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RESEARCH DIRECTED TOWARD THE STUDY OF SEISMICITY IN THE SOUTHEASTERN UNITED STATES

John W. Minear

Research Triangle Institute
P. O. Box 12194
Research Triangle Park, North Carolina 27709

Contract No. AF 19(628)-3892

Project No. 8652
Task No. 865207

FINAL REPORT
January 2, 1964 thru December 15, 1966
January 1967

Work sponsored by Advanced Research Projects Agency, Project VELA-UNIFORM
ARPA Order No. 292, Project Code No. 8100 Task 2

Prepared for
AIR FORCE CAMBRIDGE RESEARCH LABORATORIES
OFFICE OF AEROSPACE RESEARCH
UNITED STATES AIR FORCE
BEDFORD, MASSACHUSETTS

RESEARCH TRIANGLE PARK, NORTH CAROLINA 27709

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Approved by:

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ABSTRACT

This is the final report covering research directed toward the study of the seismicity of the Southeastern United States. Travel-times determined from local earthquake and refraction data are presented which indicate a crustal structure of $h_1 = 33.0$ km ($\alpha = 5.88$ km/sec), $h_2 = 10.8$ km ($\alpha = 6.58$ km/sec), and an upper mantle velocity of 8.10 km/sec. Fundamental and first higher order Rayleigh group-velocity data determined by digital bandpass filtering are presented for the Southern Appalachian region. The Dunkin modification of the Thomson-Haskell matrix method is used to compute theoretical Rayleigh dispersion curves for comparison with the observed curves. A slight velocity reversal in the upper crust centered at about 15 km, a general increase of crustal velocities and densities with depth below this zone, and an upper mantle low velocity zone beginning at a depth of 70 km are indicated beneath the Southern Appalachians. The Appalachian foreland has crustal structure similar to the Gutenberg-Birch II continental model with a total thickness of 40 km.

A sin x/x analysis of the Bouguer gravity data yields a total crustal thickness of about 50 km beneath the Southern Appalachians.

P-residuals computed at Chapel Hill, North Carolina and McMinnville, Tennessee show a systematic deviation of as much as $\pm$ 3 sec.

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1.0 INTRODUCTION

This final report covers the work done on contract AF 19(628)-3892, "Research Directed Toward the Study of the Seismicity of the Southeastern United States". The report is mainly concerned with the work done over the past year; the determination of crustal and upper mantle structure in the Southeastern United States. The First Annual Technical Report [Minear, 1965] covers the development, installation, and calibration of the short-period displacement seismograph at the University of North Carolina and the RTI field refraction system. The Second Annual Technical Report [Minear, 1966] includes the location of local epicenters, the results of the field refraction studies, the results of the computation of P-residuals, the calculations of magnitude, focal depth, and energy release for several local earthquakes, and preliminary crustal structure estimates from gravity data.

Research accomplishments during the performance of the contract are briefly summarized below.

1) During the first year of work a short-period displacement seismograph system was designed, constructed, and placed on routine operation at the University of North Carolina seismograph vault at Chapel Hill, North Carolina. Although the system performed well [Minear, 1965], the background noise level at the UNC station was too high to permit recording of local earthquakes located mainly in the Southern Appalachians. At present, a remote vault is currently under construction by the University to provide an up-to-date seismic facility.

2) Refraction work was carried out using local quarry blasts as energy sources.

3) Local travel-time curves were developed using several of the major local earthquakes which were well recorded by portable and permanent stations in the region.

4) P-residuals were computed for several hundred epicenters recorded at the Cumberland Plateau Seismological Observatory and Chapel Hill, North Carolina. A systematic deviation of the residuals similar to that noted by other investigators was found. This deviation cannot be explained by crustal velocity variations and must indicate a real error in the Jeffreys-Bullen travel-times.

5) Estimation of focal depth, magnitude, and energy release from previous intensity studies of four Southeastern earthquakes were made.

6) Total thickness of the Southern Appalachian crust was determined using Bouguer gravity anomalies and the sin x/x method of computing the mass anomaly producing a given gravity anomaly.

7) Crustal and upper mantle structure was determined using fundamental and first higher Rayleigh mode group velocity dispersion.

8) Computer programs were written for bandpass filtering, computation of P-residuals, least squares epicenter location, computation of theoretical travel-times from a given velocity structure, computation of theoretical Rayleigh dispersion curves and modal shape, computation of the variation of phase velocity with layer parameter variations, and the computation of the mass anomaly from a given gravity anomaly profile.

At the start of the project, it was anticipated to do considerable work on phase and amplitude spectra of both seismic signals and background noise. Also, it was hoped that more work could have been done on general seismicity, distribution of epicenters, focal depth, and energy release. Failure to acquire a digital system and the fact that much work had been done on background noise did not make the study of spectra appear worthwhile. The general seismicity study was frustrated by the poor recording station distribution in the region. Therefore, crustal and upper mantle structural studies utilizing refraction, gravity, and surface wave dispersion were concentrated on.

This report specifically covers the local travel-time curves for the Southern Appalachian region (Sec. 2), the determination of crustal thickness from gravity data (Sec. 3), the computation of theoretical dispersion curves (Sec. 4), the determination of Rayleigh group velocities (Sec. 5), and the crustal and upper mantle structure in the Southeastern United States (Sec. 6).

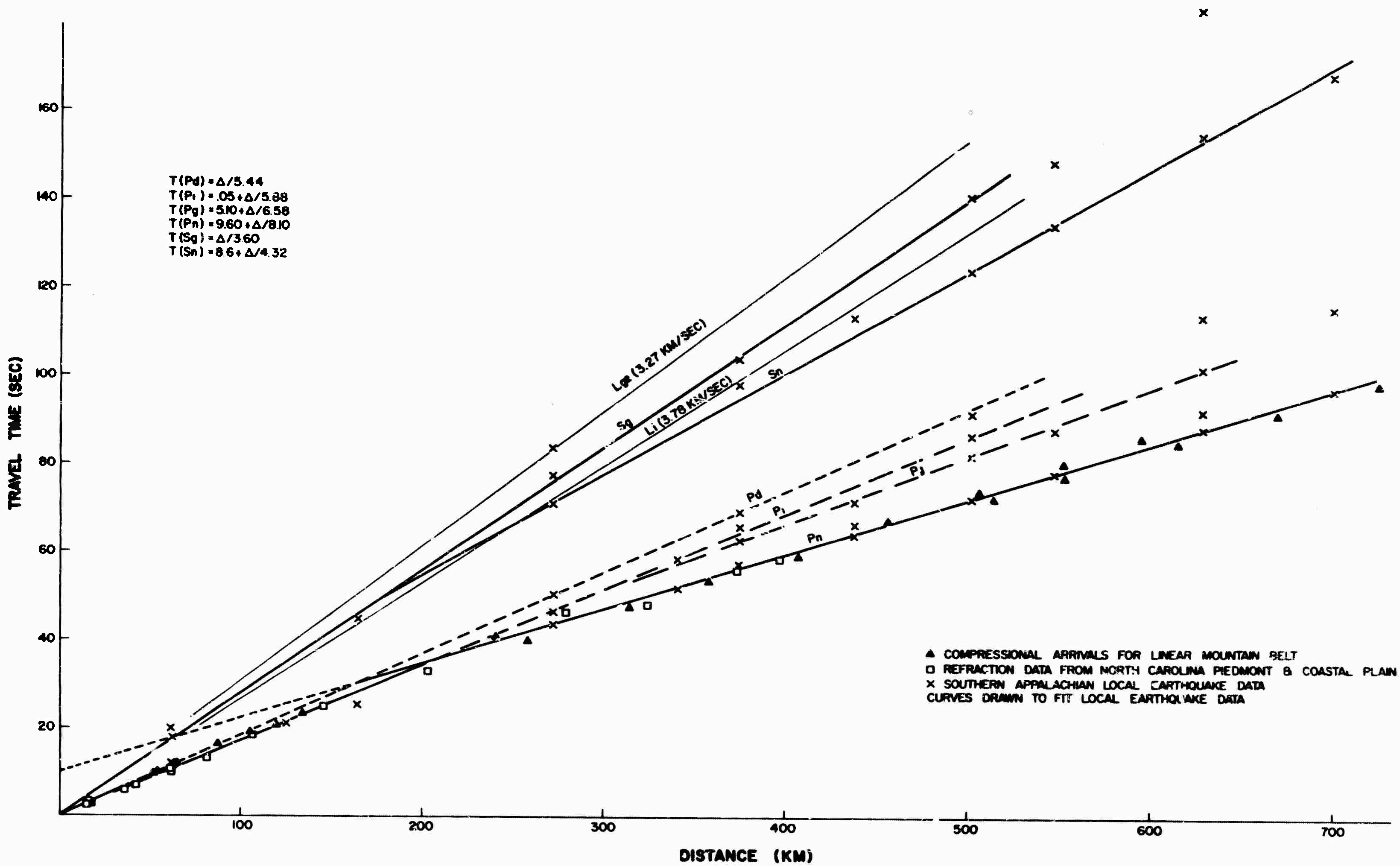
Numerical computational methods, tables, charts, and computer program listings are presented in the Appendices.

2.0 TRAVEL-TIME CURVES FOR THE SOUTHEASTERN UNITED STATES

Travel-time curves were determined by using data from three local earthquakes which were well recorded by Worldwide Standard Seismograph Stations and portable Vela stations operating in the Southeastern United States [Minear, 1966]. Fig. 4 shows the location of the epicenters and recording stations. Travel-time curves drawn from the local earthquake data are shown in Fig. 1. Refraction data obtained from quarry blasts and during the East Coast Onshore Offshore Seismic Experiment and theoretical travel-times computed for a typical linear mountain from the Herglotz-Wiechert equations are also plotted in Fig. 1.

Travel-time curves corresponding to arrivals from the first crustal layer and from the crust mantle boundary are drawn from first arrivals and are estimated accurate to within ± .1 km/sec. Second arrivals were used to define curves corresponding to two major crustal layers. No major third layer in the crust is indicated by the refraction and earthquake data. However, first arrivals from the refraction profiles and second arrivals from 250 to 550 km, indicate that the crustal velocity may increase rather continuously from about 10 km to around 45 km. The local travel-time data yields a crustal model of $h_1 = 33.0$ km ($\alpha = 5.88$ km/sec), $h_2 = 10.8$ km ($\alpha = 6.58$ km/sec), and an upper mantle velocity of 8.10 km/sec. As can be seen from Fig. 4, the epicenters and recording stations are generally located to the west of the core of the Appalachians. Crustal structure determined from the travel-time data thus corresponds to the crust beneath the Appalachian foreland.

Velocity structures of the crust and upper mantle for the Appalachian foreland, the Northern Alps (N), Central Alps (C), and Northern Alpine foreland (F) [Knopoff, et al, 1966], and a linear mountain belt are shown in Fig. 2. Crustal velocities for the Appalachian foreland generally agree



T(Pd) = Δ/5.44
T(Pi) = .05 + Δ/5.88
T(Pg) = 5.10 + Δ/6.58
T(Pn) = 9.60 + Δ/8.10
T(Sg) = Δ/3.60
T(Sn) = 8.6 + Δ/4.32
Lg (3.27 KM/SEC)
Sg
Li (3.78 KM/SEC)
Sn
Pd
Pi
Pi
Pn
▲ COMPRESSIONAL ARRIVALS FOR LINEAR MOUNTAIN BELT
□ REFRACTION DATA FROM NORTH CAROLINA PIEDMONT & COASTAL PLAIN
× SOUTHERN APPALACHIAN LOCAL EARTHQUAKE DATA
CURVES DRAWN TO FIT LOCAL EARTHQUAKE DATA
TRAVEL TIME (SEC)
DISTANCE (KM)

Fig. 1

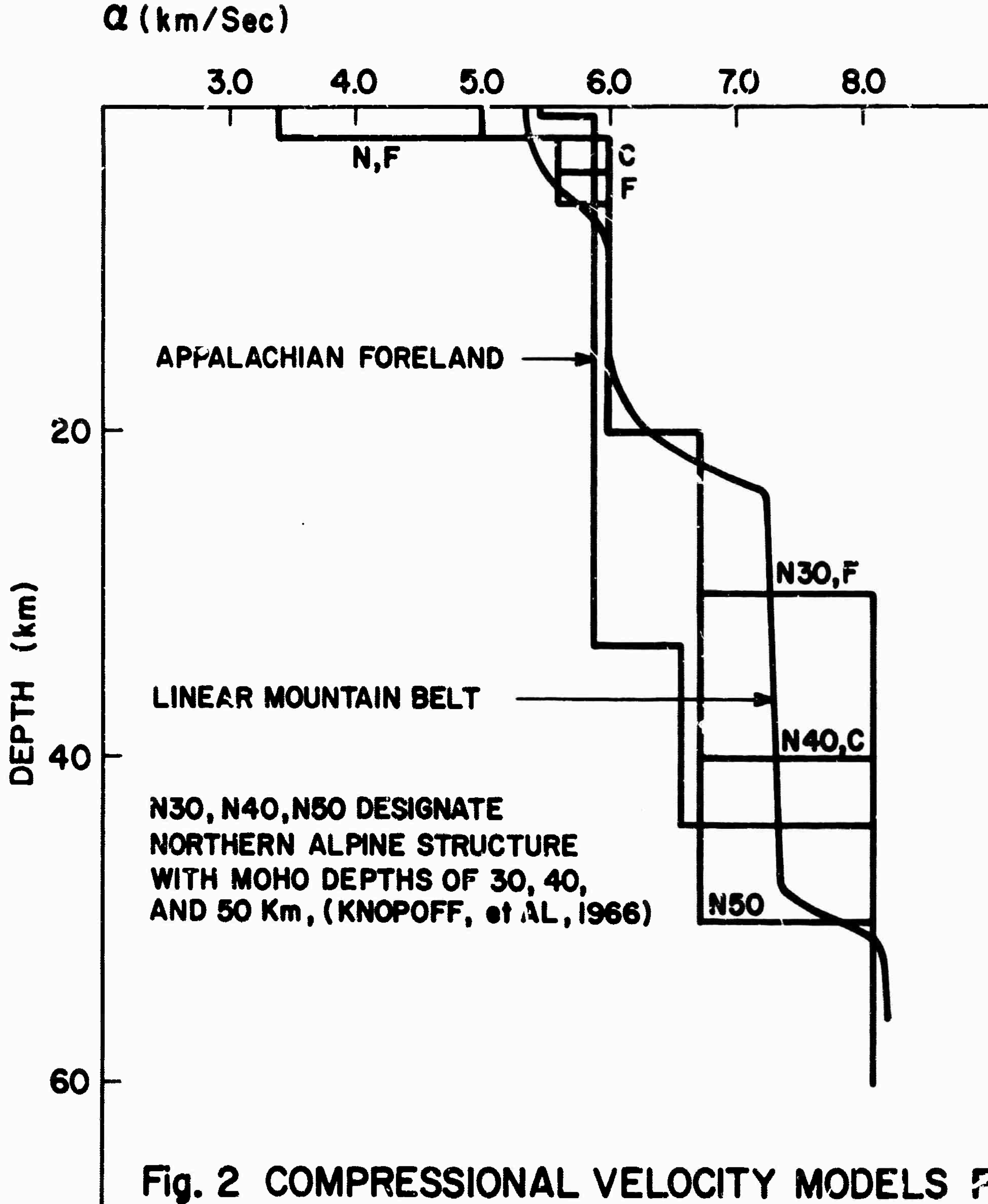


Fig. 2 COMPRESSIONAL VELOCITY MODELS FOR LINEAR MT. BELT, APPALACHIAN FORELAND, AND ALPINE STRUCTURE (KNOPOFF, et AL, 1966)

with those for the Northern Alps at depths greater than about 2 km. The disagreement for depths less than 2 km can probably be accounted for by the sedimentary cover present in the Northern Alps. The 5.85 km/sec layer in the Appalachian foreland, extending to a depth of 33 km, is thicker than any of the Alpine structures. However, as mentioned, the Appalachian foreland velocities may increase rather continuously from about 10 to 40 km. Upper mantle depth in the Appalachian foreland is greater than beneath the foreland to the north of the Alps by 14 km and greater than beneath the central Alps by 4 km.

A preliminary summary of seismic refraction work in the vicinity of the Cumberland Plateau Seismological Observatory [Borcherdt et al, 1966] indicates a crustal model of $h_1 = 12$ km ($v_1 = 6.1$ km/sec); $h_2 = 28$ km ($v_2 =$ 6.7 km/sec) and an upper mantle velocity of 8.0 + km/sec.

3.0 CRUSTAL THICKNESS FROM GRAVITY DATA

Total crustal thickness was computed from Bouguer gravity values along a Northwest-Southeast profile extending from about 460 km off the North Carolina coast (33°N, 73°W) to the Kentucky-Illinois border (38°N, 88°W). The sin x/x method of Tomoda and Aki [1955] was used to compute the depth to a mass anomaly producing the observed gravity anomalies. Bouguer gravity values were taken from the American Geophysical Union Bouguer Gravity Anomaly Map of the United States. Fig. 3 shows the gravity profile values, the total crustal thickness computed from these anomalies, and regional subsurface geology. The subsurface geological information was obtained from McGuire and Howell [1963] in Kentucky, Hersey, et al [1959] for the North Carolina continental margin, and from the geologic map of North Carolina. Crustal structure to the crust-mantle boundary at location H', and to 2 km at 12-13 is based on refraction profiles of Hersey, et al The subsurface geology in North Carolina is intended only to indicate possible near surface relations between geology and Bouguer gravity anomalies.

In the sin x/x method, crustal thickness is computed from

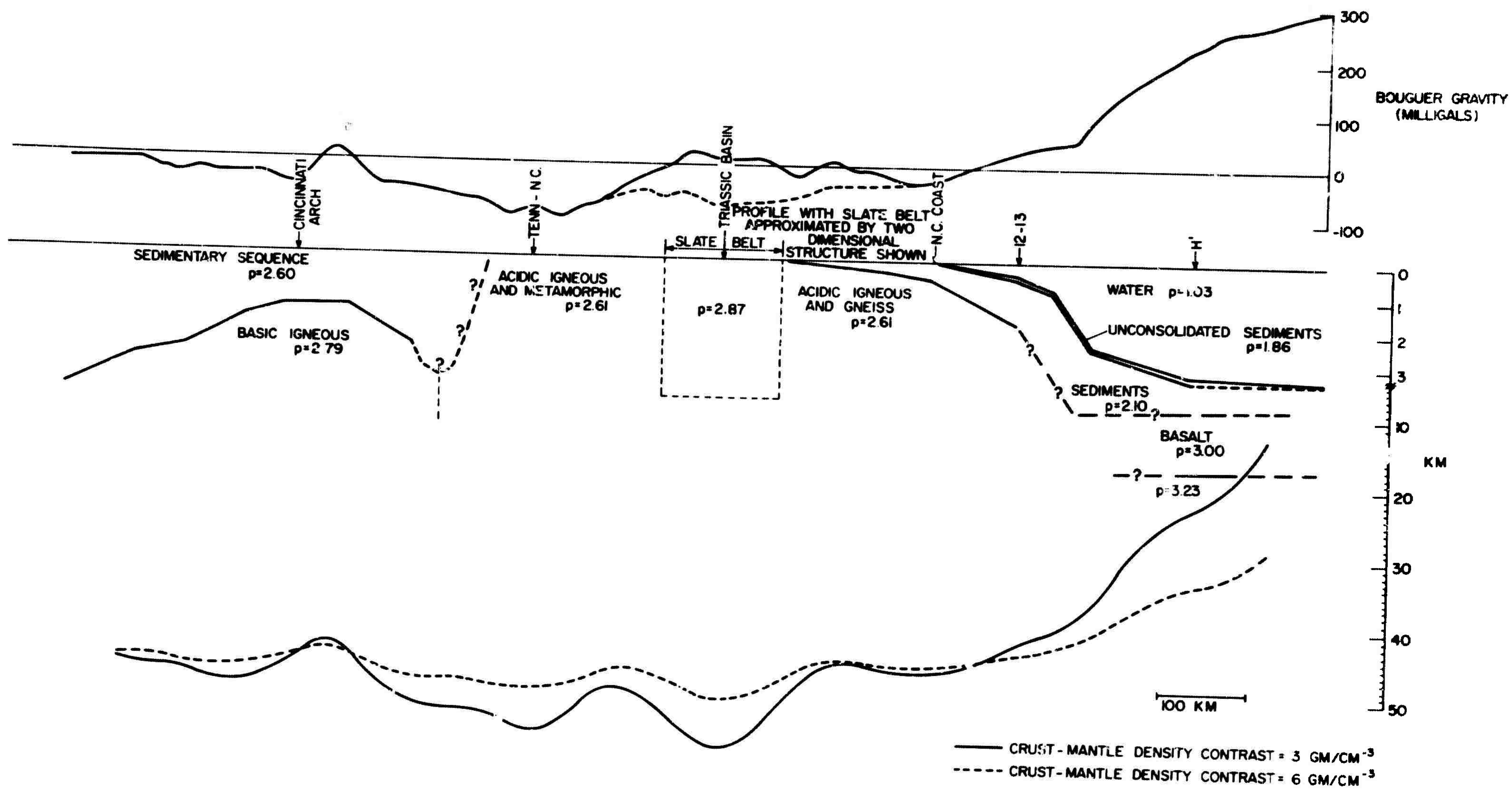
$$d(nx) = d - d'(nx) \qquad 3.0\text{-}1$$

where d is an assumed thickness and d'(nx) is a correction to this thickness given by

$$d'(nx) = \frac{M(nx)}{\Delta\rho} . \qquad 3.0\text{-}2$$

M(nx) is the convolution of the observed gravity anomalies $\Delta g(q\Delta x)$ with a symmetric function Φ_n which is a function of assumed crustal thickness and station spacing. Thus,

$$M(nx) = \frac{1}{2\pi^2 k^2} \sum_{q=-m}^{q=m} \Delta g(q\Delta x)\, \Phi_{n-q} \qquad 3.0\text{-}3$$



CRUSTAL THICKNESS FROM BOUGUER GRAVITY ANOMALIES

Fig 3

Oscillations in the gravity values $\Delta q(q\Delta x)$ due to near surface density irregularities can result in $\Delta q(q\Delta x)$ going positive and negative, such as in the region over the slate belt in Fig. 3. Thus, $M(nx)$ may oscillate between positive and negative values which in turn yields positive and negative oscillations of $d'(nx)$. The ultimate effect is that in the regions of local near surface perturbations, the total crustal thickness, $d(nx)$, may oscillate widely about the assumed thickness, d , as can be seen from 3.0-1. Introduction of a density contrast $\Delta\rho$ which varies with depth will not eliminate these oscillations. One must either choose the station spacing wide enough so that the convolution of the anomalies with the function Φ_n effectively filters out the local perturbations or smooth the total depth function $d(nx)$.

From Fig. 3, it can be seen that the Carolina Slate Belt is associated with a gravity high which effectively introduces a positive perturbation on the regionally decreasing gravity. If the crustal thickness is computed without removing this perturbation, the thickness oscillates about the assumed thickness beneath the belt. As shown in Fig. 3, the local gravity high over the slate belt can be largely accounted for if the belt is approximated by a two-dimensional block with lateral extent equal to the slate belt, an 8 km depth and a density contrast of .26 gm/cm^{-3}.

Crustal thickness was computed from 3.0-1 using 3.0-2 and 3.0-3 with an assumed crustal thickness of 45 km and a station spacing of 60 km. The thickness values were then smoothed with a three point moving average filter (.25, .50, .25) resulting in the smoothed crustal thickness curves shown in Fig. 3. Two curves are plotted, corresponding to crustal-upper mantle density contrasts of .3 and .6 gm/cm^{-3}. Since the ocean's crust is denser than the continental crust, the $\Delta\rho = .3$ curve approximates the crustal thickness better under the continental margin. The agreement with the thickness as determined by refraction work [Hersey, et.al., 1959] at point H' is good. Both curves indicate a crustal thickness of at least 50 km under the Southern Appalachians. Perturbations in the crustal thickness are caused by the Cincinnati Arch and the Carolina Slate Belt. The thinning of the crust necessary to produce the Bouguer anomaly over the Cincinnati arch and the slate belt is about 9 km for the $\Delta\rho = .6$ curve and about 4.5 km for the $\Delta\rho = .3$ curve. Due to the magnitude of the crustal thickness changes, it appears that the sources of the local highs over the Cincinnati Arch and the Carolina Slate Belt are relatively near surface.

The crust thus thickens from about 83 km at the North Carolina coast to about 51 km beneath the core of the Appalachians and then thins to about 43 km at the Kentucky-Illinois border.

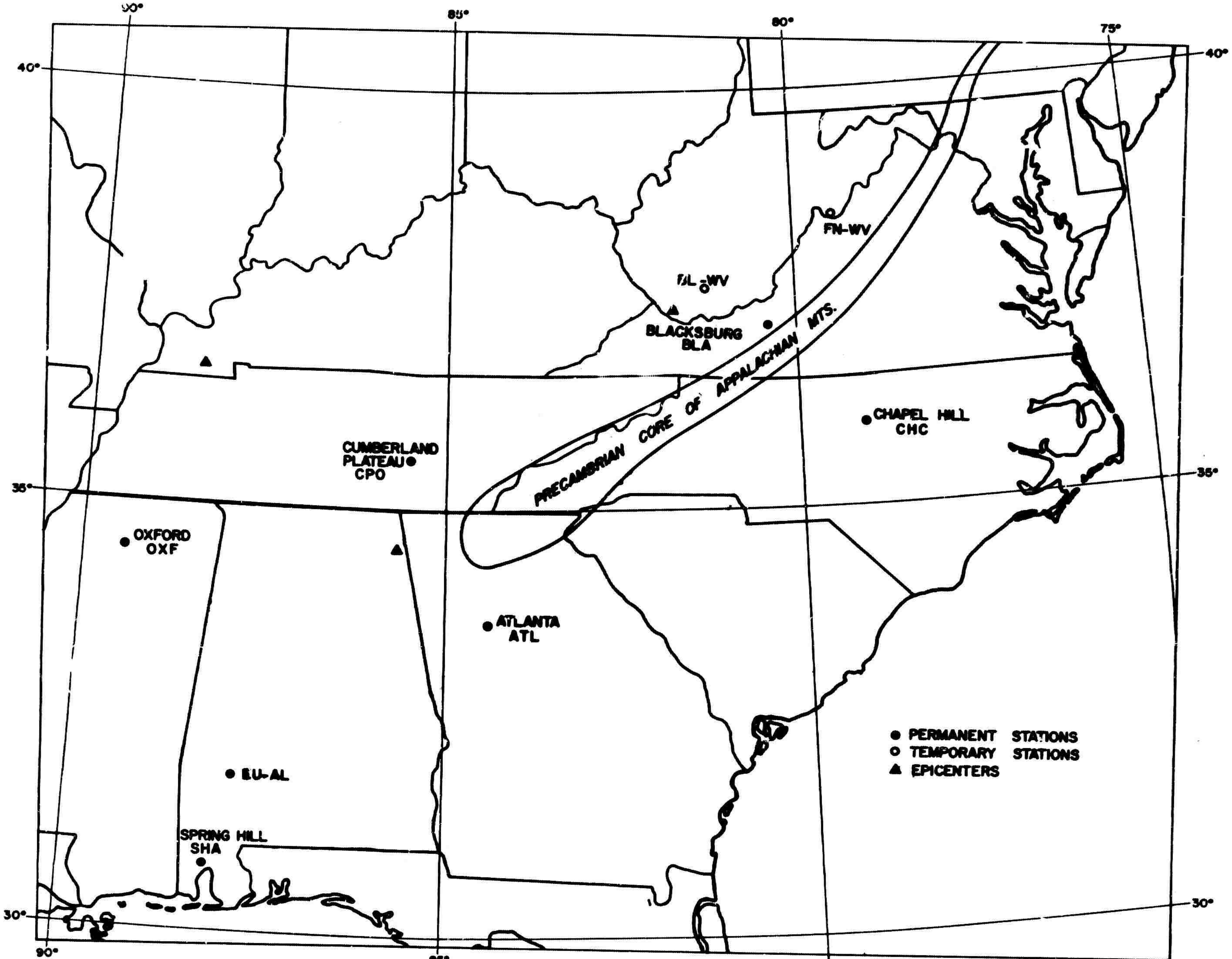
4.0 DETERMINATION OF RAYLEIGH GROUP VELOCITY

The locations of permanent recording stations in the Southeastern United States lie principally along the Appalachian trend (See Fig. 4). Permanent worldwide standard stations capable of recording long-period seismic signals are located at Spring Hill, Alabama (SHA); Atlanta, Georgia (ATL); McMinnville, Tennessee (CPO) ; Blacksburg, Virginia (BLA); and Oxford, Mississippi (OXF). Portable long-period units have been operated by the Geotechnical Corporation under project Vela, but these stations are also located along the Appalachians. Because of the widely space station, it was impossible to calculate phase velocities directly using triangular arrays of stations. Therefore, epicenters were selected to give travel paths parallel or perpendicular to the Appalachian trend. It was hoped that variations in crustal and upper mantle structure between stations located along the Appalachians could be detected by observing the variation of group velocity of a wave train traveling the station sequence SHA -ATL -BLA parallel to the Appalachian trend or by comparing group velocities at BLA and ATL from waves arriving perpendicular to the Appalachian with those of a normal continental structure.

On the basis of epicentral location, signal amplitude, and availability of records, two epicenters were selected for study. Table II gives the information pertinent to these epicenters.

Table II

Epicenter Location (USGS)	Date	Time (USGS)	Magnitude (USGS)	Focal Depth (km) (USGS)	Distance (km) to station
Jalisco, Mex. 17.8N, 105.9W	11 Oct., 1963	10:17:07.6	5.0	33	BLA - 3286.3 OXF - 2467.3 SHA - 2291.0 ATL - 2757.6
S. Alaska 62.7N, 152.0W	29 June, 1964	07:21:32.8	5.6	33	BLA - 5488.0 OXF - 5276.3 SHA - 5110.0 ATL - 5633.9



LOCATION MAP OF PERMANENT AND TEMPORARY SEISMOGRAPH STATIONS USED IN TRAVEL-TIME AND RAYLEIGH WAVE DISPERSION STUDIES

Fig. 4

Records from the standard stations were hand digitized at two second intervals using a plastic grid overlay. This digital data was stored on magnetic tape for processing. Since the azimuths of the epicenters measured from the recording stations did not coincide with either of the horizontal component seismograph orientations, Rayleigh wave motion was contaminated by Love wave motion. In order to separate the Rayleigh and Love wave motions, radial and transverse seismograms were generated from the North-South and East-West components at each station. The relations used in the transformations were

$$\text{radial component} = r = \overline{OE}\cos\theta + \overline{ON}\sin\theta \text{ , and}$$
$$\text{transverse component} = t = \overline{OE}\sin\theta - \overline{ON}\cos\theta \text{ .}$$

where

θ = azimuth of epicenter from station measured counterclockwise from east,

$\overline{OE}$ = east-west component amplitude, positive toward east,

$\overline{ON}$ = north-south component amplitude, positive toward north

$r>0$ => radial motion toward epicenter, and

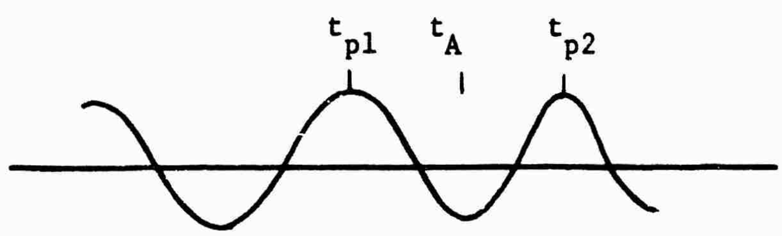
$t>0$ => transverse motion to right of propagation direction.

Sections of Rayleigh wave motion were then determined from visual inspection of plots of the radial and transverse components. Determination of the particle motion of small amplitude high frequency motion in the presence of large amplitude low frequency motion is difficult. Ideally, the radial and transverse components should be band-pass filtered to separate the frequencies and particle motion then determined for specified frequency intervals. However, due to the computer time involved, this was not feasible for this study. Since only the first higher Rayleigh mode was present on the recordings, the particle motion for the frequencies involved could be fairly well obtained from the unfiltered radial and transverse components.

Vertical component records were convolved with 101 point, digital, band-pass filters described by Minear [1966]. Pass bands of 60-100, 25-62, 16-30, 10-17, and 7-13 sec were used successively. The filtered data was plotted by using a Calcomp plotter. Period was obtained by reading the peak-to-peak period from the Calcomp plots. Arrival time for the period was taken as the time defined by

$$t_A = t_{p1} + \frac{t_{p2} - t_{p1}}{2}$$

with the quantities defined in the following figure.



Group velocity was then obtained by dividing the epicentral distance by the arrival time.

Group velocity vs. period data for the Southern Appalachians is presented in Table III and in Figs. 8-11.

TABLE III. RAYLEIGH WAVE GROUP VELOCITIES FOR THE SOUTHERN APPALACHIANS

EPICENTER-S. ALASKA
STATION-ATLANTA, GA.

Fundamental Rayleigh	
Period(sec)	Group Velocity(km/sec)
59.66	3.72
52.25	3.72
41.11	3.53
36.90	3.62
32.81	3.4[illegible]
30.62	3.38
26.69	3.29
27.00	3.24
27.00	3.19
24.02	3.14
19.31	3.05
18.68	3.02
18.84	2.96
18.21	2.99
18.06	2.86
18.06	2.31
17.90	2.91
16.80	2.94
16.17	2.88
15.70	2.83
15.17	3.04
14.13	2.98
13.25	3.02
13.03	3.07
12.09	3.13
12.09	3.00
10.99	3.11
10.99	3.09
9.26	3.01
7.54	3.01
7.54	3.00
7.22	2.99
6.59	2.98

First Higher Rayleigh	
Period(sec)	Group Velocity(km/sec)
8.64	3.52
7.85	3.56
7.85	3.55
7.85	3.51
7.07	3.49
7.07	3.47
7.07	3.49
6.28	3.46
6.28	3.45
6.91	3.50
6.12	3.54
6.12	3.47
5.65	3.52
5.34	3.43
4.55	3.53

EPICENTER-S. ALASKA
STATION-OXFORD, MISS.

Fundamental Rayleigh	
Period(sec)	Group Velocity(km/sec)
67.04	3.84
54.79	3.67
40.04	3.57
34.54	3.49
31.40	3.41
29.05	3.35
26.85	3.14
24.81	3.29
24.65	3.24
23.71	3.09
23.08	3.05
21.51	2.97
20.25	3.01
19.94	2.94
19.94	2.93
18.37	2.91
18.06	2.85
17.74	2.90
17.43	2.88
16.64	2.73
16.17	2.83
16.17	2.76
16.01	2.78
15.86	2.80
14.97	2.88
14.86	2.86
14.70	2.83
13.56	2.81
13.19	3.03
12.87	2.98

Fundamental Rayleigh	
Period(sec)	Group Velocity(km/sec)
12.40	3.01
12.40	2.96
11.78	3.07
11.62	3.05
10.52	2.98
9.89	2.97
9.89	2.92
9.89	2.70
9.73	2.95
9.73	2.88
8.32	2.93
8.16	2.86
8.01	2.89

EPICENTER-JALISCO, MEX.
STATION-ATLANTA, GA.

Fundamental Rayleigh	
Period(sec)	Group Velocity(km/sec)
55.89	3.76
44.90	3.52
38.78	3.34
33.76	3.20
27.48	3.20
22.45	3.11
19.94	3.03
17.11	2.97
16.80	2.92
12.40	2.89
12.09	2.70
11.93	2.66
11.78	2.85
11.78	2.82
11.62	2.78
10.99	2.69
10.99	2.66
10.36	2.75
10.21	2.75
10.05	2.72
10.00	2.69
9.89	2.73
8.90	2.62
8.8[illegible]	2.6[illegible]
8.20	2.64
8.19	2.60
8.16	2.58
8.16	2.56
7.69	2.64
6.44	2.5[illegible]

First Hi[illegible]ayleigh	
Period(sec)	Group Velocity(km/sec)
10.05	3.25
9.89	3.29
9.89	3.22
8.79	3.16
8.32	2.98
8.01	3.13
8.01	3.05
8.01	3.00
7.85	3.08
7.07	3.18
6.28	3.10
5.81	3.03

EPICENTER-JALISCO, MEX.
STATION-BLACKSBURG, VA.

Fundamental Rayleigh	
Period(sec)	Group Velocity(km/sec)
65.16	4.11
62.02	3.81
50.08	3.55
39.41	3.38
32.34	3.26
31.87	3.26
29.67	3.07
26.38	2.99
25.90	2.92
25.40	2.87
19.80	2.77
19.50	2.79
18.10	2.69
14.29	2.77
13.50	2.74
12.56	2.70
12.09	2.63
11.93	2.65
11.78	2.60
9.89	2.58
8.01	2.56
6.12	2.55

First Higher Rayleigh	
Period(sec)	Group Velocity(km/sec)
18.37	3.76
16.17	3.84
16.01	3.98
14.13	3.32
13.97	3.23
13.82	3.91
13.66	3.59
13.35	3.70
13.35	3.54
13.03	3.49
12.25	3.64
12.09	3.36
11.93	3.45
11.93	3.27
11.78	3.19
11.46	3.40
10.68	3.13
10.05	3.16
9.73	3.09
9.26	3.10
8.16	3.05
8.01	3.07
7.85	3.01
5.97	3.03

TABLE III (CONT'D)

EPICENTER-S. ALASKA
STATION-BLACKSBURG, VA.

Fundamental Rayleigh		First Higher Rayleigh	
Period(sec)	Group Velocity(km/sec)	Period(sec)	Group Velocity(km/sec)
66.88	4.06	33.13	4.27
62.64	3.88	32.97	4.16
58.88	3.72	31.09	4.43
49.14	3.73	30.93	4.32
39.41	3.62	30.46	4.54
39.41	3.53	28.73	4.09
31.40	3.45	20.72	4.23
27.00	3.20	20.10	4.35
25.43	3.15	17.11	4.18
25.28	3.10	15.54	4.29
21.82	3.06	15.23	3.94
19.94	3.03	14.44	3.98
19.94	2.97	13.50	4.12
18.68	2.89	13.03	4.20
18.06	3.00	13.03	4.16
18.06	2.94	13.03	4.02
17.58	2.87	12.72	3.90
17.58	2.84	12.72	3.80
16.17	2.89	12.40	3.86
15.86	2.92	12.25	3.83
15.39	2.87	11.93	3.76
13.97	2.84	11.46	3.73
11.62	3.08	11.78	3.70
11.62	3.04	9.89	3.49
11.30	3.06	8.01	3.52
10.21	3.10	7.85	3.46
9.42	3.02	7.69	3.60
9.26	3.00	6.12	3.58
8.32	2.98	6.12	3.54
8.16	2.92	6.12	3.51
8.01	2.93	6.12	3.47
7.85	3.06	5.97	3.55
7.85	2.94	5.65	3.57
7.22	3.00		
7.22	2.99		
7.22	2.97		
6.91	3.07		
6.91	3.04		
6.75	3.05		
6.75	3.03		
6.28	2.96		
6.28	2.90		
6.12	3.02		
5.65	2.91		

5.0 THEORETICAL RAYLEIGH DISPERSION CURVES

The basic Thomson-Haskell matrix method was used to compute phase and group velocity vs. period curves for layered earth models. A computer program was written to compute Rayleigh wave dispersion curves and mode shape using the modified formulation of the Thomson-Haskell method presented by Dunkin [1965]. A program was also written to compute mode shape using the Thomson-Haskell method A discussion and comparison of the computation methods used is given in this section. Appendix I and II contain detailed descriptions of the actual mechanics of computation and computer programming. Fortran listings of the programs are given in Appendix III.

5.1 Thomson-Haskell Matrix Method

As is well known, the Thomson-Haskell matrix method consists of evaluating the roots of a determinant formed by the repeated multiplication of 4 x 4 layer matrices which are functions of the layer parameters of density, thickness, compressional velocity, shear velocity, as well as phase velocity and period.

Using Haskell's notation, the displacement-stress matrices at the top and bottom of the m^{th} layer are given by

$$\begin{bmatrix} \frac{\dot{u}_m}{c} \\ \frac{\dot{w}_m}{c} \\ \sigma_m \\ \tau_m \end{bmatrix} = a_m \begin{bmatrix} \frac{\dot{u}_{m-1}}{c} \\ \frac{\dot{w}_{m-1}}{c} \\ \frac{\sigma_{m-1}}{c} \\ \frac{\tau_{m-1}}{c} \end{bmatrix} \qquad (5.1\text{-}1)$$

where $a_m = D_m F_m^{-1}$ is the $m^{\underline{th}}$ layer matrix. By repeated application of (5.1-1), Haskell shows that, assuming no stresses at the free surface ($\sigma_o = \tau_o = 0$) and no sources at infinity,

$$\begin{bmatrix} \Delta_n' \\ \Delta_n'' \\ \omega_n' \\ \omega_n'' \end{bmatrix} = J \begin{bmatrix} \frac{\dot{u}_o}{c} \\ \frac{\dot{w}_o}{c} \\ 0 \\ 0 \end{bmatrix} \tag{5.1-2}$$

where $J = F_n^{-1} a_{n-1} \cdots a_1$ is the matrix product of the 4 x 4 layer matrices a_m, eliminating Δ_n' and ω_n' between the four equations yields

$$\frac{\dot{u}_o}{\dot{w}_o} = \frac{J_{22} - J_{12}}{J_{11} - J_{21}} = \frac{J_{42} - J_{32}}{J_{31} - J_{41}} \tag{5.1-3}$$

Since the J_{ij}, are functions of phase velocity and wave number , (5.1-3) is an implicit relation between c and k and thus the phase velocity dispersion function. The layer matrix elements of a_m are either trigonometric or hyperbolic functions depending on whether the phase velocity is greater than or less than the layer compressional and/or shear velocities. The multiplication of real and imaginary components of matrices on a computer which does not have complex number subroutines would in general add considerable complexity to the problem. However, as is shown in Appendix I, the form of the layer matrices leads to a simple solution by which the multiplication of the matrices with real and imaginary elements can be accomplished by the multiplication of certain elements by $\pm$ 1.

5.2 Numerical Difficulties in the Thomson-Haskell Method

In practice, numerical computational difficulties are encountered in the repeated matrix multiplication required to evaluate the roots of (5.1-3). These difficulties are encountered as the product of kH, where k is the wave number, and H is the total thickness of the layered earth model, becomes large. Dorman, Ewing, and Oliver [1960] have used an upper limit of about 30 for kH. When the value of kH reaches about 30, the number of layers can be reduced and the computation continued with a reduced thickness. Little error is introduced by this technique. However, for higher modes and hence, higher frequencies, the product kH may be relatively large even for layered earth models of small total thickness, H.

Dunkin [1965] has shown the numerical difficulties are caused by the computation of large exponentials and a resulting loss of singificant figures. Dunkin's development is briefly repeated below in order to show the effect of loss of significance due to the addition of large and small quantities on the Haskell matrix. The agrument is applied directly to the Haskell dispersion equation (5.1-3) rather than to the secular equation used by Dunkin.

Let the scalar and vector potential functions for an elastic body have the form

$$\begin{aligned}
\phi_n &= \exp ik(ct-x)\ [A_n \exp(ikz\sqrt{c^2/\alpha^2-1}) + B_n \exp(-ikz\sqrt{c^2/\alpha^2-1})\ ] \\
&= \exp ik(ct-x)\ [\phi_n^+ + \phi_n^-\ ] \\
\psi_n &= \exp ik(ct-x)\ [C_n \exp(ikz\sqrt{c^2/\beta^2-1}) + D_n \exp(-ikz\sqrt{c^2/\beta^2-1})\ ] \\
&= \exp ik(ct-x)\ [\psi_n^+ - \psi_n^-\ ]
\end{aligned} \tag{5.2-1}$$

Using (5.2-1) and the equation

$$\bar{S}\ (u,v,w) = \nabla \cdot \phi + \nabla \times \psi\ (\psi_1, \psi_2, \psi_3) \qquad (5.2\text{-}2)$$

the displacement-stress vector, S_n, can be expressed as

$$S_n\ (z) = T_n\ \Phi_n\ (z) \qquad (5.2\text{-}3)$$

where

$$\Phi_n(z) = \begin{bmatrix} \phi_n^+ \\ \psi_n^+ \\ \phi_n^- \\ \psi_n^- \end{bmatrix}$$

and T_n is a 4 x 4 matrix function of c, k, and the layer parameters.

Taking the origin at the z_{n-1} interface (5.2-1) gives

$$\phi_{n-1} = A_n + B_n = \phi_n^+\ (z_{n-1}) + \phi_n^-\ (z_{n-1})$$

$$\psi_{n-1} = C_n + D_n = \psi_n^+\ (z_{n-1}) + \psi_n^-\ (z_{n-1})$$

At the z_n interface

$$\phi_n = A_n \exp ikd_n r_{\alpha n} + B_n \exp -ikd_n r_{\alpha n} = \phi_n^+ (z_n) + \phi_n^- (z_n)$$

$$\psi_n = C_n \exp ikd_n r_{\beta n} + D_n \exp -ikd_n r_{\beta n} = \psi_n^+ (z_n) + \psi_n^- (z_n)$$

The relation between $\Phi_n(z_n)$ at the z_n interface and $\Phi_n(z_{n-1})$ at the z_{n-1} interface is then

$$\Phi_n (z_n) = E_n \Phi_n (z_{n-1}) \tag{5.2-4}$$

where

$$E_n = \begin{bmatrix} \exp ikd_n r_{\alpha n} & 0 & 0 & 0 \\ 0 & \exp ikd_n r_{\beta n} & 0 & 0 \\ 0 & 0 & \exp -ikd_n r_{\alpha n} & 0 \\ 0 & 0 & 0 & \exp -ikd_n r_{\beta n} \end{bmatrix} \tag{5.2-5}$$

At the n-1 interface

$$S_n (z_{n-1}) = T_n \Phi_n (z_{n-1})$$

or

$$\Phi_n (z_{n-1}) = T_n^{-1} S_n(z_{n-1}) . \tag{5.2-6}$$

Now, by the boundary conditions of the continuity of stress and displacement

$$S_n(z_n) = S_{n+1} (z_n) = T_n \Phi_n (z_n) \tag{5.2-7}$$

Substituting (5.2-4) for $\Phi_n(z_n)$ and (5.2-6) for $\Phi_n(z_{n-1})$ in (5.2-7) gives

$$S_{n+1}(z_n) = T_n E_n T_n^{-1} S_n(z_{n-1}) . \qquad (5.2\text{-}8)$$

This equation is equivalent to Haskell's equation

$$S_{n+1}(z_n) = a_n S_n(z_{n-1}) , \qquad (5.2\text{-}9)$$

with $a_n = T_n E_n T_n^{-1}$.

By (5.2-8) the displacement-stress vector is converted into Φ_n , continued through the layer z_n by E_n , and converted back into $S_n(z_n)$ at the interface n+1 which is equal to $S_{n+1}(z_n)$. The Haskell layer matrix carries the displacement-stress vector from the $n^{\underline{th}}$ interface, through the layer, and across the n+1 interface in one operation. Eq. (5.2-8) brings into evidence the effect of the "continuing" matrix E_n.

Consider the matrix linking the displacement-stress vectors at the free surface and the last layer of an assumed layered sequence.

$$S_{n-1} = G_{n-1} \ldots G_1 S_o = P S_o , \qquad (5.2\text{-}10)$$

where

$$G_n = T_n E_n T_n^{-1}$$

In the Haskell formulation, the matrix from which the dispersion relation is obtained is given by $J = F_n^{-1}P$ and in the Dunkin formulation this matrix is $T_n^{-1}P$. However, considerations of the numerical evaluation of the P matrix will yield results valid to both developments since the T_n^{-1} or F_n^{-1} do not contain exponential powers.

Let P be written as

$$P = A \, T_m E_m T_m^{-1} \, B \tag{5.2-11}$$

where

$$A = G_{n-1} \cdots G_{m+1}$$

$$B = G_{m-1} \cdots G_1$$

Using the definitions of T_m and E_m , it can be shown that the components of P are of the form

$$P_{ij} = B_{ij} \exp ikd_m r_{\alpha m} + C_{ij} \exp ikd_m r_{\beta m} + D_{ij} \exp -ikd_m r_{\alpha m} + E_{ij} \exp -ikd_m r_{\beta m} \tag{5.2-12}$$

For the Haskell development, F_n^{-1} is of the form

$$F_n^{-1} = \begin{bmatrix} F_{11} & 0 & F_{23} & 0 \\ 0 & F_{22} & 0 & F_{24} \\ F_{31} & 0 & F_{33} & 0 \\ 0 & F_{42} & 0 & F_{44} \end{bmatrix}$$

which gives for the two components J_{12} and J_{22}

$$J_{12} = F_{11} P_{12} + F_{13} P_{32} \text{ , and}$$

$$J_{22} = F_{22} P_{22} + F_{24} P_{42} \tag{5.2-13}$$

Now suppose that for the m^{th} layer $r_{\alpha m}$ and $r_{\beta m}$ are negative imaginary ($c<\alpha$) so that $\exp idk_m r_{\alpha m}$ and $\exp ikd_m r_{\beta m}$ may be large depending on the value of k. If the exponential term is large enough, the effect of the smaller terms in P_{ij} will be neglected in computing the P_{ij}, because of loss of significance. In the evaluation of the roots of (5.1-3) the difference ($J_{12} - J_{22}$) must be taken. Although the P_{ij} are large, their differences may be small. Therefore, terms which were lost because of loss of significance in computing the P_{ij} would be important in the difference of J_{12} and J_{22}. Mode shape is computed from repeated applications of (5.1-1) using the starting values of $\dot{u}_o$ and $\dot{w}_o$ from (5.1-3). Therefore, the same problem with loss of significance is inherent in the Haskell method of computing modal shape.

5.3 Dunkin Modification of the Haskell Method

Dunkin derives the secular or period equation in the form

$$\text{Det } R_{11} = 0 \tag{5.3-1}$$

Where

$$R = \begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix} = T_p^{-1} \, G_{p-1} \cdots G_1 \tag{5.3-2}$$

He shows that Det R_{11} can be expanded as a product of the second order subdeterminants of T_p^{-1} and G_p yielding

$$\text{Det } R_{11} = t^{p-1} \begin{vmatrix} 12 \\ ab \end{vmatrix} g^{p-1} \begin{vmatrix} ab \\ cd \end{vmatrix} \cdots g^{1} \begin{vmatrix} ef \\ 12 \end{vmatrix} , \tag{5.3-3}$$

where $g^p \left|{}^{ij}_{k\ell}\right.$ is the second order subdeterminant of G_p involving rows i and j and columns k and ℓ. Dunkin has shown that by using algebraic expressions for the subdeterminants of the G_p , numerical difficulties with loss of significance can be avoided since the products of like exponentials normally occurring in the secular function are excluded at the start. Products of unlike exponentials for a given layer effectively increase the magnitude of Det R_{11}. To prevent machine overflow, the secular function can be divided by the two largest exponents when these exponenets become real and the exponential expression becomes hyperbolic. This results in no loss of significance.

Explicit expressions for the $g^{ij}_{k\ell}$ and the g_{ij} are given in Appendix II for real frequencies and wave numbers. These are slightly different from the definitions of Dunkin,since he assumes complex frequencies.

Mode shapes are computed using the following relation of Dunkin discussed in Appendix II ,

$$R_n^{\,m}(z;a) = r_{11}^{\,-1}\,\check{t}^{p}_{1r}\;g^{p-1}_{rs}\cdots g^{n}_{vb}(z_n-z)\;g^{n}(z-z_{n-1})\left|{}^{ab}_{cd}\right.\cdots g^{1}\left|{}^{ef}_{21}\right. \qquad (5.3\text{-}4)$$

5.4 Computational Procedure

The Dunkin method was programmed in Fortran II for the Bunker-Ramo 340 computer. Equation (5.3-1) was used for the determination of Rayleigh wave dispersion curves. Equation (5.3-4) was used to compute the mode shape once the roots of (5.3-1) were obtained. A double precision program was written in Fortran II for the IBM 360-75 using the Haskell method for computing mode shape. Equation (5.1-1) was used for this computation. Figs. 5 and 6 are flow charts of the computer program FLATRAY used in the computation of Rayleigh dispersion and modal shape. A computer listing of the program is given in Appendix III.

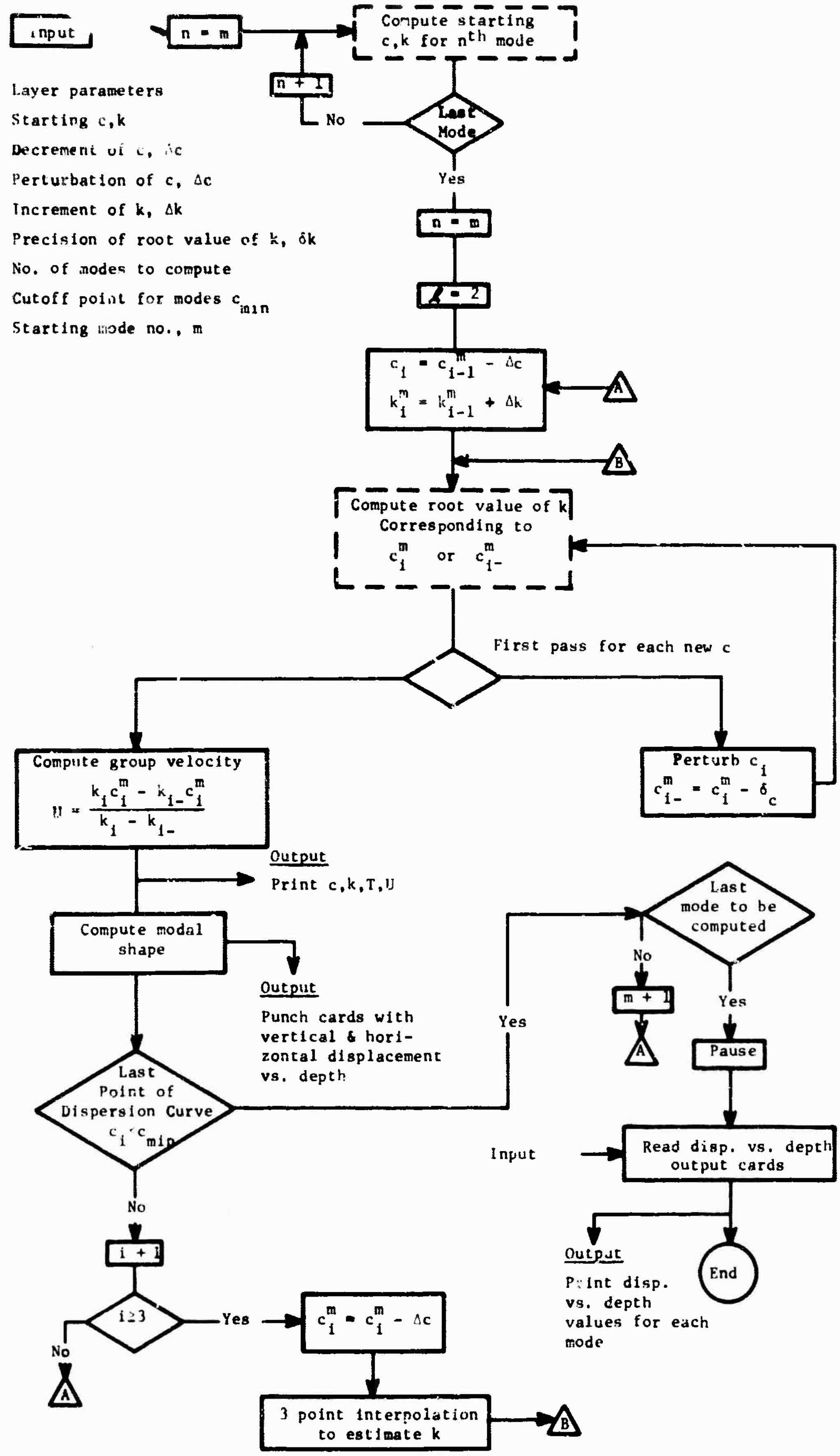


Fig. 5. Flow chart of FLATRAY

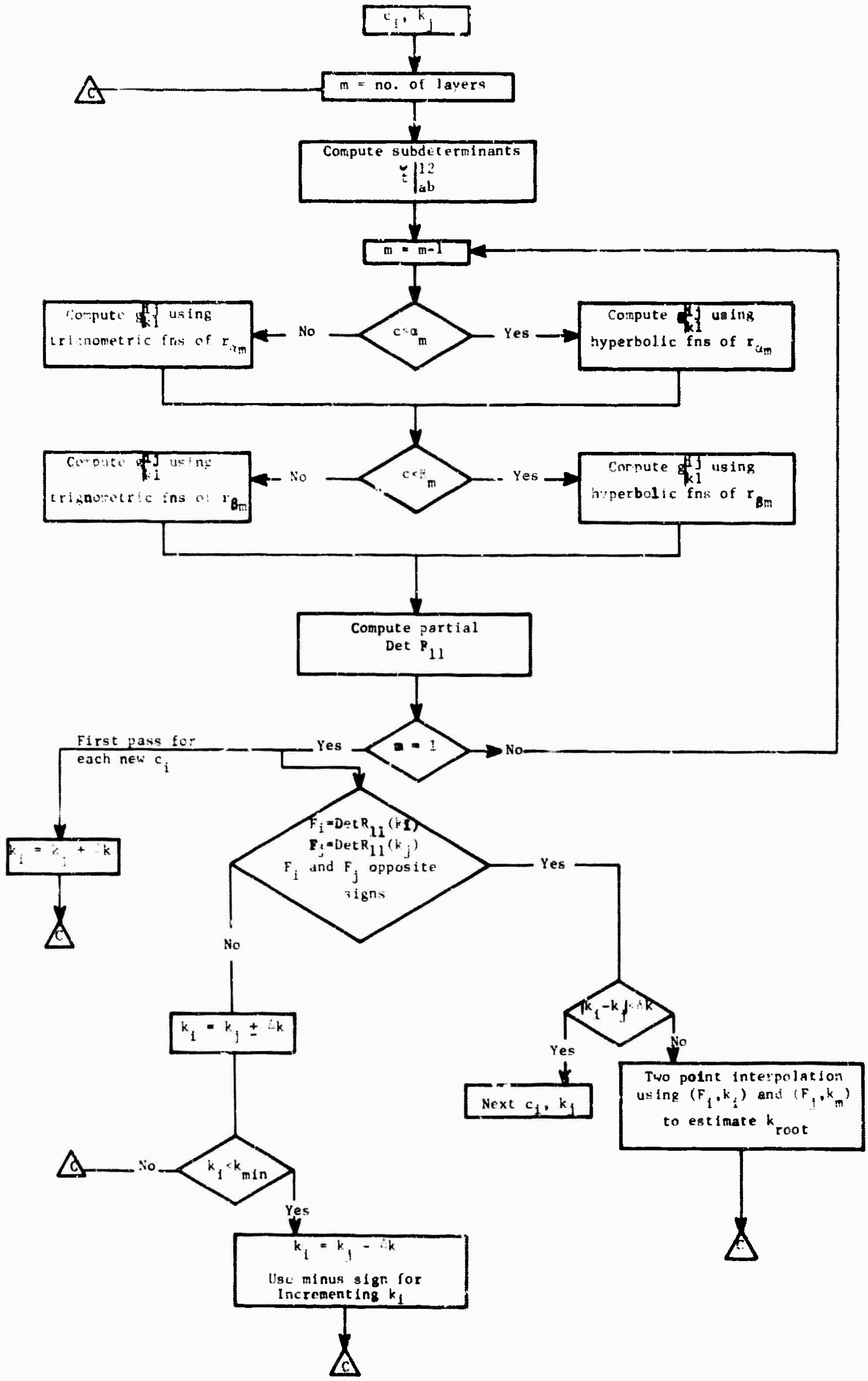


Fig. 6. Flow chart for determining roots of secular function, Det R_{11} (Dashed boxes of Fig. 5)

Root Finding Scheme

The technique used for finding the roots of (5.3-1) consists of two steps. In order to determine roots for different modes, a given phase velocity, c_o, and starting wave number, k_{io}, are specified. K_{io} is then incremented using the constant c_o value until a root is bracketed. A two point interpolation scheme is then used until the difference between two values of k which successively bracket the root is less than an input value. If more than one mode is to be investigated, k is incremented from its value at the last root found using the same value of c_o until the next root is found. Thus, the roots along a constant c curve are found which correspond to different modes. In order to define a particular mode, computation begins at c_o and the k_{io} corresponding to the desired mode. C is decremented and k incremented by values specified as input parameters. K is varied at the new - Δc until the root is bracketed. Two point interpolation is then used until k is obtained with the desired accuracy. The process of decrementing c and incrementing k is continued until three points on a given modal dispersion curve are found. A three point Gregory-Newton interpolation scheme is then used to estimate the next root on the curve. The process of interpolation and bracketing continues until a dispersion curve is defined to some minimum specified value of c.

Group velocity values are computed by perturbing c a small amount from a value at which a corresponding k has been found. K is found for the perturbed c and a two point difference scheme used to evaluate group velocity, U , according to

$$U = \frac{d\omega}{dk} = \frac{\frac{2\pi}{T_2} - \frac{2\pi}{T_1}}{k_2 - k_1}$$

From Table VI it can be seen that the group velocity curves computed by the Dunkin method for many layered models agree with those computed for models with reduced thickness to within a few tenths of a per cent. However, the section must be taken thick enough to include the entire depth to which appreciable particle motion extends. This is illustrated by the first higher mode for the Gutenberg-Birch II model. The period value at 5.00 km/sec is 50.0301 sec for the 400 km section, and 46.4060 sec for the 1000 km section. However, reference to the displacement vs. depth curves in Appendix IV shows that the 400 km section does not include the total depth to which vertical particle motion extends at this period.

Although thick sequences can be used to compute shorter period group velocity curves, it is considerably more economical in computer time to use thinner sequences of fewer layers. Displacement depth curves, such as those in Appendix IV, can be used to indicate the necessary total thickness to be used at given periods.

Modal Shape

After a point (c,k) was found on a given dispersion curve, the mode shape for the given (c,k) was computed using (5.3-4). Horizontal and vertical displacements vs. depth values were then punched out on cards to be used in computing the variation of phase velocity due to variation in layer parameters from INTEGRAL. Modal shape was computed using both the Dunkin and Haskell methods. A double precision program was used in computing modal shape by the Haskell Method. Table IV shows the comparative results for the two methods. Values for horizontal and vertical particle amplitudes agree very well (<.03%) for the first four layers in each case considered in Table IV. After this, the differences between the two methods increase rapidly. The rapid increase of particle amplitude with depth in

TABLE IV. Fundamental Rayleigh mode vertical and horizontal Particle amplitudes computed by Dunkin and Haskell Methods for the Gutenberg-Birch II model. Displacement normalized to the vertical displacement at the surface

Period = 25.[illegible]440
Phase Velocity = 3.8000
7 Layer model

Section Thickness = 140 km

HASKELL METHOD		DUNKIN METHOD	
Vertical	Horizontal	Vertical	Horizontal
1.000000	.695087i	1.000000	-.695087i
.886979	.008458i	.886972	.008445i
.518503	.056959i	.518524	.056825i
.303885	.106318i	.304132	.040085i
.164817	.076121i	.062881	.012639i
.078571	.050969i	.014165	.003463i
.018264	.048431i	.002693	.000995i

Period = 25.1380
Phase Velocity 3.8000
17 Layer model

Section Thickness = 400 km

1.000000	-.695044i	1.000000	-.695044i
.886837	.008611i	.886883	.008583i
.518193	.056916i	.518278	.056948i
.303773	.105662i	.303749	.040119i
.165958	.073450i	.062646	.012667i
.084840	.041298i	.013984	.003497i
.043072	.015424i	.002541	.001030i
.034665	-.021212i	-.000083	.000406i
.087474	-.119106i	-.000634	.000246i
.324986	-.435748i	-.000713	.000199i
1.117859	-1.484503i	-.000685	.000179i
4.078959	-4.927086i	-.000643	.000165i
13.757256	-16.190590i	-.000595	.000153i
45.749210	-53.018184i	-.000553	.000143i
150.983339	-173.243598i	-.000512	.000133i

TABLE IV (Cont'd)

Period = 25.1380 (Cont'd)
Phase Velcoity=3.8000
17 Layer model

Section Thickness = 400 km

HASKELL METHOD		DUNKIN METHOD	
Vertical	Horizontal	Vertical	Horizontal
8.176407	-567.580725i	-.000474	.000123i
10070.040164	-11362.129728i	-.001556	.000026i

Period = 106.4400
Phase Velocity=4.2000
35 Layer model

Section Thickness = 2898 km

1.000000	-.838062i	1.000000	-.838062i
1.047271	-.599088i	1.047270	-.599012i
1.054375	-.406096i	1.054340	-.405972i
1.048108	-.190905i	1.048030	-.182671i
1.010899	-.052715i	.968018	-.048623i
.954535	.043822i	.883244	.039389i
.886795	.109889i	.793526	.095305i
.812528	.152015i	.703175	.127442i
.735438	.175957i	.615440	.142587i
.658577	.186915i	.532770	.146331i
.583974	.187969i	.456412	.142087i
.513373	.182590i	.387350	.133150i
.447853	.173001i	.325880	.121633i
.387824	.160789i	.271911	.108862i
.333572	.147617i	.225107	.096133i
.284944	.134126i	.184894	.083957i
.184929	.099159i	.115100	.048651i
.1[illegible]4067	.072333i	.055227	.027497i
.024591	.043435i	.008158	.006575i
-.005338	.039627i	-.001498	.004761i
-.034168	.043999i	-.004566	.004231i
-.117644	.086439i	-.011717	.003220i
-.299456	.195575i	-.010744	.002691i

TABLE IV (Cont'd)

Period = 106.4400 (Cont'd)
Phase Velocity=4.2000
35 Layer model

Section Thickness = 2898 km

HASKELL METHOD		DUNKIN METHOD	
Vertical	Horizontal	Vertical	Horizontal
-.734195	.438798i	-.009416	.002297i
-1.730194	.869893i	-.008134	.001934i
-7.174500	-1.427081i	-.023747	.000546i
67.316536	-162.893231i	-.006543	.000141i
2727.220784	-3866.697396i	-.001763	.000036i
58523.227239	-72877.252431i	-.000463	.000009i
$.108 \times 10^{7}$	$-.127 \times 10^{7}\ i$	-.000119	.000002i
$.188 \times 10^{8}$	$-.214 \times 10^{8}\ i$	-.000030	.000000i
$.319 \times 10^{9}$	$-.355 \times 10^{9}\ i$	-.000007	.000000i
$.525 \times 10^{10}$	$-.576 \times 10^{10} i$	-.000002	.000000i
$.855 \times 10^{11}$	$-.928 \times 10^{11} i$	-.000000	.000000i
$.329 \times 10^{12}$	$-.352 \times 10^{12} i$	-.000000	.000000i

the Haskell Method results from the repeated multiplication of 4 x 4 layer matrices and gradual loss of significance in the matrix multiplication. Since the Haskell Method starts at the top layer and works down, and the Dunkin Method starts at the bottom layer and works up, the agreement of the two methods in the near surface layers indicates that the Dunkin Method is yielding correct displacement values over the entire layered sequence. In some cases when relative high frequency points on a dispersion curve were being computed with a many layered model, displacements computed by the Dunkin Method showed some slight perturbations with depth rather than a smooth decrease. Displacement values also tended to change signs as they decreased to very small quantities with depth. This is seen in the vertical displacements for the 35 layer case in Table IV. Horizontal displacement curves generally have one more lobe than the corresponding vertical displacement curves.

Modal shapes for the Gutenberg-Birch model are shown graphically in Appendix IV.

Earth Flattening Approximation

The earth flattening approximation introduced by Alterman, Jarosch, and Pekeris [1961] was used to modify the layer velocities. As has been shown by Kovach & Anderson [1964], the effect of sphericity is not negligible even for higher modes. The linear increase in velocity introduced in the earth flattening approximation is specified by the parameter

$$\xi = \left(\frac{r_m}{a}\right)^2$$

The value of r_m for a layer was taken as the radius to the center of the layer; a is the mean radius of the earth, 6371 km. Layer velocities approximately corrected for sphericity are then given by

$$\alpha'_m = \alpha_m \; \xi^{-1/2}$$

$$\beta'_m = \beta_m \; \xi^{-1/2}$$

5.5 Variation of Phase Velocity with Layer Parameters

The energy integrals for elastic wave propagation [Meissner, 1926; Jeffreys, 1934] have been used by several authors, notably Anderson [1964] and Takeuchi and Dorman [1964] to derive explicit relations between the variation of phase velocity and the variation of layer parameters. Necessary data for the evaluation of the partial derivatives of phase velocity with respect to layer parameters are horizontal and vertical particle amplitude vs. depth values.

For Rayleigh waves, the potential and kinetic energy averaged over a cycle are

$$4T = \int \rho\omega^2 \, (u^2 + w^2) \, dz$$

$$4V = \int [\lambda(ku - w')^2 + \mu(2k^2u^2 + 2w'^2) + k^2w^2 + u'^2 + 2ku'w] \, dz$$

where

u = horizontal displacement

w = vertical displacement

$u' = \frac{\partial u}{\partial z}$, and the integration extends over the entire depth.

Using the fact that the kinetic and potential energy averaged over a period are equal we obtain

$$\omega^2 I_1 = k^2(I_2 + I_5) + k(I_3 + I_6) + (I_4 + I_2) \quad , \qquad (5.5\text{-}1)$$

where

$$\begin{aligned} I_1 &= \int \rho (u^2 + w^2) dz & I_2 &= \int \lambda u^2 dz \\ I_3 &= \int 2\lambda u w' dz & I_4 &= \int \lambda w'^2 dz \\ I_5 &= \int \mu (2u^2 + w^2) dz & I_6 &= -\int 2\mu u' w dz \\ I_7 &= \int \mu (2w'^2 + u^2) dz \end{aligned} \qquad (5.5\text{-}2)$$

For a layered sequence of n layers, one can define

$$I_{1m} = \int_{z_m}^{z_{m+1}} \rho \, (u^2 + w^2) dz \qquad (5.5\text{-}3)$$

and similarly for the other integrals. Thus,

$$I_i = \sum_{m=1}^{n} I_{im} \qquad (5.5\text{-}4)$$

A perturbation of a layer parameter in the $m^{\underline{th}}$ layer will cause a perturbation in the integral, I_i , for the entire layered sequence of

$$\delta I_i = \delta I_{im} \qquad (5.5\text{-}5)$$

Differentiating (5.5-1) with respect to the layer parameters, the partial derivatives of c with respect to the layer parameters are obtained. Thus,

$$\left(\frac{\partial c}{\partial \mu_m}\right)_{\rho,\lambda,d,\omega} = \left(ck^2 \frac{\partial I_{5m}}{\partial \mu} + ck \frac{\partial I_{6m}}{\partial \mu} + c \frac{\partial I_{7m}}{\partial \mu}\right) / D \tag{5.5-6}$$

where

$$D = k[2k(I_2 + I_5) + I_3 + I_6]$$

with the integration of the integrals in D extending over the entire layered sequence.

$$\left(\frac{\partial c}{\partial \rho_m}\right)_{\lambda,\mu,d,\omega} = -c\omega^2 \frac{\partial I_{1m}}{\partial \rho} / D \tag{5.5-7}$$

$$\left(\frac{\partial c}{\partial \lambda_m}\right)_{\mu,\rho,d,\omega} = \left(ck^2 \frac{\partial I_{2m}}{\partial \lambda} + ck \frac{\partial I_{3m}}{\partial \lambda} + c \frac{\partial I_{4m}}{\partial \lambda}\right) / D \tag{5.5-8}$$

$$\left(\frac{\partial c}{\partial \beta_m}\right)_{\rho,\alpha,d,\omega} = 2\rho\beta\left[\left(\frac{\partial c}{\partial \mu_m}\right)_{\rho,\lambda,d,\omega} - 2\left(\frac{\partial c}{\partial \lambda_m}\right)_{\mu,\rho,d,\omega}\right] \tag{5.5-9}$$

Assuming ρ,α,β as independent and ρ,λ,μ as dependent

$$\left(\frac{\partial c}{\partial \rho_m}\right)_{\alpha,\beta,d,\omega} = \left(\frac{\partial c}{\partial \rho_m}\right)_{\lambda,\mu} + \left(\frac{\partial c}{\partial \lambda_m}\right)_{\rho,\mu} \left(\frac{\partial \lambda_m}{\partial \rho_m}\right)_{\alpha,\beta} + \left(\frac{\partial c}{\partial \mu_m}\right)_{\rho,\lambda} \left(\frac{\partial \mu_m}{\partial \rho_m}\right)_{\alpha,\beta} .$$

Substituting

$$\left(\frac{\partial \lambda_m}{\partial \rho_m}\right)_{\alpha,\beta} = \alpha_m^2 - 2\beta_m^2 \quad \text{and}$$

$$\left(\frac{\partial \mu_m}{\partial \rho_m}\right)_{\alpha,\beta} = \beta_m^2 ,$$

gives

$$\left(\frac{\partial c}{\partial \rho_m}\right)_{\alpha,\beta,d,\omega} = -c\omega^2 \frac{\partial I_{1m}}{\partial \rho} / D + \left(\frac{\partial c}{\partial \lambda_m}\right)_{\mu,\rho,\omega,d} (\alpha^2_m - 2\beta^2_m) + \beta^2 \left(\frac{\partial c}{\partial \mu_m}\right)_{\rho,\lambda,d,\omega}$$

(5.5-10)

$$\left(\frac{\partial c}{\partial \alpha_m}\right)_{\rho,\beta,d,\omega} = 2\rho_m \alpha_m \left(\frac{\partial c}{\partial \lambda_m}\right)_{\rho,\mu,d,\omega} \tag{5.5-11}$$

$$\left(\frac{\partial c}{\partial d}\right)_{\mu,\lambda,\rho,\omega} = \left[-c\omega^2 \frac{\partial I_1}{\partial d} + ck^2 \left(\frac{\partial I_2}{\partial d} + \frac{\partial I_5}{\partial d}\right) + ck \left(\frac{\partial I_3}{\partial d} + \frac{\partial I_6}{\partial d}\right) + c\left(\frac{\partial I_4}{\partial d} + \frac{\partial I_7}{\partial d}\right)\right]/D$$

The group velocity can be expressed in terms of the integrals I_i by

$$U = \frac{2(I_2 + I_5)k + (I_3 + I_6)}{\cdots 1} \tag{5.5-12}$$

Equations (5.5-6), (5.5-8), (5.5-9), (5.5-10), (5.5-11), (5.5-12), and (5.5-13) with the definitions of (5.5-2) were programmed for the IBM 360-75 computer. The integrals of (5.5-2) were evaluated numerically using polynomial approximations to the particle displacements obtained from the modal shape calculations. A sliding fitting procedure was used in determining the polynomials. In this procedure, a polynomial is fitted to say n points at the depths z_i, z_{i+1}, ... z_{i+n}; and the integrals and their derivatives evaluated over the interval $z_{i+n} - z_i$. The polynomial fit is then shifted to drop one layer and pick up one layer, i.e., to the points at depths

z_{i+1} , z_{i+2} , ... z_{i+1+n} . By using polynomial approximations, the integrals of the derivatives are simply the integrals of the derivatives of the polynomials. Third degree polynomials were found to give adequate fits to the displacement data.

A Fortran listing of the program, INTEGRAL, for computing the partial derivatives of phase velocity , c , with respect to layer parameters is given in Appendix III. Results of computation of the partial derivatives for the Gutenberg-Birch II continental model are given in Appendix IV.

6.0 CRUSTAL AND UPPER MANTLE STRUCTURE IN THE SOUTHEASTERN UNITED STATES

Gravity, local travel-time, and Rayleigh wave dispersion data were used to determine crustal and upper mantle structure for the Southeastern United States. Results of the Rayleigh dispersion study and a comparison with the gravity and travel-time results are given in this section.

Observed dispersion curves could be constructed only for periods less than about 50 seconds. Therefore, crustal structure was concentrated on and the upper mantle structure below 70 km was assumed to be that of the Gutenberg-Birch II continental model. The Gutenberg-Birch II crustal structure was used as the basic model for estimating the Southern Appalachian structure. Variations of phase velocity with layer parameters computed for the Gutenberg-Birch II model (See Appendix IV) were used to vary this basic model to yield dispersion curves fitting the observed data. Velocity and density structure of the models considered are given in Table V and Fig. 7. Values of the "earth flattening" velocities for the Gutenberg-Birch II are also given to indicate the effective increase of velocity with depth.

Fundamental and first higher Rayleigh mode group velocity vs. period data observed in the Southern Appalachians (Table III) are shown in Figs. 8-11. Rayleigh wave dispersion curves computed as described in Section 5.0 are given in Table VI and Figs. 8-11 for the models Gutenberg-Birch II, 310, 314, 315, and 320 defined in Table V.

Group velocity curves for waves traveling approximately perpendicular (perpendicular waves) to the Appalachians are quite similar (Figs. 8-11). They all have a local minimum of about 2.85 km/sec at a period of 17 sec, and a local maximum of about 3.10 km/sec at around 12 sec. For periods shorter than about 10 seconds the curves flatten out at about 3.0 km/sec. First higher Rayleigh mode curves indicate a broad minimum of 3.5 km/sec

TABLE V. Layer Parameters for Crustal and Upper Mantle Models

GUTENBERG-BIRCH II

FLAT EARTH				EARTH FLATTENING			
α	β	ρ	d	α	β	ρ	d
6.1400	3.5500	2.7500	19.00	6.1486	3.5550	2.7500	19.00
6.5800	3.8000	2.9000	19.00	6.6090	3.8167	2.9000	19.00
8.0800	4.6000	3.5700	22.00	8.1422	4.6354	3.5700	22.00
7.8700	4.5100	3.5100	20.00	7.9574	4.5601	3.5100	20.00
7.8000	4.4500	3.4900	20.00	7.9115	4.5136	3.4900	20.00
7.8300	4.4200	3.5000	20.00	7.9670	4.4974	3.5000	20.00
7.8900	4.4000	3.5100	20.00	8.0541	4.4915	3.5100	20.00
7.9400	4.3900	3.5300	20.00	8.1314	4.4958	3.5300	20.00
8.0000	4.4000	3.5500	20.00	8.2192	4.5206	3.5500	20.00
8.0600	4.4200	3.5600	20.00	8.3074	4.5557	3.5600	20.00
8.1200	4.4500	3.5800	20.00	8.3961	4.6013	3.5800	20.00
8.2000	4.4800	3.6100	20.00	8.5067	4.6476	3.6100	20.00
8.2700	4.5200	3.6300	20.00	8.6074	4.7044	3.6300	20.00
8.3500	4.5700	3.6500	20.00	8.7191	4.7720	3.6500	20.00
8.4300	4.6100	3.6800	20.00	8.8313	4.8294	3.6800	20.00
8.5100	4.6600	3.7000	50.00	8.9670	4.9102	3.7000	50.00
8.7500	4.8100	3.7700	50.00	9.2969	5.1106	3.7700	50.00
9.0000	4.9500	3.8500	100.00	9.6840	5.3262	3.8500	100.00
9.4900	5.2200	4.0000	50.00	10.3422	5.6888	4.0000	50.00
9.7400	5.3600	4.0700	50.00	10.7062	5.8917	4.0700	50.00
9.9900	5.5000	4.1500	100.00	11.1249	6.1248	4.1500	100.00
10.5000	5.7700	4.3000	100.00	11.9007	6.5397	4.3000	100.00
10.9000	6.0400	4.4200	100.00	12.5775	6.9696	4.4200	100.00
11.3000	6.3000	4.5400	100.00	13.2798	7.4038	4.5400	100.00
11.4000	6.3500	4.5700	200.00	13.7780	7.6746	4.5700	200.00
11.8000	6.5000	4.6900	200.00	14.8243	8.1660	4.6900	200.00
12.0500	6.6000	4.7700	200.00	15.7602	8.6321	4.7700	200.00
12.3000	6.7500	4.8500	200.00	16.7759	9.2063	4.8500	200.00
12.5500	6.8500	4.9200	200.00	17.8824	9.7606	4.9200	200.00
12.8000	6.9500	5.0000	200.00	19.0925	10.3666	5.0000	200.00
13.0000	7.0000	5.0600	200.00	20.3437	10.9543	5.0600	200.00
13.2000	7.1000	5.1200	200.00	21.7245	11.6852	5.1200	200.00
13.4500	7.2000	5.1900	200.00	23.3411	12.4949	5.1900	200.00
13.7000	7.2500	5.2700	98.00	24.7819	13.1145	5.2700	98.00
13.6500	7.2000	5.2500	∞	25.0395	13.2077	5.2500	∞

TABLE V. (Cont'd)

MODEL 310

α	β	ρ	d
5.8800	3.3800	2.6700	10.00
6.1400	3.5500	2.7600	10.00
6.5800	3.8000	2.9000	10.00
6.5800	3.8000	2.9000	10.00
8.0800	4.6000	3.5700	20.00

Same as Gutenberg-Birch II to 400 km

MODEL 314

α	β	ρ	d
5.8800	3.3800	2.6700	10.00
5.6000	3.2400	2.7600	10.00
6.1000	3.5000	2.9000	10.00
6.6000	3.8000	2.9000	10.00
7.0000	4.1000	3.1000	10.00
8.0800	4.6000	3.5700	20.00

Same as Gutenberg-Birch II to 810 km

MODEL 315

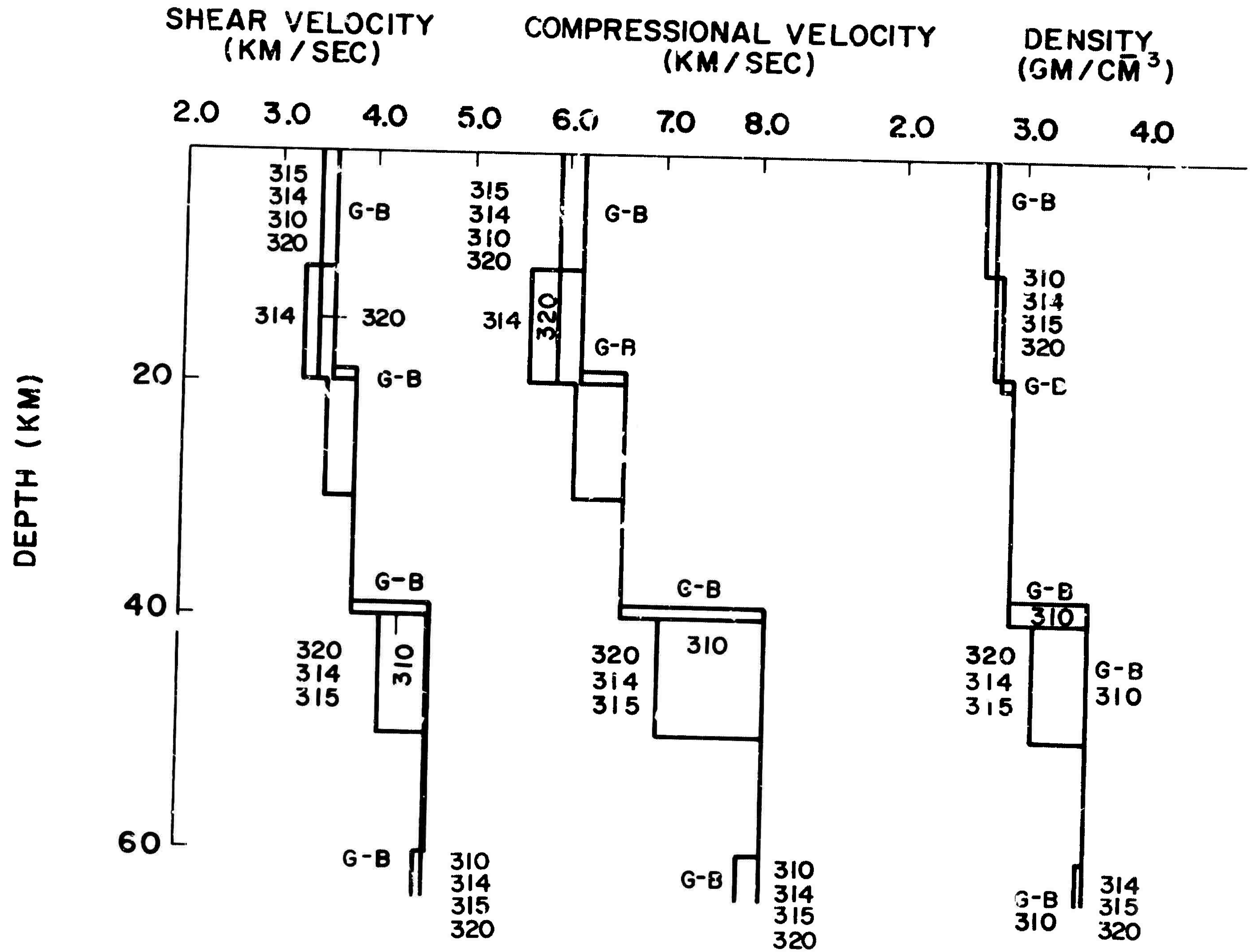
α	β	ρ	d
5.8800	3.3800	2.6700	10.00
6.1400	3.5500	2.7600	10.00
6.5800	3.8000	2.9000	10.00
6.5800	3.8000	2.9000	10.00
7.0000	4.1000	3.1000	10.00
8.0800	4.6000	3.5700	20.00

Same as Gutenberg-Birch II to 810 km

MODEL 320

α	β	ρ	d
5.8800	3.3800	2.6700	10.00
5.8800	3.3800	2.7600	10.00
6.1000	3.5000	2.9000	10.00
6.6000	3.8000	2.9000	10.00
7.0000	4.1000	3.1000	10.00
8.0800	4.6000	3.5700	20.00

Same as Gutenberg-Birch II to 810 km



CRUSTM AND UPPER MANTLE MODELS

Fig. 7

TABLE VI. Rayleigh wave dispersion curves for Gutenberg-Birch II models, and models 310, 314, and 315

GUTENBERG-BIRCH II

FUNDAMENTAL RAYLEIGH MODE

Section Thickness 2898.00

c	k	T	U	T
4.5000	.00850	164.2671		
4.4000	.00961	148.5391	3.6990	143.8940
4.3000	.01131	129.1709	3.7695	123.5369
4.1999	.01405	106.4402	3.8361	99.2251
4.0999	.01984	77.2556	3.9112	65.9224
3.9999	.03815	41.1794	3.7346	37.1515
3.8999	.05419	29.7316	3.4039	28.3452
3.7999	.06624	24.9623	3.1759	24.0856
3.6999	.07815	21.7314	3.0493	21.0107
3.5998	.09202	18.9673	2.9510	18.3398
3.4998	.10996	16.3264	3.0336	15.5185
3.3998	.14161	13.0509	3.1084	11.9219

Section Thickness 400.00

c	k	T	U	T
4.0000	.03801	41.3293		
3.9000	.05394	29.8693	3.4253	28.4013
3.8000	.06578	25.1380	3.2096	24.1929
3.7000	.07808	21.7493	3.0305	21.0535
3.5999	.09207	18.9572	2.9639	18.3140
3.4999	.11010	16.3054	3.0331	15.4998
3.3999	.14220	12.9958	3.1018	11.9019

Section Thickness 140.00

c	k	T	U	T
4.0000	.03730	42.1107		
3.9000	.05409	29.7861	3.4066	28.3880
3.8000	.06576	25.1423	3.1768	24.2579
3.7000	.07762	21.8791	3.0923	21.0892
3.5999	.09150	19.0747	2.9947	18.3862
3.4999	.11001	16.3184	3.0426	15.4971
3.3999	.14249	12.9695	3.0988	11.8907

FIRST HIGHER RAYLEIGH MODE

Section Thickness 2898.00

c	k	T	U	T
6.0000	.01020	102.6767		
5.9000	.01091	97.6155	4.5030	96.2532
5.8000	.01174	92.2541	4.4644	90.8938
5.6999	.01269	86.8711	4.4340	85.5044
5.5999	.01377	81.4673	4.4137	80.0789
5.4999	.01503	75.9839	4.3971	74.5686
5.3999	.01653	70.4033	4.3821	68.9585
5.2999	.01832	64.7021	4.3720	63.2178

TABLE VI. (Cont'd)

FIRST HIGHER RAYLEIGH MODE (Cont'd)

Section Thickness 2898.00 (Continued)

c	k	T	U	T
5.1998	.02053	5[illegible]578	4.3631	57.3296
5.0998	.02331	52.3603	4.3539	51.2881
4.9999	.02690	46.7207	4.3448	45.1023
4.8999	.03172	40.4258	4.3363	38.7565
4.7999	.03854	33.9652	4.3316	32.2275
4.6998	.04900	27.2834	4.3340	25.4277
4.5998	.06780	20.1477	4.3586	17.9272

Section Thickness 1000.00

c	k	T	U	T
5.4000	.01661	70.0684		
5.3000	.01837	64.5405	4.4587	62.8706
5.2000	.02062	58.5864	4.3608	57.0707
5.0999	.02345	52.5303	4.3509	50.9751
4.9999	.02708	46.4060	4.2904	44.9442
4.8999	.03199	40.0905	4.3225	38.4821
4.7999	.03879	33.7500	4.3062	32.1265
4.6998	.04933	27.1034	4.3338	25.2612

Section Thickness 400.00

c	k	T	U	T
5.0000	.02512	50.0301		
4.9000	.03111	41.2115	4.4079	39.2169
4.8000	.03846	34.0337	4.3302	32.2991
4.6999	.04883	27.3780	4.3458	25.4449
4.5999	.06819	20.0320	4.3531	17.8824
4.4999	.11357	12.2945	4.1959	11.2612
4.4000	.15149	9.4265		
4.3000	.17634	8.2863	3.5422	8.0520
4.2000	.20128	7.4324	3.3111	7.2619
4.0999	.22937	6.6814	3.3545	6.4906
3.9999	.25763	5.8694	3.4094	5.6467
3.8999	.32599	4.9423	3.4869	4.6556
3.7999	.43139	3.8330	3.4694	3.5446
3.6999	.61350	2.7681	3.4616	2.4623

Section Thickness 140.00

c	k	T	U	T
3.9000	.32410	4.9709	3.9114	8.4005
3.8000	.43142	3.8327	3.4763	3.5371
3.7000	.61704	2.7521	3.4545	2.4586

TABLE VI. (Cont'd)

SECOND HIGHER RAYLEIGH MODE

Section Thickness 400.00

c	k	T	U	T
4.9000	.05537	23.1581	4.2866	22.2932
4.8000	.06549	19.9891	4.2627	19.1184
4.6999	.08015	16.6789	4.2938	15.6739
4.5999	.10807	12.6396	4.3836	11.0562
4.5000	.25165	5.5484	3.9105	5.3337
4.3999	.29310	4.8721	4.4642	6.2623
4.2999	.32739	4.4632	3.2661	4.3787
4.1999	.36497	4.0990	3.1949	4.0190
4.0999	.40722	3.7634	3.4253	3.6417
3.9999	.47463	3.3096	3.4644	3.1683
3.8999	.59153	2.7237	3.5669	2.5201
3.7998	.84393	1.9593	3.4754	1.8086

Section Thickness 140.00

c	k	T	U	T
4.6000	.09402	14.5273		
4.5000	.25218	5.5369	3.8832	5.3339
4.3999	.29289	4.8756	3.5369	4.7581
4.2999	.32734	4.4640	3.3107	4.3744
4.1999	.36532	4.0951	3.3051	4.0019
4.0999	.40970	3.7406	3.3130	3.6409
3.9999	.47503	3.3069	3.4600	3.1671
3.8999	.58812	2.7395	3.5842	2.5214
3.7998	.84289	1.9617	3.4812	1.8076

TABLE VI. (Cont'd)

MODEL 314

FUNDAMENTAL RAYLEIGH MODE

Section Thickness 810 km

c	k	T	U	T
4.5000	.00800	174.4980	3.6546	152.3970
4.4000	.00910	156.9458	3.6739	133.7163
4.3000	.01054	138.6728	3.7362	111.4856
4.1999	.01273	117.4773	3.8025	84.3855
4.0999	.01661	92.2917		

Section Thickness 310

c	k	T	U	T
4.0000	.02547	61.6759	3.4695	39.4730
3.9000	.03856	41.7795	3.1635	33.1033
3.8000	.04824	34.2777	2.9694	29.2182
3.7000	.05646	30.0793	2.3125	26.2169
3.5999	.06486	26.9112	2.7168	23.4925
3.4999	.07445	24.1143	2.7635	20.7638
3.3999	.08602	21.4831	2.8034	17.8086
3.2999	.10206	18.6555	2.8919	13.9951
3.1999	.12901	15.2204		

FIRST HIGHER RAYLEIGH MODE

Section Thickness 310 km

c	k	T	U	T
5.0000	.02685	46.8048	4.4483	36.2523
4.9000	.02886	44.4386	4.3098	27.9880
4.8000	.03353	39.0360	4.2401	21.5914
4.6999	.04475	29.8741	4.1392	16.2429
4.5999	.05888	23.1967	3.8165	13.1554
4.4999	.08004	17.4449	3.5115	11.5744
4.3999	.10431	13.6899	3.3171	10.4853
4.2999	.12285	11.8943	3.2139	9.5683
4.1998	.13938	10.7339	3.1330	8.6651
4.0993	.15650	9.7925	3.1344	7.7859
3.9999	.17704	8.8727	3.1097	6.8751
3.8999	.20128	8.0045	3.1614	5.9217
3.7999	.23306	7.0948		
3.6999	.27476	6.1807		

TABLE VI. (Cont'd)

MODEL 315

FUNDAMENTAL RAYLEIGH MODE

Section Thickness 810 km

c	k	T	U	T
4.5000	.00820	170.2393		
4.4000	.00930	153.5283	3.6992	148.7251
4.3000	.01086	134.5723	3.7313	129.1714
4.1999	.01334	112.1333	3.8018	105.2905
4.0999	.01812	84.5767	3.8692	74.8110

Section Thickness 310 km

c	k	T	U	T
4.0000	.03053	51.4482		
3.9000	.04399	36.6250	3.4296	34.8049
3.8000	.05404	30.5995	3.2226	29.4172
3.7000	.06472	26.2377	3.0726	25.3270
3.5999	.07614	22.9218	3.0533	21.9835
3.4999	.09221	19.4693	3.0333	18.5069
3.3999	.11718	15.7708	3.0416	14.7044
3.2999	.16256	11.7130	3.0354	10.5790
3.1999	.25847	7.5969	3.0323	6.3232

FIRST HIGHER RAYLEIGH MODE

Section Thickness 310 km

c	k	T	U	T
5.0000	.02700	46.5387		
4.9000	.02898	44.2546		
4.8000	.03375	38.7906	4.4580	35.9338
4.6999	.04593	29.1047	4.3170	27.2273
4.5999	.06249	21.8596	4.3459	19.5919
4.4999	.09907	14.0942	4.1502	13.0875
4.3999	.12798	11.1578	3.7883	10.7461
4.2999	.14814	9.8638	3.4936	9.6058
4.1998	.16823	8.8928	3.4019	8.6584
4.0998	.19220	7.9737	3.4011	7.7268
3.9999	.22623	6.9435		
3.8999	.27637	5.8296	3.4963	5.4822
3.7999	.36302	4.5549	3.4527	4.2315
3.6999	.49743	3.4140	3.3824	3.1453
3.5998	.72066	2.4220	3.3589	2.1583
3.4998	1.18182	1.5191	3.3175	1.2885
3.3998	3.20689	.5763		

TABLE VI. (Cont'd)

MODEL 310

FUNDAMENTAL RAYLEIGH MODE

Section Thickness 400 km

c	k	T	U	T
4.0000	.02439	64.4070		
3.9000	.04255	37.8649	3.5912	34.7723
3.8000	.05805	28.4813	3.2801	27.2310
3.7000	.07023	24.1816	3.1144	23.2675
3.5999	.08334	20.9422	2.9759	20.2145
3.4999	.09981	17.9868	2.9384	17.2780
3.3999	.12365	14.9456	2.9835	14.1014
3.2999	.16535	11.5152	3.0190	10.4775

FIRST HIGHER RAYLEIGH MODE

Section Thickness 140 km

c	k	T	U	T
4.4000	.12476	11.4457		
4.3000	.16172	9.0352	3.6088	8.7490
4.2000	.18471	8.0990	3.3507	7.9020
4.0999	.21052	7.2797	3.2917	7.0921
3.9999	.24138	6.5077	3.4153	6.2579
3.8999	.29117	5.5333	3.4070	5.2734
3.7999	.36986	4.4707	3.4199	4.1854
3.6999	.49765	3.4125	3.3786	3.1475

MODEL 320

FUNDAMENTAL RAYLEIGH MODE

Section Thickness 310 km

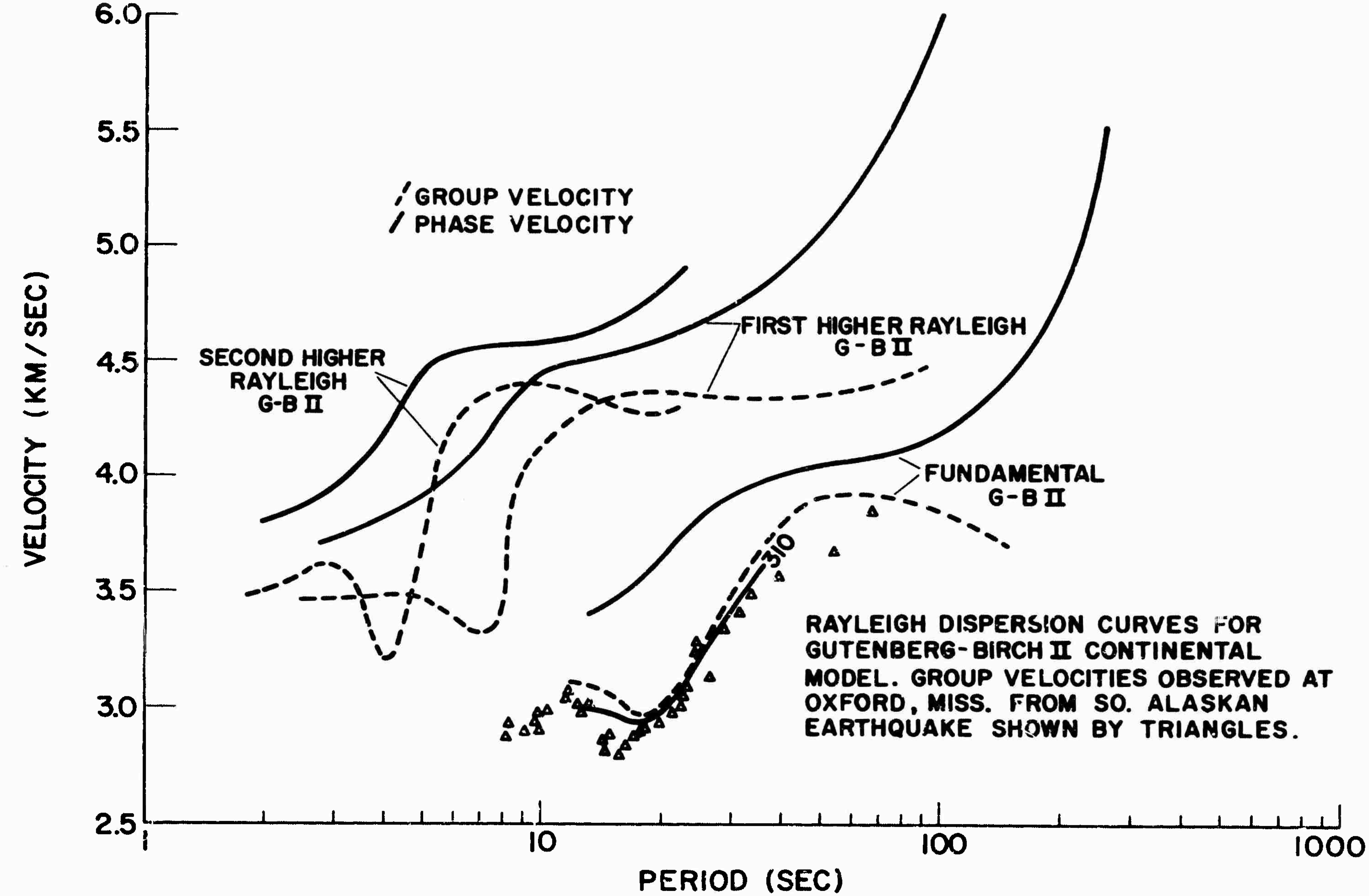
c	k	T	U	T
4.0000	.02676	58.6913		
3.9000	.03996	40.3191	3.4514	38.1989
3.8000	.04957	33.3568	3.1758	32.1857
3.7000	.05820	29.1794	2.9781	28.3310
3.5999	.06716	25.9897	2.8662	25.2547
3.4999	.07764	23.1217	2.7526	22.4878
3.3999	.09056	20.4061	2.8481	19.5889
3.2999	.11217	16.9749	2.8878	16.0079
3.1999	.15391	12.7579	3.0158	10.8511

TABLE VI. (Cont'd)

FIRST HIGHER RAYLEIGH MODE

Section Thickness 310 km

C	K	T	U	T
5.0000	.02694	46.6523		
4.9000	.02888	44.4016		
4.8000	.03358	38.9835	4.4617	36.0753
4.6999	.04490	29.7720	4.2997	27.9473
4.5999	.05957	22.9299	4.2645	21.2086
4.5000	.08333	16.7565		
4.4000	.11006	12.9742	3.8455	12.4357
4.3000	.12959	11.2758	3.5636	10.9456
4.1999	.14771	10.1280	3.3148	9.8943
4.0999	.16573	9.2470	3.2854	9.0110
3.9999	.18724	8.3893	3.1800	8.1781
3.8999	.21377	7.5367	3.1997	7.3062
3.7999	.24984	6.6182	3.2181	6.3649
3.6999	.30270	5.6103	3.2358	5.3289



GROUP VELOCITY
PHASE VELOCITY
SECOND HIGHER RAYLEIGH G-B II
FIRST HIGHER RAYLEIGH G-B II
FUNDAMENTAL G-B II
310
RAYLEIGH DISPERSION CURVES FOR GUTENBERG-BIRCH II CONTINENTAL MODEL. GROUP VELOCITIES OBSERVED AT OXFORD, MISS. FROM SO. ALASKAN EARTHQUAKE SHOWN BY TRIANGLES.
VELOCITY (KM/SEC)
6.0
5.5
5.0
4.5
4.0
3.5
3.0
2.5
1
10
100
1000
PERIOD (SEC)

Fig. 8

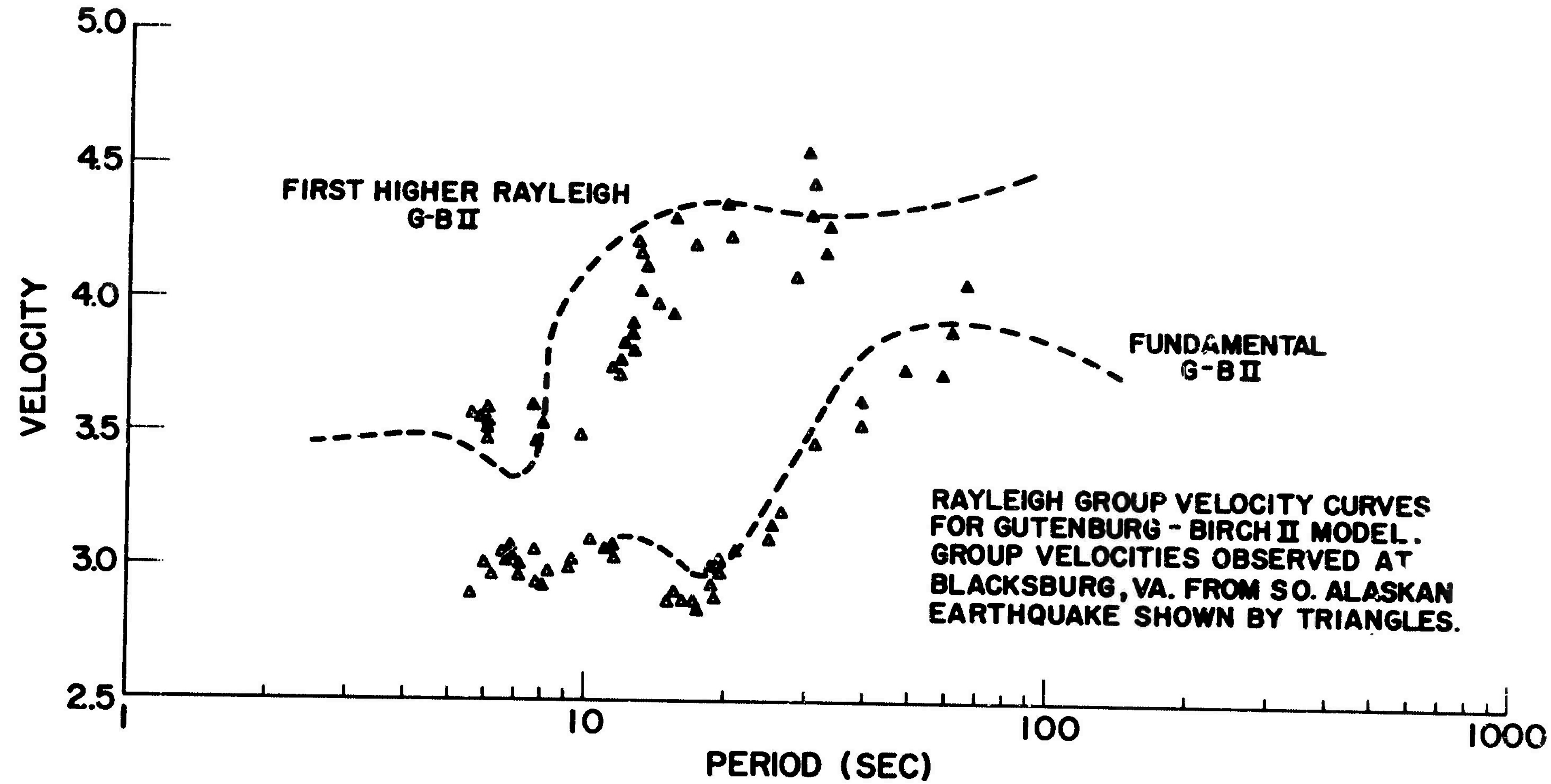
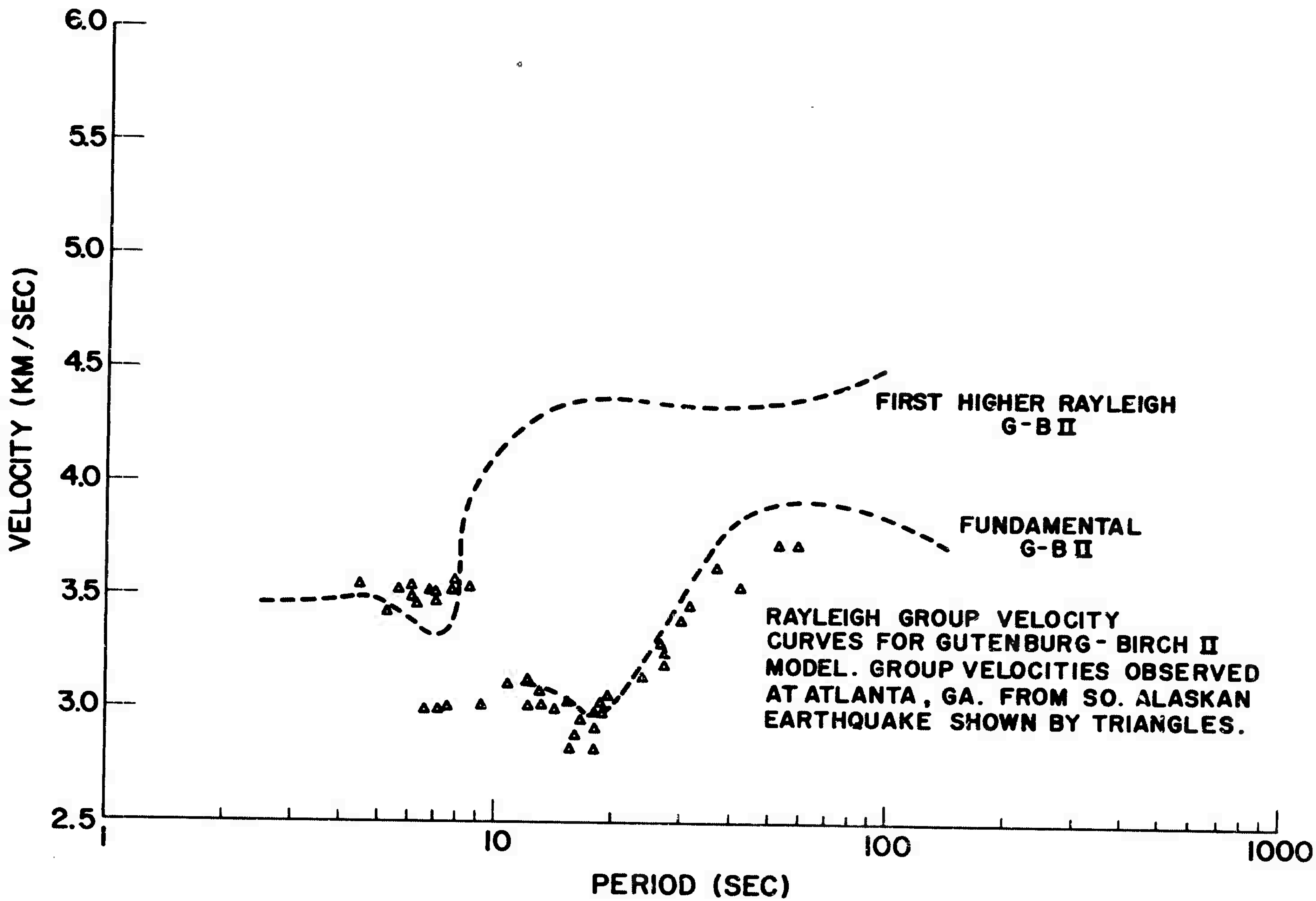

5.0
4.5
4.0
3.5
3.0
2.5
VELOCITY
FIRST HIGHER RAYLEIGH
G-B II
FUNDAMENTAL
G-B II
RAYLEIGH GROUP VELOCITY CURVES
FOR GUTENBURG - BIRCH II MODEL.
GROUP VELOCITIES OBSERVED AT
BLACKSBURG, VA. FROM SO. ALASKAN
EARTHQUAKE SHOWN BY TRIANGLES.
1
10
100
1000
PERIOD (SEC)

Fig. 9



6.0
5.5
5.0
4.5
4.0
3.5
3.0
2.5
VELOCITY (KM / SEC)
1
10
100
1000
PERIOD (SEC)
FIRST HIGHER RAYLEIGH
G-B II
FUNDAMENTAL
G-B II
RAYLEIGH GROUP VELOCITY
CURVES FOR GUTENBURG - BIRCH II
MODEL. GROUP VELOCITIES OBSERVED
AT ATLANTA, GA. FROM SO. ALASKAN
EARTHQUAKE SHOWN BY TRIANGLES.

Fig. 10

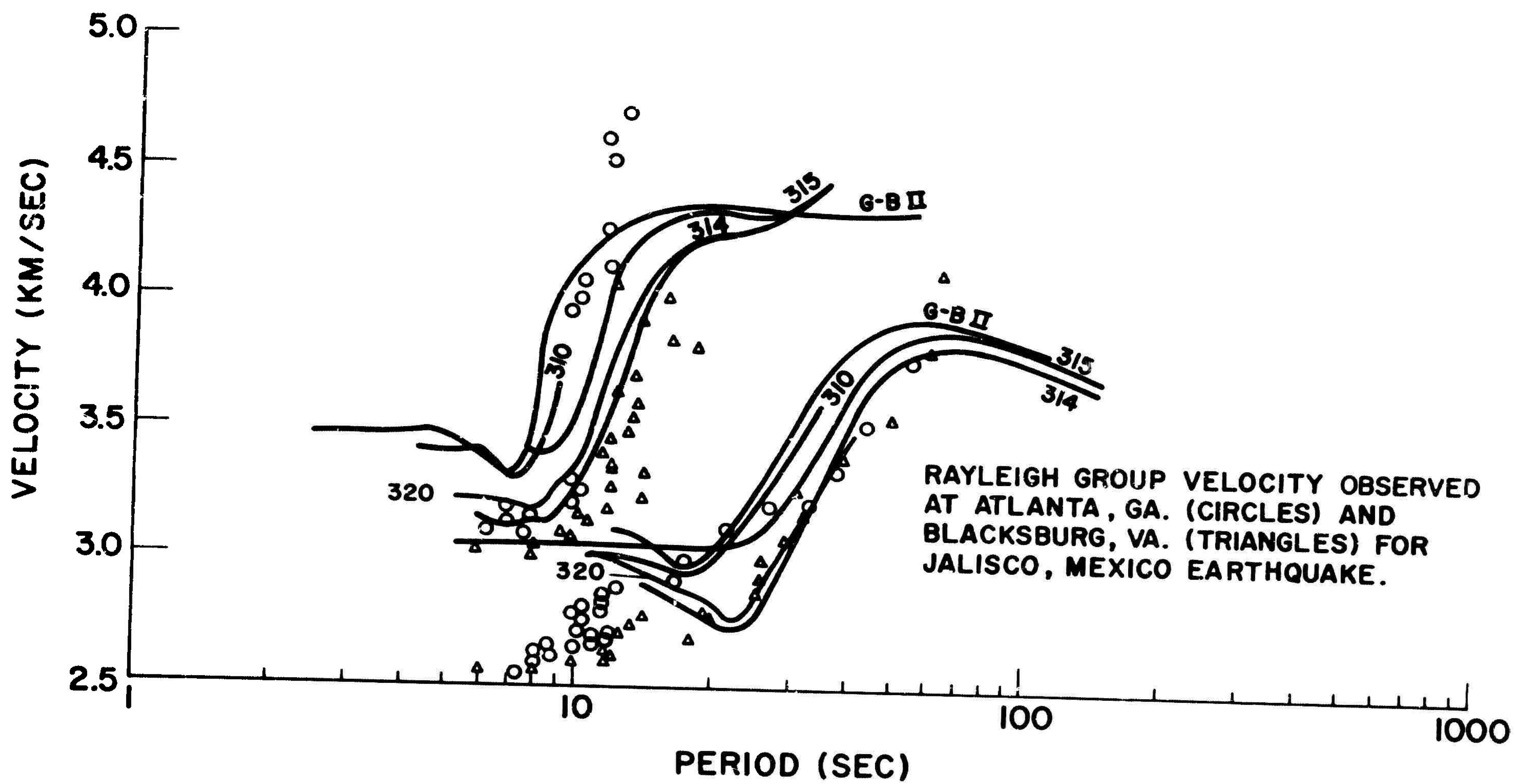

VELOCITY (KM/SEC)
5.0
4.5
4.0
3.5
3.0
2.5
1
10
100
1000
PERIOD (SEC)
310
314
315
320
G-B II
RAYLEIGH GROUP VELOCITY OBSERVED AT ATLANTA, GA. (CIRCLES) AND BLACKSBURG, VA. (TRIANGLES) FOR JALISCO, MEXICO EARTHQUAKE.

Fig. 11

from 6-10 sec. Group velocities recorded at Oxford indicate a slightly shorter period for the minimum and a gradual decrease in velocity for periods shorter than 12 sec. However, in general the curves are quite similar to those for Atlanta and Blacksburg. Waves arriving at Atlanta from Jalisco, Mexico, (parallel waves) have both fundamental and first higher Rayleigh mode group velocities considerably lower than do the perpendicular wave trains for periods less than about 20 sec. The rapidly "tailing-off" of group velocities below 15 sec is probably due to a thick sedimentary sequence (approximately 400 km of the travel path for these waves lie across the Gulf Coastal Plain).

The most significant difference is between the Blacksburg and Atlanta group velocities for parallel wave trains. A minimum of 2.75 km/sec occurs at a period of 22 sec and a maximum of 2.80 km/sec occurs at a period of 16 sec for the Blacksburg velocities. Group velocities at periods greater than 30 sec trend toward those for the perpendicular waves. First higher mode Rayleigh group velocities for Blacksburg are lower by about .1 km/sec than those for Atlanta.

Several models were constructed using the variation of phase velocity with layer parameter curves for the Gutenberg-Birch II model. Basic differences in the models are:

1) 314, 315, and 320 have a crustal thickness of 50 km, while 310 and G-B (Gutenberg-Birch II) have a crustal thickness of 40 km;

2) 314 has a low velocity zone centered at 15 km in the upper crust. Except for this low velocity zone, 320 is identical to 314;

3) G-B and 310 have a low velocity zone in the upper mantle beginning at 60 km while the low velocity begins at 70 km for 314, 315, and 320.

4) Velocities in the first 10 km of G-B are slightly higher than those in the other models.

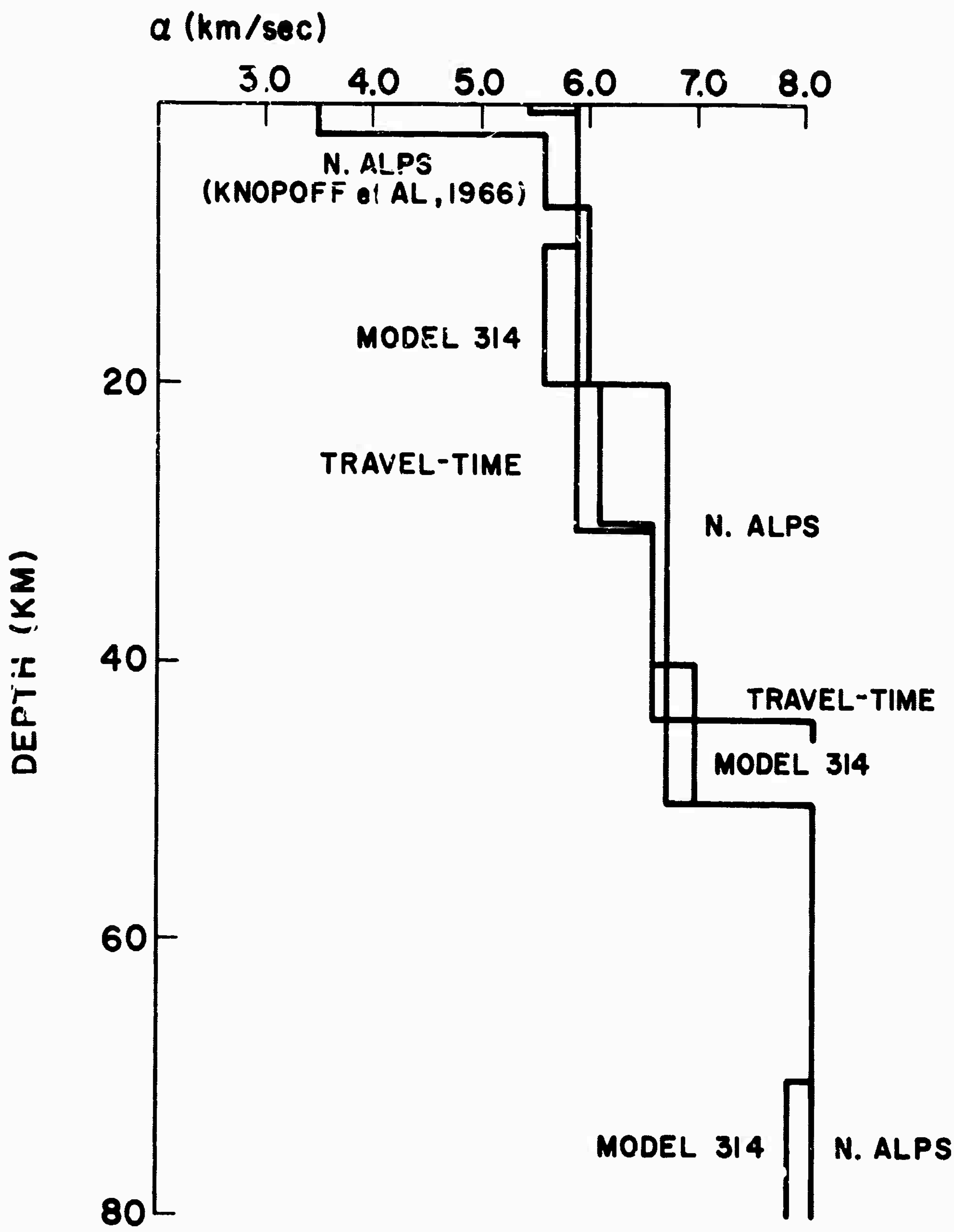
For waves traveling perpendicular to the Appalachians, the group velocity data agree well with model 310 or G-B values for the fundamental Rayleigh mode at periods less than 30 sec. At longer periods, the values fall below those for 310 and approach those for 314 and 315. First higher Rayleigh mode values observed at Blacksburg lie between those for 314 and 315 for periods greater than 10 sec. Thus, it appears that waves arriving at Atlanta, Oxford, and Blacksburg from the Southern Alaska epicenter are just beginning to "feel" the Appalachian structure. The length of the "Appalachian path" is about 240 km in the Appalachian foreland in each case,assuming that the Appalachian structure extends beneath the Mississippi Embayment (Oxford station).

Waves arriving at Atlanta from Jalisco, Mexico yield group velocities considerably below those of G-B for periods less than 20 sec (See Fig. 11). Fundamental mode group velocities at Blacksburg show fair agreement with the theoretical curves for either models 314 or 320 being considerably below curves for G-B, 315, and 310 at all periods. On the basis of fundamental mode group velocities it is impossible to determine which of models 314 or 320 most closely approximate the crustal structure. However, first higher Rayleigh mode group velocities from model 314 give a considerably better fit to the observed data than do those from model 320 in the period range from 6 to 15 sec. In this period range, the group velocity curve for 314 lies between that for 320 and the observed group velocities. Thus, a slight velocity reversal in the crust is indicated by the first higher mode group velocities.

An alternative to the low velocity zone is to lower the velocities and/or densities in the first 10 km of the crust. However, refraction data give a compressional velocity of 5.88 km/sec for the upper crust which has been used in models 314 and 320. The shear velocity value of 3.88 km/sec in models 314 and 320 corresponds to a Poisson ratio of .25.

Group velocities for periods greater than about 30 sec indicate a mantle low velocity zone beginning at about 70 km.

Compressional velocity crustal and upper mantle structure to the west of the Southern Appalachians determined from the travel times of local earthquakes and for the Southern Appalachian structure as determined from Rayleigh wave dispersion are shown in Fig. 12. Gravity and Rayleigh wave dispersion data indicate a total crustal thickness of about 50 km beneath the Southern Appalachians. Travel-time and dispersion data indicate an upper mantle velocity of 8.10 km/sec. Dispersion data indicate a slight low velocity zone in the upper crust and a general increase of velocity and density with depth below this zone. With the exception of the higher crustal velocity in the first 2 km, the Southern Appalachian s ture approximated by model 314 is similar to the Northern Alpine structure with a 50 km crust reported by Knopoff et al [1966].



CONPRESSIONAL VELOCITY STRUCTURE FOR THE SOUTHEASTERN UNITED STATES AND THE NORTHERN ALPS.

Fig. 12

7.0 CONCLUSIONS

The conclusions which have been drawn from this study are:

1) Focal depths of local earthquakes in the Southeastern United States are shallow ranging from about 7 to 18 km.

2) Systematic deviations in P-residuals observed at Chapel Hill, North Carolina, and McMinnville, Tennessee, have magnitudes from + 3 to -3 sec. The deviations indicate systematic errors in the Jeffreys-Bullen travel-times.

3) Travel-time curves constructed from local earthquakes and refraction data indicate a crustal structure for the Appalachian foreland of h_1 = 33.0 km (α = 5.88 km/sec), h_2 = 10.8 km (α = 6.58 km/sec), and an upper mantle velocity of 8.10 km/sec.

4) Rayleigh wave dispersion data indicate a crustal low velocity zone centered at about 15 km, an upper mantle low velocity zone beginning at 70 km, and a total crustal thickness of 50 km beneath the core of the Southern Appalachians.

5) Gravity data indicate a total crustal thickness beneath the Southern Appalachians of at least 50 km which is in agreement with the Rayleigh dispersion data. Gravity data indicate a crustal thickness of about 45 km in central Kentucky thickening eastward to about 50 km beneath the core of the Appalachian and thinning to 23 km beneath the North Carolina continental margin at about the 2400 fathom contour.

6) Higher Rayleigh modes can be observed using digital filtering techniques. Care must be taken to isolate Rayleigh type motion and to remove interference effects. This is particularly true for higher modes than the first.

7) It should be possible to use higher modes than the first to delineate fine detail in the crust.

ACKNOWLEDGEMENTS

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APPENDIX I

EVALUATION OF THE SECULAR EQUATION IN COMPUTING RAYLEIGH DISPERSION

In the Dunkin Method, the Rayleigh dispersion equation is the secular equation

$$\text{Det } R_{11} = r\Big|^{12}_{12} = t^{p}\Big|^{12}_{ab}\; g^{p-1}\Big|^{ab}_{cd} \cdots g^{1}\Big|^{mn}_{12} \tag{I-1}$$

Writing out (I-1) explicitly yields

$$\begin{aligned} \text{Det } R_{11} = {} & \check{t}\Big|^{12}_{12}\; g^{p-1}\Big|^{12}_{cd} \cdots \\ & + \check{t}\Big|^{12}_{13}\; g^{p-1}\Big|^{13}_{cd} \cdots \\ & + \check{t}\Big|^{12}_{14}\; g^{p-1}\Big|^{14}_{cd} \cdots \\ & + \check{t}\Big|^{12}_{23}\; g^{p-1}\Big|^{23}_{cd} \cdots \\ & + \check{t}\Big|^{12}_{24}\; g^{p-1}\Big|^{24}_{cd} \cdots \\ & + \check{t}\Big|^{12}_{34}\; g^{p-1}\Big|^{34}_{cd} \cdots \end{aligned} \tag{I-2}$$

where the ... signifies multiplication of the form $g^{p-2}\Big|^{cd}_{ef} \cdots g^{1}\Big|^{mn}_{12}$.

Continuing the process

$$\text{Det } R_{11} = a_1 g^{p-1}\Big|^{12}_{12}\; g^{p-2}\Big|^{12}_{ef} \cdots + a_1 g^{p-1}\Big|^{12}_{13}\; g^{p-2}\Big|^{13}_{ef} \cdots +$$

+ 4 more similar terms with a_1 as a factor

+ 5 more expressions with a_2, a_3, a_4, a_5, and a_6 replacing a_1

where

$$a_1 = \breve{t}\Big|^{12}_{12}$$

$$a_2 = \breve{t}\Big|^{12}_{13} \text{ , etc.} \tag{I-3}$$

Collecting terms with like coefficients $g^{p-1}\Big|^{12}_{ef}$, $g^{p-1}\Big|^{13}_{ef}$, etc., gives

$$\text{Det } R_{11} = A_1\, g^{p-2}\Big|^{12}_{ef} \cdots + A_2\, g^{p-2}\Big|^{13}_{ef} \cdots + A_3\, g^{p-2}\Big|^{14}_{ef} \cdots$$

$$+ A_4\, g^{p-2}\Big|^{23}_{ef} \cdots + A_5\, g^{p-2}\Big|^{24}_{ef} \cdots + A_6\, g^{p-2}\Big|^{34}_{ef} \cdots \tag{I-4}$$

where

$$A_1 = (a_1 b_{11} + a_2 b_{21} + a_3 b_{31} + a_4 b_{41} + a_5 b_{51} + a_6 b_{61}) ,$$

$$b_{11} = g^n\Big|^{12}_{12} ,$$

$$b_{21} = g\Big|^{13}_{12} ,$$

$$b_{31} = g\Big|^{14}_{12} , \quad \text{etc.}$$

(I-4) is of the form (I-2); the sum of six terms which are each products of the second order subdeterminants. The process of evaluating the second order subdeterminants of a given layer, multiplying by six previously determined coefficients, a_i, and summing to obtain six new coefficients, A_i, is repeated until the entire layered sequence has been traversed from bottom to top. This can be quickly and easily adapted to computer processing as is indicated in the following considerations of the real and imaginary structure of $\breve{t}\Big|^{12}_{ab}$ and $g^{p}\Big|^{12}_{ab}$. $t\Big|^{12}_{ab}$ is of the form of a 6 x 1 row matrix

$$[R\ R\ I\ I\ R\ R] \qquad \begin{array}{l} I = \text{pure imaginary quanitity} \\ R = \text{pure real quanitity} \end{array} \tag{I-5}$$

The real and imaginary structure of the $g\Big|^{ab}_{cd}$ is of the form of a 6 x 6 matrix

$$g\Big|^{ab}_{cd} = \begin{bmatrix} R & R & I & I & R & R \\ R & R & I & I & R & R \\ -I & -I & R & R & -I & -I \\ -I & -I & R & R & -I & -I \\ R & R & I & I & R & R \\ R & R & I & I & R & R \end{bmatrix} \tag{I-6}$$

The ab indices have been taken as the row indices and the cd indices the column indices with 1=12, 2=13, 3=14, 4=23, 5=24, and 6=34; $g\Big|^{12}_{34} = g_{16}$, etc. Multiplication of $\breve{t}\Big|^{12}_{ab}$ by the $g^{p-1}\Big|^{ab}_{ef}$ in (I-2) yields a matrix of the form

$$\begin{bmatrix} R & R & I & I & R & R \\ R & R & I & I & R & R \\ R & R & I & I & R & R \\ R & R & I & I & R & R \\ R & R & I & I & R & R \\ R & R & I & I & R & R \end{bmatrix} \tag{I-7}$$

Elements of this matrix correspond to the terms $a_1 b_{11}$, $a_2 b_{21}$, etc. in (I-4). Thus, the six new coefficients A_i are simply the sum of the elements in a column of (I-7). The six new coefficients can then be considered as just another 6 x 1 row matrix of the form of (I-5). Minus ones have been inserted in (I-6) to account for the multiplication of two like-signed imaginary quantities.

APPENDIX II

MODAL SHAPE COMPUTATIONS

In the Dunkin Method, mode shape is computed from the relation

$$R_n^{m}(z;a) = r_{11}^{-1}\, \check{t}_{1r}^{p}\, g_{rs}^{p-1} \ldots g_{vb}^{n}\, (z_n - z)\, g^{n}(z - z_{n-1}) \Big|_{cd}^{ab} \ldots g^{1} \Big|_{21}^{ef} \qquad \text{(II-1)}$$

where a denotes the component of displacement. $R_n^{m}(z;a)$ is the a^{th} component of the displacement-stress vector, at the depth z, normalized to the vertical displacement at the free surface. Note that r_{11}^{-1} is simply a constant multiplier for all of the displacement-stress components. Thus, it may be dropped and the $R_n^{m}(z;a)$ at different depths divided by the vertical displacement at the surface to normalize the displacements. The equation actually used for the determination of mode shape was

$$R_n^{m}(z;a) = \check{t}_{1r}^{p}\, g_{rs}^{p-1} \ldots g_{vb}^{n}\, (z_n - z) g^{n}\, (z - z_{n-1}) \Big|_{cd}^{ab} \ldots g^{1} \Big|_{21}^{ef} \qquad \text{(II-2)}$$

Writing (II-2) explicitly

$$\begin{aligned} R_n^{m}(z;a) = {} & \check{t}_{11}^{1}\, g_{1s}^{p-1}\, g_{st}^{p-2} \ldots \\ & + \check{t}_{12}^{p}\, g_{2s}^{p-1}\, g_{st}^{p-2} \ldots \\ & + \check{t}_{13}^{p}\, g_{3s}^{p-1}\, g_{st}^{p-2} \ldots \\ & + \check{t}_{14}^{p}\, g_{4s}^{p-1}\, g_{st}^{p-2} \ldots \end{aligned} \qquad \text{(II-3)}$$

When the dots denote the product of g_{ij}^{n} post multiplying g_{st}^{p-2} .

Sum over s $\longrightarrow$

$$R_n^{\,m}(z;a) = a_1 g_{11}^{p-1}\; g_{1t}^{p-2} \cdots + a_1 g_{12}^{p-1}\; g_{2t}^{p-2} \cdots + a_1 g_{13}^{p-1}\; g_{3t}^{p-2} \cdots + a_1 g_{14}^{p-1}\; g_{4t}^{p-2} \cdots$$

Sum over r $\downarrow$

$$\begin{array}{l} a_2 g_{21}^{p-1}\; g_{1t}^{p-2} \cdots + a_2 g_{22}^{p-1}\; g_{2t}^{p-2} \cdots + \cdots \\ \\ a_4 g_{41}^{p-1}\; g_{1t}^{p-2} \cdots + \cdots \end{array} \qquad \text{(II-4)}$$

where

$$a_1 = \check{t}_{11}^{p}\,, \quad a_2 = \check{t}_{12}^{p}\,, \quad a_3 = \check{t}_{13}^{p}\,, \text{ and } \quad a_4 = \check{t}_{14}^{p}\,.$$

$R_n^{\,m}\,(z;a)$ has been expanded into 16 terms of the form $a_i g_{ij}^{p-1}\; g_{jt}^{p-2} \ldots$ Now the g^{p-1} which have explicit subscripts can be included in the constant factor a_i. Collecting terms with common factors of g_{ij}^{p-2} yields

$$R_n^{\,m}(z;a) = (a'_1 + a'_2 + a'_3 + a'_4)\; g_{1t}^{p-2} \cdots + (a'_1 + a'_2 + a'_3 + a'_4)\; g_{2t}^{p-2} \cdots$$

$$+ \cdots + (a'_1 + a'_2 + a'_3 + a'_4)\; g_{4t}^{p-2} \cdots \qquad \text{(II-5)}$$

$$a'_i = a_i\; g_{ik}^{p-1}\,, \text{ etc.}$$

which is of the form of (II-3).

Continuing this process yields

$$\begin{aligned} R_n^{\,m}(z;a) = {} & a_1 g^n(z-z_{n-1}) \left|\begin{matrix} a1 \\ cd \end{matrix}\right. \cdots\; g^1 \left|\begin{matrix} ef \\ 12 \end{matrix}\right. \\ & + a_2 g^n(z-z_{n-1}) \left|\begin{matrix} a2 \\ cd \end{matrix}\right. \cdots\; g^1 \left|\begin{matrix} ef \\ 12 \end{matrix}\right. \\ & + a_3 g^n(z-z_{n-1}) \left|\begin{matrix} a3 \\ cd \end{matrix}\right. \cdots\; g^1 \left|\begin{matrix} ef \\ 12 \end{matrix}\right. \\ & + a_4 g^n(z-z_{n-1}) \left|\begin{matrix} a4 \\ cd \end{matrix}\right. \cdots\; g^1 \left|\begin{matrix} ef \\ 12 \end{matrix}\right. \end{aligned} \qquad \text{(II-6)}$$

The horizontal component, u , of displacement corresponds to $a=1$ and the vertical component, w , to $a=2$,so that

$$u = a_2\, g^n\Big|^{12}_{cd} \ldots + a_3\, g^n\Big|^{13}_{cd} \ldots + a_4\, g^n\Big|^{14}_{cd} \ldots \text{ and} \tag{II-7}$$

$$w = a_1\, g^n\Big|^{21}_{cd} \ldots + a_3\, g^n\Big|^{23}_{cd} \ldots + a_4\, g^n\Big|^{24}_{cd} \ldots \tag{II-8}$$

Note that $g^n\Big|^{ij}_{kl} = 0$, if $i=j$ or $k=l$.

Writing (II-7) explicitly

$$\begin{aligned} u = {} & a_2\, g^n\Big|^{12}_{12}\, g^{n-1}\Big|^{12}_{ef} \ldots + a_2\, g^n\Big|^{12}_{13}\, g^{n-1}\Big|^{13}_{ef} \ldots + a_2\, g^n\Big|^{12}_{14}\, g^{n-1}\Big|^{14}_{ef} \ldots \\ & + a_2\, g^n\Big|^{12}_{23}\, g^{n-1}\Big|^{23}_{ef} \ldots + a_2\, g^n\Big|^{12}_{24}\, g^{n-1}\Big|^{24}_{ef} \ldots + a_2\, g^n\Big|^{12}_{34}\, g^{n-1}\Big|^{34}_{ef} \ldots \\ & + a_3\, g^n\Big|^{13}_{12}\, g^{n-1}\Big|^{12}_{ef} \ldots + 5 \text{ more terms in } a_3 \\ & + a_4\, g^n\Big|^{14}_{12}\, g^{n-1}\Big|^{12}_{ef} \ldots + 5 \text{ more terms in } a_4 \end{aligned} \tag{II-9}$$

u is expressed in terms of the sum of 18 terms of the form $a_m\, g^n\Big|^{ij}_{kl}\, g^{n-1}\Big|^{kl}_{ef} \ldots$ Collecting terms with common $g^{n-1}\Big|^{kl}_{ef}$ gives u as the sum of six terms of the form $a_m\, g^{n-1}\Big|^{ij}_{ef} \ldots$

$$\begin{aligned} u = {} & a'_1\, g^{n-1}\Big|^{12}_{ef} \ldots + a'_2\, g^{n-1}\Big|^{13}_{ef} \ldots + a'_3\, g^{n-1}\Big|^{14}_{ef} \ldots + \\ & a'_4\, g^{n-1}\Big|^{23}_{ef} \ldots + a'_5\, g^{n-1}\Big|^{24}_{ef} \ldots + a'_6\, g^{n-1}\Big|^{34}_{ef} \ldots , \end{aligned} \tag{II-10}$$

where

$$a_1' = a_2 \, g^n \Big|{}^{12}_{12} + a_3 \, g \Big|{}^{13}_{12} + a_4 \, g \Big|{}^{14}_{12} \text{, etc.}$$

The process of evaluating the displacements u and w now proceeds exactly as for the evaluation of Det R_{11} described in Appendix I.

Let us consider the real and imaginary forms of the factors appearing in (II-4), (II-6), and (II-10). From the definition of T_p^{-1} given by Dunkin, t_{1r} is of the form

$$t_{1r} = [I \; R \; R \; I] \text{ , } I = \text{pure imaginary quantity} \quad \text{(II-11)}$$
$$R = \text{pure real quantity}$$

The layer matrices G_n are of the form

$$G_n = \begin{bmatrix} R & I & I & R \\ I & R & R & I \\ I & R & R & I \\ R & I & I & R \end{bmatrix} \quad \text{(II-12)}$$

Thus $\breve{t}_{1r} \, g_{rs}^{p-1} = [I \; R \; R \; I]$ (II-13)

Each successive multiplication by a layer matrix G_n yields a row matrix of the form [I R R I] until the first multiplication by a second order subdeterminant is encountered. In order to account for the multiplication of two imaginaries in the evaluation of column 2 and 3 in (II-13) minus ones are inserted in (II-12) to give

$$G_n = \begin{bmatrix} R & -I & -I & R \\ I & R & R & I \\ I & R & R & I \\ R & -I & -I & R \end{bmatrix} \qquad \text{(II-14)}$$

By (II-9) and the definitions of $g^n \left[\begin{matrix} ij \\ kl \end{matrix}\right.$, the form of the matrix by which a_2, a_3, and a_4 of (II-7) are multiplied is

$$\begin{bmatrix} R & R & I & I & R & R \\ R & R & I & I & R & R \\ I & I & R & R & I & I \end{bmatrix} \qquad \text{(II-15)}$$

and for a_1, a_3, a_4 in the case of the vertical component

$$\begin{bmatrix} R & R & I & I & R & R \\ I & I & R & R & I & I \\ R & R & I & I & R & R \end{bmatrix} \qquad \text{(II-16)}$$

For the u component

$$[R\ R\ I] \cdot \begin{bmatrix} R & R & I & I & R & R \\ R & R & I & I & R & R \\ -I & -I & R & R & -I & -I \end{bmatrix} = [R\ R\ I\ I\ R\ R] \qquad \text{(II-17)}$$

and for the w component

$$[I\ R\ I] \cdot \begin{bmatrix} R & R & -I & -I & R & R \\ I & I & R & R & I & I \\ R & R & -I & -I & R & R \end{bmatrix} = [I\ I\ R\ R\ I\ I] \qquad \text{(II-18)}$$

Minus ones have been inserted to account for multiplication of two imaginaries. To continue the process to the surface, the form of the matrices composed of the second order subdeterminant elements is

$$\begin{bmatrix} R & R & I & I & R & R \\ R & R & I & I & R & R \\ -I & -I & R & R & -I & -I \\ -I & -I & R & R & -I & -I \\ R & R & I & I & R & R \\ R & R & I & I & R & R \end{bmatrix} \qquad \text{(II-19)}$$

for the u component and

$$\begin{bmatrix} R & R & -I & -I & R & R \\ R & R & -I & -I & R & R \\ I & I & R & R & I & I \\ I & I & R & R & I & I \\ R & R & -I & -I & R & R \\ R & R & -I & -I & R & R \end{bmatrix} \qquad \text{(II-20)}$$

and for the w component. Minus ones have been inserted in the appropriate elements to account for the multiplication of two pure imaginary quantities. Since the lower subscripts in (II-1) for the surface layer matrix are 21, the layer matrix of the surface layer is a 6 x 1 column matrix. The final multiplications are then of the forms:

u component

$$[R\ R\ I\ I\ R\ R] \cdot \begin{bmatrix} R \\ R \\ -I \\ -I \\ R \\ R \end{bmatrix} = [R\ R\ R\ R\ R\ R]$$

w component

$$[I\ I\ R\ R\ I\ I] \cdot \begin{bmatrix} R \\ R \\ I \\ I \\ R \\ R \end{bmatrix} = [I\ I\ I\ I\ I\ I]$$

EXPLICIT FORMS OF LAYER MATRIX COMPONENTS g_{ij}

$$g_{11} = g_{44} = \frac{2\beta^2}{c^2} \cosh P - \frac{p\beta^2}{k^2c^2} \cosh Q$$

$$g_{12} = g_{34} = \frac{i\beta^2 p}{r_\alpha c^2 k^2} \sinh P + \frac{2ir_\beta \beta^2}{c^2} \sinh Q$$

$$g_{13} = g_{24} = \frac{i\beta^2}{\mu k c^2} \cosh P - \frac{i\beta^2}{k\mu c^2} \cosh Q$$

$$g_{14} = \frac{\beta^2}{kr_\alpha c^2 \mu} \sinh P + \frac{p\beta^2 r_\beta}{\mu k c^2} \sinh Q$$

$$g_{21} = g_{43} = -\frac{2ir_\alpha \beta^2}{c^2} \sinh P - \frac{ip\beta^2}{k^2 r_\beta c^2} \sinh Q$$

$$g_{22} = g_{33} = -\frac{p\beta^2}{k^2c^2} \cosh P + \frac{2\beta^2}{c^2} \cosh Q$$

$$g_{23} = \frac{r_\alpha \beta^2}{\mu k c^2} \sinh P + \frac{\beta^2}{\mu r_\beta c^2 k} \sinh Q$$

$$g_{31} = g_{42} = \frac{2i\mu\beta^2 p}{kc^2} \cosh P - \frac{2ip\mu\beta^2}{kc^2} \cosh Q$$

$$g_{32} = -\frac{\mu p^2 \beta^2}{k^3 c^2 r_\alpha} \sinh P - \frac{4\mu k r_\beta \beta^2}{c^2} \sinh Q$$

$$g_{41} = -\frac{4\mu k r_\alpha \beta^2}{c^2} \sinh P - \frac{\mu p^2 \beta^2}{r_\beta c^2 k^3} \sinh Q$$

SUMMARY OF EXPLICIT FORMS OF SECOND ORDER SUBDETERMINANTS $g\left|\begin{smallmatrix}ij\\kl\end{smallmatrix}\right.$

$$g\left|\begin{smallmatrix}12\\12\end{smallmatrix}\right. = g\left|\begin{smallmatrix}34\\34\end{smallmatrix}\right. = -2\gamma(\gamma-1) + (2\gamma^2-\gamma+1)\overline{CP}\;\overline{CQ} - \left[\frac{(\gamma-1)^2}{r_\alpha r_\beta} + \gamma^2 r_\alpha r_\beta\right]\overline{SQ}\;\overline{SP}$$

$$g\left|\begin{smallmatrix}12\\13\end{smallmatrix}\right. = g\left|\begin{smallmatrix}24\\34\end{smallmatrix}\right. = +(\rho\lambda^2 k)^{-1}\left[\frac{\overline{SQ}\;\overline{CP}}{r_\beta} + r_\alpha\,\overline{SP}\;\overline{CQ}\right]$$

$$g\left|\begin{smallmatrix}12\\14\end{smallmatrix}\right. = g\left|\begin{smallmatrix}12\\23\end{smallmatrix}\right. = g\left|\begin{smallmatrix}14\\34\end{smallmatrix}\right. = g\left|\begin{smallmatrix}23\\34\end{smallmatrix}\right. = i(\rho\lambda^2 k)^{-1}\left\{(2\gamma-1)(1-\overline{CP}\;\overline{CQ}) + \left[\frac{(\gamma-1)}{r_\alpha r_\beta} + \gamma r_\alpha r_\beta\right]\overline{SP}\;\overline{SQ}\right\}$$

$$g\left|\begin{smallmatrix}12\\24\end{smallmatrix}\right. = g\left|\begin{smallmatrix}13\\34\end{smallmatrix}\right. = -(\rho\lambda^2 k)^{-1}\left[r_\beta\,\overline{CP}\;\overline{SQ} + \frac{1}{r_\alpha}\,\overline{SP}\;\overline{CQ}\right]$$

$$g\left|\begin{smallmatrix}12\\34\end{smallmatrix}\right. = -(\rho\lambda^2 k)^{-2}\left[2(1-\overline{CP}\;\overline{CQ}) + \left(\frac{1}{r_\alpha r_\beta} + r_\alpha r_\beta\right)\overline{SQ}\;\overline{SP}\right]$$

$$g\left|\begin{smallmatrix}13\\12\end{smallmatrix}\right. = g\left|\begin{smallmatrix}34\\24\end{smallmatrix}\right. = -(\rho\lambda^2 k)\left[\gamma^2 r_\beta\,\overline{CP}\;\overline{SQ} + \frac{(\gamma-1)^2}{r_\alpha}\,\overline{SP}\;\overline{CQ}\right]$$

$$g\left|\begin{smallmatrix}13\\13\end{smallmatrix}\right. = g\left|\begin{smallmatrix}24\\24\end{smallmatrix}\right. = \overline{CQ}\;\overline{CP}$$

$$g\left|\begin{smallmatrix}13\\14\end{smallmatrix}\right. = g\left|\begin{smallmatrix}13\\23\end{smallmatrix}\right. = g\left|\begin{smallmatrix}14\\24\end{smallmatrix}\right. = g\left|\begin{smallmatrix}23\\24\end{smallmatrix}\right. = i\left(\gamma r_\beta\,\overline{CP}\;\overline{SQ} + \frac{(\gamma-1)}{r_\alpha}\,\overline{SP}\;\overline{CQ}\right)$$

$$g\left|\begin{smallmatrix}13\\24\end{smallmatrix}\right. = +\frac{r_\beta}{r_\alpha}\,\overline{SQ}\;\overline{SP}$$

$$g\Big|_{12}^{14} = g\Big|_{12}^{23} = g\Big|_{14}^{34} = g\Big|_{23}^{34} = i(\rho k\lambda^2)\Big\{ \gamma(\gamma-1)\ (2\gamma-1)\ (1-\overline{CQ}\ \overline{CP})$$

$$+ [\frac{(\gamma-1)^3}{r_\alpha r_\beta} + r_\alpha r_\beta \gamma^3]\ \overline{SQ}\ \overline{SP}\Big\}$$

$$g\Big|_{13}^{14} = g\Big|_{13}^{23} = g\Big|_{14}^{24} = g\Big|_{23}^{24} = -i(\frac{(\gamma-1)}{r_\beta}\ \overline{CP}\ \overline{SQ} + r_\alpha\gamma\ \overline{CQ}\ \overline{SP})$$

$$g\Big|_{14}^{14} = g\Big|_{23}^{23} = 1 + 2\gamma(\gamma-1)\ (1-\overline{CQ}\ \overline{CP}) + (\frac{(\gamma-1)^2}{r_\alpha r_\beta} + \gamma^2\ r_\alpha r_\beta)\ \overline{SP}\ \overline{SQ}$$

$$g\Big|_{23}^{14} = g\Big|_{14}^{23} = g\Big|_{14}^{14} - 1$$

$$g\Big|_{12}^{24} = g\Big|_{13}^{34} = + (\rho\lambda^2 k)\ [\frac{(\gamma-1)^2}{r_\beta}\ \overline{CP}\ \overline{SQ} + r_\alpha\gamma^2\ \overline{SP}\ \overline{CQ}]$$

$$g\Big|_{13}^{24} = + \frac{r_\alpha}{r_\beta}\ \overline{SP}\ \overline{SQ}$$

$$g\Big|_{12}^{34} = (\rho\lambda^2 k)^2\ [2\gamma^2(\gamma-1)^2\ (\overline{CP}\ \overline{CQ} - 1) - [\frac{(\gamma-1)^4}{r_\alpha r_\beta} + \gamma^4 r_\alpha r_\beta]\ \overline{SP}\ \overline{SQ}$$

EXPLICIT FORMS OF $t\Big|^{12}_{ab}$

$$t\Big|^{12}_{12} = -\frac{\beta^2\mu}{2\omega^2}\,(4k^4\, r_\alpha r_\beta + p^2)$$

$$t\Big|^{12}_{13} = -\frac{r_\alpha}{r_\beta} \qquad \breve{t}\Big|^{12}_{24} = \frac{i\beta^2}{2\omega^2}\; r_\alpha k(2k^2 - p)$$

$$\breve{t}\Big|^{12}_{14} = \breve{t}\Big|^{12}_{23} = \frac{i\beta^2 k}{2\omega^2}\,(2k^2 r_\alpha r_\beta + p)$$

$$\breve{t}\Big|^{12}_{34} = -\frac{\beta^2 k^2}{2\mu\omega^2}\,(r_\alpha r_\beta + 1)$$

$$P = kdr_\alpha \qquad\qquad Q = kdr_\beta$$

$$\gamma = \frac{2\beta^2}{c^2} \qquad\qquad p = 2k^2 - \frac{\omega^2}{\beta^2}$$

$$r_\alpha = \sqrt{\frac{c^2}{\alpha^2} - 1}$$

$SP = \sin P$ $\qquad\qquad$ $c \geq \alpha$

$CP = \cos P$

$$r_\beta = \sqrt{\frac{c^2}{\beta^2} - 1}$$

$SQ = \sin Q$ $\qquad\qquad$ $c \geq \beta$

$CQ = \cos Q$

$$r_\alpha = -i\sqrt{1 - \frac{c^2}{\alpha^2}}$$

$$SP = \sinh P \qquad c < \alpha$$

$$CP = \cosh P$$

$$r_\beta = -i\sqrt{1 - \frac{c^2}{\beta^2}}$$

$$SQ = \sinh Q \qquad c < \beta$$

$$CP = \cosh Q$$

The g_{ij} expressions are written for the hyperbolic functions ($c<\alpha$, $c<\beta$). For $c>\alpha$, $c>\beta$, the hyperbolic functions transform according to

$$\begin{array}{llll}
\cosh P & \longrightarrow & \cos P & c>\alpha \\
\cosh Q & \longrightarrow & \cos Q & c>\beta \\
\sinh P & \longrightarrow & -i \sin P & c>\alpha \\
\sinh Q & \longrightarrow & -i \sin Q & c>\beta
\end{array}$$

APPENDIX III

FORTRAN LISTINGS OF COMPUTER PROGRAMS
FLATRAY, STRESS, INTEGRAL, TRAVEL, VARGRAV, AND PRESID

INPUT DATA AND FORMAT FOR FLATRAY

CARD	FORMAT	DATA DESCRIPTION
1	I4	Model identification no.
	I4	No. of layers <41
	I4	No. of modes to be computed <11
	F10.4	Precision of root values of k
	F10.4	Cutoff k value to automatically stop computation
2	8F10.6	Layer parameters stored in sequence α, β, ρ, d; two layers to a card
3	F10.6	Starting c value
	F10.6	Minimum c value
	F10.6	Decrement of c
	F10.6	Starting k value
	F10.6	k increment for individual modes
	F10.6	k increment to find starting points of different modes
	F10.6	c perturbation
	F10.6	k perturbation
4	20I4	Mode no. to be found stored sequentially

```
      DIMENSION ALPHA(40),BETTA(40),RHO(40),THICK(40),ROOTS(15,40),MODE
     1(10),PERIOD(15,40),B(6),E(8),G(6,6),A(6),UDISP(40),WDISP(40)
     1,H(4)
      COMMON UDISP,WDISP,H
      READ 150,MODEL,NLAYER,NMODE,XKMIN ,XKMAX
  150 FORMAT (3I4,2F10.4)
      READ 200,(ALPHA(I),BETTA(I),RHO(I),THICK(I),I=1,NLAYER)
  200 FORMAT(8F10.6)
      PRINT 900,MODEL
  900 FORMAT(31H-LAYER PARAMETERS FOR MODEL RTI  I4 // 6H LAYER,5X,4H AL
     1PHA,5X,6H BETTA,7X,4H RHO,5X,14H THICKNESS(KM),3X, 6H H(KM) /)
      TTHICK=0.
      DO 902 I=1,NLAYER
      TTHICK=TTHICK +THICK(I)
      PRINT 902,I,ALPHA(I),BETTA(I),RHO(I),THICK(I),TTHICK
  902 FORMAT(1X,I3,3X,F10.4,4X,F8.4,4X,F8.4,5X,F8.2,5X,F8.2)
      TOTAL= TTHICK
      READ 200, CMAX,CMIN,DELC,AK,DELK,DELK1,PTRBC,PTRB K
      READ 350, (MODE(I),I=1,NMODE)
  350 FORMAT(20I4)
      IPUN=0
      NEWMOD = 1
      CMAX1 =CMAX
      NMOD = 10
      IM=MODE(1)
      ISTRT =10
      IX= NLAYER-1
      IN=1
      XK=ROOTS(IM,IN)=AK
      IPTRB =1
      C = CMAX
      JJMP1=1
  500 IDISP= 1
      JJMP=1
      IDP1=1
      UP=XK
      SIGN= 1.
      IBRAC = 10
      INDX2=1
  100 I=IX
      OMEGA=XK*C
      RALPH = SQRT(ABS((C/ALPHA(IX+1))**2-1.))
      RBET=SQRT(ABS((C/BETTA(IX+1))**2-1.))
      AL=2.*AK**2-OMEGA**2/(BETTA(IX+1)**2)
      BEFAC= BETTA(IX+1)**2/(2.*OMEGA**2)
      AMU=BETTA(IX+1)**2*RHO(IX+1)
      B(1)=T11=-BEFAC*AMU*(-4.*AK**4*RALPH*RBET+AL**2)
      B(2)=T12=BEFAC*RALPH*XK*(2.*XK**2-AL)
      B(3)=T13=BEFAC*XK*(-2.*XK**2*RALPH*RBET+AL)
      B(4)=T14=T13
      B(5)=T15=-RBET*T12/RALPH
      B(6)=T16=-BEFAC*XK**2*(-RALPH*RBET+1.)/AMU
   51 GAM=2.* BETTA(I) **2/C**2
      DIVS=1.
      RHOC=RHO(I)*C**2*XK
      RALPH=SQRT(ABS((C/ALPHA(I))**2-1.))
      RBET =SQRT(ABS((C/BETTA(I))**2-1.))
      P=XK*RALPH*THICK(I)
      Q=XK*RBET*THICK(I)
      IF(C-ALPHA(I))1,2,2
    2 FACA=1.
      CP=COS(P)
      SP=SIN(P)
      GO TO 3
    1 FACA =-1.
      CP=(EXP(P)+EXP(-P))/2.
      SP=(EXP(P)-EXP(-P))/2.
    3 IF(C-BETTA(I))5,4,4
    4 FACB=1.
      CQ=COS(Q)
      SQ=SIN(Q)
      GO TO 6
    5 FACB=-1.
      CQ=(EXP(Q)+EXP(-Q))/2.
      SQ=(EXP(Q)-EXP(-Q))/2.
      DIVS=CP*CQ
    6 G(1,1)=(GAM-1.)**2/(RALPH*RBET)+GAM**2*RALPH*RBET*FACA*FACB
      G(1,1)=-G(1,1)*SQ*SP-2.*GAM*(GAM-1.)+(2.*GAM**2-2.*GAM+1.)*CP*CQ
      G(1,2)=(SQ*CP/RBET+RALPH*SP*CQ*FACA)/RHOC
      G(1,3)=((GAM-1.)/(RALPH*RBET)+GAM*RALPH*RBET*FACA*FACB)*SP*SQ+(2.*
     1GAM-1.)*(1.-CP*CQ)
      G(1,3)=G(1,3)/RHOC
      G(1,4)=G(1,3)
      G(1,5)=-(RBET*CP*SQ*FACB+SP*CQ/RALPH)/RHOC
      G(1,6)=-(2.*(1.-CP*CQ)+(1./(RALPH*RBET)+RALPH*RBET*FACB*FACA)*SQ*S
     1P)/(RHOC**2)
      G(2,1)=-RHOC*(GAM**2*RBET*FACB*CP*SQ+(GAM-1.)**2*SP*CQ/RALPH)
      G(2,2)=CQ*CP
      G(2,3)=GAM*RBET*FACB*CP*SQ+(GAM-1.)*SP*CQ/RALPH
      G(2,4)=G(2,3)
      G(2,5)=RBET*SQ*SP*FACB/RALPH
      G(2,6)=G(1,5)
      G(3,1)=((GAM-1.)**3/(RALPH*RBET)+RALPH*RBET*GAM**3*FACB*FACA)*SQ
     1*SP
      G(3,1)=- RHOC*(GAM*(GAM-1.)*(2.*GAM-1.)*(1.-CQ*CP) +G(3,1))
      G(3,2)= ((GAM-1.)*CP*SQ/RBET+RALPH*GAM*FACA*CQ*SP)
      G(3,3)=1.+2.*GAM*(GAM-1.)*(1.-CP*CQ)+((GAM-1.)**2/(RALPH*RBET)+GAM
     1**2*RALPH*RBET*FACA*FACB)*SP*SQ
      G(3,4)=G(3,3)-1.
      G(3,5)=-G(2,3)
      G(3,6)=-G(1,3)
      G(4,1)=G(3,1)
      G(4,2)=G(3,2)
      G(4,3)=G(3,3)-1.
      G(4,4)=G(3,3)
```

```
      G(4,5)=-G(2,3)
      G(4,6)=-G(1,3)
      G(5,1)=((GAM-1.)**2*CP*SQ/RBET+RALPH*GAM*GAM*SP*CQ*FACA)*RHOC
      G(5,2)=RALPH*SP*SQ*FACA/RBET
      G(5,3)=-G(3,2)
      G(5,4)=-G(3,2)
      G(5,5)=G(2,2)
      G(5,6)=G(1,2)
      G(6,1)=((GAM-1.)**4/(RALPH*RBET)+GAM**4*RALPH*RBET*FACA*FACB)*SP*S
     1Q
      G(6,1)=RHOC**2*(2.*GAM*GAM*(GAM-1.)**2*(CP*CQ-1.)-G(6,1)   )
      G(6,2)=G(5,1)
      G(6,3)=-G(3,1)
      G(6,4)=-G(3,1)
      G(6,5)=G(2,1)
      G(6,6)=G(1,1)
      IF(IDISP-1)62,62,8012
   62 IF(I-1)402,402,101
  101 DO 30 J=1,6
      SUM=0.
      DO 40 K=1,6
      SUM=SUM+B(K)*G(K,J)
   40 CONTINUE
      A(J)=SUM
   30 CONTINUE
      DO 8 J=1,6
      B(J)=A(J)/DIVS
    8 CONTINUE
      I=I-1
      GO TO 51
  402 F=B(1)*G(1,1)+B(2)*G(2,1)+B(3)*G(3,1)+B(4)*G(4,1)+B(5)*G(5,1)+B(6)
     1*G(6,1)
      F=F/DIVS
      IF(ISTRT-1) 603,301,603
C **  COMPUTE FIRST ROOT OF EACH MODE
  603 IF(INDX2-1)605,604,605
  604 INDX2 =10
      XK1=XK
      F1=F
      XK= XK+DELK1
      GO TO 100
  605 AF =XBS(F)
      AF1=XBS(F1)
      IF(AF-AF1) 606,607,607
  607 IF(XBS(F-F1)-AF) 608,609,609
  606 IF(ABS(F1-F)-AF1) 608,609,609
  609 IF(XBS(XK1-XK)-XKMIN) 308,308,610
  610 GO TO (703,704),JUMP
  703 F2=F
      JUMP=2
      XK2=XK
      XK= XBS((XK*F1-XK1*F2)/(F1-F2))
      IBRAC = 1
      GO TO (100,60),JUMP1
  706 IBRAC=0
      F1=F2
      XK1=XK2
  704 IBRAC=0
      IF(XK-XK1) 614,613,613
  614 XK=XK+XKMIN
      F2=F
      XK2=XK
      GO TO (100,60),JUMP1
  613 F2=F
      XK2=XK
      XK=XK-XKMIN
      GO TO (100,60),JUMP1
  608 IF(IBRAC-1)306,706,707
  707 IF(NMOD-1)617,618,617
  617 IF(AF-AF1) 619,619,620
  620 XK = XK-2.*DELK1
      SIGN =-1.
      IF(XK-0.)1014,1014,100
 1014 XK=XK+DELK1
      NMOD=1
      GO TO 500
  619 XK1=XK
      F1=F
      XK=XK +SIGN*DELK1
      IF(XK-0.)1014,1014,100
  308 IF(ISTRT-1)3000,508,3000
 3000 IF(IBRAC-10) 300,708,708
  708 F2=F1
      XK2=XK1
  300 F1=F
      XK= XBS((XK2*F1-XK*F2)/(F1-F2))
      ROOTS(IM,IN)=XK
      PERIOD(IM,IN)= (2.*3.1416)/(XK*C)
      IF(ISTRT-1) 950,901,950
  950 IM=IM+1
      IF(IM-MODE(NMODE)) 615,615,616
  615 F1=F
      XK1=XK
      XK=XK+DELK1
      GO TO 100
  615 XK=XK+2.*XKMIN
      DELK1= ROOTS(IM-1,1)
      XKMIN= .01*DELK1
      ISIM=10
      NMOD =1
      GO TO 500
  616 ISIM=1
      JUMP=1
      IMOD=1
      IM   =MODE(IMOD)
```

```
      IDISP=10
      JUMP1=2
      XK=ROOTS(IM   ,1)
      IL=1
      XKMIN= .01*XK
      IJMP=1
      IN=1
 8000 I=L=IX
      AL=2.*XK*XK-(XK*C)**2/(BETTA(IX+1))**2
      ALSQ=AL*AL
      RALPH=SQRT(ABS((C/ALPHA(IX+1))**2-1.))
      AMU=BETTA(IX+1)**2*RHU(IX+1)
      B(1)=BETTA(IX+1)**2/(AK*C**2)
      B(2)=-AL*BETTA(IX+1)**2/(2.*AK**3*C*C*RALPH)
      B(3)=-BETTA(IX+1)**2/(2.*AMU*(AK*C)**2)
      B(4)=BETTA(IX+1)**2/(2.*AMU*RALPH*(C*XK)**2)
 8001 IF(I-L) 8010,8010,8009
 8009 BETSQ=BETTA(I)**2
      AMU= BETTA(I)**2*RHO(I)
      RALPH=SQRT(ABS((C/ALPHA(I))**2-1.))
      RBET=SQRT(ABS((C/BETTA(I))**2-1.))
      P=AK*RALPH*THICK(I)
      Q=XK*RBET*THICK(I)
      IF(C-ALPHA(I)) 8002,8003,8003
 8003 FACA=1.
      CP=COS(P)
      SP=SIN(P)
      GO TO 8030
 8002 FACA=-1.
      CP=(EXP(P)+EXP(-P))/2.
      SP=(EXP(P)-EXP(-P))/2.
 8030 IF(C-BETTA(I)) 8004,8005,8005
 8005 FACB=1.
      CQ=COS(Q)
      SQ=SIN(Q)
      GO TO 8006
 8004 FACB=-1.
      CQ=(EXP(Q)+EXP(-Q))/2.
      SQ=(EXP(Q)-EXP(-Q))/2.
 8006 BIG = BETTA(I)**2/(XK*C*C)
      AL=2.*XK*AK-(XK*C)**2/BETTA(I)**2
      G(1,1)=2.*BETSQ*CP/(C*C)-AL*BIG*CQ/XK
      G(1,2)=-(AL*BIG*SP/(RALPH*XK)+2.*BIG*RBET*SQ*FACB*AK)
      G(1,3)=-BIG*(CP-CQ)/AMU
      G(1,4)=BIG*SP/(AMU*   RALPH)+BIG*SQ*RBET*FACB/AMU
      G(2,1)=-2.*BIG*RALPH*SP*FACA*XK-AL*BIG*SQ/(XK*RBET)
      G(2,2)=-AL*BIG*CP/XK+2.*BIG*CQ*XK
      G(2,3)=RALPH*BIG*SP*FACA/AMU+BIG*SQ/(RBET*AMU)
      G(2,4)=-G(1,3)
      G(3,1)=2.*AMU*AL*BIG*(CP-CQ)
      G(3,2)=-AMU*AL*AL*BIG*SP/(RALPH*XK*XK)-4.*AMU*RBET*SQ*FACB*BIG*AK*
     1XK
      G(3,3)=G(2,2)
      G(3,4)= -G(1,2)
      G(4,1)=-4.*AMU*AK*AK*RALPH*BIG*SP*FACA-AMU*AL*AL*BIG*SQ/
     1(XK*XK*RBET)
      G(4,2)=-G(3,1)
      G(4,3)=-G(2,1)
      G(4,4)=G(1,1)
      DO 8007 J=1,4
      SUM=0.
      DO 8008 K=1,4
      SUM=SUM+B(K)*G(K,J)
 8008 CONTINUE
      A(J)=SUM
 8007 CONTINUE
      DO 8011 J=1,4
      E(J)= B(J)
      B(J)=B(J)*A(J)
 8011 CONTINUE
      IF(IL-1)89,83,84
   83 I=I-1
      GO TO 8001
 8010 E(5)=B(1)
      E(6)=B(2)
      E(7)=B(3)
      E(8)=B(4)
      IF(I-1)81,81,82
   81 HOLD=THICK(I)
      THICK(I)=0.
      IL=10
      ILI=1
      GO TO 8009
   84 IF(ILI-1) 85,85,86
   85 THICK(I)=HOLD
      DIVS=1.
      GO TO 51
 8012 IF(ILI-1)90,87,88
   87 UDISP(2    )=-A(2)*G(1,1)-A(3)*G(2,1)-A(4)*G(3,1)
      WDISP(2    )=A(1)*G(1,1)+A(3)*G(4,1)-A(4)*G(5,1)
      UDISP(2)= UDISP(2)/DIVS
      WDISP(2)= WDISP(2)/DIVS
      IL1=10
      B(1)=E(1)
      B(2)=E(2)
      B(3)=E(3)
      B(4)=E(4)
      GO TO 8009
   86 THICK(I)=0.
      DIVS=1.
      GO TO 51
   88 UDISP(1    )=-A(2)*G(1,1)-A(3)*G(2,1)-A(4)*G(3,1)
      WDISP(1    )=A(1)*G(1,1)+A(3)*G(4,1)-A(4)*G(5,1)
      UDISP(1)= UDISP(1)/DIVS
      WDISP(1)= WDISP(1)/DIVS
```

```
      IDISP=1
      THICK(I)=HOLD
      L=IX+1
      PUNCH 2701,MODEL,IM,L,TOTAL,C,PERIOD(IM,IN)
      PUNCH 1032,(UDISP(J),WDISP(J),J=1,L)
      DIV=WDISP(1)
      DO 1001 J=1,IX+1
      UDISP(J)=UDISP(J)/DIV
      WDISP(J)=WDISP(J)/DIV
 1001 CONTINUE
      IPUN=IPUN+2
      PUNCH 2701,MODEL,IM,L,TOTAL,C,PERIOD(IM,IN)
      PUNCH 1032,(UDISP(J),WDISP(J),J=1,L)
 2701 FORMAT(3I4,3F9.3)
 1032 FORMAT(6E13.6)
      GO TO (8025,901,8026),IJMP
   82 HOLD=THICK(I)
      THICK(I)=0.
      IL=0
      GO TO 8009
   89 THICK(I)=HOLD
      ILI=0
      M=1
      DIVS=1.
      GO TO 51
   90 GO TO(92,91,93,96),M
   92 DO 8014 K=1,6
      SUM=0.
      DO 8015 J=2,4
      SUM=SUM+B(J)*G(J-1,K)
 8015 CONTINUE
      A(K)=SUM
 8014 CONTINUE
      DO 8031 J=1,6
      B(J)=A(J)/DIVS
 8031 CONTINUE
      I=I-1
      M=2
      GO TO 51
   91 IF(I-1) 94,94,95
   95 DO 8016 J=1,6
      SUM=0.
      DO 8017 K=1,6
      SUM=SUM+B(K)*G(K,J)
 8017 CONTINUE
      A(J)=SUM
 8016 CONTINUE
      DO 8018 J=1,6
      B(J)=A(J)/DIVS
 8018 CONTINUE
      I=I-1
      GO TO 51
   94 UDISP(L+1    )=-A(1)*G(1,1)-A(2)*G(2,1)-A(3)*G(3,1)-A(4)*G(4,1)
     1-A(5)*G(5,1)-A(6)*G(6,1)
      UDISP(L+1)= UDISP(L+1)/DIVS
      B(1)= H(1)
      B(2)= H(2)
      B(3)=H(3)
      B(4)=H(4)
      I=L
      M=3
      DIVS=1.
      GO TO 51
   93 A(1)=-B(1)*G(1,1)-B(3)*G(4,1)+B(4)*G(5,1)
      A(2)=-B(1)*G(1,2)-B(3)*G(4,2)+B(4)*G(5,2)
      A(3)=B(1)*G(1,3)+B(3)*G(4,3)-B(4)*G(5,3)
      A(4)=B(1)*G(1,4)+B(3)*G(4,4)-B(4)*G(5,4)
      A(5)=-B(1)*G(1,5)-B(3)*G(4,5)+B(4)*G(5,5)
      A(6)=-B(1)*G(1,6)-B(3)*G(4,6)+B(4)*G(5,6)
      DO 8019 J=1,6
      B(J)=A(J)
 8019 CONTINUE
      M=4
      I=I-1
      GO TO 51
   96 IF(I-1) 97,97,98
   98 G(1,3)=-G(1,3)
      G(1,4)=-G(1,4)
      G(2,3)=-G(2,3)
      G(2,4)=-G(2,4)
      G(3,1)=-G(3,1)
      G(3,2)=-G(3,2)
      G(3,5)=-G(3,5)
      G(3,6)=-G(3,6)
      G(4,1)=-G(4,1)
      G(4,2)=-G(4,2)
      G(4,5)=-G(4,5)
      G(4,6)=-G(4,6)
      G(5,3)=-G(5,3)
      G(5,4)=-G(5,4)
      G(6,3)=-G(6,3)
      G(6,4)=-G(6,4)
      DO 8020 J=1,6
      SUM=0.
      DO 8021 K=1,6
      SUM=SUM+ B(K)*G(K,J)
 8021 CONTINUE
      A(J)=SUM
 8020 CONTINUE
      DO 8022 J=1,6
      B(J)=A(J)/DIVS
 8022 CONTINUE
      I=I-1
      GO TO 51
   97 WDISP(L+1    )=-A(1)*G(1,1)-A(2)*G(2,1)+A(3)*G(3,1)+A(4)*G(4,1)
```

```
     1-A(5)*G(5,1)-A(6)*G(6,1)
      WDISP(L+1)= WDISP(L+1)/DIVS
      B(1)=E(5)
      B(2)=E(6)
      B(3)=E(7)
      B(4)=E(8)
      I=L
      L=L-1
      IL=1
      GO TO 8001
 8025 XK=ROOTS(IM,  1)+DELK
      IN=2
      IM=MODE(IMOD)
      C= CMAX-DELC
 2000 PRINT 1030,IM,TOTAL
 1030 FORMAT(6H-MODE I3 /19H SECTION THICKNESS F8.2 //11X, 15H PHASE VEL
     1OCITY,4X,2H K, 8X,10H PERIOD(S),4X,15H GROUP VELOCITY,8X,10H PERIO
     1D(S))
      PRINT 1031,CMAX,ROOTS(IM,1),PERIOD(IM,1)
 1031 FORMAT(12X,F10.4,3X,F10.5,4X,F10.4)
      GO TO 500
  301 IF(INDX2-1) 502,501,502
  501 INDX2 = 10
      XK1=XK
      F1=F
      XK =XK + DELK
      GO TO 100
  502 AF=ABS(F)
      AF1=ABS(F1)
      IF(AF-AF1) 504,503,503
  503 IF(ABS(F1-F) -AF) 507,506,506
  504 IF(ABS(F1-F)-AF1)507,506,506
  506 IF(ABS(XK1-XK)-XKMIN)508,508,610
  507 IF(IBRAC-1) 508,706,513
  513 IF(AF-AF1) 510,510,511
  511 IF(IUP-1) 2004,2004,510
 2004 IF(IUP1-1) 2006,2006,2005
 2005 XK1=XK
      F1=F
      GO TO 2006
 2005 SIGN=-1.
      XK=XK-2.* DELK
      IUP1=10
      IF(XK-ROOTS(IM,IN-1)) 2002,2002,60
 2002 XK=ROOTS(IM,IN-1)
      IUP=10
      SIGN=1.
      GO TO 500
  510 XK1=XK
      F1=F
      XK=XK+DELK*SIGN
      IF(XK-ROOTS(IM,IN-1)) 2002,2002,60
   60 IF(XK-XKMAX) 100,100,520
  508 IF(IBRAC-10) 709,710,710
  710 F2=F1
      XK2=XK1
  709 F1=F
      XK= ABS((XK2*F1-XK*F2)/(F1-F2))
      IF(IPTNB-1)530,529,530
  529 ROOTS(IM,IN)=XK
      PERIOD(IM,IN)= (2.*3.1416)/(XK*C)
      IDISP=10
      IJMP=2
      IL=1
      GO TO 8000
  901 IF(IPTNB -1)531,528,531
  528 C=C-PTRBC
      IF(C-CMIN)520,520,546
  546 IF(IN-3)532,533,533
  532 XK= ROOTS(IM,IN)
      IPTNB =10
      IUP=1
      IF(XK-XKMAX) 500,500,61
  533 XK1= ROOTS(IM,IN-2)
      XK3=ROOTS(IM,IN)
      XK2=ROOTS(IM,IN-1)
      C1= CMAX1-C
      D1= -(XK3-XK1)/(2.*DELC)
      D2=(XK3 -2.*XK2 +XK1)/DELC**2
      XK= XK1- D1*C1 +(D2*C1*(C1-DELC))/2.
      IPTNB=10
      IUP=1
      IF(XK-XKMAX) 500,500,61
  531 C=C-DELC
      IPTNB=1
      IF(C-CMIN)520,521,521
  521 IF(IN-3) 522,523,523
  522 XK=ROOTS(IM,IN)+.5*(ROOTS(IM,IN)-ROOTS(IM,IN-1))
      IN=IN+1
      IUP=1
      IF(XK-XKMAX)500,500,520
  523 XK1=ROOTS(IM,IN-2)
      XK2= ROOTS(IM,IN-1)
      XK3= ROOTS(IM,IN)
      C1=CMAX1-C
      CMAX1 =CMAX1-DELC
      D1= -(XK3-XK1)/(2.*DELC)
      D2= (XK3 -2.*XK2 + XK1)/ DELC **2
      IN=IN+1
      XK= XK1 -D1*C1 +(D2*C1*(C1-DELC))/2.
      IUP=1
      IF(XK-XKMAX)500,500,520
  520 IF(IM-MODE(NMODE))524,525,525
  524 IMOD=IMOD+1
      NEWMOD = 1
```

```
      IM=MODE (IMOD)
      XK=ROOTS(IM,1)
      XKMIN=.01*XK
      XKMAX=10.*ROOTS(IM,1)
      IPTRB=1
      CMAX1 =CMAX
      C=CMAX
      IDISP=10
      IL=1
      IJMP=3
      IN=1
      IX=NLAYER-1
      TOTAL=TTHICK
      GO TO 8000
 8026 C=CMAX-DELC
      IN=2
      IX=NLAYER-1
      IOP=1
      GO TO 2000
  530 W1=ROOTS(IM,IN)*(C+PTRBC)
      W2=XK*C
      DK= ROOTS(IM,IN) -XK
      R=-(W2-W1)/DK
      C= C+PTRBC
      TGRR=2.*3.1416/((W2-W1)/2.+W1)
      IPTRB=1
      XKMAX=ROOTS(IM,IN)+10.*(ROOTS(IM,IN)-ROOTS(IM,IN-1))
      XKMIN=.01*ROOTS(IM,IN)
      PRINT 1020,C,ROOTS(IM,IN),PERIOD(IM,IN),R,TGRR
 1020 FORMAT(12X,F10.4,3X,F10.5,4X,F10.4,6X,F10.4,12X,F10.4)
      GO TO 531
   61 C=C+PTRBC
      PRINT 1031,C,ROOTS(IM,IN),PERIOD(IM,IN)
      GO TO 520
  525 PUNCH 2701,IPUN
      PAUSE 7
      READ 350,IPUN
      DO 556 I=1,IPUN
      READ 2701,MODEL,IM,L,TOTAL,C,PERIOD(1,1)
      READ 1032,(UDISP(J),WDISP(J),J=1,L)
      PRINT 557,MODEL,IM,C,PERIOD(1,1),TOTAL
  557 FORMAT(10H-MODEL RTI I4 /, 5H MODE I4/ ,15H PHASE VELOCITY F8.3 /,
     1 7H PERIOD F9.3/ ,22H SECTION THICKNESS(KM) F9.3 /// )
      PRINT 2703
 2703 FORMAT ( 52H DISPLACEMENTS AT LAYER INTERFACES NORMALIZED TO THE/
     1 51H VERTICAL DISPLACEMENT AT THE SURFACE (INTERFACE 0)  / )
      PRINT 2704
 2704 FORMAT(10H INTERFACE ,5X,13H HORIZ. DISP.,5X,12H VERT. DISP.)
      DO 556  J=0,L-1
      PRINT 556 ,J,UDISP(J+1),WDISP(J+1)
  556 FORMAT (5X,I3,11X,E13.6,9X,E13.6)
      PRINT 1024
 1024 FORMAT (19H END OF COMPUTATION )
      PAUSE
      END
```

INPUT DATA AND FORMAT FOR STRESS

CARD	FORMAT	DATA DESCRIPTION
1	I5	No. of layers in model
	I5	No. of (c,k) pairs for which modal shape is to be computed
2	8F10.4	Layer parameters stored in sequence, α, β, ρ, d ; two layers to a card
3	E13.6	c
	E13.6	k
	E13.6	Horizontal to vertical surface displacement ratio, $\frac{u_o}{w_o}$

Repeat card 3 for each set of c,k, $\frac{u_o}{w_o}$.

```
C      PROGRAM STRESS.          J.W.DUNN    GU-163
       DIMENSION ALPA%50<,BETA%50<,D%50).RHO%50<,LAMBDA%50<,AM%4,4<,
      1 MU(50)
       REAL LAMBDA,MU
       DOUBLE PRECISION AM,RATIO,SIGU TAUU,UNDO,RRWO,NEWSIG,NEWTAU,RHUM1,
      1COSPM,COSQM,SINPM,SINQM,UDOT,UDOTN,WDOT,WDOTN,X,Y,PM,QM,RALPAM
       DOUBLE PRECISION DEXPPM,RECIPP,DEXPQM,RECIPQ,RBETAM,GM1,RMC2,ALPA
       DOUBLE PRECISION RHO,ROM,PERIOD
       DOUBLE PRECISION C,ALPHA,BETA,BATEA,K,D,DM,GAMMAM,TESTER,CSQARE
       DOUBLE PRECISION NEWMAT%4<, OLDMAT%4<
  1966 CONTINUE
       READ(1,3) LAYERS,NCASES
C      NCASES IS THE NUMBER OF CASES OF C,K,RATIO FOR A GIVEN SET OF
C  LAYER PARAMETERS, ALPA, BETA, RHO, D.
   561 READ%1,1<(ALPA%I<,BETA%I<,RHO(I),D(I) , Y = 1, LAYERS)
       WRITE(3,1) (ALPA(I),BETA(I),RHO(I),D(I),I=1,LAYERS)
     1 FORMAT(8F10.4)
     3 FORMAT(2I5)
       KWITR = LAYERS - 1
       DO 200 LO=1,NCASES
       READ%1,2<C,K,RATIO
       CSQARE = C*C
       PERIOD# 6.2831853072/%K*C<
       UDOT#RATIO
       WDOT#1.0
       OLDMAT %1< # UDOT
       OLDMAT %2< # WDOT
     2 FORMAT %3E13.6<
       WRITE%1,9<C,PERIOD
     9 FORMAT%1H1,5X,15HPHASE-VELOCITY#,E20.12,/,5X,7HPERIOD#,8X,E20.12,
      1//,10X,9HINTERFACE,1X,10X,12HHORIZ.DISP.,8X,10X,11HVERT.DISP.,/<
       M#0
       WRITE %3,10<M,UDOT,WDOT
    10 FORMAT%10X,2X,I5,7X,1X,6X,E20.12,6X,E20.12<
       DO 100 M#1,LAYERS
       MU%M<#BETA%M< **2 * RHO%M<
       LAMBDA%M<# RHO%M< * %ALPA%M<**2-2.0*BETA%M<**2<
   100 CONTINUE
  1201 SIGU#0.
  1202 TAUU#0.
  1203 OLDMAT %3< # SIGU
  1204 OLDMAT %4< # TAUU
  1205 DO 200 M = 1, KWITR
  1206 M1 = M + 1
  1207 ALPHA # ALPA%M<
  1208 ROM # RHO%M<
  1209 BATEA # BETA%M<
  1210 DM # D%M<
  1211 X=DABS((C/ALPHA)**2-1.)
  1212 Y = DABS((C/BATEA)**2-1.)
  1213 RALPAM = DSQRT(X)
  1214 RBETAM = DSQRT(Y)
  1215 GAMMAM # 2.*%BATEA/C<**2

  1216 PM # K * RALPAM * DM
  1217 QM # K * RBETAM * DM
  1218 TESTER # C-ALPHA
  1219 IF (C-ALPHA) 225,220,220
   220 COSPM = DCOS(PM)
       SINPM #  DSIN%PM<
       APASYN#1.0
       GO TO 230
   225 DEXPPM # DEXP%PM<
       RECIPP # 1.0/DEXPPM
       COSPM # %DEXPPM & RECIPP</2.0
       SINPM # %DEXPPM-RECIPP</2.0
       APASYN#-1.0
   230 CONTINUE
       TESTER = C - BATEA
       IF (C-BATEA) 233,237,237
   233 DEXPQM#DEXP%QM<
       RECIPQ# 1.0/DEXPQM
       COSQM # %DEXPQM & RECIPQ</2.0
       SINQM# %DEXPQM-RECIPQ</2.0
       BTASIN#-1.0
       GO TO 239
   237 SINQM #  DSIN%QM<
       COSQM = DCOS(QM)
       BTASIN#1.0
   239 CONTINUE
       GM1 # GAMMAM-1.
       RMC2 = RHO(M)*CSQARE
  1111 AM%1,1< = GAMMAM* COSPM-GM1*COSQM
  1112 AM%1,2<#%GM1 * SINPM/RALPAM & GAMMAM* RBETAM * SINQM*BTASYN<
  1113 AM%1,3< # -%COSPM - COSQM< / RMC2
  1114 AM%1,4< # %SINPM/RALPAM & RBETAM * SINQM*BTASYN< / RMC2
  1115 AM%2,1< # -%GAMMAM * RALPAM * SINPM *APASYN & GM1 / RBETAM *SINQM<
  1116 AM%2,2< # -GM1*COSPM & GAMMAM * COSQM
  1117 AM%2,3< # %RALPAM * SINPM * APASYN & SINQM / RBETAM< / RMC2
  1118 AM%2,4< # AM%1,3<
  1119 AM% ,1< # RMC2 * GAMMAM * GM1 * %COSPM - COSQM<
  1120 A   ,2< # RMC2 * %GM1 * GM1 * SINPM / RALPAM & GAMMAM **2 * RBETAM
      1* SINQM*BTASYN<
  1121 AM%3,3< # AM%2,2<
  1122 AM%3,4< # AM%1,2<
  1123 AM%4,1<# RMC2 *%GAMMAM**2*RALPAM*SINPM*APASYN& GM1*GM1 / RBETAM*
      1SINQM<
  1124 AM%4,2< # AM%3,1<
  1125 AM%4,3< # AM%2,1<
  1126 AM%4,4< # AM%1,1<
       WRITE(1,3030)
  3030 FORMAT('  MATRIX OF A-S.     ')
       DO 301 MY=1,4
   301 WRITE(3,3333) (AM(MY,L),L=1,4)
  3333 FORMAT(5X,6E20.12)
       DO 8888 IMP # 1,4
       MODIMP # MOD%IMP,2<
       MIMP1# MODIMP-1
       NEWMAT %IMP< # 0.0
```

```
      DO 8989 KKY # 1,4
      MODKKY # MOD%KKY,2<
      NEXP # NIMP1*MODKKY
      NFAC # %-1<**NEXP
      FAC # FLOAT%NFAC<
      NEWMAT%INP<#FAC*AN%INP,KKY<*OLDMAT%KKY<&NEWMAT%INP<
 8988 CONTINUE
      UOOTH # NEWMAT %1<
      WDOTH # NEWMAT %2<
      WRITE %3,10<N,UOOTH,WOOTH
      DO 8989 KLJ # 1,4
 8989 OLDMAT %KLJ< # NEWMAT %KLJ<
  200 CONTINUE
      GO TO 1967
      END
```

STORAGE MAP   VARIABLES (TAGS: C=COMMON, E=EQUIVALENCE)

NAME	TAG	REL ADR	NAME	TAG	REL ADR	NAME	TAG	REL ADR	NAME	TAG	REL ADR
[illegible]		000158	X		000219	Y		0002F0	C		0002F8
[illegible]		000300	AN		000308	PN		000388	QN		000390
[illegible]		000398	RHO		0003A0	ON1		000530	NOM		000536
[illegible]		000540	BETA		000600	SIGU		000860	TAOO		000868
[illegible]		000870	WMWO		00087N	UOOT		000880	WDOT		00088N
[illegible]		000890	RATIO		00089N	WNON1		0008A0	COSPN		0008A8
[illegible]		0008B0	SINPN		0008B8	SINQN		0008C0	UOOTN		0008C8
[illegible]		0008D0	ALPHA		0008D8	BATEA		0008E0	NENSIO		0008E8
[illegible]		0008F0	RALPAN		0008F8	OEXPPN		000900	RECIPP		00090N
[illegible]		000910	RECIPQ		000918	NBETAN		000920	PERIOD		000928
[illegible]		000930	TESTFR		000938	CSQARE		000940	NEWMAT		000948
[illegible]		000968	I		000988	N		0009NC	L		000990
[illegible]		000994	IO		000A5C	N1		000A60	NK		000A64
[illegible]		000A68	KXY		000A6C	FAC		000A70	KLJ		000A74
[illegible]		000A78	NFAC		000A7C	KWITN		000A80	NINP1		000A84
[illegible]		000A89	LAYERS		000B50	NCASES		000B54	APASYN		000B58
[illegible]		000B5C	MODIMP		000B60	MODKXY		000B64			

EXTERNAL REFERENCES

NAME	REL ADR	NAME	REL ADR	NAME	REL ADR	NAME	REL ADR
[illegible]	000B68	DCOS	000B6C	DSIN	000B70	OEXP	000B74
[illegible]	000B78						

CONSTANTS

NAME	REL ADR	NAME	REL ADR	NAME	REL ADR	NAME	REL ADR
[illegible]	000B80	00000000	000BB4	00000002	000BB8	41100000	000B9C
[illegible]	000BC0	00000000	000BC4	41648?ED51121B7F	000BC8		
[illegible]	000B18						

IMPLIED EXTERNAL REFERENCES

INPUT DATA AND FORMAT FOR INTEGRAL

CARD	FORMAT	DATA DESCRIPTION
1	I10	No. of layers in model
	F10.4	Factor for singular matrix criteria in curve fitting subroutine, generally = .99.
	I10	degree +1 of the polynomial fit to the displacement data
	I10	Read option; 1 if read every set of displacements; 0 if read every other set
2	8F10.6	Layer parameters stored in sequence α, β, ρ, d; two layers to a card
3	20A4	Text identification $\leq$ 80 characters
4	I4	Model identification no.
	I4	Mode no.
	I4	No. of layers
	F9.3	Total section thickness
	F9.3	Phase velocity in km/s
	F9.3	Period in sec
5	6E13.6	Displacements stored in pair sequence v_1, w_1, u_2, w_2 ...;3 pairs to a card

For computations involving more than one set of displacements corresponding to a (c,T) pair, repeat cards 4 & 5 for each set.

```
C         MANY # NO. OF LAYERS IN THE MODEL
C         MANYP1 # NO. OF INTERFACES IN MODEL, # MANY & 1
C        LAYERS # NO OF LAYERS ACROSS WHICH U AND W ARE APPROXIMATED
C     THE NUMBER OF INTERFACES ON WHICH U AND W ARE APPROXIMATED # LAYERS & 1
C        OVERHANG OCCURS WHEN %LAYERS< IS NOT AN EVEN DIVISOR OF %MANY-1<.
C        IF THERE IS NO OVERHANG, MAMA#0.
C        IF THERE IS OVERHANG, MAMA#1, AND %STOP< IS GREATER THAN %MANY<
C     AT STATEMENT NO. 3318.
C        THICK# THICKNESS OF LAYER%J<
C        Z%J< # COORDINATE OF INTERFACE OF J-TH AND %J&1<-ST. LAYER
C        A%L< # COEFFICIENTS OF M-TH. DEGREE POLYNOMIAL APPROXIMATION
C        TO U%Z<, L # 1,2,....,M.
C        MM # DEGREE OF POLYNOMIALS APPROXIMATING U%Z< AND W%Z<
C        2*MM # DEGREE OF U**2
C        MM-1 # DEGREE OF DU/DZ
C        2*%MM-1< # DEGREE OF %DU/DZ<**2
C        2*MM&1 # DEGREE INTEGRAL U**2
C        2*%MM-1<&1 # DEGREE INTEGRAL %DU/DZ<**2
C        KA COUNTS LAYERS
C        II COUNTS COEFFICIENTS
S.0031     DIMENSION ALPHA%40<, BATEA%40<,TEXT%20<
S.0032     DIMENSION U%40<, W%40<, A%10<, B%10<, C%10<, D%10<, OPRIM%10<,
          1WPRIM%10<, WPRIM2%10<, UWPRIM%10<, UPRIMW%10<,UPRIM2%10<, RHO%40<,
          2Z%40<, ZZ%10,10<, F%10<, S%10<, DIDO%7<, THICK%40<,OI%10<
          3 , R%40<,LAMBDA%40<,KERNEL%7<,INTEGL%7<
S.0033     DIMENSION X%40<,SUM%30,7<,DERIV%30,7<,DRIVDD%30,7<
S.0034     DIMENSION SUMINT%7<
S.0035     DOUBLE PRECISION KERNEL,DERIV,MM,F,RHO,WPRIM2,C,U,XRQ,DCDBTA
          1DELTAM,O,XMM1,XC,INTEGL,SUM,DZ,Z,UPRIM2,WPRIM,B,DCDRHO,OCDMU,
          2DRIVDD,OI,BIGTHK,X,SUMINT,Y,THICK,ZZ,OPRIMW,UPRIM,A,BATEA,DCDD,
          3BETA,DENOM,SAVE,DRIVOD,LAMBDA,DIDD,Z,UWPRIM,D,W,ALPHA,DCOALF,ALFA,
          4OMEGA,ZZ,EPS,FACTOR
S.0036     INTEGER OVMM1, START, STOP ,TWOMM
S.0037     EQUIVALENCE %KERNEL%1<,INTEGL%1<<
C        A AND B ARE FOUND BY INVERSION OF Z MATRIX. THE GAUSS-JORDAN-RUTIS
C        HAUSER SUB-ROUTINE IS USED FOR INVERSION. THIS PERMITS APPROXIMATI
C        ON OF U AND W BY HIGHER ORDER POLYNOMIALS WITHOUT MODIFICATIO  OF
C        THE PROGRAM.
S.0038   1 FORMAT%1H1<
S.0039     READ(1,5001) MANY,FACTOR,LAYERS,NSENSE
S.0040     WRITE(3,5001) MANY,FACTOR,LAYERS,NSENSE
S.0041 5001 FORMAT(I10,F10.4,2I10)
S.0042     MM # LAYERS
S.0043     MM1 # MM&1
S.0044     MMP1#MM&1
S.0045     MANYP1 # MANY & 1
S.0046     READ(1,5002) (ALPHA(I),BATEA(I),RHO(I),THICK(I),I=1,MANY)
S.0047     WRITE(3,5002) (ALPHA(I),BATEA(I),RHO(I),THICK(I),I=1,MANY)
S.0048 5002 FORMAT%4F10.4<
S.0049     READ(1,1776) (TEXT(I),I=1,20)
S.0050 1776 FORMAT%20A4<
S.0051     DO 20 NN # 1,MANY
S.0052     [illegible]

S.0053     LAMBDA%NN<#RHO%NN<*%ALPHA%NN<**2-2.0*BATEA%NN<**2<
S.0054  20 X%NN< # THICK%NN<
S.0055 2000 FORMAT %5X,2F15.4<
S.0056     Z%1< # 0.0
S.0057     DO 21  JJ # 2,MANYP1
S.0058  21 Z%JJ< # Z%JJ-1<&THICK%JJ-1<
S.0059   2 FORMAT %5F15.2<
S.0060     WRITE(3,2)  (Z(JJ),JJ=1,MANY)
S.0061     WRITE(3,5)
S.0062   5 FORMAT (1H1)
S.0063 3366 CONTINUE
S.0064     MAMA # 0
S.0065     TIMP#1
S.0066 6004 READ(1,3003) MODEL,MODE,MANY,BIGTHK,XC,T
S.0067     W#2.0*3.1416/%T*XC<
S.0068     READ(1,3) (U(I),W(I),I=1,MANY)
S.0069     IF(NSENSE)6001,6001,6000
S.0070 6001 GO TO %6002,6000<,TIMP
S.0071 6002 TIMP#2
S.0072     GO TO 6000
S.0073 6000 DO 9030 I # 1,MANY
S.0074 9030 U%I< # -U%I<
S.0075   3 FORMAT %6E13.6<
S.0076     DO 23 KI#1,7
S.0077  23 INTEGL%KI<#0.0
S.0078   4 FORMAT %2E10.4<
S.0079     NZ # 0
S.0080     DO 330 KOUNT#1,MANY,LAYERS
S.0081     START # KOUNT
S.0082     STOP # START & LAYERS
S.0083     IF %STOP-MANY< 3317,3317,3318
S.0084 3318 STOP # MANY
S.0085     START # STOP-LAYERS
S.0086     MAMA # 1
C        SET Z MATRIX AND INVERT IT.
S.0087 3317 NZ # 0
S.0088     DO 110 JJ # START, STOP
S.0089     NZ # NZ&1
S.0090     MMP1M1 # MMP1 - 1
S.0091     DO 110 II#1,MMP1M1
C                 NZ     ROW,       LL  COLUMN ******
S.0092 4033 FORMAT(3I5,F15.4,2I5)
S.0093     WRITE(3,4033) JJ,II,NZ,Z(JJ),MM1,LL
S.0094     ZZ%NZ,II< # Z%JJ<**%MM1-LL<
S.0095 110 CONTINUE
S.0096     ZZ%NZ,MMP1< # 1.00
S.0097 100 CONTINUE
S.0098     EPS # .7E-3
S.0099 5003 FORMAT%2I4,3F9.3<
S.0100     CALL [illegible] %ZZ,MM1,EPS<
S.0101     L1 # LAYERS & 1
S.0102     DO 200 KI#1,L1
S.0103     A%KI<#0.0
S.0104     B%KI<#0.0
S.0105     KI # 0
```

```
      DO 211 IL•START,XTOP
      XL     •1
      A%K<<   (%Z<EZZ%X7,XL<•U%LL<
      E%NZ< • B%XZ<EZZ%KZ,XL<•W%LL<
  210 CONTINUE
  220 CONTINUE
C     THE COEFFICIENTS OF U**2 C%N<
C     THE COEFFICIENTS OF W**2 D%N<
C     MM • DEGREE OF U
C     U • A%1<•Z**%MM<&A%2<•Z**%MM-1<&...&A%MM *Z&A%MM&1<,%MM&1< TERMS
C     U**2 • C%1<•Z**%2*MM<&C%2<•Z**%2*MM-1<&...&C%2*MM<*Z&C%2*MM&1<,
C     %2*MM&1< TERMS.
      TWOMM1 • %2*MM<&1
      DO 99 IR • 1,TWOMM1
      D%IR< • 0.0
      C%IR< • 0.0
      IR1 • IR&1
      DO 99 KR • 1,IR
      N • IR1 - KR
      D%IR< • D%IR<&B%KR<•B%N<
      C%IR< • C%IR<&A%KR<•A%N<
   99 CONTINUE
C     THE COEFFICIENTS OF DU/DZ UPRIM%K<
C     THE COEFFICIENTS OF DW/DZ WPRIM%K<
C     MM-1 • DEGREE OF DU/DZ
C     MM • %DEGREE OF DU/DZ< &1, FOR INDEXING
C     DU/DZ • MM*A%1<•Z**%MM-1<&%MM-1<•A%2<•Z**%MM-2<&...&2*A%MM-1<•Z&
C     A%MM<,MM TERMS
C     UPRIM%K< • %MM&1-K<•A%K<,K•1,2,...,MM.MM&1 & MM1.
      DO 98 K•1,MM
      XMM1 • FLOAT%MM1-K<
      UPRIM%K< • XMM1•A%K<
      WPRIM%K< • XMM1•B%K<
   98 CONTINUE
C     THE COEFFICIENTS OF %DU/DZ<**2 UPRIM2
C     THE COEFFICIENTS OF U•DWDZ UWPRIME
C     THE COEFFICIENTS OF %DW/DZ<**2 WPRIM2
C     MM-1 • DEGREE OF DU/DZ
C     2%MM-1<•DEGREE OF %DU/DZ<**2
C     MM121 • 2%MM-1<&1, FOR INDEXING
C     DUDZ • UPRIM%1<•Z%MM-1<&UPRIM%2<•Z%MM-2< &   &UPPIM%MM<, MM TERMS
C     DEGREE OF %DUDZ<**2 • 2%MM-1<,&1 FOR INDEXING.
      TWOMM • 2•MM&1
      MM121 • 2•%MM-1<&1
      MX • MM121 & 1
      DO 97 IL • 1,MX
      UPRIM2%IL< • 0.0
      WPRIM2%IL< • 0.0
      UWPRIM%IL< • 0.0
      UPRIMW%IL< • 0.0
      ILP1 • IL&1
      DO 97 KL • 1,IL
      N • ILP1 - KL
      UPRIM2%IL< • UPRIM2%IL<&UPRIM%KL<•UPRIM%N<
      WPRIM2%IL< • WPRIM2%IL<&WPRIM%KL<•WPRIM%N<

      UWPRIM%IL< • UWPRIM%IL<&A%KL<•WPRIM%N<
      UPRIMW%IL< • UPRIMW%IL<&B%KL<•UPRIM%N<
   97 CONTINUE
C     EVALUATE INTEGRALS
      KSTOP • KTOP - 1
      DO 301 KA • START,KSTOP
      KP • KA&1
      DO 1110 I•1,10
      DTORTS • 0.0
 1110 CONTINUE
      Z1 • ZZ%KA<
      KAL • KA&1
      Z2 • ZZ%KAL<
      JJ • KA
      IF %RAM%< 1322,1322,1321
 1321 Z1 • Z%MZ<
      Z2 • Z%MZ & 1<
 1322 DO 333 KOPSIV • 1,2
      DO 333 I • 1,10
  333 INTEGL%I< • 0.0
C     I%1< • INTEGRAL U**2<DZ  FROM Z1 TO Z2.
      M1 • %2*MM< & 1
      DO 1201 II • 1,M1
      INTEGL%1< • C%II<•%Z2**%M1-II&1<  <•%Z2**%M1-II&1  <-Z1**%
     1M1-II&1<<&INTEGL%1<
 1201 CONTINUE
      INTEGL%1< • INTEGL%1<•RHO%JJ<
C     I%2< • INTEGRAL OF X U**2 DZ FROM Z1 TO Z2.
      M2 • %2*MM<&1
      DO 1202 II • 1,M2
      INTEGL%2< • C%II</FLOAT%M2-II&1  <•%Z2**%M2-II&1  <-Z1**%M2-II&1<<
     1 & INTEGL%2<
 1202 CONTINUE
      INTEGL%2< • INTEGL%2<•LAMBDA%JJ<
C     I%3< • INTEGRAL OF 2LAMBDA•U•WPRIME DZ
      M3 • %MM&MM-1<&1<
      DO 1203 II • 1,M3
      INTEGL%3< • UWPRIM%II</FLOAT%M3-II&1<•%Z2**%  M3-II&1<-Z1**%M3-II&
     11<< & INTEGL%3<
 1203 CONTINUE
      INTEGL%3< • 2.0*LAMBDA%JJ<•INTEGL%3<
C     I%4< • INTEGRAL OF LAMBDA WPRIME SQUARE DZ FROM Z1 TO Z2
      M4 • %2*%MM-1<<&1
      DO 1204 II • 1,M4
      INTEGL%4< • WPRIM2%II</FLOAT%M4-II&1        <•%Z2**%M4-II&1
     1 <-Z1**%M4-II&1        <<&INTEGL%4<
 1204 CONTINUE
      INTEGL%4< • LAMBDA%JJ<•INTEGL%4<
C     I%5< • INTEGRAL OF %   MU•%2•USQUARE&WSQUARE<•DZ FROM Z1 TO Z2.
      M5 • %2*MM<&1
      DO 1205 II • 1,M5
      INTEGL%5< • INTEGL%5<&%2.0•C%II<&D%II<</FLOAT%M5    -II&1<•%Z2**%
     1 M5   -II&1<- Z1**%M5    -II&1<<
 1205 CONTINUE
      INTEGL%5< • MU%JJ<•INTEGL%5<
```

```
              I5<  = INTEGRAL OF -2**U*UPRIME*W*DZ FROM Z1 TO Z2.
S,0154          M6 = [illegible]-1<FMM<F1
S,0155          DO 1206 LL = 1,M6
S,0156          INTEGL%6< = INTEGL%6<FUPRIM%LL</FLOAT%M6     -LL%1<*Z2**
               1*M6     -LL%1<-Z1**%M6     -LL%1<<
S,0157    1206 CONTINUE
S,0158          INTEGL%6< = -2.0*MU%J<*INTEGL%6<
              I7<  = INTEGRAL OF PRIME SQUARE 5 UPRIME SQUARE<*OZ FROM Z1 TO Z2.
S,0159          M7 = [illegible]%MM-1<<F1
S,0160          DO 1207 LL = 1,M7
S,0161          INTEGL%7<=INTEGL%7<F%2.0*WPRIM2%LL<&OPRIM2%LL<</FLOAT%M7   -LL%1<
               1*Z2****M7    -LL%1<-Z1****M7    -LL%1< <
S,0162    1207 CONTINUE
S,0163          INTEGL%7< = MU%J<*INTEGL%7<
S,0164          IF%KOPRTV-2< 1209,1311,1313
S,0165    1209 DI%1< = INTEGL%1</RHO%J<
S,0166          DI%2< = INTEGL%2</LAMBDA%J<
S,0167          DI%3< = INTEGL%3</LAMBDA%J<
S,0168          DI%4< = INTEGL%4</LAMBDA%J<
S,0169          DI%5< = INTEGL%5</MU%J<
S,0170          DI%6< = INTEGL%6</MU%J<
S,0171          DI%7< = INTEGL%7</MU%J<
S,0172          Z1 = FACTOR*Z1
S,0173          Z2 = FACTOR*Z2
S,0174    1313 DO 310 KO = 1,7
S,0175          SAVE = DIDD%KO<
S,0176          DIDD%KO< = INTEGL%KO< - DIDD%KO<
S,0177          INTEGL%KO< = SAVE
S,0178     310 CONTINUE
S,0179    5004 FORMAT%1H1 [illegible] F.T.T. ,T5,20A4,//,
               1 5H MODE ,I5,//,
               2 15H PHASE VELOCITY ,F10.4,10X,6HPERIOD ,F10.4,10X,14HGROUP VELOCIT
               3Y  ,F10.4,//,1X,
               4 [illegible] DEPTH       DC/DRHO       DC/DMU        DC/DBETA
               5   DC/DALPHA      DC/DLAMBDA      DC/DD    ,//<
[illegible]     DELTAD = [illegible] = FACTOR-1.0
[illegible]     DO 1311 [illegible] = 1,7
[illegible] 1311 DIDD%KO< = DIDD%KO</DELTAD
[illegible]     DO 2000 I = 1,7
[illegible]     SETMNZ,I< = INTEGL%I<
[illegible]     [illegible],I<=DI%I<
[illegible]     [illegible],I<=DIDD%I<
[illegible] 2000 CONTINUE
[illegible] [illegible] CONTINUE
[illegible]     [illegible]
[illegible]     DO [illegible] I = 1,7
[illegible]     SUMINT%I< = 0.0
[illegible]     DO [illegible] [illegible]
[illegible] [illegible] SUMINT%I< = SUMINT%I< & SETMNZ,I<
[illegible] [illegible] FORMAT%[illegible]
[illegible]     [illegible]SUMINT%[illegible]<<SUMINT%5<<*[illegible] SUMINT%3<&SUMINT%6<</%2.0*XC*
               1[illegible]1<<
[illegible]     WRITE( ,5004) [illegible],(TEXT(I),I=1,20),MODE,XC,T,GRUPVL
[illegible]     DO [illegible] [illegible]=1,[illegible]
[illegible]     [illegible] = [illegible]

[illegible]     [illegible]
[illegible]     DO [illegible] I = 1,7
[illegible]     [illegible]%I<=SETMNZ,I<
[illegible]     DI%I<=[illegible],I<
[illegible]     DIDD%I<=[illegible],I<
[illegible] [illegible] CONTINUE
[illegible]     [illegible]SUMINT%3<&SUMINT%6<<
               DC/[illegible],I=1,2,3,4
S,[illegible]   [illegible] = [illegible]     SXC*DI%6<     SXC*DI%7<     </DENOM
[illegible]     DCDLAM = [illegible]     [illegible]     SXC*DI%4<     </DENOM
[illegible]     ALT[illegible] = [illegible]
[illegible]     [illegible]
[illegible]     [illegible]     </[illegible]
               1[illegible]
S,0211          DCDBTA = 2.0*[illegible]-2.0*DCDLAM<
S,0212          DCDALF = 2.0*[illegible]
S,0213          DCDD%5-XC*[illegible]<&XC*K*%DIDD%3<<
               1[illegible]</DENOM
[illegible]     [illegible]
[illegible] [illegible] FORMAT%I6,[illegible],F10.4,7E15.5<
[illegible]     [illegible] = [illegible]-1
[illegible]     [illegible]/2.0
[illegible] 5005 WRITE(3,5006) ZL,[illegible],DCDRHO,DCDMU,DCDBTA,DCDALF,DCDLAM,DCDD
[illegible] [illegible] CONTINUE
[illegible]     GO TO 1466
[illegible]     STOP
[illegible]     END
```

STORAGE MAP   VARIABLES   (TAGS: C=COMMON, E=EQUIVALENCE)

NAME	TAG	REL ADR	NAME	TAG	REL ADR	NAME	TAG	REL ADR	NAME	TAG	REL ADR
[illegible]		[illegible]	W		000298	A		0003D4	[illegible]		000428
[illegible]		[illegible]	[illegible]		0004C8	Z		000518	E		00065C
[illegible]		[illegible]	X		0006F4	Q		000838	[illegible]		000840
[illegible]		[illegible]	[illegible]		000B68	MU		000998	KC		[illegible]
[illegible]		[illegible]	Z2		000D08	RHO		000D10	SET		[illegible]
[illegible]		[illegible]	[illegible]		0014E8	DIDD		001470	XMM1		[illegible]
[illegible]		001530	BETA		001538	SAVE		001540	ALFA		001548
[illegible]		[illegible]	[illegible]		001690	UPRIM		0017D0	WPRIM		001920
[illegible]		[illegible]	DERIV		00198C	DCDMU		002040	DENOM		002048
[illegible]		[illegible]	[illegible]		0020A0	UPRIMW		0020F0	UPRIM2		002140
[illegible]		[illegible]	[illegible]	[illegible]	0022D0	DRIVOD		002108	SUMINT		002388
[illegible]		[illegible]	DCDLAM		0029D8	DCDRHO		0029E0	DELTAD		0029E8
[illegible]		[illegible]	DCDALF		0029F8	GRUPVL		002A00	FACTOR		002A08
[illegible]		[illegible]	T		002A14	N		002A18	MM		002A1C
[illegible]		[illegible]	I1		002A24	I0		002A28	MG		002A2C
[illegible]		[illegible]	K2		002A34	LI		002A38	L1		002A3C
[illegible]		[illegible]	K8		002A44	KP		002A48	MX		002A4C
[illegible]		[illegible]	KA		002A54	M1		002A58	M2		002A5C
[illegible]		002A60	M4		002A64	M5		002A68	M6		002A6C
[illegible]		[illegible]	K9		002A74	MM1		002A78	IR1		002A7C
[illegible]		002A80	JJ1		002A84	TEXT		002AB8	STOP		002AD8
[illegible]		[illegible]	[illegible]		002AE0	NAMA		002AE4	[illegible]		002AE8
[illegible]		[illegible]	[illegible]		002AF0	START		002AF4	[illegible]		002AF8

```
S.0001           SUBROUTINE GJR3 (A,N,EPS)
S.0002           DIMENSION A(10, 2),B(20),C(20),L(20),Q(20)
S.0003           DOUBLE PRECISION A,B,C,EPS,PIVOT,Z
S.0004           INTEGER P,Q
S.0005           DO 10 K=1,N
          C      DETERMINATION OF THE PIVOT ELEMENT
S.0006           PIVOT=0.
S.0007           DO 20 I=K,N
S.0008           DO 20 J=K,N
S.0009           IF(DABS(A(I,J))-DABS(PIVOT))20,20,30
S.0010        30 PIVOT=A(I,J)
S.0011           P(K)=I
S.0012           Q(K)=J
S.0013        20 CONTINUE
S.0014           IF(DABS(PIVOT)-EPS)40,40,50
          C      EXCHANGE OF THE PIVOTAL ROW WITH THE KTH ROW
S.0015        50 IF(P(K)-K)60,80,60
S.0016        60 DO 70 J=1,N
S.0017           L=P(K)
S.0018           Z=A(L,J)
S.0019           A(L,J)=A(K,J)
S.0020        70 A(K,J)=Z
          C      EXCHANGE OF THE PIVOTAL COLUMN WITH THE KTH COLUMN
S.0021        80 IF(Q(K)-K)85,90,85
S.0022        85 DO 100 I=1,N
S.0023           L=Q(K)
S.0024           Z=A(I,L)
S.0025           A(I,L)=A(I,K)
S.0026       100 A(I,K)=Z
S.0027        90 CONTINUE
          C      JORDAN STEP
S.0028           DO 110 J=1,N
S.0029           IF(J-K)130,120,130
S.0030       120 B(J)=1./PIVOT
S.0031           C(J)=1.
S.00[illegible]           GO TO 140
S.00[illegible]       130 B(J)=-A(K,J)/PIVOT
S.00[illegible]           C(J)=A(J,K)
S.00[illegible]       140 A(K,J)=0.
S.00[illegible]       110 A(J,K)=0.
S.00[illegible]           DO 10 I=1,N
S.00[illegible]           DO 10 J=1,N
S.00[illegible]        10 A(I,J)=A(I,J)+C(I)*B(J)
          C      [illegible]
[illegible]
[illegible]
[illegible]
[illegible]       160 [illegible]
S.[illegible]           [illegible]
S.[illegible]           [illegible]
S.[illegible]           [illegible]
S.[illegible]       180 [illegible]
S.[illegible]       170 [illegible]

[illegible]       190 [illegible]
[illegible]           [illegible]
S.0051           [illegible]
S.[illegible]           [illegible]
S.[illegible]       150 [illegible]
[illegible]       155 [illegible]
[illegible]       151 RETURN
S.[illegible]        40 WRITE(3,45)P(K),Q(K),PIVOT
S.0057        45 FORMAT(21H0SINGULAR MATRIX3H I=I3,3H J=I3,7H PIVOT=E16.8/)
S.[illegible]           GO TO 151
[illegible]           END
```

STORAGE MAP VARIABLES (TAGS: C=COMMON, E=EQUIVALENCE)

NAME	TAG	REL ADR	NAME	TAG	REL ADR	NAME	TAG	REL ADR	NAME	TAG	REL ADR
A		000158	B		000160	C		000200	Z		0002A0
[illegible]		[illegible]	PIVOT		0002B0	N		000278	P		[illegible]
[illegible]		[illegible]	K		00035C	I		000360	J		[illegible]
L		[illegible]	[illegible]		00036C						

EXTERNAL REFERENCES

NAME	REL ADR	NAME	REL ADR	NAME	REL ADR	NAME	REL ADR

CONSTANTS

NAME	REL ADR	NAME	REL ADR	NAME	REL ADR	NAME	REL ADR
[illegible]	[illegible]	00000000	0003D4	41100000	000[illegible]		
[illegible]	[illegible]	00000050	000440				

IMPLIED EXTERNAL REFERENCES

NAME	REL ADR	NAME	REL ADR	NAME	REL ADR	NAME	REL ADR
[illegible]	000444						

STATEMENT NUMBER	REL ADR	STATEMENT NUMBER	REL ADR	STATEMENT NUMBER	REL ADR	STATEMENT NUMBER	REL ADR
[illegible]	[illegible]	00020	0004F4	00050	00052C	00060	[illegible]
[illegible]	[illegible]	00080	0005B6	00085	[illegible]	00100	[illegible]
[illegible]	[illegible]	00120	000655	00130	[illegible]	00140	[illegible]
[illegible]	[illegible]	00010	000732	00160	000706	00180	00082E
[illegible]	[illegible]	00190	000860	00150	0008B8	00155	000904
[illegible]	[illegible]	00040	00090E	00045	0003F0		

SIZE OF COMMON 000000 PROGRAM 002379

END OF COMPILATION GJR3

INPUT DATA AND FORMAT FOR TRAVEL

Card	Format	Data Description
1	I4	No. of models for which T-D curves are to be computed
2	20A4	Identification for models $\leq$ 80 characters
3	I4	NLAYER = no. of velocity layers in model
	I4	NR; depths of penetration for which T-D values are to be computed = (NR)*(DELR)
	F8.4	DELR = Interval in (km) for numerical integration
	F8.4	DEPTH = maximum depth (km) of penetration $\leq$ depth of last velocity value
	F8.4	EPSLON = distance (km) from maximum depth to begin using modified trapezoidal rule of integration; usually .3km
	F8.4	DELTA = distance from maximum depth at which integration is stopped; usually .01km
4	10F8.4	Velocities in km/sec
5	10F8.4	Depths (km) from surface of given velocities

```
      DIMENSION T(200),DEL(200),R(200),VEL(200),Z(900),ANGLE(200),V(900)
     1,DZ(500),TEXT(20)
      DOUBLE PRECISION T,DEL,R,VEL,Z,V,DZ,DELR,DEPTH,EPS,DELTA,R1,R2,R3,
     1DEL1,A,B,C,DEPTH1,RO,XR,P,P2,SUM,SUM1,SUM2,ETA,FACT,FACT1,FACT2,
     2TEST,VV1,DD,DEL1,DEPTH2,FACT5,FACT6,FACT3,FACT4,TAD,SLOP,SKD,A1,A2
     3,B1,B2,C1,C2
      DOUBLE PRECISION ZMAT(4,4),NDUMMY,VVEC(4),AVEC(4),D
   10 FORMAT(4E25.16)
      READ (1,100) NRUN
      DO 300 IRUN = 1, NRUN
      READ (1,400) (TEXT(I),I=1,20)
      READ (1,100)   NLAYER,NR,DELR,DEPTH,EPSLON,DELTA
      READ (1,200) (VEL(I),I=1,NLAYER)
      READ (1,200) (R(I),I=1,NLAYER)
  100 FORMAT (2I4,4F8.4)
  200 FORMAT (10F8.4)
      Z(1)=0
      I=1
   19 I=I+1
      Z(I)=Z(I-1)+DELR
      IF (Z(I)-DEPTH) 19,19,20
   20 J=1
      K=1
      I=1
      IJMP=1
    7 J=J+3
      NRR=J
   42 CONTINUE
      DO 710 LI=1,4
      NDUMMY = NRR - 4 + LI
      DUMMY = R(NDUMMY)
      WRITE (3,5000) DUMMY
      ZMAT(LI,1) = 1.
      DO 705 LJ=2,4
      ZMAT(LI,LJ) = DUMMY**(LJ-1)
  705 CONTINUE
  710 CONTINUE
      NDUMMY = 4
      EPS = 0.001
      CALL GJR4(ZMAT,NDUMMY,EPS)
      DO 715 LJ=1,4
      NDUMMY = NRR - 4 + LJ
      VVEC(LJ) = VEL(NDUMMY)
  715 CONTINUE
      DO 725 LI=1,4
      DUMMY = 0.
      DO 720 LJ=1,4
      DUMMY = DUMMY + ZMAT(LI,LJ)*VVEC(LJ)
  720 CONTINUE
      AVEC(LI) = DUMMY
  725 CONTINUE
      A = AVEC(1)
      B = AVEC(2)
      C = AVEC(3)
      D = AVEC(4)
      GO TO(31,40),IJMP
   31 V(I) = A + B*Z(I) + C*Z(I)**2 + D*Z(I)**3
      I=I+1
      IF(Z(I)-R(J)) 31,31,30
   30 IF(Z(I)-DEPTH)33,33,34
   33 IF(R(J+3)-DEPTH) 7,7,32
   32 R(J)=DEPTH
      GO TO 31
   34 DEPTH1=6371.-DEPTH
      I=0
   35 I=I+1
      Z(I)=6371.-Z(I)
      IF(Z(I)-DEPTH1)36,36,35
   36 II=I
      RO=6371.0
      XR=NR
      INC= DEPTH/(XR*DELR)
      INC= INC*NR
      N=1
      NRP1 = NR + 1
      INCP1 = INC + 1
      NRR = 4
      DO 8 J = NRP1,INCP1,NR
      DEL1=DELR
      IJMP=2
      DZ(N) = 6371.-Z(J)
      XR = XR*DELR
      IF (6371.0-Z(J)-R(NRR  )) 42,42,45
   45 NRR = NRR + 3
      GO TO 42
   40 P=Z(J)/V(J)
      P2=P**2
      IF ((RO/V(1))**2-P2) 4000,405,405
  405 CONTINUE
      SUM=DSQRT((RO/V(1))**2-P2)
      IF(J-2)11,11,10
11    SUM1=0.
      SUM2=0.
       GO TO 12
   10 SUM1=1./(RO*SUM)
      SUM2=SUM1*(RO/ V(1))**2
      K=2
   24 ETA=Z(K)/V(K)
      IF (ETA-P) 4000,240,240
  240 CONTINUE
      FACT=DSQRT((ETA-P)*(ETA+P))
      FACT1=1./(FACT*Z(K))
      FACT2= FACT1 *(Z(K)/V  (K))**2
      SUM1=SUM1+2.*FACT1
```

```
      SUM2=SUM2+2.*FACT2
      K =K+1
      IF(Z(K)-Z(J)-EPSLON)22,23,24
   22 TEST=Z(J)+EPSLON-.01
      IF(Z(K)-TEST)12,23,23
   23 DEPTH1=Z(J)+EPSLON
      DEPTH1 =6371.-DEPTH1
      VV1=A+DEPTH1*(B+DEPTH1*(C+DEPTH1*D))
      DEPTH1 =6371.-DEPTH1
      DD=EPSLON
      ETA=DEPTH1/VV1
      IF (ETA-P) 4000,230,230
  230 CONTINUE
      FACT=DSQRT((ETA-P)*(ETA+P))
      FACT1=1./(FACT*DEPTH1)
      FACT2= FACT1*(DEPTH1/VV1)**2
      SUM1 =SUM1*DELR + FACT1*DEL1
      SUM2 = SUM2*DELR + FACT2*DEL1
      GO TO 21
   12 DEL1=-Z(J) - EPSLON +Z(K-1)
      GO TO 23
   21 I=1
      POW=0.
      DD =EPSLON
      DEPTH2=Z(J)+DD
   14 POW=POW+1.
      IF(I-1)16,15,17
   16 DEPTH1=DEPTH2
      DEPTH1 =6371.-DEPTH1
      VV1=A+D PTH1*(B+DEPTH1*(C+DEPTH1*D))
      DEPTH1 =6371.-DEPTH1
      ETA=DEPTH1/VV1
      IF (ETA-P) 4000,160,160
  160 CONTINUE
      FACT=DSQRT((ETA-P)*(ETA+P))
      FACT1=1./(FACT*DEPTH1)
      FACT2=FACT1*(DEPTH1/VV1)**2
      I=10
      GO TO 18
   17 FACT5 = FACT1
      FACT6 = FACT2
      FACT1=FACT3
      FACT2=FACT4
   18 DEPTH2=Z(J)+DD/(2.**POW)
      DEPTH2 =6371.-DEPTH2
      VV2=A+DEPTH2*(B+DEPTH2*(C+DEPTH2*D))
      DEPTH2 =6371.-DEPTH2
      ETA=DEPTH2/VV2
      IF (ETA-P) 4000,180,180
  180 CONTINUE
      FACT=DSQRT((ETA-P)*(ETA+P))
      FACT3=1./(FACT*DEPTH2)
      FACT4=FACT3*(DEPTH2/VV2)**2
      SUM1=SUM1+DD*2.*(FACT1+FACT3)/(2.**POW)
      SUM2=SUM2+DD*2.*(FACT2+FACT4)/(2.**POW)
      IF(DD/(2.**POW)-.03   )15,15,14
   15 IF(DD/2.**POW- DELTA)60,60,61
61     TAD = DD/(2.**POW)
      SLOP = (V(J)-VV2)/TAD
   62  FACT5 =FACT1
      FACT6 = FACT2
      FACT1 = FACT3
       FACT2 = FACT4
       TAD = TAD/2.
      DEPTH1 = Z(J)+TAD
      SKD = SLOP*TAD
      VV1 = V(J)-SKD
      FACT = 2.*P*(V(J)*TAD + Z(J)*SKD) /(V(J)**2-2.*V(J)*SKD)
      FACT3 = 1./(DSQRT(FACT)*DEPTH1)
       FACT4 = FACT3*(DEPTH1/VV1)**2
      SUM1 = SUM1 +TAD*(FACT1 +FACT3)*2.
      SUM2 = SUM2 + TAD*(FACT2 +FACT4) *2.
       IF(TAD-DELTA)60,60,62
   60 A1 =FACT3
      A2 = FACT4
      B1 = FACT1
       B2=FACT2
       C1=FACT5
       C2=FACT6
      ERROR= TAD*(FACT4+3*A2-5.*B2/2.+.5*C2)/SUM2
      WRITE(3,1050) ERROR
      SUM1=  TAD*(FACT3+3.*A1-5.*B1/2.+.5*C1)+SUM1
      SUM2=  TAD*(FACT4+3.*A2-5.*B2/2.+.5*C2)+SUM2
      DEL(N)=P*SUM1*6371.
      T(N)=SUM2
      SINID = P*V(1)/RD
      COSID= SQRT(1. - SINID**2)
      ANGLE(N)  = ATAN(SINID/COSID)*180./3.1416
      N=N+1
      GO TO 8
 4000 CONTINUE
      DEL(N) = 0.
      T(N) = 0.
      ANGLE(N) = 0.
      N = N + 1
    8 CONTINUE
      N=N-1
      WRITE(3,401) (TEXT(I), I=1,20)
      WRITE(3,1000)
 1000 FORMAT ( 5X,10H DEPTH(KM) , 10X,18HVELOCITY(KM/SEC)    )
      WRITE(3,1050) (Z(J),V(J),J=1,II)
 1050 FORMAT (2X,E13.6,6X,E13.6)
      WRITE(3,401) (TEXT(I), I=1,20)
      WRITE(3,2000)
 2000 FORMAT (2X,18H TRAVEL TIME (SEC) ,5X,11H DELTA (KM),5X,21H DEPTH O
     1F PENETRATION  ,5X ,19H ANGLE OF INCIDENCE   )
      WRITE(3,2050) (T(J),DEL(J),DZ(J),ANGLE(J),J=1,N)
 2050 FORMAT  ( 5X,F8.3,13X,F8.1,14X,F7.1,16X,F7.2)
  401 FORMAT (1H1   ////,20A4,///)
  400 FORMAT (20A4)
  300 CONTINUE
      STOP
      END
```

INPUT DATA AND FORMAT FOR VARGRAV

CARD	FORMAT	DATA DESCRIPTION
1	I3	No. stations
	F10.5	Station spacing in km, Staspa
	F6.1	Depth to mass sheet in km , Depth
	I3	No. of Density layers ≤ 50
	I3	No. of Density profiles ≤ 100
	F10.5	Minimum ϕ value
	F10.5	Delta z spacing for integration
	F10.5	Subcrustal Density

$$\phi_{min} \geq \frac{[\exp(\pi \cdot \text{Depth/Staspa}) - 1.] * \text{Depth/Staspa}}{\left(\frac{\text{Depth}}{\text{Staspa}}\right)^2 + k^2}$$

$$k = \frac{\text{No. of } \phi\text{'s to be used}}{2}$$

CARD	FORMAT	DATA DESCRIPTION
2	10F8.2	Gravity values in milligals
3	40I2	No. of Density profile to be used at each station
4	16F5.2	Density values stored by profile

```
      DIMENSION GRAV(100),DENSTY(50,50),XCORR(100),DEPDIF(100),PHI(50),I
     1PROF(100)
      READ 100,NOSTA,STASPA,DEPTH,IDENS,NPROF,PHIMIN,DELZ ,SUBCRU
  100 FORMAT(I3,F10.5,F6.1,2I3,3F10.5)
      READ 200,(GRAV(I),I=1,NOSTA)
  200 FORMAT(10F8.2)
      READ 150,(IPROF(I),I=1,NOSTA)
  150 FORMAT(40I2)
      READ 300, ((DENSTY(I,J),J=1,IDENS),I=1,NPROF)
  300 FORMAT(16F5.2)
      DO 1492 I=1,NPROF
      DO 1492 J=1, IDENS
      DENSTY (I,J)= SUBCRU-DENSTY (I,J)
 1492 CONTINUE
      EXPODD = (-EXP(3.1416* DEPTH/STASPA)-1.)*DEPTH/STASPA
      EXPEVN= (EXP(3.1416*DEPTH/STASPA)-1.)*DEPTH/STASPA
      AI=0.
      I=1
      PART =(DEPTH**2)/(STASPA**2)
    6 IF( (I +1)/2 - I/2)1,1,2
    1 PHI(I)=EXPODD/(PART+AI**2)
      PRINT 101,PHI(I)
  101 FORMAT (F10.5)
      GO TO 3
    2 PHI(I)=EXPEVN/(PART + AI**2)
  102 FORMAT(F10.5)
    3 IF(ABS(PHI(I))-PHIMIN)4,4,5
    5 I=I+1
      AI =AI+1.0
      GO TO 6
    4 IX1 = 2*I-1
      IX = I+1
      IX2=I
      J=IX
      DO18 I=2,IX2
      PHI(J)=PHI(I)
      J=J+1
   18 CONTINUE
      PHI(IX2   )=PHI(1)
      J=IX1
      IX2=IX2-1
      DO 7 I=1,IX2
      PHI(I)=PHI(J)
      J=J-1
    7 CONTINUE
      PRINT 400,DEPTH,STASPA
  400 FORMAT(23H-COMPUTED PHI FACTORS/10H DEPTH(KM),F6.1,2X,20H STATION
     1SPACING(KM),F10.5)
      PRINT 401,(PHI(I),I=1,IX1)
  401 FORMAT(2X,E13.6)
      M=IX1-1
      IX=K =N=IX-1
      IX2=K-1
      DO8 I=1,IX2
      XCORR(I)=0.
      DO 9 J=1,IX
      XCORR(I)= GRAV(J)*PHI(K) +XCORR(I)
      K=K+1
    9 CONTINUE
  500 FORMAT(2X,F10.2)
      IX=IX+1
      K= N =N-1
    8 CONTINUE
      IX3=NOSTA-IX1+1
      J=IX2+1
      DO 10 I=1,IX3
      XCORR(J)=0.
      K=I
      DO 11 L=1,IX1
      XCORR(J)=GRAV(K)*PHI(L)  +XCORR(J)
      K=K+1
   11 CONTINUE
  600 FORMAT(2X,F10.2)
      J=J+1
   10 CONTINUE
      IX3=IX3+1
      N=NOSTA-IX2
      DO 12 I=IX3,N
      K=I
      XCORR(J)=0.
      DO 13 L=1,M
      XCORR(J)=GRAV(K)*PHI(L) +XCORR(J)
      K=K+1
   13 CONTINUE
  700 FORMAT(2X,F10.2)
      J=J+1
      M=M-1
   12 CONTINUE
      FACT=2.*(3.1416**2)*6.67
      DO 14 I=1,NOSTA
      XCORR(I)= XCORR(I)/FACT
   14 CONTINUE
      DO 30 J=1,NOSTA
      SUM=0.
      AMAX=XCORR(J)
      IF(AMAX-0.)50,51,52
   50 SGN=-1.0
      GO TO 53
   52 SGN=1.0
      GO TO 53
   51 DEPDIF(J)=DEPTH
      GO TO 30
   53 I=IPROF(J)
      DEPDIF(J)= 0.
      K=0
```

```
   40 K=K+1
      DENS = DENSTY(I,K)
      DENS1=DENSTY(I,K-1)
      IF (K-IDENS)43,43,44
   43 DEPDIF(J)=DEPDIF(J)+DELZ
      SUM= SUM+DENS*DELZ
      IF(ABS(SUM)-ABS(AMAX))40,41,42
   42 IF(K-1)54,55-54
   55 DENS1=DENS
   54 R=ABS(AMAX)-SUM+DENS*DELZ
      DEPDIF(J)=DEPTH-SGN*(DEPDIF(J)+R/DENS1-DELZ)
      GO TO 30
   41 DEPDIF(J)=DEPTH-SGN* DEPDIF(J)
      GO TO 30
   44 DENS = DENSTY(I,K-1)
      DENS1=DENSTY(I,K-1)
      K=K-1
      GO TO 43
   30 CONTINUE
      PRINT 2000
 2000 FORMAT(1H-,43X,27H THICKNESS OF CRUSTAL LAYER /)
      PRINT 3000
 3000 FORMAT(17H DENSITY PROFILES // 2X,10H DEPTH(KM),5X, 9H DENSITY )
      DEPTH= -DELZ
      DO 3500 I=1, NPROF
      DEPTH= DEPTH + DELZ
      PRINT 3600,(DENSTY (I,J),J=1,IDENS)
 3500 CONTINUE
 3600 FORMAT (      10F9.3)
      PRINT 1966
 1966 FORMAT(8H-STATION,5X,19H GRAVITY(MILLIGALS),2X,10H DEPTH(KM),
     14X,21H DENSITY PROFILE USED)
      DO 17 I=1,NOSTA
      PRINT 4000,I,GRAV(I), DEPDIF(I),IPROF(I)
 4000 FORMAT(2X,I3,13X,F8.2,9X,F8.2,13X,I3)
   17 CONTINUE
      PAUSE 7
      END
```

INPUT DATA AND FORMAT FOR PRESID

CARD	FORMAT	DATA DESCRIPTION
1	I3	No. of stations at which residuals will be computed
2	16F5.1	Jeffreys-Bullen P-travel times in sec
3	20F4.1	Latitude corrections for converting to geocentric coordinates
4	I3	No. of epicenter locations for each station
5	F6.1	Station latitude (geocentric)
	F6.1	Station longitude
6	18F4.1	USGS origin times for earthquakes stored in sequence as HH:MM:SS.S
7	18F4.1	Record arrival times and store in sequence as HH:MM:SS.S
8	16F5.1	USGS focal depths in km
9	12F6.1	USGS latitude and longitude of epicenters stored in pair sequence Lat_1, $Long_1$, Lat_2, $Long_2$, ... Sign Convention East Long - West Long + North Lat + South Lat -

```
      DIMENSION QUALOC(200),ORTIM(300), DEPTH(100),STATIM(300),RESID(100
     1),XITEM(100), DELTA(100),AZIMUT(100), P(14,103),LOCOEL(100),GEOC(9
     11),GEQUA(200)
      READ 100, LIMSTA
100   FORMAT(I3)
      READ 150,((P(I,J),I=1,14),J=1,103)
150   FORMAT(16F5.1)
      READ 1112,(GEOC(J),J=1,91)
1112  FORMAT(20F4.1)
C     SET STATION INDEX
      DO 1  I=1,LIMSTA
      READ 200, LIMQUA
200   FORMAT(I3)
      IX=2*LIMQUA
      READ 700,STALAT, STALNG
700   FORMAT(F6.1,F6.1)
      IX2 = 3* LIMQUA
      READ 600, (ORTIM(J),J=1,IX2)
600   FORMAT (18F4.1)
      READ 400, (STATIM(J),J=1,IX2)
400   FORMAT(18F4.1)
      READ 500,(DEPTH(J), J=1,LIMQUA)
500   FORMAT(16F5.1)
      READ 300,(QUALOC(J),J=1,IX)
300   FORMAT(12F6.1)
      DO 1111 J=1,IX,2
      K=2
      ALAT =1.0
502   IF(QUALOC(J)-ALAT)565,501,501
501   ALAT =ALAT +1.0
      K=K+1
      GO TO 502
  565 CORR=GEOC(K) -(GEOC(K)-GEOC(K-1))*(ALAT-QUALOC(J))
1111  GEQUA(J) =QUALOC(J) - CORR
      DO 2 J=1,IX,2
      INDX=J/2+1
      STAANG = STALAT*3.1416/180.
      QUANG =GEQUA(J)*3.1416/180.
      POLANG = STALNG - QUALOC(J+1)
      IF (POLANG-180.)3,3,4
   4  POLANG= POLANG -360.
   3  POLANG =POLANG*3.1416/180.
      COSDEL = SIN(STAANG)*SIN(QUANG)+COS(   AANG)*COS(QUANG)*COS(POLANG)
      SINDEL=SQRT(1.-COSDEL**2)
      DELTA(INDX)=ATAN(SINDEL/COSDEL) *180./3.1416
      DELTA(INDX)= DELTA(INDX)+90. -SIGN(90.,DELTA(INDX))
      SINBET=COS(QUANG)*SIN(POLANG)/SINDEL
      COSBET=(SIN(QUANG)*COS(STAANG)-COS(QUANG)*SIN(STAANG)*COS(POLANG))
     1/SINDEL
      ASIN=ABS(SINBET)
      BETTA=ATAN(ASIN/COSBET)*180./3.1416
      BETTA= BETTA + 90.-SIGN(90.,BETTA)
      IF(SINBET -0.)82,84,83
   84 IF(COSBET -0.)85,83,83
   85 BETTA =180.
   83 AZIMUT(INDX)=360.-BETTA
      GO TO 81
   82 AZIMUT(INDX)=BETTA
   81 CONTINUE
2     CONTINUE
      M=1
      DO 12 J=1,LIMQUA
      K=2
      DEP = 33.0
      IF(DELTA(J)-102.) 13,13,14
13    IF(DEPTH(J)-DEP)15,15,16
16    DEP = DEP + 63.0
      K=K+1
      GO TO 13
15    L=2
      DIST = 1.0
19    IF(DELTA(J)-DIST)17,17,18
18    DIST=DIST + 1.0
      L=L+1
      GO TO 19
   17 DIF 1 = P(K,L)- (P(K,L) -P(K,L-1))*(DIST-DELTA(J))
      DIF 2 =P(K-1,L)-(P(K-1,L) -P(K-1,L-1))*(DIST -DELTA(J))
      IF(DEPTH(J)-  .0) 90,90,91
 90   THEOTM = DIF1 -(DIF1-DIF2) * (DEP-DEPTH(J))/33.0
      GO TO 92
 91   THEOTM = DIF1 -(DIF1 -DIF2)*(DEP-DEPTH(J))/63.0
  92  OTIME=ORTIM(M)*3600.+ORTIM(M+1)*60.+ORTIM(M+2)
      STIME = STATIM(M)*3600. + STATIM(M+1)*60.+STATIM(M+2)
      M=M+3
      IF(OTIME - 85562.6)20,4000,4000
 4000 IF(STIME - 85562.6) 21,20,20
21    STIME = STIME + 86400.
20    RESID(J)= STIME -OTIME - THEOTM
      GO TO 12
C     PKP BRANCH OF RESIDUAL TIME
 14   RESID(J)=0.
      M=M+3
12    CONTINUE
C     ORDER EPICENTERS BY DISTANCE
      XITEM(1)=DELTA(1)
      XITEM(2)=DELTA(2)
      LOCOEL(1)=1
      LOCOEL(2)=2
      DO 80 K=3, LIMQUA
      M=1
      L=1
50    IF(DELTA(K)-XITEM(L))22,22,23
```

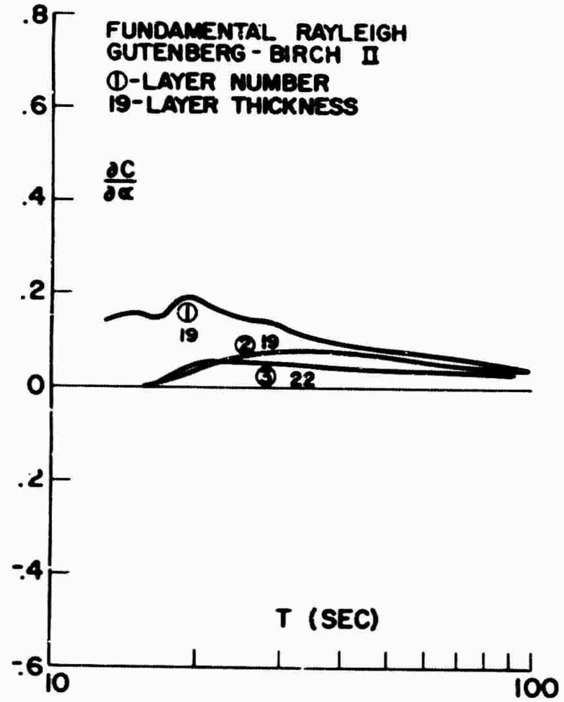
```
23     L=L+1
       IF (L-K)24,25,25
24     M=M+1
       GO TO 30
25     XITEM(L)=DELTA(K)
       LOCDEL(M+1)=K
       GO TO 80
22     IX1=K
51     XITEM(IX1)= XITEM(IX1-1)
       LOCDEL(IX1)=LOCDEL(IX1-1)
       IX1=IX1-1
       IF(IX1-M)27,27,51
27     XITEM(IX1)=DELTA(K)
       LOCDEL(IX1)=K
   80 CONTINUE
       PRINT 800
800    FORMAT(25H-RECORDING STATION COORD   /26X,9H LATITUDE,4X,10H LONGI
      2TUDE)
       PRINT 900, STALAT,STALNG
900    FORMAT(27X,F6.1,9X,F6.1)
       PRINT 1000
1000   FORMAT(/88H EPICENTER PARAMETERS ORDERED BY DISTANCE -AZIMUTH MEAS
      1URED COUNTER-CLOCKWISE FROM NORTH  //)
       PRINT 2000
2000   FORMAT(3X,9H DISTANCE,3X,9H LATITUDE,3X,10H LONGITUDE,3X,6H DEPTH,
      13X,8H AZIMUTH,3X,33H P-RESIDUAL(OBSERVED-THEORETICAL))
       DO 60 K=1,IX,2
       IX1=K/2+1
       L=LOCDEL(IX1)
       M= 2*L-1
       PRINT 3000,XITEM(IX1),QUALOC(M),QUALOC(M+1),DEPTH( ),AZIMUT(L),RES
      1ID(L)
 3000 FORMAT(5X,F7.2,5X,F6.1,7X,F6.1,7X,F6.1,7X,F6.1,12X,F7.1)
60     CONTINUE
C      ORDER EPICENTERS BY AZIMUTH
       XITEM(1)=AZIMUT(1)
       XITEM(2)= AZIMUT(2)
       LOCDEL(1)=1
       LOCDEL(2)=2
       DO 93 K=3,LIMQUA
       M=1
       L=1
 105  IF(AZIMUT(K)-XITEM(L)) 101,101,102
 102  L=L+1
       IF(L-K) 103,104,104
 103  M=M+1
       GO TO 105
 104  XITEM (L)=AZIMUT(K)
       LOCDEL(M+1)=K
       GO TO 93
 101  IX1=K
 107  XITEM(IX1)=XITEM(IX1-1)
       LOCDEL(IX1)=LOCDEL(IX1-1)
       IX1=IX1-1
       IF(IX1-M)108,10?,107
 108  XITEM(IX1)=AZIMUT( )
       LOCDEL(IX1)=K
  93  CONTINUE
       PRINT 5000
5000   FORMAT ( 25H-RECORDING STATION COORD.  /32X,9H LATITUDE,9X,10H LON
      1GITUDE)
       PRINT 5001,STALAT,STALNG
 5001 FORMAT(36X,F6.1,23X,F6.1)
       PRINT 5002
 5002 FORMAT(/87H ERICENTER PARAMETERS ORDERED BY AZIMUTH-AZIMUTH MEASUR
      1ED COUNTER-CLOCKWISE FROM NORTH    )
       PRINT 5003
 5003 FORMAT(//3X,8H AZIMUTH,3X,9H LATITUDE,3X,10H LONGITUDE,7X,6H DEPTH
      1,7X,18H DISTANCE(DEGREES) ,12X,33H P-RESIDUAL(OBSERVED-THEORETICAL
      1)  )
       DO 109 K=1,IX,2
       IX1=K/2 +1
       L=LOCDEL (IX1)
       M=2*L-1
       PRINT 5004,XITEM(IX1),QUALOC(M),QUALOC(M+1),DEPTH(L),DELTA(L),
      1RESID(L)
 5004 FORMAT(5X,F6.1,5X,F6.1,7X,F6.1,7X,F6.1,11X,F6.1,27X,F7.1)
 109  CONTINUE
       XITEM (1)=DEPTH(1)
       XITEM(2) = DEPTH(2)
       LOCDEL(1)=1
       LOCDEL(2)=2
       DO 207 K=3,LIMQUA
       M=1
       L=1
  202 IF (DEPTH(K)-XITEM(L)) 203,203,204
  204 L=L+1
       IF(L-K)205,206,206
  205 M=M+1
       GO TO 202
  206 XITEM(L)= DEPTH(K)
       LOCDEL(M+1)=K
       GO TO 207
  203 IX1=K
  209 XITEM(IX1)=XITEM(IX1-1)
       LOCDEL(IX1)=LOCDEL (IX1-1)
       IX1=IX1-1
       IF(IX1-M) 209,209,208
  208 XITEM(IX1) =DEPTH(K)
       LOCDEL (IX1)=K
  207 CONTINUE
       PRINT 801
  801 FORMAT (25 H-RECORDING STATION COORD. /26X,9H LATITUDE,4X,10H LONG
      1ITUDE)
       PRINT 802,STALAT,STALNG
```

```
802 FORMAT (27X,F6.1,9X,F6.1)
    PRINT 803
803 FORMAT (/85H EPICENTER PARAMETERS ORDERED BY DEPTH-AZIMUTH MEASUR
   1ED COUNTER-CLOCKWISE FROM NORTH //)
    PRINT 804
804 FORMAT(3X,6H DEPTH,3X,9H DISTANCE,3X,9H LATITUDE,3X,10H LONGITUDE,
   13X,3H AZIMUTH,3X,33H P-RESIDUAL (OBSERVED-THEORETICAL)
    DO 805 K=1,IX,2
    IX1=K/2+1
    L=LOCDEL(IX1)
    M=2*L-1
    PRINT 806,XITEM(IX1),DELTA(L),QUALOC(M),QUALOC(M+1),AZIMUT(L),
   1RESID(L)
806 FORMAT (5X,F6.1,5X,F6.1,7X,F6.1,7X,F6.1,7X,F6.1,27X,F7.1)
805 CONTINUE
  1 CONTINUE
    PAUSE 7
    END
```

APPENDIX IV

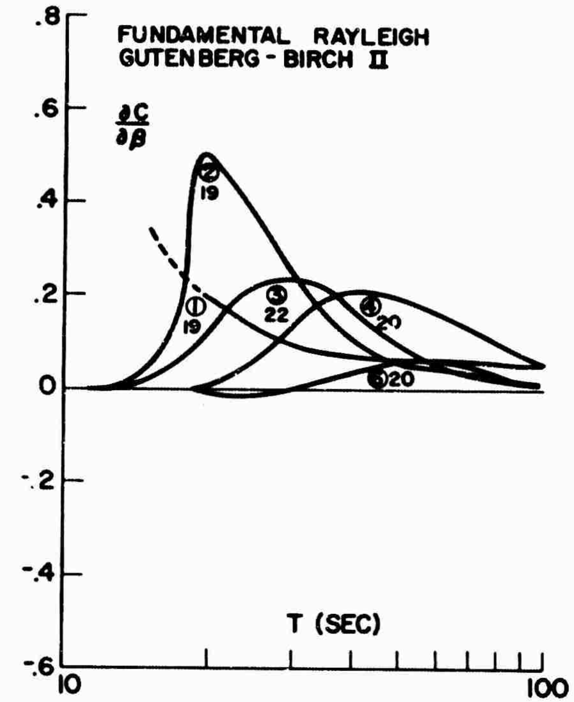
CURVES OF PARTICLE DISPLACEMENT

CURVES OF PARTIAL DERIVATIVES OF PHASE VELOCITY WITH RESPECT TO LAYER PARAMETERS



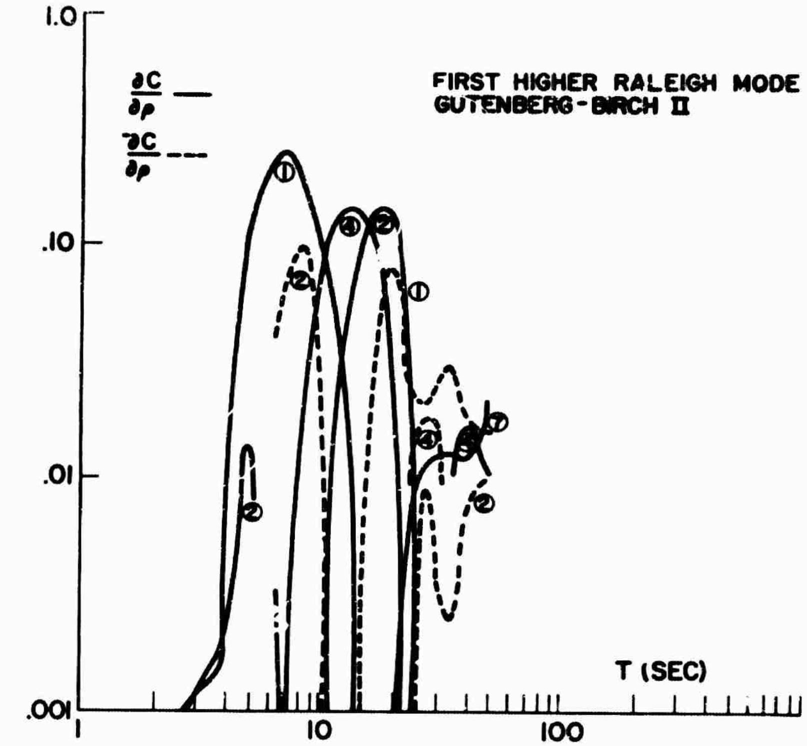
VARIATION OF PHASE VELOCITY WITH COMPRESSIONAL WAVE VELOCITY.

Fig. IV-1



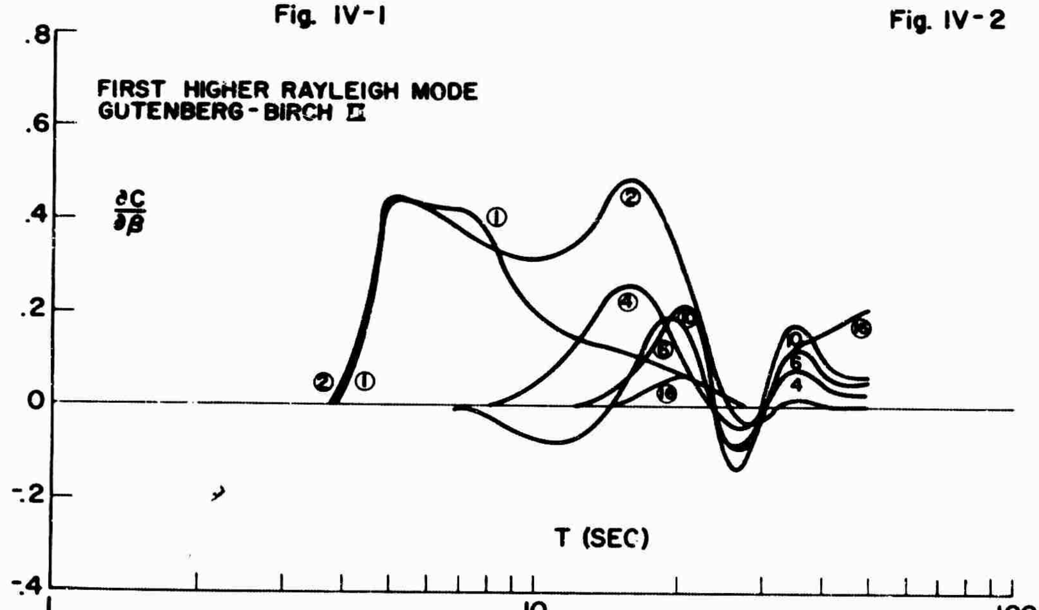
VARIATION OF PHASE VELOCITY WITH SHEAR WAVE VELOCITY.

Fig. IV-2



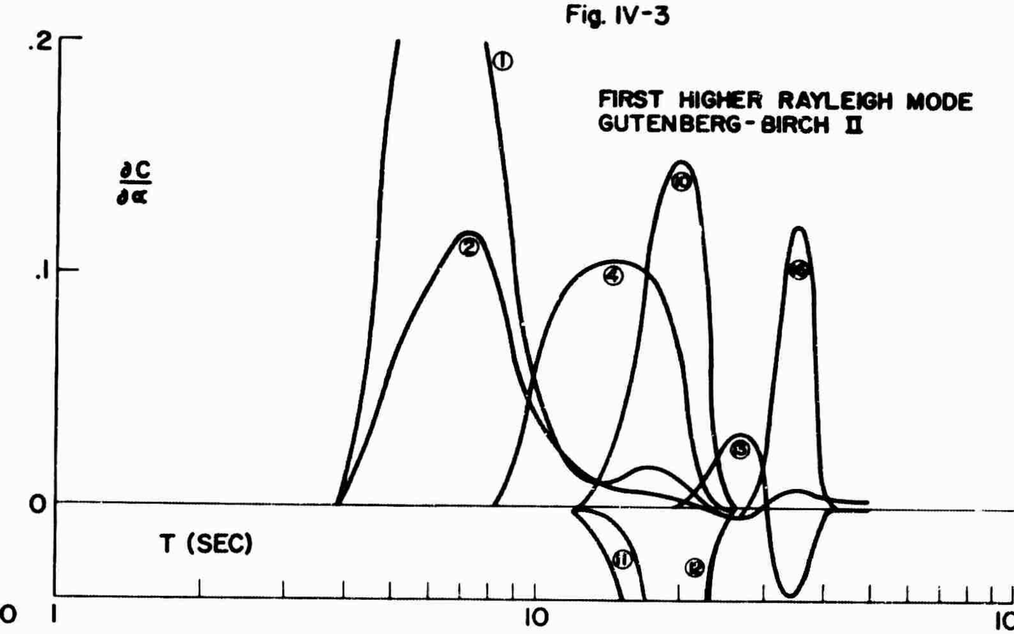
VARIATION OF PHASE VELOCITY WITH DENSITY.

Fig. IV-3



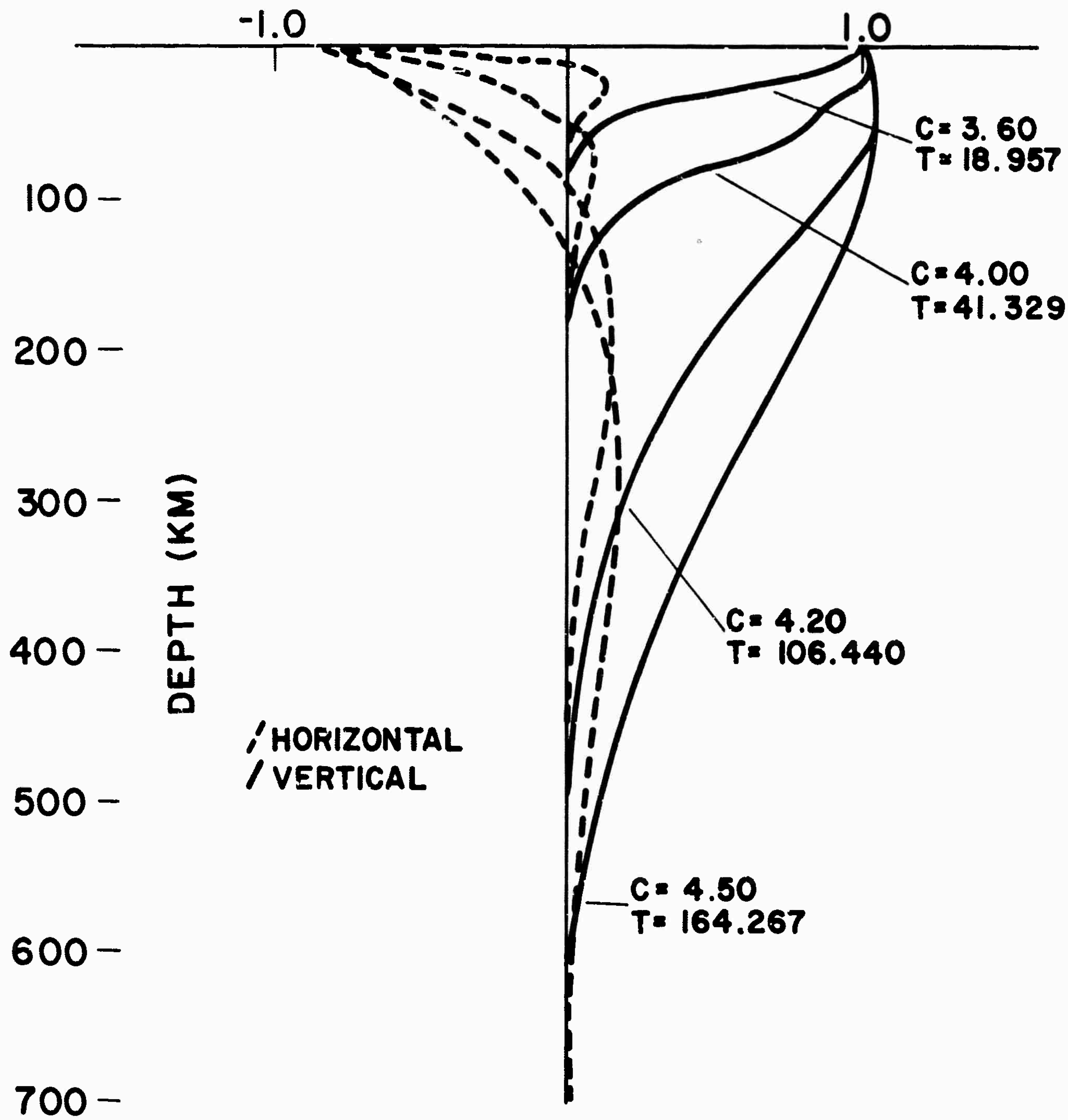
VARIATION OF PHASE VELOCITY WITH SHEAR WAVE VELOCITY.

Fig. IV-4



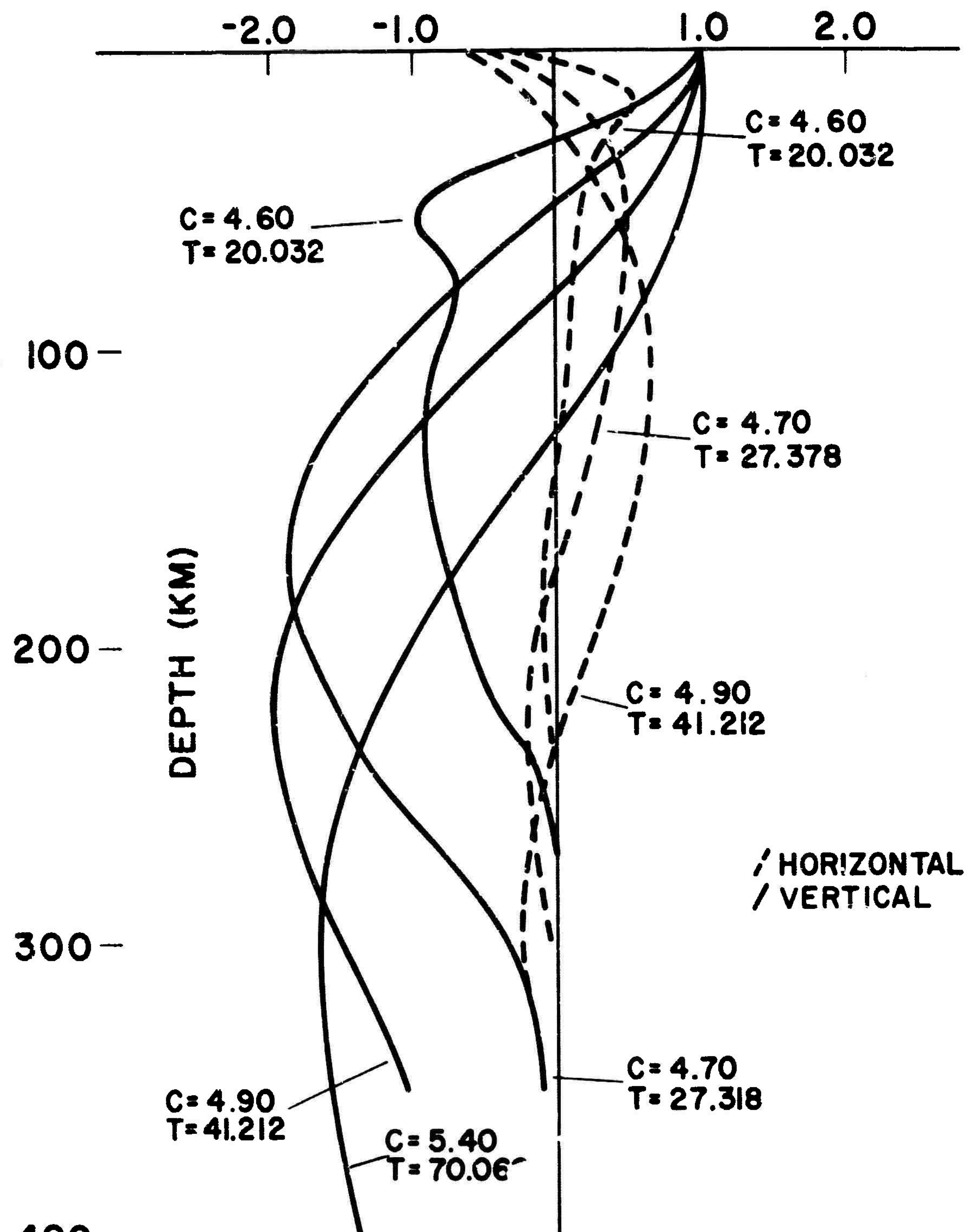
VARIATION OF PHASE VELOCITY WITH COMPRESSIONAL WAVE VELOCITY.

Fig. IV-5



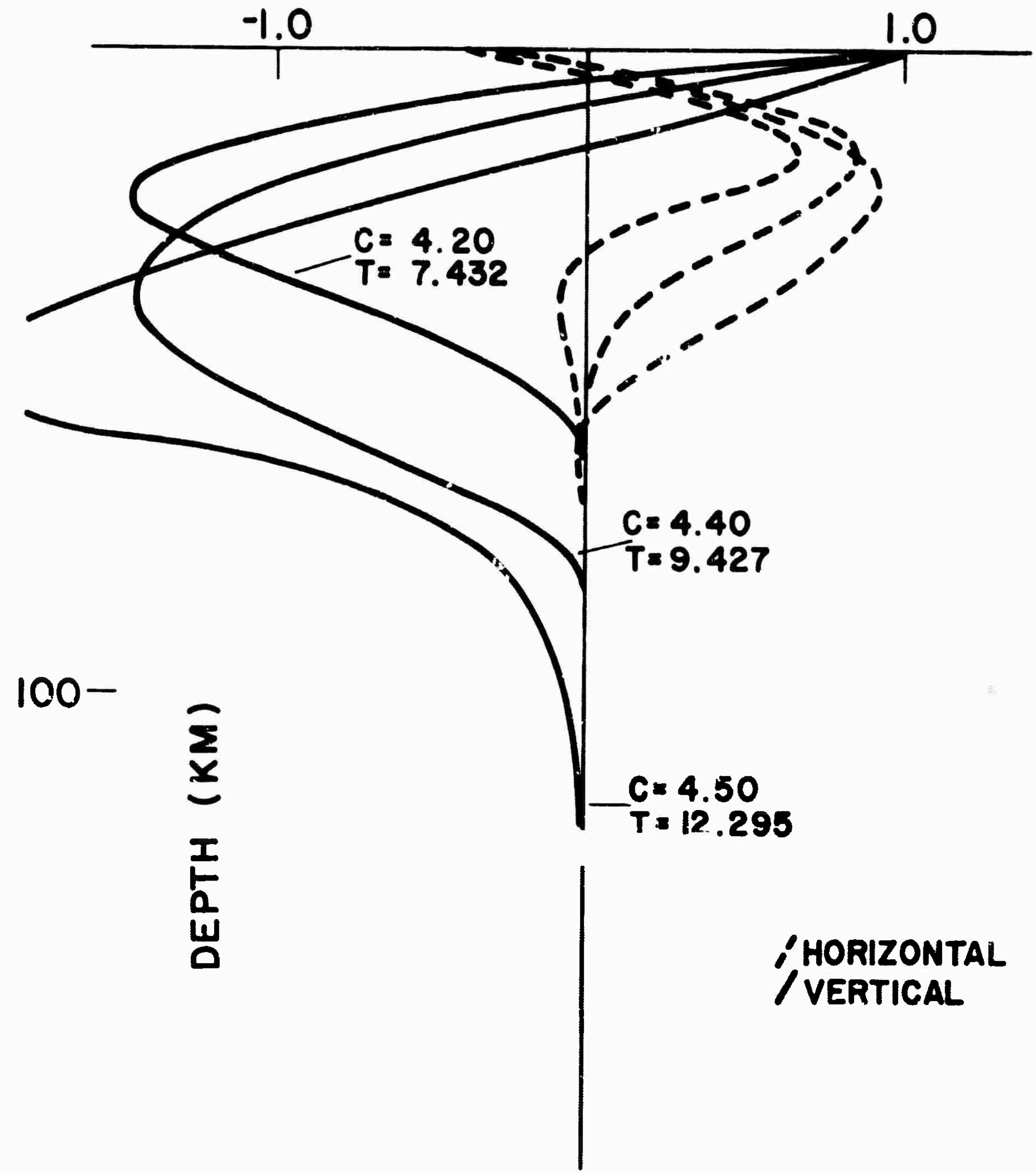
HORIZONTAL AND VERTICAL PARTICLE AMPLITUDES NORMALIZED TO THE VERTICAL AMPLITUDE AT THE SURFACE FOR FUNDAMENTAL RAYLEIGH MODE, GUTENBERG-BIRCH II MODEL.

Fig. IV-6



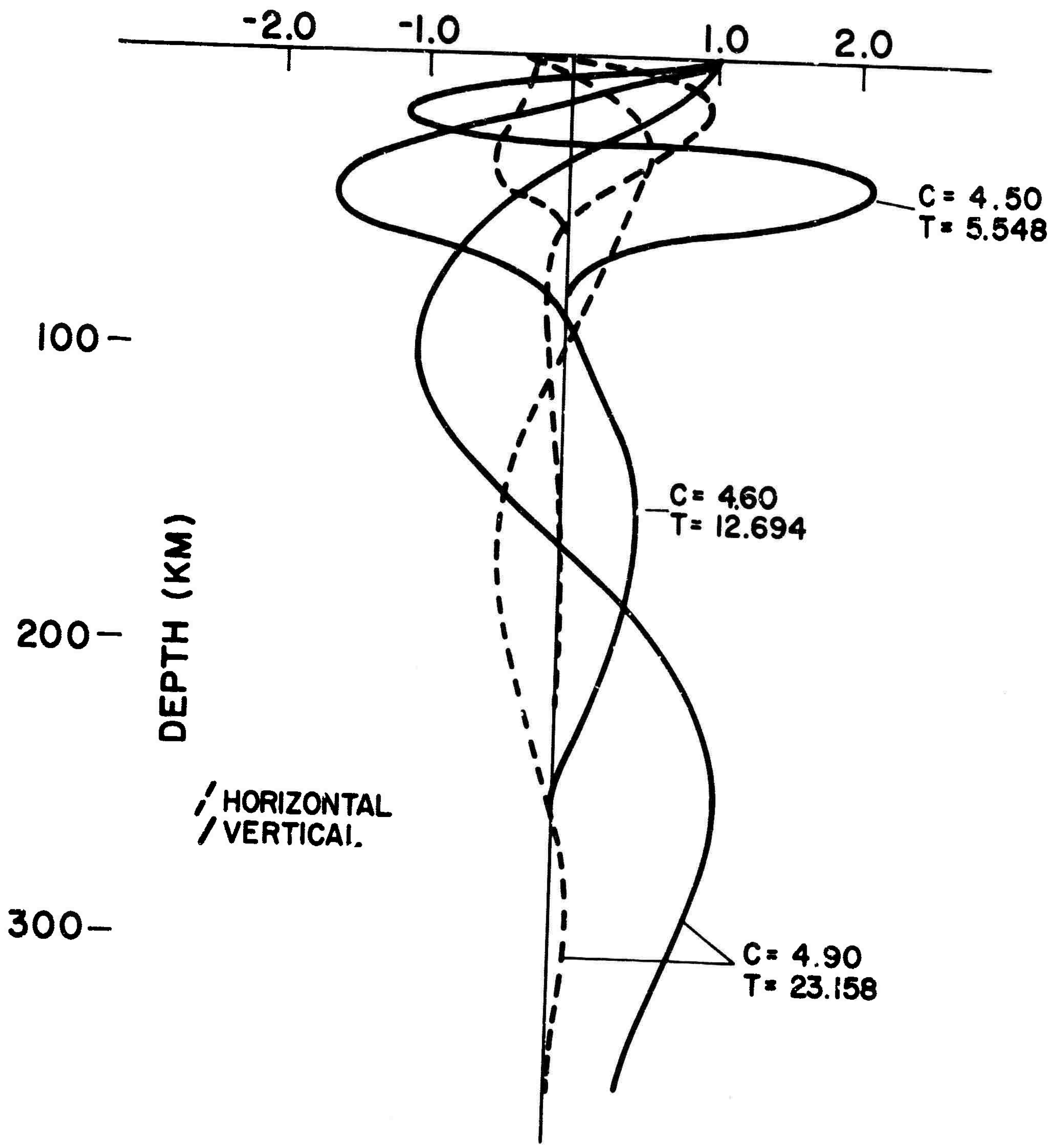
FIRST HIGHER RAYLEIGH MODE PARTICLE AMPLITUDES NORMALIZED TO THE VERTICLE AMPLITUDE AT THE SURFACE FOR THE GUTENBERG-BIRCH II MODEL.

Fig. IV-7



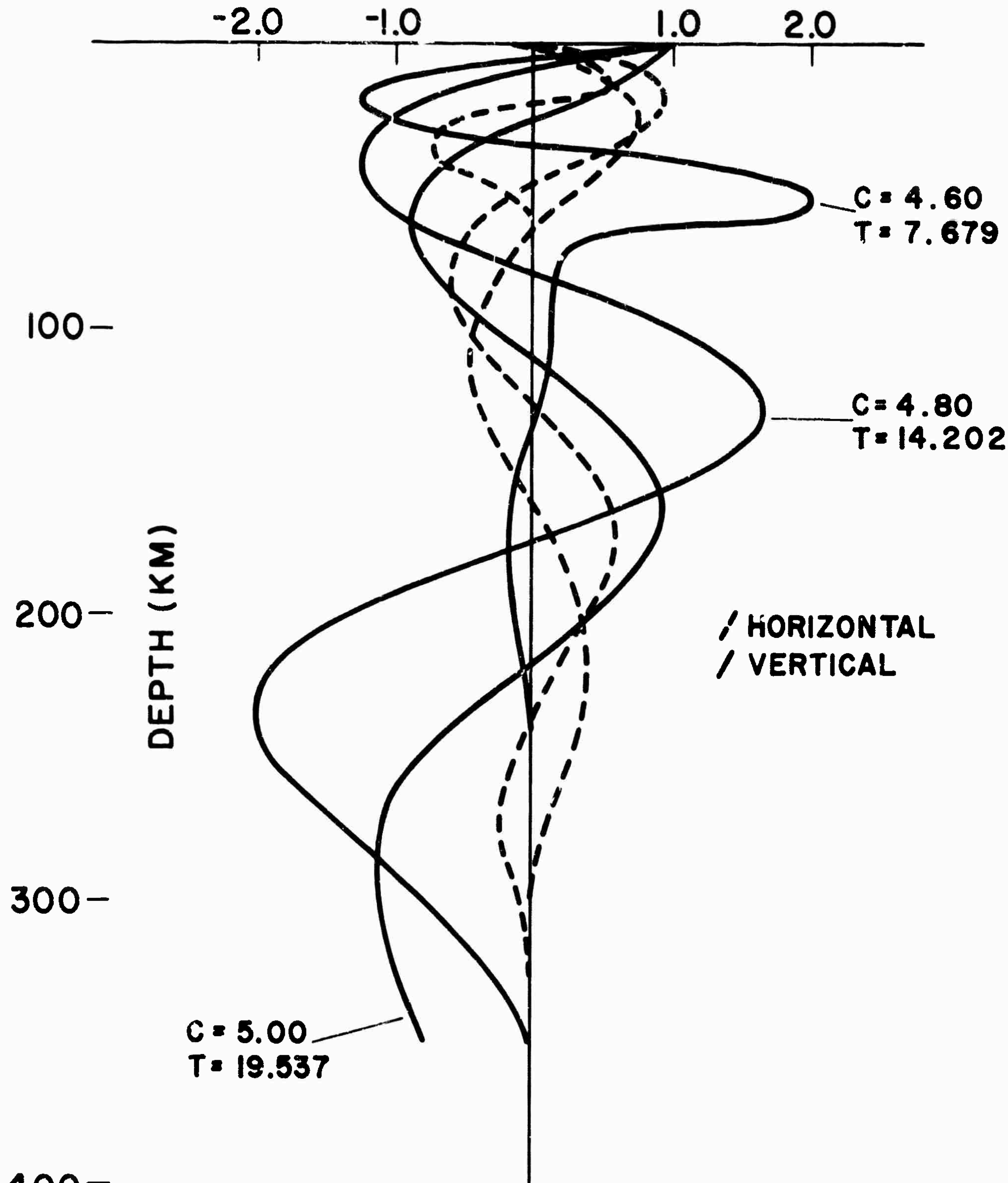
FIRST HIGHER RAYLEIGH MODE PARTICLE AMPLITUDES NORMALIZED TO THE VERTICAL AMPLITUDE AT THE SURFACE FOR THE GUTENBERG-BIRCH II MODEL.

Fig. IV-8



SECOND HIGHER RAYLEIGH MODE PARTICLE AMPLITUDES NORMALIZED TO THE VERTICLE DISPLACEMENT AT THE SURFACE FOR THE GUTENBERG - BIRCH II MODEL.

Fig. IV-9



THIRD HIGHER RAYLEIGH MODE NORMALIZED TO THE VERTICAL AMPLITUDE AT THE SURFACE FOR THE GUTENBERG-BIRCH II MODEL.

Fig. IV-10