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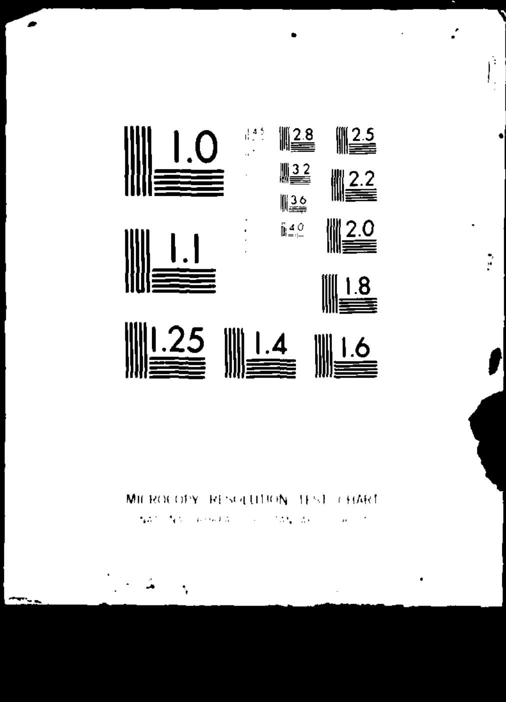

LEVEL II

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~~Final Report~~

Investigation of the Computational Utility of Geometric Programming Formulations and Algorithms

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Table of Contents

Objectives

This project proposes to investigate the effectiveness of prototype and generalized geometric programming (GP) developments in the routine solution of inequality constrained algebraic programs. The research seeks to elucidate under what conditions which of the equivalent prototype GP problem forms: primal, convexified primal, transformed primal, and, or transformed dual, as well as their generalized GP analogous are best solved and under what conditions. Moreover, direct and sequential strategies will be examined to identify the most efficient strategies for the solution of GGP's. The computational tests in the study will be performed using the leading specialized GP and GGP codes as well as a selection of general nonlinear programming codes. Statistical tests will be employed to deduce suitable measures of prototype and generalized GP problem difficulty.

Executive Summary

The research performed under this project is divided into two parts. Part I involved the study of prototype GP problems. Part II concerned the study of generalized GP problems.

In Part I fourteen test series involving ten codes or code variants were carried out to solve the five equivalent posynomial GP problem formulations. Four of the codes used were general NLP codes; six were specialized GP codes. Codes were selected on the basis of previous comparative studies as well as preliminary studies carried out as part of this project. A total of fourty-two test problems representing both engineering applications and artificially constructed problems were assembled. Each test series involved solution of each test problem from up to twenty randomly generated starting points. Starting point replication was shown to be essential to producing statistically justifiable rankings. On the basis of statistical tests, the convex primal formulation was shown to be intrinsically easiest to solve for general problems. The difference between the primal and convex primal formulations were found to lie mainly in scaling and function evaluation time. Yet these differences typically led to differences in solution times by factors of two to ten or more. In problems with special combinations of characteristics (low degree of difficulty) and mostly tight constraints, dual approaches can be competitive. A general purpose GRG code applied to the convex primal was shown to be highly competitive with the reputedly best specialized GP codes currently available. The effectiveness of the highly regarded specialized codes GGP and GPKTC appear largely to be due to the fact that these codes solve the convex primal formulation. These results therefore do cast considerable doubts on the computational significance of many years of research into prototype GP solution algorithms. Finally, a correlation analysis was carried out to show that posynomial GP problem difficulty as measured in solution time is best correlated to an exponential of the number of variables in the formulation being solved and is proportional to the total number of multi-term primal constraints.

In Part II of this study, ten test series involving five codes were carried out to solve four generalized GP problem formulations. Four of the codes used were specialized GP codes, one code was a general NLP program. The latter was the code proven most effective in Part I of this project. A total of twenty-five test problems, again representing both engineering applications and artificially constructed problems, was selected and both signomial and reversed posynomial formulations developed. Each test series involved solution of each test problem from up to twenty randomly generated starting points as in Part I. On the basis of statistical tests the preferred solution approach was shown to involve: use of the quotient form of the signomial functions; condensation of the denominators of the quotients using Duffin's geometric mean construction; and solution of the condensed subproblems in their convexified subproblem form. The code GGP which employed this strategy was shown to be most effective. Direct GRG solution of the exponential form of the primal was shown to be next best. On the basis of the Part I results, it is certain that use of the GRG code with the above three part strategy will be competitive with GGP. A variation of this strategy in which the condensed subproblems are solved in their transformed dual form was shown to be effective for problems in which the degree of freedom is much smaller than the number of primal variables. The use of condensation and the quotient representation of signomial functions thus appears to be the most computationally significant development arising from generalized GP research. As in Part I a correlation analysis was performed to deduce a measure of generalized GP problem difficulty. The number primal variables and multi-term constraints were shown to be strongly exponentially correlated. Depending upon the primal form used, the solution time is strongly correlated to the number of negative terms or the number of reversed constraints. It appears that the number of multi-term constraints is more significant than the division between posynomial and signomial constraints.

In addition to these GP oriented results, this project has developed significant methodological advances in the field of numerical evaluation of NLP software. Progress in the work of this project was reported at several symposia including the COAL Sessions held at the Montreal Mathematical Programming Symposium. One paper has been published, one will appear in a special Mathematical Programming Study, and a, third, is under review.

Part I. Posynomial Study

1. Introduction

Geometric Programming (GP) is a body of theoretical and algorithmic results concerned with constrained optimization problems involving a class of nonlinear algebraic functions [1]. Since the initial work of Zener some 16 years ago, GP has undergone considerable theoretical development, has experienced a proliferation of proposals for numerical solution algorithms, and has enjoyed considerable practical application. At the present time the field is undergoing a period of consolidation and thus a reappraisal of the practical and computational significance of the developed theory appears to be appropriate.

This paper is the second of a series of studies on the computational utility of GP formulations and developments. The overall goals of this research are to determine:

i) whether the constructions resulting from GP theory offers any computational advantages over conventional NLP methodology

ii) which of the various equivalent GP problem formulations are preferable and under what conditions

iii) which GP algorithm/formulation combination is most likely to be successful for a given problem

iv) whether a criteria can be defined by means of which GP problem difficulty can be gauged.

While the overall scope of our research encompasses both prototype and generalized GP, the present paper is confined to the prototype problem. By way of review for the novice in GP, in the next section we summarize the five equivalent prototype GP problems formulations.

2. Equivalent GP problem structures

2.1 The Primal Problem

The prototype geometric programming problem (P) is

Minimize: $g_o(x)$

Subject to: $g_k(x) \leq 1 \qquad k = 1, \ldots. K$

$\underset{\sim}{x} \geq 0 \qquad x \epsilon E^N$

where the posynomial functions $g_k(x)$ are defined as

$$g_k(\underset{\sim}{x}) = \sum_{t=S_k}^{T_k} c_t \prod_{n=1}^{N} x_n^{a_{nt}}$$

with specified positive coefficients c_t and specified real exponents a_{nt}. The term indices t are defined consecutively as

$S_o = 1$

$S_{k+1} = T_k + 1$

$T_K \equiv T$

The above problem is in general a non-convex programming problem which because of the nonlinearities of the constraints can be expected to severely tax conventional nonlinear programming codes. However, despite the apparent difficulty of the primal problem, there are structural features of the generalized posynomial functions which can be exploited to facilitate direct primal solutions.

2.2 The Convexified Primal Problem

An interesting property of the primal functions is that with the change of variable,

$x_n = \exp(z), \; n = 1, \ldots, N$

they are transformed to convex functions. This underlying convexity of the posynomial functions implies that the primal problem is basically a convex programming problem in the convex functions,

$$g_k(\underset{\sim}{z}) = \sum_t c_t \exp\left(\sum_n a_{nt} z_n\right)$$

This feature can be used to great advantage in computation since it permits application of any of a number of convex programming algorithms. Note that the variables in the convex form of the primal are unrestricted in sign whereas the original primal variables must be positive.

2.3 The Transformed Primal Problem

The convexified primal can with a further change of variable,

$$\underset{\sim}{w} = A^T \underset{\sim}{z} + \ln \underset{\sim}{c}$$

be converted to a transformed primal auxiliary problem which has the following revealing structure:

$$\text{Minimize:} \qquad g_o(\underset{\sim}{w}) = \sum_{t=1}^{T_o} e^{w_t}$$

$$\text{Subject to:} \qquad g_k(\underset{\sim}{w}) = \sum_{t=S_k}^{T_k} e^{w_t} \leq 1 \qquad k=1,\ldots,K$$

$$L(\underset{\sim}{w} - \ln\underset{\sim}{c}) = 0$$

where the rows of the matrix L are any set of linearly independent vectors spanning the null space of the exponent matrix A and where $\ln\underset{\sim}{c} = (\ln c_1, \ln c_2, \ldots, \ln c_t)$. It is readily shown that this transformed primal is in general a reduced equivalent and, if A has full rank, is exactly equivalent to the primal problem [2,3]. Given a solution, $\underset{\sim}{w}^*$, of TP a primal solution can be recovered by solving the linear system,

$$\underset{\sim}{w}^* = A^T \underset{\sim}{z} + \ln\underset{\sim}{c}$$

for the transformed primal variables $\underset{\sim}{z}^*$ and simply exponentiating the result.

2.4 The Dual Problem

As shown by Duffin, et. al., [1], the primal GP problem has associated with it a dual problem (D),

$$\text{Maximize:}\quad \nu(\underset{\sim}{\delta}) = \prod_{t=1}^{T} (c_t/\delta_t)^{\delta_t} \prod_{k=1}^{K} \lambda_k^{\lambda_k}$$

$$\text{Subject to:}\quad \sum_{t=1}^{T_o} \delta_t = 1$$

$$A\underset{\sim}{\delta} = 0$$

$$\underset{\sim}{\delta} \geq 0$$

where

$$\lambda_k = \sum_{t=S_k}^{T_k} \delta_t \qquad k=1,\ldots,K$$

and where by definition

$$\lim_{\delta_t \to 0} (c_t\lambda_k/\delta_t)^{\delta_t} = 0$$

It is well known that at their respective optima, $\underset{\sim}{\delta}^*$ and $\underset{\sim}{x}^*$,

$$\nu(\underset{\sim}{\delta}^*) = g_o(\underset{\sim}{x}^*)$$

and that the primal and dual solutions are related by the following log-linear equations which are defined for those t with $\delta_t^* > 0$,

$$\sum_{n=1}^{N} a_{nt} \ln x_n^* = \begin{cases} \ln (\delta_t^* \nu(\delta^*)/c_t) & 1 \leq t \leq T_o \\ \ln (\delta^* t/c_t\lambda_k^*) & S_m \leq t \leq T_m \\ & m=1,\ldots,M \end{cases}$$

It is further known that the logarithm of $\nu(\delta)$ is a concave function which is continuously differentiable over the positive orthant. Hence, the dual problem with logarithmic objective function is a linearly constrained concave program.

There are, however, three major complications associated with the direct maximization of the dual:

1) The gradient of $\ln(\nu(\delta))$ is not defined when any dual variable $\delta_t=0$.
2) If $\delta_t^* = 0$ for some t, $S_k \leq t \leq T_k$, and k, $1 \leq k \leq K$, then all dual variables δ_t^*, associated with constraint k must equal zero.
3) It is possible that the system of log linear equations which must be solved to determine the optimal primal variables may lead to inaccurate solutions or that its rank may be less than N.

The second of these difficulties can be mitigated if the definitions of the variables λ_k are explicitly incorporated as constraints into the problem formulation and the λ_k are explicitly treated as independent variables. The first part of the third complication can be avoided, as pointed out by Dembo [5] , if the primal variables are recovered as Kuhn-Tucker multipliers of the dual constraints. If the rank is less than N, a subsidiary maximation of the dual problem must be undertaken prior to the recovery of the primal solutions [4] . The problem of non-differentiability when $\delta_t = 0$ has to date been treated by either setting arbitrarily small lower bounds $\delta_t \geq \varepsilon$ or by introducing penalized slack variables [6] .

2.5 The Transformed Dual Problem

An alternate way of formulating the dual program is to eliminate the linear equality constraints by solving them for the dual variables in parametric form. Using this device the dual variable δ_t can be expressed as the sum of a particular solution and a linear combination of T-N-1 homogeneous solutions of the N+1 dual constraints. Thus, all feasible values of the dual variables will be given by the parametric equations.

$$\delta_t = b_t^o + \sum_{j=1}^{T-N-1} b_t^j y_j$$

where $\underset{\sim}{b}^o$ is any particular dual feasible solution and the $\underset{\sim}{b}^j$ are any basis of the solution space of the homogeneous form of the dual constraints. In this manner, the following Transformed Dual Program can be constructed.

Maximize: $$v(y) = \prod_{t=1}^{T} \left(\frac{c_t}{\delta_y(y)}\right)^{\delta_t(y)} \prod_{k=1}^{K} \lambda_k^{\lambda_k}$$

Subject to: $$\delta(y) = b^o + \sum_{j=1}^{D} \underset{\sim}{b}^j y_j \geq 0$$

where $$\lambda_k = \sum_{t=S_k}^{Th} \delta_t(y)$$

and where the y_j, j=1,.., D (=T-N-1) are unrestricted in sign.

The above is a maximization problem in D unrestricted variables subject to T linear inequality constraints. The following properties of this problem are well known:

i) The logarithm of v(y) is a continuously differentiable concave function within the positive orthant.

ii) if all primal constraints are active at the optimum, then at the corresponding dual optimum all T transformed dual inequality constraints will be inactive. Hence, the transformed dual will take on an unconstrained maximum.

iii) if primal constraint k is inactive at the optimum, then $\delta_t(y)=0$, $t=S_k,\ldots,T_k$, and the corresponding transformed dual constraints must be satisfied simultaneously as equalities.

While, from a computational point of view, the second is a very desirable property, the third very definitely is not since it requires implementation of forcing strategies to ensure that blocks of TD constraints become tight simultaneously.

3. Scope of the Present Study

From the preceding review and from the summary given in Table 1, it is evident that the various GP problem formulations differ in variable dimensionality; in number, type, and functional form of their constraints, as well as in the particular regularity conditions which must be satisfied. Numerous algorithms have been reported in the GP literature for these various formulations exploiting their peculiarities. In the present work we gathered experimental data on the performance of ten codes or code variants in solving a battery of 42 test problems each solved from up to 20 different starting points. Four previous comparative studies of prototype GP solution approaches have been reported in the literature. Two of these, Rijckaert and Martens [7] and Dembo [5] primarily focused on generalized GP's but did include prototype problems in their test slate. The study by Dinkel, et. al. [8] was restricted to the examination of alternative cutting plane methods used for the solution of the convex primal. Sarma, et. al. [9], in what may be viewed as a pilot to the present work, considered primal, dual, and transformed primal solution approaches and attempted to draw conclusions about the preferred approach.

Rijckaert and Marten's tests were restricted to eight prototype problems, used single starting points, and generally employed penalized slack variables to avoid difficulties with loose constraints. The algorithms tested included

i) dual and transformed dual maximization approaches: convex simplex method, successive LP solution, separable programming, gradient projection, modified Newton solution of the transformed dual.

ii) various strategies for solving the Kuhn-Tucker optimality conditions (in either primal or dual form) using Newton-Raphson methods

iii) two convex primal solution algorithms, GPKTC and GGP

Although the test results were quite limited, the convex simplex method adaptation due to Beck and Ecker [10] appeared to be the most reliable, if not always the fastest, dual maximization approach; the convex primal approach GPKTC appeared to be the best overall with GGP second. Dembo [5] included six prototype problems in his testing but these six were parameter variants of only three original problems. The codes tested consisted of several good general NLP codes applied directly to the primal as well as several specialized GP codes, including GGP and GPKTC but not the Beck and Ecker program. The test problems were run by the code authors on their own machines; using a single set of starting points; allowing tuning of programs by the authors; but requiring the solutions to meet fixed tolerances. Solution times were compared using Colville standard times. For the prototype problems, GGP and GPKTC gave the fastest times, often by nearly a factor of two better than the best of the NLP times. However, the validity of using Colville standardized times has since been seriously questioned by Eason [11] who showed that standardized times for a given program on different machines can differ by an order of magnitude.

Sarma, et. al. [9], solved 21 prototype problems using two variants of the Beck and Ecker code, a penalty method specialized to the GP primal, a code which solves the GP transformed primal (DAP), and GGP. One to three starting points were used and solutions were timed to achieve a specified constraint tolerance (.001) as well as a specified objective function tolerance (within 1/4% of optimum value). The last three codes used starting points generated by DAP but suitably transformed; while the Beck and Ecker code was allowed to generate its own starting points. The study indicated that the penalized slack variable device was generally less effective than the block strategy used in the unmodified Beck and Ecker code (MCS). The GGP code was often faster than DAP and significantly faster than the direct primal approach. In retrospect the experimental procedure in this study was lacking in three respects. Code timing included starting point generation in the case of MCS and DAP but not with the others; the constraint tolerance was the primary tolernace parameter in two of the codes, hence, the objective function tolerance could not be precisely controlled; and, run replication was too small.

In the present work we will attempt to rectify some of the experimental inadequacies of the previous studies: a large number of problems will be used; up to 20 replications using different starting points will be run; appropriate statistical tests will be used for comparisons; results will be obtained at several precise error levels; and code timing will be controlled to exclude starting point generation and extraneous I/O. In addition, the experiments will be designed such that, at least for the various primal formulations, formulation effects can be separated from algorithm effects.

4. Experimental Procedure

4.1 Test Codes

Ten codes or code variants form the basis of this study. The first four are general purpose NLP codes which were selected on the basis of an extensive evaluation of NLP codes carried out under another project [12].

i) OPT is a generalized reduced gradient code which employs the Fletcher-Reeves direction updating formula. [13] It has proven to be comparable to the code developed by Abadie and more robust than the code developed by Lasdon.

ii) MAYNE is a conventional interior penalty function method which employs the DFP updating formula for the unconstrained search phase. [14]

iii) RALP is a linearization based algorithm which uses successive linear programming subproblems supplemented with Newton type iterations to maintain constraint feasibility. [15]

iv) BIAS is an implementation of a variant of Hestenes' Method of Multipliers developed by Schuldt [16, 17] to accomodate nonlinear inequality constraints. Unconstrained optimization is carried out by means of the DFP algorithm.

On the basis of the studies reviewed in the previous section, the specialized GP codes selected for use in the present work were: GGP, GPKTC, DAP and MCS. In addition, a promising transformed dual code not appearing in these studies was also chosen in order to be able to generate comparative data with that GP problem formulation [20]. Finally, one general purpose NLP code modified to accept the special features of the convex primal form was also included. Details of these codes follow:

i) BIAS-SV is a variant of the Method of Multipliers specialized for the convexified GP primal. It employs a modified Newton method for the unconstrained optimization phase since for the convexified primal functions analytic second derivations are readily calculatable. BIAS SV further exploits this property by using a second derivative based line search. [17]

ii) GPKTC, a code developed by Martens and Rijckaert [18], solves the convexified primal by iterative solution of the Kuhn-Tucker optimality conditions for that problem. The iterations follow essentially the Newton-Raphson algorithm.

iii) GGP is a code developed by Dembo [19] which also solves the convexified primal although the version used in this study does not fully exploit the problem structure. The code is an implementation of Kelley's cutting place method, a venerable and well known convex programming algorithm. The linear subproblems which arise in this method are solved using the dual simplex method with provisions for upper bounded variables.

iv) MCS is a GP specialization of the convex Simplex Method proposed by Zangwill. Beck and Ecker revised the conventional direction generation machinery to insure that all dual variables associated with a given constraint reach zero simultaneously. The code also includes provisions for solving subsidiary maxmimization problems if difficulties in recovering primal solutions are encountered. [10]

v) QUADGP is a specialized GP code developed by Bradley [20] which solves the transformed dual as a series of quadratic programs. Dynamically adjusted lower bounds in the dual variables are used to accomodate the nondifferentiability of the objective function.

vi) DAP: The DAP code in an adaptation of the Differential Algorithm of Beightler and Wilde. [2] This in turn, may be viewed as a generalized reduced gradient technique which varies one variable at a time and uses an active constraint strategy to accomodate inequality constraints.

4.2 Test Problems

Of the large number of application problems available in the literature, some 42 prototype GP problems have been selected for use in this study. About half of these problems arise from engineering applications, the remainder are literature test problems. Problem references are given in Appendix B.

The problems and their characteristic dimensions are summarized in Table 2. From this table it is apparent that the test problems cover the following wide range of problem dimensions:

$$2 \leq \text{Primal Variables} \leq 30$$

$$8 \leq \text{Primal Terms} \leq 197$$

$$1 \leq \text{Number of Constraints} \leq 73$$

$$3.3\% \leq \text{Exponent matrix density} \leq 83\%$$

The lower limit on density is not as low as might be desirable. However, it should be noted that the lower limit of problem density is limited by the number of primal variables. This comes about because the sparsest possible problem involving N variables and T terms will be the one in which each term contains only a single variable (as, for example, in a linear programming problem). Thus the lower limit to the density of the exponent matrix will be,

$$\frac{T}{NXT} = \frac{1}{N}$$

It would be desirable to include large and sparse GP problems in this study. However, there are only one or two posynomial problems with more than thirty variables which have, to our knowledge, appeared in the literature. Hence we have restricted our range of investigation to problems with $N \leq 30$ and % Density $\geq 3.3\%$.

4.3 Starting Point Generation

One of the key elements in comparative numerical studies is the selection of starting points for each problem. In this study primal and dual starting points for each problem were generated randomly by sampling from the surface of an N-dimensional sphere whose center is the actual optimal solution to the problem. The choice of radius is arbitrary; in general, a reasonably large value was selected so that generation of points close to the optimum was avoided. The points generated were tested for feasibility and only feasible points retained. Typically, two different radii were used and some 10 points retained for each radius and for each problem. In some cases the feasible region was so tightly constrained that it was not possible to generate multiple and sufficiently distinct starting points, even after thousands of trials. The same primal points suitably transformed were used in the convex primal and transformed primal computations. A similar starting point procedure generation was employed for dual and transformed dual starting points. In this case, the sign unrestricted transformed dual variables were randomly sampled and the transformed dual constraints were checked for feasibility.

The need for multiple starting points cannot be overstated. As can be seen from Table 3, the variation in solution times, obtained for problem 13 using OPT started from 10 different points, can be considerable. For instance for primal solution the range of times is from 1.776 secs to 5.72 secs with a mean of 3.65 and standard deviation of 1.42. Similarly, for convex primal solution the range is from 0.83 to 1.1 secs with mean of 0.878 and standard deviation of 0.165.

Note that for a given starting point the ratio of primal to convex primal solution times changes substantially. For instance, it is 1.8 for starting point 5 and 8.3 for starting point 8. Yet these computation times are obtained using the same code with identical termination parameters. A similar although less pronounced variation can be noted when the OPT convexified primal solution times are compared to those obtained with the GGP program. The solution time ratio ranges from 0.53 for problem 8 to 1.0 for problem 10.

These results indicate that code ranking based on performance with a single starting point, as is commonly done in the literature, is a very questionable procedure. If at all possible, a sufficiently large number of points must be used so that reliable means and standard deviations can be computed. Comparisons must then be made via statistical tests such as those of Student.

The obvious difficulty with multiple starting points is the tremendous increase in computational effort. The testing carried out in the present report required some 10,000 separate test runs of which some 6,000 resulted in useful data.

4.4 Test Procedure

To carry out code tests on this scale it is imperative that the assembly, execution, and analysis of runs be automated as much as possible. In the present study we found it convenient to prepare computer files of test problem subroutines and test problem starting points for each of the five GP problem formulations. These files are accessed through an executive program which calls the desired routines, retrieves the required starting point data, executes the test run, and saves the intermediate and final results of each run in appropriate result files. The result files are separately analyzed by a data post-processing program which calculates the desired error functions, performs error function interpolations, computes means and standard deviations for each code-formulation-test problem combination at several error levels.

The primary error function used in this study is the pseudo-Lagrangian function,

$$\text{ABS}\left[\left(\frac{g_o^n - g_o^*}{g_o^*}\right) + \sum_k \frac{\lambda_k^* \,\text{ABS}\,(g_k^n - g_k^*)}{g_o^*}\right]$$

The starred quantities in this expression are the known optimal values of the problem functions and Lagrange Multipliers. The multipliers were obtained by a direct solution of the optimality conditions at x^* using a linear equation solver. The sum in the above expression is only over those constraints which are active at the optimum solution. Either because imposed constraint tolerances effectively allowed slight constraint relaxations or because the algorithms themselves generated primal exterior points, all of the codes tested except MAYNE would produce solutions with slight infeasibilities. The relative error function defined above allowed a correction to be applied to the objective function errors when slightly infeasible points are generated. Values of this error function were computed for the intermediate iteration points recorded during each run. The intermediate solution times (which excluded all I/O time required for recording test data) and error function values were fitted to polynomials and these used as interpolating functions to determine solution times at specified error function levels. Mean solution times and standard deviations were tabulated for each problem/code or formulation combination at relative error levels of 10^{-2}, 10^{-3}, 10^{-4}, for all successful runs and at termination for all runs.

Each code was run with a single fixed set of program parameters, the values of which are given in Appendix A. These parameters were selected in advance by experimentation so that a relative error of at least 10^{-4} could be attained on trial runs with a few moderately sized problems. In many cases the program parameters correspond to values recommended by the program author. No readjustment of program parameters were undertaken during the main test runs. As a result, some runs did not achieve error levels of 10^{-4}. Similarly, only a minimum of program parameter retuning was undertaken if the run failed to make any progress. The decision to avoid extensive parameter retuning resulted in gaps in the solution time data. However, because of the large number of test problems used, we believe these gaps do not seriously affect the conclusions of the study.

5. Results

An overall summary of the number of problems and number of runs attempted with each code-formulation pair is given in Table 4. As shown all problems were not run with each code. For some of the larger problems some of the codes required in excess of 150K to load and hence could not be run under the normal priority system used with the CDC6500 at Purdue University. Problem 42 had to be excluded from the study primarily for that reason. In other cases, particularly the direct primal runs, the trend was sufficiently obvious that runs with larger problems or with the complete set of starting points was not deemed necessary. This was particularly the case with the direct primal runs, the OPT transformed primal runs, and the BIAS-SV runs. Finally, in some cases, particularly the QUADGP run, the differences in the solution times between the starting points for a given problem were sufficiently small, that the number of points used per problem could be substantially reduced. Even with these economies, the number of runs which were ultimately counted in the study were nearly 6,000.

As can be seen from the last column of Table 4, the percentage of unsuccessful runs, defined as the number of attempted runs that failed to achieve any significant progress away from the starting point, varied quite substantially. The conventional NLP codes RALP and MAYNE seemed to be particularly prone to failure. The solution of the transformed primal using OPT was also unreliable, presumeably because of difficulties caused by the large number of constraints which are required by the transformed primal formulations. Most surprising was the erratic performance of GPKTC which, at times produced extremely fast solutions but in other cases failed completely. Since GPKTC basically uses the Newton

Raphson equation solving algorithm, this erratic performance may well simply reflect the often reported sensitivity of this algorithm to initial estimates. Similarly surprising is the high number of failures of the special version of BIAS, especially in view of the high reliability of the regular version of BIAS. Both solved the convexified form of the primal: the former used a modified Newton algorithm with analytic derivatives; the latter a DFP algorithm with numerical derivatives. The most reliable performance seems to have been achieved by the general NLP codes OPT and BIAS when applied to the convexified primal formulation. The next best performance was attained by the specialized codes GGP and DAP.

In order to facilitate the presentation of the more detailed test data we will aggregate these results into several series:

1) Comparison of solution times of various algorithms for a given GP problem formulation.

2) Cross-comparison of solution times for the various formulations all solved using the same algorithm.

3) Cross-comparisons of the most successful algorithms found for each GP formulation type.

4) Examination of how solution time varies with problem characteristic dimensions for each of the various formulations.

The data reported in Tables 5 through 12 is all based only on the successful runs. Moreover, the solution time for all runs of problems 15 had to be excluded because of errors introduced during post processing of the results.

5.1 Intra-formulation Comparisons

This series of runs consists of primal, convex primal, transformed primal and dual comparisons. Tables 5A, B, and C give mean solution times at relative error levels of 10^{-2}, 10^{-3}, and 10^{-4}, respectively, obtained using the codes OPT, BIAS, MAYNE and RALP to solve the primal directly. Each column of results also indicates the number of successful runs upon which the mean is based. From Tables 5A, B, and C it is clear, even without statistical testing, that OPT is generally faster than the other codes, yielding solution times less than 1/2 of the next best competitor's in the majority of cases: 20 of 35 at the 10^{-2} level, 17 of 29 at the 10^{-3}, and 13 of 21 at the 10^{-4}. Note that quite a high proportion of the direct primal attempts failed to advance the starting points to even the 10^{-2} relative error level.

The general results are quite in agreement with the conclusions of the general NLP code comparison recently completed by Sandgren [1], in which the GRG based codes outperformed all other algorithms, including MAYNE, RALP, and BIAS.

The corresponding comparisons involving solution of the convex primal are shown in Tables 6A, B, and C. The codes involved in this comparison are: the general NLP codes OPT, BIAS, and RALP; the version of BIAS specialized for GP's, and the specialized GP programs GGP and GPKTC. Again the general purpose GRG code dominates the others in mean solution times at all three error levels. Based on mean times GGP is second and GPKTC third. However, in this series of runs there is less difference between the means and, hence, statistical testing is necessary to provide a more definitive ranking. It is clear, however, that overall OPT is at least as effective as the specialized programs GGP and GPKTC. This in itself is quite surprising in view of the fact that the latter two codes are specially designed for polynomial problems and use analytic derivatives rather than difference approximations as does OPT. It should be noted that the version of GGP used in these tests performs some unnecessary computations whose exclusion would have somewhat reduced the GGP solution times. In setting up each new linear subproblem a conversion from the z to the x space is made, the original constraints are evaluated in the x space, the cut is generated, and the subproblem is reconverted to the z space for solution. Timing estimates have shown that depending upon the problem characteristics 6 to 16% savings in CPU time could have been attained if all calculations had been performed in only the z space. Such mean CUP time reductions would not, however, substantively affect the observed comparisons.

The transformed primal results summarized in Table 7 indicate quite clearly that the specialized algorithm DAP is faster: of eighteen problems for which solution times are available for both, DAP has solution times 1/2 or less those of OPT in 11 cases. OPT predominated in only 4 cases by the same margin. In the remaining three cases the mean times were too close to differentiate without statistical tests. This performance is as might be anticipated, since in the OPT version used, the T-N tranformed primal linear equality constraints are not accorded special handling. Moreover, the transformed single term constraints

$$g_k(w) = \exp(w_t) \leq 1$$

are not simplified to the form, $W_t \leq 0$, which would allow implicit rather than explicit handling of such constraints. Both of these structural features are exploited in DAP. However, it is interesting to note that the differences in the solution times decrease (e.g. problem 1, 2, 11, 12, 13, 20, 25) or occassionally are reversed at the higher accuracy level (e.g. problem 16). This indicatesthat the GRG constraint adjustment strategy employing Newton's method is more efficient than the line search based methods used in DAP. Since both algorithms essentially employ a similar direction generation method (the reduced gradient), it is thus likely that a specialized OPT transformed primal version will obtain a superior performance to that obtained by DAP.

Separate dual and transformed dual intra-formulation comparisons were not carried out as part of this study. Earlier work, cited in Section 3, indicated that the dual based MCS algorithm was preferrable to a variety of other dual and transformed dual approaches. A comparison of MCS solution times with those obtained using QUADGP, which solves the transformed dual, is shown in Table 8. Using a 2 to 1 time ratio as being significantly different, QUADGP is clearly superior, at the 10^{-2} error level, in 23 of 32 cases with MCS being superior in only 5 cases. However, at the 10^{-3} level this slips to 15 vs 10 and at the 10^{-4} level to 11 vs 14. This swing is largely due to the fact that QUADGP failed to solve problems to the lower error tolerances. Thus Table 8 tends to confirm the conclusions obtained in earlier studies about the general robustness of MCS.

The intra-formulation comparisons thus indicate that OPT is the most effective for both primal and convex primal solution with GGP a convex primal second. The specialized code DAP is better than OPT for the transformed primal formulation. Intra-formulation comparisons are not given for the dual and transformed dual.

5.2 Intra-formulation Comparison Using the Same Code

In order to elucidate which primal form is most efficiently solved, we present a comparison of solution times for the primal formulations when the same code is used for each formulation. Relative times are given for OPT in Table 9 and for BIAS in Table 10.

From the OPT and BIAS primal to convex primal ratios, it is obvious that the primal times are almost always larger by at least a factor of two. This indicates quite clearly that all of the differences reported by Dembo [4] between the best NLP solution times and the best specialized GP solution times is due to the fact that the NLP codes (several of which were GRG codes, as is OPT) solved the primal and the best specialized codes (GGP and GPKTC) solved the convex primal. A significant portion of the difference in solution times appears to be due to differences in function evaluation times. Evaluation of the term $\prod_n x_n^{a_{nt}}$ is carried out on the machine using logarithms, summing the results, and taking anti-logarithms. The term $\exp(\sum_k a_{nt} z_n)$, on the other hand, can be carried out via a simple sum and a single exponentiation. The former is much more time consuming. We have observed numerous BIAS runs in which objective and constraint function values at the successive unconstrained optimization stages as well as the actual number of functional evaluations taken were nearly identical for both the primal and the convex primal iterations of a given problem: yet, the solution times were very much different.

A second commonly cited difficulty with GP primals is scaling, that is, both the sensitivities of the various problem functions with respect to variable changes are substantially different as well as the sensitivities of any given function with respect to different variables varies substantially. Undoubtedly, the favorable scaling introduced by the transformation $z_n = \ln(x_n)$ is reflected in the primal to convex primal solution ratios. However, in our experience GRG codes are less sensitive to scaling than other NLP algorithms; while the version of BIAS employed in our tests incorporate automatic scaling of both constraints and variables based on the composite Jacobian. Thus, we can not on the basis of our results conclude to what extent scaling is a factor.

In comparing the convex and transformed primal results, the main difference in solution times can be explained in terms of a trade-off between number of variables and constraints and the reduction of variable interactions in the non-linear functions. Since for any problem T>N, the transformed primal always has higher dimensionality than the convex primal. Moreover, while both formulations always have the same number of non-linear constraints, the transformed primal will in addition have T-N linear equality constraints. When T-N is small, as in problem 8, then solution times are close. However, when T-N is large as in problem 12, then the solution times are substantially different. Because problem dimensionality will be shown to be the predominant variable in determining problem solution time, it seems unlikely that solution via transformed primal can be made significantly more efficient than convex primal solution, even if special provisions are made for the T-N linear equality constraints.

Finally, it is of interest to note from Table 10, that the special GP version of BIAS which uses analytic second derivations for unconstrained optimization and line searching is considerably slower than the normal BIAS code when both solve the convex primal form. This is particularly noticeable as problem size increases. There is to be sure some reduction in the time ratio in going from the 10^{-2} to the 10^{-3} error level, reflecting the expected faster convergence rate of the modified Newton algorithm. However, the computational time required to evaluate the second derivatives apparently is not balanced by increased efficiency in the search. This finding is consistent with the results reported by Sarma, et. al. [9] in which a primal approach using analytical second derivatives was found to be quite inefficient.

5.3 Intra-Formulation Comparison Using Different Codes

Finally, we compare the solution times for the various formulations when each is solved using the code shown to be the most effective for that formulation. Tables 11A, B, and C summarize the mean times for the best primal (OPT), convex primal (OPT), transformed primal (DAP), dual (MCS), and transformed dual (QUADGP) codes. The mean times for GGP are also included because they were sufficiently close to those of OPT. As can be seen from these tables for any given problem the mean solution times of the two fastest codes often differ by less than a factor of two. Moreover, as can be seen from Table 12, the standard deviations of the means frequently are quite substantial. Thus comparisons of the mean solution times necessitate the application of statistical tests.

Assuming that the solution times x and y of two codes for any given problem are normally distributed variables each with their own variances σ_x^2 and σ_y^2, then code solution time comparison is equivalent to the problem of testing whether the true mean solution times, M_x and M_y, of the two codes for the given problem are equal. This is the Behrens-Fisher problem of statistics [21]. It can be shown that if s_x^2 and s_y^2 are the unbiased sample estimates of the variances, then the variable

$$t = \frac{(\bar{x}-\bar{y}) - (M_x-M_y)}{\left(s_x^2/n_x + s_y^2/n_y\right)^{1/2}}$$

will possess an approximate student t distribution with degree of freedom

$$\nu = \frac{\left(\frac{s_x^2}{n_x} + \frac{s_y^2}{n_y}\right)^2}{\frac{\left(\frac{s_x^2}{n_x}\right)^2}{n_x + 1} + \frac{\left(\frac{s_y^2}{n_y}\right)^2}{n_y + 1}} - 2$$

In these expressions, $\bar{x}$ and $\bar{y}$, are the sample means and n_x and n_y are the sizes of the two samples. These formulas were applied to test the difference in the means of the solution times given in Tables 11A, B, and C. Sample results for the 10^{-2} error level are given in Table 13. The Student t value was calculated using the means given in Table 11A and standard deviations given in Table 12. The significance level of the differences in the means can be determined using standard tables of the Student t distribution [21, Appendix 2]. These significance levels are given in the last column of Table 13 for those means which were based on at least three runs. As can be seen from the Table, for several problems, e.g. problem 2, the means were not significantly different. In the case of problem 2, it was necessary to proceed to the fourth best mean time before a significant statistical difference from the best mean time could be established.

These calculations were repeated at the 10^{-3} and 10^{-4} relative error levels. The results were used to determine the number of problems for which each code achieved the best or second best solution times. An 90% significance level was required before means were considered to be different. In the case of differences below that significance level, both codes were ranked equally. The results of this ranking are shown in Table 14. It is quite clear that OPT applied to the convex primal is the best GP solution approach overall. At the highest error level of 10^{-2}, DAP and QUADGP are competitive However, at lower error levels these two codes fade out to be decisively overtaken by MCS at the 10^{-4} level. The better ranking of MCS at the 10^{-4} error level, however, is largely a result of its robustness rather than its speed. In 9 out of 13 cases in which it is first, it is the only code to solve the problem to that accuracy level and in several of those instances the solution times exceed 50 sec., a very large time within the framework of this study.

In general, the dual based codes are only competitive with the convex primal solution methods when T-N is small and when the multi-term constraints are active at the optimum (eg. problems 2, 8, 13, 16, 32, and 34). The specialized codes, GGP, DAP, as well as GPKTC, overall do not appear competitive with OPT. Quite clearly the successes of GGP and GPKTC against other GP solution approaches, as reported in several recent comparative studies [5, 7, and 9] are largely due to the fact that these codes are based on the convex primal formulation rather any insights offered by GP theory. Thus these results do cast doubts on the computational significance of many years of research into non-zero degree of difficulty, prototype GP solution algorithms.

It should be noted that significant improvements in the performance of OPT would in all likelihood be achieved if some structural features of the convex primal are exploited. For instance, single term constraints need not be treated as normal constraints but can be converted to linear inequality constraints. Also, computation of derivatives could be made much more efficient by saving the term values for each posynomial during function evaluation and then calculating the partial derivatives analytically as a simple weighted sum of these term values, i.e.,

$$\frac{\partial gk}{\partial z_n} = \sum_t a_{nt} \, [c_t \exp (\sum_n a_{nt} z_n)]$$

where the quantities in brackets are the already calculated term values. In OPT derivatives are calculated numerically by differences and single term constraints are treated as explicit nonlinear inequality constraints.

5.4 Effect of Problem Dimensions on Solution Time

The remaining objective of this study was to attempt to deduce which characteristic dimensions of a prototype GP problem could best be used as a measure of solution difficulty. To that end pairwise correlation coefficients were computed between solution difficulty as measured by mean solution time and seven different, but not necessarily independent, problem characteristic dimensions. For purposes of these computations the total solution time to termination of a run was used rather than the solution time to a specified relative error tolerance. This is appropriate because the runs used to calculated a given correlation coefficient all involve the same code run with the same set of program parameters.

The results for two types of assumed relationships are given in Tables 15A and 15B. Table 15A contains the correlation coefficients obtained by assuming that solution time is proportional to an exponential function of the particular problems characteristic, that is,

$$\text{time} \propto b^y$$

where y is the problem characteristic such as number of variables, number of constraints, etc. Table 15B gives the correlation coefficients when a linear relationship is assumed. For comparative purposes, the last column of the tables lists the critical value of the correlation coefficient for a 0.05 significance level ([21], p. 167).

For the primal solution approach (OPT-P) the solution time correlates most strongly to the exponential of the number of the number of primal variables. However, significant linear correlation also exists with the number of constraints and the degree of difficulty. Note that the degree of difficulty correlates better than either the number of terms or the number of primal variables separately. Also the number of constraints correlates more strongly than either the number of multi-term constraints or the number of tight constraints. The dependence on the exponential of the number of variables and the less than exponential dependence on the total number of constraints is as might be expected for a GRG code. The correlation to degree of difficulty, on the other hand, is a GP problem characteristic which probably reflects the time required to evaluate the problem functions.

For the convex primal solution approaches (OPT-CP, GGP, and GPKTC) the situation is less coherent. The strongest exponentially correlated variable seems to be the number of primal terms both for GPKTC and for OPT. For both the correlation coefficient is higher for primal terms than for either primal variables or degree of difficulty. In the case of GGP, however, the number of primal variables shows a higher correlation. The strongest linear correlation appears to involve the degree of difficulty for GGP and GPKTC but the number of constraints for OPT. Apparently the differences in the operations of the algorithms used to solve the convex primal serve to obscure the trends. Nonetheless, it is clear that the number of primal variables becomes less significant and the number of primal terms as well as the density more important in going from the primal to the convex primal.

In the transformed primal case, the exponential dependence on the number of primal terms becomes even more pronounced. This is to be expected since for the transformed primal the latter becomes equal to the number of problem variables. The linear dependence on the number of multi-term constraints becomes more pronounced because the effect of the single term constraints is minimized since they are uncoupled as a result of the problem transformation.

In the case of the dual approaches, the most significant correlation is found to the exponential of the degree of difficulty for the transformed dual (QUADGP) and to the exponential of the number of primal terms for the dual (MCS). This is consistent since these quantities correspond to the number of variables in these formulations. Strong linear correlation is shown to the number of primal multi-term constraints - significantly stronger than to either the total number of constraints or to the number of tight constraints. Apparently, this reflects the overhead of having to deal with the λ_k variables in the problem formulations, regardless of whether or not these vanish. Curiously the density of the exponent matrix shows no significant correlati for either the dual approaches or the transformed primal approaches, but does show a reverse correlation in the primal and convex primal cases. The density correlation in the primal case is probably spurious since it is unlikely that primal computation time will decrease with increased density for problems of the same dimensionality. After all, in the primal case, density reflects the degree of coupling of the program variables. Most likely the reverse correlation is induced because of the basically inverse relationship between density and number of primal variables noted in Section 4.2. We are thus led to conclude that density does not appear to be a reliable primary indication of problem difficulty.

In summary, in all cases the key exponentially correlated problem characteristic appears to be the number of variables in the problem formulation. In going from primal to convex primal to transformed primal to dual, the key linearly correlated problem characteristic shifts from number of constraints to number of multi-term constraints. Number of tight primal constraints is in all cases only a secondary factor. Density of the exponent matrix does not appear to be a reliable primary indicator of problem difficulty as measured in solution time.

6. Conclusions

Within the limits of the experimental design of this study, a key feature of which is the use of fixed code parameters, the following overall conclusions may be drawn:

i) the convex primal is inherently the most advantageous formulation for solution.

ii) a general purpose GRG code applied to the convex primal is competitive with the reputedly best specialized GP codes currently available.

iii) the differences between the primal and convex primal formulations lie mainly in scaling and function evaluation time.

iv) transformed primal solution approaches are not likely to lead to more efficient GP solution than the convex primal

v) the dual approaches are only likely to be competitive for small degree of difficulty, tightly constrained problems.

vi) posynomial GP problem difficulty as measured in solution time is best correlated to an exponential of the number of variables in the formulation being solved and is proportional to the total number of multi-term primal constraints.

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TABLE 1: PROBLEM CHARACTERISTICS

	PRIMAL	CONVEX PRIMAL	TRANSFORMED PRIMAL	DUAL	TRANSFORMED DUAL
NO. OF VARIABLES	N	N	T	T(+K)	T-N-1
NO. OF CONSTRAINTS					
LINEAR EQUALITY	-	-	T-N	N+1(+K)	-
LINEAR INEQUALITY	-	-	-	-	T
NON-LINEAR INEQUALITY	K	K	K	-	-
SIGN RESTRICTION ON VARIABLES	YES	NO	NO	YES	NO
PROBLEM CONVEX (CONCAVE)	NO	YES	YES	YES	YES
PROBLEM SEPARABLE	NO	NO	YES	NO(YES)	NO

Table 2. Posynomial Test Problems and Their Characteristics

Problem ID	No. of Primal Variables	No. of Primal Terms	Degree of Difficulty	No. of Constraints							Density
				Total	Multiterm			Single Term			
					Total	Loose	Tight	Total	Loose	Tight	
1	2	12	9	8	3	2	1	5	4	1	83
2	3	6	2	2	2	1	1	-			78
3	3	6	2	2	2		2	-			78
4	4	8	3	3	1		1	2		2	58
5	3	9	5	1	1		1	-			33
6	3	11	7	1	1		1	-			57
7	4	6	1	2	1		1	1		1	71
8	4	6	1	2	2		2	-			38
9	4	8	3	3	1		1	2		2	44
10	4	12	7	1	1		1	-			42
11	4	13	8	2	2		2	-			30
12	4	21	16	4	4	2	2	-			78
13	5	9	3	3	1		1	2	1	1	33
14	5	9	3	2	2	1	1	-			60
15	5	10	4	3	3	1	2	-			72
16	6	13	6	5	1		1	4	4		23
17	6	17	10	4	4		4	-			53
18	6	19	12	2	1		1	1		1	58
19	7	11	3	2	2		2	-			72
20	7	13	5	1	1		1				44
21	7	18	10	4	4	2	2	-			70
22	7	18	10	4	4	1	3	-			70
23	7	18	10	4	4		4	-			70
24	7	18	10	3	3		3	-			33
25	7	18	10	3	3	2	1	-			48
26	7	18	10	4	4		4	-			70
27	7	18	10	12	1		1	11	6	5	23

Table 2. (continued)

Problem ID	No. of Primal Variables	No. of Primal Terms	Degree of Difficulty	No. of Constraints							Density
				Total	Multiterm			Single Term			
					Total	Loose	Tight	Total	Loose	Tight	
28	7	48	40	9	9	5	4	-			42
29	7	48	40	7	7	5	2	-			35
30	7	12	4	7	1		1	6	-	6	33
31	7	12	4	7	1		1	6	1	5	33
32	8	17	8	7	1		1	6	6		19
33	9	17	7	8	5		5	3	1	2	31
34	10	13	2	3	3		3	-			30
35	12	31	18	3	3		3	-			17
36	12	31	18	3	3		3	-			17
37	17	40	22	19	11	1	10	8	3	5	16
38	22	45	22	13	3		3	10		10	9.6
39	22	73	50	36	5	3	2	31	21	10	7.2
40	24	50	25	22	1		1	21		21	4
41	24	81	56	42	5	4	1	37	16	21	8
42	30	197	166	73	13	8	5	60			3.3

Table 3. The Effect of Starting Points on Solution Times

Starting Point (R = .5)	OPT Primal (sec)	OPT P. / OPT C.P.	OPT Convexified Primal (sec)	OPT C.P. / GGP	GGP (sec)
1	2.991	2.7	1.094	.87	1.253
2	4.224	6.2	.684	.58	1.174
3	2.161	2.8	.770	.58	1.335
4	3.323	4.4	.748	.58	1.296
5	1.776	1.8	.995	.87	1.141
6	2.115	2.6	.824	.63	1.305
7	4.309	4.1	1.045	.68	1.535
8	5.673	8.3	.683	.53	1.298
9	5.722	6.8	.837	.65	1.320
10	4.214	3.8	1.100	1.00	1.104

Table 4. Number of Solutions Attempted and Solved

Code	Problems Attempted	Runs Attempted	Runs Failed	% Unsuccessful Attempts
OPT-P	40	399	27	6.77
OPT-CP	41	616	1	0.16
OPT-TP	25	452	124	27.43
GGP	41	598	24	4.01
GPKTC	39	589	240	40.75
MAYNE	31	379	61	16.09
RALP-P	34	446	146	32.74
RALP-CP	37	552	115	20.83
BIAS-P	39	260	1	0.38
BIAS-CP	40	457	0	0.0
BIAS-SV	29	166	21	12.65
MCS	26	412	60	14.56
DAP	40	406	14	3.45
QUAD-GP	34	149	13	8.72

Table 5A Direct Primal Results

10^{-2} Relative Error

AVG. CPU Time (Secs)

Problem Number	OPT		BIAS		MAYNE		RALP	
	#	TIME	#	TIME	#	TIME	#	TIME
1	20	0.6319	20	1.0935	20	3.095	10	0.1270
2	20	0.3434	20	1.1410	20	1.8085		-----
3	14	0.2402	20	0.1898	18	1.3724	18	0.6950
4	19	1.2517	2	33.35		-----	20	0.8865
5	20	0.2363	20	3.458		-----	20	0.3370
6		-----		-----	1	13.886		-----
7	19	0.6629	2	31.29	2	3.651	1	0.7210
8	20	0.7549	2	3.882	20	6.369		-----
9	20	0.5805	20	3.571	2	4.425	20	1.328
10	20	0.5316	2	12.251	2	3.062	20	3.991
11	20	0.4586	20	2.455	20	4.389	20	1.9260
12	20	0.7237	10	5.090	17	6.609	20	2.640
13	20	0.3718	20	2.236	20	4.281	20	2.759
14	2	0.4648	14	0.5830	14	2.864	14	2.041
16	20	0.4371	18	4.266	18	5.5353	10	8.538
17	20	2.978	2	8.161	19	18.927		-----
18	19	1.8140		-----		-----		-----
19		-----	2	21.22	2	14.434		-----
20	2	0.7904	2	7.286	20	9.449	20	9.631
21		-----	2	29.76	2	21.31		-----
22	2	20.98	2	24.64	2	26.70		-----
23	2	17.692		-----		-----		-----
24		-----	3	18.171	14	21.80	7	35.77
25		-----	2	10.773	13	16.591	7	17.661
26		-----	2	27.313	11	19.848		-----
27	1	4.248	1	49.15		-----	1	1.6140
28	2	8.622	2	54.41	17	51.18	1	-----
29	2	6.965	2	21.64		-----		-----
30	1	5.308		-----		-----		-----
31	2	4.359		-----	20	42.03		-----
32		-----	2	15.454		-----		-----
34	2	5.730	2	28.07		-----		-----
35	2	1.4072		33.70		-----		-----
39	1	22.12		-----		-----		-----
40	1	46.03		-----		-----		-----

Table 5B Direct Primal Results

10^{-3} Relative Error

AVG. CPU TIME (Secs)

Problem Number	OPT		BIAS		MAYNE		RALP	
	#	TIME	#	TIME	#	TIME	#	TIME
1	20	0.6475	20	1.613	20	3.461	10	0.130
2	20	0.4953	20	1.3[illegible]	20	2.509		-----
3	14	0.2956	20	0.209	20	1.5565	20	0.768
4	19	1.3514	1	33.74		-----	20	1.191
5	20	0.3895	20	4.093		-----	20	0.6185
9	20	0.6671	20	3.978	2	5.161	1	1.496
10	19	1.1177	2	13.51	2	4.576	20	7.08
11	20	0.6414	20	2.7	20	5.717	20	2.289
12	20	1.0209	10	5.501	17	8.647	20	3.22
13	20	0.9925	20	3.717	20	5.769	17	3.986
14	3	0.5036	19	0.7427	19	3.836	19	2.569
16	20	0.9849	20	5.937	20	9.293	10	10.036
17	20	3.4559	2	24.515	19	21.35		-----
18	2	3.4367		-----		-----		-----
20	2	1.9572	2	9.416	20	11.38	13	11.66
22		-----	2	37.715	2	27.88		-----
23		-----		-----		-----		-----
24		-----	2	27.452	14	27.35		-----
25		-----	2	14.147	13	19.726	4	21.20
26		-----	2	29.80	11	23.79		-----
27	1	5.1514		-----		-----	1	4.325
28		-----		-----	17	63.98		-----
29	2	13.316	2	35.65		-----		-----
30	1	5.576		-----		-----		-----
31	2	5.2618		-----	20	56.54		-----
32		-----	2	19.625		-----		-----
34	1	6.3759	2	36.14		-----		-----
35		-----	2	41.63		-----		-----
39	1	26.1704		-----		-----		-----
40	1	54.8196		-----		-----		-----

Table 5C Direct Primal Results

10^{-4} Relative Error

AVG. CPU Time (Secs)

Problem Number	OPT #	OPT TIME	BIAS #	BIAS TIME	MAYNE #	MAYNE TIME	RALP #	RALP TIME
1		-----		-----		-----	3	0.129
2	20	0.6147	1	1.747	20	2.762		-----
3	14	0.3323	20	0.213	20	1.591	10	0.8352
4	11	1.3717		-----		-----	20	1.3685
9	4	0.6862	20	4.510	2	6.084		-----
10	18	1.5322	2	14.229	2	4.986	20	8.035
11	16	0.7926	20	3.298	20	6.651	18	2.516
12	20	1.5076	10	6.579	17	10.085	19	3.499
13	18	1.5786	20	4.793	20	7.890		-----
14		-----		-----		-----	11	3.021
16	20	1.8454	20	7.202	20	10.751	3	11.183
17	1	4.109		-----		-----		-----
20	2	3.273	2	9.9185	20	16.707		-----
25		-----	2	14.6965	13	27.06		-----
27	1	5.242		-----		-----	1	6.654
28		-----		-----		-----	1	6.654
29	2	17.767	2	48.134		-----		-----
30	1	5.959		-----		-----		-----
31		-----		-----	20	64.01		-----
32		-----	2	22.10		-----		-----
35		-----	2	52.49		-----		-----
39	1	26.58		-----		-----		-----
40	1	55.74		-----		-----		-----

Table 6A: Conve[illegible] Primal Results
10^{-2} Relative Error

AVG CPU TIME (secs)

Prob. No.	OPT #	OPT TIME	BIAS #	BIAS TIME	BIAS-SV #	BIAS-SV TIME	GGP #	GGP TIME	GPKTC #	GPKTC TIME	RALP #	RALP TIME
1	20	0.4842	20	0.6925	4	0.6545	20	0.2998		-----	20	0.1075
2	20	0.1753	20	0.5005	1	0.3350	20	0.2714	20	0.3320		-----
3	18	0.0873	18	0.0818	18	0.0708	18	0.2243	18	0.2510	18	0.3650
4	20	0.2656	2	20.686	4	11.779	20	0.2828	20	0.4940	20	0.8775
5	20	0.0972	20	1.0305	20	1.5750	20	0.3747	20	0.2845	20	0.3220
6	20	0.2963		-----		-----	20	0.4283		-----	20	4.999
7	20	0.2081	2	5.973		-----	20	0.3126		-----	19	0.6582
8	20	0.2005	20	0.7731		-----	20	0.3160		-----		-----
9	20	0.2774	20	1.253	10	2.653	7	0.3166	20	0.4900	20	0.9635
10	20	0.1292	20	2.853	2	9.511	20	0.7512	20	0.3935	20	1.244
11	20	0.2643	20	1.634	10	3.512	20	0.5520	18	0.6185	20	0.9100
12	20	0.4001	20	2.284	10	5.881	20	0.6240	16	1.0940	20	1.402
13	20	0.1820	20	0.6625		-----	20	0.6530	20	0.4590	20	1.187
14	14	0.0963	14	0.1637	14	0.5308	18	0.6005	9	0.8852	8	0.9828
16	18	0.1944	18	1.127	2	9.639	18	0.8927	18	0.7353	18	5.588
17	.20	0.8506	20	2.099	1	31.32	20	0.7925	20	1.077	20	2.750
18	20	0.4696		-----		-----		-----		-----	1	5.259
19	20	0.5193	20	5.605	2	63.39	20	1.651		-----	15	9.463
20	20	0.3175	20	2.360	2	18.324	19	2.932		-----	20	5.620
21		-----	10	7.506	2	110.66	20	2.293	9	3.983	13	8.643
22	20	1.3209	10	6.635		-----	20	2.206		-----	9	8.216
23	20	1.5894		-----		-----		-----		-----		-----
24		-----	10	7.750	2	40.38	20	1.1495	10	6.707		-----
25		-----	20	3.602	2	45.87	20	3.450	1	3.075	20	8.213
26		-----	10	11.334		-----	20	1.837		-----	7	7.165
27	1	1.0102		-----		-----	1	0.456	1	2.210	1	6.302
28	20	2.516	2	24.04		-----	20	1.979		-----	11	12.14
29	20	1.4573	8	4.571	2	91.65	20	3.245	8	35.28	10	15.74
30	1	2.6070		-----		-----	1	0.440		-----		-----
31	2	1.6350		-----		-----		-----		-----		-----
32		-----	15	3.337	2	83.59	15	1.6345	15	1.1870	15	
33		-----		-----		-----		-----		-----		-----
34	2	0.8279	2	6.290		-----	2	2.177	1	3.476		-----
35	20	0.7272	2	5.022		-----	2	7.570	19	2.229	6	
36	2	0.7605		-----		-----	2	4.225		-----	1	82.43
37		-----		-----		-----	1	1.543		-----		-----
3[illegible]		-----		-----		-----		-----	1	13.559	1	31.45
[illegible]	1	33.05		-----		-----	1	8.751		-----		-----
[illegible]	1	4.944		-----		-----		-----		-----		-----
	1	12.800		-----		-----		-----		-----		-----

Table 6B: Convexified Primal Results

10^{-3} Relative Error

AVG CPU TIME (secs)

Prob.	OPT		BIAS		BIAS-SV		GGP		GPKTC		RALP	
No.	#	TIME	#	TIME	#	TIME	#	TIME	#	TIME	#	TIME
1	20	0.4948	20	0.959	4	0.9717	20	0.2878			20	0.112
2	20	0.2296	20	0.6145	1	1.451	20	0.3293	20	0.332		-----
3	18	0.1491	20	0.0925	20	0.0815	20	0.2707	20	0.2565	15	0.3965
4	20	0.2993	20	21.52	3	12.625	20	0.3246		-----	20	0.9115
5	20	0.1809	20	1.2365	20	2.248	20	0.4702	20	0.3015	20	0.490
6	19	0.3503		-----		-----	20	0.4659		-----	17	5.109
7		-----		-----		-----		-----		-----		-----
8		-----		-----		-----		-----		-----		-----
9	20	0.3174	20	1.4025	7	3.592	7	0.4119		-----	20	10.343
10	20	0.1872	20	3.291	2	12.047	20	0.9647	20	0.4385	20	1.471
11	20	0.3244	20	1.8085	3	3.920	20	0.647	18	0.6265	19	1.051
12	20	0.5014	20	2.474	10	6.582	20	0.752	18	1.11[illegible]	15	1.615
13	20	0.3613	20	1.0985		-----	20	0.8475	20	0.[illegible]	20	1.649
14	20	0.1446	19	0.2166	19	0.6365	19	0.8097	14	0.8[illegible]	10	1.328
16	20	0.4082	20	1.5855	2	10.463	20	1.3205	20	0.856	20	7.790
17	20	0.9798	20	2.914		-----	20	0.995		-----	20	3.092
18	4	0.7012		-----		-----		-----		-----		-----
19	12	0.8774		-----	2	64.15		-----		-----		-----
20	20	0.5275	20	2.917	2	20.36	19	3.782		-----	18	5.757
21		-----		-----		-----		-----		-----		-----
22	20	1.6684	8	7.633		-----	20	2.681		-----	1	8.157
23	20	1.9682		-----		-----		-----		-----		-----
24		-----	10	8.755	1	44.14	20	1.4345	10	6.766		-----
25		-----	20	4.359	2	51.24	20	4.435	1	3.097	20	8.597
26		-----	10	12.102		-----	20	2.408		-----	3	8.311
27	1	1.3333		-----		-----	1	0.577	1	0.4882	1	6.305
28		-----	2	25.475		-----	20	2.774		-----	10	13.052
29	20	2.233	8	8.171		-----	20	4.614	8	37.03	7	16.190
30	1	2.843		-----		-----		-----		-----		-----
31	2	1.5293		-----		-----		-----		-----		-----
32		-----	20	4.367	2	89.77	20	2.234	20	1.4365	20	13.598
33		-----		-----		-----		-----		-----		-----
34	1	0.8861	2	7.382		-----	2	2.905		-----		-----
35	18	1.1286	2	8.095		-----	2	10.050		-----	6	11.371
36	2	1.0415		-----		-----	2	7.276		-----	1	83.12
37		-----		-----		-----		-----		-----		-----
38		-----		-----		-----		-----		13.559	1	196.366
39	1	40.46		-----		-----		8.751		-----	1	79.72
40	1	6.037		-----		-----		-----		-----		-----
41	1	16.006		-----		-----		-----		-----		-----

Table 6C: Convexified Primal Results

10^{-4} Relative Error

AVG CPU TIME (secs)

Prob.	OPT		BIAS		BIAS-SV		GGP		GPKTC		RALP	
No.	#	TIME	#	TIME	#	TIME	#	TIME	#	TIME	#	TIME
1		-----		-----		-----		-----		-----		-----
2	20	0.3058	20	0.762	1	1.572	20	0.3763		-----		-----
3	18	0.1617	20	0.095	20	0.083	20	0.3115	20	0.2575	2	0.390
4	12	0.3875		-----	1	12.316	20	0.4179		-----	17	1.067
5		-----		-----		-----		-----		-----		-----
6	8	0.3509		-----		-----	20	0.5841		-----		-----
9	7	0.3544	20	1.544		-----	7	0.433		-----	15	10.910
10	16	0.2643	20	3.34	2	12.520	20	1.148	20	0.451	20	1.574
11	9	0.3726	20	2.001		-----	20	0.7905	1	0.641	2	1.192
12	7	0.5498	20	2.876	1	4.966	20	0.897	1	1.151		-----
13	19	0.4472	20	1.3795		-----	20	0.994	19	0.9346	16	1.9319
14	9	0.1968		-----		-----	14	1.1887	9	1.1792	3	1.128
16	20	0.6257	20	1.8845	2	10.629	20	1.68	20	1.0045	10	7.401
17	17	1.229	20	3.919		-----	20	1.178		-----	12	3.230
20	5	0.7546	20	3.054		-----	3	5.604		-----		-----
25		-----	20	4.486	2	51.46	6	6.027	1	3.169		-----
27	1	1.7049		-----		-----	1	0.705		-----	1	6.305
28		-----		-----		-----	8	3.026		-----	1	15.439
29	19	2.978	2	15.047		-----	20	7.031	1	15.183	1	18.364
[illegible]	1	3.102		-----		-----		-----		-----		-----
32		-----	20	5.465	2	90.93	20	3.516	20	1.7865		-----
35	1	1.2891	1	9.198		-----	2	13.205		-----	6	11.393
36	1	1.2695		-----		-----	2	11.785		-----	1	83.19
38		-----		-----		-----		-----		-----	1	212.9
39	1	45.4		-----		-----	1	20.99		-----	1	85.75
40	1	6.153		-----		-----		-----		-----		-----
41	1	16.346		-----		-----		-----		-----		-----

Table 8 Dual Results

CPU Time (Seconds)

Prob.	Relative Error 10^{-2}				Relative Error 10^{-3}				Relative Error 10^{-4}			
	MCS		QUADGP		MCS		QUADGP		MCS		QUADGP	
No.	#	TIME	#	TIME	#	TIME	#	TIME	#	TIME	#	TIME
1	20	0.4950	1	0.8530	20	0.542		-----	20	0.546		-----
2	6	0.1430	4	0.1533	18	0.2081	4	0.17883	20	0.2275	4	0.1896
4	20	0.6140	5	0.1979	20	0.768	5	0.2689	20	0.873	5	0.2904
5	15	0.8070	5	0.1923	20	1.995	5	0.3070	20	4.07		-----
6		-----	5	0.8166		-----	5	0.9524		-----	5	1.0253
7		-----	5	0.4284		-----		-----		-----	5	1.0253
8	20	0.1666	5	0.08015	20	0.1724		-----	20	0.1731		-----
9	20	0.5370	5	0.1647	20	0.7755	5	0.2377	20	1.1980	5	0.2463
10	20	0.7160	5	0.5392	20	1.7185	5	0.5823	20	2.40	5	0.6012
11		-----	4	1.4333		-----	4	1.6942		-----	4	1.8505
12	10	10.485	4	15.43	10	15.931	3	16.387	10	16.471	2	15.275
13	15	0.2263	5	0.1444	19	0.3658	3	0.3026	20	0.3915	3	0.3292
16	9	0.3100	2	0.5986	19	0.3755	2	0.6175	20	0.4605	2	0.9315
17	20	2.554	5	1.3402	20	5.0	5	1.9298	20	5.331	5	2.086
18		-----	5	4.694		-----		-----		-----		-----
19	20	1.2655	4	0.3454	20	1.4365		-----	20	1.8135		-----
[illegible]	20	5.420	4	0.8676	20	7.476	5	1.2255	20	8.856	5	1.6259
[illegible]		-----	5	1.2101		-----		-----		-----		-----
22		-----	4	2.615		-----	4	10.049		-----		-----
23	1	167.53	4	6.707	1	177.43		-----	1	178.43		-----
24	20	10.394	5	2.707	20	15.421	5	3.287	20	16.622		-----
25	20	11.621	3	5.062	20	12.784	2	4.486	20	12.901	1	5.541
26	3	13.106	5	3.000	3	78.99	5	3.79	3	117.99		-----
27	10	1.2995	3	5.230	10	3.913		-----	10	7.786		-----
28	10	1.349		-----	10	7.638		-----	10	10.359		-----
30		-----	5	0.6464		-----		-----		-----		-----
31	10	0.9825	5	0.7738	10	1.3435		-----	10	1.767		-----
32	5	0.4750	2	1.9732	10	0.607	2	2.135	10	0.673	2	2.452
33	8	28.89	4	3.035	8	40.63		-----	8	51.81		-----
34	9	0.3250	5	0.1225	10	0.4805	5	0.1314	10	0.543		-----
38		-----	3	26.597		-----	3	31.977		-----	3	35.06
40		-----	4	19.067		-----	4	31.18		-----	4	35.15

Table 9: OPT Primal Form Comparisons

(CPU Times Divided by Convex Primal Times)

Problem No.	Error 10^{-2} P/CP	Error 10^{-2} TP/CP	Error 10^{-3} P/CP	Error 10^{-3} TP/CP
1	1.31	8.26	1.31	8.36
2	1.96	2.95	2.16	3.06
3	2.75	---	1.98	---
4	4.71	2.90	4.52	2.79
5	2.43	---	2.15	---
7	3.19	4.00	---	---
8	3.77	1.33	---	---
9	2.09	3.13	2.10	3.09
10	4.10	---	5.97	---
11	1.74	7.69	1.98	8.18
12	1.81	19.64	2.04	20.47
13	2.04	3.38	2.75	3.00
14	4.83	---	3.48	---
16	2.25	4.59	2.41	6.26
17	3.50	9.15	3.53	8.75
18	3.86	---	4.90	---
19	---	7.89	---	---
20	2.49	19.82	3.71	21.74
22	15.88	10.81	---	10.67
23	11.13	---	---	---
27	4.21	---	3.86	---
28	3.43	---	---	---
29	4.78	---	5.96	---
30	2.04	---	1.96	---
31	2.67	---	3.44	---
34	6.49	---	7.20	---
35	1.94	---	---	---
39	0.670	---	0.647	---
40	9.31	---	9.08	---

Table 10: BIAS Primal Form Comparisons

(CPU Times Divided by Convex Primal CPU Times)

Problem No.	Error = 10^{-2}		Error = 10^{-3}	
	P/CP	CP-SV/CP	P/CP	CP-SV/CP
1	1.58	0.945	1.68	1.02
2	2.28	1.77	2.21	2.36
3	2.32	0.866	2.26	0.881
4	1.60	0.564	1.57	0.586
5	3.36	1.53	3.31	1.82
7	5.24	---	---	---
8	5.02	---	---	---
9	2.85	2.12	2.84	2.56
10	4.29	3.33	4.10	3.66
11	1.50	2.15	1.49	2.17
12	2.23	2.57	2.22	2.66
13	3.38	---	3.38	---
14	3.56	3.24	3.43	2.94
16	3.79	8.55	3.74	6.60
17	3.89	14.92	8.41	---
19	3.79	11.31	---	---
20	3.09	7.76	3.23	6.98
21	3.97	14.74	-----	-----
22	3.71	---	4.94	---
24	2.34	5.21	3.14	5.04
25	2.99	12.73	3.25	11.75
26	2.41	---	2.46	---
28	2.26	---	---	---
29	4.74	20.05	4.36	---
32	4.63	25.05	4.49	20.56
34	4.46	---	4.90	---
35	6.71	---	5.14	---

Table 11A: Summary Cross Comparison: Relative Error 10^{-2}

CPU Times in Seconds

	PRIMAL (OPT)	C-PRIMAL (OPT)	C-PRIMAL (GGP)	T-PRIMAL (DAP)	DUAL (MCS)	T-DUAL (QUAD GP)
1	0.6319	0.4842	0.299	0.2471	0.4950	0.8530
2	0.3434	0.1753	0.2714	0.1440	0.1430	0.1533
3	0.2402	0.0873	0.2243	0.0595	-----	-----
4	1.2517	0.2656	0.2823	0.4625	0.6140	0.1979
5	0.2363	0.0972	0.3747	0.2306	0.8070	0.1923
6	-----	0.2963	0.4283	0.2190	-----	0.8166
7	0.6629	0.2081	0.3126	0.3538	-----	0.4284
8	0.7544	0.2005	0.3160	0.4006	0.1666	0.08015
9	0.5805	0.2774	0.3166	1.332	0.5370	0.1647
10	0.5316	0.1292	0.7512	0.1747	0.7160	0.5392
11	0.4586	0.2643	0.5520	0.5076	-----	1.4333
12	0.7237	0.4001	0.6240	1.1840	10.485	15.43
13	0.3718	0.1820	0.653	0.3081	0.2263	0.1444
14	0.4648	0.0963	0.6005	1.0804	-----	-----
16	0.4371	0.1944	0.8927	0.1699	0.3100	0.5986
17	2.978	0.8506	0.7925	11.027	2.554	1.3402
18	1.8140	0.4692	-----	0.3685	-----	4.694
19	-----	0.5193	1.651	7.590	1.2655	0.3454
20	0.7904	0.3175	2.932	2.287	5.420	0.8676
21	-----	-----	2.293	3.220	-----	12.101
22	20.98	1.3209	2.206	15.083	-----	2.615
23	17.692	1.5894	-----	-----	167.54	6.707
24	-----	-----	1.1495	9.548	10.394	2.7078
25	-----	-----	3.450	1.6304	11.621	5.062
26	-----	-----	1.837	29.96	13.106	3.000
27	4.248	1.0102	0.456	1.3162	1.2995	5.230
28	8.622	2.516	1.979	8.533	1.349	-----
29	6.965	1.4573	3.245	-----	-----	-----
30	5.308	2.607	0.440	0.4090	-----	0.6464
31	4.359	1.6350	-----	-----	0.9825	0.7738
32	-----	-----	1.6345	0.2249	0.4750	1.9732
33	-----	-----	-----	-----	28.89	3.035
34	5.370	0.8279	2.127	3.805	0.3250	0.1225
35	1.4072	0.7272	7.570	0.2962	-----	-----
36	-----	0.7605	4.225	0.4649	-----	-----
37	-----	-----	1.543	-----	-----	-----
38	-----	-----	-----	2.475	-----	26.58
39	22.13	33.05	8.751	5.016	-----	-----
40	46.03	4.944	-----	1.731	-----	19.067
41	-----	12.800	-----	-----	-----	-----

Table 11B: Summary Cross-Comparison (Relative Error 10^{-3})

CPU Time in Seconds

	PRIMAL (OPT)	C-PRIMAL (OPT)	C-PRIMAL (GGP)	T-PRIMAL (DAP)	DUAL (MCS)	T-DUAL (QUADGP)
1	0.6475	0.4948	0.2878	0.3590	0.542	-----
2	0.4953	0.2296	0.3293	0.2167	0.2081	1.7883
3	0.2956	0.1491	0.2707	0.0690	-----	-----
4	1.3514	0.2993	0.3246	0.4925	0.7680	0.2689
5	0.3895	0.1809	0.4702	0.6149	1.995	0.3070
6	-----	0.3503	0.4659	0.4038	-----	0.9524
7	-----	-----	-----	-----	-----	-----
8	-----	-----	-----	-----	0.1724	-----
9	0.6671	0.3174	0.4119	2.021	0.7755	0.2377
10	1.1177	0.1872	0.9647	0.2903	1.7185	0.5823
11	0.6414	0.3244	0.647	1.3407	-----	1.6942
12	1.0209	0.5014	0.752	2.393	15.93	16.387
13	0.9925	0.3613	0.8425	0.7693	0.3658	0.3026
14	0.5036	0.1446	0.8097	1.4042	-----	-----
16	0.9848	0.4082	1.3205	1.0381	0.3755	0.6175
17	3.455	0.9798	0.995	12.992	5.00	1.9298
18	3.436	0.7012	-----	0.3399	-----	-----
19	-----	0.8774	-----	-----	1.4365	-----
20	1.9572	0.5275	3.782	10.22	7.476	1.2255
21	-----	-----	-----	-----	-----	-----
22	-----	1.6684	2.681	-----	-----	10.05
23	-----	1.9682	-----	-----	177.4	-----
24	-----	-----	1.4345	17.44	15.42	3.288
25	-----	-----	4.435	9.638	12.78	4.486
26	-----	-----	2.408	30.75	78.99	3.790
27	5.151	1.333	0.577	10.158	3.914	-----
28	-----	-----	2.775	13.403	7.638	-----
29	13.316	2.234	4.614	-----	-----	-----
30	5.576	2.843	-----	-----	-----	-----
31	5.262	1.5293	-----	-----	1.3435	-----
32	-----	-----	2.235	1.242	0.607	2.135
33	-----	-----	-----	-----	40.63	-----
34	6.379	0.8861	2.905	27.98	0.4805	0.1314
35	-----	1.1286	10.051	3.672	-----	-----
36	-----	1.0415	7.276	0.6723	-----	-----
37	-----	-----	-----	-----	-----	-----
38	-----	-----	-----	2.777	-----	31.97
39	26.17	40.47	8.751	5.016	-----	-----
40	54.82	6.038	-----	1.731	-----	31.18
41	-----	16.01	-----	-----	-----	-----

Table 11C: Summary Cross-Comparison: (Relative Error 10^{-4})

CPU Time in Seconds

	PRIMAL (OPT)	C-PRIMAL (OPT)	C-PRIMAL (GGP)	T-PRIMAL (DAP)	DUAL (MCS)	T-DUAL (QUADGP)
1	-----	-----	-----	0.4729	0.5460	-----
2	0.6147	0.3058	0.3763	0.3073	0.2275	0.1896
3	0.3323	0.1617	0.3115	0.0706	-----	-----
4	1.3717	0.3875	0.4179	0.5065	0.8730	0.2904
5	-----	-----	-----	-----	4.070	-----
6	-----	0.3509	0.5841	0.7386	-----	1.0253
7	-----	-----	-----	-----	-----	-----
8	-----	-----	-----	-----	0.1731	-----
9	0.6862	0.3544	0.4330	0.9290	1.198	0.2463
10	1.5322	0.2643	1.1480	0.6237	2.400	0.6012
11	0.7926	0.3726	0.7905	1.7970	-----	1.8505
12	1.5076	0.5498	0.8970	3.2332	16.47	15.28
13	1.5786	0.4472	0.9940	1.1689	0.3915	0.3292
14	-----	0.1968	1.189	-----	-----	-----
16	1.8454	0.6257	1.600	2.7803	0.4605	0.9315
17	4.110	1.227	1.178	16.78	5.331	2.086
18	-----	-----	-----	-----	-----	-----
19	-----	-----	-----	-----	1.814	-----
20	3.273	0.7546	5.604	20.28	8.857	1.626
21	-----	-----	-----	-----	-----	-----
22	-----	-----	-----	-----	-----	-----
23	-----	-----	-----	-----	178.4	-----
24	-----	-----	-----	-----	16.62	-----
25	-----	-----	6.028	30.81	12.90	5.541
26	-----	-----	-----	-----	118.00	-----
27	5.242	1.7049	0.705	10.24	7.786	-----
28	-----	-----	3.026	-----	10.36	-----
29	17.77	2.978	7.032	-----	-----	-----
30	5.959	3.102	-----	-----	-----	-----
31	-----	-----	-----	-----	1.767	-----
32	-----	-----	3.516	3.200	0.6730	2.453
33	-----	-----	-----	-----	51.81	-----
34	-----	-----	-----	-----	0.5430	-----
35	-----	1.2891	13.21	12.70	-----	-----
36	-----	1.2695	11.79	12.90	-----	-----
37	-----	-----	-----	-----	-----	-----
38	-----	-----	-----	7.659	-----	35.06
39	26.59	45.4	21.00	-----	-----	-----
40	55.74	6.153	-----	-----	-----	35.15
41	-----	16.346	-----	-----	-----	-----

Table 12. Sample Standard Deviations Data (Relative Error of 10^{-2})

Problem Number	OPT-CP	GGP	DAP	MCS	QUADGP
1	0.0866	0.0077	0.0318	0.475	—
2	0.0455	0.0154	0.0522	0.0655	0.116
3	0.0356	0.0411	0.0097	0.0070	—
4	0.0122	0.0177	0.416	0.0725	0.00369
5	0.0404	0.0547	0.142	0.364	0.0273
6	0.0542	0.0501	0.0498	—	0.1703
7	0.0177	0.0336	0.0555	—	0.00204
8	0.0176	0.0185	0.1602	0.0106	0.00178
9	0.0178	0.0220	2.50	0.177	0.00129
10	0.0393	0.0884	0.0160	0.445	0.00823
11	0.0251	0.0584	0.248	—	0.629
12	0.0416	0.0784	1.302	7.54	4.360
13	0.0324	0.1118	0.0795	0.1123	0.00188
14	0.0492	0.0805	2.10	—	—
16	0.0406	0.1389	0.0814	0.1120	0.00172
17	0.0776	0.0552	6.45	0.936	0.395
18	0.1132	—	0.267	—	1.077
19	0.1908	0.2040	8.93	0.206	0.1220

Table 12. (continued)

Problem Number	OPT-CP	GGP	DAP	MCS	QUADGP
20	0.0314	0.435	2.44	2.72	0.261
21	——	0.358	1.562	——	4.26
22	0.228	0.276	——	——	0.395
24	——	0.125	10.95	3.30	0.710
25	——	0.337	2.10	7.91	1.680
26	——	0.189	2.58	0.983	1.245
28	0.336	0.460	——	0.1835	——
29	0.1320	0.572	——	——	——
30	——	——	——	——	0.0933
31	——	——	——	0.513	0.127
32	——	0.397	0.1581	0.213	0.478
33	——	——	——	3.76	0.862
34	——	——	——	0.202	0.00188
35	0.228	——	0.1250	——	——

Table 13. Sample Student Test Results
(Relative Error 10^{-2})

Problem Number	Code with Best Avg. Time	Next Best Code	Student's t Value	Degree of Freedom	Significance Level of Difference (%)
1	DAP	GGP	7.203	21.45	99+
2	MCS	DAP	0.053	38.01	10-
		QUADGP	0.172	3.65	15-
		OPT-CP	1.811	35.44	90+
3	DAP	OPT-CP	3.208	19.54	99+
4	QUADGP	OPT-CP	21.23	24.68	99+
5	OPT-CP	QUADGP	6.26	11.23	99+
6	DAP	OPT-CP	4.70	39.70	99+
7	OPT-CP	GGP	12.31	29.82	99+
8	QUADGP	MCS	34.6	22.90	99+
9	QUADGP	OPT-CP	28.02	19.86	99+
10	OPT-CP	DAP	4.795	25.78	99+
11	OPT-CP	OPT-P	12.02	24.65	99+
12	OPT-CP	GGP	11.28	29.96	99+
13	QUADGP	OPT-CP	5.08	19.54	99+
14	OPT-CP	OPT-P	6.18	21.11	99+
16	DAP	OPT-CP	1.205	28.84	75+
		MCS	4.525	36.35	99+
17	GGP	OPT-CP	2.730	35.92	99+
18	DAP	OPT-CP	0.903	5.77	60-
		OPT-P	11.62	9.50	99+
19	QUADGP	OPT-CP	2.34	8.49	95+
20	OPT-CP	OPT-P	N.A.		N.A.
21	GGP	DAP	1.662	7.380	85-
		QUADGP	5.144	4.021	99+

Table 13. (continued)

Problem Number	Code with Best Avg. Time	Next Best Code	Student's t Value	Degree of Freedom	Significance Level of Difference (%)
22	OPT-CP	GGP	11.06	38.56	99+
23	OPT-CP	QUADGP	N.A.	N.A.	——
24	GGP	QUADGP	3.518	6.093	98+
25	DAP	GGP	2.854	10.33	98+
26	GGP	QUADGP	2.083	4.069	90-
		MCS	30.14	6.208	99+
27	GGP	OPT-CP	N.A.	N.A.	
28	MCS	GGP	5.725	25.52	99+
29	OPT-CP	GGP	13.62	21.23	99+
30	DAP	GGP	N.A.	N.A.	——
31	QUADGP	MCS	1.214	11.49	80-
		OPT-CP	N.A.	N.A.	
32	DAP	MCS	3.288	15.21	99+
33	QUADGP	MCS	18.50	8.78	99+
34	QUADGP	MCS	3.170	9.004	98+
35	DAP	OPT-CP	5.344	10.55	99+

Table 14. CODE Ranking

Relative Error	10^{-2}		10^{-3}		10^{-4}	
Ranking	1st	2nd	1st	2nd	1st	2nd
OPT-CP	13	14	18	8	14	7
DAP	16	1	8	5	4	2
GGP	6	9	7	7	3	8
QUADGP	10	5	5	7	5	6
MCS	3	6	7	4	13	1
OPT-P	0	4	0	4	0	3

Table 15A. Correlation Coefficient of log(CPU time) VS.

	Number of Primal Variables	Number of Primal Constraints	Number of Primal Terms	Degree of Difficulty	Number of Primal Multi-Term Constraints	Number of Primal Tight Constraints	Density (% nonzero)	Critical Value
OPT-P	0.8387	0.5813	0.6718	0.6664	0.5511	0.5827	-0.4967	0.327
OPT-CP	0.7177	0.7774	0.8185	0.7856	0.5313	0.7449	-0.5933	0.326
OPT-TP	0.7010	0.3501	0.9255	0.8156	0.6768	0.1352	0.1024	0.475
GGP	0.7524	0.6157	0.7269	0.7079	0.6489	0.4647	-0.5499	0.327
GPKTC	0.6615	0.4333	0.8000	0.7115	0.5004	0.4094	-0.3929	0.402
DAP	0.2811	0.1402	0.4453	0.3623	0.4580	0.1384	-0.2567	0.238
QUADGP	0.4635	0.5470	0.8594	0.9064	0.6802	0.3556	-0.1436	0.350
MCS	0.4424	0.1725	0.7374	0.6108	0.4627	0.3029	0.1194	0.402

Table 15B. Correlation Coefficients of (CPU time) VS.

	Number of Primal Variables	Number of Primal Constraints	Number of Primal Terms	Degree of Difficulty	Number of Primal Multi-Term Constraints	Number of Primal Tight Constraints	Density (% nonzero)	Critical Value
OPT-P	0.8056	0.8379	0.8170	0.8366	0.4523	0.7463	-0.4389	0.327
OPT-CP	0.7024	0.8121	0.7771	0.7305	0.2926	0.5955	-0.4615	0.326
OPT-TP	0.6719	0.2068	0.8266	0.7037	0.7019	0.0256	0.1596	0.475
GGP	0.6524	0.8207	0.9120	0.9225	0.6322	0.4706	-0.3673	[illegible]
GPKTC	0.5088	0.2488	0.7748	0.7495	0.5162	0.2115	-0.3362	0.402
DAP	0.2475	0.1948	0.3451	0.2933	0.5857	0.2286	-0.1091	0.238
QUADGP	0.1457	0.2935	0.8298	0.8930	0.7178	0.0819	-0.1286	0.350
MCS	0.2684	0.0638	0.4233	0.3511	0.4607	0.1248	0.3361	0.402

Appendix

OPT	EPSIS	10^{-3}
	EPSBD	10^{-5}
	EPS	10^{-6}
	CRIT	10^{-4}
BIAS	EPSLS	10^{-4}
	EPSI	10^{-5}
MAYNE	TT	10^{-1}
	ALL	10^{-6}
	FF	10^{-6}
	FC	10^{-1}
RALP	QC(1)	1000
	QC(2)	10^{-2}
	QC(3)	10^{-5}
	QC(4)	10^{-5}
	QC(5)	.8
GGP	EPSCON	10^{-5}
	EPSCGP	10^{-5}
	EPSLP	10^{-11}
	EPSPN	10^{-11}
MCS	TTOL	10^{-5}
	YTOL	10^{-7}
	DUTOL	10^{-6}
	OFTOL	5×10^{-3}
	CTOL	10^{-5}

(Appendix A continued)

	TAUTOL	5×10^{-3}
	OPTOL	10^{-5}
GPKTC	EPSCON	10^{-5}
	EPSDO	10^{-5}
	BETA	10^{-2}
	BS	1
	IREF	1
	No other options	
QUADGP	EPSCOV	10^{-6}
	EPSTOL	10^{-3}
	VELTOL	2×10^{-6}
	EPS	10^{-3}
	TOLCON	10^{-4}
	EPSEQ	10^{-7}
	EPSVAR	10^{-3}
DAP	EPS	10^{-7}
	EPS1	10^{-6}
	EPS2	10^{-4}
	EPS3	10^{-5}
	EPS4	10^{-4}
	R	10^{4}

Appendix B

Problem Number	Reference
1	L.J. Mancini and R.L. Pizialli, "Optimal Design of Helical Springs by GP," Engr. Opt., 2, 73(1976).
2	P.A. Beck and J.G. Ecker, "Some Computational Experience with a Modified Convex Simplex Algorithm for GP," USAF, Armanent Development and Test Center, Report ADTC-72-20, Elgin AFB, Florida, April, 1972., Problem 1.
3	Beck and Ecker, Problem 2.
4	F. Neghabat and R.M. Stark, "A Cofferdam Design Optimization," Math. Progr. 3, 263 (1972).
5	Problem 2 of Ref. [6].
6	Beck and Ecker, Problem 3.
7	C.S. Beighther and D.T. Phillips, "Applied Geometric Programming," Wiley, New Yor, 1976, p. 84.
8	Problem 1 of Ref [6].
9	W. Gochet and V. Smeers, "On the Use of Linear Programs to Solve Prototype GP's," CORE Discussion Paper No. 7229, November, 1972.
10	V.P. Loonkar and S.D. Robinson, "Minimization of Capital Investment for Batch Probesses," I&EC Proc. Des. Dev. 9, 625 (1970).
11	Beck and Ecker, Problem 4A.
12	Beck and Ecker, Problem 4.
13	Beck and Ecker, Problem 5.
14	Beck and Ecker, Problem 6.
15	Beck and Ecker, Problem 7.
16	Beck and Ecker, Problem 9.
17	Beck and Ecker, Problem 8.
18	Problem 2 of Ref. [19].
19	Beck and Ecker, Problem 10A

(Appendix B, continued)

Problem Number	Reference
20	Beck and Ecker, Problem 12A.
21	R.S. Dembo, "A Set of GP Test Problems and Their Solution," Math. Progr., 10, 192 (1976).
22	ibid, Problem 8B.
23	ibid, Problem 8C.
24	Beck and Ecker, Problem 11.
25	Beck and Ecker, Problem 12.
26	Beck and Ecker, Problem 14A.
27	U. Passy, "Modular Design: An Application of Structural GP," Opns. Res., 18, 441 (1970).
28	Beck and Eckar, Problem 14.
29	Beck and Ecker, Problem 13.
30	H. Mine and K. Ohno, "Decomposition of Mathematical Programs and its Application to Block Diagonal GP," J. Math. And. Appl., 32, 370 (1970) (Problem 6 of Ref. [6]).
31	Problem 7 of Ref. [6].
32	Beck and Ecker, Problem 15.
33	J.S. Folkers, "Ship Operation and Design," in Optimization and Design, Avriel, Rijckaert, and Wilde (eds), Prentice-Hall, 1973.
34	M. Heyman and M. Avriel, "On the Decomposition for a Special Class of GP Problems," J.O.T.A., 3, 392 (1969).
35	Dembo, Problem 1B.
36	Dembo, Problem 1A.
37	J.G. Ecker and R.D. Weibling, "Optimal Design of a Dry Type Natural Draft Cooling Tower by GP," CORE Discussion Paper No.T610, 5-76.
38	J.G. Ecker, "A GP Model for Optimal Allocation of Stream Dissolved Oxygen," Manag. Science, 21, 658(1975), loose constraints deleted.

(Appendix B, continued)

Problem Number	Reference
39	ibid, loose constraints retained.
40	Beck and Ecker, Problem 16C.
41	Beck and Ecker, Problem 16.
42	W.B. Cheng and R.S.H. Mah, "Optimal Design of Pressure Relieving Piping Networks by Discrete Merging," AIChE. J., 22, 471 (1976).

Part II. Generalized GP Study

1. Introduction

Geometric Programming (GP) is a body of theoretical and algorithmic results concerned with constrained optimization problems involving a class of nonlinear algebraic functions. The pioneering work in this field was performed by Duffin, Peterson, and Zener [1] who developed a duality theory for nonlinear programs, now called prototype GP's, that consist of an objective function and upper bounded inequality constraints involving posynomial functions, that is, functions of the form,

$$g_k(x) = \sum_{t=S_k}^{T_k} C_t \prod_{r=1}^{N} x_n^{a_{nt}}$$

where all $C_t > 0$ and the a_{nt} are arbitrary real numbers. Soon after this development, extensions of the methodology were reported by Passy and Wilde [2) which allowed the sign restrictions on the coefficients C_t to be dropped and could accommodate both upper and lower bounded inequality constraints. Since this extension was reported, a considerable number of applications involving such generalized geometric programs (GGP) have been published (see the bibliography reported by Rijckaert [3]) and numerous algorithms for solving both GP's and GGP's have been proposed in the literature (see [4] and [5] for reviews). However, relatively little attention has been given to an appraisal of the computational significance of the various theoretical and algorithmic GGP developments. This paper is the third in a series of studies on the computational utility of GP formulations and developments. The overall goals of this research has been to determine:

i) whether the constructions resulting from GP developments offer any computational advantages over conventional NLP methodology

ii) which of the various equivalent GP problem formulations are preferrable and under what conditions

iii) which GP algorithm/formulation combination is most likely to be successful for a given problem

iv) whether a criteria can be defined by means of which GP problem difficulty can be gauged.

While in previous two papers [6,7], these questions were addressed in the context of prototype GP problems, the present work will specifically be addressed to generalized GP problems. By way of review, we briefly summarize the alternate GGP formulations and key computationally significant features in the next section.

2. Equivalent GGP Problem Structures

2.1 The Primal Problem

The generalized GP primal problem (P) in the form initially presented by Passy and Wilde [2] is,

$$\text{Minimize:} \quad g_o(x)$$

$$\text{Subject to:} \quad \sigma_m' \, (g_m(x))^{\sigma_m'} \leq 1 \qquad m = 1,..,M$$

$$\underset{\approx}{x} > 0$$

where the signomial functions $g_m(x)$ are defined as

$$g_m(x) = \sum_{t=S_m}^{T_m} \sigma_t C_t \prod_{n=1}^{N} x_n^{a_{nt}}$$

with specified positive coefficients C_t and arbitrary exponents a_{nt}. The coefficients σ_t and σ'_m take on the specified values ± 1 and are known as signum functions. The term indices t are defined consecutively as,

$$S_o = 1$$

$$S_{m+1} = T_m + 1$$

$$T_M \equiv T$$

As in the posynomial case, the difference T-N-1 is referred to as the degree of freedom of the problem.

The above problem is in general a non-convex nonlinear program which may possess multiple local minima. A structurally more revealing but not necessarily computationally more advantageous form of the primal can be obtained by rewriting each signomial function as the difference of two posynomials, i.e.

$$g_m(x) = P_m(x) - Q_m(x)$$

where,
$$P_m(x) = \sum_{t \in \mathcal{P}m} C_t \prod_{n=1}^{N} x_n^{a_{nt}}$$

$$Q_m(x) = \sum_{t \in \mathcal{N}m} C_t \prod_{n=1}^{N} x_n^{a_{nt}}$$

and $\mathcal{P}m$ is the subset of term indices of signomial m whose signum functions are positive, and $\mathcal{N}m$ is the subset of term indices of signomial m whose signum functions are negative.

As shown by Avriel and Williams [9], the generalized GP primal can then be written in the complementary or quotient form, (QP)

Minimize: x_o

Subject to: $f_m(x) \leq 1 \qquad m = 1,.., M+1$

$x_o, \underset{\sim}{x} > 0$

where each function $f_m(x)$ is a quotient of posynomials,

$$f_m(x) = \frac{P_m(x)}{Q_m(x) + 1} \qquad m = 1,..,M$$

and $f_{M+1}(x)$ is given by

$$f_{M+1}(x) = \frac{P_o(x)}{x_o + Q_o(x)}$$

Note that the variable x_o is simply a device used to transform the signomial objective function to a constraint. Furthermore, the positivity of x_o is guaranteed by if necessary, including a positive constant of suitable magnitude as one of the terms of $P_o(x)$.

Alternatively, Duffin and Peterson [9] have shown that since each signomial can be written as the difference of two posynomial functions, each signomial constraint can be replaced by two posynomial constraints one of which is a lower bounded constraint. Specifically, by introducing an artificial variable y_m, each constraint

$$P_m(x) - Q_m(x) \leq 1$$

can be replaced by,

$$P_m(x) \leq y_m \leq 1 + Q_m(x)$$

or,

$$y_m^{-1} P_m(x) \leq 1 \text{ and } (1 + Q_m(x))y_m^{-1} \geq 1$$

Therefore, at the expense of increasing the number of variables and constraints, the signomial program can be converted to a <u>reversed</u> GP, a problem in which all functions are posynomials but some are involved in upper bounded or normal constraints while others are involved in lower bounded or <u>reversed</u> constraints.

Thus, the reversed primal GP (RP) is defined as follows,

$$\begin{array}{lll} \text{Minimize:} & h_0(x) & \\ \text{Subject to:} & h_k(x) \leq 1 & k=1,..,K \\ & h_k(x) \geq 1 & k=K+1,..,L \\ & \underset{\sim}{x} < \underset{\sim}{0} & \end{array}$$

where all $h_k(x)$, k=0,1,..,L, are posynomials.

Finally, Duffin and Peterson [9] have suggested continued application of a similar construction to reduce all multi-term constraints to two term constraints, alluding to possible computational advantages. Thus, if u_t denotes a posynomial term, then

$$u_1 + u_2 + u_3 \leq 1$$

Could be replaced by,

$$(u_1 + u_2)\, y_1^{-1} \leq 1 \text{ and } y_1 + u_3 \leq 1$$

Moreover, the reversed constraint,

$$u_1 + u_2 + u_3 \geq 1$$

could be replaced by,

$$(u_1 + u_2)y_1^{-1} \geq 1 \text{ and } y_1 + u_3 \geq 1.$$

Presumably each of the four primal formulations could be solved directly by the application of suitably specialized NLP techniques and, presumably, one ought to be preferred over the others.

2.2 The Exponential Primal Problem

As in the prototype case, each signomial function can be recast to a sum of exponentials via the transformation $x_n = \exp(z_n)$. Thus, the signomial,

$$g_m(x) = \sum_{t=S_m}^{T_m} \sigma_t C_t \prod_{n=1}^{N} x_n^{a_{nt}}$$

can be replaced by,

$$g_m(z) = \sum_t \sigma_t C_t \exp\left(\sum_n a_{nt} z_n\right) .$$

Whereas the original variables x_n are constrained to be positive, the z_n are unrestricted in sign. In the posynomial case, these exponential functions are convex functions and use of this form of the primal in computation proved to be much preferrable to direct primal solution [7]. In the signomial case, the transformed functions are in general nonconvex, hence, some of the computational advantages may well be diminished. However, application of this transformation to the reversed primal, results in a problem in which all functions,

$$h_k(z) = \sum_t C_t \exp\left(\sum_n a_{nt} z_n\right)$$

are convex but the feasible region is the intersection of a convex set, generated by the inequalities

$$h_k(z) \leq 1 \qquad k=1,..,K$$

and a reverse convex set, generated by the inequalities

$$h_k(z) \geq 1 \qquad k=K+1,..,L$$

A reverse convex set is simply the complement of a convex set. The exponential form of the reverse primal thus clearly reveals the underlying structure of GGP problems and clarifies the reason for the possible occurrence of multiple local minima.

2.3 The Transformed Primal Problem

The exponential form of the reversed primal, can with the further change of variable,

$$\underset{\sim}{w} = A^T \underset{\sim}{z} + \ln \underset{\sim}{c}$$

where A is the matrix of primal variable exponents and $\ln \underset{\sim}{c}$ the vector of logs of the term coefficients, be rewritten to the transformed primal form,

$$\begin{array}{lll} \text{Minimize:} & h_o(\underset{\sim}{w}) & \\ \text{Subject to:} & h_k(\underset{\sim}{w}) \leq 1 & k=1,..,K \\ & h_k(\underset{\sim}{w}) \geq 1 & k=K+1,..,L \\ & L(\underset{\sim}{w}-\ln \underset{\sim}{c}) = 0 & \end{array}$$

where,

$$h_k(w) = \sum_{t=S_k}^{T_k} e^{w_t}$$

and, as in the prototype case, the rows of the matrix L are any set of linearly independent vectors spanning the null space of A [10]. This form of the primal offers very attractive structural features for computation, but it is not clear whether the considerable increases in variable and constraint dimensionality which these transformations impose are adequately compensated by increased computational efficiency.

2.4 The Dual Problem

As shown in [3], the GGP problem has associated with it a dual problem, (D)

$$\text{Extremize:} \quad v(\delta) = \sigma_o^1 \left\{ \prod_{t=1}^{T} \left(\frac{C_t}{\delta_t}\right)^{\sigma_t \delta_t} \prod_{m=1}^{M} \lambda_m^{\sigma_m^1 \lambda_m} \right\}^{\sigma_o^1}$$

$$\text{Subject to:} \quad \sum_{t=1}^{T_o} \sigma_t \delta_t = \sigma_o^1$$

$$\sum_{t=1}^{T} \sigma_t a_{nt} \delta_t = 0 \qquad n=1,..,N$$

$$\delta \geq \underset{\sim}{0}$$

$$\underset{\sim}{\lambda} \geq \underset{\sim}{0}$$

where,
$$\lambda_m = \sigma_m^1 \sum_{t=S_m}^{T_m} \sigma_t \delta_t$$

and,
$$\sigma_0^1 = \frac{\min g_0(x)}{|\min g_0(x)|}$$

Although relationships between primal and dual variables at corresponding stationary points can again be given, as in the prototype case, the bounding relationship between the primal and objective functions no longer holds. Hence, maximization of the dual must be replaced by a search for dual stationary points [3]. These properties of the dual are made quite apparent if the primal is formulated in the reversed GP form. In that form, it becomes clear that $\ln v(\delta)$ is concave with respect to the dual variables associated with upper bounded constraints and convex in the dual variables associated with lower bounded constraints [9]. Solution of the dual thus amounts to locating equilibrium or saddle points. This feature disallows direct maximization and thus GGP dual solution requires numerical solution of the Lagrangian conditions applied to the dual. As in the prototype case, however, this approach must be used with great care because of difficulties presented by vanishing dual variables [5].

2.5 The Transformed Dual Problem

A reduction of the generalized dual to a transformed dual similar to that developed for prototype GP's [1], can also be carried out and is reported in [9]. This transformation does not, however, ameliorate the difficulties posed by the search for saddle or equilibrium points. Numerical solution of the Lagrangian condition appears to be the only route for solving GGP's via the transformed dual.

3. Solution Approaches

The solution methods proposed for generalized GP's have generally been of two types: sequential methods employing a series of approximating problems and direct approaches to one of the equilvalent GGP forms. Available fragmentary evidence indicates that the sequential methods are superior to direct approaches.

3.1 Sequential Minimization

The QP and RP forms of the primal suggest that if the denominators and reversed constraints, respectively, could be replaced by approximating single term posynomials, then the resulting approximating problems would reduce to prototype GP's. Such approximations can readily be obtained via the condensation device proposed by Avriel and Williams [8] and Duffin [11].

Given a posynomial, $P(x) = \sum_{t=1}^{T} u_t(x)$, and a set of non-negative, normalized parameters α_t, $t=1,\ldots,T$, then from the inequality between the arithmetic and geometric means, it follows that

$$P(x) = \sum_t u_t(x) \geq \prod_t \left(\frac{u_t}{\alpha_t}\right)^{\alpha_t} \equiv \tilde{P}(x,\alpha) \tag{1}$$

Thus, a multi-term posynomial $P(x)$ is approximated by a single term posynomial $\tilde{P}(x,\alpha)$. Using this construction, Avriel and Williams [8] have proposed replacing the constraints of a QP,

$$f_m(x) = \frac{P_m(x)}{Q_m(x)+1} \leq 1$$

with the approximation,

$$\tilde{f}_m(x) = P_m(x)\ (\tilde{Q}(x\ \alpha))^{-1} \leq 1$$

,

Similarly, Duffin proposed replacing the reversed constraint

$$h_k(x) \geq 1$$

with the posynomial approximation,

$$(\tilde{h}_k(x_1 a))^{-1} \leq 1$$

Note that since inequality (1) is an equality if and only if, $\alpha_t = u_t/\sum u_t$, the parameters α_t of the approximating functions are in both the QP and RP cases updated by setting,

$$\alpha_t^k = \frac{u_t(x^{k-1})}{P(x^{k-1})}$$

where x^{k-1} is the solution of the k-1 $\underline{st}$ approximating *problem. Thus* a series of approximating problems is generated and solved until, the difference

$$(\alpha_t^k - \alpha_t^{k-1})$$

becomes sufficiently small for all t. It can be shown [8] that,

(i) any feasible point of an approximating problem will also be a feasible point of the GGP

(ii) the sequence of approximating problem solutions will converge to a local minimum of the GGP under mild assumptions

It can further be shown [12] that condensation of P(x) is equivalent to a Taylor series linearization of lnP(x) with respect to the variables $\ln x_n$. Thus condensation may be viewed as a special type of partial linearization. The advantage of condensation as opposed to direct Taylor series linearization is that it leads to a closer approximation of the original posynomial [12].

Note that since,

$$\prod_t \left(\frac{u_t}{\alpha_t}\right)^{\alpha_t} = \prod_t \left(\frac{c_t \prod_n x_n^{a_{nt}}}{\alpha_t}\right)^{\alpha_t} = \prod_t \left(\frac{c_t}{\alpha_t}\right)^{\alpha_t} \prod_n x_n^{\sum_t a_{nt}\alpha_t},$$

the condensation calculation will result in changes in the exponent matrix of the variables. To avoid recomputation of the exponent matrix, Duffin and Peterson [13] have proposed an alternate condensation construction which employs the harmonic mean. Using this construction the reversed inequality,

$$\sum u_t \geq 1$$

is replaced by the approximation,

$$\sum \alpha_t^2 (u_t)^{-1} \geq 1$$

where the α_t's are updated as before. It can be shown [13] that the harmonic mean condensation of a reversed constraint can always be bounded by the geometric mean condensation,

$$\sum u_t \geq \Pi \left(\frac{u_t}{\alpha_t}\right)^{\alpha_t} \geq [\sum \alpha_t^2 (u_t)^{-1}]^{-1} \geq 1$$

Thus, the savings in exponent matrix recomputations are obtained at the price of poorer approximating functions.

These alternative primal approximation schemes reduce the solution of GGP's to the solution of a series of prototype GP's but leave open the choices of which prototype formulation to solve and what solution algorithm to use. Proposals which have been made include:

i) solution of the exponential form of the primal using Kelley's cutting plane method [14].

ii) solution of the transformed using successive quadratic programming construction [12].

iii) solution of the transformed primal using a form of reduced gradient method which employs an active constraint strategy to accommodate nonlinear inequalities [10]

iv) solution of the dual in which the linear dual constraints are used to explicitly eliminate variables. Templeman, et.al.

[15] used condensation of the QP form and a modified conjugate gradient method. Jefferson [16] chose the harmonic mean approximation of the RP and a modified Newton method.

In principle, any approach suitable for prototype GP's can be employed in conjunction with condensation constructions including direct minimization of the primal approximating problem.

3.2 Direct GGP Solution

Direct approaches to the solution of GGP's can be of two types: direct minimization of one of the primal forms (P, QP, RP, exponential, or transformed) or solution of the Kuhn-Tucker conditions corresponding to one of the GGP formulations. For instance, Lasdon, et.al. [17] reported on the use of GRG for direct primal minimization with some success. Rijckaert and Martens [18] developed a specialized Newton-Raphson adaptation to solve the Kuhn-Tucker conditions of the primal in exponential form. The linearized equations employed in the N-R iterations were generated using the condensation construction. Blau and Wilde [19] solved the Kuhn-Tucker conditions of the dual using a specialized N-R method which exploited the structure of the linear/log-linear equation set to reduce the set of iteration variables.

In all of these approaches to the solution of GGP's, no attempt was made to locate global minima. Attempts along these lines were reported by Passy [20], Falk [21], and others using branch and bound procedures. However, no generally available software seems to have been produced as yet.

4. Scope of This Study

Three previous comparative studies of generalized GP solution approaches have been reported in the literature. The study by Rijckart and Martens [22] is the most comprehensive of the three. It involved 16 generalized GP test problems, used up to five starting points for each test problem, and investigated both direct and sequential approaches. The direct approaches involved various Newton-Raphson strategies applied to solving either the exponential primal, the dual, or the transformed dual Lagrangian (Kuhn-Tucker) conditions. The sequential algorithms considered included SIGNOPT [15], GPROG [16] and GGP [24]. The reported results indicated that the direct Kuhn-Tucker condition solver GPKTC [25] and the sequented minimizer GGP were fastest and most robust. While it is an important contribution, the study was flawed in that different starting point generation procedures were employed for different codes and in that the time to achieve a specified relative error between successive iterates rather than the deviation from the known solution was used as ranking criterion. Furthermore no attempt was made to extract information about the relative computational advantages of the alternative GGP formulations.

The study reported by Dembo [5] involved six generalized GP problems solved using a single starting point. The codes employed were GGP, GPKTC, GPROG, several additional specialized codes representing alternate implementations of the same sequential strategies, as well as five general NLP codes applied directly to the generalized GP primal. Again GGP and GPKTC emerged as fastest and most robust. The test problems were run by the code authors on their own machines; allowing tuning of programs by the authors; but requiring that the solutions meet fixed tolerances. Solution times were reported using Colville Standard times. The use of Colville standardized times is known to lead to considerable error [26] as is the use of single starting points [7]. Hence, the results of this study must be accepted with considerable reservation.

The study by Dinkel, et al. [23] investigated the relative advantages of the use of geometric mean condensation with the QP and RP forms and harmonic mean condensation with the RP form. The posynomial subproblems were solved using a Newton-Raphson method applied to the transformed dual. Twelve problems were employed and each was run with a single starting point. The conclusion was drawn that the difference between geometric mean condensation applied to the QP form and the RP form was insignificant. The harmonic mean approximation was found to be inferior. This performance was attributed to the increased dimensionality of the subproblem obtained using the harmonic mean and the poorer approximation to the reversed constraints which that approximation yields. A similar conclusion was reported by Bradley [12] on the basis of limited testing using this transformed dual based code.

In this study we will seek to rectify some of the experimental inadequacies of the previous studies. Twenty five generalized GP problems will be solved in both their signomial and their RP forms; using up to 20 different starting point replications; code timing will be obtained at several precise error levels excluding phase I procedure overheads. A series of experiments will be included which will allow investigation of primal formulation effects and of the relative merits of direct primal minimization versus sequential minimization. Appropriate statistical tests will be used in the performance comparisons and the correlation between solution time and various problem characteristic dimensions will be tested.

5. Experimental Procedure

5.1 Test Codes

Five test codes are employed in this study. The four specialized codes GGP [24], GPKTC [25], QUAPGP [12], and DAP [10] as well as the general

NLP Code OPT [27]. The first two codes were selected because of their superior performance, relative to other specialized GP programs employing the direct or sequential strategies, in previous studies [5,22]. The second two codes were selected because of their successes with posynomial problems in reference [7]. These four codes basically span the range of solution approaches:

i) sequential solution using convexified (exponential) primal subproblems (GGP)

ii) sequential solution - using transformed dual subproblems (QUADGP)

iii) sequential solution using transformed primal subproblems (DAP)

iv) direct solution of the Kuhn-Tucker conditions of the exponential primal (GPKTC)

Moreover, the sequential codes employ between them all three condensation formulations:

i) GGP employs the QP form and geometric mean condensation

ii) QUADGP has the option of either employing the QP form and geometric mean condensation on the RP form and harmonic mean condensation

iii) DAP employs the RP form and geometric mean condensation.

The general purpose GRG based NLP Code OPT was selected because its superior performance in a general NLP comparative study [28] as well as its outstanding performance in solving posynomial problems in our earlier study [7]. A round of tests were carried out using the successive LP code RALP and Method of Multipliers Code BIAS used in the posynomial study [7]. However, as in that study, these codes proved to be significantly less effective than OPT and, hence, no results with these codes will be reported.

5.2 Test Problems

For purposes of this study, a set of 25 generalized GP test problems was selected from among those reported in the previous comparative studies [5,22,23], various engineering applications discussed in [9] or referenced in [3], and unpublished problems available to the authors. The characteristic dimensions of these problems are summarized in Table 1 and the problem formulations are given in Appendix A. As evident from the Table the problems range from 4 to 16 primal variables, 1 to 12 multi-term constraints, and 6 to 69 primal terms. Problems with less than 4 variables were excluded because in the posynomial study such problems often did not yield meaningful comparative data. With short run times, program overheads dominate the rankings. The upper end of the test problem size range was limited by the size of the RP form which could be accommodated by some of the codes. In transforming a signomial problem to RP form, two variables, two constraints, and two terms are added to eliminate a signomial objective function and one variable, one constraint, and one term are added in converting each signomial constraint. Consequently, as shown in Table 2, transformation will frequently more than double the exponent matrix size (e.g. see problems 3,8,14,19,24-26). All of the codes used in this study are written for dense matrices and do not employ storage saving (sparse matrix) methods. Thus, in the posynomial study [7], problems with exponent matrices exceeding 2000 elements typically could not be run within the 150 K octal word memory limit set on the Purdue system. As it is, several of the 25 problems could not be run in RP form.

5.3 Testing Conditions

The basic testing procedure followed that employed in the companion study [7]; hence, it will be reiterated here only in outline. The key elements of the experimental design are the use of:

i) Fixed code parameters.

After selective tests to study the effects of code parameters, a fixed set of parameters was chosen for use in all subsequent testing.

The code parameters are given in Appendix C.

ii) Feasible starting points.

In order to separate algorithm performance from the effectiveness of various phase I procedures, only feasible points were used. Admittedly, in practice the costs of generating starting points can be significant; however, a thorough investigation of this question is a major study in itself.

iii) Multiple, randomized starting points.

As in [7], up to 20 starting points were generated for each test problem both in the signomial and the RP form. The points were generated by random sampling from the surface of an N-dimensional sphere whose center is the actual problem optimal solution. Normally, two different radii were used and only the feasible points retained. In some cases, because the feasible region was very tightly constrained, it was not possible to generate a full compliment of sufficiently distinct feasible points even after 1000 sec Cpu time (CDC 6500). In such cases a third, shorter radius was used. The number of starting points used for each problem in its signomial form are summarized in Table 3A; the number used for the RP form in Table 3B.

iv) Pseudo-Lagrangian error function to measure solution accuracy.

The function used is,

$$\mathrm{ABS}\left[\left(\frac{g_o^n - g_o^*}{g_o^*}\right) + \sum_m \lambda_m^* \, \mathrm{ABS}\left(\frac{g_m^n - g_m^*}{g_o^*}\right)\right]$$

where g_o^*, g_m^*, λ_m^* are the values of the objective function, constraints, and multipliers at the optimum.

The sum over m only includes the constraints active at the optimum solution.

v) Performance data accumulation at specified error levels.

Intermediate solution times were obtained at error function values of 10^{-2}, 10^{-3}, 10^{-4}, and 10^{-5}. Means and standard deviations were computed for the runs with each code-problem combination.

As described in [7], the testing, data accumulation, and statistical analysis of the resulting performance data were automated to a large degree using appropriate pre- and post-processing programs, as well as an efficient system of problem, starting point, intermediate result, and reduced data files. A typical intermediate data summary for a test problem run is shown in Fig. 1. Note that mean times and standard deviations are calculated only for successful runs. All runs were carried out on the Purdue University dual CDC 6500 System with its MACE operating system using the MNF (Version 5.3) compiler.

5.4 Test Runs

The runs were grouped into two main test series; alternate minimization strategies employing OPT and solution using the specialized GP codes. The OPT runs were grouped into two sub-series, identified as follows:

A) Direct Minimization

1. Signomial form of the primal (OPTPD)
2. Signomial form of the exponential primal (OPTCPD)
3. Signomial form of the transformed primal (OPTTPD)
4. Reversed form of the exponential primal (OPTCPR)

B) Sequential Minimization

Sequential Solution of the RP using convex (exponential) primal subproblems (OPTCPS)

The specialized generalized GP runs consisted of the following:

1. Direct solution of the exponential primal KT conditions (GPKTC)
2. Sequential solution of the QP using convex primal subproblems (GGP)
3. Sequential solutions of the transformed primal (DAP)
4. Sequential solution using transformed dual subproblems in two series: QP form and geometric mean condensation (designated BRADAW) and RP form using the harmonic mean condensation (designated BRADHA).

A total of ten test series will thus be reported.

6. Results

6.1 Primary Data

The primary data for the ten test series can be condensed to four tables. Table 4 indicates the number of runs attempted and successfully solved for each series. A run was deemed successful if it reached the 10^{-2} error level. A run was labelled unsuccessful if either it did not reach the 10^{-2} error level or the problem could not be run because the memory requirements exceeded the 150K octal maximum allowed on the Purdue University system. As evident from the Table there were 338 possible runs per series if the code used the signomial form input files and 274 possible runs per series if the code used the RP data files. The smaller number in the RP case arises because the increased size of the PR form problems prohibited solution of some of these problems (8,24,26) or forced a reduction in the number of sufficiently different starting points which could be generated.

The most startling result evident from the table is the generally higher level of unsuccessful runs, especially by the specialized codes, when compared to the % unsuccessful attempts obtained in the posynomial study [7]. For instance, GGP 8.0% versus 4%; GPKTC 51% vs. 41%; DAP 90.5% vs. 3.5%; and, QUADGP 60% vs. 9%. The OPT runs also exhibit this trend: primal solution 27.8 vs. 6.8%; convex (exponential) primal 3.3% vs. 0.16%; and transformed primal 38.8%

vs. 27.4%. These results seem to indicate that the non-convexity of the signomial problems does in fact make them more difficult to solve. This conclusion is a tenuous one, however, because the signomial problems used in this study may simply constitute a subset of more difficult problems and thus may not be representative of the class as a whole.

The mean times in CP seconds and number of successful runs upon which these means are based for the ten series are given in Tables 5A, 5B, and 5C. Results are separately tabulated for the 10^{-2}, 10^{-3}, and 10^{-4} error leads. Results are not shown for the 10^{-5} error level because with the code parameter values selected in this study most runs terminated at error levels between 10^{-4} and 10^{-5}.

As can be seen from Tables 5A through 5C, two problems (24 and 28) could not be solved by any code from any starting point. Moreover, several problems, noteably, 8,14, and 18, were only solved in a few of the series. A cursory study of the mean times indicates that GGP generally performed best, followed by OPTCPD, with occassionally very good times by the QUADGP series, BRADAW and BRADHA. Standard deviation values corresponding to the reported mean times are not listed separately. However, they are used in the Student's t tests, the results of which will be detailed later. Since standard deviation values typically ranged from 50% to 10% of the magnitude of the mean, a rigorous comparison of mean solution times must take the standard deviations into account.

6.2 Analysis of Primary Data

In order to compare the mean times of alternate run series, a modified Student t test was employed as described in [7]. This test assumes that the solution times for two series for any given problem are normally distributed each with its own variance. Code time comparison is then equivalent to testing whether the true mean times are equal. A 90% significance level will be required before means will be considered to be different. These comparisons will be presented in six groupings:

1. OPT Direct Primal Form Minimization
2. OPT Strategies involving the Exponential (Convexified) Primal
3. Convexified (Exponential) Primal Strategies
4. Harmonic vs. Geometric Means
5. Transformed Primal Strategies
6. Overall Comparison

For the first set of comparison, the three series in which OPT is used to directly minimize the signomial problem in its primal, exponential primal, and transformed primal form will be considered. As shown in Tables 6A, 6B, 6C at the 10^{-2}, 10^{-3}, as well as the 10^{-4} error levels, the exponential primal approach is faster at virtually 100% significance level in 23 out of 23 cases, 20 out of 22 cases, and 15 out of 18 cases respectively. This dominance is quite substantial: from Table 5A, for instance, the ratio of mean times of OPTPD/OPTCPD is always at least 2, often 5 or more, and sometimes 10 or more. These results are similar to those observed in the posynomial case.

In the next set of comparisons, the three OPT series involving the use of the exponential primal form in direct signomial solution (OPTCPD), direct reversed posynomial solution (OPTCPR) and sequential reversed posynomial solution (OPTCPS) will be tested. From Tables 7A, B, and C it is evident that at all three error levels direct signomial solution dominates: in 23 of 23 cases at 10^{-2}, in 20 of 21 cases at 10^{-3}, and in 15 of 18 cases at 10^{-3}. This also is not surprising in view of the larger dimensionality of the reversed form relative to the signomial form. For problems with larger dimensionality differences (3,5,10,14,18,19,20,21,25) the mean time ratios often exceed four. It is, however, interesting to note from Tables 5A, B, and C that the sequential and reversed times are generally fairly close to each other. Since, the OPT sequential implementation does not take advantage of the single term constraints generated by condensation, this indicates that sequential solution can readily be made more efficient than direct solution.

A cross comparison of three series which use the exponential (convexified) primal: the OPT Sequential series which uses the RP form, GGP which uses the QP form for sequential solution, and GPKTC which solves the Kuhn-Tucker conditions is given in Tables 8A,B,C. GGP dominates overwhelmingly with OPTCPS generally second. GPKTC does not appear to be competitive, primarily because of frequent failure to converge. When it does converge, it apparently has no difficulty achieving high accuracy solutions. Note that the principal difference between GGP and OPTCPS lies in the dimensionality of the convex primal subproblem which is solved and in the treatment of single term constraints. Because GGP uses the QP form while OPTCPS uses the RP form, the GGP condensed subproblem will always be smaller in both variables and number of constraints. Moreover, in GGP single term constraints are converted and treated directly as linear constraints while in OPTCPS this was not done. This comparison thus clearly indicates that condensation of the QP form is to be preferred.

Next we compare the difference between condensation using the geometric mean and condensation using the harmonic mean. In series BRADAW and BRADHA, the solution of the posynomial subproblems is carried out in the transformed dual using QUADGP. As evident from Tables 9A, B, and C, the performance of the two strategies is generally similar despite the differences in the dimensionality of the associated subproblems. The geometric mean series dominates in 10 of 17 cases at the 10^{-2}, level, 9 of 14 cases at the 10^{-3} level, and 8 of 13 cases at the 10^{-4} level. These results seem to anomalous in view of the substantial dominance of the geometric mean approach reported in [23] and [12].

Next, the two transformed primal series are compared. DAP uses the transformed primal and condenses the reversed constraints. In the OPTTPD series the transformed primal is minimized directly. Tables 10A, B, and C indicate that OPT is faster in 12 of 21 cases at the 10^{-2} level, 12 of 19 cases at the 10^{-3} level, and 10 of 14 cases at the 10^{-4} level. This despite the fact that DAP exploits single term constraints. In general DAP proved unreliable in solving signomial problems, in startling contrast to its performance in the posynomial study.

Finally, we will use the Student t test to perform an overall comparison of all ten series. As shown in Tables 11 A, B, and C, the GGP, OPTCPD and the QUADGP series dominate the rankings. GGP is first in 13 of 23 cases at the 10^{-2} level, 12 of 23 cases at 10^{-3}, and 10 of 21 cases at 10^{-4}. The performance of the two QUADGP series (BRADAW and BRADHA) is quite close: when one is first, the other invariably is a close second. It is important to note, however, that the problems in which the QUADGP times rank first are precisely those in which the degree of freedom of the condensed problem is much less than the number of primal variables. Specifically, for problem 1, the condensed degree of freedom is 1; for problem 15, it is 1; for problem 16, 3; for problem 17, 4; for problem 20, 2; and for problem 21, it is 4. For the other problems the degree of freedom is much larger than the number of primal variables and for none of these is QUADGP competitive with GGP or OPTCPD. In view of this factor and from the summary given in Table 12, it appears that OPTCPD can be ranked second on the basis of its numerous second best times, while QUADGP is third only in the basis of the above six problems.

6.3 Effect of Problem Dimension on Solution Time

To help clarify which generalized GP problem characteristic could best be used as measure of solution difficulty, correlation coefficients were computed between the problem solution time and each of eight problem characteristics.

Since each such correlation coefficient only involves data from the same code run with the same set of program parameters, the mean termination time of a run was used as the characteristic problem solution time. The resulting correlation coefficients are summarized in Tables 13 and 14. In the former it is assumed that time is to the exponential of the particular problem characteristic; in the latter, a direct linear relation is assumed. The last column of both Tables lists the critical correlation coefficient value for a 0.05 significance level.

From the tables it is evident that, for direct primal solution (OPTPD), there is a stronger correlation of time to the exponential of the number of primal variables and number of multi-term constraints than to the corresponding linear relationships. Moreover, the correlation is considerably higher to the number of multi-term constraints than to either the total number or number of tight constraints. On the other hand, the linear correlation to the number of primal terms and negative terms is stronger than the exponential. The correlation to the number of terms apparently reflects the effort involved in function evaluations, while that to the negative terms could well reflect some measure of the difficulty introduced by these non-convex elements.

For direct exponential primal solution (OPTCPD), the exponential form correlation coefficients for the number of variables, number of multi-term constraints, and number of tight constraints are highest and very nearly equal. The linear coefficient is higher than the corresponding exponential only for the number of negative terms. The correlation to the number of terms weakens, possibly because in the exponential primal term evaluations are less time consuming.

The reversed exponential primal (OPTCPR) correlation coefficients are highest for the exponential of the number of variables, number of multi-term constraints,

A SAMPLE STUDENT TEST RESULTS (RELATIVE ERROR 1.E-2)

PROBLEM NUMBER	CODE WITH BEST AVG. TIME	NEXT BEST CODE	STUDENTS T VALUE	DEGREE OF FREEDOM	SIGNIFICANCE LEVEL OF DIF(_)
1	OPTCPD	OPTCPS	39.906	25.150	100.000
2	OPTCPD	OPTCPR	53.557	14.020	100.000
3	OPTCPD	OPTCPR	21.200	4.148	100.000
5	OPTCPD	OPTCPS	89.021	21.921	100.000
6	OPTCPD	OPTCPS	367.699	19.000	100.000
7	OPTCPD	OPTCPS	16.207	9.740	100.000
8	OPTCPD	OPTCPD	0	0	100.000
9	OPTCPD	OPTCPR	31.622	14.776	100.000
10	OPTCPD	OPTCPS	67.027	2.945	99.999
11	OPTCPD	OPTCPR	70.178	13.208	100.000
12	OPTCPD	OPTCPS	1.867	20.928	92.404
12	OPTCPD	OPTCPR	9.243	21.817	100.000
13	OPTCPD	OPTCPS	32.697	6.001	100.000
14	OPTCPD	OPTCPS	580.218	13.000	100.000
15	OPTCPD	OPTCPS	137.274	23.252	100.000
16	OPTCPD	OPTCPS	42.744	28.637	100.000
17	OPTCPD	OPTCPS	86.500	20.730	100.000
18	OPTCPD	OPTCPR	22.631	1.971	99.993
19	OPTCPD	OPTCPS	200.076	4.000	100.000
20	OPTCPD	OPTCPS	0	0	100.000
21	OPTCPD	OPTCPS	0	0	100.000
22	OPTCPD	OPTCPD	0	0	100.000
23	OPTCPD	OPTCPR	86.777	27.641	100.000
25	OPTCPD	OPTCPR	562.654	9.000	100.000

B SAMPLE STUDENT TEST RESULTS (RELATIVE ERROR 1.E-3)

PROBLEM NUMBER	CODE WITH BEST AVG. TIME	NEXT BEST CODE	STUDENTS T VALUE	DEGREE OF FREEDOM	SIGNIFICANCE LEVEL OF DIF(_)
1	OPTCPD	OPTCPS	48.144	22.062	100.000
2	OPTCPD	OPTCPR	33.173	9.236	100.000
3	OPTCPD	OPTCPS	26.992	19.152	100.000
5	OPTCPD	OPTCPR	437.256	35.695	100.000
6	OPTCPD	OPTCPS	551.989	19.000	100.000
7	OPTCPD	OPTCPR	53.137	26.576	100.000
9	OPTCPD	OPTCPR	95.246	19.727	100.000
10	OPTCPD	OPTCPR	78.958	3.625	100.000
11	OPTCPD	OPTCPS	303.266	16.000	100.000
12	OPTCPD	OPTCPS	2.131	14.281	94.908
12	OPTCPD	OPTCPR	11.730	13.771	100.000
13	OPTCPD	OPTCPS	22.968	4.586	100.000
14	OPTCPD	OPTCPD	0	0	100.000
15	OPTCPD	OPTCPS	80.958	20.204	100.000
16	OPTCPD	OPTCPS	35.223	14.943	100.000
17	OPTCPD	OPTCPS	100.043	19.921	100.000
19	OPTCPD	OPTCPR	441.208	4.000	100.000
20	OPTCPS	OPTCPS	0	0	100.000
21	OPTCPD	OPTCPS	0	0	100.000
22	OPTCPD	OPTCPD	0	0	100.000
23	OPTCPD	OPTCPR	68.721	28.907	100.000
25	OPTCPD	OPTCPR	1061.446	9.000	100.000

Table 7. Student t Comparisons: OPT Convex Primal Approaches

and number of primal terms. The first two correspond to the results obtained for the primal and exponential primal. The reason for the emergence of the latter is not obvious. Also, anomalous is the negative correlation coefficient for the number of tight constraints. The strong linear correlation to the number of reversed constraints apparently indicates that this measure of non-convexity is a significant measure of problem difficulty, since its value is much larger than the linear coefficient for both total number of constraints and number of multi-term constraints.

The transformed primal results indicate very strong correlation to the number of terms and degree of difficulty. Since the latter is related to the former, the number of terms is clearly the significant parameter. The total number of constraints also has a stronger exponential correlation, while the number of reversed constraints has a stronger linear correlation. These results are reasonable because, in the transformed case, the number of actual problem variables is equal to the number of terms. In this case, the number of reversed constraints also apparently acts as measure of problem difficulty arising from nonconvexities. Thus, the results for the transformed and reversed primal complement each other in this regard.

For sequential transformed dual solution (BRADAW) the key exponentially related properties are the degree of difficulty and the number of primal terms. The latter is equal to the number of transformed dual constraints while the former is a measure of the number of transformed dual variables. The strongest linear correlation is to the number of primal constraints. This is not unreasonable since the number of primal constraints does not directly influence the dimensionality of the transformed dual - it only adds to the complexity of the objective function. Anomalously, although the problem negative terms impose the need for sequential solution, the correlation with respect to that parameter is not significant.

For direct solution of the exponential primal Kuhn-Tucker conditions (GPKTC), the correlation is strongest to the exponential of the number of primal variables, constraints, and multi-term constraints. Since the primal variables and linearization weights for the multi-term functions are the primary iteration variables in the Newton-Raphson algorithm, the observed strong correlation can be rationalized. The stronger linear correlation to the number of primal constraints may reflect the fact that the constraint multipliers are less significant variables in the iterations since they enter linearly.

Finally, consider the sequential approaches using convex primal subproblems. The sequential QP series (GGP) indicates a strong exponential correlation to only the primal variables. Curiously there is no significant correlation to either the number of negative terms or the number of tight constraints. In the sequential RP series, the correlations to the number of primal variables and multi-term constraints are strongest for the exponential relation, while those for the number of constraints, number of primal terms, and number of reversed constraints is strongest for the linear relation. These results are consistent with those obtained for direct signomial exponential primal and reversed exponential primal solution. The stronger linear correlations to the number of reversed constraints again suggests that this parameter may be a valid measure of generalized GP problem difficulty.

7. Conclusions

On the basis of the results of this comparative study, it can be concluded that, given the computational state of the art represented by the test codes employed in this study, the preferred solution approach for generalized GP's involves the following elements:

i) use of the quotient form of the signomial functions

ii) condensation of the denominators of the quotients using the geometric mean construction

iii) solution of the condensed subproblems in their convexified primal form.

The best method which should be employed for the subproblems minimizations is not obvious from the results of this study. The Kelley's Cutting Plane method employed by the best code in this study (GGP), obviously performs superlatively. However, in the posynomial study [7], the general purpose GRG Code OPT proved at least as effective as GGP in solving prototype GP problems even without incorporating devices appropriate for GP's which would enhance its effectiveness. A separate test series in which OPT was used within a strategy employing the above three elements was not executed in this study because, on the basis of the evidence accumulated, the results were clearly predictable. Hence, the cost of developing additional problem formulation files and conducting another full test series was not justifiable. Since direct OPT minimization of the exponential primal was second only to GGP in effectiveness and since sequential OPT minimization of reversed exponential primals was at least as effective as direct minimization of reversed exponential primal, it follows that sequential QP minimization using OPT will be competitive with GGP.

It is furthermore clear that a variation of the above strategy which combines quotient form condensation with transformed dual subproblem solution will be effective for problems in which the degree of freedom is much smaller than the number of primal variables. Since in practice it is normal to impose upper and lower variable bounds and multiple constraints when formulating models, the practical significance of the above subclass of generalized GP problems appears to be small.

The correlation coefficient analysis performed in this study indicates that the primary exponentially correlated problem characteristic is the number of variables in the problem formulation being solved. For primal approaches the number of multi-term constraints also is strongly exponentially correlated. For signomial form problems solved in the various primal formulations, the solution time is strongly correlated to the number of negative terms. For reversed posynomial problems solved in the various primal formulations, the solution time is generally strongly correlated to the number of reversed constraints. It thus appears that in judging the difficulty of a generalized GP the number of variables and multi-term constraints is more significant than the number of negative terms or reversed constraints.

Acknowledgement

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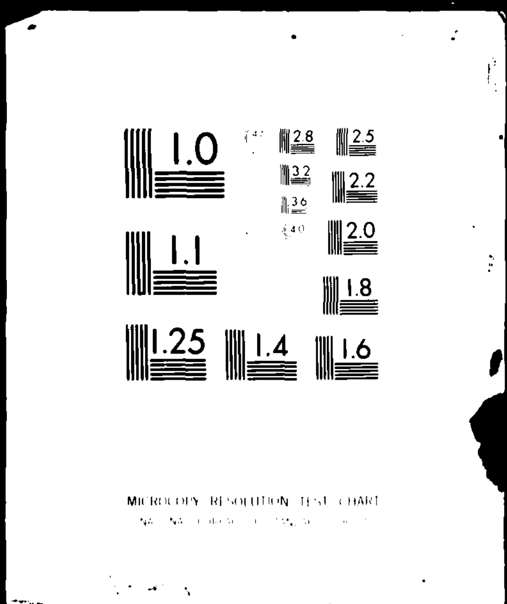

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PROBLEM 14-1

  OBJ FUNC = -3.77970732E+02

        ERF              TIME            A. ERF
   3.10859262E-06 1.18560000E+01 6.33999008E-06
  -4.35963677E-05 1.04100000E+01 1.02675373E-04
  -4.95658933E-06 1.31630000E+01 3.86761749E-05
   8.81929268E-05 1.21470000E+01 1.84564869E-04
   4.09208848E-06 1.20060000E+01 2.56980454E-05
   6.62356564E-06 1.12550000E+01 1.52060806E-05
   8.00077784E-06 1.17990000E+01 1.62907410E-05
   3.98691867E-05 9.36600000E+00 7.93403142E-05
   3.09013056E-05 1.19790000E+01 6.18276597E-05
   2.25150681E-05 1.41390000E+01 4.50254447E-05
  FINAL VALUES,    AVE.ERROR = 1.54750555E-05  STAN. DEV.  3.41468945E-05
  FINAL VALUES,    A.AVE.ERROR = 5.75644693E-05  STAN. DEV.  5.40654154E-05
  FINAL VALUES,    AVE.TIME = 1.18120000E+01  STAN. DEV.  1.31988021E+00
   .01000      NO. PTS.= 10      MEAN TIME= 5.41581661E+00      STANDARD DEV.= 8.64575516E-01
   .00100      NO. PTS.= 10      MEAN TIME= 7.22900845E+00      STANDARD DEV.= 9.56717259E-01
   .00010      NO. PTS.=  9      MEAN TIME= 8.20045084E+00      STANDARD DEV.= 1.18815908E+00
   .00001      NO. PTS.=  1      MEAN TIME= 8.61237092E+00      STANDARD DEV.=              0
```

Fig. 1. Typical Intermediate Result Summary for a test Problem run.

TABLE 1. SIGNOMIAL TEST PROBLEM CHARACTERISTICS

oblem umber	Number of Primal Variables	Number of Primal Terms	Objective Negative Terms	Total Number of Constraints	Number of Posynomial Constraints	Number of Signomial Constraints	Number of Single Term Constraints	Comments
1	4	7	1	2	2			Unit Design
2	4	6	1	1	1			Gas Transmission Line
3	4	29	4	4		3	1	All quadratic
5	5	31	0	16	2	4	10	Colville Quadratic
6	7	19	1	10	4	0	6	Multi-Media Filter
7	7	24	0	9	3	1	5	Welded Beam
8	7	58	1	28	3	11	14	Alkylation Process
9	8	32	2	20	4		16	Reactor Design
10	8	35	0	22	1	5	16	Condenser Design
11	8	26	2	12	5	1	6	Fuel Blending Opt.
2	9	22	0	11	3	1	7	Mod. Multi-Media Filter
3	9	21	0	11	5	3	3	Tanker Bulkhead Design
4	9	57	2	10	1	8	1	Test Problem
5	10	15	0	7	1	3	3	Test Problem
6	10	16	0	7	4	2	1	Process Problem
7	10	18	0	7	4	3		Test Problem
8	10	38	1	22	2	5	15	Modified Alklyation Plant
9	10	49	3	23	0	8	15	Membrane Separation
0	11	19	0	9	4	5		Test Problem
1	13	22	0	9	3	4	2	Reactor Problem
2	13	30	2	11	2	4	5	Recycle Process Opt.
3	8	28	2	5	5			CSTR Reactor Loop
4	13	65	0	39		11	28	Mod. Membrane Separation
5	14	33	0	10	1	9		Test Problem
5	16	69	5	27		12	15	5 Stage Separation Problem

TABLE 2. CHARACTERISTICS OF TEST PROBLEMS IN REVERSED POSYNOMIAL FORM

Problem Number	Number of Variables	Number of Terms	Normal	Number of Constraints Reversed	Single Term	Total
1	6	10	2	1	1	4
2	6	9	2	1		3
3	9	38	4	4	1	9
5	9	39	3	4	13	20
6	9	22	5	1	6	12
7	8	26	3	1	6	10
8	20	83	12	12	17	41
9	10	35	5	1	16	22
10	13	45	5	5	17	27
11	11	31	6	2	7	15
12	10	24	3	1	8	12
13	12	27	5	3	6	14
14	19	76	9	9	2	20
15	13	21	1	3	6	10
16	12	20	4	2	3	9
17	13	24	4	3	3	10
18	17	51	6	6	17	29
19	20	68	8	9	16	33
10	16	29	4	5	5	14
21	17	30	4	4	5	13
22	19	41	5	5	7	17
23	10	31	6	1		7
24	24	87	7	11	32	50
25	23	51	9	9	1	19
26	34	96	12	13	16	41

TABLE 3A. STARTING POINT REPLICATION SUMMARY
Signomial Form

Problem Number	Number of Starting Points		
	Set 1	Set 2	Set 3
1	10	10	
2	10	10	
3	10	10	
5	10	10	
6	10	10	
7	10	9	
8	0	2	
9	10	10	
10	10	3	
11	10	10	
12	1	10	
13	10	10	
14	10	4	
15	10	10	
16	10	5	
17	10	10	
18	0	0	2
19	1	5	
20	1	1	
21	1	1	
22	10	9	
23	10	10	
24	0	0	2
25	10	0	
26	2	2	1

TABLE 3B. STARTING POINT REPLICATION SUMMARY
Reversed Primal Form

Problem Number	Number of Starting Points	
	Set 1	Set 2
1	8	10
2	10	2
3	10	10
5	10	10
6	1	1
7	10	9
8*	—	—
9	10	4
10	1	3
11	10	10
12	1	10
13	1	4
14	10	4
15	10	10
16	10	5
17	10	10
18	2	0
14	1	5
20	1	1
21	1	1
22	10	8
23	10	10
24*	—	—
25	10	0
26*	—	—

*Problem too large

Table 4: Number of Solutions Attempted and Solved

Test Series	Runs Attempted	Runs Successful	% Unsuccessful Attempts
GGP	338	311	8.0
GPKTC	274	134	51.1
DAP	274	26	90.5
BRADAW	338	136	59.8
BRADHA	338	135	60.1
OPTCPS	274	174	36.5
OPTCPD	338	327	3.3
OPTCPR	274	201	26.6
OPTPD	338	244	27.8
OPTTPD	338	207	38.8

[illegible]	[illegible]	#	[illegible] TIME	#	[illegible] TIME	#	[illegible] TIME	#	[illegible] TIME	#	[illegible] TIME
[illegible]	[illegible]	2	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.5574	20	[illegible]
[illegible]	[illegible]	0	0	20	1.7872	10	1.7795	0	0	20	.1033
[illegible]	[illegible]	1	[illegible]	1	5.4148	10	5.1515	20	2.0265	20	.3645
[illegible]	0	0	0	0	0	0	0	0	0	0	0
[illegible]	[illegible]	0	[illegible]	0	0	0	0	20	[illegible]	20	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	0	0	0	0	2	[illegible]	20	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	17	[illegible]	[illegible]	[illegible]	10	2.7224	19	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	0	0	0	0	0	0	1	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	0	0	0	0	14	[illegible]	20	3.2324
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	2	[illegible]	3	[illegible]	12	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	5	[illegible]	[illegible]	[illegible]	19	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	0	0	0	0	11	[illegible]	11	1.5760
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	20	[illegible]	5	[illegible]	19	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	0	0	0	0	1	[illegible]	14	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	10	[illegible]	10	[illegible]	20	[illegible]	20	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	15	[illegible]	10	.2721	15	[illegible]	14	[illegible]
[illegible]	[illegible]	3	[illegible]	[illegible]	[illegible]	10	[illegible]	20	[illegible]	19	[illegible]
[illegible]	[illegible]	0	0	0	0	0	0	0	0	2	[illegible]
[illegible]	[illegible]	0	0	0	0	0	0	1	[illegible]	0	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	2	[illegible]	1	[illegible]	2	[illegible]
[illegible]	[illegible]	1	[illegible]	[illegible]	[illegible]	2	.4755	2	[illegible]	[illegible]	3.7742
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	6	[illegible]	0	0	13	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	2	[illegible]	[illegible]	[illegible]	0	0	20	1.5754
[illegible]	[illegible]	[illegible]	[illegible]	0	0	0	0	0	0	0	0
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	5	[illegible]	2	[illegible]	10	[illegible]
[illegible]	0	[illegible]	[illegible]	[illegible]	[illegible]	0	0	0	0	0	0

Table 5A. Mean times at 10^{-2} Error Level

CRITRD TIME	#	CRITRD TIME	#	CRITRD TIME	#	CRITRD TIME
.1940	17	.4413	20	.3343	20	.4217
.1031	12	.3557	20	.3520	20	.2530
.3643	5	1.79[illegible]	[illegible]	1.0822	10	9.3522
2.3632	23	6.333[illegible]	20	5.1332	[illegible]	67.1523
1.7375	2	2.[illegible]7	20	4.[illegible]7	[illegible]	10.[illegible]
1.2355	13	3.[illegible]	[illegible]	[illegible].3[illegible]1	14	31.[illegible]
6.3375	[illegible]	[illegible]	2	4[illegible].[illegible]4[illegible]	[illegible]	4[illegible].[illegible]
3.2324	14	5.31[illegible]	14	11.1[illegible]5	[illegible]	[illegible]
3.5013	4	19.3526	11	22.10[illegible]1	[illegible]	34.[illegible]1
2.5470	13	[illegible].3332	15	[illegible]	10	14.3753
1.5760	11	2.0358	11	4.5773	15	23.3175
2.7[illegible]3	[illegible]	[illegible].0317	20	11.3342	0	[illegible]
4.7652	[illegible]	[illegible]	[illegible]	2[illegible].3295	[illegible]	4.37[illegible]
1.5547	15	4.5455	20	3.4422	[illegible]	5.3374
1.8452	14	3.0455	15	7.355[illegible]	24	[illegible].49[illegible]
2.5232	23	3.[illegible]	1	4.0753	20	[illegible]
12.0540	[illegible]	4[illegible].3352	[illegible]	[illegible]	[illegible]	3210.2[illegible]
4.5575	[illegible]	44.0[illegible]1	[illegible]	17.231[illegible]	[illegible]	1[illegible].1[illegible]
3.4355	[illegible]	12.5721	[illegible]	[illegible]	2	10.[illegible]1
3.7742	[illegible]	11.[illegible]34[illegible]	[illegible]	[illegible]	4	8[illegible].4713
5.5535	[illegible]	[illegible]	[illegible]	[illegible]	8	27.9[illegible]
1.5754	2	2.9[illegible]47	3	2[illegible].[illegible]55	[illegible]	[illegible]
5.8557	1	41.7317	3	43.7594	2	6[illegible].3407

. 2

#	TIME	#	TIME	#	TIME	#	[illegible] TIME	#	[illegible] TIME	#	[illegible] TIME
17	[illegible]	2	[illegible]	[illegible]	[illegible]	[illegible]	.1533	[illegible]	[illegible]	[illegible]	.3112
					1.7311		1.8135				.1593
	[illegible]							23	4.3923		[illegible]
	[illegible]								[illegible]		[illegible]
			[illegible]						[illegible]		[illegible]
			[illegible]						[illegible]		[illegible]
17	[illegible]		[illegible]	17	[illegible]	[illegible]	[illegible]				
[illegible]	[illegible]							[illegible]	12.2315	[illegible]	[illegible]
										12	[illegible]
					[illegible]		[illegible]	2	[illegible]	17	[illegible]
					[illegible]		[illegible]	11	[illegible]	11	[illegible]
			[illegible]	[illegible]	[illegible]		[illegible]		[illegible]	[illegible]	[illegible]
	[illegible]										[illegible]
					[illegible]	[illegible]	.2211		[illegible]	11	[illegible]
	[illegible]		[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	14	[illegible]
[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	10	[illegible]	[illegible]	[illegible]		[illegible]
									[illegible]		[illegible]
									[illegible]		
					[illegible]		[illegible]		[illegible]		[illegible]
					[illegible]		[illegible]			12	[illegible]
											[illegible]
17	[illegible]		[illegible]		[illegible]		[illegible]				
					[illegible]		[illegible]			[illegible]	[illegible]

Table 5B. Mean times at 10^{-3} Error Level

[illegible]	#	[illegible]	#	[illegible]	#	[illegible]	#	[illegible]
.5[illegible]	20	.3132	2	.5[illegible]	2[illegible]	.7[illegible]18	20	.5[illegible]3
0	20	.1593	10	.7814	13	.9933	20	.4[illegible]
4.5923	20	.55[illegible]4	0	0	5	2.[illegible]	10	1[illegible].[illegible]4
0	0	0	0	0	0	0	0	0
[illegible].[illegible]2[illegible]1	20	2.5[illegible]7	22	7.0[illegible]0	2[illegible]	[illegible].5[illegible]	5	[illegible]2.[illegible]
3.1[illegible]2	20	1.7[illegible]	2	[illegible].2[illegible]19	20	[illegible].[illegible]75	[illegible]0	1[illegible].[illegible]
3.73[illegible]7	19	2.2[illegible]	15	[illegible].1[illegible]7	5	1[illegible].27[illegible]1	14	33.3[illegible]
0	0	0	0	0	2	4[illegible].[illegible]	0	0
12.2[illegible]15	20	4.[illegible]112	14	[illegible].0512	1[illegible]	1[illegible].1[illegible]7[illegible]	4	6[illegible].1[illegible]17
0	12	7.[illegible]75	4	23.0[illegible]	11	[illegible].[illegible]7	0	[illegible]
10.17[illegible]4	17	3.[illegible]700	0	0	19	[illegible].2[illegible]27	5	4[illegible].3[illegible]1[illegible]
2.[illegible]3[illegible]3	11	2.3[illegible]7	11	2.7[illegible]	11	7.[illegible]40	10	1[illegible].1[illegible]1
5.[illegible]2[illegible]4	19	3.[illegible]0	5	7.[illegible]4[illegible]	1[illegible]	1[illegible].[illegible]527	1[illegible]	2[illegible].7[illegible]
0	14	[illegible].7151	0	[illegible]	5	41.[illegible]5	[illegible]	[illegible]
[illegible].[illegible]0	11	2.0[illegible]9	10	[illegible].3[illegible]	15	[illegible].37[illegible]2	14	[illegible].[illegible]2
4.[illegible]	14	2.2[illegible]	14	4.[illegible]2[illegible]5	13	1[illegible].2[illegible]3	1[illegible]	7.[illegible]
[illegible].7[illegible]	[illegible]	2.[illegible]2[illegible]0	[illegible]	[illegible].03[illegible]	13	1[illegible].2425	14	1[illegible].[illegible]1[illegible]1
0	0	0	0	0	0	0	0	0
55.5[illegible]1[illegible]	0	[illegible].6479	1	[illegible].[illegible]0	0	0	[illegible]	[illegible]33.[illegible]
15.4[illegible]0	0	0	0		1	2[illegible].43[illegible]2	0	0
11.[illegible]830	2	[illegible].[illegible]51	1	14.8[illegible]33	0	0	2	21.27[illegible]
0	12	[illegible].[illegible]44	0	0	0	0	5	[illegible].[illegible]5[illegible]
0	2[illegible]	1.72[illegible]7	2[illegible]	3.7[illegible]	5	[illegible].73[illegible]4	[illegible]	[illegible].43[illegible]
0	0	0	[illegible]	[illegible]	0	0	0	0
[illegible]	10	[illegible].27[illegible]	1	55.[illegible]744	3	[illegible].415[illegible]	2	[illegible]3.[illegible]
0	0	0	0	0	0	0	0	0

Table 60. Mean times at 10^{-4} Error Level

A SAMPLE STUDENT TEST RESULTS
(RELATIVE ERROR 1.E-2)

PROBLEM NUMBER	CODE WITH BEST AVG. TIME	NEXT BEST CODE	STUDENTS T VALUE	DEGREE OF FREEDOM	SIGNIFICANCE LEVEL OF DIF(_)
1	OPTCPD	OPTTPD	47.816	25.335	100.000
2	OPTCPD	OPTTPD	33.972	24.747	100.000
3	OPTCPD	OPTPD	25.036	8.329	100.000
5	OPTCPD	OPTPD	70.335	21.619	100.000
6	OPTCPD	OPTPD	462.422	25.817	100.000
7	OPTCPD	OPTPD	47.333	7.031	100.000
8	OPTCPD	OPTPD	59.174	1.000	98.924
8	OPTCPD	OPTCPD	0	0	100.000
9	OPTCPD	OPTPD	52.451	17.408	100.000
10	OPTCPD	OPTPD	32.920	10.430	100.000
11	OPTCPD	OPTPD	98.128	21.130	100.000
12	OPTCPD	OPTPD	28.432	11.733	100.000
13	OPTCPD	OPTPD	101.395	23.841	100.000
14	OPTCPD	OPTPD	15.772	5.047	100.000
15	OPTCPD	OPTTPD	131.801	20.687	100.000
16	OPTCPD	OPTTPD	148.959	27.560	100.000
17	OPTCPD	OPTPD	278.871	20.174	100.000
18	OPTCPD	OPTCPD	0	0	100.000
19	OPTCPD	OPTTPD	50.427	2.015	99.999
20	OPTCPD	OPTTPD	0	0	100.000
21	OPTCPD	OPTTPD	0	0	100.000
22	OPTCPD	OPTTPD	60.332	3.018	100.000
23	OPTCPD	OPTTPD	91.479	7.013	100.000
25	OPTCPD	OPTPD	52.122	2.062	99.999

B SAMPLE STUDENT TEST RESULTS
(RELATIVE ERROR 1.E-3)

PROBLEM NUMBER	CODE WITH BEST AVG. TIME	NEXT BEST CODE	STUDENTS T VALUE	DEGREE OF FREEDOM	SIGNIFICANCE LEVEL OF DIF(_)
1	OPTCPD	OPTTPD	50.324	24.389	100.000
2	OPTCPD	OPTTPD	42.020	22.141	100.000
3	OPTCPD	OPTPD	16.851	2.031	99.981
5	OPTCPD	OPTPD	80.341	20.059	100.000
6	OPTCPD	OPTPD	502.351	25.830	100.000
7	OPTCPD	OPTPD	42.577	5.015	100.000
8	OPTPD	OPTPD	0	4.000	0
8	OPTPD	OPTPD	0	4.000	0
9	OPTCPD	OPTPD	54.989	17.269	100.000
10	OPTCPD	OPTPD	32.677	10.182	100.000
11	OPTCPD	OPTPD	83.275	22.405	100.000
12	OPTCPD	OPTPD	49.369	10.433	100.000
13	OPTCPD	OPTPD	140.413	24.631	100.000
14	OPTCPD	OPTPD	10.906	4.004	99.995
15	OPTCPD	OPTTPD	182.075	16.630	100.000
16	OPTCPD	OPTTPD	119.854	14.237	100.000
17	OPTCPD	OPTPD	228.520	12.549	100.000
19	OPTCPD	OPTTPD	56.992	2.005	100.000
20	OPTPD	OPTPD	0	0	100.000
21	OPTCPD	OPTTPD	0	0	100.000
22	OPTCPD	OPTTPD	47.235	2.103	99.999
23	OPTCPD	OPTTPD	128.488	7.346	100.000
25	OPTCPD	OPTPD	19.942	2.001	99.989

Table 6. Student t Comparisons: OPT Primal Approaches

C SAMPLE STUDENT TEST RESULTS
(RELATIVE ERROR 1.E-4)

PROBLEM NUMBER	CODE WITH BEST AVG. TIME	NEXT BEST CODE	STUDENTS T VALUE	DEGREE OF FREEDOM	SIGNIFICANCE LEVEL OF DIF( )
1	OPTCPD	OPTTPD	22.622	11.224	100.000
2	OPTCPD	OPTTPD	20.012	25.842	100.000
3	OPTCPD	OPTPD	372.707	19.000	100.000
5	OPTCPD	OPTPD	58.731	16.222	100.000
6	OPTCPD	OPTPD	369.448	19.059	100.000
7	OPTCPD	OPTPD	46.564	5.010	100.000
9	OPTCPD	OPTPD	31.583	10.251	100.000
10	OPTPD	OPTPD	0	18.000	0
10	OPTPD	OPTPD	0	18.000	0
11	OPTCPD	OPTPD	219.276	1.000	99.710
12	OPTCPD	OPTTPD	172.578	15.310	100.000
13	OPTCPD	OPTPD	68.762	18.739	100.000
14	OPTCPD	OPTPD	9.459	3.005	100.000
16	OPTCPD	OPTTPD	58.984	8.364	100.000
17	OPTTPD	OPTTPD	0	0	100.000
19	OPTCPD	OPTTPD	0	0	100.000
20	OPTPD	OPTPD	0	0	100.000
23	OPTCPD	OPTTPD	710.828	7.000	100.000
25	OPTCPD	OPTPD	285.396	3.000	100.000

Table 6. Student t Comparisons: OPT Primal Approaches

A SAMPLE STUDENT TEST RESULTS (RELATIVE ERROR 1.E-2)

PROBLEM NUMBER	CODE WITH BEST AVG. TIME	NEXT BEST CODE	STUDENTS T VALUE	DEGREE OF FREEDOM	SIGNIFICANCE LEVEL OF DIF(_)
1	OPTCPD	OPTCPS	39.906	25.150	100.000
2	OPTCPD	OPTCPR	53.557	14.020	100.000
3	OPTCPD	OPTCPR	21.200	4.144	100.000
5	OPTCPD	OPTCPS	89.021	21.921	100.000
6	OPTCPD	OPTCPS	367.699	19.000	100.000
7	OPTCPD	OPTCPS	16.207	9.740	100.000
8	OPTCPD	OPTCPD	0	0	100.000
9	OPTCPD	OPTCPR	31.622	14.776	100.000
10	OPTCPD	OPTCPS	67.027	2.945	99.999
11	OPTCPD	OPTCPR	70.178	13.208	100.000
12	OPTCPD	OPTCPS	1.867	20.928	92.404
12	OPTCPD	OPTCPR	9.243	21.817	100.000
13	OPTCPD	OPTCPS	32.697	6.001	100.000
14	OPTCPD	OPTCPS	580.218	13.000	100.000
15	OPTCPD	OPTCPS	137.274	23.252	100.000
16	OPTCPD	OPTCPS	42.744	28.637	100.000
17	OPTCPD	OPTCPS	86.500	20.730	100.000
18	OPTCPD	OPTCPR	22.631	1.971	99.993
19	OPTCPD	OPTCPS	200.096	4.000	100.000
20	OPTCPD	OPTCPS	0	0	100.000
21	OPTCPD	OPTCPS	0	0	100.000
22	OPTCPD	OPTCPD	0	0	100.000
23	OPTCPD	OPTCPR	86.777	27.641	100.000
25	OPTCPD	OPTCPR	562.854	9.000	100.000

B SAMPLE STUDENT TEST RESULTS (RELATIVE ERROR 1.E-3)

PROBLEM NUMBER	CODE WITH BEST AVG. TIME	NEXT BEST CODE	STUDENTS T VALUE	DEGREE OF FREEDOM	SIGNIFICANCE LEVEL OF DIF(_)
1	OPTCPD	OPTCPS	48.144	22.062	100.000
2	OPTCPD	OPTCPR	33.173	9.236	100.000
3	OPTCPD	OPTCPS	26.992	19.152	100.000
5	OPTCPD	OPTCPR	437.256	35.695	100.000
6	OPTCPD	OPTCPS	551.989	19.000	100.000
7	OPTCPD	OPTCPR	53.137	26.576	100.000
9	OPTCPD	OPTCPR	95.246	19.727	100.000
10	OPTCPD	OPTCPR	78.958	3.625	100.000
11	OPTCPD	OPTCPS	303.266	16.000	100.000
12	OPTCPD	OPTCPS	2.131	14.281	94.908
12	OPTCPD	OPTCPR	11.730	13.771	100.000
13	OPTCPD	OPTCPS	22.968	4.586	100.000
14	OPTCPD	OPTCPD	0	0	100.000
15	OPTCPD	OPTCPS	80.358	20.204	100.000
16	OPTCPD	OPTCPS	35.223	14.943	100.000
17	OPTCPD	OPTCPS	100.043	19.921	100.000
19	OPTCPD	OPTCPR	441.208	4.000	100.000
20	OPTCPS	OPTCPS	0	0	100.000
21	OPTCPD	OPTCPS	0	0	100.000
22	OPTCPD	OPTCPD	0	0	100.000
23	OPTCPD	OPTCPR	68.721	28.907	100.000
25	OPTCPD	OPTCPR	1661.446	9.000	100.000

Table 7. Student t Comparisons: OPT Convex Primal Approaches

SAMPLE STUDENT TEST RESULTS
(RELATIVE ERROR 1.E-4)

PROBLEM NUMBER	CODE WITH BEST AVG. TIME	NEXT BEST CODE	STUDENTS T VALUE	DEGREE OF FREEDOM	SIGNIFICANCE LEVEL OF DIF(%)
1	OPTCPD	OPTCPD	0	0	100.000
2	OPTCPD	OPTCPD	0	0	100.000
3	OPTCPD	OPTCPD	0	0	100.000
5	OPTCPD	OPTCPR	320.059	20.967	100.000
6	OPTCPD	OPTCPD	0	0	100.000
7	OPTCPD	OPTCPR	38.638	12.571	100.000
9	OPTCPD	OPTCPD	0	0	100.000
10	OPTCPR	OPTCPS	0	0	100.000
11	OPTCPD	OPTCPD	0	0	100.000
12	OPTCPD	OPTCPS	5.801	11.364	99.999
13	OPTCPD	OPTCPS	40.839	4.000	100.000
14	OPTCPD	OPTCPS	0	0	100.000
15	OPTCPR	OPTCPS	0	0	100.000
16	OPTCPD	OPTCPR	67.218	6.628	100.000
17	OPTCPR	OPTCPF	0	4.000	0
17	OPTCPR	OPTCPF	0	4.000	0
19	OPTCPD	OPTCPD	0	0	100.000
23	OPTCPD	OPTCPR	2.758	1.028	99.328
25	OPTCPD	OPTCPR	259.972	3.000	100.000

Table 7. Student t Comparisons: OPT Convex Primal Approaches

A

SAMPLE STUDENT TEST RESULTS
(RELATIVE ERROR 1.E-2)

PROBLEM NUMBER	CODE WITH BEST AVG. TIME	NEXT BEST CODE	STUDENTS T VALUE	DEGREE OF FREEDOM	SIGNIFICANCE LEVEL OF DIF(_)
1	OPTCPS	GGP	19.345	27.202	100.000
2	GGP	GPKTC	65.138	33.887	100.000
3	GGP	OPTCPS	47.421	34.647	100.000
5	GGP	OPTCPS	133.361	19.485	100.000
6	GGP	OPTCPS	2715.364	19.000	100.000
7	GGP	OPTCPS	44.318	9.847	100.000
8	GGP	GGP	0	0	100.000
9	GGP	OPTCPS	43.063	18.123	100.000
10	GGP	OPTCPS	91.579	2.696	100.000
11	GGP	OPTCPS	43.098	20.316	100.000
12	GGP	OPTCPS	25.733	10.995	100.000
13	GGP	OPTCPS	68.373	4.160	100.000
14	GGP	OPTCPS	1594.288	13.000	100.000
15	GGP	OPTCPS	107.813	39.980	100.000
16	GGP	OPTCPS	44.392	23.502	100.000
17	GGP	OPTCPS	79.475	35.938	100.000
18	GGP	GGP	0	0	100.000
19	GGP	OPTCPS	653.577	5.000	100.000
20	GGP	GPKTC	0	0	100.000
21	GGP	OPTCPS	0	0	100.000
22	GGP	GGP	0	0	100.000
23	GGP	GPKTC	22.178	17.379	100.000
24	GPKTC	OPTCPS	.974	1.000	49.172
25	GPKTC	GPKTC	0	0	100.000

SAMPLE STUDENT TEST RESULTS
(RELATIVE ERROR 1.E-3)

PROBLEM NUMBER	CODE WITH BEST AVG. TIME	NEXT BEST CODE	STUDENTS T VALUE	DEGREE OF FREEDOM	SIGNIFICANCE LEVEL OF DIF(_)
1	OPTCPS	GGP	18.721	34.141	100.000
2	GGP	GPKTC	.311	19.305	24.059
2	GGP	GPKTC	.311	19.305	24.059
3	GGP	GPKTC	57.343	29.432	100.000
5	GGP	OPTCPS	189.126	19.112	100.000
6	GGP	OPTCPS	5667.490	19.000	100.000
7	GGP	OPTCPS	23.078	4.065	100.000
8	GGP	GGP	0	0	100.000
9	GGP	OPTCPS	38.596	10.169	100.000
10	GGP	GGP	0	0	100.000
11	GGP	OPTCPS	57.347	16.000	100.000
12	GGP	OPTCPS	37.983	10.231	100.000
13	GGP	OPTCPS	44.986	4.017	100.000
14	GGP	GGP	0	0	100.000
15	GGP	OPTCPS	42.689	34.382	100.000
16	GGP	OPTCPS	22.401	27.536	100.000
17	GGP	OPTCPS	56.816	36.778	100.000
18	GGP	GGP	0	0	100.000
19	GGP	OPTCPS	2053.044	5.000	100.000
20	GGP	OPTCPS	0	0	100.000
21	GGP	OPTCPS	0	0	100.000
22	GGP	GGP	0	0	100.000
23	GGP	GPKTC	17.140	21.445	100.000

Table 8. Student t Comparisons: Convexified Primal Strategies

C SAMPLE STUDENT TEST RESULTS
(RELATIVE ERROR 1.E-4)

PROBLEM NUMBER	CODE WITH BEST AVG. TIME	NEXT BEST CODE	STUDENTS T VALUE	DEGREE OF FREEDOM	SIGNIFICANCE LEVEL OF DIF(_)
1	GGP	GPKTC	43.261	36.869	100.000
2	GPKTC	GGP	15.536	18.044	100.000
3	GGP	GPKTC	66.794	31.154	100.000
5	GGP	OPTCPS	68.911	5.004	100.000
6	GGP	GGP	0	0	100.000
7	GGP	OPTCPS	138.569	3.864	100.000
9	GGP	GPKTC	4.476	2.061	99.043
10	GGP	GGP	0	0	100.000
11	GGP	GGP	0	0	100.000
12	GGP	OPTCPS	54.245	7.089	100.000
13	GGP	OPTCPS	3113.649	19.000	100.000
14	GGP	GGP	0	0	100.000
15	GGP	GPKTC	14.135	5.272	100.000
16	GGP	OPTCPS	3.357	2.387	97.155
16	GGP	GPKTC	24.242	26.728	100.000
17	GGP	GPKTC	1.439	1.001	91.158
17	GGP	GPKTC	1.439	1.001	91.158
18	GGP	GGP	0	0	100.000
19	GGP	GGP	0	0	100.000
20	GGP	GGP	0	0	100.000
21	GGP	GGP	0	0	100.000
23	GGP	GPKTC	2.261	25.079	96.732
23	GGP	GGP	0	40.000	0

Table 8. Student t Comparisons: Convexified Primal Strategies

A SAMPLE STUDENT TEST RESULTS (RELATIVE ERROR 1.E-2)

PROBLEM NUMBER	CODE WITH BEST AVG. TIME	NEXT BEST CODE	STUDENTS T VALUE	DEGREE OF FREEDOM	SIGNIFICANCE LEVEL OF DIF( )
1	BRADAW	BRADHA	1.221	37.888	77.026
2	BRADAW	BRADHA	14.448	18.749	100.000
3	BRADAW	BRADHA	144.916	9.008	100.000
7	BRADHA	BRADAW	17.980	34.972	100.000
10	BRADAW	BRADHA	0	0	100.000
11	BRADAW	BRADHA	315.362	5.002	100.000
13	BRADAW	BRADHA	.475	33.494	[illegible]
15	BRADHA	BRADAW	10.947	14.927	100.000
16	BRADAW	BRADHA	15.287	25.370	100.000
17	BRADAW	BRADHA	8.029	18.755	100.000
20	BRADHA	BRADAW	0	0	100.000
21	BRADHA	BRADAW	0	0	100.000
22	BRADHA	BRADAW	2.020	7.748	92.078
23	BRADAW	BRADHA	0	0	100.000
25	BRADAW	BRADHA	9.559	2.636	99.829

B SAMPLE STUDENT TEST RESULTS (RELATIVE ERROR 1.E-3)

PROBLEM NUMBER	CODE WITH BEST AVG. TIME	NEXT BEST CODE	STUDENTS T VALUE	DEGREE OF FREEDOM	SIGNIFICANCE LEVEL OF DIF( )
1	BRADAW	BRADHA	.142	37.747	11.246
2	BRADAW	BRADHA	10.307	13.883	100.000
7	BRADHA	BRADAW	17.482	31.636	100.000
10	BRADAW	BRADHA	0	0	100.000
11	BRADAW	BRADHA	211.413	3.659	100.000
[illegible]	BRADHA	BRADAW	4.323	20.472	99.968
[illegible]	BRADHA	BRADAW	7.593	14.230	100.000
16	BRADAW	BRADHA	14.331	24.678	100.000
17	BRADAW	BRADHA	8.125	17.844	100.000
20	BRADAW	BRADHA	0	0	100.000
21	BRADHA	BRADAW	0	0	100.000
23	BRADAW	BRADHA	0	0	100.000
25	BRADAW	BRADHA	5.972	4.857	99.829

C SAMPLE STUDENT TEST RESULTS (RELATIVE ERROR 1.E-4)

PROBLEM NUMBER	CODE WITH BEST AVG. TIME	NEXT BEST CODE	STUDENTS T VALUE	DEGREE OF FREEDOM	SIGNIFICANCE LEVEL OF DIF( )
1	BRADHA	BRADAW	1.212	36.990	76.663
2	BRADAW	BRADAW	0	20.000	0
7	BRADHA	BRADAW	17.[illegible]	32.249	100.000
10	BRADAW	BRADHA	0	0	100.000
11	BRADAW	BRADHA	208.235	3.566	100.000
13	BRADAW	BRADHA	.519	10.147	38.491
15	BRADAW	BRADHA	4.715	19.636	99.996
16	BRADAW	BRADHA	17.141	25.777	100.000
17	BRADAW	BRADHA	11.960	15.074	100.000
20	BRADHA	BRADAW	0	0	100.000
21	BRADHA	BRADAW	0	0	100.000

Table 9. Student t Comparison: Harmonic vs Geometric Mean

A SAMPLE STUDENT TEST RESULTS
(RELATIVE ERROR 1.E-2)

PROBLEM NUMBER	CODE WITH BEST AVB. TIME	NEXT BEST CODE	STUDENTS T VALUE	DEGREE OF FREEDOM	SIGNIFICANCE LEVEL OF DIF(_)
1	OPTTPD	DAP	119.272	19.000	100.000
2	OPTTPD	OPTTPD	0	0	100.000
3	DAP	OPTTPD	26.559	9.000	100.000
5	DAP	OPTTPD	87.679	5.000	100.000
6	DAP	OPTTPD	95.609	19.000	100.000
7	DAP	OPTTPD	8.635	5.063	100.000
9	OPTTPD	OPTTPD	0	[illegible].000	0
11	OPTTPD	DAP	17.268	8.000	100.000
12	DAP	OPTTPD	56.839	9.000	100.000
13	DAP	OPTTPD	206.321	18.000	100.000
15	DAP	OPTTPD	106.628	19.000	100.000
16	OPTTPD	OPTTPD	0	28.000	0
17	OPTTPD	DAP	.987	2.001	57.252
19	OPTTPD	OPTTPD	0	0	100.000
20	OPTTPD	OPTTPD	0	0	100.000
21	OPTTPD	DAP	0	0	100.000
22	OPTTPD	OPTTPD	0	0	100.000
23	DAP	OPTTPD	.178	3.026	12.984
25	OPTTPD	OPTTPD	0	4.000	0

B SAMPLE STUDENT TEST RESULTS
(RELATIVE ERROR 1.E-3)

OBLEM NUMBER	CODE WITH BEST AVB. TIME	NEXT BEST CODE	STUDENTS T VALUE	DEGREE OF FREEDOM	SIGNIFICANCE LEVEL OF DIF(_)
1	OPTTPD	DAP	143.591	19.000	100.000
2	OPTTPD	OPTTPD	0	0	100.000
3	OPTTPD	OPTTPD	0	0	100.000
5	DAP	OPTTPD	307.815	5.000	100.000
6	DAP	OPTTPD	229.008	19.000	100.000
7	DAP	OPTTPD	17.498	3.192	100.000
9	OPTTPD	OPTTPD	0	8.000	0
11	OPTTPD	OPTTPD	0	18.000	0
12	DAP	OPTTPD	556.208	9.000	100.000
13	DAP	OPTTPD	230.448	14.000	100.000
15	OPTTPD	DAP	53.751	13.000	100.000
16	OPTTPD	OPTTPD	0	0	100.000
17	OPTTPD	DAP	.880	1.000	48.110
19	OPTTPD	OPTTPD	0	0	100.000
21	OPTTPD	OPTTPD	0	0	100.000
22	OPTTPD	OPTTPD	0	0	100.000
23	DAP	OPTTPD	24.750	7.000	100.000
25	OPTTPD	OPTTPD	0	4.000	0

Table 10. Student t Comparison: Transformed Primal Strategies

C SAMPLE STUDENT TEST RESULTS
(RELATIVE ERROR 1.E-4)

PROBLEM NUMBER	CODE WITH BEST AVG. TIME	NEXT BEST CODE	STUDENTS T VALUE	DEGREE OF FREEDOM	SIGNIFICANCE LEVEL OF DIF(_)
1	OPTTPD	CAP	1.582	5.361	82.939
2	OPTTPD	OPTTPD	0	0	100.000
3	OPTTPD	OPTTPD	0	0	100.000
5	DAP	OPTTPD	335.393	2.000	99.999
6	OPTTPD	OPTTPD	0	16.000	0
7	DAP	OPTTPD	9.890	2.038	99.905
9	OPTTPD	OPTTPD	0	0	100.000
12	OPTTPD	OPTTPD	0	18.000	0
13	DAP	OPTTPD	126.665	7.000	100.000
16	OPTTPD	OPTTPD	0	16.000	0
17	OPTTPD	OPTTPD	0	0	100.000
19	OPTTPD	OPTTPD	0	0	100.000
23	OPTTPD	OPTTPD	0	0	100.000

Table 10. Student t Comparison: Transformed Primal Strategies

A SAMPLE STUDENT TEST RESULTS
(RELATIVE ERROR 1.E-2)

PROBLEM NUMBER	CODE WITH BEST AVG. TIME	NEXT BEST CODE	STUDENTS T VALUE	DEGREE OF FREEDOM	SIGNIFICANCE LEVEL OF DIF(_)
1	BRADAW	BRADHA	1.221	37.888	77.026
1	BRADAW	OPTCPD	60.239	19.267	100.000
2	OPTCPD	OPTTPD	33.972	24.747	100.000
3	OPTCPD	GGP	14.150	27.263	100.000
5	GGP	OPTCPD	147.347	25.758	100.000
6	GGP	OPTCPD	411.901	22.591	100.000
7	GGP	OPTCPD	107.181	37.819	100.000
8	GGP	OPTCPD	192.121	1.000	99.669
9	GGP	OPTCPD	22.153	10.210	100.000
10	GGP	OPTCPD	31.879	24.038	100.000
11	GGP	OPTCPD	13.683	18.913	100.000
12	GGP	DAP	1.294	10.000	77.524
12	GGP	OPTCPD	18.443	10.640	100.000
13	GGP	BRADAW	157.347	13.601	100.000
14	GGP	OPTCPD	51.780	17.656	100.000
15	BRADHA	BRADAW	10.947	14.927	100.000
16	BRADAW	BRADHA	15.287	25.370	100.000
17	BRADAW	BRADHA	8.029	18.755	100.000
18	GGP	OPTCPD	12.925	2.355	99.940
19	GGP	OPTCPD	16.009	5.172	100.000
20	BRADHA	BRADAW	0	0	100.000
21	BRADHA	BRADAW	0	0	100.000
22	GGP	OPTCPD	17.418	35.642	100.000
23	OPTCPD	GGP	32.891	23.074	100.000
25	OPTCPD	BRADAW	59.728	9.359	100.000

B SAMPLE STUDENT TEST RESULTS
(RELATIVE ERROR 1.E-3)

PROBLEM NUMBER	CODE WITH BEST AVG. TIME	NEXT BEST CODE	STUDENTS T VALUE	DEGREE OF FREEDOM	SIGNIFICANCE LEVEL OF DIF(_)
1	BRADAW	BRADHA	.142	37.747	11.246
1	BRADAW	OPTCPD	74.748	19.596	100.000
2	OPTCPD	OPTTPD	42.020	22.141	100.000
3	OPTCPD	GGP	28.520	26.702	100.000
5	GGP	OPTCPD	272.556	23.683	100.000
6	GGP	OPTCPD	450.713	20.671	100.000
7	GGP	OPTCPD	102.005	37.953	100.000
8	GGP	OPTPD	34.121	1.013	100.000
9	GGP	OPTCPD	17.228	10.098	100.000
10	GGP	OPTCPD	39.401	24.651	100.000
11	GGP	OPTCPD	5.900	17.032	100.000
12	GGP	DAP	309.697	10.000	100.000
13	GGP	OPTCPD	96.717	19.144	100.000
14	GGP	OPTCPD	66.200	16.567	100.000
15	BRADHA	BRADAW	7.593	14.230	100.000
16	BRADAW	BRADHA	14.331	24.678	100.000
17	BRADAW	BRADHA	8.125	17.844	100.000
18	GGP	GGP	0	0	100.000
19	GGP	OPTCPD	31.599	4.807	100.000
20	BRADAW	BRADHA	0	0	100.000
21	BRADHA	BRADAW	0	0	100.000
22	OPTCPD	GGP	2.127	20.315	95.417
22	OPTCPD	OPTTPD	47.235	2.103	99.999
23	OPTCPD	OPTCPR	68.721	28.907	100.000
25	OPTCPD	BRADAW	247.694	12.699	100.000

Table 11. Student t Comparisons: Overall

C SAMPLE STUDENT TEST RESULTS
(RELATIVE ERROR 1.E-4)

PROBLEM NUMBER	CODE WITH BEST AVG. TIME	NEXT BEST CODE	STUDENTS T VALUE	DEGREE OF FREEDOM	SIGNIFICANCE LEVEL OF DIF(_)
1	BRADHA	BRADAW	1.212	36.990	76.663
1	BRADHA	OPTCPD	33.996	5.027	100.000
2	OPTCPD	OPTTPD	20.012	25.842	100.000
3	OPTCPD	GGP	38.635	31.394	100.000
5	GGP	OPTCPD	168.539	9.968	100.000
6	GGP	OPTCPD	372.576	4.296	100.000
7	GGP	OPTCPD	126.197	37.957	100.000
9	OPTCPD	GGP	.692	9.761	49.516
9	OPTCPD	GPKTC	4.554	2.002	99.148
10	GGP	OPTCPR	230.654	12.000	100.000
11	GGP	OPTCPD	4.306	16.991	99.952
12	GGP	OPTCPD	26.134	6.029	100.000
13	DAP	GGP	162.754	19.000	100.000
14	GGP	OPTCPD	41.471	16.713	100.000
15	BRADAW	BRADHA	4.715	19.636	99.996
16	BRADAW	BRADHA	17.141	25.777	100.000
17	BRADAW	BRADHA	11.960	15.074	100.000
18	GGP	GGP	0	0	100.000
19	GGP	OPTCPD	177.864	5.000	100.000
20	BRADHA	BRADAW	0	0	100.000
21	BRADHA	BRADAW	0	0	100.000
23	OPTCPD	OPTCPR	2.759	1.028	99.928
25	OPTCPD	OPTCPR	259.972	3.000	100.000

Table 11. Student t Comparisons: Overall

	Frequency					
Error Level	10^{-2}		10^{-3}		10^{-4}	
Ranking	1st	2nd	1st	2nd	1st	2nd
Series						
GGP	13	2	12	2	10	2
GPKTC	0	0	0	0	0	1
DAP	1	0	0	1	1	0
BRADAW	3	5	4	3	4	2
BRADHA	4	2	3	3	3	3
OPTCPS	0	0	0	0	0	0
OPTCPD	4	13	5	12	5	8
OPTCPR	0	0	0	1	0	3
OPTPD	0	0	0	0	0	0
OPTTPD	0	1	0	1	0	1

Table 12: Test Series Ranking Summary

Series	Number of Primal Variables	Number of Primal Constraints	Number of Primal Terms	Degree of Difficulty	Number of Negative Terms	Number of Multi-Term Constraints	Number of Tight Constraints	Number of Reversed Constraints	Critical Value
GGP	0.638	0.290	0.333	0.217	0.211	0.318	0.316	---	
GPKTC	0.717	0.730	0.495	0.313	0.032	0.688	-0.141	---	
BRADAW	0.145	0.652	0.805	0.910	0.385	0.229	0.182	---	
OPTCPS	0.805	0.843	0.829	0.619	---	0.870	-0.352	0.725	
OPTCPD	0.797	0.694	0.675	0.512	0.472	0.819	0.820	---	
OPTCPR	0.898	0.833	0.902	0.690	---	0.901	-0.419	0.866	
OPTPD	0.757	0.546	0.706	0.579	0.527	0.767	0.599	---	
OPTTPD	0.583	0.784	0.929	0.825	---	0.693	0.644	0.559	

Table 13. Correlation Coefficient of LOG(CPu Time) vs. Problem Characteristics.

eries	Number of Primal Variables	Number of Primal Constraints	Number of Primal Terms	Degree of Difficulty	Number of Negative Terms	Number of Multi-term Constraints	Number of Tight Constraints	Number of Reversed Constraints	Critical Value
GP	0.577	0.194	0.238	0.127	0.140	0.199	0.303	---	
PKTC	0.597	0.796	0.441	0.299	-0.016	0.535	-0.240	---	
RADAW	0.014	0.835	0.701	0.818	0.373	0.107	0.042	---	
PTCPS	0.755	0.863	0.880	0.685	---	0.764	-0.358	0.833	
PTCPD	0.723	0.587	0.672	0.518	0.595	0.749	0.689	---	
PTCPR	0.888	0.726	0.867	0.620	---	0.800	-0.448	0.954	
PTPD	0.541	0.394	0.726	0.628	0.665	0.744	0.354	---	
PTTPD	0.338	0.766	0.866	0.815	---	0.523	0.468	0.753	

able 14: Correlation Coefficient of CPu time vs. Problem Characteristics.

APPENDIX A

Test Problem File: Signomial Form

PROBLEM NO 1 [22], No.11

MINIMIZE :

```
GO(X) =  - (X1) + 0.400000(X1** 0.67)(X3**-0.67)
```

SUBJECTED TO:

```
G1(X) =  0.058820(X3)(X4) + 0.100000(X1) <= 1

G2(X) =  4.0000(X2)(X4**-1.00) + 2.0000(X2**-0.71)(X4**-1.00)
         + 0.058820(X2**-1.30)(X3) <= 1
```

PROBLEM NO 2 [4], Section 11.2

MINIMIZE :

```
G0(X) =  861000.(X1** 0.50)(X2)(X3**-0.67)(X4**-0.50)
         + 36900.(X3) +  7.7E+008(X1**-1.00)(X2** 0.22)
         -  7.7E+008(X1**-1.00)
```

SUBJECTED TO:

```
G1(X) =  (X2**-2.00)(X4) + (X2**-2.00) <= 1
```

PROBLEM NO 3 private files

MINIMIZE :

```
G0(X) =  (X1** 2.00) + (X2** 2.00) + 2.0000(X3** 2.00) + (X4** 2.00) - 5.0000(X1)
         - 5.0000(X2) - 21.0000(X3) - 7.0000(X4)
```

SUBJECTED TO:

```
G1(X) =  0.125000(X1** 2.00) + 0.125000(X2** 2.00)
         + 0.125000(X3** 2.00) + 0.125000(X4** 2.00)
         + 0.125000(X1) - 0.125000(X2) + 0.125000(X3) + 0.125000(X4) <= 1

G2(X) =  0.100000(X1** 2.00) + 0.200000(X2** 2.00)
         + 0.100000(X3** 2.00) + 0.200000(X4** 2.00)
         - 0.100000(X1) + 0.100000(X4) <= 1

G3(X) =  0.400000(X1** 2.00) + 0.200000(X2** 2.00)
         + 0.200000(X3** 2.00) + 0.400000(X1) - 0.200000(X2)
         + 0.200000(X4) <= 1

G4(X) =  0.000100(X1**-1.00) <= 1
```

PROBLEM NO 5 [5], No.2

MINIMIZE :

G0(X) = 5.3578(X3** 2.00) + 0.835700(X1)(X5) + 37.2392(X1)

SUBJECTED TO:

G1(X) = 2.6E-005(X3)(X5) - 6.7E-005(X2)(X5) - 7.3E-005(X1)(X4) <= 1

G2(X) = 0.000853(X2)(X5) + 9.4E-005(X1)(X4) - 0.000331(X3)(X5) <= 1

G3(X) = 1330.33(X2**-1.00)(X5**-1.00) - 0.420000(X1)(X5**-1.00)
- 0.305860(X2**-1.00)(X3** 2.00)(X5**-1.00) <= 1

G4(X) = 2275.13(X3**-1.00)(X5**-1.00) - 0.266800(X1)(X5**-1.00)
- 0.405840(X4)(X5**-1.00) <= 1

G5(X) = 0.000242(X2)(X5) + 0.000102(X1)(X2) + 7.4E-005(X3** 2.00) <= 1

G6(X) = 0.000300(X3)(X5) + 8.0E-005(X1)(X3) + 0.000122(X3)(X4) <= 1

G7(X) = 78.0000(X1**-1.00) <= 1

G8(X) = 0.009804(X1) <= 1

G9(X) = 33.0000(X2**-1.00) <= 1

G10(X) = 0.022222(X2) <= 1

G11(X) = 27.0000(X3**-1.00) <= 1

G12(X) = 0.022222(X3) <= 1

G13(X) = 27.0000(X4**-1.00) <= 1

G14(X) = 0.022222(X4) <= 1

G15(X) = 27.0000(X5**-1.00) <= 1

G16(X) = 0.022222(X5) <= 1

PROBLEM NO 6 private files

MINIMIZE :

```
G0(X) =  0.003496(X1)(X3**-1.00)(X4) + 0.001886(X1)(X3**-1.00)
         - 0.001475(X2)(X3**-1.00) + 2345.00(X3**-1.00)(X7**-1.00)
```

SUBJECTED TO:

```
G1(X) =  0.007353(X1)(X4) + 0.004760(X2) + 0.021500(X1) <= 1

G2(X) =  0.108600(X1**-1.00)(X2)(X4**-1.00)
         + 0.400000(X4**-1.00) <= 1

G3(X) =  0.619600(X1**-1.00)(X2) + 0.748000(X4) <= 1

G4(X) =  (X5)(X6**-1.00) + 1.3330(X6**-1.00) <= 1

G5(X) =  142.86(X1** 0.53)(X3**-0.34)(X4**-0.51)(X7**-1.70) <= 1

G6(X) =  0.002480(X1** 0.80)(X3** 1.40)(X4**-1.10)(X5**-1.00)(X7) <= 1

G7(X) =  40.0000(X7**-1.00) <= 1

G8(X) =  0.125000(X6) <= 1

G9(X) =  3.0000(X3**-1.00) <= 1

G10(X) =  (X1**-1.00)(X3)(X4** 3.00)(X6) <= 1
```

PROBLEM NO 7 [4], Section 11.3

MINIMIZE :

```
G0(X) =  1.1047(X1** 2.00)(X2) + 0.673500(X3)(X4) + 0.048110(X2)(X3)(X4)
```

SUBJECTED TO:

```
G1(X) =  12.0000(X2**-2.00)(X6) - 3.0000(X2**-2.00)(X3** 2.00)
         - 6.0000(X1)(X2**-2.00)(X3) - 3.0000(X1** 2.00)(X2**-2.00) <= 1

G2(X) =  0.094280(X3**-1.00)(X4**-3.00) + 0.027760(X3) <= 1

G3(X) =   1.8E+007(X1**-2.00)(X2**-2.00)(X5**-1.00)
         +  2.5E+008(X1**-2.00)(X2**-1.00)(X5**-1.00)(X6**-1.00)
         +  9.0E+006(X1**-2.00)(X5**-1.00)(X6**-1.00)
         +  3.5E+009(X1**-2.00)(X2**-2.00)(X5**-1.00)(X6**-2.00)(X7)
         +  2.5E+008(X1**-2.00)(X2**-1.00)(X5**-1.00)(X6**-2.00)(X7)
         +  4.5E+006(X1**-2.00)(X5**-1.00)(X6**-2.00)(X7) <= 1

G4(X) =  0.250000(X2** 2.00)(X7**-1.00) + 0.250000(X3** 2.00)(X7**-1.00)
         + 0.500000(X1)(X3)(X7**-1.00) + 0.250000(X1** 2.00)(X7**-1.00) <= 1

G5(X) =  16.8000(X3**-2.00)(X4**-1.00) <= 1

G6(X) =  (X1)(X4**-1.00) <= 1

G7(X) =  0.125000(X1**-1.00) <= 1

G8(X) =  9.0800(X3**-3.00)(X4) <= 1

G9(X) =   7.4E-005(X5** 0.50) <= 1
```

PROBLEM NO 8 [5], No.3

MINIMIZE :

```
G0(X) =  1.7150(X1) + 0.035000(X1)(X6) + 4.0565(X3) + 10.0000(X2)
         + 3000.00 - 0.063000(X3)(X5)
```

SUBJECTED TO:

```
G1(X) =  0.005955(X6** 2.00) + 0.883929(X1**-1.00)(X3)
         - 0.117563(X6) <= 1

G2(X) =  1.1088(X1)(X3**-1.00) + 0.130353(X1)(X3**-1.00)(X6)
         - 0.006603(X1)(X3**-1.00)(X6** 2.00) <= 1

G3(X) =  0.000662(X6** 2.00) + 0.017240(X5) - 0.005660(X4)
         - 0.019121(X6) <= 1

G4(X) =  56.8507(X5**-1.00) + 1.0870(X5**-1.00)(X6)
         + 0.321750(X4)(X5**-1.00) - 0.037620(X5**-1.00)(X6** 2.00) <= 1

G5(X) =  0.006198(X7) + 2462.31(X2)(X3**-1.00)(X4**-1.00)
         - 25.1256(X2)(X3**-1.00) <= 1

G6(X) =  161.19(X7**-1.00) + 5000.00(X2)(X3**-1.00)(X7**-1.00)
         - 489550.(X2)(X3**-1.00)(X4**-1.00)(X7**-1.00) <= 1

G7(X) =  44.3333(X5**-1.00) + 0.330000(X5**-1.00)(X7) <= 1

G8(X) =  0.022556(X5) - 0.007595(X7) <= 1

G9(X) =  0.000610(X3) - 0.000500(X1) <= 1

G10(X) =  0.819672(X1)(X3**-1.00) + 0.819672(X3**-1.00) <= 1

G11(X) =  24500.(X2)(X3**-1.00)(X4**-1.00)
          - 255.00(X2)(X3**-1.00) <= 1

G12(X) =  0.010204(X4) +  1.2E-005(X2**-1.00)(X3)(X4) <= 1

G13(X) =   6.2E-005(X1)(X6) +  6.2E-005(X1) -  7.6E-005(X3) <= 1

G14(X) =  1.2200(X1**-1.00)(X3) + (X1**-1.00) - (X6) <= 1
```

```
G15(X) =  0.000500(X1) <= 1

G16(X) =  (X1**-1.00) <= 1

G17(X) =  0.008333(X2) <= 1

G18(X) =  (X2**-1.00) <= 1

G19(X) =  0.000200(X3) <= 1

G20(X) =  (X3**-1.00) <= 1

G21(X) =  0.010753(X4) <= 1

G22(X) =  85.0000(X4**-1.00) <= 1

G23(X) =  0.010526(X5) <= 1

G24(X) =  90.0000(X5**-1.00) <= 1

G25(X) =  0.083333(X6) <= 1

G26(X) =  3.0000(X6**-1.00) <= 1

G27(X) =  0.006173(X7) <= 1

G28(X) =  145.00(X7**-1.00) <= 1
```

PROBLEM NO 9 [5], No.4

MINIMIZE :

```
G0(X) =  0.400000(X1** 0.67)(X7**-0.67) + 0.400000(X2** 0.67)(X8**-0.67)
         + 10.0000 - (X1) - (X2)
```

SUBJECTED TO:

```
G1(X) =  0.058800(X5)(X7) + 0.100000(X1) <= 1

G2(X) =  0.058800(X6)(X8) + 0.100000(X1) + 0.100000(X2) <= 1

G3(X) =  4.0000(X3)(X5**-1.00) + 2.0000(X3**-0.71)(X5**-1.00)
         + 0.058800(X3**-1.30)(X7) <= 1

G4(X) =  4.0000(X4)(X6**-1.00) + 2.0000(X4**-0.71)(X6**-1.00)
         + 0.058800(X4**-1.30)(X8) <= 1

G5(X) =  0.100000(X1) <= 1

G6(X) =  0.100000(X1**-1.00) <= 1

G7(X) =  0.100000(X2) <= 1

G8(X) =  0.100000(X2**-1.00) <= 1

G9(X) =  0.100000(X3) <= 1

G10(X) =  0.100000(X3**-1.00) <= 1

G11(X) =  0.100000(X4) <= 1

G12(X) =  0.100000(X4**-1.00) <= 1

G13(X) =  0.100000(X5) <= 1

G14(X) =  0.100000(X5**-1.00) <= 1

G15(X) =  0.100000(X6) <= 1

G16(X) =  0.100000(X6**-1.00) <= 1

G17(X) =  0.100000(X7) <= 1
```

```
G18(X) =  0.100000(X7**-1.00) <= 1

G19(X) =  0.100000(X8) <= 1

G20(X) =  0.100000(X8**-1.00) <= 1
```

PROBLEM NO 10 [5], No.5

MINIMIZE :

```
G0(X) =  (X1) + (X2) + (X3)

SUBJECTED TO:

G1(X) =  833.33(X1**-1.00)(X4)(X6**-1.00)
         + 100.0000(X6**-1.00) - 83333.(X1**-1.00)(X6**-1.00) <= 1

G2(X) =  1250.00(X2**-1.00)(X5)(X7**-1.00)
         + (X4)(X7**-1.00) - 1250.00(X2**-1.00)(X4)(X7**-1.00) <= 1

G3(X) =  1.2E+006(X3**-1.00)(X8**-1.00) + (X5)(X8**-1.00) - 2500.00(X3**-1.00)(X5)(X8**-1.00) <= 1

G4(X) =  0.002500(X4) + 0.002500(X6) <= 1

G5(X) =  0.002500(X5) + 0.002500(X7) - 0.002500(X4) <= 1

G6(X) =  0.010000(X8) - 0.010000(X5) <= 1

G7(X) =  0.000100(X1) <= 1

G8(X) =  100.0000(X1**-1.00) <= 1

G9(X) =  0.000100(X2) <= 1

G10(X) =  1000.00(X2**-1.00) <= 1

G11(X) =  0.000100(X3) <= 1

G12(X) =  1000.00(X3**-1.00) <= 1

G13(X) =  0.001000(X4) <= 1

G14(X) =  10.0000(X4**-1.00) <= 1

G15(X) =  0.001000(X5) <= 1

G16(X) =  10.0000(X5**-1.00) <= 1

G17(X) =  0.001000(X6) <= 1

G18(X) =  10.0000(X6**-1.00) <= 1
```

```
G19(X) =  0.001000(X7) <= 1

G20(X) =  10.0000(X7**-1.00) <= 1

G21(X) =  0.001000(X8) <= 1

G22(X) =  10.0000(X8**-1.00) <= 1
```

PROBLEM NO 11 private files

MINIMIZE :

G0(X) = - (X3)(X5) - (X4)(X6) + (X5) + (X6)

SUBJECTED TO:

G1(X) = 50.0000(X1**-1.00) - (X1**-1.00)(X2) <= 1

G2(X) = 1.4609(X5**-1.00) + 0.151860(X1)(X5**-1.00)
 + 0.001450(X1** 2.00)(X5**-1.00) <= 1

G3(X) = 0.800800(X6**-1.00) + 0.203100(X2)(X6**-1.00)
 + 0.000916(X2** 2.00)(X6**-1.00) <= 1

G4(X) = 0.100000(X3)(X7) + 0.100000(X4)(X8) <= 1

G5(X) = 1.5742(X7**-1.00) + 0.163100(X1)(X7**-1.00)
 + 0.001358(X1** 2.00)(X7**-1.00) <= 1

G6(X) = 0.726600(X8**-1.00) + 0.225600(X2)(X8**-1.00)
 + 0.000778(X2** 2.00)(X8**-1.00) <= 1

G7(X) = 18.0000(X1**-1.00) <= 1

G8(X) = 0.033333(X1) <= 1

G9(X) = 14.0000(X2**-1.00) <= 1

G10(X) = 0.040000(X2) <= 1

G11(X) = (X3) <= 1

G12(X) = (X4) <= 1

PROBLEM NO 12 private files

MINIMIZE :

```
G0(X) =  0.005650(X1)(X4**-1.00) + 0.002189(X2** 2.00)(X4**-1.00)
         + 0.032840(X3)(X4**-1.00) + 2345.00(X4**-1.00)(X5**-1.00)
```

SUBJECTED TO:

```
G1(X) =  0.747940(X1**-1.00)(X6)(X7) - 0.380400(X1**-1.00)(X2)
         - 0.299180(X1**-1.00)(X3) <= 1

G2(X) =  0.031330(X1) + 0.030000(X2) + 0.024400(X3) <= 1

G3(X) =  (X8)(X9**-1.00) + 1.3330(X9**-1.00) <= 1

G4(X) =  (X1)(X7**-1.00) + (X2)(X7**-1.00) + (X3)(X7**-1.00) <= 1

G5(X) =  0.007000(X4** 0.34)(X5** 1.70)(X6** 0.51)(X7**-0.53) <= 1

G6(X) =  0.002480(X4** 1.40)(X5)(X6**-1.10)(X7** 0.80)(X8**-1.00) <= 1

G7(X) =  40.0000(X5**-1.00) <= 1

G8(X) =  0.125000(X9) <= 1

G9(X) =  3.0000(X4**-1.00) <= 1

G10(X) =  (X4)(X6** 3.00)(X7**-1.00)(X9) <= 1

G11(X) =  0.050000(X3**-1.00)(X7) <= 1
```

PROBLEM NO 13 [4], Section 10.1

MINIMIZE :

```
G0(X) =  5.8850(X1)(X4)(X5**-1.00) + 5.8850(X3)(X4)(X5**-1.00)
```

SUBJECTED TO:

```
G1(X) =  (X3**-2.00)(X6** 2.00) + (X2** 2.00)(X3**-2.00) <= 1

G2(X) =  8.9400(X1)(X2**-1.00)(X4**-1.00)(X7**-1.00)
         + 8.9400(X2**-1.00)(X4**-1.00)(X7**-1.00)(X8) <= 1

G3(X) =  8.9400(X1)(X9**-2.00) + 8.9400(X8)(X9**-2.00) <= 1

G4(X) =  0.015600(X1)(X4**-1.00) + 0.150000(X4**-1.00) <= 1

G5(X) =  0.015600(X3)(X4**-1.00) + 0.150000(X4**-1.00) <= 1

G6(X) =  (X1**-1.00)(X5) - (X1**-1.00)(X6) <= 1

G7(X) =  2.5000(X1**-1.00)(X7) - 0.416667(X1**-1.00)(X3) <= 1

G8(X) =  (X2**-2.00)(X3** 2.00) - (X2**-2.00)(X8** 2.00) <= 1

G9(X) =  4.4000(X2**-2.00)(X4**-1.00)(X7**-1.00)(X9** 2.67) <= 1

G10(X) =  1.0500(X4**-1.00) <= 1

G11(X) =  (X2)(X3**-1.00) <= 1
```

PROBLEM NO 14 [22], No.12

MINIMIZE :

```
G0(X) =  2.8485(X1) - 22.4990(X1)(X2) + 2.8952(X1)(X3)
         + 0.305700(X1)(X4) - 4.4318(X1)(X5) + 0.140000(X1)(X5** 2.00)
         + 3.5974(X1)(X6) + 0.050000(X1)(X7)
```

SUBJECTED TO:

```
G1(X) =  100.0000(X3)(X9**-1.00) - 100.0000(X3)(X8** 0.01)(X9**-1.01)
         + (X8)(X9**-1.00) <= 1

G2(X) =  0.474400(X1)(X4**-1.00)(X8**-1.00)
         + 0.875640(X1**-1.00)(X4)(X6)(X8**-1.00)
         + 0.012152(X1)(X8**-1.00) + 0.139100(X1)(X6)(X8**-1.00)
         + 0.397900(X1)(X6** 2.00)(X8**-1.00)
         - 5.7222(X6) <= 1

G3(X) =  10.4351(X1**-1.00)(X4)(X5**-1.00)(X9**-1.00)
         - 72.5476(X5**-1.00) + 5.6303(X3)(X5**-1.00)
         + 0.127900(X4)(X5**-1.00) - 1.8459(X6) - 133.91(X5**-1.00)(X6)
         + 10.3930(X3)(X5**-1.00)(X6) + 0.236200(X4)(X5**-1.00)(X6)
         + 19.2611(X1**-1.00)(X4)(X5**-1.00)(X6)(X9**-1.00) <= 1

G4(X) =  0.003309(X1) - 0.006910(X1)(X3) - 0.000486(X1)(X4)
         + 0.010090(X1)(X5) -  1.3E-006(X1** 3.00) -  1.5E-005(X1** 3.00)(X6)
         -  4.2E-005(X1** 3.00)(X6** 2.00)(X8**-1.00)
         - 0.000253(X1)(X5** 2.00) <= 1

G5(X) =  21.3351(X4**-1.00) - 1.8458(X6) <= 1

G6(X) =  0.002017(X1) + 0.004878(X1)(X2) + 0.005735(X1)(X5)
         - 0.000744(X1)(X3) -  6.3E-005(X1)(X4) -  1.9E-005(X1)(X7) <= 1

G7(X) =  0.001817(X1) + 0.011287(X1)(X2) + 0.010795(X1)(X5)
         +  1.3E-005(X1)(X7) - 0.003304(X1)(X3) - 0.000471(X1)(X4)
         - 0.000363(X1)(X5** 2.00) <= 1

G8(X) =  0.025616(X1** 2.00)(X7**-1.00) + 0.293164(X1** 2.00)(X6)(X7**-1.00)
```

```
        + 0.838770(X1** 2.00)(X6** 2.00)(X7**-1.00) <= 1

G9(X) =  - 4.4400(X5**-1.00) + 41.0400(X2)(X5**-1.00)
        + 5.6300(X3)(X5**-1.00) + 0.122800(X4)(X5**-1.00) <= 1

G10(X) =  0.400000(X6**-1.00) <= 1
```

PROBLEM NO 15 [22], No.15

MINIMIZE :

G0(X) = 0.050000(X1) + 0.050000(X2) + 0.050000(X3) + (X9)

SUBJECTED TO:

G1(X) = (X7**-1.00)(X10) - 0.500000(X1)(X4)(X7**-1.00) <= 1

G2(X) = (X7)(X8**-1.00) - 0.500000(X2)(X5)(X8**-1.00) <= 1

G3(X) = (X8)(X9**-1.00) - 0.500000(X3)(X6)(X9**-1.00) <= 1

G4(X) = 0.500000(X9)(X10**-1.00) + 0.250000(X10**-1.00) <= 1

G5(X) = 0.796810(X4)(X7**-1.00) <= 1

G6(X) = 0.796810(X5)(X8**-1.00) <= 1

G7(X) = 0.796810(X6)(X9**-1.00) <= 1

PROBLEM NO 16 [22], No. 14

MINIMIZE :

G0(X) = (X6) + 0.400000(X4** 0.67) + 0.400000(X9** 0.67)

SUBJECTED TO:

G1(X) = 10.0000(X3**-1.00) - (X1)(X3**-1.00) <= 1

G2(X) = (X1)(X8**-1.00) - (X6)(X8**-1.00) <= 1

G3(X) = (X1**-1.00)(X2**-1.50)(X3)(X4**-1.00)(X5**-1.00)
+ 5.0000(X1**-1.00)(X2**-1.00)(X3)(X5** 1.20) <= 1

G4(X) = 0.050000(X3) + 0.050000(X2) <= 1

G5(X) = (X6**-1.00)(X7**-1.50)(X8)(X9**-1.00)(X10**-1.00)
+ 5.0000(X6**-1.00)(X7**-1.00)(X8)(X10** 1.20) <= 1

G6(X) = (X2**-1.00)(X7) + (X2**-1.00)(X8) <= 1

G7(X) = 10.0000(X10) <= 1

PROBLEM NO 17 [22], No. 16 .

MINIMIZE :

GO(X) = 0.050000(X1) + 0.050000(X2) + 0.050000(X3) + (X9)

SUBJECTED TO:

G1(X) = 0.500000(X9)(X10**-1.00) + 0.250000(X10**-1.00) <= 1

G2(X) = (X7**-1.00)(X10) - 0.500000(X1)(X4)(X7**-1.00) <= 1

G3(X) = (X7)(X8**-1.00) - 0.500000(X2)(X5)(X8**-1.00) <= 1

G4(X) = (X8)(X9**-1.00) - 0.500000(X3)(X6)(X9**-1.00) <= 1

G5(X) = 0.700329(X4)(X7**-1.00) + 0.307795(X7) <= 1

G6(X) = 0.700329(X5)(X8**-1.00) + 0.307795(X8) <= 1

G7(X) = 0.700329(X6)(X9**-1.00) + 0.307795(X9) <= 1

PROBLEM NO 18 [22], NO. 21.

MINIMIZE :

```
G0(X) =  - 0.063000(X4)(X7) + 5.0400(X1) + 0.035000(X2) + 10.0000(X3)
         + 3.3500(X5)
```

SUBJECTED TO:

```
G1(X) =  0.892860(X1**-1.00)(X4) - 0.117560(X8) + 0.005955(X8** 2.00) <= 1

G2(X) =  0.017410(X7) - 0.019130(X8) + 0.000662(X8** 2.00)
         - 0.005660(X6) <= 1

G3(X) =  35.8200(X9**-1.00) - 0.222100(X9**-1.00)(X10) <= 1

G4(X) =  1.2200(X4)(X5**-1.00) - (X1)(X5**-1.00) <= 1

G5(X) =  (X1)(X2**-1.00)(X8) - 1.2300(X2**-1.00)(X4)
         + (X1)(X2**-1.00) <= 1

G6(X) =  0.330000(X7**-1.00)(X10) + 44.3330(X7**-1.00) <= 1

G7(X) =   1.0E-005(X3**-1.00)(X4)(X6)(X9) + 0.010202(X6) <= 1

G8(X) =  0.000500(X1) <= 1

G9(X) =   5.2E-005(X2) <= 1

G10(X) =  0.008333(X3) <= 1

G11(X) =  0.000200(X4) <= 1

G12(X) =  0.000500(X5) <= 1

G13(X) =  0.010753(X6) <= 1

G14(X) =  85.0000(X6**-1.00) <= 1

G15(X) =  0.010526(X7) <= 1

G16(X) =  90.0000(X7**-1.00) <= 1

G17(X) =  0.083333(X8) <= 1
```

```
G18(X) =  3.0000(X8**-1.00) <= 1

G19(X) =  0.250000(X9) <= 1

G20(X) =  1.2000(X9**-1.00) <= 1

G21(X) =  0.006173(X10) <= 1

G22(X) =  145.00(X10**-1.00) <= 1
```

PROBLEM NO 19 [22], No. 24

MINIMIZE :

```
G0(X) =  1.2626(X8) + 1.2626(X9) + 1.2626(X10) - 1.2311(X1)(X8)
         - 1.2311(X2)(X9) - 1.2311(X3)(X10)
```

SUBJECTED TO:

```
G1(X) =  0.034745(X1)(X4**-1.00) + 0.975000(X1) - 0.009800(X1** 2.00)(X4**-1.00) <= 1

G2(X) =  0.034745(X2)(X5**-1.00) + 0.975000(X2) - 0.009800(X2** 2.00)(X5**-1.00) <= 1

G3(X) =  0.034745(X3)(X6**-1.00) + 0.975000(X3) - 0.009800(X3** 2.00)(X6**-1.00) <= 1

G4(X) =  (X1)(X5**-1.00)(X7**-1.00)(X8) + (X4)(X5**-1.00) - 1.1000(X4)(X5**-1.00)(X7**-1.00)(X8) <= 1

G5(X) =  0.002000(X2)(X9) + 0.002000(X5)(X8) + (X5) + (X6) - 0.002000(X1)(X8)
         - 0.002100(X6)(X9) <= 1

G6(X) =  (X2**-1.00)(X3)(X9**-1.00)(X10) + (X2**-1.00)(X6) + 500.00(X9**-1.00)
         - 1.1000(X9**-1.00)(X10) - 510.00(X2**-1.00)(X6)(X9**-1.00) <= 1

G7(X) =  0.900000(X2**-1.00) + 0.002000(X10) - 0.002100(X2**-1.00)(X3)(X10) <= 1

G8(X) =  0.002000(X7) - 0.002100(X8) <= 1

G9(X) =  (X2)(X3**-1.00) <= 1

G10(X) =  (X1)(X2**-1.00) <= 1

G11(X) =  (X1) <= 1

G12(X) =  0.100000(X1**-1.00) <= 1

G13(X) =  (X2) <= 1

G14(X) =  0.100000(X2**-1.00) <= 1

G15(X) =  (X3) <= 1

G16(X) =  0.900000(X3**-1.00) <= 1

G17(X) =  10.0000(X4) <= 1
```

```
G18(X) =  0.100000(X5**-1.00) <= 1

G19(X) =  1.1111(X6) <= 1

G20(X) =  0.100000(X6**-1.00) <= 1

G21(X) =  500.00(X9**-1.00) <= 1

G22(X) =  0.002000(X10) <= 1

G23(X) =  0.100000(X10**-1.00) <= 1
```

PROBLEM NO 20 [22], No. 17 .

MINIMIZE :

```
G0(X) =  (X3**-1.00)
```

SUBJECTED TO:

```
G1(X) =  0.999000(X4**-1.00) - 100.10(X7)(X10) <= 1

G2(X) =  (X10)(X11**-1.00) - 10.0200(X8) <= 1

G3(X) =  (X5**-1.00) - 10.2000(X1**-1.00)(X8)(X11) <= 1

G4(X) =  10.0000(X11) - 10.0200(X9) <= 1

G5(X) =  (X6**-1.00) - 1.2000(X2**-1.00)(X9) <= 1

G6(X) =  0.098000(X10) + 0.980000(X7)(X10) <= 1

G7(X) =  9.8000(X1)(X4) + 9.9000(X1)(X4)(X7** 2.00) <= 1

G8(X) =  0.980000(X1**-1.00)(X2)(X5) + 0.990000(X1**-1.00)(X2)(X5)(X8** 2.00) <= 1

G9(X) =  0.970000(X2**-1.00)(X3)(X6) + 0.980000(X2**-1.00)(X3)(X6)(X9** 2.00) <= 1
```

PROBLEM NO 21 [22], No. 18

MINIMIZE :

G0(X) = (X9**-1.00)

SUBJECTED TO:

G1(X) = - (X2)(X4**-1.00)(X11) + (X4**-1.00)(X5) + 0.010000(X4**-1.00)(X5)(X11)(X13**-1.00)
+ 0.010000(X4**-1.00)(X5)(X11) <= 1

G2(X) = - 0.010000(X5)(X7**-1.00)(X11) + (X7**-1.00)(X8) <= 1

G3(X) = - 2100.00(X3)(X5**-1.00) + 26.2000(X5**-1.00)(X6) <= 1

G4(X) = - 21.1300(X6)(X8**-1.00) + (X8**-1.00)(X9) <= 1

G5(X) = (X1) + (X1)(X10) + (X1)(X10)(X12) <= 1

G6(X) = (X1**-1.00)(X4)(X10**-1.00) + 0.009000(X1**-1.00)(X4)(X12**-1.00)
+ 0.009000(X1**-1.00)(X4) <= 1

G7(X) = 0.990000(X1**-1.00)(X2) + (X1**-1.00)(X2)(X11) + (X1**-1.00)(X2)(X11)(X13) <= 1

G8(X) = 94.0000(X4**-1.00)(X7)(X10**-1.00) <= 1

G9(X) = 9301.00(X2**-1.00)(X3) <= 1

PROBLEM NO 22 [22], No. 20 .

MINIMIZE :

```
G0(X) =  - 0.280000(X1)(X6**-1.00) + 0.673200(X2)(X6**-1.00)
         + 1.1200(X3)(X6**-1.00) - 31047.(X6**-1.00)
         + 0.007400(X5)(X6**-1.00)
```

SUBJECTED TO:

```
G1(X) =  0.639926(X4**-0.25)(X8)(X10**-1.00)
         - 0.156564(X4** 0.42)(X9**-1.00)(X11)
         - 0.100000(X10)(X13**-1.00) <= 1

G2(X) =  (X1)(X4**-1.00)(X7**-1.00)(X8**-1.00)(X9**-1.00)
         - 0.312540(X4** 0.25)(X10)(X13**-1.00) <= 1

G3(X) =  (X11**-1.00)(X13) + 1.2501(X4** 1.25)(X7)(X9)(X10)(X11**-1.00)
         - 0.244660(X4** 1.67)(X7)(X10) <= 1

G4(X) =  (X5**-1.00)(X12) + 0.733980(X4** 1.67)(X5**-1.00)(X7)(X10)(X11)
         + (X5**-1.00)(X11) - (X5**-1.00)(X13) <= 1

G5(X) =  3809.97(X4**-1.25)(X7**-1.00)(X9**-1.00)(X10**-1.00)
         + 0.195706(X4** 0.42)(X9**-1.00)(X11) <= 1

G6(X) =  (X10)(X12**-1.00) + 0.244660(X4** 0.67)(X9**-1.00)(X10)(X11)(X12**-1.00)
         + 0.156270(X4** 0.25)(X10** 2.00)(X12**-1.00)(X13**-1.00)
         + (X9)(X12**-1.00) + 11.0000(X12**-1.00)(X13) + 1.5628(X4** 0.25)(X10)(X12**-1.00) <= 1

G7(X) =  0.733980(X3**-1.00)(X4** 0.67)(X7)(X10)(X11) <= 1

G8(X) =  0.312540(X2**-1.00)(X4** 1.25)(X7)(X9)(X10)(X12)(X13**-1.00) <= 1

G9(X) =  0.020000(X5** 2.00)(X6**-1.00)(X7) <= 1

G10(X) =  0.007720(X4) <= 1

G11(X) =  6.1800(X4**-1.00) <= 1
```

PROBLEM NO 23 [22], No. 19

MINIMIZE :

```
G0(X) =  2.0425(X1** 0.78) + 52.2500(X2) + 192.85(X2** 0.90)
         + 5.2500(X2** 3.00) + 61.4650(X6** 0.47)
         + 0.017480(X3** 1.33)(X4**-0.80) + 100.70(X4** 0.55)
         +  3.7E-010(X3** 2.85)(X4**-1.70) + 0.009450(X5) +  1.1E-010(X4**-1.80)(X5** 2.80)
         + 116.00(X6) - 205.00(X6)(X7) - 278.00(X2** 3.00)(X7)
```

SUBJECTED TO:

```
G1(X) =  129.40(X2**-3.00) + 105.00(X6**-1.00) <= 1

G2(X) =  103000.(X2** 3.00)(X3**-1.00)(X7)(X8**-1.00)
         +  1.2E+006(X3**-1.00)(X8**-1.00) <= 1

G3(X) =  4.6800(X1**-1.00)(X2** 3.00) + 61.3000(X1**-1.00)(X2** 2.00)
         + 160.50(X1**-1.00)(X2) <= 1

G4(X) =  1.7900(X7) + 3.0200(X2** 3.00)(X6**-1.00)(X7)
         + 35.7000(X6**-1.00) <= 1

G5(X) =  0.001220(X3)(X4**-0.20)(X5**-0.80)(X8)
         + 0.001670(X3** 0.40)(X4**-0.43)(X8)
         +  3.6E-005(X3)(X4**-1.00)(X8) + 0.002000(X3)(X5**-1.00)(X8)
         + 0.004000(X8) <= 1
```

PROBLEM NO 24 [5], No. 6

MINIMIZE :

G0(X) = (X11) + (X12) + (X13)

SUBJECTED TO:

G1(X) = 1.2626(X8)(X11**-1.00) - 1.2311(X1)(X8)(X11**-1.00) <= 1

G2(X) = 1.2626(X9)(X12**-1.00) - 1.2311(X2)(X9)(X12**-1.00) <= 1

G3(X) = 1.2626(X10)(X13**-1.00) - 1.2311(X3)(X10)(X13**-1.00) <= 1

G4(X) = 0.034750(X2)(X5**-1.00) + 0.975000(X2) - 0.009800(X2** 2.00)(X5**-1.00) <= 1

G5(X) = 0.034750(X3)(X6**-1.00) + 0.975000(X3) - 0.009750(X3** 2.00)(X6**-1.00) <= 1

G6(X) = (X1)(X5**-1.00)(X7**-1.00)(X8) + (X4)(X5**-1.00) - 1.1000(X4)(X5**-1.00)(X7**-1.00)(X8) <= 1

G7(X) = 0.002000(X2)(X9) + 0.002000(X5)(X8) + (X6) + (X5) - 0.002110(X1)(X8)
- 0.002000(X6)(X9) <= 1

G8(X) = (X2**-1.00)(X3)(X9**-1.00)(X10) + (X2**-1.00)(X6) + 500.00(X9**-1.00)
- (X9**-1.00)(X10) - 501.00(X2**-1.00)(X6)(X9**-1.00) <= 1

G9(X) = 0.900000(X2**-1.00) + 0.002000(X10) - 0.002200(X2**-1.00)(X3)(X10) <= 1

G10(X) = 0.002000(X7) - 0.002100(X8) <= 1

G11(X) = 0.034750(X1)(X4**-1.00) + 0.975000(X1) - 0.009800(X1** 2.00)(X4**-1.00) <= 1

G12(X) = 0.980000(X2)(X3**-1.00) <= 1

G13(X) = (X1)(X2**-1.00) <= 1

G14(X) = 0.100000(X1**-1.00) <= 1

G15(X) = (X1) <= 1

G16(X) = 0.100000(X2**-1.00) <= 1

G17(X) = (X2) <= 1

G18(X) = 0.900000(X3**-1.00) <= 1

```
G19(X) =  (X3) <= 1

G20(X) =  0.000100(X4**-1.00) <= 1

G21(X) =  10.0000(X4) <= 1

G22(X) =  0.100000(X5**-1.00) <= 1

G23(X) =  1.1111(X5) <= 1

G24(X) =  0.100000(X6**-1.00) <= 1

G25(X) =  1.1111(X6) <= 1

G26(X) =  0.100000(X7**-1.00) <= 1

G27(X) =  0.001000(X7) <= 1

G28(X) =  0.100000(X8**-1.00) <= 1

G29(X) =  0.001000(X8) <= 1

G30(X) =  500.00(X9**-1.00) <= 1

G31(X) =  0.001000(X9) <= 1

G32(X) =  0.100000(X10**-1.00) <= 1

G33(X) =  0.002000(X10) <= 1

G34(X) =  (X11**-1.00) <= 1

G35(X) =  0.006667(X11) <= 1

G36(X) =  0.000100(X12**-1.00) <= 1

G37(X) =  0.006667(X12) <= 1

G38(X) =  0.000100(X13**-1.00) <= 1

G39(X) =  0.006667(X13) <= 1
```

PROBLEM NO 25 [23], No. 11

MINIMIZE :

G0(X) = (X14) + (X13) + (X12) + (X11) + (X10)

SUBJECTED TO:

G1(X) = 0.002000(X9) + 0.002000(X8) <= 1

G2(X) = 0.001000(X7) + 0.001000(X6) - 0.001000(X9) <= 1

G3(X) = 0.002000(X5) + 0.002000(X4) - 0.002000(X7) <= 1

G4(X) = 0.001400(X3) + 0.001400(X2) - 0.001400(X5) <= 1

G5(X) = 0.003000(X1) - 0.003000(X3) <= 1

G6(X) = 0.010000(X9) + 0.001750(X14) - 1.2E-005(X8)(X14) <= 1

G7(X) = (X7)(X9**-1.00) + 0.001150(X13) - 0.000800(X6)(X9**-1.00)(X13) <= 1

G8(X) = (X5)(X7**-1.00) + 0.000364(X12) - 0.000400(X4)(X7**-1.00)(X12) <= 1

G9(X) = (X3)(X5**-1.00) + 0.000332(X11) - 0.000200(X2)(X5**-1.00)(X11) <= 1

G10(X) = 700.00(X3**-1.00) + 0.000103(X10) - 0.000100(X1)(X3**-1.00)(X10) <= 1

PROBLEM NO 26 [5] No. 7 ·

MINIMIZE :

```
G0(X) =  1.2626(X12) + 1.2626(X13) + 1.2626(X14) + 1.2626(X15)
         + 1.2626(X16) - 1.2311(X1)(X12) - 1.2311(X2)(X13)
         - 1.2311(X3)(X14) - 1.2311(X4)(X15) - 1.2311(X5)(X16)
```

SUBJECTED TO:

```
G1(X) =  0.034750(X1)(X6**-1.00) + 0.975000(X1) - 0.009800(X1** 2.00)(X6**-1.00) <= 1

G2(X) =  0.034750(X2)(X7**-1.00) + 0.975000(X2) - 0.009800(X2** 2.00)(X7**-1.00) <= 1

G3(X) =  0.034750(X3)(X8**-1.00) + 0.975000(X3) - 0.009800(X3** 2.00)(X8**-1.00) <= 1

G4(X) =  0.034750(X4)(X9**-1.00) + 0.975000(X4) - 0.009800(X4** 2.00)(X9**-1.00) <= 1

G5(X) =  0.034750(X5)(X10**-1.00) + 0.975000(X5) - 0.009800(X5** 2.00)(X10**-1.00) <= 1

G6(X) =  (X6)(X7**-1.00) + (X1)(X7**-1.00)(X11**-1.00)(X12) - (X6)(X7**-1.00)(X11**-1.00)(X12) <= 1

G7(X) =  (X7)(X8**-1.00) + 0.002000(X7)(X8**-1.00)(X12)
         + 0.002000(X2)(X8**-1.00)(X13) - 0.002000(X13) - 0.002000(X1)(X8**-1.00)(X12) <= 1

G8(X) =  (X8) + 0.002000(X8)(X13) + 0.002000(X3)(X14) + (X9) - 0.002100(X2)(X13)
         - 0.002100(X9)(X14) <= 1

G9(X) =  (X3**-1.00)(X9) + (X3**-1.00)(X4)(X14**-1.00)(X15) + 500.00(X3**-1.00)(X10)(X14**-1.00)
         - 501.00(X3**-1.00)(X9)(X14**-1.00)
         - (X3**-1.00)(X8)(X14**-1.00)(X15) <= 1

G10(X) =  0.990000(X4**-1.00)(X5)(X15**-1.00)(X16)
         + 0.990000(X4**-1.00)(X10) + 499.00(X15**-1.00)
         - 1.1000(X15**-1.00)(X16) - 501.00(X4**-1.00)(X10)(X15**-1.00) <= 1

G11(X) =  0.880000(X4**-1.00) + 0.002000(X16) - 0.002200(X4**-1.00)(X5)(X16) <= 1

G12(X) =  0.002000(X11) - 0.002000(X12) <= 1

G13(X) =  (X11**-1.00)(X12) <= 1

G14(X) =  (X4)(X5**-1.00) <= 1
```

```
G15(X) =  (X3)(X4**-1.00) <= 1

G16(X) =  (X2)(X3**-1.00) <= 1

G17(X) =  (X1)(X2**-1.00) <= 1

G18(X) =  (X9)(X10**-1.00) <= 1

G19(X) =  (X8)(X9**-1.00) <= 1

G20(X) =  1.1111(X3) <= 1

G21(X) =  1.1111(X4) <= 1

G22(X) =  0.900000(X5**-1.00) <= 1

G23(X) =  10.0000(X6) <= 1

G24(X) =  0.100000(X7**-1.00) <= 1

G25(X) =  500.00(X14**-1.00) <= 1

G26(X) =  500.00(X15**-1.00) <= 1

G27(X) =   1.0E-006(X16**-1.00) <= 1
```

PROBLEM NO 1-1

SOLUTION :

(8.1301, 0.615368, 0.564042, 5.6362)

STARTING POINTS :

NO 1 = (8.1057, 0.628351, 0.364381, 5.6574)

NO 2 = (6.2548, 0.719659, 0.427716, 5.6274)

NO 3 = (6.2207, 0.649534, 0.415851, 5.6541)

NO 4 = (6.5015, 0.573547, 0.400944, 5.6468)

NO 5 = (5.6104, 0.550418, 0.476718, 5.6167)

NO 6 = (8.1632, 0.663106, 0.366412, 5.6526)

NO 7 = (5.4532, 0.659151, 0.494682, 5.6462)

NO 8 = (5.7133, 0.645669, 0.457506, 5.6611)

NO 9 = (6.6075, 0.531670, 0.402689, 5.6361)

NO 10 = (6.5931, 0.734362, 0.413470, 5.6514)

PROBLEM NO 1-2

SOLUTION :

(8.1301, 0.615368, 0.564042, 5.6362)

STARTING POINTS :

NO 1 = (7.7815, 0.634553, 0.522380, 6.6402)

NO 2 = (7.7292, 0.590641, 0.477555, 6.2649)

NO 3 = (7.3389, 0.640407, 0.640926, 6.2081)

NO 4 = (7.2063, 0.668051, 0.616745, 6.2281)

NO 5 = (7.7566, 0.560861, 0.466279, 5.6270)

NO 6 = (7.4947, 0.696034, 0.524641, 6.2492)

NO 7 = (7.7469, 0.670657, 0.497031, 6.3400)

NO 8 = (7.0609, 0.526088, 0.559544, 5.8610)

NO 9 = (7.1573, 0.560656, 0.522456, 6.2624)

NO 10 = (7.7294, 0.685576, 0.540562, 6.4877)

PROBLEM NO 2-1

SOLUTION :

(52.6009, 1.1869, 24.7980, 0.408687)

STARTING POINTS :

NO 1 = (68.2065, 1.6134, 29.0510, 0.384794)

NO 2 = (41.6520, 1.5211, 18.7360, 0.302417)

NO 3 = (39.9813, 1.4775, 29.3659, 0.280381)

NO 4 = (38.4519, 1.5797, 25.2471, 0.302295)

NO 5 = (42.3744, 1.5675, 32.9390, 0.392482)

NO 6 = (46.9631, 1.2794, 20.2188, 0.590697)

NO 7 = (62.9266, 1.5409, 22.0829, 0.544536)

NO 8 = (76.8066, 1.2291, 28.2350, 0.463155)

NO 9 = (39.8044, 1.3076, 33.2485, 0.512354)

NO 10 = (37.6591, 1.4409, 30.5907, 0.515989)

PROBLEM NO 2-2

SOLUTION :

(52.6009, 1.1869, 24.7980, 0.408587)

STARTING POINTS :

NO 1 = (58.8431, 1.3575, 26.4992, 0.399130)

NO 2 = (48.2213, 1.3206, 22.3732, 0.366179)

NO 3 = (47.5531, 1.3031, 26.6251, 0.357365)

NO 4 = (46.9413, 1.3440, 24.9776, 0.366130)

NO 5 = (48.5103, 1.3391, 28.0544, 0.402205)

NO 6 = (50.3458, 1.2239, 22.9663, 0.481491)

NO 7 = (56.7312, 1.3285, 23.7119, 0.463027)

NO 8 = (62.2832, 1.2038, 26.1728, 0.430474)

NO 9 = (47.4823, 1.2352, 28.1782, 0.450154)

NO 10 = (46.6242, 1.2885, 27.1150, 0.451608)

PROBLEM NO 3-1

SOLUTION :

(0.000100, 0.999973, 1.9999, 1.0001)

STARTING POINTS :

NO 1 = (0.000125, 0.926877, 1.2426, 1.1960)

NO 2 = (0.000111, 1.0976, 1.0764, 0.877413)

NO 3 = (0.000113, 0.973347, 1.6504, 0.550537)

NO 4 = (0.000122, 0.724077, 1.5599, 0.722890)

NO 5 = (0.000124, 0.948827, 1.4740, 0.656020)

NO 6 = (0.000137, 1.1670, 1.4386, 0.904726)

NO 7 = (0.000125, 1.3477, 1.6913, 0.794910)

NO 8 = (0.000119, 1.2882, 1.2834, 0.968813)

NO 9 = (0.000127, 1.3852, 1.6942, 1.0519)

NO 10 = (0.000127, 1.3504, 1.9215, 0.773129)

PROBLEM NO 3-2

SOLUTION :

(0.000100, 0.999973, 1.9999, 1.0001)

STARTING POINTS :

NO 1 = (0.000110, 0.970735, 1.6970, 1.0785)

NO 2 = (0.000104, 1.0390, 1.6305, 0.951014)

NO 3 = (0.000105, 0.989323, 1.8601, 0.820264)

NO 4 = (0.000109, 0.889615, 1.8239, 0.889205)

NO 5 = (0.000110, 0.979515, 1.7896, 0.862457)

NO 6 = (0.000115, 1.0668, 1.7754, 0.961940)

NO 7 = (0.000110, 1.1391, 1.8765, 0.918013)

NO 8 = (0.000108, 1.1153, 1.7133, 0.987574)

NO 9 = (0.000111, 1.1540, 1.8776, 1.0208)

NO 10 = (0.000111, 1.1401, 1.9686, 0.909301)

PROBLEM NO 5-1

SOLUTION :

(78.0000, 33.0000, 29.9957, 45.0000, 36.7753)

STARTING POINTS :

NO 1 = (100.50, 35.4915, 38.8361, 37.8710, 28.6243)

NO 2 = (82.0295, 33.3329, 37.8173, 28.0470, 29.6721)

NO 3 = (86.8346, 37.0309, 41.5272, 32.9536, 38.7201)

NO 4 = (90.9677, 40.5841, 40.0037, 35.6493, 32.2644)

NO 5 = (89.7803, 40.4919, 40.4457, 38.4262, 30.1009)

NO 6 = (99.5187, 33.3920, 37.9493, 31.6024, 41.2494)

NO 7 = (78.0605, 33.3289, 39.6590, 29.1210, 31.3703)

NO 8 = (98.0024, 35.5475, 39.5616, 32.8626, 34.4998)

NO 9 = (78.4972, 35.3481, 43.2623, 36.4196, 32.5942)

NO 10 = (86.1427, 38.1545, 39.1833, 31.9787, 29.6865)

PROBLEM NO 5-2

SOLUTION :

(78.0000, 33.0000, 29.9957, 45.0000, 36.7753)

STARTING POINTS :

NO 1 = (87.0003, 33.9966, 33.5319, 42.1484, 33.5149)

NO 2 = (79.6118, 33.1332, 33.1244, 38.2188, 33.9340)

NO 3 = (81.5339, 34.6123, 34.6083, 40.1814, 37.5532)

NO 4 = (82.7121, 35.9968, 34.1757, 42.3705, 34.1055)

NO 5 = (86.6075, 33.1568, 33.1772, 39.6410, 38.5649)

NO 6 = (78.0242, 33.1316, 33.8610, 38.6484, 34.6133)

NO 7 = (86.0010, 34.0190, 33.8221, 40.1451, 35.8651)

NO 8 = (79.7649, 35.8989, 34.0212, 42.9372, 32.8043)

NO 9 = (78.1989, 33.9393, 35.3024, 41.5678, 35.1029)

NO 10 = (88.5190, 34.9989, 31.4040, 40.5922, 33.8459)

PROBLEM NO 6-1

SOLUTION :

(1.5360, 1.0E-008, 3.0000, 0.400000, 6.6670, 8.0000, 149.51)

STARTING POINTS :

NO 1 = (1.5882, 6.9E-009, 3.0337, 0.401476, 6.5773, 7.9927, 108.29)

NO 2 = (1.5719, 7.8E-009, 3.0176, 0.403252, 6.4190, 7.8101, 103.08)

NO 3 = (1.5856, 1.0E-008, 3.0276, 0.402255, 6.3897, 7.7998, 98.1192)

NO 4 = (1.5758, 7.2E-009, 3.0178, 0.403329, 6.5429, 7.8943, 105.18)

NO 5 = (1.5731, 7.6E-009, 3.0286, 0.402185, 6.4691, 7.8173, 103.60)

NO 6 = (1.6752, 7.9E-009, 3.0617, 0.404248, 6.3600, 7.8975, 115.14)

NO 7 = (1.6192, 8.8E-009, 3.1592, 0.401502, 6.5086, 7.8824, 103.69)

NO 8 = (1.6623, 7.4E-009, 3.0355, 0.402302, 6.2152, 7.7183, 123.27)

NO 9 = (1.6117, 6.9E-009, 3.0452, 0.405690, 6.3023, 7.9221, 115.60)

NO 10 = (1.5835, 7.7E-009, 3.0882, 0.401267, 6.5345, 7.8943, 103.14)

PROBLEM NO 6-2

SOLUTION :

(1.5360, 1.0E-008, 3.0000, 0.400000, 6.6670, 8.0000, 149.51)

STARTING POINTS :

NO 1 = (1.5569, 8.8E-009, 3.0135, 0.400590, 6.6311, 7.9931, 133.02)

NO 2 = (1.5504, 9.1E-009, 3.0071, 0.401301, 6.5678, 7.9240, 130.94)

NO 3 = (1.5558, 1.0E-008, 3.0110, 0.400902, 6.5561, 7.9199, 128.95)

NO 4 = (1.5519, 8.9E-009, 3.0071, 0.401332, 6.6173, 7.9577, 131.78)

NO 5 = (1.5508, 9.0E-009, 3.0114, 0.400874, 6.5878, 7.9269, 131.14)

NO 6 = (1.5917, 9.2E-009, 3.0247, 0.401699, 6.5442, 7.9590, 135.76)

NO 7 = (1.5693, 9.5E-009, 3.0637, 0.400601, 6.6037, 7.9530, 131.18)

NO 8 = (1.5865, 9.0E-009, 3.0142, 0.400921, 6.4863, 7.8873, 139.01)

NO 9 = (1.5663, 8.8E-009, 3.0181, 0.402276, 6.5211, 7.9688, 135.94)

NO 10 = (1.5550, 9.1E-009, 3.0353, 0.400507, 6.6140, 7.9577, 130.96)

PROBLEM NO 7-1

SOLUTION :

(0.245485, 6.1970, 8.2726, 0.245488, 1.8E+008, 21.3393, 27.7408)

STARTING POINTS :

NO 1 = (0.283507, 6.8854, 9.2618, 0.287684, 1.4E+008, 23.9525, 34.8307)

NO 2 = (0.282461, 5.9345, 9.0908, 0.320009, 1.8E+008, 21.7391, 33.1972)

NO 3 = (0.275239, 7.4562, 9.0813, 0.278507, 1.4E+008, 21.4388, 35.9301)

NO 4 = (0.286742, 6.2486, 9.2512, 0.301732, 1.5E+008, 23.2965, 34.4532)

NO 5 = (0.301457, 7.0203, 8.0919, 0.304828, 1.6E+008, 18.2918, 32.9435)

NO 6 = (0.299984, 7.1783, 8.1551, 0.301242, 1.5E+008, 22.0897, 33.0936)

NO 7 = (0.294854, 6.4566, 9.5608, 0.299750, 1.8E+008, 22.7052, 35.0714)

NO 8 = (0.290556, 5.6665, 9.2453, 0.300622, 1.5E+008, 24.4893, 32.5990)

NO 9 = (0.312850, 5.9828, 8.2076, 0.316598, 1.7E+008, 19.1277, 28.1352)

NO 10 = (0.275709, 7.6027, 8.0918, 0.311877, 1.7E+008, 18.5721, 32.4813)

PROBLEM NO 7-2

SOLUTION :

(0.245485, 6.1970, 8.2726, 0.245488, 1.8E+008, 21.3393, 27.7408)

STARTING POINTS :

NO 1 = (0.261432, 6.0822, 8.9636, 0.265791, 1.7E+008, 22.2948, 30.6096)

NO 2 = (0.261337, 6.8330, 8.2467, 0.262402, 1.7E+008, 20.9365, 30.3124)

NO 3 = (0.254943, 6.5314, 8.6373, 0.270589, 1.8E+008, 21.7892, 30.7604)

NO 4 = (0.263330, 6.1633, 8.3471, 0.274898, 1.8E+008, 21.4009, 30.2702)

NO 5 = (0.267137, 6.3108, 8.4364, 0.269286, 1.8E+008, 21.9383, 30.9559)

NO 6 = (0.240239, 6.3362, 8.7328, 0.271734, 1.8E+008, 22.7511, 30.5737)

NO 7 = (0.261711, 6.3810, 8.5919, 0.266452, 1.7E+008, 22.9564, 30.0083)

NO 8 = (0.264054, 5.9783, 8.7974, 0.268570, 1.8E+008, 22.2546, 30.6400)

NO 9 = (0.257028, 6.0372, 8.6365, 0.273977, 1.8E+008, 22.2574, 30.2022)

PROBLEM NO 8-2

SOLUTION :

(1699.01, 53.3569, 3031.94, 90.0714, 95.0000, 10.5456, 153.53)

STARTING POINTS :

NO 1 = (1530.54, 52.6886, 2708.00, 90.1608, 95.0000, 11.5003, 153.42)

NO 2 = (1538.12, 55.7571, 2702.05, 90.1291, 95.0000, 11.4783, 153.38)

PROBLEM NO 9-1

SOLUTION :

(6.4810, 2.2175, 0.666969, 0.595832, 5.9300, 5.5271, 1.0092, 0.400467)

STARTING POINTS :

NO 1 = (5.0175, 2.1079, 0.799084, 0.502201, 6.0199, 6.6925, 0.947077, 0.388499)

NO 2 = (5.1677, 2.0075, 0.679073, 0.515244, 6.0351, 6.7144, 0.988460, 0.302427)

NO 3 = (5.0117, 1.7069, 0.658526, 0.633554, 5.9508, 6.2398, 0.814804, 0.384846)

NO 4 = (4.3080, 1.9720, 0.598054, 0.546640, 6.0914, 5.8912, 1.0248, 0.405226)

NO 5 = (5.4415, 2.0320, 0.796953, 0.623089, 6.0175, 6.1605, 0.788410, 0.317377)

NO 6 = (5.5488, 2.5601, 0.629372, 0.717721, 5.8717, 6.5761, 0.856932, 0.476052)

NO 7 = (4.8749, 2.1847, 0.814495, 0.516258, 6.0583, 6.4503, 0.924717, 0.377815)

NO 8 = (5.5205, 1.8535, 0.533308, 0.577560, 6.0660, 6.4712, 0.797346, 0.376559)

NO 9 = (6.2818, 2.2707, 0.734797, 0.531583, 6.0387, 7.0198, 0.814931, 0.340567)

NO 10 = (4.8394, 1.8952, 0.608771, 0.449090, 6.0878, 5.9106, 0.976795, 0.465350)

PROBLEM NO 9-2

SOLUTION :

(6.4810, 2.2175, 0.666969, 0.595832, 5.9300, 5.5271, 1.0092, 0.400467)

STARTING POINTS :

NO 1 = (5.9530, 2.1752, 0.717996, 0.559669, 6.3725, 5.9772, 1.0434, 0.395845)

NO 2 = (5.9900, 2.3890, 0.705763, 0.536767, 6.0274, 5.9755, 1.0128, 0.380027)

NO 3 = (6.3227, 2.1131, 0.605508, 0.595316, 6.0218, 6.0899, 0.956342, 0.433144)

NO 4 = (5.8526, 1.9603, 0.663048, 0.613354, 6.2914, 5.8582, 0.940396, 0.393211)

NO 5 = (5.9683, 2.1781, 0.616980, 0.546914, 6.3918, 5.5302, 0.946361, 0.369038)

NO 6 = (6.3375, 2.1392, 0.721875, 0.607346, 6.4628, 5.7947, 0.941621, 0.365370)

NO 7 = (5.8909, 2.2052, 0.722532, 0.565862, 6.4523, 5.8748, 1.0307, 0.391936)

NO 8 = (6.3208, 2.0758, 0.614925, 0.588718, 6.4829, 5.8947, 0.940173, 0.391158)

NO 9 = (5.9226, 2.1770, 0.705116, 0.560157, 6.0754, 6.0701, 1.0236, 0.432864)

NO 10 = (5.9127, 2.1614, 0.659951, 0.520887, 6.1542, 5.9899, 1.0288, 0.398622)

PROBLEM NO 10-1

SOLUTION :

(574.80, 1364.79, 5109.67, 181.64, 295.61, 218.36, 286.03, 395.61)

STARTING POINTS :

NO 1 = (752.99, 1505.55, 6004.25, 176.15, 293.34, 213.30, 280.07, 386.04)

NO 2 = (722.53, 1608.63, 6404.44, 177.75, 294.16, 215.74, 283.48, 392.55)

NO 3 = (653.54, 1642.10, 6933.35, 177.08, 289.47, 218.23, 287.37, 389.26)

NO 4 = (762.12, 1477.24, 5676.54, 178.51, 286.90, 212.83, 287.83, 380.93)

NO 5 = (718.71, 1682.31, 5665.41, 183.33, 287.65, 215.74, 277.43, 384.99)

NO 6 = (607.48, 1771.52, 6480.63, 180.75, 296.77, 213.89, 279.23, 393.08)

NO 7 = (721.04, 1665.43, 5920.24, 184.57, 289.94, 211.37, 285.03, 389.68)

NO 8 = (574.56, 1488.15, 7223.82, 174.92, 285.22, 219.26, 281.65, 372.07)

NO 9 = (713.87, 1604.23, 6529.52, 181.80, 293.31, 215.04, 280.98, 387.37)

NO 10 = (626.34, 1832.32, 5447.06, 176.70, 297.20, 218.86, 277.06, 396.77)

PROBLEM NO 10-2

SOLUTION :

(574.80, 1364.79, 5109.67, 181.64, 295.61, 218.36, 286.03, 395.61)

STARTING POINTS :

NO 1 = (639.47, 1493.87, 5419.32, 172.71, 291.18, 227.28, 281.24, 389.69)

NO 2 = (576.88, 1520.40, 5880.29, 179.19, 295.23, 218.76, 280.89, 392.41)

NO 3 = (670.75, 1351.61, 5478.78, 177.61, 291.76, 217.86, 285.41, 387.53)

PROBLEM NO 11-1

SOLUTION :

(30.0000, 20.0000, 1.000000, 0.416339, 8.0435, 5.2292, 7.6894, 5.5498)

STARTING POINTS :

NO 1 = (29.9046, 20.4201, 0.813318, 0.376984, 7.5564, 6.5176, 7.8453, 6.5153)

NO 2 = (29.9889, 20.1459, 0.706576, 0.356304, 7.4495, 6.0150, 7.9648, 6.0155)

NO 3 = (29.8050, 20.6638, 0.780375, 0.336272, 7.8523, 6.1912, 7.9352, 6.3889)

NO 4 = (29.7289, 20.6271, 0.831167, 0.362491, 8.7355, 6.6466, 7.9139, 6.1502)

NO 5 = (29.9196, 20.0826, 0.747964, 0.325960, 7.5908, 5.7924, 7.9065, 6.5143)

NO 6 = (29.9138, 20.3698, 0.784867, 0.281819, 8.9610, 5.8803, 7.7622, 6.0680)

NO 7 = (29.5380, 20.5802, 0.755409, 0.517750, 7.3934, 6.0694, 7.7616, 6.0825)

NO 8 = (29.7658, 20.4716, 0.843403, 0.358561, 7.7600, 6.4206, 7.9081, 6.6864)

NO 9 = (29.7998, 20.3500, 0.855039, 0.327134, 9.8411, 5.6459, 7.8052, 6.8423)

NO 10 = (29.8622, 20.5159, 0.977180, 0.332574, 9.4592, 6.1633, 7.8601, 6.9010)

PROBLEM NO 11-2

SOLUTION :

(30.0000, 20.0000, 1.000000, 0.416339, 8.0435, 5.2292, 7.6894, 5.5498)

STARTING POINTS :

NO 1 = (29.9203, 20.1512, 0.910265, 0.383625, 7.9654, 5.6222, 7.7298, 5.8926)

NO 2 = (29.9094, 20.1417, 0.945025, 0.410216, 7.6388, 5.6910, 7.6786, 6.0923)

NO 3 = (29.8662, 20.2234, 0.917054, 0.416298, 7.5987, 5.3888, 7.6858, 6.1646)

NO 4 = (29.9041, 20.2446, 0.931523, 0.424867, 8.0743, 5.8155, 7.7883, 5.8727)

NO 5 = (29.9396, 20.1162, 0.891552, 0.394634, 7.9113, 5.2899, 7.7587, 6.0457)

NO 6 = (29.9135, 20.0894, 0.902909, 0.362024, 8.3745, 5.4479, 7.6946, 5.6693)

NO 7 = (29.8816, 20.1310, 0.946223, 0.376730, 8.2800, 5.7197, 7.7494, 5.9177)

NO 8 = (29.8885, 20.1157, 0.886221, 0.417565, 8.3800, 5.4367, 7.7190, 5.9910)

NO 9 = (29.9241, 20.0952, 0.925786, 0.442552, 7.6010, 5.6279, 7.7067, 6.0058)

NO 10 = (29.8997, 20.1858, 0.935117, 0.434939, 8.8731, 5.4002, 7.6871, 6.0942)

PROBLEM NO 12-1

SOLUTION :

(17.8264, 1.0E-008, 1.2323, 3.2032, 40.0000, 0.987081, 24.6453, 6.6670, 8.0000)

STARTING POINTS :

NO 1 = (17.7765, 8.8E-009, 1.6432, 3.0061, 40.0157, 0.960220, 24.3516, 6.4257, 7.8299)

PROBLEM NO 12-2

SOLUTION :

(17.8264, 1.0E-008, 1.2323, 3.2032, 40.0000, 0.987081, 24.6453, 6.6670, 8.0000)

STARTING POINTS :

NO 1 = (19.2499, 1.1E-008, 1.3467, 3.0886, 40.0116, 0.995226, 24.8031, 6.6263, 7.9637)

NO 2 = (19.4109, 9.1E-009, 1.3051, 3.0154, 40.1442, 0.979916, 24.8986, 6.5824, 7.9309)

NO 3 = (19.5246, 1.0E-008, 1.3417, 3.0069, 40.2178, 0.987671, 24.6850, 6.6064, 7.9412)

NO 4 = (19.6469, 9.9E-009, 1.3314, 3.0161, 40.0262, 0.996480, 24.6479, 6.5715, 7.9927)

NO 5 = (19.6378, 9.5E-009, 1.3041, 3.0049, 40.6374, 0.985578, 25.0366, 6.6337, 7.9990)

NO 6 = (19.9444, 9.6E-009, 1.2255, 3.0115, 40.3502, 0.982768, 24.3216, 6.5634, 7.9169)

NO 7 = (19.8493, 9.7E-009, 1.3270, 3.0711, 40.1777, 0.984030, 24.4706, 6.6130, 7.9694)

NO 8 = (19.4276, 1.0E-008, 1.3514, 3.0249, 40.1484, 0.990211, 24.6132, 6.6059, 7.9859)

NO 9 = (18.5611, 1.1E-008, 1.3400, 3.0880, 40.3801, 0.977291, 24.7013, 6.5662, 7.9886)

NO 10 = (19.7258, 9.0E-009, 1.2382, 3.0209, 40.1159, 0.980227, 24.3241, 6.6254, 7.9711)

PROBLEM NO 13-1

SOLUTION :

(57.6923, 34.1476, 57.6923, 1.0500, 104.19, 46.5010, 32.6923, 46.5010, 30.5203)

STARTING POINTS :

NO 1 = (58.9067, 32.9226, 56.5912, 1.3583, 87.7983, 45.6421, 32.0687, 47.9642, 31.5034)

NO 2 = (58.5523, 34.4512, 57.1795, 1.1811, 70.3982, 45.0163, 32.5326, 48.2006, 31.0288)

NO 3 = (59.8054, 36.0838, 57.6802, 1.3405, 94.0966, 43.6916, 32.4197, 47.1088, 31.4522)

NO 4 = (59.8708, 33.1053, 56.4510, 1.3407, 84.0568, 45.3695, 33.3511, 46.7462, 31.1486)

NO 5 = (58.9144, 33.5194, 57.3664, 1.3662, 84.3079, 45.7239, 32.6248, 46.6473, 31.3681)

NO 6 = (57.2811, 33.8340, 56.6057, 1.3570, 84.8876, 44.7901, 31.7143, 45.5124, 30.3585)

NO 7 = (59.8326, 33.8312, 56.9527, 1.3877, 93.8653, 45.1650, 31.3129, 46.4069, 30.9239)

NO 8 = (59.3187, 35.2483, 59.5257, 1.2109, 74.1607, 47.8050, 31.8593, 48.0410, 31.0029)

NO 9 = (55.7180, 33.9708, 57.7023, 1.4078, 99.7801, 46.0123, 31.7312, 47.9576, 31.1371)

NO 10 = (58.1089, 33.1897, 55.9499, 1.3361, 83.0471, 44.8339, 31.9253, 45.6980, 30.8671)

PROBLEM NO 13-2

SOLUTION :

(57.6923, 34.1476, 57.6923, 1.0500, 104.19, 46.5010, 32.6923, 46.5010, 30.5203)

STARTING POINTS :

NO 1 = (56.6722, 35.7528, 58.4369, 1.1143, 94.1526, 46.0833, 32.0307, 48.0737, 30.7377)

NO 2 = (57.8992, 35.6660, 58.1578, 1.1807, 103.21, 45.7836, 32.5498, 48.5147, 30.8722)

NO 3 = (56.2808, 36.0506, 58.4706, 1.1032, 94.9450, 46.0082, 32.0995, 48.3862, 30.6385)

NO 4 = (57.2510, 35.4277, 58.3429, 1.1429, 100.82, 45.7911, 31.7561, 47.6668, 31.3719)

NO 5 = (57.5982, 33.8818, 58.7557, 1.1403, 95.7659, 47.2409, 32.1105, 48.6787, 30.8854)

NO 6 = (57.3298, 34.1492, 57.9[illegible]11, 1.1715, 96.0602, 46.3063, 32.4609, 48.2191, 30.7998)

NO 7 = (58.8491, 35.2363, 58.9637, 1.1575, 99.0499, 45.7137, 32.8700, 47.3196, 30.9586)

NO 8 = (57.7223, 34.4269, 57.8394, 1.2015, 102.84, 46.2782, 32.7084, 47.2173, 30.8242)

NO 9 = (57.9579, 33.9261, 57.7951, 1.1566, 93.5229, 46.1033, 32.7807, 46.8569, 30.8175)

NO 10 = (57.3232, 34.2993, 58.4439, 1.1511, 94.5383, 47.0958, 32.4643, 48.2202, 30.7881)

PROBLEM NO 14-1

SOLUTION :

(11.7660, 0.371346, 0.342312, 12.2734, 14.2344, 0.400000, 38.3594, 0.719656, 0.130695)

STARTING POINTS :

NO 1 = (11.6031, 0.272973, 0.400766, 12.5913, 14.3238, 0.418139, 41.5309, 0.732878, 0.137098)

NO 2 = (11.5949, 0.303689, 0.377265, 11.9999, 13.7886, 0.425037, 45.6996, 0.757215, 0.136657)

NO 3 = (11.5315, 0.293268, 0.407190, 12.4605, 13.8009, 0.405423, 41.2724, 0.766394, 0.138342)

NO 4 = (11.8989, 0.273110, 0.362774, 12.3646, 13.7709, 0.403951, 46.4365, 0.744475, 0.134899)

NO 5 = (11.8169, 0.346776, 0.418303, 12.1494, 13.7972, 0.416651, 47.8849, 0.738932, 0.133668)

NO 6 = (11.6754, 0.256354, 0.353487, 12.1755, 14.0045, 0.411820, 44.8922, 0.731053, 0.133870)

NO 7 = (11.6030, 0.317193, 0.406245, 12.2783, 14.5157, 0.400389, 48.9057, 0.728756, 0.132975)

NO 8 = (11.8438, 0.301142, 0.373331, 12.2411, 14.0348, 0.406035, 49.7014, 0.736292, 0.132025)

NO 9 = (11.8091, 0.308244, 0.421226, 12.5948, 14.2612, 0.400001, 46.5934, 0.721742, 0.134824)

NO 10 = (11.8944, 0.274492, 0.407973, 12.3616, 13.9607, 0.407146, 43.9402, 0.737737, 0.133902)

PROBLEM NO 14-2

SOLUTION :

(11.7660, 0.371346, 0.342312, 12.2734, 14.2344, 0.400000, 38.3594, 0.719656, 0.130695)

STARTING POINTS :

NO 1 = (11.8202, 0.331271, 0.350659, 12.3106, 14.1010, 0.401612, 41.6544, 0.729780, 0.132410)

NO 2 = (11.6267, 0.364469, 0.385829, 12.2366, 14.3226, 0.404812, 40.2729, 0.725684, 0.132428)

NO 3 = (11.6222, 0.349662, 0.350410, 12.2958, 14.3614, 0.402612, 43.1436, 0.722622, 0.132648)

NO 4 = (11.6630, 0.346663, 0.361218, 12.3157, 14.3286, 0.400536, 42.0408, 0.733716, 0.133581)

PROBLEM NO 15-1

SOLUTION :

(0.724310, 0.723675, 0.724185, 0.257577, 0.177137, 0.121792, 0.205240, 0.141145, 0.097045, 0.298522)

STARTING POINTS :

NO 1 = (0.885143, 0.898727, 0.701987, 0.251369, 0.173597, 0.125460, 0.203351, 0.144152, 0.107233, 0.306945)

NO 2 = (0.847967, 0.925640, 0.825330, 0.253377, 0.171897, 0.121671, 0.202682, 0.141248, 0.102269, 0.302391)

NO 3 = (0.811277, 0.868714, 0.868784, 0.252688, 0.173085, 0.120379, 0.210527, 0.140165, 0.113611, 0.306958)

NO 4 = (0.782414, 0.866258, 0.926057, 0.254303, 0.180736, 0.118876, 0.211617, 0.144361, 0.096973, 0.299525)

NO 5 = (0.856169, 0.876012, 0.880297, 0.253798, 0.180010, 0.123228, 0.204776, 0.144632, 0.103387, 0.304918)

NO 6 = (0.826233, 0.828637, 0.935552, 0.249805, 0.174911, 0.121709, 0.210384, 0.140583, 0.097922, 0.302780)

NO 7 = (0.939807, 0.824469, 0.813875, 0.253083, 0.176404, 0.123863, 0.204196, 0.145098, 0.102201, 0.306165)

NO 8 = (0.925514, 0.736728, 0.885658, 0.251061, 0.174433, 0.124138, 0.201883, 0.142942, 0.099368, 0.301411)

NO 9 = (0.867507, 0.896304, 0.852750, 0.258599, 0.177316, 0.122037, 0.210377, 0.143764, 0.102638, 0.302967)

NO 10 = (0.847150, 0.879610, 0.878932, 0.253062, 0.174458, 0.119477, 0.209721, 0.139915, 0.098055, 0.310303)

PROBLEM NO 15-2

SOLUTION :

(0.724310, 0.723675, 0.724185, 0.257577, 0.177137, 0.121792, 0.205240, 0.141145, 0.097045,
0.298522)

STARTING POINTS :

NO 1 = (0.800602, 0.795195, 0.746508, 0.249655, 0.173970, 0.112559, 0.204009, 0.140455, 0.101273,
0.302040)

NO 2 = (0.798166, 0.798868, 0.747273, 0.241045, 0.166837, 0.120657, 0.206634, 0.141999, 0.100567,
0.301148)

NO 3 = (0.810508, 0.775783, 0.777489, 0.250296, 0.165424, 0.116225, 0.204962, 0.142273, 0.098811,
0.301150)

NO 4 = (0.794393, 0.810902, 0.652504, 0.258428, 0.174113, 0.123130, 0.206741, 0.140477, 0.101210,
0.302097)

NO 5 = (0.821059, 0.789757, 0.766432, 0.253334, 0.172824, 0.114974, 0.207392, 0.142241, 0.098591,
0.301217)

NO 6 = (0.808189, 0.775342, 0.746325, 0.238810, 0.170457, 0.114329, 0.206155, 0.141290, 0.099063,
0.300627)

NO 7 = (0.804499, 0.757050, 0.790558, 0.257872, 0.173470, 0.111330, 0.205957, 0.142077, 0.099405,
0.302036)

NO 8 = (0.806478, 0.806894, 0.743752, 0.239014, 0.173142, 0.117592, 0.204745, 0.140699, 0.097075,
0.300608)

NO 9 = (0.801489, 0.810308, 0.675563, 0.255067, 0.169238, 0.118907, 0.204216, 0.141145, 0.101231,
0.302012)

NO 10 = (0.807950, 0.739402, 0.812145, 0.250976, 0.175483, 0.120921, 0.204556, 0.140637, 0.103286,
0.302986)

PROBLEM NO 16-1

SOLUTION :

(2.0952, 12.0952, 7.9048, 0.459381, 0.357935, 0.454747, 10.4547, 1.6405, 1.1975, 0.100000)

STARTING POINTS :

NO 1 = (2.1540, 11.8235, 8.0440, 0.540846, 0.339613, 0.516851, 10.1317, 1.6711, 1.5143, 0.099980)

NO 2 = (2.1645, 11.9773, 7.8813, 0.583513, 0.358778, 0.532515, 10.1133, 1.6526, 1.3498, 0.096613)

NO 3 = (2.1436, 12.0699, 7.9221, 0.509337, 0.321325, 0.605544, 10.1927, 1.6796, 1.2208, 0.099819)

NO 4 = (2.0679, 11.9307, 7.9958, 0.591329, 0.388459, 0.519907, 10.1137, 1.6302, 1.3527, 0.097010)

NO 5 = (2.0824, 12.0195, 7.9573, 0.566162, 0.410826, 0.514167, 10.2230, 1.6522, 1.4671, 0.099657)

NO 6 = (2.0794, 11.9641, 8.0008, 0.577907, 0.395934, 0.444950, 10.1410, 1.6544, 1.4676, 0.096622)

NO 7 = (2.0765, 11.9925, 7.9853, 0.591599, 0.403974, 0.534048, 10.1633, 1.6614, 1.2611, 0.096762)

NO 8 = (2.0733, 11.8203, 8.1261, 0.584185, 0.440265, 0.517903, 10.1416, 1.6605, 1.2429, 0.099188)

NO 9 = (2.0987, 11.9102, 7.9132, 0.584064, 0.433543, 0.493334, 10.2393, 1.6170, 1.3919, 0.098671)

NO 10 = (2.1205, 11.8943, 7.9855, 0.541232, 0.308966, 0.536208, 10.1381, 1.5982, 1.4648, 0.098102)

PROBLEM NO 16-2

SOLUTION :

(2.0952, 12.0952, 7.9048, 0.459381, 0.357935, 0.454747, 10.4547, 1.6405, 1.1975, 0.100000)

STARTING POINTS :

NO 1 = (2.0830, 12.0193, 7.9331, 0.488574, 0.392031, 0.475835, 10.3626, 1.6272, 1.3236, 0.097835)

NO 2 = (2.1251, 11.9678, 7.9687, 0.512452, 0.342468, 0.498969, 10.3244, 1.6421, 1.2376, 0.097539)

NO 3 = (2.1117, 12.0705, 7.9009, 0.492736, 0.335943, 0.502195, 10.3660, 1.6579, 1.2464, 0.094423)

NO 4 = (2.1017, 12.0165, 7.9515, 0.515554, 0.395673, 0.474221, 10.3099, 1.6422, 1.2777, 0.099861)

NO 5 = (2.1068, 12.0463, 7.9424, 0.489870, 0.367188, 0.490971, 10.3794, 1.6272, 1.2956, 0.092790)

PROBLEM NO 17-1

SOLUTION :

(0.731055, 0.712511, 0.702707, 0.265162, 0.182000, 0.124031, 0.197735, 0.132896, 0.089318, 0.294659)

STARTING POINTS :

NO 1 = (0.893386, 0.884862, 0.681166, 0.258771, 0.178363, 0.127766, 0.195915, 0.135728, 0.098694, 0.302972)

NO 2 = (0.855864, 0.911360, 0.800852, 0.260838, 0.1766[illegible]6, 0.123908, 0.195271, 0.132994, 0.094126, 0.298477)

NO 3 = (0.818833, 0.855313, 0.843017, 0.260129, 0.177837, 0.122592, 0.202829, 0.131974, 0.104564, 0.302985)

NO 4 = (0.789701, 0.852895, 0.898591, 0.261792, 0.185698, 0.121061, 0.203879, 0.135924, 0.089251, 0.295648)

NO 5 = (0.864143, 0.862497, 0.854188, 0.261271, 0.184952, 0.125493, 0.197288, 0.136180, 0.095155, 0.300971)

NO 6 = (0.833928, 0.815854, 0.907804, 0.257161, 0.179713, 0.123946, 0.203269, 0.132367, 0.090125, 0.298861)

NO 7 = (0.948560, 0.811750, 0.789736, 0.260536, 0.181246, 0.126140, 0.196730, 0.136618, 0.094064, 0.302202)

NO 8 = (0.934134, 0.725362, 0.859390, 0.258454, 0.179222, 0.126420, 0.194501, 0.134589, 0.091456, 0.297510)

NO 9 = (0.875586, 0.882477, 0.827458, 0.266214, 0.182184, 0.124280, 0.202684, 0.135362, 0.094465, 0.299046)

NO 10 = (0.855040, 0.866040, 0.852864, 0.260514, 0.179248, 0.121674, 0.202053, 0.131739, 0.090247, 0.306287)

PROBLEM NO 17-2

SOLUTION :

(0.731055, 0.712511, 0.702707, 0.265162, 0.182000, 0.124031, 0.197735, 0.132896, 0.089318
0.294659)

STARTING POINTS :

NO 1 = (0.808058, 0.782928, 0.724368, 0.257006, 0.178746, 0.114628, 0.196549, 0.132246, 0.093209
0.298131)

NO 2 = (0.805598, 0.786544, 0.725109, 0.248143, 0.171418, 0.122875, 0.199078, 0.133700, 0.092558
0.297250)

NO 3 = (0.818056, 0.763815, 0.754429, 0.257667, 0.169966, 0.118361, 0.197467, 0.133958, 0.090943,
0.297252)

NO 4 = (0.786672, 0.791415, 0.781939, 0.254536, 0.175525, 0.123532, 0.196299, 0.134643, 0.091809,
0.296316)

NO 5 = (0.801792, 0.798392, 0.633151, 0.266038, 0.176893, 0.125393, 0.199181, 0.132268, 0.093152,
0.298187)

NO 6 = (0.828705, 0.777574, 0.743701, 0.260794, 0.177569, 0.116986, 0.199809, 0.133928, 0.090741,
0.297319)

NO 7 = (0.815716, 0.763381, 0.724190, 0.245842, 0.175136, 0.116431, 0.198616, 0.133033, 0.091176,
0.296736)

NO 8 = (0.846754, 0.736165, 0.701488, 0.249112, 0.179053, 0.119138, 0.196054, 0.131068, 0.091556,
0.295850)

NO 9 = (0.811991, 0.745371, 0.767111, 0.2[illegible]5466, 0.179232, 0.113377, 0.198426, 0.133774, 0.091490,
0.298127)

NO 10 = (0.813989, 0.794446, 0.721694, 0.246052, 0.177895, 0.119753, 0.197258, 0.132476, 0.089345,
0.296717)

PROBLEM NO 18-3

SOLUTION :

(1766.29, 18676., 96.9866, 3085.58, 1999.42, 92.0220, 95.0000, 11.7051, 2.0422, 152.15)

STARTING POINTS :

NO 1 = (1783.14, 19031., 98.0549, 2931.33, 1909.74, 92.0062, 94.9048, 11.6694, 2.0801, 152.89)

NO 2 = (1781.02, 18653., 100.87, 2937.52, 1952.36, 92.0205, 94.8560, 11.1183, 2.0304, 152.93)

PROBLEM NO 19-1

SOLUTION :

(0.804085, 0.899972, 0.991282, 0.100000, 0.190423, 0.900000, 538.84, 36.9856, 500.00,
0.100000)

STARTING POINTS :

NO 1 = (0.797310, 0.900011, 0.920699, 0.098460, 0.193580, 0.882403, 440.21, 44.6834, 599.85,
0.111278)

PROBLEM NO 19-2

SOLUTION :

(0.804085, 0.899972, 0.991282, 0.100000, 0.190423, 0.900000, 538.84, 36.9866, 500.00,
0.100000)

STARTING POINTS :

NO 1 = (0.801375, 0.839987, 0.963049, 0.099384, 0.191686, 0.892961, 499.39, 40.0653, 539.94,
0.104511)

NO 2 = (0.796980, 0.900433, 0.995813, 0.099710, 0.192626, 0.895091, 508.11, 40.8669, 526.53,
0.105950)

NO 3 = (0.801537, 0.900152, 0.914192, 0.099522, 0.190839, 0.895719, 489.77, 37.9526, 521.81,
0.108639)

NO 4 = (0.796544, 0.900126, 0.929714, 0.099005, 0.190821, 0.891303, 484.00, 38.2368, 535.02,
0.104370)

NO 5 = (0.799779, 0.900187, 0.907476, 0.099255, 0.191440, 0.898918, 494.82, 38.4526, 537.72,
0.105919)

PROBLEM NO 20-3

SOLUTION :

(7.0037, 7.6458, 7.3183, 0.012445, 0.811659, 0.955586, 0.381392, 0.358090, 0.352934,
2.0764, 0.452995)

STARTING POINTS :

NO 1 = (6.9661, 7.6257, 7.0708, 0.012733, 0.807539, 0.954640, 0.379406, 0.379690, 0.354385,
2.0791, 0.452616)

PROBLEM NO 21-3

SOLUTION :

(0.392000, 0.094000, 1.0E-005, 0.481000, 0.643000, 0.025132, 0.007000, 0.022006, 0.553037,
1.3800, 2.3800, 0.123000, 0.335000)

STARTING POINTS :

NO 1 = (0.392174, 0.094083, 9.9E-006, 0.483292, 0.638449, 0.025015, 0.006901, 0.021984, 0.529723,
1.3784, 2.3779, 0.118818, 0.317082)

AD-A098 245 PURDUE UNIV LAFAYETTE IN SCHOOL OF CHEMICAL ENGINEERING F/G 12/1
INVESTIGATION OF THE COMPUTATIONAL UTILITY OF GEOMETRIC PROGRAM--ETC(U)
JAN 81 G V REKLAITIS N00014-76-C-0551

UNCLASSIFIED NL

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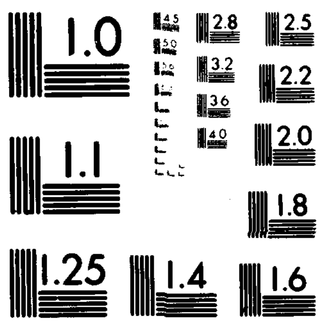

MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS 1963 A

PROBLEM NO 22-2

SOLUTION :

(13115., 36471., 3212.00, 112.19, 370699., 31.5088, 1.1E-008, 47399., 146959., 7794.88, 19327., 362648., 14563.)

STARTING POINTS :

NO 1 = (11899., 40037., 2918.34, 113.33, 374387., 33.0766, 1.1E-008, 46201., 147286., 7770.63, 19823., 362820., 14539.)

NO 2 = (12774., 40095., 3368.11, 113.19, 378852., 34.7106, 1.1E-008, 44339., 147742., 7807.66, 19729., 366445., 14446.)

PROBLEM NO 23-1

SOLUTION :

(5154.08, 6.6200, 169485., 743.89, 87999., 189.54, 0.125831, 29.2653)

STARTING POINTS :

NO 1 = (6403.17, 6.6254, 168985., 887.56, 79862., 223.87, 0.104181, 28.9273)

NO 2 = (5806.06, 6.7472, 168289., 921.16, 81850., 206.44, 0.095633, 29.7279)

NO 3 = (6811.50, 6.6364, 171127., 890.93, 78086., 222.21, 0.118323, 29.6090)

NO 4 = (5015.05, 6.5385, 169885., 856.75, 108247., 242.42, 0.122467, 28.3349)

NO 5 = (5943.29, 6.5904, 165565., 885.96, 103255., 236.82, 0.117769, 29.8528)

NO 6 = (5531.11, 6.7031, 172647., 865.98, 73258., 236.82, 0.105344, 28.9416)

NO 7 = (6040.07, 6.6350, 171116., 794.10, 103261., 237.98, 0.102906, 28.8511)

NO 8 = (6123.78, 6.5865, 172906., 814.59, 98530., 229.34, 0.096418, 29.2088)

NO 9 = (6110.92, 6.7922, 167735., 868.28, 98339., 233.35, 0.106042, 28.9788)

NO 10 = (6364.69, 6.5230, 166074., 784.07, 108436., 227.84, 0.103923, 29.2060)

PROBLEM NO 23-2

SOLUTION :

(5154.08, 6.6200, 169485., 743.89, 87999., 189.54, 0.125831, 29.2653)

STARTING POINTS :

NO 1 = (5590.88, 6.6036, 169295., 793.15, 96443., 213.25, 0.121369, 29.5905)

NO 2 = (5690.28, 6.5770, 169765., 707.70, 97051., 202.92, 0.116128, 29.2390)

NO 3 = (5703.28, 6.5897, 172564., 804.27, 95168., 211.25, 0.122959, 29.4826)

NO 4 = (5213.21, 6.6090, 171391., 823.52, 86300., 201.12, 0.111734, 29.5902)

NO 5 = (5492.60, 6.5876, 180109., 842.34, 81416., 200.74, 0.125415, 28.8854)

NO 6 = (5661.00, 6.7165, 168486., 800.56, 83882., 187.91, 0.113152, 28.8940)

NO 7 = (5768.59, 6.6125, 169203., 818.79, 95808., 201.67, 0.120203, 29.2225)

NO 8 = (5618.18, 6.6137, 178747., 829.11, 91094., 197.42, 0.116461, 29.4211)

NO 9 = (5602.61, 6.7064, 181492., 795.94, 94296., 198.74, 0.117106, 29.0616)

NO 10 = (5635.48, 6.5511, 168006., 780.37, 87094., 208.71, 0.113898, 29.4058)

X

PROBLEM NO 24-3

SOLUTION :

(0.804050, 0.901863, 0.997200, 0.100000, 0.193638, 0.899983, 552.30, 72.9389, 500.00,
0.100001, 19.8948, 76.1710, 0.003497)

STARTING POINTS :

NO 1 = (0.800259, 0.900012, 0.933849, 0.099325, 0.193836, 0.883556, 552.30, 73.8278, 501.65,
0.100182, 20.6268, 80.9022, 0.015613)

NO 2 = (0.803022, 0.900686, 0.925804, 0.099558, 0.193080, 0.879287, 562.39, 72.6221, 501.86,
0.100089, 21.0665, 78.2407, 0.016072)

PROBLEM NO 25-1

SOLUTION :

(679.40, 532.66, 346.07, 435.55, 164.45, 1000.00, 100.00, 1.0E-005, 100.0000,
10959., 3502.59, 467.63, 1.0E-005, 1.0E-005)

STARTING POINTS :

NO 1 = (666.44, 516.68, 346.74, 444.37, 151.64, 906.60, 98.4436, 8.5E-006, 99.3388,
13721., 3705.64, 512.85, 8.2E-006, 1.1E-005)

NO 2 = (670.55, 524.52, 349.69, 423.67, 164.32, 973.69, 96.8481, 7.5E-006, 99.7027,
12051., 4085.23, 567.92, 9.1E-006, 1.0E-005)

NO 3 = (667.97, 524.11, 349.70, 437.67, 160.96, 794.11, 99.0721, 1.2E-005, 99.8687,
12198., 4012.98, 480.44, 1.2E-005, 8.4E-006)

NO 4 = (666.17, 534.26, 336.41, 428.09, 168.70, 976.11, 98.6953, 9.5E-006, 98.7308,
12342., 3751.45, 574.99, 1.2E-005, 7.8E-006)

NO 5 = (668.07, 531.21, 339.13, 428.60, 164.61, 868.02, 98.0965, 1.2E-005, 98.6388,
12497., 4304.11, 504.70, 8.6E-006, 1.1E-005)

NO 6 = (666.14, 518.90, 336.79, 432.89, 142.93, 962.08, 97.4475, 1.1E-005, 98.6568,
12799., 3519.78, 526.08, 1.3E-005, 1.2E-005)

NO 7 = (666.15, 520.44, 346.33, 424.97, 157.70, 870.94, 97.4398, 9.6E-006, 99.8639,
13445., 3957.99, 496.69, 7.9E-006, 1.1E-005)

PROBLEM NO 25-2

SOLUTION :

(679.40, 532.66, 346.07, 435.55, 164.45, 1000.00, 100.00, 1.0E-005, 100.0000,
10959., 3502.59, 467.63, 1.0E-005, 1.0E-005)

STARTING POINTS :

NO 1 = (676.31, 527.55, 343.05, 426.77, 162.95, 890.24, 99.3425, 9.4E-006, 99.8500,
11300., 3590.86, 509.72, 1.0E-005, 9.8E-006)

NO 2 = (674.38, 530.26, 344.92, 411.45, 161.69, 990.23, 98.1455, 1.1E-005, 99.5782,
11971., 3692.86, 499.60, 9.8E-006, 1.1E-005)

PROBLEM NO 26-3

SOLUTION :

(0.803772, 0.817513, 0.900000, 0.900000, 0.900000, 0.100000, 0.107884, 0.190837, 0.190837,
0.190837, 505.66, 5.6651, 72.4752, 500.00, 500.00, 1.0E-006)

STARTING POINTS :

NO 1 = (0.774701, 0.802429, 0.888789, 0.899360, 0.901714, 0.099219, 0.107788, 0.190048, 0.190172,
0.194380, 501.10, 5.5035, 70.9103, 515.84, 500.66, 1.0E-006)

APPRNDIX C: Code Parameters

OPT:	EPSIS	10^{-6}
	EPSBD	10^{-4}
	EPS	10^{-6}
	CRIT	10^{-4}
GGP:	EPSCON	10^{-8}
	EPSCGP	10^{-6}
	EPSLP	10^{-11}
	EPSPN	10^{-11}
GPKTC:	EPSCON	10^{-4}
	EPSDO	10^{-3}
	BETA	10^{-2}
	BS	1.0
QUADGP:	EPSCOV	10^{-5}
	EPSTOL	10^{-3}
	VELTOL	10^{-7}
	EPS	10^{-3}
	TOLCON	10^{-6}
	EPSEQ	10^{-6}
	EPSEQ	10^{-6}
	EPSVAR	10^{-3}
	GENTOL	10^{-8}
	RLOWR	10^{-1}
	RUPPER	10^{+1}
DAP:	EPS	10^{-6}
	EPSI	10^{-4}
	EPS2	10^{-4}
	EPS3	10^{-6}
	EPS4	10^{-4}
	RRR	10^{4}