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Mining Research Centre established at University of Pretoria

A multidisciplinary Mining Resilience Research Centre (MRRC) has been established at the University of Pretoria (UP) to focus on multi-disciplinary research activities within the mining industry and to develop lasting partnerships with leading international research and academic institutions.

Prof Jan du Plessis, Sasol Chair in Health, Safety and Environment in the Department of Mining Engineering at UP, says although mining faces severe challenges under the current economic conditions, it remains an important sector for growth and transformation in Africa. 'Issues around legacy, responsibility, impact and innovation need to be addressed in order to achieve a resilient mining industry in Africa. At the heart of any strategy to achieve resilience in African Mining lies the requirement for appropriate knowledge, capability, attitude and behaviour of the mining leaders of the future. The establishment of the MRRC is the result of thorough industry consultation and the main aim of this Centre is to provide modern approaches, world class facilities and globally relevant topics, making it possible for researchers to excel and for the industry to build capacity.'

According to the World Bank (2016), Africa is home to about 30% of the world's mineral reserves, 10% of the world's oil, and 8% of the world's natural gas. In South Africa the mining industry is responsible for an estimated 19% of all economic activity and supports at least another 25 % of up and downstream economic activities. Despite this considerable wealth on the continent, it is plagued by poverty, social inequality, and slow economic development. However, mining remains a key driver for growth and is inextricably linked to Africa's future - with mining comes employment and skills development, investment in education, the construction of infrastructure and the generation of much-needed revenue.

Amongst others the MRRC is establishing multidisciplinary collaborations that will address:

➢ future mining education with the aim to co-create, develop and adopt a Resilient African Mining Model as the blueprint for mine designs of the future;
➢ future new technologies including mechanised mining, automation, robotics and the associated workplace order and culture;
➢ future socio-economic aspects of mining, on how empowered, technologically enabled and educated communities relate to mining operations; and
➢ future mining governance in Africa to meet the highest standards.

Faculties that currently form part of the MRRC are the Faculties of Humanities, Natural and Agricultural Sciences, Economic and Management Sciences, Engineering, Built Environment and Information Technology and Law. The intention is that more faculties at the University of Pretoria that are involved in mining research will eventually also form part of the MRRC activities, making it a fully interdisciplinary mining research centre. The MRRC is currently busy with six research projects in Engineering, Built Environment and Humanities.

As part of the Centre strategic intend it has and will continue to form partnerships with leading International and local universities. This will include student and lecturer exchanges, joint research activities and the opportunities for post graduate studies in the different speciality fields.

L. Swart
Department of University Relations, University of Pretoria
Contact: (012) 420-3650

The Southern African Institute of Mining and Metallurgy

OFFICE BEARERS AND COUNCIL FOR THE 2015/2016 SESSION

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Journal Comment

Danie Krige Geostatistical Conference 2015

One might ask what benefit the Danie Krige Geostatistical Conference imparted to the delegates. Principally, it drew us together and confirmed again the importance of the work being done in the field of geostatistics. A significant concern over the past decade and a half has been the declining numbers of local geostatistics practitioners and the need for ongoing education of the geostatistical fraternity. Unfortunately, there are many geostatisticians working in South Africa who have become 'transparent' to the professional institutions in that they are not affiliated in any way. All participants at the Conference were urged to enrol as members of the Southern African Institute of Mining and Metallurgy (SAIMM), and the Geostatistical Association of Southern Africa (GASA).

Papers published in this issue of the Journal arose from the Proceedings of the Danie Krige Geostatistical Conference, which in turn was based on submissions of original geostatistical research presented in the Danie Krige Commemorative Volume. The intention for the Danie Krige Geostatistical Conference was to provide the authors of papers in the Commemorative Volume with a platform from which to present their research. However, most of the twenty-two papers contained in the Proceedings were original items of research that relate to, or are extensions of, work published in the Commemorative Volume. The international call for papers in honour of Professor Krige through the SAIMM resulted in three issues of the Journal, published in March 2014, in August 2014, and in January 2015, and included 35 papers submitted by 83 authors from 17 countries around the world. The theme of the Danie Krige Geostatistical Conference, '*Geostatistical Geovalue - Rewards and Returns for Spatial Modelling*', highlighted the role of geostatistics in optimizing financial returns from mineral extraction by minimizing uncertainty. 'Geovalue' refers to the capitalized value of the Earth's primary natural resources, and only the diligent and correct application of geostatistics can maximize this value. The Conference went a long way in presenting new and innovative ways to improve 'geovalue', but it is felt necessary to briefly explain the history underlying the development of geostatistics.

In 1644 Descartes used a method, later to be referred to as the Voronoi diagram after the Ukrainian mathematician Georgy Voronoy (1868–1908), in a strictly geometric or polygonal method of estimation; a similar method employs what are known as Thiessen polygons. Others who investigated spatial variability include Bertil Matern (1917–2007), a Swedish statistician whose research applied to forestry, and Lev Gandin (1921–1997), a Russian mathematician whose work centred on climatology and the best way to average scattered meteorological data to give a spatial average. Georges Matheron (1930–2000), who knew of the work of Matern and Gandin, drew heavily on the work done by Andrey Kolmogorov (1903–1987), a Russian

mathematician who made significant contributions to the mathematics of probability theory as well as other areas. Matheron, intrigued by the pioneering work of Danie Krige and Herbert Sichel in the late 1940s and early 1950s on topics specific to mining and mineral resource evaluation, went on to formalize Krige's evaluation methods at Ecole des Mines de Paris in Fontainebleau, France. It was Krige's work in particular that became known as geostatistics, and the technique for estimating values at unsampled localities using nearby samples that Matheron referred to as *'kriging'*. Evidence presented by Noel Cressie indicates that both Georges Matheron and Lev Gandin independently developed ordinary kriging as we know it today.

What was particularly important in the advancement of geostatistics and the spreading of this idea through industry as an estimation technique in mineral resource evaluation was the parallel development of computing technology. Computing power grew quickly from its inception in the late 1940s and early 1950s, without which the science of geostatistics would simply not have been possible. In addition, a growing range of software for application to geostatistical problems also found a place on the stage. The application of geostatistics grew not only in mining- and mineral resource-related problems, but also in soil science, meteorology, environmental science, the oil and gas industries, and more recently in ecology and image analysis.

The simple concepts lying at the foundations of geostatistics need some air-time lest readers relegate the contents of this volume to paths less well trodden. Krige's main aim was to convince South African mining engineers in particular to use multiple regressions to predict the grade of mining blocks from the huge amount of assay values that had already been collected. Spatial modelling and the need to predict point or block estimates in space from surrounding data has given rise to what is today known as geostatistics. This is a significant break from classical statistics, which demands that data values be both random and independent. Geostatistics has taken a more pragmatic view, with its fundamental premise being that while data values may be random, they are not independent of one another. Closer sample values will be more similar than those farther apart. If one imagines a point in space for which you would like to estimate a value (or grade) from surrounding samples, one could simply calculate their average and assign it as the estimate. If the concept of spatial dependence between variables is allowed to ferment, the next logical step would be to weight the contributions of local data values to the estimate based on their distance away from the point being estimated; nearer data points contribute a greater proportion of their value to the estimate than points further away. Hence, inverse distance estimation – a weighted linear combination of nearby sample values. Immediately, questions arise about the nature, validity, and confidence one might place in the estimate produced in this way. What confidence do we have in the

estimate? Is it a single point estimate or only one value from a probability distribution; is the inverse distance function appropriate; how many samples should we use; what is the maximum distance for including samples in the estimate; what should we do if the samples are clustered; how do we manage anisotropy; how do we deal with outliers; what is the data from a skewed distribution; how do we manage the regression effect; and how will the estimate change if we consider an area or volume rather than a point estimate? Almost half these questions can be answered by resorting to the standard geostatistical approach, which is to use a variogram, a graph which shows how the variance of the difference between data points changes as the distance between them increases. Georges Matheron succeeded in answering the balance of the questions by developing the concepts of ordinary kriging for the mining industry.

It is now sixteen years since the last significant geostatistical conference, Geostats 2000, which was held in South Africa. The length of time between geostatistical conferences rang a note of concern amongst all delegates and acted as a reminder that we should be in regular contact to share ideas. The Danie Krige Geostatistical Conference provided geostatisticians with just such an opportunity and I trust that the momentum for good quality research generated by this Conference will be carried into the future.

R.C.A. Minnitt
Conference Convener

35TH INTERNATIONAL GEOLOGICAL CONGRESS

27 AUGUST - 4 SEPTEMBER 2016 | CAPE TOWN, SOUTH AFRICA

The International Geological Congress (IGC) is the principal event of the International Union of Geological Sciences (IUGS) and the 35th IGC will be held in Cape Town, South Africa between 28 August and 2 September 2016.

Close to 4800 oral and poster presentations in 200 symposia have been accepted from delegates from 115 countries. Geoscience in the Economy, Geoscience in Society and Fundamental Geoscience are the three major strands of the technical program, each compromising a third of the accepted presentations. The technical program under Geoscience in the Economy will be of direct interest to SAIMM members. Geostatistics and Mathematical Geoscience, Mining Geology and Earth Resource Engineering, Applied Mineralogy and Geometallurgy, Mineral Resource Evaluation, Resourcing Future Generations, and Global Geoscience Professionalism and Geoethics are just some of the 48 major themes and symposia to be addressed.

For information on the full program, including the workshop and field trip offerings, see www.35igc.org, and follow the meeting events on Facebook (https://www.facebook.com/35thigc/?ref=aymt_homepage_panel) and Twitter (https://twitter.com/35thigc).

Day Registration Packages are available.

www.35igc.org

resourcing future generations

Celebrating the ordinary

Exceptionalism comes easily to South Africans. We are used to living in a country with wonderful weather, spectacular scenery, and the richest collection of mineral wealth in our ground. There is no other country in the world where you have two Nobel Peace Prize winners who lived in the same street. We are the Rainbow Nation of Desmond Tutu; the country where Gandhi formulated his ideas of passive resistance; and the people led by Nelson Mandela that practised reconciliation instead of a civil war. Johannesburg is the city where all of these great leaders lived and worked; it is also the location of the world's greatest deposit of gold; and is even claimed to be the world's largest manmade urban forest. I was born in Germiston (now regarded as part of greater Johannesburg; both cities were founded in 1886), and I grew up feeling proud of the accomplishments of the industrialists of my father's generation. The city was home to the Rand Refinery (the world's largest refinery of gold, which has refined 30% of all the gold mined in the world since antiquity), and the largest railway junction in the Southern Hemisphere.

South Africa, as a country, does not do things in half measures. For a few decades in the 20th century, South Africans were pariahs because of our discriminatory apartheid laws, then during the Mandela years we went to being one of the world's favourite nations and were a shining example to the world in how to overcome discrimination, and how to unite divided societies. Unfortunately, more recently, our reputation has been sullied and we have become known as one of the world's more violent, lawless, unequal, corrupt, and ineffective countries. Our national psyche seems to demand that we are either at the top or at the bottom of the pile. Surely there has to be a better way – maybe we could try to be just a normal and peaceful place.

The writer of the book of Ecclesiastes (usually assumed to be King Solomon) was someone in a position to test what made for a successful life. He pursued great wealth and found that unsatisfying; he pursued a life of hedonistic pleasures and found that to be like 'chasing after the wind'; he attained great wisdom and knowledge and found even that to be 'utterly meaningless'. Eventually he concluded that the secret to a happy and successful life was to find pleasure in the simple things – a shared meal with friends, the satisfaction of work, laughing together, and enjoying the beauty around us – a celebration of the ordinary. Despite this really good advice, we do seem to pay special attention to people who achieve first place and to things that are bigger or better than other things like them.

Charles Schulz, the creator of the *Peanuts* comic strip said 'Nobody remembers who came in second'. Andrew Carnegie said something similar: 'The first man gets the oyster, the second man gets the shell'. Most people I know will remember that Neil Armstrong was the first man who walked on the moon, but it is probably true that fewer will remember that it was Buzz Aldrin who was the second, and even fewer still that Pete Conrad was third. By the way, all three of these astronauts were born in the same year, 1930.

If you drive to the top of Northcliff Hill in Johannesburg, apart from the spectacular view of the World Cup soccer stadium to the south and the Sandton skyline to the north, you can see a signboard that states 'At 1807 metres above sea level the ridge is only 1 metre lower than the highest point in the Johannesburg municipal area'. Being of a curious turn of mind, I find this kind of statement drives me to distraction. I think that it should be against the law to say what is in second place without saying what is in the first place. After seeing this sign for the first time, it took me a little while to find out what the actual highest point of ground in Johannesburg is. In case you are wondering too, it is on the Observatory Ridge (to the east of the city centre), just above the site of the old observatory and the home of some technical societies.

That got me thinking about how things are measured and ranked. Many lists and rankings are contentious because they don't make explicit all of the factors that are included in the evaluation. Even if the factors are listed, different people might weight them differently.

For example, Victoria Falls is undoubtedly one of the world's greatest waterfalls. Yet it is not the highest. That honour belongs to the Angel Falls (979 m) in Venezuela, followed by the Tugela Falls (948 m) in South Africa. It is also not the widest; that title belongs to Iguazu Falls (2700 m) between Argentina and Brazil. It also does not have the largest mean annual flow rate; which goes to Niagara Falls (2407 m^3/a) between Canada and the USA. What distinguishes the Victoria Falls (apart from its spectacular natural beauty) is that it is the largest falling sheet or curtain of water in the world, being 1.7 km wide and with a single drop of 108 m.

The largest lake in the world is also subject to definition. If saltwater lakes are included, then the Caspian Sea is the largest by surface area and by volume, but if we restrict the category to freshwater bodies only, then Lake

Celebrating the ordinary *(continued)*

Superior (North America) has the largest surface area (followed by Lake Victoria in Africa), while Lake Baikal in Asia is the largest by volume (containing approximately 20% of Earth's fresh surface water), followed by Lake Tanganyika in Africa. The deepest lake in the world is Lake Baikal, followed by Lake Tanganyika, then the Caspian Sea. So, if anyone asks me the ambiguous question 'What is the largest lake?', they will get quite an earful in response.

Even greater complexities occur when rating universities. This month saw the release of a list of 1000 top universities worldwide by the Center for World University Rankings (CWUR). The South African universities included in this list were the University of the Witwatersrand (176th), University of Cape Town (265th), Stellenbosch University (329th), University of KwaZulu-Natal (467th), and University of Pretoria (697th). This is quite impressive, given that there are approximately 20 000 universities internationally. The factors taken into account include quality of education, alumni employment, quality of faculty, publications, influence, citations, broad impact, and patents. The highest scoring position of these universities went to Wits University, being rated 35th in the world for alumni employment.

There are numerous university ranking systems, each with a different emphasis. Four of the most prominent are the Times Higher Education (THE) World University Rankings (perhaps the most widely accepted), the CWUR, QS World University Rankings, and the Academic Ranking of World Universities (ARWU/Shanghai). Earlier this year, the THE ranking of South African universities showed UCT (1), Wits (2), Stellenbosch (3), UKZN (4), Pretoria (5), and Unisa (6). The top five universities appearing in both these lists also appear in the top five positions in the QS ranking. These institutions are well known for providing graduates to the mining and metallurgical industries.

South Africa was once overwhelmingly dominant in gold production. However, this was achieved at a high social cost, with the introduction of the migrant labour system that has been so damaging to large portions of our society, and we continue to reap the cost of this today. In recent years, South Africa's gold production has decreased as many mines have become depleted, and the remaining ores are deeper, and extraction of the gold has become more expensive. South Africa no longer holds the dominant position it once did in gold.

Chromium (seen as essential to the production of stainless steel) is another element that South Africa has in great abundance. South Africa's chromite reserve base has been calculated at more than 70% of the world's total. World production of chromite is dominated by South Africa, with Kazakhstan in second place. Chromite production clearly depends significantly on what is in the ground, but is also affected by policies and infrastructure development within a country, as can be seen by the significant increase in chromite production in Turkey, and the significant decline in chromite production in Zimbabwe during the past decade.

Ten years ago, 90% of the chromite that South Africa produced was converted to ferrochromium (FeCr) in South Africa, making SA by far the world's largest producer of this ferro-alloy. China, by comparison, has very little chromite, and has to either import it (much of it from South Africa) to produce FeCr, or has to import the FeCr necessary for its stainless steel production. Thirty years ago, China was in seventh place for FeCr production, producing only 120 kt/a. By 2006 (ten years ago), China's FeCr production had grown to 1.0 Mt/a, and they had moved up to third place (after South Africa with 3.0 Mt/a, and Kazakhstan with 1.2 Mt/a). China continued to grow rapidly, and South Africa's production of FeCr declined as a result of power shortages and higher costs. China overtook South Africa as the world's leading producer of ferrochromium in 2012. I suppose that the question has to be asked whether it really matters if our country is in first or second place, but it certainly does matter whether the minerals in the ground contribute in the best way possible to the prosperity of the people of our region.

The Greek historian Polybius said: 'Those that know how to win are much more numerous than those who know how to make proper use of their victories'. I hope that the mining industry in southern Africa will strive towards making 'proper use' of our natural resources to the benefit of our people.

R.T. Jones
President, SAIMM

PAPERS IN THIS EDITION

These papers have been refereed and edited according to internationally accepted standards and are accredited for rating purposes by the South African Department of Higher Education and Training

Papers – Danie Krige Geostatistical Conference

AngloGold Ashanti (AGA) utilizes a technique termed macro cokriging (MCK), which entails the estimation of mixed support size data together with the application of four-parameter distribution models. The gold value estimation for the Carbon Leader Reef (CLR) on the AGA TauTona Mine is used in this paper as a case study of the technique and to demonstrate its effectiveness through production reconciliation.

A study was undertaken to determine how valid the predicted grade patterns of a typical Birimian- style gold deposit, with high nugget effect and strong short-range variability, might be expected to be. The results showed a satisfactory correlation between the direct kriging ranking of the smallest mining units (based on sparse data) and the grade control model ranking. The LUC technique is therefore considered to be useful for this type of deposit.

This paper outlines the use of uniform conditioning (UC) to provide a robust estimate for use in underground mine planning and published Mineral Resource statements. The process and validation techniques employed for UC are discussed using a case study from an orebody in Tanzania. The UC model, in spite of the inherent lack of local spatial accuracy, can be used to estimate the potential underground stopes with a lower error of estimation than other techniques.

This paper demonstrates how expert opinion is used to generate a composite probability map for diamond concentration using a greyscale hand-sketching technique. The resultant grade simulation is used to test different sample scenarios and is a first step towards determining an appropriate sampling strategy for the target area.

This paper provides a summary of the first Professor Danie Krige Memorial Lecture that was held in 2014, and is a testimony to a man who became a giant not only in the South African mining industry, but indeed worldwide.

This paper presents an overview of uniform conditioning and localized uniform conditioning, with discussions around results of the research work that was carried out.

These papers will be available on the SAIMM website
http://www.saimm.co.za

PAPERS IN THIS EDITION

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Papers of General Interest

Resource estimation for deep tabular orebodies the AngloGold Ashanti way

by T. Flitton* and R. Peattie*

Synopsis

The extreme depths and consequent expense of drilling and sampling the gold-bearing reefs of the Witwatersrand Basin have resulted in limited data being available for estimation of grade ahead of the mining face. There is, however, a wealth of information from mined-out areas of these deposits. This estimation challenge resulted in the development of a unique method of Mineral Resource estimation.

AngloGold Ashanti (AGA) utilizes a technique termed macro cokriging (MCK), which allows for the integration of the limited advanced borehole data with the large chip sample data-sets from the previously mined-out areas by adopting a Bayesian geostatistical approach. The MCK process, in short, is the estimation of mixed support size data together with the application of four-parameter distribution models.

The gold value estimation for the Carbon Leader Reef (CLR) on the AGA TauTona Mine is used in this paper as a case study of the process and to demonstrate the effectiveness of this technique through production reconciliation. The current method has been proven over the last 20 years and is now an established part of the Mineral Resource evaluation process within AGA.

Keywords
resource evaluation, Bayesian assumption, macro cokriging, mixed support, four-parameter distribution.

Introduction

Owing to the extreme depths of the gold-bearing reefs at the AngloGold Ashanti (AGA) Witwatersrand operations and the prohibitive expense and time involved in drilling boreholes to the required depth, only limited drilling and sampling data is available ahead of the mining face. Whereas traditional estimation techniques are interpolative, within the Witwatersrand the estimation is primarily extrapolative, with the majority of the data being sourced from the mined-out areas. This significant challenge to estimation resulted in the development of a unique method of Mineral Resource estimation and evaluation.

The process

Macro cokriging (MCK) was first introduced in 1994 and has been is use for 20 years by AGA. There are three key aspects to this technique; namely a Bayesian approach to estimation, the estimation of mixed support size data (MCK), and the utilization of four-parameter distributional models.

(1) The Bayesian process followed allows for the integration of the limited advanced data with the large data-set from the previously mined-out areas. Krige *et al.* (1990), and later Dohm (1995), proposed the use of a Bayesian geostatistical approach where '*the geological, statistical and spatial characteristics observed in the known population area hold for the virgin areas*'

(2) Estimation by MCK of two different sampling support sizes, one at block support, representing the dense underground chip sampling data and the other cluster support, representing the widely spaced borehole data (Dohm, 1995; Chamberlain, 1997). The MCK technique is not strictly cokriging, but the modification of the diagonal kriging matrix to reflect different nugget effects related to different data supports. The methodology also assumes the same spatial covariance structure for the different data supports and thus does not use cross-variograms

(3) The continued development of distributional models beyond the use of the two- and three-parameter lognormal models led to the development of four-parameter distributional models by Sichel (1990) and Sichel *et al.* (1992) that are more applicable for the gold reefs of the Witwatersrand. Estimation is done in natural logarithmic (Ln) space because of the highly skewed gold distribution. The final gold estimates are calculated by back-transforming the estimates using four-parameter distribution models (Dohm, 1995; Chamberlain, 1997).

* *AngloGold Ashanti.*

Resource estimation for deep tabular orebodies the AngloGold Ashanti way

Many of the processes developed and used by AGA are based on the work completed and detailed by Dohm (1995) and Chamberlain (1997).

The unique estimation method followed does not, however, detract from the criticality of having a sound geological model. It is imperative for this and any other estimation process that the geological model accurately represents the understanding of the deposit.

Location and data

AGA's TauTona Mine lies on the West Wits Line, just south of Carletonville in North West Province, about 70 km southwest of Johannesburg (Figure 1). Mining at this operation commenced over 50 years ago and currently takes place at depths ranging from 2000 m to 3640 m below surface. The mine has a three-shaft system and employs a sequential and/or scattered grid mining method to extract the gold in the deep, narrow, tabular orebody. The grid is pre-developed through a series of haulages and crosscuts. Stoping takes place by means of breast mining using conventional drill-and-blast techniques. The smallest mining unit (SMU) is 100 m × 100 m.

The CLR is the principal economic horizon at TauTona. The CLR is located near the base of the Johannesburg Subgroup, which forms part of the Central Rand Group of the Witwatersrand Supergroup. The CLR is a thin (on average 20 cm thick) tabular, auriferous quartz pebble conglomerate.

The sampling data is comprised of underground chip sample sections (425 917 points), underground boreholes, and surface boreholes from TauTona and neighbouring mines. Underground sampling is in the form of chip sampling taken on the mining face using a hammer and chisel. All sample locations are reported as a composite over a mineralized width, resulting in a single channel width (cm) and gold metal accumulation value (cm.g/t) (Figure 2). The natural logarithms (Ln) of this metal accumulation is used in the estimation.

Bayesian assumption and the geological model

An upfront Bayesian assumption is made that the mined-out areas are from the same statistical population as the areas yet to be mined – these are generally down-dip or along strike. The underlying assumption is that the Ln mean and Ln variance of the metal accumulation of a deposit will vary from locality to locality but the shape of the distribution will remain constant (Dohm, 1995; Chamberlain, 1997). Thus, with appropriate consideration of support differences, the distribution of data for the mined-out area can be equated to the unmined area, with the sparse surface boreholes being a subset of the whole.
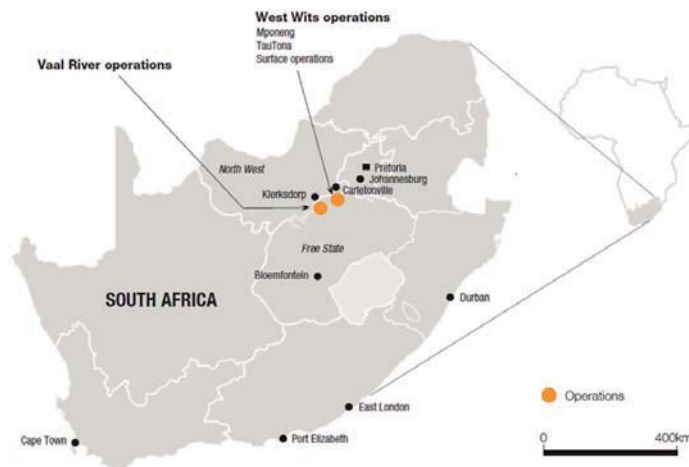
Figure 1—Locality of AGA TauTona Mine, South Africa
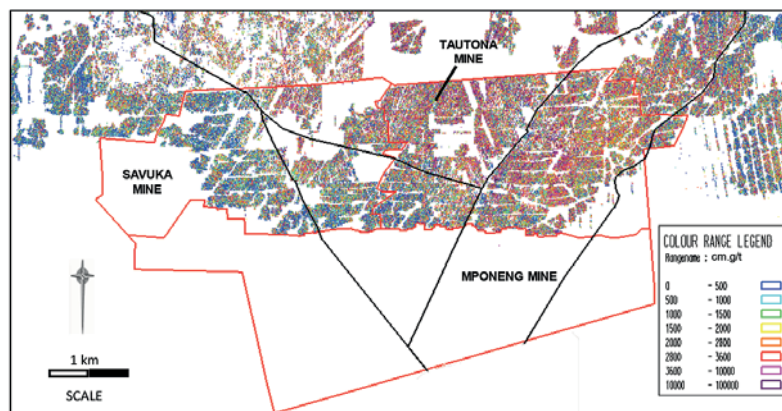
Figure 2—Colour-coded value (cm.g/t) chip sample plot of the CLR

Resource estimation for deep tabular orebodies the AngloGold Ashanti way

The geological model that underlies the estimation process is crucial input to effective estimation and the validity of the above assumptions. The individual domains (geozones) within the geological model must not only be geologically homogeneous but also define the gold grade distribution. The geozones subdivide the data into distinct populations and the parameters of these populations play a critical role in the development of the estimates (Dohm, 1995; Chamberlain, 1997). It is thus important to identify, separate, and validate geozones on an ongoing basis so that the geological model is robust and stationarity is maintained as far as possible.

Determining and validating geozone boundaries is done using a combination of statistical techniques such as classical comparative statistics, histograms, and quantile-quantile scatter plots as well as geostatistical techniques such as trend, channel width, and boundary analysis. Many of these techniques were described by both Dohm (1995) and Chamberlain (1997). Comparative semivariograms and bivariate statistical scatter plots are also used to further refine geozones.

In recent years, extensive work has been done on refining the geozone model for the CLR, supported by new thinking in geochemistry and spectral scanning in addition to the traditional geostatistical techniques. Five geozones have been identified in the CLR (Figure 2). All geozone boundaries for estimation are treated as 'soft', with a skin of overlapping data being selected as the result of boundary analysis work.

Analysis of borehole sampling data

The prohibitive cost involved in deep drilling means that boreholes are normally drilled on a wide spacing, resulting in very low data support. To ensure that as much of this data as possible is available for the estimation process AGA uses 'clusterizing' and 'acceptorizing' processes to try to optimize its availability. The surface boreholes usually consist of multiple reef intersections that are drilled from a single parent diamond borehole. These intersections can range from less than one metre apart up to tens of metres for 'long deflections' (Figure 3).
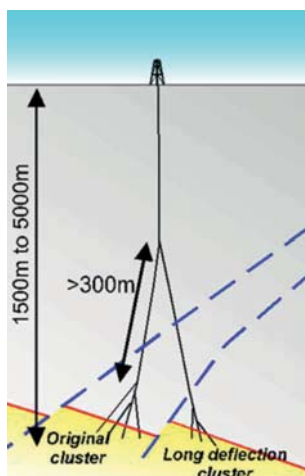
Borehole cluster analysis, also known as 'clusterization', aims to determine whether the gold values within the original cluster are sufficiently different from the gold values in the long deflection of the same borehole and as such can be treated independently (O'Brien, 1996; Chamberlain, 1997). The borehole intersection clusters from long deflections are compared to those obtained from original closely spaced intersection values, using the standard statistical analysis of variance approach (O'Brien, 1996). If the analysis of variance shows that the samples from clusters under consideration are not significantly different, then all samples are combined in a 'super cluster' for further use.

The acceptability of all cored borehole reef intersections is classified according to their mechanical acceptability (completeness of cut, identification of missing chips) and geological acceptability (complete reef, presence of faulting or shearing). The classification of mechanical acceptability is a subjective process and traditionally, if a sample was classified as mechanically unacceptable, the entire intersection would not have been used in estimation. This significantly reduced the number of intersections that could be used and reduced the size of the very limited data-set even further. The 'acceptorizing' process aims to statistically identify which of the unacceptable intersections can be retained and which need to be removed, so as to maximize the number of borehole samples used in the resource estimation process.

The statistical basis of the 'acceptorizing' process is derived from Heyns (1958) and also as expanded and discussed in Dohm (1995), O'Brien (1996), and Chamberlain (1997). Generally there is a mixture of acceptable and non-acceptable intersections within any particular cluster. The logarithms of the ratios of the non-acceptable to acceptable intersection pairs are calculated and 95% confidence limits are set up around the mean of the logarithms. The individual intersections are then plotted against the mean for each cluster (Figure 4). Non-acceptable outliers are then reviewed and removed and the process repeated until the amount of acceptable data that lies outside these confidence limits is at most 5%. Acceptable intersections are not discarded without

Figure 3—Schematic cross-sectional representation of surface borehole deflections (after O'Brien, 1996)
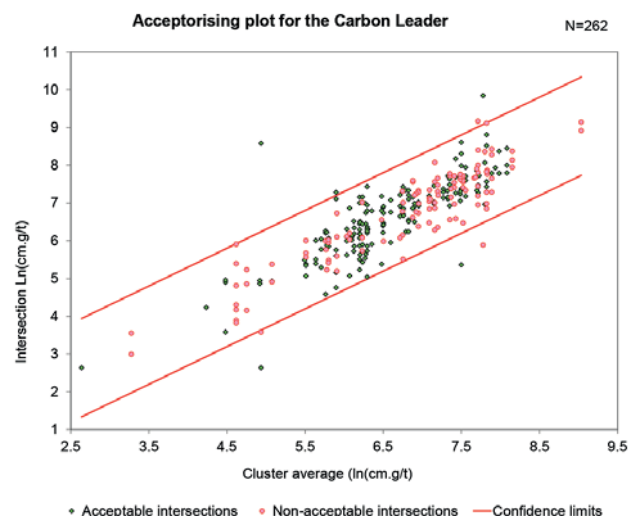
Figure 4—Acceptorizing plot for CLR – non-acceptable intersections outside limits are removed from the cluster

evidence of significant mechanical loss or geological unacceptability (O'Brien, 1996).

The CLR borehole data-set consists of 58 clusters after the clusterizing and acceptorizing processes have been carried out.

Block size selection and why it is important

The underground chip sampling and surface boreholes are on very different densities, with the chip sampling spacing typically around 5 m × 5 m and the surface boreholes spacing on anything up to 1000 m × 1000 m. The process of preparing the data for estimation of two different sampling support sizes is a critical aspect of the MCK process. The process taken is to first regularize the chip sampling data into a predetermined block size. The method used to calculate this optimum block size is referred to as the variance size of area analysis (VSOA). The approach ensures that the block size selected is such that the within-block variance is effectively maximized and the between-block variance minimized using the linear extrapolation of dispersion variance. Regularization of chip sampling data on the CLR is performed into 420 m × 420 m-sized blocks as determined by the VSOA (Figure 5).

Those 420 m × 420 m blocks that are not fully informed by the chip sampling, whether due to having too few data points or due to the chip sampling not having a good spatial distribution within the block, are rejected. This data is not, however, lost as it is then created in 'pseudo' boreholes known as clusters. Clusters are created on a 30 m × 30 m block size for the CLR (approximating a parent borehole and its short deflections) by regularizing the samples within that area. The cluster support data, which now approximates a borehole, is then combined with the real borehole data.

In this way the total data-set is split into block support data and cluster data (inclusive of the boreholes).

Relationship of the mean and variance

Estimation in MCK is done in natural logarithmic space and therefore the key components to allow the final back-transform are Ln mean and Ln variance. The Ln mean and Ln variance are compared on a scatter plot for the chosen block support data in order to determine whether there is a significant relationship between the two. Ln variance can be

estimated from the established relationship using the estimated Ln mean value if a significant relationship is demonstrated. The Ln variance is estimated independently, however, if the relationship is poor.

In some instances a linear relationship, although significant, does not produce reliable results at the final stages of reconciliation and thus it becomes necessary to estimate Ln mean and Ln variance separately (Figure 6). Both Ln mean and Ln variance are estimated for all geozones of the CLR using MCK since the relationship between Ln mean and Ln variance is poor.

Variography

Two sets of variograms are required for estimation, one for block support and one for cluster support. Both sets of variograms are done on Ln mean (value) and Ln variance if the relationship described above is poor.

The block support (420 m × 420 m) data based on the VSOA process therefore presumes that most or all of the variance is constrained within the block and thus results in a block support variogram model with zero nugget variance. The block support variography is generally characterized by longer ranges, in the order of 1000 m or more (Figure 7), due to the large block sizes.

Figure 6—Relationship between natural logarithm mean and variance for one of the geozones of the CLR

Figure 5—Relationship between variance and size of area for the CLR
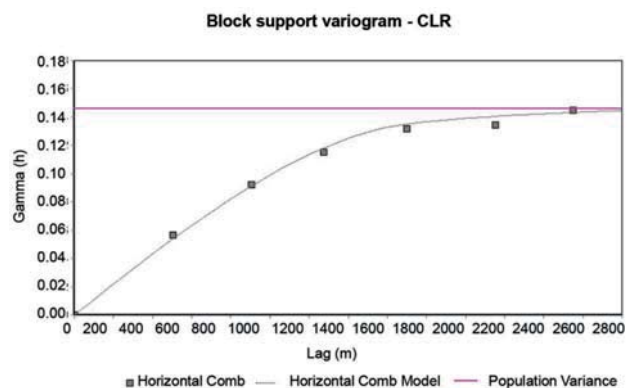
Figure 7—Block support variogram for one of the geozones of the CLR

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Cluster support variograms (inclusive of the boreholes) are calculated and modelled to determine the nugget variance (Figure 8), with the final cluster variogram used in MCK being a combination of the nugget as modelled from the cluster variogram and the sill and ranges from the block variogram.

Estimation

MCK of the two support sizes is performed by modification of the kriging matrix to allow different nugget effects; this allows the weighting of the block data differently to the clusters using the combined block and cluster variogram and thus accounting for their support difference (Chamberlain, 1997). The estimation employed for MCK, while termed cokriging, is not strictly cokriging as the data does not need to be collocated nor does it require cross-variograms, with the data for the two support sizes (blocks and clusters) not existing at the same locations. The block size estimated is the same as the block size determined by the VSOA process, in this case 420 m × 420 m.

The number of samples used in MCK has a large influence on the resulting estimate. If the number of samples used is too small (*i.e.* from a restrictive search neighbourhood), conditional bias could be introduced. Conversely, too many samples could cause undesirable smoothing levels and introduce significant amounts of negative weights, which will also increase the processing time. The amount of samples used in MCK is also controlled by the search neighbourhood. Search parameters used in the MCK process are established through a process of optimization similar to the quantitative kriging neighbourhood analysis (QKNA) process described by Vann *et al.* (2003). The discretization, numbers of samples, and neighbourhood searches are determined by analysing the kriging variance, regression slope, and percentage of negative weights. This is an iterative process and usually needs to be done for a number of iterations on spatially separated blocks.

The 95% confidence limits are calculated using kriging variance, *i.e.* Ln 95% lower limit = Ln mean – 1.96 * Ln kriging variance. This methodology is used because the distribution of variances within the 420 m × 420 m blocks is assumed to approach normality. These values are then input into the CLN model to calculate the limits (Chamberlain, 1997).
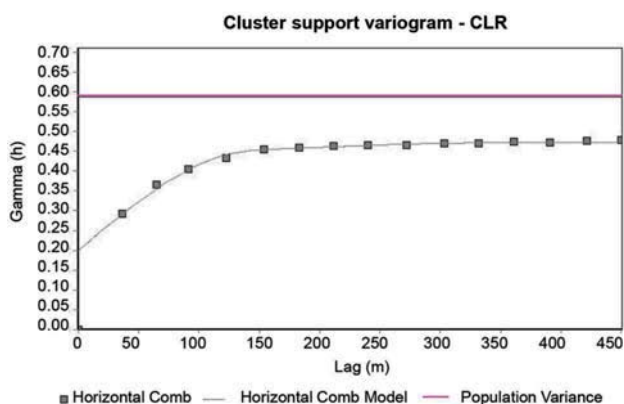
Back-transforming the estimates using four-parameter models

Traditional lognormal distributions have been found to be sub-optimal. Sichel (1990) and Sichel *et al.* (1992) suggested possible alternatives to lognormal distribution models. The suggested four-parameter distributional models were tested against more traditional models by Dohm (1995) and successfully shown to be a more accurate estimation technique than traditional techniques using lognormal theory by Chamberlain (1997).

The distribution needs to be defined and fitted once the most appropriate model is determined. Either a four-parameter compound lognormal (CLN) or logarithmic generalized inverse Gaussian distribution (LNGIG) model is used, depending on which distribution model best fits the gold grades and a theoretical test on the shape parameters of the log-transformed values. A theoretical test to differentiate between the two was detailed by Dohm (1995) and can be graphically represented. All the geozones of the CLR follow the CLN distributional model (Figure 9).

The process of distributional fitting is followed by using classical statistics on the observed data and modelling the distribution using both the histograms of Ln(value) as well as the value (Figure 10). Generally, the fitted CLN model maximum difference (from the observed frequency of the data) needs to be less than the test value for the model fit to be acceptable and so that the Kolmogorov-Smirnov test for goodness of fit does not reject the null hypothesis that the model describes the distribution of the data at the 1% level of significance. The model parameters in the case of the CLN calculated from fitting the distribution used in back-transforming are location (mean), spread (variance), skewness, and kurtosis.

Figure 8—Cluster support variogram for one of the geozones of the CLR
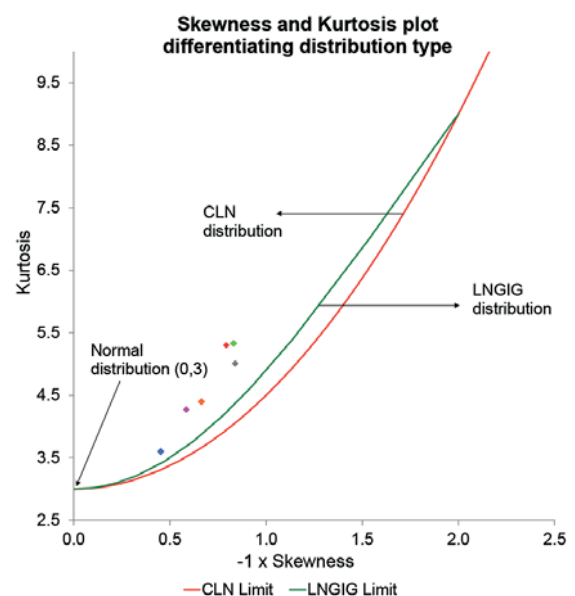
Figure 9—Skewness and kurtosis plot for differentiating distribution type – all geozones of the CLR are plotted as diamonds and are CLN distributions

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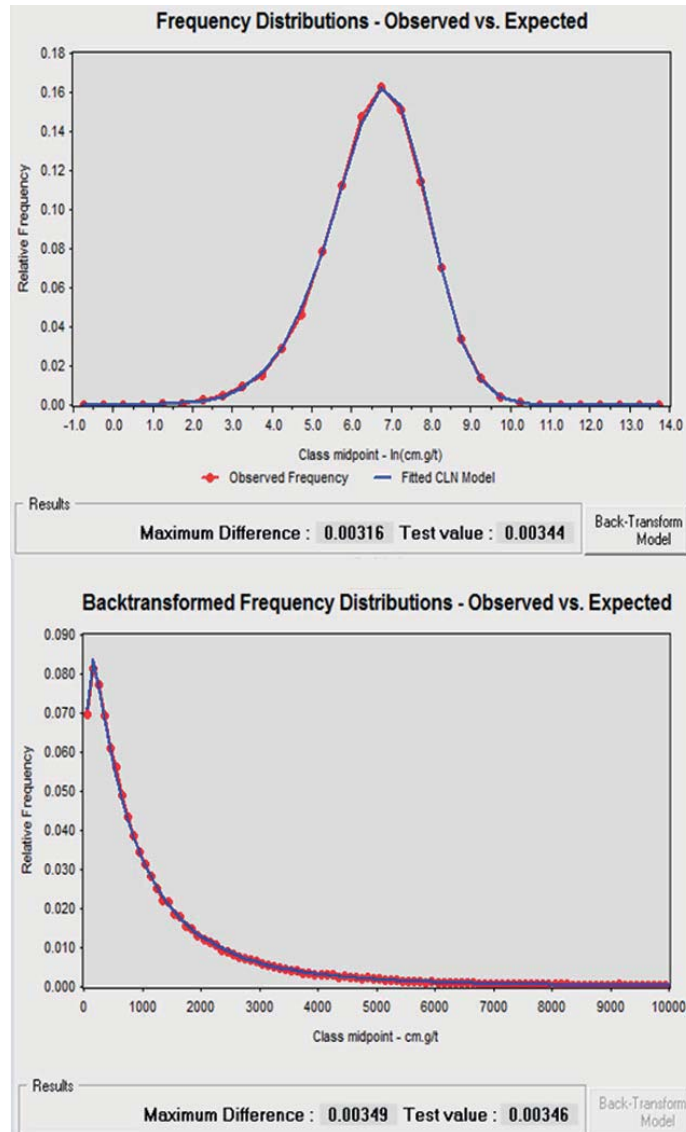
Figure 10—Example of a CLN model fit for CLN distribution for one of the geozones in the CLR

Estimation is done in natural logarithmic (Ln) space because of the highly skewed gold distribution. The final gold estimates are calculated by back-transforming the estimates using the CLN model in the case of the CLR. The value is estimated by MCK, as is the variance. The skewness and kurtosis parameters are derived from the distribution model of the *a priori* data as per the Bayesian assumptions.

Regression

The mean block value of the actual input sampling data at block support is then compared with CLN estimated block values in mined-out areas to determine if a regression effect is present. There is generally a small regression effect still present (Figure 11), thus the back-transformed estimates are regressed using the linear regression observed. Upper and lower limits of the linear regression are identified and the regression is applied to the range of estimates over which the regression is valid. The regression-corrected block estimates are used further in the long-range forecasts of value for the CLR.
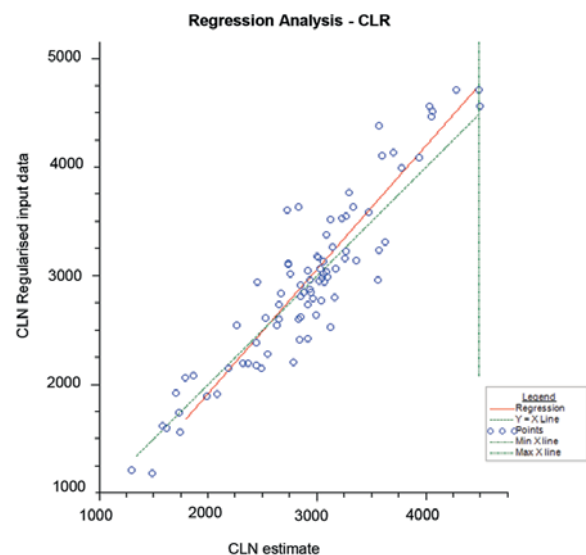
Figure 11—Regression analysis for one of the geozones of the CLR

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Reconciliations

Numerous methods of reconciliation performed by Chamberlain (1997) validated and demonstrated the effectiveness of MCK. As a final step in the validation process, a similar exercise was followed for the CLR by comparing block estimates over a ten-year period.

The underground chip sample database from TauTona and neighbouring mines in 2005 consisted of 353 072 points, and 425 917 points in 2015 (Figure 12), reflecting a notable 72 845 increase in samples. The 420 m × 420 m block estimates were compared for the two periods for a selected number of blocks where there had been the largest change in data (Figure 12). There was an initial need to ensure that the estimates used the same geozones, as there has been extensive work on the geological model over time. Thus the 2005 data was re-estimated using the variography, estimation parameters, and distributional models from 2015. This again highlights the importance of accurate and appropriate geological modelling. The 420 m × 420 m blocks for the two periods are compared in Figure 13. There is a very close correlation between the 2005 and 2015 block estimates for the 18 blocks.

As this reconciliation process provides common critical parameter inputs into the two estimates for 2005 and 2015, it would be a best-case result and could bias the 2005 estimates. Therefore a further reconciliation was done taking the 2005 estimates as done in 2005 *vs* the 2015 estimates. Figure 14 shows the comparison between the two sets of results, together with the 95% confidence limits from 2005. The MCK estimates from 2015 are well within the 95% confidence limits for the 2005 estimation, indicating that the estimation process used is acceptable and robust for un-informed areas.

Conclusion

MCK has a proven and reliable track record and the estimates have been shown to reconcile well over a long timeframe and distance from mining area. Adopting and using a Bayesian approach together with MCK and an appropriate distribution model has resulted in effective and appropriate long-range value forecasts for the CLR. The process is still highly dependent, however, on an accurate geological model as well as a full understanding of the statistical and spatial parameters of the known data. The MCK estimation method
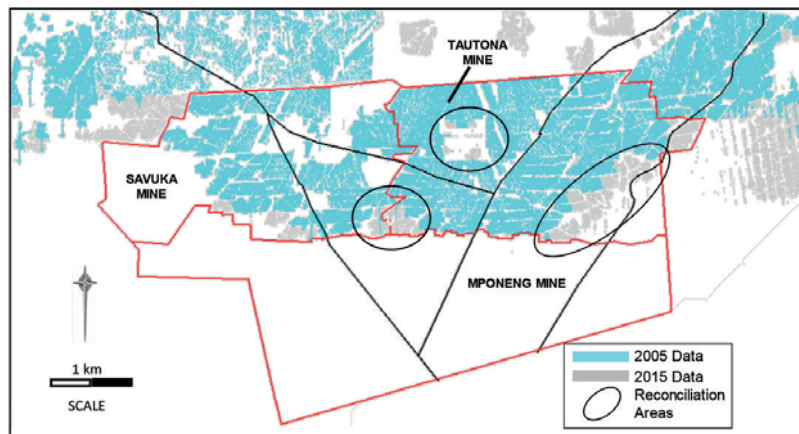
Figure 12—Chip samples plot for the CLR showing data as at 2005 and 2015
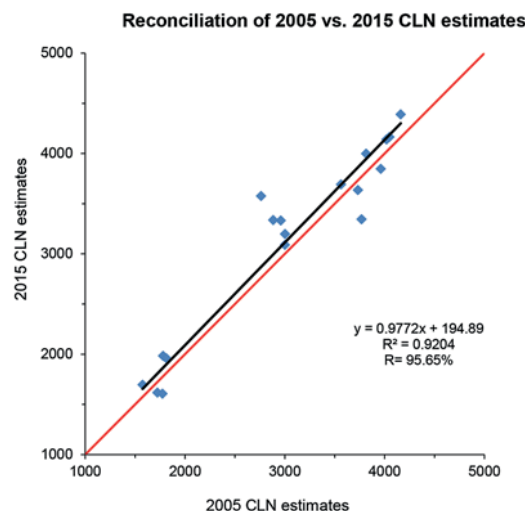
Figure 13—Reconciliation of 2005 and 2015 CLN estimates for CLR (2005 data re-estimated using the variography, estimation parameters, and distributional models from 2015)

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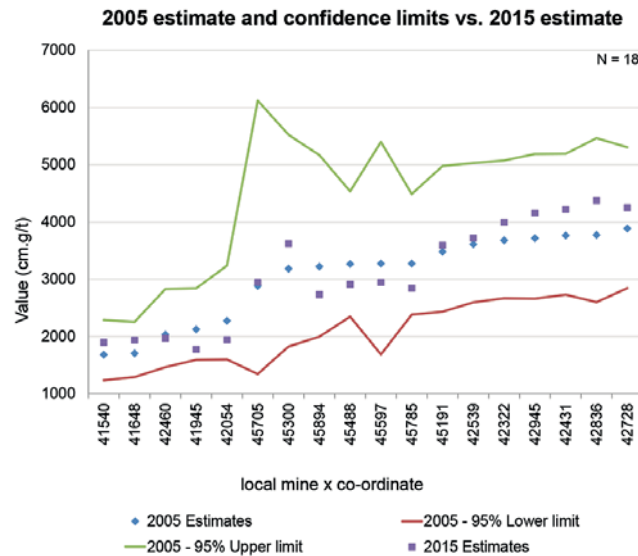
Figure 14—Comparison of the 2005 and 2015 estimates with the 95% confidence limits from the 2005 estimation (taking the 2005 estimates as done in 2005 *vs.* the 2015 estimates)

has undergone intense scrutiny by a number of external auditors over the past couple of years and proved to be appropriate for the CLR.

Acknowledgements

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A test of the appropriateness of the LUC technique in high-nugget Birimian-style gold deposits

by E. Maritz*

Synopsis

The localized uniform conditioning (LUC) technique was proposed by Marat Abzalov in 2006. The technique converts conventional uniform conditioning (UC) grade-tonnage curves into single grade values attached to each smallest mining unit (SMU). This is achieved by ranking the SMUs within a panel in increasing order of their grade (based on the direct kriging of SMUs). This ranking is then used to localize the conventional UC grade-tonnage curves for each panel by dividing them into classes and computing their mean grades, which are assigned to the SMUs. The quality of this localization process will depend heavily on the validity of the grade patterns predicted by the direct kriging of the SMUs. Abzalov noted that where the distribution of data available for the direct kriging of the SMU is characterized by strong short-range variability, the advantages of using the LUC approach may be more limited. Consequently, a study was undertaken to determine how valid the predicted grade patterns of a typical Birimian-style gold deposit (with high nugget effect and strong short-range variability) might be expected to be. This was determined by comparing the direct SMU kriging ranking (based on sparse data) with the grade control model ranking (based on close-spaced data and the *best available estimate* of the deposit). The results showed a satisfactory correlation between these rankings and it was concluded that, although the grade patterns predicted by the direct kriging of the SMUs may be less meaningful for deposits exhibiting strong short-range continuity, there is nevertheless a convincing relationship with the actual (or *best available*) rankings. Therefore, the LUC technique is still considered to be useful for this style of deposit.

Keywords
localized uniform conditioning; geostatistics; recoverable resources.

Introduction

For adequate technical and financial evaluation of a project, attempts should be made to estimate the *recoverable resources* – the portion of the *in situ* resource that can be economically extracted by mining. To achieve this, the estimates of the tonnage and grade of the mineralization should be produced above a given economic cut-off and should take into consideration the proposed mining selectivity.

At the early stages of exploration, we often have only broad-spaced sample data to estimate with. Ordinary kriging (OK), a commonly used linear interpolator, may be used to estimate grades into larger panels (estimation into smaller panels that are not adequately supported by dense data may result in smoothed and conditionally biased estimates). These larger panels, which are suitable for the broadly spaced data, often do not adequately represent the selectivity expected at the time of mining. The mining selectivity (represented by the smallest mining unit or SMU) is based on the deposit type and the chosen mining equipment.

Nonlinear techniques, such as uniform conditioning (UC) and multiple indicator kriging (MIK), are commonly used to generate estimates at SMU scale reflecting the proposed mining selectivity. With these techniques, the portion of the mineralization that can be economically extracted is estimated by determining the distribution of SMUs within each panel based on a change-of-support model. Estimates of the grades and proportions extractable above a given cut-off are provided for each panel without specifying precise spatial locations for this recoverable mineralization. A better understanding of the actual spatial locations of the SMUs would significantly simplify the manipulation of the results for mine planning purposes and would enhance the technical and financial evaluation of the project.

In 2006, Marat Abzalov (Abzalov, 2006) proposed a method called localized uniform conditioning (LUC) for predicting the spatial locations of the economically extractable mineralization by assigning a single grade to each SMU-sized block. LUC enhances the UC approach by localizing the model results. The grades of the SMUs are derived from the conventional UC grade-tonnage relationships. For each panel, the UC grade-tonnage function is divided into grade classes and the mean grades of the grade classes are assigned to the

* AngloGold Ashanti.

SMUs in the panel. The method of mean grade assignment is based on a predicted grade pattern within each panel. The grade pattern is determined by OK of the SMUs from the sparse data-set and is used to rank the SMUs within each panel in increasing order of their grade before assigning the mean grades of the UC grade classes.

Abzalov (2006) noted that spatial grade distribution patterns are often recognized by geoscientists in deposits even when drill spacing is still too broad for direct accurate modelling of small block grades, but sufficient for identification of the major distribution trends. He suggested that, even when drill spacing is too broad to avoid a smoothed SMU grade estimate, direct kriging of the small blocks can be used to obtain reliable grade patterns and the resultant SMU ranking within the panels. Abzalov deemed that this was particularly applicable to continuous mineralization characterized by a low nugget effect, such as disseminated base-metal sulphides, bauxites, and iron oxide deposits. He cautioned that where the data is sparse and not close to a panel, or its distribution is characterized by strong short-range variability, there could be less of a meaningful pattern. Accordingly, if the predictions of the SMU rankings by OK (or any other technique) are inadequate, the advantages of using the LUC approach will be more limited, or LUC may even be entirely unsuitable. A basic assumption of the conventional UC approach is that the locations of ore and waste within the panels are unknown (the SMUs are distributed randomly within the panels). The LUC method aims to overcome this theoretical constraint by attempting to predict the spatial locations of the SMUs, but its validity is strongly dependent on the ability to confidently estimate the rankings of the SMUs within the panels.

As a result of this, a study was undertaken to determine how meaningful the predicted grade patterns of a typical Birimian-style gold deposit (with high nugget effect and strong short-range variability) might be expected to be. This was determined by investigating the relationship between the LUC ranking (based on the direct kriging of the SMUs from sparse data) and the grade control model ranking (based on close-spaced data and the best available estimate of the deposit).

Case study

The northern pit of the Tambali gold deposit was chosen for the case study. The deposit forms part of the Sadiola gold mine located in Mali close to the border with Senegal and approximately 440 km northwest of the capital Bamako (Figure 1).

The Sadiola gold deposits lie within the Kenieba Kedougou Birimian greenstone belt of southwestern Mali (2.17–2.18 Ga). The deposits are hosted by the Kofi Formation – a dominantly metasedimentary unit. At Tambali, the host rocks consist of moderately-sorted meta-sandstone with minor meta-siltstone interbeds and a finely bedded siltstone-shale unit with minor sandstone interbeds. These metasedimentary units are north-trending, but are intruded by numerous NNE-trending quartz-feldspar porphyry (QFP) dykes and plugs. The mineralization is developed in all host rocks and the mineralization trends are associated with structural corridors (shear zones) marked by veining, alteration, and weathering. The dominant ore mineral is arsenopyrite, although pyrite, and to a lesser extent pyrrhotite, have also been observed in drill core. Antimony-bearing minerals are present in trace to minor amounts. The pathfinder element association of the ore typically comprises As-Au-Sb ± Ag-Bi-Mo.

Gold grade and structural trends were used to interpret the mineralization using Leapfrog© software. The interpretation was generated using the implicit Leapfrog© Grade Interpolation technique, which involves the 3D contouring of grades while taking into account a chosen grade threshold and defined structural trends. The output envelope based on a threshold (or lower grade limit) of 0.35 g/t was selected as it was deemed to best represent the mineralization. Before finalizing, it was adjusted by a few manual edits where required. The domain used for the study included all material
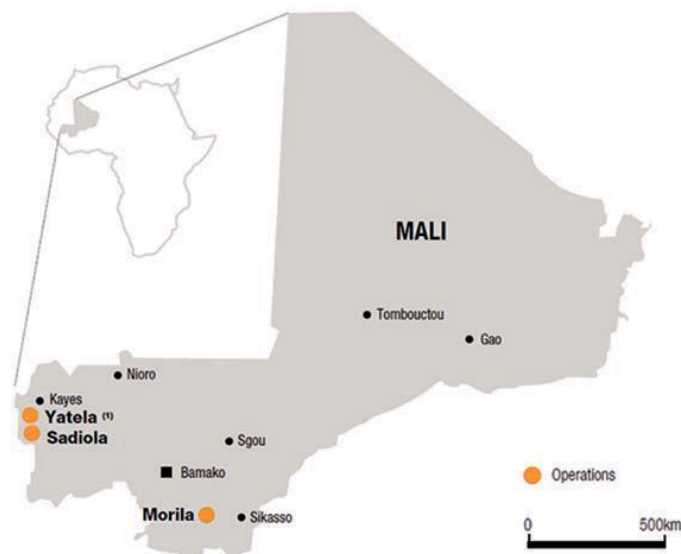
Figure 1—Locality map of the Sadiola gold mine in Mali

occurring within the mineralization envelope and represented the north- to northeast- trending shear fabric to which the mineralization is related (Figure 2).

All available exploration and grade control data from the mined-out portion of the Tambali North pit informed the study. The exploration drill-hole spacing was approximately 25 m E by 25 m N and the grade control drill-hole spacing approximately 6.25 m E by 12.5 m N (Figure 2).

The study area contained 4851 composited grade control plus exploration samples (all available data, *i.e.* the dense data-set) and 806 composited exploration samples (the sparse data-set). All composites were approximately 2 m long (Figure 3).

LUC estimation

A grade capping exercise showed that capping the exploration data-set to 15 g/t and the total data-set to 25 g/t would be appropriate for estimation. The investigation of histograms, log probability plots, and mean and variance plots was used to determine suitable grade cap values. A total of four values were capped for the exploration data-set (representing about 0.5% of the data-set) and eleven values for the total data-set (representing about 0.2% of the total data-set). The two data-sets were de-clustered with the ISATIS© software, which makes use of a moving window to assign de-c1 (perpendicular to the major and semi-major planes). The experimental variogram was modelled with a nugget effect and two spherical structures (Figure 5). The

relative nugget effect of this variogram, calculated as a ratio of nugget to the global sill, is approximately 33%. This variogram model has been used further in this study for all the block grade estimation using OK and UC techniques.

The optimal set of estimation parameters was determined by a kriging neighbourhood analysis (KNA). The kriging efficiency and slope of regression were used to investigate conditional bias for a given set of estimation parameters (Figure 6). At the chosen block size of 30 m N by 30 m E by 10 m RL and a maximum number of composites of 80, the slope of regression and kriging efficiency were satisfactory at about 0.95 and 0.82 respectively.

The final set of estimation parameters used for kriging are summarized in Table I.

The Tambali mining equipment supports selectivity (SMU size) of 10 m N by 10 m E by 3.33 m RL (mining of 10 m benches in 3.33 m flitches). In total, 27 SMUs fit within each panel of size 30 m N by 30 m E by 10 m RL.
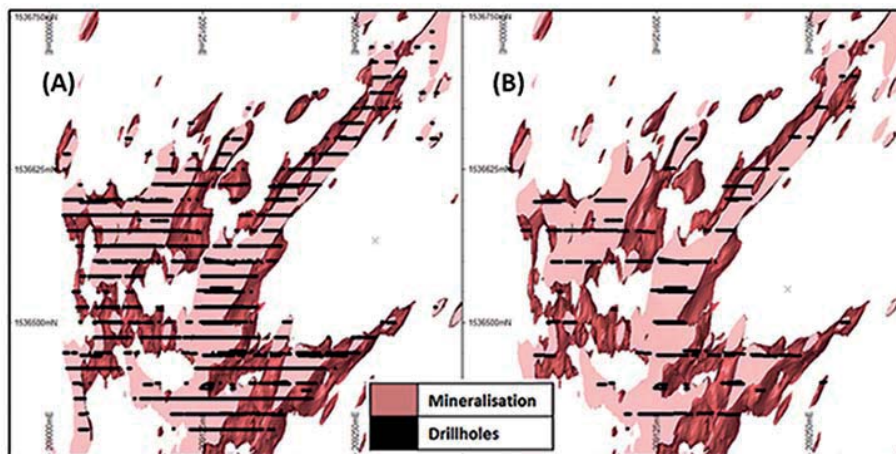
Figure 2—Plan view showing (A) all samples and (B) exploration samples distributed in the study area
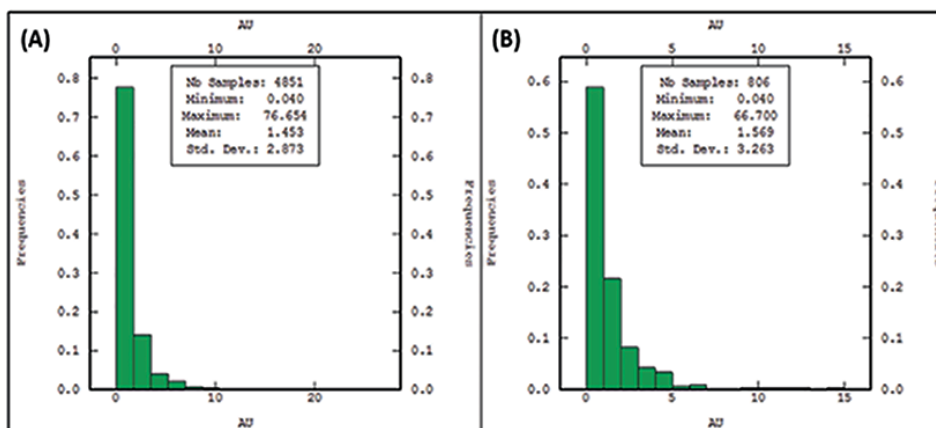
Figure 3—Histograms of gold grade – all data (A) and exploration data (B)
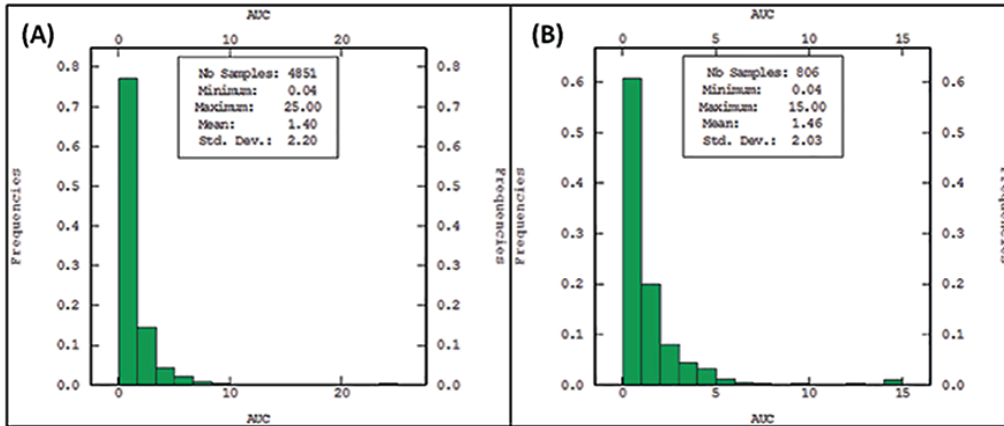
Figure 4—Histograms of de-clustered and top-capped gold grade – all data (A) and exploration data (B)

Parameters of the variogram model

Structure 1 (Nugget)	1.7	
	Structure 2 (Spherical)	Structure 3 (Spherical)
Sill	1.38	2.09
Range Major (A)	7	45
Range Semi-Major (B)	17	30
Range Minor (C)	5	12
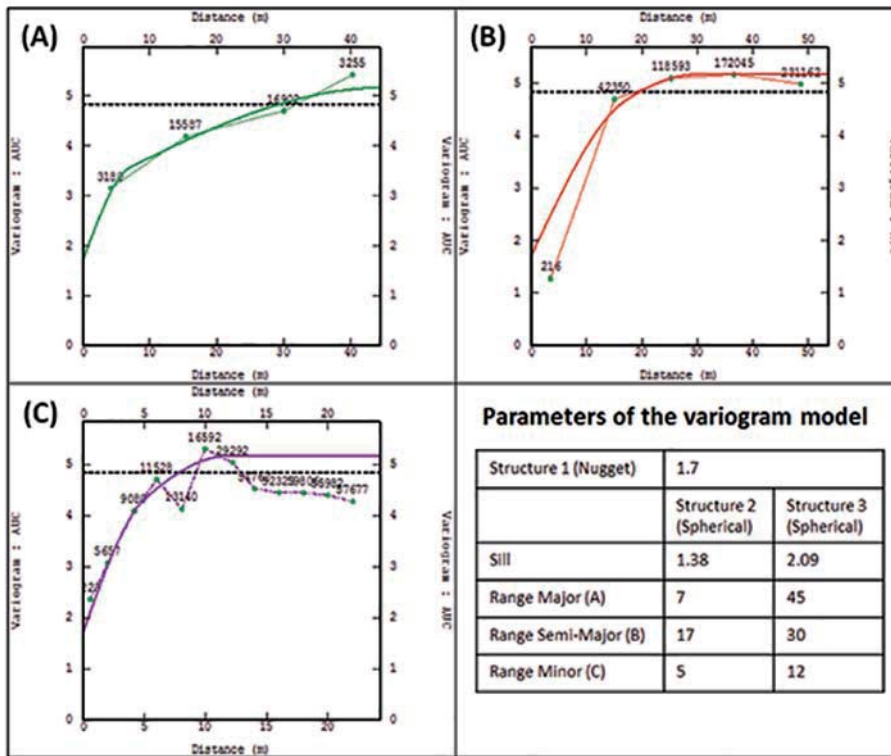
Figure 5—Directional variograms of gold composite grades along the major (A), semi-major (B), and minor (C) directions of continuity
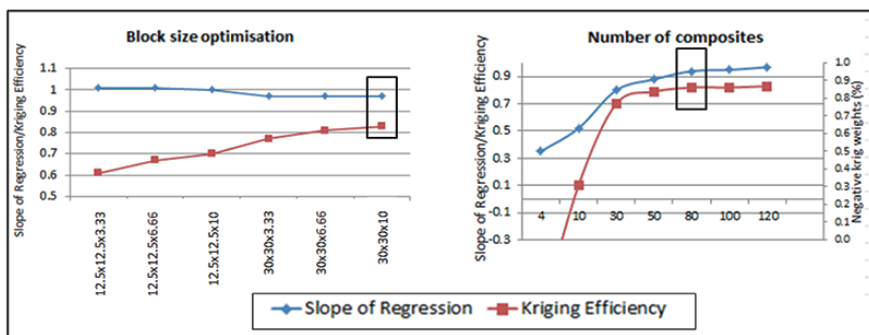
Figure 6—Slope of regression and kriging efficiency for block size and number of composites

A test of the appropriateness of the LUC technique

Table I

Estimation parameters for ordinary kriging

Parameter	OK panels: sparse data	OK SMU: sparse data	OK SMU: all data ('truth')
Minimum number of composites	10	10	10
Maximum number of composites	80	80	40
Search ellipsoid rotation	Azimuth: 35 Dip: 75 Dip direction: 125	Azimuth: 35 Dip: 75 Dip direction: 125	Azimuth: 35 Dip: 75 Dip direction: 125
Search ellipse dimensions	70×50×20	70×50×20	35×25×15
Discretization	5×5×5	5×5×5	5×5×5
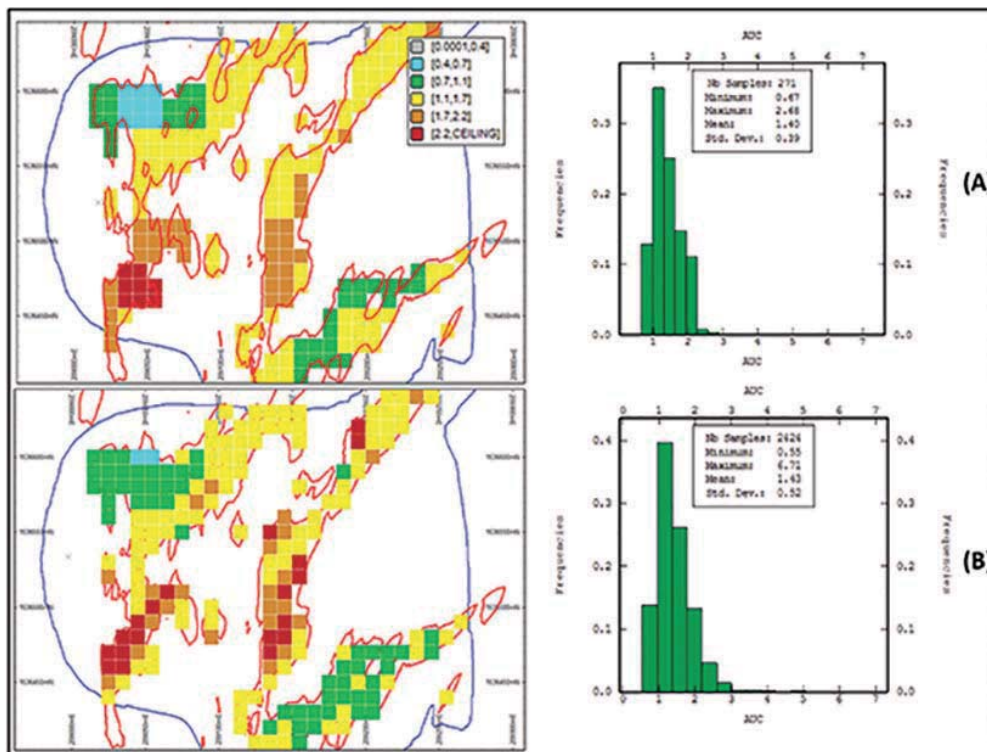
Figure 7—Panel (A) and SMU (B) grades estimated by ordinary kriging with sparse exploration data

The sparse data-set (early stage/exploration) was used for kriging both the SMUs and the panels. The same variogram model and the same search neighbourhoods were used for both kriging runs (Table I). The distributions of the OK grades of the SMU and panels are compared in Figure 7 (represented by a 3.33 m horizontal slice through the block models).

Ordinary kriging estimates of the SMUs based on all available data (dense data-set: grade control plus exploration samples) were also generated and were considered to represent the best available estimate of the SMU grades. For the purposes of this study; they are referred to as the 'true' SMU grades. The SMU estimates from sparse data were excessively smoothed in comparison with these 'true' SMU grades, as shown in Figure 8. The global mean grades were similar, but the variances differed markedly with the 'true' grade standard deviation of 0.75 much greater than the standard deviation of the sparse data estimates (0.52). As noted by Abzalov (2006), an attempt to use SMU grades

obtained by kriging with the sparsely distributed data can lead to very inaccurate assumptions regarding the optimal mining scenarios.

ISATIS© software was used to model the recoverable resources from the sparse data using the conventional UC method. Correction for the information effect was made during the change of support procedure. The information effect takes account of the fact that the SMUs will ultimately be selected on an estimated grade (based on the grade control samples) instead of the real grade. Hence, some ore blocks will be misclassified as waste and *vice versa*. In order to obtain a more realistic recoverable estimate that takes account of this misclassification, a correction for the information effect was made by assuming that the final sampling mesh will be 6.25 m E by 12.5 m N by 2 m RL (*i.e.* the production or grade control sample spacing).

The grade-tonnage curves of the OK panel grades, the block anamorphosis function (at SMU support), and the UC grades are shown in Figure 9. Compared with the panel

estimate, the block anamorphosis and the UC estimate showed greater selectivity (initial lower tons at higher grade).

The conventional UC grade-tonnage relationships corresponded significantly better with the grade-tonnage relationships of the 'true' SMU grades than that obtained with the OK estimates from sparse data (Figure 10). The UC model represents a significant improvement in comparison with the 'unconditioned' OK estimates from sparse data.

The conventional UC results were localized by the LUC technique, which involved ranking the SMU blocks within each panel (based on the OK SMU grades from sparse data) and deriving the grades of the SMU ranks from the UC model and assigning them to the corresponding SMU blocks (Figure 11).

The grade-tonnage curves of the LUC estimate were very similar to those of the UC estimate (Figure 12). The good match between the grade-tonnage curves derived from UC and LUC is expected as the LUC algorithm simply localizes the UC results, maintaining the grade–tonnage relationships predicted by the conventional UC model.
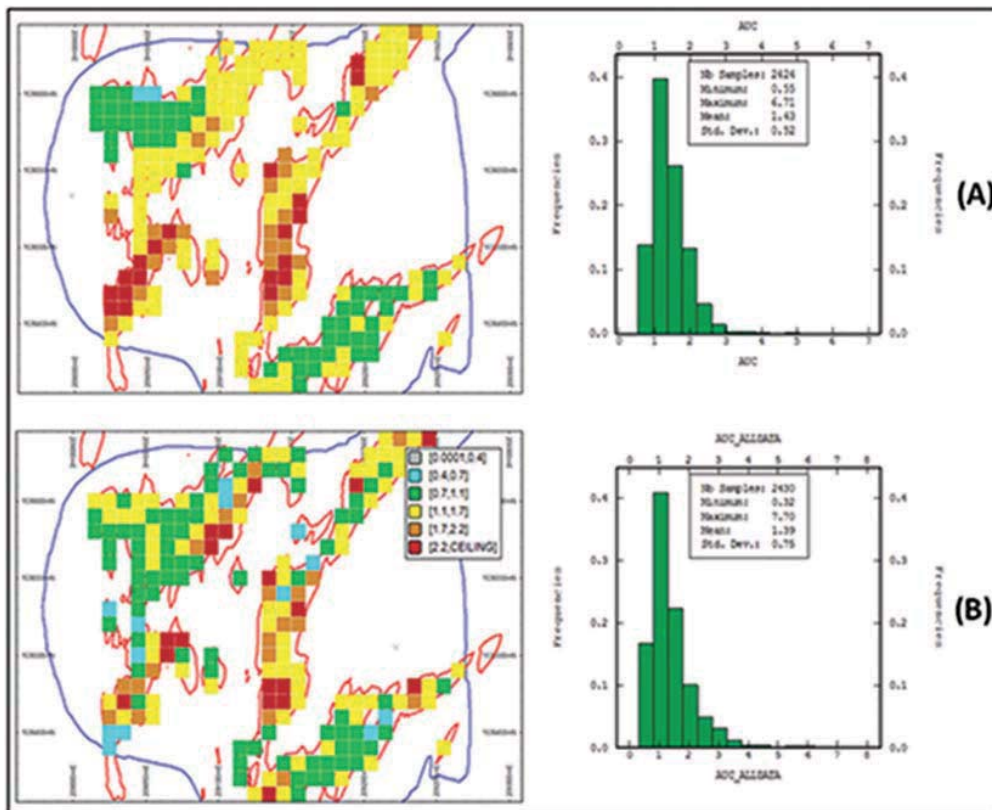
Figure 8—SMU grades estimated by ordinary kriging with (A) sparse exploration data and (B) dense exploration plus grade control data ('true' grades)
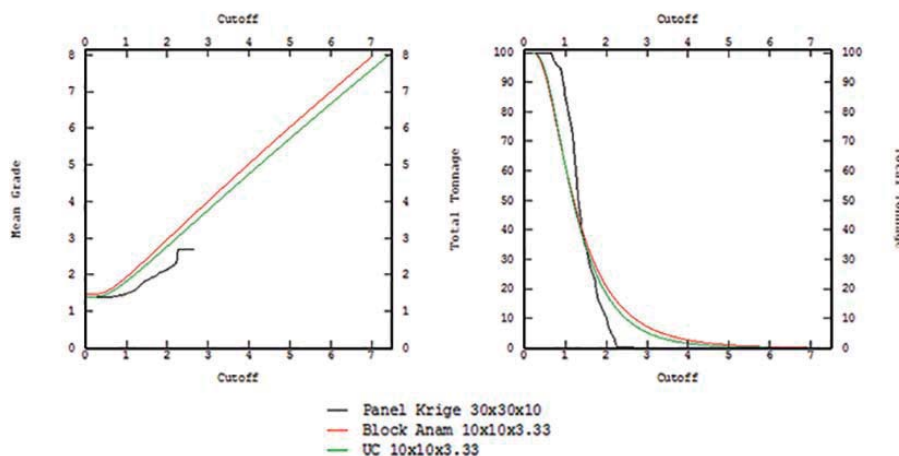
Figure 9—Mean grade and total tonnage curves: panel kriging *vs.* UC

A test of the appropriateness of the LUC technique
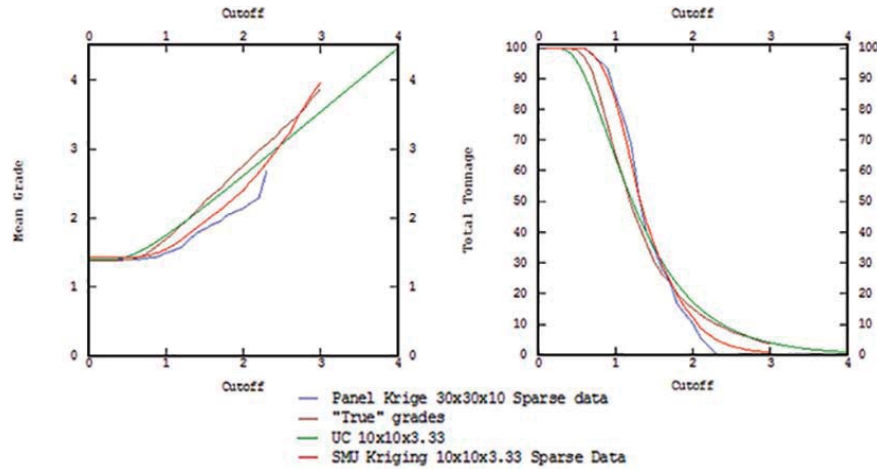
Figure 10—Mean grade and total tonnage curves: 'true' grades *vs.* UC and OK grades
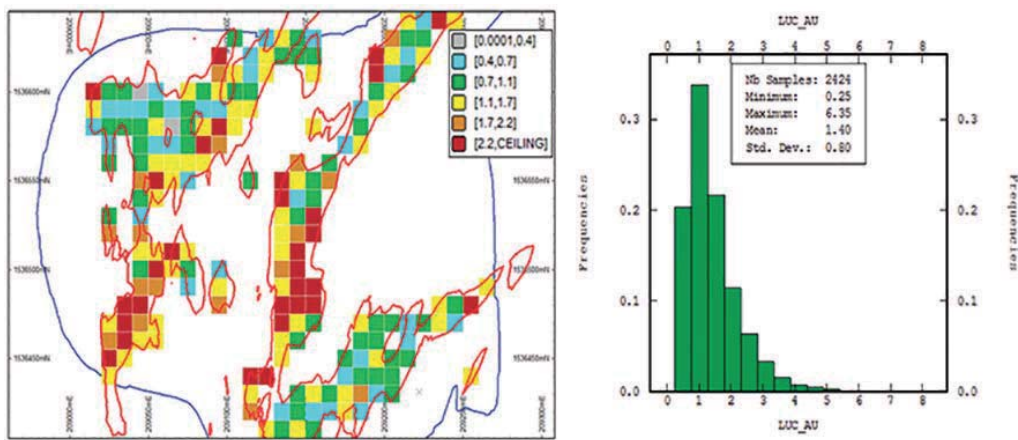
Figure 11—SMU grades estimated by the LUC technique with sparse data
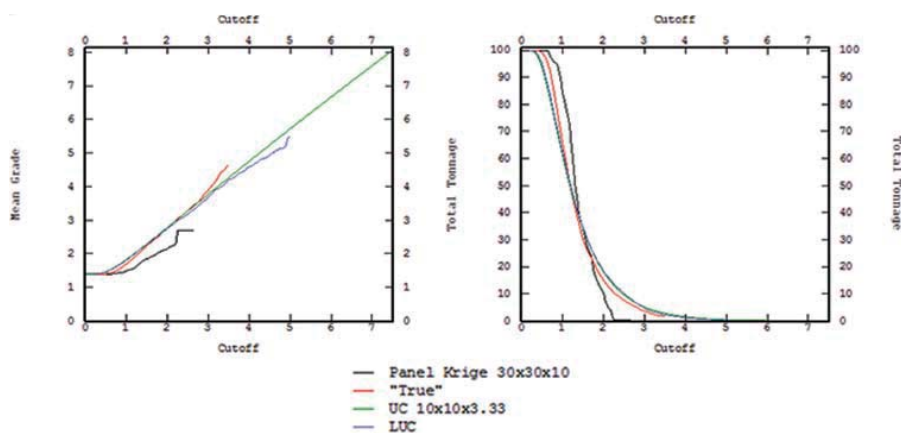
Figure 12—Mean grade and total tonnage curves: 'true' grades *vs.* UC, LUC and OK grades

The grade distribution of the LUC estimates was less smoothed than that of the sparse data OK estimates and, compared with the 'true' SMU grades, it better represented the variability of the deposit (Figure 13). The standard deviation of the SMU grades modelled by the LUC method

(SD = 0.80) was closer to that of the 'true' grades (SD = 0.75) and significantly larger than that obtained by kriging from a sparse data grid (SD = 0.52).

It is evident that the LUC estimate is a significantly better estimate of the recoverable resources than the OK estimates

A test of the appropriateness of the LUC technique
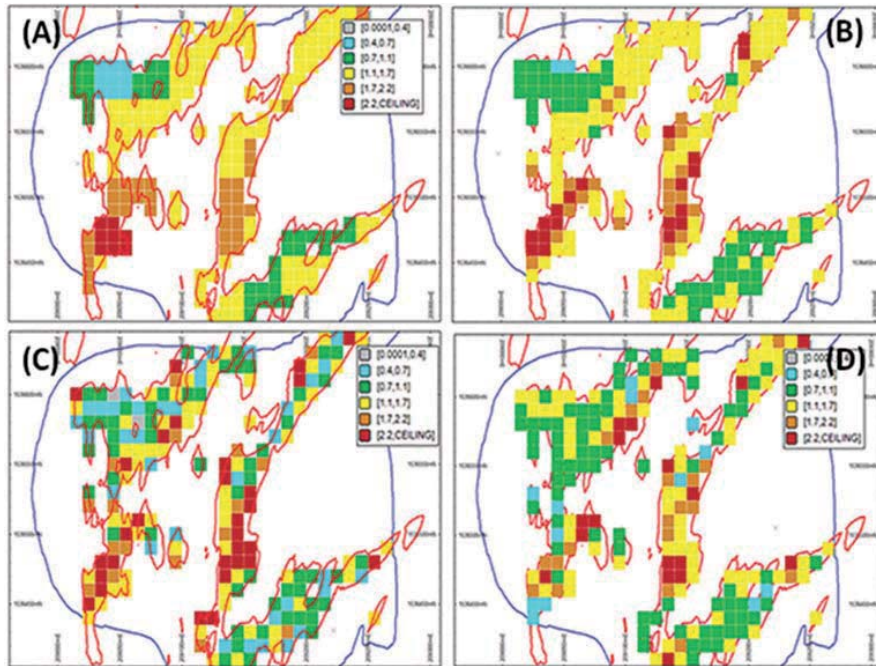
Figure 13—Plan view comparison of the panel kriging (A), the direct SMU kriging with sparse data (B), the LUC model (C), and the 'true' grades (D)

(from sparse data) and better represents the variability expected at the time of mining. The LUC estimate is still noticeably different from the 'true' grades. The technique itself does not make up for the fact that the LUC estimate is based on sparse (incomplete) data and that the LUC result depends heavily on the grade pattern predicted by the direct SMU kriging (also from sparse data).

Quality of the localization

The quality of the LUC localization is dependent on the meaningfulness of the grade pattern predicted by the direct kriging of the SMU (Abzalov, 2006). The resultant grade pattern is used for ranking of the SMUs into increasing order of their grade, which determines the order in which the mean grades of the UC grade classes are assigned to the SMUs.

For the case study, the quality of the localization was assessed by comparing the rankings of the 'true' grades with the LUC rankings. For both data-sets, the 27 SMUs within each panel were sorted in increasing order of grade. Thus, each SMU was assigned a 'true' ranking as well as a 'predicted' (or LUC) ranking between 1 and 27. The SMUs that fell outside of the estimation domain were disregarded (the affected panels therefore had fewer ranking pairs). A scatter plot showed a reasonable correlation between the 'true' and LUC rankings with a correlation coefficient of 0.6 (Figure 14).

The number of occurrences of each ranking combination ('true' *vs.* LUC) was subsequently counted across all panels. For example, counting the number of instances where the actual and predicted ranks were both 1; then the number of instances where the actual rank was 1, but the predicted rank was 2; and so forth. The result is presented in Figure 15 and shows all possible ranking combinations for up to 27 SMUs.

The actual (or 'true') ranking is shown on the X-axis and the predicted (or LUC) ranking on the Y-axis. The colouring is based on the number of instances that a rank pair occurred.

Overall, the results showed a reasonable relationship between the actual and predicted rankings, with a significantly greater amount of predicted SMU rankings being closer to the actual rankings than further away. It can be concluded that, even though we are dealing with a deposit exhibiting high nugget effect and strong short-range variability, there nevertheless appears to be some confidence in the local positioning achieved by the LUC technique, *i.e.* it does not appear to be random, but shows a relationship with the 'true' positioning.
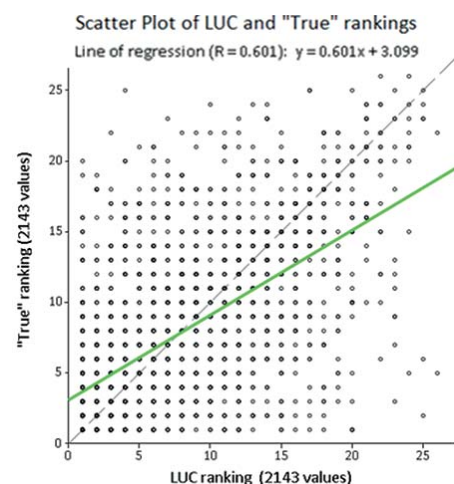
Figure 14—Scatter plot of LUC *vs.* true rankings

A test of the appropriateness of the LUC technique
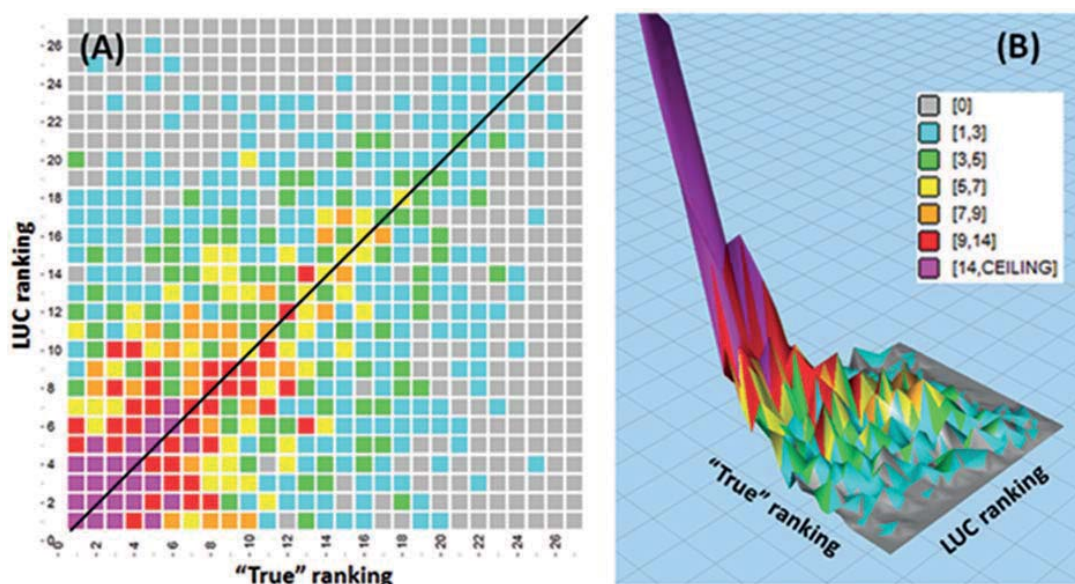
Figure 15—Plot of the number of occurrences of 'true' *vs.* LUC rankings shown in (A) plan and (B) three-dimensional view

Ordinary kriging *versus* inverse distance weighting for SMU ranking

LUC can easily incorporate external information such as high-resolution geophysical data or other estimation techniques, as pointed out by Abzalov (2006). The LUC ranking determined by OK was compared with rankings obtained by inverse distance weighting (IDW) to evaluate the robustness of the OK estimation technique for determination of the rankings (Figure 16). Two IDW estimates were produced - one to the power of 2 (IDW²) and one to the power of 5 (IDW⁵).

Visually, the LUC results from the OK and IDW rankings look similar, with the LUC model based on IDW rankings slightly more smoothed in comparison with that based on OK rankings. However, when comparing the rank count plots for the three scenarios (counting the number of occurrences of each ranking combination) the LUC ranking based on OK appears to be better correlated with the 'true' rankings than those based on IDW² and IDW⁵ (Figure 17). In turn, compared with the IDW² rankings, the IDW⁵ rankings show a better relationship with the 'true' rankings.

Grade control reconciliation

As a last check of the reliability of the LUC estimate, it was compared with the grade control model estimate over the study area (Table II). For confidentiality purposes, the grades have been factored with a constant value.

The grade control and LUC models compared well, with tons and metal within about 4–9% of each other and grades within 1–2%.

Summary and conclusions

A basic assumption of the conventional UC approach is that the locations of ore and waste within the panels are unknown. The LUC method aims to overcome this theoretical constraint by attempting to predict the spatial locations of the

SMUs, but its validity is strongly dependent on the ability to confidently estimate the rankings of the SMUs within the panels.

Since 2006, the LUC method has been implemented in commercial software and has been commonly used for the estimation of recoverable resources. The LUC technique is an enhancement of the conventional UC technique and it reproduces the conventional UC grade-tonnage relationships. Even though this is the case, the validity of the localization is heavily reliant on the ability to reasonably predict SMU rankings from sparse data and the accuracy of this localization depends on the techniques used for the SMU ranking (Abzalov, 2014). It is considered that, when using direct kriging of the SMU for ranking, the presence of a high nugget effect and strong short-range variability could potentially result in inadequate localization. Accordingly, if the predictions of the SMU rankings by OK (or any other technique) are inadequate, the advantages of using the LUC approach will be more limited or LUC may even be entirely unsuitable. It is therefore deemed necessary to assess the quality of the localization before accepting a LUC result. In the mined-out area of an active open pit, one could achieve this by comparing the rankings of the SMUs based on close-spaced grade control data with the rankings based on sparse exploration data (as was done in this study). In an unmined pit with no close-spaced data, it is more difficult to assess the quality of the localization. However, one could attempt to improve the rankings from the direct kriging of the SMUs by integrating them with auxiliary data such as geophysical or geochemical information as proposed by Abzalov (s2014).

In the current study, the LUC technique was implemented for the mined-out portion of a typical Birimian-style gold deposit (mined by open pit methods) to model the grades of SMU-sized blocks from sparse, early-stage data. The LUC grade-tonnage relationships closely matched the conventional UC grade-tonnage relationships and better predicted the grade-tonnage relationship of the 'true' grades than those derived from ordinary kriging. In order to assess the quality
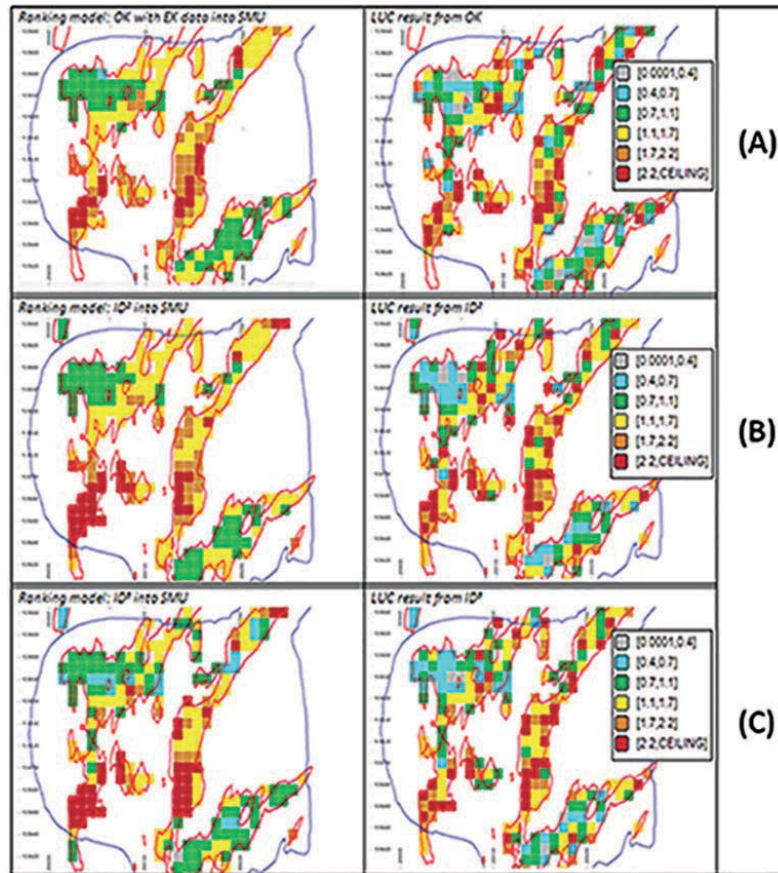
Figure 16—Plan view comparison showing the ranking models (left) and the corresponding LUC result (right) for ordinary kriging (A), IDW² (B), and IDW⁵ (C)
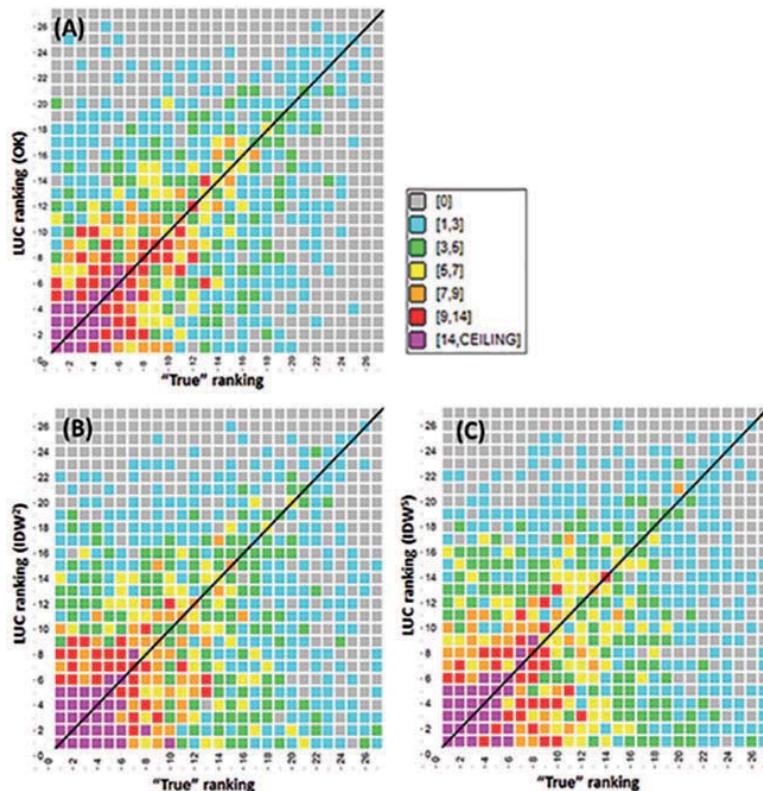
Figure 17—Comparison of the number of occurrences of 'true' vs. LUC rankings based on rankings from (A) ordinary kriging, (B) IDW², and (C) IDW⁵

A test of the appropriateness of the LUC technique

Table II

Tons, grade, and metal comparisons of grade control and LUC within the study area

Cut-off grade (g/t)	Grade control model			LUC model			Percentage difference		
	Tons	Grade (g/t)	Metal (g)	Tons	Grade (g/t)	Metal (g)	Tons	Grade (g/t)	Metal (g)
0.0	778 083	1.39	1 084 858	713 083	1.41	1 005 692	-8%	1%	-7%
0.4	777 472	1.40	1 084 658	711 667	1.41	1 005 208	-8%	1%	-7%
0.5	775 528	1.40	1 083 744	706 583	1.42	1 002 855	-9%	2%	-7%
0.6	757 944	1.42	1 073 911	691 083	1.44	994 202	-9%	2%	-7%
0.7	718 417	1.46	1 048 128	650 417	1.49	967 707	-9%	2%	-8%
0.8	651 805	1.53	998 432	590 333	1.56	922 518	-9%	2%	-8%
0.9	575 139	1.62	933 577	529 833	1.64	871 178	-8%	1%	-7%
1.0	510 944	1.71	872 971	470 417	1.73	814 883	-8%	1%	-7%
1.1	445 139	1.81	804 089	412 833	1.83	754 347	-7%	1%	-6%
1.2	384 000	1.91	733 918	355 000	1.94	687 977	-8%	1%	-6%
1.3	330 444	2.02	667 239	308 000	2.04	629 348	-7%	1%	-6%
1.4	286 139	2.12	607 876	267 833	2.15	575 153	-6%	1%	-5%
1.5	242 917	2.24	545 271	231 250	2.26	522 203	-5%	1%	-4%
2.0	121 722	2.78	337 940	115 917	2.80	324 365	-5%	1%	-4%

of the LUC localization, the direct SMU kriging rankings (based on sparse data) were compared with the grade control model rankings (based on close-spaced data and the best available estimate of the deposit). The results showed a reasonable relationship between the actual and predicted rankings and it was concluded that, even though the grade patterns predicted by the direct kriging of the SMUs may be less meaningful for deposits exhibiting strong short-range continuity, there nevertheless appears to be some confidence in the local positioning achieved by the LUC technique. Therefore, it is considered that the use of the LUC technique may be useful for this style of deposits.

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The practical implementation of uniform conditioning at AngloGold Ashanti African Operations, and a case study as applied for potential underground mining at Nyankanga pit, Geita gold mine, Tanzania

by V. Govindsammy*

Synopsis

The use of uniform conditioning (UC) as an estimation technique to produce robust recoverable resource models has been implemented across various operations within the AGA Continental Africa Region. This paper outlines the relevance of using UC as an estimation technique to provide a robust estimate for use in underground mine planning and published Mineral Resource statements. The change-of-support model forms the basis on which planning and financial decisions are made, and it is therefore imperative that appropriate validations and checks against 'reality' are carried out prior to implementation. The process and validation techniques employed for UC will be discussed using a case study from the Nyankanga orebody at Geita gold mine in Tanzania. The deeper portions of the orebody constitute potential underground mining areas, and by using appropriate reconciliations it can be shown that the UC model, in spite of the inherent lack of local spatial accuracy, can be used to estimate the potential underground stopes with a lower error of estimation than other techniques.

Keywords
discrete Gaussian model, selective mining unit, dispersion variance, population variance, block support correction, information effect.

Introduction

The successful implementation of uniform conditioning (UC) requires a good understanding of both project-scale and local-scale geological controls on mineralization. On a project scale, orientation of the mineralization as well as the nature of ore/waste contacts, together with an understanding of size and distribution of high- and low-grade ore blocks, forms a key component of the UC process. Any geological feature that disturbs the main mineralization must also be understood and modelled correctly. In addition to the basics of any estimation technique like proper database protocols, quality assurance and control (QA/QC) for both laboratory and drilling practices, and sound geological models, UC is heavily reliant on proper domaining and robust kriged estimates. This differs from some statements in the literature, that UC performs well when domains are not strictly stationary (in theory, loose domaining can be used, but experience has found that this is not the case in practice). UC is a variation of Gaussian disjunctive kriging that is better adapted to situations where stationarity is not very good (Vann, 1998). The change-of-support model for the Nyankanga project uses a drill pattern of $40 \times 40 \times 1$ m for Indicated Resources and $40 \times 80 \times 1$ m for Inferred Resources to produce grade-tonnage relationships on a support size of $10 \times 10 \times 3.33$ m. The behaviour of this grade-tonnage relationship is a reflection of the local geological controls on mineralization. Practical reconciliation studies have shown that improvements in the geological and statistical domains of the Nyankanga orebody results in better local accuracy while still preserving the global accuracy that comes with a UC model. This allows for the UC model to be used for both underground and open pit mine planning scenarios.

The Nyankanga deposit forms the southwestern limit of the current known resources along the Geita central trend and suboutcrops in low ground below 10 to 15 m of barren, transported laterite cover. The main orebody ranges up to 100 m in thickness in the central part of the deposit and dips sub-parallel to the stratigraphy. Two phases of syn- to post-mineralization dykes occur throughout the deposit and are thought to represent late felsic evolution of the intrusive system at Nyankanga. Initial emplacement of en-echelon felsic porphyry (FP) dykes 50 to 100 m apart occurred dominantly along pre-existing joint sets dipping 40 to 50 degrees in a northwesterly direction with some dilation parallel to the ore zone where they crosscut the stratigraphy. A barren, final-stage quartz porphyry (QP) dyke crosscuts and displaces all lithologies and mineralization and has been emplaced along similar joint-related structures.

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* AngloGold Ashanti.

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Another late-stage QP dyke dips 70 to 80 degrees to the southeast. All dykes follow the same sigmoidal drag-folded pattern as the bedding and orebody (Figure 1).

On a local scale, the mineralization model follows a complex interplay between structure and lithology. The higher grade mineralization occurs mostly in the banded iron formation (BIF) and lower grade mineralization mostly occurs in the microdiorite (MD) units. High-grade mineralization can traverse into the MD unit manifested as brecciated zones. The felsic intrusive that cuts through the orebody is mostly barren, but some high-grade samples occur in the contact areas due to remobilization of gold.

The initial UC model for the Nyankanga deposit at Geita gold mine was produced in 2006 (Gaunt, 2006) with guidance and technical support provided by V.A Chamberlain from the corporate office. Mineral Resource models created prior to this used deterministic wireframes of the ore zones that were subdivided into BIF and MD type mineralization. Kriged block models with block sizes generally much smaller than the drill-hole spacing were used as a basis for the annual published Mineral Resource and mine planning processes. Over the last 7 years, numerous geologists worked on gaining a better understanding of the local geological controls and the Resource model using UC as a primary technique evolved through regular analyses of reconciliations, allowing for continuous improvements in the practical implementation of this technique.

Key checks prior to UC

Statements like 'UC does not work' are common in the industry, and learning experience from Geita has shown that insufficient time spent on data validation, QA/QC checks, improper domaining, and substandard optimization of kriging parameters can lead to poor-quality or biased kriged estimates, resulting in poor reconciliations between the UC and grade control models.

During the exploratory data analysis stage it is important to ensure that all drill-hole information is representative (*i.e.* sampled across the entire orebody width) and sample intervals reflect the variability at an appropriate width. Compositing of sampling data was done on a 1 m interval since this was the accepted sampling interval that represents the local grade-lithological relationships.

Is your deposit suited for UC?

Two basic models exist that describe the spatial distribution of the variable to be estimated – the mosaic- and diffusion-type models. In a mosaic-type model the edges of the high-grade material are not systematically lower than central areas, while in a diffusion-type model, high and low grades are separated by intermediate grade material (Deraisme, 2012). Gaussian-based methods like the discrete Gaussian model (DGM) are applicable when the orebody displays characteristics of the diffusion model.

When using UC a Gaussian distribution is employed. The Gaussian transformed grades, however, have to show properties of bi-Gaussianality. This simply means that linear combinations of data pairs separated by a vector h follow a normal distribution. This is an important assumption when using UC, since the DGM change of support assumes that point values within a block are correlated with the block values (Deraisme, 2012).

In addition to the test for bi-Gaussianality one should initially test for normality of the transformed variable (Harley, 2009). This is conveniently done using a normal probability plot, which must show that the standard Gaussian data-set has a mean close to zero and standard deviation close to unity. These tests were conducted in Isatis and confirmed that UC is an appropriate estimation method.

Domaining: a key component of exploratory data analysis

The histograms of samples together with the corresponding variograms and the subsequent DGM change-of-support model from points to blocks are fundamental to the UC process. Effective domaining is therefore critical, since the characteristics of each population (in the form of histograms and semivariograms) can be adequately determined. The local geology at Nyankanga is complex and hence numerous
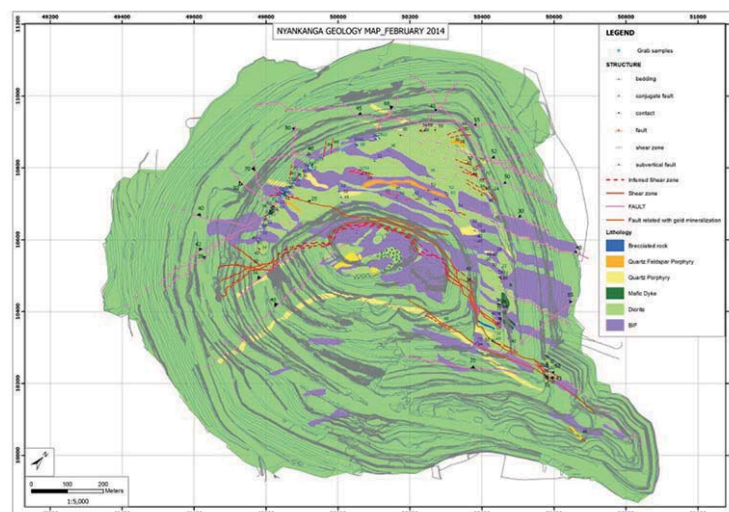
Figure 1—Plan view of Nyankanga pit showing mapped geological features

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domains have been constructed for the entire project. The domaining process considered all geological features on strike of the orebody as major domain boundaries. Subdomains to reflect the local geological variation were then created by analysing grade variation in relation to local geological features. This step is critical to ensure that robust kriged estimates are achieved.

Strong relationships between lithology and grade were identified, and by conducting trend analyses across the three dimensions of the orebody suitable subdomains were created. The orebody limit was chosen at 0.5 g/t (boundary of ore wireframe). This corresponded in some instances with a structural contact along a thrust surface. The grade rapidly decreases to below 0.5 g/t as one move across the orebody limit. Subdomains are then created based on BIF- and MD-hosted rock types, since the grade signatures within these units are significantly different (Figure 2). This key aspect of domaining forms an integral part of separating high- and low-grade units of the orebody, which aids in planning for underground mining scenarios (Table I). Historical reconciliations to grade control models were poor when these domains based on BIF and MD was not created. Analyses of these reconciliations showed a significant overstatement of high-grade tonnages (Figure 3). In Figure 3, 'Old Model' refers to combined BIF and MD domains and 'ResMod' refers to the new resource model where BIF and MD were treated as separate domains. 'GC MOD' refers to the grade control model that was generated after close-spaced drilling.

Statistics within the domains

Table I shows significant differences in mean grade and variance between the BIF and MD units. The coefficient of variation is also significantly high, suggesting highly skewed distributions. The high-grade tails form a significant portion of the gold above the economic cut-off. The orebody is up to 100 m wide in the central portion and therefore multiple cutbacks are planned to access the deeper portions of the ore. The economic viability of these cutbacks depends on large amounts of capital spending and the relatively higher grade of ore to pay back this capital and generate profits. UC was therefore deemed an appropriate technique to estimate these high-grade units from relatively widely spaced drill-holes (40 × 40 × 1 m). It was therefore deemed important to validate the change-of-support model that translates point distributions as shown in Figure 2 to mineable block units.

Validation of the kriged estimates

A robust semivariogram, optimal kriging parameters, and a validated data-set may be used to produce panel estimates per geological/geostatistical homogenous zone. The standard checks like stepping through sections and comparing kriged estimates to sample information, checking the effect of high-grade outliers (especially in poorly informed blocks), and comparing local kriged estimates with local sample averages were performed. In addition to this, the DGM change-of-support distribution was validated against the distribution of panel kriged blocks. An example of this validation is shown
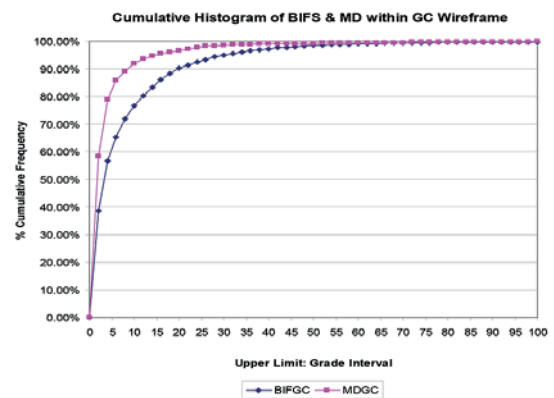
Figure 2—Cumulative percentage histogram of BIF and MD in grade control (GC) wireframe
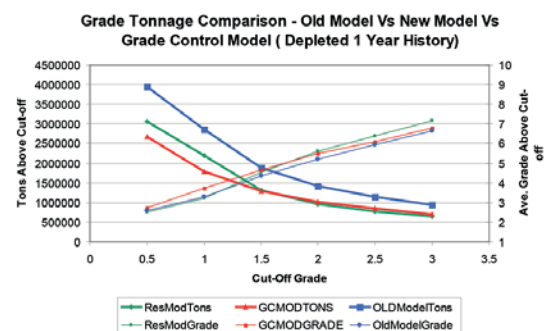
Figure 3—Reconciliation of old and new resource models in relation to grade control model

Table I

Univariate statistics of the domains at Nyankanga

STATS	Z1BIF&MD	Z2BIF	Z2MD	BLOCK1 BIF	BLOCK1 MD	BLOCK1 EAST BIF	BLOCK1 EAST MD	BLOCK1 WEST BIF	BLOCK1 WEST MD
Mean	1.20	4.61	1.83	8.00	2.88	11.68	3.76	6.11	2.14
Standard Error	0.04	0.07	0.03	0.30	0.14	0.60	0.27	0.31	0.13
Median	0.45	1.12	0.50	3.03	0.68	6.75	0.77	2.18	0.65
Mode	0.07	0.06	0.08	0.06	0.02	0.06	0.01	0.06	0.02
Standard Deviation	3.12	10.54	6.05	11.51	7.37	13.67	9.40	9.70	4.96
Sample Variance	9.75	111.07	36.65	132.40	54.33	186.99	88.43	94.11	24.60
CV	8.11	24.08	20.06	16.55	18.86	16.01	23.51	15.39	11.48
Kurtosis	283.79	311.30	744.33	12.87	49.89	5.30	34.38	25.63	49.28
Skewness	13.22	11.03	19.21	2.83	6.10	1.91	5.22	3.83	5.87
Range	96.00	496.00	378.88	110.00	93.00	101.00	93.00	110.00	72.82
Minimum	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.01
Maximum	96.00	496.00	378.88	110.00	93.00	101.00	93.00	110.00	72.82
Sum	5,914	107,050	61,384	12,117	7,848	5,991	4,671	6,126	3,177
Count	4,921	23,206	33,604	1,515	2,724	513	1,242	1,002	1,482

for one of the domains. The change of support using the point variogram and declustered sample point file was conducted to produce a global grade-tonnage relationship on the same support as the panel kriged blocks. The global distribution which shows the downside risk due to information effect closely resembles the kriged distribution on 40 × 40 ×10 m panel support (Figure 4).

Deviations in the above relationships can be due to a number of reasons and hence this forms a good validation technique. Practical issues that could contribute to such deviations are summarized as follows:

1. Significant nugget effect and ranges that may be shorter than the panel size to be estimated. Kriged panel size is much smaller than the drill-hole spacing
2. Poor choice of search neighbourhood and number of samples used to estimate panels
3. Closer and wider spaced drill-hole areas being estimated with the same estimation parameters (block size, minimum/maximum number of samples, block size, search neighbourhood)
4. In areas of wider spaced drill-hole information, the presence of a few high-grade outliers causes significant smearing of grade in the kriged distribution.

Prior to the adoption of UC, all of the above were checked and modified if necessary to produce robust kriged estimates of panel grades. This ensures that the local accuracy of the kriged estimates has not compromised the equivalent global accuracy.

Uniform conditioning

UC was first developed to relax the hypothesis of strict stationarity needed for disjunctive kriging and conditional expectation (Rivorard, 1994). In practice, however, it was observed that effective application of domaining resulted in improved reconciliations (Figure 3).

UC is performed for each domain and requires:

1. Gaussian anamorphosis modelling on point support, which requires a normal-score transformation
2. A change-of-support correlation (or coefficient) is determined for a selective mining unit (SMU) support (r). During this stage it is also possible to account for the information effect resulting from the final estimates of the SMU
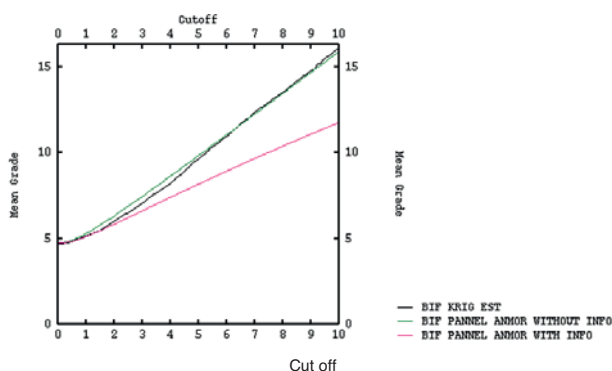
3. A change-of-support coefficient at a panel support (R) is determined from the panel dispersion variance and Gaussian variance properties of the anamorphosis model.

Gaussian anamorphosis modelling on point support

Knowing a representative histogram of point samples allows prediction of the recoverable resources by applying a cut-off grade. A model of the distribution with an appropriate change of support is needed to compute the global recoverable resources for any cut-off. Isatis uses the discrete Gaussian model (Poisson, 2008).

1. This model corresponds to a diffusion type of model
2. Grade distributions are rarely Gaussian. The raw distribution Z(x) is transformed into a Gaussian distribution Y(x): this transformation is called Gaussian anamorphosis and is noted: Z(x) = (Y(x)), as presented in Figure 5
3. The anamorphosis is bijective and invertible: knowing the anamorphosis is equivalent to knowing the histogram, hence recoverable resources may be computed by applying a cut-off on the Gaussian distribution
4. The Gaussian grades need to be bi-Gaussian. Checks for this have been discussed in the preceding section.

The following practical issues need to considered and implemented effectively.

1. Point samples should be declustered to prevent misrepresentation of population histogram characteristics. Drilling data occurs within grid patterns of 20 × 20 m and 40 × 40 m for Indicated Resources at Geita; hence a declustered mean was obtained at these drill spacings and compared with the ordinary kriged estimate. On a global basis, the mean grades of point samples and panels are in close agreement (Table II)
2. The declustered mean and variance of the raw data should be in close agreement with the mean and variance of the model fit, and in Gaussian space these should be close to 0 and 1 respectively
3. The tail end of the distribution is modelled up to an acceptable upper limit that represents the domain. In this instance an outlier analysis should be performed and the upper value modelled accordingly. This upper value is further validated after the change of support is performed.

Figure 4—Comparison of panel kriged block distribution to the equivalent DG change-of-support distribution for zone 2 BIF
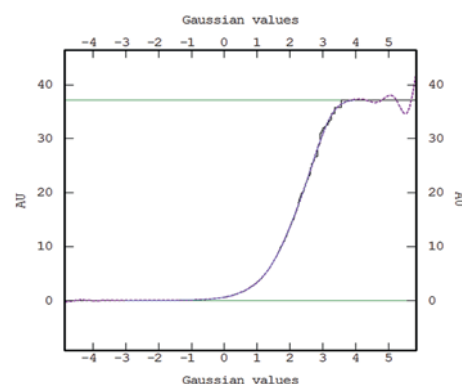
Figure 5—Gaussian anamorphosis modelling

Table II

Comparison of declustered and kriged mean values

Zone	Panel estimates				Declustered exploration samples			
	Min. samples	N	Mean	Variance	N	Mean	Variance	% diff. in mean
BIF	5	4025	6.45	32.07	9308	6.52	119.8	1.0
MD	5	6037	1.64	2.83	22563	1.73	52.0	5.5

Table III

Change-of-support parameters for BIF and MD domains in zone 2

	Selective mining unit		Real block correction	Kriged block correction	Kriged block real block correction	Kriged panel correction	SMU to kriged panel
	Var Z*	Cov Z|Z*	r	s	rho	S	(S/r*rho)
BIF AU<80 TC	26.250	26.261	0.702	0.677	0.964	0.430	0.634
MD AU<100	6.378	6.379	0.746	0.715	0.958	0.374	0.522

Discrete Gaussian change of support

The theory behind the change of support as used in the discrete Gaussian model is covered in detail by Neufeld (2005). The change-of-support model is applied after the anamorphosis of point data. During this process a function defined by Hermite polynomial expansion is fitted to the data, which provides a mapping of the point variable Z to the Gaussian variable Y and *vice versa*: $(z(\mathbf{x}) = \Phi(y(\mathbf{x})))$.

The change of support was conducted for BIF and MD as part of zone 2 in the Nyankanga resource model (Table III). Outlier analyses were performed on these point distributions and 80 g/t and 100 g/t were chosen as appropriate top capping limits for the BIF and MD respectively. Maximum values of 496 g/t and 378 g/t occur within the sampled distributions of BIF and MD respectively.

An error variance associated with the 'future' estimates of SMU in the absence of grade control drilling was computed by applying the information effect. This was determined by using the same semivariograms and realistic grade control drill patterns.

The distributions of SMUs obtained from UC were compared to the global (DGM) distribution obtained from the points to SMU and were found to be in close agreement.

Uniform conditioning – change of support from panel to SMU

As explained above, a change-of-support correlation (r) from points to blocks (SMU) can be computed, and in a similar manner a change-of-support correlation (R) from points to panels is computed. In both instances the variance of SMUs and panels are used and therefore it is important to ensure these variances are accurate.

In areas with wider spaced drill-hole information, the variance of panel estimates (VarZ*, or shown as Bvar in Figure 6) tends to be lower due to the smoothing effect of kriging. In this instance it is advisable to use appropriate dispersion variance (Var ZV*, or shown as Dvar in Figure 6) grouped per geographical areas. In Isatis this can be achieved by using two or three intervals of dispersion variance, and in

this case the SMU distributions are accordingly computed to reflect the appropriate dispersion variance (Figure 6).

Checking the dimension of SMU for open pit mining

The risk of selection of the SMU dimensions must, given its importance, be quantified by conducting a selectivity study that takes into account the spatial structure of the mineralization, the size of the SMU, and the data spacing (Chamberlain, 2009).

The SMU is defined as the smallest block that could be mined by the mining fleet, and an ore block marked out in the field will often consist of a number of contiguous SMUs. The equipment fleet used or being evaluated, as well as practicalities like finding maximum value by comparing economic factors against the orebody geometry with minimal dilution, need to be taken into account when defining the SMU.

During the initial project phase (Figure 7), the SMU can only be determined empirically. However it is critical that during the latter stages of the Pre-feasibility Study or early in the Feasibility Study a representative volume of up to one year's production volume is drilled out to grade control spacing. This will allow for a reconciliation of the initial Recoverable Resource model and a further adjustment to reflect the reality of additional information. It will also serve to significantly reduce the level of risk in the early mining phase.

A 10 × 10 × 3.33 m SMU was used in the resource model. The current mining fleet at Geita is capable of mining to a selectivity of 10 × 10 × 3.33 m, and larger mining perimeters are normally made up of a series of 10 × 10 m SMUs based on the orebody shape and grade distribution.

Validating the selectivity used for underground mining (UG) Mineral Resources

The current resource model is constructed on a 0.5 g/t resource envelope (wireframe). A kriged block model (40 × 40 × 10 m) is sub-celled within this wireframe. Each kriged block is informed with a grade-tonnage curve depicting the

The practical implementation of uniform conditioning at AngloGold Ashanti
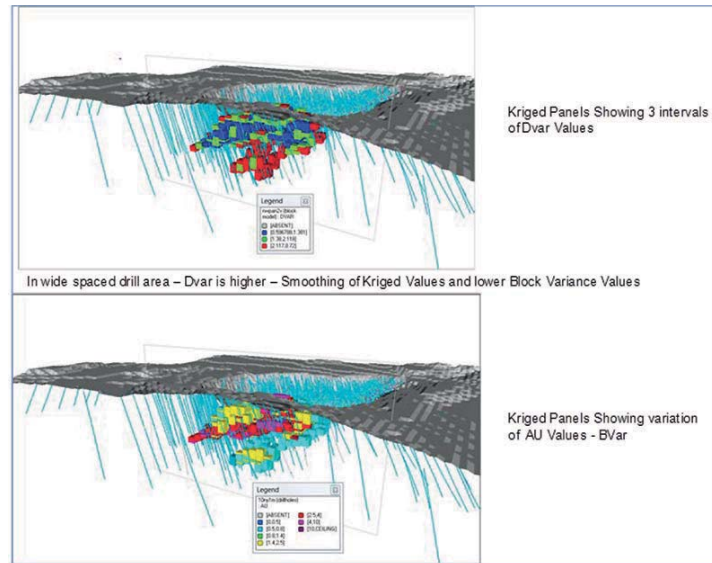
Figure 6—Plots showing variation of dispersion variance in relation to variance of kriged panel grades (Dvar = Var ZV* and Bvar = Var Z*)
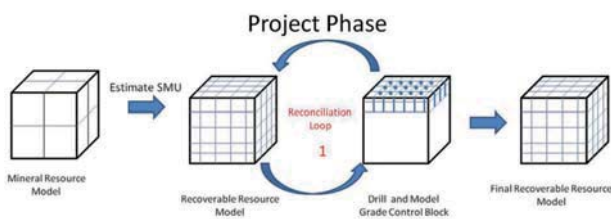
Figure 7—SMU reconciliation chain in the project phase

grades and tonnages of smaller mineable units (10 × 10 × 3.33 m). This is computed using UC. This model is appropriate for open pit planning since complete benches are planned and mined. Ore (according to the various cut-offs, where marginal cut-offs are in the order of 0.9 g/t and the high-grade ore cut-off in the order of 1.1 to 1.3 g/t) and waste are identified via grade control drilling to a 10 × 5 m spaced grid. All mined material is then appropriately stockpiled. Reconciliations to the original SMU resource model are reasonable and acceptable when done over an annual mined volume for open pit mining.

In the underground mining scenario, mining is carried out according to a higher cut-off (3–4 g/t). From a planning perspective, larger stopes will have to be designed on the higher grade kriged blocks. However since the current resource model is based on a 0.5 g/t resource envelope and 40 m spaced data in the underground areas, the kriged estimates tend to under-represent the variability of the higher grade stope-scale mineable units and the full underground potential may not be realized if kriged blocks are used to identify the higher grade zones. This becomes more evident in the wider portion of the orebody, where a mixture of low- and high-grade blocks tends to result in an average kriged block estimate. In order to assess whether the current UC model is adequate to predict the higher grade tonnage behaviour as can be expected from underground stopes, a study was conducted in a relatively closer-drilled area

(Robins, 2010). The placement of the underground stopes will be based on a deterministic high-grade wireframe using 20 × 20 m spaced drilling. Although this still forms part of the open pit mining area, the geological characteristics are expected to extend into the underground mineable portion. The process followed in this study area can be summarized as follows.

1. Based on mapping and drilling information, structural wireframes were computed. Using all relevant geological information, a high-grade (5 g/t) wireframe was generated
2. The high-grade wireframe was then used to guide the string interpretation and produce a resource envelope that geologically holds potential high-grade mineable blocks. The cut-off for which the kriged resources contain all the high-grade material is seen at approximately 2 to 2.5 g/t
3. The resource envelope (produced by 20 × 20 m spaced drilling) was then estimated using appropriate ordinary kriging parameters. The grade and tonnages were reconciled to the grade control model (produced by 10 × 5 m spaced drilling) and the UC model (produced by 20 × 20 m to 40 × 40 m spaced drilling).

Generally, the geological model was used to infer continuity according to the directions of the structural features.

The exploration data was extracted within the 2 g/t resource envelope. A variogram was generated, and using optimized kriging parameters relevant to 20 × 20 m spaced drilling the envelope was estimated using ordinary kriging into 20 × 20 × 10 m parent block sizes (Figure 8). This model was then reconciled to the grade control and original UC models.

As can be seen in Figures 8 and 9, the grade control model is more variable than the 20 m high-grade resource model. Both of the above models were then reconciled to the original UC model in the mined-out volume (Figure 10).

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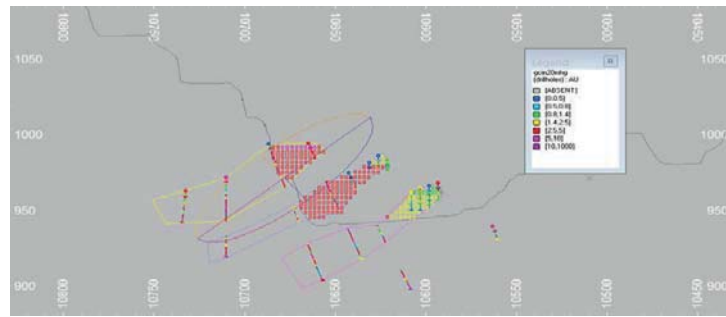
Figure 8—For reconciliation, a 20 m high-grade wireframe was estimated and cut into mined-out volume for comparison with grade control
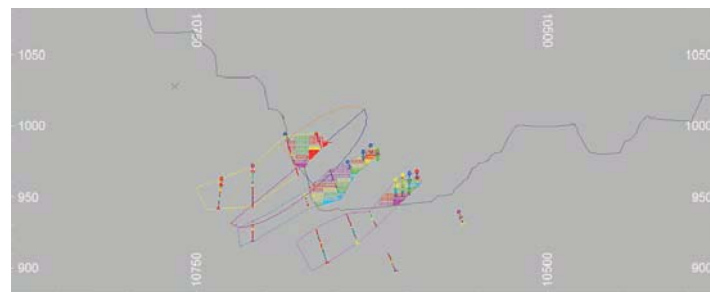
Figure 9—For reconciliation, a 20 m high-grade wireframe was estimated using grade control data into a 10 ×10 × 3.33 m block model and cut into the equivalent mined-out area as the resource model

Description of underground mining method

The planned mining methodology is bottom-up longitudinal stoping, and backfilling with a combination of cemented rock fill (CRF) and loose rock fill (LRF). The CRF negates the need to leave pillars. Slot rises would be excavated either by using airleg mining techniques or longhole drill-and-blast rises with uphole production drilling. (Rees, 2013)

A typical cross-section of the stope design is shown in Figure 11. To ensure a practical mining shape the footwall angle has been maintained at a minimum 40°. It is possible to mine stopes with footwall angles less than 40° but in practice this results in ore being left on the footwall (lower mining recovery).

Based on the above study, which represents a wider portion of the orebody (up to 80 m wide), the following salient conclusions were established.

1. Using all geological information it is possible to delineate relatively higher grade wireframes based on 20 m drill spacing. The shapes of these wireframes are coherent enough to mimic larger underground stopes (1.2 Mt, which represents an annual volume of underground mining, was delineated in this exercise)
2. During the wireframing process the core high-grade units (5 g/t) occur within a broader envelope of >2 g/t material. This process of modelling the lower grade halo around the core high-grade envelope is subjective, and the geological controls on mineralization at this scale are complex and cannot be easily correlated using 20 m spaced data. Closer spaced information is required, as can be seen in Figures 8 and Figure 9
3. Due to the above geological complexities a portion of the internal waste zones tends to be modelled with the
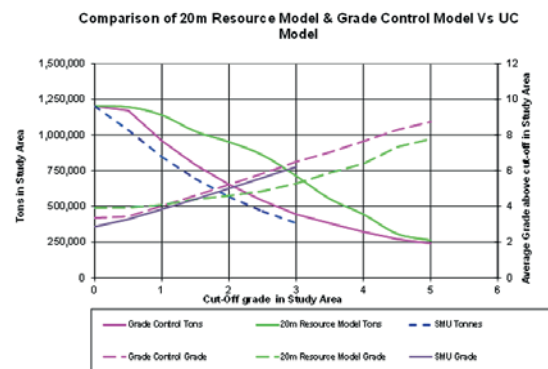
Figure 10—Reconciliation of ordinary kriged resource and grade control models to the UC (SMU) resource model
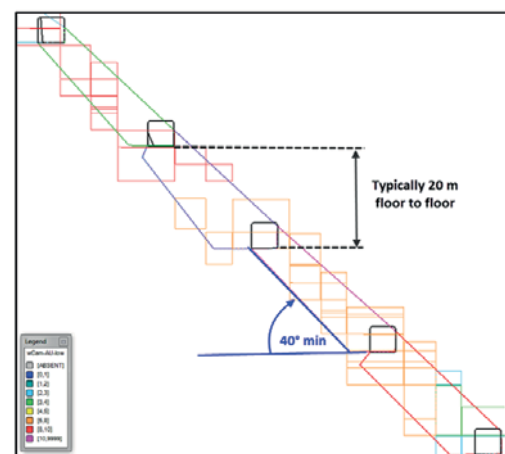
Figure 11—Typical stope cross-section showing resource block model

high-grade envelope, and the amount of this material is known after grade control has been applied in these areas. Also, the contact of the high-grade stopes tends to more variable than that delineated by using 20 m spaced data. In addition, due to the differences in block support size between the ordinary kriged (20 × 20 × 10 m) and the grade control blocks (10 × 10 × 3.33 m), a poor correlation in the shapes of the grade and tonnages can be expected between these two models. Therefore incorporating an ordinary kriged model into these high-grade wireframes does not reconcile well with the grade control model and an overstatement of gold of up to 25% at a 2.5 g/t cut-off occurs, as seen in Figure 10

4. The UC model performs better, with closer grade and tonnage reconciliations to grade control. It is therefore suggested that the potential underground stope areas be drilled to a 20 × 20 m grid as per the original drilling strategy. Higher grade stopes can then be wireframed using the same principles as used in this exercise. Care should be taken, however, to incorporate the lower grade halo (up to 2 g/t) around the high-grade core. This will ensure that all potential high grades are captured. A UC model can then be used for estimating the grade-tonnage relationship. The exact shapes of the high-grade stopes will have to be determined by mapping and drilling information obtained from underground development crosscuts across the width of the orebody

5. The uncertainty in the UC estimate can be quantified based on the reconciliation to grade control. Appropriate planned ore loss and dilution factors can be computed using this reconciliation and applied appropriately in the mine planning process. An alternative to this approach would be to conduct a simulation study and quantify the uncertainty.

Conclusions

The geological nature of the orebody at Geita is conducive to the usage of uniform conditioning (UC) methods, and historical reconciliations have shown this to be a robust technique to produce recoverable resource models for quantifying published Mineral Resources and for use in both medium- and long-term planning. Planning in the open pit environment implies a free selection of the SMU, while that in the underground environment implies a fixed selection of the SMU. The spatial positions of high-grade units are important for underground planning, and reconciliation studies for the Nyankanga deposit have shown that a tighter drill spacing will be required to indicate the position of these high-grade units. The accuracy of local predictions of the grades and tonnages within these high-grade wireframes is better if a UC model is used rather than a deterministic ordinary kriged model.

The importance of the geological model, understanding of local grade variation, and improved domaining between the banded iron formation (BIF) and microdiorite (MD) units have been highlighted in this paper. This assisted in improving local SMU accuracy to aid in the underground evaluation of stopes.

The highly positive skewed sample distribution present in the Geita orebodies was adequately represented in the UC model, and this ensured that the total orebody potential was captured prior to any financial decisions to start new and capital-intensive cutbacks or underground development decline systems. Appropriate underground mine planning modifying factors can be computed from the reconciliation study and used in the Resource conversion process. This potential could have been easily under- or overestimated if linear kriging methods were employed based on the initial wide-spaced exploration data.

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Construction of an expert-opinion-based virtual orebody for a diamondiferous linear beach deposit

by J. Jacob* and C. Prins†

Synopsis

During early-stage diamond exploration projects, hard data underpinning spatial continuity is often very limited. An extreme example of this is a submerged diamondiferous marine placer target area alongside a current onshore mining area in southwestern Namibia. Although an abundance of geological and grade data exists for the adjacent onshore mining area, the target area itself contains no such information. Despite this apparent abundance of data, it is extremely difficult to obtain a variogram (Prins and Jacob, 2014) for use in this study area. The use of traditional simulation techniques is further hindered by the fact that diamond entrapment within the highly gullied footwall is non-stationary. An alternative approach for creating a simulated virtual orebody (VOB) is thus required in order to enable the assessment of sampling strategies.

This paper demonstrates how expert opinion is used to generate a composite probability map for diamond concentration using a greyscale hand-sketching technique. The probability map is subsequently calibrated and populated using the diamond distribution for different raised beaches obtained from analog data based on sample results adjacent to the target area. The resultant grade simulation is used to test different sample scenarios and is a first step towards determining an appropriate sampling strategy for the target area. The VOB is used to analyse and rank the efficiency of different sampling strategies for grade determination of submerged diamondiferous linear beach exploration targets.

Keywords
Expert opinion, simulation, diamondiferous marine placer, non-stationary.

Introduction

Expert opinion is used in various fields such as engineering, biological research, economics *etc.* (Kuhnert *et al.*, 2009, Pearce *et al.*, 2001) to assess uncertainty where limited or no hard data is available. Several approaches exist for combining probabilities obtained from expert opinion. The linear opinion pool (Stone, 1961; Winkler, 1968) is a weighted combination of expert opinion probabilities that satisfies the marginalization property. This requires that the combined probability is the same for combining either the marginal distributions or the joint distributions and then calculating the marginal distribution (Clemen and Winkler, 1999). Game theory is applied in deciding on when to combine which probabilities obtained from different methods that are equally appropriate based on the available data (Bickel, 2012).

A hierarchical modelling framework for combining expert opinion data and actual observed data for inferential purposes in a spatial context demonstrated that even a misleading expert opinion can be useful in cases where hard data refutes the expert opinion data (Lele and Das, 2000). The expert opinion data is influenced by hard data, hence the data-sets are not independent; for example, probabilities generated from different likelihood functions (Journel, 1986). Truong *et al.* (2014) illustrate how expert opinion is used as input to determine variogram parameters for downscaling from block support observations to point support where no point support observations are available.

Spatially, it is often very difficult to obtain or access hard data for a sampling optimization strategy. This paper demonstrates how expert geological opinion is firstly used to generate a composite probability map for diamond concentration using a greyscale hand-sketching technique; secondly, how the probability map is then calibrated to the correct sample support size; and thirdly, how the map is populated using the diamond grade distri-bution histogram obtained from observed analog data. Figure 1 shows the orientation and size of the virtual orebody (VOB) for a submerged diamondiferous marine placer target area in southwestern Namibia.

Background

Onshore diamondiferous linear beaches along the Namibian coast (Figure 1) have been the mainstay of Namdeb's diamond production for

* *Namdeb Diamond Corporation.*

† *Anglo American plc.*

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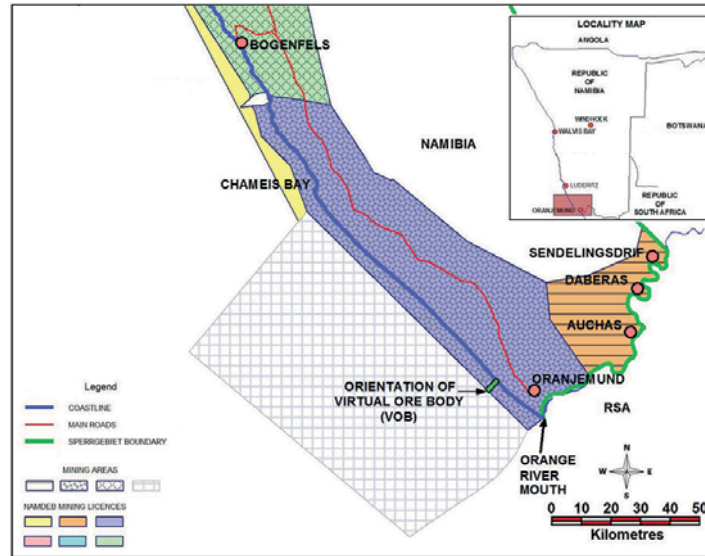
Figure 1—Locality map showing the size and orientation of the virtual orebody (VOB), southwestern Namibia

more than 80 years. These beaches are developed at different elevations, occurring as high as 30 m above sea level and as low as 30 m below sea level. A modern day analogue would be the present-day gravel beaches along the Black Sea in the Bolshoi Sochi region, which are perhaps good examples of what the study area's beaches would have looked like thousands of years ago (Spaggiari *et al.*, 2006). The fossil gravel deposits of the study area extend continuously for approximately 100 km northward from the Orange River mouth along the coastline. Most of the onshore beaches have been mined out to date, but great potential exists in remaining areas that are currently submerged under water. During 2010, the concept of planned, controlled shoreline accretion was initiated; until then, accretion was a consequence of the stripping and dredging processes. Beach accretion is a natural process resulting from overabundance in sediment supply, which in effect builds the coastline outwards. The stripping and dredging processes are used to remove overburden sand in order to gain access to the diamondiferous basal marine gravel. At present, focused deliberate beach accretion is inherent in the mine plan and is also considered a possible means for mining of the submerged beaches. Therefore it is imperative to find a way to determine the diamond grade to justify the accretion for future mining areas.

The sea's high-energy swash zone makes obtaining upfront data well in advance of mining a challenge. In addition to the high-energy swash zone aerated water column, a sand overburden sequence must be penetrated before the diamondiferous basal gravel sequence is reached. At present, a probe drill platform (PDP) is the only implemented technology that successfully withstands these energies in the vigorous swash zone, and it provides geological data only. Furthermore, the sample size is too small for grade determination in the submerged target areas, due to the low-grade nature of the deposit (Jacob *et al.*, 2013). The PDP is restricted by a land-bound base station and can operate only within 300 m from the current

shoreline. The potential of the offshore linear beaches, however, extends up to 4 km seaward, and the challenge is thus to generate a VOB that can be used for sampling strategy and risk studies in the absence of any hard data.

Nature of the analog data

The initial delineation of the onshore raised beaches was done between the 1930s and 1960s by a comprehensive 1 m-wide trench campaign. The 1 m trenches were spaced 500 m apart along the coast covering the 100 km from the Orange River mouth northwards to Chameis Bay (Figure 1). These 1 m trenches, orientated normal to the coastline, spanned six distinct onshore raised beaches. Continuous trench paddocks of 5 m lengths resulted in more than 26 000 samples at 5 m^2 support. Diamonds are concentrated in both gravel lenses (mobile trapsites) suspended above the bedrock footwall and in the highly gullied footwall. An example of the very detailed methodical mapping (1930s to 1960s) of the 1 m trench sections where the locations of individual diamonds are recorded is shown in Figure 2. The morphology of the marine erosion pattern on the bedrock surface (fixed trapsites) dominates the distribution of alluvial diamonds (Jacob *et al.*, 2006). From detailed sampling results it is evident that two-thirds of the diamonds occur in the fixed trapsites, with one-third in the mobile lenses; this observation is incorporated in the construction of the VOB.

Methodology

Prins (2011) developed a method using an expert opinion-based hand sketch to simulate the occurrence of diamonds in a VOB in cases where no hard data (spatial diamond grade) is available. The hand sketch is constructed on the principle that darker areas in the sketch represent areas with a higher probability of containing diamonds. This VOB is designed to form a strip connecting the current onshore area to the edge of the mining license approximately 4 km offshore (Figure 1). The methodology of developing the probability map based on

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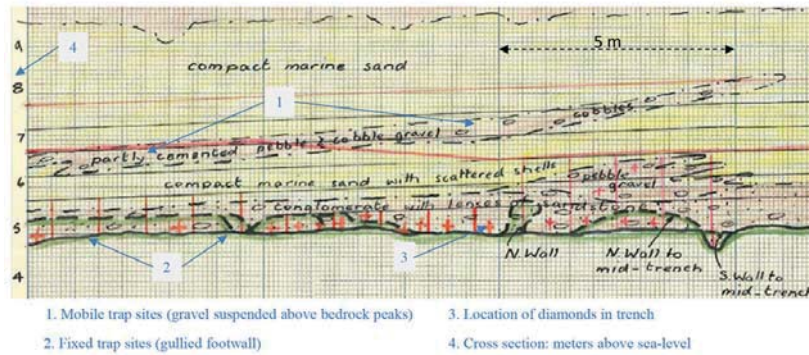
Figure 2—Vertical cross-section of a 1 m sample trench showing the spatial relation of fixed and mobile trapsites. Each sample paddock comprises a 5 × 1 m (5 m²) support size
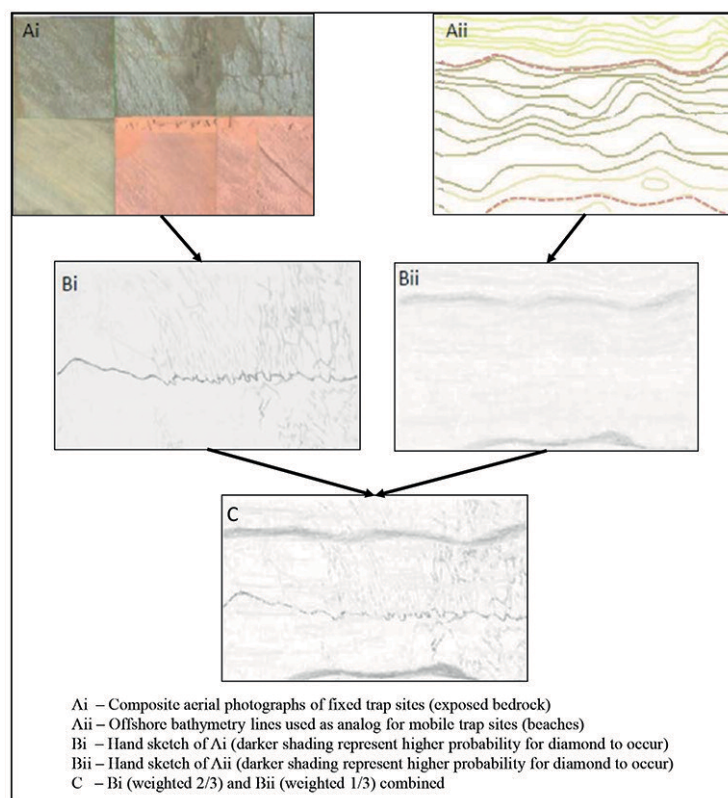
Figure 3—Steps involved in constructing a probability map for potential diamond entrapment

expert opinion is shown in Figure 3. The first step is the bedrock morphology sketch by the expert. The bedrock morphology is based on actual bedrock patterns currently exposed by onshore mining activities and exposed bedrock patterns obtained from bathymetry surveys in water depths greater than 30 m. Secondly, sea-floor bathymetry contour lines are used as a proxy for gravel beach location suspended above the bedrock fixed trapsites. These two sketches are merged in such a way that the shading reflects the ²/₃:¹/₃ proportion of diamond potential of the two trapsite types, resulting in a single combined probability model that can be used as a diamond potential entrapment map for the 1 × 4 km strip.

Geologists with combined experience in the order of 50 years were asked to assess the resultant sketch after the

merging described above. All agreed that the proposed model reflects current understanding of what could be a reasonable representation of diamond distribution in the study area, based on their extensive onshore production experience.

This paper introduces an additional aspect to the method proposed by Prins (2011), by applying the grade (stones per m²) profile observed across the different beaches, resulting in a grade-distance profile in the VOB (Figure 4). This profile is based on the average grade of the six onshore linear beaches obtained from the 26 000 sample results spanning roughly 100 km from the Orange River mouth to Chameis Bay (Figure 1).

Statistical manipulation to align the hand sketch with the proxy grade data requires four steps. The first step is to regularize the sketch, mapped to scaled coordinates, into

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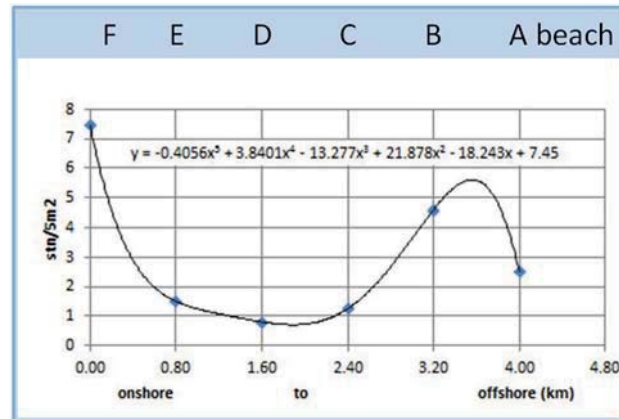
Figure 4—Grade profile used to adjust the diamond potential entrapment map
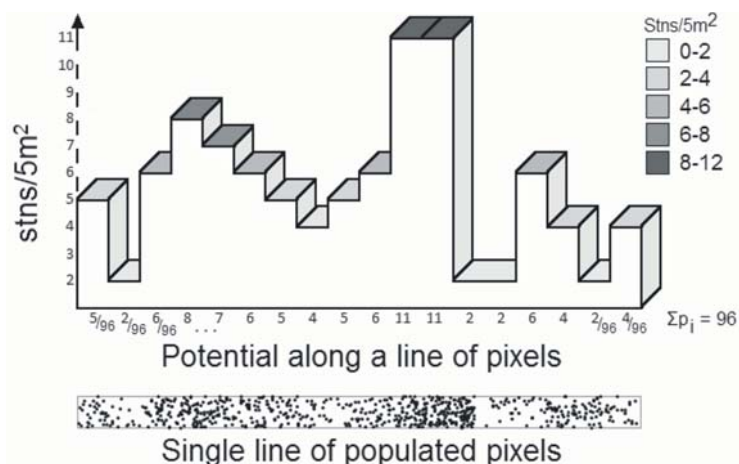
Figure 5—Diamond potential (greyscale), with a line of simulated stone locations

pixels representing 5 m² data so that it is aligned with the support size of the proxy data. Secondly, a normal score transformation is applied to the potential obtained from the sketch. Thirdly, a back-transformation of the regularized sketch data is done using the distribution of the proxy data. Finally, a 5th-degree polynomial fitted to the grade in stones per 5 m² per beach (Figure 4), with the y-axis as independent variable, is used to adjust the grade prior to seeding stones into the VOB. The grade-distance profile observed is thus matched to the grade-distance of the hand-sketched diamond entrapment potential map.

The outcome is finally adjusted by shifting the greyscale potential through histogram transformation, adjustment of individual histogram classes, and the random removal of stones (decimation). This is done in order to honour the zero proportions tail characteristics of the histogram, and univariate statistics of the proxy data.

Once the grade for a particular line of pixels is determined, the total number of stones for that line can be seeded. The individual stone locations simulated into the VOB based on the 5 m² pixels' greyscale potential is shown in Figure 5. For example, the first pixel will have the potential to randomly receive 5/96th (the second pixel 2/96th, the third

pixel 6/96th and so forth) of the number of stones for that line.

The process is graphically depicted in Figure 6, which shows how the two hand sketches are merged and the resulting 1×4 km VOB. The three diagrams on the left-hand side in Figure 6 represent probability maps, while the right-hand side diagram represents the locations of populated individual stones. Figure 7 (b, c, and d) is a zoomed-in version of the right-hand diagram in Figure 6, and individual dots representing the location of stones can be observed on this scale.

This simulation is one realization that is sampled, evaluated, and used to rank different sample spacing/size combinations to determine the optimum sampling strategy.

Creating different grade scenarios using the VOB

To create multiple realizations to facilitate decision-making for the exploration project, the hand sketch is also populated with the grade characteristics of individual beaches. This is done by regularizing the sketch to a sample support size of 5 m² so that it is directly comparable to the support of the proxy sample data. A Gaussian transform of the potential

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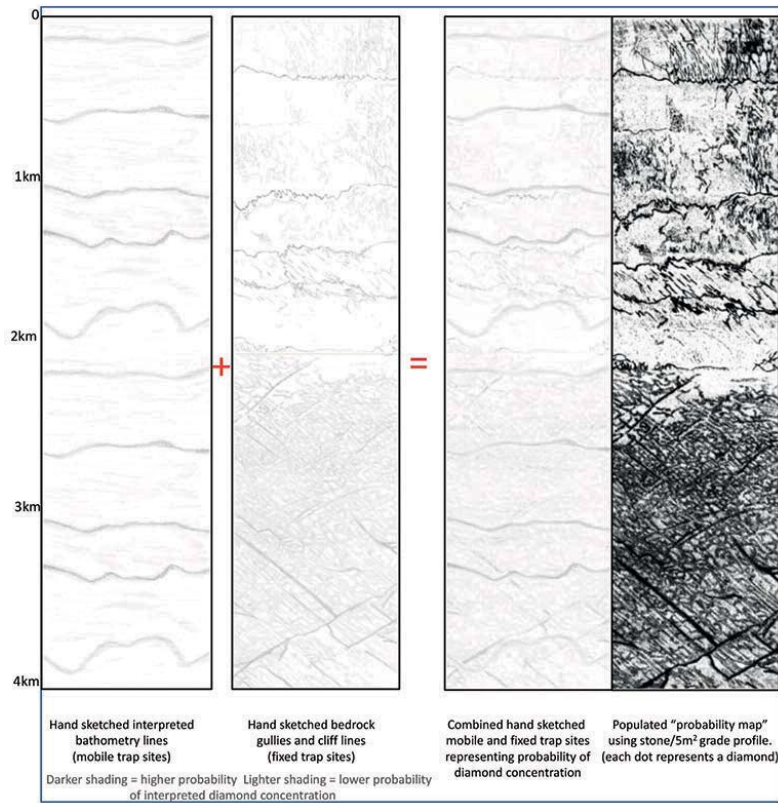
Figure 6—Process showing the combining of probability maps and grade profile to seed stones
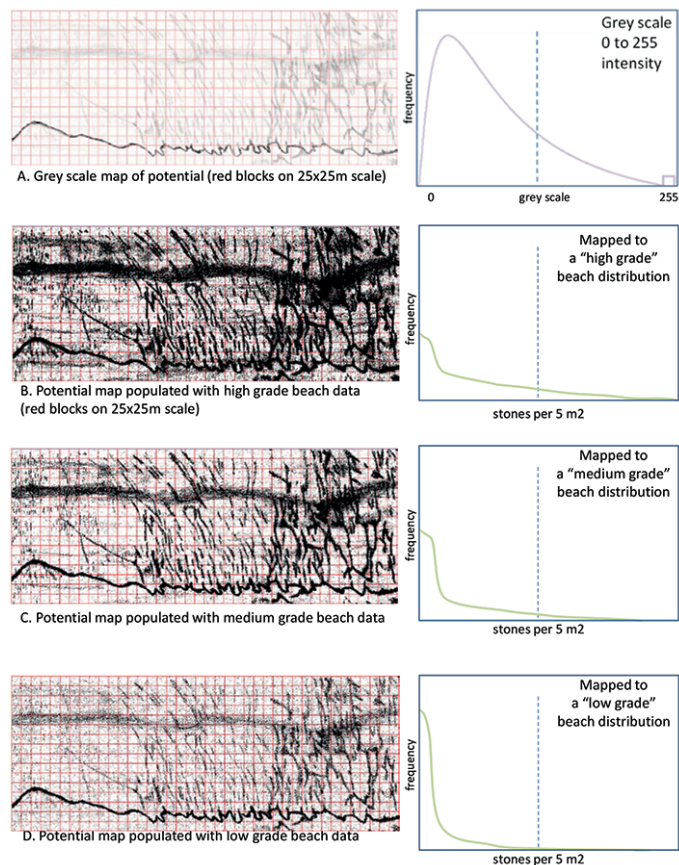
Figure 7—The simulated outcome for a 400 × 1000 m portion of the study area, populated respectively by low, medium, and high (schematically presented) beach grades

Construction of an expert-opinion-based virtual orebody

map is back-transformed using the transformation table of the proxy data. The underlying potential sketch remains unchanged but different results are obtained when back-transforming as the resultant realizations will have low, medium, and high grades with different higher order statistics (Figure 7).

In this way a range of possible outcomes is generated as realizations available for analysis. Since grade (the spatial intensity of the stones per unit area) affects the confidence of the resource estimates for a specific sample campaign, the realizations give an indication of the variability in the outcome of the assessments of the sample campaign effectiveness. The benefit of this work is that it can be used in risk rankings and assessments. Based on acceptable levels of uncertainty and taking into account the cost to execute a sampling campaign and the confidence obtained for the block estimates, the sampling scenarios can be ranked and a decision made on the appropriate course of action.

Discussion and conclusions

The authors admit to an acute awareness of the challenges of this approach. The quality of the hand sketch, paper texture, grey shading, and scan quality all impact on the final result of the VOB. In highly non-stationary environments where kriging is sometimes not possible, sample size and spacing are the most influential factors on the outcome of the final estimate. The VOB presented in this paper provides a first attempt at ranking different sampling scenarios. It also provides a preliminary risk quantification tool until such time when hard data becomes available for incorporation into conditional simulations. As data becomes available, the sample size and spacing should be re-assessed to confirm the outcome of the sketching technique, using well-established geostatistical simulation methods (Kleingeld *et al.*, 1996) developed for discrete particles.

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The basic tenets of evaluating the Mineral Resource assets of mining companies, as observed through Professor Danie Krige's pioneering work over half a century

by W. Assibey-Bonsu*

Synopsis

This paper constitutes a write-up of the first Professor Danie Krige memorial lecture in 2014, which was organized by the University of the Witwatersrand in collaboration with the Southern African Institute of Mining and Metallurgy (SAIMM) and the Geostatistical Association of Southern Africa, at which his wife, Mrs Ansie Krige, the SAIMM, and Professor RCA Minnitt also spoke. The memorial lecture was presented by his previous PhD graduate student, Dr Winfred Assibey-Bonsu.

During that inaugural memorial lecture, the SAIMM highlighted three activities that the Institute would undertake going forward, so as to remember this great South African mining pioneer:

➤ The publication of a Danie Krige Commemorative Volume of the SAIMM Journal
➤ An annual Danie Krige Memorial Lecture to be facilitated by the School of Mining Engineering at the University of the Witwatersrand
➤ The annual award of a Danie Krige Medal.

What follows is both a tribute to his work and a testimony to the great man's deep personal integrity, belief in family, humility, and faith in Christ: all of which led him to become a giant not only in the South African mining industry, but indeed worldwide.

Keywords
geostatistics, kriging, conditional bias, block model, regionalized variables, regression, ore evaluation.

Introduction

It has been said that 'we make a living by what we receive, but we make a life by what we give'. Professor Krige epitomized this in both thought and deed, by showing that true success in life does not revolve around material accomplishments accrued as an individual, but is defined by that which one does and leaves for others.

It was the author's privilege to be associated with Professor Krige for over 20 years, both initially as a student during doctoral studies at the University of the Witwatersrand and later with him as mentor, counsellor, and 'father figure' for the period that followed.

This paper will cover the two aspects that defined Professor Krige; firstly his personal life and career, including the achievements of both; while the second part will briefly touch on his immense contribution to industry and the world for over half a century, through his pioneering work in ore deposit evaluation, economics, and of course geostatistics. Indeed, his passing was recorded in Wikipedia under 'notable persons', a distinction he shared with renowned figures such as Margaret Thatcher.

The great man – Professor Danie Krige

Family and faith

This memorial lecture would be incomplete without firstly throwing light on some of the things Professor Krige held very dear in his life, as told in his interview in 2012 with Professor R.C.A. Minnitt of the University of the Witwatersrand.

Professor Krige was born in Bothaville in the Free State and was the youngest of nine children born to a pastor.

Professor Krige was a devout Christian, who always emphasized that what made a difference in his life was his belief in Jesus Christ. He also acknowledged that he had been the recipient of gifts of grace from the Creator –

A 1930 photograph of Professor Krige aged 11 (front middle) with family

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'grace given to him' – drawing attention to six specific areas, in which he could identify the grace of the Almighty at work in his life and career:

The first gift of grace

It was a tribute to his parents for the practical application of a godly lifestyle, the establishment of a firm foundation, and a life philosophy that was modelled by them in every area of life – an example being, that even with the limited resources at their disposal, they ensured that seven of the nine siblings received a tertiary education.

The second gift of grace

The second of the gifts of grace that he acknowledged was the support he received from his two spouses. He was happily married for 45 years to his first wife (until her death), and thereafter for 20 years to Ansie.

The third gift of grace

The third gift of grace was the way in which his career developed, and the various changes in direction that it took, as his research unfolded.

The fourth gift of grace

The fourth gift of grace was that when he returned to work at Anglovaal, the company began to apply his advanced methods of evaluation on their mines.

The fifth gift of grace

The fifth gift of grace was that on retirement from Anglovaal at the age of 60, he received the unexpected opportunity of taking up the Chair of Professor of Mineral Economics at the University of the Witwatersrand, which he occupied for the next 10 years. This enabled him to teach and undertake extensive consulting work for mining companies both locally and internationally, and was, in his opinion, a great blessing.

The sixth gift of grace

The final gift of grace was that after leaving the University of the Witwatersrand, he was still able to undertake extensive national and international consulting work, which kept him occupied and young for the following 20 years.

He also acknowledged with deep gratitude that while the opportunities were presented to him, it was his responsibility to make good use of them and that without these gifts of grace, his life's work would not have been possible.

The photos that follow bear testimony to his strong belief in family values, those same ones he was blessed with as a young boy.

Professor Krige with great-grandchildren

Celebrating his 90th birthday with wife Ansie

University graduate   **Learning the trade, 1939**

Career, achievements, and awards

Professor Krige matriculated from Monument High School at the age of 15 and in 1938, at the age of 19, graduated as a Mining Engineer from the University of the Witwatersrand. It was clear early on that he was destined for great achievements.

The two photographs above show the great difference between the robe of a university graduate and typical clothes of an underground miner, and provide a perfect illustration of Professor Krige's values regarding theoretical developments aimed at solving practical problems.

Professor Krige and family

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Career

Professor Krige worked with Anglo Transvaal on a number of gold mines on the Witwatersrand until 1943, and thereafter joined the Government Mining Engineering Department, where he worked for a further eight years. He spent time studying data and developing mathematical models. He returned to industry as Group Financial Engineer of the Anglovaal Group until 1981, when he 'retired'. He then spent another ten years (of his 'retirement') as Professor of Mineral Economics at the University of the Witwatersrand.

Professor Krige's seminal papers, published in the *Journal of the Chemical, Metallurgical and Mining Society of South Africa*, led to additional fundamental research in France on 'regionalized variables' by Professor George Matheron and his team. Professor Matheron named the new method of linear estimation of the regionalized variables using a spatial model 'kriging', in recognition of Professor Krige's distinguished pioneering work.

His 1951 paper, based on his MSc (Eng.) thesis at the University of the Witwatersrand, expounded his pioneering work in geostatistics in more detail. His research and paper covered and assisted with the statistical explanation of conditional biases in block evaluation. It stimulated the use of regression corrections for routine ore reserve evaluations by several mines, and the technique was essentially the first elementary basis of what is now known as kriging. The paper introduced, *inter alia*, the basic geostatistical concepts of support, spatial structure, selective mining units, and grade-tonnage curves. The concept of recoverable resources/reserves in current use is based on what is known as 'Krige's relationship'.

Kriging is currently applied worldwide in the fields of exploration, ore evaluation, environmental studies, petroleum, agriculture, fisheries, and other disciplines. Professor Krige's outstanding influence on the worldwide mining industry is visible every day, as shown by the decision-making processes followed by international mining companies.

Over the course of his career, he published some 96 technical papers, including the Geostatistics Monograph, the first in the Monograph Series of the SAIMM. A complete record of all his publications is available digital format from the SAIMM.

Dedicated service

As a professional engineer, Professor Krige served for many years on the Mining Committee of the Engineering Council of South Africa and on the Council of the SAIMM. He was a co-founder of the International Association of Mathematical Geology, the Geostatistical Association of Southern Africa, the Geostatistical Association of Australia, and the Statistical Association of South Africa.

He also served as a director of several companies, on the sub-committee of the South African Prime Minister's Economic Advisory Council during 1967/8, as well as on various committees of the South African Chamber of Mines. He was a member of the SAMREC Working Committee for The South African Code for Reporting of Exploration Assets, Mineral Resources and Mineral Reserves (SAMREC Code) as first published in 2000.

Amongst all of this, he still managed to find time to design the State aid formula, which assisted a large number of gold mines to survive the period of low gold prices; establish the original South African uranium contracts; and in 1955 and writing in Afrikaans, publish probably one of the first papers on risk analysis for new mining investment. He also gave major inputs in the fields of financial analysis and taxation.

Professor Krige was especially committed to the Application of Computers and Operations Research in the Mineral Industry (APCOM) symposia. He was South Africa's representative on the International APCOM Council from its inception, served as the Chairman of Council, and was the first member outside of the USA to be elected to this position. He initiated and was directly involved with all arrangements for the APCOM symposia held in South Africa (in 1972, 1987, and 2003), and is believed to have attended all APCOM symposia until he was almost 90 years old. In 2003, two weeks after a major operation, he managed to convince his medical doctors to allow him to attend the 2003 APCOM in Cape Town, South Africa, where he was a keynote speaker and also presented two other papers.

During his time as a Professor of Mineral Economics at the University of the Witwatersrand, he was responsible for postgraduate courses in geostatistics and mineral economics, and supervised many masters and doctoral theses. While at the university and afterwards, he presented courses in geostatistics and lectured at South African universities as well as universities in Australia, Germany, Taiwan, Chile, Russia, and China, to name but a few. He also still found the time to undertake valuable consultancy work locally and internationally, and participated in and contributed to many international congresses all over the world.

Achievements and awards

Over his lifetime, Professor Krige was the recipient of numerous local and international awards, too many to mention all. His academic achievements and awards included:

➤ DSc (Eng.) 1963, University of the Witwatersrand

➤ DIng (HC) 1981, Honorary Degree, University of Pretoria

➤ Honorary Doctorate from Moscow State Mining University

➤ Honorary Doctorate from the University of South Africa (UNISA)

➤ Order of Meritorious Service Class 1, Gold, awarded by the South African State President

➤ The highest award of the SAIMM, the Brigadier Stokes Award, in 1984

➤ Many other merit awards from the SAIMM, including two gold medals in 1966 and 1980 and two silver medals in 1979 and 1993

➤ International Association of Mathematical Geology – William Krumbein Medal, 1984

➤ One of the highest awards from the American Society of Mining Engineers – the Daniel Jackling Award

➤ Several awards from APCOM International Council, including the Distinguished Achievement Award, 1989

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➤ Elected as Foreign Associate of the US National Academy of Engineers (NAE) 2010, the first South African to ever receive this award, for his distinguished contributions to Engineering

➤ Order of the Baobab in silver – awarded by President Jacob Zuma.

Professor Danie Krige's work on essential tenets in evaluating the mineral resource assets of mining companies

Although it is impossible to provide a comprehensive list in this paper, the author will try to detail at least some of the many principles that Professor Krige brought forth over half a century.

Historical background and motivation – the capital intensiveness of mining

The mining industry requires very capital-intensive investments. Figures 1 and 2 provide some examples in this regard.

Figure 1 shows that in 2007 Rio Tinto's acquisition of ALCAN, a Canadian aluminium company, cost US$38.1 billion. It further shows that the estimated cost for Billiton's Olympic Dam Project in Australia was US$27 billion.

Figure 2 illustrates that in 2007 Gold Fields Limited acquired the South Deep Gold Mine in South Africa at a cost of US$2.5 billion (the equivalent of R22.2 billion at the then-prevailing exchange rate). It also shows that in 2011 Newmont's acquisition of Fronteer Gold Inc. cost US$2.3 billion, and that Barrick's ongoing development of the Pascua–Lama gold mine in South America was estimated at US$8.5 billion.

Mineral Resources and Mineral Reserves are the fundamental assets of mining companies and capital-intensive investments are made with respect to these. The strategic objective is to explore, acquire, develop, and ultimately mine them, but one critical risk exists in the uncertainty of the estimation of Resources and Reserves. If, after intensive capital investments, it is subsequently found that the expected Mineral Resources and Mineral Reserves were inefficiently estimated or valued, billions of dollars may be lost. Professor Krige's pioneering research work provides technical solutions to mitigate these technical and financial risks when evaluating these fundamental assets.

Essential tenets in evaluating Mineral Resource assets of mining companies based on over half a century of Professor Krige's pioneering work

Data integrity

Professor Krige emphasized the critical importance of data integrity as the lifeblood of Mineral Resource and Reserve
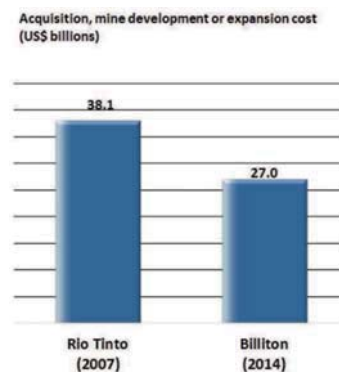
Figure 1—Capital investment example 1
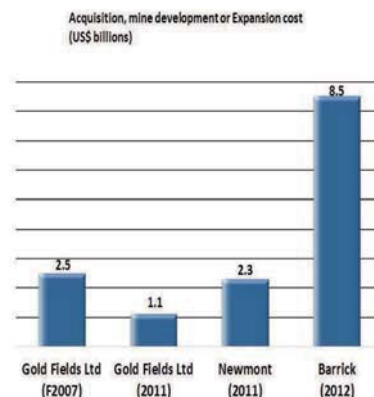
Figure 2—Capital investment example 2

Being awarded the Order of the Baobab in silver by President Jacob Zuma

US NAE Award (Professor Krige 5th from left in front row)

evaluation. Ensuring data integrity includes data validation and authorization, use of standards and blanks with approved laboratories, and also database safety and security, which are all critical requirements of the Sarbanes–Oxley Act of 2002 (SOX) that is necessary for compliance with the New York Stock Exchange regulations.

Geology models

Professor Krige highlighted geology as the foundation of Mineral Resource and Reserve modelling. He emphasized that orebodies differ and that the main geological characteristics, including lithological and structural features, mode of origin and formation, as well as controls of mineralization, are critical inputs in orebody modelling.

He further warned against the dangerous practice of subdividing orebodies, not on geological grounds, but directly on grade only, as this can lead to serious biases, particularly where the data in one or more subdivisions is insufficient to allow proper geostatistical analysis.

Geostatistics technology – technique selection and optimal application

In the field of Mineral Resource and Reserve evaluation, geology and geostatistics are two inseparable sides of the same coin. As stressed above, on the one side geology concentrates on the physical features of the orebody, such as structures, source, deposition, and type of mineralization. Geostatistics is the other side of the coin, and provides mathematical, statistical, and geostatistical models for the analytical data available from sampling, in order to introduce efficient evaluation techniques for Resource and Reserve estimate, and to attach confidence limits to these estimates.

Uncertainty is fundamental in all branches of science and in human life itself. Uncertainty is the reason for the introduction of mathematical and statistical techniques in geology and is behind the birth of geostatistics over half a century ago.

Frequency distribution

The initial efforts in applying classical statistical procedures to orebody evaluation in South Africa date back to 1919 (Watermeyer) and 1929 (Truscott). It was only in the late 1940s and early 1950s that Sichel (1947, 1952) introduced the lognormal model for gold values, and using this model developed the 't' estimator. Departures from the usual lognormal model were largely overcome with the introduction in 1960 (Krige, 1960) of the three-parameter lognormal model, which requires an additive constant before taking logarithms. However, there were still cases that could not be covered by the three-parameter lognormal model, and Sichel (1992) introduced the more flexible compound lognormal distribution, originally developed by him for diamond distributions.

Spatial concepts and the birth of geostatistics and kriging

Geostatistics as such did not really originate until the basic concept of ore grades as a spatial variable, with a spatial structure, was introduced in 1951/52 by Professor Krige.

This arose firstly from his endeavour to explain the phenomenon experienced on the South African gold mines for many decades, where ore reserve block estimates consis-

tently showed significant undervaluation in the lower grade categories, and overvaluation for estimates in the higher grade categories, during subsequent mining, *i.e.* what is now known as conditional biases, illustrated in the form of a simple diagram in Figure 3. Professor Krige's pioneering work provided the geostatistical explanation of conditional biases as unavoidable errors resulting from the use of limited data on the periphery of blocks, which was used in evaluating ore reserve blocks. He proposed and implemented corrective measures to eliminate these significant conditional biases. The regression corrections were applied routinely to block estimates on several mines in the early 1950s and represented the actual birth of kriging. The regressed estimate was, in effect, a weighted average of the peripheral estimate and the global mean of the mine section – it was the first application of kriging. It could be called 'simple elementary kriging', being based on the spatial correlation between the peripheral values and the actual grades of the ore in the blocks, and giving proper weight to the data outside the block periphery via the mean. In this way, the spatial concept and kriging were introduced. The concept of 'support' is very basic to geostatistics, and was first covered by Ross (1950), and further developed by Krige (1951), including Krige's variance-size of area relationship.

Spatial structure and variograms

Professor Krige's pioneering work in the early 1950s aroused interest worldwide, particularly in France where, at the instigation of Professor Maurice Allais, Professor Krige's papers were republished in French (Krige, 1955). One of Professor Allais's students, later to become world renowned as Professor George Matheron, started the development of the theory of regionalized variables. Matheron also proposed the use of the variogram to define the spatial structure. This model is an extension and refinement of the concept covered by De Wijs (1951,1953). Professor Krige's regressed estimates were then still called 'weighted moving averages' until Matheron's insistence in the middle 1960s on the term 'kriging' in recognition of Professor Krige's pioneering work.

During 1963 to 1966 (Krige, 1963, 1966) the spatial patterns were defined in far more detail. These studies covered the spatial correlations between individual 'point' sample values, as well as those between regularized data
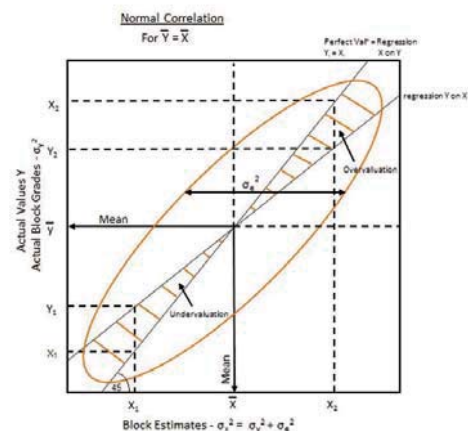
Figure 3—Conditional biases

blocks. The corresponding correlograms or covariograms were used on a simple kriging basis for block evaluations. Kriging on a routine basis for ore reserve evaluation was, therefore, already in use on some Anglovaal gold mines more than 50 years ago.

Conditional unbiasedness

It is instructive to observe that on the South African gold mines, the improvement in the standard of block evaluations due to the elimination of conditional biases accounts for some 70% of the total level of improvement achievable today, using the most sophisticated geostatistical techniques. It is for this reason that Professor Krige placed so much emphasis on the 'proper' implementation of the methods to mitigate conditional biases. Thus, the elimination of conditional biases is not only the major contributor to the reduction of uncertainty in assessing the Mineral Resources of mining companies; it is also an integral and fundamental part of any kriging and Mineral Resource and Reserve assessment process.

Conditional biases

The elimination of conditional biases is basic to ore evaluation and all geostatistical procedures, as emphasized by David (1977) in his popular geostatistics book '*Geostatistical Ore Reserve Estimation*'. As David states, conditional unbiasedness is the 'key point of Krige's 1951 paper, one of the key points of his 1976 paper but even then, still appeared as a revelation to many people'.

What contributes to conditional biases

Any increase in knowledge and available data relevant to any uncertainty being studied will reduce the level of uncertainty, provided that the knowledge is applied properly. Knowledge will never be perfect and data never complete, and therefore uncertainty will never be entirely eliminated. However, any procedure or technique that does not use all relevant data in order to provide the 'best' perspective on the remaining uncertainty must not be accepted. Professor Krige reported that in his worldwide experience, he unfortunately encountered many cases where practitioners had erred in respect of this fundamental concept. In too many cases, Mineral Resources and Mineral Reserves were estimated from limited data, and further relevant data was ignored. Use of insufficient data can still be a problem today. In 1950 only the peripheral data for each block was used, while now, with the use of geostatistics, the data search routine is still often inadequate, even with the complete database available on the computer. This is often compounded with no advance analysis to determine the minimum search routine required to eliminate the biases, and no follow-up studies to record the presence of these biases and the need to eliminate them.

Practical examples of outcomes of conditional biases

The graphs and tables that follow, some which are taken from Professor Krige's historical and practical work, illustrate the effect and outcomes of conditional biases.

Figure 4 illustrates feasibility block estimates *versus* final production blast-hole averages for an aluminium deposit. There is no correlation between the feasibility block estimates and those observed during production, as demonstrated by

the regression trend, which could lead to significant risk in invested capital. Figure 5 illustrates similar conditional bias problems and demonstrates why they are important, and shows how they result in misclassification of ore blocks, resulting in levels of profit well below what can be achieved. Figure 6 demonstrates the improved estimates for the data in Figure 5 that can be achieved through using 'proper' kriging with an adequate search.

More recent practical examples of conditional biases are illustrated in Tables I and II and Figure 7. Table I, a case study of a historically mined-out open pit, demonstrates that even the latest sophisticated geostatistics method used to estimate recoverable resources can suffer from inherent conditional biases. Table II shows the effect of conditional biases over time, from a historically mined-out case study, with consistently large negative percentage errors for tons and positive errors for grade, over various time periods and cut-offs. Figure 7 illustrates the financial impact of the errors over the respective cut-offs and time periods.
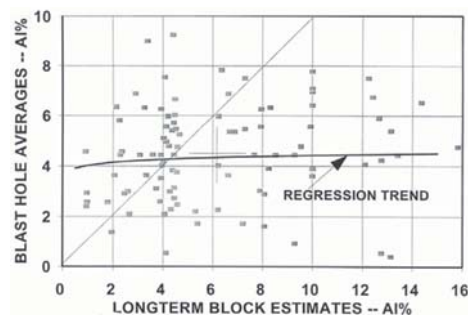
Figure 4—Positively skewed, block estimates *versus* blast-hole averages
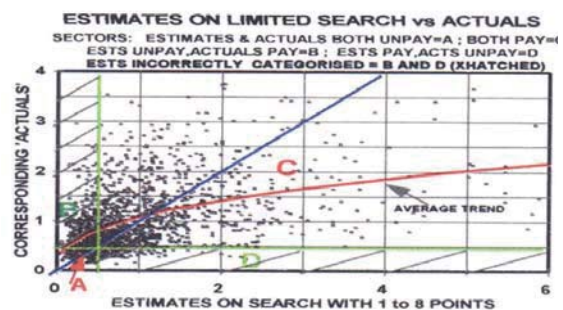
Figure 5—Misclassification of blocks
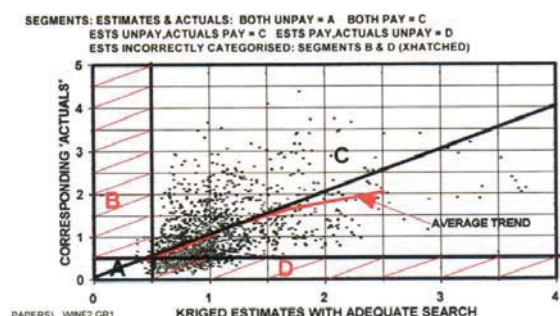
Figure 6—Proper kriged estimates *versus* 'actuals'

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Table I

Geostatistical estimate with conditional biases – grades (g/t)

Recoverable LUC SMU estimates of blocks for grade categories (g/t)	Corresponding average grade for actual blocks (g/t)
12.0	3.0
6.0	2.3
0.3	1.7

Table II

Geostatistical estimate with conditional biases – percentage errors

Cut-off (g/t)	6 Months		1 Year		3 Years	
	Tonnes (%)	Grade (%)	Tonnes (%)	Grade (%)	Tonnes (%)	Grade (%)
0.6	-15%	31%	-15%	20%	-15	16%
0.7	-14%	31%	-16%	22%	-16%	18%
1.0	-3%	27%	-16%	24%	-17%	19%
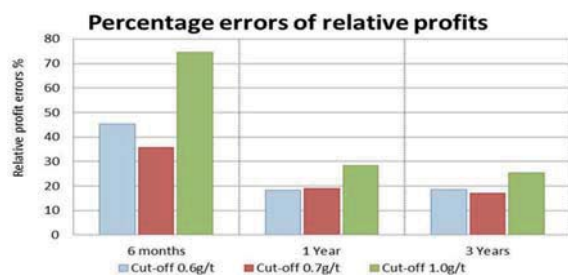
Figure 7—Financial impact measured on the basis of relative profit

Conditional biases – testing tools

Mineral Resource estimation for a new or an existing mine covers two major stages:

➤ At the initial or first stage, the data is limited and is obtained either from a broad drill-hole grid or from an initial main development grid

➤ During the second or final stage, more data becomes available from grade control drilling or from stope faces and auxiliary developments.

Apart from providing a basis for short- and longer term mine planning and viability studies, evaluations are frequently required to provide Resource and Reserve classification figures (Measured/Indicated/Inferred and Proven/Probable), to substantiate a major capital investment and/or the raising of finance. At both stages of evaluation, the evaluation technique should ensure minimum error variances/uncertainty. These requirements are linked closely to the expected slopes of regression of the eventual follow-up values on the original block estimates. Slopes of less than unity indicate the presence of conditional biases, with blocks in the upper grade categories overvalued and low-grade blocks undervalued.

The efficiency of block evaluations

Block evaluations subject to conditional biases have lower efficiencies. Professor Krige in 1996 proposed to define and measure the efficiency as follows:

$$\text{Efficiency} = \frac{(BV-KV)}{BV} \text{ expressed as a percentage}$$

where:

BV = block variance (*i.e.* the variance of actual block values, calculated from a variogram)

KV = kriging variance (*i.e.* the error variance of respective block estimates).

For perfect evaluations: KV = 0, the dispersion variance (DV) of the estimates (calculated from the observed kriged model) = BV, and then:

$$\text{Efficiency} = \frac{(BV-0)}{BV} = 100\%.$$

Where only a global estimate of all blocks is practical, all blocks will be valued at the global mean, *i.e.*:

$$DV = 0, KV = BV, \text{ and efficiency} = \frac{(BV-BV)}{BV} = 0\%.$$

Usually, blocks are valued imperfectly. With no conditional biases:

$$DV = BV - KV, \text{ and efficiency} = \frac{(BV-KV)}{BV} = \frac{DV}{BV}.$$

However, with conditional biases present, this relationship does not hold and then:

$$DV > (BV-KV)$$

because of insufficient smoothing, and

$$\text{Efficiency} = \frac{(BV-KV)}{BV} < \frac{DV}{BV}$$

The efficiency of a block evaluation can even be negative if *KV > BV*. As stressed by Professor Krige, such a situation is unacceptable and the block evaluations will be worthless; yet he encountered several such cases in practice, where the data accessed per block was inadequate.

Critical control limit test for kriged block evaluations

In order to avoid unacceptable negative efficiency for block estimates, the following critical control limit test is proposed for the regression slope to test for conditional biases (Assibey-Bonsu, 2014):

Regression slope can be written as:

$$\text{Regression slope} = \frac{(BV - KV + |LM|)}{(BV - KV + 2|LM|)} \quad [1]$$

where LM is the respective Lagrange multiplier for ordinary kriging, and BV and KV are as defined above.

Where only a global estimate of all blocks is practical, all blocks will be valued at the global or sub-domain mean, *i.e.*, KV = BV and efficiency = 0

Substituting KV = BV into Equation [1]

$$\text{Regression slope} = |LM|/2|LM| = 0.5$$

Thus, a regression slope of less than 0.5 will always lead to a negative block efficiency estimate (*i.e.* worthless kriged

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estimates). This highlights the danger of accepting block estimates that have a slope of regression less than 0.5.

The critical regression slope limit of 0.5 should only be used to identify blocks that will result with negative kriging efficiencies. Ideal slopes of regression should be greater than 0.95, as proposed by Krige (1996).

An extensive study of some 70 cases by Professor Krige, covering a wide range of spatial and data patterns, indicated a correlation between kriging efficiency and the regression slope (actuals on estimates) of 87.5% (Krige, 1996). Thus the slope (or the extent of conditional biases present) effectively incorporates all the major factors affecting the efficiency of block evaluations.

Smoothing effect of kriging

The absence of conditional biases is unavoidably accompanied by some smoothing, and it is a fallacy to use the data search routine for block evaluation in an endeavour to reduce or eliminate it. Smoothing is inevitable and essential for conditionally unbiased estimates.

This can be explained in terms of a theoretical approach by reviewing the definition of the slope of the line of regression of actuals (Y) on estimated block values (X):

$$Slope = r \, (\sigma_y/\sigma_x)$$

where σ_y and σ_x are the standard deviations of actuals and estimates respectively, and r is the correlation coefficient.

If the slope is to be unity, (*i.e.* slope = approx. 1) for unbiased block estimates and (r) is less than unity, because estimates are never perfect, then:

$$(\sigma_y/\sigma_x) > 1$$

I.e., the standard deviation (or variance) of the actual or real block values must be larger than that of the estimated block values. The gap between these two variances (the smoothing effect) can therefore be reduced only by increasing the correlation (r) between block estimates and actual values, *i.e.* by improving the efficiency of the estimation technique or by providing more data. *No mathematical manoeuvring can achieve this objective.*

Various post-processing techniques are available to remove smoothing effects (*e.g.*, Krige and Assibey-Bonsu, 1999; Journel *et al.*, 2000) and should be applied only to block estimates that are conditionally unbiased.

Conclusion

Professor Danie Krige's basic points of advice for the practitioner

Ensure data integrity

Disastrous errors and critical risks will result from using erroneous data. Various processes that are usually set out in company or mineral resource regulatory body standards and protocols should be followed to ensure overall data integrity. The SAMREC Code, Table 1, provides good guidelines as to those aspects that should be considered and reported on in the relevant Competent Person reports.

Establish all the necessary geological and geostatistical models and parameters

Geology should always be recognized as a vital element in deposit modelling. Experience has shown that geostatistical

Mineral Resource and Reserve assessment, without proper geological input, can also be disastrous and constitute a critical risk. A robust geological model is therefore a prerequisite, and if the geostatistical model does not agree with the geological one, there are grounds for serious concern. Either one or both models should be critically re-examined, so as to establish the essential correlation and validation.

Technique selection and optimal application: Choose an appropriate geostatistical technique and determine the search routine required to eliminate conditional biases and to ensure optimal application

Use effective tools, including slope of regression and block efficiencies, in this regard.

As blocks are mined out, conduct follow-up (reconciliation) studies to validate the Mineral Resource estimates

This is a further important aspect, not only to ensure that estimates have the quality required and that no biases are present, but also to timeously record the differences and facilitate corrective action.

Research new techniques and applications, but validate new techniques properly by way of (follow-up) checks to confirm the absence of biases and the practical advantages to be gained when they are applied in practice.

Final thoughts

The industry seems to be going backwards in certain areas, with a widespread misunderstanding of the causes and consequences of conditional biases. The following are some of the possible causes.

➤ In certain universities, as well as training provided elsewhere in the industry, geostatistics is taught using commercially available computer programs, with the emphasis being how to use the programs

➤ Unfortunately, this is what many mining companies expect: graduates or practitioners who are good at operating programs (the 'black box approach'). This does not allow much time for teaching the fundamentals of geostatistics and the consequences of misusing the technology

➤ What complicates matters is that the universities rarely have large databases to demonstrate the strengths and weaknesses of various methods in different environments, and research is by its own nature geared towards only the development of theoretical geostatistics, often based on strong stationarity assumptions.

'... *after half a century of phenomenal developments in geostatistics, conditional biases which gave birth to this subject, are still encountered in practical applications... the main concern is that this record will be tarnished by the all too ready acceptance (in certain cases) of estimates, which are still conditionally biased. For the future, I would like to see geostatistics continue to grow from strength to strength with new models, techniques and applications, but where these are all validated properly by way of (follow-up) checks to confirm the absence of biases and the practical advantages to be gained when they are applied in practice*' Professor Danie Krige.

The basic tenets of evaluating the Mineral Resource assets of mining companies

Professor D. Krige was indeed a pioneering giant, and the South African mining industry is blessed to have had the benefit conferred by his immense contributions. He always gave willingly and unselfishly, with the rewards being not gold, platinum, and diamonds, but the tools for others to utilize in finding and evaluating Mineral Resources, so as to achieve a positive financial return, while minimizing the associated risk. He took the industry far along the road, but the journey is not over and it now remains the responsibility of those that follow to adhere to his principles, and indeed continue to build on them, to ensure his legacy lives on.

Acknowledgements

Professor Danie Gerhardus Krige: He made the world a better place

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Disciplines unite at Wits to prepare mining for the 21st century

29 June 2016 – Johannesburg: Innovative technology solutions for the struggling mining sector are the focus of a new unit at Wits University, bringing together various disciplines and headed by former School of Mining Engineering head Professor Fred Cawood.

The Wits Mining Institute (WMI) will house the school's Digital Mine project – already well advanced in developing a mock mine within the Chamber of Mines building on Wits University's West Campus – and a college network to develop 21st century skills at artisan and technician level.

'The institute's mission is to make mining safer and more sustainable by harnessing fast-developing technologies and practices from different sectors – which are sadly not always incorporated into mining applications quickly enough to address the industry's many challenges,' said Professor Cawood.

He said the breakthrough that the WMI had made was to forge working links across the university's schools and research units, so that mining issues could be addressed in an integrated manner.

'It has taken some time to achieve this, but the WMI now draws upon a formidable battery of expertise and insights from disciplines like architecture, public health, law, global change, population migration, urban development, electronics and computer science,' he said. 'These now augment the already substantial work being done within the School of Mining Engineering through its Centre for Mechanised Mining Systems and the Centre for Sustainability in Mining and Industry.'

He said that South Africa's deep level ore bodies posed particularly difficult challenges to mining operations, but argued that encouraging progress was already being made to show the path forward for both established and new operations.

'Work on converting 'indoor' positioning systems to underground applications is already underway, for instance, paving the way to developing an automated tunnel for mining at depths no longer viable or safe for humans to operate,' said Professor Cawood.

Cutting edge software, sensors and related high-tech infrastructure were allowing developments like real time underground airflow modeling, and access systems that could automatically exclude personnel restricted by health issues or legal compliance requirements.

'This kind of intervention brings us closer to the concept of the intelligent mine, where the data required for good decisions is available in real time – and in many cases can inform automated responses that removes the risk of human error,' he said. 'The vision of safe and more efficient operations is reachable, if we can adapt and apply the remarkable technologies available to us.'

The mock mine at Wits University currently includes a 67 m life-size mine tunnel called 'Nick's Tunnel' a stairwell equipped as a mock vertical shaft, the NCM Stope, Lamp room and Control room – which are used for both teaching and research into aspects such as security, systems integration and video analytics.

Skills development by the WMI will focus on modern skills required to install and maintain the various new technologies being implemented or considered by mechanised and digital mines.

'Mines that are already mechanised find themselves in a difficult position, as last century's skills are unable to properly manage and advance the modern technologies that they have installed in their operations,' said Professor Cawood.

Professor Fred Cawood, director of the Wits Mining Institute

The· major funders of the digital mining infrastructure to date are Gold Fields, Aveng Mining, the Minerals and Education Trust Fund, Wits University, New Concept Mining and Sibanye Gold, who is currently the largest sponsor. The research agenda is significant, with a total of 16 postgraduates who use/used the facility for their research and 10 undergraduate students who will graduate at the end of 2016 with a digital mining competence. The research projects are sponsored by different companies who are partnering with the WMI in its objective to prepare the sector for 21st century mining.

S. Braham
Account Director
Contact: (011) 646-9322

When should uniform conditioning be applied?

by K. Hansmann*

Synopsis

Blindly applying any methodology to estimate the recoverable resources of a mineral deposit without considering the suitability of the approach to the deposit being evaluated can render misleading results. While 'running the software' provides an answer, one should, amongst numerous other considerations, understand the impact the underlying distributions and assumptions have on the validity of the result.

Uniform conditioning (UC) is a nonlinear estimation method that models the conditional distribution of smallest mining unit (SMU) block grades within panels, and localized uniform conditioning (LUC) places these SMU at plausible locations within a panel. The localization process does not improve the accuracy of the UC result, but rather presents the result in a more practical format; particularly for use in mine planning.

A case study was carried out to compare the suitability of UC and LUC on two hypothetical data-sets. The data-sets are simulated realizations of a normal grade distribution and a highly skewed lognormal grade distribution which are akin to grade distributions found in mineral deposits. The estimation methods were applied to both data-sets, and the results compared with the actual grades of the simulated realizations. This paper presents an overview of UC and LUC, with discussions around the case study results.

Keywords

change of support, Gaussian anamorphosis, localized uniform conditioning, lognormal distribution, normal distribution, uniform conditioning.

Introduction

Uniform conditioning (UC) is a nonlinear estimation technique that estimates the conditional distribution of metal and tonnage above cut-off within a mining panel. It does not directly estimate grade, although grade is a typical outcome from the estimated metal-tonnage distribution or the results produced by localized uniform conditioning (LUC). UC results are typically presented as a recoverable resource above multiple cut-off grades. The advantage of UC is that it can be used on widely spaced data, across domains that are not strictly stationary, provided that there is sufficient data for a conditionally unbiased estimate of the panel mean grade (Rivoirard, 1994).

Previous studies where UC has been applied to porphyry copper deposits (Deraisme et al., 2008; Deraisme and Assibey-Bonsu, 2011; Millad and Zammit, 2014) show the application of the method to normal grade distributions. Additional studies have applied this approach to gold deposits (Assibey-Bonsu, 1998; Humphreys, 1998) and an iron ore deposit (De-Vitry et al., 2007), indicating that the method is applicable on skew, lognormal distributions. While UC has been practically applied to different types of deposit, it is not known how well UC predicts actual grades for the underlying grade distribution.

This paper discusses the UC estimation method as well as the popular add-on, LUC, presented by Abzalov (2006). A case study is presented that compares UC and LUC estimates of two hypothetical data-sets, referred to herein as scenario 1 and scenario 2. Scenario 1 is a normally distributed grade distribution and scenario 2 is a skew, lognormally distributed grade distribution, both of which are compared against the simulated realizations that represent the actual grades.

The conditions found in the two data-sets are similar to those found in naturally occurring mineral deposits. The aim of this investigation is to determine the underlying conditions of the grade distribution that produces favourable results when applying UC, and subsequently LUC, to such data-sets.

Uniform conditioning workflow

The following section describes a UC with LUC workflow, which follows the process outlined in Figure 1.

Data preparation and declustering

This initial part of the UC workflow is to prepare and carry out exploratory data analyses, including histograms, to understand the sample grade distribution and variability in the deposit. The data must be appropriately

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* *University of the Witwatersrand, Johannesburg, South Africa.*

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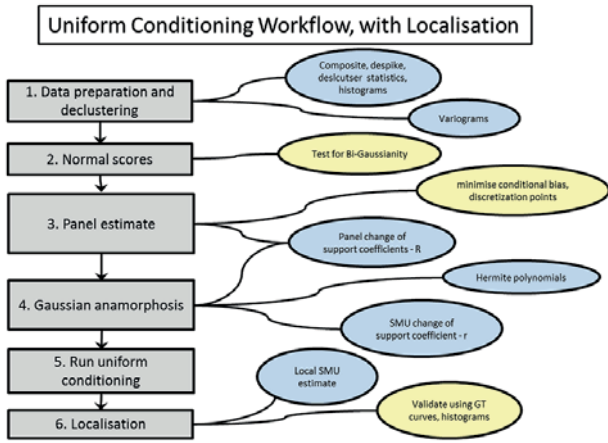
Figure 1—Uniform conditioning workflow, with localization

declustered, so that the normal score transforms of the grades are an accurate representation of the grade distribution. This is important for the Gaussian anamorphosis function (to be discussed later), and failure to correctly complete this will affect the results of the UC estimate.

Normal scores

As the discrete Gaussian model (DGM) for change of support is used for UC, the data must be transformed to the equivalent Gaussian (or normal score) values using declustered weights. This is performed by transforming the cumulative distribution frequency (CDF) of the original grades to a Gaussian probability CDF, on a percentile to percentile basis for the entire data-set. Figure 2 shows the CDF of a lognormal grade distribution (data from scenario 2) on the left, with the green and red lines showing percentile paired mapping of values to the equivalent normal score values on the right.

The DGM relies upon the assumption of bivariate Gaussianity of the transformed grades (Rivoirard, 1994). Bivariate Gaussianity means that any linear combination of the Gaussian transformed data is also Gaussian. Several tests exist to determine if the transformed data conforms to such conditions, and is therefore suitable for use with the DGM. Schofield (1988), Rivoirard (1994) and Humphreys (1998) give practical examples on how these tests may be run.

Panel estimates

The quality of the panel estimate determines the success of the UC estimation (Rivoirard, 1994). A panel estimate should be conditionally unbiased (Rivoirard, 1994; De-Vitry *et al.*, 2007), so that the UC conditional grade distribution will be an accurate estimate of the actual grade distribution. The panel estimate can be carried out using any linear estimator, but conventionally ordinary kriging (OK) is used.

The panel size should be chosen relative to the spacing of the sample data. De Vitry *et al.* (2007) suggested that the panel should be as small as possible to ensure an accurate estimate, but large enough for minimal conditional bias of the estimate. The number of smallest mining units (SMU) within the panel is linked to the resolution of the grade-tonnage relationship, as the number of SMU discretizes the grade-tonnage curve of the panel (Harley and Assibey-Bonsu, 2007).

Gaussian anamorphosis

The DGM can be used to derive the marginal histograms at different supports. The Gaussian anamorphosis function is modelled by a set of Hermite polynomials, weighted with an accompanying set of Hermite coefficients. A full description of Hermite polynomials and how these may be calculated is given by Rivoirard (1994).

A point and fitted model anamorphosis function for a normal and lognormal distribution are shown in Figure 3. The anamorphosis function for the lognormal distribution is constructed from the normal score data, by plotting pairs of grade and Gaussian transformed grade values.
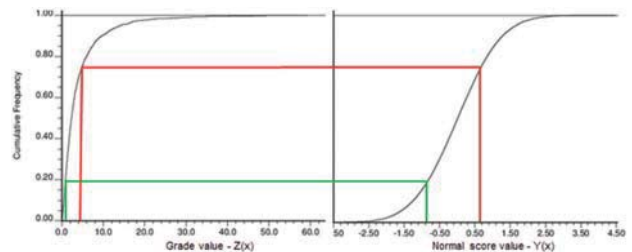
Figure 2—Cumulative distribution frequency of grade values to Gaussian transformed values for a lognormal distribution
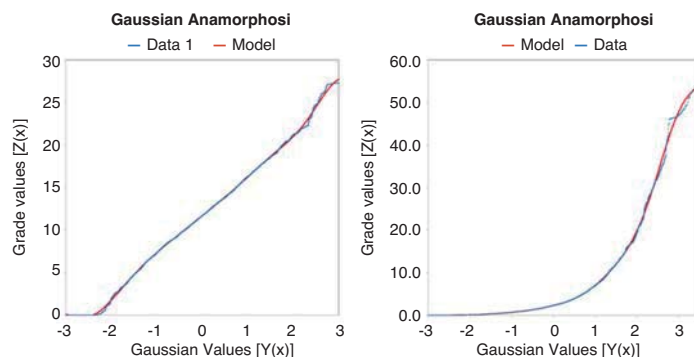
Figure 3—(Left) Gaussian anamorphosis experimental data and model for scenario 1 with an underlying normal distribution, and (right) Gaussian anamorphosis experimental data and model for scenario 2 with an underlying lognormal distribution

When should uniform conditioning be applied?

The Hermite polynomials are functions of the standard Gaussian distribution, and therefore they express probabilities and have the properties of a standard Gaussian distribution (data is symmetrically distributed around the mean value of zero, and has a unit variance). An additional property (see Table I) shows the variance of the grade $Z(x)$ expressed by the sum of Hermite coefficients, excluding the 0th, which describes the mean.

The number of Hermite coefficients used to fit the anamorphosis model can vary, and the optimal number depends on the how well the polynomial set fits the underlying distribution. Neufeld (2005) recommends using less than 100 coefficients, although 20 to 30 coefficients are usually sufficient.

The variance of grades depends on the support that the grade represents, and a change-of-support model, like the DGM, is used to predict the distribution of grade at different supports. Grades at a point support have a higher variance than grades of SMU, which similarly have a higher variance than grades of panels. As the support of a grade increases, so the grade values tend towards the population mean, have less deviation from it, and are more symmetrical around it (Figure 4).

There is a correlation between the distribution of grades seen at a point support and the distribution of grades seen at a SMU support, named the SMU change-of-support co-efficient (r). Similarly, there is a correlation between the distribution of grades seen at a point support and the distribution of grades seen at a panel support, named the panel change-of-support coefficients (R). The ratio R/r is the correlation of SMU grades and panel grades. The R and r change-of-support coefficients are determined by solving the variance equations and the Gaussian anamorphosis equations at SMU and panel supports, shown in Table I (Rivoirard, 1994).

Uniform conditioning

The schematic in Figure 5 shows the relationship between grades at a SMU support, grades at panel support, and how a distribution of SMU grades is conditional on panel grade. A low R/r ratio indicates a weak correlation between the SMU and panel grades, which is caused by a high-nugget semivariogram and/or short semivariogram ranges relative to the data spacing. A high R/r ratio is indicative of a strong correlation between SMU and panel grades, which indicates good grade continuity in the deposit.

Localization

The result of a UC estimate is presented as a distribution of grades, shown as metal content and tonnages reported for a series of cut-off grades. While this is insightful information about the grade-tonnage distribution, it is not a particularly practical data format as the SMU location is not provided.

Abzalov (2006) presents LUC as a simple extension to UC that provides a practical solution for visualizing grades at the SMU level. A UC grade-tonnage distribution is decomposed to
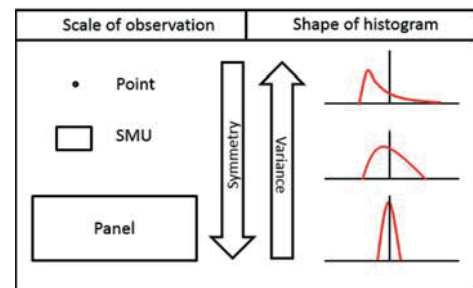
Figure 4—Schematic showing support effects for point samples, SMU, and panels
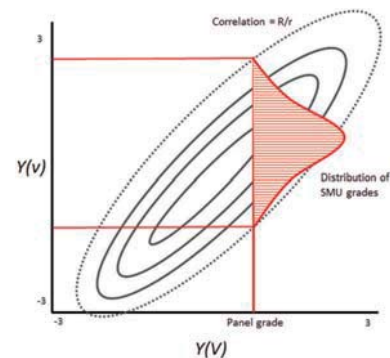
Figure 5—Conditional SMU grade distribution ($Y(v)$) from a panel grade ($Y(V)$). The correlation of the grades is R/r

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Table I

(Top) Equations for calculating variance from Hermite polynomial coefficients point, SMU, and panel support. (Bottom) Equations for Gaussian anamorphosis function at point support, and adjusted for SMU and panel support. After Rivoirard (1994)

	Point support	SMU support	Panel support
Variance equations	$\sigma^2_{Z(x)} = \sum_{i=1}^{n} \phi_i^2$	$\sigma^2_{Z(v)} = \sum_{i=1}^{n} r^{2i}\phi_i^2$	$\sigma^2_{Z(V)} = \sum_{i=1}^{n} R^{2i}\phi_i^2$
Gaussian anamorphosis equations	$Z(x) = \sum_{i=0}^{n} \phi_i H_i[Y(x)]$	$Z(v) = \sum_{i=0}^{n} r^i\phi_i H_i[Y(v)]$	$Z(V) = \sum_{i=0}^{n} R^i\phi_i H_i[Y(V)]$

ϕ_i Hermite coefficient
H_i Hermite polynomial evaluated for Gaussian value
^{2} Dispersion variance
n Number of Hermite polynomial terms
R Panel change-of-support coefficient $[0 \leq R \leq 1]$

r SMU change-of-support coefficient $[0 \leq r \leq 1]$
$Z(V)$ Grade at panel support V
$Z(v)$ Grade at SMU support v
$Z(x)$ Grade at point location x

When should uniform conditioning be applied?

a series of grade values that reproduce the grade-tonnage relationships. These plausible SMU grades are located into a SMU model based on the rank location of grades from a linear estimate of equally sized blocks. This results in a direct grade model at the SMU resolution that respects the grade-tonnage distributions of the UC panels and attempts to reflect the localized spatial grade distribution within the panel.

Case study

The objective of this case study is to assess the suitability of UC and LUC for two data-sets with different grade distributions, namely scenario 1 and scenario 2. The two scenarios are distinctly different and represent two end-members of the range of grade distributions that may typically be seen in mineral occurrences, being a symmetrical distribution and a positively skewed distribution. The grade distributions were synthetically generated and sampled to mimic how this would be done in a mineral exploration project.

The UC with localization procedure described in this paper was followed for both data-sets, as outlined in Figure 1.

Description of case study data

Two sets of simulated data were generated, which were used as base data for the assessment. A single realization was simulated on a 2 m × 2 m × 2 m point grid, over a 800 m × 600 m area, with a thickness of 200 m, for each distribution. A plan view at surface through both simulations is shown in Figure 6 and Figure 7.

Statistics and variography

A spatially representative subset of data was taken from both simulations, which makes up the sample database used for this project. A total of 417 pseudo drill-holes, each containing 100 composites, was taken over the study area. The area is densely sampled, and this drilling grid would be consistent with that of a feasibility-stage project.
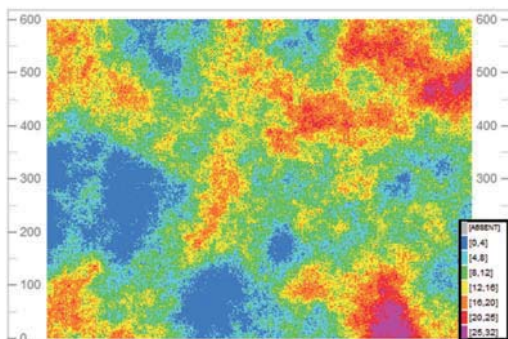
Although both scenarios are equally sampled (at a density of 1%), the drill-hole spacing relative to the semivariogram ranges for scenario 1 is closer than for scenario 2. For scenario 1, the samples are spaced at approximately one-third of the semivariogram ranges (120 m) in X and Y, while for scenario 2 the samples are spaced at approximately the maximum semivariogram range (40 m). Statistics of both scenarios are compared, but there is no correlation between them as the simulations were run independently. Declustered statistics are presented in Table II, histograms in Figure 8 and Figure 9, and modelled semivariograms in Figure 10 and Figure 11.

In scenario 1, the grade distribution is symmetrical, with a comparatively low nugget effect (12%) and well-defined continuity up to distances of 170–300 m. Slight anisotropy was evident, and possibly some zonal anisotropy seen in the Y-direction where the variance does not reach the sill value. The distribution is approximately normal, and is similar to what one would find in a porphyry copper mineral occurrence. The first distribution has a smaller range of grade values than the second (approximately half).

In scenario 2, the grade distribution is approximately lognormal, supported by the shape of the histogram. This distribution has the characteristics of being asymmetrical, strongly positively skewed, with a long tail. There is a higher nugget effect (26%), with long-range continuity of approximately 60–90 m. The coefficient of variation (CoV) for scenario 2 is higher (1.3) than that of scenario 1 (0.4), showing a wider spread and higher variability of grade values.

Panel estimates

Panel grade estimates, using OK, were produced for both scenarios with the intent of minimizing conditional bias while retaining some local variability. The block sizes were chosen

Figure 6—Plan view of the simulation of scenario 1 (normal distribution)
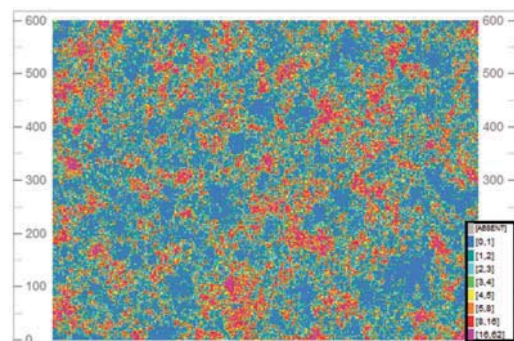
Figure 7—Plan view of the simulation of scenario 2 (lognormal distribution)

Table II

Descriptive statistics for scenario 1 and scenario 2

	Number of samples	Number of boreholes	Min g/t	Max g/t	Mean g/t	Variance g/t²	Standard deviation g/t	Skewness	Kurtosis	Coefficient of variation
Scenario 1	41 700	417	0.0	29.6	11.8	21.5	4.6	0.1	0.3	0.4
Scenario 2	41 700	417	0.0	59.3	4.1	30.3	5.5	3.7	19.5	1.3
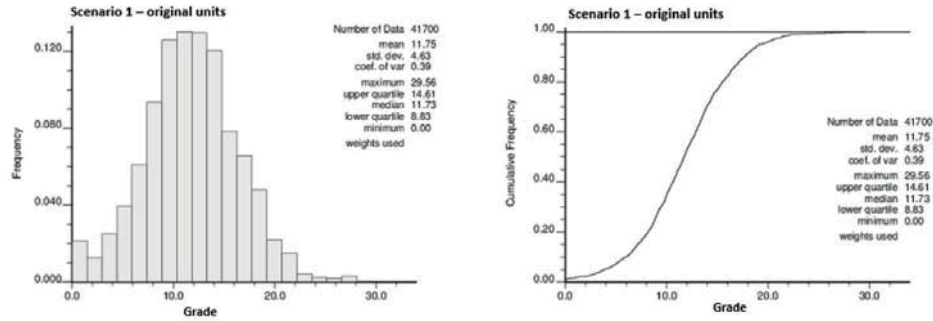
Figure 8—Sample histogram and cumulative frequency histogram for scenario 1
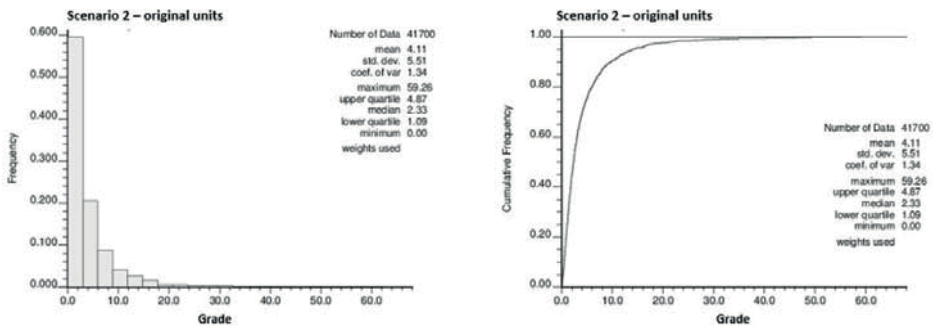
Figure 9—Sample histogram and cumulative frequency histogram for scenario 2
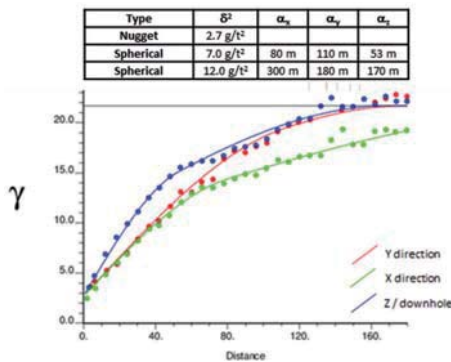
Figure 10—Experimental semivariogram, model semivariogram, and parameters for scenario 1
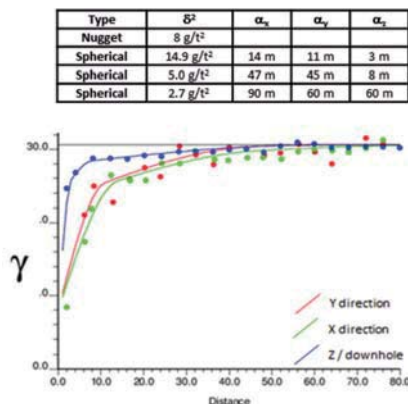
Figure 11—Experimental semivariogram, model semivariogram, and parameters for scenario 2

relative to the average sample spacing, at 50 m × 50 m × 20 m. Ten discretization points were chosen in the X and Y directions, based on a quantitative kriging neighbourhood analysis (QKNA) optimization of the block variance. Discretization points in the Z direction were chosen to be equal to the compositing length. A sufficiently large search neighbourhood was chosen for the panel estimate to ensure high slopes of regression, without the introduction of too many (>5%) negative kriging weights.

The panel model estimation for scenario 1 had a mean slope of regression of 0.97; while that of the scenario 2 panel model estimation was 0.72. Figure 12 and Figure 13 show plan views of the respective scenarios, at surface elevation. Comparing these to the simulated data (Figure 6 and Figure 7), the grade smoothing effect and reduction of variance from the OK is evident.

Change of support

Sample data was converted to normal scores and tested for bivariate Gaussianity. For both scenarios, the test results were consistent with bivariate Gaussian conditions of the Gaussian transformed data. Change of support was carried out for each scenario using the DGM, with parameters shown in Figure 3.

In scenario 1 the SMU change-of-support coefficient indicates a strong correlation between point and SMU grades. For scenario 2, the SMU change-of-support coefficient implies a weak correlation between point and SMU grades and is a result of the high nugget effect.

Panel change-of-support coefficients are measured from the direct variance of estimated panel grades. Well-informed

When should uniform conditioning be applied?
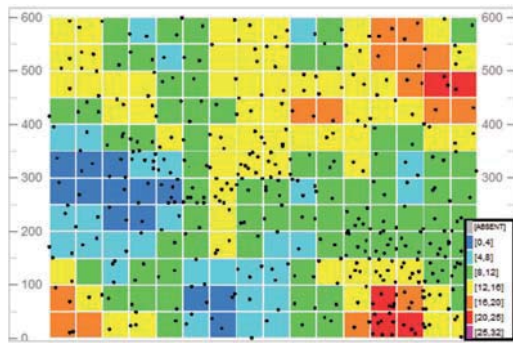
Figure 12—Surface plan view of ordinary kriged panel estimate and location of pseudo drill-holes for scenario 1
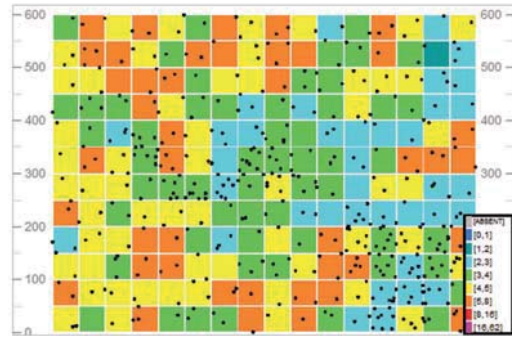
Figure 13—Surface plan view of ordinary kriged panel estimate and location of pseudo drill-holes for scenario 2

Table III

SMU and panel dispersion variances and resulting change -of -support coefficients. Ranged values are for minima and maxima of isovariance groupings

	Scenario 1	Scenario 2
Sample variance (sill)	21.7 g/t^2	30.6 g/t^2
Dispersion variance in SMU (theoretical value)	4.22 g/t^2	24.45 g/t^2
SMU change of support coefficient, r	0.903	0.503
Dispersion variance in panel (measured values)	6.17 g/t^2–13.39 g/t^2	0.91 g/t^2–1.04 g/t^2
Panel change of support coefficient, R	0.537–0.791	0.230–0.215
Change of support correlation R/r	0.595–0.876	0.427–0.457

panels, as defined by the number of samples in the neighbourhood and semivariogram ranges, will have better estimation confidence. To account for this, different panel change-of-support correlations are used for panels with similar confidence in the estimation. Three such isovariance groupings were used for each scenario, and the ranges of these results are shown in Table III. Relatively higher variances of estimated panel grade groupings are typically from better-informed areas where there is better grade continuity that results in a greater spread of estimated panel grades. Conversely, relatively lower estimated panel grade variance groupings are for worse-informed areas, where there is more evidence of smoothing and panel estimates are closer to the mean, giving rise to less variance

Running uniform conditioning and localization

UC was carried out for both data-sets using the panel model and DGM. After completion of UC, the model was localized using a local SMU model, which was estimated using smaller kriging neighbourhoods to reflect local variability.

Discussion on uniform conditioning

The performance of UC may be assessed by how closely the UC grade-tonnage estimate conforms to the actual simulated model and an OK model, as a benchmark for a linear estimator. This comparison was made globally, to demonstrate the effects of incorrectly predicting the extractable tonnage of the deposit, and locally, to demonstrate the effects of getting individual panel grades right/wrong.

Global grade-tonnage assessment

The grade-tonnage relationship for the normally distributed scenario is shown in Figure 14. The global UC prediction of tons and grades is very close to the actual grades, and shows a slight improvement on the OK grade-tonnage curve.

In the case of the normally distributed grades (scenario 1), where there is good data coverage (relative to the semivariogram ranges), OK performs well for determining recoverable resources. Slopes of regression for the OK model were, on average, close to unity, indicating a very low conditional bias (which is reflected in the OK estimates being close to the actual values). However, UC marginally outperforms OK in terms of estimating a recoverable resource, as it more closely predicts the grade and tonnage of the simulated reality. For low cut-off grades, there is slightly less tonnage than predicted for the both the OK and UC model, but the UC estimates are closer to the actual values.

Where the grade data has an underlying lognormal distri-bution (scenario 2) with relatively poor data coverage, the simulated model shows a decline in tonnage (or volume) as the cut-off grade increases (Figure 15). OK generates a moderate estimation of the grade and tonnage extractable for any cut-off grade. This lack of adherence to the grade-tonnage curve can be explained by a grade smoothing, which was expected, as the slopes of regression of the panel estimate were, on average, poor. UC gives a better result than OK, but the resultant estimation of grades and tonnage does not closely conform to the actual values. As the selectivity increases (*i.e.* high-grade areas are targeted), the average grade of the actual material will be higher than the OK model predicts.

When should uniform conditioning be applied?
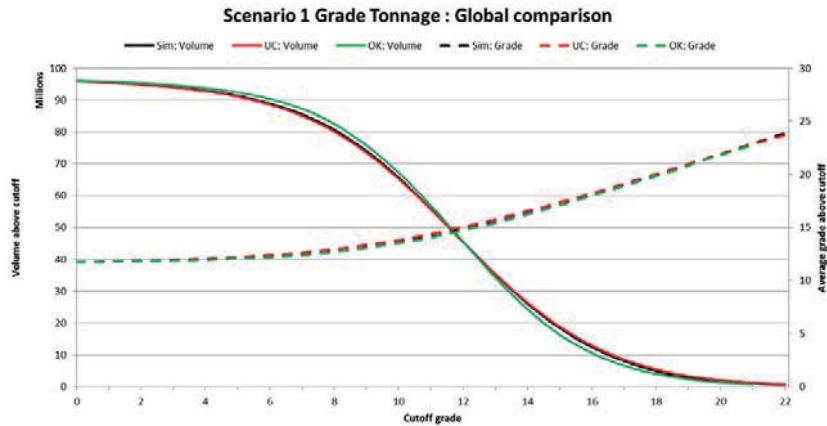
Figure 14—Global grade-tonnage curves for scenario 1
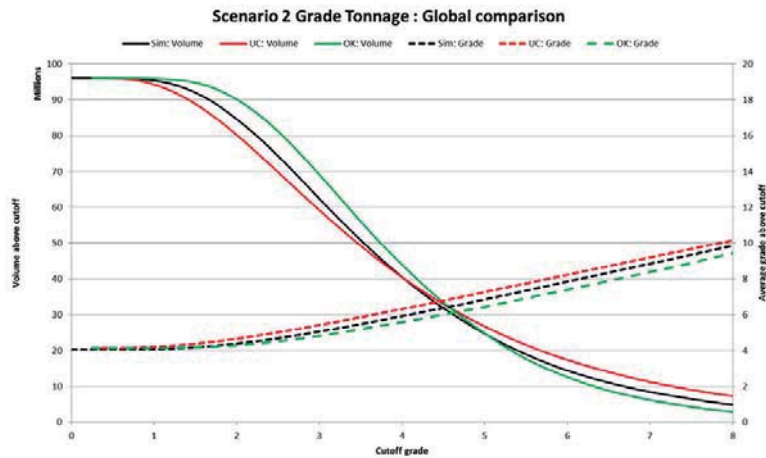
Figure 15—Global grade-tonnage curves for scenario 2

At low cut-off grades, a linear estimated model frequently shows an overestimation of volume or payable ground. This is referred to as the 'vanishing tons' problem as described by David (1977), which is seen when mining commences and less material is recovered than was predicted. This is caused by a conditional bias and/or smoothing in the estimate, which are reflected respectively by low slopes of regression and/or higher estimation variance in the estimated result. This phenomenon is amplified by a high nugget effect and small block sizes used for estimation.

In order to resolve a conditional bias, one can estimate grades into larger blocks. However, estimating into larger blocks can produce an over-smoothed histogram, or too much average material, and does not provide the accuracy required to select blocks for mining. This is the 'kriging oxymoron' (Isaaks, 2004), which states that a kriged estimate cannot be conditionally unbiased and accurate at the same time. UC uses the 'conditionally unbiased' large block estimator to condition the average of a distribution of small blocks, thereby maintaining the correct grade-tonnage curves and applying a conditional distribution to obtain an accurate histogram of small block (SMU) grades. This attempts to satisfy the apparent contradiction embodied in the kriging oxymoron.

Local grade-tonnage assessment

An assessment was done to compare the grade-tonnage results of panels that are well estimated and did not contain a conditional bias (as determined by the slope of regression) against poorly estimated panels. Panels chosen for this assessment are shown in Figure 16, where values with the better slopes of regression fall on or close to the 1:1 regression line.

For the normally distributed data, if the mean panel grade estimate is correct, the UC accurately predicts the distribution of grades and tonnage (Figure 17). For the lognormally distributed data, UC predicts the grade-tonnage relationship (Figure 19 and Figure 20) fairly well. For both distributions, if the mean panel grade is wrong, the distribution of SMU grades will not necessarily match the simulated distribution (Figure 18 and Figure 21). It appears that, in addition to an unbiased panel estimate, UC performs slightly better on an individual panel basis when the underlying distribution is normal.

Localization ranking assessment

The localization of the UC result places individual SMU grades (derived from the SMU grade-tonnage curve within

When should uniform conditioning be applied?
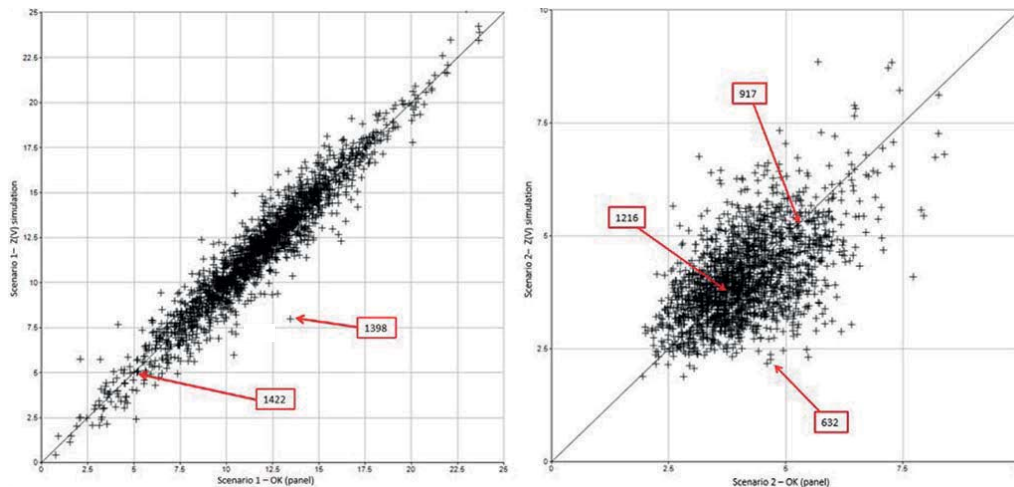
Figure 16—Actual *versus* OK panel scatter plot, showing blocks for LUC analysis for scenario 1 (left) and scenario 2 (right)
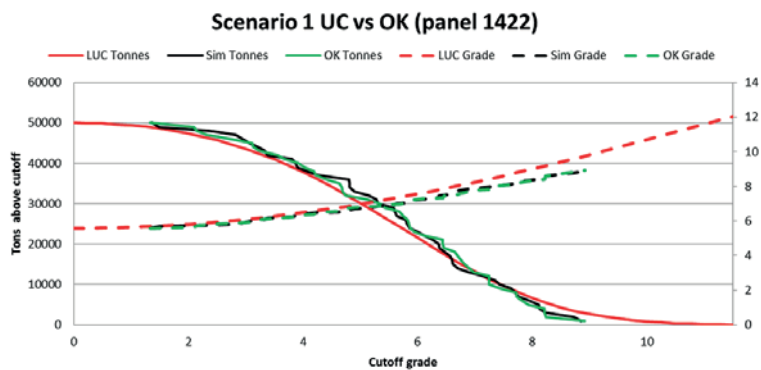
Figure 17—Grade-tonnage curve for panel 1422 (normal distribution: scenario 1)
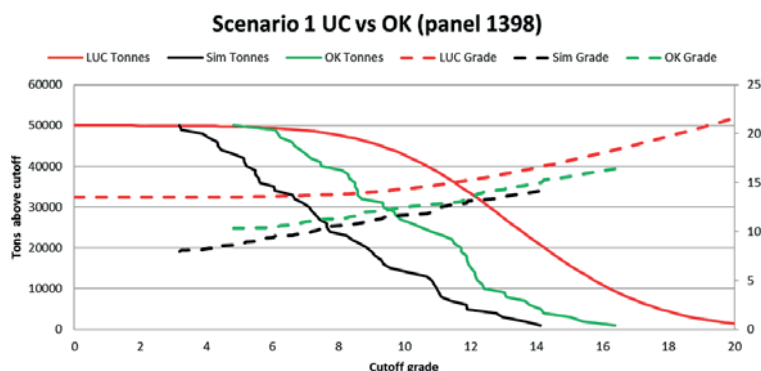
Figure 18—Grade-tonnage curve for panel 1398 (normal distribution: scenario 1)

the panel) at specific locations within the panel, based on the estimated grades of the OK SMU model. The success of the localization is verified by visual comparison and by statistically comparing the rank values of the actual *versus* the UC ranking (Figure 22).

The success of localization depends entirely on the reliability of the OK SMU estimate. However, the smoothing and inaccuracy of this estimate is the prime motivation to use

UC in preference to linear estimates. If the OK SMU estimate provides a good spatial representation of the local grades, then the location of the UC grades within the panel will be more accurate. This confirms Abzalov's (2006) findings that the localization success is dependent on available data (among other factors).

If the data is closely spaced enough to provide accurate localization, then it is also likely that the data is sufficiently

When should uniform conditioning be applied?
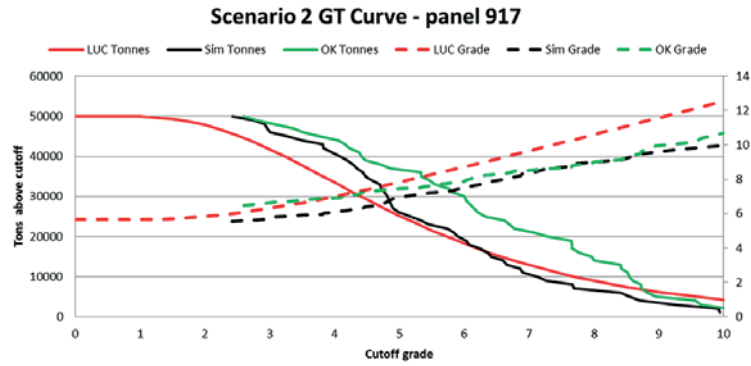
Figure 19—Grade-tonnage curve for panel 917 (lognormal distribution: scenario 2)
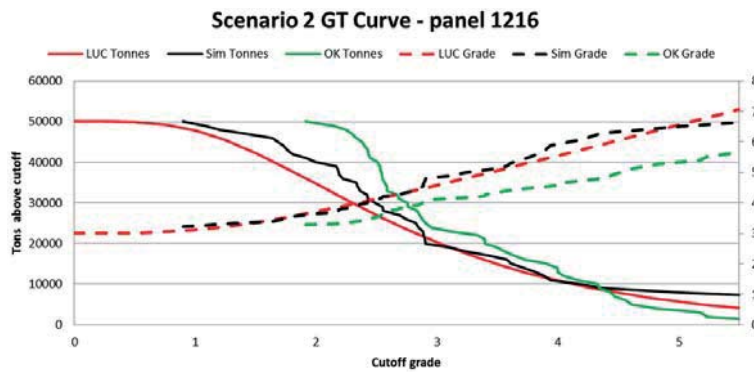
Figure 20—Grade-tonnage curve for panel 1216 (lognormal distribution: scenario 2)
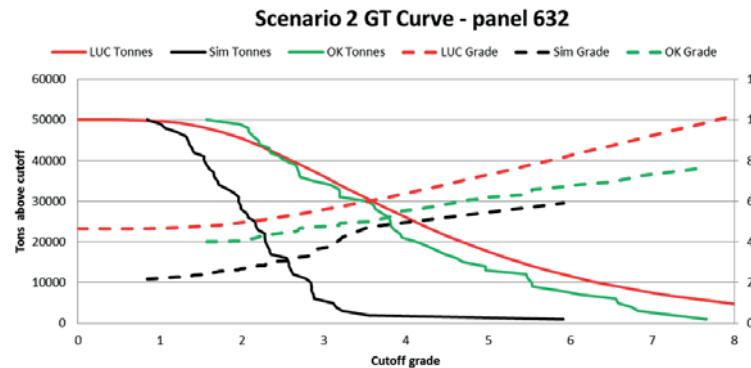
Figure 21—Grade-tonnage curve for panel 632 (lognormal distribution: scenario 2)
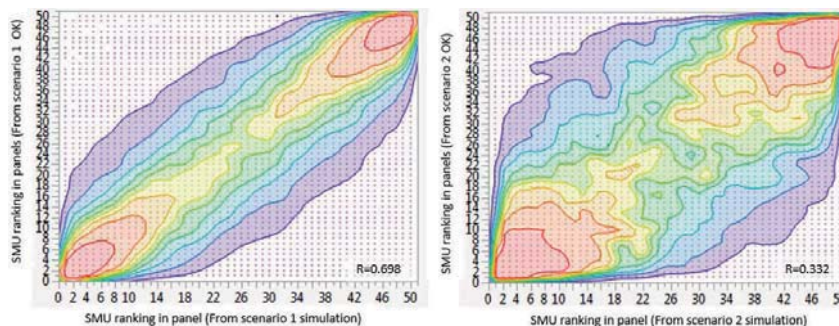
Figure 22—Comparison of SMU within-panel ranking for different data distributions

When should uniform conditioning be applied?

closely spaced for a linear estimation to accurately predict the model grade value. In this circumstance, the benefit of using a nonlinear UC estimator over a linear estimator is not as significant as the benefit seen with widely spaced data. This is evident in the grade-tonnage predictions for the well-estimated data, where the OK and UC results are similar; and the predictions for the poorly estimated data, where the UC results show a significant improvement over the OK results.

Although LUC is a useful addition to UC, it does not improve the accuracy of the UC estimate and the localization algorithm cannot predict the placements of SMU beyond the available data. This is the main problem: one cannot simultaneously know the local mean and the local variability from limited local data. The single largest contribution of the localization approach is to present a UC model in a more accessible and immediately useful format for mine planning.

Conclusions

UC performs well in terms of estimating grades and tonnages when there is a normal underlying grade distribution and good sample coverage relative to the variogram ranges, which result in low conditional biases. In such circumstances a linear estimator can also closely predict recoverable resources and provide a spatially representative grade model, although the UC estimate of tons and grades is slightly better.

When there is an underlying lognormal distribution and poor sample coverage relative to variogram ranges, conditional biases of a linear panel estimate will occur. This results in UC providing a more accurate global estimate of grades and tonnage than a linear estimate. The individual panel results predict the actual grade and tonnage distribution when there is no evidence for conditional bias for that panel.

LUC results are favourable when there is sufficient closely spaced data, in which case it is likely that a linear estimation could also accurately predict the model grade values. Therefore, the benefit of using a nonlinear UC estimator over a linear estimator is more significant when the data is widely spaced.

Acknowledgments

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Optimizing open-pit block scheduling with exposed ore reserve

by J. Saavedra-Rosas*, E. Jélvez†, J. Amaya‡, and N. Morales†

Synopsis

A crucial problem in the open pit mining industry is to determine the *optimal block scheduling*, defining how the orebody will be sequenced for exploitation. An orebody is often comprised of several thousand or million blocks and the scheduling models for this structure are very complex, giving rise to very large combinatorial linear problems. Operational mine plans are usually produced on a yearly basis and further scheduling is attempted to provide monthly, weekly, and daily schedules. A portion of the ore reserve is said to be exposed if it is readily available for extraction at the start of the period. In this paper, an integer programming (IP) model is presented to generate pit designs under exposed ore reserve requirements, as an extension of the classical optimization models for mine planning. For this purpose, we introduce a set of new binary variables, representing which blocks can be declared as exposed ore reserve, in addition to the extraction and processing decisions. The model has been coded and tested in a set of standard instances, showing very encouraging results in the generation of operational block schedules.

Keywords

block scheduling, surface mining, open pit planning, optimization model, exposed ore reserve.

Introduction

Open pit mines are characterized by high production levels and low operational costs compared to other exploitation methods. Unfortunately, when using this technique, material with poor economic value (waste) usually has to be removed to give access to more economically profitable material. Each block is characterized by its metal content, density, lithology, and other relevant attributes that are derived by using estimation techniques specifically designed to deal with the spatial nature of the mineralization. The value of a mine plan is thus determined by the value contained in the blocks that are extracted at certain periods and has a clear dependency on the *block schedule*, the order in which the material is extracted and processed.

Generally speaking, different problems are usually considered by mining planners for the economic valuation, design, and planning of open-pit mines, as pointed out by Hustrulid and Kuchta (2006); for example, the 'final pit problem', also called the 'ultimate pit limit problem', which aims to find the region of maximal undiscounted economic value for

exploitation under certain geotechnical stability constraints. Another example is known as the open pit production scheduling problem, which aims to find an optimal sequence of extraction in a certain finite time horizon with bounded capacities (for example, extraction and processing) at each period and where the usual optimality criterion is the total discounted profit. A common practice for the formulation of these problems consists of describing an ore reserve via the construction of a three-dimensional block model from the orebody with each block corresponding to the basic volume of extraction, characterized by several geological and economic properties that are estimated from sample data. For this reason, the open pit production scheduling problem is also known as the block scheduling problem. Block models can be represented as directed graphs where nodes are associated with blocks, while arcs correspond to the precedence of these blocks, induced by physical and operational requirements derived from the geomechanics of slope stability. This discrete approach gives rise to huge combinatorial problems, the mathematical formulations of which are special large-scale instances of integer programming (IP) optimization problems; see for instance Caccetta (2007). Precedence between blocks is one of the most important sets of constraints, as the extraction process proceeds from surface down to the bottom of the mineralization. This idea applies to every block in the model: it is not possible to access a given block in a certain time unless the blocks that are above have already been extracted, because stability of the pit walls

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* Department of Mineral and Energy Economics, Curtin University, WA, Australia.

† Delphos Mine Planning Lab., AMTC & Department of Mining Engineering, Universidad de Chile, Santiago, Chile.

‡ Center for Mathematical Modeling & Department of Mathematical Engineering, Universidad de Chile, Santiago, Chile.

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must be ensured. The amount of material to be transported and processed in each period is subject to upper and lower bounds, resulting from transportation and plant capacities, usually expressed in tons.

Regarding the block scheduling problem, a very general formulation was proposed by Johnson (1968, 1969), who presented a linear programming model under slope, capacity, and blending constraints (the latter given by ranges of the processed ore grade) within a multi-destination setting, *i.e.*, the optimization model decides the best process to apply on a block-by-block basis. Unfortunately, the complexity of the model was too great to be solved in realistic case studies, and therefore the industry preferred an approach based on nested pits, computed as a parametrization of the ultimate pit solved with the algorithm proposed by Lerchs and Grossmann (1965) and subsequent improvements (see, for example, Picard, 1976; Hochbaum and Chen, 2000; Amankwah *et al.*, 2014).

Caccetta and Hill (2003) proposed a model containing additional constraints on the mining extraction sequence and used a customized version of the branch-and-cut algorithm to solve it up to a few hundreds of thousands of blocks. Bley *et al.* (2010) used a similar model, but considering a fixed cut-off grade and incorporating additional cuts based on the capacity constraints that strengthen the formulation of the problem. This strategy was also used by Fricke (2006) in order to find inequalities that improve various integer formulations of the same model. Gaupp (2008) also reduced the number of variables by deriving minimum and maximum extraction periods for each block, therefore eliminating some of the variables and reducing the original MIP model size. Bienstock and Zuckerberg (2010) used Lagrangian relaxation on all constraints except the precedence constraints, reducing the model to the final pit problem. Chicoisne *et al.* (2012) focused on the case of one destination and one capacity constraint per period, developing a customized algorithm for the linear relaxation and a heuristic based on topological sorting to obtain integer feasible solutions. Cullembine *et al.* (2011) proposed a heuristic procedure using Lagrangian relaxation on lower and upper capacity constraints and a sliding time window strategy, in which late periods are also relaxed while variables of early periods are fixed incrementally. Lambert and Newman (2013) employed a tailored Lagrangian relaxation, which uses information obtained while generating the initial solution to select a dualization scheme for the resource constraints. Dagdelen *et al.* (1986, 1999) and Ramazan *et al.* (2005) worked on a model with fixed cut-off grades and upper and lower bounds for blending. Boland *et al.* (2009) proposed a different model, in which they aggregate blocks for the extraction decisions, including slope constraints, while the processing decisions are modelled at the level of individual blocks. Jélvez *et al.* (2016) used heuristics based on incremental and aggregation approaches in order to solve the open pit block scheduling problem. The model considers upper and lower capacity constraints, but the application considers only upper bounds. Zhang (2006) used a genetic algorithm combined with a block aggregation technique. Another approach is developed by Tabesh and Askari-Nasab (2011), who presented an algorithm that aggregates blocks into mining units and uses Tabu search to calibrate the number of final units; the

resulting problem is then solved using standard IP algorithms. The aggregation technique is interesting, because it is based on a similarity index that considers attributes like rock type, ore grade, and the distance between the blocks.

The problem that this paper addresses is the design of a block schedule, but with the additional constraint of leaving enough exposed ore reserve that is readily available at the start of every period. Usually, once the phases are designed, their scheduling is adjusted so that there is always enough exposed ore to feed the processes for a few months. Unfortunately, these considerations are not included in the strategic optimization model that generates the phases and therefore operational delays may impact production. While this could be addressed using stocks, this is theoretically more complex (because of potential nonlinearity) and in practice requires material re-handling, which is more expensive and more difficult to track than material coming straight from the mine. On the other hand, as we show in this work, these constraints can be included in the block scheduling without these shortcomings. This may prove particularly relevant in the case of mines with disseminated or irregular ore distribution, and may be used as a tool to reduce stock sizes and therefore the operation footprint.

In terms of the model itself, as we will see later, the only *cost* to pay in the model is the introduction of new variables representing the exposed blocks and the corresponding new constraints, to be described in the next section.

Mathematical model

In this section the conceptual model is presented using an IP framework. It is important to stress that this new mathematical model is an extension of those considered as *classical*, but adds new variables and constraints that identify the blocks as exposed ore reserve.

Let B be the set of blocks, and $\#B$ denote the number of blocks. Each block has a certain number of attributes such as tonnage and ore grade; these attributes permit the economic value of every block in B to be determined. The slope requirements for the set of blocks are described by a set of precedence arcs $A \subset B \times B$, in such a manner that the pair $(i,j) \in A$ means that block i must be extracted by period t if block j needs to be extracted at period t.

In this model, a decision whether the extracted material should be sent to a processing plant or to the waste dump is included, thereby defining a variable cut-off grade. For each block i it is assumed that the tonnage τ_i, the ore grade λ_i, and the net discounted value, given by $b_i^t - p_i^t$ if block i is sent to processing plant at period t, and $-m_i^t$ if block i is sent to waste dump at period t, are known.

For every period t, maximum limits are imposed on the amount of material that is mined (M^t), and on the amount of ore that is milled (P^t). Moreover, in each period a minimum exposed ore reserve F^t made available for the start of the next period must be guaranteed. For this, we will not allow every block to contribute to this minimum exposed reserve requirement, but only those above a certain cut-off grade λ_{cg}. In order to do this, it will be convenient to introduce the parameter

$$\bar{\lambda}_i = \begin{cases} \lambda_i & if\ \lambda_i \geq \lambda_{cg} \\ 0 & otherwise. \end{cases}$$

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The reason for doing this is to give the model flexibility to choose mineralization from waste, but to prevent it from using many very low-grade blocks to comply with the constraint, therefore defeating the purpose of the model.

Table I summarizes the indexes, sets, and parameters used in the IP model.

Three types of variables are used in the model, all of them binary. The first type is the variable associated to the extraction for processing purposes for each block:

$$x_i^t = \begin{cases} 1 & \text{if block } i \text{ is extracted and processed at period } t \\ 0 & \text{otherwise} \end{cases}$$

The second variable type describes the decision relating to the disposal of a block by sending it to the waste dump:

$$w_i^t = \begin{cases} 1 & \text{if block } i \text{ is extracted and sent to waste dump at } t \\ 0 & \text{otherwise} \end{cases}$$

The third variable type is used to identify exposed blocks; throughout the paper it will be called the 'visibility' or 'exposure' variable:

$$y_i^t = \begin{cases} 1 & \text{if block } i \text{ is exposed at period } t \\ 0 & \text{otherwise} \end{cases}$$

The objective function for the model is the usual maximization of net present value (NPV). The formulation of the mathematical model is as follows:

$$\text{(OPBSEO)} \quad max \sum_{i \in B} \sum_{t=1}^{T} [(b_i^t - p_i^t)x_i^t - m_i^t w_i^t] \qquad [1]$$

$$\sum_{t=1}^{T} (x_i^t + w_i^t) \leq 1 \quad \forall i \in B \qquad [2]$$

$$\sum_{i \in B} \tau_i(x_i^t + w_i^t) \leq M^t \quad \forall t \in \{1, \dots, T\} \qquad [3]$$

$$\sum_{i \in B} \tau_i x_i^t \leq P^t \quad \forall t \in \{1, \dots, T\} \qquad [4]$$

$$y_j^t + \sum_{s=1}^{t} (x_j^s + w_j^s) \leq \sum_{s=1}^{t} (x_i^s + w_i^s)$$
$$\forall (i,j) \in A, \ t \in \{1, \dots, T\} \qquad [5]$$

$$y_i^t \leq x_i^{t+1} \quad \forall i \in B, \ t \in \{1, \dots, T-1\} \qquad [6]$$

$$\sum_{i \in B} \tau_i \bar{\lambda}_i y_i^t \geq F^t \quad \forall t \in \{1, \dots, T-1\} \qquad [7]$$

$$x_i^t, w_i^t, y_i^t \in \{0,1\} \quad \forall i \in B, \ t \in \{1, \dots, T\} \qquad [8]$$

The objective function [1] represents the maximization of the cumulative discounted cash flow. Constraint [2] simply expresses that it is not possible to choose two different destinations for a block, *i.e.*, a block can be sent either to process or to the waste dump, but not to both at the same period. Constraints [3] and [4] establish an upper bound on mining and ore production for each period. Analogous constraints related to other capacities of the system could also be established (*e.g.* water, energy, *etc.*). Constraint [5] is the usual slope constraint of open pit planning models, but written in a manner consistent with the identification of blocks that can be declared exposed for the start of the next

period. Constraint [6] ensures that once exposed a block needs to be extracted and sent to the processing plant in the next period. Constraint [7] ensures that a minimum exposed reserve (in terms of units of extractable metal) must be available for the start of the next period (therefore, this is not imposed for period *T*). Finally, constraint [8] declares the nature of the variables involved in the model.

The model proposed here (which we name OPBSEO for 'open pit block scheduling with exposed ore reserve') contains #*B*×*T* new binary variables with respect to the standard open pit scheduling model (see, for example, CPIT in Espinoza *et al.* (2013)). The model has also #*B*×(*T*-1) additional constraints of type [6] and *T*-1 constraints of type [7]. While in this work we focus on the model and the properties of its solutions, it is worth noting that the problem not only has more variables and constraints, but the structure is harder to exploit. In the CPIT problem (and also PCPSP), the vast majority of constraints correspond to precedences and they involve only two variables, one with coefficient 1 and other with coefficient -1. This corresponds to flow conservation constraints in a MAXFLOW problem, which allows the use of adapted methods like the BZ algorithm (see Bienstock and Zuckerberg, 2010). This is not the case for OPBSEO, as the precedence constraints do not have this simple structure.

It is also interesting to point out than an alternative way to approach the issue of exposed ore reserves is to use the standard formulation but with shorter time periods, so there is more control of the production at each moment in time. This means duplicating or triplicating the number of periods in which to handle ore reserve, and therefore implies that the total number of variables grows in the same order of magnitude, and in addition, this may produce a large number of requirements in terms of ore at every period, leading to infeasibilities.

Computational experiments

In this section a description of the application of OPBSEO to some instances is provided. The aim of these experiments is

Table I

Main notation used in IP models

Symbol	Description
B	Set of blocks
A	Set of precedence arcs
T	Time horizon (number of periods)
b_i^t	Discounted profit resulting from the mining of block i at period t
p_i^t	Cost of mining and processing block i at period t
m_i^t	Cost of mining block i at period t
M_t	Maximum mining capacity for period t
P^t	Maximum processing capacity for period t
F^t	Minimum exposed ore reserve required at period t (as metal)
τ_i	Tonnage of block i
λ_i	Ore grade of block i
λ_{cg}	Cut-off grade to define minimum exposed ore reserve requirement

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essentially to evaluate the performance of the proposed model and compare it with the equivalent IP model, but without exposed ore reserve requirements in terms of extraction geometries, exposed mineralization, and (to a lesser extent) NPV. In the first subsection the implementation of the model on a hypothetical two-dimensional orebody is described. Then two more realistic three-dimensional instances (block models and parameters) are presented, and finally the experiments on these instances as case studies are described and compared.

Example: two-dimensional data-set

The instance considered here is a two-dimensional orebody (a slice of a three-dimensional deposit) that requires mining with a 45° slope angle. The block model contains 399 regular blocks, each of which has attributes such as tonnage and copper grade. Economic values associated with the extraction and destination (processing plant or waste dump) of the blocks are also given. The model decides the best destination for each block and defines exposed mineralization for each period while maximizing the NPV of the entire project. The planning horizon for this hypothetical case study is 3 years (considering annual periods) and the discount rate is set to 10%. Mining and processing capacities are fixed to a maximum of 4 Mt and 2.8 Mt per year, respectively. For each period, a minimum of 12 kt of exposed ore reserve is required and the minimum ore grade for a block to be considered exposed ore reserve is set to 0.3%.

The schedule obtained is shown in Figure 1. We can distinguish three important groups of blocks: unmined blocks, which are represented by code 0 (white); blocks mined in each period, encoded by numbers 1-2-3 (cyan, yellow, and brown, respectively); and exposed blocks within each period, in order to be mined and sent to processing at the next period (orange). An important aspect to highlight from the schedule is the geometry obtained at the bottom of the pit per period, which is quite satisfactory in terms of operational spaces.

Block models and parameters for two more realistic cases

The instances considered for this study were obtained from Minelib (Espinoza *et al.*, 2013), a publicly available library of test problem instances for open pit mining problems, some of which correspond to real-world mining projects, making it
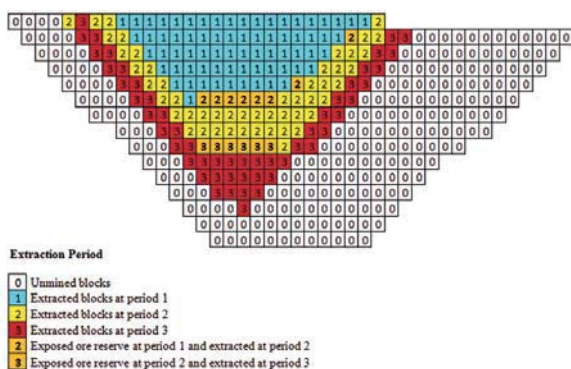
very interesting for comparison. In our case, we used the Newman1 and Marvin instances described below, because they were more appropriate for our study in terms of the size of the instances (as mentioned before, the resulting problem is quite complex and required long computational times).

Newman1

The block model contains 1060 regular blocks. For each block there are attributes such as rock type, tonnage, ore grade, and economic values associated the destination of the blocks (waste dump or processing plant). In this instance the wall slope requirements are not given by an angle as usual, but to remove a given block, five blocks above must be extracted.

Annual periods with a yearly discount rate equivalent to an 8% are considered. The planning horizon is 6 years, but capacities permit completing the exploitation in 3 years. Mining and processing capacities are fixed to a maximum of 2 Mt and 1.1 Mt per year, respectively. For each period, a minimum of 5 kt of exposed ore reserve is required (as metal, calculated as tonnage multiplied by ore grade) to ensure production in the first half of the period, and the minimum grade for a block to be considered exposed ore reserve is set to 0.3%.

Marvin

This block model contains 53 271 blocks of $30 \times 30 \times 30$ m, but this can be reduced when blocks that are not accessible are removed from the block model. The wall slope requirements are given by a 45° slope angle and by using seven levels of precedence above a given block. This deposit contains two metals of interest, copper and gold, and for each block there are attributes such as tonnage and grade. In order to consider a single-element deposit instead of multi-element one, we use a copper equivalent grade:

$$EG_{cu} = \lambda^{cu} + \lambda^{au} \cdot \frac{P_{au} \cdot R_{au}}{P_{cu} \cdot R_{cu}}$$

where EG_{cu} is the copper equivalent grade, λ^{cu} and λ^{au} are the copper and gold grades, P_{cu} and P_{au} are the copper and gold prices, and R_{cu} and R_{au} are the copper and gold recoveries, respectively. The model contains some economic parameters that are used to obtain more realistic economic values for each block. The block value was obtained according the following expression:

$$EV_i = [(P - C_s) \cdot f \cdot R \cdot \lambda_i - C_m^{ref} \cdot C_m^{caf} - C_p] \cdot \tau_i$$

where EV_i is the economic value of block i and P is the price of the element of interest. Coefficients C_s, C_m^{ref}, and C_p are the selling, reference mining, and processing costs, respectively. The term C_m^{caf} is the block mining cost adjustment factor associated with the position (depth) of the block and f is an appropriated unit conversion factor. R is the metallurgical recovery, λ_i is the equivalent ore grade, and τ_i is the tonnage of block i. We consider annual periods with a yearly discount rate of 10%, and the planning horizon is 7 years. Mining and processing capacities are fixed to a maximum of 60 Mt and 20 Mt per year, respectively. For each period, a minimum of 100 kt of exposed ore reserve (as metal) is required to ensure the first 4 months of production, and the minimum grade in

Extraction Period

0	Unmined blocks
1	Extracted blocks at period 1
2	Extracted blocks at period 2
3	Extracted blocks at period 3
2	Exposed ore reserve at period 1 and extracted at period 2
3	Exposed ore reserve at period 2 and extracted at period 3

Figure 1—Block scheduling obtained through our exposed ore reserve model

Optimizing open-pit block scheduling with exposed ore reserve

order to a block for be considered exposed ore reserve is set to 0.25%. Table II summarizes the main economic and technical parameters.

Implementation/instance

In this subsection further detail is provided about the implementation of the exposed ore reserve model and other strategy to compare them. In the following cases no operational spatial constraints were considered. The following cases were implemented:

➤ Open pit block scheduling model with exposed ore reserve OPBSEO as detailed previously.
   This is the main experiment in the article. The objective is to analyse the block schedule obtained in terms of extraction geometry, exposed ore reserve, and NPV. Newman1 and Marvin cases were implemented using the parameters explained in the previous subsection.
➤ Open pit block scheduling model without exposed mineralization, that is, OPBSEO but without binary variable y_i^t and without constraints [6] and [7]. This model is denoted as OPBS.

OPBS considers precedence constraints and limited mining and processing capacities only. Newman1 and Marvin instances were implemented using the parameters (if corresponding) detailed in the previous subsection. The comparison between OPBS and OPBSEO models is interesting because it allows the evaluation of the insertion within the same formulation of the exposed ore reserve concept.

In order to implement cases (a) and (b), PuLP was used (see Mitchell, 2009), which is a free open-source software written in Python that allows optimization problems to be set and solved with different optimization engines. In our case, we used GUROBI version 5.6.0 to solve the resulting IP models. Integer instances are solved up to a maximum 5% gap.

In all cases the resolution of the instances was performed on an Intel Core i5-3570 CPU machine with 16 GB running Windows XP version 2003. This machine has one processor with four cores, and is clocked at 3.4 GHz.

Results and discussion

Our interest is to evaluate the introduction of a new model with exposed ore reserve requirements, comparing the results obtained from OPBSEO and OPBS models, in terms of production plans, but also in terms of geometries and NPV. The results and discussion for two case studies, named Newman1 and Marvin, are presented.

Newman1 case study

The pits obtained when scheduling the Newman1 case are shown in Figure 2, which presents XZ and YZ section views for both schedules, OPBS (left) and OPBSEO (right). The colours correspond to the periods at which the blocks are extracted. There are three extraction periods (cyan, yellow, and brown), unextracted blocks appear in dark blue.

First notice that this data-set has a very special shape. It is not a 'box full of blocks', but it consists of different disjoint parts. Also, the slopes at the borders are very steep. This is a property of the data-set as available in Minelib, and has nothing to do with the schedules. The most interesting

property regarding the obtained geometries is that those with exposed ore reserve are better in terms of operational spaces and regularity. They are closer to a *worst case* and therefore suffer from fewer operational problems.

The production plans for the schedules are presented in Table III, which contains the following information: *period* indicates the year of the production, *grade* is the average grade of processed blocks, *ore tons* and *total tons* are respectively the processed and extracted material per period (in tons). Finally, exposed tons is the material exposed in that period and made available for the next period (in metal tons). The production plans are similar and both saturate the mining capacity, but the grades and exposed tonnages differ significantly. Indeed, OPBSEO extracts 9% less ore tonnage, but ensures that there is sufficient exposed ore for at least 6 months at the beginning of each period.

Regarding the NPVs (see Table IV), the block schedule computed using OPBS extracts blocks with higher grades as soon as possible, but OPBSEO has to deal with additional constraints of exposed ore reserves, which forces it to delay some high-grade blocks for future extraction. Still, the difference in NPVs is only 4% in favour of OPBS.

Marvin case study

The pits obtained when scheduling the Marvin case are presented in Figure 3, showing a YZ-section view and a horizontal plane for both schedules, obtained from OPBS (left) and OPBSEO (right). The colours correspond to the periods at which the blocks are extracted.

First of all, there is a large difference between the extraction geometries; it is worth noting that for this case, the pits obtained using the OPBSEO model are more operational than those obtained using OPBS.

Both models aim to maximize NPV and therefore to extract blocks with higher grades as soon as possible, but OPBSEO must ensure a minimum exposed ore reserve at the beginning of each period, allowing a large horizontal surface in the bottom of each pit.

Production plans obtained from models OPBS and OPBSEO are shown in Table V, where the impact of the exposed ore reserve constraints appears in terms of ore and

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Table II

Economic and technical parameters in the Marvin instance

Parameter	Value	Unit
Copper price	3.02	US$/lb
Gold price	1132	US$/oz
Copper recovery	0.88	-
Gold recovery	0.60	-
Selling cost	0.60	US$/lb
Processing cost	10	US$/ton
Reference mining cost	1.8	US$/ton
Increment mining cost	0.002	US$/ton · m
Slope angle	45	degrees
Time horizon	7	Years
Discount rate	0.10	-
Mining capacity	60 000 000	t/a
Processing capacity	20 000 000	t/a
Minimum exposed ore (as metal)	100 000	t/a
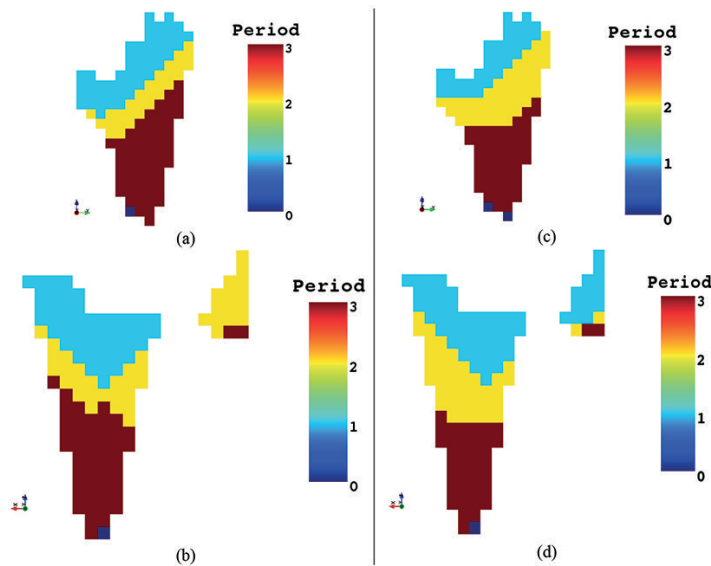
Figure 2—Block schedules for the Newman1 data-set. On the left, geometries obtained using the standard OPBS model: (a) YZ-section view, (b) XZ-section view. On the right, geometries obtained using the exposed ore reserve model OPBSEO: (c) YZ-section view and (d) XZ-section view. Colours show extraction periods

Table III

Comparison of production plans for the Newman1 data-set

Period	OPBS				OPBSEO			
	Grade %	Ore tons	Total tons	Exposed tons	Grade %	Ore tons	Total tons	Exposed tons
1	1.77	785 471	1 999 937	475	1.44	729 864	1 998 338	5 005
2	1.48	1 099 749	1 973 150	424	1.74	903 975	1 999 944	5 014
3	1.70	1 094 495	1 620 287	-	1.77	1 099 498	1 541 290	-
Total		2 979 715	5 593 374	899		2 733 337	5 539 572	10 019

total tonnages, grade profile over time, and exposed tonnage (as metal) per period, as in Table III. It is interesting to observe that there are no differences in terms of ore tonnage, but the OPBSEO model reported 11% less total tonnage than the solution obtained using the OPBS model (recall that in this case the model must ensure at least 4 months' supply of exposed ore at the beginning of each period), but about the same amount of ore (the difference is less than 0.1%).

Table VI shows the discounted values for both models. The results are similar to the other example: OPBS is able to provide a schedule with higher value by extracting high-grade blocks as soon as possible, while OPBSEO delays some of these blocks to ensure minimum exposed ore reserves in order to keep ore available for future exposure. The difference in NPVs in this case is about 5%, but we observe that the solutions are optimal only within the time horizon considered here. Indeed, the production plans suggest that additional periods may lead to different solutions and higher NPVs.

Conclusions and open questions

In this paper, the concept of exposed ore reserve is introduced in a mathematical model and defined at a given period as the

Table IV

Cumulative discounted values for the Newman1 data-set

Period	Discounted value	
	OPBS	OPBSEO
1	7 616 322	4 086 113
2	6 655 602	7 767 913
3	7 987 006	9 436 000
Total NPV	22 258 930	21 290 026

set of blocks for which all its preceding blocks have been already extracted, but not the block itself. A new integer programming model, named OPBSEO, to generate block schedules under exposed ore reserve requirements has been presented and tested in a set of standard instances.

First, the model was tested on a two-dimensional instance to validate the solutions that it provided, with satisfactory results. The model was then tested on two realistic instances, providing comparisons between the solutions from the OPBSEO model and the equivalent one, but without exposed ore requirement, named OPBS. The

Optimizing open-pit block scheduling with exposed ore reserve
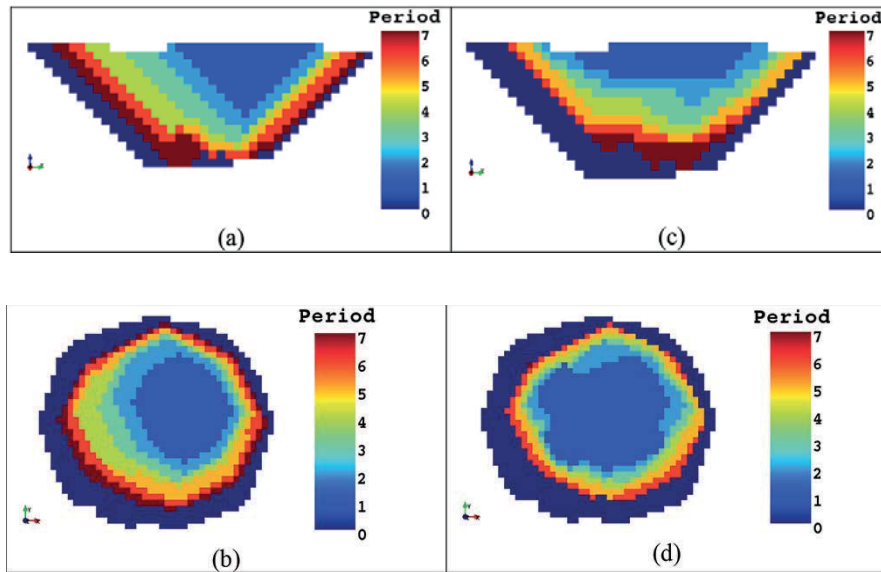
Figure 3—Block schedules for the Marvin data-set. On the left, geometries obtained using the standard OPBS model: (a) X-section view, (b) XY-horizontal plane. On the right, geometries obtained using the exposed ore reserve model OPBSEO: (c) YZ-section view and (d) XY-horizontal plane

Table V

Comparison of production plans for the Marvin case study

Period	OPBS				OPBSEO			
	Grade %	Ore tons	Total tons	Exposed tons	Grade %	Ore tons	Total tons	Exposed tons
1	1.03	19 994 670	59 989 965	0	0.72	19 999 638	59 891 451	100 134
2	1.14	19 971 010	49 490 568	359	0.92	19 966 980	45 939 352	100 055
3	1.05	19 980 880	40 052 602	1 462	1.14	19 985 370	39 798 429	100 026
4	1.02	19 985 340	44 985 249	4 719	1.09	19 987 840	42 930 865	100 099
5	0.97	19 999 920	49 166 274	7 569	1.14	19 990 580	52 232 416	100 362
6	0.92	19 999 290	52 191 380	6 254	1.14	19 992 410	46 784 718	100 159
7	0.88	15 535 850	46 060 989	-	1.47	15 516 740	16 587 549	-
Total		135 466 960	341 937 027	20 363		135 439 558	304 164 781	600 835

Table VI

Cumulative discounted values for the Marvin case study

Period	Discounted value	
	OPBS	OPBSEO
1	582 605 720	329 142 653
2	629 312 551	463 833 658
3	523 828 567	586 467 752
4	444 162 523	494 561 419
5	371 571 924	398 567 122
6	304 876 581	339 961 783
7	191 140 416	273 394 271
Total NPV	3 047 498 282	2 885 928 658

solutions obtained with the aid of the proposed model are consistent with those obtained by other optimization models for mine planning that do not have exposed ore reserve requirements in their formulation.

The new model, compared with similar optimization models without the exposed ore reserve requirement, suffers from a small reduction in the value of the solution due to the additional requirements included. However, the geometrical nature of the new solutions obtained in the tests performed exhibits better operational spaces and regularity. While it is not possible to generalize this property of the solutions to other instances, it is believed that the model has the potential to produce results that are better suited to the type of operational spacing needed in mining operations, thus adding a valuable alternative to the current tools used by mine planners. Besides, this new requirement ensures the quantity of metal produced from the beginning of each period in the schedule, making compliance with medium- and short-term mine planning stages easier.

A potential research path to pursue in the future relates to the definition of the exposed ore reserve. In the present version we only consider a minimum amount of metal readily available at the beginning of every period, but other metal components could also be considered, expressed in terms of tonnage, grades, or economic value; or this requirement could

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be relaxed, for example, to consider blocks at a maximum distance from surface that is larger than only one bench. Other important questions for future study relate to assessing the suitability of the model for different types of deposits and mining operations, and which conditions are required in order to produce solutions with the geometrical properties that have been observed in the present study.

The proposed model exhibits great potential in terms of applicability, but further algorithmic research is required to improve the computational execution time in the case of very large instances. However, it is important to mention here that the present paper has focused on introducing the concept and presenting the associated model. As the results obtained in the instances tested are encouraging enough to warrant additional research, it is believed that the first step taken in this study will serve as a solid foundation for new ways of thinking about mine planning, to the benefit of industry and practitioners.

Acknowledgments

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Increasing the value and feasibility of open pit plans by integrating the mining system into the planning process

by N. Morales* and P. Reyes†

Synopsis

We present a model that allows us to consider mine production scheduling coupled with the mining system at different levels of detail: from the standard origin-destination approach to a network considering different processing paths. Each of these is characterized by variable costs, capacities, and geometallurgical constraints.

We then apply this model to a real mine, comparing the results with those obtained by traditional methodology: the destination of materials defined a *priori*, before computing the schedules, using standard criteria like cut-off grades.

As expected, using optimization to schedule and define dynamically the best processing alternatives shows a big opportunity for potential value improvement. However, the main result is that using only origin-destination and fixed cut-off grades may produce schedules that are not feasible when the actual constraints of the mining system are taken into account. Therefore, it is essential to include the considerations proposed in the planning process.

Keywords
mine planning, optimization, open pit, cheduling, multi-destination, mining system.

Introduction

Mine planning is defined as the process of mining engineering that transforms the mineral resource into the best productive business. A central point of this process is the *production plan*, which is a bankable document that sets the production goals over the planning horizon (short, medium, or long) (Rubio, 2006). In turn, this production plan is supported by a *production scheduling* that indicates which part of the resource must be extracted in each period and what to do with these portions of the resource in order to reach the production goals indicated in the production plan.

In order to deliver a feasible production scheduling, the mine planning process must deal with complex issues like operational and metallurgical limitations, slope angles, stock handling, design, mining system selection, fleet considerations, *etc.* This means that it is not possible to construct the production scheduling in a single step considering all the elements involved, but that the process is split into different stages, which depending on the time horizon and level of decision, rely on different levels of information and must comply with different levels of precision and detail.

Among the relevant aspects that are traditionally left out by optimization tools for mine planning, it is possible to mention (Espinoza *et al.*, 2013): optimal mine design of phases, roads, and operational space; equipment considerations like location, capabilities, and optimal fleet size; optimal processing capabilities; inventory management; and stochastic data. As the planning horizon decreases from long-term to shorter periods, the length of this list increases as it must comply with additional considerations of more detailed decision levels (see, for example, Newman *et al.*, (2010) for a more detailed presentation of the different decisions levels).

One example of the increasing complexity of the information and planning constraints that we address in the model used in this study is that long-term production scheduling is constructed without taking into account the variability of the ore content within a phase bench. Indeed, in long-term mine planning, large portions of material (called bench phases, see Figure 1 for a graphical example) are scheduled assuming a homogeneous distribution of materials within the bench. Indeed, the actual attributes of the rock (like grades or hardness) change within the bench and therefore, the availability of ore for processing is a function of the schedule within the bench, which in turn is limited, for example, by the location of the ramps and the type of equipment used in the mine.

* *DELPHOS Mine Planning Lab., Mining Engineering Department & Advanced Mining Technology Center, University of Chile.*
† *Technological Institute for Industrial Mathematics (ITMATI), University of Santiago de Compostela, Santiago de Compostela, Spain.* (Research was conducted while visiting DELPHOS Mine Planning Lab., Mining Engineering Department & Advanced Mining Technology Center, University of Chile.)

Increasing the value and feasibilitly of open pit plans

Figure 1—Phase benches and access ramps in an open-pit mine

A result of the example described above is that constructing medium- and short- term production scheduling can be very difficult, because the planner must comply with production goals set in longer-term decisions which are based on less restrictive constraints. Furthermore, there are very few computational optimizing tools to aid the short-term planner, so the process is a manual -trial-and- error procedure with an important time investment that consists of finding a feasible scheduling, leaving little room for optimality in terms of recoverable metal, fleet utilization, and reserves consumption among other items. Hence, at the end of the day, the short-term plans are unfeasible (within the parts of the mine scheduled for production) or the costs are higher than the optimal.

The approach that we propose aims to advance the solution of this and other issues related to short-term mine planning. For this, we integrate considerations related to the mine, but also elements related to the mining system downstream.

Mining system considerations

The standard way of looking at the mining system from a mine scheduling point of view can be illustrated as in Figure 2, in which material is evaluated in terms of the net profit with regard to a certain process choice. For example, the value of a block sent to the plant is different from the value of the same block sent to the waste dump. In fact, in long-term mine planning, this potential choice is reduced further on by using cut-off grades. Indeed, the actual destination of a block is chosen before scheduling simply by assuming that it will be sent to the most profitable process (plant or waste).

In this paper, we develop a different approach that aims for a more balanced (and hopefully more realistic) view of the mining system. We understand the mine planning scheduling from a *pull* perspective in which schedules are developed to maximize value and, therefore, must comply with consider-ations in terms of capacities, recoveries, and ultimately value.

Our approach considers that there are many *processing alternatives* or *processing paths*. For example, as presented in Figure 3, an extracted block can go straight to a waste dump, or it can be sent to either of the two available crushers. In the latter case, the possibilities depend on the transportation system (whether the connection exists and its capacity) as well as the processing limitations of each facility (type of material, grades, *etc*).

The modelling we propose assigns material coming from the mine to the possible processing path. Depending on this decision, the corresponding material will have a certain economic value, but it will use certain processing resources like crushing time or transportation along the processing path.

For example, if one portion of material from a mine is assigned the highlighted path in Figure 3, then its actual economic value will be:

$$V = -MiningCost - TranspCostMine1ToCrusher1 - CrushingCost1 - TranspCostCrusher1ToMill1 - MillingCost1 - TranspCostMill1ToPlant2 - ProcessingCost Plant2 + IncomePlant2.$$

Certainly, the economic value in this case is different from that for any other path, and thus the optimal value will depend on the processing path choices.

Notice that the selection of the best processing path is not only dependent on the material itself, as assumed when using cut-offs to make such decisions. Indeed, the best processing path changes at each moment depending on the available processing capacities or the blending properties that impact the recovery at each plant. To account for these elements, we consider that:

➤ At each node in the mining system, a certain (possibly limited) number of resources are used. In this case, the resource is shared by all the paths going through the node. For example, in Figure 3 the total milling capacity at *Mill1* is shared by the material coming from either *Crusher1* or *Crusher2*

➤ At each node in the mining system there may be blending constraints that limit, for example, the maximum content of certain pollutant. This constraint is applicable mostly at the plant nodes.

Mine considerations

Mine considerations are among the most studied in the literature on open pit scheduling. They refer to the techno-
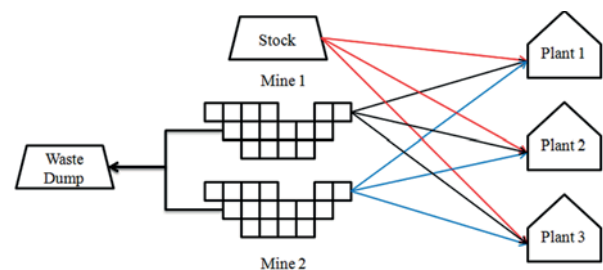
Figure 2—Standard simplified version of the mining system
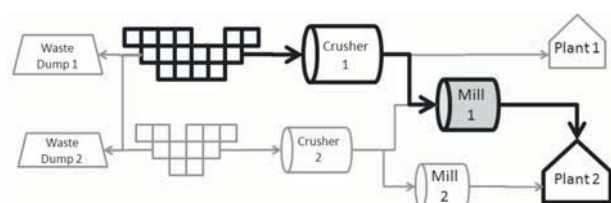
Figure 3—An example of a mining system with multiple mines, and processing alternatives or paths

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logical aspects needed to comply with a certain slope angle required for the pit. Furthermore, they also refer to overall operation size, which translates into mining capacities.

In terms of the mine, we consider that the material is discretized into mining reserve units (MRUs) (we use the term reserve as the MRUs are already scheduled for extraction in the long-term plans). Each MRU may correspond to a block in the original block model or it can represent other structures, either larger (drilling polygons) or smaller (shovel buckets) ones. This depends on long-term schedules. Notice that the actual set of available MRUs for scheduling is an input for the model.

For each MRU, there are some attributes that must be considered in the mining system constraints. For example, nodes could have capacity constraints. In that case, those MRUs will have a tonnage attribute stating the maximum tonnage allowed.

MRUs are also related by precedences referred to the slope angle or accessibility constraints. Slope precedence constraints relate to the fact that in an open pit mine there is a minimum angle to ensure the stability of the pit walls, hence it is not possible to excavate vertically as much as desired. Accessibility precedences relate to the fact that in the short term, there are already ramps constructed, and extraction will start in those ramps and will be propagated through the bench. This is, indeed, overlooked in long-term planning because, as explained previously, scheduling is done at the bench-phase level of aggregation.

Related work

The strategic decision of determining the optimum final pit has been effectively treated using the Lerchs-Grossman algorithm (Lerchs and Grossman, 1965) or the Picard flow networks method (Picard, 1976; Cacceta and Giannini, 1986). These methods are based on a block model that characterizes a mineralized body only in terms of the economic value of each block and the slope precedence constraints. As a result, these models are very aggregated and ignore key elements like mining and processing capacities or the possibility of selecting the optimal block destination in a dynamic setting.

The problem of generating a production schedule in open pit mines has also been studied at different levels of detail. For example, Johnson (1968) introduces a mixed integer problem for scheduling under capacity, slope angle, and grade constraints. The mathematical formulation shares several elements with ours (processing and mining capacity, grade control, multiple periods and destinations, slope controls), but the focus is quite different. Johnson (1968) focuses on strategic planning and introducing the mathematical formulation. We are more interested in the impact of more complex models on the planning results.

The problem introduced by Johnson (1968) and variations of that method have been widely studied, because it has proven to be very difficult to solve in practical cases due to the size of the problem. For example, some papers that provide techniques to speed the resolution of the model are those by Cacceta and Giannini (1986, 1988), and more recently Gaupp (2008), Cullenbine, Wood, and Newman (2011), Bienstock and Zuckerberg (2010), Chicoisne *et al.* (2012); and others that propose heuristics are Cacceta,

Kelsey, and Giannini (1998), Gershon (1983, 1987), or Dagdelen and Johnson (1986) and Whittle Programming (1998) for parametric methods. Nevertheless, all these studies have a different focus than ours, as they concentrate on the resolution of the problem.

Works that are closer to ours are Kumral (2012, 2015), which schedule blocks under slope constraints and capacity constraints (with upper and lower bounds) and where the model decides the final destination of the blocks. Kumral (2014) also considers bounds on the grades sent to the plant and uses conditional simulations to incorporate ore grade variability and compares the results using a *priori* cut-off grades (which is the standard procedure in mine planning) rather than allowing the model to decide such destinations. Kumral (2015) includes constraints to reduce production variation between consecutive periods. Both papers are, nevertheless, oriented to long-term scheduling, do not consider the short-term accessibility constraints included in this work, and focus on block destination and not alternative processing paths.

Regarding open pit block scheduling for the short term, Smith (1988) poses a mixed integer programming model that is responsible for block extraction scheduling in the short term, with the objective of maximizing the production of the material of interest, subject to certain constrains on blending, while ensuring a simple scheme of horizontal and vertical constrains without considering the presence of stocks. In the same line, Morales and Rubio (2010) present a model to maximize the production of metal in a copper mine, subject to certain geometallurgical constraints. Eizavy and Askari-Nasab (2012) present aggregation techniques to help solving a mixed integer model for short-term open pit mining. The model considered there includes, as ours does, stocks, blending, and capacity constraints, but with fixed block destinations and directional constraints in bench accessibility, which is very different from our approach.

Mathematical modelling

In this section we present the main notation and the mathematical formulation of the optimization model used in this work. A brief introduction to the mathematical model follows, and then a discussion about its use, limits, and potential extensions for real applications.

Notation

To begin, we consider a set of MRUs $B = \{1, 2, \ldots, N\}$. Time is divided in T time periods, given in advance, hence the production is scheduled in periods $t = 1, 2, \ldots, T$. Nevertheless, we do not assume that the periods have the same length.

There exists a set of processing paths D and we denote by $d \in D$ the potential processing paths in the mining system. In this way, the economic value perceived if MRU i is sent through processing path $d \in D$ is denoted as $v(i, d)$.

While the case study is short-term, we still consider that value of money over time is adjusted using an discount factor $\rho(t)$ so that $\rho(t)$ is the present value of a dollar perceived during time period t. For example, if the time periods had the same length (a year) and the yearly discount rate is α, then we would have that $\rho(t) = \frac{1}{(1+\alpha)^t}$.

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We consider two sets of attributes A and $\bar{A}$. A refers to the block attributes that participate in capacity constraints like tonnage or processing times. $\bar{A}$ relates to the attributes that are averaged, like grades or pollutant contents. The value of attribute $a \in A$ (or $a \in \bar{A}$) in MRU i is denoted by $g(i, a)$.

We model the precedence constraints as a set of arcs $E \subset B \times B$, where $(i, j) \in E$ means that MRU j has to be extracted before MRU i. (We provide extensive examples of this in the case study.)

We consider a set of pairs $N = \{(a_\ell, D_\ell)\}_{\ell=1}^{n}$ where $a_\ell \in A$ and $D_\ell \subset D$. For any $(a, D) \in N$, and for each $t = 1, 2, \ldots, T$ we are given a minimum capacity (thus a demand) $M^-(a, D, t) \in \mathbb{R} \cup \{-\infty\}$ and and maximum capacity $M^+(a, D, t) \in \mathbb{R} \cup \{\infty\}$. Similarly, we have a second set of pairs $\tilde{N} = \{(a_\ell, D_\ell)\}_{\ell=1}^{n}$ where $a_\ell \in \tilde{A}$ and $D_\ell \subset D$. For any $(a, D) \in \tilde{N}$, and for each $t = 1, 2, \ldots, T$ we are given minimum and maximum average allowed values $B^-(a, D, t) \in \mathbb{R} \cup \{-\infty\}$ and $B^+(a, D, t) \in \mathbb{R} \cup \{\infty\}$, respectively. For simplicity, we assume that attribute ton (i), the tonnage of MRU i, is the average of the weights. Extending the model to arbitrary weights is trivial.

The idea of having lower and upper bounds defined on sets of processing paths instead of bounds to each of the processing paths is that, as mentioned in the Introduction, the associated constraints aim to model limits at the nodes and, therefore, they will apply over the set of paths that go through them.

The model also considers a set S of stocks. Each stock can be seen as a MRU in the sense that all attributes $a \in A$ have to be defined and we denote $h(s, a)$ the value of the attribute $a \in A$ for *one ton* of stock $s \in S$. We define $h(s, \tilde{a})$ analogously for $\tilde{a} \in \tilde{A}$. Finally, we consider $u(s, d)$ to be the value of sending *one ton* of stock s through processing path $d \in D$.

Note that the above definitions of value and attributes of stocks are per ton of material. This is because while stocks are treated similarly to MRUs, they are also different as they are not subject to precedence constrains. Moreover, it is possible to extract fractions of them and to send different fractions through different processing paths.

Given the above, in this model we use stocks only as possible sources of mineral, and not possible destinations. This will be discussed further.

A summary of this notation can be found in Table I

Mathematical formulation

This subsection introduces the decision variables, the objective function, and the constraints expressed in mathematical terms.

The decision variables are related to the decision whether to mine a given MRU, when to do so, and what processing path is chosen for that MRU; and similarly for material at the stockpiles. The constraints considered are those described above: capacity, blending, and precedence. Other constraints include scheduling constraints and constraints related to the nature of the variables.

Variables

We consider, for each MRU $i \in B$, processing path $d \in D$, and period $t = 1, 2, \ldots, T$, the variable

$$x_{idt} = \begin{cases} 1 & \text{MRU } i \text{ is sent through processing path } d \text{ at period } t, \\ 0 & \text{otherwise;} \end{cases} \quad [1]$$

Similarly, for processing path $d \in D$, period $t = 1, 2, \ldots, T$ and stock $s \in S$:

$$y_{sdt} = \text{tonnage sent from stock } s \text{ through processing path } d \text{ at period } t. \quad [2]$$

Objective function

The goal function considers the overall gain obtained from extracting and processing the MRUs, discounted over the planning horizon:

$$F = \sum_{t=1}^{T} \rho(t) \sum_{d \in \mathcal{D}} \left(\sum_{i \in \mathcal{B}} v(i, d)\, x_{idt} + \sum_{s \in \mathcal{S}} u(s, d)\, y_{sdt} \right) \quad [3]$$

Constraints

Finite mass—The following constraint simply ensures that each MRU is processed at most once and sent through at most one processing path. For all MRU $i \in B$:

$$\sum_{t=1}^{T} \sum_{d \in \mathcal{D}} x_{idt} \leq 1. \quad [4]$$

Similarly, for each stock $s \in S$.

$$\sum_{t=1}^{T} \sum_{d \in \mathcal{D}} y_{sdt} \leq \text{ton}(s). \quad [5]$$

where ton(i) is the weight (in tonnage) of stock $s \in S$.
Precedence constraints—As described before, these constraints are encoded in sets of arcs. They read, for each $t = 1, 2, \ldots, T$ and for $(i, j) \in E$:

Table I

Notation for the mathematical formula

Symbol	Description
B	Set of MRUs
T	Number of periods
A	Set of capacity attributes
$\tilde{A}$	Set of blending attributes
D	Set of processing paths
S	Set of stocks
E	Set of precedence arcs
i, j	MRUs, elements of B
t	Time period, $t \in \{1, 2, \ldots, T\}$
$v(i, d)$	Economic value perceived if processing path $d \in D$ is chosen for MRU $i \in B$
$g(i, a)$	Value of attribute $a \in A$ (or $\tilde{A}$) of MRU $i \in B$
$M^-(a, D, t)$ $M^+(a, D, t)$	Minimum and maximum capacities for attribute for processing path $d \in D \subset D$ at period $t \in \{1, 2, \ldots, T\}$
$B^-(a, D, t)$ $B^+(a, D, t)$	Minimum and maximum average values for attribute for processing paths $d \in D \subset D$ at period $t \in \{1, 2, \ldots, T\}$
$\rho(t)$	Update factor for period $t \in \{1, 2, \ldots, T\}$
$u(s, d)$	Economic value perceived if one ton of stock $s \in S$ is sent through processing path $d \in D$
$h(s, a)$	Value of attribute $a \in A$ (or $\tilde{A}$) of stock $s \in S$

$$\sum_{t' \le t} \sum_{d \in \mathcal{D}} x_{idt'} \le \sum_{t' \le t} \sum_{d \in \mathcal{D}} x_{jdt'} \quad . \tag{6}$$

Capacity—For any time period $t = 1, \ldots, T$ and $(a, D) \in N$, we have that:

$$M^-(a, D, t) \le \sum_{i \in \mathcal{B}} \sum_{d \in D} g(i, a) \, x_{idt} +$$

$$\sum_{s \in \mathcal{S}} \sum_{d \in D} h(s, a) \, y_{sdt} \le M^+(a, D, t). \tag{7}$$

Blending—For any time period $t = 1, \ldots, T$ and $(a, D) \in \tilde{N}$, we have:

$$B^-(a, D, t) \le \frac{\sum_{d \in D} \left(\sum_{i \in \mathcal{B}} g(i, a) \mathrm{ton}(i) x_{idt} + \sum_{s \in \mathcal{S}} h(s, a) y_{sdt} \right)}{\sum_{d \in D} \left(\sum_{i \in \mathcal{B}} \mathrm{ton}(i) x_{idt} + \sum_{s \in \mathcal{S}} y_{sdt} \right)} \le B^+(a, D, t). \tag{8}$$

In the actual formulation of the problem, we transform these constraints to their equivalent linear representation.

Scheduling constraints—We also consider several constraints that allow fixing or limiting the schedule of the MRUs. This is useful, for example, to construct heuristics for resolution of the problem or to compare the solutions obtained with other approaches (see later for examples). For this, we consider for each MRU i, time periods $\tau^-(i)$, $\tau^+(i)$ $\{1, 2, \ldots, T\}$ where MRU i cannot be extracted before $\tau^-(u)$ and has to be extracted by period $\tau^+(u)$.

These constraints read:

$$x_{idt} = 0 \quad (\forall t < \tau^-(u)), \tag{9}$$

and

$$\sum_{d \in \mathcal{D}} x_{idt} = 1 \quad (\forall t \ge \tau^+(u)). \tag{10}$$

Additional comments on the model

Capacity or blending constraints that affect specific material sources

The above formulation can be used, for example, to impose capacity or blending constraints that apply only to a certain set $\mathcal{B} \subset \mathcal{B}$ of MRUs. For this, it suffices to create an attribute $\bar{a}$ such that $g(i, \bar{a}) = 0$ if MRU $i \notin \mathcal{B}$. In this way, the MRUs outside B do not participate in the capacity or blending constraints. An example of this is the simulating the assignment of loading equipment to certain parts of the mine (see below for more details). The same technique can be used, for example, to impose capacities on the stocks.

Fixing or limiting MRU destinations or processing paths

Similarly to the example above, by creating dummy attributes, it is possible to place a restriction such that MRUs in a given set $\mathcal{B} \subset \mathcal{B}$ (and/or from given stocks) are not sent through a certain processing path $d \in \mathcal{D}$.

Using the above two techniques it is possible preset the processing paths for certain MRUs or, for example, to fix cut-off grades for a given process path, or to force the condition that the material of a given stock follows some specific path.

Bench-by-bench extraction and minimum and maximum lead constraints

Precedence constraints can be used, for example, to model bench-by-bench extraction. This means that, within each phase, it is not possible to start the extraction of a lower bench unless the one immediately overlying it has been completely mined. For this, it suffices to mark as predecessors all the MRUs located in the bench phase immediately above a given one.

Another type of constraint that can be modelled using precedence constraints is related to the minimum/maximum distance (in benches) that must/can exists between two contiguous phases in the mine. For example, if we consider a minimum lead of three benches between hypothetical phases 1 and 2, this can be represented as imposing the requirement that MRUs in bench 1 of phase 2 are predecessors of MRUs in bench 4 of phase 1.

Optimizing equipment assignment

The model presented does not consider the capacities of the shovels or transportation fleet explicitly. Therefore, it cannot be used, in a direct manner, to optimize the assignment of loading equipment to different sectors of the mine over time. However, assignments of loading equipment can be modelled as capacities to specific parts of the mine or time periods (as described previously). Conversely, given a certain assignment of the loading equipment, it is possible to model it by considering capacities in specific parts of the mine, at specific time periods. This approach allows us to use the model to optimize loading fleet assignments by looking at different potential scenarios.

Modelling of stocks

In general, it is difficult to model stocks, because the attributes of the stocks (grades, tonnage) change with the schedule being computed. For example, one possible (but extreme) model consists of assuming that the grade of the stock is the average grade of its current content. This is highly nonlinear assumption, and thus difficult to solve. Even worse, it is not realistic, because mixing is not homogeneous, thus the stock ends up with layers of different grade.

In the case of the model proposed in this paper it is possible to consider stocks as an input, as described before, but also as a possible destination for the MRUs. The actual scheduled process could then be split into several small stages: no mixing in the stocks occurs for a number of periods, then the stocks are updated accordingly to the material sent to them, and the scheduling process continues.

Other objective functions

An advantage of using a mathematical formulation of the problem is the fact that the objective function is very flexible. For example, it suffices to set $\rho(t) = 1$ in case of undiscounted cash flows. Moreover, the economic value could refer only to costs that need to be minimized. Further on, considering the fact that different processing paths may lead to different metallurgical recoveries, it allows consideration of the case of maximizing the net mineral production.

Alternative formulations

Depending on the choice of variables, it is possible to write equivalent formulations of the problem above that may be better suited for computational purposes. For example, it is

possible to use a by-formulation of the variables that allows the use of efficient solving schemes based on relaxation of certain constraints. A more detailed discussion on these topics can be found in in Espinoza *et al.* (2013).

Numerical experiences

In this section we introduce the case study used for applying the model described in the previous sections, as well as the experiments that were carried out.

Case study

For the case study, and in order to keep actual data confidential, we show only the information that is relevant for presenting the results. We have also normalized values like production tonnage, as described later.

The case study is of a porphyry copper mining complex consisting of two open pits or sub-mines, Mine1 and Mine2. The actual mining system consists of several crushers (including some in-pit), conveyor belts, intermediate stockpiles, and uses trucks for transportation. There are three different final processes, in some cases with more than one location each. Each block has more than 20 different processing paths.

All the alternatives mentioned previously are included in the modelling of the problem and its resolution, but for the sake of simplicity and to keep information confidential, we report for an aggregated system as shown in Figure 4. Note that standard planning procedures work at this level of aggregation, therefore it is also useful for comparison purposes.

In our case study, the most profitable plant is Plant1, but it is also the one with more geometallurgical constraints. Plant2 and Plant3 have lower mineral recoveries and take longer to process the ore. Indeed, although no discount rate is applied between different periods in the planning, the economic values of MRUs sent to Plant2 and Plant3 are penalized according to the time required to produce the final product.

The planning horizon consists of a trimester, split into 12 periods with different lengths (from 4 days to 11 days). See, for example, Table II for details.

The capacity and blending constraints that affect the system are discussed below.

Capacity constraints

Capacity constraints are summarized in Table II. We are given three different mining capacities regarding the transportation
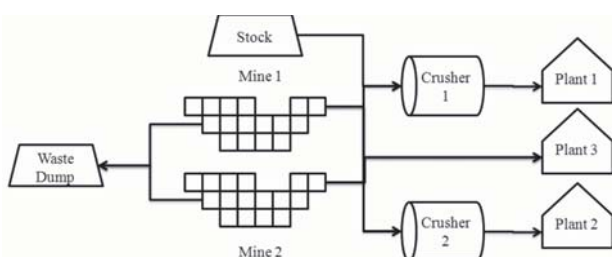
limits: one upper bound for each mine (Mine1 and Mine2) and one for the entire mining complex. It should be noted that the joint capacity of both pits is larger than the global capacity of the mine, which allows some flexibility in the process.

Blending constraints

The case study included only one geometallurgical constraint, on Plant1, referring to a minimum Cu/Fe ratio of 0.5 for material fed to that plant in order to guarantee the efficiency of the process.

Precedence constraints

In terms of precedence constraints, we consider slope constraints, accessibility constraints at each bench, and bench-by-bench constraints.

Slope constraints

These constraints are given by a certain slope angle α and a tolerance height Δz. Given two MRUs i, j, we consider that j is a predecessor of i if the mass centre of j lies in the upper cone with the vertex at the mass centre of i and angle α, and the vertical distance between these mass centres is at most Δz. For example, in Figure 5a, MRU j is a predecessor of MRU i but j^t is not, because although it is within the cone of angle α, it is farther than Δz above the mass centre of i.

Notice that the threshold Δz is used only to limit the number of arcs created in this way. Using this parameter is very common and does not mean that slope constraints are violated.

As seen in Figure 5, due to transitivity, the MRU j^t is a predecessor of i even though the arc itself is not explicitly in E.

Accessibility within a phase bench

At each bench phase we consider at least one special MRU i_0 representing the location of a ramp (see Figure 5b). Then, from this MRU i_0, we compute the shortest Euclidean path that goes through MRUs in that bench phase, and add the

Figure 4—An aggregated view of the mining system considered in the case study

Table II

Mining and processing capacities. Except for *Period Length* all quantities are in tons and normalized by the maximum total mining tonnage. Plant3 has no capacity constraints

Period	Length	Mining limits			Processing limits		
	(days)	Mine1	Mine2	Total	Plant1	Plant2	Total
1	6	35	36	59	12	6	17
2	7	41	42	64	10	3	13
3	7	41	42	69	12	5	17
4	11	64	66	100	20	10	30
5	10	58	60	92	20	10	29
6	7	41	42	63	14	3	17
7	7	41	42	70	14	6	20
8	4	23	24	44	9	4	12
9	10	58	60	94	20	6	26
10	7	41	42	67	14	5	18
11	7	41	42	62	15	7	21
12	7	41	42	66	14	7	21

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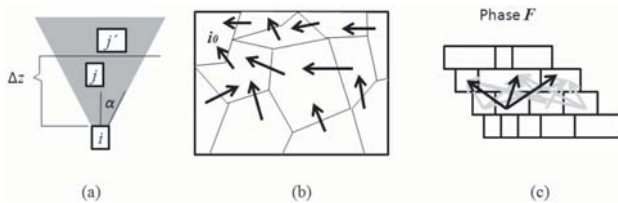
Figure 5—Example of precedences considered in the case study. (a) Precedences for slope constraints, (b) precedences for accessibility from a ramp in a bench phase, and (c) precedences for bench-by-bench extraction (a few highlighted for clarity). In all cases, arcs go from successor to predecessor

precedence constraints accordingly. This allows us to guarantee that extraction within a bench phase (a) is propagated from the ramps, and (b) remains always continuous.

Bench-by-bench extraction

We also use precedence constraints to ensure that the extraction of a bench phase occurs only when the overlying bench has already been extracted. For this, we include precedence constraints between the MRUs of the lower bench and the MRUs at the top, as explained previously (see Figure 5c).

Notice that as these constraints relate only MRUs in the same phase, they cannot substitute slope precedences, which may affect MRUs from different phases.

Types of schedule

The experiments generate production schedules using the optimization model and we compare them to the standard approach. The results of the experiments allow us to compare three scenarios: considering all the relevant short-term constraints (MODEL), with the MRU destinations defined beforehand (FIXED), or the standard approach (STD).

➤ STD corresponds to schedules and production plans obtained using the standard methodology available at the mine. In this case, the schedule is done at the beginning. The method takes into account the properties (attributes) of each individual MRU to define its final destination (and not the processing path that leads to it). Then the schedule is reviewed to satisfy mining and blending constraints at the final destination

➤ FIXED is a special case in which we preset the final destination of the MRU in the same way as in STD, and then schedule using the same considerations about mining and final destination capacities and blending constraints. In this case, the schedule used is the optimization model presented previously, but collapsing all the processing paths. Therefore, there is a unique aggregated path per destination, so as to mimic the STD case (See Figure 2 as an example)

➤ MODEL represents the resulting schedules and plans of our model. In this case, the economic values of the MRUs are computed for each possible processing path, and capacities and blending constraints are set at the node levels in the complete mining system. The scheduling process is carried out by the model, but also including the scheduling and processing path decisions as variables and not inputs.

As is mentioned in the description, schedules STD and FIXED do not take into account the mining system, but only the source and destination capacities. This is, indeed, the standard procedure in most mines (including this case study). This scheme is shown in Figure 2. In this case, intermediate nodes (including their associated capacities and constraints) are ignored in favour of an aggregated view that takes into account only the source and potential destinations.

For this reason, it was important to see whether these new elements introduced any changes in the resulting schedules or the assignment of MRU destinations. For this, we set two instances in which we considered the complete mining system but forced the model to comply with the STD and FIXED schedules, respectively. It was required to force the extraction of the MRUs at the same time periods of the corresponding schedules, and also to assign the actual processing path and comply with all the constraints along these paths.

Results and analysis

In this section we present and discuss the results obtained from the numerical experiments.

Firstly, we show the production plans obtained and some general conclusions. Next, we present the main result of this work, which shows the relevance of integrating the mining system in the mine scheduling. Finally, we analyse the differences in the production plans obtained.

Production plans

The resulting production plans are presented in Figure 6, in which we have de-aggregated the production for each schedule in terms of the final destination of the material.

We recall that the planning periods have different lengths, which translate into capacities that change from period to period.

Overall, these production plans show that the results obtained improve as more detail of the mining system is considered. More specifically:

➤ As expected, production schedules originating from optimization models tend to fulfill capacity constraints easily. However, the STD plan fails to comply with the mining tonnage constraint in some periods (see Figure 6, for example at periods 5 and 9). This is due to the fact that STD doesn't consider those constraints in the model

➤ FIXED and MODEL schedules tend to make better use of the processing capacities. For example, as there is no imposed capacity constraint on Plant3, both schedules make greater use of processing paths to that destination

➤ As there are no lower bounds imposed in the total mining tonnage, MODEL is able to produce higher production plans with less material extraction. For example, in periods 4 and 5 the total mined tonnage (stocks included) is about 85% of total capacity.

Economic value

The economic values of the three schedules are presented in Table III, normalized by the value obtained using the

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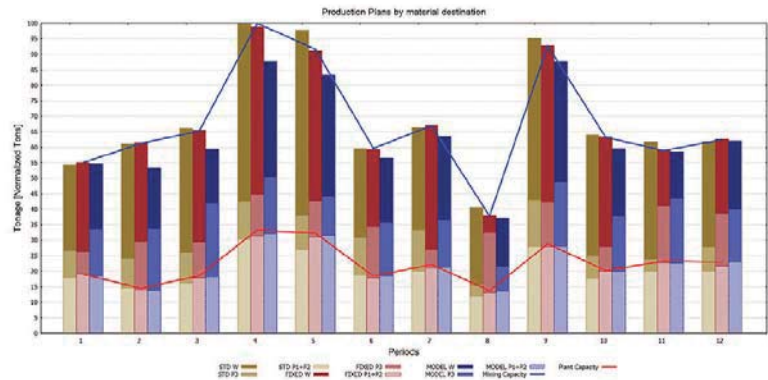
Figure 6—Production plan for STD, FIXED, and MODEL solutions, de-aggregated by destination: P1=Plant1, P2=Plant2, P3=Plant3, W=WstDump. Plant Capacity represents the processing capacity of Plant1 and Plant2 combined. Mine Capacity is the overall mining capacity

Table III

Economic values of each plan, normalized by the value of STD

Plan	Normalized economic value
STD	1.00
FIXED	1.44
MODEL	1.58

standard methodology. The numbers presented here must be regarded only as a reference because, as discussed later, there are several considerations to take into account.

The scheduling FIXED (fixed destinations) results in a plan with value 44% higher than the traditional methodology (STD). Moreover, if we consider the destinations as part of the decision process (MODEL), the value is 14% higher than FIXED. In total, we obtain an increase of 58% with respect to STD.

The increase in value using optimization models for scheduling is expected and well documented in many case studies (see, for example, Caccetta and Giannini, 1988; Chicoisne *et al.*, 2012). The increase due to selecting the optimal destination is also expected (see, for example, Kumral, 2013). Using an optimization model tends to produce superior results compared with manual or semi-automated procedures, within the corresponding model. Indeed, it is very important to note that these increases in value are not always reflected, because there are operational constraints not considered in the model. In our case it is important to take into account the following:

➤ STD is the only scheduling considered that satisfies the requirements of the planning procedure. Accordingly, it is the more realistic plan. Recall that STD considers only an aggregated level of constraints (for example, as in Figure 2) and therefore the model has less constraints than the corresponding MODEL version (as in Figure 4)

➤ Conversely, while FIXED and MODEL can be seen as good guides for constructing an operational plan, these models do not take into account all the complexities of a scheduling. Therefore, their economic values tend to be over-optimistic. Previous experiences comparing

results from STD and FIXED show that about half of the increase by using a scheduling algorithm is lost in operational considerations (see Vargas, Morales, and Rubio, 2009a, 2009b for example). Then, we could expect a value increase of about 22%

➤ The economic value of MODEL has been evaluated under the same parameters as STD and FIXED, that is, at the origin/destination level using average costs. Nevertheless, the actual optimization process used costs to each processing path and the scheduling is optimized accordingly.

Despite the above considerations, we conclude that there is a huge potential value increase to be gained by means of optimized schedules. Furthermore, this approach allows the optimization model to choose the actual processing alternative for the material.

It may be also tempting to conclude that the potential of value optimization depends on scheduling instead of grade optimization. This requires further analysis because, as discussed in the following section, our experiments show that STD and FIXED cannot be applied because they do not satisfy the mining system constraints.

Scheduling considering the mining system versus not considering the mining system

We would like to point out how feasible it is to let the model choose the schedule. Thus, we set up an experiment taking into account the following considerations:

1. We used the same mining system modeling as in MODEL, *i.e.*, we included all the intermediate nodes with the corresponding constraints
2. We included an additional constraint indicating that the extraction period of each MRU must be the same as in STD and FIXED
3. We allowed the model to schedule: the model chooses the processing path (and therefore, the final destination).

We expected that the model would produce a schedule with different choices for the destinations and with higher value. As it turned out, the resulting schedules were unfeasible. This means that there was no possible assignment of processing paths to the MRUs that would comply with the capacities and blending constraints (at the

detailed mining system), but also respecting the schedule of STD or FIXED. Therefore, we conclude that these schedules were unfeasible from the very beginning.

We would like to stress that in the above experiment, the decision of which processing paths to use (and therefore destination) was made by the model. That is, even with this added flexibility the problem still remained unfeasible.

In the following sections we provide some analysis to illustrate how the schedules differ.

Changes in the schedule

The changes in the production plans presented in the previous section also suppose changes in the production schedule. Table IV shows how the FIXED and MODEL schedules make different decisions regarding the extraction time for individual MRUs. The rows of the table are separated for each mine and correspond to the following.

➤ *Same*—This corresponds to the MRUs that are scheduled at the same time period as in the STD case
➤ *Schedule Before*—This corresponds to the fraction of MRUs that were scheduled at a period strictly before the period in the STD case
➤ *Schedule After*—This is the fraction of MRUs scheduled for extraction at a period strictly after the period in STD
➤ *New Schedule*—This corresponds to MRUs that were not scheduled for extraction in STD, but are now scheduled at a certain period
➤ *Not scheduled*—This is the case of MRUs that were originally scheduled for extraction by STD, but now they are left unmined by the corresponding schedule.

We observe from Table IV that, overall, the scheduled period remained the same in only about half of the cases, and that the decision for MRUs that were mined in all schedules is significant and accounts for about 20% of the tonnage.

What is more interesting is that there are very large differences in terms of the MRU being mined in the schedules. Indeed, if the destination decisions are the same as in the STD case (which is the FIXED plan), there is a difference of about 20% in terms of areas scheduled for mining, but this difference increases to 30% when destinations can be changed (MODEL case).

Changes in MRU destination

In order to compare the results obtained using a predefined destination for the MRUs with those obtained by letting the model choose the best option, we compare the actual destinations assigned to the MRUs in Table V.

Table V is constructed as follows. On each row we have the tonnage that the solution for STD or FIXED sent to the corresponding destination, but split according to the solution for MODEL. For example, the row Plant1 corresponds to material sent to Plant1 by FIXED. It indicates that 6.1% of the total mineral content was sent by the fixed destination to the plant (Total column), but the MODEL solution did the same for only 5% of this material, while 1.1% remained unextracted.

A first conclusion from Table V is that the there is a change of approximately 13% in the chosen destination of MRUs, as there are values that differ from zero outside the diagonal of the table. The only exception to this is the Plant2 destination. Indeed, both solutions send exactly the same 0.4% of the total tonnage to this destination.

We also observe that, while the total tonnage of material sent to Plant1 is not significantly different, MODEL mines less material by reducing the material sent to the waste dump while still increasing production, in particular the usage of Plant3.

Conclusions and future work

We have successfully modelled the mining system and extraction of an open pit operation under complex considerations and constraints of short-term planning that include accessibility, blending constraints, transportation and processing costs, slope angles, mining and processing capacities, *etc.*

Table IV

Changes in schedule with respect to STD plan. Values in percentage of tons of MRUs

Mine	Decision	FIXED	MODEL
Mine1	Same	50	50
	Schedule Before	13	9
	Schedule After	8	5
	New Schedule	11	10
	Not Scheduled	18	26
Mine2	Same	46	43
	Schedule Before	15	11
	Schedule After	4	6
	New Schedule	21	22
	Not Scheduled	14	18

Table V

Comparison of final destinations for the STD, FIXED, and MODEL schedules. Values are normalized over the total MRU tonnage. WstDump stands for waste dump

		MODEL					
		Not mined	WstDump	Plant1	Plant2	Plant3	Total
STD and	Not mined	55.3	3.0	0.9	0.0	2.2	61.4
	WstDump	5.6	15.6	0.0	0.0	1.9	23.1
	Plant1	1.1	0.0	5.0	0.0	0.0	6.1
FIXED	Plant2	0.0	0.0	0.0	0.4	0.0	0.4
	Plant3	0.7	0.1	0.0	0.0	8.2	8.9
	Total	62.7	18.6	6.0	0.4	12.3	100.0

Increasing the value and feasibilitly of open pit plans

We applied our model to a real case study, for which we conducted a series of experiments, in particular to compare the effect of using fixed rules to determine the destination of the blocks or MRUs (using, for example, cut-off grades) rather than leaving the optimal decision to the model.

The results show a high potential increase in the final value of a schedule under similar operational constraints, which is expected when comparing output from an optimization model with practice. Even better, the results also show an important potential in terms of using dynamic cut-off grades in order to optimize the final value of a mine operation, even under the hard limitations of volumes and design imposed by medium- and long-term decisions.

Even though the obtained schedule is not fully operational, it is close enough to provide a good guide for planners. Even more important, it allows the planner to make more robust decisions by exploring different scenarios. For example, although it is not reported in this article, we have used the model to study how the scheduling and processing path decisions change in the case of operational failure of the crushers, as well as considering different MRU definitions to study the impact of dilution of the scheduling process.

In terms of extensions of this work, we see at least the following research topics and applications:

(1) Improving the computation time, in order to be able to tackle more scenarios and larger case studies
(2) Improving the geometrical constraints imposed in the model so as to obtain schedules closer to operational ones
(3) Extending the models in order to deal with operational variability or geological and operational uncertainties.

Acknowledgments

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An improved meta-heuristic approach to extraction sequencing and block routing

by Y.A. Sari* and M. Kumral*

Synopsis

Mine production scheduling can be solved through many different techniques that have the drawbacks of either producing sub-optimal solutions or taking a long time. In this paper, a new approach based on a meta-heuristic is proposed. Meta-heuristic approaches use processing, inference, and memory at the same time in order to learn how to improve the solution. Different meta-heuristic techniques and their applications to mine production scheduling are discussed. A meta-heuristic approach, a combination of heuristic memory and simulated annealing, as demonstrated by means of a case study, takes a sub-optimal solution and improves it over time; thus it provides the best solution that it finds in the given time.

Keywords

Simulated annealing, mine production scheduling, open pit mining, mine planning, heuristic memory.

Introduction

Mineral deposits are most commonly represented by a block model that divides the orebody into a three-dimensional array of blocks. Each block consists of a cluster of similar characteristics such as rock type and ore grade, and has attributes such as tonnage of ore contained within the block and an expected economic value (Bley *et al.*, 2010). For each block, the mine production scheduling problem consists of the decisions of (1) whether to mine a block, (2) when to mine that block, and (3) how to process the mined block. The overall objective is to maximize the net present value (NPV) while meeting feasibility constraints such as production, blending, sequencing, and pit slope (Dagdelen, 2001).

Three main sub-problems of scheduling are the determination of production rates, discrimination between ore and waste, and block sequencing (Kumral, 2013a). These problems are interdependent; one sub-problem cannot be solved if the others have not been solved previously. However, in common applications, production rates are usually assumed and the other sub-problems are solved under this assumption (Menabde *et al.*, 2004; Nehring *et al.*, 2010; Asad and Topal, 2011). This leads to sub-optimal results. Our approach introduces a concept of cut-off range,

which regards the cut-off grade as guidance and optimizes it within the range provided. This is a step toward simultaneously optimizing production rates along with process destination discrimination and extraction sequencing.

Exact methods such as mixed integer programming (MIP) have been used for the block sequencing problem to obtain an optimal result for various cases (Kumral, 2013b; Little *et al.*, 2013; Nehring *et al.*, 2012; de Carvalho Jr. *et al.*, 2012) and yields a deterministic plan. However, MIP suffers from certain drawbacks. The size of the problem increases exponentially as the level of complexity (such as multiple metals, process destinations, rock types) increases (Rothlauf, 2011). To overcome the data size problem in MIP, block aggregation is suggested (Tabesh and Askari-Nasab, 2011; Topal, 2011) but naturally, this results in loss of optimality. Also, given that the block model is based on drill-hole data but is usually generated by geostatistical simulation, it is impossible in practice for the generated schedule to be optimal. Considering the amount of time MIP takes with large data-sets and that MIP is unnecessarily precise in our case, a faster, approximately-optimal algorithm is much more suited to the practical need.

Another widely used exact method is the Lerchs-Grossman algorithm (Lerchs and Grossman, 1964), which yields the ultimate pit. This is an algorithm based on graph theory that converts each block to nodes. Although faster than MIP, in addition to the problems in MIP, when using Lerchs-Grossman algorithm it is difficult to assign varying pit slopes at different points and determine mining and processing capacities for each period. Dagdelen and Johnson (1986) attempted to handle the

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* *McGill University, Department of Mining and Materials Engineering, Montreal Canada.*

capacity constraints problem by incorporating the Lagrangean multiplier. The selection of the Lagrengean multiplier is a significant problem and the viability of the sequence generated depends on this selection. There is no clear way to determine the multiplier such that the NPV of the project is maximized.

Meta-heuristic approach to the mine production scheduling problem

In this research, simulated annealing (SA) meta-heuristic with addition of heuristic memory is utilized to solve mine production scheduling. The addition of heuristic memory helps to reduce the randomness of SA and improves computational efficiency. Heuristic memory learns the path of search in SA in such a way as to accelerate escape from local optima. As such, this addition can be seen as the incorporation of machine learning into the optimization process. Machine learning takes existing data a step further by automatically learning and improving the performance based on the data (Witten and Frank, 2005). Machine learning consists of many different techniques based on mathematical and empirical methods. These methods can be used to enhance the optimization and are especially easy to integrate with meta-heuristic approaches.

The application of SA to the mine production scheduling problem was developed by Kumral and Dowd (2005). This approach gradually improves an initial non-optimal solution by making several changes at each step and observing the effects of the changes. Although reaching the near-optimal solution takes time, the advantage of this technique is that it can be stopped at any time to obtain the most profitable solution so far.

The application of genetic algorithms was first introduced by Clement and Vagenas (1994). Based on the principles of natural selection, multiple feasible solutions are mixed by involving randomization. Similar to SA, solutions are improved gradually and the process can be stopped to obtain the best solution so far.

Ant colony optimization is a population-based metaheuristic method first developed by Dorigo and Birattari (2010) to imitate the foraging mechanism of ants. Ant colony optimization was proposed to solve the mine production scheduling problem by Sattarvand and Niemann-Delius (2013), Sattarvand (2009), and Shishvan and Sattarvand (2015). Using the Lerchs-Grossman method to produce an initial solution, the schedule was improved through iterations based on pheromone trails.

Reinforcement learning is similar to SA and genetic algorithms in terms of being an algorithm for searching the parameter space using the concept of reward; which in our case will be the improvement in the NPV. However, it yields better immediate results by applying a trial-and-error search having a memory-like system by incorporating historical error into its search mechanism (Sutton and Barto, 1998). Combined with dynamic programming, this type of learning can be used to adapt incoming updated information, for example during mine exploration.

Bayesian inference assumes the quantities of interest and parameters have an underlying probability distribution. By combining these probability distributions and observed data, optimal decisions can be made (Mitchell, 1997). Bayesian

learning can be used to estimate the parameters, their relations to other parameters, and update their values with the incoming new drilling data.

Method

SA was developed initially by Kirkpatrick *et al.* (1983) and Cerny (1985). The method was applied to open pit mine production scheduling by Kumral and Dowd (2005) and Kumral (2013a) by the following steps:

Step 1: Start with a non-optimal feasible solution

Step 2: Select a portion of the blocks

Step 3: Possibility 1: modify ore-waste discrimination. Change ore blocks to waste, and waste to ore by some probability
Possibility 2: modify the period of the given block to a previous or following period by some probability

Step 4: Recalculate NPV for the newly found solution

Step 5: Apply the Metropolis criterion as the acceptance criterion; accept the new solution with the probability yielded by Metropolis

Step 6: If the NPV has not increased for last n steps, terminate. Otherwise, go to Step 2.

The Metropolis criterion (Metropolis *et al.*, 2004), shown in Equation [1], is a criterion that takes two solutions and a temperature T as inputs and outputs a probability of acceptance between 0 and 1, where E_0 is the current solution's NPV and E is the newly found solution's NPV.

$$\min\left(1, exp\left\{-\frac{(E-E_0)}{T}\right\}\right) \tag{1}$$

T should be chosen high at first and then decreased slowly. If T decreases slowly enough, theoretically a global minimum will be reached (Lundy and Mees, 1986). According to how the Metropolis criterion is set up, at higher temperatures the criterion tends to accept solutions that are not improving as well as those are improving. This stage is called 'exploration of the parameter space', as shown in Figure 1a. As T is lowered, there is less chance of accepting solutions that are not improving. If T is not decreased slowly enough, there is a chance of becoming stuck at a local minimum as shown in Figure 1b.

a) Searching for solutions in parameter space (Red arrow: current position in the search, Grey arrow: search direction)

b) Landing on local optimum
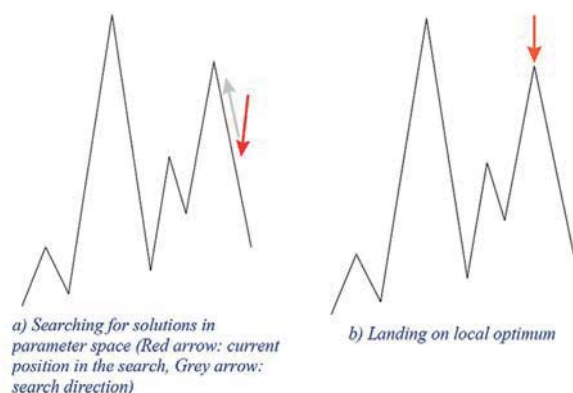
Figure 1—Parameter space

An improved meta-heuristic approach to extraction sequencing and block routing

As SA takes time, ideally the initial solution should provide very fast, although sub-optimal, results. The ranked positional weight (RPW) algorithm is a heuristic algorithm that draws a downward cone from each block and the block gains a score according to the economic values of the blocks in the downward cone (Gershon, 1987b). This approach follows the logic that if a block is underlain by a valuable block, it should gain more score as the removal of this block leads the way to the underlying valuable block. After the scoring has been completed, a schedule is generated such that starting from the first level, the highest, scored blocks will be mined. The RPW fits our purpose well because it produces a feasible solution rapidly.

A computer program was written to perform RPW and SA as demonstrated in Algorithm 1 to perform mine production scheduling. First, RPW is run to generate an initial feasible, sub-optimal solution. Then this solution is transferred to SA, which needs an initial input. SA gradually improves this solution at each iteration and outputs the result. A feasible solution respects the slope constraints, mining capacity constraints, and processing capacity constraints.

```
Algorithm 1 Mine Production scheduling Using Ranked Positional Weight and
Simulated Annealing
Input: Grade of each block
    Determine cutOff grade using Osanloo and Ataei's method(Osanloo & Ataei,
    2003)
    function RANKED POSITIONAL WEIGHT(blockGrades, cutOff)
        Call Ranked positional weight algorithm to produce initial output
        return initialOutput
    end function
    function SIMULATED ANNEALING(blockGrades, initialOutput, cutOff)
        Call Simulated annealing algorithm to optimize the initial output
        return finalOutput
    end function
Output: finalOutput
```

In this paper, in addition to SA, the developed SA variant method with memory is used to solve a mine production scheduling problem. In SA, only improvement is tracked and the decisions are made based on improving the objective function. Thus, SA is memoryless (Glover and Kochenberger, 2003). Tabu search attempted to improve SA by creating a dynamic list of forbidden solutions, thus introducing a concept of memory (Glover, 1989, 1990). However, this is very specific and limited. Tabu search only attempts to decrease re-visitation of the same solutions; it does not attempt to utilize the information in the solutions in some way.

Our proposed SA variant method, improved SA with heuristic memory, deducts information from the 'big data' produced by SA through inputting a heuristic (a quantifiable component of the solution that is thought to influence the objective function) and recording the heuristics value and the corresponding objective functions value. This method adds a memory on top of SA, with the intent of making it faster to find the global optimum. The representation of heuristic memory-added SA can be followed through the pseudocode demonstrated in Algorithm 2. The algorithm contemplates whether the provided heuristics indeed have an effect on the objective function by collecting data and looking at the relationship between the heuristic and objective function. If the heuristic has an effect, the value of the heuristic, along

with a balancing parameter k_1, is also added into the objective function to increase its effect. With this approach, if there is a known or reasoned important component of the problem, it can be put forward rather than performing a wholly random search. A comparison of the SA flow chart and SA with the heuristic memory-added flow chart is given in Figure 2.

```
Algorithm 2 Improved Simulated Annealing by Adding Heuristic Memory
Input: Define n measurable heuristics suspected to influence the objective func-
    tion, stored in array h[n]
    while SA produces a solution do
        for i = 0 to n do
            Save the value of h[i] and the objective function for the current solution
            Perform function fitting using all recorded values of h[i] vs objective
            function for all hitherto obtained solutions
        end for
        Objective function is modified to be submitted to Metropolis criterion
        such that objective function ← objective function + k_1 × fitted function ×
        heuristic
    end while
```
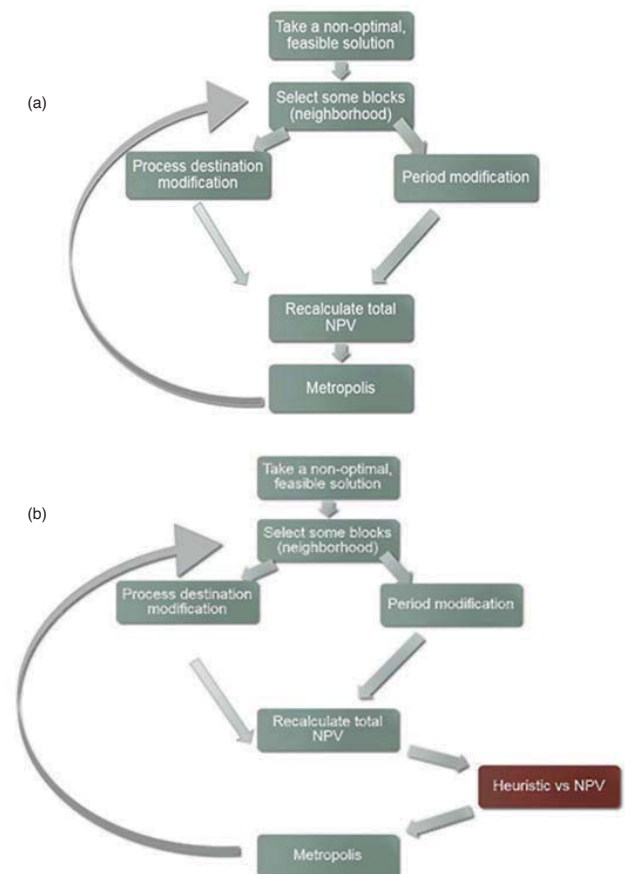
Figure 2—(a) Flow chart of SA from Kumral and Dowd (2005). Randomly selected blocks have either their process destination or period modified. If feasible, the new model's NPV is calculated and the objective function is evaluated by the Metropolis criterion. (b) Flow chart of modified SA with heuristic memory added. The modification is that after a new model is obtained and its NPV is calculated, the heuristic *vs* NPV data is saved. An updated objective function is submitted to the Metropolis criterion that maximizes NPV, pushes the number of blocks to capacity, and at the same time maximizes or minimizes the heuristic, depending on how it influences the NPV

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The program is able to work with multiple metals, process destinations, and rock types. Other details of the program are as follows:

➤ *Cooling schedule*—If initial temperature is too high, all new solutions are accepted. This will lead to undirected search. If initial temperature is too low, only improved solutions will be accepted and the annealing process will be reduced to a local search. Therefore, the initial temperature is set by taking the first two solutions and finding T in Equation [1] such that the equation will be equal to 0.5. This sets T such that, in the beginning, a solution will have a 50% chance of being accepted even if it is not improving. Decrementing T is accomplished by $T \leftarrow T \times 0.9999$ to ensure it decreases slowly enough to accept more solutions. Each time a fixed number of solutions are found (40 solutions), T is updated as described

➤ *Stopping criteria*—There are two conditions that can stop the annealing loop:
  • When the Metropolis Criterion does not accept the solution for a pre-set, empirically selected amount of iterations (in our case, four iterations)
  • When the program loops for a user-set amount of value. The second condition exists to produce solution under limited time. However, the longer the program is allowed to run, the better the results

➤ *Maximum number of solutions at each temperature*—This is a parameter that sets the number of generated solutions before decreasing the temperature. This should depend on the size of the data, so in our program we set it to 200 solutions

➤ *Cut-off range*—SA uses the guidance of the cut-off grades. However, it does not adhere to them strictly. During the generation of transition destinations process, the cut-off range is used to decide to which extent the blocks out of the limits of the cut-off grade could be accepted. This parameter may have a major effect on the results. If set high enough, it can remove cut-off grade boundaries altogether

➤ *Number of iterations*—SA terminates either when there is no improvement or when the given number of iterations is reached. Mining problem sizes are very large and thus the number of iterations is usually reached sooner than settling on the ideal solution. This parameter should be selected as large as possible as time permits

➤ *Short-coming process blocks effect*—This parameter is used as the balance between maximizing NPV and satisfying capacity constraints in the objective function. The parameter specifies how important it is to fulfill the process capacities. This value ranges between zero where it is not considered and unity where this criterion is all that matters

➤ *Mining cost adjustment factor*—The modifiable mining cost adjustment factor (MCAF) is used to reflect the increased cost of transport in deeper levels of the deposit. MCAF is entered by the user and the MCAF is added to the mining cost using the Equation [2].

$$miningCost \leftarrow miningCost + level \times MCAF \qquad [2]$$

➤ *Heuristic memory*—For each heuristic, the heuristic value and the objective function value are stored. When enough data is produced, a function-fitting method (Equation [3]) is performed to deduct information of how this heuristic influences the objective function. In our case, the number of blocks was the heuristic used and the fitting function was linear regression. This influence, along with a balancing parameter (k), is included in the objective function. The balancing parameter depends on the coefficient of determination, R^2, of the fitting function. Possible heuristics include the number of blocks, number of ore blocks, block grade *versus* process destination, coordinates of the main ore clusters, mine depth, and mine life.

$$\text{Objective Function} \leftarrow \\ \text{Objective Function} + heuristic \times R^2 \times k \qquad [3]$$

Case study

To demonstrate an application of meta-heuristic optimization on mine production scheduling, a program has been written using SA with the heuristic method approach.

The case study considers a copper and molybdenum deposit generated from a public drill-hole data-set in http://www.kriging.com/datasets/ Using sequential Gaussian simulation, a 3D block model of 595 046 blocks was created, where each block is 10×10×10 m in size. The mining company has one waste dump and three process destinations (low-, middle-, and high-grade processing), where the ore is processed by different procedures and thus their costs and recoveries are different. The slopes are 45 degrees in four directions (north, south, east, and west). Parameters for the case study are given in Table I. With 595 046 blocks, four periods, and four total destinations there are 595 046 × 4 × (4 + 1) = 11 900 920 decision variables. In the calculation, destinations are incremented by one because the the decision can also be taken not to extract the block.

Cut-off grades were calculated using the method of Osanloo and Ataei (2003) for finding the equivalent cut-off grade for multiple metal deposits, yielding 0.4859%, 0.6006%, and 0.7257 % respectively for each process. First, the RPW algorithm (Gershon, 1987a,b) was run to output a sub-optimal initial result. This result was input to the SA and SA with heuristic memory as an initial solution. All solutions respect the slope, mining, and process capacity constraints.

The resultant NPV of each algorithm is given in Table II. Using SA improved the RPW results by $75 743 914, which is 4.70%. SA with heuristic memory, on the other hand, improved the NPV by $77 386 239, which is a 4.80% improvement, when run for the same amount of time.

Figures 3 and 4 show various cross-sections of the orebody. These figure also compare the RPW and SA outputs, with each colour corresponding to an extraction period (1: light blue, 2: green, 3: orange, 4: red, dark blue: not extracted). It can be seen from these figures that compared to the RPW algorithm, SA is inclined to mine the blocks in the earlier periods to increase the NPV. However, the SA results

Table I

Parameters of the case study

Values	Explanation
10 10 10	Block dimensions (m)
98 132 46	No. of blocks in x, y, z directions
4	No. of total destinations
3	No. of ore destinations
1	No. of waste destinations
4	No. of periods
2	No. of rock types
3 3 3 3	Mining cost of each destination rock type 1 ($ per ton
4 4 4 4	Mining cost of each destination rock type 2 ($ per ton)
0 15 30 50	Mineral processing cost of each destination rock type 1 ($ per ton)
0 18 38 53	Mineral processing cost of each destination rock type 2 ($ per ton)
0 60 70 80	Sales cost of each destination ($/ton concentrate)
5 5 5 5	Specific gravity of each destination (t/m^3)
60	Mining capacity (ore and waste) (in thousands of number of blocks)
25 20 15	Processing capacity of each destination (in thousands of number of blocks)
0.01	Cut-off range
2	Number of metals (Cu and Mo)
10000 30000	Ore price ($ per ton) (Cu and Mo)
0 40 70 95	Recovery for each destination metal 1 rock type 1 (%)
0 30 40 75	Recovery for each destination metal 2 rock type 1 (%)
0 40 80 95	Recovery for each destination metal 1 rock type 2 (%)
0 30 40 75	Recovery for each destination metal 2 rock type 2 (%)
0.1	Discount rate
300	Number of iterations
0.9	Shortcoming process block effect
0.1	MCAF
0.600 0.680 0.780	Copper grade requirement (%)
0.043 0.063 0.010	Molybdenum grade requirement (%)

(a) Output from Ranked Positional Weight

Figure 3—Cross-sections (sections: 48, 52, 0)
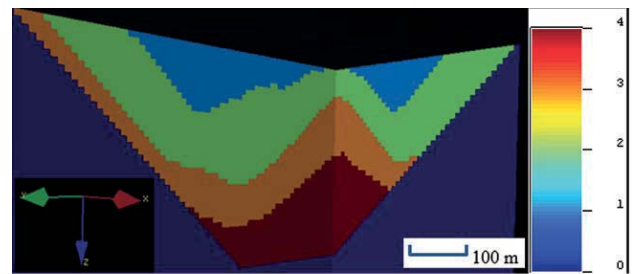
(a) Output from Ranked Positional Weight
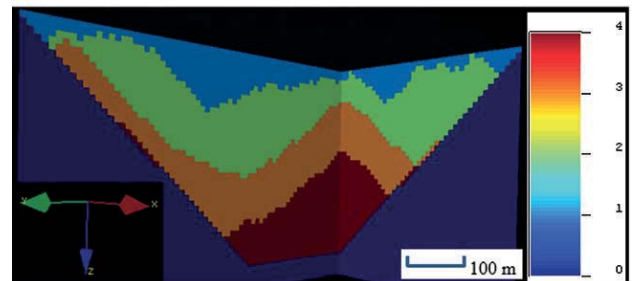
Figure 4—Cross-sections (sections: 56, 65, 0)

Table II

Resultant NPVs for each applied technique

Algorithm	Resultant NPV
Ranked positional weight	$1 613 205 645
Ranked positional weight + simulated annealing	$1 688 949 559
Ranked positional weight + simulated annealing with heuristic memory	$1 690 591 884

look less smooth than the ranked positional algorithm's result. This is mainly because of the structure of annealing, where blocks are switched between the periods one by one, causing the sections to look rugged.

Figure 5 shows the average grade of Cu and Mo at each period for each process, as well as the number of blocks extracted at each period. These results belong to the SA
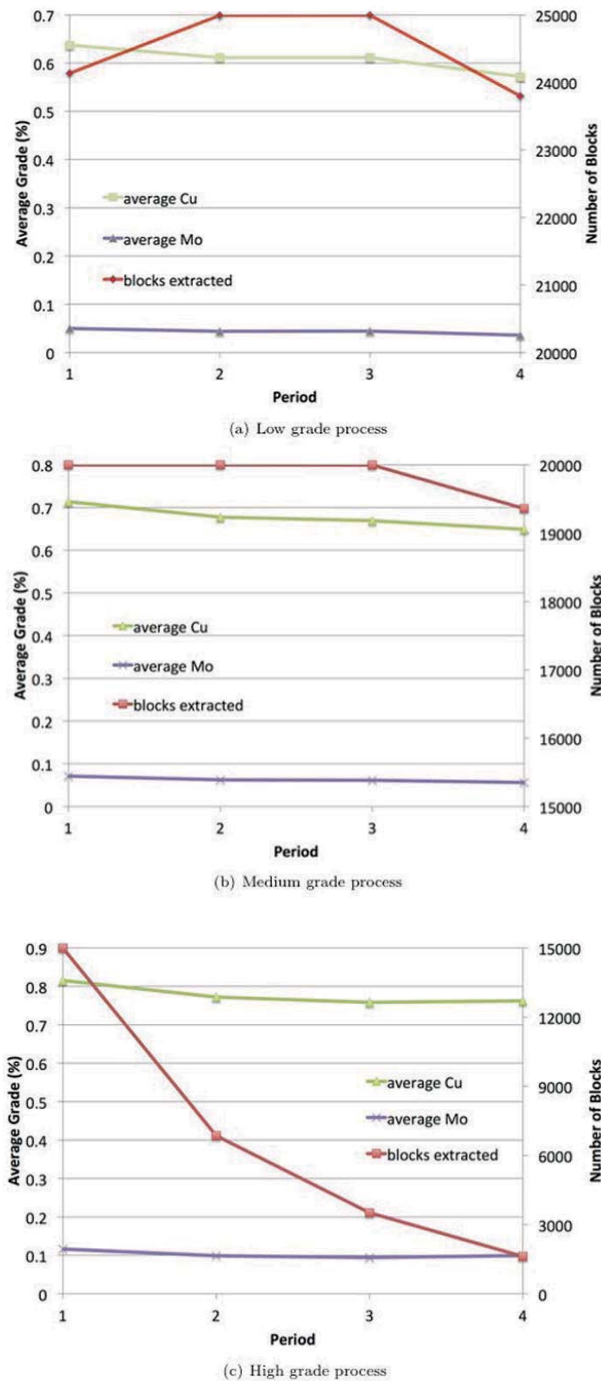
Figure 5—Average grades and number of blocks in each destination and period

method, but the SA with heuristic memory results are very similar and not distinctive and thus yield the same figures because of the process capacity push mechanism. On the other hand, the intelligent search mechanism through heuristic memory also reduces the running time by about 25%. To maximize NPV, the approach forces to reach the capacities. Therefore, the results are similar but the NPVs are different. This can be observed in the NPV increases in Table II. While the number of blocks extracted and the average grades are similar, the block configurations are different with these two methods. It should also be noted

that average grade is persistent within a range throughout the periods at each destination. Therefore, there are no distinctive changes in average grades. This is important for the processes to choose and maintain a recovery, where fluctuations in the block grades affect the recovery process negatively. Another point to note is that process capacity push is working well, except for the high-grade process. The reason for this is the grade is not homogeneous and there are not enough high-grade blocks for the later periods. Most of the high-grade material is near the surface, thus the capacity during the first period is completely filled. It can also be

observed from Figure 5 that number of blocks sent to each process at each period is below the corresponding capacity, satisfying the process capacity constraints. The processing capacities are 25 000, 20 000, and 15 000 blocks for the low-, medium-, and high-grade processes, respectively (Table I). As can be seen from Figure 5, for the low- and medium-grade processes, capacity satisfaction is quite good. Since the number of high-grade blocks is low, there is a decreasing order of number of blocks. In this case, there may be a few solutions: establishing stockpiles, changing the high-grade process design to meet the grade requirement, or re-installed high-grade process capacities. This is a common problem in mining operations because the capacity installation ignores ore material heterogeneity. As can be also seen from Figure 5, the grades at each destination are consistent in terms of periods. Mining capacity was 60 000 and was also satisfied, as the numbers of blocks extracted in each period are 60 000, 60 000, 59 703, and 46 609 respectively.

Lastly, the overall average grades of low-, medium-, and high-grade processes are compared in Figure 6. Average Cu and Mo grades obtained at each process destination are given as 0.6081% and 0.043 275% for low-grade processing, 0.677 325% and 0.062 778 25% for medium-grade processing, and 0.776 868 25% and 0.101 760 25% for high-grade processing. The average grades are highly compatible with the grade requirements. As expected, the high-grade process has the highest average grade, followed by the medium and low grades.

Conclusions and future work

The use of SA after a heuristic-based method guarantees that it will either produce a better solution or return the initial solution. It is true that SA takes time to reach the optimal value. However, unlike exact methods, it can be stopped at any point and best solution found so far can be returned. Moreover, almost all parameters can be integrated into SA, such as process capacity, transportation cost, and multiple process destinations, which are impossible to integrate in some other techniques. In exact methods, as the number of parameters increases, the problem size increases exponentially, whereas with SA the problem size increases proportionally; only as much as the expansion of the search space. SA is also more convenient to apply to our problem than

other meta-heuristic methods such as genetic algorithms, particle swarm optimization, and evolutionary search because these types of algorithms require a pool of initial solutions. In our case, we used RPW to generate the initial solution, which can provide only one solution. For such a large problem, generating more than one solution is hard and time-consuming.

It is observed from the case study that usage of SA can add large gains to the revenue compared to RPW. The average grade and number of blocks sent to destinations were overall stable. Moreover, the case study has shown that the revenue of the solution obtained in the same amount of time has been increased by SA with heuristic memory. As the running time increases, further improvement can be achieved.

In the case study of the heuristic-memory-based SA, a linear fitting function was used. Efficiency of the memory enhancement can be increased through improving this fitting function. Also, in our case most parameters related to heuristic-memory-based SA were chosen empirically, such as when to produce the first function, how often to update the function, and how to balance the optimal function with the heuristic. Research can be conducted on how to optimize these parameters.

The main issue in all meta-heuristic applications is the parameter selection. This is also true for all types of SA. Selection of SA-related parameters such as the temperature, number of iterations, and maximum number of solutions at each temperature can affect the running time of the program to a great extent. If the parameters are poorly set and the program is run for a short time, the results may not be optimal.

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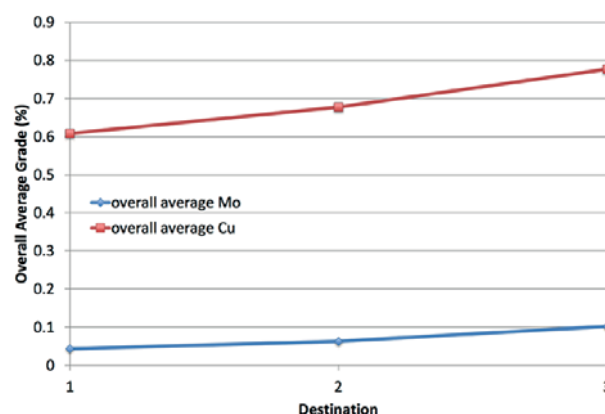
Figure 6—Overall average grades compared between low-, medium-, and high-grade process destinations

An improved meta-heuristic approach to extraction sequencing and block routing

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Multiple cut-off grade optimization by genetic algorithms and comparison with grid search method and dynamic programming

by E. Cetin* and P.A. Dowd†

Synopsis

Optimization of cut-off grades is a fundamental issue for mineral deposits. Determination of optimum cut-off grades, instead of application of a static cut-off grade for the life of a mine, maximizes the net present value. The authors describe the general problem of cut-off grade optimization for multi-mineral deposits and outline the use of genetic algorithms, the grid search method, and dynamic programming for optimal cut-off grade schedules for deposits with up to three constituent minerals. The methods are compared by assessing the results of the implications involved in using them.

Keywords
cut-off grade, sequencing, optimization, multi-mineral deposit, genetic algorithms.

Introduction

Determination of optimum cut-off grades is a fundamental issue in mineral extraction as it assigns the boundaries between ore and waste over time.

The profit from a mining operation is a direct function of the sequences of cut-off grades and associated ore tonnages that define the life-of-mine production schedule. As profit varies with these sequences there will be a sequence, or sequences, that optimize any specified profit criterion. The most widely used cut-off grade optimization criterion is maximum net present value (NPV) of profits. The NPV can be maximized by maximizing profit per unit time. This process necessitates applying, in the early years of operation, the highest cut-off grade that can provide sufficient ore to satisfy the requirements of the processing plant. As time passes, the cut-off grade must be lowered, thereby lowering the opportunity cost. Hence, the highest NPV is achieved.

The objectives of this paper are to develop general methods for determining optimal sequences of cut-off grades for multi-mineral deposits by means of genetic algorithms, to implement this method in computer programs, and to assess the performance of the method. In order to assess the performance of the

genetic algorithms method, the grid search method and the dynamic programming method are used and compared with the results of the case of genetic algorithms. The computer programs developed for this purpose are capable of determining optimal sequences of cut-off grades for multi-mineral deposits that contain up to three valuable minerals.

Optimization of cut-off grades for multi-mineral deposits

Mine planning and the financial evaluation of mineral deposits that contain more than one valuable mineral are generally done on the basis of parametric cut-off grades or the equivalents. However, because of problems related to this method, an alternative method of individually optimizing the cut-off grades of the component minerals has been used. The main problem arises from the fact that the revenue and the costs must be calculated on the basis of the average grades of the individual minerals from the calculated equivalent grade. If the constituent minerals are highly correlated, the average grades can be estimated by iteration and by defining some additional parameters for the equivalent grade-tonnage data (Dowd and Xu, 1999). However, if there is very little correlation between the minerals, the validity of the equivalents method is not obvious. Because of the problems the equivalents method brings about, in order to optimize a multi-mineral deposit, the constituent minerals are best dealt with separately.

* Engineering Faculty, Mining Engineering Department, Dicle University, Turkey.
† Faculty of Engineering, Computer and Mathematical Sciences, University of Adelaide, Australia.

Multiple cut-off grade optimization by genetic algorithms

Optimization by genetic algorithms

Genetic algorithms constitute a class of stochastic algorithms that use a search method based on the principles of biological genetics and natural evolution. Holland (1975) proposed the basic principles of genetic algorithms. In this approach, individuals of a population are represented as chromosomes and an expanded set of genetic operations takes place. It is presumed that the potential solution of any problem is an individual and can be represented by a set of parameters.

The vocabulary of genetic algorithms is borrowed from genetics science. In nature, each cell of every living organism has a set of chromosomes that make up DNA. Chromosomes are made up of genes, which control different characteristics of an organism. In genetic algorithms, a potential solution to a problem is called an individual or chromosome. Individuals make up a population. Genetic operations, such as crossover, mutation, and reproduction, are also used in genetic algorithms.

Genetic algorithms are particularly suited to the solution of large-scale optimization problems. They belong to the class of probabilistic algorithms but are very different from random algorithms as they combine directed and stochastic searches. Another important property of genetic-based search methods is that they maintain a population of potential solutions. Genetic algorithms can also easily escape from local optima by using genetic operators, such as mutation.

A genetic algorithms flow chart is given in Figure 1. The basic principles of genetic algorithms are as follows:

1. A set of strings composed of finite elements, generally a binary code, is assigned. Each string refers to a point in the search space or a solution to the problem among the alternatives. Genetic algorithms work on these strings, which are called chromosomes or individuals

2. A first generation, *i.e.* a population, of individuals, is selected. Generally, the selection is done at random

3. The individuals are evaluated on the basis of their return values. Fitness values are assigned to the individuals in order to rank them on the basis of their return values. The values assigned to better solutions result in higher fitness values

4. Some of the individuals are selected on the basis of their fitness values. The individuals with lower fitness values lose in competition

5. Parents are chosen from among the selected individuals. They are crossed over by pairs. The result is two new individuals from each parent

6. Some chromosomes enter a mutation process. That is, one or more digits of a string are changed at random. A new population is ready

7. The process is repeated from step 3 until it converges to a stable value or an assigned number of generations is reached.
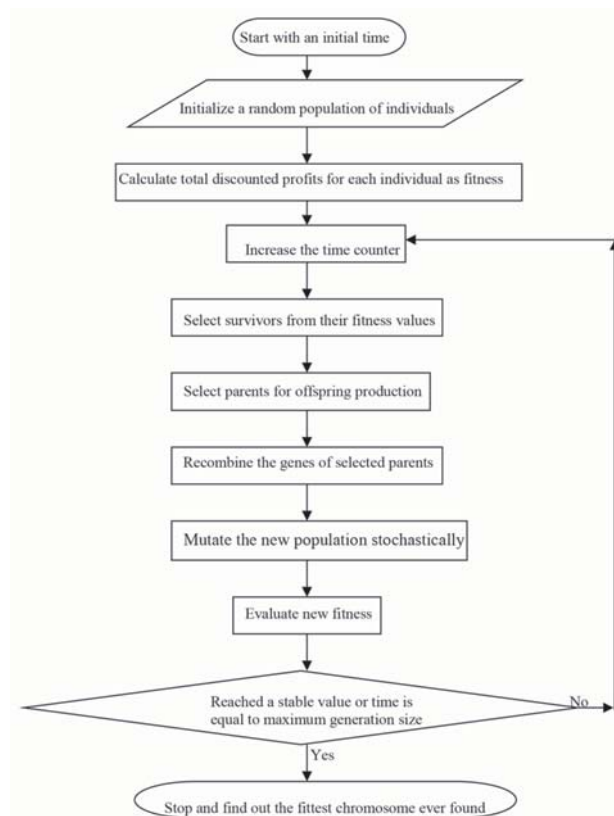
Figure 1—A genetic algorithms flow chart

Multiple cut-off grade optimization by genetic algorithms

Major elements of genetic algorithms

Encoding

Genetic algorithms work with representations of solutions. The representation is a symbol string, which carries all the information about the individual. The string has a fixed length and is called a chromosome or individual. The length of the string of an individual depends on the precision requirements. The string can be composed of real decimal numbers or characters, but the most widely used string representation is binary numbers.

The mapping value of a binary string into a real number is straightforward. The binary string is converted into a real number, which is an integer. Then a corresponding real number, which is the mapping value, is found.

In order to initialize an individual composed of binary strings, all bits are to be initialized randomly.

Population

In genetics science, individuals make up a population. The bigger the population, the more extended the search area. However, the number of individuals adversely affects the speed of a computer program based on genetic algorithms.

Evaluation

After initialization of a population made up of binary or real number strings, an evaluation process takes place. Each individual is assigned a fitness value, which is calculated on the basis of objective function for the problem.

Selection

Good individuals with better fitness values are selected in a selection process. Each generation produces new individuals from the current population. Selection is a process of finding how many times each individual from the current population should be copied to generate a new set of solutions or a new population. The process resembles natural selection in that individuals that give better results in the evaluation process have a greater chance of reproducing. The selection process consists of determining the number of times that a particular individual is chosen to have offspring. The selection process can be deterministic or probabilistic.

In deterministic selection, better individuals are determined to have more offspring than poor ones. Individuals with very low fitness values have no chance of survival. Deterministic selection helps to get rid of poor individuals and to generate a quick result.

The roulette wheel method is the most widely used selection method in genetic algorithms. It is a probabilistic method in which individuals with better fitness values are more likely to reproduce, although weak individuals still have a chance of survival. Individuals are represented on the wheel as a proportion of their fitness values.

There are other parameters that can be applied to the deterministic and probabilistic approaches. Scaling is one of these. When the fitness values of individuals of a population are sufficiently distinct, there is no need for any kind of scaling. But if the fitness values are close to each other, which is generally the case as generations pass and most of the individuals have relatively good fitness values, good individuals will lose competitiveness. Scaling is used to improve the situation. The individuals are scaled in order to improve the competition abilities of the good individuals during the selection process. This is generally achieved by subtracting the same number from all the fitness values of the individuals. Consider a problem with only two individuals. Suppose that their fitness values, based on their performances, are 495 and 497. If one of them is to be selected randomly, the chance of the first individual being selected is 49.9% and that of the second is 50.1%. Although one of the individuals is obviously better, the chances of selection are almost the same. However, if the fitness values of the individuals are scaled by subtracting 490 from both, the chances of being selected change to 41.7% and 58.3% respectively.

Another commonly used method of improving the performance of a genetic algorithms process is elitist selection. In genetic algorithms there is always a risk of losing the best individual when generations pass. In elitist selection, the most fit individual, or individuals, after each evaluation phase could be carried to the next generation unchanged (Zalzala and Fleming, 1977).

Genetic operators

As in nature, there are mainly two types of classical genetic operators in genetic algorithms: crossover and mutation.

Crossover

Crossover is the basic operator for the production of new chromosomes. It mimics the sexual reproduction of living organisms. Two parents come together and they produce two infants whose genes resemble those of the parents. The common forms of crossover are 1-point crossover, 2-point crossover, n-point crossover, and uniform crossover (Green, 1999).

In 1-point crossover, a crossover point is randomly selected for each couple. Each half of the chromosomes then crosses over to find two new offspring that resemble both parents.

The basic difference in 2-point crossover is that there are two crossover points assigned for each couple. The sections between the two crossover points are swapped for the new individuals.

In n-point crossover, there are n crossover points. The parts of the strings of the parents between every two crossover points are swapped for the new infants. As a result, the infants would get the parts of the string of a parent between every successive crossover point.

In uniform crossover, a number of points are selected at random. Each selected point is swapped over rather than swapping a part of the string.

Parents are chosen from among the selected individuals randomly according to an explicitly assigned crossover probability.

Multiple cut-off grade optimization by genetic algorithms

Mutation

In nature, copying DNA to create offspring can sometimes result in errors. These errors, called mutations, generally do not have a positive effect on the fitness of the individual, although they can sometimes result in beneficial features and they can be passed to succeeding generations via reproduction. Mutation is so important for the evolution of living organisms that without it nature would have been in a vicious circle rather than an evolutionary process.

Genetic algorithms are very different from other stochastic search methods in the method used for searching. Searching starts as a randomly selected population and future solutions depend on mutual relationships of the individuals. Without the mutation process, the search area would be so restricted that finding the global maximum point would be almost impossible in large-scale problems. The search area can be widened gradually by the mutation process and deepened by the crossover process; the features of individuals are improved by the selection process. Individuals evolve gradually until the solutions converge to a maximum point or a predetermined number of generations is reached.

As in the crossover process, mutation points are selected randomly. However, the probability of mutation should be comparatively low, since it is not a common process like crossover. There are different types of mutation. In bit-by-bit mutation, random numbers are generated for each digit of the whole population and, depending on the assigned mutation probability, the digit might be changed. In binary code this is a trivial exercise. If the original digit value were 0, the changed value would be 1, and *vice versa*. In string mutation, however, the mutation probability value is assigned on the string basis. After random numbers have been generated, if a string is mutated, another random number is generated and assigned to the mutation point for the string.

Application of genetic algorithms to cut-off grade optimization

Many cut-off grade optimization problems have huge numbers of local optimum values, which are widely separated from the global optimum point and from each other. Stochastic search methods can easily fail to find the global optimum point for such problems. The real challenge in such problems is finding solutions close to the global optimum point for a restricted time. Genetic algorithms are more robust in this context than many other existing search methods.

Yun *et al.* (1998) applied genetic algorithms to the Jingtieshan iron ore mine in China in order to optimize cut-off grade and minimum average grade, which is a criterion used in China to define ore for mining purposes. They used net present value as a fitness value, binary representation, roulette wheel selection, and 100 iterations (number of generations).

Encoding and evaluation processes used in this paper for the application of genetic algorithms to cut-off grade optimization are described below.

Encoding

Encoding of an individual for the optimization of a single cut-off grade for an ore deposit with only one valuable mineral is straightforward. The string is composed of only one gene, which represents a cut-off grade. The size of the string depends on the number of cut-off grades to be evaluated (searched). If the binary representation is used, the string will be long. A 5-bit string can represent $2^5 = 32$ cut-off grades. To derive real values from the binary code (*i.e.* mapping) the string the formula is:

$$X = X_{min} + \frac{X_{max} - X_{min}}{2^L - 1} * Y$$

where

X:	the mapping value
X_{min}:	the minimum cut-off grade to be searched for
X_{max}:	the maximum cut-off grade to be searched for
L:	the length of the binary string
Y:	the value of binary representation.

The value of the binary representation for a 5-bit string would be an integer between 0 for string 00000 and 31 for string 11111.

The application of genetic algorithms to the solution of the optimum cut-off grade problem requires a crucial increase in the length of the string. Since in each year in the mine life there might be a different optimum cut-off grade, there should be different genes in the same string. If the mine life is 20 years, the string will be composed of 20 genes, each with a length of five bits, making the total length of the chromosome 100.

Evaluation

In genetic algorithms, every individual is assigned to a fitness value depending on its performance. In cut-off grade optimization, the objective function is maximum NPV. The higher the discounted profit, the better the individual.

Optimization of cut-off grades for multi-mineral deposits by genetic algorithms

The optimization of cut-off grades for multi-mineral deposits is significantly more complex than for single-mineral deposits. A multivariate grade distribution must be used and consequently the dimension of the data increases. This increase in dimension causes an exponential increase in the area to be searched for the optimum.

Besides, genetic algorithms work on representatives of solutions, known as chromosomes. The structures of chromosomes for single-mineral deposits and for multi-mineral deposits differ in that as the number of minerals increases, the length of the related chromosomes increases arithmetically.

The application of genetic algorithms to the optimization of cut-off grades for multi-mineral deposits brings about a further increase of the length of the string. Since for each year of mine life there might be a different optimum cut-off grade, there should be different genes in the same string. In the case of a two-mineral deposit, if the mine life is 20 years, the string would be composed of 40 genes, each five bits in length, making the total length of the chromosome 200.

Multiple cut-off grade optimization by genetic algorithms

The genetic algorithms computer program developed in this research work is capable of optimizing cut-off grades for mineral deposits that contain up to three minerals. Binary representation is used and if 32 different cut-off grades are to be searched for each mineral, 5-bit genes must be used. Three minerals require a 15-bit string length. Therefore, if the maximum mine life is 20 years, an ore deposit that contains three minerals requires a string size of 300.

The process of scaling is used in order to improve the selection process. Fitness values are scaled by subtracting the fitness value of the worst individual from the fitness values of all the individuals of the population.

One safeguard has proved necessary to improve the computation results. We know that true maximization of NPV necessitates a sequence of declining cut-off grades. However, only a very small part of randomly selected populations can have cut-off grades in declining order for the life of the mine. Consequently, the algorithm has been changed in such a way that if the depletion rate for a specific year is more than that of the previous year, the cut-off grade for the specified year is set deterministically to that of the previous year. This policy enables the program to search for the optimum among the alternatives that are limited to sequences of declining cut-off grades, and brings about a substantial improvement in the performance of the algorithm.

With respect to the other two methods used in this research, genetic algorithms use four additional parameters that are not directly related to technical or economic constraints. These parameters are population size, generation size, crossover rate, and mutation rate. These parameters have been tested in order to determine an optimum range of control values that will generate the highest discounted profit. As a result of the tests, a population size between 250 and 500, a generation size between 400 and 500, a crossover rate of 20% to 50%, and a mutation rate of 40% to 100% are proved to be reasonable.

For the sake of comparison, the results have been tested by other methods that were used in multi-mineral cut-off grade optimization. These are the grid search method and the dynamic programming method. The grid search method used here is explained by Cetin and Dowd (2013). The use of dynamic programming in multi-mineral cut-off grade optimization used in this work is explained by Cetin and Dowd (2011).

Case study

A case study has been included here to illustrate the application of the software for determining optimal cut-off grades for multi-mineral deposits.

The case study is of a gold, lead, and zinc deposit. The technical and economic data are shown in Table I and Figure 2. The results are given in Table II.

For the sake of comparison, the deposit shown in Figure1 is applied to the grid search method and dynamic programming method. The technical and economic data for the grid search method are shown in Table III and Figure 2. The results are given in Table IV. The technical and economic data for dynamic programming method are shown in Table V and Figure 2. The results are given in Table VI.

Table VII compares the results of the three methods.

The results indicate that all three method give reasonable results but genetic algorithms deliver a better result. Genetic algorithms is a more robust search engine since it can easily escape from a local optimum point by means of crossover and mutation tools, and its natural selection environment.

Conclusions

The paper shows the applicability and robustness of genetic algorithms methods to multi-mineral cut-off grade optimization.

Determination of a complete mine production schedule requires complex modelling of an orebody and the inclusion of access constraints. The work described serves to find broad indications of optimum cut-off grades and a mining sequence that gives optimum discounted profits by using technical and economic constraints only. Detailed mine scheduling that includes physical, or access, constraints is beyond the scope of this research. The orebody is defined by a grade-tonnage distribution, which gives the ore tonnage for different grade intervals. Access constraints are not included, so that any parcel of the orebody is assumed to be immediately accessible. In other words, the grade-tonnage

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Table I

Technical and economic data for genetic algorithms for the gold, lead, and zinc deposit

Description	Value
Lower limit of cut-off grades for gold (%)	0
Upper limit of cut-off grades for gold (%)	0.009
Lower limit of cut-off grades for zinc (%)	0
Upper limit of cut-off grades for zinc (%)	3
Lower limit of cut-off grades for lead (%)	0
Upper limit of cut-off grades for lead (%)	1.5
Mining capacity (t/a)	1 200 000
Mineral processing capacity (t/a)	1 000 000
Marketing and/or refining capacity for gold (t/a)	10
Marketing and/or refining capacity for zinc (t/a)	11000
Marketing and/or refining capacity for lead (t/a)	1600
Selling price for gold (dollars per ton)	11 000 000
Selling price for zinc (dollars per ton)	1 400
Selling price for lead (dollars per ton)	600
Marketing and/or refining cost for gold (dollars per ton)	3 000 000
Marketing and/or refining cost for zinc (dollars per ton)	300
Marketing and/or refining cost for lead (dollars per ton)	150
Recovery rate for gold (%)	46
Recovery rate for zinc (%)	80
Recovery rate for lead (%)	85
Variable mining cost of material mined (dollars per ton)	0.4
Variable concentration cost of material processed (dollars per ton)	0.4
Fixed costs (dollars per year)	1 000 000
Discount rate (%)	10
Population size (number of individuals in the population)	500
Number of generations	500
Crossover rate (%)	50
Mutation rate (%)	60

Multiple cut-off grade optimization by genetic algorithms
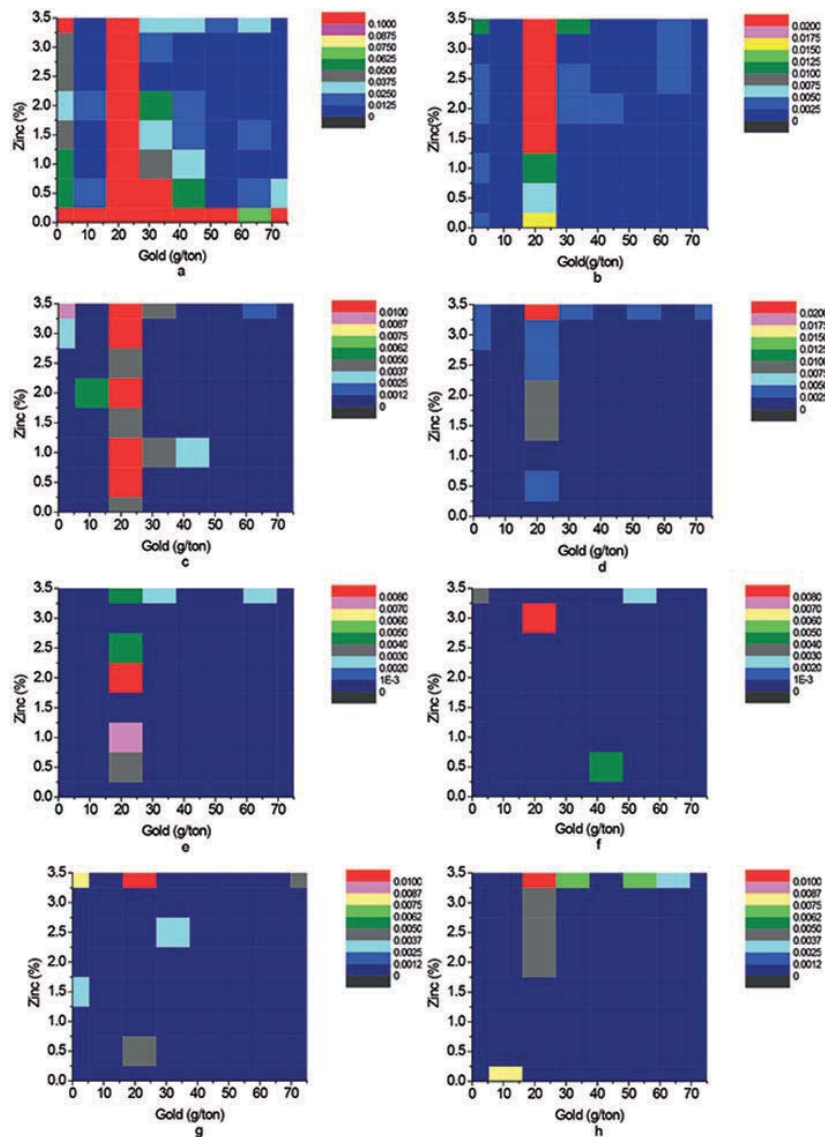
Figure 2—Grade-tonnage distribution for the gold, lead, and zinc deposit. Distributions from the first to the last lead grade intervals can be seen from sections (a) to (h). Figure 2a shows the lead grade interval between 0 and 0.2%, Figure 2b shows the lead grade interval between 0.2% and 0.4%, and Figure 2h shows the lead grade interval 1.4% and higher. The numbers next to the colours indicate the reserve tonnages (in millions of tons)

Table II

Output file for genetic algorithms

				The optimum discounted profit is 439.8e + 6 $						
Year	Profit ($)	Discounted profit ($)	Depletion (t)	Production (t)	Gold (t)	Zinc (t)	Lead (t)	Gold cut-off grade (%)	Lead cut-off grade (%)	Zinc cut-off grade (%)
1	82 319 950	74 836 319	1 164 693	1 000 000	9	10 564	1526	0.0012	3 000	0.800
2	82 319 950	68 033 017	1 164 693	1 000 000	9	10 564	1526	0.0012	3 000	0.800
3	82 020 659	61 623 335	1 158 748	1 000 000	9	10 582	1526	0.0012	2 600	0.800
4	82 020 659	56 021 214	1 158 748	1 000 000	9	10 582	1526	0.0012	2 600	0.800
5	82 020 659	50 928 376	1 158 748	1 000 000	9	10 582	1526	0.0012	2 600	0.800
6	79 839 492	45 067 312	1 114 896	1 000 000	9	10 342	1507	0.0012	0.800	0.700
7	79 839 492	40 790 283	1 114 896	1 000 000	9	10 342	1507	0.0012	0.800	0.700
8	72 691 453	33 911 099	1 000 000	1 000 000	8	9482	1462	0.0000	2 400	1 100
9	18 589 355	8 463 256	255 730	255 730	2	2425	374	0.0000	2 200	0.800

Multiple cut-off grade optimization by genetic algorithms

<table>
<tr><td colspan="2">Table III

Technical and economic data for the grid search method for the gold, lead and zinc deposit</td></tr>
<tr><td>Description</td><td>Value</td></tr>
<tr><td>Lower limit of cut-off grades for gold (%)</td><td>0</td></tr>
<tr><td>Upper limit of cut-off grades for gold (%)</td><td>0,009</td></tr>
<tr><td>Interval between cut-off grade decisions gold (%)</td><td>0,0006</td></tr>
<tr><td>Lower limit of cut-off grades for zinc (%)</td><td>0</td></tr>
<tr><td>Upper limit of cut-off grades for zinc (%)</td><td>3</td></tr>
<tr><td>Interval between cut-off grade decisions zinc (%)</td><td>0,2</td></tr>
<tr><td>Lower limit of cut-off grades for lead (%)</td><td>0</td></tr>
<tr><td>Upper limit of cut-off grades for lead (%)</td><td>1,5</td></tr>
<tr><td>Interval between cut-off grade decisions lead (%)</td><td>0,1</td></tr>
<tr><td>Mining capacity (tons per year)</td><td>1 200 000</td></tr>
<tr><td>Mineral processing capacity (t/a)</td><td>1 000 000</td></tr>
<tr><td>Marketing and/or refining capacity for gold (t/a)</td><td>10</td></tr>
<tr><td>Marketing and/or refining capacity for zinc (t/a)</td><td>11000</td></tr>
<tr><td>Marketing and/or refining capacity for lead (t/a)</td><td>1600</td></tr>
<tr><td>Selling price for gold (dollars per ton)</td><td>11 000 000</td></tr>
<tr><td>Selling price for zinc (dollars per ton)</td><td>1 400</td></tr>
<tr><td>Selling price for lead (dollars per ton)</td><td>600</td></tr>
<tr><td>Marketing and/or refining cost for gold (dollars per ton)</td><td>3 000 000</td></tr>
<tr><td>Marketing and/or refining cost for zinc (dollars per ton)</td><td>300</td></tr>
<tr><td>Marketing and/or refining cost for lead (dollars per ton)</td><td>150</td></tr>
<tr><td>Recovery rate for gold (%)</td><td>46</td></tr>
<tr><td>Recovery rate for zinc (%)</td><td>80</td></tr>
<tr><td>Recovery rate for lead (%)</td><td>85</td></tr>
<tr><td>Variable mining cost of material mined (dollars per ton)</td><td>0,4</td></tr>
<tr><td>Variable concentration cost of material processed (dollars per ton)</td><td>0,4</td></tr>
<tr><td>Fixed costs (dollars per year)</td><td>1 000 000</td></tr>
<tr><td>Discount rate (%)</td><td>10</td></tr>
</table>

<table>
<tr><td colspan="2">Table V

Technical and economic data for dynamic programming for the gold, lead, and zinc deposit</td></tr>
<tr><td>Description</td><td>Value</td></tr>
<tr><td>Lower limit of cut-off grades for gold (%)</td><td>0</td></tr>
<tr><td>Upper limit of cut-off grades for gold (%)</td><td>0.009</td></tr>
<tr><td>nterval between cut-off grade decisions gold (%)</td><td>0.0006</td></tr>
<tr><td>Lower limit of cut-off grades for zinc (%)</td><td>0</td></tr>
<tr><td>Upper limit of cut-off grades for zinc (%)</td><td>3</td></tr>
<tr><td>nterval between cut-off grade decisions zinc (%)</td><td>0.2</td></tr>
<tr><td>Lower limit of cut-off grades for lead (%)</td><td>0</td></tr>
<tr><td>Upper limit of cut-off grades for lead (%)</td><td>1.5</td></tr>
<tr><td>Interval between cut-off grade decisions lead (%)</td><td>0.1</td></tr>
<tr><td>Mining capacity (t/a)</td><td>1 200 000</td></tr>
<tr><td>Mineral processing capacity (t/a)</td><td>1 000 000</td></tr>
<tr><td>Tonnage interval between decisions to mine or not (t/a)</td><td>20 000</td></tr>
<tr><td>Marketing and/or refining capacity for gold (t/a)</td><td>10</td></tr>
<tr><td>Marketing and/or refining capacity for zinc (t/a)</td><td>11000</td></tr>
<tr><td>Marketing and/or refining capacity for lead (t/a)</td><td>1600</td></tr>
<tr><td>Selling price for gold (dollars per ton)</td><td>11 000 000</td></tr>
<tr><td>Selling price for zinc (dollars per ton)</td><td>1 400</td></tr>
<tr><td>Selling price for lead (dollars per ton)</td><td>600</td></tr>
<tr><td>Marketing and/or refining cost for gold (dollars per ton)</td><td>3 000 000</td></tr>
<tr><td>Marketing and/or refining cost for zinc (dollars per ton)</td><td>300</td></tr>
<tr><td>Marketing and/or refining cost for lead (dollars per ton)</td><td>150</td></tr>
<tr><td>Recovery rate for gold (%)</td><td>46</td></tr>
<tr><td>Recovery rate for zinc (%)</td><td>80</td></tr>
<tr><td>Recovery rate for lead (%)</td><td>85</td></tr>
<tr><td>Variable mining cost of material mined (dollars per ton)</td><td>0.4</td></tr>
<tr><td>Variable concentration cost of material processed (dollars per ton)</td><td>0.4</td></tr>
<tr><td>Fixed costs (dollars per year)</td><td>1 000 000</td></tr>
<tr><td>Discount rate (%)</td><td>10</td></tr>
</table>

Table IV

Output file for the grid search

				The optimum discounted profit is 439.1e+6						
Year	Profit ($)	Discounted profit ($)	Depletion (t)	Production (t)	Gold (t)	Zinc (t)	Lead (t)	Gold cut-off grade (%)	Lead cut-off grade (%)	Zinc cut-off grade (%)
1	82 423 897	74 930 816	1 167 041	1 000 000	9	10 572	1527	0.0012	2,600	1,500
2	82 423 897	68 118 923	1 16 7041	1 000 000	9	10 572	1527	0.0012	2,600	1,500
3	82 423 897	61 926 294	1 167 041	1 000 000	9	10 572	1527	0.0012	2,600	1,500
4	82 344 016	56 242 071	1 165 144	1 000 000	9	10 564	1526	0.0012	2,600	1,300
5	82 344 016	51 129 155	1 165 144	1 000 000	9	10 564	1526	0.0012	2,600	1,300
6	80 753 096	45 583 017	1 131 110	1 000 000	9	10 377	1513	0.0006	2,600	1,300
7	80 75 3096	41 439 107	1 131 110	1 000 000	9	10 377	1513	0.0006	2,600	1,300
8	80 30 1619	37 46 1298	1 122 803	1 000 000	9	10 374	1510	0.0006	1,600	1,100
9	5 350 339	2 269 066	74 718	66 991	1	693	101	0.0006	0.800	1,400

Multiple cut-off grade optimization by genetic algorithms

Table VI

Output file for dynamic programming

				The optimum discounted profit is 434.1e+6							
Year	Profit ($)	Discounted profit ($)	Depletion (t)	Production (t)	Gold (t)	Zinc (t)	Lead (t)	Gold cut-off grade (%)	Lead cut-off grade (%)	Zinc cut-off grade (%)
1	81 144 899	73 76 8090	1 160 000	999 881	9	10584	1525	0.0024	3 000	0.300
2	81 144 899	67 06 1900	1 160 000	999 881	9	10584	1525	0.0024	3 000	0.300
3	81 144 899	60 96 5364	1 160 000	999 881	9	10584	1525	0.0024	3 000	0.300
4	81 14 4899	55 42 3058	1 160 000	999 881	9	10584	1525	0.0024	3 000	0.300
5	79 199 360	49 17 6571	1 120 000	997 909	9	10349	1507	0.0012	2 800	0.200
6	79 19 9360	44 70 5974	1 120 000	997 909	9	10349	1507	0.0012	2 800	0.200
7	79 19 9360	40 64 1795	1 120 000	997 909	9	10349	1507	0.0012	2 800	0.200
8	71 81 4509	33 50 1999	1 000 000	1 000 000	8	9482	1462	0.0000	3 000	0.000
9	20 90 8899	8 867 414	291 151	29 1151	2	2761	426	0.0000	3 000	0.000

Table VII

Cut-off grades over time for the three methods and for the three constituent minerals of the case study

Year	Genetic algorithms gold cut-off grade (%)	Genetic algorithms lead cut-off grade (%)	Genetic algorithms zinc cut-off grade (%)	Grid search gold cut-off grade (%)	Grid search lead cut-off grade (%)	Grid search zinc cut-off grade (%)	Dynamic programm grade (%)	Dynamic programm grade (%)	Dynamic programm grade (%)
1	0.0012	3 000	0.800	0.0012	2 600	1 500	0.0024	3 000	0.300
2	0.0012	3 000	0.800	0.0012	2 600	1 500	0.0024	3 000	0.300
3	0.0012	2 600	0.800	0.0012	2 600	1 500	0.0024	3 000	0.300
4	0.0012	2 600	0.800	0.0012	2 600	1 300	0.0024	3 000	0.300
5	0.0012	2 600	0.800	0.0012	2 600	1 300	0.0012	2 800	0.200
6	0.0012	0.800	0.700	0.0006	2 600	1 300	0.0012	2 800	0.200
7	0.0012	0.800	0.700	0.0006	2 600	1 300	0.0012	2 800	0.200
8	0.0000	2 400	1 100	0.0006	1 600	1 100	0.0000	3 000	0.000
9	0.0000	2 200	0.800	0.0006	0.800	1 400	0.0000	3 000	0.000

distribution is identical for all parts of the orebody and for all parcels of ore.

It is very clear from this work, and that done by others, that maximum NPV can be achieved only by a declining cut-off grades policy. That is, the mining operation should start with a relatively high cut-off grade that declines gradually over the life of the mine. For that reason, and in order to increase the speed of the computations, production schedules that do not have declining cut-off grades are eliminated explicitly in the computer program.

The genetic algorithms method is a very robust search engine. The crossover, mutation and natural selection behaviour of the method ensures that it escape from a local optimum point.

The software developed for this study includes programs for the determination of optimum cut-off grades for multi-mineral deposits by means of the genetic algorithms, grid search method, and dynamic programming are written in C++ code. Although all the programs written are basically for cut-off grade optimization, they are slightly different in terms of data requirements.

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In situ mining through leaching: experimental methodology for evaluating its implementation and economic considerations

by C. Bahamóndez*, R. Castro*, T. Vargas†, and E. Arancibia‡

Synopsis

Rising costs in the mining industry have necessitated a search for alternative methods for the recovery of metals from deposits that are no longer economically or environmentally exploitable by conventional mining. These alternative methods include *in situ* mining.

A laboratory model was developed and an experimental programme undertaken to determine the effect of temperature, aeration, material compression, and material extraction on copper recovery by *in situ* leaching using H_2SO_4 and Fe^{3+}. Recovery was estimated using the shrinking core model. Based on the experimental results and recovery estimations, an economic evaluation was completed comparing *in situ* mining with conventional mining methods.

Keywords
mining methods; *in situ* leaching; copper; laboratory model; experimental protocol.

Introduction

The decrease in average copper grades, increasing depth of deposits, depletion of copper oxide ores, and increasing energy costs have caused an overall increase in operational and capital costs of copper mining (Bearman, 2007; Harmsen *et al.*, 2013). This has led to a continuous search for new mining methods and technologies that would complement or replace conventional mining methods and allow metal recovery at lower costs. *In situ* leaching (ISL) has been proposed to recover metals more cheaply, with less environmental damage and less energy usage than conventional mining (O Gorman *et al.*, 2004). In principle, ISL removes the metals while leaving the deposits essentially undisturbed by conventional mining (Schlitt, 1992). Metal is extracted from the host rock by the injection of a chemical solution into the orebody. The pregnant solution is then pumped to the surface, where the metals are recovered (IAEA, 2001). ISL is the major method for uranium mining and has been successfully used since the mid-1970s in the USA and the former Soviet Union (Akin *et al.*, 1996; Mudd, 2001). However, in the case of copper, the use of ISL remains in the experimental stage due to insufficient metal recoveries and long leaching times, mainly because of poor solution contact with the ore and poor aeration (Gorman *et al.*, 2004), which makes the method uneconomic. Therefore, an *in situ* mining (ISM) method has recently been proposed, which integrates conventional underground mining method(s) and ISL in a novel way (Castro *et al.*, 2013).

In the ISM method, blasted rock in a large stope is irrigated with a leaching solution, if the sublevel stoping (SLS) method was employed to access the orebody. The leaching process is carried out taking into consideration, for example, the ventilation and extraction system of a large stope, where the ore could be compacted over time due to the characteristics of the SLS method. It is expected that ISM will improve on the metallurgical recovery obtained with ISL through the incorporation of mining procedures and the control of metallurgical variables such as degree of compression, removal of material, temperature, and ventilation.

In this study, a laboratory model and an experimental methodology were developed to evaluate the influence of the main operational variables on the ISM method for copper sulphide deposits. Continuous samples of the pregnant leach solution (PLS) were measured and analysed for pH, redox potential, and dissoluble copper by atomic absorption spectrometry. During leaching of a copper sulphide sample, the gallery was aerated using an air pump. The adjustment of copper curves within the shrinking core kinetic model over time showed that all experiments exhibited a diffusional control mechanism.

* *Department of Mining Engineering, Advanced Mining Technology Center, University of Chile, Chile.*
† *Department of Chemical Engineering and Biotechnology, Advanced Mining Technology Center, University of Chile, Chile.*
‡ *CODELCO Chile, Chile.*

In situ mining through leaching

In situ leaching

ISL involves irrigating an ore deposit with a leaching solution through injection wells. The injected solution permeates through channels in the ore and solubilizes the metal(s) of interest. The PLS is then returned to the surface through recovery wells to be processed (Figure 1) (NRC, 1997a).

According to the literature (Morais *et al.*, 2008; Venter *et al.*, 2009), there are many similarities in the leaching processes for copper and uranium deposits. Solutions used in both cases are either acid or alkaline, with the most common being sulphuric acid with the addition of an oxidizing agent. Moreover, for both copper and uranium, injection and recovery wells are used to irrigate the orebody and recover the PLS (Llorente, 1991). In 2013, 47% of the world's uranium produced was produced through the ISL method (WNA, 2014). Despite the resemblances to uranium extraction, the recovery of copper by ISL is less satisfactory. Some of the more likely reasons include the difference in particle size, control of solutions leaks, and mineralogy.

The literature and case study show (Gorman *et al.*, 2004; Pradhan *et al.*, 2008) that the variability of metallurgical recoveries through the ISL is associated with low or no control of geometallurgical variables such as fragmentation, aeration, and temperature. To overcome these deficiencies, the ISM method, in which leaching and conventional underground mining methods are integrated in a novel way, was proposed (Castro *et al.*, 2013).

In situ mining method

As shown in Figure 1, the development for ISM is similar to the SLS method, but the broken material is irrigated inside the stope instead of being loaded and hauled to the surface (Castro *et al.*, 2013). ISM considers that only the material corresponding to the increase in volume due to blasting will be transferred to the surface. Consequently, mining and processing costs would decrease significantly due to the reduction in the amount of material that needs to be moved and elimination of the need for comminution processes. In addition, the lower impact on the surface due to less waste production and better prevention and monitoring of solution leakages (compared with ISL, since the irrigation of the mineral is delimited by the stope and is located underground), delivers environmental benefits. Furthermore, ISM allows a better control of fragmentation by blasting than

ISL, by increasings the mineral surface exposed to the leaching solution, improving the permeability, and thus enabling control of other variables: temperature, aeration, and material extraction.

The ISM method consists of the following unit operations: drilling and blasting, loading and hauling, ventilation, and stope irrigation.

Drilling and blasting is the first fragmentation process that will increase the mineral surface exposed to the leaching solution as well as the permeability of the ore to be irrigated. The blasting design should be aimed at obtaining the degree of fragmentation that minimizes the costs and maximizes the copper recovery. Loading and haulage is limited to removing the swell material. In the case of channelling or reduction in permeability, constant extraction of material at drawpoints could probably improve recovery by reordering the fragments and creating high-permeability zones.

The application of air flow provides the required oxygen to increase sulphide copper oxidation and to improve the activity of leaching microorganisms (Lorca, 2004; Ghorbani *et al.*, 2011). This effect has been observed at the Miami mine in Arizona, USA, where maintaining an efficient ventilation system greatly contributed to the copper recovery (Herrera, 1987). The aeration requirements should therefore be evaluated with respect to the leaching process parameters (*i.e.* mineralogy) (Wu *et al.*, 2006).

The irrigation system must allow homogenous irrigation of the material as well as consistent concentration of the leaching solution inside the stope. For the irrigation, it is important to consider the drilling of broken material and intubation of the wells. The set-up proposed for ISM is shown in Figure 1. The influence of mining variables was studied (air flow, temperature, and granular density) through controlled experiments, and the results are described in subsequent sections.

Experimental methodology

Model design

A laboratory model (Figure 2) was designed and constructed to evaluate the effect of various mining and metallurgical parameters on copper recovery using the ISM method. The main structure of the model was a high-density polyethylene cylinder 700 mm in height and 400 mm in diameter. A drawbell, made of Robalon, and a gallery were attached to the
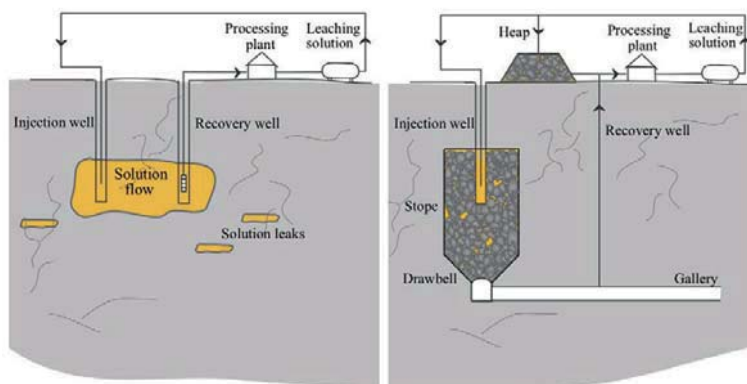

Figure 1—Schematic representation of ISL (left) and ISM operations (right). Ventilation is not shown
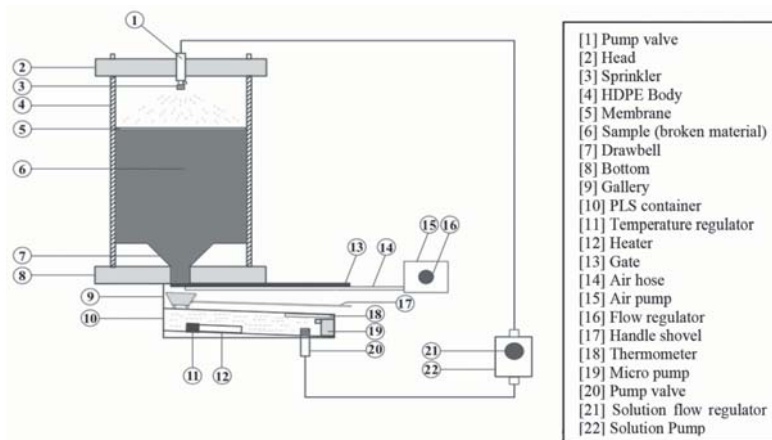
In situ mining through leaching

Figure 2—Schematic representation of the laboratory model

base of the cylinder for extraction of material. A container for the PLS was placed under the gallery. To ensure the constant recirculation of the PLS, a pump was incorporated next to the PLS container. To investigate the effect of ventilation, an air pump was added to the model. A heater and temperature regulator were used for temperature control, and a hydraulic press to investigate the effect of compression of the material. Figure 3 shows photographs of the laboratory model.

As shown in Figure 4A, a stainless steel plug closed the drawbell and prevented movement of material during leaching, but enabled drip-feeding of PLS. An acrylic gallery was placed under the drawbell. The gallery, designed with a 5.5° inclination, included a set of holes to allow the drainage of the PLS into the container.

The laboratory model was designed to allow a vertical load to be applied to the confined material. The model dimensions also minimized the effect of the wall, avoiding the solution flowing through the pipe rather than the material (Llorente, 1991). The model had a capacity to contain 75–80 kg of crushed sample.

Mineral samples

An ore sample was obtained from a copper mine located in the Atacama region, Chile. The sample was crushed using a roller crusher and then classified using four sized sieves (Table I). The size distribution was arbitrarily defined for this research.

The samples granulometry shown in Figure 5 was obtained by crushing. The uniformity index ($d60/d10$) of 2.4 reveals a well-graded distribution of samples.

Copper grade was estimated at 0.6% by X-ray fluorescence spectrometry (Table II). Among all copper species observed by X-ray diffraction (1.9% of the total), 1.2% corresponded to secondary sulphides and 0.7% to primary sulphides (Figure 6). No copper oxides were identified in the sample. Optical microscopy revealed the presence of molybdenum as MoS_2 and titanium, as TiO_2.

Leaching experiments

The leaching experiments were performed using 75 kg of sample. Leaching solutions were prepared with reagent-grade chemicals and distilled water. The leaching solution

contained 20 g/L H_2SO_4 and 3 g/L of Fe (III) as the oxidizing agent. Sulphuric acid was employed to maintain the desired pH (1.8–2.0). Samples of the solution were continuously taken from the PLS container to determine the pH, Eh, and copper concentration by atomic absorption spectrometry.

Figure 3—Lateral (A) and front view (B) of the laboratory model, and sample compression by hydraulic press (C)

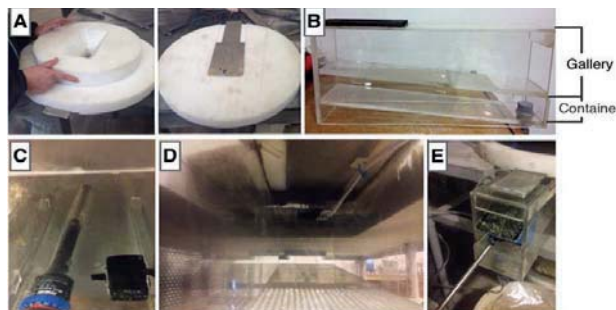**Figure 4—Drawbell (A), gallery and PLS container (B), heater and micro pump incorporated into the solution container (C), air hose connected to the gallery ceiling (D), and handle shovel inside the gallery during mineral extraction (E)**

Table I

Sieve sizes for sample classification

Sieve	Size [mm]
-8+10	2.36
-4+8	4.75
-1/4+4	6.4
-3/8+1/4	9.5

Table II

Elemental composition of the samples

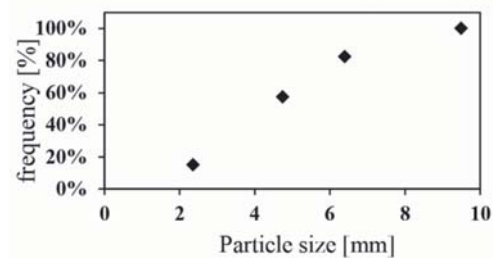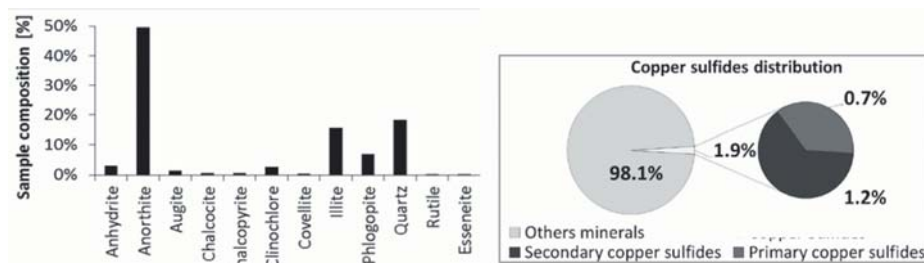| Element	Si	Al	Ca	Fe	Mg	S	K	Ti	Cu	Sr	Cr	V	Mo	Mn	Zn	Zr	Rb
Composition (%)	51.7	21.3	6.92	6.43	1.98	5.59	4.26	0.96	0.63	0.08	0.05	0.04	0.03	0.04	0.03	0.02	0.01

Figure 5—Cumulative frequency of sample granulometry

Figure 6—(A) Mineralogical composition of the samples and (B) copper sulphide content distribution

Table III

Experimental cases and conditions

Case	Sample (kg)	H$_2$SO$_4$ (g/L)	Fe^{3+} (g/L)	Initial density (t/m³)	T (°C)	Artificial aeration	Material extraction
Base case	75	20	3	1.15	20	No	Yes
Case 1a	75	20	3	1.27	20	No	Yes
Case 1b	75	20	3	1.27	20	No	No
Case 2	75	20	3	1.15	29	No	No
Case 3	75	20	3	1.15	20	Yes	No

Five experimental cases were developed (Table III). The base case considered the leaching of the sample without previous compression, at room temperature (20°C average) and without artificially induced air flow. The effects of four variables were studied: density, extraction, temperature of the leaching solution, and air incorporation. To study the effect of compression, the density of the sample was increased by 10.4% from 1.15 to 1.27 t/m³. In the case of material extraction, a removal rate of 1.5 kg/d for 7 days was considered. To study the influence of temperature, the temperature was increased incrementally up to an average of 29°C, and to evaluate the effect ventilation, the air pump was used to provide 270 L/h of air to the model. The laboratory protocol is shown in Figure 7.

Results and discussion

Effect of compression

The natural compression of the material by the overburden weight results in an increase in density and decreasing porosity with increasing depth (Fatt, 1952; Bass, 1980). The increase in pressure due to taller leach columns also leads to a reduction in the porosity and, therefore, a decrease in leaching efficiency (Dixon, 2007). During the laboratory experiments, increasing the material density by 10.4% led to a reduction of the recovery: 16.5% in case 1a and 13.4% in case 1b, in comparison to the 19.9% in the base case within the same period of time (Figure 8A). This represents a reduction of 17.1% and 32.6% in copper recovery for cases
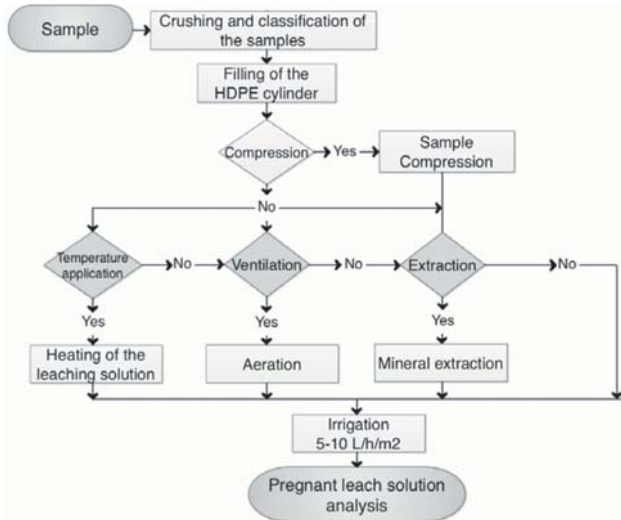
In situ mining through leaching

Figure 7—Experimental work flow

1a and 1b, respectively. Therefore, the compression of the ore being leached must be considered in the future work to prevent over-estimation of the recovery through the leaching operation. The applied vertical stress must be defined according to the height of the stope.

Effect of material extraction

The effect of material extraction was studied in two cases, with and without initial compression of the sample and after 750 hours of irrigation. As indicated in Figure 8B, only the base case shows an increase in copper recovery after material extraction. Even though the results from the laboratory testing were inconclusive with regard to the effect of material extraction on copper recovery, the benefits of extraction cannot be dismissed as the removal of material had been shown to improve the permeability by creating zones of higher porosity (Kvapil, 1992) and could contribute to reordering of the fragments and channeling of the solution. Therefore, it is recommended to continue the study by extending the range of extraction and initial permeability.

Effect of temperature

During the leaching process, the temperature can rise naturally due to the exothermic nature of the sulphide oxidation reactions. The effect of the geothermal gradient (1°C for each 100 m) should also be considered. The temperature may also be maintained artificially through heating of the leaching solution. High dependence of the metallurgical recovery on the temperature was observed by Lorca (2004). The increase in the temperature reduces the passivation of the sulphide minerals and makes the passivation layer less stable (Lorca, 2004; Pradhan *et al.*, 2008). The activity of leaching microorganisms also improves with increasing temperature (Kelly *et al.*, 2008). In the present study, copper recovery increased from 19.9% to 29.8% when the temperature of the leaching solution was increased from 20°C to 29ºC, representing a 49.7% improvement over the base case scenario (Figure 8C).

Effect of aeration

An air flow of 270 L/h into the drawbell base increased the recovery from 19.9% to 33.8%, which represents a 69.8% improvement over the base case scenario (Figure 8D).

Summary of effects of selected variables

The copper recovery in cases 2 and 3, (Figure 8D) shows the necessity of considering the effects of variables when designing the ISM method. The decision should be made based on the economic evaluation of the project. It is envisioned that there will be a trade-off between increasing mining and processing costs and increasing copper recovery, which will influence the net present value (NPV) estimation.

In addition to pH, Eh, and copper measurements, the PLS was examined under the microscope, and microorganisms were detected. Considering that no chemical oxidant were added after the experiments began, the presence of microorganisms in the PLS allowed the ferric iron to regenerate during the leaching process.

Table IV shows a resumé of the experimental variables and copper recoveries.

Kinetic analysis and modelling

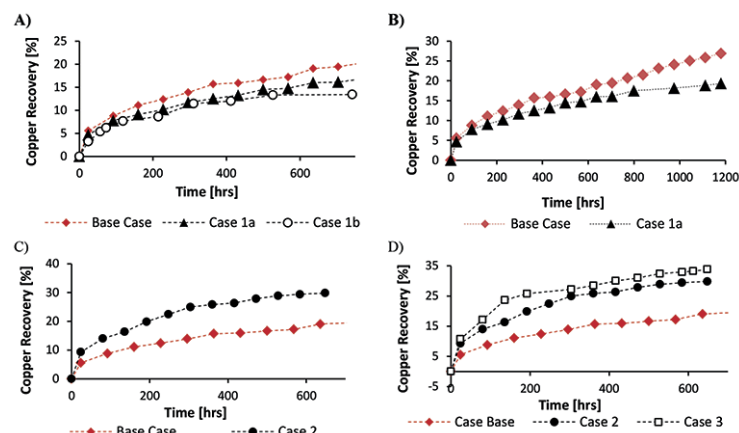The leaching kinetics were described by the shrinking core

Figure 8—Influence of (A) compression, (B) material extraction, (C) temperature, and (D) aeration on copper recovery

In situ mining through leaching

Copper recovery for each experimental case

Case	Test duration (hours)	Fixed variables	Changed variable	Final copper recovery (%)
Base case	1180	Temperature, aeration, density	-	23.1
Case 1a	1180	Temperature, aeration	Density increased from 1.15 to 1.27 t/m³	16.5
Case 1b	743	Temperature, aeration	Density increasedfrom 1.15 to 1.27 t/m³	13.4
Case 2	648	Aeration, density	Temperature increased from 20°C to 29°C average	29.8
Case 3	648	Temperature, density	Aeration: from natural aeration to an artificial flow of 215 L/min	33.8

Table V

Chemical- or difussional-controlled reaction parameters according the shrinking core model

	R^2 chemical controlled	R^2 diffusion controlled	Control type	K_{dif}	τ (years)	$\Delta\% \ \tau$*
Base case	0.88	0.98	Diffusion	2×10^{-5}	6.6	-
Case 1a	0.81	0.94	Diffusion	1×10^{-5}	10.7	↑**62.1**
Case 1b	0.80	0.92	Diffusion	9×10^{-6}	11.1	↑**68.2**
Case 2	0.85	0.96	Diffusion	5×10^{-5}	2.2	↓**66.7**
Case 3	0.80	0.94	Diffusion	7×10^{-5}	1.6	↓**75.8**

* Variation between the registered value in the base case and the respective case study

model. The reaction-controlled process is expressed by Equations [1] and [2] (Levenspiel, 1962). Equation [1] presents the relationship between the copper recovery (X), the time of reaction (t), and the theoretical time for the leaching reaction to proceed to completion (τ).

$$t = \tau \cdot (1 - (1 - X)^{1/3}) \tag{1}$$

$$\tau = (\rho \cdot R) / b \cdot k_s \cdot C_{AL} \tag{2}$$

where τ is a function of the density of the solid (ρ the particle radius (R), the molar mass(b), the kinetic coefficient (k_s), and the concentration of the leaching solution (C_{AL}). On the other hand, the diffusional-controlled process is described by Equations [3] and [4]; in this case (τ) is also a function of the effective diffusion (D_{eff}).

$$t = \tau \cdot [1 - 3 \cdot (1 - X_B)^{2/3} + 2(1 - X_B)] \tag{3}$$

$$\tau = (\rho_B \cdot R^2) / 6b \cdot D_{eff} \cdot C_{AL} \tag{4}$$

Table V shows the diffusional reaction controlling mechanism, which best describes the experimental results. Also shown are the experimentally determined K_{diff} values and the theoretical time (τ) for completion.

Based on the diffusional-controlled equations, the copper recovery with time can be improved by decreasing the fragment size, increasing the temperature (increasing the effective diffusion through the product layer), increasing the concentration gradient, or decreasing the thickness of the diffusion layer, among other possibilities.

Using the shrinking core model, the maximum copper recovery is estimated at t = 10 years (Figure 9). The estimation is developed for six different granulometries, considering the experimental results of the base case and case 3. The effect of fragment size is greater in the case base

than for case 3. Case 3 (aertion study) resulted in the best experimental recovery during the leaching tests.

As shown in Figure 9, the difference in the estimated copper recovery (X) between the base case and case 3 (including air flow) increases with particle size. This relationship is shown in Table VI.

Business model and economic evaluation

The second part of the investigation was focused on the feasibility and applicability of the ISM method. A business model was developed to investigate the feasible point of applying ISM. The case study was conducted on an orebody that could be considered as a mid-size mining operation in Chile. The orebody was exploited by conventional SLS, which was economically compared with the same orebody exploited by ISM.

Mining and processing costs and investment

Both initial cases considered a production rate of 1 000 kt/a. The capital and operational costs for the SLS were derived from a preliminary economic assessment developed for an ore deposit located in the north of Chile.

As indicated in Table VII, the mining cost will reach US$16.28 per ton for conventional mining and US$9.16 per ton for ISM. In the case of ISM, an additional 15% is incorporated for the leasing of the mining equipment.

According to Table VII, the main difference between the mining costs for these methods is due to the reduction in loading and hauling of ore, since in ISM only the portion corresponding to swelling is sent to the surface after blasting.

In terms of the operational processing plant costs, the conventional mining case includes primary and secondary crushing, agglomeration, stacking, leaching, solvent extraction (SX), and electrowinning (EW). Considering these
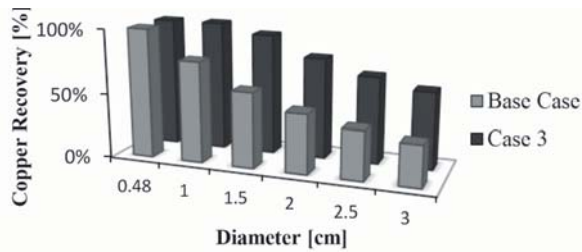
In situ mining through leaching

Figure 9—Estimations of copper recovery applying the shrinking core model for six different particle sizes. Estimations made for two cases: base case and case 3, at _t_ = 10 years

Table VI

Estimated copper recovery (%) for the base case and case 3 for six different fragment sizes at _t_ = 10 years

Particle diameter (cm)	XCase 3–XBase Case (%)
0.48	0
1.0	22
1.5	35
2.0	32
2.5	29
3.0	27

Table VII

Operating costs in conventional mining and ISM

	Conventional US$ per ton	ISM US$ per ton
Production drilling	0.53	0.79
Production blasting	0.38	0.56
Haulage	0.62	0.12
Transport	0.93	0.18
Production fortification	0.85	0.55
Production drill-holes	1.09	1.42
Mining services	2.12	0.86
Sub-total (considering ISM leasing)	6.52	4.50
Maintenance cost	7.26	1.42
Stope preparation	2.50	3.25
Total	16.28	9.16

operations, the processing cost in conventional mining method would be US$15.3 per ton. It is expected that ISM would not require such operations as crushing, agglomeration, and stacking. By benchmarking of different operations in Chile, it was established that approximately 40.9% of the processing costs correspond to SX-EW

operations and 59.1% to the other operations (comminution processes, agglomeration, stacking, and leaching). Thus, the plant cost for ISM reaches approximately US$6.26 per ton. Based on the Florence project in Arizona (SRK, 2010), the additional operational cost of solution injection and recovery in ISM is considered at US$0.28 per pound Cu.

The mine investment includes the development of ramps, ventilation equipment, shafts, and mining equipment. The mine investment has been estimated as US$6.334 million for conventional mining. For ISM, it is estimated that less investment would be required, due to the less intensive use of equipment as only a fraction of the material is hauled to the surface, which will required an investment of US$526 000.

The capital expenses of the processing plant for conventional mining include the costs of equipment for comminution and agglomeration, leaching, SX, EW, tank farm, and civil works among other operations. Therefore, the processing plant investment for conventional mining has been estimated at US$38.54 million. ISM does not entail comminution and agglomeration, but includes the development of injection and recovery wells. The total plant investment for ISM corresponds to US$19.11 million. This includes a scaled average cost of US$3.27 million for the wellfield, based on the case studies of San Manuel, Florence, and Gunnison mines (Williamson, 1998; SRK, 2010; M3, 2011).

The capital and operating costs of mining and processing plant are summarized in Table VIII. The copper recovery in the conventional mining method, for this economic evaluation, is considered constant and equal to 85%, while copper recovery in ISM is dependent on the leaching kinetics (Figure 10A). The volume of PLS considered to be sent to the processing plant per year in ISM is presented in Figure 10B.

Table VIII

Main economic parameters used to compare ISM and conventional mining

| Parameter	Unit	Conventional	ISM
Maximum copper recovery	%	85	60
Copper price	US$/lb	2.80	2.80
Discount rate	%	10	10
Mine operational expenses	US$/ton	16.3	9.2
Processing plant operational expenses	US$/ton	15.28	6.82
Wellfield operational expenses	US$/lb Cu	0	0.28
Mine capital expenses	US$ million	6.334	0.526
Processing plant capital expenses	US$ million	38.5	15.8
Wellfield capital expenses	US$ million	0	3.3
Production rate	kt/a	1,000	1,000
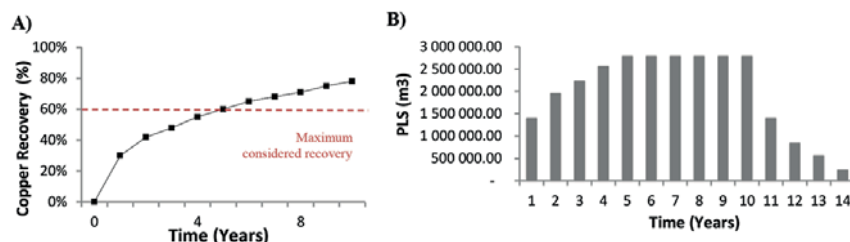
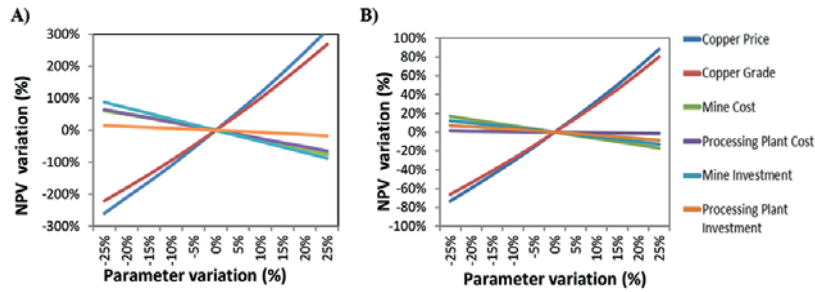**Figure 10—(A) Estimated metallurgical copper recovery in ISM, (B) volume of PLS sent to plant**

Figure 11—Sensitivity analysis of NPV for (A) conventional mining and (B) for ISM

Table IX

Economic indices estimated for conventional and ISM perations

	kt/a	kt Cu / year	Recovery (%)	Initial capex (US$ million)	NPV (US$ million)	NPVI	IRR (%)	Average cash cost (US$/lb)	Life of mine (years)
Conventional	1000	10.9	0.85	64.4	60.95	0.95	27	1.39	10
ISM case A	1000	Variable	Max. 60%	23.1	62.05	2.68	37	1.19	14
ISM case B	Variable	10.9	Max. 60%	27.5	87.45	3.19	62	1.07	10
ISM case C	1000	Variable	Maax. 67%	24.8	94.28	4.07	60	1.12	11

NPV

According to the economic evaluation, the net present value (NPV) of both the conventional mining and ISM operations is on average, equal to US$61 million (US$60.95 million and US$62.05 million respectively). According to the NPV, internal rate of return (IRR), and NPV/investment (NPVI), both methods are profitable. The average cash cost per pound copper is US$1.39 for SLS and US$1.19 for ISM. ISM also presents a better IRR (27% *versus* 37%) and NPVI (0.95 *versus* 2.68) than conventional mining. Different procedures should be evaluated to enhance the ISM method by increasing the copper recovery, which will improve the NPV, making it more profitable and cost-effective compared to conventional mining.

Sensitivity analyses

Drilling and blasting can increase the leaching efficiency due to the increase in the leachable surface. A sensitivity analysis was carried out to evaluate the impact of this operation on the operating expenses (OPEX). The analysis showed that OPEX is more sensitive to drilling and blasting than to hauling and loading and mine services.

A sensitivity analysis for copper price, grade, costs, and investment was also completed. As shown in Figure 11, the NPV of both methods is more sensitive to the price of copper and grade than to the OPEX and capital expenditure (CAPEX) of the project. A price decrease of 25%, which corresponds to a drop of US$2.17 per pound, results in a negative NPV for the conventional mining case. In the same scenario, ISM maintains a positive NPV of US$18.1 million. When the copper grade is decreased by 25%, the NPVs are US$2.2 million for SLS and US$26.4 million for ISM. Accordingly, under unfavourable price and grades conditions, the ISM method represents a better alternative.

After evaluation of conventional mining and ISM at equal tonnages of blasted material per year (ISM case A), two additional cases were considered:

➤ *Case B*—equal tonnages of copper per year were assumed for ISM and SLS. Based on the leaching kinetics, to recover the same tonnage of copper for ISM, a higher tonnage of ore should be blasted in a shorter period of time
➤ *Case C*—the estimated copper recovery, based on the shrinking core model, in ISM is assumed to be increase up to a maximum of 67% by leaching at elevated temperature. The recovery in SLS is maintained at 85%. To achieve the increase in copper recovery in ISM, additional technology will be required to heat the leaching solution.

Table IX shows the summary of the economic indices estimated for the conventional and ISM cases.

As shown in Table IX, the NPV of the SLS case study is less than that for ISM. The registered NPVs for ISM are A < B < C; Case B considers an accelerated mine plan (a greater tonnage of ore must be blasted and irrigated per year than in case A), and the life of mine is shorter. This results in the cash flow being less affected by the discount rate. A greater NPV is achieved in case C due to the increase in the leaching kinetics by heating of the leaching solution.

The capital expenses of ISM are less than those estimated for conventional mining. ISM considers leasing of mining equipment as is less hauling- and loading-intensive than SLS. The increases of CAPEX in cases B and C are due to the larger number of stopes required in a shorter period of time (case B) and additional technology required in the processing plant (case C).

Based on the results, to optimize the NPV of the project, it is essential to improve control of the parameters that affect copper recovery. This can be achieved, for example, by increasing the surface area of the mineral in contact with the leaching solution by blasting to increase fragmentation, increasing the temperature or the aeration, and increasing the activity of leaching microorganisms. There will be a trade-off between increased recovery of copper over time and increased operational cost.

In situ mining through leaching

As shown schematically in Figure 12A, the increase in the OPEX of ISM (heating of solutions or more extensive drilling and blasting) increases the recovery of copper over time. This effect was observed in case C. The rise in OPEX will have a negative impact on the NPV (Figure 12B). Considering the effects of operational expenditures and that the copper recovery does not increase steadily over time, the ratio of OPEX to copper recovery will decrease as maximum recovery is approached.

Based on Figure 12, Figure 13 shows the NPV–OPEX relationship for ISM. Unlike conventional mining, the increase in OPEX will results in a higher NPV for ISM (green zone) as a consequence of higher copper recovery over time. However, the OPEX/recovery ratio will increase when a slower leaching kinetic is reached or maximum recovery of copper is achieved. When the increase in the OPEX adversely affects the net value, the project's NPV will be negatively affected (red zone). The maximum recovery rate depends on several factors, such as mineralogy and the operational parameters considered in the project.

Conclusion

A laboratory model and an experimental protocol to evaluate in situ mining (ISM) were developed. These allowed a preliminary estimate to be made of the recovery of copper from sulphide deposits. The model design allowed compression and extraction of the mineral sample contained in the leaching column as well as the application of aeration to the ore sample and heating of the leaching solution. The experimental parameters used for air flow and the irrigation rate were taken from the literature, while the size distribution was arbitrarily defined. To evaluate the efficiency of leaching for a given deposit, the post-blasting size fragmentation should be considered in the experimental evaluation.

The results of the experimental test work showed that increasing the material density reduced the copper recovery on average by 24.8%. Thus, ignoring the effect of compression may lead to overestimation of the copper recovery in an ISM operation. With respect to the mineral extraction operation, only the base case showed an increase in the copper recovery. Further studies should be conducted to assess the benefits of, for example, improving the permeability, on recovery. Increasing the temperature of the leaching solution as well aeration of the model through the gallery improved the copper recovery in comparison to the base case by 49.7% and 69.8% respectively. Stope ventilation and heating of the leaching solution should therefore be economically evaluated. In relation to the shrinking core model, a similar kinetic behaviour was observed in the leaching of the copper sulphide samples, where the kinetic data best fitted the diffusion-controlled reaction in all the study cases.

According to the economic evaluation, ISM effectively had lower mining and processing costs and required lower investment than the conventional mining case. In the case studied, with a maximum metallurgical recovery of 60%, ISM and SLS present a NPV of approximately US$61–62 million. By accelerating stope development and irrigation of the ore, the NPV reaches US$87.4 million, representing an increase of 40.9% for ISM. On the other hand, the acceleration of leaching kinetics at elevated temperature results in a NPV of US$94.3 million, an increase of 51.9%. According to the economic index and the sensitivity analysis, the NPV of ISM is more sensitive to metallurgical recovery than to costs. This makes it possible to maximize the project NPV by increasing the copper recovery, even when this results in increased operational and capital costs. However, the relation between costs, recovery, and NPV is neither constant over time nor equivalent among different mineral deposits, since it will depend on the leaching kinetics. The relationship between these variables will not be persistent since the achievable recovery is subject to an upper limit. The maximum OPEX and CAPEX that lead to a profitable increase in metallurgical recovery should be calculated for each case to establish the cost limits under which it is possible to optimize the benefit of the ISM method.
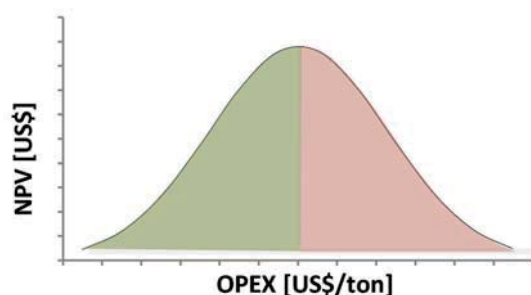
Acknowledgments

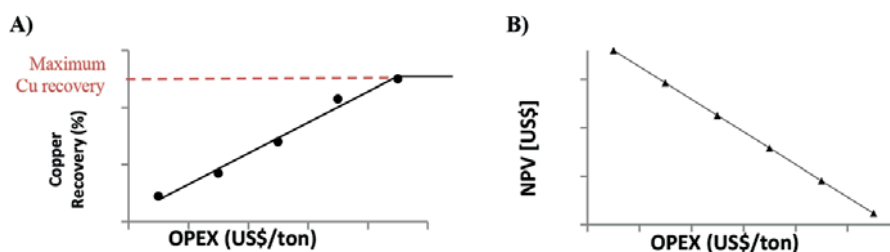Figure 13—Schematic curve of the OPEX *versus* NPV for the *in situ* mining method

Figure 12—Effect of OPEX on (A) copper recovery and (B) NPV

In situ mining through leaching

Abbreviations

CAPEX   Capital expenditure
ISL   *In situ* leaching
ISM   *In situ* mining
IRR   Internal rate of return
NPV   Net present value
OPEX   Operating expenditure
PLS   Pregnant leach solution
SLS   Sublevel stoping

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Coal quality management model for dome storage (DS-CQMM)

by M.A. Badani-Prado*, V. Kecojevic*, and D. Bogunovic†

Synopsis

Coal quality (ash, sulphur, moisture, and heating value) is one of the fundamental concerns for both coal mines and power plants. In order to deliver uniform coal quality to the power plant, there is a need for real-time monitoring of coal quality from the mine to the coal stockpiles. The specific problem represents the process of stacking the coal inside an enclosed facility such as a dome. The objective of this research was to develop a custom-made and integrated coal quality management model for dome storage (DS-CQMM). The DS-CQMM merges existing technology in surface mines, such as coal analysers, together with automation technologies, information technologies (IT), and mathematical models. The DS-CQMM is organized into four major sections: Delay Time application, Stacker application, Reclaimer application, and Live Stockpile application. A sub-process called Volume Calculation is embedded in Stacker application, while an additional feature called Forecast tool is included in the Reclaimer application. The DS-CQMM model was developed for a surface coal mine in the southern USA.

Keywords
coal quality; coal analyser; mathematical modelling; dome storage; stacker, reclaimer.

Introduction

Knowledge of coal quality properties (ash, sulphur, moisture, and heating value) is one of the key aspects for mines and power plants. Modern mines invest time and economic and technological resources in order to manage coal quality for final use in power plants. Since the coal market is characterized by the need for a uniform product of particular specifications (Keleher *et al.*, 1998), it is critical that the delivered product meet the quality requirements imposed by contracts. Oman *et al.* (2001) state that boiler efficiency at power plants is affected mainly by the changes in coal quality. Therefore, coal quality management is a fundamental concern.

In order to deliver a uniform product, mines usually use coal blending in order to homogenize mixtures of coal in such a way that the properties of the final blend satisfy particular specifications. Coal blending is also called coal mixing (Arnold and Smith, 1994). Mines blend coal to not only meet the customer's requirements, but also to optimize the life of their high-quality reserves (Reeves, 1995).

Coal blending can be performed at the mine site, on stockpiles, and in bins, bunkers, and silos (Arnold and Smith, 1994). There are a number of techniques that enhance the blending process, such as different ways of storage (circular stockpiles, longitudinal stockpiles), different procedures for stacking, and different processes of reclaiming. Wolpers (2014) indicates that stockpile homogenization systems equalize variations in chemical and physical properties of the raw materials and transform low-quality grades into a uniform mixture of higher material quality. Therefore, there is a need to know coal quality before stacking coal at the stockpiles so that potential problems can be blended out (France, 1999). Schott (2004) also stated that input properties of coal are a main factor for performance of homogenization of bulk materials in mammoth silos.

Blankenship (1995) used online coal quality data for fuel analysis in order to obtain better coal blending. The fundamental components of online coal quality studies are coal analysers, and they are employed in monitoring, blending, or sorting applications (Laurila, 1995). An example of integration of online coal analysers and a control algorithm is given by Ganguli *et al.* (1998). Prompt gamma neutron activation analysis (PGNAA) is considered to give the best online analysis precision for reporting coal parameters such as ash, moisture, sulphur, and heating value, by determining the concentration of primary elements (France, 1999).

* *Department of Mining Engineering, West Virginia University, Morgantown, WV, USA.*
† *North American Coal Corporation, De Kalb, MS, USA.*

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An example of the importance of coal management is demonstrated by a mine in central Mississippi, which provides coal to a 440 MW power plant. The mine experimented with multiple coal quality analysers in order to find an accurate and reliable solution. The mine and adjacent power plant were challenged with quality deliveries; therefore, a suitable solution was to acquire coal analysers that would ensure that the product delivered to the power plant did not exceed a certain ash content. A dual gamma ash gauge was installed, with an associated microwave moisture meter to monitor coal quality. At first, the results were encouraging, but soon the operators found that the variation in coal flow rate was causing disparities in ash and moisture results. Additionally, the performance of microwave moisture meters is diminished with an increase in moisture concentration, and coal operators are wary of using them where moisture exceeds 30% (Foster and Heger, 2014). Due to these problems, the dual gamma ash gauge was removed from service. According to the mine and power plant operators, two lessons were learned from this experience: 'Real-time coal quality data is extremely valuable, but incorrect real-time data is worse than no real-time data at all.' The mine then addressed the problem by installing PGNAA analysers on the conveyor belt system. The analysers were used to monitor coal flow to the silos and boilers in such a way that if low-quality coal was arriving, control room operators could make adjustments to the blend ratio, depending on the real-time data retrieved from the analysers (Foster and Heger, 2014). However, since silos were used only for temporary storage, the entire system had limited impact on the blending process and therefore on the final quality of the coal being delivered.

A power plant near Castle Dale, Utah used a coal analyser to control the ash fusion temperature of the coal blend (Snider *et al.*, 2005). Low ash fusion temperatures were the primary cause of slagging and unplanned outages. Engineers from the plant studied the relationship between certain coal ash minerals and the softening temperature of the ash, and developed formulae that estimated the ash-softening temperatures of the coal blend as a function of the ash components. They realized that PGNAA analysers could help determine the chemistry of the six major ash components and the ash-softening temperature (Snider *et al.*, 2005). The analyser monitors blend coal conveyed from the stockpiles to the screening transfer building and then to a second transfer tower, which is connected to the storage barn (Snider *et al.*, 2005). This solution was effective in dealing with ash challenges and led to reduced slagging, but lost generation and frequency of forced outages indicated that the rest of the quality tags should be given more attention.

According to Woodward (2008), the number of analysers purchased by utilities is growing. This shows that the coal mining industry has realized the advantages of using analysers for quality management purposes.

France (1999) investigated the utilization of coal analysers in a coal management system already installed in a coal mine. The coal analysers automated the data with a two-dimensional image of the stockpiles by accepting the coal quality data into stockpiling modelling software. The software allows users to visualize and analyse the coal stockpile stacked by the stacker/reclaimer. The model also provides the ability to predict the quality of the coal in case of reclaiming a specific area of the stockpile. The stockpile management system provides a visual output that allows graphic analysis of the content of ash and sulphur in the stockpile. This is an example of how online coal analysers can enhance the automation of coal quality management using computer technologies. Nonetheless, to the best knowledge of the authors, the visual output shows only two quality parameters (ash and sulphur) of the stacked coal.

There is a lack of real-time coal quality management models for dome storage. The specific problem represents the process of stacking the coal inside the enclosed facility such as a dome. This process generates the unique geometry of the stockpile due to the physical constraints caused by the circular shape and walls of dome and height and length of stacker. The reclaiming process also creates a particular shape of stockpile. The specific challenge to the mine is to know spatial distribution of coal quality parameters, volume, and tonnage in such a stockpile.

The objective of this research study was to develop a user-friendly interface for a coal quality management model for a dome storage (DS-CQMM) by using existing technologies in a surface coal mine in southern USA, merging automation technologies with information technologies (IT) and mathematical modelling. Specific aims were as follows:

i   Create multiple user-friendly applications based on the Windows® OS for the process of stacking and reclaiming coal flow into a dome storage

ii Establish the connection between the applications, the coal analyser, and the distributed control system (DCS) room databases in order to retrieve and store necessary data for building a DS-CQMM model

iii Develop an algorithm for retrieving data from the DCS room database containing the velocities of different conveyor belts, and calculate the time remaining for a given batch of coal coming from the crusher and belt conveyors to the boom of the stacker inside the dome

iv Formulate a three-dimensional mathematical model for developing a stacking algorithm that will assign shape and relative position of the coal stockpile inside the dome

v   Create an algorithm for calculating the coal volume that is being stacked into the dome and assign quality properties for presenting values in tons for the user interface

vi Formulate a mathematical model for developing a reclaiming algorithm that will show the operator the different ranges of values of different quality tags of the remaining coal inside the dome (in addition, it needs to show the remaining shape of the coal stockpile after reclaiming

vii Build a tool that implements a set of tables with numerical values of coal quality and tonnage for forecasting future reclaiming processes

vii Develop multiple simulators for each created application in order to test the model.

Methodology

The proposed technical approach for the development of the DS-CQMM is based on the technological process designed at a

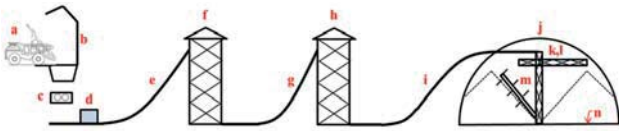# Coal quality management model for dome storage (DS-CQMM)

Figure 1—System components: (a) truck, (b) hopper, (c) crusher, (d) coal analyser, (e) belt conveyor 1, (f) transfer tower 1, (g) belt conveyor 2, (h) transfer tower 2, (i) belt conveyor 3, (j) dome, (k) stacker, (i) stacker's belt conveyor, (m) reclaimer, (n) emergency reclaimer

surface coal mine operated by one of the largest coal companies in the USA. Figure 1 shows the basic concept of the system and the flow of the coal from the dumping point to the dome. The system consists of a hopper, crusher, coal analyser, conveyor belt system, transfer towers, dome, stacker, reclaimer, and emergency reclaimer.

Truck (a) dumps the coal through the hopper (b) into the crusher (c). Crushed coal is transported by the belt conveyor 1 (e) to the transfer tower 1 (f) and belt conveyor 2 (g) and then to transfer tower 2 (h), which is designed to direct coal to either the emergency stockpile, power plant silos, or dome (j). The process at the transfer towers is conducted by the actuation of fast-acting proportional flop gates located on the towers. Coal quality and timestamps are recorded by the coal analyser (d), which is located above the belt conveyor 1 (e).

The coal flow directed to the dome (j) by the belt conveyor 3 (i) is delivered to the stacker (k). The boom of the stacker (k) rotates and steers the coal to its final location through a built-in belt conveyor (l). Finally, the coal is retrieved by the reclaimer (m) and transported outside the dome to the silos located at the power plant. In addition to the reclaimer (m), an emergency reclaimer (n) is installed at floor level in order to reclaim coal in case of the reclaimer's (m) failure or maintenance.

The PGNAA coal analyser provides coal quality information in real time, every minute for each batch of coal. A stacker stores the coal inside the dome and has one degree of freedom in the rotational (azimuthal) angle. A reclaimer that reclaims coal from the dome has two degrees of freedom: one in the rotational angle, and the other one in the elevation angle. Ultrasonic sensors are used for measuring the level of the coal stockpile, and encoders for determining the angular position of the stacker and the angular position and elevation of the reclaimer.

Figure 2 shows the concept of the DS-CQMM that is developed through this research. This model is organized into four major sections: (i) Delay Time application; (ii) Stacker application; (iii) Reclaimer application; and (iv) Live Stockpile application. A sub-process called Volume Calculation is embedded in the Stacker application, while an additional feature called Forecast tool is included in the Reclaimer
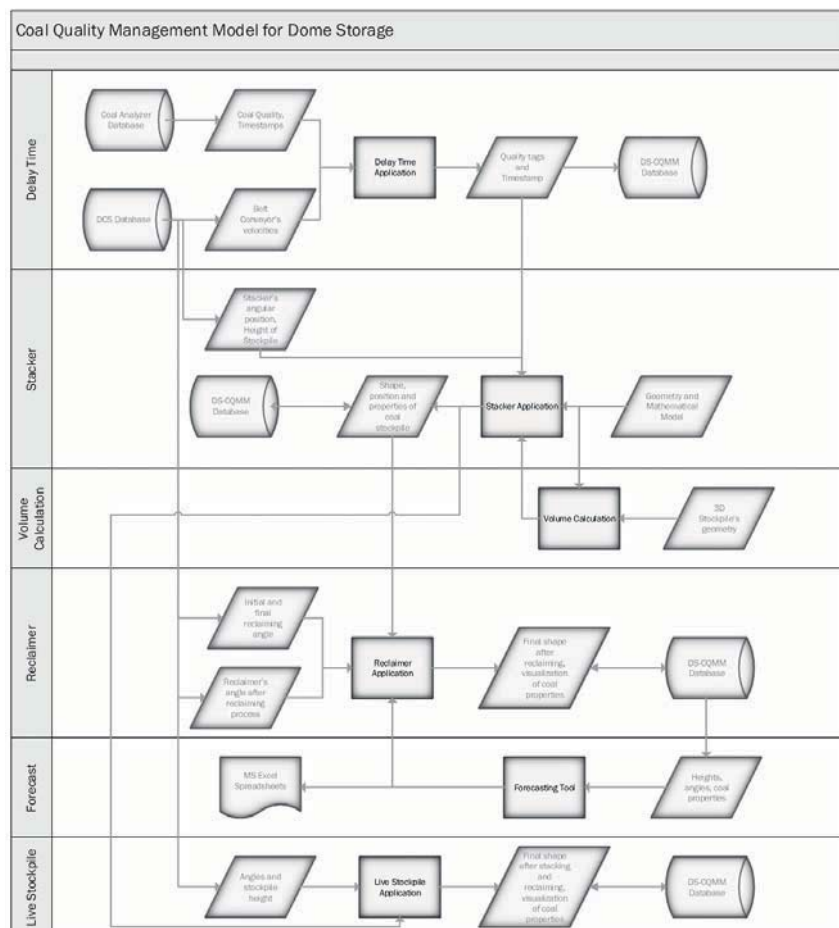

Figure 2—The concept for a coal quality management model for dome storage (DS-CQMM)

Coal quality management model for dome storage (DS-CQMM)

application. All sections are related to each other in such a way that the data must be sequentially generated so that the DS-CQMM works properly in a specific coal mine.

One of the most important components of the DS-CQMM is database synchronization. All applications are designed to retrieve, parse, and store values from and into the three different databases and their tables. These databases include: 'Coal Analyser' database, 'DCS' database, and 'DS-CQMM' database. Figure 3 shows the structure of these databases and their tables that are needed for a proper performance of the DS-CQMM.

Delay time application

Coal has to be transported between the coal analyser (starting point of the system) and the final point inside the dome. The coal analyser provides the instantaneous quality input of the current batch of coal that is passing under the detector at the beginning of the first belt conveyor. Therefore, it is necessary to know, for each batch, what its quality tags and timestamp are, and when will it reach the boom of the stacker in order to assign the angular position of the boom and the quality tags.

Mine operators can set and change the velocities of each belt conveyor, and this is achieved by the variable frequency drives (VFDs). After setting or changing the velocities, the DCS will retrieve the values from the programmabler logic controllers (PLCs) and store them along with a timestamp into the DCS database.

By accessing the DCS database, belt conveyor velocities can be queried every second or less to compare them with previous values and observe whether the belt conveyor has changed its velocity. Along with this information, querying the Coal Analyser database, and the absolute time from the DCS database clock, one is able to program a routine that calculates the coal delay time. For example, if it is supposed that there is only one batch of coal being transported from the coal analyser to the dome, its trajectory can be analysed along the belt conveyor system, the two towers, and finally

the boom of the stacker. First, the coal is analysed by the PGNAA at the beginning of the conveyor system. Then, it is carried by the belt conveyor 1 and, depending on its velocity, it takes a certain time to travel along this conveyor. After that, it passes through the transfer tower 1 and it is directed towards belt conveyor 2. This process takes a fixed time. It is then carried through belt conveyor 2, transfer tower 2, and belt conveyor 3, repeating the described process. When the coal reaches the dome, it takes a certain time to be transferred to the stacker from belt conveyor 3. This can be considered as a delay time similar to the transfer tower's delay time. Finally, the stacker's belt conveyor meets the same criteria as the other belt conveyors. Once the batch of coal has reached the boom of the stacker, it is ready to be stacked inside the dome.

The process of the Delay Time application starts 'attaching' the quality tags to the batch of coal during transportation through the conveyor system. In this way, a query to the Coal Analyser database is performed and stored into local memory. By retrieving the belt conveyors' velocities, the Delay Time application calculates the time a batch of coal takes to be transferred from the coal analyser to the boom of the stacker inside the dome. When the coal reaches that point (*i.e.* when the coal is at the boom of the stacker and is ready to be stacked), the coal quality information with its timestamp is passed to the next application for further analysis and database storage.

Stacker application

Mine operators need to know and visualize the actual quantity (volume and tonnage), location, and properties (heating value, moisture, ash, and sulphur) of stacked coal inside the dome. In order to accomplish this task, the first step is to determine the angular position of the boom of the stacker inside the dome. This enables the assignment of a relative position to the coal inside the dome, along with its particular properties. For this purpose, absolute rotary encoders record the angular position of the boom of the stacker into the DCS database through the PLCs. This is then retrieved by the Stacker application in order to build the virtual stockpile. Another important piece of information is the height of the stockpile, which is measured by an ultrasonic sensor located at the boom of the stacker. This value is stored into the DCS database.

The geometry of the dome and coal stockpile is designed by using trigonometry equations. Discretization is conducted by a three-dimensional method using the cylindrical coordinate system. This ad-hoc model considers the ground of the dome as reference plane and the centre of the base circumference as the origin of the coordinate system. Discretization of the dome is performed as follows (Figure 4):

➤ Polar axis (θ): the dome is divided into 360 equal parts, each one corresponding to one degree on the angular coordinate. This is performed in order to retrieve the rotational angle of the boom of the stacker and the rotational angle of the reclaimer from the DCS database with a resolution given by the PLCs of one degree

➤ Radial axis (ρ): the dome is also divided on the radial coordinate, forming concentric circumferences with 0.3 m (1 ft) of radius-length difference. The discretization on this axis provides the third dimension
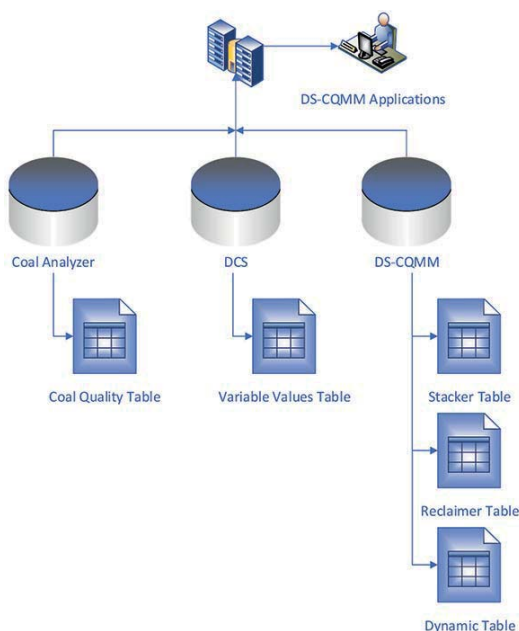

Figure 3—Database structure
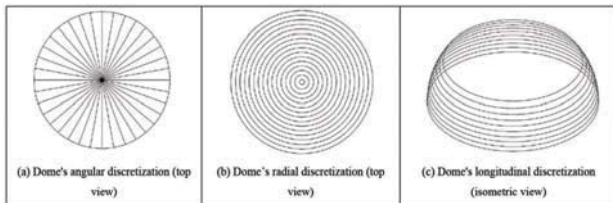
Coal quality management model for dome storage (DS-CQMM)

Figure 4 – Discretization of the dome

➤ Longitudinal axis (*z*): the dome is divided on the longitudinal axis into 90 equal parts with 0.3 m (1 ft) of distance. Since the maximum height of the coal stockpile is 26 m (85 ft), there is no need to extend the division further than 27.4 m (90 ft).

The model for the coal stockpile in the Stacker application is based on the following assumptions: (i) the stockpile's shape is approximately pyramidal; (ii) the analysis is performed in a plane that 'cuts' the pyramid in half; and (iii) the analysis is conducted in a two-dimensional plane. These three assumptions simplify the mathematical modelling and make analysis feasible.

The first assumption is made because of the admissible approximation between the shape of the coal stockpile and the shape of the pyramid observed in perpendicular planes. The stockpile's peak is located at a given (and constant) distance from the centre of the dome. Using this information, the second assumption is formulated. Since the stockpile's height and stacker's angular position are key inputs of the DS-CQMM, the location of the stockpile's peak seems to be a reasonable main reference for the mathematical model and further graphical user interface (GUI). Therefore, the plane where the radius for the stockpile's peak is constant ($\rho = \rho_{peak}$ = constant) is used for developing the model. The third assumption is an extension of the second assumption, *i.e.* given that the plane of study results in a cylindrical shape, it can be 'unfolded' and the mathematical model developed in a conventional plane with angle measurements in the abscissae and height measurements in the ordinates. Additionally, the mathematical model needs a point as an absolute reference (origin). At that point, angles 0° and 360° concur, establishing a rotational symmetry (Figure 5).

Since the dome is discretized in three dimensions, then the stockpile can be built as a pyramid and one can calculate the volume based on geometrical shape. Recall that the divisions are performed on the three axes (ρ, θ, z) at given distances. A closer observation of the resulting unit division of the intersection of these three divisions allows us to find the primary unit of volume, which is shown in Figure 6. Sides *a* and *b* measurements are each 0.3 m (1 ft) in length. Side *c* has no linear shape because it depends on the position of the prism, being larger if it is located farther from the centre of the dome. In other words, side *c* depends on the radius; therefore, there are two measurements of this side in each prism, therefore their average is taken for calculations.

The shape of the actual stockpile is plotted on the screen by querying the DS-CQMM database so the mine operator can know the actual shape of the coal stockpile inside the dome. Simultaneously, the screen shows an update of the remaining time until the next batch of coal along with its quality tags

will arrive to the boom of the stacker. This enables the operator to rotate the boom of the stacker to a specific location in the dome. This is achieved by using the Delay Time application that queries the Coal Analyser database.

The Stacker application retrieves the angle of the boom of the stacker from the DCS database along with the height of the stockpile measured after the stacking process. The rotational angle and the stockpile's height are the key inputs for this application. After retrieving these measurements, the Stacker application calculates and generates data that is finally stored into the Stacker and Dynamic tables in the DS-CQMM database.

The stockpile's shape is plotted in a plane that follows the criteria explained in Figure 5 where a plane (θz) is extracted from the dome at a constant radius. Along with the quality tags, the Stacker application assigns the corresponding tonnage to each block using the volume and coal density.

Reclaimer application

One of the objectives of the DS-CQMM was to develop a user-friendly interface for determining the coal quality distribution inside the dome. Therefore, the Reclaimer application should have a configuration that allows the operator to navigate through the applications with minimal training. The Reclaimer application should also be able to obtain the correct inputs in order to show the previous and later status of the coal stockpile at the reclaiming process. In addition, numerical values of different coal quality tags should be available in tables for more accurate studies and analysis. Since the dome is used for storing purposes, the operator should be able to observe how much coal can be reclaimed, from which point of the dome, and the average of each quality tag that the reclaimed coal would have. All these objectives are achieved with the Reclaimer application and the Forecast tool.

The Reclaimer application shows the actual state of the coal stockpile inside the dome using the same interface layout as the Stacker application; which means that the interface is implemented in a two-dimensional plane where the radius of the peak of the stockpile is located. The operator
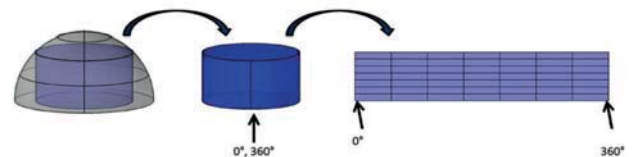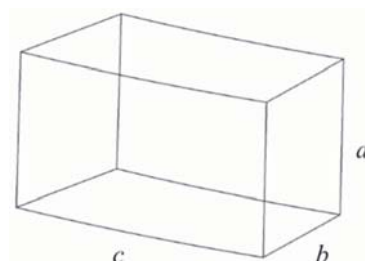

Figure 5—Dome's 2D implementation plane

Figure 6—Volume prism

has three options while the shape of the stockpile is being displayed: (i) to see the distribution of the different coal quality tags of the stockpile differentiated by colours; (ii) to perform the reclaiming process; and (iii) to start the Forecast tool.

The Forecast tool shows interactive tables; by selecting different cells or range of cells, the operator can display different values of coal quality and tonnage.

The operator is able to observe the coal quality distribution of the actual stockpile differentiated by colours. These colours are associated with a range of values for each quality tag. In other words, different ranges of coal quality are shown by colours for each coal quality tag following the actual shape of the stockpile.

The program retrieves the values of the different coal quality tags along with their position from the dynamic table in the DS-CQMM database so that the operator can decide which of the quality tags (ash, heating value, moisture, or sulphur) will be shown in the display area.

The process of plotting the quality distribution is the same for all tags. First, ranges of quality are set for each colour. Then, each point's quality value is compared with the ranges and the program decides which colour is assigned to that particular point. Finally, points are plotted into the display area.

It is important to recall that the points plotted correspond to the blocks located in the plane $\rho = \rho_{peak}$ (*i.e.* the stockpile's peak radius). This situation is similar to the Stacker application where the two-dimensional implementation was performed. However, the database contains the entire data from the stockpile stored for every block: the location (ρ,θ,z) of the block, its quality tags, and the tonnage.

For this particular section of the DS-CQMM, three inputs are needed: initial angle of the reclaiming process (θ_s), final angle of the reclaiming process (θ_f), and final angle of the reclaimer after the reclaiming process (φ).

For historical records, the program stores the reclaimed coal properties and tonnage divided by blocks into reclaimer and dynamic tables in the DS-CQMM database; it then deletes the blocks that are located above the line defined by the reclaimer angle (φ) from the Dynamic table in the DS-CQMM database.

It is important to recall that the Dynamic table is used as a bridge table between the Stacker and Reclaimer tables. This table contains the actual state of the dome's stockpile. In that sense, the blocks corresponding to reclaimed coal are deleted from the Dynamic table but not from the reclaimer table.

The process of calculating the height and deleting the points above is repeated for every radius point ρ_k at every angle θ_k contained in the θ_s, θ_f range.

The final shape of the stockpile is shown in the Reclaimer application display area after the reclaiming process. It corresponds to the plane where the radius is constant ($\rho = \rho_{peak}$); however, there is coal stacked above that height 'behind' that plane. This is not reflected at the application display area, but remains stored in the DS-CQMM database and is reflected in numerical values at the Forecast tool.

Reclaimer application is a useful tool for graphical determination of coal quality distribution inside the dome. It gives the operator a good approximation of quality values and relative position of the stacked coal. However, it is

necessary to obtain numerical values of coal quality data for different purposes such as forecasting, mine planning, blending, *etc.* The Forecast tool is developed for giving the operator information about the numerical values within a given range of angle and height for each coal quality tag, presented in tables. It also gives the operator a quick reference of average values of all the quality tags by selecting specific cells. Each quality tag and tonnage includes its own table of values that are shown on a display grid. These tables are divided using the same concept as used for Stacker and Reclaimer applications, *i.e.* by angles on the abscissae and heights on the ordinates. The user can select one of the following tables: Ash; Heating Value; Moisture; Sulphur; or Tonnage. The average values of the selected quality tag are shown in the table within a range of angles and heights.

The Forecast tool provides a dynamic way to obtain quality tag averages and a total summation of the tonnage of selected cells. The program obtains the initial and final value of the angle by retrieving the initial and final selected column. Once these values are obtained, it queries the Dynamic table in the DS-CQMM database for retrieving the averages and summation values.

After selecting the table of interest (*i.e.* any quality tag or tonnage table), the operator can select any range of cells from the table and the Forecast tool provides the averages and summation of the rest of the quality tag values. This flexible tool provides the user with the ability to forecast the reclaiming process based on one decision variable and to know the value of the rest of the variables in the system in a dynamic way.

Live Stockpile application

The emergency reclaimer is installed at the dome's ground level. This reclaimer is designed to work when the mobile reclaimer is undergoing maintenance or suffers a possible failure. It is located at 290° from the origin and the area of the live stockpile covers from angle 200° to angle 360° (Figure 7).

The coal will spend less time in this area of the dome than in the rest of the dome. In fact, the quantity of coal stored in the live stockpile increases and decreases constantly. By locating the stacker at angle 290° and activating the emergency reclaimer, this stockpile will dynamically change its shape.
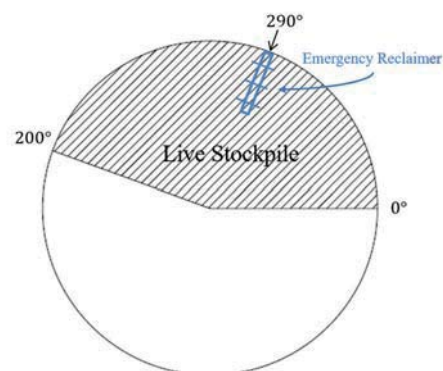

Figure 7—Schematic view of live stockpile and emergency reclaimer

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Above the emergency reclaimer, an ultrasonic sensor is installed in order to provide the value of the stockpile's height constantly. In this way, the Live Stockpile application can retrieve that value from the DCS database in order to build the model.

The Live Stockpile application consists of a stacking part and reclaiming part. The operator decides what action is taken by switching between different available options: stacking, reclaiming, or coal quality display.

The Coal Quality Distribution display functions in the same way as in the Reclaimer application. This process shows the quality tags distributed within the coal stockpile differentiated within quality ranges by different colours. It is performed in the θz plane where the radius is constant and coincides with the radius of the peak of the stockpile (*i.e.* $\rho = \rho_{peak}$). The difference is that coal quality distribution is shown in this section from angle 200° to angle 360°.

For implementation of the stacking part on the Live Stockpile application, the same inputs (*i.e.* stacker angular position (θ_0) and stockpile height (z_0)), and same volume calculation concepts as Stacker application are used. The data generated is also stored in Stacker and Dynamic tables in the DS-CQMM database. Additionally, this section includes the same graphical outputs as the Stacker application.

The display area shows the shape of the actual coal stockpile in a plane θz where the radius is constant and coincides with the radius of the peak of the stockpile (*i.e.* $\rho = \rho_{peak}$).

Although the concept and development of the stacking part in the Live Stockpile application is similar to the Stacker application, the reclaiming concept and development is completely different. In the Reclaimer application, the reclaimer performs the reclaiming process on the surface of the stockpile. In other words, it reclaims from top to bottom and from side to side. In this case, the emergency reclaimer is located in a fixed position at ground level and reclaims from the bottom and from only one point of the stockpile. These are opposing concepts.

The reclaiming process at the live stockpile can be compared with an hourglass when it is running empty. The shape of the remaining stockpile will contain a hole with a shape of an inverted pyramid.

For implementation of the mathematical model, it is necessary to build an inverted pyramid that works as the reclaimed volume by the emergency reclaimer. For that purpose, the mathematical model is implemented on plane θz where the radius is constant and coincides with the stockpile's peak radius ($\rho = \rho_{peak}$). The width of the emergency reclaimer is also taken into consideration for accuracy.

The length of the emergency reclaimer is considered in the third dimension (*i.e.* ρz plane). The influence of the emergency reclaimer's length on the live stockpile is reflected in the reclaiming process: the remaining stockpile has a dead volume originated by the emergency reclaimer's length. The dead volume has a triangular shape in plane ρz determined by the angle of repose of coal (α).

Results and discussion

It should be noted that the DS-CQMM model was designed for a US-based surface coal mine and all units in Figures 8–16

are given in US units. However, in the text of this paper, all these units were converted to the metric system.

The screen of the Delay Time application test program is shown in Figure 8. The actual delay time (g) in seconds and the calculated delay time (f) are shown for each belt conveyor. The scroll bar (e) can be used to increase or decrease the velocity rate of the belt conveyor with ranges between 1.52 m/s (5 ft/second) and 3.35 m/s (11 ft/second) with increments of 0.3 m/s (1 ft/second). The velocity for each belt conveyor can be changed independently. The transfer towers are represented by a labelled square (c) that contains the actual residence time of coal (d) inside the tower. Raw data retrieved from the Coal Analyser database (k) is passed to the next stage for further analysis and processes. The total time that coal remains on the conveyor belts and in the towers (h) is the summation of all final delay times. The program starts by activating the 'Start' button (i), and closes database connections and the delay time applications through the 'Close' button (j).

The two key inputs of the Stacker application (θ_0 and z_0) are shown as keyboard inputs. In the mine, these two inputs are retrieved automatically from the DCS database.

Suppose that a stacking process is taking place. The boom of the stacker is positioned at angle 65° and coal is stacked until the stockpile reaches the maximum height, 26 m (85 ft). The boom of the stacker then rotates to angle 70° and later to 75°, and stacks coal up to 26 m (85 ft). It is important to emphasize that the rotational angle of the stacker (θ_0) is stored in the DCS database by the PLC and the height of the stockpile (z_0) is measured by the ultrasonic sensor and stored in the same database.

In case the operator decides to stack coal in a different location than the continuous stockpile, because the quality has a given characteristic or for any other reason, the Stacker application is able to build that stockpile. Figure 9 shows the final shape of the stockpile with coal stacked at 160° and 20° up to the maximum height. It should be noted that the left-hand branch of the stockpile does not overlap the existing branches.
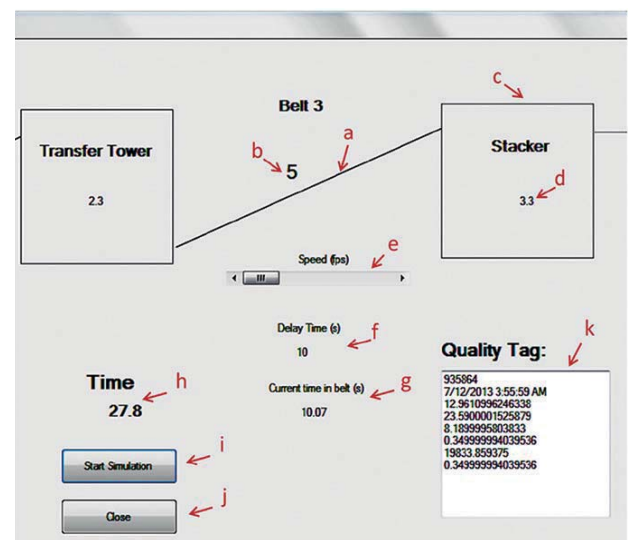

Figure 8—Detail of Delay application
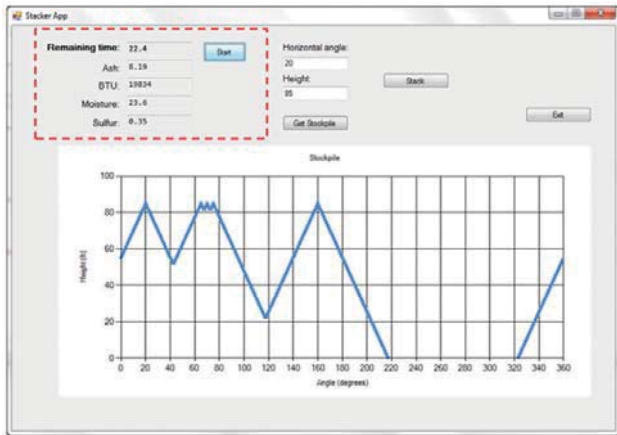
Coal quality management model for dome storage (DS-CQMM)

Figure 9—Remaining time and analyser real-time output window

Integration between the Stacker application and the Delay Time application is implemented in order to allow the operator to know the exact time remaining for that particular batch of coal to reach the boom of the stacker. This feature provides an extra benefit to the stacking process.

The 'Start' button that initiates the timer and triggers the queries is implemented for testing purposes; that action is automatically started when new data is available from the Coal Analyser database.

After stacking coal inside the dome, it is important to know the range of coal quality and its location. It is also important that the operators have an intuitive interface. For that purpose, the Reclaimer application provides the location of coal and, depending on the quality tag; it shows the range of quality distribution differentiated by colours.

Additionally, the Reclaimer application provides numerical values of coal quality, dividing the dome by angle and height ranges and displaying them in a table. This is implemented in a separate table for each quality tag and tonnage so that when the operator needs to know or 'forecast' the other quality tags based on that table, all that is required is to make a simple cells selection.

The example starts from the stockpile that has been created with the Stacker application. When the Reclaimer application is launched, it displays the actual shape of the coal stockpile. The display area shares the same concept as the Stacker application, where the stockpile is shown in two dimensions on plane z where the radius is constant and coincides with the radius where the peak of the stockpile is located ($\rho = \rho_{peak}$).

Now that the stockpile's actual shape is known, one can see the distribution of coal quality on the chart. Four coal quality tags are available in different colours for display. By selecting the corresponding buttons, the Reclaimer application displays the quality distribution associated with the quality tag. Figure 10 shows distribution of the heating value.

Once the operator decides what portion of the stockpile will be reclaimed – either using a graphical (Reclaiming application) or numerical (Forecast tool) approach – the reclamation process is performed. After finishing reclaiming coal, the Reclaimer application retrieves the required inputs from the DS-CQMM and DCS databases and displays the remaining shape of coal stockpile inside the dome. This is

shown in Figure 11, where the reclaimed coal has been removed from the dome starting at angle 5° and ending at angle 90° and where the final angle of the reclaimer (φ) is 30°.

The Forecast tool is used for a more accurate approach and for an interactive way of calculating the average quality of a specific portion of coal. This tool displays an expandable menu where the quality tag tables are contained. Each table consists of cells that show the calculated average of quality within the range of angle and height that are shown on the headers of rows and columns. There is also a table for the summation of the tonnage using the same range division.

By selecting one of the tables, the Forecast tool queries the Dynamic table in the DS-CQMM database and retrieves the average of the quality within the ranges if the option chosen is a quality tag, and retrieves the summation of the tonnage within the range if tonnage is chosen.

The division ranges of the rows and columns of the tables are performed as follows: angle divisions (every 20°) and height divisions (every 1.5 m (5 ft)). These values could be changed internally in the program code.

There is an essential difference between the graphical and numerical approaches. The highest height value shown in the charts for the stockpile is different, as shown in the tables. This situation is explained as follows lines. Recall that the GUI in two dimensions is developed in a plane that corresponds to a constant radius coincident with the
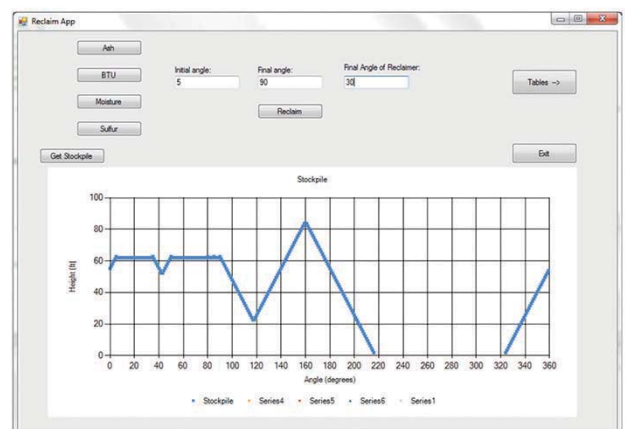

Figure 10—Distribution of heating value

Figure 11— Stockpile's shape after coal reclaiming process from 5° to 90° and final reclaimer angle of 30°

Coal quality management model for dome storage (DS-CQMM)

stockpile's peak radius (*i.e.* $\rho = \rho_{peak}$), and given that the reclaiming process is performed in a perpendicular plane, it cannot be completely reflected in the charts. However, this is avoided in the tables where the database is queried for the entire content of the dome. For this reason, the heights of the stockpile in the charts and in the tables are not consistent due to the existence of remaining coal 'behind' the charts' plane. That coal is stacked to a greater height than that shown in the graphical approach.

The Tonnage table uses a different concept. Figure 12 shows that it shares the same layout as the Quality-tag tables; the difference is that the queries against the DS-CQMM database are different. The value displayed in cells is a total summation of the tonnage contained in the ranges shown in the headers of the rows and columns.

Assume that the operator needs to forecast a given portion of coal for reclaiming and its location inside the dome based on the quantity of coal. The Tonnage table should be selected for starting the process. By selecting which part of the dome's coal will be reclaimed, the Forecast tool returns the total value of tonnage and the average of the four coal quality tags within the selected cells. Figure 13 shows a forecast process (in the 'Forecast' square) by selecting a range of cells.

The Live Stockpile application is a specific part of the dome that covers the area from angle of 200° to 360°. The emergency reclaimer is installed at ground level at angle of 290°. The live stockpile is characterized by its dynamic behaviour of continuous stacking and reclaiming processes, which increase and decrease its height regularly.

When the Live Stockpile application is launched, the stockpile's actual shape is displayed for the operator. Suppose that coal has been stacked at angle 240° up to 26 m (85 ft). Its height is measured by the ultrasonic sensor located at the boom of the stacker and stored in the DCS database. At this point, the blocks containing the tonnage and quality tags created by stacking part of the Live Stockpile application are stored in the Dynamic and Stacker tables in the DS-CQMM database.

If one assumes that the boom of the stacker moves to angle 290° (which is the position where the emergency reclaimer is installed) and stacks up to 26 m (85 ft), which is the maximum height designed to be stacked inside the dome, the shape of the stockpile is created and its quality tags are as shown in Figure 14. The quality distribution display part of this application is similar to the previous Reclaimer application; it shows the distribution of the coal quality inside the dome by selecting quality tags buttons. An example of moisture distribution is shown in Figure 15.

This stockpile behaves dynamically, increasing and decreasing its height constantly. An ultrasonic sensor is installed above the emergency reclaimer at the dome for measuring the live stockpile's height, even if the boom of the stacker is not located at this angle. If the operator activates

Figure 12—Forecast tonnage table

Figure 13—Reclaiming forecast for specific selected angles and heights

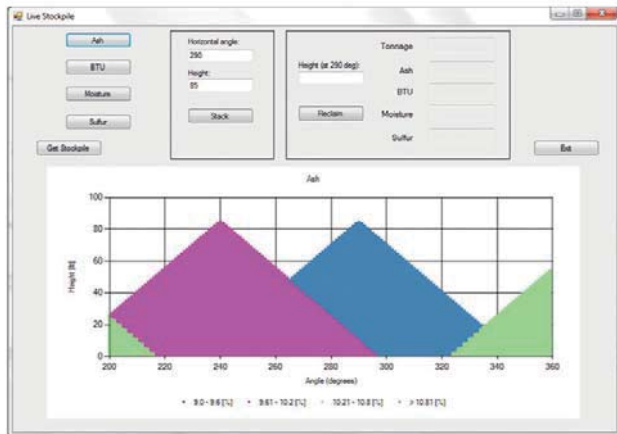# Coal quality management model for dome storage (DS-CQMM)

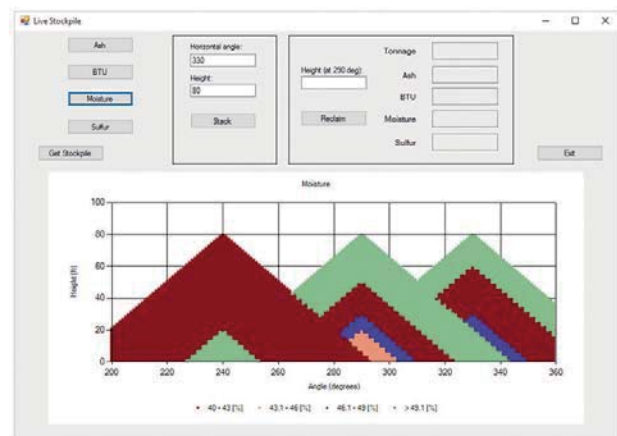**Figure 14—Ash distribution for the live stockpile**

Figure 15—Moisture distribution for the live stockpile

the emergency reclaimer, the stockpile will decrease in height due to reclaiming process. The ultrasonic sensor provides the height and the Live Stockpile application builds the stockpile. Figure 16 shows the shape of the stockpile after being reclaimed to 12 m (40 ft). It has a flat zone due to the width of the emergency reclaimer. The slope of the reclaimed part has the same slope as the rest of the stockpile, which is the angle of repose of coal (α).

It is important to note that, along with the final shape of the stockpile, the numerical values of the average of the quality tags and the total summation of the tonnage of the reclaimed coal are shown.

Conclusions

The coal quality management for dome storage (DS-CQMM) model developed through this research provides a graphical and numerical distribution of coal quality and its relative position inside the dome by integration of a variety of technologies. Mathematical models for different DS-CQMM applications were developed and interaction between these applications and databases was established. Algorithms for three-dimensional assignment of coal properties during the stacking process and reclaiming process based on the reclaimer's operations were developed, and a useful tool to forecast coal reclaiming process was designed.

This model can be helpful in the process of managing coal quality as follows:

Figure 16—Stockpile's shape after reclaiming to 12 m (40 ft)

➤ Storing coal inside the dome is an advantage for reclaiming purposes. If the mine needs a specific tonnage with a specific quality in a relatively short time-frame, then a dome with an appropriate quality management system is enormously helpful

➤ If we know the quality of the coal stored inside the dome and, most importantly, where it is stored and how many tons are available, then the process of blending the coal incoming from the mine and retrieved from the dome can be accomplished.

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INTERNATIONAL ACTIVITIES

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31 July–3 August 2016 — **Hydrometallurgy Conference 2016**
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collaboration with MinProc and the Western Cape Branch
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E-mail: raymond@saimm.co.za, Website: http://www.saimm.co.za

8 August 2016 — **South African Underground Coal
Gasification Association**
2nd Underground Coal Gasification Network Workshop
CDH Sandton, Johannesburg
Contact: Shehzaad Kauchali
Email: shehzaad.kauchali@wits.ac.za
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9–12 August 2016 — **Thirty Third Annual International
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12–14 September 2016 — **8th International Symposium on
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19–21 October 2016 — **AMI Ferrous and Base Metals
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Southern Sun Elangeni Maharani, KwaZulu-Natal, South Africa
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27–29 June 2017 —**4th Mineral Project Valuation School**
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the Witwatersrand, Johannesburg
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Tel: +27 11 834-1273/7, Fax: +27 11 838-5923/833-8156
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2–7 October 2017 — **AfriRock 2017: ISRM International
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Contact: Raymond van der Berg
Tel: +27 11 834-1273/7, Fax: +27 11 838-5923/833-8156
E-mail: raymond@saimm.co.za, Website: http://www.saimm.co.za

25–27 October 2017 — **AMI Precious Metals 2017**
'The Precious Metals Development Network (PMDN)'
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Contact: Raymond van der Berg
Tel: +27 11 834-1273/7, Fax: +27 11 838-5923/833-8156
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31 August–2 September 2016, Emperors Palace, Hotel Casino Convention Resort,

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For further information contact:
Conferencing, SAIMM
P O Box 61127, Marshalltown 2107
Tel: (011) 834-1273/7
Fax: (011) 833-8156 or (011) 838-5923
E-mail: raymond@saimm.co.za