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ABSTRACT

We discuss the use of homogenization techniques to derive approximate models with simple geometry for physical models of grids and trusses which have a complex geometry that gives rise to computational difficulties. Our presentation is in the context of inverse or parameter estimation problems for composite material structures with unknown characteristics such as stiffness and internal damping. We present the necessary theoretical foundations for this approach and discuss comparison of modal properties of the resulting homogenization model for a two-dimensional grid structure with modal properties observed in experiments with this grid.

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1. Introduction

We report here on a part of our continuing efforts on the development of high fidelity dynamic models for composite material structures. The focus of our investigations has been on models to be used in estimation and control of large flexible structures mainly intended for use in space (e.g., antennas, platforms, solar panels, experimental arrays, etc.). At present there is a reasonably adequate understanding of the dynamics of beams and plates made from known materials such as aluminum alloys. Our recent efforts (see [BIn] and the references therein) involved models and methods to determine material parameters in composite material structures with simple geometry (beams with attached solid bodies or solid plates). These result in inverse problems that are computationally tractable as long as the physical geometry is relatively simple. However, substantial difficulties arise in cases involving more complex geometries such as grids (which may be viewed as plates with many holes) and trusses (solid columns from which most of the material is removed in some periodic, regular fashion). In these cases the difficulties associated with unknown composite material characteristics such as stiffness and internal damping are combined with severe difficulties related to computational grid selection for a domain that is mostly holes or perforations.

The purpose of this paper is to outline one possible methodology for dealing with these structures of complex geometry and to report on some of our initial investigations in this regard. This methodology is based on ideas from homogenization and requires that the structures be highly periodic (many perforations repeated in a regular pattern) and sparse in material (i.e., a grid or truss with thin members).

We formulate our ideas in the context of an inverse problem approximation framework that has been developed for problems involving simple geometries. A summary of this theoretical framework is presented in Section 2. In Section 3 we describe a particular grid structure which we have used in our experiments and indicate how to formulate a direct physical model for this grid within the framework of Section 2. This model leads to computational methods that are inherently intractable for estimation of parameters (and also for the ultimately desired control investigations) due to the underlying computational domain.

In Section 4 we present a summary of results for a homogenization procedure that approximates (in a nonstandard way) the original direct physical model on a perforated domain by a homogenized model on a domain that is very simple (the perforated domain with all of the perforations filled in). The resulting approximate model is also in a form to fit into the inverse problem framework of Section 2. Finally, in Section 5 we report on our initial efforts on validation of this approximate model by comparing experimentally observed modal properties of the grid with those possessed by the homogenized model.

2. Review of Theory for Second Order Systems

In this section we give a brief summary of the theoretical background necessary for a rigorous discussion of estimation problems for second order systems. Detailed discussions can be found in [BI], [BR1], [BR2], [BW], [B], [BK].

Let V and H be complex Hilbert spaces with V continuously and densely embedded in H. We may then formulate a Gelfand triple $V \hookrightarrow H \hookrightarrow V^*$ with duality pairing $\langle\ ,\ \rangle_{V^*,V}$ (e.g., see [W]), which we shall denote by $\langle\ ,\ \rangle$ when no confusion results. Let (Q, d) be a compact metric space of admissible parameters q. We consider the parameter dependent abstract second order system

$$\ddot{u}(t) + A_2(q)\dot{u}(t) + A_1(q)u(t) = f(t,q) \quad t > 0 \tag{2.1}$$

$$u(0) = u_0, \quad \dot{u}(0) = v_0,$$

where the operators $A_i(q) \in \mathcal{L}(V, V^*), i = 1, 2$, arise from parameter dependent sesquilinear forms $\sigma_i(q) : V \times V \to \mathbf{C}$ which represent generalized stiffness (σ_1) and damping (σ_2), respectively. More precisely, we consider the equation

$$\langle \ddot{u}(t), \phi\rangle + \sigma_2(q)(\dot{u}(t), \phi) + \sigma_1(q)(u(t), \phi) = \langle f(t,q), \phi\rangle \tag{2.2}$$

for all $\phi \in V$, where σ_1 is symmetric and σ_1, σ_2 satisfy the following conditions (a theory for damping forms σ_2 that satisfy weaker conditions than *(B)*, namely H-semiellipticity, can be found in [BI]):

(A) **Parameter Continuity:** *For each $\phi \in V$ there is a function $\varepsilon(\cdot,\cdot;\phi)$ on $Q \times Q$ satisfying $\varepsilon(q,p;\phi) \to 0$ as $q \to p$ in Q such that for each $\psi \in V$ we have*

$$|\sigma(q)(\phi,\psi) - \sigma(p)(\phi,\psi)| \le \varepsilon(q,p;\phi)|\psi|_V.$$

(B) **V-ellipticity:** *There exists $c_1 > 0$ such that for all $q \in Q$ and all $\phi \in V$ we have*

$$Re\ \sigma(q)(\phi,\phi) \ge c_1|\phi|_V^2.$$

(C) **Boundedness:** *There exists $c_2 > 0$ such that for all $q \in Q$ and all $\phi, \psi \in V$ we have*

$$|\sigma(q)(\phi,\psi)| \leq c_2 |\phi|_V |\psi|_V.$$

Under these conditions on σ_1 and σ_2, there exist operators $A_i(q) \in \mathcal{L}(V, V^*)$ such that

$$\sigma_i(q)(\phi,\psi) = \langle A_i(q)\phi, \psi\rangle \quad \phi, \psi \in V.$$

Hence equation (2.1), or equivalently, (2.2), is to be interpreted as an equation in V^*. Following standard practice, we may rewrite (2.1) in first order vector form in the coordinates $w = (u, \dot{u})$ and use semigroup considerations when discussing solutions. To this end, we define $\mathcal{H} = V \times H$ and $\mathcal{V} = V \times V$ and note that $\mathcal{V} \hookrightarrow \mathcal{H} \hookrightarrow \mathcal{V}^*$ also forms a Gelfand triple. We also define the sesquilinear form $\sigma(q) : \mathcal{V} \times \mathcal{V} \to \mathbf{C}$ by

$$\sigma(q)((\phi,\psi),(\eta,\xi)) = -\langle\psi,\eta\rangle_V + \sigma_1(q)(\phi,\xi) + \sigma_2(q)(\psi,\xi) \tag{2.3}$$

so that (2.2) may be written

$$\begin{aligned} \langle \dot{w}(t), \chi\rangle + \sigma(q)(w(t),\chi) &= \langle F(t,q), \chi\rangle \\ w(0) &= (u_0, v_0) \end{aligned} \tag{2.4}$$

for $w(t) = (u(t), \dot{u}(t))$ and $\chi = (\phi,\psi)$ in $\mathcal{V}$ with $F(t,q) = (0, f(t,q))$. Equivalently, we may write the equation in $\mathcal{V}^*$ as

$$\dot{w}(t) = \mathcal{A}(q)w(t) + F(t,q) \tag{2.5}$$

where $\sigma(q)(\chi,\zeta) = \langle -\mathcal{A}(q)\chi, \zeta\rangle$ with

$$\mathcal{A}(q) = \begin{bmatrix} 0 & I \\ -A_1(q) & -A_2(q) \end{bmatrix}.$$

Since σ_1 and σ_2 satisfy *(B)* and *(C)*, it is readily established that $\mathcal{A}(q)$ is the infinitesimal generator of an analytic semigroup $\mathcal{T}(t;q)$ on $\mathcal{V}$, $\mathcal{H}$ and $\mathcal{V}^*$.

Moreover, the unique solution $w \in L_2(0,T;\mathcal{H})$ of (2.5) for $w_0 = (u_0, v_0) \in \mathcal{H} = V \times H$ and $f \in L_2(0,T;H)$ is given by

$$(2.6) \qquad w(t;q) = \mathcal{T}(t;q)w_0 + \int_0^t \mathcal{T}(t-s;q)F(s,q)ds.$$

For computational purposes, one must consider approximation schemes for (2.6). Let H^N be a family of finite dimensional subspaces of H satisfying $H^N \subset V$ and the condition

(C1) For each $\phi \in V$, there exists $\phi^N \in H^N$ such that $|\phi - \phi^N|_V \to 0$ as $N \to \infty$.

Let $\mathcal{H}^N = H^N \times H^N$ and let P^N be the orthogonal projection of $\mathcal{H}$ onto $\mathcal{H}^N$. Moreover, let $\mathcal{A}^N(q) \in \mathcal{L}(\mathcal{H}^N)$ denote the operator obtained by restricting $\sigma(q)$ of (2.3) to $\mathcal{H}^N \times \mathcal{H}^N$; i.e., for $\chi, \zeta \in \mathcal{H}^N$,

$$\sigma(q)(\chi,\zeta) = \langle -\mathcal{A}^N(q)\chi, \zeta\rangle.$$

We denote by $\mathcal{T}^N(t;q) = e^{\mathcal{A}^N(q)t}$ the corresponding analytic semigroups on $\mathcal{H}^N$. The approximating systems for (2.6) are then defined by

$$(2.7) \qquad w^N(t;q) = \mathcal{T}^N(t;q)P^N w_0 + \int_0^t \mathcal{T}^N(t-s;q)P^N F(s,q)ds.$$

From an application of the Trotter-Kato theorem on convergence of semigroups, one can readily obtain that under conditions *(C1), (A), (B), (C)*, we have for each $\chi \in \mathcal{H}, \mathcal{T}^N(t;q^N)P^N\chi \to \mathcal{T}(t;q)\chi$ in $\mathcal{H}$, uniformly in t on compact intervals, whenever $\{q^N\}$ is an arbitrarily chosen sequence in Q with $q^N \to q$. Moreover, using the theory of analytic semigroups, one can argue that for each $\chi \in \mathcal{H}$ and each positive integer $k, \mathcal{A}^N(q^N)^k\mathcal{T}^N(t;q^N)P^N\chi \to \mathcal{A}(q)^k\mathcal{T}(t;q)\chi$ in $\mathcal{H}$, again uniformly in t on compact intervals.

It follows immediately that $w^N(t;q^N)$ of (2.7) converges in $\mathcal{H}$ to $w(t;q)$ of (2.6), with the convergence being uniform in t on compact intervals. Furthermore, for f sufficiently smooth (e.g., $f \in C^1([0,T],H)$ suffices), we find that $\dot{w}^N(t;q^N) \to \dot{w}(t;q)$ in $\mathcal{H}$ as $N \to \infty$, uniformly in t on compact intervals. Thus, we see that the first component $u_t^N(t;q^N)$ converges to $u_t(t;q)$ in V while the approximate acceleration $u_{tt}^N(t;q^N)$ converges in H to $u_{tt}(t;q)$. With more smoothness on f (e.g., $f \in C^2([0,T],H)$), we obtain that $u_{tt}^N(t;q^N)$ converges in V to $u_{tt}(t;q)$ as $N \to \infty$.

If V embeds continuously in $C(\Omega)$ (this is the case, for example, if $V \subset H^2(\Omega)$ where $\Omega \subset R^1$ or $\Omega \subset R^2$), then the above results lead to pointwise convergence (in the spatial variables as well as time) of approximations to the displacement, velocity or acceleration in dynamic problems involving structures such as beams and plates. These approximation results are precisely those needed to treat certain questions arising in inverse or parameter estimation problems for such structures.

To be more specific, suppose we are given a set of observations z in the observation space $\mathcal{Z}$ along with an observation map $\Gamma(q)$ from $\mathcal{D} \subset C([0,T],H)$ to $\mathcal{Z}$ for the system (2.1). Thus, z is a set of observations or data for $\Gamma u(q)$ where $u(q)$ is the solution to (2.1). A least squares estimation problem then consists of finding $\bar{q} \in Q$ which minimizes over Q the criterion

$$\Phi(q) \equiv |\Gamma u(q) - z|^2_{\mathcal{Z}}, \tag{2.8}$$

subject to (2.1). Typical examples included in this setting are problems for beams and plates where acceleration measurements (accelerometers) are available. In the case of a beam of length a, we might have observations z_{jk} of the acceleration $u_{tt}(t_j, x_k; q)$ at several times $t_j, j = 1, 2, \ldots, J$, in $[0,T]$ and at several locations $x_k \in (0,a], k = 1, 2, \ldots, K$, along the beam. Then $V \subset H^2(0,a)$ in the usual formulation and $\mathcal{D} = C^2([0,T],V), \mathcal{Z} = R^{JK}$ with $\Gamma u(q) = \{u_{tt}(t_j, x_k; q) : j = 1, \ldots, J,\ k = 1, \ldots, K\}$ so that

$$\Phi(q) = \sum_{j,k} |u_{tt}(t_j, x_k; q) - z_{jk}|^2.$$

For a rectangular plate occupying the region $(x,y) \in \Omega = (0,a) \times (0,b)$, use of accelerometers at locations $(x_k, y_k), k = 1, \ldots, K$, would lead to observations $\{z_{jk}\}$ for $\{u_{tt}(t_j, x_k, y_k; q)\}$. With $V \subset H^2(\Omega), \mathcal{D} = C^2([0,T],V)$ and $\mathcal{Z} = R^{JK}$ we have $\Gamma u(q) = \{u_{tt}(t_j, x_k, y_k; q) : j = 1, \ldots, J,\ k = 1, \ldots, K,\}$ and

$$\Phi(q) = \sum_{j,k} |u_{tt}(t_j, x_k, y_k; q) - z_{jk}|^2.$$

The corresponding approximate problems are formulated using the criterion

$$\Phi^N(q) = |\Gamma u^N(q) - z|^2_{\mathcal{Z}} \tag{2.9}$$

for solutions $w^N(q) = (u^N(q), \dot{u}^N(q))$ of (2.7). One then seeks a parameter $\bar{q}^N \in Q$ that minimizes $\Phi^N(q)$ over $q \in Q$ subject to (2.7).

Among the important questions related to such approximate problems are those pertaining to *parameter convergence*; i.e., for a given fixed set of data $z \in \mathcal{Z}$, do optimal parameters $\bar{q}^N$ for (2.9) converge in some sense to an optimal parameter for (2.8)? More generally, one can also incorporate continuous dependence of the optimal estimates on the data by employing the concept of *method stability.* If one is given a sequence $\{z^m\}$ of data sets that converge in $\mathcal{Z}$ to a data set z^0, and one denotes by $\bar{q}^N(z^m)$ and $\bar{q}(z^0)$ the optimal parameters (in general, these are sets) for (2.9) and (2.8) corresponding to z^m and z^0, respectively, then method stability requires that $\bar{q}^N(z^m)$ converges (in some appropriate setwise sense) to $\bar{q}(z^0)$ as $N, m \to \infty$. These issues are carefully discussed in [B], [BK], where it is shown that to insure both parameter convergence and method stability, it suffices to argue that $\Gamma u^N(q^N) \to \Gamma u(q)$ in $\mathcal{Z}$ for *any* sequence $\{q^N\}$ in Q with $q^N \to q$. From the discussions above one thus has that the conditions *(A), (B), (C)* on σ_1 and σ_2 and *(C1)* along with a compact parameter space (Q, d) are sufficient to treat these questions in the case of accelerometer data for beams and plates.

3. The AFAL Grid Structure

We consider a rectangular plate perforated with rectangular holes as depicted in Figure 3.1. A similar plate has been the subject of numerous experimental investigations at the Air Force Astronautics Lab (now a part of the Phillips Lab) at Edwards Air Force Base. Using the Love-Kirchhoff plate theory (the 2-dimensional analogue of the Euler-Bernoulli theory for beams), one can model such a grid structure as a second order system of the form (2.1) so that conditions $(A), (B), (C)$ of the previous section are satisfied. We summarize here previous findings and refer the reader to [BR1], [BR2], [R] and the references therein for more detailed discussions.

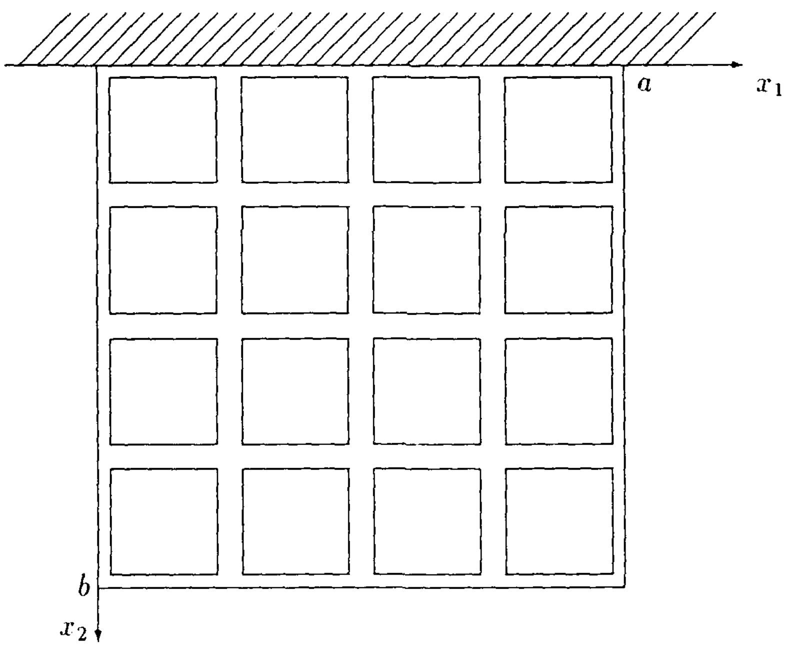

Figure 3.1

We assume that the plate is cantilevered along the x_1 axis (at $x_2 = 0$) as depicted in Figure 3.1 and is free on the other edges as well as the edges of the holes (perforations). If we use the Love-Kirchhoff theory and assume Kelvin-Voigt damping as well as viscous (air) damping, the transverse displacements

$u(t, x_1, x_2)$ at time t and location $x = (x_1, x_2)$ in the perforated domain $\Omega_{\text{per}} = (0, a) \times (0, b) - \{$ perforations $\}$ satisfy a second order (in time) system:

$$(3.1) \quad \rho h \frac{\partial^2 u}{\partial t^2} + \gamma \frac{\partial u}{\partial t} + \frac{\partial^2 M^1}{\partial x_1^2} + 2 \frac{\partial^2 M^{12}}{\partial x_1 \partial x_2} + \frac{\partial^2 M^2}{\partial x_2^2} = f \quad t > 0, (x_1, x_2) \in \Omega_{\text{per}},$$

where ρ is the mass density, h is the thickness of the plate, and M^1, M^2, and M^{12} are the bending moment about the x_1 axis, the bending moment about the x_2 axis and the twisting moment, respectively. If we include Kelvin-Voigt damping in these moments, we find

$$(3.2) \quad \begin{aligned} M^1 &= \frac{EI}{1-\nu^2}\left\{\frac{\partial^2 u}{\partial x_1^2} + \nu \frac{\partial^2 u}{\partial x_2^2}\right\} + \frac{c_D I}{1-\nu^2}\left\{\frac{\partial^3 u}{\partial x_1^2 \partial t} + \nu \frac{\partial^3 u}{\partial x_2^2 \partial t}\right\} \\ M^2 &= \frac{EI}{1-\nu^2}\left\{\frac{\partial^2 u}{\partial x_2^2} + \nu \frac{\partial^2 u}{\partial x_1^2}\right\} + \frac{c_D I}{1-\nu^2}\left\{\frac{\partial^3 u}{\partial x_2^2 \partial t} + \nu \frac{\partial^3 u}{\partial x_1^2 \partial t}\right\} \\ M^{12} &= \frac{EI}{1+\nu}\frac{\partial^2 u}{\partial x_1 \partial x_2} + \frac{c_D I}{1+\nu}\frac{\partial^3 u}{\partial x_1 \partial x_2 \partial t} \end{aligned}$$

where EI and $c_D I$ are the usual stiffness and Kelvin-Voigt damping coefficients and ν is Poisson's ratio. The plate is clamped along the x_1 axis where we must have the (essential) boundary conditions

$$(3.3) \quad u = \frac{\partial u}{\partial x_2} = 0, \text{ for } x_2 = 0, \quad 0 < x_1 < a, \quad t > 0.$$

Since the plate is free on its other edges and the hole edges, it must satisfy zero moment and zero shear conditions on these edges. These result in natural boundary conditions given by

$$(3.4) \quad M^1 = 0, \quad \frac{\partial M^1}{\partial x_1} + 2\frac{\partial M^{12}}{\partial x_2} = 0$$

on edges parallel to the x_2 axis, and

$$(3.5) \quad M^2 = 0, \quad \frac{\partial M^2}{\partial x_2} + 2\frac{\partial M^{12}}{\partial x_1} = 0$$

on edges parallel to the x_1 axis.

The system (3.1) - (3.5) can be readily formulated in the sesquilinear framework of Section 2 so that the results given there are applicable. To that end, define

$$V_{\text{per}} = H_b^2(\Omega_{\text{per}}) \equiv \{\phi \in H^2(\Omega_{\text{per}}) : \phi = \frac{\partial \phi}{\partial x_2} = 0 \text{ on } x_2 = 0\}$$

and

$$H_{\text{per}} = L_2(\Omega_{\text{per}}).$$

Then $V_{\text{per}} \hookrightarrow H_{\text{per}} \hookrightarrow V_{\text{per}}^*$ forms a Gelfand triple. Moreover, we may define sesquilinear forms $\sigma_i^{\text{per}}(q) : V_{\text{per}} \times V_{\text{per}} \to \mathbf{C}$, $i = 1, 2$ by

$$\sigma_i^{\text{per}}(q)(\phi, \psi) =$$

$$\int_{\Omega_{\text{per}}} \{q_i(\phi_{,11}\psi_{,11} + \phi_{,22}\psi_{,22}) + q_i q_3(\phi_{,11}\psi_{,22} + \phi_{,22}\psi_{,11}) + 2q_i(1 - q_3)\phi_{,12}\psi_{,12}\} dx, \tag{3.6}$$

where $q = (q_1, q_2, q_3, q_4) = (\frac{EI}{1 - \nu^2}, \frac{c_D I}{1 - \nu^2}, \nu, \gamma)$ and $\phi_{,ij} = \frac{\partial^2 \phi}{\partial x_i \partial x_j}$. It is not difficult to verify that $\sigma_i^{\text{per}}(q)$ satisfies conditions $(A), (B), (C)$ whenever Q is a compact subset of $C(\Omega_{\text{per}}, R^4) \cap \{q : q_i \geq c > 0, i = 1, 2, 3; q_4 \geq 0, q_3 < 1\}$.

The system may then be written for $u(t) \in V_{\text{per}}$ as

$$\langle \rho h u_{tt}, \psi \rangle + \sigma_1^{\text{per}}(q)(u(t), \psi) + \sigma_2^{\text{per}}(q)(\dot{u}(t), \psi) + \langle q_4 \dot{u}(t), \psi \rangle = \langle f, \psi \rangle \tag{3.7}$$

for all $\psi \in V_{\text{per}}$. Given initial data $u(0) = u_0, \dot{u}(0) = v_0$ with $u_0 \in V_{\text{per}}, v_0 \in H_{\text{per}}$, the methods outlined in Section 2 can be used to develop direct numerical methods as discussed in detail in [BR2]. While the theoretical aspects of approximation in parameter estimation problems are relatively straightforward conceptually given the theory of Section 2, the problems are computationally quite demanding. This is true even though the computational domain Ω_{per} is among the simplest and most regular of those arising in grid and truss structures.

The computational difficulties alluded to here motivate one to develop alternative methods for modeling of grid and truss structures for the purposes of identification and control.

4. Homogenization for Grid Structures

Given the difficulties associated with the model for the grid described in the previous section, it is of interest to approximate grid-like and truss-like structures with a model that retains physical fidelity but is computationally more tractable than a direct physical model. One such approximate model can be formulated using homogenization techniques which were originally developed to model composite material structures. Here we outline results for a grid structure similar to that of Section 3. Details for this grid and other lattice/truss derivations can be found in [BCR], [R], [BLP], [CS], [CD], and [SP].

The basic idea in our use of homogenization is to approximate a model such as (3.1) - (3.5) or, equivalently, (3.7), by a homogenized model *(HE)* on a domain in which the "holes" or perforations have been "filled in". That is, instead of a perforated domain $\Omega_{\rm per}$ as described in Section 3, we wish to compute on the domain $\Omega = (0, a) \times (0, b)$. The model associated with this domain, even if it is plate-like in form, will, of course, be nonphysical. The coefficients in the distributed system will be nonphysical parameters.

The homogenization procedures we outline here require periodicity in the structure (many regularly placed "holes") as well as sparseness of material in the structure. Thus we consider a grid structure like that in Figure 3.1, but with many "holes" or bays. We

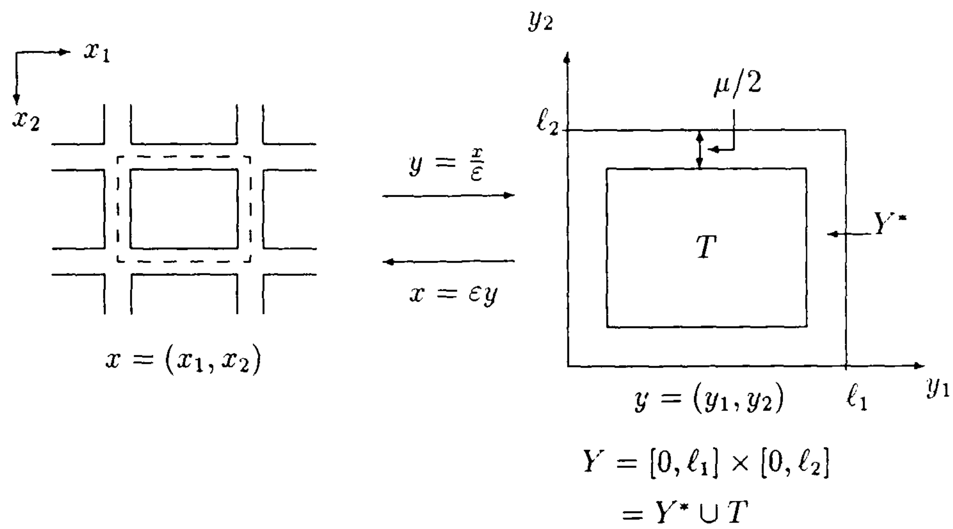

Figure 4.1

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consider a typical cell as depicted in Figure 4.1. We map this typical cell to a fixed size cell $Y = [0, \ell_1] \times [0, \ell_2]$ via a mapping $y = x/\varepsilon$ so that the original cell has dimension $\varepsilon\ell_1$ by $\varepsilon\ell_2$. As $\varepsilon \to 0$, periodicity in the original fixed domain $\Omega_{\rm per}$ increases. As $\mu \to 0$, the thickness of the members in the grid decreases. Our approximate equation on Ω will be obtained by taking limits as $\varepsilon \to 0, \mu \to 0$. Of course, the approximate limit model can be expected to be a better approximation to the actual grid structure if the grid has a large number of bays and thin members.

To facilitate our discussions, we parameterize the grid structure, denoting the perforated region by $\Omega_{\varepsilon,\mu}$, and consider a model of which (3.7) is a special case. (For convenience, we drop the viscous damping term for the discussions in this section.) We define $H_{\varepsilon,\mu} = L_2(\Omega_{\varepsilon,\mu})$ and $V_{\varepsilon,\mu} = H_b^2(\Omega_{\varepsilon,\mu}) = \{\phi \in H^2(\Omega_{\varepsilon,\mu}) : \phi = \frac{\partial\phi}{\partial x_2} = 0 \text{ on } x_2 = 0\}$. The generalized stiffness and damping sesquilinear forms are defined as

$$\sigma_1^{\varepsilon,\mu}(\phi, \psi) \equiv \int_{\Omega_{\varepsilon,\mu}} \sum_{i,j,k,h} a_{ijkh}^{\varepsilon,\mu}(x)\phi_{,kh}(x)\psi_{,ij}(x)dx \tag{4.1}$$

$$(4.2) \qquad \sigma_2^{\varepsilon,\mu}(\phi,\psi) \equiv \int_{\Omega_{\varepsilon,\mu}} \sum_{i,j,k,h} b_{ijkh}^{\varepsilon,\mu}(x)\phi_{,kh}\,\psi_{,ij}(x)dx$$

for $\phi, \psi \in V_{\varepsilon,\mu}$. Here and throughout we adopt the notation $\phi_{,ij}$ for $\frac{\partial^2\phi}{\partial x_i \partial x_j}$. We assume that the functions defined by

$$a_{ijkh}\left(\frac{x}{\varepsilon}\right) \equiv a_{ijkh}^{\varepsilon,\mu}(x) \text{ and } b_{ijkh}\left(\frac{x}{\varepsilon}\right) \equiv b_{ijkh}^{\varepsilon,\mu}(x) \text{ for } x \in \Omega_{\varepsilon,\mu}$$

are Y-periodic and that initial data

$$u_0^{\varepsilon,\mu} \in V_{\varepsilon,\mu}, v_0^{\varepsilon,\mu} \in H_{\varepsilon,\mu}$$

are given. We then consider the system for $u^{\varepsilon,\mu}(t) \in V_{\varepsilon,\mu}$ satisfying for $\psi \in V_{\varepsilon,\mu}$

(4.3)

$$\langle \rho h u_{tt}^{\varepsilon,\mu}(t), \psi \rangle + \sigma_1^{\varepsilon,\mu}(u^{\varepsilon,\mu}(t), \psi) + \sigma_2^{\varepsilon,\mu}(u_t^{\varepsilon,\mu}(t), \psi) = \langle f, \psi \rangle \quad t \in (0,T),$$

$$u^{\varepsilon,\mu}(0) = u_0^{\varepsilon,\mu}, \quad u_t^{\varepsilon,\mu}(0) = v_0^{\varepsilon,\mu}.$$

We wish to take limits as $\varepsilon \to 0, \mu \to 0$ in this system. Note however that as ε and μ change so does the domain $\Omega_{\varepsilon,\mu}$; hence limit taking is not a straightforward process. We need to first extend functions such as $u^{\varepsilon,\mu}(t), u_0^{\varepsilon,\mu}, f$, etc. to the entire domain Ω. We can do this by assigning the value zero to functions in the region $\Omega - \Omega_{\varepsilon,\mu}$. We shall denote by $\tilde{g}$ this extension by zero of any function $g \in L_2(\Omega_{\varepsilon,\mu})$. This results in $\tilde{g} \in L_2(\Omega)$. Such an extension is not so useful if we wish to deal with functions $g \in H^2(\Omega_{\varepsilon,\mu})$ (such as $u^{\varepsilon,\mu}(t)$) for which the H^2 smoothness is to be preserved. For such an extension, special extension operators must be formulated.

We shall proceed in two steps. We fix μ at some value and take limits as $\varepsilon \to 0$, obtaining an intermediate "homogenized" solution u^μ satisfying a certain intermediate equation. We then take the limit as $\mu \to 0$, obtaining the desired homogenized system *(HE)*.

Let $\mu > 0$ be fixed and let $u^\varepsilon = u^{\varepsilon,\mu}$ be the solution to (4.3) where we shall in our notation temporarily suppress the dependence on the fixed value of μ. Then we have

Lemma 4.1. *There exists an extension operator P^ε*

$$P^\varepsilon \in \mathcal{L}(L^\infty(0,T;V_{\varepsilon,\mu}), L^\infty(0,T;H_b^2(\Omega)))$$

and a function $u = u^\mu$ *such that for some sequence* $\varepsilon_n \to 0$ *we have*

$$P^{\varepsilon_n} u^{\varepsilon_n} \to u \text{ in } L^\infty(0,T; H_b^2(\Omega)) \text{ weak }^*$$

$$(P^{\varepsilon_n} u^{\varepsilon_n})_t = P^{\varepsilon_n} u_t^{\varepsilon_n} \to u_t \text{ in } L^\infty(0,T; L_2(\Omega)) \text{ weak }^*.$$

With additional assumptions on the initial data in (4.3), we can determine a limit equation that $u = u^\mu$ of Lemma 4.1 satisfies. Define $\theta = |Y^*|/|Y|$, where $|Y|$ denotes the measure (area) of Y. Then we have

Theorem 4.1. *Let* $u^\varepsilon = u^{\varepsilon,\mu}$ *be the solution of (4.3) and suppose that* $f \in L_2(\Omega_{\varepsilon,\mu})$ *so that* $\tilde{f} \in L_2(\Omega)$. *Moreover, suppose there exist* $u_0 \in H_b^2(\Omega)$ *and* $v_0 \in L_2(\Omega)$ *such that the initial data for (4.3) satisfy*

$$\tilde{u}_0^{\varepsilon,\mu} \to u_0 \quad \text{as } \varepsilon \to 0, \text{ weakly in } L_2(\Omega)$$

$$\tilde{v}_0^{\varepsilon,\mu} \to v_0 \quad \text{as } \varepsilon \to 0, \text{ weakly in } L_2(\Omega).$$

Then the convergence of Lemma 4.1 holds with the limit function $u(t) = u^\mu(t) \in H_b^2(\Omega)$ *satisfying the homogenized system*

(HE^μ)

$$\langle \theta \rho h u_{tt}(t), \psi\rangle + \sigma(t)(u(\cdot),\psi) = \langle \theta \tilde{f}, \psi\rangle \quad \text{for } t \in (0,T) \text{ and all } \psi \in H_b^2(\Omega)$$

$$u(0) = u_0/\theta, \quad u_t(0) = v_0/\theta,$$

corresponding to the homogenized hysteresis sesquilinear form

$$\sigma(t)(u(\cdot),\psi) = \int_\Omega \int_0^t \sum_{i,j,k,h} p_{ijkh}(t-\tau) u_{,kh}(\tau,x) \psi_{,ij}(x)\, d\tau\, dx. \tag{4.4}$$

The coefficients $p_{ijkh}(t) = L^{-1}[\hat{p}_{ijkh}(s)]$ *are the inverse Laplace transforms of the functions*

(4.5)

$$\hat{p}_{ijkh}(s) = L[p_{ijkh}](s) \equiv$$

$$\frac{1}{|Y|}\int_{Y^*} \{a_{ijkh}(y) + s b_{ijkh}(y) - \sum_{\ell,m} (a_{\ell mkh}(y) + s b_{\ell mkh}(y)) \chi_{,\ell m}^{ij}(s,y)\} dy$$

where the functions $\chi^{ij}(s,y)$ are Y-periodic solutions of

$$\int_{Y^*} \sum_{\ell,m,k,h} (a_{\ell mkh}(y) + s b_{\ell mkh}(y))(\chi^{ij}(s,y) - \mathcal{P}^{ij}(y))_{,\ell m}\, \psi_{,kh}(y)dy = 0$$

for all Y-periodic $\psi \in H^2(Y^*)$. Here $\mathcal{P}^{ij}(y) \equiv \frac{1}{2}y_i y_j$.

We note that the general homogenized plate equation (for μ fixed) for a grid with damping involves a time hysteresis functional of the solution. It is instructive to consider the special case of the grid with no damping; i.e., $b_{ijkh} = 0$. In this case we find that $\hat{p}_{ijkh}(s) = L[p_{ijkh}](s)$ is actually a constant, say $\hat{p}_{ijkh}(s) = \bar{a}_{ijkh}$ with

$$\bar{a}_{ijkh} = \frac{1}{|Y|}\int_{Y^*} \{a_{ijkh}(y) - \sum_{\ell,m} a_{\ell mkh}(y)\chi^{ij}_{,\ell m}(y)\}dy \tag{4.6}$$

where χ^{ij} is the solution of

$$\int_{Y^*} \sum_{\ell,m,k,h} a_{\ell mkh}(y)(\chi^{ij}(y) - \mathcal{P}^{ij}(y))_{,\ell m}\, \psi_{,kh}(y)dy = 0$$

for all Y-periodic $\psi \in H^2(Y^*)$. Recalling that the inverse Laplace transform of a constant is that constant times the Dirac delta function (i.e., $L^{-1}[\bar{a}_{ijkh}] = \bar{a}_{ijkh}\delta$), we find the sesquilinear form (4.4) reduces to

$$\sigma(t)(u(\cdot),\psi) = \int_\Omega \sum_{i,j,k,h} \bar{a}_{ijkh} u_{,kh}(t,x)\psi_{,ij}(x)dx,$$

or

$$\sigma(\phi,\psi) = \int_\Omega \sum_{i,j,k,h} \bar{a}_{ijkh}\phi_{,kh}(x)\psi_{,ij}(x)dx. \tag{4.7}$$

Thus, the hysteresis in the model is a result of nontrivial damping terms in the grid model. Moreover, we point out that the coefficients in (4.7), and hence the coefficients in *(HE $^\mu$)* for grids without damping, are *not* simple averages of the original coefficients a_{ijkh} of (4.3). Rather, the homogenized coefficients are averages of the original coefficients over the structure plus some correction terms as shown in (4.6).

We proceed to the next step by letting $\mu \to 0$ in *(HE $^\mu$)* and the associated equations given in Theorem 4.1. To facilitate our discussions, we restrict our considerations to the constant coefficient case; i.e., a_{ijkh} and b_{ijkh} are constants. Moreover, we take $\ell_1 = \ell_2 = 1$; hence $Y = [0,1] \times [0,1]$ and $|Y| = 1, |Y^*| = \mu(2-\mu)$ so that $\theta = \mu(2-\mu)$. It follows that

$$\frac{\theta}{\mu} \to 2 \quad \text{as } \mu \to 0.$$

One can establish the following.

Theorem 4.2. *For the constant coefficient problem with $\ell_1 = \ell_2 = 1$, there exists a sequence $\mu_n \to 0$ such that:*

(i) The coefficients $p^{\mu_n}_{ijkh}$ of (4.5) converge in the sense that

$$\frac{1}{\mu_n} p^{\mu_n}_{ijkh} \to p^*_{ijkh}$$

where

(4.8)

$$\hat{p}^*_{ijkh}(s) = L[p^*_{ijkh}](s) = 2(a_{ijkh} + sb_{ijkh}) - \sum_{\ell=1}^{2} \frac{(a_{ij\ell\ell} + sb_{ij\ell\ell})(a_{\ell\ell kh} + sb_{\ell\ell kh})}{a_{\ell\ell\ell\ell} + sb_{\ell\ell\ell\ell}}.$$

(ii) The solutions u^{μ_n} of (HE^{μ_n}) satisfy

$$u^{\mu_n} \to u^* \text{ in } L^\infty(0,T; H^2_b(\Omega)) \text{ weak}^*$$

where u^* is the solution to

(HE)

$$\langle 2\rho h u^*_{tt}(t), \psi\rangle + \sigma^*(t)(u^*(\cdot), \psi) = \langle 2\tilde{f}, \psi\rangle \quad t \in (0,T) \text{ and all } \psi \in H^2_b(\Omega),$$

$$u^*(0) = \lim_{\mu \to 0} \frac{\mu}{\theta} u_0 = u_0/2$$

$$u^*_t(0) = \lim_{\mu \to 0} \frac{\mu}{\theta} v_0 = v_0/2,$$

with

$$\sigma^*(t)(u^*(\cdot), \psi) = \int_\Omega \int_0^t \sum_{i,j,k,h} p^*_{ijkh}(t-\tau) u^*_{,kh}(\tau, x) \psi_{,ij}(x) d\tau \, dx. \tag{4.9}$$

We return to the AFAL grid structure of Section 3 corresponding to a direct physical model using the Love-Kirchhoff theory with Kelvin-Voigt damping which is given by equation (3.7) with the sesquilinear forms given by (3.6). This was the abstract variational form of (3.1) - (3.5). We specialize the results of this section to that example. We have the coefficients $a_{ijkh}^{\varepsilon,\mu} = a_{ijkh}$ given by

$$a_{1111} = a_{2222} = \frac{EI}{1-\nu^2} = q_1$$

$$a_{1122} = a_{2211} = \nu\frac{EI}{1-\nu^2} = q_1 q_3$$

$$a_{1212} = a_{2121} = a_{1221} = a_{2112} = \frac{EI}{2(1+\nu)} = \frac{1}{2}q_1(1-q_3)$$

with all other a_{ijkh} zero. The b_{ijkh} are given by

$$b_{1111} = b_{2222} = \frac{c_D I}{1-\nu^2} = q_2$$

$$b_{1122} = b_{2211} = \nu\frac{c_D I}{1-\nu^2} = q_2 q_3$$

$$b_{1212} = b_{2121} = b_{1221} = b_{2112} = \frac{c_D I}{2(1+\nu)} = \frac{1}{2}q_2(1-q_3)$$

with all other b_{ijkh} zero. In this case (3.6) and (4.1), (4.2) agree. Moreover, using (4.8) of Theorem 4.2, we find

$$\hat{p}^*_{1111}(s) = \hat{p}^*_{2222}(s) = EI + sc_D I = q_1(1-q_3^2) + sq_2(1-q_3^2)$$

$$\hat{p}^*_{1212}(s) = \hat{p}^*_{2121}(s) = \hat{p}^*_{2112}(s) = \hat{p}^*_{1221}(s) = \frac{EI}{1+\nu} + s\frac{c_D I}{1+\nu} = q_1(1-q_3) + sq_2(1-q_3)$$

with all other $\hat{p}^*_{ijkh}$ zero. It follows that σ^* from (4.9) can be written as the sum of two sesquilinear forms σ_1^* and σ_2^* given by

$$\sigma_i^*(\phi,\psi) = \int_\Omega \{q_i(1-q_3^2)(\phi_{,11}\psi_{,11} + \phi_{,22}\psi_{,22}) + 4q_i(1-q_3)\phi_{,12}\psi_{,12}\}dx.$$

We observe that σ_1^*, σ_2^* also satisfy the conditions (A), (B) and (C) of Section 2 so that the convergence results summarized in that section are readily applicable to the homogenized model in this case. If we write the homogenized model system in strong form analogous to (3.1) - (3.5), we obtain (we include the viscous damping term so as to compare with (3.1))

$$\rho h \frac{\partial^2 u^*}{\partial t^2} + \gamma \frac{\partial u^*}{\partial t} + \frac{\partial^2 M_*^1}{\partial x_1^2} + 2\frac{\partial^2 M_*^{12}}{\partial x_1 \partial x_2} + \frac{\partial^2 M_*^2}{\partial x_2^2} = \tilde{f} \quad t \in (0,T), x \in \Omega.$$

(4.11)

$$u^*(0) = u_0/2,\; u_t^*(0) = v_0/2$$

where $\tilde{u}_0^{\varepsilon,\mu} \to u_0$, $\tilde{v}_0^{\varepsilon,\mu} \to v_0$ weakly in $L_2(\Omega)$ as $\varepsilon \to 0$ as stated in Theorem 4.1 and the moments M_*^1, M_*^2, M_*^{12} are given by

(4.12)

$$\begin{aligned} M_*^1 &\equiv \frac{EI}{2}\frac{\partial^2 u^*}{\partial x_1^2} + \frac{c_D I}{2}\frac{\partial^3 u^*}{\partial x_1^2 \partial t} \\ M_*^2 &\equiv \frac{EI}{2}\frac{\partial^2 u^*}{\partial x_2^2} + \frac{c_D I}{2}\frac{\partial^3 u^*}{\partial x_2^2 \partial t} \\ M_*^{12} &\equiv \frac{EI}{1+\nu}\frac{\partial^2 u^*}{\partial x_1 \partial x_2} + \frac{c_D I}{1+\nu}\frac{\partial^3 u^*}{\partial x_1 \partial x_2 \partial t}. \end{aligned}$$

We note that M_*^{12} is the same as M^{12} of (3.2) while the form of equation (4.11) is the same as that of (3.1), with only the moments being different. Of course u^* must also satisfy the clamped boundary condition (3.3) along the edge $x_2 = 0$ of Ω and the free boundary conditions (3.4) (with M_*^1, M_*^{12}) along $x_1 = 0$ and $x_1 = a$ and (3.5) (with M_*^2, M_*^{12}) along $x_2 = b$.

5. Comparison with Experimental Results for the AFAL Grid

The homogenized model (4.11) - (4.12) for the AFAL Grid is, of course, a type of approximation to the physical model (3.1) - (3.5). Unlike many approximate models, we cannot increase the order of approximation by changing some parameters or mesh sizes. Even though *(HE)* is the result of a limiting procedure $\varepsilon, \mu \rightarrow 0$, the periodicity of bays and the thickness of members in a given grid are fixed and hence ε and μ are fixed. We can expect the homogenized model to be a better approximation for structures with more bays and thinner members than for structures with a small number of bays and relatively thick members.

Before using the model (4.11) - (4.12) for estimation and control problems for the AFAL Grid, we performed some initial experiments to test how well the homogenized model described quantitative properties of vibrational characteristics of the grid. One such investigation involved vibrational experiments (the grid was displaced initially from equilibrium and free release vibrations were observed; the data consisted of accelerometer measurements at several locations on the grid). Analysis of the data included experimental determination of the fundamental frequencies (which, of course, depend on the internal damping) for the grid. We then compared these values with those predicted by the analysis of the homogenization model (4.11) - (4.12).

The AFAL Grid used in our experiments was a $5ft.$ square ($a = b = 5$) with $16 = 4 \times 4$ square bays measuring $12'' \times 12''$. This corresponds to a calculated $\frac{\varepsilon\mu}{2} \simeq 1.2''$ with $\varepsilon \ell_i \simeq 14.4''$. The grid was constructed from aluminum alloy 6061-T6 for which hand book values for stiffness, etc., were available. We used $\rho h = .0550132 slugs/ft^2$, $\nu = \frac{1}{3}, EI = 135.64 lb \cdot ft, c_D = 1.728 \times 10^5 lb \cdot sec/ft^2$ and $\gamma = .02 slugs/ft^2 \, sec$ in our calculations with the model (4.11) - (4.12). We calculated the first eight frequencies ω_i^* for the homogenized model using two different approaches: an approximate mode shape technique based on textbook approximations [G], [L] and an eigenvalue analysis (MATLAB) for a finite element (bicubic B-splines modified to satisfy the essential boundary conditions) Galerkin approximation to (4.11) - (4.12).

We obtained essentially the same results with both approaches. These values are listed along with the experimentally obtained frequencies ω_i^{EXP} in the table below. We observe that for $i = 1, 2, 3$, there is good agreement while ω_4^* is approximately the average of ω_4^{EXP} and ω_5^{EXP}. There is reasonably good agreement between the pairs ω_i^* and $\omega_{i+1}^{\text{EXP}}$ for $i = 5, 6, 7$. This comparison suggests that use of the homogenized model is a reasonable approximation at least for vibrations involving modes with frequencies less than $16hz$.

Mode i	ω_i^*	ω_i^{EXP}
1	.785	.781
2	2.52	2.15
3	4.93	4.69
4	6.84	6.35
5	8.44	7.23
6	13.8	8.11
7	15.1	13.67
8	15.5	15.5

Table 4.1

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Final Report

16. Abstract

We discuss the use of homogenization techniques to derive approximate models with simple geometry for physical models of grids and trusses which have a complex geometry that gives rise to computational difficulties. Our presentation is in the context of inverse or parameter estimation problems for composite material structures with unknown characteristics such as stiffness and internal damping. We present the necessary theoretical foundations for this approach and discuss comparison of modal properties of the resulting homogenization model for a two-dimensional grid structure with modal properties observed in experiments with this grid.

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