
-SIZZ
-SIRZ
-UR
-UZ
VARIABLE-AXIS
TIME-AXIS
DURATION TIME OF LOAD = PERMANENT
WIDTH OF LOAD = 0.04

Figure 38

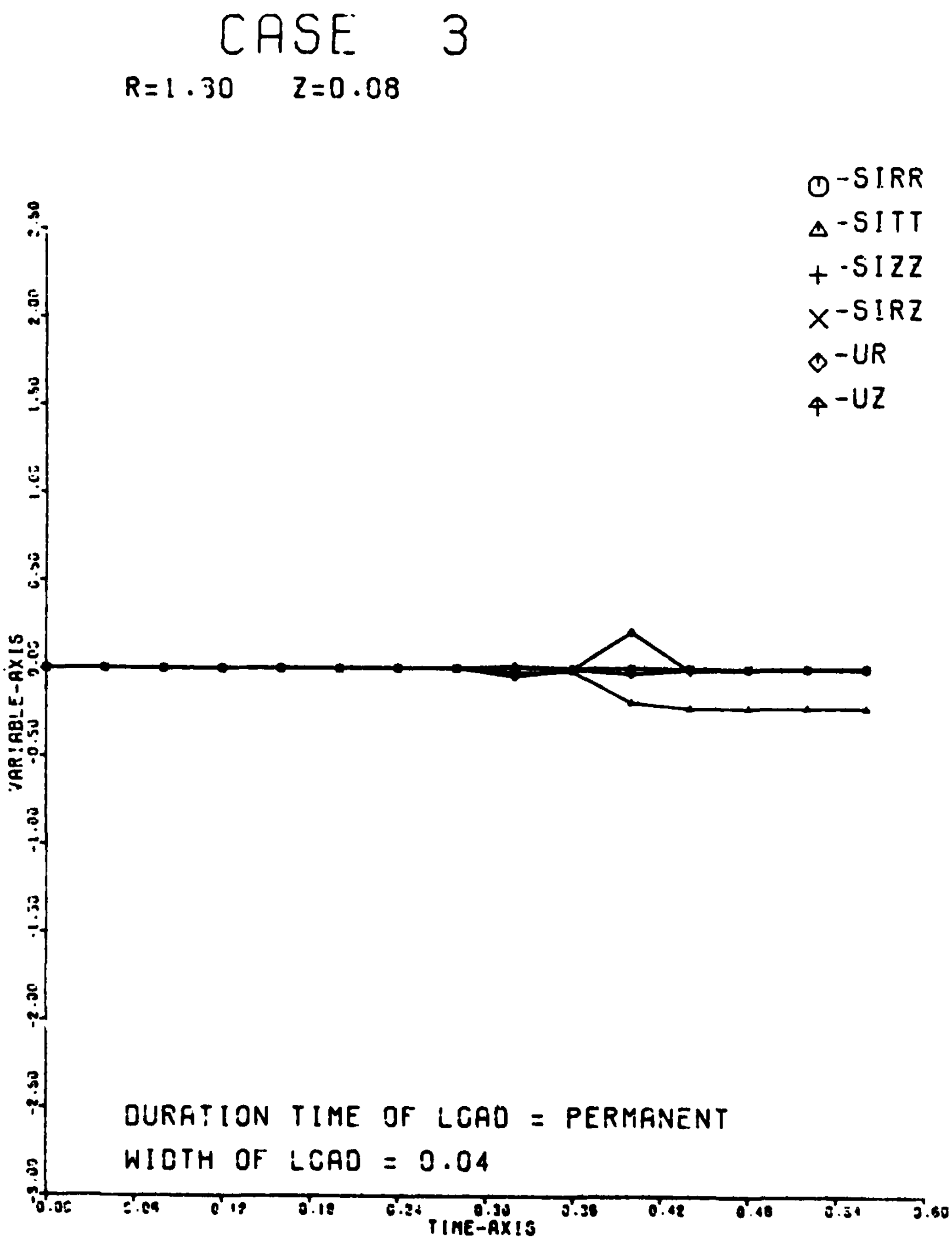

CASE 3
R=1.30 Z=0.08
-SIRR
-SITT
-SIZZ
-SIRZ
-UR
-UZ
VARIABLE-AXIS
TIME-AXIS
DURATION TIME OF LOAD = PERMANENT
WIDTH OF LOAD = 0.04

Figure 39

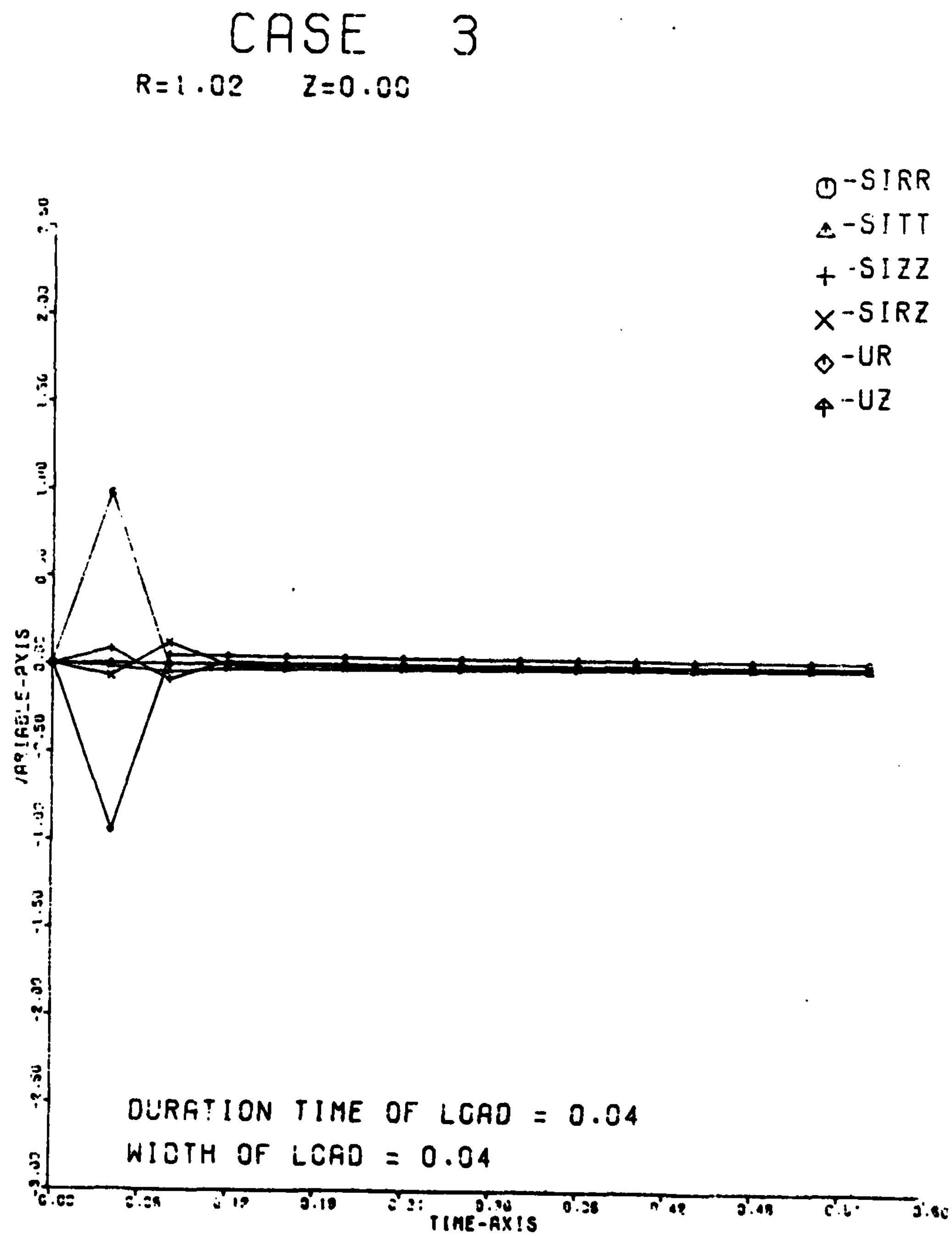

CASE 3
R=1.02 Z=0.00
-SIRR
-SITT
-SIZZ
-SIRZ
-UR
-UZ
VARIABLE-AXIS
TIME-AXIS
DURATION TIME OF LOAD = 0.04
WIDTH OF LOAD = 0.04

Figure 40

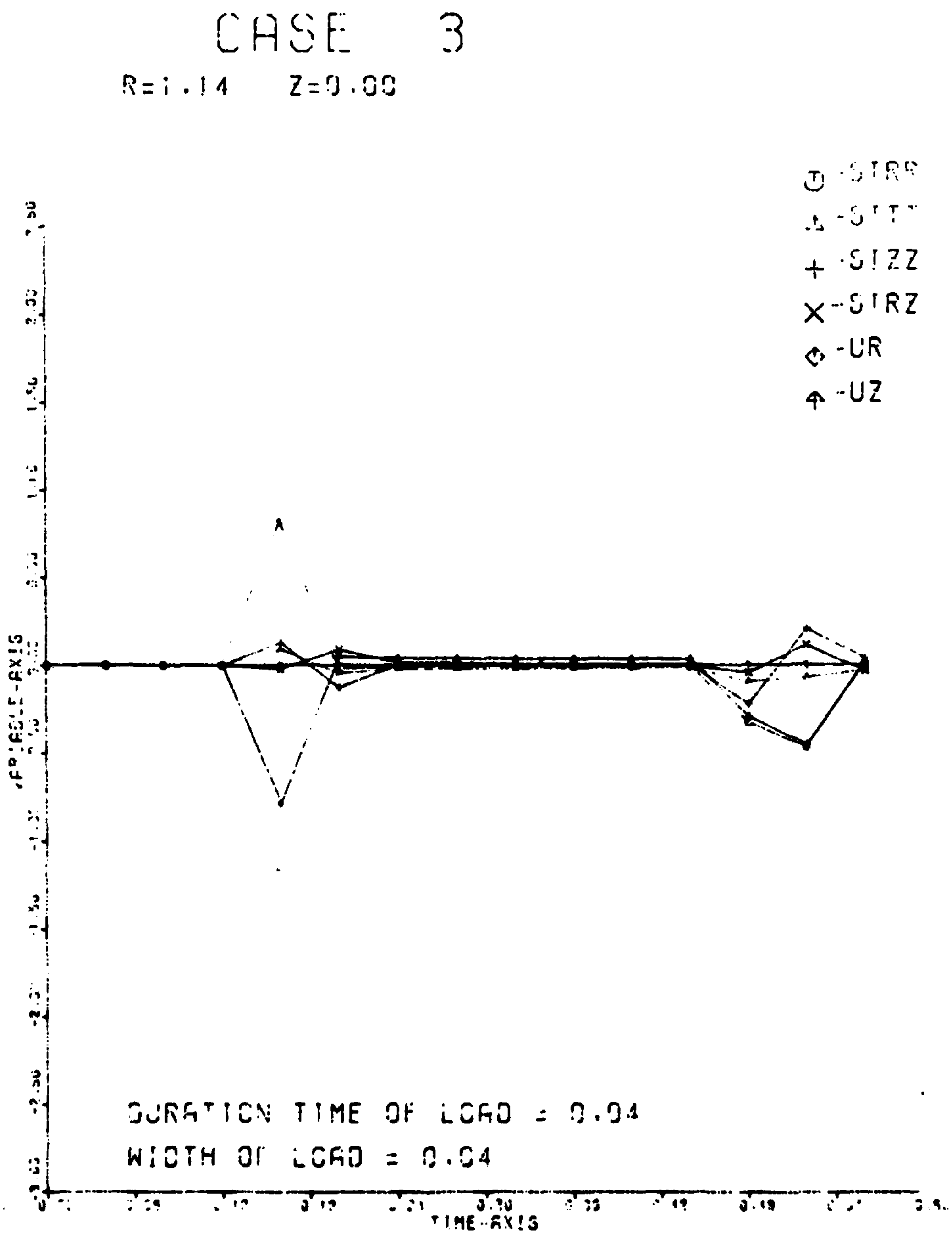


Figure 41

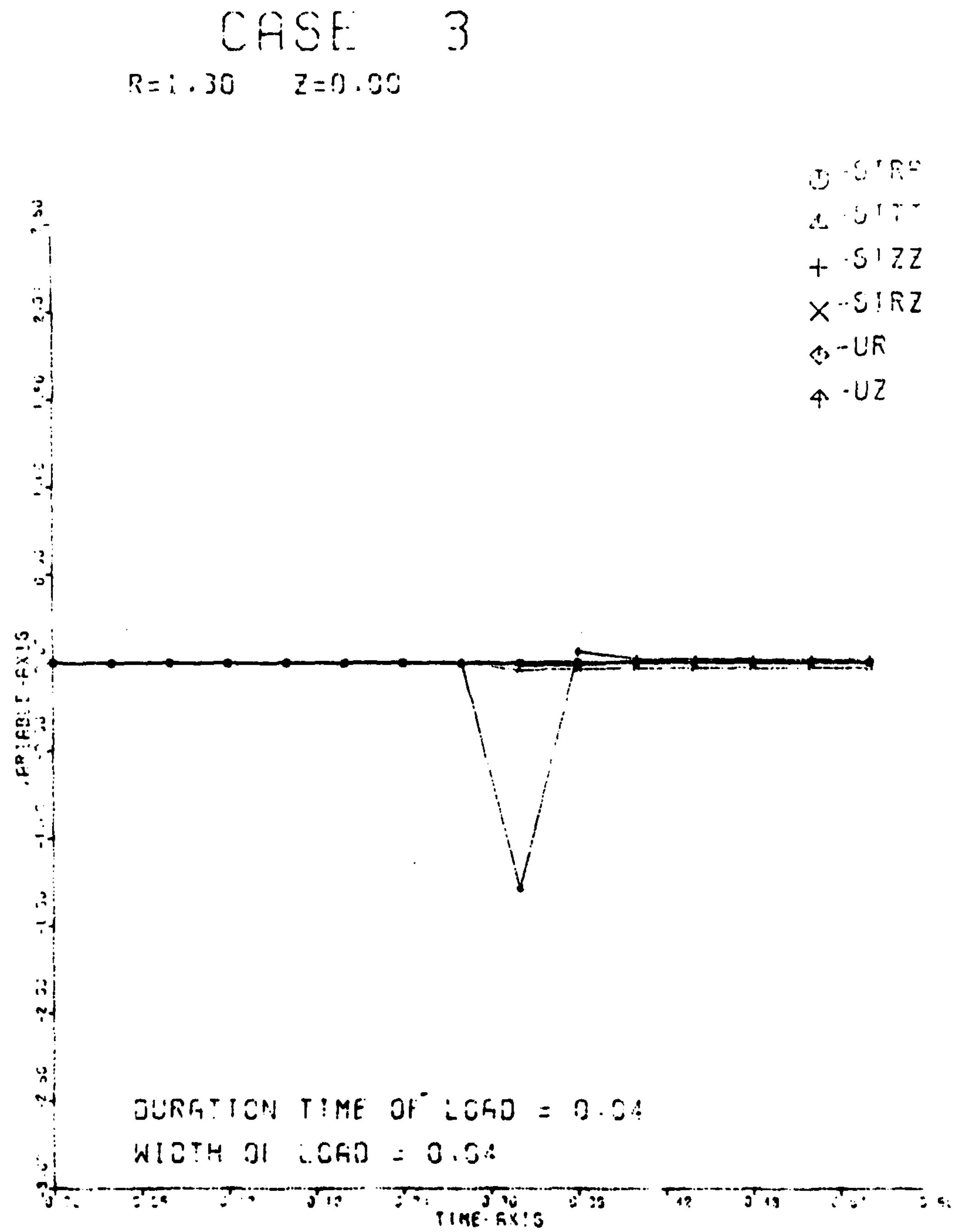

CASE 3
R=1.30 Z=0.00
-SIZZ
-SIRZ
-UR
-UZ
TIME-AXIS
DURATION TIME OF LOAD = 0.04
WIDTH OF LOAD = 0.04

Figure 42

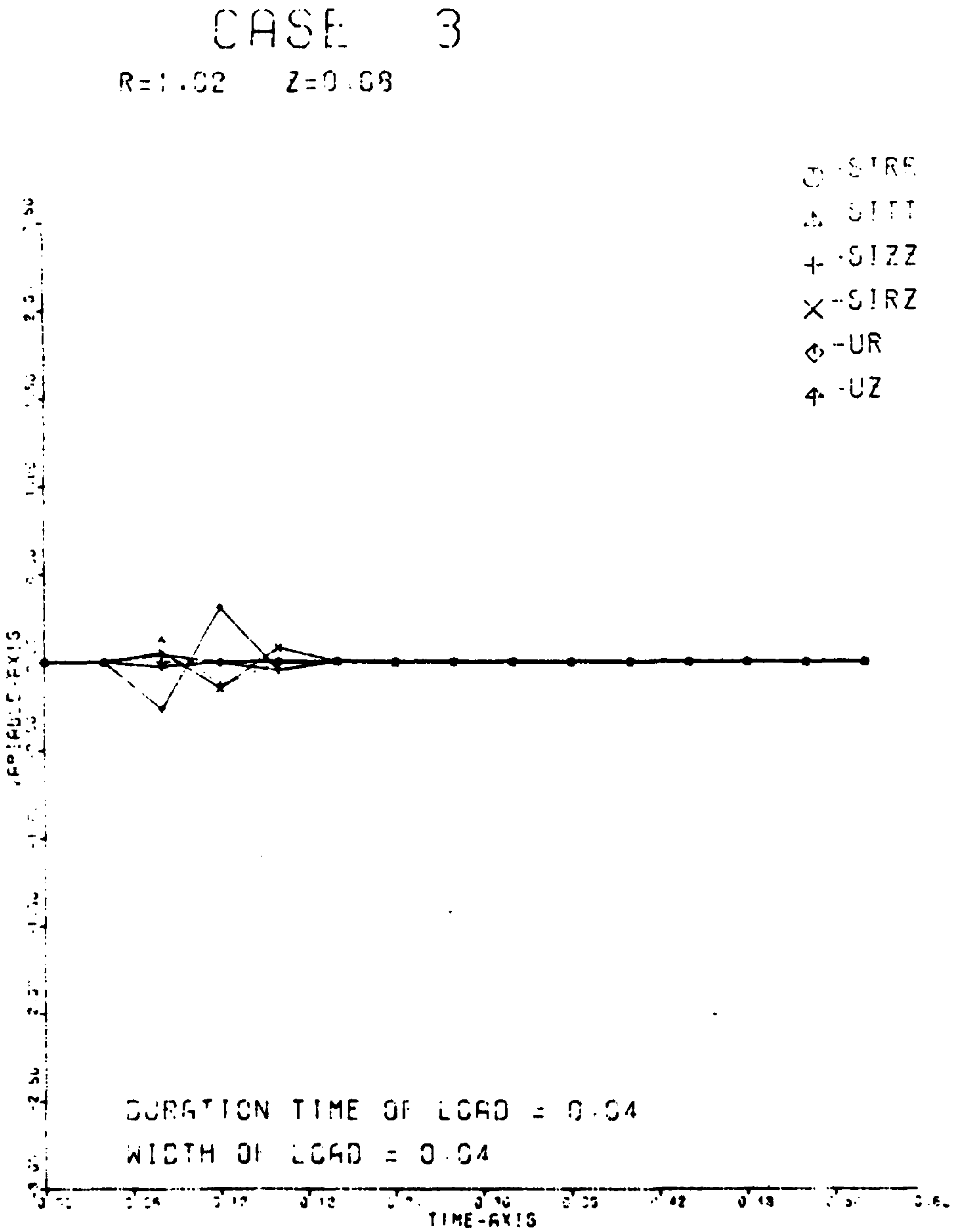


Figure 43

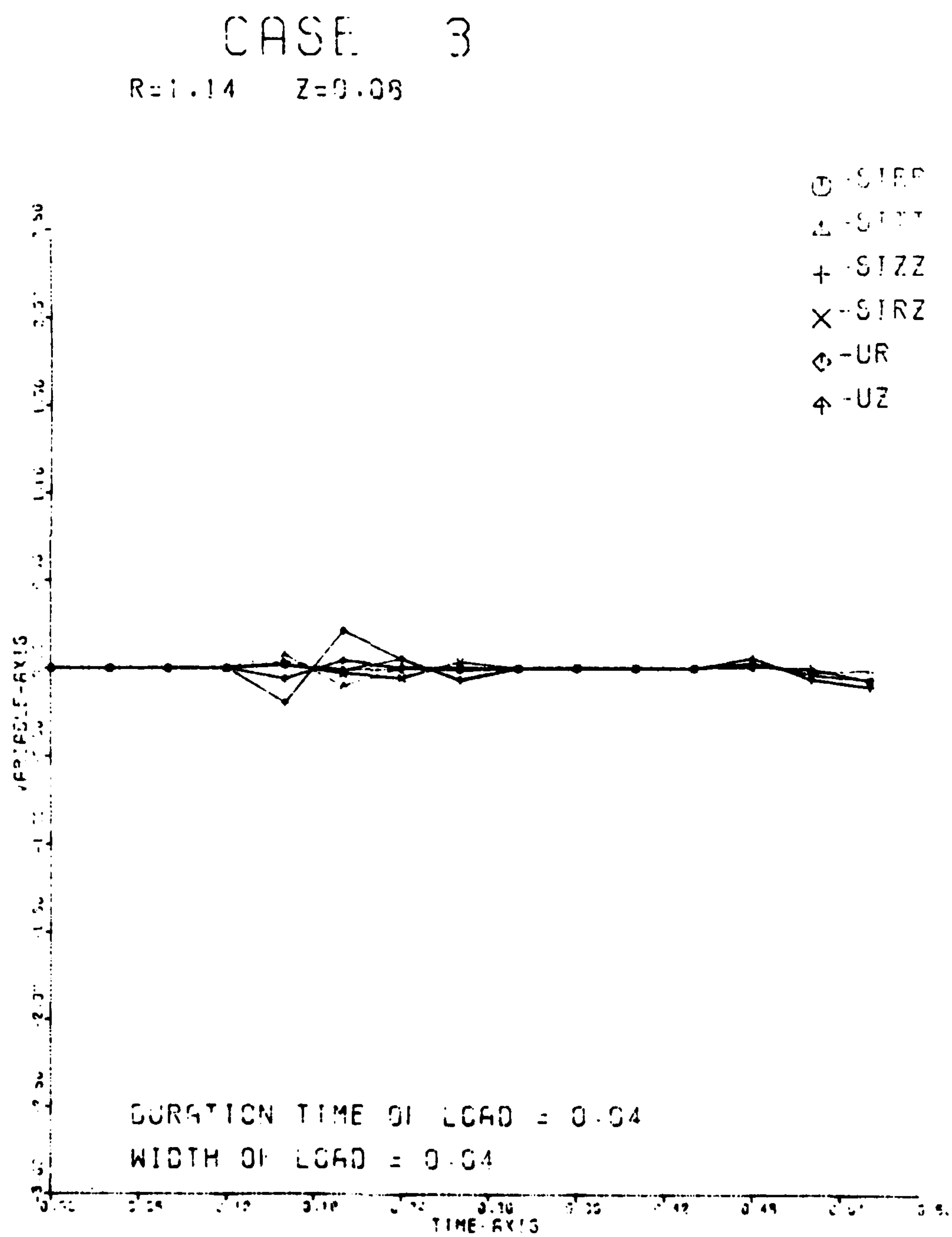

CASE 3
R=1.14 Z=0.08
-SIZZ
-SIRZ
-UR
-UZ
TIME-AXIS
DURATION TIME OF LOAD = 0.04
WIDTH OF LOAD = 0.04

Figure 44

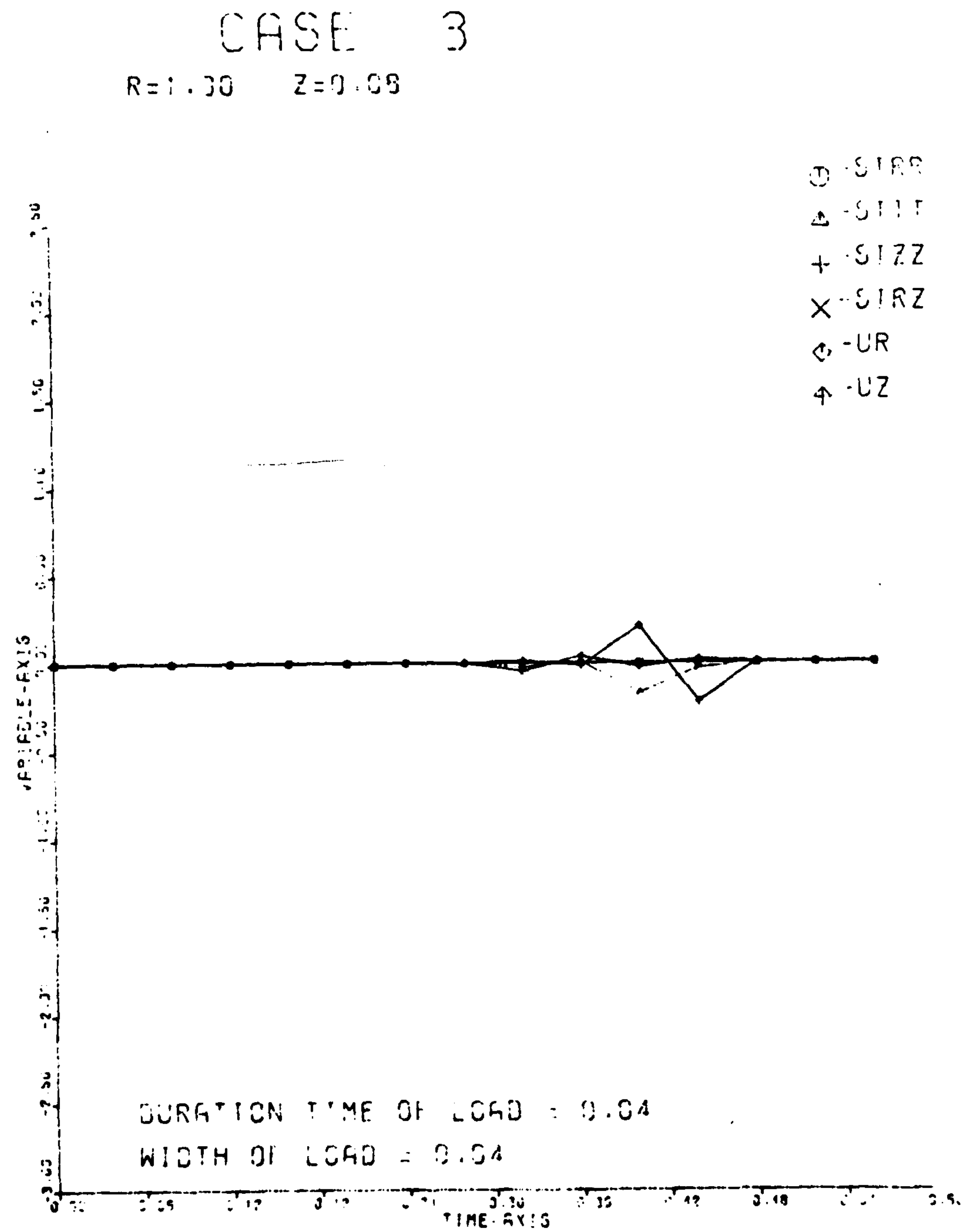

CASE 3
R=1.30 Z=0.08
-SIRR
-SITT
-SIZZ
-SIRZ
-UR
-UZ
VARIABLE-AXIS
TIME-AXIS
DURATION TIME OF LOAD = 0.04
WIDTH OF LOAD = 0.04

Figure 45

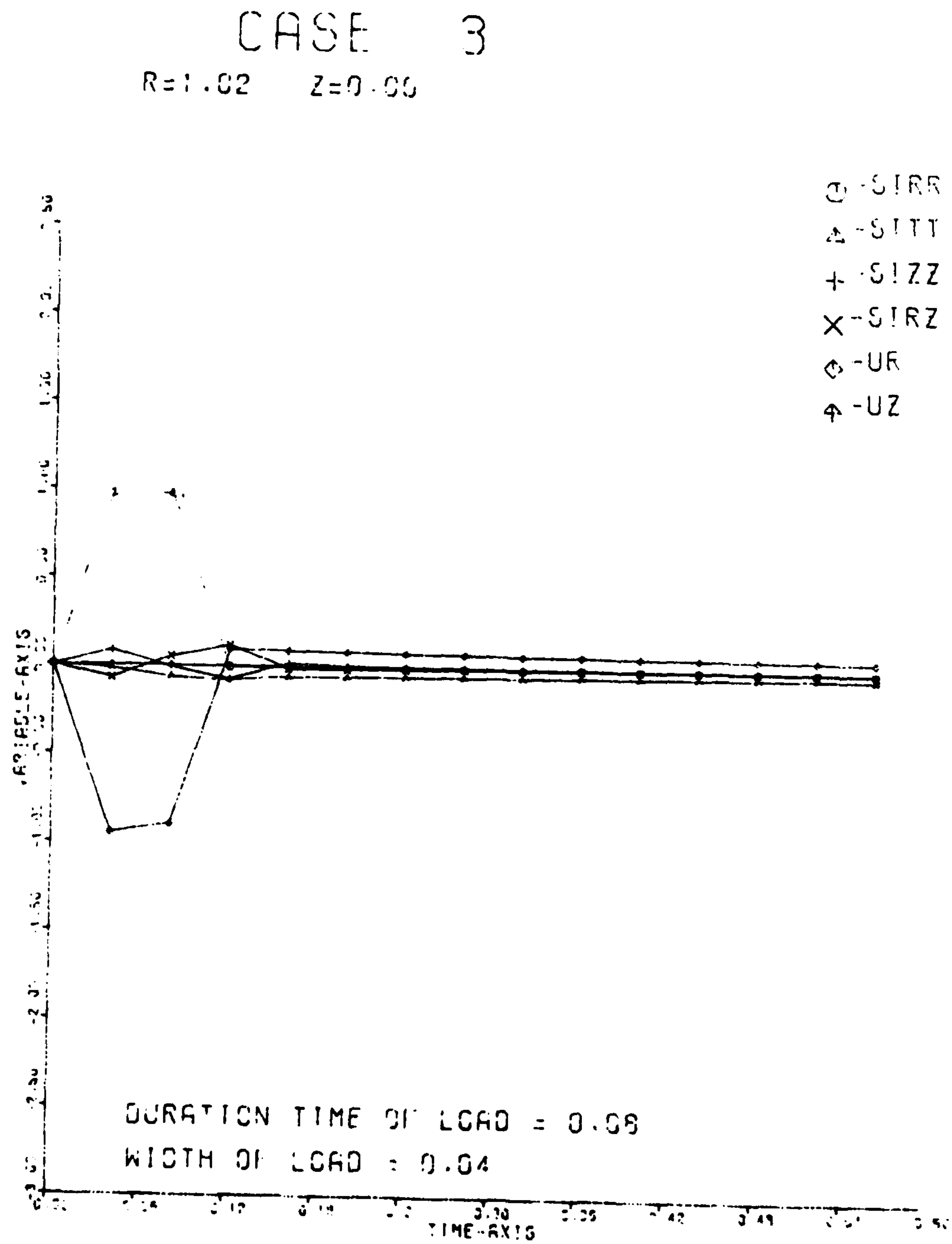

CASE 3
R=1.02 Z=0.00
-SIRR
-SITT
-SIZZ
-SIRZ
-UR
-UZ
VARIABLE-AXIS
TIME-AXIS
DURATION TIME OF LOAD = 0.08
WIDTH OF LOAD = 0.04

Figure 46

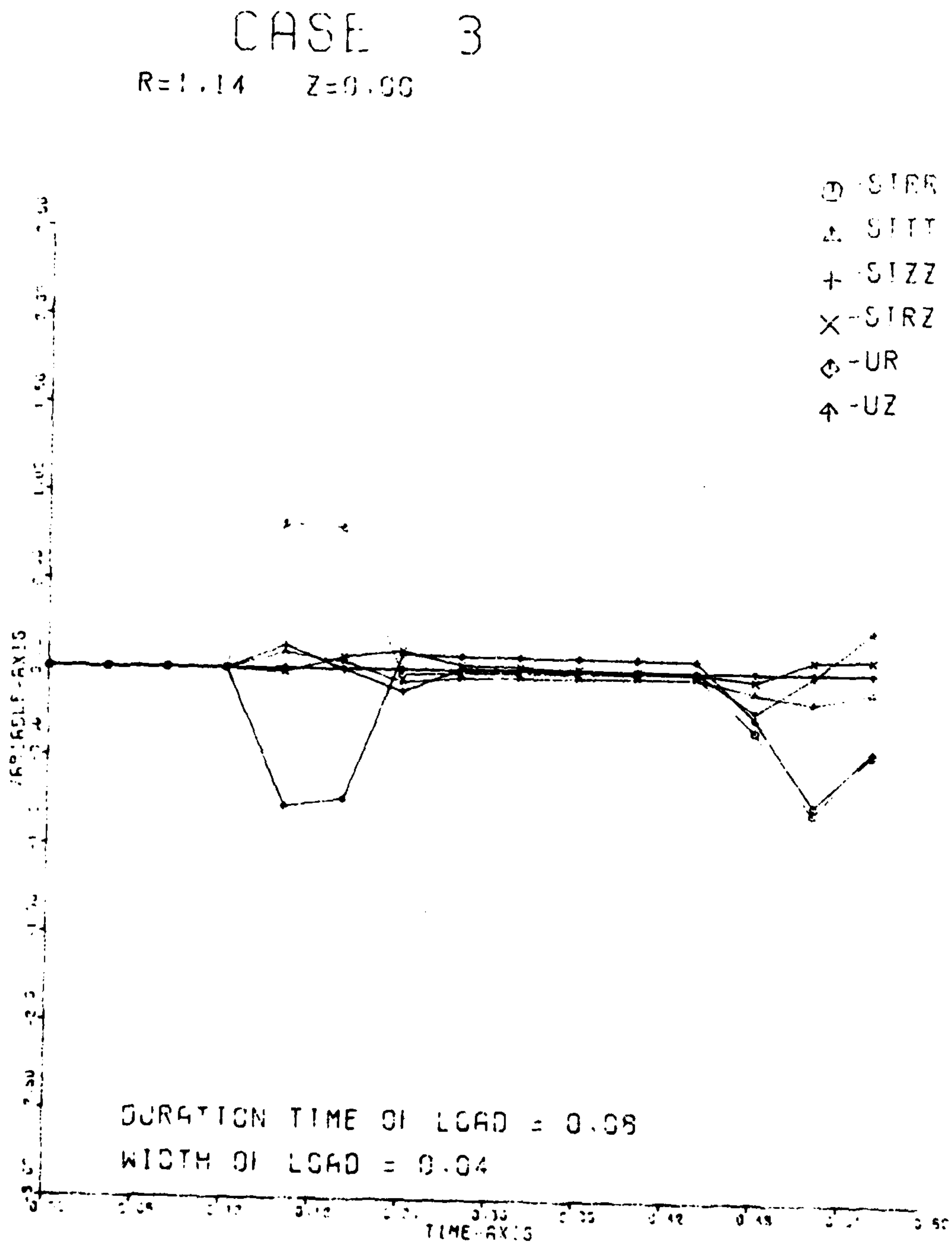

CASE 3
R=1.14 Z=0.00
-SIRR
-SITT
-SIZZ
-SIRZ
-UR
-UZ
VARIABLE-AXIS
DURATION TIME OF LOAD = 0.08
WIDTH OF LOAD = 0.04
TIME-AXIS

Figure 47

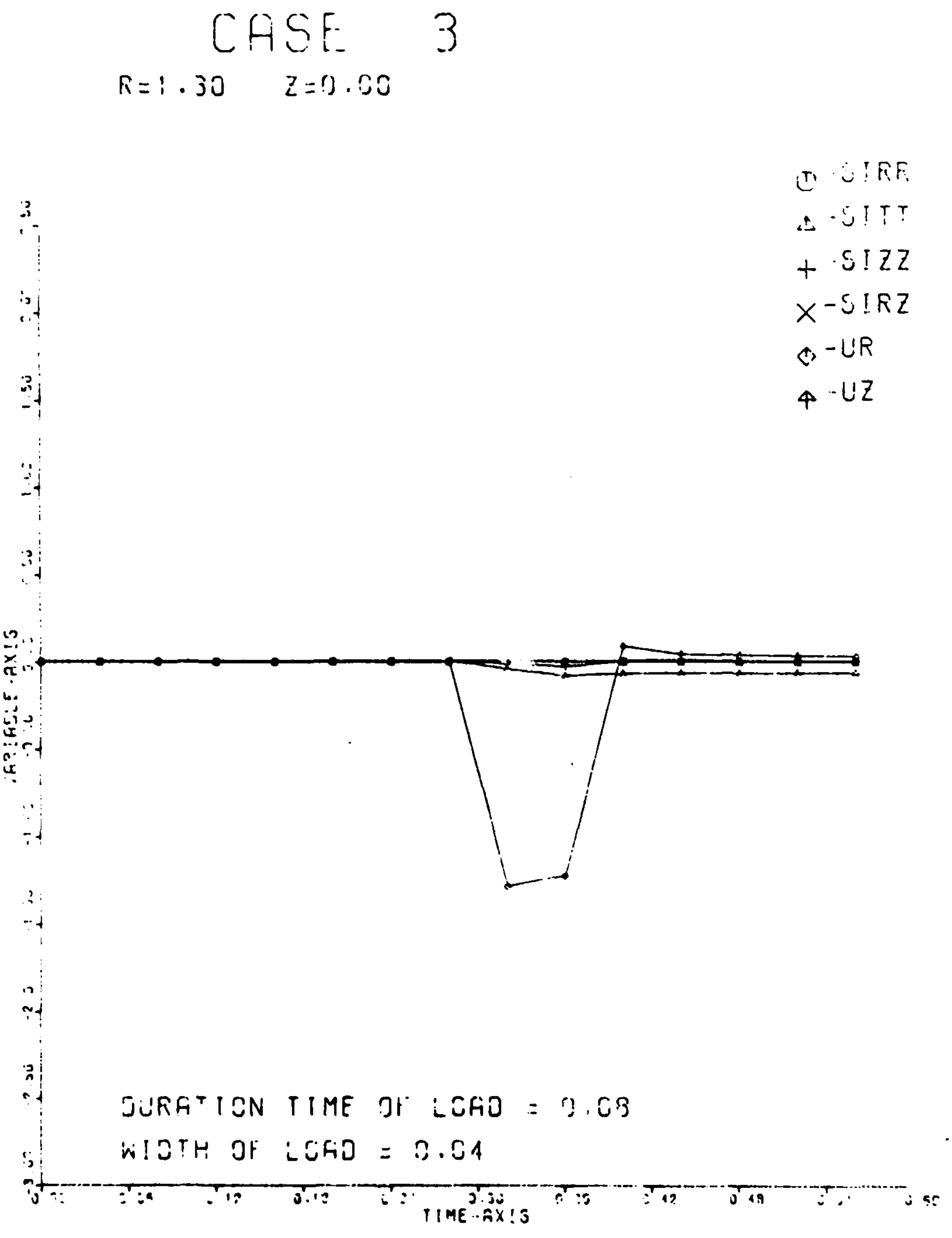

CASE 3
R=1.30 Z=0.00
-SIRR
-SITT
-SIZZ
-SIRZ
-UR
-UZ
VARIABLE-AXIS
TIME-AXIS
DURATION TIME OF LOAD = 0.08
WIDTH OF LOAD = 0.04

Figure 48

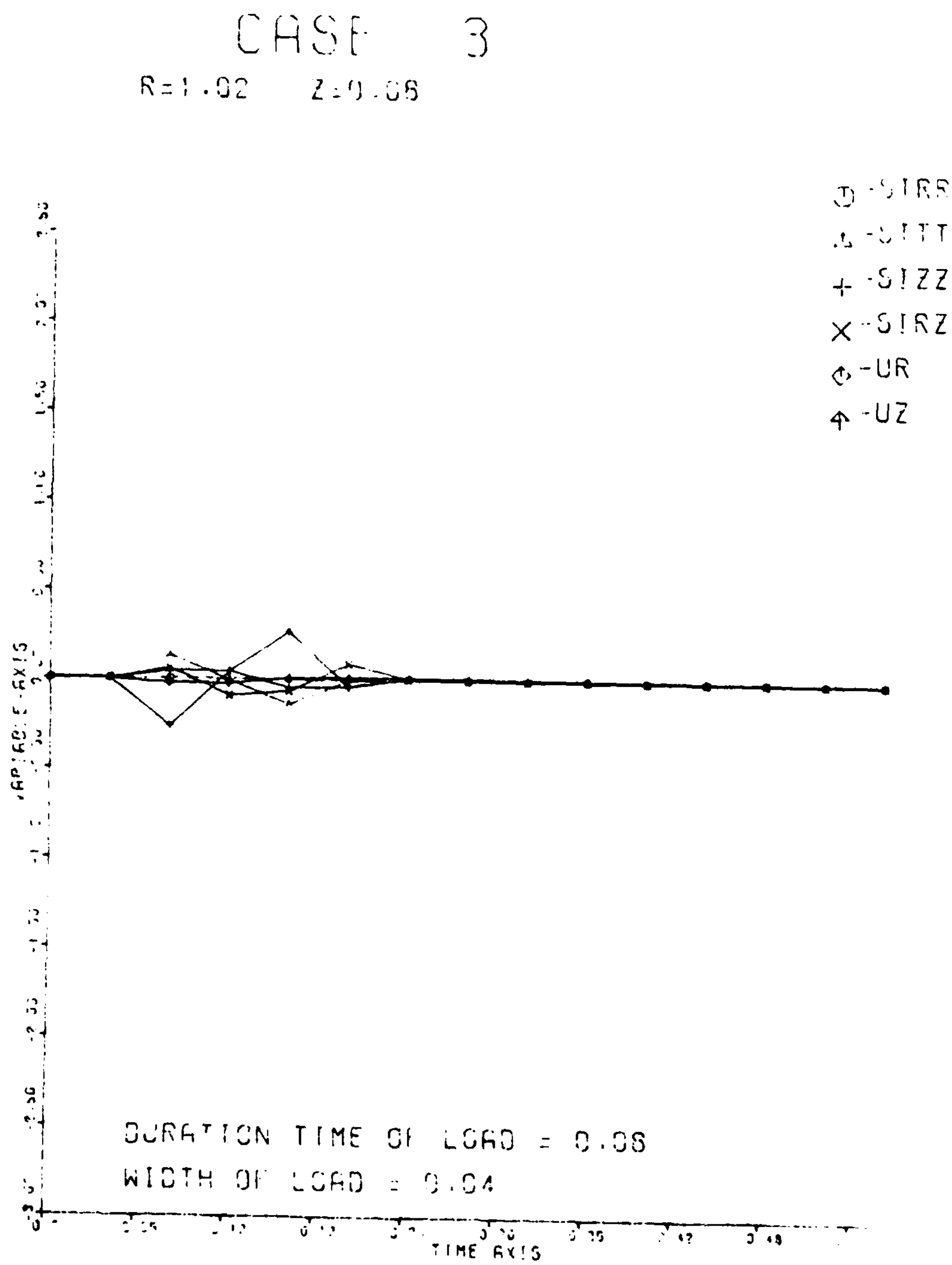

CASE 3
R=1.02 Z=0.06
-SIRR
-SITT
-SIZZ
-SIRZ
-UR
-UZ
DURATION TIME OF LOAD = 0.08
WIDTH OF LOAD = 0.04
TIME AXIS

Figure 49

AD-A118 600 MASSACHUSETTS UNIV AMHERST DEPT OF CIVIL ENGINEERING F/G 20/14
RESPONSE OF THICK CYLINDRICAL SHELLS TO TRANSIENT INTERNAL LOAD--ETC(U)
AUG 82 T HAN-URA, W A NASH DAAG29-77-G-0095

UNCLASSIFIED ARO-14700.2-EG NL

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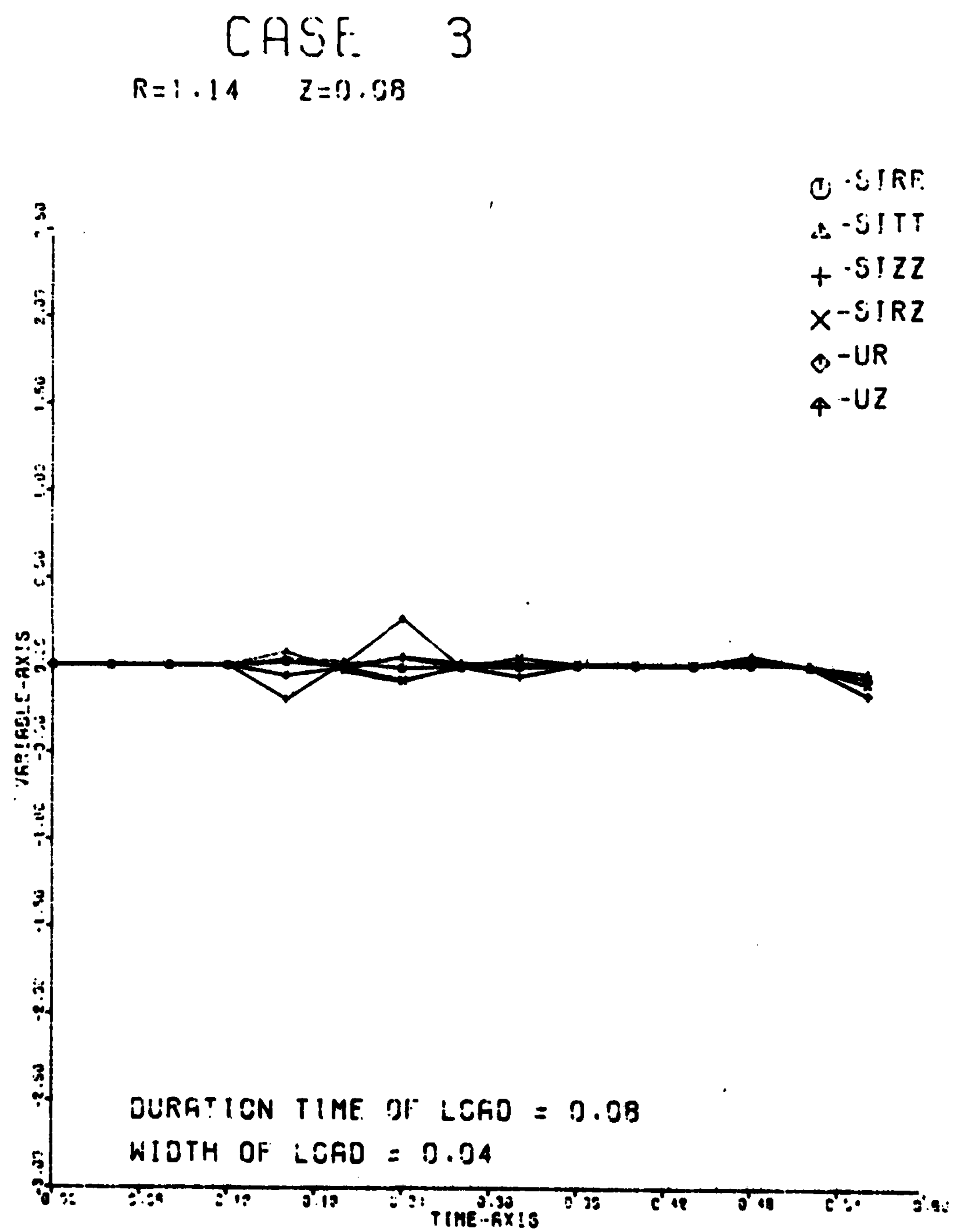

CASE 3
R=1.14 Z=0.08
-SIRR
-SITT
-SIZZ
-SIRZ
-UR
-UZ
VARIABLE-AXIS
TIME-AXIS
DURATION TIME OF LOAD = 0.08
WIDTH OF LOAD = 0.04

Figure 50

CASE 3

R=1.30 Z=0.08

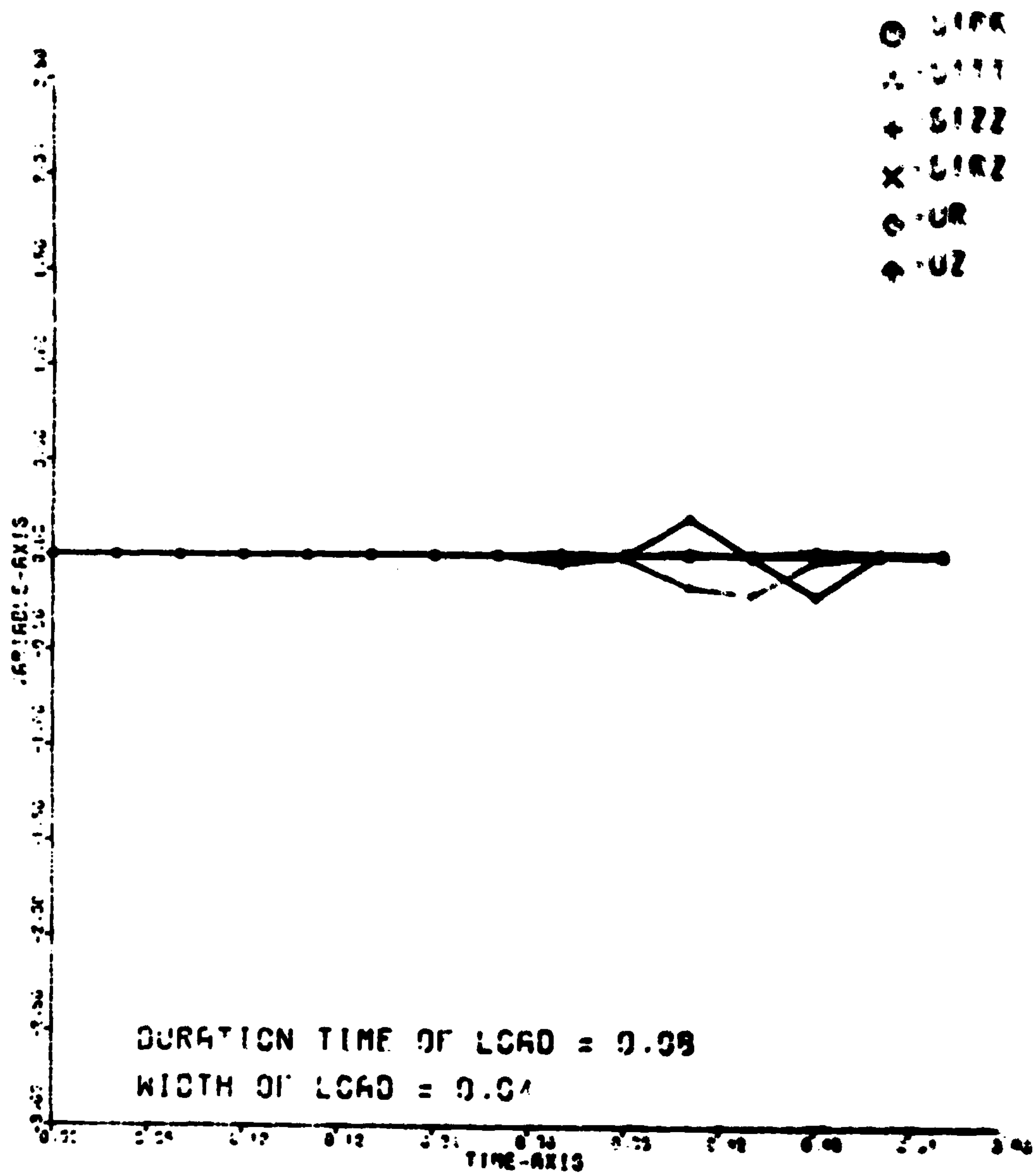


Figure 51

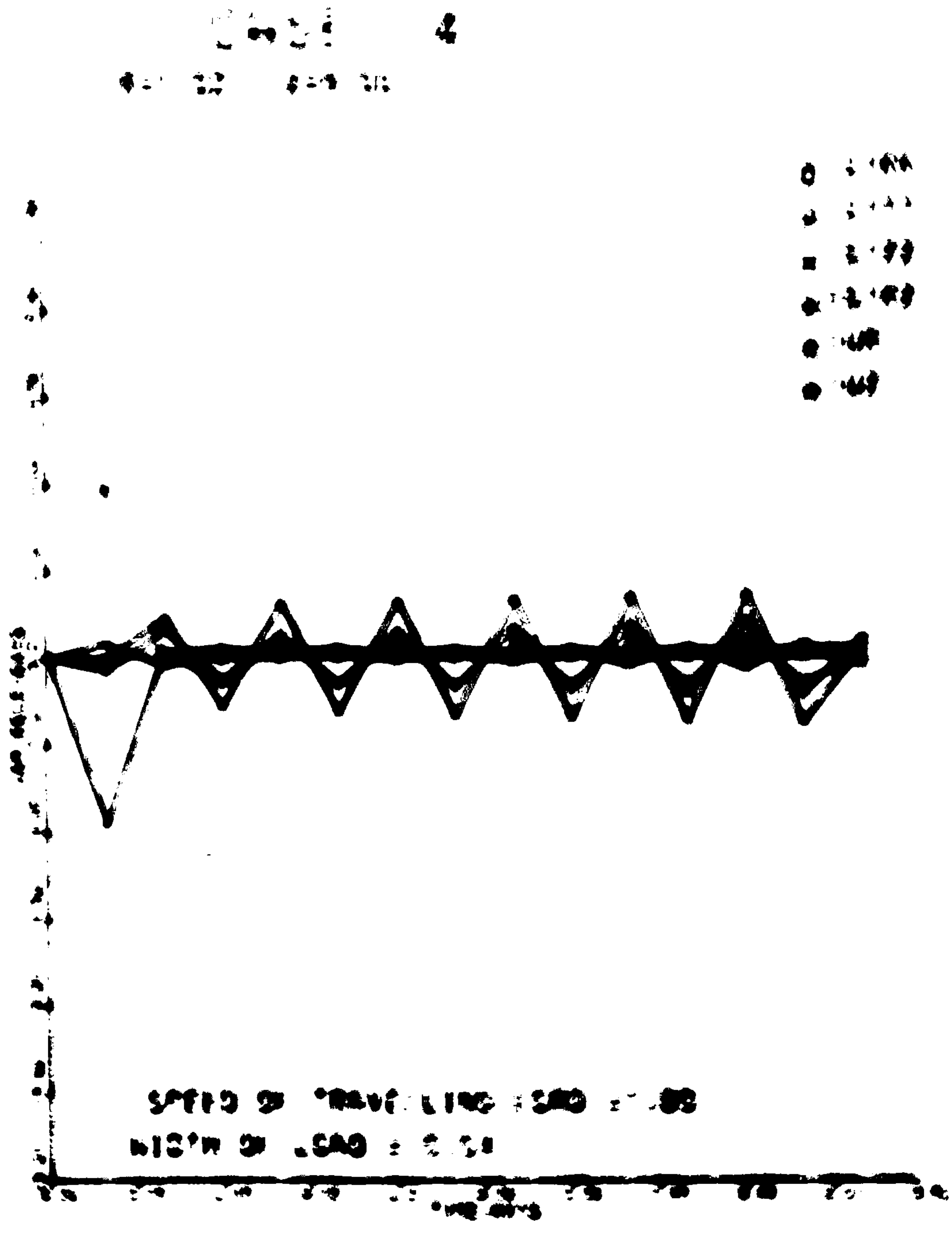
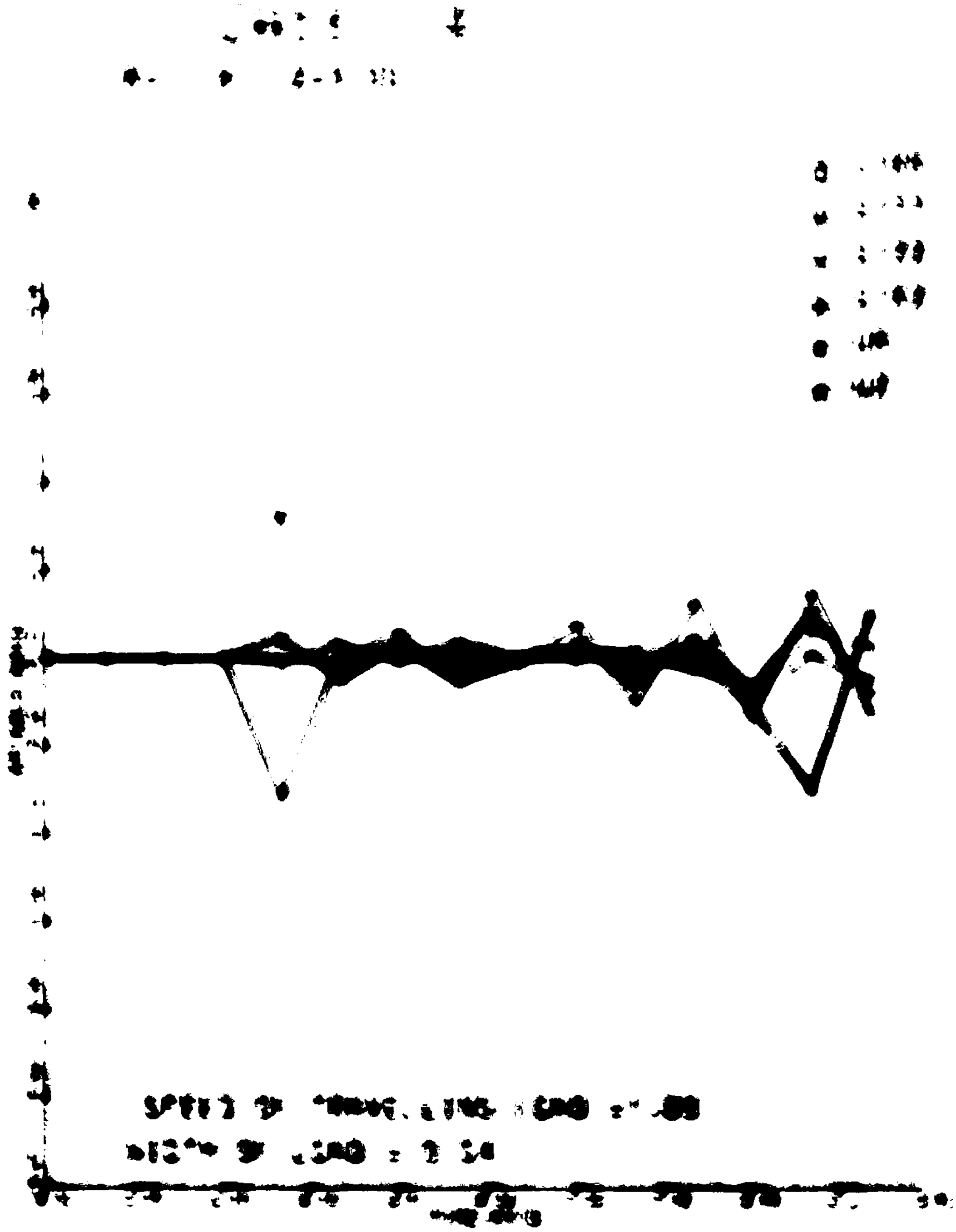
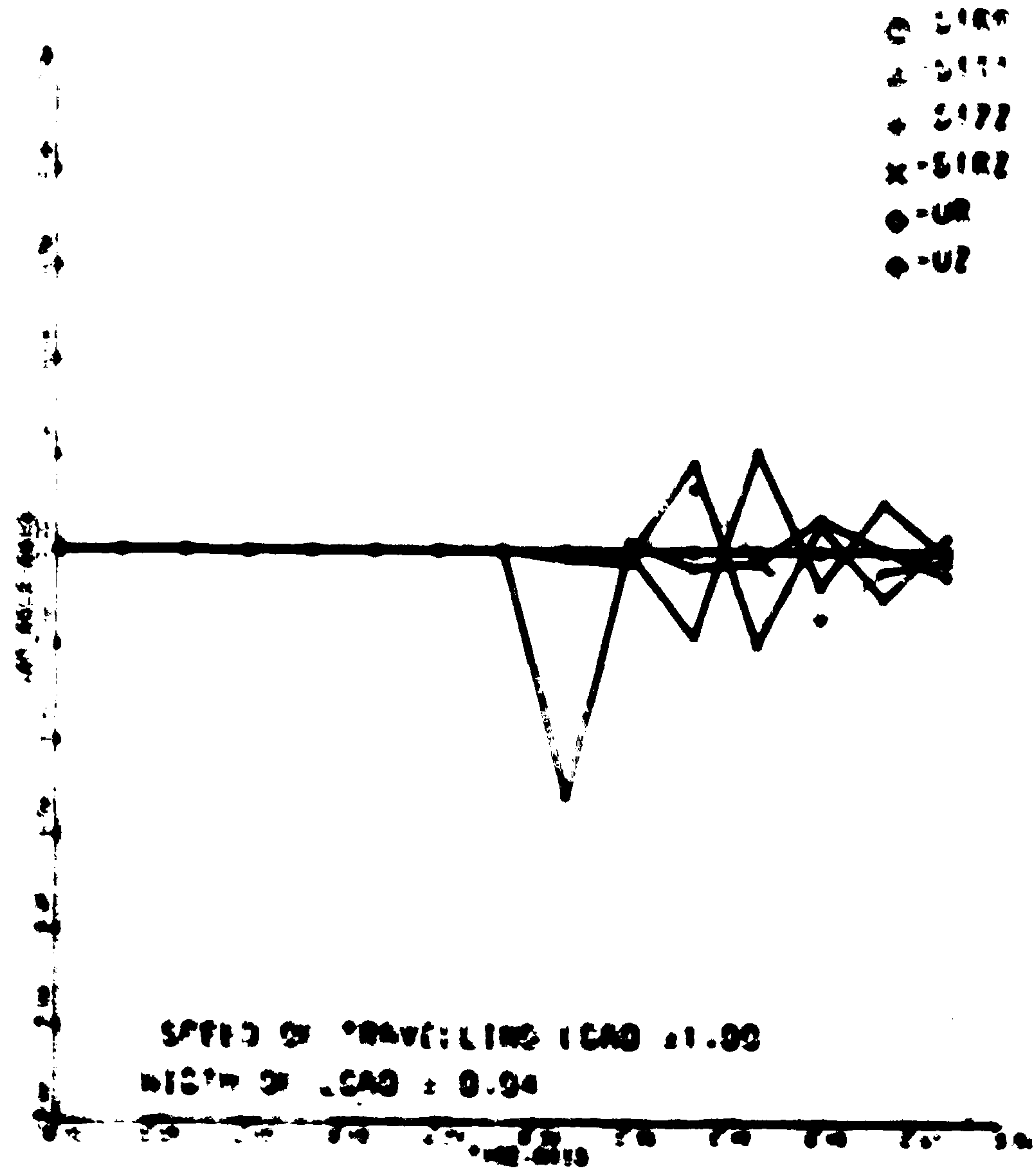


Figure 22



CASE 4



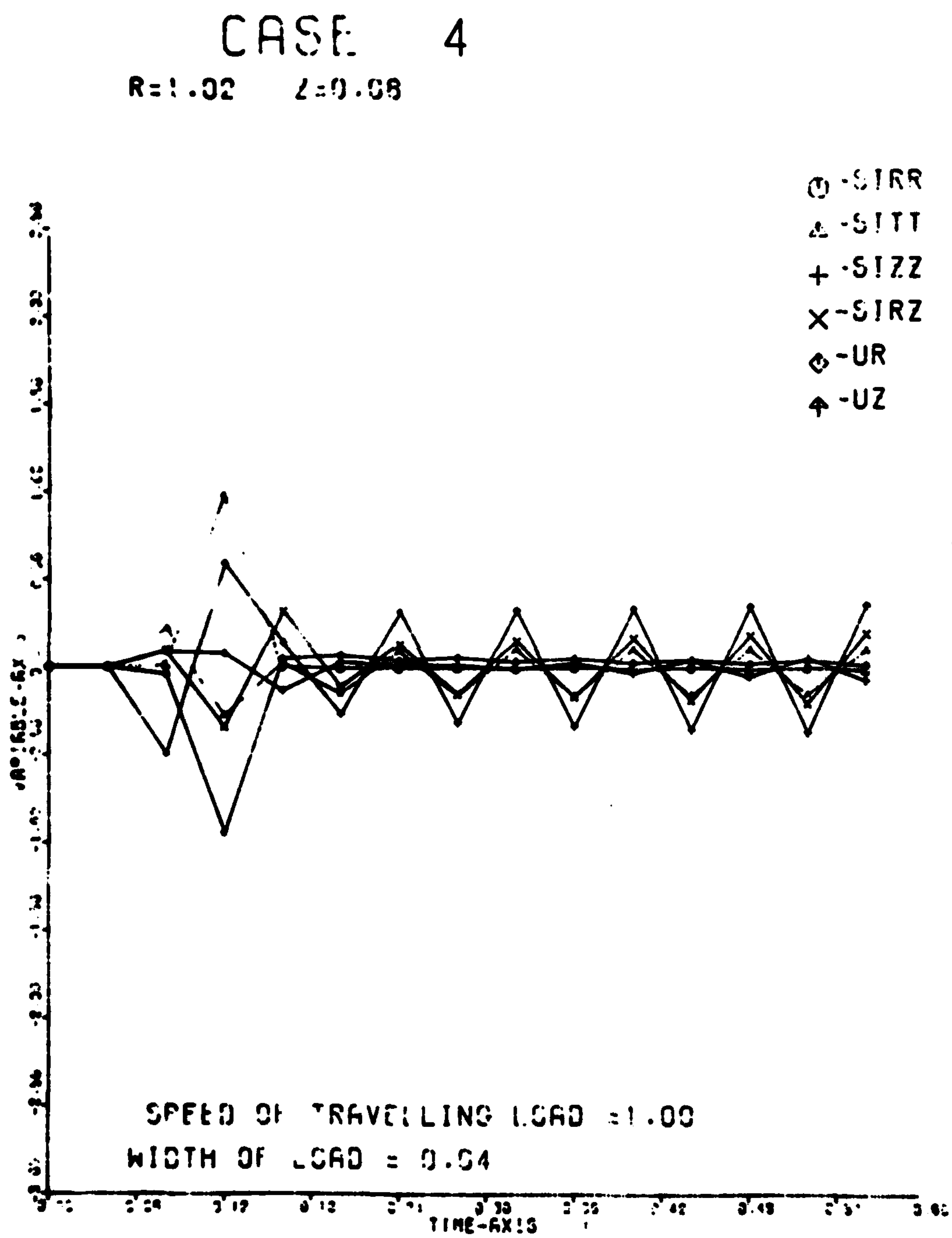

CASE 4
R=1.02 Z=0.08
-SIRR
-SITT
-SIZZ
-SIRZ
-UR
-UZ
SPEED OF TRAVELLING LOAD =1.00
WIDTH OF LOAD = 0.04
TIME-AXIS

Figure 55

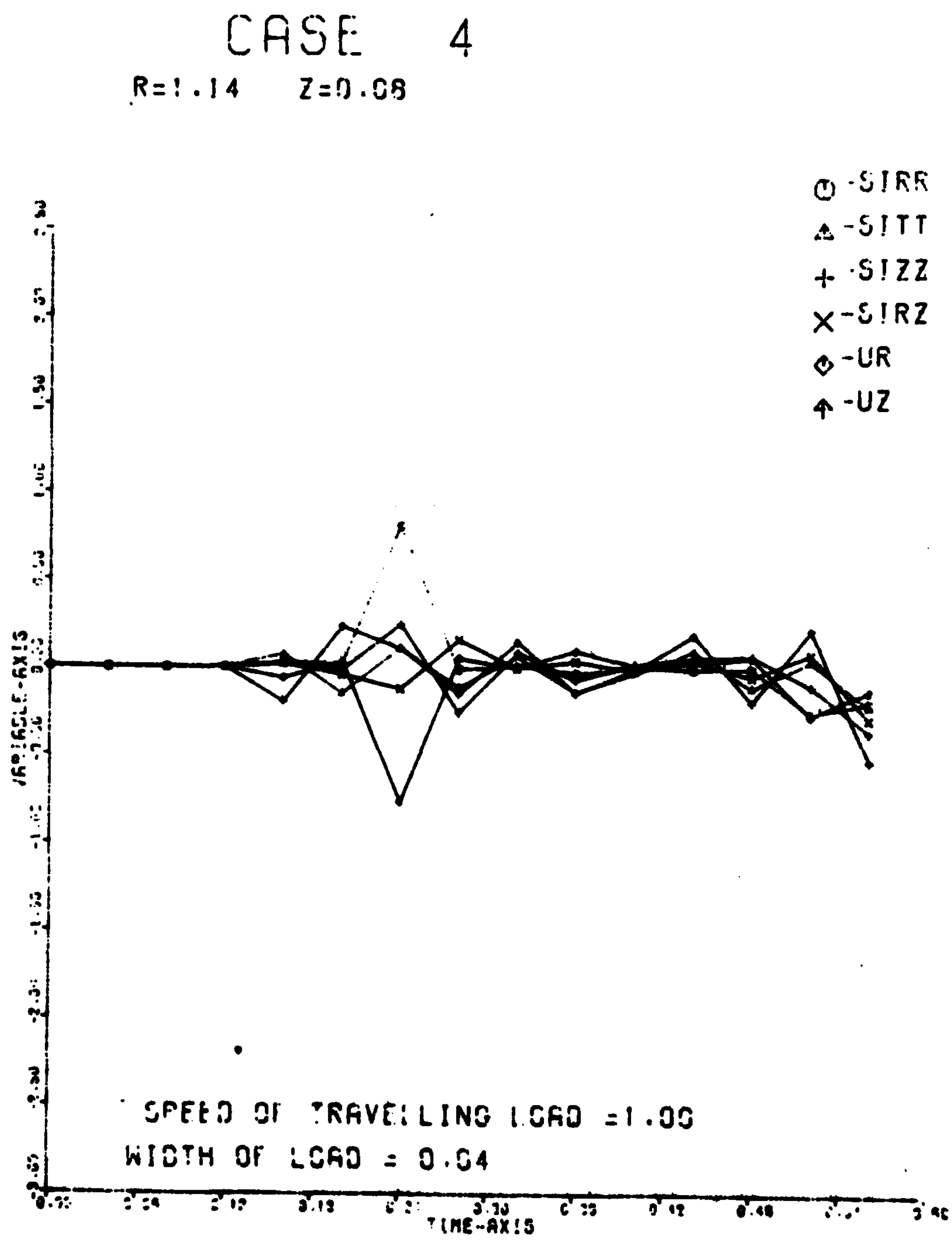

CASE 4
R=1.14 Z=0.08
-SIRR
-SITT
-SIZZ
-SIRZ
-UR
-UZ
VARIABLE-AXIS
TIME-AXIS
SPEED OF TRAVELLING LOAD =1.00
WIDTH OF LOAD = 0.04

Figure 56

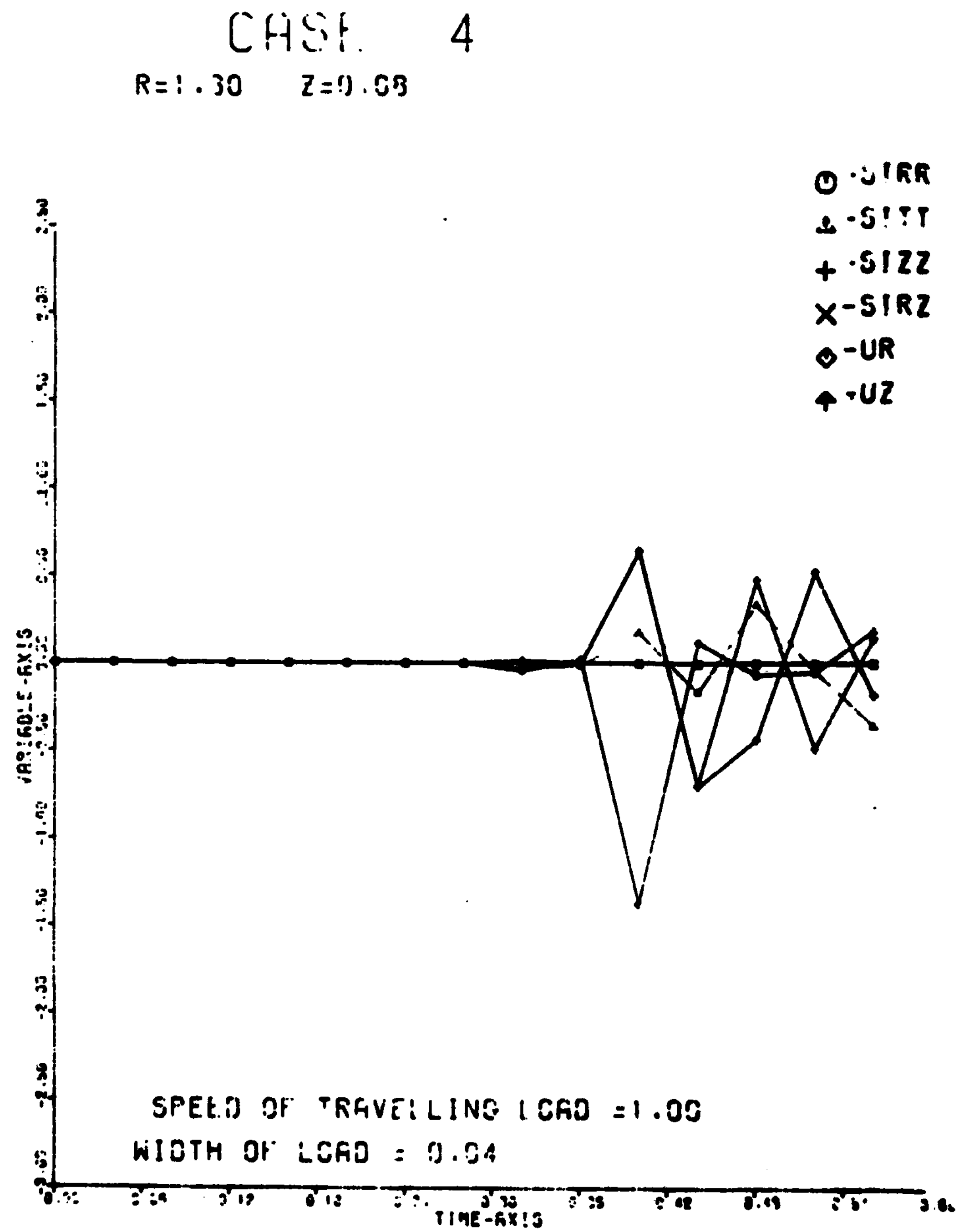

CASE 4
R=1.30 Z=0.08
-SIRR
-SITT
-SIZZ
-SIRZ
-UR
-UZ
VARIABLE-AXIS
TIME-AXIS
SPEED OF TRAVELLING LOAD =1.00
WIDTH OF LOAD = 0.04

Figure 57

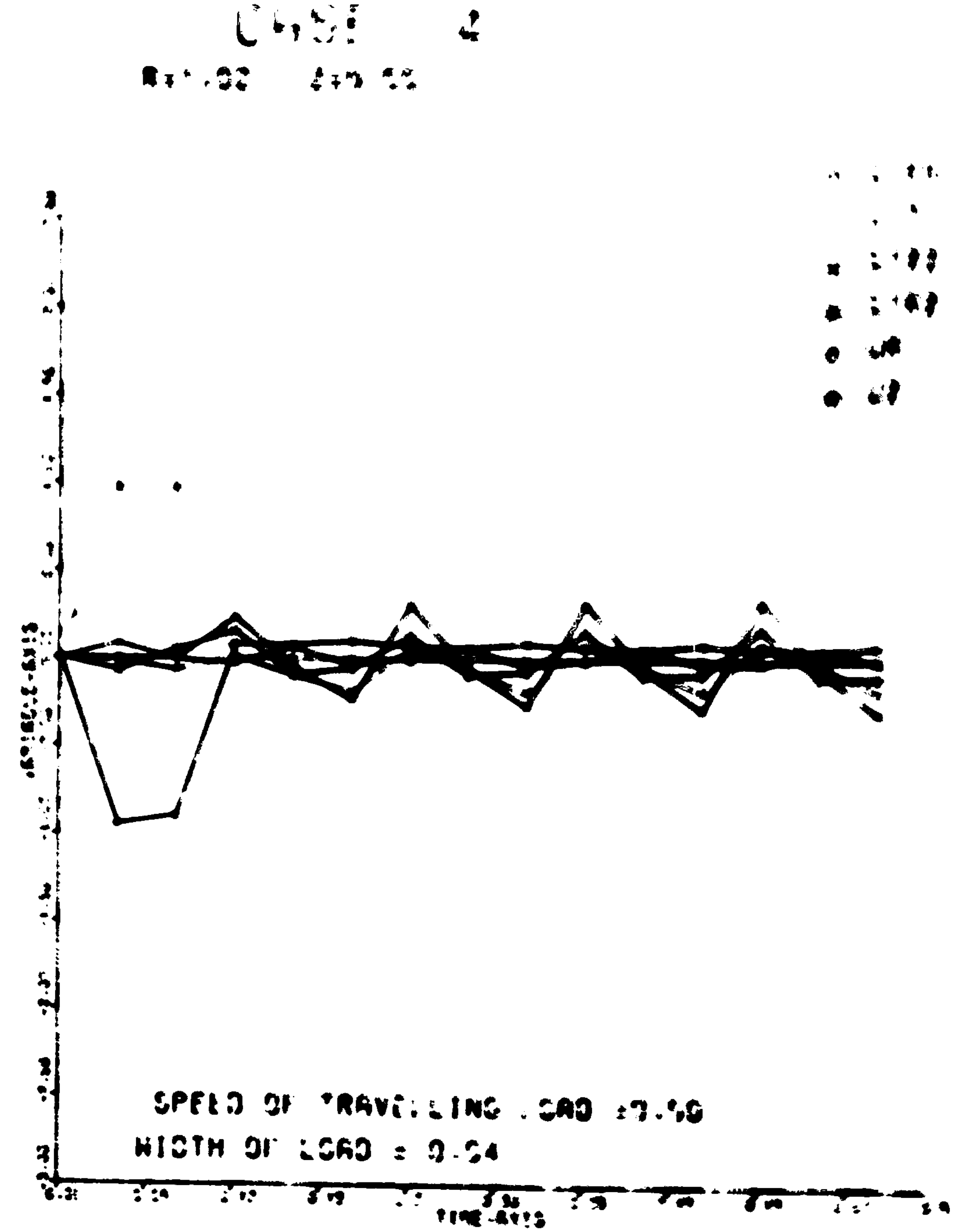
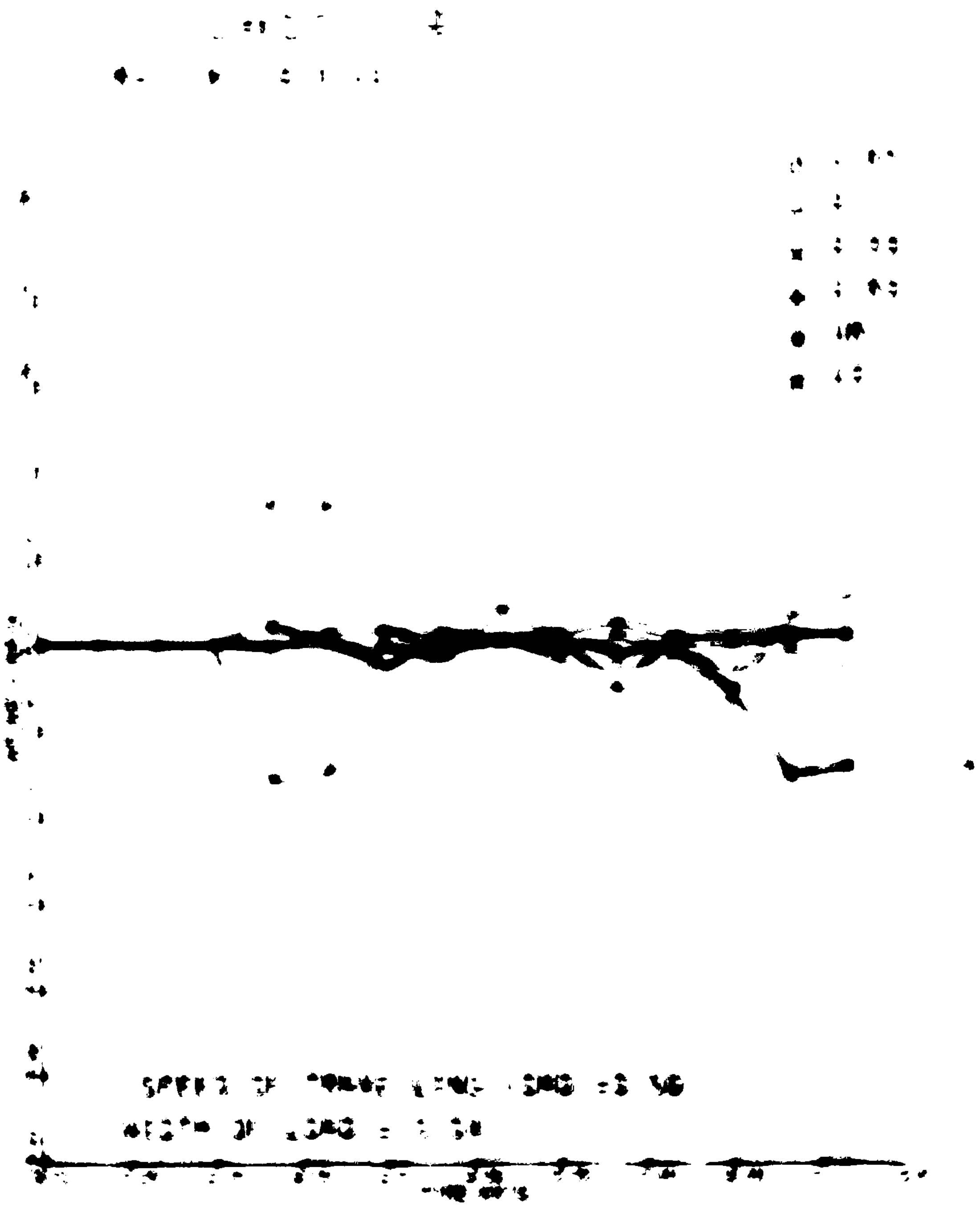


Figure 58



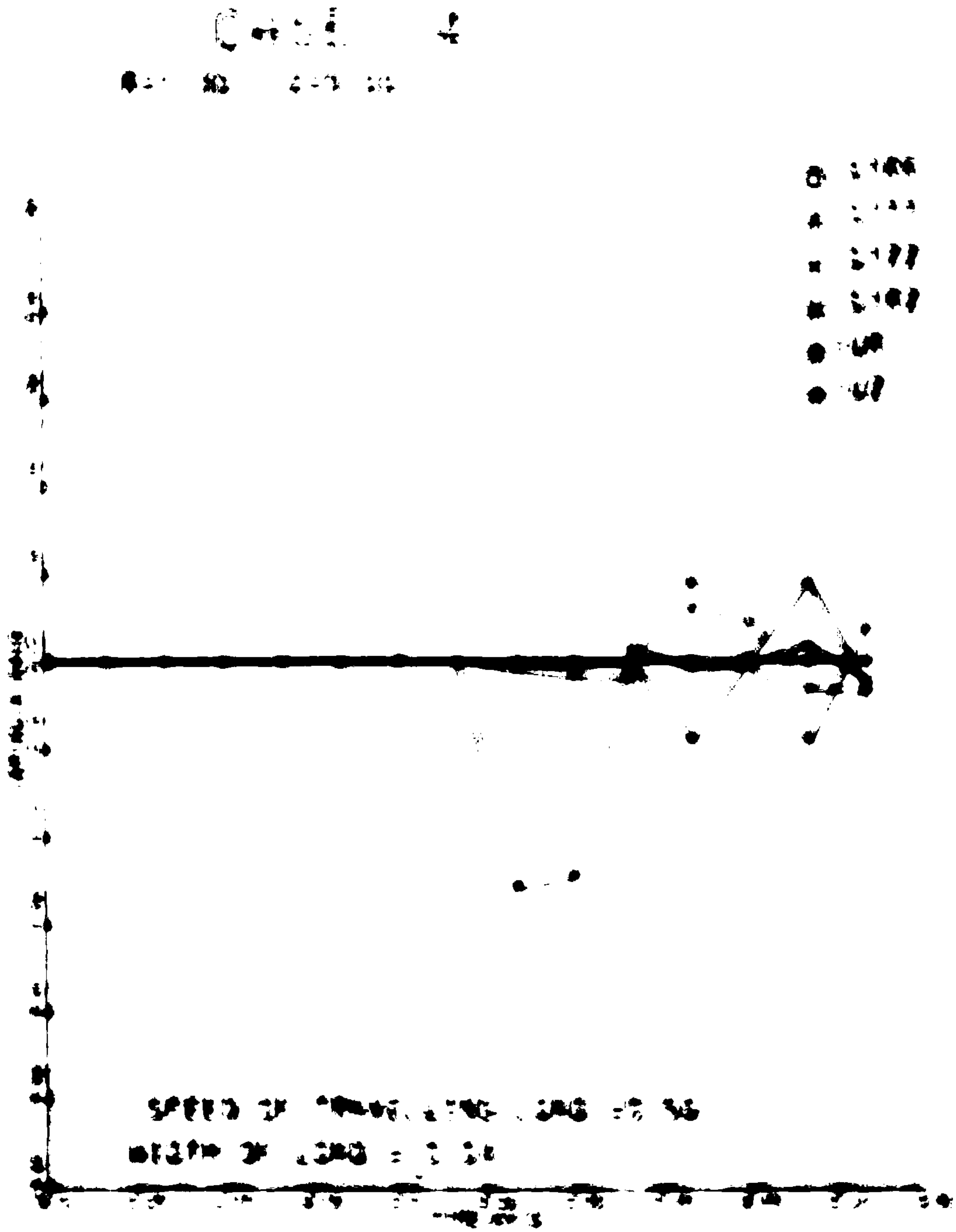


Figure [illegible]

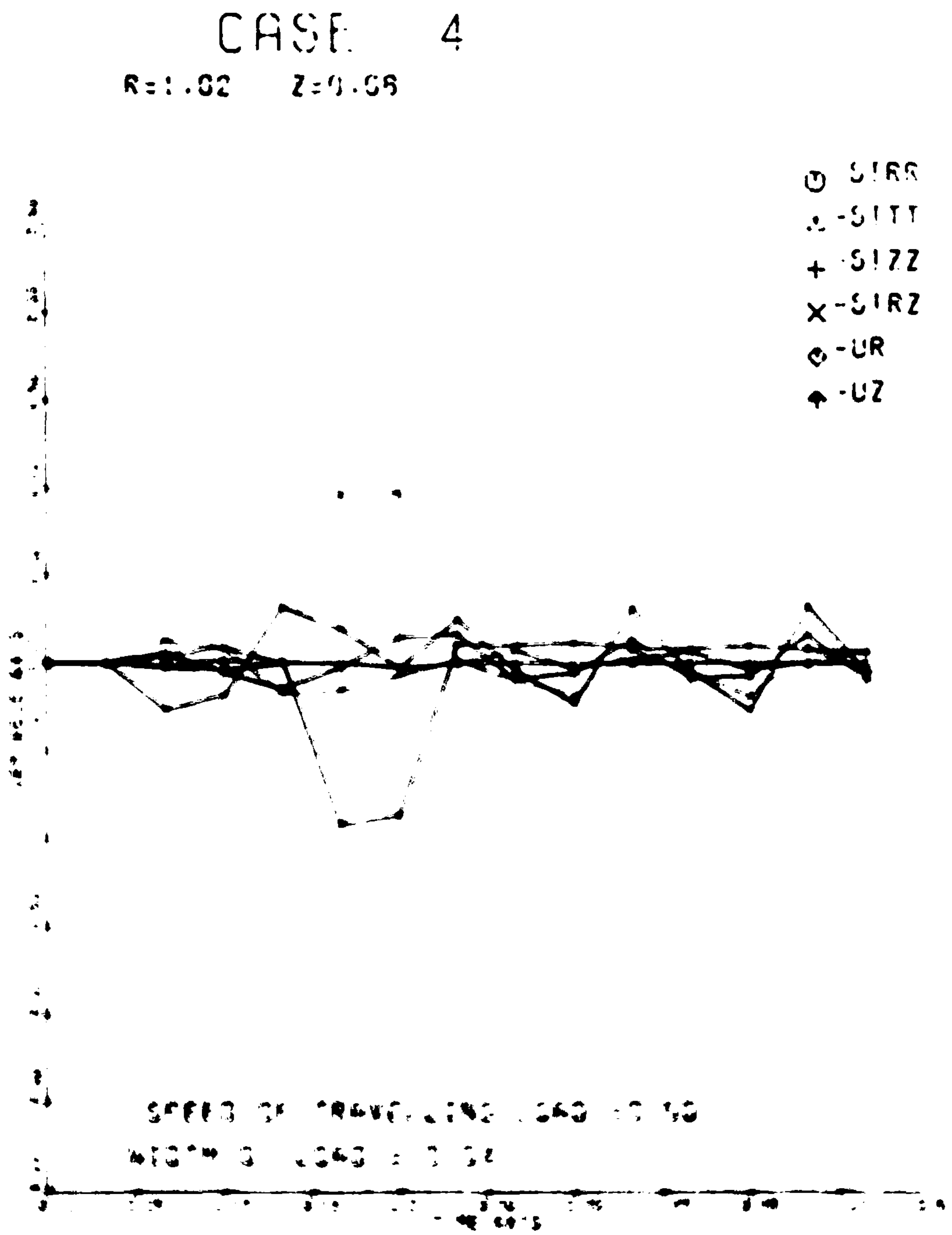


Figure 61

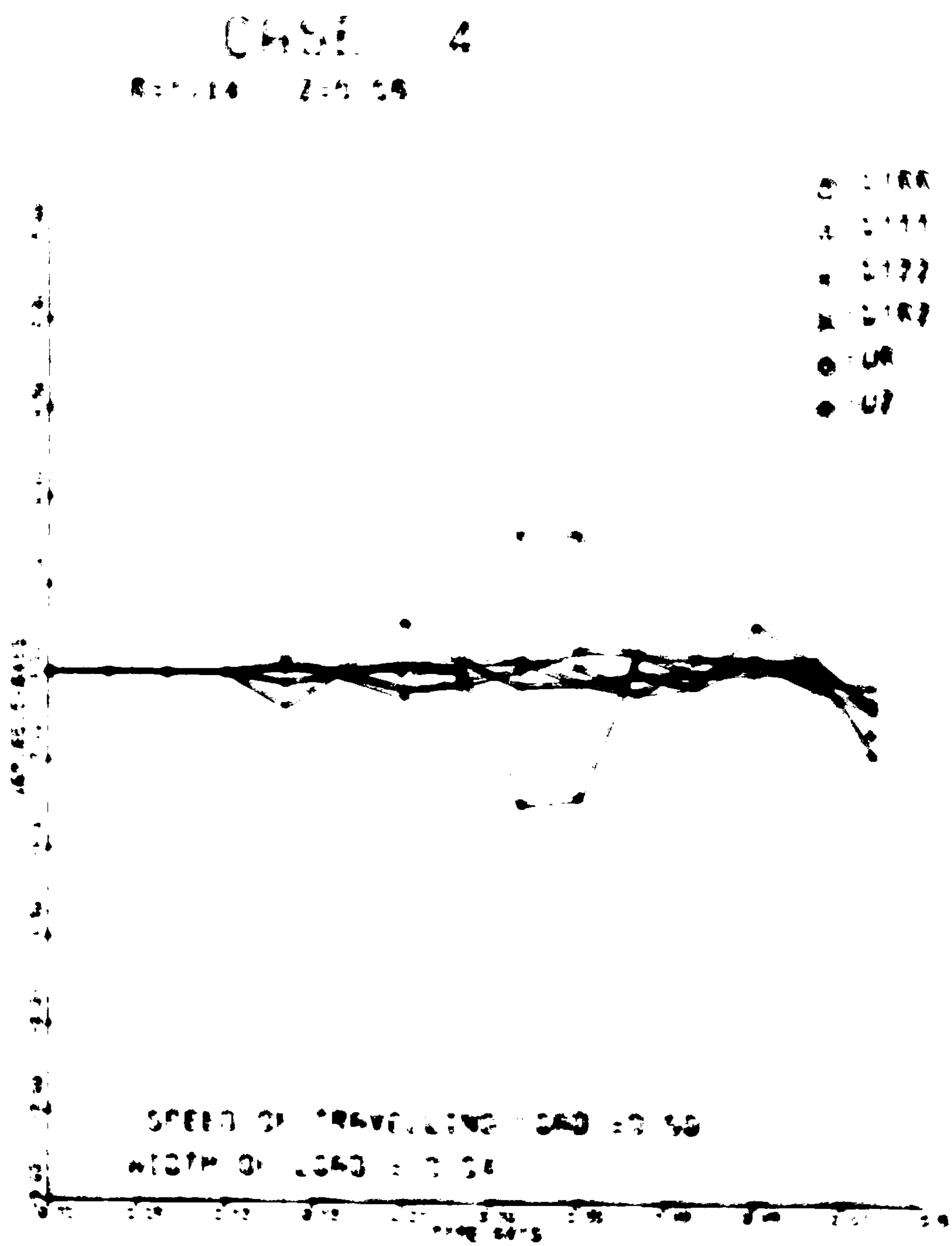
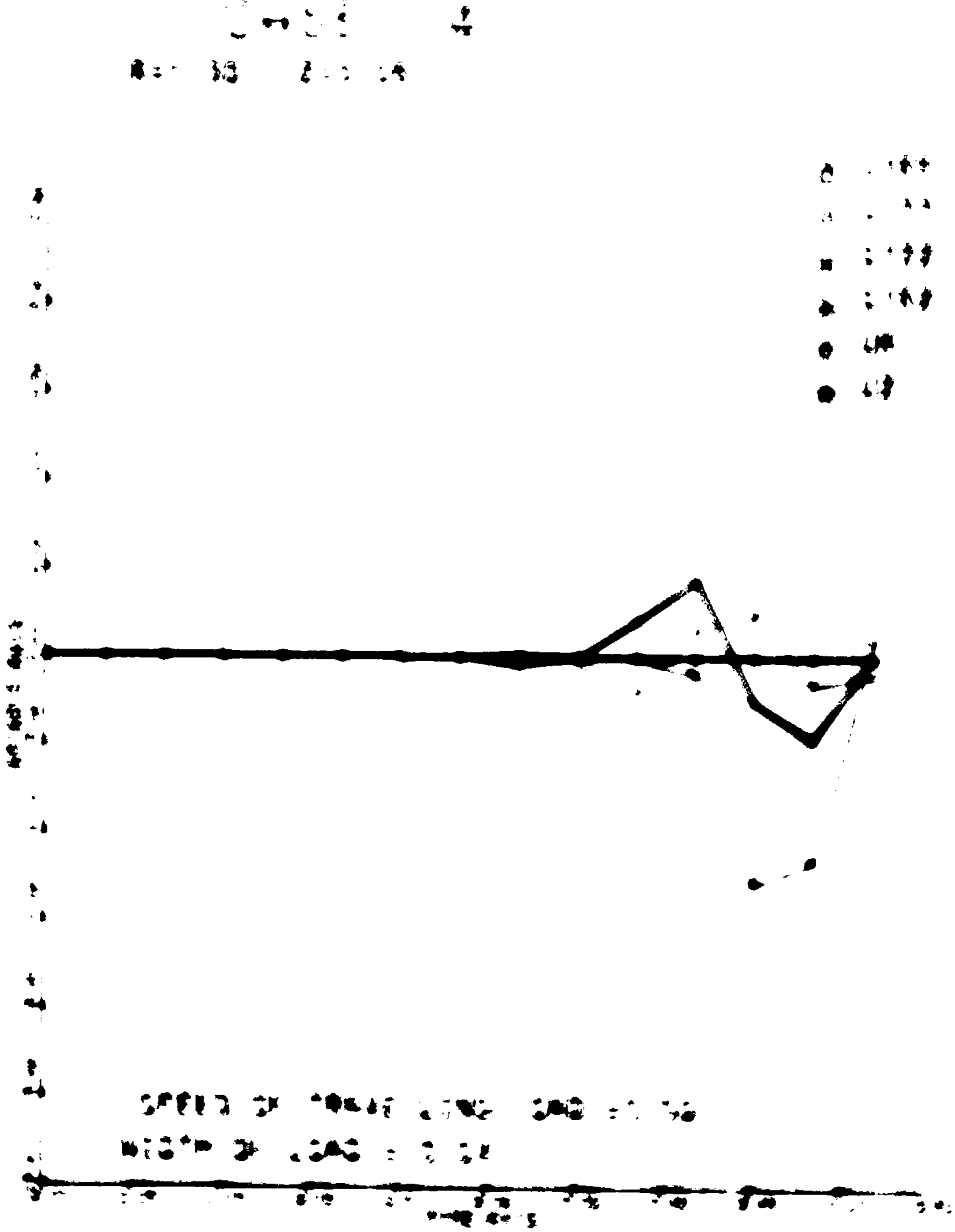


Figure 62



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