AN ENVIRONMENTAL HEAT TRANSFER STUDY OF A ROCKET MOTOR STORAGE CONTAINER SYSTEM

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Monterey, California
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' The heat transfer characteristics of a rocket motor storage container system have been investigated using analytical and experimental techniques. Analytically, both closed form and numerical solutions have been developed. These solutions may be used to determine maximum temperatures and temperature gradients within the rocket motor. Comparison between theoretical and experimental values of temperature are within the estimated experimental uncertainties of $\pm 3^{\circ} \mathrm{F}$. It is proposed that the theoretical solutions can be used to thermally optimize container design.
A secondary investigation was carried out to determine the feasibility of using choiesteric liquid crystals, a temperature sensitive material, to thermally map the surface of the container. The crystals appear to remain stable under desert type conditions and produce brilliantly colored displays of the temperature field.
$i$

An Environmental Heat Transfer Study of
A Rocket Motor Storage Container System
by

Allen Henry Wirzburger Lieutenant, United States Navy
S.B., Massachusetts Institute of Technology, 1964

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MASTER OF SCIENCE IN MECHANICAL ENGINEERING


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## TABLE OF CONTENTS

II. BACKGROUND 14

III, EXPERIMENTAL PROCEDURE 17
IV. THEORETICAL ANALYSIS ..... 28
A. ONE DIMENSIONAL ANALY'IICAL MODEL ..... 28
B. TRUMP MODEL ..... 36
V. RESULTS ..... 43
A. ANALYTICAL MODEL ..... 43
B. TRUMP MODEL ..... 47

1. One Dimensional ..... 47
2. Two Dimensional ..... 52
C. GENERAL ..... 55
D. LIQUID CRYSTALS ..... 58
VI. CONCLUSIONS ..... 62
VII. RECOMMENDATTONS ..... 64
APPENDIX A: INTRODUCTION TO LIQUID CRYSTALS ..... 66
APFENDIX B: ANA: YTICAL SOLUTION ..... 72
APPENDIX C: TRUMI: SOUUTYION ..... 99
APPENDIX D: EXIERIMELTAL IAATA ..... 151
APPENDIX E: UNCSRMAINEX AVALYSIS ..... 161
LIST OF REFERENCES ..... 164
INITIAL DISTRIBUTION LIST ..... 166
FORM DD 1473 ..... 167

## LIST OF TABLES

I. Calibration of Liquid Crystals ..... 25
II. Thermal Properties of Materials ..... 100
III.Matrix Form of Energy Balance Equations ..... 110
IV. Radiosities at Nodes ..... 111
V. Change in Parameters Due to Changes in Thermal Properties ..... 161

## LIST OF ILLUSTRATIONS

Figure

1. Simulated Storage Dump at China Lake ..... 15
2. Thermocouple Locations on Expeximental System ..... 18
3. Top View of Rocket Motor Storage Container System ..... 19
4. Stevenson Shelter ..... 20
5. Rocket Motor Mounted in Storage Container ..... $2 ?$
6. Experimental System at Dump Storage Site ..... 26
7. Comparison of Sinusoidal Temperature Variation to Bulk Temperature ..... 29
8. Variation in Time Delay with Change
in Biot Modulus ..... 33
9. Variation in Relative Amplitude with Change in Biot Modulus ..... 34
10. Analytical Prediction of Temperature Variation with 'rime ..... 38
11. Comparison of Bulk Temperature to Two TRUMP Approximations ..... 39
12. Comparison of Analytical and Experimental Temperatures at Surface of Rocicet Motor ..... 44
13. Comparison of Analytical and Experimental
Temperatures at Center of Rocket Motor ..... 45
14. Comparison of $1-\mathrm{D}$ TRUMP and Experimental Temperatures at Surface of Rocket Motor ..... 48
15. Comparison of $1-D$ TRUMP and Experimental
Temperatures at Center of Rocket Motor ..... 49
16. Comparison of Temperatures from Four
TRUMP Variations at Surface of Rocket Motor ..... 50
17. Comparison of Temperatures from Four TRUMP Variations at Center of Rocket Motor ..... 51
18. Comparison of 2-D TRUMP and Experimental Temperatures at Surface of Rocket Motor ..... 53
19. Comparison of 2-D TRUMP and Experimental Temperatures at Center of Rocket Motor ..... 54
20. Temperature Distribution at Surface of Storage Container at Maximum Bulk Temperature ..... 56
21. Temperature Distribution at Surface of the Rocket Motor at Maximum Bulk Temperature ..... 57
22. Thermal Mapping with Liquid Crystals ..... 59
23. Liquid Crystals Feasible Under Hostile Environment ..... 60
24. Molecular Structure of Cholesteric Ester ..... 67
25. Light Reflection from Liquid Crystals ..... 67
26. Analytical Model of Experimental System ..... 73
27. Location of Nodes Jor One Dimensional TRUMP Model ..... 101
28. Location of Nodes for Two Dimensional TRUMP Mode1 ..... 104
29. Graphical Construction for Crossed- Strirgs Method ..... 105
30. Radiation Network ..... 108
31. Equivalent Radiation Network ..... 112
32. Thermocouple Locations for Experimental Data ..... 152

## TABLE OF SYMBOLS



|  | $=\text { effective thermal conductivity } \frac{B T U}{\operatorname{hr} f t^{\circ} \mathrm{F}}$ |
| :---: | :---: |
| $x_{n}$ | ```= radial distance from center of rocket motor to point n in``` |
| $r_{0}$ | = inner radius of rocket motnr in |
| $S_{n}$ | $=$ length of surface $n \quad$ in |
| t | $=$ time min |
| $\mathbf{T}$ | $=$ temperature of position $r$ at time $t \quad{ }^{\circ} \mathrm{R}$ |
| $\mathrm{T}_{\infty}$ | $=$ storage container temperature ${ }^{\circ} \mathrm{R}$ |
| $\mathrm{T}_{\mathbf{M}}$ | $=$ maximum temperature of storage container ${ }^{\circ} \mathrm{R}$ |
| $\mathrm{T}_{\mathrm{A}}$ | $=$ average temperature of storage container ${ }^{\circ} \mathrm{R}$ |
| Z | $=\sqrt{\frac{i \omega r_{o}^{2}}{\alpha}} \xi=\text { dimensionless distance parameter }$ |
| $\alpha$ | $=\text { thermal diffusivity } \quad \frac{f t^{2}}{h r}$ |
| $\beta$ | $=\frac{\bar{h} r_{0}}{k}=\text { Biot modulus }$ |
| $\delta$ | $=$ width of air gaij in |
| $\varepsilon$ | = emissivity |
| $\xi$ | $=\frac{x}{r_{0}}=\text { dimensionless distance }$ |
| $\theta$ | $=\frac{T-T_{A}}{T_{M}^{-} T_{A}}=\text { dimensionless temperature }$ |
| $\theta *$ | = dimensionless temperature for supplementary problem |
| $\theta^{2}$ | $=\text { construction angle for crossed-strings method }$ radians |
| $\theta_{5}$ | ```= relative amplitude of maximbs temperature at point of interest to the maximum temperature of the storage container``` |
| $\mu$ | $=$ dynamic viscosity $\frac{1 \mathrm{bm}}{\mathrm{ft-hr}}$ |

$$
\begin{aligned}
& \text { = density } \\
& \frac{1 b m}{f t^{3}} \\
& \sigma \quad=\text { Stefan-Boltzman constant } 0.171 \times 10^{-8} \frac{\mathrm{BTU}}{\mathrm{hr} \mathrm{ft}} \mathrm{ft}^{2-0} \mathrm{R}^{4} \\
& \tau \quad=\tau(t)=\text { solvtion of } \psi ; \\
& =e^{i m \omega t} \text { for large values of time } \\
& =\phi(r)=\text { solution of } \psi \\
& \psi \quad=\text { complex temperature }=\theta *(r, t)+i \theta(x, t) \\
& \omega \quad=\text { frequency of sinusoldal variation } \frac{2 \pi}{24 \text { hours }} \\
& \omega_{T} \quad \text { resulting uncertainty in calculated temperature } \\
& { }_{\mathrm{C}}{ }_{\mathrm{C}} \quad=\text { uncertainty in calculated temperature due to } \\
& \text { variation in volumetric heat capacity } \\
& \omega_{K} \quad=\quad \begin{array}{l}
\text { uncertainty in calculated temperature due to } \\
\quad \text { variation in conductivity }
\end{array} \\
& \omega_{\varepsilon} \quad=\quad \begin{array}{l}
\text { uncertainty in calculated temperature due to } \\
\\
\text { variation in emissivity }
\end{array} \\
& \text { Gr }=\frac{\rho^{2} g B(\Delta T) \delta^{3}}{\mu^{2}}=\text { Grashof Number } \\
& \text { Pr }=\frac{c \mu}{k}=\text { Prandtl Number } \\
& I_{0}, F_{0}, K_{0}, B E R, B E I \\
& X_{R}=\operatorname{BER}_{0}(a)+\frac{a}{\sqrt{2} \beta} B E R_{1}(a)+\frac{a}{\sqrt{2} \beta} B E I_{1}(a) \\
& X_{1}=B E I_{0}(a)+\frac{a}{\sqrt{2} \beta} B E I_{1}(a)-\frac{a}{\sqrt{2} \beta} B E R_{1}(a) \\
& \delta *=\tan ^{-1} \frac{B E 1_{0}(a \xi) X_{R}-\operatorname{BER}_{0}(a \xi) X_{i}}{\operatorname{BER}_{0}(a \xi) X_{R}+B E I_{0}(a \xi) X_{i}}=t i m e d e l a y
\end{aligned}
$$

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## I. "INTRODUCTION

The purpose of this investigation was to develop a heat transfer model that will fllow prediction of the temperature distribution in a container stored rocket motor placed in a hostile thermal environment such as the desert. It is proposed that such a model would be a useful tool for thermally optimizing future contaiier designs. As extreme variations In the rocket motor temperature may lead to large thermal stresses in the propellant which could result in fracture, or otherwise degrade the performance of the motor, a major objective of this study was to design a model that could reliably predict the thermal gradient in the motor. The predictions would be based on the surface temperature distribution, the thermal properties and the geometrical details of the system. The model may also be used to predict a critical temperature range over which the propellant must be chemically stable when in a storage situation. The upper limit of this temperature range is referred to as the design temperature of the system. As the design temperature for most weapon development projects is derived from dump storage conditions, a dump storage situation was used to obtain the experimental data for this project.

Several approaches were taken to predict the rocket motor temperature distribution from a knowledge of only the surface temperature distribution of the storage container and the thermal properties and geometrical details of the
experimental model. The experimental model used in this test was a once-fired Navy antisubmarine rocket (ASROC) motor, filled with dry desert blow sand to simulate the propellant, and placed in its storage container. This container system was placed in a dump storage site at the Naval Weapons Center, China Lake, California to simulate a desert environment.

The method of complex temperatures [Ref. 1] was used to de:elop an analytical prediction of the transient temperature field that exists in a container stored rocket motor. The analytical model assumes that heat is transferred only in the radial direction and that the container surface temperature variation is sinusoidal with time. Comparison between theory and experiment is within experimental uncertainty when temperature is interpreted as "bulk" temperature. The analytical model is especially useful for studying gemetrical and thermal physical property effects on rocket motor temperature. Such parameter studies have been carried out and the results are presented in a form that will be useful from a container design point of view.

TKUMP [Refs. 2 and 3], a computer program for transient and steady-state temperature distributions in multidimensional systems, was used to obtain detailed information about the thermal state of the rocket motor. TRUMP allows actual container surface temperature distributions to be used as well as sinusoidal variations. In addition, both one dimensional (radial) and two dimensional (radial and circumferential)
heat transfer were modeled with TRUMP, using both the sinusoidal and actual teinperature distributions. The actual. temperature distributions were obtained from the experimental data of the motor container system.

Comparisons between the experimental values and those predicted by the models were in good agreement, with those predicted $b y$ TRUMP using the actual temperature distribution as the boundary condition being the closest. However, the sinusoidal variations used in both the analytical model and the TRUMP model are also suitable for design purposes.

Another aspect of this project was to obtain the storage container surface temperature distribution using cholesteric 1iquid crystals, a material that undergoes brilliant changes in color over known, well defined temperature ranges. Color slides and movies were taken of the liquid crystals demonstrating the feasibility of using them for on site temperature measurements.

## II.: BACKGROUND

In 1959 the Naval Weapons Center, China Lake recognized the need for a concerted attack on the problem of thermail criteria assignment for new weapon systems. In 1963 a task force was established to study the complete environmental criteria determination problem. The key to this problem seemed to be the thermal area in the storage and transportation events of any item. It was realized that transportation was a short term situation compared to the storage situation. Therefore, the major portion of the life of an item must be in storage. There are three types of storage; covered, igloo and dump. The dump storage situation leads to the more extreme thermal exposure situations which then leads to the design temperature.

As data was not available for the dump storage situation, instrumented storage dumps were created at representative places on a woridwide basis so that statistical data could be derived on a variety of ordnance. The first site was at China Lake, California, in the middle of the Mojave Desert. This site now has the capability to return about 250 channels of information on a continuous time-temperature basis (Figure 1). Other arctic and tropical sites were set up to study extreme conditions.

The dump storage situation was reproduced to study the extreme situation. The ordnance was exposed singly, directly situated on the ground, with the long axis aligned in the

north-south direction to allow maximum normal exposure of the container surface to the sun's rays. In actual practice, ordnance is usually stacked and oriented in other than a north-south direction, thereby avoiding the extreme situation. Ordnance sitting on the ground receives reflected radiation from the ground, cannot quickly give off heat by conduction to the soil, and is not as apt to be cooled by the prevailing breeze; therefore, extreme temperatures result.

The most important source of heat to the ordnance is the direct radiation from the sun, with reflected radiation of secondary importance. For extreme conditions to occur the wind must be calm (less than 5 knots), the sky clear, and the outside air temperature high. After sunrise, the ordnance skin temperature rises much more rapidly than the ambient air temperature; therefore, the surrounding air cools the ordnance, rather than heats it.

The rocket motors used for the tests were military sur-plus. Even though the material had served its intended inFleet purpose, it was still representative of new hardware, when viewed in a thermodynamic context. When inert rocket motors were available, they were used intact; however, in most cases, once-fired hardware was used. Thoroughly dried desert blown sand, being similar in thermal properties to most propellants, was used to backfill empty rockei motors. It was assumed that the thermal response of the sand filled motors was essentially the same as actual propellant filled motors.

Although Naval Weapons Center, China Lake had accumulated vast amounts of data in the past, it was decided to in $u$ ment a rocket motor storage container system especially for this project. This would allow base data to be taken exactly where it was required. It also allowed variations in the system without interfering with one of China Lake's ongoing projects. An ASROC system was chosen for this study. The outer storage container was 75 inches long with an inner diameter of 18 inches and a wall thickness of $1 / 16$ inch. The rocket motor was 57 inches long with an outside diameter of 12 inches and a wall thickness of $1 / 4$ inch. Both the container and motor wexe made of steel.

The rocket motor storage container system was instrumented with 20 gage copper-constantan insulated thermocouple wire which has an ISA Calibration of $\pm 1-1 / 2^{\circ} \mathrm{F}$ over the range -75 to $+200^{\circ} \mathrm{F}$. Twenty-one thermocouples were originally placed on the systen with positions indicated in Figures 2 and 3. The ambient air temperature was measured with thermocouple number 19 which was located in a Stevenson shelter about 60 feet away from the system (Figure 4). The thermocouples were mounted intrinsically on the motor and storage container. Two small holes were drilled approximately $1 / 8$ inch apart in the metal and the individual wires were inserted in the holes. The metal was then hammered around the wires until a snug fit was obtaincd. Bead thermocouples were mounted at the


Figure 2. Thermocouple Locations on Experimental System.

Five thermocouples were located under the section painted with the liquid wiystals. Their locations corresponding to the ones shown above are: $\# 14=\# 1,\|15=\| 2, \# 16=\# 8, \# 17=\# 9$, and $18=\# 3$ (See Figure 3).

Figure 3. Top View of Rocket Motor Storage Container System.


Figure 4. Stevenson Sbelter.
center of the motor and in the air gap. The thermocouples located at the center of the motor were supported by small pleces of wood several inches from the kead. The use of these supports was necessary to keep. the thermocouples in position when the motor was being filled with sand. After all the thernocouples on the rocket motor were in place, the rocket motor was filled with dry desert blown sand. The wires from the two thermocouples located in the center of the motor were led out a hole in the end cap. To avoid settling of the sand after the motor was in place on the site, with a resulting air gap being formed between the sand and the motor skin, the sand was compacted by striking the sides of the motor with small sledge hammers and then adding additional sand through the hole in the end cap. This was continued until the sand was tightly packed. The hole in the end cap was then sealed. The rocket motor was carefully placed in its storage container (Figure 5) which had previeusly been instrumented with thermocouples. The thermocouples in the air gap were mounted by affixing the lead wire to the rocket motor at the desired position and then putting a 90 degree bend in the wire so that it placed the bead of the thermocouple approximately 1.5 inches into the air gap. Neither the thermocouples in the center of the motor nor those in the air gap could be considered accurately positioned; however, every effort was made to minimize positioning erxors. All thermocouple wires were located inside the storage container and were led through a

hole in one end. This hole was then sealed. The two halves of the storage container were then bolted shut.

The outer surface of the rocket motor and the inner and outer surfaces of the storage container were all painted various shades of haze gray. Weathering had caused the painted surfaces to appear fairly rough. This is typical of the conditions of a storage dump. From the condition of the surfaces, it was estimated that the emissivity was approximately 0.9.

Prior to loading the rocket motor into the storage container, it was decided to apply liquid crystals (See Appendix A) to part of the storage container surface in order to obtain a thermal mapping of the surface temperature at any instant of time. Liquid crystals are temperature sensitive materials that produce immediate thermal images in a pattern of colors which respond rapidiy to minute changes in substrate surface temperatures. A second reason for applying the crystals to the container surface was to determine the feasibility of using the crystals under adverse environmental conditions (desert atmosphere). Prinr to applying the crystals, a 15 inch strip of the storage container, 20 inches from one end, was sprayed with two coats of Testors Spray Pla E. crystals. A one inch strip of 11 different ranges of crystal, with approximately $1 / 2$ inch of black paint between them, was applied over the black paint. Two coats of each crystal were applied, using a small paint brush. The first coat was allowed to dry completejy before the second coat
was applied. After the crystals were dry, two coats of Rez polyurethane (gloss clear plastic coating, interior-exterior 77-5) coating were applied by brush completely covering the crystals and black painted area. The polyurethane coating was applied to protect the crystals from wind blown sand and from the ultraviolet rays of the sun. Ten of the eleven crystals had been previously calibrated [Ref. 4]. Using the constant temperature bath procedure recommended in Ref. 4, R-27 was calibrated and the compls :e calibration results are shown in Table $I$.

The rocket motor storage container system was then moved to the China Lake dump storage site. The system was aligned in a north-south direction, well away from the influence of other ordnance (Figure 6). The thermocouple leads were connected through a junction box and underground cable to a Honeywell Electronik 25 Recorder which had been calibrated to read the thermocouple output directly in degrees Fahrenheit to an accuracy of $\pm 1^{\circ} \mathrm{F}$. The recorder was located in an air-conditioned building about 60 feet from the system.

Initial data indicated that the number 7 thermocouple was not responding properly and therefore this data was neglected. Initial color photographs were taken of the liquid crystal.s and it was immediately apparent that good thermal mappings could be obtainad if the crystals were stable under the adverse desert environment. The brilliance of the colors exhibited by the crystals under the bright desert sun was much better than had been expected. The

TABLE I


system was allowed two weeks to reach a periodic steady state before additional photographic data was obtained.

Extensive photographic data was collected on 27 and 28 July 1972 after two weeks of exposure to the desert environment. Both super 8 mm and 16 mm color movies and 35 mm color slides were taken of the liquid crystals. No colored filters were used on any of the cameras, although standard haze filters were used to take the super 8 mm movies and most of the 35 mm slides.

At inis time, a second storage containe:, this one without a rocket motor inside, was instrumented with intrinsic thermocouples in the same manner as, the previous container. Ás only three data channels remained open on the recorder, only three thermocouples were applied to this new container. The three thermocouples were applied at the 0300,0900 , and 1200 positions at the midpoint of the container. This container was set end to end with the system that was already in place at the site. The purpose of this study was to determine if the inclusion of the rocket motor in the container had a significant effect on the surface temperature of the container. Thermocouple \#j was connected at the 0900 position, \#23 at the 1200 position, and \#24 at the 0300 position. It was immediately apparent that thermocouple \#7 was continuing to give unreliable readings and therefore the data taken on channel 77 was neglected.

## IV. THEORETICAL ANALYSIS

A. ONE-DIMENSIONAL ANALYTICAL MODEL

The first step was to try to devise an analytical model that would simulate the actual rocket motor storage container experimental system. The first simplifying assumption was that the storage container temperature could be modeled by a sine wave which had a period of 24 hours. A comparison of the sinusoidal variation to the average (bulk) storage container temperature [obtained by averaging the four thermocouple readings on the surface of the container (1, 8, 10 , and 12 ) as shown in Appendix $D$ ] is given in Figure 7.

The method of complex temperature as presented by Arpaci [Ref. 1] was used to find the steady periodic solution of a body experiencing a periodic sinusoidal disturbance. A complete analytical derivation is given in Appendix B. The general heat conduction equation $\AA n$ cylindrical coordinates was the basis for this derivation. It was assumed that there was one dimensional radial heat flow with no conduction in the axial or circumferential directions, that no heat sources existed in the model, that the rocket motor storage container system was infinitely long, and that the sinusoidally varying surface temperature was spacially uniform over the entire container surface. The storage container temperature is assumed to vary as

$$
T_{\infty}=\left(T_{M}-T_{A}\right) \sin \omega t+T_{A}
$$


where $T_{M}=$ maxinum bulk temperature of the storage container
$T_{A}=$ average bulk temperature of the storage container
$\omega=$ frequency of tie sinusoidal variation ( $2 \pi / 24$ hours)
$t=$ time (hours)
It was assumed that all the thermal properties remained constant over the temperature range of the problem. The effective heat transfer coefficient, $\bar{h}$, across the air gap between the storage container and the rocket motor combines the heat transfer effects of radiation, convection, and conduction into one coefficient. The radiation coefficient was linearized by assuming constant representative temperatures in the equation

$$
h_{\mathrm{RAD}}=\mathcal{F}_{1-2} \sigma\left(\mathrm{~T}_{1}+\mathrm{T}_{2}\right)\left(\mathrm{T}_{1}^{2}+\mathrm{T}_{2}^{2}\right)
$$

where $\sigma$ is the stefan-Boltzmann constant and $\mathcal{F}_{1-2}$ is the radiation exchange factor. The convection coefficient is the effective conductivity of air, obtained from the Beckmann correlations [Ref. 5], djvided by the width of the air gap. In the analytical model, the effective conductivity was assumed to equal the conductivity, thereby treating it as pure conduction and giving the equation

$$
\bar{h}=h_{R A D}+h_{C O N}
$$

An initial condition was not specified in this derivation as the only concern was the steady-state, periodic behavior. The steady-state solution is (Appendix B)

$$
\begin{aligned}
& \theta(r, t)=\frac{T(r, t)-T_{A}}{T_{M}-T_{A}}=\frac{\sqrt{B_{E R}^{o}}{ }_{0}^{2}(a \xi)+B E_{i_{0}}{ }^{2}(a \xi)}{\sqrt{X_{R}^{2}+X_{i}^{2}}} \sin (\omega t+\delta *) \\
& \theta(r, t)=\theta_{r} \sin \left(\omega t+\delta^{*}\right)
\end{aligned}
$$

where $T(x, t)=$ the temperature of a point $x$ in the rocket motor at time $t$

$$
\begin{aligned}
& a=\sqrt{\frac{\omega r_{o}^{2}}{\alpha}}=\text { conduction parameter } \\
& \xi=\frac{r}{r_{0}}=\begin{array}{l}
\text { dimensionless } \\
\text { of }
\end{array} \\
& x_{0}=\text { inner radius of the rocket motor } \\
& r=\text { distance from the centex of the rocket motor } \\
& \alpha=\frac{k}{p c}=\text { thermal diffusivity } \\
& \rho=\text { density } \\
& \mathrm{k}=\text { thermal conductivity } \\
& \text { c = specific heat } \\
& B E R=\text { real Bessel Function } \\
& \text { BEi = imaginary Bessel Function } \\
& X_{R}=\operatorname{BER}_{0}(a)+\frac{a}{\sqrt{2} \beta} \mathrm{BER}_{1}(a)+\frac{a}{\sqrt{2} \beta} \mathrm{BEI}_{1}(a) \\
& X_{i}=B E i_{0}(a)+\frac{a}{\sqrt{2} \beta} \mathrm{BEX}_{1}(a)-\frac{a}{\sqrt{2} \beta} \mathrm{BER}_{1}(a) \\
& \beta=\frac{\overline{\mathrm{h}} \mathrm{r}_{\mathrm{o}}}{\mathrm{k}}=\text { Biot modulus } \\
& \delta *=\tan ^{-1} \frac{\operatorname{BEX}_{0}(a \xi) X_{R}-B E R_{0}(a \xi) X_{i}}{\operatorname{BER}_{0}(a \xi) X_{R}+B E i_{0}(a \xi) X_{i}}
\end{aligned}
$$

Two computer studies were done based on the steady state solution. The first study was a completely dimensionless situation which served as a parameter study of the effects of varying a and $\beta$ on the temperature and the time lag of the temperature at various positions in the model.

$$
a=\sqrt{\frac{\omega r_{0}^{2}}{\alpha}}=\text { conduction parameter }
$$

and

$$
\beta=\frac{\bar{h} r_{o}}{k}=\text { Biot modulus }
$$

Parameter a was varied from 1.0 to 5.0 and $\beta$ was varied from 0.1 to 100. These were the only values studied, as
only values within this range are of interest in this type problem. The computer program and its output are given at the end of Appendix 5 . The output lists the following values:

1) a, the conduction parameter
2) $\beta$, the Biot modulus
3) $\xi$, the non-dimensional distance from the center of the motor
4) $\delta *$, the time delay between the maximum storage container temperature and the maximum temperature reached at the point of interest in the motor
5) $\theta_{r}$ the relative amplitude of the maximum temperature at the point of interest compared to the maximum temperature of the storage container

The time delay is given in radians, where $2 \pi$ radians equals one complete cycle. A graph of the time delay versus $\beta$ for a constant value of "a" is given in Figure 8 at three different positions within the motor. A graph of the relative amplitudes of the temperatures versus $\beta$ for a constant value of "a" is given in Figure 9. It was noted that for a constant value of "a", the time delay decreased as Became zarger. As the point of interest approaches the center of the rocket motor, the time delay increases. The relative amplitude of the temperatures also becomes larger as $\beta$ is increased when the value of " $a$ " is held constant. If $\beta$ is held constant and "a" is varied, the time delay increases and the relative amplitude decreases as "a" increases.

The second study was obtaining the analytical solution to the particular rocket motor storage container system

studied at China Lake. The thermodynamic properties of dry sand were obtained from Ref. 6 as

$$
\begin{aligned}
& \rho=94.8 \mathrm{lbm} / \mathrm{ft}^{3} \\
& \mathrm{k}=0.188 \mathrm{BTU} / \mathrm{hr} \mathrm{ft} \\
& c=0.195 \mathrm{BTU} / \mathrm{Jbm}{ }^{\circ} \mathrm{F}
\end{aligned}
$$

Substituting these values and using 1440 minutes (24 hours) as a complete cycle, the parameters a and $\beta$ were calculated for this model as

$$
a=\sqrt{\frac{\omega r_{0}^{2}}{\alpha}}=2.43
$$

where $r_{0}=5.75$ inches, the inner radius of the rocket motor.

$$
\beta=\frac{\overline{\mathrm{h}} \mathrm{r}_{0}}{\mathrm{k}}=2.90
$$

where $\bar{h}=h_{\text {CON }}+h_{\text {RAD }}$
and $h_{C O N}=\frac{k_{\text {AJR }}}{\Delta r}=6.48 \times 10^{-2} \frac{\mathrm{BTU}}{\mathrm{hr}-\mathrm{ft}^{2}{ }_{\mathrm{O} F}}$
where $\Delta r=2.94$ inches, the distance across the air gap and $k_{A I R}=1.62 \times 10^{-2} \frac{B T U}{h r f t^{\circ} \mathrm{F}}$

$$
h_{R A D}=7_{1-2} \sigma\left(T_{1}+T_{2}\right)\left(T_{1}{ }^{2}+T_{2}{ }^{2}\right)=1.09 \frac{\mathrm{BTU}}{\mathrm{hr} \mathrm{ft}}{ }^{2 \circ} \mathrm{~F}
$$

where $\sigma$ is the Stefan-Boltzmann constant, $\mathcal{F}_{1-2}$ is the radiation exchange factor which for this gecmetry is

$$
\exists_{1-2}=\frac{1}{\frac{1}{c_{1}}+\frac{r_{1}}{r_{2}}\left(\frac{1}{\varepsilon_{2}}-1\right)}=0.84
$$

when

$$
\varepsilon_{1}=\varepsilon_{2}=.9, \quad r_{1}=6.0, \quad r_{2}=8.94
$$

therefore $\overline{\mathrm{h}}=1.15 \mathrm{BTU} / \mathrm{hr} \mathrm{ft}{ }^{20} \mathrm{~F}$
The average surface temperature of the storage container was found to be $104^{\circ} \mathrm{F}$ for a particular day at China Lake,
with a maximum temperature of $138^{\circ} \mathrm{F}$. These values were obtained by averaging the readings of thermocouples $1,8,10$, and 12 as shown in Appendix $D$ which give the bulk temperature.

The temperatures of seven positions within the rocket motor were calculated and the results are printed at 30 minute intervals for one complete cycle in Appendix B. A graph of temperature versus time was plotted by the computer showing the relationship between the surface temperature of the storage container (TINF), the temperature on the outer skin of the rocket motor (TEDG), and the temperature at the center of the motor (TCEN). This graph is Figure 10 .
B. TRUMP MODEL

The rocket motor storage container system at China Lake was modeled on TRUMP, a numerical conduction code, (See Appendix $C$ for a description of the TRUMP program) to predict the temperature at any point in the system from a knowledge of the storage container surface temperature variation, the thermal properties and the geometrical details of the system. Two models were used to simulate the rocket motor storage container system and several variations of each model were investigated.

The first model assumed one dimensional heat transfer (radial). The system was modeled as two infinitely long concentric steel cylinders, the inner of which was filled with dry sand. A 2.94 inch air gap separated the ryinders. The model was subdivided into concentric volumetric elements
with representative nodal points as given in Figure 27, Appendix $C$. It was assumed that the storage container surface temperature was spacially uniform. From the data given In Appendix $D$ and the observation of the liquid crystals' thermal mapping, it was obvious that the temperature distribution on the storage container was not spacially uniform. In order to sfmulate a spacially uniform condition, the readings of the thermocouples located at the 1200,0300 , 0600 , and 0900 positions (\#1, 8,10 , and 12) were averaged and this average value of the surface temperature (referred to as the bulk temperature) was used as the spacially unform temperature distribution. Two methods were used to describe the container temperature. The first method used the maximum bulk temperature $\left(138^{\circ} \mathrm{F}\right)$ and the average bulk temperature $\left(104^{\circ} \mathrm{F}\right)$ of the storage container to generate a sine wave with a period of 24 hours ( 1440 minutes). The second method took the bulk temperature readings at two hour intervals and fed this data into the TRUMP program in a tabular (temperature versus time) form. The version of TRUMP used In this problem was limited to able length of 12 tabular values. TRUMP intexpolated between the tabular points. Figure 11 compares the actual bulk data with the sinusoidal approximation and the interpolated tabular values.

Several assumptions were made to simplify the solution of this problem. As the thermocouple data from the storage container gave an average value of the temperature across the $1 / 16$ inch steel wall, node 12 was modeled as a zero


volume boundary node with a known temperature impressed upon it. It was also assumed that heat transfer across the air gap occurred by radiation and conduction alone. Free convection effects were initially neglected. This assumption was later modified to investigate the free convection effects. All surfaces of the storage container and the outside surface of the rocket motor werc painted various shades of haze gray and it was estimated that the emissivity of these surfaces. was, 0.9. The radiation exchange factor, $\mathcal{F}_{1-2}$, for this model was the same as that for the analytical solution $\mathcal{F}_{1-2}$ $=0.84$ ). It was also assumed that there was perfect thermal contact between the rocket motor and the sand that filled it. This neglects the possibility that the sand might slightly settle after being on the site for a long period of time.

The second model assumed two dimensional heat transfer (radial and circumferential). The same physical model was used as in the one dimensional case with the sole exception that 48 nodes were used instead of 12 . The representative nodal points and an example of the thermal connections from one of the nodal points are shown in Figure 28 in Appendix $C$. The four nodes on the surface of the storage container were modeled as zero volume boundary nodes. The sinusoidal and tabular representations were used to describe the surface temperature of the storage container at each boundary node. Actual data taken at each position, rather than bulk data, were used as the input data for these representations.

The same assumptions made in the one dimensional case were also applicable to the two dimensional model. A complete discussion of the calculation of the radiation exchange factors in the two dimensional case is given in Appendix $C$.

The effect of natural convection was studied in both the one and two dimensional models. References 5 and 7 give correlations between the Grashof number based on the gap width and the effective thermal conductivity. The Grashof number was calculated from the equation

$$
G r=\frac{\rho^{2} g B(\Delta T) \delta^{3}}{\mu^{2}}
$$

where $\delta=$ width of the air gap

$$
\begin{aligned}
\Delta T= & \text { maximum temperature difference at any instant } \\
& \text { of time in the air gap } \\
B= & \frac{1}{T} \text { where } T=565^{\circ} \mathrm{R}
\end{aligned}
$$

At $T=565^{\circ} \mathrm{R}$, air has the following properties $\rho=0.071 \mathrm{bm} / \mathrm{ft}^{3}$ $\mu=0.046 \mathrm{Ibm} / \mathrm{hr}-\mathrm{ft}$

The maximum Grashof number for this experiment was calculated to be $1.25 \times 10^{6}$. The diameter ratio was approximately 1.5 and the $\log G r=6.1$. From the Beckmann correlation [Ref. 5], the effective thermal conductivity ratio ( $\frac{k_{c}}{k}$ ) was approximately 3.2. Using the Liu correlation [Ref. 7]

$$
\frac{k_{c}}{k}=0.135\left(\frac{P_{r}^{2} G r_{\delta}}{1.36+P_{r}}\right)^{0.278}=4.5
$$

where the Prandtl Number $=0.707$. An effective thermal conductivity of 4.0 was assumed as the average value of these
two correlations and it was used to study the effects of free convection. This change was placed into the TRUMP program by increasing the value of the thermal conductivity of air by a factor of 4 in each of the TRUMP runs.

## V. RESULTS

A. ANALYTICAL MODEL

Using the sinusoidal temperature distribution as an approximation to the actual average experimental data as shown in Figure 7, comparisons were made between predicted temperatures and actual temperatures for two radial loca~ tions in the rocket motor. Figure 12 compares the results on the surface of the rocket motor and Figure 13 does the comparison at the center of the rocket motor. An uncertainty analysis is given in Appendix E which establishes the uncertainty bounds for both the predicted and the actual temperatures. These uncertainty bounds are included.in Figures 12 and 13.

It is readily seen from Figure 7 that a sine wave was not an ideal fit as an approximation to the experimental data, as it varies as much as $20^{\circ} \mathrm{F}$ during part of the cycle. However, it was also noted that the sine wave closely approximated the experimental data during the heating phase of the cycle and only during the cooling phase were there large variations. As the main purpose of this study was to design a model that would be useful in optimizing storage container design, the errors in the cooling phase are not critical as long as the temperatures reach the same minimum point before beginning another cycie. Figure 12 shows that the maximum surface temperature of the rocket motor predicted by the analytical model is a good approximation to the actual


experimental data. Again it is noted that, in actuality, the motor cools faster than the predicted value. The maximum difference in temperatures on the surface of the motor is $15^{\circ}$ F. Figure 13 shows that the predicted value and the experimental value of the temperature at the center of the rocket motor were in close approximation except during the early stages of the cooling phase where a maximum temperature variation of about $5^{\circ} \mathrm{F}$ occurred.

One of the reasons the system cools faster than predicted could be the Jight breeze that is usually evident in the early afternoon hours at China Lake that is not present during the morning. No attempt was made to shield the system from the wind to study the effects of a light breeze on the surface temperature of the storage container.

Another point not taken into account by the analytical model is the fact that the time delay at any point in the system is not constant throughout the day as predicted in Figure 10, but varies as given by the data in Appendix $D$. Time delays between the peak temperature on the container surface and the peak temperature at the center of the rociket motor vary from about 250 to 400 minutes, whereas the low temperature on the surface of the container and the low temperature at the center of the rocket motor vary from about 150 to 250 minutes. The analytical model predicts a constant variation of 388 minutes at the center of the rocket motor and 159 minutes at the surface.
B. TRUMP MODEL

1. One Dimensional

Four variations of the one dimensional TRUMP model were investigated and compared to the experimental data. Figures 14 and 15 compare the TRUMP predictions to the actual experimental data at the surface and the center of the rocket motor, respectively. The TRUMP variation used for this comparison modeled the storage container temperature with tabular aata (See Figure ll) and assumed convection was present $\left(\frac{k_{c}}{k}=4.0\right)$. The uncertainty analysis (Appendix E) established the uncertainty bounds for both the experimental and the analytical data in these Figures. The variation between the bulk temperature predicted'by TRUMP and the experimental data ciosely matches with only two experimental points in Figure 14 falling outside the uncertainty bounds for this one dimensional model. Figure 11 shows that the tabular data that TRUMP interpolates is a good approximation to the averaged experimental data. At the center of the motor, as shown in Figure 15, all experimental points fall within the predicted error bounds. A comparison of the four one-dimensional TRUMP variations are given in Figures 16 and 17 at the surface and the center of the rocket motor respectively. It is clearly seen from these Figures that the convection assumption results in an increase of $2^{\circ} F$ in the maximum temperature and a decrease of $2^{\circ} \mathrm{F}$ in the minimum temperature on the surface of the rocket motor. This temperature change drops to $\pm 1.5^{\circ} \mathrm{F}$ at the center of the




motor as shown in Figure 17. The differences between the sinusoidal approximation anc the tabular approximation of the experimental data was clearly shown in Figure 11. The data in Figures 16 and 17 can be easily correlated to that in Figure 11, thereby explaining the differences in the predicted values.

## 2. Two Dimensional

Four variations of the two dimensional TRUMP model were investigated and compared to the experimental data. Comparisons of each TRUMP variation to the experimental data are given in Figures 18 and 19 for node 8 (located on the skin of the rocket motor at the 1200 position) and node 1 (at the center of the rocket motor) respectively. These Figures show that the TRUMP variations that used tabular data to model the suxface temperature of the storage container predicted temperatures that more closely approximated the experimental values than were those predicted by TRUMP variations using sinusoidal data to model the surface temperature. Appendix $D$ shows that all points on the surface of the storage container reach their minimum temperature at the same time; however, these points reach their maximum temperature as much as 200 minutes apart. Whereas, all the points on the surface of the storage container are in phase at the minimum temperature, they rapidly become out of phase as the container temperature rises. This varying phase shift makes it difficult to model the four boundary nodes with sinusoidal approximations which must have constant


phase shifts. Sizable exrors in the input data during some parts of the cycle were caused by these varying phase shifts. These errors in the input data led to the variations in the predicted temperature values. As noted in the one dimensional section, the inclusion of convection effects does not produce large variations in the predicted temperatures.

Figures 20 and 21 show the actual temperature distributions on the surface of the storage container and on the surface of the rocket, motor respectively at maximum bulk temperature compared to a two dimensional TRUMP program. The TRUMP variation used for this comparison assumed no free convection in the air gap and used tabular data to approximate the surface temperature of the storage container.
c. General

A comparison was made between surface temperatures on the storage container that contained the rocket motor and the storage container that was empty. The low temperature was about $4^{\circ} \mathrm{F}$ colder in the empty container; whereas the high temperature was about $4^{\circ} \mathrm{F}$ higher on the container that contained the rocket motor. The empty container had a faster response time than the one containing the motor. The differences in heat capacities, radiation effects, and natural convection all contribute to the changes in temperature noted.


Figure 20: Temperature Distribution at Surface of Storage Container at Maximum Bulk Temperature at approximately 1500 on 2 August 1972 .


Figure 2l: Temperature Distribution at Surface of. the Rocket Motor at Maximum Bulk Temperature at approximately 1500 on 2 August 1972 .

## D. LIQUID CRYSTALS

The encapsulated cholesteric liquid crystals applied to the surface of the storage container gave brilliant colors under the intense desert sun. These colors were much clearer and brighter than the same crystals viewed under laboratory lighting conditions. The liquid crystals photographed well in both the color movies and the color slides. The movies showed by time lapse photography the rapidly changing surface temperature of the storage container. Two sample color prints mate from the color slides are enclosed as Figures 22 and 23 to show the brilliance of the colors and the feasibility of obtaining data from color photos. The only photographic problem encountered was the intense reflection of the sunlight from the polyurethane film. This problem was partially overcome by taking the photographs from angles where the reflection was less intense. Qualitatively the liquid crystals were not adversely affected by the sun's rays after two weeks of desert exposure. No accurate quantitative determination was attempted; however, rough approximations were made at the site. These approximations were made by noting the color exhibited by a crystal at a certain time and then comparing the calibration of the crystal (Table 1) to the temperature recorded by the thermocouple located directly beneath the region of color change. The readings were within $\pm 2^{\circ} \mathrm{F}$, which was very encouraging, especially considering the approximations made while taking these measurements. Although photos were taken only during




[^0]the initial čwo weeks of the study, on site observations indicate that the cuystals are still showing brilliant colors after $3-1 / 2$ months. Preliminary evidence indicates that the polyurethane film did protect the crystals from decomposing from the sun's rays and from being worn away by the wind blown sand.

It was noted that the surface temperature of the storage container under the liquid cyrstals reached temperatures up to $15^{\circ} \mathrm{F}$ higher than a similar point not under the crystals. This $15^{\circ} \mathrm{F}$ difference was only evident when maximum temperatures were obtained. During sunlight. hours the temperature under the liquid crystals was always somewhat higher; however, at night both temperatures were equal. The difference in the container surface temperatures led to a difference of $4^{\circ} \mathrm{F}$ on the surface of the rocket motor and $1^{\circ} \mathrm{F}$ at the center of the motor. It is believed that the difference in emissivities of the gray and black surfaces resulted in the difference in container surface temperature.

## VI. CONCLUSIONS

From the results of this investigation, the following conclusions were drawn:

1. Although a sine wave is not a perfect fit for the experimental data at all points, it is useful in predicting bulk temperatures in the rocket motor, especially if only the high and low bulk temperatures are of concern. This is especially true in the one dimensional case. In the two dimensional case, the problem of phase shift variations make the method of sinusoidar variation less desirable, although still useful.
2. The simulation of the actual data by a table of temperatures gave the most accurate predictions of the experimental data. This method should be used whenever tabulated data are available; however, this will generally not be the case for design work, in which case the sinusoidal approximations must be used.
3. The flexibility of both the analytical and computer simulations allow the changing of many parameters. The resulting effects of these changes on rocket motor temperature can be studied with the models.
4. The convection assumption for this system resulted in only small changes in temperature and can be neglected when predicting design temperatures. Either the Liu or Beckmann correlation should be used to determine if convection can be neglected in a particular system.
5. The use of an empty storage container to obtain surface temperature data is a good approximation to using one with a rocket motor inside.
6. It is feasible to use liquid crystals for thermal mapping under desert conditions. Color photography with standard equipment gives excellent results since brilliant colors were observed.
7. The liquid crystals appear to be stable for at least two weeks under the desert conditions when protected with a polyurethane coating.
8. The application of the liquid crystal system to the surface of the storage container resulted in-large increases in the surface temperature of the container throughout the hottest part of the day. Care must be taken in applying and interpreting thermal readings from liquid crystals when exposed to radiant heating.

## VII. RECOMMENDATIONS

From the results of this basic study, the following recommendations for future work are offered:

1. To refine the results of this project, a second rocket motor storage container system should be instrumented with the following changes:
a. Liquid crystals snould not be applied to the system used as the experimental model. As steel is a good thermal conductor, axial conduction on the surface of the storage container may be significant. Heat flow from the area where the crystals axe applied may lead to higher than normal temperatures at other points on the surface of the container.
b. The rocket motor should be weighed before and after the loading of the dry sand so that an accurate determination of the density of the propellant simulant can be determined.
c. Four additional thermocouples should be located on the surface of the storage container to better enable the averaging of data. At present, the \#l thermocouple which was used as the average temperature reading of the top quarter of the surface of the container, in actuality is its hotest point; likewise the $\$ 10$ thermocouple was used as the average temperature of the bottom quarter of the surface, in actuality it's the coldest point. For averaging data, it is recommended that thermocouples be placed at 0130,

0430,0730 , and 1030 and the quarters of the system be divided at $0300,0600,0900$, and 1200 to give a more realistic bulk temperature. Thermocouples at 1200 and 0600 will provide the maximum and minimum temperature of the system.
2. The TRUMP program should be rerun in both the one and two dimensional form, varying the mesh sizes to determine the optimum number of nodes.
3. A long term study of the effects of the desert environment on liquid crystals should be done. The crystals should be calibrated before being placed in the desert and then brought to a laboratory for recalibration at specific intervals.
4. Several modifications should be made to the TRUMP program to make it comparable to the version used at Lawrence Radiation Laboratory. The variable conductivity section (BLOCK 2) and the PJOT subroutine (BLOCK 1I) need to be corrected. The TIMEP subroutine which allows the setting of the problem time interval between data output should be added to this version of TRUMP. It would also be advantageous to increase the amount of tabular data that could be read in as boundary temperatures.

5:. From an acadenfc standpoint, the effects of free convection in an air gap with varying boundary temperatures should be investigated.

APPENDIX /.

## Introduction to Liquid Crystals

Liquid crystals were first discovered in 1889 by Refaitm zer [Ref. 8] and the investigations of Lehmann which continued to 1915. Liquid crystals were considered to be laboratory curiosities with no scientific or practical merit until the $1950^{\prime}$ s. They share some of the properties of both liquids and crystals; for example, a typical liquid crystal substance scatters light in symmetrical patterns and reflects different colors depending on the angle from which it is viewed. Studies in the last few years have helped to clarify the unusual molecular structure of liquid cyystals. Many applications arise from their ability to detect minute fluctuations in cemperature, mechanical stress, electronagnetic radiation and chemical environment by changes in their color.

Liquid crystals are divided into three classes; smectic, nematic, and cholesteric, depending on the degree of spatial arrangement of the molecules in the mass of the material and the type of the material [Ref. 9]. In this project only cholesteric liquid crystals were used and therefore only their properties will be mentioned. The molecular structure of cholesteric liquid crystals is characteristic of the esters of cholesterol (Figure 24). The molecular layers are very thin with the long axis of the molecules parallel to the plane of the layers. The individual molecules are


Figure 24: Molecular Structure of Cholesteric Estex


Figure 25. Light Reflection from Liquid Crystals.
basically flat, with a side chain of methyl groups ( $-\mathrm{CH}_{3}$ ) projecting upward from the plane of each molecule. This configuration causes the direction of the long axis of the molecules in each layer to be displaced slightly from the corresponding direction in adjacent layers. This displacement, which averages about fifteen minutes of arc per layer, is cumulative through successive layers, and the overall displacement traces out a helical path.

The molecular structure of cholesteric liquid crystals gives rise to many peculiar optical properties. If linearly polarized light is transmitted perpendicularly to the molecular layers, the direction of the electric vector of the light will be rotated to the left in a helical path. Therefore, the plane of polarization will also be rotated to the left, through an angle proportional to the thickness of the transmitting material. Liquid crystals are the most optically active substances known. Another strictly crystalline optical property exhibited by cholesteric liquid crystals is circular dichroism. When ordinary white light is incident to a cholesteric material, the light is separated into two components, one with the electric vector rotating clockwise, the other rotating counterclockwise. Depending on the material, one of these components is transmitted, and the other is reflected. It is this property that gives the cholesteric phase its iridescent color when it is illuminated by white light. The particular combination of colors depends on the material, the temperature, and the angle of the incident light.

The molecular structure of a cholesteric substance is very delicately balanced and is easily upset. Any small disturbance that interferes with the weak forces between the molecules can produce marked changes in optical properties such as reflection, transmission, birefringence, circular dichroism, optical activity and color. The most striking optical transformation that occurs in a cholesteric substance, in response to small changes in its environment, is the variation of color with temperature. The crystal lattice is disrupted by the thermal vibrations giving successive transitions between the solid, the mesophase, and the isotropic liquid with rising temperature. The change from the thre dimensional order of the crystal lattice to the disorder of the isotrepic 1 iquid occurs via one or more intermediate states, each of which has a particular temperature range at which it is stable [Ref. 10].

A cholesteric liquid crystal system responds to changes in temperature by sequentially passing through the complete visual spectrum (red through violet) in fractions or multidegrees, depending on which cholesterol esters comprise the formulation. This color phenomenon is reversible and has been reported to function over a temperature range of $-20^{\circ} \mathrm{C}$ to $250^{\circ} \mathrm{C}$. A very important point to note is that at a certain temperature a given material or combination of materials will alwiye extrotit the same aolor. Also, the rate of change from color to color as well as the exact temperature at which the specific color changes occur are invariable. By
mixing cholesteric substances in various proportions, any desired temperature combination can be obtained. The thickness of the cholesteric film does not affect the predominant wave length of the reflected light; the light becomes circularly polarized [Ref. 11].

The colors scattered by the liquid crystals represent only a fraction of the incident light (Figure 25 ). The remaining portion of the incident light is transmitted by the liquid crystals. Therefore, an absorptive black background must be used to prevent reflection of the transmitted light, thereby enhancing the resolution of the scatteres colors or wavelengths reflected by the liquid crystal system.

The cholesteric liquid crystal systems often present a number of problems due to the fact that they are viscous liquids. Some problems associated with the handling and the use of these materials are:

1. The tendency of the liquid crystal system to flow during application can cause variations in applied film thickness. This may result in non-uniform thermal patterns.
2. Direct exposure of liquid crystals to adverse environmental effects can cause variations in their sensitivity and deteriorate their color response in a few days.

These problems can be partially overcome by using an encapsulated liquid crystal material system. The capsules are 20-30 microns in diameter and are a water-based siurry suitable for application by conventional coating techniques such as brushing or spraying.

Encapsulated liquid crystals offer several advantages:

1. They convert the liquid crystal system to a pseudosolid, which provides for easier handling, application, and use.
2. They provide longer shelf life by minimizing surface contamination and giving protection from ultraviolet light [Ref. 12].
3. They exhibit relatively unlimited fatigue life.
4. They reduce the angular dependence of the color observed.

## APPENDIX B

Analytical Solution

The method of complex temperature as presented by Arpaci [Ref. 1] was used to find the steady periodic solution of a body experiencing a periodic sinusoidal disturbance. The general heat conduction equation in cylindrical coordinates was the basis for this derivation. It was assumed that no heat sources existed in this problem, that the rocket motor storage container system was infinitely long, that there was no heat conduction in the axial or circumferential directions, and that the container surface temperature was spacially uniform. Figure 26 gives a basic sketch of the system. The assumptions reduced the heat conduction equation to

$$
\begin{equation*}
\frac{1}{r} \frac{\partial\left(r \frac{\partial T}{\partial r}\right)}{\partial r}=\frac{1}{\alpha} \frac{\partial T}{\partial t} \tag{I}
\end{equation*}
$$

where $r$ is the radial distance from the center of the rocket motor, $T$ is the temperature of the rocket motor at time $t$ and position $r$, and $\alpha$ is the thermal diffusivity, a property of the conducting material.

$$
\begin{equation*}
\alpha=\frac{k}{\rho c} \tag{2}
\end{equation*}
$$

where $k$ is the thermal conductivity of the conducting material, $\rho$ is the density of the material, end $c$ is the specific heat. All thermal properties were assumed to be constant over the temperature range of this problem.


Figure 26. Analytical Model of Experimental System.

The boundary conditions used in this derivation were

$$
\frac{\partial \dot{T}}{d r}=0 \quad \text { at } r=0
$$

and $\frac{\partial T}{d r}=-\frac{\vec{h}}{k}\left(T-T_{\infty}\right) \quad$ at $r=r_{0}$
where $r_{0}$ is the inner radius of the rocket motor.
$T_{\infty}$ is the known storage container temperature which is assumed to vary as

$$
T_{\infty}=\left(T_{M}-T_{A}\right) \sin \omega t+T_{A}
$$

where
$T_{M}=$ maximum bulk temperature of the storage container $T_{A}=$ average bulk temperature of the storage container $\omega^{A}=$ frequency of the sinusoidal varizition ( $\frac{2 \pi}{24 \text { hours }}$ )
$t=t i m e$
$\bar{h}$ is the effective heat transfer coefficient across the air gap between the storage container and the rocket motor. It combines the heat transfer effects of radiation, convection, and conduction into one coefficient. The radiation coefficient was linearized by assuming constant temperatures ( $T_{1}, T_{2}$ ), representative of the average temperatures expected in the problem, in the equation

$$
h_{R A D}=7_{1-2} \sigma\left(T_{1}+T_{2}\right)\left(T_{1}^{2}+T_{2}^{2}\right)
$$

where $\sigma$ is the Stefan-Boltzmann constant and $\mathcal{F}_{1-2}$ is the radiation exchange factor between surfaces 1 and 2 . The convection coefficient is

$$
\mathbf{h}_{\mathrm{CON}}=\frac{\mathbf{k}_{\mathrm{c}}}{\delta}
$$

where $k_{c}$ is the effective conductivity of air as obtained from the Beckmann and Liu correlations [Ref. 5 and 7] and $\delta$ is the width of the air gap. In the analytical model,
the effective conductivity was assumed to equal the conductivity, thereby treating it as pure conduction and

$$
\bar{h}=h_{\mathrm{RAD}}+\mathrm{h}_{\mathrm{CON}}
$$

Equation (1) was non-dimensionalized using the following relationships

$$
\begin{aligned}
& \theta=\frac{T-T_{A}}{T_{M}-T_{A}} \quad \text { (a non-dimensional temperature) } \\
& \xi=\frac{r}{r_{0}} \quad \text { (a non-dimensional distance) }
\end{aligned}
$$

to give

$$
\frac{1}{\xi} \quad \frac{d\left(\xi \frac{\partial \theta}{\partial \xi}\right)}{\partial \xi}=\frac{r_{o}^{2}}{\alpha} \frac{\partial \theta}{\partial t}
$$

with boundary conditions

$$
\frac{d \theta}{d \xi}=0 \quad \text { at } \quad \xi=0
$$

and $\frac{d \theta}{d \xi}=-\beta(\theta-\sin \omega t)$ at $\xi=1$
where $\beta=\frac{\overline{\underline{h}} r_{0}}{k}$ is the Biot modulus (which compares the relative magnitudes of the effective heat transfer coefficient across the air gap and the internal conduction resistances to heat transfer).

An initial condition was not specified as the only
concera was with the steady state, periodic behavior. Following Arpaci [Ref. l], a complex temperature was defined as

$$
\psi(r, t)=\theta^{*}(r, t)+i \theta(r, t)
$$

where $\psi(x, t)$ satisfied

$$
\begin{equation*}
\frac{1}{\xi} \frac{\alpha\left(\xi \frac{\partial \psi}{\partial \xi}\right)}{\partial \xi}=\frac{r_{0}^{2}}{\alpha} \frac{\partial \psi}{\partial t} \tag{3}
\end{equation*}
$$

with boundary conditions

$$
\frac{\partial \psi}{\partial \xi}=\frac{\partial \theta^{*}}{\partial \xi}+i \frac{\partial \theta}{\partial \xi}=0 \quad \text { at } \xi=0
$$

and $\frac{d \psi}{d \xi}=-\beta\left(\psi-e^{i \omega t}\right)=-\beta\left(\theta^{*}-\cos \omega t\right)+i\{-\beta(\theta \cdot \sin \omega t)\}$ at $\xi=1$
This leads to $\theta(r, t)$ which satisfied

$$
\frac{1}{\xi} \frac{\alpha\left(\xi \frac{d \theta}{d \xi}\right)}{\partial \xi}=\frac{r_{0}^{2}}{\alpha} \frac{d \theta}{d t}
$$

with boundary conditions

$$
\frac{d \theta}{d \xi}=0
$$

$$
\text { at } \xi=0
$$

and $\quad \frac{d \theta}{d \xi}=-\beta(\theta-\sin \omega t) \quad$ at $\xi=1$
also $\theta^{*}(\underline{r}, t)$ which satisfied

$$
\frac{1}{\xi} \frac{d\left(\xi \frac{d \theta}{d \xi}\right)^{*}}{d \xi}=\frac{r_{0}^{2}}{\alpha} \frac{d \theta^{*}}{d t}
$$

with boundary conditions

$$
\begin{array}{ll} 
& \frac{d \theta^{*}}{d \xi}=0 \quad \text { at } \xi=0 \\
\text { and } \quad \frac{d \theta^{*}}{d \xi}=-\beta\left(\theta^{*}-\cos \omega t\right) \text { at } \xi=1
\end{array}
$$

A solution of the form

$$
\psi(x, t)=\phi(r) \tau(t)
$$

was assumed, where for large values of time $\tau(t)$ was assumed to equal $e^{\text {t wt }}$; therefore,

$$
\begin{equation*}
\psi(r, t)=\phi(r) e^{i \omega t} \tag{4}
\end{equation*}
$$

Equation (4) was then substituted into equation (3)

$$
\begin{equation*}
\frac{1}{\xi} \frac{d\left(\frac{\bar{\xi}}{} \frac{d \phi}{d \xi}\right)}{d \xi}-\frac{i \omega r_{o}^{2} \phi}{\alpha}=0 \tag{5}
\end{equation*}
$$

with boundary conditions

$$
\begin{array}{ll}
\frac{d \phi}{d \xi}=0 & \text { at } \xi=0 \\
\text { and } & \frac{d \phi}{d \xi}=-\beta(\phi-1)
\end{array} \text { at } \xi=1
$$

Equation (5) was expanded to give

$$
\begin{equation*}
\frac{\partial^{2} \phi}{\partial \xi^{2}}+\frac{1}{\xi} \frac{\partial \phi}{\partial \xi}-\frac{i \omega r_{o}^{2}}{\alpha} \phi=0 \tag{6}
\end{equation*}
$$

$$
\text { Now, let } z=\sqrt{\frac{i \omega r_{0}^{2}}{a}} \xi
$$

with boundary conditions

$$
\frac{d \phi}{d z}=0 \quad . \quad \text { at } z=0
$$

and

$$
\frac{d \phi}{d z}=-\frac{\beta}{\sqrt{\frac{i \omega r_{0}}{\alpha}}}(\phi-1) \text { at } z=\sqrt{\frac{i \omega r_{o}^{2}}{\alpha}}
$$

and substitute into equation (6)

$$
\begin{equation*}
\frac{\mathrm{d}^{2} \phi}{\mathrm{~d} z^{2}}+\frac{1}{z} \frac{\mathrm{~d} \phi}{\mathrm{~d} Z}-\phi=0 \tag{7}
\end{equation*}
$$

a

The general solution of equation (7) is

$$
\begin{equation*}
\phi=C_{1} I_{0}(z)+C_{2} K_{0}(z) \tag{8}
\end{equation*}
$$

as given in Ref. 13 with

$$
I_{0}(Z)=1+\left(\frac{1}{2} Z\right)^{2}+\frac{\left(\frac{1}{2} Z\right)^{4}}{(2!)^{2}}+\cdots=\sum_{n=0}^{\infty} \frac{\left(\frac{1}{2} Z\right)^{2 n}}{(n!)^{2}}
$$

and

$$
K_{v}(z)=-\left\{\gamma+\log \left(\frac{1}{2} z\right)\right\} I_{0}(z)+\sum_{n=1}^{\infty} \frac{\left(\frac{1}{2} z\right)^{2 n}}{(n!)^{-2}}\left\{1+\frac{1}{2}+\frac{1}{3}+=-\frac{1}{n}\right\}
$$

Now using the boundary condition

$$
\frac{\partial \phi}{\partial z}=0 \quad \text { at } z=0
$$

and differentiating equation (8) yields

$$
\frac{\partial \phi}{d z}=c_{1} \frac{d\left(I_{0}(z)\right)}{\partial z}+c_{2} \frac{\partial\left(K_{0}(z)\right)}{d z}
$$

where

$$
\frac{d\left(I_{0}(Z)\right)}{d z}=0 \quad \text { at } z=0
$$

and

$$
\frac{d\left(K_{0}(z)\right)}{d z} \neq 0 \quad \text { at } z=0
$$

therefore $\mathrm{C}_{2} \equiv 0$
and

$$
\begin{equation*}
\phi=c_{1} I_{0}(Z) \tag{9}
\end{equation*}
$$

Now using the second boundary condition that

$$
\begin{equation*}
\frac{d \phi}{d Z}=-\frac{\beta}{\sqrt{\frac{i \omega r_{0}^{2}}{\alpha}}}(\phi-1) \text { at } z=\sqrt{\frac{i \omega r_{0}^{2}}{\alpha}} \tag{10}
\end{equation*}
$$

and differentiating equation (9) gives

$$
\frac{d \phi}{d Z}=c_{1} \frac{d\left(I_{0}(Z)\right)}{\partial Z}
$$

Noting that $\frac{d\left(I_{o}(Z)\right)}{d Z}=I_{1}(Z)$ and substituting into equation (10)

$$
C_{1} I_{1}\left(\sqrt{\frac{i \omega r_{0}^{2}}{\alpha}}\right)=\frac{B}{\sqrt{\frac{i \omega r_{0}^{2}}{\alpha}}}\left(C_{1} I_{0}\left(\sqrt{\frac{i \omega r_{0}^{2}}{\alpha}}\right)-1\right)
$$

Rearranging and solving for $C_{1}$

$$
C_{1}=\frac{1}{\sqrt{\frac{1 \omega r_{0}^{2}}{\alpha \beta^{2}}} I_{-1}\left(\sqrt{\frac{i \omega r_{0}^{2}}{\alpha}}\right)+I_{0}\left(\sqrt{\frac{i \omega r_{0}^{2}}{\dot{\alpha}}}\right)}
$$

and then substituting into equation (9)

$$
\begin{equation*}
\phi=\frac{I_{0}(Z)}{I_{0}\left(\sqrt{\frac{i \omega r_{0}{ }^{2}}{\alpha}}\right)+\frac{1}{\beta} \sqrt{\frac{i \omega E_{0}^{2}}{\alpha}} I_{1}\left(\sqrt{\frac{i \omega r_{0}{ }^{2}}{\alpha}}\right)} \tag{11}
\end{equation*}
$$

Now as

$$
\begin{aligned}
& J_{V}(i m x)=i^{V} I_{V}(m x) \quad[R e f \cdot 3, p \cdot 135] \\
& I_{0}\left(\sqrt{\frac{\omega r_{0}^{2}}{\alpha}} i^{1 / 2} \xi\right)=J_{0}\left(\sqrt{\frac{\omega r_{0}^{2}}{\alpha}} i^{3 / 2} \xi\right)
\end{aligned}
$$

and

$$
I_{1}\left(\sqrt{\frac{\omega r_{o}^{2}}{\alpha}} i^{1 / 2} \xi\right)=\frac{1}{i} J_{1}\left(\sqrt{\frac{\omega r_{0}^{2}}{\alpha}} i^{3 / 2} \xi\right)
$$

Let $a=\sqrt{\frac{\omega r_{o}^{2}}{\alpha}}$ and substitute into equation (11)

$$
\begin{equation*}
\phi=\frac{J_{0}\left(i^{3 / 2} a \xi\right)}{J_{0}\left(i^{3 / 2} a\right)-\frac{a}{\beta} i^{3 / 2} J_{1}\left(i^{3 / 2} a\right)} \tag{12}
\end{equation*}
$$

Now $i^{3 / 2}=e^{i \frac{3 \pi}{4}}=\cos \frac{3 \pi}{4}+i \sin \frac{3 \pi}{4}=\frac{1}{\sqrt{2}}(-1+i)$
Substitutig this into equation (12)

$$
\begin{equation*}
\phi=\frac{J_{0}\left(i^{3 / 2} a \xi\right)}{J_{0}\left(i^{3 / 2} a\right)+\frac{a}{\sqrt{2} \beta}(1-i) J_{1}\left(i^{3 / 2} a\right)} \tag{13}
\end{equation*}
$$

As $J_{0}\left(a \xi i^{3 / 2}\right)=J_{0}\left(a \xi e^{i \frac{3 \pi}{4}}\right)=\operatorname{BER}_{0}(a \xi)+i B E i_{0}(a \xi)$
and

$$
J_{1}\left(a \xi i^{3 / 2}\right)=J_{1}\left(a \xi e^{i \frac{3 \pi}{4}}\right)=B E R_{1}(a \xi)+i B E i_{1}(a \xi)
$$

Substituting these rusults into equation (13) yields

$$
\begin{equation*}
\phi=\frac{\operatorname{BER}_{0}(a \xi)+i B E i_{0}(a \xi)}{B E R_{0}(a)+i B E i_{0}(a)+\frac{a}{\sqrt{2} \beta}(1-i)\left(B E R_{1}(a)+i B E i_{1}(a)\right)} \tag{14}
\end{equation*}
$$

After rearrangement.

$$
X_{R}=B E R_{0}(a)+\frac{a}{\sqrt{2} \beta} B E R_{1}(a)+\frac{a}{\sqrt{2} \beta} B E i_{1}(a)
$$

and

$$
X_{i}=B E i_{0}(a)+\frac{a}{\sqrt{3} \beta} B E i_{1}(a)-\frac{a}{\sqrt{2} \beta} B E R_{1}(a) .
$$

and substituting into.equation. (14) gives

$$
\phi=\frac{B E R_{o}(a \xi)+1 B E i_{0}(a \xi)}{X_{R}+i X_{i}}
$$

PEtionalizing the Aerominator yields

$$
\begin{equation*}
\phi=\frac{B E R_{0}(a \xi)+i B E i_{0}(a \xi)}{x_{R}^{2}+x_{i}^{2}}\left(x_{R}-i X_{i}\right) \tag{15}
\end{equation*}
$$

Now
$\phi=\frac{\left(B E R_{0}(a \xi) X_{R}+B E i_{0}(a \xi) X_{i}\right)+i\left(B E i_{0}(a \xi) X_{R}-B E R_{0}(a \xi) X_{i}\right)}{X_{R}^{2}+X_{i}{ }^{2}}$
which after rearrangement gives.
$\phi=\sqrt{\frac{B E R_{0}{ }^{2}(a \xi)+B E i_{o}{ }^{2}(a \xi ;}{X_{R}^{2}+x_{i}^{2}}} e^{i \delta}{ }^{*}$
where
$\delta^{\dot{*}}=\tan ^{-1} \frac{B E 1_{0}(a \xi) X_{R}-B E R_{0}(a \xi) X_{i}}{B E R_{0}(a \xi) X_{R}+B E i_{0}(a \xi) X_{i}}$

Substituting into equation (4). gives

$$
\psi(x, t)=\sqrt{\frac{\mathrm{BER}_{0}^{2}(\mathrm{a} \xi)+\mathrm{BEi}_{0}^{2}(a \xi)}{\mathrm{X}_{R}^{2} \cdot+\mathrm{X}_{i}^{2}}} e^{i\left(\omega t+\delta^{*}\right)}
$$

which also equals
$\psi(r, t)=\sqrt{\frac{B E R_{o}{ }^{2}(a \xi)+B E i_{o}{ }^{2}(a \xi)}{X_{R}{ }^{2}+X_{i}{ }^{2}}}\left[\cos \left(\omega t+\delta^{*}\right)+i \sin \left(\omega t+\delta^{*}\right)\right]$
As this problem was modeled as a sine wave, the imaginary part of $\psi(r, t)$ was used.
$I(\psi(r, t))=\theta(r, t)=\sqrt{\frac{B E R_{0}^{2}(a \xi)+B E I_{0}^{2}(a \xi)^{2}}{X_{R}^{2}+X_{i}^{2}}} \sin \left(\omega t+\delta^{*}\right)$
which is the analytical solution of infinitely long concen-
tric cylinders experiencing a periodic sinusoidal temperature variation on its outermost surface when seat conduction is assumed to be radial only.

In summary

$$
\theta(r, t)=\frac{T-T_{A}}{T_{M} T_{A}}=\sqrt{\frac{\operatorname{BER}_{0}^{2}(a \xi)+B E i_{o}^{2}(a \xi)}{X_{R}^{2}+x_{i}^{2}}} \sin \left(\varphi t+\delta^{*}\right)
$$

where

$$
\begin{aligned}
& T=\text { the temperarure of a point } r \text { in the rocket motor at } \\
& T_{A}=\text { average bulk temperature of the storage container } \\
& T_{M}=\text { maximum bulk temperature of the storage container } \\
& \omega=\text { frequency of the sinusoidal variation }(2 \pi / 24 \text { hours }) \\
& t=\text { time } \\
& \xi=\frac{r}{r_{0}}=\text { dimensionless distance from the center of the } \\
& x_{0}=\text { distance to the surface of the rocket motor } \\
& r=d i s t a n c e ~ f r o m ~ t h e ~ c e n t e r ~ o f ~ t h e ~ r o c k e t ~ m o t o r ~
\end{aligned}
$$

$$
\begin{aligned}
& \alpha=\frac{k}{\rho_{c}}=\text { thermal diffusivity } \\
& \rho=\text { density } \\
& k=\text { thermal conductivity } \\
& c=\text { specific heat } \\
& \text { BER }=\text { real Bessel Function } \\
& \text { BEi = imaginary Bessel Function }
\end{aligned}
$$

The following computer prograns were used to investigate a wide variety of parameters. The outputs are samples of

PFI AIIVE AMPLITUDE













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# APPENDIX C <br> <br> TRUMP Solution 

 <br> <br> TRUMP Solution}

TRIMP is a computer program for solving transient and steady-state temperature distributions in multidimensional systems. This program was developed in 1965 at the Lawrence Radiation Laboratory by A. L. Edwards [Ref. 2] for their CDC/3600 computer. The program was adapted to the Naval Postgraduate $S$ chool $I B M / 360$ Model 67 computer system in 1971 by C. Erbayrum [Ref. 3] from a version used by the B. F. Goodrich Corporation.

TRUMP is a malt: ourpose program able to solve a wide variety of problems involving flow in various kinds of potential fields such as heat flow in a temperature field. TRUNP allows the solution of general nonlinear parabolic partial differential equations both in steady-state and transient problems. Complex geometric configurations with multidimensional flow may be solved using various coordinate systems. Initial conditions may vary with spatial position. Material properties, boundary conditions, and other problem parameters may vary with spatial position, time, or the primary dependent variable.

Input data are fed to TRUMP in "Block" form through its 12 input data blocks, A complete description of each of these blocks is gi on in Ref. 2. A model of the problem must be constructed and data from this model read into TRUMP through the data blocks.

Two models were used to simulate the rocket motor storage container system and several variations of each model were investigated.

The first model assumed one dimensional heat transfer (radial) with the assumptions that the system was infinitely long and that the container surface temperature was spatially uniform. The system was modeled as two infiniteiy long concentric cylinders separated by a 2.94 inch air gap. The inner cylinder was constructed of 4130 steel and was filled with dry wind blown rand. The thermal properties of the materials used in the experimental system are given in Table II with units most easily compared to the actual data obtained from the system at China Lake.

TABLE II
Thermal Properties of Materials

| Material | Density 3 | Specific Heat | Thermal. Conductivity |
| :---: | :---: | :---: | :---: |
| Sand | $0.05486 \mathrm{lbm} / \mathrm{in}^{3}$ | $0.195 \mathrm{BTU} / 1 \mathrm{bm}{ }^{\circ} \mathrm{F}$ | 0.00026BTU/min-in- ${ }^{\circ} \mathrm{F}$ |
| Steel | $0.2807 \mathrm{lbm} / \mathrm{in}^{3}$ | $0.109 \mathrm{BTU} / 1 \mathrm{bm}^{\circ} \mathrm{F}$ | $0.364 \mathrm{Br} \mathrm{B}^{\prime} / \mathrm{min}-\mathrm{in}-{ }^{0} \mathrm{~F}$ |
| Air | 0.0000436 " | $0.240 \mathrm{Br}^{\text {TU }} / 1 \mathrm{bm}^{\circ} \mathrm{F}$ | 0.0000225 |

The modil was subdivided into volume elements or nodes with the representative nodal points given in Figure 27 . Although the representative nodal point may be located anywhere in the node or on the surface of the node, in transient problems it is usualiy located so that the lines connecting the nodal points are perpendicularly bisected by the connected area. This gives maxinum accuracy. Two boundary conditions were given to the surface node. The first was a sinusoidal disturbance which closely modeled the actual


Figure 27. Location of Nodes for One Dimensional TRUMP Model.
average experimental data obtained at China Lake as given in Appendix D. The sine wave exhibited a maximum temperature of $138^{\circ} \mathrm{F}$ and an average temperature of $104^{\circ} \mathrm{F}$. Its period was 24 hours ( 1440 minutes). The second boundary condition was the actual average surface temperature of the storage container given at two hour intervals. Both these boundary conditions are approxinations of the actual surface temperature. Two hour intervals were the minimum allowable for the tabular data as this version of TRUMP has a maximum table size of 12.

Several assumptions were initially made. The thermocouple data obtained from the experiment at China Lake gave the averaga temperature at a point on the storage container and not the actual outside surface temperature. As this container wall was only $1 / 16$ of an inch thick and made of a good thermal conductor, it was decided to model this data as a zero volume boundary node with a known temperature impressed on it. It was also assumed that heat transfer across the air gap occurred only by radiation and conduction, neglecting the effects of free convection.

It was estimated that the surface emissivities for the rocket motor and the storage container were 0.9 [Ref. 6] based on their haze gray surfaces. The radiation exchange factor for this geometry is given by

$$
\mathcal{F}_{1-2}=\frac{1}{\frac{1}{\varepsilon_{1}}+\frac{r_{1}}{r_{2}}\left(\frac{1}{\varepsilon_{2}}-1\right)}=0.84
$$

A sample input deck for the tabular approximation of the boundary condition is given at the end of this appendix. Several cycles of output data for the one dimensional model are also given.

The second model assumed two dimensional heat transfer (radial and circumferential) with the assumption that the system was infinitely long. The same physical model was assumed for the system except 48 nodes were used instead of 12. The representative nodal points are given in Figure 28. The four surface nodes ( $12,24,36$, and 48 ) each had two different temperature approximations applied, a sinusoidal representation and a tabular input taken at two hour intervals. The four surface nodes were also modeled as zero volume boundary nodes. Each internai thermal connection between nodes is described in the input data by specifying the two node identification numbers, two connector lengths, and two interface dimensional factors. An example of the thermal connections of node 4 is shown in Figure 28 and the input data in BLOCK 5 of the two dimensional TRUMP program.

The calculation of the radiosities in the two dimensional case was accomplished by using a radiation-network and the method of crossed-strings.

The radiation shape factors for the two dimensional system were determined by the method of crossed-strings [Ref. 14]. The graphical construction for this method is given in Figure 29.


Figure 28: Location of Nodes for Two Dimensional TRUMP Model.


Figure 29: Graphical Construction for Crossed-Strings Method.
$F_{m-n}$ is defined as the fraction of energy leaving surface $m$ which directly reaches surface $n$. From the physical dimensions of the model $A_{1}=A_{4}=A_{6}=9.45 \frac{\text { sg.in }}{i n \cdot,} A_{2}=A_{3}=A_{5}$ $=14.05 \frac{\mathrm{sq} . \mathrm{in}}{\mathrm{in}}, A_{2^{\prime}}=12.65 \frac{\mathrm{sg}, \mathrm{in}}{\operatorname{in}}$, assuming unit depth. Let $S_{i}$ equal the length of $A_{i}$.

The crossed-string method lets each surface represent the efeective surface obtained by stretching a string tightly over the radiating face between the bounding edges, to produce a surface that cannot see any of itself, For example, surface $2^{\text {: }}$ in Figure 29 stretched over surface 2. By definition $F_{2}^{\prime}{ }^{\prime} \equiv 1$, which by reciprocity leads to

$$
F_{22^{\prime}}=\frac{A_{2}^{\prime}}{A_{2}} F_{2^{\prime} 2}=0.9
$$

and therefore since

$$
F_{22}+F_{22}=1 \quad \text { then } F_{22}=0.10
$$

To calculate the direct'radiant heat exchanged between surfaces 1 and 2 , a minimum-length line was stretched connecting edge of $A_{1}$ to edge $E$ of $A_{2}$ and a second minimum length line from eure $L$ af $A_{1}$. to edge $F$ of $A_{2}$. These lines are labeled $L_{1}$ in Fagure 29 and are equal to the width of the air gap, $L_{1}=2.9375$ in. Minimum length lines were also stretched from point $B$ on $A_{1}$ to $F$ on $A_{2}$ and $L$ on $A_{1}$ to $E$ on $A_{2}$. The length of these lines is $D_{1}$ and is made up of two parts; $D_{1}$ ', the tangential distance fxom $F$ to surface $A_{1}$ and $D_{1}$ ", the arc length from the point the tangent hits $A_{1}$ to $B$, From geometry $\quad D_{1}=\sqrt{x_{1}^{2}-r_{0}^{2}}=6.62^{\prime \prime}$

$$
D_{1}^{\prime \prime}=r_{0} \theta_{a}=4.42^{\prime \prime}
$$

therefore
. $\quad D_{1}=D_{1}^{\prime}+D_{1}^{\prime \prime}=11.04^{\prime \prime}$
Now $\quad F_{12}=\frac{2 \mathrm{D}_{1}-2 \mathrm{~L}_{1}}{2 \mathrm{~S}_{1}}=0.86$
From reciprocity, $A_{1} F_{12}=A_{2} F_{21}$

$$
F_{21}=\frac{r_{0}}{r_{1}} F_{12}=0.578
$$

Now $\mathrm{F}_{13}$ is calculated from

$$
\mathrm{F}_{12}+2 \mathrm{~F}_{13}=1
$$

therefore $\mathrm{F}_{13}=0.07$.
From symmetry $\mathrm{F}_{42}=\mathrm{F}_{13}$
and then by reciprocity

$$
F_{24}=\frac{r_{0}}{r_{1}} F_{42}=0.047
$$

Now $F_{23}$ was calculated $y$ stretching minimum length lines from $F$ to $E$, from $E$ tc $G$, from $F$ to $G$ and from $E$ to $E$ where the length of the line from $F$ to $E=$ from $E$ to $G=S_{2}{ }^{\prime}$, from $F$ to $G=2 D_{1}$, and from $E$ to $E=0$

$$
F_{23}=\frac{2 S_{2}^{\prime}-2 D_{1}}{2 S_{2}}=.113
$$

$\mathrm{F}_{25}$ was now calculated from

$$
F_{21}+F_{22}+F_{23}+F_{24}+F_{25}=.1
$$

therefore $P_{25}=0.002$
As $F_{25}$ was much smaller than the other $F_{m-n}$, it was not included in the radiation-network diagram in Figure 30 [Ref. 15].


Figure 30: Radiation Network.

To calculate $F_{1-n}$, the blackbody potential of area 1 is set to unity and all other blackbody potentials are set as zero. An energy balance was written at each node giving a set of six simultaneous equations as follows:

$$
\begin{aligned}
& \text { Node } 1 \\
& \left.E_{1} A_{1}\left(1-J_{1}\right)=A_{1} F_{12}\left(J_{1}-J_{2}\right)+2 A_{1} F_{13}\left(J_{1}-J_{3}\right)\right)
\end{aligned}
$$

Node 2
$E_{2} A_{2}\left(0-J_{2}\right)=A_{1} F_{12}\left(J_{2}-J_{1}\right)+2 A_{2} F_{24}\left(J_{2}-J_{4}\right)+2 A_{2} F_{23}\left(J_{2}-J_{3}\right)$
Node 3

$$
\begin{aligned}
E_{2} A_{2}\left(0-J_{3}\right)= & A_{1} F_{12}\left(J_{3}-J_{4}\right)+A_{2} F_{23}\left(J_{3}-J_{2}\right)+A_{1} F_{13}\left(J_{3}-J_{1}\right)+ \\
& A_{1} F_{13}\left(J_{3}-J_{6}\right)+A_{2} F_{23}\left(J_{3}-J_{5}\right)
\end{aligned}
$$

Node 4
$E_{1} A_{1}\left(0-J_{4}\right)=A_{2} F_{24}\left(J_{4}-J_{2}\right)+A_{1} F_{12}\left(J_{4}-J_{3}\right)+A_{2} F_{24}\left(J_{4}-J_{5}\right)$
Node 5

$$
E_{2} A_{2}\left(0-J_{5}\right)=2 A_{2} F_{23}\left(J_{5}-J_{3}\right)+2 A_{2} F_{24}\left(J_{5}-J_{4}\right)+A_{1} F_{12}\left(J_{5}-J_{6}\right)
$$

Node 6

$$
E_{1} A_{1}\left(0-J_{6}\right)=2 A_{1} F_{13}\left(J_{6}-J_{3}\right)+A_{1} F_{12}\left(J_{6}-J_{5}\right)
$$

where $J_{n}=$ radiosity of node $n$.

$$
\begin{aligned}
& E_{1} A_{1}=\frac{\varepsilon_{1}}{1-\varepsilon_{1}} A_{1} \\
& E_{2} A_{2}=\frac{\varepsilon_{2}}{1-\varepsilon_{2}} A_{2}
\end{aligned}
$$

$\mathbf{F}_{\mathrm{nm}}=$ radiation shape factors previously calculated. Now the values of $A_{1}, A_{2}$ and $F_{n m}$ were substituted into the energy balance equations which were then put into matrix form as shown in Table III.

TABLE III
Matrix Form of Energy Balance Equations


|  | $\overparen{J}$ | 0 |  |  | $\overparen{J}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | $\ddots$ | $\infty$ | $H$ | 0 | $\ddots$ |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | $H$ | 0 | 1 |

$(-0.14)$
$(-0.33)$
$\left(1.33+\frac{3 \varepsilon_{2}}{2\left(1-\varepsilon_{2}\right)}\right)$
$(-0.86)$
$(-0.14)$
$(-0.33)$



Letting $\varepsilon_{1}=\varepsilon_{2}=0.9$, a standard computer solution for matrix problems gave the radiosities as listed in Table IV.

TABLE IV

$$
\begin{aligned}
& \text { Radiosities at Nodes } \\
& \mathrm{J}_{1}=.9046 \\
& \mathrm{~J}_{2}=.0529 \\
& \mathrm{~J}_{3}=.00495 \\
& \mathrm{~J}_{4}=.000797 \\
& \mathrm{~J}_{5}=.000125 \\
& \mathrm{~J}_{6}=.000080
\end{aligned}
$$

Now to find the radiation exchange factors from node 1 to nodes $n$, the radiation network shown in Figure 30 was reduced to the equivalent network shown in Figure 31. Where the nodal equations are

$$
7_{1-2} A_{1}(1-0)=E_{2} A_{2}\left(J_{2}-0\right)
$$

where

$$
\begin{aligned}
\mathcal{F}_{1-2} & =\frac{E_{2} A_{2}}{A_{1}} J_{2}=.71 \\
\mathcal{F}_{13 A_{1}}(1-0) & =E_{2} A_{2}\left(J_{3}-0\right) \\
\mathcal{F}_{1 . .3} & =\frac{E_{2} A_{2}}{A_{1}} J_{3}=.066
\end{aligned}
$$

These values of the radiation exchange factor are used in the two-dimensional program. A sample input deck for the sinusoidal boundary condition is included at the end of this appendix. Several cycles of output data for the tabular boundary condition are also given.


Figure 31: Equivalent Radiation Network
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TRUMP GUTPUT RATA

TRUMP DUTPUT DATA
OATA DECK 1

trump gutput jata






trump output cata


[^2]






APPENDIX D

Experimental Data

The data presented in this appendix were obtained from the thermocouples on the rocket motor storage container system located at China Lake, California. The thermocouple output was read out on a Honeywell Electronik 25,24 channel recorder which had been calibrated at 50,100 and $150^{\circ} \mathrm{F}$. The data was taken on two consecutive, typical summer days (August 1 and 2, 1972) at China Lake. Each thermocouple was read once every 24 minutes. The first set of data presents the storage container temperature at four locations plus three different ways of averaging this data. It also presents the ambient temperature and the approximate time of day. The second set of data presents the surface temperature of the rocket motor and three ways to average this data. It also presents the temperature at the center of the rocket motor and the approximate time of day. Figure 32 shows the location of the thermocouples used. to collect this temperature data.


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| Time（Approximate） <br> Aug．1， 1972 | Ambient $\left(\begin{array}{l}\text { O } \\ \text { F }\end{array}\right.$ | $\binom{\# 1}{{ }^{\circ} \mathrm{F}}$ | $\# 8$ $\binom{\circ}{$ ¢ } | $\# 10$ $\left({ }^{\circ} \mathrm{F}\right)$ | $\# 12$ $\left({ }^{\circ} \mathrm{F}\right)$ | $\begin{gathered} \text { Avg•非1 \& 据10 } \\ (0, F) \end{gathered}$ | Avg•非 8 \＆$\# 12$ （ ${ }^{\circ} \mathrm{F}$ ） | $\begin{gathered} \text { Avg. all } 4 \\ \text { "Bulk } \\ \hline \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1800 | 103 | 118 | 131 | 113 | 123 | －115．5 | 127 | 121．25 |
|  | 101 | 113 | 127 | 111 | 120 | 112 | 123．5 | 117.75 |
|  | 99 | 107 | 121 | 108 | 117 | 107．5 | 119 | 113.25 |
|  | 96 | 101 | 106 | 103 | 111 | 102 | 108.5 | 105.25 |
|  | 95 | 91 | 93 | 98 | 104 | 94．5 | 98.5 | 96.5 |
| 2000 | 93 | 88 | 91 | 95 | 101 | 91．5 | 96 | 93.75 |
|  | 91 | 86 | 89 | 94 | 99 | 90 | 94 | 92 |
|  | 90 | 85 | 88 | 92 | 97 | 88.5 | 92.5 | 90.5 |
|  | 89 | 84 | 87 | 91 | 96 | 87.5 | 91． 5 | 89.5 |
|  | 87 | 83 | 85 | 89 | 94 | 86 | 89.5 | 87.75 |
| 2200 | 87 | 81 | 84 | 88 | 93 | 84.5 | 88.5 | 86.5 |
|  | 85 | 80 | 83 | 87 | 91 | 83．5 | 87 | 85.25 |
|  | 86 | 79 | 82 | 86 | 91 | 82．5 | 86.5 | 84.5 |
|  | 86 | 79 | 82 | 87 | 90 | 83 | 86 － | 84.5 |
|  | 84 | 79 | 82 | 86 | 89 | 82.5 | 85.5 | 84 |
| 00002 Aug． | 82 | 77 | 80 | 85 | 88 | 81 | 84 | 82.5 |
|  | 81 | 74 | 78 | 84 | 86 | 79 | 82 | 80.5 |
|  | 81 | 72 | 76 | 83 | 85 | 77.5 | 80.5 | 79 |
|  | 81 | 72 | 77 | 83 | 85 | 77.5 | 81 | 79.25 |
|  | 79 | 72 | 75 | 81 | 83 | 76.5 | 79 | 77.75 |
| 0200 | 79 | 72 | 75 | 80 | 83 | 76 | 79 | 77.5 |
|  | 79 | 72 | 74 | 80 | 82 | 76 | 78 | 77 |
|  | 78 | 72 | 74 | 79 | 82 | 75.5 | 78 | 76.75 |
|  | 77 | 71 | 73 | 79 | 81 | 75 | 77 | 76 |
|  | 75 | 70 | 72 | 78 | 80 | 74 | 76 | 75 |
| 0400 | 73 | 67 | 69 | 77 | 78 | 72 | 73.5 | 72.75 |
|  | 72 | 66 | 69 | 75 | 77 | 70.5 | 73 | 71.75 |
|  | 73 | 65 | 68 | 75 | 76 | 70 | 72 | 71 |
|  | 69 | 64 | 67 | 73 | 75 | 68．5 | 71 | 69.75 |
|  | 69 | 63 | 66 | 72 | 74 | $67 \cdot 5$ | 70 | 68.75 |
| 0600 | 68 | 63 | 66 | 71 | 73 | 67 | 69.5 | 68.25 |
|  | 71 | 65 | 67 | 75 | 76 | 70 | 71.5 | 70.75 |
|  | 77 | 74 | 71 | 80 | 79 | 77 | 75 | $76$ |
|  | 79 | 84 | 77 | 34 | 84 | 84 | 80.5 | 82． 2.5 |



Series 2





## APPENDIX E <br> Uncertainty Analysis

An uncertainty analysis was carried out on both the analytical solution and on a one dimensional TRUMP model of the rocket motor storage contixiner system. In both models, the volumetric heat capacity of the sand ( $\rho c$ ), the conductivity of the sand (k), and the emissivity of the surfaces were each varied by ten percent to determine the sensitivity of the sysiem temperature response to each variatior. Although other factors may also be varied, it was theorized that these three had the greatest effect on the heat transfer of the system. These factors were also known with the least accuracy; the maximum uncertainty of each was estimated to be plus or minus ten percent (odds 20 to 1).

In the analyidical solution, varying the volumetric heat capacity changed parameter a, varying the emissivity changed parameter $\beta$, and varying the conductivity changed both parameters and $\beta$. The effects on each parameter from each variation are given in Table V.

TABLE V
Change in Parameters due to Changes in Thermal Properties

Change in Property
Volumetric Heat Capacity $+10 \%$
Volumetric Heat Capacity - $10 \%$
Emissivity $+10 \%$
Emissivity - 10\%
Conductivity $+10 \%$
Conductivity - 10\%

Change in Parameters
$a+.12$
a -.12
$8+.49$
B-. 37
a - .11, $\beta-.22$
$a+.13, \beta+.35$

Each factor was varied holding the other factors constant. The changes in cemperature and time delay were computed from the difference between these new values and those previously obtained from the analytical solution. To obtain uncertainty bounds on the analytical curve, the second power equation [Ref. 16] was used, namely

$$
\omega_{T}=\sqrt{\omega_{C}^{2}+\omega_{k}^{2}+\omega_{\varepsilon}^{2}}
$$

where
$\omega_{\mathrm{T}}$. $=$ resulting uncertainty in the calculated temperature due to uncertainties in temperature caused by
$\omega_{C}=$ estimated uncertainty in volumetric heat capacity
$\omega_{k}=$ estimated uncertainty in conductivity
$\omega_{\varepsilon}=$ estimated uncertainty in emissivity
An identical calculation was carried out to calculate the uncertainty in time delay. The results of these calculations are shown in Figures 12 and 13 for the surface and center of the rocket motor respectively. The uncertainty in temperature varied with time with a maximum variation of $\pm 2.75^{\circ} \mathrm{F}$ at the senter of the motor and a maximum variation of $\pm 1.85^{\circ} \mathrm{F}$ at the surface of the rocket motor. The time delay varied by $\pm 31$ minutes at the center of the motor and $\pm 11$ minutes at the surface. The actual experimental data was also plotted on these Figures for comparison.

The experimental data also had an uncertainty bound. Three primary factors made up this uricercainty bound; the accuracy of the thermocouple wire (i. $1.5^{\circ} \mathrm{F}$ ), the readability of the recorder ( $1-1^{\circ} \mathrm{F}$ ) , and the yariman in temnerature
caused by inaccuracy in the placement of the thermocouples $\left(+1^{\circ} F\right.$, estimated). The overall uncertainty in the experimental data was also calculated from the second power equation as

$$
\omega_{T}=\sqrt{\omega_{W I R E}^{2}+\omega_{R E A D}^{2}+\omega_{P L A C E}^{2}} \quad \approx 2^{\circ} \mathrm{F}
$$

These uncertainty bounds are also shown in Figures 12 and 13.
A procedure, similar to that used to find the uncertainties of the analytical solution, was used to analyze the resulting uncertainty in the TRUMP numerical calculation. The results of these calculations are shown in Figures 14 and 15. The uncertainty in temperature varied with time with a maximum variation of $\pm 2.95^{\circ} \mathrm{F}$ at the center of the rocket motor and a maximum variation of $\pm 1.95^{\circ} \mathrm{F}$ at the surface of the motor. The time delay varied from $\pm 20$ minutes at the center of the motor to +9 minutes at the surface of the motor.

On the basis of the propagation of uncertainty analysis, it was determined that the solutions vere most sensitive, in order of importance, to changes in the volumetric heat capacity, emissivity, and the conductivity.

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