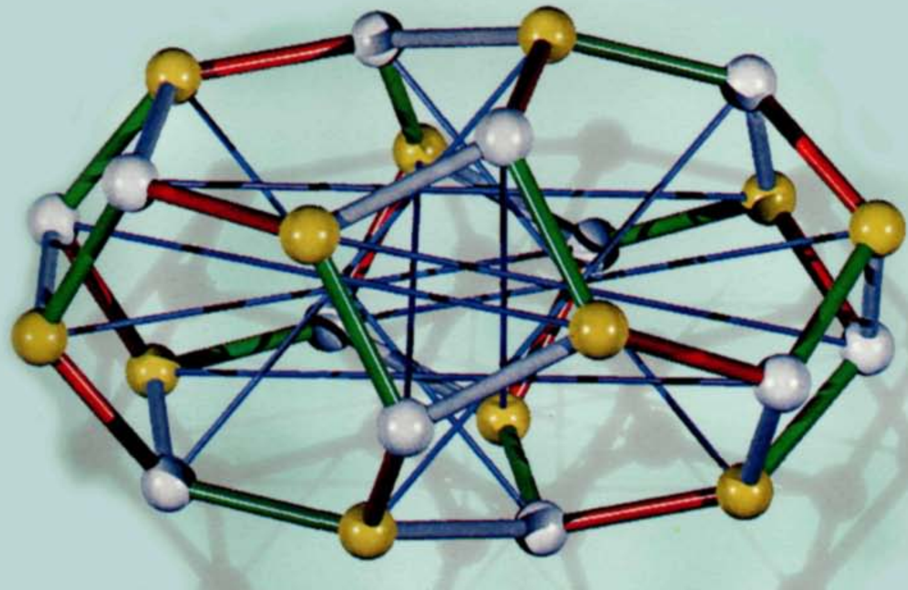
Series on Knots and Everything – Vol. 5

GEMS, COMPUTERS AND ATTRACTORS FOR 3-MANIFOLDS

Sóstenes Lins



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Gems, Computers and Attractors for 3-Manifolds

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World Scientific
Singapore • New Jersey • London • Hong Kong

Published by

World Scientific Publishing Co. Pte. Ltd.

P O Box 128, Farrer Road, Singapore 9128

USA office: Suite 1B, 1060 Main Street, River Edge, NJ 07661

UK office: 57 Shelton Street, Covent Garden, London WC2H 9HE

Library of Congress Cataloging-in-Publication Data

Lins, Sóstenes.

Gems, computers, and attractors for 3-manifolds / Sóstenes Lins.

p. cm. -- (K & E series on knots and everything ; vol. 5)

ISBN 9810219075

1. Three-manifolds (Topology) -- Data processing. I. Title.

II. Series.

QA613.2.L56 1995

514'.223--dc20 95-31016

CIP

British Library Cataloguing-in-Publication Data

A catalogue record of this book is available from the British Library.

This book is printed on acid-free paper.

Printed in Singapore by Uto-Print

Preface

Crystallizations (and more generally, *gems*) have been around for almost two decades. Personally I rediscovered them in 1979. Since this time I have been playing with these objects. *Gems* (*G*raph *e*ncoded *m*anifolds) encode closed manifolds by means of colored graphs. The prototype for realizing this encoding arises by taking the dual of the barycentric subdivision of a cell complex whose underlying topological space is a closed n-manifold. The 1-skeleton of such an object is an $(n+1)$-regular graph whose edges can be *properly colored* with $n+1$ colors.

The construction is simple, but by itself has a serious drawback. The size of the gems become unecessarily big. This may cause concern among combinatorial and geometric topologists. Why care about objects that seem to carry an intrinsic redundancy? Why care about objects which are difficult to manipulate and hard to remember?

In this book, I try to show that 3-gems are worthwhile objects. In the first place, gems are very easy to manipulate with a computer because they are purely combinatorial and not topological objects. However, they do encode the relevant topology. From the data structure viewpoint they are (basically) graphs, one of the simplest of such structures. The isomorphism problem between gems can be efficiently solved (see Section 2.6) and, thus, computers *"remember"* them very well. By far the most important of their properties is that they have a rich internal simplification theory and a companion horizontal set of moves (which do not change the size of the gem) providing a combinatorial dynamics that makes them ideal for the explicit recognition of 3-manifolds.

In this respect they have been accomplishing what no other theory of 3-manifolds has been able to do: a successful computational classification theory. Indeed, the homeomorphism problem for orientable 3-manifolds induced by 3-gems up to 30 vertices has been solved. The techniques employed seem to work further and suggest a possible necessary and sufficient condition for two closed 3-manifolds to be homeomorphic: *iff any two 3-gems inducing them are linked by a u_*^1-move.* There is a finite number of such moves and one can try them all. See Section 0.5.

The 3-gems also behaves as bridges among other combinatorial presentations of 3-manifolds: Heegaard diagrams, facial identifications schemes, surgery descriptions, etc. These presentations can be comfortably transformed into a 3-gem whose rich combinatorial dynamics has so far provided us via computer (the TS_ρ^U-algorithm of Chapter 4) with one of the smallest possible 3-gems inducing the same 3-manifold. The 3-manifold is recognized at once, if it is in the realm of our tables or if it comes from a known topological construction.

The *attractor for* M^3 is defined as the set of all minimum 3-gems inducing M^3. My main discovery along these years is that the attractors seem to have the following nice computational properties: (i) if a 3-gem is not in an attractor, it can be simplified to one inducing the same 3-manifold; (ii) if two 3-gems are in an attractor, they are linked by moves which can be effectively found. Unfortunately, up to now, these properties seem difficult to prove in general. However, evidence from the data gathered so far has motivated me to state (i) and (ii) as the *Strong Conjecture on 3-Gems* (Section 0.5).

Many well-known 3-manifolds have an attractor formed only by a single 3-gem. In this case the 3-gem is named the *superattractor for the 3-manifold.* In Section 6.7 I conjecture that each *"plane manifold"* arising from a 3-connected plane graph has a superattractor (Conjecture 7). The set of such manifolds constitutes an infinite and highly structured class of closed oriented 3-manifolds: the 2-fold branched coverings over S^3 whose branch set is an alternating link. Another infinite family of manifolds whose members seem to have superattractors is the one formed by the Cartesian products of a circle by an orientable surface of genus n (Fig. 46, Conjecture 3).

Being grounded in such a small number of postulates and having such an easily implementable flavor the theory that is presented in this book suits well for further computer explorations. I do believe that it has something new to say most of which was not yet found. I also hope that it sparks the interest of some young topologists or combinatorialists who are computer oriented. They can quickly learn this material and be enchanted, as I was, by the mistery of why 3-manifolds are so complicated. This book puts a definite order in the simplest orientable closed 3-manifolds (from the point of view of 3-gems): it classifies them! However, a general solution seems out of reach by this simple minded approach. Nevertheless, the good behavior of the big (and complete) chunk of data that have been dealt with cannot be disregarded. Maybe, in an effort of extending the computations to disprove the Strong Conjecture some of those young people may find theoretical ways to deal with the general *Weak Conjecture on 3-Gems* (stated in Section 0.5).

Acknowledgements: It gives me great pleasure to acknowledge the following people who helped me on various aspects related to the research and the production of this

book. They appear, approximately, in chronological order of interaction.

Daniel Younger	Arnaldo Mandel	Getúlio Katz	Said Sidki
Manoel Lemos	Massimo Ferri	Carlo Gagliardi	Cassiano Durand
Nadja Lins	Paul Schweitzer	Cleide Martins	Louis Kauffman
Jeffrey Weeks	Dale Rolfsen	Marcus Wanderley	Oscar Pereira
Marco Carnut	Alexandre Brainer	Michel Boileau	Michele Mulazzani
Fernando Souza	Pedro Cruz	Jorge Stolfi	Ramon Mendoza

Of these people I want to mention six to whom the work contained in this book owes a lot: Massimo Ferri and Carlo Gagliardi, for proving their theorem on the factorization of homeomorphism between closed manifolds into simple combinatorial moves (the dipole moves). This is the basic stone for the whole theory. My former student Cassiano Durand, for his implementations of the routines generating the catalogue and of the TS_ρ-algorithm. Said Sidki for his great expertise in combinatorial group theory, making sense of very raw combinatorial data. The computations of Section 5.3 are due to him. My student Pedro Cruz, who developed the mass-spring heuristic which attains the 3-dimensional drawings of the gems (which we use freely along the book). He also implemented the ray-tracing routine which is responsible for the colored picture in the cover of the book, namely, *the superattractor for Poincaré's homology sphere*. To Lou Kauffman my gratitude for his encouragement of my work along the last ten years.

I also thank Ms. H. M. Ho, from World Scientific, for her patience with my delay of almost two years in the preparation of this book. The financial supports of UFPE, CNPq (process number 30.1103/80) and FINEP were essential to the development of this project.

Recife, Brazil **S.L.L.**
July 1995

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Contents

Gems, Computers and Attractors for 3-Manifolds

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Chapter 0

Introduction

This chapter discusses the motivations and provides an overview of the central issues treated in the book.

An $(n+1)$-*graph* is an edge-colored finite graph, regular of degree $n+1$, such that the edges incident to each vertex receive distinct colors and where the total number of colors painting the edges is also $n+1$. The colors are named $0, 1, \ldots, n$ and are depicted by the corresponding number of marks in the figures.

The $(n+1)$-graphs are our basic data structure. In spite of their simplicity, they permit the construction and convenient manipulation of PL n-manifolds. We restrict our focus to dimensions $n = 2$ and $n = 3$.

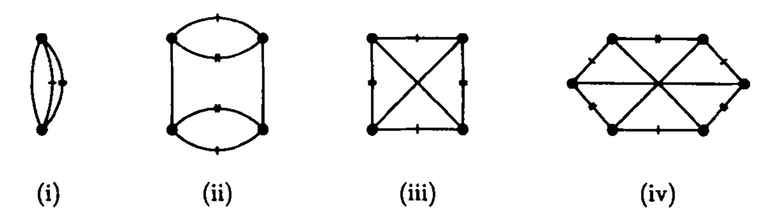
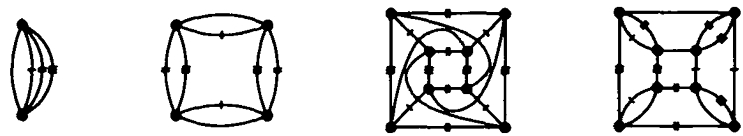


Fig. 1: Examples of $(2+1)$-graphs

The $(2+1)$-graphs are also named *2*-dimensional *g*raph-*e*ncoded *m*anifolds or *2-gems*. We shall see how these objects encode closed compact surfaces (Section 1.1) and how they enable a topological classification of them (Section 3.1).



(i) (ii) (iii) (iv)

Fig. 2: Examples of $(3+1)$-graphs

A subclass of the $(3+1)$-graphs encoding closed compact 3-manifolds named *3-gems* (defined in the next section) is our central object of study. All the above examples of $(3+1)$-graphs are 3-gems and they induce, respectively, S^3, S^3, RP^3 and $S^1 \times S^2$.

0.1 Motivations and Objectives

In this book we provide a combinatorial approach to the theory of 3-manifolds, focused on solving the homeomorphism problem for the manifolds formed by pasting a small number of "colored" tetrahedra. The choice of using only combinatorial techniques may seem an anacronism: 3-manifolds are too complicated to be dealt by these simple-minded techniques. However, now there is a factor that justifies such point of view: namely, the widespread use of computers.

We treat each homeomorphic class of 3-manifold as an equivalence class of edge-colored graphs named *3-gems*. An *n-residue* $(0 \leq n \leq 3)$ in a $(3+1)$-graph G is a connected component of a subgraph of G induced by all the edges of n chosen colors. Thus the 2-residues are *bicolored polygons* in G, also called *bigons*. A 3-residue which does not use color k is also called a $\hat{k}$-residue. A *3-gem* is a $(3+1)$-graph G in which $v+t=b$, where v is the number of vertices, b is the number of 2-residues and t is the number of 3-residues, all relative to G. It follows easily from the Triangulation Theorem for 3-manifolds of Moise [Moi52] that every closed compact 3-manifold can be induced by a 3-gem. See Theorem 2.

The equivalence classes (which are the combinatorial counterpart of homeomorphism among 3-manifolds) are generated by two simple moves due to Ferri and Gagliardi [FG82]:

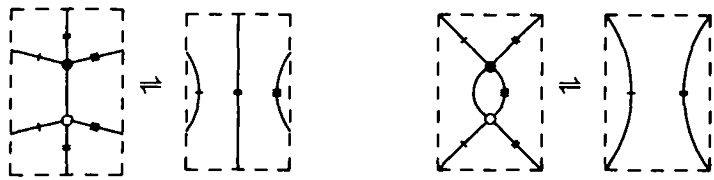


Fig. 3: 1-Dipole and 2-Dipole Moves

We suppose, in the first picture, that the two vertices are in distinct $\hat{0}$-residues and in the third one that they are in distinct 12-gons. Under this hypothesis, the configurations are called an *1-dipole involving color 0* and a *2-dipole involving colors 0 and 3*, respectively. There are, of course, four types of 1-dipoles and six types of 2-dipoles. The *involved colors* are the colors of the i edges linking the two vertices which form an i-dipole.

After the Ferri-Gagliardi Theorem, the problem of deciding whether two given 3-manifolds are homeomorphic becomes the one of deciding whether two 3-gems inducing them are *linked* (i.e. transformable one into the other) by a finite number of cancellations and creations of 1- and 2-dipoles. Of course this is, still, an exceedingly difficult problem, which is not satisfactorily solved even if one of the manifolds is the 3-sphere.

The central points of this book are to develop the theory of 3-gems from scratch, to show that it has enough internal structure to provide a topologically complete catalogue of the *simplest* 3-manifolds and to accomplish the corresponding topological classification.

In dual form 3-gems are just colored triangulations, i.e., the 3-manifold is dissected into a collection of tetrahedra whose vertices are painted with four colors such that each tetrahedron has four differently colored vertices. Indeed, given a 3-gem G with n vertices, to form the corresponding 3-manifold start with n disjoint tetrahedra in 1-1 correspondence with the vertices of G. The four vertices of each tetrahedron are labelled with colors $0, 1, 2, 3$. If vertices v and w of G are linked by an edge of color i, identify the boundaries of the tetrahedra corresponding to v and w along the face that misses color i such as to match the other three colors. If G is a 3-gem, the resulting quotient space is a 3-manifold. This is what we name *dual construction*. There is a primal construction yielding the same manifold. We treat this construction in Chapter 1.

Orientable 3-manifolds induced by 3-gems up to 28 vertices are generated, displayed and classified in this book. Except for computing time and memory requirement, the methods developed could go on. Recently, in a joint work with C. Durand and S. Sidki, we have extended the classification of the 3-gems with 30 vertices. *A* *general question arises: do the techniques employed break down at some level? If they never break, this would imply a general algorithm to classify closed compact 3-manifolds.*

The classification was achieved by the computational possibility of identifying the *attractor* for each of the 3-manifolds involved. We define *the attractor of* M^3 as the set of *all* 3-gems inducing M^3 and having the minimum number of vertices. Clearly, the attractor exists and is unique, being formed by a finite number of 3-gems.

One nice structural property of the attractors which was observed for the manifolds that we have studied so far is the following: given two 3-gems in one of these attractors they are linked by a few types of moves *which do not increase the number of vertices*. This implies that the whole set of 3-gems in the attractor can be obtained from one of its members. The fact that the size of the objects (in the case the number of vertices) do not increase is in contrast with the unlimited need of refinements in combinatorial topology. These refinements increase the size of the objects in an explosive way. This is unimportant for classical combinatorial topology. However, not increasing these sizes is our main issue in 3-gem theory because we aim at a monotone simplification theory.

0.2 Attractor at Work: the Case of Poincaré's Homology Sphere $S^3/\langle 5,3,2\rangle$

The reason for the terminology *attractor for* M^3 is the empirical fact that this set is the output of a combinatorial simplifying dynamics that starts with any 3-gem inducing M^3. This was observed for all the manifolds induced by 3-gems in the realm of our catalogue and for many others. Often the attractor is formed by a unique 3-gem, which in this case is called *the superattractor* for the associated 3-manifold. Poincaré's original homology sphere, the spherical dodecahedral space, $S^3/\langle 5,3,2\rangle$, has a superattractor. This notation is used because this space is the quotient of the tridimensional sphere S^3 (seen as the topological group of unit quaternions) by the the binary dodecahedral group $\langle 5,3,2\rangle = \langle a,b,c \mid a^5 = b^3 = c^2 = abc\rangle$. S^3 admits an action based on a subgroup isomorphic to $\langle 5,3,2\rangle$. This group is the fundamental group of the space and has 120 elements. It is the only finite *perfect group* known. A group is *perfect* if it is generated by its commutators. Hence the 3-manifold has the same homology as S^3 and is called a homology sphere. It was the first homology sphere to be discovered.

All sort of interesting manifestations of $S^3/\langle 5,3,2\rangle$ are easily transformed into a 3-gem inducing the same 3-manifold. By feeding these 3-gems to the simplification algorithm (named TS_ρ-algorithm), it invariably converges to this superattractor. It follows a list of examples of the phenomenon. The first description is given by the quotient of a solid dodecahedron by pasting pairs of opposite faces as indicated below by small lines with a distinguished end (a small black circle).

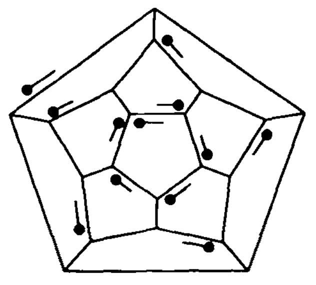


Fig. 4: Description of $S^3/\langle 5,3,2\rangle$ as a facial identification scheme

The opposite faces are identified so as to match these lines and their corresponding black-circle ends. From this scheme it is possible to obtain, in a natural way, a 3-gem with 120 vertices also inducing $S^3/\langle 5,3,2\rangle$. See Section 1.3 for a general construction of this type.

Fig. 5: $S^3/\langle 5,3,2\rangle$ given by a framed link and by a charged string

Surgery instructions over a *framed link* (link with a rational coefficient attached to each of its components — see [Rol76]) in S^3 is one of the most elegant and useful ways to describe a 3-manifold. The description on the left is taken from page 310 of [Rol76] and it induces $S^3/\langle 5,3,2\rangle$. From it we get in a straightforward way a 3-gem with 336 vertices inducing the same space which is absorbed by the superattractor. See Section 2.9 for the general construction of 3-gem from a framed link.

The description on the right is named a *(charged) string presentation.* It is a coded form of some K_4-symmetric 3-gems, named *σ-gems,* which we discuss in Section 2.5. The above charged string is in fact a 3-gem with 32 vertices absorbed by the superattractor for $S^3/\langle 5,3,2\rangle$.

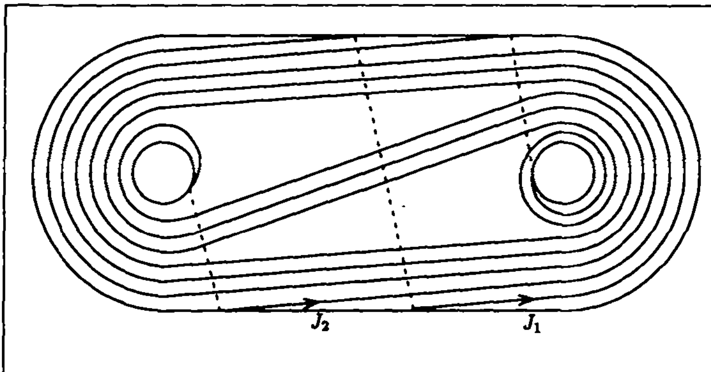


Fig. 6: Genus 2 Heegaard diagram inducing $S^3/\langle 5,3,2\rangle$

Above we show a Heegaard diagram inducing $S^3/\langle 5,3,2\rangle$ that was taken from page 19 of [Hem76]. A 3-gem with 40 vertices H also inducing $S^3/\langle 5,3,2\rangle$ is easily obtained from this diagram (see the next section). H is absorbed by the superattractor for $S^3/\langle 5,3,2\rangle$.

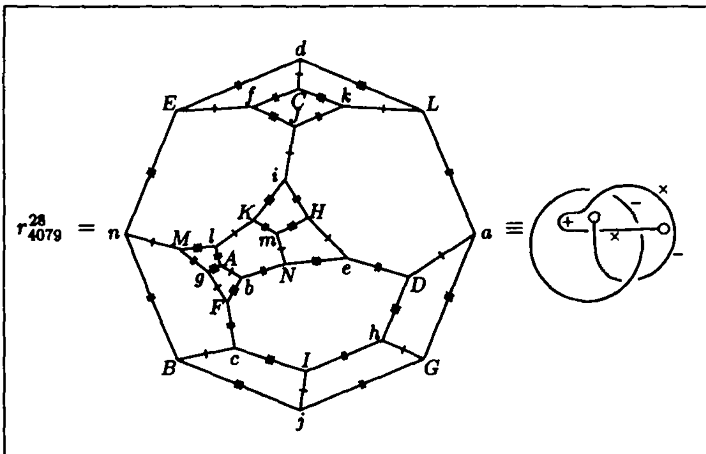


Fig. 7: The smallest σ-gem inducing $S^3/\langle 5,3,2\rangle$

This 3-gem is the 4079^{th} *rigid 3-gem* (see Section 0.4) with 28 vertices in the catalogue at the Appendix A. In general, the $i-th$ rigid 3-gem with n vertices is denoted r^n_i. See Tables 4A — 4F in Section 5.1. We note that for the 3-gems presented, as above, with labels in the vertices and without edges of color 0, these are implicitly given by a pair of labels of type xX.

$\mathcal{S}(5,3,2,1) = r^{30}_{24555} =$

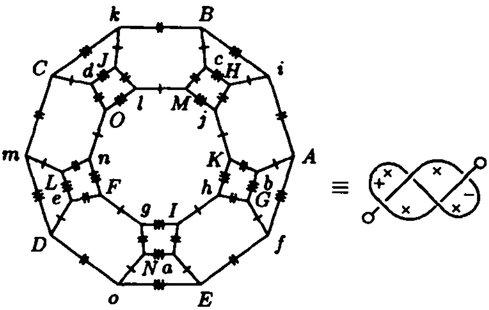


Fig. 8: A σ-gem with 30 vertices in the family $\mathcal{S}(b,l,t,c)$

This 30-vertex 3-gem also induces $S^3/\langle 5,3,2\rangle$ since it is absorbed by r_2^{24}. See Section 2.2, where we introduce the family $\mathcal{S}(b,l,t,c)$.

$\mathcal{S}(3,5,4,1) = r^{30}_{54668}$

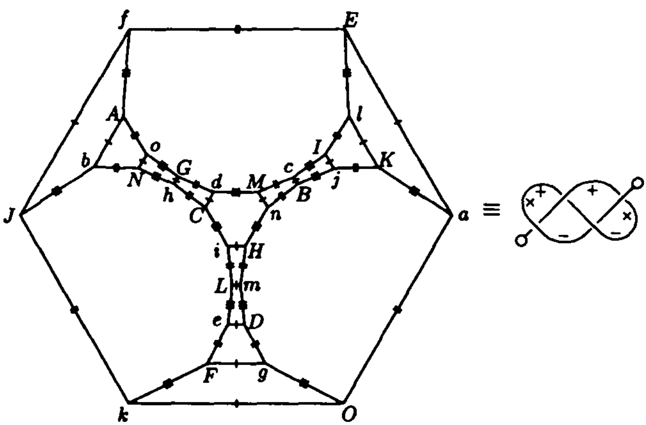


Fig. 9: Another σ-gem with 30 vertices in the family $\mathcal{S}(b,l,t,c)$

Once more, the above 3-gem induces $S^3/\langle 5,3,2\rangle$: it gets absorbed by r_2^{24}.

Fig. 10: Blackboard framed link and blinks inducing $S^3/\langle 5,3,2\rangle$

The left picture is a description of $S^3/\langle 5,3,2\rangle$ as surgery instructions over a blackboard framed link in S^3 (and associated blink). This description is the simplest possible. There is an associated 3-gem with 60 vertices that gets attracted by the superattractor. See the next section to get a 3-gem from a blackboard framed link. A *blink* is a plane graph with a bipartite coloration of its edges. These objects are in 1-1 correspondence with blackboard framed links — see Section 2.9. From the second blink we get an associated 3-gem with 84 vertices that gets attracted by r_2^{24}.

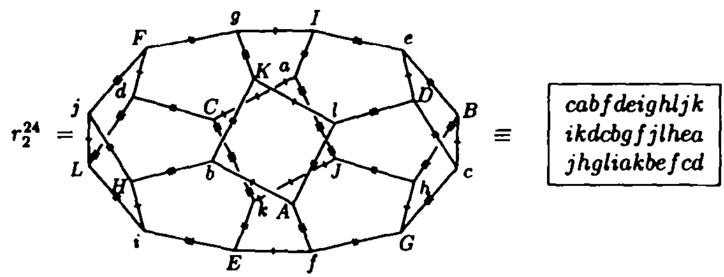


Fig. 11: Superattractor and respective code for $S^3/\langle 5,3,2\rangle$

This 24-vertex 3-gem r_2^{24} absorbs, under the simplification TS_ρ-algorithm given in Chapter 4, all the manifestations of $S^3/\langle 5,3,2\rangle$ that have been tested.

We finish this section by emphasizing that *there is nothing special about the 3-manifold $S^3/\langle 5,3,2\rangle$. Indeed, to illustrate the point of this section, it could have been replaced by any one of the 100 orientable 3-manifolds which are induced by 3-gems up to 28 vertices. That is, the attractors for all these 3-manifolds really attract them! This is a useful empirical phenomenon which deserves and awaits an appropriate theoretical explanation.*

0.3 Translating from Other Presentations into 3-Gems

We discuss the way to get a 3-gem from a facial identification scheme in Section 1.3 and to get the 3-gem associated with a string presentation in Section 2.5. Here we briefly discuss how to get a 3-gem from a Heegaard diagram and from a blackboard framed link. These discussions are resumed in more detailed form latter on. See Sections 2.7 and 2.9.

We examplify one way to get a 3-gem from Heegaard diagram (in a canonically embedded genus n handlebody) using the example on Fig. 6. Start by drawing the

$(n + 1)$ canonical meridian curves:

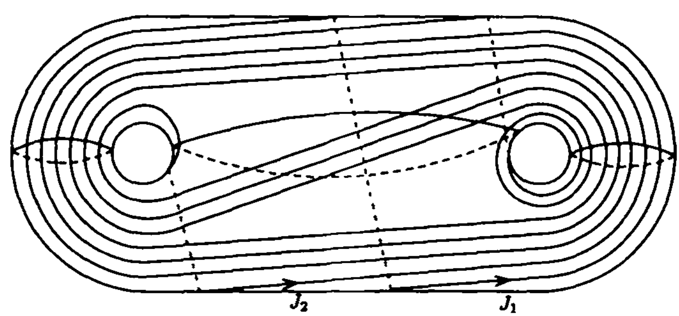


Fig. 12: First step in obtaining a 3-gem from a Heegaard diagram

Note that cutting along these meridian disks one splits the handlebody into an upper and a lower balls having $(n + 1)$ disks in common. Draw, in the surface, a parallel to each of the original curves. After that, each meridian curve meets the original curves in an even number of crossings. Color alternately with 1 and 2 the segments of the meridian curves between crossings in such a way that a 1-colored segment links two parallel curves. Color with 3 the segments of the original curves in the upper part of the handlebody and with 0 the ones in the lower part. The $(3 + 1)$-graph whose vertices are the crossings and whose edges are the colored segments is a 3-gem inducing the same manifold as the Heegaard diagram. In the above example, before the doubling of the curves there are $6 + 7 + 7 = 20$ crossings. Thus the 3-gem formed has 40 vertices. See Section 2.7 for more details on this and an alternative construction in the *complementary handlebody*. This construction is more natural but it only applies to handlebodies embedded in S^3 such that its complement is also a handlebody.

Now we provide a general recipe to go from a blackboard framed link with n crossings to a 3-gem with $12n$ vertices inducing the same 3-manifold. Each crossing is replaced by a fixed partial 3-gem, as shown below. To get the corresponding replacement for the other crossing rotate the crossing and the partial 3-gem of 90 degrees or, alternatively, reflect the whole picture in a vertical (or horizontal) mirror, keeping the colors of the partial 3-gem.

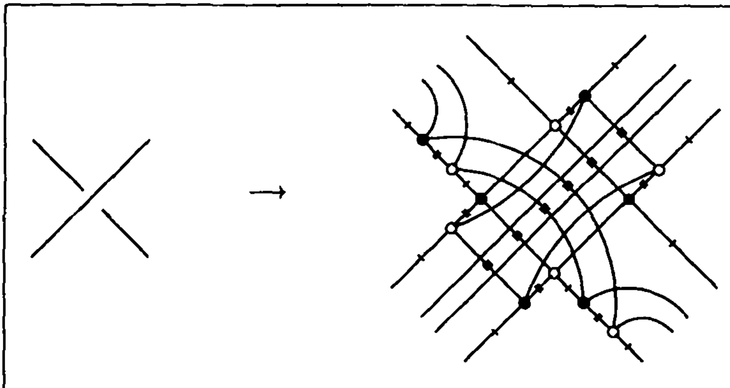


Fig. 13: From blackboard framed link to 3-gem

This is an essential construction because it provides an immediate bridge between blackboard framed links and 3-gems. The construction embodied by the above figure has been implemented and is crucial for the recognition of manifolds from blinks given in Section 7.2.

0.4 The Elements of the Simplifying Dynamics

We give a brief presentation of all the operations that we employ to achieve the simplifying dynamics. These operations are discussed with details in Section 4.1.

0.4.1 1-Dipole Cancellation, ρ-Pair Switching and ρ-Move

Beyond looking for 1-dipoles in order to cancel them, our simplifying dynamics look for *ρ-pairs*. A *ρ-pair* in a $(3+1)$-graph is a pair of equaly colored edges that appear together in exactly 2 or 3 bigons. The *switching of a ρ-pair* is the passage from a gem G to a $(3+1)$-graph G' obtained by replacing $\{a_1, a_2\}$ by new edges $\{a'_1, a'_2\}$ having the same ends and preserving the $(\circ, \bullet)$-bipartition (this coloring can be locally defined even if G is not bipartite — see Subsection 2.3.2):

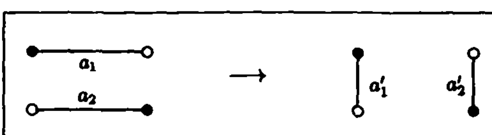


Fig. 14: Switching a ρ-pair

A ρ-pair which is in i bigons, $(i = 2, 3)$, is called a ρ_i-pair. The result of switching a ρ-pair $\{a, b\}$ in a gem G is another 3-gem, denoted $G^{swt}_{a,b}$. The switching of a ρ-pair causes the appearance of 1-dipoles and so, smaller 3-gems inducing the same manifolds (up to connected sums with $S^1 \times S^2$, in the case of ρ_3-pairs — see Proposition 20). Thus we may suppose 3-gems with dipoles or with ρ-pairs as irrelevant and concentrate in 3-gems without them. These are named *rigid 3-gems*. A *ρ-move* is either the cancellation of a 1-dipole or else the switching of a ρ-pair (which creates 1-dipoles) followed by the cancellation of a 1-dipole.

0.4.2 The TS-Configurations and the TS-Moves

If a 3-gem is rigid our simplifying dynamics starts looking for the availability of *TS-moves.* These moves are based on the configurations below:

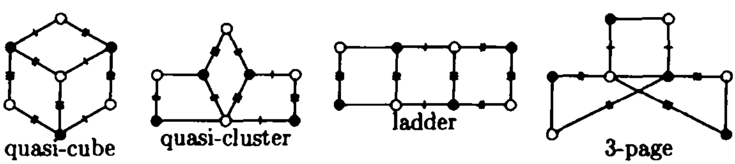


Fig. 15: The TS-configurations

The first three TS-moves, TS_1, TS_2 and TS_3 , are available when the first configuration of three squares occurs. The configuration is named a *quasi-cube.*

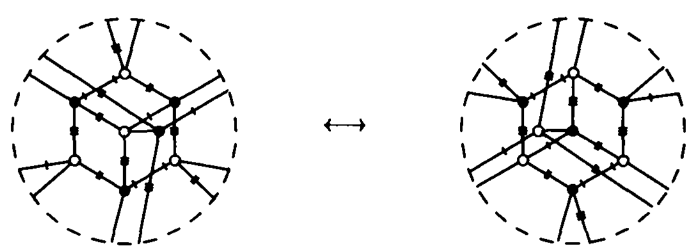


Fig. 16A: Definition of TS_1-move up to edge-color permutation

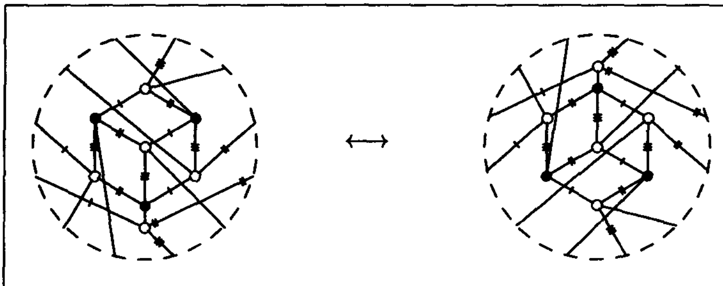


Fig. 16B: Definition of TS_2-move up to edge-color permutation

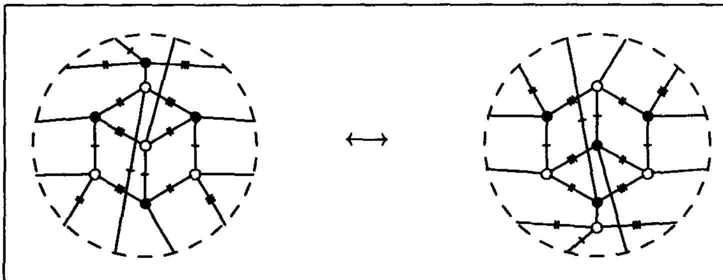


Fig. 16C: Definition of TS_3-move up to edge-color permutation

The fourth TS-move, TS_4, is available whenever three squares meet as in Fig. 15b. The configuration is called a *quasi-cluster* and the corresponding move is defined below:

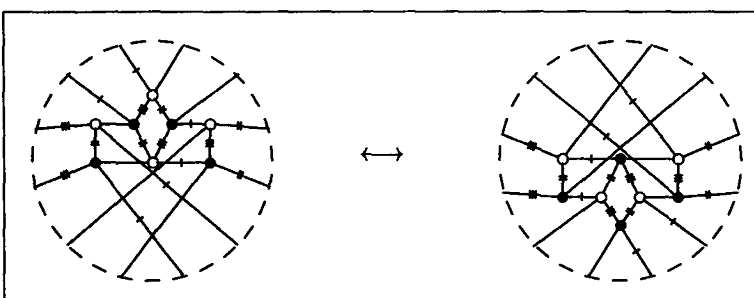


Fig. 16D: Definition of TS_4-move up to edge-color permutation

The fifth TS-move, TS_5, is available whenever three squares meet as in Fig. 15c. The configuration is named a *ladder* and the corresponding move is defined below:

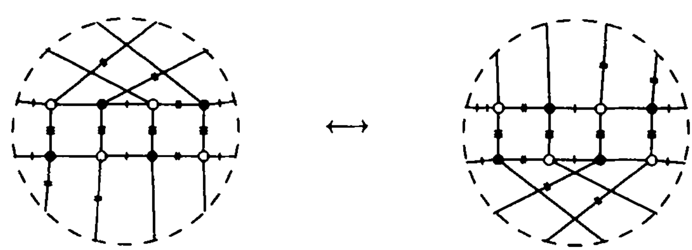


Fig. 16E: Definition of TS_5-move up to edge-color permutation

The sixth TS-move, TS_6, is available whenever three squares meet as in Fig. 15d. The configuration is called a *3-page* and the corresponding move is defined below:

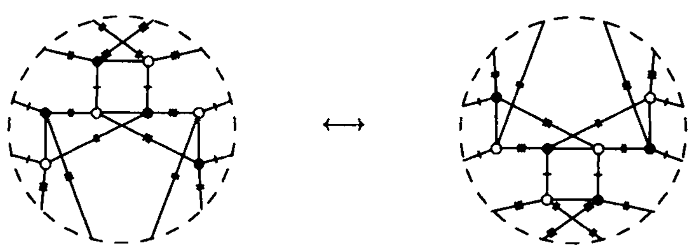


Fig. 16F: Definition of TS_6-move up to edge-color permutation

Starting with a (rigid) 3-gem G we form a finite graph denoted Γ_G^{TS}. The vertices of this graph are in 1-1 correspondence with the 3-gems that are obtained from G by a finite number of TS-moves. An edge in this graph corresponds to a single TS-move. All the vertices of Γ_G^{TS} are 3-gems with the same number of vertices inducing the same 3-manifold: the one induced by G. If there is a vertex H in Γ_G^{TS} which is not a rigid 3-gem, a smaller 3-gem inducing the same manifold is easily produced from H. See Subsections 2.3.2 and 3.2.5.

0.4.3 The U-Move

The last element in our simplifying dynamics is a move, named *U-move*, which increases the number of vertices (!). Whenever two bigons of complementary colors meet in a single vertex v, a U-move can be applied. The vertex v is called a *monopole*. Below we present how a U-move looks in the $\hat{0}$-residue. Vertex v is the single meeting of a 01-gon of 6 edges and a 23-gon of 6 edges. The 0-colored edges are presented in a dashed form:

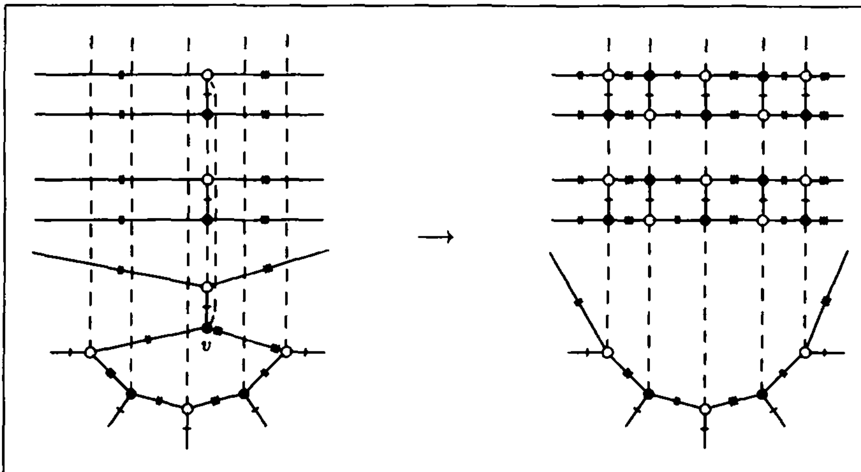


Fig. 17: An example of a U-move at a single meeting v of two hexagonal bigons

The U-move is the counterpart in 3-gem theory of the Reidemeister-Singer stabilization move in the theory of Heegaard decompositions (Section 2.7). It corresponds to the cancellation of a pair of complementary handles.

The force of this move is that it induces many TS-configurations which in turn may provide various simplifications. In conjunction with the ρ-moves and the TS-moves, the U-moves achieve the complete topological classification of 3-gems up to 30 vertices. In particular, we have concretely obtained all the attractors for the orientable 3-manifolds induced by 3-gems up to 30 vertices. In this book we provide in Section 5.1 and in the Appendix of Section 8.1 a complete catalogue of all rigid bipartite 3-gems up to 28 vertices. From this catalogue and the classification performed in Chapter 5 one can explicitly display all the attractors up to this level.

0.5 Main Theorem and Main Conjectures

The U-moves form a nice computational counterpart of the TS and ρ-moves. As we show in Chapter 5, the uncertainties remaining with the TS_ρ-algorithm were resolved when we put a single U-move into scene.

A *u^0-move* on a 3-gem is either a ρ-move or a TS-move, whereas a *$u^0_\bullet$-move* is the identity or a finite sequence of u^0-moves. A *u^1-move* is a move of type $Uu^0_\bullet$ *which may decrease but does not increase the number of vertices.* A *$u^1_\bullet$-move* on a 3-gem is a finite sequence of u^1- and u^0-moves. In general, let a *u^n-move* be a move of the type $Uu^{n-1}_\bullet$, *which may decrease but does not increase the number of vertices.* Let finally a *$u^n_\bullet$-move* be a finite sequence of u^m's moves with $m \leq n$.

A 3-gem is u^n*-essential* if it cannot reach another with less vertices by u^n-moves. Otherwise the 3-gem is called u^n*-reducible*. A u^n*-class* is a maximal set of 3-gems such that given any ordered pair in the set the second 3-gem is obtained from the first by a u^n_*-move. Note that all the 3-gems in a u^n-class have the same number of vertices and induce the same 3-manifold.

An u^n-class is *essential* if each of its members is u^n-essential. In general, for a given n, the attractor of a 3-manifold is the union of some essential u^n-classes.

The u^n*-classification* of the set of all u^n-essential 3-gems of a given number of vertices is the process of partitioning the set into u^n-classes. Theorem 12 and other issues contained in this book provide a definite finite procedure to achieve the u^n-classification for a given n. See Subsection 4.1.9 where we describe the U^n-algorithm. The graphs generated by this algorithm perform, in particular, the u^n-classification.

We conclude this section by stating the Main Theorem which this book proves and with two Main Conjectures suggested by the Main Theorem. A *horizontal move* on 3-gems is one that maintains the induced 3-manifold and does not change the number of vertices.

Theorem 1 *Two bipartite 3-gems with at most 28 vertices induce the same orientable 3-manifold if and only if they are linked by a* u^1_**-move. In particular any two 3-gems belonging to an attractor for a 3-manifold induced by a 3-gem up to 28 vertices are linked by a horizontal* u^1_**-move.*

This Theorem can be rephrased simply as: *For orientable 3-manifolds induced by 3-gems up to 28 vertices the attractor is formed by a single essential* u^1*-class.* The Theorem has been extended to all the bipartite 3-gems up to 30 vertices.

Conjecture 1 (Weak Conjecture on 3-Gems) *There exists a computable function* $f : \mathbb{N} \times \mathbb{N} \longrightarrow \mathbb{N}$ *(which might be constant — maybe always 1) so that two 3-gems having* m *and* n *vertices respectively induce the same 3-manifold if and only if they are linked by a* u^k_**-move where* $k=f(m,n)$.

We remark that the number of relevant u^k_*-moves is finite. Therefore, if true, the Conjecture implies a finite algorithm to decide homeomorphism among 3-manifolds.

The most optimistic result that is suggested by the theory and data gathered so far would rephrase the above conjecture into a stronger form:

Conjecture 2 (Strong Conjecture on 3-Gems) *(i) A 3-gem which is not in the attractor of its induced 3-manifold is* u^1*-reducible. (ii) The attractor of any 3-manifold is formed by a single essential* u^1*-class.*

Chapter 1

Graph-Encoded Manifolds

In this chapter we provide the basic constructions to get surfaces and 3-manifolds from 2- and 3-gems. We establish a general construction to go from a facial identification scheme on the boundary of a solid 3-ball to a 3-gem inducing the same 3-manifold. Since every 3-manifold is formed as such a scheme, we use this construction to show that every closed compact 3-manifold admits a 3-gem inducing it. This is adequate computationally. However a much simpler proof of the universality of the 3-gem model is possible and is given at the end of the chapter.

1.1 Surfaces from $(2+1)$-Graphs

There is a natural construction which associates to each $(2+1)$-graph a compact closed surface. The surfaces corresponding to the $(2+1)$-graphs given in Fig. 1 are, respectively: S^2, S^2, RP^2 and $S^1 \times S^1$ (or 2-torus). Here is the construction: start with the $(2+1)$-graph and a disjoint collection of 2-disks in 1-1 correspondence with the components of the $(2+1)$-graph induced by 2 colors. Attach each 2-disk in the collection to its corresponding component induced by 2 colors. Indeed, each such component, named a *bigon* (for *bi*colored poly*gon*) is (topologically) an S^1 and can become the boundary of the corresponding disk. Unless where the distinction is important, we let the term "bigon" mean either the polygon (an S^1) or the disk which it bounds. Note that an edge of color 0, for instance, appears in the boundary of two disks: one bounded by bigon in colors 0 and 1 (01-gon) and another bounded by a 02-gon. Thus, each edge e has two ocurrences and we paste the two bigons containing e along e. Here are these constructions for the surfaces associated to the $(2+1)$-graphs of Fig. 1:

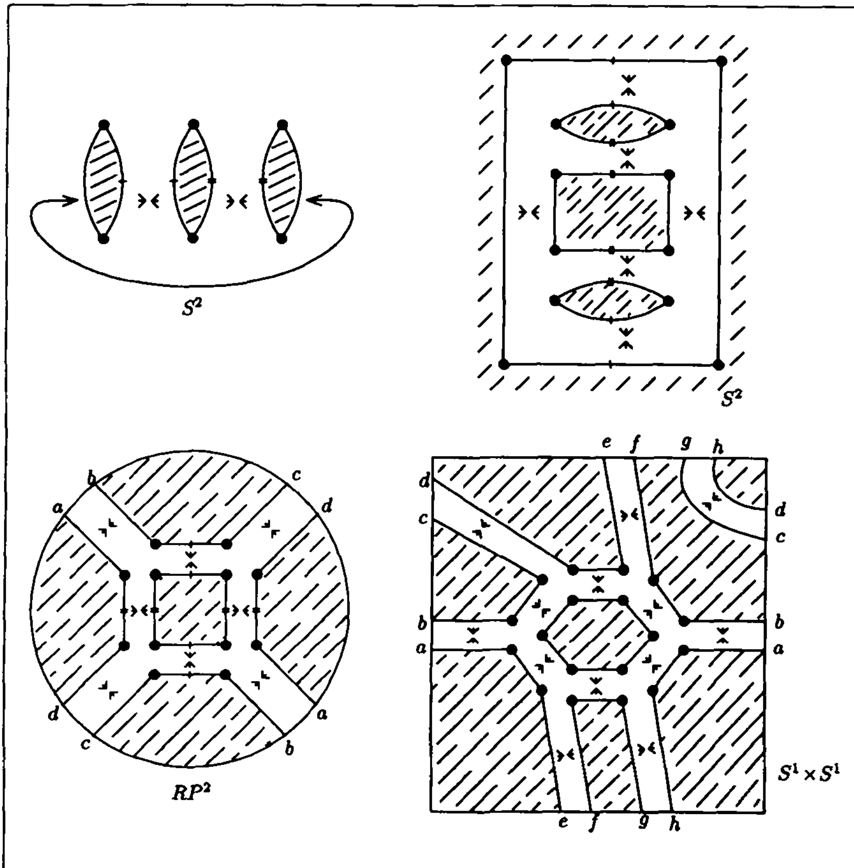


Fig. 18: The surfaces induced by the 2-gems of Fig 1

From $(2+1)$-graph, we can get at once the Euler characteristic of its surface and whether the surface is orientable or not. This means that we can topologically decide which is the surface (without constructing it explicitly).

Proposition 1 *The Euler characteristic of the surface S associated with a $(2+1)$-graph G with v vertices and b bigons is $\chi = b - v/2$. Moreover, this surface is orientable if and only if G has no odd polygon (it is a bipartite graph).*

Proof: In general $\chi = v + f - e$ is the Euler characteristic of a colored surface S where v, e are the numbers of vertices and edges of a graph G embedded into S so that $S \backslash G$ is a collection of f open disks named *faces*. Here, $f = b$, $e = 3v/2$ and any component of $S - G$ is an open disk. (In fact, the closure of such component is always a closed disk in our context.) Thus, $\chi = b - v/2$.

Assume that the surface S is orientable. Consider disjoint disks in the plane bounded by the bigons of G, as for example:

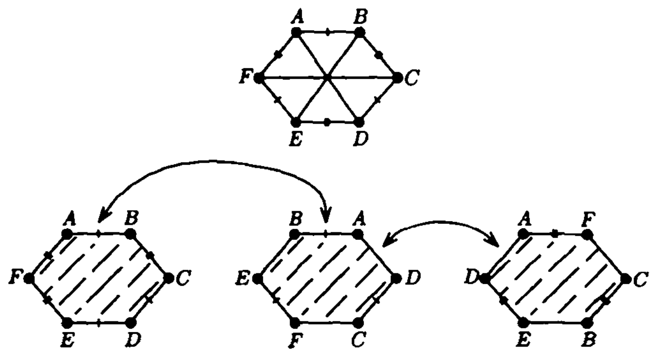


Fig. 19: Bipartiteness and global orientation

By the orientability of S we can globally choose orientations for these embeddings so that given any oriented edge e of G we can slide one disk containing one ocurrence of e so as to glue with the other ocurrence of e without the need of turning over one of the disks. Note that after identification,

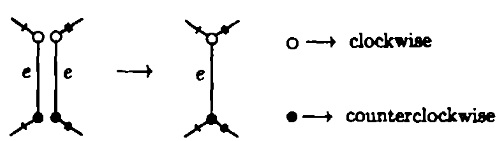


Fig. 20: Bipartition of vertices from an orientable surface

edge e has an end in which the cyclic sequence of colors of edges around it is 0-1-2 clockwise and an end which is 0-1-2 anticlockwise. Thus the vertices of G are of two types clockwise and anticlockwise and every edge has ends of distinct type. Thus G is bipartite.

Conversely assume that G is bipartite. Orient the edges consistently from one class of the bipartition to the other (from black to white).

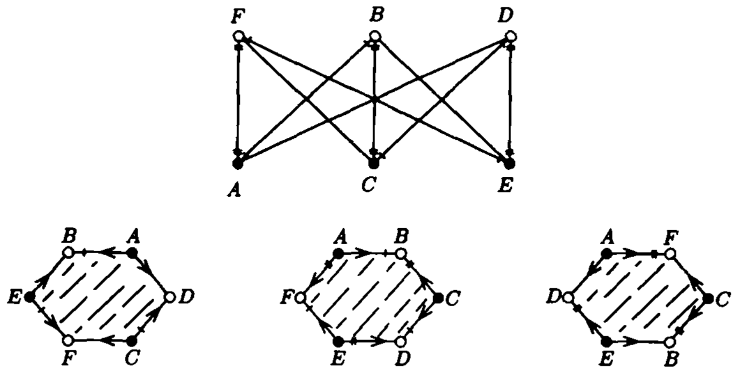


Fig. 21: Orientable embedding from the bipartition

Embed the 01-gons so that the oriented 0-colored edges (0-darts) point clockwise; embed the 12-gons so that the 1-darts point clockwise; finally, embed the 20-gons so that the 2-darts point clockwise. In this way each edge has its two occurrences pointing in distinct clock directions, proving that S is orientable. ∎

Remark 1 *A good understanding for the orientable issue above is obtained if one tries to follow the proof with the* $(2+1)$*-graph of Fig. 1(iii), which is non-bipartite.*

Recall that we have defined a *2-gem* for 2-dimensional graph encoded manifold simply as a $(2+1)$-graph: there are no further restrictions since each of such graphs induces a 2-manifold.

1.2 3-Manifolds from 3-Gems

We want to move one dimension up in the ideas leading from $(2+1)$-graphs to closed surfaces. Unfortunately, not all $(3+1)$-graphs will induce 3-manifolds. We need to restrict the class of $(3+1)$-graphs.

Recall that a *k-residue* $(0 \leq k \leq n+1)$ in an $(n+1)$-graph is a connected component of a subgraph induced by k colors. In the tridimensional case the difficulty which we get is that whereas the 2-residues (the bigons) are always topologically 1-dimensional spheres and can bound 2-disks, the 3-residues induce surfaces which are not always 2-spheres. For the generalization we need each 3-residue of a $(3+1)$-graph to induce the only surface capable of bounding a 3-ball: namely, the 2-sphere.

We define a *3-gem* (for 3-dimensional *g*raph *e*ncoded *m*anifold) to be a $(3+1)$-graph in which each one of its 3-residues induces an S^2. With this restriction, the ideas generalize for getting a 3-manifold from a (3+1)-graph. Indeed, we attach a 3-ball for each 3-residue and there is a topologically unique way to do the pasting of these 3-balls, named *triballs*, so as to form a 3-manifold: there are two ocurrences of each bigon. For example, one 12-gon appears once in the boundary of a triball induced by a 3-residue involving colors 0,1,2 and once in the boundary of a triball induced by a 3-residue involving colors 1,2,3. These two occurrences of the same bigon bound disks in distinct triballs and must be identified. If we put auxiliary labels we know exactly how:

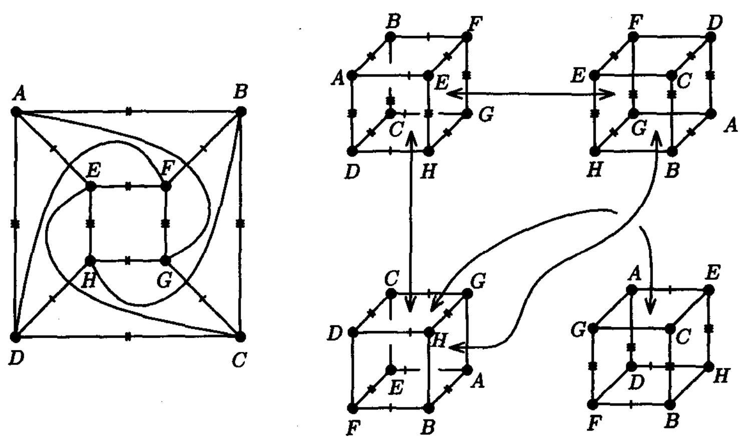


Fig. 22: Primal construction of a 3-manifold from a 3-gem

By effecting all the identifications above we produce a compact closed 3-manifold. In the case above, the real projective space RP^3, as we shall show. In general, after the pairwise identifications of the faces in the triballs of a 3-gem G, we get a quotient space $|G| \cong M^3$ which is a closed 3-manifold, since it has no singularities: each point has a neighborhood homeomorphic to a 3-ball. This is obvious if the point is in the interior of a triball. A point in the interior of a bigon (here bigon means the disk) has a ball neighborhood in M^3 formed by two half 3-balls, each contained in one of the 2 triballs sharing the bigon. A point in the interior of an edge e of G has a ball neighborhood in M^3 formed by 3 third balls. Each one of such third balls is contained in one of the 3 triballs sharing e. Finally, a point which is also a vertex of G is in M^3

is the center of a small tetrahedron formed by four tetrahedra. Each one of these is contained in only one of the four triballs which share the vertex.

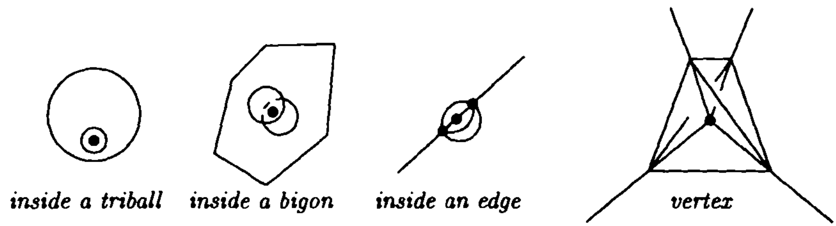


Fig. 23: Types of points a 3-gem induces in the associated 3-manifold

Note that these 4 types of points constitute a partition of the 3-manifold $|Q|$.

Whereas in the bidimensional case it is fairly easy to recognize which 2-manifold a 2-gem induces, in the tridimensional case this is difficult. A surprise of using 3-gems is that the attractors could be effectively computed (for 3-manifolds induced by up to 30 vertices). After the attractor has been computed, we have been able to effectively recognize the induced 3-manifolds by applying our simplification dynamics to 3-gems arising directly from classical topological descriptions.

1.3 Constructing 3-Gems from Facial Identifications

There are a profusion of ways to describe a 3-manifold. Some of these directly translate into 3-gems. Here we study a general construction. Let G be a graph embedded into the boundary S^2 of a 3-ball B^3 so that $S^2 \backslash G$ is a collection of open disks each named a *face* of the *cellular embedding* (G, S^2). Any 3-manifold M^3 arises from B^3 by the pairwise identification of the faces of an appropriate cellular embedding (G, S^2): see the second proof of Theorem 2.

We follow the theory with two examples. The tridimensional torus $S^1 \times S^1 \times S^1$ is obtained by identifying the opposite faces of a solid cube by translations parallel to the sides of the cube. A simpler case is the real projective space RP^3 which arises from a solid ball B^3 by the identifications of the pairs of antipodal points in its surface S^2. To put this under our scheme in the most economical way, we consider the graph G formed by two edges linking two vertices embedded into the boundary S^2 of a 3-ball B^3, so as to separate it into two disks.

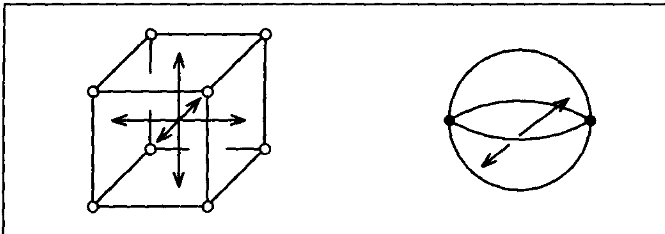


Fig. 24: $S^1 \times S^1 \times S^1$ and RP^3 as facial identification schemes

Henceforth in the pictures we use the plane of the paper as if it were an S^2. (Mentally, do the inverse of a stereographical projection to fix the situation.) The cellular embedding (G, S^2) is a 2-dimensional ball complex and can be refined and dualized in the usual way. Take the barycentric subdivision (G', S^2) of (G, S^2). Consider the dual of (G', S^2), naming it (H, S^2). In the second (simplest) case above we get:

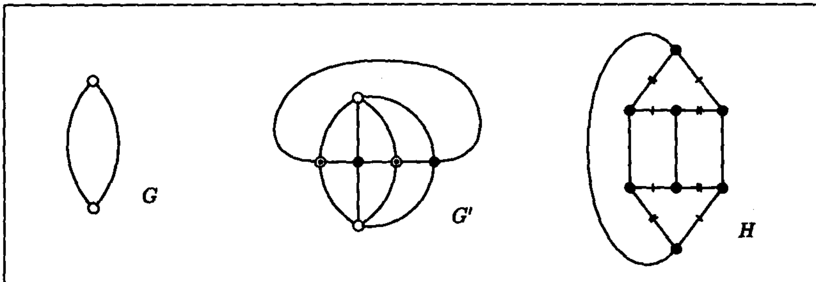


Fig. 25A: Taking the dual H of barycentric subdivision G'

Given any cellular embedding (G, S^2), the vertices of G' can be colored with three colors in such a way that each triangle has vertices of distinct colors: use color 0 to paint the original vertices, the 0-cells of the complex; use color 1 to paint the vertices corresponding to the edges of G, the 1-cells of the complex; use color 2 to paint the vertices corresponding to the faces of (G, S^2). This coloration makes the graph H of the dual complex (H, S^2) a $(2+1)$-graph in a natural way: paint an edge with color i if the dual edge misses color i in its ends.

To enable a combinatorial description, which paves the way to a computer implementation, we attach labels for the i-cells of (G, S^2), that is, for the vertices of G'. In our example, here is a possible labelling:

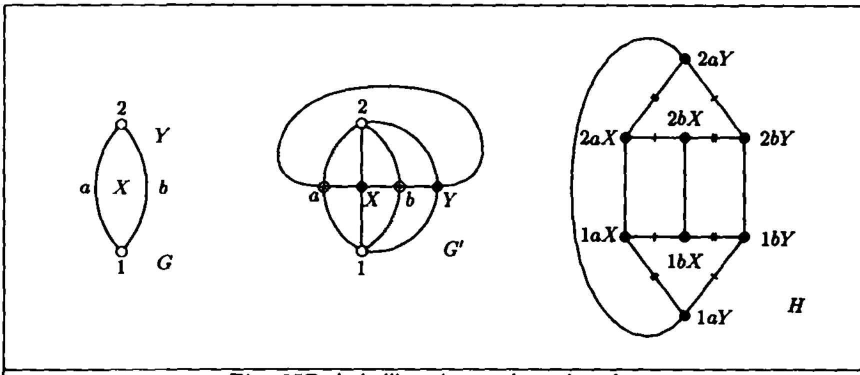


Fig. 25B: Labelling the previous situation

Let a vertex of H be labelled by the three labels of the vertices of the triangular region to which it corresponds. We order the three labels by dimensions, so that a vertex (x, y, z) has 0-cell of (G, S^2), x, is incident to 1-cell y, which is incident to 2-cell z. Observe that, in H, an edge is colored i iff the labels of their ends differ exactly in the $(i+1)$-entry.

Here is the basic observation: *if a pseudo 3-manifold M^3 arises from B^3 by pairwise identifications of the 2-cells of (G, S^2), then for each vertex (a, b, c) of H, there is exactly another vertex (a', b', c') such that the individual i-cells of (G, S^2) differ at least in the third entry, i.e., $c \neq c'$ before the identifications, but become identified, $a \equiv a'$, $b \equiv b'$, $c \equiv c'$ after them.*

This means that the identifications in (G, S^2) induce a perfect matching of the vertices of H. In the above example: $\{1bX, 2aY\}$, $\{1bY, 2aX\}$, $\{1aY, 2bX\}$, $\{1aX, 2bY\}$. Add new edges, of color 3 to H linking these mates, naming the resulting graph H^*. In the example at hand, H^* is isomorphic, up to a switching of colors 0 and 3, to

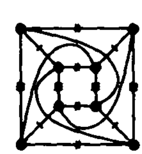


Fig. 26: The superattractor for RP^3

We prove, in Lemma 1 of next section, that this general construction produces in this case a 3-gem inducing RP^3. Since there are no other 3-gems with eight or less vertices inducing RP^3, the above 3-gem is the superattractor for RP^3.

In practice we can do the construction to get H^* over the complex (G, S^2) itself, without the need of building (G', S^2) or (H, S^2). Instead of using a triple label (v, e, f) we use only one, x, positioned near vertex v, towards edge e and inside of face f. The perfect matching of the labels providing the third color of H^* is given by a pair (x, X) of lower-upper case equal letters. Here is how to get a 3-gem H^* for the 3-torus by identifying opposite faces of a solid cube.

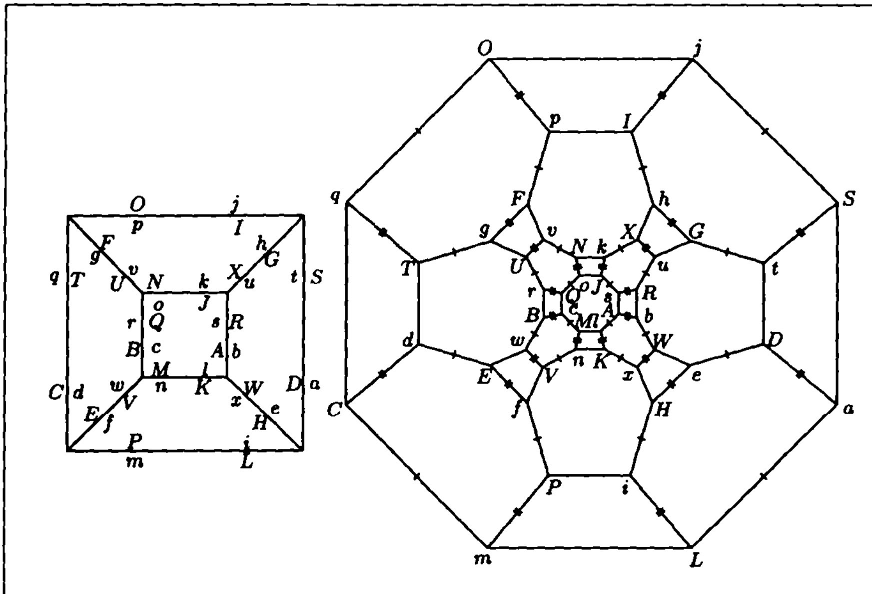


Fig. 27: A 3-gem for the 3-torus obtained from the facial identification scheme

The justification that this construction provides a 3-gem inducing the same 3-manifold is given in the next section.

This drawing/labelling approach is easier to work by hand and for understanding. However, the defining construction, using triple labels and a matching, enables an easy general machine implementation, since it leads to a combinatorial approach not depending on *proximity*.

1.4 3-Gems from 3-Manifolds

We start the section by providing a useful arithmetic characterization of 3-gems. After that, we use the construction of the previous section to prove in Lemma 1 that it provides an economical way to get a 3-gem for a 3-manifold given by facial identifications on the boundary of a solid 3-ball.

1.4.1 Arithmetic Characterization of 3-Gems

Before we give the proof of Lemma 1, we pause to provide a counting based characterization of 3-gems and an easy corollary.

Proposition 2 *Let G be a $(3+1)$-graph having b_G 2-residues, t_G 3-residues and v_G vertices. Then G is a 3-gem if and only if*

$$v_G + t_G = b_G \ .$$

Proof: If G is a 3-gem, then for each 3-residue T we have $b_T - v_T/2 = 2$. Note that $\sum_T v_T = 4v_G$, since each vertex appears in 4 distinct triballs. Also, $\sum_T b_T = 2b_G$, because each bigon appears in two distinct triballs. Thus, $2t_G = \sum_T (b_T - v_T/2) = 2b_G - 2v_G$, or $v_G + t_G = b_G$.

Conversely, suppose that G is not a 3-gem. Then some 3-residue T' induces a surface $S_{T'}$ which is not a 2-sphere. Therefore, $\chi(S_{T'}) = b_{T'} - v_{T'}/2$ is less than 2. Since for each 3-residue T, $b_T - v_T/2 \leq 2$, it follows that $\sum_T (b_T - v_T/2) < 2t_G$. As the sum on the left is $2b_G - 2v_G$, we get $b_G < v_G + t_G$. ∎

Corollary 1 *Let G be a $(3+1)$-graph having b_G 2-residues, t_G 3-residues and v_G vertices. Then,*

$$v_G + t_G \geq b_G.$$

Proof: A direct consequence of the proof of the previous proposition. ∎

1.4.2 A Basic Lemma

Since the construction of the H^* below is easily implementable, the next lemma provides a useful decision tool for testing whether a given identification scheme produces a 3-manifold.

Lemma 1 *Suppose that a pseudo-manifold M^3 arises from B^3 by a pairwise identification of the faces of the cellular embedding (G, S^2). Let H^* be the $(3+1)$-graph as constructed in the previous section. Then M^3 is a manifold if and only if H^* is a 3-gem inducing it.*

Proof: We follow the proof with the example of the $(3+1)$-graph for the 3-torus, as given above.

M^3 is a 3-manifold iff its Euler characteristic is 0, [ST80]. The Euler characteristic can be obtained from the quotient 3-ball complex Q, obtained from B^3 by the identifications in the cellular embedding (G, S^2). Let v', f', and e' be the number of 0-cells, 2-cells and 1-cells of Q, respectively. Note that Q has only one 3-cell. Thus, to say that M^3 has Euler characteristic 0 means the same as $v' + f' = e' + 1$. We need to show that this equality is equivalent to the one which characterizes 3-gems for H^*: $v_{H^*} + t_{H^*} = b_{H^*}$ (see Proposition 2).

Let b_{ij} be the number of ij-gons of H^*. Let t_k be the number of 3-residues of H^* which do not use color k. Recall that such a 3-residue is named a $\hat{k}$*-residue*. From the construction we obtain enough relations among these parameters to show the equivalence. Note first that $t_0 = v'$: the vertices of Q are in 1-1 correspondence with the $\hat{0}$-residue's of H^*. In our example this corresponds to the unique triball

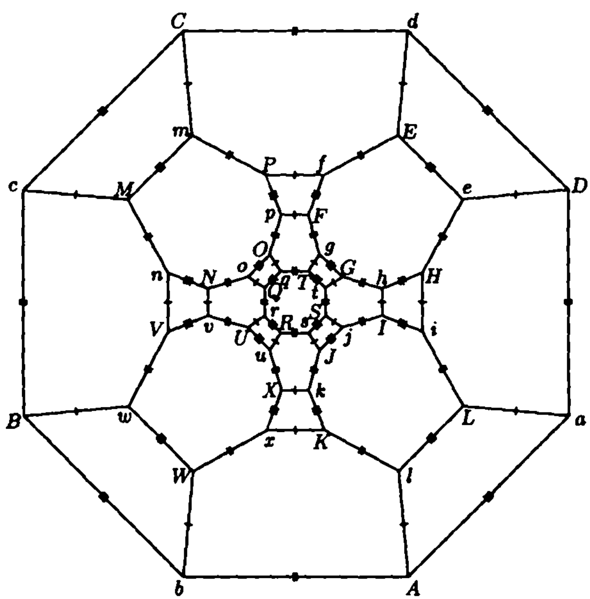


Fig. 28: Unique $\hat{0}$-residue of 3-gem of Fig. 27 inducing the 3-torus

The construction also implies that every 02-gon, every 03-gon and every 13-gon has four edges. Thus, $v_{H^*} = 4b_{02} = 4b_{03} = 4b_{13}$. These four-edge-bigons imply that each $\hat{1}$-residue and each $\hat{2}$-residue is the 1-skeleton of a prism. Thus, $t_1 = e' = b_{23}/2$

and $t_2 = f' = b_{01}/2$.

Here is a typical $\hat{1}$-residue (all of them are isomorphic) in our ongoing example. The associated triball, when embedded in $|Q|$ is a prism which we draw elongated, since it corresponds to a 1-cell of Q:

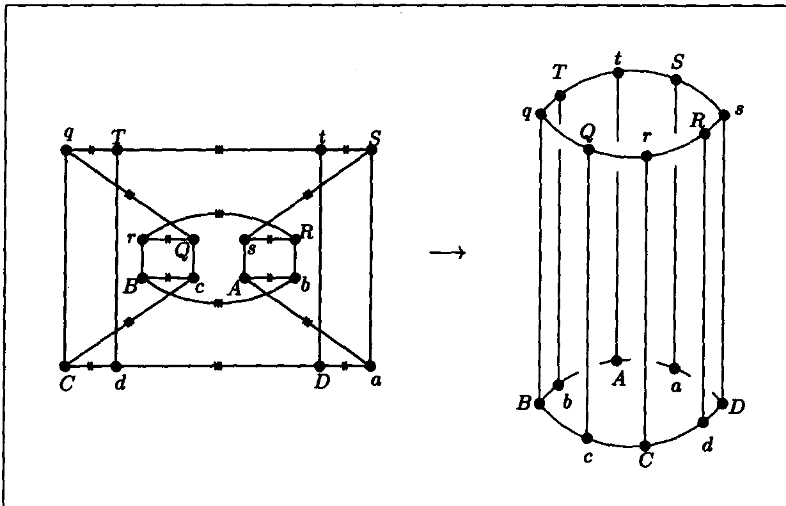


Fig. 29: A typical $\hat{1}$-residue

Here is also a typical $\hat{2}$-residue and its associated triball, which is also a prism when embedded in $|Q|$. This time we draw it flat, since it corresponds to a 2-cell of Q:

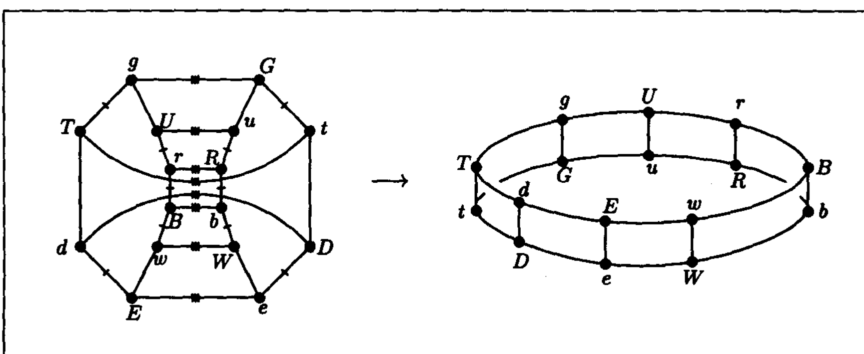


Fig. 30: A typical $\hat{2}$-residue

Finally, observe that $t_3 = 1$. The unique triball is the one of Fig. 27.

From these relations we get the following set of equivalent identities:

$$
\begin{aligned}
v_{H^*} + t_{H^*} &= b_{H^*} \\
4b_{02} + (v' + e' + f' + 1) &= 3b_{02} + b_{01} + b_{12} + b_{23} \\
v' + f' + e' + 1 &= -2b_{02} + (b_{02} + b_{01}) + b_{12} + b_{23} \\
v' + f' + e' + 1 &= -v_{H^*}/2 + (v_{H^*}/2 + 2 - b_{12}) + b_{12} + 2e' \\
v' + f' + e' + 1 &= 2 + 2e' \\
v' + f' &= e' + 1
\end{aligned}
$$

From the third to the fourth row we have used that the Euler characteristic of the surface induced by the $\hat{3}$-residue is 2, since it induces a 2-sphere. The other passages are straightforward from the above discussion. This proves that the $M^3 \cong |Q|$ is a manifold if and only if H^* is a 3-gem.

In the case of a manifold M^3 we show now that the 3-manifold induced by H^* is $|Q|$. Let $|H^*|$ be dissected by the cells obtained from the 3-gem H^*. Start by contracting to points the triballs associated to the $\hat{0}$-residues. Since these balls are disjoint, no singularity arises. Only edges of color 0 and bigons involving color 0 survive. Also, at this point, an elongated prism associated to a $\hat{1}$-residue has their two bases contracted to points. Retract each such pinched prism to its medial line. Again, no singularities arise since the interiors are disjoint. The only disks corresponding to the original bigons which survive are those of the 01-gons. The space bounded by each original flat prism corresponding to the $\hat{2}$-residues is now bounded by a pair of 2-cells, having an S^1 in common. Retract these disjoint lens shaped open 3-cells to their medial 2-cells. The resulting complex is Q. Since the 3-manifold did not change, the proposition is established. ∎

The above method, to produce a 3-gem from a planar description of a facial identification scheme, is useful and it has been extensively used due to its simplicity.

We use the Basic Lemma to show the universality of the 3-Gem Model.

Theorem 2 *Given any 3-manifold M^3 there is a 3-gem H^* which induces it.*

Proof: By Moise's Theorem [Moi52], there exists a triangulation T of M^3. Start by deleting triangular 2-cells of T as long as these 2-cells separates distinct 3-cells. At the end of this process we get to a ball complex having a unique 3-cell B^3. The 2-skeleton of the complex obtained by cutting along each remaining 2-cell is a 2-sphere denoted S^2 bounding B^3. The 1-skeleton is a graph G embedded on S^2. Use the construction of Proposition 1 on (G, S^2) to get H^*. ∎

Only in rare occasions the 3-gem H^* which we obtain with this construction leads to the smallest possible one inducing M^3. The main force of the above theorem is that it provides a more or less economical and computationally simple way to get 3-gems for arbitrary 3-manifolds. Indeed, the number of vertices of H^* is just four times the number of edges of the graph G. This is a linear increase and can be dealt easily via computers. If we do not care about number of vertices, then there is much simpler proof of this theorem, which does not use the construction of Proposition 1.

Another proof of Theorem 2: This is a straightforward consequence of Moise's triangulation theorem for 3-manifolds [Moi77]. Let T be a triangulation for M^3 and T' its barycentric subdivision. The vertices (0-cells) of T' are naturally partitioned into four classes V_0, V_1, V_2, V_3. The class V_0 is formed by the original vertices of T. For $i = 1, 2, 3$, the class V_i is formed by those vertices which are in 1–1 correspondence with the i-cells of T, situated in the barycentre of an i-cell. Note that each tetrahedron of T' has the four vertices in distinct classes. This 4-partition of the vertices of T' induces a 4-coloration of its triangular 2-cells: color with i the face of a tetrahedron in T' whose opposite vertex is in V_i.

Let G be the 1-skeleton of the cell decomposition dual to T'. The 4-coloration of the 2-cells of T' is inherited, under geometric duality, by the edges (1-cells) of G. Graph G with the edges so colored is a 3-gem and satisfies $|G| \cong M^3$. ∎

1.4.3 The Superattractor for $S^1 \times S^1 \times S^1$

The *size* (i.e. number of vertices) of the 3-gem obtained from the construction of Section 1.3 is unecessarily large. This bring again to consider our main issue: to reach the smallest possible 3-gem which induces a given 3-manifold. As a further instance on how 3-gems simplify, consider the 48-vertex 3-gem inducing $S^1 \times S^1 \times S^1$ of Fig. 27. It is absorbed under the TS_ρ-algorithm, by the 24-vertex 3-gem

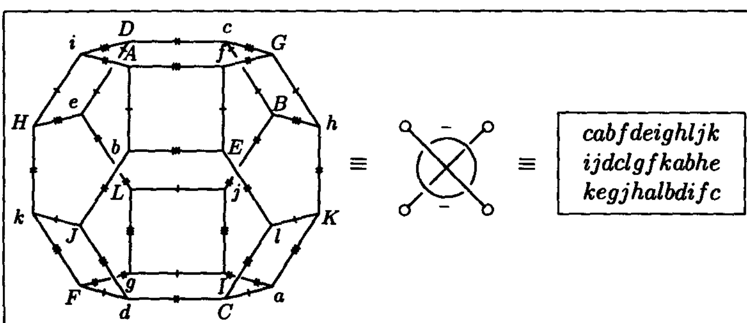


Fig. 31: Superattractor, string and code for $S^1 \times S^1 \times S^1$

The above 3-gem (where, as always, when we do not draw them, the 0-colored edges are given by a lower/upper case pair of equal letters) is the unique minimum gem, denoted $s^1 \times s^1 \times s^1$, inducing $S^1 \times S^1 \times S^1$. Therefore it is the superattractor for this 3-manifold. This is a fact which follows from our catalogue of rigid 3-gems, treated in Chapter 5.

The small diagram on the center is called a *string presentation* for the 3-gem on its left. Under some implicit symmetries we can recover the 3-gem from its string presentation (see Section 2.5). By using the idea of *code of a 3-gem* (see Section 2.6), even the specific vertex labelling can be recovered from the string presentation. The literal matrix on the right is the code itself. This is an instance on how the computer stores a 3-gem. The specific vertex labelling associated to the code is recoverable from a vertex unlabelled (3 + 1)-graph (see Section 2.6).

Chapter 2

Elements of the Theory of 3-Gems

In this chapter we provide ways to get families of 3-manifolds directly from 3-gems. We start with the lens spaces (Section 2.1) and generalize them to a 4-parametric family of 3-manifolds (defined by 4 integers — Section 2.2) which contains many interesting examples of 3-manifolds. We present with details the elementary simplification techniques based on cancellation of dipoles and on the switching of ρ-pairs (Sections 2.3 and 2.4).

We consider a Kleinian K_4-symmetry which appear often in 3-gems. This kind of symmetry yields a contract presentation named *string presentation* (Section 2.5). The computationally essential concept of the *code of a 3-gem* is dealt with in Section 2.6.

Sections 2.7, 2.8, 2.9 provide connections among 3-gems, Heegaard diagrams, framed links and blinks. Algorithms to get presentations of the fundamental group and of the homology group of a 3-manifold from 3-gems are discussed. Sections 2.10 and 2.11 are formed by examples which we feel are interesting to mention.

2.1 Lens Spaces

The construction to get RP^3 (which is the lens space $L_{2,1}$), given in the previous chapter generalizes to provide a standard 3-gem for the lens space $L_{p,q}$. For natural numbers $p, q > 0$, consider the following description of the lens space $L_{p,q}$. Take a 3-cell B^3 bounded by two 2-cells with a polygon of p sides as common boundary. Identify the two faces in such a way that an edge of its boundary is mapped onto the qth edge after it and in the same direction along the polygon. These facial identification schemes are denoted by $L(p, q)$ and they induce, under the construction of Section 1.3, the lens spaces $L_{p,q}$. The corresponding 3-gem H^* given by the construction is denoted $\ell_{p,q}$.

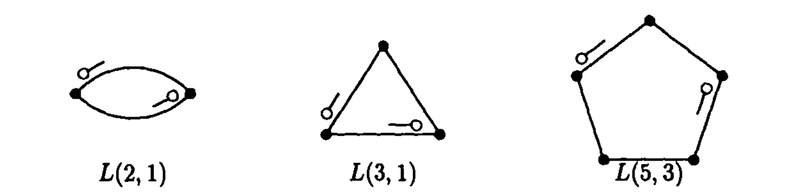


Fig. 32A: Some lens spaces, given by stereographical identification schemes

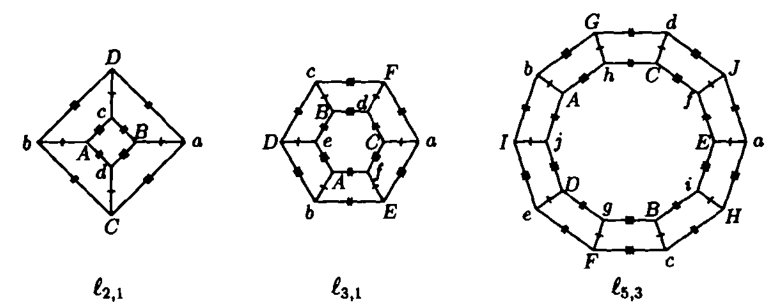


Fig. 32B: Respective 3-gems H^*, under color permutation $(0123) \mapsto (3210)$

There are easy ways for a computer program to produce a cell decomposition of the lens space $L_{p,q}$. Here is one which provides the 3-gem $\ell_{p,q}$ inducing it. The method that follows is important because it generalizes providing 3-gems inducing more complex 3-manifolds. See the next section. Let Z_n denote the set of integers mod n. The vertices of $\ell_{p,q}$ are the elements of the group $Z_2 \times Z_{2p}$. Let $x' = 1 - x$ in Z_2. An *involution* is a map of a set into itself so that its square is the identity. Consider the following four involutions, free of fixed points, defined on $Z_2 \times Z_{2p}$:

$$\begin{aligned}
\epsilon_0(i,j) &= (i', 1 - j + 2q) \\
\epsilon_1(i,j) &= (i, j - (-1)^j) \\
\epsilon_2(i,j) &= (i, j + (-1)^j) \\
\epsilon_3(i,j) &= (i', 1 - j)\,.
\end{aligned}$$

These involutions induce the edges of $\ell_{p,q}$ in the sense that, for $0 \le k \le 3$, we define an edge of color k between (i,j) and $\epsilon_k(i,j)$. (Note that $\epsilon_k^2(i,j) = (i,j)$.) By implementing this construction we get an easy way to input arbitrary lens spaces on a computer. As an example, the above description for $\ell_{3,1}$ is the following 3-gem with the auxiliary labels, which permit the input for computers. These labels are, otherwise, irrelevant:

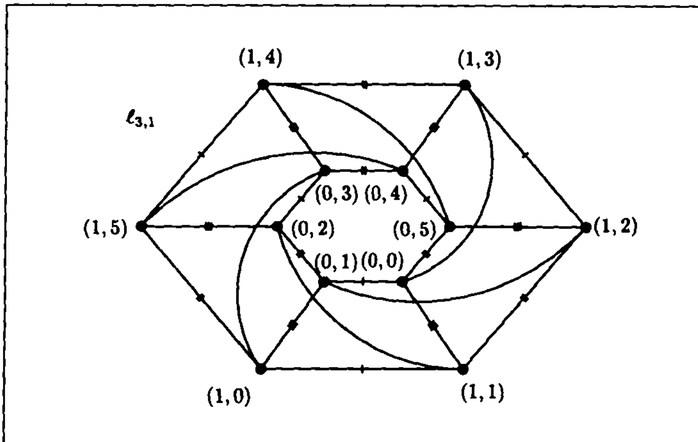


Fig. 33: Combinatorial labelling of $\ell_{3,1}$: just an input for computers

2.2 A 4-Parametric Family of 3-Manifolds

Generalizing the construction used for the lens spaces in the previous section we describe a 4-regular 4-edge-colored graph denoted $\mathcal{S}(b, \ell, t, c)$.

2.2.1 Defining the Family

The vertices of this graph are the elements of $Z_b \times Z_{2\ell}$. Let μ be a function from $Z_{2\ell}$ onto $\{-1, +1\}$: $\mu(j) = 1$ if $1 \leq j \leq \ell$ and $\mu(j) = -1$ if $\ell < j \leq 2\ell$. We let the argument j of μ be normalized to the range $1 \leq j \leq 2\ell$. To define the edges of the graph we consider four fixed point-free involutions on the set of vertices. The arithmetic below is *mod* b in the first coordinates and *mod* 2ℓ in the second coordinates.

$$\begin{aligned}
\epsilon_0(i,j) &= (i + c\mu(j-t), 1 - j + 2t) \\
\epsilon_1(i,j) &= (i, j - (-1)^j) \\
\epsilon_2(i,j) &= (i, j + (-1)^j) \\
\epsilon_3(i,j) &= (i + \mu(j), 1 - j)\ .
\end{aligned}$$

As before, these involutions induce the edges of $\mathcal{S}(b, \ell, t, c)$, in the sense that, for $0 \leq k \leq 3$, we define an edge of color k between (i,j) and $\epsilon_k(i,j)$.

When $c = 1$ the geometrical shape of $\mathcal{S}(b, \ell, t, c)$ can be informally described as follows. It consists of b 12-gons cyclically set on the plane and numbered $1, 2, \ldots, b$.

There are ℓ links colored 3 between two adjacent 12-gons. Given (i, j) its neighbor by color 0 is obtained by going t edges counter-clockwise around its 12-gon, jumping to the next or previous 12-gon by using the edge colored 3 and finally going t edges clockwise around the new 12-gon. The generalization for arbitrary c is that we jump, instead of one, c 12-gons forward or backward to obtain the 0-neighbor of a vertex.

Below we present (with most of the 0-colored edges missing) $\mathcal{S}(3,3,2,1)$, one of the simplest members of this 4-parametric family. It is a 3-gem inducing a 3-manifold known as the *quaternionic space*, since its fundamental group is the 8-element discrete group of quaternions

$$Q_8 = \{\pm 1, \pm i, \pm j, \pm k\}.$$

Indeed, this manifold is homeomorphic to the quotient S^3/Q_8, if we consider S^3 as the set of all quaternions of norm 1. We shall prove that this space can be obtained from a solid cube by identifying opposite faces, where each identification is preceded of a $\pi/2$-rotation of one of its faces (the one which incides to a fixed vertex v of the cube).

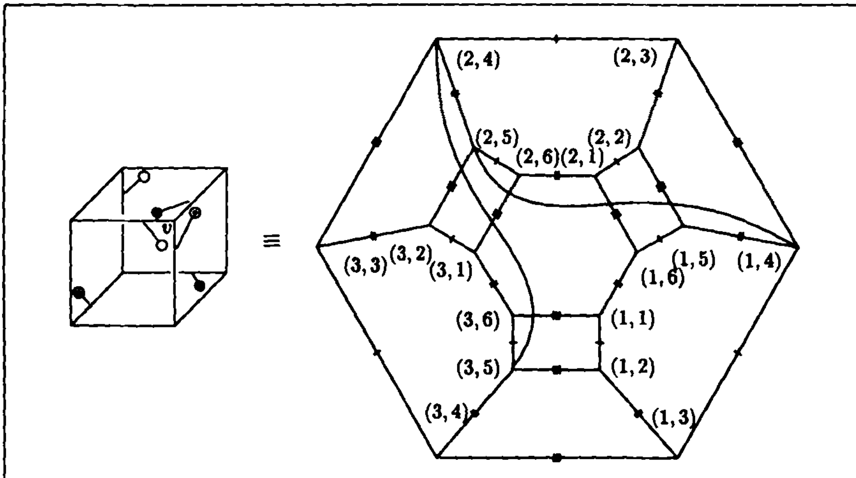


Fig. 34: $\mathcal{S}(3,3,2,1)$ induces the same 3-manifold as a 90-degree twisted solid cube

How do we know that the 3-gem on the right figure above induces the 3-manifold described by the identification scheme given in the left figure? How do we know that its fundamental group is Q_8? We discuss the recognition issue latter in the next section. We show how to get, from a 3-gem, a presentation for the fundamental group of the induced 3-manifold in Section 2.8.

2.2.2 1-Dipoles

Recall that a k-colored edge α in a 3-gem is called a *1-dipole* if the ends of α belong to distinct $\hat{k}$-residues. A 3-gem which has a 1-dipole directly simplifies to one which has less vertices and induces the same 3-manifold. Indeed, the operation shown below, named *cancellation of a* 1-*dipole*, creates a triangular tunnel connecting the 2 triballs associated to the distinct $\hat{k}$-residues. These triballs coalesce into one but the topology of the 3-manifold remains intact.

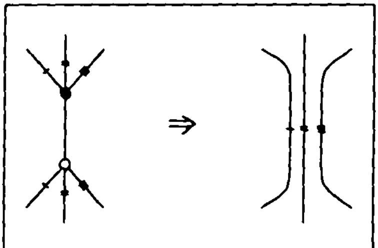


Fig. 35: Cancellation of a 1-dipole

The *creation of a 1-dipole* is the inverse operation. The cancellation or creation of a 1-dipole maintains the induced 3-manifold. Therefore, 3-gems with 1-dipoles can be easily simplified. Thus, it is of interest to have constructions yielding families of 3-gems without them.

2.2.3 Crystallizations in $S(b, \ell, t, c)$

Our next Theorem provides an easy such construction. If a 3-gem is free of 1-dipoles it is called a *3-crystallization* and have been extensively studied. See the survey article [FGG86]. As with gems, *crystallizations* can be defined in every dimension. They are always gems without 1-dipoles.

Theorem 3 *Suppose that b, ℓ, t, c are non-negative integers satisfying $gcd(b, c) = 1$, $gcd(\ell, t) = 1$ and that ℓ is odd, implies $c = (-1)^t$. Then $S(b, \ell, t, c)$ is a crystallization inducing an orientable 3-manifold.*

We need to establish two simple lemmas before the proof of this Theorem. But first we deal with the orientability of the manifolds which is transparent from a 3-gem inducing it. Indeed the equivalence between orientability and bipartiteness goes through the dimensions. It is not only for dimension 2. We leave to the reader the proof of this fact in dimension 3.

Proposition 3 *The 3-manifold induced by a 3-gem G is orientable if and only if G is bipartite.*

Note that the second coordinates of the ends of any edge of $\mathcal{S}(b, \ell, t, c)$ have distinct parity. Therefore this graph is bipartite and if it is a 3-gem it induces an orientable 3-manifold.

Lemma 2 *Assume the conditions of Theorem 3. Then* $(\epsilon_0\epsilon_3)^\ell = Id$*, the identity permutation, but* $(\epsilon_0\epsilon_3)^m \neq Id$*, if* $m < \ell$*.*

Proof: We have

$$\epsilon_0\epsilon_3(i,j) = (i + \mu(j) + c\mu(1 - j - t), j + 2t) = (i + \mu(j) - c\mu(j + t), j + 2t),$$

because, in general, $\mu(1 - k) = -\mu(k)$. Now, it is a simple matter to establish by induction that, for $n > 1$,

$$(\epsilon_0\epsilon_3)^n(i,j) = (i + \sum_{k=0}^{n-1}[\mu(j + 2t) - c\mu(j + t + 2kt)], j + 2nt).$$

Since $gcd(\ell, t) = 1$, the second coordinates of $(\epsilon_0\epsilon_3)^h(i,j)$, $0 \leq h \leq \ell - 1$, assume all the ℓ distinct values of the same parity as j in the interval $[1, 2\ell]$. In this way, $(\epsilon_0\epsilon_3)^m(i,j) \neq Id$ if $m < \ell$. Observe that the second coordinate of $(\epsilon_0\epsilon_3)^\ell(i,j)$ is j. Its first coordinate is i if and only if

$$\sum_{k=0}^{\ell-1} \mu(j + 2kt) = c\sum_{k=0}^{\ell-1} \mu(j + t + 2kt).$$

If ℓ is even, both summands are zero. Suppose that ℓ is odd and thus $c = (-1)^t$. If t is odd, the two summands differ in sign. If t is even, they are equal. Therefore, the equality holds in every case, concluding the proof. ∎

Lemma 3 *Assume the conditions of Theorem 3. Then for* $0 \leq k \leq 3$*, there is only one* $\hat{k}$*-residue.*

Proof: The lemma is straighforward for $k = 0$ and is implied by $gcd(b, c) = 1$ for $k = 3$. To prove that there is only one $\hat{1}$-residue note initially that vertex representatives for the 03-gons can be chosen to be: $(1,1), (2,1), \ldots, (b,1)$. We show that $(i,1)$ and $(i+1,1)$ are in the same $\hat{1}$-residue:

$$\epsilon_2\epsilon_3(i,1) = \epsilon_2(i+1, 2\ell) = (i+1, 1).$$

Thus, there is a unique $\hat{1}$-residue. In order to prove that there is also a unique $\hat{2}$-residue, we consider two cases. First suppose ℓ to be odd. Take $(1,\ell),(2,\ell),\ldots,(b,\ell)$ as representatives of the 03-gons. Note that

$$\epsilon_1\epsilon_3(i,\ell) = \epsilon_1(i+1, 1+\ell) = (i+1,\ell),$$

which implies that there is just one $\hat{2}$-residue. Suppose now that ℓ is even. Since $gcd(\ell,t) = 1$ this implies that t is odd. This time take $(1,t),(2,t),\ldots,(b,t)$ as representatives of the 03-gons. Consider

$$\epsilon_0\epsilon_1(i,\ell) = \epsilon_0(i,t+1) = (i+c,t).$$

Since $gcd(b,c) = 1$, there is a unique $\hat{2}$-residue and the proof is complete. ∎

Proof of Theorem 3 The first lemma says that permutation $\epsilon_0\epsilon_3$ has b cycles of length ℓ, and so there are b 03-gons (each with 2ℓ edges). By construction, the same is true for the 12-gons. The construction also implies that the unique $\hat{0}$-residue and $\hat{3}$-residue are triballs, i.e., induce each an S^2. Therefore $b_{12}+b_{13}+b_{23} = v/2+2$ and $b_{01}+b_{02}+b_{12} = v/2+2$. Since $b_{03} = b_{12} = b$, the addition of these two equalities gives that the total number of bigons is the number of vertices plus 4. By the second lemma, 4 is the total number of 3-residues. According to Proposition 2, this proves that $\mathcal{S}(b,\ell,t,c)$ is a crystallization. Since the graph is bipartite, it induces an orientable 3-manifold. ∎

The family $\mathcal{S}(b,\ell,t,c)$ has been extensively studied by M. Mulazzani. In his Dissertation [Mul94] he extends it with a fifth parameter and topologically characterizes the induced spaces. In particular, he gets complete arithmetical conditions on the five parameters which ensure we shall get a 3-gem.

2.3 2-Dipoles and ρ_2-Pairs

In this section we prepare the stage to present the detailed example of attraction in the next one. We show that, as with the 1-dipole, the cancellation of 2-dipoles and the switching of ρ_2-pairs in a 3-gem maintain the induced 3-manifold. It is easier to observe the invariance under 2-dipole cancellation in the dual construction. Therefore, in what follows, we detail this construction.

2.3.1 Dipoles in the Dual Construction

Two edges of colors $i,j \in \{0,1,2,3\}$ form a *2-dipole* if they have the same ends and these ends are in distinct $k\ell$-gons, where $\{k,\ell\}$ is the complement of $\{i,j\}$ in the set of colors. The colors i and j are said to be *involved* in the 2-dipole. The presence of a 2-dipole enables a trivial simplification. The *cancellation of a 2-dipole* with ends v, w is defined as follows: remove vertices v and w together with the two edges that form the dipole and weld the free ends along edges of the same colors. For instance, if the black and white vertices below are in distinct 13-gons, then they form a 2-dipole which can be cancelled as shown.

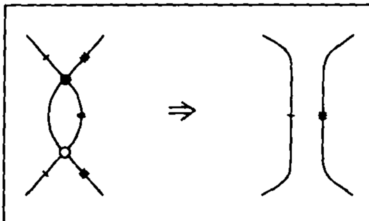


Fig. 36: Cancellation of a 2-dipole

The *creation of a 2-dipole* is the inverse operation. The cancellation (or creation) of a 2-dipole maintains the associated 3-manifold. We leave as an exercise for the reader to convince himself of this basic fact, given our primal construction of the induced 3-manifold. However, there is a dual construction to get the manifold $|G|$ associated to a 3-gem G. Under this construction, the invariance of 2-dipole cancellation or creation becomes straightforward.

Consider a collection of tetrahedra, each with the colors $\{0,1,2,3\}$ labelling its four vertices. These tetrahedra are in 1-1 correspondence with the vertices of G. For each i-colored edge of G we glue the pair of tetrahedra corresponding to its ends via the triangular face not containing i so as to match the other 3 colors, $\{j,k,l\}$. Do this for every edge of G and the result is a manifold $|G|$ if G is a 3-gem, see [Gag79a]. This tridimensional dual complex associated to a 3-gem G is denoted D. This construction produces the same manifold in the case of a 3-gem G, but it asssociates a topological space to every subgraph of G, contrary to the primal construction, where the space is only defined for the whole G and only in the case that it is a 3-gem. Nevertheless, the graphic nature of the primal model makes it substantially better for computations. Triangulations which admit a $(n+1)$-coloration of the vertices of its tetrahedra are called *colored triangulations*.

Both constructions are important for the topological interpretation of the fundamental objects and operations in the theory. In fact, it is convenient to consider the primal and the dual complexes at hand. Observe the following correspondence between dual cells:

- a vertex in v in $G \rightleftharpoons$ a solid tetrahedron T_v in D whose vertices are labelled $0,1,2,3$;

- an i-colored edge e_i in $G \rightleftharpoons$ a triangular 2-cell E_i in D whose vertices are labelled with the 3 colors distinct from i;
- a bigon B_{ij} using colors i, j in $G \rightleftharpoons$ an edge b_{ij} in D whose ends are labelled h, k, where (h, i, j, k) is a permutation of $(0, 1, 2, 3)$;
- a $\hat{i}$-residue V_i in $G \rightleftharpoons$ a vertex of D labelled i.

Here is what happens, in dual language, when a 2-dipole is created:

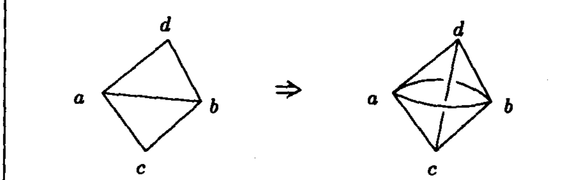


Fig. 37: 2-Dipole creation in the dual: the *pillow* move

Two triangular regions a, b, c and a, b, d (edges in the 3-gem) sharing a common edge ab (bigon in the 3-gem) are inflated into a *pillow*: two curved tetrahedra sharing two faces situated in the plane of the paper. One tetrahedron is below the plane of the paper and the other above it. Both have the same four vertices. This dual manifestation of 2-dipole creation/cancellation is called *a pillow move* and it clearly does not change the 3-manifold associated to the complex.

In the case of a 1-dipole creation, in dual language, we have:

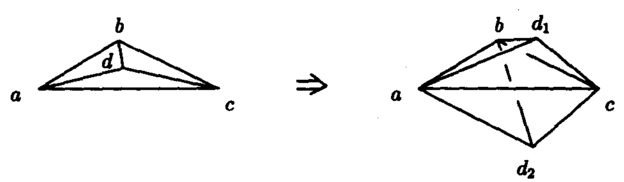


Fig. 38: 1-Dipole creation in the dual: the *baloon* move

Three triangular regions a, b, d and a, c, d and b, c, d (differently colored edges in the 3-gem) sharing a common point d (triball in the 3-gem) are inflated into two tetrahedra sharing a face (an edge of the fourth color in the 3-gem). This configuration is named a *baloon*. Note that a baloon has **five** distinct vertices.

Dipoles are important in the theory of 3-gems. The Fundamental Theorem of this theory is:

Theorem 4 (Ferri and Gagliardi [FG82]) *Two 3-gems induce the same 3-manifold if and only if there is a finite number of h-dipole creations/cancellations* $h = 1, 2$ *which transforms one 3-gem in the other.* ∎

We do not prove the theorem in this book. The proof is rather technical and relies implicitly on fundamental ideas of Alexander in the 20's [Ale30]. In [FG82] the proof is given in terms of crystallizations and is general for the dimensions $n \geq 2$.

2.3.2 The Switching of ρ_2-Pairs

Recall that a *ρ-pair* in a $(3+1)$-graph G is a pair of equally colored edges $\{a_1, a_2\}$ which are contained in exactly two or three bigons. Assume that G is a 3-gem. Even if G is not bipartite, we can partition the vertices of one of the bigons of a ρ-pair by labelling them alternatively °vertex and •vertex. This labels the four vertices of the ρ-pair. This labelling can be consistently extended to all the vertices of the other(s) bigon(s) associated to the ρ-pair: if not we would detect an odd polygon in a 3-residue contrary to the hypothesis that G is a 3-gem. Fig. 14 displays the operation of *switching* a ρ-pair.

If a ρ_2-pair is in the same ij-gon, in the same ik-gon and in distinct $i\ell$-gons, then it can be checked that its switching produces a $(3+1)$-graph which has one more $\hat{\ell}$-residue. Moreover, as we prove next, if the starting point is a 3-gem, the result of switching a ρ_2-pair is also a 3-gem (containing 1-dipoles) inducing the same 3-manifold. In this way, since it does not change the induced 3-manifold nor the number of vertices and it produces 1-dipoles, which can be cancelled, a ρ_2-pair is as good as a 1- or 2-dipole regarding simplifying the 3-gem. Note also that a pair of edges of color k incident to the vertices of a 2-dipole involving colors i and j is a ρ_2-pair. Thus a 2-dipole induces ρ_2-pairs.

We denote by $t_c(G)$ the number of $\hat{c}$-residues of a $(3+1)$-graph G.

Lemma 4 (Lemma 8 of [FL91]) *Let G be a 3-gem and (h, i, j, k) a permutation of its edge colors $(0, 1, 2, 3)$. Suppose that $\{a_1, a_2\}$ is a ρ_2-pair of color h appearing together in the same hi-gon, in the same hj-gon and belonging to distinct hk-gons. Let G' be the resulting $(3+1)$-graph after switching the pair. Then G' is a connected 3-gem, $|G'| \cong |G|$, $t_k(G') = t_k(G) + 1$ and $t_c(G') = t_c(G)$ for $c \neq k$.*

Proof: For this proof we have fixed (h, i, j, k) to be the identity. Because a_1, a_2 are in the same 03-gon of G', this is a connected $(3+1)$-graph. Note that $\{a_1, a_2\}$ when deleted from G increases the number of components of its $\hat{3}$-residue: embed the $\hat{3}$-residue of G containing a_1 and a_2 in the plane so that the bigons are the boundaries of the faces. Let β_c be the region bounded by the $0c$-gon containing (a_1, a_2), $c \in \{1, 2\}$. It follows that there is a closed path π in the plane contained in $\beta_1 \cup \beta_2$ and crossing once only the edges a_1 and a_2. Thus, their removal disconnects that 3-residue. A small perturbation in the closed path above, so as to miss a_1 and cross a_4 and a_3 instead, shows that the removal of $\{a_2, a_3, a_4\}$ also disconnects the $\hat{3}$-residue containing these edges.

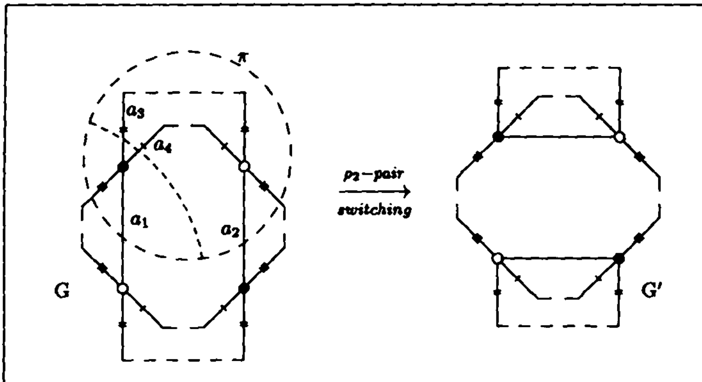


Fig. 39A: 3-Gem G before and $(3+1)$-graph G' after the switching

Therefore, we can create a 1-dipole D of color 3 by subdividing these three edges of G, as shown in graph H below.

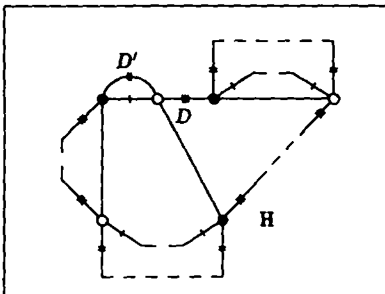


Fig. 39B: An intermediary 3-Gem H

This process also induces a 2-dipole D' involving colors 1 and 2. Cancellation of D' also produces the graph G'. Since G' is linked to G by dipole moves, G' is a 3-gem and $|G'| \cong |G|$.

Note that G' has the same number of $\hat{c}$-residues for $c \in \{0,1,2,3\}$ as H: the cancellation of a 2-dipole does not change the number of $\hat{c}$-residues. The comparison between the 3-residues of G and H is easy, since the first is obtained from the last by a cancellation of a 1-dipole of color 3. ■

2.4 A Detailed Example of Attraction

This section is a very detailed account of some basic aspects of the simplification theory for 3-gems. The reader has to have in mind that the procedures here presented are to be carried out by a computer. With a usual PC it would take only a few seconds. After the presentation of this case study we will feel free to display the output of the TS_ρ-algorithm without further explanations.

We have claimed that $S(3,3,2,1)$ induces the manifold S^3/Q_8, given by the identification scheme on the left.

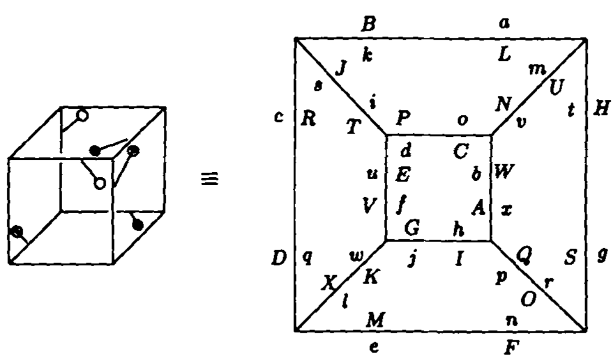


Fig. 40A: A 3-gem for the space of the 90-degree twisted solid cube

To prove this claim we use the construction of Section 1.3 to get a 3-gem on the right which induces the same 3-manifold as the one of the scheme. The *reduced adjacency matrix* of this 3-gem is given below. What characterizes these matrices are the bipartiteness of the 3-gem and that all the edges of one of the colors (in the case color 3) link vertices labelled with the same letter, lower and upper case, as x and X. A reduced adjacency matrix informs the neighbors of the capital labelled vertices by the four colors, thus the bipartite 3-gem can be recovered from such a matrix.

	A	B	C	D	E	F	G	H	I	J	K	L
0	b	a	d	c	f	e	h	g	j	i	l	k
1	h	c	b	e	d	g	f	a	p	k	j	m
2	x	k	o	q	u	n	j	t	h	s	w	a
3	a	b	c	d	e	f	g	h	i	j	k	l

	M	N	O	P	Q	R	S	T	U	V	W	X
0	n	m	p	o	r	q	t	s	v	u	x	w
1	l	o	n	i	x	s	r	u	t	w	v	q
2	e	v	r	d	p	c	g	i	m	f	b	l
3	m	n	o	p	q	r	s	t	u	v	w	x

2.4.1 Cancelling 1-Dipoles

This 3-gem has two $\hat{0}$-residues, depicted below. Consequently, we can eliminate a 1-dipole to coalesce these 3-residues into one. As a matter of fact, each 0-colored edge is a 1-dipole and we can cancel any of them. Let us choose 0-edge uV.

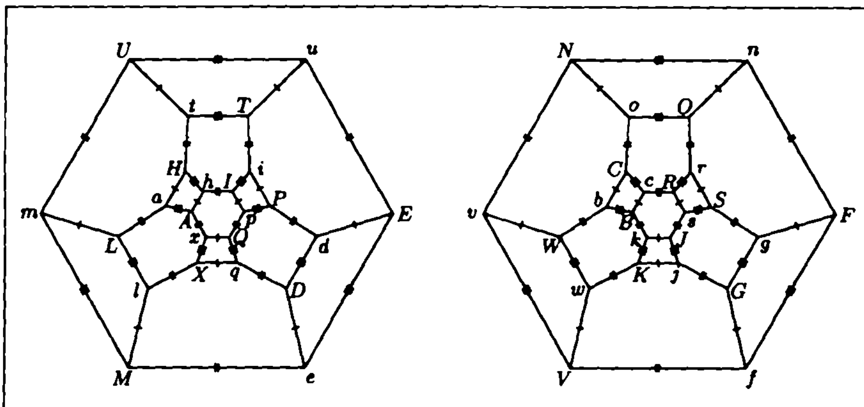


Fig. 40B: The two $\hat{0}$-residues of the 3-gem of Fig. 40A

After the cancellation, which is realized by the following local modification,

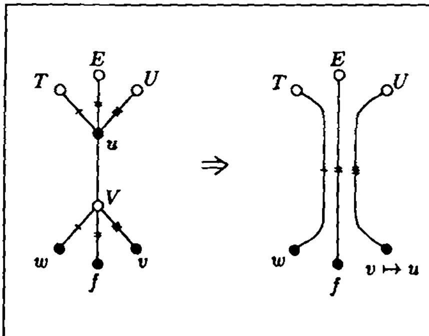


Fig. 40C: Cancelling the first 1-dipole

the vertices u and V disappear. To keep the lower/upper case compatibility associated with color 3, we rename v by u. Thus, the new adjacency matrix is

	A	B	C	D	E	F	G	H	I	J	K
0	b	a	d	c	f	e	h	g	j	i	l
1	h	c	b	e	d	g	f	a	p	k	j
2	x	k	o	q	f	n	j	t	h	s	w
3	a	b	c	d	e	f	g	h	i	j	k

	L	M	N	O	P	Q	R	S	T	U	W	X
0	k	n	m	p	o	r	q	t	s	u	x	w
1	m	l	o	n	i	x	s	r	w	t	u	q
2	a	e	u	r	d	p	c	g	i	m	b	l
3	l	m	n	o	p	q	r	s	t	u	w	x

The new 3-gem, G_{46}, has four $\hat{1}$-residues:

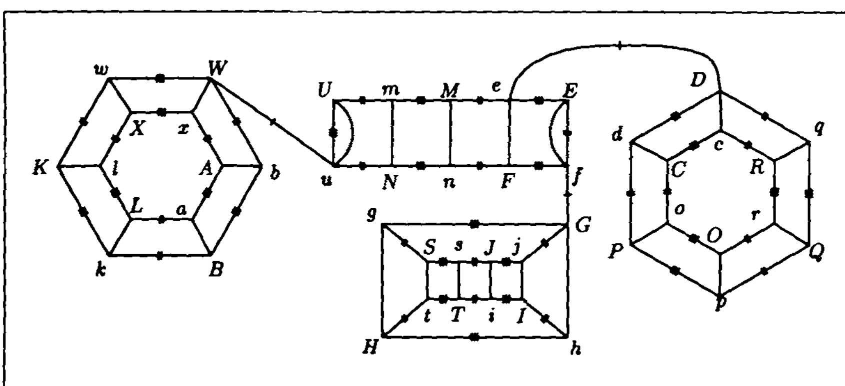


Fig. 40D: The four $\hat{1}$-residues of G_{46}

After the cancellation of 1-colored 1-dipoles eD, uW and fG, and the relabelling $d \mapsto e$, $w \mapsto u$ and $g \mapsto f$ we get a unique $\hat{1}$-residue of a 3-gem G_{40}:

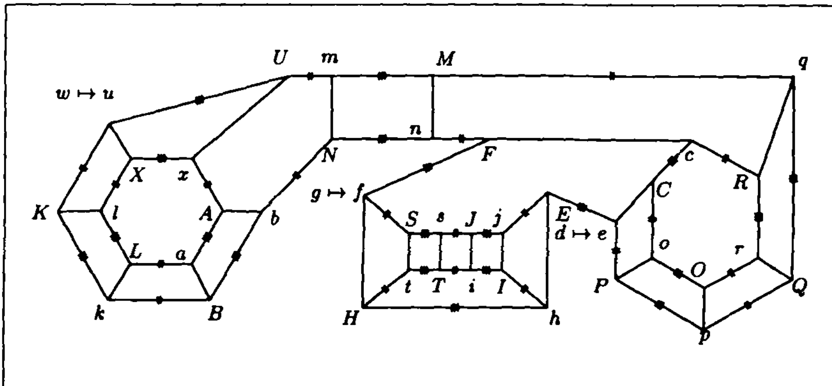


Fig. 40E: The unique $\hat{1}$-residue of G_{40}

The corresponding adjacency matrix is

	A	B	C	E	F	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	X
0	b	a	e	h	c	f	j	i	l	k	n	m	p	o	r	q	t	s	x	u
1	h	c	b	e	f	a	p	k	j	m	l	o	n	i	x	s	r	u	t	q
2	x	k	o	j	n	t	h	s	u	a	q	b	r	e	p	c	f	i	m	l
3	a	b	c	e	f	h	i	j	k	l	m	n	o	p	q	r	s	t	u	x

The new 3-gem G_{40} still has three $\hat{2}$-residues:

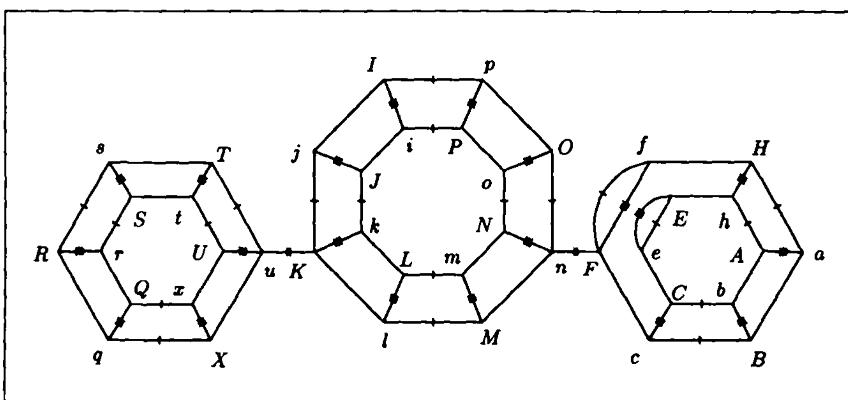


Fig. 40F: The three $\hat{2}$-residues of G_{40}

After the cancellation of 2-colored 1-dipoles uK and nF, and the relabelling $k \mapsto u$ and $f \mapsto n$ we get a 3-gem G_{36}

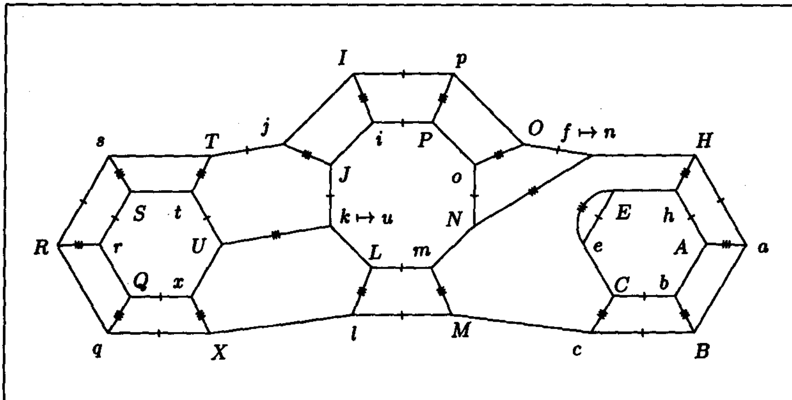


Fig. 40G: The unique $\hat{2}$-residue of G_{36}

The adjacency matrix now is

	A	B	C	E	H	I	J	L	M	N	O	P	Q	R	S	T	U	X
0	b	a	e	h	n	j	i	u	c	m	p	o	r	q	t	s	x	l
1	h	c	b	e	a	p	u	m	l	o	n	i	x	s	r	j	t	q
2	x	u	o	j	t	h	s	a	q	b	r	e	p	c	n	i	m	l
3	a	b	c	e	h	i	j	l	m	n	o	p	q	r	s	t	u	x

This 3-gem is a crystallization: there are no further 1-dipoles. However, 2-dipoles are present, and the simplification continues.

2.4.2 Cancelling 2-Dipoles

From the last adjacency matrix in our ongoing example we see that vertices X and l are common ends of edges of colors 0 and 2. Moreover they are in distinct 13-gons. Therefore, they form a 2-dipole which can be cancelled:

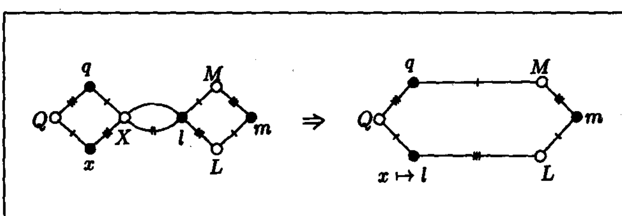


Fig. 40H: Cancelling the first 2-dipole producing G_{34}

The adjacency matrix for G_{34} becomes

	A	B	C	E	H	I	J	L	M	N	O	P	Q	R	S	T	U
0	b	a	e	h	n	j	i	u	c	m	p	o	r	q	t	s	l
1	h	c	b	e	a	p	u	m	q	o	n	i	l	s	r	j	t
2	l	u	o	j	t	h	s	a	q	b	r	e	p	c	n	i	m
3	a	b	c	e	h	i	j	l	m	n	o	p	q	r	s	t	u

Now vertices e and E form a 2-dipole, which can be cancelled as shown below:

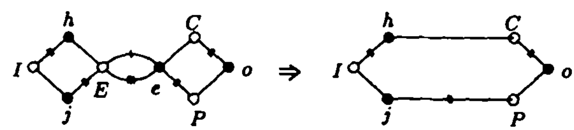


Fig. 40I: Cancelling the second 2-dipole producing G_{32}

Adjacency matrix for G_{32}:

	A	B	C	H	I	J	L	M	N	O	P	Q	R	S	T	U
0	b	a	h	n	j	i	u	c	m	p	o	r	q	t	s	l
1	h	c	b	a	p	u	m	q	o	n	i	l	s	r	j	t
2	l	u	o	t	h	s	a	q	b	r	j	p	c	n	i	m
3	a	b	c	h	i	j	l	m	n	o	p	q	r	s	t	u

See that vertices q and M form a 2-dipole, which can be cancelled as shown below:

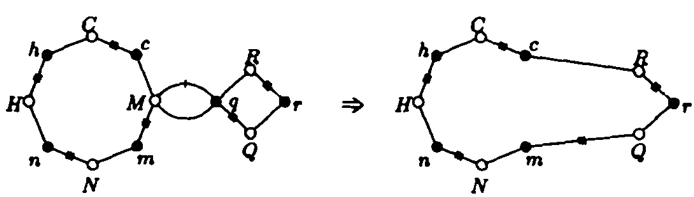


Fig. 40J: Cancelling the third 2-dipole producing G_{30}

Adjacency matrix for G_{30}:

	A	B	C	H	I	J	L	N	O	P	Q	R	S	T	U
0	b	a	h	n	j	i	u	q	p	o	r	c	t	s	l
1	h	c	b	a	p	u	q	o	n	i	l	s	r	j	t
2	l	u	o	t	h	s	a	b	r	j	p	c	n	i	q
3	a	b	c	h	i	j	l	n	o	p	q	r	s	t	u

Cancelling the 2-dipole formed by vertices c and R:

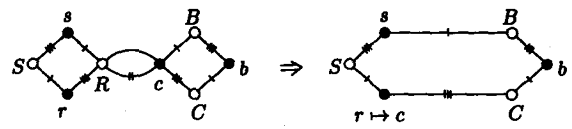


Fig. 40K: Cancelling the fourth 2-dipole producing G_{28}

Adjacency matrix for G_{28}:

	A	B	C	H	I	J	L	N	O	P	Q	S	T	U
0	b	a	h	n	j	i	u	q	p	o	c	t	s	l
1	h	s	b	a	p	u	q	o	n	i	l	c	j	t
2	l	u	o	t	h	s	a	b	c	j	p	n	i	q
3	a	b	c	h	i	j	l	n	o	p	q	s	t	u

This 3-gem is a crystallization and, as can be seen from the adjacency matrix, it does not have 2-dipoles.

2.4.3 Switching a ρ_2-Pair

Our current crystallization do have ρ_2-pairs. This can be better appreciated if we present its $\hat{1}$-residue:

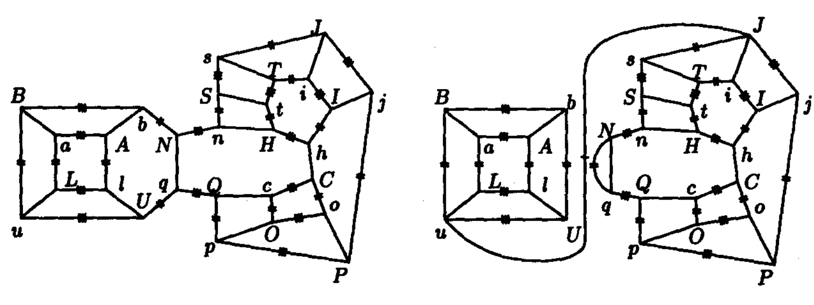


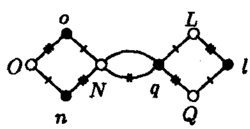
Fig. 40L: Switching a ρ-pair in G_{28}

Indeed, pairs of edges $\{bN, qU\}$ of color 2 and $\{nN, qQ\}$ of color 3 are ρ_2-pairs. We have chosen to switch the 2-colored one to continue our simplification towards

$S(3,3,2,1)$. After the switching, we cancel the 1-dipole uJ, making as usual the substitution $j \mapsto u$, to arrive at G_{26} which has the adjacency matrix:

	A	B	C	H	I	L	N	O	P	Q	S	T	U
0	b	a	h	n	u	i	q	p	o	c	t	s	l
1	h	s	b	a	p	q	o	n	i	l	c	u	t
2	l	s	o	t	h	a	q	c	u	p	n	i	b
3	a	b	c	h	i	l	n	o	p	q	s	t	u

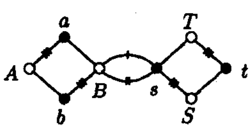
Now vertices q and N form a 2-dipole involving colors 0 and 2 whose elimination produces G_{24}, with adjacency matrix (after the relabelling $n \mapsto q$):



	A	B	C	H	I	L	O	P	Q	S	T	U
0	b	a	h	q	u	i	p	o	c	t	s	l
1	h	s	b	a	p	o	q	i	l	c	u	t
2	l	s	o	t	h	a	c	u	p	q	i	b
3	a	b	c	h	i	l	o	p	q	s	t	u

Fig. 40M: Eliminating a fifth 2-dipole and obtaining G_{24}

Vertices s and B are the ends of a 2-dipole involving colors 1 and 2. Its cancellation provides (under $b \mapsto s$) a smaller crystallization given by the matrix below:



	A	C	H	I	L	O	P	Q	S	T	U
0	s	h	q	u	i	p	o	c	t	a	l
1	h	s	a	p	o	q	i	l	c	u	t
2	l	o	t	h	a	c	u	p	q	i	s
3	a	c	h	i	l	o	p	q	s	t	u

Fig. 40N: Eliminating a sixth 2-dipole and obtaining G_{22}

2.4.4 Final Simplifications to Get $S(3,3,2,1)$

Now there are no dipoles nor ρ_2-pairs. However we can create a 1-dipole of color 3, and after cancelling another 1-dipole, a 2-dipole that arises can be cancelled. The net effect is that the number of vertices goes down by 2.

Here is how this program is performed in our current crystallization, which is given by the last adjacency matrix. A drawing of its $\hat{3}$-residue is depicted below.

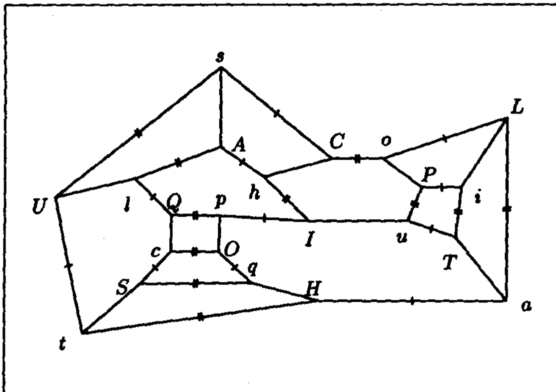


Fig. 40O: The unique $\hat{3}$-residue of G_{22}

Note that we can create a 1-dipole by subdividing the edges uI of color 0, aH of color 1 and oC of color 2. Let the new vertices of the 1-dipole which we create be denoted by z and Z and the new 3-gem be denoted by H_{24}. Here are two $\hat{3}$-residues of H_{24}:

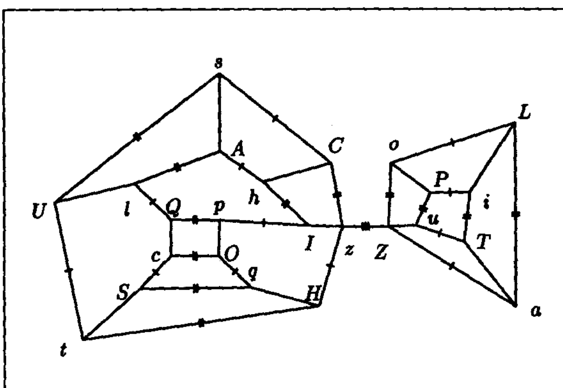


Fig. 40P: After the creation of zZ G_{22} becomes H_{24}

Now cancel the 1-dipole oO, obtaining the crystallization H_{22} given by the following matrix. Note that p and P become vertices of a 2-dipole.

	A	C	H	I	L	P	Q	S	T	U	Z
0	s	h	q	z	i	p	c	t	a	l	u
1	h	s	z	p	q	i	l	c	u	t	a
2	l	z	t	h	a	u	p	q	i	s	c
3	a	c	h	i	l	p	q	s	t	u	z

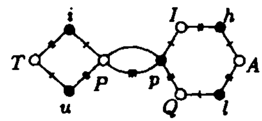


Fig. 40Q: Cancelling a seventh 2-dipole and obtaining H_{20}

Here is the resulting matrix, after the elimination of this 2-dipole. Note also that i and I are vertices of another 2-dipole:

	A	C	H	I	L	Q	S	T	U	Z
0	s	h	q	z	i	c	t	a	l	u
1	h	s	z	i	q	l	c	u	t	a
2	l	z	t	h	a	u	q	i	s	c
3	a	c	h	i	l	q	s	t	u	z

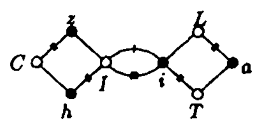


Fig. 40R: Cancelling an eighth 2-dipole and obtaining r_1^{18}

After the elimination of this final 2-dipole and exchanging colors 0 and 3 we get to the adjacency matrix which corresponds, to precisely the crystallization $\mathcal{S}(3,3,2,1)$, up to vertex relabelling.

	A	C	H	L	Q	S	T	U	Z
0	a	c	h	l	q	s	t	u	z
1	h	s	z	q	l	c	u	t	a
2	l	z	t	a	u	q	h	s	c
3	s	h	q	z	c	t	a	l	u

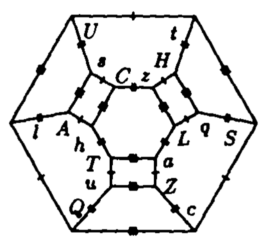


Fig. 40S: Finally arriving at $\mathcal{S}(3,3,2,1)$

This finishes our proof that $\mathcal{S}(3,3,2,1)$ induces the same 3-manifold as the solid cube under opposite face identifications preceded of 90-degree twists. As a matter of fact $\mathcal{S}(3,3,2,1)$ is also identified as the first rigid 3-gem with 18 vertices in our catalogue of Chapter 5. We summarize the process of attraction by

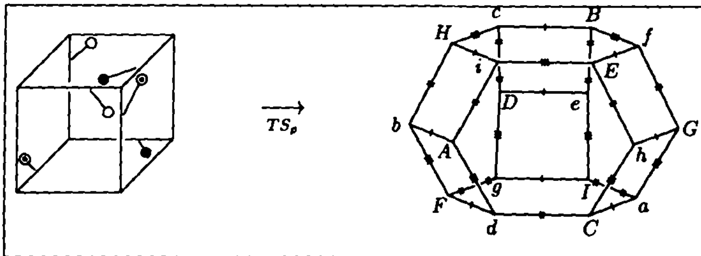


Fig. 40T: $\mathcal{S}(3,3,2,1)$: the superattractor r_1^{18} for S^3/Q_8

In Chapter 4 we describe an algorithm, named TS_ρ-algorithm, which incorporates the essence of the method here presented. This algorithm is the computational basis of the topological classification of 3-gems.

$\mathcal{S}(3,3,2,1)$ is indeed the superattractor for S^3/Q_8 . The complete catalogue of bipartite rigid 3-gems up to 30 vertices proves this fact since there are no other 3-gems up to 18 vertices inducing S^3/Q_8.

We next deal with a special kind of symmetry that often arises on superattractors.

2.5 K_4-Symmetry and Strings Presenting 3-Manifolds

We isolate a type of symmetry that arises frequently in a number of gems, particularly in superattractors. From this symmetry we get a contracted presentation named *string presentation*.

2.5.1 Identifying a Special K_4-Symmetry

The four 3-residues of $\mathcal{S}(3,3,2,1)$ look precisely the same:

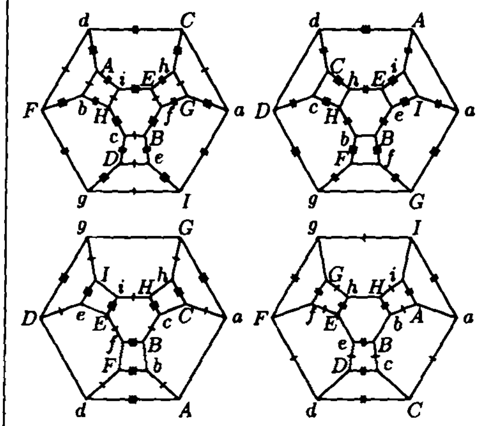


Fig. 41: The four 3-residues of $\mathcal{S}(3,3,2,1)$

The reader can see that the symmetries come not only from automorphic 3-residues but are indeed involutory automorphisms of the full 3-gem (involving the four colors). These automorphisms commute and each of them interchanges two pairs of colors. We identify a first involutory automorphism, σ^+, which interchanges pairs of colors (0,1) and (2,3); a second σ^-, which interchanges pairs of colors (0,2) and (1,3). These automorphisms are obtained by translations which maps the $\hat{0}$-residue over the $\hat{1}$-residue (σ^+) and the $\hat{0}$-residue over the $\hat{2}$-residue (σ^-). They commute and their product, also an involution, is denoted $\sigma^\times$. Note that $\sigma^\times$ is obtained by a translation which maps the $\hat{0}$-residue over the $\hat{3}$-residue. In this way the identity automorphism, σ^+, σ^- and $\sigma^\times$ form a Kleinian group of automorphisms which permit the simplification of the presentation of the 3-gem, as we now discuss.

The topological notion of *preserving the orientation* means, in terms of graphs, to preserve the bipartition. In general, a 3-gem G is a *σ-gem* if there are orientation preserving commuting involutions σ^+ and σ^- acting on the vertices of G and inducing automorphisms of G which satisfy:

σ^+ interchanges the pairs of colors $(0,1)$ and $(2,3)$;
σ^- interchanges the pairs of colors $(0,2)$ and $(1,3)$.

As a consequence of the definition, the composition

$$\sigma^\times = \sigma^+ \circ \sigma^- = \sigma^- \circ \sigma^+ ,$$

denoted by justaposition, $\sigma^+\sigma^-$, is again an automorphic involution of G, now interchanging the pairs of colors $(0,3)$ and $(1,2)$. The orientation preserving pairwise commuting involutions σ^+, σ^- and $\sigma^\times$ are called *σ-symmetries*. Suppose that we have a triple (G, σ^+, σ^-), where the σ^+ and σ^- are σ-symmetries of a 3-gem G. To derive the string presentation, γ_G, associated to this triple we first define the *expanded gist* Γ_G for (G, σ^+, σ^-). This is the (combinatorial) graph whose vertices are those of G and whose edges are of four types:

- the original 0-colored edges;
- $^+$labelled edges which link vertices v and $\sigma^+(v)$ for each vertex v of G;
- $^-$labelled edges which link vertices v and $\sigma^-(v)$ for each vertex v of G;
- $^\times$labelled edges which link vertices v and $\sigma^\times(v)$ for each vertex v of G.

Note that the signed labelled edges can be loops, since the corresponding involutions can have fixed points.

From the expanded gist it is easy to recover the 3-gem. Observe that the colors of a 3-gem may be thought as fixed point free involutions $\{\epsilon_0, \epsilon_1, \epsilon_2, \epsilon_3\}$ acting on their vertices. Note also that ϵ_0 is the only present in Γ_G. Of course, any ϵ_i can take the place of ϵ_0 in the definition of expanded gist, in what concerns the recoverability — see the proof of Proposition 4. Also we can drop edges of type $+$, of type $-$, or of type $\times$, since a σ-symmetry equals the (commuting) product of the other two.

Proposition 4 *A 3-gem is recoverable from any of its expanded gist.*

Proof: To recover a 3-gem G from one of its expanded gist (G, σ^+, σ^-) just note that the color exchanging symmetries means $\sigma^+\epsilon_1 = \epsilon_0\sigma^+$, $\sigma^-\epsilon_2 = \epsilon_0\sigma^-$ and $\sigma^\times\epsilon_3 = \epsilon_0\sigma^\times$. Since all of the permutations are involutions these equations are equivalent to

$$\epsilon_1 = \sigma^+\epsilon_0\sigma^+, \quad \epsilon_2 = \sigma^-\epsilon_0\sigma^-, \quad \epsilon_3 = \sigma^\times\epsilon_0\sigma^\times.$$

As ϵ_0, σ^+, σ^- and $\sigma^\times$ are present in the expanded gist, the result follows. ∎

2.5.2 Getting a String Presentation from the σ-Symmetries

The expanded gist associated to the σ-symmetries which we displayed for $S(3, 3, 2, 1)$ is shown below, on the left:

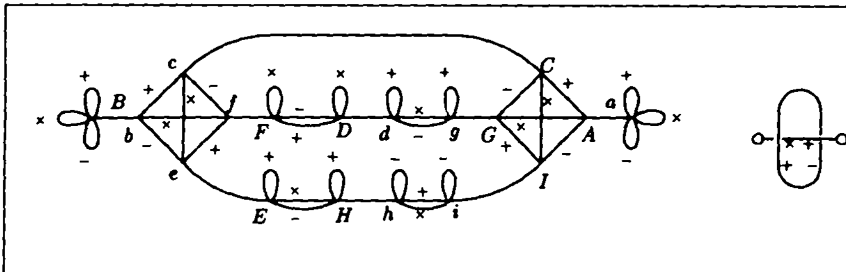


Fig. 42: From gist to string presentation

The small figure on the right is a *string presentation* the 3-gem $\mathcal{S}(3,3,2,1)$. By means of the generic local conventions:

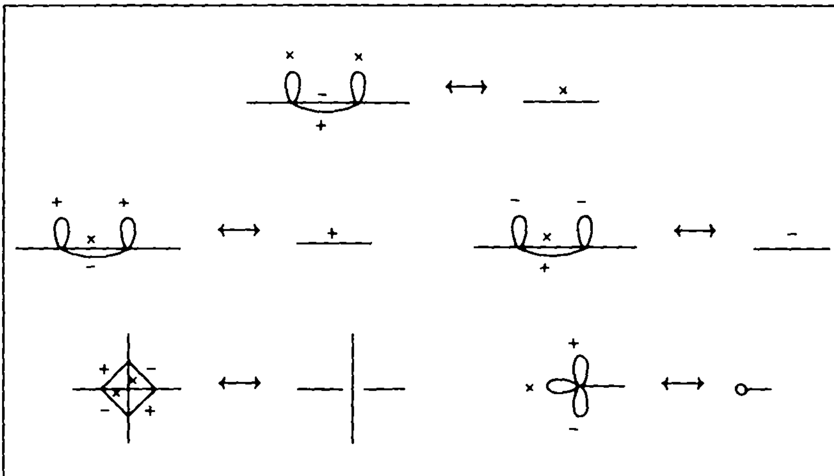


Fig. 43: Getting from the gist to the string presentation ... and back

it holds all the information for obtaining the gist, and hence the 3-gem. Note that the $^+$angle in the gist corresponds, in the string presentation, to the angle swept by the overpass under an anticlockwise rotation to coincide with the underpass. The main use of the string presentation in this book is its great convenience in presenting σ-gems (which appears frequently). The (finite) set of all string presentations associated with a 3-gem are said to be *equivalent*. A string presentation is also said to be *equivalent* to the associated 3-gem.

A theory of these charged strings presenting 3-manifold appears in [Lin95]. In particular, from such an object inducing an M^3 some decompositions of M^3 along 2-spheres and tori are transparent.

To have isomorphic 3-residues is not enough to qualify a 3-gem as a σ-gem. For instance, the superattractor r_2^{24} for $S^3/\langle 5,3,2\rangle$ has isomorphic 3-residues:

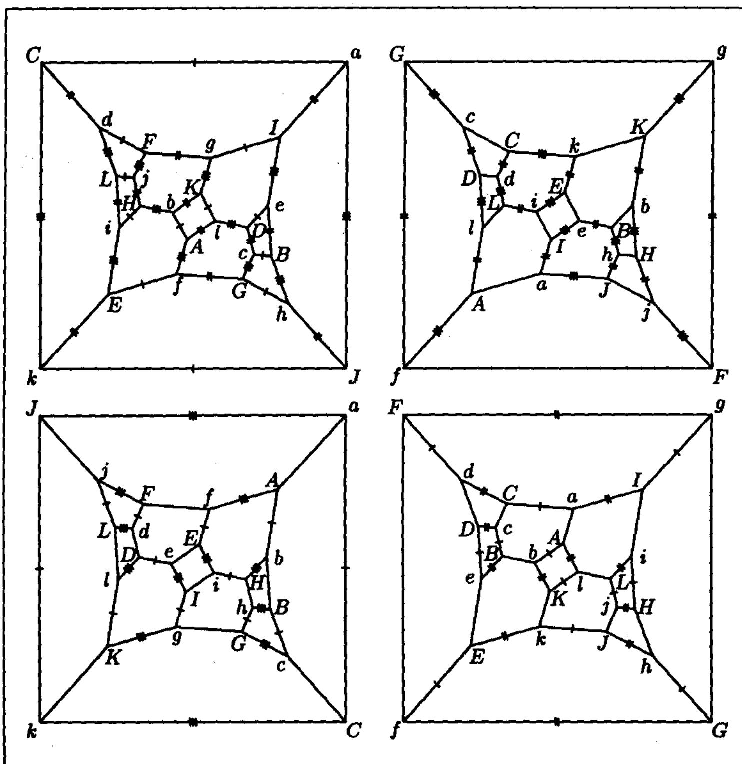


Fig. 44: The four 3-residues of r_2^{24}, superattractor for $S^3/\langle 5,3,2\rangle$

However, these automorphisms are not involutions; so r_2^{24} is not a σ-gem and it does not admit a string presentation. The Poincaré sphere is also induced by $\mathcal{S}(5,3,2,1) = r_{24555}^{30}$ and this is a σ-gem:

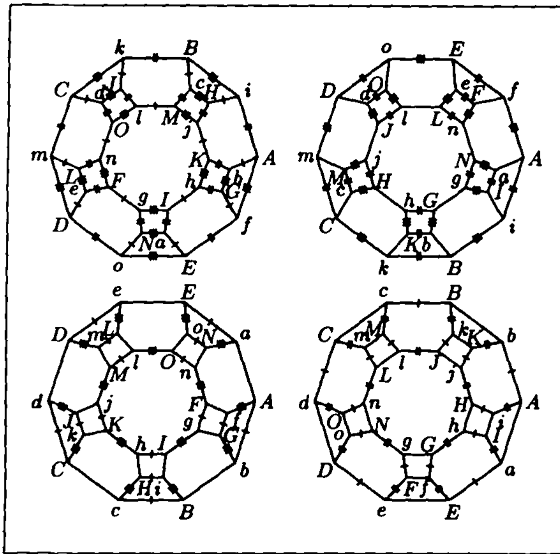


Fig. 45A: The four 3-residues of $\mathcal{S}(5,3,2,1) = r^{30}_{24555}$

The string presentation associated to the above σ-symmetries is simply:

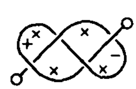


Fig. 45B: A string presentation equivalent to r^{30}_{24555}

As we have seen in the introduction Poincaré's homology sphere, the spherical dodecahedral space, is also induced by $\mathcal{S}(3,5,4,1) = r^{30}_{54668}$ because it is absorbed by r^{24}_1. This gem is also a σ-gem having the string presentation shown in Fig. 9.

2.5.3 An Example of Montesinos and Boileau-Zieschang

One of the important findings of last decade was the discovery that the Heegaard genus of a 3-manifold can be greater than the minimum number of generators of its fundamental group. In [BZ84] Boileau and Zieschang displayed an infinite family of 3-manifolds (Seifert spaces) having Heegaard genus 3 whose fundamental group is 2-generated.

The simplest of these manifolds is depicted in the Heegaard diagram (which we discuss in more details in Section 2.7) below. This diagram of genus 3 has been obtained by Montesinos, see Fig. 6 of [Mon89]:

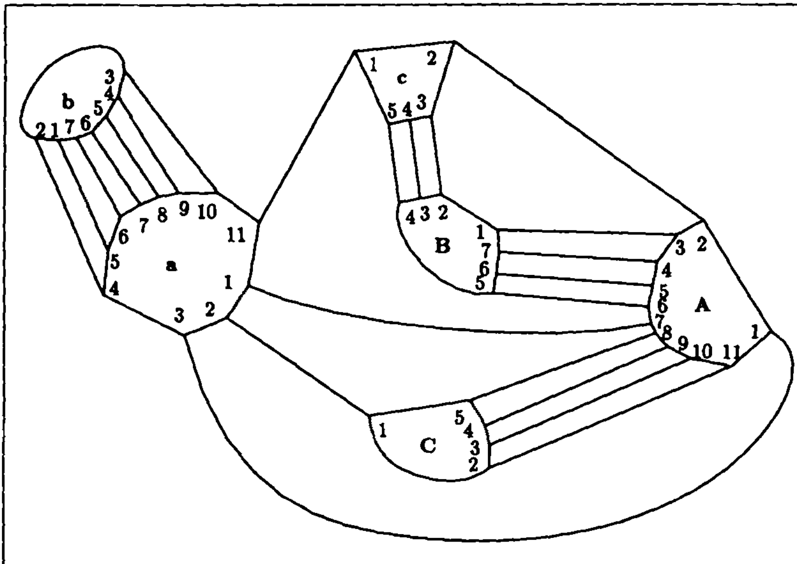


Fig. 45C: An important Heegaard diagram

A presentation for the fundamental group of the 3-manifold can be directly read from the Heegaard diagram. It is

$$\langle a, b, c \mid a^2 = (ba^{-1})^2, a = cac, (ba^{-1}c)^3 = (ab^{-1})^2 \rangle .$$

As it can be seen, even the first example of the Boileau-Zieschang family is very complicated. As it happen, a simplified 3-gem inducing the above 3-manifold admits a string presentation.

Given a Heegaard diagram in a solid ball with handles as above, it is straightforward to obtain a 3-gem inducing the same 3-manifold: double all the edges between the handles; these edges are of colour 1; there exists now an even number of edges through each handle; let the boundary of the disks of the handles to be the 23-gons; the edges of color 0 realize the passage through the handles. The resulting $(3+1)$-graph is a 3-gem. It is not difficult to show that this 3-gem induces the same 3-manifold as the one given by the Heegaard diagram. See Section 2.7. This algorithm applied to the above diagram produces a 3-gem with 92 vertices. By using the simplification algorithm TS_ρ of Chapter 4, this 3-gem produces a u^0-class of sixteen 3-gems each with 32 vertices, and each inducing the same 3-manifold. The fourteenth 3-gem in this set is a σ-gem whose string presentation we reproduce below:

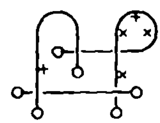


Fig. 45D: A string presentation for the same 3-manifold

We have included this example here to show that the string presentation, when available, provides a much more compact presentation than Heegaard diagrams. Indeed, it is the internal theory of 3-gems which provides the string presentation: by squeezing as much as we can the 3-gems inducing a certain 3-manifold, usually we get highly symmetric objects. The σ-symmetries (which are not uncommon) produce the string presentation.

It is easy to read a presentation of the fundamental group of a 3-manifold from a string presentation for it using the 01-algorithm of Subsection 2.8.1 for the corresponding crystallization.

2.5.4 A Conjecture on $S^1\times$ Orientable Surfaces

We finish this section with a conjecture motivated by the regular shape of the following string presentations and the fact that it holds for $n = 1$ and seems to hold for small n's. By T^2_n we mean the orientable surface homeomorphic to the connected sum of n copies of $T^2 = S^1 \times S^1$.

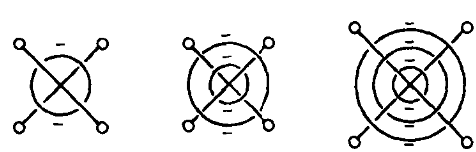


Fig. 46: String presentations for $T^2_n \times S^1, n = 1, 2, 3$

Conjecture 3 *The family of string presentations $t^2_n \times s^1$, above depicted for $n = 1, 2, 3$, are the superattractors for the spaces $T^2_n \times S^1$.*

We next deal on how to recover a distinguished vertex labelling from a vertex-unlabelled 3-gem.

2.6 A Recoverable Vertex Labelling: The Code

Usually the specific vertex labelling of a 3-gem (or more generally of an $(n+1)$-graph) is irrelevant. Beyond being used for reference purposes in a explanation, its main use is to permit the input of the object into a computer. It is a very important property of the $(n+1)$-graphs the possibility of providing a distinguished labelling, i.e., one which is quickly recoverable solely from the topology of the edge colored graph. The importance of this property is that it solves the isomorphism problem for $(n+1)$-graphs, objects that includes the 3-gems.

2.6.1 DFS-Numbering Algorithms

Let $\Delta_n = \{0, 1, 2, \ldots, n\}$ be the set of colors attached to the edges of an $(n+1)$-graph G. Given a permutation π of Δ_n and a vertex r of G we define a bijection:

$$N_\pi^r : V(G) \longrightarrow \{1, 2, \ldots, |V(G)|\}$$

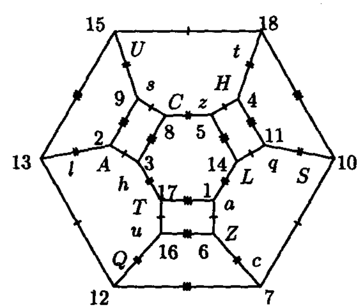
by a standard algorithm, which is a slight variation of the well known technique to explore a graph named *depth-first search (DFS)* , [Gol80]. In fact we need only a simplified DFS, because we only need to number the vertices in the order that DFS finds them. The important point for us is that the list of adjacent vertices to v, Adj(v) is ordered according to π: the *first neighbor* of v is $\epsilon_{\pi_0}(v)$, its *second neighbor* is $\epsilon_{\pi_1}(v)$, and so one ... the $i+1$-th neighbor is $\epsilon_{\pi_i}(v)$.

In DFS we start at a *root vertex* r and then visit the first neighbor of r which was not yet visited, say a. Then we proceed with the same idea: visit the first neighbor b of a which was not yet visited; then repeat with b and the next ones. As we go deeper into the graph we will eventually come to a vertex z which has no unvisited neighbors. At this point we return to the vertex y immediately preceding z and revisit y, trying for not yet visited neighbors of y. Note that if the graph is connected, then each vertex will be visited. This algorithm has complexity proportional to the number of edges of the graph. Its main subroutine, $DFSEARCH(v)$, fits very well for a recursive approach, in which the compiler takes care of the backtracking by means of an internal stack. Since we only work with connected graphs, a single call to $DFSEARCH(r)$ will produce the DFS-numbering.

Algorithm 1 (DFS-Numbering in General Connected Graphs) .

```
procedure DFSEARCH(v):
begin
    Mark v "visited"; i ←— i + 1; DFSNUMBER(v) ←— i;
    for each w ∈ Adj(v) do
        if w is marked "unvisited" do
            DFSEARCH(w);
end;
begin
    i ←— 0;
    Mark all vertices "unvisited";
    DFSEARCH(r);
end.
```

Here is the numbering of the a-rooted DFS with $\pi = Id$ when applied to the crystallization for $\mathcal{S}(3,3,2,1)$ (with the final vertex labelling after the simplification) and the adjacency matrix based on the numbering:



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
0	2	1	4	3	6	5	8	7	10	9	12	11	14	13	16	15	18	17
1	6	3	2	5	4	1	10	9	8	7	14	13	12	11	18	17	16	15
2	14	13	17	18	8	7	6	5	15	11	10	16	2	1	9	12	3	4
3	17	9	8	11	14	16	12	3	2	18	4	7	15	5	13	6	1	10

Fig. 47A: Example of DFS-numbering and respective matrix

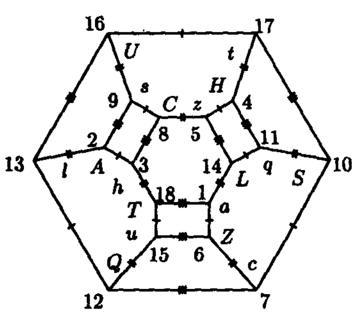
Note that the matrix depends only on the root vertex and on the permutation π, which orders the neighbors.

Since we deal only with bipartite $(n+1)$-graphs, we are going to use a small conceptual modification of the DFS-approach. We want the (odd/even)-numbering to respect the bipartition. This is not the case with the general approach: note that vertices 13 and 15 are neighbors. One simple solution, is in the backtracking, to look only for even numbered vertices. A single parity test added to the previous algorithm provides the convenient approach.

Algorithm 2 (Bipartite DFS-Numbering in Bipartite Connected Graphs)

```
procedure DFSEARCH(v):
begin
     Mark v "visited"; i ← i + 1; N_π^r(v) ← i;
          for each w ∈ Adj(v) do
               if w is marked "unvisited"
                    and i, N_π^r(v) have the same parity do
                         DFSEARCH(w);
end;
begin
     i ← 0;
     Mark all vertices "unvisited";
     DFSEARCH(r);
end.
```

The bipartite numbering corresponding to our example and respective reduced adjacency matrix are:



	1	3	5	7	9	11	13	15	17
1	6	2	4	10	8	14	12	18	16
2	14	18	8	6	16	10	2	12	4
3	18	8	14	12	2	4	16	6	10

Fig. 47B: Bipartite DFS-numbering on the same example

2.6.2 The Code of a Connected Bipartite $(n+1)$-Graph

The easiest way to present a $(n+1)$-graph G, is to use a $(n+1)\times|V(G)|)$-matrix, also denoted G, where $G(c,v)$ is the c-colored neighbor of v. The N_π^r*-code* of a connected bipartite $(n+1)$-graph G is the $(n\times\frac{1}{2}|V(G)|)$-matrix whose (i,j)-entry is

$$\frac{1}{2}N_\pi^r(G(\pi(i),(N_\pi^r)^{-1}(2j-1))).$$

Here, $1\leq i\leq n$ and $1\leq j\leq\frac{1}{2}|V(G)|$. Thus, the neighbors by color $\pi(0)$ are not explicitly given by the code. However, we know that the number of the $\pi(0)$-neighbor of vertex number i is $i+1$ if i is odd and $i-1$ if i is even. Also we take advantage of the bipartiteness and present explicitly only the neighbors of the odd numbered vertices. These are even numbers and we use such numbers divided by two in the codes. In the above example we get for the N_{Id}^a-code:

3	1	2	5	4	7	6	9	8
7	9	4	3	8	5	1	6	2
9	4	7	6	1	2	8	3	5

$\rightleftharpoons$

cabedgfih
gidcheafb
idgfabhce

We may also replace the numerical labellings given by the bipartite DFS algorithm by the literal ones: $1,2,3,4,5,6,\ldots$ becomes $a,A,b,B,c,C,\ldots$. Instead of dividing by two we replace the upper letters by the corresponding lower case ones.

Observe that from any of its N_π^r-codes, we can recover G up to color permutation. In fact, the permutation π is lost and we *canonically* color the edges by assuming the permutation to be the identity.

The *code for a connected bipartite* $(n+1)$*-graph* G is the lexicographic maximum among the N_π^r-codes for G, where we write the rows in sequence and consider each N_π^r-code as a single vector with $\frac{1}{2}n|V(G)|$ coordinates.

Henceforth all the vertex labellings of the diagrams presenting 3-gems by its $\hat{0}$-residues and an lower/upper case implicitly given 0-colored edges are the ones which attain the code. A diagram like this is called *canonical presentation* of the 3-gem. Thus the code of the 3-gem inducing S^3/Q_8 (and for any other 3-gem) is read directly from its canonical presentation:

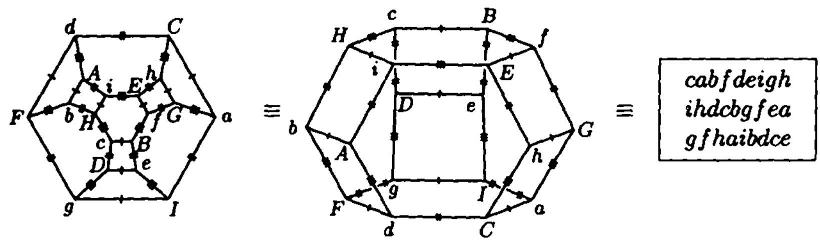


Fig. 48: Bijection between canonical presentation of a 3-gem and its code

Theorem 5 *Two connected bipartite $(n+1)$-graphs are isomorphic up to color permutation if and only if their codes coincide.*

Proof: This is a straighforward consequence of the bipartite DFS-algorithm which permits defining the N_π^r-codes and from the definition of the code as the lexicographic maximum. ∎

2.6.3 The Code of Non-Bipartite Gems

The code of a bipartite disconnected $(n+1)$-graph G is obtained as follows. Take the code of each component and order them first by non-decreasing number of vertices and then by their codes (thinking of them as a single number on base $|V_G|+1$). Add to the code-number of each vertex of component i the total number of vertices of the previous $i-1$ components. These code numbers so modified form an adjacency matrix which is, by definition, *the code of the bipartite non-connected $(n+1)$-graph G*.

Recall that n-Gems are characterized by the property that their n-residues induce the sphere S^{n-1}. Therefore these residues are bipartite. The *code of a non-bipartite n-gem* is obtained as follows. Take the codes of their $\hat{k}$-residues for all the colors k. Choose the greatest of them, say the one corresponding to the missing color i inducing the bipartite (in general disconnected) n-graph G_i. The code of G is the code of G_i followed by a permutation of the numbers $1, 2, \ldots, |V_G|$. The m-entry of this permutation is the code-number of the neighbor of the vertex with code-number m by the last color n. Clearly G is recoverable from its code and G and H are isomorphic if and only if their codes coincide.

Note that, in particular, the code of a non-bipartite 3-gem with n vertices has $n/2 + n/2 + n = 2n$ entries. The first and second group of $n/2$ entries form permutations of $1, 2, \ldots, |V_G/2|$ and the last n entries form a permutation of $1, 2, \ldots, |V_G|$.

Our implemented routines do not include facilities to deal with non-orientable 3-manifolds. Therefore we only consider bipartite 3-gems.

2.7 Heegaard Splittings of 3-Manifolds

At this point is convenient to recall and discuss in the context of 3-gems the well known Heegaard splittings of 3-manifolds [Hem76],[Sti80]. From them it is easy to get a presentation for the fundamental group of the associated 3-manifold. The best way we get a similar presentation from a 3-gem owes its justification to Heegaard diagrams. However, before discussing these we first derive a few useful arithmetic properties.

2.7.1 Arithmetic Properties of $(3+1)$-Graphs

The *connectivity of a closed connected surface* is defined as 2 minus its Euler characteristic. If the surface is not connected, its connectivity is the sum of those of its components. Given a $(3+1)$-graph G, let $\xi(G) \equiv \xi$ be the sum of the connectivities of the surfaces associated to the 3-residues. As before, t_c denotes the number of $\hat{c}$-residues. Let $b_{\bar{c}}$ denote the number of bigons not using color c and let ξ_c be the sum of the connectivities of the surfaces associated with the $\hat{c}$-residues. Finally, let $v = |V(G)|$, b and t the total number 2- and 3-residues of G.

Proposition 5 *For each $(3+1)$-graph G and each color c we have*

1. $\xi_c = v/2 + 2t_c - b_{\bar{c}}$
2. $v + t = b + \xi/2$.

Proof: Let R be a $\hat{c}$-residue. By definition, ξ_R equals $e_R + 2 - v_R - b_r$. By adding these expressions for all $\hat{c}$-residues we obtain

$$\xi_c = 3v/2 + 2t_c - v - b_{\bar{c}},$$

which yields the first part. To obtain the second just add the equality for all colors c. ∎

In the next proposition, b_{xy} denotes the total number xy-gons.

Proposition 6 *For each $(3+1)$-graph G and every permutation (i, j, k, h) of the colors we have*

$$b_{ij} + t_i + t_j - \xi_i/2 - \xi_j/2 = b_{kh} + t_k + t_h - \xi_k/2 - \xi_h/2.$$

Proof: Consider the four equations obtained from the first part of previous proposition by using i, j, k, h. By subtracting the sum of the last two from the sum of the first two and simplifying we obtain the above equality. ∎

Corollary 2 *For each crystallization G and every color permutation (i, j, k, h), we have*

$$b_{ij} = b_{kh}$$

Proof: In a 3-gem all the connectivities are 0, moreover, in a crystallization, $t_c = 1$ for every color. The result then follows from the previous proposition. ∎

2.7.2 Heegaard Splittings

A *handlebody* is a tubular neigborhood of a graph, i. e., the topological product of a (small) disk with a graph. A *Heegaard splitting* of a closed connected compact 3-manifold is a partition of it into two open handlebodies and a surface. This surface is the common boundary of the two handlebodies; if the manifold is oriented, it is an orientable surface of genus g, called the *genus of the splitting.*

Proposition 7 *Each closed, compact, connected 3-manifold M^3 has a Heegaard splitting.*

Proof: We know already that there are 3-gems G so that $|G| \cong M^3$. By eliminating 1-dipoles we may suppose that G is a crystallization. Therefore, we may suppose that M^3 is the union of the four triballs of G. Note that the union of any two of these triballs is a handlebody. Let (i, j, k, h) be any permutation of colors. Chosen any pair of triballs, say the ones B_i associated with the $\hat{i}$-residue and B_j associated to the $\hat{j}$-residue, the handlebody associated with the complementary pair $B_k \cup B_h$ has the same genus: the first has genus $b_{kh} - 1$ and the second genus $b_{ij} - 1$. From the above corollary, $b_{ij} = b_{kh}$. In this way, associated to a crystallization there are three natural Heegaard splittings:

$$\begin{array}{l}(\mathcal{H}_{23} = B_0 \cup B_1, \mathcal{H}_{01} = B_2 \cup B_3, \partial\mathcal{H}_{01})\\ (\mathcal{H}_{13} = B_0 \cup B_2, \mathcal{H}_{02} = B_1 \cup B_3, \partial\mathcal{H}_{02})\\ (\mathcal{H}_{12} = B_0 \cup B_3, \mathcal{H}_{03} = B_1 \cup B_2, \partial\mathcal{H}_{03}),\end{array}$$

establishing the proposition. ∎

An economical way to present a Heegaard splitting is to use the so called *Heegaard diagram.* It consists in a drawing of a handlebody of genus g together with g

disjoint closed curves in its boundary. The convention is that each one of these curves is going to be pasted against a distinct meridian of a canonical genus g complementary handlebody. It is easy to prove (proof left to the reader) that up to homeomorphism this scheme completely defines the mapping between the surfaces of the two handlebodies and so, the diagram defines the 3-manifold. In the discussion associated to Fig. 12 we have described a way to get a 3-gem from a Heegaard diagram in a canonically embedded handlebody. Now we consider the Heegaard diagram in the complement in S^3 of an embedded handlebody provided this complement is also a handlebody. The reason for this shift is because that is the way that we get Heegaard diagrams in Section 2.8, when deriving 3-gems from blackboard framed links.

For instance, RP^3 is defined by the Heegaard diagram in the complementary handlebody below on the left.

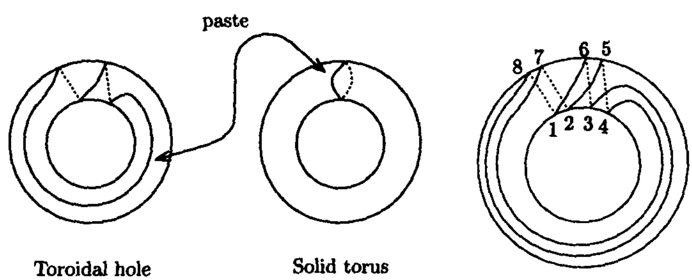


Fig. 49: A 3-manifold from a Hegaard diagram in the complementary handlebody

To get a 3-gem from a Heegaard diagram in a complementary handlebody in such a way that the induced 3-manifolds coincide proceed as follows:

Algorithm 3 (Heegaard diagram, complementary handlebody $\mapsto$ 3-gem) :

1. Draw a parallel to each curve in the diagram.
2. By effecting isotopies in the curves make sure that each one of them has transition points between the upper and the lower level of the diagram of the handlebody. Label these transition points which become the vertices of the 3-gem.
3. Since we doubled each curve, the boundary of the diagram are circles which have an even number of labels. Color with 1 the arcs of these circles which are between two parallel curves and with 2 the other arcs. Color with 3 the arcs of the curves which are above and with color 0 the ones which are below (dotted arcs in the above example). This coloring produces a $(3+1)$-graph. If this graph is not connected, make further isotopies in step 2 which will ensure a connected graph.

The result is not only a connected $(3+1)$-graph, but a 3-gem inducing the same 3-manifold as the original Heegaard diagram. The simplicity of this construction is an evidence on the naturality of our primal construction for 3-gems. Indeed, it realizes the pasting instructions in a straightforward way:

Justification: The handlebody is formed by B_0 and B_3, 3-balls corresponding to the $\hat{0}$-residue and to the $\hat{3}$-residue. There are as many $\hat{2}$-residues's as there are curves in the Heegaard diagram. The 3-balls corresponding to those are prisms whose lateral surfaces are to be pasted to the strips between the doubled curves. Finally, there is an extra 3-ball, B_1, induced by the unique $\hat{1}$-residue. Note that this justification is the same for the algorithm associated to Fig. 12.

In the above example we get a crystallization, since there is only one curve,

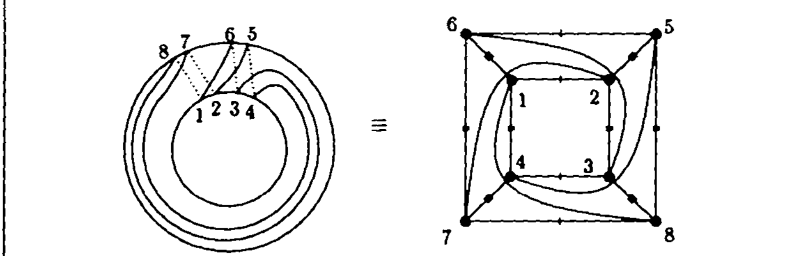


Fig. 50: A 3-gem from a Heegaard diagram in the complementary handlebody

which we recognize as the superattractor for RP^3. In general this construction provides a 3-gem but not a crystallization.

From a crystallization it is easy to reverse the process getting a Heegaard diagram. We can choose any one of the 6 handlebodies $\mathcal{H}_{ij}$'s to obtain such a diagram. Let (i,j,k,ℓ) be a permutation of $(0,1,2,3)$.

Algorithm 4 (Getting a Heegaard diagram from a crystallization G) :

1. Consider the handlebody $\mathcal{H}_{ij} = B_k \cup B_\ell$ where G is naturally embedded.
2. Drop all the edges of color i and of color j.
3. Drop all the edges of a $k\ell$-gon (any of them). The remaining $k\ell$-gons in $\partial\mathcal{H}_{ij}$ are the curves which defines the Heegaard diagram.

Justification: Let g be the genus of the handlebody $\mathcal{H}_{ij}$. A *system of meridian disks in a handlebody of genus g* is a set of g disks, each properly embedded (i.e. exactly the boundary of the disk is in the boundary of the handlebody) so that the closure of the handlebody minus the set of disks is a 3-ball. The justification of the algorithm is straightforward, since the remaining curves are the boundary of a system of meridian disks in the complementary handlebody. Thus, the instructions to get the 3-manifold from the Heegaard splitting matches with those to get the 3-manifold from the crystallization. ∎

Example: For the 3-gem below we get the Heegaard splitting depicted on the right. The Heegaard diagram is to be considered on the complementary handlebody. In particular, the two holes are in fact solid cylinders.

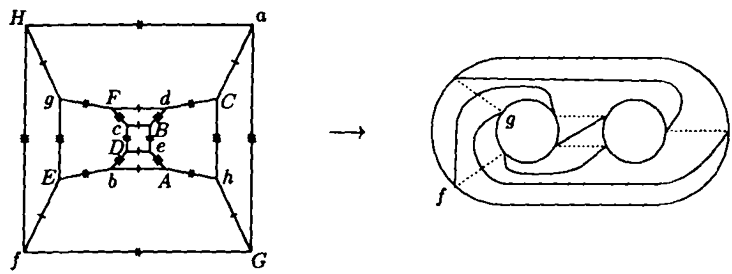


Fig. 51: From a 3-gem to a Heegaard diagram

We have used the two curves corresponding to the two 01-gons containing f and g, leaving out the one which contains a. The above 16-vertex 3-gem is, as a matter of fact, the superattractor for the lens space $L_{5,2}$. In Chapter 5 we justify this claim. This attractor shows that, in general, the minimum Heegaard genus ($L_{5,2}$ has it one) does not coincide with the minimum 3-gem (whose associated three Heegaard splittings have genus at least 2).

2.8 3-Gems and π_1, the Fundamental Group

In this section we discuss a number of ways to get a presentation for the fundamental group $\pi_1(M^3)$ from a 3-gem inducing M^3.

2.8.1 Getting 01-Presentation for π_1

From a crystallization G it is rather easy to obtain a presentation for the fundamental group of the induced 3-manifold based on a bipartition of the four colors into two sets of two colors. In what follows we use the partition $\{\{0,1\},\{2,3\}\}$, but any of the six possibilities produces the same group. This approach was introduced in [Gag79b] and independently in [Lin88]. For groups given by generators and relators we refer to [MKS76].

Algorithm 5 *[01-Presentation for $\pi_1(|G|)$ from a crystallization G]:*

1. Label the m 01-gons with the symbols $\ell_1, \ell_2, \ldots, \ell_m$. These are the generators.
2. Go around each 23-gon (in any sense) writing for each vertex v the symbol $\ell_i^{\pm 1}$ where ℓ_i is the symbol of the 01 gon to which v belongs. The exponent is $+1$ if v is the transition between an edge of color 2 followed by an edge of color 3, according to the choice of the sense of traversal. Otherwise the exponent is -1. These sequences of symbols are to be considered cyclic words and form the relators.
3. Choose one symbol (any symbol) ℓ_i and make it equal to the identity. (Otherwise the group would have a free extra factor.)
4. Choose one relator (any relator) and drop it. (It is implied by the others.)

We exemplify this procedure in r_2^{24}. The generators correspond to the 01-gons:

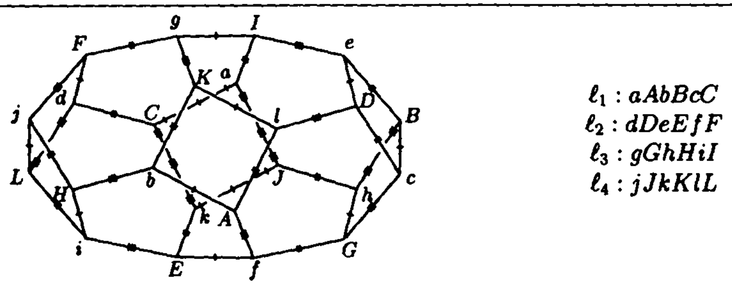


Fig. 52: Starting to get the 01-presentation for $\pi_1(|r_2^{24}|)$

The relators are given by the 23-gons, where each vertex is replaced by the symbol of the 01-gon it belongs to with the adequate exponent:

$$bKgFjH \mapsto \ell_1^{-1}\ell_4\ell_3^{-1}\ell_2\ell_4^{-1}\ell_3$$

$$cDlAfG \mapsto \ell_1^{-1}\ell_2\ell_4^{-1}\ell_1\ell_2^{-1}\ell_3$$

$$eBhJaI \mapsto \ell_2^{-1}\ell_4\ell_3^{-1}\ell_2\ell_4^{-1}\ell_3$$

$$kEiLdC \mapsto \ell_4^{-1}\ell_2\ell_3^{-1}\ell_4\ell_2^{-1}\ell_1$$

We must make one of the generators equal to the identity, say, $\ell_3 = Id$. Moreover any relator is implied by the others. So we may drop any one of them, say, the fourth relator. Let then $x = \ell_1^{-1}$, $y = \ell_2^{-1}$, $\ell_3 = Id$ and $z = \ell_4^{-1}$. We then get

$$\pi_1(r_2^{24}) = \langle x, y, z \mid xz^{-1}y^{-1}z, xy^{-1}zx^{-1}y, yx^{-1}z^{-1}x\rangle.$$

Using the third relator we get $z = xyx^{-1}$. We can therefore eliminate the generator z, obtaining

$$\begin{aligned}\pi_1(r_2^{24}) &= \langle x, y \mid xxy^{-1}x^{-1}y^{-1}xyx^{-1}, xy^{-1}xyx^{-1}x^{-1}y\rangle \\ &= \langle x, y \mid y^{-1}xy = xyx^{-1}, x^2 = yxy^{-1}xy\rangle \\ &= \langle x, y \mid y^{-1}xy = xyx^{-1}, x^3 = yx^2y\rangle.\end{aligned}$$

The last relation is obtained with the replacement of $y^{-1}xy$ by xyx^{-1} in the second relation and multiplying by x to the right. Now introduce a new generator $w = xy$, and use it to eliminate y:

$$\begin{aligned}\pi_1(r_2^{24}) &= \langle x, y, w \mid w = xy, y^{-1}xy = xyx^{-1}, x^3 = yx^2y\rangle \\ &= \langle x, w \mid w^{-1}xw = wx^{-1}, x^3 = x^{-1}wxw\rangle \\ &= \langle x, w \mid xwx = w^2, x^5 = (xw)^2\rangle \\ &= \langle x, w \mid (xw)^2 = w^3 = x^5\rangle.\end{aligned}$$

This is the binary dodecahedral group $\langle 5, 3, 2\rangle$ and it has 120 elements, [CM72b]. This was expected, since it is known that this group is the fundamental group of the Poincaré homology sphere, $S^3/\langle 5, 3, 2\rangle$.

Justification for the Above Algorithm: It is a simple consequence of getting a presentation for π_1 from a Heegaard diagram. Indeed, consider $\mathcal{H}_{01}$ and all but one 23-gons in its surface. We know that these curves form a Heegaard diagram for the 3-manifold and the generators for π_1 are given by r_{01}-based loops ℓ_i in the interior or $\mathcal{H}_{01}$ as shown below. Loop ℓ_i crosses transversally the 23-gons b_{23}^0 and $0b_{23}^i$. The base point r_{01} is in the interior of disk b_{23}^0. The symbol which is made equal to the identity is, here, ℓ_0 the one that corresponds to b_{23}^0.

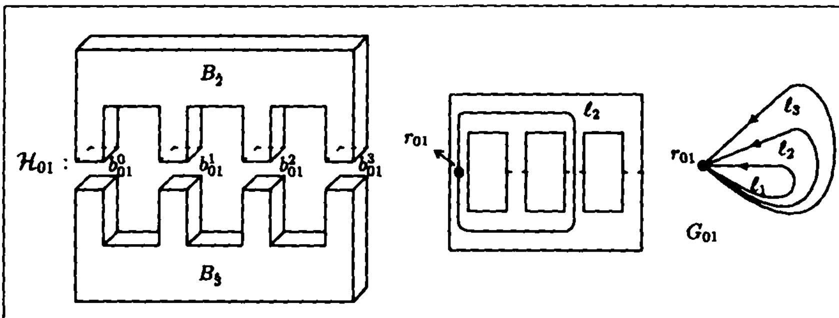


Fig. 53A: Handlebody $\mathcal{H}_{01}$ and its deformation retract

It is straightforward to note that each surviving 23-gon is isotopic to a word in the ℓ_i's which coincides with the words given by the algorithm (in which we make ℓ_0 equal to the identity).

The fact that the word corresponding to one (any one) of the 23-gons is redundant can be seen in the complementary handlebody:

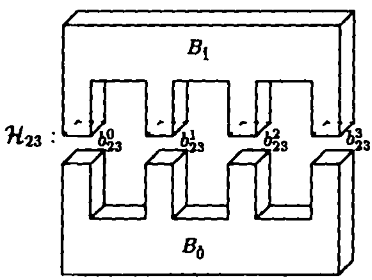
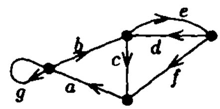


Fig. 53B: Complementary handlebody explains redundancy

The set of 23-gons corresponds to one more disk than a system of meridians. As a consequence, one of the 23-gons (any one) may not be used, since its associated word is implied by the others. This is a geometric reason of this redundancy.

2.8.2 Combinatorial Explanations

There is also a simple combinatorial explanation for the above redundancy based on planar graphs. Let M be a connected graph embedded into the surface of a 2-sphere. Consider the edges of M oriented arbitrarily. The group Γ_M is defined as follows: the generators are symbols in $1-1$ correspondence with the edges of M. The relators are in $1-1$ correspondence with the vertices of M. To obtain a relator from a vertex we list in clockwise order (starting at any edge) the symbols for edges around the vertex. A generator receives a negative exponent if the edge is directed towards the vertex and $+1$ (no exponent), otherwise. For instance, if M is the graph below (considered embedded into an S^2)



$$\Gamma_M = \langle a, b, c, d, e, f, g \mid a^{-1}gg^{-1}b, b^{-1}ed^{-1}c, ac^{-1}f^{-1}, de^{-1}f \rangle$$

Fig. 54: A group Γ_M from a graph M in S^2

Lemma 5 *Let M be any connected graph embedded into an S^2. Then each relator of Γ_M is implied by the others.*

Proof: Note that if M has a unique vertex, then the unique relator is redundant: it freely reduces to the identity. (A *free reduction* is the replacement of a pair like xx^{-1} by the identity.) We proceed by induction. Let M have $n+1$ vertices. Choose an edge x linking two distinct vertices of M. Symbol x appears twice in two distinct relators. We can take its value in terms of the other symbols in one of the relators and use it in the other to accomplish the complete elimination of x. The geometric manifestation of this elimination is the contraction of edge x, producing a graph M' having n vertices. Let r be the vertex which we want to prove redundant. If it is possible to choose x not incident to r, then since the relator r in Γ_M is still a relator in $\Gamma_{M'}$ we are done by induction.

Otherwise, every non-loop is incident to r, as in the example below.

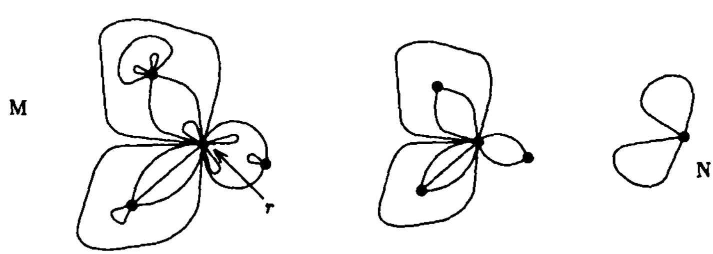


Fig. 55: Geometric manifestation of free reduction

Observe that the loops incident to a $t \neq r$, as well as those incident to r which do not enclose any other t, freely reduces in the corresponding relators and behave as if did not existed. Now, the product of the edge-symbols corresponding to each $t \neq r$ is the identity and this fact can be used to simplify the relator r up to the point where it comes from a single vertex (while, except for free reduction, maintaining all the other relators intact). As we already have point out, a relator coming from a graph N with a single vertex is redundant. ∎

Proposition 8 *Let $\pi_1(G)$ be the group obtained from a crystallization by Algorithm 5, in which we do not drop any of the relators corresponding to the 23-gons (step 4). Then each one of the relators is implied by the others.*

Proof: Consider the $\hat{0}$-residue G_0 embedded into an S^2. Let M be the embedded graph obtained from G_0 by contracting to a single vertex each disk bounded by a 23-gon. Observe that $\pi_1(G)$ is a quotient of the group Γ_M, above defined: indeed, we must make the symbol of each 1-colored edge (a generator of Γ_M) equal to the symbol of the 01-gon to which it belongs (a 01-gon is a generator for $\pi_1(G)$). After that make one of the symbol for one 01-gon equal to the identity. Since, by the previous lemma, any relator is implied by the others in Γ_M it clearly will be so in $\pi_1(G)$, a quotient. ∎

We now present a simple combinatorial proposition which explains the fact that the quotient group obtained by making one generator equal to the identity in Step 3 of Algorithm 5 is independent of the generator chosen. A presentation for a group is called an *alternating presentation* if in all the relators the generators have alternatively $+1$ and -1 as exponents.

Proposition 9 (Lemma 5 of [Lin89]) *If $P = \langle X \mid R\rangle$ is an alternating presentation for a group P and $a, b \in X$, then the quotient groups $\langle X \mid R \cup \{a\}\rangle$ and $\langle X \mid R \cup \{b\}\rangle$ are isomorphic.*

Proof: Let $\hat{X}$ be a set disjoint form G in bijective correspondence with it via $g \mapsto \hat{g}$. Note that

$$\langle X \mid R \cup \{a\}\rangle \equiv \langle X \cup \hat{X} \mid \hat{R} \cup \{a\} \cup \{\hat{x}bx^{-1} \mid x \in X\}\rangle.$$

Since R is alternating, R and $\hat{R}$, obtained by replacing $x \mapsto \hat{x}$ for each $x \in X$, are equivalent. By using the relator $\hat{a}ba^{-1}$ to eliminate a we obtain the isomorphic group

$$\langle \hat{X} \cup X - \{a\} \mid \hat{R} \cup \{\hat{a}b\} \cup \{\hat{x}bx^{-1} \mid x \in X - \{a\}\rangle.$$

Now use the relator $\hat{a}b$ to eliminate b, obtaining

$$\langle \hat{X} \cup X - \{a, b\} \mid \hat{R} \cup \{\hat{x}\hat{a}^{-1}x^{-1} \mid x \in X - \{a, b\}\} \cup \{\hat{b}\}\rangle.$$

Note that each symbol in $X - \{a, b\}$ appears isolated in one relator. By using these relators we can eliminate all the symbols in $X - \{a, b\}$. We get simply,

$$\langle \hat{X} \mid \hat{R} \cup \{\hat{b}\}\rangle \equiv \langle X \mid R \cup \{b\}\rangle,$$

proving the proposition. ∎

Now we show that a from group having an alternating presentation we can extract a free factor. This is a combinatorial explanation on why we must make one generator equal to the identity in Algorithm 5.

Proposition 10 *If $P = \langle X \mid R \rangle$ is an alternating presentation for a group P and $a \in X$. Let R_a be the set of relators obtained from R by making $a = Id$. Then $\langle X \mid R \rangle$ and $\langle a \rangle * \langle X - \{a\} \mid R_a \rangle$ are isomorphic.*

Proof: Let $\hat{X}$ be a disjoint copy of $X - \{a\}$, via $x \mapsto \hat{x}$. Then,

$$\langle X \mid R \rangle \equiv \langle X \cup \hat{X} \mid R \cup \{\hat{x}ax^{-1} \mid x \in X - \{a\}\} \rangle.$$

Note that $x = \hat{x}a$, relation derived from the new relators. By using these relations, we eliminate all the symbols in $X - \{a\}$. Note that by free substitutions and conjugations the relators in R become simply $\hat{R}_a$ (i.e. make $a = Id$ and put a hat at each symbol of R). We get

$$\langle \hat{X} \cup \{a\} \mid \hat{R}_a \rangle \equiv \langle a \rangle * \langle \hat{X} \mid \hat{R}_a \rangle \equiv \langle a \rangle * \langle X - \{a\} \mid R_a \rangle,$$

establishing the proposition. ∎

2.8.3 Getting a 0-Presentation for π_1

We now treat an alternative way to get $\pi_1(|G|)$ from a crystallization G. It is based on the choice of one of the colors in $\{0, 1, 2, 3\}$. We choose color 0, but the group would be isomorphic with any other choice.

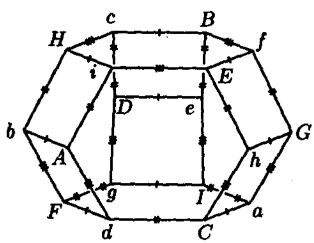
Algorithm 6 *[0-Presentation for $\pi_1(|G|)$ from a crystallization G]:*

1. For $i = 1, 2, \ldots, \frac{1}{2}|V(G)|$ orient the i-th 0-colored edge from vertex $2i - 1$ to vertex $2i$, assigning to it the symbol a_i; these are the generators.
2. For each $c \in \{1, 2, 3\}$, starting at each odd labeled vertex, go around the $(0, c)$-colored bigons recording the symbol a_i for each 0-colored edge traversed. The cyclic words so obtained are the relators of the presentation.

Justification of the algorithm: Contract to a point the unique 3-cell bounded by the sphere of the 3-residue not involving color 0. In the new cell complex the 0-colored edges are the only edges and become loops. The new 2-skeleton has pinched disks bounded by loops in the same order as they are found in step 2. Since the fundamental group depends only on the 2-skeleton of a cell decomposition, the proof is complete. ∎

The above algorithm yields less compact presentations than others based on two colors. However, it is useful not only because it is very simple to justify but mainly, because it relates nicely with the algorithm for constructing covering 3-gems, which is treated in Subsection 5.4.5.

We exemplify the above presentation in the 3-gem for S^3/Q_8; the fundamental group is generated by $\{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9\}$ with the 11 relators given on the right:



$a_1a_2a_3$, $a_4a_5a_6$, $a_7a_8a_9$ (from 01-gons)
a_1a_9, $a_2a_5a_8$, a_3a_4, a_6a_7 (from 02-gons)
$a_1a_4a_7$, a_2a_6, a_3a_8, a_5a_9 (from 03-gons)

Fig. 56: Obtaining the 0-presentation for $\pi_1(|r_1^{18}|)$

Of course, the 0-presentation that we get can be considerably simplified. We illustrate the algorithm and its typical simplifications in the above 3-gem, r_1^{18}.

Start by listing the 01-relators, then the 02-relators, then the 03-relators, each in lexicographical order, as above. As long as there is a relator $r = ua_iv$ with a generator a_i appearing once (*isolated generator*), we use the Tietze substitution $a_i \to u^{-1}v^{-1}$ into the other relators to eliminate the generator a_i from the presentation. To ensure an orderly way, we have chosen the first such relator, and in it, the isolated generator which has the highest index. After the substitution of all the occurrences of the isolated generator, we effect free simplifications as well as the ones coming from conjugation. This procedure yields the presentation $< a_1, a_2 | a_2^2a_1^2, a_2^{-1}a_1a_2a_1, a_2^{-1}a_1^{-1}a_2^{-1}a_1^{-3} >$ for the above example. The last relator is redundant, as we show in the following Proposition.

Proposition 11 *In the 0-presentation of π_1 of a rigid 3-gem G, any two relations produced from differently colored bigons are implied by the rest of the relators in the presentation.*

Proof: Let (i,j,k) be any permutation of $(1,2,3)$. For $h \in \{i,j,k\}$ let R_h be the set of all relators coming from the $(0,h)$-colored bigons. The relators in $R_i \cup R_k$ can be read from the faces of a graph embedded into a 2-sphere. Namely, consider the 2-complex formed by the 2-disks attached to the (i,k)-, the $(0,i)$- and $(0,k)$-colored bigons. This forms a 2-sphere where the subgraph of G not involving color j embeds with its bigons being the faces. Now contract to points the disks bounding by (i,k)-bigons. Only the 0-colored edges remain and each member of $R_i \cup R_k$ is read from the boundary of a face in the order and sense of traversal in which the corresponding edges appear in the embedding. In this situation the relator coming from any such face is (because we are in the 2-sphere) implied by the others. (See the dual form of Lemma 1 of [Lin88] for a detailed proof.) Thus, any r_i in R_i is implied by $(R_i \backslash \{r_i\}) \cup R_k$. Replacing i by j, any r_j is implied by $(R_j \backslash \{r_i\}) \cup R_k$. Therefore, both r_i and r_j are implied by $(R_i \backslash \{r_i\}) \cup (R_j \backslash \{r_j\}) \cup R_k$. ∎

At the end of the simplification, the relator originated by the last 02-gon becomes empty and is deleted. The other relator which is deleted is the longest one which is originated by a 03-bigon.

To complete the explanation of the implemented simplifications routines, which yield Tables 9A and 9B of Section 5.3, we "tidy" the numbering of the remaining generators: for instance, if only generators a_3, a_{10} and a_{19} have survived, we rename them a, b and c. Finally, we invert a relator, if doing so the number of negative exponents decreases. This is the justification for the presentations given in those Tables.

2.9 Framed Links, Blinks and 3-Gems

One of the most interesting and important presentations of a 3-manifold is the one that produces it from a *framed link*. This was put at our disposal by Lickorish as consequence of his fundamental work [Lic62]. Further, Kirby [Kir78] discovered moves on these objects which are enough to replace homeomorphism of 3-manifolds by a simple calculus on framed links. Our quicky review of the moves below follows the presentation of Rolfsen [Rol76].

A *framed link* is a link in S^3 with an irreducible fraction attached to each component. These objects encode 3-manifolds via a standard construction. Below we exemplify this construction in the case of one unknotted component with framing $-3/2$.

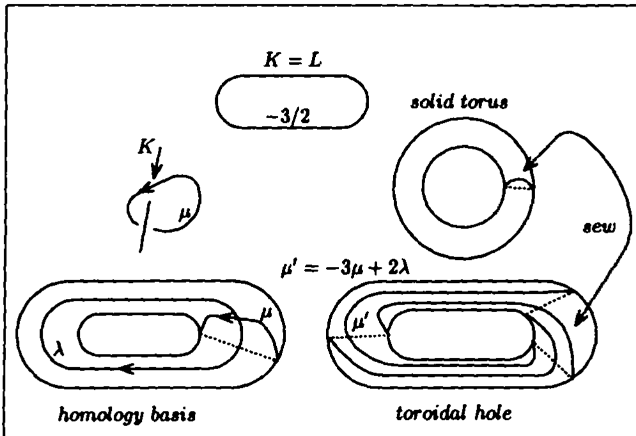


Fig. 57: 3-manifold associated to unknot with framing -3/2

In general, given a framed link L with n components, consider a tubular neighborhood of L in S^3. Each component K of the link is enlarged to a thin solid torus T_K. The fraction p/q associated to K is now used to specify an S^1 in the boundary of T_K: it is the curve of type homology type $\mu' = p\mu + q\lambda$, where μ is the boundary of a meridian of T_K and λ is a curve parallel to K in the surface of T_K so that $\ell kn(K, \mu) = +1$. The curves μ' are to become identified with the meridians of n new disjoint solid tori: one for each component. The 3-manifold is formed by the exterior of the tubular neighborhood of L together with the new tori with pasting specified by the μ'-curves. See [Rol76] for more details.

It is possible to get, in a straightforward way, a 3-gem inducing the same 3-manifold as a framed link *if the framings are integers.* Therefore we briefly review how it is possible to modify the link so as to get integer framings without changing the 3-manifold. The modification of the link and of the surgery coefficients below are justified in Chapter 9 of [Rol76]. Here are the two permissible operations that we can do on L:

- Create or cancell a component with framing $\pm\infty$.
- Effect a $\pm t$-twist about an unknotted component changing judiciously the link and the framings r_k's.

  The framings change as follows:

  Component L_i of the twist: $r'_i = 1/(t + (1/r_i))$

  Other components L_j: $r'_j = r_j + t(\ell kn(L_i, L_j))^2$.

The link changes as follows:

Effect t full twists in the cable of parallel lines encircled by L_i (a left handed twist is a (-1)-twist).

2.9.1 The Smallest Known Hyperbolic 3-Manifold

This example is due to Jeffrey Weeks. It appears in [HW94]. Consider the 3-manifold given by the following framed link which induces the hyperbolic 3-manifold $H^3_{0.94}$ of smallest known volume (which is 0.942707362776928...):

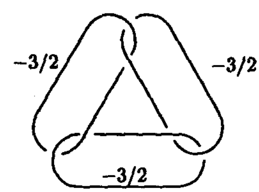


Fig. 58A: $H^3_{0.94}$: a hyperbolic 3-manifold of volume 0.94...

Our immediate objective is to get an equivalent framed link which has *integer* coefficients.

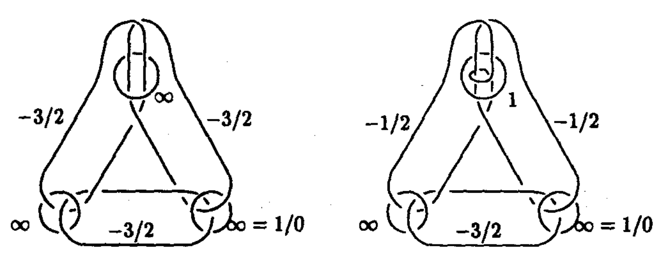


Fig. 58B: Create ∞-components; do a 1-twist about the upper one to switch a crossing

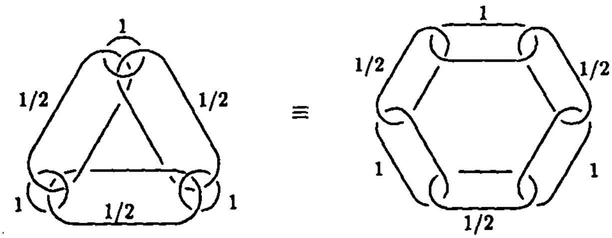


Fig. 58C: Repeat 1-twists about bottom ∞-components getting a special 6-ring

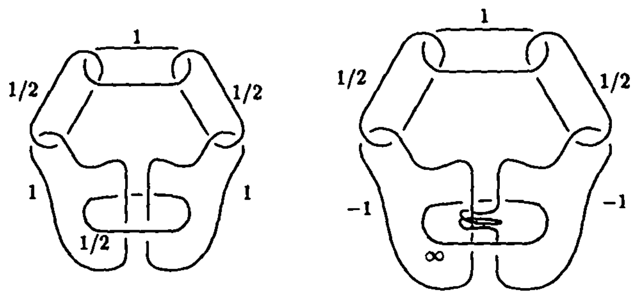


Fig. 58D: Do a (-2)-twist about the lower (1/2)-component

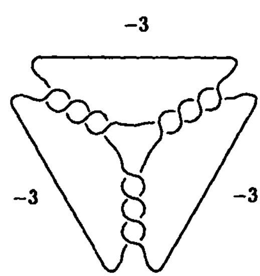


Fig. 58E: Result after effecting (-2)-twists about the remaining (1/2)-components

We have achieved our goal to produce an equivalent link having only integers framings. Now we introduce some curls in the link so that the (integer) framing of component each equals the algebraic sum of self-crossings of the component. Thus the mention to the framing becomes redundant but the drawing in the plane (particularly the curls) becomes important. This is what is called a *blackboard framed link*.

A more condensed form of a blackboard framed link is a *blink*. To form such an object, note that the faces of a diagram of a link can be bicolored, the exterior face being white its neighbors sharing a segment between 2 crossings being black and so one. The *blink* associated to a blackboard framed link is a plane graph with a bicoloration of its edges. The vertices correspond to the black faces and are fixed arbitrary interior points on them. These points are said to *represent* its black face. The edges correspond to the crossings: there is an edge contained in the black faces and across a crossing starting and ending at the vertices (which may coincide) representing the two black faces incident to the crossing. An edge of the blink is *plain* if in going along the edge the overpass of the crossing is from northeast to southwest. It is *dotted* otherwise. In our example there are only plain edges.

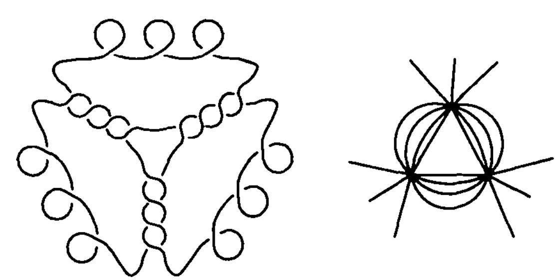


Fig. 58F: Blackboard framed link and associated blink inducing $H^3_{0.94}$

2.9.2 From Blinks to 3-Gems

Suppose we are given a blink. Draw the associated blackboard framed link. Then it is rather easy to construct a Heegaard diagram (in the complementary handlebody) yielding the same manifold as the one given by the surgery presentation. Consider each component in the surface of a thin solid torus. At each crossing make a bridge linking upper and lower tori with a solid cylinder, as shown in the picture below.

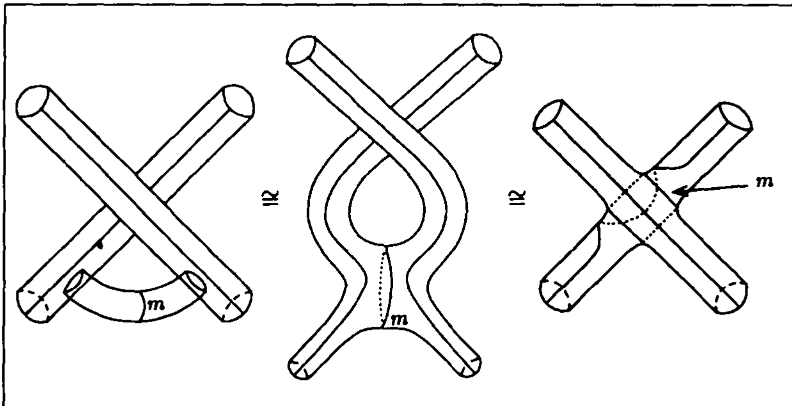


Fig. 59: Getting Heegaard diagram in the complementary handlebody

After the addition of one cylinder for each crossing we get a handlebody. An easy, but important fact about the situation is that this handlebody considered naturally embedded into S^3 has a complement which is, clearly, also a handlebody. Our diagrams will be considered in this complementary handlebody. It is a tricky fact, but a same system of curves considered as Heegaard diagrams in the common boundary of two complementary handlebodies in S^3 usually induces two distinct 3-manifolds. The following proposition provides a bridge between blackboard framed links and Heegaard diagrams in the complementary handlebody. Since we already know (Algorithm 3) how to go from such a diagram to a 3-gem, this proposition is the missing link to achieve our objective.

Proposition 12 *The complementary handlebody above constructed and the original link components embedded on its boundary together with the meridian curves of the added cylinders form a Heegaard diagram for the same manifold given by the link with blackboard framing.*

Proof: The proof is straightforward from the definitions of surgery on framed links and of Heegaard diagrams. ■

We can produce the passage from the projection of a blackboard framed link to a Heegaard diagram in the complementary handlebody inducing the same 3-manifold by repeating at each crossing the local substitution depicted below:

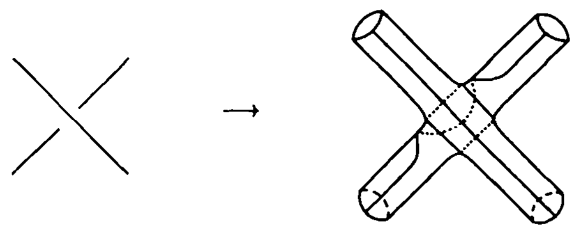


Fig. 60: Blackboard framed link $\mapsto$ Heegaard diagram in complementary handlebody

Here is an example for S^3. We get a genus two Heegaard diagram of S^3 in the complementary handlebody.

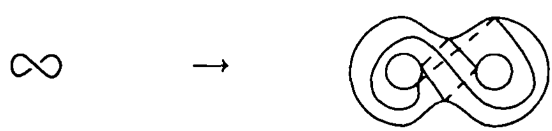


Fig. 61: The simplest example of the above situation

We have seen in Algorithm 3 how to obtain a 3-gem inducing the same 3-manifold as the one induced by a Heegaard diagram in a complementary handlebody. We briefly recall this algorithm: start by drawing a parallel curve to each curve in the Heegaard diagram. Each vertex of the 3-gem is defined from a transition of a curve from an arc behind to one in front. Let the behind arcs be edges of color 3 and frontal arcs be edges of color 0. There are now an even number of vertices around each hole. Color the boundary of each such hole alternatively with colors 1 and 2, in such a way that the 3-residue not involving color 3 has bicolored boundaries of faces.

It is straightforward to prove that the resulting $(3+1)$-graph is a 3-gem inducing the same 3-manifold as the one given by the Heegaard diagram. See the justification of Algorithm 3. In the above example, after the doubling we get

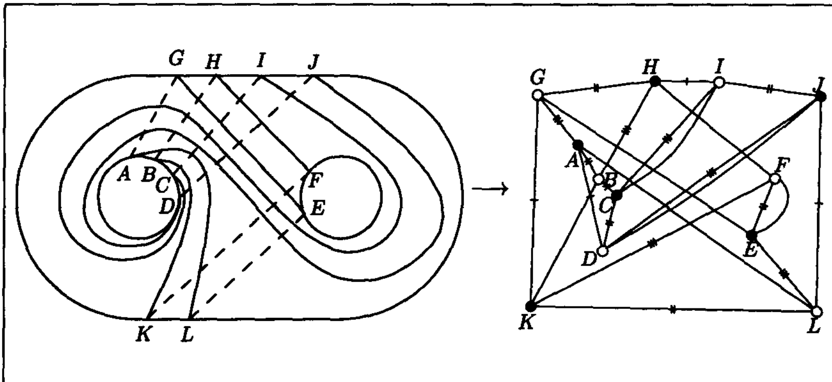


Fig. 62: Getting 3-gem from doubled Heegaard diagram

A sequence of dipole cancellations gets us to the superattractor for S^3: the 3-gem s^3 with two vertices.

From the above discussion it is straightforward, in general, to go from a blackboard framed link with n crossings to a 3-gem with $12n$ vertices inducing the same 3-manifold. Each crossing is replaced by a fixed partial 3-gem, as shown below:

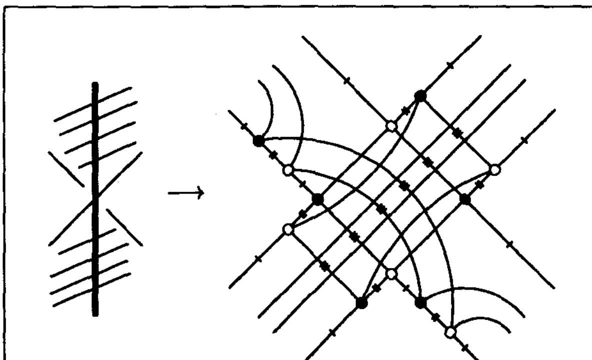


Fig. 63: Local generic modification: from blink to 3-gem

The algorithm realizing the transformation *blink* $\mapsto$ *3-gem* has been implemented and is used to recognize the 3-manifolds induced by simple blinks. The 3-gems have a richer simplification theory implying that it is easier to recognize 3-manifolds given by small 3-gems than by small blinks. Therefore, the above *"multiplication by 12"* pays off.

There are two observations about the above transformation: (i) blinks can only induce connected 3-manifolds. A disconnected blink induces the 3-manifold given by the connected sum of the disjoint 3-manifolds given by each connected component of the blink. Thus, we need to ambient isotope the link inducing the blink so it becomes connected. (ii) if the blink is a single vertex, there is no way to perform the above transformation. We need to deform the inducing circle to get crossings in the projection. Indeed, from the second blackboard framed link below we get a 3-gem for $S^1 \times S^2$.

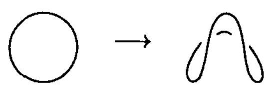


Fig. 64: Forcing crossings to permit the construction

2.10 From Blinks to Superattractors: Examples

We say that a *blink gets absorbed* by an attractor if the 3-gem associated to the blink, as constructed in the previous section simplifies, under algorithm TS_ρ (detailed in Chapter 4) to a member of the attractor.

In this subsection we provide some examples of simple blinks getting absorbed by superattractors. The last example is an attractor having four 3-gems.

2.10.1 The Closed Hyperbolic 3-Manifold: $H^3_{0.94}$

The first example is the hyperbolic manifold of smallest known volume $H^3_{0.94}$.

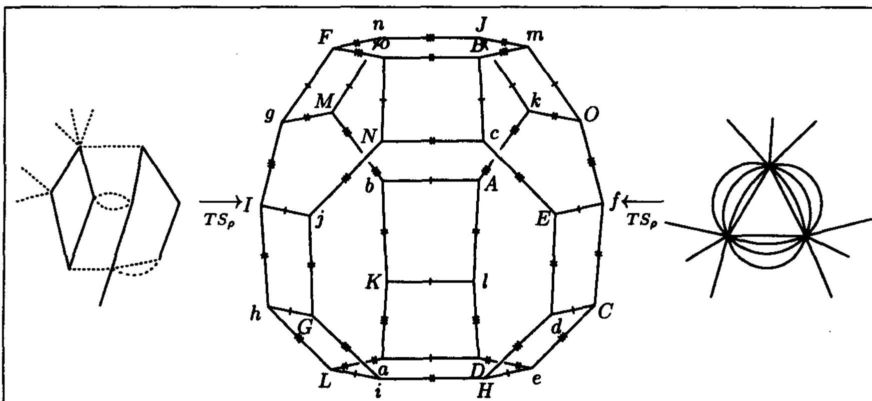


Fig. 65: Blinks getting absorbed by the superattractor r^{30}_{24481} for $H^3_{0.94}$

The second blink is the one that we have obtained in Subsection 2.8.1. The less symmetric first blink for $H^3_{0.94}$ was the first obtained for this manifold. We got it with the help of D. Rolfsen and J. Weeks.

There are no other 3-gems up to 30 vertices inducing $H^3_{0.94}$. Therefore, r^{30}_{24481} is the superattractor for this space. This is a σ-gem. Here are the σ-symmetries:

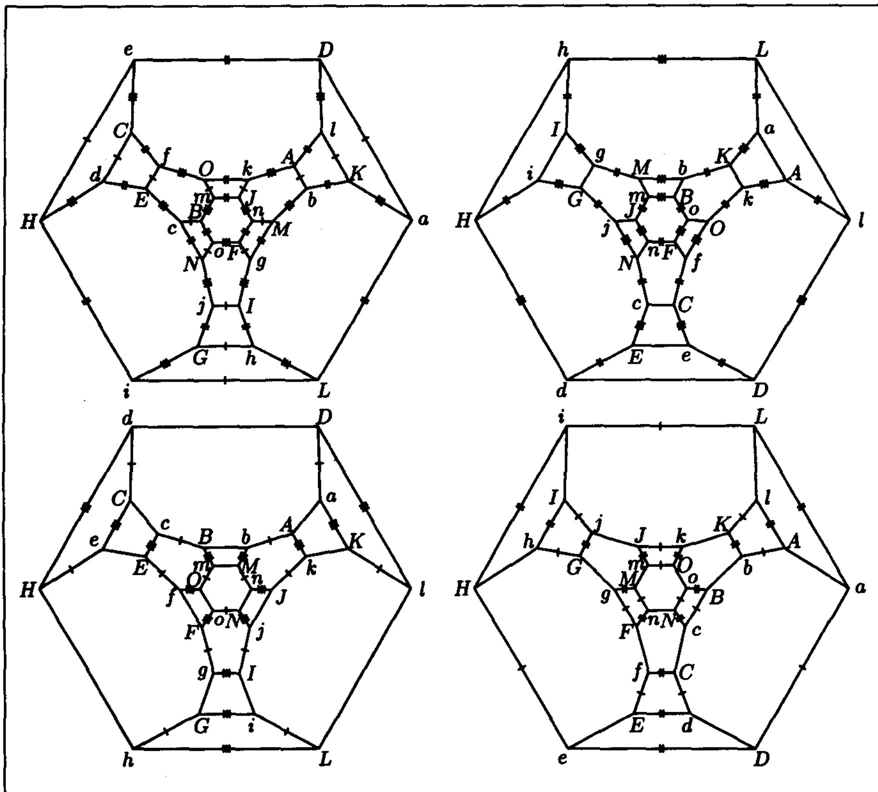


Fig. 66A: σ-Symmetries on r^{30}_{24481}

The string presentation associated with the above σ-symmetries is

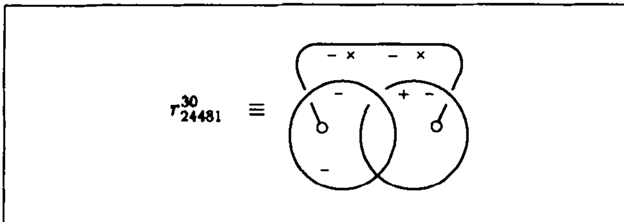


Fig. 66B: String presentation induced by σ-symmetries above

Note that the lower crossing is a $^{\times}$crossing corresponding to the usual crossing of two 0-colored edges. It is not a $^{\div}$crossing and it has no underpass (and no overpass).

2.10.2 Square-Blink and Tetrahedron-Blink

The quaternionic 3-manifold S^3/Q_8, whose superattractor is $\mathcal{S}(3,3,2,1)$, admits a surprisingly simple blink which induces it:

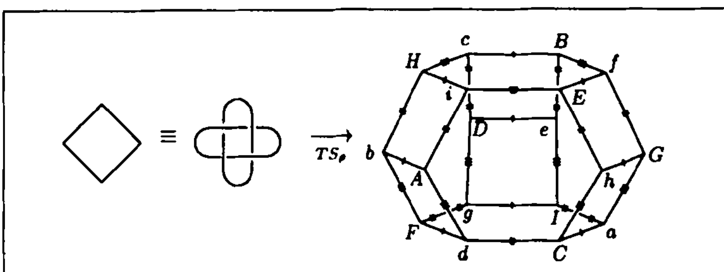


Fig. 67: Square-blink getting absorbed by the superattractor for S^3/Q_8

The 3-torus $S^1 \times S^1 \times S^1$ has the 1-skeleton of a tetrahedron as its simplest inducing blink. All edges are plain edges. Indeed, this blink corresponds to the Borromean rings with 0 as framings. They produce a 3-gem with 72 vertices which gets absorbed by r_1^{24}, the superattractor for the 3-torus.

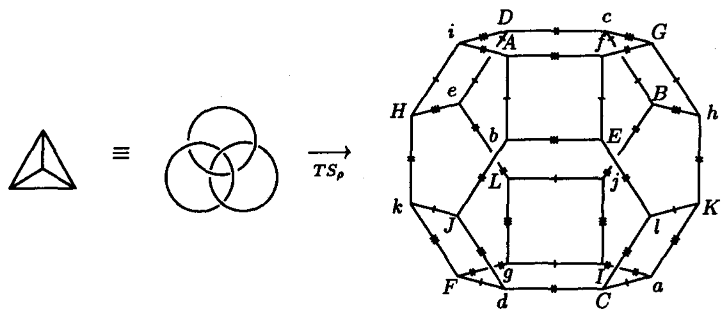


Fig. 68: Tetrahedron-blink getting absorbed by the superattractor for $S^1 \times S^1 \times S^1$

2.10.3 Another Closed Hyperbolic 3-Manifold

The closed orientable hyperbolic 3-manifold $H^3_{1.01}$ of third lowest known volume which is

$$\mathrm{Vol}(H^3_{1.01}) = 1.014941606409654...$$

See #3 of [HW94], where it appears induced by a specific framed link. By effecting the admissible manipulations of the last section over this link we find that $H^3_{1.01}$ is induced by the following equivalent blink:

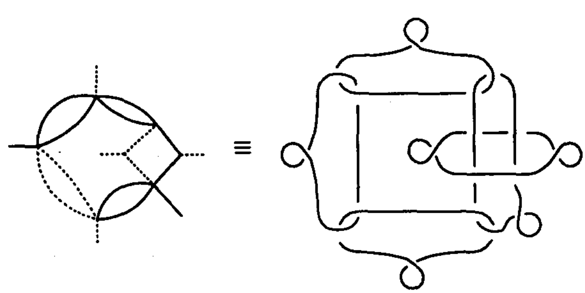


Fig. 69A: Blink inducing the hyperbolic 3-manifold $H^3_{1.01}$

The 3-gem associated to this blink gets absorbed by an attractor (as our catalogue and the various invariants that we applied prove) having four 3-gems: r^{30}_{646}, r^{30}_{1802}, r^{30}_{916} and r^{30}_{1857}. These are depicted below.

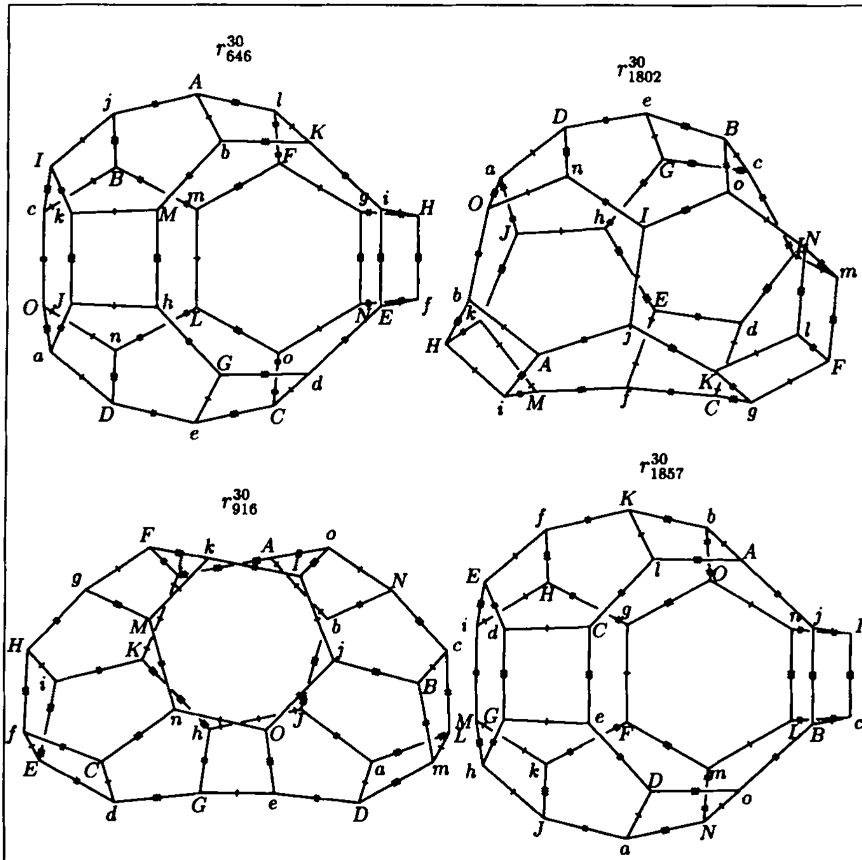


Fig. 69B: The attractor for the hyperbolic 3-manifold $H^3_{1.01}$

The 3-gems r^{30}_{646} and r^{30}_{1802} are linked by one TS-move and r^{30}_{916} and r^{30}_{1857} are also linked by one TS-move. These pairs of 3-gems form two u^0-classes. There is a u^1-move from the first to second class and another in the opposite direction. Therefore, any two of these 3-gems in the attractor are linked by a single u^1_*-move.

An attractor so that each one of its n 3-gems is linked to any other by a u^i_*-move is called an *u^i_n-attractor*. Thus, the above one is a u^1_4-attractor. The *u^1_n-attractor* is the 3-gem with the smallest code belonging to a *u^1_n-attractor*. Therefore, r^{30}_{646} is *the u^1_4-attractor* for $H^3_{1.01}$.

2.11 Interesting 3-Manifolds Induced by the Family $\mathcal{S}(b, \ell, t, c)$

In this section we give a few more examples of 3-manifolds induced by crystallizations in $\mathcal{S}(b, \ell, t, c)$'s, establish a proposition on isomorphisms on this family and finally give the homologies of its smallest members.

We have seen that some members of $\mathcal{S}(b, \ell, t, c)$ induce quotients of S^3, like $\mathcal{S}(3,3,2,1)$ which induces S^3/Q_8 and $\mathcal{S}(5,3,2,1)$ or $\mathcal{S}(3,5,4,1)$ both of which induce $S^3/\langle 5,3,2\rangle$. We also have quotients of the Euclidean space R^3 and of the hyperbolic space H^3.

2.11.1 EUCLID$_1$ an Euclidean Orientable 3-Manifold

$\mathcal{S}(3,5,2,1)$ induces EUCLID$_1$, one of the six orientable Euclidean closed 3-manifolds.

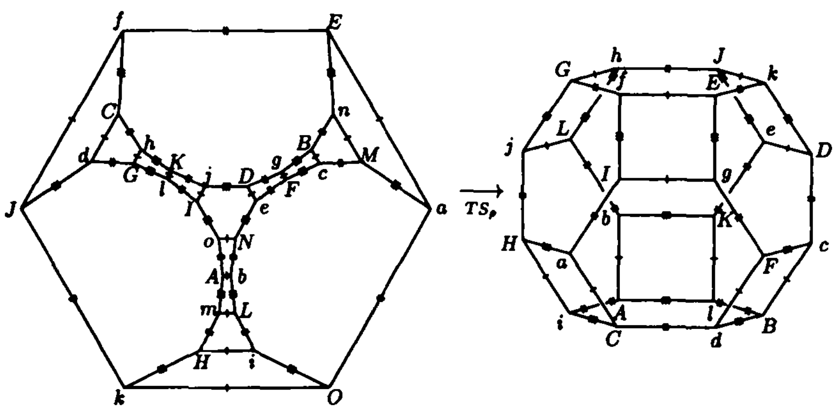


Fig. 70A: $\mathcal{S}(3,5,2,1)$ getting absorbed by r_5^{24}, the superattractor for EUCLID$_1$

The superattractor is the σ-gem r_5^{24}. A string presentation equivalent to it is given below:

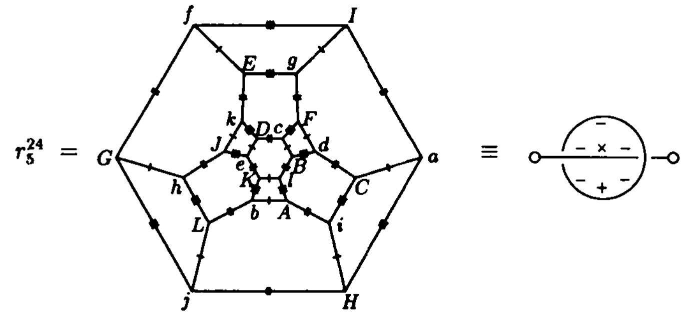


Fig. 70B: String presentation equivalent to the superattractor for EUCLID_1

More information about EUCLID_1 is given in Section 4.2, where we get it as an identification of the faces of a solid cube.

2.11.2 The Hyperbolic 3-Manifold $H^3_{0.94}$

The hyperbolic 3-manifold $H^3_{0.94}$ which we have seen in previous section is induced by $\mathcal{S}(3,7,4,1)$. Indeed, r^{30}_{24481} is the attractor for $H^3_{0.94}$ and it absorbs $\mathcal{S}(3,7,4,1)$.

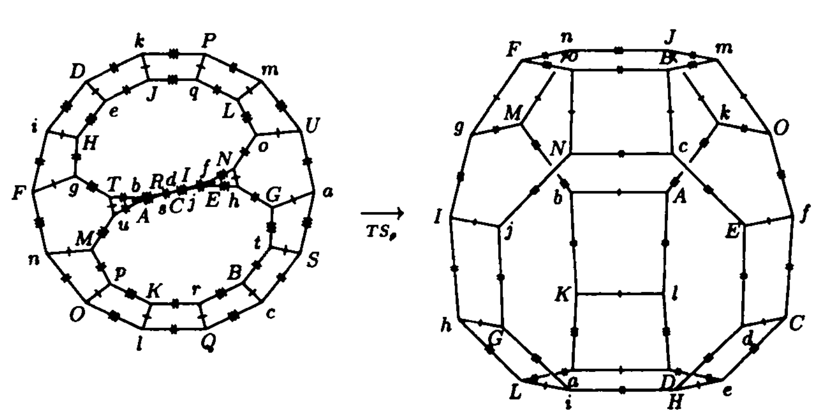


Fig. 71: $\mathcal{S}(3,7,4,1)$ getting absorbed by the superattractor for $H^3_{0.94}$

2.11.3 The Hyperbolic Dodecahedral Space

As one last example, consider the hyperbolic dodecahedral space H^3_{dod}, also known as the Seifert-Weber 3-manifold, obtained from the antipodal facial identification in a solid dodecahedron, as shown below, see [ST80]. This is a very interesting 3-manifold having a lot of unusual properties. For instance, it is a hyperbolic 3-manifold which is not *sufficiently large* in the sense of Hacken. This means that it does not contain an embedded *essential orientable surface*. Such a surface is characterized by the fact that its fundamental group injects into the fundamental group of the 3-manifold. In particular, the homology group of the 3-manifold has to be infinite to exist such a surface. The homology group of the Seifert-Weber space has Betti number 0 and three torsion coefficients equal to 5.

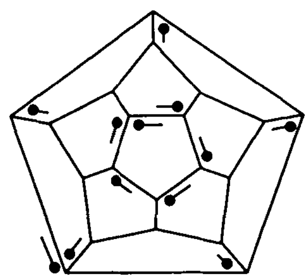


Fig. 72: Antipodal facial identifications yielding the Seifert-Weber 3-manifold also known as the hyperbolic dodecahedral space H^3_{dod}

By feeding TS_ρ with the 120-vertex 3-gem associated to the facial identification scheme above we get to the left 3-gem below. No TS-moves are available on it. This 3-gem is also the output of TS_ρ when we give as input either $\mathcal{S}(5,8,3,2)$ or $\mathcal{S}(5,8,3,3)$. The 3-gem on the right is the output when TS_ρ is fed with $\mathcal{S}(5,8,3,1)$ or $\mathcal{S}(5,8,3,4)$. The 3-manifolds induced by these 3-gems have been very difficult to discriminate. They have the same homology groups, and the same linking sequence, as studied in Chapter 6. Also their fundamental groups may coincide. In conjunction with S. Sidki we have made tests which did not decide the isomorphism question.

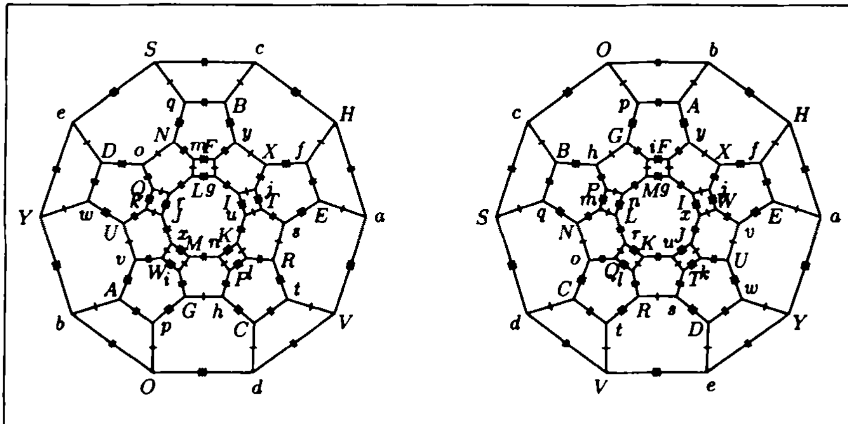


Fig. 73A: A 3-gem inducing H^3_{dod} and a cousin inducing $\tilde{H}^3_{dod}$

Conjecture 4 *The 3-manifolds H^3_{dod} and $\tilde{H}^3_{dod}$ are non-homeomorphic. The above 50-vertex 3-gems are the superattractors for them.*

It is conceivably that the new quantum invariants [KL94] distinguish these 3-manifolds. However in order to compute these invariants we would need blinks inducing them. And at the present, these are not available. The first 3-gem above is not a σ-gem, but the second is.

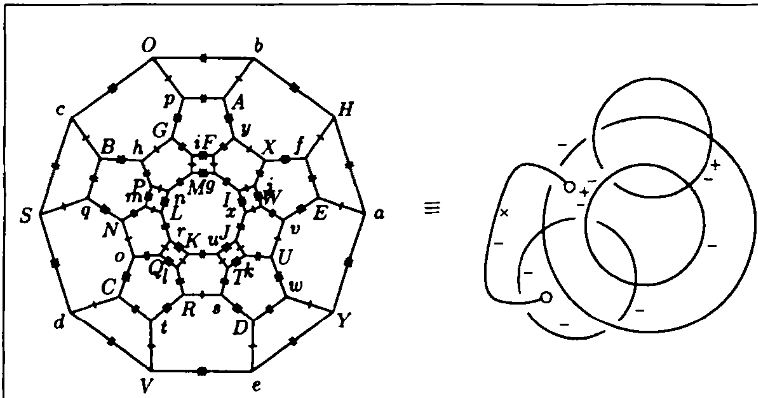


Fig. 73B: A string presentation for the image of $S(5, 8, 3, 1)$ under TS_ρ

2.11.4 Homology Tables and Isomorphisms in $\mathcal{S}(b,\ell,t,c)$

We provide tables of the first homology groups of the smallest $|\mathcal{S}(b,\ell,t,c)|$'s. Before that is convenient to identify some isomorphic objects among the inducing $(3+1)$-graphs.

Proposition 13 *The graphs $\mathcal{S}(b,\ell,t,c)$, $\mathcal{S}(b,\ell,2\ell-t,c)$ and $\mathcal{S}(b,\ell,\ell-t,b-c)$ are isomorphic.*

Proof: The isomorphism between the first and the third of these graphs is in fact the identity. To show it note that ϵ_1, ϵ_2 and ϵ_3 depend only on b and ℓ. Let ϵ_0' be the first involution on the third graph. By definition,

$$\epsilon_0'(i,j) = (i+(b-c).\mu(j-t+\ell), 1-j+2t-2\ell) = (i+c\mu(j-t), 1-j+2t) = \epsilon_0(i,j).$$

Now we prove that $(i,j) \mapsto \beta(i,j) = (i, 1+\ell-j)$ is an isomorphism between the first and the second graphs. Let ϵ_0, ϵ_1, ϵ_2 and ϵ_3 refer to the first graph and ϵ_0', ϵ_1', ϵ_2' and ϵ_3' refer to the second graph. Note that, except for $i=0$, $\epsilon_i' = \epsilon_i$. It is also clear that β is a bijection. We have the following relations:

$$\begin{aligned}\beta\epsilon_3(i,j) &= \beta(i+\mu(j), 1-j) = (i+\mu(j), \ell+j) = \epsilon_3(i, 1+\ell-j)\\ &= \epsilon_3\beta(i,j) = \epsilon_3'\beta(i,j)\end{aligned}$$

$$\begin{aligned}\beta\epsilon_0(i,j) &= \beta(i+c.\mu(j-t), 1-j+2t) = (i+c.\mu(j-t), \ell+j-2t)\\ &= \epsilon_0'(i,1+\ell-j) = \epsilon_0'\beta(i,j)\end{aligned}$$

Therefore $\beta\epsilon_3 = \epsilon_3'\beta$ and $\beta\epsilon_0 = \epsilon_0'\beta$. To continue, let us suppose that ℓ is even.

$$\begin{aligned}\beta\epsilon_1(i,j) &= \beta(i, j-(-1)^j) = (i, 1+\ell-j+(-1)^j)\\ &= (i, 1+\ell-j-(-1)^{1+\ell-j}) = \epsilon_1(i,1+\ell-j) = \epsilon_1'\beta(i,j)\end{aligned}$$

$$\begin{aligned}\beta\epsilon_2(i,j) &= \beta(i, j+(-1)^j) = (i, 1+\ell-j-(-1)^j)\\ &= (i, 1+\ell-j+(-1)^{1+\ell-j}) = \epsilon_2(i,1+\ell-j) = \epsilon_2'\beta(i,j)\end{aligned}$$

Thus if ℓ is even, β is an isomorphism that preserves the colors. Suppose now that ℓ is odd.

$$\begin{aligned}\beta\epsilon_1(i,j) &= \beta(i, j-(-1)^j) = (i, 1+\ell-j+(-1)^j)\\ &= (i, 1+\ell-j+(-1)^{1+\ell-j}) = \epsilon_2(i,1+\ell-j) = \epsilon_2'\beta(i,j)\end{aligned}$$

$$\begin{aligned}\beta\epsilon_2(i,j) &= \beta(i, j+(-1)^j) = (i, 1+\ell-j-(-1)^j)\\ &= (i, 1+\ell-j-(-1)^{1+\ell-j}) = \epsilon_1(i,1+\ell-j) = \epsilon_1'\beta(i,j)\end{aligned}$$

Therefore, in the case ℓ odd, β is an isomorphism that preserves colors 0 and 3 and interchanges colors 1 and 2. This concludes the proof. ∎

In order to present a glimpse of the richness of the family $\mathcal{S}(b,\ell,t,c)$ we display tables of the first Z-homology group for the smallest members of this family. We use the fact that, as proved above,

$$\mathcal{S}(b,\ell,t,c) \cong \mathcal{S}(b,\ell,2\ell-t,c) \cong \mathcal{S}(b,\ell,\ell-t,b-c).$$

We present tables in the range

$$H_1(|\mathcal{S}(b,\ell,t,c)|), 3 \le b \le 9, 3 \le \ell \le 9, 1 \le t \le \ell, 1 \le c \le b$$

The number in parenthesis is the Betti number. Follow the torsion coefficients in a multiplicative notation.

$b=3$			
3321(0)2^2	3611(0)3.9	3761(Hom.sphere)	3832(0)3.9
3411(0)2.6	3612(2)(Tors.free)	3811(0)4.12	3921(0)7^2
3412(0)3	3712(Hom.sphere)	3812(0)3	3941(0)7^2
3521(0)4^2	3741(0)5^2	3831(0)3.9	3921(0)2^2
3541(Hom.sphere)			

Table 1A: Homologies of the first $\mathcal{S}(3,\ell,t,c)$'s

$b=4$			
4321(0)3	4611(0)3^2.12	4761(0)7	4833(0)2^2.16
4411(0)2^2.8	4613(0)12	4811(0)4^2.16	4921(0)5.45
4413(2)2	4721(0)3.21	4813(2)4	4941(0)5.45
4521(0)3.15	4741(0)3.21	4831(0)2^2.16	4981(0)9
4541(0)5			

Table 1B: Homologies of the first $\mathcal{S}(4,\ell,t,b)$'s

$b=5$			
5321(Hom.sphere)	5611(0)3^3.15	5761(Hom.sphere)	5832(0)5^3
5411(0)2^3.10	5612(0)5	5811(0)4^3.20	5833(0)5^3
5412(0)5	5613(0)5	5812(0)5	5834(0)5^3
5413(0)5	5614(0)5	5813(0)5	5921(0)31^2
5414(0)5	5721(0)11^2	5814(0)5	5941(0)31^2
5521(0)11^2	5741(0)11^2	5831(0)5^3	5981(Hom.sphere)
5541(0)2^4			

Table 1C: Homologies of the first $\mathcal{S}(5,\ell,t,c)$'s

$b=6$			
6321(Tors.free)	6611(0)3^4.18	6761(0)7	6835(0)3.72
6411(0)2^4.12	6615(4)2	6811(0)4^4.24	6921(0)21.189
6415(0)12	6721(0)5.35	6815(0)24	6941(0)21.189
6521(0)8.40	6741(0)5.35	6831(0)3.72	6981(2)3
6541(0)5			

Table 1D: Homologies of the first $\mathcal{S}(6,\ell,t,c)$'s

$b=7$			
7321(Hom.sphere)	7611(0)3^5.21	7761(0)2^6	7832(0)7^3
7411(0)2^5.14	7612(0)7	7811(0)4^5.28	7833(0)7^3
7412(0)7	7613(0)7	7812(0)7	7834(0)7
7413(0)7	7614(0)7	7813(0)7	7835(0)7^3
7414(0)7	7615(0)7	7814(0)7	7836(0)7^3
7415(0)7	7616(0)7	7815(0)7	7921(0)127^2
7416(0)7	7721(0)13^2	7816(0)7	7941(0)127^2
7521(0)29^2	7741(0)13^2	7831(0)7^3	7981(Hom.sphere)
7541(Hom.sphere)			

Table 1E: Homologies of the first $\mathcal{S}(7,\ell,t,c)$'s

$b=8$			
8321(0)2^2	8611(0)3^6.24	8761(0)7	8833(0)4^2.32
8411(0)2^6.16	8613(0)24	8811(0)4^6.32	8835(0)4^2.32
8413(2)4	8615(0)3^2.24	8813(6)2	8837(0)4^2.32
8415(4)4	8617(0)24	8815(4)2^2.8	8921(0)85.765
8417(2)4	8721(0)3.21	8817(6)2	8941(0)85.765
8521(0)21.105	8741(0)3.21	8831(0)4^2.32	8981(0)9
8541(0)5			

Table 1F: Homologies of the first $\mathcal{S}(8,\ell,t,c)$'s

$b=9$			
9321(0)2^2	9611(0)3^7.27	9761(Hom.sphere)	9832(0)3.9.27
9411(0)2^7.18	9612(2)3	9811(0)4^7.36	9834(0)3.9.27
9412(0)9	9614(6)3	9812(0)9	9835(0)3.9.27
9414(0)2.18	9615(2)3	9814(0)4.36	9837(0)3.9.27
9415(0)9	9617(6)3	9815(0)9	9838(0)3.9.27
9417(0)2.18	9618(2)3	9817(0)4.36	9921(0)511^2
9418(0)9	9721(0)5^2	9818(0)9	9941(0)511^2
9521(0)76^2	9741(0)5^2	9831(0)3.9.27	9981(0)2^8
9541(Hom.sphere)			

Table 1G: Homologies of the first $\mathcal{S}(9,\ell,t,c)$'s

Chapter 3

Decomposition Theory: Handles

In this chapter we study combinatorial manifestations of *handles* in dimensions 2 and 3. We show that these combinatorial handles induce a standard decomposition theory at the topological level. This theory is strong enough to provide a gem based classification of the closed compact surfaces (orientable and non-orientable). This follows because there are only two building pieces, the superattractors $s^1 \times s^1$ and p^2, inducing the only two irreducible 2-manifolds: the 2-torus, $S^1 \times S^1$, and the real projective plane, RP^2. In contrast, starting in dimension 3, the theory becomes very weak because the irreducible 3-manifolds are very complicated. For this reason we switch, in the next chapter, to a different approach that seems not to need, in practice, the decomposition induced by handles (which nevertheless is useful).

3.1 A Gem-Based Classification of Surfaces

In this section we provide a gem-based topological classification of closed compact surfaces. It reflects the sort of thing we might naively hope for in the case of 3-manifolds.

3.1.1 Basic Properties and the Classification Theorem

Recall that a 2-gem is simply a $(2+1)$-graph and that the way to get a surface from a 2-gem G is as follows. Consider (i, j, k) as a permutation of the colors $(0, 1, 2)$ of the edges of a 2-gem. We let each bigon of a 2-gem G be the boundary of an attached disk. Then, clearly, the resulting topological space is a closed surface: note that each i-colored edge appears in an ij-gon and in an ikgon. The disks attached to these bigons are identified along the edge and so, globally, a closed surface, S_G, is formed. We say that G *induces* the surface S_G. For the simplest 2-gem, s^2, inducing the

2-sphere S^2 we have:

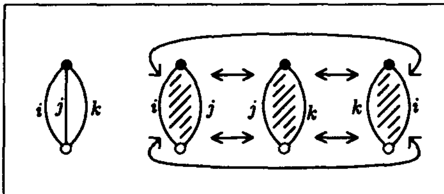


Fig. 74: s^2 inducing S^2

A *1-dipole* in an 2-gem is an edge of color i so that their ends are in distinct jk-gons. Here, The *cancellation of an 1-dipole* consists of deleting the i-colored edge and its two ends; following that the four free ends are pairwise pasted along edges of the same color j and k. The *creation of a 1-dipole* is the inverse operation.

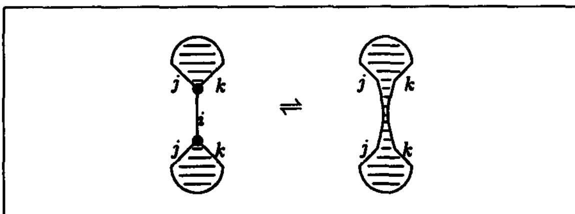


Fig. 75: Cancellation/creation of a 1-dipole in a 2-gem

The resulting graph is a 2-gem. The inverse operation is named *1-dipole creation* .

From the definition of 1-dipole and the construction of S_G follows at once the following proposition:

Proposition 14 *If G' is obtained from G by the cancellation of a 1-dipole, then S_G and $S_{G'}$ are homeomorphic surfaces, $S_G \cong S_{G'}$.*

Proof: Consider Figure 75. It shows the unique local changes from the embedding of G in S_G to the embedding of G' in the same surface: the two distinct jk-gons coalesce into one. ■

Remark 2 *Observe that the above proposition is just not true if the edge is not a 1-dipole.*

Each closed compact is induced by some 2-gem. We have the following easy consequence of the triangulation Theorem.

Proposition 15 *Let S be a connected closed surface. Then there is a 2-gem G inducing S.*

Proof: Let T be a triangulation for S. From T we construct G as follows: its vertices are triples of the form (v, e, f) where vertex v is a vertex of T incident to an edge e (of T) in turn incident to a face f. Each vertex (v, e, f) of G is embedded into S with the following instructions: choose a small distance ϵ; start at v go ϵ in the direction of e and after that ϵ towards the interior of f.

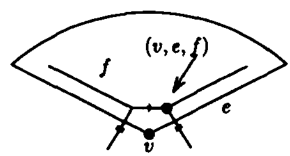


Fig. 76: Constructing 2-gem G from triangulation T

Two vertices of G are linked if they differ in precisely one of the three coordinates. If they differ in the $i-th$ coordinate the linking edge has color $i-1$. Edges of color 0 are parallel to the original edges; those of color 1 corresponds to an angle at a face of T; finally, those of color 2 crosses transversally the original edge e. This embedded G clearly induces S. Note that if we keep the colors, we do not need the embedding. It can be recovered, since the faces are just disks bounded by the bigons of G. ∎

Remark 3 *Note that the 2-complex defined by the 2-gem G of the above proof is simply the dual of the barycentric division of T.*

Motivated by Proposition 14 we introduce the following definition: two 2-gem are *equivalent* if one is obtained from the other by means of a finite number of *1-dipole moves*, i.e., cancellations and creations of 1-dipoles. The remaining of this section is devoted to provide an algorithmic proof of the following Theorem:

Theorem 6 (Classification Theorem) *Let S_1 and S_2 be closed connected surfaces, G_1 and G_2 be 2-gems such that $S_{G_i} \cong S_i$, $i = 1, 2$. Then $G_1 \equiv G_2$ if and only if $S_1 \cong S_2$.*

By the Proposition 14, equivalent 2-gems imply homeomorphic surfaces. Thus, only the converse implication needs to be established.

3.1.2 The 2-Dimensional Walking Lemma

The notation $v_1 \equiv_{ij} v_2$ means that vertices v_1 and v_2 are in the same ij-gon. In the case that edges e_1 and e_2 have distinct colors, we use $e_1 \equiv e_2$ to mean that they are in the same bigon. The two colors of this bigon are specified by the two colors of the edges. If edges e_1 and e_2 of the same color i appear in the same ij-gon, we use the notation $e_1 \equiv_j e_2$. Suppose (i, j, k) is an arbitrary permutation of the colors $(0, 1, 2)$ of a 2-gem G and that a, b, x are vertices of G satisfying the conditions:

- a is linked to b by an ij-path: an i-colored edge followed by a j-colored edge;
- $a \not\equiv_{ik} x$, $a \not\equiv_{jk} x$, $b \not\equiv_{ik} x, b \not\equiv_{jk} x$

In this situation, we name $(a, b; x)$ an *ij-walking triplet* of G. [1]

Let p, q be vertices of a 2-gem G. We denote by $G^{fus}_{p,q}$ the 2-gem obtained from G by the *p, q-fusion*, operation defined as follows: the vertices of $G^{fus}_{p,q}$ are those of G except p and q; two vertices of $G^{fus}_{p,q}$ are linked by an i-colored edge if they are so in G or if one is linked to p and the other to q by i-colored edges. The edges linking p to q (if there are any) disappear in the p, q-fusion. Note that this operation includes the 1-dipole cancellation when p, q are the ends of a 1-dipole. The following Lemma is central in the proof of Theorem 6 and also for the combinatorial handle theory of 3-manifolds.

Lemma 6 (2-Dim Walking Lemma) *If $(a, b; x)$ is an ij-walking triplet in a 2-gem G, then $G^{fus}_{a,x} \equiv G^{fus}_{b,x}$.*

Proof: We take (i, j, k) to be the identity (0,1,2). The proof is easily followable in the figures below:

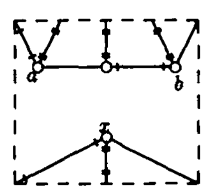


Fig. 77A: 2-Gem G: the initial situation

[1] The definition of walking triplet in dimension 2 is slightly more complex than in dimension 3 (given in the next section) because while 2-gems may be non-bipartite every 3-residue in a 3-gem induces the 2-sphere and so is bipartite.

Let $G_1 = G^{fus}_{a,x}$ and $G_2 = G^{fus}_{b,x}$:

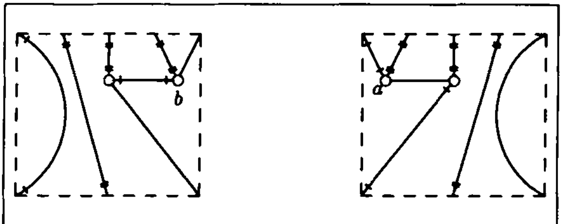


Fig. 77B: 2-Gems G_1 and G_2

Consider G'_1 and G'_2 given below. From G_1 to G_2 and from G'_1 to G'_2 we have created 1-dipoles. It follows from the hypothesis that $(a, b; x)$ is an 01-walking triplet in G that the two leftmost edges in the first figure above and the two rightmost edges in the second figure above are in the same bigons.

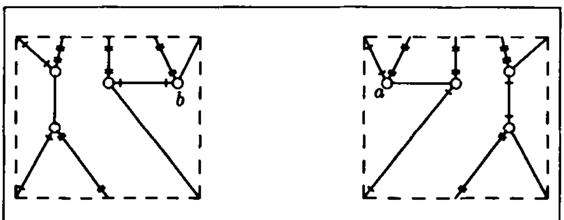


Fig. 77C: 2-Gems G'_1 and G'_2

Denote by G''_1 and G''_2 the two 2-gems below. The fact that (a, b, x) is a walking triplet also implies that the vertical (horizontal) 0-colored edge and the horizontal (vertical) 1-colored edge in the first (second) figures are in the same bigons. Therefore we get G''_1 and G''_2 by 1-dipole creations from G'_1 and G'_2.

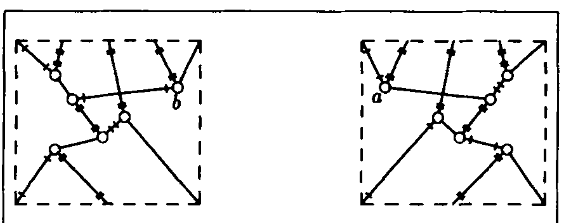


Fig. 77D: 2-Gems G''_1 and G''_2

Note that the 0-colored (1-colored) edge incident to the lowest left (right) vertex is a 1-dipole. By cancelling these 1-dipoles in 2-gems G''_1 and G''_2 produce, both, the same gem H.

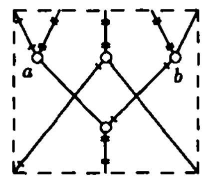


Fig. 77E: A symmetric final equivalent 2-gem H

This establishes the 2-dimensional Walking Lemma. ∎

An important special case of the Walking Lemma occurs when vertex x is in a distinct component of the one of vertices a and b. In the case that p and q are in different components of a gem we also denote $G_{p,q}^{fus}$ by $G_1^p\#^q G_2$, where G_1, G_2 is a partition of G with p in G_1 and q in G_2. This notation should be read as the *connected sum of G_1 and G_2 along p, q*.

Let G_1 and G_2 be connected bipartite 2-gems. The classes of the bipartition are labelled $^\bullet$class and $^\circ$class. Then, from the Walking Lemma, we have at most four classes of connected sums, denoted $G_1^\bullet\#^\bullet G_2$, $G_1^\bullet\#^\circ G_2$, $G_1^\circ\#^\bullet G_2$ and $G_1^\bullet\#^\circ G_2$: we replace the labels of the vertices by their classes. The following proposition divides by 2 these 4 possibilities.

Proposition 16 *Let G_1 and G_2 be disjoint 2-gems. Then*

$$G_1^\bullet\#^\bullet G_2 \equiv G_1^\circ\#^\circ G_2$$

$$G_1^\bullet\#^\circ G_2 \equiv G_1^\circ\#^\bullet G_2$$

Proof: In view of the Walking Lemma, it is enough to show that for arbitrary vertices, a, b (resp. c, d) ends of an i-colored edge of G_1 (of G_2) then $G_1^a\#^c G_2$ and $G_1^b\#^d G_2$ are equivalent. This follows because the fourth 2-gem below is obtained from either the second or the third one by the creation of one 1-dipole.

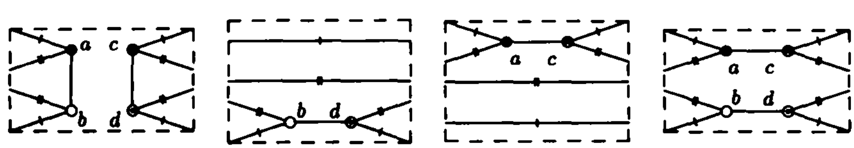


Fig. 78: Coalescing two classes of connected sums

This establishes the proposition. ∎

So, we might expect at most two classes of connected sums. However, by the symmetry of the **unique** minimum 2-gem inducing the **unique** *orientable prime 2-manifold*, the torus, we shall prove that there is in dimension two a unique connected sum. In fact this simplicity occurs only in dimension 2. As we shall see, for bipartite 3-gems G_1 and G_2 it might be the case that the two classes of possibly inequivalent connected sums $G_1^+\#^+G_2$ and $G_1^+\#^-G_2$ are indeed inequivalent. In fact, the 3-manifolds induced by such classes are topologically distinct. Specific examples are given in Chapter 6 about linking invariants for 3-manifolds.

A *polygon* in a graph is a non-null connected subgraph in which each vertex has degree 2. An easy fact is that a graph is bipartite if and only if there are no odd polygons. As a consequence of the Walking Lemma and of the previous proposition we show now that the existence of an odd polygon simplifies the situation.

Corollary 3 *If at least one of the connected gems G_1, G_2 has an odd polygon, then for arbitrary vertices a,b of G_1 and c,d of G_2, holds*

$$G_1^a\#^cG_2 \equiv G_1^b\#^dG_2.$$

Proof: Assume that G_1 has an odd polygon. Initially we claim that $G_1^w\#^yG_2 \equiv G_1^x\#^yG_2$ for w,x ends of an arbitrary i-colored edge of G_1 and an arbitrary vertex y of G_2. From the existence of an odd polygon in G_1 it follows that there is a sequence of vertices $w = v_0, v_1, \ldots, v_n = x$ such that v_i and v_{i+1} are ends of a path with 2 edges in graph G_1. This *2 by 2 connection*, is exemplified below:

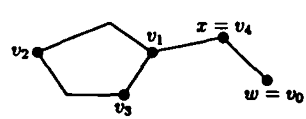


Fig. 79: An odd polygon provides parity changing

The claim follows by n applications of Lemma 6. Now let z be so that y,z are the ends of an i-colored edge in G_2. By the previous proposition, $G_1^w\#^zG_2 \equiv G_1^x\#^yG_2$. Together with the claim this implies $G_1^w\#^yG_2 \equiv G_1^w\#^zG_2$. The claim permits the "walking" along G_1 edge by edge. The last equivalence does the same in graph G_2. Therefore, by using a path in G_1 between a and b and a path in G_2 between c and d, we get $G_1^a\#^cG_2 \equiv G_1^b\#^cG_2 \equiv G_1^b\#^dG_2$. ∎

The (unique) common class of 2-gems given by the class of connected sums of 2-gems of Corollary 3 is denoted simply by $G_1\#G_2$, since a, b, c, d are irrelevant to define the class.

3.1.3 Special Sequences of 2-Gems Inducing the Closed Surfaces

We let the terminology attractor to apply also for the surfaces: the *attractor of a surface* is the set of 2-gems with minimum vertices inducing the surface. There are only three superattractors in dimension 2. They are important in the classification of closed surfaces and appear quite naturally in the theory. They are named s^2, p^2 and $t^2 = s^1 \times s^1$, inducing respectively the 2-sphere S^2, the real projective plane $RP^2 = P^2$ and the 2-dimensional torus, $T^2 = S^1 \times S^1$. The superattractors are depicted below:

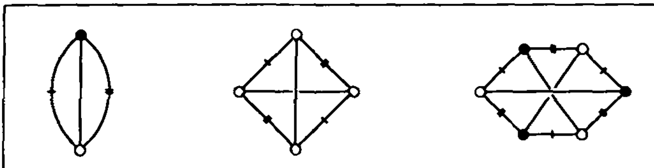


Fig. 80: The only superattractors in dimension 2: s^2, p^2 and t^2

Even being bipartite the 2-gem t^2 admits a color preserving automorphism which reverses the classes: simply take the reflection along the vertical axis passing through the center of t^2 in the way it is described above. This fact is explored later to show that there is only one class of connected sums in dimension two.

When we feel relevant, in the figures vertices in the $^\bullet$class and the $^\circ$-class shown explicitly. From the above reversion we get,

Corollary 4 *Let G be any connected 2-gem. Then any gem in $G^\bullet\#^\bullet T^2$ is equivalent to any other in $G^\bullet\#^\circ T^2$. Therefore the two classes coincide.*

∎

As before, the unique class of 2-gems defined by the connected sums of Corollary 4 is denoted by $G\#T^2$. More generally, the above corollaries enable us to define the classes of 2-gems

$$\underbrace{t^2\#t^2\#\ldots\#t^2}_{n} = t^2_n$$

$$\underbrace{p^2\#p^2\#\ldots\#p^2}_{m} = p^2_m$$

as the unique classes obtained by connected sums along arbitrary vertices of the n (resp. m) summands. Representative of these two sequences of classes are depicted below:

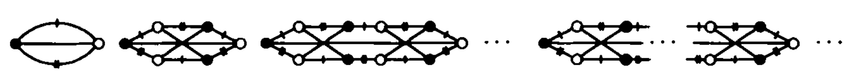


Fig. 81A: Sequence of 2-gems inducing the orientable surfaces

Fig. 81B: Sequence of 2-gems inducing the non-orientable surfaces

The first sequence induces the 2-sphere, the usual torus, the torus with two holes, ..., the torus with n holes, ... As for the second, it induces the real projective plane, the Klein bottle (connected sum of two projective planes), ..., connected sum of m projective planes ... The *Euler characteristic* of a 2-gem G, denoted by $\chi(G)$, is the number $v + b - e = b - \frac{v}{2}$, where v, b and e stand for the number of vertices, bigons and edges of G. The *bipartiteness character* of a graph G, denoted $\beta(G)$, is a boolean value which is true or false accordingly.

It is rather easy to show that the Euler characteristic and the bipartiteness character of a 2-gem are preserved under 1-dipole creation. In this way we define $\chi(t_n^2)$ and $\chi(p_m^2)$ as the Euler characteristic of any member of these classes. Follows that, $\chi(t_n^2) = (2+4n)+3-(3+6n) = 2-2n$ and $\chi(p_m^2) = (2+2m)+3-(3+3m) = 2-m$.

We also observe that the classes t_n^2 and p_m^2 are proper subsets of the attractors for T_n^2 and P_m^2. For instance, two 2-gems form the attractor for T_2^2. However, t_2^2 is formed by a single 2-gem, the one shown below on the left.

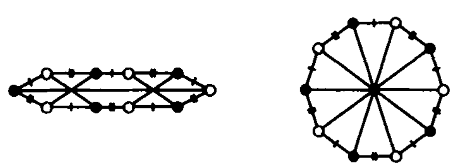


Fig. 82: The attractor for T_2^2: t_2^2 is only part of it

The following result is a stronger reformulation of Theorem 6. It shows that χ and β are complete invariants.

Theorem 7 (Classification Theorem) *Let G_1 and G_2 be connected 2-gems. Then $G_1 \equiv G_2$ if and only if $\chi(G_1) = \chi(G_2)$ and $\beta(G_1) = \beta(G_2)$.*

The necessity part is trivial. The proof of the suficiency is postponed to the final subsection of the section.

3.1.4 Completeness of the Special Sequences

Consider an arbitrary orientation of the edges of a graph G with m vertices and n edges, making it a *digraph*. The *incidence matrix* of G is an $m \times n$ matrix whose rows are indexed by its vertices and whose columns are indexed by its edges. Each column has (at most) two non-zero entries. If e is not a *loop*, i.e., its origin and terminus do not coincide, then the column indexed by e has a -1 in the row corresponding to the origin of e, a $+1$ in the row corresponding to the terminus of e, and zeros in the other rows. If the e is a loop the terminus and origin coincide, and the common row gets a zero. The column corresponding to a loop e has only null entries.

The real vector space of $\mathbb{R}^n$ generated by the rows of the incidence matrix of a digraph G is named the *bond space* of G, denoted $BS(G)$. The dimension of $BS(G)$ is $m - p$, where p is the number of components of G. See chapter 12 of [BM76]. The orthogonal complement of $BS(G)$ is named the *cycle space* of G and is denoted $CS(G)$. The dimension of $CS(G)$ is therefore $n - m + p$. The cycle space of G can be also described by the subspace generated by the characteristic vectors of the polygons of G. For more details see [BM76].

The next two lemmas are tools to show that the two lists of 2-gems presented are indeed complete sets of representatives for the closed surfaces.

Lemma 7 *Let G be a connected 2-gem. Then $\chi(G) \leq 2$. Moreover, if equality is attained, then G is bipartite.*

Proof: Since G is connected, the dimension of $CS(G)$ is, in the usual notation for 2-gems, $e - v + 1$. The subspace generated by the characteristic vectors of the bigons is a subspace of $CS(G)$ of dimension $b - 1$. This follows because this space coincides with $BS(H)$, where H is the *geometric dual* of G embedded in S_G: we consider a vertex of H for and inside each face. We link two vertices of H if the correponding bigons share an edge. This is a *dual edge,* crossing transversally once each primal one. From this discussion we get $b - 1 = \dim BS(H) \leq \dim CS(G) = e - v + 1$. This inequality translates into $\chi(G) = v + b - e \leq 2$.

If equality is attained, then $BS(H) = CS(G)$. Therefore, all polygons of G have an even number of edges since their characteristic vectors are sum of those of the bigons. Thus, G is bipartite. ∎

Lemma 8 *Let G be a connected bipartite 2-gem, with $v = 2k$ vertices and b bigons. Then k and b have the same parity and so, $\chi(G) = b - k$ is even.*

Proof: Denote the colors of G by 0, 1, and 2. Let $\{V^+, V^-\}$ be the bipartition of G. Consider the permutations π_{ij} on V^+ given by $\pi_{ij}(v)$ = terminus of a length 2 (i, j)-path starting at v. Observe that b is the sum of the number of cycles of the permutations π_{01}, π_{12} and π_{20}. The composition $\pi_{01} \circ \pi_{12} \circ \pi_{20}$ is the identity, therefore an even permutation. This implies that the total number of even cycles in the three permutations is even. (Recall that an even cycle is an odd parity permutation and vice-versa.) Thus the parity of the total number of cycles is the parity of the total number of odd cycles. Make three copies of V^+ and use each copy to represent the cycles of each permutation as disjoint polygons. The even polygons use an even number of the $3k$ vertices. Therefore, b has the same parity as the total number of odd polygons, which has the same parity as $3k$, or simply the one of k. ■

If a 2-gem G is equivalent to some member of t^2_n or p^2_m we write, by abuse of language, that $G \equiv t^2_n$ or $G \equiv p^2_m$.

Corollary 5 (Completion & no duplicates in the special sequences) *Let G be a connected 2-gem and assume Theorem 7 holds. Then there exists a unique n such that $G \equiv t^2_n$, or there exists a unique m such that $G \equiv p^2_m$. (Recall that we consider T^2_0 as S^2.)*

Proof: Recall that 1-dipole creations and cancellations do not change χ or β. Since $\chi(t^2_n) = 2 - 2n$ and $\chi(p^2_m) = 2 - m$, all the members in the same list are inequivalent. Members of distinct lists are distinguished by β. Therefore G might be equivalent to at most one member of one list. No bipartite 2-gem can have odd χ, by Lemma 8. No connected 2-gem can have $\chi > 2$, by Lemma 7. Together with Theorem 7 these facts imply that either there is an $n \geq 0$ with $\chi(t^2_n) = \chi(G)$ or else an $m > 0$ with $\chi(p^2_m) = \chi(G)$. Therefore, the lists are complete and contain once a representative of a class of homeomorphism of closed surfaces. ■

3.1.5 The Classification Theorem

Three edges (e_0, e_1, e_2) of a gem G is a *trio* if e_i is i-colored and $e_i \equiv e_{i+1}$, i mod 3. For the sort of 2-gems which are relevant for the proof of Theorem 7, namely those with exactly only three bigons, any three edges with distinct colors form a trio.

The crucial operation which heads towards the proof of Theorem 7 is the *breaking of a trio* which we now describe. Let the ends of e_i be u_i, v_i, $i = 0, 1, 2$. The notation can be adjusted so that for i=0 and i=1 we have: between u_i and u_{i+1} (whence between v_i and v_{i+1}) along their $(i, i+1)$-gon there is an even number of edges. To *break the trio* each edge e_i of the trio is subdivided into two: $\dot{e}_i$ and $\ddot{e}_i$; two new vertices p incident to $\dot{e}_i$ and q incident to $\ddot{e}_i$ are created $(i = 0, 1, 2)$. Note that fusion at the new vertices reproduces the original gem, before breaking the trio:

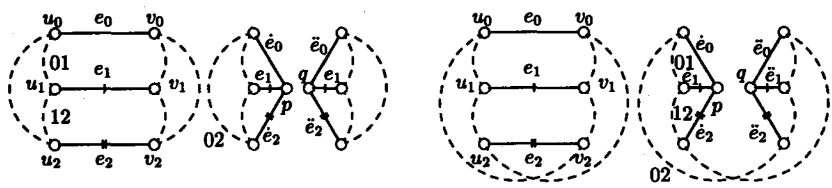


Fig. 83: Breaking of trios

Trios are of three types according to how they break:

- A type 0 trio is one whose breaking increases the number of components of the gem;
- A type 1 trio is one such that in the gem after the breaking $p \equiv_{20} q$;
- A type 2 trio is one which is not of type 0 and is such that in the gem after its breaking, $p \not\equiv_{20} q$.

On the left part of the above figure we may have a type 0 or type 2 trio. On the right part we depict a type 1 trio.

Lemma 9 (Type 1 trios) *Let G be a 2-gem containing a type 1 trio. The breaking of the type 1 trio produces a 2-gem G′ satisfying $G \equiv G' \# p^2$. (This operation is the combinatorial manifestation of the removal of a cross cap.)*

Proof: Observe that in graph G', the $(0, 2)$-path between p and q must have an even number of edges: so, we arrive at q by a 2-colored edge.

The graph H shown below is to be considered as obtained from G' by creating two 1-dipoles along the 2-colored edge incident to p, so four new vertices arise: a, b, c and d.

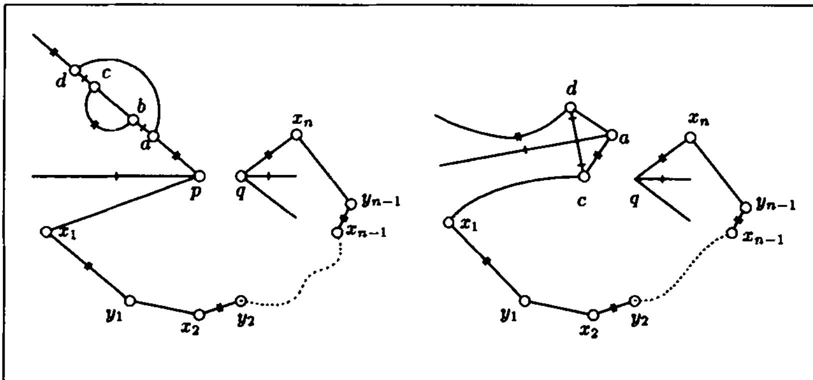


Fig. 84: 2-Gems H and $H^{fus}_{p,b}$

Since $p \not\equiv_{01} q$ and $p \not\equiv_{12} q$ and the same is true with b in place of p, it follows that $(p, b; q)$ is a 20-walking triplet. By Lemma 6, $H^{fus}_{p,q} \equiv H^{fus}_{b,q}$. Note that $(q, y_{n-1}; b)$, $(y_{n-1}, y_{n-2}; b)$, $\ldots (y_2, y_1; b)$ and $(y_1, p; b)$ are also 20-walking triplets. Therefore,

$$G \equiv G'^{fus}_{p,q} \equiv \ldots \equiv H^{fus}_{p,b} \equiv H\#p^2 \equiv G'\#p^2,$$

concluding the proof. ∎

As a consequence of the above Lemma we establish, in the following corollary, a fact needed to finish the proof of the Classification Theorem.

Corollary 6 *The classes $p^2\#t^2$ and p_3^2 are the same.*

Proof: Consider the type 1 trio $\{e_0, e_1, e_2\}$ shown on the left figure below, which is the unique member G of the class $p^2\#t^2$:

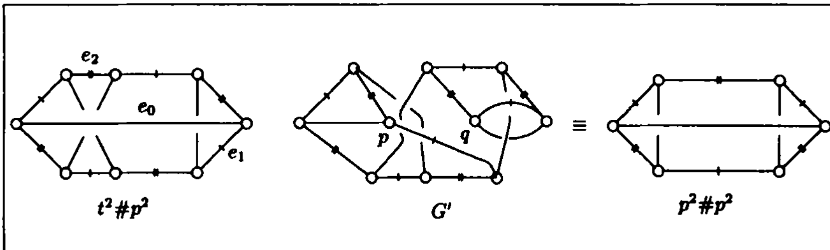


Fig. 85: Breaking a trio and cancelling two 1-dipoles

The breaking of the trio produces G', and this 2-gem is equivalent to $p^2\#p^2$. By Lemma 9 we get

$$G = p^2\#t^2 \equiv G'^{fus}_{p,q} \equiv G'\#p^2 \equiv p^2_3$$

establishing the corollary. ■

Lemma 10 (Type 2 trios) *Let G be a 2-gem and containing a type 2 trio and let p, q be the two new vertices of the 2-gem G' obtained by breaking the trio. If there is a path in G' with an odd number of edges between p and q, then $G \equiv G'\#t^2_1$. If there is a path with an even number of edges between those two vertices, then $G \equiv G'\#p^2_2$. (The two cases are combinatorial manifestations of the removal of orientable and non-orientable handles and are not exclusive: both can occur.)*

Proof: Assume the existence of an odd path between p and q. Consider the 2-gem H as depicted below, obtained from G' by introducing three 1-dipoles along the 0-colored edge incident to p, therefore creating six new vertices a, b, c, d, e and f.

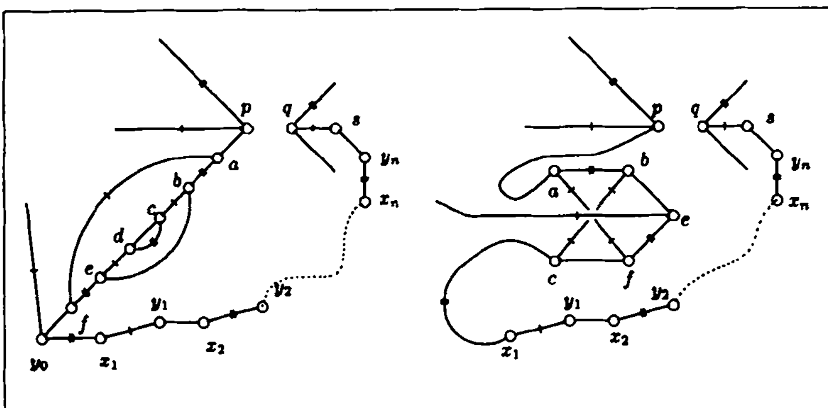


Fig. 86: Case of path of odd lenght: 2-Gems H and $H^{fus}_{y_0,d}$

Since we have broken a type 2 trio, in 2-gem H, $p \not\equiv_{ij} q$, for any pair of colors i,j. Note that $(p,b;q)$ is a 02-walking triplet and $(b,d;q)$ is a 12-walking triplet. Therefore $H^{fus}_{p,q} \equiv H^{fus}_{b,q} \equiv H^{fus}_{d,q}$. Also $(q,y_n;d)$ is an ij-walking triplet and thus $H^{fus}_{d,q} \equiv H^{fus}_{y_n,d}$. There are also colors i_k, j_k such that $(y_k, y_{k-1}; d)$ is an i_kj_k-walking triplet, for $k = n, n-1, \ldots, 1$. In this way,

$$G \equiv G'^{fus}_{p,q} \equiv H^{fus}p,q \equiv H^{fus}_{y_0,d} \equiv H\#t^2 \equiv G'\#t^2,$$

establishing the first part.

Suppose now there is an even path between p and q in G'. This time we subdivide, with three 1-dipoles, the 0-colored edge incident to p to get 2-gem H, as shown below:

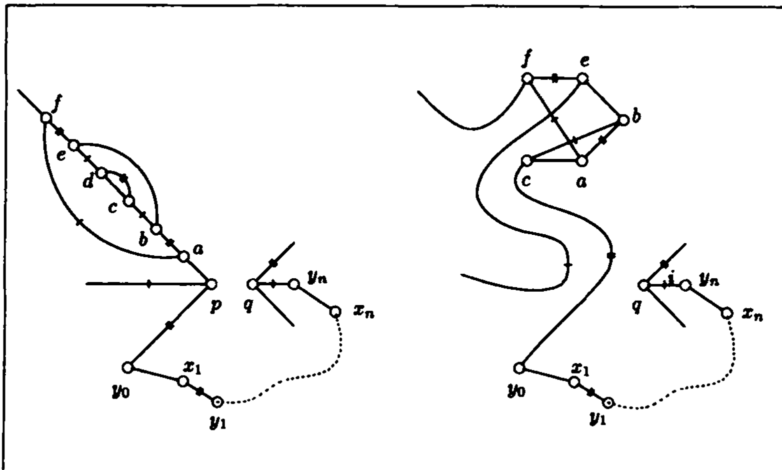


Fig. 87: Case of path of even lenght: 2-Gems H and $H^{fus}_{p,b}$

By the same sort of arguments this time we get

$$\begin{aligned} G \equiv G'^{fus}_{p,q} &\equiv H^{fus}_{p,q} \equiv H^{fus}_{b,q} \\ \equiv H^{fus}_{d,q} &\equiv H^{fus}_{x_n,d} \equiv \ldots \equiv H^{fus}_{x_1,d} \\ \equiv H^{fus}_{p,d} &\equiv H\#p_2^2 \equiv G'\#p_2^2, \end{aligned}$$

finishing the proof. ∎

Proof of the Classification Theorem: Let G be a 2-gem. It is enough to prove that there is either an $n \geq 0$ with $G \equiv t_n^2$ or else an $m > 0$ with $G \equiv p_m^2$. We may suppose that G is a 2-gem with three bigons and with more than six vertices. If it has more than three bigons, then we may cancell 1-dipoles.

We claim, and prove by induction on the number of vertices, that if a connected 2-gem G has three bigons, then there are non-negative integers r and s so that $G \equiv t_r^2\#p_s^2$. For this matter we consider $t_0^2 = s^2 = p_0^2$. It is simple to list the 2-gem having three bigons and at most six vertices: if it has two vertices, it is s^2. If it has four vertices is p^2. If it has six vertices it is either t^2 or p_2^2. Note that this proves the claim if the 2-gem has less than 8 vertices, and establishes the basis of the induction.

Let the above claim be true for all 2-gems with less vertices than G. Choose three differently colored edges (e_0, e_1, e_2) in G which are not all incident to a vertex. Since

G has three bigons this triple is a trio. Let G' be the 2-gem obtained by breaking the trio.

If we have a type 0 trio, then G' has two components G_1 and G_2 both having three bigons and less vertices than G (since both have at least four vertices). By induction there are non-negative integers r_k, s_k, $k = 1, 2$, such that $G_k \equiv t^2_{r_k} \# p^2_{s_k}$. It follows that $G \equiv t^2_{r_1+r_2} \# p^2_{s_1+s_2}$.

Now suppose the trio is of type 1. Then G' has two 01-gons, two 12-gons and one 20-gon. We may cancel two 1-dipoles from G' therefore arriving to a 2-gem H equivalent to G' having two less vertices than G. Thus there are r and s so that $H \equiv t^2_r \# p^2_s$. By Lemma 9, $G \equiv G' \# p^2 \equiv H \# p^2_1$. It follows that $G \equiv t^2_r \# p^2_{s+1}$.

Finally suppose the trio is of type 2. Then G' has six bigons, two of each type and we can eliminate three 1-dipoles to get a 2-gem H equivalent to G' with four less vertices than G. By induction, there are r and s so that $H \equiv t^2_r \# p^2_s$. By Lemma 10 we might have $G \equiv G' \# t^2_1$ or $G \equiv G' \# p^2_2$. We then get accordingly, $G \equiv t^2_{r+1} \# p^2_s$ or $G \equiv t^2_r \# p^2_{s+2}$.

Observe that if $s > 0$, then from repeated use of Corollary 6 we get $t^2_r \# p^2_s = p^2_{2r+s}$. This fact enables us to conclude: there is an integer n, $n \geq 0$, so that $G \equiv t^2_n$ or there is an integer m, $m > 0$, so that $G \equiv p^2_m$. ∎

A *cut-and-glue move* in 2-gems which are crystallizations is the breaking of a trio, followed by cancellations of enough 1-dipoles to provide a crystallization again, followed by the connected sum with p^2_1, p^2_2 or t^2_1 so as not to change β, χ nor the number of vertices.

We conclude this section with the following very simple characterization of bidimensional attractors.

Theorem 8 *Two gems belong to the attractor for T^2_n (for P^2_m) if they are bipartite (non-bipartite) crystallizations having $2 + 4n$ $(2 + 2m)$ vertices. Moreover any two 2-gems in an attractor are linked by a finite number of cut-and-glue moves.*

Proof: The proof has been given in the preceding discussion. ∎

We finish this section by tracing a parallel on what happens in dimensions 2 and 3. In both cases we have a finite number of vertex-increasing moves followed by a sequence of vertex-decreasing ones so that gems at the minimum level are linked by these moves. The vertex increasing moves in dimension 2 are the breaking of trios. The vertex-decreasing ones the cancellation of 1-dipoles. In dimension 3 we have the correspondence to the breaking of trios, namely breaking of quartets (which are always handles — see next section). However these moves are rather weak. What seems best is to consider the U-moves as the vertex-increasing moves. The vertex-non-increasing ones are the ρ-moves and the TS-moves. This strategy works to the level of 3-gems up to 30 vertices. When does it break down?

3.2 Handles in Dimension 3

In this section we develop a combinatorial theory of 1-handles in 3-manifolds. Essentially this theory is about connected sums with S^2-bundles over S^1.

3.2.1 The 3-Dimensional Walking Lemma

Two vertices in an $(n+1)$-graph are *n-separated* if any path between them has edges of all the $n+1$ colors. Vertices in disjoint components are automatically n-separated. Note that in the dual the simplexes corresponding to n-separated vertices are entirely disjoint.

In dimension $n \geq 3$, the definition of a *ij-walking triplet* is simpler. A triple of vertices $(u, v; x)$ in an $(n+1)$-graph is a *ij-walking triplet* if u and v are linked by an i-colored edge followed by a j-colored edge; moreover vertex x is n-separated from u (and from v).

A 3-manifold M^3 is *prime* if $M^3 = M_1^3 \# M_2^3$ implies that either $M_1^3 = M^3$ and $M_2^3 = S^3$ or that $M_2^3 = M^3$ and $M_1^3 = S^3$.

The following Lemma is central for our combinatorial theory of 3-dimensional handles. In practice the theory behaves well by identifying places where connected sums occur. However, on the contrary of the 2-dimensional case, it is rather useless to help in the general homeomorphism problem. Its weakness is due to the fact that the prime 3-manifolds are not just two as in dimension 2, but form an infinite set which has been, so far, defying any sensible classification.

Lemma 11 (3-Dim Walking Lemma) *If $(v, u; x)$ is an i, j-walking triplet in a 3-gem G, then $G_{v,x}^{fus} \equiv G_{u,x}^{fus}$.*

Proof: The proof follows the steps of the bidimensional case taken care in the previous section. In fact it generalizes easily for all dimensions $n \geq 2$. For completeness we do it again, with details, for the case n=3. Let $G_1 = G_{v,x}^{fus}$ and $G_2 = G_{u,x}^{fus}$. We present all the needed passages in the single figure below:

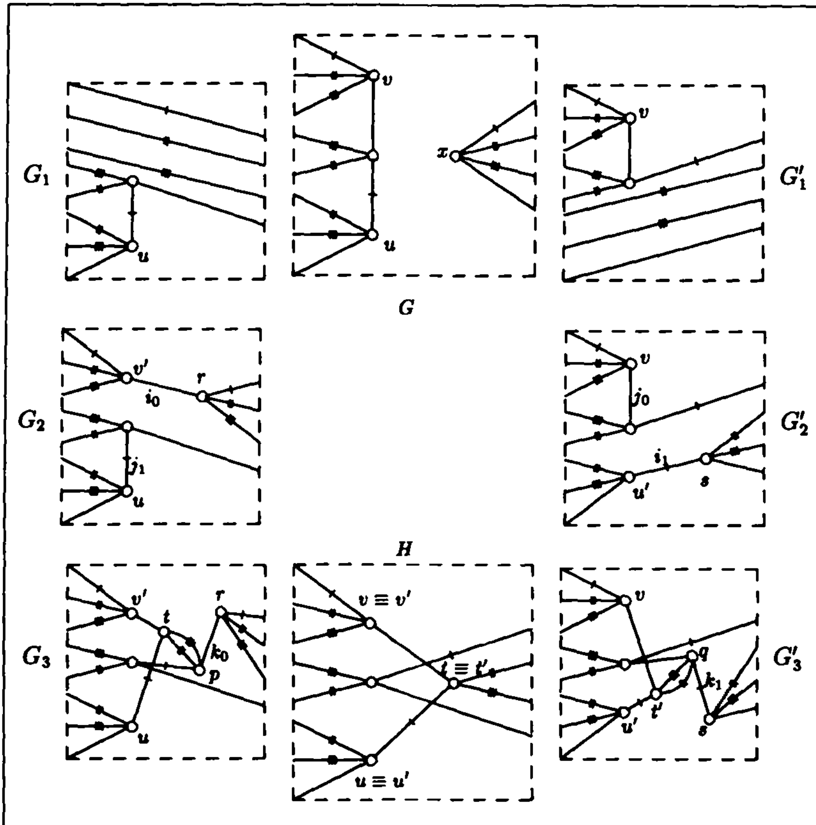


Fig. 88: Proving the 3-dimensional Walking Lemma

From G_1 to G_2 and from G'_1 to G'_2 we have created 1-dipoles, which follow from the hypothesis that $(v, u; x)$ is an i, j-walking triplet: whence v and x as well as u and x are 3-separated. These facts also imply that i_0 and j_1 are in a same bigon of G_2 and that i_1 and j_0 are in a same bigon of G'_2. Therefore we get G_3 and G'_3 by 2-dipole creations. The ends of edge k_0 of G_3 are in distinct $\hat{0}$-residue's because so are v and x in G. Analogously the ends of edge k_1 are in distinct $\hat{1}$-residues. The cancellation of 1-dipole k_0 in G_3 and of 1-dipole k_1 in G'_3 produce, both, the same gem H. ■

An important special case of the Walking Lemma occurs when vertex x is in a distinct component of the component of vertices v and u. In the case that p and q are in different components of a gem we also denote $G^{fus}_{p,q}$ by $G_1^p \#^q G_2$, where G_1, G_2 is a partition of G with p in G_1 and q in G_2. As with 2-gems, this notation should be read as the *connected sum of* G_1 *and* G_2 *along* p, q.

Let G_1 and G_2 be connected bipartite gems. The classes of the bipartition are labelled °class and •class. Then, from the Walking Lemma, we have at most four classes of connected sums, denoted $G_1^\circ \#^\circ G_2$, $G_1^\circ \#^\bullet G_2$, $G_1^\bullet \#^\circ G_2$ and $G_1^\bullet \#^\bullet G_2$. However, below we show that $G_1^\circ \#^\circ G_2 \equiv G_1^\bullet \#^\bullet G_2$, and that $G_1^\circ \#^\bullet G_2 \equiv G_1^\bullet \#^\circ G_2$.

Proposition 17 *Let G_1 and G_2 be disjoint gems, a,b (resp. c,d) ends of an i-colored edge of G_1 (of G_2). Then $G_1^a \#^c G_2$ and $G_1^b \#^d G_2$ are equivalent.*

Proof: Consider the situation below:

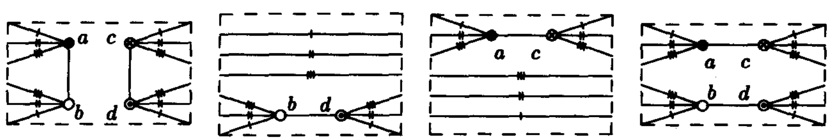


Fig. 89: Coalescing the four classes of connected sums into two

The second and third gems are equivalent to the fourth one, estabilishing the proof. ■

So, we might expect at most two classes of connected sums. And, on the contrary of dimension 2, they do occur in dimension 3: there are two non-homeomorphic classes of connected sums of the lens space $L_{3,1}$ with itself. In Chapters 6 and 7 we deal with invariants that distinguish these connected sums: the self-linking sequence and the new quantum invariants.

As a consequence of the Walking Lemma and of the above proposition we show that the existence of odd polygons again simplifies the situation.

Corollary 7 *If at least one of the connected gems G_1, G_2 has an odd polygon, then for arbitrary vertices a,b of G_1 and c,d of G_2, holds*

$$G_1^a \#^c G_2 \equiv G_1^b \#^d G_2.$$

Proof: The proof is exactly the same as in the 2-dimensional case of the same corollary, given in the previous section. ■

3.2.2 The Superattractors for $S^1 \times S^2$ and $S^1 \tilde{\times} S^2$

We present the smallest 3-gems inducing (the attractors for) $S^1 \times S^2$ (and $S^1 \tilde{\times} S^2$ – the non-orientable S^2 - bundle over S^1). These specific 3-gems play a distinguished role in our combinatorial theory of handles.

Two vertices in a 3-gem are *3-separated* if any path between them involves edges of the four colors. In particular, vertices in distinct components are 3-separated.

The manifold $S^1 \times S^2$ is obtained from S^3 by removing two disjoint tetrahedra and identifying their boundary with distinct orientations. This is easily achievable in terms of small gems. Start with the 2-vertex gem for S^3 and create 1-dipoles to get 3-separated vertices v and w (w'), as shown below:

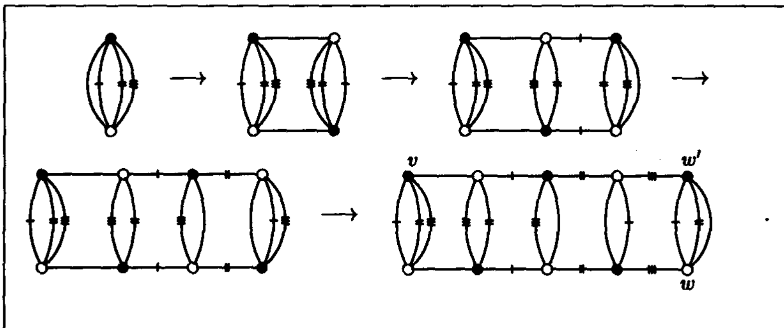


Fig. 90: Obtaining 3-separated vertices from s^3

In the dual interpretation vertices v and w correspond to tetrahedra which are entirely disjoint. Moreover, their boundary have distinct orientations, since the vertices are in different classes of the bipartition. Therefore, to obtain a gem for $S^1 \times S^2$ we just have to effect fusion along v and w in the last gem above. This fusion corresponds to the identifications along the boundaries of the tetrahedra. Note that the interior of the tetrahedra are no longer present since v and w disappear:

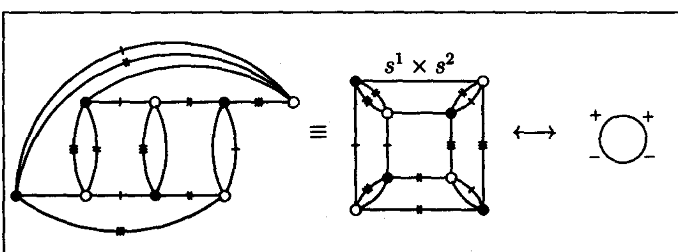


Fig. 91: $s^1 \times s^2$: Superattractor for $S^1 \times S^2$

The above gem is named $s^1 \times s^2$ and is the superattractor for $S^1 \times S^2$. It is a σ-gem and on the right we display a string presentation for it.

To obtain a 8-vertex gem inducing $S^1 \tilde{\times} S^2$ we just have to effect the fusion along v and w. These vertices are in the same class. Accordingly we get:

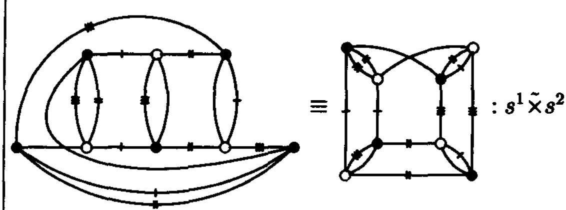


Fig. 92: $s^1 \tilde{\times} s^2$: Superattractor for $S^1 \tilde{\times} S^2$

This gem, named $s^1 \tilde{\times} s^2$, is the superattrator for $S^1 \tilde{\times} S^2$.

An automorphism of a bipartite gem is said to be *orientation reversing* if the colors of its edges are permuted by an even permutation and there is a $(\circ, \bullet)$ interchange of the vertex classes or if there is no such interchange but the edge color permutation is of odd class. Note that this definition accords to the topological notion of orientability of the induced manifold: this follows straightforward from the construction of the manifold from the gem.

The next proposition shows that the class of $G^a \#^b (s^1 \times s^2)$ and the class of $G^a \#^b (s^1 \tilde{\times} s^2)$ are independent of a and b. So there is no ambiguity in denoting these classes simply by $G\#(s^1 \times s^2)$ and $G\#(s^1 \tilde{\times} s^2)$.

Proposition 18 *Let G be a 3-gem, a and b vertices on it. Let c and d be vertices of either $s^1 \times s^2$ or $s^1 \tilde{\times} s^2$. Then*

- $|G^a \#^c (s^1 \times s^2)| \cong |G^b \#^d (s^1 \times s^2)|$;
- $|G^a \#^c (s^1 \tilde{\times} s^2)| \cong |G^b \#^d (s^1 \tilde{\times} s^2)|$.

Proof: The second fact follows from Proposition 7, since $s^1 \tilde{\times} s^2$ has odd polygons. The first follows from the fact that $s^1 \times s^2$ admits an orientation reversing automorphism. Indeed, the bijection which sends vertex i onto vertex 9-i, as numbered below, is an automorphism that preserves the colors but interchanges the $(\circ, \bullet)$-classes.

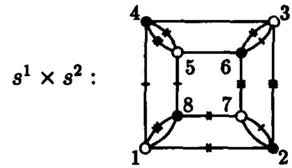


Fig. 93: A color preserving, (•, ∘)-reversing automorphism in $s^1 \times s^2$

This establishes the independence of a, b, c, d. ∎

Note that having an orientation reversing automorphism implies a *symmetric space*, that is, a 3-manifold which cannot be distinguished from itself with opposite orientation. The simplest example of non-symmetric space is the lens space $L_{3,1}$, whose superattractor is seen below:

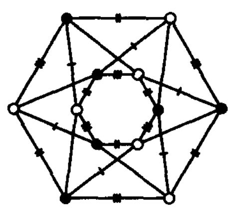


Fig. 94: The superattractor $\ell_{3,1}$ for $L_{3,1}$, a non-symmetric 3-manifold

The fact that this space is non-symmetric implies that not only its superattractor, but any gem inducing it has no orientation reversing automorphism. This fact is easily proved by means of the linking invariants of Chapter 6.

3.2.3 3-Separation, $s^1 \times s^2$ and $s^1 \tilde{\times} s^2$

We can now state and prove, solely by combinatorial means, our main tool to deal with 3-dimensional handles. A similar theorem holds for all dimensions n and the proof is a straightforward generalization of the cases $n = 2$ and $n = 3$.

Theorem 9 *Let a and b be 3-separated vertices in a connected 3-gem G. Then,*

- *if there is a path π with an odd number of vertices between a and b,*

$$|G^{fus}_{a,b}| \cong |G\#(s^1 \times s^2)| \ ;$$

- *if there is a path π with an even number of vertices between a and b,*

$$|G^{fus}_{a,b}| \cong |G\#(s^1 \tilde{\times} s^2)| \ .$$

Proof: Consider the 3-gem H obtained from G by creating four 1-dipoles in distinct colors along one of the edges incident to a, as depicted below:

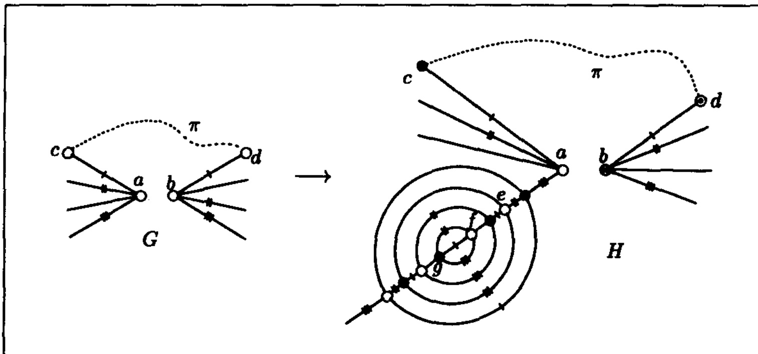


Fig. 95: Inducing 3-separation to form superattractors $s^1 \times s^2$ and $s^1 \tilde{\times} s^2$

Observe that $G \equiv H$, and so $G^{fus}_{a,b} \equiv H^{fus}_{a,b}$. By two applications of the Walking Lemma, $H^{fus}_{a,b} \equiv H^{fus}_{e,b} \equiv H^{fus}_{f,b}$. If π has an odd number of edges, the Walking Lemma implies $H^{fus}_{f,b} \equiv H^{fus}_{f,c} \equiv H^{fus}_{g,a}$, the last equivalence following from Proposition 17. The gem $H^{fus}_{g,a}$ is equal to $G^a\#^x(s^1 \times s^2)$, for some vertex x as shown below:

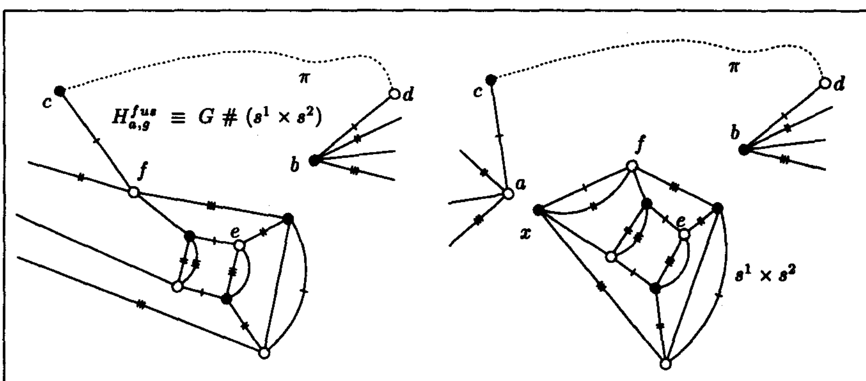


Fig. 96: Case π odd: extracting an $s^1 \times s^2$-summand

Therefore $G^{fus}_{a,b} \equiv G\#(s^1 \times s^2)$.

To establish the second part, note that if π has an even number of edges, by various applications of the Walking Lemma along π, $H^{fus}_{f,b} \equiv H^{fus}_{f,a}$. The gem $H^{fus}_{f,a}$ is equal to $G^a\#^x(s^1 \tilde{\times} s^2)$, for some vertex x, as indicated below:

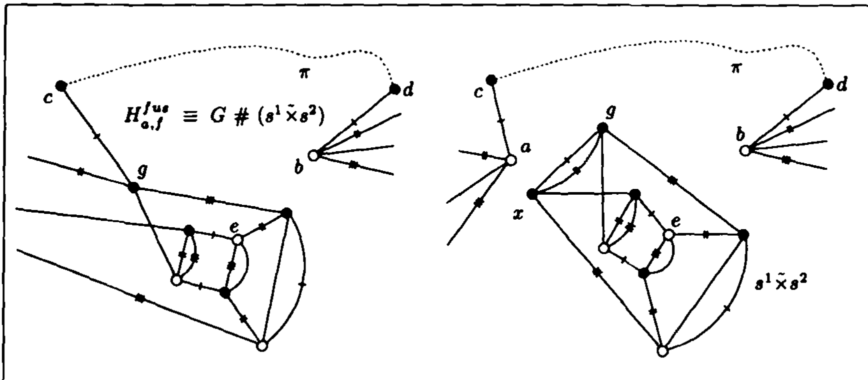


Fig. 97: Case π even: extracting an $s^1 \tilde{\times} s^2$-summand

Therefore $G^{fus}_{a,b} \equiv G\#(s^1 \tilde{\times} s^2)$, establishing the theorem. ■

3.2.4 Quartets and Handles

A *quartet* in a $(3+1)$-graph is a set of 4 distinctly colored edges $\{e_0, e_1, e_2, e_3\}$ which are pairwise in the same bigon. Let the ends of e_i be u_i and v_i, $i = 0, 1, 2, 3$. The notation can be adjusted so that, for $i = 0, 1, 2$, between u_i and u_{i+1} there is an even number of edges along the $(i, i+1)$-gon. In general $(3+1)$-graphs we might have 8 types of quartets depending on

- whether the path π_{02} between v_0 and v_2 along the 02-gon not including u_0 includes vertex u_2 or not;
- whether the paths π_{13} between v_1 and v_3 along the 13-gon not including u_1 includes vertex u_3 or not;
- whether the paths π_{03} between v_0 and v_3 along the 03-gon not including u_0 includes vertex u_3 or not.

In the case that these three paths do not include u_i, $i = 2, 3$, the quartet is called a *handle*. In a handle the paths π_{ij}'s connect as shown below:

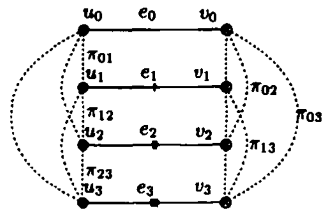


Fig. 98: Connections in a quartet that yields a handle

Of course in a generic (3 + 1)-graph the eight types of quartets are possible. However in 3-gems we have:

Proposition 19 *Any quartet* $\{e_0, e_1, e_2, e_3\}$ *in a gem is a handle.*

Proof: Assume one of the paths π_{02}, π_{13}, or π_{03} is of the wrong type. These wrong connections are shown below:

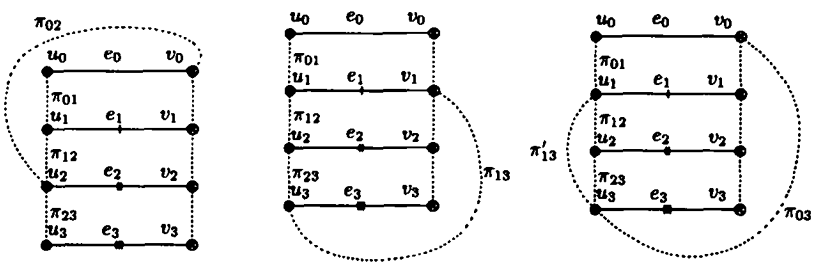


Fig. 99: Impossible connections in a 3-gem: the only quartet is a handle

Observe that each dotted path in these figures has an even number of edges. If π_{02} is as shown, then π_{02} followed by path π_{12}^{-1}, by path π_{01}^{-1} and by edge e_0 is an odd polygon in a $\hat{3}$-residue. This contradicts the fact that each 3-residue is a 2-gem for a 2-sphere, which is bipartite. If path π_{13} is as shown, then consider the odd polygon: $\pi_{13}, \pi_{23}^{-1}, \pi_{12}^{-1}, e_1$. This odd polygon is in a $\hat{0}$-residue, contradicting the fact that this residue is bipartite. Therefore, we might suppose that the path π'_{13} has the type of connection shown in the third figure. Finally, if path π_{03} has the type of connection shown, then the odd polygon $\pi_{03}, \pi'^{-1}_{13}, \pi_{01}^{-1}, e_0$ is in a $\hat{2}$-residue. A similar contradiction arises, establishing the proposition. ∎

Handles are nice configurations because they induce a natural partition of the gem and associated manifold. To recover the manifold, we just have to do connected sums with the pieces. In the orientable case, it is enough to keep track of the orientation of the components by a $(\circ, \bullet)$-partition of their vertices. Do the connected sums *always* along a °vertex and a •vertex. To get the partition we just have to *break the handle*, similarly as we have done in dimension 2:

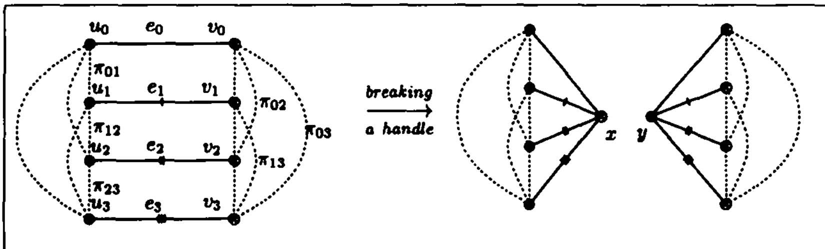


Fig. 100: The operation of breaking a handle

Even if the breaking of a handle does not disconnects the 3-gem, we have a non-trivial partition at the topological level.

Theorem 10 (Breaking Handle Theorem) *Let G be a connected 3-gem having a handle $\{e_0, e_1, e_2, e_3\}$. Let H be the $(3+1)$-graph obtained by breaking this handle, x and y the new vertices. Then H is a 3-gem and vertices x and y are 3-separated. Moreover, if H is connected then $|G| \cong |H\#(s^1 \times s^2)|$ or $|G| \cong |H\#(s^1 \tilde{\times} s^2)|$. If H has two components, H_1 and H_2 then $G = H^{fus}_{x,y} = H_1^x \#^y H_2$.*

Proof: Denote by v', b' and t' the number of vertices, bigons and 3-residues of H, and by v, b and t the same numbers for G. We have $v' = v + 2$ and $b' = b + 6$. Since $b' \leq v' + t'$, by Corollary 1, we must have $t' \geq t + 4$. On the other hand, as the fusion along x, y (which transforms H back into G) can only decrease by at most one the number of $\hat{i}$-residues $(0 \leq i \leq 3)$ we also conclude that $t' \leq t + 4$. Thus equality holds implying that $b' = v' + t'$, showing that H is a 3-gem.

Observe that x and y are 3-separated in H, because they belong to distinct $\hat{i}$-residues for each color i. Note that $G^{fus}_{v,w} \equiv H$. If H is connected the result follows at once when applying to it Theorem 17 with x, y in place of a, b. If H has two components H_1 and H_2, then clearly, $G = H^{fus}_{x,y} = H_1^x \#^y H_2$. ∎

If after breaking a handle, the number of connected pieces increase, the original manifold is expressible as a connected sum of manifolds induced by smaller gems, and this is desirable. If the resulting gem H is connected, then we can cancell at least four 1-dipoles in it. This gives us a gem with six less vertices than the original one. To recover the manifold we just have to effect connected sum of the final gem with $s^1 \times s^2$ or $s^1 \tilde{\times} s^2$, according to Theorem 9. These are constant 3-gems and we may keep track of just the number of these summands.

3.2.5 ρ_3-Pairs and Handles

We now study another operation which implies a handle, whence a connected sum partition. Recall that a *ρ_3-pair* in a $(3+1)$-graph G is a pair of edges $\{a_1, a_2\}$ equally colored which is contained in three bigons. For the switching of a ρ_i-pair, $i = 2, 3$, we refer to Fig. 14. Assume that G is a 3-gem. As mentioned in Section 2.3.2 even if G is not bipartite, we can partition the vertices of one of the bigons of a ρ_3-pair by labeling them alternatively °vertex and •vertex. This labels the four vertices of the ρ-pair. This labeling can be consistently extended to all the vertices of the other(s) bigon(s) associated to the ρ_3-pair: if not we would detect an odd polygon in a 3-residue contrary to the hypothesis that G is a 3-gem. Recall that $G^{swt}_{a_1,a_2}$ denotes the $(3+1)$-graph obtained from 3-gem G by switching a ρ_3-pair $\{a_1, a_2\}$.

Proposition 20 (Lemma 9 of [FL91]) *Let G be a connected 3-gem, (h,i,j,k) a permutation of $(0,1,2,3)$, $\{a_1,a_2\}$ a ρ_3-pair of color h. Then, if $G' = G^{swt}_{a_1,a_2}$ is a connected 3-gem, $|G| \cong |G'\#(s^1 \times s^2)|$ or $|G| \cong |G'\#(s^1 \tilde{\times} s^2)|$. If G' has components G_1 and G_2, then $|G| \cong |G_1\#G_2|$. Moreover, $t_h(G') = t_h(G)$ and $t_c(G') = t_c(G) + 1$, for $c \neq h$.*

Proof: We prove the proposition for (h,i,j,k) the identity. Let a_3, a_4, a_5 be the non 0-colored edges incident to one of the ends of a_1. Observe that $\{a_2, a_3, a_4, a_5\}$ is a handle. Let H be the $(3+1)$-graph obtained by breaking this handle.

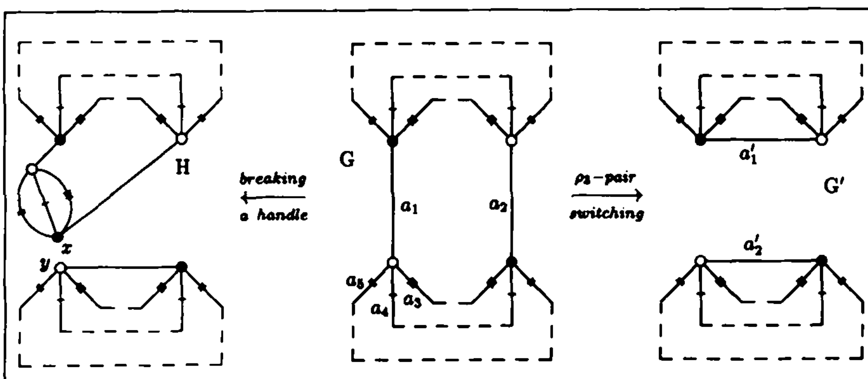


Fig. 101: Breaking a handle, switching a ρ_3-pair

The result follows trivially from the comparison between H and G' and from the Breaking Handle Theorem. ∎

Corollary 8 *Let G be a crystallization and G' obtained from G by a ρ_3-pair switching. G' is a connected 3-gem and we can cancell three 1-dipoles on it. Moreover,* $|G| \cong |G'\#(s^1 \times s^2)|$ *or* $|G| \cong |G'\#(s^1 \tilde{\times} s^2)|$.

Proof: This is an easy consequence of previous proposition because if the switching disconnects G, it would have at least two $\hat{h}$-residue, where h is the color of the ρ_2-pair. This contradicts the fact that G is a crystallization. ■

In virtue of this corollary, the existence of ρ_3-pairs in crystallizations means that the 3-manifold is induced by a connected sum in which the summands are induced by smaller 3-gems.

3.2.6 Double Diagonals and Handles

A pair of edges of colors h, i having the same ends v, w is called a *double diagonal* if these ends are vertices of the same jk-gon and the distance between v and w along the jk-gon is at least 3. Here (h, i, j, k) is a permutation of $(0, 1, 2, 3)$. The *cancellation of a double diagonal* with end points v and w is defined to be the fusion along v and w.

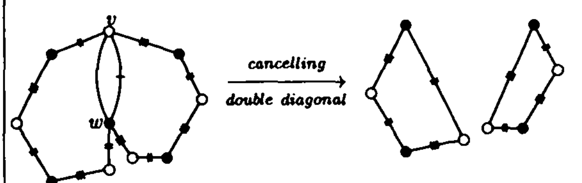


Fig. 102: Cancelling a double diagonal

We can now establish a proposition which show that except in $s^1 \times s^2$ and $s^1 \tilde{\times} s^2$ double diagonals are unecessary. This means that, in the presence of double diagonals, we get a smaller 3-gem and a number of copies of connected sums with $s^1 \times s^2$ and $s^1 \tilde{\times} s^2$ inducing the same 3-manifold.

Proposition 21 *Let G be a connected 3-gem having a double diagonal with end points v and w. Cancelling the double diagonal produces a 3-gem $G^{fus}_{v,w}$. If $G^{fus}_{v,w}$ is connected, then* $|G^{fus}_{v,w}| \cong |G\#(s^1 \times s^2)|$ *or* $|G^{fus}_{v,w}| \cong |G\#(s^1 \tilde{\times} s^2)|$. *If $G^{fus}_{v,w}$ has two components G_1 and G_2, then $G \equiv G_1^x \#^y G_2$, for some vertices x of G_1 and y of G_2.*

Proof: Note that $\{e_0, e_1, e_2, e_3\}$ as shown in the central figure below is a handle. Consider the $(3+1)$-graph H obtained from G by breaking the handle. Note that $H^{fus}_{a,b} = G$.

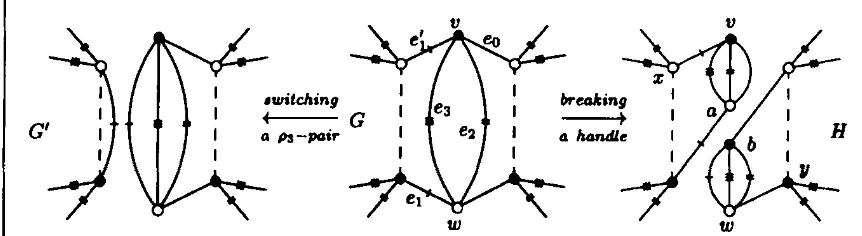


Fig. 103: Switching a ρ_3-pair, breaking a handle

The result follows from the Breaking Handle Theorem, if we note that the final graph $G^{fus}_{v,w}$ is obtained from H by cancelling two 1-dipoles.

This result also follows from Proposition 20, because e_1, e'_1 is a ρ_3-pair. Its switching produces gem G' from which we get $G^{fus}_{v,w}$ by cancelling a 1-dipole.

Just to be precise with the vertices x and y: if $G^{fus}_{v,w}$ has two components G_1 and G_2, let H_1 and H_2 be the corresponding components of H. Then with a, b, x, y as shown in the above figure,

$$G = H^{fus}_{a,b} = H^a_1 \#^b H_2 \equiv H^x_1 \#^y H_2 \equiv G^x_1 \#^y G_2,$$

proving the proposition. ∎

We finish this chapter with a conjecture motivated by all the examples so far. An *irreducible 3-manifold* is one in which every 2-sphere is the boundary of a 3-ball in the 3-manifold. A *reducible 3-manifold* is one which is not irreducible. The only reducible prime 3-manifolds are $S^1 \times S^2$ and $S^1 \tilde{\times} S^2$ ([Hem76], Lemma 3.13).

Conjecture 5 *Let G be a rigid 3-gem inducing a reducible 3-manifold. Then there are in G four differently colored edges in G which form a handle.*

Thus, if true, the Conjecture implies that the topological connected sum could be detected at the combinatorial level, as discussed on Theorem 10.

Chapter 4

Simplifying Dynamics: the TS_ρ^U-Algorithm

In this chapter we give detailed proofs that the TS-moves and that the U-move maintain the induced 3-manifold. We give some other reducing configurations (like the clusters of four squares) and discuss in detail a specific algorithm, the TS_ρ-algorithm. This algorithm has been implemented by Cassiano Durand and is the basis for the topological classification of the bipartite 3-gems up to 30 vertices. In the final section, illustrating the capability of the TS_ρ-algorithm, we use it to recognize all orientable 3-manifolds which are formed by pairwise identifications of the six faces of a solid cube. This is achieved because the TS_ρ-algorithm in all but one case produces the attractors for these 3-manifolds. The exception is the lens space $L_{12,5}$, for which we need to introduce one U-move in order to obtain the attractor.

4.1 ρ-Move, TS-Moves and U-Move

In this section we discuss the ρ-algorithm, and show that the TS-moves and the U-move do not alter the 3-manifold because they are factorable as dipole moves. We restrict the discussion for the bipartite case, but the generalization to the non-orientable case is straightforward.

4.1.1 The ρ-Algorithm

We describe a basic simplification algorithm that takes any bipartite 3-gem G and a non-negative integer n as input and produces a rigid crystallization H and another integer m, so that $m \geq n$ as output. The crucial property of the algorithm is that the input induces the 3-manifold $|G|\#(S^1 \times S^2)_n$ and the output induces the homeomorphic 3-manifold $|H|\#(S^1 \times S^2)_m$ or $|H|\#(S^1 \tilde{\times} S^2)_m$. We have the first possibility if and only if G is bipartite.

A *rigid 3-gem* is a crystallization free of ρ-pairs. Let $\mathcal{G}$ denote the set of all 3-gems and $\mathcal{RG}$ the set of the rigid 3-gems. The ρ-algorithm defines a function $\rho_* : \mathcal{G} \times \mathbb{N} \to \mathcal{RG} \times \mathbb{N}$. This function depends on the code. Consider a 3-gem G with *code-numbered* vertices and *code-colored edges*. This means that the numbering and the coloring is given by the bipartite DFS-numbering which attains the code. Thus the vertices are numbered from 1 to v_G and the edge colors from 0 to 3. The edge of color c incident to odd numbered vertex $2i-1$ gets the number $cv_G/2+i$. We consider the set of dipoles, if non-empty, ordered by the number of their edges. We also consider the set of ρ-pairs, if non-empty, by the lexicographical order of the pairs of numbers associated to their two edges. We are assuming, in particular, that each time a 3-gem is modified, its code is immediately computed to have these orders at hand. Also, in describing the algorithm which defines ρ_* we do not distinguish between a vertex, an edge and their canonical numbers. Recall that a *ρ-move* is either the cancellation of a 1-dipole or the switching of a ρ-pair (which creates 1-dipoles) followed by the cancellation of a 1-dipole. We note that a 2-dipole is never cancelled directly: its cancellation is factored as a ρ_2-pair (2 edges of a same color not involved in the 2-dipole but incident to it) switching followed by a 1-dipole cancellation.

Algorithm 7 (ρ-algorithm, getting function $\rho_*(G,n)=(H,m)$) :

$m \leftarrow n$;
$H \leftarrow G$;
repeat
    repeat
        if H has 1-dipoles then begin
            $\{p,q\} \leftarrow$ pair of numbers of ends of smallest 1-dipole;
            $H \leftarrow H^{fus}_{p,q}$;
        end;
    until H has no 1-dipoles;
    if H has ρ-pairs then begin
        $\{e,f\} \leftarrow$ pair of numbers of edges in the smallest ρ-pair;
        $H \leftarrow H^{swt}_{e,f}$;
        if $\{e,f\}$ is a ρ_3-pair then $m \leftarrow m+1$;
    end;
until H has no ρ-moves available.

In the output of the above algorithm we have a pair (H,m), with H a connected crystallization so that if G is bipartite,

$$G \equiv H \;\#\; \underbrace{((s^1 \times s^2) \;\#\; (s^1 \times s^2) \;\#\; \dots \;\#\; (s^1 \times s^2))}_{m-n},$$

and if G is not bipartite,

$$G \equiv H \ \# \ \underbrace{((s^1 \tilde{\times} s^2) \ \# \ (s^1 \tilde{\times} s^2) \ \# \ \ldots \ \# \ (s^1 \tilde{\times} s^2))}_{m-n}$$

This is because each time we switch a ρ_3-pair in a crystallization we get a connected gem and a summand $s^1 \times s^2$ or $s^1 \tilde{\times} s^2$, arises according to Corollary 8.

We define $\rho_*(G, n)$ to be (H, m), given as the output of Algorithm 7. In case that $m = n$, then we may write only $\rho_*(G) = H$. If $n = 0$, by a further abuse of language, we write $\rho_*(G) = (H, m)$.

If G induces S^3 there is a good chance that H be s^3, the 3-gem with two vertices. Note that this gem is rigid, according to our definition. Indeed, s^3 is the only rigid 3-gem which has the same pair of vertices linked by more than one edge. Unfortunately there are other rigid 3-gems inducing S^3. The smallest of them has 24 vertices. In fact there are 3 rigid 3-gems with 24 vertices inducing S^3. They are r^{24}_{20}, r^{24}_{61} and r^{24}_{199}. Of these only the first is a σ-gem with corresponding string presentation given below. The 0-colored edges link the vertices $2i-1$ and $2i$. Some are indicated in a dashed way.

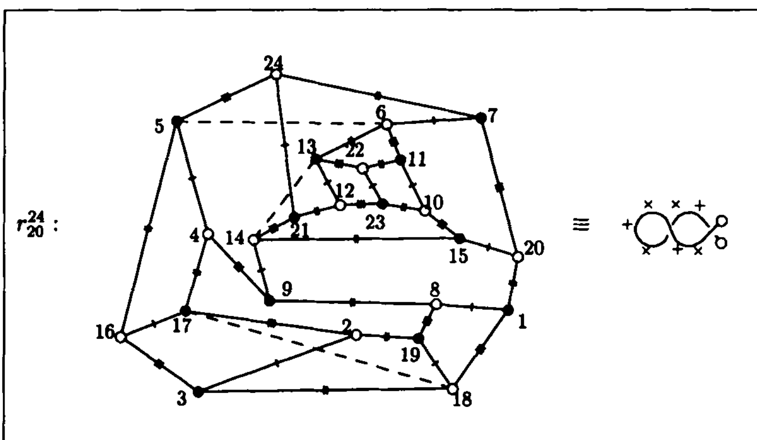


Fig. 104: A rigid σ-gem inducing S^3

Every rigid 3-gem G satisfies $\rho_*(G) = G$, being a fixed point for ρ_*. We shall now introduce more elements in the theory, which cause, in particular, the above 3-gem to collapse into s^3.

4.1.2 The TS-Moves

Let G be a 3-gem and $B(G)$ the set of its bigons. For $a \in B(G)$ let s_a denote the number of edges in the bigon a, or the *size* of a. Let also t_G be the number of 3-residues of G.

Proposition 22 (Lemma 2 of [Lin86]) *For any 3-gem G the following equality holds:*

$$\sum_{a \in B(G)} (3 - \frac{1}{2} s_a) = 3 \times t_G.$$

Proof: The sum is $3b_G - 3e_G/2$, where b_G and e_G are respectively the number of bigons and edges of G. This follows because the sum of the sizes of the bigons is equal to 3 times the number of edges. As $e_G = 2v_G$ and $b_G = v_G + t_G$ we get

$$3b_G - 3e_G/2 = 3b_G - 3v_G = 3t_G,$$

establishing the claim. ∎

The above proposition implies that a 3-gem without multiple edges with the same ends, must contain at least 12 square bigons. If there are bigons a with $s_a \geq 6$, the number of square bigons increases to compensate for the negative summands that arise. Thus, a rigid 3-gem must have many square bigons. This explains the frequent appearance of the configurations below formed by three square bigons which are called, for this reason, *TS-configurations*:

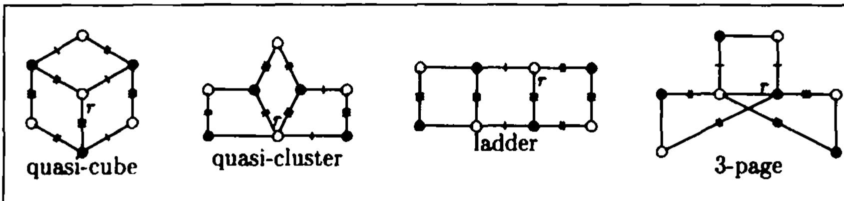


Fig. 105: Rooted TS-configurations

In the above configurations a *root vertex* r (given by the code-number of the vertex) and a sequence of colors named *color specification* are used to specify the configuration and, latter, the TS-moves acting on them.

The *root vertex of a quasi-cube* is the code-number r of the vertex incident to its three squares. The *color specification of a quasi-cube* is formed by a single color: the one which is missing in the configuration. In the above example, the quasi-cube is specified by 0_r: 0 is its color specification and the integer r is the root vertex. We emphasize that r is the number that the code algorithm attaches to the vertex.

The *root vertex of a quasi-cluster* is the code-number r of the vertex incident to its three squares. The *color specification of a quasi-cluster* is an ordered pair of colors ij. These two are the colors of the central square-bigon. Let $\{h, k\}$ be the complement of $\{i, j\}$ in $\{0, 1, 2, 3\}$. In principle there are two alternatives for the colors of the lateral square-bigons: hi and kj or hj and ik. If ij is decreasing we take the alternative that 0 and the greatest color in the other set are the colors of a lateral square-bigon. If ij is increasing we take the other possibility. In the above figure, the quasi-cluster is specified by 23_r.

The *root vertex of a ladder* is the smallest code-number r of the four vertices incident to two of the square-bigons of the ladder. The *color specification of a ladder* is a sequence of three colors ijk, where i is the color of the rungs of the ladder, j is such that the central bigon is an ij-gon and k is the third color incident to r in the ladder. The above ladder is specified by 312_r.

The *root vertex of a 3-page* is the smallest code-number r of the two vertices incident to the edge α belonging to the three square-bigons.The *color specification of a 3-page* is a single color i which is the color of α. The above 3-page is specified by 0_r.

The TS-configurations imply six involutions named TS-moves. These moves are factorable into dipole moves. This was first proved in [LD91]. Below we reproduce this proof. Therefore the TS-moves maintain the induced 3-manifold *and do not change the number of vertices*. Moves like these which are defined at local specific configurations are highly important for us. This is so because they are computationally easy to find and provide ways to get 3-gems of the same size inducing the same manifold. Last but not least they may cause the appearance of ρ-pairs which yields smaller 3-gems. The TS-moves provide the basis for our simplifying dynamics on 3-gems.

In order to describe the TS-moves as geometrical involutions, we draw the outgoing edges of a TS-configuration in an adequate way.

The first configuration, the quasi-cube, provides three TS-moves, which we define below, up to color permutation. The first TS-move consists of a local 180º-rotation and a $(\circ, \bullet)$-exchange of the vertices in the quasi-cube. Observe that even refering to vertices in $1 - 1$ correspondence, the numbers r and s are code dependent and are usually different.

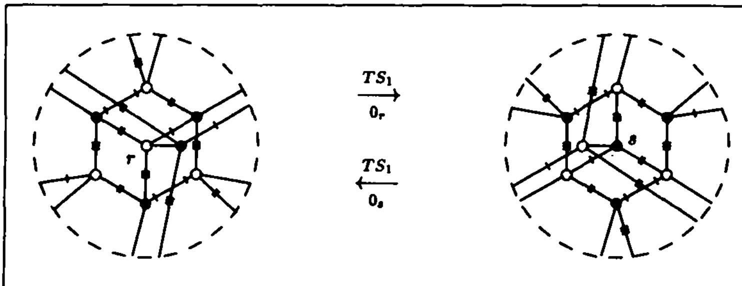


Fig. 106: Examples of TS_1-moves, first parts of A-moves

In the implemented applications a TS_i-move is always followed by the ρ-algorithm. The composite move $\rho_* \circ TS_1$ is named an *A-move.* The specification of an A-move is given as A_r^i, where i_r is the specification of the TS_1-move. The A-moves corresponding to the above two TS_1-moves are A_r^0 and A_s^0.

The second TS-move consists of a local reflection in the horizontal diameter of a quasi-cube:

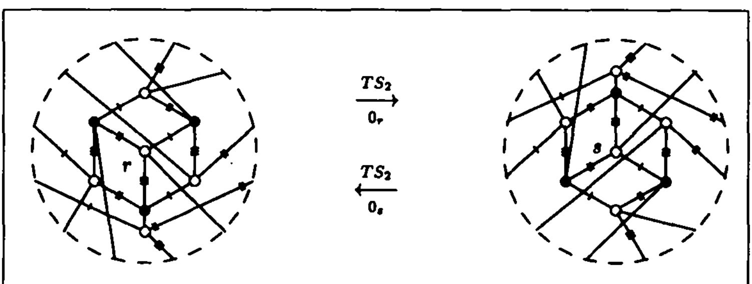


Fig. 107: Examples of TS_2-moves, first parts of B-moves

The composite move $\rho_* \circ TS_2$ is named a *B-move.* The specification of a B-move is given as B_r^i, where i_r is the specification of the TS_2-move. The B-moves corresponding to the above two TS_2-moves are B_r^0 and B_s^0.

The third TS-move consists of a local reflection in the horizontal diameter and a local $(\circ, \bullet)$-vertex reversal in the quasi-cube:

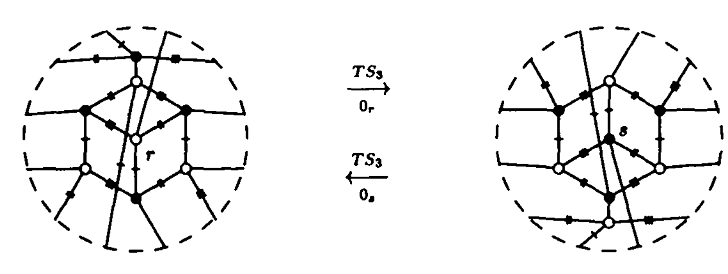


Fig. 108: Examples of TS_3-moves, first parts of C-moves

The composite move $\rho_s \circ TS_3$ is named a *C-move*. The specification of a C-move is given as C_r^i, where i_r is the specification of the TS_3-move. The C-moves corresponding to the above two TS_3-moves are C_r^0 and C_s^0.

The fourth TS-move consists of a local reflection in the horizontal diameter, a local $(\circ, \bullet)$-vertex exchange and a local exchange of edge-colors 0 and 1 in the quasi-cluster:

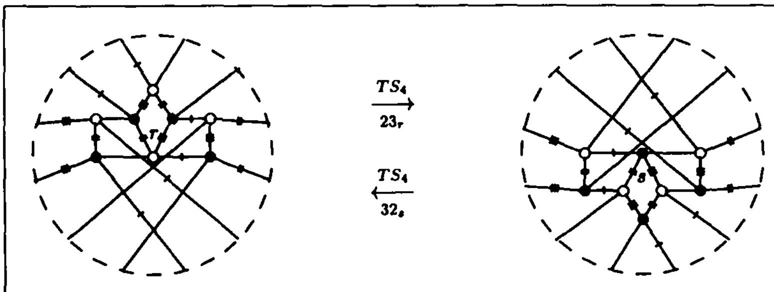


Fig. 109: Examples of TS_4-moves, first parts of D-moves

The composite move $\rho_s \circ TS_4$ is named a *D-move*. The specification of a D-move is given as D_r^{ij}, where ij_r is the specification of the TS_4-move. The D-moves corresponding to the above two TS_4-moves are D_r^{23} and D_s^{32}.

The fifth TS-move is a local 180º-rotation of the ladder:

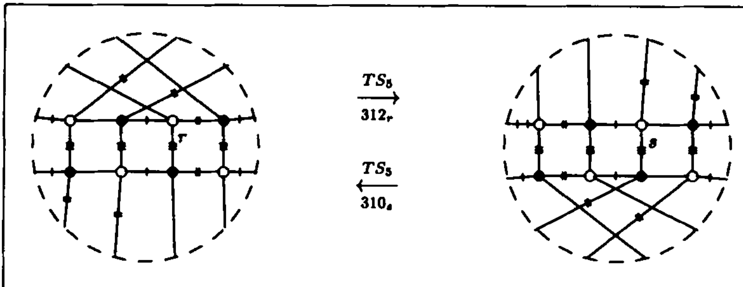


Fig. 110: Examples of TS_5-moves, first parts of E-moves

The composite move $\rho_* \circ TS_5$ is named an *E-move*. The specification of an E-move is given as E_r^{ijk}, where ijk_r is the specification of the TS_5-move. The E-moves corresponding to the above two TS_5-moves are E_r^{312} and E_s^{310}.

The sixth TS-moves is a local 180º-rotation of the crossing cluster:

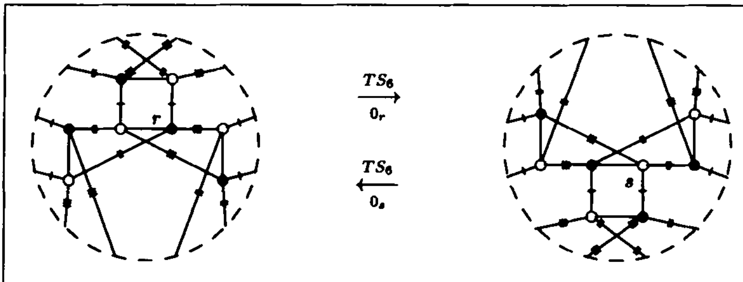


Fig. 111: Examples of TS_6-moves, first parts of F-moves

The composite move $\rho_* \circ TS_6$ is named an *F-move*. The specification of an F-move is given as F_r^i, where i_r is the specification of the TS_6-move. The F-moves corresponding to the above two TS_6-moves are F_r^0 and F_s^0.

4.1.3 Factoring the TS-Moves into Dipole Moves

The objective of this subsection is to establish the following Theorem:

Theorem 11 *Two 3-gems which differ by a TS-move induce the same 3-manifold.*

Proof: To get the proof we factor each TS-move into dipole moves.

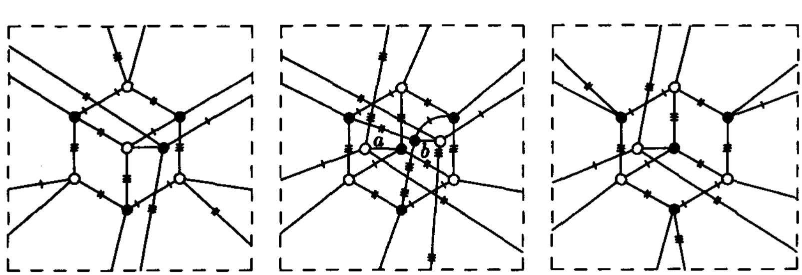


Fig. 112: Factoring TS_1 into dipole moves

TS-move of type 1, TS_1: The first and last configurations are obtained from the central one by the cancellation of 1-dipoles a and b, respectively. The passage from the first to the third is, by definition, a TS_1-move.

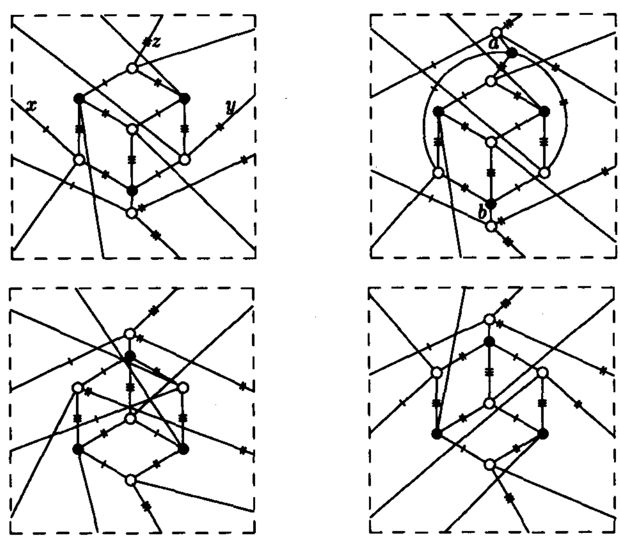


Fig. 113: Factoring TS_2 into dipole moves

TS-move of type 2, TS_2: From the first to the second configuration above we create a 1-dipole a by breaking the trio x, y and z. The third is obtained from the second by the cancellation of 1-dipole b. The third and fourth configurations are isomorphic up to a left-right flipping. The global passage from the first to the fourth configurations constitutes a generic TS_2-move.

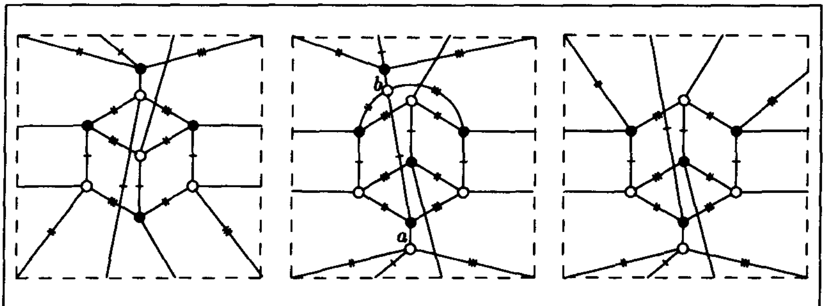


Fig. 114: Factoring TS_3 into dipole moves

TS**-move of type 3,** TS_3**:** The first and third configurations above are obtained from the central one by the cancellation of 1-dipoles a and b, respectively. The passage from the first to the third is a TS_3-move.

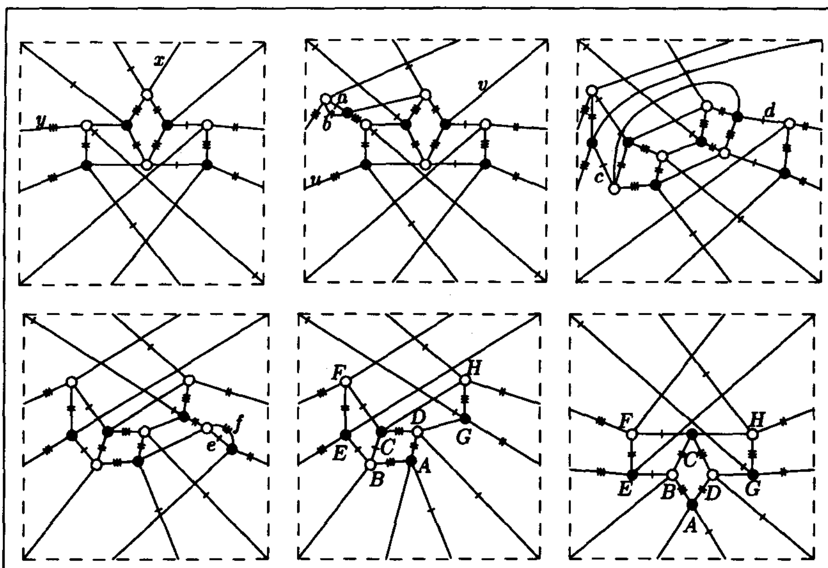


Fig. 115: Factoring TS_4 into dipole moves

TS**-move of type 4,** TS_4**:** We first create a 2-dipole formed by edges a and b by subdividing the 03-gon containing x and y. Next, we create a 1-dipole c, by breaking the trio u, b and v. The fourth configuration is obtained from the third by the cancellation of 1-dipole d, and the fifth from the fourth by the cancellation of 2-dipole formed by e and f. Finally, the sixth configuration is just a proper redrawing of the fifth. The passage from the first to the sixth is, by definition, a TS_4-move.

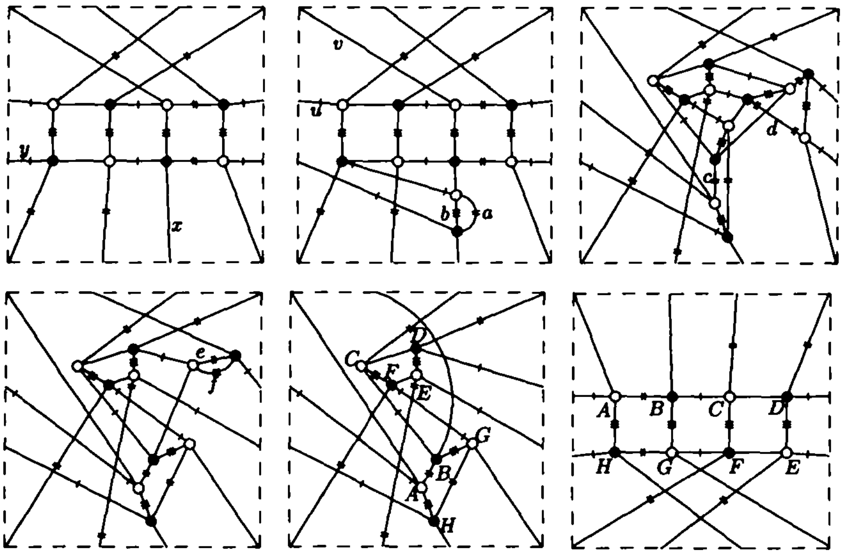


Fig. 116: Factoring TS_5 into dipole moves

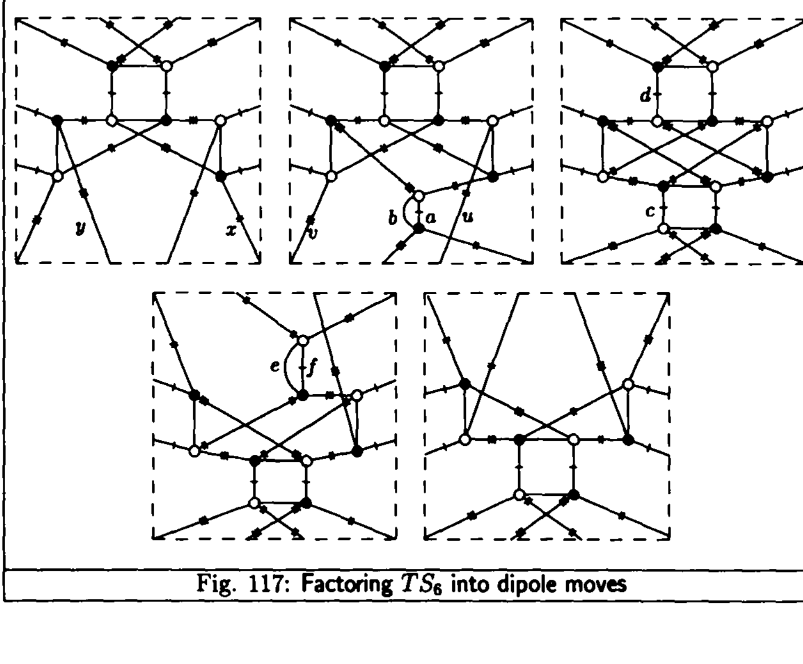


Fig. 117: Factoring TS_6 into dipole moves

TS-move of type 5, TS_5: The passages are similar to the ones of the previous move. We first create a 2-dipole formed by edges a and b by subdividing the 01-gon containing x and y. Next, we create a 1-dipole c, by breaking the trio b, u and v. The fourth configuration is obtained from the third by the cancellation of 1-dipole d, and the fifth from the fourth by the cancellation of 2-dipole formed by e and f. Finally, the sixth configuration is just a proper redrawing of the fifth. The passage from the first to the sixth is, by definition, a TS_5-move.

TS-move of type 6, TS_6: The passages here are a bit simpler than the ones of the previous two moves. We first create a 2-dipole formed by edges a and b by subdividing the 23-gon containing x and y. Next, we create a 1-dipole c, by breaking the trio v, b and u. The fourth configuration is obtained from the third by the cancellation of 1-dipole d. The fifth from the fourth by the cancellation of 2-dipole formed by e and f. Note that the passage from the first configuration to the fifth is, up to color permutation, a generic TS_6-move. ∎

4.1.4 Clusters of Four Squares

If a vertex is incident to four square bigons, then as we show next, a straightforward simplification is available. These configurations often occur and are useful reduction tools. However, as shown below in the proof of the simplification moves, the reductions are easy consequences of the TS-moves and are only treated indirectly in our algorithms. As an explanation of why so often there are vertices incident to clusters of squares we establish the following characterization of a 3-gem. Let b_{ij}^v be the number of edges of the ij-gon incident to v.

Proposition 23 *Let G be an $(3+1)$-graph. Then G is a 3-gem if and only if*

$$\sum_{v \in V_G} (1/b_{01}^v + 1/b_{02}^v + 1/b_{03}^v + 1/b_{12}^v + 1/b_{13}^v + 1/b_{23}^v - 1) = t_G.$$

Proof: The above sum equals $b_G - v_G$. This number is t_G if and only if G is a 3-gem. ∎

Call the *contribution of a vertex v* the number

$$1/b_{01}^v + 1/b_{02}^v + 1/b_{03}^v + 1/b_{12}^v + 1/b_{13}^v + 1/b_{23}^v - 1.$$

The proposition says that the sum of these contributions is t_G, at least 4. Vertices which are not incident to bigons of sizes 2 or 4 have non-positive contribution. The above proposition says that a 3-gem ought to have many squares to compensate for the vertices with negative contribution. If the gem has no bigons of size two, then the appearance of squares is forced to yield positive contribution.

Proposition 24 *If at least four of the six bigons incident to a vertex v of a 3-gem G are squares, then there exists a smaller gem H inducing the same 3-manifold.*

Proof: There are essentially two cases: the two bigons incident to v which are possibly not squares have one color in common or none. If they have, then up to color permutation, we may suppose the possibly non-square bigons are the 01- and the 03-gons. Then we have:

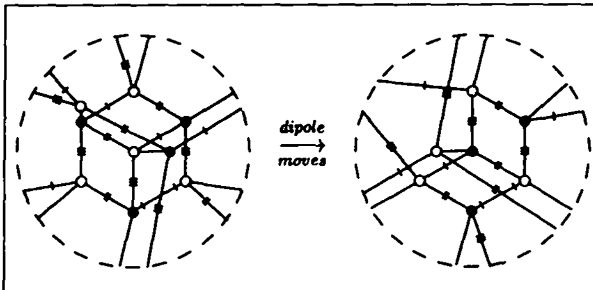


Fig. 118: Simplification in the first case

The proof that the final configuration is obtainable from the first by dipole moves follows from the TS_1 move applied to part of the configurations:

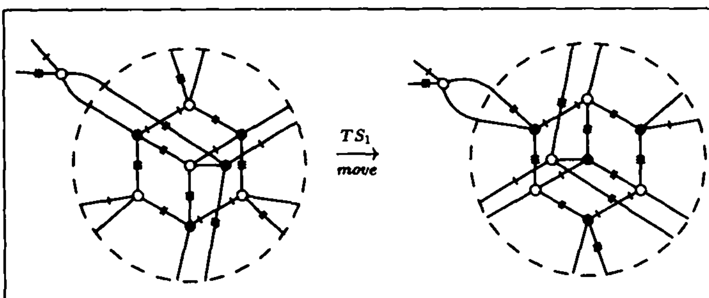


Fig. 119: A proof for the first case using a TS_1-move

Note that the configuration on the left is the same as the initial configuration, and that by cancelling the 2-dipole of the one on the right, yields the final configuration.

The other possibility, up to color permutation, is to have the 03- and 12-gons incident to v as the ones which are possibly not squares. Then we can draw the configuration as shown on the left. We get

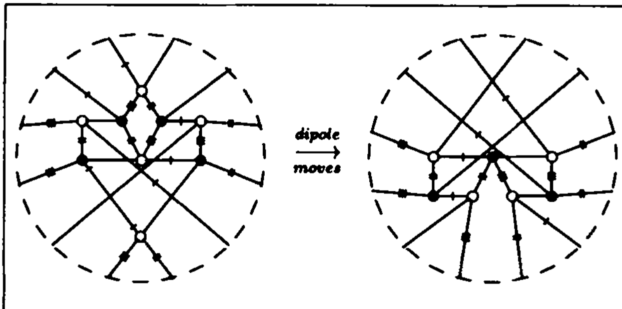


Fig. 120: Simplification in the second case

To prove the above passage note that we can use TS_4 to switch part of the configuration and what remains is to cancell a 2-dipole:

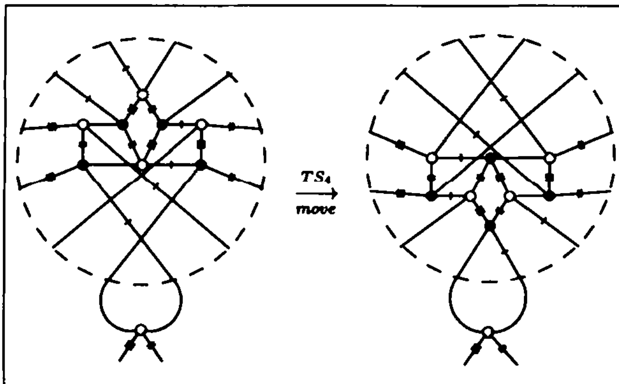


Fig. 121: A proof for the second case using a TS_4-move

This proves the proposition. ∎

A better way to depict the second move of the previous proposition is

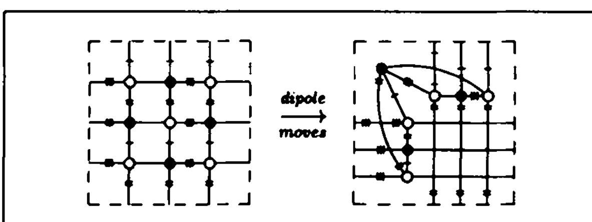


Fig. 122: Simplification of a cluster of four square bigons

As an application of this move, consider the standard 3-gem $\ell_{5,2}$ inducing the lens space $L_{5,2}$, as constructed in Section 2.1. There are no dipoles no ρ-pairs. However it has a 9-vertex cluster of four squares implying the fact that the standard gem for $L_{5,2}$ can be reduced.

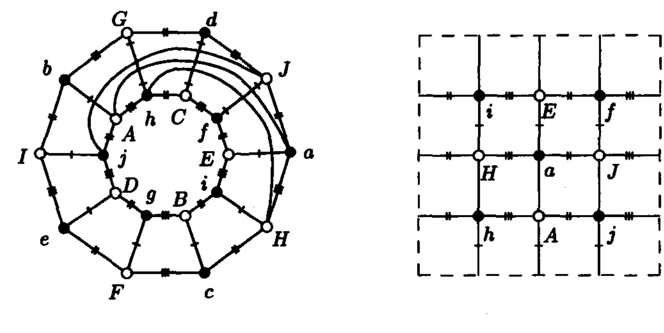


Fig. 123: Identifying a cluster of four square bigons in $\ell_{5,2}$

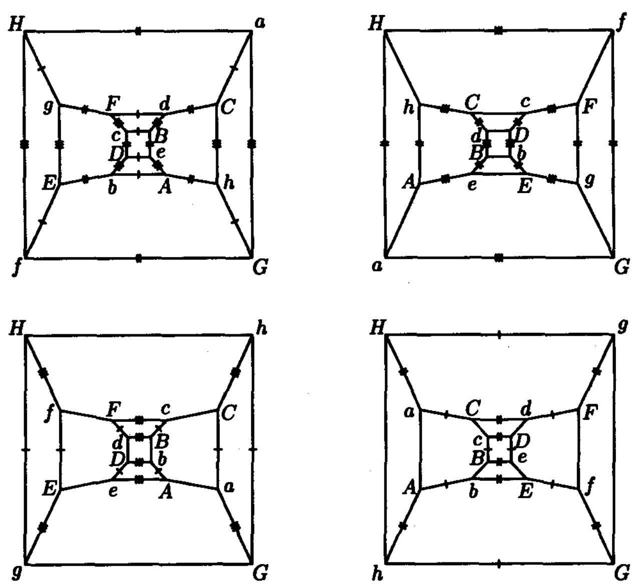


Fig. 124A: 3-residues of the superattractor r_1^{16} for $L_{5,2}$

After applying the move of Fig. 122 to $\ell_{5,2}$ we get a 2-dipole in the resulting 3-gem. Cancelling this 2-dipole we arrive to the superattractor for the lens space $L_{5,2}$. This is a σ-gem and we display above the σ-symmetries among its 3-residues. The gist of these symmetries is shown below with the corresponding string presentation:

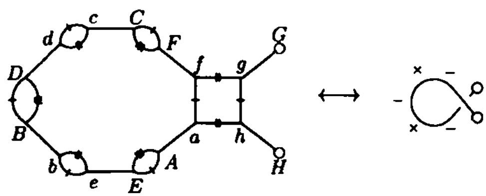


Fig. 124B: Gist and string presentation relative to the above σ-symmetries

The simplification given by clusters of four square-bigons in Proposition 24 applies frequently. However, in the TS_ρ-algorithm discussed below we only use it indirectly: clusters of four square-bigons have clusters of three square-bigons as subconfigurations and these are the ones which we deal with in the TS_ρ-algorithm. The simplification will eventually occur, because 2-dipoles are formed by the TS-moves which involve four square-bigons, as in the proof of Proposition 24.

4.1.5 The TS_ρ-Algorithm

Let G be a 3-gem. The TS-graph of G, denoted Γ_G^{TS}, is the digraph in whose vertices are in $1-1$ correspondence with gems reachable from G by a finite number of TS-moves. There is a directed edge from G_1 to G_2 if G_2 is obtained from G_1 by a single TS-move. Note that this is a symmetric digraph, since each TS-move is an involution.

We go forming Γ_G^{TS} in breadth first [Gol80] fashion. To this end we order the TS-moves, first by type of move: $TS_1, \ldots, TS_6$. Second by the code-number of the root vertex and last by the lexicographical order of the sequence of color specification. With this convention, an order to draw the edges of Γ_G^{TS} is fixed and is used to define a function $ts_{\rho_*} : \mathcal{G} \times \mathbb{N} \to \mathcal{RG} \times \mathbb{N}$, based on the TS_ρ-algorithm below. We denote by $\lambda(\Gamma_H^{TS})$ the 3-gem of smallest code which is a vertex of Γ_H^{TS}.

Algorithm 8 (TS_ρ-algorithm, getting function $ts_{\rho_*}(G,n) = (H,m)$) :

```
m ← n;
H ← G;
repeat
    go forming Γ_H^TS in a breadth first search;
    if a non-rigid vertex of Γ_H^TS is found begin
        H ← the first non-rigid 3-gem found;
        (H, n) ← ρ(H, 0);
        m ← m + n;
    end;
until Γ_H^TS is complete and all its vertices are u_0−essential;
H ← λ(Γ_H^TS);
```

In an actual implementation we do not need to keep all edges of Γ_H^{TS}. A spanning tree is enough. The final gem H, is a rigid gem and is called the *u_0-representative* of G. As before we have in the output of the above algorithm a pair (H, m), with H a connected crystallization so that if G is bipartite,

$$G \equiv H \ \# \ \underbrace{((s^1 \times s^2) \ \# \ (s^1 \times s^2) \ \# \ \ldots \ \# \ (s^1 \times s^2))}_{m-n},$$

and if G is not bipartite,

$$G \equiv H \ \# \ \underbrace{((s^1 \tilde{\times} s^2) \ \# \ (s^1 \tilde{\times} s^2) \ \# \ \ldots \ \# \ (s^1 \tilde{\times} s^2))}_{m-n}$$

Let us define $ts_{\rho_*}(G,n) = (H,m)$. If $m = n$ then we may write simply $ts_{\rho_*}(G) = H$ and if $n = 0$ then we may write $ts_{\rho_*}(G) = (H,m)$.

In Section 5.2 and Appendix B (Section 8.2) we present tables of the function ts_{ρ_*} for the rigid bipartite 3-gems up to 28 vertices. These tables form a crucial part of the Classification Theorem.

4.1.6 Invariance under the U-Move

A *monopole* in a (3+1)-graph is a vextex which is the only intersection of an hi-gon and a jk-gon, (h,i,j,k) a permutation of $(0,1,2,3)$. This defines a configuration which induces a fundamental move in the classification algorithm of the next chapter. A U-move is defined on a monopole, by making the hi-gon of size $2m$ and the jk-gon of size $2n$ disappear, being replaced by a U-shaped cluster of squares with $(2m-1)\times(2n-1)$ vertices:

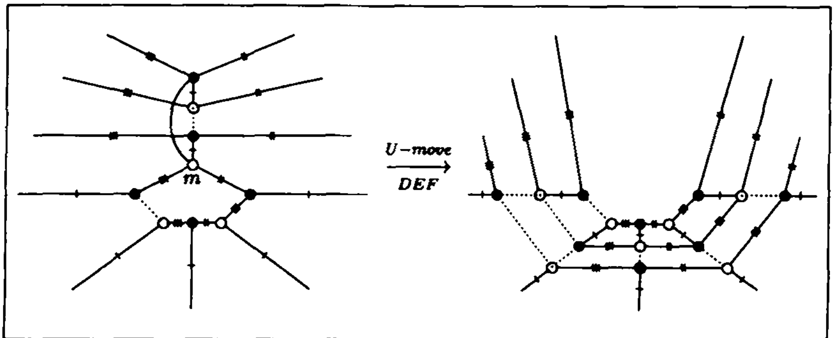


Fig. 125: Definition of U-move

Proposition 25 *A U-move in a 3-gem is factorable as dipole moves. Thus it maintains the induced 3-manifold.*

Proof: We suppose the permutation (h, i, j, k) associated to the monopole m at which the U-move is performed is the identity. If the 23-gon incident to m has $2n$ edges, then by creating $2n - 2$ 2-dipoles we get a trio a, b, c.

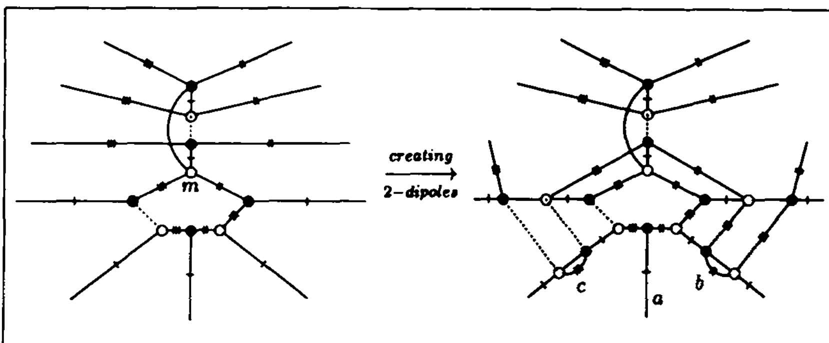


Fig. 126A: Creating $2n - 2$ 2-dipoles to get a trio a, b, c

Breaking the trio $\{a, b, c\}$ produces a 0-colored 1-dipole d. By cancelling d we reproduce the initial situation: a monopole m, but now the 01-gon incident to m has two fewer edges. The new vertices form the bottom layer of square bigons.

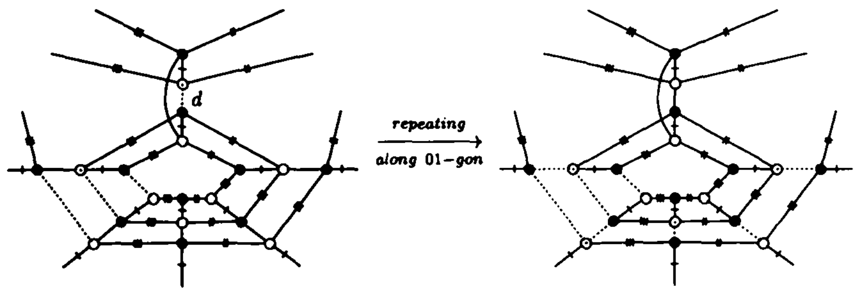


Fig. 126B: Breaking the trio and repeating $m - 1$ times

If the initial 01-gon has $2m$ edges, repeating the whole process $m - 1$ times, yields the situation below, in which appears a 01-gon with two edges:

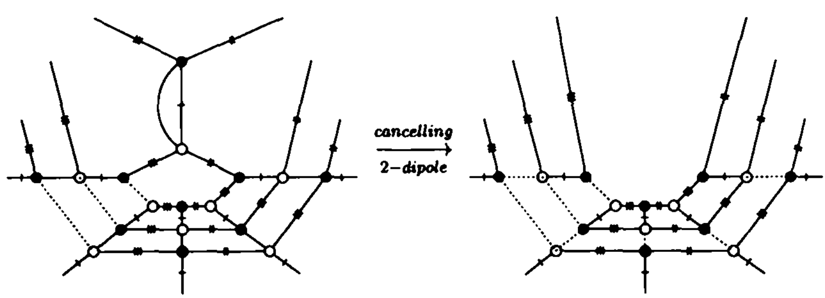


Fig. 126C: Cancelling a final 2-dipole

This 01-gon defines a 2-dipole and its cancellation produces the final configuration of the U-move. ∎

Below we exemplify how a U-move on a monopole m can give birth to new TS-configurations. This is a reason why the composition of U-moves and TS-moves work well in practice reaching gems in the attractor.

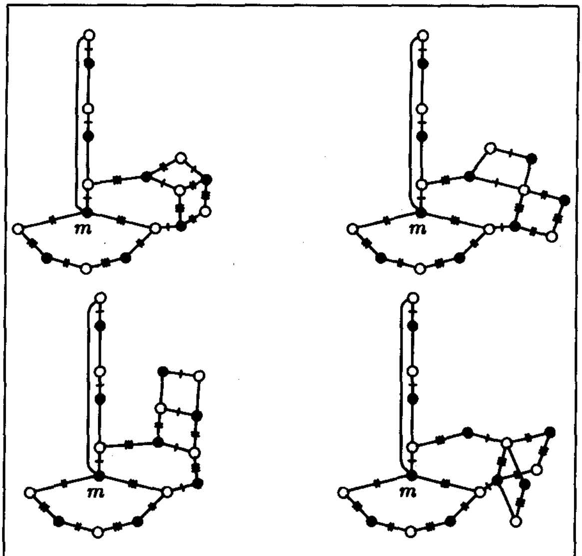


Fig. 127: U-move giving birth to new quasi-cube, new quasi-cluster, new ladder and new 3-page

In the next subsection we deal with a detailed example of this more sophisticated attraction where a U-move is necessary.

4.1.7 A $u^1_\downarrow$-Move on r^{26}_3: A Detailed Example

A *$u^i_\downarrow$-move* is a u^i_*-move which decreases the number of vertices. A U-move on a monopole with code-number m of a 3-gem G which is the single meeting of a $0i$-gon and a complementary bigon is completely characterized and we refer to it as *the U^i_m-move on G*. Here we show how a $u^1_\downarrow$-move can simplify the 3-gem r^{26}_3, which is u^0-essential, forming by itself a u^0-class. This shows that r^{26}_3 is not u^1-essential. The example is needed to resolve one of the 12 uncertainties left by the u^0-classification. What follows is a very detailed account intended to simulate the steps in the order that the implementation of TS_ρ-algorithm finds them.

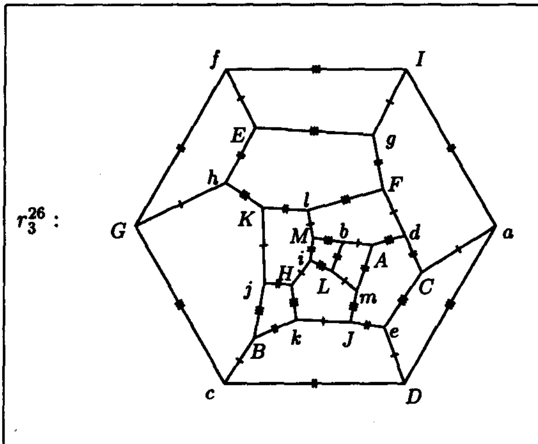


Fig. 128A: Initial 26-vertex 3-gem, r_3^{26}

Consider the U_3^1-move on r_3^{26}:

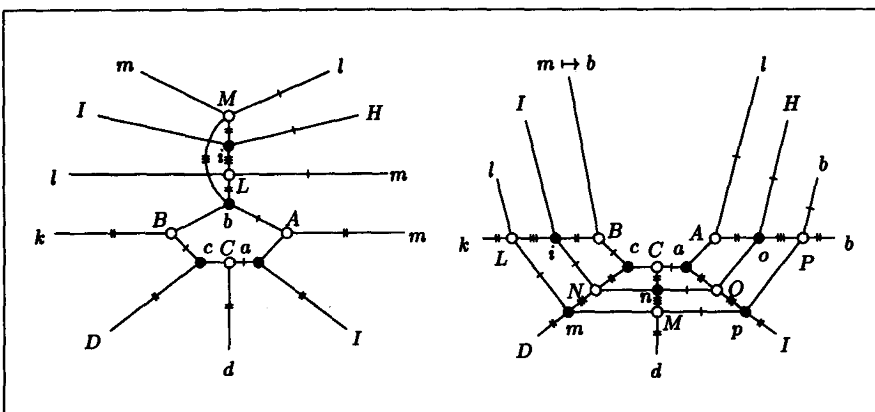


Fig. 128B: U_3^1-move on r_3^{26}

After the U-move appears a 2-dipole $\{b, P\}$, which can be cancelled producing a rigid 3-gem G_1.

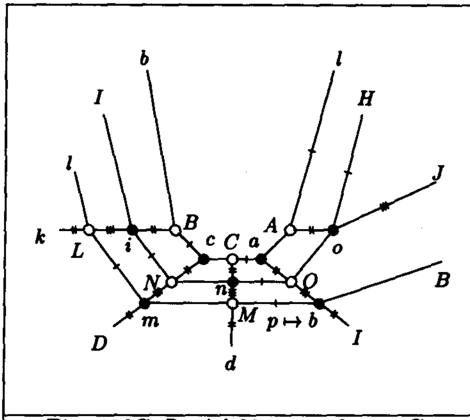


Fig. 128C: Partial 30-vertex 3-gem G_1

A *tidy matrix* for a (3+1)-graph is an incidence matrix in which the vertex labels are $a, A, b, B, \ldots$. Color 0 is given by a pair x, X and every edge links a lower case labelled vertex to an upper case labelled one. We have the following tidy incidence matrix for G_1:

Tidy Incidence Matrix for a 30-Vertex ρ-Free Gem G_1 (which induces 3-manifold $\lvert r_3^{26} \rvert$)															
	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o
1	C	M	B	F	D	E	I	G	N	K	J	A	L	O	H
2	O	I	N	M	J	G	F	E	B	H	L	K	D	C	A
3	D	O	G	A	C	I	E	K	L	B	H	F	N	M	J

In order to follow the computer steps, we must compute the code for G_1. This is attained from the above matrix by the identity edge-color permutation (0123) ↦ (0123) and vertex relabelling as follows:

$a \mapsto C$	$A \mapsto c$	$b \mapsto E$	$B \mapsto e$	$c \mapsto D$	$C \mapsto d$	$d \mapsto M$	$D \mapsto m$
$e \mapsto L$	$E \mapsto l$	$f \mapsto K$	$F \mapsto k$	$g \mapsto J$	$G \mapsto j$	$h \mapsto I$	$H \mapsto i$
$i \mapsto F$	$I \mapsto f$	$j \mapsto N$	$J \mapsto n$	$k \mapsto O$	$K \mapsto o$	$l \mapsto B$	$L \mapsto b$
$m \mapsto A$	$M \mapsto a$	$n \mapsto G$	$N \mapsto g$	$o \mapsto H$	$O \mapsto h$		

Computing the code and comparing with our catalogue tell us that G_1 is r^{30}_{54348}.

Code for $G_1 = r^{30}_{54348}$
eabcdjfghimklon
mohgfedcnkjialb
gfmlnkaeodbjchi

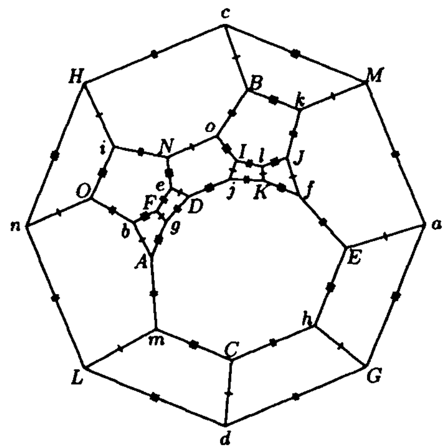


Fig. 128D: Code-labelled $G_1 = r^{30}_{54348}$

A TS_1-move which induces a 2-dipole is available in G_1:

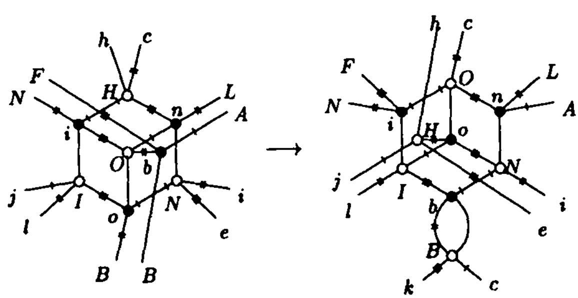


Fig. 128E: First part of the A^2_{30}-move on $G_1 = r^{30}_{54348}$

The A-move effects the TS_1-move and cancels the 2-dipole $\{b, B\}$ yielding 28-vertex 3-gem G_2. By replacing $\{o, O\}$ by $\{b, B\}$ we get a tidy incidence matrix for G_2:

Tidy Incidence Matrix for a 28-Vertex ρ-Free Gem G_2 (which induces 3-manifold $\lvert r_3^{26}\rvert$)														
	a	b	c	d	e	f	g	h	i	j	k	l	m	n
1	E	I	N	C	D	J	F	G	B	H	M	K	L	A
2	M	H	B	G	F	E	D	C	N	K	J	I	A	L
3	G	N	M	L	H	K	A	E	F	D	I	J	C	B

We must now compute the code for G_2. This is attained from the above matrix by the identity edge-color permutation $(0123) \mapsto (0123)$ and vertex relabelling as follows:

$a \mapsto a$	$A \mapsto A$	$b \mapsto m$	$B \mapsto M$	$c \mapsto c$	$C \mapsto C$	$d \mapsto d$	$D \mapsto D$
$e \mapsto e$	$E \mapsto E$	$f \mapsto f$	$F \mapsto F$	$g \mapsto g$	$G \mapsto G$	$h \mapsto h$	$H \mapsto H$
$i \mapsto n$	$I \mapsto N$	$j \mapsto i$	$J \mapsto I$	$k \mapsto j$	$K \mapsto J$	$l \mapsto k$	$L \mapsto K$
$m \mapsto l$	$M \mapsto L$	$n \mapsto b$	$N \mapsto B$				

Computing the code enable us to identify G_2 as r_{6775}^{28}.

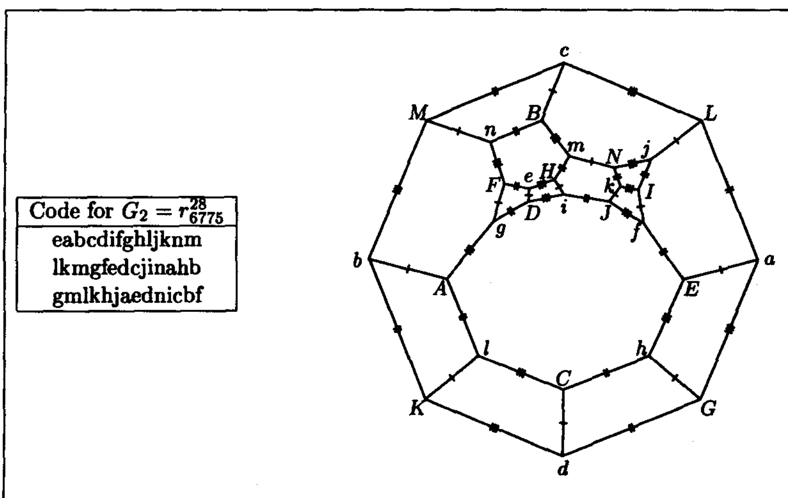


Fig. 128F: Code-labelled $G_2 = r_{6775}^{28}$

A TS_4-move which induces a 2-dipole is available in G_2:

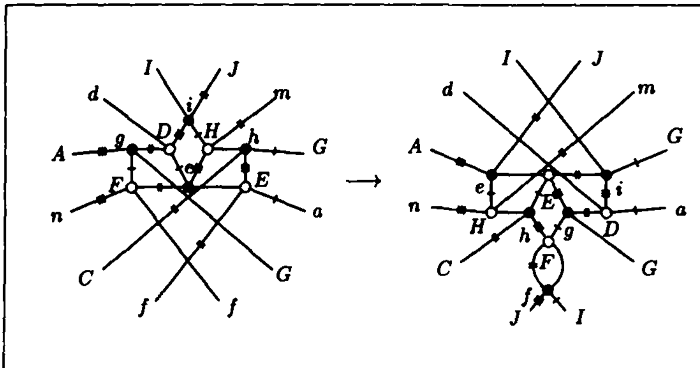


Fig. 128G: First part of the D_9^{31}-move on $G_2 = r_{6775}^{30}$

The D-move effects the TS_4-move and cancels the 2-dipole $\{f, F\}$ yielding a rigid 26-vertex 3-gem G_3. By replacing $\{n, N\}$ by $\{f, F\}$ we get a tidy incidence matrix for G_3:

Tidy Incidence Matrix for a 26-Vertex ρ-Free Gem G_3 (which induces 3-manifold $\lvert r_3^{26}\rvert$)													
	a	b	c	d	e	f	g	h	i	j	k	l	m
1	D	A	B	C	H	M	I	E	G	L	J	K	F
2	L	K	M	G	J	B	D	C	E	I	F	A	H
3	G	M	L	K	A	H	E	J	D	F	I	C	B

The code for G_3 is attained from the above matrix by the edge-color permutation $(0123) \mapsto (1023)$ and vertex relabelling as follows:

$a \mapsto C$	$A \mapsto d$	$b \mapsto D$	$B \mapsto a$	$c \mapsto A$	$C \mapsto b$	$d \mapsto B$	$D \mapsto c$
$e \mapsto J$	$E \mapsto k$	$f \mapsto I$	$F \mapsto h$	$g \mapsto L$	$G \mapsto m$	$h \mapsto K$	$H \mapsto j$
$i \mapsto M$	$I \mapsto l$	$j \mapsto F$	$J \mapsto g$	$k \mapsto G$	$K \mapsto e$	$l \mapsto E$	$L \mapsto f$
$m \mapsto H$	$M \mapsto i$						

Gem G_3 is recognized as r_{43}^{26} in our catalogue.

Code for $G_3 = r^{26}_{43}$
dabcgefihkjml
ikledcjgahmfb
hemjbakgdilgc

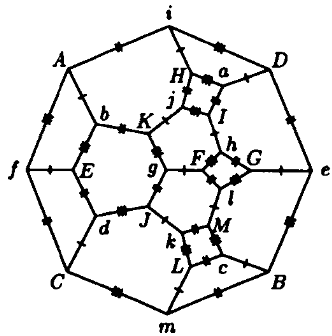


Fig. 128H: Code-labelled $G_3 = r^{26}_{43}$

Again there exists, in G_3, a TS_4-move inducing a 2-dipole:

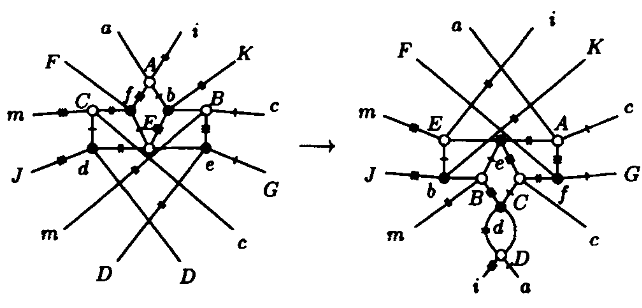


Fig. 128I: First part of the D^{31}_{10}-move on $G_3 = r^{26}_{43}$

The D-move effects the TS_4-move and follows by cancelling the 2-dipole $\{d, D\}$, yielding 24-vertex 3-gem G_4. By replacing $\{m, M\}$ by $\{d, D\}$ we get a tidy incidence matrix for G_4:

Tidy Incidence Matrix for a 24-Vertex ρ-Free Gem G_4 (which induces 3-manifold $\|r^{26}_3\|$)												
	a	b	c	d	e	f	g	h	i	j	k	l
1	C	E	A	L	B	G	F	I	H	K	J	D
2	I	K	L	B	A	C	J	G	E	H	D	F
3	H	J	D	E	C	A	K	F	B	I	L	G

The code for G_4 is attained from the above matrix by the edge-color permutation $(0123) \mapsto (0213)$ and vertex relabelling as follows:

$a \mapsto A$	$A \mapsto a$	$b \mapsto D$	$B \mapsto d$	$c \mapsto I$	$C \mapsto i$	$d \mapsto F$	$D \mapsto f$
$e \mapsto C$	$E \mapsto c$	$f \mapsto H$	$F \mapsto h$	$g \mapsto J$	$G \mapsto j$	$h \mapsto L$	$H \mapsto l$
$i \mapsto B$	$I \mapsto b$	$j \mapsto K$	$J \mapsto k$	$k \mapsto E$	$K \mapsto e$	$l \mapsto G$	$L \mapsto g$

Computing the code permits us the recognition of G_4 as r_5^{24}.

Code for for $G_4 = r_5^{24}$
cabfdeighljk
ildckgfjaheb
hkfbjielcgda

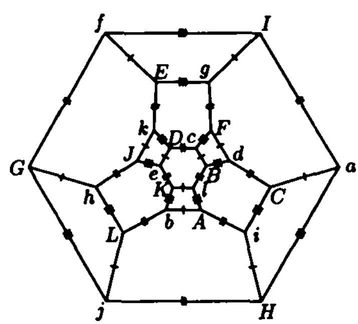


Fig. 128J: Code-labelled $G_4 = r_5^{24}$

Summarizing,

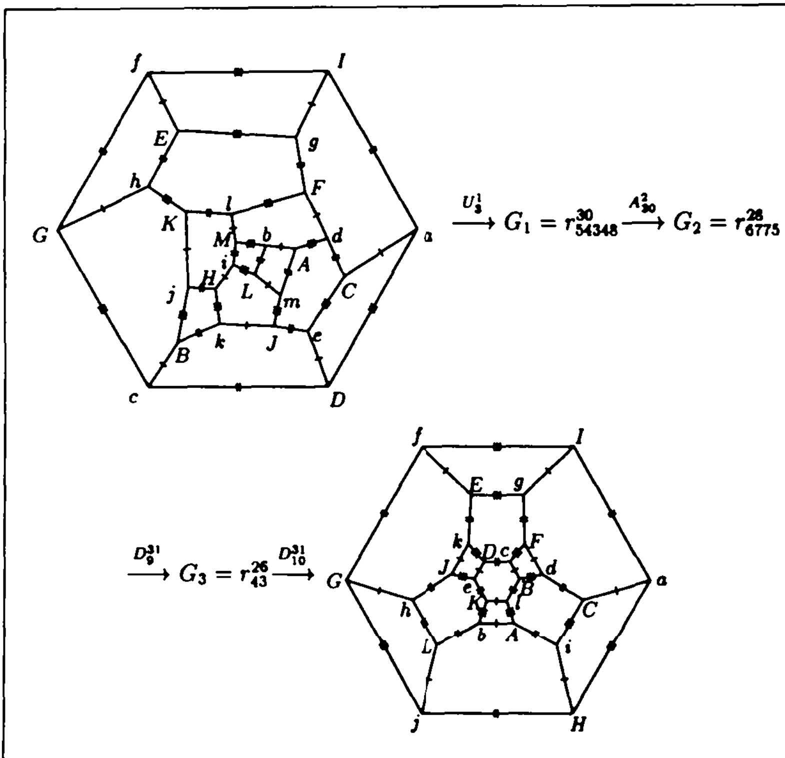


Fig. 128K: From r_3^{28} to r_5^{24} by a u_1^1-move

4.1.8 The TS_ρ^U-Algorithm

Below we present the TS_ρ^U-algorithm, which is enough to achieve the topological classification of the bipartite 3-gems up to 28 vertices, as we show in Chapter 5. Recently, this classification, using the same algorithm, was extended to 3-gems with 30 vertices in a joint work with C. Durand and S. Sidki.

Recall that v_G denotes the number of vertices of a graph G and that the code of a 3-gem is ordered first by the number of vertices and then, lexicographically among the 3-gems with the same number of vertices.

Algorithm 9 (TS^U_ρ-algorithm, getting function $ts^U_{\rho.}(G,n) = (H,m)$) :

1 For each J_i obtained from a U-move on G let $(H_i, n_i) \leftarrow ts_{\rho.}(J_i, n)$
2 Let i_0 be such that H_{i_0} has smallest code among the H_i's.
3 If $v_{H_{i_0}} > v_G$, then $(H,m) \leftarrow (G,n)$ else $(H,m) \leftarrow (H_{i_0}, n_{i_0})$

Define $ts^U_{\rho.}(G,n) = (H,m)$. If $m = n$ then we may write simply $ts^U_{\rho.}(G) = H$ and if $n = 0$ then we may write, by abuse of language, $ts^U_{\rho.}(G) = (H,m)$.

Of course, the interesting inputs for the above algorithm are the u^0-essential 3-gems, because the TS^U_ρ-algorithm is only used when the u^0-class of the output of the TS_ρ-algorithm seems not to coincide with the attractor. For the bipartite rigid 3-gems up to 28 vertices there are only 12 such uncertainties (see the next chapter).

Consider the bipartite 3-gems up to $n = 30$ vertices. The function $ts^U_{\rho.}$ maps each one of these in the 3-gem of smallest code belonging to the attractor of the induced manifold. We have empirical reasons to suspect that this fact goes on for at least a few higher values of n. Checking beyond $n = 30$ awaits a better implementation of the techniques discussed. It is an interesting problem to find at which n the TS^U_ρ-algorithm fails to identify the attractor, if it ever fails.

4.1.9 Sketching the U^i-Algorithms

There is a sense in which the sequence of algorithms ρ-, TS_ρ- and TS^U_ρ- are weak. Namely, they choose a specific path for the simplification and attains a single 3-gem. This is artificial but it was a time-space compromise to attain the topological classification.

The hierarchical U^i-algorithms which we now outline are of more theoretical flavor. They are simpler conceptually, potentially more powerful but rather time/space consuming for actual implementation. We include them here as a guide for a future implementation of this kind, which nevertheless can be done since machines are becoming more and more powerful, and parallelism can be explored.

Let us recall some definitions given in Section 0.5. A *u^0-move* on a 3-gem is either a ρ-move or a TS-move, whereas a *u^0_*-move* is the identity or a finite sequence of u^0-moves. A *u^1-move* is a move of type Uu^0_* *which may decrease but does not increase the number of vertices.* A *u^1_*-move* is a finite sequence of u^1- and u^0-moves. In general, let a *u^n-move* be a move of the type Uu^{n-1}_*, *which may decrease but does not increase the number of vertices.* Let finally a *u^n_*-move* be a finite sequence of u^m's moves with $m \leq n$.

The basic property of the U^i-algorithms should be that they provide concretely *all the 3-gems* which are linked to an input 3-gem by u^i_*-moves. Note that there are only finitely of these, since an $u^i_\bullet$-move does not increase the number of vertices. We use some data structure to hide all but the highest level of a hierarchy of moves.

The input of the U^i-algorithm is a 3-gem G and the output is a finite directed graph U^i_G, which is acyclic, that is, has no directed polygon.

The vertices of U^0_G are the TS-classes which contain 3-gems linked to G by $u^0_\bullet$-moves. A directed edge of U^0_G links TS-class A to TS-class B if there is a member of B which is reached from a member of A by a single ρ-move. This finishes the description of U^0_G.

The vertices of an auxiliary graph $\hat{U}^1_G$ are the TS-classes which contain 3-gems reachable from G by $u^1_\bullet$-moves. A directed edge of $\hat{U}^1_G$ links TS-class A to TS-class B if there is a member of B which is reached from a member of A by a single u^0-move or by a single u^1-move. Graph $\hat{U}^1_G$ may have directed cycles. A *strong component of a directed graph* is a maximal set of vertices so that there is a directed path joining any ordered pair of them. To define U^1_G, simply take the quotient of $\hat{U}^1_G$ by its strong components, that is, each strong component is replaced by a single vertex, which is a u^1-class, and the graph becomes acyclic. Therefore, the vertices of U^1_G are u^1-classes of 3-gems reachable from G by a single $u^1_\bullet$-move.

Having defined U^{n-1}_G, to form U^n_G form first $\hat{U}^n_G$ as follows. The vertices of $\hat{U}^n_G$ are the u^{n-1}-classes reachable from G by $u^n_\bullet$-moves. A directed edge of $\hat{U}^n_G$ links u^{n-1}-class A to u^{n-1}-class B if there is a member of B which is reached from a member of A by a single u^m-move, $m \leq n$. Define U^n_G, as the quotient of $\hat{U}^n_G$ by its strong components. The vertices of U^n_G are the u^n-classes of 3-gems reachable from G by a single $u^n_\bullet$-move.

The U^i-algorithm is an algorithm which implements the construction of graph U^i_G providing book-keeping mechanisms to recover all the 3-gems reachable from G by a $u^i_\bullet$-move. We do not elaborate more on this topic.

A stronger form of Conjecture 1, equivalent to Conjecture 2, can be stated as follows:

Conjecture 6 *For every 3-gem G, U^1_G has a single sink whose associated u^1-class is the attractor for 3-manifold $|G|$.*

4.2 All Oriented Identifications of a Solid Cube

As an application of the TS_ρ-algorithm and of the TS^U_ρ-algorithm we present below all the orientable 3-manifolds obtainable as an orientable identification scheme of the faces of a solid cube. It is not difficult to write a computer program which exhausts all these possibilities. We have done this and by applying TS_ρ to each 3-gem obtained as in Section 1.3, we get many duplicates and exactly 17 distinct orientable 3-manifolds. All of these manifolds are easily recognizable, because TS_ρ^U sends the 3-gems into the attractor for the respective manifolds. We need the full power of TS_ρ^U only in the case of the lens space $L_{12,5}$. In the other cases TS_ρ suffice. One can be surprised by the facts that lens spaces such as $L_{14,3}$ or non-trivial connected sums are obtainable by identifying properly the faces of a solid cube.

The 17 possibilities are naturally grouped into families. First we have all but one of the Euclidean orientable 3-manifolds:

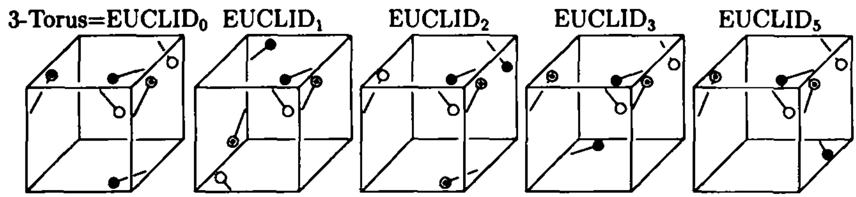


Fig. 129: The orientable Euclidean 3-manifolds obtained from a solid cube

Next comes the lens spaces that are obtained by identifying the faces of a solid cube. We consider $S^1 \times S^2$ as the lens space $L_{0,1}$ and S^3 as the lens space $L_{1,1}$.

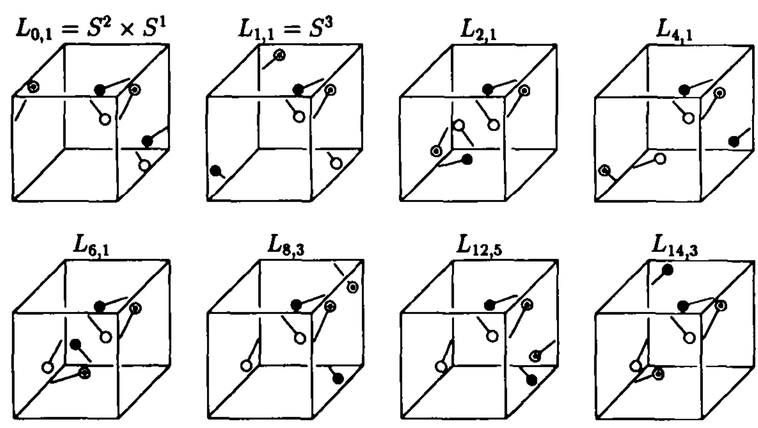


Fig. 130: The lens spaces obtained from a solid cube

We have two non-trivial connected sums:

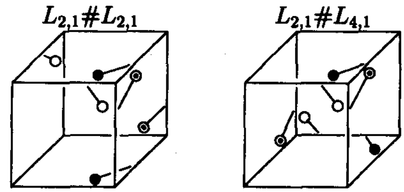


Fig. 131: Non-trivial connected sums of 3-manifolds obtained from a solid cube

Finally we have two manifolds which are finite non-cyclic quotients of S^3.

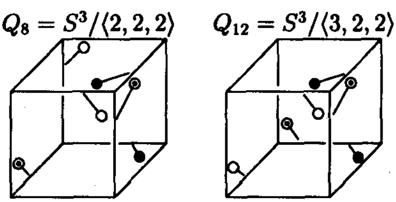


Fig. 132: Non-cyclic quotients of S^3 from a solid cube

4.2.1 The Orientable Euclidean 3-Manifolds

Clearly, the first cube identification induces the 3-dimensional torus. This manifold, $S^1 \times S^1 \times S^1$, has the superattractor which we display below together with one of the string presentations inducing it. From the method discussed in Section 1.3 we may consider the facial identification schemes as 3-gems. Hence we are saying by the first arrow below that the image of the gem on the left under TS_ρ is the gem in the center (with h, the second element of the image, being 0).

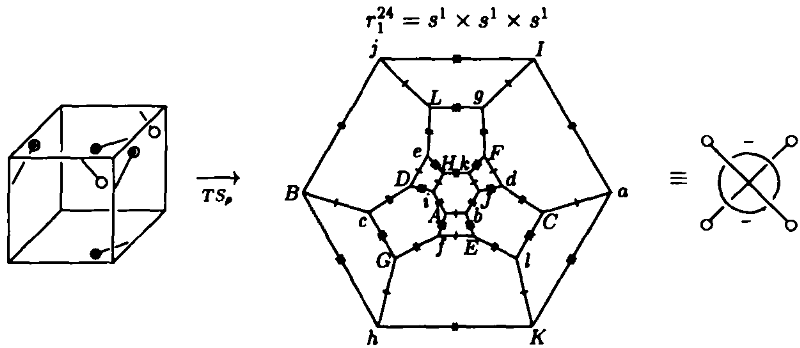


Fig. 133: Identification scheme absorbed by the superattractor r_1^{24} for $S^1 \times S^1 \times S^1$

Here and in the other examples, the way we know the subsets of gems to be attractors is justified by the material presented in Chapter 5. All rigid orientable 3-gems up to 30 vertices have been generated. Thus, we know that no 3-gem with fewer than 24 vertices induces $S^1 \times S^1 \times S^1$. Moreover, $s^1 \times s^1 \times s^1 = r_1^{24}$, is the only vertex of $\Gamma^{TS}_{r_1^{24}}$.

The manifold EUCLID$_1$ has fundamental group given by

$$\pi_1(\text{EUCLID}_1) = \langle\ a, b \mid a = b^2ab^2, b = a^2ba^2\ \rangle.$$

This group is faithfully represented by the subgroup of translation matrices generated by:

$$a \mapsto \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad b \mapsto \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Therefore, the manifold is the quotient of R^3, identified by the group of translations generated by the above matrices, [LS95]. It also can be obtained from a facial identification scheme on a polyhedron formed by a triangular prism at whose bases we grow two triangular pyramides, as shown below.

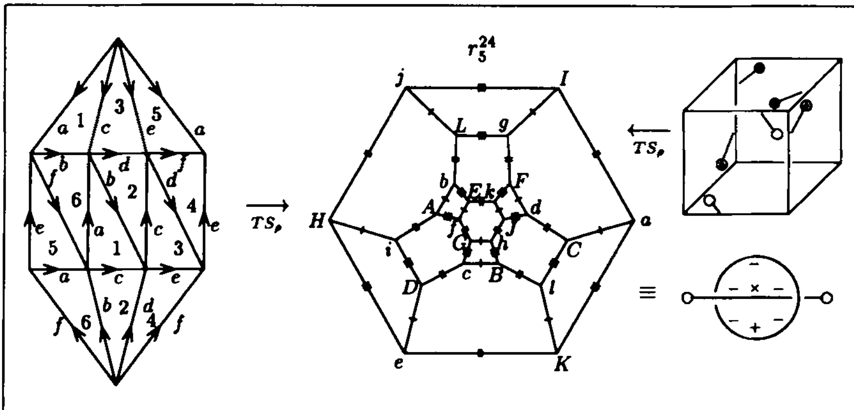


Fig. 134: Identification schemes absorbed by the superattractor r_5^{24} for EUCLID_1

The first identification scheme above is provided in [HKM89]. By using TS_ρ we discover that EUCLID_1 can also be obtained by the much simpler facial identification in a solid cube according to above scheme. The 3-gems associated to the two identification schemes have the same attractor, r_5^{24}.

The other four orientable Euclidean manifolds are given by rather simple identifications schemes on a 4-prism (cube) and a 6-prism. Each lateral face of the prism is identified with its opposite by a translation along the common normal to the planes containing the faces. The superior and inferior bases are identified in this way but only after a twist indicated in the superior faces:

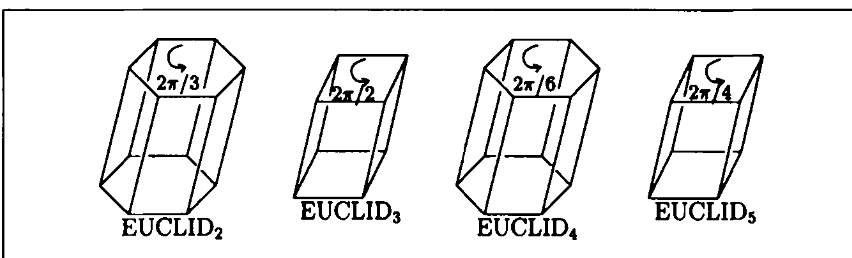


Fig. 135: Identification schemes for the other four orientable Euclidean 3-manifolds

The alternative description of EUCLID_2 given by identifications on a solid cube instead of a solid hexagonal prism was a surprise provided by TS_ρ. The superattractor r_7^{24} is a σ-gem and admits a string presentation, also depicted.

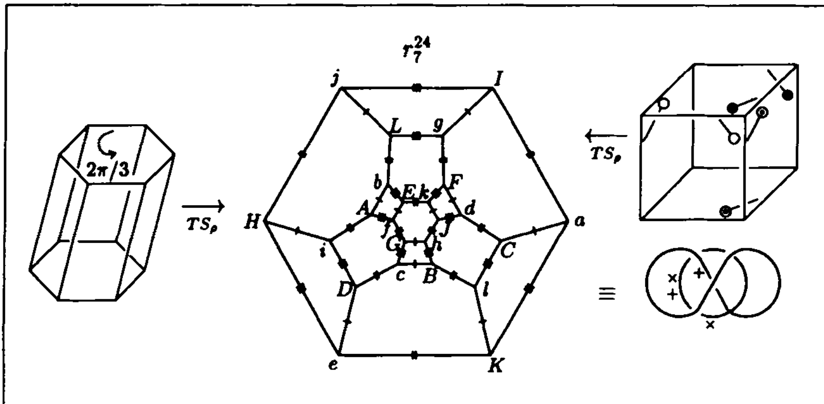


Fig. 136: Identification schemes absorbed by the superattractor r_7^{24} for $EUCLID_2$

The image of $EUCLID_3$ given by the identification scheme below (considered as a 3-gem) is r_6^{24}, the superattractor for the manifold. It admits a curiously simple string presentation.

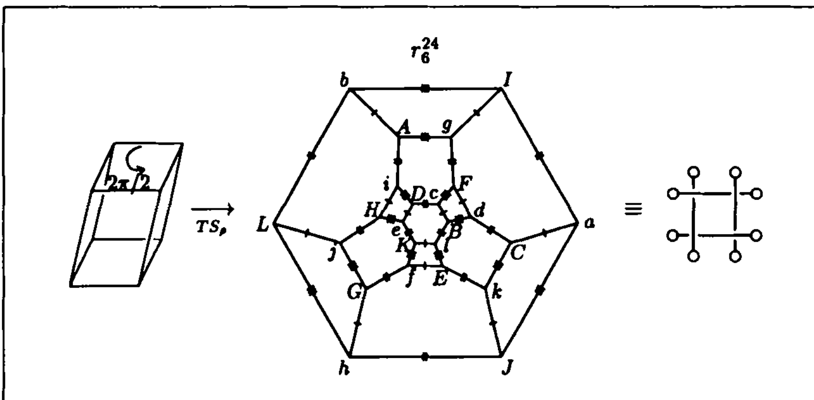


Fig. 137: Identification scheme absorbed by the superattractor r_6^{24} for $EUCLID_3$

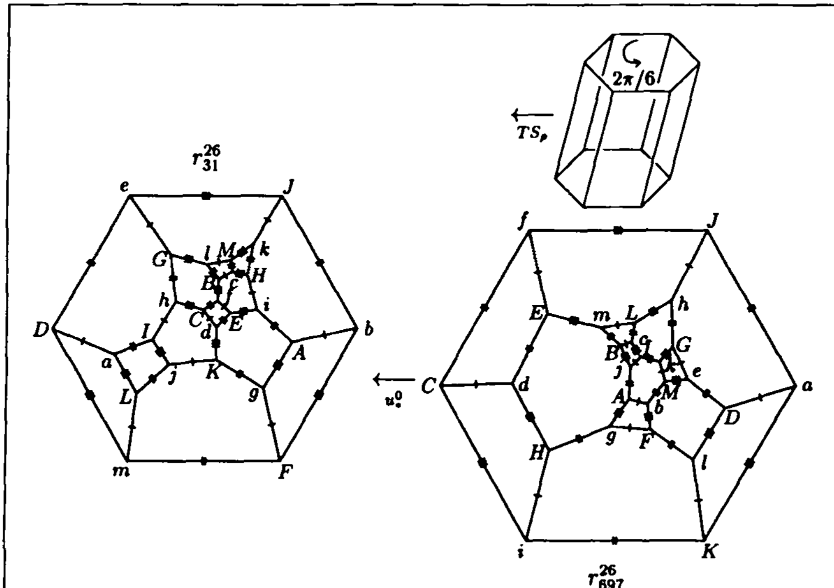


Fig. 138: Identification scheme absorbed by the u^0_2-attractor r^{26}_{31} for EUCLID$_4$

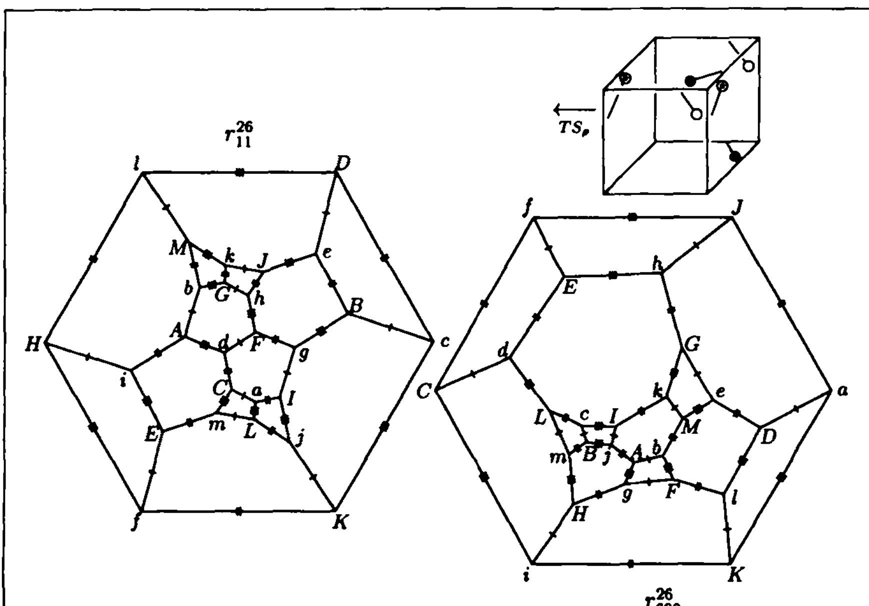


Fig. 139: Identification scheme absorbed by the u^0_2-attractor r^{26}_{11} for EUCLID$_5$

EUCLID$_4$ is the only orientable Euclidean manifold which is not formed by identifying the faces of a cube. The image under TS_ρ of the gem associated with its identification scheme is the u_2^0-attractor formed by r_{31}^{26} and r_{697}^{26}. None of these is a σ-gem.

EUCLID$_5$ also has an attractor formed by 3-gems, r_{11}^{26} and r_{698}^{26}, which are depicted above. Again, none of these is a σ-gem.

4.2.2 The Lens Spaces

We present below the lens spaces that appear as an identification scheme in a solid cube. The first is $S^1 \times S^2$ which is the lens space $L_{0,1}$.

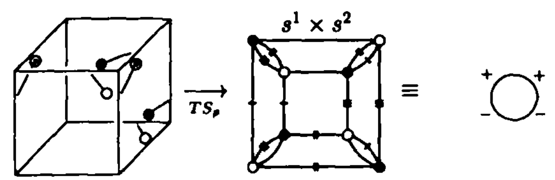


Fig. 140: Identification scheme absorbed by the superattractor for $S^1 \times S^2$

The 3-sphere S^3, which is also the lens space $L_{1,1}$, can be realized as an identification scheme in a solid cube:

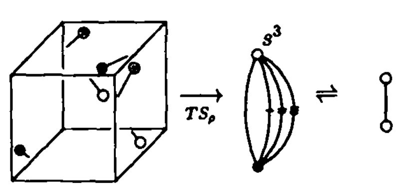


Fig. 141: Identification scheme absorbed by the superattractor for S^3

Next comes the real projective space RP^3, which equals the lens space $L_{2,1}$:

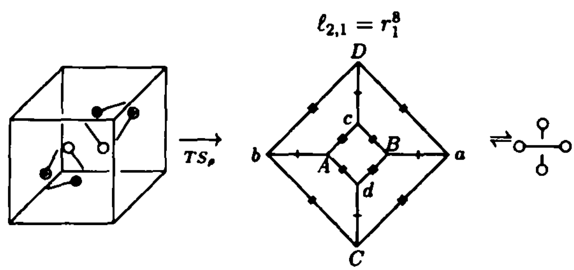


Fig. 142: Identification scheme absorbed by the superattractor r_1^8 for $L_{2,1}$

The lens space $L_{4,1}$ also appears as an identification scheme on the faces of a solid cube. The attractor for this lens space has two 3-gems: r_2^{16} and r_3^{16}.

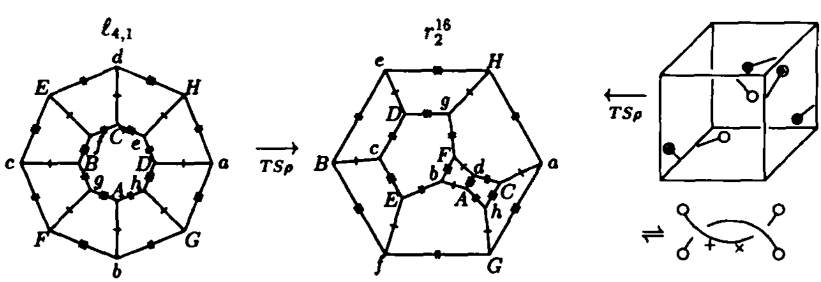


Fig. 143: Identification scheme absorbed by the u_2^0-attractor r_2^{16} for $L_{4,1}$

The next lens space to appear is $L_{6,1}$. It has an attractor formed by twenty 3-gems. The u_{20}^0-attractor is r_{33}^{24}.

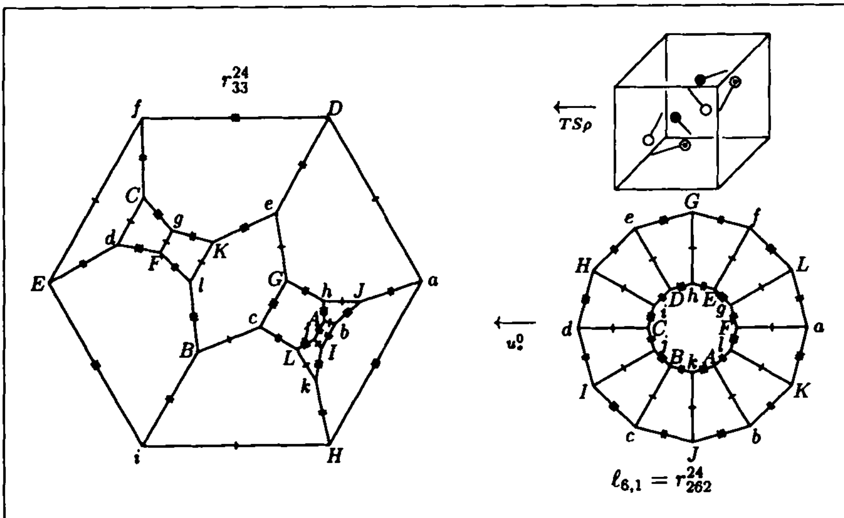


Fig. 144: Identification scheme absorbed by the u^0_{20}-attractor for $L_{6,1}$

The lens space $L_{8,3}$ appears by the identification of a solid cube, as shown below. The attractor for this 3-manifold is formed by three 3-gems at the level of 20 vertices: r^{20}_1, r^{20}_6 and r^{20}_{17}.

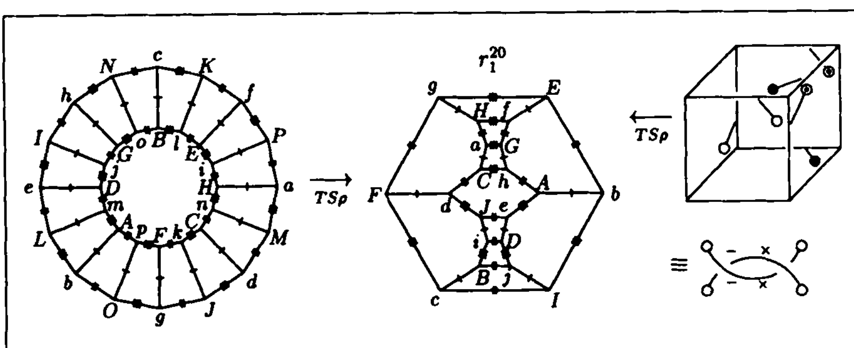


Fig. 145: Identification scheme absorbed by r^{20}_1, the u^0_3-attractor for $L_{8,3}$

Next comes the lens space $L_{12,5}$. The attractor for this space has nine 3-gems. This is a very special case, because it is the smallest case where we need u^1-moves. The u^0-classification does not attain the topological classification. Indeed, the graph $\Gamma^{TS}_{r^{24}_{32}}$ has six vertices, the 3-gems: r^{24}_{32}, r^{24}_{37}, r^{24}_{115}, r^{24}_{121}, r^{24}_{133} and r^{24}_{165}. The graph $\Gamma^{TS}_{r^{24}_{34}}$ has three vertices, the 3-gems: r^{24}_{34}, r^{24}_{44} and r^{24}_{82}. However, as we show with details in Proposition 27 (Chapter 5), starting with the U^1_1-move on r^{24}_{32} we can get by a few u^0-moves to r^{24}_{34}. Reciprocally, starting with the U^1_{21}-move on r^{24}_{34} we reach r^{24}_{32} by a few u^0-moves. This implies that each member of the attractor can be reached from any other member by means of a single u^1_*-move.

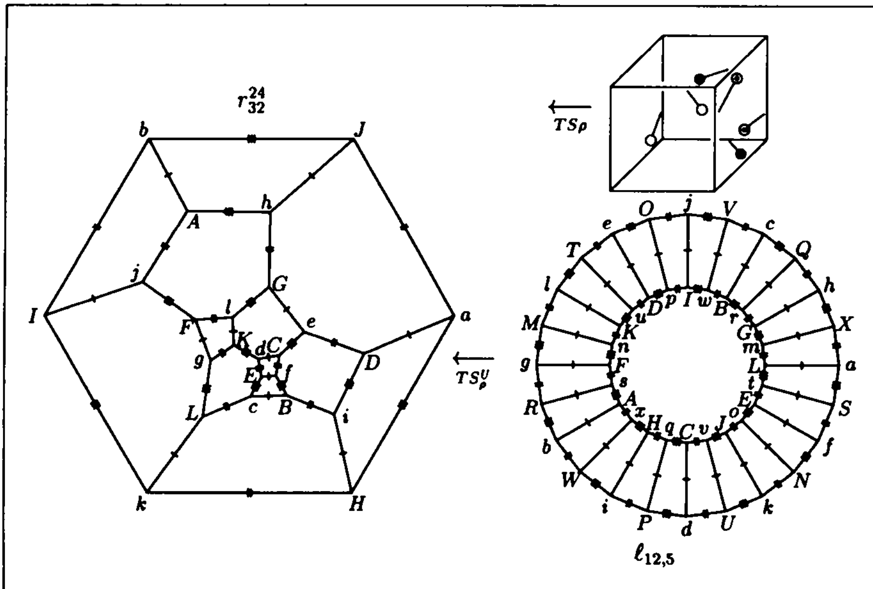


Fig. 146: Identification scheme absorbed by r^{24}_{32}, the u^1_9-attractor for $L_{12,5}$

The last lens space which arises by identifying the faces of a solid cube is $L_{14,3}$. It has an attractor formed by fifty two 3-gems at the level of 28 vertices.

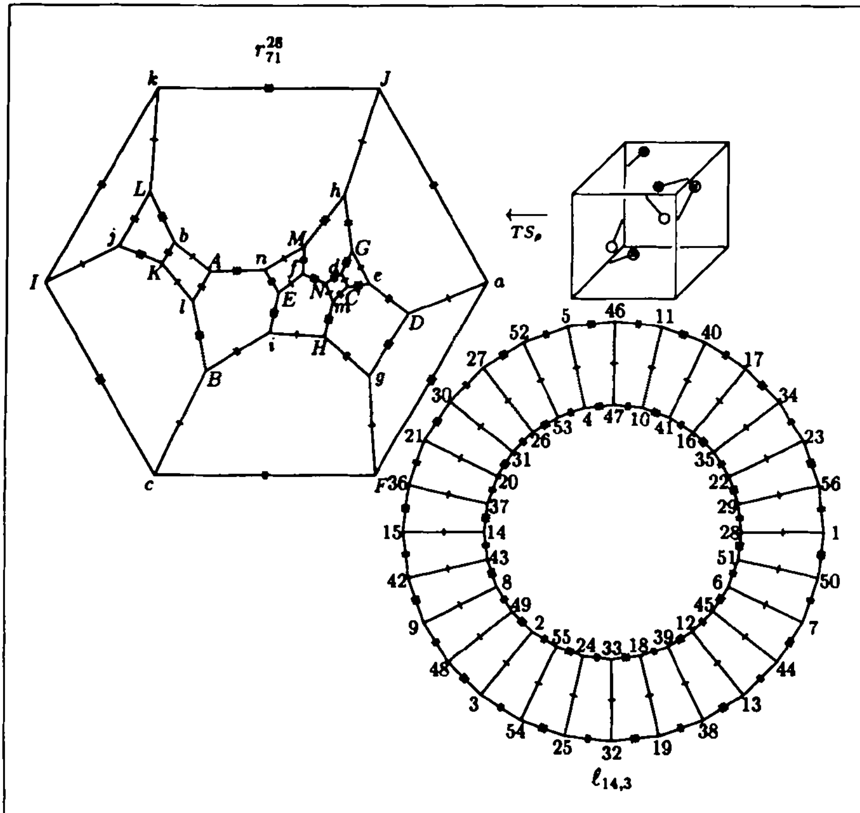


Fig. 147: Identification scheme absorbed by r_{71}^{28}, the u_{52}^0-attractor for $L_{14,3}$

4.2.3 Non-Trivial Connected Sums

Here are the attractors for the two non-trivial connected sums that appear by identifying faces in a solid cube. The first is the connected sum of two copies of $L_{2,1}$.

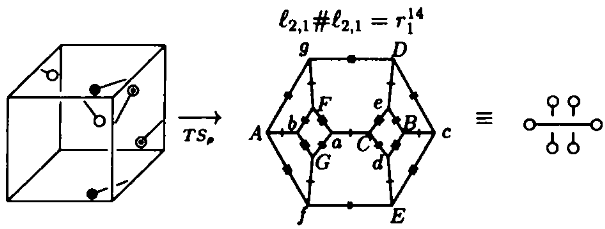


Fig. 148: Identification scheme absorbed by r_1^{14}, the superattractor for $L_{2,1}\#L_{2,1}$

The other 3-manifold is the connected sum of $L_{2,1}$ and $L_{4,1}$. This manifold has an attractor formed by five 3-gems at the level of 22 vertices.

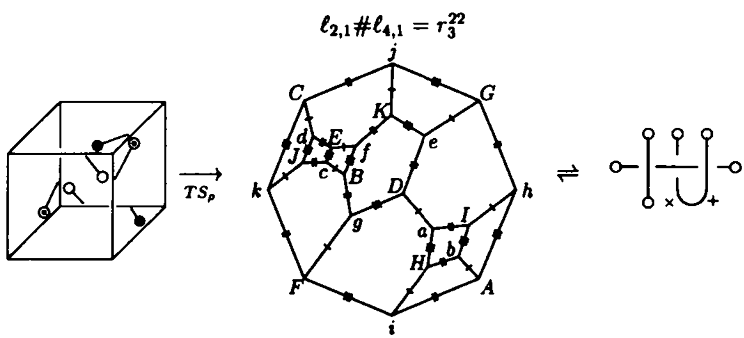


Fig. 149: Identification scheme absorbed by r_3^{22}, the u_5^0-attractor for $L_{2,1}\#L_{4,1}$

4.2.4 Non-Cyclic Quotients of S^3

Now we prove that the quaternionic space, $S^3/\langle 2,2,2\rangle$ and the space $S^3/\langle 3,2,2\rangle$ arise by identifying faces in a solid cube.

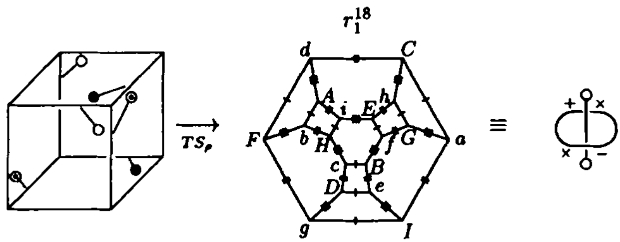


Fig. 150: Identification scheme absorbed by r_1^{18}, the superattractor for $S^3/\langle 2,2,2\rangle$

Here is the proof that $S^3/\langle 3,2,2\rangle$ is obtained from an identification of the faces of a solid cube.

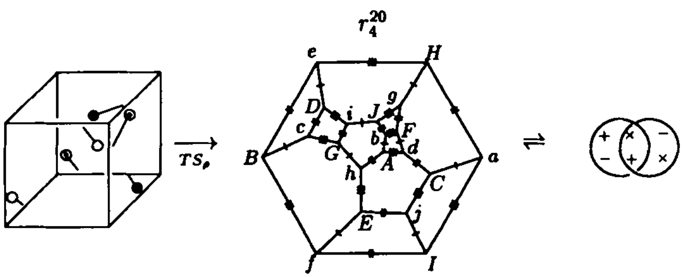


Fig. 151: Identification scheme absorbed by r_4^{20}, the superattractor for $S^3/\langle 3,2,2\rangle$

These spaces complete the list of all the orientable 3-manifolds which arise by identifying the faces of a solid cube.

4.2.5 A Remarkably Symmetric 3-Gem

The last superattractor of the previous subsection, r_4^{20}, has a remarkable group of symmetries. *For each permutation of the colors of the edges there is precisely one symmetry realizing it.* We cannot help in displaying all these symmetries. The pictures below are 24 snapshots of **the same vertex labelled** $(3+1)$**-graph.**

Here are the symmetries on the $\hat{0}$-residue and on the $\hat{1}$-residue of r_4^{20}:

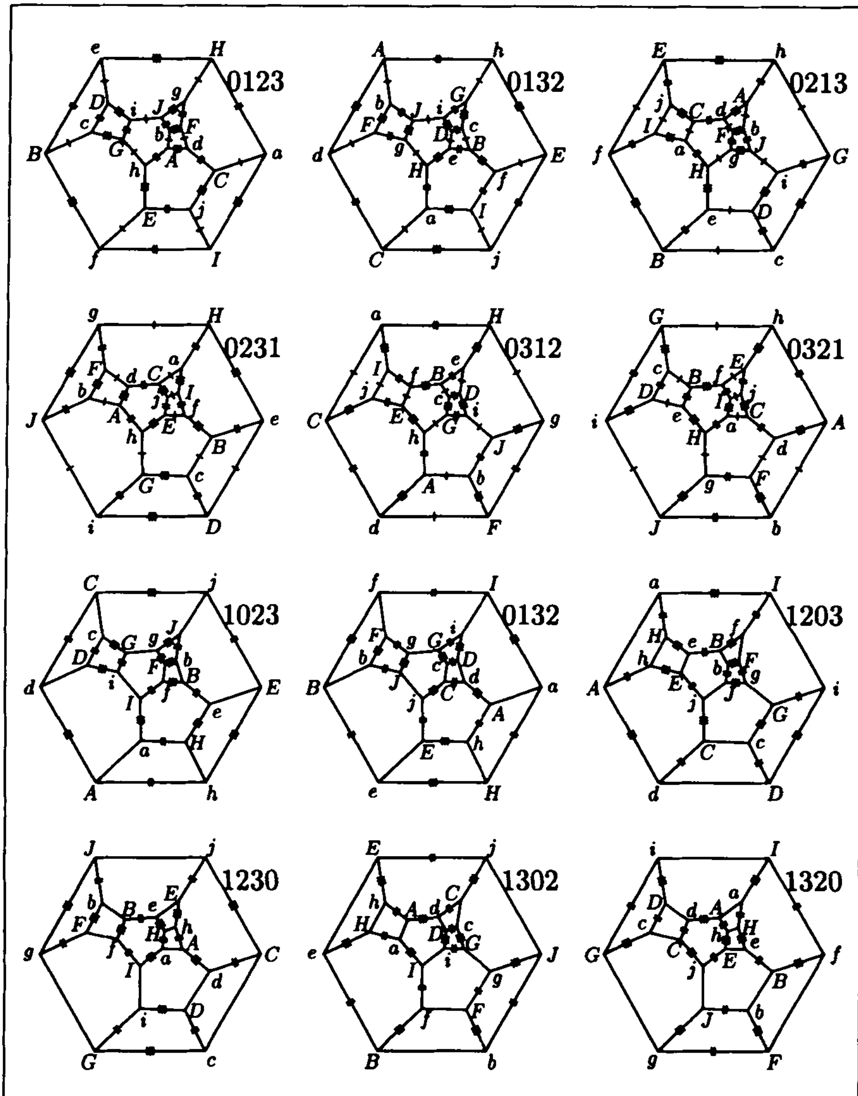


Fig. 152A: Symmetries on the $\hat{0}$-residue and on the $\hat{1}$-residue of r_4^{20}

Now we depict the symmetries on the $\hat{2}$-residue and on the $\hat{3}$-residue of r_4^{20}:

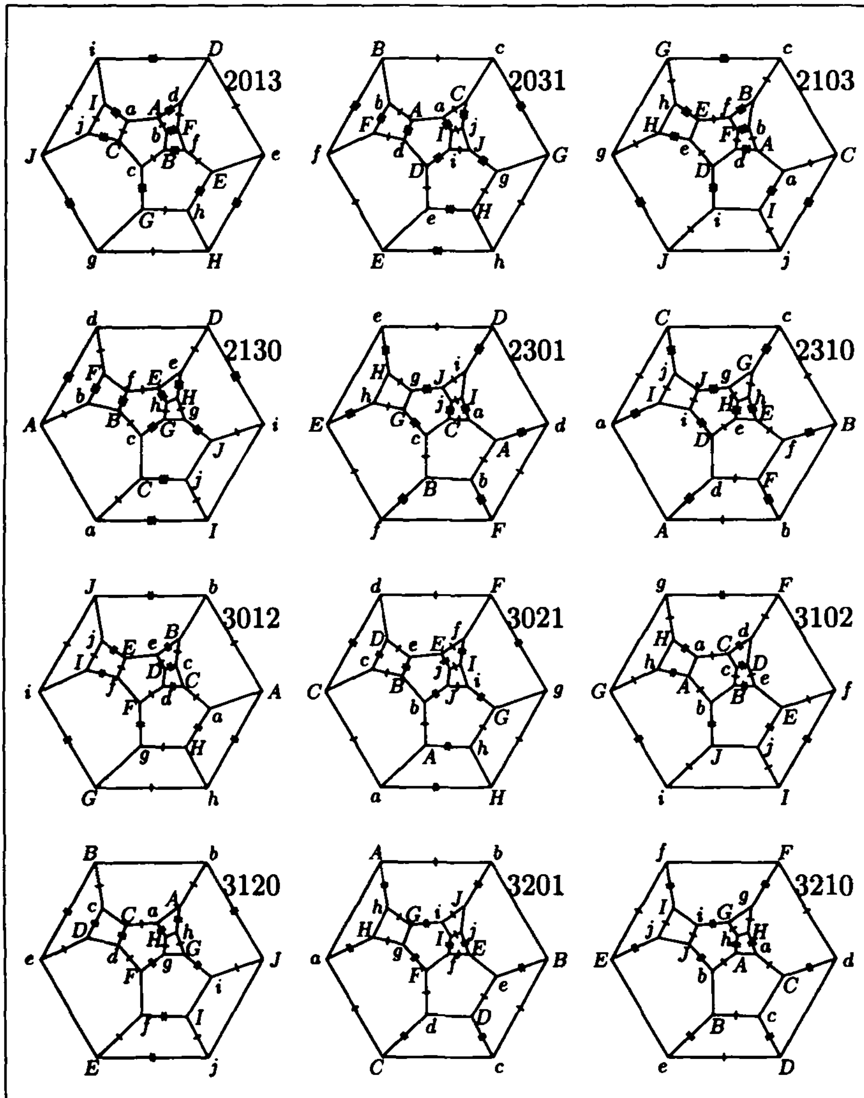


Fig. 152B: Symmetries on the $\hat{2}$-residue and on the the $\hat{3}$-residue of r_4^{20}

We note that r_4^{20} is the smallest 3-gem having the above kind of symmetry. Another 3-gem which has this property is r_2^{26}, which induces the 3-torus $S^1 \times S^1 \times S^1$.

Chapter 5

The Generation and Classification of 3-Manifolds

In the first part of this chapter we provide a method for generating a catalogue of all the rigid 3-gems of $2n$ vertices, $n \geq 1$. This theory was implemented for the orientable case and we have obtained in this way all the bipartite rigid 3-gems up to 30 vertices. Complete tables up to 28 vertices of our computations are provided, to illustrate the general approach.

Next we consider the problem of the topological classification of the objects in our catalogue up to 28 vertices. The u^0-classification leaves 11 topological uncertainties which are resolved with the u^1-classification, in the sense that each attractor is formed by a single essential u^1-class. Therefore, the u^1-classification coincides with the topological classification. The same result holds for the rigid bipartite 3-gems with 30 vertices. However, in this book we do not list nor classify these 3-gems here by lack of space to do so.

We leave open the question: what is the first counter-example for the observed fact that the minimal u^1-classes of 3-gems coincide with the homeomorphism classes of 3-manifolds?

5.1 A Catalogue of 3-Manifolds

In this section we develop a theory that generates recursively all the rigid 3-gems. It is an easy fact that each 3-manifold which is not a connected sum with $S^1 \times S^2$ or with $S^1 \tilde{\times} S^2$ is induced by a rigid 3-gem.

In the implementation we restrict ourselves to the orientable case, just to go further in practice. The theory is easily extended to the non-bipartite gems and the non-orientable manifolds could be generated as well.

5.1.1 Constructing the Rigid S^2-Gems.

Let G be a 2-gem inducing S^2 free of ρ_2-pairs. We call these 2-gems *rigid S^2-gems.* The 1-skeletons of the even prisms whose base polygons have an even number, $2n, n \geq 2$, of sides, form a simple family of S^2-gems, denoted by P_{2n}.

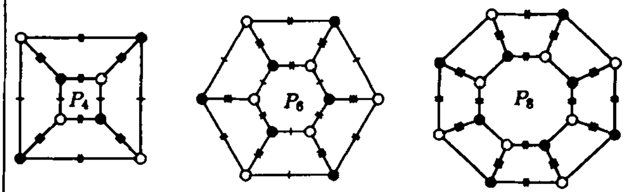


Fig. 153: Family of even prisms: all edges non-fusible

Our strategy to produce all rigid orientable 3-gems is to start with a rigid S^2-gem and try for all the possibilities of introducing edges of the fourth color, keeping those $(3+1)$-graphs that yield rigid orientable 3-gems. In this way, to get the rigid orientable 3-gems with $2n$ vertices, the first step is to be able to generate all rigid S^2-gems with $2n$ vertices.

An edge with ends v and u in a rigid S^2-gem G is called *fusible* if G_{uv}^{fus} is another rigid S^2-gem. Note that no edge of P_{2n} is fusible. However, the following theorem holds and it provides a recursive generation of all S^2-gems with $2n+2$ vertices from the ones with $2n$ vertices.

Theorem 12 (Lins [Lin84]) *A rigid S^2-gem G with $2n$ vertices has a fusible edge or else $n \geq 4$ is even and G is isomorphic to P_n.*

The proof follows from three simple lemmas.

Lemma 12 *Let $G \neq P_{2m}$, $m > 2$, be a rigid S^2-gem. There exists in G an edge which is not part of a square bigon.*

Proof: If every bigon of G is a square, then G is P_4, a contradiction. Suppose β is a bigon of G with more than 4 edges, say $2n$, $n \geq 3$. For each edge of G there are two bigons containing it. For α in β, denote by β_α the bigon distinct of β containing α.

If the lemma is not true for G then each β_α is a square. Since G is cubic, this implies $G = P_{2n}$, again a contradiction. ∎

Lemma 13 *Let G be a rigid S^2-gem and α be an edge not contained in a square bigon. If α is not fusible, then at least one of its ends is not a vertex of a square bigon.*

Proof: Suppose the lemma is not true.

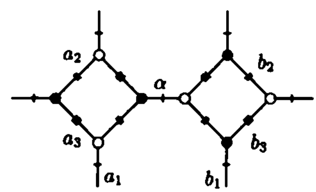


Fig. 154: If both ends of α incide to square faces ...

Since edge α is not fusible, the pair $\{a_2, b_2\}$ is in the same 12-gon or else the pair $\{a_3, b_3\}$ is in the same 13-gon. Up to symmetry we may assume that the last possibility holds. It follows that the pair $\{a_1, b_1\}$ is in the same 12-gon and in the same 13-gon, and so is a ρ_2-pair. This is a contradiction with the fact that G is a rigid S^2-gem. ∎

Proof of Theorem 12: Assume that G is a rigid S^2-gem, has no fusible edge and that is not a P_{2n}, for some n. We will produce a contradiction. From the two previous lemmas we may assume that G has a vertex v so that its three incident bigons have more than four edges. Denote by H_1, H_2, H_3 the bigons incident to v and by J_1, J_2, J_3 the third bigons incident to the other ends of the 3 edges a_1, a_2, a_3, incident to v, as depicted below.

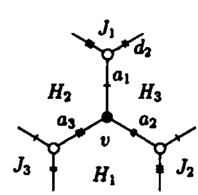


Fig. 155: Vertex v not incident to a square face

If $H_1 = J_1$, then $\{a_2, d_2\}$ is a ρ_2-pair, a contradiction. Thus $H_1 \neq J_1$, and similarly, $H_i \neq J_i$, for $i = 2, 3$.

Since a_i is not fusible, there are edges b_i in J_i and edges c_i in H_i so that $\{b_i, c_i\}$ becomes a ρ_2-pair after the fusion of a_i. In particular, b_i and c_i have the same color. Thus b_i and c_i are in a same bigon X_i of G. We claim that $X_1 = X_2 = X_3$.

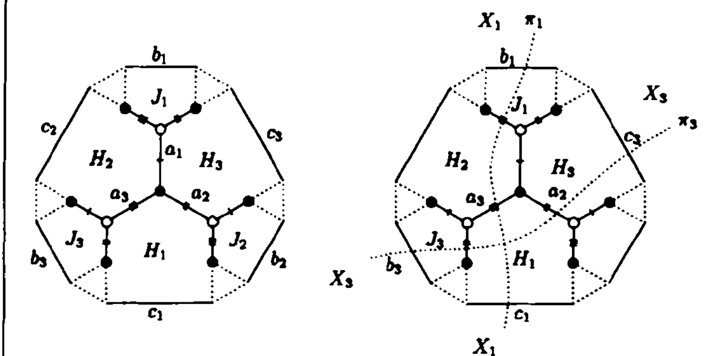


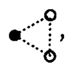
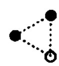
Fig. 156: The structure of a counter-example

To prove this consider the closed paths π_1 and π_3:

- π_1 is in the surface of S^2; it visits the interior of X_1, of H_2, of H_1 and back to X_1 crossing G only at the interior of 4 edges, as indicated above.
- π_3 is in the surface of S^2; it visits the interior of X_3, of H_3, of H_1 and back to X_3 crossing G at the interior of 4 edges, as indicated above.

Paths π_1 and π_3 must cross in the interior of H_1. They must therefore cross another time and the only possibility left is to have $X_1 = X_3$. In an analogous way, we could define a path π_2 and show that $X_2 = X_3$. Therefore, the six edges $\{b_1, b_2, b_3, c_1, c_2, c_3\}$ are in a same bigon, which above bounds the exterior face.

Now we have to consider the six dotted triangular regions. These regions have necessarily an internal structure to take into account the bipartiteness of the whole embedded graph. The true number of vertices in the boundary of this triangular region is even (there is no odd polygon in G). We distinguish two types of these regions: a *homogeneous region* is one in which the three vertices that constitute the corners of the region are all in the same class of the bipartition. The other type of region, a *non-homogeneous region* is one which has two corners in one class and one corner in the other. However, the lemma below shows that this type of region is impossible. Assuming the lemma, it follows that the three corners of the triangular regions above are, say, •vertices. So, there is an even number of edges along the dotted paths of the boundary of the triangular regions. The case of zero edges may occur and, in this case, the triangular region collapses to a unique vertex. The dotted paths having an even number of edges imply that the boundaries of the H_i's and the J_i's have an odd number of edges. Contradiction, because these are bigons. The theorem is proved modulo the lemma below. ∎

Lemma 14 *Non-homogeneous triangular regions,* , , *do not occur.*

Proof: A non-homogeneous triangular region is dissected by a bipartite graph H (the restriction of G to the interior of the region and its boundary). Assume that H has p •vertices and q °vertices. Assume also that there are two •corners and one °corner. By counting total number of edges in H, considering edges incident to the •class and to the °class we have: $3p - 2 + 2 \times 2 = 3(q - 1) + 2 \times 1$. This is equivalent to $3(p - q) = 1$, a contradiction, since p and q are natural numbers. A similar contradiction is obtained if there is one •corner and two °corners. ∎

5.1.2 Generation of the Bipartite Rigid 3-gems

The generation process consists of two phases: (i) the generation of all rigid S^2-gems; (ii) for each S^2-gem, the insertion of the edges of the fourth colour in all possible ways.

Let a *duet* in a $(n + 1)$-graph be a pair of edges of distinct colors which are members of a same bigon. The *breaking of a duet* is the inverse operation of the fusion of two vertices linked by $(n - 1)$ edges. For $(2 + 1)$-graphs, the breaking of a duet is the creation of a 1-dipole.

To generate all rigid S^2-gems with $2n$ vertices: We use the ones with $2n - 2$ vertices to effect breaking of duets in all possible ways. In 2-gems, this is simply the inverse of the fusion of a 1-dipole. In P_6 we have essentially a unique way to apply the operation:

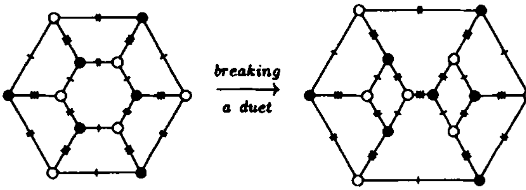


Fig. 157: The unique (up to symmetry) breaking of a duet in P_6

We must be careful in two aspects: (i) it is easily seen that the operation may produce ρ_2-pairs; (ii) is each such triball obtainable from a smaller one by breaking duets? The first aspect we tackle explicitly: once an S^2-gem is generated we only consider it if it has no ρ_2-pairs. As for the second, the answer is no. The 1-skeletons of the prisms with $2n$ edges in the base provide counter-examples. Fortunately, the theorem proved in the last subsection shows that those are the only possible ones.

From Theorem 12 we can extract an algorithm to produce all the rigid triballs with $2n$ vertices by breaking duets on rigid S^2-gems with $2n-2$ vertices. In our implementation, we took advantage of the symmetry and considered only those pairs of duets which are distinct under the automorphism group of the S^2-gem. A completely independent algorithm was used to generate the same class of graphs in [HMM85]. Our numbers agree with the ones published in that paper:

No. Vertices	No. Rigid S^2-gems	No. Vertices	No. Rigid S^2-gems	No. Vertices	No. Rigid S^2-gems
2	1	14	1	24	32
4	0	16	2	26	57
6	0	18	2	28	185
8	1	20	8	30	466
10	0	22	8	32	1543
12	1				

Table 2: Number of rigid S^2-gems

To obtain the bipartite rigid 3-gems from a rigid S^2-gem: We use a simple minded back-tracking procedure which tries to insert edges of the fourth colour sequentially and in all possible ways. During this process we make three time saving restrictions: (i) avoid inserting an edge connecting vertices in the same class of the bipartition; (ii) avoid connections between vertices of the same bigon; (iii) avoid connections which destroy planarity of some 3-residue. When we get all vertices incident to the fourth colour, we check whether all the 3-residues are in the catalogue of rigid

S^2-gems. If it is the case, we compute the code and check whether the 3-gem was already generated. If not, we keep it. Restriction (i) is justified because we want to generate only bipartite 3-gems; the second because those connections would create ρ-pairs; the third restriction because those connections would generate non-gems. The actual implementation of this method was done by Cassiano Durand [Dur92].

We display a summary of the catalogue and of the combinatorial and topological classifications. **There are exactly 100 non-homeomorphic closed orientable 3-manifolds induced by 3-gems up to 28 vertices.** Of these 93 are *handle-free* and 7 are not. A 3-manifold is *handle-free* if it is not a connected sum of some other 3-manifold and $S^1 \times S^2$ or $S^1 \tilde{\times} S^2$.

No. vertices	Bipart. rigid 3-gems	Bipart. u^0-classes	Bipart. u^1-classes	Handle-free topological classes
2	1	1	1	1
4	0	0	0	0
6	0	0	0	0
8	1	1	1	1
10	0	0	0	0
12	1	1	1	1
14	1	1	1	1
16	3	2	2	2
18	4	2	2	2
20	23	5	5	5
22	44	6	6	6
24	262	18	17	17
26	1252	20	18	18
28	7760	50	39	39
30	56912			

Table 3: Summary of the catalogue and classifications

The manifolds which are not handle-free are not induced, at the lowest level, by rigid 3-gems. For instance there is only one rigid bipartite 3-gem with 8 vertices, but there are two distinct classes of homeomorphic orientable 3-manifolds which have complexity 3: $L_{2,1}$ and $S^1 \times S^2$. We consider the rigid representative of $S^1 \times S^2$ to be the pair $(s^3, 1)$, meaning the connected sum of s^3 and one copy of $s^1 \times s^2$. Note that s^3 is a rigid 3-gem. There are seven non-handle-free orientable 3-manifolds induced by 3-gems up to 28 vertices. These are listed in Table 10C.

5.1.3 Codes for Rigid Bipartite 3-Gems

Below we present the codes for all rigid bipartite 3-gems up to 24 vertices. Up to 14 vertices there is only one entry at the levels of 2, 8, 12 and 14 vertices:

r_1^2	a	a	a
r_1^8	badc	dcba	cdab
r_1^{12}	cabfde	fedcba	edfbac
r_1^{14}	cabedgf	gfdcbea	fgebcad

Table 4A: Rigid bipartite 3-gems up to 14 vertices

In the above table and subsequent ones, the first row of a code appears under and overlined; the second row appears only underlined; the third row is unmarked, i.e., has no under nor overline. For the next tables we avoid repeating the first and second lines of a code as long as this is possible. For instance, to get r_{41}^{22} we start at the line headed by 39 in the gems with 22 vertices (Table 4E). The first unmarked entry in this row is the third row of the code for r_{39}^{22}. From this entry, starting with 39, we go counting *skipping marked lines* up to 41 arriving at jikhgcedbaf. This is the third row of r_{41}^{22}. To get the second recede until the first underlined entry, in the case, kjigfedchba. Finally, to get the first keep receding until finding the first under and overlined entry, namely, eabcdhfgkij.

Here are the codes for the rigid bipartite 3-gems from 16 up to 24 vertices. The *complexity of a 3-gem* is defined to be half of the number of vertices minus one. The *complexity of a closed 3-manifold* is the complexity of a 3-gem in its attractor. In Appendix A we display similar tables for the 3-gems with 26 and 28 vertices.

01	cabfdehg	hedcbgfa	gdfbahec	gfeahbdc	dabchefg	hgfedcba
03	gfehcbad					

Table 4B: Rigid bipartite 3-gems with 16 vertices – Complexity 7 – Three 3-gems

01	cabfdeigh	ihdcbgfea	gfhaibdce	dabcgefih	ihfedcbga	fehibadcg
03	hegibacfd	higfbdcae				

Table 4C: Rigid bipartite 3-gems with 18 vertices – Complexity 8 – Four 3-gems

01	cabfdehgji	heicjgfadb	gifjahecbd	hjdcbifage	gfeicbjdha	ifeahbjcdg
04	ifgahbjedc	jheicbafdg	jhfbicadeg	jhibgaedcf	hjicagfedb	digacjebhf
09	jihgbcdfea	dabcfehgji	higedcfjba	gjhfcdieab	hjgedcfiab	gfhiabejdc
12	ghjibdaefc	jhegcbifda	jigedcfbha	gfjhabidec	ijhfcdbeag	dabcgefjhi
16	jifedchgba	ifejhbacgd	ihegcbjfad	ihgfbjcead	dabchefgji	jgfedcbiha
19	gfjhciadbe	ifehcbajgd	ihgfajcbed	eabcdjfghi	jihgfedcba	hgfjicbaed
23	ihgfjdcbae					

Table 4D: Rigid bipartite 3-gems with 20 vertices – Complexity 9 – 23 3-gems

01	cabfdeighkj	ikdcjgfeahb	jehkiadcbfg	kdigahbfjce	dabcgefihkj
03	ihjedkbgacf	hiejkbdafgc	ihkedjbgafc	fjhibkdcgea	higkbdjaecf
06	hkjfaicedgb	kihgcajebdf	ikfedcjgahb	hegjbakfdic	kfeihbacjdg
10	kfijchabedg	kigfbjcaedh	kjghbdifcea	khfedcbgjia	hegkbajfdci
14	hekfjicadbg	hfkgcjiadbe	hkegcbdjfai	jiegcbdakhf	jigfbdcakhe
19	jihgcakebfd	kjfedchgbia	egihadkjcfb	fejkhadbgci	fhekjidbacg
23	hegfbkcjdai	hegkbajfdci	hegkbijadcf	hfegcbkjdai	hgjfaikedcb
28	jegkbaifchd	jeikhacbgfd	jhifakcbged	jhigcakbefd	jkegcbifdah
33	jkeihbdcgaf	jkgfbicedah	jkifhdcbgae	jkigchdbeaf	dabchefgkij
37	kjfedcbihga	ihgjakcbedf	eabcdhfgkij	kjigfedchba	ikfhjcbdaeg
39	jgfikcbadhe	jikfhdcebag	jikhgcedbaf	eabcdifghkj	kjhgfedcbia
42	hgjkidbaecf	jfihkbadcge	jkihgbedcaf		

Table 4E: Rigid bipartite 3-gems with 22 vertices – Complexity 10 – 44 3-gems

001	cabfdeighljk	ijdclgfkabhe	kegjhalbdifc	ikdcbgfjlhea	jhgliakbefcd
003	ikjchgfeldba	kjigchlfebad	ildcbkfjahge	kfhaibjlegdc	ildckgfjaheb
005	hkfbjielcgda	jifbhkaldgce	kegjhalbdifc	ilkcjgfeahdb	dglackbfjihe
009	dkgaclebjihf	gdfbijalkceh	gehlijkcbfad	hdfbkijlcgea	kiljbhafedgc
014	lkigbhdfjcea	dabcfehgjilk	hjledkfagbic	kfgiahjblced	hljedifakcgb
016	kigfclebhajd	dabcgefihljk	lkjedchgfiba	fjegchdklbai	kfegcbijlhad
019	dabcgefjhilk	gjfedclakbih	jegfbkcdlahi	jihldkcgbafe	ihljbckeadgf
021	jikedchglafb	kjigchdlebaf	jikedclgbafh	ehlfidkbagjc	ejlfhdkagbic
024	fhlgkidbaejc	khjfldcbagie	khjgcldbaeif	jiledchgkabf	ijgfbdckleha
028	kfhjibacelgd	jiledckgbahf	egihakblcfjd	egikadjlcbfh	ekihadlfcbjg
032	hjekcbladfig	hjgfkdcaelib	ihegckdlafjb	ihekcblfadjg	ihgfkdclaejb
037	ikegcbjladfh	ljifkdcagbeh	ljigckdaebfh	jkfedchgblia	fekjbaicldhg
041	hfelcbjadkgi	hjelcbkadfgi	hjlfbikadegc	hlegcbiadkjf	hleicbjkdgaf
046	ifelhbacgkjd	ihefkjlbadcg	ihegcblfakjd	ihelkjdbafcg	ihjkcdlbaefg
051	ihlfkdcbaejg	kfejhblcgdai	kfjihbaelcgd	kfjilbaedcgh	khegcbjfldai
056	khgfbjceldai	kjegcbildfah	kjifhdclgbae	kjigchdlebaf	kjihldafcbge
061	jkiedchgflba	flegchdabkji	jliedchgfkab	efjilhkbdcga	egjiakbflchd
064	hegkbjcadlif	hjegcblakfid	hjgfblcakeid	hjgkblcadeif	ifejhbkcldga
069	lehjikdcbgfa	lfhjkbacedig	lheicbjfkgda	lhgibjcekfda	ljehcbikgfda
074	lhfedcbgakji	eilfadckbghj	hkegcbdaflij	hkgfbdcaelij	hljgcidkfeab
078	hlkicajfdgeb	kehjbaicglfd	kfighjlecbad	lifedchgbkja	hjglbikadecf
082	hlegcbikdfaj	ilgfbjcekdah	kfejhbacglid	kgihaljfcbed	khegcbjfalid
087	khgfbjcealid	lkiedchgfbja	egjfadiklchb	ehjfkdcblgai	eigfhdckblaj
091	eilkadjfbghc	fhjgckdbleai	fhjikadblceg	fhkliadbegcj	fiegchdkblaj
096	flhjiadckgeb	fljgcadbkeih	hegiblckdfaj	hgjialbkdcef	hgkiajbedlcf
101	ieklbhjfadcg	ighjaklcedfb	igljakbcedfh	igljhkbcadfe	ihelkjdbafcg
106	kegfbjcaldhi	kfegcbjaldhi	klgibjcedfah	klhjibdcegaf	dabchefgjilk
110	jgfedcbkhlia	gfjhckadlebi	gjehcladfkbi	ifehcbalgkjd	jgkedclihafb
113	fkhjblicgdea	ihlfajkbedgc	ikehclajgdbf	jgledckihabf	ehgiakcblfjd
117	elgiadjkfchb	iekgljdcafhb	ihgfkdcbleja	jgledkbihafc	ekgildjbfcha
121	ifkhclajgdeb	khifalcjgbed	lejgiakcbhdf	jigedklbhafc	egjildbkfcha
125	egklidbjahcf	ifehcbalgkjd	ifhjklacgdeb	lfihcjkdebga	jigedlkbhacf

Table $4F_1$: Rigid bipartite 3-gems with 24 vertices – Complexity 11 – 262 3-gems

129	egjildbkfcha	gelkbjidafhc	gfejlbadkhic	ijeglbdkafhc	jkfedcblhagi
133	elgfidckahjb	ihgkalcbedjf	ilhgbkdcafje	jkfedclihgba	elifkdcjghab
137	fleickdjghab	iekgljdcafhb	ijeglbdkafhc	ilehcbkjadgf	jlfedcbkhaig
141	ehgkadjblcfi	ekgiadjlfcbh	fjkgbidlaech	fjlgiadcekbh	iehkbjdlafgc
146	ihgfkdcbleja	ihlfajkbedgc	ikgfajcledbh	kfehcbajldgi	kfelibajchgd
151	kfhlijacebgd	kfjhibalecgd	khgfajcbldei	khgiajcblfed	khifaljbcged
156	khigaldjcbef	kjehcialfbgd	lehjbaicgkdf	lfjgibkcehda	ljehcbkdafgi
161	ljekcbiagfdh	ljgfkdcbahei	jlfedckihagb	ihgkajlbedfc	ihlfajckedbg
165	ikehcbljgdfa	kjgfilcbahed	ljehcbikafdg	jlfedckihgab	ehgjakcbflid
169	ejlfidckabhg	feikbaljghdc	fhjikldbgcea	fjelcidkabhg	fjlgkadcebih
174	fkjgladcbhie	ihgkajlbefdc	ljehcbikgfda	ljgfikcbehda	ljhgikdcabfe
179	jlkedcaihgfb	ehlfadcjkbig	eilfadckbhjg	ekgfadcjblih	ekifadclgbjh
183	felgiadcbkjh	fhgjklibcdea	fihgbldcekja	fkhgiadclbje	iehgbjkcaldf
188	igehclkjadbf	ijhfbdklaegc	kgjfldiabche	kgjildbafche	khigaldjcbef
193	khjfalibgced	khjialdbgcef	lehjbaicgkdf	ljehcikdabgf	ljgkiacbehdf
198	lkifbdcjghea	lkjhcibdfega	lgfedcbihkja	kfehcbajglid	khgfajcbelid
202	khgialjbfced	khifalcjgbed	lkfedcbihgja	iehlbjdkafcg	ihgjalcbkdef
206	ihkfajlbedcg	klhgbjdcefai	kljgibdcehaf	dabchefglijk	lkfedcjihgba
209	ihgjalkbedcf	kjehcbilgfad	kjgfilcbehad	eabcdhfgjilk	jigkfelbhadc
212	ifkhjladgecb	ijhlgbekafcd	lfijgbkdchea	lgijhdbkcfea	jiglfekbhacd
216	lgjihdbkfcea	jikgfedlhacb	ijlhgkedafbc	jilgfedkhabc	ijfhkcblaged
219	ljihgckdfbea	jilkfedbhagc	ijflkcbeaghd	ijklhdbeagcf	kfhijblcdgae
223	klijhdbecfag	ljihkcadfbeg	lkijgbedchfa	jkigfedchlba	fhgjialbkdec
227	glfijckedhab	ilfhjckdgeab	iljhgcedkfab	jligfedchkab	fhgjialbkdec
231	gijlkcaebfhd	igfhjcbkleda	ihgfjkcbleda	ihgjkacbfled	ijlkgbedafhc
236	ijlkhdbeagfc	lgjihdbkfcea	ljfhicbdkgea	ljgfidcbkhea	lkigfedchbja
240	fhljiaebkdgc	gifklcjebadh	glkijcaedhfb	ifklgbjdaech	igklhdbjaecf
245	ihgljacbkefd	ilkhjcadgefb	klfhjcbdgeai	klfijcbedhag	kljhgcedbfai
250	kljihdbefcag	eabcdifghljk	lkhgfedcjiba	hgfjlkbaedci	jlgfidcbkaeh
253	khgfljcbadie	kjgfidcblhae	kjihgbldcfae	eabcdjfghilk	lihgfedcbkja
256	hgkjiclaedfb	ihlfjdckaebg	kfjihbaldcge	kgfjicbaldhe	khgfjdcbalie
261	kjihgaldcbfe	fabcdelghijk	lkjihgfedcba	kjihgledcbaf	

Table $4F_2$: Rigid bipartite 3-gems with 24 vertices – Complexity 11 – 262 3-gems

5.2 Understanding Data in the Catalogue

In this section we discuss various issues related to the better comprehension of the catalogue. The codes seen in the previous section are raw combinatorial data, and by themselves rather useless. Their importance are in their conciseness and their completion. To make them useful we need to employ them as inputs in various computations. These will enable us to achieve the topological classification of the bipartite 3-gems up to 28 vertices.

5.2.1 Displaying the u^0-Forests

The first step in a classification is to form the *u^0-forests* of the rigid bipartite 3-gems with n vertices. These are formed by the tree edges in a breadth first search ([Gol80]) on the various Γ_G^{TS}. We need to consider only the G's which have the smallest code among the vertices of Γ_G^{TS}. Clearly, the point to consider these forests is that the 3-gems which are vertices of a tree in such forests *induce the same 3-manifold.*

Follow the u^0-forests for the rigid bipartite gems up to 24 vertices. We label the edges by the six moves $A, \ldots, F$, specifying them as in Subsection 4.1.2. Recall that these six moves are defined as

$$A = \rho_* \circ TS_1, \quad B = \rho_* \circ TS_2, \quad C = \rho_* \circ TS_3,$$

$$D = \rho_* \circ TS_4, \quad E = \rho_* \circ TS_5, \quad F = \rho_* \circ TS_6.$$

In the actual entries below, the superscript on the move symbol is the color specification and its subscript is the root of the TS-move. See Subsection 4.1.2.

The u^0-forests of the rigid bipartite 3-gems of 2, 8, 12 and 14 are isolated vertices. The first such forest which deserves attention is the one relative to 16 vertices. Let $\mathcal{F}_n^0$ be the u^0-forest of the rigid bipartite 3-gems with n vertices. Here is how we display $\mathcal{F}_{16}^0$:

01	root	root	$2D_{05}^{31}01$

Table 5A: Compressed tabular form of $\mathcal{F}_{16}^0$

The three entries correspond to the three 3-gems in $\mathcal{RBG}_{16}$. The table says that the first and second 3-gems are roots of trees in $\mathcal{F}_{16}^0$. The third entry specifies that if we apply the move D_{05}^{31} to r_3^{16} we get r_2^{16}. This is read as the *prefix* 02 before the move (in such entries there are no change in the number of vertices, i.e. the TS-move produces a rigid 3-gem - see second entry of $\mathcal{F}^0_{18}$). The *suffix* 01 following the move symbol is the *height in the tree*, i.e. the distance in edges to the root of the tree, of the corresponding entry. In the case, it says that r_3^{16} is at height 1 (from the root). Follows $\mathcal{F}^0_{18}$, in a similar form:

01	root	14/1.0B^3_{17}	16/1.0B^3_{17}	root

Table 5B: Compressed tabular form of $\mathcal{F}^0_{18}$

01	root	root	02E^{32}_{20}01	root	root
06	17D^{31}_{05}02	18/01.0B^2_{14}	18/04.0A^3_{14}	02/01.1A^3_{14}	root
11	08/01.1A^3_{10}	16/02.0A^3_{10}	12C^3_{10}01	12C^1_{07}01	10A^3_{19}01
16	07D^{30}_{04}01	01D^{30}_{06}01	05D^{21}_{11}01	02D^{21}_{10}01	05D^{30}_{12}01
21	02D^{30}_{05}01	18/03.0D^{31}_{01}	21D^{31}_{19}02		

Table 5C: Compressed tabular form of $\mathcal{F}^0_{20}$

01	root	root	root	20/08.0B^3_{17}	root
06	20/21.0A^3_{17}	20/06.0A^3_{17}	07D^{01}_{21}01	10D^{23}_{02}01	20/04.0B^2_{19}
11	20/18.0A^1_{06}	23B^2_{21}02	23D^{21}_{11}02	15D^{21}_{07}02	04A^0_{04}01
16	root	root	16D^{23}_{18}01	23C^0_{07}02	16/02.0A^3_{21}
21	04D^{23}_{08}01	06C^2_{16}01	11C^0_{21}01	15A^3_{21}02	22C^0_{06}02
26	29C^0_{21}03	18/01.0A^3_{21}	23D^{23}_{12}02	31C^2_{21}02	23C^1_{15}02
31	07B^3_{21}01	05A^2_{19}01	36A^3_{21}02	03D^{23}_{09}01	32A^0_{03}02
36	03A^2_{19}01	09D^{30}_{05}02	31D^{21}_{04}02	20D^{31}_{21}01	16D^{21}_{08}01
41	17D^{31}_{08}01	15D^{20}_{05}02	11D^{31}_{16}01	03D^{21}_{15}01	

Table 5D: Compressed tabular form of $\mathcal{F}^0_{22}$

The first and fourth 3-gems in $\mathcal{RBG}_{18}$ are roots of trees in $\mathcal{F}^0_{18}$. Whenever a slash bar "/" appears in an entry this means the move specified by the final part of the entry is followed by an effective ρ-algorithm which decreases the number of vertices. For instance, the second entry above specifies that the move B^3_{17} applied to r_2^{18} becomes r_1^{14} with handle number 0. Similarly, the third entry means that r_3^{18}

under the move B^3_{17} becomes r^{16}_1 with handle number 0. This discussion finishes all the types of entries in the compressed tabular forms of the forests $\mathcal{F}^0_n$'s.

001	root	root	root	root	root
006	root	root	$181D^{31}_{08}01$	$179D^{31}_{08}01$	$22/22.0D^{31}_{04}$
011	$067D^{31}_{05}01$	$22/02.0D^{31}_{04}$	root	root	$100D^{01}_{15}03$
016	$096D^{23}_{01}01$	$12/01.0A^{3}_{16}$	$20/13.0A^{3}_{16}$	$22/05.0D^{31}_{01}$	$02/01.0A^{3}_{22}$
021	root	root	$086D^{32}_{24}03$	$22/10.0B^{0}_{12}$	$059D^{31}_{12}02$
026	$022D^{30}_{04}01$	$208C^{2}_{21}02$	root	$066D^{20}_{10}02$	$024D^{30}_{09}01$
031	$098D^{20}_{02}03$	root	root	root	$013D^{30}_{18}01$
036	$057D^{21}_{02}02$	$032D^{21}_{18}01$	$087D^{10}_{22}02$	$055D^{31}_{07}02$	$22/42.0A^{0}_{21}$
041	$250B^{1}_{16}03$	$057E^{03}_{13}02$	$134E^{12}_{18}02$	$034D^{30}_{10}01$	$228A^{1}_{13}02$
046	$024D^{21}_{10}01$	$22/33.0A^{2}_{23}$	$087D^{32}_{18}02$	$141D^{30}_{16}02$	$047D^{30}_{15}01$
051	$142D^{31}_{02}02$	$065D^{21}_{06}02$	$22/13.0D^{31}_{06}$	$194D^{31}_{19}02$	$003D^{30}_{06}01$
056	$087D^{32}_{23}02$	$014D^{21}_{06}01$	$022D^{31}_{19}01$	$021D^{21}_{06}01$	$024D^{31}_{19}01$
061	$020D^{31}_{07}01$	$156B^{2}_{21}02$	$053D^{30}_{09}01$	$024D^{20}_{10}01$	$096B^{3}_{24}01$
066	$024D^{21}_{07}01$	$22/01.0A^{3}_{24}$	$053D^{31}_{11}01$	$096A^{3}_{24}01$	$102A^{3}_{24}03$
071	$22/07.0A^{3}_{24}$	$096C^{3}_{24}01$	$20/16.0A^{3}_{24}$	$090D^{21}_{11}04$	root
076	$075D^{21}_{05}01$	$062D^{20}_{12}03$	$138D^{21}_{12}03$	$138C^{3}_{22}03$	$190C^{3}_{22}04$
081	$043D^{23}_{04}03$	$044D^{23}_{21}02$	$021D^{21}_{10}01$	$065D^{30}_{12}02$	$024D^{30}_{05}01$
086	$055D^{30}_{06}02$	$022D^{01}_{22}01$	$08/01.0A^{3}_{23}$	$071C^{3}_{23}01$	$062C^{3}_{23}03$
091	$063C^{2}_{23}02$	$073C^{3}_{23}01$	$151B^{2}_{22}02$	$102D^{30}_{01}03$	$189C^{1}_{23}03$
096	$22/02.0A^{0}_{07}$	$089D^{31}_{12}02$	$016D^{21}_{16}02$	$102C^{1}_{14}03$	$011D^{30}_{14}02$
101	$089C^{0}_{19}02$	$156A^{2}_{21}02$	$064B^{0}_{18}02$	$187B^{2}_{22}03$	$054A^{0}_{06}03$
106	$099D^{21}_{16}04$	$193C^{0}_{16}02$	root	$108A^{1}_{01}01$	$051D^{30}_{14}03$
111	$047D^{20}_{05}01$	$056D^{31}_{05}03$	$126D^{31}_{05}03$	$014D^{30}_{16}01$	$165D^{32}_{21}03$
116	$147E^{13}_{20}02$	$033D^{30}_{01}01$	$20/10.0A^{1}_{07}$	$146D^{31}_{04}02$	$191D^{23}_{02}03$
121	$115D^{23}_{22}04$	$134D^{23}_{03}02$	$194D^{23}_{05}02$	$18/02.0A^{3}_{19}$	$20/09.0A^{3}_{19}$
126	$128E^{03}_{21}02$	$22/30.0A^{3}_{19}$	$071B^{1}_{07}01$	$20/11.0A^{3}_{19}$	$129B^{0}_{15}01$
131	$053D^{31}_{14}01$	$118B^{0}_{05}01$	$032D^{20}_{12}01$	$033D^{20}_{12}01$	$118E^{10}_{22}01$

Table 5E$_1$: First part of compressed tabular form of $\mathcal{F}^0_{24}$

136	$217A^{3}_{18}03$	$217A^{2}_{24}03$	$089B^{1}_{08}02$	$151C^{1}_{08}02$	$071D^{30}_{10}01$
141	$033E^{30}_{04}01$	$014D^{30}_{01}01$	$123D^{01}_{19}03$	$113E^{01}_{17}04$	$227C^{0}_{04}02$
146	$033D^{31}_{12}01$	$033D^{21}_{10}01$	$036D^{21}_{10}03$	$021D^{30}_{06}01$	$193B^{2}_{23}02$
151	$053D^{20}_{06}01$	$157D^{20}_{05}02$	$146E^{32}_{16}02$	root	$154D^{20}_{06}01$
156	$010D^{20}_{07}01$	$047D^{20}_{05}01$	$096D^{20}_{06}01$	$183A^{2}_{24}04$	$022D^{21}_{10}01$
161	$024A^{2}_{18}01$	$014D^{20}_{06}01$	$117D^{01}_{23}02$	$057D^{21}_{10}02$	$133D^{23}_{20}02$
166	$057D^{30}_{13}02$	$161D^{23}_{15}02$	$186D^{31}_{02}02$	$12/01.1A^{3}_{18}$	$227A^{1}_{16}02$
171	$16/01.0A^{3}_{18}$	$12/01.1A^{3}_{18}$	$151B^{3}_{18}02$	$237C^{3}_{24}03$	$12/01.0A^{3}_{18}$
176	$073D^{21}_{06}01$	$237C^{3}_{18}03$	$129B^{1}_{03}01$	$22/17.0A^{3}_{18}$	$181D^{31}_{10}01$
181	$22/16.0A^{3}_{18}$	$179D^{21}_{07}01$	$258D^{03}_{20}03$	$178A^{2}_{21}02$	$053D^{31}_{05}01$
186	$053F^{3}_{12}01$	$092D^{21}_{07}02$	$20/13.0A^{3}_{18}$	$151A^{3}_{23}02$	$222C^{2}_{20}03$
191	$060D^{21}_{07}02$	$040D^{21}_{10}01$	$053D^{30}_{03}01$	$040D^{30}_{16}01$	$138A^{3}_{18}03$
196	$222C^{3}_{23}03$	$040D^{20}_{07}01$	$250A18^{3}03$	$020D^{20}_{07}01$	$149D^{23}_{20}02$
201	$146D^{30}_{13}02$	$117D^{01}_{18}02$	$117D^{21}_{04}02$	$241C^{0}_{04}03$	$092C^{0}_{03}02$
206	$064D^{31}_{03}02$	$135A^{0}_{04}02$	$019A^{2}_{21}01$	$100D^{30}_{12}03$	$035D^{21}_{18}02$
211	$065D^{30}_{17}02$	$22/026.0A^{3}_{19}$	$137A^{2}_{24}04$	$131A^{2}_{17}02$	$171A^{2}_{24}01$
216	$139C^{3}_{24}03$	$249A^{0}_{03}02$	$170A^{3}_{19}03$	$246C^{1}_{08}03$	$208A^{2}_{17}02$
221	$217A^{0}_{03}03$	$156A^{2}_{17}02$	$169D^{20}_{09}01$	$138B^{3}_{19}03$	$241C^{1}_{09}03$
226	$124C^{2}_{19}01$	$118C^{2}_{17}01$	$019D^{30}_{11}01$	$075D^{31}_{08}01$	$129A^{3}_{24}01$
231	$192C^{3}_{24}02$	$089D^{31}_{06}02$	$092D^{31}_{13}02$	$088D^{30}_{14}01$	$196C^{3}_{24}04$
236	$186A^{2}_{17}02$	$128A^{0}_{09}02$	$190C^{3}_{24}04$	$150C^{3}_{24}03$	$226C^{0}_{03}02$
241	$192C^{3}_{23}02$	$197A^{2}_{17}02$	$099D^{21}_{07}04$	$185A^{3}_{23}02$	$205D^{30}_{14}03$
246	$128B^{3}_{23}02$	$208A^{2}_{21}02$	$227A^{0}_{03}02$	$075A^{3}_{23}01$	$208A^{2}_{21}02$
251	$156D^{21}_{22}02$	$059D^{30}_{14}02$	$161D^{21}_{22}02$	$087D^{31}_{07}02$	$057D^{21}_{15}02$
256	$185D^{21}_{04}02$	$146D^{21}_{04}02$	$128D^{30}_{18}02$	$151D^{21}_{09}02$	$114D^{30}_{14}02$
261	$146D^{30}_{20}02$	$261D^{31}_{23}03$			

Table 5E$_2$: Final part of compressed tabular form of $\mathcal{F}^{0}_{24}$

As an example of the use of the above tables we can get the u^0-tree of *all* the 3-gems up to 24 vertices inducing $L_{5,2}\#L_{2,1}$. Indeed, for this 3-manifold and at the level considered, the u^0-class coincides with the homeomorphism class.

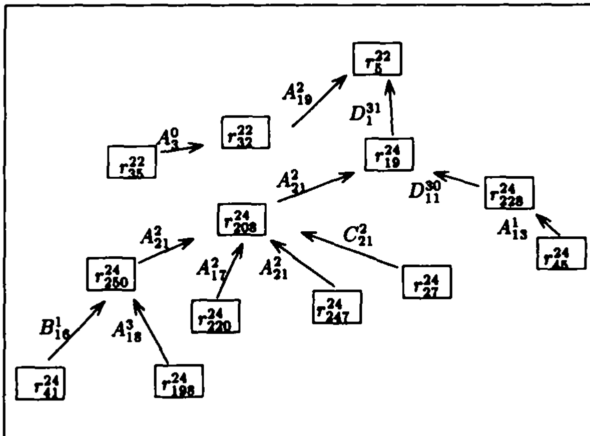


Fig. 157B: u^0-tree of all the rigid bipartite 3-gems up to 24 vertices inducing $L_{5,2}\#L_{2,1}$

5.2.2 Tables of the ts_{ρ_*}-Function

Below we present tables of the ts_ρ-function for each gem in $\mathcal{RBG}_n$, $n \leq 24$. The rigid bipartite 3-gems G up to 14 vertices satisfy $ts_\rho(G,0) = (G,0)$. Note that the value of the ts_{ρ_*}-function on (G,n) is simply the root of the ts_{ρ_*}-tree in the ts_{ρ_*}-forest which contains G. We display the values in a tabular form. The second argument of the function is implicitly considered to be 0. One can see, from Table 6E, for instance, that r^{24}_{20}, r^{24}_{61}, r^{24}_{199} all have as image, 02/001.0. This is the 3-gem with 2 vertices, s^3. The suffix .0 means that the handle number is zero. Thus, those three gems induce the 3-sphere. In the same way r^{24}_{129}, r^{24}_{130}, r^{24}_{178}, r^{24}_{184}, r^{24}_{230}, have as representative r^8_1 with handle number 1. Since r^8_1 induces RP^3, the five gems induce the connected sum of RP^3 with $S^1 \times S^2$.

01	16/01.0	16/02.0	16/02.0

Table 6A: Values of ts_{ρ_*} on $\mathcal{RBG}_{16}$

01	18/01.0	14/01.0	16/01.0	18/04.0

Table 6B: Values of ts_{ρ_*} on $\mathcal{RBG}_{18}$

01	20/01.0	20/02.0	20/02.0	20/04.0	20/05.0	20/01.0	18/01.0
08	18/04.0	02/01.1	20/10.0	08/01.1	16/02.0	16/02.0	16/02.0
15	20/10.0	18/01.0	20/01.0	20/05.0	20/02.0	20/05.0	20/02.0
22	16/01.0	20/02.0					

Table 6C: Values of $ts_{\rho.}$ on $\mathcal{RBG}_{20}$

01	22/01.0	22/02.0	22/03.0	18/04.0	22/05.0	20/02.0	20/01.0
08	20/01.0	20/04.0	20/04.0	20/05.0	20/05.0	20/05.0	18/04.0
15	18/04.0	22/16.0	22/17.0	22/16.0	20/05.0	16/02.0	18/04.0
22	20/02.0	20/05.0	18/04.0	20/02.0	20/01.0	18/01.0	20/05.0
29	20/01.0	20/05.0	20/01.0	22/05.0	22/03.0	22/03.0	22/05.0
36	22/03.0	20/04.0	20/01.0	16/02.0	22/16.0	22/17.0	18/04.0
43	20/05.0	22/03.0					

Table 6D: Values of $ts_{\rho.}$ on $\mathcal{RBG}_{22}$

01	24/001.0	24/002.0	24/003.0	24/004.0	24/005.0	24/006.0
07	24/007.0	22/016.0	22/017.0	20/002.0	22/001.0	22/002.0
13	24/013.0	24/014.0	22/001.0	22/002.0	12/001.0	16/002.0
19	22/005.0	02/001.0	24/021.0	24/022.0	24/003.0	20/004.0
25	24/021.0	24/022.0	22/005.0	24/028.0	20/004.0	20/004.0
31	22/002.0	24/032.0	24/033.0	24/034.0	24/013.0	24/014.0
37	24/032.0	24/022.0	24/003.0	18/004.0	22/005.0	24/014.0
43	24/033.0	24/034.0	22/005.0	20/004.0	22/003.0	24/022.0
49	24/033.0	22/003.0	24/014.0	22/002.0	20/005.0	18/004.0
55	24/003.0	24/022.0	24/014.0	24/022.0	24/021.0	20/004.0
61	02/001.0	20/002.0	20/005.0	20/004.0	22/002.0	20/004.0
67	22/001.0	20/005.0	22/002.0	20/002.0	20/001.0	22/002.0
73	18/001.0	20/002.0	24/075.0	24/075.0	20/002.0	20/001.0

Table 6E$_1$: First part of $ts_{\rho.}$ on $\mathcal{RBG}_{24}$

79	20/001.0	20/002.0	24/033.0	24/034.0	24/021.0	22/002.0
85	20/004.0	24/003.0	24/022.0	08/001.0	20/001.0	20/002.0
91	20/005.0	18/001.0	20/005.0	20/002.0	20/005.0	22/002.0
97	20/001.0	22/002.0	20/002.0	22/001.0	20/001.0	20/002.0
103	20/004.0	18/001.0	18/004.0	20/002.0	20/005.0	24/108.0
109	24/108.0	24/014.0	22/003.0	24/022.0	20/001.0	24/014.0
115	24/032.0	24/033.0	24/033.0	20/010.0	24/033.0	20/004.0
121	24/032.0	24/033.0	18/004.0	14/001.0	02/001.1	20/001.0
127	20/005.0	20/001.0	08/001.1	08/001.1	20/005.0	20/010.0
133	24/032.0	24/033.0	20/010.0	24/075.0	24/075.0	20/001.0
139	20/005.0	20/001.0	24/033.0	24/014.0	18/004.0	20/001.0
145	20/010.0	24/033.0	24/033.0	24/014.0	24/021.0	20/005.0
151	20/005.0	22/003.0	24/033.0	24/154.0	24/154.0	20/002.0
157	22/003.0	22/002.0	20/001.0	24/022.0	20/004.0	24/014.0
163	24/033.0	24/014.0	24/032.0	24/014.0	20/004.0	20/005.0
169	12/001.1	20/010.0	16/001.0	12/001.1	20/005.0	20/001.0
175	12/001.0	18/001.0	20/001.0	08/001.1	22/017.0	22/016.0
181	22/016.0	22/017.0	20/001.0	08/001.1	20/005.0	20/005.0
187	18/001.0	16/002.0	20/005.0	20/002.0	20/004.0	18/004.0
193	20/005.0	18/004.0	20/001.0	20/002.0	18/004.0	22/005.0
199	02/001.0	24/021.0	24/033.0	24/033.0	24/033.0	18/004.0
205	18/001.0	20/004.0	20/010.0	22/005.0	22/001.0	24/013.0
211	22/002.0	20/001.0	24/075.0	20/005.0	16/001.0	20/005.0
217	24/075.0	20/010.0	20/001.0	22/005.0	24/075.0	20/002.0
223	12/001.1	20/001.0	18/004.0	14/001.0	20/010.0	22/005.0
229	24/075.0	08/001.1	18/004.0	20/001.0	18/001.0	08/001.0
235	20/002.0	20/005.0	20/001.0	20/002.0	20/005.0	14/001.0
241	18/004.0	18/004.0	20/002.0	20/005.0	18/001.0	20/001.0
247	22/005.0	20/010.0	24/075.0	22/005.0	20/002.0	24/021.0
253	20/004.0	24/022.0	24/014.0	20/005.0	24/033.0	20/001.0
259	20/005.0	24/014.0	24/033.0	24/033.0		

Table 6E$_2$: Final part of ts_{ρ_*} on $\mathcal{RBG}_{24}$

Similar tables for $\mathcal{RBG}_{26}$ and $\mathcal{RBG}_{28}$ appear in the Appendix B, Section 8.2.

5.2.3 TS_ρ-Classes Ordered by Homology

In the following tables, we number and list the TS_ρ*-classes*. A TS_ρ-class is a partial u^0-class given by the TS_ρ-algorithm as follows. When TS_ρ builds a tree of Γ_G^{TS} (G a 3-gem of smallest code) in a breadth first search [Gol80] and it finds a non-rigid 3-gem, the set of nodes of the tree so far is considered a TS_ρ-class. In general it corresponds to a partial u^0-class. It coincides with a full u^0-class if all their 3-gems are u^0-essential. Note that the TS_ρ-classes have all its members with the same number of vertices.

The order in which we list the TS_ρ-classes is by the first homology group of the 3-manifold associated to the class in the following way: make a vector whose first entry is the Betti number, the second the number of torsion coefficients and the following these coefficients in increasing order. The classification is by lexicographic ordering of these vectors. The ties are broken by the codes of the representatives of each TS-class (which is taken as the gem with smaller code in the class). The homology groups associated to the TS_ρ-classes are given in the next subsection.

In the tables, each TS_ρ-class is preceded by a blank entry and has the prefix C before its number. A TS_ρ-class which is under and overlined is u_0-essential. Only these are relevant for the topological classification. If a TS_ρ-class is only underlined we can produce by TS-moves a 3-gem with ρ-pairs which is equivalent up to handles to a 3-gem with less vertices. After the entry heading, each TS_ρ-class has a number preceded by an "M". This number is the *mutability* of the TS_ρ-class, defined as its cardinality. The subsequent entries in a TS_ρ-class are the indices of the codes of its members. There is a single TS_ρ-class at the levels 2, 8, 12 and 14 vertices. Thus, we start our tabulations with 16 vertices. At this level there are two TS_ρ-classes, both u^0-essential.

$\overline{\underline{\text{C1}}}$	M2	02	03		$\overline{\underline{\text{C2}}}$	M1	01

Table 7A: TS_ρ-classes on $\mathcal{RBG}_{16}$

With 18 vertices there are four TS_ρ-classes, two of which are u^0-essential.

$\underline{\text{C1}}$	M1	03		$\overline{\underline{\text{C2}}}$	M1	04		$\overline{\underline{\text{C3}}}$	M1	01		$\underline{\text{C4}}$	M1	02

Table 7B: TS_ρ-classes on $\mathcal{RBG}_{18}$

With 20 vertices there are eleven TS_ρ-classes, five of which are u^0-essential.

C1	M1	04		C2	M3	12	13	14		C3	M5	02	03	19
21	23		C4	M1	22		C5	M1	08		C6	M3	05	18
20		C7	M3	01	06	17		C8	M2	07	16		C9	M2
10	15		C10	M1	09		C11	M1	11					

Table 7C: TS_ρ-classes on $\mathcal{RBG}_{20}$

With 22 vertices there are thirteen TS_ρ-classes, six of which are u^0-essential.

C1	M1	01		C2	M3	09	10	37		C3	M2	20	39	
C4	M3	06	22	25		C5	M6	04	14	15	21	24	42	
C6	M8	11	12	13	19	23	28	30	43		C7	M1	02	
C8	M6	07	08	26	29	31	38		C9	M3	05	32	35	
C10	M1	27		C11	M5	03	33	34	36	44		C12	M3	16
18	40		C13	M2	17	41								

Table 7D: TS_ρ-classes on $\mathcal{RBG}_{22}$

With 24 vertices there are forty TS_ρ-classes, eighteen of which are u^0-essential.

C01	M01	002		C02	M03	020	061	199		C03	M01
028		C04	M02	088	234		C05	M05	011	015	067
100	209		C06	M02	017	175		C07	M02	018	188
	C08	M15	024	029	030	046	060	064	066	085	103
120	161	167	191	206	253		C09	M19	010	062	070
074	077	080	090	094	099	102	106	156	190	196	222
235	238	243	251		C10	M02	171	215		C11	M20
033	043	049	081	116	117	119	122	134	141	146	147
153	163	201	202	203	257	261	262		C12	M13	040
054	105	123	143	192	194	197	204	225	231	241	242
	C13	M25	053	063	068	091	093	095	107	127	131
139	150	151	168	173	185	186	189	193	214	216	236
239	244	256	259		C14	M01	004		C15	M12	012
016	031	052	065	069	072	084	096	098	158	211	
C16	M24	071	078	079	089	097	101	113	126	128	138
140	144	159	174	177	183	195	212	219	224	232	237
246	258		C17	M14	014	036	042	051	057	110	114
142	148	162	164	166	255	260		C18	M10	019	027
041	045	198	208	220	228	247	250		C19	M07	021
025	059	083	149	200	252		C20	M10	022	026	038
048	056	058	087	112	160	254		C21	M06	032	037
115	121	133	165		C22	M03	034	044	082		C23
M05	003	023	039	055	086		C24	M08	073	092	104
176	187	205	233	245		C25	M03	124	226	240	
C26	M02	154	155		C27	M05	047	050	111	152	157
	C28	M03	013	035	210		C29	M09	075	076	136
137	213	217	221	229	249		C30	M03	008	180	181
	C31	M03	009	179	182		C32	M01	005		C33
M02	108	109		C34	M09	118	132	135	145	170	207
218	227	248		C35	M01	125		C36	M05	129	130
178	184	230		C37	M01	007		C38	M03	169	172
223		C39	M01	006		C40	M01	001			

Table 7E: TS_ρ-classes on $\mathcal{RBG}_{24}$

Similar tables for $\mathcal{RBG}_{26}$ and $\mathcal{RBG}_{28}$ appear in the Appendix C. There are 50 TS_ρ-classes on $\mathcal{RBG}_{26}$, 20 of which u^0-essential. There are 109 TS_ρ-classes on $\mathcal{RBG}_{28}$, 50 of which u^0-essential.

5.2.4 Tabulating the Homology Groups

The tables are indexed by the TS_ρ-classes in the order given by the previous subsection. Up to 14 vertices we have S^3, RP^3, $L_{3,1}$ and $RP^3 \# RP^3$. The homology groups are respectively Z_1, Z_2, Z^3 and $Z_2 + Z_2$. For the next tables, the number in parenthesis means the Betti number; it is followed by the torsion coefficients in a multiplicative notation.

01	$(0)4^1$	$(0)5^1$

Table 8A: Homology of TS_ρ-classes in $\mathcal{RBG}_{16}$

01	$(0)5^1$	$(0)6^1$	$(0)2^2$	$(0)2^2$

Table 8B: Homology of TS_ρ-classes in $\mathcal{RBG}_{18}$

01	$(0)4^1$	$(0)4^1$	$(0)5^1$	$(0)5^1$	$(0)6^1$	$(0)7^1$	$(0)8^1$	$(0)2^2$
09	$(0)2^3$	(1)	$(1)2^1$					

Table 8C: Homology of TS_ρ-classes in $\mathcal{RBG}_{20}$

01	$(0)3^1$	$(0)4^1$	$(0)4^1$	$(0)5^1$	$(0)6^1$	$(0)7^1$	$(0)8^1$	$(0)8^1$
09	$(0)10^1$	$(0)2^2$	$(0)2^1 4^1$	$(0)3^2$	$(0)3^2$			

Table 8D: Homology of TS_ρ-classes in $\mathcal{RBG}_{22}$

01	(0)	(0)	$(0)2^1$	$(0)2^1$	$(0)3^1$	$(0)3^1$	$(0)4^1$	$(0)4^1$
09	$(0)5^1$	$(0)5^1$	$(0)6^1$	$(0)6^1$	$(0)7^1$	$(0)8^1$	$(0)8^1$	$(0)8^1$
17	$(0)9^1$	$(0)10^1$	$(0)10^1$	$(0)11^1$	$(0)12^1$	$(0)12^1$	$(0)13^1$	$(0)2^2$
25	$(0)2^2$	$(0)2^2$	$(0)2^1 4^1$	$(0)2^1 6^1$	$(0)2^1 6^1$	$(0)3^2$	$(0)3^2$	$(0)4^2$
33	$(0)2^3$	$(0)2^3$	(1)	$(1)2^1$	$(1)3^1$	$(1)3^1$	$(1)2^2$	(3)

Table 8E: Homology of TS_ρ-classes in $\mathcal{RBG}_{24}$

01	(0)	(0)	$(0)2^1$	$(0)2^1$	$(0)3^1$	$(0)3^1$	$(0)4^1$	$(0)4^1$
09	$(0)5^1$	$(0)5^1$	$(0)6^1$	$(0)6^1$	$(0)7^1$	$(0)7^1$	$(0)8^1$	$(0)8^1$
17	$(0)8^1$	$(0)9^1$	$(0)9^1$	$(0)10^1$	$(0)10^1$	$(0)10^1$	$(0)11^1$	$(0)12^1$
25	$(0)12^1$	$(0)12^1$	$(0)12^1$	$(0)13^1$	$(0)14^1$	$(0)15^1$	$(0)15^1$	$(0)15^1$
33	$(0)2^2$	$(0)2^2$	$(0)2^2$	$(0)2^14^1$	$(0)2^14^1$	$(0)2^16^1$	$(0)2^16^1$	$(0)2^16^1$
41	$(0)2^18^1$	$(0)3^2$	$(0)3^2$	$(0)4^2$	$(0)4^2$	$(0)4^2$	$(0)2^3$	$(0)2^3$
49	$(0)2^4$	(1)	(1)	$(1)2^1$	$(1)2^1$	$(1)3^1$	$(1)3^1$	$(1)4^1$
57	$(1)2^2$	$(1)2^2$	(2)	(2)	(3)			

Table 8F: Homology of TS_ρ-classes in $\mathcal{RBG}_{26}$

001	(0)	(0)	(0)	$(0)2^1$	$(0)2^1$	$(0)2^1$	$(0)3^1$
008	$(0)3^1$	$(0)3^1$	$(0)3^1$	$(0)4^1$	$(0)4^1$	$(0)4^1$	$(0)5^1$
015	$(0)5^1$	$(0)5^1$	$(0)6^1$	$(0)6^1$	$(0)6^1$	$(0)7^1$	$(0)7^1$
022	$(0)7^1$	$(0)8^1$	$(0)8^1$	$(0)8^1$	$(0)8^1$	$(0)9^1$	$(0)9^1$
029	$(0)10^1$	$(0)10^1$	$(0)10^1$	$(0)11^1$	$(0)11^1$	$(0)12^1$	$(0)12^1$
036	$(0)12^1$	$(0)12^1$	$(0)12^1$	$(0)13^1$	$(0)13^1$	$(0)14^1$	$(0)14^1$
043	$(0)15^1$	$(0)15^1$	$(0)15^1$	$(0)16^1$	$(0)16^1$	$(0)16^1$	$(0)17^1$
050	$(0)17^1$	$(0)18^1$	$(0)19^1$	$(0)19^1$	$(0)20^1$	$(0)21^1$	$(0)21^1$
057	$(0)21^1$	$(0)24^1$	$(0)2^2$	$(0)2^2$	$(0)2^2$	$(0)2^2$	$(0)2^14^1$
064	$(0)2^14^1$	$(0)2^16^1$	$(0)2^16^1$	$(0)2^16^1$	$(0)2^16^1$	$(0)2^16^1$	$(0)2^16^1$
071	$(0)2^18^1$	$(0)2^18^1$	$(0)2^110^1$	$(0)2^110^1$	$(0)2^110^1$	$(0)3^2$	$(0)3^2$
078	$(0)3^2$	$(0)3^16^1$	$(0)3^16^1$	$(0)3^16^1$	$(0)3^16^1$	$(0)3^16^1$	$(0)3^16^1$
085	$(0)4^2$	$(0)4^2$	$(0)4^2$	$(0)4^2$	$(0)2^3$	$(0)2^3$	$(0)2^24^1$
092	$(0)2^24^1$	(1)	(1)	(1)	$(1)2^1$	$(1)2^1$	$(1)3^1$
099	$(1)3^1$	$(1)4^1$	$(1)4^1$	$(1)5^1$	$(1)5^1$	$(1)2^2$	$(1)2^2$
106	$(1)2^2$	(2)	$(2)2^1$	(3)			

Table 8G: Homology of TS_ρ-classes in $\mathcal{RBG}_{28}$

5.3 The Fundamental Groups of 3-Manifolds $|G|$, $\kappa(G) \leq 13$

This section appears originally in the article [LS95] and the group computations are due to Said Sidki.

5.3.1 An Overview of the Tables

In the next tables we give simplified 0-presentations for the fundamental groups of the closed orientable 3-manifolds M^3 of complexity up to 13, which are neither cyclic nor nontrivial free products. All the M^3's which have cyclic fundamental groups were proven to be lens spaces, by the u^1-classification. All the M^3's whose fundamental groups are nontrivial free products were found to have explicit non-trivial connected sum decompositions at the level of 3-gems. Thus we omit these classes of spaces from our list. With these omissions the space of smallest complexity is the quaternionic space S^3/Q_8, of complexity 8.

The manifolds are indexed by the u^0-classes of rigid 3-gems [LD89] which induce them. An entry like $C_n(m)$ refers to the m-th u^0-class with n vertices. We only consider essential u^0-classes. They are ordered by their homology groups in basically a lexicographic ordering [LD89].

The entry r^n_m in the column headed by "Rep." refers to the m-th rigid 3-gem with n vertices in the catalogue. The listed 3-gems are the representatives of their u^0-classes. This means that they are the ones with minimal code. (The codes of are ordered lexicographically.)

The presentations come from the 0-presentation given in Subsection 2.8.3. The simplifications performed are also explained in this subsection and we arrive at the presentations displayed.

Finally, we describe the entries under "*Mnemonic for Groups*", explaining the notation used for the various families of groups which appear.

1. C_n is the cyclic group of order n.

2. $H \rtimes_t K$ is the semi-direct product of the group H by the group K, with K inducing an automorphism group of order t on H. When $H = C_n$, $K = C_{2m}$ and $t = 2$ we write the group as $C_n \rtimes_i C_{2m}$.

3. $Q_{2^{m+1}} = \langle a, b \mid a^{2^m} = b^2, b^{-1}ab = a^{-1} \rangle$ is the quaternion group of order 2^{m+1}.

4. Let $m, n, k > 0$.

$(m,n,k) = \langle a,b,c \mid a^m = b^n = c^k = e\rangle$ is the orientation preserving transformations of a triangle with angles $\pi/m, \pi/n, \pi/q$ in the Euclidean, spherical or hyperbolic plane P, depending upon $1/m + 1/n + 1/k$ being equal to 1, greater than 1 or less than 1.

$(m,n,k)^* = \langle a,b,c \mid a^m = b^n = c^k\rangle$.

$\langle m,n,k\rangle = \langle a,b,c \mid a^m = b^n = c^k = abc\rangle$ is the polyhedral group which is known to be the fundamental group of the 3-manifold $\mathcal{L}/(m,n,k)$, where $\mathcal{L}$ is the connected Lie group of orientation preserving isometries of the plane P [Mil75]. We note the coincidence $(m,n,2)^* = \langle m,n,2\rangle$.

$\langle m,n,k\rangle t$ indicates that the group has a normal subgroup isomorphic to $\langle m,n,k\rangle$ of index t.

5. Let $g, r \geq 1$.

$$S_g^r = \langle a_1,\ldots,a_g,b_1,\ldots b_g,c \mid [c,a_i] = e = [c,b_i](1 \leq i \leq g), c^r = \prod_{i=1}^{g}[a_i,b_i]\rangle$$

which is the fundamental group of a Seifert manifold over a surface of genus g [Orl72].

6. The euclid$_i$ $(0 \leq i \leq 5)$ are the fundamental groups of the six Euclidean closed connected orientable 3-manifolds. Four of them appear in $\mathcal{RBG}_{24}$ and the other two in $\mathcal{RBG}_{26}$.

7. $Z\ M$, where M is a 2×2 matrix over the integers, indicates the semi-direct product group

$$(Z \times Z) \rtimes_M Z = H \rtimes \langle a\rangle$$

with a acting on the canonical basis of H as the matrix M.

8. $E_2(i,j) = \langle a,b \mid (a^2)^b = b^{-2i}a^{-2}, (b^2)^a = a^{-2j}b^{-2}\rangle$.

9. $[H_1,\ldots,H_s; G^{(s)}]$ denotes factors of the derived series of the group G, where $G/G' \cong H_1, \ldots G^{(s-1)}/G^{(s)} \cong H_s$. Since H_i is abelian, we denote it by its numerical invariants.

5.3.2 Tables of Fundamental Groups

u^0-class	Rep.	Presentation	Mnemonic for Group
$C_{18}(3)$	r_1^{18}	$\langle a, b \mid b^2a^2, b^{-1}aba \rangle$	Q_8 $\langle 2, 2, 2 \rangle$
$C_{20}(1)$	r_4^{20}	$\langle a, b \mid b^{-1}aba, a^3b^2 \rangle$	$\langle 3, 2, 2 \rangle$ $C_3 \rtimes_i C_4$
$C_{22}(1)$	r_1^{22}	$\langle a, b \mid a^{-1}ba^2b, ab^2ab^{-1} \rangle$	$\langle 3, 3, 2 \rangle$
$C_{22}(7)$	r_2^{22}	$\langle a, b \mid ab^{-1}ab, a^{-1}baba^{-1}b^2 \rangle$	$C_3 \rtimes_i C_8$ $\langle 3, 2, -2 \rangle$
$C_{24}(1)$	r_2^{24}	$\langle a, b \mid ba^{-1}bab^{-1}a, b^{-2}a^{-1}baba^{-1} \rangle$	$\langle 5, 3, 2 \rangle$
$C_{24}(3)$	r_{28}^{24}	$\langle a, b \mid a^{-2}b^{-1}ab^{-1}, ab^{-1}ab^3 \rangle$	$\langle 4, 3, 2 \rangle$
$C_{24}(14)$	r_4^{24}	$\langle a, b \mid b^{-1}a^{-1}ba^{-1}, ba^{-1}ba^2b^2a^2 \rangle$	$C_5 \rtimes_i C_8$
$C_{24}(26)$	r_{154}^{24}	$\langle a, b \mid abab^{-1}, a^{-2}ba^2b \rangle$	Q_{16}
$C_{24}(28)$	r_{13}^{24}	$\langle a, b \mid b^2a^2, a^5ba^3b \rangle$	$C_3 \times Q_8$
$C_{24}(32)$	r_5^{24}	$\langle a, b \mid b^2ab^2a^{-1}, a^2b^{-1}a^2b \rangle$	euclid$_1$ $E_2(1, 1)$
$C_{24}(37)$	r_7^{24}	$\langle a, b \mid ba^{-1}b^2a^{-1}bab^{-1}a, b^{-1}ab^{-1}a^{-2}b^{-1}a \rangle$	euclid$_2$ $\langle 3, 3, 3 \rangle$
$C_{24}(39)$	r_6^{24}	$\langle a, b, c \mid baba^{-1}, abc^2ab, bcbc^{-1} \rangle$	euclid$_3$ $Z^{-1\ 0}_{0\ -1}$
$C_{24}(40)$	r_1^{24}	$\langle a, b, c \mid bc^{-1}b^{-1}c, b^{-1}a^{-1}c^{-1}abc, aba^{-1}b^{-1} \rangle$	$Z \times Z \times Z$ euclid$_0$
$C_{26}(13)$	r_{10}^{26}	$\langle a, b \mid ba^3ba^{-2}, a^{-2}baba^{-2}b^{-1} \rangle$	$\langle 5, 3, 2 \rangle \times C_7$
$C_{26}(18)$	r_8^{26}	$\langle a, b \mid a^2ba^{-1}ba^2b, a^2bab^{-1}ab \rangle$	$Q_8 \rtimes_3 C_9$ $\langle 3, 2, -3 \rangle$
$C_{26}(24)$	r_6^{26}	$\langle a, b \mid b^2a^2, abab^{-1}aba^3bab^{-1} \rangle$	$C_5 \rtimes_i C_{12}$
$C_{26}(30)$	r_4^{26}	$\langle a, b \mid b^3a^2, a^3ba^3bab \rangle$	$Q_8 \rtimes_3 C_{15}$ $(3, 3, 4)^*$
$C_{26}(32)$	r_{696}^{26}	$\langle a, b \mid bab^2a^3b^2a, b^2ab^{-1}ab^2a^4 \rangle$	$Q_8 \rtimes_3 C_{15}$
$C_{26}(39)$	r_{65}^{26}	$\langle a, b \mid ababab^{-1}ab, a^{-2}b^2 \rangle$	$C_3 \times Q_{16}$ $(2, 2, 4)^*$
$C_{26}(44)$	r_3^{26}	$\langle a, b \mid ba^2b^{-1}a^2, b^2ab^2a^{-1} \rangle$	euclid$_1$
$C_{26}(45)$	r_5^{26}	$\langle a, b \mid b^{-1}a^2ba^2, b^2aba^{-2}ba \rangle$	$[4^2, S_1^4]$ $E_2(0, 2)$
$C_{26}(50)$	r_{31}^{26}	$\langle a, b \mid a^{-2}bab^{-1}ab, abab^{-1}a^{-2}b^{-1} \rangle$	euclid$_4$ $\langle 6, 3, 2 \rangle$
$C_{26}(52)$	r_{11}^{26}	$\langle a, b \mid ab^{-1}ab^3, a^3ba^{-1}b \rangle$	euclid$_5$ $\langle 4, 4, 2 \rangle$
$C_{26}(56)$	r_{13}^{26}	$\langle a, b \mid a^2b^3a^2b, ab^{-1}a^{-2}b^{-1}ab^2 \rangle$	$E_2(2, 1)$
$C_{26}(59)$	r_{14}^{26}	$\langle a, b \mid ab^2ab^{-1}a^{-2}b^{-1}, ab^{-1}a^{-1}baba^{-1}b^{-1} \rangle$	$Z^{1\ 0}_{1\ 1}$

Table 9A: Presentations for $\pi_1(G)$, $|G|$ of complexity ≤ 12

u^0-class	Rep.	Presentation	Mnemonic for Group
$C_{28}(3)$	r^{28}_{172}	$\langle a, b \mid a^{-1}bab^{-1}a^{-2}b^{-1}ab,$ $a^2b^{-1}a^{-1}bababa^{-1}b^{-1}\rangle$	$\langle 7, 3, 2\rangle$
$C_{28}(6)$	r^{28}_{202}	$\langle a, b \mid a^{-1}bab^{-1}a^{-2}b^{-1}ab,$ $aba^{-1}b^{-1}ab^2ab^{-1}a^{-1}b\rangle$	$\langle 5, 5, 2\rangle 2$
$C_{28}(7)$	r^{28}_{2}	$\langle a, b \mid b^2ab^{-1}ab^{-1}a,$ $ab^{-1}aba^{-1}bab^{-1}a^2b^{-1}aba^{-1}b\rangle$	$[3, 5^2, S^5_6]$
$C_{28}(8)$	r^{28}_{9}	$\langle a, b \mid b^{-1}ab^2ab^2a, a^3ba^{-1}b^{-2}a^{-1}b\rangle$	$\langle 4, 3, 3\rangle$ $[3, 4^2, S^4_3]$
$C_{28}(13)$	r^{28}_{2418}	$\langle a, b \mid ab^{-1}ab^4, ab^3ab^{-2}\rangle$	$C_5 \rtimes_i C_4$
$C_{28}(15)$	r^{28}_{203}	$\langle a, b \mid b^2a^{-1}b^2ab^{-2}a, b^{-1}a^{-1}b^2ab^2a^{-1}\rangle$	$\langle 7, 3, 2\rangle 5$
$C_{28}(26)$	r^{28}_{2314}	$\langle a, b \mid a^{-2}ba^5b, a^3ba^{-4}b\rangle$	$C_7 \rtimes_i C_8$
$C_{28}(38)$	r^{28}_{402}	$\langle a, b \mid ba^{-1}ba, a^2b^2a^2b^2a^2b^3\rangle$	$C_7 \rtimes_i C_{12}$
$C_{28}(47)$	r^{28}_{27}	$\langle a, b \mid b^{-5}ab^3a^3b^3a, b^{-2}ab^3a^2b^3a\rangle$	$C_3 \rtimes_i C_{16}$
$C_{28}(54)$	r^{28}_{29}	$\langle a, b \mid b^2ababab^2a, aba^2bab^{-1}\rangle$	$C_3 \rtimes_i C_{20}$
$C_{28}(58)$	r^{28}_{1}	$\langle a, b \mid a^3ba^3b^{-1}, a^{-2}ba^{-1}ba^{-2}ba^{-4}b\rangle$	$[24, S^1_2]$
$C_{28}(59)$	r^{28}_{3}	$\langle a, b, c \mid a^2b^3, abc^{-1}abc, c^2b^3\rangle$	$[2^2, 3^2.6, S^9_{10}]$
$C_{28}(66)$	r^{28}_{56}	$\langle a, b \mid bab^{-1}ab^{-1}abab^{-1}aba^{-1}bab^{-1}a, b^{-2}a^2\rangle$	$C_3 \times Q_{32}$
$C_{28}(73)$	r^{28}_{7}	$\langle a, b \mid a^{-1}b^5ab^5, b^4a^{-1}b^6a^{-1}\rangle$	$C_5 \times Q_8$
$C_{28}(74)$	r^{28}_{59}	$\langle a, b \mid b^3a^{-1}ba^{-1}, ab^2ab^4\rangle$	$C_5 \times Q_8$
$C_{28}(76)$	r^{28}_{34}	$\langle a, b \mid a^2ba^{-1}ba^{-1}b, b^{-1}aba^{-1}bab^{-1}a^2\rangle$	$[3^2, S^3_1]$
$C_{28}(86)$	r^{28}_{19}	$\langle a, b \mid a^{-1}b^2ab^2, ab^3a^2b^3ab^{-2}\rangle$	$(3,3,3)^*$ $[4^2, S^8_1]$ $E_2(4,0)$
$C_{28}(87)$	r^{28}_{25}	$\langle a, b \mid a^{-2}bab^2ab, a^2b^{-1}a^2b\rangle$	$[4^2, S^4_1]$
$C_{28}(91)$	r^{28}_{6}	$\langle a, b, c \mid ab^{-1}ab, b^{-1}a^{-1}cabc, bcb^3c\rangle$	$[2^2.4, S^4_1]$
$C_{28}(93)$	r^{28}_{5}	$\langle a, b \mid b^2ab^{-1}a^{-3}b^{-1}a, a^2ba^{-1}b^{-1}ab^{-1}a^{-1}b\rangle$	$Z^{0\ 1}_{-1\ 3}$
$C_{28}(102)$	r^{28}_{10}	$\langle a, b \mid aba^{-2}bab^3, b^2aba^{-1}ba^{-1}ba\rangle$	$Z^{0\ 1}_{-1\,-3}$
$C_{28}(104)$	r^{28}_{42}	$\langle a, b, c \mid a^2bcbc^{-1}, c^{-1}b^{-1}abca, a^{-1}b^{-1}ab\rangle$	$Z^{-1\ 0}_{-2\,-1}$
$C_{28}(108)$	r^{28}_{280}	$\langle a, b, c \mid b^2c^{-1}abcb^{-1}a^{-1}, cbc^{-1}b^{-1}, abca^{-1}bc^{-1}\rangle$	$Z^{1\,-2}_{0\ 1}$

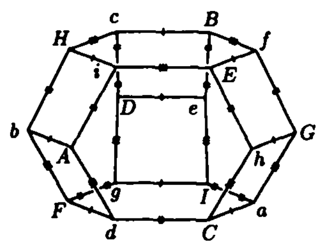
Table 9B: Presentations for $\pi_1(G)$, $|G|$ of complexity $= 13$

Next we provide a thorough identification for the fundamental groups of the u^0-essential classes listed above. This analysis of the fundamental groups is important to discover the u^0-classes which are either u^1-reducible or else only part of an attractor. There are four such cases up to 28 vertices (discounting cyclic groups and non-trivial free products). The members of these u^0-classes with smallest codes are: r^{24}_5 and r^{26}_3, r^{26}_4 and r^{26}_{696}, r^{26}_5 and r^{28}_{25}, r^{28}_7 and r^{28}_{59}.

5.3.3 Manifolds of Complexity ≤ 10

The computations presented in the remaining of this section appear in [LS95] and are due to S. Sidki.

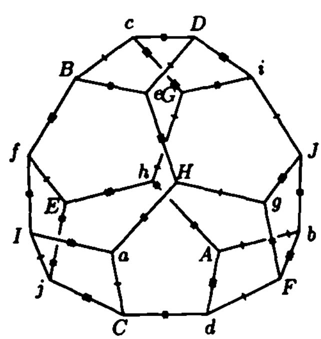
Homology: $Z_2 + Z_2$



$r_1^{18} \in C_{18}(3) \to \langle a, b | b^2a^2, b^{-1}aba \rangle$.
$r_1 : a^2 = b^{-2}, r_2 : b^{-1}ab = a^{-1}$;
a presentation for Q_8,
which is easily expressed as $\langle 2, 2, 2 \rangle$.
By GDV and TS_ρ the manifold is a quotient of S^3 by this group of order 8.

Fig. 158: $\pi_1(S^3/\langle 2, 2, 2\rangle)$, induced by its superattractor

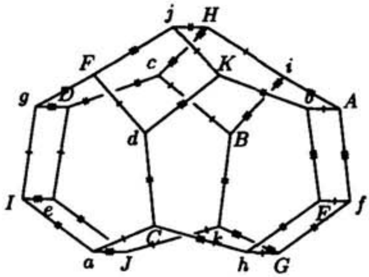
Homology: Z_4



$r_4^{20} \in C_{20}(1) \to \langle a, b | b^{-1}aba, a^3b^2 \rangle$.
$r_1 : b^{-1}ab = a^{-1}, r_2 : a^3 = b^{-2}$(central)
$\Rightarrow b^{-1}a^3b = a^{-3} = a^3 \Rightarrow a^6 = e = b^4$;
$a_1 := a^2 \Rightarrow b^{-1}a_1b = a_1^{-1}, a_1^3 = e = b^4$;
a presentation for $C_3 \rtimes_i C_4$. The original presentation easily transforms to $\langle 3, 2, 2 \rangle$.
By GDV and TS_ρ the manifold is a quotient of S^3 by this group of order 12.

Fig. 159: $\pi_1(S^3/\langle 3, 2, 2\rangle)$, induced by its superattractor

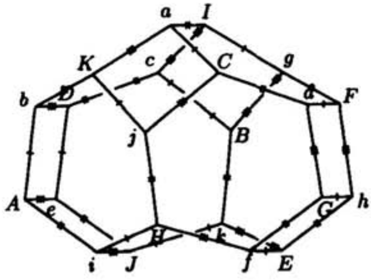
Homology: Z_3



$r_1^{22} \in C_{22}(1) \to \langle a, b | a^{-1}ba^2b, ab^2ab^{-1}\rangle$.
$r_1 : b^{-1}a^{-2}b^{-1}a$
$\Rightarrow a^{-2}b^{-1}ab^{-1} \Rightarrow a^{-3}(ab^{-1})^2$
$\Rightarrow a^3 = (ab^{-1})^2$
$r_2 : ab^2ab^{-1} \Rightarrow (ab^2)^2b^{-3} \Rightarrow$
$b^3 = (ab^2)^2$ (central) $\Rightarrow$
$b^3 = (ab^{-1}b^3)^2 = (ab^{-1})^2b^6 \Rightarrow$
$b^{-3} = (ab^{-1})^2$;
$b_1 := b^{-1} \Rightarrow a^3 = b_1^3 = (ab_1)^2$,
which is the definition of $\langle 3, 3, 2\rangle$.
By GDV and TS_ρ the manifold
is a quotient of S^3 by this group
of order 24.

Fig. 160: $\pi_1(S^3/\langle 3, 3, 2\rangle)$, induced by its superattractor

Homology: Z_8

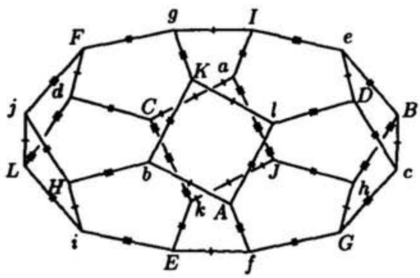


$r_2^{22} \in C_{22}(7) \to$
$\langle a, b | ab^{-1}ab, a^{-1}baba^{-1}b^2\rangle$.
$r_1 : ab^{-1}ab \Rightarrow b^{-1}ab = a^{-1}$
$\Rightarrow b^2$ central,
$r_2 : b^{-2}ab^{-1}a^{-1}b^{-1}a$
$\Rightarrow_{r_1} b^{-4}a^3 \Rightarrow a^3 = b^4$,
$b^{-1}a^3b = a^3 = a^{-3} \Rightarrow a^6 = e = b^8$;
$a_1 := a^2 \Rightarrow a_1^3 = e = b^8, b^{-1}a_1b = a_1^{-1}$;
a presentation for $C_3 \rtimes_i C_8$.
By GDV and TS_ρ the manifold is a
quotient of S^3 by this group
of order 24.

Fig. 161: $\pi_1(S^3/(C_3 \rtimes_i C_8))$, induced by its superattractor

5.3.4 Manifolds of Complexity 11

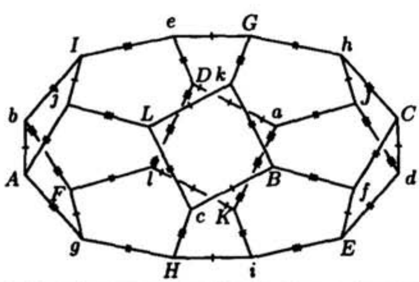
Homology: Trivial



$r_2^{24} \in C_{24}(1) \to$
$\langle a, b | ba^{-1}bab^{-1}a,$
$b^{-2}a^{-1}baba^{-1}\rangle.$
$r_1 : ba^{-1}bab^{-1}a$
$\Rightarrow ba^{-1}ba = a^{-1}b$
$\Rightarrow (ab^{-1})^2 = b^{-1}ab^{-2}$
$\Rightarrow_{r_1} (ab^{-1})^3 = (ab^{-2})^2,$
$r_2 : b^{-2}a^{-1}baba^{-1}$
$\Rightarrow_{r_1} b^{-3}(ba^{-1}ba)ba^{-1}$
$\Rightarrow b^{-3}a^{-1}b^2a^{-1}$
$\Rightarrow b^{-5}(b^2a^{-1})^2$
$\Rightarrow (ab^{-2})^2 = b^{-5},$
$a_1 := ab^{-1}, b_1 := b^{-1}$
$\Rightarrow a_1^3 = (a_1b_1)^2 = b_1^5,$
the definition of $\langle 5, 3, 2\rangle$.
The manifold is the original Poincaré sphere.

Fig. 162: $\pi_1(S^3/\langle 5, 3, 2\rangle)$, induced by its superattractor

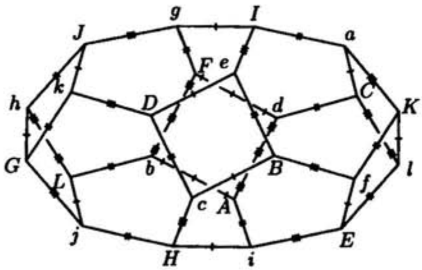
Homology: Z_2



$r_{28}^{24} \in C_{24}(3) \to$
$\langle a, b | a^{-2}b^{-1}ab^{-1}, ab^{-1}ab^3\rangle.$
$r_1 : a^{-2}b^{-1}ab^{-1} \Rightarrow a^3 = (ab^{-1})^2,$
$r_2 : ab^{-1}ab^3 \Rightarrow (ab^{-1})^2 = b^{-4},$
$b_1 := b^{-1} \Rightarrow a^3 = (ab_1)^2 = b_1^4;$
the definition of $\langle 4, 3, 2\rangle$.
By GDV and TS_ρ the manifold is a quotient of S^3 by this group, the binary octahedral group of order 48.

Fig. 163: $\pi_1(S^3/\langle 4, 3, 2\rangle)$, induced by its superattractor

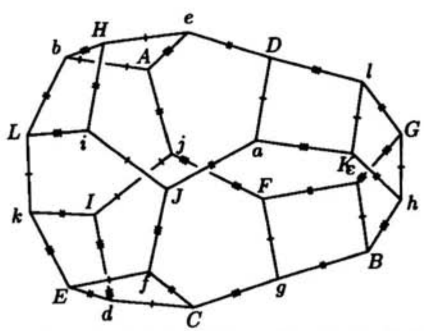
Homology: Z_8



$r_4^{24} \in C_{24}(14) \rightarrow$
$\langle a, b | b^{-1}a^{-1}ba^{-1}, ba^{-1}ba^2b^2a^2 \rangle$.
$r_1 : b^{-1}a^{-1}ba^{-1}$
$\Rightarrow a^b = a^{-1}, b^2$ central,
$r_2 : ba^{-1}ba^2b^2a^2$
$\Rightarrow_{r_1} a^5 = b^{-4}$,
$\Rightarrow a^{10} = e = b^8$.
The group is isomorphic to $C_5 \rtimes_i C_8$. By GDV and TS_ρ the manifold is a quotient of S^3 by this group of order 40.

Fig. 164: $\pi_1(S^3/C_5 \rtimes_i C_8)$, induced by its superattractor

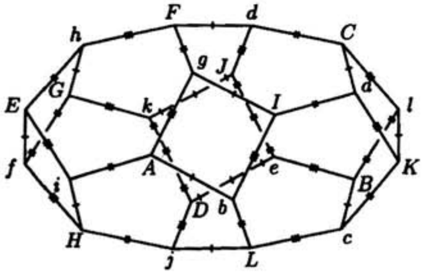
Homology: $Z_2 + Z_2$



$r_{154}^{24} \in C_{24}(26) \rightarrow$
$\langle a, b | abab^{-1}, a^{-2}ba^2b \rangle$.
$r_1 : abab^{-1}$
$\Rightarrow a^b = a^{-1}, b^2$ central,
$r_2 : a^{-2}ba^2b$
$\Rightarrow_{r_1} a^4 = b^2$,
$a^4 = b^2, a^b = a^{-1}$.
This is a presentation for Q_{16}. By GDV and TS_ρ the manifold is a quotient of S^3 by this group of order 16.

Fig. 165: $\pi_1(S^3/Q_{16})$, induced by its u_2^0-attractor

Homology: $Z_2 + Z_6$

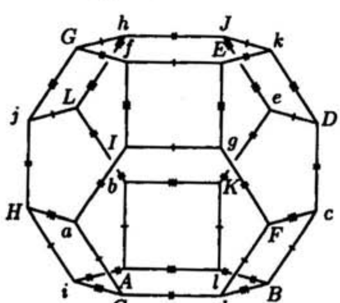


The group is $C_3 \times Q_8$.
By GDV and TS_ρ the manifold is a quotient of S^3 by this group of order 24.

$r^{24}_{13} \in C_{24}(28) \rightarrow$
$\langle a, b | b^2a^2, a^5ba^3b \rangle$.
$r_1 : b^2a^2$
$\Rightarrow a^2 = b^{-2}$ central,
$r_2 : b^{-1}a^{-3}b^{-1}a^{-5}$
$\Rightarrow_{r_1} (ab)^2 = a^{-6}$,
$a^b = b^{-2}.baba.a^{-1}$
$= a^2.a^{-6}.a^{-1} = a^{-5}$,
$a^2 = b^{-1}a^2b = a^{-10}$
$\Rightarrow a^{12} = e = b^{12}$,
$a_3 := a^4, a_4 := a^{-3}$,
$b_3 := b^4, b_4 := b^{-3}$
$\Rightarrow a = a_3a_4, b = b_3b_4$,
$a_4^b = a^{15} = a^3 = a_4^{-1}$
$\Rightarrow a_4^{b_4} = a_4^{-1}$
$\Rightarrow a_4^2 = b_4^{-2} = b_4^2$
$\Rightarrow \langle a_4, b_4 \rangle \cong Q_8$.

Fig. 166: $\pi_1(S^3/C_3 \times Q_8)$, induced by its u_2^0-attractor

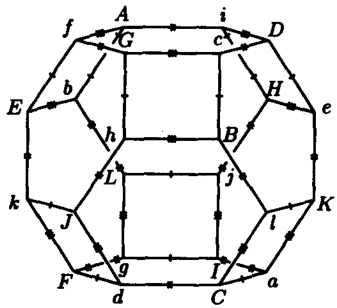
Homology: $Z_4 + Z_4$



$r^{24}_5 \in C_{24}(32) \rightarrow \langle a, b | b^2ab^2a^{-1}, a^2b^{-1}a^2b \rangle$.
$r_1 : b^2ab^2a^{-1} \Rightarrow (b^2)^a = b^{-2}$,
$r_2 : b^{-1}a^{-2}ba^{-2} \Rightarrow (a^2)^b = a^{-2}$.
The manifold is EUCLID_1.
We name this group euclid_1.

Fig. 167: $\pi_1(\text{EUCLID}_1)$, induced by its superattractor

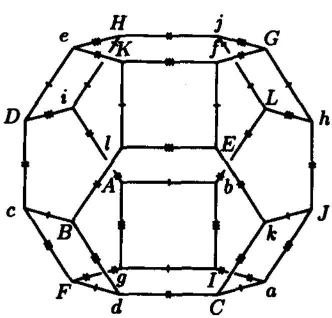
Homology: $Z + Z_3$



$r_7^{24} \in C_{24}(37) \to$
$\langle a, b | ba^{-1}b^2a^{-1}bab^{-1}a, b^{-1}ab^{-1}a^{-2}b^{-1}a \rangle$.
$r_1 : ba^{-1}b^2a^{-1}bab^{-1}a$
$\Rightarrow (ba^{-1}b)^2ab^{-1}a$,
$r_2 : b^{-1}ab^{-1}a^{-2}b^{-1}a$
$\Rightarrow (b^{-1}a)^3 = a^3, a^3$ central,
$\Rightarrow b^{-1}ab^{-1} = a^{-1}ba^{-1}.a^3, ab^{-1}a$
$= ba^{-1}b.a^3$,
substitute into r_1,
$(ba^{-1}b)^3 = a^{-3}, (a^{-1}b^2)^3 = a^{-3}$,
$(a^{-1}b)^3 = (a^{-1}b^2)^3 = a^{-3}$;
$a_1 := a^{-1}, b_1 := a_1 b, c := a_1^{-1} b_1^2 \Rightarrow$
$a_1^3 = b_1^3 = c^3 = a_1 c b_1$, the definition of $\langle 3, 3, 3 \rangle$. The manifold is EUCLID$_2$.
We name this group euclid$_2$.

Fig. 168: π_1(EUCLID$_2$), induced by its superattractor

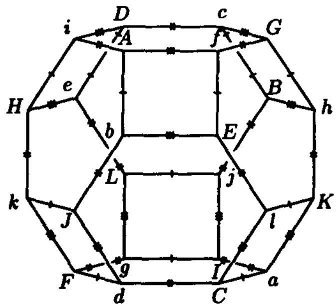
Homology: $Z + Z_2 + Z_2$



$r_6^{24} \in C_{24}(39) \to$
$\langle a, b, c | baba^{-1}, abc^2ab, bcbc^{-1} \rangle$.
$r_1 : baba^{-1} \Rightarrow b^a = b^{-1}$,
$r_2 : b^{-1}a^{-1}c^{-2}b^{-1}a^{-1} \Rightarrow (b^{-1}a^{-1})^2c^{-2}$
$\Rightarrow (ab)^2 = c^{-2} \Rightarrow_{r_1} a^2 = c^{-2}$,
$r_3 : cb^{-1}c^{-1}b^{-1} \Rightarrow b^c = b^{-1}$;
$a_1 := ac \Rightarrow b^{a_1} = b, a_1^c = c^{-1}ac^2 =$
$c^{-1}a.a^{-2} = a_1^{-1}$;
we have: $a_1 b = b a_1, a_1^c = a_1^{-1}, b^c = b^{-1}$,
which is a definition of $(Z \times Z) \rtimes_i Z$.
The manifold is EUCLID$_3$.
We name this group euclid$_3$.

Fig. 169: π_1(EUCLID$_3$), induced by its superattractor

Homology: $Z + Z + Z$

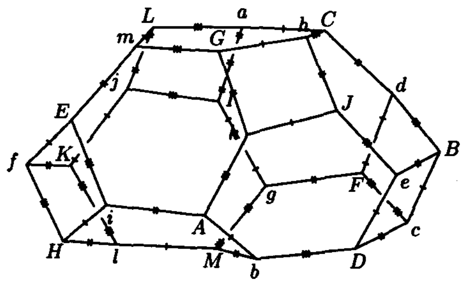


$r_1^{24} \in C_{24}(40) \rightarrow$
$\langle a, b, c | bc^{-1}b^{-1}c, b^{-1}a^{-1}c^{-1}abc, aba^{-1}b^{-1}\rangle.$
$r_1 : bc^{-1}b^{-1}c \Rightarrow bc = cb,$
$r_2 : b^{-1}a^{-1}c^{-1}abc$
$\Rightarrow_{r_1} b^{-1}a^{-1}c^{-1}acb \Rightarrow ac = ca,$
$r_3 : aba^{-1}b^{-1} \Rightarrow ab = ba.$
This is a presentation of $Z \times Z \times Z$.
The manifold is the 3-torus.

Fig. 170: $\pi_1(S^1 \times S^1 \times S^1)$, induced by its superattractor

5.3.5 Manifolds of Complexity 12

Homology: Z_7



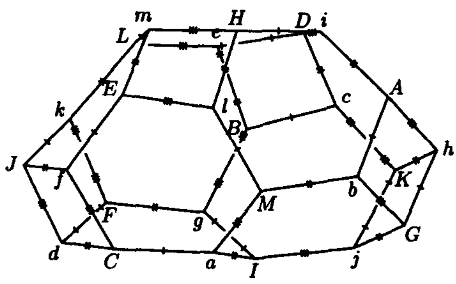
By GDV and TS_ρ this manifold is a quotient of $|r_2^{24}|$ by the monodromy group of order 7.

$r_{10}^{26} \in C_{26}(13) \rightarrow$
$\langle a, b | ba^3ba^{-2}, a^{-2}baba^{-2}b^{-1}\rangle$.
$r_1 : ba^3ba^{-2}$
$\Rightarrow (ba^{-2})^2 = a^{-5}$, central
$r_2 : a^{-2}baba^{-2}b^{-1}$
$\Rightarrow_{r_1} a^{-2}ba.a^2b^{-1}a^{-5}.b^{-1}$
$\Rightarrow a^{-2}ba^{-2}a^5b^{-1}a^{-5}b^{-1}$
$\Rightarrow (a^{-1}b)^2b^{-3} \Rightarrow_{r_1} a^{-5}b^{-3}$;
$b_1 := b^{-1} \Rightarrow a^5 = b_1^3 = (a^2b_1)^2$;
by SPAS and [GAP], this group has order 840 and is isomorphic to $\langle 5,3,2\rangle \times C_7$, where $\langle 5,3,2\rangle$ is the fundamental group of the original Poincaré sphere $|r_2^{24}|$.

Fig. 171: $\pi_1(S^3/(\langle 5,3,2\rangle \times C_7))$, induced by its superattractor

$r_8^{26} \in C_{26}(18) \rightarrow$
$\langle a, b | a^2ba^{-1}ba^2b,$
$a^2bab^{-1}ab\rangle$.

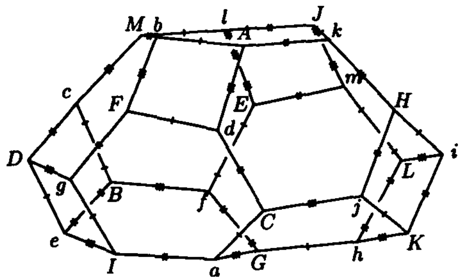
Homology: Z_9



$b_1 := ab \Rightarrow$
$r_1 : ab_1a^{-2}b_1ab_1 \Rightarrow a^{-3}(ab_1)^3$,
$r_2 : ab_1ab_1^{-1}ab_1 \Rightarrow b_1^{-2}(ab_1)^3$,
$b_2 := ab_1 \Rightarrow$
$a^3 = b_2^3 = (a^{-1}b_2)^2$.
By SPAS this is a presentation of $Q_8 \rtimes_3 C_9$. By GDV and TS_ρ the manifold is a quotient of S^3 by this group of order 72.

Fig. 172: $\pi_1(S^3/Q_8 \rtimes_3 C_9)$, induced by its u_{11}^0-attractor

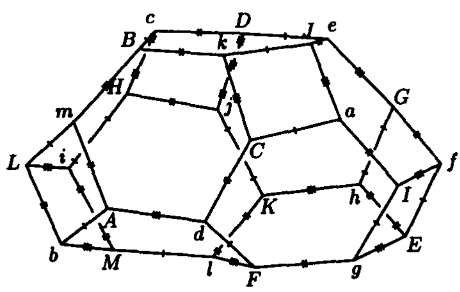
Homology: Z_{12}



$r_6^{26} \in C_{26}(24) \to$
$\langle a, b | b^2a^2, abab^{-1}aba^3bab^{-1}\rangle$.
$r_1 : b^2a^2 \Rightarrow b^2 = a^{-2}$ central,
$r_2 : abab^{-1}aba^3bab^{-1}$
$\Rightarrow_{r_1} (ab)^5a^6$;
$b_1 := ab \Rightarrow b_1^5 = a^{-6}, ab_1a^{-1} = a^2ba^{-1} = b_1^{-1} \Rightarrow b_1^{10} = e = a^{12}$;
we have:
$b_1^a = b_1^{-1}, a^6 = b_1^5, a^{12} = e = b_1^{10}$,
which is a presentation of
$C_5 \rtimes_i C_{12}$.
By GDV and TS_ρ the manifold is a quotient of S^3 by this group of order 60.

Fig. 173: $\pi_1(S^3/C_5 \rtimes_i C_{12})$, induced by its u_4^0-attractor

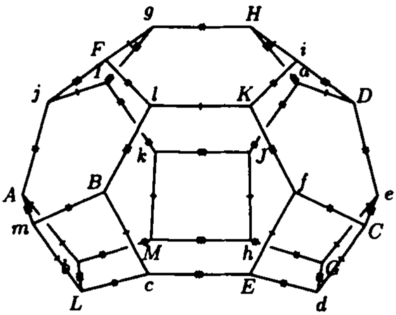
Homology: Z_{15}



$r_4^{26} \in C_{26}(30) \to$
$\langle a, b | b^3a^2, a^3ba^3bab\rangle$.
$r_1 : b^3a^2 \Rightarrow a^2 = b^{-3}$ central,
$r_2 : a^3ba^3bab \Rightarrow_{r_1} (ab)^3 = a^{-4}$;
By SPAS and GAP this is a presentation of
$(Q_8 \rtimes_3 C_3) \times C_5$.
By GDV and TS_ρ this manifold is a quotient of S^3 by this group of order 120.

Fig. 174: $\pi_1(S^3/((Q_8 \rtimes_3 C_3) \times C_5)$, induced by its u_2^1-attractor

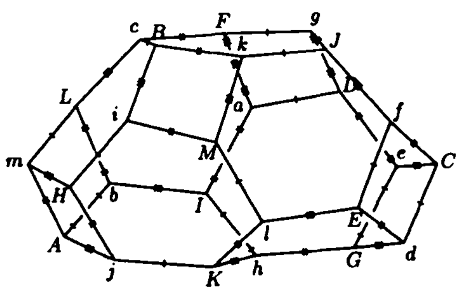
Homology: Z_{15}



$r^{26}_{696} \in C_{26}(32) \rightarrow$
$\langle a, b | bab^2a^3b^2a, b^2ab^{-1}ab^2a^4 \rangle$.
By SPAS and GAP this is a presentation of $(Q_8 \rtimes_3 C_3) \times C_5$.
By GDV and TS_ρ this manifold is a quotient of S^3 by this group of order 120. The previous 3-gem and the current one define the same manifold although they are in distinct u^0-classes.

Fig. 175: $\pi_1(S^3/((Q_8 \rtimes_3 C_3) \times C_5)$, induced by the other u^1_2-attractor

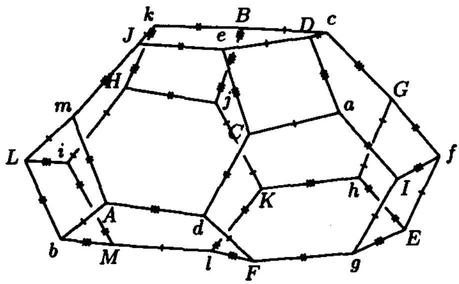
Homology: $Z_2 + Z_6$



$r^{26}_{65} \in C_{26}(39) \rightarrow$
$\langle a, b | ababab^{-1}ab, a^{-2}b^2 \rangle$.
$r_1 : ababab^{-1}ab$
$\Rightarrow b^2 = (ab)^4$, central,
$r_2 : a^{-2}b^2 \Rightarrow a^2 = b^2$.
We have $a^2 = b^2 = (ab)^4$.
By SPAS and GAP the group is isomorphic to $Q_{16} \times C_3$. By GDV and TS_ρ this manifold is a quotient of S^3 by this group of order 48.

Fig. 176: $\pi_1(S^3/Q_{16} \times C_3)$, induced by its u^0_3-attractor

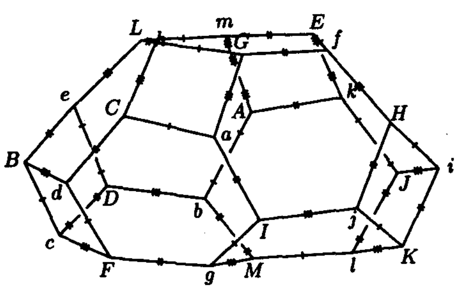
Homology: $Z_4 + Z_4$



$r_3^{26} \in C_{26}(44) \to$
$\langle a, b | ba^2b^{-1}a^2, b^2ab^2a^{-1} \rangle$.
$r_1 : ba^2b^{-1}a^2 \Rightarrow (a^2)^b = a^{-2}$,
$r_2 : b^2ab^2a^{-1} \Rightarrow (b^2)^a = b^{-2}$.
Another instance of EUCLID_1.

Fig. 177: $\pi_1(\text{EUCLID}_1)$, induced by a u^0-irreducible but u^1-reducible 3-gem

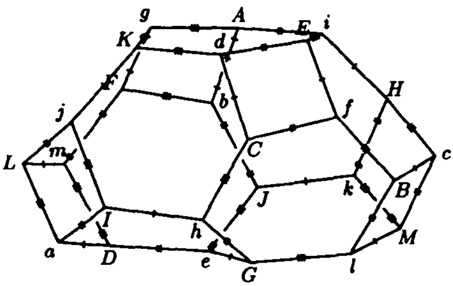
Homology: $Z_4 + Z_4$



$r_5^{26} \in C_{26}(45) \to$
$\langle a, b | b^{-1}a^2ba^2, b^2aba^{-2}ba \rangle$.
$r_1 : b^{-1}a^2ba^2$
$\Rightarrow (a^2)^b = a^{-2}$,
$r_2 : b^2aba^{-2}ba$
$\Rightarrow (b^2)^a = a^{-4}b^{-2}$.
$H := \langle a^2, b^2 \rangle$ is a commutative normal subgroup.
$G/H \cong C_2 * C_2, G$ is soluble.
By SPAS $G/G' \cong C_4^2, G' \cong S_1^4$.
We note that G' is a 3-generated nilpotent group of class 2. The group is $E_2(0, 2)$. We also denote it by $[4^2, S_1^4]$.

Fig. 178: $\pi_1(|r_5^{26}|) = [4^2, S_1^4]$, induced by its superattractor

Homology: Z

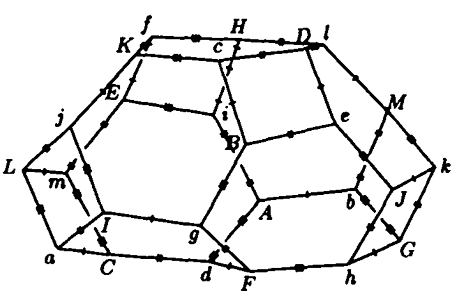


The manifold is EUCLID$_4$, see Section 4.2.1.

$r_{31}^{26} \in C_{26}(50) \to$
$\langle a, b | a^{-2}bab^{-1}ab, abab^{-1}a^{-2}b^{-1} \rangle$.
$r_1 : a^{-2}bab^{-1}ab$
$\Rightarrow b^{a^{-1}} = bb^{-a}$,
$r_2 : abab^{-1}a^{-2}b^{-1}$
$\Rightarrow (b^{-1})^{a^{-1}} = b^{-1}b^a$,
$\Rightarrow H := \langle b, b^a \rangle$
is a commutative normal subgroup, on which a induces $a : b \mapsto b^a$, $b^a \mapsto b^{-1}b^a$.
This is a presentation for the group
$(Z \times Z) \rtimes_M Z$, where
$M = \begin{pmatrix} 0 & 1 \\ -1 & 1 \end{pmatrix}$.
We note that on letting $x := a^2b^{-1}, y := a$,
the presentation becomes:
$x^3 = (xy)^2 = y^6$,
which is $\langle 6, 3, 2 \rangle$.

Fig. 179: π_1(EUCLID$_4$), induced by its u_2^0-attractor

Homology: $Z + Z_2$

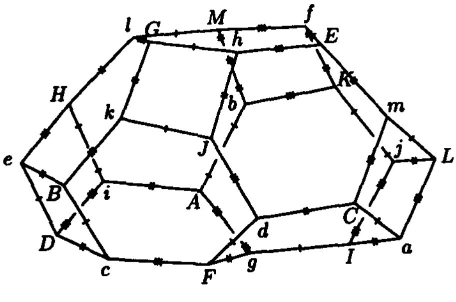


The manifold is EUCLID$_5$, see Section 4.2.1.

$r_{11}^{26} \in C_{26}(52) \to$
$\langle a, b | ab^{-1}ab^3, a^3ba^{-1}b \rangle$.
$r_1 : ab^{-1}ab^3$
$\Rightarrow (ba^{-1})^2 = b^4$, central,
$r_2 : a^3ba^{-1}b$
$\Rightarrow (ba^{-1})^2 = a^{-4}$,
$a_1 := a^{-1} \Rightarrow a_1^4 = b^4 = (a_1 b)^2$,
which is $\langle 4, 4, 2 \rangle$.
We note that on letting $v_1 := [a, b], v_2 := v_1^a$, we have that $H := \langle v_1, v_2 \rangle$
is a normal commutative subgroup. Follows that the group is $(Z \times Z) \rtimes_M Z$,
where $M = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$.

Fig. 180: π_1(EUCLID$_5$), induced by its u_2^0-attractor

Homology: $Z + Z_4$

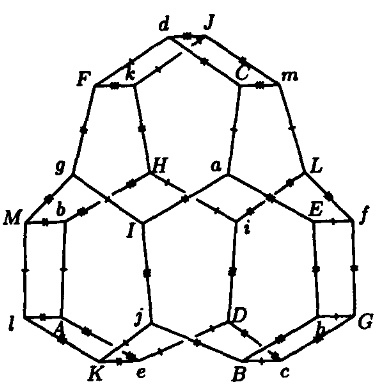


The group is *soluble*: modulo H it is the infinite dihedral group $C_2 * C_2$. The group is $E_2(2,1)$.

$r^{26}_{13} \in C_{26}(56) \to$
$\langle a, b | a^2b^3a^2b, ab^{-1}a^{-2}b^{-1}ab^2\rangle$.
$r_1 : a^2b^3a^2b \Rightarrow (a^2)^b = b^{-4}a^{-2}$, $(a^2b)^2 = b^{-2}$, central,
$r_2 : ab^{-1}a^{-2}b^{-1}ab^2 \Rightarrow_{r_1} a(a^2b.b^2)b^{-1}ab^2 \Rightarrow a^3b^2ab^2 \Rightarrow a^4.a^{-1}b^2a.b^2 \Rightarrow (b^2)^a = a^{-4}b^{-2}, (ab^2)^2 = a^{-2}$.
We have that $H = \langle a^2, b^2\rangle$ is a commutative normal subgroup, on which a and b have the following actions: $(a^2)^b = a^{-2}b^{-4}, (b^2)^a = a^{-4}b^{-2}$

Fig. 181: $\pi_1(|r^{26}_{13}|) = E_2(2,1)$, induced by its u^0_3-attractor

Homology: $Z + Z$

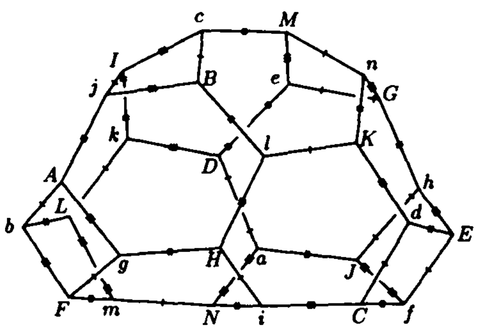


$r^{26}_{14} \in C_{26}(59) \to$
$\langle a, b | ab^2ab^{-1}a^{-2}b^{-1}, ab^{-1}a^{-1}baba^{-1}b^{-1}\rangle$.
$r_1 : ab^2ab^{-1}a^{-2}b^{-1} \Rightarrow aa^{b^{-2}}(a^{-2})^{b^{-1}} \Rightarrow a^{b^2} = (a^2)^b a^{-1}$,
$r_2 : ab^{-1}a^{-1}baba^{-1}b^{-1} \Rightarrow aa^{-b}aa^{-b^{-1}} \Rightarrow a^b a^{-b^2} a^b a^{-1} \Rightarrow_{r_1} aa^b = a^b a$.
We have that $H = \langle a, a^b\rangle$ is a commutative normal subgroup. With a proper choice of a Z-basis for H, b can be represented on it as $\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$. This is a presentation of the *nilpotent* group $(Z \times Z) \rtimes_M Z$, where $M = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$. We have denoted this manifold by $Z^{1\,0}_{1\,1}$.

Fig. 182: $\pi_1(\ Z^{1\,0}_{1\,1}\ )$, induced by its u^0_2-attractor

5.3.6 Manifolds of Complexity 13

Homology: Trivial



$r^{28}_{172} \in C_{28}(3) \to$
$\langle a, b | a^{-1}bab^{-1}a^{-2}b^{-1}ab,$
$a^2b^{-1}a^{-1}bababa^{-1}b^{-1}\rangle.$
$r_1 : a^{-1}bab^{-1}a^{-2}b^{-1}ab \Rightarrow$
$b^{-1}ab = a^2ba^{-1}b^{-1}a,$
$[b,a] = a^{-1}[b^{-1}a^{-1}] =$
$a^{-1}[a^{-1},b]^{b^{-1}},$
$r_2 : a^2b^{-1}a^{-1}bababa^{-1}b^{-1}$
$\Rightarrow a^2[b,a]b[a^{-1},b^{-1}] \Rightarrow$
$[b,a] = a^{-2}[a^{-1},b]^{b^{-1}}b^{-1};$
$c := [b,a], d := [b,a^{-1}]$
into r_1 and r_2
$\Rightarrow c = a^{-1}(d^{-1})^{b^{-1}},$
$c = a^{-2}(d^{-1})^{b^{-1}}b^{-1}$
$\Rightarrow d^{b^{-1}} = c^{-1}a^{-1},$
$d^{b^{-1}} = b^{-1}c^{-1}a^{-2}$
$\Rightarrow a = cb^{-1}c^{-1} \Rightarrow$
$d^{b^{-1}} = bc^{-1} \Rightarrow b = cd$
$\Rightarrow a = c(d^{-1}c^{-1})c^{-1} =$
$cd^{-1}c^{-2}.$
Thus, $a = cd^{-1}c^{-2}, b = cd.$

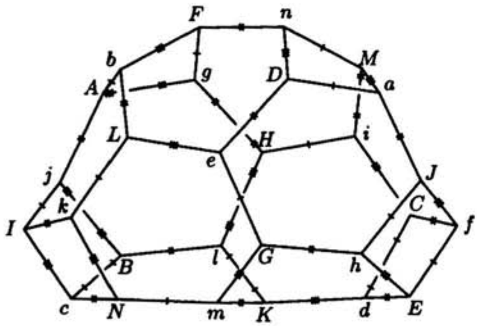
Fig. 183: $\pi_1(|r^{28}_{172}|) = \langle 7,3,2 \rangle$, induced by its u^0_6-attractor

Complement of analysis:
Relations for c, d:
$c = [b,a] = d^{-1}c^{-1}.c^2dc^{-1}.cd.cd^{-1}c^{-2},$
$c^3 = d^{-1}cd^2cd^{-1} \Rightarrow dc^3d = cd^2c, d = [b,a^{-1}] = d^{-1}c^{-1}.cd^{-1}c^{-2}.cd.c^2dc^{-1} \Rightarrow$
$d^3 = c^{-1}dc^2dc^{-1} \Rightarrow cd^3c = dc^2d.$

Thus, $dc^3d = cd^2c, cd^3c = dc^2d$. Rewrite: $c^3 = d^{-1}cd^2cd^{-1}$, $c^2 = d^{-1}cd^3cd^{-1} \Rightarrow cd^{-1}c = d^{-1}cd^{-1}, d^3 = c^{-1}dc^2dc^{-1}$. Rewrite $cd^{-1}c = d^{-1}cd^{-1}as(d^{-1}c)^3 = (d^{-2}c)^2$, central; $d^3 = c^{-1}dc^2dc^{-1} \Rightarrow d^5 = (dc^{-1}d)c^2(dc^{-1}d) = c^{-1}d^2c^{-1} \Rightarrow d^7 = (d^2c^{-1})^2$.

Thus, for $f := dc^{-1}$, we have the presentation: $d^7 = (df)^2 = f^3$, the definition of $\langle 7,3,2 \rangle$.

Homology: Z_2



$r^{28}_{202} \in C_{28}(6) \to$
$\langle a, b | a^{-1}bab^{-1}a^{-2}b^{-1}ab,$
$aba^{-1}b^{-1}ab^2ab^{-1}a^{-1}b\rangle.$
$r_1 : a^{-1}bab^{-1}a^{-2}b^{-1}ab \Rightarrow$
$a^{-1}a^{b^{-1}}a^{-2}a^b \Rightarrow$
$a^{b^{-1}} = aa^{-b}a^2 \Rightarrow$
$a^{b^2} = (a^2)^b a^{-1} a^b,$
$r_2 : aba^{-1}b^{-1}ab^2ab^{-1}a^{-1}b \Rightarrow$
$a^b a^{-1} b^{-2} a^{-1} a^{b^{-1}} a^1 =_{r_1}$
$b^2 = (a^{-1})^b a a^b a^{-1};$
$a_1 := a, a_2 := a^b, H := \langle a_1, a_2\rangle$
$\Rightarrow a_2^b =_{r_1} a_2^2 a_1^{-1} a_2, b^2 =_{r_2}$
$a_2^{-1}a_1a_2a_1^{-1} \Rightarrow$
H is a normal subgroup.

Fig. 184: $\pi_1(|r^{28}_{202}|) = \langle 5, 5, 2\rangle 2$, induced by its superattractor

Complement of analysis:
From $a_2^{b^2} = (a_2^2)^b a_2^{-1} a_2^b$, we have

$$a_2^{b^2} = a_1a_2^{-1}a_1^{-1}a_2.a_2.a_2^{-1}a_1a_2a_1^{-1} = (a_2^2a_1^{-1}a_2)^2a_2^{-1}(a_2^2a_1^{-1}a_2)$$
$$a_1a_2^{-1}a_1^{-1}a_2a_1a_2a_1^{-1} = a_2^2a_1^{-1}a_2^3a_1^{-1}a_2^2a_1^{-1}a_2.$$

The last relation implies

$$a_1^{-1}a_2^2a_1^{-1}a_2a_1a_2^{-1}a_1^{-1}(=: u) = a_2^{-3}a_1a_2^{-2}a_1a_2^{-1}a_1^{-1}a_2.$$

From $(a_2^b)^{b^{-2}} = a_2^{b^{-1}} = a_1$, we have

$$a_1 = a_2^{-1}a_1a_2a_1^{-1}.a_2^2a_1^{-1}a_2.a_1a_2^{-1}a_1^{-1}a_2 = a_2^{-1}a_1a_2ua_2,$$

which implies $u = a_2^{-1}a_1^{-1}a_2a_1a_2^{-1}$. From the definition of u we get

$$a_2^{-1}a_1^{-1}a_2a_1a_2^{-1} = a_2^{-3}(a_1a_2^{-2}a_1a_2^{-1}a_1^{-1})a_2,$$
$$(a_2^{-2}a_1a_2^{-1})^{a_1^{-1}} = a_2^2a_1^{-1}a_2a_1a_2^{-2} = a_2^{a_1a_2^{-2}},$$
$$(a_1a_2^{-3})^{a_2^2a_1^{-1}} = a_2^{a_1a_2^{-2}}.$$

Define $v := a_1a_2^{-2}$, then $a_2^{v^2} = va_2^{-1}, v^5 = (a_2v^2)^2$. Define $w := a_1a_2^{-1}$, then $a_1 = wv^{-1}w, a_2 = v^{-1}w$. Thus, $H = \langle v, w\rangle$, and we note that

$$v^{b^{-1}} = a_1^{b^{-1}}a_1^{-2} = a_1a_2^{-1} = w$$
$$b^2 = a_2^{-1}a_1a_2a_1^{-1} = w^{-1}vwv^{-1}$$

Note that $v^5 = (wv)^2 =: z$ is central in H. Now,

$$w^{b^{-1}} = a_1^{b^{-1}} a_1^{-1} = a_1 a_2^{-1} a_1 = w^2 v^{-1} w =$$
$$w^2(v^{-1}w^{-1})w^2 = w^3 v w^2 z^{-1} = w^5 . w^{-2} v w^2 . z^{-1} = w^{-2} v w^2.$$

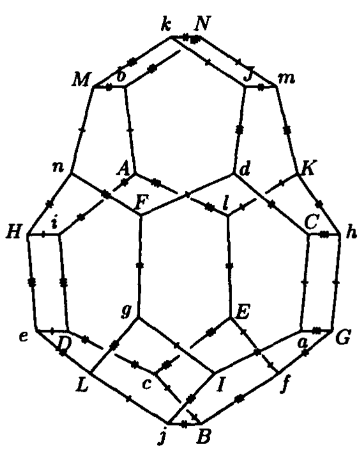
Define $b_1 := w^2 b$, then $w^{b_1} = v, v^{b_1} = w, v^5 = w^5 = (vw)^2 (= z)$.

We also have

$$(w^2 b)^2 = w^2 (w^2)^{b^{-1}} b^2 = w^2 (v^2)^{w^2} b^2 = v^2 w^2 . w^{-1} v w v^{-1},$$
$$b_1^2 = v^2 (wv)^2 v^{-2} = z.$$

Therefore, we get $v^5 = w^5 = (vw)^2 = b_1^2$. That is the group is an extension of $\langle 5, 5, 2\rangle$ by an infinite cyclic group generated by b_1, which interchanges v and w and $b_1^2 = (vw)^2$.

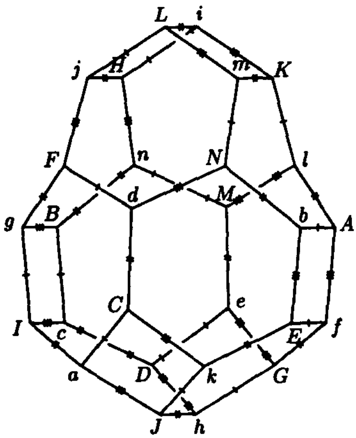
Homology: Z_3



$r_2^{28} \in C_{28}(7) \to$
$\langle a, b | b^2 a b^{-1} a b^{-1} a,$
$a b^{-1} a b a^{-1} b a b^{-1} a^2 b^{-1} a b a^{-1} b\rangle.$
$r_1 : b^2 a b^{-1} a b^{-1} a \Rightarrow b^3 = (a^{-1} b)^3 := z,$
z central,
$r_2 : a b^{-1} a b a^{-1} b a b^{-1} a^2 b^{-1} a b a^{-1} b \Rightarrow$
$a^{-2} b = (b^{-1} a b a^{-1} b a)^2 \Rightarrow_{r_1}$
$a^{-2} b = (b^{-1} a . a b^{-1} a z . a)^2 \Rightarrow$
$(a^{-2} b)^5 = z^2.$
$a_1 := a^{-1} b \Rightarrow a_1^3 = b^3 = z, (a_1^2 b^{-1})^5 = z^2,$
and the presentation can be written as
$a_1^3 = b^3, (a_1 b)^5 = a_1^9.$
The derived series of the group has the following form:
$G/G' \cong C_3, G'/G'' \cong C_5^2$ and $G'' \cong S_6^5$
This information was obtained by SPAS and independently by GDV and TS_p.

Fig. 185: $\pi_1(|r_2^{28}|) = [3, 5^2, S_6^5]$, induced by its superattractor

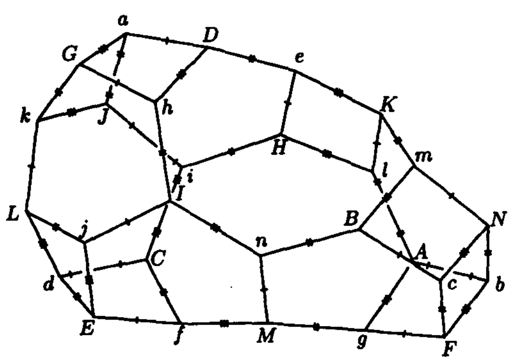
Homology: Z_3



$r_9^{28} \in C_{28}(8) \rightarrow$
$\langle a, b | b^{-1}ab^2ab^2a, a^3ba^{-1}b^{-2}a^{-1}b \rangle$.
$r_1 : b^{-1}ab^2ab^2a \Rightarrow (a^{-1}b^{-2})^3 = b^{-3}$,
$r_2 : a^3ba^{-1}b^{-2}a^{-1}b \Rightarrow a^3b(a^{-1}b^{-2})^2b^3 \Rightarrow_{r_1}$
$a^3b^3a \Rightarrow a^4 = b^{-3}; b_1 := b^{-1} \Rightarrow$
$a^4 = b_1^3 = (a^{-1}b_1^2)^3$,
which is equivalent to the definition of $\langle 4,3,3 \rangle$.The derived series of this group has the following form:
$G/G' \cong C_3, G'/G'' \cong C_4^2$ and $G'' \cong S_3^4$.
This information was obtained by SPAS and independently by GDV and TS_ρ.

Fig. 186: $\pi_1(|r_9^{28}|) = [3, 4^2, S_3^4]$, induced by its u_3^0-attractor

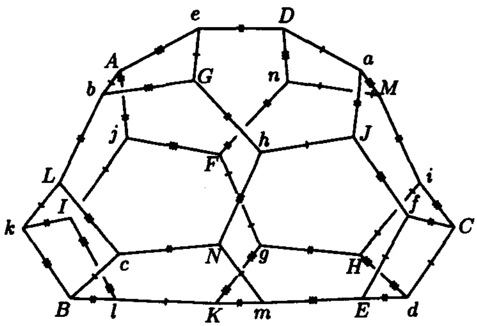
Homology: Z_4



$r_{2418}^{28} \in C_{28}(13) \rightarrow$
$\langle a, b | ab^{-1}ab^4, ab^3ab^{-2} \rangle$.
$r_1 : ab^{-1}ab^4$
$\Rightarrow b^{-5} = (ab^{-1})^2$,
$r_2 : b^{-5} = (b^{-3}a^{-1})^2$;
$a_1 := ab^3$
$\Rightarrow b^5 = a_1^2 = (a_1b)^2$,
which is the definition of $\langle 5,2,2 \rangle$. By SPAS and GAP this is a presentation for $C_5 \rtimes_i C_4$.
By GDV and TS_ρ the manifold is a quotient of S^3 by this group of order 20.

Fig. 187: $\pi_1(S^3/\langle 5,2,2 \rangle)$, induced by its u_{18}^0-attractor

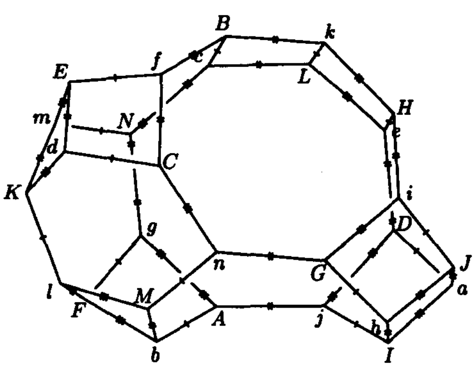
Homology: Z_5



$r^{28}_{203} \in C_{28}(15) \to$
$\langle a, b | b^2a^{-1}b^2ab^{-2}a,$
$b^{-1}a^{-1}b^2ab^2a^{-1}\rangle.$
$r_1 : b^2a^{-1}b^2ab^{-2}a$
$\Rightarrow (b^2)^a = (a^{-1})^{b^2},$
$r_2 : b^{-1}a^{-1}b^2ab^2a^{-1}$
$\Rightarrow b^{-1}(b^2)^a b^2 a^{-1}$
$\Rightarrow_{r_1} b^{-3}a^{-1}b^4a^{-1}$
$\Rightarrow b^{-7}(b^4a^{-1})^2$
$\Rightarrow (b^4a^{-1})^2 = b^7$, central,
$\Rightarrow_{r_1} b^{14} = a^{-7}$, central.
G/G' is isomorphic to C_5. By using GDV we get a 140-vertex 3-gem whose π_1 is G'. It gets attracted under TS_ρ to r^{28}_{172} whose π_1 is $\langle 7, 3, 2\rangle$. A long hand computation shows that this group is isomorphic to $\langle 7, 3, 2\rangle \times \langle c\rangle$ with c^5 amalgamated with an element of the center of the first group.

Fig. 188: $\pi_1(|r^{28}_{203}|) = \langle 7, 3, 2\rangle 5$, induced by its superattractor

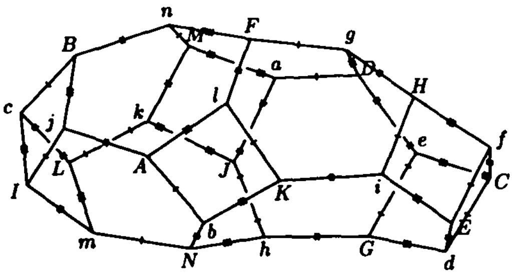
Homology: Z_8



$r^{28}_{2314} \in C_{28}(26) \to$
$\langle a, b | a^{-2}ba^5b, a^3ba^{-4}b\rangle.$
$r_1 : a^{-2}ba^5b \Rightarrow a^7 = (a^2b^{-1})^2,$
$r_2 : a^3ba^{-4}b \Rightarrow a^7 = (b^{-1}a^4)^2;$
$b_1 := a^2b^{-1}$
$\Rightarrow a^7 = b_1^2 = (a^2b_1)^2.$
This is a presentation for $C_7 \rtimes_i C_8$. By GDV and TS_ρ the manifold is a quotient of S^3 by this group of order 56.

Fig. 189: $\pi_1(S^3/C_7 \rtimes_i C_8)$, induced by its u_6^0-attractor

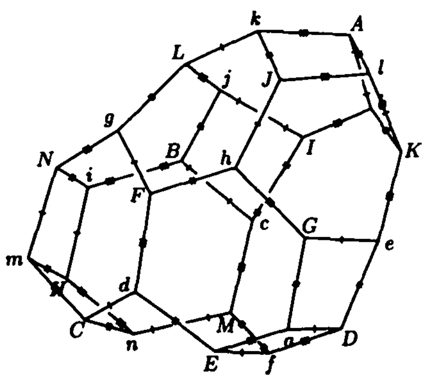
Homology: Z_{12}



$r^{28}_{402} \in C_{28}(38) \to$
$\langle a, b | ba^{-1}ba,$
$a^2b^2a^2b^2a^2b^3 \rangle$.
$r_1 : b^a = b^{-1}, a^2$ is central,
$r_2 : a^2b^2a^2b^2a^2b^3$
$\Rightarrow b = (b^{-2}a^{-2})^3$
$\Rightarrow_{r_1} b^7 = a^{-6}$.
This group is isomorphic to $C_7 \rtimes_i C_{12}$. By GDV and TS_ρ the manifold is a quotient of S^3 by this group of order 84.

Fig. 190: $\pi_1(S^3/C_7 \rtimes_i C_{12})$, induced by its u^0_2-attractor

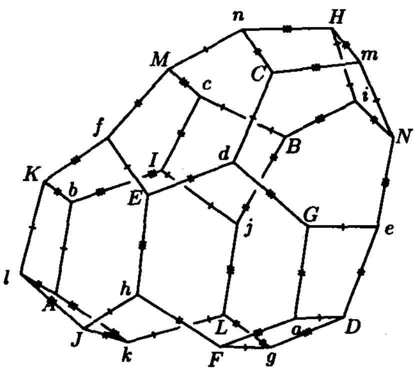
Homology: Z_{16}



$r^{28}_{27} \in C_{28}(47) \to$
$\langle a, b | b^{-5}ab^3a^3b^3a,$
$b^{-2}ab^3a^2b^3a \rangle$.
$r_1 : b^{-5}ab^3a^3b^3a$
$\Rightarrow b^3ab^{-3} = a^{-3}b^{-3}a^{-1}b^2,$
$r_2 : b^{-2}ab^3a^2b^3a$
$\Rightarrow a^{-1}b^3a = a^{-3}b^{-3}a^{-1}b^2$
$\Rightarrow_{r_1} a^{-1}b^3a = b^3ab^{-3}$.
By SPAS and GAP, the group is isomorphic to $C_3 \rtimes_i C_{16}$.
By GDV and TS_ρ the manifold is a quotient of S^3 by this group of order 48.

Fig. 191: $\pi_1(S^3/C_3 \rtimes_i C_{16})$, induced by its u^0_{18}-attractor

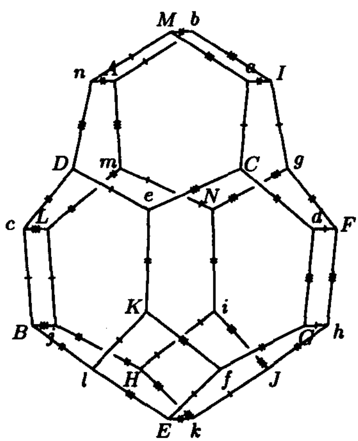
Homology: Z_{20}



$r^{28}_{29} \in C_{28}(54) \to$
$\langle a, b | b^2ababab^2a, aba^2bab^{-1}\rangle$.
$r_1 : b^2ababab^2a \Rightarrow$
$(a^{-1}b^{-1})^2 = (b^2a)^2$,
$r_2 : aba^2bab^{-1} \Rightarrow b = (aba)^2$.
By SPAS and GAP, the group is isomorphic to $C_3 \rtimes_i C_{20}$.
By GDV and TS_ρ the manifold is a quotient of S^3 by this group of order 60.

Fig. 192: $\pi_1(S^3/C_3 \rtimes_i C_{20})$, induced by its u^0_{14}-attractor

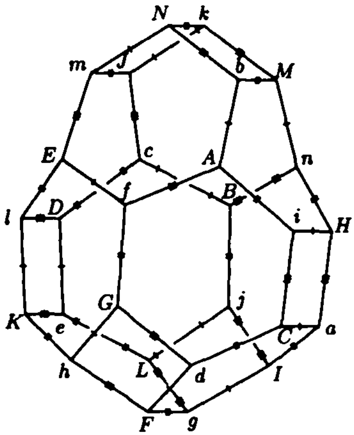
Homology: Z_{24}



$r^{28}_1 \in C_{28}(58) \to$
$\langle a, b | a^3ba^3b^{-1}, a^{-2}ba^{-1}ba^{-2}ba^{-4}b\rangle$.
$r_1 : a^3ba^3b^{-1} \Rightarrow (a^3)^b = a^{-3}$,
$r_2 : a^{-2}ba^{-1}ba^{-2}ba^{-4}b$
$\Rightarrow_{r_1} a^3 = (aba^{-1}b)^2$;
$b_1 := ab \Rightarrow (a^3)^{b_1} = a^{-3}, a^3 = (a^1b_1^2)^2$.
By SPAS $G/G' \cong C_{24}, G' \cong S^1_2$
Using GDV and TS_ρ we arrive to a 46-vertex 3-gem with $\pi_1 \cong G'$

Fig. 193: $\pi_1(|r^{28}_1|) = [24, S^1_2]$, induced by its u^0_2-attractor

Homology: $Z_2 + Z_2$

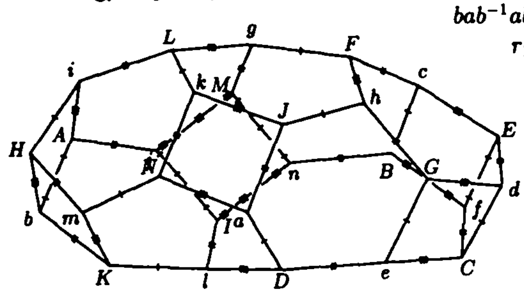


$r_3^{28} \in C_{28}(59) \to$
$\langle a, b, c | a^2b^3, abc^{-1}abc, c^2b^3\rangle$.
$r_1 : a^2b^3 \Rightarrow a^2 = b^{-3}$,
$r_2 : abc^{-1}abc \Rightarrow (abc)^2 = c^2$,
$r_3 : c^2b^3 \Rightarrow c^2 = b^{-3}$.
By SPAS $G/G' \cong C_2^2, G'/G'' \cong C_3^2 \times C_6$,
$G'' \cong S_{10}^9$.
By using GDV and TS_p we get a 40-vertex 3-gem with $\pi_1 \cong G,'$ and also one with 74 vertices whose $\pi_1 \cong G''$.

Fig. 194: $\pi_1(|r_3^{28}|) = [2^2, 3^2 \times 6, S_{10}^9]$, induced by its u_2^0-attractor

$r_{56}^{28} \in C_{28}(66) \to$
$\langle a, b | b^{-2}a^2,$
$bab^{-1}ab^{-1}abab^{-1}aba^{-1}bab^{-1}a, \rangle$.

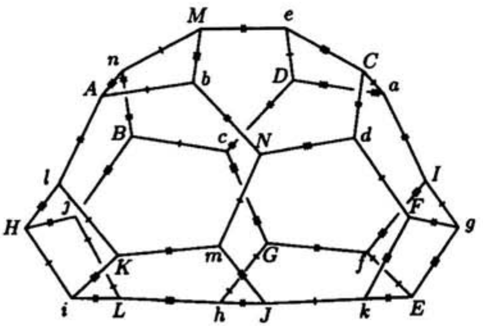
Homology: $Z_2 + Z_6$



$r_1 : b^{-2}a^2 \Rightarrow a^2 = b^2$, central, into $r_2 \Rightarrow (ab)^8 = a^{22}$;
By SPAS and GAP, the group is isomorphic to $Q_{32} \times C_3$.
By GDV and TS_p the manifold is a quotient of S^3 by this group of order 96.

Fig. 195: $\pi_1(S^3/Q_{32} \times C_3)$, induced by its u_2^0-attractor

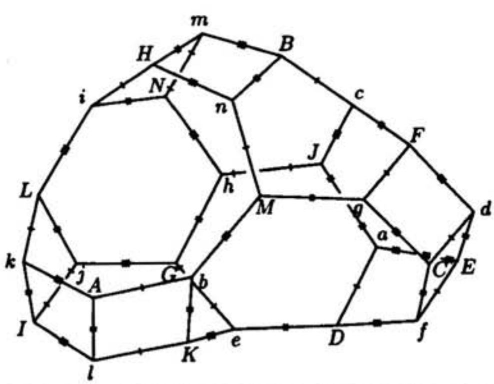
Homology: $Z_2 + Z_{10}$



$r_7^{28} \in C_{28}(73) \to$
$\langle a, b | a^{-1}b^5ab^5, b^4a^{-1}b^6a^{-1} \rangle$.
$r_1 : a^{-1}b^5ab^5 \Rightarrow (b^5)^a = b^{-5}$,
$r_2 : b^4a^{-1}b^6a^{-1}$
$\Rightarrow (b^6a^{-1})^2 = b^2$, central.
This group is isomorphic to $Q_8 \times C_5$. By GDV and TS_ρ the manifold is a quotient of S^3 by this group of order 40.

Fig. 196: $\pi_1(S^3/Q_8 \times C_5)$, induced by its u^1_{10}-attractor

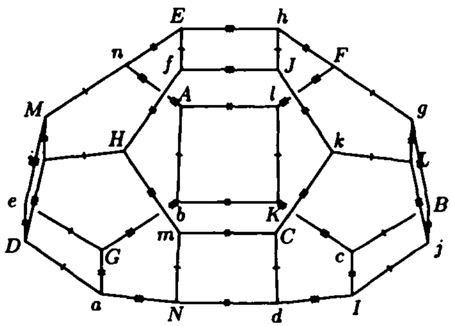
Homology: $Z_2 + Z_{10}$



$r_{59}^{28} \in C_{28}(74) \to$
$\langle a, b | b^3a^{-1}ba^{-1}, ab^2ab^4 \rangle$.
$r_1 : b^3a^{-1}ba^{-1}$
$\Rightarrow (ba^{-1})^2 = b^{-2}$, central,
$r_2 : ab^2ab^4 \Rightarrow (b^{-2}a^{-1})^2 = b^2$
$\Rightarrow_{r_1} b^6 = a^{-2}$;
This group is isomorphic to $Q_8 \times C_5$. This 3-gem also induces the previous manifold. See next section.

Fig. 197: $\pi_1(S^3/Q_8 \times C_5)$, induced by another u^1_{10}-attractor (in other u^0-class)

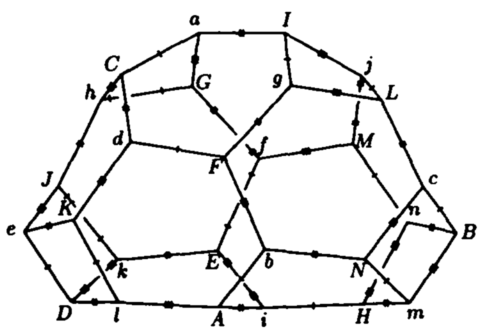
Homology: $Z_3 + Z_3$



$r^{28}_{34} \in C_{28}(76) \to$
$\langle a, b | a^2ba^{-1}ba^{-1}b,$
$b^{-1}aba^{-1}bab^{-1}a^2\rangle.$
$r_1 : a^2ba^{-1}ba^{-1}b$
$\Rightarrow a^3 = (ab^{-1})^3$, central,
$r_2 : b^{-1}aba^{-1}bab^{-1}a^2$
$\Rightarrow b^{-1}a^2(a^{-1}b)^2ab^{-1}a^2$
$\Rightarrow b^{-1}a^2.a^{-3}b^{-1}a.ab^{-1}a^2$
$\Rightarrow a^3 = (b^{-1}a^2)^3;$
$b_1 := ab^{-1} \Rightarrow a^3 = b_1^3 = (ab_1)^3$, the definition of $(3,3,3)^*$.
By SPAS $G/G' \cong C_3^2, G' \cong S_1^3$.
We note that G' is a 3-generated nilpotent group of class 2, therefore solvable.
From r^{28}_{34} using GDV and TS_ρ we get a 30-vertex 3-gem with $\pi_1 \cong G'$.

Fig. 198: $\pi_1(|r^{28}_{34}|) = [3^2, S_1^3])$, induced by its u_5^0-attractor

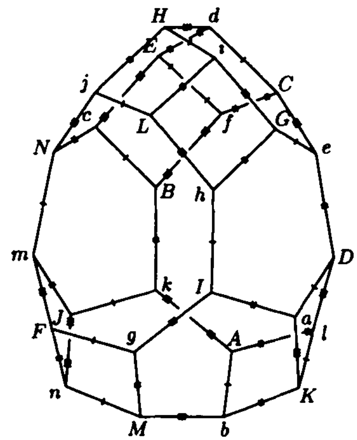
Homology: $Z_4 + Z_4$



$r^{28}_{19} \in C_{28}(86) \to$
$\langle a, b | a^{-1}b^2ab^2, ab^3a^2b^3ab^{-2}\rangle.$
$r_1 : a^{-1}b^2ab^2$
$\Rightarrow (b^2)^a = b^{-2},$
$r_2 : ab^3a^2b^3ab^{-2}$
$\Rightarrow ab.b^2a^2b.b^2ab^{-2}$
$\Rightarrow_{r_1} ba^2b = a^{-2}b^{-6}$
$\Rightarrow (a^2)^b = a^{-2}b^{-8};$
$H : \langle a^2, b^2\rangle$ is a commutative normal subgroup. Therefore, $G/H \cong C_2 * C_2$, and G is soluble.
By SPAS $G/G' \cong C_4^2$ and $G' \cong S_1^8$.

Fig. 199: $\pi_1(|r^{28}_{19}|) = [4^8, S_1^8]$, induced by its superattractor

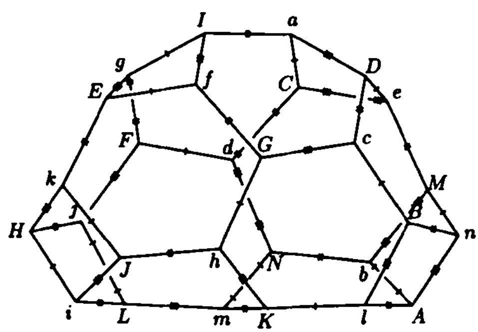
Homology: $Z_4 + Z_4$



$r_{25}^{28} \in C_{28}(87) \to$
$\langle a, b | a^{-2}bab^2ab, a^2b^{-1}a^2b \rangle$.
This is another instance of group euclid$_1$ and of the manifold EUCLID$_1 \cong |r_5^{26}|$ See next section.

Fig. 200: $\pi_1(|r_{25}^{28}|) = \pi_1(|r_5^{26}|) = [4^2, S_1^4]$, induced by a u^1-reducible 3-gem

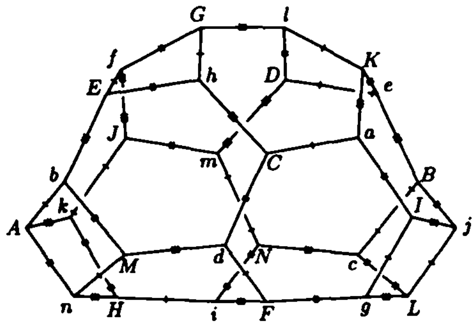
Homology: $Z_2 + Z_2 + Z_4$



$r_6^{28} \in C_{28}(91) \to$
$\langle a, b, c | ab^{-1}ab,$
$b^{-1}a^{-1}cabc, bcb^3c \rangle$.
$r_1 : ab^{-1}ab$
$\Rightarrow a^b = a^{-1}$,
$r_2 : b^{-1}a^{-1}cabc$
$\Rightarrow c^{ab} = c^{-1}$,
$r_3 : bcb^3c$
$\Rightarrow (b^{-1}c^{-1})^2 = b^2$;
therefore b^2 is central, and from r_3 we derive $[c, a] = b^4$. The subgroup $H := \langle a, c \rangle$ is nilpotent and normal; therefore G is a solvable group. By SPAS,
$G/G' \cong C_2^2 \times C_4$,
$G' \cong S_1^4$.

Fig. 201: $\pi_1(|r_6^{28}|) = [2^2 \times 4, S_1^4]$, induced by its u_3^0-attractor

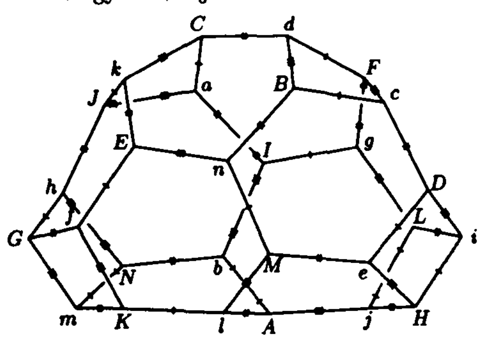
Homology: Z



$r_5^{28} \in C_{28}(93) \rightarrow$
$\langle a, b | b^2ab^{-1}a^{-3}b^{-1}a,$
$a^2ba^{-1}b^{-1}ab^{-1}a^{-1}b\rangle.$
$r_1 : b^2ab^{-1}a^{-3}b^{-1}a$
$\Rightarrow a^{b-1} = (a^{-1})^b a^3,$
$r_2 : a^2ba^{-1}b^{-1}ab^{-1}a^{-1}b$
$\Rightarrow a^2(a^{-1})^{b^{-1}} a(a^{-1})^b$
$\Rightarrow a^{b^{-1}} = a(a^{-1})^b a^2$
$\Rightarrow_{r_1} [a, a^b] = e;$
$H := \langle a, a^b\rangle$ is a commutative normal subgroup; the group is isomorphic to $(Z \times Z) \rtimes_M Z$, where $M = \begin{pmatrix} 0 & 1 \\ -1 & 3 \end{pmatrix}$.
We have denoted this manifold by $Z_{\begin{smallmatrix}0&1\\-1&3\end{smallmatrix}}$.

Fig. 202: $\pi_1(Z_{\begin{smallmatrix}0&1\\-1&3\end{smallmatrix}})$, induced by its u_8^0-attractor

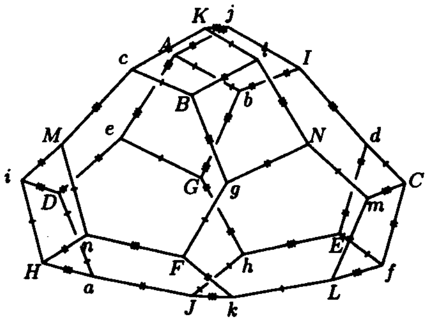
Homology: $Z + Z_5$



$r_{10}^{28} \in C_{28}(102) \rightarrow$
$\langle a, b | aba^{-2}bab^3,$
$b^2aba^{-1}ba^{-1}ba\rangle.$
$r_1 : aba^{-2}bab^3$
$\Rightarrow b^{a^{-1}} = b^{-3}(b^{-1})^a,$
$r_2 : b^2aba^{-1}ba^{-1}ba$
$\Rightarrow b^2b^{a^{-1}}bb^a$
$\Rightarrow_{r_1} [b, b^a] = e; H := \langle b, b^a\rangle$ is a normal commutative subgroup; the group is isomorphic to $Z \times Z) \rtimes_M Z$, where $M = \begin{pmatrix} 0 & 1 \\ -1 & -3 \end{pmatrix}$.
We have denoted this manifold by $Z_{\begin{smallmatrix}0&1\\-1&-3\end{smallmatrix}}$.

Fig. 203: $\pi_1(Z_{\begin{smallmatrix}0&1\\-1&-3\end{smallmatrix}})$, induced by its u_8^0-attractor

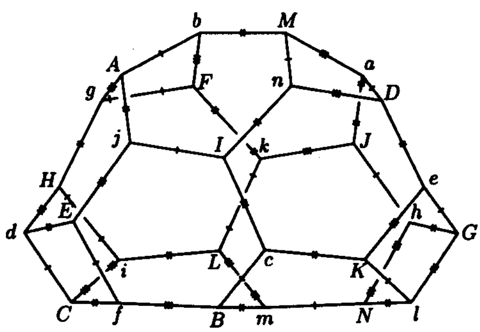
Homology: $Z + Z_2 + Z_2$



$r^{28}_{42} \in C_{28}(104) \rightarrow$
$\langle a, b, c | a^2bcbc^{-1}, c^{-1}b^{-1}abca, a^{-1}b^{-1}ab \rangle$.
$r_1 : a^2bcbc^{-1}$
$\Rightarrow cbc^{-1} = b^{-1}a^{-2}$,
$r_3 : a^{-1}b^{-1}ab \Rightarrow [a, b] = e$,
into $r_2 \Rightarrow a^c = a^{-1}$;
$H := \langle a, b \rangle$ is a commutative normal subgroup. The group is isomorphic to $(Z \times Z) \rtimes_M Z$,
where $M = \begin{pmatrix} -1 & 0 \\ -2 & -1 \end{pmatrix}$.
We have denoted this manifold by $Z^{-1\ 0}_{-2\ -1}$.

Fig. 204: $\pi_1(Z^{-1\ 0}_{-2\ -1})$, induced by its u^0_8-attractor

Homology: $Z + Z + Z_2$



$r^{28}_{280} \in C_{28}(108) \rightarrow$
$\langle a, b, c | b^2c^{-1}abcb^{-1}a^{-1}, cbc^{-1}b^{-1}, abca^{-1}bc^{-1} \rangle$.
$r_2 : cbc^{-1}b^{-1} \Rightarrow [c, b] = e$,
$r_3 : abca^{-1}bc^{-1}$
$\Rightarrow_{r_2} ba^{-1}b = c^{-1}a^{-1}c$;
r_2 into r_1
$\Rightarrow c^{-1}a^{-1}c = a^{-1}b^2$
$\Rightarrow_{r_3} [a, b] = e, b$ is central.
$H := \langle a, b \rangle$ is a commutative normal subgroup. The group is isomorphic to
$(Z \times Z) \rtimes_M Z$,
where $M = \begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}$.
We have denoted this manifold by $Z^{1\ -2}_{0\ 1}$.

Fig. 205: $\pi_1(Z^{1\ -2}_{0\ 1})$, induced by its u^0_5-attractor

5.4 Topological Classification

In this section we provide the topological classification for the closed orientable 3-manifolds which are induced by gems with less than 30 vertices.

In our analysis we divide the 3-manifolds into the following families:

1. Manifolds with cyclic finite fundamental groups (24 manifolds).
2. Manifolds which are non-trivial connected sums (25 manifolds).
3. Manifolds with handle number greater than zero (7 manifolds).
4. Manifolds with finite non-cyclic fundamental groups (21 manifolds).
5. Euclidean manifolds (6 manifolds).
6. Other 3-manifolds (17 manifolds).

There is a total of exactly 24+25+7+21+6+17 = 100 non-homeomorphic orientable 3-manifolds induced by orientable 3-gems up to 28 vertices. We show that in all these families the u_1-classification coincides with the topological classification. The next and subsequent tables are a summary of the u^0-classification performed by the TS_ρ-algorithm on these families. This classification relies on the ts_{ρ_*}-function displayed in the appendix B of Section 8.2.

Our strategy to complete the classification was first to isolate the uncertainties by identifying the fundamental groups. When the fundamental groups coincide in two distinct u^0-classes, we tried, with success, the TS_ρ^U-algorithm. This enabled us to prove that from the 12 uncertainties 11 were indeed homeomorphic. The only remaining case,

$$S^3/\langle 2,2,2\rangle \# L_{3,1} \quad \text{and} \quad S^3/\langle 2,2,2\rangle \# L_{3,2}$$

can proved to be non-homeomorphic by the, so called, *quantum invariants*, see Subsection 7.1.4.

In the tables of u^0-classification we also display the self-linking sequences, which are discussed in Chapter 6.

5.4.1 Manifolds with Cyclic Finite Fundamental Groups

κ	Manifold	Rep.	u^0-class	μ	Non-null self-linking sequence: σ
0	S^3	r_1^2	$C_2(1)$	1	*
3	$L_{2,1}$	r_1^8	$C_8(1)$	1	$\pm1^1$
5	$L_{3,1}$	r_1^{12}	$C_{12}(1)$	1	$+1^2$
7	$L_{4,1}$	r_2^{16}	$C_{16}(1)$	2	$+1^2$
	$L_{5,2}$	r_1^{16}	$C_{16}(2)$	1	$+2^2\ -2^2$
9	$L_{5,1}$	r_2^{20}	$C_{20}(3)$	5	$+1^2\ -1^2$
	$L_{7,2}$	r_5^{20}	$C_{20}(6)$	3	$+1^2\ +2^2\ -3^2$
	$L_{8,3}$	r_1^{20}	$C_{20}(7)$	3	$+3^4\ \pm4^2$
11	$L_{6,1}$	r_{33}^{24}	$C_{24}(11)$	20	$+1^2\ \pm3^1\ -2^2$
	$L_{9,2}$	r_{14}^{24}	$C_{24}(17)$	14	$+1^2\ +4^2\ -2^2$
	$L_{10,3}$	r_{21}^{24}	$C_{24}(19)$	7	$+2^2\ +3^2\ \pm5^1\ -3^2\ -2^2$
	$L_{11,3}$	r_{22}^{24}	$C_{24}(20)$	10	$+1^2\ +3^2\ +4^2\ +5^2\ -2^2$
	$L_{12,5}$	r_{32}^{24}	$C_{24}(21)$	6	$+3^2\ +4^4\ -5^4$
	$L'_{12,5}$	r_{34}^{24}	$C_{24}(22)$	3	$+3^2\ +4^4\ -5^4$
	$L_{13,5}$	r_3^{24}	$C_{24}(23)$	5	$+2^2\ +5^2\ +6^2\ -6^2\ -5^2\ -2^2$
13	$L_{7,1}$	r_{230}^{28}	$C_{28}(22)$	119	$+1^2\ +2^2\ -3^2$
	$L_{11,5}$	r_{49}^{28}	$C_{28}(32)$	95	$+1^2\ +3^2\ +4^2\ +5^2\ -2^2$
	$L_{13,3}$	r_{33}^{28}	$C_{28}(39)$	67	$+1^2\ +3^2\ +4^2\ -4^2\ -3^2\ -1^2$
	$L_{14,3}$	r_{71}^{28}	$C_{28}(41)$	52	$+1^2\ +2^2\ +4^2\ \pm7^1\ -6^2\ -5^2\ -3^2$
	$L_{15,4}$	r_{70}^{28}	$C_{28}(44)$	28	$+1^4\ +4^4\ +6^2\ -6^2\ -5^2$
	$L_{16,7}$	r_{14}^{28}	$C_{28}(46)$	22	$+1^4\ +4^4\ -7^4$
	$L'_{16,7}$	r_{113}^{28}	$C_{28}(48)$	19	$+1^4\ +4^4\ -7^4$
	$L_{17,5}$	r_{65}^{28}	$C_{28}(49)$	26	$+3^2\ +5^2\ +6^2\ +7^2\ -7^2\ -6^2\ -5^2\ -3^2$
	$L'_{17,5}$	r_{106}^{28}	$C_{28}(50)$	20	$+3^2\ +5^2\ +6^2\ +7^2\ -7^2\ -6^2\ -5^2\ -3^2$
	$L_{18,5}$	r_{54}^{28}	$C_{28}(51)$	33	$+1^2\ +4^2\ +7^2\ \pm9^3\ -8^2\ -5^2\ -2^2$
	$L_{19,7}$	r_{41}^{28}	$C_{28}(52)$	20	$+1^2\ +4^2\ +5^2\ +6^2\ +7^2\ +9^2\ -8^2\ -3^2\ -2^2$
	$L'_{19,7}$	r_{81}^{28}	$C_{28}(53)$	16	$+1^2\ +4^2\ +5^2\ +6^2\ +7^2\ +9^2\ -8^2\ -3^2\ -2^2$
	$L'_{21,8}$	r_{13}^{28}	$C_{28}(55)$	6	$+2^4\ +8^4\ +9^2\ -10^4\ -7^2\ -6^2\ -3^2$
	$L_{21,8}$	r_{31}^{28}	$C_{28}(56)$	8	$+2^4\ +8^4\ +9^2\ -10^4\ -7^2\ -6^2\ -3^2$
	$L''_{21,8}$	r_{60}^{28}	$C_{28}(57)$	3	$+2^4\ +8^4\ +9^2\ -10^4\ -7^2\ -6^2\ -3^2$

Table 10A: Manifolds with cyclic fundamental groups

Proposition 26 *The manifolds in $\mathcal{RBG}_n$ $(n \leq 28)$ which have finite cyclic fundamental group, as listed in the above table, are lens spaces.*

Proof: $S^3 = L_{1,1}$ is clearly induced by r_1^2.

$L_{2,1}$ is induced by r_1^8: $\mathcal{S}(2,2,1,1) = G_0 = r_1^8$

$L_{3,1}$ is induced by r_1^{12}: $\mathcal{S}(2,3,1,1) = G_0 = r_1^{12}$

We use the notation described in Subsection 4.1.2.

$L_{4,1}$ is induced by r_2^{16}: $\mathcal{S}(2,4,1,1) = r_3^{16} = G_0 \xrightarrow{D_1^{31}} G_1 = r_2^{16}$

$L_{5,1}$ is induced by r_2^{20}: $\mathcal{S}(2,5,1,1) = r_{23}^{20} = G_0 \xrightarrow{D_1^{31}} G_1 \xrightarrow{D_5^{21}} G_2 = r_2^{20}$

$L_{5,2}$ is induced by r_1^{16}: $\mathcal{S}(2,5,2,1) = G_0 \xrightarrow{D_1^{31}} G_1 \xrightarrow{A_{17}^{3}} G_2 = r_1^{16}$

$L_{6,1}$ is induced by r_{33}^{24}:

$$\mathcal{S}(2,6,1,1) = r_{262}^{24} = G_0 \xrightarrow{D_1^{31}} G_1 \xrightarrow{D_5^{21}} G_2 \xrightarrow{D_{10}^{21}} G_3 = r_{33}^{24}$$

$L_{7,1}$ is induced by r_{230}^{28}:

$$\mathcal{S}(2,7,1,1) = r_{7760}^{28} = G_0 \xrightarrow{D_1^{31}} G_1 \xrightarrow{D_7^{21}} G_2 \xrightarrow{D_5^{31}} G_3 \xrightarrow{D_7^{21}} G_4 = r_{230}^{28}$$

$L_{7,2}$ is induced by r_5^{20}:

$$\mathcal{S}(2,7,2,1) = G_0 \xrightarrow{D_1^{31}} G_1 \xrightarrow{A_{25}^{3}} G_2 \xrightarrow{D_4^{31}} G_3 \xrightarrow{A_{17}^{2}} G_4 = r_5^{20}$$

$L_{8,3}$ is induced by r_1^{20}:

$$\mathcal{S}(2,8,3,1) = G_0 \xrightarrow{D_1^{31}} G_1 \xrightarrow{A_{29}^{3}} G_2 \xrightarrow{A_{25}^{3}} G_3 \xrightarrow{D_4^{13}} G_4 \xrightarrow{A_{15}^{2}} G_5 \xrightarrow{D_5^{31}} G_6 \xrightarrow{D_2^{21}} G_7 = r_1^{20}$$

$L_{9,2}$ is induced by r_{14}^{24}:

$$\mathcal{S}(2,9,2,1) = G_0 \xrightarrow{D_1^{31}} G_1 \xrightarrow{A_{33}^{3}} G_2 \xrightarrow{D_4^{31}} G_3 \xrightarrow{A_{29}^{3}} G_4 \xrightarrow{D_{12}^{31}} G_5 \xrightarrow{A_{22}^{2}}$$
$$G_6 \xrightarrow{A_1^{1}} G_7 \xrightarrow{D_1^{30}} G_8 = r_{14}^{24}$$

$L_{10,3}$ is induced by r_{21}^{24}:

$$\mathcal{S}(2,10,3,1) = G_0 \xrightarrow{D_1^{31}} G_1 \xrightarrow{A_{37}^{3}} G_2 \xrightarrow{A_{33}^{3}} G_3 \xrightarrow{D_4^{31}} G_4 \xrightarrow{A_{25}^{2}} G_5 \xrightarrow{B_{23}^{3}}$$
$$G_6 \xrightarrow{D_6^{21}} G_7 = r_{21}^{24}$$

$L_{11,3}$ is induced by r_{22}^{24}:

$$\mathcal{S}(2,11,3,1) = G_0 \xrightarrow{D_1^{31}} G_1 \xrightarrow{A_{41}^{3}} G_2 \xrightarrow{A_{37}^{3}} G_3 \xrightarrow{A_{33}^{3}} G_4 \xrightarrow{D_4^{31}} G_5 \xrightarrow{A_{30}^{2}} G_6 \xrightarrow{A_{23}^{2}}$$
$$G_7 \xrightarrow{B_{25}^{3}} G_8 \xrightarrow{D_6^{21}} G_9 = r_{22}^{24}$$

$L_{11,5}$ is induced by r^{28}_{49}:

$$\begin{aligned}\mathcal{S}(2,11,5,1) = \ & G_0 \xrightarrow{D^{31}_1} G_1 \xrightarrow{A^3_{41}} G_2 \xrightarrow{D^{31}_4} G_3 \xrightarrow{A^3_{33}} G_4 \xrightarrow{D^{31}_4} G_5 \xrightarrow{A^2_{30}} G_6 \xrightarrow{D^{30}_1} G_7 \xrightarrow{B^3_{27}} \\ & G_8 \xrightarrow{A^2_{22}} G_9 \xrightarrow{B^2_{23}} G_{10} \xrightarrow{D^{21}_6} G_{11} \xrightarrow{E^{312}_3} G_{12} = r^{28}_{49}\end{aligned}$$

$L_{12,5}$ is induced by r^{24}_{32}:

$$\begin{aligned}\mathcal{S}(2,12,5,1) = \ & G_0 \xrightarrow{D^{31}_1} G_1 \xrightarrow{A^3_{45}} G_2 \xrightarrow{A^3_{41}} G_3 \xrightarrow{A^3_{37}} G_4 \xrightarrow{A^3_{33}} G_5 \xrightarrow{D^{31}_4} G_6 \xrightarrow{A^3_{25}} G_7 \\ & G_8 \xrightarrow{D^{31}_{13}} G_9 \xrightarrow{A^3_{26}} G_{10} = r^{24}_{32}\end{aligned}$$

$L_{13,3}$ is induced by r^{28}_{33}:

$$\begin{aligned}\mathcal{S}(2,13,3,1) = \ & G_0 \xrightarrow{D^{31}_1} G_1 \xrightarrow{A^3_{49}} G_2 \xrightarrow{A^3_{45}} G_3 \xrightarrow{A^3_{41}} G_4 \xrightarrow{D^{31}_4} G_5 \xrightarrow{A^2_{33}} G_6 \xrightarrow{B^2_{31}} \\ & G_7 \xrightarrow{A^2_{34}} G_8 \xrightarrow{B^2_{26}} G_9 \xrightarrow{A^2_{30}} G_{10} \xrightarrow{D^{21}_6} G_{11} \xrightarrow{D^{30}_{11}} G_{12} = r^{28}_{33}\end{aligned}$$

$L_{13,5}$ is induced by r^{24}_3:

$$\begin{aligned}\mathcal{S}(2,13,5,1) = \ & G_0 \xrightarrow{D^{31}_1} G_1 \xrightarrow{A^3_{49}} G_2 \xrightarrow{A^3_{45}} G_3 \xrightarrow{A^3_{41}} G_4 \xrightarrow{A^3_{37}} G_5 \xrightarrow{D^{31}_4} G_6 \xrightarrow{A^3_{33}} \\ & G_7 \xrightarrow{A^3_{25}} G_8 \xrightarrow{D^{31}_6} G_9 \xrightarrow{D^{13}_{19}} G_{10} \xrightarrow{A^3_{23}} G_{11} \xrightarrow{D^{32}_{24}} G_{12} \xrightarrow{D^{23}_{24}} G_{13} \xrightarrow{D^{21}_6} G_{14} = r^{24}_3\end{aligned}$$

$L_{14,3}$ is induced by r^{28}_{71}:

$$\begin{aligned}\mathcal{S}(2,14,3,1) = \ & G_0 \xrightarrow{D^{31}_1} G_1 \xrightarrow{A^3_{53}} G_2 \xrightarrow{A^3_{49}} G_3 \xrightarrow[0123]{A^3_{45}} G_4 \xrightarrow{A^3_{41}} G_5 \xrightarrow{D^{31}_4} G_6 \xrightarrow{A^2_{38}} \\ & G_7 \xrightarrow{A^2_{34}} G_8 \xrightarrow{A^2_{34}} G_9 \xrightarrow{A^2_{27}} G_{10} \xrightarrow{B^3_{29}} G_{11} \xrightarrow{D^{31}_5} G_{12} \xrightarrow{D^{31}_{24}} \\ & G_{13} \xrightarrow{E^{031}_7} G_{14} \xrightarrow{D^{20}_9} G_{15} = r^{28}_{71}\end{aligned}$$

$L_{15,4}$ is induced by r^{28}_{70}:

$$\begin{aligned}\mathcal{S}(2,15,4,1) = \ & G_0 \xrightarrow{D^{31}_1} G_1 \xrightarrow{A^3_{57}} G_2 \xrightarrow{A^3_{53}} G_3 \xrightarrow{A^3_{49}} G_4 \xrightarrow{D^{31}_4} G_5 \xrightarrow{A^3_{41}} G_6 \xrightarrow{A^3_{41}} \\ & G_7 \xrightarrow{A^3_{33}} G_8 \xrightarrow{D^{31}_9} G_9 \xrightarrow{A^1_{12}} G_{10} \xrightarrow{D^{31}_{12}} G_{11} \xrightarrow{D^{31}_3} G_{12} \xrightarrow{B^1_{29}} G_{13} \xrightarrow{A^2_{23}} \\ & G_{14} \xrightarrow{D^{31}_5} G_{15} \xrightarrow{D^{23}_{24}} G_{16} \xrightarrow{D^{21}_6} G_{17} \xrightarrow{D^{30}_5} G_{18} = r^{28}_{70}\end{aligned}$$

$L_{16,7}$ is induced by r^{28}_{14}:

$$\begin{aligned}\mathcal{S}(2,16,7,1) = \ & G_0 \xrightarrow{D_1^{31}} G_1 \xrightarrow{A_{61}^{3}} G_2 \xrightarrow{A_{57}^{3}} G_3 \xrightarrow{A_{53}^{3}} G_4 \xrightarrow{A_{49}^{3}} G_5 \xrightarrow{A_{45}^{3}} G_6 \xrightarrow{A_{41}^{3}} \\ & G_7 \xrightarrow{D_4^{31}} G_8 \xrightarrow{A_{33}^{3}} G_9 \xrightarrow{D_{11}^{31}} G_{10} \xrightarrow{A_{10}^{0}} G_{11} \xrightarrow{A_{20}^{1}} G_{12} \xrightarrow{B_4^{0}} G_{13} \xrightarrow{D_{19}^{23}} \\ & G_{14} \xrightarrow{D_{25}^{23}} G_{15} \xrightarrow{D_{20}^{13}} G_{16} \xrightarrow{D_{12}^{21}} G_{17} \xrightarrow{D_7^{21}} G_{18} \xrightarrow{D_{11}^{21}} G_{19} = r_{14}^{28}\end{aligned}$$

$L_{17,5}$ is induced by r_{65}^{28}:

$$\begin{aligned}\mathcal{S}(2,17,5,1) = \ & G_0 \xrightarrow{D_1^{31}} G_1 \xrightarrow{A_{65}^{3}} G_2 \xrightarrow{A_{61}^{3}} G_3 \xrightarrow{A_{57}^{3}} G_4 \xrightarrow{A_{53}^{3}} G_5 \xrightarrow{A_{49}^{3}} G_6 \xrightarrow{A_{45}^{3}} \\ & G_7 \xrightarrow{D_4^{31}} G_8 \xrightarrow{A_{37}^{3}} G_9 \xrightarrow{A_{37}^{3}} G_{10} \xrightarrow{D_{10}^{31}} G_{11} \xrightarrow{B_5^{0}} G_{12} \xrightarrow{D_8^{31}} G_{13} \xrightarrow{B_{30}^{3}} \\ & G_{14} \xrightarrow{D_6^{31}} G_{15} \xrightarrow{A_{12}^{0}} G_{16} \xrightarrow{E_8^{201}} G_{17} = r_{65}^{28}\end{aligned}$$

$L_{18,5}$ is induced by r_{54}^{28}:

$$\begin{aligned}\mathcal{S}(2,18,5,1) = \ & G_0 \xrightarrow{D_1^{31}} G_1 \xrightarrow{A_{69}^{3}} G_2 \xrightarrow{A_{65}^{3}} G_3 \xrightarrow{A_{61}^{3}} G_4 \xrightarrow{A_{57}^{3}} G_5 \xrightarrow{A_{53}^{3}} G_6 \xrightarrow{A_{49}^{3}} \\ & G_7 \xrightarrow{D_4^{31}} G_8 \xrightarrow{A_{45}^{3}} G_9 \xrightarrow{A_{37}^{3}} G_{10} \xrightarrow{A_{33}^{3}} G_{11} \xrightarrow{D_9^{31}} G_{12} \xrightarrow{A_{29}^{3}} G_{13} \xrightarrow{A_{31}^{2}} \\ & G_{14} \xrightarrow{F_{21}^{0}} G_{15} \xrightarrow{B_{23}^{3}} G_{16} \xrightarrow{D_{26}^{32}} G_{17} \xrightarrow{D_{28}^{23}} G_{18} \xrightarrow{D_6^{21}} G_{19} \xrightarrow{D_7^{02}} G_{20} = r_{54}^{28}\end{aligned}$$

$L_{19,7}$ is induced by r_{41}^{28}:

$$\begin{aligned}\mathcal{S}(2,19,7,1) = \ & G_0 \xrightarrow{D_1^{31}} G_1 \xrightarrow{A_{73}^{3}} G_2 \xrightarrow{A_{69}^{3}} G_3 \xrightarrow{A_{65}^{3}} G_4 \xrightarrow{A_{61}^{3}} G_5 \xrightarrow{A_{57}^{3}} G_6 \xrightarrow{A_{53}^{3}} \\ & G_7 \xrightarrow{A_{49}^{3}} G_8 \xrightarrow{D_4^{31}} G_9 \xrightarrow{A_{41}^{3}} G_{10} \xrightarrow{A_{37}^{3}} G_{11} \xrightarrow{D_7^{31}} G_{12} \xrightarrow{A_{29}^{2}} G_{13} \xrightarrow{B_{31}^{3}} \\ & G_{14} \xrightarrow{D_{12}^{31}} G_{15} \xrightarrow{D_{21}^{21}} G_{16} \xrightarrow{B_{27}^{2}} G_{17} \xrightarrow{D_{27}^{12}} G_{18} \xrightarrow{D_8^{21}} G_{19} \xrightarrow{D_{11}^{21}} G_{20} = r_{41}^{28}\end{aligned}$$

$L_{21,8}$ is induced by r_{31}^{28}:

$$\begin{aligned}\mathcal{S}(2,21,8,1) = \ & G_0 \xrightarrow{D_1^{31}} G_1 \xrightarrow{A_{81}^{3}} G_2 \xrightarrow{A_{77}^{3}} G_3 \xrightarrow{A_{73}^{3}} G_4 \xrightarrow{A_{69}^{3}} G_5 \xrightarrow{A_{65}^{3}} G_6 \xrightarrow{A_{61}^{3}} \\ & G_7 \xrightarrow{A_{57}^{3}} G_8 \xrightarrow{D_4^{31}} G_9 \xrightarrow{A_{49}^{3}} G_{10} \xrightarrow{A_{45}^{3}} G_{11} \xrightarrow{A_{45}^{3}} G_{12} \xrightarrow{A_{37}^{3}} G_{13} \xrightarrow{D_5^{31}} \\ & G_{14} \xrightarrow{A_{38}^{2}} G_{15} \xrightarrow{C_{33}^{3}} G_{16} \xrightarrow{D_5^{31}} G_{17} \xrightarrow{B_{23}^{3}} G_{18} \xrightarrow{D_4^{20}} G_{19} \xrightarrow{D_9^{20}} G_{20} \xrightarrow{A_{30}^{2}} \\ & G_{21} \xrightarrow{D_7^{21}} G_{22} \xrightarrow{D_{22}^{32}} G_{23} \xrightarrow{D_{10}^{21}} G_{24} = r_{31}^{28}\end{aligned}$$

This concludes the proof that all manifolds in the above table are indeed lens spaces. ∎

We now treat the cases of manifolds with finite cyclic fundamental groups let unresolved by the u_0-classification.

Proposition 27 *(a) $L_{12,5} = |r^{24}_{32}|$ and $L'_{12,5} = |r^{24}_{34}|$ are homeomorphic. (b) $L_{16,7} = |r^{28}_{14}|$ and $L'_{16,7} = |r^{28}_{113}|$ are homeomorphic. (c) $L_{17,5} = |r^{28}_{65}|$ and $L'_{17,5} = |r^{28}_{106}|$ are homeomorphic. (d) $L_{19,7} = |r^{28}_{41}|$ and $L'_{19,7} = |r^{28}_{81}|$ are homeomorphic. (e) $L_{21,8} = |r^{28}_{31}|$, $L'_{21,8} = |r^{28}_{13}|$ and $L''_{21,8} = |r^{28}_{60}|$ are homeomorphic. Moreover, any two 3-gems in the attractor of these five lens spaces are linked by a single u^1_*-move.*

Proof:

$$r^{24}_{32} = G_0 \xrightarrow[\rho]{U^1_2} G_1 \xrightarrow{D^{31}_2} G_2 \xrightarrow{A^3_{35}} G_3 \xrightarrow{A^3_{30}} G_4 \xrightarrow{D^{21}_{24}} G_5 \xrightarrow{D^{23}_{21}} G_6 \xrightarrow{D^{21}_6} G_7 = r^{24}_{44} \equiv r^{24}_{34}$$

$$r^{24}_{34} = G_0 \xrightarrow[\rho]{U^1_{21}} G_1 \xrightarrow{D^{31}_6} G_2 \xrightarrow{A^3_{25}} G_3 \xrightarrow{D^{21}_{10}} G_4 = r^{24}_{115} \equiv r^{24}_{32}$$

In this way, both 3-manifolds in (a) are the lens space $L_{12,5}$. The attractor for this manifold is formed by the 9 gems obtained as the union of its two u^0-classes.

$$r^{28}_{14} = G_0 \xrightarrow[\rho]{U^1_2} G_1 \xrightarrow{D^{31}_8} G_2 \xrightarrow{A^0_8} G_3 \xrightarrow{A^3_{29}} G_4 \xrightarrow{D^{31}_{12}} G_5 \xrightarrow{D^{31}_{10}} G_6 = r^{28}_{7323} \equiv r^{28}_{113}$$

$$\begin{aligned} r^{28}_{113} = {} & G_0 \xrightarrow[\rho]{U^1_{17}} G_1 \xrightarrow{D^{31}_7} G_2 \xrightarrow{A^2_{25}} G_3 \xrightarrow{D^{31}_8} G_4 \xrightarrow{B^3_{29}} G_5 \xrightarrow{A^3_{29}} G_6 \xrightarrow{D^{23}_{19}} G_7 \xrightarrow{D^{23}_{25}} \\ & G_8 \xrightarrow{D^{13}_{20}} G_9 \xrightarrow{D^{21}_{12}} G_{10} \xrightarrow{D^{21}_7} G_{11} \xrightarrow{D^{21}_{11}} G_{12} = r^{28}_{131} \equiv r^{28}_{14} \end{aligned}$$

Therefore, both 3-manifolds in (b) are the lens space $L_{16,7}$. The attractor for this manifold is formed by the 41 gems obtained as the union of its two u^0-classes.

$$\begin{aligned} r^{28}_{65} = {} & G_0 \xrightarrow[\rho]{U^2_{28}} G_1 \xrightarrow{D^{31}_5} G_2 \xrightarrow{A^3_{40}} G_3 \xrightarrow{A^3_{24}} G_4 \xrightarrow{A^1_{28}} G_5 \xrightarrow{A^2_{30}} G_6 \xrightarrow{D^{31}_3} G_7 \xrightarrow{D^{30}_{13}} \\ & G_8 = r^{28}_{7069} \equiv r^{28}_{106} \end{aligned}$$

$$\begin{aligned} r^{28}_{106} = {} & G_0 \xrightarrow[\rho]{U^1_{15}} G_1 \xrightarrow{A^2_{39}} G_2 \xrightarrow{D^{31}_2} G_3 \xrightarrow{A^3_{33}} G_4 \xrightarrow{A^2_{34}} G_5 \xrightarrow{D^{31}_7} G_6 \xrightarrow{B^2_{26}} G_7 \xrightarrow{E^{201}_8} \\ & G_8 = r^{28}_{6692} \equiv r^{28}_{65} \end{aligned}$$

It follows that both 3-manifolds in (c) are the lens space $L_{17,5}$. The attractor for this manifold is formed by the 46 gems obtained as the union of its two u^0-classes.

$$\begin{aligned} r^{28}_{41} = {} & G_0 \xrightarrow[\rho]{U^1_{16}} G_1 \xrightarrow{D^{31}_7} G_2 \xrightarrow{A^3_{29}} G_3 \xrightarrow{B^2_{25}} G_4 \xrightarrow{D^{31}_4} G_5 \xrightarrow{A^3_{25}} G_6 \xrightarrow{D^{13}_{21}} G_7 \xrightarrow{D^{31}_3} \\ & G_8 \xrightarrow{D^{12}_{10}} G_9 \xrightarrow{D^{31}_{21}} G_{10} = r^{28}_{1355} \equiv r^{28}_{81} \end{aligned}$$

$$r_{81}^{28} = G_0 \xrightarrow[\rho]{U_1^1} G_1 \xrightarrow{D_6^{31}} G_2 \xrightarrow{A_{38}^3} G_3 \xrightarrow{A_{35}^2} G_4 \xrightarrow{A_{34}^3} G_5 \xrightarrow{D_8^{31}} G_6 \xrightarrow{D_4^{31}} G_7 \xrightarrow{B_{29}^3}$$
$$G_8 \xrightarrow{D_{10}^{21}} G_9 = r_{86}^{28} \equiv r_{41}^{28}$$

In this way, both 3-manifolds in (d) are the lens space $L_{19,7}$. The attractor for this manifold is formed by the 36 gems obtained as the union of its two u^0-classes.

$$r_{13}^{28} = G_0 \xrightarrow[\rho]{U_6^1} G_1 \xrightarrow{D_7^{31}} G_2 \xrightarrow{A_{38}^3} G_3 \xrightarrow{D_5^{31}} G_4 \xrightarrow{D_6^{31}} G_5 \xrightarrow{A_9^1} G_6 \xrightarrow{D_{24}^{32}} G_7 \xrightarrow{D_{23}^{01}}$$
$$G_8 \xrightarrow{A_{23}^2} G_9 \xrightarrow{D_9^{31}} G_{10} = r_{60}^{28}$$

$$r_{60}^{28} = G_0 \xrightarrow[\rho]{U_3^1} G_1 \xrightarrow{D_2^{31}} G_2 \xrightarrow{A_{29}^3} G_3 \xrightarrow{D_{12}^{31}} G_4 \xrightarrow{D_{30}^{10}} G_5 \xrightarrow{A_{28}^3} G_6 \xrightarrow{D_7^{21}} G_7 \xrightarrow{D_{22}^{32}}$$
$$G_8 \xrightarrow{D_{10}^{21}} G_9 = r_{1673}^{28} \equiv r_{31}^{28}$$

$$r_{31}^{28} = G_0 \xrightarrow[\rho]{U_{28}^1} G_1 \xrightarrow{D_2^{31}} G_2 \xrightarrow{D_{11}^{31}} G_3 \xrightarrow{D_{13}^{20}} G_4 \xrightarrow{B_{25}^3} G_5 \xrightarrow{D_{24}^{13}} G_6 \xrightarrow{D_3^{31}} G_7 \xrightarrow{D_6^{21}} G_8 = r_{53}^{28} \equiv r_{13}^{28}$$

The 3-manifolds in (e) are, therefore, the lens space $L_{21,8}$. The attractor for this manifold is formed by the 17 gems obtained as the union of its three u^0-classes. This concludes the proof of the proposition. ∎

5.4.2 Non-Trivial Connected Sums

We list now the orientable 3-manifolds induced by 3-gems with less than 30 vertices which are non-trivial connected sums, i.e., those which have summands distinct from S^3 and from $S^1 \times S^2$.

κ	Manifold	Rep.	u^0-class	μ	Hom	sequence: σ
6	$L_{2,1}\#L_{2,1}$	r_1^{14}	$C_{14}(1)$	1	$(0)2^2$	$\pm 2^2$
8	$L_{2,1}\#L_{3,1}$	r_4^{18}	$C_{18}(2)$	1	$(0)6^1$	$+1^2\ \pm 3^1\ -2^2$
9	$\#_3 L_{2,1}$	r_{10}^{20}	$C_{20}(9)$	2	$(0)2^3$	$\pm 4^4$
10	$L_{2,1}\#L_{5,2}$	r_5^{22}	$C_{22}(9)$	3	$(0)10^1$	$+1^2\ +4^2\ \pm 5^1\ -4^2\ -1^2$
	$L_{2,1}\#L_{4,1}$	r_3^{22}	$C_{22}(11)$	5	$(0)2^1 4^1$	$2^2\ \pm 4^2\ -2^2$
	$L_{3,1}^+\#L_{3,1}^+$	r_{16}^{22}	$C_{22}(12)$	3	$(0)3^2$	$+3^4\ -3^4$
	$L_{3,1}^+\#L_{3,1}^-$	r_{17}^{22}	$C_{22}(13)$	2	$(0)3^2$	$+3^2\ -3^2$
11	$L_{2,1}\#L_{2,1}\#L_{3,1}$	r_{75}^{24}	$C_{24}(29)$	9	$(0)2^1 6^1$	$+2^4\ \pm 6^2\ -4^4$
	$L_{2,1}\#\mathrm{QUAT}$	r_{108}^{24}	$C_{24}(33)$	2	$(0)2^3$	$\pm 4^4$
12	$L_{2,1}\#L_{5,1}$	r_{49}^{26}	$C_{26}(22)$	32	$(0)10^1$	$+2^2\ +3^2\ \pm 5^1\ -3^2\ -2^2$
	$L_{3,1}^+\#L_{4,1}^+$	r_{254}^{26}	$C_{26}(26)$	15	$(0)12^1$	$+1^4\ +4^4\ -3^2$
	$L_{3,1}^+\#L_{4,1}^-$	r_{256}^{26}	$C_{26}(27)$	15	$(0)12^1$	$+3^2\ +4^4\ -5^4$
	$L_{2,1}\#L_{7,2}$	r_{48}^{26}	$C_{26}(29)$	29	$(0)14^1$	$+1^2\ +2^2\ +4^2$ $\pm 7^1\ -6^2\ -5^2\ -3^2$
	$L_{3,1}\#L_{5,2}$	r_{227}^{26}	$C_{26}(31)$	15	$(0)15^1$	$+1^4\ +4^4\ +6^2\ -6^2\ -5^2$
	$L_{2,1}\#C_3^i C_4$	r_{397}^{26}	$C_{26}(37)$	2	$(0)2^1 4^1$	$+2^2\ \pm 4^2\ -2^2$
	$L_{2,1}\#L_{8,3}$	r_{25}^{26}	$C_{26}(41)$	14	$(0)2^1 8^1$	$+2^4\ \pm 8^4\ -6^4$
	$\#_4 L_{2,1}$	r_{22}^{26}	$C_{26}(49)$	7	$(0)2^4$	$\pm 8^8$
13	$L_{2,1}\#\mathrm{BINTET}$	r_{453}^{28}	$C_{28}(19)$	6	$(0)6^1$	$+1^2\ \pm 3^1\ -2^2$
	$L_{3,1}^+\#\mathrm{QUAT}$	r_{1473}^{28}	$C_{28}(69)$	6	$(0)2^1 6^1$	$+4^8$
	$L_{3,1}^-\#\mathrm{QUAT}$	r_{1474}^{28}	$C_{28}(70)$	6	$(0)2^1 6^1$	$+4^8$
	$L_{2,1}\#C_3^i C_8$	r_{177}^{28}	$C_{28}(72)$	10	$(0)2^1 8^1$	$+2^4\ \pm 8^4\ -6^4$
	$L_{2,1}\#L_{2,1}\#L_{5,2}$	r_{473}^{28}	$C_{28}(75)$	27	$(0)2^1 10^1$	$+2^4\ +8^4\ \pm 10^2\ -8^4\ -2^4$
	$L_{2,1}\#(L_{3,1}^+\#L_{3,1}^-)$	r_{514}^{28}	$C_{28}(79)$	6	$(0)3^1 6^1$	$+3^2\ +6^2\ \pm 9^5\ -6^2\ -3^2$
	$L_{2,1}\#(L_{3,1}^+\#L_{3,1}^+)$	r_{517}^{28}	$C_{28}(80)$	6	$(0)3^1 6^1$	$+3^4\ +6^4\ \pm 9^1\ -6^4\ -3^4$
	$L'_{2,1}\#(L_{3,1}^+\#L_{3,1}^-)$	r_{3659}^{28}	$C_{28}(81)$	15	$(0)3^1 6^1$	$+3^2\ +6^2\ \pm 9^5\ -6^2\ -3^2$
	$L'_{2,1}\#(L_{3,1}^+\#L_{3,1}^+)$	r_{3661}^{28}	$C_{28}(82)$	15	$(0)3^1 6^1$	$+3^4\ +6^4\ \pm 9^1\ -6^4\ -3^4$
	$L''_{2,1}\#(L_{3,1}^+\#L_{3,1}^+)$	r_{6372}^{28}	$C_{28}(83)$	3	$(0)3^1 6^1$	$+3^4\ +6^4\ \pm 9^1\ -6^4\ -3^4$
	$L''_{2,1}\#(L_{3,1}^+\#L_{3,1}^-)$	r_{6378}^{28}	$C_{28}(84)$	2	$(0)3^1 6^1$	$+3^2\ +6^2\ \pm 9^5\ -6^2\ -3^2$
	$L_{2,1}\#L_{2,1}\#L_{4,1}$	r_{485}^{28}	$C_{28}(92)$	55	$(0)2^2 4^1$	$+4^4\ \pm 8^4\ -4^4$

Table 10B: Manifolds which are non-trivial connected sums

The u_0-classification leaves open the homeomorphism decision for the following connected sums:

1. $L_{2,1}\#(L^+_{3,1}\#L^+_{3,1}) = |r^{28}_{514}|$, $L'_{2,1}\#(L^+_{3,1}\#L^+_{3,1}) = |r^{28}_{3659}|$ and $L''_{2,1}\#(L^+_{3,1}\#L^+_{3,1}) = |r^{28}_{6378}|$
2. $L_{2,1}\#(L^+_{3,1}\#L^-_{3,1}) = |r^{28}_{517}|$, $L'_{2,1}\#(L^+_{3,1}\#L^-_{3,1}) = |r^{28}_{3661}|$ and $L''_{2,1}\#(L^+_{3,1}\#L^-_{3,1}) = |r^{28}_{6372}|$

By using the Walking Lemma is not difficult to prove that the manifolds in (1) and (2) are homeomorphic. However, we can also prove this only by single u^1_*-moves, providing a uniform classification.

Proposition 28 *(a) The three 3-manifolds given in (1) are homeomorphic. (b) The three 3-manifolds given in (2) are homeomorphic. Any two gems in the attractor of these two 3-manifolds are linked by a single u^1_*-move.*

Proof:

$$r^{28}_{514} = G_0 \xrightarrow[\rho]{U^1_{19}} G_1 \xrightarrow{D^{31}_2} G_2 \xrightarrow{A^3_{24}} G_3 = r^{28}_{6378}$$

$$r^{28}_{6378} = G_0 \xrightarrow[\rho]{U^1_3} G_1 \xrightarrow{D^{31}_5} G_2 \xrightarrow{A^3_{38}} G_3 \xrightarrow{B^{32}_{30}} G_4 \xrightarrow{D^{31}_4} G_5 = r^{28}_{4814} \equiv r^{28}_{514}$$

$$r^{28}_{514} = G_0 \xrightarrow[\rho]{U^1_{16}} G_1 \xrightarrow{D^{31}_3} G_2 \xrightarrow{A^3_{35}} G_3 \xrightarrow{B^3_{31}} G_4 \xrightarrow{A^1_{23}} G_5 \xrightarrow{A^3_{27}} G_6 \xrightarrow{D^{31}_2} G_7 = r^{28}_{5018} \equiv r^{28}_{3659}$$

$$r^{28}_{3659} \equiv r^{28}_{3682} = G_0 \xrightarrow[\rho]{U^2_{27}} G_1 \xrightarrow{D^{31}_3} G_2 \xrightarrow{A^1_{14}} G_3 \xrightarrow{B^1_{12}} G_4 \xrightarrow{A^0_3} G_5 \xrightarrow{B^0_2} G_6 \xrightarrow{B^1_{11}} G_7 \xrightarrow{B^3_{29}}$$
$$G_8 \xrightarrow{D^{31}_4} G_9 = r^{28}_{4814} \equiv r^{28}_{514}$$

This proves part (a) of the proposition.

$$r^{28}_{517} = G_0 \xrightarrow[\rho]{U^2_{16}} G_1 \xrightarrow{D^{31}_3} G_2 \xrightarrow{A^3_{35}} G_3 \xrightarrow{B^3_{31}} G_4 \xrightarrow{A^1_{23}} G_5 \xrightarrow{A^3_{27}} G_6 \xrightarrow{D^{31}_3} G_7 = r^{28}_{5009} \equiv r^{28}_{3661}$$

$$r^{28}_{3661} \equiv r^{28}_{3681} = G_0 \xrightarrow[\rho]{U^2_{27}} G_1 \xrightarrow{D^{31}_3} G_2 \xrightarrow{A^1_{14}} G_3 \xrightarrow{B^1_{12}} G_4 \xrightarrow{A^0_3} G_5 \xrightarrow{B^0_2} G_6 \xrightarrow{B^1_{11}} G_7 \xrightarrow{B^3_{29}}$$
$$G_8 \xrightarrow{D^{23}_{22}} G_9 \xrightarrow{A^3_{24}} G_{10} = r^{28}_{6372}$$

$$r^{28}_{6372} = G_0 \xrightarrow[\rho]{U^1_2} G_1 \xrightarrow{D^{31}_5} G_2 \xrightarrow{A^3_{38}} G_3 \xrightarrow{A^3_{29}} G_4 \xrightarrow{B^3_{30}} G_5 \xrightarrow{D^{31}_4} G_6 = r^{28}_{4813} \equiv r^{28}_{517}$$

Proving part (b) and establishing the proposition. ∎

Proposition 29 *$L^+_{3,1}\#QUAT$ and $L^-_{3,1}\#QUAT$ are non-homeomorphic.*

At first, the invariants at our disposal were insuficient for differentiate them and the u_0-classification is uncapable of proving that they are equal. However new quantum invariants prove them to be different. See Subsection 7.1.5, where these spaces are denoted $L_{3,1}\#S^3/\langle 2,2,2\rangle$ and $L_{3,2}\#S^3/\langle 2,2,2\rangle$. ∎

5.4.3 Manifolds with Handle Number Greater than 0

κ	Manifold	Rep.	u^0-class	μ	Self-linking sequence: σ
9	$S^1 \times S^2$	r^{20}_9	$C_{20}(10)$	1	*
	$S^1 \times S^2\#L_{2,1}$	r^{20}_{11}	$C_{20}(11)$	1	$\pm 1^1$
11	$S^1 \times S^2\#L_{3,1}$	r^{24}_{169}	$C_{24}(38)$	3	$+1^2$
12	$S^1 \times S^2\#L_{2,1}\#L_{2,1}$	r^{28}_{29}	$C_{26}(57)$	6	$\pm 2^2$
	$S^1 \times S^2\#S^1 \times S^2$	r^{26}_{89}	$C_{26}(60)$	5	*
13	$S^1 \times S^2\#L_{4,1}$	r^{28}_{3383}	$C_{28}(101)$	16	$+1^2$
	$S^1 \times S^2\#L_{5,2}$	r^{28}_{490}	$C_{28}(103)$	10	$+2^2\ -2^2$

Table 10C: Manifolds with handle number greater than 0

As we have mentioned these manifolds are the only ones in which the smallest rigid 3-gems inducing them have more vertices than the members of the attractors for the corresponding 3-manifold. This is because connected sums with $s^1 \times s^2$ are adequately described just by the number of summands: by the Walking Lemma and the fact that $s^1 \times s^2$ admits an orientation reversing automorphism, follow that all the connected sums are independent of where they occur.

5.4.4 Manifolds with Finite Non-Cyclic Fundamental Groups

κ	Manifold	Rep.	u^0-class	μ	Hom	sequence: σ
8	QUAT	r_1^{18}	$C_{18}(3)$	1	$(0)2^2$	*
	$S^3/\langle 2,2,2\rangle$					
9	$S^3/\langle 3,2,2\rangle$	r_4^{20}	$C_{20}(1)$	1	$(0)4^1$	$+1^2$
10	BINTET	r_1^{22}	$C_{22}(1)$	1	$(0)3^1$	$+1^2$
	$S^3/\langle 3,3,2\rangle$					
	$S^3/(C_3 \rtimes_i C_8)$	r_2^{22}	$C_{22}(7)$	1	$(0)8^1$	$+1^4 \pm 4^2$
11	BINDOD	r_2^{24}	$C_{24}(1)$	1	(0)	*
	$S^3/\langle 5,3,2\rangle$					
	BINOCT	r_{28}^{24}	$C_{24}(3)$	1	$(0)2^1$	$\pm 1^1$
	$S^3/\langle 4,3,2\rangle$					
	$S^3/(C_5 \rtimes_i C_8)$	r_4^{24}	$C_{24}(14)$	1	$(0)8^1$	$+1^4 \pm 4^2$
	S^3/Q_{16}	r_{154}^{24}	$C_{24}(26)$	2	$(0)2^2$	$\pm 2^2$
	$S^3/(C_3 \times Q_8)$	r_{13}^{24}	$C_{24}(28)$	2	$(0)2^1 6^1$	$+4^8$
12	$S^3/(C_7 \times \langle 5,3,2\rangle)$	r_{10}^{26}	$C_{26}(13)$	1	$(0)7^1$	$+1^2 +2^2 -3^2$
	$S^3/(Q_8 \rtimes_3 C_9)$	r_8^{26}	$C_{26}(18)$	11	$(0)9^1$	$+1^2 +4^2 -2^2$
	$S^3/(C_5 \rtimes_i C_{12})$	r_6^{26}	$C_{26}(24)$	4	$(0)12^1$	$+1^4 +4^4 -3^2$
	$S^3/(Q_8 \rtimes_3 C_{15})$	r_4^{26}	$C_{26}(30)$	1	$(0)15^1$	$+2^4 +3^2 +5^2 -7^4 -3^2$
	$S^3/(Q_8 \rtimes_3' C_{15})$	r_{696}^{26}	$C_{26}(32)$	1	$(0)15^1$	$+2^4 +3^2 +5^2 -7^4 -3^2$
	$S^3/(C_3 \times Q_{16})$	r_{65}^{26}	$C_{26}(39)$	3	$(0)2^1 6^1$	$+2^4 \pm 6^2 -4^4$
13	$S^3/(C_5 \rtimes_i C_4)$	r_{2418}^{28}	$C_{28}(13)$	18	$(0)4^1$	$+1^2$
	$S^3/(C^7 \rtimes_i C_8)$	r_{2314}^{28}	$C_{28}(26)$	6	$(0)8^1$	$+3^4 \pm 4^2$
	$S^3/(C_7 \rtimes_i C_{12})$	r_{402}^{28}	$C_{28}(38)$	2	$(0)12^1$	$+1^4 +4^4 -3^2$
	$S^3/(C_3 \rtimes_i C_{16})$	r_{27}^{28}	$C_{28}(47)$	18	$(0)16^1$	$+3^4 -5^4 -4^4$
	$S^3/(C_3 \rtimes_i C_{20})$	r_{29}^{28}	$C_{28}(54)$	14	$(0)20^1$	$+3^4 +7^4 +8^4 -8^4 -5^2$
	$S^3/(C_3 \times Q_{32})$	r_{56}^{28}	$C_{28}(66)$	2	$(0)2^1 6^1$	$+2^4 \pm 6^2 -4^4$
	$S^3/(C_5 \times Q_8)$	r_7^{28}	$C_{28}(73)$	6	$(0)2^1 10^1$	$+8^8 -8^8$
	$S^3/(C_5 \times' Q_8)$	r_{59}^{28}	$C_{28}(74)$	4	$(0)2^1 10^1$	$+8^8 -8^8$

Table 10D: Manifolds with finite non-cyclic fundamental groups

We show that the two *primed'* manifolds above are indeed duplicates.

Proposition 30 *(a) $S^3/(Q_8 \rtimes_3 C_{15}) = |r_4^{26}|$ and $S^3/(Q_8 \rtimes_3' C_{15}) = |r_{696}^{26}|$ are homeomorphic. (b) $S^3/(C_5 \times Q_8) = |r_7^{28}|$ and $S^3/(C_5 \times' Q_8) = |r_{59}^{28}|$ are homeomorphic. Any two gems in the attractor of these two 3-manifolds are linked by a single u_*^1-move.*

Proof:

$$r_4^{26} = \ G_0 \xrightarrow[\rho]{U_1^1} G_1 \xrightarrow{A_1^1} G_2 \xrightarrow{D_6^{31}} G_3 \xrightarrow{A_{32}^3} G_4 \xrightarrow{B_{25}^3} G_5 \xrightarrow{D_6^{31}} G_6 \xrightarrow{D_{22}^{10}} G_7 \xrightarrow{D_{13}^{23}} G_8 \xrightarrow{A_{26}^3} G_9 = r_{696}^{26}$$

$$r^{26}_{696} = G_0 \underset{\rho}{\xrightarrow{U^1_{12}}} G_1 \xrightarrow{D^{31}_3} G_2 \xrightarrow{A^2_{34}} G_3 \xrightarrow{B^3_{32}} G_4 \xrightarrow{B^2_{30}} G_5 \xrightarrow{D^{31}_5} G_6 \xrightarrow{A^0_7} G_7 = r^{26}_4.$$

From the above two lines we get a $\overline{u}^1$-move between r^{26}_4 and r^{26}_{696}, which proves (a).

$$r^{28}_7 = G_0 \underset{\rho}{\xrightarrow{U^1_{16}}} G_1 \xrightarrow{D^{31}_6} G_2 \xrightarrow{A^3_{33}} G_3 \xrightarrow{A^1_1} G_4 \xrightarrow{A^3_{26}} G_5 \xrightarrow{D^{13}_{21}} G_6 \xrightarrow{D^{31}_4} G_7 = r^{28}_{1373} \equiv r^{28}_{59}$$

$$r^{28}_{59} = G_0 \underset{\rho}{\xrightarrow{U^1_{14}}} G_1 \xrightarrow{A^3_{37}} G_2 \xrightarrow{D^{31}_7} G_3 \xrightarrow{A^0_9} G_4 \xrightarrow{A^0_9} G_5 \xrightarrow{D^{21}_{18}} G_6 \xrightarrow{D^{23}_{17}} G_7 \xrightarrow{D^{21}_6} G_8 = r^{28}_{2269} \equiv r^{28}_7.$$

From the above two lines we get a $\overline{u}^1$-move between r^{28}_7 and r^{28}_{59}, which proves (b). ∎

Before proving that the above 3-manifolds are quotients of S^3 we need to introduce the concept of *derived 3-gem.*

5.4.5 Covering 3-Gems, Derived 3-Gems

This subsection originally appears in [LS95]. In the tables of the previous subsections the groups are finite and so one would expect them to correspond to spaces which are elliptic 3-manifolds; i.e., quotients of the 3-sphere S^3. Since we are dealing with a catalogue without omissions, an exotic 3-manifold could show up. From the 3-gems themselves, we cannot be certain, for their generating process was completely blind to geometrical descriptions. However, up to the present level examined, $\kappa \leq 13$, the universal covering of the 3-manifolds with finite π_1 have been verified to be S^3, by a combination of two computer packages, named GDV and TS_ρ. In this subsection we explain the algorithm GDV.

Let J be a normal subgroup of finite index n of $\pi_1(M^3)$ for some closed 3-manifold M^3. It is well known that there exists a covering space N^3 such that $\pi_1(N^3)$ is J, [ST80]. Thus, the manifold M^3 is a quotient of N^3. The GDV algorithm applies to the case where J is the derived subgroup in the case where it is of finite index in π_1; that is, when π_1 has finite homology.

In the general case the procedure starts with a permutation representation of π_1 on the n left cosets of J in π_1. The output is an n-covering 3-gem H so that $|H| \cong N^3$ and $\pi_1(|H|) \simeq J$. The number of vertices of H will be n times that of G. However, by using TS_ρ H usually simplifies a lot.

The group $\frac{\pi_1(G)}{J}$ has order n. By using the Todd-Coxeter algorithm, [CM72a], as implemented in [SPA] or [CAY], one can obtain a transitive permutation group Q, on the set of left cosets of J in π_1, which is isomorphic to the quotient $\frac{\pi_1(G)}{J}$. The group Q is normally called the monodromy group of the pair of complexes (G^3, H^3) [ST80].

The construction which we give next is based on the 0-presentation algorithm and it works for general 3-gems. Each generator a_i related to the i-th 0-colored oriented (from odd to even) edge of G is mapped, by the quotient map, into a permutation q_i of Q. These permutations are the only data we need to construct H.

The construction of H is as follows. Make n copies of the 0-missing 3-residues of G. Let (v_i, k) denote the vertex of H which arises as the *k-th* copy of the *i*-th vertex v_k of G. We are assuming that the 0-colored edges of G connect vertices v_{2i-1} and v_{2i}. To complete the definition of H link vertex (v_{2i-1}, k) to vertex $(v_{2i}, q_i(k))$ by a 0-colored edge.

Proposition 31 *Let G be a rigid 3-gem and let J be a normal subgroup of index n of $\pi_1(|G|)$. Suppose that Q is the image of the representation of $\pi_1(|G|)$ on the left cosets of J in $\pi_1(|G|)$. Then the $(3+1)$-graph H described above is a 3-gem and*

$$\pi_1(|H|) \simeq J.$$

Proof: Consider the covering mapping from the vertices of H onto the vertices of G, $(v,k) \mapsto v$. This mapping when restricted to a bigon of H is an isomorphism of colored graphs. This is clear if the bigon does not involve color 0 and it follows from the fact that, the 0-colored edges around a $(0,c)$-bigon define a relator (see the 0-presentation algorithm). So the product of the permutations q_i along the $(0,c)$-bigons is the identity, and the isomorphism follows. Therefore, not only the number of vertices, but also the number of bigons of H is n times those of G. We have for general $(3+1)$-graphs the inequality $v + t \geq b$, see Corollary 1. These facts imply that the number of 3-residues of H is at least n times that of G.

Note that the pre-image of any 3-residue of G is a set of at most n 3-residues of H: if more than n 3-residues of H go to the same 3-residue of G, then some vertex would be covered more than n times. If some 3-residue of G has less than n 3-residues of H in its pre-image, some other would have more than n to compensate for. This would be a contradiction. Whence it follows that each pre-image of a 3-residue is formed by n disjoint isomorphic 3-residues of H. The result follows from the construction of the manifolds $|G|$ and $|H|$ and from the usual topological theory [ST80]. ∎

We have implemented this construction for the case where J is the derived subgroup of π_1. Whence, when the first Z-homology of a 3-manifold is finite, we can get at once a 3-gem whose fundamental group is its derived subgroup (generated by all commutators). The 3-gem H in this case is called the *derived 3-gem* of G. This package is named GDV, for *derived gems.* The simplification package, also named TS_ρ, implements the TS_ρ-algorithm and ends by producing a u^0-essential TS-class. Both packages run under Turbo Pascal for DOS. The second one was implemented by Cassiano Durand at UFPE. To get general finite index normal subgroups and the permutation representations of the group on their cosets we have used the package SPAS installed at UNB and running on a VAX.

Theorem 13 (Quotients of S^3) *The universal covering of all the closed orientable 3-manifolds with complexity at most 13 having finite fundamental groups is S^3.*

Proof: The proof consists in taking each one of these manifolds and getting a 3-gem which covers a 3-gem inducing it and which has smaller fundamental group, iterating the process. This was done with the help of the GDV, ad-hoc extensions of it (for the non-solvable groups) and TS_ρ. In all but two cases (related to Poincaré sphere) the iterations ended at a 3-gem which collapses to r_1^2. ∎

5.4.6 Quotients of S^3

Proposition 32 *Each of the 3-manifolds in $\mathcal{RBG}_n$, $n < 30$, which have finite fundamental group is a quotient of S^3.*

Proof: The manifolds in $\mathcal{RBG}_n$, $n < 30$, which have cyclic fundamental group are the lens spaces of table 10A and the ones in Table 10D. For the those lens space we have, in all cases:

$$r_m^n \xrightarrow[der]{h} g^{nh} \xrightarrow[\rho_*]{} s^3,$$

where r_m^n is the gem with minimum code in the attractor, h is the order of the fundamental group and g^{nh} is a specific 3-gem with nh vertices, which is the derived 3-gem of r_m^n. As for the other manifolds, listed above, we have:

$$S^3/\langle 2,2,2\rangle : \quad r_1^{18} \xrightarrow[der]{4} g_1^{72} \xrightarrow[\rho_*]{} r_1^8 \xrightarrow[der]{2} g_2^{16} \xrightarrow[\rho_*]{} s^3$$

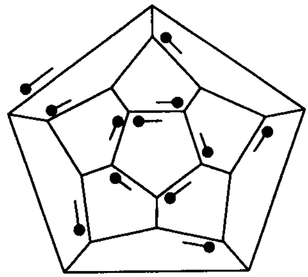
$$S^3/\langle 3,2,2\rangle : \quad r_4^{20} \xrightarrow[der]{4} g_1^{80} \xrightarrow[\rho_*]{} r_1^{12} \xrightarrow[der]{3} g_2^{36} \xrightarrow[\rho_*]{} s^3$$

$$S^3/\langle 3,3,2\rangle : \quad r_1^{22} \xrightarrow[der]{3} g_1^{66} \xrightarrow[\rho_*]{} r_7^{20} \xrightarrow[der]{4} g_2^{80} \xrightarrow[\rho_*]{} r_1^8 \xrightarrow[der]{2} g_3^{16} \xrightarrow[\rho_*]{} s^3$$

$$S^3/(C_3 \rtimes_i C_8) : \quad r_2^{22} \xrightarrow[der]{8} g_1^{176} \xrightarrow[\rho_*]{} r_1^{12} \xrightarrow[der]{3} g_2^{36} \xrightarrow[\rho_*]{} s^3$$

Since $\langle 5,3,2\rangle$ is a perfect group (trivial homology), the derived gem of r_2^{24} is itself. To identify Poincaré sphere as $|r_2^{24}|$ we use Poincaré original identification scheme in conjunction with ts_{ρ_*}. The scheme below identifies opposite faces.

$$S^3/\langle 5,3,2\rangle =$$



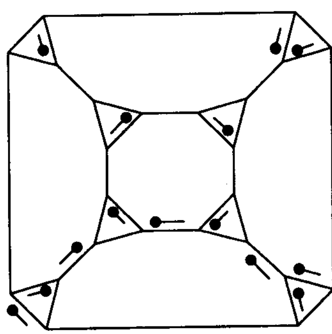
$$\xrightarrow[\rho_*]{} r^{28}_{1993} \xrightarrow{A^3_{28}} r^{24}_2$$

Fig. 206: An identification scheme for $S^3/\langle 5,3,2\rangle$ being attracted by r^{24}_2

$$S^3/\langle 4,3,2\rangle : \quad r^{24}_{28} \xrightarrow[der]{2} g^{48}_1 \xrightarrow[\rho_*]{} g^{28}_2 \xrightarrow[der]{3} g^{84}_3 \xrightarrow[\rho_*]{} g^{38}_4 \xrightarrow[der]{4} g^{152}_5 \xrightarrow[\rho_*]{} r^8_1 \xrightarrow[der]{2} g^{16}_2 \xrightarrow[\rho_*]{} s^3$$

The space $S^3/\langle 4,3,2\rangle$ also appears as an identification scheme in the truncated cube. Pairs of opposite faces are identified, as indicated below:

$$S^3/\langle 4,3,2\rangle =$$



$$\xrightarrow[\rho_*]{} r^{28}_{1692} \xrightarrow{A^0_{27}} r^{26}_{774} \xrightarrow{A^2_{26}} r^{24}_{28}$$

Fig. 207: An identification scheme for $S^3/\langle 4,3,2\rangle$ being attracted by r^{24}_{28}

$$S^3/(C_5 \rtimes_i C_8) : \quad r^{24}_4 \xrightarrow[der]{8} g^{192}_1 \xrightarrow[\rho_*]{} r^{20}_3 \xrightarrow[der]{5} g^{100}_2 \xrightarrow[\rho_*]{} s^3$$

$$S^3/Q_{16} : \quad r^{24}_{154} \xrightarrow[der]{4} g^{96}_1 \xrightarrow[\rho_*]{} r^{16}_2 \xrightarrow[der]{4} g^{64}_2 \xrightarrow[\rho_*]{} s^3$$

$$S^3/(C_3 \times Q_8) : \quad r^{24}_{13} \xrightarrow[der]{12} g^{288}_1 \xrightarrow[\rho_*]{} r^8_1 \xrightarrow[der]{2} g^{16}_2 \xrightarrow[\rho_*]{} s^3$$

$$S^3/(C_7 \times \langle 5,3,2\rangle) : \quad r^{26}_{10} \xrightarrow[der]{7} g^{182}_1 \xrightarrow[ts_{\rho_*}]{} r^{24}_2 \quad \text{(induces Poincaré sphere)}$$

$$S^3/(Q_8 \rtimes_3 C_9) : \quad r^{26}_8 \xrightarrow[der]{9} g^{234}_1 \xrightarrow[\rho_*]{} r^{20}_{16} \xrightarrow[der]{4} g^{80}_2 \xrightarrow[\rho_*]{} r^8_1 \xrightarrow[der]{2} g^{16}_3 \xrightarrow[\rho_*]{} s^3$$

$S^3/(C_5 \rtimes_i C_{12}):\ r^{26}_6 \xrightarrow[der]{12} g^{312}_1 \xrightarrow[\rho_*]{} r^{20}_3 \xrightarrow[der]{5} g^{100}_2 \xrightarrow[\rho_*]{} s^3$

$S^3/(Q_8 \rtimes_3 C_{15}):\ r^{26}_4 \xrightarrow[der]{15} g^{390}_1 \xrightarrow[\rho_*]{} r^{28}_{2865} \xrightarrow[der]{4} g^{112}_2 \xrightarrow[\rho_*]{} r^8_1 \xrightarrow[der]{2} g^{16}_3 \xrightarrow[\rho_*]{} s^3$

$S^3/(C_3 \times Q_{16}):\ r^{26}_{65} \xrightarrow[der]{12} g^{312}_1 \xrightarrow[\rho_*]{} g^{40}_2 \xrightarrow[der]{4} g^{160}_2 \xrightarrow[\rho_*]{} s^3$

$S^3/(C_5 \rtimes_i C_4):\ r^{28}_{2418} \xrightarrow[der]{4} g^{112}_1 \xrightarrow[\rho_*]{} g^{32}_2 \xrightarrow[der]{5} g^{160}_3 \xrightarrow[\rho_*]{} s^3$

$S^3/(C_7 \rtimes_i C_8):\ r^{28}_{2314} \xrightarrow[der]{8} g^{224}_1 \xrightarrow[\rho_*]{} g^{34}_2 \xrightarrow[der]{7} g^{238}_3 \xrightarrow[\rho_*]{} s^3$

$S^3/(C_7 \rtimes_i C_{12}):\ r^{28}_{402} \xrightarrow[der]{12} g^{336}_1 \xrightarrow[\rho_*]{} r^{28}_{7313} \xrightarrow[der]{7} g^{196}_2 \xrightarrow[\rho_*]{} s^3$

$S^3/(C_3 \rtimes_i C_{16}):\ r^{28}_{27} \xrightarrow[der]{16} g^{448}_1 \xrightarrow[\rho_*]{} r^{12}_1 \xrightarrow[der]{3} g^{36}_2 \xrightarrow[\rho_*]{} s^3$

$S^3/(C_3 \rtimes_i C_{20}):\ r^{28}_{29} \xrightarrow[der]{20} g^{560}_1 \xrightarrow[\rho_*]{} r^{12}_1 \xrightarrow[der]{3} g^{36}_2 \xrightarrow[\rho_*]{} s^3$

$S^3/(C_3 \times Q_{32}):\ r^{28}_{56} \xrightarrow[der]{12} g^{336}_1 \xrightarrow[\rho_*]{} g^{32}_2 \xrightarrow[der]{8} g^{256}_3 \xrightarrow[\rho_*]{} s^3$

$S^3/(C_5 \times Q_8):\ r^{28}_7 \xrightarrow[der]{20} g^{560}_1 \xrightarrow[\rho_*]{} r^8_1 \xrightarrow[der]{2} g^{16}_2 \xrightarrow[\rho_*]{} s^3$, establishing the proposition.■

5.4.7 The Euclidean Manifolds

κ	Manifold and its π_1	Rep.	u^0-class	μ	Hom	Non-null self-linking sequence: σ
11	EUCLID_1 $E_2(1,1)$	r^{24}_5	$C_{24}(32)$	1	$(0)4^2$	$\pm 8^{12}$
	EUCLID_2 $\langle 3,3,3\rangle$	r^{24}_7	$C_{24}(37)$	1	$(1)3^1$	$+1^2$
	EUCLID_3 $Z^{\begin{smallmatrix}-1&0\\0&-1\end{smallmatrix}}$	r^{24}_6	$C_{24}(39)$	1	$(1)2^2$	*
	EUCLID_0 $Z+Z+Z$	r^{24}_1	$C_{24}(40)$	1	(3)	*
12	EUCLID'_1	r^{26}_3	$C_{26}(44)$	1	$(0)4^2$	$\pm 8^{12}$
	EUCLID_4 $\langle 6,3,2\rangle$	r^{26}_{31}	$C_{26}(50)$	2	(1)	*
	EUCLID_5 $\langle 4,4,2\rangle$	r^{26}_{11}	$C_{26}(52)$	2	$(1)2^1$	$\pm 1^1$

Table 10E: The euclidean manifolds

Proposition 33 *$EUCLID_1 = |r_5^{24}|$ and $EUCLID_1' = |r_3^{26}|$ are homeomorphic.*

Proof:

$r_3^{26} = G_0 \xrightarrow[\rho_*]{U_3^1} G_1 \xrightarrow{A_{30}^2} G_2 \xrightarrow{D_9^{31}} G_3 \xrightarrow{D_{10}^{31}} G_4 = r_5^{24}$, establishing the proposition. ∎

This reduction was given in detail in Subsection 4.1.7.

5.4.8 The Other 3-Manifolds

κ	π_1 of Manifold	Rep.	u^0-class	μ	Hom	Non-null self-linking sequence: σ
12	$[4^2, S_1^4]$	r_5^{26}	$C_{26}(45)$	1	$(0)4^2$	$+4^8 \pm 8^4$
	$E_2(0,2)$					
	$E_2(2,1)$	r_{13}^{26}	$C_{26}(56)$	3	$(1)4^1$	$+1^2$
	$z^{1\ 0}_{1\ 1}$	r_{14}^{26}	$C_{26}(59)$	2	(2)	*
13	$\langle 7,3,2\rangle$	r_{172}^{28}	$C_{28}(3)$	6	(0)	*
	$\langle 5,5,2\rangle 2$	r_{202}^{28}	$C_{28}(6)$	1	$(0)2^1$	$\pm 1^1$
	$[3, 5^2, S_6^5]$	r_2^{28}	$C_{28}(7)$	1	$(0)3^1$	$+1^2$
	$[3, 4^2, S_3^4]$	r_9^{28}	$C_{28}(8)$	3	$(0)3^1$	$+1^2$
	$\langle 4,3,3\rangle$					
	$\langle 7,3,2\rangle 5$	r_{203}^{28}	$C_{28}(15)$	1	$(0)5^1$	$+2^2\ -2^2$
	$[24, S_2^1]$	r_1^{28}	$C_{28}(58)$	2	$(0)24^1$	$+3^4\ +8^4\ +11^8\ \pm 12^2\ -4^4$
	$[2^2, 3^2{\times}6, S_{10}^9]$	r_3^{28}	$C_{28}(59)$	2	$(0)2^2$	*
	$[3^2, S_1^3]$	r_{34}^{28}	$C_{28}(76)$	5	$(0)3^2$	$+3^2\ -3^2$
	$(3,3,3)^*$					
	$[4^2, S_1^8]$	r_{19}^{28}	$C_{28}(86)$	1	$(0)4^2$	$\pm 8^4$
	$E_2(4,0)$					
	$[4^2, S_1^4]'$	r_{25}^{28}	$C_{28}(87)$	7	$(0)4^2$	$+4^8 \pm 8^4$
	$[2^2 \times 4, S_1^4]$	r_6^{28}	$C_{28}(91)$	3	$(0)2^2 4^1$	$+4^8$
	$z^{0\ 1}_{-1\ 3}$	r_5^{28}	$C_{28}(93)$	6	(1)	*
	$z^{0\ 1}_{-1\,-3}$	r_{10}^{28}	$C_{28}(102)$	8	$(1)5^1$	$1^2\ -1^2$
	$z^{-1\ 0}_{-2\,-1}$	r_{42}^{28}	$C_{28}(104)$	8	$(1)2^2$	$\pm 2^2$
	$z^{1\,-2}_{0\ 1}$	r_{280}^{28}	$C_{28}(108)$	5	$(2)2^1$	± 1

Table 10F: The Other 3-manifolds

Proposition 34 *$[4^2, S_1^4] = |r_5^{26}|$ and $[4^2, S_1^4]' = |r_{25}^{28}|$ are homeomorphic.*

Proof:

$$r_{25}^{28} = G_0 \xrightarrow[\rho_*]{U_{10}^1} G_1 \xrightarrow{D_4^{31}} G_2 \xrightarrow{A_{32}^3} G_3 \xrightarrow{A_{32}^3} G_4 \xrightarrow{D_5^{31}} G_5 = r_5^{26},$$ proving the proposition. ∎

5.4.9 Summary of the u_0-Uncertainties

Theorem 14 (Classification Theorem) *The u^1-classification coincides with the topological classification for $\mathcal{RBG}_n$, $n < 30$.*

Proof: The u_0-classification let unresolved the following cases:

1. $L_{2,1}\#(L_{3,1}^+\#L_{3,1}^+) = |r_{514}^{28}|$, $L'_{2,1}\#(L_{3,1}^+\#L_{3,1}^+) = |r_{3659}^{28}|$ and $L''_{2,1}\#(L_{3,1}^+\#L_{3,1}^+) = |r_{6378}^{28}|$
2. $L_{2,1}\#(L_{3,1}^+\#L_{3,1}^-) = |r_{517}^{28}|$, $L'_{2,1}\#(L_{3,1}^+\#L_{3,1}^-) = |r_{3661}^{28}|$ and $L''_{2,1}\#(L_{3,1}^+\#L_{3,1}^-) = |r_{6372}^{28}|$
3. $L_{12,5} = |r_{32}^{24}|$ and $L'_{12,5} = |r_{34}^{24}|$
4. $L_{16,7} = |r_{14}^{28}|$ and $L'_{16,7} = |r_{113}^{28}|$
5. $L_{17,5} = |r_{65}^{28}|$ and $L'_{17,5} = |r_{106}^{28}|$
6. $L_{19,7} = |r_{41}^{28}|$ and $L'_{19,7} = |r_{81}^{28}|$
7. $L_{21,8} = |r_{31}^{28}|$, $L'_{21,8} = |r_{13}^{28}|$ and $L''_{21,8} = |r_{60}^{28}|$
8. $\mathrm{EUCLID}_1 = |r_5^{24}|$ and $\mathrm{EUCLID}'_1 = |r_3^{26}|$
9. $S^3/(Q_8 \rtimes_3 C_{15}) = |r_4^{26}|$ and $S^3/(Q_8 \rtimes'_3 C_{15}) = |r_{696}^{26}|$
10. $S^3/(C_5 \times Q_8) = |r_7^{28}|$ and $S^3/(C_5 \times' Q_8) = |r_{59}^{28}|$
11. $[4^2, S_1^4] = |r_5^{26}|$ and $[4^2, S_1^4]' = |r_{25}^{28}|$
12. $L_{3,1}^+\#QUAT = |r_{1473}^{28}|$ and $L_{3,1}^-\#QUAT = |r_{1474}^{28}|$

We have seen above that in all but the last case the 3-manifolds are homeomorphic. As for the last pair of 3-manifolds, we prove them to be non-homeomorphic in Subsection 7.1.4. In this way, in $\mathcal{RBG}_n$, $n < 30$, the u^1-classification coincides with the topological classification. ∎

Remark 4 *Recently, in a joint work with C. Durand and S. Sidki we have extended this Theorem for $n = 30$.*

Chapter 6

Getting Linking Numbers of 3-Manifolds

In the tables of the previous chapter we have listed for the 3-manifolds which appear their *sequence of non-null linking numbers*. In this chapter we define and develop an easily implementable algorithm to obtain these invariants. Indeed, from the combinatorics of a 3-gem inducing M^3 we provide a simple recipe to get a special kind of intersection matrix $\mathcal{M}$; from this matrix and from the torsion coefficents of the first Z-homology group of M^3 we provide a formula which yields its linking numbers.

6.1 Introduction

The linking numbers of a 3-manifold M^3 are defined for any pair of division null homologous 1-chains. Easily presented invariants for 3-manifolds are very rare. The full sequence of self-linking numbers for a closed orientable 3-manifold, denoted $\sigma(M^3)$ and defined in the next section is such an invariant. Different sequences are, of course, easily distinguishable and provide a simple proof that the corresponding manifolds are not homeomorphic.

Effectively obtaining these simple-looking sequences is rather laborious. Unless the manifolds are very simply structured, as the lens spaces, it is impossible to get the sequences *"by hand"*. In this chapter we give an algorithm which efficiently does the job. We get, in polynomial time complexity on the number of vertices of a 3-gem G, a matrix $\mathcal{M}(G)$. The self-linking sequence is derived from $\mathcal{M}(G)$ and from the torsion coefficients of the first homology group of the manifold $|G| \cong M^3$. The amount of work to get the sequence from these data depends linearly on the order of the torsion subgroup of the homology group of $|G|$.

In Section 6.2 we give the basic definitions and properties of linking numbers.

These are taken from [ST80]. These revisions are intended to make our exposition self-contained. In Section 6.3 we present a dual pair of open cell decompositions for a closed orientable 3-manifold $|G|$. In Section 6.4 we recall how to obtain a presentation of the fundamental group and how to get a basis for the torsion subgroup. In Section 6.5 appears the crucial point which characterize our approach: the possibility of efficiently presenting, for each generator of the fundamental group, a pair of homotopically equivalent dual words. From this fact, in Section 6.6 we obtain a formula for the linking numbers. Finally, in Section 6.7 we discuss a class of 3-manifolds, present a conjecture related with planar 3-connected graphs. We also provide the self-linking sequences of two not so small examples in this class of graphs. These examples are intended as control cases for checking independent implementations of our algorithm.

6.2 Linking Numbers and Self-Linking Sequences

Let K be a cell complex which decomposes an orientable closed connected 3-dimensional manifold M^3. The dual cell complex is denoted by K^*. Suppose that the first torsion subgroup of M^3 has order $t > 1$. Given a pair x, y^* of division-null homologous 1-chains (over Z) in K and K^*, respectively we can define an integer number $\lambda(x, y^*)$ satisfying the following property: if x and x' are homologous then $\lambda(x, y^*) \equiv \lambda(x', y^*) \bmod t$.

This property is invariant by refinements of K and therefore, by the Triangulation Theorem and the Hauptvermutung of Moise [Moi77], the number $\lambda(x, y^*)$ depends only on the homology classes of x and y^*. Thus, they are truly topological invariants for 3-manifolds. Our issue is to provide an easily implementable effective way to compute $\lambda(x, y^*)$.

We now restrict the definitions to $n = 3$ and recall some of the basic facts given in [ST80]. The fundamental property is the following formula about the *intersection numbers*, $\Im(X, y^*)$'s, between a 2-chain X and a 1-chain y^* in the dual complex. See page 256 of [ST80].

Proposition 35 *Let X and Y^* be arbitrary 2-chains in K and K^*. Then we have* $\Im(X, \partial Y^*) = \Im(\partial X, \Im Y^*)$.

This property follows from the generalization of the geometric situation depicted below for elementary 2-chains:

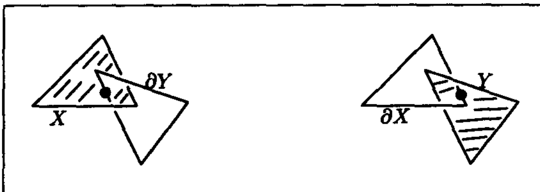


Fig. 208: Geometric explanation for the proposition

From the above basic fact, it is easy to derive the three lemmas that follow (see pages 289-290 of [ST80]).

Lemma 15 *If X is a closed 2-chain and y^* is a division-null homologous 1-chain, then*

$$\Im(X, y^*) = 0.$$

Proof: Let Y^* be such that $\partial Y^* = cy^*$ for some integer c. Then

$$\Im(X, y^*) = \frac{1}{c}\Im(X, cy^*) = \frac{1}{c}\Im(X, \partial Y^*).$$

By property (*) the latter expression is $\frac{1}{c}\Im(\partial X, Y^*)$. Since X is closed, $\partial X = 0$ and the Lemma is established. ∎

Suppose that x, y^* are division-null homologous 1-chains and that X is a 2-chain satisfying $\partial X = cx$ for some integer c. We define

$$\lambda(x, y^*) = \frac{t}{c}\Im(X, y^*).$$

Next we show that $\lambda(x, y^*)$ is a well defined integer which is invariant mod t of the homology classes of x and y^*.

Lemma 16 *The definition of $\lambda(x, y^*)$ does not depend on the choice of the pair (X, c) with $\partial X = cx$.*

Proof: Let (X', c') be such that $\partial X' = c'x$. We have that

$$c.c'[\frac{1}{c}\Im(X, y^*) - \frac{1}{c'}\Im(X', y^*)] =$$

$$\Im(c'X, y^*) - \Im(cX, y) = \Im(c'X - cX', y^*).$$

However, $c'X - cX'$ is closed: $\partial(c'X - cX') = c'cx - cc'x = 0$. Therefore the expression in square brackets is null, proving the Lemma. ∎

Recall that $t > 1$ is the order of the torsion subgroup of the first Z-homology group of M^3.

Lemma 17 *Assume that x_1 and x_2 are homologous division-null homologous 1-chains in K and y^* is a division-null homologous 1-chain in K^*. Then $\lambda(x_1, y^*), \lambda(x_2, y^*)$ are integers and congruent mod t.*

Proof: Let (X_i, c_i) be such that $\partial X_i = c_i x_i$, $i = 1, 2$. We get

$$\lambda(x_1, y^*) - \lambda(x_2, y^*) = \frac{t}{c_1}\Im(X_1, y^*) - \frac{t}{c_2}\Im(X_2, y^*) = \frac{t}{c_1 c_2}\Im(c_2 X_1 - c_1 X_2, y^*).$$

Since x_1 and x_2 are homologous, we have a 2-chain X such that $x_1 - x_2 = \partial X$. We want to replace $c_2 X_1 - c_1 X_2$ by $c_1 c_2 X$ in the above expression. To justify the replacement is enough, by Lemma 1, to show that $c_2 X_1 - c_1 X_2 - c_1 c_2 X$ is closed: $\partial(c_2 X_1 - c_1 X_2 - c_1 c_2 X) = c_2 c_1 x_1 - c_1 c_2 x_2 - c_1 c_2 \partial X = c_1 c_2 (x_1 - x_2 - \partial X) = 0$. Therefore the difference is

$$\frac{t}{c_1 c_2}\Im(c_1 c_2 X, y^*) = t.\Im(X, y^*).$$

Since $\Im(X, y^*) \in \mathbb{Z}$, $\lambda(x_1, y^*) \equiv \lambda(x_2, y^*) mod\ t$. Now choose (X_1, c_1) so that $|c_1|$ is the smallest possible. Since $\partial X_1 = c_1 x_1$, it follows that $c_1 | t$, because x_1 generates a subgroup of order c_1 of the first torsion subgroup. Then

$$\lambda(x_1, y^*) = \frac{t}{c_1}\Im(X_1, y^*) \in \mathbb{Z}.$$

∎

The above Lemmas, the Triangulation Theorem and the Hauptvermutung for 3-manifolds show that $\lambda(x, y^*)$ *mod t* depend only on the homology classes of x and y^* and so, is a topological invariant. However, in order to have an easily presented invariant it is usual [ST80] to restrict to the diagonal $\lambda(x, x^*)$, naming them *self-linking numbers*. We need to consider the whole set of self-linking numbers and so let x run over a set of representatives of the homology classes in the first torsion subgroup. We normalize the numbers in the interval $[\lfloor -\frac{t}{2} \rfloor, \lfloor \frac{t}{2} \rfloor]$ and order them in the following way: the positive ones in non- decreasing order followed by the negative ones in non-increasing order, letting out the zeros. If t is even and $\frac{t}{2}$ appears, then we may consider it positive or negative and indicate this by writing $\pm\frac{t}{2}$. By $\sigma(M^3)$ we indicate the sequence of self-linking numbers normalized in the way explained and written in a multiplicative form where exponents indicate number of repetitions. For instance,

$$\sigma(L_{2,1} \# L_{5,2}) = (+1^2 + 4^2 \pm 5^1 - 4^2 - 1^2).$$

The (+/−, left-right)-symmetry is a necessary condition for the space to be symmetric, i.e., to admit an orientation preserving homeomorphism onto itself which reverses the orientation. This property motivated the above normalizations. The fact that

$$\sigma(L_{3,1} \# L_{5,2}) = (+1^4 + 4^4 + 6^2 - 6^2 - 5^2)$$

proves that $L_{3,1} \# L_{5,2}$ is not symmetric.

6.3 An Adequate Pair of Dual Decompositions

Given a closed orientable 3-manifold M^3, we can get a 3-gem G inducing M^3, see Section 1.4. Cancelling 1-dipoles we may suppose that G is a crystallization. Therefore, the graph $G_{\hat{i}}$ $(i = 0, 1, 2, 3)$ obtained from crystallization G by deleting all the edges of color i, is connected.

Below we present a drawing for the crystallization $\mathcal{G} = \mathcal{S}(4, 5, 2, 1)$, see Section 2.2. It induces a 3-manifold whose torsion coefficients are 3 and 15. Indeed, its homology group is $Z_3 + Z_{15}$. We will use $\mathcal{G}$ throughout to illustrate the algorithm to get the linking numbers, refering to it as $\mathcal{G}$-example.

$\mathcal{G}$:

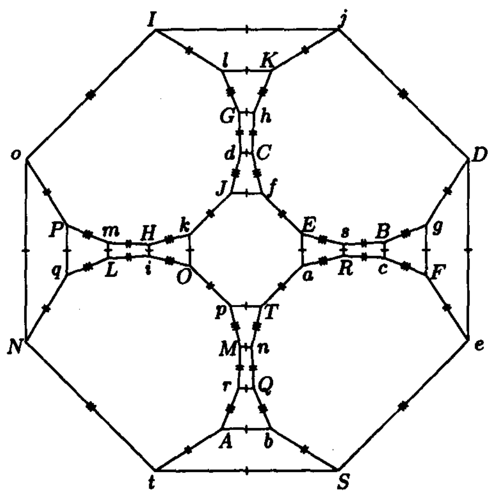


Fig. 209: Ongoing example to illustrate algorithm for linking numbers

Let $b^0_{ij}, b^1_{ij}, \ldots, b^m_{ij}$ denote the $\{i, j\}$-colored bigons. The manifold M^3 is formed by the union of four 3-balls. The 3-ball $B_{\hat{i}}$ is bounded by a 2-sphere where the graph $G_{\hat{i}}$ is embedded so that the polygons induced by 2 colors (the bigons) are the boundaries of the faces. Each bigon appears in two of the four 3-balls and we identify the disks bounded by these twin bigons. After we effect the identifications in $B_{\hat{2}}$ and $B_{\hat{3}}$ we have a handlebody $\mathcal{H}_{01}$ of genus $\beta_{01} - 1$ (β_{ij} is the number of bigons in colors i, j). For the above 3-gem the two balls with dissected boundaries forming $\mathcal{H}_{01}$ are:

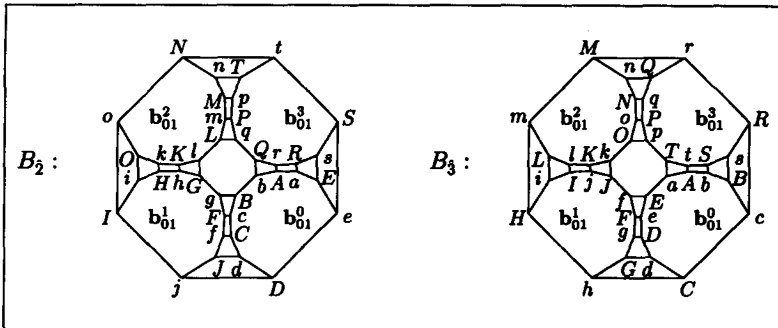


Fig. 210: 3-balls forming $\mathcal{H}_{01}$ in the $\mathcal{G}$-example

Analogously, from $B_{\hat 0}$ and $B_{\hat 1}$ we get a handlebody $\mathcal{H}_{23}$ of genus $\beta_{23} - 1$.

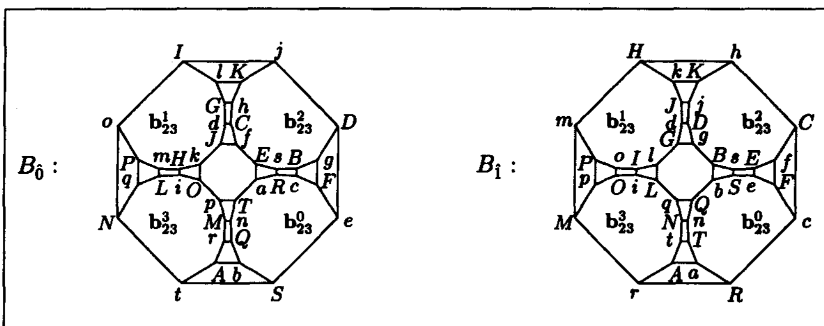


Fig. 211: 3-balls forming $\mathcal{H}_{23}$ in the $\mathcal{G}$-example

In a crystallization, $\beta_{ij} = \beta_{hk}$ for every partition $\{i, j\}, \{h, k\}$ of $\{0, 1, 2, 3\}$. See the Corollary 2 in Subsection 2.7.1. Therefore the boundaries of $\mathcal{H}_{01}$ and $\mathcal{H}_{23}$ are in general homeomorphic. If the two handlebodies are identified by their boundaries, as prescribed by the crystallization, the common boundary becomes a Heegaard surface for the induced 3-manifold M^3.

We use the Heegaard splitting $(\mathcal{H}_{01}, \mathcal{H}_{23}, \partial\mathcal{H}_{01})$ to get an adequate dual pair of decompositions by open cells. Let G_{01} be a bouquet of $m = \beta_{01} - 1$ loops ℓ_i, $1 \le i \le m$, based at a point r_{01}, which is contained in the interior of $\mathcal{H}_{01}$. More specifically, let r_{01} be inside the 2-cell bounded by b^0_{01} (see cross section below). Suppose, for $1 \le i \le m$, that loop ℓ_i starting at r_{01} proceeds internally to $B_{\hat 3}$, crosses transversally once the disk bounded by b^i_{01} and then comes back to r_{01} by an arc inside $B_{\hat 2}$.

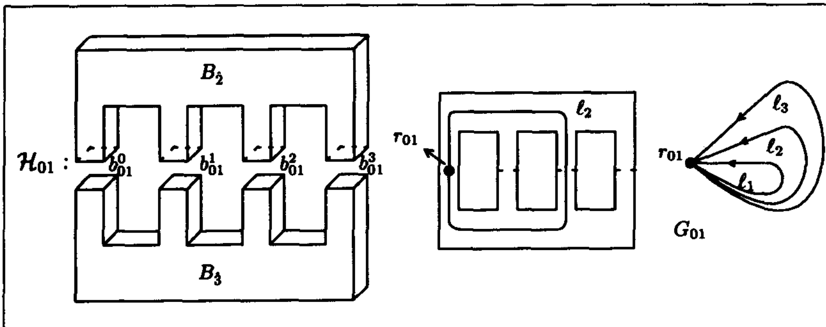


Fig. 212: Forming graph G_{01}

The graph G_{01} is a deformation retract of $\mathcal{H}_{01}$. If this retraction is in conjunction with the corresponding expansion of the complement $\mathcal{H}_{23}$, at the end of the process $\mathcal{H}^{\circ}_{23}$ becomes $M^3\backslash G_{01}$ which is denoted by J°_{23}. Let $B^j_{23}, 1 \leq j \leq m$, be the open cell in J°_{23} which is the expansion of the 2-cell bounded by the 2-residue b^j_{23}.

We also define a graph G_{23}, as a bouquet of $m = \beta_{01} - 1$ loops ℓ^*_j, $1 \leq j \leq m$, based at a point r_{23}, which is contained in the interior of $\mathcal{H}_{23}$, inside the 2-cell bounded by b^0_{23}. Suppose, for $1 \leq i \leq m$, that loop ℓ^*_j starting at r_{23} proceeds internally to $B_{\hat{0}}$, crosses transversally once the disk bounded by b^j_{23} and then comes back to r_{23} by an arc internal to $B_{\hat{1}}$.

Similarly, graph G_{23} is a deformation retract of $\mathcal{H}_{23}$. If this retraction is in conjunction with the corresponding expansion of the complement $\mathcal{H}_{01}$, at the end of the process $\mathcal{H}^{\circ}_{01}$ becomes $M^3\backslash G_{23}$ which is denoted by J°_{01}. Let $B^i_{01}, 1 \leq i \leq m$, be the open cell in J°_{01} which is the expansion of the 2-cell bounded by the 2-residue b^i_{01}.

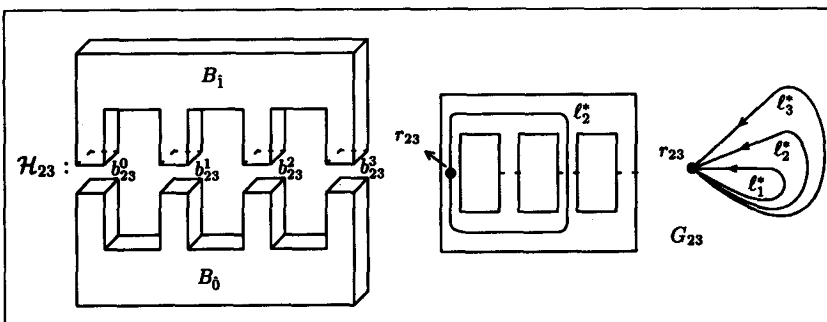


Fig. 213: Forming graph G_{23}

From this analysis we can derive two dual cellular decompositions K, K^* of M^3, whose cells are open balls. They have a single 0-cell, m 1-cells, m 2-cells and a single

3-cell and are defined as follows:

$$K = (r_{01}, \cup_{i=1}^{m} \ell_i \backslash r_{01}, \cup_{j=1}^{m} B_{23}^{j}, J_{23}^{\circ} \backslash B_{23}^{0}),$$

$$K^* = (r_{23}, \cup_{j=1}^{m} \ell_j^* \backslash r_{23}, \cup_{i=1}^{m} B_{01}^{i}, J_{01}^{\circ} \backslash B_{01}^{0}).$$

Note that each ℓ_i pierces once B_{01}^i and no other B_{01}^k. Also that each ℓ_j^* pierces once B_{23}^j and no other B_{23}^k. This dual pair of decompositions is used to produce all the data we need: a basis for the torsion subgroup, the torsion coefficients and the intersection matrix $\mathcal{M}$.

6.4 The Fundamental Group and a Basis for the Torsion Subgroup

From a crystallization G it is rather easy to obtain a presentation for the fundamental group of the induced 3-manifold.

We write a word w_i in terms of the $\ell_j's$ for the boundary curve (which lies on the graph G_{01}) of the open disk B_{23}^i, as follows: label each vertex of G with ℓ_j if it belongs to the j-th 01-gon; go around the vertices of b_{23}^i writing the label of each vertex with the exponent of ± 1 according to the class to which the vertex belongs: lower case -1, upper case $+1$. In this process, ℓ_0 is to be taken equal to the identity. In fact the set of relators

$$P \begin{cases} \partial B_{23}^1 = \omega_1(\ell_1, \ldots, \ell_m) = 1 \\ \partial B_{23}^2 = \omega_2(\ell_1, \ldots, \ell_m) = 1 \\ \vdots \\ \partial B_{23}^m = \omega_m(\ell_1, \ldots, \ell_m) = 1 \end{cases}$$

is a presentation P of the fundamental group $\pi_1(M^3)$ in terms of $\ell_1, \ldots, \ell_m$. We note that $\partial B_{23}^0 = 1$ is obtained as a consequence of the above relations, see Algorithm 5 in Section 2.8 and also [Lin88]. This follows because these curves on G_{01} are the meridians of the complementary open handlebody J_{23}°. Thus, our technique is a combinatorial manifestation of the standard procedure to get a presentation of the fundamental group of a 3-manifold from a Heegaard diagram for it, [Hem76].

Exemplifying this procedure in our example $\mathcal{G}$ we get

$$\ell_1^{-2}\ell_2\ell_1^{-1}\ell_2\ell_3^{-1}\ell_2\ell_1^{-1}\ell_2 = 1, \ell_1^2\ell_3\ell_1^2\ell_2^{-1} = 1, \ell_1\ell_2^{-1}\ell_3\ell_2^{-1}\ell_3^2\ell_2^{-1}\ell_3\ell_2^{-1} = 1$$

By abelianizing the presentation P, and using an additive notation we obtain the homology relations among the $\ell_i's$ via the matrix equation $A\ell = 0$, where ℓ is a column vector of the $\ell_i's$. In the case at hand, we get

$$A = \begin{pmatrix} -4 & 4 & -1 \\ 4 & -1 & 1 \\ 1 & -4 & 4 \end{pmatrix}$$

There are unimodular matrices R and C such that $RAC = S$, where S is the *Smith normal form* of A : S is a diagonal matrix and each non-null term $t_i = s_{ii}$ divides the next t_{i+1}. Note that if

$$R = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 1 & 0 \\ 5 & 4 & 4 \end{pmatrix} \text{ and } C = \begin{pmatrix} 1 & 4 & -4 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

then we get

$$RAC = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 15 \end{pmatrix} = S.$$

The number of $0's$ in the diagonal is the *Betti number of* M^3 and each $t_i \notin \{0,1\}$ is a *torsion coefficient.* The general process to get R, C and S from A is algorithmically efficient [NW88]. Also note that t, the order of the first torsion subgroup, is the product of the non-null $t_i's$ in the diagonal of S. Consider the equation

$$RA = SC^{-1}.$$

The row i of RA is a linear combination of the rows of A, i.e., $(RA)_i = \sum_{j=1}^{m} r_{ij}\partial B_{23}^j$. (Here we interpret ∂B_{23}^j as the image under the boundary operator of the 2-chain B_{23}^j.) On the other side $(SC^{-1})_i = \sum_{j=1}^{n} s_{ij}(C^{-1})_j = t_i(C^{-1})_i$. Let $h_i = \sum_{j=1}^{m} c'_{ij}\ell_j$, where c'_{ij} is the (i,j)-entry of C^{-1}. It follows that the $h_i's$ corresponding to $t_i \notin \{0,1\}$ with the relations $t_i h_i = 0$ form a presentation of the first torsion subgroup of M^3. Also, let H_i be the 2-chain $\sum_{j=1}^{m} r_{ij}B_{23}^j$. From the above equation and from the definitions of H_i and h_i we get $\partial H_i = t_i h_i$. Note that the set of h_i's such that $t_i \notin \{0,1\}$ is a basis for the torsion subgroup.

6.5 Getting Loop Words from Paths in a 3-Gem

Since our basic 2-chains are combinations of the B_{23}^i's, a way to compute the intersection numbers, is to replace the $h_i's$ by homologous h_i^*'s, written in terms of the ℓ_j^*'s. The justification is that ℓ_j^* cross transversally once the disk bounded by b_{23}^i (whence also B_{23}^i) and no other similar disks.

The possibility of the above replacement is the crucial point where the 3-gem approach is adequate. Indeed, consider any closed path p based at a point a of the gem. We assume without loss of generality, that point a is a common point in b^0_{01} and b^0_{23}. Recall that the gem G is naturally embedded into $\partial\mathcal{H}_{01} = \partial\mathcal{H}_{23}$. In the expansion of $\mathcal{H}_{23}$ to produce J°_{23} path p is deformed into a well determined path p' of G_{01}, which is defined, up to homotopy, by a sequence of loops ℓ_i. In particular, the base point a is deformed into r_{01}. Analogously, in the expansion of $\mathcal{H}_{01}$ to produce J°_{01} path p is deformed into a well determined path p'' of G_{23}, which is r_{23}-based and is defined, up to homotopy, by a sequence of loops ℓ^*_j.

The well determined sequence of loops forming p' and p'' are easily obtained at the combinatorial level. Suppose, the a-based counterclockwise closed paths p_1, p_2, p_3 depicted below. They are paths in the crystallization $\mathcal{G}$ and are relevant to proceed with our example.

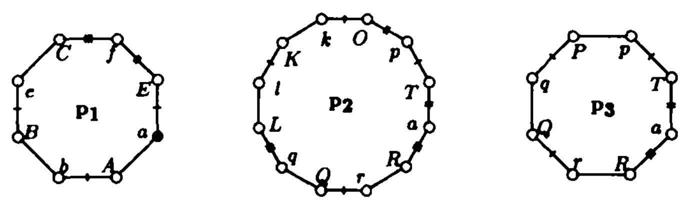


Fig. 214: Paths p_i, $i = 1, 2, 3$, in the $\mathcal{G}$-example

In general, to obtain p'_k start at a and proceed traversing p_k. A vertex v in this traversal is called a *positive transition* at v if p_k leaves B_3 and enters B_2. It is a *negative transition* if the roles of 2 and 3 are interchanged. Mark a transition vertex with ℓ_i if it is a positive transition in b^i_{01}. Mark a transition vertex with $(\ell_i)^{-1}$ if it is a negative transition in bigon b^i_{01}. The path p'_k is defined in G_{01} by the sequence of loops corresponding to these markers. The procedure to get the presentation of the fundamental group by reading in G_{01} the boundary of the B^j_{23} is a special case of this method. In it all the vertices are transitions. The justification is straighforward: to check it in general, just think of a path parallel to p_k in the interior of $\mathcal{H}_{01}$. Such a path is clearly deformed into p'_k. For the three paths above, displaying the root and the transition vertices in black, we get:

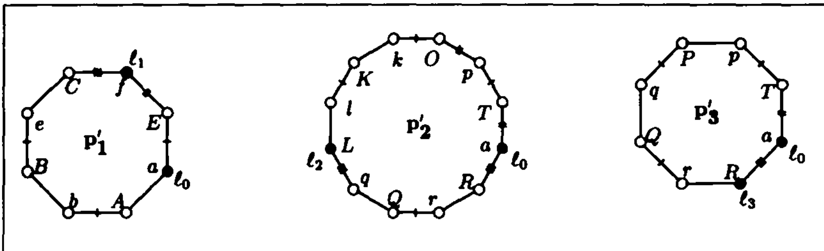


Fig. 215: Paths p'_i, $i = 1, 2, 3$, in the $\mathcal{G}$-example

Note that indeed, $p_k (1 \leq k \leq 3)$ is homotopically equivalent up to conjugation to the loop ℓ_k. We must define ℓ_0 as the identity.

Now we get the paths p''_k for the same paths p_k's. It is enough to consider this time the transition vertices between $B_{\hat{0}}$ and $B_{\hat{1}}$. Mark a transition vertex with ℓ^*_j if it is a transition vertex from $B_{\hat{0}}$ to $B_{\hat{1}}$ and it belongs to bigon b^j_{23}. A similar transition in the other direction is marked with $(\ell^*_j)^{-1}$. The path p''_k is defined in G_{01} by the sequence of loops corresponding to these markers. In the $\mathcal{G}$-example we get:

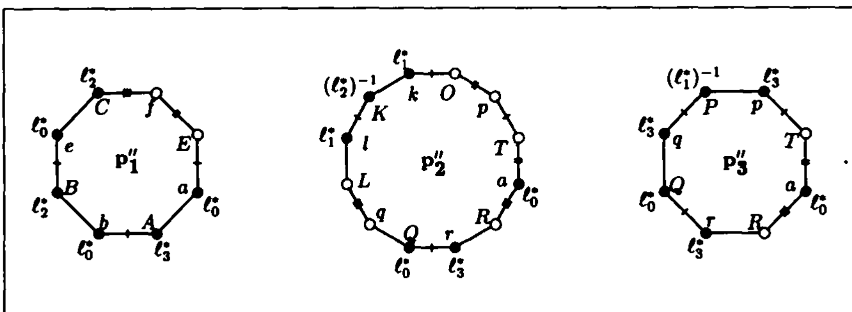


Fig. 216: Paths p''_i, $i = 1, 2, 3$, in the $\mathcal{G}$-example

and therefore, since ℓ^*_0 is the identity, $p''_1 = \ell^*_2\ell^*_2\ell^*_3$, $p''_2 = \ell^*_1(\ell^*_2)^{-1}\ell^*_1\ell^*_3$, $p''_3 = \ell^*_3(\ell^*_1)^{-1}\ell^*_3\ell^*_3$.

Since in general we can read the paths p' and p'' from an a-based closed gem path p, it remains to discuss how we can find a path p_k which is homotopic up to conjugation to ℓ_k, for $1 \leq k \leq m$. Choose a vertex v_k for each b^k_{01} $k = 0, 1, \ldots, m$. Define p_k to be any closed path in G that goes from v_0 to v_k without using 3-colored edges and after comes back to v_0 without using 2-colored edges. Observe that there exists such a path, since G is a crystallization and that it is homotopic up to conjugation to ℓ_k.

We have used this method to obtain these paths p_1, p_2, p_3 in our illustrative $\mathcal{G}$-example. Our choices were: a in b^0_{01}, f in b^1_{01}, k in b^2_{01}, p in b^3_{01}. The above analysis constitutes the essence of the proof of the following Lemma.

Lemma 18 *We can obtain, directly from the gem G, an $m \times m$ matrix ${}^tQ = (q_{ji})$ such that ℓ_i is homologous to $\sum_{j=1}^m q_{ji}\ell_j^*$.*

Proof: Take ${}^tQ$ be the matrix defined by the paths p_i'': row i of the matrix ${}^tQ$ is the abelianized form of the path p_i''. ∎

In the $\mathcal{G}$-example, we have $$\left.\begin{array}{l} p_1'' = \ell_2^*\ell_2^*\ell_3^* \\ p_2'' = \ell_1^*(\ell_2^*)^{-1}\ell_1^*\ell_3^* \\ p_3'' = \ell_3^*(\ell_1^*)^{-1}\ell_3^*\ell_3^* \end{array}\right\} \Rightarrow \quad {}^tQ = \begin{pmatrix} 0 & 2 & 1 \\ 2 & -1 & 1 \\ -1 & 0 & 3 \end{pmatrix}$$

6.6 Obtaining the Linking Numbers

Let ${}^tq_{ji}$ be q_{ij}. From the previous Lemma we get that h_i is homologous to

$$h_i^* = \sum_{j=1}^m c'_{ij}\left(\sum_{k=1}^m {}^tq_{jk}\ell_k^*\right) = \sum_{k=1}^m\left(\sum_{j=1}^m (c'_{ij}\,{}^tq_{jk})\right)\ell_k^* = \sum_{k=1}^m (C^{-1\,t}Q)_{ik}\ell_k^*.$$

Thus, the i-th row of $(C^{-1})({}^tQ)$ are the coordinates of h_i^* in terms of the $\ell_j^{*\prime}s$.

It is rather easy to get $\Im(H_i, h_j^*)$: it is just the internal product of the coordinate vectors of H_i in terms of the $B_{23}^{k}{}'s$ and of h_j^* in terms of the $\ell_k^{*\prime}s$. We are not interested in the indices i, for which $t_i \in \{0,1\}$. Therefore let f be the first i with $t_i \neq 1$ and let g be the last i for which t_i is non-null. Since we assumed that $t > 1$, it follows that $1 \le f \le g \le m$. The positive integer $n = g - f + 1$ is the number of torsion coefficients.

Given a matrix X, let $X_q^p (p \le q)$ be the submatrix of X formed by rows from p to q. Let also $X_{p,q}$ denote the submatrix of X formed by columns from p to q. Our main result provides a well determined matrix $\mathcal{M}$ which[1] yields the relevant intersection numbers.

Theorem 15 *Let $\mathcal{M}$ be the $n \times n$ matrix whose (i,j)-entry is $\Im(H_{i+f-1}, h_{j+f-1}^*)$. Then, $\mathcal{M} = R_g^f.Q.({}^tC^{-1})_{f,g}$.*

Proof: Let $\mathcal{M}'$ be the $m \times m$ matrix whose (i,j)-entry is $\Im(H_i, h_j^*)$. It follows that $\mathcal{M}' = R.{}^t(C^{-1}.{}^tQ) = R.Q.{}^tC^{-1}$. The restriction to the relevant rows and columns $f,\ldots,g$ are accomplished by similar restrictions in R and in ${}^tC^{-1}$. ∎

[1]P. Cristofori in her thesis (under A. Costa, [Cos92]) in UNED, Spain, has used our matrix $\mathcal{M}$ and the dipole moves on 3-gems to define a topologically invariant matrix equivalence. This class of matrices is a 3-manifold invariant which holds more information than the self-linking numbers.

Note that for the $\mathcal{G}$-example

$$ {}^tC^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 4 & 0 & 1 \end{pmatrix} \quad R_3^2 = \begin{pmatrix} 1 & 1 & 0 \\ 5 & 4 & 4 \end{pmatrix} \quad {}^tC_{2,3}^{-1} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}, $$

$$ \text{and} \quad \mathcal{M}(\mathcal{G}) = \begin{pmatrix} 1 & -1 \\ 10 & 7 \end{pmatrix}. $$

An element of the first torsion subgroup of M^3 is conveniently described by a vector $\rho = (\rho_1, \dots, \rho_n)$, with $0 \le \rho_i < t_{i+f-1}$: we make the association $\rho \leftrightarrow \sum_{i=1}^n \rho_i h_{i+f-1}$ or, since h_j^* is homologous to h_j, $\rho \leftrightarrow \sum_{i=1}^n \rho_j h_{j+f-1}^*$. As a Corollary, we state a formula for the linking number between two 1-chains belonging to dual cell decompositions:

Corollary 9 *Let ρ and ρ' be elements of the first torsion subgroup of M^3. Then*

$$ \lambda(\rho, \rho') \equiv \sum_{i=1}^n \frac{t}{t_{i+f-1}} \rho_i (\sum_{j=1}^n \rho_j' m_{i,j}) \quad mod\ t, $$

where $m_{i,j}$ is the (i,j)-entry of the intersection matrix M.

Proof:

$$ \lambda(\rho, \rho') = \lambda \left(\sum_{i=1}^n \rho_i h_{i+f-1}, \sum_{j=1}^n \rho_j' h_{j+f-1}^* \right). $$

Observe that

$$ \partial \left(\sum_{i=1}^n \frac{t}{t_{i+f-1}} \rho_i H_{i+f-1} \right) = \sum_{i=1}^n \frac{t}{t_{i+f-1}} \rho_i \partial H_{i+f-1} = $$

$$ \sum_{i=1}^n \frac{t}{t_{i+f-1}} \rho_i . t_{i+f-1} . h_{i+f-1} = t \sum_{i=1}^n \rho_i h_{i+f-1} $$

Therefore, from the definition of λ,

$$ \lambda(\rho, \rho') = \frac{t}{t} \Im \left(\sum_{i=1}^n \frac{t}{t_{i+f-1}} \rho_i H_{i+f-1}, \sum_{j=1}^n \rho_j' h_{j+f-1}^* \right) = $$

$$ \sum_{i=1}^n \frac{t}{t_{i+f-1}} \rho_i \sum_{j=1}^n \rho_j' \Im(H_{i+f-1}, h_{j+f-1}^*) = \sum_{i=1}^n \frac{t}{t_{i+f-1}} \rho_i \left(\sum_{j=1}^n \rho_j' m_{i,j} \right), $$

finishing the proof. ∎

By applying this formula to our $\mathcal{G}$-example we obtain, after the proper ordering,

$$ \sigma(\mathcal{G}) = (+6^8 + 9^2 + 15^4 + 21^8 - 21^8 - 15^4 - 9^2 - 6^8) $$

and there is only $45 - 2 \times (8 + 2 + 4 + 8) = 1$ null self-linking number. The (+/-, left/right)-symmetry of $\sigma(\mathcal{G})$ suggests that $|\mathcal{S}(4,5,2,1)|$ could be symmetric. A proof that this is so is provided by the picture below:

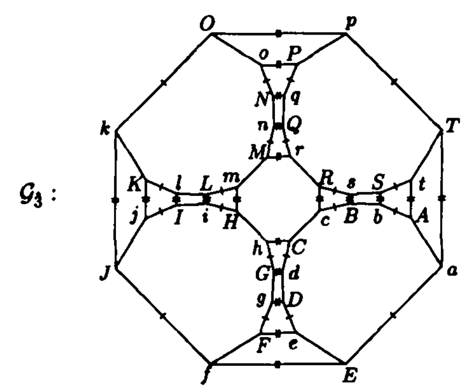


Fig. 216: Proving that $|\mathcal{S}(4,5,2,1)|$ is symmetric

Indeed, with regard to the first drawing of $\mathcal{G}$ (Fig. 209) the above picture defines an automorphism which permutes the edge-colors as the 4-cycle $(0,3,1,2)$, while preserving the bipartition of the vertices. Thus it induces a reversal of orientation, proving that the 3-manifold is symmetric.

6.7 Remarks on a Class of 3-Manifolds

We want to finish this chapter by mentioning the class of *planar 3-manifolds* of [Lin88]. They constitute a highly structured class of 3-manifolds in which the order of the first homology group is the number of trees of a generating planar graph. In this way, small graphs give rise to a rich amount of linking invariants.

Given a graph G embedded in the plane, we can proceed to get a crystallization $\Psi(G)$ as follows. Take the dual of the baricentric subdivision of the 2-cell complex induced by the plane embedding of G. This dual is a cubic bipartite plane graph which is naturally 3-edge colored. Indeed, the original edges correspond to square 2-residues which we paint with colors 1 and 2. We use color 3 to paint the other edges which correspond to adjacent pairs of the original edges. To get the associated bipartite crystallization introduce the 0-colored edges, as follows: the other end of a black vertex by a 0-colored edge, is the terminus of a 3-edge path which starting at the vertex uses a 1-colored edge, followed by a 2-colored edge, followed by a 3-colored edge. The $(3+1)$-graph so obtained is a crystallization ([Lin88]) which we denote by $\Psi(G)$.

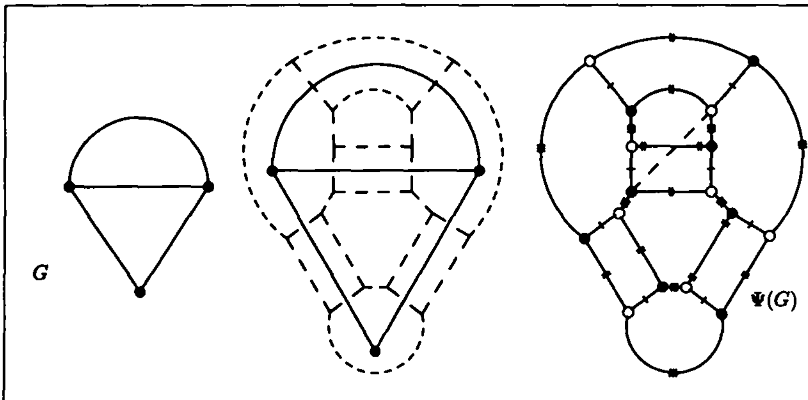


Fig. 217: Obtaining crystallization $\Psi(G)$

As mentioned, this construction has the property that the first homology group of the $|\Psi(G)|$ is finite and its order equals the number of spanning trees of the originating, plane graph G, 5 in the above case. This property is proved in [Lin88]. We also have the following

Conjecture 7 *If G is 3-connected, then $\Psi(G)$ is the superattractor for $|\Psi(G)|$.*

The associated class of 3-manifolds has been recognized by M. Ferri [Fer85] by using the central idea in [Fer79] as composed of 2-fold branched coverings of S^3 branched along the alternating links, $L(G)$, defined by the *medial of the plane graphs*, G.

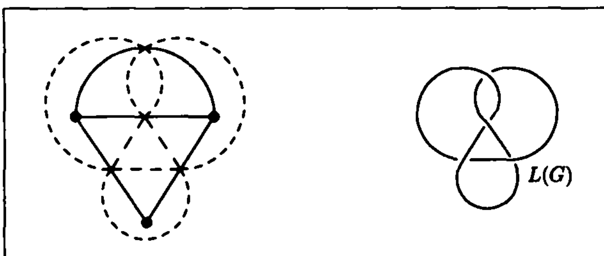


Fig. 218: Alternating link from the medial of plane graphs

Thus, the 3-manifold induced by the above $\Psi(G)$ is homeomorphic to the 2-fold branched covering over the figure 8 knot. In this case, the manifold is the lens space $L_{5,2}$.

Two random examples of self linking sequences on 3-manifolds given by the Ψ-construction follow. They are intended as control cases for checking independent implementations of the algorithm we have presented.

G_1 :

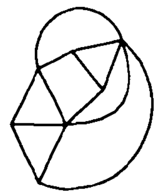


Fig. 219: Plane graph G_1

The number of trees of G_1 is equal to the order of torsion subgroup of $|\Psi(G_1)|$, namely 585. The number of its null self-linking numbers is 3. The self-linking sequence of $|\Psi(G_1)|$ follows:

$+2^8$	$+5^4$	$+8^8$	$+18^{12}$	$+20^4$	$+32^8$	$+45^6$	$+47^8$
$+50^4$	$+36^{12}$	$+65^2$	$+72^{12}$	$+80^4$	$+83^8$	$+98^8$	$+110^4$
$+117^6$	$+122^8$	$+125^4$	$+128^8$	$+135^6$	$+137^8$	$+143^4$	$+158^8$
$+162^{12}$	$+167^8$	$+180^6$	$+182^4$	$+188^8$	$+197^8$	$+200^4$	$+203^8$
$+215^4$	$+227^8$	$+242^8$	$+245^4$	$+252^{12}$	$+260^2$	$+275^4$	$+278^8$
$+288^{12}$	-292^8	-288^{12}	-280^4	-268^8	-265^4	-262^8	-253^8
-252^{12}	-247^4	-232^8	-223^8	-208^4	-202^8	-193^8	-190^4
-187^8	-180^6	-175^4	-163^8	-162^{12}	-148^8	-145^4	-135^6
-130^2	-117^6	-115^4	-112^8	-97^8	-85^4	-73^8	-72^{12}
-70^4	-67^8	-63^{12}	-58^8	-52^4	-45^6	-37^8	-28^8
-18^{12}	-13^4	-7^8					

Table 11: Self-linking sequence of $|\Psi(G_1)|$

In the above example, there is a unique torsion coefficient: 585. The matrix $\mathcal{M}(\Psi(G_1))$ is a 1×1 matrix defined by the single entry 323. Therefore, $|\Psi(G_1)|$ and the lens space $L_{585,323}$ have the same self-linking sequence. The manifolds have distinct π_1's.

G_2 :

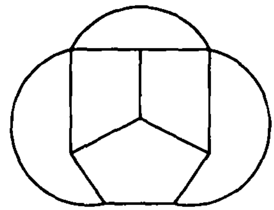


Fig. 220: Plane graph G_2

The number of trees of G_2 is equal to the order of torsion subgroup of $|\Psi(G_2)|$, namely 644. The number of its null self-linking numbers is 2. The self-linking sequence of $|\Psi(G_2)|$ follows:

$+2^8$	$+4^8$	$+8^8$	$+16^8$	$+18^8$	$+32^8$	$+36^8$	$+46^4$
$+50^8$	$+58^8$	$+64^8$	$+70^4$	$+72^8$	$+78^8$	$+92^4$	$+98^4$
$+100^8$	$+116^8$	$+128^8$	$+140^4$	$+142^8$	$+144^8$	$+154^4$	$+156^8$
$+162^8$	$+170^8$	$+184^4$	$+186^8$	$+190^8$	$+196^4$	$+200^8$	$+210^4$
$+232^8$	$+238^4$	$+242^8$	$+246^8$	$+254^8$	$+256^8$	$+266^4$	$+280^4$
$+282^8$	$+284^8$	$+288^8$	$+294^4$	$+302^8$	$+308^4$	$+312^8$	$\pm 322^2$
-320^8	-318^8	-314^8	-306^8	-304^8	-290^8	-286^8	-276^4
-272^8	-264^8	-258^8	-252^4	-250^8	-244^8	-230^4	-224^4
-222^8	-206^8	-194^8	-182^4	-180^8	-178^8	-168^4	-166^8
-160^8	-152^8	-138^4	-136^8	-132^8	-126^4	-122^8	-112^4
-90^8	-84^4	-80^8	-76^8	-68^8	-66^8	-56^4	-42^4
-40^8	-38^4	-34^8	-28^4	-20^8	-14^4	-10^8	

Table 12: Self-linking sequence of $|\Psi(G_2)|$

In the above example, there are two torsion coefficients: 2 and 322. The matrix $\mathcal{M}(\Psi(G_2))$ is

$$\mathcal{M}(\Psi(G_2)) = \begin{pmatrix} 57 & 8 \\ 0 & 177 \end{pmatrix}.$$

In the case of an M^3 given induced by a $\Psi(G)$ for an arbitrary plane graph G, the one row one column extension of the matrix A, from which we derive the presentation of the first homology group (see Section 6.4), is the product $A'(G) = \mathcal{I}(G).{}^t\mathcal{I}(G)$ of the incidence matrix of G by its transpose. This leads to the natural graph-theoretical question, which does not seem to be yet considered: *what are, in terms of G, the interpretations of the the number of torsion coefficients and of these coefficients in matrix $A'(G)$?*

Chapter 7

Computing the Quantum Invariants from Blinks

In this chapter we provide a recipe on how to compute the Witten-Reshetikhin-Turaev invariants. The computation is effected over a blink by means of a partition function. The full theory which justifies the formulas that we use appear in [KL94].

7.1 A Partition Function

We describe the partition function associated with the combinatorial definition and computation of the Witten-Reshetikhin-Turaev invariants for 3-manifolds. See [Wit89] and [RT91]. The recoupling theory developed in [KL94] which supports the invariant formulas here presented is a direct analog of the corresponding theory for q-deformed angular momentum recoupling using the quantum group $SL(2)_q$, [KR88].

7.1.1 Algebraic Ingredients

Let A be a primitive $4r$-th root of unity, $r \geq 3$, and $\mathcal{I} = \{0, 1, ..., r-2\}$. For n in $\mathcal{I}$, let

$$\Delta_n = (-1)^n \frac{A^{2n+2} - A^{-2n-2}}{A^2 - A^{-2}},$$

$$[n] = \frac{A^{2n} - A^{-2n}}{A^2 - A^{-2}} = (-1)^{n-1}\Delta_{n-1}.$$

Letting $q = A^2$, for reasons inherited from the physics we call $[n]$ the *q-deformed quantum integer* and

$$[n]! = \prod_{0 \leq m \leq n} [m]$$

the *q-deformed quantum factorial*. Note that even though A is complex, Δ_n and $[n]$ assume only real values.

Three numbers $a, b, c \in \mathcal{I}$ form an *r-admissible triple* if $a+b+c \leq 2r-4$ and the three numbers $a+b-c, b+c-a, c+a-b$ are non-negative and even. Let $\theta : \mathcal{I}^3 \mapsto \mathbb{R}$, defined on the r-admissible triples by means of

$$\theta(a,b,c) = \frac{(-1)^{m+n+p}[m+n+p+1]![n]![m]![p]!}{[m+n]![n+p]![p+m]!}, \text{where}$$

$m = (a+b-c)/2, n = (b+c-a)/2, p = (c+a-b)/2$. We define $\theta(a,b,c) = 0$ if (a,b,c) fails to be r-admissible.

Let $\lambda : \mathcal{I}^3 \mapsto \mathbb{C}$, be defined on the r-admissible triples by

$$\lambda(a,b,c) = \lambda_c^{ab} = (-1)^{(a+b-c)/2} A^{[a(a+2)+b(b+2)-c(c+2)]/2}$$

We define $\lambda_c^{ab} = 0$ if (a,b,c) fails to be r-admissible.

Finally, define $Tet : \mathcal{I}^6 \mapsto \mathbb{R}$, as follows. If (A,B,F), (A,D,E), (C,D,F), (B,C,E) are r-admissible, define

$$Tet(A,B,C,D,E,F) = Tet\begin{bmatrix} A & B & E \\ C & D & F \end{bmatrix} = \frac{\mathcal{I}nt!}{\mathcal{E}xt!} \sum_{m \leq s \leq M} \frac{(-1)^s[s+1]!}{\prod_i [s-a_i]! \prod_j [b_j - s]!}$$

where,

$$\begin{aligned} \mathcal{I}nt! &= \textstyle\prod_{i,j}[b_j - a_i]! \\ \mathcal{E}xt! &= [A]![B]![C]![D]![E]![F]! \\ a_1 &= \tfrac{1}{2}(A+D+E) \qquad b_1 = \tfrac{1}{2}(B+D+E+F) \\ a_2 &= \tfrac{1}{2}(B+C+E) \qquad b_2 = \tfrac{1}{2}(A+C+E+F) \\ a_3 &= \tfrac{1}{2}(A+B+F) \qquad b_3 = \tfrac{1}{2}(A+B+C+D) \\ a_4 &= \tfrac{1}{2}(C+D+F) \qquad m = \max\{a_i\} \quad M = \min\{b_j\} \end{aligned}$$

If one of the four triples above fails to be r-admissible, define the value of Tet as null.

These are all the algebraic ingredients we need to define a partition function on a shaded blackboard framed link. The partition function turns out to be an invariant of the oriented 3-manifold induced by the blackboard framed link. The outer face is not shaded, their neighboors by a segment between two crossings are shaded and so on, in a way to form a bicoloration of the link diagram. This shading is unique and could be disregarded, except that it is convenient to encode the partition function to be defined. Specifically, the λ_c^{ab}'s part of the algebraic data associated to the crossings need the shading.

Suppose that we label the faces of the link diagram and their components with elements of $\mathcal{I}$ in all possible ways, except that the outer face always gets a zero. Such a choice is named an *state*. From each such state we produce a complex number and the value of the partition function is the sum of all the *r-admissible* states. A *state is r-admissible* if all the triples of numbers associated to a black face, a neighboring white face and to a component separating them are r-admissible.

7.1.2 An Example for $L_{3,1}$ and the General Case

We display an example of the partition function in the case of a $\mathcal{I}$-labelled shaded blackboard framed link inducing $L_{3,1}$,

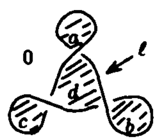


Fig. 221: Blackboard framed link for $L_{3,1}$, shading and variables for computing quantum invariants

Here is the (raw) partition function that we associate to the above diagram. [1] The sum is over all the r-admissible states. The bar over a λ_c^{ab} indicates its multiplicative inverse.

$$\sum_{a,b,c,d,\ell} \frac{Tet\begin{bmatrix} 0 & a & \ell \\ 0 & d & \ell \end{bmatrix} \lambda_0^{d\ell}\overline{\lambda_a^{0\ell}}\, Tet\begin{bmatrix} 0 & b & \ell \\ 0 & d & \ell \end{bmatrix} \lambda_0^{d\ell}\overline{\lambda_b^{0\ell}}\, Tet\begin{bmatrix} 0 & c & \ell \\ 0 & d & \ell \end{bmatrix} \lambda_0^{d\ell}\overline{\lambda_c^{0\ell}}\, \Delta_0\Delta_a\Delta_b\Delta_c\Delta_d\Delta_\ell}{\theta(0,d,\ell).\theta(0,d,\ell).\theta(0,d,\ell).\theta(0,a,\ell).\theta(0,b,\ell).\theta(0,c,\ell)}$$

The state is r-admissible if $\theta(0,d,\ell)$, $\theta(0,a,\ell)$, $\theta(0,b,\ell)$ and $\theta(0,c,\ell)$ are r-admissible. Note that this implies $a = b = c = d = \ell$.

The summation simplifies to

$$\sum_{\ell} \frac{Tet\begin{bmatrix} 0 & \ell & \ell \\ 0 & \ell & \ell \end{bmatrix} \lambda_0^{\ell\ell}\overline{\lambda_\ell^{0\ell}}\, Tet\begin{bmatrix} 0 & \ell & \ell \\ 0 & \ell & \ell \end{bmatrix} \lambda_0^{\ell\ell}\overline{\lambda_\ell^{0\ell}}\, Tet\begin{bmatrix} 0 & \ell & \ell \\ 0 & \ell & \ell \end{bmatrix} \lambda_0^{\ell\ell}\overline{\lambda_\ell^{0\ell}}\, \Delta_\ell^5}{\theta(0,\ell,\ell)^6}$$

It happens that $Tet\begin{bmatrix} 0 & \ell & \ell \\ 0 & \ell & \ell \end{bmatrix} = \theta(0,\ell,\ell)$ and that $\theta(0,\ell,\ell) = \Delta_\ell$. Thus we have the summation assuming the simple form

$$\sum_{\ell} \lambda_0^{\ell\ell}\overline{\lambda_\ell^{0\ell}}\lambda_0^{\ell\ell}\overline{\lambda_\ell^{0\ell}}\lambda_0^{\ell\ell}\overline{\lambda_\ell^{0\ell}}\Delta_\ell^2$$

[1] Indeed to have an invariant this partition function has to be multiplied by a normalization factor which we discuss briefly.

Fig. 222: Blinks and blackboard framed links for $L_{3,1}$ and $L_{3,2}$

In a way explained in the next subsection, from the blink inducing a 3-manifold we get a recipe on how to compute the quantum invariants. For the 3-star blink inducing $L_{3,1}$ we get, according to the above formula, conveniently normalized:

r	$S^1 \times S^2 - normalization$	$S^3 - normalization$	C^{++}, 486 *time*	$r-$ *states*
3	$-0.70710678i$	$-1.00000000i$	0.003*sec*	2
4	$-0.35355339 - 0.35355339i$	$-0.70710678 - 0.70710678i$	0.002*sec*	3
5	$-0.57206140 + 0.18587402i$	$-1.53884177 + 0.50000000i$	0.003*sec*	4
6	$+0.50000000i$	$+1.73205081i$	0.003*sec*	5
7	$+0.32673246 + 0.26056044i$	$+1.40881165 + 1.12348980i$	0.004*sec*	6

Table 13A: Values of the quantum invariants at $A = e^{\pi i/2r}$ for $L_{3,1}$

As for $L_{3,2}$ we get the following table:

r	$S^1 \times S^2 - normalization$	$S^3 - normalization$	C^{++}, 486 *time*	$r-$ *states*
3	$+0.70710678i$	$+1.00000000i$	0.003*sec*	2
4	$-0.35355339 + 0.35355339i$	$-0.70710678 + 0.70710678i$	0.002*sec*	3
5	$-0.57206140 - 0.18587402i$	$-1.53884177 - 0.50000000i$	0.003*sec*	4
6	$-0.50000000i$	$-1.73205081i$	0.003*sec*	5
7	$+0.32673246 - 0.26056044i$	$+1.40881165 - 1.12348980i$	0.004*sec*	6

Table 13B: Values of the quantum invariants at $A = e^{\pi i/2r}$ for $L_{3,2}$

Have in mind that $L_{3,2}$ is $L_{3,1}$ with opposite orientation. The effect of changing the orientation is to conjugate the invariants (which are complex numbers). This is due to the λ_c^{ab} factors, which conjugate under the reversal of crossings. The tables appear with two normalizations. The $S^1 \times S^2$-*normalization* assigns 1 to all the invariants of $S^1 \times S^2$. The S^3-*normalization* assigns 1 to all the invariants of S^3.

Given a blackboard framed link L, the S^3-normalization factor is

$$(\alpha/\mu)^{-n(L)}$$

and the $S^1 \times S^2$-normalization factor is

$$\mu^{|L|+1}\alpha^{-n(L)}$$

where[2]

$$\mu = sin(\pi/r)\sqrt{2/r}, \quad \alpha = (-i)^{r-2}e^{3i\pi(r-2)/4r},$$

$|L|$ is the number of components of the link L, and $n(L)$ is the number of positive eigenvalues minus the number of negative eigenvalues of a symmetric matrix, named the *linking matrix of* L, $N = N(L)$. To get this matrix, choose an orientation for the components $K_1, K_2, \ldots, K_k$ and define

$$N_{ij} = \begin{cases} link(K_i, K_j), & \text{for } i \neq j \\ writhe(K_i), & \text{for } i = j \end{cases}$$

Here $link(K_i, K_j)$ is half of the algebraic sum of the crossing between components K_i and K_j and $writhe(K_i)$ is the algebraic sum of the self-crossings of the component K_i. Each crossing contributes with a ± 1 according to the rule:

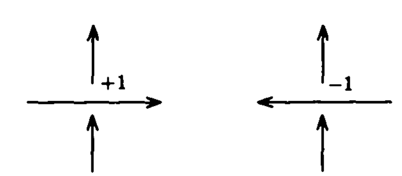


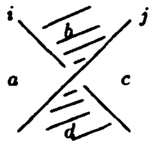
Fig. 223: Positive and negative crossings

Indeed, the effect on N of changing orientation of a component K_i is to multiply the corresponding row and the column by -1. This has no effect on the eigenvalues of N. Therefore, the precise orientation that we use does not matter.

We consider now how to obtain the partition function from a $\mathcal{I}$-labelled shaded blackboard framed link.

To each slash crossing as shown below we associate the product of a Tet by two λ's. There are two possibilities for the λ-terms.

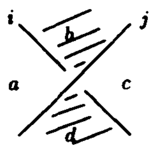
[2]These values of μ and α only are valid for $A = e^{i\pi/2r}$. For other primitive $4r$-roots of unity a similar complicated computation of section 12.8 of [KL94] has to be performed.



$$\mapsto Tet\begin{bmatrix} a & b & i \\ c & d & j \end{bmatrix} \lambda_b^{ai}\overline{\lambda}_c^{di} \quad \text{or} \quad Tet\begin{bmatrix} a & b & i \\ c & d & j \end{bmatrix} \lambda_b^{cj}\overline{\lambda}_a^{dj}$$

Fig. 224A: Algebraic factors from a slash crossing

For the backslash crossings we simply invert the λ-factors:

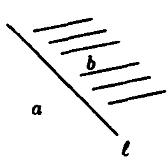


$$\mapsto Tet\begin{bmatrix} a & b & i \\ c & d & j \end{bmatrix} \overline{\lambda}_b^{ai}\lambda_c^{di} \quad \text{or} \quad Tet\begin{bmatrix} a & b & i \\ c & d & j \end{bmatrix} \overline{\lambda}_b^{cj}\lambda_a^{dj}$$

Fig. 224B: Algebraic factors from a backslash crossing

One of the various *"miracles"* which make this partition function work is that its value does not depend on the choices for the λ-terms.

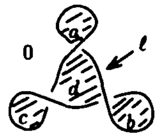
We associate the inverse of a θ-term to each line segment between two crossings (which may coincide). The parameters of the θ-function are the labels of the white face, the black face and the link component which supports the segment:



$$\mapsto \quad \frac{1}{\theta(a,b,\ell)}$$

Fig. 225: Algebraic factor from a segment between crossings

Finally we associate a Δ_a to each face or link component labelled with an a. In the example inducing $L_{3,1}$ we get:



$\mapsto \quad \Delta_0\Delta_a\Delta_b\Delta_c\Delta_d\Delta_\ell$

Fig. 226: Algebraic factors from the faces and link components

7.1.3 An Example for $S^3/\langle 2,2,2\rangle$

From the square blink inducing $S^3/\langle 2,2,2\rangle$ we get at once from the associated blackboard framed link all that we need to compute the quantum invariants.

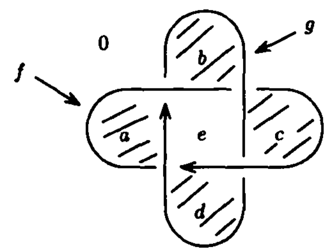


Fig. 227: All data needed to yield the quantum invariants for $S^3/\langle 2,2,2\rangle$

The matrix N is $\begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix}$. The eigenvalues are ± 2, therefore $n(L) = 0$. There are two components, and so the factor of the $S^1 \times S^2$-normalization is simply $\mu^3\alpha^0 = \mu^3$. As for the S^3-normalization, it is simpler yet, $(\alpha/\mu)^0 = 1$.

The partition function assumes the form

$$\sum_{\substack{a,b,c,d,e,f,g \in \mathcal{I} \\ \text{and } r\text{-admissible}}} TET.\lambda.\Delta/\theta,$$

where the four crossings yield

$$TET = Tet\begin{bmatrix} 0 & a & f \\ e & b & g \end{bmatrix} Tet\begin{bmatrix} 0 & b & f \\ e & c & g \end{bmatrix} Tet\begin{bmatrix} 0 & c & f \\ e & d & g \end{bmatrix} Tet\begin{bmatrix} 0 & d & f \\ e & a & g \end{bmatrix}$$

The four crossings (which are slash crossings) also yield

$$\lambda = (\lambda_b^{0g}.\overline{\lambda}_e^{ag}).(\lambda_c^{0f}.\overline{\lambda}_e^{bf}).(\lambda_d^{0g}.\overline{\lambda}_e^{cg}).(\lambda_a^{0f}.\overline{\lambda}_e^{df}).$$

The eight segments between crossings give us

$$\theta = \theta(0,f,a).\theta(a,g,e).\theta(0,g,b).\theta(b,f,e).\theta(0,f,c).\theta(c,g,e).\theta(0,g,d).\theta(d,g,e).$$

Finally, the four black faces, the two white faces and the two components yield

$$\Delta = \Delta_a.\Delta_b.\Delta_c.\Delta_d.\Delta_0.\Delta_e.\Delta_f.\Delta_g$$

Implementing the above formulas we get

r	$S^1 \times S^2 - normalization$	$S^3 - normalization$	C^{++}, 486 *time*	$r-$ *states*
3	$+1.41421356$	$+2.00000000$	0.008*sec*	4
4	$+1.50000000 + 0.50000000i$	$+3.00000000 + 1.00000000i$	0.012*sec*	10
5	$+1.11524410 + 1.14412281i$	$+3.00000000 + 3.07768354i$	0.025*sec*	20
6	$+0.28867513 + 1.50000000i$	$+1.00000000 + 5.19615242i$	0.044*sec*	35
7	$-0.66117206 + 1.26706919i$	$-2.85085508 + 5.46337461i$	0.070*sec*	56
8	$-1.27373392 + 0.46193977i$	$-6.65685425 + 2.41421356i$	0.105*sec*	84
9	$-1.21205194 - 0.55851656i$	$-7.51754097 - 3.46410162i$	0.190*sec*	120
10	$-0.47983739 - 1.27597621i$	$-3.47213595 - 9.23305061i$	0.257*sec*	165

Table 14: Values of the quantum invariants at $A = e^{\pi i/2r}$ for $S^3/\langle 2,2,2\rangle$

7.1.4 Distinguishing $S^3/\langle 2,2,2\rangle \# L_{3,1}$ and $S^3/\langle 2,2,2\rangle \# L_{3,2}$

As it can be seen from the last table, since the invariants are not real numbers, that the space $S^3/\langle 2,2,2\rangle$ is not symmetric. It follows from Milnor's Unique Decomposition Theorem for 3-manifolds [Mil62] that

$$L_{3,1}\#S^3/\langle 2,2,2\rangle \quad \text{and} \quad L_{3,2}\#S^3/\langle 2,2,2\rangle$$

are not homeomorphic, because this theorem applies to oriented 3-manifolds: since $L_{3,1}$ is not symmetric, the only way that the above 3-manifolds could be homeomorphic would be through an orientation reversing homeomorphism if the piece $S^3/\langle 2,2,2\rangle$ were symmetric. We know that this is not the case since there are values of the previous table which are not real. Recall that change of orientation conjugates the invariants and if such an invariant has non-zero imaginary part, it distinguishes the oriented forms of the 3-manifold.

Here are the first quantum invariants of the above connected sums. The invariants confirm that the 3-manifolds are not homeomorphic:

r	$S^1 \times S^2$ – *normalization*	S^3 – *normalization*	C^{++}, 486 *time*	$r-$ *states*
3	$-1.41421356i$	$-2.00000000i$	0.016*sec*	8
4	$-0.70710678 - 1.41421356i$	$-1.41421356 - 2.82842712i$	0.065*sec*	34
5	$-2.28824561 - 1.20300191i$	$-6.15536707 - 3.23606798i$	0.211*sec*	104
6	$-2.59807621 + 0.50000000i$	$-9.00000000 + 1.73205081i$	0.547*sec*	259
7	$-2.35500622 + 1.04224178i$	$-10.15436351 + 4.49395921i$	1.226*sec*	560
8	$-2.41421356 + 2.20710678i$	$-12.61728812 + 11.53489592i$	2.501*sec*	1092

Table 15A: Values of the quantum invariants at $A = e^{\pi i/2r}$ for $S^3/\langle 2,2,2\rangle \# L_{3,1}$

r	$S^1 \times S^2$ – *normalization*	S^3 – *normalization*	C^{++}, 486 *time*	$r-$ *states*
3	$+1.41421356i$	$+2.00000000i$	0.016*sec*	8
4	$-1.41421356 + 0.70710678i$	$-2.82842712 + 1.41421356i$	0.065*sec*	34
5	$-1.14412281 - 2.31824601i$	$-3.07768354 - 6.23606798i$	0.211*sec*	104
6	$+2.59807621 - 0.50000000i$	$+9.00000000 - 1.73205081i$	0.547*sec*	259
7	$+0.49207242 + 2.52788191i$	$+2.12172782 + 10.89977241i$	1.226*sec*	560
8	$-3.26776695 - 0.14644661i$	$-17.07817311 - 0.76536686i$	2.452*sec*	1092

Table 15B: Values of the quantum invariants at $A = e^{\pi i/2r}$ for $S^3/\langle 2,2,2\rangle \# L_{3,2}$

The first of the above tables is derived from the partition function given by the following blackboard framed link (we display the relevant variables but not the shading, which is implicit):

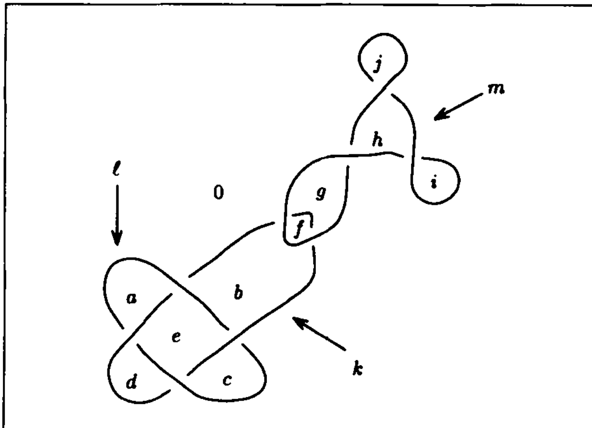


Fig. 228: Blackboard framed link for $S^3/\langle 2,2,2\rangle \# L_{3,1}$ and variables for computing its quantum invariants

7.1.5 A Brief Explanation for the Invariance

We briefly display diagramatic moves on blackboard framed links so that the invariance at those imply invariance at the 3-manifold level, according to basic Theorems of Lickorish [Lic62] and Kirby [Kir78]. That is what the normalized partition function of the previous subsection accomplishes [KL94].

By a Theorem of Reidemeister, ambient isotopy of links can be generated by the following three moves:

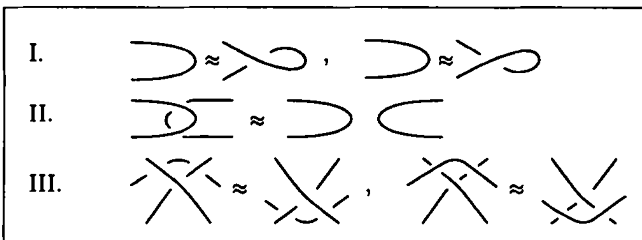


Fig. 229: Ambient and regular isotopy

To our purposes, move I should not be considered at all, since blackboard framed links differing by curls, induce distinct 3-manifolds. The equivalence relation generated by moves II and III on link diagrams is named *regular isotopy*. The equivalence generated by all the three moves is called *ambient isotopy*.

Blackboard framed links differing by regular isotopy induce the same 3-manifold. Another move to which any invariant should be blind is the *ribbon move* defined below:

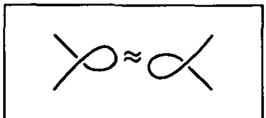


Fig. 230: Ribbon move

The reason for the name and to impose the invariance under the ribbon move is because the framings corresponding to these two curls are ambient isotopic. (They are not regularly isotopic because have distinct winding number in the plane.)

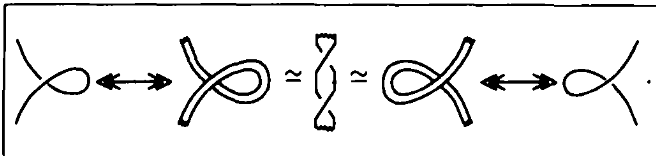


Fig. 231: Why to impose ribbon move invariance

The crucial move to which any such invariant must be blind to is named *handle slide* and is exemplified below.

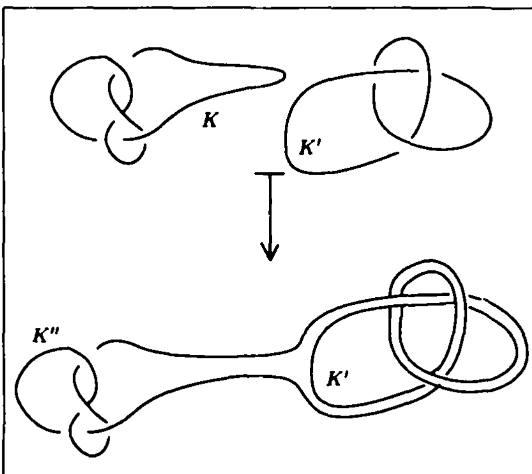


Fig. 232: An example of handle slide

The above move is the manifestation at a link level of the sliding of handles in a 4-manifold induced by the framed link. See Section 12.1 of [KL94]. In this move (at link level) one component (K) is replaced by a new component (K'') obtained by taking a connected sum of K with a parallel push-off (in the blackboard framing) of another component K'. The operation is reversible, in the sense that another adequate handle slide can bring back the original situation. The connected sum is obtained by moving the diagrams by regular isotopy so that an arc of K is contiguous with an arc of K' as shown in the left figure below. Thus, we can indicate a general handle slide symbolically via the hieroglyph:

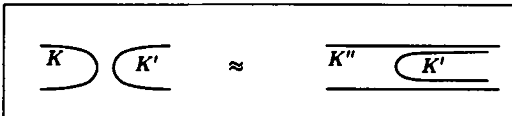


Fig. 233: The general handle slide

The addition of a disjoint component of type shown below corresponds to effecting the connected sum of the 3-manifold induced by the rest of the link with S^3. Therefore, it is possible to add or subtract disjoint components of this type.

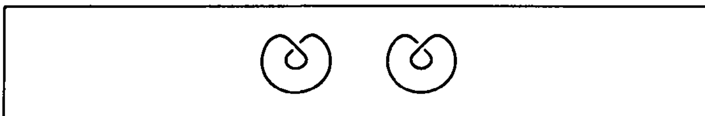


Fig. 234: S^2-separated components that can be added or subtracted

We have the following central results.

Theorem 16 *[Lickorish [Lic62]]. Every compact orientable 3-manifold can be induced as surgery on a blackboard framed link $L \subset S^3$.*

By $M^3(L)$ we denote the 3-manifold induced by a blackboard framed link L.

Theorem 17 *[Kirby [Kir78]]. $M^3(L)$ is homeomorphic to $M^3(L')$ if and only if L' can be obtained from L by combinations of*

1) regular isotopy,

2) ribbon moves,

3) handle slides,

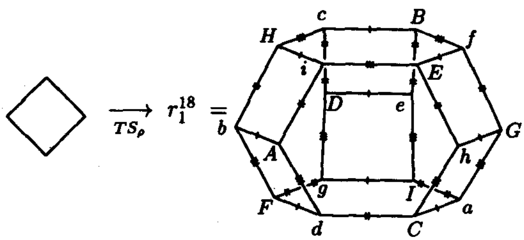
4) addition or subtraction of S^2-separated components of the form  *or* .

The original statements and proofs of these Theorems did not use blackboard framed links. This specialization is straighforward and resulted crucial for our combinatorial approach of the quantum invariants in [KL94]. Also, the direct construction of a 3-gem inducing the same 3-manifold as the one of a framed link (Figure 13) is possible due to the use of blackboard framing.

7.2 Tables of Quantum Invariants

In this section we provide blinks and quantum invariants for some of the 3-manifolds. Our purposes are (i) to provide a glimpse of these new and interesting invariants; (ii) to present blinks (as much as we could) inducing 3-manifolds in our catalogue. It is a computationally interesting open problem to obtain an implementable algorithm which produces a blink inducing the same 3-manifold as a given 3-gem.

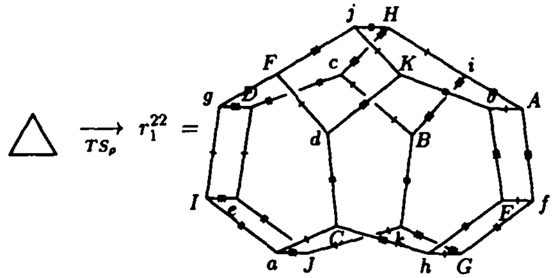
In the following tables a blank entry in a row headed by r means that the invariants vanish at that level. We also include the number of r-states and the time it took for each computation (in a PC-486 at 50 Mhz).



$\pi_1 = \langle 2,2,2\rangle$
Quaternion
group:Q_8
3-manifold
$S^3/\langle 2,2,2\rangle$

r	$S^1 \times S^2 - normalization$	$S^3 - normalization$	C^{++}, 486 *time*	*r–states*
3	+1.41421356	+2.00000000	0.008*sec*	4
4	+1.50000000 + 0.50000000*i*	+3.00000000 + 1.00000000*i*	0.012*sec*	10
5	+1.11524410 + 1.14412281*i*	+3.00000000 + 3.07768354*i*	0.025*sec*	20
6	+0.28867513 + 1.50000000*i*	+1.00000000 + 5.19615242*i*	0.044*sec*	35
7	−0.66117206 + 1.26706919*i*	−2.85085508 + 5.46337461*i*	0.070*sec*	56
8	−1.27373392 + 0.46193977*i*	−6.65685425 + 2.41421356*i*	0.105*sec*	84
9	−1.21205194 − 0.55851656*i*	−7.51754097 − 3.46410162*i*	0.190*sec*	120
10	−0.47983739 − 1.27597621*i*	−3.47213595 − 9.23305061*i*	0.257*sec*	165
11	+0.54864472 − 1.30064086*i*	+4.56704736 − 10.82683953*i*	0.280*sec*	220
12	+1.31698730 − 0.60566243*i*	+12.46410162 − 5.73205081*i*	0.363*sec*	286
13	+1.39389429 + 0.43709222*i*	+14.84962173 + 4.65648949*i*	0.465*sec*	364
14	+0.72362901 + 1.25143152*i*	+8.60387547 + 14.87939376*i*	0.582*sec*	455
15	−0.33004844 + 1.37759673*i*	−4.34739800 + 18.14570407*i*	0.716*sec*	560
16	−1.18011531 + 0.73694061*i*	−17.10935797 + 10.68419374*i*	0.867*sec*	680
17	−1.34643031 − 0.31525559*i*	−21.36324070 − 5.00202729*i*	1.045*sec*	816
18	−0.73197896 − 1.18531940*i*	−12.64589651 − 20.47794707*i*	1.240*sec*	969
19	+0.31603521 − 1.37761916*i*	+5.91809212 − 25.79736918*i*	1.462*sec*	1140
20	+1.19986899 − 0.77996840*i*	+24.25500606 − 15.76683657*i*	1.704*sec*	1330
21	+1.41193001 + 0.26799755*i*	+30.69717235 + 5.82661111*i*	1.977*sec*	1540
22	+0.82747903 + 1.16619329*i*	+19.28426777 + 27.17795011*i*	2.274*sec*	1771
23	−0.22067076 + 1.39770880*i*	−5.49569930 + 34.80926625*i*	2.600*sec*	2024
24	−1.13109425 + 0.82740158*i*	−30.01869094 + 21.95883521*i*	2.948*sec*	2300
25	−1.37942070 − 0.21792867*i*	−38.91217463 − 6.14756501*i*	3.343*sec*	2600
26	−0.82173517 − 1.13608224*i*	−24.58013857 − 33.98303963*i*	3.762*sec*	2925
27	+0.22049987 − 1.39655149*i*	+6.97861918 − 44.19957801*i*	4.216*sec*	3276
28	+1.14470151 − 0.84789570*i*	+38.25392652 − 28.33519461*i*	4.698*sec*	3654
29	+1.41525872 + 0.19320608*i*	+49.84455775 + 6.80460156*i*	5.232*sec*	4060
30	+0.87423073 + 1.12407384*i*	+32.39195304 + 41.64912732*i*	5.789*sec*	4495

Table 16: Values of the quantum invariants at $A = e^{\pi i/2r}$ for $S^3/\langle 2,2,2\rangle$

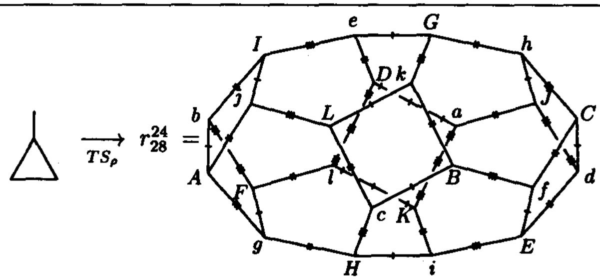


$\pi_1 = \langle 3,3,2 \rangle$
Binary tetrahedral group

r	$S^1 \times S^2$ – *normalization*	S^3 – *normalization*	C^{++}, 486 *time*	r– *states*
3	$+0.70710678i$	$+1.00000000i$	0.004*sec*	2
4	$+0.35355339 - 0.35355339i$	$+0.70710678 - 0.70710678i$	0.004*sec*	4
5	$-0.21850801 + 0.90225143i$	$-0.58778525 + 2.42705098i$	0.006*sec*	6
6	$-0.50000000i$	$-1.73205081i$	0.009*sec*	9
7	$-0.50805530 + 0.53386681i$	$-2.19064313 + 2.30193774i$	0.012*sec*	12
8	$+0.32322330 - 0.78033009i$	$+1.68924640 - 4.07820156i$	0.015*sec*	16
9	$-0.35355339 + 0.34590729i$	$-2.19285330 + 2.14542968i$	0.019*sec*	20
10	$+0.76631190 - 0.39429833i$	$+5.54508497 - 2.85316955i$	0.024*sec*	25
11	$-0.56088532 + 0.45576490i$	$-4.66894094 + 3.79389393i$	0.029*sec*	30
12	$+0.61237244$	$+5.79555496$	0.034*sec*	36
13	$-0.88793114 + 0.01176234i$	$-9.45942723 + 0.12530814i$	0.046*sec*	42
14	$+0.60899921 + 0.02025142i$	$+7.24093881 + 0.24078731i$	0.054*sec*	49
15	$-0.58835672 - 0.45271780i$	$-7.74983465 - 5.96319882i$	0.054*sec*	56
16	$+0.71509707 + 0.42546890i$	$+10.36750525 + 6.16846475i$	0.061*sec*	64
17	$-0.34952089 - 0.44947463i$	$-5.54570031 - 7.13162409i$	0.069*sec*	72
18	$+0.25000000 + 0.78712518i$	$+4.31907786 + 13.59861971i$	0.077*sec*	81
19	$-0.27176891 - 0.67277595i$	$-5.08915893 - 12.59843795i$	0.086*sec*	90
20	$-0.13110234 + 0.60923960i$	$-2.65019609 + 12.31560305i$	0.096*sec*	100
21	$+0.24789667 - 0.80524083i$	$+5.38959200 - 17.50697019i$	0.105*sec*	110
22	$-0.24163484 + 0.57131097i$	$-5.63126173 + 13.31431167i$	0.116*sec*	121
23	$+0.59467394 - 0.40121973i$	$+14.81006873 - 9.99218475i$	0.126*sec*	132
24	$-0.64145656 + 0.47091587i$	$-17.02394497 + 12.49787809i$	0.138*sec*	144
25	$+0.55934839 - 0.15204332i$	$+15.77869769 - 4.28900045i$	0.150*sec*	156

Table 17: Values of the quantum invariants at $A = e^{\pi i/2r}$ for $|r_1^{22}| = S^3/\langle 3,3,2 \rangle$

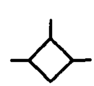
Other blink inducing the same oriented 3-manifold:  .

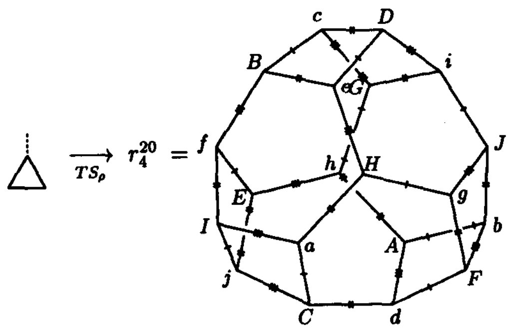


$\pi_1 = \langle 4,3,2 \rangle$
Space of truncated cube

r	$S^1 \times S^2$ – *normalization*	S^3 – *normalization*	C^{++}, 486 *time*	$r-$ *states*
3			0.005*sec*	2
4	$+0.92387953i$	$+1.84775907i$	0.005*sec*	4
5			0.007*sec*	6
6	$-0.28867513 - 0.86602540i$	$-1.00000000 - 3.00000000i$	0.011*sec*	9
7			0.014*sec*	12
8	$+0.31015268 + 0.86572292i$	$+1.62093605 + 4.52448601i$	0.019*sec*	16
9			0.024*sec*	20
10	$-0.23229208 - 0.90311247i$	$-1.68088129 - 6.53498326i$	0.030*sec*	25
11			0.037*sec*	30
12	$+0.09904576 + 0.94205477i$	$+0.93737914 + 8.91570205i$	0.044*sec*	36
13			0.050*sec*	42
14	$+0.06946844 - 0.95931590i$	$+0.82597275 - 11.40616859i$	0.058*sec*	49
15			0.067*sec*	56
16	$-0.25780158 + 0.94054485i$	$-3.73761739 + 13.63605600i$	0.076*sec*	64
17			0.086*sec*	72
18	$+0.45110530 - 0.87781652i$	$+7.79343563 - 15.16543162i$	0.096*sec*	81
19			0.107*sec*	90
20	$-0.63463263 + 0.76853006i$	$-12.82891579 + 15.53561380i$	0.118*sec*	100
21			0.130*sec*	110
22	$+0.79426308 - 0.61478742i$	$+18.51017542 - 14.32752358i$	0.143*sec*	121
23			0.157*sec*	132
24	$-0.91732847 + 0.42283990i$	$-24.34545122 + 11.22196504i$	0.171*sec*	144
25			0.185*sec*	156

Table 18: Values of the quantum invariants at $A = e^{\pi i/2r}$ for $|r_{28}^{24}| = S^3/\langle 4,3,2 \rangle$

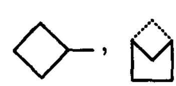
Other blinks inducing the same oriented 3-manifold: , .



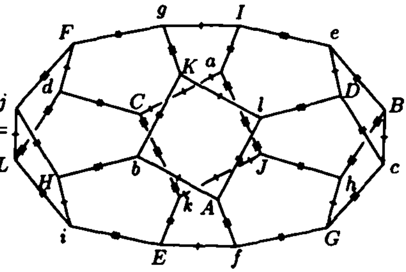
$\pi_1 = \langle 3,2,2\rangle$
Non-abelian group of order 12
3-manifold: $S^3/\langle 3,2,2\rangle$

r	$S^1 \times S^2$ – normalization	S^3 – normalization	C^{++}, 486 time	r– states
3	$+0.70710678 + 0.70710678i$	$+1.00000000 + 1.00000000i$	0.005sec	2
4	$-0.65328148 + 0.27059805i$	$-1.30656296 + 0.54119610i$	0.005sec	4
5	$-0.01443935 - 1.12968345i$	$-0.03884177 - 3.03884177i$	0.007sec	6
6	$+1.15470054$	$+4.00000000$	0.011sec	9
7	$-0.40843813 + 0.76906498i$	$-1.76111181 + 3.31606994i$	0.014sec	12
8	$-0.55368450 - 0.97130504i$	$-2.89369463 - 5.07628477i$	0.019sec	16
9	$+1.07791490 - 0.37080812i$	$+6.68557934 - 2.29987274i$	0.024sec	20
10	$-0.17082039 + 0.85065081i$	$-1.23606798 + 6.15536707i$	0.030sec	25
11	$-0.70274421 - 0.78095730i$	$-5.84980763 - 6.50087175i$	0.037sec	30
12	$+0.96574120 - 0.48374643i$	$+9.13987282 - 4.57822536i$	0.044sec	36
13	$-0.08661209 + 0.84632810i$	$-0.92270758 + 9.01621614i$	0.050sec	42
14	$-0.74851543 - 0.70449548i$	$-8.89977241 - 8.37637968i$	0.058sec	49
15	$+0.93234396 - 0.52296927i$	$+12.28083457 - 6.88855126i$	0.067sec	56
16	$-0.04958464 + 0.87691959i$	$-0.71888002 + 12.71361459i$	0.076sec	64
17	$-0.80735362 - 0.66456400i$	$-12.80993866 - 10.54435615i$	0.086sec	72
18	$+0.91959016 - 0.57735027i$	$+15.88712603 - 9.97448307i$	0.096sec	81
19	$+0.00567097 + 0.90666400i$	$+0.10619484 + 16.97823809i$	0.107sec	90
20	$-0.85510915 - 0.60727946i$	$-17.28578518 - 12.27597942i$	0.119sec	100
21	$+0.88425814 - 0.62233291i$	$+19.22490812 - 13.53031708i$	0.130sec	110
22	$+0.05331135 + 0.90434040i$	$+1.24241256 + 21.07550999i$	0.143sec	121
23	$-0.86738023 - 0.55732947i$	$-21.60168809 - 13.88002266i$	0.157sec	132
24	$+0.85151984 - 0.63474837i$	$+22.59892222 - 16.84591270i$	0.171sec	144
25	$+0.07082544 + 0.89722193i$	$+1.99791990 + 25.30979574i$	0.185sec	156

Table 19: Values of the quantum invariants at $A = e^{\pi i/2r}$ for $|r_4^{20}| = S^3/\langle 3,2,2\rangle$

Other blinks inducing the same oriented 3-manifold: .

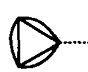
$\xrightarrow[TS_p]{}$ r_2^{24} =

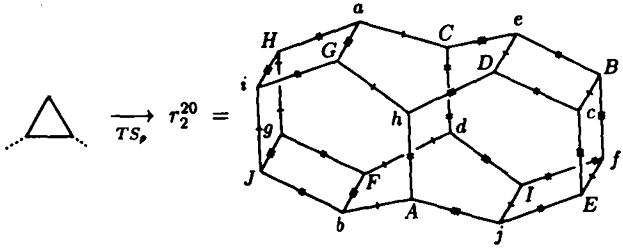


$\pi_1 = \langle 5,3,2\rangle$
Poincaré
sphere

r	$S^1 \times S^2$ – *normalization*	S^3 – *normalization*	C^{++}, 486 *time*	$r-$ *states*
3	$+0.70710678$	$+1.00000000$	0.006*sec*	2
4	-0.50000000	-1.00000000	0.006*sec*	4
5	$-0.30075048 + 0.92561479i$	$-0.80901699 + 2.48989828i$	0.009*sec*	6
6	$+0.28867513$	$+1.00000000$	0.014*sec*	9
7	$-0.84603445 + 0.04478304i$	$-3.64794847 + 0.19309643i$	0.018*sec*	12
8	$+0.73253782i$	$+3.82842712i$	0.024*sec*	16
9	$-0.17612688 - 0.40204608i$	$-1.09239627 - 2.49362077i$	0.030*sec*	20
10	$-0.76631190 + 0.55675818i$	$-5.54508497 + 4.02874005i$	0.038*sec*	25
11	$+0.29986112 + 0.15573689i$	$+2.49611427 + 1.29639039i$	0.047*sec*	30
12	$-0.78867513 - 0.18301270i$	$-7.46410162 - 1.73205081i$	0.056*sec*	36
13	$-0.11486097 + 0.74265244i$	$-1.22365232 + 7.91172464i$	0.064*sec*	42
14	$-0.10744239 - 0.39775226i$	$-1.27747907 - 4.72923401i$	0.074*sec*	49
15	$-0.77709557 + 0.534440395i$	$-10.23590284 + 7.03916882i$	0.085*sec*	56
16	$+0.31417116 + 0.17622148i$	$+4.55486585 + 2.554865585i$	0.097*sec*	64
17	$-0.78042634 - 0.142853305i$	$-12.38269491 - 2.26658898i$	0.109*sec*	72
18	$-0.05909505 + 0.76366977i$	$-1.02094453 + 13.19339680i$	0.123*sec*	81
19	$-0.13018472 - 0.37301190i$	$-2.43784591 - 6.98504059i$	0.137*sec*	90
20	$-0.70858278 + 0.62543139i$	$-14.32379682 + 12.64291553i$	0.152*sec*	100
21	$+0.34104884 + 0.14952903i$	$+7.41483989 + 3.25095320i$	0.167*sec*	110
22	$-0.78546018 - 0.02481144i$	$-18.30502518 - 0.57822665i$	0.184*sec*	121
23	$+0.06003894 + 0.77496127i$	$+1.49524085 + 19.30003825i$	0.201*sec*	132
24	$-0.18144700 - 0.33767686i$	$-4.81551515 - 8.96177952i$	0.219*sec*	144
25	$-0.58950598 + 0.74415703i$	$-16.62941518 + 20.99197757i$	0.237*sec*	156

Table 20: Values of the quantum invariants at $A = e^{\pi i/2r}$ for $|r_2^{24}| = S^3/\langle 5,3,2\rangle$

Other blink inducing the same oriented 3-manifold: ...... .

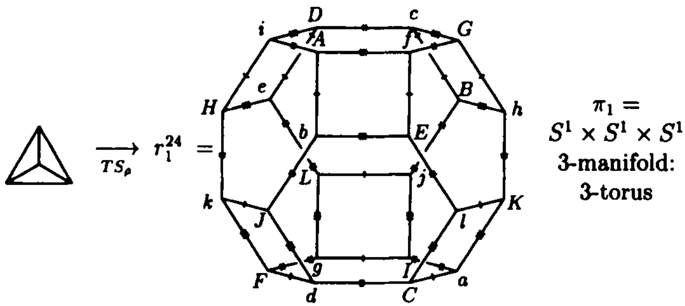


$\pi_1 = Z_5$
Lens space
$L_{5,1}$

r	$S^1 \times S^2$ – *normalization*	S^3 – *normalization*	C^{++}, 486 *time*	$r-$ *states*
3	+0.70710678	+1.00000000	0.005*sec*	2
4	+0.50000000i	+1.00000000i	0.006*sec*	4
5	−0.41562694 + 0.57206140i	−1.11803399 + 1.53884177i	0.009*sec*	6
6	−0.28867513	−1.00000000	0.013*sec*	9
7	+0.11596031 − 0.50805530i	+0.50000000 − 2.19064313i	0.017*sec*	12
8	+0.32664074 − 0.32664074i	+1.70710678 − 1.70710678i	0.023*sec*	16
9	+0.15150649 + 0.26241694i	+0.93969262 + 1.62759536i	0.028*sec*	20
10	−0.15450850 + 0.47552826i	−1.11803399 + 3.44095480i	0.035*sec*	25
11	−0.22119193 + 0.06494781i	−1.84125353 + 0.54064082i	0.044*sec*	30
12	−0.39433757i	−3.73205081i	0.052*sec*	36
13	+0.20833408 − 0.30182402i	+2.21945257 − 3.21543215i	0.059*sec*	42
14	+0.14692973 + 0.18424406i	+1.74697960 + 2.19064313i	0.069*sec*	49
15	−0.08487959 + 0.39932709i	−1.11803399 + 5.25993637i	0.079*sec*	56
16	−0.18147187 + 0.07516811i	−2.63098631 + 1.08979021i	0.090*sec*	64
17	−0.03043965 − 0.32849616i	−0.48297310 − 5.21210981i	0.102*sec*	72
18	+0.15661544 − 0.27126589i	+2.70573706 − 4.68647407i	0.114*sec*	81
19	+0.13496658 + 0.14661279i	+2.52739140 + 2.74547893i	0.127*sec*	90
20	−0.05530794 + 0.34920056i	−1.11803399 + 7.05898879i	0.141*sec*	100
21	−0.15662834 + 0.07542823i	−3.40530138 + 1.63990671i	0.155*sec*	110
22	−0.04117140 − 0.28635338i	−0.95949297 − 6.67342021i	0.170*sec*	121
23	+0.12783256 − 0.24670564i	+3.18360854 − 6.14408558i	0.186*sec*	132
24	+0.12426291 + 0.12426291i	+3.29787706 + 3.29787706i	0.203*sec*	144
25	−0.03963385 + 0.31373422i	−1.11803399 + 8.85015032i	0.220*sec*	156

Table 21: Values of the quantum invariants at $A = e^{\pi i/2r}$ for $|r_2^{20}| = L_{5,1}$

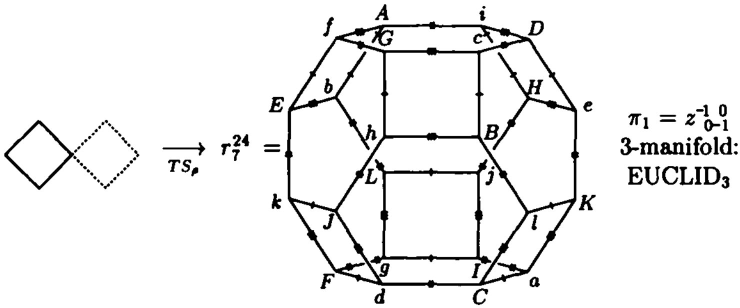
Other blink inducing the same oriented 3-manifold: 



r	$S^1 \times S^2$ – normalization	S^3 – normalization	C^{++}, 486 time	$r-$ states
3	+2.00000000	+2.82842712	0.019sec	8
4	+3.00000000	+6.00000000	0.106sec	40
5	+4.00000000	+10.75997619	0.462sec	152
6	+5.00000000	+17.32050808	1.563sec	475
7	+6.00000000	+25.87092151	4.395sec	1280
8	+7.00000000	+36.58376302	10.707sec	3072
9	+8.00000000	+49.61860604	23.417sec	6720
10	+9.00000000	+65.12461180	47.095sec	13629
11	+10.00000000	+83.24234537	88.351sec	25960
12	+11.00000000	+104.10511777	156.477sec	46904
13	+12.00000000	+127.84001069	265.411sec	81016
14	+13.00000000	+154.56868004	431.670sec	134615
15	+14.00000000	+184.40799974	678.699sec	216256
16	+15.00000000	+217.47058712	1036.553sec	337280
17	+16.00000000	+253.86523847	1542.887sec	512448
18	+17.00000000	+293.69729464	2240.732sec	760665
19	+18.00000000	+337.06895185	3197.289sec	1105800
20	+19.00000000	+384.07952830	4062.767sec	1577608

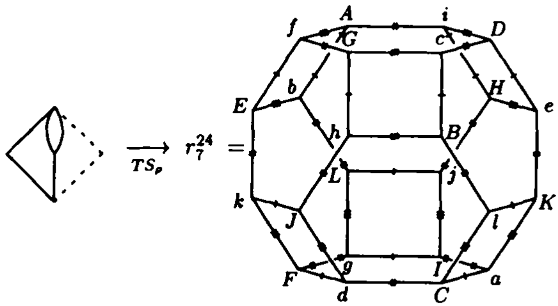
Table 22: Values of the quantum invariants at $A = e^{\pi i/2r}$ for $S^1 \times S^1 \times S^1$

We have shown in [KL94] that the invariants for the 3-torus and $EUCLID_3$ coincide for all the primitive $4r$-root of unity. This is a rather curious fact because the homology suffices to distinguish these 3-manifolds.



r	$S^1 \times S^2$ − *normalization*	S^3 − *normalization*	C^{++}, 486 *time*	r− *states*
3	+2.00000000	+2.82842712	0.022*sec*	8
4	+3.00000000	+6.00000000	0.088*sec*	34
5	+4.00000000	+10.75997619	0.286*sec*	104
6	+5.00000000	+17.32050808	0.744*sec*	259
7	+6.00000000	+25.87092151	1.668*sec*	560
8	+7.00000000	+36.58376302	3.336*sec*	1092
9	+8.00000000	+49.61860604	6.166*sec*	1968
10	+9.00000000	+65.12461180	10.633*sec*	3333
11	+10.00000000	+83.24234537	17.410*sec*	5368
12	+11.00000000	+104.10511777	27.229*sec*	8294
13	+12.00000000	+127.84001069	41.172*sec*	12376
14	+13.00000000	+154.56868004	60.227*sec*	17927
15	+14.00000000	+184.40799974	85.847*sec*	25312
16	+15.00000000	+217.47058712	119.348*sec*	34952
17	+16.00000000	+253.86523847	163.003*sec*	47328
18	+17.00000000	+293.69729464	218.227*sec*	62985
19	+18.00000000	+337.06895185	287.746*sec*	82536
20	+19.00000000	+384.07952830	373.536*sec*	106666
21	+20.00000000	+434.82569506	479.430*sec*	136136
22	+21.00000000	+489.40167759	607.487*sec*	171787
23	+22.00000000	+547.89943286	761.888*sec*	214544
24	+23.00000000	+610.40880603	945.156*sec*	265420
25	+24.00000000	+677.01766985	1164.446*sec*	325520

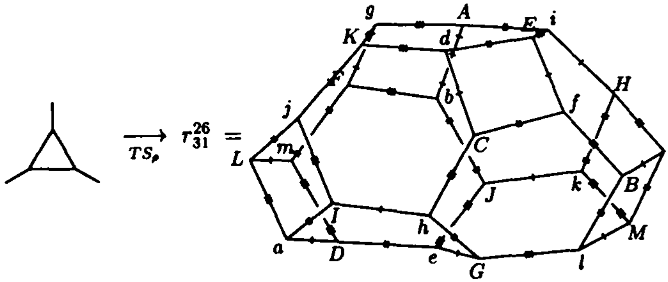
Table 23: Values of the quantum invariants at $A = e^{\pi i/2r}$ for EUCLID$_3$



$\pi_1 = \langle 3, 3, 3\rangle$
3-manifold:
$EUCLID_2$

r	$S^1 \times S^2$ – *normalization*	S^3 – *normalization*	C^{++}, 486 *time*	$r-$ *states*
3	$-1.00000000i$	$-1.41421356i$	0.012*sec*	4
4			0.046*sec*	18
5	$-0.58778525 - 0.80901699i$	$-1.58113883 - 2.17625090i$	0.150*sec*	52
6	$-0.50000000 + 0.86602540i$	$-1.73205081 + 3.00000000i$	0.416*sec*	141
7			0.971*sec*	320
8	$+0.38268343 + 0.92387953i$	$+2.00000000 + 4.82842712i$	2.049*sec*	680
9	$+0.64278761 - 0.76604444i$	$+3.98677815 - 4.75125718i$	3.934*sec*	1312
10			7.092*sec*	2405
11	$-0.28173256 - 0.95949297i$	$-2.34520788 - 7.98704455i$	12.037*sec*	4148
12	$-0.70710678 + 0.70710678i$	$-6.69213043 + 6.69213043i$	19.551*sec*	6882

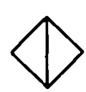
Table 24: Values of the quantum invariants at $A = e^{\pi i/2r}$ for $EUCLID_2$

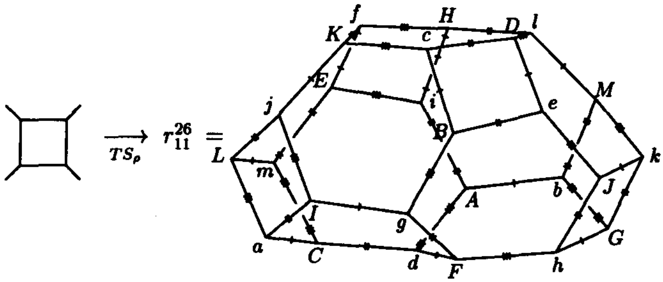


$\pi_1 = \langle 6, 3, 2 \rangle$
3-manifold:
$EUCLID_4$

r	$S^1 \times S^2$ – *normalization*	S^3 – *normalization*	C^{++}, 486 *time*	$r-$ *states*
3	+1.00000000	+1.41421356	0.006*sec*	2
4			0.007*sec*	4
5	−0.30901699 − 0.95105652*i*	−0.83125388 − 2.55833637*i*	0.010*sec*	6
6	+0.50000000 − 0.86602540*i*	+1.73205081 − 3.00000000*i*	0.015*sec*	9
7			0.020*sec*	12
8	−0.70710678 − 0.70710678*i*	−3.69551813 − 3.69551813*i*	0.026*sec*	16
9	+0.17364818 − 0.98480775*i*	+1.07702256 − 6.10809849*i*	0.033*sec*	20
10			0.041*sec*	25
11	−0.84125353 − 0.54064082*i*	−7.00279171 − 4.50042096*i*	0.049*sec*	30
12	−1.00000000*i*	−9.46410162*i*	0.059*sec*	36
13			0.069*sec*	42
14	−0.90096887 − 0.43388374*i*	−10.71242836 − 5.15883360*i*	0.080*sec*	49
15	−0.10452846 − 0.99452190*i*	−1.37684892 − 13.09984239*i*	0.092*sec*	56
16			0.105*sec*	64
17	−0.93247223 − 0.36124167*i*	−14.79514281 − 5.73166886*i*	0.118*sec*	72
18	−0.17364818 − 0.98480775*i*	−3.00000000 − 17.01384546*i*	0.132*sec*	81
19			0.147*sec*	90
20	−0.95105652 − 0.30901699*i*	−19.22533359 − 6.24668955*i*	0.164*sec*	100
21	−0.22252093 − 0.97492791*i*	−4.83789099 − 21.19618535*i*	0.180*sec*	110
22			0.198*sec*	121
23	−0.96291729 − 0.26979677*i*	−23.98099253 − 6.71915900*i*	0.216*sec*	132
24	−0.25881905 − 0.96592583*i*	−6.86893149 − 25.63520132*i*	0.235*sec*	144
25			0.255*sec*	156

Table 25: Values of the quantum invariants at $A = e^{\pi i/2r}$ for $|r_{31}^{26}| = EUCLID_4$

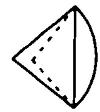
Other blink inducing the same oriented 3-manifold: 

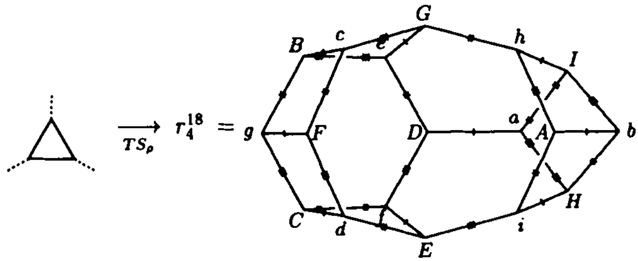


$\pi_1 = \langle 4, 4, 2 \rangle$
3-manifold
EUCLID_5

r	$S^1 \times S^2 - normalization$	$S^3 - normalization$	C^{++}, 486 time	$r-$ states
3			0.011*sec*	4
4	$-0.38268343 + 0.92387953i$	$-0.76536686 + 1.84775907i$	0.022*sec*	10
5			0.044*sec*	20
6	$-1.00000000i$	$-3.46410162i$	0.076*sec*	35
7			0.122*sec*	56
8	$+0.19509032 + 0.98078528i$	$+1.01959116 + 5.12583090i$	0.183*sec*	84
9			0.263*sec*	120
10	$-0.30901699 - 0.95105652i$	$-2.23606798 - 6.88190960i$	0.361*sec*	165
11			0.481*sec*	220
12	$+0.38268343 + 0.92387953i$	$+3.62175489 + 8.74368978i$	0.625*sec*	286
13			0.797*sec*	364
14	$-0.43388374 - 0.90096887i$	$-5.15883360 - 10.71242836i$	0.995*sec*	455
15			1.225*sec*	560
16	$+0.47139674 + 0.88192126i$	$+6.83432834 + 12.78612901i$	1.486*sec*	680
17			1.786*sec*	816
18	$-0.50000000 - 0.86602540i$	$-8.63815572 - 14.96172460i$	2.120*sec*	969
19			2.495*sec*	1140
20	$+0.52249856 + 0.85264016i$	$+10.56215801 + 17.23587538i$	2.910*sec*	1330
21			3.371*sec*	1540
22	$-0.54064082 - 0.84125353i$	$-12.59954872 - 19.60528049i$	3.875*sec*	1771
23			4.430*sec*	2024
24	$+0.55557023 + 0.83146961i$	$+14.74456359 + 22.06679884i$	5.030*sec*	2300
25			5.692*sec*	2600

Table 26: Values of the quantum invariants at $A = e^{\pi i/2r}$ for EUCLID_5

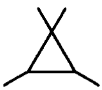
Another blink inducing the same orientable 3-manifold: 



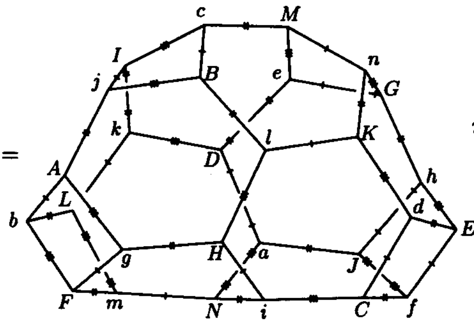
$\pi_1 = Z_2 * Z_3$
$L_{2,1} \# L_{3,1}$

r	$S^1 \times S^2 - normalization$	$S^3 - normalization$	$C^{++}, 486$ *time*	$r-$ *states*
3			0.006*sec*	2
4	$+0.27059805 + 0.27059805i$	$+0.54119610 + 0.54119610i$	0.007*sec*	4
5			0.010*sec*	6
6	$+0.36602540i$	$+1.26794919i$	0.015*sec*	9
7			0.020*sec*	12
8	$-0.30768885 + 0.12744889i$	$-1.60805943 + 0.66608002i$	0.026*sec*	16
9			0.033*sec*	20
10	$-0.20955371 - 0.15224969i$	$-1.51634492 - 1.10168907i$	0.041*sec*	25
11			0.049*sec*	30
12	$-0.25215724i$	$-2.38644174i$	0.059*sec*	36
13			0.069*sec*	42
14	$+0.21832071 - 0.10513771i$	$+2.59581102 - 1.25007670i$	0.080*sec*	49
15			0.091*sec*	56
16	$+0.17367171 + 0.11604372i$	$+2.51789919 + 1.682240645i$	0.105*sec*	64
17			0.118*sec*	72
18	$+0.20490387i$	$+3.53998301i$	0.132*sec*	81
19			0.147*sec*	90
20	$-0.17806871 + 0.090073054i$	$-3.59960778 + 1.83409177i$	0.163*sec*	100
21			0.180*sec*	110
22	$-0.15126916 - 0.09721479i$	$-3.52530393 - 2.26557527i$	0.198*sec*	121
23			0.216*sec*	132
24	$-0.17715600i$	$-4.70163402i$	0.235*sec*	144
25			0.256*sec*	156

Table 27: Values of the quantum invariants at $A = e^{\pi i/2r}$ for $|r_4^{18}| = L_{2,1} \# L_{3,1}$



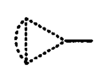
$\xrightarrow[TS_\rho]{} r^{28}_{172} =$

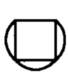


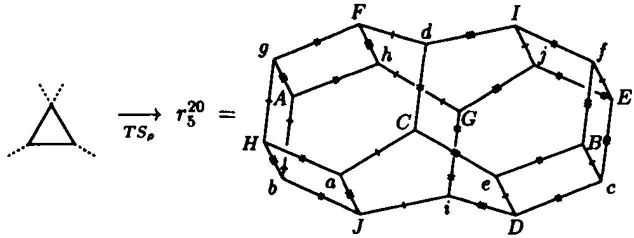
$\pi_1 = \langle 7,3,2 \rangle$ homology sphere

r	$S^1 \times S^2 - normalization$	$S^3 - normalization$	C^{++}, 486 *time*	$r-$ *states*
3	$+0.70710678$	$+1.00000000$	$0.006sec$	2
4	-0.50000000	-1.00000000	$0.008sec$	4
5	$-0.71637742 - 0.35355339i$	$-1.92705098 - 0.95105652i$	$0.011sec$	6
6	$+0.28867513$	$+1.00000000$	$0.017sec$	9
7	$+0.58547400 + 0.73416118i$	$+2.52445867 + 3.16557105i$	$0.022sec$	12
8	$+0.19134172i$	$+1.00000000i$	$0.030sec$	16
9	$+0.09888884 - 0.87838728i$	$+0.61334080 - 5.44804403i$	$0.037sec$	20
10	$+0.38819660 - 0.50655533i$	$+2.80901699 - 3.66546879i$	$0.046sec$	25
11	$-0.36522184 + 0.40677809i$	$-3.04019223 + 3.38611619i$	$0.056sec$	30
12	$-0.78867513 + 0.18301270i$	$-7.46410162 + 1.73205081i$	$0.067sec$	36
13	$+0.24724342 - 0.10586787i$	$+2.63396677 - 1.12784583i$	$0.078sec$	42
14	$+0.85966457 + 0.41399264i$	$+10.22132448 + 4.92233044i$	$0.090sec$	49
15	$+0.08219527 + 0.20412415i$	$+1.08267606 + 2.68872324i$	$0.103sec$	56
16	$-0.26138010 - 0.81695034i$	$-3.78949898 - 11.84417797i$	$0.118sec$	64
17	$+0.16713092 - 0.59182253i$	$+2.65179560 - 9.39019802i$	$0.133sec$	72
18	$-0.19211727 + 0.53326852i$	$-3.31907786 + 9.21291311i$	$0.149sec$	81
19	$-0.67439974 + 0.46666858i$	$-12.62884527 + 8.73886052i$	$0.166sec$	90
20	$+0.19737596 - 0.16561496i$	$+3.98989828 - 3.34785876i$	$0.184sec$	100
21	$+0.94865073 + 0.12384016i$	$+20.62488555 + 2.69244420i$	$0.203sec$	110
22	$+0.14411067 + 0.18165006i$	$+3.35847636 + 4.23332596i$	$0.223sec$	121
23	$-0.50508391 - 0.67914673i$	$-12.57887210 - 16.91382302i$	$0.243sec$	132
24	$-0.02388795 - 0.60181010i$	$-0.63397460 - 15.97174719i$	$0.265sec$	144
25	$-0.01964222 + 0.57823771i$	$-0.55408868 + 16.31154775i$	$0.287sec$	156

Table 28: Values of the quantum invariants at $A = e^{\pi i/2r}$ for $|r^{28}_{172}|$

Other blinks inducing the same oriented 3-manifold: , ,

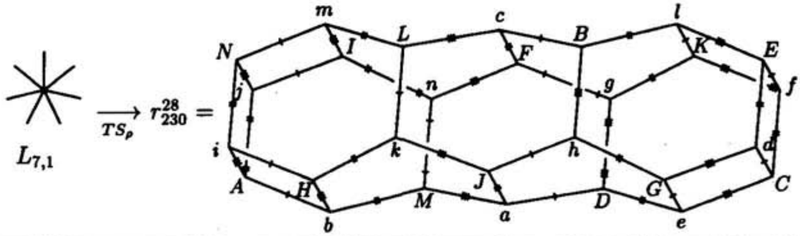
, .



$\pi_1 = Z_7$
Lens space
$L_{7,2}$

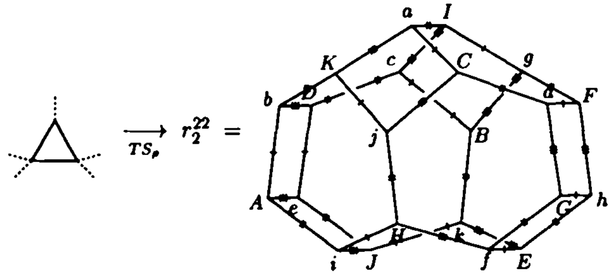
r	$S^1 \times S^2$ – *normalization*	S^3 – *normalization*	C^{++}, 486 *time*	$r-$ *states*
3	$+0.70710678i$	$+1.00000000i$	0.006*sec*	2
4	$-0.35355339 - 0.35355339i$	$-0.70710678 - 0.70710678i$	0.008*sec*	4
5	$+0.35355339 - 0.48662449i$	$+0.95105652 - 1.30901699i$	0.011*sec*	6
6	$+0.28867513$	$+1.00000000$	0.017*sec*	9
7			0.023*sec*	12
8	$-0.17677670 + 0.07322330i$	$-0.92387953 + 0.38268343i$	0.030*sec*	16
9	$-0.40204608 - 0.23212141i$	$-2.49362077 - 1.43969262i$	0.037*sec*	20
10	$+0.11180340 - 0.34409548i$	$+0.80901699 - 2.48989828i$	0.046*sec*	25
11	$+0.17422296 + 0.27109622i$	$+1.45027281 + 2.25666855i$	0.056*sec*	30
12	$-0.27883877 + 0.27883877i$	$-2.63895843 + 2.63895843i$	0.067*sec*	36
13	$-0.18095040 - 0.02197136i$	$-1.92772512 - 0.23406821i$	0.078*sec*	42
14			0.090*sec*	49
15	$+0.14125017 - 0.04589496i$	$+1.86054730 - 0.60452846i$	0.103*sec*	56
16	$+0.28832037 + 0.19264951i$	$+4.18008002 + 2.79304017i$	0.118*sec*	64
17	$-0.09887822 + 0.25523411i$	$-1.56885894 + 4.04969173i$	0.133*sec*	72
18	$-0.12767407 - 0.22113798i$	$-2.20573706 - 3.82044866i$	0.149*sec*	81
19	$+0.23139619 - 0.21301520i$	$+4.33313728 - 3.98893387i$	0.166*sec*	90
20	$+0.14179689 + 0.02245842i$	$+2.86638321 + 0.45399050i$	0.184*sec*	100
21			0.203*sec*	110
22	$-0.12017874 + 0.03528766i$	$-2.80074651 + 0.82237337i$	0.223*sec*	121
23	$-0.23587240 - 0.16649692i$	$-5.87428880 - 4.14652583i$	0.243*sec*	132
24	$+0.08764269 - 0.21158817i$	$+2.32599430 - 5.61544699i$	0.265*sec*	144
25	$+0.10499053 + 0.19097707i$	$+2.96168522 + 5.38728542i$	0.287*sec*	156

Table 29: Values of the quantum invariants at $A = e^{\pi i/2r}$ for $|r_5^{20}| = L_{7,2}$



r	$S^1 \times S^2$ – *normalization*	S^3 – *normalization*	C^{++}, 486 *time*	$r-$ *states*
3	$-0.70710678i$	$-1.00000000i$	0.004*sec*	2
4	$+0.35355339 - 0.35355339i$	$+0.70710678 - 0.70710678i$	0.004*sec*	3
5	$-0.60150096i$	$-1.61803399i$	0.005*sec*	4
6	$+0.28867513$	$+1.00000000$	0.006*sec*	5
7	$+0.55283834 - 0.44087387i$	$+2.38373956 - 1.90096887i$	0.007*sec*	6
8	$+0.07322330 - 0.17677670i$	$+0.38268343 - 0.92387953i$	0.009*sec*	7
9	$+0.40204608 + 0.23212141i$	$+2.49362077 + 1.43969262i$	0.010*sec*	8
10	$+0.36180340$	$+2.61803399$	0.011*sec*	9
11	$+0.09078908 + 0.30919920i$	$+0.75574957 + 2.57384668i$	0.012*sec*	10
12	$+0.27883877 + 0.27883877i$	$+2.63895843 + 2.63895843i$	0.014*sec*	11
13	$-0.15001303 + 0.10354652i$	$-1.59813890 + 1.10311564i$	0.015*sec*	12
14	$-0.21694187 + 0.45048443i$	$-2.57941680 + 5.35621418i$	0.016*sec*	13
15	$+0.14851922i$	$+1.95629520i$	0.017*sec*	14
16	$-0.34009707 - 0.06764951i$	$-4.93074057 - 0.98078528i$	0.019*sec*	15
17	$-0.26326866 + 0.07490639i$	$-4.17717250 + 1.18850805i$	0.020*sec*	16
18	$-0.12767407 - 0.22113798i$	$-2.20573706 - 3.82044866i$	0.021*sec*	17
19	$-0.26330140 - 0.17202339i$	$-4.93059587 - 3.22131905i$	0.022*sec*	18
20	$+0.10151536 - 0.10151536i$	$+2.05210381 - 2.05210381i$	0.024*sec*	19
21	$+0.12033330 - 0.39011096i$	$+2.61620045 - 8.48151356i$	0.025*sec*	20
22	$-0.01782527 - 0.12397745i$	$-0.41541501 - 2.88927488i$	0.026*sec*	21
23	$+0.28804308 + 0.01970269i$	$+7.17357461 + 0.49068595i$	0.027*sec*	22
24	$+0.21158817 - 0.08764269i$	$+5.61544699 - 2.32599430i$	0.029*sec*	23
25	$+0.12809842 + 0.17631235i$	$+3.61353735 + 4.97360748i$	0.030*sec*	24

Table 30: Values of the quantum invariants at $A = e^{\pi i/2r}$ for $L_{7,1}$

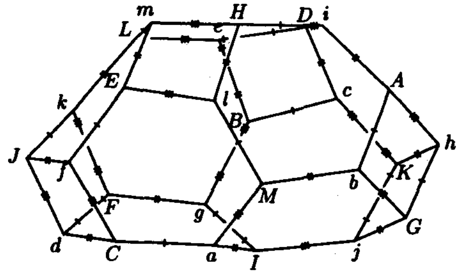


$\pi_1 = Z_3 \rtimes_i Z_8$
3-manifold:
S^3/π_1

r	$S^1 \times S^2$ − *normalization*	S^3 − *normalization*	C^{++}, 486 *time*	$r-$ *states*
3	+0.70710678 + 0.70710678i	+1.00000000 + 1.00000000i	0.007sec	2
4	−0.38268343 + 0.92387953i	−0.76536686 + 1.84775907i	0.008sec	4
5	−1.12075944 + 0.68374342i	−3.01483624 + 1.83926573i	0.013sec	6
6	−1.15470054	−4.00000000	0.019sec	9
7	−0.80125141 − 0.89081749i	−3.45485204 − 3.84104490i	0.025sec	12
8	−0.27589938 − 1.38703985i	−1.44191964 − 7.24901957i	0.033sec	16
9	+0.45659672 − 1.16370351i	+2.83196163 − 7.21766823i	0.042sec	20
10	+1.17082039 − 0.52573111i	+8.47213595 − 3.80422607i	0.052sec	25
11	+1.36673518 + 0.12326067i	+11.37702419 + 1.02605077i	0.062sec	30
12	+0.91608553 + 0.70293715i	+8.66992654 + 6.65266861i	0.074sec	36
13	+0.23851182 + 1.14228866i	+2.54094609 + 12.16918282i	0.087sec	42
14	−0.29898978 + 1.16399264i	−3.55495813 + 13.83975429i	0.101sec	49
15	−0.70774332 + 0.69631236i	−9.32239493 + 9.17182644i	0.116sec	56
16	−0.99518473 + 0.09801714i	−14.42822712 + 1.42105634i	0.132sec	64
17	−0.95812101 − 0.30791397i	−15.20210114 − 4.88554082i	0.149sec	72
18	−0.56486424 − 0.57735027i	−9.75877048 − 9.97448307i	0.167sec	81
19	−0.12038497 − 0.80850144i	−2.25433538 − 15.14004066i	0.186sec	90
20	+0.16771251 − 0.85126901i	+3.39026001 − 17.20815780i	0.206sec	100
21	+0.42535821 − 0.60556782i	+9.24783398 − 13.16582244i	0.227sec	110
22	+0.73390466 − 0.25826673i	+17.10353195 − 6.01886530i	0.250sec	121
23	+0.88783335 + 0.03093762i	+22.11106303 + 0.77048656i	0.272sec	132
24	+0.73229353 + 0.36112720i	+19.43471380 + 9.58414022i	0.297sec	144
25	+0.40314014 + 0.77349759i	+11.37220816 + 21.81964732i	0.322sec	156

Table 31: Values of the quantum invariants at $A = e^{\pi i/2r}$ for $S^3/(Z_3 \rtimes_i Z_8)$

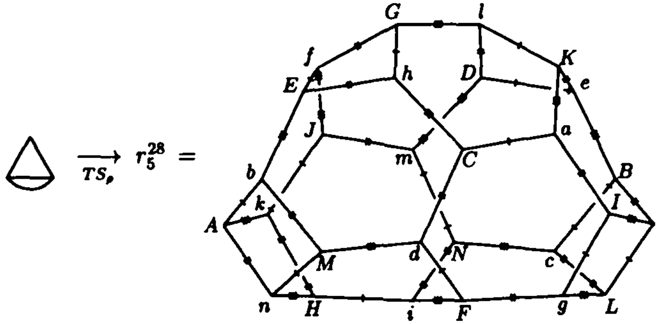
$\xrightarrow[TS_\rho]{} r_8^{26} =$



$\pi_1 = Q_8 \rtimes_3 Z_9$
3-manifold:
$\pi_1 = S^3/\pi_1$

r	$S^1 \times S^2$ – *normalization*	S^3 – *normalization*	C^{++}, 486 *time*	$r-$ *states*
3	+0.70710678	+1.00000000	0.007*sec*	2
4	+0.50000000	+1.00000000	0.009*sec*	4
5	+0.55762205 – 0.57206140*i*	+1.50000000 – 1.53884177*i*	0.014*sec*	6
6	–0.50000000*i*	–1.73205081*i*	0.021*sec*	9
7	–0.23759308 – 0.22610588*i*	–1.02445867 – 0.97492791*i*	0.028*sec*	12
8	–0.73253782	–3.82842712	0.037*sec*	16
9	–0.45451948 + 0.54167522*i*	–2.81907786 + 3.35964617*i*	0.046*sec*	20
10	–0.01631190 + 0.63798811*i*	–0.11803399 + 4.61652531*i*	0.058*sec*	25
11	+0.42528742 + 0.82454339*i*	+3.54019223 + 6.86369258*i*	0.069*sec*	30
12	+0.75000000 + 0.25000000*i*	+7.09807621 + 2.36602540*i*	0.082*sec*	36
13	+0.54755931 – 0.24768552*i*	+5.83333237 – 2.63867661*i*	0.096*sec*	42
14	+0.37220062 – 0.67122029*i*	+4.42542754 – 7.98074105*i*	0.112*sec*	49
15	–0.31622777 – 0.60150096*i*	–4.16535213 – 7.92297057*i*	0.128*sec*	56
16	–0.62454508 – 0.09754516*i*	–9.05467898 – 1.41421356*i*	0.147*sec*	64
17	–0.78334078 + 0.30632948*i*	–12.42893717 + 4.86040035*i*	0.165*sec*	72
18	–0.29619813 + 0.81379768*i*	–5.11721119 + 14.05942220*i*	0.185*sec*	81
19	+0.33228574 + 0.73207401*i*	+6.22240040 + 13.70885654*i*	0.206*sec*	90
20	+0.71173789 + 0.54104636*i*	+14.38757654 + 10.93709629*i*	0.229*sec*	100
21	+0.80016926 – 0.16756750*i*	+17.39670765 – 3.64313270*i*	0.251*sec*	110
22	+0.31632111 – 0.62931365*i*	+7.37181345 – 14.66605512*i*	0.277*sec*	121
23	–0.09314593 – 0.74612841*i*	–2.31975475 – 18.58196973*i*	0.302*sec*	132
24	–0.69290965 – 0.36626891*i*	–18.38948486 – 9.72059856*i*	0.329*sec*	144
25	–0.74124332 + 0.31864771*i*	–20.90978428 + 8.98875537*i*	0.357*sec*	156

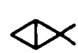
Table 32: Values of the quantum invariants at $A = e^{\pi i/2r}$ for $S^3/(Q_8 \rtimes_3 Z_9)$



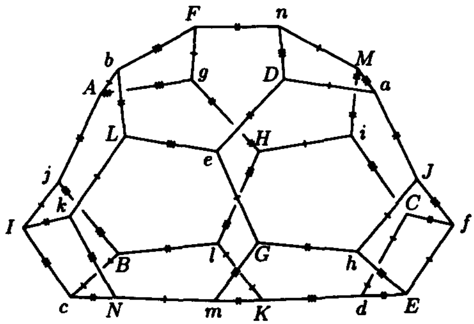
$\pi_1 = z^{\;0\;1}_{-1\;3}$
3-manifold:
$Z^{\;0\;1}_{-1\;3}$

r	$S^1 \times S^2$ – *normalization*	S^3 – *normalization*	C^{++}, 486 *time*	$r-$ *states*
3	+1.00000000	+1.41421356	0.005*sec*	2
4			0.008*sec*	6
5	−0.61803399	−1.66250775	0.014*sec*	10
6			0.027*sec*	19
7	+1.00000000	+4.31182025	0.042*sec*	28
8	+1.00000000	+5.22625186	0.065*sec*	44
9			0.091*sec*	60
10	−0.61803399	−4.47213595	0.129*sec*	85
11			0.171*sec*	110
12	+1.00000000	+9.46410162	0.226*sec*	146
13	+1.00000000	+10.65333422	0.285*sec*	182
14			0.362*sec*	231
15	−0.61803399	−8.14074369	0.444*sec*	280
16			0.546*sec*	344
17	+1.00000000	+15.86657740	0.653*sec*	408
18	+1.00000000	+17.27631145	0.783*sec*	489
19			0.920*sec*	570
20	−0.61803399	−12.49337910	1.081*sec*	670
21			1.250*sec*	770
22	+1.00000000	+23.30484179	1.448*sec*	891
23	+1.00000000	+24.90451968	1.654*sec*	1012
24			1.889*sec*	1156
25	−0.61803399	−17.43416379	2.134*sec*	1300

Table 33: Values of the quantum invariants at $A = e^{\pi i/2r}$ for $Z^{\;0\;1}_{-1\;3}$

Other blinks inducing the same oriented 3-manifold: , , , .

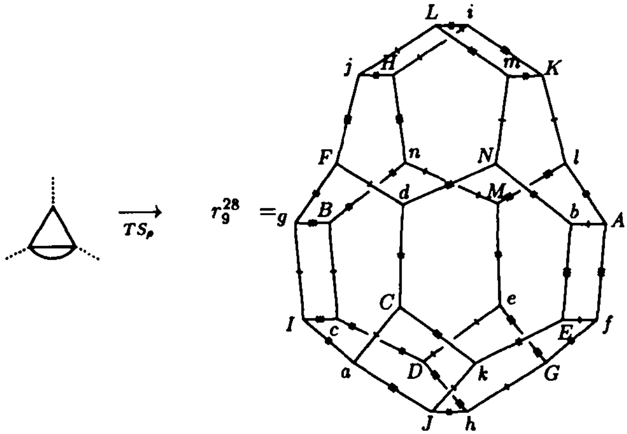
$\xrightarrow[TS_\rho]{} r^{28}_{202} =$



$\pi_1 = \langle 5,5,2\rangle 2$
$\pi_1/\pi_1' = Z_2$
$\pi_1'/\pi_1'' = Z_5$
$\pi_1''/\pi_1''' = Z_2^4$

r	$S^1 \times S^2$ – *normalization*	S^3 – *normalization*	C^{++}, 486 *time*	$r-$ *states*
3			0.006*sec*	2
4	$+0.92387953i$	$+1.84775907i$	0.011*sec*	6
5			0.019*sec*	10
6	-0.78867513	-2.73205081	0.037*sec*	19
7			0.055*sec*	28
8	$-0.02288733 - 0.80858229i$	$-0.11961494 - 4.22585467i$	0.086*sec*	44
9			0.120*sec*	60
10	$+0.88122428 + 0.09409548i$	$+6.37659882 + 0.68088129i$	0.170*sec*	85
11			0.223*sec*	110
12	$-0.27059805 + 0.84300901i$	$-2.56096744 + 7.97832291i$	0.296*sec*	146
13			0.373*sec*	182
14	$-0.74645259 - 0.36848811i$	$-8.87524547 - 4.38128627i$	0.474*sec*	231
15			0.579*sec*	280
16	$+0.42852736 - 0.70122874i$	$+6.21280645 - 10.16644168i$	0.712*sec*	344
17			0.850*sec*	408
18	$+0.66819110 + 0.54060056i$	$+11.54387757 + 9.33958364i$	1.020*sec*	489
19			1.196*sec*	570
20	$-0.66610546 + 0.56747261i$	$-13.46512996 + 11.47129529i$	1.404*sec*	670
21			1.624*sec*	770
22	$-0.43273130 - 0.73122858i$	$-10.08473452 - 17.04116648i$	1.878*sec*	891
23			2.141*sec*	1012
24	$+0.76267264 - 0.32903797i$	$+20.24096074 - 8.73250747i$	2.448*sec*	1156
25			2.762*sec*	1300

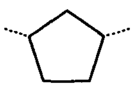
Table 34: Values of the quantum invariants at $A = e^{\pi i/2r}$ for $|r^{28}_{202}|$



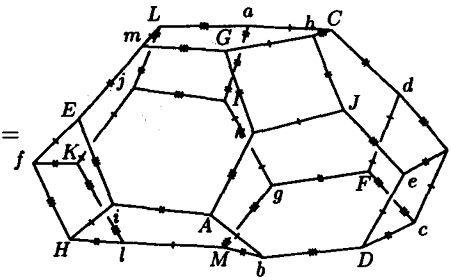
$\pi_1 = \langle 4,3,3 \rangle$
$\pi_1/\pi_1' = Z_3$
$\pi_1'/\pi_1'' = Z_4^2$
$\pi_1''/\pi_1''' = S_3^4$

r	$S^1 \times S^2$ – *normalization*	S^3 – *normalization*	C^{++}, 486 *time*	$r-$ *states*
3	$+0.70710678i$	$+1.00000000i$	0.007*sec*	2
4	$+0.35355339 - 0.35355339i$	$+0.70710678 - 0.70710678i$	0.012*sec*	6
5	$+0.35355339 + 0.48662449i$	$+0.95105652 + 1.30901699i$	0.021*sec*	10
6	$-0.50000000i$	$-1.73205081i$	0.040*sec*	19
7	$+0.91548402 + 0.02296736i$	$+3.94740253 + 0.09903113i$	0.061*sec*	28
8	$-0.03033009 + 0.07322330i$	$-0.15851267 + 0.38268343i$	0.095*sec*	44
9	$+0.66446302 - 0.38362791i$	$+4.12121613 - 2.37938524i$	0.132*sec*	60
10	$+0.11180340 + 0.34409548i$	$+0.80901699 + 2.48989828i$	0.187*sec*	85
11	$+0.04958903 - 0.73276764i$	$+0.41279068 - 6.09972967i$	0.245*sec*	110
12	$+0.61237244$	$+5.79555496$	0.325*sec*	146
13	$-0.21008981 - 0.44423541i$	$-2.23815698 - 4.73258825i$	0.409*sec*	182
14	$+0.73721382 - 0.40998160i$	$+8.76539748 - 4.87463956i$	0.520*sec*	231
15	$-0.22473227 + 0.11387792i$	$-2.96017343 + 1.50000000i$	0.635*sec*	280
16	$+0.15748644 - 0.79173781i$	$+2.28324459 - 11.47864572i$	0.781*sec*	344
17	$+0.10339902 + 0.10059154i$	$+1.64058852 + 1.59604352i$	0.932*sec*	408
18	$-0.35286853 - 0.78107901i$	$-6.09626666 - 13.49416426i$	1.117*sec*	489
19	$+0.48809268 - 0.25653424i$	$+9.14004929 - 4.80387381i$	1.309*sec*	570
20	$-0.50908292 - 0.17912251i$	$-10.29096452 - 3.62090996i$	1.540*sec*	670
21	$+0.20895325 - 0.61140173i$	$+4.54291217 - 13.29265917i$	1.778*sec*	770
22	$-0.35484862 + 0.20922635i$	$-8.26969084 + 4.87598694i$	2.059*sec*	891
23	$-0.41066396 - 0.78948595i$	$-10.22738859 - 19.66176842i$	2.347*sec*	1012
24	$+0.10854344 + 0.03790317i$	$+2.88069001 + 1.00593173i$	2.681*sec*	1156
25	$-0.71809015 - 0.36973436i$	$-20.25665494 - 10.42986231i$	3.025*sec*	1300

Table 35: Values of the quantum invariants at $A = e^{\pi i/2r}$ for $|r_9^{28}|$



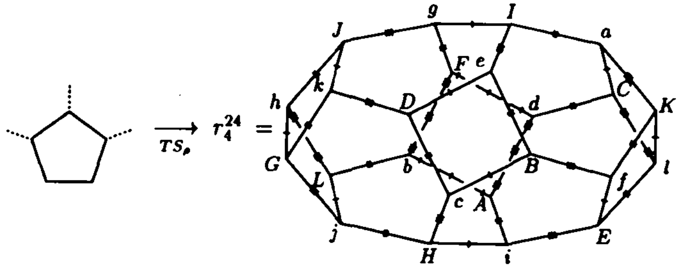
$\xrightarrow[TS_\rho]{} r_{10}^{26} =$



$\pi_1 =$
$\langle 5,3,2\rangle \times Z_7$
$\pi_1/\pi_1' = Z_7$
$\pi_1'' = \pi_1'$
3-manifold:
S^3/π_1

r	$S^1 \times S^2$ – *normalization*	S^3 – *normalization*	C^{++}, 486 *time*	r– *states*
3	$+0.70710678i$	$+1.00000000i$	0.006*sec*	2
4	$-0.35355339 - 0.35355339i$	$-0.70710678 - 0.70710678i$	0.008*sec*	4
5	$-0.21850801 - 0.07099756i$	$-0.58778525 - 0.19098301i$	0.011*sec*	6
6	$+0.28867513$	$+1.00000000$	0.017*sec*	9
7	$-0.68937814 + 0.15734606i$	$-2.97247462 + 0.67844793i$	0.022*sec*	12
8	$+0.67677670 - 0.28033009i$	$+3.53700546 - 1.46507563i$	0.030*sec*	16
9	$-0.26241694 + 0.55975478i$	$-1.62759536 + 3.47178151i$	0.037*sec*	20
10	$-0.04270510 + 0.03102707i$	$-0.30901699 + 0.22451399i$	0.046*sec*	25
11	$+0.06494781 - 0.48892915i$	$+0.54064082 - 4.06996095i$	0.056*sec*	30
12	$+0.33353367 + 0.63239216i$	$+3.15659652 + 5.98502365i$	0.067*sec*	36
13	$-0.30327274 - 0.62336697i$	$-3.23086585 - 6.64093665i$	0.078*sec*	42
14	$+0.39091574 + 0.31174490i$	$+4.64794847 + 3.70661522i$	0.090*sec*	49
15	$-0.14320045 - 0.06445060i$	$-1.88623634 - 0.84894331i$	0.103*sec*	56
16	$-0.44606695 - 0.21509707i$	$-6.46709609 - 3.11848568i$	0.118*sec*	64
17	$+0.73481939 - 0.14051572i$	$+11.65906874 - 2.22950356i$	0.133*sec*	72
18	$-0.71449816 + 0.37152172i$	$-12.34389279 + 6.41852487i$	0.149*sec*	81
19	$+0.27090101 - 0.28224845i$	$+5.07290656 - 5.28539933i$	0.166*sec*	90
20	$-0.08618550 + 0.08998111i$	$-1.74221509 + 1.81894222i$	0.184*sec*	100
21	$-0.17713229 + 0.36781900i$	$-3.85108366 + 7.99685762i$	0.203*sec*	110
22	$+0.14342329 - 0.67722979i$	$+3.34245703 - 15.78273311i$	0.223*sec*	121
23	$+0.21375515 + 0.85898038i$	$+5.32346941 + 21.39249379i$	0.243*sec*	132
24	$-0.28481147 - 0.39892058i$	$-7.55875786 - 10.58715816i$	0.265*sec*	144
25	$+0.10429884 - 0.03567316i$	$+2.94217335 - 1.00630676i$	0.288*sec*	156

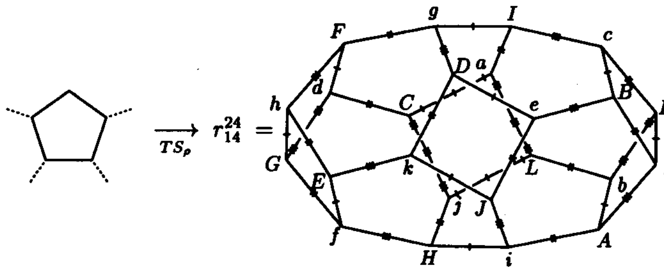
Table 36: Values of the quantum invariants at $A = e^{\pi i/2r}$ for $S^3/(\langle 5,3,2\rangle \times Z_7)$



$\pi_1 = Z_5 \rtimes_i Z_8$
$\pi_1/\pi_1' = Z_8$
3-manifold:
$S^3/(Z_5 \rtimes_i Z_8)$

r	$S^1 \times S^2$ − *normalization*	S^3 − *normalization*	C^{++}, 486 *time*	r− *states*
3	+0.70710678 + 0.70710678i	+1.00000000 + 1.00000000i	0.007*sec*	2
4	−0.38268343 + 0.92387953i	−0.76536686 + 1.84775907i	0.008*sec*	4
5	−0.22975292 + 0.22975292i	−0.61803399 + 0.61803399i	0.013*sec*	6
6	−0.28867513 + 0.50000000i	−1.00000000 + 1.73205081i	0.019*sec*	9
7	−1.34746553 − 0.16996242i	−5.81002915 − 0.73284740i	0.025*sec*	12
8	−0.27589938 − 1.38703985i	−1.44191964 − 7.24901957i	0.033*sec*	16
9	+1.03939610 − 0.51456222i	+6.44667323 − 3.19148250i	0.042*sec*	20
10	+0.55278640	+4.00000000	0.052*sec*	25
11	+0.82933635 + 0.17212458i	+6.90359028 + 1.43280537i	0.062*sec*	30
12	+0.30732410 + 1.49629049i	+2.90854651 + 14.16104524i	0.075*sec*	36
13	−1.30928196 + 0.75415770i	−13.94821831 + 8.03429404i	0.087*sec*	42
14	−0.81184821 − 0.75500169i	−9.65279281 − 8.97689340i	0.101*sec*	49
15	−0.28686458 − 0.51661750i	−3.77858026 − 6.80488572i	0.116*sec*	56
16	−0.09801714 − 0.99518473i	−1.42105634 − 14.42822712i	0.132*sec*	64
17	+1.43385083 − 0.66988891i	+22.75030515 − 10.62884417i	0.149*sec*	72
18	+0.99757535 + 1.09881447i	+17.23442238 + 18.98346096i	0.167*sec*	81
19	−0.49989842 + 0.85457535i	−9.36112418 + 16.00282312i	0.186*sec*	90
20	−0.35900605 + 0.42034208i	−7.25720394 + 8.49709409i	0.206*sec*	100
21	−0.96299110 + 0.32391336i	−20.93666366 + 7.04229257i	0.227*sec*	110
22	−0.83897085 − 1.24533599i	−19.55208302 − 29.02235823i	0.250*sec*	121
23	+0.88512569 − 1.01185801i	+22.04363028 − 25.19983775i	0.272*sec*	132
24	+0.73229353 + 0.36112720i	+19.43471380 + 9.58414022i	0.297*sec*	144
25	+0.40030708 + 0.17570712i	+11.29229040 + 4.95653442i	0.322*sec*	156

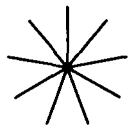
Table 37: Values of the quantum invariants at $A = e^{\pi i/2r}$ for $|r_4^{24}| = S^3/(Z_5 \rtimes_i Z_8)$



$\pi_1 = Z_9$
3-manifold:
$L_{9,2}$

r	$S^1 \times S^2$ − normalization	S^3 − normalization	C^{++}, 486 time	r− states
3	+0.70710678	+1.00000000	0.007sec	2
4	+0.50000000	+1.00000000	0.009sec	4
5	−0.11487646 − 0.35355339i	−0.30901699 − 0.95105652i	0.014sec	6
6	−0.50000000i	−1.73205081i	0.021sec	9
7	+0.11596031 − 0.50805530i	+0.50000000 − 2.19064313i	0.028sec	12
8	+0.19134172	+1.00000000	0.037sec	16
9			0.046sec	20
10	−0.11180340 − 0.08122992i	−0.80901699 − 0.58778525i	0.058sec	25
11	+0.17533059 − 0.38392045i	+1.45949297 − 3.19584383i	0.069sec	30
12	+0.25000000 − 0.25000000i	+2.36602540 − 2.36602540i	0.083sec	36
13	+0.23030543 − 0.12087362i	+2.45352077 − 1.28770704i	0.096sec	42
14	−0.14692973 − 0.18424406i	−1.74697960 − 2.19064313i	0.112sec	49
15	−0.09771975 − 0.30075048i	−1.28716460 − 3.96148528i	0.128sec	56
16	−0.34675996i	−5.02733949i	0.147sec	64
17	+0.12179515 − 0.02276746i	+1.93247223 − 0.36124167i	0.165sec	72
18			0.185sec	81
19	−0.09264941 − 0.05013935i	−1.73495775 − 0.93891218i	0.206sec	90
20	+0.09651666 − 0.29704774i	+1.95105652 − 6.00473452i	0.229sec	100
21	+0.16663466 − 0.20895325i	+3.62285157 − 4.54291217i	0.252sec	110
22	+0.16610378 − 0.10674842i	+3.87102225 − 2.48775494i	0.277sec	121
23	−0.12702156 − 0.13600688i	−3.16341089 − 3.38718599i	0.302sec	132
24	−0.09567086 − 0.23096988i	−2.53905801 − 6.12982829i	0.329sec	144
25	−0.01744527 − 0.27728455i	−0.49211470 − 7.82193919i	0.357sec	156

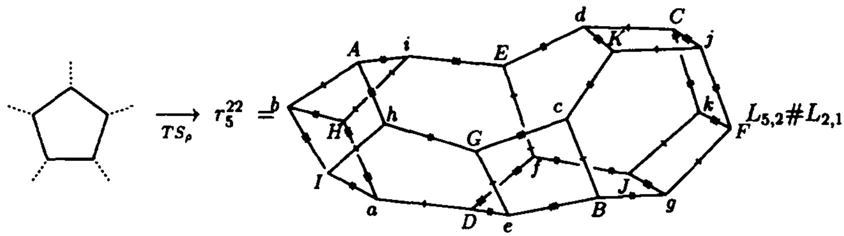
Table 38: Values of the quantum invariants at $A = e^{\pi i/2r}$ for $|r^{24}_{14}| = L_{9,2}$



$\pi_1 = Z_9$
3-manifold:
$L_{9,1}$

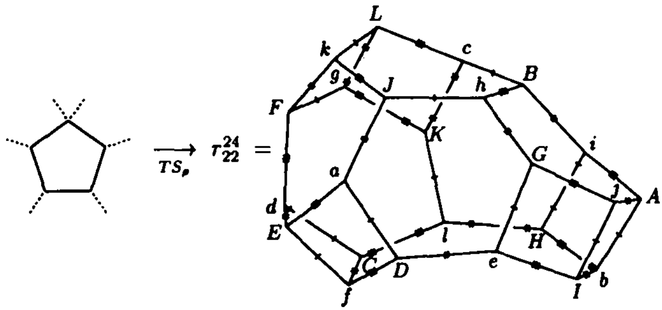
r	$S^1 \times S^2$ − normalization	S^3 − normalization	C^{++}, 486 time	$r-$ states
3	+0.70710678	+1.00000000	0.007*sec*	2
4	−0.50000000	−1.00000000	0.007*sec*	3
5	−0.11487646 − 0.35355339i	−0.30901699 − 0.95105652i	0.010*sec*	4
6	+0.50000000i	+1.73205081i	0.012*sec*	5
7	−0.52112089	−2.24697960	0.014*sec*	6
8	+0.19134172i	+1.00000000i	0.017*sec*	7
9	−0.61237244 + 0.35355339i	−3.79813333 + 2.19285330i	0.019*sec*	8
10	−0.11180340 − 0.08122992i	−0.80901699 − 0.58778525i	0.021*sec*	9
11	−0.27639136 + 0.31897263i	−2.30074651 + 2.65520301i	0.024*sec*	10
12	−0.25000000 − 0.25000000i	−2.36602540 − 2.36602540i	0.026*sec*	11
13	+0.03135136 + 0.25820170i	+0.33399654 + 2.75070898i	0.029*sec*	12
14	−0.23565699	−2.80193774	0.031*sec*	13
15	+0.25583364 + 0.18587402i	+3.36984066 + 2.44833255i	0.033*sec*	14
16	−0.24519632 + 0.24519632i	−3.55486585 + 3.55486585i	0.036*sec*	15
17	+0.07466937 + 0.09887822i	+1.18474727 + 1.56885894i	0.038*sec*	16
18	−0.25000000 + 0.43301270i	−4.31907786 + 7.48086230i	0.040*sec*	17
19	−0.10390963 − 0.01733945i	−1.94581724 − 0.32469947i	0.043*sec*	18
20	−0.09651666 + 0.29704774i	−1.95105652 + 6.00473452i	0.045*sec*	19
21	−0.24079406 − 0.11596031i	−5.23517219 − 2.52112605i	0.047*sec*	20
22	+0.08202284 + 0.17960496i	+1.91152928 + 4.18566527i	0.050*sec*	21
23	−0.17919668 + 0.05020855i	−4.46280712 + 1.25041991i	0.052*sec*	22
24	+0.23096988 + 0.09567086i	+6.12982829 + 2.53905801i	0.055*sec*	23
25	−0.14887025 + 0.23458198i	−4.19949134 + 6.61733950i	0.057*sec*	24
26	+0.07361581 + 0.06521791i	+2.20203152 + 1.95082970i	0.060*sec*	25
27	−0.13962914 + 0.38362791i	−4.41913461 + 12.14147256i	0.062*sec*	26
28	−0.08827079	−2.94985582	0.064*sec*	27
29	−0.04192634 + 0.25573939i	−1.47662056 + 9.00698693i	0.067*sec*	28
30	−0.21266270 − 0.06909830i	−7.87956771 − 2.56022675i	0.069*sec*	29

Table 39: Values of the quantum invariants at $A = e^{\pi i/2r}$ for $L_{9,1}$



r	$S^1 \times S^2$ − *normalization*	S^3 − *normalization*	C^{++}, 486 *time*	$r-$ *states*
3			0.008*sec*	2
4	−0.38268343	−0.76536686	0.010*sec*	4
5			0.015*sec*	6
6	−0.21132487	−0.73205081	0.023*sec*	9
7			0.031*sec*	12
8	+0.33304001	+1.74055098	0.041*sec*	16
9			0.051*sec*	20
10			0.063*sec*	25
11			0.076*sec*	30
12	−0.28124486	−2.66172990	0.091*sec*	36
13			0.106*sec*	42
14	+0.16768905	+1.99380581	0.124*sec*	49
15			0.141*sec*	56
16	+0.13956460	+2.02341303	0.161*sec*	64
17			0.182*sec*	72
18	−0.22233372	−3.84110662	0.204*sec*	81
19			0.227*sec*	90
20			0.251*sec*	100
21			0.277*sec*	110
22	+0.20508713	+4.77952307	0.305*sec*	121
23			0.333*sec*	132
24	−0.12452953	−3.30495321	0.362*sec*	144
25			0.392*sec*	156

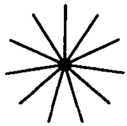
Table 40: Values of the quantum invariants at $A = e^{\pi i/2r}$ for $|r_5^{22}| = L_{5,2}\#L_{2,1}$



$\pi_1 = Z_{11}$
3-manifold:
$L_{11,3}$

r	$S^1 \times S^2$ – normalization	S^3 – normalization	C^{++}, 486 time	r– states
3	$+0.70710678i$	$+1.00000000i$	0.008*sec*	2
4	$-0.35355339 + 0.35355339i$	$-0.70710678 + 0.70710678i$	0.011*sec*	4
5	$-0.21850801 + 0.30075048i$	$-0.58778525 + 0.80901699i$	0.017*sec*	6
6	-0.28867513	-1.00000000	0.025*sec*	9
7	$-0.40742872 + 0.09299295i$	$-1.75675939 + 0.40096887i$	0.033*sec*	12
8	$-0.42677670 - 0.17677670i$	$-2.23044250 - 0.92387953i$	0.044*sec*	16
9	$-0.46424283i$	$-2.87938524i$	0.055*sec*	20
10	$+0.04270510 - 0.13143278i$	$+0.30901699 - 0.95105652i$	0.069*sec*	25
11			0.083*sec*	30
12	$-0.07471462 - 0.07471462i$	$-0.70710678 - 0.70710678i$	0.099*sec*	36
13	$-0.18095040 - 0.34477219i$	$-1.92772512 - 3.67297334i$	0.116*sec*	42
14	$+0.21231962 - 0.26624038i$	$+2.52445867 - 3.165557105i$	0.134*sec*	49
15	$+0.25807691 - 0.08385427i$	$+3.39938907 - 1.10452846i$	0.154*sec*	56
16	$+0.16332037 - 0.10912718i$	$+2.36782513 - 1.58213017i$	0.176*sec*	64
17	$+0.17367180 + 0.04941389i$	$+2.75557705 + 0.78402927i$	0.197*sec*	72
18	$+0.25534815$	$+4.41147413$	0.222*sec*	81
19	$+0.24873564 + 0.16250710i$	$+4.65783675 + 3.04311663i$	0.247*sec*	90
20	$-0.04885988 + 0.30848912i$	$-0.98768834 + 6.23601876i$	0.274*sec*	100
21	$-0.03946755 + 0.08195521i$	$-0.85807523 + 1.78181158i$	0.301*sec*	110
22			0.331*sec*	121
23	$+0.05020855 + 0.06171460i$	$+1.25041991 + 1.53697255i$	0.361*sec*	132
24	$+0.10952609 + 0.26441938i$	$+2.90676926 + 7.01756176i$	0.394*sec*	144
25	$-0.17519185 + 0.18656038i$	$-4.94199920 + 5.26269462i$	0.427*sec*	156

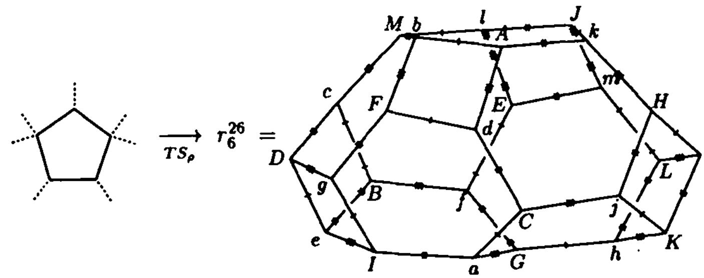
Table 41: Values of the quantum invariants at $A = e^{\pi i/2r}$ for $|r^{24}_{22}| = L_{11,3}$



$\pi_1 = Z_{11}$
3-manifold:
$L_{11,1}$

r	$S^1 \times S^2$ – *normalization*	S^3 – *normalization*	C^{++}, 486 *time*	$r-$ *states*
3	$-0.70710678i$	$-1.00000000i$	0.008*sec*	2
4	$+0.35355339 + 0.35355339i$	$+0.70710678 + 0.70710678i$	0.009*sec*	3
5	$+0.37174803i$	$+1.00000000i$	0.011*sec*	4
6	-0.28867513	-1.00000000	0.014*sec*	5
7	$-0.18132284 - 0.37652075i$	$-0.78183148 - 1.62348980i$	0.017*sec*	6
8	$+0.17677670 + 0.42677670i$	$+0.92387953 + 2.23044250i$	0.020*sec*	7
9	$+0.46424283i$	$+2.87938524i$	0.023*sec*	8
10	$+0.13819660$	$+1.00000000$	0.026*sec*	9
11	$+0.64320695 - 0.29374277i$	$+5.35420553 - 2.44518373i$	0.028*sec*	10
12	$+0.07471462 - 0.07471462i$	$+0.70710678 - 0.70710678i$	0.031*sec*	11
13	$-0.25820170 - 0.29144947i$	$-2.75070898 - 3.10490859i$	0.034*sec*	12
14	$-0.07577599 - 0.33199632i$	$-0.90096887 - 3.94740253i$	0.037*sec*	13
15	$+0.27135812i$	$+3.57432919i$	0.040*sec*	14
16	$-0.19264951 + 0.03832037i$	$-2.79304017 + 0.55557023i$	0.043*sec*	15
17	$+0.03317868 - 0.17749028i$	$+0.52643216 - 2.81616334i$	0.046*sec*	16
18	$+0.25534815$	$+4.41147413$	0.048*sec*	17
19	$-0.28802443 - 0.07293769i$	$-5.39356066 - 1.36583505i$	0.051*sec*	18
20	$-0.22085383 - 0.22085383i$	$-4.46449651 - 4.46449651i$	0.054*sec*	19
21	$-0.07111807 + 0.05671477i$	$-1.54619814 + 1.23305187i$	0.057*sec*	20
22	$-0.27032041 + 0.42062677i$	$-6.29977436 + 9.80264024i$	0.060*sec*	21
23	$-0.02146469 + 0.07660847i$	$-0.53456774 + 1.90789725i$	0.063*sec*	22
24	$+0.26441938 + 0.10952609i$	$+7.01756176 + 2.90676926i$	0.066*sec*	23
25	$+0.15042820 + 0.20704665i$	$+4.24343953 + 5.84059345i$	0.069*sec*	24
26	$-0.07361581 - 0.19410893i$	$-2.20203152 - 5.80627993i$	0.072*sec*	25
27	$+0.12952051 - 0.07477870i$	$+4.09920569 - 2.36667751i$	0.074*sec*	26
28	$+0.01592040 + 0.14129748i$	$+0.53203208 + 4.72191528i$	0.077*sec*	27
29	$-0.19285796 + 0.05354675i$	$-6.79234101 + 1.88588420i$	0.080*sec*	28
30	$+0.23587642$	$+8.73968132$	0.083*sec*	29

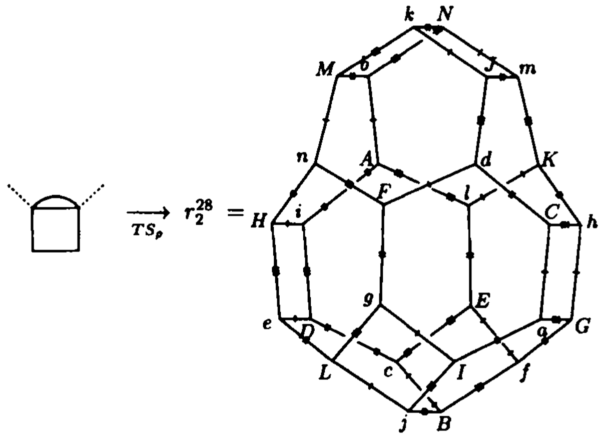
Table 42: Values of the quantum invariants at $A = e^{\pi i/2r}$ for $L_{11,1}$



$\pi_1 =$
$Z_5 \rtimes_i Z_{12}$
3-manifold:
S^3/π_1

r	$S^1 \times S^2$ − *normalization*	S^3 − *normalization*	C^{++}, 486 *time*	$r-$ *states*
3	+0.70710678 + 0.70710678*i*	+1.00000000 + 1.00000000*i*	0.009*sec*	2
4	+0.27059805 + 0.65328148*i*	+0.54119610 + 1.30656296*i*	0.012*sec*	4
5	−0.14751046 − 0.28950557*i*	−0.39680225 − 0.77876826*i*	0.018*sec*	6
6	−1.00000000*i*	−3.46410162*i*	0.027*sec*	9
7	+0.10047727 − 1.45741117*i*	+0.43323994 − 6.28409501*i*	0.036*sec*	12
8	+0.13983543 − 1.10925473*i*	+0.73081516 − 5.79724459*i*	0.048*sec*	16
9	−0.14942925 − 0.55767754*i*	−0.92680886 − 3.45889774*i*	0.060*sec*	20
10	−0.44721360 + 0.32491970*i*	−3.23606798 + 2.35114101*i*	0.075*sec*	25
11	−0.66952203 + 0.87164357*i*	−5.57325842 + 7.25576549*i*	0.090*sec*	30
12	−0.54119610 + 1.30656296*i*	−5.12193489 + 12.36544467*i*	0.107*sec*	36
13	−0.21207665 + 1.19662730*i*	−2.25932341 + 12.74807052*i*	0.125*sec*	42
14	+0.23565699 + 0.88651357*i*	+2.80193774 + 10.54055637*i*	0.146*sec*	49
15	+0.54598069 + 0.08647485*i*	+7.19165759 + 1.13904666*i*	0.167*sec*	56
16	+0.72624364 − 0.67165733*i*	+10.52910870 − 9.73771425*i*	0.190*sec*	64
17	+0.69474126 − 1.38634844*i*	+11.02316602 − 21.99660476*i*	0.214*sec*	72
18	+0.50000000 − 1.44337567*i*	+8.63815572 − 24.93620766*i*	0.240*sec*	81
19	+0.08054409 − 1.06656254*i*	+1.50827288 − 19.97250661*i*	0.267*sec*	90
20	−0.42856110 − 0.14801415*i*	−8.66323932 − 2.99206340*i*	0.297*sec*	100
21	−0.89127685 + 0.65935624*i*	−19.37750378 + 14.33525167*i*	0.327*sec*	110
22	−1.04298146 + 1.29452487*i*	−24.30651798 + 30.16869738*i*	0.359*sec*	121
23	−0.80246532 + 1.29945912*i*	−19.98501336 + 32.36240531*i*	0.392*sec*	132
24	−0.19509032 + 0.98078528*i*	−5.17760220 + 26.02956400*i*	0.427*sec*	144
25	+0.49367554 + 0.25495612*i*	+13.92612771 + 7.19207490*i*	0.463*sec*	156

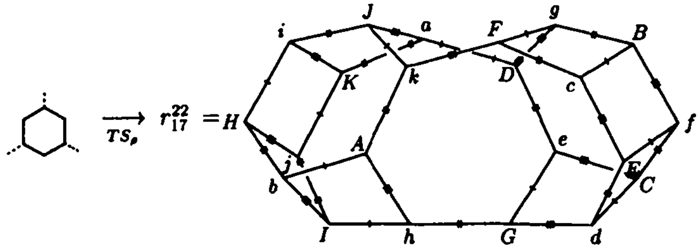
Table 43: Values of the quantum invariants at $A = e^{\pi i/2r}$ for $|r_6^{26}| = S^3/(Z_5 \rtimes_i Z_{12})$



$$\pi_1 = [3, 5^2, S_6^5]$$
$$\pi_1/\pi_1' = Z_3$$
$$\pi_1'/\pi_1'' = Z_5^2$$
$$\pi_1''/\pi_1''' = S_6^5$$

r	$S^1 \times S^2$ – *normalization*	S^3 – *normalization*	C^{++}, 486 *time*	$r-$ *states*
3	$-0.70710678i$	$-1.00000000i$	0.007*sec*	2
4	$-0.35355339 - 0.35355339i$	$-0.70710678 - 0.70710678i$	0.012*sec*	6
5	$+0.13504538 + 0.18587402i$	$+0.36327126 + 0.50000000i$	0.020*sec*	10
6	$+0.50000000i$	$+1.73205081i$	0.038*sec*	19
7	$-0.10062658 - 0.73007414i$	$-0.43388374 - 3.14794847i$	0.058*sec*	28
8	$-0.42677670 + 0.17677670i$	$-2.23044250 + 0.92387953i$	0.090*sec*	44
9	$+0.66446302 + 0.38362791i$	$+4.12121613 + 2.37938524i$	0.125*sec*	60
10	$+0.01631190 + 0.05020285i$	$+0.11803399 + 0.36327126i$	0.177*sec*	85
11	$-0.45323982 - 0.53732629i$	$-3.77287459 - 4.47283007i$	0.232*sec*	110
12	$+0.35355339i$	$+3.34606521i$	0.308*sec*	146
13	$+0.66385087 - 0.28872185i$	$+7.07222514 - 3.07585041i$	0.388*sec*	182
14	$-0.55747609 + 0.24598896i$	$-6.62833414 + 2.92478374i$	0.492*sec*	231
15	$-0.27673476 + 0.13077456i$	$-3.64515022 + 1.72256245i$	0.600*sec*	280
16	$+0.48996111 - 0.02617184i$	$+7.10347538 - 0.37944039i$	0.738*sec*	344
17	$+0.15243122 - 0.63539756i$	$+2.41856172 - 10.08158459i$	0.881*sec*	408
18	$-0.35286853 + 0.78107901i$	$-6.09626666 + 13.49416426i$	1.057*sec*	489
19	$+0.08353579 - 0.13115072i$	$+1.56429571 - 2.45593527i$	1.237*sec*	570
20	$-0.06138127 - 0.38754606i$	$-1.24080462 - 7.83413206i$	1.455*sec*	670
21	$-0.14719050 - 0.05137243i$	$-3.20011056 - 1.11690264i$	1.679*sec*	770
22	$+0.51457322 + 0.78507066i$	$+11.99204758 + 18.29594764i$	1.944*sec*	891
23	$-0.18670214 - 0.60057694i$	$-4.64972720 - 14.95708028i$	2.215*sec*	1012
24	-0.43301270	-11.49194636	2.530*sec*	1156
25	$+0.25740336 - 0.00013030i$	$+7.26110917 - 0.00367576i$	2.857*sec*	1300

Table 44: Values of the quantum invariants at $A = e^{\pi i/2r}$ for $|r_2^{28}|$

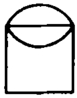


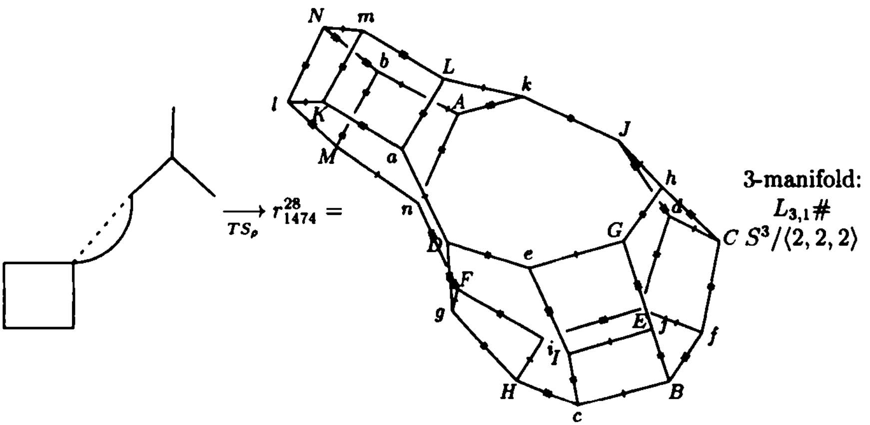
$\pi_1 = Z_3 * Z_3$
3-manifold
$L_{3,1}\#L_{3,2}$

r	$S^1 \times S^2 - normalization$	$S^3 - normalization$	C^{++}, 486 *time*	$r-$ *states*
3	+0.70710678	+1.00000000	0.012*sec*	4
4	+0.50000000	+1.00000000	0.024*sec*	10
5	+0.97324899	+2.61803399	0.048*sec*	20
6	+0.86602540	+3.00000000	0.084*sec*	35
7	+0.75304150	+3.24697960	0.135*sec*	56
8	+1.11522125	+5.82842712	0.203*sec*	84
9	+1.03372096	+6.41147413	0.290*sec*	120
10	+0.94721360	+6.85410197	0.399*sec*	165
11	+1.25231353	+10.42455157	0.532*sec*	220
12	+1.18301270	+11.19615242	0.690*sec*	286
13	+1.11008143	+11.82606850	0.880*sec*	364
14	+1.37879492	+16.39373162	1.100*sec*	455
15	+1.31720000	+17.35015835	1.353*sec*	560
16	+1.25288741	+18.16441068	1.643*sec*	680
17	+1.49578391	+23.73297122	1.974*sec*	816
18	+1.43969262	+24.87257811	2.342*sec*	969
19	+1.38148540	+25.86976870	2.757*sec*	1140
20	+1.60483108	+32.44119814	3.215*sec*	1330
21	+1.55294891	+33.76310448	3.723*sec*	1540
22	+1.49936468	+34.94245664	4.284*sec*	1771
23	+1.70723766	+42.51793381	4.896*sec*	2024
24	+1.65871958	+44.02161041	5.556*sec*	2300
25	+1.60879629	+45.38264647	6.290*sec*	2600

Table 45: Values of the quantum invariants at $A = e^{\pi i/2r}$ for $L_{3,1}\#L_{3,2}$

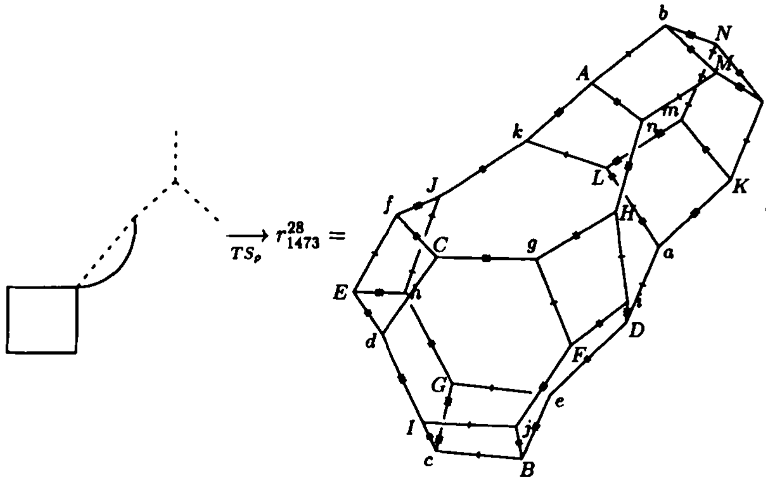
The manifold is homeomorphic to the connected sum $L_{3,1}\#L_{3,2}$. Note that since the summands are homeomorphic with distinct orientations, the space is symmetric, hence, the values of the invariant are real. Another blink inducing the same orientable

3-manifold: 



r	$S^1 \times S^2$ – *normalization*	S^3 – *normalization*	C^{++}, 486 *time*	$r-$ *states*
3	$-1.41421356i$	$-2.00000000i$	0.016*sec*	8
4	$-0.70710678 - 1.41421356i$	$-1.41421356 - 2.82842712i$	0.065*sec*	34
5	$-2.28824561 - 1.20300191i$	$-6.15536707 - 3.23606798i$	0.211*sec*	104
6	$-2.59807621 + 0.50000000i$	$-9.00000000 + 1.73205081i$	0.547*sec*	259
7	$-2.35500622 + 1.04224178i$	$-10.15436351 + 4.49395921i$	1.226*sec*	560
8	$-2.41421356 + 2.20710678i$	$-12.61728812 + 11.53489592i$	2.501*sec*	1092
9	$-1.41421356 + 3.06902325i$	$-8.77141320 + 19.03508193i$	4.523*sec*	1968
10	$-0.94721360 + 3.44095480i$	$-6.85410197 + 24.89898285i$	7.799*sec*	3333
11	$+0.13315494 + 4.55577007i$	$+1.10841293 + 37.92329853i$	12.757*sec*	5368
12	$+2.02658600 + 4.40672539i$	$+19.17981582 + 41.70569685i$	19.953*sec*	8294

Table 46: Values of the quantum invariants at $A = e^{\pi i/2r}$ for $S^3/\langle 2,2,2\rangle \# L_{3,1}$



r	$S^1 \times S^2$ − normalization	S^3 − normalization	C^{++}, 486 time	$r-$ states
3	$+1.41421356i$	$+2.00000000i$	0.016*sec*	8
4	$-1.41421356+0.70710678i$	$-2.82842712+1.41421356i$	0.065*sec*	34
5	$-1.14412281-2.31824601i$	$-3.07768354-6.23606798i$	0.211*sec*	104
6	$+2.59807621-0.50000000i$	$+9.00000000-1.73205081i$	0.547*sec*	259
7	$+0.49207242+2.52788191i$	$+2.12172782+10.89977241i$	1.226*sec*	560
8	$-3.26776695-0.14644661i$	$-17.07817311-0.76536686i$	2.452*sec*	1092
9	$+1.41421356-3.06902325i$	$+8.77141320-19.03508193i$	4.523*sec*	1968
10	$+2.97983739+1.96416717i$	$+21.56230590+14.21284718i$	7.800*sec*	3333
11	$-3.35582335+3.08402672i$	$-27.93466061+25.67216170i$	12.757*sec*	5368

Table 47: Values of the quantum invariants at $A = e^{\pi i/2r}$ for $S^3/\langle 2,2,2\rangle \# L_{3,2}$

Chapter 8

Appendices

In this chapter we provide three appendices which are needed in the Classification Theorem (Theorem 14). They are (a) the codes for the rigid bipartite 3-gems, (b) tables of the function ts_{ρ_*} and (c) the TS-classes. These appendices are given for the rigid bipartite 3-gems of 26 and 28 vertices.

The programs which generated these tables were implemented by Cassiano Durand in the context of his Master Thesis [Dur92].

The purpose of these tables are the following:

- To make the proof of the Classification Theorem meaningful, by presenting, in the first place, what we are classifying and providing ways to verify the u^0-classification (function ts_{ρ_*}), which leads to the 11 uncertainties resolved in Section 5.4.
- To permit a quick and exact topological identification of any orientable 3-manifold induced by a 3-gem up to 28 vertices.
- To act as control data for independent implementations of our algorithms.

The fact that there are no missing 3-gems in the first Appendix is assured, at a theoretical level, by Theorem 12. At an implementational level the routines are simple minded and have been independently checked in various aspects.

To perform the identification of a rigid 3-gem, the reader has to obtain its code (which is easy to implement). If the gem is not rigid, simplifications must be performed.

Detailed information on how to read the tables which follow are given in Sections 5.1 and 5.2 where we display the similar tables for 3-gems up to 24 vertices. The tables are available from the author as Pascal or Tex files.

8.1 Appendix A - Codes of Rigid Bipartite 3-Gems

8.1.1 3-Gems with 26 Vertices

0001	cabfdeighkjml	fidclkjemgahb	jhmkclfbaidge	fmkchabljideg
0002	hjlmbkfacegid	ildcjgfemhbka	dmgacieklbhfj	jmhagiekldcfb
0005	imdcbgflkhaje	gdfblhmcjieka	gfmaibdklchej	mdiglhbfjceka
0008	imdcbjfkaglhe	mgkjlcbadifeh	imdcbkfjalghe	gfmakbdcjileh
0010	ldfbjhmceiakg	lgkajhbfeimdc	ljhmkadcbiefg	lkfjhmaedibgc
0014	imdckgfelbhaj	ehgjalmbdifkc	imhckjfdlgeab	hfiklbejcamgd
0016	imlcakfjehgdb	hljgkbdmciefa	lehgkadmjibfc	miehkbldfgcja
0019	mifhljadbcekg	mikbhlaedfcjg	mlijbcdkfghea	dabcgefihkjml
0022	ihjkdlmgacefb	hikjbmlaegcdf	ihkedlbgafmjc	fjhibmdcgelka
0024	fklmbhdajiecg	higkbdlaemfcj	kihgcalebmfdj	ihledjbgamckf
0027	leimkacjghbfd	ihledkbgacmjf	fkmlbhdajiegc	ihmldjkgafbec
0029	lmhibkjcgdfae	ijledckgamhfb	mehkbajcgildf	ijmedckgalhbf
0031	lfhkjbaceimgd	ijmldbkgafhec	hmljakcedifgb	ikfldmjgahbec
0033	liemckafjbhgd	mekijldcgbhfa	ikfmdljgahbce	flijcmkbedgha
0036	kimfbjlaedhgc	klejciabmdghf	lekgjidambchf	ikhedmlgjabcf
0039	ejihmdbfcglka	ikhmdlcgjabfe	mhkibcjelgfda	ikledcjgahmfb
0041	fiemjhdabclkg	heglbjckmiadf	hemjbakfdilgc	jimglakebfdhc
0045	jlghmaikcfdbe	kiglbjcaemhdf	mghjaibkflcde	iklmdbjgahfce
0048	hemlbjdkfiagc	hgmljdbkeiafc	kimlbjdaefhgc	ikmedcjgahlbf
0051	eikmhlbagfcjd	ikmjdahgfelbc	hflgkbiadcmje	ikmldbjgahfec
0053	jgikacmlfedhb	jlfkacibmedhg	jlikacmbfedhg	jmkhcdifgleab
0057	kmlihcaejdgfb	likmchafjbegd	lmgjbdikcfhae	lmjhbdkfcigae
0061	ilfedcjgmhbka	eiglhkcabdmjf	eilfjdckbamgh	fielchkabdmjg
0064	fieljhdabcmgk	filgcjdkbameh	hegmbakflicjd	helfbikadcmjg
0068	hiemkbdaflcjg	higmbdkaelcjf	himgjbdaflcek	jemfbkicladgh
0072	jghfalicbemdk	jihgcalebfmdk	keilhjmbcdafg	khemliabfdgcj
0076	legibkcfdamjh	lekijmdcgbhaf	lfeihbmcjdgak	lfigkbmejchad
0080	lfigkhmbjcead	lfijchmbedgak	lfimcjakebhgd	lhikajcbgfmed
0084	ligfbjcmedhak	ligjhmkabdcef	lihgjmdkbacef	ilfedckgmbhja
0087	hkgmjdlaeibcf	ilhedjkgmfcba	hilgkbdafcmje	ilhkdjcgmfeba
0089	jhklbiaefgmdc	iljedchgmfbka	fmhkjadcligeb	jghmkdilbaecf

0092	kmifhdclgeajb	iljedcmgafbkh	fjmglkdebiahc	hgljakbedimcf
0095	iljkdbhgmcefa	ehlfadjmgickb	hikfbdlaegmjc	likjbhdmegcaf
0098	ljigckdbehmaf	ilkedchgmbfja	hklmjdbaeigcf	ilkedcjgmhfba
0100	emgjldikcfhab	kmifjdclgehab	kmigcjdlefhab	ilkedcmgabfjh
0103	fklgmhdaeibcj	ilkjdbhgmecfa	egjhamklcibdf	eklmhdbajigcf
0106	ekmfhdcalibjg	emhkadjclifgb	fmlhcajkgideb	hmlkcaifjgdeb
0110	lkhmacifjgbed	ilkmdahgfbcje	mfikjbaechldg	ilmedkhgfcabj
0112	emlhjdifgackb	imfedclgkhajb	ehjladikgmbfc	femiladkgchbj
0115	fimjchkaldgbe	fkeiljdmgabch	heglbamfjidkc	hemflikadcgbj
0119	hieklbdafmgcj	hijglbdafmekc	hijlbdkaemgfc	hkgimacfdelbj
0123	megibkcflahdj	mfeihbacjdlkg	mhklcjabfiedg	imfjdklgaechb
0126	egjhalkmcibfd	femkladcjigbh	imfkdjlgacehb	mekfhljbgicda
0129	imhedjkglfcab	fijglhdakmbec	hgilakjembfdc	jgfhkibmcaled
0132	mhikcalbegdjf	imhjdkcglefab	eimlkdbjghafc	ejimadlbchgkf
0135	ekmlhdbajigfc	eljhadmfcibkg	fkmlbhdajiegc	jiemcbkafhlgd
0139	imhkdjcglfeab	fhmlbkdejiagc	hkilabjemgfdc	jemlbidakhfgc
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0951	giljhdaemfckb	jgfilcbamhekd	jifhlcbmgaekd	klmgfedihcabj
0954	fmljgiedahckb	gmljhdieafckb	kmigfedchlajb	fhjilaekmcbgd
0957	fhkjiaelmdcbg	fhkmiaebgdlcj	fkjhmielacbdg	fkjmgledacbih
0961	gfjihlaemcbkd	gfjimlaedcbkh	gfjlkbaedmhic	gfmiklaedchbj
0965	gikmhdjebalcf	gkjfmlieacbdh	gkjmhdleacbif	glkjhdaebmcif
0969	igfhjcblmedka	ihgkjalbmedcf	ihgkmacbfeldj	ikfhmcblaegdj
0973	ikfmgljdaebch	iljfhdcemgbka	iljhgcedmfbka	jgflkcbadmeih
0977	jgmikcladhebf	jifmglkdbaech	jigmhdlkbaecf	jlkfhdcebmgia
0981	jlkhgcedbmfia	lhgjiakbmdcfe	lhjigamkdcbfe	lkfhjcbdamgie
0985	lkgfjdcbamhie	lkjfhdceambig	lkjhgcedambif	lkjihdamfcbge
0989	mgfikjbadcleh	kmjgfedihclab	fhgkjalbcmdie	flkjgaedbmcih
0992	gfkjliaebmchd	glkjhdaebmcif	ifhjlbakgmced	jhgfklcbdmeia
0996	kmlgfedihcajb	fhkjiaelmdcbg	fiklgmedbachj	flmkgjedcahbi
0999	gfmlkjaedbhic	gikflmjebachd	giklhdmebacfj	ihglmakbfecdj
1003	ikfhmcblaegdj	ikgfhdcmaelbj	ilgfhdcjmebka	jimfhdclbaekg

1007	jimhgcldbaekf	jlmfkicadhebg	lhgkjaibcmdfe	lkfhmcbjadgie
1011	mfhjibakldceg	mfikgjldcbhea	mgikhdjlcbfea	mgikldjacbfeh
1015	mlkfhdcjbegia	mlkhgcjdbefia	mjigfedchblka	fiklgaedbhmjc
1018	fikmgledbacjh	fkjmgledacbih	fmkjgledbhcai	gfjimbaelchdk
1022	gfkjmlaebdcih	gflikbaedcmjh	gikfmljebacdh	giklhdaebfmjc
1026	giklmcjebafdh	gikmhdlebacjf	gkjfmlieacbdh	gkjlhdaefmbic
1030	gmjikbledchaf	gmkjhdlebfcai	ikfhlcbdaemjg	ikfhmcjlaebdg
1034	ikfmgljdaebch	ikgmhdljaebcf	iklfhdceagmjb	iklhgcedafmjb
1038	iklmhdbjaegcf	imfhjcbdlegak	imjfhdcelgbak	imjhgcedlfbak
1042	jifhmcblgaedk	jifhmcklbaedg	jifmglkdbaech	jigfmlckbaedh
1046	jigmhdlkbaecf	jimfhdcelagbk	jimhgcedlafbk	jimhkcldgaebf
1050	jmkfhdceblgai	jmkhgcedblfai	lgfikcbadhmje	lhgkjacbfmdie
1054	lhkjiambgdcfe	likfhdcebamjg	likhgcedbamjf	likjhdambfcge
1058	lkjfhdceambig	lkjhgcedambif	lkjihdamfcbge	mljgfedihcbka
1061	fhikjaemcldgb	fhkjiaemldcgb	fhkliaebgdmjc	fhmkjaebcldgi
1065	fikjlaembhcgd	fikmgledbacjh	filmgjedbahck	fjiklaemcbhgd
1069	gfilmjaecbhdk	gfklmiaebhcdj	gikfmljebacdh	gilmhdjebafck
1073	gjifmlkecbadh	ihgkjambledfc	ihgklacbfemjd	jhgfmicblaedk
1077	jhgmkacbdlefi	jhgmkaiblfecd	jhgmkalbdfeci	jhgmkilbdaecf
1081	jmlfhdckgaeib	jmlfkicadhegb	jmlhgckdfaeib	lfhjmbakgdcie
1085	lfikmjadcbhge	lfkjmiadbhcge	likfhdcmbagje	likhgcmdbafje
1089	likjgamdbhcfe	likjhdambfcge	ljikgamdcbhfe	ljikhdamcbfge
1093	lmgfkicbdheaj	lmhjibekgdcaf	lmifhdckgbeaj	lmihgckdfbeaj
1097	lmikhdjecbfag	lmkfhdcjbegai	lmkjgiedbhcaf	lmkjhdiebfcag
1101	eabcdifghkjml	kjhlfemcbiadg	gkjilcaemfbhd	lgjkidbamcfhe
1103	kjhmfedlbiacg	jfmhkbadlgeic	kjhmfelcbiagd	glijkbmecdhfa
1105	hmfjkcblediag	jflhkmadcgeib	jglikcmadhefb	jkglidmbeahfc
1109	lgjimckadfbhe	lgjkidbamcfhe	mfjkhbaldcieg	kjlgfedmbiach
1112	jgflkmbadheic	jkgflmcbeahid	lhgkmdjbacfie	kjmgfedlbiahc
1115	fmlhjaekcgdib	gfljkmaecdhib	glihkbmjcdefa	gmjkhbledciaf
1119	hgljkcmaedifb	hljkidbmecfga	jfimkbadlgech	jflhkmadcgeib
1123	jkgmidlbeahcf	jkihlbemcafgd	jkimgbedlafch	jmlhgakdcfeib
1127	jmlihcakdgefb	lkjihcamdfbge	mhgkidjblcfea	mkjfidcalgbeh
1131	klmgfedcjiabh	fmlhjaekcgdib	fmljgaedkhcib	hgjflmkaecbid
1134	jgfmlcbakheid	kmhgfedcliajb	hlfjkcbmediga	jfihlbamcgekd
1137	jgmikcladhebf	lgfijcbakmdhe	lgjkidbamcfhe	mfijlbadkhceg

1141	mfkhgjldcbiea	mfkihjaldbceg	mhgfkjcbadlei	mhgkldjbacfei
1145	mhkfldcjabgei	mkihgbldcafej	mlihjbekcgdfa	mlijgbedkhcfa
1149	kmlgfedcjiahb	fijhlaemkcbgd	glihkbmjcdefa	glijhbmekdcfa
1152	hgjimckalfbde	hgmlkjbaedifc	hlmfkjcaedibg	hlmkidjaecfbg
1156	jgmlidbakhefc	jlihgbkdcmefa	jlmfidcakhebg	lijhgamdkcbfe
1160	mfihkbajcdleg	mgjkidbalcfeh	mkihlbjdcafeg	mlkhgjedcbifa
1164	mjhgfedcbilka	fmihjlekcgdab	fmijgledkhcab	gfijmbaeclhdk
1167	gfilkbaecdmjh	gfmjhlaekdcbi	gkilmbjecafdh	gmijkblecdhaf
1171	hgjfmlkaecbdi	hgjmidbaelfck	hgjmilkaefbcd	hglkidbaecmjf
1175	hkgfmlcjeabdi	hkgmidljeabcf	hklfidcaegmjb	hklmidbjeagcf
1179	hmjfidclegbak	hmjkidblecfag	jfihmbalcgedk	jfihmbklcaedg
1183	jfimgbkdlaech	jfmhglkdcaebi	jgfmilbakhecd	jhgfmlcbkaedi
1187	jhgmidlbkaecf	jhmfidclkaebg	jmgfidcbelhak	jmgfidclkheab
1191	jmihgbedclfak	jmihglkdcfeab	jmihkbldcgeaf	lfihkbadcgmje
1195	lkgfidcbeamjh	lkihgbedcamjf	lkjfidcamgbhe	mlhgfedcjibka
1198	fikjgaedmlchb	filhjaekmgdcb	fjkhlaemcbigd	gfijhbaemlcdk
1202	gfilmbaekhcdj	gfklmjaecbidh	gjilhbkemfadc	hglmidjaebfck
1206	hglmkjbaedicf	hjgfmlckebadi	hjgmidlkebacf	hmlfkjcaedigb
1210	hmlkidjaecfgb	jfihgbmdcleak	jfihmbalcgedk	jfihmbklcaedg
1214	jfilhbakmgedc	jfimgbkdlaech	jgfikcbamlehd	jglikcmadhefb
1218	jhgfidcbmleak	jilhgakdmfecb	lfihmbakcgdje	lfijmbadkhcge
1222	lfkhmjadcbige	lgfjkcbamdihe	lgkjidbamfche	ljkfidcambghe
1226	ljkhgamdcbife	lmgfidcbkheaj	lmgfkjcbediah	lmgkidjbecfah
1230	lmihgbkdcfeaj	lmihjbekcgdaf	lmijgbedkhcaf	lmkfidcjebgah
1234	lmkhgjedcbiaf	eabcdifghmjkl	mlkgfedcjihba	lhgjmkcbadfie
1236	eabcdjfghimkl	mlhgfedcbkjia	ihkmjdlbaecgf	kgfjlcbamdhei
1238	kjihlamdcbfeg	fabcdejghimkl	mlkihgfedcjba	gjimlkafcbhed
1240	kjihmladcbgfe	kmgjilcbedafh	lhgjkmcbadeif	lkgjimcbedhaf
1244	lkmhgjedcfbai	lkmjihcfedbag	fabcdekghijml	mljihgfedcbka
1246	jihglmdcbafek	jilmgkedbafch	lgkjimbaedchf	lhgkjmcbaedif
1250	lihgkmdcbaejf	lmhgkjdcbfeai	lmkjihbfedcag	

8.1.2 3-Gems with 28 Vertices

0001	cabfdeighljknm	imdckgfjnhebla	milgcknfjbhead	imdclgfkabnejh
0002	gnejhblcdimakf	hnjgkalfcimdeb	imkcjgfenhdbla	jehmilncbagfdk
0005	imlckgfenbhdja	kenmbjlcfiagdh	indcmgfjlheakb	dmgnciekjfhbla
0007	dmgncielkbfhja	hlfnjieacgdmbk	jeingabdkfcmlh	jifbhklndacmge
0011	ndiglkbfmahecj	indcmgflkeahjb	negmhalbjidckf	injchgfeldmakb
0013	kmigchlfebndja	inkchgfejadmlb	kinjgaedmhfcbl	inlchgfekmadjb
0015	nhkjiadlegmbcf	inlckgfjamedhb	dkgnclebjihmfa	dlngckafjihmbe
0018	gfkalmdcjihnbe	gfnkjmlceidabh	gkfbjmlneidach	mdjkanlfcibgeh
0022	inlckjfbmgedah	hdkbamnejicflg	dabcgefihljknm	iknedcmglhbajf
0023	ehkmadilgncbfj	khemcbnfjiadlg	kmehcbilgnadfj	dabcgefjhilknm
0026	gkiedmlfnbjahc	eilfndkgbmhcja	eimfkdngblajch	filgkndebmhcja
0029	fimgnkdeblajch	ihgkmjcnaldfeb	gkimdhlfnbjaec	fihgnkdmblajce
0032	nhkglidbmecjfa	gkimdnlfcbjaeh	milfndkgbahcje	gkindhlfmbjace
0034	ngkimjbedlcfha	gmhedknlbfjcia	feinkamgclbjhd	gmhldiacnkjebf
0036	keminalgdcfhjb	gnhldiacmkjefb	ikemnblgadfhjc	gnhldmacbkjefi
0038	emikndhglbfcja	ikemnbhgldfajc	jihldncgbamekf	ihljbcneadgmfk
0041	jikedchgnafmlb	kjifhdcngbmeal	jikedcngmafblh	hgmialbedkjncf
0043	jiledckgnamfhb	fmhjnadckgible	jimedckglahnfb	hjegcnlakmidbf
0045	nglimjbedkfcha	jinedcmgkabhlf	hlemcknadfjbig	mgkinlbedcjfah
0048	jinedcmglahbkf	njigcmdlebahfk	jkfedchgbnimla	hjnfbikademcgl
0050	kemjbalcndfihg	kfejhbncgdmial	kfjimbaencgdhl	khegcbjfndmial
0054	khgfbjcendmial	jkfedcmglanihb	ifekmbalgdjnch	nhegcblfamjdki
0057	jkfedcmgnliahb	elifmdcngkjbah	eljfndckmghbia	emjfkdcnlgaibh
0060	hlegcbnamkjfid	ifelmbangkjdch	imegcnlfakjdbh	kfejhbncmdailg
0064	kjegcbnlmfahid	kjifhdnlmbaecg	kjifmdcngbaelh	jkfmdnhgbliaec
0067	kfmjhblngdaice	khmgnbjfldaice	mgkhndjflbciae	mjkfndcalbgihe
0071	jkfndmhgbliace	fligcndmekjbha	igejhnlcadmbkf	ilegcnjmadhbkf
0074	kjnfhdmlgbaeic	jkiednmgclbahf	nheikblfmcjgda	jkledchgnabmfi
0076	ifnmhbalgdjekc	nfihmbalcejdkg	jkmedchgnliafb	elnfhdcamkjbig
0079	elnhadjfmkgbic	flngchdamkjbie	hlegcniadkjmbf	ifnlhbamgkjdec
0083	ihegcnlmakjdbf	kfnjhblmgdaiec	khngmbjfldaiec	kjnfhdclmbaeig
0087	kjngchdlmbafie	kmejhnlcgdaibf	ngljmkbcedfhia	jkmedcngbliafh
0090	ejnfhdmalbgikc	elnfhdmagkjbic	feijbadlngmhkc	ihlfndcbamjgke
0094	kjnfhdmlgbaeic	ngljmkbcedfhia	jknedchglamibf	mfhnkiacbejgld

0097	jknedchgmabilf	emihnkblcfjdga	feihnaklcmjdgb	jknedchgmliabf
0099	enkhadjflmcigb	jknedcmgbliahf	egkmadjnlbcifh	ejknhmcalbgifd
0102	elimadjnckgbfh	elinhmcagkjbfd	hgknalbemcjfid	hjilcdnaebmfkg
0106	ihegcmdnakjflb	ihemcbnfakjdlg	ihgfmdcnakjelb	ilegckjnadmbfh
0110	ilgfkjcnadmbeh	ilmfkjneadhbcg	imegcblnakjdfh	kjemcbnldfahig
0114	kjifmdclgbaneh	kjinhmclgbaefd	kmegcbjnldaifh	mlegcbindkjfah
0118	njkfmdcalbgieh	njkgcmdalbeifh	nlifmdcagkjbeh	nligcmdaekjbfh
0122	jknedmhgfabilc	ihmfndclaejbgk	mfhknbaledjcgi	mfihnbalcejdgk
0125	mfkinbaldcjegh	jlfedchgbnmika	egkiahnldcjmfb	egnhakmlcfjdbi
0128	hkincdmlebjagf	mkifhdcngbjale	jlfedcmgkainhb	ejilhdkambnfcg
0131	ekjfndcblghmia	hjefmikadbnclg	hjegcbkamfnild	hjgfbkcamenild
0135	hjimcdkaebnflg	hjknmldafbeicg	kfhjnbacemgdli	kjegcbnalfhmid
0139	kjenmidalbhfcg	kjgfbncalehmid	kjnfmdcalbheig	kjngcmdalbhfie
0143	nfkjibaclmhedg	ngjiahklmcbfde	jlfedcngkaimbh	hegnbalfdmjck
0146	lfjmkbaednhgic	jlhmdncgkaibef	hmjlnabfdkcige	mhljbckeangfid
0149	mjlhndafckbige	mkfjhcbelngaid	jlhndmcgkaibfe	hkjmlabfdngeic
0152	hnjlabmedkcigf	kemhbajlcnfdig	kgihmdjfcbnela	kjmgcidalnhfeb
0156	lhnjbcmeakdigf	jliednhgmkabcf	hjgfmkcadnielb	jliednmgckabhf
0158	enhjmkbcflidag	lenjmakcgdifbh	jlmedchgkainfb	fmkjbnicldhgea
0161	kfhjnbacemgdli	kjencbialfhmgd	lhencbjfakmigd	mfihkbancejgld
0165	jlmedchgnaibfk	ejnfhdcamkbilg	fenjkadcmgbilh	fjngchdamkbile
0168	kgimanjfcbedlh	mkifhdcngbjale	mkigchdnebjalf	nfihmjakcbdelg
0172	nfikmjaecbdhlg	jlmednhgfaibck	nfihmjakcbdelg	nfikmjaecbdhlg
0175	jlmednhgkaifcb	emlhndjfckgiba	fkjmiadblnhegc	hgnlajmedkfibc
0178	imlgckjfndheba	khjfidcbmgnela	kjnfmdcalbheig	nkifhmclgbjeda
0182	jlmndbhgkaifce	ikfjmcbeldnahg	lnfjmcbeakdihg	jlnedchgkaimbf
0184	hjgnbmladefcki	hngibmjldcfake	jlnedchgmaibkf	egjnahkldmbfci
0187	ehjnilcbagdmfk	enlhadjfcmbigk	fnhjbmlcgkdiae	helnbajfdkgmci
0191	hemlnjcadkfigb	ihjmcdkbaenflg	ikejhmacgdnflb	imegcbjnldhafk
0195	kjemcbianfhdlg	kjifhdcmgbnela	kjigchdmebnfla	leihnajkcmdfgb
0199	lfmjibacnkdghe	lhgnbjceamdifk	ljgfbicndehmak	mfikljaecbnhgd
0203	mglhajkncfbied	mjgnbikalecfhd	mjigkanelbhcfd	nfihkbalcejmdg
0207	ngejhlkcambfdi	jlnedmhgfaibkc	leihnajkcmdfgb	mgikahjflbnced
0210	mkgibhjfnceald	mkjfincbeghald	jlnedmhgkaifbc	ejlnhdmagkbcif
0213	mghkajilbfnced	mglhajbnckfied	jlnmdbhgkaifec	fhjlcimbaegnkd
0216	fkjihmdlgcbnae	hkjgcinlmebafd	ikfgnmjbldhace	kgmhajnlcfedib

0220	khjgimlbncfdae	jmfedckglnhbia	mhegcblfnkjdai	jmfedckgnahbli
0222	hkegcinlmbjafd	hkgmidnlebjafc	hknfmdclebjaig	inlfkdcbmejgah
0226	inlgckdbmfjeah	meghbailcnjdfk	mhegcblfanjdik	mhgfblceanjdik
0230	mhlgckdbanjeif	jmfedclgknbhia	hlenckmadfjbgi	jmfedclgnabhki
0232	nfhlkiacbejmdg	jmfedcngkliabh	eghnadilbmjckf	jmfkdlhgbnecia
0234	megibkcndfjalh	mekibalndcjgfh	jmfldkhgbnceia	ehlfidnbamjckg
0237	enlfidkmagjchb	hlgmkicanejbdf	hlnfidcaemjbkg	ihlfkdnbaejmcg
0241	ingkmlceadjfhb	mhgkblcendjfai	jmhkdlcgbnefia	hlemckianfjbdg
0244	mekibnlfdcjgah	jmhldkcgbnfeia	hlemcbiankjfdg	hlfmicbankjgde
0247	hnfikcbadejmlg	ignkhlbeadjmfc	mfeicbkndgjalh	jmhldkcgnafebi
0250	hnfimcbldejakg	jmiedckglnhfba	ilejnkamgdfbhc	jmiedclgknfhba
0252	fehliancbmjdkg	fihgkadnbejmlc	jmiedkhglnfcba	ehmjldinkfgacb
0255	eimhldjfnkgacb	fhmjildnkgeacb	flhjiadcmgenkb	heglbajfmcinkd
0259	hjmglbiadfcnke	igflahbcemjnkd	inhgcjkemldfab	lfmjknacedighb
0263	lhgibjnekmdafc	ligjbhnmekdafc	ljfhinbkmgdaec	ljghbinkmedafc
0267	mehjildcbgankf	jmiedlhgkncfba	egjiaknmlchdfb	fehniadlbmjckg
0270	hjlfbimadegnkc	hlgfbncadmjeki	hngfbicldmjake	inejhbmcldgakf
0274	inlfkjcemdhgab	mijfancklghbed	jmiedlkgcnhfba	eghjakncmlidfb
0277	ehmjkdlnafigcb	elmjkincfdaghb	limfakjcngdehb	jmkedchglafnbi
0280	mfihkbancejgld	jmkedchglnfbia	fkemcinlabjhdg	mfhlibncekjgad
0283	jmkedchgnafbli	fhlgcmdbanjeik	fhlnmidbaejckg	jmkedclgbnfhia
0285	ehlfadnbgkjmci	ehlfidnbakjmcg	ehlfmdkbngjcai	enlfidkmagjchb
0289	fhlgkmdbnejcai	fhlmcinbakjedg	fnlgkidmaejchb	hleicbnadmjfkg
0293	hlencbmadkjfgi	igelhmncakjdfb	ignlambcekjdfh	ingkmlceadjfhb
0297	mhgkblcendjfai	jmkedlhgfacnbi	feihkamncbjgld	jmkldbhgnacefi
0299	ehmfidknagjclb	jmledchgkanfbi	nfihmbalcejdkg	jmledchgknbfia
0301	elmfhdcankjbgi	flmgchdankjbei	jmledchgknifba	elmhadjfnkgicb
0304	hlmgckiadfjneb	ilmgnkjfadhecb	kjelnbiamfhdgc	kjghbinlmeadfc
0308	jmledchgnabfki	enkfadilmcjbgh	ifnkhbalgdjmec	nfhlkiacbejmdg
0311	nfihkbalcejmdg	jmledckgbnhfia	ekmhadlfnbjgci	enihadlfcmjgkb
0314	hlemckianfjbdg	hlnfidcaemjbkg	iemlbancfkjdhg	inekcmlfadjghb
0318	mhekcblfndjgai	jmlkdbhgnaecfi	ekihadmncbjglf	jnfedchgblmaki
0320	fhmgnidlaejbck	hlegcbmadnjfik	meihbanlcgjdfk	mhegcblfanjdik
0324	mhgfblceanjdik	mhgfknclaejbid	mhlfkncbaejgid	jnfedcmgkliahb
0327	elhmkdjnbgaifc	eljfkdcnmgaibh	fjnklhdaemgibc	hginajlemkfbcd
0331	ifenmbalgkjdch	kfihljancbgmed	kinfaljcmgedbh	kmghbjilcfndea

0335	jnhkdlcgbmefai	hkegcidnfbjmla	igfmhcbnakjdle	jnhkdlcgmaefib
0337	neihbamlcgjdkf	jnhldkcgbmfeai	hgfikcbndejmla	jnhldkcgmafeib
0339	egihamkncbjfld	egknahbldmjfci	emkinhbldcjfga	fkinmldecbjahg
0343	hgfialbedmjnkc	hgfnkmbldejaci	hlfnkmbadejgci	nfkimlaedcjbhg
0347	jniedckglmhfab	eljinhbfdcmgka	nehjlmdckgibfa	nheicblfmkjgda
0350	njehcblmkfigda	nkeicbjlmgfhda	jniedclgkmfhab	fihlkamcbejngd
0353	hlngmbiadkjfec	nhgiblcemkjfda	jniedkhglmfcab	hjgmkdnaelibfc
0356	hjkgcinamlebfd	hjkmidnaelgbfc	ifjmhlakgnebdc	igejhlmcadbnkf
0360	nhgibjkemlcfda	jniedklgcmfhab	lhegcbjfmdinka	nhgiblkemcjfda
0363	jniedlhgkmcfab	ekjiadmbnchglf	fhgmkinlaejbdc	hgfmknbldejaic
0366	nheilbjfmkgcda	jniedlkgcmhfab	ekhjadmcbniglf	fekmiadlbnjhgc
0369	fknjiadcmgbelh	hgfmknbldejaic	hjeklbiamfncgd	imhknablfdjcge
0373	imlfnjceadbgkh	kejmhaibgnfdlc	nehjiamcbkdglf	jnkedchglafmib
0376	fehniadlbmjckg	fkenmidlabjhcg	mleicbkangjfhd	nfihmbalcejdkg
0380	jnkedchglmfbai	ekgfidcnabjmlh	jnkedlhgmacbif	feimbaklngjdhc
0382	imlfkdcbnejgha	mgfhkibncejald	jnkldbhgmaceif	eghnamilbfjckd
0385	eknfidclamjhbg	felihadbgmjnkc	mgfhkibncejald	mlfnkibacejghd
0389	jnledchgkamfib	eghmakinbfjdlc	egnhakmlcfjdbi	ehkmadnlgcjbfi
0392	fkimhadlnbjegc	hfeikbmndcjgla	hgnfaimldkjebc	hgnfkimldejabc
0396	hkegcimlnbjadf	hkgnidmlebjacf	hkincdmlebjagf	mfhlkiacbejngd
0400	mfihkbancejgld	mkifhdcngbjale	mkigchdnebjalf	nmegckildfjbha
0404	jnledchgkmifab	ehkjadmlgncbif	hekmljcadngbif	hjegcbnamkfild
0407	hjgfbncamkeild	kjghbnmlceadif	kjgmbicanehdlf	kjgmbiclneadhf
0411	lmjkciabnedghf	nfkjmbacldheig	nhelcbjfmkgida	nhglbjcemkfida
0415	njehcblmgkfida	njekcbimlfhgda	nlejhkmcgdabif	jnledchgmabfik
0418	fmkihadngcjble	jnledkhgfamcib	mfihkbalcejngd	mfkinlaedcjbgh
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7631	hjlgmndkfbacei	hjlnmieafbgckd	hnmkjlcafedigb	jgihmnblcafdek
7635	kihnjlacbemfgd	kihnjlambegfcd	kjmngiedfbahcl	kjmnihdfebagcl
7639	kjmnildbehafcg	knhgmldcbeafji	knmgildcehafjb	knmgjldcaeifhb
7643	mgilknbachedjf	mgjlknbaecidhf	mjlgindcebahkf	mjlngiedfbackh
7647	mjlnihdfebackg	mngkjlcbfediah	mnhgjldcbeifak	mnihlkbdcgfeaj
7651	mnilkjbfchedag	mnjgildcehbfak	mnjlkiebfcgdah	mnlgikdcebfhaj
7655	mnlkgiedfbhcaj	mnlkihdfebgcaj	mnlkjiebfgdcah	fabcdejghilknm
7658	lkimhgfenbjadc	kgmjilnaedhfcb	klhgmjncbfaied	kljnihbfemagcd
7661	ngklihbmedcjfa	niklgjedbmchfa	lkinhgfembjacd	hlkjimcandgbfe
7664	ilkgjmdcanhbfe	nihkljecbmdgfa	nilkgjedbmhcfa	lkinmgfecbjahd
7667	hmnjklcafdeibg	kgnmilbaedhfjc	khnmjldbaeifgc	klhmnjecbfaidg
7671	milkgnedbahcjf	mnjklhbfecdiag	ngklimbaedcjfh	nmjklhbfecdiga
7675	lkmihgfednjacb	klhmgjenbfaidc	klnjihmfedagbc	ngjkihbmeldcfa
7678	lkmnhgfedbjaci	klhgnmdcbfaije	lknihgfedmjabc	ihgklmcbandjfe
7680	klgjimcbndahfe	klhgmjncbfaied	ngklihbmedcjfa	nlkjihcmedgbfa
7684	lknmhgfedbjaic	hngjklcmfdeiab	klhnmjecbfaigd	klmngjedbfaich
7687	knmgjldcaeifhb	mgjklhbnecdiaf	mnhgkldcbfejai	mnklihbfedcjag

7691	nhgklmcbaedjfi	nlkjimcaedgbfh	nmklihbfedcjga	nmkljidbfecgha
7695	lmkihgfedcjnba	gjikmlafcbnhed	gjilkhafnbmdec	inhkljecmfdgab
7698	khgmjlnbaeifdc	kngjilcmedhfab	knljihcfedmgab	lnkihgfedcjmab
7701	gjilkhafnbmdec	gjilmkafcbhned	hgnmklbafdeijc	hkljnmcafdbgie
7705	ihgkmlcbafnjed	iklnmjecafbhgd	khgjilcbmdnfea	khgjmlcbanifed
7709	kjihglmdcbnfea	kjihlmadcbgnfe	klgnmicbfeahjd	klhgnmdcbfaije
7713	klnmgjedbfaihc	klnmihbfedagjc	nhgklicbmedjfa	nilkgjedbmhcfa
7717	nlgjikcbedmhfa	nlihgkedcbmjfa	nmkihgfedcjbla	gjinlkafcbmehd
7720	gjnlkhafebmdic	hgmnkibafledcj	hkgjmnclfdbaei	hnmjklcafdeigb
7724	ihgmnlcbafejdk	ihlmnjebafgcdk	ihnmkldbafejgc	ikhmnjeclfbadg
7728	inmkljecafdghb	kgmnihbledafcj	khnmjldbaeifgc	kihgmndcblafej
7732	kimngjedblafch	kjihnladcbmfge	knmjilcaedhfgb	mngjilcbedhfak
7736	mngjklcbfdeiah	mnhkljecbfdgai	mnljihcfedbgak	mnlkgjedbfhcai
7740	fabcdejghinklm	nmlihgfedkjcba	mhgjkncbaledif	mihgknlcbaedjf
7742	mlihgnkdcbejaf	mlkhgjedcnibaf	mlkjihcnedgbaf	fabcdekghijnlm
7745	nmjihgfedclkba	jihlnkembafdcg	lngkjicbmedafh	lnhgkjdcbmeafi
7748	lnihgkedcbmafj	mhgkjncbaldeif	mjihgnldcbaekf	mlgkjicbnedhaf
7752	mlhgkjdcbneiaf	mlihgkedcbnjaf	mlkjihbnedcgaf	fabcdelghijknm
7755	nkjihgfedcbmla	kjingledmbafch	kjnhgledcmafbi	mjihgledcbankf
7758	mlkjihanedcbgf	gabcdefnhijklm	nmlkjihgfedcba	lkjihnmedcbagf
7760	mlkjihnfedcbag			

8.2 Appendix B - Function ts_{ρ_*} on Rigid Bipartite 3-Gems

8.2.1 3-Gems with 26 Vertices

01	22/0017.0	24/0001.0	26/0003.0	26/0004.0	26/0005.0	26/0006.0
07	24/0003.0	26/0008.0	26/0008.0	26/0010.0	26/0011.0	24/0007.0
13	26/0013.0	26/0014.0	24/0108.0	22/0002.0	20/0002.0	20/0005.0
19	14/0001.0	22/0001.0	08/0001.1	26/0022.0	22/0005.0	18/0004.0
25	26/0025.0	24/0003.0	20/0004.0	22/0005.0	14/0001.1	24/0006.0
31	26/0031.0	24/0022.0	24/0033.0	24/0022.0	24/0033.0	24/0021.0
37	24/0154.0	24/0022.0	02/0001.0	20/0004.0	24/0022.0	24/0013.0
43	24/0005.0	24/0003.0	20/0005.0	24/0032.0	24/0002.0	26/0048.0
49	26/0049.0	26/0025.0	24/0004.0	02/0001.0	24/0154.0	22/0001.0
55	24/0028.0	16/0001.0	16/0002.0	20/0004.0	22/0003.0	22/0005.0
61	22/0016.0	26/0006.0	22/0017.0	24/0021.0	26/0065.0	24/0003.0
67	24/0022.0	20/0005.0	24/0022.0	22/0017.0	24/0033.0	22/0001.0
73	24/0014.0	22/0002.0	24/0028.0	24/0022.0	24/0003.0	26/0008.0
79	24/0022.0	26/0008.0	26/0008.0	22/0002.0	22/0003.0	24/0032.0
85	18/0004.0	22/0002.0	20/0004.0	12/0001.0	02/0001.2	26/0008.0
91	22/0001.0	24/0014.0	22/0005.0	24/0154.0	24/0003.0	22/0003.0
97	26/0048.0	24/0004.0	20/0004.0	26/0049.0	26/0048.0	26/0025.0
103	24/0032.0	22/0002.0	12/0001.0	22/0002.0	24/0022.0	24/0033.0
109	24/0014.0	20/0005.0	20/0002.0	26/0022.0	22/0001.0	24/0003.0
115	24/0022.0	24/0021.0	24/0032.0	24/0003.0	24/0022.0	22/0016.0
121	24/0014.0	24/0014.0	24/0032.0	24/0154.0	24/0154.0	22/0002.0
127	24/0022.0	24/0021.0	20/0005.0	14/0001.1	20/0004.0	20/0001.0
133	08/0001.1	22/0002.0	08/0001.1	20/0005.0	20/0002.0	20/0005.0
139	02/0001.0	08/0001.0	20/0005.0	22/0003.0	22/0003.0	16/0001.0
145	18/0004.0	24/0003.0	20/0004.0	20/0001.0	20/0002.0	22/0002.0
151	22/0003.0	24/0022.0	24/0022.0	22/0001.0	24/0003.0	12/0001.1
157	20/0005.0	24/0014.0	24/0003.0	20/0002.0	24/0003.0	24/0075.0
163	22/0001.0	18/0004.0	20/0001.0	22/0002.0	20/0002.0	24/0004.0
169	24/0154.0	24/0028.0	20/0005.0	24/0003.0	22/0002.0	24/0028.0
175	22/0002.0	20/0001.0	24/0004.0	12/0001.0	24/0033.0	24/0033.0
181	24/0014.0	24/0014.0	18/0004.0	24/0014.0	24/0032.0	20/0005.0

187	22/0002.0	16/0002.0	20/0002.0	16/0002.0	18/0004.0	20/0001.0
193	20/0002.0	20/0002.0	08/0001.1	20/0005.0	20/0002.0	20/0001.0
199	20/0002.0	20/0005.0	18/0004.0	18/0004.0	20/0002.0	20/0005.0
205	24/0032.0	24/0021.0	22/0003.0	24/0154.0	24/0033.0	24/0014.0
211	24/0021.0	22/0005.0	20/0001.0	24/0022.0	20/0005.0	24/0021.0
217	22/0005.0	22/0003.0	20/0010.0	22/0003.0	22/0005.0	22/0003.0
223	22/0005.0	24/0022.0	24/0014.0	20/0001.0	26/0227.0	22/0003.0
229	20/0010.0	22/0005.0	24/0021.0	24/0021.0	26/0227.0	24/0013.0
235	22/0003.0	24/0022.0	24/0021.0	22/0002.0	20/0005.0	22/0003.0
241	20/0001.0	24/0033.0	20/0005.0	24/0014.0	22/0003.0	22/0017.0
247	24/0154.0	22/0003.0	22/0005.0	20/0001.0	22/0003.0	22/0003.0
253	22/0005.0	26/0254.0	22/0003.0	26/0256.0	24/0154.0	22/0017.0
259	22/0003.0	22/0016.0	22/0005.0	14/0001.0	18/0004.0	26/0048.0
265	26/0049.0	24/0075.0	24/0022.0	24/0022.0	24/0003.0	24/0003.0
271	24/0033.0	20/0005.0	24/0014.0	18/0004.0	12/0001.1	20/0002.0
277	02/0001.0	20/0001.0	20/0004.0	24/0033.0	22/0002.0	20/0001.0
283	20/0004.0	24/0014.0	22/0005.0	24/0033.0	20/0002.0	24/0033.0
289	20/0002.0	24/0014.0	24/0014.0	20/0001.0	24/0022.0	20/0005.0
295	24/0003.0	24/0033.0	22/0016.0	24/0022.0	12/0001.0	20/0005.0
301	24/0022.0	24/0022.0	26/0048.0	26/0049.0	24/0075.0	26/0025.0
307	26/0025.0	26/0048.0	24/0022.0	24/0003.0	22/0017.0	24/0022.0
313	24/0003.0	24/0022.0	18/0001.0	24/0021.0	20/0005.0	24/0022.0
319	20/0002.0	20/0004.0	20/0005.0	12/0001.1	22/0003.0	24/0021.0
325	22/0017.0	24/0022.0	22/0016.0	24/0014.0	22/0005.0	22/0016.0
331	16/0001.0	12/0001.1	16/0002.0	22/0001.0	24/0033.0	24/0014.0
337	22/0001.0	18/0004.0	20/0005.0	24/0154.0	22/0016.0	24/0014.0
343	20/0004.0	24/0014.0	24/0033.0	24/0032.0	26/0049.0	26/0049.0
349	24/0075.0	26/0049.0	24/0075.0	26/0048.0	24/0014.0	22/0017.0
355	24/0033.0	18/0004.0	18/0004.0	22/0003.0	08/0001.1	02/0001.0
361	20/0005.0	16/0001.0	14/0001.0	22/0002.0	22/0001.0	22/0003.0
367	22/0003.0	20/0010.0	22/0003.0	22/0003.0	24/0108.0	22/0003.0
373	20/0004.0	16/0002.0	20/0002.0	16/0001.0	16/0001.0	22/0003.0
379	16/0002.0	22/0003.0	08/0001.1	22/0005.0	02/0001.0	20/0002.0
385	08/0001.1	02/0001.0	20/0001.0	20/0004.0	18/0001.0	02/0001.0
391	12/0001.0	20/0005.0	18/0001.0	24/0022.0	26/0049.0	26/0049.0
397	26/0397.0	26/0048.0	26/0049.0	24/0022.0	24/0021.0	16/0002.0

403	20/0004.0	24/0002.0	20/0002.0	24/0014.0	18/0004.0	20/0002.0
409	20/0004.0	24/0013.0	20/0001.0	24/0033.0	22/0003.0	24/0154.0
415	24/0014.0	22/0017.0	24/0014.0	24/0032.0	24/0108.0	26/0049.0
421	26/0025.0	24/0108.0	24/0028.0	22/0001.0	22/0003.0	02/0001.0
427	22/0003.0	22/0005.0	18/0001.0	24/0014.0	24/0032.0	12/0001.0
433	20/0005.0	24/0108.0	16/0002.0	20/0004.0	18/0004.0	20/0004.0
439	22/0003.0	24/0075.0	20/0004.0	24/0028.0	20/0002.0	20/0002.0
445	24/0033.0	24/0014.0	24/0032.0	08/0001.0	24/0014.0	18/0001.0
451	20/0004.0	18/0004.0	18/0004.0	20/0005.0	20/0002.0	22/0003.0
457	20/0005.0	24/0032.0	20/0001.0	24/0014.0	18/0001.0	18/0004.0
463	20/0004.0	24/0032.0	24/0014.0	24/0033.0	24/0032.0	24/0014.0
469	18/0004.0	20/0005.0	24/0032.0	24/0014.0	24/0032.0	26/0048.0
475	26/0048.0	26/0025.0	24/0075.0	26/0049.0	26/0049.0	26/0397.0
481	26/0049.0	26/0049.0	26/0048.0	26/0025.0	24/0075.0	24/0108.0
487	26/0048.0	26/0048.0	26/0048.0	24/0075.0	22/0016.0	22/0003.0
493	24/0014.0	26/0008.0	22/0001.0	20/0004.0	22/0002.0	24/0032.0
499	22/0016.0	20/0004.0	20/0004.0	24/0028.0	22/0001.0	22/0002.0
505	24/0004.0	22/0017.0	12/0001.1	20/0001.0	22/0002.0	20/0002.0
511	14/0001.0	20/0004.0	22/0001.0	20/0004.0	20/0002.0	20/0005.0
517	20/0005.0	22/0005.0	20/0001.0	14/0001.0	16/0002.0	16/0001.0
523	20/0005.0	20/0002.0	22/0002.0	20/0004.0	20/0005.0	20/0005.0
529	22/0002.0	08/0001.1	22/0003.0	24/0075.0	24/0075.0	08/0001.1
535	18/0004.0	14/0001.0	08/0001.0	20/0002.0	22/0003.0	02/0001.0
541	22/0003.0	08/0001.0	16/0001.0	20/0004.0	20/0002.0	16/0001.0
547	22/0003.0	24/0013.0	20/0001.0	20/0002.0	26/0256.0	22/0002.0
553	26/0254.0	22/0002.0	20/0005.0	16/0001.0	20/0010.0	08/0001.1
559	08/0001.0	22/0016.0	20/0001.0	22/0017.0	24/0014.0	24/0014.0
565	22/0017.0	20/0005.0	24/0003.0	22/0003.0	22/0005.0	22/0003.0
571	22/0003.0	22/0005.0	22/0017.0	24/0021.0	24/0014.0	22/0017.0
577	24/0021.0	20/0005.0	24/0014.0	22/0003.0	22/0016.0	20/0001.0
583	22/0016.0	24/0032.0	22/0005.0	22/0003.0	24/0022.0	24/0032.0
589	22/0005.0	24/0022.0	20/0001.0	20/0005.0	24/0021.0	22/0003.0
595	20/0005.0	20/0005.0	20/0005.0	22/0002.0	20/0005.0	20/0001.0
601	24/0022.0	24/0003.0	22/0005.0	22/0003.0	24/0021.0	24/0021.0
607	18/0004.0	24/0014.0	26/0227.0	26/0227.0	24/0032.0	20/0005.0
613	24/0022.0	20/0010.0	20/0004.0	22/0002.0	24/0033.0	22/0003.0

619	26/0254.0	24/0014.0	26/0256.0	18/0004.0	24/0033.0	24/0014.0
625	24/0033.0	22/0003.0	18/0004.0	22/0003.0	22/0003.0	24/0033.0
631	22/0005.0	24/0033.0	22/0002.0	20/0002.0	18/0004.0	20/0010.0
637	20/0002.0	20/0005.0	20/0001.0	20/0005.0	22/0005.0	26/0256.0
643	24/0022.0	24/0013.0	24/0032.0	24/0032.0	26/0254.0	26/0227.0
649	24/0003.0	24/0032.0	24/0014.0	22/0005.0	24/0014.0	24/0014.0
655	24/0003.0	22/0002.0	24/0014.0	24/0032.0	22/0005.0	24/0022.0
661	22/0005.0	22/0005.0	20/0002.0	20/0005.0	08/0001.1	08/0001.1
667	02/0001.1	18/0004.0	18/0004.0	20/0002.0	20/0001.0	02/0001.0
673	20/0004.0	20/0001.0	18/0001.0	20/0004.0	20/0001.0	22/0002.0
679	18/0004.0	22/0003.0	20/0001.0	20/0005.0	20/0001.0	20/0005.0
685	20/0005.0	22/0003.0	20/0005.0	08/0001.1	18/0001.0	18/0004.0
691	20/0001.0	20/0001.0	22/0003.0	12/0001.0	26/0014.0	26/0696.0
697	26/0031.0	26/0011.0	26/0013.0	26/0013.0	26/0006.0	26/0065.0
703	26/0006.0	20/0002.0	24/0006.0	24/0032.0	24/0004.0	24/0003.0
709	24/0032.0	24/0028.0	24/0004.0	22/0002.0	24/0004.0	20/0005.0
715	24/0014.0	20/0004.0	20/0002.0	22/0001.0	20/0002.0	24/0032.0
721	24/0013.0	22/0005.0	18/0004.0	22/0005.0	20/0005.0	18/0004.0
727	24/0014.0	26/0256.0	26/0227.0	20/0002.0	22/0002.0	24/0022.0
733	26/0254.0	26/0227.0	16/0002.0	18/0004.0	26/0025.0	22/0002.0
739	24/0108.0	22/0002.0	24/0004.0	24/0022.0	24/0014.0	22/0001.0
745	20/0004.0	24/0003.0	24/0022.0	24/0002.0	20/0002.0	24/0028.0
751	22/0003.0	22/0003.0	26/0065.0	26/0008.0	24/0022.0	24/0021.0
757	24/0003.0	22/0001.0	24/0028.0	22/0016.0	22/0001.0	26/0008.0
763	24/0022.0	24/0003.0	24/0154.0	24/0154.0	24/0004.0	24/0004.0
769	24/0003.0	24/0022.0	22/0002.0	24/0003.0	20/0004.0	24/0028.0
775	26/0008.0	22/0005.0	26/0008.0	22/0002.0	20/0004.0	22/0001.0
781	22/0005.0	24/0032.0	22/0017.0	24/0032.0	18/0004.0	26/0048.0
787	26/0048.0	24/0003.0	20/0001.0	20/0001.0	22/0002.0	24/0014.0
793	22/0002.0	22/0002.0	24/0033.0	24/0014.0	24/0033.0	24/0014.0
799	24/0033.0	20/0001.0	20/0001.0	26/0049.0	24/0154.0	26/0049.0
805	26/0048.0	24/0033.0	22/0003.0	22/0003.0	20/0010.0	20/0005.0
811	24/0022.0	18/0004.0	22/0003.0	24/0154.0	24/0154.0	22/0002.0
817	24/0013.0	24/0022.0	24/0014.0	24/0032.0	22/0005.0	22/0005.0
823	22/0003.0	20/0005.0	24/0014.0	22/0001.0	22/0001.0	24/0014.0
829	22/0002.0	22/0001.0	24/0014.0	22/0002.0	24/0154.0	24/0154.0

835	22/0003.0	24/0032.0	24/0022.0	24/0022.0	24/0022.0	24/0021.0
841	22/0003.0	24/0013.0	22/0002.0	24/0154.0	24/0154.0	22/0003.0
847	24/0022.0	24/0021.0	20/0004.0	20/0005.0	22/0001.0	24/0021.0
853	24/0032.0	22/0005.0	22/0003.0	22/0002.0	22/0003.0	24/0003.0
859	24/0028.0	20/0004.0	22/0001.0	24/0022.0	20/0005.0	20/0002.0
865	20/0005.0	20/0005.0	22/0005.0	22/0003.0	24/0032.0	24/0022.0
871	20/0004.0	22/0002.0	22/0002.0	24/0021.0	26/0025.0	26/0048.0
877	26/0048.0	24/0108.0	26/0025.0	24/0108.0	02/0001.2	26/0022.0
883	08/0001.1	14/0001.1	20/0002.0	02/0001.2	18/0004.0	12/0001.0
889	26/0022.0	18/0001.0	22/0003.0	20/0004.0	22/0003.0	14/0001.1
895	20/0001.0	20/0001.0	20/0005.0	22/0003.0	22/0003.0	20/0001.0
901	26/0022.0	12/0001.0	22/0003.0	20/0005.0	26/0022.0	02/0001.2
907	20/0002.0	16/0002.0	20/0005.0	20/0005.0	08/0001.0	14/0001.1
913	18/0004.0	22/0003.0	14/0001.1	18/0004.0	20/0005.0	20/0005.0
919	22/0003.0	22/0002.0	20/0005.0	22/0003.0	08/0001.1	20/0002.0
925	22/0003.0	20/0005.0	22/0003.0	20/0001.0	16/0002.0	20/0005.0
931	02/0001.2	20/0005.0	18/0004.0	22/0003.0	26/0022.0	24/0022.0
937	24/0022.0	26/0227.0	26/0256.0	26/0227.0	26/0254.0	20/0002.0
943	24/0013.0	22/0002.0	22/0002.0	24/0003.0	24/0032.0	22/0001.0
949	20/0002.0	20/0004.0	24/0022.0	20/0002.0	24/0014.0	26/0025.0
955	26/0048.0	22/0001.0	20/0005.0	20/0005.0	20/0001.0	24/0022.0
961	22/0002.0	20/0005.0	18/0001.0	20/0001.0	24/0022.0	24/0021.0
967	24/0021.0	24/0003.0	20/0001.0	20/0002.0	20/0002.0	24/0032.0
973	22/0016.0	26/0254.0	26/0256.0	20/0005.0	20/0002.0	22/0017.0
979	22/0016.0	26/0227.0	26/0227.0	20/0002.0	22/0001.0	24/0013.0
985	24/0032.0	26/0254.0	26/0256.0	24/0014.0	20/0002.0	18/0004.0
991	24/0013.0	22/0005.0	24/0032.0	22/0003.0	20/0004.0	22/0002.0
997	22/0005.0	26/0048.0	20/0004.0	22/0003.0	22/0003.0	12/0001.0
1003	22/0003.0	24/0032.0	24/0014.0	24/0032.0	24/0014.0	24/0108.0
1009	18/0004.0	22/0002.0	20/0002.0	22/0002.0	20/0004.0	22/0001.0
1015	24/0032.0	24/0014.0	24/0014.0	24/0003.0	24/0022.0	24/0032.0
1021	20/0005.0	20/0001.0	20/0002.0	24/0022.0	24/0032.0	22/0005.0
1027	24/0022.0	24/0021.0	24/0014.0	20/0005.0	24/0013.0	24/0022.0
1033	22/0016.0	22/0016.0	22/0017.0	26/0254.0	26/0256.0	24/0014.0
1039	24/0003.0	26/0227.0	26/0227.0	24/0014.0	22/0017.0	22/0017.0
1045	22/0016.0	22/0016.0	26/0256.0	26/0254.0	24/0014.0	26/0256.0

1051	26/0254.0	20/0001.0	14/0001.0	18/0001.0	26/0227.0	26/0227.0
1057	24/0032.0	26/0254.0	26/0256.0	24/0014.0	20/0005.0	20/0002.0
1063	20/0004.0	18/0004.0	24/0022.0	24/0032.0	20/0001.0	24/0021.0
1069	22/0003.0	22/0005.0	24/0014.0	18/0001.0	24/0033.0	18/0004.0
1075	18/0004.0	20/0004.0	20/0004.0	20/0004.0	22/0001.0	20/0004.0
1081	24/0003.0	22/0002.0	24/0022.0	22/0002.0	24/0022.0	24/0003.0
1087	24/0003.0	24/0022.0	24/0022.0	24/0003.0	24/0021.0	24/0022.0
1093	24/0108.0	24/0108.0	26/0025.0	26/0048.0	26/0049.0	26/0048.0
1099	26/0025.0	26/0048.0	08/0001.1	20/0002.0	24/0014.0	22/0016.0
1105	08/0001.0	24/0014.0	02/0001.0	26/0048.0	20/0001.0	20/0005.0
1111	20/0002.0	18/0004.0	26/0049.0	24/0154.0	24/0033.0	22/0003.0
1117	18/0004.0	12/0001.0	12/0001.0	22/0017.0	24/0032.0	24/0014.0
1123	26/0049.0	26/0049.0	26/0048.0	24/0033.0	20/0005.0	20/0005.0
1129	24/0033.0	24/0022.0	26/0049.0	26/0049.0	24/0033.0	22/0005.0
1135	20/0002.0	24/0022.0	20/0005.0	22/0002.0	20/0005.0	22/0003.0
1141	24/0021.0	22/0002.0	24/0154.0	24/0154.0	24/0154.0	24/0021.0
1147	24/0014.0	24/0014.0	24/0033.0	20/0001.0	20/0001.0	18/0004.0
1153	24/0014.0	26/0049.0	26/0049.0	24/0032.0	24/0022.0	26/0048.0
1159	24/0033.0	20/0005.0	20/0004.0	20/0005.0	24/0021.0	24/0014.0
1165	24/0014.0	22/0017.0	22/0016.0	18/0004.0	22/0016.0	22/0017.0
1171	18/0004.0	22/0016.0	20/0001.0	22/0017.0	24/0014.0	24/0014.0
1177	24/0021.0	22/0017.0	24/0022.0	22/0016.0	24/0021.0	22/0003.0
1183	22/0003.0	22/0003.0	20/0001.0	22/0003.0	22/0003.0	22/0003.0
1189	26/0256.0	24/0022.0	26/0254.0	24/0014.0	24/0021.0	24/0022.0
1195	26/0254.0	26/0256.0	24/0021.0	18/0001.0	20/0004.0	24/0014.0
1201	20/0005.0	22/0003.0	22/0005.0	20/0005.0	22/0003.0	22/0003.0
1207	24/0033.0	24/0033.0	24/0014.0	24/0014.0	24/0014.0	22/0003.0
1213	24/0033.0	20/0005.0	24/0033.0	20/0001.0	20/0001.0	24/0022.0
1219	20/0004.0	24/0014.0	24/0014.0	24/0032.0	20/0001.0	20/0001.0
1225	24/0022.0	24/0014.0	26/0048.0	26/0049.0	26/0049.0	26/0049.0
1231	26/0049.0	26/0049.0	26/0049.0	26/0048.0	22/0001.0	24/0154.0
1237	22/0002.0	24/0154.0	14/0001.0	18/0001.0	24/0022.0	20/0005.0
1243	24/0021.0	26/0256.0	26/0254.0	18/0004.0	22/0003.0	24/0014.0
1249	20/0005.0	20/0001.0	22/0005.0	26/0049.0		

8.2.2 3-Gems with 28 Vertices

01	28/0001.0	28/0002.0	28/0003.0	26/0014.0	28/0005.0	28/0006.0
07	28/0007.0	24/0004.0	28/0009.0	28/0010.0	26/0696.0	28/0009.0
13	28/0013.0	28/0014.0	24/0014.0	26/0013.0	26/0005.0	26/0696.0
19	28/0019.0	26/0008.0	28/0010.0	22/0002.0	26/0013.0	28/0006.0
25	28/0025.0	26/0065.0	28/0027.0	26/0006.0	28/0029.0	12/0001.1
31	28/0031.0	26/0696.0	28/0033.0	28/0034.0	28/0025.0	28/0034.0
37	26/0014.0	26/0013.0	26/0014.0	12/0001.0	28/0041.0	28/0042.0
43	24/0006.0	28/0013.0	28/0042.0	28/0025.0	24/0006.0	28/0029.0
49	28/0049.0	20/0005.0	26/0006.0	22/0002.0	28/0013.0	28/0054.0
55	26/0008.0	28/0056.0	28/0029.0	24/0004.0	28/0059.0	28/0060.0
61	26/0008.0	28/0041.0	26/0008.0	28/0054.0	28/0065.0	28/0027.0
67	26/0696.0	28/0031.0	22/0001.0	28/0070.0	28/0071.0	22/0001.0
73	28/0070.0	28/0070.0	22/0002.0	24/0006.0	24/0001.0	28/0029.0
79	28/0025.0	28/0027.0	28/0081.0	22/0001.0	28/0054.0	26/0008.0
85	28/0054.0	28/0041.0	28/0065.0	26/0696.0	26/0011.0	28/0054.0
91	28/0031.0	18/0004.0	28/0033.0	28/0054.0	26/0031.0	26/0031.0
97	26/0011.0	26/0013.0	26/0013.0	22/0001.0	28/0070.0	26/0008.0
103	28/0054.0	18/0004.0	26/0048.0	28/0106.0	28/0027.0	28/0033.0
109	28/0065.0	28/0033.0	28/0106.0	28/0065.0	28/0113.0	28/0070.0
115	28/0070.0	28/0041.0	28/0014.0	28/0070.0	28/0054.0	28/0054.0
121	28/0013.0	24/0006.0	24/0007.0	24/0001.0	24/0007.0	20/0002.0
127	24/0028.0	26/0049.0	28/0071.0	26/0696.0	28/0014.0	26/0048.0
133	28/0041.0	28/0065.0	26/0048.0	28/0049.0	24/0002.0	28/0029.0
139	28/0113.0	28/0027.0	28/0059.0	28/0113.0	26/0010.0	26/0013.0
145	26/0006.0	18/0004.0	22/0003.0	24/0154.0	02/0001.0	24/0004.0
151	22/0003.0	22/0005.0	20/0004.0	22/0001.0	26/0025.0	08/0001.0
157	24/0002.0	20/0001.0	24/0021.0	24/0032.0	26/0031.0	26/0065.0
163	28/0070.0	28/0010.0	28/0029.0	28/0025.0	28/0027.0	28/0034.0
169	28/0071.0	28/0033.0	28/0005.0	28/0172.0	26/0014.0	26/0031.0
175	24/0013.0	26/0696.0	28/0177.0	22/0002.0	26/0008.0	26/0003.0
181	18/0004.0	26/0006.0	26/0065.0	26/0006.0	24/0013.0	24/0032.0
187	24/0003.0	24/0004.0	24/0022.0	24/0032.0	24/0032.0	26/0048.0
193	26/0065.0	26/0696.0	26/0696.0	28/0071.0	28/0033.0	28/0010.0
199	26/0010.0	26/0696.0	20/0004.0	28/0202.0	28/0203.0	24/0007.0

205	24/0014.0	28/0010.0	24/0033.0	26/0013.0	26/0010.0	26/0005.0
211	24/0022.0	12/0001.0	24/0002.0	26/0010.0	24/0028.0	20/0004.0
217	18/0001.0	02/0001.0	26/0010.0	22/0001.0	28/0081.0	26/0227.0
223	26/0256.0	26/0227.0	28/0014.0	28/0049.0	24/0014.0	28/0071.0
229	28/0033.0	28/0230.0	24/0004.0	28/0172.0	22/0001.0	24/0033.0
235	12/0001.1	28/0033.0	28/0033.0	24/0014.0	20/0004.0	26/0254.0
241	28/0106.0	28/0081.0	24/0003.0	16/0001.0	22/0002.0	18/0004.0
247	22/0002.0	26/0008.0	22/0002.0	22/0016.0	24/0021.0	22/0001.0
253	22/0001.0	24/0003.0	16/0002.0	26/0696.0	26/0006.0	24/0021.0
259	18/0004.0	18/0004.0	26/0025.0	26/0008.0	26/0013.0	26/0011.0
265	26/0011.0	26/0014.0	26/0065.0	24/0022.0	26/0013.0	24/0028.0
271	22/0001.0	26/0010.0	24/0033.0	28/0177.0	20/0005.0	20/0001.0
277	24/0032.0	02/0001.0	24/0014.0	28/0280.0	22/0003.0	26/0010.0
283	22/0001.0	28/0034.0	24/0022.0	26/0008.0	28/0081.0	28/0071.0
289	24/0004.0	22/0001.0	24/0154.0	24/0022.0	24/0004.0	24/0154.0
295	24/0002.0	28/0033.0	28/0071.0	26/0011.0	26/0696.0	26/0031.0
301	26/0006.0	26/0065.0	24/0006.0	26/0696.0	26/0003.0	20/0004.0
307	26/0031.0	26/0025.0	28/0042.0	28/0005.0	28/0280.0	26/0008.0
313	26/0696.0	24/0032.0	20/0004.0	20/0002.0	28/0027.0	28/0029.0
319	24/0021.0	24/0022.0	28/0106.0	20/0002.0	28/0071.0	28/0033.0
325	24/0014.0	26/0008.0	26/0008.0	24/0013.0	24/0022.0	08/0001.0
331	08/0001.0	22/0001.0	26/0008.0	26/0008.0	26/0049.0	24/0028.0
337	18/0004.0	20/0002.0	24/0004.0	22/0016.0	22/0017.0	24/0014.0
343	18/0004.0	22/0017.0	24/0014.0	12/0001.0	24/0022.0	26/0008.0
349	24/0032.0	22/0002.0	24/0013.0	24/0002.0	24/0014.0	24/0004.0
355	20/0004.0	26/0008.0	22/0001.0	22/0003.0	24/0014.0	26/0005.0
361	20/0001.0	26/0025.0	26/0006.0	18/0001.0	24/0002.0	24/0003.0
367	24/0021.0	20/0002.0	22/0002.0	24/0028.0	18/0004.0	02/0001.0
373	24/0003.0	20/0002.0	24/0014.0	24/0014.0	24/0014.0	24/0022.0
379	24/0007.0	22/0005.0	26/0008.0	22/0002.0	26/0031.0	26/0008.0
385	24/0032.0	24/0021.0	26/0011.0	26/0013.0	24/0007.0	26/0010.0
391	24/0014.0	24/0014.0	22/0016.0	26/0008.0	26/0010.0	26/0010.0
397	24/0028.0	26/0397.0	28/0172.0	28/0042.0	28/0056.0	28/0402.0
403	26/0005.0	22/0002.0	26/0013.0	26/0696.0	26/0008.0	24/0002.0
409	24/0007.0	26/0011.0	20/0004.0	24/0014.0	26/0003.0	26/0696.0
415	22/0002.0	20/0004.0	12/0001.0	24/0075.0	24/0007.0	26/0010.0

421	26/0696.0	26/0065.0	22/0001.0	22/0001.0	20/0004.0	24/0022.0
427	20/0005.0	22/0005.0	20/0004.0	24/0022.0	22/0017.0	24/0033.0
433	26/0696.0	24/0022.0	24/0021.0	26/0008.0	26/0013.0	22/0005.0
439	28/0177.0	26/0014.0	26/0013.0	24/0006.0	24/0007.0	26/0011.0
445	24/0013.0	26/0011.0	28/0177.0	24/0028.0	26/0031.0	18/0004.0
451	26/0011.0	26/0010.0	28/0453.0	18/0004.0	24/0021.0	26/0048.0
457	14/0001.0	24/0014.0	24/0022.0	24/0003.0	24/0022.0	24/0003.0
463	22/0002.0	24/0022.0	26/0049.0	24/0028.0	24/0154.0	24/0022.0
469	24/0022.0	24/0033.0	20/0002.0	24/0003.0	28/0473.0	22/0003.0
475	20/0005.0	24/0014.0	24/0033.0	12/0001.1	22/0017.0	20/0002.0
481	22/0017.0	22/0017.0	24/0014.0	12/0001.1	28/0485.0	24/0028.0
487	24/0033.0	24/0022.0	20/0005.0	16/0001.1	24/0003.0	22/0002.0
493	20/0001.0	24/0028.0	26/0008.0	24/0033.0	08/0001.0	18/0004.0
499	24/0022.0	22/0001.0	24/0021.0	26/0008.0	24/0021.0	24/0022.0
505	24/0033.0	22/0003.0	24/0022.0	20/0001.0	20/0005.0	24/0033.0
511	20/0002.0	22/0005.0	08/0001.1	28/0514.0	20/0010.0	28/0514.0
517	28/0517.0	22/0005.0	24/0032.0	24/0022.0	22/0005.0	24/0014.0
523	24/0014.0	24/0032.0	26/0008.0	20/0004.0	24/0022.0	22/0003.0
529	24/0108.0	22/0003.0	24/0003.0	24/0022.0	26/0006.0	20/0005.0
535	22/0005.0	24/0022.0	24/0004.0	12/0001.0	22/0001.0	26/0065.0
541	02/0001.1	22/0017.0	24/0022.0	24/0003.0	24/0022.0	24/0033.0
547	24/0014.0	24/0022.0	22/0016.0	24/0003.0	24/0004.0	26/0006.0
553	26/0008.0	24/0032.0	26/0006.0	20/0002.0	08/0001.0	18/0004.0
559	24/0014.0	24/0022.0	22/0016.0	24/0014.0	18/0004.0	20/0004.0
565	20/0005.0	24/0014.0	24/0028.0	20/0002.0	24/0004.0	20/0004.0
571	24/0154.0	24/0021.0	24/0003.0	20/0004.0	20/0002.0	20/0001.0
577	24/0032.0	08/0001.0	28/0473.0	24/0003.0	24/0014.0	24/0014.0
583	24/0003.0	24/0033.0	18/0001.0	24/0021.0	22/0003.0	20/0005.0
589	24/0033.0	24/0032.0	12/0001.0	24/0075.0	22/0003.0	24/0028.0
595	22/0001.0	22/0001.0	18/0004.0	24/0033.0	20/0005.0	24/0028.0
601	18/0004.0	24/0022.0	28/0485.0	24/0022.0	24/0022.0	24/0022.0
607	24/0022.0	24/0004.0	08/0001.1	12/0001.0	28/0070.0	22/0016.0
613	24/0022.0	28/0060.0	28/0054.0	22/0002.0	24/0014.0	26/0008.0
619	28/0071.0	28/0033.0	28/0071.0	26/0025.0	28/0033.0	28/0065.0
625	28/0230.0	02/0001.0	24/0022.0	28/0081.0	28/0106.0	22/0016.0
631	28/0014.0	28/0049.0	24/0022.0	28/0042.0	26/0025.0	28/0042.0

637	28/0280.0	24/0007.0	26/0008.0	24/0007.0	22/0001.0	02/0001.0
643	24/0014.0	26/0008.0	22/0001.0	20/0005.0	22/0001.0	26/0008.0
649	24/0032.0	22/0016.0	20/0005.0	24/0154.0	26/0013.0	26/0008.0
655	26/0008.0	22/0017.0	24/0021.0	24/0154.0	26/0006.0	26/0008.0
661	26/0065.0	24/0033.0	22/0005.0	24/0032.0	24/0108.0	24/0154.0
667	24/0014.0	26/0048.0	24/0032.0	22/0003.0	24/0075.0	24/0154.0
673	26/0049.0	28/0049.0	28/0081.0	24/0021.0	22/0003.0	26/0049.0
679	24/0022.0	22/0003.0	24/0003.0	24/0108.0	26/0256.0	24/0032.0
685	28/0054.0	22/0002.0	24/0014.0	24/0022.0	24/0003.0	28/0054.0
691	28/0031.0	26/0254.0	28/0230.0	28/0071.0	26/0025.0	26/0049.0
697	26/0696.0	24/0032.0	24/0021.0	26/0008.0	24/0003.0	26/0065.0
703	26/0008.0	24/0003.0	24/0032.0	24/0013.0	24/0021.0	24/0108.0
709	24/0028.0	24/0002.0	24/0021.0	28/0033.0	26/0025.0	26/0049.0
715	26/0049.0	26/0048.0	20/0010.0	28/0230.0	24/0022.0	22/0003.0
721	20/0004.0	26/0254.0	22/0001.0	26/0048.0	24/0028.0	28/0230.0
727	24/0075.0	24/0154.0	24/0004.0	24/0154.0	24/0003.0	24/0003.0
733	24/0022.0	26/0256.0	28/0049.0	24/0154.0	22/0003.0	24/0154.0
739	22/0003.0	22/0003.0	24/0014.0	24/0021.0	22/0003.0	22/0005.0
745	24/0014.0	24/0032.0	20/0004.0	22/0002.0	24/0013.0	20/0004.0
751	18/0004.0	28/0485.0	22/0003.0	28/0473.0	26/0049.0	22/0001.0
757	28/0485.0	28/0473.0	24/0033.0	28/0485.0	18/0004.0	24/0022.0
763	24/0014.0	24/0022.0	26/0008.0	24/0022.0	08/0001.1	02/0001.1
769	24/0033.0	24/0032.0	24/0033.0	20/0004.0	22/0001.0	22/0003.0
775	22/0003.0	20/0004.0	24/0022.0	22/0001.0	24/0033.0	20/0005.0
781	20/0005.0	22/0001.0	20/0004.0	26/0025.0	26/0397.0	20/0001.0
787	20/0004.0	20/0005.0	20/0005.0	26/0048.0	26/0025.0	24/0075.0
793	24/0075.0	22/0001.0	22/0005.0	08/0001.0	16/0002.0	02/0001.0
799	22/0003.0	20/0005.0	16/0002.0	18/0004.0	26/0049.0	24/0075.0
805	20/0002.0	18/0004.0	24/0021.0	26/0025.0	24/0022.0	22/0002.0
811	20/0002.0	26/0048.0	08/0001.0	20/0005.0	26/0049.0	24/0033.0
817	20/0005.0	02/0001.0	26/0048.0	24/0108.0	26/0048.0	26/0048.0
823	26/0048.0	26/0025.0	26/0025.0	24/0014.0	24/0033.0	20/0001.0
829	24/0028.0	22/0001.0	26/0008.0	28/0041.0	28/0029.0	28/0071.0
835	24/0022.0	24/0021.0	24/0108.0	28/0065.0	24/0022.0	24/0013.0
841	28/0027.0	24/0022.0	28/0033.0	24/0021.0	26/0005.0	28/0106.0
847	24/0032.0	24/0154.0	22/0001.0	28/0070.0	22/0003.0	26/0025.0

853	28/0070.0	24/0022.0	26/0008.0	26/0008.0	26/0008.0	22/0001.0
859	22/0002.0	22/0002.0	26/0008.0	28/0054.0	28/0054.0	26/0696.0
865	24/0003.0	28/0010.0	28/0071.0	28/0033.0	24/0154.0	28/0081.0
871	28/0106.0	24/0004.0	26/0013.0	22/0005.0	20/0004.0	22/0003.0
877	28/0005.0	26/0014.0	24/0028.0	26/0048.0	26/0025.0	24/0004.0
883	22/0001.0	26/0048.0	24/0022.0	26/0049.0	26/0049.0	22/0002.0
889	26/0065.0	24/0003.0	24/0075.0	22/0001.0	24/0003.0	22/0002.0
895	26/0048.0	26/0048.0	24/0003.0	24/0075.0	24/0075.0	26/0025.0
901	26/0048.0	28/0071.0	28/0033.0	24/0154.0	26/0025.0	26/0048.0
907	22/0005.0	28/0172.0	24/0002.0	24/0154.0	22/0017.0	12/0001.1
913	24/0013.0	24/0033.0	22/0002.0	20/0004.0	20/0005.0	20/0002.0
919	24/0004.0	08/0001.0	24/0032.0	24/0013.0	22/0005.0	24/0021.0
925	24/0003.0	26/0397.0	20/0002.0	24/0154.0	24/0014.0	24/0033.0
931	24/0028.0	24/0014.0	24/0032.0	24/0033.0	22/0001.0	24/0028.0
937	24/0002.0	20/0002.0	20/0010.0	02/0001.1	24/0033.0	24/0013.0
943	24/0002.0	24/0028.0	24/0013.0	18/0001.0	12/0001.0	18/0004.0
949	22/0005.0	18/0001.0	24/0003.0	02/0001.1	24/0014.0	26/0065.0
955	24/0032.0	24/0003.0	22/0017.0	24/0021.0	26/0065.0	22/0017.0
961	24/0032.0	24/0003.0	26/0006.0	24/0013.0	24/0003.0	22/0002.0
967	26/0008.0	22/0002.0	24/0007.0	26/0013.0	20/0005.0	24/0013.0
973	22/0001.0	26/0008.0	24/0032.0	24/0003.0	24/0022.0	22/0002.0
979	22/0005.0	22/0002.0	24/0013.0	26/0696.0	24/0003.0	24/0075.0
985	20/0005.0	20/0004.0	22/0002.0	20/0002.0	22/0005.0	24/0006.0
991	24/0014.0	26/0013.0	26/0008.0	20/0002.0	24/0003.0	12/0001.1
997	22/0001.0	22/0002.0	24/0004.0	24/0033.0	24/0013.0	02/0001.0
1003	20/0001.0	24/0022.0	24/0032.0	20/0004.0	20/0005.0	24/0007.0
1009	26/0011.0	26/0031.0	26/0010.0	24/0013.0	22/0002.0	20/0004.0
1015	24/0006.0	20/0004.0	22/0017.0	18/0004.0	22/0002.0	22/0002.0
1021	24/0075.0	26/0397.0	28/0177.0	18/0001.0	24/0007.0	26/0011.0
1027	26/0013.0	24/0007.0	26/0013.0	20/0005.0	24/0154.0	24/0004.0
1033	26/0010.0	26/0011.0	24/0001.0	26/0014.0	26/0031.0	26/0014.0
1039	24/0014.0	24/0004.0	24/0154.0	20/0002.0	20/0001.0	12/0001.1
1045	12/0001.0	08/0001.0	20/0004.0	24/0014.0	24/0022.0	20/0002.0
1051	26/0008.0	24/0003.0	22/0002.0	20/0004.0	26/0008.0	24/0022.0
1057	24/0021.0	24/0022.0	24/0154.0	24/0032.0	22/0001.0	22/0002.0
1063	24/0014.0	22/0005.0	22/0003.0	24/0032.0	22/0003.0	24/0154.0

1069	24/0021.0	24/0022.0	26/0397.0	26/0397.0	26/0025.0	20/0004.0
1075	14/0001.0	20/0004.0	24/0032.0	24/0013.0	24/0033.0	24/0032.0
1081	24/0021.0	24/0014.0	24/0022.0	12/0001.0	24/0033.0	26/0008.0
1087	24/0033.0	22/0002.0	20/0005.0	20/0004.0	16/0002.0	20/0004.0
1093	20/0004.0	24/0154.0	20/0004.0	22/0005.0	20/0004.0	26/0048.0
1099	26/0397.0	24/0004.0	24/0028.0	12/0001.0	20/0004.0	24/0022.0
1105	24/0003.0	02/0001.1	24/0022.0	22/0002.0	20/0004.0	24/0007.0
1111	26/0011.0	26/0008.0	24/0022.0	24/0003.0	26/0696.0	20/0002.0
1117	24/0033.0	22/0005.0	24/0014.0	24/0014.0	22/0002.0	26/0065.0
1123	26/0397.0	26/0049.0	28/0177.0	26/0397.0	28/0453.0	24/0007.0
1129	24/0004.0	02/0001.0	08/0001.0	22/0001.0	24/0028.0	26/0014.0
1135	26/0031.0	26/0013.0	26/0011.0	20/0005.0	12/0001.1	22/0001.0
1141	26/0065.0	20/0004.0	22/0002.0	24/0022.0	20/0002.0	24/0021.0
1147	20/0002.0	18/0004.0	24/0028.0	22/0001.0	24/0013.0	24/0022.0
1153	24/0003.0	24/0021.0	02/0001.1	18/0004.0	26/0008.0	26/0696.0
1159	26/0008.0	22/0001.0	22/0001.0	24/0014.0	24/0033.0	26/0696.0
1165	24/0022.0	26/0008.0	24/0032.0	22/0001.0	24/0003.0	08/0001.0
1171	24/0014.0	24/0033.0	24/0022.0	24/0022.0	20/0005.0	24/0022.0
1177	22/0016.0	18/0004.0	22/0002.0	26/0696.0	24/0003.0	26/0008.0
1183	22/0002.0	24/0003.0	26/0003.0	26/0025.0	28/0453.0	28/0177.0
1189	24/0075.0	28/0177.0	24/0032.0	24/0022.0	20/0004.0	24/0028.0
1195	26/0010.0	26/0008.0	22/0001.0	24/0032.0	24/0028.0	20/0002.0
1201	26/0031.0	20/0002.0	26/0008.0	24/0003.0	12/0001.1	22/0002.0
1207	24/0014.0	24/0014.0	24/0033.0	22/0002.0	24/0021.0	16/0002.0
1213	24/0022.0	20/0005.0	20/0002.0	18/0001.0	24/0022.0	24/0021.0
1219	24/0021.0	26/0013.0	24/0006.0	22/0005.0	24/0021.0	20/0010.0
1225	24/0022.0	20/0004.0	24/0022.0	20/0010.0	24/0022.0	24/0022.0
1231	26/0003.0	28/0453.0	28/0177.0	26/0025.0	28/0177.0	26/0025.0
1237	24/0032.0	24/0013.0	18/0004.0	26/0696.0	24/0032.0	20/0002.0
1243	26/0013.0	24/0014.0	26/0008.0	16/0002.0	16/0001.0	18/0004.0
1249	20/0002.0	24/0004.0	24/0028.0	22/0003.0	24/0022.0	18/0001.0
1255	26/0005.0	24/0004.0	20/0002.0	22/0005.0	24/0022.0	20/0002.0
1261	24/0022.0	26/0008.0	26/0696.0	24/0003.0	24/0014.0	28/0054.0
1267	22/0003.0	24/0108.0	26/0048.0	26/0008.0	26/0003.0	22/0005.0
1273	24/0022.0	22/0005.0	22/0002.0	18/0004.0	28/0070.0	22/0002.0
1279	24/0154.0	24/0028.0	24/0021.0	24/0014.0	24/0003.0	28/0054.0

1285	24/0075.0	24/0108.0	24/0021.0	24/0075.0	12/0001.1	24/0004.0
1291	24/0022.0	24/0021.0	24/0022.0	28/0106.0	24/0022.0	24/0021.0
1297	24/0022.0	22/0005.0	28/0106.0	28/0081.0	26/0025.0	26/0049.0
1303	26/0048.0	26/0008.0	24/0154.0	12/0001.0	24/0021.0	24/0028.0
1309	26/0049.0	28/0071.0	22/0003.0	26/0049.0	28/0230.0	22/0016.0
1315	22/0016.0	28/0071.0	28/0033.0	24/0014.0	24/0033.0	24/0154.0
1321	28/0033.0	24/0003.0	24/0003.0	24/0022.0	28/0025.0	24/0004.0
1327	20/0002.0	24/0022.0	26/0048.0	28/0081.0	28/0049.0	28/0106.0
1333	22/0005.0	24/0032.0	24/0003.0	26/0048.0	28/0014.0	24/0108.0
1339	24/0022.0	26/0008.0	24/0154.0	24/0108.0	26/0254.0	26/0256.0
1345	28/0041.0	24/0075.0	26/0048.0	24/0003.0	24/0154.0	22/0002.0
1351	26/0008.0	26/0010.0	22/0001.0	28/0081.0	28/0081.0	24/0004.0
1357	24/0004.0	24/0006.0	28/0106.0	28/0031.0	26/0003.0	26/0696.0
1363	26/0065.0	22/0001.0	26/0048.0	24/0022.0	28/0029.0	24/0075.0
1369	28/0027.0	24/0075.0	26/0049.0	28/0230.0	28/0059.0	26/0011.0
1375	26/0031.0	24/0154.0	26/0006.0	22/0001.0	24/0075.0	24/0075.0
1381	20/0002.0	26/0006.0	24/0022.0	24/0014.0	24/0003.0	24/0014.0
1387	24/0022.0	22/0002.0	26/0065.0	24/0014.0	24/0013.0	22/0002.0
1393	22/0002.0	20/0004.0	24/0033.0	20/0010.0	24/0014.0	20/0005.0
1399	08/0001.0	22/0016.0	24/0028.0	24/0033.0	24/0004.0	22/0001.0
1405	24/0004.0	22/0003.0	24/0003.0	24/0022.0	24/0033.0	24/0021.0
1411	20/0002.0	24/0022.0	22/0003.0	02/0001.0	22/0003.0	24/0022.0
1417	26/0008.0	24/0033.0	24/0014.0	24/0014.0	16/0002.0	24/0021.0
1423	24/0108.0	20/0001.0	24/0022.0	26/0006.0	24/0004.0	24/0108.0
1429	24/0108.0	24/0154.0	24/0028.0	24/0028.0	24/0108.0	22/0005.0
1435	24/0108.0	24/0108.0	24/0003.0	24/0022.0	20/0005.0	20/0002.0
1441	22/0002.0	22/0001.0	22/0002.0	22/0003.0	20/0010.0	24/0003.0
1447	24/0003.0	22/0003.0	24/0021.0	20/0004.0	22/0001.0	24/0022.0
1453	24/0014.0	26/0065.0	24/0032.0	24/0032.0	24/0032.0	22/0002.0
1459	26/0006.0	22/0016.0	24/0022.0	22/0001.0	24/0014.0	24/0004.0
1465	24/0014.0	24/0022.0	20/0004.0	20/0001.0	24/0032.0	24/0014.0
1471	20/0001.0	22/0017.0	28/1473.0	28/1474.0	24/0032.0	22/0016.0
1477	24/0154.0	24/0033.0	24/0003.0	24/0022.0	20/0004.0	24/0004.0
1483	22/0017.0	22/0016.0	24/0022.0	22/0017.0	22/0016.0	24/0032.0
1489	18/0004.0	24/0021.0	28/1474.0	26/0006.0	20/0005.0	28/1473.0
1495	24/0004.0	26/0065.0	24/0014.0	20/0005.0	20/0004.0	08/0001.1

1501	24/0032.0	18/0004.0	20/0005.0	02/0001.0	20/0005.0	24/0033.0
1507	20/0004.0	24/0004.0	24/0032.0	18/0004.0	20/0002.0	20/0001.0
1513	20/0005.0	24/0033.0	24/0033.0	24/0033.0	24/0004.0	24/0014.0
1519	22/0002.0	24/0004.0	24/0154.0	24/0028.0	24/0033.0	20/0002.0
1525	24/0154.0	20/0004.0	12/0001.1	24/0032.0	24/0032.0	26/0025.0
1531	26/0397.0	08/0001.1	18/0004.0	18/0004.0	20/0005.0	22/0005.0
1537	24/0014.0	24/0013.0	24/0014.0	20/0001.0	22/0002.0	24/0014.0
1543	26/0048.0	18/0004.0	24/0075.0	24/0032.0	24/0014.0	24/0014.0
1549	20/0005.0	22/0002.0	24/0033.0	24/0014.0	24/0033.0	20/0001.0
1555	18/0004.0	28/0485.0	28/0473.0	24/0014.0	22/0002.0	08/0001.0
1561	24/0021.0	18/0001.0	24/0022.0	24/0021.0	24/0022.0	24/0003.0
1567	24/0022.0	20/0004.0	20/0005.0	20/0005.0	20/0004.0	22/0002.0
1573	24/0022.0	22/0002.0	26/0008.0	26/0010.0	24/0014.0	20/0002.0
1579	26/0025.0	22/0001.0	24/0028.0	24/0028.0	24/0002.0	24/0033.0
1585	26/0065.0	22/0017.0	26/0048.0	24/0004.0	20/0005.0	22/0001.0
1591	24/0075.0	24/0075.0	26/0049.0	24/0014.0	26/0008.0	24/0014.0
1597	24/0033.0	24/0028.0	20/0002.0	24/0033.0	24/0154.0	22/0017.0
1603	24/0013.0	24/0032.0	22/0001.0	24/0004.0	22/0002.0	24/0032.0
1609	24/0004.0	28/0071.0	26/0008.0	28/0071.0	28/0041.0	26/0008.0
1615	28/0081.0	22/0005.0	24/0154.0	24/0154.0	28/0033.0	24/0014.0
1621	22/0003.0	22/0003.0	28/0033.0	28/0065.0	24/0013.0	22/0005.0
1627	26/0008.0	24/0022.0	22/0005.0	28/0106.0	24/0022.0	26/0048.0
1633	26/0025.0	20/0010.0	24/0004.0	24/0108.0	26/0049.0	28/0049.0
1639	28/0230.0	26/0049.0	28/0230.0	26/0048.0	28/0014.0	28/0049.0
1645	26/0048.0	24/0032.0	28/0049.0	28/0049.0	22/0005.0	18/0004.0
1651	22/0016.0	22/0002.0	20/0004.0	24/0007.0	26/0011.0	22/0001.0
1657	24/0154.0	22/0002.0	20/0002.0	24/0075.0	24/0003.0	24/0075.0
1663	24/0022.0	26/0065.0	26/0006.0	24/0021.0	26/0065.0	24/0021.0
1669	20/0005.0	22/0001.0	22/0002.0	28/0054.0	28/0031.0	28/0070.0
1675	28/0054.0	24/0003.0	24/0003.0	26/0048.0	24/0003.0	24/0003.0
1681	28/0049.0	28/0071.0	28/0033.0	24/0154.0	24/0021.0	24/0014.0
1686	24/0003.0	26/0008.0	24/0032.0	26/0065.0	22/0001.0	24/0028.0
1693	26/0011.0	24/0007.0	24/0028.0	26/0031.0	26/0011.0	24/0075.0
1699	28/0054.0	20/0004.0	28/0070.0	24/0003.0	24/0075.0	20/0002.0
1705	24/0028.0	24/0028.0	24/0022.0	20/0005.0	24/0014.0	26/0008.0
1711	24/0004.0	24/0014.0	24/0033.0	26/0025.0	26/0048.0	24/0154.0

1717	24/0003.0	22/0017.0	26/0065.0	24/0022.0	24/0021.0	24/0022.0
1723	26/0006.0	24/0022.0	24/0022.0	24/0022.0	20/0005.0	02/0001.0
1729	18/0004.0	20/0002.0	24/0013.0	28/0014.0	28/0013.0	26/0006.0
1735	26/0006.0	26/0013.0	28/0054.0	26/0065.0	24/0014.0	24/0004.0
1741	28/0081.0	28/0106.0	20/0002.0	24/0004.0	24/0022.0	24/0022.0
1747	22/0005.0	24/0013.0	22/0002.0	26/0256.0	26/0254.0	26/0008.0
1753	22/0001.0	20/0004.0	22/0001.0	22/0002.0	12/0001.0	24/0033.0
1759	24/0028.0	22/0002.0	24/0014.0	24/0033.0	26/0008.0	24/0022.0
1765	24/0013.0	20/0001.0	22/0002.0	20/0002.0	24/0021.0	18/0004.0
1771	02/0001.0	24/0022.0	24/0003.0	20/0004.0	22/0003.0	22/0005.0
1777	20/0002.0	20/0001.0	12/0001.1	24/0021.0	16/0001.0	24/0032.0
1783	20/0005.0	22/0003.0	24/0032.0	24/0021.0	24/0032.0	20/0005.0
1789	24/0013.0	20/0005.0	22/0005.0	08/0001.0	24/0022.0	26/0008.0
1795	24/0003.0	22/0002.0	24/0022.0	24/0022.0	26/0025.0	24/0154.0
1801	20/0002.0	08/0001.0	20/0010.0	24/0022.0	24/0033.0	24/0022.0
1807	26/0025.0	24/0003.0	16/0002.0	20/0001.0	22/0002.0	22/0002.0
1813	20/0005.0	22/0002.0	26/0003.0	26/0008.0	20/0002.0	20/0002.0
1819	22/0002.0	20/0005.0	20/0002.0	20/0004.0	22/0002.0	24/0014.0
1825	22/0002.0	20/0001.0	20/0001.0	24/0021.0	24/0033.0	24/0014.0
1831	24/0033.0	24/0003.0	24/0033.0	20/0002.0	20/0002.0	24/0108.0
1837	24/0004.0	24/0154.0	26/0008.0	24/0108.0	24/0154.0	22/0001.0
1843	02/0001.0	08/0001.0	20/0005.0	24/0003.0	24/0021.0	24/0022.0
1849	26/0397.0	24/0004.0	24/0154.0	22/0002.0	24/0022.0	26/0048.0
1855	24/0022.0	20/0005.0	20/0005.0	24/0032.0	24/0021.0	18/0004.0
1861	20/0002.0	24/0028.0	22/0001.0	20/0001.0	20/0002.0	24/0004.0
1867	24/0014.0	20/0002.0	24/0028.0	02/0001.0	24/0032.0	22/0002.0
1873	18/0004.0	22/0001.0	12/0001.0	22/0001.0	26/0696.0	26/0008.0
1879	26/0003.0	22/0002.0	20/0004.0	18/0004.0	20/0004.0	22/0002.0
1885	24/0033.0	22/0001.0	24/0028.0	24/0033.0	24/0033.0	24/0022.0
1891	22/0005.0	24/0003.0	22/0002.0	22/0003.0	24/0022.0	24/0032.0
1897	08/0001.1	24/0003.0	18/0004.0	24/0022.0	26/0003.0	24/0003.0
1903	24/0021.0	24/0022.0	26/0008.0	22/0002.0	24/0032.0	22/0002.0
1909	20/0004.0	24/0022.0	26/0006.0	24/0003.0	26/0065.0	22/0002.0
1915	20/0004.0	24/0033.0	24/0033.0	20/0002.0	16/0002.0	18/0004.0
1921	24/0154.0	20/0004.0	24/0002.0	20/0004.0	24/0028.0	26/0010.0
1927	24/0014.0	26/0696.0	24/0033.0	26/0010.0	26/0008.0	24/0014.0

1933	24/0028.0	24/0002.0	24/0003.0	22/0002.0	22/0003.0	20/0002.0
1939	22/0002.0	24/0003.0	26/0696.0	24/0003.0	24/0003.0	22/0005.0
1945	24/0033.0	24/0007.0	26/0013.0	24/0014.0	24/0022.0	20/0004.0
1951	26/0011.0	24/0033.0	26/0008.0	24/0004.0	24/0108.0	26/0065.0
1957	26/0065.0	26/0006.0	22/0002.0	26/0008.0	20/0002.0	22/0002.0
1963	24/0003.0	26/0065.0	24/0022.0	26/0006.0	24/0033.0	24/0032.0
1969	22/0016.0	24/0022.0	24/0004.0	20/0005.0	20/0002.0	22/0002.0
1975	22/0002.0	20/0004.0	22/0002.0	20/0001.0	16/0002.0	20/0001.0
1981	20/0005.0	22/0002.0	18/0004.0	22/0002.0	24/0003.0	02/0001.0
1987	20/0001.0	20/0002.0	24/0014.0	18/0004.0	08/0001.0	22/0001.0
1993	24/0002.0	18/0004.0	20/0004.0	24/0154.0	24/0033.0	24/0013.0
1999	22/0017.0	22/0017.0	20/0001.0	22/0005.0	22/0005.0	20/0005.0
2005	22/0003.0	24/0033.0	20/0005.0	24/0014.0	22/0002.0	24/0032.0
2011	22/0002.0	22/0003.0	20/0004.0	14/0001.0	24/0032.0	18/0001.0
2017	18/0004.0	02/0001.0	20/0004.0	24/0022.0	24/0014.0	24/0014.0
2023	02/0001.0	26/0008.0	24/0022.0	24/0003.0	24/0022.0	24/0022.0
2029	22/0002.0	16/0002.0	16/0001.0	24/0013.0	24/0032.0	24/0021.0
2035	20/0002.0	20/0001.0	24/0014.0	24/0014.0	20/0005.0	20/0005.0
2041	02/0001.0	20/0004.0	16/0001.0	08/0001.1	16/0001.0	12/0001.1
2047	16/0002.0	18/0004.0	02/0001.0	20/0002.0	20/0005.0	24/0021.0
2053	22/0001.0	24/0021.0	20/0001.0	12/0001.1	12/0001.1	02/0001.0
2059	08/0001.0	24/0022.0	20/0005.0	20/0005.0	20/0005.0	24/0022.0
2065	16/0002.0	20/0004.0	20/0004.0	20/0002.0	24/0028.0	14/0001.0
2071	20/0004.0	22/0002.0	20/0001.0	08/0001.0	22/0002.0	24/0004.0
2077	26/0025.0	26/0397.0	22/0003.0	24/0022.0	24/0003.0	20/0002.0
2083	24/0022.0	24/0032.0	24/0003.0	20/0001.0	22/0005.0	20/0002.0
2089	24/0075.0	24/0021.0	24/0022.0	24/0022.0	20/0005.0	02/0001.0
2095	22/0002.0	22/0002.0	24/0003.0	20/0005.0	24/0022.0	26/0008.0
2101	20/0004.0	12/0001.0	16/0001.0	16/0002.0	14/0001.0	02/0001.0
2107	16/0002.0	20/0002.0	02/0001.0	24/0003.0	26/0048.0	26/0048.0
2113	24/0021.0	20/0005.0	20/0001.0	22/0001.0	18/0004.0	20/0005.0
2119	20/0002.0	16/0002.0	20/0005.0	20/0001.0	18/0004.0	20/0005.0
2125	20/0005.0	20/0005.0	20/0004.0	20/0005.0	24/0021.0	02/0001.0
2131	24/0014.0	24/0033.0	20/0005.0	24/0154.0	20/0005.0	24/0033.0
2137	22/0003.0	24/0021.0	22/0002.0	08/0001.0	20/0005.0	20/0005.0
2143	20/0001.0	24/0021.0	20/0001.0	22/0002.0	24/0003.0	20/0002.0

2149	02/0001.0	16/0001.0	16/0001.0	20/0005.0	24/0022.0	02/0001.0
2155	24/0033.0	20/0004.0	20/0001.0	28/0070.0	20/0001.0	22/0002.0
2161	28/0071.0	28/0041.0	28/0049.0	28/0049.0	22/0002.0	24/0014.0
2167	22/0003.0	26/0008.0	24/0022.0	28/0041.0	28/0230.0	28/0230.0
2173	28/0230.0	22/0017.0	22/0016.0	26/0049.0	26/0049.0	22/0003.0
2179	24/0013.0	22/0001.0	20/0005.0	28/0007.0	24/0154.0	24/0003.0
2185	28/0172.0	24/0028.0	26/0256.0	24/0003.0	28/0007.0	24/0004.0
2191	24/0022.0	16/0001.1	24/0002.0	24/0154.0	24/0004.0	24/0022.0
2197	24/0014.0	24/0003.0	24/0033.0	28/0007.0	28/0049.0	22/0002.0
2203	24/0154.0	28/0230.0	28/0230.0	18/0001.0	08/0001.0	16/0002.0
2209	28/0033.0	24/0021.0	28/0071.0	24/0013.0	28/0049.0	24/0013.0
2215	24/0032.0	28/0033.0	26/0049.0	28/0071.0	28/0065.0	24/0022.0
2221	24/0033.0	24/0014.0	24/0032.0	24/0004.0	26/0397.0	26/0025.0
2227	08/0001.0	24/0014.0	28/0230.0	28/0014.0	24/0154.0	22/0003.0
2233	22/0003.0	22/0003.0	28/0065.0	28/0041.0	28/0049.0	28/0065.0
2239	24/0022.0	24/0154.0	24/0002.0	24/0021.0	24/0022.0	28/0014.0
2245	08/0001.1	28/0041.0	24/0154.0	28/0230.0	16/0001.1	16/0002.1
2251	22/0001.0	18/0001.0	20/0001.0	24/0108.0	20/0001.0	26/0008.0
2257	24/0154.0	28/0041.0	24/0154.0	24/0154.0	24/0033.0	24/0033.0
2263	28/0230.0	24/0014.0	24/0014.0	24/0028.0	26/0254.0	28/0230.0
2269	28/0007.0	24/0033.0	24/0033.0	28/0230.0	28/0230.0	28/0049.0
2275	24/0003.0	22/0005.0	28/0473.0	22/0002.0	24/0003.0	26/0025.0
2281	24/0028.0	24/0003.0	26/0049.0	26/0048.0	24/0154.0	08/0001.0
2287	24/0014.0	20/0005.0	20/0004.0	22/0001.0	24/0033.0	22/0002.0
2293	18/0004.0	24/0033.0	24/0154.0	20/0002.0	26/0008.0	12/0001.0
2299	14/0001.0	24/0003.0	28/0113.0	24/0021.0	28/0049.0	24/0075.0
2305	20/0005.0	08/0001.0	24/0032.0	22/0001.0	20/0002.0	24/0013.0
2311	28/0071.0	24/0032.0	24/0014.0	28/2314.0	24/0014.0	20/0004.0
2317	22/0002.0	24/0003.0	22/0017.0	24/0014.0	22/0002.0	22/0017.0
2323	22/0016.0	24/0003.0	22/0002.0	18/0004.0	24/0075.0	24/0022.0
2329	22/0002.0	24/0003.0	26/0048.0	28/0230.0	26/0049.0	24/0022.0
2335	28/0473.0	26/0048.0	22/0002.0	28/0485.0	26/0048.0	22/0003.0
2341	20/0005.0	24/0022.0	24/0033.0	22/0002.0	28/0230.0	22/0017.0
2347	28/0230.0	28/0230.0	20/0010.0	24/0004.0	20/0010.0	22/0003.0
2353	28/0049.0	28/0049.0	24/0075.0	20/0004.0	22/0001.0	24/0028.0
2359	22/0003.0	22/0003.0	22/0003.0	24/0154.0	24/0033.0	24/0028.0

2365	24/0154.0	28/0071.0	28/0071.0	24/0032.0	28/0014.0	26/0048.0
2371	26/0048.0	28/0049.0	28/0113.0	22/0002.0	24/0032.0	22/0002.0
2377	24/0003.0	20/0001.0	22/0017.0	24/0022.0	22/0016.0	22/0003.0
2383	24/0014.0	28/0027.0	24/0032.0	22/0016.0	26/0008.0	24/0014.0
2389	28/0033.0	28/0033.0	24/0014.0	28/0230.0	28/0049.0	26/0065.0
2395	24/0021.0	22/0003.0	24/0033.0	24/0028.0	24/0004.0	24/0032.0
2401	22/0002.0	24/0003.0	24/0022.0	26/0065.0	24/0014.0	22/0002.0
2407	24/0014.0	22/0001.0	20/0004.0	26/0008.0	24/0033.0	28/0033.0
2413	24/0033.0	28/0065.0	28/0027.0	26/0008.0	28/0230.0	28/2418.0
2419	28/2418.0	24/0033.0	28/0230.0	28/0230.0	28/0049.0	28/2314.0
2425	28/0049.0	28/0049.0	28/2314.0	28/0049.0	28/0113.0	28/0049.0
2431	22/0016.0	18/0004.0	24/0014.0	28/0049.0	24/0013.0	22/0017.0
2437	24/0014.0	22/0002.0	24/0033.0	24/0014.0	22/0001.0	24/0028.0
2443	24/0004.0	24/0003.0	24/0003.0	28/0033.0	28/0071.0	28/0033.0
2449	28/0041.0	24/0022.0	24/0022.0	24/0032.0	24/0033.0	18/0004.0
2455	26/0008.0	24/0033.0	26/0049.0	26/0008.0	24/0032.0	24/0014.0
2461	26/0008.0	24/0014.0	24/0028.0	24/0014.0	20/0001.0	26/0049.0
2467	26/0049.0	26/0049.0	26/0008.0	26/0048.0	20/0004.0	28/0065.0
2473	24/0013.0	24/0032.0	22/0002.0	22/0002.0	28/2418.0	28/0230.0
2479	26/0048.0	26/0048.0	28/0014.0	26/0048.0	28/0049.0	28/0049.0
2485	28/0071.0	28/0049.0	28/0014.0	22/0002.0	22/0001.0	28/0033.0
2491	28/0041.0	24/0022.0	24/0013.0	28/0033.0	28/0033.0	28/0230.0
2497	28/0049.0	24/0022.0	20/0004.0	28/0065.0	24/0013.0	28/0230.0
2503	26/0008.0	24/0032.0	28/0033.0	28/0071.0	24/0014.0	24/0032.0
2509	26/0049.0	24/0014.0	26/0008.0	26/0008.0	22/0002.0	24/0032.0
2515	28/0071.0	28/0065.0	24/0154.0	20/0004.0	24/0004.0	24/0003.0
2521	24/0002.0	22/0001.0	26/0008.0	24/0003.0	24/0032.0	26/0008.0
2527	22/0017.0	22/0003.0	28/0014.0	26/0049.0	28/0049.0	26/0048.0
2533	24/0003.0	28/0049.0	28/0113.0	26/0049.0	28/0049.0	24/0003.0
2539	20/0002.0	24/0154.0	22/0016.0	22/0017.0	22/0016.0	24/0013.0
2545	24/0032.0	20/0002.0	20/0002.0	22/0002.0	24/0021.0	22/0002.0
2551	24/0032.0	22/0005.0	28/0230.0	26/0008.0	28/0230.0	22/0005.0
2557	24/0004.0	28/0049.0	28/0049.0	20/0005.0	28/0230.0	28/0113.0
2563	28/0049.0	22/0003.0	24/0154.0	28/0014.0	24/0154.0	24/0014.0
2569	28/0049.0	28/0049.0	26/0048.0	28/0049.0	28/0230.0	28/0049.0
2575	28/0113.0	28/0230.0	26/0049.0	28/0049.0	26/0048.0	24/0022.0

2581	28/0230.0	24/0022.0	28/0230.0	24/0033.0	20/0002.0	08/0001.0
2587	24/0022.0	24/0033.0	24/0028.0	24/0021.0	08/0001.0	22/0002.0
2593	16/0001.0	24/0022.0	16/0002.0	20/0005.0	24/0032.0	24/0003.0
2599	24/0022.0	24/0003.0	20/0002.0	24/0014.0	24/0021.0	22/0003.0
2605	24/0032.0	24/0014.0	08/0001.0	20/0001.0	22/0003.0	24/0003.0
2611	24/0003.0	24/0021.0	24/0003.0	16/0002.0	22/0002.0	24/0014.0
2617	24/0004.0	26/0048.0	26/0048.0	08/0001.0	24/0022.0	24/0033.0
2623	24/0003.0	28/0049.0	28/0230.0	24/0022.0	24/0014.0	24/0032.0
2629	24/0014.0	24/0022.0	24/0003.0	22/0002.0	22/0002.0	08/0001.0
2635	24/0033.0	24/0003.0	22/0003.0	18/0004.0	26/0008.0	24/0013.0
2641	24/0033.0	22/0003.0	24/0028.0	22/0001.0	28/2418.0	28/2314.0
2647	24/0014.0	28/0049.0	28/2314.0	26/0008.0	24/0022.0	22/0005.0
2653	20/0005.0	24/0075.0	24/0022.0	08/0001.0	26/0049.0	20/0004.0
2659	24/0154.0	24/0028.0	28/0485.0	24/0154.0	16/0002.0	22/0017.0
2665	24/0021.0	26/0227.0	26/0227.0	20/0005.0	24/0004.0	24/0154.0
2671	26/0254.0	26/0256.0	28/0113.0	28/0049.0	24/0154.0	24/0033.0
2677	28/0014.0	28/0049.0	24/0013.0	24/0002.0	28/0049.0	28/0049.0
2683	26/0048.0	24/0013.0	24/0033.0	24/0002.0	24/0028.0	24/0021.0
2689	28/0049.0	24/0004.0	28/0049.0	26/0048.0	28/0113.0	20/0002.0
2695	18/0004.0	28/0049.0	24/0014.0	24/0032.0	24/0014.0	24/0032.0
2701	24/0032.0	24/0013.0	24/0032.0	12/0001.0	24/0032.0	24/0022.0
2707	24/0022.0	22/0002.0	20/0005.0	08/0001.0	20/0001.0	24/0022.0
2713	18/0004.0	26/0031.0	24/0022.0	24/0021.0	24/0021.0	24/0014.0
2719	24/0032.0	24/0013.0	22/0003.0	24/0022.0	24/0154.0	16/0002.0
2725	22/0001.0	22/0016.0	22/0001.0	26/0049.0	22/0003.0	26/0008.0
2731	26/0049.0	20/0002.0	24/0028.0	24/0003.0	24/0022.0	24/0022.0
2737	22/0005.0	22/0003.0	22/0017.0	26/0049.0	24/0021.0	26/0049.0
2743	26/0048.0	24/0014.0	22/0002.0	24/0021.0	26/0008.0	22/0002.0
2749	26/0048.0	18/0004.0	24/0014.0	24/0032.0	24/0032.0	24/0014.0
2755	02/0001.0	16/0002.0	24/0032.0	24/0032.0	24/0013.0	24/0013.0
2761	22/0017.0	22/0016.0	26/0227.0	26/0227.0	26/0227.0	26/0227.0
2767	24/0014.0	24/0014.0	02/0001.0	26/0256.0	26/0254.0	26/0254.0
2773	26/0256.0	22/0002.0	22/0017.0	22/0016.0	08/0001.0	20/0005.0
2779	02/0001.0	08/0001.0	08/0001.0	24/0154.0	24/0004.0	20/0002.0
2785	24/0022.0	24/0003.0	24/0013.0	24/0006.0	22/0002.0	24/0022.0
2791	26/0003.0	22/0002.0	24/0154.0	20/0001.0	24/0022.0	18/0004.0

2797	24/0032.0	24/0032.0	24/0003.0	24/0014.0	24/0013.0	24/0013.0
2803	24/0032.0	22/0001.0	20/0005.0	22/0003.0	12/0001.0	20/0002.0
2809	22/0002.0	24/0003.0	24/0014.0	20/0004.0	24/0022.0	20/0004.0
2815	24/0032.0	24/0022.0	26/0008.0	24/0004.0	24/0154.0	24/0154.0
2821	20/0001.0	24/0014.0	26/0008.0	20/0002.0	24/0021.0	24/0032.0
2827	18/0004.0	22/0002.0	08/0001.0	24/0022.0	26/0397.0	24/0003.0
2833	24/0022.0	20/0004.0	20/0002.0	20/0002.0	22/0002.0	24/0003.0
2839	24/0022.0	24/0075.0	24/0022.0	24/0022.0	24/0003.0	18/0004.0
2845	08/0001.0	22/0003.0	22/0016.0	08/0001.0	22/0003.0	02/0001.0
2851	08/0001.0	20/0004.0	20/0002.0	24/0004.0	20/0002.0	20/0001.0
2857	24/0033.0	22/0002.0	24/0014.0	24/0014.0	24/0003.0	12/0001.1
2863	12/0001.0	22/0016.0	18/0001.0	08/0001.0	24/0022.0	08/0001.0
2869	16/0002.0	22/0003.0	16/0002.0	24/0004.0	26/0397.0	26/0008.0
2875	22/0003.0	20/0002.0	22/0001.0	02/0001.0	22/0005.0	20/0001.0
2881	20/0001.0	08/0001.0	20/0005.0	20/0005.0	20/0002.0	22/0002.0
2887	28/0071.0	28/0230.0	22/0003.0	28/0049.0	26/0049.0	26/0049.0
2893	24/0021.0	24/0004.0	24/0014.0	24/0022.0	28/0230.0	24/0021.0
2899	22/0003.0	22/0003.0	22/0002.0	18/0004.0	22/0002.0	28/0054.0
2905	28/0049.0	28/0071.0	24/0022.0	24/0022.0	24/0003.0	22/0003.0
2911	28/0049.0	24/0022.0	24/0032.0	24/0032.0	28/0106.0	24/0021.0
2917	24/0003.0	22/0017.0	28/0230.0	28/0049.0	26/0049.0	26/0049.0
2923	26/0049.0	28/0230.0	28/0049.0	26/0049.0	28/0230.0	28/0230.0
2929	24/0014.0	24/0021.0	28/0049.0	24/0013.0	22/0002.0	24/0032.0
2935	24/0014.0	24/0022.0	24/0022.0	28/0230.0	28/0230.0	22/0016.0
2941	24/0021.0	24/0021.0	22/0003.0	24/0021.0	22/0003.0	28/0070.0
2947	22/0003.0	22/0003.0	24/0014.0	24/0021.0	28/0033.0	24/0021.0
2953	24/0154.0	22/0003.0	22/0003.0	24/0021.0	24/0154.0	24/0021.0
2959	22/0003.0	18/0004.0	26/0065.0	24/0013.0	24/0022.0	28/0230.0
2965	28/0230.0	26/0049.0	22/0005.0	28/0033.0	28/0230.0	28/0230.0
2971	28/0230.0	26/0049.0	28/0049.0	28/0049.0	28/0065.0	24/0022.0
2977	24/0022.0	28/0230.0	22/0002.0	24/0032.0	22/0003.0	28/0230.0
2983	28/2418.0	28/0230.0	28/2418.0	20/0001.0	28/0230.0	28/0230.0
2989	26/0049.0	28/0049.0	28/0230.0	24/0022.0	28/0230.0	28/0230.0
2995	24/0014.0	28/0230.0	28/0230.0	26/0049.0	28/0049.0	28/0230.0
3001	22/0005.0	28/0230.0	28/0230.0	24/0014.0	24/0014.0	08/0001.0
3007	18/0004.0	02/0001.0	26/0048.0	24/0014.0	24/0022.0	22/0016.0

3013	08/0001.0	20/0001.0	20/0005.0	24/0022.0	20/0005.0	24/0014.0
3019	24/0014.0	24/0032.0	20/0005.0	24/0003.0	24/0003.0	24/0032.0
3025	22/0017.0	24/0014.0	24/0032.0	24/0032.0	20/0001.0	24/0014.0
3031	24/0075.0	02/0001.0	22/0003.0	24/0022.0	24/0003.0	24/0014.0
3037	20/0005.0	20/0005.0	22/0005.0	24/0022.0	20/0005.0	08/0001.0
3043	24/0003.0	22/0016.0	24/0003.0	24/0022.0	24/0033.0	24/0022.0
3049	24/0022.0	24/0021.0	26/0048.0	24/0021.0	22/0003.0	24/0022.0
3055	22/0003.0	24/0014.0	22/0005.0	22/0005.0	24/0075.0	18/0004.0
3061	24/0022.0	24/0032.0	24/0003.0	24/0003.0	24/0032.0	20/0005.0
3067	24/0022.0	24/0003.0	22/0003.0	24/0032.0	22/0005.0	24/0003.0
3073	22/0017.0	24/0022.0	24/0014.0	26/0048.0	20/0001.0	12/0001.0
3079	24/0013.0	24/0022.0	22/0005.0	22/0005.0	24/0032.0	24/0032.0
3085	22/0005.0	22/0005.0	24/0014.0	22/0017.0	22/0016.0	24/0003.0
3091	24/0014.0	24/0022.0	22/0017.0	24/0022.0	24/0014.0	20/0002.0
3097	24/0022.0	22/0016.0	24/0033.0	02/0001.0	24/0022.0	24/0003.0
3103	24/0014.0	22/0017.0	20/0004.0	22/0002.0	12/0001.0	12/0001.1
3109	12/0001.1	22/0017.0	24/0021.0	24/0033.0	20/0005.0	20/0005.0
3115	24/0022.0	20/0005.0	22/0016.0	22/0016.0	08/0001.0	24/0022.0
3121	24/0021.0	18/0004.0	18/0004.0	24/0021.0	24/0014.0	24/0022.0
3127	24/0003.0	24/0021.0	24/0022.0	24/0003.0	20/0002.0	20/0002.0
3133	24/0075.0	22/0005.0	24/0022.0	24/0003.0	24/0014.0	20/0005.0
3139	22/0005.0	22/0016.0	24/0032.0	24/0033.0	18/0004.0	20/0005.0
3145	22/0002.0	24/0075.0	24/0022.0	22/0001.0	24/0033.0	24/0022.0
3151	22/0016.0	22/0005.0	18/0004.0	24/0154.0	24/0033.0	12/0001.1
3157	12/0001.1	08/0001.0	22/0005.0	22/0003.0	26/0256.0	22/0003.0
3163	26/0254.0	22/0005.0	24/0021.0	22/0003.0	26/0227.0	22/0005.0
3169	20/0005.0	24/0014.0	24/0014.0	22/0002.0	24/0003.0	18/0004.0
3175	24/0013.0	22/0017.0	24/0021.0	22/0003.0	22/0017.0	20/0005.0
3181	20/0004.0	24/0032.0	24/0003.0	26/0256.0	24/0014.0	24/0014.0
3187	26/0256.0	22/0016.0	22/0016.0	20/0001.0	18/0004.0	22/0002.0
3193	22/0002.0	22/0001.0	02/0001.0	08/0001.0	22/0002.0	24/0014.0
3199	24/0032.0	08/0001.0	22/0017.0	22/0017.0	24/0014.0	18/0004.0
3205	20/0005.0	24/0033.0	24/0033.0	24/0014.0	24/0154.0	12/0001.0
3211	18/0004.0	26/0227.0	22/0005.0	26/0254.0	22/0005.0	26/0256.0
3217	24/0075.0	24/0075.0	24/0022.0	22/0005.0	24/0021.0	22/0005.0
3223	22/0003.0	16/0001.0	22/0005.0	20/0001.0	24/0014.0	20/0001.0

3229	20/0001.0	20/0001.0	26/0048.0	24/0032.0	22/0005.0	24/0032.0
3235	22/0005.0	02/0001.0	22/0005.0	20/0005.0	22/0003.0	24/0032.0
3241	24/0013.0	26/0227.0	24/0014.0	24/0014.0	26/0025.0	08/0001.0
3247	22/0016.0	26/0048.0	24/0075.0	22/0016.0	20/0001.0	22/0005.0
3253	24/0014.0	02/0001.0	24/0022.0	24/0032.0	26/0227.0	26/0025.0
3259	02/0001.1	24/0022.0	26/0254.0	26/0227.0	26/0048.0	22/0016.0
3265	24/0032.0	22/0005.0	22/0016.0	20/0005.0	20/0001.0	24/0014.0
3271	14/0001.0	20/0004.0	24/0032.0	22/0003.0	20/0005.0	12/0001.1
3277	12/0001.1	12/0001.1	18/0004.0	08/0001.0	02/0001.0	20/0001.0
3283	24/0033.0	22/0003.0	24/0014.0	20/0002.0	22/0005.0	24/0033.0
3289	24/0022.0	24/0003.0	18/0004.0	20/0005.0	24/0014.0	20/0001.0
3295	20/0002.0	18/0004.0	24/0032.0	24/0021.0	24/0022.0	24/0003.0
3301	02/0001.0	22/0003.0	22/0005.0	24/0022.0	22/0016.0	24/0022.0
3307	24/0003.0	24/0032.0	24/0003.0	24/0021.0	24/0022.0	20/0001.0
3313	24/0075.0	24/0022.0	08/0001.0	24/0004.0	24/0022.0	24/0021.0
3319	24/0033.0	08/0001.0	24/0014.0	22/0005.0	18/0001.0	20/0001.0
3325	20/0005.0	20/0001.0	24/0022.0	24/0003.0	24/0032.0	22/0017.0
3331	20/0001.0	24/0021.0	24/0013.0	24/0003.0	24/0032.0	26/0025.0
3337	20/0001.0	24/0075.0	28/0485.0	26/0397.0	28/0485.0	28/0473.0
3343	26/0025.0	24/0014.0	24/0021.0	24/0014.0	24/0032.0	24/0022.0
3349	24/0021.0	24/0014.0	24/0021.0	12/0001.1	24/0014.0	24/0032.0
3355	24/0014.0	24/0014.0	24/0014.0	24/0032.0	22/0005.0	20/0005.0
3361	20/0005.0	18/0001.0	24/0014.0	24/0032.0	24/0032.0	24/0032.0
3367	24/0013.0	24/0022.0	08/0001.0	24/0022.0	24/0033.0	24/0014.0
3373	24/0021.0	24/0013.0	24/0022.0	20/0004.0	20/0005.0	24/0022.0
3379	24/0013.0	18/0001.0	20/0005.0	22/0002.0	16/0002.1	16/0002.1
3385	16/0001.1	12/0001.1	24/0014.0	24/0022.0	24/0032.0	20/0004.0
3391	20/0005.0	22/0003.0	20/0004.0	24/0021.0	24/0022.0	24/0033.0
3397	24/0154.0	12/0001.1	24/0033.0	24/0014.0	24/0033.0	20/0005.0
3403	24/0021.0	24/0033.0	24/0014.0	20/0005.0	20/0005.0	20/0005.0
3409	18/0004.0	20/0005.0	24/0021.0	24/0022.0	24/0022.0	24/0021.0
3415	12/0001.1	20/0001.0	20/0004.0	22/0005.0	22/0017.0	24/0022.0
3421	22/0017.0	22/0016.0	24/0022.0	24/0022.0	24/0075.0	24/0075.0
3427	24/0075.0	26/0049.0	24/0003.0	02/0001.1	22/0003.0	24/0022.0
3433	22/0003.0	22/0003.0	20/0004.0	22/0001.0	22/0003.0	22/0002.0
3439	18/0004.0	24/0021.0	24/0032.0	22/0001.0	20/0004.0	20/0004.0

3445	24/0014.0	22/0003.0	22/0017.0	22/0003.0	26/0256.0	26/0227.0
3451	18/0004.0	20/0004.0	24/0021.0	20/0004.0	22/0002.0	20/0001.0
3457	24/0022.0	24/0021.0	24/0014.0	24/0003.0	20/0002.0	22/0002.0
3463	24/0021.0	26/0256.0	26/0227.0	24/0003.0	24/0108.0	24/0075.0
3469	26/0025.0	24/0033.0	22/0003.0	22/0002.0	24/0021.0	22/0002.0
3475	24/0022.0	22/0002.0	18/0004.0	20/0002.0	08/0001.0	12/0001.1
3481	18/0004.0	24/0021.0	24/0014.0	22/0002.0	20/0001.0	20/0004.0
3487	20/0002.0	20/0001.0	24/0028.0	24/0002.0	20/0004.0	24/0032.0
3493	24/0004.0	24/0154.0	24/0033.0	24/0033.0	18/0004.0	20/0004.0
3499	24/0028.0	08/0001.0	12/0001.1	12/0001.1	12/0001.0	26/0254.0
3505	26/0227.0	22/0003.0	22/0005.0	22/0003.0	22/0017.0	24/0021.0
3511	20/0005.0	24/0014.0	20/0002.0	26/0049.0	24/0033.0	18/0004.0
3517	24/0022.0	22/0003.0	20/0005.0	08/0001.0	24/0022.0	24/0033.0
3523	24/0154.0	22/0016.0	22/0003.0	24/0033.0	20/0001.0	20/0001.0
3529	18/0001.0	18/0001.0	20/0005.0	22/0017.0	22/0016.0	24/0022.0
3535	24/0021.0	24/0022.0	24/0022.0	24/0022.0	24/0022.0	24/0003.0
3541	24/0075.0	24/0075.0	26/0048.0	26/0397.0	26/0049.0	26/0048.0
3547	28/0473.0	26/0048.0	26/0049.0	26/0254.0	26/0227.0	24/0003.0
3553	02/0001.0	08/0001.0	26/0254.0	26/0227.0	18/0001.0	22/0003.0
3559	12/0001.1	26/0256.0	26/0227.0	24/0033.0	22/0016.0	22/0017.0
3565	18/0004.0	24/0014.0	18/0004.0	20/0002.0	22/0003.0	18/0004.0
3571	20/0005.0	20/0002.0	22/0003.0	22/0001.0	20/0002.0	20/0005.0
3577	22/0005.0	20/0004.0	20/0005.0	16/0001.0	22/0003.0	22/0003.0
3583	22/0003.0	22/0005.0	22/0005.0	22/0003.0	26/0048.0	24/0075.0
3589	22/0001.0	14/0001.0	20/0004.0	22/0002.0	24/0004.0	24/0154.0
3595	22/0003.0	26/0008.0	24/0033.0	24/0032.0	24/0014.0	26/0025.0
3601	26/0048.0	26/0048.0	22/0002.0	20/0005.0	26/0048.0	26/0049.0
3607	26/0048.0	20/0005.0	20/0001.0	20/0005.0	02/0001.0	26/0049.0
3613	26/0025.0	24/0022.0	24/0154.0	24/0154.0	24/0075.0	24/0075.0
3619	28/0230.0	28/0230.0	22/0001.0	24/0014.0	20/0005.0	28/0473.0
3625	22/0017.0	22/0016.0	24/0002.0	22/0001.0	18/0004.0	20/0001.0
3631	24/0022.0	26/0025.0	24/0003.0	24/0014.0	24/0014.0	20/0001.0
3637	24/0013.0	24/0013.0	24/0022.0	28/0473.0	24/0032.0	28/0485.0
3643	22/0002.0	20/0002.0	20/0005.0	24/0108.0	22/0003.0	20/0005.0
3649	22/0002.0	24/0022.0	24/0028.0	26/0031.0	22/0002.0	20/0001.0
3655	24/0014.0	20/0005.0	24/0032.0	24/0075.0	28/3659.0	26/0048.0

3661	28/3661.0	24/0022.0	24/0003.0	24/0014.0	24/0032.0	24/0014.0
3667	24/0014.0	24/0032.0	24/0014.0	24/0022.0	20/0001.0	22/0005.0
3673	22/0016.0	20/0002.0	24/0032.0	20/0005.0	24/0021.0	22/0005.0
3679	22/0017.0	20/0005.0	28/3661.0	28/3659.0	26/0048.0	20/0001.0
3685	20/0001.0	22/0002.0	24/0003.0	24/0003.0	24/0022.0	24/0022.0
3691	22/0017.0	20/0005.0	24/0014.0	24/0022.0	18/0001.0	24/0003.0
3697	22/0002.0	24/0022.0	24/0075.0	26/0048.0	24/0075.0	24/0003.0
3703	16/0001.0	22/0016.0	20/0004.0	24/0021.0	24/0022.0	24/0003.0
3709	24/0075.0	22/0002.0	22/0005.0	20/0001.0	20/0004.0	24/0022.0
3715	18/0004.0	20/0005.0	20/0004.0	20/0005.0	26/0227.0	24/0003.0
3721	26/0256.0	26/0254.0	24/0022.0	26/0254.0	20/0001.0	26/0227.0
3727	22/0016.0	22/0017.0	24/0075.0	26/0048.0	26/0025.0	26/0048.0
3733	08/0001.0	20/0005.0	26/0048.0	24/0022.0	22/0005.0	20/0005.0
3739	22/0003.0	22/0016.0	22/0017.0	26/0048.0	24/0075.0	16/0001.0
3745	22/0003.0	22/0003.0	22/0005.0	22/0003.0	24/0075.0	20/0010.0
3751	20/0002.0	20/0001.0	24/0022.0	22/0003.0	24/0032.0	20/0005.0
3757	24/0154.0	20/0005.0	20/0002.0	12/0001.0	24/0021.0	20/0005.0
3763	20/0001.0	28/0485.0	26/0256.0	18/0001.0	12/0001.0	22/0001.0
3769	24/0154.0	24/0002.0	24/0028.0	24/0154.0	24/0022.0	24/0014.0
3775	24/0022.0	20/0001.0	24/0014.0	24/0021.0	24/0032.0	24/0003.0
3781	22/0003.0	16/0002.0	22/0005.0	24/0022.0	20/0001.0	24/0033.0
3787	22/0002.0	20/0002.0	20/0005.0	26/0008.0	24/0003.0	24/0021.0
3793	24/0033.0	22/0003.0	22/0003.0	22/0005.0	28/0033.0	26/0049.0
3799	26/0048.0	20/0001.0	24/0154.0	28/0033.0	26/0049.0	26/0049.0
3805	24/0022.0	24/0021.0	28/0033.0	28/0070.0	24/0022.0	22/0003.0
3811	24/0021.0	24/0003.0	22/0002.0	24/0033.0	22/0003.0	20/0002.0
3817	26/0065.0	24/0014.0	22/0002.0	22/0002.0	20/0005.0	24/0021.0
3823	24/0021.0	24/0154.0	24/0002.0	24/0028.0	24/0002.0	20/0004.0
3829	22/0001.0	24/0028.0	20/0004.0	24/0002.0	24/0028.0	22/0001.0
3835	16/0002.0	24/0021.0	24/0108.0	24/0004.0	24/0021.0	22/0005.0
3841	24/0014.0	22/0017.0	24/0032.0	22/0003.0	26/0696.0	24/0003.0
3847	24/0003.0	24/0022.0	24/0003.0	22/0016.0	24/0013.0	24/0032.0
3853	24/0032.0	24/0022.0	26/0065.0	24/0108.0	24/0108.0	26/0254.0
3859	28/0071.0	22/0001.0	28/0071.0	28/0054.0	26/0048.0	26/0008.0
3865	22/0005.0	26/0008.0	24/0032.0	24/0003.0	24/0022.0	24/0003.0
3871	22/0003.0	24/0003.0	24/0003.0	24/0014.0	26/0008.0	18/0004.0

3877	24/0022.0	28/0071.0	26/0227.0	28/0230.0	28/2418.0	28/2418.0
3883	24/0033.0	28/0230.0	28/0033.0	22/0005.0	12/0001.0	24/0014.0
3889	24/0013.0	24/0022.0	24/0022.0	20/0004.0	20/0004.0	24/0022.0
3895	20/0001.0	26/0008.0	24/0032.0	24/0032.0	24/0075.0	22/0005.0
3901	22/0005.0	20/0001.0	22/0005.0	24/0022.0	22/0005.0	24/0022.0
3907	24/0022.0	24/0033.0	24/0033.0	24/0075.0	20/0010.0	24/0022.0
3913	24/0032.0	28/0033.0	28/0065.0	26/0254.0	28/0230.0	28/0230.0
3919	28/0230.0	28/0230.0	24/0033.0	28/2418.0	28/0230.0	28/0230.0
3925	22/0002.0	22/0005.0	20/0004.0	24/0032.0	28/0049.0	28/0049.0
3931	22/0005.0	24/0032.0	28/0049.0	24/0032.0	24/0033.0	24/0033.0
3937	24/0032.0	24/0014.0	20/0005.0	24/0032.0	26/0049.0	28/2418.0
3943	28/2418.0	28/2418.0	24/0033.0	24/0033.0	28/2418.0	24/0014.0
3949	26/0049.0	26/0049.0	22/0005.0	24/0014.0	26/0049.0	24/0014.0
3955	22/0017.0	24/0014.0	26/0008.0	26/0008.0	24/0014.0	22/0017.0
3961	24/0022.0	24/0022.0	22/0016.0	22/0016.0	24/0022.0	28/0070.0
3967	22/0001.0	28/0033.0	24/0014.0	22/0002.0	26/0008.0	24/0014.0
3973	24/0022.0	24/0022.0	26/0256.0	22/0017.0	24/0021.0	20/0004.0
3979	18/0004.0	24/0003.0	24/0003.0	24/0022.0	22/0002.0	16/0001.1
3985	24/0003.0	24/0022.0	24/0022.0	22/0016.0	22/0003.0	22/0017.0
3991	22/0016.0	22/0016.0	20/0001.0	20/0005.0	22/0017.0	20/0002.0
3997	20/0002.0	20/0004.0	22/0001.0	18/0004.0	20/0004.0	20/0002.0
4003	22/0001.0	02/0001.0	20/0004.0	18/0004.0	08/0001.0	24/0033.0
4009	24/0013.0	24/0032.0	24/0013.0	22/0005.0	24/0032.0	18/0004.0
4015	22/0005.0	22/0002.0	16/0001.0	20/0005.0	22/0005.0	24/0014.0
4021	16/0001.1	16/0002.1	24/0022.0	18/0004.0	24/0014.0	26/0049.0
4027	22/0002.0	24/0003.0	20/0002.0	26/0048.0	22/0003.0	22/0002.0
4033	24/0014.0	20/0005.0	18/0001.0	20/0001.0	20/0001.0	20/0005.0
4039	20/0004.0	24/0021.0	16/0002.0	26/0397.0	24/0013.0	26/0006.0
4045	24/0022.0	26/0008.0	24/0022.0	20/0002.0	20/0001.0	24/0022.0
4051	24/0003.0	22/0005.0	26/0048.0	26/0025.0	24/0022.0	20/0005.0
4057	20/0005.0	12/0001.0	12/0001.0	24/0014.0	22/0003.0	22/0005.0
4063	24/0033.0	24/0021.0	22/0002.0	20/0001.0	12/0001.0	24/0022.0
4069	24/0022.0	24/0003.0	26/0025.0	26/0025.0	08/0001.0	26/0048.0
4075	20/0002.0	22/0001.0	20/0005.0	24/0028.0	24/0002.0	24/0028.0
4081	20/0005.0	24/0154.0	22/0002.0	24/0154.0	20/0004.0	24/0154.0
4087	24/0154.0	24/0028.0	20/0004.0	24/0028.0	22/0002.0	20/0005.0

4093	12/0001.0	18/0004.0	18/0004.0	20/0005.0	22/0002.0	20/0002.0
4099	24/0004.0	22/0002.0	22/0002.0	16/0002.0	24/0022.0	22/0001.0
4105	20/0002.0	24/0033.0	26/0008.0	24/0021.0	24/0022.0	20/0002.0
4111	24/0003.0	26/0025.0	26/0003.0	24/0021.0	24/0003.0	24/0013.0
4117	08/0001.0	20/0004.0	26/0025.0	20/0005.0	18/0004.0	20/0002.0
4123	22/0001.0	20/0004.0	22/0001.0	24/0028.0	20/0004.0	20/0005.0
4129	26/0397.0	26/0008.0	26/0008.0	26/0048.0	22/0002.0	26/0048.0
4135	26/0048.0	26/0397.0	26/0008.0	22/0001.0	26/0008.0	20/0004.0
4141	26/0397.0	20/0004.0	18/0004.0	20/0002.0	20/0004.0	24/0032.0
4147	20/0001.0	24/0022.0	22/0002.0	22/0002.0	24/0014.0	26/0048.0
4153	24/0003.0	24/0033.0	22/0016.0	24/0021.0	24/0022.0	24/0032.0
4159	22/0017.0	24/0014.0	22/0017.0	24/0021.0	24/0022.0	12/0001.0
4165	24/0032.0	24/0022.0	22/0005.0	24/0032.0	22/0005.0	22/0005.0
4171	24/0014.0	22/0005.0	20/0005.0	20/0001.0	20/0005.0	20/0005.0
4177	20/0001.0	20/0001.0	24/0014.0	22/0016.0	24/0014.0	12/0001.0
4183	20/0001.0	18/0004.0	12/0001.0	20/0005.0	24/0032.0	24/0014.0
4189	24/0014.0	24/0013.0	24/0032.0	24/0032.0	20/0010.0	22/0001.0
4195	16/0002.0	18/0004.0	22/0017.0	24/0032.0	24/0014.0	24/0014.0
4201	20/0005.0	20/0002.0	20/0004.0	02/0001.0	08/0001.0	24/0033.0
4207	16/0001.0	08/0001.0	24/0021.0	24/0032.0	20/0004.0	22/0002.0
4213	20/0002.0	24/0003.0	24/0022.0	24/0022.0	20/0001.0	24/0022.0
4219	08/0001.0	22/0005.0	24/0033.0	26/0256.0	24/0075.0	26/0254.0
4225	24/0075.0	24/0032.0	26/0227.0	24/0075.0	26/0254.0	26/0227.0
4231	20/0002.0	24/0014.0	24/0003.0	22/0017.0	24/0075.0	26/0256.0
4237	24/0014.0	26/0256.0	22/0016.0	24/0014.0	24/0032.0	18/0004.0
4243	26/0256.0	24/0014.0	24/0032.0	26/0254.0	24/0013.0	22/0003.0
4249	26/0227.0	26/0227.0	24/0032.0	22/0002.0	24/0014.0	24/0022.0
4255	24/0021.0	24/0022.0	24/0003.0	22/0005.0	24/0022.0	24/0003.0
4261	22/0017.0	24/0032.0	22/0002.0	20/0001.0	22/0002.0	22/0017.0
4267	24/0003.0	20/0001.0	02/0001.0	18/0004.0	22/0005.0	24/0014.0
4273	24/0014.0	22/0005.0	22/0005.0	22/0005.0	24/0003.0	22/0002.0
4279	22/0005.0	26/0048.0	20/0004.0	20/0004.0	20/0005.0	22/0005.0
4285	08/0001.1	12/0001.1	20/0001.0	20/0010.0	22/0003.0	22/0005.0
4291	24/0033.0	24/0014.0	22/0017.0	20/0005.0	18/0001.0	22/0017.0
4297	20/0004.0	18/0004.0	24/0033.0	20/0002.0	12/0001.1	08/0001.0
4303	22/0005.0	24/0003.0	24/0022.0	24/0022.0	20/0004.0	22/0003.0

4309	26/0025.0	22/0003.0	22/0002.0	22/0002.0	02/0001.0	20/0002.0
4315	20/0004.0	22/0001.0	24/0033.0	12/0001.1	08/0001.0	20/0010.0
4321	22/0003.0	22/0003.0	02/0001.0	24/0014.0	24/0032.0	20/0001.0
4327	08/0001.0	22/0016.0	16/0002.0	22/0005.0	24/0014.0	24/0003.0
4333	20/0005.0	18/0004.0	22/0003.0	18/0004.0	22/0001.0	18/0004.0
4339	18/0004.0	24/0014.0	22/0005.0	22/0003.0	24/0021.0	24/0022.0
4345	22/0003.0	22/0003.0	22/0005.0	22/0002.0	24/0021.0	12/0001.0
4351	02/0001.1	20/0001.0	24/0013.0	24/0032.0	20/0001.0	22/0003.0
4357	22/0003.0	24/0033.0	22/0016.0	24/0033.0	22/0005.0	22/0003.0
4363	20/0001.0	22/0003.0	22/0003.0	26/0227.0	26/0227.0	24/0032.0
4369	02/0001.0	08/0001.0	02/0001.0	08/0001.0	24/0014.0	12/0001.1
4375	26/0048.0	26/0025.0	26/0254.0	24/0021.0	24/0075.0	24/0021.0
4381	24/0022.0	24/0032.0	18/0004.0	18/0004.0	24/0014.0	20/0002.0
4387	20/0005.0	12/0001.1	24/0022.0	20/0005.0	20/0001.0	20/0001.0
4393	20/0001.0	20/0001.0	20/0001.0	20/0005.0	20/0005.0	20/0001.0
4399	24/0014.0	24/0032.0	20/0001.0	20/0005.0	28/0485.0	20/0004.0
4405	20/0002.0	20/0005.0	20/0004.0	22/0016.0	24/0033.0	18/0004.0
4411	24/0022.0	24/0003.0	22/0005.0	24/0003.0	22/0016.0	22/0017.0
4417	26/0254.0	22/0016.0	24/0022.0	26/0256.0	22/0002.0	24/0014.0
4423	24/0013.0	20/0001.0	26/0256.0	24/0003.0	24/0022.0	26/0254.0
4429	26/0025.0	20/0004.0	08/0001.0	20/0002.0	22/0002.0	18/0004.0
4435	12/0001.1	08/0001.1	26/0049.0	20/0005.0	20/0005.0	24/0022.0
4441	24/0022.0	20/0005.0	22/0002.0	20/0004.0	22/0002.0	20/0004.0
4447	20/0005.0	20/0004.0	22/0002.0	20/0004.0	26/0048.0	28/0054.0
4453	28/0054.0	22/0017.0	24/0033.0	24/0007.0	22/0002.0	24/0006.0
4459	26/0003.0	28/0009.0	24/0032.0	24/0032.0	26/0006.0	28/0029.0
4465	28/0071.0	28/0280.0	28/0042.0	26/0031.0	24/0028.0	24/0007.0
4471	26/0005.0	26/0008.0	28/0006.0	28/0025.0	28/0113.0	28/0059.0
4477	28/0014.0	26/0014.0	26/0065.0	24/0004.0	24/0154.0	24/0004.0
4483	24/0154.0	28/0081.0	28/0041.0	28/0029.0	24/0154.0	26/0010.0
4489	22/0001.0	28/0517.0	28/0517.0	28/0514.0	18/0004.0	20/0004.0
4495	24/0028.0	24/0033.0	20/0004.0	18/0004.0	28/0049.0	28/0049.0
4501	26/0008.0	22/0002.0	26/0003.0	20/0004.0	26/0008.0	24/0033.0
4507	28/0033.0	26/0031.0	28/0027.0	28/0402.0	28/2314.0	24/0154.0
4513	24/0028.0	24/0028.0	26/0005.0	28/0034.0	26/0013.0	28/0001.0
4519	26/0014.0	28/0003.0	26/0011.0	26/0696.0	24/0014.0	24/0004.0

4525	24/0007.0	24/0001.0	26/0013.0	18/0004.0	24/0006.0	24/0007.0
4531	24/0032.0	26/0006.0	26/0256.0	28/0054.0	26/0254.0	26/0696.0
4537	26/0011.0	28/0005.0	28/0280.0	28/0042.0	26/0013.0	28/0010.0
4543	20/0004.0	20/0004.0	26/0008.0	24/0006.0	26/0696.0	26/0006.0
4549	26/0003.0	26/0010.0	24/0021.0	24/0022.0	24/0022.0	24/0003.0
4555	28/0172.0	28/0005.0	28/0010.0	24/0004.0	28/0033.0	28/0065.0
4561	26/0696.0	28/0106.0	28/0070.0	28/0031.0	26/0065.0	26/0013.0
4567	22/0016.0	18/0004.0	24/0013.0	18/0004.0	22/0005.0	18/0004.0
4573	24/0033.0	20/0004.0	18/0004.0	08/0001.1	02/0001.1	18/0004.0
4579	18/0004.0	22/0005.0	20/0005.0	20/0004.0	16/0001.0	02/0001.1
4585	28/0230.0	22/0003.0	28/0230.0	28/0065.0	20/0005.0	22/0001.0
4591	22/0017.0	22/0016.0	24/0004.0	26/0006.0	20/0004.0	24/0028.0
4597	26/0008.0	22/0005.0	24/0004.0	24/0154.0	26/0008.0	28/0049.0
4603	24/0154.0	20/0002.0	28/0007.0	28/0113.0	24/0004.0	28/0014.0
4609	20/0001.0	08/0001.1	18/0004.0	16/0002.0	24/0022.0	20/0001.0
4615	18/0001.0	26/0227.0	26/0006.0	26/0065.0	24/0028.0	26/0010.0
4621	24/0108.0	26/0025.0	28/0029.0	26/0031.0	24/0007.0	26/0011.0
4627	28/0027.0	24/0108.0	24/0021.0	22/0003.0	24/0028.0	24/0108.0
4633	24/0021.0	26/0025.0	24/0108.0	26/0008.0	22/0003.0	26/0696.0
4639	24/0032.0	28/0029.0	26/0006.0	26/0008.0	28/0027.0	26/0065.0
4645	16/0002.0	24/0033.0	24/0003.0	24/0154.0	24/0154.0	20/0010.0
4651	24/0022.0	24/0028.0	26/0008.0	24/0028.0	20/0002.0	22/0002.0
4657	24/0154.0	20/0001.0	24/0004.0	24/0004.0	26/0008.0	24/0032.0
4663	26/0008.0	20/0004.0	26/0008.0	24/0028.0	24/0021.0	28/0027.0
4669	26/0065.0	26/0696.0	24/0028.0	26/0008.0	28/0029.0	26/0006.0
4675	16/0001.0	24/0022.0	20/0004.0	24/0014.0	24/0032.0	24/0033.0
4681	20/0004.0	20/0004.0	24/0033.0	24/0032.0	24/0002.0	24/0028.0
4687	22/0002.0	24/0033.0	18/0004.0	24/0108.0	24/0032.0	22/0002.0
4693	20/0004.0	20/0004.0	22/0001.0	22/0002.0	20/0004.0	22/0001.0
4699	26/0397.0	26/0397.0	28/0485.0	24/0022.0	20/0004.0	22/0002.0
4705	20/0001.0	24/0032.0	18/0001.0	24/0075.0	22/0001.0	24/0022.0
4711	24/0014.0	28/0485.0	24/0154.0	24/0002.0	24/0154.0	26/0049.0
4717	16/0001.1	20/0002.0	26/0048.0	20/0005.0	26/0397.0	24/0033.0
4723	22/0002.0	24/0003.0	24/0154.0	24/0022.0	16/0001.0	24/0033.0
4729	26/0003.0	20/0002.0	24/0154.0	16/0002.0	24/0003.0	24/0022.0
4735	24/0022.0	24/0022.0	20/0005.0	24/0033.0	24/0014.0	22/0005.0

4741	24/0022.0	24/0022.0	22/0005.0	22/0002.0	24/0022.0	24/0003.0
4747	24/0154.0	20/0005.0	24/0014.0	22/0003.0	24/0033.0	22/0002.0
4753	24/0013.0	24/0003.0	26/0025.0	28/0473.0	22/0003.0	26/0048.0
4759	22/0001.0	24/0075.0	20/0002.0	24/0033.0	22/0005.0	26/0049.0
4765	26/0048.0	24/0028.0	24/0014.0	24/0033.0	24/0014.0	22/0005.0
4771	22/0005.0	24/0033.0	20/0002.0	26/0048.0	26/0025.0	24/0022.0
4777	16/0001.0	24/0022.0	24/0014.0	24/0033.0	26/0025.0	26/0025.0
4783	24/0028.0	12/0001.0	22/0001.0	24/0014.0	26/0008.0	24/0028.0
4789	24/0014.0	24/0033.0	24/0003.0	20/0004.0	26/0008.0	22/0001.0
4795	22/0002.0	16/0002.0	24/0013.0	24/0022.0	24/0033.0	24/0154.0
4801	08/0001.0	26/0397.0	22/0001.0	24/0014.0	18/0004.0	18/0004.0
4807	22/0002.0	24/0013.0	24/0003.0	22/0005.0	28/0514.0	28/0517.0
4813	28/0517.0	28/0514.0	26/0025.0	22/0003.0	24/0003.0	24/0022.0
4819	28/0485.0	28/0485.0	24/0022.0	24/0022.0	22/0001.0	24/0108.0
4825	18/0004.0	28/0485.0	26/0397.0	20/0004.0	24/0022.0	26/0048.0
4831	28/0473.0	28/0473.0	26/0049.0	26/0025.0	22/0003.0	28/0485.0
4837	28/0485.0	26/0048.0	24/0022.0	24/0003.0	22/0003.0	26/0048.0
4843	26/0008.0	24/0033.0	24/0022.0	22/0002.0	16/0001.0	16/0001.0
4849	08/0001.0	02/0001.0	20/0005.0	24/0022.0	24/0021.0	20/0004.0
4855	24/0014.0	16/0002.0	18/0004.0	24/0032.0	24/0013.0	24/0003.0
4861	24/0033.0	24/0021.0	24/0022.0	24/0022.0	22/0003.0	24/0022.0
4867	20/0004.0	24/0021.0	22/0003.0	24/0004.0	02/0001.0	08/0001.0
4873	24/0032.0	20/0002.0	24/0033.0	24/0014.0	24/0021.0	24/0022.0
4879	24/0014.0	24/0032.0	20/0002.0	12/0001.1	02/0001.0	18/0001.0
4885	20/0002.0	20/0005.0	12/0001.1	02/0001.0	12/0001.0	24/0022.0
4891	18/0001.0	22/0005.0	24/0003.0	22/0002.0	20/0004.0	14/0001.0
4897	22/0005.0	20/0005.0	24/0003.0	24/0033.0	24/0022.0	26/0049.0
4903	02/0001.1	20/0004.0	24/0004.0	24/0033.0	24/0004.0	26/0065.0
4909	26/0006.0	24/0033.0	24/0014.0	24/0013.0	24/0022.0	24/0032.0
4915	24/0108.0	24/0033.0	24/0022.0	24/0014.0	24/0003.0	24/0022.0
4921	24/0003.0	26/0008.0	24/0021.0	08/0001.0	22/0002.0	24/0022.0
4927	22/0001.0	22/0003.0	26/0397.0	24/0022.0	20/0004.0	20/0005.0
4933	20/0002.0	22/0002.0	26/0025.0	20/0001.0	20/0004.0	24/0021.0
4939	24/0003.0	22/0002.0	26/0025.0	26/0008.0	22/0002.0	26/0397.0
4945	24/0154.0	24/0004.0	24/0021.0	20/0001.0	22/0002.0	24/0022.0
4951	26/0397.0	26/0397.0	12/0001.0	12/0001.0	26/0008.0	08/0001.0

4957	20/0001.0	22/0003.0	22/0005.0	24/0028.0	24/0032.0	18/0004.0
4963	08/0001.0	20/0004.0	22/0002.0	20/0004.0	22/0001.0	22/0005.0
4969	24/0014.0	24/0022.0	24/0022.0	24/0022.0	14/0001.0	26/0048.0
4975	24/0014.0	20/0001.0	24/0022.0	28/0473.0	22/0001.0	18/0004.0
4981	22/0002.0	22/0003.0	24/0014.0	22/0003.0	22/0005.0	20/0005.0
4987	20/0001.0	22/0002.0	24/0022.0	24/0022.0	20/0002.0	24/0022.0
4993	24/0021.0	20/0005.0	20/0001.0	24/0022.0	24/0075.0	24/0003.0
4999	24/0075.0	24/0013.0	24/0022.0	22/0002.0	24/0032.0	24/0075.0
5005	22/0003.0	24/0075.0	24/0075.0	24/0022.0	28/3661.0	26/0025.0
5011	24/0032.0	28/3659.0	26/0048.0	24/0022.0	24/0003.0	24/0003.0
5017	24/0075.0	28/3659.0	28/3661.0	26/0048.0	20/0005.0	22/0016.0
5023	24/0022.0	20/0001.0	22/0017.0	24/0021.0	20/0001.0	16/0002.0
5029	24/0014.0	12/0001.1	24/0033.0	22/0005.0	26/0048.0	26/0025.0
5035	26/0048.0	24/0075.0	24/0075.0	24/0075.0	24/0013.0	26/0025.0
5041	26/0048.0	22/0003.0	22/0016.0	24/0033.0	22/0005.0	22/0005.0
5047	26/0048.0	24/0033.0	12/0001.0	12/0001.0	20/0001.0	16/0001.0
5053	12/0001.0	22/0002.0	24/0014.0	24/0021.0	24/0021.0	20/0001.0
5059	24/0022.0	20/0005.0	20/0001.0	24/0021.0	20/0002.0	22/0005.0
5065	22/0005.0	22/0003.0	20/0001.0	22/0003.0	18/0004.0	24/0022.0
5071	24/0032.0	24/0014.0	24/0014.0	24/0013.0	26/0227.0	24/0022.0
5077	22/0003.0	22/0017.0	24/0014.0	26/0227.0	24/0032.0	24/0033.0
5083	24/0022.0	24/0003.0	24/0022.0	24/0003.0	20/0005.0	24/0014.0
5089	24/0003.0	24/0022.0	18/0004.0	24/0032.0	24/0033.0	22/0005.0
5095	12/0001.0	22/0016.0	12/0001.1	22/0002.0	20/0004.0	22/0016.0
5101	22/0017.0	24/0022.0	20/0002.0	24/0075.0	24/0075.0	24/0003.0
5107	24/0075.0	24/0075.0	20/0005.0	24/0003.0	24/0022.0	26/0048.0
5113	24/0075.0	24/0075.0	24/0003.0	24/0022.0	24/0032.0	24/0033.0
5119	26/0025.0	22/0017.0	26/0048.0	24/0014.0	20/0005.0	24/0032.0
5125	24/0003.0	22/0017.0	24/0033.0	24/0033.0	22/0016.0	16/0001.0
5131	16/0002.0	14/0001.0	12/0001.1	26/0048.0	24/0032.0	26/0048.0
5137	24/0075.0	24/0003.0	28/3659.0	26/0048.0	26/0048.0	24/0033.0
5143	22/0005.0	24/0014.0	26/0025.0	28/3661.0	22/0017.0	22/0001.0
5149	18/0004.0	18/0004.0	20/0004.0	22/0016.0	24/0032.0	22/0017.0
5155	20/0001.0	20/0001.0	16/0001.0	24/0032.0	26/0227.0	24/0075.0
5161	24/0032.0	26/0227.0	24/0075.0	24/0022.0	22/0016.0	24/0022.0
5167	24/0014.0	24/0033.0	24/0032.0	24/0033.0	20/0010.0	20/0010.0

5173	22/0003.0	20/0001.0	24/0003.0	22/0005.0	26/0048.0	20/0002.0
5179	18/0004.0	22/0003.0	24/0013.0	24/0013.0	24/0022.0	22/0005.0
5185	20/0005.0	22/0003.0	24/0075.0	26/0227.0	24/0003.0	24/0033.0
5191	24/0014.0	28/3661.0	26/0025.0	24/0021.0	20/0005.0	20/0010.0
5197	22/0016.0	22/0017.0	22/0005.0	20/0005.0	22/0003.0	26/0227.0
5203	28/3659.0	24/0033.0	24/0014.0	20/0005.0	24/0014.0	24/0014.0
5209	22/0016.0	24/0033.0	12/0001.1	12/0001.0	12/0001.0	22/0017.0
5215	20/0004.0	20/0001.0	22/0002.0	24/0013.0	20/0001.0	24/0032.0
5221	22/0017.0	22/0016.0	24/0003.0	24/0032.0	26/0048.0	24/0032.0
5227	24/0014.0	24/0032.0	20/0005.0	20/0001.0	12/0001.0	20/0005.0
5233	16/0002.0	24/0032.0	24/0022.0	24/0014.0	20/0005.0	22/0016.0
5239	26/0048.0	22/0003.0	26/0025.0	20/0001.0	18/0001.0	20/0001.0
5245	24/0154.0	24/0033.0	24/0003.0	26/0025.0	24/0003.0	22/0005.0
5251	24/0022.0	24/0003.0	26/0048.0	26/0025.0	24/0075.0	28/0473.0
5257	26/0048.0	28/0485.0	28/0485.0	26/0049.0	26/0048.0	28/0473.0
5263	26/0025.0	26/0048.0	28/0485.0	26/0025.0	24/0075.0	24/0075.0
5269	24/0022.0	22/0002.0	24/0014.0	24/0032.0	20/0005.0	18/0004.0
5275	20/0001.0	24/0033.0	20/0004.0	22/0001.0	24/0004.0	24/0033.0
5281	24/0028.0	24/0014.0	22/0017.0	24/0022.0	20/0010.0	22/0002.0
5287	22/0002.0	24/0014.0	24/0032.0	26/0048.0	26/0025.0	28/0485.0
5293	26/0048.0	28/0485.0	28/0485.0	28/0473.0	26/0397.0	18/0004.0
5299	16/0002.1	16/0002.1	20/0005.0	20/0004.0	22/0003.0	20/0001.0
5305	24/0032.0	22/0017.0	18/0004.0	24/0033.0	24/0022.0	26/0049.0
5311	26/0048.0	24/0014.0	20/0005.0	20/0005.0	24/0004.0	24/0014.0
5317	20/0005.0	16/0002.0	22/0005.0	24/0032.0	20/0004.0	18/0004.0
5323	22/0005.0	24/0014.0	22/0003.0	24/0033.0	24/0022.0	22/0016.0
5329	22/0017.0	24/0014.0	24/0032.0	24/0003.0	22/0002.0	20/0004.0
5335	22/0002.0	22/0005.0	20/0010.0	24/0021.0	24/0014.0	22/0017.0
5341	20/0010.0	20/0005.0	20/0002.0	24/0032.0	24/0014.0	22/0002.0
5347	24/0075.0	26/0048.0	24/0075.0	26/0049.0	26/0048.0	24/0108.0
5353	26/0049.0	26/0048.0	24/0022.0	24/0003.0	20/0004.0	24/0014.0
5359	12/0001.1	26/0008.0	18/0004.0	24/0003.0	24/0032.0	18/0004.0
5365	12/0001.1	24/0022.0	12/0001.1	20/0001.0	24/0032.0	24/0033.0
5371	24/0014.0	16/0002.1	22/0003.0	16/0001.1	22/0005.0	20/0005.0
5377	24/0022.0	22/0003.0	18/0004.0	20/0001.0	20/0004.0	22/0001.0
5383	24/0154.0	24/0014.0	20/0001.0	22/0005.0	24/0032.0	22/0005.0

5389	20/0004.0	24/0022.0	24/0021.0	16/0001.0	24/0013.0	24/0032.0
5395	24/0032.0	20/0004.0	24/0021.0	02/0001.1	26/0048.0	24/0075.0
5401	26/0049.0	26/0048.0	26/0025.0	26/0048.0	26/0025.0	24/0108.0
5407	26/0049.0	26/0048.0	22/0003.0	14/0001.0	24/0022.0	24/0021.0
5413	24/0021.0	20/0005.0	02/0001.0	16/0002.0	24/0021.0	24/0014.0
5419	24/0022.0	20/0005.0	22/0005.0	24/0032.0	22/0005.0	20/0001.0
5425	20/0004.0	24/0004.0	24/0028.0	24/0033.0	12/0001.0	22/0005.0
5431	22/0005.0	24/0033.0	24/0033.0	24/0154.0	24/0075.0	26/0397.0
5437	08/0001.1	22/0003.0	18/0001.0	20/0005.0	20/0004.0	20/0005.0
5443	24/0033.0	24/0014.0	08/0001.1	22/0002.0	18/0004.0	24/0032.0
5449	20/0005.0	24/0108.0	26/0025.0	26/0049.0	26/0025.0	26/0048.0
5455	26/0025.0	22/0003.0	20/0001.0	22/0001.0	22/0002.0	16/0001.0
5461	16/0002.0	20/0001.0	16/0002.0	20/0010.0	24/0022.0	22/0016.0
5467	22/0002.0	24/0014.0	02/0001.0	08/0001.1	22/0003.0	20/0010.0
5473	22/0005.0	20/0010.0	22/0003.0	20/0005.0	24/0075.0	28/0485.0
5479	20/0004.0	22/0001.0	18/0004.0	20/0004.0	24/0033.0	20/0002.0
5485	24/0033.0	24/0014.0	24/0033.0	24/0014.0	26/0048.0	22/0001.0
5491	22/0002.0	12/0001.0	20/0002.0	08/0001.1	22/0003.0	22/0003.0
5497	20/0010.0	22/0003.0	22/0005.0	24/0014.0	22/0003.0	24/0032.0
5503	22/0005.0	24/0014.0	24/0033.0	24/0014.0	18/0004.0	20/0005.0
5509	22/0002.0	20/0005.0	20/0001.0	24/0021.0	20/0002.0	22/0002.0
5515	20/0001.0	24/0021.0	20/0005.0	20/0001.0	24/0021.0	22/0005.0
5521	22/0005.0	24/0003.0	24/0014.0	24/0075.0	26/0048.0	26/0049.0
5527	28/0473.0	26/0025.0	28/0485.0	26/0048.0	28/0473.0	26/0048.0
5533	26/0048.0	24/0075.0	26/0049.0	20/0005.0	24/0022.0	20/0001.0
5539	22/0002.0	20/0005.0	22/0002.0	18/0004.0	16/0002.0	20/0001.0
5545	08/0001.0	20/0004.0	20/0005.0	20/0005.0	18/0001.0	26/0025.0
5551	26/0025.0	26/0048.0	24/0108.0	26/0048.0	28/0485.0	28/0473.0
5557	28/0485.0	20/0004.0	20/0005.0	22/0002.0	24/0022.0	24/0014.0
5563	24/0032.0	20/0004.0	24/0154.0	24/0033.0	24/0014.0	24/0002.0
5569	24/0028.0	24/0014.0	22/0002.0	24/0075.0	26/0049.0	24/0108.0
5575	24/0033.0	26/0048.0	26/0025.0	26/0048.0	26/0025.0	28/0485.0
5581	28/0473.0	28/0485.0	28/0485.0	20/0005.0	20/0004.0	20/0005.0
5587	24/0014.0	16/0002.1	16/0001.1	16/0002.0	20/0004.0	20/0005.0
5593	20/0001.0	26/0048.0	24/0075.0	24/0075.0	22/0003.0	26/0049.0
5599	26/0049.0	26/0025.0	26/0048.0	12/0001.0	24/0028.0	22/0001.0

5605	22/0001.0	20/0002.0	20/0004.0	20/0005.0	24/0022.0	20/0004.0
5611	20/0002.0	20/0001.0	24/0004.0	12/0001.0	08/0001.0	08/0001.0
5617	18/0004.0	22/0016.0	18/0001.0	24/0032.0	24/0033.0	24/0003.0
5623	22/0005.0	14/0001.0	24/0021.0	24/0032.0	24/0154.0	20/0002.0
5629	20/0004.0	24/0033.0	20/0001.0	16/0001.0	20/0005.0	20/0005.0
5635	22/0002.0	22/0002.0	24/0022.0	24/0033.0	24/0022.0	24/0003.0
5641	24/0022.0	22/0002.0	24/0032.0	24/0003.0	26/0025.0	26/0048.0
5647	24/0075.0	26/0048.0	26/0025.0	26/0397.0	26/0397.0	26/0397.0
5653	26/0397.0	26/0048.0	26/0048.0	26/0048.0	26/0025.0	26/0025.0
5659	26/0048.0	26/0048.0	20/0001.0	24/0032.0	22/0002.0	20/0004.0
5665	24/0021.0	24/0014.0	20/0002.0	22/0002.0	24/0033.0	24/0014.0
5671	24/0032.0	22/0003.0	20/0002.0	20/0004.0	20/0004.0	20/0002.0
5677	22/0002.0	24/0003.0	24/0022.0	24/0004.0	22/0002.0	20/0004.0
5683	24/0014.0	24/0022.0	24/0021.0	20/0002.0	16/0002.0	22/0001.0
5689	24/0028.0	24/0154.0	20/0002.0	26/0008.0	24/0003.0	24/0022.0
5695	24/0032.0	26/0008.0	12/0001.1	22/0002.0	18/0004.0	24/0033.0
5701	24/0014.0	18/0004.0	18/0004.0	20/0005.0	20/0005.0	20/0001.0
5707	26/0049.0	24/0021.0	24/0022.0	22/0003.0	26/0025.0	18/0001.0
5713	22/0003.0	22/0003.0	24/0033.0	12/0001.0	24/0003.0	20/0001.0
5719	22/0002.0	24/0003.0	20/0005.0	18/0001.0	16/0002.0	24/0003.0
5725	24/0075.0	24/0108.0	26/0049.0	26/0025.0	26/0048.0	24/0075.0
5731	26/0049.0	26/0048.0	26/0025.0	26/0048.0	28/0473.0	28/0485.0
5737	12/0001.0	20/0005.0	24/0033.0	24/0014.0	24/0033.0	24/0014.0
5743	20/0001.0	18/0001.0	12/0001.1	12/0001.1	24/0014.0	20/0005.0
5749	12/0001.0	16/0001.1	16/0002.1	16/0002.1	16/0002.1	22/0003.0
5755	18/0004.0	18/0001.0	20/0004.0	22/0002.0	22/0002.0	22/0002.0
5761	22/0005.0	12/0001.0	02/0001.0	08/0001.1	18/0004.0	08/0001.1
5767	12/0001.0	16/0001.0	20/0001.0	18/0001.0	18/0004.0	22/0003.0
5773	02/0001.0	22/0005.0	18/0001.0	20/0010.0	20/0010.0	22/0005.0
5779	22/0005.0	22/0003.0	20/0002.0	22/0003.0	20/0001.0	22/0005.0
5785	24/0014.0	18/0004.0	24/0033.0	24/0033.0	20/0005.0	24/0032.0
5791	20/0001.0	22/0002.0	22/0002.0	20/0005.0	22/0005.0	20/0010.0
5797	24/0022.0	22/0005.0	20/0004.0	24/0003.0	24/0032.0	24/0032.0
5803	20/0001.0	22/0016.0	16/0001.0	20/0004.0	24/0003.0	24/0022.0
5809	26/0227.0	24/0075.0	26/0227.0	24/0033.0	24/0003.0	20/0001.0
5815	24/0014.0	22/0017.0	20/0001.0	20/0001.0	22/0005.0	02/0001.0

5821	24/0032.0	26/0227.0	24/0032.0	24/0075.0	20/0001.0	26/0227.0
5827	24/0022.0	26/0227.0	24/0003.0	26/0227.0	24/0003.0	22/0003.0
5833	24/0021.0	08/0001.0	18/0004.0	08/0001.0	20/0004.0	08/0001.0
5839	24/0021.0	22/0002.0	26/0048.0	24/0022.0	20/0005.0	16/0001.0
5845	22/0005.0	20/0004.0	24/0075.0	24/0075.0	26/0048.0	26/0048.0
5851	24/0075.0	26/0025.0	24/0075.0	18/0004.0	20/0001.0	20/0005.0
5857	20/0005.0	24/0003.0	24/0075.0	26/0048.0	24/0075.0	24/0022.0
5863	24/0075.0	24/0075.0	24/0022.0	28/3661.0	24/0032.0	24/0022.0
5869	28/3659.0	24/0014.0	26/0048.0	24/0003.0	24/0022.0	24/0022.0
5875	24/0013.0	24/0021.0	24/0003.0	24/0032.0	22/0016.0	22/0017.0
5881	20/0005.0	20/0001.0	20/0001.0	24/0003.0	24/0022.0	24/0032.0
5887	22/0003.0	22/0003.0	28/3659.0	28/3661.0	26/0048.0	22/0002.0
5893	20/0002.0	20/0010.0	20/0002.0	24/0013.0	24/0032.0	22/0002.0
5899	24/0021.0	24/0022.0	24/0014.0	24/0032.0	20/0004.0	08/0001.0
5905	16/0002.0	12/0001.1	18/0004.0	24/0013.0	24/0032.0	24/0014.0
5911	22/0003.0	26/0048.0	02/0001.1	24/0014.0	12/0001.1	24/0032.0
5917	24/0033.0	24/0033.0	24/0075.0	26/0049.0	26/0048.0	22/0002.0
5923	20/0001.0	08/0001.1	20/0005.0	20/0002.0	18/0004.0	20/0005.0
5929	24/0022.0	24/0022.0	26/0049.0	24/0075.0	24/0108.0	24/0003.0
5935	24/0022.0	24/0003.0	24/0022.0	24/0003.0	24/0022.0	02/0001.0
5941	20/0001.0	20/0004.0	20/0001.0	22/0016.0	24/0033.0	22/0017.0
5947	26/0025.0	20/0001.0	26/0048.0	26/0025.0	28/0473.0	28/0485.0
5953	28/0485.0	28/0485.0	28/0473.0	28/0485.0	22/0002.0	12/0001.1
5959	24/0022.0	20/0004.0	24/0021.0	24/0021.0	24/0032.0	24/0013.0
5965	24/0022.0	24/0033.0	24/0014.0	20/0004.0	26/0048.0	26/0025.0
5971	22/0003.0	26/0048.0	26/0025.0	24/0075.0	26/0048.0	24/0075.0
5977	26/0048.0	20/0010.0	20/0010.0	22/0005.0	26/0025.0	26/0048.0
5983	26/0048.0	16/0002.0	26/0048.0	20/0004.0	22/0003.0	24/0022.0
5989	24/0022.0	26/0048.0	24/0032.0	24/0013.0	12/0001.0	28/0485.0
5995	28/0485.0	12/0001.1	12/0001.1	12/0001.0	20/0005.0	12/0001.0
6001	22/0002.0	08/0001.1	08/0001.0	24/0013.0	22/0005.0	20/0001.0
6007	22/0003.0	24/0014.0	22/0005.0	22/0003.0	24/0032.0	24/0003.0
6013	22/0005.0	24/0032.0	22/0005.0	24/0014.0	24/0014.0	24/0013.0
6019	24/0032.0	20/0002.0	12/0001.1	12/0001.0	18/0004.0	22/0002.0
6025	24/0021.0	24/0021.0	24/0022.0	22/0016.0	24/0014.0	22/0017.0
6031	22/0017.0	22/0016.0	14/0001.0	08/0001.0	24/0022.0	24/0003.0

6037	18/0004.0	24/0021.0	22/0002.0	12/0001.0	24/0033.0	24/0014.0
6043	22/0003.0	24/0014.0	20/0005.0	24/0032.0	20/0001.0	16/0001.0
6049	16/0002.0	24/0075.0	24/0075.0	24/0022.0	16/0002.0	24/0003.0
6055	14/0001.0	24/0028.0	22/0001.0	20/0002.0	18/0004.0	24/0021.0
6061	02/0001.0	08/0001.0	24/0032.0	18/0004.0	20/0004.0	22/0003.0
6067	20/0005.0	22/0005.0	22/0002.0	22/0003.0	24/0108.0	24/0033.0
6073	22/0003.0	22/0003.0	24/0004.0	20/0004.0	24/0108.0	22/0005.0
6079	24/0032.0	22/0003.0	22/0003.0	22/0003.0	22/0005.0	22/0005.0
6085	22/0003.0	24/0022.0	24/0022.0	22/0002.0	24/0003.0	24/0022.0
6091	24/0022.0	24/0021.0	20/0001.0	24/0021.0	22/0002.0	18/0004.0
6097	24/0003.0	22/0001.0	20/0004.0	20/0004.0	22/0001.0	20/0004.0
6103	18/0004.0	22/0003.0	24/0033.0	24/0033.0	24/0014.0	22/0003.0
6109	24/0032.0	22/0005.0	24/0003.0	24/0003.0	24/0014.0	24/0032.0
6115	24/0014.0	24/0022.0	24/0003.0	08/0001.0	08/0001.1	20/0004.0
6121	14/0001.0	24/0022.0	24/0003.0	24/0021.0	24/0022.0	24/0033.0
6127	24/0014.0	12/0001.0	22/0002.0	18/0004.0	20/0004.0	20/0002.0
6133	24/0028.0	20/0002.0	18/0004.0	26/0006.0	26/0065.0	28/1473.0
6139	28/1474.0	24/0004.0	24/0154.0	24/0004.0	24/0022.0	24/0032.0
6145	24/0003.0	24/0014.0	24/0021.0	26/0008.0	24/0022.0	24/0014.0
6151	24/0022.0	26/0008.0	20/0002.0	20/0004.0	24/0033.0	24/0022.0
6157	24/0004.0	24/0014.0	22/0003.0	24/0033.0	24/0033.0	28/1474.0
6163	26/0006.0	28/1473.0	26/0065.0	24/0004.0	24/0014.0	24/0032.0
6169	24/0013.0	20/0004.0	24/0032.0	22/0002.0	24/0022.0	22/0005.0
6175	24/0032.0	22/0005.0	12/0001.1	20/0005.0	08/0001.0	20/0005.0
6181	12/0001.1	08/0001.0	24/0033.0	22/0016.0	20/0010.0	20/0004.0
6187	22/0001.0	20/0004.0	20/0004.0	20/0004.0	18/0001.0	22/0003.0
6193	22/0017.0	24/0014.0	20/0001.0	20/0004.0	24/0022.0	22/0005.0
6199	24/0003.0	24/0013.0	20/0005.0	22/0002.0	24/0003.0	20/0005.0
6205	24/0022.0	24/0032.0	22/0003.0	24/0014.0	24/0014.0	24/0033.0
6211	24/0022.0	24/0021.0	24/0021.0	20/0001.0	24/0032.0	20/0002.0
6217	20/0005.0	22/0002.0	24/0033.0	24/0014.0	24/0032.0	24/0014.0
6223	24/0033.0	18/0001.0	24/0154.0	20/0004.0	22/0005.0	16/0002.0
6229	20/0002.0	22/0003.0	22/0005.0	22/0005.0	24/0003.0	24/0075.0
6235	22/0005.0	22/0005.0	20/0005.0	22/0003.0	22/0003.0	22/0005.0
6241	22/0005.0	20/0010.0	24/0075.0	24/0075.0	22/0003.0	22/0005.0
6247	22/0003.0	22/0005.0	20/0010.0	22/0005.0	26/0227.0	26/0227.0

6253	24/0075.0	26/0048.0	24/0075.0	24/0075.0	24/0075.0	20/0001.0
6259	20/0004.0	26/0227.0	26/0227.0	24/0003.0	26/0025.0	16/0002.0
6265	24/0154.0	20/0004.0	24/0014.0	22/0002.0	20/0005.0	20/0005.0
6271	22/0002.0	22/0001.0	22/0001.0	20/0004.0	22/0002.0	22/0003.0
6277	24/0021.0	24/0022.0	22/0003.0	20/0004.0	20/0005.0	22/0002.0
6283	08/0001.0	20/0002.0	22/0003.0	24/0033.0	22/0017.0	24/0014.0
6289	20/0005.0	26/0049.0	24/0075.0	20/0005.0	22/0002.0	20/0005.0
6295	24/0003.0	22/0016.0	20/0002.0	24/0014.0	22/0002.0	26/0025.0
6301	26/0048.0	24/0075.0	26/0049.0	24/0108.0	20/0005.0	24/0003.0
6307	24/0003.0	22/0002.0	02/0001.0	24/0022.0	16/0002.0	16/0001.0
6313	24/0003.0	26/0048.0	26/0048.0	26/0025.0	26/0025.0	12/0001.0
6319	20/0005.0	20/0004.0	24/0033.0	18/0004.0	18/0004.0	12/0001.1
6325	24/0014.0	24/0033.0	20/0001.0	18/0004.0	24/0022.0	22/0005.0
6331	24/0021.0	24/0022.0	22/0001.0	20/0004.0	20/0002.0	24/0022.0
6337	24/0021.0	20/0001.0	18/0004.0	24/0022.0	20/0004.0	22/0017.0
6343	24/0014.0	22/0016.0	24/0032.0	24/0022.0	24/0033.0	24/0033.0
6349	18/0004.0	20/0004.0	20/0004.0	24/0032.0	24/0032.0	24/0032.0
6355	24/0032.0	24/0014.0	24/0032.0	22/0005.0	22/0003.0	20/0010.0
6361	22/0005.0	20/0010.0	22/0003.0	22/0003.0	22/0005.0	22/0005.0
6367	22/0005.0	22/0003.0	22/0005.0	26/0227.0	24/0075.0	28/6372.0
6373	26/0227.0	24/0032.0	16/0002.0	26/0227.0	24/0075.0	28/6378.0
6379	28/6372.0	26/0227.0	24/0022.0	18/0004.0	24/0003.0	22/0005.0
6385	22/0003.0	22/0005.0	22/0005.0	20/0010.0	22/0003.0	12/0001.0
6391	22/0003.0	24/0014.0	22/0005.0	24/0022.0	24/0032.0	22/0005.0
6397	24/0022.0	22/0003.0	24/0032.0	22/0005.0	20/0004.0	18/0004.0
6403	24/0014.0	22/0017.0	22/0016.0	24/0032.0	24/0032.0	24/0014.0
6409	20/0004.0	20/0002.0	24/0003.0	22/0005.0	24/0075.0	24/0075.0
6415	22/0002.0	24/0033.0	24/0014.0	24/0003.0	24/0032.0	20/0005.0
6421	24/0022.0	24/0003.0	26/0227.0	26/0227.0	24/0003.0	24/0032.0
6427	26/0025.0	26/0048.0	24/0075.0	24/0075.0	24/0075.0	24/0075.0
6433	24/0075.0	28/3659.0	26/0048.0	28/3661.0	26/0048.0	26/0048.0
6439	26/0025.0	26/0048.0	28/3661.0	26/0025.0	28/3659.0	26/0048.0
6445	26/0048.0	26/0025.0	24/0075.0	20/0002.0	20/0001.0	18/0004.0
6451	16/0002.0	18/0001.0	02/0001.1	22/0016.0	24/0014.0	22/0017.0
6457	20/0002.0	22/0005.0	22/0005.0	20/0005.0	20/0005.0	24/0014.0
6463	22/0017.0	22/0016.0	24/0032.0	22/0005.0	24/0014.0	20/0001.0

6469	24/0013.0	22/0002.0	20/0002.0	20/0004.0	22/0002.0	22/0003.0
6475	20/0005.0	20/0004.0	24/0022.0	24/0021.0	20/0001.0	24/0022.0
6481	24/0032.0	24/0003.0	20/0001.0	24/0032.0	20/0001.0	26/0048.0
6487	26/0048.0	24/0075.0	26/0049.0	26/0025.0	24/0075.0	26/0025.0
6493	22/0003.0	24/0108.0	26/0049.0	26/0048.0	26/0025.0	26/0048.0
6499	28/0473.0	28/0485.0	28/0485.0	28/0485.0	28/0473.0	28/0485.0
6505	26/0048.0	22/0001.0	26/0014.0	24/0002.0	26/0696.0	26/0008.0
6511	26/0011.0	22/0001.0	22/0001.0	24/0006.0	28/0070.0	28/0060.0
6517	28/0054.0	24/0001.0	24/0007.0	26/0031.0	26/0031.0	24/0006.0
6523	26/0013.0	24/0007.0	24/0004.0	24/0014.0	20/0002.0	26/0006.0
6529	22/0017.0	26/0008.0	24/0022.0	26/0065.0	28/0070.0	24/0021.0
6535	22/0016.0	26/0013.0	22/0001.0	26/0065.0	26/0006.0	24/0022.0
6541	22/0005.0	28/0013.0	28/0054.0	28/0014.0	28/0081.0	28/0106.0
6547	12/0001.0	02/0001.1	24/0032.0	20/0005.0	22/0001.0	24/0004.0
6553	24/0028.0	24/0013.0	24/0022.0	20/0002.0	24/0014.0	24/0033.0
6559	24/0004.0	24/0021.0	24/0033.0	16/0001.0	26/0006.0	22/0002.0
6565	28/0081.0	28/0106.0	22/0017.0	26/0065.0	28/0070.0	22/0016.0
6571	26/0006.0	24/0004.0	28/0031.0	28/0054.0	20/0005.0	24/0032.0
6577	24/0003.0	24/0033.0	20/0005.0	18/0004.0	24/0014.0	24/0013.0
6583	24/0004.0	24/0032.0	26/0006.0	24/0013.0	22/0002.0	24/0032.0
6589	24/0014.0	24/0014.0	22/0001.0	22/0002.0	24/0033.0	24/0004.0
6595	22/0001.0	20/0002.0	22/0001.0	20/0004.0	24/0028.0	24/0002.0
6601	26/0065.0	26/0006.0	24/0033.0	22/0002.0	18/0004.0	24/0032.0
6607	22/0002.0	22/0002.0	24/0033.0	24/0014.0	24/0014.0	28/1474.0
6613	28/1473.0	24/0014.0	24/0004.0	20/0001.0	20/0002.0	28/1473.0
6619	28/1474.0	24/0033.0	28/0033.0	28/0230.0	22/0001.0	28/0065.0
6625	28/0049.0	16/0001.0	24/0003.0	22/0003.0	20/0005.0	24/0075.0
6631	22/0016.0	22/0017.0	26/0048.0	02/0001.0	08/0001.0	24/0004.0
6637	24/0022.0	24/0022.0	24/0021.0	24/0033.0	24/0033.0	20/0010.0
6643	20/0001.0	28/0027.0	24/0021.0	20/0005.0	24/0014.0	24/0033.0
6649	28/0113.0	28/0054.0	24/0022.0	24/0021.0	26/0048.0	24/0003.0
6655	28/0070.0	24/0021.0	28/0071.0	24/0021.0	22/0016.0	22/0017.0
6661	28/0049.0	24/0014.0	24/0021.0	24/0022.0	24/0154.0	28/0033.0
6667	28/0065.0	24/0003.0	24/0003.0	24/0154.0	24/0004.0	28/0071.0
6673	28/0014.0	28/0041.0	28/0049.0	24/0003.0	24/0154.0	22/0003.0
6679	24/0154.0	22/0003.0	26/0008.0	22/0003.0	24/0014.0	24/0021.0

6685	28/0049.0	28/0230.0	22/0003.0	28/0049.0	24/0033.0	28/0049.0
6691	24/0033.0	28/0065.0	22/0005.0	28/0041.0	28/0049.0	24/0154.0
6697	24/0022.0	20/0004.0	22/0005.0	24/0032.0	20/0001.0	24/0003.0
6703	28/3659.0	24/0013.0	08/0001.0	28/3661.0	24/0075.0	22/0002.0
6709	22/0002.0	20/0005.0	20/0001.0	02/0001.0	20/0001.0	20/0001.0
6715	24/0003.0	24/0032.0	08/0001.0	22/0003.0	24/0022.0	28/3661.0
6721	28/3659.0	18/0001.0	24/0003.0	08/0001.0	24/0013.0	08/0001.1
6727	20/0010.0	08/0001.1	22/0002.0	24/0075.0	20/0002.0	20/0002.0
6733	16/0002.0	24/0014.0	02/0001.0	24/0032.0	26/0227.0	22/0005.0
6739	24/0075.0	20/0002.0	26/0227.0	02/0001.1	12/0001.0	20/0002.0
6745	08/0001.0	22/0003.0	24/0032.0	08/0001.0	24/0014.0	18/0004.0
6751	26/0227.0	20/0005.0	24/0003.0	24/0014.0	22/0003.0	26/0025.0
6757	26/0227.0	24/0032.0	26/0227.0	26/0227.0	26/0227.0	24/0032.0
6763	20/0002.0	22/0002.0	24/0003.0	28/0054.0	26/0008.0	22/0002.0
6769	24/0022.0	24/0003.0	20/0004.0	24/0032.0	22/0002.0	20/0004.0
6775	26/0003.0	22/0002.0	16/0002.0	22/0003.0	26/0008.0	28/0071.0
6781	24/0014.0	20/0005.0	20/0001.0	24/0013.0	28/0033.0	28/0033.0
6787	26/0254.0	26/0256.0	24/0022.0	22/0005.0	24/0003.0	24/0154.0
6793	24/0154.0	24/0154.0	24/0003.0	24/0003.0	28/0071.0	28/0071.0
6799	26/0227.0	26/0227.0	28/0071.0	24/0004.0	18/0004.0	28/0033.0
6805	24/0022.0	24/0154.0	24/0028.0	24/0002.0	20/0002.0	26/0065.0
6811	28/0065.0	28/0041.0	24/0033.0	24/0022.0	22/0002.0	22/0001.0
6817	24/0032.0	28/0230.0	28/0049.0	28/0070.0	28/0033.0	22/0003.0
6823	28/0033.0	28/0230.0	28/0071.0	28/0049.0	24/0021.0	26/0065.0
6829	20/0002.0	20/0005.0	24/0003.0	22/0001.0	26/0397.0	26/0008.0
6835	20/0001.0	24/0003.0	24/0022.0	24/0003.0	22/0002.0	24/0022.0
6841	24/0003.0	24/0003.0	24/0075.0	20/0004.0	22/0001.0	24/0154.0
6847	24/0014.0	24/0028.0	24/0033.0	22/0003.0	26/0008.0	22/0001.0
6853	24/0033.0	26/0397.0	26/0025.0	24/0022.0	24/0014.0	20/0004.0
6859	24/0028.0	22/0005.0	20/0004.0	20/0004.0	24/0022.0	26/0049.0
6865	24/0022.0	24/0032.0	24/0014.0	24/0032.0	26/0048.0	26/0048.0
6871	24/0002.0	24/0154.0	20/0004.0	24/0154.0	22/0001.0	22/0001.0
6877	28/0033.0	24/0032.0	28/0033.0	28/0071.0	28/0071.0	26/0008.0
6883	20/0005.0	22/0003.0	22/0003.0	24/0014.0	20/0004.0	22/0005.0
6889	24/0022.0	24/0003.0	24/0033.0	20/0002.0	24/0003.0	22/0005.0
6895	24/0032.0	20/0004.0	22/0002.0	24/0003.0	24/0032.0	24/0004.0

6901	24/0075.0	28/0033.0	28/0065.0	26/0254.0	26/0254.0	28/0230.0
6907	22/0002.0	24/0032.0	24/0003.0	22/0003.0	24/0022.0	18/0004.0
6913	20/0002.0	24/0075.0	24/0032.0	28/0106.0	28/0027.0	26/0256.0
6919	26/0256.0	28/0049.0	28/0049.0	24/0003.0	28/0014.0	24/0013.0
6925	28/0065.0	28/0049.0	26/0008.0	22/0002.0	22/0001.0	24/0033.0
6931	24/0028.0	26/0008.0	24/0022.0	24/0022.0	24/0028.0	20/0004.0
6937	26/0049.0	26/0048.0	20/0010.0	24/0033.0	24/0032.0	20/0001.0
6943	22/0002.0	18/0004.0	24/0013.0	28/0054.0	26/0008.0	28/0071.0
6949	24/0003.0	18/0004.0	28/0033.0	24/0022.0	18/0004.0	28/0033.0
6955	28/0065.0	24/0154.0	24/0004.0	28/0071.0	28/0049.0	28/0014.0
6961	28/0041.0	28/0033.0	28/0071.0	22/0001.0	26/0008.0	24/0033.0
6967	24/0028.0	24/0004.0	12/0001.0	12/0001.0	22/0002.0	02/0001.1
6973	22/0001.0	20/0002.0	08/0001.0	24/0032.0	08/0001.0	16/0002.0
6979	24/0028.0	22/0002.0	24/0013.0	08/0001.0	22/0001.0	22/0003.0
6985	24/0033.0	24/0014.0	24/0014.0	22/0002.0	24/0003.0	24/0014.0
6991	26/0008.0	24/0022.0	22/0005.0	20/0002.0	22/0002.0	22/0002.0
6997	24/0003.0	24/0022.0	24/0033.0	24/0014.0	22/0001.0	24/0014.0
7003	22/0001.0	20/0005.0	22/0003.0	22/0002.0	08/0001.0	22/0002.0
7009	22/0002.0	20/0002.0	24/0014.0	24/0032.0	22/0005.0	24/0003.0
7015	18/0004.0	22/0005.0	24/0032.0	24/0003.0	08/0001.0	20/0004.0
7021	20/0005.0	22/0005.0	24/0033.0	24/0014.0	24/0022.0	24/0075.0
7027	24/0022.0	22/0003.0	22/0005.0	24/0154.0	20/0002.0	20/0004.0
7033	22/0005.0	20/0001.0	24/0002.0	18/0004.0	24/0022.0	24/0022.0
7039	22/0016.0	22/0017.0	22/0003.0	24/0033.0	24/0014.0	24/0021.0
7045	24/0022.0	24/0033.0	22/0003.0	22/0016.0	24/0033.0	22/0017.0
7051	24/0021.0	22/0003.0	26/0049.0	22/0003.0	24/0022.0	24/0022.0
7057	24/0003.0	24/0013.0	24/0022.0	24/0032.0	26/0048.0	24/0032.0
7063	22/0005.0	24/0022.0	28/0033.0	28/0230.0	28/0230.0	24/0014.0
7069	28/0106.0	26/0048.0	26/0049.0	26/0008.0	24/0022.0	22/0002.0
7075	24/0022.0	24/0014.0	22/0001.0	22/0001.0	26/0254.0	24/0014.0
7081	22/0016.0	26/0256.0	28/0070.0	28/0230.0	24/0022.0	28/0230.0
7087	22/0017.0	24/0021.0	26/0256.0	26/0254.0	24/0022.0	28/0033.0
7093	20/0004.0	18/0004.0	24/0033.0	24/0014.0	24/0022.0	24/0032.0
7099	24/0003.0	22/0005.0	22/0003.0	24/0021.0	26/0008.0	26/0008.0
7105	22/0002.0	20/0005.0	26/0008.0	20/0002.0	22/0002.0	28/0054.0
7111	28/0071.0	28/0033.0	28/0033.0	28/0070.0	24/0013.0	24/0022.0

7117	22/0001.0	08/0001.1	08/0001.0	26/0397.0	16/0002.0	20/0004.0
7123	26/0397.0	18/0004.0	20/0004.0	22/0005.0	22/0002.0	24/0014.0
7129	20/0004.0	24/0022.0	20/0005.0	24/0033.0	24/0003.0	20/0002.0
7135	24/0028.0	22/0001.0	24/0108.0	18/0004.0	22/0001.0	26/0065.0
7141	22/0002.0	24/0021.0	24/0154.0	24/0028.0	20/0004.0	24/0021.0
7147	20/0002.0	24/0022.0	24/0003.0	22/0002.0	02/0001.0	24/0032.0
7153	24/0013.0	24/0022.0	24/0075.0	24/0022.0	20/0005.0	24/0004.0
7159	26/0008.0	24/0003.0	24/0022.0	20/0004.0	24/0003.0	24/0004.0
7165	22/0005.0	22/0002.0	20/0002.0	24/0033.0	24/0033.0	24/0075.0
7171	02/0001.1	20/0002.0	20/0005.0	18/0001.0	24/0075.0	20/0002.0
7177	20/0005.0	22/0017.0	22/0016.0	20/0001.0	24/0021.0	24/0022.0
7183	24/0033.0	24/0033.0	26/0048.0	20/0005.0	20/0005.0	28/0485.0
7189	28/0485.0	22/0005.0	24/0022.0	24/0033.0	24/0022.0	24/0022.0
7195	18/0004.0	20/0005.0	24/0033.0	24/0033.0	24/0154.0	26/0065.0
7201	24/0033.0	24/0014.0	24/0021.0	24/0022.0	24/0003.0	24/0014.0
7207	24/0021.0	20/0004.0	24/0021.0	24/0022.0	24/0021.0	24/0154.0
7213	28/0033.0	24/0022.0	28/0033.0	28/0230.0	28/0230.0	28/0106.0
7219	28/0070.0	22/0001.0	28/0033.0	24/0022.0	24/0003.0	22/0002.0
7225	26/0008.0	24/0154.0	28/0054.0	28/0071.0	18/0004.0	14/0001.0
7231	24/0028.0	22/0001.0	20/0002.0	24/0032.0	24/0033.0	24/0021.0
7237	20/0002.0	22/0005.0	22/0005.0	16/0002.0	08/0001.0	20/0005.0
7243	24/0022.0	20/0004.0	20/0004.0	26/0008.0	22/0002.0	20/0004.0
7249	24/0021.0	20/0004.0	26/0008.0	24/0022.0	26/0008.0	20/0005.0
7255	22/0002.0	20/0001.0	22/0001.0	20/0002.0	24/0028.0	22/0002.0
7261	24/0003.0	22/0002.0	20/0005.0	20/0004.0	20/0004.0	22/0001.0
7267	20/0010.0	22/0003.0	22/0003.0	24/0014.0	20/0010.0	20/0001.0
7273	24/0033.0	24/0004.0	20/0002.0	24/0021.0	26/0025.0	26/0397.0
7279	26/0397.0	26/0397.0	26/0397.0	26/0025.0	22/0005.0	24/0032.0
7285	22/0005.0	24/0014.0	28/0049.0	26/0049.0	26/0049.0	24/0003.0
7291	24/0032.0	28/0071.0	22/0002.0	28/0113.0	28/0230.0	22/0002.0
7297	28/0230.0	24/0022.0	24/0022.0	28/0230.0	24/0013.0	22/0005.0
7303	24/0032.0	28/0049.0	28/0230.0	28/0049.0	28/0113.0	24/0013.0
7309	24/0014.0	28/0230.0	28/0230.0	28/0230.0	28/0230.0	22/0005.0
7315	24/0014.0	28/2418.0	24/0014.0	24/0014.0	28/0230.0	24/0014.0
7321	24/0154.0	24/0014.0	28/0113.0	28/0049.0	28/0230.0	24/0003.0
7327	22/0003.0	28/0230.0	24/0032.0	16/0002.0	16/0002.0	24/0022.0

7333	20/0001.0	24/0021.0	24/0021.0	20/0001.0	28/0485.0	24/0022.0
7339	22/0003.0	24/0014.0	16/0002.1	08/0001.0	22/0003.0	24/0014.0
7345	24/0033.0	08/0001.0	12/0001.1	16/0002.0	20/0002.0	02/0001.0
7351	24/0075.0	24/0022.0	02/0001.1	02/0001.1	12/0001.1	24/0021.0
7357	24/0022.0	24/0021.0	20/0005.0	22/0017.0	20/0005.0	22/0003.0
7363	22/0002.0	22/0017.0	22/0016.0	20/0001.0	24/0033.0	28/0230.0
7369	28/0113.0	22/0002.0	22/0002.0	24/0013.0	24/0014.0	28/0230.0
7375	20/0005.0	28/0485.0	20/0001.0	24/0032.0	28/0485.0	20/0004.0
7381	24/0014.0	24/0032.0	18/0001.0	14/0001.1	22/0016.0	22/0017.0
7387	28/0230.0	28/2418.0	28/0230.0	28/0049.0	28/0049.0	24/0033.0
7393	22/0002.0	24/0014.0	24/0032.0	24/0013.0	24/0032.0	24/0022.0
7399	20/0004.0	22/0003.0	22/0003.0	20/0004.0	22/0003.0	24/0022.0
7405	24/0003.0	22/0005.0	24/0003.0	22/0005.0	24/0014.0	24/0032.0
7411	22/0005.0	28/0230.0	28/0230.0	28/0230.0	24/0032.0	28/0049.0
7417	20/0002.0	24/0022.0	22/0005.0	24/0014.0	24/0004.0	20/0005.0
7423	28/0033.0	24/0021.0	24/0021.0	26/0049.0	26/0049.0	28/0230.0
7429	28/2418.0	28/2418.0	24/0033.0	28/2418.0	26/0049.0	22/0005.0
7435	24/0021.0	24/0021.0	24/0022.0	24/0022.0	24/0032.0	28/0071.0
7441	24/0154.0	28/0049.0	24/0003.0	22/0002.0	24/0013.0	22/0002.0
7447	24/0013.0	28/0049.0	28/0230.0	28/0230.0	28/0049.0	28/0113.0
7453	24/0013.0	24/0032.0	24/0032.0	28/0049.0	22/0002.0	24/0154.0
7459	22/0002.0	22/0003.0	24/0014.0	16/0002.1	24/0075.0	12/0001.1
7465	20/0005.0	20/0005.0	24/0075.0	20/0004.0	20/0001.0	22/0003.0
7471	16/0002.1	12/0001.1	20/0010.0	20/0002.0	16/0001.0	22/0005.0
7477	08/0001.1	24/0032.0	20/0004.0	24/0022.0	20/0001.0	20/0005.0
7483	22/0016.0	26/0254.0	26/0256.0	26/0256.0	24/0014.0	26/0254.0
7489	20/0001.0	12/0001.1	14/0001.1	22/0002.0	20/0005.0	20/0001.0
7495	22/0016.0	22/0017.0	02/0001.0	22/0016.0	20/0005.0	22/0017.0
7501	22/0017.0	20/0005.0	22/0016.0	26/0048.0	12/0001.1	18/0001.0
7507	20/0001.0	12/0001.1	24/0021.0	20/0005.0	20/0005.0	22/0017.0
7513	20/0005.0	24/0033.0	24/0154.0	24/0014.0	18/0004.0	20/0001.0
7519	24/0075.0	24/0033.0	22/0016.0	20/0005.0	20/0001.0	18/0004.0
7525	26/0048.0	08/0001.0	02/0001.0	24/0014.0	28/0230.0	28/0230.0
7531	24/0022.0	24/0022.0	24/0014.0	24/0014.0	24/0014.0	28/0033.0
7537	28/0230.0	28/0230.0	28/0230.0	28/0230.0	20/0004.0	20/0001.0
7543	20/0004.0	24/0154.0	24/0154.0	22/0005.0	18/0004.0	26/0048.0

7549	24/0075.0	24/0028.0	26/0065.0	28/0027.0	26/0008.0	28/0514.0
7555	28/0517.0	24/0032.0	18/0004.0	18/0004.0	24/0032.0	26/0227.0
7561	26/0227.0	28/6372.0	28/6378.0	18/0004.0	20/0005.0	08/0001.0
7567	22/0002.0	20/0004.0	20/0002.0	24/0014.0	24/0003.0	24/0022.0
7573	24/0003.0	24/0033.0	26/0025.0	26/0048.0	22/0005.0	22/0003.0
7579	26/0227.0	26/0227.0	28/3661.0	28/3659.0	24/0032.0	24/0033.0
7585	24/0014.0	12/0001.0	12/0001.0	24/0013.0	22/0005.0	24/0032.0
7591	22/0003.0	20/0002.0	20/0010.0	20/0005.0	20/0002.0	20/0004.0
7597	20/0010.0	24/0075.0	24/0075.0	22/0002.0	26/0227.0	26/0227.0
7603	22/0002.0	22/0003.0	20/0004.0	24/0003.0	24/0022.0	22/0003.0
7609	28/3661.0	28/3659.0	08/0001.1	18/0004.0	22/0005.0	24/0032.0
7615	24/0033.0	24/0014.0	22/0002.0	24/0075.0	20/0001.0	20/0005.0
7621	24/0032.0	24/0013.0	24/0003.0	24/0022.0	24/0032.0	18/0004.0
7627	20/0002.0	24/0022.0	22/0005.0	20/0001.0	22/0005.0	24/0003.0
7633	20/0002.0	18/0004.0	20/0002.0	20/0004.0	24/0075.0	24/0075.0
7639	22/0005.0	20/0005.0	24/0032.0	20/0001.0	20/0001.0	24/0032.0
7645	24/0032.0	26/0227.0	26/0227.0	22/0003.0	22/0003.0	22/0003.0
7651	22/0003.0	26/0048.0	26/0025.0	26/0025.0	28/3659.0	28/3661.0
7657	26/0048.0	24/0032.0	26/0048.0	28/0485.0	24/0014.0	20/0005.0
7663	02/0001.1	22/0005.0	22/0003.0	24/0014.0	18/0004.0	24/0032.0
7669	18/0004.0	28/0485.0	24/0033.0	16/0002.1	20/0004.0	22/0003.0
7675	26/0048.0	28/0485.0	24/0033.0	24/0075.0	20/0010.0	24/0075.0
7681	24/0075.0	22/0003.0	24/0022.0	20/0005.0	26/0048.0	28/0485.0
7687	20/0005.0	24/0033.0	12/0001.1	16/0002.1	20/0010.0	24/0022.0
7693	22/0003.0	14/0001.1	08/0001.1	16/0002.0	24/0075.0	20/0001.0
7699	26/0048.0	28/0485.0	08/0001.0	08/0001.1	20/0002.0	22/0017.0
7705	20/0010.0	22/0016.0	24/0032.0	22/0005.0	20/0004.0	14/0001.0
7711	20/0001.0	20/0005.0	24/0021.0	24/0033.0	20/0002.0	24/0022.0
7717	24/0033.0	24/0021.0	16/0002.0	16/0002.0	20/0002.0	22/0017.0
7723	22/0016.0	20/0010.0	22/0005.0	20/0001.0	22/0016.0	22/0017.0
7729	24/0033.0	20/0001.0	20/0005.0	24/0021.0	20/0004.0	24/0022.0
7735	26/0048.0	24/0075.0	24/0075.0	28/0485.0	26/0048.0	18/0004.0
7741	20/0004.0	22/0001.0	28/0070.0	28/0033.0	24/0033.0	24/0022.0
7747	24/0014.0	28/0033.0	22/0002.0	24/0154.0	24/0003.0	24/0032.0
7753	28/0071.0	28/0049.0	28/0230.0	28/0230.0	28/0049.0	28/0230.0
7759	20/0005.0	28/0230.0				

8.3 Appendix C - TS-Classes of Rigid Bipartite 3-Gems

8.3.1 3-Gems with 26 Vertices

C001	M012	0039	0052	0139	0277	0360	0383	0386	0390
0426	0540	0672	1107	C002	M003	0047	0404	0748	C003
M012	0055	0075	0170	0174	0423	0442	0502	0710	0750
0759	0774	0859	C004	M007	0140	0448	0537	0542	0559
0911	1105	C005	M030	0020	0054	0072	0091	0113	0154
0163	0334	0337	0365	0424	0495	0503	0513	0718	0744
0758	0761	0780	0826	0827	0830	0851	0861	0948	0956
0983	1014	1079	1235	C006	M012	0088	0105	0178	0299
0391	0432	0694	0888	0902	1002	1118	1119	C007	M050
0027	0040	0058	0087	0099	0131	0147	0279	0283	0320
0343	0373	0388	0403	0409	0436	0438	0441	0451	0463
0496	0500	0501	0512	0514	0526	0544	0615	0673	0676
0716	0745	0773	0779	0849	0860	0871	0892	0950	0995
0999	1013	1063	1076	1077	1078	1080	1161	1199	1219
C008	M012	0057	0188	0190	0333	0374	0379	0402	0435
0521	0735	0908	0929	C009	M056	0017	0111	0137	0149
0160	0167	0189	0193	0194	0197	0199	0203	0276	0287
0289	0319	0375	0384	0405	0408	0443	0444	0455	0510
0515	0524	0538	0545	0550	0634	0637	0663	0670	0704
0717	0719	0730	0749	0864	0885	0907	0924	0942	0949
0952	0970	0971	0977	0982	0989	1011	1023	1062	1102
1111	1135	C010	M010	0056	0144	0331	0362	0376	0377
0522	0543	0546	0556	C011	M048	0024	0085	0145	0164
0183	0191	0201	0202	0263	0274	0338	0356	0357	0407
0437	0452	0453	0462	0469	0535	0607	0622	0627	0635
0668	0669	0679	0690	0723	0726	0736	0785	0812	0887
0913	0916	0933	0990	1009	1064	1074	1075	1112	1117
1152	1168	1171	1246	C012	M039	0033	0035	0071	0108
0179	0180	0209	0242	0271	0280	0286	0288	0296	0335
0345	0355	0412	0445	0466	0617	0623	0625	0630	0632
0795	0797	0799	0806	1073	1115	1126	1129	1133	1149

1159	1207	1208	1213	1215	C013	M001	0010	C014	M087
0018	0045	0068	0110	0129	0136	0138	0141	0157	0171
0186	0196	0200	0204	0215	0239	0243	0272	0294	0300
0317	0321	0339	0361	0392	0433	0454	0457	0470	0516
0517	0523	0527	0528	0555	0566	0578	0592	0595	0596
0597	0599	0612	0638	0640	0664	0682	0684	0685	0687
0714	0725	0810	0824	0850	0863	0865	0866	0897	0904
0909	0910	0917	0918	0921	0926	0930	0932	0957	0958
0962	0976	1021	1030	1061	1110	1127	1128	1137	1139
1160	1162	1201	1204	1214	1242	1249	C015	M056	0016
0074	0082	0086	0104	0106	0126	0134	0150	0166	0173
0175	0187	0238	0281	0364	0497	0504	0509	0525	0529
0552	0554	0598	0616	0633	0656	0678	0712	0731	0738
0740	0771	0778	0791	0793	0794	0816	0829	0832	0843
0856	0872	0873	0920	0944	0945	0961	0996	1010	1012
1082	1084	1138	1142	1237	C016	M011	0051	0098	0168
0177	0505	0707	0711	0713	0741	0767	0768	C017	M055
0132	0148	0165	0176	0192	0198	0213	0226	0241	0250
0278	0282	0292	0387	0411	0459	0508	0519	0549	0561
0582	0591	0600	0639	0671	0674	0677	0681	0683	0691
0692	0789	0790	0800	0801	0895	0896	0900	0928	0959
0964	0969	1022	1052	1067	1109	1150	1151	1173	1185
1216	1217	1223	1224	1250	C018	M011	0008	0009	0078
0080	0081	0090	0494	0754	0762	0775	0777	C019	M084
0073	0092	0109	0121	0122	0158	0181	0182	0184	0210
0225	0244	0273	0284	0290	0291	0328	0336	0342	0344
0353	0406	0415	0417	0430	0446	0449	0460	0465	0468
0472	0493	0563	0564	0575	0579	0608	0620	0624	0651
0653	0654	0657	0715	0727	0743	0792	0796	0798	0819
0825	0828	0831	0953	0988	1005	1007	1016	1017	1029
1038	1042	1049	1060	1071	1103	1106	1122	1147	1148
1153	1164	1165	1175	1176	1192	1200	1209	1210	1211
1220	1221	1226	1248	C020	M043	0023	0028	0060	0093
0212	0217	0221	0223	0230	0249	0253	0261	0285	0329
0382	0428	0518	0569	0572	0585	0589	0603	0631	0641
0652	0659	0661	0662	0722	0724	0776	0781	0821	0822

0854	0867	0992	0997	1026	1070	1134	1203	1251	C021
M036	0036	0064	0116	0128	0206	0211	0216	0231	0232
0237	0316	0324	0401	0574	0577	0593	0605	0606	0756
0840	0848	0852	0874	0966	0967	1028	1068	1091	1141
1146	1163	1177	1181	1193	1197	1243	C022	M032	0049
0100	0265	0304	0347	0348	0350	0395	0396	0399	0420
0478	0479	0481	0482	0802	0804	1097	1113	1123	1124
1131	1132	1154	1155	1228	1229	1230	1231	1232	1233
1252	C023	M074	0032	0034	0038	0041	0067	0069	0076
0079	0107	0115	0119	0127	0152	0153	0214	0224	0236
0267	0268	0293	0298	0301	0302	0309	0312	0314	0318
0326	0394	0400	0587	0590	0601	0613	0643	0660	0732
0742	0747	0755	0763	0770	0811	0818	0837	0838	0839
0847	0862	0870	0936	0937	0951	0960	0965	1019	1024
1027	1032	1065	1083	1085	1088	1089	1092	1130	1136
1157	1179	1190	1194	1218	1225	1241	C024	M004	0006
0062	0701	0703	C025	M047	0046	0084	0103	0117	0123
0185	0205	0346	0418	0431	0447	0458	0464	0467	0471
0473	0498	0584	0588	0611	0645	0646	0650	0658	0706
0709	0720	0782	0784	0820	0836	0853	0869	0947	0972
0985	0993	1004	1006	1015	1020	1025	1057	1066	1121
1156	1222	C026	M015	0254	0553	0619	0647	0733	0941
0974	0986	1036	1048	1051	1058	1191	1195	1245	C027
M015	0256	0551	0621	0642	0728	0939	0975	0987	1037
1047	1050	1059	1189	1196	1244	C028	M038	0007	0026
0044	0066	0077	0095	0114	0118	0146	0155	0159	0161
0172	0269	0270	0295	0310	0313	0567	0602	0649	0655
0708	0746	0757	0764	0769	0772	0788	0858	0946	0968
1018	1039	1081	1086	1087	1090	C029	M029	0048	0097
0101	0264	0303	0308	0352	0398	0474	0475	0483	0487
0488	0489	0786	0787	0805	0876	0877	0955	0998	1096
1098	1100	1108	1125	1158	1227	1234	C030	M001	0004
C031	M015	0227	0233	0609	0610	0648	0729	0734	0938
0940	0980	0981	1040	1041	1055	1056	C032	M001	0696
C033	M008	0019	0262	0363	0511	0520	0536	1053	1239
C034	M026	0037	0053	0094	0124	0125	0169	0208	0247

0257	0340	0414	0765	0766	0803	0814	0815	0833	0834
0844	0845	1114	1143	1144	1145	1236	1238	C035	M014
0315	0389	0393	0429	0450	0461	0675	0689	0890	0963
1054	1072	1198	1240	C036	M093	0059	0083	0096	0142
0143	0151	0207	0218	0220	0222	0228	0235	0240	0245
0248	0251	0252	0255	0259	0323	0358	0366	0367	0369
0370	0372	0378	0380	0413	0425	0427	0439	0456	0492
0531	0539	0541	0547	0568	0570	0571	0580	0586	0594
0604	0618	0626	0628	0629	0680	0686	0693	0751	0752
0807	0808	0813	0823	0835	0841	0846	0855	0857	0868
0891	0893	0898	0899	0903	0914	0919	0922	0925	0927
0934	0994	1000	1001	1003	1069	1116	1140	1182	1183
1184	1186	1187	1188	1202	1205	1206	1212	1247	C037
M002	0397	0480	C038	M012	0042	0234	0410	0548	0644
0721	0817	0842	0943	0984	0991	1031	C039	M003	0065
0702	0753	C040	M011	0162	0266	0305	0349	0351	0440
0477	0485	0490	0532	0533	C041	M014	0025	0050	0102
0306	0307	0421	0476	0484	0737	0875	0879	0954	1095
1099	C042	M024	0001	0063	0070	0246	0258	0311	0325
0354	0416	0506	0562	0565	0573	0576	0783	0978	1035
1043	1044	1120	1166	1170	1174	1178	C043	M024	0061
0120	0260	0297	0327	0330	0341	0491	0499	0560	0581
0583	0760	0973	0979	1033	1034	1045	1046	1104	1167
1169	1172	1180	C044	M001	0003	C045	M001	0005	C046
M001	0043	C047	M012	0015	0371	0419	0422	0434	0486
0739	0878	0880	1008	1093	1094	C048	M007	0219	0229
0368	0557	0614	0636	0809	C049	M007	0022	0112	0882
0889	0901	0905	0935	C050	M002	0031	0697	C051	M001
0667	C052	M002	0011	0698	C053	M016	0021	0133	0135
0195	0359	0381	0385	0530	0534	0558	0665	0666	0688
0883	0923	1101	C054	M001	0012	C055	M005	0156	0275
0322	0332	0507	C056	M003	0013	0699	0700	C057	M006
0029	0130	0884	0894	0912	0915	C058	M002	0030	0705
C059	M002	0014	0695	C060	M005	0089	0881	0886	0906
0931	C061	M001	0002						

8.3.2 3-Gems with 28 Vertices

C001	M035	0137	0157	0213	0295	0352	0365	0408	0710
0909	0937	0943	1583	1923	1934	1993	2193	2241	2521
2680	2686	3490	3627	3770	3825	3827	3832	4079	4685
4714	5568	6508	6600	6808	6871	7035	C002	M069	0149
0218	0278	0372	0626	0642	0798	0818	1002	1130	1414
1504	1728	1771	1843	1870	1986	2018	2023	2041	2049
2058	2094	2106	2109	2130	2149	2154	2755	2769	2779
2850	2878	3008	3032	3100	3195	3236	3254	3281	3301
3553	3611	4004	4204	4269	4313	4323	4369	4371	4850
4871	4883	4888	5415	5469	5763	5773	5820	5940	6061
6309	6634	6712	6735	7151	7350	7497	7527	C003	M006
0172	0232	0399	0908	2185	4555	C004	M108	0127	0215
0270	0336	0370	0397	0448	0466	0486	0494	0567	0594
0600	0709	0725	0829	0879	0931	0936	0944	1101	1133
1149	1194	1199	1251	1280	1308	1401	1431	1432	1522
1581	1582	1598	1692	1695	1705	1706	1759	1862	1869
1887	1925	1933	2069	2186	2266	2281	2358	2364	2398
2442	2463	2589	2643	2660	2687	2733	3489	3499	3651
3771	3826	3830	3833	4078	4080	4088	4090	4126	4469
4495	4513	4514	4596	4619	4631	4652	4654	4666	4671
4686	4766	4783	4788	4960	5281	5427	5569	5603	5689
6056	6133	6553	6599	6807	6848	6859	6931	6935	6967
6979	7135	7144	7231	7259	7550	C005	M109	0156	0330
0331	0497	0557	0578	0796	0813	0920	1046	1131	1170
1399	1560	1792	1802	1844	1991	2059	2074	2140	2207
2227	2286	2306	2586	2591	2607	2620	2634	2656	2710
2777	2780	2781	2829	2845	2848	2851	2866	2868	2882
3006	3013	3042	3119	3158	3196	3200	3246	3280	3315
3320	3369	3479	3500	3520	3554	3733	4007	4073	4117
4205	4208	4219	4302	4319	4327	4370	4372	4431	4801
4849	4872	4924	4956	4963	5545	5615	5616	5834	5836
5838	5904	6003	6034	6062	6118	6179	6182	6283	6635
6705	6717	6724	6745	6748	6975	6977	6982	7007	7019
7119	7241	7342	7346	7526	7566	7701	C006	M001	0202

C007	M001	0002	C008	M003	0009	0012	4460	C009	M068
0040	0212	0346	0417	0538	0591	0610	0947	1045	1084
1102	1306	1757	1875	2102	2298	2704	2807	2863	3078
3107	3210	3503	3760	3767	3887	4058	4059	4067	4093
4164	4182	4185	4350	4784	4889	4953	4954	5049	5050
5053	5095	5212	5213	5231	5429	5492	5602	5614	5716
5737	5749	5762	5767	5993	5998	6000	6022	6040	6128
6318	6390	6547	6743	6969	6970	7586	7587	C010	M165
0069	0072	0082	0100	0154	0220	0233	0252	0253	0271
0283	0290	0332	0357	0423	0424	0500	0539	0595	0596
0641	0645	0647	0723	0756	0773	0778	0782	0794	0830
0849	0858	0883	0892	0935	0973	0997	1061	1132	1140
1150	1160	1161	1168	1197	1353	1364	1378	1404	1442
1451	1462	1580	1590	1605	1656	1670	1691	1753	1755
1842	1863	1874	1876	1886	1992	2053	2116	2180	2251
2290	2308	2357	2408	2441	2489	2522	2644	2725	2727
2804	2877	3148	3194	3436	3442	3574	3589	3621	3628
3768	3829	3834	3860	3967	3999	4003	4076	4104	4123
4125	4138	4194	4316	4337	4489	4590	4695	4698	4709
4759	4785	4794	4803	4823	4927	4967	4979	5148	5278
5382	5458	5480	5490	5604	5605	5688	6057	6098	6101
6187	6272	6273	6333	6506	6512	6513	6537	6551	6591
6595	6597	6623	6816	6832	6845	6852	6875	6876	6929
6964	6973	6983	7001	7003	7077	7078	7117	7136	7139
7220	7232	7257	7266	7742	C011	M271	0153	0201	0216
0239	0306	0315	0355	0411	0416	0425	0429	0526	0564
0570	0574	0721	0747	0750	0772	0776	0783	0787	0875
0916	0986	1006	1014	1016	1047	1054	1074	1076	1090
1092	1093	1095	1097	1103	1109	1142	1193	1226	1394
1450	1467	1481	1499	1507	1526	1568	1571	1653	1700
1754	1774	1822	1881	1883	1909	1915	1922	1924	1950
1976	1995	2013	2019	2042	2066	2067	2071	2101	2127
2156	2289	2316	2356	2409	2471	2499	2518	2658	2812
2814	2834	2852	3105	3181	3272	3376	3390	3393	3417
3435	3443	3444	3452	3454	3486	3491	3498	3578	3591
3705	3713	3717	3828	3831	3892	3893	3927	3978	3998

4001	4005	4039	4085	4089	4118	4124	4127	4140	4142
4145	4203	4211	4281	4282	4297	4307	4315	4404	4407
4430	4444	4446	4448	4450	4494	4497	4504	4543	4544
4574	4582	4595	4664	4677	4681	4682	4693	4694	4697
4703	4792	4828	4854	4867	4895	4904	4931	4937	4964
4966	5099	5151	5215	5277	5302	5321	5334	5357	5381
5389	5396	5425	5441	5479	5482	5546	5558	5564	5585
5591	5607	5610	5629	5664	5674	5675	5682	5757	5799
5806	5837	5846	5903	5942	5960	5968	5986	6065	6076
6099	6100	6102	6120	6131	6154	6170	6186	6188	6189
6190	6196	6226	6259	6266	6274	6280	6320	6334	6341
6350	6351	6401	6409	6472	6476	6598	6698	6771	6774
6844	6858	6861	6862	6873	6887	6896	6936	7020	7032
7093	7122	7125	7129	7145	7162	7208	7244	7245	7248
7250	7264	7265	7380	7399	7402	7468	7479	7541	7543
7568	7596	7605	7636	7673	7709	7733	7741	C012	M066
0255	0797	0801	1091	1212	1246	1421	1809	1919	1979
2030	2047	2065	2104	2107	2120	2208	2595	2614	2663
2724	2756	2869	2871	3782	3835	4041	4102	4195	4329
4612	4645	4732	4796	4856	5028	5131	5233	5318	5416
5461	5463	5543	5590	5687	5723	5905	5984	6049	6053
6228	6264	6311	6375	6451	6733	6777	6978	7121	7240
7330	7331	7348	7696	7719	7720	C013	M018	2418	2419
2477	2645	2983	2985	3881	3882	3922	3942	3943	3944
3947	7316	7388	7429	7430	7432	C014	M205	0126	0316
0322	0338	0368	0374	0471	0480	0511	0556	0568	0575
0805	0811	0918	0927	0938	0988	0994	1042	1050	1116
1145	1147	1200	1202	1215	1242	1249	1257	1260	1327
1381	1411	1440	1511	1524	1578	1599	1659	1704	1730
1743	1768	1777	1801	1817	1818	1821	1834	1835	1861
1865	1868	1918	1938	1961	1973	1988	2035	2050	2068
2082	2088	2108	2119	2148	2296	2309	2539	2546	2547
2585	2601	2694	2732	2784	2808	2824	2835	2836	2853
2855	2876	2885	3096	3131	3132	3286	3295	3461	3478
3487	3513	3568	3572	3575	3644	3674	3751	3759	3788
3816	3996	3997	4002	4029	4048	4075	4098	4105	4110

4122	4144	4202	4213	4231	4300	4314	4386	4405	4432
4604	4655	4718	4730	4761	4773	4874	4881	4885	4933
4991	5063	5103	5178	5343	5484	5493	5513	5606	5611
5628	5667	5673	5676	5686	5691	5781	5893	5895	5926
6020	6058	6132	6134	6153	6216	6229	6284	6297	6335
6410	6448	6457	6471	6527	6556	6596	6617	6731	6732
6740	6744	6763	6809	6829	6892	6913	6974	6994	7010
7031	7108	7134	7147	7167	7172	7176	7233	7237	7258
7275	7349	7417	7474	7569	7592	7595	7627	7633	7635
7703	7715	7721	C015	M001	0203	C016	M036	0244	1247
1781	2031	2043	2045	2103	2150	2151	2593	3224	3580
3703	3744	4017	4207	4583	4675	4727	4777	4847	4848
5052	5130	5157	5392	5460	5632	5768	5805	5844	6048
6312	6562	6626	7475	C017	M200	0092	0104	0146	0181
0246	0259	0260	0337	0343	0371	0450	0454	0498	0558
0563	0597	0601	0751	0761	0802	0806	0948	1018	1148
1156	1178	1239	1248	1276	1489	1502	1510	1533	1534
1544	1555	1650	1729	1770	1860	1873	1882	1899	1920
1983	1990	1994	2017	2048	2117	2123	2293	2326	2432
2454	2638	2695	2713	2750	2796	2827	2844	2902	2960
3007	3060	3122	3123	3143	3153	3174	3191	3204	3211
3279	3291	3296	3409	3439	3451	3477	3481	3497	3516
3565	3567	3570	3629	3715	3876	3979	4000	4006	4014
4024	4094	4095	4121	4143	4184	4196	4242	4270	4298
4334	4336	4338	4339	4383	4384	4410	4434	4493	4498
4528	4568	4570	4572	4575	4578	4579	4611	4689	4805
4806	4825	4857	4962	4980	5069	5091	5149	5150	5179
5274	5298	5307	5322	5361	5364	5379	5447	5481	5507
5542	5617	5699	5702	5703	5755	5765	5771	5786	5835
5854	5907	5927	6023	6037	6059	6064	6096	6103	6130
6135	6322	6323	6328	6339	6349	6382	6402	6450	6580
6605	6750	6803	6912	6944	6950	6953	7015	7036	7094
7124	7138	7195	7229	7517	7524	7547	7557	7558	7564
7612	7626	7634	7667	7669	7740	C018	M283	0207	0234
0273	0432	0470	0477	0487	0496	0505	0510	0546	0584
0589	0598	0662	0759	0769	0771	0779	0816	0827	0914

0930	0934	0941	1000	1079	1085	1087	1117	1163	1172
1209	1319	1395	1402	1409	1418	1478	1506	1514	1515
1516	1523	1551	1553	1584	1597	1600	1713	1758	1762
1805	1829	1831	1833	1885	1888	1889	1916	1917	1929
1945	1952	1967	1997	2006	2132	2136	2155	2199	2221
2261	2262	2270	2271	2291	2294	2343	2363	2397	2411
2413	2420	2439	2453	2456	2584	2588	2622	2635	2641
2676	2685	2857	3047	3099	3112	3142	3149	3155	3206
3207	3283	3288	3319	3371	3396	3399	3401	3404	3470
3495	3496	3515	3522	3526	3562	3597	3786	3793	3814
3883	3908	3909	3921	3935	3936	3945	3946	4008	4063
4106	4154	4206	4221	4291	4299	4317	4358	4360	4409
4455	4496	4506	4573	4646	4680	4683	4688	4722	4728
4738	4751	4762	4768	4772	4780	4790	4799	4844	4861
4875	4900	4906	4910	4916	5031	5044	5048	5082	5093
5118	5127	5128	5142	5168	5170	5190	5204	5210	5246
5276	5280	5308	5326	5370	5428	5432	5433	5443	5483
5485	5487	5505	5566	5575	5621	5630	5638	5669	5700
5715	5739	5741	5787	5788	5812	5917	5918	5945	5966
6041	6072	6105	6106	6126	6155	6160	6161	6183	6210
6219	6223	6286	6321	6326	6347	6348	6416	6558	6561
6578	6593	6603	6609	6620	6640	6641	6648	6689	6691
6813	6849	6853	6891	6930	6940	6966	6985	6999	7023
7042	7046	7049	7095	7132	7168	7169	7183	7184	7192
7197	7198	7201	7235	7273	7345	7367	7392	7431	7514
7520	7574	7584	7615	7671	7677	7688	7714	7717	7729
7745	C019	M004	0453	1127	1187	1232	C020	M305	0050
0275	0427	0475	0489	0509	0534	0565	0588	0599	0646
0651	0780	0781	0788	0789	0800	0814	0817	0917	0971
0985	1007	1030	1089	1138	1175	1214	1398	1439	1493
1498	1503	1505	1513	1535	1549	1569	1570	1589	1669
1708	1727	1783	1788	1790	1813	1820	1845	1856	1857
1972	1981	2004	2007	2039	2040	2051	2061	2062	2063
2093	2098	2114	2118	2121	2124	2125	2126	2128	2133
2135	2141	2142	2152	2181	2288	2305	2341	2560	2596
2653	2668	2709	2778	2805	2883	2884	3015	3017	3021

3037	3038	3041	3066	3113	3114	3116	3138	3144	3169
3180	3205	3238	3268	3275	3292	3325	3360	3361	3377
3381	3391	3402	3406	3407	3408	3410	3511	3519	3531
3571	3576	3579	3604	3608	3610	3623	3645	3648	3656
3676	3680	3692	3716	3718	3734	3738	3756	3758	3762
3789	3821	3939	3994	4018	4034	4038	4056	4057	4077
4081	4092	4096	4120	4128	4173	4175	4176	4186	4201
4283	4294	4333	4387	4390	4396	4397	4402	4406	4438
4439	4442	4447	4581	4589	4720	4737	4748	4851	4886
4898	4932	4986	4994	5021	5060	5087	5109	5123	5185
5195	5200	5206	5229	5232	5237	5273	5301	5313	5314
5317	5342	5376	5414	5420	5440	5442	5449	5476	5508
5510	5517	5536	5540	5547	5548	5559	5584	5586	5592
5608	5633	5634	5704	5705	5721	5738	5748	5789	5794
5843	5856	5857	5881	5925	5928	5999	6045	6067	6178
6180	6201	6204	6217	6237	6269	6270	6281	6289	6292
6294	6305	6319	6420	6460	6461	6475	6550	6575	6579
6629	6646	6710	6752	6782	6830	6883	7004	7021	7106
7131	7157	7173	7177	7186	7187	7196	7242	7254	7263
7359	7361	7375	7422	7465	7466	7482	7493	7499	7502
7510	7511	7513	7522	7565	7594	7620	7640	7662	7684
7687	7712	7731	7759	C021	M022	0143	0199	0209	0214
0219	0272	0282	0390	0395	0396	0420	0452	1011	1033
1195	1352	1576	1926	1930	4488	4550	4620	C022	M119
0230	0625	0693	0718	0726	1313	1372	1639	1641	2171
2172	2173	2204	2205	2229	2248	2263	2268	2272	2273
2332	2345	2347	2348	2392	2417	2421	2422	2478	2496
2502	2553	2555	2561	2573	2576	2581	2583	2625	2888
2897	2919	2924	2927	2928	2938	2939	2964	2965	2969
2970	2971	2978	2982	2984	2987	2988	2991	2993	2994
2996	2997	3000	3002	3003	3619	3620	3880	3884	3917
3918	3919	3920	3923	3924	4585	4587	6622	6686	6818
6824	6906	7066	7067	7084	7086	7216	7217	7295	7297
7300	7305	7310	7311	7312	7313	7319	7325	7328	7368
7374	7387	7389	7412	7413	7414	7428	7449	7450	7529
7530	7537	7538	7539	7540	7755	7756	7758	7760	C023

M111	0008	0058	0150	0188	0231	0289	0293	0339	0354
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1040	1100	1129	1250	1256	1290	1326	1356	1357	1403
1405	1427	1464	1482	1495	1508	1517	1520	1588	1606
1609	1635	1711	1740	1744	1837	1850	1866	1954	1971
2076	2190	2195	2224	2350	2399	2443	2519	2557	2617
2669	2690	2783	2818	2854	2872	2894	3316	3493	3593
3838	4099	4480	4482	4524	4558	4593	4599	4607	4659
4660	4870	4905	4907	4946	5279	5315	5426	5613	5680
6075	6140	6142	6157	6166	6525	6552	6559	6572	6583
6594	6615	6636	6671	6802	6900	6957	6968	7158	7164
7274	7421	C024	M329	0022	0052	0075	0178	0245	0247
0249	0350	0369	0382	0404	0415	0463	0492	0616	0686
0748	0810	0859	0860	0888	0894	0915	0966	0968	0978
0980	0987	0998	1013	1019	1020	1053	1062	1088	1108
1121	1143	1179	1183	1206	1210	1275	1278	1350	1388
1392	1393	1441	1443	1458	1519	1541	1550	1559	1572
1574	1607	1652	1658	1671	1749	1756	1760	1767	1796
1811	1812	1814	1819	1823	1825	1852	1872	1880	1884
1893	1906	1908	1914	1936	1939	1959	1962	1974	1975
1977	1982	1984	2009	2011	2029	2072	2075	2095	2096
2139	2146	2160	2165	2202	2278	2292	2317	2321	2325
2329	2337	2344	2374	2376	2401	2406	2438	2475	2476
2488	2513	2548	2550	2592	2615	2632	2633	2708	2745
2748	2774	2789	2792	2809	2828	2837	2858	2886	2901
2903	2933	2979	3106	3145	3172	3192	3193	3197	3382
3438	3455	3462	3472	3474	3476	3484	3592	3603	3643
3649	3653	3686	3697	3710	3787	3813	3819	3820	3925
3970	3983	4016	4027	4032	4065	4083	4091	4097	4100
4101	4133	4149	4150	4212	4252	4263	4265	4278	4311
4312	4348	4421	4433	4443	4445	4449	4457	4502	4656
4687	4692	4696	4704	4723	4744	4752	4795	4807	4846
4894	4925	4934	4940	4943	4949	4965	4981	4988	5002
5054	5098	5217	5270	5286	5287	5333	5335	5346	5446
5459	5467	5491	5509	5514	5539	5541	5560	5571	5635
5636	5642	5663	5668	5677	5681	5698	5719	5758	5759

5760	5792	5793	5840	5892	5898	5922	5957	6001	6024
6039	6069	6088	6095	6129	6172	6202	6218	6268	6271
6275	6282	6293	6299	6308	6415	6470	6473	6564	6587
6592	6604	6607	6608	6708	6709	6729	6764	6768	6773
6776	6815	6839	6897	6907	6928	6943	6971	6980	6988
6995	6996	7006	7008	7009	7074	7105	7109	7127	7141
7150	7166	7224	7247	7255	7260	7262	7293	7296	7363
7370	7371	7393	7444	7446	7457	7459	7492	7567	7600
7603	7617	7749	C025	M218	0158	0276	0361	0493	0508
0576	0786	0828	1003	1043	1424	1468	1471	1512	1540
1554	1766	1778	1810	1826	1827	1864	1978	1980	1987
2001	2036	2055	2073	2086	2115	2122	2143	2145	2157
2159	2253	2255	2378	2465	2608	2711	2794	2821	2856
2880	2881	2986	3014	3029	3077	3190	3226	3228	3229
3230	3251	3269	3282	3294	3312	3324	3326	3331	3337
3416	3456	3485	3488	3527	3528	3609	3630	3636	3654
3671	3684	3685	3712	3725	3752	3763	3776	3785	3800
3895	3902	3993	4036	4037	4049	4066	4147	4174	4177
4178	4183	4217	4264	4268	4287	4326	4352	4355	4363
4391	4392	4393	4394	4395	4398	4401	4424	4609	4614
4658	4705	4936	4948	4957	4976	4987	4995	5024	5027
5051	5058	5061	5067	5155	5156	5174	5216	5219	5230
5242	5244	5275	5304	5368	5380	5385	5424	5457	5462
5511	5515	5518	5538	5544	5593	5612	5631	5661	5706
5718	5743	5769	5783	5791	5803	5814	5817	5818	5825
5855	5882	5883	5923	5941	5943	5948	6006	6047	6093
6195	6214	6258	6327	6338	6449	6468	6479	6483	6485
6616	6643	6701	6711	6713	6714	6783	6835	6942	7034
7180	7256	7272	7333	7336	7366	7377	7469	7481	7489
7494	7507	7518	7523	7542	7619	7630	7642	7643	7698
7711	7726	7730	C026	M006	2314	2424	2427	2646	2649
4511	C027	M409	0015	0205	0227	0238	0279	0325	0342
0345	0353	0359	0375	0376	0377	0391	0392	0412	0458
0476	0483	0522	0523	0547	0559	0562	0566	0581	0582
0617	0643	0667	0687	0741	0745	0763	0826	0929	0932
0953	0991	1039	1048	1063	1082	1119	1120	1162	1171

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1419	1420	1453	1463	1465	1470	1497	1518	1537	1539
1542	1547	1548	1552	1558	1577	1594	1596	1620	1686
1709	1712	1739	1761	1824	1830	1867	1927	1932	1948
1989	2008	2021	2022	2037	2038	2131	2166	2197	2222
2228	2264	2265	2287	2313	2315	2320	2383	2388	2391
2405	2407	2433	2437	2440	2460	2462	2464	2507	2510
2568	2602	2606	2616	2627	2629	2647	2697	2699	2718
2744	2751	2754	2767	2768	2800	2811	2822	2859	2860
2895	2929	2935	2949	2995	3004	3005	3010	3018	3019
3026	3030	3036	3056	3075	3087	3091	3095	3103	3125
3137	3170	3171	3185	3186	3198	3203	3208	3227	3243
3244	3253	3270	3285	3293	3321	3344	3346	3350	3353
3355	3356	3357	3363	3372	3387	3400	3405	3445	3459
3483	3512	3566	3599	3622	3634	3635	3655	3664	3666
3667	3669	3693	3774	3777	3818	3841	3874	3888	3938
3948	3952	3954	3956	3959	3969	3972	4020	4025	4033
4060	4151	4160	4171	4179	4181	4188	4189	4199	4200
4232	4237	4240	4244	4253	4272	4273	4292	4324	4331
4340	4373	4385	4399	4422	4523	4678	4711	4739	4749
4767	4769	4779	4786	4789	4804	4855	4876	4879	4911
4918	4969	4975	4983	5029	5055	5072	5073	5079	5088
5122	5144	5167	5191	5205	5207	5208	5227	5236	5271
5282	5288	5312	5316	5324	5330	5339	5345	5358	5371
5384	5418	5444	5468	5486	5488	5500	5504	5506	5523
5562	5567	5570	5587	5666	5670	5683	5701	5740	5742
5747	5785	5815	5870	5901	5910	5914	5967	6008	6016
6017	6029	6042	6044	6107	6113	6115	6127	6146	6150
6158	6167	6194	6208	6209	6220	6222	6267	6288	6298
6325	6343	6356	6392	6403	6408	6417	6455	6462	6467
6526	6557	6581	6589	6590	6610	6611	6614	6647	6662
6683	6734	6749	6754	6781	6847	6857	6867	6886	6986
6987	6990	7000	7002	7011	7024	7043	7068	7076	7080
7096	7128	7202	7206	7270	7286	7309	7315	7317	7318
7320	7322	7340	7344	7373	7381	7394	7409	7420	7461
7487	7516	7528	7533	7534	7535	7570	7585	7616	7661

7666	7747	C028	M165	0020	0055	0061	0063	0084	0102
0179	0248	0262	0286	0312	0326	0327	0333	0334	0348
0356	0381	0384	0394	0407	0436	0495	0502	0525	0553
0618	0639	0644	0648	0654	0655	0660	0700	0703	0765
0831	0855	0856	0857	0861	0967	0974	0993	1051	1055
1086	1112	1157	1159	1166	1182	1196	1203	1245	1262
1270	1304	1340	1351	1417	1575	1595	1611	1614	1627
1688	1710	1752	1763	1794	1816	1839	1878	1905	1931
1953	1960	2024	2100	2168	2256	2297	2387	2410	2416
2455	2458	2461	2469	2503	2511	2512	2523	2526	2554
2639	2650	2730	2747	2817	2823	2874	3596	3790	3864
3866	3875	3896	3957	3958	3971	4046	4107	4130	4131
4137	4139	4472	4501	4505	4545	4597	4601	4636	4642
4653	4661	4663	4665	4672	4787	4793	4843	4922	4942
4955	5360	5692	5696	6148	6152	6510	6530	6681	6767
6779	6834	6851	6882	6927	6932	6947	6965	6991	7072
7103	7104	7107	7159	7225	7246	7251	7253	7553	C029
M098	0128	0335	0465	0673	0678	0696	0714	0715	0755
0803	0815	0886	0887	1124	1302	1309	1312	1371	1593
1637	1640	2176	2177	2217	2283	2333	2457	2466	2467
2468	2509	2530	2536	2577	2657	2728	2731	2740	2742
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3949	3950	3953	4026	4437	4716	4764	4833	4902	5260
5310	5350	5353	5401	5407	5452	5526	5535	5573	5598
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6864	6937	7053	7071	7288	7289	7426	7427	7433	C030
M253	0152	0380	0428	0438	0512	0518	0521	0535	0663
0744	0795	0874	0907	0923	0949	0979	0989	1064	1096
1118	1222	1258	1272	1274	1298	1333	1434	1536	1616
1626	1629	1649	1747	1776	1791	1891	1944	2002	2003
2087	2276	2552	2556	2652	2737	2879	2967	3001	3039
3057	3058	3071	3081	3082	3085	3086	3134	3139	3152
3159	3164	3168	3213	3215	3220	3222	3225	3233	3235
3237	3252	3266	3287	3303	3322	3359	3418	3507	3577
3584	3585	3672	3678	3711	3737	3747	3783	3796	3840

3865	3886	3900	3901	3903	3905	3926	3931	3951	4012
4015	4019	4052	4062	4167	4169	4170	4172	4220	4258
4271	4274	4275	4276	4279	4284	4290	4303	4330	4341
4347	4361	4413	4571	4580	4598	4740	4743	4763	4770
4771	4810	4892	4897	4959	4968	4985	5032	5045	5046
5064	5065	5094	5143	5176	5184	5199	5250	5319	5323
5336	5375	5386	5388	5421	5423	5430	5431	5473	5499
5503	5520	5521	5623	5761	5774	5778	5779	5784	5795
5798	5819	5845	5980	6005	6009	6013	6015	6068	6078
6083	6084	6110	6174	6176	6198	6227	6231	6232	6235
6236	6240	6241	6246	6248	6250	6330	6358	6361	6365
6366	6367	6369	6384	6386	6387	6393	6396	6400	6412
6458	6459	6466	6541	6693	6699	6738	6790	6860	6888
6894	6993	7013	7016	7022	7029	7033	7063	7100	7126
7165	7190	7238	7239	7283	7285	7302	7314	7406	7408
7411	7419	7434	7476	7546	7577	7589	7613	7629	7631
7639	7664	7708	7725	C031	M227	0159	0251	0258	0319
0367	0386	0435	0455	0501	0503	0572	0586	0657	0676
0699	0707	0711	0742	0807	0836	0844	0924	0958	1057
1069	1081	1146	1154	1211	1218	1219	1223	1281	1287
1292	1296	1307	1410	1422	1449	1490	1561	1564	1666
1668	1685	1721	1769	1780	1786	1828	1847	1859	1903
2034	2052	2054	2090	2113	2129	2138	2144	2210	2242
2302	2395	2549	2590	2603	2612	2665	2688	2716	2717
2741	2746	2825	2893	2898	2916	2930	2941	2942	2944
2950	2952	2956	2958	3050	3052	3111	3121	3124	3128
3165	3177	3221	3298	3310	3318	3332	3345	3349	3351
3373	3394	3403	3411	3414	3440	3453	3458	3463	3473
3482	3510	3535	3677	3706	3761	3778	3792	3806	3811
3822	3823	3836	3839	3977	4040	4064	4108	4114	4156
4162	4209	4255	4343	4349	4378	4380	4551	4629	4633
4667	4853	4862	4868	4877	4923	4938	4947	4993	5026
5056	5057	5062	5194	5338	5391	5397	5412	5413	5417
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6147	6212	6213	6277	6331	6337	6478	6534	6560	6639

6645	6652	6656	6658	6663	6684	6827	7044	7051	7088
7102	7142	7146	7181	7203	7207	7209	7211	7236	7249
7276	7334	7335	7356	7358	7424	7425	7435	7436	7509
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0735	1331	1638	1644	1647	1648	1681	2163	2164	2201
2213	2237	2274	2303	2353	2354	2372	2393	2423	2425
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2537	2558	2559	2563	2569	2570	2572	2574	2578	2624
2648	2674	2678	2681	2682	2689	2691	2696	2890	2905
2911	2920	2925	2931	2973	2974	2990	2999	3929	3930
3933	4499	4500	4602	6625	6661	6675	6685	6688	6690
6695	6819	6826	6920	6921	6926	6959	7287	7304	7306
7324	7390	7391	7416	7442	7448	7451	7456	7754	7757
C033	M532	0189	0211	0268	0285	0292	0320	0329	0347
0378	0426	0430	0434	0459	0461	0464	0468	0469	0488
0499	0504	0507	0520	0527	0532	0536	0543	0545	0548
0560	0602	0604	0605	0606	0607	0613	0627	0633	0679
0688	0719	0733	0762	0764	0766	0777	0809	0835	0839
0842	0854	0885	0977	1004	1049	1056	1058	1070	1083
1104	1107	1113	1144	1152	1165	1173	1174	1176	1192
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1408	1412	1416	1425	1438	1452	1461	1466	1480	1485
1563	1565	1567	1573	1628	1631	1663	1707	1720	1722
1724	1725	1726	1745	1746	1764	1772	1793	1797	1798
1804	1806	1848	1853	1855	1890	1895	1900	1904	1910
1949	1965	1970	2020	2025	2027	2028	2060	2064	2080
2083	2091	2092	2099	2153	2169	2191	2196	2220	2239
2243	2328	2334	2342	2380	2403	2450	2451	2492	2498
2580	2582	2587	2594	2599	2621	2626	2630	2651	2655
2706	2707	2712	2715	2722	2735	2736	2785	2790	2795
2813	2816	2830	2833	2839	2841	2842	2867	2896	2907
2908	2912	2936	2937	2963	2976	2977	2992	3011	3016
3034	3040	3046	3048	3049	3054	3061	3067	3074	3080
3092	3094	3097	3101	3115	3120	3126	3129	3135	3147
3150	3219	3255	3260	3289	3299	3304	3306	3311	3314

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3536	3537	3538	3539	3614	3631	3639	3650	3662	3670
3689	3690	3694	3698	3707	3714	3723	3736	3753	3773
3775	3784	3805	3809	3848	3854	3869	3877	3890	3891
3894	3904	3906	3907	3912	3961	3962	3965	3973	3974
3982	3986	3987	4023	4045	4047	4050	4055	4068	4069
4103	4109	4148	4157	4163	4166	4215	4216	4218	4254
4256	4259	4305	4306	4344	4381	4389	4411	4419	4427
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4734	4735	4736	4741	4742	4745	4776	4778	4798	4818
4821	4822	4829	4839	4845	4852	4863	4864	4866	4878
4890	4901	4913	4917	4920	4926	4930	4950	4970	4971
4972	4977	4989	4990	4992	4996	5001	5008	5014	5023
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5166	5183	5235	5251	5269	5284	5309	5327	5355	5366
5377	5390	5411	5419	5465	5537	5561	5609	5637	5639
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5865	5868	5873	5874	5885	5900	5929	5930	5935	5937
5939	5959	5965	5988	5989	6027	6035	6052	6086	6087
6090	6091	6116	6122	6125	6143	6149	6151	6156	6173
6197	6205	6211	6278	6310	6329	6332	6336	6340	6346
6381	6394	6397	6421	6477	6480	6531	6540	6555	6637
6638	6651	6664	6697	6719	6769	6789	6805	6814	6837
6840	6856	6863	6865	6889	6911	6933	6934	6952	6992
6998	7025	7027	7037	7038	7045	7055	7056	7059	7064
7073	7075	7085	7091	7097	7116	7130	7148	7154	7156
7161	7182	7191	7193	7194	7204	7210	7214	7222	7243
7252	7298	7299	7332	7338	7352	7357	7398	7404	7418
7437	7438	7480	7531	7532	7572	7607	7624	7628	7683
7692	7716	7734	7746	C034	M042	0028	0051	0145	0182
0184	0257	0301	0363	0533	0552	0555	0659	0963	1377
1382	1426	1459	1492	1665	1723	1734	1735	1911	1958
1966	4044	4463	4532	4548	4594	4617	4641	4674	4909
6136	6163	6528	6539	6563	6571	6585	6602	C035	M330
0160	0186	0190	0191	0277	0314	0349	0385	0519	0524

0554	0577	0590	0649	0664	0669	0684	0698	0705	0746
0770	0847	0921	0933	0955	0961	0975	1005	1060	1066
1077	1080	1167	1191	1198	1237	1241	1334	1455	1456
1457	1469	1475	1488	1501	1509	1528	1529	1546	1604
1608	1646	1689	1782	1785	1787	1858	1871	1896	1907
1968	2010	2015	2033	2084	2215	2223	2307	2312	2368
2375	2385	2400	2452	2459	2474	2504	2508	2514	2525
2545	2551	2597	2605	2628	2698	2700	2701	2703	2705
2719	2752	2753	2757	2758	2797	2798	2803	2815	2826
2913	2914	2934	2980	3020	3024	3027	3028	3062	3065
3070	3083	3084	3141	3182	3199	3232	3234	3240	3256
3265	3273	3297	3308	3329	3335	3347	3354	3358	3364
3365	3366	3389	3441	3492	3598	3641	3657	3665	3668
3675	3755	3779	3843	3852	3853	3867	3897	3898	3913
3928	3932	3934	3937	3940	4010	4013	4146	4158	4165
4168	4187	4191	4192	4198	4210	4226	4241	4245	4251
4262	4325	4354	4368	4382	4400	4461	4462	4531	4639
4662	4679	4684	4691	4706	4858	4873	4880	4914	4961
5003	5011	5071	5081	5092	5117	5124	5135	5153	5158
5161	5169	5220	5224	5226	5228	5234	5272	5289	5305
5320	5331	5344	5363	5369	5387	5394	5395	5422	5448
5502	5563	5620	5626	5643	5662	5671	5695	5790	5801
5802	5821	5823	5867	5878	5886	5897	5902	5909	5916
5963	5991	6011	6014	6019	6046	6063	6079	6109	6114
6144	6168	6171	6175	6206	6215	6221	6345	6352	6353
6354	6355	6357	6374	6395	6399	6406	6407	6419	6426
6465	6481	6484	6549	6576	6584	6588	6606	6700	6716
6736	6747	6758	6762	6772	6817	6866	6868	6878	6895
6899	6908	6915	6941	6976	7012	7017	7060	7062	7098
7152	7234	7284	7291	7303	7329	7378	7382	7395	7397
7410	7415	7439	7454	7455	7478	7556	7559	7583	7590
7614	7621	7625	7641	7644	7645	7658	7668	7707	7752
C036	M033	0223	0683	0734	1344	1750	2187	2672	2770
2773	3161	3184	3187	3216	3449	3464	3560	3721	3765
3975	4222	4236	4238	4243	4420	4425	4533	6788	6918
6919	7082	7089	7485	7486	C037	M033	0240	0692	0722

1343	1751	2267	2671	2771	2772	3163	3214	3261	3504
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4417	4428	4535	6787	6904	6905	7079	7090	7484	7488
C038	M002	0402	4510	C039	M067	0033	0093	0108	0110
0170	0197	0229	0236	0237	0296	0324	0620	0623	0712
0843	0868	0903	1317	1321	1619	1623	1683	2209	2216
2389	2390	2412	2446	2448	2490	2494	2495	2505	2951
2968	3797	3802	3807	3885	3914	3968	4507	4559	6621
6666	6785	6786	6804	6821	6823	6877	6879	6902	6951
6954	6962	7065	7092	7112	7113	7213	7215	7221	7423
7536	7744	7748	C040	M315	0187	0243	0254	0366	0373
0460	0462	0472	0491	0531	0544	0550	0573	0580	0583
0681	0689	0701	0704	0731	0732	0865	0890	0893	0897
0925	0951	0956	0962	0965	0976	0983	0995	1052	1105
1114	1153	1169	1181	1184	1204	1264	1283	1322	1323
1335	1348	1385	1407	1437	1446	1447	1479	1566	1661
1676	1677	1679	1680	1687	1702	1717	1773	1795	1808
1832	1846	1892	1898	1902	1912	1935	1940	1942	1943
1963	1985	2026	2081	2085	2097	2110	2147	2184	2188
2198	2275	2279	2282	2300	2318	2324	2330	2377	2402
2444	2445	2520	2524	2533	2538	2598	2600	2610	2611
2613	2623	2631	2636	2734	2786	2799	2810	2832	2838
2843	2861	2909	2917	3022	3023	3035	3043	3045	3063
3064	3068	3072	3090	3102	3127	3130	3136	3173	3183
3290	3300	3307	3309	3328	3334	3429	3460	3466	3540
3552	3633	3663	3687	3688	3696	3702	3708	3720	3780
3791	3812	3846	3847	3849	3868	3870	3872	3873	3980
3981	3985	4028	4051	4070	4111	4115	4153	4214	4233
4257	4260	4267	4277	4304	4332	4412	4414	4426	4554
4647	4724	4733	4746	4754	4791	4809	4817	4840	4860
4893	4899	4919	4921	4939	4998	5015	5016	5084	5086
5089	5106	5110	5115	5125	5138	5175	5189	5223	5247
5249	5252	5332	5356	5362	5522	5622	5640	5644	5678
5693	5717	5720	5724	5800	5807	5813	5829	5831	5858
5872	5877	5884	5934	5936	5938	6012	6036	6054	6089
6097	6111	6112	6117	6123	6145	6199	6203	6233	6262

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6577	6627	6654	6668	6669	6676	6702	6715	6723	6753
6765	6770	6791	6795	6796	6831	6836	6838	6841	6842
6890	6893	6898	6909	6922	6949	6989	6997	7014	7018
7057	7099	7133	7149	7160	7163	7205	7223	7261	7290
7326	7405	7407	7443	7571	7573	7606	7623	7632	7751
C041	M052	0071	0129	0169	0196	0228	0288	0297	0323
0619	0621	0694	0834	0867	0902	1310	1316	1610	1612
1682	2161	2211	2218	2311	2366	2367	2447	2485	2506
2515	2887	2906	3859	3861	3878	4465	6657	6672	6780
6797	6798	6801	6825	6880	6881	6948	6958	6963	7111
7228	7292	7440	7753	C042	M201	0105	0132	0135	0192
0456	0668	0716	0724	0790	0812	0819	0821	0822	0823
0880	0884	0895	0896	0901	0906	1098	1269	1303	1329
1336	1347	1365	1543	1587	1632	1642	1645	1678	1715
1854	2111	2112	2284	2331	2336	2339	2370	2371	2470
2479	2480	2482	2532	2571	2579	2618	2619	2683	2692
2743	2749	3009	3051	3076	3231	3248	3263	3543	3546
3548	3587	3601	3602	3605	3607	3660	3683	3700	3730
3732	3735	3742	3799	3863	4030	4053	4074	4132	4134
4135	4152	4280	4375	4451	4719	4758	4765	4774	4830
4838	4842	4974	5013	5020	5033	5035	5041	5047	5112
5121	5134	5136	5140	5141	5177	5225	5239	5253	5257
5261	5264	5290	5293	5311	5348	5351	5354	5399	5402
5404	5408	5454	5489	5525	5530	5532	5533	5552	5554
5576	5578	5594	5601	5646	5648	5654	5655	5656	5659
5660	5729	5732	5734	5841	5849	5850	5860	5871	5891
5912	5921	5949	5969	5972	5975	5977	5982	5983	5985
5990	6254	6301	6314	6315	6428	6435	6437	6438	6440
6444	6445	6486	6487	6496	6498	6505	6633	6653	6869
6870	6938	7061	7070	7185	7504	7525	7548	7576	7652
7657	7659	7675	7685	7699	7735	7739	C043	M039	0011
0018	0032	0067	0088	0130	0176	0194	0195	0200	0256
0299	0304	0313	0406	0414	0421	0433	0697	0864	0982
1115	1158	1164	1180	1240	1263	1362	1877	1928	1941
3845	4522	4536	4547	4561	4638	4670	6509	C044	M028

0070	0073	0074	0101	0114	0115	0118	0163	0611	0850
0853	1277	1674	1701	2158	2946	3808	3966	4563	6515
6533	6569	6655	6820	7083	7114	7219	7743	C045	M068
0222	0224	2666	2667	2763	2764	2765	2766	3167	3212
3242	3257	3262	3450	3465	3505	3551	3556	3561	3719
3726	3879	4227	4230	4249	4250	4366	4367	4616	5075
5080	5159	5162	5188	5202	5809	5811	5822	5826	5828
5830	6251	6252	6260	6261	6370	6373	6376	6380	6423
6424	6737	6741	6751	6757	6759	6760	6761	6799	6800
7560	7561	7579	7580	7601	7602	7646	7647	C046	M022
0014	0117	0131	0225	0631	1337	1643	1732	2230	2244
2369	2481	2487	2529	2566	2677	4477	4608	6544	6673
6923	6960	C047	M018	0027	0066	0080	0107	0140	0167
0317	0841	1369	2384	2415	4509	4627	4643	4668	6644
6917	7552	C048	M019	0113	0139	0142	2301	2373	2429
2535	2562	2575	2673	2693	4475	4606	6649	7294	7307
7323	7369	7452	C049	M026	0065	0087	0109	0112	0134
0624	0838	1624	2219	2235	2238	2414	2472	2500	2516
2975	3915	4560	4588	6624	6667	6692	6811	6903	6925
6955	C050	M020	0106	0111	0241	0321	0629	0846	0871
1294	1299	1332	1359	1630	1742	2915	4562	6546	6566
6916	7069	7218	C051	M033	0054	0064	0083	0085	0090
0094	0103	0119	0120	0615	0685	0690	0862	0863	1266
1284	1672	1675	1699	1737	2904	3862	4452	4453	4534
6517	6543	6574	6650	6766	6946	7110	7227	C052	M020
0041	0062	0086	0116	0133	0832	1345	1613	2162	2170
2236	2246	2258	2449	2491	4485	6674	6694	6812	6961
C053	M016	0081	0221	0242	0287	0628	0675	0870	1300
1330	1354	1355	1615	1741	4484	6545	6565	C054	M014
0029	0048	0057	0078	0138	0165	0318	0833	1367	4464
4486	4623	4640	4673	C055	M006	0013	0044	0053	0121
1733	6542	C056	M008	0031	0068	0091	0691	1360	1673
4564	6573	C057	M003	0060	0614	6516	C058	M002	0001
4518	C059	M002	0003	4520	C060	M141	0148	0291	0294
0467	0571	0652	0658	0666	0672	0728	0730	0736	0738
0848	0869	0904	0910	0928	1031	1041	1059	1068	1094

1279	1305	1320	1341	1349	1376	1430	1477	1521	1525
1601	1617	1618	1657	1684	1716	1800	1838	1841	1851
1921	1996	2134	2183	2194	2203	2231	2240	2247	2257
2259	2260	2285	2295	2362	2365	2517	2540	2565	2567
2659	2662	2670	2675	2723	2782	2793	2819	2820	2953
2957	3154	3209	3397	3494	3523	3594	3615	3616	3757
3769	3772	3801	3824	4082	4084	4086	4087	4481	4483
4487	4512	4600	4603	4648	4649	4657	4713	4715	4725
4731	4747	4800	4945	5245	5383	5434	5565	5627	5690
6141	6225	6265	6665	6670	6677	6679	6696	6792	6793
6794	6806	6846	6872	6874	6956	7030	7143	7199	7212
7226	7321	7441	7458	7515	7544	7545	7750	C061	M044
0217	0364	0585	0946	0950	1024	1216	1254	1562	2016
2206	2252	2865	3323	3362	3380	3529	3530	3557	3695
3766	4035	4295	4615	4707	4884	4891	5243	5439	5549
5619	5712	5722	5744	5756	5770	5775	6191	6224	6452
6722	7174	7383	7506	C062	M018	0457	1075	2014	2070
2105	2299	3271	3590	4896	4973	5132	5410	5624	6033
6055	6121	7230	7710	C063	M277	0147	0151	0281	0358
0474	0506	0528	0530	0587	0593	0670	0677	0680	0720
0737	0739	0740	0743	0753	0774	0775	0799	0851	0876
1065	1067	1252	1267	1311	1406	1413	1415	1444	1448
1621	1622	1775	1784	1894	1937	2005	2012	2079	2137
2167	2178	2232	2233	2234	2340	2352	2359	2360	2361
2382	2396	2528	2564	2604	2609	2637	2642	2721	2729
2738	2806	2846	2849	2870	2875	2889	2899	2900	2910
2943	2945	2947	2948	2954	2955	2959	2981	3033	3053
3055	3069	3160	3162	3166	3178	3223	3239	3274	3284
3302	3392	3431	3433	3434	3437	3446	3448	3471	3506
3508	3518	3525	3558	3569	3573	3581	3582	3583	3586
3595	3647	3739	3745	3746	3748	3754	3781	3794	3795
3810	3815	3844	3871	3989	4031	4061	4248	4289	4308
4310	4321	4322	4335	4342	4345	4346	4356	4357	4362
4364	4365	4586	4630	4637	4750	4757	4816	4835	4841
4865	4869	4928	4958	4982	4984	5005	5042	5066	5068
5077	5173	5180	5186	5201	5240	5303	5325	5373	5378

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5672	5710	5713	5714	5754	5772	5780	5782	5832	5887
5888	5911	5971	5987	6007	6010	6043	6066	6070	6073
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6230	6238	6239	6245	6247	6276	6279	6285	6359	6363
6364	6368	6385	6389	6391	6398	6474	6493	6628	6678
6680	6682	6687	6718	6746	6755	6778	6822	6850	6884
6885	6910	6984	7005	7028	7041	7047	7052	7054	7101
7268	7269	7327	7339	7343	7362	7400	7401	7403	7460
7470	7578	7591	7604	7608	7648	7649	7650	7651	7665
7674	7682	7693	C064	M044	0398	0785	0926	1022	1071
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M051	0026	0162	0183	0193	0267	0302	0422	0540	0661
0702	0889	0954	0959	1122	1141	1363	1389	1454	1496
1585	1664	1667	1690	1719	1738	1913	1956	1957	1964
2394	2404	2961	3817	3855	4479	4565	4618	4644	4669
4908	6137	6165	6532	6538	6568	6601	6810	6828	7140
7200	7551	C066	M002	0056	0401	C067	M114	0175	0185
0328	0351	0445	0706	0749	0840	0913	0922	0942	0945
0964	0972	0981	1001	1012	1078	1151	1238	1391	1538
1603	1625	1731	1748	1765	1789	1998	2032	2179	2212
2214	2310	2435	2473	2493	2501	2544	2640	2679	2684
2702	2720	2759	2760	2787	2801	2802	2932	2962	3079
3175	3241	3333	3367	3374	3379	3637	3638	3851	3889
4009	4011	4043	4116	4190	4247	4353	4423	4569	4753
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5393	5875	5896	5908	5964	5992	6004	6018	6169	6200
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7058	7115	7153	7301	7308	7372	7396	7445	7447	7453
7588	7622	C068	M166	0418	0592	0671	0727	0792	0793
0804	0891	0898	0899	0984	1021	1189	1285	1288	1346
1368	1370	1379	1380	1545	1591	1592	1660	1662	1698
1703	2089	2304	2327	2355	2654	2840	3031	3059	3133

3146	3217	3218	3249	3313	3338	3425	3426	3427	3468
3541	3542	3588	3617	3618	3658	3699	3701	3709	3729
3743	3749	3899	3910	4223	4225	4228	4235	4379	4708
4760	4997	4999	5004	5006	5007	5017	5036	5037	5038
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5255	5267	5268	5347	5349	5400	5435	5477	5524	5534
5572	5595	5596	5647	5725	5730	5810	5824	5847	5848
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6050	6051	6234	6243	6244	6253	6255	6256	6257	6291
6302	6371	6377	6413	6414	6429	6430	6431	6432	6433
6447	6488	6491	6630	6707	6730	6739	6843	6901	6914
7026	7155	7170	7175	7351	7463	7467	7519	7549	7598
7599	7618	7637	7638	7678	7680	7681	7697	7736	7737
C069	M006	1473	1494	6138	6164	6613	6618	C070	M006
1474	1491	6139	6162	6612	6619	C071	M112	0155	0261
0308	0362	0622	0635	0695	0713	0784	0791	0808	0824
0825	0852	0881	0900	0905	1073	1186	1234	1236	1301
1530	1579	1633	1714	1799	1807	2077	2226	2280	3245
3258	3336	3343	3469	3600	3613	3632	3731	4054	4071
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4781	4782	4815	4834	4935	4941	5010	5034	5040	5119
5145	5193	5241	5248	5254	5263	5266	5291	5403	5405
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5973	5981	6263	6300	6316	6317	6427	6439	6442	6446
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C072	M010	0177	0274	0439	0447	1023	1125	1188	1190
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C074	M004	0059	0141	1373	4476	C075	M027	0473	0579
0754	0758	1557	2277	2335	3342	3547	3624	3640	4756
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0630	0650	1177	1314	1315	1400	1460	1476	1484	1487
1651	1969	2175	2323	2381	2386	2431	2541	2543	2726
2762	2776	2847	2864	2940	3012	3044	3089	3098	3117

3118	3140	3151	3188	3189	3247	3250	3264	3267	3305
3422	3524	3533	3563	3626	3673	3704	3727	3740	3850
3963	3964	3988	3991	3992	4155	4180	4239	4328	4359
4408	4415	4418	4567	4592	5022	5043	5096	5100	5129
5152	5165	5197	5209	5222	5238	5328	5466	5618	5804
5879	5944	6028	6032	6184	6296	6344	6405	6454	6464
6535	6570	6631	6659	7039	7048	7081	7179	7365	7385
7483	7495	7498	7503	7521	7706	7723	7727	C078	M115
0341	0344	0431	0479	0481	0482	0542	0656	0911	0957
0960	1017	1472	1483	1486	1586	1602	1718	1999	2000
2174	2319	2322	2346	2379	2436	2527	2542	2664	2739
2761	2775	2918	3025	3073	3088	3093	3104	3110	3176
3179	3201	3202	3330	3419	3421	3447	3509	3532	3564
3625	3679	3691	3728	3741	3842	3955	3960	3976	3990
3995	4159	4161	4197	4234	4261	4266	4293	4296	4416
4454	4591	5025	5078	5101	5120	5126	5147	5154	5198
5214	5221	5283	5306	5329	5340	5816	5880	5946	6030
6031	6193	6287	6342	6404	6456	6463	6529	6567	6632
6660	7040	7050	7087	7178	7360	7364	7386	7496	7500
7501	7512	7704	7722	7728	C079	M006	0514	0516	4492
4811	4814	7554	C080	M006	0517	4490	4491	4812	4813
7555	C081	M015	3659	3682	5012	5018	5139	5203	5869
5889	6434	6443	6703	6721	7582	7610	7655	C082	M015
3661	3681	5009	5019	5146	5192	5866	5890	6436	6441
6706	6720	7581	7609	7656	C083	M003	6372	6379	7562
C084	M002	6378	7563	C085	M008	0017	0210	0360	0403
0845	1255	4471	4515	C086	M001	0019	C087	M007	0025
0035	0046	0079	0166	1325	4474	C088	M017	0180	0305
0413	1185	1231	1271	1361	1815	1879	1901	2791	4113
4459	4503	4549	4729	6775	C089	M051	0515	0717	0939
1224	1228	1396	1445	1634	1803	2349	2351	3750	3911
4193	4288	4320	4650	5171	5172	5196	5285	5337	5341
5464	5472	5474	5497	5776	5777	5796	5894	5978	5979
6185	6242	6249	6360	6362	6388	6642	6727	6939	7267
7271	7473	7593	7597	7679	7691	7705	7724	C090	M045
0529	0665	0682	0708	0820	0837	1268	1286	1338	1342

1423	1428	1429	1433	1435	1436	1636	1836	1840	1955
2254	3467	3646	3837	3856	3857	4621	4628	4632	4635
4690	4824	4915	5352	5406	5450	5553	5574	5726	5933
6071	6077	6304	6494	7137	C091	M003	0006	0024	4473
C092	M055	0485	0603	0752	0757	0760	1556	2338	2661
3339	3341	3642	3764	4403	4701	4712	4819	4820	4826
4836	4837	5258	5259	5265	5292	5294	5295	5478	5529
5555	5557	5580	5582	5583	5736	5952	5953	5954	5956
5994	5995	6500	6501	6502	6504	7188	7189	7337	7376
7379	7660	7670	7676	7686	7700	7738	C093	M006	0005
0171	0310	0877	4538	4556	C094	M021	0095	0096	0161
0174	0300	0307	0383	0449	1010	1037	1135	1201	1375
1696	2714	3652	4468	4508	4624	6520	6521	C095	M022
0541	0768	0940	0952	1106	1155	3259	3430	4351	4577
4584	4903	5398	5913	6453	6548	6742	6972	7171	7353
7354	7663	C096	M024	0089	0097	0264	0265	0298	0387
0410	0444	0446	0451	1009	1026	1034	1111	1137	1374
1655	1693	1697	1951	4521	4537	4626	6511	C097	M028
0513	0609	0767	1500	1532	1897	2044	2245	4285	4436
4576	4610	5437	5445	5470	5494	5764	5766	5924	6002
6119	6726	6728	7118	7477	7611	7695	7702	C098	M066
0030	0235	0478	0484	0912	0996	1044	1139	1205	1289
1527	1779	2046	2056	2057	2862	3108	3109	3156	3157
3276	3277	3278	3352	3386	3398	3415	3480	3501	3502
3559	4286	4301	4318	4374	4388	4435	4882	4887	5030
5097	5133	5211	5359	5365	5367	5697	5745	5746	5906
5915	5958	5996	5997	6021	6177	6181	6324	7347	7355
7464	7472	7490	7505	7508	7689	C099	M026	0123	0125
0204	0379	0389	0409	0419	0443	0638	0640	0969	1008
1025	1028	1110	1128	1654	1694	1946	4456	4470	4525
4530	4625	6519	6524	C100	M030	0016	0023	0038	0098
0099	0144	0208	0263	0269	0388	0405	0437	0441	0653
0873	0970	0992	1027	1029	1136	1220	1243	1736	1947
4517	4527	4541	4566	6523	6536	C101	M016	2250	3383
3384	4022	5299	5300	5372	5588	5751	5752	5753	7341
7462	7471	7672	7690	C102	M008	0010	0021	0164	0198

0206	0866	4542	4557	C103	M010	0490	2192	2249	3385
3984	4021	4717	5374	5589	5750	C104	M008	0042	0045
0309	0400	0634	0636	4467	4540	C105	M016	0043	0047
0076	0122	0303	0442	0990	1015	1221	1358	2788	4458
4529	4546	6514	6522	C106	M003	7384	7491	7694	C107
M013	0004	0037	0039	0173	0266	0440	0878	1036	1038
1134	4478	4519	6507	C108	M005	0280	0311	0637	4466
4539	C109	M005	0077	0124	1035	4526	6518		

Bibliography

[Ale30] J.W. Alexander. The combinatorial theory of complexes. *Ann. Math.*, 31:292–320, 1930.

[BM76] J. A. Bondy and U. S. R. Murty. *Graph Theory with Applications.* American Elsevier, 1976.

[BZ84] M. Boileau and H. Zieschang. Heegaard genus of closed orientable Seifert manifolds. *Invent. Math.*, 76:455–468, 1984.

[CAY] CAYLEY. V.3.7 export, Computational Algebra Group, J. Brownie et al. User's Manual, Univ. Sydney - Australia (1989).

[CM72a] H. S. M. Coxeter and W. O. Moser. *Generators and relations for the discrete groups.* Springer, 1972.

[CM72b] H.S.M. Coxeter and W.O. Moser. *Generators and relations for the discrete groups.* Springer, 1972.

[Cos92] A. Costa. Personal communication. 1992.

[Dur92] C. Durand. Geração e classificação de 3-variedades. Master Thesis presented at UFPE, 1992.

[Fer79] M. Ferri. Crystallizations of 2-fold branched covering of S^3. *Proc. Amer. Math. Soc.*, 73:271–276, 1979.

[Fer85] L. Ferri. Personal communication. 1985.

[FG82] M. Ferri and C. Gagliardi. Crystallization moves. *Pacific J. Math.*, 100:85–103, 1982.

[FGG86] S. Ferri, C. Gagliardi, and L. Graselli. A graph-theoretical representation of PL-manifolds: a survey on crystallizations. *Aequationes Mathematicæ*, 31:121–141, 1986.

[FL91] M. Ferri and S. L. Lins. Topological aspects of edge fusions in 4-graphs. *J. Combin. Th. (B)*, 51:227–243, 1991.

[Gag79a] C. Gagliardi. A combinatorial characterization of 3-manifold crystalizations. *Boll. Un. Mat. Ital.*, 16-A:441–449, 1979.

[Gag79b] C. Gagliardi. How to deduce the fundamental group of a closed n-manifold from a contracted triangulation. *J. Inform. Syst. Sci.*, 4:237–252, 1979.

[GAP] GAP. Groups, Algorithms and Programming. M. Schoenert et al. User's Manual, Lehrstuhl D. fuer Mathematik, RWTH, Aachen - R.F.A. (1993), 850pp.

[Gol80] M. C. Golumbic. *Algorithmic Graph Theory and Perfect Graphs.* Academic Press, 1980.

[Hem76] J. Hempel. *3-Manifolds.* Princeton University Press, 1976.

[HKM89] H. Helling, A. C. Kim, and J. L. Mennicke. On Fibonacci groups. *Preprint*, 1989.

[HMM85] D. A. Holton, B. Manvel, and B.D. McKay. Hamiltonean cycles in cubic 3-connected bipartite planar graphs. *Journal of Combin. Theory (B)*, 38:379–297, 1985.

[HW94] C. Hodgson and J. Weeks. Symmetries, isometries and length spectra of closed hyperbolic 3-manifolds. *Res. Report GCG67, Univ. Minnesota*, 1994.

[Kir78] R. Kirby. A calculus for framed links in S^3. *Inv. Math.*, 45:35–56, 1978.

[KL94] L. H. Kauffman and S. L. Lins. Temperley-Lieb recoupling theory and invariants of 3-manifolds. Number 134 in *Annals of Mathematical Studies.* Princeton Univ. Press, 1994.

[KR88] A.N. Kirillov and N.Y. Reshetikhin. *Representations of the algebra $U_q(sl_2)$, q-orthogonal polynomials and invariants of links. In Infinite Dimensional Lie Algebras and Groups*, volume 7. Ed. by V.G. Kac. Ser. in Math. Phys., 1988.

[LD89] S. Lins and C. Durand. A complete catalogue of rigid graph-encoded orientable 3-manifolds up to 28 vertices. *Notas Com. Mat. UFPE*, 168, 1989.

[LD91] S. Lins and C. Durand. Topological classification of small graph-encoded orientable 3-manifolds. *Notas Com. Mat. UFPE*, 177, 1991.

[Lic62] W. B. R. Lickorish. A representation of oriented combinatorial 3-manifolds. *Annals of Mathematics*, 76:531–540, 1962.

[Lin84] S. Lins. Towards a catalogue of 3-manifold crystallizations. *Atti. Sem. Mat. Fis. Univ. Modena*, 33:369–378, 1984.

[Lin86] S. Lins. Paintings: A planar approach to higher dimensions. *GeometriæDedicata*, 20:1–25, 1986.

[Lin88] S. Lins. On the fundamental group of 3-gems and a "planar" class of 3–manifolds. *Europ. J. Combinatorics*, 9:291–305, 1988.

[Lin89] S. Lins. Invariants groups on equivalent crystallizations. *Europ. J. Combinatorics*, 10:575–584, 1989.

[Lin95] S. Lins. A string theory for 3-manifolds with a Kleinian action. *To appear in the J. Knot Th. & Ramifications*, 1995.

[LS95] S. Lins and S. Sidki. The fundamental groups of closed orientable 3-manifolds admitting colored triangulations with less than 30 tetrahedra. *Int. J. Algebra and Computation*, 5:205–250, 1995.

[Mil62] J. Milnor. A unique factorization theorem for 3-manifolds. *Amer. J. Math.*, 84:1–7, 1962.

[Mil75] J. Milnor. On the 3-dimensional Brieskorn manifolds $m(p, q, r)$. In P.L.Neuwirth, editor, *Annals of Mathematical Studies*, pages 175–225. Princeton Univ. Press, 1975.

[MKS76] W. Magnus, A. Karass, and D. Solitar. *Combinatorial Group Theory*. Dover, 1976.

[Moi52] E. E. Moise. Affine structures in 3-manifolds: V. The triangulation theorem and Hauptvermutung. *Ann. Math.*, 56:96–114, 1952.

[Moi77] E. E. Moise. Graduate texts in mathematics 47. In *Geometric Topology in Dimensions 2 and 3*. Springer-Verlag, 1977.

[Mon89] J. Montesinos. La discrepancia entre el rango y el genero de Heegaard de una 3-variedad. *Note di Matematica*, 9:101–117, 1989.

[Mul94] M. Mulazzani. Caratterizzazione topologica degli spazi di Lins e Mandel. Tesi di Dottorato — University of Bologna, 1994.

[NW88] G. Nemhauser and L. Wolsey. *Integer and combinatorial optimization*. John Wiley and Sons, 1988.

[Orl72] P. Orlik. Lecture notes in mathematics. In *Seifert manifolds*, volume 291. Springer-Verlag, 1972.

[Rol76] D. Rolfsen. *Knots and links*. Publish or Perish, 1976.

[RT91] N. Yu Reshetikhin and V. Turaev. Invariants of three-manifolds via link polynomials and quantum groups. *Invent. Math.*, 103:547–597, 1991.

[SPA] SPAS. Subgroup Presentation Algorithm System, V. Felsch, J. Neubueser et al. User's Reference Manual, Lehrstuhl D. fuer Mathematik, RWTH, Aachen - R.F.A. (1989), 160pp.

[ST80] H. Seifert and W. Threlfall. *A textbook on topology.* Academic Press, 1980.

[Sti80] J. Stillwell. Graduate texts in mathematics 72. In *Classical topology and combinatorial group theory.* Springer-Verlag, 1980.

[Wit89] E. Witten. Quantum field theory and the Jones polynomial. *Commun. Math. Phys.*, 121:351–399, 1989.

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ISBN 981-02-1907-5
9 789810 219079