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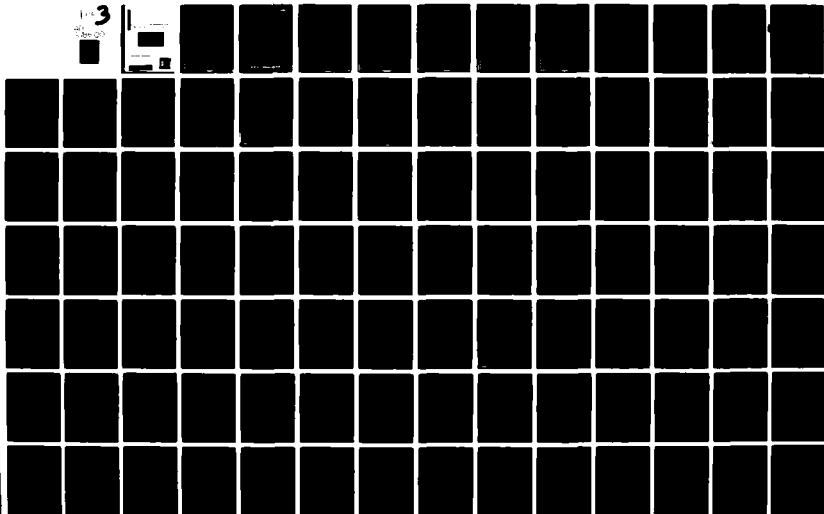
MASSACHUSETTS UNIV AMHERST DEPT OF CIVIL ENGINEERING F/G 20/14
RESPONSE OF THICK CYLINDRICAL SHELLS TO TRANSIENT INTERNAL LOAD--ETC(U)
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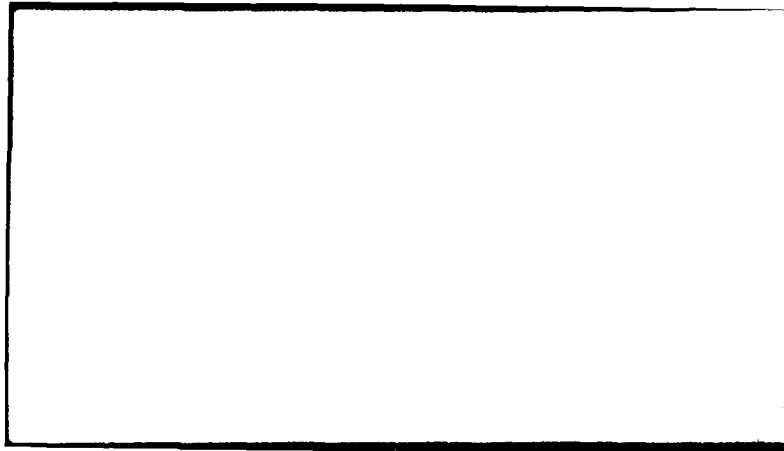
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FINAL TECHNICAL REPORT

TO

DEPARTMENT OF THE ARMY
U.S. ARMY RESEARCH OFFICE
Grant DAAG 29-77-G-0095

ARO Project Number P-14700-E

RESPONSE OF THICK CYLINDRICAL
SHELLS TO TRANSIENT INTERNAL LOADINGS

Takeshi Han-Ura
William A. Nash



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August, 1982

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Amherst, MA 01003

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER 14700.2-EG	2. GOVT ACCESSION NO. AD-A18600	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) Response of Thick Cylindrical Shells to Transient Internal Loadings		5. TYPE OF REPORT & PERIOD COVERED Final: 1 Feb 77 - 31 Dec 81
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Takeshi Han-Ura William A. Nash		8. CONTRACT OR GRANT NUMBER(s) DAAG29 77 G 0095
9. PERFORMING ORGANIZATION NAME AND ADDRESS University of Massachusetts at Amherst Amherst, MA 01003		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS U. S. Army Research Office Post Office Box 12211 Research Triangle Park, NC 27709		12. REPORT DATE Aug 82
		13. NUMBER OF PAGES 16 + reprints
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report) Approved for public release; distribution unlimited.		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report) NA		
18. SUPPLEMENTARY NOTES The view, opinions, and/or findings contained in this report are those of the author(s) and should not be construed as an official Department of the Army position, policy, or decision, unless so designated by other documentation.		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) elastic bodies wave propagation cylindrical shells shell geometries thick-walled shells		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) The case of wave propagation in a thick-walled elastic cylindrical shells subject to dynamically applied internal pressure is examined for various shell geometries and modes of application of the internal loading. Shells of both infinite as well as semi-infinite length are treated. In both cases the loading is considered to be axisymmetric. The investigation culminates in the determination of dynamic behavior of the thick shell subject to a band of constant intensity pressure moving at constant velocity along the inner surface of the shell. Displacement and strain components at any point in the shell may be evaluated in terms of stress variables from the computer program presented.		

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ABSTRACT

The case of wave propagation in a thick-walled elastic cylindrical shell subject to dynamically applied internal pressure is examined for various shell geometries and modes of application of the internal loading. Shells of both infinite as well as semi-infinite length are treated. In both cases the loading is considered to be axisymmetric. The investigation culminates in the determination of dynamic behavior of the thick shell subject to a band of constant intensity pressure moving at constant velocity along the inner surface of the shell. Displacement and strain components at any point in the shell may be evaluated in terms of dimensionless variables from the computer program presented.




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NOMENCLATURE

$r, z,$	Cylindrical coordinates
σ_{rr}	Radial stress component
σ_{zz}	Vertical stress component
$\sigma_{\theta\theta}$	Tangential stress component
σ_{rz}	Vertical shear stress component
U_r	Radial particle velocity component
U_z	Vertical particle velocity component
λ, μ	Lame's constants
t	Time dimension
ρ	Material density
n_r, n_z	Cylindrical components of the unit normal vector to the wave surface
G_L	Longitudinal wave velocity
G_S	Shear wave velocity
R	Internal radius
R_0	External radius
α	Dimensionless ratio = $\frac{\lambda}{\lambda + 2\mu}$
β	Dimensionless ratio = $\frac{G_S}{G_L}$
τ	Dimensionless time = $\frac{G_L t}{R}$
A_i	Known quantity at a point i
B_i	Dimensionless time = $\tau - \tau_i$
[]	Bracket notation = [The value at the rear - The value at the front of the wave front of the wave front
,	Partial differentiation

CHAPTER I
INTRODUCTION

Background

Transient motion in solids has been of interest principally in mechanical, mining, and geophysical engineering. In recent years, however, stress wave propagation problems are becoming increasingly important in aeronautical and nuclear engineering in problems associated with structures under impulsive loading. In the current study a thick cylindrical shell subjected to impulsive loads applied at the interior is considered by the application of the method of characteristics.

It is known that when an elastic body is disturbed by a load which is abruptly or gradually applied on a portion of it, elastic waves propagate through the body with two different velocities which depend upon elastic properties. Waves involving dilatation (or voluminal change) are usually called dilatation or longitudinal waves and those involving rotation (or no voluminal change) are called transverse or shear waves.

In such a boundary value problem as considered in this study nonplanar elastic waves, like spherically and cylindrically expanding waves, are propagating through the medium. These nonplanar waves, in contrast to plane waves, change the shape of the wave front as they propagate, thus varying the distribution of stresses and velocities of the disturbed particle in the elastic body.

Various kinds of wave propagation problems which can be represented in terms of a set of partial differential equations have been studied previously using different approaches such as the integral transform method, the finite element method, and the method of characteristics which is relevant to this study. When the theory of characteristics is applied to elastic wave propagation problems it involves the solution of a system of first-order partial differential equations which governs the deformation in the dynamic field. It is more advantageous to apply the method of characteristics rather than the integral transform method to solve complex boundary value problems, since those first-order partial differential equations involve stresses and particles as dependent variables.

In solving a problem expressed in terms of a system of partial differential equations which involves more than two independent variables certain integral or solution surfaces may exist in the solution space. The integral surfaces consist of what are called the characteristics (or the characteristic curves or waves), and the relations governing the dependent variables along these characteristics will be called characteristic equations.

If only discontinuities in the first partial derivatives of continuous dependent variables on the wave fronts are assumed, the characteristic equations can be derived by the employment of Hadamard's kinematical discontinuity relations (1949) (so called "weak" discontinuity relations). In their work on cylindrical and spherical wave propagation by the method of characteristics [6], Chou and Koenig

determined the distribution of stresses behind the wave fronts of the one-dimensional case where the load is abruptly applied. Their computational procedure is that they first derive the characteristic equations compatible along the leading wave front by the employment of the weak discontinuity relations and then evaluate the solution domain behind the wave front by numerical stepwise integration of the characteristic equations. Their method, however, is restricted to one dimensional problems.

Hadamard's work was successfully extended by T.Y. Thomas[2] to kinematical discontinuity relations across a wave front where the dependent variables themselves are discontinuous (so called "strong" discontinuity relations). Subsequently these strong discontinuity relations provided the capability for solving impulsive loading problems of two-dimensional cases by the method of characteristics. Recently in a series of his works[3][4][5][6], M. Ziv successfully applied the theory of characteristics to two-dimensional wave propagation problems where the load is applied abruptly to the boundary. In this case the discontinuity of the dependent variables will occur on the wave front. A computational method with a computer code was presented in his work[6] for the transient motion of a half-space subject to an impulsive load applied radially and uniformly at the boundary of a cylindrical cavity, and his work is extended in the present study to the case of stress propagation in a thick cylindrical shell due to internal axisymmetric impulsive loading.

The computational procedure for solving an impulsive loading problem of the two-dimensional case by the method of characteristics can be roughly divided into three stages as follows:

1. Characteristic Formulation

Combining Newton's laws of motion with the elastic relations obtained from Hooke's law gives us a system of first-order partial differential equations involving the variables of stresses and particle velocities, and after certain mathematical procedures involving the application of weak discontinuity relations we obtain a system of characteristic equations along the characteristics.

2. Application of Strong Discontinuity relations

Strong discontinuity relations are superimposed on the characteristic equations obtained in the previous stage. Then, we can compute the decay of the stresses along the wave front.

3. Numerical Integration

The solution domain behind the wave front is divided into a grid system by the characteristics along which the characteristic equations expressed in finite difference form will be integrated in a stepwise manner.

An incident longitudinal wave generated by an abruptly applied load will emanate from the inner surface of the tube through the medium and after some time elapses it strikes the outer free surface. Thus we must consider a reflection of waves from the outer free surface as well as from the free top surface (of all four cases to be considered only in CASE 1 does a free top surface exist). Since all action is linear elastic we can apply the principle of superposition to the interacted region of incident and reflected waves.

If an incident longitudinal wave strikes a free surface normally, it is totally reflected with a 180° change in phase, namely a compression wave will be reflected as a tensile wave and vice versa. It should be noted here that near a free surface where an incident compression wave was reflected and transformed into a tensile wave spectacular fracture caused by so called "spalling"[7](p203) may occur. Spalling occurs when a high intensity compressive wave reflects at a free surface and will play a main role in fracture of structures such as thick cylindrical concrete shells under impulsive loading. Furthermore it is a fact that after reflection at the outer free surface incident spherical and cylindrical waves change the nature of divergent waves into that of convergent waves, namely the intensity of the reflected tensile wave will increase as it propagates towards the inner surface. Therefore it is likely that spalling might occur not only near the outer free surface but also in the region far from the reflected surface.

Present Boundary Value Problems

Before presenting the characteristic formulation the boundary value problems to be considered are discussed here in detail. Two basic structures are shown in Figure 1 and Figure 2 while we consider four cases of loading and geometry as follows:

1. CASE 1

A semi-infinitely long thick cylindrical shell subjected to a uniformly and radially applied impulsive load on the entire inner surface of the shell.

2. CASE 2

An infinitely long thick cylindrical shell subjected to a uniformly and radially applied impulsive load along the semi-infinite length of the interior (from $z=0$ to $z+\infty$).

3. CASE 3

The same shell as CASE 2 subjected to a uniformly and radially applied impulsive load along the finite length of the interior (from $z=-a$ to $z+a$).

4. CASE 4

The same shell with the same initial conditions as CASE 3, however, the load of finite length is travelling with a constant speed in the direction of the z -axis.

Loading in CASE 1 and CASE 2 is axisymmetric and is given in terms of a step radial-stress input which is an abruptly applied permanent constant load. We also will consider the case of a rectangular input which is equivalent to a case involving a loading and unloading process. The rectangular input case which is essential to calculate the travelling-load case (CASE 4) can be evaluated by means of the superposition of two step-input cases.

CHAPTER II

CHARACTERISTIC FORMULATION

Basic Dynamic Field Equations

The basic dynamic field equations governing the deformation in the solid stem from combining the equations of motion with the equations of the stress-strain relationships. The equations of motion for linear elastic, isotropic, homogeneous material in cylindrical coordinates are written as:

$$\frac{\partial \sigma_{rr}}{\partial r} + \frac{\partial \sigma_{rs}}{\partial s} + \frac{1}{r}(\sigma_{rr} - \sigma_{\theta\theta}) = \rho \frac{\partial u_r}{\partial t}$$
$$\frac{\partial \sigma_{rs}}{\partial r} + \frac{\partial \sigma_{ss}}{\partial s} + \frac{\sigma_{rs}}{r} = \rho \frac{\partial u_s}{\partial t},$$

and the equations of the stress-strain relationships after differentiation with respect to time are given as:

$$\frac{\partial \sigma_{rr}}{\partial t} = (\lambda + 2\mu) \frac{\partial u_r}{\partial r} + \lambda \left(\frac{u_r}{r} + \frac{\partial u_s}{\partial s} \right)$$
$$\frac{\partial \sigma_{ss}}{\partial t} = (\lambda + 2\mu) \frac{\partial u_s}{\partial s} + \lambda \left(\frac{u_r}{r} + \frac{\partial u_r}{\partial r} \right)$$
$$\frac{\partial \sigma_{\theta\theta}}{\partial t} = (\lambda + 2\mu) \frac{u_r}{r} + \lambda \left(\frac{\partial u_r}{\partial r} + \frac{\partial u_s}{\partial s} \right)$$
$$\frac{\partial \sigma_{rs}}{\partial t} = \mu \left(\frac{\partial u_s}{\partial r} + \frac{\partial u_r}{\partial s} \right).$$

These symbols are defined in the nomenclature for the geometry shown in Figure 1 and Figure 2.

Characteristic Equations

The derivation of the characteristic equations from the above dynamic field equations was presented in detail by Siv in references[3][5]. The general orthogonal scheme for the characteristic formulation[6], however, is repeated here for convenience.

When stress waves propagate in the x,z,t space, they form cones as shown in Figure 3 and the cones consist of, or are characterized by, the infinite number of the characteristics whose origin is the apex of the cone. In our scheme, however, we consider only four characteristic curves which are formed by the intersection of the surface of the cone with two planes passing through the t -axis. These planes intersect orthogonally with each other at the t -axis and each plane has two intersection lines with the cone to form two characteristic curves (or lines). We define these orthogonal characteristic curves as bicharacteristic curves or the bicharacteristics.

The characteristic equations governing the variation of stresses and particle velocities along the bicharacteristic curves are reduced by the orthogonal scheme, in which the bicharacteristic curves are separated by 90° from each other. These characteristic equations along the integration paths shown in Figure 4 are written as follows according to reference[6]:

$$d\sigma_{rr} - \rho G_L dU_r = \left(\lambda \left(\frac{\partial U_s}{\partial s} + \frac{U_r}{r} \right) - G_L \left(\frac{\partial \sigma_{rs}}{\partial s} - \frac{\sigma_{\theta\theta} - \sigma_{rr}}{r} \right) \right) dt$$

for the bicharacteristic curve $(n_r=1, n_s=0) \theta=0^\circ$ along $\frac{dr}{dt} = G_L, \frac{ds}{dt} = 0$

$$d\sigma_{rr} + \rho G_L dU_r = \left(\lambda \left(\frac{\partial U_s}{\partial s} + \frac{U_r}{r} \right) + G_L \left(\frac{\partial \sigma_{rs}}{\partial s} - \frac{\sigma_{\theta\theta} - \sigma_{rr}}{r} \right) \right) dt$$

for the bicharacteristic curve $(n_r=-1, n_s=0) \theta=0^\circ$ along $\frac{dr}{dt} = -G_L, \frac{ds}{dt} = 0$,

$$d\sigma_{rs} - \rho G_S dU_s = \left(\mu \frac{\partial U_r}{\partial s} - G_S \left(\frac{\partial \sigma_{ss}}{\partial s} + \frac{\sigma_{rr}}{r} \right) \right) dt$$

for the bicharacteristic curve $(n_r=1, n_s=0) \theta=90^\circ$ along $\frac{dr}{dt} = G_S, \frac{ds}{dt} = 0$,

$$d\sigma_{rs} + \rho G_S dU_s = \left(\mu \frac{\partial U_r}{\partial s} + G_S \left(\frac{\partial \sigma_{ss}}{\partial s} + \frac{\sigma_{rr}}{r} \right) \right) dt$$

for the bicharacteristic curve $(n_r=-1, n_s=0) \theta=90^\circ$ along $\frac{dr}{dt} = -G_S, \frac{ds}{dt} = 0$,

$$d\sigma_{ss} - \rho G_L dU_s = \left(\lambda \left(\frac{\partial U_r}{\partial r} + \frac{U_r}{r} \right) - G_L \left(\frac{\partial \sigma_{rs}}{\partial r} + \frac{\sigma_{rr}}{r} \right) \right) dt$$

for the bicharacteristic curve $(n_r=0, n_s=1) \theta=90^\circ$ along $\frac{dr}{dt} = 0, \frac{ds}{dt} = G_L$,

$$d\sigma_{ss} + \rho G_L dU_s = \left(\lambda \left(\frac{\partial U_r}{\partial r} + \frac{U_r}{r} \right) + G_L \left(\frac{\partial \sigma_{rs}}{\partial r} + \frac{\sigma_{rr}}{r} \right) \right) dt$$

for the bicharacteristic curve $(n_r=0, n_s=-1) \theta=90^\circ$ along $\frac{dr}{dt} = 0, \frac{ds}{dt} = -G_L$,

$$d\sigma_{rs} - \rho G_S dU_r = \left(\mu \frac{\partial U_s}{\partial r} - G_S \left(\frac{\partial \sigma_{rr}}{\partial r} - \frac{\sigma_{\theta\theta} - \sigma_{rr}}{r} \right) \right) dt$$

for the bicharacteristic curve $(n_r=0, n_s=1) \theta=90^\circ$ along $\frac{dr}{dt} = 0, \frac{ds}{dt} = G_S$,

$$d\sigma_{rs} + \rho G_S dU_r = \left(\mu \frac{\partial U_s}{\partial r} + G_S \left(\frac{\partial \sigma_{rr}}{\partial r} - \frac{\sigma_{\theta\theta} - \sigma_{rr}}{r} \right) \right) dt$$

for the bicharacteristic curve $(n_r=0, n_s=-1) \theta=90^\circ$ along $\frac{dr}{dt} = 0, \frac{ds}{dt} = -G_S$,

$$d\sigma_{rr} = \left((\lambda + 2\mu) \frac{\partial U_r}{\partial r} + \lambda \left(\frac{U_r}{r} + \frac{\partial U_s}{\partial s} \right) \right) dt$$

$$d\sigma_{ss} = \left((\lambda + 2\mu) \frac{\partial U_s}{\partial s} + \lambda \left(\frac{U_r}{r} + \frac{\partial U_r}{\partial r} \right) \right) dt$$

$$d\sigma_{rz} = \mu \left(\frac{\partial U_z}{\partial r} + \frac{\partial U_r}{\partial z} \right) dt$$

$$d\sigma_{\theta\theta} = \left((\lambda + 2\mu) \frac{U_r}{r} + \lambda \left(\frac{\partial U_r}{\partial r} + \frac{\partial U_z}{\partial z} \right) \right) dt$$

$$dU_r = \frac{1}{\rho} \left(\frac{\partial \sigma_{rr}}{\partial r} + \frac{\partial \sigma_{rz}}{\partial z} - \frac{\sigma_{\theta\theta} - \sigma_{rr}}{r} \right) dt$$

$$dU_z = \frac{1}{\rho} \left(\frac{\partial \sigma_{rz}}{\partial r} + \frac{\partial \sigma_{zz}}{\partial z} + \frac{\sigma_{rz}}{r} \right) dt.$$

The last six equations are for the static line $dr=dz=0$. These equations apparently form a system of fourteen simultaneous equations with the fourteen unknowns σ_{rr} , $\sigma_{\theta\theta}$, σ_{zz} , σ_{rz} , $\frac{\partial \sigma_{rr}}{\partial r}$, $\frac{\partial \sigma_{zz}}{\partial z}$, $\frac{\partial \sigma_{rz}}{\partial r}$, $\frac{\partial \sigma_{rz}}{\partial z}$, U_r , U_z , $\frac{\partial U_r}{\partial r}$, $\frac{\partial U_r}{\partial z}$, $\frac{\partial U_z}{\partial r}$, and $\frac{\partial U_z}{\partial z}$.

For convenience, dimensionless variables are now introduced as follows:

$$\begin{aligned} \bar{U}_r &= \frac{U_r}{G_L R}, & \bar{\sigma}_{rr} &= \frac{\sigma_{rr}}{\lambda + 2\mu}, \\ \bar{U}_z &= \frac{U_z}{G_L R}, & \bar{\sigma}_{\theta\theta} &= \frac{\sigma_{\theta\theta}}{\lambda + 2\mu}, \\ \bar{\tau} &= \frac{G_L t}{R}, & \bar{\sigma}_{zz} &= \frac{\sigma_{zz}}{\lambda + 2\mu}, \\ \bar{r} &= \frac{r}{R}, & \bar{\sigma}_{rz} &= \frac{\sigma_{rz}}{\lambda + 2\mu}, \\ \bar{z} &= \frac{z}{R}. \end{aligned}$$

also, let $\alpha = \frac{\lambda}{\lambda + 2\mu}$ and $\frac{G_S}{G_L} = \beta = \left(\frac{\mu}{\lambda + 2\mu} \right)^{\frac{1}{2}}$.

Rewriting the fourteen characteristic equations in terms of the dimensionless variables after the bars have been removed

$$d\sigma_{rr} - dU_r = \left(a \left(\frac{\partial U_s}{\partial s} + \frac{U_r}{r} \right) - \frac{\partial \sigma_{rs}}{\partial s} + \frac{\sigma_{\theta\theta} - \sigma_{\phi\phi}}{r} \right) dr \quad (1)$$

along $dr = dr$,

$$d\sigma_{rr} + dU_r = \left(a \left(\frac{\partial U_s}{\partial s} + \frac{U_r}{r} \right) + \frac{\partial \sigma_{rs}}{\partial s} - \frac{\sigma_{\theta\theta} - \sigma_{\phi\phi}}{r} \right) dr \quad (2)$$

along $dr = -dr$,

$$d\sigma_{rs} - \beta dU_s = \beta \left(\frac{\partial U_r}{\partial s} - \frac{\partial \sigma_{ss}}{\partial s} - \frac{\sigma_{rs}}{r} \right) dr \quad (3)$$

along $dr = \beta dr$,

$$d\sigma_{rs} + \beta dU_s = \beta \left(\frac{\partial U_r}{\partial s} + \frac{\partial \sigma_{ss}}{\partial s} + \frac{\sigma_{rs}}{r} \right) dr \quad (4)$$

along $dr = -\beta dr$,

$$d\sigma_{ss} - dU_s = \left(a \left(\frac{\partial U_r}{\partial r} + \frac{U_r}{r} \right) - \frac{\partial \sigma_{rs}}{\partial r} - \frac{\sigma_{rs}}{r} \right) ds \quad (5)$$

along $ds = dr$,

$$d\sigma_{ss} + dU_s = \left(a \left(\frac{\partial U_r}{\partial r} + \frac{U_r}{r} \right) + \frac{\partial \sigma_{rs}}{\partial r} + \frac{\sigma_{rs}}{r} \right) ds \quad (6)$$

along $ds = -dr$,

$$d\sigma_{rs} - \beta dU_r = \beta \left(\frac{\partial U_s}{\partial r} - \frac{\partial \sigma_{rr}}{\partial r} + \frac{\sigma_{\theta\theta} - \sigma_{\phi\phi}}{r} \right) dr \quad (7)$$

along $ds = \beta dr$,

$$d\sigma_{rs} + \beta dU_r = \beta \left(\frac{\partial U_s}{\partial r} + \frac{\partial \sigma_{rr}}{\partial r} - \frac{\sigma_{\theta\theta} - \sigma_{\phi\phi}}{r} \right) dr \quad (8)$$

along $ds = -\beta dr$,

$$d\sigma_{rr} = \left(\frac{\partial U_r}{\partial r} + a \left(\frac{U_r}{r} + \frac{\partial U_s}{\partial s} \right) \right) dr \quad (9)$$

along $dr = ds = 0$,

$$d\sigma_{ss} = \left(\frac{\partial U_s}{\partial s} + a \left(\frac{U_r}{r} + \frac{\partial U_r}{\partial r} \right) \right) ds \quad (10)$$

along $dr = ds = 0$,

$$d\sigma_{rs} = \beta^2 \left(\frac{\partial U_s}{\partial r} + \frac{\partial U_r}{\partial s} \right) dr \quad (11)$$

along $dr-dz=0$,

$$d\sigma_{\theta\theta} = \left(\frac{U_r}{r} + a \left(\frac{\partial U_r}{\partial r} + \frac{\partial U_z}{\partial z} \right) \right) dr \quad (12)$$

along $dr-dz=0$,

$$dU_r = \left(\frac{\partial \sigma_{rr}}{\partial r} + \frac{\partial \sigma_{rz}}{\partial z} - \frac{\sigma_{\theta\theta} - \sigma_{rr}}{r} \right) dr \quad (13)$$

along $dr-dz=0$,

$$dU_z = \left(\frac{\partial \sigma_{rz}}{\partial r} + \frac{\partial \sigma_{zz}}{\partial z} + \frac{\sigma_{rz}}{r} \right) dr \quad (14)$$

along $dr-dz=0$.

In order to investigate the behavior of the shell at a point (r, z, τ) by the finite difference technique these characteristic equations are expressed in the finite difference form. Letting f be any variable of the equation, df can be written as $df = f - f_1$ where f_1 denotes the known variable at the point 1 and f denotes the unknown variable at the point (r, z, τ) . If we take the first characteristic equation along $dr-dz=0$ we obtain two equations as follows:

$$\sigma_{rr} - \sigma_{rr1} - U_r + U_{r1} = \left(a \left(\frac{\partial U_z}{\partial z} + \frac{U_r}{r} \right) - \frac{\partial \sigma_{rz}}{\partial z} + \frac{\sigma_{\theta\theta} - \sigma_{rr}}{r} \right) (\tau - \tau_1)$$

$$\sigma_{rr} - \sigma_{rr1} - U_r + U_{r1} = \left(a \left(\frac{\partial U_z}{\partial z_1} + \frac{U_{r1}}{r} \right) - \frac{\partial \sigma_{rz}}{\partial z_1} + \frac{\sigma_{\theta\theta_1} - \sigma_{rr1}}{r} \right) (\tau - \tau_1).$$

After adding these two equations, dividing by 2, and gathering the unknown terms on the left hand side we finally come up with

$$\left(1 + \frac{B_1}{2r} \right) \sigma_{rr} - \left(1 + a \frac{B_1}{2r} \right) U_r - \frac{B_1}{2r} \sigma_{\theta\theta} - a \frac{B_1}{2} \frac{\partial U_z}{\partial z} + \frac{B_1}{2} \frac{\partial \sigma_{rz}}{\partial z} = A_1$$

where

$$A_1 = \left(\sigma_{rr} \left(1 - \frac{B}{2r} \right) - U_r \left(1 - \frac{aB}{2r} \right) + \left(a \frac{\partial U}{\partial z} - \frac{\partial \sigma_{rz}}{\partial z} + \frac{\sigma_{\theta\theta}}{r} \right) \frac{B}{2} \right) \frac{1}{r}$$

$$B_1 = r - r_1.$$

Likewise the characteristic equations (2) through (14) are transformed and written in accordance with reference[6]:

	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}	A_{11}	A_{12}	A_{13}	A_{14}
	σ_{xx}	σ_{yy}	σ_{zz}	σ_{xz}	$\frac{\partial \sigma_{xx}}{\partial x}$	$\frac{\partial \sigma_{yy}}{\partial y}$	$\frac{\partial \sigma_{zz}}{\partial z}$	$\frac{\partial \sigma_{xz}}{\partial z}$	U_x	U_y	$\frac{\partial U_x}{\partial x}$	$\frac{\partial U_x}{\partial y}$	$\frac{\partial U_y}{\partial x}$	$\frac{\partial U_y}{\partial z}$
B_1	$-\frac{B_1}{2}$	$\frac{B_2}{2}$	0	0	0	0	0	0	$\frac{B_9}{2}$	$-\frac{B_9}{2}$	0	$\frac{B_9}{2}$	0	
	0	0	0	0	0	0	$\frac{B_7}{2}$	$-\frac{B_8}{2}$	0	0	0	0	0	
	0	0	$\frac{B_3}{2}$	$\frac{B_4}{2}$	0	0	0	0	0	$-\frac{B_9}{2}$	0	0	0	
	0	0	0	0	$-\frac{B_5}{2}$	$-\frac{B_6}{2}$	0	0	$\frac{B_9}{2}$	$-\frac{B_9}{2}$	0	$-\frac{B_9}{2}$	0	
	0	0	0	0	0	0	0	0	0	0	0	0	0	
	0	0	0	0	$-\frac{B_5}{2x}$	$-\frac{B_6}{2x}$	0	0	$-\frac{B_9}{2x}$	$\frac{B_9}{2x}$	0	$-\frac{B_9}{2x}$	0	
	0	0	0	0	0	0	0	0	0	0	0	0	0	
	$1 - \frac{B_1}{2x}$	$\frac{B_2}{2x}$	$1 + \frac{B_3}{2x}$	$1 - \frac{B_4}{2x}$	$\frac{B_5}{2x}$	$-\frac{B_6}{2x}$	0	0	0	0	0	0	0	
	$\frac{B_1}{2x}$	0	0	0	0	0	$\frac{B_7}{2}$	$\frac{B_8}{2}$	0	0	0	0	0	
	0	0	0	0	0	0	0	0	0	0	0	$\frac{B_9}{2}$	0	
	0	0	0	0	0	0	0	0	0	0	0	0	$-\frac{B_9}{2}$	
	0	0	0	0	0	0	0	0	0	0	0	0	0	
	0	0	0	0	0	0	0	0	0	0	0	0	0	
	0	0	0	0	0	0	0	0	0	0	0	0	0	
	0	0	0	0	0	0	0	0	0	0	0	0	0	

Matrix 1

where

$$A_1 = (\sigma_{rr} (1 - \frac{B}{2r}) - U_r (1 - \frac{B}{2r}) + (\alpha \frac{\partial U_z}{\partial z} - \frac{\partial \sigma_{rz}}{\partial z} + \frac{\sigma_{\theta\theta}}{r}) \frac{B}{2})_1$$

$$A_2 = (\sigma_{rr} (1 + \frac{B}{2r}) + U_r (1 + \frac{B}{2r}) + (\alpha \frac{\partial U_z}{\partial z} + \frac{\partial \sigma_{rz}}{\partial z} - \frac{\sigma_{\theta\theta}}{r}) \frac{B}{2})_2$$

$$A_3 = (\sigma_{rz} (1 - \frac{B}{2r}) - \beta U_z + \beta (\beta \frac{\partial U_r}{\partial z} - \frac{\partial \sigma_{zz}}{\partial z}) \frac{B}{2})_3$$

$$A_4 = (\sigma_{rz} (1 + \frac{B}{2r}) + \beta U_z + \beta (\beta \frac{\partial U_r}{\partial z} + \frac{\partial \sigma_{zz}}{\partial z}) \frac{B}{2})_4$$

$$A_5 = (\sigma_{zz} - U_z + (\alpha (\frac{\partial U_r}{\partial z} + \frac{U_r}{r}) - \frac{\partial \sigma_{rz}}{\partial z} - \frac{\sigma_{rz}}{r}) \frac{B}{2})_5$$

$$A_6 = (\sigma_{zz} + U_z + (\alpha (\frac{\partial U_r}{\partial z} + \frac{U_r}{r}) + \frac{\partial \sigma_{rz}}{\partial z} + \frac{\sigma_{rz}}{r}) \frac{B}{2})_6$$

$$A_7 = (\sigma_{rz} - \beta U_r + \beta (\beta \frac{\partial U_z}{\partial r} - \frac{\partial \sigma_{rz}}{\partial r} + \frac{\sigma_{\theta\theta} - \sigma_{rr}}{r}) \frac{B}{2})_7$$

$$A_8 = (\sigma_{rz} + \beta U_r + \beta (\beta \frac{\partial U_z}{\partial r} + \frac{\partial \sigma_{rz}}{\partial r} - \frac{\sigma_{\theta\theta} - \sigma_{rr}}{r}) \frac{B}{2})_8$$

$$A_9 = (\sigma_{rr} + (\frac{\partial U_r}{\partial r} + \alpha (\frac{U_r}{r} + \frac{\partial U_z}{\partial z})) \frac{B}{2})_9$$

$$A_{10} = (\sigma_{zz} + (\frac{\partial U_z}{\partial z} + \alpha (\frac{U_r}{r} + \frac{\partial U_z}{\partial z})) \frac{B}{2})_{10}$$

$$A_{11} = (\sigma_{rz} + \beta^2 (\frac{\partial U_z}{\partial r} + \frac{\partial U_r}{\partial z}) \frac{B}{2})_{11}$$

$$A_{12} = (\sigma_{\theta\theta} + (\frac{U_r}{r} + \alpha (\frac{\partial U_r}{\partial r} + \frac{\partial U_z}{\partial z})) \frac{B}{2})_{12}$$

$$A_{13} = (U_r + (\frac{\partial \sigma_{rr}}{\partial r} + \frac{\partial \sigma_{rz}}{\partial z} - \frac{\sigma_{\theta\theta} - \sigma_{rr}}{r}) \frac{B}{2})_{13}$$

$$A_{14} = (U_z + (\frac{\partial \sigma_{rz}}{\partial r} + \frac{\partial \sigma_{zz}}{\partial z} + \frac{\sigma_{rz}}{r}) \frac{B}{2})_{14}$$

and $B_1 = r - r_1$, $B_2 = r - r_2$, $B_3 = r - r_3$, $B_4 = r - r_4$, $B_5 = r - r_5$, $B_6 = r - r_6$, $B_7 = r - r_7$,
 $B_8 = r - r_8$, $B_9 = r - r_9$. The subscripts 1 through 9 denote the points where
 the quantities are already known.

At this stage the applicability of the above characteristic equations is confined to the case where the dependent variables σ_{rr} , $\sigma_{\theta\theta}$, σ_{zz} , σ_{rz} , U_r , and U_z are continuous while their first partial derivatives may be discontinuous across the wave front. In the present boundary value problems, however, the load is abruptly applied at the boundary. Therefore discontinuities in the dependent variables themselves exist across the bicharacteristic curves. Furthermore the discontinuities may occur not only due to the impulsive load but also due to the reflections from the boundaries. This means that strong discontinuities must be considered for the reflected region even if the load is applied gradually. In the next section the strong discontinuity relations will be applied to the characteristic equations to be compatible with the strong motion of the present boundary value problems.

Strong Discontinuity Relations

If strong discontinuities which mean the discontinuities in the dependent variables themselves exist along the bicharacteristic curves, strong discontinuity relations must be imposed on the characteristic equations along the discontinuities. Observing the characteristic equations (1) through (14) it is seen that every equation apparently involves partial derivatives of the dependent variables which are not differentiable along the bicharacteristic curves carrying discontinuities.

Now we assume that all characteristic equations except the static equations (9) through (14) would be along the bicharacteristic curves which are carrying discontinuities. Accordingly in order to superimpose strong discontinuity relations the characteristic equations (1) (8) should be first rewritten using the well known bracket notation as follows:

$$d[\sigma_{rr}] - d[U_r] = (\alpha([U_{z;z}] + \frac{[U_r]}{r}) - [\sigma_{rz;z}] + \frac{[\sigma_{\theta\theta}] - [\sigma_{rr}]}{r}) d\tau \quad (1a)$$

along $dr = d\tau$,

$$d[\sigma_{rr}] + d[U_r] = (\alpha([U_{z;z}] + \frac{[U_r]}{r}) + [\sigma_{rz;z}] - \frac{[\sigma_{\theta\theta}] - [\sigma_{rr}]}{r}) d\tau \quad (2a)$$

along $dr = -d\tau$,

$$d[\sigma_{rz}] - \beta d[U_z] = \beta(\beta[U_{r;z}] - [\sigma_{zz;z}] - \frac{[\sigma_{rz}]}{r}) d\tau \quad (3a)$$

along $dr = \beta d\tau$,

$$d[\sigma_{rz}] + \beta d[U_z] = \beta(\beta[U_{r;z}] + [\sigma_{zz;z}] + \frac{[\sigma_{rz}]}{r}) d\tau \quad (4a)$$

along $dr = -\beta d\tau$,

$$d[\sigma_{zz}] - d[U_z] = (\alpha([U_{r;r}] + \frac{[U_r]}{r}) - [\sigma_{rz;r}] - \frac{[\sigma_{rz}]}{r}) d\tau \quad (5a)$$

along $dz = d\tau$,

$$d[\sigma_{zz}] + d[U_z] = (\alpha([U_{r;r}] + \frac{[U_r]}{r}) + [\sigma_{rz;r}] + \frac{[\sigma_{rz}]}{r}) d\tau \quad (6a)$$

along $dz = -d\tau$,

$$d[\sigma_{rz}] - \beta d[U_r] = \beta(\beta[U_{z;r}] - [\sigma_{rr;r}] + \frac{[\sigma_{\theta\theta}] - [\sigma_{rr}]}{r}) d\tau \quad (7a)$$

along $dz = \beta d\tau$,

$$d[\sigma_{rz}] + \beta d[U_r] = \beta(\beta[U_{z;r}] + [\sigma_{rr;r}] - \frac{[\sigma_{\theta\theta}] - [\sigma_{rr}]}{r}) d\tau \quad (8a)$$

along $dz = -\beta d\tau$,

where

$$[f] = f \left| \begin{array}{l} \text{The value of } f \\ \text{at the rear of} \\ \text{the wave front} \end{array} \right. - f \left| \begin{array}{l} \text{The value of } f \\ \text{at the front of} \\ \text{the wave front} \end{array} \right.$$

, = Partial differentiation.

The strong discontinuity relations to the relevant case are derived by Ziv by means of the kinematical compatibility relations given in reference[2] by Thomas. Those strong discontinuity relations are presented in reference[6] as follows:

$$[U_{z;z}]_{1a} = [U_{z;z}]_{2a} = \frac{[U_z]}{r} \quad (15)$$

$$[\sigma_{rz;z}]_{1a} = [\sigma_{rz;z}]_{2a} = \frac{[\sigma_{rr}] - [\sigma_{zz}]}{r} \quad (16)$$

$$[U_{r;z}]_{3a} = [U_{r;z}]_{4a} = \frac{[U_z]}{r} \quad (17)$$

$$[\sigma_{zz;z}]_{3a} = [\sigma_{zz;z}]_{4a} = \frac{2[\sigma_{rz}]}{r} \quad (18)$$

$$[U_{r;r}]_{5a} = \frac{[U_z]}{R+z} \quad (19)$$

$$[\sigma_{rz;r}]_{5a} = \frac{[\sigma_{zz}] - [\sigma_{rr}]}{R+z} \quad (20)$$

$$[U_{r;r}]_{6a} = [\sigma_{rz;r}]_{6a} = 0 \quad (21)$$

$$[U_{z;r}]_{7a} = \frac{[U_z]}{R+z} \quad (22)$$

$$[\sigma_{rr;r}]_{7a} = \frac{2[\sigma_{rz}]}{R+z} \quad (23)$$

$$[U_{z;r}]_{8a} = [\sigma_{rr;r}]_{8a} = 0, \quad (24)$$

where the subscripts refer to the corresponding characteristic equations.

Substituting these relations (15) (24) into the corresponding equations (1a) (8a) gives us

$$d[\sigma_{rr}] - d[U_r] = (2\alpha[U_r] - 2[\sigma_{rr}] + [\sigma_{\theta\theta}] + [\sigma_{zz}]) \frac{dr}{r} \quad (1b)$$

$$d[\sigma_{rr}] + d[U_r] = (2\alpha[U_r] + 2[\sigma_{rr}] - [\sigma_{\theta\theta}] - [\sigma_{zz}]) \frac{dr}{r} \quad (2b)$$

$$d[\sigma_{rz}] - \beta d[U_z] = -\beta(\beta[U_z] + 3[\sigma_{rz}]) \frac{dr}{r} \quad (3b)$$

$$d[\sigma_{rz}] + \beta d[U_z] = \beta(-\beta[U_z] + 3[\sigma_{rz}]) \frac{dr}{r} \quad (4b)$$

$$d[\sigma_{zz}] - d[U_z] = \left(\alpha \left(\frac{U_r}{R+z} + \frac{U_r}{r} \right) - \frac{[\sigma_{zz}] - [\sigma_{rr}]}{R+z} - \frac{[\sigma_{rz}]}{r} \right) dr \quad (5b)$$

$$d[\sigma_{zz}] + d[U_z] = (\alpha[U_r] + [\sigma_{rz}]) \frac{dr}{r} \quad (6b)$$

$$d[\sigma_{rz}] - \beta d[U_r] = \beta \left(-\beta \frac{U_r}{R+z} - \frac{2[\sigma_{rz}]}{R+z} + \frac{[\sigma_{\theta\theta}] - [\sigma_{rr}]}{r} \right) dr \quad (7b)$$

$$d[\sigma_{rz}] + \beta d[U_r] = \beta([\sigma_{rr}] - [\sigma_{\theta\theta}]) \frac{dr}{r} \quad (8b)$$

or in the finite difference form

$$\left(1 + \frac{B_1}{r}\right) [\sigma_{rr}] - \frac{B_1}{2r} [\sigma_{\theta\theta}] - \frac{B_1}{2r} [\sigma_{zz}] - \left(1 + \alpha \frac{B_1}{r}\right) [U_r] = [A_1] \quad (1c)$$

$$\left(1 - \frac{B_2}{r}\right) [\sigma_{rr}] + \frac{B_2}{2r} [\sigma_{\theta\theta}] + \frac{B_2}{2r} [\sigma_{zz}] + \left(1 - \alpha \frac{B_2}{r}\right) [U_r] = [A_2] \quad (2c)$$

$$\left(1 + 3\beta \frac{B_3}{2r}\right) [\sigma_{rz}] - \beta \left(1 - \beta \frac{B_3}{2r}\right) [U_z] = [A_3] \quad (3c)$$

$$\left(1 - 3\beta \frac{B_4}{2r}\right) [\sigma_{rz}] + \beta \left(1 + \beta \frac{B_4}{2r}\right) [U_z] = [A_4] \quad (4c)$$

$$-\frac{B_5}{2(R+z)} [\sigma_{rr}] + \left(1 + \frac{B_5}{2(R+z)}\right) [\sigma_{zz}] - \alpha \frac{B_5}{2r} [U_r] - \left(1 + \frac{\alpha B_5}{2(R+z)}\right) [U_z] + \frac{B_5}{2r} [\sigma_{rz}] = [A_5] \quad (5c)$$

$$[\sigma_{zz}] - \alpha \frac{B_6}{2r} [U_r] + [U_z] - \frac{B_6}{2r} [\sigma_{rz}] = [A_6] \quad (6c)$$

$$\left(1 + \frac{\beta B_7}{R+z}\right) [\sigma_{rz}] - \beta \left(1 - \frac{\beta B_7}{2(R+z)}\right) [U_r] - \beta \frac{B_7}{2r} [\sigma_{\theta\theta}] + \beta \frac{B_7}{2r} [\sigma_{rr}] = [A_7] \quad (7c)$$

$$[\sigma_{rz}] + \beta [U_r] - \beta \frac{B_8}{2r} [\sigma_{rr}] + \beta \frac{B_8}{2r} [\sigma_{\theta\theta}] = [A_8] \quad (8c)$$

where

$$[A_1] = \left(\left(1 - \frac{B}{r}\right) [\sigma_{rr}] - \left(1 - \alpha \frac{B}{r}\right) [U_r] + \left(\frac{[\sigma_{zz}] + [\sigma_{\theta\theta}]}{r} \right) \frac{B}{2} \right)_1$$

$$[A_2] = \left(\left(1 + \frac{B}{r}\right) [\sigma_{rr}] + \left(1 + \alpha \frac{B}{r}\right) [U_r] - \left(\frac{[\sigma_{zz}] + [\sigma_{\theta\theta}]}{r} \right) \frac{B}{2} \right)_2$$

$$[A_3] = \left(\left(1 - \frac{3\beta B}{2r}\right) [\sigma_{rz}] - \beta \left(1 + \frac{\beta B}{2r}\right) [U_z] \right)_3$$

$$[A_4] = \left(\left(1 + \frac{3\beta B}{2r}\right) [\sigma_{rz}] + \beta \left(1 - \frac{\beta B}{2r}\right) [U_z] \right)_4$$

$$[A_5] = \left(\left(1 - \frac{B}{2(R+z)}\right) [\sigma_{zz}] - \left(1 - \frac{\alpha B}{2(R+z)}\right) [U_z] + \frac{\alpha B}{2r} [U_r] + \frac{B}{2(R+z)} [\sigma_{rr}] - \frac{B}{2r} [\sigma_{rz}] \right)_5$$

$$[A_6] = \left([\sigma_{zz}] + [U_z] + \frac{\alpha B}{2r} [U_r] + \frac{B}{2r} [\sigma_{rz}] \right)_6$$

$$[A_7] = \left(\left(1 - \frac{\beta B}{R+z}\right) [\sigma_{rz}] - \beta \left(1 + \frac{\beta B}{2(R+z)}\right) [U_r] + \frac{\beta B}{2r} [\sigma_{\theta\theta}] - \frac{\beta B}{2r} [\sigma_{rr}] \right)_7$$

$$[A_8] = \left([\sigma_{rz}] + \beta [U_r] + \frac{\beta B}{2r} [\sigma_{rr}] - \frac{\beta B}{2r} [\sigma_{\theta\theta}] \right)_8$$

CHAPTER III

COMPUTATIONAL METHOD FOR CASE 1

Characteristic Equations along Strong Discontinuity

The discussion in reference[6] on the behavior of waves and their discontinuities must hold for CASE 1 as long as the incident wave front does not reach the outer free surface of the shell, since the initial and boundary conditions in the present case are exactly same as those in [6]. Therefore the characteristic equations (1b),(2b),(3b),(5b), and (6b) should be employed for the two dimensional region in which every grid point of integration mesh has strong nature of discontinuity in dependent variables due to the impulsive load and the reflected waves from the top free surface. In other words those five bicharacteristic curves which construct the orthogonal integration mesh in the two dimensional region carry strong discontinuities so that we should employ the characteristic equations (1b),(2b),(3b),(5b), and (6b) instead of (1),(2),(3),(5), and (6). The characteristic equations (1c) (6c) and static equations (9),(10), and (12) are rewritten in matrix form as follows after the bracket notation has been removed:

$1 + \frac{B_1}{r}$	$-\frac{B_1}{2r}$	$-\frac{B_1}{2r}$	0	$-1 - \frac{B_1}{r}$	0	0	0	σ_{rr}	$[A_1]$
$1 - \frac{B_2}{r}$	$\frac{B_2}{2r}$	$\frac{B_2}{2r}$	0	$1 - \frac{B_2}{r}$	0	0	0	$\sigma_{\theta\theta}$	$[A_2]$
0	0	0	$1 + \frac{3B_3}{2r}$	0	$-B + B^2 \frac{B_3}{2r}$	0	0	σ_{zz}	$[A_3]$
$-\frac{B_5}{2(R+z)}$	0	$1 + \frac{B_5}{2(R+z)}$	$\frac{B_5}{2r}$	$-\frac{B_5}{2r}$	$-\frac{B_5}{2(R+z)}$	0	0	σ_{rz}	$[A_5]$
0	0	1	$-\frac{B_6}{2r}$	$-\frac{B_6}{2r}$	1	0	0	U_r	$[A_6]$
1	0	0	0	$-\frac{B_9}{2r}$	0	$-\frac{B_9}{2}$	$-\frac{B_9}{2}$	U_z	A_9
0	0	1	0	$-\frac{B_9}{2r}$	0	$-\frac{B_9}{2}$	$-\frac{B_9}{2}$	$\frac{\partial U_r}{\partial z}$	A_{10}
0	1	0	0	$-\frac{B_9}{2r}$	0	$-\frac{B_9}{2}$	$-\frac{B_9}{2}$	$\frac{\partial U_z}{\partial r}$	A_{12}

Matrix 2

where

$$[A_1] = \left(\left(1 - \frac{B}{r}\right) \sigma_{rr} - \left(1 - \frac{B}{r}\right) U_r + \left(\frac{\sigma_{zz} + \sigma_{\theta\theta}}{r} \right) \frac{B}{2} \right)_1$$

$$[A_2] = \left(\left(1 + \frac{B}{r}\right) \sigma_{rr} + \left(1 + \frac{B}{r}\right) U_r - \left(\frac{\sigma_{zz} + \sigma_{\theta\theta}}{r} \right) \frac{B}{2} \right)_2$$

$$[A_3] = \left(\left(1 - \frac{3B}{2r}\right) \sigma_{rz} - B \left(1 + \frac{B}{2r}\right) U_z \right)_3$$

$$[A_4] = \left(\left(1 - \frac{B}{2(R+z)}\right) \sigma_{zz} - \left(1 - \frac{B}{2(R+z)}\right) U_z + \left(\frac{U_r}{r} + \frac{\sigma_{rr}}{R+z} - \frac{\sigma_{rz}}{r} \right) \frac{B}{2} \right)_4$$

$$\begin{aligned}
 [A_6] &= (\sigma_{zz} + U_z + (aU_r + \sigma_{rz}) \frac{B}{2r})_6 \\
 A_9 &= (\sigma_{rr} + (\frac{\partial U_r}{\partial r} + a(\frac{U_r}{r} + \frac{\partial U_z}{\partial z})) \frac{B}{2})_9 \\
 A_{10} &= (\sigma_{zz} + (\frac{\partial U_z}{\partial z} + a(\frac{U_r}{r} + \frac{\partial U_r}{\partial r})) \frac{B}{2})_{10} \\
 A_{12} &= (\sigma_{\theta\theta} + (\frac{U_r}{r} + a(\frac{\partial U_r}{\partial r} + \frac{\partial U_z}{\partial z})) \frac{B}{2})_{12}
 \end{aligned}$$

It is worthwhile to note here that the bracketed value of the dependent variables in (1b), (2b), (3b), (5b), and (6b) represent "jumps" across the corresponding bicharacteristic curves carrying strong discontinuities so that the jumps are equivalent to the values of the dependent variables which are discontinuous across the bicharacteristics. However there are no jumps of the dependent variables in the static equation (9), (10), and (12) since the dependent variables are continuous across the bicharacteristics $dr=dz=0$. It can be noted that the static equations (9) through (12) are merely a restatement of the constitutive equations in the basic field equations.

It is known that in reflections of spherical longitudinal waves at the boundary both longitudinal and shear waves are generated as reflected waves [6] (P158). Therefore in CASE 1 after the reflection at the outer free surface we must consider not only reflected longitudinal waves but also reflected shear waves which emanate from the outer surface into the two-dimensional region. These reflected shear waves are characterized by the bicharacteristic curve $dr=-\beta dt$. Accordingly we should employ the characteristic equation (4c) and

superimpose it on Matrix 2. The characteristic equation (4c) is superimposed on (3c) to obtain

$$(2 + \frac{3B}{2r}(B_3 - B_4))[\sigma_{rz}] + \frac{B^2}{2r}(B_3 + B_4)[U_z] = [A_{34}] \quad (3c')$$

where

$$[A_{34}] = ((1 - \frac{3B}{2r})[\sigma_{rz}] - B(1 + \frac{B}{2r})[U_z])_3 + ((1 + \frac{3B}{2r})[\sigma_{rz}] + B(1 - \frac{B}{2r})[U_z])_4.$$

Regions of Influence and Typical Recurring Points

A region in the solution domain where all points are under the same type of motion is defined here as a region of influence. In other words all points of each region of influence are affected by the same boundary and initial conditions which induce the same type of particle motion in the region. All points in each region of influence are categorized into three groups which are defined as follows:

1. Leading Points

Points that are located on a leading wave front and do not comply with the orthogonal scheme.

2. Intermediary Points

Points that have an immediate contact with one or more leading points via their bicharacteristic curves and require a special orthogonal mesh configuration.

3. Regular Points

Points that comply with the perfect symmetric mesh configuration.

Four regions of influence and twenty four recurring points in the entire solution domain are shown in Figure 5. Before the leading wave reflects at the outer surface the solution domain is divided into two regions of influence in accordance with the case of reference[6]. One is a one-dimensional region r, τ , influenced by the loaded boundary and the other is a two-dimensional region r, z, τ , influenced by the free top surface as well as by the loaded boundary. After the reflection of the leading wave from the outer surface, however, the entire domain is divided into four regions of influence. In addition to the two previous regions of influence we must add two reflected regions which are influenced by the outer boundary. One is a reflected one-dimensional region r, τ , influenced not only by the loaded boundary but also by the outer boundary. The other is a reflected two-dimensional region r, z, τ , influenced by the outer boundary as well as the loaded boundary and the top surface.

Accordingly all points in each region of influence are categorized into three groups previously defined with respect to their computational schemes. Therefore twenty four recurring points a through p in the solution domain can be defined as shown in Figure 5. All recurring points are categorized into the three groups in the following way: Points a, a', b, c, c', d, e, e', f, n and p belong to

regular points, points g, g', i, i', j, k, k' , and m belong to the leading points and points h, h', l, l' , and o belong to the intermediary points.

In the next section the boundary and initial conditions will be prescribed and the computational scheme of integration for each recurring point will be discussed in addition to the derivation of the decays of the leading waves in terms of the closed-form solutions.

Computational Scheme for Each Recurring Point

The recurring points were defined with respect to the region of influence, the group of points, and the boundary conditions. Subsequently we must detect twenty four recurring points to determine the dependent variables in the entire solution domain. Once the neighborhood of the leading wave which implies the intermediary points as well as the leading points is determined, we can apply the perfect symmetric scheme to the rest of the solution domain, since all points remaining are regular points which comply with the perfect symmetric mesh.

The following dimensionless values of the boundary conditions are prescribed when both the intensity of σ_{rr} -input and the inner radius of the shell are considered to be unity.

The boundary conditions for the loaded boundary ($r, s > 0, \tau$) are

given as $\sigma_{rr} = -1$ and $\sigma_{rz} = 0$. The initial conditions for the loaded boundary are given as $U_r = -1$ due to $[U_r] = -[\sigma_{rr}]$ the dynamical conditions of discontinuities across a wave front (8) (P140), $U_z = \partial U_z / \partial z = 0$, $\sigma_{\theta\theta} = \sigma_{zz} = -\sigma_{rr}$ due to static equations (9), (10), (12), and $\partial U_r / \partial r = 0.5$ due to the known decay expression for a one-dimensional cylindrical leading wave.

The boundary conditions for point $(R, 0, \tau)$ are given as $\sigma_{rr} = -1$ and $\sigma_{zz} = \sigma_{rz} = 0$. The initial conditions for the corner point are given as $U_r = -1$, $\sigma_{\theta\theta} = -\sigma_{rr}$, $\partial U_z / \partial z = 0$ and $\partial U_r / \partial r = 0.5$.

The boundary conditions for points $(R < r < R_0, 0, \tau)$ on the free top surface are given as $\sigma_{zz} = \sigma_{rz} = 0$.

The boundary conditions for points $(R_0, z > 0, \tau)$ on the outer free surface are given as $\sigma_{rr} = \sigma_{rz} = 0$.

All particles in the shell except for those on the loaded boundary are considered to be at rest at $\tau = 0$.

Stresses and particle velocities at the typical recurring points a through p are evaluated in the following way:

a. Regular Inner Two-Dimensional Points a(r, z, \tau)

These points comply with the perfect symmetric mesh configuration referring to Matrix 2.

a'. Reflected Regular Inner Two-Dimensional Points $a'(x, y, z)$

After replacing (3c) in Matrix 2 with (3c') due to reflected shear waves from the outer surface, Matrix 2 becomes

$1 + \frac{B_1}{2r}$	$-\frac{B_1}{2r}$	$-\frac{B_1}{2r}$	0	$-1 - \frac{aB_1}{r}$	0	0	0	σ_{rr}	$[A_1]$
$1 - \frac{B_2}{r}$	$\frac{B_2}{2r}$	$\frac{B_2}{2r}$	0	$1 - \frac{aB_2}{r}$	0	0	0	$\sigma_{\theta\theta}$	$[A_2]$
0	0	0	$2 + \frac{3B_3}{2r}(B_3 - B_4)$		$\frac{B_3^2}{2r}(B_3 + B_4)$	0	0	σ_{zz}	$[A_{34}]$
$-\frac{B_5}{2R+2}$	0	$1 + \frac{B_5}{2R+2}$	$\frac{B_5}{2r}$	$-\frac{aB_5}{2r}$	$-1 - \frac{aB_5}{2R+2}$	0	0	σ_{rz}	$[A_5]$
0	0	1	$-\frac{B_6}{2r}$	$-\frac{aB_6}{2r}$	1	0	0	u_r	$[A_6]$
1	0	0	0	$-\frac{aB_9}{2r}$	0	$-\frac{B_9}{2}$	$-\frac{B_9}{2}$	u_z	A_9
0	0	1	0	$-\frac{aB_{10}}{2r}$	0	$-\frac{B_{10}}{2}$	$-\frac{B_{10}}{2}$	$\frac{\partial u_r}{\partial r}$	A_{10}
0	1	0	0	$-\frac{B_{10}}{2r}$	0	$-\frac{B_{10}}{2}$	$-\frac{B_{10}}{2}$	$\frac{\partial u_z}{\partial r}$	A_{12}

Matrix 3

b. Regular Loaded Boundary Two-Dimensional Points $b(R, z > 0, \tau)$

In order to meet the boundary conditions $\sigma_{rr} = 1$ and $\sigma_{rz} = 0$
Matrix 2 becomes

$\frac{B_2}{2r}$	$\frac{B_2}{2r}$	$1 - \frac{a}{r}$	$\frac{B_2}{r}$	0	0	0	$\sigma_{\theta\theta}$	$[A_2] - (1 - \frac{a}{r})$
0	$1 + \frac{B_5}{2(R+z)}$	$-\frac{a}{2r}$	$\frac{B_5}{r}$	$-1 - \frac{a}{2(R+z)}$	0	0	σ_{zz}	$[A_5] + \frac{B_5}{2(R+z)}$
0	1	$-\frac{a}{2r}$	$\frac{B_6}{r}$	1	0	0	U_r	$[A_6]$
0	0	$-\frac{a}{2r}$	$\frac{B_9}{r}$	0	$-\frac{B_9}{2}$	$-\frac{a}{2} \frac{B_9}{r}$	U_z	$A_9 + 1$
0	1	$-\frac{a}{2r}$	$\frac{B_9}{r}$	0	$-\frac{B_9}{2}$	$-\frac{B_9}{2}$	$\frac{\partial U_r}{\partial z}$	A_{10}
1	0	$-\frac{a}{2r}$	$\frac{B_9}{r}$	0	$-\frac{a}{2} \frac{B_9}{r}$	$-\frac{a}{2} \frac{B_9}{r}$	$\frac{\partial U_z}{\partial r}$	A_{12}

Matrix 4

c. Regular Free Top Boundary Points $C(R < R_0, 0, r)$

After applying the boundary conditions $\sigma_{zz} = \sigma_{rz} = 0$ Matrix 2 becomes

$1 + \frac{B_1}{r}$	$-\frac{B_1}{2r}$	$-1 - \frac{a}{r} \frac{B_1}{r}$	0	0	0	σ_{rr}	$[A_1]$
$1 - \frac{B_2}{r}$	$\frac{B_2}{2r}$	$1 - \frac{a}{r} \frac{B_2}{r}$	0	0	0	$\sigma_{\theta\theta}$	$[A_2]$
0	0	$-\frac{a}{r} \frac{B_0}{r}$	1	0	0	U_r	$[A_6]$
1	0	$-\frac{a}{r} \frac{B_0}{r}$	0	$-\frac{B_0}{2r}$	$-\frac{a}{r} \frac{B_0}{r}$	U_z	A_9
0	0	$-\frac{a}{r} \frac{B_0}{r}$	0	$-\frac{a}{r} \frac{B_0}{r}$	$-\frac{B_0}{2r}$	$\frac{\partial U_r}{\partial r}$	A_{10}
0	1	$-\frac{a}{r} \frac{B_0}{r}$	0	$-\frac{a}{r} \frac{B_0}{r}$	$-\frac{a}{r} \frac{B_0}{r}$	$\frac{\partial U_z}{\partial r}$	A_{12}

Matrix 5

c'. Reflected Regular Free Top Boundary Points $c'(R < r < R_0, 0, \tau)$

We apply the same boundary conditions as the points c to

Matrix 3 to obtain

$1 + \frac{B_1}{r}$	$-\frac{B_1}{2r}$	$-1 - \frac{aB_1}{r}$	0	0	0	σ_{rr}	$[A_1]$
$1 - \frac{B_2}{r}$	$\frac{B_2}{2r}$	$1 - \frac{aB_2}{r}$	0	0	0	$\sigma_{\theta\theta}$	$[A_2]$
0	0	$-\frac{aB_6}{2r}$	1	0	0	U_r	$[A_6]$
1	0	$-\frac{aB_9}{2r}$	0	$-\frac{B_9}{2}$	$-\frac{aB_9}{2}$	U_z	A_9
0	0	$-\frac{aB_9}{2r}$	0	$-\frac{aB_9}{2}$	$-\frac{B_9}{2}$	$\frac{\partial U_r}{\partial r}$	A_{10}
0	1	$-\frac{B_9}{2r}$	0	$-\frac{aB_9}{2}$	$-\frac{aB_9}{2}$	$\frac{\partial U_z}{\partial z}$	A_{12}

Matrix 6

d. Regular Inner Edge Point d(R.O.P)

When $\sigma_{rr} = 1$, $\sigma_{\theta\theta} = \sigma_{rz} = 0$, Matrix 2 becomes

$\frac{B}{2r}$	$1 - \frac{aB}{r}$	0	0	0	$\sigma_{\theta\theta}$	$(A_2) - (1 - \frac{B}{r})$
0	$-\frac{aB}{2r}$	1	0	0	U_r	(A_6)
0	$-\frac{aB}{2r}$	0	$-\frac{B}{2}$	$-\frac{aB}{2}$	U_z	$A_9 - 1$
0	$-\frac{aB}{2r}$	0	$-\frac{aB}{2}$	$-\frac{B}{2}$	$\frac{\partial U_r}{\partial r}$	A_{10}
1	$-\frac{aB}{2r}$	0	$-\frac{aB}{2}$	$-\frac{aB}{2}$	$\frac{\partial U_z}{\partial z}$	A_{12}

Matrix 7

e. Regular Inner One-Dimensional Points $\theta(r, z)$

Since these points are located in the region of weak one-dimensional motion, the matrix for these points is obtained from Matrix 1 as follows:

$1 + \frac{B_1}{2r}$	$-\frac{B_1}{2r}$	0	$-1 - \frac{B_1}{2r}$	0	•	σ_{rr}	A_1
$1 - \frac{B_2}{2r}$	$\frac{B_2}{2r}$	0	$1 - \frac{B_2}{2r}$	0		$\sigma_{\theta\theta}$	A_2
1	0	0	$-\frac{B_3}{2r}$	$-\frac{B_3}{2r}$		σ_{zz}	A_9
0	0	1	$-\frac{B_4}{2r}$	$-\frac{B_4}{2r}$		U_r	A_{10}
0	0	0	$-\frac{B_5}{2r}$	$-\frac{B_5}{2r}$		$\frac{\partial U_r}{\partial r}$	A_{12}

Matrix 8

f. Regular Loaded Boundary One-Dimensional Points f(R,r)

To be compatible with the boundary conditions $\sigma_{rr}=1$, $\sigma_{rz}=0$

Matrix 8 becomes

$$\begin{array}{|c|c|c|c|} \hline \frac{B}{2r^2} & 0 & 1 - \frac{a}{2r} \frac{B}{2r^2} & 0 \\ \hline 0 & 0 & -\frac{a}{2r} \frac{B}{2r^2} & -\frac{B}{2r} \\ \hline 0 & 1 & -\frac{a}{2r} \frac{B}{2r^2} & -\frac{B}{2r} \\ \hline 1 & 0 & -\frac{B}{2r} & -\frac{a}{2r} \frac{B}{2r^2} \\ \hline \end{array} \cdot \begin{array}{|c|} \hline \sigma_{\theta\theta} \\ \hline \sigma_{zz} \\ \hline U_r \\ \hline \frac{\partial U_r}{\partial r} \\ \hline \end{array} = \begin{array}{|c|} \hline A_2 - (1 - \frac{B}{2r}) \\ \hline A_9 - 1 \\ \hline A_{10} \\ \hline A_{11} \\ \hline \end{array}$$

Matrix 9

g. Leading Inner One-Dimensional Points g(R+r,r)

The geometric decay of the incident leading wave in the medium can be obtained in a closed-form solution. It is a cylindrical longitudinal wave front carrying strong discontinuities. The wave front appears as a line in the r-z plane, this implies that the decay is a function of the independent variable r only. Therefore the characteristic equation(1a) along $dr=dz$ is reduced to

$$d[\sigma_{rr}] - d[U_r] = (a[U_r] + [\sigma_{\theta\theta}] - [\sigma_{rr}]) \frac{dr}{r}.$$

From the equations (2a) as $dz=0$ or from the dynamical conditions for discontinuities across a wave surface[8](p141) we obtain

$$[U_r] = -[\sigma_{rr}]$$

and also the static equations (9), (10), and (12) give us the

following expression:

$$[\sigma_{\theta\theta}] = [\sigma_{zz}] = a[\sigma_{rr}].$$

Substituting these relations into the former equation gives us (with $dt = dr$) the well-known expression

$$\frac{d[\sigma_{rr}]}{[\sigma_{rr}]} = -\frac{1}{2} \frac{dr}{r} \quad \text{or} \quad [\sigma_{rr}] = Kr^{-\frac{1}{2}}.$$

When the initial condition $\sigma_{rr} = 1$ is applied K becomes unity. Finally we can summarize the quantities of leading one-dimensional points $g(R+r, \tau)$ as follows:

$$\begin{aligned} \sigma_{rr} &= r^{-\frac{1}{2}}, \quad \sigma_{\theta\theta} = \sigma_{rr}, \quad \sigma_{zz} = a\sigma_{rr}, \quad \sigma_{rz} = 0, \\ u_r &= -\sigma_{rr}, \quad u_z = 0, \quad \frac{\partial u_r}{\partial r} = 0.5r^{-\frac{3}{2}} \quad \text{and} \quad \frac{\partial u_z}{\partial z} = 0. \end{aligned}$$

g'. Reflected Leading Inner One-Dimensional Points g'(R_0; R+r, \tau)

This is a cylindrical longitudinal tensile wave which propagates via the bicharacteristic curve $dr = -dt$. Therefore we employ the characteristic equation (2a). Since the stresses are irrespective of the z -coordinate the equation (2a) becomes

$$d[\sigma_{rr}] + d[u_r] = (a[u_r] - [\sigma_{\theta\theta}] + [\sigma_{rr}]) \frac{dr}{r},$$

as $dt = 0$ in (1a) we obtain

$$[\sigma_{rr}] = [u_r]$$

and from the static equations (9), (10), and (12)

$$[\sigma_{\theta\theta}] = [\sigma_{zz}] = a[\sigma_{rr}].$$

Using these relations (with $dt = -dr$) we finally come up with the reverse decay expression as follow:

$$\frac{d[\sigma_{rr}]}{[\sigma_{rr}]} = -\frac{1}{2} \frac{dr}{r} \quad \text{or} \quad [\sigma_{rr}] = Kr^{-\frac{1}{2}}.$$

This expression is same as that of the incident wave, but we should notice that the value of r decreases as the reflected wave propagates towards the inner surface. This implies that after the reflection at the outer surface the incident divergent wave turns to the reflected convergent wave.

h. and h' Intermediary One-Dimensional Points
 $h.(R_0 < R+r-At, r)$ and $h'(R_0 < R+r-At, r)$

Although Matrix B is valid for points h and h' , B_2 should be replaced by $At/2$ for h and B should be replaced by $At/2$ for h' . B_1 for both h and h' is replaced by At .

i. Leading Free Top Surface Points $i(R+r, 0, r)$

These leading points are sheared by the incident wave and all longitudinal spherical waves reflected from the free top surface. Those waves on the points i are characterized by the bicharacteristic curve $dr=dt$. Therefore we use the characteristic equation (1b) and by equations (2b), (9), and (12) as $dt=0$, we have $[U_r] = -[\sigma_{rr}]$ and $[\sigma_{\theta\theta}] = a[\sigma_{rr}]$, and (1b) becomes $2d[\sigma_{rr}] = -((2+a)[\sigma_{rr}] - [\sigma_{zz}])\frac{dr}{r}$. But the boundary condition is $[\sigma_{zz}] = 0$, thus the decay is given by

$$\frac{d[\sigma_{rr}]}{[\sigma_{rr}]} = -(1 + \frac{a}{2}) \quad \text{or} \quad [\sigma_{rr}] = r^{-(1 + \frac{a}{2})}$$

and in summary

$$\frac{\partial U_r}{\partial r} = (1 + \frac{a}{2})r^{-(2 + \frac{a}{2})}, \quad \text{and} \quad \frac{\partial U_z}{\partial z} = 0, \quad \sigma_{rr} = r^{-(1 + \frac{a}{2})}, \quad \sigma_{\theta\theta} = a\sigma_{rr}, \quad \sigma_{rz} = 0, \quad U_r = -\sigma_{rr}, \quad U_z = ar^{-(\frac{1}{2})}$$

The particles at points i are subject to the compressive stress

component σ_{zz} which refers to the inner leading one-dimensional points g. Since $[\sigma_{zz}] = -[U_z]$ (from (6b) as $dr=0$), U_z at points i is equal to $\sigma_{zsg} = \sigma_{rrg} = cr^{-\frac{1}{2}}$.

1'. Reflected Leading Free Top Surface Points i'(R_0 < R + r, r)

The reflected leading wave front is propagating towards the inner surface via the bicharacteristic curve $dr = -dr$. We substitute $[U_r] = [\sigma_{rr}]$ and $[\sigma_{\theta\theta}] = a[\sigma_{rr}]$ that come from (1b), (9), and (12) as $dr=0$ into the characteristic equation (2b) to obtain $2d[\sigma_{rr}] = ((2+a)[\sigma_{rr}] - [\sigma_{zz}]) \frac{dr}{r}$. But the free surface condition leads us to (with $dr = -dr$)

$$\frac{d[\sigma_{rr}]}{[\sigma_{rr}]} = -(1 + \frac{a}{2}) \frac{dr}{r} \quad \text{or} \quad [\sigma_{rr}] = r^{-(1 + \frac{a}{2})}$$

and in summary

$$\sigma_{rr} = r^{-(1 + \frac{a}{2})}, \quad \sigma_{\theta\theta} = a\sigma_{rr}, \quad \sigma_{zz} = 0, \quad U_r = \sigma_{rr}, \quad U_z = cr^{-\frac{1}{2}},$$

$$\frac{\partial U_r}{\partial r} = -(1 + \frac{a}{2})r^{-(2 + \frac{a}{2})}, \quad \text{and} \quad \frac{\partial U_z}{\partial z} = 0.5cr^{-\frac{3}{2}}.$$

The particles at points i' are subjected to the tensile stress component σ_{zz} which refers to the reflected inner leading one-dimensional points g'. Since $[\sigma_{zz}] = -[U_z]$ (from (6b) as $dr=0$), U at points i' becomes $\sigma_{zsg'} = \sigma_{rrg'} = cr^{-\frac{1}{2}}$.

1. Leading Two-Dimensional Loaded Boundary Points j(R, r, r)

We can refer to Reference[6] for the calculation of these points. The dependent variables are integrated by the following matrix:

0	$1 + \frac{B_5}{2(R+z)}$	$-1 - \frac{aB_5}{2(R+z)}$	0	0		$\sigma_{\theta\theta}$	$[A_5] + \frac{B_5}{2} \left(\frac{\sigma_{rr}}{R+z} + \frac{U_{rj}}{r} \right)_j$
0	1	1	0	0		σ_{zz}	$[A_6] + \frac{aB_6}{2r} U_{rj}$
0	0	0	$-\frac{B_9}{2}$	$-\frac{aB_9}{2}$	•	u_z	$A_9 - \sigma_{rr} + \frac{aB_9}{2r} U_{rj}$
0	1	0	$-\frac{B_9}{2}$	$-\frac{B_9}{2}$		$\frac{\partial U_r}{\partial r}$	$A_{10} + \frac{aB_9}{2r} U_{rj}$
1	0	0	$-\frac{aB_9}{2}$	$-\frac{B_9}{2}$		$\frac{\partial U_z}{\partial z}$	$A_{12} + \frac{B_9}{2r} U_{rj}$

Matrix 10

k. Leading Inner Two-Dimensional Points $k(r, z) \rightarrow r+R-z, r$

These points are not compatible with the orthogonal scheme for the characteristic equations. Points k, however, lie on the reflected spherical wave which passes through two known points so that we can apply a linear interpolation between these two points which are known from previous calculations. One of the known points is a point i and the other is possibly a point l. If the latter point does not coincide with a point l and appears between two points, another interpolation between the two l points is necessary. All calculations of these interpolations are carried out in the same manner as in reference[6].

k'. Reflected Leading Inner Two-Dimensional Points $k'(r, z) \rightarrow R+r-2R+z, r$

According to Figure 6 we can apply a similar interpolation scheme

to evaluate stresses and particle velocities at these points k' . Two known points from which a point k' is calculated by interpolation exist in the r -constant plane. One is a point i' and the other is possibly a point l' . If the latter point does not coincide with a point l' which is a known grid point of integration, it is calculated by another interpolation that can refer to [6].

l. Intermediary Two-Dimensional Points $l(x, R+r-x, r)$

These points are located on the reflected longitudinal cones and directly on the integration grid points. But these points do not comply with the orthogonal mesh configuration, since the bicharacteristic curves drawn from one of these points must terminate on the preceding reflected longitudinal cone $(r-\Delta t, 0, r-R+\Delta t)$. It should be noted that every reflected longitudinal wave generated by the incident wave on its free top surface carries new strong discontinuities. Accordingly the integration paths must be drawn from the preceding reflected cone other than ordinary grid points. The quantities at points are evaluated by Matrix 2. The details of the calculations for B_1 and $[A_1]$ are presented in reference[6].

l'. Reflected Intermediary Two-Dimensional Points
 $l'(x, R+r-2R+x, r)$

Although the similar calculation scheme for B_1 and $[A_1]$ can be applied to these reflected points, the governing matrix is Matrix 3 due to the reflected shear wave.

m. and o. Outer boundary Two-Dimensional Points
 $m(R, z, R+r-R_0, r)$, $o(R, R+r-R_0, r)$

After applying the boundary conditions $\sigma_{rr} = \sigma_{rz} = 0$ Matrix 2 becomes

$-\frac{B}{2r_1}$	0	$-1 - \frac{aB}{r_1}$	0	0	0	$\sigma_{\theta\theta}$	$[A_1]$
0	$\frac{B}{2r_2}$	$-\frac{aB}{2r_2}$	$-1 - \frac{aB}{2(R+z)}$	0	0	σ_{zz}	$[A_5]$
0	$-\frac{B}{2r_1}$	$-\frac{aB}{2r_1}$	1	0	0	u_r	$[A_6]$
0	0	$-\frac{aB}{2r_1}$	0	$-\frac{B}{2}$	$-\frac{aB}{2}$	u_z	A_9
0	0	$-\frac{aB}{2r_1}$	0	$-\frac{aB}{2}$	$-\frac{B}{2}$	$\frac{\partial u_r}{\partial z}$	A_{10}
1	0	$-\frac{B}{2r_1}$	0	$-\frac{aB}{2}$	$-\frac{aB}{2}$	$\frac{\partial u_z}{\partial r}$	A_{12}

Matrix 11

The evaluations of B_1 and $[A_1]$ at these points m and o can be done by the same methods as those for the points k and l respectively.

n. Regular Outer Corner Point $n(R_0, 0, r)$

When $\sigma_{rr} = \sigma_{rz} = 0$ Matrix 2 becomes

$-\frac{B}{R_1^2}$	$-1 - \frac{a^2 B}{R_1^2}$	0	0	0
0	$-\frac{a^2 B}{R_1^2}$	1	0	0
0	$-\frac{a^2 B}{R_1^2}$	0	$-\frac{B}{2l_0}$	$-\frac{a^2 B}{2l_0}$
0	$-\frac{a^2 B}{R_1^2}$	0	$-\frac{a^2 B}{2l_0}$	$-\frac{B}{2l_0}$
1	$-\frac{a^2 B}{R_1^2}$	0	$-\frac{a^2 B}{2l_0}$	$-\frac{a^2 B}{2l_0}$

$\sigma_{\theta\theta}$
u_r
u_z
$\frac{\partial u_r}{\partial z}$
$\frac{\partial u_z}{\partial z}$

 \bullet

$[A_1]$
$[A_6]$
A_9
A_{10}
A_{12}

Matrix 12

p. Regular One-Dimensional outer Surface Points p(Re.1)

When $\sigma_{rr} = \sigma_{rz} = 0$ Matrix 8 becomes

$-\frac{B}{R_1^2}$	0	$-1 - \frac{a^2 B}{R_1^2}$	0
0	0	$-\frac{a^2 B}{R_1^2}$	$-\frac{B}{2l_0}$
0	1	$-\frac{a^2 B}{R_1^2}$	$-\frac{a^2 B}{2l_0}$
1	0	$-\frac{a^2 B}{R_1^2}$	$-\frac{a^2 B}{2l_0}$

$\sigma_{\theta\theta}$
σ_{zz}
u_r
$\frac{\partial u_r}{\partial z}$

 \bullet

$[A_1]$
A_9
A_{10}
A_{12}

Matrix 13

CHAPTER IV

COMPUTATIONAL METHODS FOR CASE 2, CASE 3 AND CASE 4

Boundary and Initial Conditions in case 2

The geometry and the loading for CASE 2 are shown in Figure 1 and discussed in detail in Chapter I. The shell considered here is infinitely long and then it is obvious that the shell does not have a free top surface at $z=0$ which in CASE 1 generates reflected two-dimensional longitudinal waves. However all discussion in CASE 1 on the characteristic formation and the computational method except on the application of the boundary conditions must hold for CASE 2. In other words we can assume that spherical longitudinal waves reflect from the plane $z=0$, accordingly the characteristic formation and all computational procedures in CASE 1 are valid for CASE 2 with the exception of the application of the following conditions at the plane $z=0$.

The following boundary conditions are prescribed when both the intensity of σ_{rr} -input and the inner radius of the shell are considered to be unity.

The boundary conditions for point $(R,0,\tau)$ are given as $\sigma_{rr}=1$ and $\sigma_{rz}=0$. The initial conditions for point $(R,0,0)$ are given as $U_r=-1$, $\sigma_{\theta\theta}=\sigma_{zz}=\sigma_{rr}$, and $\partial U_z/\partial z = -\frac{\alpha(1+\alpha)}{2}$, $\partial U_r/\partial r=1$ as will be shown later.

The boundary conditions for points $(R, z < 0, z)$ are $\sigma_{rr} = \sigma_{rz} = 0$. The initial conditions for those points are $\sigma_{\theta\theta} = \sigma_{zz} = U_r = U_z = 0$.

The boundary and initial conditions for loaded boundary points $(R, z > 0)$ and outer surface points $(R_0, -\infty < z < +\infty)$ are the same as those in CASE 1.

Recurring Points in CASE 2

Figure 6 shows the four regions of influence and twenty five recurring points in the r - z plane. The quantities at all recurring points except points $c, d, i, i', n,$ and q can be evaluated by the same procedure as in CASE 1. The quantities at the recurring points where we can not apply the same computational method as in CASE 1 are computed in the following way:

c. Regular Two-Dimensional Outer surface Points $C(R_0 \leq r < R_1, z = R_0, r)$

When $\sigma_{rr} = \sigma_{rz} = 0$ Matrix 2 becomes

$-\frac{B_1}{2r}$	$-\frac{B_1}{2r}$	$-1 - \frac{B_1}{r}$	0	0	0	$\sigma_{\theta\theta}$	$\{A_1\}$
0	$1 + \frac{B_5}{2(R+z)}$	$-\frac{B_5}{2r}$	$-1 - \frac{B_5}{2(R+z)}$	0	0	σ_{zz}	$\{A_5\}$
0	1	$-\frac{B_6}{2r}$	1	0	0	U_r	$\{A_6\}$
0	0	$-\frac{B_9}{2r}$	0	$-\frac{B_9}{2}$	$-\frac{B_9}{2}$	U_z	A_9
0	1	$-\frac{B_{10}}{2r}$	0	$-\frac{B_{10}}{2}$	$-\frac{B_{10}}{2}$	$\frac{\partial U_r}{\partial z}$	A_{10}
1	0	$-\frac{B_{10}}{2r}$	0	$-\frac{B_{10}}{2}$	$-\frac{B_{10}}{2}$	$\frac{\partial U_z}{\partial r}$	A_{12}

Matrix 14

d. Regular Two-Dimensional Edge Point d(R, 0, r)

When $\sigma_{rr} = 1$, $\sigma_{rz} = 0$ Matrix 2 becomes

$\frac{B_2}{2r}$	$\frac{B_2}{2r}$	$1 - \frac{aB_2}{r}$	0	0	0	$\sigma_{\theta\theta}$	$[A_2] - (1 - \frac{B_2}{r})$
0	$1 + \frac{B_5}{2(R+z)}$	$-\frac{aB_5}{2r}$	$-1 - \frac{aB_5}{2(R+z)}$	0	0	σ_{zz}	$[A_5] + \frac{B_5}{2(R+z)}$
0	1	$-\frac{aB_6}{2r}$	1	0	0	U_r	$[A_6]$
0	0	$-\frac{aB_9}{2r}$	0	$-\frac{B_9}{2}$	$-\frac{aB_9}{2}$	U_z	$A_9 - 1$
0	1	$-\frac{aB_{10}}{2r}$	0	$-\frac{aB_9}{2}$	$-\frac{B_9}{2}$	$\frac{\partial U_r}{\partial r}$	A_{10}
1	0	$-\frac{B_9}{2r}$	0	$-\frac{aB_9}{2}$	$-\frac{B_9}{2}$	$\frac{\partial U_z}{\partial z}$	A_{12}

Matrix 15

1. Leading Two-Dimensional Incident Points i(R+r, 0, r)

The leading wave front at points i carries strong discontinuities via the bicharacteristic curve $dr=dr$. Therefore we employ the characteristic equation (1b)

$$d[\sigma_{rr}] - d[U_r] = (2a[U_r] - 2[\sigma_{rr}] + [\sigma_{\theta\theta}] + [\sigma_{zz}]) \frac{dr}{r},$$

substituting the dynamical condition for wave surface $[\sigma_{rr}] = -[U_r]$ (or from (2b) as $dr=0$) to obtain

$$d[\sigma_{rr}] = -(a+1)[\sigma_{rr}] + \frac{1}{2}([\sigma_{\theta\theta}] + [\sigma_{zz}]) \frac{dr}{r},$$

and from the static equations (9), (10), and (12), $[\sigma_{\theta\theta}] = [\sigma_{zz}] = a[\sigma_{rr}]$.

We finally come up with (as $d\sigma=dr$)

$$\frac{d[\sigma_{rr}]}{[\sigma_{rr}]} = -\frac{dr}{r} \quad \text{or} \quad [\sigma_{rr}] = Kr^{-1}.$$

Since $\sigma_{rr} = 1$ at $(R, 0, 0)$ K is taken to be unity. This is the well-known expression for the geometric decay of the spherical incident leading wave. All quantities for points i are obtained as the following closed-form solution:

$$\sigma_{rr} = r^{-1}, \sigma_{\theta\theta} = -\alpha\sigma_{rr}, \sigma_{zz} = \alpha\sigma_{rr},$$

$$\sigma_{rz} = 0, U_r = -\sigma_{rr}, U_z = -\sigma_{zz}, \frac{\partial U}{\partial r} = r^{-2}, \text{ and } \frac{\partial U}{\partial z} = 0.$$

i'. Reflected Leading Two-Dimensional Points i' ($R < R+r, 0, r$)

The reflected leading wave propagates along the bicharacteristic curve $dr = -dr$. Therefore we take the characteristic equation (2b) to determine the decay of the reflected spherical wave,

$$d[\sigma_{rr}] + d[U_r] = (2\alpha[U_r] + 2[\sigma_{rr}] - [\sigma_{\theta\theta}] - [\sigma_{zz}]) \frac{dr}{r}.$$

From (1b) as $dt = 0$ we have $[\sigma_{rr}] = [U_r]$ and from the static equations $\sigma_{\theta\theta} = \sigma_{zz} = \alpha\sigma_{rr}$. We substitute these relations into (2b) to obtain (as $dr = -dr$)

$$\frac{d[\sigma_{rr}]}{[\sigma_{rr}]} = -\frac{dr}{r} \text{ or } [\sigma_{rr}] = Kr^{-1}.$$

This reverse decay expression implies the convergent spherical wave. In summary the closed-form solutions are as follows:

$$\sigma_{rr} = r, \sigma_{\theta\theta} = -\alpha\sigma_{rr}, \sigma_{zz} = \alpha\sigma_{rr},$$

$$\sigma_{rz} = 0, U_r = \sigma_{rr}, U_z = \sigma_{zz}, \frac{\partial U}{\partial r} = -r^{-2}, \text{ and } \frac{\partial U}{\partial z} = 0.$$

n. Regular Inner Surface Points n ($0, -r < z < 0, r$)

When $\sigma_{rr} = \sigma_{rz} = 0$ Matrix 2 becomes

$\frac{B_2}{2r}$	$\frac{B_2}{2r}$	$1 - \frac{a}{r}$	0	0	0	$\sigma_{\theta\theta}$	$[A_2]$
0	$1 + \frac{B_5}{2(R+z)}$	$-\frac{aB_5}{2r}$	$-\frac{aB_5}{2(R+z)}$	0	0	σ_{zz}	$[A_5]$
0	1	$-\frac{aB_6}{2r}$	1	0	0	U_r	$[A_6]$
0	0	$-\frac{aB_9}{2r}$	0	$-\frac{B_9}{2}$	$-\frac{aB_9}{2}$	U_z	A_9
0	1	$-\frac{aB_9}{2r}$	0	$-\frac{aB_9}{2}$	$-\frac{B_9}{2}$	$\frac{\partial U_r}{\partial r}$	A_{10}
1	0	$-\frac{B_9}{2r}$	0	$-\frac{aB_9}{2}$	$-\frac{B_9}{2}$	$\frac{\partial U_z}{\partial z}$	A_{12}

Matrix 16

q. Leading Inner-Surface Points $q(R, -r, r)$

If the boundary conditions for points j are set to the same value as those for points q , the closed-form solution of σ_{zz} for points j is equal to that for points q . It follows that in order to calculate the decay of σ_{zz} at points q we can employ the characteristic equation (5b) along the bicharacteristic curve $dz=d$ which characterizes the leading spherical wave front at points j under the boundary conditions $\sigma_{rr} = \sigma_{rz} = 0$. The characteristic equation (5b) with $\sigma_{rr} = \sigma_{rz} = 0$ is written as

$$d[\sigma_{zz}] - d[U_z] = (a[U_z] - [\sigma_{zz}]) \frac{dr}{R+z}$$

From the dynamical condition or (6b) as $dr=0$ we have $[\sigma_{zz}] = -[U_z]$, then the equation becomes

$$2d[\sigma_{zz}] = -(1+a)[\sigma_{zz}] \frac{dr}{R+z}$$

as $dr=dz$ we obtain

$$\frac{d[\sigma_{zz}]}{[\sigma_{zz}]} = -\frac{1+a}{2} \frac{dz}{(R+z)} \quad \text{or} \quad [\sigma_{zz}] = K(R+z)^{-\frac{1+a}{2}}$$

Since $\sigma_{zz} = a$ at $(R, 0, 0)$ and if $R=1$, K becomes a . It follows that

$$[\sigma_{zz}] = a(1+z)^{-\frac{1+a}{2}}$$

Finally the quantities for points q are obtained as the following closed-form solution:

$$\begin{aligned} \sigma_{rr} = 0, \quad \sigma_{zz} = a(1+z)^{-\frac{1+a}{2}}, \quad \sigma_{\theta\theta} = \sigma_{zz}, \\ \sigma_{rz} = 0, \quad U_r = 0, \quad U_z = -\sigma_{zz}, \quad \frac{\partial U_r}{\partial r} = 0, \quad \text{and} \quad \frac{\partial U_z}{\partial z} = \frac{a(1+a)}{2}(1+z)^{-\frac{3+a}{2}}. \end{aligned}$$

Superposition of Shifted Loads for CASE 3

Our aim here is to obtain the transient stress distribution in an infinitely long thick cylindrical shell subject to an impulsive constant load of finite length. It is obtained by applying the principle of superposition since all action is linear elastic.

We now consider two cases of loading that can be readily obtained from the previous calculation for CASE 2. One of those loading cases is, as shown in Figure 6, that its semi-infinite load along the inner surface shifts upwards (in the negative direction of z) by a half of the load width. The other is that the semi-infinite load shifts downwards also by a half of the load width and the load is applied in the reverse direction, namely the input stress is changed to tensile stress while the stress intensity remains unity ($\sigma_{rr} = -1$). After the superposition we obtain the stress distribution for a case of a finite-width load whose

center line lies on the r-axis. In the practical computation, however, we have to shift the load of the latter case one grid point lower than a half of the load width as shown in Figure 6.

Consecutive superposition Scheme for CASE 4

Up to here we considered only step constant input shown in Figure 8(a) for CASE 1, CASE 2, and CASE 3. However, in order to obtain a stress distribution of the travelling load case(CASE 4) rectangular inputs which have various duration of loading time must be considered. A rectangular σ_{rr} input can be obtained by superimposing a negative unit step input whose striking time is shifted to τ on a unit step input as shown in Figure 8. Once a stress distribution of case 3 due to a rectangular input load which has certain duration of loading is obtained, we can compute the stress distribution of the travelling load case by means of the consecutive superposition of the previous result. The speed of the travelling load can be controlled by the duration of loading time in CASE 3.

In view of Figure 9 it is seen that a load of finite length is applied at $\tau=0$ as shown in (a) and then at $\tau=\tau_a$ the load is removed, however at the same time another load which is shifted downwards by one grid point distance is applied and again at $\tau=\tau_b$ the load is removed when another shifted load is applied. This process is repeated until the incident wave due to the first loading reaches the inner surface after reflecting at the outer surface. The stress distribution at $\tau=\tau_a$

is obtained by superimposing the stresses due to the first loading at $t=t_1$ on the stresses due to the second load applied at $t=t_2$. Thus what we obtained by superposition is an accumulated stress distribution due to many rectangular input loads that account for a travelling load of finite length.

CHAPTER V

RESULTANT MOTION IN THICK SHELL

Computer programs

The transient motion in a thick shell was obtained with aid of CDC CYBER computer. Five separate programs are utilized to obtain the stress distribution of the present boundary value problems. The first and second programs, TRES1 and TRES2 are an extension of the coputer code CHAR2DZ developed by M. Ziv[6]. Next three programs, RECTINP, TRES3, and TRES4 were developed during the course of this work to culminate the computation of the transient motion in a thick shell subject to an impulsive travelling load of finite length on the interior. Also another program PLOTALL is utilized to plot the time history of the stresses and the particle velocities which are computed by the above five programs. These programs are written in FORTRAN IV language.

The program TRES1 determines the stress distribution of CASE 1 in which the load is applied as a step input, so the duration of loading is permanent. The output of TRES1 stored on TAPE1 can be plotted on the CALCOMP plotting machine by using the program PLOTALL.

The second program TRES2 is used to determine the stress distribution of CASE 2 in which the duration of loading is permanent. The output is stored on TAPES which will be retrieved separately by the programs, RECTINP, TRES3, and PLOTALL for their computation and

plotting purposes. RECTINP is a program to compute various cases of a rectangular input load for CASE 2. This program retrieves TAPK5 and stores its output on TAPE6.

The program TRES3 carry out the calculation of the superposition scheme for CASE 3. This program retrieves TAPK5 obtained by TRES2 or RECTINP. Since the program RECTINP stores its output on TAPE6, before attempting to run TRES3 of a rectangular input loading case TAPE6 should be renamed as TAPK5 which is compatible with the format in TRES3.

The last program TRES4 determines the stress distribution of a travelling load case (CASE 4). TAPE10 obtained by TRES3 is retrieved by TRES4 which will store the output on TAPE15.

All program listings are presented in Appendix A-F.

Recorded Plots

Nine groups of plots were obtained as shown in Figures 10-63 which represent the time history of the stresses and particle velocities for all cases. The following data common to all cases are prescribed during the computation: inner radius of the shell $R=1.00$, outer radius $R_o=1.30$, Poisson's ratio $\nu=0.15$, intensity of input $\sigma_{TT}=-1.0$, step size of integration $\Delta r=0.02$, and number of time steps is 30.

The first group of plots shows the time history of the stresses and the particle velocities of CASE 1 at six detected points. Observing Figure 14 one can see that three jumps occur during the entire time of record. Those jumps are due to the arrivals of the incident longitudinal wave, the first reflected longitudinal wave from (1.0,0,0), and the reflected wave from the outer surface in the order of arrival time.

The next three groups of plots Figure 16 through 33 show the time history of the stresses and the particle velocities of CASE 2. Three cases of the duration of loading are computed for CASE 2, that are 0.04, 0.08, and permanent cases. It should be noticed that in Figure 23 or 29 of the rectangular input cases a high tensile stress wave which is due to the reflected longitudinal wave from the outer free surface passes through the points located around the half way of width of the shell. If the duration of loading is permanent a reflected tensile stress wave will be cancelled by a compression wave issued from the loaded boundary.

The next three groups of plots are recorded for CASE 3 and the last two groups of plots are obtained for CASE 4 in which speeds of the travelling load are 0.5 and 1.0. Observing Figure 52 through 63 one can see that in those plots many jumps in the stresses and the particle velocities occur during the entire time of record. This is because that many discontinuous wave fronts due to the loading and unloading process exist in the solution domain. For example in Figure 63 the first jump occurs at $\tau=0.08$ and after that a jump is

observed step by step until the last time step, since, in this case, the speed of the travelling load is 1.0 which implies that the travelling load is moving step by step.

Thus, displacement and strain components at any point in the shell may be evaluated in terms of dimensionless valuables from the computer program presented.

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APPENDIX A

FIGURES IN CHAPTERS I-IV

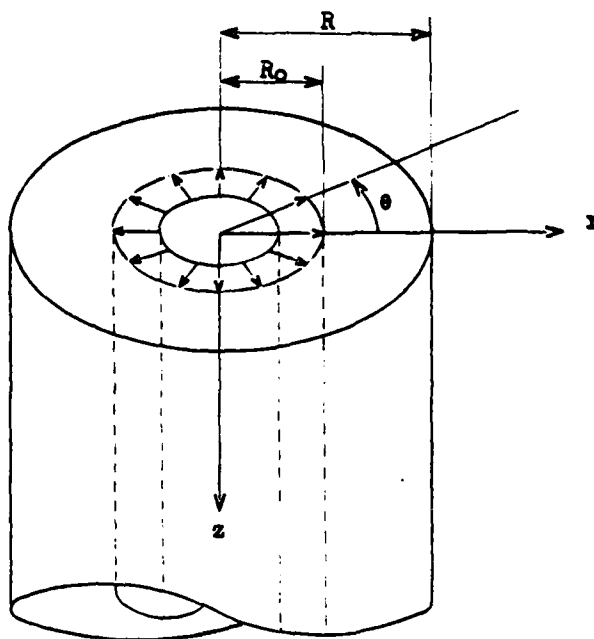


Figure 1. Geometry, loading, and coordinate system for CASE 1.

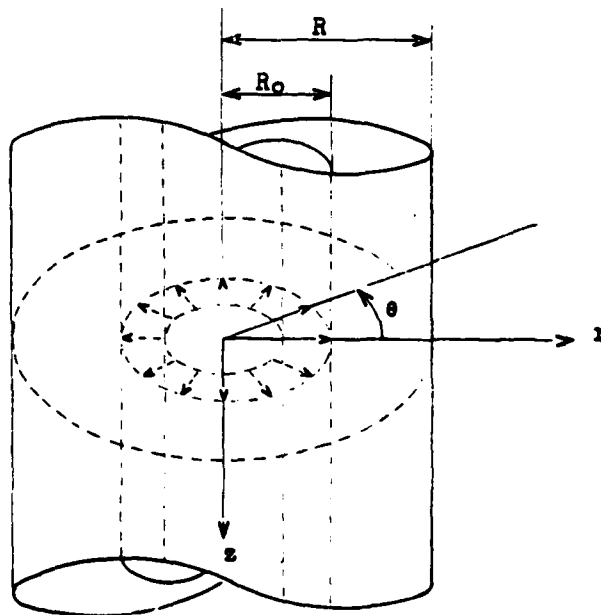


Figure 2. Geometry, loading, and coordinate system for CASE 2.

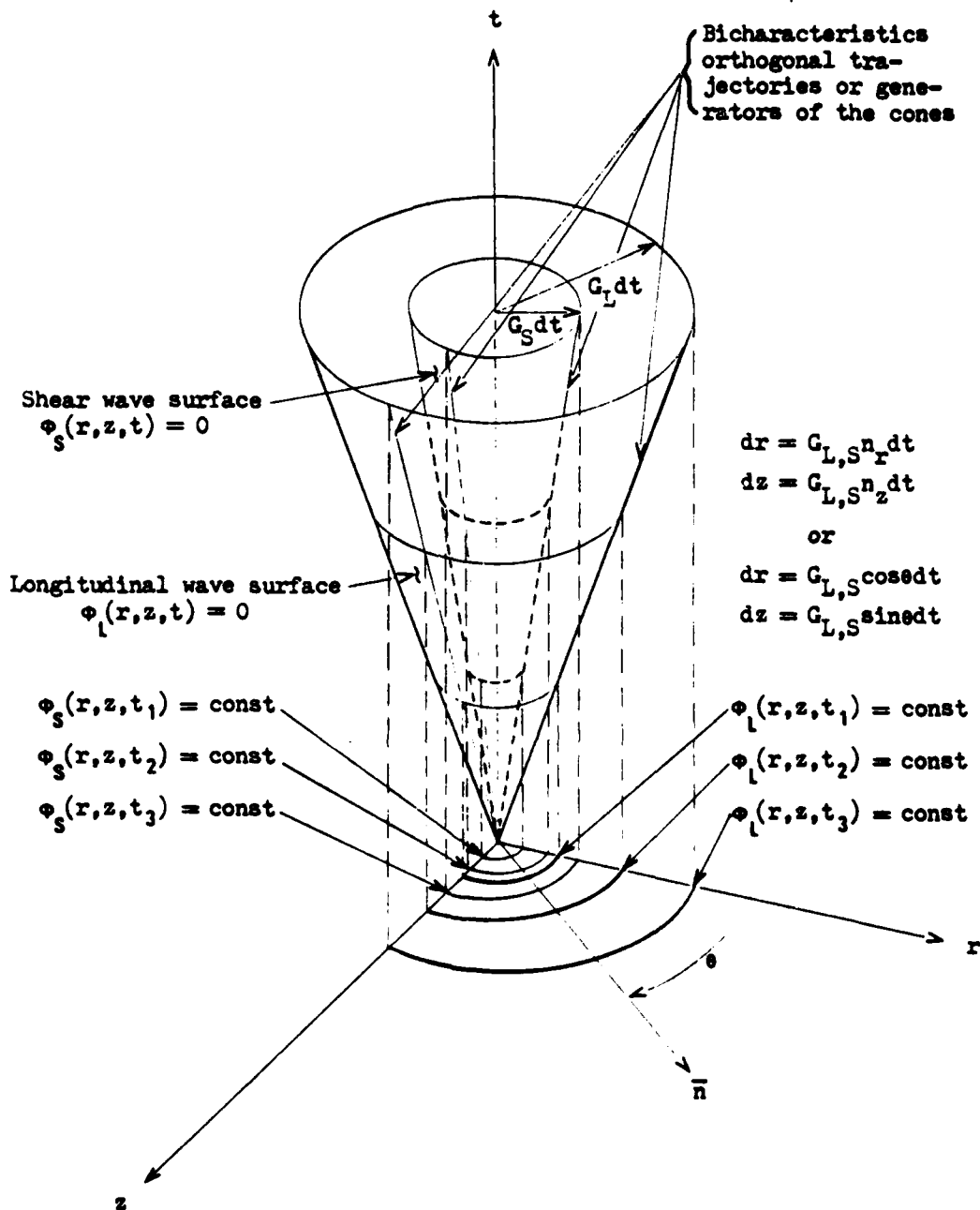


Figure 3. Longitudinal and shear wave cones and the bicharacteristic curves.

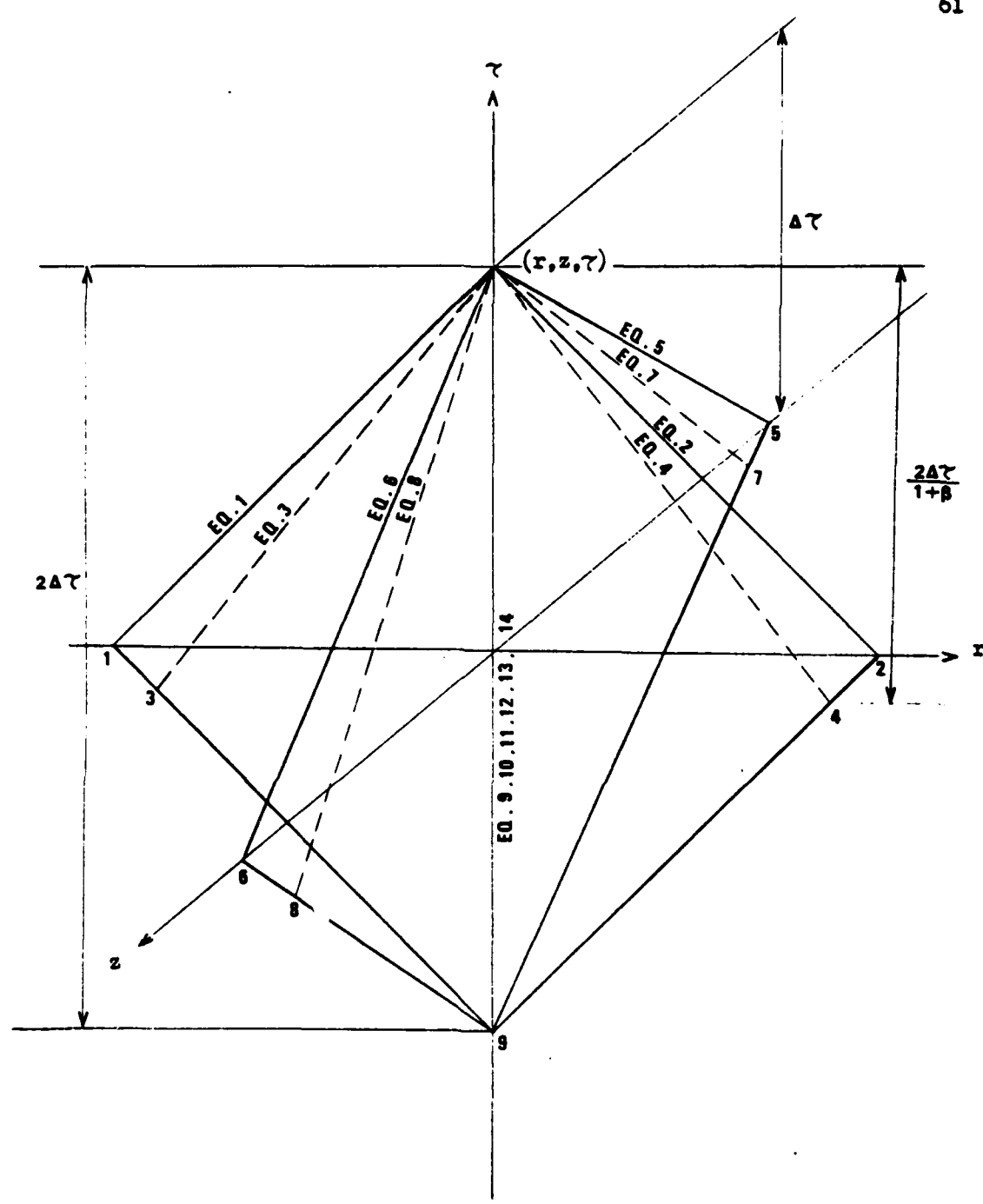


Figure 4. Orthogonal symmetric mesh element in the r, z, τ space.

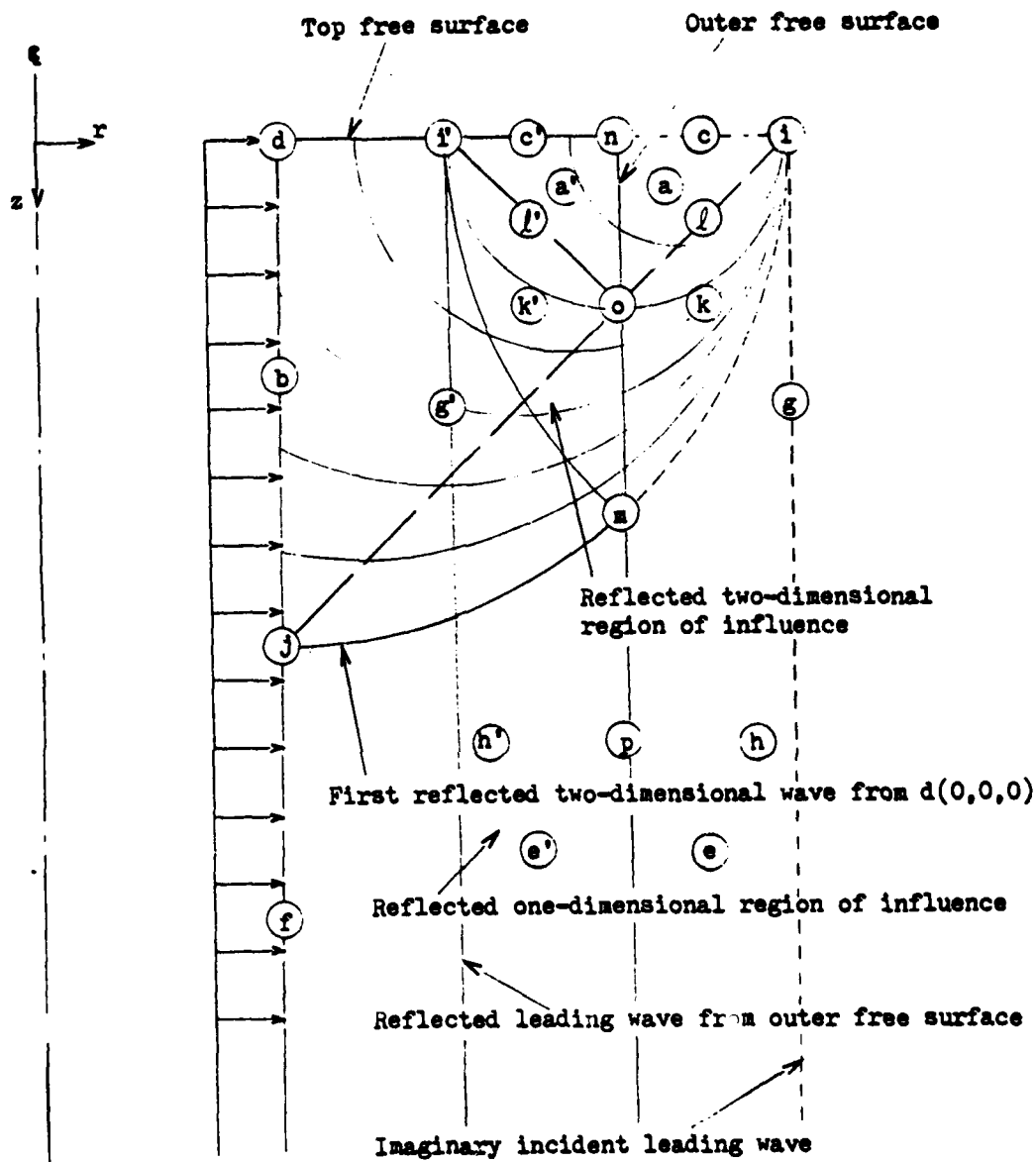


Figure 5. Influence regions and twenty four recurring points in CASE 1.

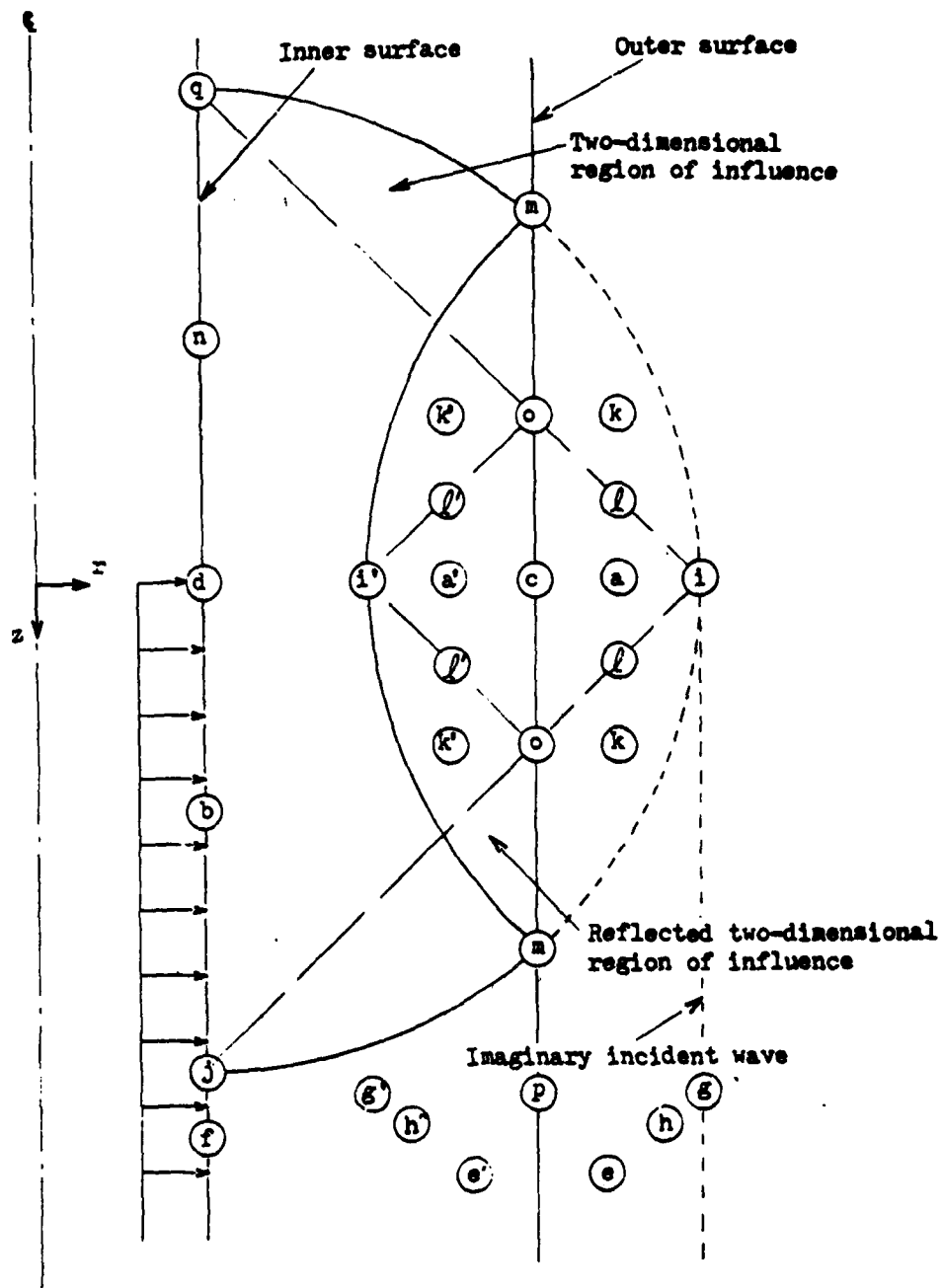


Figure 6. Influence regions and twenty five recurring points in CASE 2.

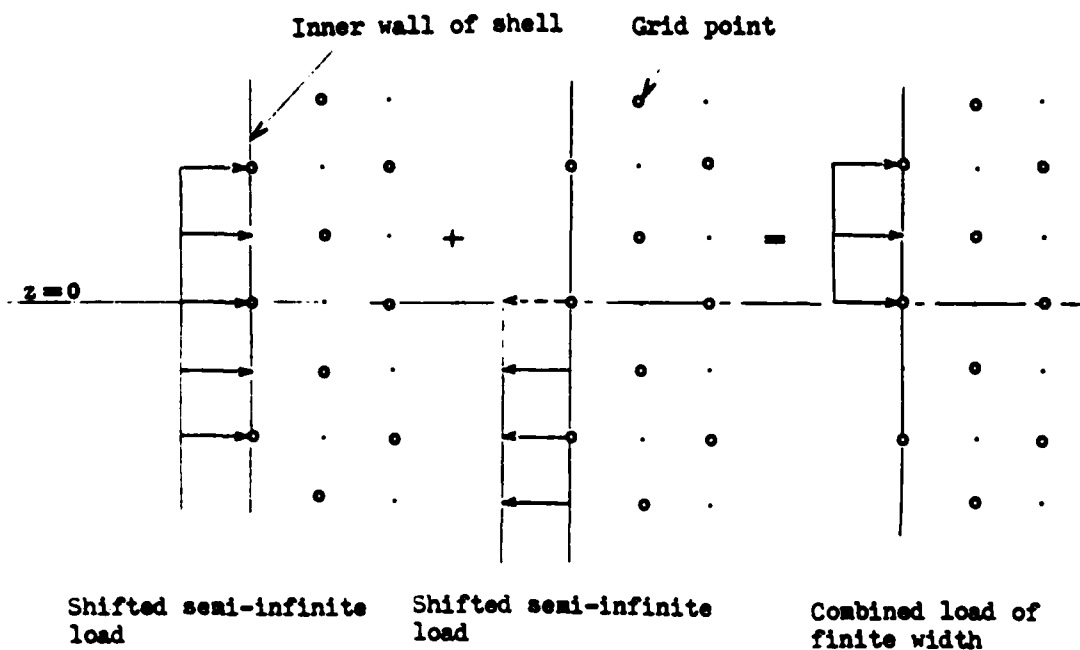


Figure 7. Superposition scheme of shifted loads for CASE 3.

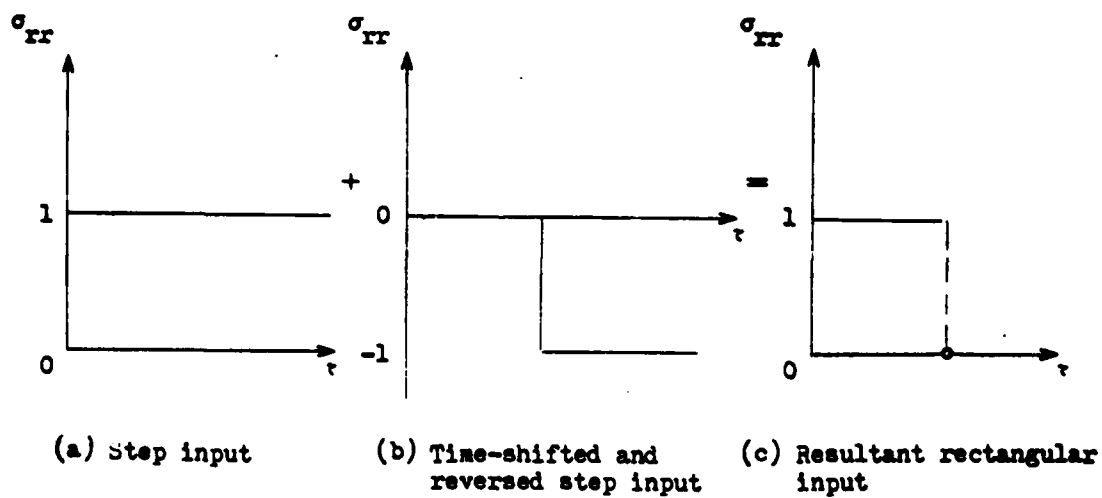
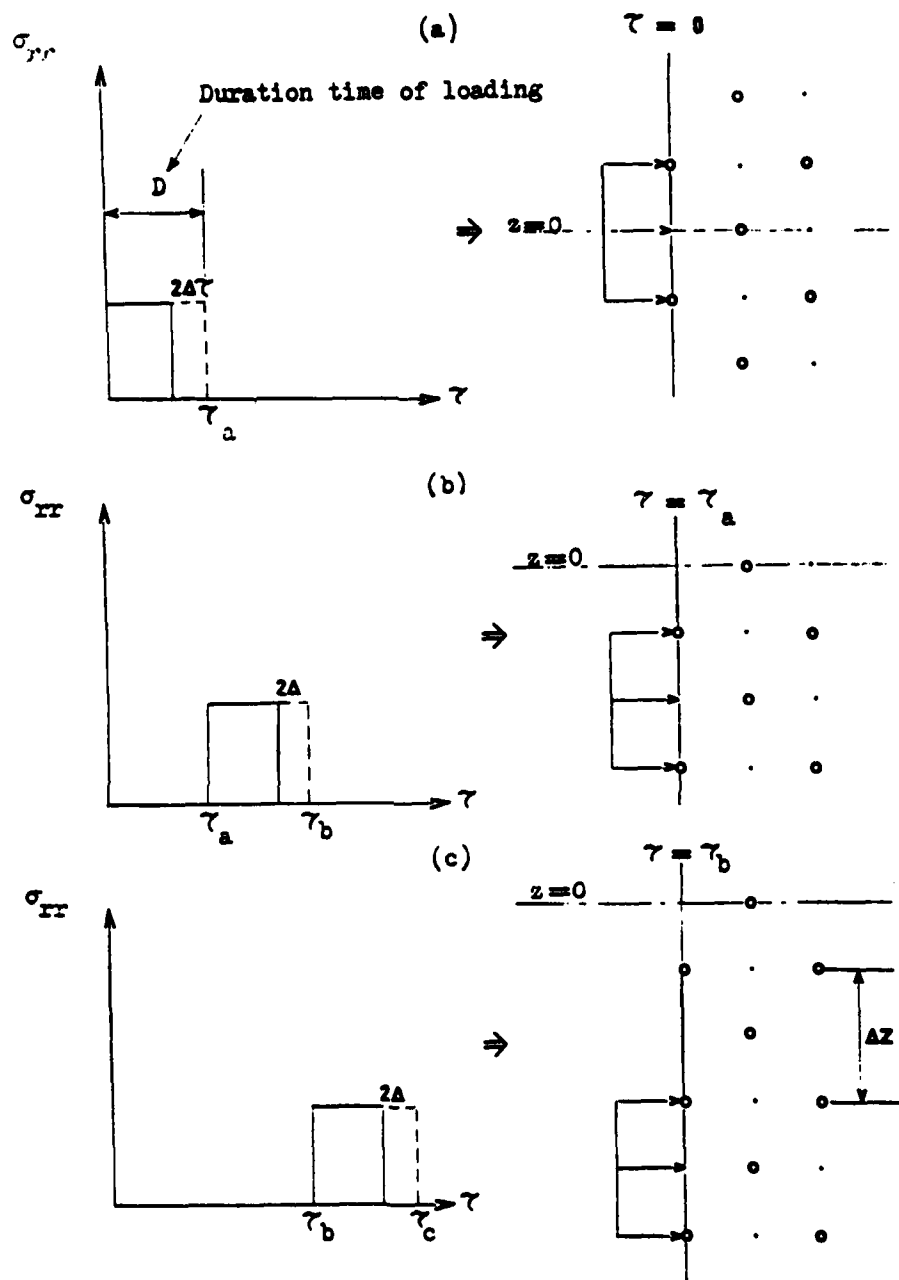


Figure 8. Impulsive rectangular-input loading.



Speed of travelling load = $\frac{\Delta Z}{D}$

Figure 9. Consecutive superposition scheme for CASE 4.

APPENDIX B

PROGRAM LISTING OF TRES1

```

PROGRAM TRES1(OUTPUT,TAPE1,TAPE2)                                00010
C *****00020
C CASE 2 : TRANSIENT RESPONSE OF A SEMI-INFINITELY LONG TUBE    00030
C   SUBJECTED TO LOAD APPLIED FROM THE INTERIOR AND ALONG      00040
C   THE LENGTH OF THE TUBE                                     00050
C                                                                 00060
C -----00070
C CO-ORDINATE SYSTEM FOR THE ARRAYS USED IN THE PROGRAM:      00080
C                                                                 00090
C J - COORDINATE FOR Z-AXIS. J=1 AT Z=0.0.                    00100
C K - COORDINATE FOR R-AXIS. K=1 AT R=DRO.                    00110
C L - COORDINATE FOR VARIABLES. 1 - SITT      5 - UZ           00120
C                                     2 - UR      6 - DUZDZ       00130
C                                     3 - DURDR     7 - SIRR        00140
C                                     4 - SIZZ      8 - SIRZ        00150
C NT - TIME COORDINATE. 1 - TOW, 2 - TOW-DT, 3 - TOW-2*DT     00160
C                                                                 00170
C -----00180
C INPUT DATA                                                  00190
C                                                                 00200
C N      ---      NUMBER OF DIVISIONS ACROSS THICKNESS OF CYLINDER 00210
C INDEX ---      TOTAL NUMBER OF INTEGRATION(MUST BE 2*N)(MAXIMUM 30) 00220
C DRO    ---      INTERNAL RADIUS                                00230
C DRI    ---      EXTERNAL RADIUS                                00240
C RNE    ---      POISSON'S RATIO                                00250
C FF     ---      DIMENSIONLESS LOADING INTENSITY                00260
C                                                                 00270
C                                                                 00280
C THE FORM OF OUTPUT IS SELECTED BY SPECIFYING IPRINT AND INPUT 00290
C THE INFORMATION REQUIRED FOR THE OUTPUT                          00300
C                                                                 00310
C IPRINT      FORM OF OUTPUT                                    00320
C -----      -----00330
C   1          PRINT FOR A SPECIFIED TIME                        00340
C   2          PRINT FOR A SPECIFIED POINT                       00350
C   3          PRINT FOR BOTH SPECIFIED TIME AND POINTS        00360
C -----00370
C IFROM ---      STARTING TIME FOR PRINTING                      00380
C ITILL ---      TIME FOR TERMINATION OF PRINTING                00390
C IWRITE ---     TIME INTERVAL OF PRINTING                       00400
C NPRINT ---     NUMBER OF POINTS SPECIFIED (MAXIMUM 9)         00410
C JPRINT ---     J - COORDINATE OF SPECIFIED POINTS             00420
C KPRINT ---     K - COORDINATE OF SPECIFIED POINTS             00430
C                                                                 00440
C *****00450
C COMMON/FO/FP,FB,FR                                           00470
C COMMON/ALL/I,TOW,R,Z,K,J,DT5,ALFA,BETA,BETA2,G2,BG,CBA,CABA,PIE5,NO0490

```

```

COMMON/RE/IPFROM, ITILL, IWRITE, IJ1, IPRINT, NPRINT          00500
COMMON/PRINT/JPRINT(9), KPRINT(9)                            00510
DIMENSION T3(31,8), TT(34,8)                                  00520
DIMENSION Q3(31,8), QJ(2,31,8)                               00530
DIMENSION ZA(41)                                              00540
COMMON/TN/T(34,31,8), T1(34,31,8), T2(34,31,8), TOTAL(34,31,8) 00550
COMMON/QN/Q(34,31,8), Q1(34,31,8), Q2(34,31,8)              00560
C                                                                00570
C INPUT TO THE PROGRAM                                       00580
C                                                                00590
DATA INDEX, N, DRO, DRI, RNE, FF/30, 15, 1., 1.30, .15, 1./  00600
DATA(JPRINT(M), M=1, 2)/2, 2/                                00610
DATA(KPRINT(M), M=1, 2)/1, 2/                                00620
DATA IPRINT, IPFROM, ITILL, IWRITE, NPRINT/1, 1, 30, 1, 2/  00630
N1=N+1                                                         00640
INDX1=INDEX+1                                                 00650
INDX4=INDEX+4                                                 00660
CALL GALIM(RNE, DRO, INDEX, INDX1, INDX4, T3, TT,            00670
+DRI, Q3, QJ, ZA, N1)                                         00680
STOP                                                           00690
END                                                            00700
C                                                                00710
C                                                                00720
SUBROUTINE GALIM(RNE, DRO, INDEX, INDX1, INDX4, T3, TT,      00730
+DRI, Q3, QJ, ZA, N1)                                         00740
C                                                                00750
C***** 00760
C PURPOSE: TO FIX THE SCHEME OF INTEGRATION * 00770
C***** 00780
C                                                                00790
COMMON/ALL/I, TOW, R, Z, K, J, DTS, ALFA, BETA, BETA2, G2, BG, CRA, CABA, PIES, NOO800
COMMON/PRINT/JPRINT(9), KPRINT(9)                            00810
COMMON/RE/IPFROM, ITILL, IWRITE, IJ1, IPRINT, NPRINT        00820
COMMON/PO/FF, FB, FR                                         00830
COMMON/AA/B1, B2, B3, B5, B6, B9                              00840
DIMENSION ZA(N1)                                              00850
COMMON/TN/T(34,31,8), T1(34,31,8), T2(34,31,8),            00860
+TOTAL(34,31,8)                                               00870
COMMON/QN/Q(34,31,8), Q1(34,31,8), Q2(34,31,8)             00880
DIMENSION T3(INDX1,8), T4(3,8), TT(INDX4,8)                 00890
DIMENSION Q3(INDX1,8), Q4(3,8), QJ(2,INDX1,8)               00900
C                                                                00910
C CALCULATION OF THE CONSTANTS                                00920
C                                                                00930
FR=-FF                                                         00940
FRZ=0.                                                         00950
DT=(DRI-DRO)/FLOAT(N)                                         00960
DT5=DT/2.0                                                    00970
ALFA=RNE/(1.0-RNE)                                           00980
BETA2=(1.0-ALFA)/2.0                                          00990
BETA=SQRT(BETA2)                                             01000
G2=DT/(1.0+BETA)                                             01010

```

	BG=2.0*BETA*G2	01020
	CBA=(1.0-BETA)/(1.0+BETA)	01030
	CABA=2.0*BETA/(1.0+BETA)	01040
	B1=DT5	01050
	B2=DT5	01060
	B3=G2	01070
	B5=DT5	01080
	B6=DT5	01090
	B9=DT	01100
	IJ=1	01110
	PIE5=2.0*ATAN(1.0)	01120
C		01130
C	SET ALL ARRAYS EQUAL TO ZERO	01140
C		01150
	DO 50 L=1,8	01160
	DO 49 NT=1,3	01170
	T4(NT,L)=0.0	01180
	Q4(NT,L)=0.0	01190
49	CONTINUE	01200
	DO 50 K=1,INDX1	01210
	T3(K,L)=0.0	01220
	Q3(K,L)=0.0	01230
	QJ(1,K,L)=0.0	01240
	QJ(2,K,L)=0.0	01250
	DO 50 J=1,INDX4	01260
	T(J,K,L)=0.0	01270
	T1(J,K,L)=0.0	01280
	T2(J,K,L)=0.0	01290
	TOTAL(J,K,L)=0.0	01300
	Q(J,K,L)=0.0	01310
	Q1(J,K,L)=0.0	01320
	Q2(J,K,L)=0.0	01330
50	CONTINUE	01340
C		01350
C	INITIAL CONDITIONS	01360
C		01370
	T1(1,1,7)=FF/DRO**(1.0+ALFA/2.0)	01380
	T1(1,1,1)=ALFA*T1(1,1,7)	01390
	T1(1,1,2)=-T1(1,1,7)	01400
	T1(1,1,3)=FF*(1.0+ALFA/2.0)/DRO**(2.0+ALFA/2.0)	01410
	T1(1,1,5)=ALFA*FF/SQRT(DRO)	01420
	DO 55 J=2,INDX4	01430
	T1(J,1,7)=FF/SQRT(DRO)	01440
	T1(J,1,1)=ALFA*T1(J,1,7)	01450
	T1(J,1,4)=-T1(J,1,1)	01460
	T1(J,1,2)=-T1(J,1,7)	01470
	T1(J,1,3)=0.5*FF/DRO**1.5	01480
55	CONTINUE	01490
	DO 60 L=1,8	01500
	T4(1,L)=T1(1,1,L)	01510
	DO 60 J=1,INDX4	01520
	TT(J,L)=T1(J,1,L)	01530

60	CONTINUE	01540
C		01550
C	THE FRAME OF INTEGRATION	01560
C		01570
	DO 2000 I=1,INDEX	01580
	SIGN=1.0	01590
	TOW=I*DT	01600
	IA=I+1	01610
	IA1=IA	01620
	IB=I+2	01630
	IC=I+3	01640
	KK=3	01650
	DO 1000 JM=2,IC	01660
	KK=5-KK	01670
	J=I+4-JM	01680
	Z=FLOAT(J)*DT-DT	01690
	IF(JM .GT. 3) GO TO 200	01700
	IF(JM .EQ. 3) GO TO 300	01710
C		01720
100	K=1	01730
	R=DRO	01740
	CALL LOADED(DRO,INDX4,INDX1,DT,T,T1,T2,SIGN,DR1)	01750
	IF(I .EQ. 1) GO TO 105	01760
	DO 110 K=2,I	01770
	R=FLOAT(K)*DT-DT+DRO	01780
	CALL GENER(DRO,INDX4,INDX1,DT,T,T1,T2,SIGN,DR1)	01790
110	CONTINUE	01800
105	K=I+1	01810
	R=FLOAT(I)*DT+DRO	01820
	CALL WAVE(DRO,INDX4,INDX1,T,SIGN,FF)	01830
	DO 115 K=1,IA	01840
	DO 115 JJ=IC,INDX4	01850
	DO 115 L=1,8	01860
	T(JJ,K,L)=T(IB,K,L)	01870
115	CONTINUE	01880
	DO 120 JJ=2,IA	01890
	DO 120 L=1,8	01900
	T(JJ,IA,L)=T(IB,IA,L)	01910
120	CONTINUE	01920
	GO TO 1000	01930
C		01940
300	K=1	01950
	R=DRO	01960
	CALL LEAD(DRO,INDX4,INDX1,DT,T,T1,T2,SIGN,DR1)	01970
	DO 305 L=1,8	01980
	T3(1,L)=T(J,1,L)	01990
	T4(3,L)=T4(2,L)	02000
	T4(2,L)=T4(1,L)	02010
305	CONTINUE	02020
	T4(1,7)=FF/(DRO+TOW)**(1.0+0.5*ALFA)	02030
	T4(1,1)=ALFA*T4(1,7)	02040
	T4(1,2)=-T4(1,7)	02050

T4(1,3)=FF*(1.0+0.5*ALFA)/(DRO+TOW)**(2.0+0.5*ALFA)	02060
T4(1,5)=FF*ALFA/SQRT(DRO+TOW)	02070
T4(1,4)=0.0	02080
T4(1,6)=0.0	02090
T4(1,8)=0.0	02100
IF(I .LE. 2) GO TO 1000	02110
DO 310 K=3,I,2	02120
DO 310 L=1,8	02130
T(J,K,L)=T(IB,K,L)	02140
310 CONTINUE	02150
320 GO TO 1000	02160
C	02170
200 IF(J.EQ. 1) GO TO 500	02180
C	02190
400 IF(KK .EQ. 2) GO TO 405	02200
K=1	02210
R=DRO	02220
CALL LOAD1(DRO,INDX4,INDX1,DT,T,T1,T2,SIGN,DR1,FRZ)	02230
405 DO 445 K=KK,I,2	02240
R=FLOAT(K)*DT-DT+DRO	02250
LM=(J-1)*(J-1)-I*I+(K-1)*(K-1)	02260
IF(LM .GT. 0) GO TO 430	02270
ML=K+J-I-2	02280
IF(ML) 410,415,425	02290
410 CALL GENER1(DRO,INDX4,INDX1,DT,T,T1,T2,SIGN,DR1)	02300
GO TO 445	02310
415 CALL DIAG(DRO,INDX4,INDX1,DT,T,T1,T2,T4,SIGN,DR1)	02320
DO 420 L=1,8	02330
T3(K,L)=T(J,K,L)	02340
420 CONTINUE	02350
GO TO 445	02360
425 CALL GINTER(DRO,INDX4,INDX1,DT,T3,T4,T,SIGN,DR1,N1)	02370
GO TO 445	02380
430 DO 440 L=1,8	02390
T(J,K,L)=T(IB,K,L)	02400
440 CONTINUE	02410
445 CONTINUE	02420
GO TO 1000	02430
C	02440
500 IF(KK .EQ. 2) GO TO 505	02450
K=1	02460
R=DRO	02470
CALL BOAX1(DRO,INDX4,INDX1,DT,T,T1,T2,SIGN,DR1)	02480
505 IF(I .LE. 2) GO TO 515	02490
IAN=I-1	02500
DO 510 K=KK,IAN,2	02510
R=FLOAT(K)*DT-DT+DRO	02520
CALL FREE(DRO,INDX4,INDX1,DT,T,T1,T2,SIGN,DR1)	02530
510 CONTINUE	02540
515 K=I+1	02550
DO 520 L=1,8	02560
T(J,K,L)=T4(1,L)	02570

520	CONTINUE	02580
1000	CONTINUE	02590
	SIGN=-1.0	02600
	IF(I-N)1985,599,604	02610
C	INITIAL REFLECTED CONDITIONS	02620
599	Q1(1,N1,7)=FR/DR1**(1.0+ALFA/2.0)	02630
	Q1(1,N1,1)=ALFA*Q1(1,N1,7)	02640
	Q1(1,N1,2)=-SIGN*Q1(1,N1,7)	02650
	Q1(1,N1,3)=SIGN*FR*(1.0+ALFA/2.0)/DR1**(2.0+ALFA/2.0)	02660
	Q1(1,N1,5)=ALFA*FR/SQRT(DR1)	02670
	DO 600 J=2,INDX4	02680
	Q1(J,N1,7)=FR/SQRT(DR1)	02690
	Q1(J,N1,1)=ALFA*Q1(J,N1,7)	02700
	Q1(J,N1,4)=ALFA*Q1(J,N1,7)	02710
	Q1(J,N1,2)=-SIGN*Q1(J,N1,7)	02720
	Q1(J,N1,3)=0.5*SIGN*FR/DR1**1.5	02730
600	CONTINUE	02740
	DO 602 L=1,8	02750
	Q4(1,L)=Q1(1,N1,L)	02760
	DO 602 M=1,N1	02770
	QJ(1,M,L)=Q4(1,L)	02780
602	CONTINUE	02790
	DO 603 J=1,INDX4	02800
	DO 603 K=1,INDX1	02810
	DO 603 L=1,8	02820
603	Q(J,K,L)=Q1(J,K,L)	02830
	GO TO 4000	02840
604	IR=I-N	02850
	KW=N1-IR	02860
	DO 606 L=1,8	02870
	Q4(3,L)=Q4(2,L)	02880
	Q4(2,L)=Q4(1,L)	02890
606	CONTINUE	02900
	R=FLOAT(2*N-I)*DT+DRO	02910
	Q4(1,7)=FR/R**(1.0+ALFA*0.5)	02920
	Q4(1,1)=ALFA*Q4(1,7)	02930
	Q4(1,2)=-SIGN*Q4(1,7)	02940
	Q4(1,3)=FR*SIGN*(1.0+ALFA*0.5)/R**(2.0+0.5*ALFA)	02950
	Q4(1,5)=FR*ALFA/R**0.5	02960
	Q4(1,4)=0.	02970
	Q4(1,6)=0.	02980
	Q4(1,8)=0.	02990
	DO 3000 JM=2,IC	03000
	J=I+4-JM	03010
	IM=MOD(JM+N,2)+2	03020
	Z=FLOAT(J-1)*DT	03030
	J1=I-N+1	03040
	J2=2+SQRT(FLOAT(I*I-N*N))	03050
	IF(J-J1)640,630,610	03060
610	IF(J.LT.J2)GO TO 620	03070
	K=N1	03080
	R=DR1	03090

FB=-T(J,N1,7)	03100
CALL LOADED(DRO,INDX4,INDX1,DT,Q,Q1,Q2,SIGN,DR1)	03110
IF(IR.EQ.1)GO TO 614	03120
DO 612 KR=2,IR	03130
K=N-KR+2	03140
R=FLOAT(K)*DT-DT+DRO	03150
CALL GENER(DRO,INDX4,INDX1,DT,Q,Q1,Q2,SIGN,DR1)	03160
612 CONTINUE	03170
614 K=KW	03180
R=FLOAT(K-1)*DT+DRO	03190
CALL WAVE(DRO,INDX4,INDX1,Q,SIGN,FR)	03200
DO 616 K=KW,N1	03210
DO 616 JJ=J2,INDX4	03220
DO 616 L=1,8	03230
Q(JJ,K,L)=Q(IB,K,L)	03240
616 CONTINUE	03250
DO 618 L=1,8	03260
DO 617 M=1,N1	03270
617 QJ(2,M,L)=QJ(1,M,L)	03280
QJ(1,1,L)=Q(J2,N1,L)	03290
618 CONTINUE	03300
DO 619 JJ=2,J2	03310
DO 619 LL=1,8	03320
619 Q(JJ,KW,LL)=Q(IB,KW,LL)	03330
CALL RBOUND(DRO,INDX4,INDX1,DT,Q,Q1,Q2,Q4,QJ,J1,SIGN,T,ZA,N1,DR1)	03340
GO TO 3000	03350
620 IF(IM.EQ.2)GO TO 622	03360
K=N1	03370
R=DR1	03380
AJM=1.0+FLOAT(I*I-N*N-(J-1)*(J-1))/FLOAT(?*(I-N))	03390
M=AJM	03400
DO 624 L=1,8	03410
624 Q(J,K,L)=(AJM-FLOAT(M))*QJ(1,M+1,L)+(FLOAT(M+1)-AJM)*QJ(1,M,L)	03420
622 DO 629 KR=IM,IR,2	03430
K=N-KR+2	03440
R=FLOAT(K-1)*DT+DRO	03450
LM=(J-1)**2-I*I+(2*N-K+1)**2	03460
IF(LM.GT.0)GO TO 626	03470
CALL RINTER(Q,QJ,Q4,INDX4,INDX1,ZA,N1)	03480
GO TO 629	03490
626 DO 625 L=1,8	03500
Q(J,K,L)=Q(IB,K,L)	03510
625 CONTINUE	03520
629 CONTINUE	03530
GO TO 3000	03540
630 K=N1	03550
R=DR1	03560
DO 633 L=1,8	03570
Q(J,K,L)=QJ(1,N1,L)	03580
Q3(N1,L)=Q(J,K,L)	03590
633 CONTINUE	03600
IF(IR.LE.2)GO TO 3000	03610

DO 636 KR=3, IR, 2	03620
K=N-KR+2	03630
LM=(J-1)**2-I*I+(2*N-K+1)**2	03640
IF(LM.GT.0)GO TO 634	03650
CALL RINTER(Q,QJ,Q4,INDX4,INDX1,ZA,N1)	03660
GO TO 636	03670
634 DO 635 L=1, 8	03680
Q(J,K,L)=Q(IB,K,L)	03690
635 CONTINUE	03700
636 CONTINUE	03710
GO TO 3000	03720
640 IF(J.EQ.1)GO TO 700	03730
IF(IM.EQ.2)GO TO 641	03740
K=N1	03750
R=DR1	03760
FB=-T(J,N1,7)	03770
FRZ=-T(J,N1,8)	03780
CALL LOAD1(DRO,INDX4,INDX1,DT,Q,Q1,Q2,SIGN,DR1,FRZ)	03790
IF(IR.LE.2)GO TO 3000	03800
641 DO 649 KR=IM, IR, 2	03810
K=N-KR+2	03820
R=FLOAT(K-1)*DT+DRO	03830
LM=(J-1)**2-I*I+(2*N-K+1)**2	03840
LA=(J-1)**2-(I-N)**2+(N-K+1)**2	03850
ML=J+2*N-K-I	03860
IF(LM.GT.0)GO TO 647	03870
IF(LA.GT.0)GO TO 646	03880
IF(ML)642, 643, 645	03890
642 CALL GENER1(DRO,INDX4,INDX1,DT,Q,Q1,Q2,SIGN,DR1)	03900
GO TO 649	03910
643 CALL DIAG(DRO,INDX4,INDX1,DT,Q,Q1,Q2,Q4,SIGN,DR1)	03920
DO 644 L=1, 8	03930
Q3(K,L)=Q(J,K,L)	03940
644 CONTINUE	03950
GO TO 649	03960
645 CALL GINTER(DRO,INDX4,INDX1,DT,Q3,Q4,Q,SIGN,DR1,N1)	03970
GO TO 649	03980
646 CALL RINTER(Q,QJ,Q4,INDX4,INDX1,ZA,N1)	03990
GO TO 649	04000
647 DO 648 L=1, 8	04010
Q(J,K,L)=Q(IB,K,L)	04020
648 CONTINUE	04030
649 CONTINUE	04040
GO TO 3000	04050
700 IF(IM.EQ.2)GO TO 705	04060
K=N1	04070
R=DR1	04080
FB=-T1(J,N1,7)	04090
CALL BOAX1(DRO,INDX4,INDX1,DT,Q,Q1,Q2,SIGN,DR1)	04100
705 IF(IR.LE.2)GO TO 715	04110
DO 710 KR=IM, IR, 2	04120
K=N-KR+2	04130

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R=FLOAT(K-1)*DT+DRO                                04140
CALL FREE (DRO,INDX4,INDX1,DT,Q,Q1,Q2,SIGN,DRI)      04150
710 CONTINUE                                         04160
715 K=KW                                              04170
      DO 720 L=1,8                                    04180
      Q(J,K,L)=Q4(1,L)                               04190
720 CONTINUE                                         04200
3000 CONTINUE                                        04210
4000 CONTINUE                                        04220
1985 CONTINUE                                        04230
      DO 1975 J=1,INDX4                              04240
      DO 1975 L=1,8                                    04250
      DO 1970 K=1,IAL                                04260
1970 TOTAL(J,K,L)=Q(J,K,L)+T(J,K,L)                04270
      IF(I.NE.2*N)GO TO 1975                         04280
      TOTAL(J,1,L)=TOTAL(J,1,L)+TT(J,L)             04290
1975 CONTINUE                                        04300
      CALL RESULT(RNE,DRO,DRI,DT,I,FF,INDX4,INDX1,TOTAL,T1,IJ,N1) 04310
      DO 1980 J=1,INDX4                              04320
      DO 1980 L=1,8                                    04330
      DO 1980 K=1,IAL                                04340
      T2(J,K,L)=T1(J,K,L)                           04350
      T1(J,K,L)=T(J,K,L)                             04360
      Q2(J,K,L)=Q1(J,K,L)                            04370
      Q1(J,K,L)=Q(J,K,L)                             04380
1980 CONTINUE                                        04390
2000 CONTINUE                                        04400
      RETURN                                          04410
      END                                            04420
C                                                    04430
C                                                    04440
      SUBROUTINE AMAT(DRO,INDX4,INDX1,DT,T,T1,T2,A,SIGN,DRI) 04450
C                                                    04460
C*****04470
C PURPOSE: TO CALCULATE THE MATRIX AND VECTOR BASED ON MATRIX 2 *04480
C                                                    *04490
C COLUMNS 1 TO 8 OF A(II,JJ) REPRESENT THE MATRIX WHILE A(II,9) IS *04500
C THE COLUMN VECTOR SUCH THAT: *04510
C      A(1,9)= [A2]          A(5,9)= [A6]          *04520
C      A(2,9)= A9           A(6,9)= [A1]          *04530
C      A(3,9)= A10          A(7,9)= [A5]          *04540
C      A(4,9)= A12          A(8,9)= [A3]          *04550
C                                                    04560
C COMMON/ALL/I,TOW,R,Z,K,J,DT5,ALFA,BETA,BETA2,G2,BG,CBA,CABA,PIE5,NO4570
COMMON/PA/B1,B2,B3,B5,B6,B9                          04580
COMMON/PO/FF,FB,FR                                    04590
DIMENSION T(INDX4,INDX1,8),T1(INDX4,INDX1,8),T2(INDX4,INDX1,8) 04600
DIMENSION A(8,9)                                     04610
F=FF                                                  04620
IF(SIGN.EQ.-1.0)F=FR                                  04630
DO 600 II=1,8                                        04640
DO 600 JJ=1,9                                        04650

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	A(II, JJ)=0.0	04660
	600 CONTINUE	04670
C		04680
C	CALCULATING THE MATRIX	04690
C		04700
	A(1,1)=B2/R	04710
	A(1,2)=1.-2.*ALFA*B2/R	04720
	A(1,4)=B2/R	04730
	A(1,7)=1.-2.*B2/R	04740
C		04750
	ALFA9=B9*ALFA	04760
	ALFA9R=ALFA9/R	04770
	A(2,2)=-ALFA9R	04780
	A(2,3)=-B9	04790
	A(2,6)=-ALFA9	04800
	A(2,7)=1.	04810
C		04820
	A(3,2)=-ALFA9R	04830
	A(3,3)=-ALFA9	04840
	A(3,4)=1.	04850
	A(3,6)=-B9	04860
C		04870
	A(4,1)=1.	04880
	A(4,2)=-B9/R	04890
	A(4,3)=-ALFA9	04900
	A(4,6)=-ALFA9	04910
C		04920
	A(5,2)=-ALFA*B6/R	04930
	A(5,4)=1.0	04940
	A(5,5)=1.0	04950
	A(5,8)=-B6/R	04960
C		04970
	A(6,1)=-B1/R	04980
	A(6,2)=-1.-2.*ALFA*B1/R	04990
	A(6,4)=-B1/R	05000
	A(6,7)=1.+2.*B1/R	05010
C		05020
	A(7,2)=-ALFA*B5/R	05030
	A(7,4)=1.0+B5/(Z+DRO)	05040
	A(7,5)=-1.0-ALFA*B5/(Z+DRO)	05050
	A(7,7)=-B5/(Z+DRO)	05060
	A(7,8)=B5/R	05070
C		05080
	A(8,5)=-SIGN*BETA+BETA2*B3/R	05090
	A(8,8)=1.0+3.0*SIGN*BETA*B3/R	05100
C		05110
C	CALCULATING THE VECTOR	05120
C		05130
	ZIR=Z*Z-TOW*TOW+(R-DRO)*(R-DRO)	05140
	IF(SIGN.EQ.-1.0)ZIR=Z*Z-TOW*TOW+(2.0*DR1-DRO-R)**2	05150
	KNEG=K-1	05160
	KPOS=K+1	05170

JNEG=J-1	05180
JPOS=J+1	05190
C	05200
IF(KNEG .EQ. 0) GO TO 620	05210
A(6,9)=(T1(J,KNEG,1)+ALFA*T1(J,KNEG,2)-T1(J,KNEG,7))	05220
+*B1/(R-DT)+T1(J,KNEG,7)-T1(J,KNEG,2)	05230
IF(ZIR .GT. 0.0) GO TO 610	05240
A(6,9)=A(6,9)+(T1(J,KNEG,4)+ALFA*T1(J,KNEG,2)-	05250
+T1(J,KNEG,7))*B1/(R-DT)	05260
C	05270
610 CONTINUE	05280
SRZ=CABA*T1(J,KNEG,8)+CBA*T2(J,K,8)	05290
UZ=CABA*T1(J,KNEG,5)+CBA*T2(J,K,5)	05300
A(8,9)=SRZ*(1.0-3.0*SIGN*BETA*B3/(R-BG))+UZ*(-SIGN*BETA-BETA2*B3/	05310
+(R-BG))	05320
C	05330
620 CONTINUE	05340
A(1,9)=(T1(J,KPOS,7)+ALFA*T1(J,KPOS,2)-T1(J,KPOS,1))	05350
+*B2/(R+DT)+T1(J,KPOS,7)+T1(J,KPOS,2)	05360
IF(ZIR .GT. 0.0) GO TO 640	05370
A(1,9)=A(1,9)+B2*(T1(J,KPOS,7)+ALFA*T1(J,KPOS,2)-	05380
+T1(J,KPOS,4))/(R+DT)	05390
C	05400
640 CONTINUE	05410
IF(JNEG .EQ. 0) GO TO 650	05420
A(7,9)=T1(JNEG,K,4)-T1(JNEG,K,5)+B5*ALFA*T1(JNEG,K,2)/R	05430
++B5*(T1(JNEG,K,7)-T1(JNEG,K,4)+ALFA*T1(JNEG,K,5))/(DRO+Z-DT)	05440
+B5*T1(JNEG,K,8)/R	05450
C	05460
650 CONTINUE	05470
A(5,9)=T1(JPOS,K,4)+T1(JPOS,K,5)+B6*ALFA*T1(JPOS,K,2)/R	05480
++B6*T1(JPOS,K,8)/R	05490
C	05500
IF(SIGN .EQ. -1 .AND. K .EQ. 2*N-I+2) GO TO 670	05510
IF(K .EQ. I .AND. SIGN .EQ. 1) GO TO 660	05520
A(2,9)=T2(J,K,7)+B9*T2(J,K,3)+ALFA9*T2(J,K,2)/R+ALFA9*T2(J,K,6)	05530
A(3,9)=T2(J,K,4)+B9*T2(J,K,6)+ALFA9*T2(J,K,2)/R+ALFA9*T2(J,K,3)	05540
A(4,9)=T2(J,K,1)+B9*T2(J,K,2)/R+ALFA9*T2(J,K,3)+ALFA9*T2(J,K,6)	05550
RETURN	05560
C	05570
660 B=B2	05580
GO TO 680	05590
670 B=B1	05600
680 CON=F/R**1.5	05610
R2=R+SIGN*DT/2.0	05620
AXE=SIGN*F*B*(1.0-2.0*ALFA)/R2**1.5	05630
A(2,9)=CON*R+SIGN*0.5*B9*CON-SIGN*ALFA*B9*CON	05640
A(3,9)=ALFA*CON*F 0.5*SIGN*B9*ALFA*CON	05650
A(4,9)=ALFA*CON*R+0.5*SIGN*B9*ALFA*CON-CON*SIGN*B9	05660
IF(SIGN .EQ. 1) A(1,9)=AXE	05670
IF(SIGN .EQ. -1) A(6,9)=AXE	05680
RETURN	05690

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END 05700
C 05710
C*****05720
C SUBROUTINES RINT AND CINT ARE USED TO ADJUST MATRIX 2 FOR EACH 05730
C CATEGORY OF POINTS 05740
C*****05750
C 05760
C SUBROUTINE RINT(A,N,M,LA,LB) 05770
C 05780
C INTERCHANGE ROWS LA AND LB 05790
C 05800
C DIMENSION A(N,M) 05810
C DO 10 II=1,M 05820
C SAVE=A(LA,II) 05830
C A(LA,II)=A(LB,II) 05840
C 10 A(LB,II)=SAVE 05850
C RETURN 05860
C END 05870
C 05880
C SUBROUTINE CINT(A,N,M,LA,LB) 05890
C 05900
C INTERCHANGE COLUMNS LA AND LB 05910
C 05920
C DIMENSION A(N,M) 05930
C DO 10 II=1,N 05940
C SAVE=A(II,LA) 05950
C A(II,LA)=A(II,LB) 05960
C 10 A(II,LB)=SAVE 05970
C RETURN 05980
C END 05990
C 06000
C 06010
C SUBROUTINE WAVE(DRO,INDX4,INDX1,T,SIGN,F) 06020
C 06030
C*****06040
C INCIDENT LEADING WAVE POINTS — G— 06050
C*****06060
C 06070
C COMMON/ALL/I,TOW,R,Z,K,J,DT5,ALFA,BETA,BETA2,G2,BG,CBA,CABA,PIE5,NO6080
C COMMON/PO/PP,FB,FR 06090
C DIMENSION T(INDX4,INDX1,8) 06100
C T(J,K,7)=F/R**0.5 06110
C T(J,K,1)=ALFA*T(J,K,7) 06120
C T(J,K,2)=-SIGN*T(J,K,7) 06130
C T(J,K,3)=F*SIGN*0.5/R**1.5 06140
C T(J,K,4)=ALFA*T(J,K,7) 06150
C RETURN 06160
C END 06170
C 06180
C 06190
C SUBROUTINE BOAX1(DRO,INDX4,INDX1,DT,T,T1,T2,SIGN,DR1) 06200
C 06210

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6 AB(II,JJ)=A(II,JJ)	06740
7 COE(II)=A(II,9)	06750
T(J,K,4)=0.0	06760
T(J,K,8)=0.0	06770
CALL MATINV(AB,COE,6,6,1,DET,KS)	06780
IF(KS.EQ.1) GO TO 4	06790
T(J,K,1)=COE(1)	06800
T(J,K,2)=COE(2)	06810
T(J,K,3)=COE(3)	06820
T(J,K,7)=COE(4)	06830
T(J,K,5)=COE(5)	06840
T(J,K,6)=COE(6)	06850
RETURN	06860
4 WRITE (1,5)	06870
5 FORMAT(*SINGULAR AT REGULAR FREE SURFACE POINT — C — *)	06880
STOP	06890
END	06900
C	06910
C	06920
SUBROUTINE LEAD(DRO,INDX4,INDX1,DT,T,T1,T2,SIGN,DR1)	06930
C	06940
C*****	06950
C LEADING LOADED BOUNDARY POINT (TWO DIMENSIONAL) — J —	06960
C*****	06970
C	06980
COMMON/ALL/I,TOW,R,Z,K,J,DT5,ALFA,BETA,BETA2,G2,BG,CBA,CABA,PIE5,NO	06990
COMMON/AA/B1,B2,B3,B5,B6,B9	07000
COMMON/FO/FF,FB,FR	07010
DIMENSION T(INDX4,INDX1,8),T1(INDX4,INDX1,8),T2(INDX4,INDX1,8)	07020
DIMENSION A(8,9),AB(5,5),COE(5)	07030
IF(K.EQ.1) B9=DT5	07040
CALL AMAT(DRO,INDX4,INDX1,DT,T,T1,T2,A,SIGN,DR1)	07050
CALL CINT(A,8,9,2,6)	07060
CALL RINT(A,8,9,7,1)	07070
DO 7 II=1,5	07080
DO 6 JJ=1,5	07090
6 AB(II,JJ)=A(II,JJ)	07100
7 COE(II)=A(II,9)	07110
T(J,K,2)=T(I+2,1,2)	07120
T(J,K,7)=T(I+2,1,7)	07130
T(J,K,8)=0.0	07140
COE(1)=COE(1)+B5*T(J,K,7)/(Z+DRO)+B5*ALFA*T(J,K,2)/R	07150
COE(2)=COE(2)-T(J,K,7)+B9*ALFA*T(J,K,2)/R	07160
COE(3)=COE(3)+B9*ALFA*T(J,K,2)/R	07170
COE(4)=COE(4)+B9*T(J,K,2)/R	07180
COE(5)=COE(5)+B6*ALFA*T(J,K,2)/R	07190
CALL MATINV(AB,COE,5,5,1,DET,KS)	07200
IF(KS.EQ.1) GO TO 3	07210
T(J,K,1)=COE(1)	07220
T(J,K,3)=COE(3)	07230
T(J,K,4)=COE(4)	07240
T(J,K,5)=COE(5)	07250

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T(J,K,6)=COE(2)                                07260
B9=DT                                            07270
RETURN                                          07280
3 WRITE (1,5)                                    07290
5 FORMAT(*SINGULAR AT LEADING LOADED BOUNDARY 2-D POINT -J- *) 07300
STOP                                            07310
END                                             07320
C
C
SUBROUTINE LOADED(DRO,INDX4,INDX1,DT,T,T1,T2,SIGN,DRI) 07330
C
C
SUBROUTINE LOADED(DRO,INDX4,INDX1,DT,T,T1,T2,SIGN,DRI) 07340
C
C
C*****07350
C REGULAR LOADED BOUNDARY POINT (ONE DIMENSIONAL) — F — 07380
C*****07390
C
COMMON/ALL/I,TOW,R,Z,K,J,DT5,ALFA,BETA,BETA2,G2,BG,CBA,CABA,PIE5,N07410
COMMON/AA/B1,B2,B3,B5,B6,B9                    07420
COMMON/FO/FF,FB,FR                             07430
DIMENSION T(INDX4,INDX1,8),T1(INDX4,INDX1,8),T2(INDX4,INDX1,8) 07440
DIMENSION A(8,9),AB(4,4),COE(4)              07450
F=FF                                            07460
IF(SIGN.EQ.-1.0)F=FB                          07470
IF(K.EQ.I .AND. SIGN .EQ. 1) GO TO 2          07480
IF(SIGN .EQ. -1 .AND. K .EQ. 2*N-I+2) GO TO 8 07490
GO TO 5                                        07500
2 B2=B1/2.                                     07510
B9=B1                                          07520
GO TO 5                                        07530
8 B9=B1                                        07540
B1=B1/2.0                                     07550
5 CALL AMAT(DRO,INDX4,INDX1,DT,T,T1,T2,A,SIGN,DRI) 07560
IF(SIGN.EQ.-1.0)CALL RINT(A,8,9,1,6)         07570
DO 7 II=1,4                                   07580
DO 6 JJ=1,4                                   07590
6 AB(II,JJ)=A(II,JJ)                         07600
7 COE(II)=A(II,9)                            07610
T(J,K,5)=0.0                                  07620
T(J,K,6)=0.0                                  07630
T(J,K,7)=F                                    07640
T(J,K,8)=0.0                                  07650
AB(1,2)=AB(1,2)+ALFA*B2/R                   07660
AB(1,4)=0.                                    07670
COE(1)=COE(1)-(1.0-2.0*B2/R)*T(J,K,7)       07680
IF(SIGN.EQ.-1.0)COE(1)=A(1,9)-(1.0+2.0*B1/R)*T(J,K,7) 07690
COE(2)=COE(2)-T(J,K,7)                      07700
CALL MATINV(AB,COE,4,4,1,DET,KS)            07710
IF(KS.EQ.1) GO TO 3.                         07720
DO 9 L=1,4                                   07730
9 T(J,K,L)=COE(L)                            07740
B1=DT5                                        07750
B2=B1                                          07760
B9=2.*B1                                      07770

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RETURN 07780
3 WRITE (1,4) 07790
4 FORMAT(*SINGULAR AT REGULAR LOADED BOUNDARY 1-D POINT - F -*) 07800
STOP 07810
END 07820
C 07830
C 07840
SUBROUTINE LOAD1(DRO,INDX4,INDX1,DT,T,T1,T2,SIGN,DR1,FRZ) 07850
C 07860
C*****07870
C REGULAR LOADED BOUNDARY POINT (TWO DIMENSIONAL) -- B -- 07880
C*****07890
C 07900
COMMON/ALL/I,TOW,R,Z,K,J,DT5,ALFA,BETA,BETA2,G2,BG,CBA,CABA,PIE5,NO7910
COMMON/AA/B1,B2,B3,B5,B6,B9 07920
COMMON/FO/FF,FB,FR 07930
DIMENSION T(INDX4,INDX1,8),T1(INDX4,INDX1,8),T2(INDX4,INDX1,8) 07940
DIMENSION A(8,9),AB(6,6),COE(6) 07950
F=FF 07960
IF(SIGN.EQ.-1.0)F=FB 07970
CALL AMAT(DRO,INDX4,INDX1,DT,T,T1,T2,A,SIGN,DR1) 07980
IF(SIGN.EQ.1.0)CALL RINT(A,8,9,6,7) 07990
IF(SIGN.EQ.-1.0)CALL RINT(A,8,9,1,7) 08000
DO 7 II=1,6 08010
DO 6 JJ=1,6 08020
6 AB(II,JJ)=A(II,JJ) 08030
7 COE(II)=A(II,9) 08040
T(J,K,7)=F 08050
T(J,K,8)=0.5*(1.0-SIGN)*FRZ 08060
COE(1)=COE(1)-(1.0-2.0*B2/R)*T(J,K,7) 08070
IF(SIGN.EQ.-1.0)COE(1)=A(1,9)+B5/(DRO+Z)*T(J,K,7) 08080
+ -B5/R*T(J,K,8) 08090
COE(2)=COE(2)-T(J,K,7) 08100
COE(6)=COE(6)+B5*T(J,K,7)/(Z+DRO) 08110
IF(SIGN.EQ.-1.0)COE(6)=A(6,9)-(1.0+2.0*B1)*T(J,K,7) 08120
IF(SIGN.EQ.-1.0)COE(5)=COE(5)+B6/R*T(J,K,8) 08130
CALL MATINV(AB,COE,6,6,1,DET,KS) 08140
IF(KS.EQ.1) GO TO 3 08150
DO 9 L=1,6 08160
9 T(J,K,L)=COE(L) 08170
RETURN 08180
3 WRITE (1,4) 08190
4 FORMAT(*SINGULAR AT REGULAR LOADED BOUNDARY 2-D POINT - B - *) 08200
STOP 08210
END 08220
C 08230
C 08240
SUBROUTINE GENER(DRO,INDX4,INDX1,DT,T,T1,T2,SIGN,DR1) 08250
C 08260
C*****08270
C REGULAR INNER ONE DIMENSIONAL POINTS -- E -- 08280
C*****08290

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C
COMMON/ALL/I,TOW,R,Z,K,J,DT5,ALFA,BETA,BETA2,G2,BG,CBA,CABA,PIE5,N08310 08300
COMMON/AA/B1,B2,B3,B5,B6,B9 08320
DIMENSION T(INDX4,INDX1,8),T1(INDX4,INDX1,8),T2(INDX4,INDX1,8) 08330
DIMENSION A(8,9),AB(5,5),COE(5) 08340
IF(SIGN.EQ.1.0.AND.K.EQ.I)GO TO 2 08350
IF(SIGN.EQ.-1.0.AND.K.EQ.2*N-I+2)GO TO 8 08360
GO TO 5 08370
2 B2=B1/2.0 08380
B9=B1 08390
GO TO 5 08400
8 B9=B1 08410
B1=B1/2.0 08420
5 CALL AMAT(DRO,INDX4,INDX1,DT,T,T1,T2,A,SIGN,DR1) 08430
CALL CINT(A,8,9,5,7) 08440
CALL RINT(A,8,9,5,6) 08450
DO 7 II=1,5 08460
DO 6 JJ=1,5 08470
6 AB(JJ,II)=A(II,JJ) 08480
7 COE(II)=A(II,9) 08490
AB(1,2)=AB(1,2)+ALFA*B2/R 08500
AB(1,4)=0. 08510
AB(1,5)=AB(1,5)+B2/R 08520
AB(5,2)=AB(5,2)+ALFA*B1/R 08530
AB(5,4)=0. 08540
AB(5,5)=AB(5,5)-B1/R 08550
CALL MATINV(AB,COE,5,5,1,DET,KS) 08560
IF(KS.EQ.1) GO TO 3 08570
DO 9 L=1,4 08580
9 T(J,K,L)=COE(L) 08590
T(J,K,5)=0.0 08600
T(J,K,6)=0.0 08610
T(J,K,7)=COE(5) 08620
T(J,K,8)=0.0 08630
B1=DT5 08640
B2=B1 08650
B9=2.0*B1 08660
RETURN 08670
3 WRITE (1,4) 08680
4 FORMAT(*SINGULAR AT REGULAR INNER ONE DIMENSIONAL POINT - E -*) 08690
STOP 08700
END 08710
C 08720
C 08730
SUBROUTINE GENER1(DRO,INDX4,INDX1,DT,T,T1,T2,SIGN,DR1) 08740
C 08750
C*****08760
C REGULAR INNER TWO DIMENSIONAL POINTS -- A -- 08770
C SOLVE MATRIX 2 WITHOUT ALTERATION 08780
C*****08790
C 08800
COMMON/ALL/I,TOW,R,Z,K,J,DT5,ALFA,BETA,BETA2,G2,BG,CBA,CABA,PIE5,N08810

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      TA=ASIN( DT*SQRT(FLOAT( (I+N-2*(M-2))*(I-N) ) )/RA )      10380
      IF(I.EQ.N1)GO TO 8      10390
      TB=ASIN( DT*SQRT(FLOAT( (I-1+N-2*(M-2))*(I-1-N) ) )/RB )      10400
8     CONTINUE      10410
      R1=R+2.0*B1      10420
      T2=ATAN( Z/(R1 - (M-2)*DT - DRO) )      10430
      CALL ENV1(R1,Z,T2,TA,TB,B1,E,QJ,Q4,DT,M,INDX1,N1)      10440
      COE(6)=E(7)*(1.-B1/R1)-E(2)*(1.-ALFA*B1/R1)+B1*E(1)/R1      10450
      + B1*(ALFA*E(2) - E(7) + E(4) )/R1      10460
      COE(6)=COE(6)-(1.+2.*B1/R)*QJ(1,M,7)      10470
      Z6=Z+2.0*B6      10480
      T6=ATAN( Z6/(R-(M-2)*DT-DRO) )      10490
      CALL ENV1(R,Z6,T6,TA,TB,B6,E,QJ,Q4,DT,M,INDX1,N1)      10500
      COE(5)=E(4)+E(5)+B6*ALFA*E(2)/R+B6*E(8)/R      10510
      COE(5)=COE(5)+B6/R*QJ(1,M,8)      10520
      T9=ATAN( Z/(R-(M-2)*DT-DRO) )      10530
      CALL ENV1(R,Z,T9,TA,TB,B9,E,QJ,Q4,DT,M,INDX1,N1)      10540
      COE(2)=E(7)+B9*( E(3)+ALFA*(E(2)/R+E(6) ) )      10550
      COE(2)=COE(2)-QJ(1,M,7)      10560
      COE(3)=E(4)+B9*( E(6)+ALFA*(E(2)/R+E(3) ) )      10570
      COE(4)=E(1)+B9*( E(2)/R+ALFA*(E(3)+E(6) ) )      10580
      IF(B5.EQ.DT5)GO TO 9      10590
      Z5=Z-2.0*B5      10600
      T5=ATAN( Z5/(R-(M-2)*DT-DRO) )      10610
      CALL ENV1(R,Z5,T5,TA,TB,B5,E,QJ,Q4,DT,M,INDX1,N1)      10620
      GO TO 11      10630
9     DO 10 L=1,8      10640
10    E(L)=(ZN-FLOAT(NZ))*Q1(NZ+1,N1,L)+(FLOAT(NZ+1)-ZN)*Q1(NZ,N1,L)      10650
11    CONTINUE      10660
      COE(1)=E(4)-E(5)+B5*ALFA*E(2)/R-B5*E(8)/R      10670
      COE(1)=COE(1)+B5/(Z+DRO)*QJ(1,M,7)      10680
      + -B5/R*QJ(1,M,8)      10690
      IF(M.EQ.N1)COE(1)=A(1,9)+B5/(Z+DRO)*QJ(1,M,7)      10700
      + -B5/R*QJ(1,M,8)      10710
      CALL MATINV(AB,COE,6,6,1,DET,KS)      10720
      DO 209 L=1,6      10730
      QJ(1,M,L)=COE(L)      10740
209  CONTINUE      10750
200  CONTINUE      10760
      B1=DT5      10770
      B5=DT5      10780
      B6=DT5      10790
      B9=DT      10800
      RETURN      10810
      END      10820
C      10830
C      10840
      SUBROUTINE ENV1(RL,ZL,TETA,TA,TB,BL,E,QJ,Q4,DT,M,INDX1,N1)      10850
C*****      10860
C      CALCULATION OF QUANTITIES AT TERMINAL POINTS BICHARACTERISTIC      10870
C      CURVES OF POINTS -- M--      10880
C*****      10890

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COMMON/ALL/I,TOW,R,Z,K,J,DT5,ALFA,BETA,BETA2,G2,BG,CBA,CABA,PIE5,N10900
DIMENSION QJ(2,INDX1,8),Q4(3,8),E(8) 10910
TTA=TETA/TA 10920
TTAL=1.0-TTA 10930
IF(I.EQ.N1)GO TO 8 10940
TTB=TETA/TB 10950
GO TO 9 10960
8 TTB=0.0 10970
9 CONTINUE 10980
TTB1=1.0-TTB 10990
C2=2.0*BL/DT 11000
C1=1.0-C2 11010
DO 10 L=1,8 11020
E(L)=C1*(TTB*QJ(2,M-1,L)+TTB1*Q4(2,L) ) 11030
+ C2*(TTA*QJ(1,M-1,L)+TTAL*Q4(1,L) ) 11040
10 CONTINUE 11050
RETURN 11060
END 11070

C 11080
C 11090
SUBROUTINE RINTER(Q,QJ,Q4,INDX4,INDX1,ZA,N1) 11100
C*****11110
C INTERMEDIATE TWO-DIMENSIONAL POINTS — N — 11120
C*****11130
COMMON/ALL/I,TOW,R,Z,K,J,DT5,ALFA,BETA,BETA2,G2,BG,CBA,CABA,PIE5,N11140
DIMENSION ZA(N1) 11150
DIMENSION QJ(2,INDX1,8),Q4(3,8),Q(INDX4,INDX1,8) 11160
Y=I-2*N-1+K 11170
X=0.5*Y+0.5*FLOAT((J-1)*(J-1))/Y 11180
A=X-FLOAT(I-N) 11190
AJAX=X*X-A*A 11200
AJ=SQRT(AJAX) 11210
AJM=(FLOAT(I*I-N*N)-AJ*AJ)/FLOAT(2*(I-N))+1.0 11220
TJK=ASIN(FLOAT(J-1)/X) 11230
TAJ=ASIN(AJ/X) 11240
TETA=TJK/TAJ 11250
M=AJM 11260
C1=(ZA(M)-AJ)/(ZA(M)-ZA(M+1) ) 11270
DO 10 L=1,8 11280
10 Q(J,K,L)=TETA*(C1*QJ(1,M+1,L)+(1.-C1)*QJ(1,M,L))+(1.-TETA)*Q4(1,L) 11290
RETURN 11300
END 11310

C 11320
SUBROUTINE MATINV(A,B,N,N1,MSUB,DET,KS) 11330
DIMENSION A(1),B(1) 11340
TOL=0.0 11350
KS=0 11360
JJ=-N 11370
DO 65 J=1,N 11380
JY=J+1 11390
JJ=JJ+N+1 11400
BIGA=0.0 11410

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AD-A118 600

MASSACHUSETTS UNIV AMHERST DEPT OF CIVIL ENGINEERING F/G 20/14
RESPONSE OF THICK CYLINDRICAL SHELLS TO TRANSIENT INTERNAL LOAD--ETC(U)
AUG 82 T HAN-URA W A NASH DAAG29-77-G-0095

UNCLASSIFIED

ARO-14700.2-EG

NL

2nd
3
of 10
10/10/82



IT=JJ-J	11420
DO 30 I=J,N	11430
IJ=IT+I	11440
IF(ABS(BIGA)-ABS(A(IJ)))20,30,30	11450
20 BIGA=A(IJ)	11460
IMAX=I	11470
30 CONTINUE	11480
IF(ABS(BIGA)-TOL)35,35,40	11490
35 KS=1	11500
RETURN	11510
40 I1=J+N*(J-2)	11520
IT=IMAX-J	11530
DO 50 K=J,N	11540
I1=I1+N	11550
I2=I1+IT	11560
SAVE=A(I1)	11570
A(I1)=A(I2)	11580
A(I2)=SAVE	11590
50 A(I1)=A(I1)/BIGA	11600
SAVE=B(IMAX)	11610
B(IMAX)=B(J)	11620
B(J)=SAVE/BIGA	11630
IF(J=N)55,70,55	11640
55 IQS=N*(J-1)	11650
DO 65 IX=JY,N	11660
IKJ=IQS+IX	11670
IT=J-IX	11680
DO 60 JX=JY,N	11690
IKJX=N*(JX-1)+IX	11700
JJX=IKJX+IT	11710
60 A(IKJX)=A(IKJX)-(A(IKJ)*A(JJX))	11720
65 B(IK)=B(IK)-(B(J)*A(IKJ))	11730
70 NY=N-1	11740
IT=N*N	11750
DO 80 J=1,NY	11760
IA=IT-J	11770
IB=N-J	11780
IC=N	11790
DO 80 K=1,J	11800
B(IB)=B(IB)-A(IA)*B(IC)	11810
IA=IA-N	11820
80 IC=IC-1	11830
RETURN	11840
END	11850
C	11860
C	11870
SUBROUTINE RESULT(RME,DRO,DRL,DT,I,FP,INDX4,INDX1,T,T1,IJ,N1)	11880
COMMON/RE/IFROM,ITILL,IWRITE,IJ1,IPRINT,NPRINT	11890
COMMON/PRINT/JPRINT(9),KPRINT(9)	11900
DIMENSION T(INDX4,INDX1,8),T1(INDX4,INDX1,8)	11910
IF(IJ.EQ.2)GO TO 90	11920
IJ1=1	11930

```

      IF(IPRINT .EQ. 1) GO TO 75
75 CALL OUTP(RNE,DRO,DRI,DT,I,FF,INDX4,INDX1,T1)
      IJ=2
90 IF(I .GT. ITILL) RETURN
      GO TO(100,200,300) IPRINT
100 IF(I .NE. IFROM ) RETURN
      IFROM=IFROM+IWRITE
      CALL OUTP1(DRO,DT,I,INDX4,INDX1,T,M1)
      RETURN
200 CALL OUTP2(DRO,DT,I,INDX4,INDX1,T,T1)
      RETURN
300 IF(I .NE. IFROM) GO TO 310
      IFROM=IFROM+IWRITE
      CALL OUTP1(DRO,DT,I,INDX4,INDX1,T,M1)
310 CALL OUTP2(DRO,DT,I,INDX4,INDX1,T,T1)
      RETURN
      END
C
C
      SUBROUTINE OUTP(RNE,DRO,DRI,DT,I,FF,INDX4,INDX1,T1)
C
C*****
C PRINTS THE VALUES OF THE INPUT CONSTANTS AND THE INITIAL *
C CONDITIONS *
C*****
C
      DIMENSION T1(INDX4,INDX1,8)
      INDEX=INDX1-1
      CALL DATE(MDATE)
      WRITE(1,9) MDATE
      WRITE (1,10) RNE,DRO,DRI,FF,DT,INDEX
9 FORMAT(A12/* TRANSIENT RESPONSE OF SEMI-INFINITELY LONG*
+* TUBE SUBJECT TO ABRUPTLY APPLIED LOAD(CASE 1)*//
+* DURATION TIME OF LOAD = PERMANENT*/* WIDTH OF LOAD*
+* = SEMI-INFINITE*//)
10 FORMAT(* THE INPUT CONSTANTS */21(1E-)//*POISSON'S RATIO =*,
+*F5.3,/*INNER RADIUS =*,F4.2/*OUTER RADIUS =*,F4.2/*WGM=*
+*DIMENSIONAL SIRR =*,F4.2/*STEP SIZE FOR INTEGRATION =*,F6.4/
+*NUMBER OF TIME STEPS =*,I3//* THE INITIAL CONDITIONS *
+*(TIME T=0.0)*/38(1E-)//)
      WRITE (1,20) (T1(1,1,L),T1(2,1,L),L=1,8)
20 FORMAT(*POINT - R=1.0 Z=0.0*,27X,*POINTS - R=1.0 Z=0.0*/
+20(1E-),27X,21(1E-)//2(* SITT =*,F7.4,32X)/2(* UR =*,F7.4,32X)/
+2(*DURDR =*,F7.4,32X)/2(* SIZR =*,F7.4,32X)/2(* UR =*,F7.4,32X)
+2(*DUZDR =*,F7.4,32X)/2(* SIRR =*,F7.4,32X)/2(* SIZR =*,F7.4,
+32X)/72(1E-)//)
      RETURN
      END
C
C
      SUBROUTINE OUTP1(DRO,DT,I,INDX4,INDX1,T,M1)
C

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C*****12660
C PRINTS VALUES OF VARIABLES L AT SPECIFIED TIME FOR POINTS (J,K) 12670
C FOR J=1, INDX+4 AND K=1, INDX+1 12680
C*****12680
C 12690
C DIMENSION T(INDX4, INDX1, 8) 12710
30 TOW=FLOAT(I)*DT 12720
WRITE (1,40) TOW 12730
40 FORMAT(// ' AT TIME T= ', F7.4/19(1H-)//5X, 'Z', 8X, 'R', 7X, 'SIR', 5X, 12740
+ 'SITT', 5X, 'SIZZ', 5X, 'SIRZ', 6X, 'UR', 7X, 'UZ'/72(1H-)) 12750
IA=I+1 12760
IC=I+3 12770
KK=1 12780
IF(MOD(I,2) .EQ. 0) KK=2 12790
DO 60 J=1, IC 12800
KK=3-KK 12810
Z=DT*FLOAT(J-1) 12820
WRITE (1,50) 12830
50 FORMAT(' ') 12840
DO 60 K=KK, N1, 2 12850
R=FLOAT(K-1)*DT+DRO 12860
WRITE (1,70) Z, R, T(J,K,7), T(J,K,1), T(J,K,4), T(J,K,8), 12870
+ T(J,K,2), T(J,K,5) 12880
60 CONTINUE 12890
70 FORMAT(8(2X, F7.4)) 12900
WRITE (1,80) 12910
80 FORMAT(72(1H-)) 12920
RETURN 12930
END 12940
C 12950
C SUBROUTINE OUTP2(DRO, DT, I, INDX4, INDX1, T, T1) 12960
C 12970
C*****12980
C PRINTS VALUES OF VARIABLES L FOR SPECIFIED POINTS (JPRINT, KPRINT) 12990
C*****13000
C 13010
C COMMON/RE/IFROM, ITILL, IWRITE, IJ1, IPRINT, NPRINT 13020
COMMON/PRINT/JPRINT(9), KPRINT(9) 13030
DIMENSION T(INDX4, INDX1, 8), T1(INDX4, INDX1, 8) 13040
DIMENSION R(9), Z(9), TOW(51), VAR(9, 51, 8) 13050
IF(IJ1 .NE. 1) GO TO 100 13060
IJ1=2 13070
DO 10 JJ=1, NPRINT 13080
R(JJ)=FLOAT(KPRINT(JJ))*DT-DT+DRO 13090
Z(JJ)=FLOAT(JPRINT(JJ))*DT-DT 13100
10 CONTINUE 13110
DO 20 L=1, 8 13120
DO 20 LL=1, NPRINT 13130
J10=JPRINT(LL) 13140
K10=KPRINT(LL) 13150
TOW(L)=0.0 13160
VAR(LL, 1, L)=T1(J10, K10, L) 13170

```

20	CONTINUE	12980
100	IF(MOD(I,2) .EQ. 1) RETURN	12990
	IP=I/2+1	13000
	TOW(IP)=FLOAT(I)*DT	13010
	DO 120 JJ=1,NPRINT	13020
	J10=JPRINT(JJ)	13030
	K10=KPRINT(JJ)	13040
	DO 110 L=1,8	13050
	VAR(JJ,IP,L)=T(J10,K10,L)	13060
110	CONTINUE	13070
120	CONTINUE	13080
300	IF(I .LT. ITILL) RETURN	13090
	DO 340 JJ=1,NPRINT	13100
	WRITE (2,310) JJ,R(JJ),Z(JJ)	13110
C 310	FORMAT(///5X,I2,5X*THE POINT: R =*,F7.4,2X*Z =*,F7.4/3X,45(1E-)	13120
C	+ //7X*TOW*,6X*SIRR*,6X*SITT*,6X*SIZZ*,6X*SIRZ*,7X*UR*,6X*UZ*/	13130
C	+72(1E-))	13140
310	FORMAT(I2,2F7.4)	13150
	DO 330 KK=1,IP	13160
	WRITE (2,320) TOW(KK),VAR(JJ,KK,7),VAR(JJ,KK,1),VAR(JJ,KK,4),	13170
	+VAR(JJ,KK,8),VAR(JJ,KK,2),VAR(JJ,KK,5)	13180
320	FORMAT(7(3X,F7.4))	13190
330	CONTINUE	13200
C	WRITE (2,335)	13210
335	FORMAT(72(1E-)/)	13220
340	CONTINUE	13230
	RETURN	13240
	END	13250

APPENDIX C

PROGRAM LISTING OF TRES2

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PROGRAM TRES2(OUTPUT,TAPES,TAPES)                                00010
C *****00020
C CASE 2: TO OBTAIN THE TRANSIENT RESPONSE OF AN INFINITELY LONG, 00030
C THICK CYLINDRICAL SHELL TO AN EMBEDDED CYLINDRICAL LOAD        00040
C BY TWO-DIMENSIONAL METHOD OF CHARACTERISTICS. THE LOAD          00050
C WHICH IS APPLIED SUDDENLY AT T=0, IS UNIFORM A CROSS THE      00060
C LENGTH OF THE SHELL FROM Z=0 TO Z-INFINITY.                    00070
C                                                                    00080
C -----00090
C COORDINATE SYSTEM FOR THE ARRAYS USED IN THE PROGRAM:          00100
C                                                                    00110
C J - COORDINATE FOR Z-AXIS. J=1 AT Z=0.0.                       00120
C K - COORDINATE FOR R-AXIS. K=1 AT R=DRO.                        00130
C L - COORDINATE FOR VARIABLES. 1 - SITT      5 - UR             00140
C                                     2 - UR      6 - DURDR        00150
C                                     3 - DURDR      7 - SIRR        00160
C                                     4 - SIZE      8 - SIRR        00170
C NT - TIME COORDINATE. 1 - TOW, 2 - TOW-T, 3 - TOW-2*DT        00180
C                                                                    00190
C -----00200
C INPUT DATA                                                    00210
C                                                                    00220
C N --- NUMBER OF DIVISIONS ACROSS THICKNESS OF CYLINDER        00230
C INDEX --- TOTAL NUMBER OF INTEGRATION(MUST BE 2*N)(MAXIMUM 90) 00240
C DRO --- INTERNAL RADIUS                                         00250
C DRI --- EXTERNAL RADIUS                                         00260
C RNE --- POISSON'S RATIO                                         00280
C FP --- DIMENSIONLESS LOADING INTENSITY                         00290
C                                                                    00300
C THE FORM OF OUTPUT IS SELECTED BY SPECIFYING IPRINT AND INPUT 00310
C THE INFORMATION REQUIRED FOR THE OUTPUT                          00320
C                                                                    00330
C IPRINT          FORM OF OUTPUT                                00340
C -----          -----
C 1          PRINT FOR A SPECIFIED TIME                         00350
C 2          PRINT FOR A SPECIFIED POINT                       00360
C 3          PRINT FOR BOTH SPECIFIED TIME AND POINTS         00370
C -----          -----
C IFROM --- STARTING TIME FOR PRINTING                          00400
C ITILL --- TIME FOR TERMINATION OF PRINTING                   00410
C IWRITE --- TIME INTERVAL OF PRINTING                         00420
C NPRINT --- NUMBER OF POINTS SPECIFIED (MAXIMUM 9)           00430
C JPRINT --- J - COORDINATE OF SPECIFIED POINTS               00440
C KPRINT --- K - COORDINATE OF SPECIFIED POINTS               00450
C                                                                    00470
C *****00480
C COMSON/FO/FP,FB,FR,FM                                         00490

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COMMON/ALL/I,TOW,R,Z,K,J,DTS,ALFA,BETA,BETA2,Q2,BG,CBA,CABA,PIES,N00600
COMMON/WE/IPFROM,ITILL,IWRITE,IJ1,IPRINT,NPRINT          00610
COMMON/PRINT/JPRINT(9),KPRINT(9)                          00620
DIMENSION T3(31,8),TT(34,8)                                00630
DIMENSION Q3(31,8),QJ(2,31,8)                              00640
DIMENSION ZA(31)                                            00650
COMMON/TH/T(34,31,8),T1(34,31,8),T2(34,31,8),TOTAL(34,31,8) 00660
COMMON/QM/Q(34,31,8),Q1(34,31,8),Q2(34,31,8)              00670
C                                                            00680
C INPUT TO THE PROGRAM                                     00690
C                                                            00700
DATA INDEX,N,DRO,DRI,RNE,FF/30,15,1.,1.30,.15,1.0/        00710
DATA(JPRINT(N),N=1,2)/2,2/                                  00720
DATA(KPRINT(N),N=1,2)/1,2/                                  00730
DATA IPRINT,IPFROM,ITILL,IWRITE,NPRINT/1,1,30,1,2/        00740
N1=N+1                                                       00750
INDX1=INDEX+1                                                00760
INDX4=INDEX+4                                                00770
CALL GALIM(RNE,DRO,INDEX,INDX1,INDX4,T3,TT,                00780
+DRI,Q3,QJ,ZA,N1)                                           00790
STOP                                                         00800
END                                                           00810
C                                                            00820
C SUBROUTINE GALIM(RNE,DRO,INDEX,INDX1,INDX4,T3,TT,        00830
+DRI,Q3,QJ,ZA,N1)                                           00840
C                                                            00850
C *****                                                    00860
C PURPOSE: TO FIX THE SCHEME OF INTEGRATION                * 00870
C *****                                                    00880
C                                                            00890
COMMON/ALL/I,TOW,R,Z,K,J,DTS,ALFA,BETA,BETA2,Q2,BG,CBA,CABA,PIES,N00810
COMMON/PRINT/JPRINT(9),KPRINT(9)                          00820
COMMON/WE/IPFROM,ITILL,IWRITE,IJ1,IPRINT,NPRINT          00830
COMMON/FO/FF,FB,FR,FM                                       00840
COMMON/AA/B1,B2,B3,B5,B6,B9                                 00850
COMMON/LINK/BD(8)                                           00860
DIMENSION ZA(N1)                                            00870
COMMON/TH/T(34,31,8),T1(34,31,8),T2(34,31,8),            00880
+TOTAL(34,31,8)                                             00890
COMMON/QM/Q(34,31,8),Q1(34,31,8),Q2(34,31,8)             00900
DIMENSION T3(INDX1,8),T4(3,8),TT(INDX4,8)                00910
DIMENSION Q3(INDX1,8),Q4(3,8),QJ(2,INDX1,8)              00920
C                                                            00930
C CALCULATION OF THE CONSTANTS                              00940
C                                                            00950
FF=FF                                                       00960
FB=0.                                                         00970
DT=(DRI-DRO)/FLOAT(N)                                       00980
DTS=DT/2.0                                                  00990
ALFA=RNE/(1.0-RNE)                                          01000
BETA2=(1.0-ALFA)/2.0                                       01010

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BETA=SQRT(BETA2)	01020
G2=DT/(1.0+BETA)	01030
BG=2.0*BETA*G2	01040
CBA=(1.0-BETA)/(1.0+BETA)	01050
CABA=2.0*BETA/(1.0+BETA)	01060
B1=DT5	01070
B2=DT5	01080
B3=G2	01090
B5=DT5	01100
B6=DT5	01110
B9=DT5	01120
IJ=1	01130
PIES=2.0*ATAN(1.0)	01140
C	01150
C	01160
C	01170
DO 50 L=1,8	01180
ED(L)=0.	01190
DO 49 NT=1,3	01200
T4(NT,L)=0.0	01210
Q4(NT,L)=0.0	01220
49 CONTINUE	01230
DO 50 K=1,INDX1	01240
T3(K,L)=0.0	01250
Q3(K,L)=0.0	01260
QJ(1,K,L)=0.0	01270
QJ(2,K,L)=0.0	01280
DO 50 J=1,INDX4	01290
T(J,K,L)=0.0	01300
T1(J,K,L)=0.0	01310
T2(J,K,L)=0.0	01320
TOTAL(J,K,L)=0.0	01330
Q(J,K,L)=0.0	01340
Q1(J,K,L)=0.0	01350
Q2(J,K,L)=0.0	01360
50 CONTINUE	01370
C	01380
C	01390
C	01400
INITIAL CONDITIONS	
F1=ALFA	01410
F2=1.0	01420
T1(1,1,7)=FF/DRO**P2	01430
T1(1,1,1)=ALFA*T1(1,1,7)	01440
T1(1,1,2)=-T1(1,1,7)	01450
T1(1,1,3)=F2*FF/DRO**(F2+1.)	01460
T1(1,1,4)=F1*T1(1,1,7)	01470
T1(1,1,5)=T1(1,1,4)	01480
T1(1,1,8)=0.	01490
T1(1,1,6)=-FF*ALFA*(1.0+ALFA)/2.0	01500
DO 55 J=2,INDX4	01510
T1(J,1,7)=FF/SQRT(DRO)	01520
T1(J,1,1)=ALFA*T1(J,1,7)	01530

T1(J,1,4)=T1(J,1,1)	01540
T1(J,1,2)=-T1(J,1,7)	01550
T1(J,1,3)=0.5*FF/DRO**1.5	01560
55 CONTINUE	01570
DO 60 L=1,8	01580
T4(1,L)=T1(1,1,L)	01590
DO 60 J=1,INDX4	01600
TT(J,L)=T1(J,1,L)	01610
60 CONTINUE	01620
C	01630
C THE FRAME OF INTEGRATION	01640
C	01650
DO 2000 I=1,INDEX	01660
SIGN=1.0	01670
TOW=I*DT	01680
IA=I+1	01690
IA1=IA	01700
IB=I+2	01710
IC=I+3	01720
KK=3	01730
DO 1000 JM=2,IC	01740
KK=5-KK	01750
J=I+4-JM	01760
Z=FLOAT(J)*DT-DT	01770
IF(JM .GT. 3) GO TO 200	01780
IF(JM .EQ. 3) GO TO 300	01790
C	01800
100 K=1	01810
R=DRO	01820
CALL LOADED(DRO,INDX4,INDX1,DT,T,T1,T2,SIGN,DR1)	01830
IF(I .EQ. 1) GO TO 105	01840
DO 110 K=2,I	01850
R=FLOAT(K)*DT-DT+DRO	01860
CALL GENER(DRO,INDX4,INDX1,DT,T,T1,T2,SIGN,DR1)	01870
110 CONTINUE	01880
105 K=I+1	01890
R=FLOAT(I)*DT+DRO	01900
CALL WAVE(DRO,INDX4,INDX1,T,SIGN,FF)	01910
DO 115 K=1,IA	01920
DO 115 JJ=IC,INDX4	01930
DO 115 L=1,8	01940
T(JJ,K,L)=T(IB,K,L)	01950
115 CONTINUE	01960
DO 120 JJ=2,IA	01970
DO 120 L=1,8	01980
T(JJ,IA,L)=T(IB,IA,L)	01990
120 CONTINUE	02000
GO TO 1000	02010
C	02020
300 K=1	02030
R=DRO	02040
CALL LEADN(F1,DRO,INDX4,INDX1,T,FF)	02050

DO 305 L=1,8	02060
T3(1,L)=T(J,1,L)	02070
T4(3,L)=T4(2,L)	02080
T4(2,L)=T4(1,L)	02090
305 CONTINUE	02100
R=DRO+TOW	02110
T4(1,7)=FF/R**P2	02120
T4(1,1)=ALFA*T4(1,7)	02130
T4(1,2)=-T4(1,7)	02140
T4(1,3)=P2*FF/R**(P2+1.)	02150
T4(1,4)=P1*T4(1,7)	02160
T4(1,5)=-T4(1,4)	02170
T4(1,6)=T4(1,2)/R	02180
T4(1,8)=0.	02190
IF(I .LE. 2) GO TO 1000	02200
DO 310 K=3,I,2	02210
DO 310 L=1,8	02220
T(J,K,L)=T(IB,K,L)	02230
310 CONTINUE	02240
320 GO TO 1000	02250
C	02260
200 IF(J.EQ. 1) GO TO 500	02270
C	02280
400 IF(KK .EQ. 2) GO TO 405	02290
K=1	02300
R=DRO	02310
FR=FF	02320
CALL LOAD1(DRO,INDX4,INDX1,DT,T,T1,T2,SIGN,DRI,FR2)	02330
405 DO 445 K=KK,I,2	02340
R=FLOAT(K)*DT-DT+DRO	02350
LM=(J-1)*(J-1)-I*I+(K-1)*(K-1)	02360
IF(LM .GT. 0) GO TO 430	02370
ML=K+J-I-2	02380
IF(ML) 410,415,425	02390
410 CALL GENER1(DRO,INDX4,INDX1,DT,T,T1,T2,SIGN,DRI)	02400
GO TO 445	02410
415 CALL DIAG(DRO,INDX4,INDX1,DT,T,T1,T2,T4,SIGN,DRI)	02420
DO 420 L=1,8	02430
T3(K,L)=T(J,K,L)	02440
420 CONTINUE	02450
GO TO 445	02460
425 CALL GINTER(DRO,INDX4,INDX1,DT,T3,T4,T,SIGN,DRI,ML)	02470
GO TO 445	02480
430 DO 440 L=1,8	02490
T(J,K,L)=T(IB,K,L)	02500
440 CONTINUE	02510
445 CONTINUE	02520
GO TO 1000	02530
C	02540
500 IF(KK .EQ. 2) GO TO 505	02550
K=1	02560
R=DRO	02570

DO 501 L=1,8	02580
501 BD(L)=T1(I+2,1,L)-T1(2,1,L)	02590
BD(5)=-BD(5)	02600
FM=FF	02610
CALL LOAD1(DRO,INDX4,INDX1,DT,T,T1,T2,SIGN,DRI,PRE)	02620
505 IF(I .LE. 2) GO TO S15	02630
IAN=I-1	02640
DO 510 K=KK,IAN,2	02650
R=FLOAT(K)*DT-DT+DRO	02660
DO 506 L=1,8	02670
506 BD(L)=T1(I+2,K,L)-T1(2,K,L)	02680
BD(5)=-BD(5)	02690
CALL GENER1(DRO,INDX4,INDX1,DT,T,T1,T2,SIGN,DRI)	02700
510 CONTINUE	02710
515 K=I+1	02720
DO 520 L=1,8	02730
T(J,K,L)=T4(1,L)	02740
520 CONTINUE	02750
1000 CONTINUE	02760
SIGN=-1.0	02770
IF(I-N)1985,598,604	02780
C INITIAL REFLECTED CONDITIONS	02790
598 CONTINUE	02800
Q1(1,N1,7)=FR/DRI**P2	02810
Q1(1,N1,1)=ALFA*Q1(1,N1,7)	02820
Q1(1,N1,2)=-SIGN*Q1(1,N1,7)	02830
Q1(1,N1,3)=SIGN*P2*FR/DRI**(P2+1.)	02840
Q1(1,N1,4)=P1*Q1(1,N1,7)	02850
Q1(1,N1,5)=-Q1(1,N1,4)	02860
Q1(1,N1,6)=Q1(1,N1,2)/DRI	02870
Q1(1,N1,8)=.0	02880
DO 600 J=2,INDX4	02890
Q1(J,N1,7)=FR/SQRT(DRI)	02900
Q1(J,N1,1)=ALFA*Q1(J,N1,7)	02910
Q1(J,N1,4)=ALFA*Q1(J,N1,7)	02920
Q1(J,N1,2)=-SIGN*Q1(J,N1,7)	02930
Q1(J,N1,3)=0.5*SIGN*FR/DRI**1.5	02940
600 CONTINUE	02950
DO 602 L=1,8	02960
Q4(1,L)=Q1(1,N1,L)	02970
DO 602 M=1,N1	02980
QJ(1,M,L)=Q4(1,L)	02990
602 CONTINUE	03000
DO 603 J=1,INDX4	03010
DO 603 K=1,INDX1	03020
DO 603 L=1,8	03030
Q(J,K,L)=Q1(J,K,L)	03040
603 CONTINUE	03050
GO TO 4000	03060
604 IR=I-N	03070
KR=N1-IR	03080
DO 606 L=1,8	03090

Q4(3,L)=Q4(2,L)	03100
Q4(2,L)=Q4(1,L)	03110
606 CONTINUE	03120
R=FLOAT(2*N-I)*DT+DRO	03130
Q4(1,7)=FR/R**P2	03140
Q4(1,1)=ALFA*Q4(1,7)	03150
Q4(1,2)=-SIGN*Q4(1,7)	03160
Q4(1,3)=SIGN*P2*FR/R**(P2+1.0)	03170
Q4(1,4)=P1*Q4(1,7)	03180
Q4(1,5)=-Q4(1,4)	03190
Q4(1,6)=Q4(1,2)/R	03200
Q4(1,8)=0.	03210
DO 3000 JM=2,IC	03220
J=I+4-JM	03230
IM=MOD(JM+N,2)+2	03240
Z=FLOAT(J-1)*DT	03250
J1=I-N+1	03260
J2=2+SQRT(FLOAT(I*I-N*N))	03270
IF(J-J1)640,630,610	03280
610 IF(J.LT.J2)GO TO 620	03290
K=N1	03300
R=DRI	03310
FB=-T(J,N1,7)	03320
CALL LOADED(DRO,INDX4,INDX1,DT,Q,Q1,Q2,SIGN,DRI)	03330
IF(IR.EQ.1)GO TO 614	03340
DO 612 KR=2,IR	03350
K=N-KR+2	03360
R=FLOAT(K)*DT-DT+DRO	03370
CALL GENER(DRO,INDX4,INDX1,DT,Q,Q1,Q2,SIGN,DRI)	03380
612 CONTINUE	03390
614 K=KW	03400
R=FLOAT(K-1)*DT+DRO	03410
CALL WAVE(DRO,INDX4,INDX1,Q,SIGN,FR)	03420
DO 616 K=KW,N1	03430
DO 616 JJ=J2,INDX4	03440
DO 616 L=1,8	03450
Q(JJ,K,L)=Q(IB,K,L)	03460
616 CONTINUE	03470
DO 618 L=1,8	03480
DO 617 M=1,N1	03490
QJ(2,M,L)=QJ(1,M,L)	03500
617 CONTINUE	03510
QJ(1,1,L)=Q(J2,N1,L)	03520
618 CONTINUE	03530
DO 619 JJ=2,J2	03540
DO 619 LL=1,8	03550
619 Q(JJ,KW,LL)=Q(IB,KW,LL)	03560
CALL RBOUND(DRO,INDX4,INDX1,DT,Q,Q1,Q2,Q4,QJ,J1,SIGN,T,ZA,N1,DRI)	03570
GO TO 3000	03580
620 IF(IM.EQ.2)GO TO 622	03590
K=N1	03600
R=DRI	03610

AJM=1.0+FLOAT(I*I-N*N-(J-1)*(J-1))/FLOAT(2*(I-N))	03620
M-AJM	03630
DO 624 L=1,8	03640
Q(J,K,L)=(AJM-FLOAT(M))*QJ(1,M+1,L)+(FLOAT(M+1)-AJM)*QJ(1,M,L)	03650
624 CONTINUE	03660
Q(J,K,7)=-T(J,K,7)	03670
Q(J,K,8)=-T(J,K,8)	03680
622 DO 629 KR=IM,IR,2	03690
K=N-KR+2	03700
R=FLOAT(K-1)*DT+DRO	03710
LM=(J-1)**2-I*I+(2*N-K+1)**2	03720
IF(LM.GT.0)GO TO 626	03730
CALL RINTER(Q,QJ,Q4,INDX4,INDX1,ZA,N1)	03740
GO TO 629	03750
626 DO 625 L=1,8	03760
Q(J,K,L)=Q(IB,K,L)	03770
625 CONTINUE	03780
629 CONTINUE	03790
GO TO 3000	03800
630 K=N1	03810
R=DR1	03820
DO 633 L=1,8	03830
Q(J,K,L)=QJ(1,N1,L)	03840
Q3(N1,L)=Q(J,K,L)	03850
633 CONTINUE	03860
IF(IR.LE.2)GO TO 3000	03870
DO 636 KR=3,IR,2	03880
K=N-KR+2	03890
LM=(J-1)**2-I*I+(2*N-K+1)**2	03900
IF(LM.GT.0)GO TO 634	03910
CALL RINTER(Q,QJ,Q4,INDX4,INDX1,ZA,N1)	03920
GO TO 636	03930
634 DO 635 L=1,8	03940
Q(J,K,L)=Q(IB,K,L)	03950
635 CONTINUE	03960
636 CONTINUE	03970
GO TO 3000	03980
640 IF(J.EQ.1)GO TO 700	03990
IF(IM.EQ.2)GO TO 641	04000
K=N1	04010
R=DR1	04020
FR=-T(J,N1,7)	04030
FRZ=-T(J,N1,8)	04040
CALL LOAD1(DRO,INDX4,INDX1,DT,Q,Q1,Q2,SIGN,DR1,FRZ)	04050
IF(IR.LE.2)GO TO 3000	04060
641 DO 649 KR=IM,IR,2	04070
K=N-KR+2	04080
R=FLOAT(K-1)*DT+DRO	04090
LM=(J-1)**2-I*I+(2*N-K+1)**2	04100
LA=(J-1)**2-(I-N)**2+(N-K+1)**2	04110
ML=J+2*N-K-I	04120
IF(LM.GT.0)GO TO 647	04130

IF(LA.GT.0)GO TO 646	04140
IF(ML)642,643,645	04150
642 CALL GENER1(DRO,INDX4,INDX1,DT,Q,Q1,Q2,SIGN,DRI)	04160
GO TO 649	04170
643 CALL DIAG(DRO,INDX4,INDX1,DT,Q,Q1,Q2,Q4,SIGN,DRI)	04180
DO 644 L=1,8	04190
Q3(K,L)=Q(J,K,L)	04200
644 CONTINUE	04210
GO TO 649	04220
645 CALL GINTER(DRO,INDX4,INDX1,DT,Q3,Q4,Q,SIGN,DRI,N1)	04230
GO TO 649	04240
646 CALL RINTER(Q,QJ,Q4,INDX4,INDX1,ZA,N1)	04250
GO TO 649	04260
647 DO 648 L=1,8	04270
Q(J,K,L)=Q(IB,K,L)	04280
648 CONTINUE	04290
649 CONTINUE	04300
GO TO 3000	04310
700 IF(IM.EQ.2)GO TO 705	04320
R=N1	04330
R=DRI	04340
FB=T(J,N1,7)	04350
DO 701 L=1,8	04360
701 BD(L)=Q1(I+2,N1,L)-Q1(2,N1,L)	04370
BD(5)=-BD(5)	04380
CALL LOAD1(DRO,INDX4,INDX1,DT,Q,Q1,Q2,SIGN,DRI,FRZ)	04390
705 IF(IR.LE.2)GO TO 715	04400
DO 710 KR=IM,IR,2	04410
K=N-KR+2	04420
R=FLOAT(K-1)*DT+DRO	04430
DO 706 L=1,8	04440
706 BD(L)=Q1(I+2,K,L)-Q1(2,K,L)	04450
BD(5)=-BD(5)	04460
CALL GENER1(DRO,INDX4,INDX1,DT,Q,Q1,Q2,SIGN,DRI)	04470
710 CONTINUE	04480
715 K=KW	04490
DO 720 L=1,8	04500
Q(J,K,L)=Q4(1,L)	04510
720 CONTINUE	04520
3000 CONTINUE	04530
4000 CONTINUE	04540
1985 CONTINUE	04550
DO 1975 J=1,INDX4	04560
DO 1975 L=1,8	04570
DO 1970 K=1,IAL	04580
TOTAL(J,K,L)=Q(J,K,L)+T(J,K,L)	04590
1970 CONTINUE	04600
IF(I.NE.2*M)GO TO 1975	04610
TOTAL(J,1,L)=TOTAL(J,1,L)+TT(J,L)	04620
1975 CONTINUE	04630
CALL RESULT(RNE,DRO,DT,I,FF,INDX4,INDX1,TOTAL,T1,IJ,N1, +TT,DRI,N)	04640
	04650


```

DO 1980 J=1,INDX4                                04660
DO 1980 L=1,8                                    04670
DO 1980 K=1,IAL                                  04680
T2(J,K,L)=T1(J,K,L)                             04690
T1(J,K,L)=T(J,K,L)                              04700
Q2(J,K,L)=Q1(J,K,L)                             04710
Q1(J,K,L)=Q(J,K,L)                              04720
1980 CONTINUE                                    04730
2000 CONTINUE                                    04740
RETURN                                           04750
END                                               04760
C                                                 04770
C                                                 04780
SUBROUTINE AMAT(DRO,INDX4,INDX1,DT,T,T1,T2,A,SIGN,DRI) 04790
C                                                 04800
C*****04810
C PURPOSE: TO CALCULATE THE MATRIX AND VECTOR BASED ON MATRIX 2 *04820
C *04830
C COLUMNS 1 TO 8 OF A(II,JJ) REPRESENT THE MATRIX WHILE A(II,9) IS *04840
C THE COLUMN VECTOR SUCH THAT: *04850
C A(1,9)= [A2] A(5,9)= [A6] *04860
C A(2,9)= A9 A(6,9)= [A1] *04870
C A(3,9)= A10 A(7,9)= [A5] *04880
C A(4,9)= A12 A(8,9)= [A3] *04890
C*****04900
C 04910
COMMON/ALL/I,TOW,R,Z,K,J,DTS,ALFA,BETA,BETA2,G2,BG,CBA,CABA,PIES,NO4920
COMMON/AA/B1,B2,B3,B5,B6,B9 04930
COMMON/FO/FF,FB,FR,FM 04940
COMMON/LINK/BD(8) 04950
DIMENSION T(INDX4,INDX1,8),T1(INDX4,INDX1,8),T2(INDX4,INDX1,8) 04960
DIMENSION A(8,9) 04970
F=FF 04980
IF(SIGN.EQ.-1.0)F=FR 04990
DO 600 II=1,8 05000
DO 600 JJ=1,9 05010
A(II,JJ)=0.0 05020
600 CONTINUE 05030
C 05040
C CALCULATING THE MATRIX 05050
C 05060
A(1,1)=B2/R 05070
A(1,2)=1.-2.*ALFA*B2/R 05080
A(1,4)=B2/R 05090
A(1,7)=1.-2.*B2/R 05100
C 05110
ALFA9=B9*ALFA 05120
ALFA9R=ALFA9/R 05130
A(2,2)=-ALFA9R 05140
A(2,3)=-B9 05150
A(2,6)=-ALFA9 05160
A(2,7)=1. 05170

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C	A(3,2)=-ALFA9R	05180
	A(3,3)=-ALFA9	05190
	A(3,4)=1.	05200
	A(3,6)=-B9	05210
C	A(4,1)=1.	05220
	A(4,2)=-B9/R	05230
	A(4,3)=-ALFA9	05240
	A(4,6)=-ALFA9	05250
C	A(5,2)=-ALFA*B6/R	05260
	A(5,4)=1.0	05270
	A(5,5)=1.0	05280
	A(5,8)=-B6/R	05290
C	A(6,1)=-B1/R	05300
	A(6,2)=-1.-2.*ALFA*B1/R	05310
	A(6,4)=-B1/R	05320
	A(6,7)=1.+2.*B1/R	05330
C	A(7,2)=-ALFA*B5/R	05340
	A(7,4)=1.0+B5/(Z+DRO)	05350
	A(7,5)=-1.0-ALFA*B5/(Z+DRO)	05360
	A(7,7)=-B5/(Z+DRO)	05370
	A(7,8)=B5/R	05380
C	A(8,5)=-SIGN*BETA+BETA2*B3/R	05390
	A(8,8)=1.0+3.0*SIGN*BETA*B3/R	05400
C		05410
C	CALCULATING THE VECTOR	05420
C		05430
	ZIR=Z*Z-TOW*TOW+(R-DRO)*(R-DRO)	05440
	IF(SIGN.EQ.-1.0)ZIR=Z*Z-TOW*TOW+(2.0*DRI-DRO-R)**2	05450
	KNEG=K-1	05460
	KPOS=K+1	05470
	JNEG=J-1	05480
	JPOS=J+1	05490
C		05500
	IF(KNEG.EQ.0)GO TO 620	05510
	A(6,9)=(T1(J,KNEG,1)+ALFA*T1(J,KNEG,2)-T1(J,KNEG,7))	05520
	+*B1/(R-DT)+T1(J,KNEG,7)-T1(J,KNEG,2)	05530
	IF(ZIR.GT.0.0)GO TO 610	05540
	A(6,9)=A(6,9)+(T1(J,KNEG,4)+ALFA*T1(J,KNEG,2)-	05550
	+T1(J,KNEG,7))*B1/(R-DT)	05560
C		05570
	610 CONTINUE	05580
	SRZ=CBA*T1(J,KNEG,8)+CBA*T2(J,K,8)	05590
	UZ=CBA*T1(J,KNEG,5)+CBA*T2(J,K,8)	05600
	A(8,9)=SRZ*(1.0-3.0*SIGN*BETA*B3/(R-BG))+UZ*(-SIGN*BETA-BETA2*B3/	05610
	+(R-BG))	05620
C		05630
		05640
		05650
		05660
		05670
		05680
		05690

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620 CONTINUE                                05700
      A(1,9)=(T1(J,KPOS,7)+ALFA*T1(J,KPOS,2)-T1(J,KPOS,1))
      +*B2/(R+DT)+T1(J,KPOS,7)+T1(J,KPOS,2)  05710
      IF(XIR .GT. 0.0) GO TO 640             05720
      A(1,9)=A(1,9)+B2*(T1(J,KPOS,7)+ALFA*T1(J,KPOS,2)-
      +T1(J,KPOS,4))/(R+DT)                 05730
C                                             05740
640 CONTINUE                                05750
      IF(JNEG .EQ. 0) GO TO 645              05760
      A(7,9)=T1(JNEG,K,4)-T1(JNEG,K,5)+B5*ALFA*T1(JNEG,K,2)/R
      ++B5*(T1(JNEG,K,7)-T1(JNEG,K,4)+ALFA*T1(JNEG,K,5))/(DRO+Z-DT)
      +-B5*T1(JNEG,K,8)/R                  05780
      GO TO 650                              05810
645 A(7,9)=BD(4)-BD(5)+B5*ALFA*BD(2)/R+B5*(BD(7)-BD(4)+ALFA*
      +BD(5))/(DRO+Z-DT)-B5*BD(8)/R        05820
C                                             05840
650 CONTINUE                                05850
      A(5,9)=T1(JPOS,K,4)+T1(JPOS,K,5)+B6*ALFA*T1(JPOS,K,2)/R
      ++B6*T1(JPOS,K,8)/R                  05860
C                                             05870
C                                             05880
      IF(SIGN .EQ. -1 .AND. K .EQ. 2*N-I+2) GO TO 670
      IF(K .EQ. I .AND. SIGN .EQ. 1) GO TO 660
      A(2,9)=T2(J,K,7)+B9*T2(J,K,3)+ALFA9*T2(J,K,2)/R+ALFA9*T2(J,K,6)
      A(3,9)=T2(J,K,4)+B9*T2(J,K,6)+ALFA9*T2(J,K,2)/R+ALFA9*T2(J,K,3)
      A(4,9)=T2(J,K,1)+B9*T2(J,K,2)/R+ALFA9*T2(J,K,3)+ALFA9*T2(J,K,6)
      RETURN                                  05900
C                                             05910
C                                             05920
660 B=B2                                    05970
      GO TO 680                              05980
670 B=B1                                    05990
680 CON=F/R**1.5                            06000
      R2=R+SIGN*DT/2.0                       06010
      AXE=SIGN*P*B*(1.0-2.0*ALFA)/R2**1.5    06020
      A(2,9)=CON*R+SIGN*0.5*B9*CON-SIGN*ALFA*B9*CON
      A(3,9)=-ALFA*CON*R-0.5*SIGN*B9*ALFA*CON
      A(4,9)=-ALFA*CON*R+0.5*SIGN*B9*ALFA*CON-CON*SIGN*B9
      IF(SIGN .EQ. 1) A(1,9)=AXE             06050
      IF(SIGN .EQ. -1) A(6,9)=AXE           06070
      RETURN                                  06080
      END                                     06090
C                                             06100
C*****06110
C      SUBROUTINES RINT AND CINT ARE USED TO ADJUST MATRIX 2 FOR EACH
C      CATEGORY OF POINTS                    06120
C*****06130
C                                             06140
C      SUBROUTINE RINT(A,N,M,LA,LB)           06150
C                                             06160
C      INTERCHANGE ROWS LA AND LB            06170
C                                             06180
C      DIMENSION A(N,M)                      06190
C      DO 10 II=1,M                          06200
C                                             06210

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SAVE=A(LA, II)	06220
A(LA, II)=A(LB, II)	06230
10 A(LB, II)=SAVE	06240
RETURN	06250
END	06260
C	06270
SUBROUTINE CINT(A, N, M, LA, LB)	06280
C	06290
C INTERCHANGE COLUMNS LA AND LB	06300
C	06310
DIMENSION A(N, M)	06320
DO 10 II=1, N	06330
SAVE=A(II, LA)	06340
A(II, LA)=A(II, LB)	06350
10 A(II, LB)=SAVE	06360
RETURN	06370
END	06380
C	06390
C	06400
SUBROUTINE WAVE(DRO, INDX4, INDX1, T, SIGN, F)	06410
C	06420
C*****	06430
C INCIDENT LEADING WAVE POINTS — G —	06440
C*****	06450
C	06460
COMMON/ALL/I, TOW, R, Z, K, J, DTS, ALFA, BETA, BETA2, G2, BG, CBA, CBA2, PIES, NO6470	06470
COMMON/FO/FF, FB, FR, FM	06480
DIMENSION T(INDX4, INDX1, 8)	06490
T(J, K, 7)=F/R**0.5	06500
T(J, K, 1)=ALFA*T(J, K, 7)	06510
T(J, K, 2)=-SIGN*T(J, K, 7)	06520
T(J, K, 3)=F*SIGN*0.5/R**1.5	06530
T(J, K, 4)=ALFA*T(J, K, 7)	06540
RETURN	06550
END	06560
C	06570
C	06580
C	06590
SUBROUTINE LEADM(P1, DRO, INDX4, INDX1, T, FF)	06600
C	06610
C*****	06620
C LEADING INNER-SURFACE POINT — Q —	06630
C*****	06640
C	06650
COMMON/ALL/I, TOW, R, Z, K, J, DTS, ALFA, BETA, BETA2, G2, BG, CBA, CBA2, PIES, NO6660	06660
DIMENSION T(INDX4, INDX1, 8)	06670
T(J, K, 7)=0.	06680
T(J, K, 8)=0.	06690
T(J, K, 3)=0.	06700
DECAY=(1.+ALFA)/2.0	06710
T(J, K, 4)=P1*FF/ABS(Z+DRO)**DECAY	06720
T(J, K, 5)=T(J, K, 4)	06730


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3 WRITE (5,4) 07260
4 FORMAT('SINGULAR AT REGULAR LOADED BOUNDARY 1-D POINT - F - ') 07270
STOP 07280
END 07290
C 07300
C 07310
SUBROUTINE LOAD1(DRO, INDX4, INDX1, DT, T, T1, T2, SIGN, DR1, FRZ) 07320
C 07330
C*****07340
C REGULAR LOADED BOUNDARY POINT (TWO DIMENSIONAL) -- B -- 07350
C*****07360
C 07370
COMMON/ALL/I, TOW, R, Z, K, J, DT5, ALFA, BETA, BETA2, G2, BG, CBA, CBA, PIES, N07380
COMMON/AA/B1, B2, B3, B5, B6, B9 07390
COMMON/TO/TF, FB, FR, FM 07400
COMMON/LINK/BD(8) 07410
DIMENSION T(INDX4, INDX1, 8), T1(INDX4, INDX1, 8), T2(INDX4, INDX1, 8) 07420
DIMENSION A(8, 9), AB(6, 6), COE(6) 07430
F=FM 07440
IF(SIGN.EQ.-1.0)F=FB 07450
CALL AMBT(DRO, INDX4, INDX1, DT, T, T1, T2, A, SIGN, DR1) 07460
IF(SIGN.EQ.1.0)CALL RINT(A, 8, 9, 6, 7) 07470
IF(SIGN.EQ.-1.0)CALL RINT(A, 8, 9, 1, 7) 07480
DO 7 II=1, 6 07490
DO 6 JJ=1, 6 07500
6 AB(II, JJ)=A(II, JJ) 07510
7 COE(II)=A(II, 9) 07520
T(J, K, 7)=F 07530
T(J, K, 8)=0.9*(1.0-SIGN)*FRZ 07540
COE(1)=COE(1)-(1.0-2.0*B2/R)*T(J, K, 7) 07550
IF(SIGN.EQ.-1.0)COE(1)=A(1, 9)+B5/(DRO+Z)*T(J, K, 7) 07560
+ -B5/R*T(J, K, 8) 07570
COE(2)=COE(2)-T(J, K, 7) 07580
COE(6)=COE(6)+B5*T(J, K, 7)/(Z+DRO) 07590
IF(SIGN.EQ.-1.0)COE(6)=A(6, 9)-(1.0+2.0*B1)*T(J, K, 7) 07600
IF(SIGN.EQ.-1.0)COE(5)=COE(5)+B6/R*T(J, K, 8) 07610
CALL MATINV(AB, COE, 6, 6, 1, DET, KB) 07620
IF(KB.EQ.1) GO TO 3 07630
DO 9 L=1, 6 07640
9 T(J, K, L)=COE(L) 07650
RETURN 07660
3 WRITE (5,4) 07670
4 FORMAT('SINGULAR AT REGULAR LOADED BOUNDARY 2-D POINT - B - ') 07680
STOP 07690
END 07700
C 07710
C 07720
SUBROUTINE GENER(DRO, INDX4, INDX1, DT, T, T1, T2, SIGN, DR1) 07730
C 07740
C*****07750
C REGULAR INNER ONE DIMENSIONAL POINTS -- E -- 07760
C*****07770

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C
COMMON/ALL/I,TOW,R,Z,K,J,DT5,ALFA,BETA,BETA2,CG,BG,CBA,CAB,PIES,NO7790 07790
COMMON/AA/B1,B2,B3,B5,B6,B9 07800
DIMENSION T(INDX4,INDX1,8),T1(INDX4,INDX1,8),T2(INDX4,INDX1,8) 07810
DIMENSION A(8,9),AB(8,5),COE(5) 07820
IF(SIGN.EQ.1.0.AND.K.EQ.I)GO TO 2 07830
IF(SIGN.EQ.-1.0.AND.K.EQ.2*N-I+2)GO TO 8 07840
GO TO 5 07850
2 B2=B1/2.0 07860
B9=B1 07870
GO TO 5 07880
8 B9=B1 07890
B1=B1/2.0 07900
5 CALL AMAT(DRO,INDX4,INDX1,DT,T,T1,T2,A,SIGN,DRI) 07910
CALL CINT(A,8,9,5,7) 07920
CALL RINT(A,8,9,5,6) 07930
DO 7 II=1,5 07940
DO 6 JJ=1,5 07950
6 AB(JI,JJ)=A(JI,JJ) 07960
7 COE(JI)=A(JI,9) 07970
AB(1,2)=AB(1,2)+ALFA*B2/R 07980
AB(1,4)=0. 07990
AB(1,5)=AB(1,5)+B2/R 08000
AB(5,2)=AB(5,2)+ALFA*B1/R 08010
AB(5,4)=0. 08020
AB(5,5)=AB(5,5)-B1/R 08030
CALL MATINV(AB,COE,5,5,1,DET,KS) 08040
IF(KS.EQ.1) GO TO 3 08050
DO 9 L=1,4 08060
9 T(J,K,L)=COE(L) 08070
T(J,K,5)=0.0 08080
T(J,K,6)=0.0 08090
T(J,K,7)=COE(5) 08100
T(J,K,8)=0.0 08110
B1=DT5 08120
B2=B1 08130
B9=2.0*B1 08140
RETURN 08150
3 WRITE(5,4) 08160
4 FORMAT('SINGULAR AT REGULAR INNER ONE DIMENSIONAL POINT - Z -') 08170
STOP 08180
END 08190
C 08200
C 08210
SUBROUTINE GENERAL(DRO,INDX4,INDX1,DT,T,T1,T2,SIGN,DRI) 08220
C 08230
C*****08240
C REGULAR INNER TWO DIMENSIONAL POINTS -- A -- 08250
C SOLVE MATRIX 2 WITHOUT ALTERATION 08260
C*****08270
C 08280
COMMON/ALL/I,TOW,R,Z,K,J,DT5,ALFA,BETA,BETA2,CG,BG,CBA,CAB,PIES,NO8290

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COMMON/AA/B1,B2,B3,B5,B6,B9                                08300
COMMON/LINK/BD(8)                                         08310
DIMENSION T(INDX4,INDX1,8),T1(INDX4,INDX1,8),T2(INDX4,INDX1,8) 08320
DIMENSION A(8,9),AB(8,8),COE(8)                          08330
CALL AMAT(DRO,INDX4,INDX1,DT,T,T1,T2,A,SIGN,DRI)          08340
DO 7 II=1,8                                                08350
DO 6 JJ=1,8                                                08360
6 AB(II,JJ)=A(II,JJ)                                       08370
7 COE(II)=A(II,9)                                          08380
CALL MATINV(AB,COE,8,8,1,DET,KS)                           08390
IF(KS.EQ.1) GO TO 3                                        08400
DO 9 L=1,8                                                 08410
9 T(J,K,L)=COE(L)                                         08420
RETURN                                                     08430
3 WRITE (5,4)                                              08440
4 FORMAT(*SINGULAR AT REGULAR INNER TWO DIMENSIONAL POINT - A --) 08450
STOP                                                       08460
END                                                         08470
C                                                         08480
C                                                         08490
SUBROUTINE GINTER(DRO,INDX4,INDX1,DT,T3,T4,T,SIGN,DRI,N1) 08500
C                                                         08510
C*****08520
C INTERMEDIATE INNER TWO DIMENSIONAL POINT -- K --      08530
C CALCULATED BY INTERPOLATION ALONG THE REFLECTED LONGITUDINAL WAVE 08540
C*****08550
C                                                         08560
COMMON/ALL/I,TOW,R,Z,K,J,DT5,ALFA,BETA,BETA2,G2,BG,CBA,CABA,PIES,N08570
DIMENSION T(INDX4,INDX1,8),T3(INDX1,8),T4(3,8)          08580
Y=TOW-(R-DRO)                                             08590
IF(SIGN.EQ.-1.0)Y=TOW-2.0*DRI+DRO+R                      08600
X=(Z*Z/Y+Y)/2.                                           08610
RK=(X-Y)/Z                                               08620
TETA1=ATAN(RK)/PIES                                     08630
TETA=1.-TETA1                                           08640
RK=(TOW-X)/DT+1.                                         08650
IF(SIGN.EQ.-1.0)RK=2*N-I+1+X/DT                          08660
KA=IFIX(RK)                                              08670
KB=KA+1                                                  08680
C2=RK-FLOAT(KA)                                          08690
C1=1.-C2                                                 08700
DO 154 L=1,8                                             08710
154 T(J,K,L)=TETA*(C1*T3(KA,L)+C2*T3(KB,L))+TETA1*T4(1,L) 08720
RETURN                                                    08730
END                                                        08740
C                                                         08750
C                                                         08760
SUBROUTINE DIAG(DRO,INDX4,INDX1,DT,T,T1,T2,T4,SIGN,DRI) 08770
C                                                         08780
C*****08790
C INTERMEDIATE TWO DIMENSIONAL POINT -- L --          08800
C*****08810

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C
COMMON/ALL/I,TOW,R,Z,K,J,DT5,ALFA,BETA,BETA2,G2,BG,CBA,CABA,PIES,NOC830 08820
COMMON/AA/B1,B2,B3,B5,B6,B9 08840
DIMENSION T(INDX4,INDX1,8),T1(INDX4,INDX1,8),T2(INDX4,INDX1,8) 08850
DIMENSION A(8,9),AB(8,8),COE(8),E(8),T4(3,8) 08860
B1=DT5 08870
B2=Z*DT/(2.*(Z+2.*DT)) 08880
IF(SIGN.EQ.1.0)GO TO 5 08890
BT=B1 08900
B1=B2 08910
B2=BT 08920
5 B3=(Z+(1.-BETA)*DT-SQRT((DT-BETA*(Z+DT))**2+(1.-BETA2)*Z**2)) 08930
+/(2.*(1.-BETA2)) 08940
B5=DT5 08950
B6=Z*DT/(2.*(2.*Z+DT)) 08960
B9=(Z+DT-SQRT(Z*Z+DT*DT))/2. 08970
CALL AMAT(DRO,INDX4,INDX1,DT,T,T1,T2,A,SIGN,DRI) 08980
DO 7 II=1,8 08990
DO 6 JJ=1,8 09000
6 AB(II,JJ)=A(II,JJ) 09010
7 COE(II)=A(II,9) 09020
R2=R+2.0*B2 09030
CALL ENV(INDX4,INDX1,DT,Z,R2,E,T,T1,T4,B2) 09040
COE(1)=E(7)*(1.+B2/R2)+E(2)*(1.+ALFA*B2/R2)-B2*E(1)/R2 09050
+B2*(ALFA*E(2)+E(7)-E(4))/R2 09060
Z6=Z+2.*B6 09070
CALL ENV(INDX4,INDX1,DT,Z6,R,E,T,T1,T4,B6) 09080
COE(5)=E(4)+E(5)+B6*ALFA*E(2)/R+B6*E(8)/R 09090
CALL ENV(INDX4,INDX1,DT,Z,R,E,T,T1,T4,B9) 09100
COE(2)=E(7)+B9*(E(3)+ALFA*(E(2)/R+E(6))) 09110
COE(3)=E(4)+B9*(E(6)+ALFA*(E(2)/R+E(3))) 09120
COE(4)=E(1)+B9*(E(2)/R+ALFA*(E(3)+E(6))) 09130
R3=R-2.0*B3*BETA 09140
CALL ENV(INDX4,INDX1,DT,Z,R3,E,T,T1,T4,B3) 09150
COE(8)=E(8)*(1.0-SIGN*3.0*BETA*B3/R3)-E(5)*(SIGN*BETA+B3*BETA2/R3) 09160
CALL MATINV(AB,COE,8,8,1,DET,KS) 09170
IF(KS .EQ. 1) GO TO 3 09180
DO 209 L=1,8 09190
209 T(J,K,L)=COE(L) 09200
B1=DT5 09210
B2=DT5 09220
B6=DT5 09230
B3=G2 09240
B9=DT 09250
RETURN 09260
3 WRITE (5,4) 09270
4 FORMAT(* SINGULAR AT INTERMEDIARY TWO DIMENSIONAL POINT - L -*) 09280
STOP 09290
END 09300
C
C
SUBROUTINE ENV(INDX4,INDX1,DT,ZL,RL,E,T,T1,T4,EL) 09310
09320
09330

```

```

C
C*****09340
C      CALCULATION OF QUANTITIES AT TERMINAL POINTS OF      09360
C      BICHARACTERISTIC CURVES OF POINTS L                  09370
C*****09380
C
C      COMMON/ALL/I,TOW,R,Z,K,J,DTS,ALFA,BETA,BETA2,G2,BG,CBA,CABA,PIES,NO9400
DIMENSION T(INDX4,INDXL,8),T1(INDX4,INDXL,8),T4(3,8),E(8)      09410
C1=2.0*BL/DT                                                    09420
C2=1.0-C1                                                        09430
THETA=ATAN(ZL/(ABS(ZL-R)+DT))/PIES                               09440
THETA1=1.0-THETA                                                09450
DO 10 L=1,8                                                      09460
E(L)=C1*(THETA*T1(J,K-1,L)+THETA1*T4(2,L))+                    09470
+C2*(THETA*T(J+1,K-1,L)+THETA1*T4(1,L))                        09480
10 CONTINUE                                                      09490
RETURN                                                            09500
END                                                                09510

C
C      SUBROUTINE RBOUND(DRO,INDX4,INDXL,DT,Q,Q1,Q2,Q4,QJ,J1,SIGN,T,EA,
+ N1,DR1)
C
C*****09560
C      INTERMEDIATE TWO-DIMENSIONAL POINTS — M —          09570
C*****09580
C      COMMON/ALL/I,TOW,R,Z,K,J,DTS,ALFA,BETA,BETA2,G2,BG,CBA,CABA,PIES,NO9590
COMMON/AA/B1,B2,B3,B5,B6,B9                                     09600
COMMON/FO/FF,FB,FR,FM                                         09610
DIMENSION ZA(N1)                                                09620
DIMENSION T(INDX4,INDXL,8),QJ(2,INDXL,8)                      09630
DIMENSION Q(INDX4,INDXL,8),Q1(INDX4,INDXL,8),Q2(INDX4,INDXL,8) 09640
DIMENSION A(8,9),AB(6,6),COE(6),E(8),Q4(3,8)                 09650
R=FLOAT(N)*DT+DRO                                              09660
DO 200 M=1,N1                                                    09670
ZN=SQRT( FLOAT( (I+N-2*M+2)*(I-N) ) )                          09680
Z=ZN*DT                                                         09690
ZA(M)=ZN                                                         09700
NZ=ZN                                                            09710
DO 12 L=1,8                                                      09720
12 QJ(1,M,L)=0.0
IF(M.EQ.1)GO TO 200
QJ(1,M,7)=-((ZN-FLOAT(NZ))*T(NZ+2,N1,7)
+ (FLOAT(NZ+1)-ZN)*T(NZ+1,N1,7) )
QJ(1,M,8)=-((ZN-FLOAT(NZ))*T(NZ+2,N1,8)
+ (FLOAT(NZ+1)-ZN)*T(NZ+1,N1,8) )
B1=0.25*(TOW-R+DRO-Z**2/(TOW+R-2.0*(M-2)*DT-DRO) )
B9=0.5*(TOW-(M-2)*DT-SQRT(Z**2+(R-(M-2)*DT-DRO)**2) )
B5=0.25*(TOW+Z-(M-2)*DT+(R-(M-2)*DT-DRO)**2/(Z-TOW+(M-2)*DT))
IF(B5.GT. DTS) B5=DTS
B6=0.25*(TOW-Z-(M-2)*DT-(R-(M-2)*DT-DRO)**2/(TOW+Z-(M-2)*DT))
CALL AMAT(DRO,INDX4,INDXL,DT,Q,Q1,Q2,A,SIGN,DR1)
CALL RIWT(A,8,9,1,7)

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```

DO 7 II=1,8
DO 6 JJ=1,8
6 AB(II, JJ)=A(II, JJ)
7 COE(II)=A(II, 9)
RA=TOW-(M-2)*DT
RB=TOW-(M-1)*DT
TA=ASIN( DT*SQRT(FLOAT( (I+N-2*(M-2))*(I-N) ) )/RA )
IF(I.EQ.N1)GO TO 8
TB=ASIN( DT*SQRT(FLOAT( (I-1+N-2*(M-2))*(I-1-N) ) )/RB )
8 CONTINUE
R1=R-2.0*B1
T1=ATAN( Z/(R+2.*B1 - (M-2)*DT - DRO) )
CALL ENV1(R, Z, T1, TA, TB, B1, E, QJ, Q4, DT, M, INDX1, N1)
COE(6)=E(7)*(1.-B1/R1)-E(2)*(1.-ALFA*B1/R1)+B1*E(1)/R1
+ +B1*(ALFA*E(2) - E(7) + E(4) )/R1
COE(6)=COE(6)-(1.+2.*B1/R)*QJ(1, M, 7)
Z6=Z+2.0*B6
T6=ATAN( Z6/(R-(M-2)*DT-DRO) )
CALL ENV1(R, Z6, T6, TA, TB, B6, E, QJ, Q4, DT, M, INDX1, N1)
COE(5)=E(4)+E(5)+B6*ALFA*E(2)/R+B6*E(8)/R
COE(5)=COE(5)+B6/R*QJ(1, M, 8)
T9=ATAN( Z/(R-(M-2)*DT-DRO) )
CALL ENV1(R, Z, T9, TA, TB, B9, E, QJ, Q4, DT, M, INDX1, N1)
COE(2)=E(7)+B9*( E(3)+ALFA*(E(2)/R+E(6) ) )
COE(2)=COE(2)-QJ(1, M, 7)
COE(3)=E(4)+B9*( E(6)+ALFA*(E(2)/R+E(3) ) )
COE(4)=E(1)+B9*( E(2)/R+ALFA*(E(3)+E(6) ) )
IF(B5.EQ.DT5)GO TO 9
Z5=Z-2.0*B5
T5=ATAN( Z5/(R-(M-2)*DT-DRO) )
CALL ENV1(R, Z5, T5, TA, TB, B5, E, QJ, Q4, DT, M, INDX1, N1)
GO TO 11
9 DO 10 L=1,8
10 E(L)=(ZN-FLOAT(NZ))*Q1(NZ+1, N1, L)+(FLOAT(NZ+1)-ZN)*Q1(NZ, N1, L)
11 CONTINUE
COE(1)=E(4)-E(5)+B5*ALFA*E(2)/R-B5*E(8)/R
COE(1)=COE(1)+B5/(Z+DRO)*QJ(1, M, 7)
+ -B5/R*QJ(1, M, 8)
IF(M.EQ.N1 )COE(1)=A(1, 9)+B5/(Z+DRO)*QJ(1, M, 7)
+ -B5/R*QJ(1, M, 8)
CALL MATINV(AB, COE, 6, 6, 1, DET, KS)
DO 209 L=1,6
QJ(1, M, L)=COE(L)
209 CONTINUE
200 CONTINUE
B1=DT5
B5=DT5

B6=DT5
B9=DT
RETURN
END

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C		10900
	SUBROUTINE MATINV(A,B,N,N1,MSUB,DET,KS)	10910
	DIMENSION A(1),B(1)	10920
	TOL=0.0	10930
	KS=0	10940
	JJ=-N	10950
	DO 65 J=1,N	10960
	JY=J+1	10970
	JJ=JJ+N+1	10980
	BIGA=0.0	10990
	IT=JJ-J	11000
	DO 30 I=J,N	11010
	IJ=IT+I	11020
	IF(ABS(BIGA)-ABS(A(IJ)))20,30,30	11030
20	BIGA=A(IJ)	11040
	IMAX=I	11050
30	CONTINUE	11060
	IF(ABS(BIGA)-TOL)35,35,40	11070
35	KS=1	11080
	RETURN	11090
40	I1=J+N*(J-2)	11100
	IT=IMAX-J	11110
	DO 50 K=J,N	11120
	I1=I1+N	11130
	I2=I1+IT	11140
	SAVE=A(I1)	11150
	A(I1)=A(I2)	11160
	A(I2)=SAVE	11170
50	A(I1)=A(I1)/BIGA	11180
	SAVE=B(IMAX)	11190
	B(IMAX)=B(J)	11200
	B(J)=SAVE/BIGA	11210
	IF(J-N)55,70,55	11220
55	IQS=N*(J-1)	11230
	DO 65 IX=JY,N	11240
	IXJ=IQS+IX	11250
	IT=J-IX	11260
	DO 60 JX=JY,N	11270
	IXJX=N*(JX-1)+IX	11280
	JJX=IXJX+IT	11290
60	A(IXJX)=A(IXJX)-(A(IXJ)*A(JJX))	11300
65	B(IX)=B(IX)-(B(J)*A(IXJ))	11310
70	NY=N-1	11320
	IT=N*N	11330
	DO 80 J=1,NY	11340
	IA=IT-J	11350
	IB=N-J	11360
	IC=N	11370
	DO 80 K=1,J	11380
	B(IB)=B(IB)-A(IA)*B(IC)	11390
	IA=IA-N	11400
80	IC=IC-1	11410

```

RETURN                                11420
END                                    11430
C                                     11440
C                                     11450
SUBROUTINE RESULT(RNE,DRO,DT,I,FF,INDX4,INDX1,T,T1,IJ,N1, 11460
+TT,DRI,N)                             11470
C                                     11480
COMMON/RE/IFROM,ITILL,IWRITE,IJ1,IPRINT,NPRINT 11490
COMMON/PRINT/JPRINT(9),KPRINT(9)          11500
DIMENSION T(INDX4,INDX1,8),T1(INDX4,INDX1,8) 11510
DIMENSION TT(INDX4,8)                    11520
IF(IJ.EQ.2)GO TO 90                       11530
IJ1=1                                      11540
IF(IPRINT.EQ.1)GO TO 75                   11550
75 CALL OUTP(RNE,DRO,DT,I,FF,INDX4,INDX1,TT,DRI,N) 11560
IJ=2                                       11570
90 IF(I.GT.ITILL)RETURN                    11580
GO TO(100,200,300) IPRINT                 11590
100 IF(I.NE.IFROM)RETURN                  11600
IFROM=IFROM+IWRITE                        11610
IF(MOD(I,2).EQ.0)CALL OUTP1(DRO,DT,I,INDX4,INDX1,T,N1) 11620
RETURN                                     11630
200 CALL OUTP2(DRO,DT,I,INDX4,INDX1,T,T1) 11640
RETURN                                     11650
300 IF(I.NE.IFROM)GO TO 310               11660
IFROM=IFROM+IWRITE                        11670
IF(MOD(I,2).EQ.0)CALL OUTP1(DRO,DT,I,INDX4,INDX1,T,N1) 11680
310 CALL OUTP2(DRO,DT,I,INDX4,INDX1,T,T1) 11690
RETURN                                     11700
END                                        11710
C                                     11720
C                                     11730
SUBROUTINE OUTP(RNE,DRO,DT,I,FF,INDX4,INDX1,TT,DRI,N) 11740
C                                     11750
C*****                                11760
C PRINTS THE VALUES OF THE INPUT CONSTANTS AND THE INITIAL * 11770
C CONDITIONS *                          11780
C*****                                11790
C                                     11800
COMMON/LINK/BD(8)                          11810
DIMENSION TT(INDX4,8),TA(8),TB(8),BDI(8) 11820
INDEX=INDX1-1                              11830
CALL DATE(NDATE)                            11840
WRITE(5,7) NDATE                            11850
WRITE(5,8)                                   11860
7 FORMAT(A12/* TRANSIENT RESPONSE OF INFINITELY LONG TUBE* 11870
+* SUBJECT TO ABRUPTLY APPLIED LOAD(CASE 2)*/) 11880
8 FORMAT(* DURATION TIME OF LOAD = PERMANENT*/ * WIDTH OF* 11890
+* LOAD = SEMI-INFINITE*/)                 11900
WRITE(5,10) RNE,DRO,DRI,FF,DT,INDEX        11910
10 FORMAT(* THE INPUT CONSTANTS */21(LH-)// *POISSONS RATIO =*,F3.3 11920
+*/INNER RADIUS =*,F4.2/*OUTER RADIUS =*,F4.2/*NON-DIMENSIONAL * 11930

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+*SIRR =*,F4.2/*STEP SIZE FOR INTEGRATION =*,F6.4/*NUMBER OF *      11940
+*TIME STEPS =*,I3/* THE INITIAL CONDITIONS (TIME T=0.0)*          11950
+/38(1H-)/)                                                         11960
DO 15 L=1,8                                                         11970
15 BDI(L)=0.                                                         11980
WRITE(5,20) (BDI(L),TT(1,L),TT(2,L),L=1,8)                          11990
20 FORMAT(*POINTS- R=1.0,Z<0.0*,6X,*POINT - R=1.0,Z=0.0*,6X,      12000
+*POINTS- R=1.0,Z>0.0*/                                           12010
+3(19(1H-),6X)//3(* SITT =*,F7.4,12X)/3(* UR =*,F7.4,12X)/      12020
+3(*DURDR =*,F7.4,12X)/3(* SIZZ =*,F7.4,12X)/3(* UZ =*,F7.4,12X) 12030
+/3(*DUZDZ =*,F7.4,12X)/3(* SIRR =*,F7.4,12X)/3(* SIRZ =*,F7.4, 12040
+12X)/72(1H-)/)                                                  12050
RETURN                                                             12060
END                                                                  12070
C                                                                    12080
C                                                                    12090
SUBROUTINE OUTP1(DRO,DT,I,INDX4,INDX1,T,N1)                          12100
C                                                                    12110
C*****12120
C PRINTS VALUES OF VARIABLES L AT SPECIFIED TIME FOR POINTS (J,K) 12130
C FOR J=1,INDEX+4 AND K=1,INDEX+1                                   12140
C*****12150
C                                                                    12160
DIMENSION T(INDX4,INDX1,8),Q(8)                                     12170
TOW=FLOAT(I)*DT                                                    12180
WRITE(5,40) TOW                                                    12190
40 FORMAT(/** AT TIME T=*,F7.4/19(1H-)//5X,*Z*,8X,*R*,7X,*SIRR*,5X, 12200
+*SITT*,5X,*SIZZ*,5X,*SIRZ*,6X,*UR*,7X,*UZ*/72(1H-))            12210
IA=I+1                                                             12220
IB=I+2                                                             12230
IC=I+3                                                             12240
KK=2                                                                12250
DO 45 JJ=1,IB                                                      12260
KK=3-KK                                                            12270
J=IC-JJ+1                                                         12280
Z=-DT*FLOAT(J-1)                                                  12290
WRITE(5,50)                                                         12300
DO 45 K=KK,N1,2                                                    12310
R=FLOAT(K-1)*DT+DRO                                               12320
DO 42 L=1,8                                                         12330
42 Q(L)=T(I+2,K,L)-T(J,K,L)                                       12340
WRITE(5,70)Z,R,Q(7),Q(1),Q(4),Q(8),Q(2),Q(5)                    12350
45 CONTINUE                                                         12360
KK=1                                                                12370
IF(MOD(I,2).EQ.0) KK=2                                           12380
DO 60 J=1,IC                                                       12390
KK=3-KK                                                            12400
Z=DT*FLOAT(J-1)                                                  12410
WRITE(5,50)                                                         12420
50 FORMAT(* *)                                                    12430
DO 60 K=KK,N1,2                                                    12440
R=FLOAT(K-1)*DT+DRO                                               12450

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TM=-T(J,K,5)	12460
WRITE (5,70) Z,R,T(J,K,7),T(J,K,1),T(J,K,4),	12470
+T(J,K,8),T(J,K,2),TM	12480
60 CONTINUE	12490
70 FORMAT(8(2X,F7.4))	12500
WRITE (5,80)	12510
80 FORMAT(72(1E-))	12520
RETURN	12530
END	12540
C	12550
C	12560
SUBROUTINE OUTP2(DRO,DT,I,INDX4,INDX1,T,T1)	12570
C	12580
C*****	12590
C PRINTS VALUES OF VARIABLES L FOR SPECIFIED POINTS (JPRINT,KPRINT)	12600
C*****	12610
C	12620
COMMON/RE/IPROM,ITILL,IWRITE,IJ1,IPRINT,NPRINT	12630
COMMON/PRINT/JPRINT(9),KPRINT(9)	12640
DIMENSION T(INDX4,INDX1,8),T1(INDX4,INDX1,8)	12650
DIMENSION R(9),Z(9),TOW(51),VAR(9,51,8)	12660
IF(IJ1.NE.1)GO TO 100	12670
IJ1=2	12680
DO 10 JJ=1,NPRINT	12690
R(JJ)=FLOAT(KPRINT(JJ))*DT-DT+DRO	12700
Z(JJ)=FLOAT(JPRINT(JJ))*DT-DT	12710
IF(JPRINT(JJ).LT.0)Z(JJ)=Z(JJ)+2.*DT	12720
10 CONTINUE	12730
DO 20 L=1,8	12740
DO 20 LL=1,NPRINT	12750
TEMP=JPRINT(LL)	12760
J10=ABS(TEMP)	12770
K10=KPRINT(LL)	12780
TOW(1)=0.0	12790
IF(JPRINT(LL).LT.0)GO TO 15	12800
VAR(LL,1,L)=T1(I+1,K10,L)-T1(J10,K10,L)	12810
GO TO 20	12820
15 VAR(LL,1,L)=VAR(J10,K10,L)	12830
20 CONTINUE	12840
100 IF(MOD(I,2).EQ.1)RETURN	12850
IP=I/2+1	12860
TOW(IP)=FLOAT(I)*DT	12870
DO 120 JJ=1,NPRINT	12880
TEMP=JPRINT(JJ)	12890
J10=ABS(TEMP)	12900
K10=KPRINT(JJ)	12910
DO 110 L=1,8	12920
IF(JPRINT(JJ).LT.0)GO TO 105	12930
VAR(JJ,IP,L)=T(J10,K10,L)	12940
GO TO 110	12950
105 VAR(JJ,IP,L)=T(I+2,K10,L)-T(J10,K10,L)	12960
110 CONTINUE	12970

120	CONTINUE	12980
300	IF(I .LT. ITILL) RETURN	12990
	DO 340 JJ=1,NPRINT	13000
	WRITE (6,310) JJ,R(JJ),Z(JJ)	13010
C 310	FORMAT(///5X,I2,5X*THE POINT: R =*,F7.4,2X*Z =*,F7.4/5X,45(1E-)	13020
C	+ //7X*TOW*,6X*SIR*,6X*SITT*,6X*SIZE*,6X*SIR*,7X*UR*,6X*UR*/	13030
C	+72(1E-))	13040
310	FORMAT(I2,2F7.4)	13050
	DO 330 KK=1,IP	13060
	WRITE (6,320) TOW(KK),VAR(JJ,KK,7),VAR(JJ,KK,1),VAR(JJ,KK,4),	13070
	+VAR(JJ,KK,8),VAR(JJ,KK,2),VAR(JJ,KK,5)	13080
320	FORMAT(7(3X,F7.4))	13090
330	CONTINUE	13100
C	WRITE (6,335)	13110
335	FORMAT(72(1E-)/)	13120
340	CONTINUE	13130
	RETURN	13140
	END	13150

APPENDIX D
PROGRAM LISTING OF TRES3

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PROGRAM TRES3(OUTPUT,TAPE10,TAPES)                                00010
C *****00020
C                                                                 00030
C CASE3 : TRANSIENT RESPONSE OF AN IMPINITELY LONG TUBE SUBJECTED 00040
C TO A LOAD OF FINITE WIDTH FROM THE INTERIOR. THE DATA OF      00050
C CASE2 IS RETRIEVED FROM TAPES FOR SUPERPOSITION.              00060
C                                                                 00070
C ARRAYS; TOTAL(J,K) - STRESSES AFTER SUPERPOSITION             00080
C TU(J,K) - DATA FROM TAPES AT EACH TIME STEP                 00090
C LOAD IS SHIFTED HALF WIDTH OF LOAD UPWARD.                   00100
C TL(J,K) - DATA FROM TAPES FOR EACH TIME STEP                00110
C LOAD IS SHIFTED DOWNWARD(Z+DIRECTION)                          00120
C                                                                 00130
C *****00140
C *****00150
C DIMENSION TOTAL(600,8),TU(600,8),TL(600,8),ROW(20),L(8)      00160
C                                                                 00170
C INPUT DATA                                                    00180
C WIDTH STANDS FOR LOAD WIDTH                                    00190
C SET NLOAD=1 IF DURATION OF LOAD IS PERMANENT                  00200
C NLOAD=2 IF DURATION OF LOAD IS FINITE                         00210
C                                                                 00220
C DATA WIDTH,NLOAD/0.04,1/                                     00230
C                                                                 00240
C CALL READ(WIDTH,HW,NLOAD,NWIDTH,NWH,JOE,DT,INDX,INDX1,RI,    00250
+          SIRR,IDT)                                           00260
C DO 1000 I=3,INDX,2                                           00270
C DO 10 J1=1,8                                                  00280
C READ(5,100)(ROW(J),J=1,20)                                   00290
10 WRITE(10,100)(ROW(J),J=1,20)                                00300
100 FORMAT(20A4)                                               00310
C NP=(INDX/2+3-JOE)/2                                          00320
C NP1=NP+1                                                      00330
C NP2=NP+2                                                      00340
C IF(JOE.EQ.1)LCI=NP1*(2*(I-1)+5)                              00350
C IF(JOE.EQ.2)LCI=(NP1+NP2)*(2*(I-1)+4)/2+NP2                00360
C LC=LCI-(2*(I-1)+5)                                           00370
C LN=2*NP+JOE-1                                                00380
C CALL RED(I,LCI,LC,LCA,NP,NP1,JOE,NWH,LN,TU,TL)               00390
C KK=2                                                           00400
C NC=2*(I-1)+5+NWIDTH                                          00410
C KT=0                                                           00420
C DO 12 J=1,NC                                                  00430
C NN=J-(NC+1)/2                                                00440
C Z=DT*FLOAT(NN)                                               00450
C KK=3-KK                                                       00460
C DO 12 K=KK,INDX1,2                                           00470
C KT=KT+1                                                       00480
C TOTAL(KT,2)=FLOAT(K-1)*DT+RI                                 00490

```

TOTAL(KT,1)=Z	00500
DO 12 M=3,8	00510
TOTAL(KT,M)=TU(KT,M)-TL(KT,M)	00520
IF(Z.EQ.HW)TOTAL(KT,M)=TOTAL(KT-LN*NWE,M)	00530
IF(Z.EQ.HW.A.M.EQ.8)TOTAL(KT,8)=TOTAL(KT,8)	00540
12 CONTINUE	00550
WRITE(10,200)	00560
JS=NP1	00570
JE=LCA-NP	00580
IF(JOE.EQ.1) GO TO 13	00590
JS=JS+MOD(NWE,2)	00600
JE=JE-MOD(NWE,2)	00610
13 DO 14 J1=JS,JE	00620
WRITE(10,201) (TOTAL(J1,M),M=1,8)	00630
IF(J1.EQ.JE) GO TO 14	00640
IF(TOTAL(J1,1).NE.TOTAL(J1+1,1)) WRITE(10,200)	00650
14 CONTINUE	00660
IF(I.EQ.INDX-1) WRITE(10,202)	00670
200 FORMAT()	00680
201 FORMAT(8F9.4)	00690
202 FORMAT(72(1E-))	00700
1000 CONTINUE	00710
STOP	00720
END	00730
C	00740
SUBROUTINE READ(WIDTH,HW,NLOAD,NWIDTH,NWE,JOE,DT,INDX,INDX1,RI,	00750
+ SIRR, IDT)	00760
C	00770
C	00780
C	00790
DIMENSION ROW(20),CI(5,8)	00800
REWIND 5	00810
IF(NLOAD.EQ.1) READ(5,100) RI,RO,SIRR,DT,INDX	00820
IF(NLOAD.EQ.2) READ(5,101) DTL,RI,RO,SIRR,DT,INDX	00830
100 FORMAT(//////////14X,F4.2/14X,F4.2/22X,F4.2/27X,F6.4/22X,I3)	00840
101 FORMAT(///24X,F5.2////////14X,F4.2/14X,F4.2/22X,F4.2/27X,F6.4/22X,	00850
+I3)	00860
NWIDTH=WIDTH/DT	00870
IF(MOD(NWIDTH,2).EQ.1)WRITE(10,199)	00880
199 FORMAT(* WIDTH OF LOAD IS UNSUITABLE *)	00890
NWE=NWIDTH/2	00900
WIDTH=2.0*DT*NWE	00910
HW=WIDTH/2.0	00920
NWIDTH=WIDTH/DT	00930
CALL DATE(NDATE)	00940
WRITE(10,200) NDATE	00950
IF(NLOAD.EQ.1) WRITE(10,201) WIDTH	00960
IF(NLOAD.EQ.2) WRITE(10,202) DTL,WIDTH	00970
200 FORMAT(A12/* TRANSIENT RESPONSE OF INFINITELY LONG *	00980
+*TUBE SUBJECT TO ABRUPTLY APPLIED LOAD(CASE 3)*//)	00990
201 FORMAT(* DURATION TIME OF LOAD = PERMANENT*/	01000
+* WIDTH OF LOAD =*,F4.2)	01010

202	FORMAT(* DURATION TIME OF LOAD =*,F3.2/ +* WIDTH OF LOAD =*,F4.2)	01020
	REWIND 5	01030
	READ(5,102)	01040
102	FORMAT(////)	01050
	DO 10 J1=1,14	01060
	READ(5,203)(ROW(J),J=1,20)	01070
10	WRITE(10,203)(ROW(J),J=1,20)	01080
203	FORMAT(20A4)	01090
	WRITE(10,204) RI,HW,RI,HW,RI,HW,HW,RI,HW,RI,HW	01100
204	FORMAT(*R=*,F3.1,*,Z<=*,F4.2,4X,*R=*,F3.1,*,Z=*,F4.2,2X, +*R=*,F3.1,*,=*,F4.2,*<Z>*,F4.2,2X,*R=*,F3.1,*,Z=*,F4.2,4X, +* R=*,F3.1,*,Z>*,F4.2/13(1H-),4X,13(1H-),2X,10(1H-),2X, +12(1H-),5X,12(1H-)/)	01110
	READ(5,103)	01120
103	FORMAT(//)	01130
	READ(5,104)((CI(J,K),J=1,3),K=1,8)	01140
104	FORMAT(3(7X,F7.4,12X))	01150
	READ(5,105)	01160
105	FORMAT()	01170
	DO 11 K=1,8	01180
	CI(4,K)=CI(2,K)	01190
11	CI(5,K)=CI(1,K)	01200
	CI(4,5)=CI(4,5)	01210
	CI(4,6)=CI(4,6)	01220
	CI(4,8)=CI(4,8)	01230
	WRITE(10,205)((CI(J,K),J=1,5),K=1,8)	01240
205	FORMAT(5(* SITT =*,F7.4,3X)/5(* UR =*,F7.4,3X)/ + 5(*DURDR =*,F7.4,3X)/5(* SIZZ =*,F7.4,3X)/ + 5(* UZ =*,F7.4,3X)/5(*DUZDZ =*,F7.4,3X)/ + 5(* SIRR =*,F7.4,3X)/5(* SIRZ =*,F7.4,3X)/82(1H-))	01250
	IDT=DTL/DT	01260
	INDX1=INDX/2+1	01270
	IND=INDX1-1	01280
	IF(MOD(IND,2).EQ.0) JOE=2	01290
	IF(MOD(IND,2).EQ.1) JOE=1	01300
	RETURN	01310
	END	01320
C		01330
	SUBROUTINE RED(I,LCI,LC,LCA,NP,NP1,JOE,NWE,IN,TU,TL)	01340
C		01350
	PURPOSE; TO READ THE VARIABLES FROM TAPES AND STORE THEM	01360
C		01370
	IN THE ARRAY T(J,K)	01380
C		01390
	DIMENSION TU(600,8),TL(600,8),TT(8)	01400
	NWH=NWE/2	01410
	IF(JOE.EQ.1)LCA=LC+2*NP*NWE	01420
	IF(JOE.EQ.2)LCA=LC+(NWE*NP+NWH)*2+MOD(NWE,2)	01430
	KF=LN*NWE+1	01440
	K1=KF	01450
10	READ(5,100)(TT(J),J=1,8)	01460
100	FORMAT(8F9.4)	01470
		01480
		01490
		01500
		01510
		01520
		01530

IF(TT(2).EQ.0.0) K1=K1-1	01540
IF(TT(2).EQ.0.0) GO TO 11	01550
DO 12 L=1,8	01560
TL(K1,L)=TT(L)	01570
TU(K1-KF+1,L)=TT(L)	01580
12 IF(K1.GT.LCA-LN=NNN) TU(K1,L)=TT(L)	01590
11 K1=K1+1	01600
LCI=LCI-1	01610
IF(LCI.GT.0) GO TO 10	01620
RETURN	01630
END	01640

APPENDIX E

PROGRAM LISTING OF TRES4

L
I
I


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IF(DP.GT.1.0E-5) WRITE(15,500)                                00500
500 FORMAT(// * DATA IN TAPE10 IS NOT SUITABLE TO THIS PROBLEM*//) 00510
    CALL DATE(NDATE)                                          00520
    WRITE(15,200) NDATE,SPEED,WIDTH                          00530
200 FORMAT(A12/* TRANSIENT RESPONSE OF INFINITELY LONG TUBE SUBJECT* 00540
    ** TO IMPULSIVE TRAVELLING LOAD (CASE 4)*//              00550
    ** SPEED OF TRAVELLING LOAD =*,F4.2/* WIDTH OF LOAD =*,F4.2) 00560
    REWIND 10                                                00570
    READ(10,103)                                             00580
103 FORMAT(////)                                           00590
    DO 1 I=1,26                                             00600
    READ(10,104)(ROW(J),J=1,20)                             00610
    1 WRITE(15,104)(ROW(J),J=1,20)                          00620
104 FORMAT(20A4)                                           00630
    RETURN                                                  00640
    END                                                      00650
C                                                            00660
    SUBROUTINE SUPO(NP,NSP1,JOE,WIDTH,RI,DT,INDX)           00670
C-----                                                    00680
C    PURPOSE; TO FIX THE SCHEME OF SUPERPOSITION           00690
C-----                                                    00700
    COMMON CI(5,6),T(600,8,5)                               00710
    DIMENSION ROW(20)                                       00720
    NSP=NSP1-1                                              00730
    NP1=NP+1                                                00740
    NP2=NP+2                                                00750
    NWIDTH=WIDTH/DT                                         00760
    DO 1000 I=3,INDX,2                                       00770
    NT=(FLOAT(I)-1.999)/FLOAT(NSP)                          00780
    NI=(I-NSP*NT)/2                                         00790
    NSM=(NSP1+1)/2                                          00800
    DO 10 L=1,8                                             00810
    READ(10,100)(ROW(J),J=1,20)                             00820
    10 WRITE(15,100)(ROW(J),J=1,20)                         00830
100 FORMAT(20A4)                                           00840
    NWH=NWIDTH/2                                           00850
    IF(JOE.LE.1)LCI=NP1*(2*(I-1)+3+NWIDTH)                 00860
    IF(JOE.EQ.2)LCI=(NP1+NP2)*(2*(I-1)+2+NWIDTH)/2+NP2   00870
    LC=LCI-(2*(I-1)+3+NWIDTH)                              00880
    CALL RED(I,NI,LCI,LC,NSP1,NSM)                          00890
    IF(I.LT.NSP1) GO TO 1000                                00900
    IF(I.GT.NSP1) GO TO 14                                   00910
    CALL INT(RI,DT,NI,NSP1,LC,WIDTH)                        00920
    GO TO 1000                                              00930
14 IF(JOE.LE.1)LCB=LC-2*NP*NSP                             00940
    IF(JOE.EQ.2)LCB=LC-(NP+NP1)*NSP                       00950
    DO 23 J1=1,LCB                                          00960
23 T(J1,1,NI)=T(J1,1,NI)+2.0*DT                          00970
    DO 24 JA=1,LC                                           00980
    DO 24 JB=1,LCB                                          00990
    IF(T(JB,1,NI).NE.T(JA,1,NSM).OR.T(JB,2,NI).NE.T(JA,2,NSM)) 01000
    +GO TO 24                                               01010

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DO 25 L=3,8	01020
25 T(JA,L,NSM)=T(JA,L,NSM)+T(JB,L,NI)	01030
24 CONTINUE	01040
WRITE(15,200)	01080
DO 26 JD=1,LC	01060
DO 27 L=1,8	01070
27 T(JD,L,NI)=T(JD,L,NSM)	01080
WRITE(15,101)(T(JD,M,NI),M=1,8)	01090
101 FORMAT(8F9.4)	01100
IF(JD.EQ.LC) GO TO 26	01110
IF(T(JD,1,NSM).NE.T(JD+1,1,NSM)) WRITE(15,200)	01120
200 FORMAT()	01130
26 CONTINUE	01140
IF(I.EQ.INDX-1) WRITE(15,201)	01150
201 FORMAT(72(1H-))	01160
1000 CONTINUE	01170
RETURN	01180
END	01190
C	01200
SUBROUTINE RED(I,NI,LCI,LC,NSP1,NSM)	01210
C	01220
C PURPOSE; TO READ DATA FROM TAPE10	01230
C	01240
COMMON CI(5,6),T(600,8,5)	01250
DIMENSION TT(8)	01260
K1=1	01270
13 READ(10,101)(TT(J),J=1,8)	01280
101 FORMAT(8F9.4)	01290
IF(TT(2).EQ.0.0) K1=K1-1	01300
IF(TT(2).EQ.0.0) GO TO 11	01310
DO 12 L=1,8	01320
IF(I.LE.NSP1) T(K1,L,NI)=TT(L)	01330
12 IF(I.GT.NSP1) T(K1,L,NSM)=TT(L)	01340
11 K1=K1+1	01350
LCI=LCI-1	01360
IF(I.LT.NSP1.AND.TT(2).NE.0.0) WRITE(15,101)(TT(J),J=1,8)	01370
IF(I.LT.NSP1.AND.TT(2).EQ.0.0) WRITE(15,200)	01380
200 FORMAT()	01390
IF(LCI.GT.0) GO TO 13	01400
RETURN	01410
END	01420
C	01430
SUBROUTINE INT(RI,DT,NI,NSP1,LC,WIDTH)	01440
C	01450
C PURPOSE; TO SUPERPOSE THE INITIAL CONDITIONS	01460
C	01470
COMMON CI(5,6),T(600,8,5)	01480
WL=WIDTH/2.0+2.0*DT	01490
WU=-WIDTH/2.0+2.0*DT	01500
DO 10 J1=1,LC	01510
IF(T(J1,2,NI).NE.RI) GO TO 10	01520
IF(T(J1,1,NI).GE.WU) GO TO 11	01530

DO 12 L=3,8	01540
12 T(J1,L,NI)=T(J1,L,NI)+CI(1,L-2)	01550
GO TO 10	01560
11 IF(T(J1,1,NI).GT.WU) GO TO 13	01570
DO 14 L=3,8	01580
14 T(J1,L,NI)=T(J1,L,NI)+CI(2,L-2)	01590
GO TO 10	01600
13 IF(T(J1,1,NI).GE.WL) GO TO 15	01610
DO 16 L=3,8	01620
16 T(J1,L,NI)=T(J1,L,NI)+CI(3,L-2)	01630
GO TO 10	01640
15 IF(T(J1,1,NI).GT.WL) GO TO 17	01650
DO 18 L=3,8	01660
18 T(J1,L,NI)=T(J1,L,NI)+CI(4,L-2)	01670
GO TO 10	01680
17 DO 19 L=3,8	01690
19 T(J1,L,NI)=T(J1,L,NI)+CI(5,L-2)	01700
10 CONTINUE	01710
WRITE(15,200)	01720
DO 20 J1=1,LC	01730
WRITE(15,101)(T(J1,J,NI),J=1,8)	01740
101 FORMAT(8F9.4)	01750
IF(J1.EQ.LC) GO TO 20	01760
IF(T(J1,1,NI).NE.T(J1+1,1,NI)) WRITE(15,200)	01770
20 CONTINUE	01780
200 FORMAT()	01790
RETURN	01800
END	01810

APPENDIX F

PROGRAM LISTING OF RECTINP

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PROGRAM RECTINP(OUTPUT,TAPES,TAPE6)                                00010
C *****00020
C                                                                 00030
C CASE 2 : TRANSIENT RESPONSE OF AN INFINITELY LONG TUBE SUBJECTED 00040
C TO A RECTANGULAR INPUT LOAD. THE DATA OF CASE 2 00050
C IS RETRIEVED FROM TAPES FOR SUPERPOSITION. 00060
C                                                                 00070
C *****00080
C COMMON CI(3,6),T(600,8,5) 00090
C                                                                 00100
C INPUT DATA 00110
C NTS = NUMBER OF TIME STEPS WHEN THE CONSTANT LOAD IS REMOVED 00120
C DURATION OF LOADING IS GIVEN BY DTL=2*DT*NTS 00130
C (WHERE DT IS STEP SIZE OF INTEGRATION) 00140
C                                                                 00150
C IN CASE 4 SPEED OF TRAVELLING LOAD IS GIVEN BY SPEED=1/NTS 00160
C                                                                 00170
C DATA NTS/2/ 00180
C                                                                 00190
C NS=2*NTS-1 00200
C CALL READ(NS,RI,RO,DT,INDX) 00210
C INDX1=INDX/2+1 00220
C NSP1=NS+2 00230
C IND=INDX1-1 00240
C LN=IND+1 00250
C IF(MOD(IND,2).EQ.0) JOE=2 00260
C IF(MOD(IND,2).EQ.1) JOE=1 00270
C NP=(INDX/2+3-JOE)/2 00280
C NP1=NP+1 00290
C CALL SUPO(NP,NSP1,JOE,RI,DT,INDX,LN,NTS) 00300
C STOP 00310
C END 00320
C                                                                 00330
C SUBROUTINE READ(NS,RI,RO,DT,INDX) 00340
C -----00350
C PURPOSE; TO READ THE INITIAL CONDITIONS FROM TAPES AND PRINT 00360
C THEM ON TAPE6 00370
C -----00380
C COMMON CI(3,6) 00390
C DIMENSION ROW(20) 00400
C REWIND 5 00410
C REWIND 6 00420
C READ(5,100)RI,RO,DT,INDX 00430
100 FORMAT(/////////14X,F4.2/14X,F4.2//27X,F6.4/22X,I3) 00440
C READ(5,101)(CI(I,2),I=1,3),(CI(I,5),I=1,3),(CI(I,3),I=1,3), 00450
C + (CI(I,6),I=1,3),(CI(I,1),I=1,3),(CI(I,4),I=1,3) 00460
101 FORMAT(/////////3(7X,F7.4,12X)/3(7X,F7.4,12X)// 00470
C +3(7X,F7.4,12X)/3(7X,F7.4,12X)//3(7X,F7.4,12X)/3(7X,F7.4,12X)) 00480
C DTL=DT*(NS+1) 00490

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CALL DATE(NDATE)                                00600
WRITE(6,200) NDATE,DTL                          00610
200 FORMAT(A12/* TRANSIENT RESPONSE OF INFINITELY LONG TUBE SUBJECT* 00620
+* TO RECTANGULAR INPUT LOAD (CASE 2)*//         00630
+* DURATION TIME OF LOAD =*,FS.2/              00640
+* WIDTH OF LOAD = SEMI-INFINITE*)             00650
REWIND 5                                         00660
READ(5,103)                                      00670
103 FORMAT(////)                                00680
DO 1 I=1,26                                     00690
READ(5,104)(ROW(J),J=1,20)                     00600
1 WRITE(6,104)(ROW(J),J=1,20)                  00610
104 FORMAT(20A4)                                00620
RETURN                                           00630
END                                              00640
C                                                00650
SUBROUTINE SUPO(NP,NSP1,JOE,RI,DT,INDX,LN,NTS)  00660
C-----00670
C PURPOSE; TO FIX THE SCHEME OF SUPERPOSITION 00680
C-----00690
COMMON CI(3,6),T(600,8,5)                      00700
DIMENSION ROW(20),TEM(600,8)                   00710
NSP=NSP1-1                                      00720
NP1=NP+1                                        00730
NP2=NP+2                                        00740
DO 1000 I=3,INDX,2                             00750
NT=(FLOAT(I)-1.999)/FLOAT(NSP)                 00760
NI=(I-NSP*NT)/2                                00770
NSM=(NSP1+1)/2                                  00780
DO 10 L=1,8                                     00790
READ(5,100)(ROW(J),J=1,20)                     00800
10 WRITE(6,100)(ROW(J),J=1,20)                  00810
100 FORMAT(20A4)                                00820
IF(JOE.LE.1)LCI=NP1*(2*(I-1)+5)                00830
IF(JOE.EQ.2)LCI=(NP1+NP2)*(2*(I-1)+4)/2+NP2   00840
LC=LCI-(2*(I-1)+5)                             00850
CALL RED(I,NI,LCI,LC,NSP1,NSM,LN,DT,NTS)       00860
IF(I.LT.NSP1) GO TO 1000                       00870
IF(I.GT.NSP1) GO TO 14                          00880
CALL INT(RI,DT,NI,NSP1,LC,NSM)                 00890
GO TO 1000                                      00900
14 IF(JOE.LE.1)LCB=LC-2*NP*NSP                 00910
IF(JOE.EQ.2)LCB=LC-(NP+NP1)*NSP               00920
DO 23 L1=1,LC                                  00930
DO 23 L=1,8                                     00940
23 TEM(L1,L)=T(L1,L,NSM)                       00950
LCB=LCB+NTS*LN                                  00960
DO 24 JA=1,LC                                  00970
DO 24 JB=1,LCB                                  00980
IF(T(JB,1,NI).NE.T(JA,1,NSM).OR.T(JB,2,NI).NE.T(JA,2,NSM)) 00990
+GO TO 24                                       01000
DO 25 JC=3,8                                    01010

```

25	T(JA,JC,NSM)=T(JA,JC,NSM)-T(JB,JC,NI)	01020
24	CONTINUE	01030
	WRITE(6,200)	01040
	DO 26 JD=1,LC	01050
	DO 27 L=1,8	01060
27	T(JD,L,NI)=TEM(JD,L)	01070
	IF(JD.LE.LC-LN) GO TO 28	01080
	DO 29 M=1,NTS	01090
	T(JD+M*LN,1,NI)=TEM(JD,1)+2*M*DT	01100
	DO 29 L=2,8	01110
29	T(JD+M*LN,L,NI)=TEM(JD,L)	01120
28	WRITE(6,101)(T(JD,M,NSM),M=1,8)	01130
101	FORMAT(8F9.4)	01140
	IF(JD.EQ.LC) GO TO 26	01150
	IF(T(JD,1,NSM).NE.T(JD+1,1,NSM)) WRITE(6,200)	01160
200	FORMAT()	01170
26	CONTINUE	01180
	IF(I.EQ.INDX-1) WRITE(6,201)	01190
201	FORMAT(72(1E-))	01200
1000	CONTINUE	01210
	RETURN	01220
	END	01230
C		01240
	SUBROUTINE RED(I,NI,LCI,LC,NSP1,NSM,LN,DT,NTS)	01250
C		01260
C	PURPOSE; TO READ DATA FROM TAPE 5	01270
C		01280
	COMMON CI(3,6),T(600,8,5)	01290
	DIMENSION TT(8)	01300
	K1=1	01310
13	READ(5,101)(TT(J),J=1,8)	01320
101	FORMAT(8F9.4)	01330
	IF(TT(2).EQ.0.0) K1=K1-1	01340
	IF(TT(2).EQ.0.0) GO TO 11	01350
	DO 12 L=1,8	01360
	IF(I.LE.NSP1) T(K1,L,NI)=TT(L)	01370
12	IF(I.GT.NSP1) T(K1,L,NSM)=TT(L)	01380
	IF(K1.LE.LC-LN) GO TO 11	01390
	DO 14 M=1,NTS	01400
	T(K1+M*LN,1,NI)=TT(1)+M*2*DT	01410
	DO 14 L=2,8	01420
14	T(K1+M*LN,L,NI)=TT(L)	01430
11	K1=K1+1	01440
	LCI=LCI-1	01450
	IF(I.LT.NSP1.AND.TT(2).NE.0.0) WRITE(6,101)(TT(J),J=1,8)	01460
	IF(I.LT.NSP1.AND.TT(2).EQ.0.0) WRITE(6,200)	01470
200	FORMAT()	01480
15	IF(LCI.GT.0) GO TO 13	01490
	RETURN	01500
	END	01510
C		01520
	SUBROUTINE INT(RI,DT,NI,NSP1,LC,NSM)	01530

C		01540
C	PURPOSE; TO SUPERPOSE THE INITIAL CONDITIONS	01550
C		01560
	COMMON CI(3,6),T(600,8,5)	01570
	DO 10 J1=1,LC	01580
	DO 17 L=1,8	01590
17	T(J1,L,NSM)=T(J1,L,NI)	01600
	IF(T(J1,2,NI).NE.RI) GO TO 10	01610
	IF(T(J1,1,NI).GE.0.) GO TO 11	01620
	DO 12 L=3,8	01630
12	T(J1,L,NSM)=T(J1,L,NI)-CI(1,L-2)	01640
	GO TO 10	01650
11	IF(T(J1,1,NI).GT.0.) GO TO 13	01660
	DO 14 L=3,8	01670
14	T(J1,L,NSM)=T(J1,L,NI)-CI(2,L-2)	01680
	GO TO 10	01690
13	DO 16 L=3,8	01700
16	T(J1,L,NSM)=T(J1,L,NI)-CI(3,L-2)	01710
10	CONTINUE	01720
	WRITE(6,200)	01730
	DO 20 J1=1,LC	01740
	WRITE(6,101)(T(J1,J,NSM),J=1,8)	01750
101	FORMAT(8F9.4)	01760
	IF(J1.EQ.LC) GO TO 20	01770
	IF(T(J1,1,NSM).NE.T(J1+1,1,NSM)) WRITE(6,200)	01780
20	CONTINUE	01790
200	FORMAT()	01800
	RETURN	01810
	END	01820
	END	

APPENDIX G

PROGRAM LISTING OF PLOTALL

```

PROGRAM PLOTALL(OUTPUT,TAPE1,TAPE3,TAPE5,TAPE7,TAPE10,TAPE12,      00010
+      TAPE15,TAPE17,TAPE20)                                     00020
C*****00030
C                                                                 00040
C   PURPOSE; TO PLOT THE VARIABLES AT SELECTED POINTS ACCORDING TO 00050
C   TIME ON THE CALCOMP PLOTTER AND TO PRINT THE VARIABLES        00060
C   AT THOSE POINTS ACCORDING TO TIME ON TAPE NTO                 00070
C                                                                 00080
C   THE DATA OF THIS PROGRAM IS RETRIEVED FROM TAPE NTI         00090
C                                                                 00100
C*****00110
C   DIMENSION Z(6),R(6),T(6,6,35),XA(35),YA(35)                   00120
C                                                                 00130
C   INPUT DATA                                                    00140
C   SET NCASE=1 IF NUMBER OF CASE IS 1                             00150
C       NCASE=2 FOR CASE2                                         00160
C       NCASE=3 FOR CASE3                                         00170
C       NCASE=4 FOR CASE4                                         00180
C                                                                 00190
C   SET NOPTION=1 IF ONLY PLOTTING IS NEEDED                      00200
C       NOPTION=2 IF ONLY PRINTING IS NEEDED                      00210
C       NOPTION=3 IF BOTH OF PLOTTING AND PRINTING ARE NEEDED    00220
C                                                                 00230
C                                                                 00240
C   SET NLOAD=1 IF DURATION OF LOAD IS PERMANENT                  00250
C       NLOAD=2 IF DURATION OF LOAD IS FINITE                     00260
C                                                                 00270
C   Z(K) STANDS FOR Z-COORDINATE OF THE SELECTED POINTS          00280
C   R(K) STANDS FOR R-COORDINATE OF THE SELECTED POINTS          00290
C   K; THE MAXIMUM NUMBER OF THE POINTS IS 6                     00300
C                                                                 00310
C   DATA NCASE,NOPTION,NLOAD/2,3,2/                              00320
C   DATA (Z(K),K=1,6)/0.00,0.00,0.00,0.08,0.08,0.08/           00330
C   DATA (R(K),K=1,6)/1.02,1.14,1.30,1.02,1.14,1.30/           00340
C                                                                 00350
C   IF(NCASE.EQ.1) NTI=1                                          00360
C   IF(NCASE.EQ.1) NTO=3                                          00370
C   IF(NCASE.EQ.2) NTI=5                                          00380
C   IF(NCASE.EQ.2) NTO=7                                          00390
C   IF(NCASE.EQ.3) NTI=10                                         00400
C   IF(NCASE.EQ.3) NTO=12                                         00410
C   IF(NCASE.EQ.4) NTI=15                                         00420
C   IF(NCASE.EQ.4) NTO=17                                         00430
C   REWIND NTI                                                     00440
C   IF(NCASE.NE.4) GO TO 1                                         00450
C   READ(15,100) DTL,WIDTH                                        00460
100 FORMAT(///27X,F4.2/16X,F4.2)                                  00470
C   GO TO 2                                                         00480
C   1 IF(NCASE.NE.2) GO TO 3                                       00490

```

	IF(NLOAD.EQ.1) READ(5,101)	00800
101	FORMAT(////)	00810
	IF(NLOAD.EQ.2) READ(5,102) DTL	00820
102	FORMAT(///25X,F4.2/)	00830
	GO TO 2	00840
	3 IF(NCASE.NE.3) GO TO 4	00850
	IF(NLOAD.EQ.1) READ(10,103) WIDTH	00860
103	FORMAT(////16X,F4.2)	00870
	IF(NLOAD.EQ.2) READ(10,104) DTL,WIDTH	00880
104	FORMAT(///25X,F4.2/16X,F4.2)	00890
	4 IF(NCASE.EQ.1) READ(1,101)	00900
	2 READ(NTI,105) RI,RO,DT,INDX	00910
105	FORMAT(////14X,F4.2/14X,F4.2//27X,F6.2/22X,I3)	00920
	INDX1=INDX/2+1	00930
	INDX2=INDX1+1	00940
	IND=INDX1-1	00950
	IF(MOD(IND,2).EQ.0) JOE=2	00960
	IF(MOD(IND,2).EQ.1) JOE=1	00970
	CALL READ(DT,INDX,INDX1,IND,JOE,WIDTH,Z,R,T,XA,	00980
	+ NCASE,NTI,NTO)	00990
	IF(NOPTION.NE.1) CALL WRIT(NLOAD,DTL,WIDTH,INDX1,IND,Z,R,T,XA,	00700
	+ NCASE,NTI,NTO)	00710
	IF(NOPTION.NE.2) CALL SBPL(NLOAD,DTL,WIDTH,DT,INDX1,IND,INDX2,	00720
	+ Z,R,T,XA,YA,NCASE,NTI,NTO)	00730
	STOP	00740
	END	00750
C		00760
	SUBROUTINE READ(DT,INDX,INDX1,IND,JOE,WIDTH,Z,R,T,XA,	00770
	+ NCASE,NTI,NTO)	00780
C		00790
C	PURPOSE; TO READ DATA FROM TAPE10	00800
C		00810
	DIMENSION Z(6),R(6),T(6,6,IND),XA(IND),TT(8),CI(5,6)	00820
C		00830
C	READ INTIAL CONDITIONS	00840
C		00850
	IF(NCASE.GT.1) GO TO 20	00860
	READ(1,100)(CI(K,2),K=2,3),(CI(K,5),K=2,3),(CI(K,3),K=2,3),	00870
	+ (CI(K,6),K=2,3),(CI(K,1),K=2,3),(CI(K,4),K=2,3)	00880
100	FORMAT(////////7X,F7.4,39X,F7.4/7X,F7.4,39X,F7.4//7X,	00890
	+ F7.4,39X,F7.4/7X,F7.4,39X,F7.4//7X,F7.4,39X,F7.4/	00900
	+ 7X,F7.4,39X,F7.4/)	00910
	GO TO 21	00920
20	IF(NCASE.GT.2) GO TO 22	00930
	READ(5,101)(CI(K,2),K=1,3),(CI(K,5),K=1,3),(CI(K,3),K=1,3),	00940
	+ (CI(K,6),K=1,3),(CI(K,1),K=1,3),(CI(K,4),K=1,3)	00950
101	FORMAT(////////7X,F7.4,2(19X,F7.4)/7X,F7.4,2(19X,F7.4)//7X,F7.4,	00960
	+ 2(19X,F7.4)/7X,F7.4,2(19X,F7.4)//7X,F7.4,2(19X,F7.4)/7X,	00970
	+ F7.4,2(19X,F7.4)/)	00980
	GO TO 21	00990
22	READ(NTI,102)(CI(K,2),K=1,5),(CI(K,5),K=1,5),(CI(K,3),K=1,5),	01000
	+ (CI(K,6),K=1,5),(CI(K,1),K=1,5),(CI(K,4),K=1,5)	01010

102	FORMAT(////////7X,F7.4,4(10X,F7.4)/7X,F7.4,4(10X,F7.4)//	01020
	+7X,F7.4,4(10X,F7.4)/7X,F7.4,4(10X,F7.4)//7X,F7.4,4(10X,F7.4)/	01030
	+7X,F7.4,4(10X,F7.4)//)	01040
21	DO 1 K=1,6	01050
	IF(R(K).NE.1.0) GO TO 1	01060
	WL=WIDTH/2.0	01070
	WU=-WL	01080
	IF(NCASE.LE.2) WU=0.0	01090
	IF(Z(K).GE.WU) GO TO 2	01100
	DO 3 L=1,6	01110
3	T(K,L,1)=CI(1,L)	01120
	GO TO 1	01130
2	IF(Z(K).GT.WU) GO TO 4	01140
	DO 5 L=1,6	01150
5	T(K,L,1)=CI(2,L)	01160
	GO TO 1	01170
4	IF(NCASE.LE.2) GO TO 23	01180
	IF(Z(K).GE.WL) GO TO 6	01190
23	DO 7 L=1,6	01200
7	T(K,L,1)=CI(3,L)	01210
	GO TO 1	01220
6	IF(Z(K).GT.WL) GO TO 8	01230
	DO 9 L=1,6	01240
9	T(K,L,1)=CI(4,L)	01250
	GO TO 1	01260
8	DO 10 L=1,6	01270
10	T(K,L,1)=CI(5,L)	01280
1	CONTINUE	01290
	NWIDTH=WIDTH/DT	01300
	NP=(INDX1+2-JOE)/2	01310
	NP1=NP+1	01320
	NP2=NP+2	01330
	DO 11 I=2,INDX	01340
	IF(NCASE.EQ.1) GO TO 40	01350
	IF(MOD(I,2).EQ.0) GO TO 11	01360
40	IF(NCASE.GT.1) GO TO 24	01370
	IF(JOE.EQ.1) LCI=NP1*(I+2)	01380
	RM=(I-2)*0.5+2.0	01390
	IM=RM	01400
	DM=RM-IM	01410
	IF(DM.EQ.0.0.A.JOE.EQ.2) LCI=(NP1+NP2)*IM	01420
	IF(DM.GT.0.0.A.JOE.EQ.2) LCI=(NP1+NP2)*IM+NP2	01430
	GO TO 25	01440
24	IF(NCASE.GT.2) GO TO 26	01450
	IF(JOE.EQ.1) LCI=NP1*(3+2*I)	01460
	IF(JOE.EQ.2) LCI=(NP1+NP2)*(I+1)+NP2	01470
	GO TO 25	01480
26	IF(NCASE.NE.3) GO TO 27	01490
	IF(JOE.EQ.1) LCI=NP1*(2*(I-1)+3+NWIDTH)	01500
	IF(JOE.EQ.2) LCI=(NP1+NP2)*(2*(I-1)+2+NWIDTH)/2+NP1	01510
	GO TO 25	01520
27	IF(JOE.EQ.1) LCI=NP1*(2*(I-1)+3+NWIDTH)	01530

	IF(JOB.EQ.2) LCI=(NP1+NP2)*(2*(I-1)+2+WIDTH)/2+NP1	01540
C		01550
C	READ STRESSES AT SPECIFIED POINTS	01560
C		01570
	25 READ(NTI,103)	01580
	103 FORMAT(////////)	01590
	12 READ(NTI,104)(TT(J),J=1,8)	01600
	104 FORMAT(8F9.4)	01610
	IH=I/2+1	01620
	DO 13 K=1,6	01630
	IF(Z(K).NE.TT(1).OR.R(K).NE.TT(2)) GO TO 13	01640
	XA(IH)=DT*(I-1)	01650
	DO 14 L=3,8	01660
	T(K,L-2,IH)=TT(L)	01670
	13 CONTINUE	01680
	LCI=LCI-1	01690
	IF(LCI.GT.0) GO TO 12	01700
	11 CONTINUE	01710
	RETURN	01720
	END	01730
C		01740
	SUBROUTINE WRIT(NLOAD,DTL,WIDTH,INDX,IND,Z,R,T,XA,NCASE,NTI,NTO)	01750
C		01760
C	PURPOSE; TO PRINT THE VARIABLES AT THE SPECIFIED POINTS	01770
C	ACCORDING TO TIME	01780
C		01790
	DIMENSION Z(6),R(6),T(6,6,IND),XA(IND)	01800
	REWIND NTO	01810
	IF(NCASE.GT.1) GO TO 10	01820
	WRITE(3,200) NCASE	01830
	200 FORMAT(5X,*CASE*,I2/5X,*DURATION TIME OF LOAD = PERMANENT*/5X,	01840
	+ *WIDTH OF LOAD = SEMI-INFINITE*)	01850
	GO TO 11	01860
	10 IF(NCASE.GT.2) GO TO 12	01870
	IF(NLOAD.EQ.1) WRITE(7,200) NCASE	01880
	IF(NLOAD.EQ.2) WRITE(7,201) NCASE,DTL	01890
	201 FORMAT(5X,*CASE*,I2/5X,*DURATION TIME OF LOAD = *,F6.3/5X,	01900
	+ *WIDTH OF LOAD = SEMI-INFINITE*)	01910
	GO TO 11	01920
	12 IF(NCASE.NE.3) GO TO 13	01930
	IF(NLOAD.EQ.1) WRITE(12,202) NCASE,WIDTH	01940
	202 FORMAT(5X,*CASE*,I2/5X,*DURATION TIME OF LOAD = PERMANENT*/5X,	01950
	+ *WIDTH OF LOAD =*,F6.3)	01960
	IF(NLOAD.EQ.2) WRITE(12,203) NCASE,DTL,WIDTH	01970
	203 FORMAT(5X,*CASE*,I2/5X,*DURATION TIME OF LOAD =*,F6.3/5X,	01980
	+ *WIDTH OF LOAD =*,F6.3)	01990
	GO TO 11	02000
	13 WRITE(17,204) NCASE,DTL,WIDTH	02010
	204 FORMAT(5X,*CASE*,I2/5X,*SPEED OF TRAVELLING LOAD =*,F6.3/5X,	02020
	+ *WIDTH OF LOAD =*,F6.3)	02030
	11 DO 1 K=1,6	02040
	WRITE(NTO,205) R(K),Z(K)	02050

APPENDIX H

PLOTS OF DEPENDENT VARIABLES

CASE 1
 DURATION TIME OF LOAD - PERMANENT
 WIDTH OF LOAD - SEMI-INFINITE

THE POINT R= 1.000 Z= 0.000

TOW	SIRR	SITT	SIZZ	SIRZ	UR	UZ
0.0000	1.0000	.1765	0.0000	0.0000	-1.0000	.1765
.0400	1.0000	.1391	0.0000	0.0000	-.9707	.1695
.0800	1.0000	.1029	0.0000	0.0000	-.9417	.1634
.1200	1.0000	.0678	0.0000	0.0000	-.9130	.1574
.1600	1.0000	.0337	0.0000	0.0000	-.8847	.1516
.2000	1.0000	.0008	0.0000	0.0000	-.8568	.1460
.2400	1.0000	-.0312	0.0000	0.0000	-.8293	.1405
.2800	1.0000	-.0621	0.0000	0.0000	-.8021	.1353
.3200	1.0000	-.0920	0.0000	0.0000	-.7755	.1302
.3600	1.0000	-.1208	0.0000	0.0000	-.7492	.1252
.4000	1.0000	-.1487	0.0000	0.0000	-.7235	.1205
.4400	1.0000	-.1757	0.0000	0.0000	-.6982	.1159
.4800	1.0000	-.2017	0.0000	0.0000	-.6734	.1115
.5200	1.0000	-.2267	0.0000	0.0000	-.6491	.1072
.5600	1.0000	-.2508	0.0000	0.0000	-.6253	.1031

THE POINT R= 1.120 Z= 0.000

TOW	SIRR	SITT	SIZZ	SIRZ	UR	UZ
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
.0400	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
.0800	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
.1200	.8840	.1560	0.0000	0.0000	-.8840	.1667
.1600	.8798	.1259	0.0000	0.0000	-.8567	.1597
.2000	.8756	.0968	0.0000	0.0000	-.8298	.1534
.2400	.8715	.0685	0.0000	0.0000	-.8032	.1474
.2800	.8674	.0412	0.0000	0.0000	-.7771	.1418
.3200	.8634	.0148	0.0000	0.0000	-.7514	.1364
.3600	.8594	-.0108	0.0000	0.0000	-.7261	.1313
.4000	.8556	-.0355	0.0000	0.0000	-.7012	.1264
.4400	.8517	-.0594	0.0000	0.0000	-.6769	.1218
.4800	-.0360	-.2385	0.0000	0.0000	-1.5369	-.0494
.5200	-.0349	-.2902	0.0000	0.0000	-1.5319	-.0247
.5600	-.0246	-.3402	0.0000	0.0000	-1.5181	-.0665

THE POINT R= 1.300 Z= .020

TOW	SIRR	SITT	SIZZ	SIRZ	UR	UZ
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
.0400	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
.0800	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
.1200	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
.1600	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
.2000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
.2400	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
.2800	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
.3200	0.0000	-.0299	.0038	0.0000	-1.4927	-.0088
.3600	0.0000	-.0743	-.0038	0.0000	-1.4704	-.0088
.4000	0.0000	-.1179	-.0110	0.0000	-1.4529	-.0236
.4400	0.0000	-.1588	-.0028	0.0000	-1.4373	-.0373
.4800	0.0000	-.2006	-.0040	0.0000	-1.4225	-.0439
.5200	0.0000	-.2419	-.0041	0.0000	-1.4085	-.0518
.5600	0.0000	-.2828	-.0041	0.0000	-1.3951	-.0597

THE POINT R= 1.000 Z= .080

TOW	SIRR	SITT	SIZZ	SIRZ	UR	UZ
0.0000	1.0000	.1765	.1765	0.0000	-1.0000	0.0000
.0400	1.0000	.1382	.1707	0.0000	-.9768	0.0000
.0800	1.0000	.0797	.0261	0.0000	-.9633	.1383
.1200	1.0000	.0442	.0268	0.0000	-.9200	.1316
.1600	1.0000	.0101	.0271	0.0000	-.8800	.1254
.2000	1.0000	-.0225	.0274	0.0000	-.8429	.1194
.2400	1.0000	-.0537	.0277	0.0000	-.8081	.1135
.2800	1.0000	-.0836	.0280	0.0000	-.7751	.1079
.3200	1.0000	-.1124	.0283	0.0000	-.7439	.1024
.3600	1.0000	-.1399	.0287	0.0000	-.7141	.0972
.4000	1.0000	-.1664	.0290	0.0000	-.6856	.0921
.4400	1.0000	-.1918	.0293	0.0000	-.6582	.0872
.4800	1.0000	-.2162	.0296	0.0000	-.6319	.0824
.5200	1.0000	-.2396	.0299	0.0000	-.6066	.0779
.5600	1.0000	-.2621	.0302	0.0000	-.5822	.0735

THE POINT R= 1.120 Z= .080

TOW	SIRR	SITT	SIZZ	SIRZ	UR	UZ
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
.0400	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
.0800	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
.1200	.9449	.1667	.1667	0.0000	-.9449	0.0000
.1600	.9107	.0909	.0181	-.0096	-.8660	.1408
.2000	.8985	.0623	.0231	-.0208	-.8244	.1299
.2400	.8895	.0338	.0237	-.0208	-.7892	.1235
.2800	.8816	.0065	.0240	-.0192	-.7562	.1177
.3200	.8746	-.0195	.0243	-.0176	-.7250	.1122
.3600	.8682	-.0445	.0245	-.0163	-.6955	.1070
.4000	.8623	-.0684	.0247	-.0153	-.6674	.1020
.4400	.8569	-.0913	.0248	-.0143	-.6406	.0972
.4800	-.0931	-.2800	-.1417	-.0135	-1.5597	.0926
.5200	-.0492	-.3358	-.0050	-.0153	-1.5203	-.0532
.5600	-.0841	-.3709	-.0180	-.0268	-1.5425	-.0482

THE POINT R= 1.300 Z= .060

TOW	SIRR	SITT	SIZZ	SIRZ	UR	UZ
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
.0400	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
.0800	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
.1200	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
.1600	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
.2000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
.2400	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
.2800	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
.3200	0.0000	-.0422	.0105	0.0000	-1.5093	-.0164
.3600	0.0000	-.2978	-.5008	0.0000	-1.5329	.4742
.4000	0.0000	-.2694	-.0174	0.0000	-1.4902	-.0148
.4400	0.0000	-.3123	-.0175	0.0000	-1.4547	-.0227
.4800	0.0000	-.3533	-.0107	0.0000	-1.4312	-.0374
.5200	0.0000	-.3949	-.0120	0.0000	-1.4102	-.0442
.5600	0.0000	-.4357	-.0120	0.0000	-1.3921	-.0521

CASE 1

R=1.00 Z=0.00

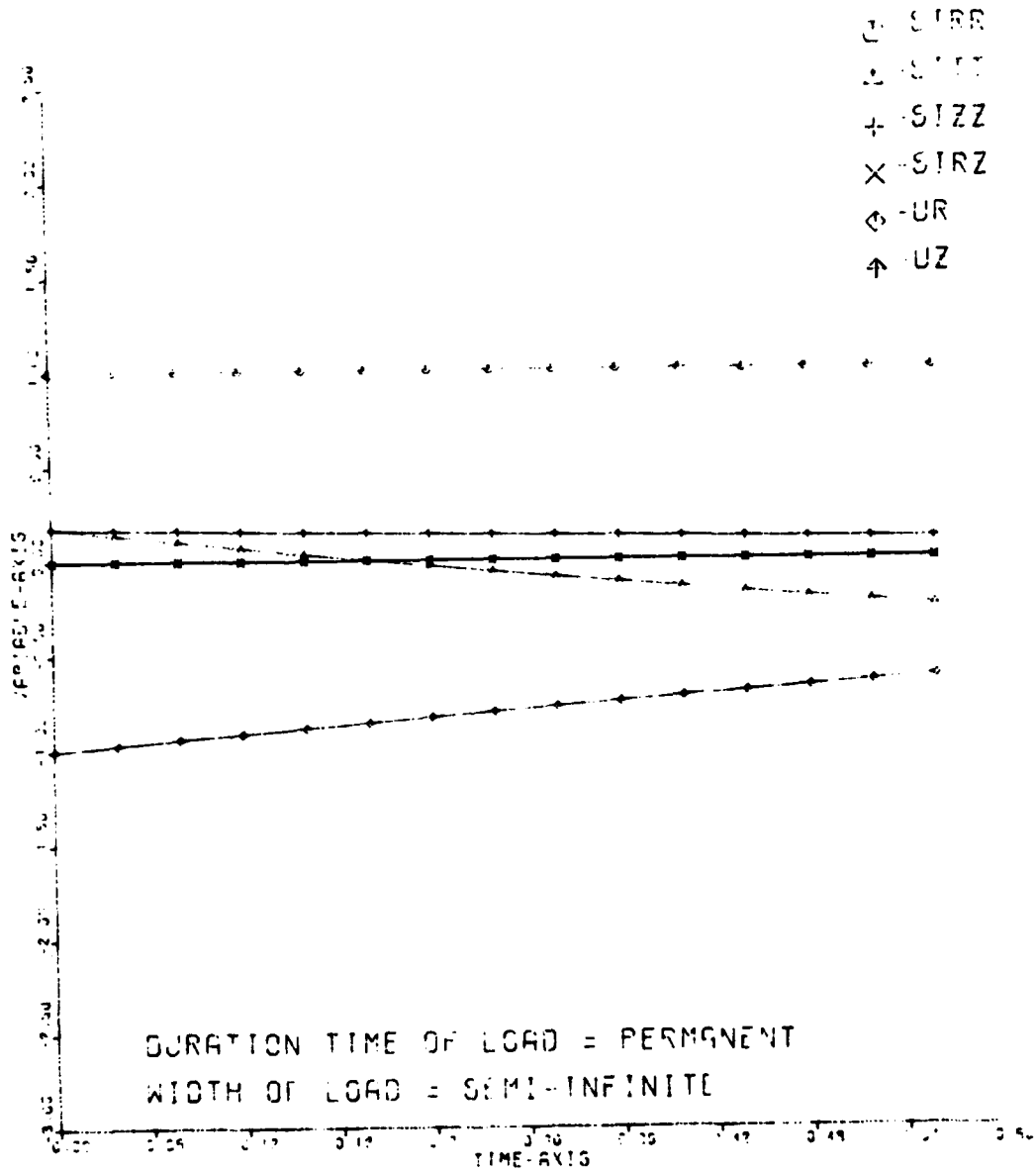


Figure 10

CASE 1

R=1.14 Z=0.00

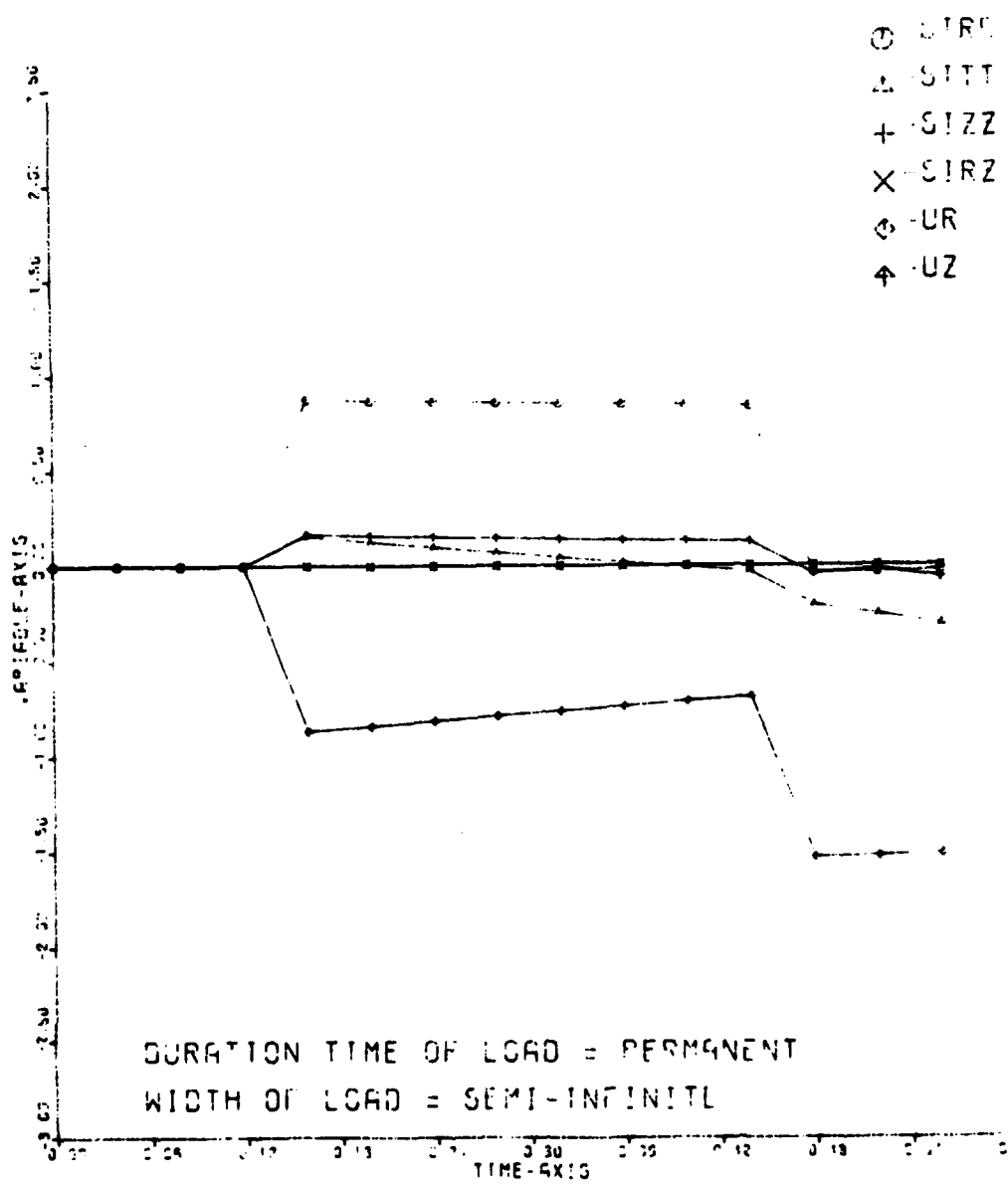


Figure 11

CASE 1

R=1.35 Z=0.00

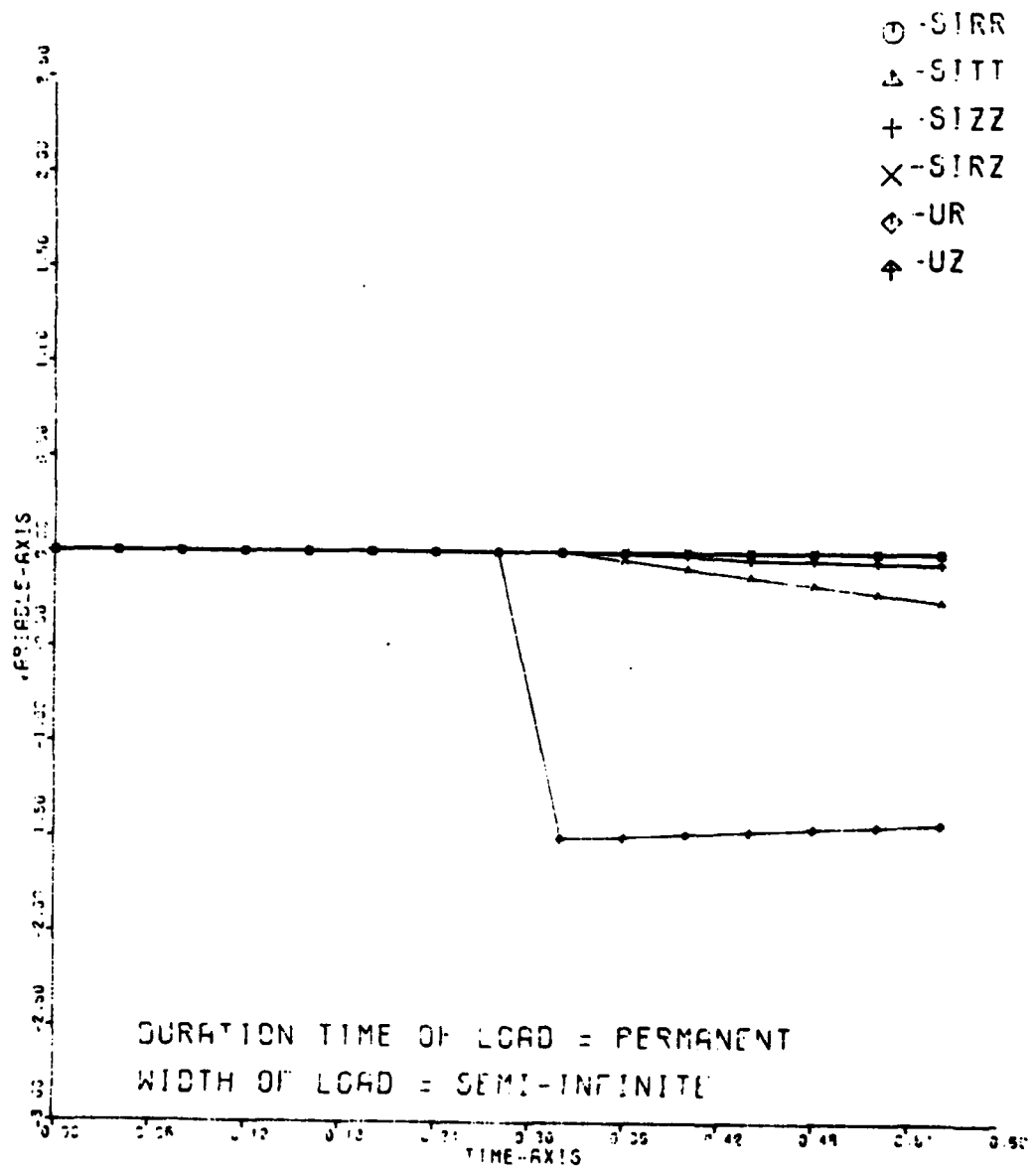


Figure 12

CASE 1

R=1.00 Z=0.08

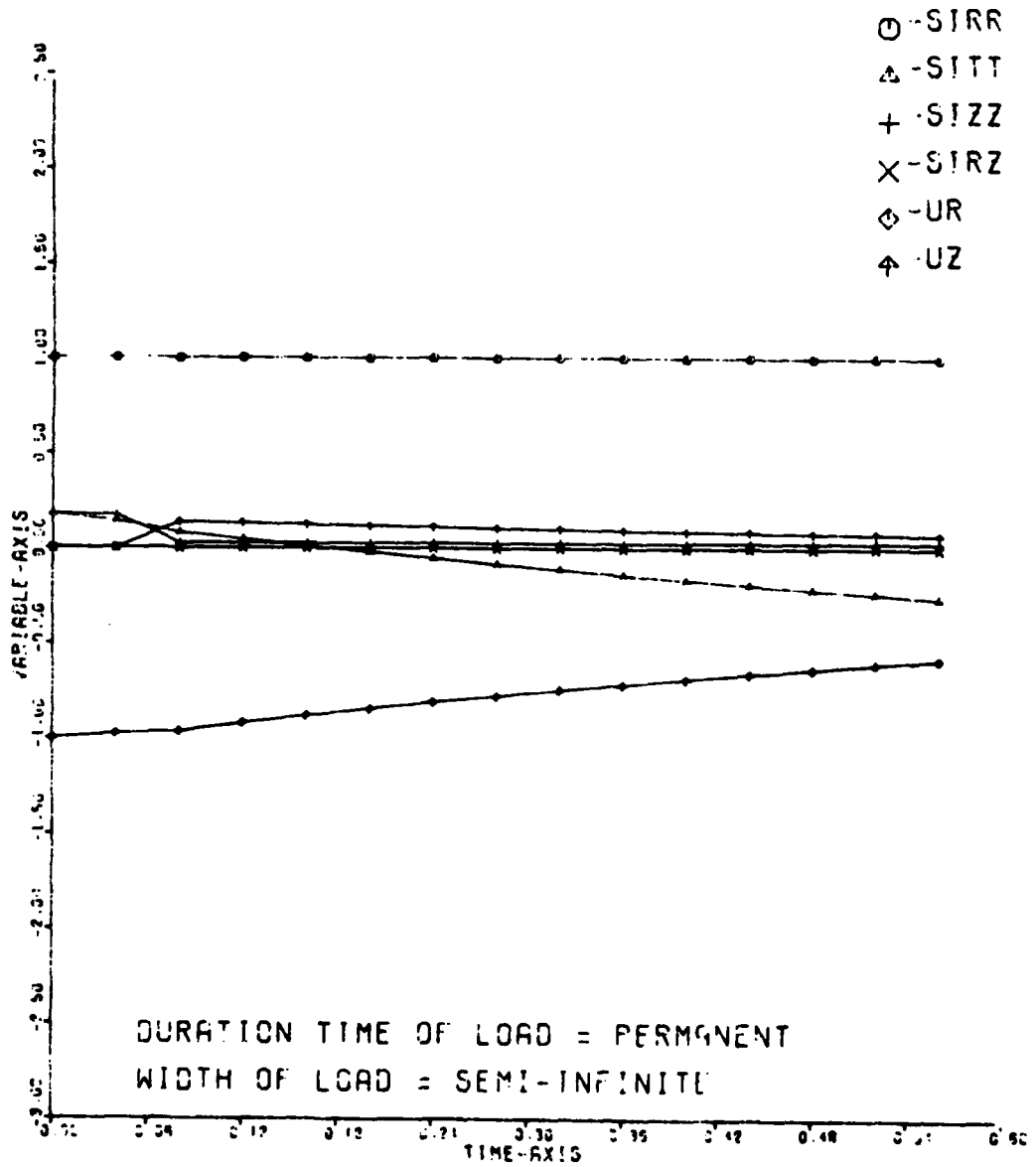


Figure 13

CASE 1

R=1.14 Z=0.08

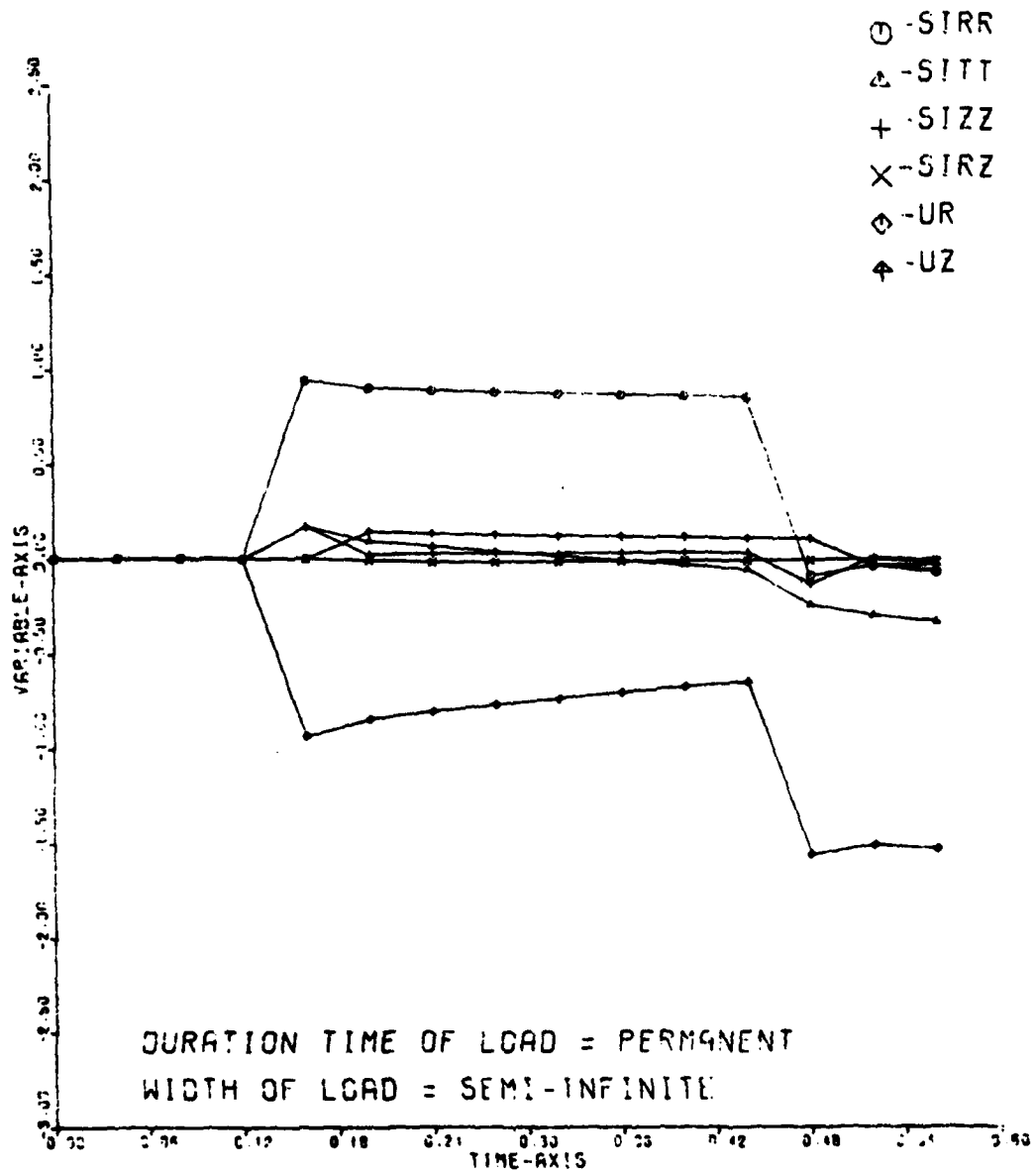


Figure 14

CASE 1

R=1.30 Z=0.08

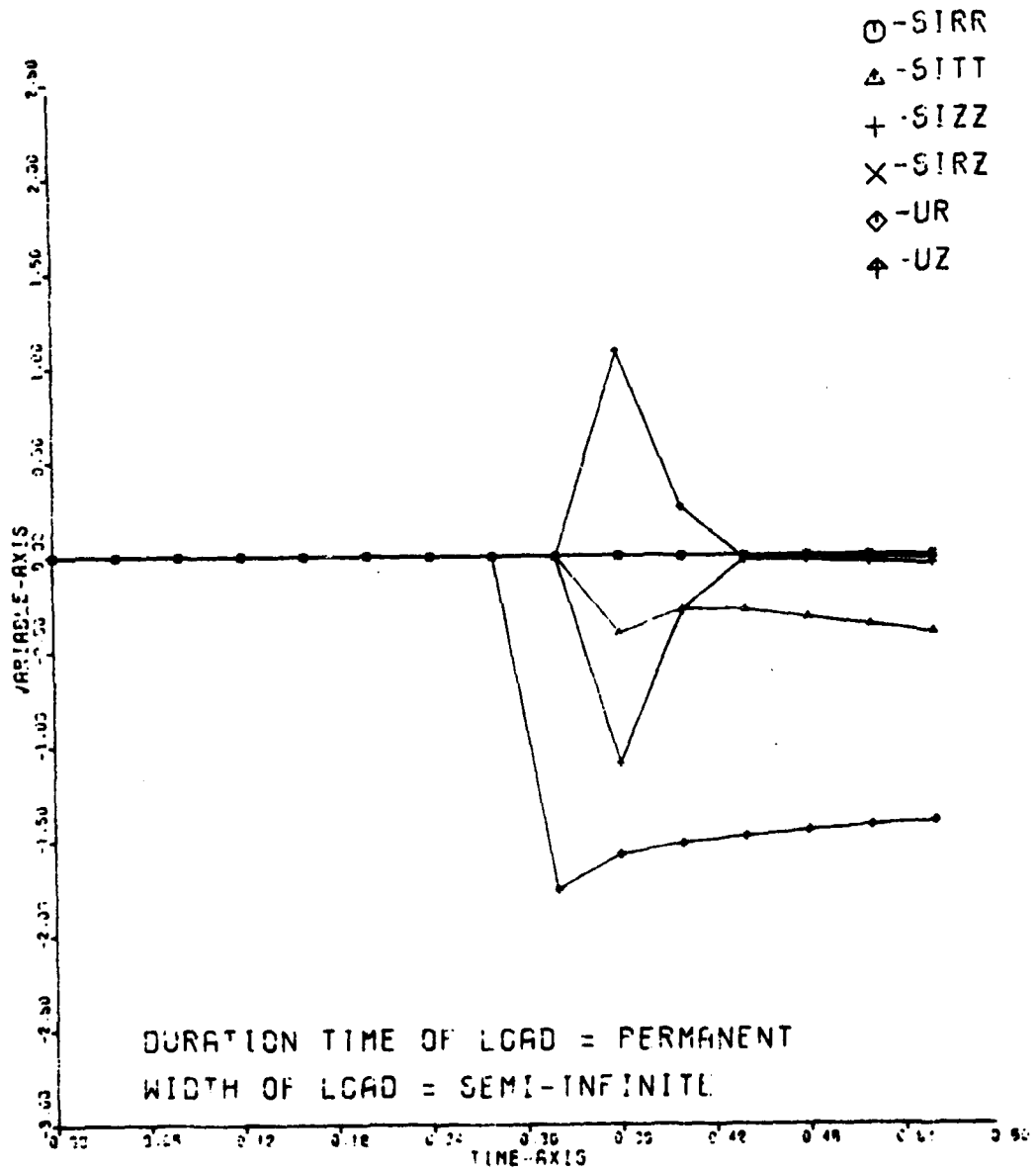


Figure 15

CASE 2

R=1.00 Z=0.00

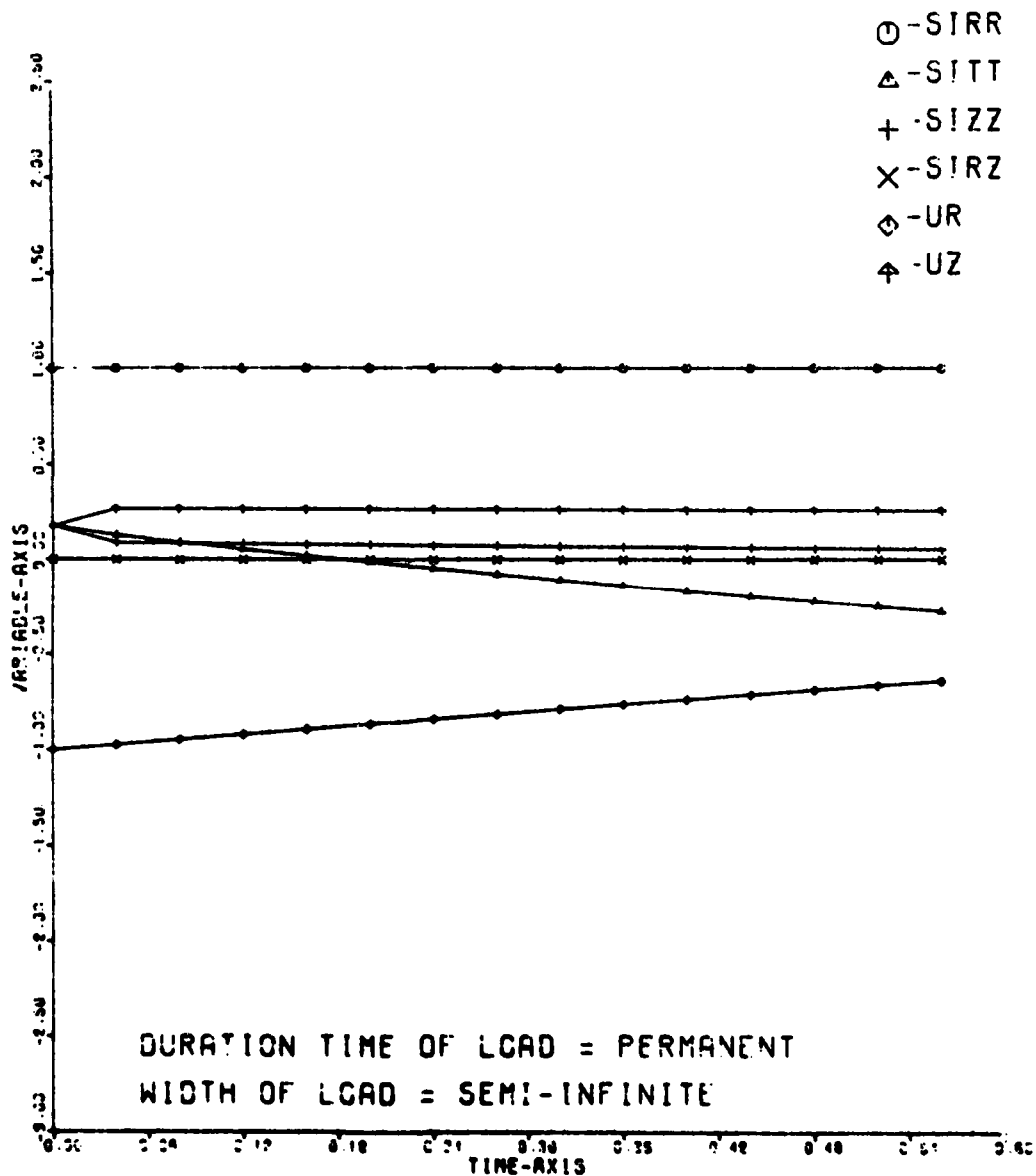


Figure 16

CASE 2

R=1.12 Z=0.00

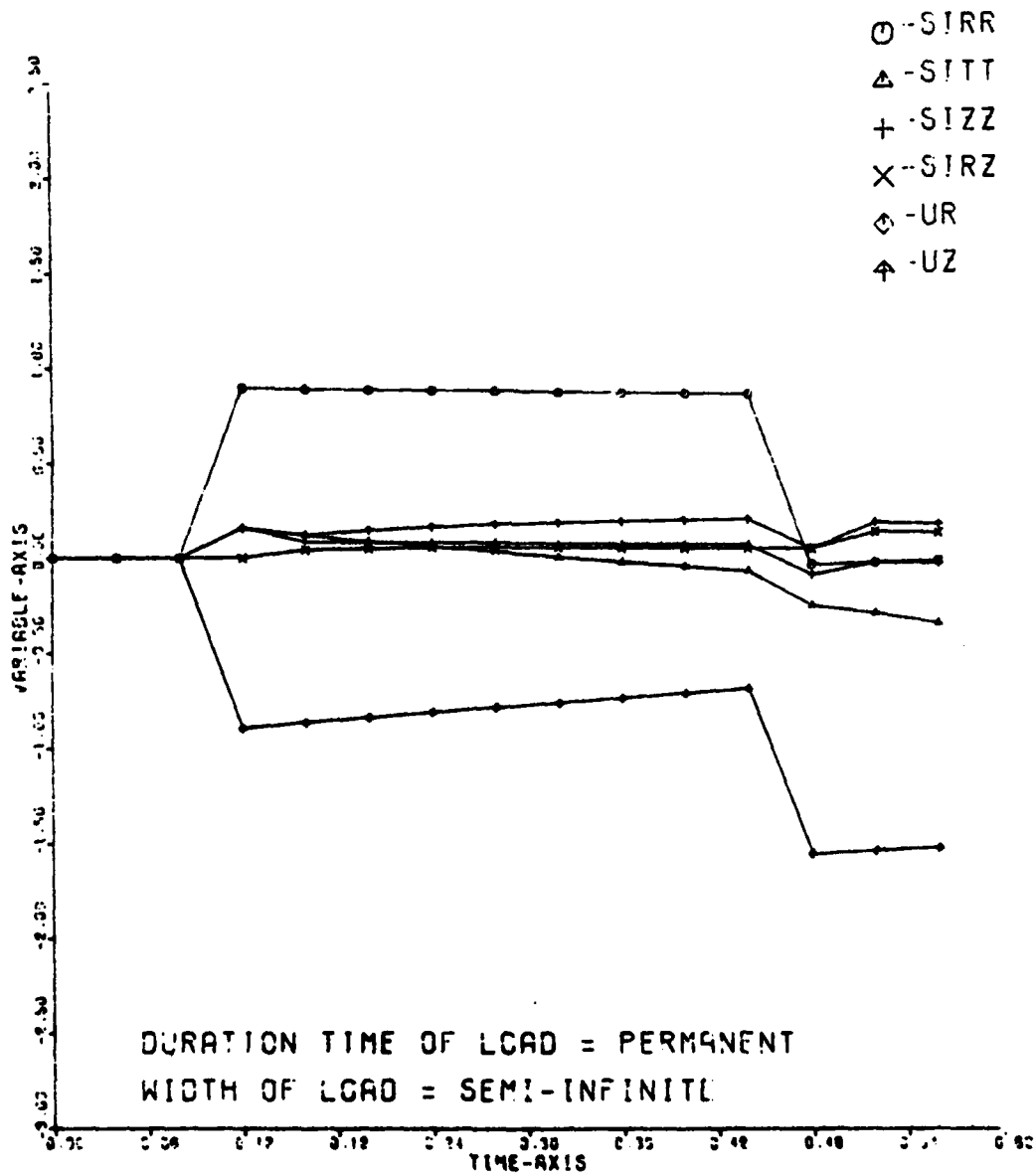


Figure 17

CASE 2

R=1.30 Z=0.02

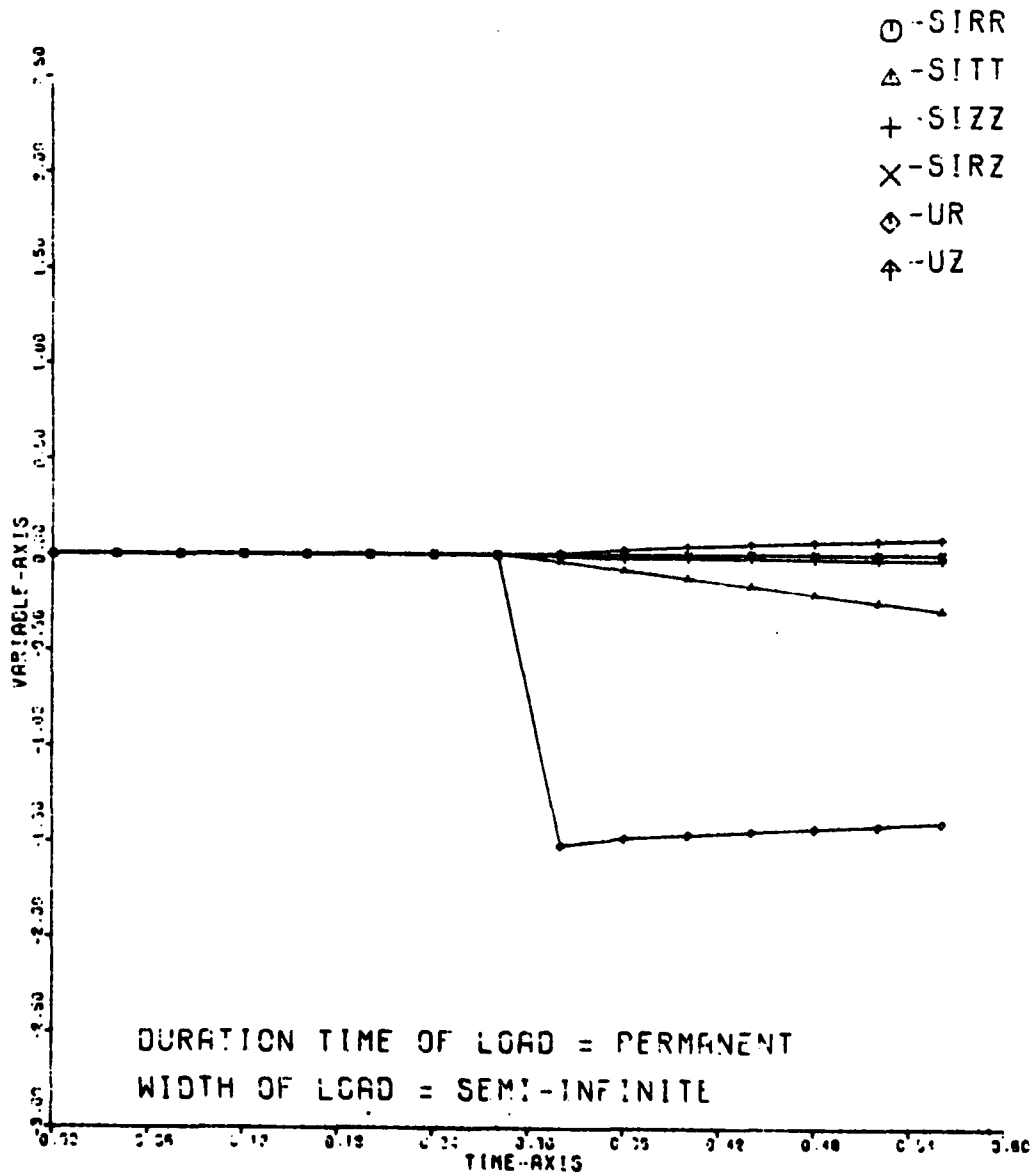


Figure 18

CASE 2

R=1.00 Z=0.08

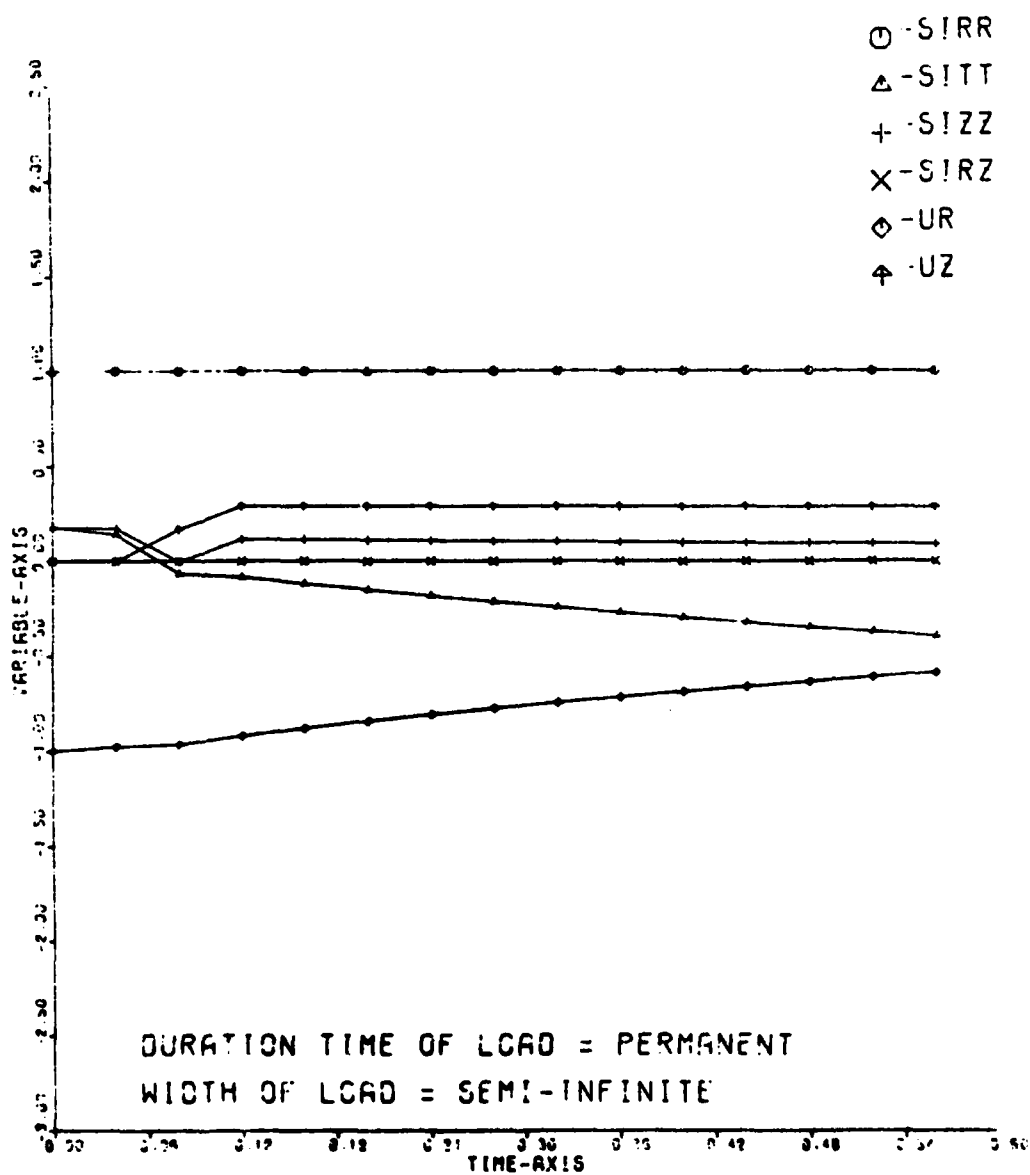


Figure 19

CASE 2

R=1.12 Z=0.08

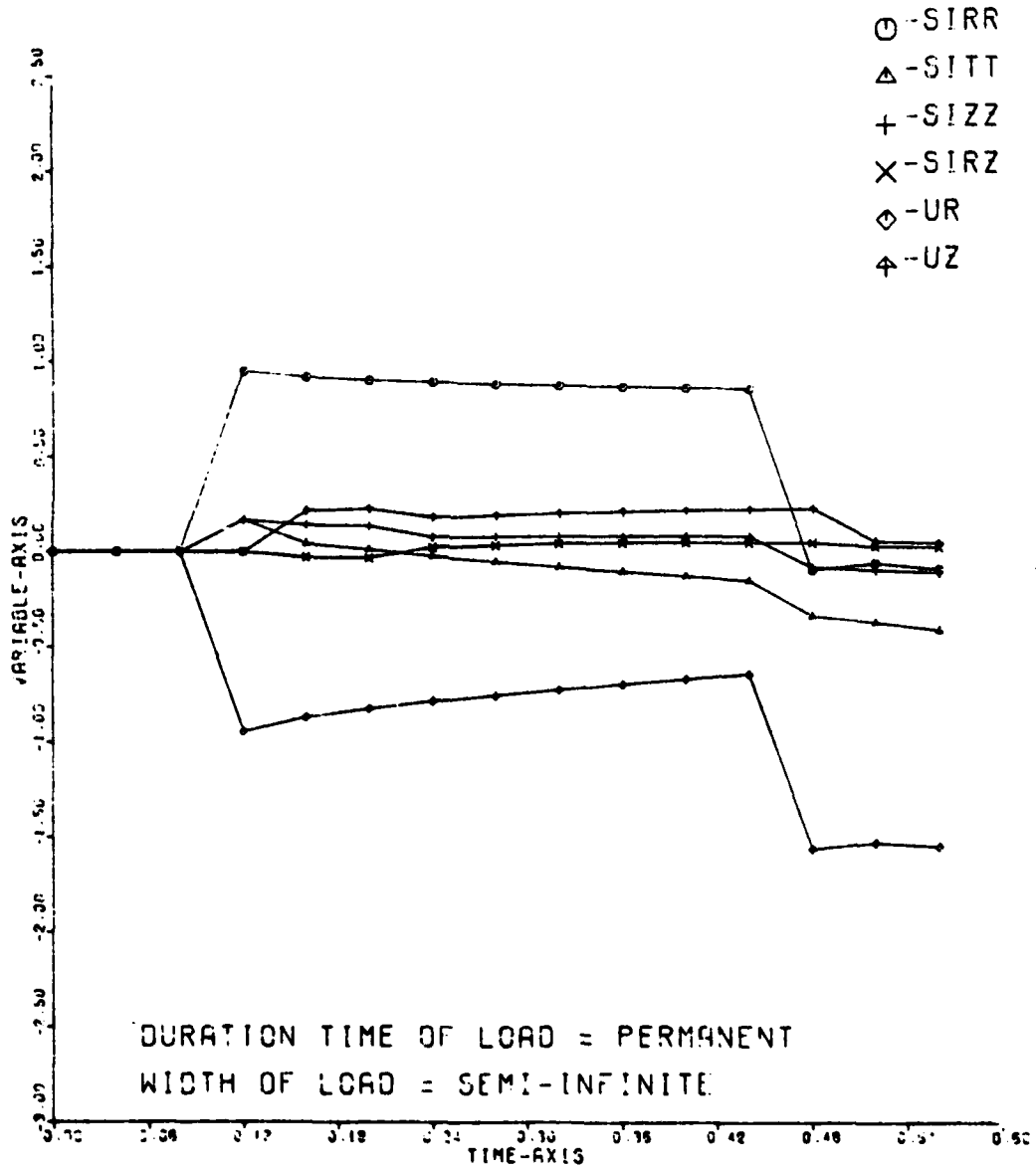


Figure 20

CASE 2

R=1.30 Z=0.06

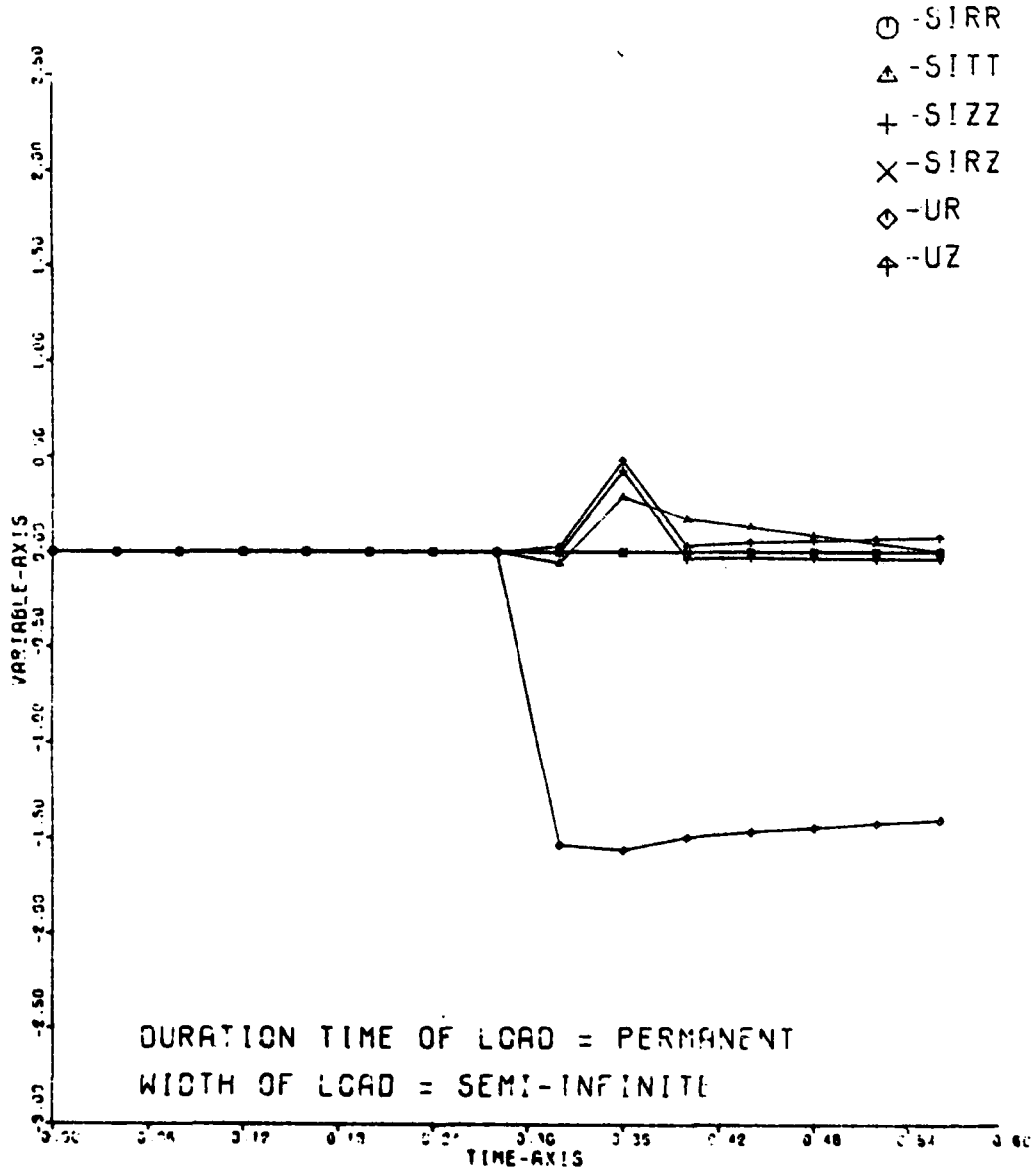


Figure 21

CASE 2

R=1.00 Z=0.00

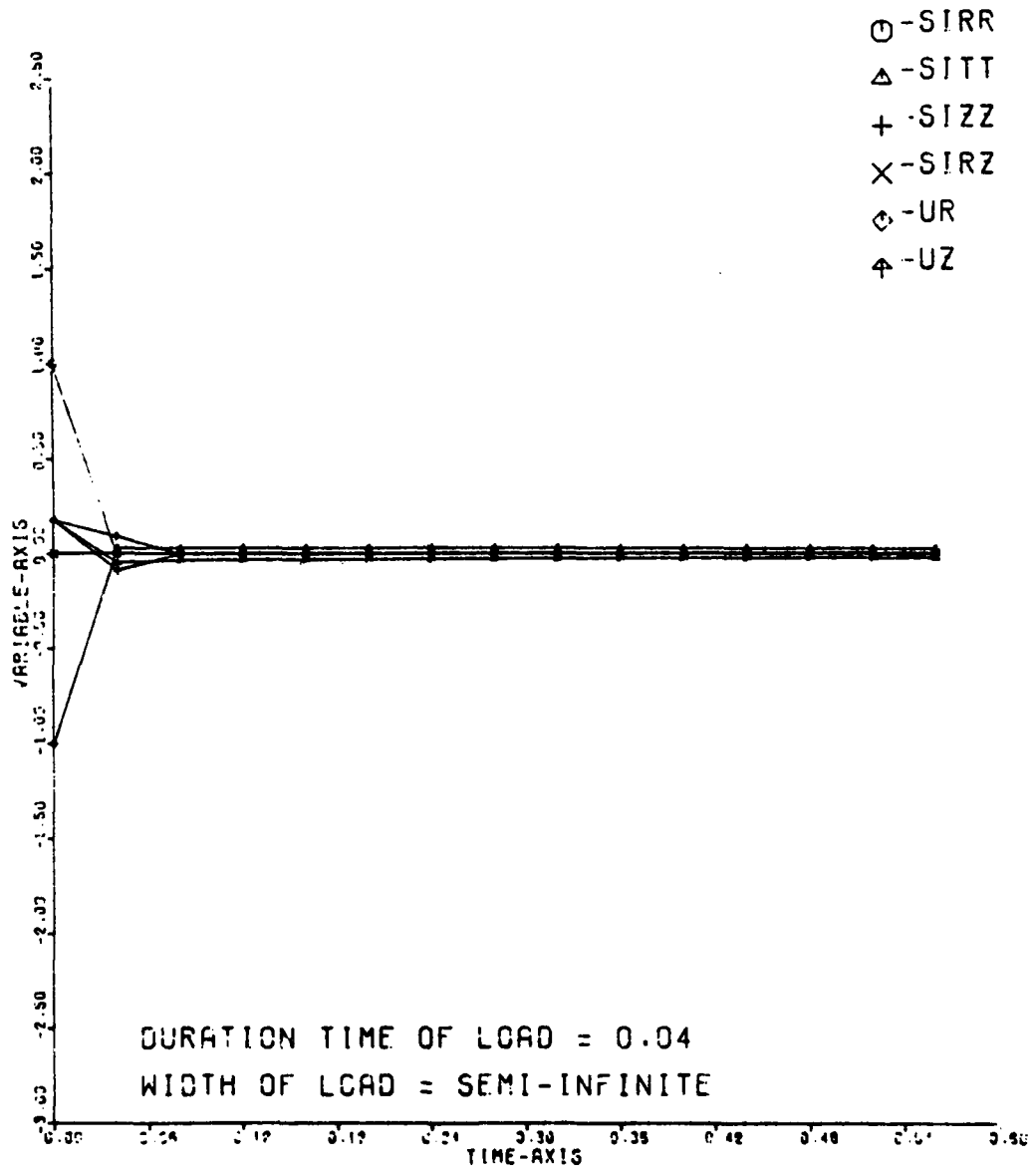


Figure 22

CASE 2

R=1.12 Z=0.00

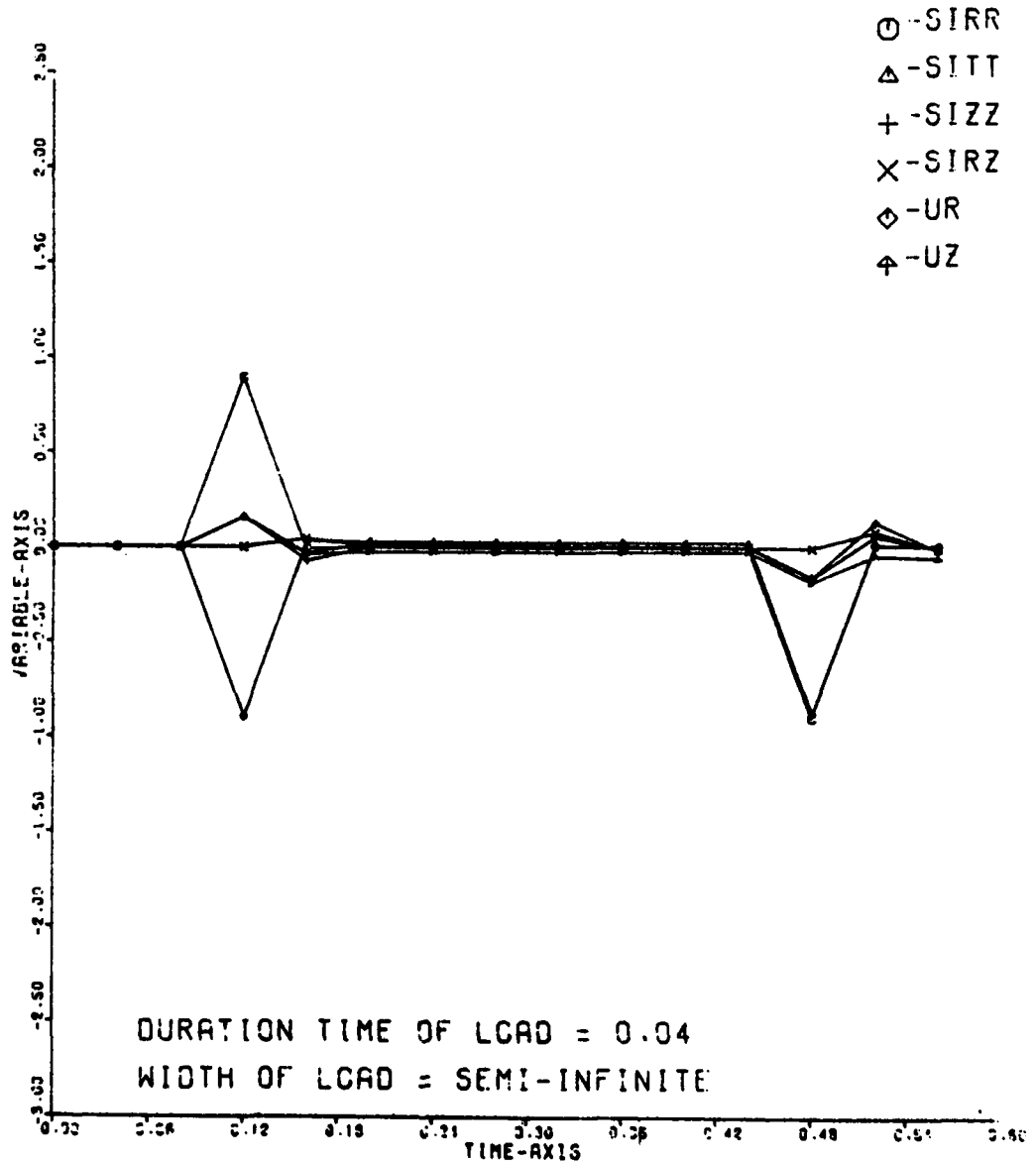


Figure 23

CASE 2

R=1.30 Z=0.02

- -SIRR
- △ -SITT
- + -SIZZ
- × -SIRZ
- ◇ -UR
- ↑ -UZ

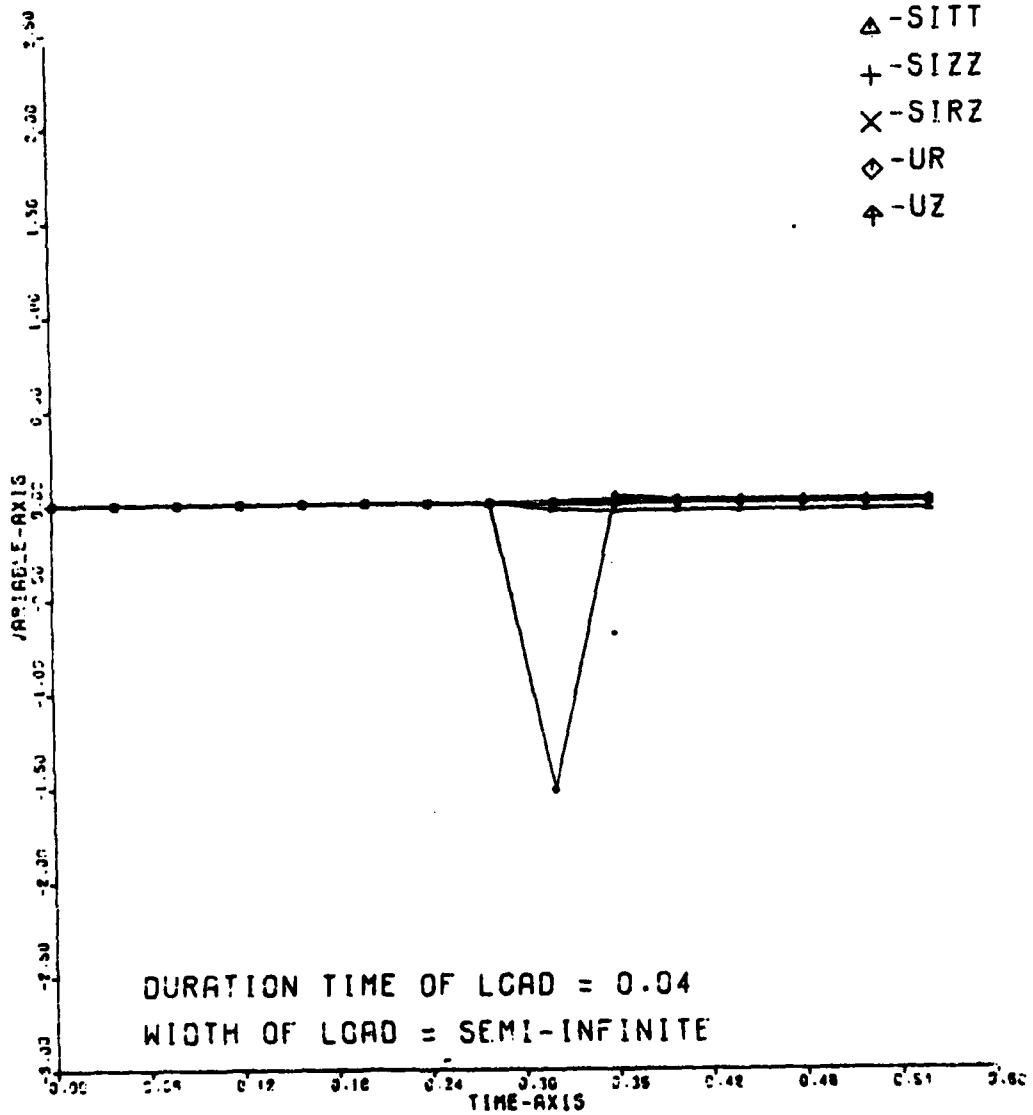


Figure 24

CASE 2

R=1.00 Z=0.08

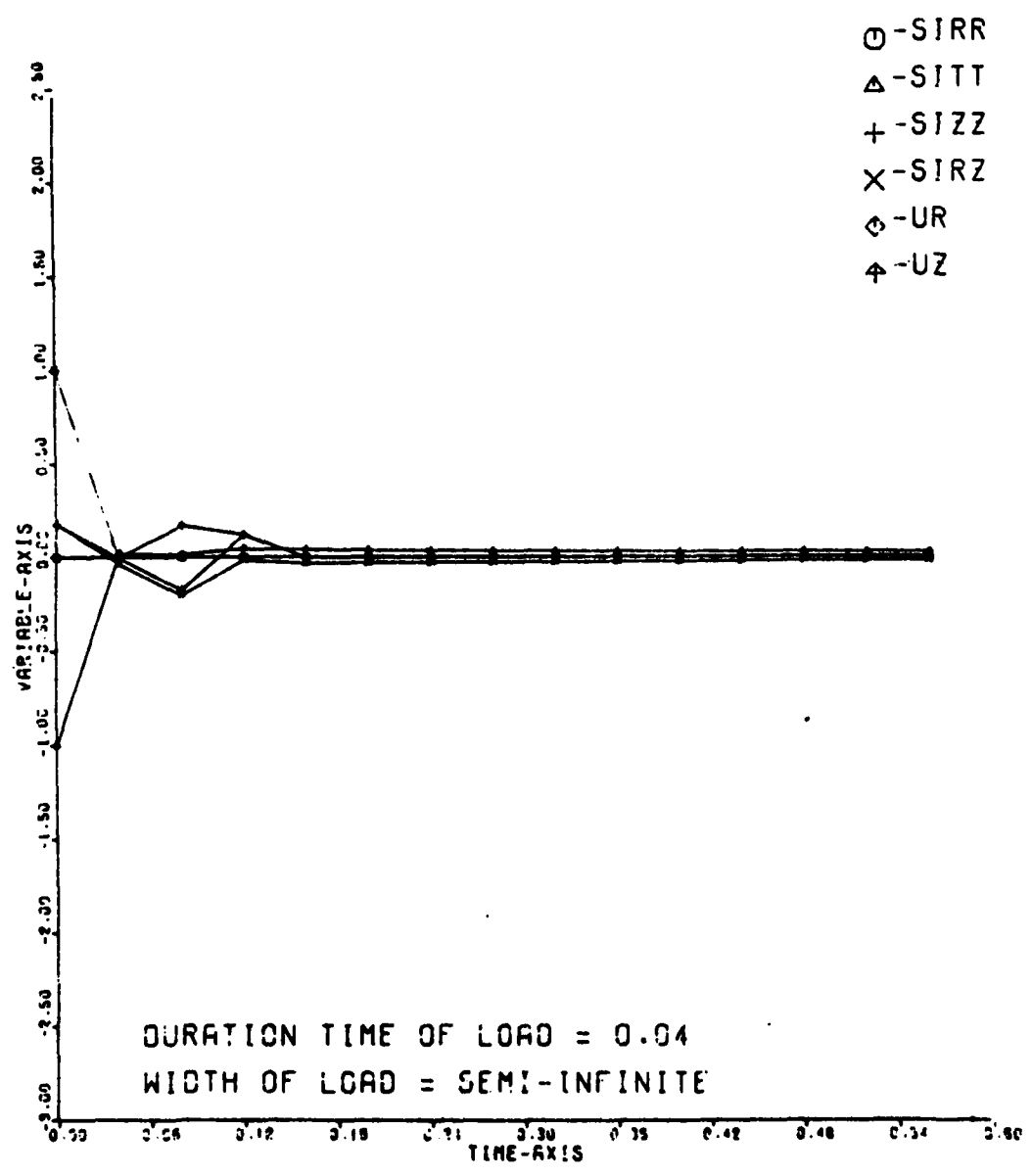


Figure 25

CASE 2

R=1.30 Z=0.05

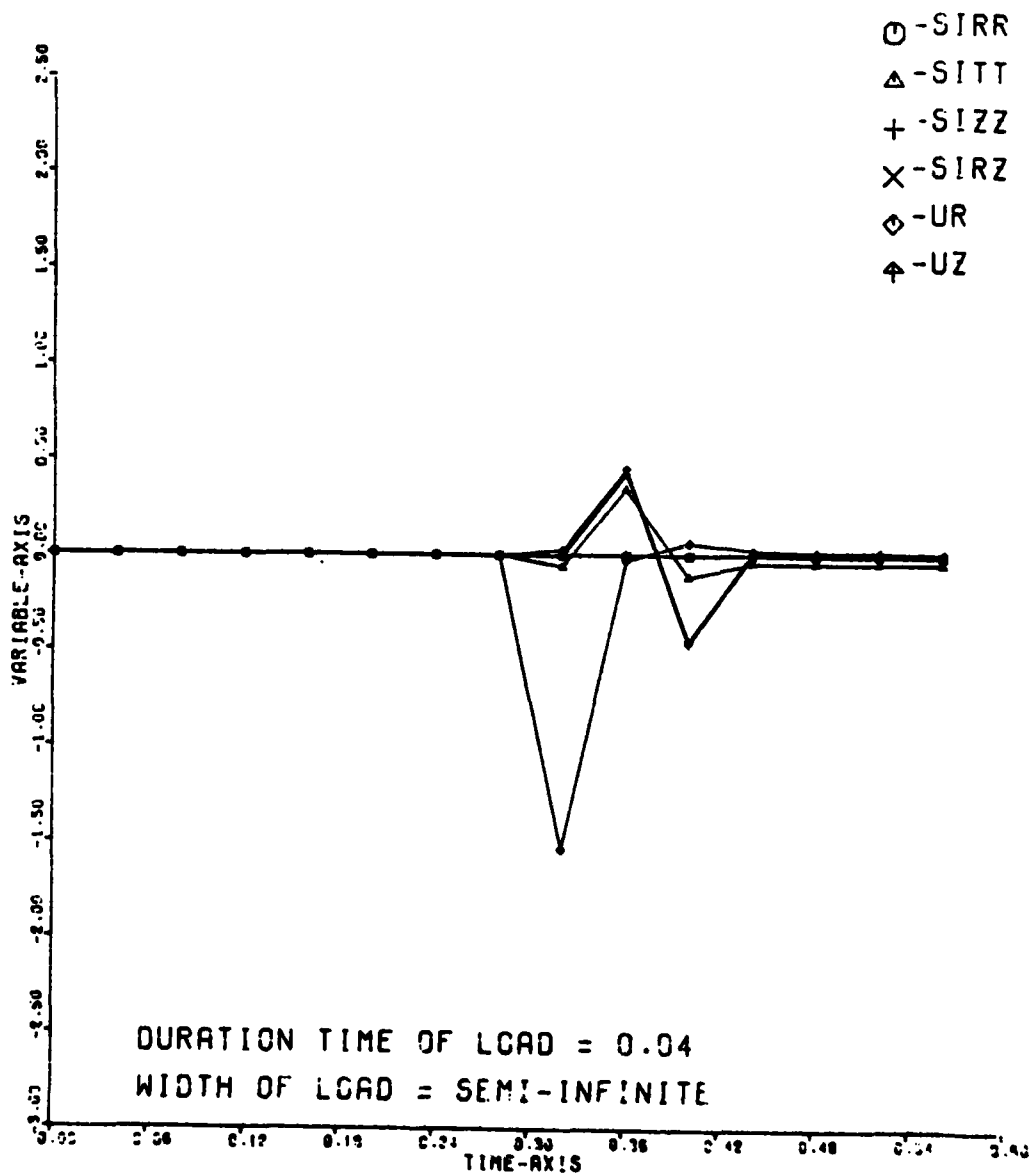


Figure 26

CASE 2

R=1.12 Z=0.08

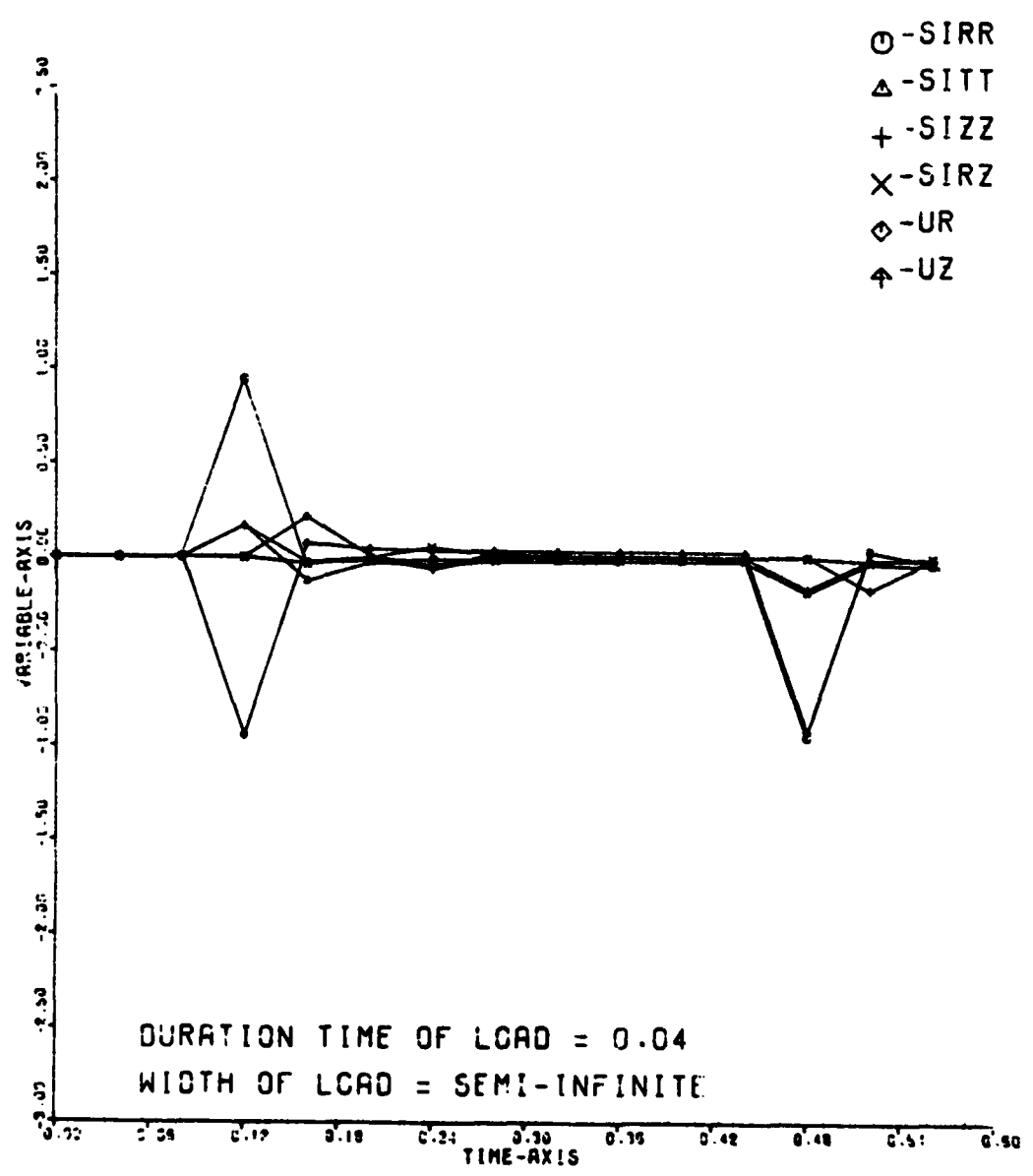


Figure 27

CASE 2

R=1.00 Z=0.00

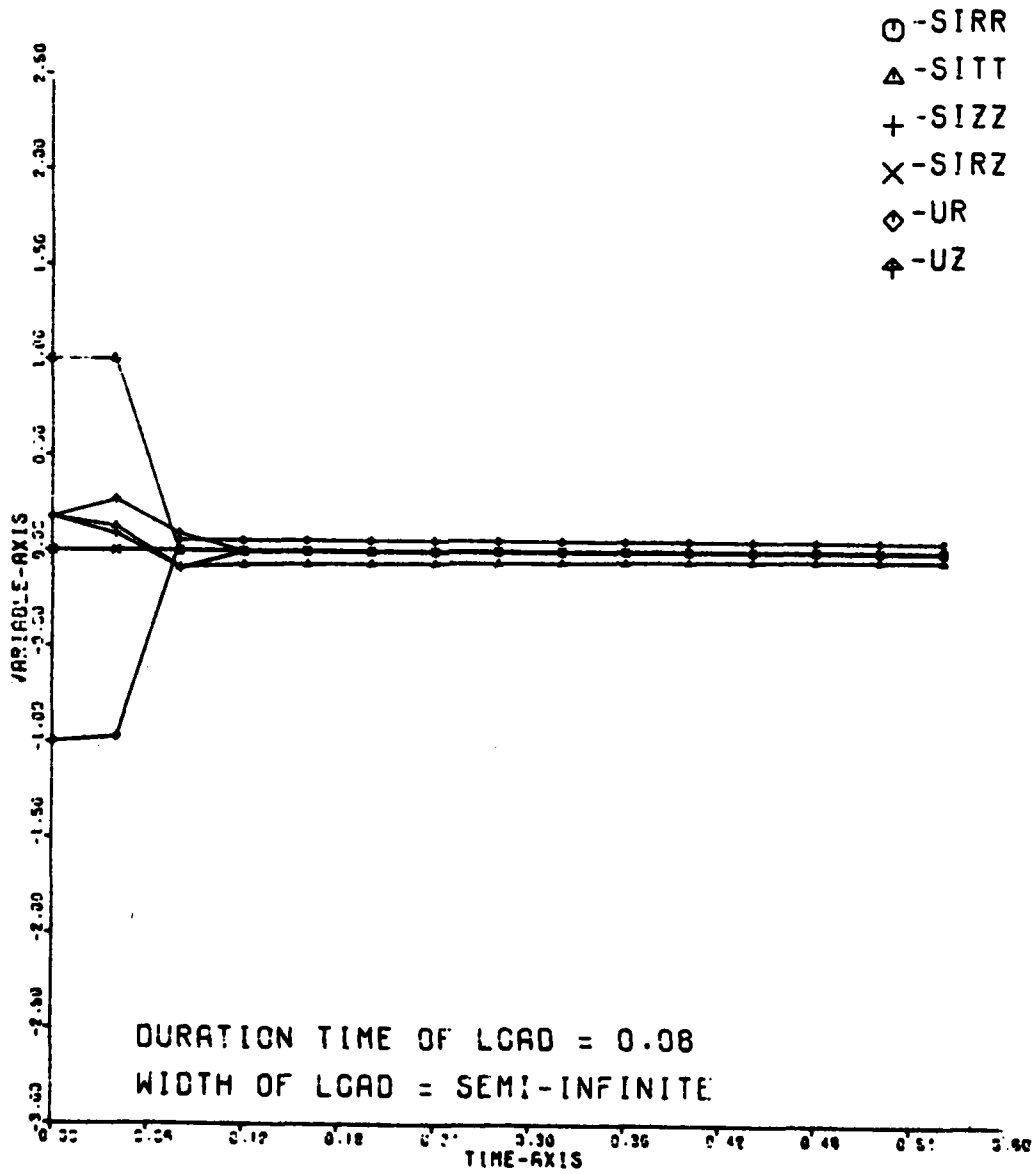


Figure 28

CASE 2

R=1.12 Z=0.00

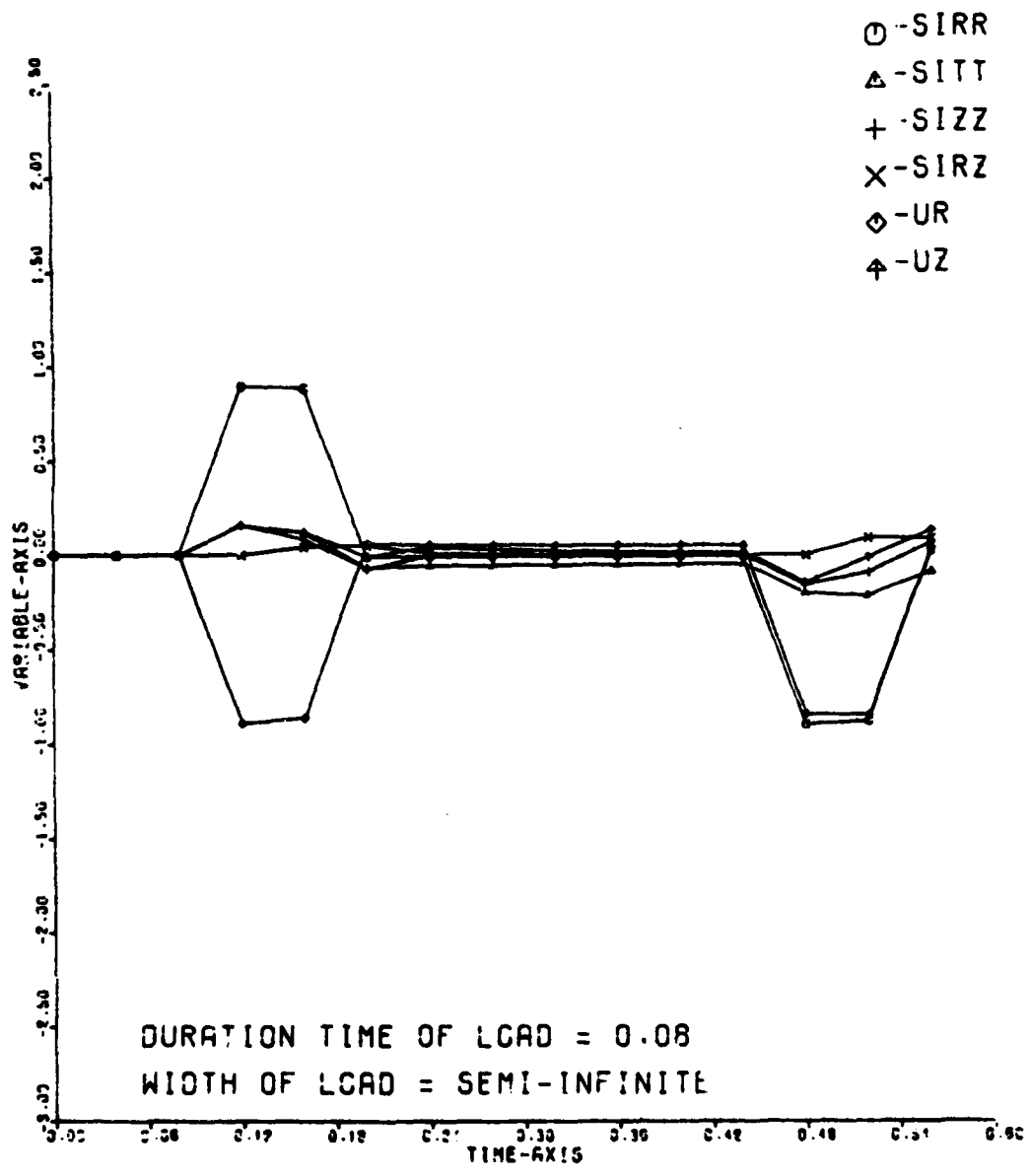


Figure 29

CASE 2

R=1.30 Z=0.02

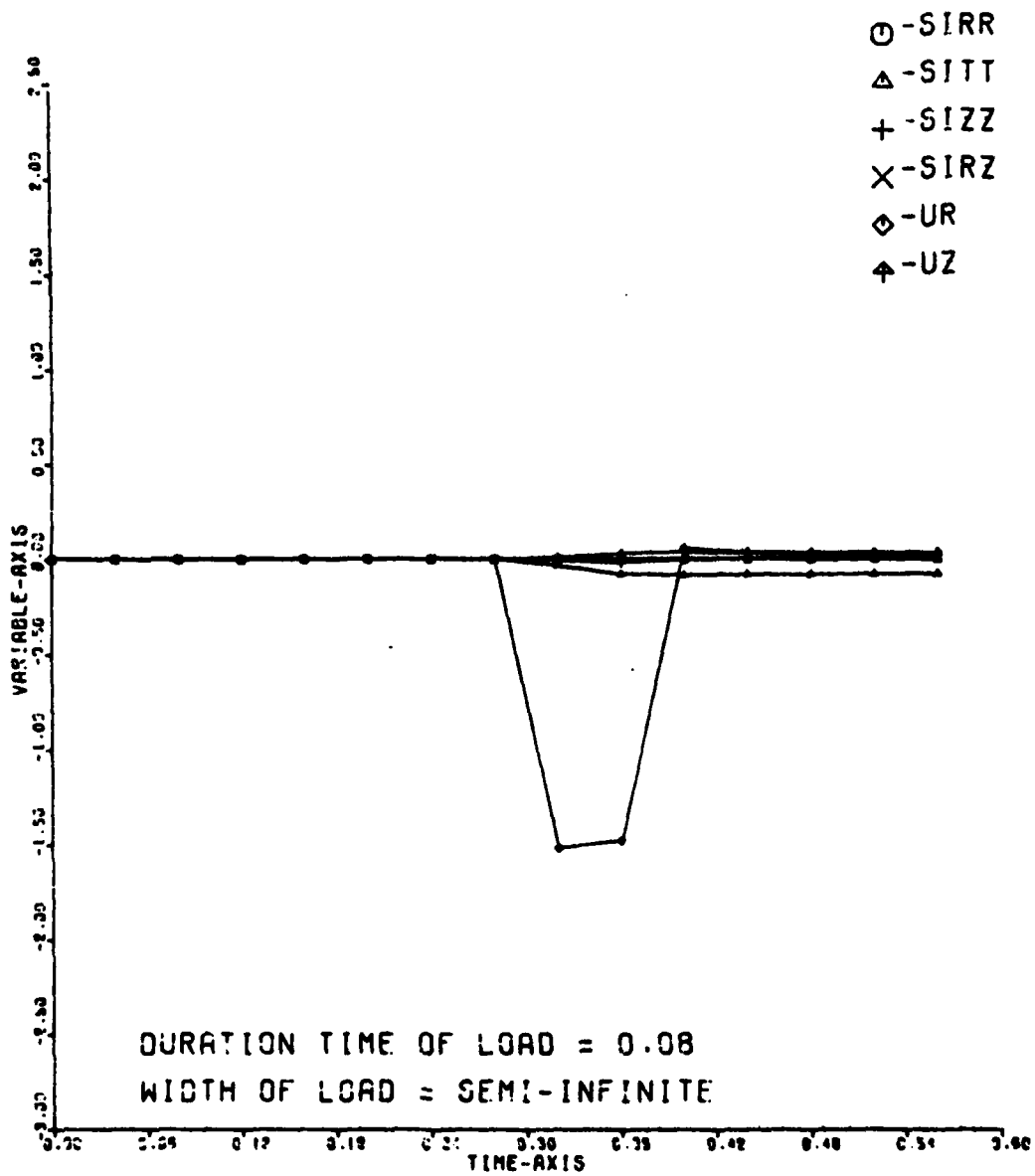


Figure 30

CASE 2

R=1.00 Z=0.08

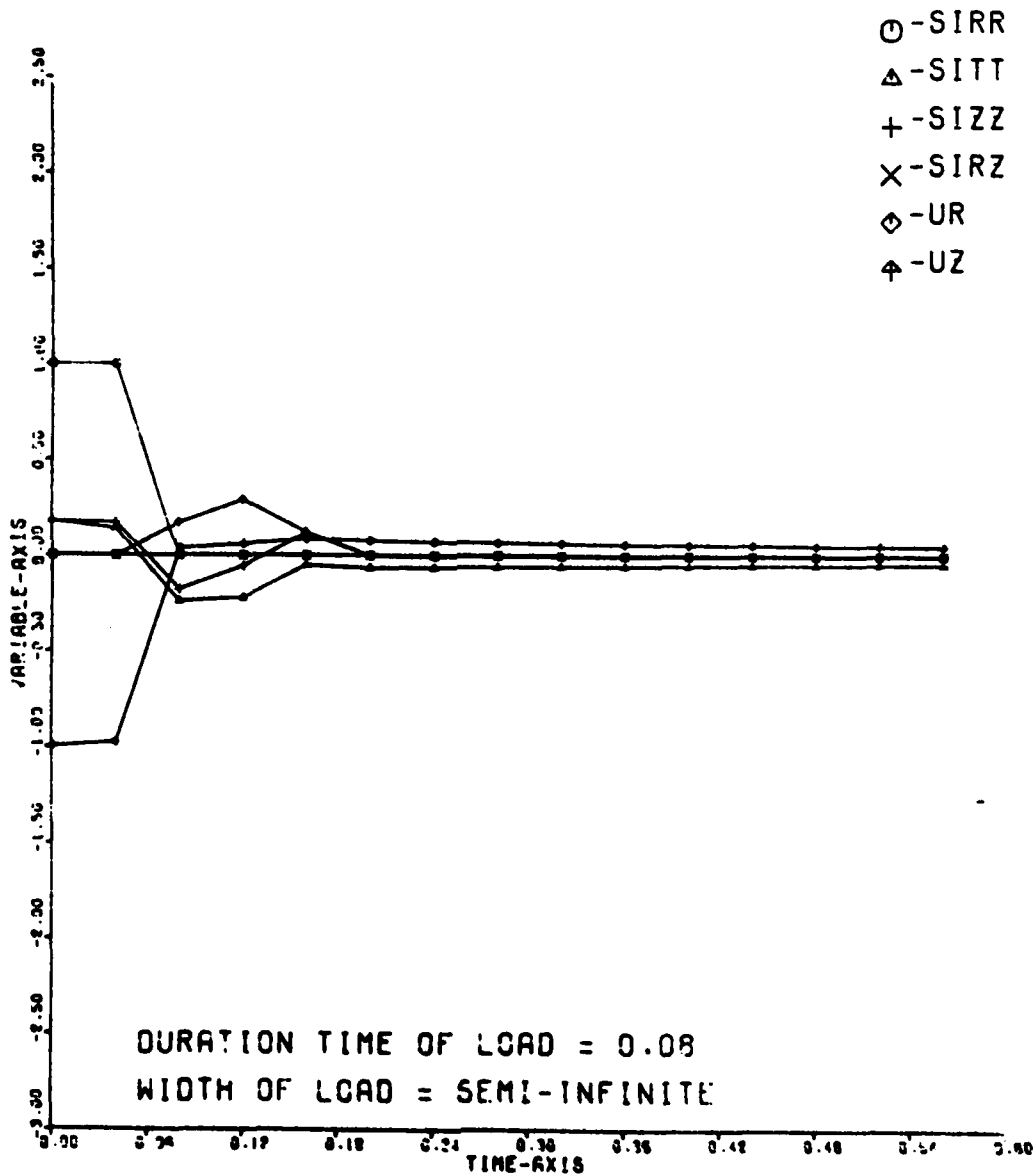


Figure 31

CASE 2

R=1.12 Z=0.08

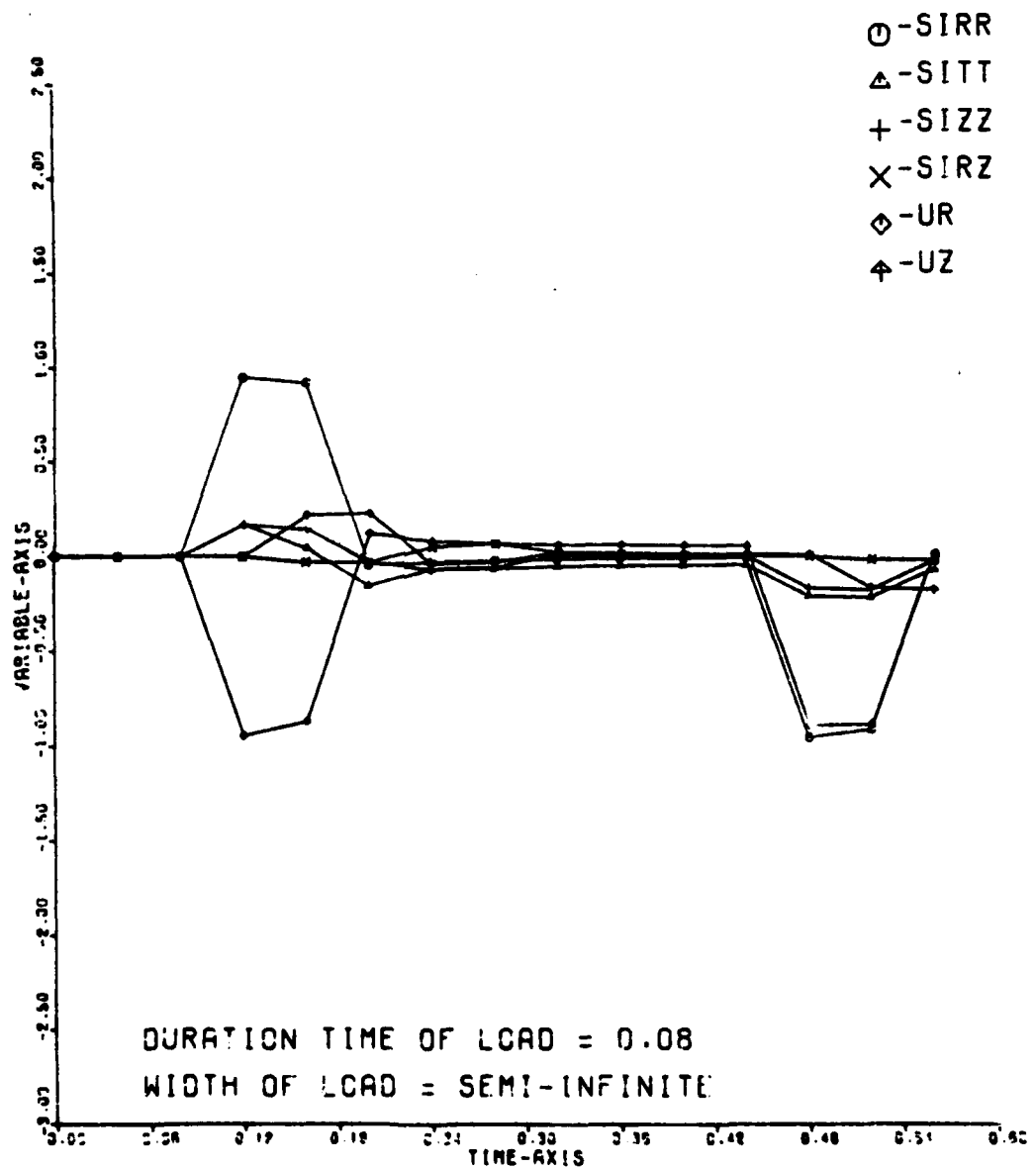


Figure 32

CASE 2

R=1.30 Z=0.06

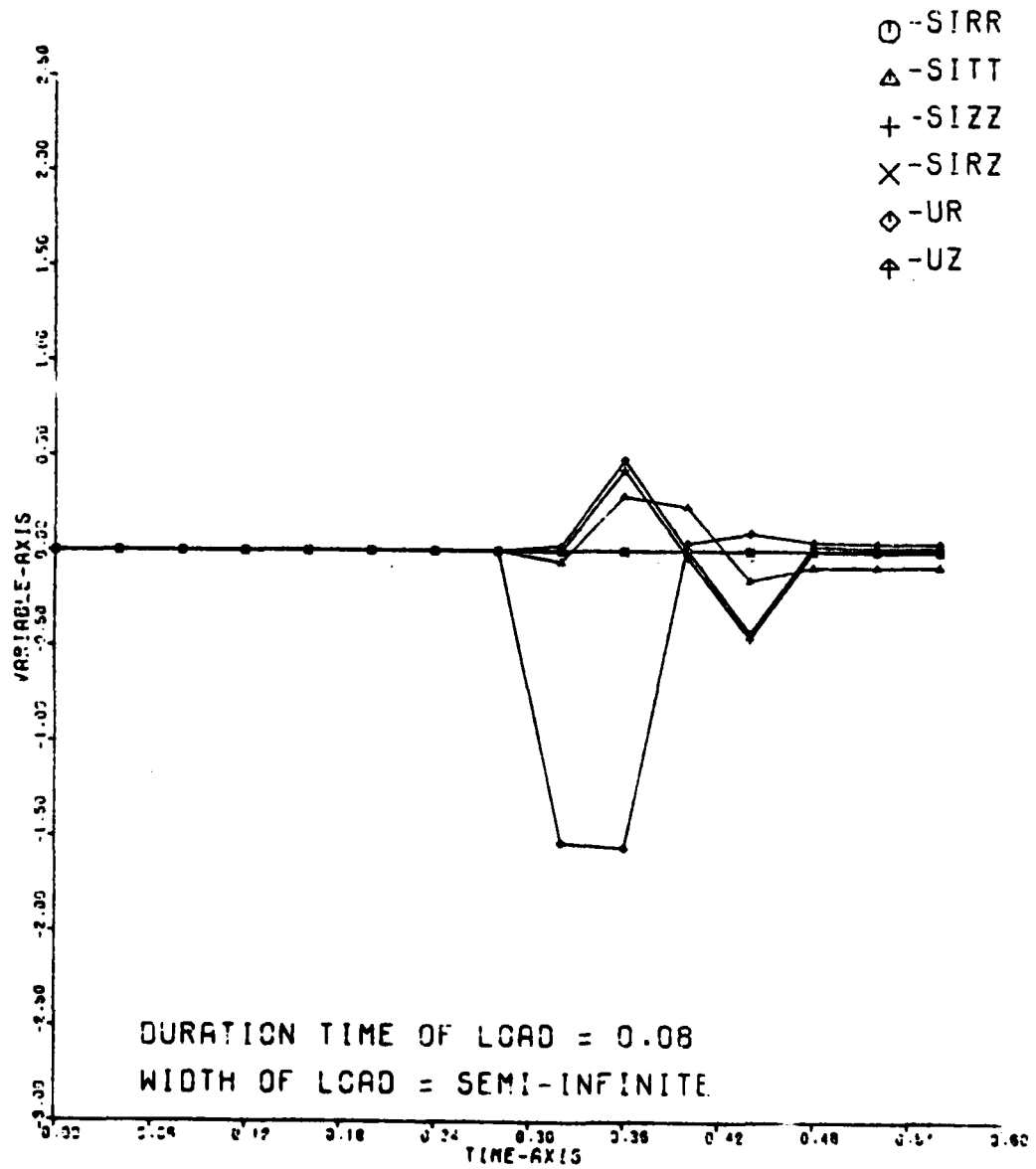


Figure 33

CASE 3

R=1.02 Z=0.00

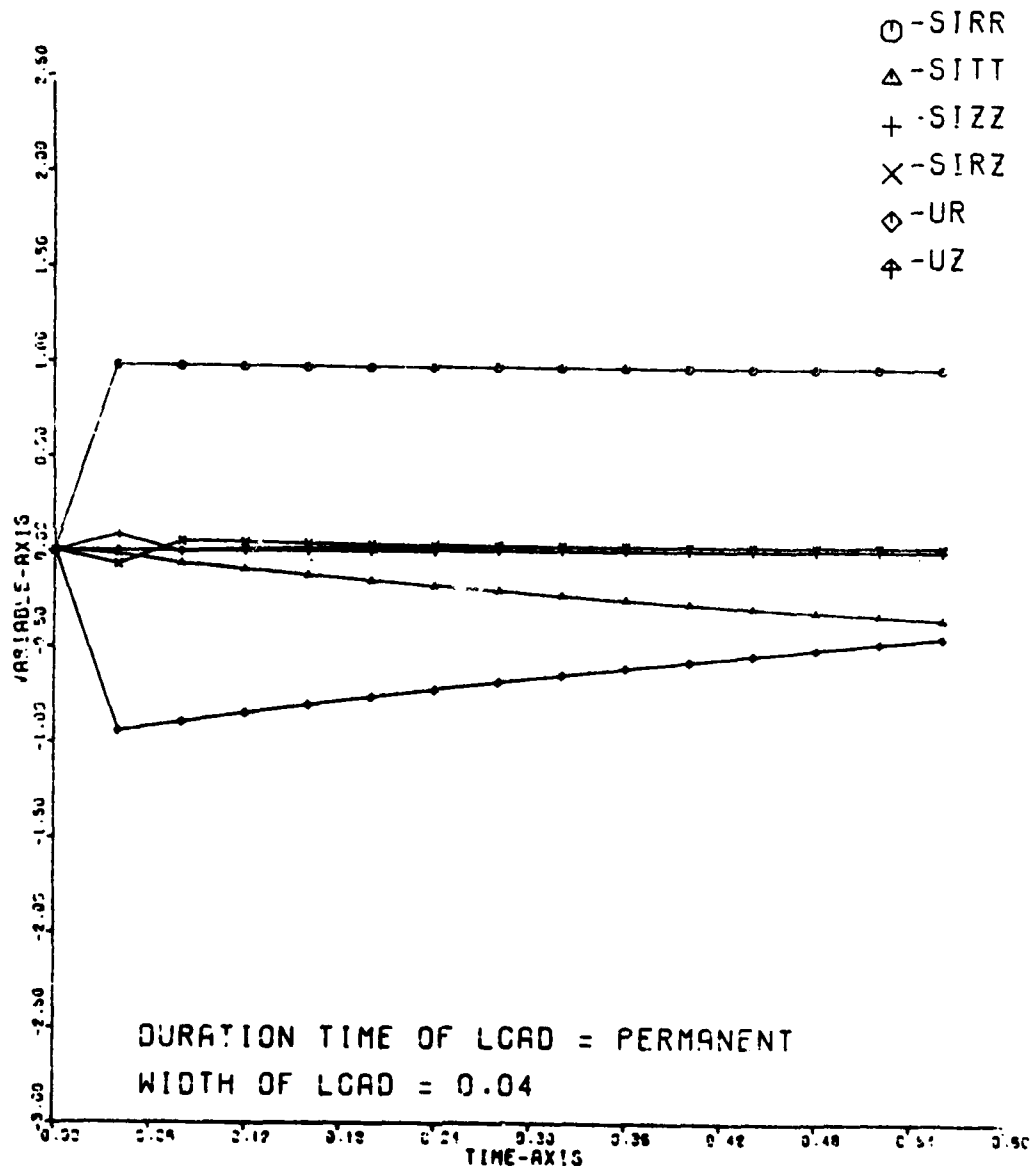


Figure 34

CASE 3

R=1.14 Z=0.00

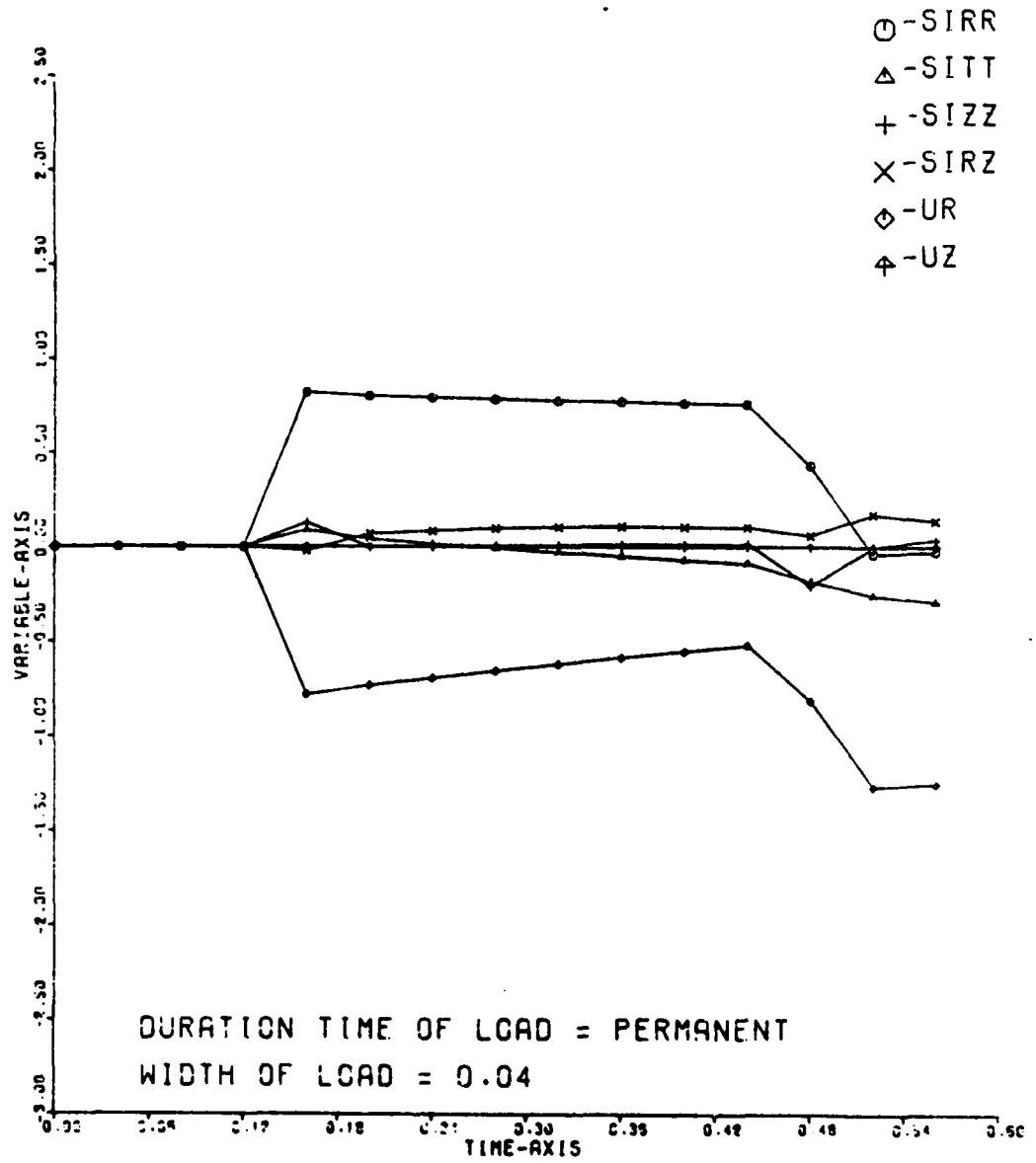


Figure 35

CASE 3

R=1.30 Z=0.00

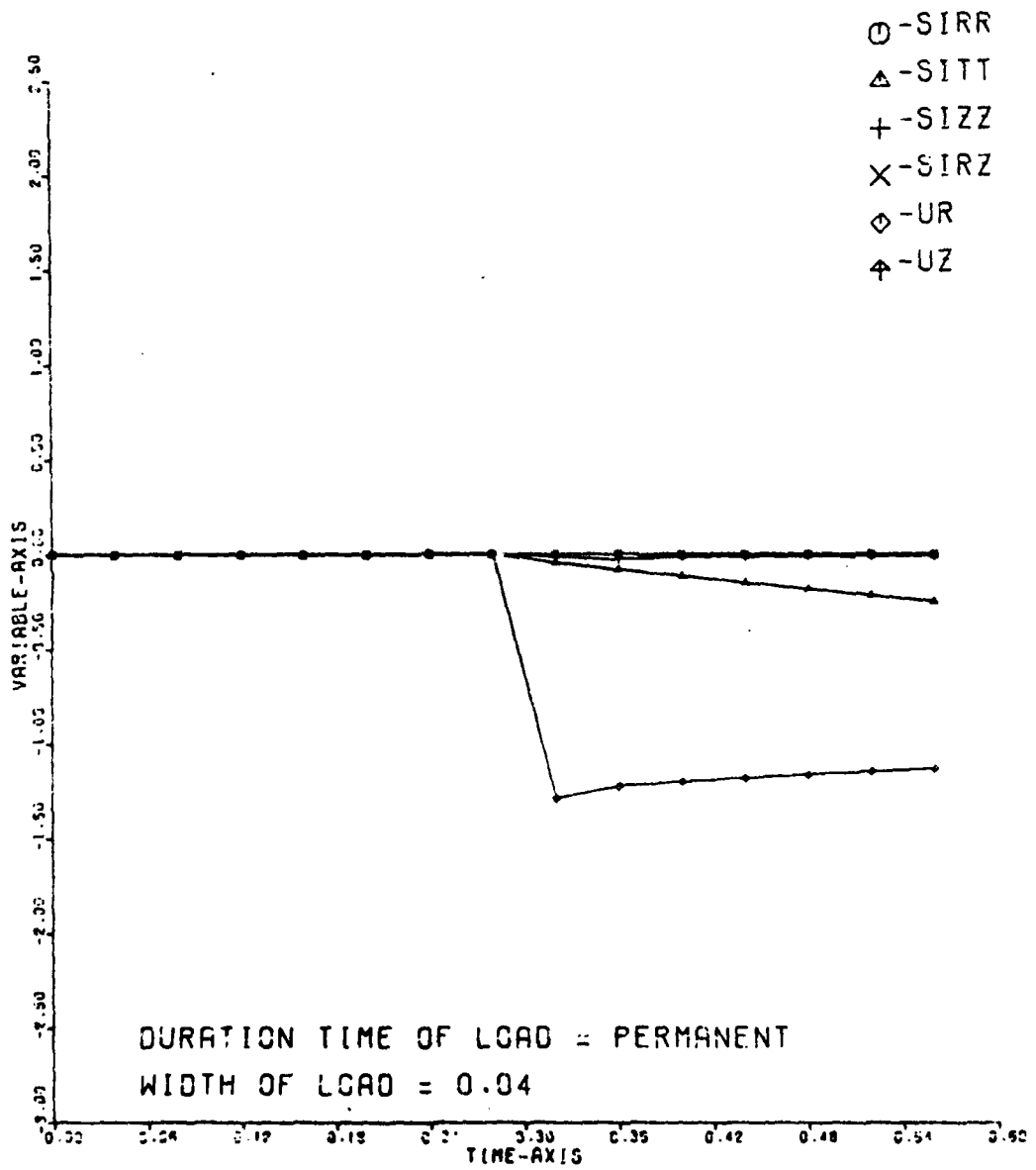


Figure 36

CASE 3

R=1.02 Z=0.08

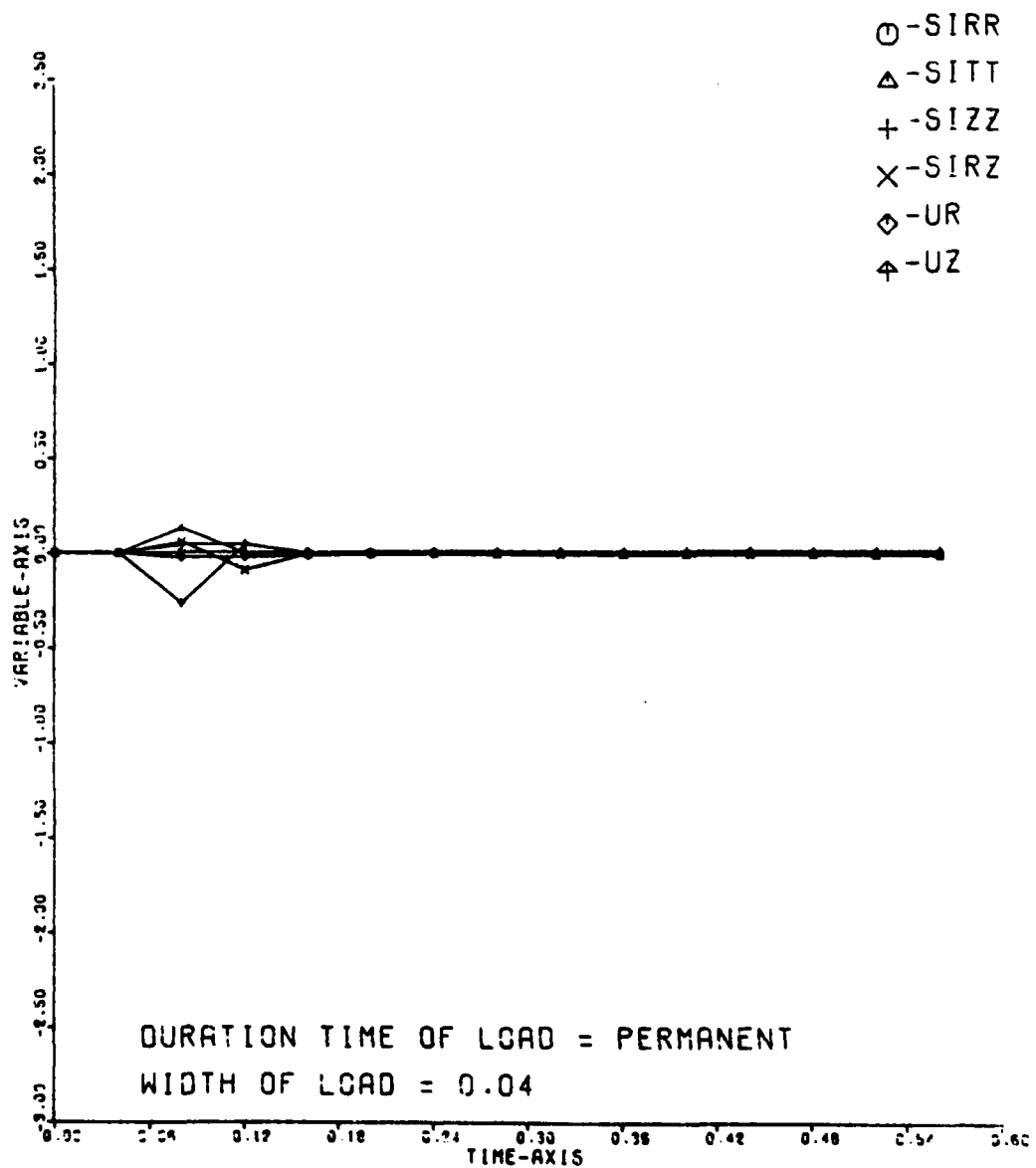


Figure 37

CASE 3

R=1.14 Z=0.08

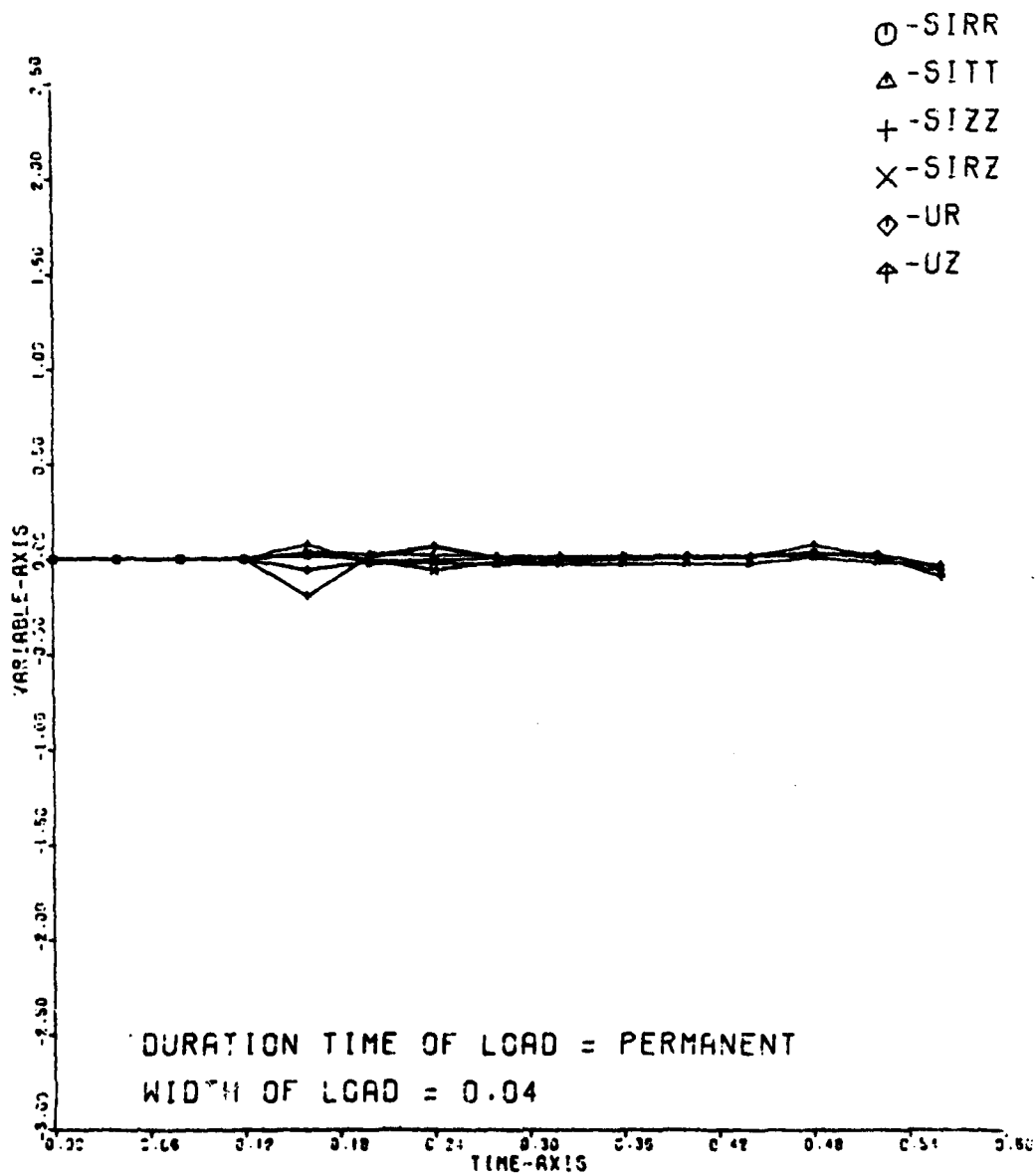


Figure 38

CASE 3

R=1.30 Z=0.08

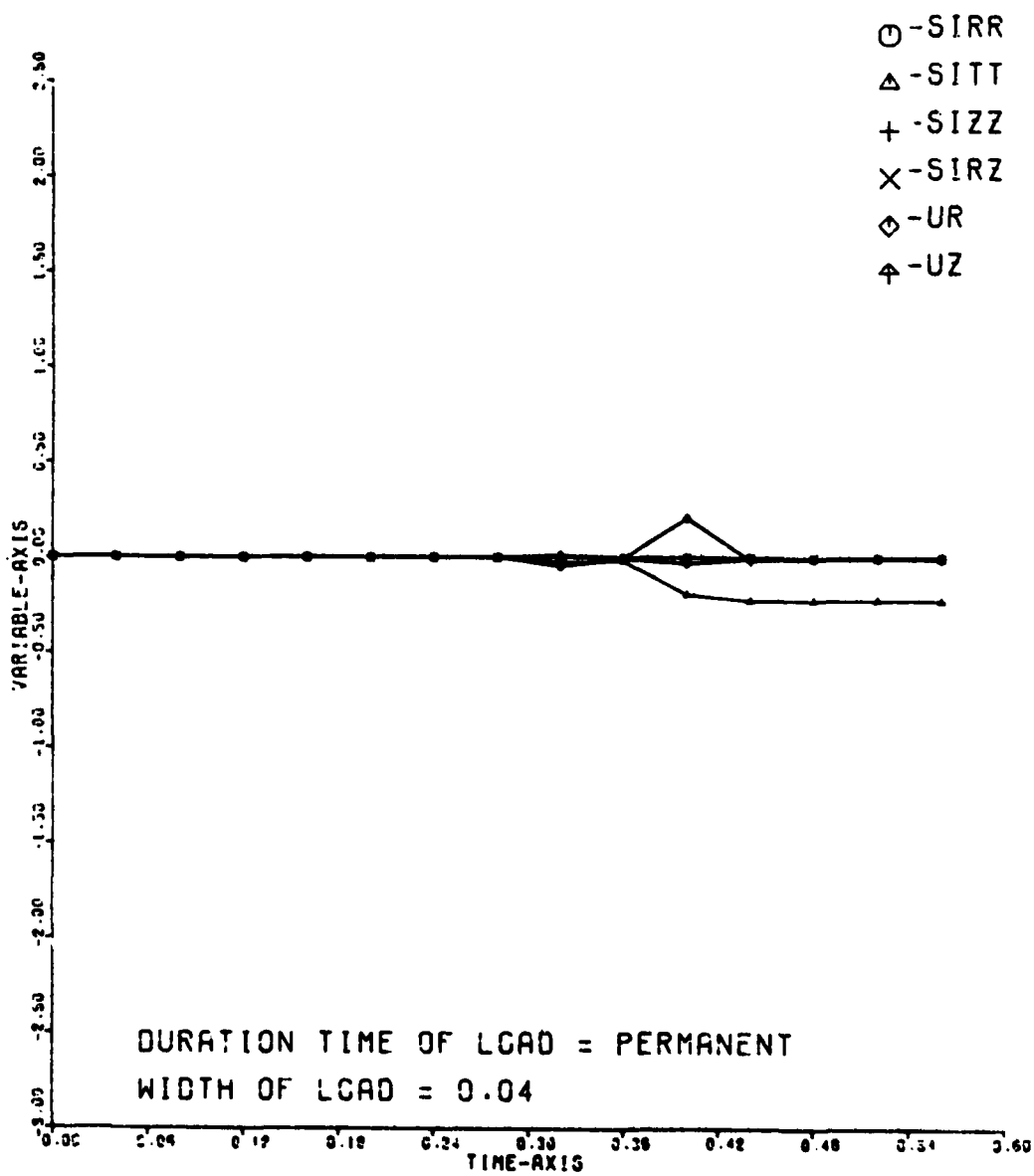


Figure 39

CASE 3

R=1.02 Z=0.00

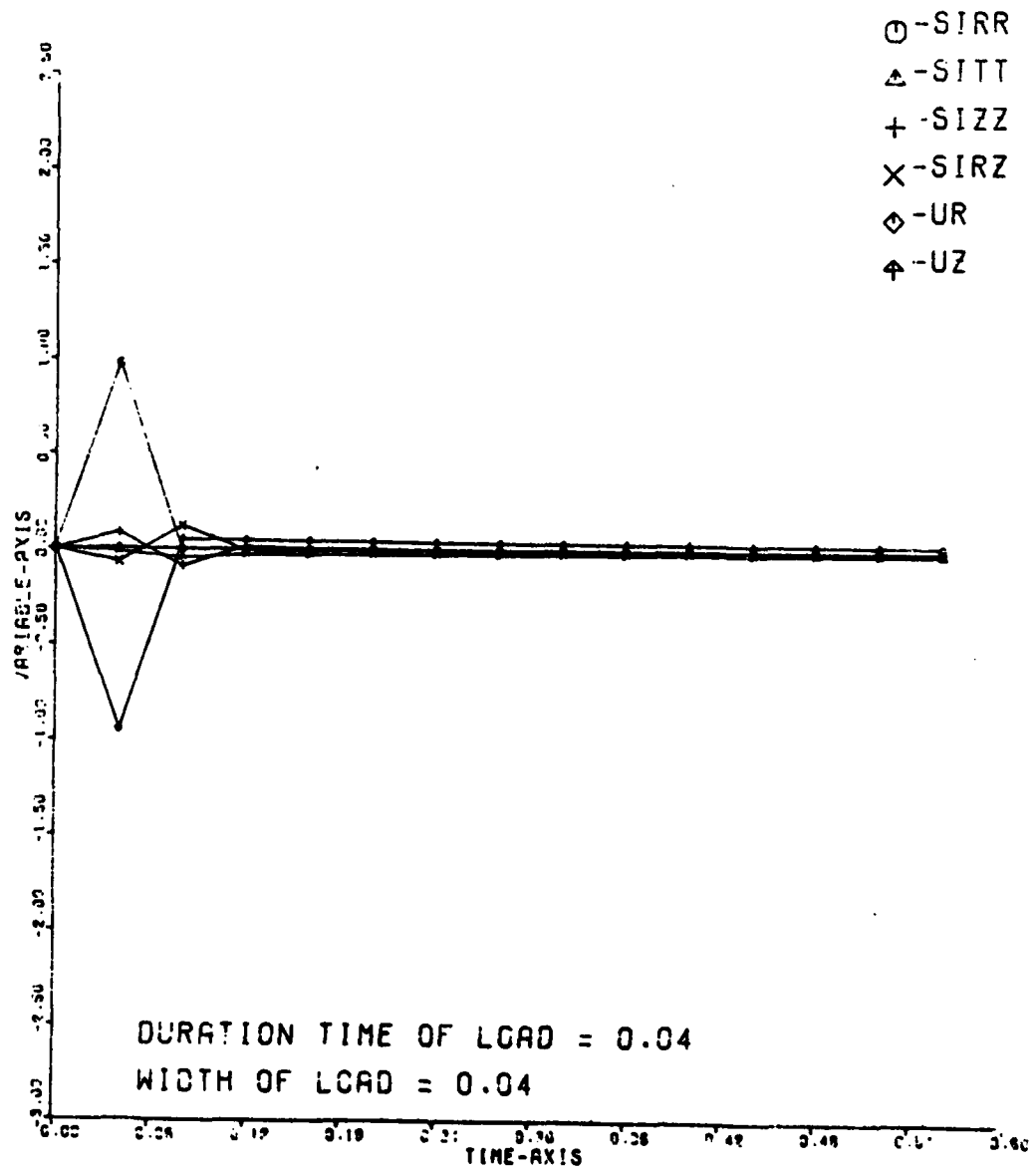


Figure 40

CASE 3

R=1.14 Z=0.00

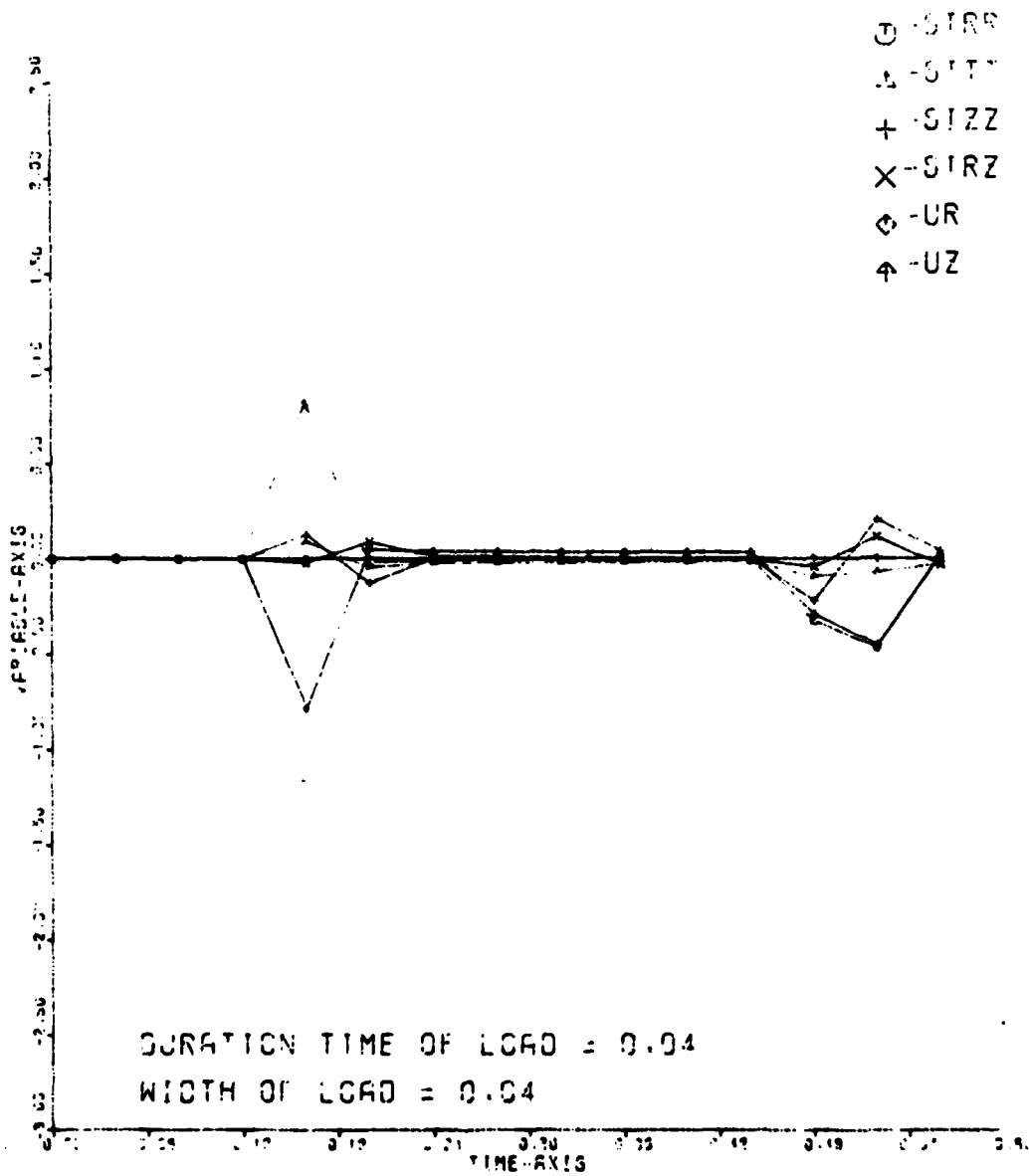


Figure 41

CASE 3

R=1.30 Z=0.99

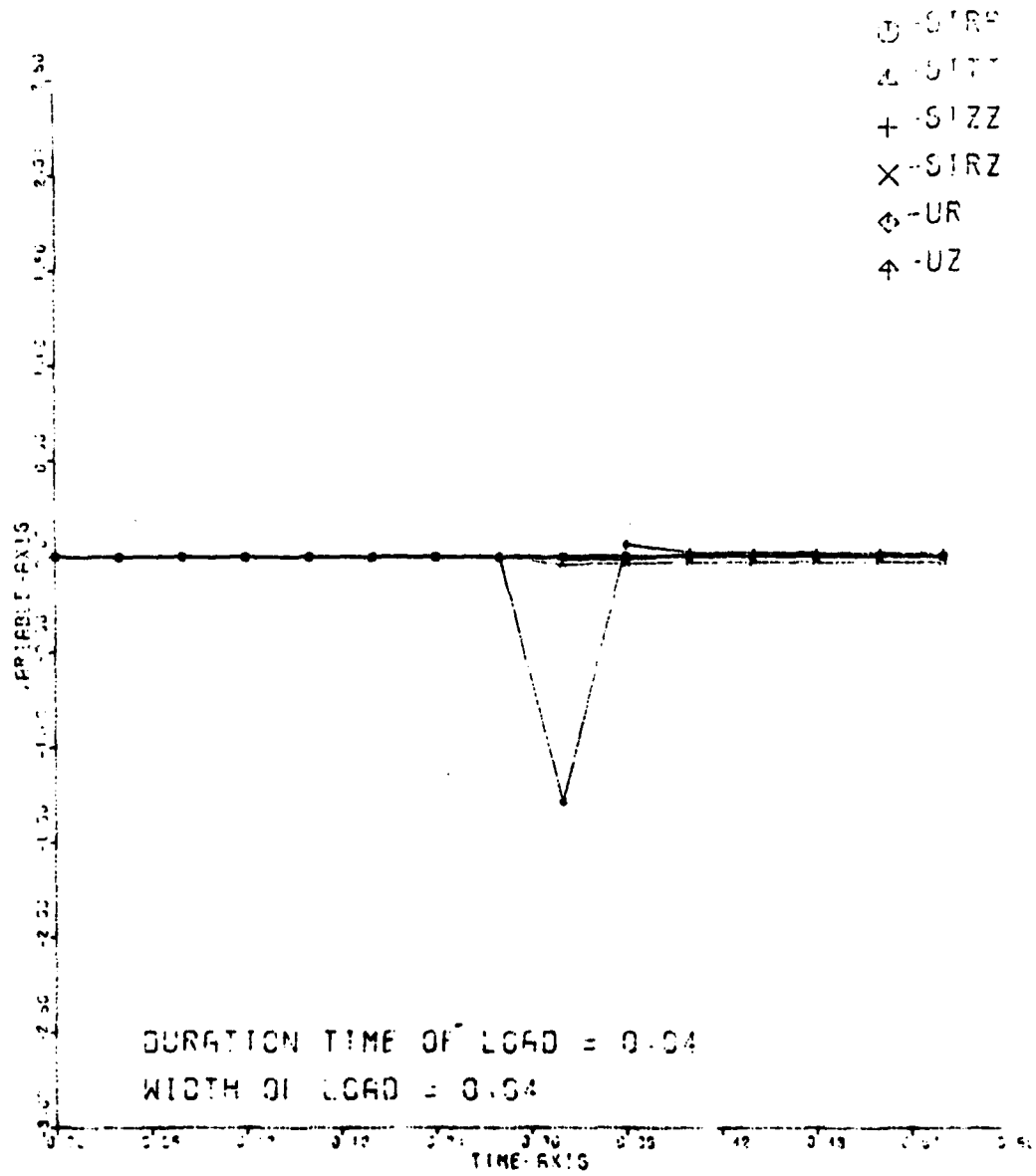


Figure 42

CASE 3

R=1.02 Z=0.08

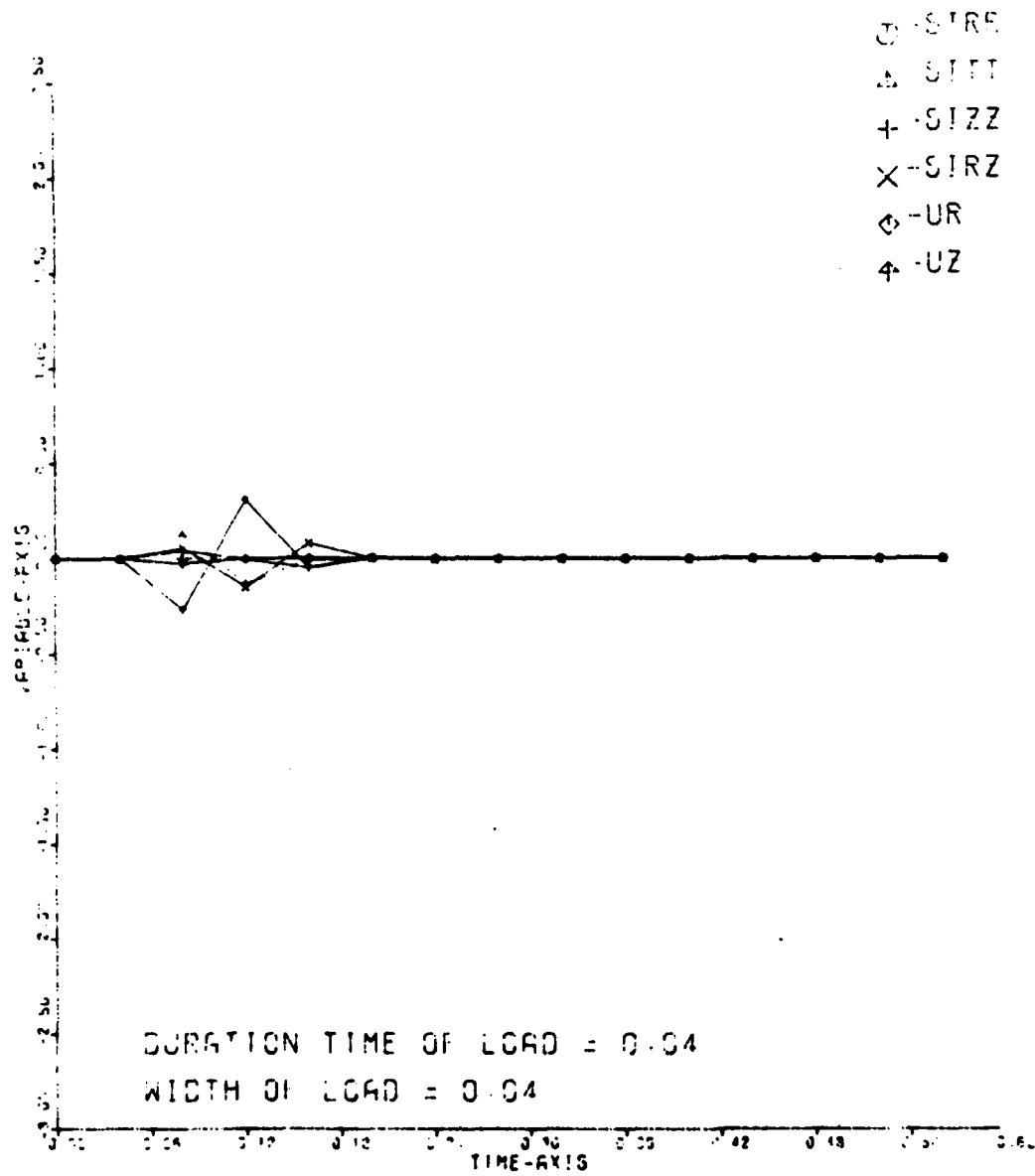


Figure 43

CASE 3

R=1.14 Z=0.08

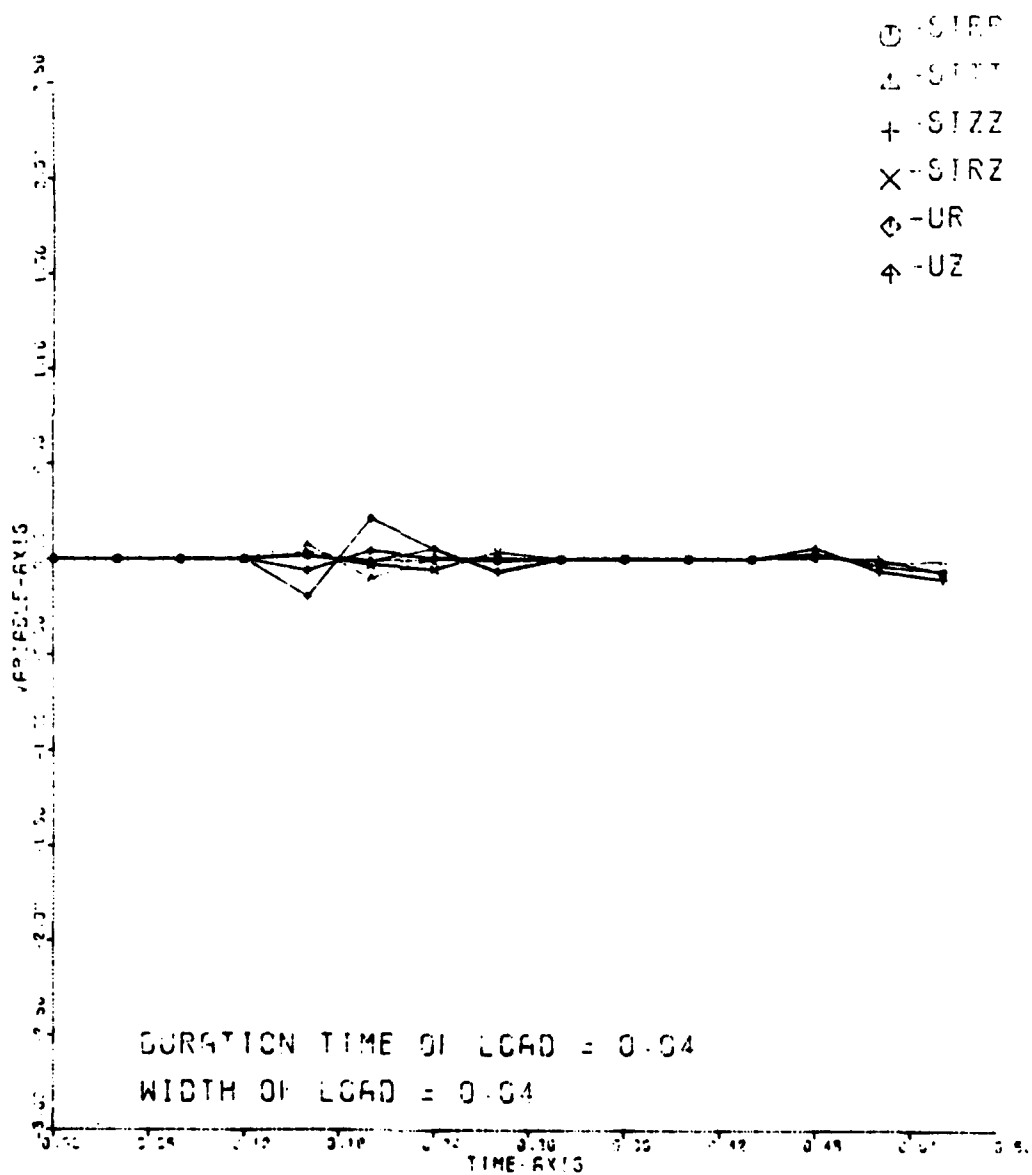


Figure 44

CASE 3

R=1.30 Z=0.08

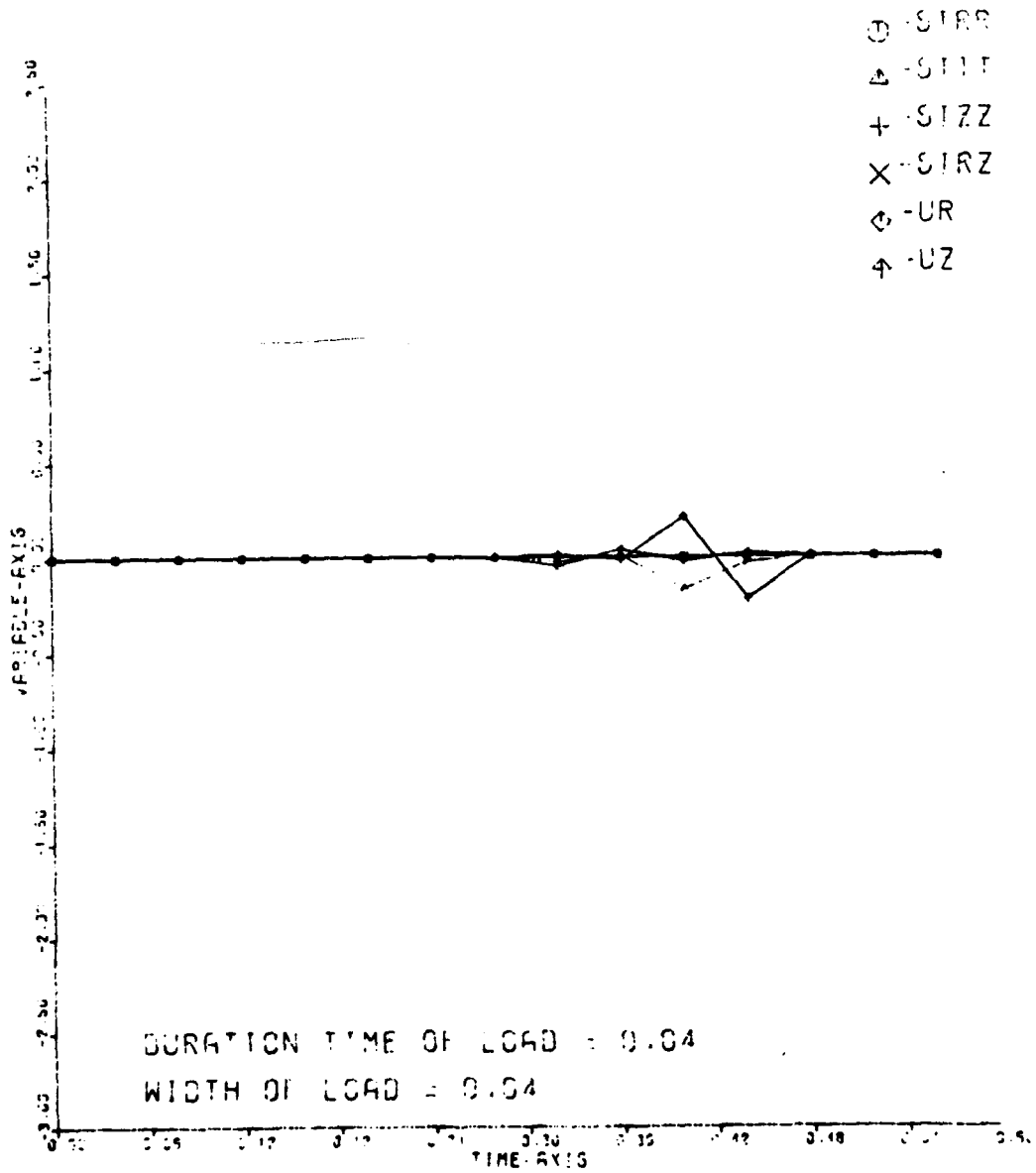


Figure 45

CASE 3

R=1.02 Z=0.00

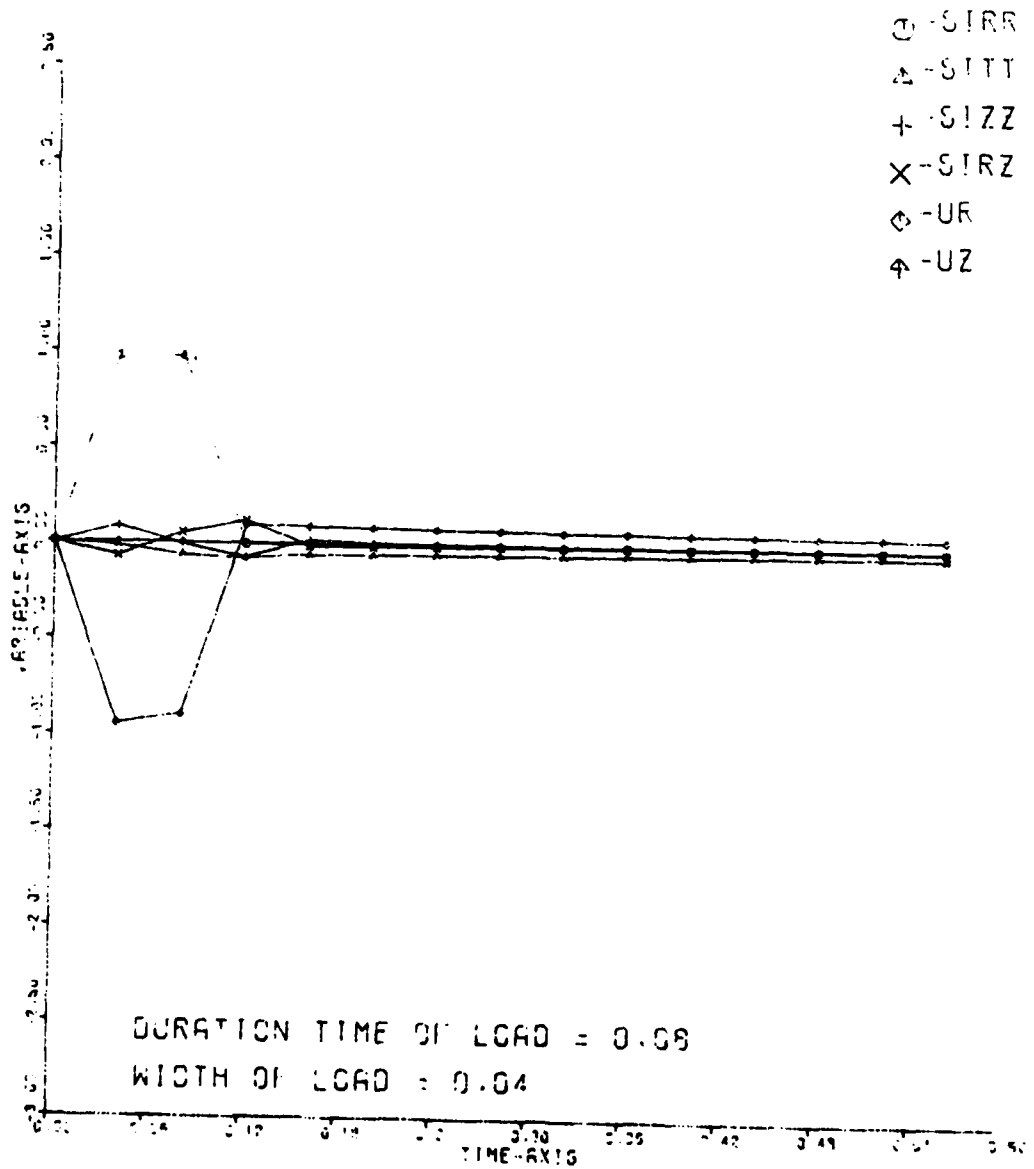


Figure 46

CASE 3

R=1.14 Z=0.00

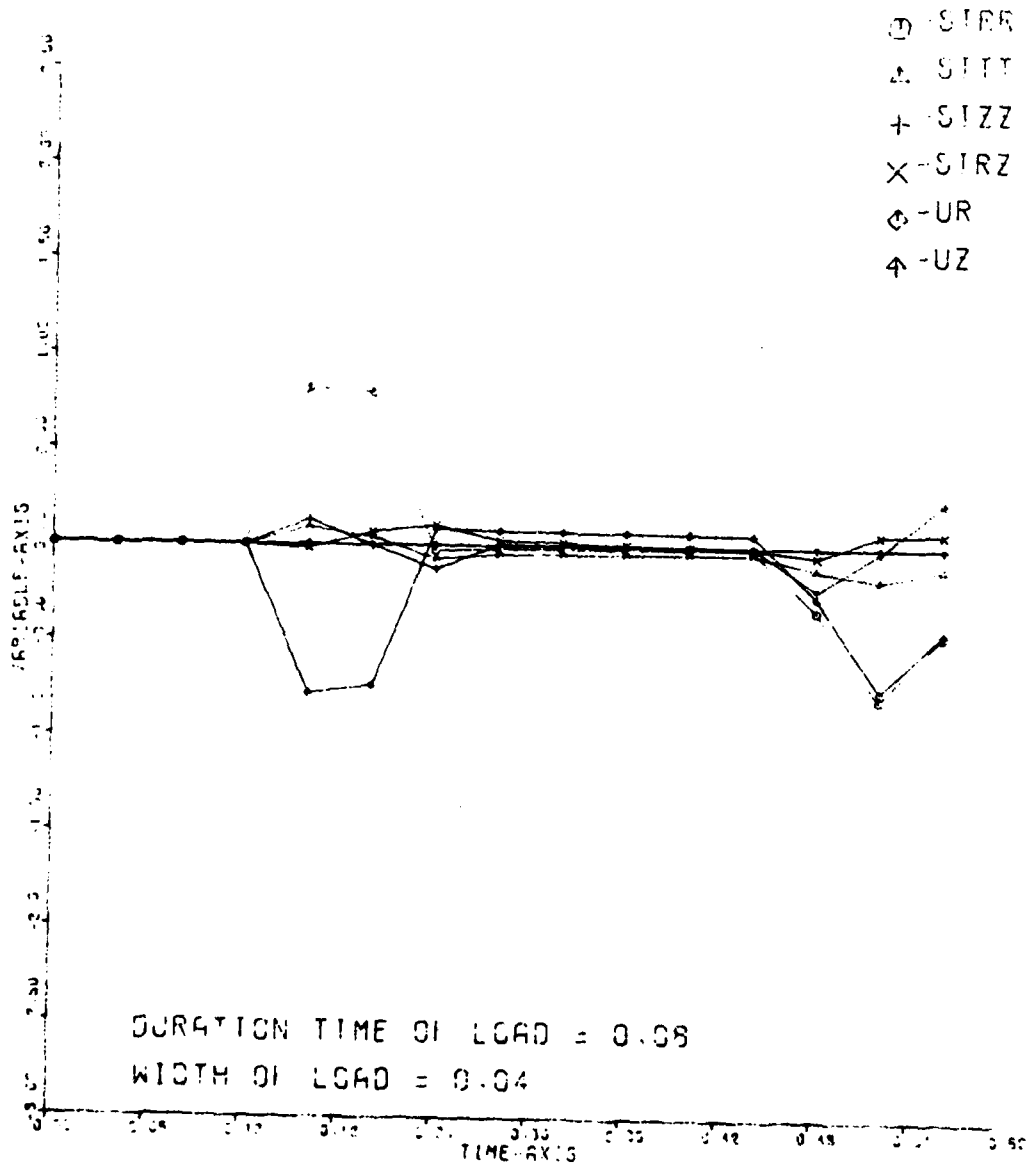


Figure 47

CASE 3

R=1.30 Z=0.00

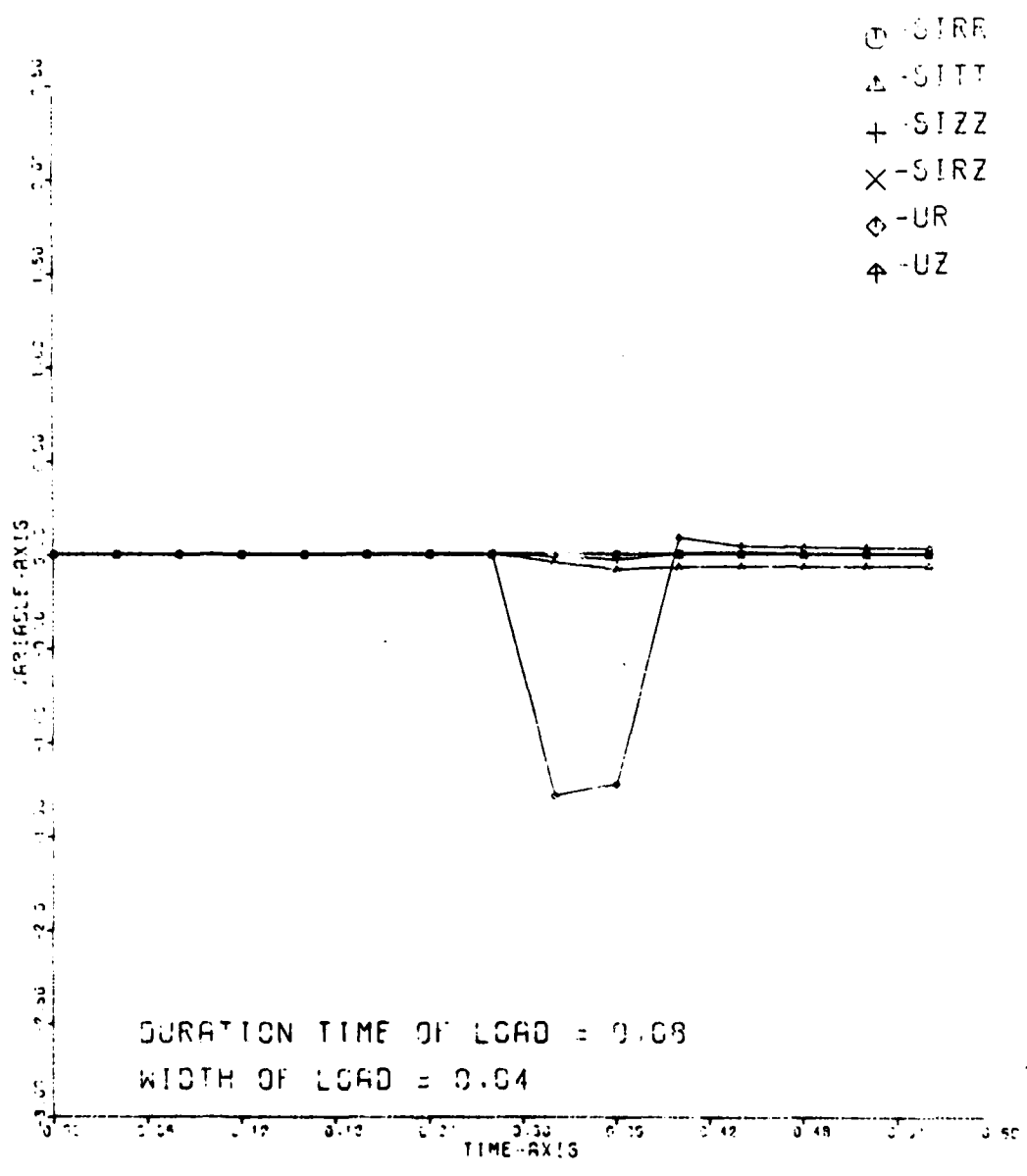


Figure 48

I

CASE 3

R=1.02 Z=0.08

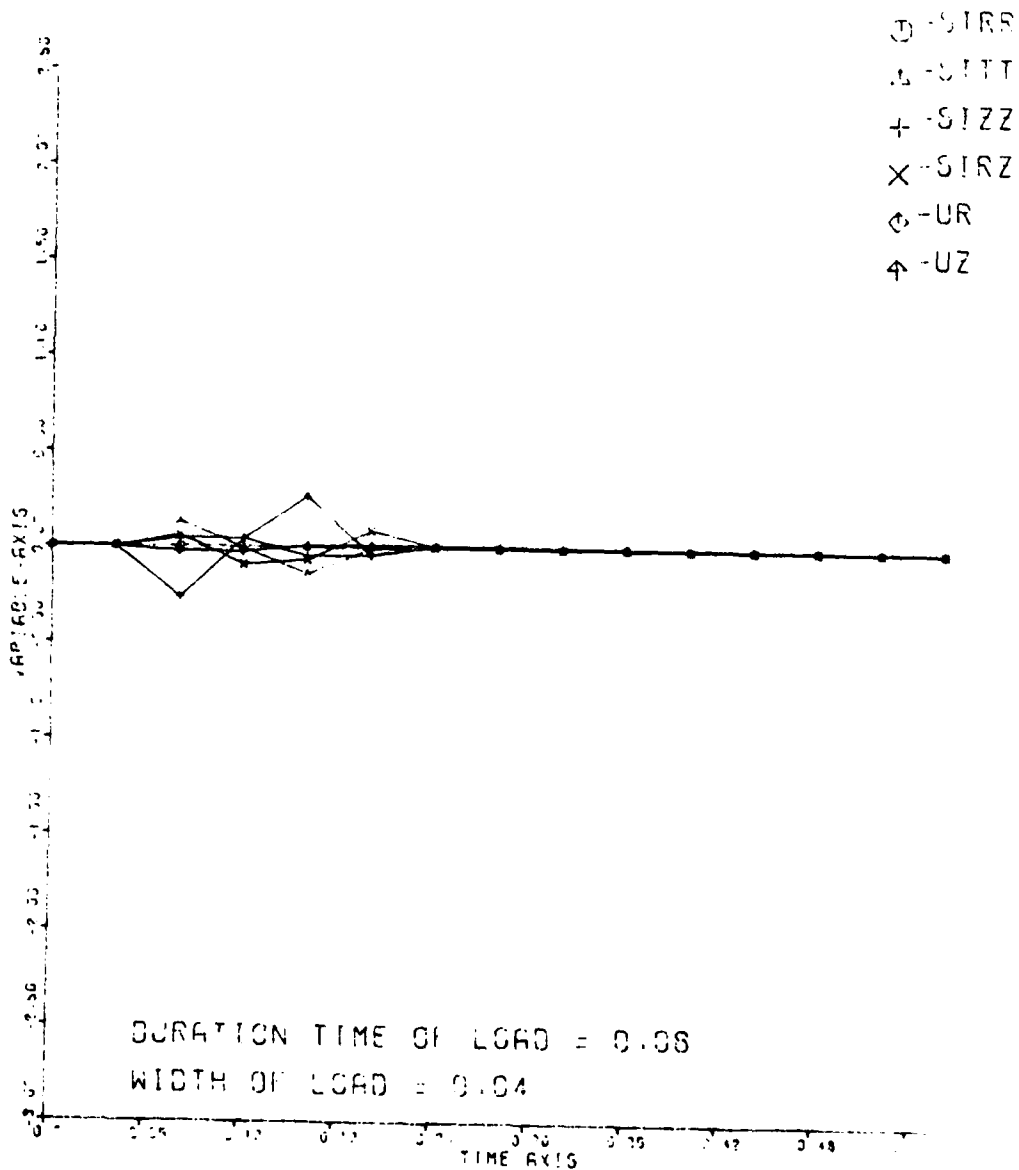


Figure 49

AD-A118 600

MASSACHUSETTS UNIV AMHERST DEPT OF CIVIL ENGINEERING F/G 20/14
RESPONSE OF THICK CYLINDRICAL SHELLS TO TRANSIENT INTERNAL LOAD--ETC(U)
AUG 82 T HAN-URA, W A NASH DAAG29-77-G-0095

ARO-14700.2-EG

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CASE 3

R=1.14 Z=0.08

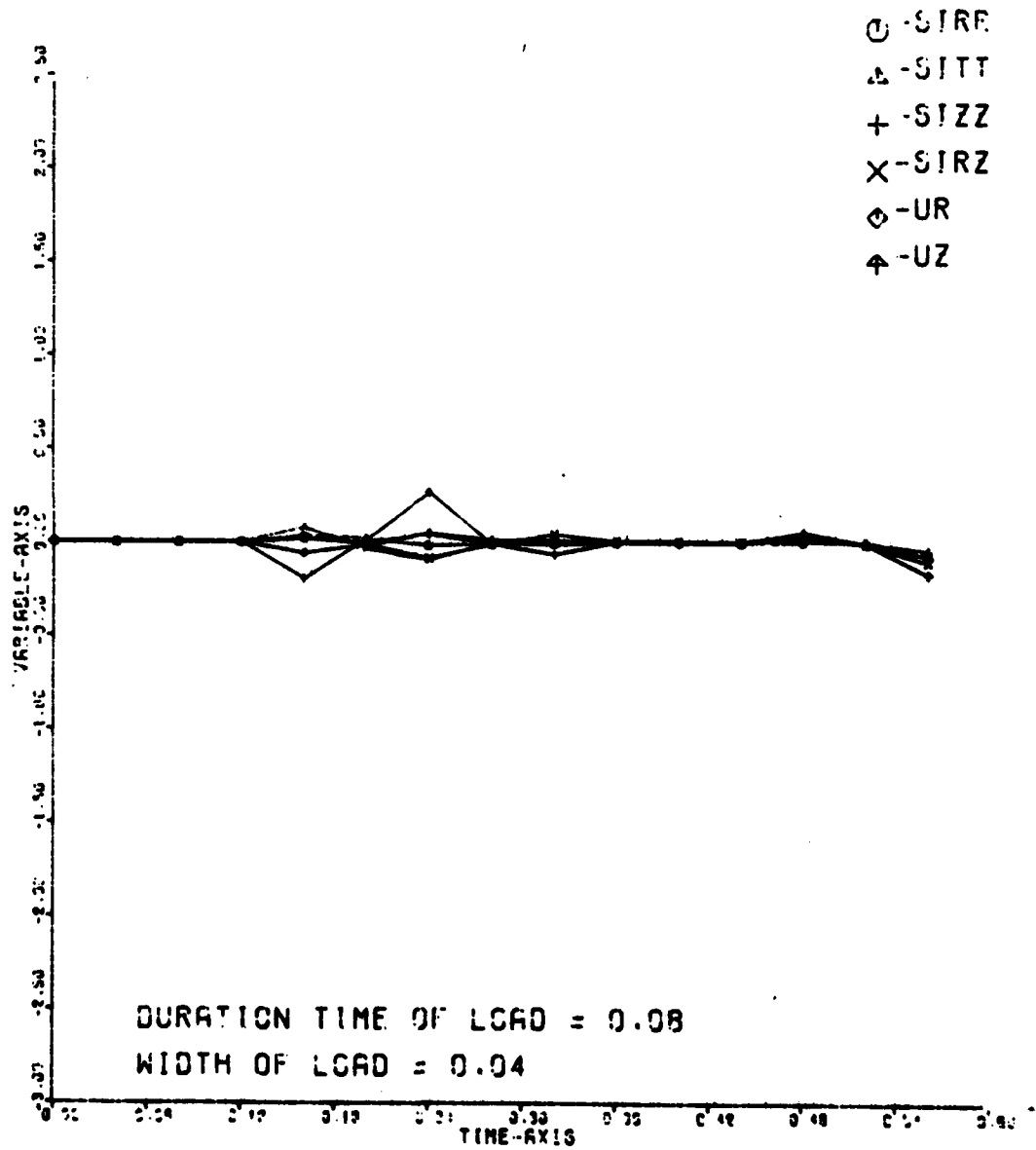


Figure 50

CASE 3

R=1.30 Z=0.09

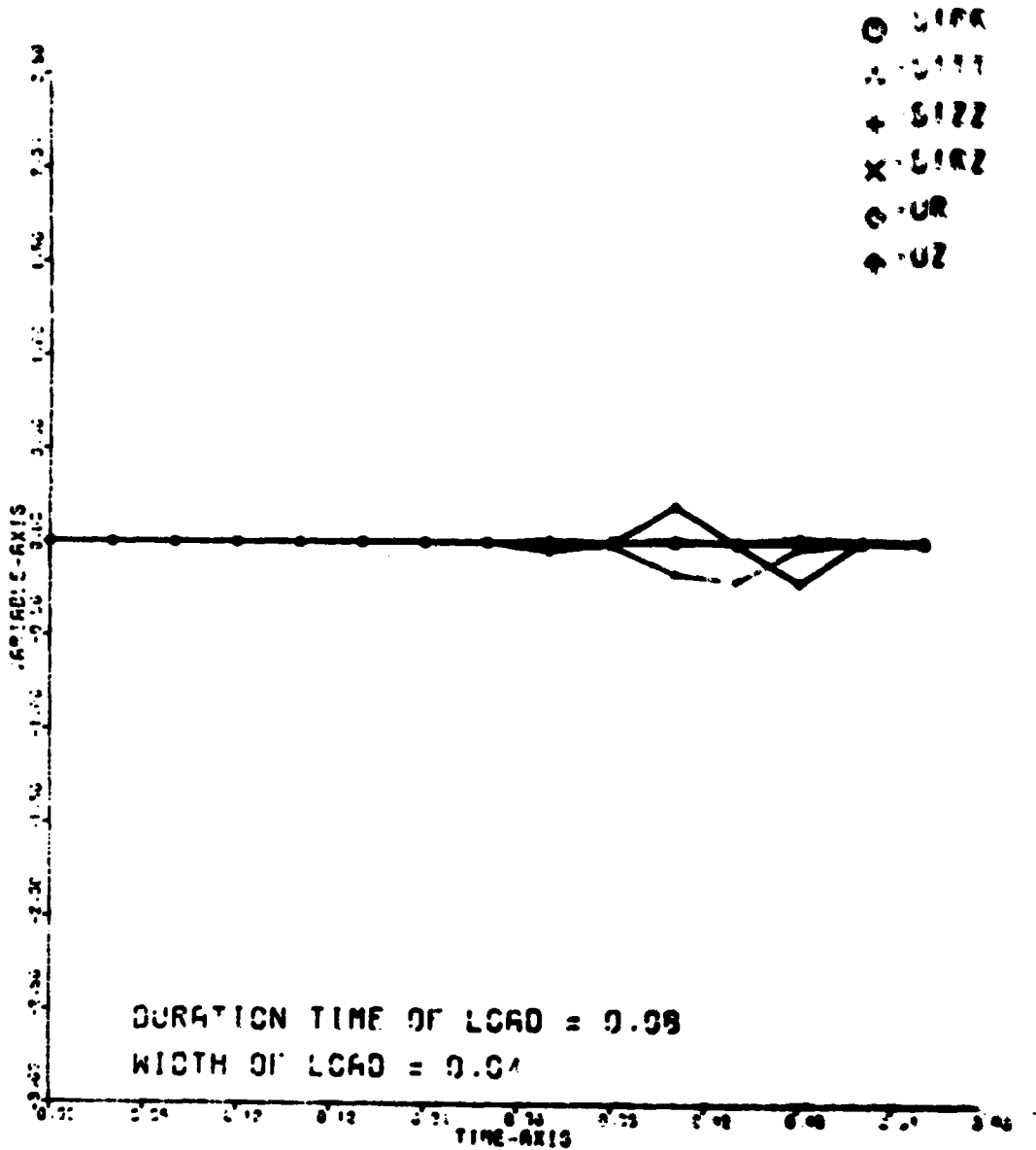
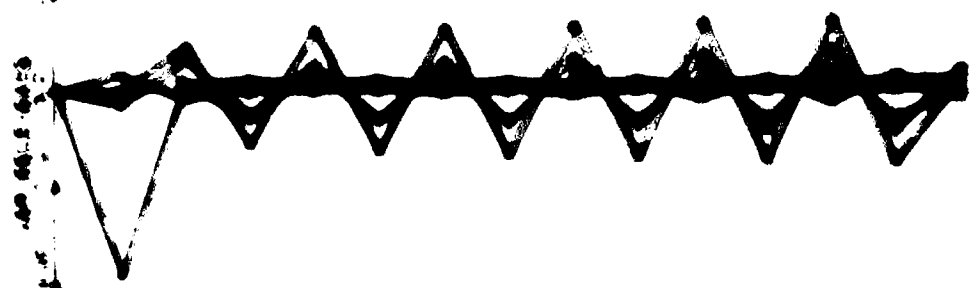


Figure 21

Figure 4
4-1-52 4-1-52

- 1.00
- 1.11
- 1.22
- ◆ 1.33
- 1.44
- 1.55



SPEED OF TRAVELLING LEAD = 0.88
DISTANCE LEAD = 0.51

Figure 4

1000 1000
1000 1000

1000
1000
1000
1000
1000
1000



SPEED OF
... ..

Figure 10

CASE 4

R=1.02 Z=0.08

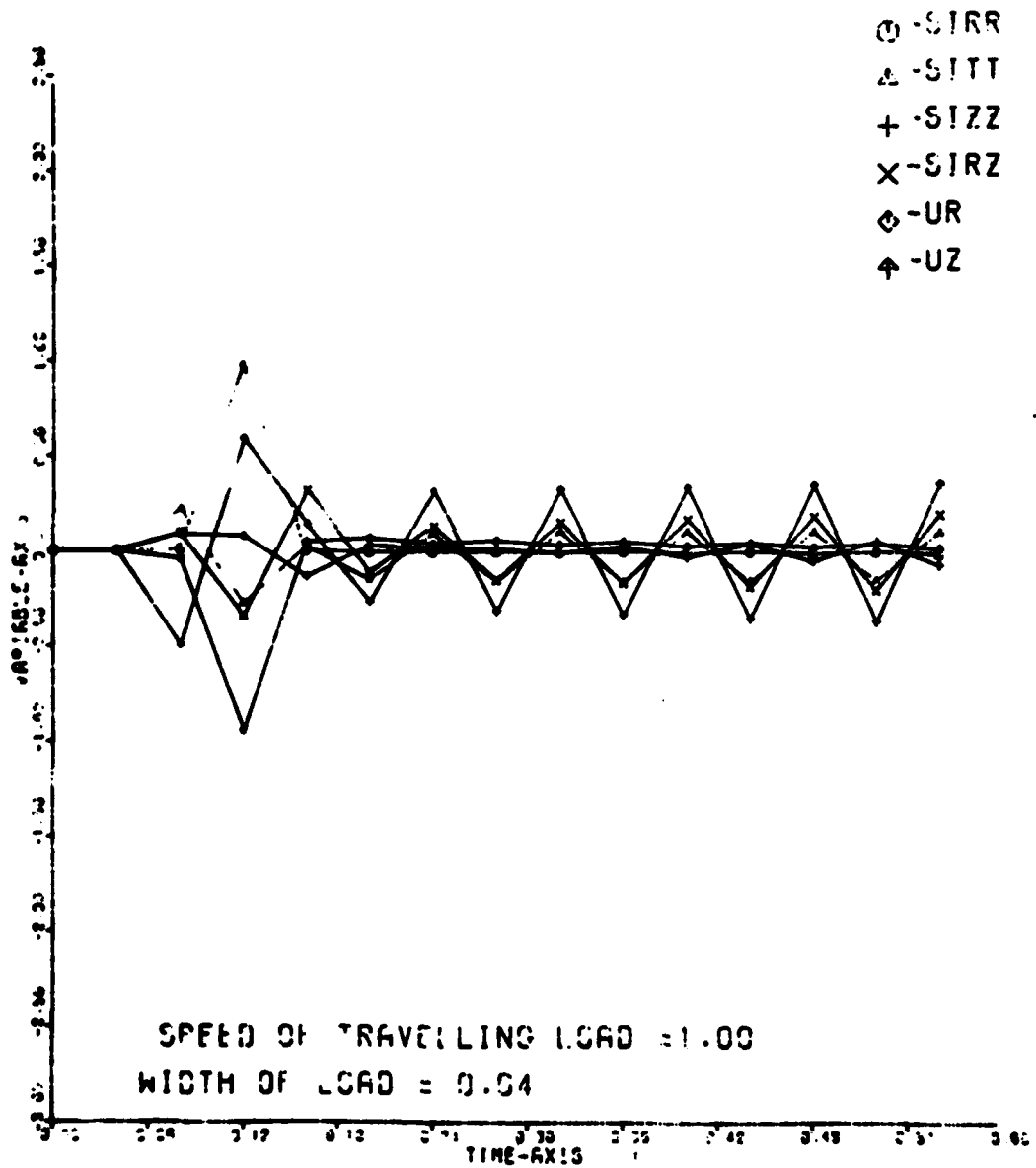


Figure 55

CASE 4

R=1.14 Z=0.08

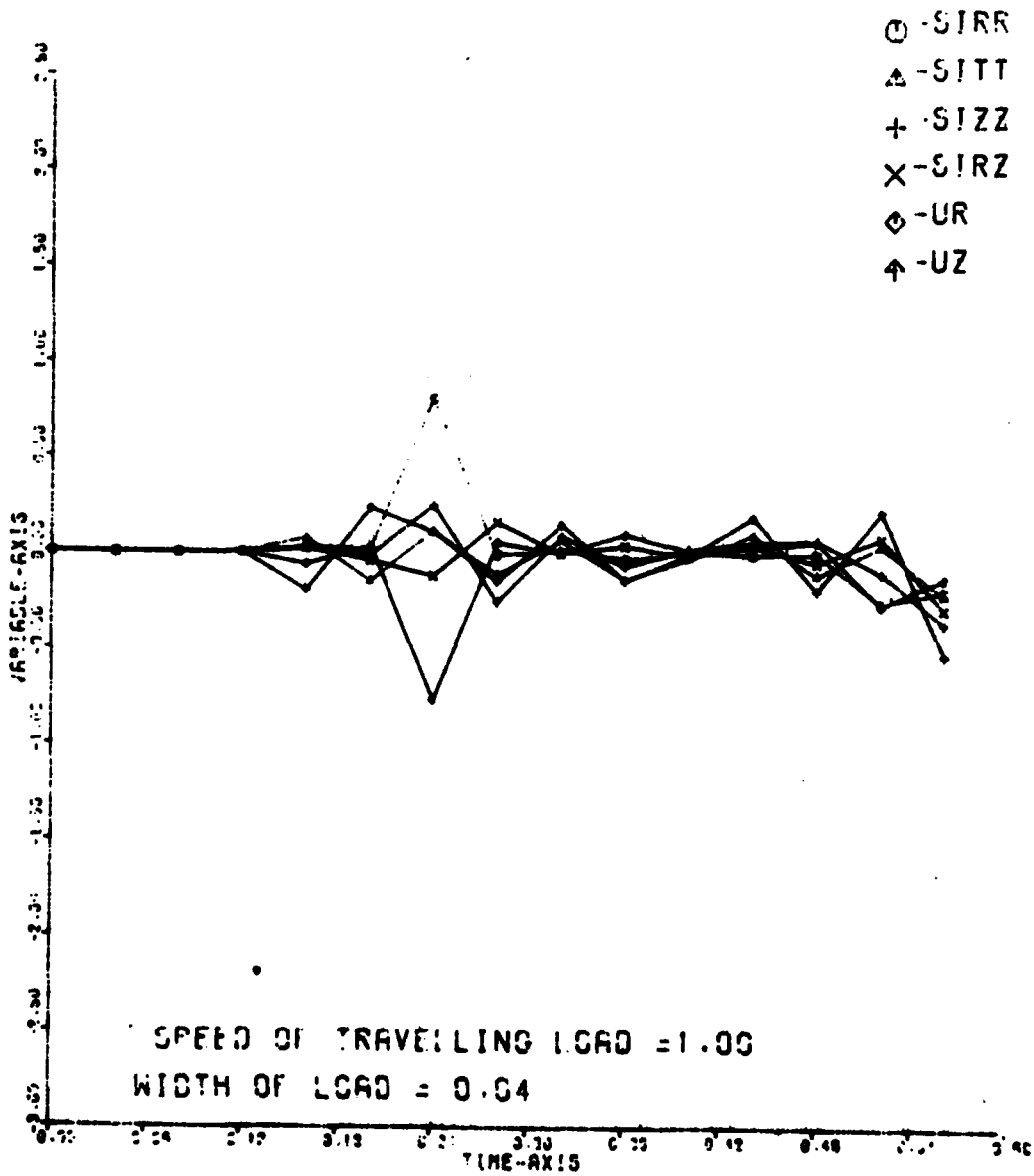


Figure 56

CASE 4

R=1.30 Z=0.08

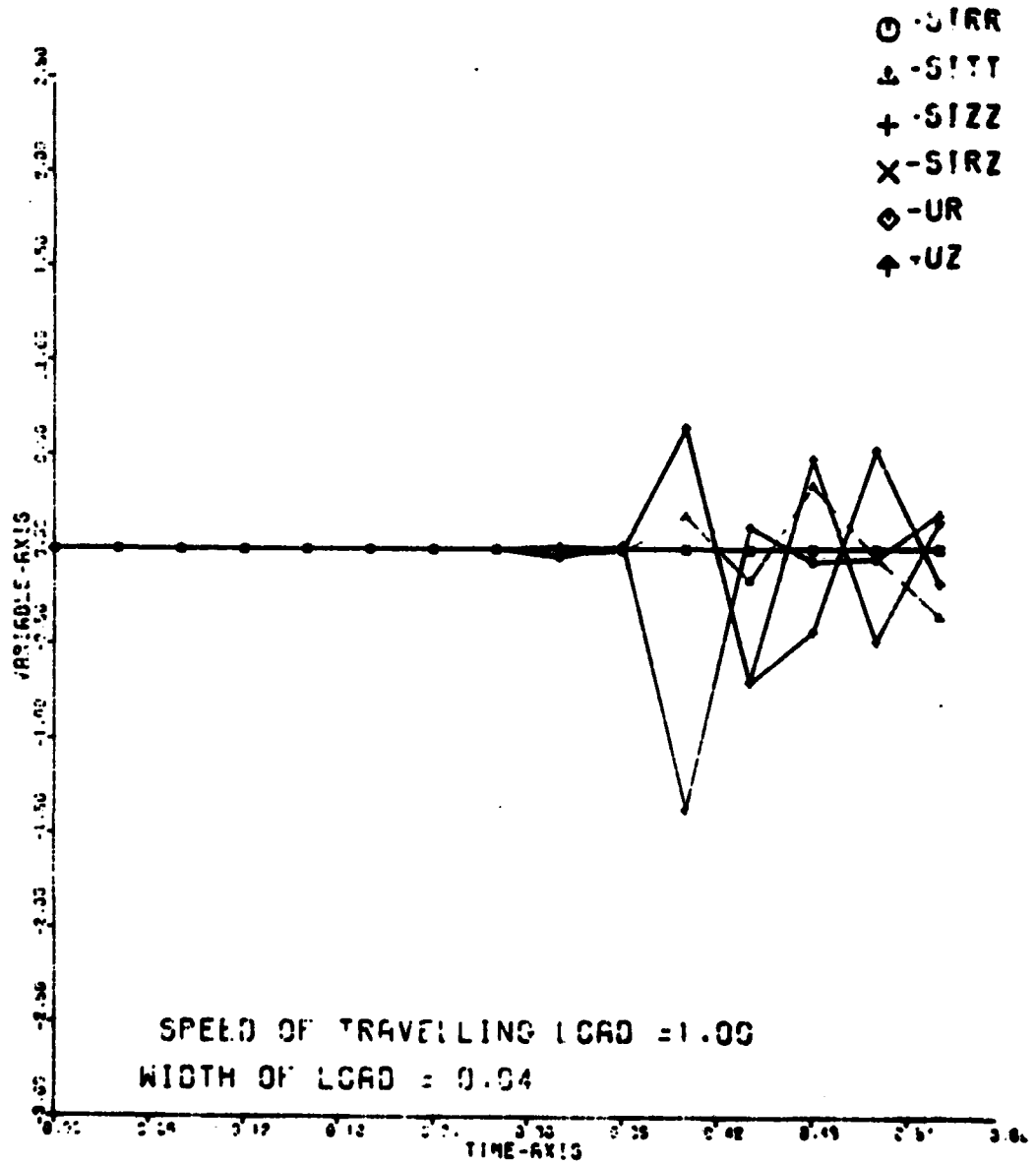


Figure 57

CASE 2

REV. 02 277 02

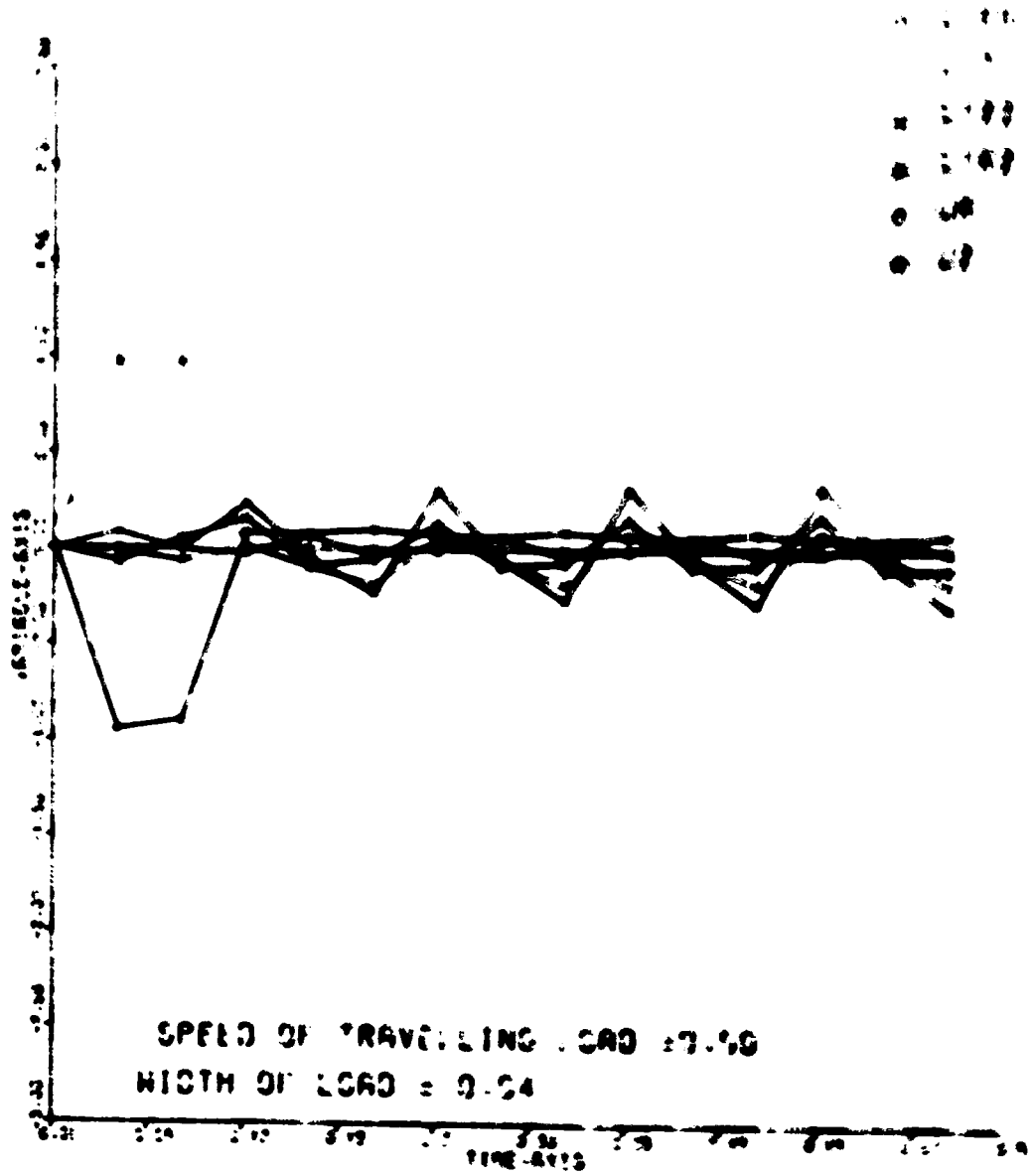


Figure 28

1947
1948

1949
1950
1951
1952
1953
1954
1955
1956
1957
1958
1959
1960



SPRINT OF ... 1947 = 1.50
SPRINT OF ... 1948 = 1.50

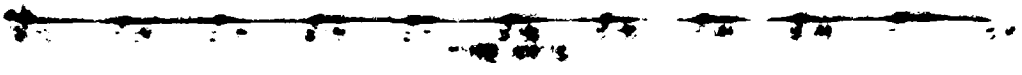
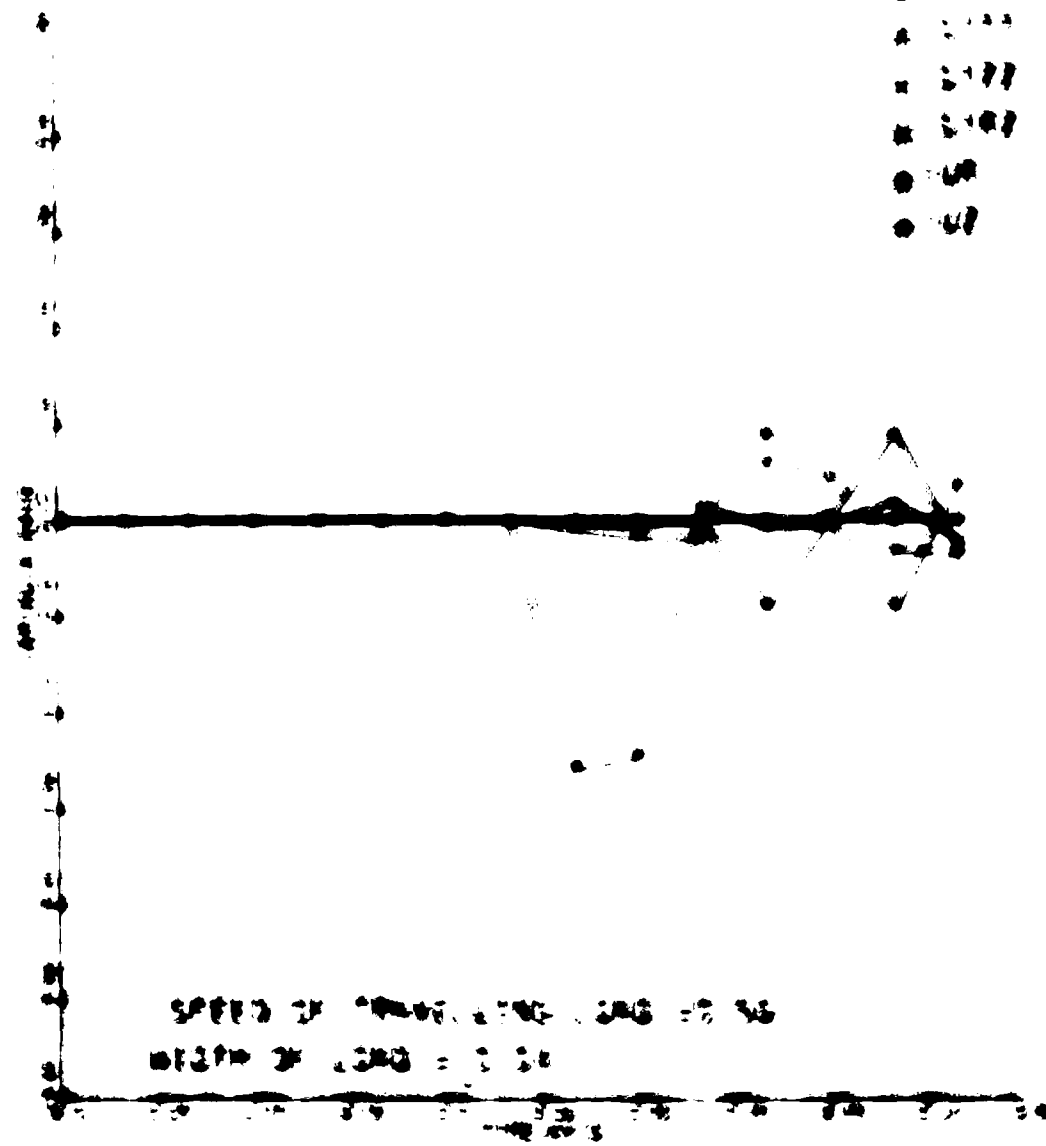


Figure 10

Chart 4

NO. 20 200 20

- 2100
- △ 2111
- × 2177
- 2187
- 2190
- 2191



SPEED OF ... 200 ... 20
 DISTANCE OF ... 200 ... 20

Figure 10

CASE 4

R=1.02 Z=9.05

- -S1R
- △ -S1T
- + -S1Z
- X -S1RZ
- ◇ -UR
- ↑ -UZ

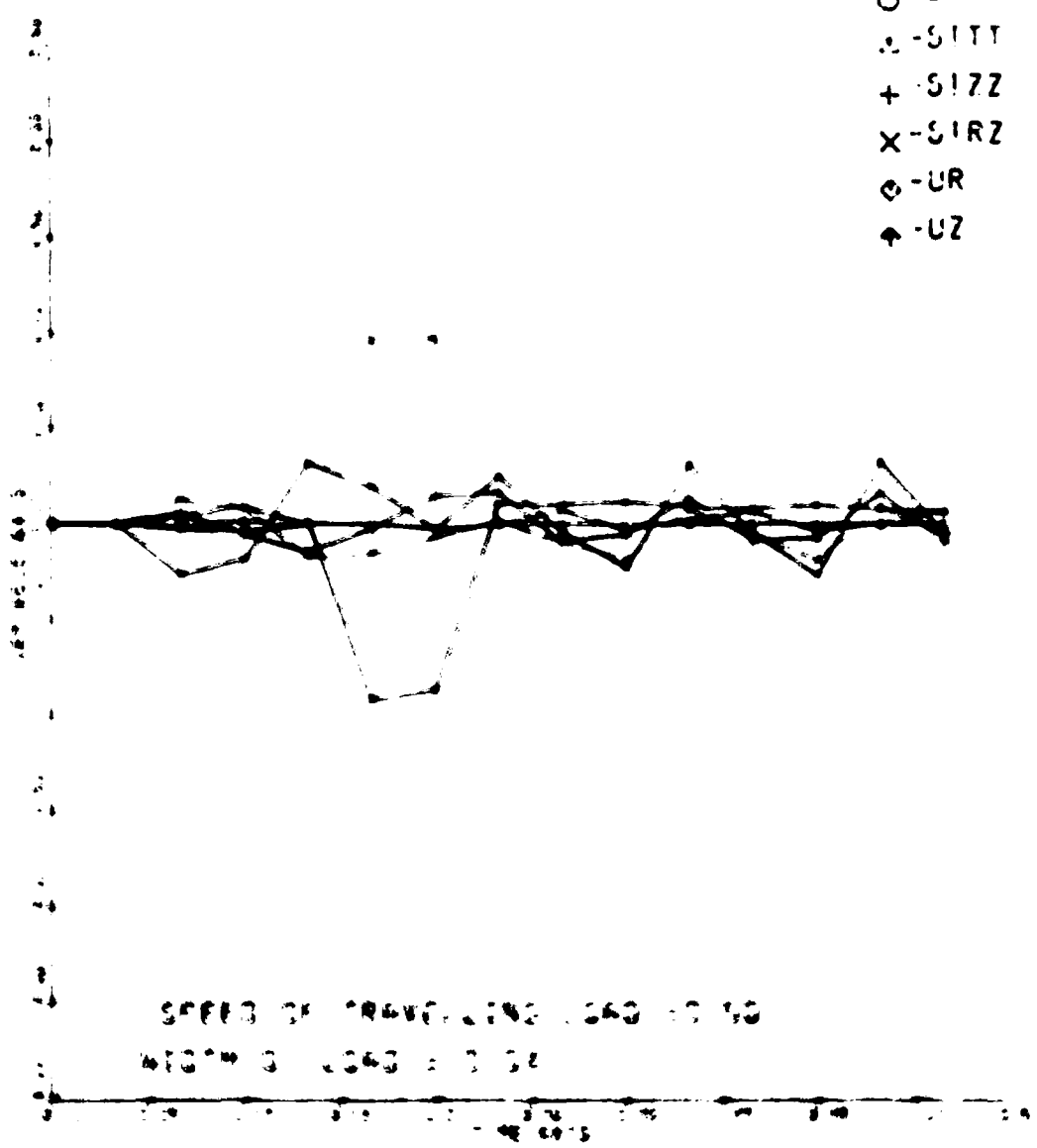


Figure 62

CASE 4

RE: 14 2-15-58

- 1188
- 1191
- 1192
- 1193
- 1194
- 1195

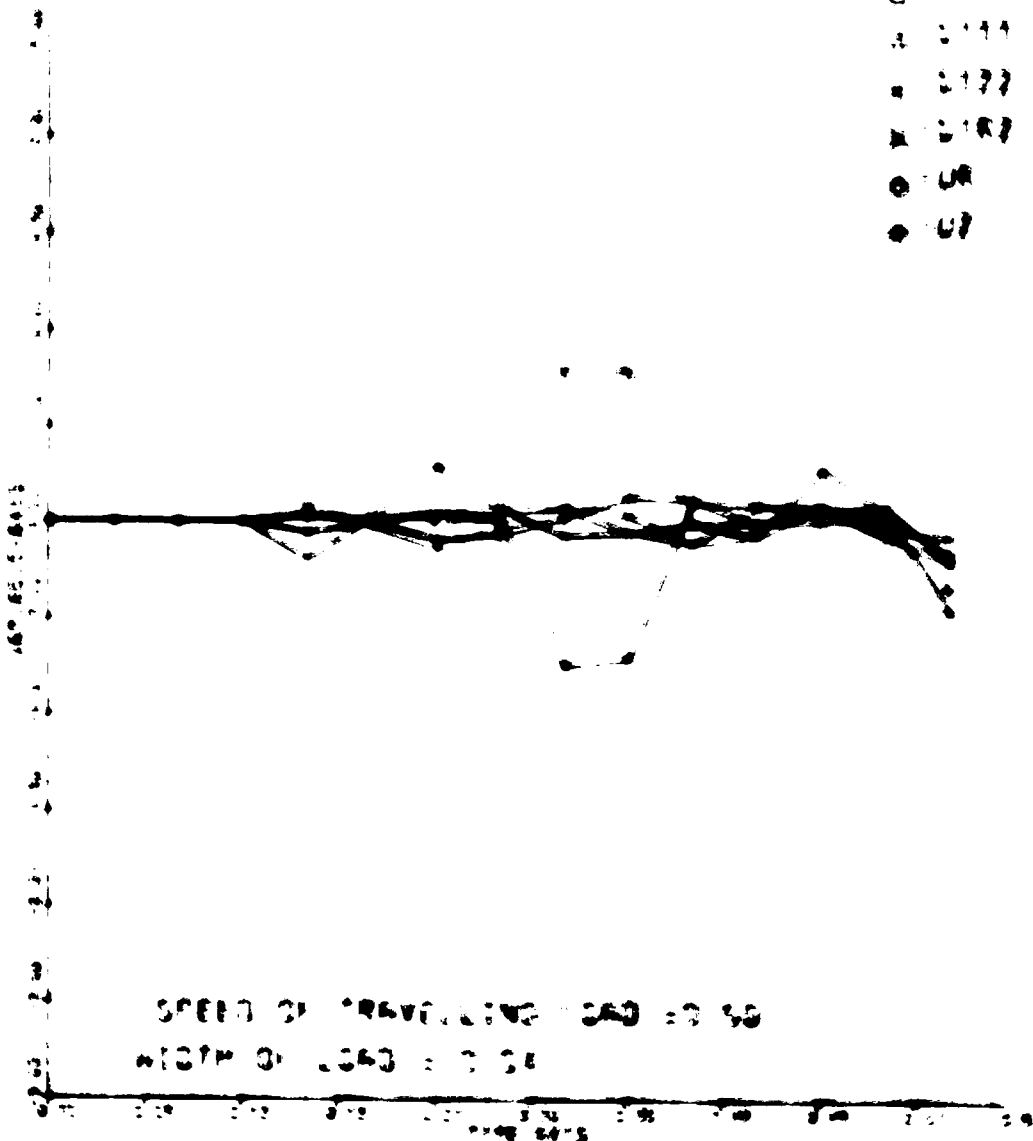
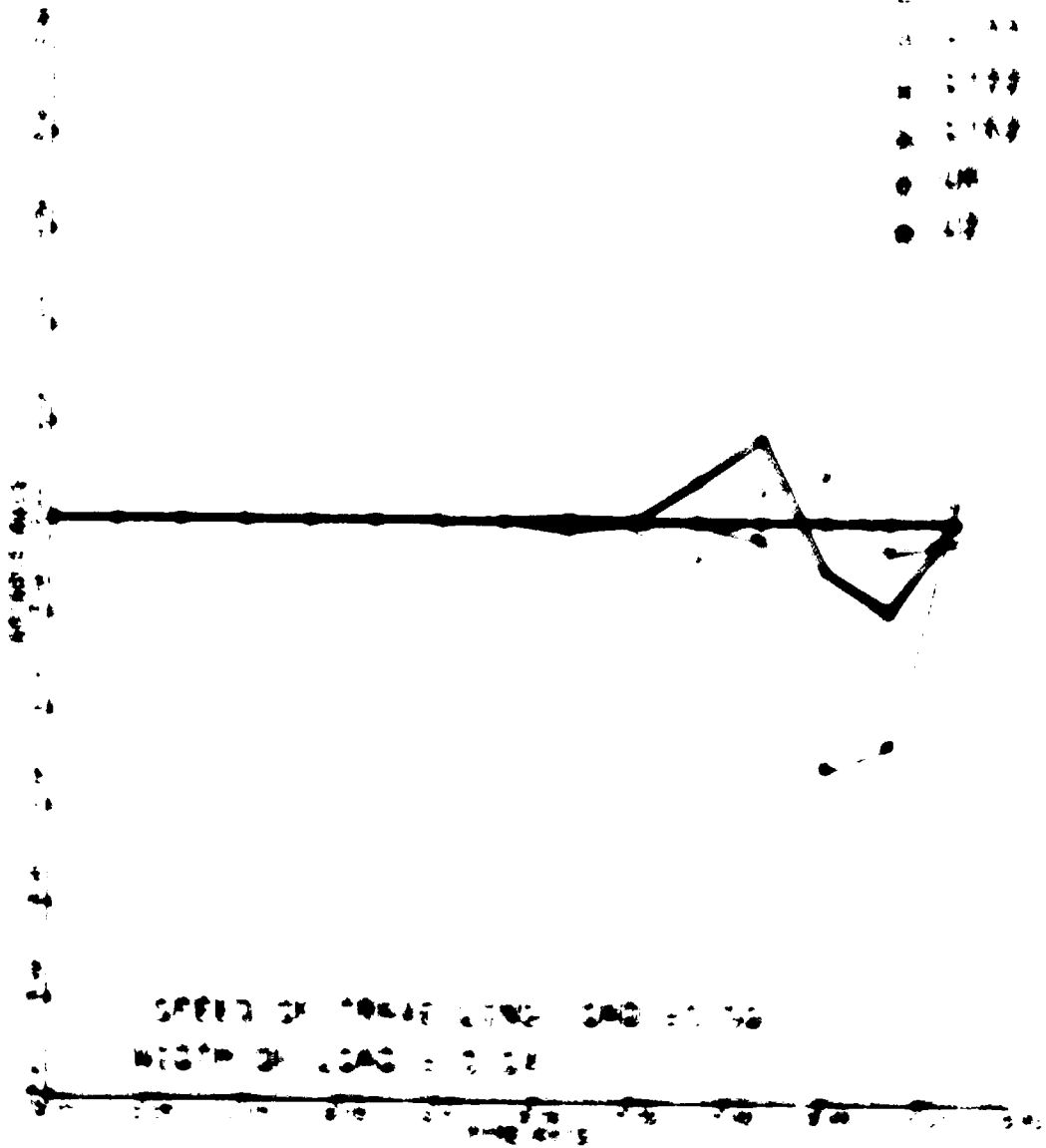


Figure 02

Case 4
No. 33 2-1-54

○	199
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○	196
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○	7
○	6
○	5
○	4
○	3
○	2
○	1



SPEED OF ...
WIND ...

Figure 40

FILME
— 8