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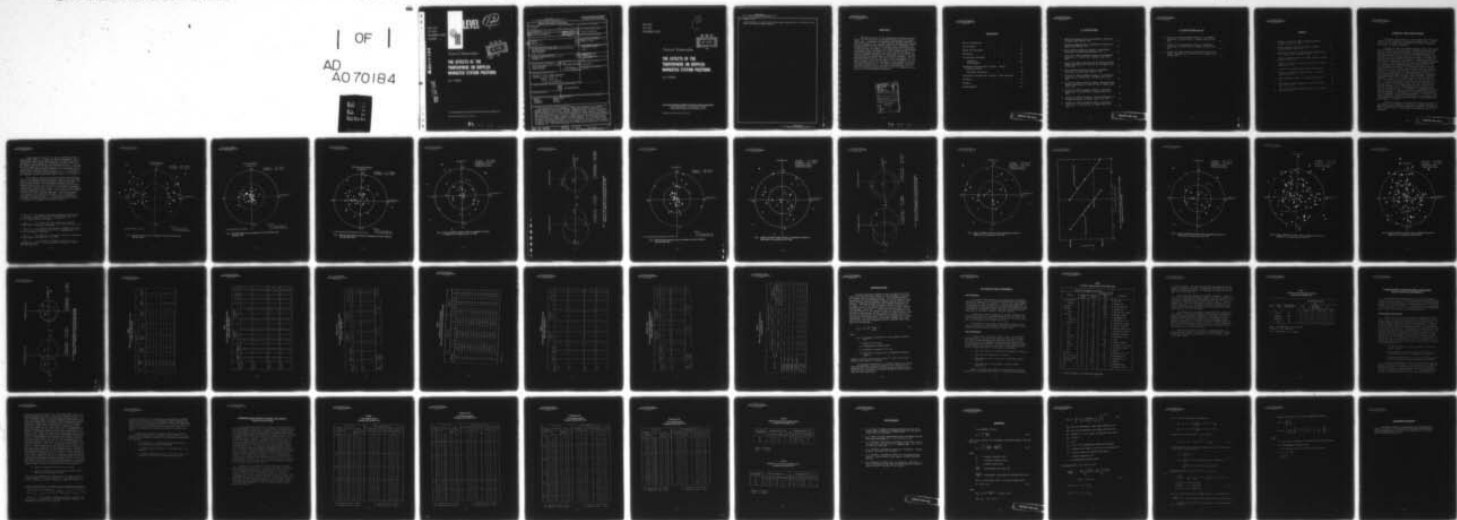
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*Technical Memorandum*

# THE EFFECTS OF THE TROPOSPHERE ON DOPPLER- NAVIGATED STATION POSITIONS

by A. EISNER

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by A. EISNER

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Johns Hopkins Road, Laurel, Maryland 20810  
Operating under Contract N00024-78-C-5384 with the Department of the Navy

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20. ABSTRACT (Cont'd)

in mean latitude, a 0.7 m shift in the mean navigated longitude, and a 4.7 m reduction in the scatter of the mean navigated longitude.

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## ABSTRACT

Ignoring the effect of the troposphere on Doppler data introduces 35 to 40 m errors in the navigated longitude of the Transit system user. Eliminating low elevation data (AN/BRN-3 strategy) reduces the errors by 70%. The BRN-3 software was modified to account for the troposphere using Black's analytic form of Hopfield's tropospheric model. Two sets of fixes were done at the APL/JHU site. In the first set (39 passes), it was demonstrated that the neglected tropospheric effect had masked a 9 m error in station radius. For the non-troposphere-corrected case, the second set (115 passes) resulted in navigated longitudes for passes east and west of the station clustered on opposite sides of the "true" longitude. The 29 m separation between the east and west clusters was reduced to 6.7 m by correcting the data for the effects of the troposphere. For the BRN-3 navigator at sea this represents, on the average, an 11 m reduction in longitude error. The implied consequences to the fixed site BRN-3 surveyor are a 2.2 m shift in mean latitude, a 0.7 m shift in the mean navigated longitude, and a 4.7 m reduction in the scatter of the mean navigated longitude.

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## CONTENTS

List of Illustrations . . . . .	7
List of Tables . . . . .	9
Summary and Conclusions . . . . .	11
Introduction . . . . .	34
The Simulation Experiment . . . . .	35
Background . . . . .	35
The Experiment . . . . .	35
Tropospheric-Corrected BRN-3 Software: Initial Navigation Results . . . . .	39
Experiment Description . . . . .	39
Tropospheric-Corrected BRN-3 Software: Final Evaluation . . . . .	42
References . . . . .	49
Appendix . . . . .	51
Acknowledgments . . . . .	55

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## ILLUSTRATIONS

1	Simulated navigation runs, no tropospheric correction, 39 passes, days 268-269, 1970 . . . . .	13
2	Simulated navigation runs, tropospheric correction, 39 passes, days 268-269, 1970 . . . . .	14
3	BRN-3 simulated navigation results, no tropospheric correction, 39 passes, days 268-269, 1970 . . . . .	15
4	Station 110, BRN-3 navigation results, no tropospheric correction, R = 6369.757 km, 31 passes, days 41-55, 1976 . . . . .	16
5	Station 110, BRN-3 navigation results showing east-west bimodal distribution, R = 6369.757 km, 31 passes, days 41-55, 1976 . . . . .	17
6	BRN-3 simulated navigation results, tropospheric correction, 39 passes, days 268-269, 1970 . . . . .	18
7	Station 110, BRN-3 navigation results, no tropospheric correction, R = 6369.766 km, 31 passes, days 41-55, 1976 . . . . .	19
8	Station 110, BRN-3 navigation results showing east-west bimodal distribution, R = 6369.766 km, 31 passes, days 41-55, 1976 . . . . .	20
9	Station 110, BRN-3 navigation results, tropospheric correction, R = 6369.766 km, 31 passes, days 41-55, 1976 . . . . .	21
10	Station 110, BRN-3 navigation, bimodal distribution in navigated longitude, 31 passes, days 41-55, 1976 . . . . .	22
11	Station 110, BRN-3 navigation results, tropospheric correction, R = 6369.757 km, 31 passes, days 41-55, 1976 . . . . .	23

### ILLUSTRATIONS (cont'd)

- |    |   |    |
|----|---|----|
| 12 | Station 110, BRN-3 navigation results, no tropospheric correction, R = 6369.757 km, 115 passes, days 183-203, 1977 . . . . .            | 24 |
| 13 | Station 110, BRN-3 navigation results, tropospheric correction, R = 6369.757 km, 115 passes, days 183-203, 1977 . . . . .               | 25 |
| 14 | Station 110, BRN-3 navigation results showing east-west bimodal distribution, R = 6369.757 km, 115 passes, days 183-203, 1977 . . . . . | 26 |



## TABLES

1	Summary of station 110 BRN-3 navigation results, 31 passes, days 41-55, 1976 . . . . .	27
2	Initial navigation results with BRN-3 software, station radius = 6369.766 km . . . . .	28
3	Corrections to NWL 10-D coordinates determined in the APL-4.5 study versus the WGS-72 study . . . . .	30
4	Initial navigation results with BRN-3 software, station radius = 6369.757 km . . . . .	31
5	Summary of station 110 navigation results, 115 passes, days 183-203, 1977 . . . . .	33
6	Locations of operational and Tranet tracking sites . . . . .	36
7	Summary of simulated navigation results, 39 passes, days 268-269, 1970 . . . . .	38
8	Final navigation results, 115 passes, days 183-203, 1977 . . . . .	43
9	BRN-3 mean east/west navigation results, 115 passes, days 183-203, 1977 . . . . .	47
10	BRN-3 mean north/south navigation results, 115 passes, days 183-203, 1977 . . . . .	47

## SUMMARY AND CONCLUSIONS

The Transit system user may or may not take into account the effects of the troposphere on the collected Doppler data. In this report, we will discuss the consequences to the user of ignoring such effects on signal propagation.

Although the study was conducted using one type of user equipment (the AN/BRN-3), the results and conclusions are equally applicable to other Transit user equipment. The tropospheric effect manifests itself as an error in the instantaneous range from the observer (navigator) to the satellite. The error is largest (typically 80 m) at low elevations and smallest (about 2.3 m) when the satellite is directly above the observer. Ignoring the effects of the troposphere introduces large errors into the navigated longitude (35 to 40 m). Multiple passes for a fixed-site navigator result in a characteristic bimodal distribution in longitude (Fig. 1). Satellite passes east and west of the navigator cluster on either side of the "true" longitude, with the means of the two clusters typically 70 m apart. This problem is totally rectified when the effects of the troposphere are taken into account (see Fig. 2) using a tropospheric model in the navigation program. Eliminating low elevation data (BRN-3 strategy) reduces east-west longitude separation from 70 m (35 m on either side of the "true" mean) to 22 m ( $\pm 11$  m relative to the "true" mean), as shown in Figs. 3, 4, and 5. The price associated with eliminating low elevation data is a 6 m increase in latitude scatter, from 13.8 m when using all the data (Fig. 2) to 19.9 m when using only data around the time when the satellite is at closest approach (tca) to the station ( $\pm 110$  s), as shown in Fig. 6. The absence of tropospheric corrections to the data can lead to errors in the fixed-site station altitude determination. A positive error in station altitude (radius, R) will compensate for the missing troposphere. A 9 m error in the radius (altitude) of station 110 eliminated a 22 m east-west longitude separation that should have been evident in the navigation results (Figs. 4, 7, and 8).

The BRN-3 software, as it exists today, does not account for the effects of the troposphere on the data. The BRN-3 strategy reduces the resulting errors in longitude by eliminating low elevation data, using only 110 s of data around the satellite's tca to the navigator.

Black's analytic form (Refs. 1 and 2) of Hopfield's tropospheric model (Refs. 3, 4, and 5) was recently incorporated into an experimental version of the BRN-3 software. The BRN-3 was used to navigate two sets of passes for station 110 (located at APL) with and without correcting the data for tropospheric effects. The first set of 39 passes led to the discovery of a 9 m error in the altitude of station 110, which partially compensated for the missing tropospheric effect (Tables 1 through 4, Figs. 4, 5, 7, 8, 9, 10 and 11). A second set of 115 passes was navigated, and a 4.7 m reduction in the longitude scatter was achieved (Table 5, Figs. 12, 13, and 14).

This, however, does not tell the whole story since the navigator at sea does not usually get multiple fixes for the same site. Figure 14 illustrates what happens when we separate the passes into east and west clusters. The troposphere-corrected passes have identical means for the two sets, whereas in the case of the uncorrected sets, the means are 22 m apart. An individual troposphere-uncorrected fix will, on the average, be 11 m in error when compared with the troposphere-corrected pass. It will fall 11 m to the east or west of the "true" position (the mean). Low elevation passes (15°) exhibit a 23 m shift when compared to their troposphere-corrected counterparts (Table 4). Viewed from this perspective, accounting for the troposphere in the navigation software results in significant improvements in navigation accuracy.

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Ref. 1. H. D. Black, "An Easily Implemented Algorithm for the Tropospheric Range Correction," J. Geophys. Res., Vol. 83, No. 34, 10 Apr 1978, pp. 1825-1828.

Ref. 2. H. D. Black, "Position determination using the TRANSIT System," Proc. Int. Geod. Symp., Vol. 1, Oct 1976, pp. 24-45.

Ref. 3. H. S. Hopfield, "Two-Quartic Tropospheric Refractivity Profile for Correcting Satellite Data," J. Geophys. Res., Vol. 74, No. 18, 1969, pp. 4487-4499.

Ref. 4. H. S. Hopfield, "Tropospheric Range Error Parameters: Further Studies," APL/JHU CP 015, Jun 1972.

Ref. 5. H. S. Hopfield, "Tropospheric Effects on Low-Elevation-Angle Signals: Further Studies, Final Report," APL/JHU SDO-4588, Aug 1976.

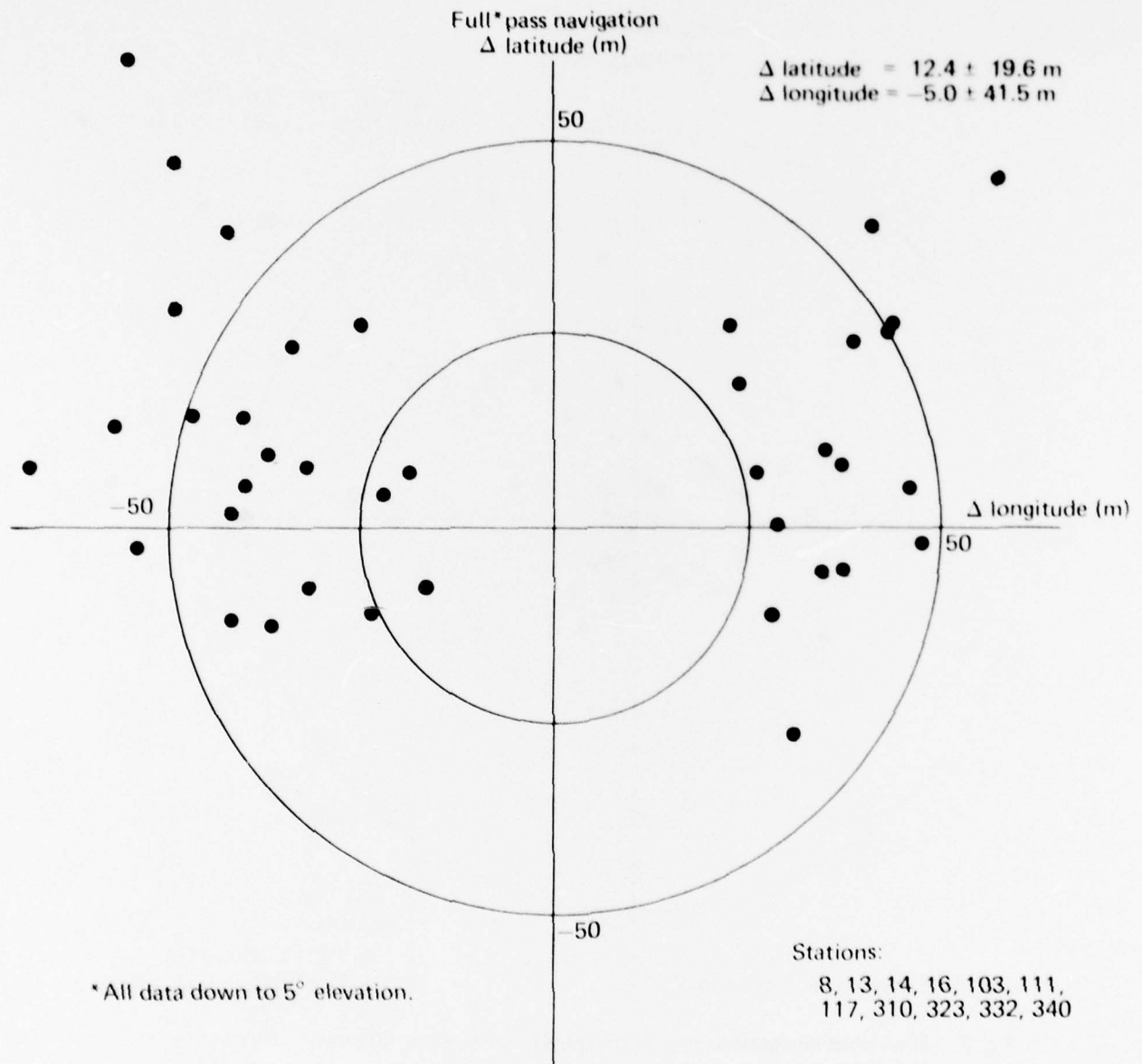


Fig. 1 Simulated navigation runs, no tropospheric correction, 39 passes, days 268-269, 1970.

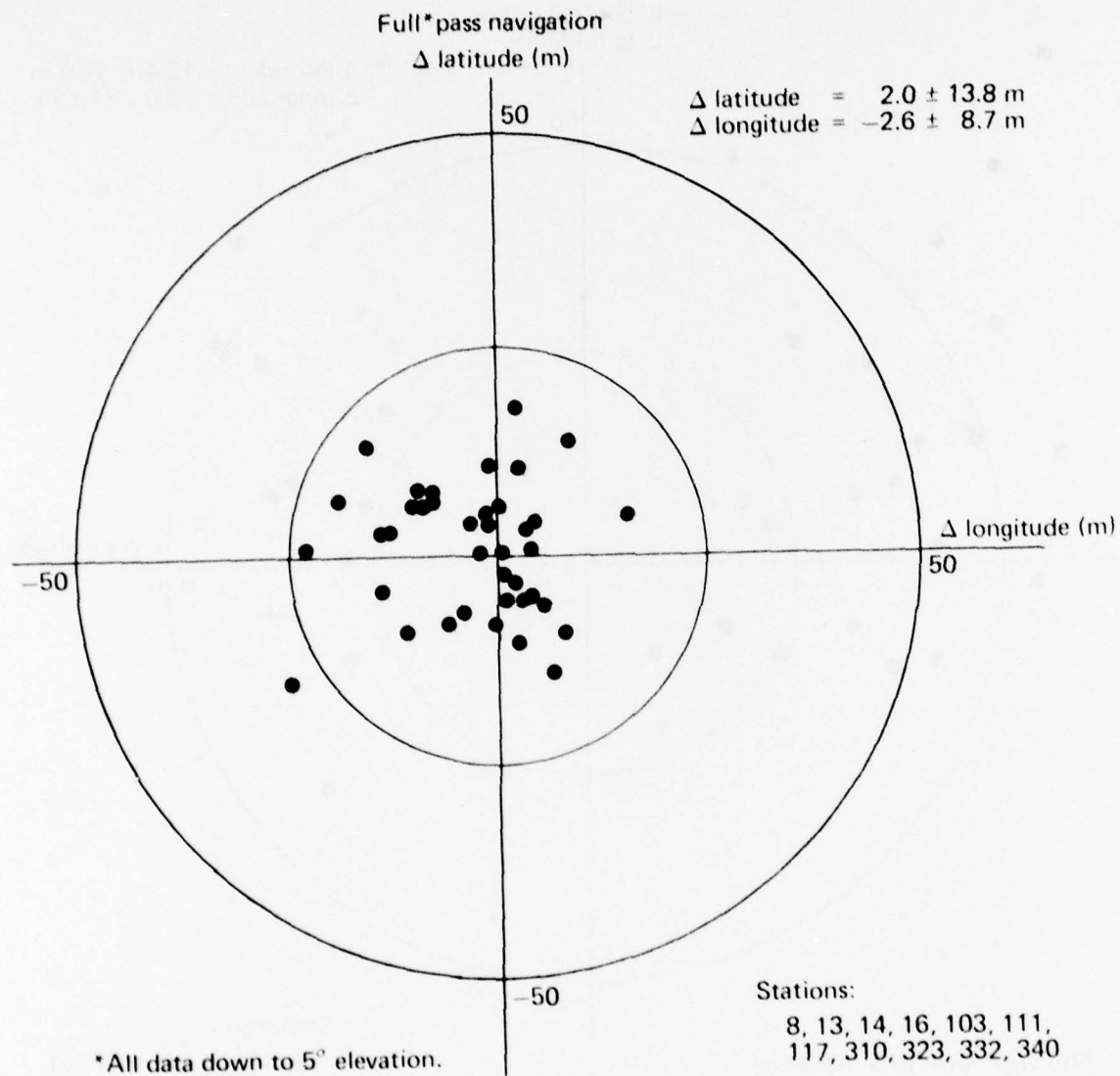


Fig. 2 Simulated navigation runs, tropospheric correction, 39 passes, days 268-269, 1970.



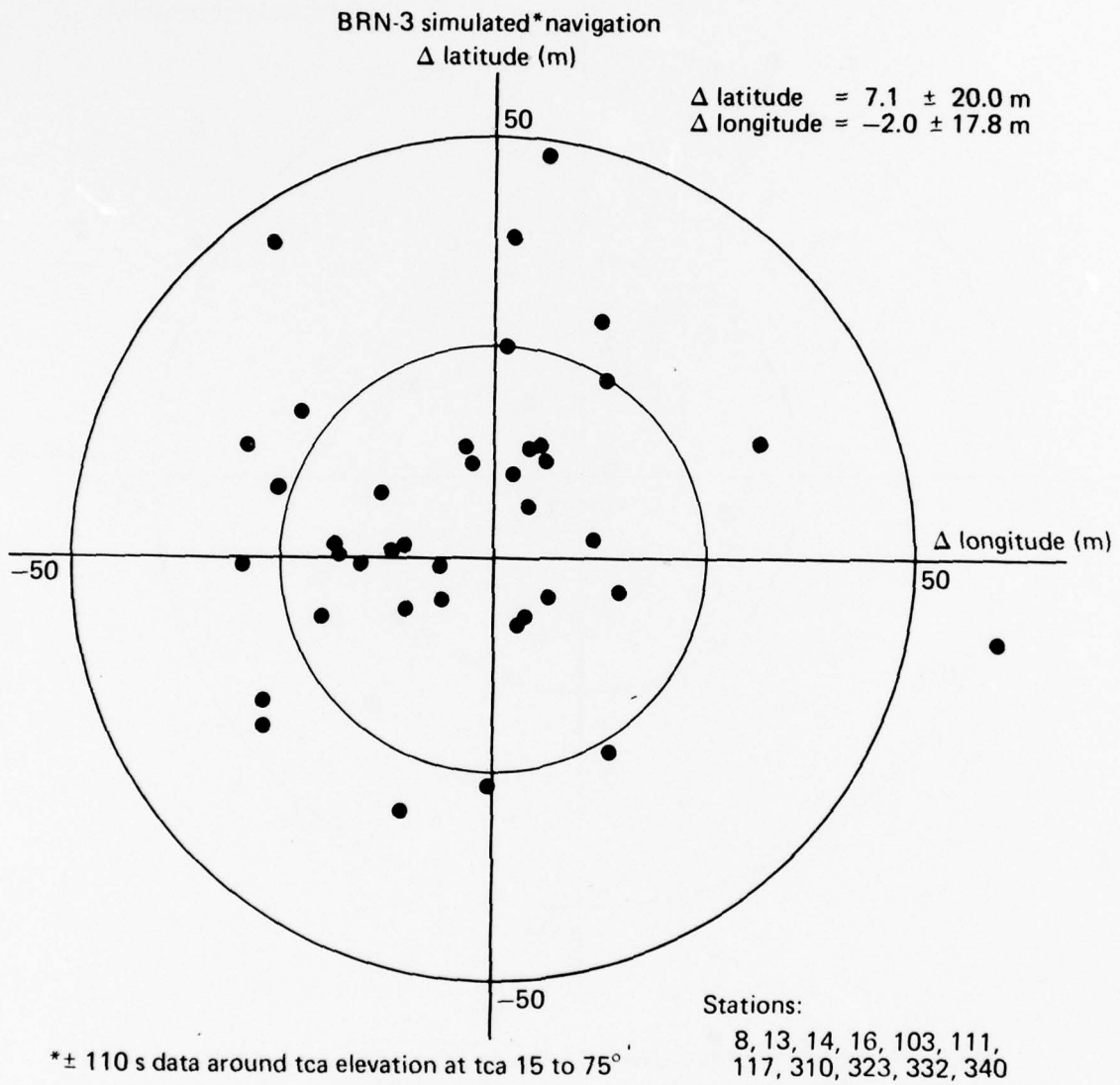


Fig. 3 BRN-3 simulated navigation results, no tropospheric correction, 39 passes, days 268--269, 1970.

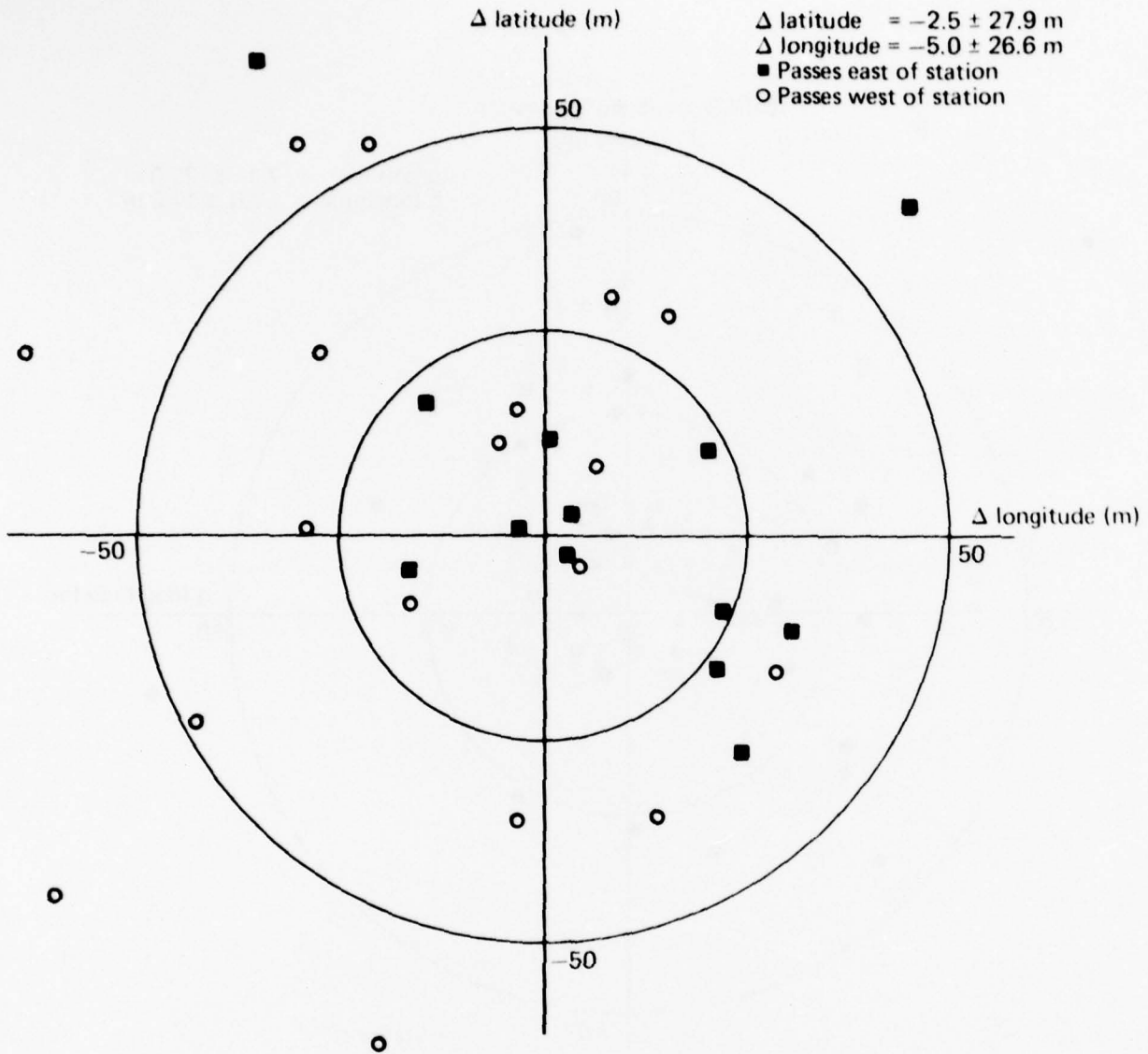


Fig. 4 Station 110, BRN-3 navigation results, no tropospheric correction,  
R = 6369.757 km, 31 passes, days 41-55, 1976.

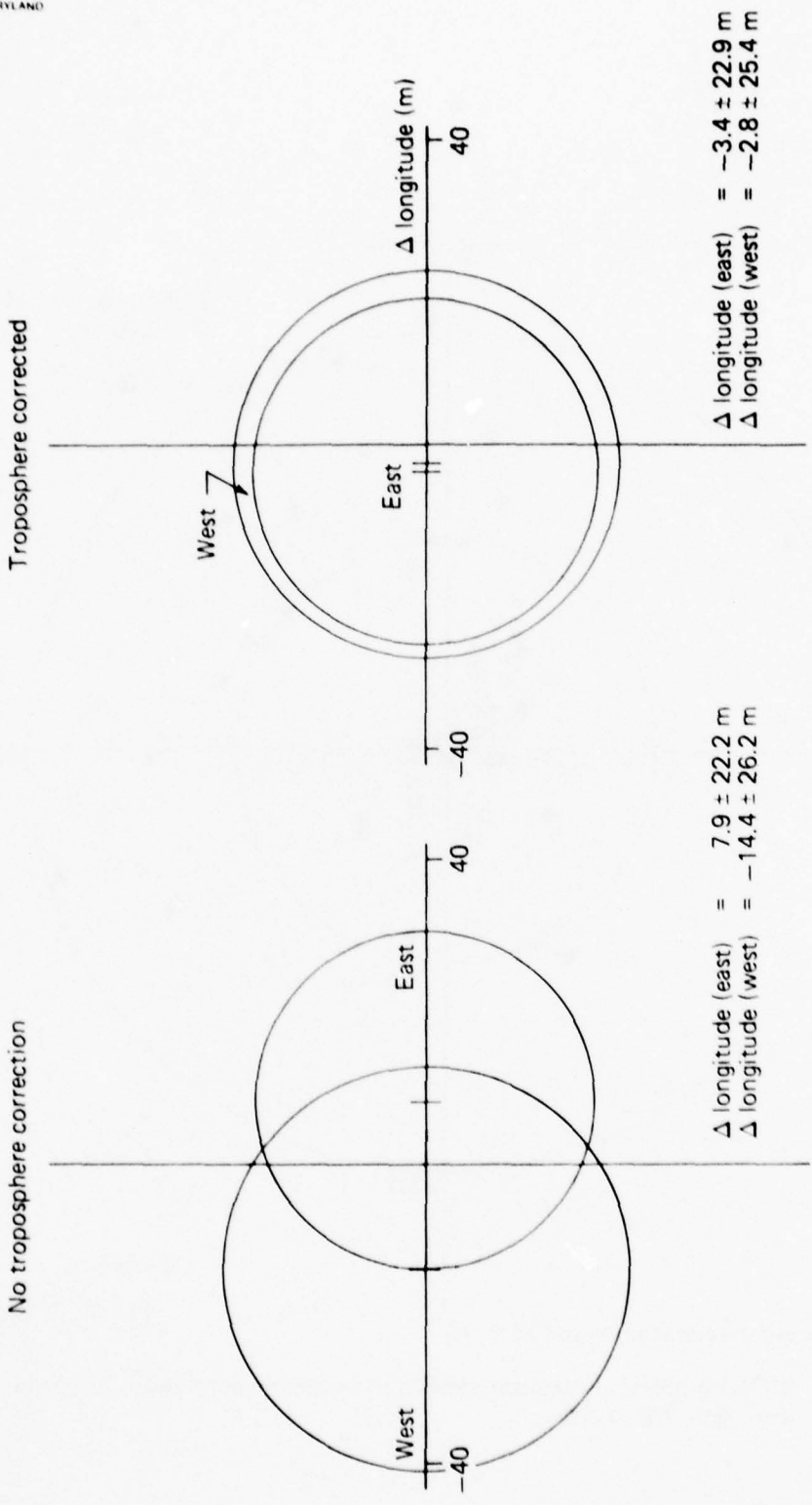
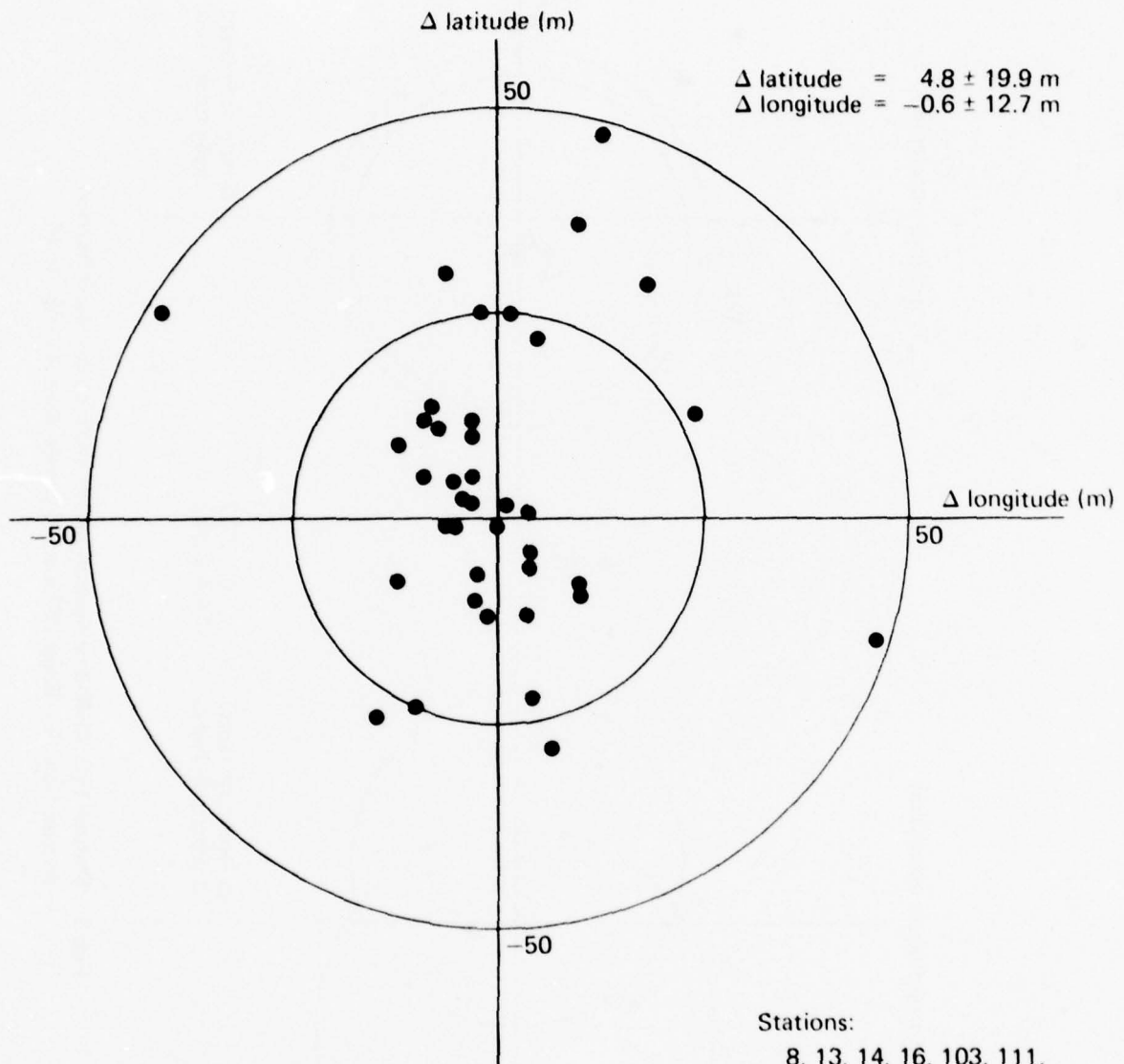


Fig. 5 Station 110, BRN-3 navigation results showing east-west bimodal distribution, R = 6369.757 km, 31 passes, days 41-55, 1976.



\*  $\pm 110 \text{ s}$  data around tca elevation at tca 15 to 75°

**Fig. 6** BRN-3 simulated navigation results, tropospheric correction, 39 passes, days 268-269, 1970.

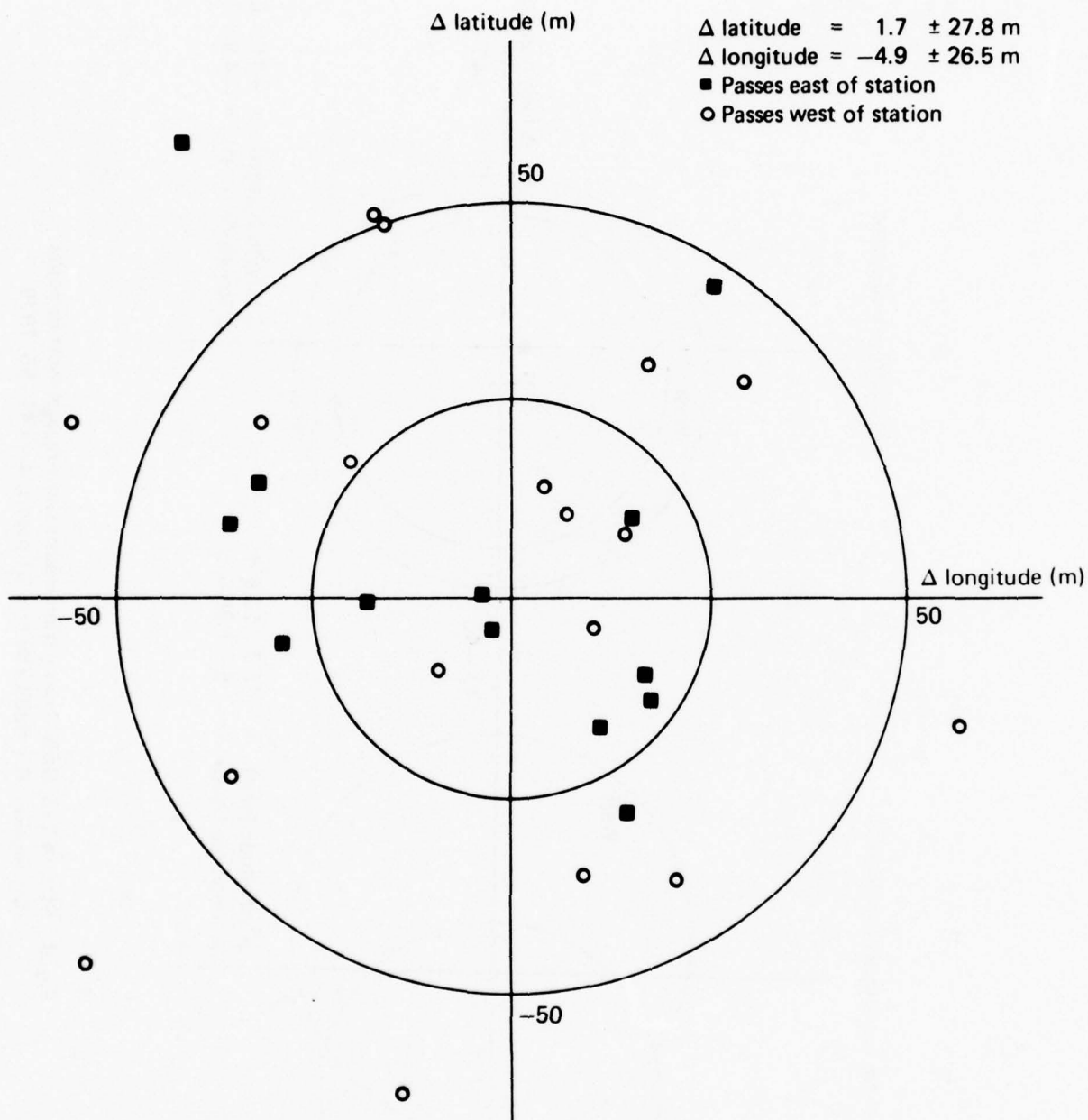


Fig. 7 Station 110, BRN-3 navigation results, no tropospheric correction, R = 6369.766 km, 31 passes, days 41-55, 1976.



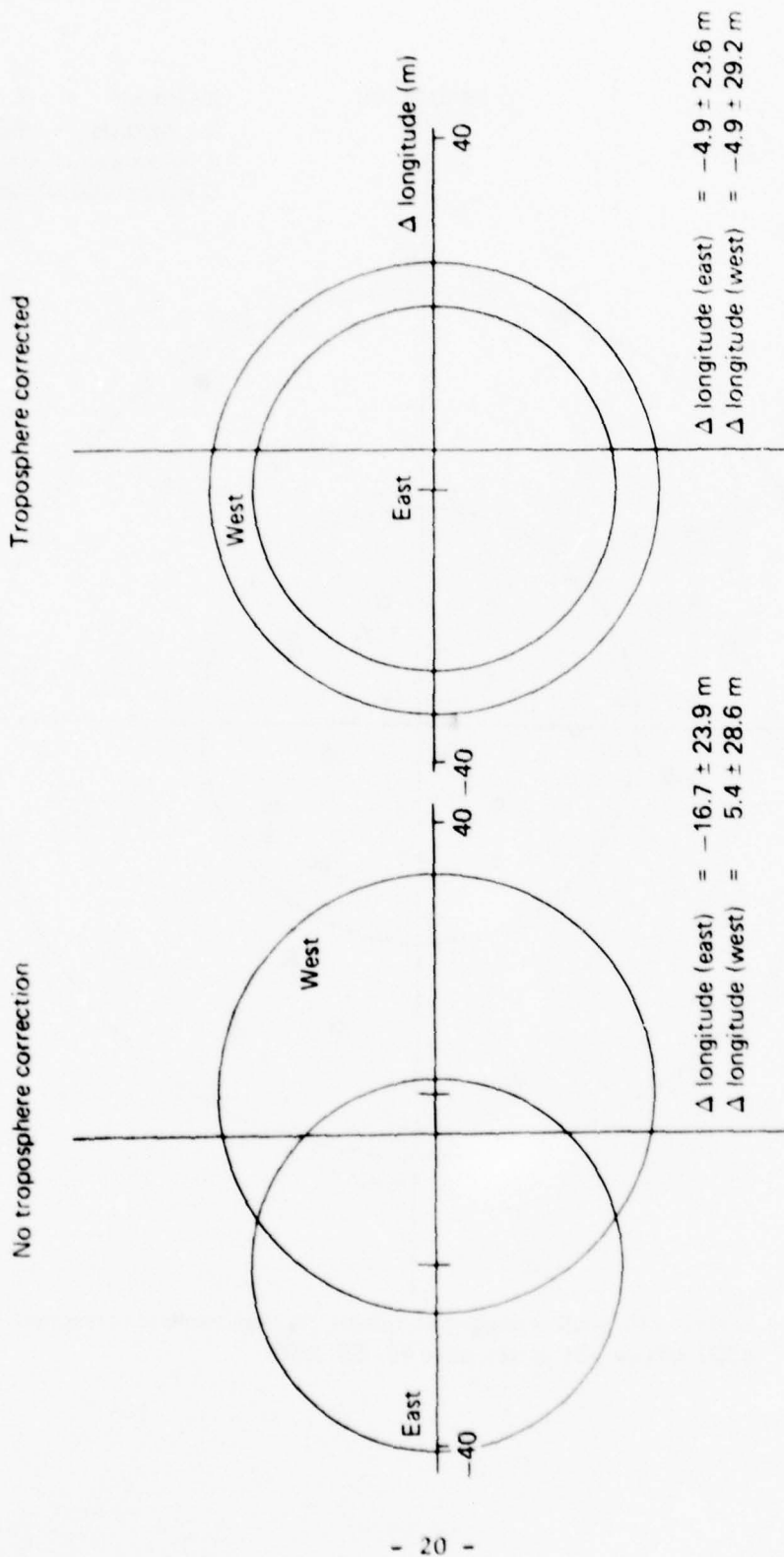


Fig. 8 Station 110, BRN-3 navigation results showing east-west bimodal distribution, R = 6369.766 km, 31 passes, days 41-55, 1976.

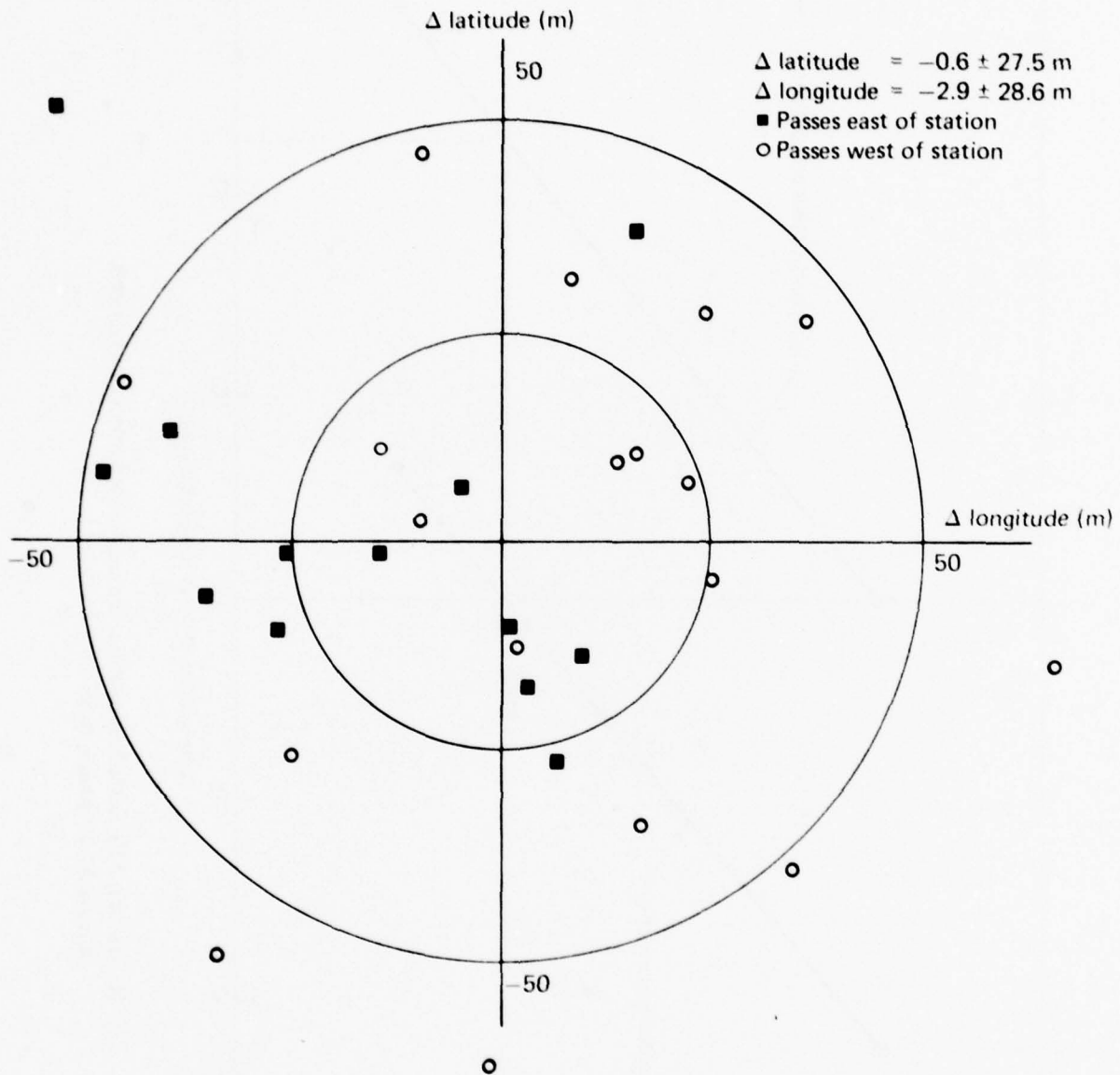


Fig. 9 Station 110, BRN-3 navigation results, tropospheric correction, R = 6369.766 km, 31 passes, days 41-55, 1976.

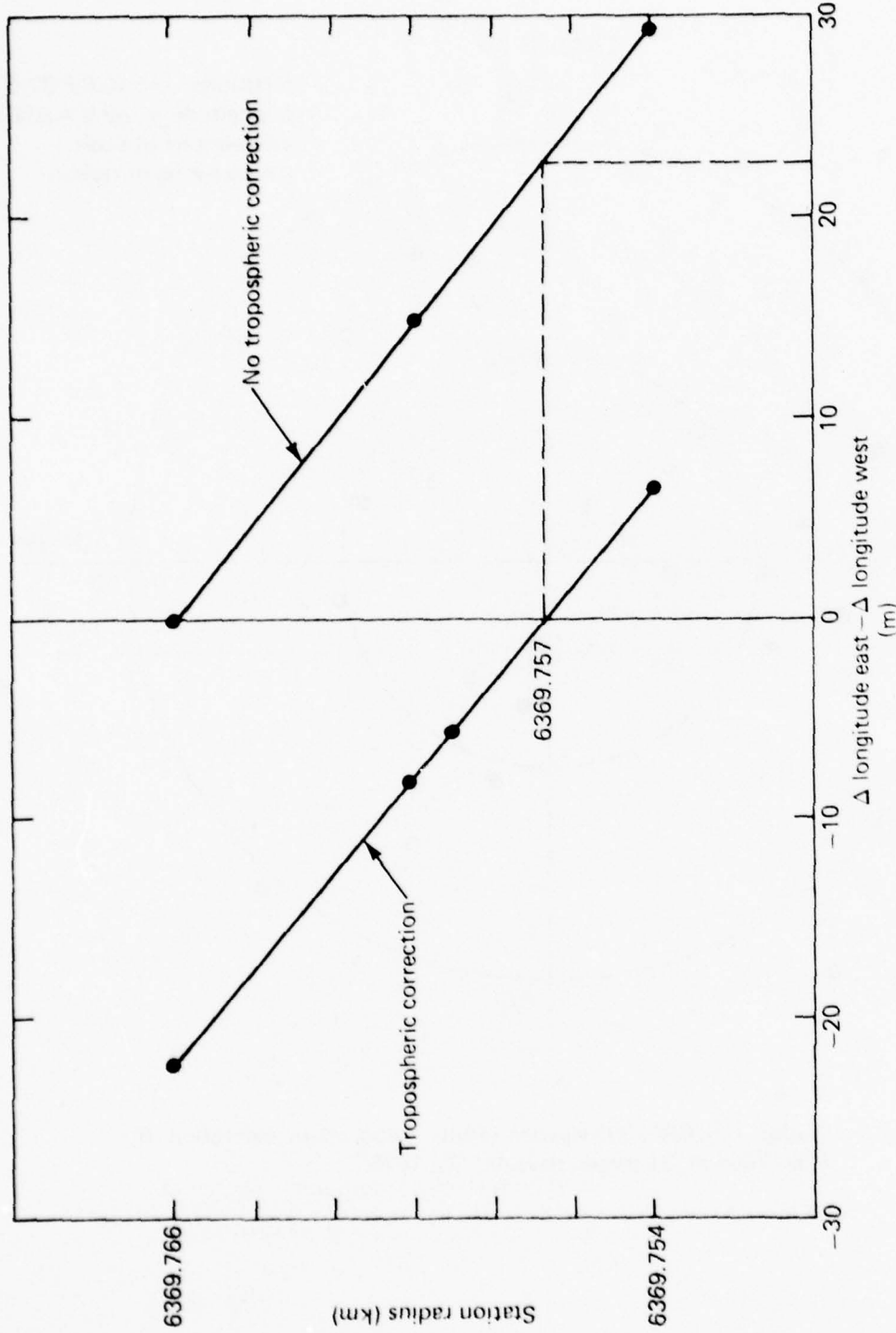


Fig. 10 Station 110, BRN-3 navigation, bimodal distribution in navigated longitude, 31 passes, days 41-55, 1976.

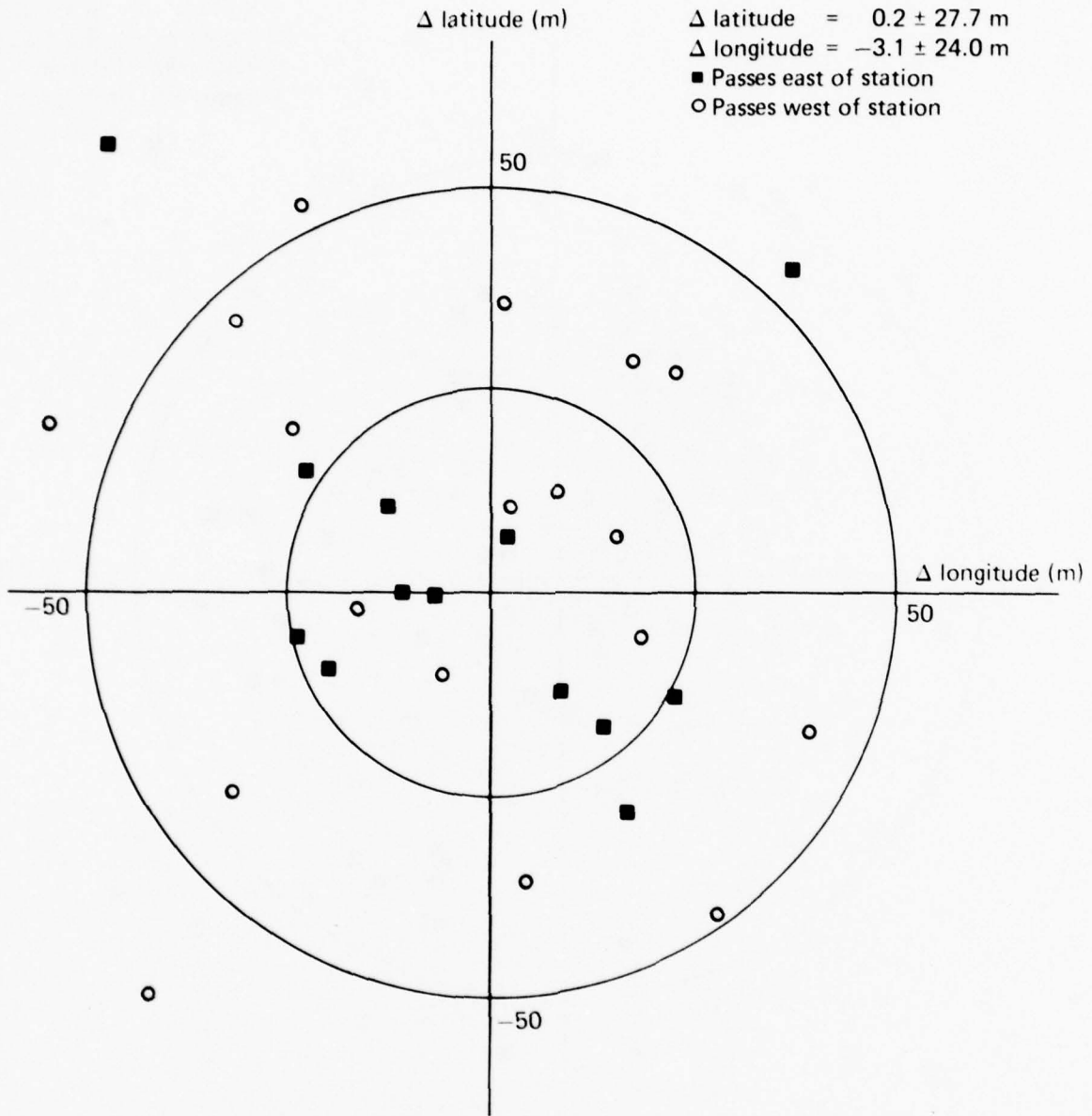


Fig. 11 Station 110, BRN-3 navigation results, tropospheric correction, R = 6369.757 km, 31 passes, days 41-55, 1976.

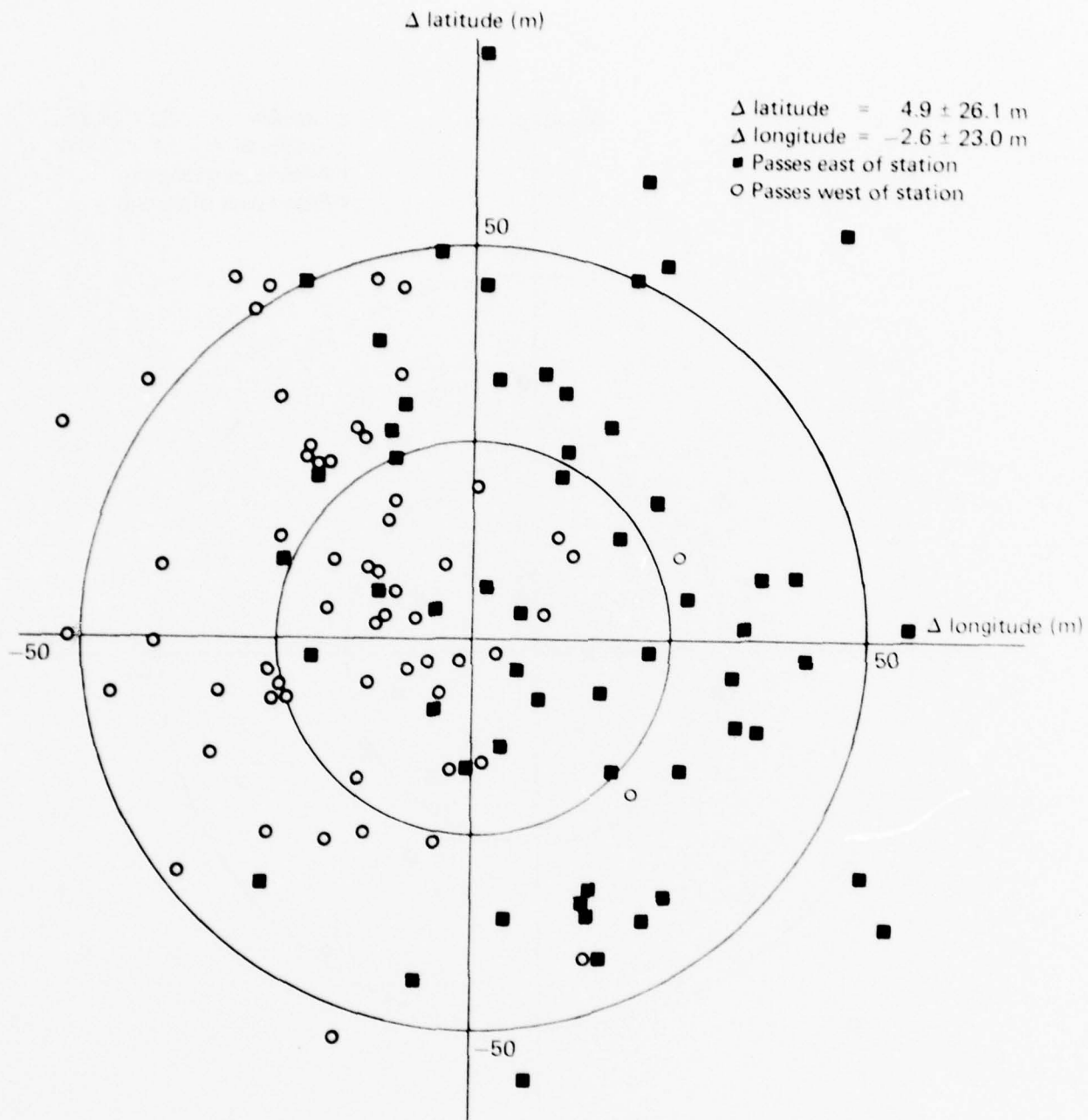


Fig. 12 Station 110, BRN-3 navigation results, no tropospheric correction, R = 6369.757 km, 115 passes, days 183-203, 1977.



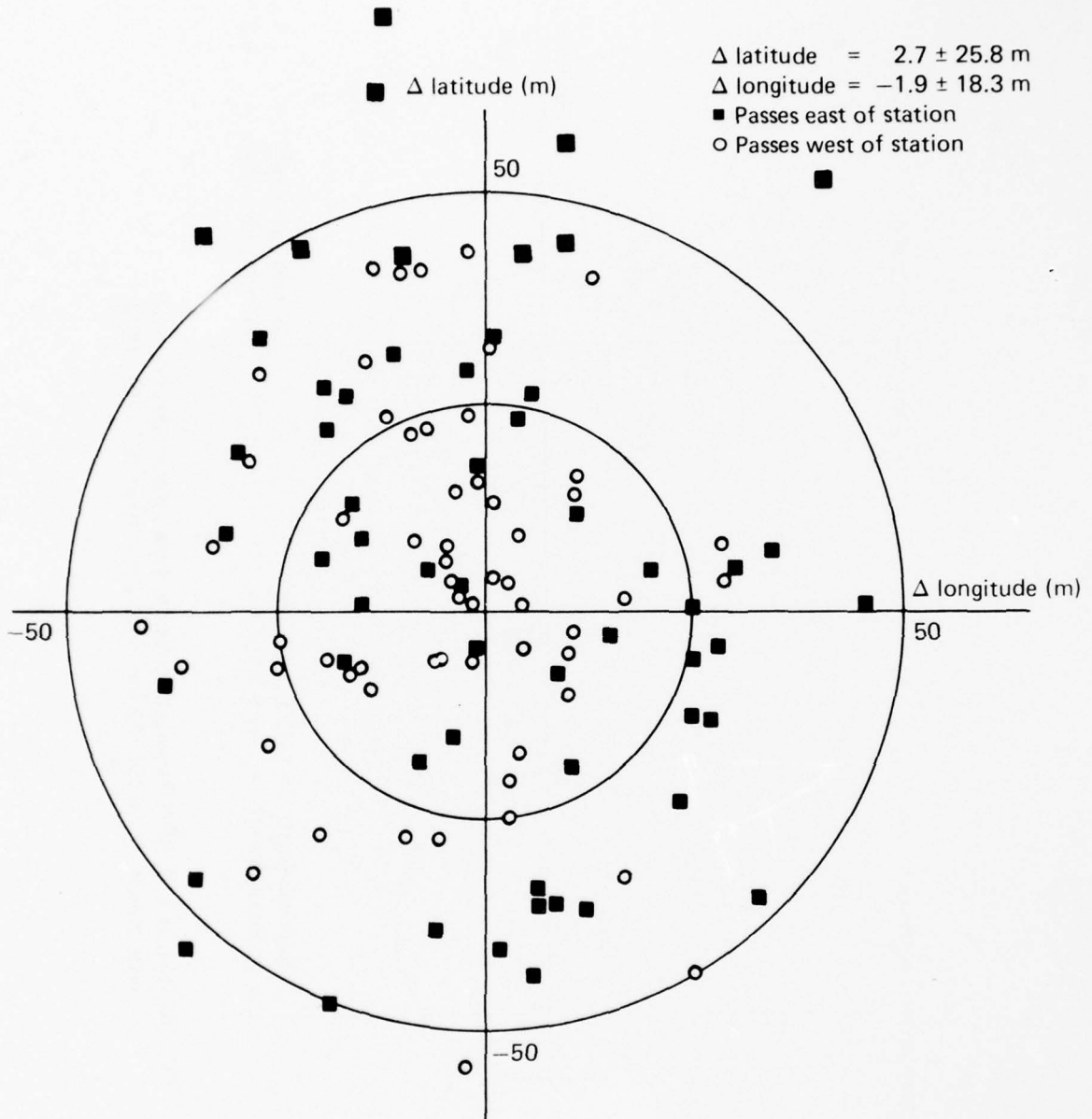


Fig. 13 Station 110, BRN-3 navigation results, tropospheric correction, R = 6369.757 km, 115 passes, days 183-203, 1977.

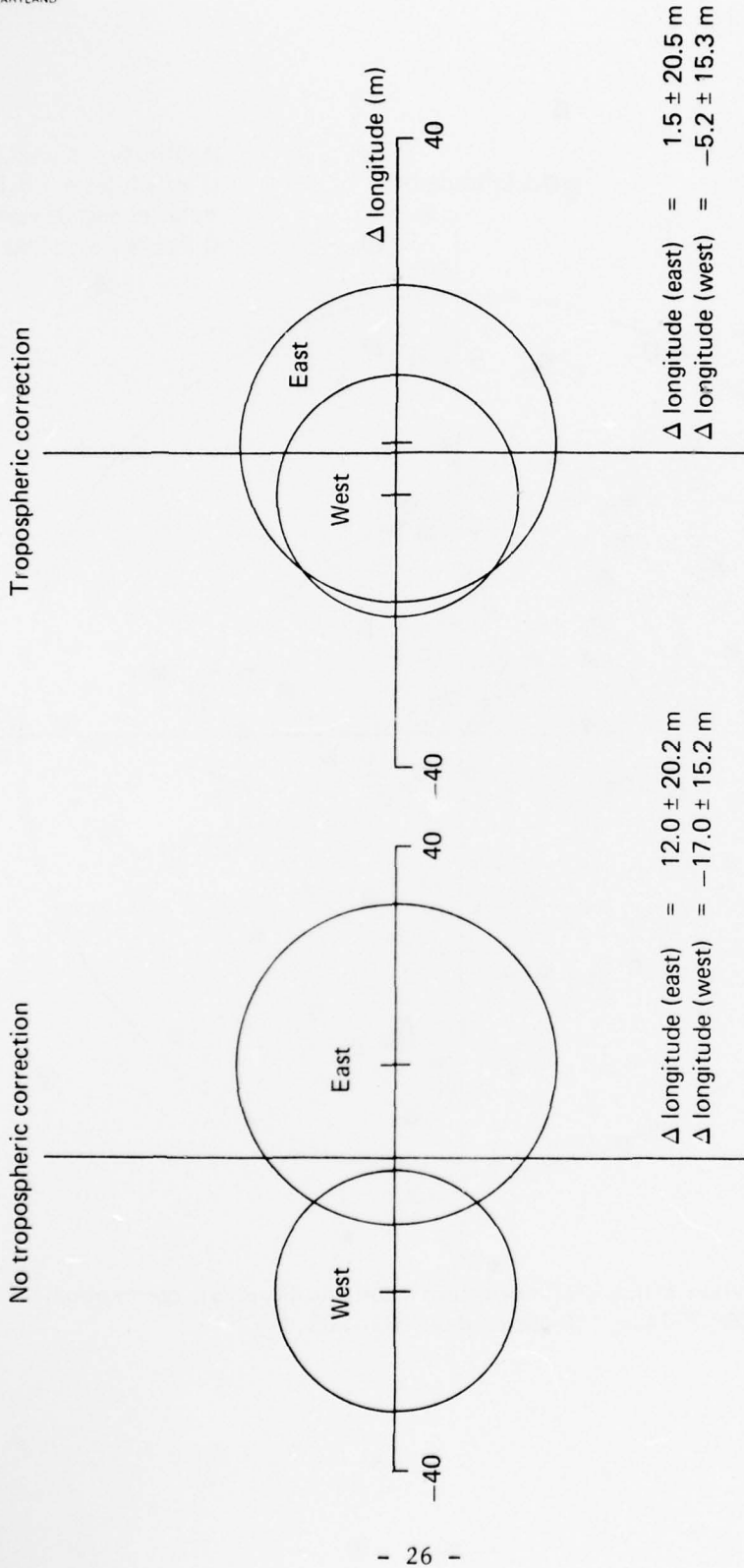


Fig. 14 Station 110, BRN-3 navigation results showing east-west bimodal distribution, R = 6369.757 km, 115 passes, days 183-203, 1977.

Table 1  
 Summary of station 110 BRN-3 navigation results,  
 31 passes, days 41-55, 1976.

Runs	Station 110 Radius (km)	Δ Long (m)			Δ Lat (m) Combined (31)	Tropo Corrected ?	Δ Long (m) East-West
		East Passes (13)	West Passes (18)	Combined (31)			
1	6369.766	- 4.9 ± 23.6	- 4.9 ± 29.2	-4.9 ± 26.5	1.7 ± 27.8	No	0
3	6369.760	3.9 ± 22.4	-11.2 ± 27.1	-4.9 ± 26.0	0.8 ± 28.9	No	15.1
8	6369.757	7.9 ± 22.2	-14.4 ± 26.2	-5.0 ± 26.6	-2.5 ± 27.9	No	22.3
6	6369.754	11.5 ± 22.4	-17.4 ± 25.4	-5.1 ± 28.0	3.0 ± 27.9	No	29.3
2	6369.766	-16.7 ± 23.9	5.4 ± 28.6	-2.9 ± 28.6	-0.6 ± 27.5	Yes	-22.1
4	6369.760	- 7.6 ± 22.9	0.4 ± 26.3	1.8 ± 25.0	-0.3 ± 27.6	Yes	- 8.0
5	6369.759	- 6.2 ± 23.1	- 0.7 ± 25.9	-3.0 ± 24.5	-0.2 ± 27.6	Yes	- 5.5
9	6369.757	- 3.4 ± 22.9	- 2.8 ± 25.4	-3.1 ± 24.0	0.2 ± 27.7	Yes	- 0.6
7	6369.754	0.7 ± 23.3	- 5.8 ± 24.7	-3.1 ± 24.0	4.8 ± 27.3	Yes	6.5

Table 2  
 Initial navigation results with BRN-3 software,  
 station radius = 6369.766 km.

Satellite	Elevation (deg)	Satellite Heading	No Tropospheric Correction (m)		With Tropospheric Correction (m)		No Tropo - Tropo (m)	
			$\Delta\phi$	$\Delta\lambda$	$\Delta\phi$	$\Delta\lambda$	$\delta\Delta\phi$	$\delta\Delta\lambda$
30120	40	SE	-26.9	13.9	-27.4	6.3	0.5	7.6
	40	NE	-16.1	10.9	-17.8	3.0	1.7	7.9
	31	NW	29.4	16.3	28.9	25.6	0.5	-9.3
	53	NE	-0.2	-18.3	-1.5	-26.5	1.3	8.2
	60	E	38.9	25.6	37.8	16.7	1.1	8.9
30130	27	SW	-23.3	-35.5	-25.9	-25.9	2.6	-9.4
	47	NE	-13.1	17.6	-14.3	9.6	1.2	8.0
	24	NE	57.2	-41.7	53.7	-54.3	3.5	12.6
	52	NW	27.2	29.6	27.0	37.4	0.2	-7.8
	72	NE	9.3	-35.6	8.1	-49.3	1.3	13.7
	21	NE	-10.0	16.3	-12.8	1.5	2.8	14.8
	15	N	-3.9	-2.4	-10.9	-27.6	7.0	25.2
30140	57	NE	14.6	-32.2	13.3	-40.6	1.3	8.4
	46	NE	-5.6	-28.9	-6.8	-36.7	1.2	7.8
	26	NW	-8.9	-9.4	-10.4	1.1	1.5	-10.5
	21	NW	22.0	-31.7	19.4	-17.6	2.6	-14.1
	50	NW	48.0	-16.9	47.6	-9.6	0.4	-7.3
30180	32	NW	8.0	14.6	6.9	23.5	1.1	-8.9
	17	SE	9.8	14.8	6.7	-4.8	3.1	19.6
	70	W	-15.9	56.7	-15.5	67.6	-0.4	-10.9
	24	NW	-62.0	-13.9	-64.4	-1.8	2.4	-12.1
	22	NW	0.6	-23.1	2.2	-9.8	-1.6	-13.4

Table 2 (cont'd)  
 Initial navigation results with BRN-3 software,  
 station radius = 6369.766 km.

Satellite	Elevation (deg)	Satellite Heading	No. Tropospheric Correction (m)		With Tropospheric Correction (m)		No Tropo - Tropo (m)	
			$\Delta\psi$	$\Delta\lambda$	$\Delta\psi$	$\Delta\lambda$	$\Delta\psi$	$\Delta\lambda$
30190	25	SW	13.9	4.1	10.6	16.3	3.3	-12.2
	28	NE	0.7	-3.9	-1.7	-14.8	2.4	10.9
	18	SW	-45.9	-54.3	-51.1	-35.2	5.2	-19.1
30200	48	NW	-34.8	8.7	-35.2	16.5	0.4	-7.8
	20	NW	-3.7	10.2	-5.2	25.6	1.5	-15.4
	15	SW	46.8	-16.1	32.2	8.9	14.6	-25.0
	31	SW	22.0	-55.4	19.4	-45.9	2.6	-9.5
	45	W	10.4	6.9	9.3	14.4	1.1	-7.5
	20	SW	-35.6	20.6	-40.7	35.0	5.1	-14.4

East - 13 passes      -4.9 ± 23.6      -16.7 ± 23.9      11.9 ± 5.7  
 West - 18 passes      -4.9 ± 29.2      5.4 ± 28.6      -11.8 ± 4.8  
 Combined - 31 passes      1.7 ± 27.8      -4.9 ± 26.5      -0.6 ± 27.5      -2.9 ± 28.6      2.3 ± 2.9



Table 3  
 Corrections to NWL 10-D coordinates  
 determined in the APL-4.5 study versus the WGS-72 study.

Station	Rad ( $R_0$ )			Meters		
	$\Delta\phi$	$\Delta\lambda$	$\Delta R$	$R\Delta\phi$	$R \cos \phi \Delta\lambda$	$\Delta R$
008	2.0875-07	-0.48853-07	-2.0154-07	+1.33	-0.29	-1.28
013	-6.2704-07	-6.7720-07	-1.790-07	-3.99	-3.28	-1.14
014	-3.8398-07	-1.2881-07	-4.2274-07	-2.44	-0.40	-2.69
018	4.7987-07	-7.2548-07	-3.7076-07	3.05	-1.08	-2.36
019	0.7726-07	-1.77049-06	-3.6032-07	0.49	-2.38	-2.29
103	-3.8859-07	-3.6719-07	-3.9096-07	-2.48	-1.98	-2.49
106	2.9150-07	-1.1269-06	3.3285-07	1.86	-4.51	2.12
111	-1.3630-07	-2.6671-07	-9.3780-07	-0.87	-1.32	-5.97
117	2.6518-07	-1.7870-07	-6.5925-07	1.69	-1.10	-4.20
121	-2.12134-07	-6.9740-07	-2.3948-07	-1.35	-4.30	-1.53
311	2.01795-07	3.9765-07	-8.7581-07	1.28	1.81	-5.58
321	-0.93158-07	1.2511-07	-6.2857-07	-0.59	0.57	-4.00
323	-2.9617-07	-0.3258-07	-6.2034-07	-1.89	-0.15	-3.95
330	-0.7753-07	3.0280-07	-4.2046-07	-0.49	1.60	-2.68
340	-4.2133-07	3.8377-07	-1.3735-07	-2.69	2.28	-0.88

From Ref. 6.

Table 4  
 Initial navigation results with BRN-3 software,  
 station radius = 6369.757 km.

Satellite	Elevation (deg)	Satellite Heading	No Tropospheric Correction (m)		With Tropospheric Correction (m)		No Tropo - Tropo (m)	
			$\Delta\phi$	$\Delta\lambda$	$\Delta\phi$	$\Delta\lambda$	$\Delta\phi$	$\Delta\lambda$
30120	40	SE	-27.01	-24.1	-27.6	16.5	0.6	7.6
	40	NE	-14.6	21.5	-16.3	13.5	1.7	8.0
	31	NW	29.5	8.5	28.5	17.4	1.0	-8.9
	53	NE	1.1	-3.0	0.0	-10.9	1.1	7.9
	60	NE	40.7	45.9	39.6	36.9	1.1	9.0
	27	SW	-21.9	-43.0	-24.6	-32.0	2.7	-11.0
30130	47	NE	-11.9	30.4	-13.0	22.6	1.1	7.8
	24	NE	58.3	-34.8	55.0	-46.9	3.3	12.1
	52	NW	27.0	15.4	26.9	23.0	0.1	-7.6
	72	NE	11.7	0.6	10.6	-12.8	1.1	13.4
	21	NE	-9.6	22.2	-12.2	8.3	2.6	13.9
	15	NE	-2.6	3.0	-9.3	-20.0	6.7	23.0
30140	57	NE	15.9	-14.6	-14.8	-23.0	1.1	8.4
	46	NE	-4.4	-16.3	-5.6	-24.1	1.2	7.8
	26	NW	-8.5	-16.5	-10.0	-5.9	1.5	-10.6
	21	NW	22.4	-37.8	20.0	-24.3	2.4	-13.5
	50	NW	48.0	-30.4	47.6	-23.0	0.4	-7.4
	32	NW	8.2	6.5	7.0	15.2	1.2	-8.7
30180	17	SE	10.2	20.4	7.0	1.9	3.2	18.5
	70	W	-17.1	28.3	-16.9	39.1	-0.3	-10.8
	24	NW	-61.7	-20.6	-64.1	-8.9	2.4	-11.7
	22	NW	0.7	-29.5	-1.9	-16.7	2.6	-12.8

Table 4 (cont'd)  
 Initial navigation results with BHN-3 software,  
 station radius = 6369.757 km.

Satellite	Elevation (deg)	Satellite Heading	No Tropospheric Correction (m)		With Tropospheric Correction (m)		No Tropo - Tropo (m)	
			Δp	Δh	Δp	Δh	Δp	Δh
30190	25	SW	15.4	-3.2	12.2	8.7	3.2	-11.9
	26	NE	2.2	3.5	-0.2	-6.9	2.4	10.4
	18	SW	-44.5	-60.4	-49.3	-42.4	4.8	-18.0
30200	48	NW	-35.2	-3.3	-35.6	4.1	0.4	-7.4
	20	NW	-3.7	4.3	-5.2	18.5	1.5	-14.2
	15	SW	48.2	-21.7	35.2	1.3	13.0	-23.0
	31	SW	23.3	-63.9	20.7	-54.6	2.6	-9.3
	45	W	11.7	-5.4	10.6	2.2	1.1	-7.6
20	SW	-34.3	13.9	-39.3	27.8	5.0	-13.9	
East	- 13 passes			7.9 ± 22.2		- 3.4 ± 22.9		11.4 ± 4.8
West	- 18 passes			-14.4 ± 26.2		- 2.8 ± 25.4		-11.6 ± 4.0
Combined	- 31 passes			-2.5 ± 27.9		0.2 ± 27.7		2.3 ± 2.5

Table 5  
 Station 110, summary of navigation results,  
 days 183-203, 1977.

Description	No. of passes	BRN-3 Mean Navigation Errors (m)												
		No Tropospheric Correction					Tropospheric Correction					Difference (no tropo - tropo)		
		Latitude		Longitude		Latitude	Longitude	Latitude		Longitude	Mean	S.D.	Mean	S.D.
		Latitude	Longitude	Latitude	Longitude			Latitude	Longitude					
South/east quadrant passes	29	6.7 ± 30.5	11.4 ± 19.0	5.3 ± 30.7	1.1 ± 18.7	1.4	-0.2	10.3	0.3					
North/east quadrant passes	28	2.2 ± 29.8	12.7 ± 21.7	0.0 ± 29.4	1.8 ± 22.6	2.2	0.4	10.9	-0.9					
South/west quadrant passes	32	7.8 ± 20.3	-16.9 ± 15.9	4.4 ± 19.3	-4.9 ± 16.1	3.4	1.0	-12.0	-0.2					
North/west quadrant passes	26	2.1 ± 23.8	-17.2 ± 14.7	0.7 ± 23.7	-5.7 ± 14.6	1.4	0.1	-11.5	0.1					
East passes	58	4.5 ± 30.0	12.0 ± 20.2	2.7 ± 29.9	1.5 ± 20.5	1.8	0.1	10.5	-0.3					
West passes	57	5.2 ± 21.9	-17.0 ± 15.2	2.7 ± 21.3	-5.2 ± 15.3	2.5	0.6	-11.8	-0.1					
All passes	115	4.9 ± 26.1	- 2.6 ± 23.0	2.7 ± 25.8	-1.9 ± 18.3	2.2	0.3	- 0.7	4.7					

## INTRODUCTION

The BRN-3 navigation programs as they are used in the field today do not account for the effects of the troposphere on the signal received from the Transit satellite. The troposphere delays the passage of an electromagnetic signal so that the measured range (data) is longer than the geometric distance between satellite and navigator. Hopfield developed and later refined tropospheric models (Refs. 3, 4, and 5) that have been successfully used in the tracking portion of the Transit system but not in the BRN-3 navigation programs. Black (Ref. 1) utilized the Hopfield model for the altitude variation of the refractive index but incorporated his own approach to the necessary integrals leading to simple analytic forms. The forms illustrate that the tropospheric correction is directly proportional to surface pressure and is quite insensitive to surface temperature. The corrections to the Doppler frequency for a navigator at sea level can be computed from the following simple equation (see the appendix):

$$\Delta f_d^T = 2.475 \frac{f_T}{c} \frac{\cos e}{\sin^2 e} \dot{e} \quad (1)$$

where

$\Delta f_d^T$  = tropospheric correction to vacuum Doppler frequency (Hz),

$c$  = speed of light (m/s),

$e$  = instantaneous elevation angle,

$f_T$  = navigator's frequency (c/s), and

$\dot{e}$  = time rate of change of the instantaneous elevation (rad/s).

Equation 1 should, in most cases, remove 85 to 90% of the tropospheric effects above 5° elevation.

The necessary modifications to the BRN-3 software to incorporate the tropospheric correction capability were done at APL by George Martin. The balance of this report will be devoted to the effects of the troposphere on the BRN-3 navigation results both before and after modification.



## THE SIMULATION EXPERIMENT

### BACKGROUND

The results presented here were obtained using the IBM/360 navigation program, with a full-precision-tracked (not extrapolated outside the data interval) ephemeris. Users of the Transit satellite navigation system navigate with an extrapolated ephemeris. As a consequence of these differences, the absolute navigated positions reported here are generally better than those the fixed-site navigator user of the Transit system is likely to achieve.

We were particularly interested in the BRN-3 navigator who (a) does not correct for troposphere range errors, (b) limits himself to passes whose elevations at tca are 15 to 75°, and (c) uses only 110 s worth of data on either side of tca.

The question we would like to investigate is what, if any, are the differences in the navigated positions resulting from data truncation and uncorrected tropospheric range errors.

### THE EXPERIMENT

We chose a 36 h span (days 268-269, 1970). All available passes (a total of 39) for Transit satellite 30140 were selected. The IBM/360 orbit determination program (ODP) was used to track the satellite, and all subsequent navigations were performed with the resulting tracked ephemeris. Eleven different fixed sites were included (Table 6), giving us a wide range of latitudes and longitudes. The following sets of navigations were performed:

1. Full data (down to 5° elevation) tropospheric correction;
2. Full data, no tropospheric correction;
3. Truncated data ( $\pm 110$  s about tca), tropospheric correction; and
4. Truncated data, no tropospheric correction (BRN-3 simulation).

Figure 2 presents the results of the world-wide fixed-site navigation, using maximum data (down to 5° elevation) and correcting

Table 6  
 Locations of operational and Tranet tracking sites.

Earth Fixed Spherical Position				Location
Station	Latitude (deg)	Longitude (deg)	Height above Geoid (m)	
001/111*/502	39	-77	148	APL, MD
002/192	30	-98	180	Austin, TX
003/103*	32	-107	1218	Las Cruces, NM
006/106	51	-1	1859	Lasham, U.K.
008*	-23	-46	609	San Jose Dos Campos
013*/027	41	141	609	Misawa, Japan
014*	61	-150	69	Anchorage, AK
016*/116	51	-1	71	Bartonstacy, U.K.
017/024/117*	14	-171	34	Tafuna, Am. Samoa
018	76	-69	91	Thule, Greenland
019	-78	167	30	McMurdo, Antarctica
020	-5	55	595	Seychelles Islands
021	51	4	1200	Brussels, Belgium
022/121	15	120	7	San Miguel, Phil.
023	13	145	162	Guam
028	45	-76	59	Ottawa, Canada
105/115	-26	-28	1580	Pretoria, S. Africa
112	-34	139	35	Smithfield, Aust.
197	53	174	0	Shemya
310*/311	44	-68	24	Prospect Har., ME
320,1,2,3*,7,8	45	-93	304	Rosemount, MN
330,2*,4,6,505	34	-119	450	Laguna Peak, CA
340*,4,511	21	-158	401	Hawaii
352	69	-105	0	Cambridge Bay
641	44	11	0	Florence, Italy

\* Sites included in the simulation experiment.

for the troposphere. The same data were next navigated without tropospheric corrections applied. As expected, the scatter in the navigation increased, particularly in longitude (rms increased from 9 m [Fig. 2] to 42 m [Fig. 1]).

We now turn our attention to the BRN-3 navigator. Figure 3 is a plot of the navigation results using  $\pm 110$  s worth of data around tca without accounting for the troposphere (note the dramatic 24 m reduction in longitude scatter from the value in Fig. 1). Figure 6 repeats Fig. 3 but with tropospheric corrections applied (note the further reduction of 5 m in longitude scatter). Comparing the results of using full data (ODP navigation) and truncated data (BRN-3 navigation), we note the following:

In the absence of tropospheric corrections, the current practice (BRN-3) of truncating the data clearly reduces the navigation errors (24 m improvement in longitude). Tropospheric correction improves navigation further (5 additional m in longitude out of a possible 9 m). However, correcting for the troposphere without adding data beyond the  $\pm 110$  s about tca does not significantly improve the latitude scatter (only 1 m out of 6). Table 7 summarizes the four sets of navigation schemes in order of increasing navigation errors, i.e., from best to worst.

Based on the simulation results, it seemed reasonable to modify the BRN-3 software to account for the troposphere. We anticipated an improvement of several meters in the longitude scatter of the BRN-3 fixes.

Table 7  
 Summary of simulated navigation results,  
 39 passes, days 268-269, 1970.

Rank	Data Used*	Tropospheric Correction	Navigation Results (m)					
			Mean		Standard Deviation		rms	
			Lat	Long	Lat	Long	Lat	Long
1	Full	Yes	2.0	-2.6	13.8	8.7	13.9	9.1
2	Reduced	Yes	4.8	-0.6	19.9	12.7	20.5	12.7
3	Reduced	No	7.1	-2.0	20.0	17.8	21.2	17.9
4	Full	No	12.4	-5.0	19.6	41.5	23.1	41.8

\* Full; All data down to 5° elevation  
 Reduced: ±110 s about tca

Note: See Figs. 1, 2, 3, and 6.

## TROPOSPHERIC-CORRECTED BRN-3 SOFTWARE: INITIAL NAVIGATION RESULTS

George Martin has implemented H. D. Black's tropospheric corrections (Ref. 1) in a special experimental version of the BRN-3 program (see the appendix). In the first attempt to evaluate the new software, Mr. Martin navigated 31 satellite passes for the APL site (station 110, days 41-55, 1976). We were looking for an improvement in the longitudinal scatter of the troposphere-corrected navigation and a possible shift in the mean longitude position.

### EXPERIMENT DESCRIPTION

The results of the first two sets of navigation runs (Runs 1 and 2, Table 1, with and without tropospheric correction) were puzzling. Based on the earlier simulations, we had expected the troposphere-corrected navigations to improve the results. We found to our great dismay (Table 2 and Figs. 7, 8, and 9) that the scatter of the troposphere-corrected navigations showed a 2.1 m degradation (26.5 versus 28.6, Table 1). We separated the passes into two groups based on their east or west locations relative to the station. The navigated longitudes of passes not corrected for troposphere were expected to show a characteristic bimodal distribution. We found instead that the troposphere-corrected set had a strong bimodal distribution (a 22.1 m separation of the means of the east and west groups) and the uncorrected run had none. In searching for a plausible explanation for these peculiar results, we were faced with the following possibilities:

1. The BRN-3 software was already allowing for the tropospheric effects.
2. There were bugs in the BRN-3 program introduced by the new modification (Black's tropospheric correction).
3. The BRN-3 was being run with an incorrect station radius that compensated for the unaccounted-for troposphere.

We quickly eliminated the first two, which left us with the question of station 110's radius. It transpired that the BRN-3 has been using the value of 6369.766 km for many years. We checked the WGS-72 station 110 coordinates (NWL 10-D) and found that the "correct" radius was indeed 6369.766 km. To confuse matters even further, we discovered that the APL-4.5 station 110 radius was



supposed to have been 6369.757 km\*, which meant that, prior to the 1976 implementation of WGS-72, station 110 should have shown a bi-model (east-west) distribution in navigated longitude due to a 9 m error in station radius. Mr. Martin informed us that there was no significant change in the character of the navigated mean longitude of station 110 following the introduction of the WGS-72 geodesy. Clearly, the answer had to be that 6369.766 was not the correct radius for station 110 for either WGS-72 or APL-4.5 geodesies. It was at this point that an earlier study (Ref. 6) comparing WGS-72 and APL-4.5 navigation results for the Tranet network came to our rescue. Table 3 lists the corrections to the nominal NWL 10-D coordinates for a number of Tranet sites as determined in that earlier study. We noted immediately that station 111 (same as 110) has a 6 m error in its radius. The "correct" station radius should be 6369.760 km and the 6 m error was, in fact, compensating for the missing tropospheric corrections. As for the 4.5 coordinates, it turns out that the "correct" 4.5 radius, again based on Ref. 6, was only 1.4 m smaller than the NWL 10-D radius, or 6369.759 km. In any case, a 1.4 m difference in altitude would not be significant when monitoring station 111 (or 110) navigation fixes. Having convinced ourselves that 6369.760 was a better value for the station 110 radius, we performed Runs 3 and 4 (Table 1), using 6369.760 km as the "true" station 110 radius. As expected, we then did see the effect of the troposphere in reducing both the bimodal (east-west) distribution of the mean navigated longitude, from 15.1 to 8 m, and the scatter, from 26 to 25 m. However, although this is an improvement, we still had an 8 m bimodal distribution in the troposphere-corrected case. We were now faced with the following choices:

1. Station 110 radius was still not quite right.
2. Modeled tropospheric errors caused the bimodal distribution in the navigated longitude.

Although both statements were probably true, we chose to trust the modeled troposphere over the station radius and conducted Runs 5, 6, and 7 (Table 1) in order to establish a radius for station 110.

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\* Based on the location of station 111 and constraints extrapolated from station 111 to station 110:  $(\varphi_{(111)} - \varphi_{(110)}) = 0.77568 \times 10^{-6}$  rad,  $\lambda_{(111)} - \lambda_{(110)} = 1.1635 \times 10^{-6}$  rad,  $R_{(111)} - R_{(110)} = 0.8$  m).

Ref. 6. B. B. Holland, A. Eisner, and S. M. Yionoulis, "The Effect of WGS-72 Geopotential in the Navy Navigation Satellite System Surveys," APL/JHU TG 1311, Aug 1977.

We defined "better" as the radius that would minimize the bimodal separation in longitude of the troposphere-corrected navigated position. Figure 10 is a plot of the station radius versus east-west separation based on the results in Table 1. A radius of 6369.757 km turned out to be the "best" radius for station 110. Runs 8 and 9 (Table 1) confirmed that choice (see also Table 4 and Figs. 4, 5, and 11).

Summarizing the above, we conclude that the new radius of 6369.757 km for station 110 is probably accurate to about 1 to 2 m. Correcting for tropospheric range errors in the BRN-3 accomplished the following:

1. It eliminated a 22 m east-west bimodal distribution in the mean navigated station longitude.
2. It shifted the mean latitude 2.7 m northward and 2.1 m westward.
3. It reduced longitude scatter by 2.6 m from 26.6 to 24.0 (latitude scatter was not significantly affected).

## TROPOSPHERIC-CORRECTED BRN-3 SOFTWARE: FINAL EVALUATION

George Martin next undertook to navigate 115 passes with the new BRN-3 software to get better statistics and to understand better the "true" radius of station 110. Passes for 20 days (days 183-203, 1977) for station 110 were navigated using the BRN-3 with and without applying tropospheric corrections to the data (Table 8). We used a radius of 6369.757 km, which had been established earlier with the smaller set of 31 passes. The plots of the results (Figs. 12, 13, and 14) clearly show the east-west bimodal distribution (29 m) of the uncorrected navigation fixes (Fig. 12). The troposphere-corrected navigation fixes (Fig. 13) also exhibit a small bimodal distribution (6.7 m). We can surmise from the magnitude and sign of the troposphere-corrected bimodal distribution, with the help of Fig. 10, that the station altitude as determined with the 39 passes was 2 m too low. It would seem that a station radius of 6369.759 is the "correct" radius for station 110. Table 5 summarizes the results. This time we find a reduction of 4.7 m in the longitude scatter of the troposphere-corrected fixes, which is very close to the simulation results of 5 m. We also note that the mean latitude is shifted by 2.2 m (important for fixed site surveying), though the mean longitude shifts by less than 1 m.

In Table 9, fix results are compared for passes separated into east and west groups, while fix results for passes separated into north and south groups are compared in Table 10. The effect of the troposphere on the bimodal distribution in longitude is clearly evident in Table 9 and absent in Table 10. There is a 5 m difference in the mean latitude of the north and south groups, which is probably due to drag since the two sets of passes are always 12 h apart; one set will always be further away in time from the injection epoch and will, therefore, navigate using an "older" segment of the injected ephemeris (12 h older and hence subject to more severe drag errors).

**Table 8**  
**Final navigation results,**  
**115 passes, days 183-203, 1977.**

Rise Time (1977)		Pass I.D. Satellite	Geometry		BRN-3 Navigation Errors (m)			
					No Tropospheric Correction		Tropospheric Correction	
Day	Hr.Min	I.D.	Elev. (deg)	Head*	Latitude	Longitude	Latitude	Longitude
183	00.09	30120	44	SE	5.18	27.37	4.81	20.02
	01.55	30120	29	SW	18.69	0.58	16.28	10.80
	03.05	30190	54	NE	- 5.00	- 9.08	- 6.11	-17.00
	04.52	30190	26	NW	-41.63	14.69	-42.74	25.35
	05.12	30130	42	NE	-11.29	35.72	-12.77	27.80
	06.25	30200	48	SE	52.18	47.97	51.81	40.77
	06.47	30140	19	NW	- 0.74	-41.63	- 3.89	-25.64
	08.10	30200	29	SW	34.23	- 9.22	31.64	0.72
12.05	30120	43	NE	-33.86	13.97	-35.34	6.19	
185	13.48	30120	27	NW	31.27	-24.92	29.61	-14.41
	16.33	30130	27	SE	-36.45	4.03	-38.12	- 6.34
	17.45	30200	27	NE	-44.04	- 7.78	-46.26	-18.73
	18.20	30130	48	SW	2.96	9.08	1.48	16.85
	19.31	30200	45	NW	9.44	- 3.60	9.07	3.89
	23.10	30120	18	SE	46.07	21.32	42.93	4.61
186	00.56	30120	17	SW	2.22	-12.39	0.93	- 1.87
	04.33	30130	26	NE	45.52	1.44	42.56	- 9.80
	06.19	30130	48	NW	- 2.04	3.31	- 2.41	10.52
	08.20	30200	20	SW	7.59	-12.82	2.78	3.03
	11.07	30120	18	NE	49.59	- 4.47	43.48	-21.90
	20.09	30200	20	NW	42.37	-28.23	41.08	-13.68
187	00.01	30120	50	SE	-35.16	14.41	-35.53	7.06
	07.18	30130	16	NW	46.07	-31.11	41.26	-10.08
	11.57	30120	49	NE	-17.21	18.29	-18.50	10.52
	13.44	30120	25	NW	6.48	-10.80	4.26	0.43
	16.17	30140	47	SE	9.99	-24.49	9.62	-31.69
	16.42	30130	36	SE	-16.84	27.37	-22.76	23.77
	17.17	30200	19	NE	-30.53	50.13	-34.05	32.55
	18.29	30130	35	SW	3.15	-11.67	1.30	- 3.17
	19.02	30200	66	NW	- 6.85	-34.72	- 6.85	-24.92
	23.06	30120	20	SE	47.74	24.92	44.78	9.94

\*NE = Quadrant north, east of station  
 NW = Quadrant north, west of station

SE = Quadrant south, east of station  
 SW = Quadrant south, west of station



**Table 8 (cont'd)**  
**Final navigation results,**  
**115 passes, days 183-203, 1977.**

Rise Time (1977)		Pass I.D. Satellite I.D.	Geometry Elev. (deg)    Head*		BRN-3 Navigation Errors (m)			
					No Tropospheric Correction		Tropospheric Correction	
Day	Hr.Min				Latitude	Longitude	Latitude	Longitude
188	02.35	30190	46	NE	-16.65	- 0.29	-18.13	- 7.92
	13.48	30190	20	SE	- 2.41	42.93	- 3.89	27.95
	15.25	30140	21	SE	33.12	3.75	30.53	-10.66
	17.12	30140	64	SW	-26.09	-19.45	-27.20	- 9.94
189	03.24	30140	18	NE	- 2.78	-20.60	- 7.96	-39.04
	04.49	30200	16	SE	-36.64	22.18	-40.52	1.87
	11.53	30120	53	NE	8.33	41.34	7.22	33.42
	16.21	30140	58	SE	- 3.89	6.63	- 4.44	- 1.44
	23.02	30120	21	SE	31.46	11.81	29.24	- 2.02
	23.57	30120	54	SE	-31.09	-27.37	-31.46	-34.86
190	02.45	30190	61	NE	22.95	- 9.94	21.83	-19.01
	04.18	30140	52	NE	24.24	12.10	23.31	4.18
	04.50	30130	48	NE	-14.06	3.46	-15.17	- 4.32
	05.38	30200	35	SE	34.23	9.36	33.49	1.15
	06.05	30140	24	NW	2.59	- 7.49	0.74	4.32
	06.37	30130	26	NW	-30.16	-38.46	-31.46	-27.95
	07.23	30200	40	SW	- 4.07	- 8.64	- 5.74	- 0.72
	10.59	30120	21	NE	37.93	-12.82	33.12	-27.23
	13.58	30190	27	SE	1.48	35.15	0.56	24.78
	16.02	30130	22	SE	58.47	22.62	56.25	9.65
	17.16	30140	52	SW	- 5.37	-13.83	- 6.29	- 5.91
17.49	30130	57	SW	- 7.77	-24.06	- 9.44	-13.68	
192	15.54	30190	36	SW	9.81	-17.72	8.33	- 9.36
	16.59	30200	20	NE	-37.19	53.01	-40.52	36.73
	17.58	30130	43	SW	9.25	-40.33	7.96	-32.55
	18.44	30200	65	NW	3.89	-13.68	4.07	- 4.32
193	02.06	30190	39	NE	- 1.48	22.62	- 2.96	14.69
	03.31	30140	28	NE	29.79	- 8.93	27.39	-19.45
	05.17	30140	45	NW	14.99	-10.80	14.62	- 3.46
	05.48	30200	50	SE	13.51	23.34	13.14	16.13
	07.33	30200	28	SW	45.89	-12.24	43.30	- 1.59

\*NE = Quadrant north, east of station  
 NW = Quadrant north, west of station

SE = Quadrant south, east of station  
 SW = Quadrant south, west of station



**Table 8 (cont'd)**  
**Final navigation results,**  
**115 passes, days 183-203, 1977.**

Rise Time (1977)		Pass I.D.	Geometry		BRN-3 Navigation Errors (m)			
Day	Hr. Min	Satellite	Geometry		No Tropospheric Correction		Tropospheric Correction	
		I.D.	Elev. (deg)	Head*	Latitude	Longitude	Latitude	Longitude
193	13.32	30120	20	NW	- 7.03	- 4.61	- 9.81	10.08
	17.37	30200	42	NE	-10.73	32.99	-12.21	25.21
	18.14	30140	15	SW	27.57	-52.87	18.13	-28.38
	18.55	30130	14	SW	27.02	-14.98	14.06	10.95
	19.23	30200	30	NW	9.07	-13.68	8.33	- 4.47
194	14.17	30190	49	SE	8.51	- 7.78	8.70	-15.13
	15.36	30140	39	SE	- 4.81	33.13	- 5.74	25.50
	16.04	30190	27	SW	0.19	-52.43	- 2.04	-41.34
	17.23	30140	34	SW	-25.16	-14.55	-27.02	- 5.76
	18.07	30130	32	SW	17.58	- 9.80	15.54	- 0.58
	23.46	30120	66	SE	2.96	6.48	2.96	- 3.03
195	01.31	30120	19	SW	10.36	12.39	4.63	28.81
	02.15	30190	52	NE	7.59	37.16	6.66	29.39
	04.02	30280	28	NW	22.57	-18.73	21.46	- 8.79
	14.44	30140	17	SE	-57.36	6.77	-61.99	-12.39
	15.15	30190	58	SW	- 4.07	-26.65	- 5.37	-18.15
	18.54	30200	44	NW	-17.21	- 3.03	-17.39	4.32
196	00.37	30120	48	SW	-25.53	-27.37	-26.83	-19.73
	01.27	30190	25	NE	20.91	10.80	17.58	- 0.86
	07.43	30200	19	SW	13.14	10.80	8.51	28.52
	15.40	30140	48	SE	- 7.03	16.28	- 7.40	8.64
	17.26	30140	28	SW	- 2.59	- 6.05	- 4.63	4.32
197	01.27	30120	18	SW	-17.95	-15.41	-24.42	2.45
	02.25	30190	69	NE	-35.53	23.62	-36.45	12.10
	04.51	30200	24	SE	27.20	17.43	26.09	5.62
	05.24	30140	30	NW	24.61	-21.18	23.31	-11.81
	06.16	30130	23	NW	25.53	-14.41	23.31	- 2.31
	06.37	30200	55	SW	-14.43	-34.43	-15.73	-26.36
201	01.19	30120	16	SW	22.57	-20.45	12.95	1.01
	02.44	30190	70	NW	- 7.22	-47.39	- 6.85	-37.02
	03.44	30140	65	NE	1.67	55.75	0.93	45.66

\*NE = Quadrant north, east of station  
 NW = Quadrant north, west of station

SE = Quadrant south, east of station  
 SW = Quadrant south, west of station

**Table 8 (cont'd)**  
**Final navigation results,**  
**115 passes, days 183-203, 1977.**

Rise Time (1977)		Pass I.D.	Geometry		BRN-3 Navigation Errors (m)			
Day	Hr.Min	Satellite	Elev. (deg)	Head*	No Tropospheric Correction		Tropospheric Correction	
		I.D.			Latitude	Longitude	Latitude	Longitude
201	05.31	30140	19	NW	-51.62	-18.29	-54.03	- 2.74
	07.25	30200	18	SW	45.33	-26.65	40.71	- 7.63
	13.57	30190	55	SE	6.85	-11.96	6.48	-19.59
	14.55	30140	32	SE	6.48	1.73	5.18	- 7.06
	15.59	30130	46	SE	26.65	-10.52	26.09	-17.72
	19.15	30200	21	NW	23.68	-21.03	21.83	- 6.91
	22.38	30120	32	SE	-32.01	14.84	-32.94	6.05
202	01.56	30190	59	NE	13.14	19.30	12.21	10.66
	02.53	30140	28	NE	-40.89	15.70	-43.30	5.33
	03.43	30190	25	NW	- 2.96	- 1.44	- 4.81	9.65
	04.33	30200	24	SE	45.70	-21.75	44.59	-33.56
	06.19	30200	54	SW	7.40	-12.52	6.11	- 4.75
	10.35	30120	31	NE	20.91	-19.59	18.32	-28.81
	12.21	30120	39	NW	- 6.11	-25.21	- 6.66	-17.57
	14.55	30190	50	SW	12.40	-25.06	11.10	-17.14
	16.23	30200	22	NE	75.31	1.44	71.42	-12.53
	16.56	30130	62	SW	-15.73	0.72	-20.54	2.88
18.08	30200	63	NW	- 7.40	-26.07	- 7.22	-17.43	
203	01.08	30190	29	NE	3.52	- 4.90	0.74	-15.13
	05.35	30140	15	NW	-26.83	- 4.90	-31.27	16.85
	15.53	30190	16	SW	44.78	- 9.08	39.60	12.82
	17.54	30130	21	SW	33.31	-41.92	28.68	-27.08

\*NE = Quadrant north, east of station  
 NW = Quadrant north, west of station

SE = Quadrant south, east of station  
 SW = Quadrant south, west of station

**Table 9**  
 BRN-3 mean east/west\* navigation results,  
 115 passes, days 183-203, 1977.

Troposphere Corrected ?	Latitude Error (m)			Longitude Error (m)		
	East	West	East-West	East	West	East-West
No	4.5	5.2	-0.7	12.0	-17.0	29.0
Yes	2.7	2.7	0	1.5	- 5.2	6.7

\*East - 58 passes  
 West - 57 passes

**Table 10**  
 BRN-3 mean north/south\* navigation results,  
 115 passes, days 183-203, 1977.

Troposphere Corrected ?	Latitude Error (m)			Longitude Error (m)		
	North	South	North-South	North	South	North-South
No	2.2	7.5	-5.3	-2.3	-2.8	0.5
Yes	0.4	4.9	-4.5	-2.0	-1.9	-0.1

\*North - 54 passes  
 South - 61 passes

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## APPENDIX

In the AN/BRN-3 we have

$$f_d = - \frac{f_T}{c} \frac{d|\bar{\rho}|}{dt}, \quad (\text{A-1})$$

which, when corrected for tropospheric refraction effects, will have the form

$$f_d = - \frac{f_T}{c} \left[ \frac{d|\bar{\rho}|}{dt} + \frac{d(\Delta\rho)^T}{dt} \right], \quad (\text{A-2})$$

where

$f_d$  = Doppler frequency (c/s),

$f_T$  = navigator frequency (c/s),

$c$  = speed of light (m/s),

$\frac{d|\bar{\rho}|}{dt}$  = vacuum range rate (m/s), and

$\frac{d(\Delta\rho)^T}{dt}$  = tropospheric corrections to the range rate (m/s).

The H. D. Black model (Ref. 1) has the following form:

$$\Delta\rho = \Delta S_d + \Delta S_w, \quad (\text{A-3})$$

where

$$\Delta S_d = 2.343 P \frac{T-4.12}{T} \cdot I(h-h_d, e) \text{ and}$$

$$\Delta S_w = k_w \cdot I(h = h_w, e).$$



$$I(h, e) = \left\{ 1 - \left[ \frac{\cos e}{1 + (1 - \ell_c) \cdot h/r_s} \right]^2 \right\}^{-1/2} \quad (A-4)$$

$\Delta S_d$  = dry term tropospheric slant range correction (m)

$\Delta S_w$  = wet term tropospheric slant range correction (m)

$h_d$  = 148.98 (T - 4.12) m above the station (about 40 km)

$h_w$  = 13 000 m

$\ell_c$  = 0.85

$k_w$  = 0.05 - 0.28 (depending on season and latitude)

$r_s$  = distance from center of the earth to the station (m)

P = surface pressure in standard atmospheres

T = surface temperature (K)

e = instantaneous elevation angle

Differentiating Eq. A-3, above, we get

$$\frac{d(\Delta\rho)^T}{dt} = - \frac{\Delta S_d \cdot I^2 (h=h_d)}{H_d^2} + \frac{\Delta S_w \cdot I^2 (h=h_w)}{H_w^2} \cdot \sin e \cdot [\cos e \dot{e}] \quad (A-5)$$

$$H_d \triangleq 1 + (1 - \ell_c) \cdot h_d/r_s$$

$$H_w \triangleq 1 + (1 - \ell_c) \cdot h_w/r_s$$

Eq. A-5 can be simplified by noting that

$$H_d \cong 1 + (1 - 0.85) \frac{40}{6000} = 1 + 10^{-4} \sim 1 \quad \text{and}$$

$$H_w \cong 1 + (1 - 0.85) \frac{13}{6000} = 1 + 0.33 \cdot 10^{-4} \sim 1 .$$

Therefore, we can rewrite Eq. A-4 as follows:

$$I(h, e) = \left[ 1 - (\cos e)^2 \right]^{-1/2} = \frac{1}{\sin e} \quad . \quad (A-6)$$

Assuming an average world surface temperature of 15°C and pressure at sea level of 1 atm we get

$$\Delta S_d = \frac{2.31}{\sin e} \text{ (m)} ; \quad (A-7)$$

$$\Delta S_w = \frac{0.05}{\sin e} \text{ (m) for winter polar region (minimum); and}$$

$$\frac{0.28}{\sin e} \text{ (m) for summer tropics (maximum).}$$

Substituting Eqs. A-6 and A-7 into Eq. A-5 we get

$$\frac{d(\Delta \rho)^T}{dt} = - \left( 2.3 \frac{1}{\sin^3 e} + 0.165 \frac{1}{\sin^3 e} \right) \sin e \cos e \dot{e}$$

$$\boxed{\frac{d(\Delta \rho)^T}{dt} = - 2.475 \frac{\cos e}{\sin^2 e} \dot{e}} \quad , \quad (A-8)$$

where we used the mean value of  $\frac{0.165}{\sin e}$  for  $\Delta S_w$ . The maximum error in  $\frac{d(\Delta \rho)^T}{dt}$  is 6% for the extreme cases (summer tropics, winter polar).

Equation A-8 can be used in place of Eq. A-5 with negligible loss of accuracy.

Finally,  $\sin e$  and  $\cos e \dot{e}$  can be computed as follows:

$$\sin e = \frac{\bar{r}_s \cdot \bar{\rho}}{|\bar{r}_s| |\bar{\rho}|} ,$$

$$\cos e \dot{e} = \frac{1}{|\bar{\rho}| |\bar{r}_s|} \left[ \bar{\rho} \cdot \dot{\bar{r}} + \bar{r}_s \cdot \left( \dot{\bar{r}} - \dot{\bar{r}}_s - \frac{\dot{\rho}}{|\rho|} \bar{\rho}' \right) \right] ,$$

where

$\bar{r}_s$  = the (vector) position of the navigator (station) antenna,

$e$  = instantaneous elevation angle,

$\bar{r}$  = the (vector) position of the satellite antenna,

$\bar{\rho} \triangleq \bar{r} - \bar{r}_s$  , and

$\rho' \triangleq \frac{d}{dt} |\bar{\rho}|$  .

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