Elementary Particle Physics III
(素粒子物理学 III)

Time Monday 14:55-16:40 (105min/lecture) R207

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Website for materials : (to be uploaded after the each lecture)
http://www.icepp.s.u-tokyo.ac.jp/~satoru/lecture/pp3/
http://www.icepp.s.u-tokyo.ac.jp/~jtanaka/lecture/pp3/

Schedule

- 4/6 Introduction, definition of Unit (1) JT
- 4/13 Quark and Hadrons, Weak and EW unification (2) SY
- 4/20 QCD, parton model (3) JT
- 4/27 CKM Matrix and CP Violation (4) JT
- **5/11, 18 Higgs Mechanism, Higgs Search and Measurements (5,6) SY**
- 6/1 Higgs Measurements and Supersymmetry(7) JT
- 6/8 Supersymmetry(8) JT
- 6/15 Neutrino Physics (9)SY
- 6/22, 6/29 New Physics Search at the Energy Frontier Experiments (10,11) JT
- 7/6 Grand Unified Theories (12) SY
- 7/13 Search for LFV and Summary of this Course (13) SY –Final Report

Evaluation:

Several short reports after lectures and Final report

Note that the final report is required for the credits.

Reference Books;

- Quarks and Leptons: An Introductory Course in Modern Particle Physics, F.Halze and A.D.Martin
- Introduction to High Energy Physics, D.H.Perkins
- Techniques for Nuclear and Particle Physics Experiments: A How-to-Approach, W.R.Leo
- 素粒子標準理論と実験的基礎, 高エネルギー物理学の発展, 長島順清

TODAY
electro-weak unification and Higgs mechanism

- Summary of the lectures so far (electro-weak mixing, chirality, weak gauge bosons and experiments).
- Idea of the Higgs mechanism.
- Experiments to search for Higgs and measurements.

- Appendix for more on theoretical explanations

Standard Model

- Fermions: Quarks and Leptons – 3 generations
- 3 interactions $U(1)_Y$, $SU(2)_L$, SU(3)
- Flavor mixing (each in quark-sector (CKM) and lepton-sector (MSN: to come later))
- U(1)Y x SU(2)L mixing → Higgs mechanism

  → $U(1)_{em}$ (electric charge) x Z^0 (neutral current)

  Mass of W and Z bosons, Mass of fermions

Out of Standard Model (beyond Standard Model):

- Structure of the Higgs field and forces (especially negative mu)
- GUT (relation between quarks and leptons, electro-weak and SU(3))
- Dark matter, dark energy
- Reasons of the CKM/MNS flavor mixing

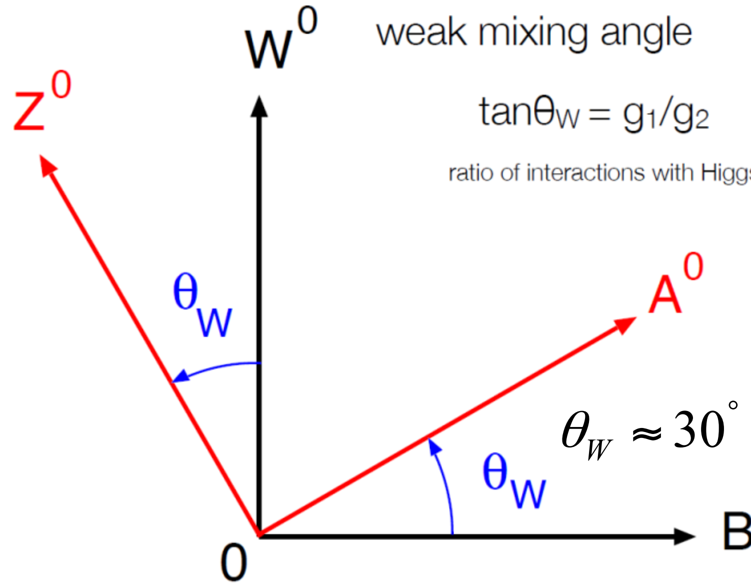
Today's topic (~2.5 weeks)

1979 Nobel Prize


S.L.Glashow


A.Salam

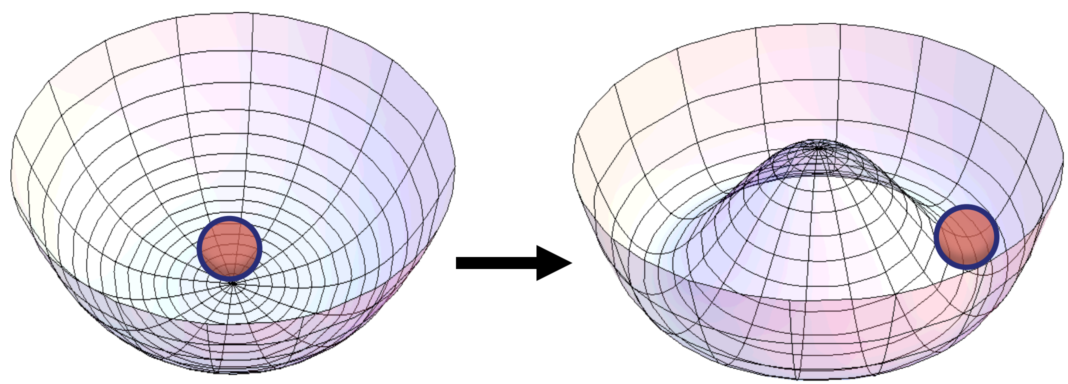

S.Weinberg



2013 Nobel Prize


F.Englert


P.W.Higgs



ATLAS+CMS seminar at CERN at the 4th July, 2012
(← Japanese news paper)


ヒッグス粒子 発見か
新素粒子検出 年内に結論
日米欧２チーム

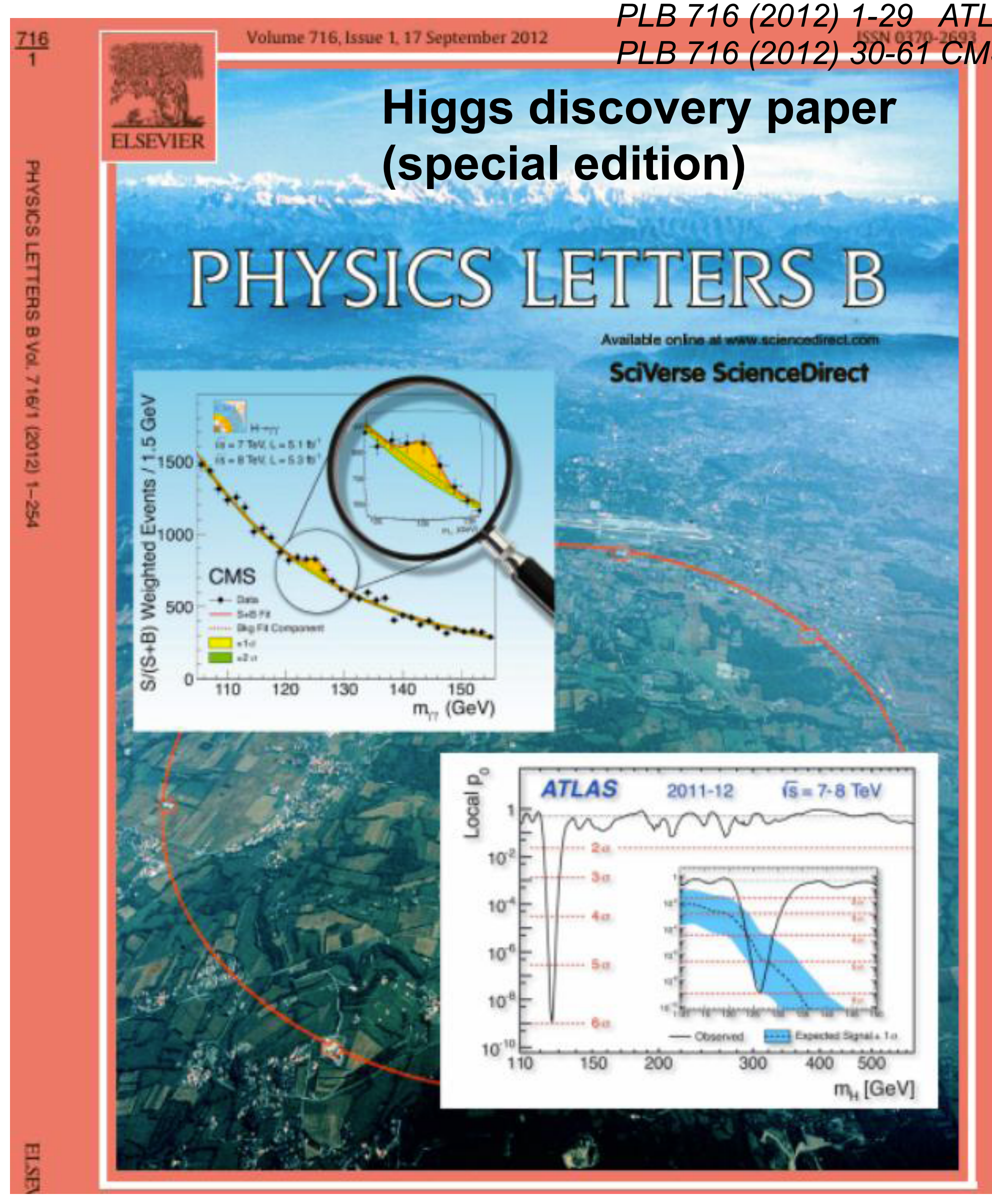
あらゆる物質に質量を与えたと考えられる仮説上の素粒子「ヒッグス粒子」を探しているジュネーブの欧州合同原子核研究所（CERN＝セルン）は4日、大型粒子加速器「LHC」による実験で、ヒッグス粒子とみられる新たな素粒子を見つけたと発表した。ヒッグス粒子は、万物に含まれる素粒子の基本理論として1960年代に提唱された「標準理論」で存在が予言された18種の素粒子のうち、唯一未発見だ。（3面にクローズアップ）

得られたデータからは発見された素粒子をヒッグス粒子と考えても矛盾がないが、ヒッグス粒子かを見極める方法は複数あり、さらなる実験や分析で確度を高める必要があるという。今年中には結論に至る見込み。

ヒッグス粒子は、英国の物理学者ピーター・ヒッグス博士(83)が提唱。宇宙誕生の大爆発（ビッグバン）直後、宇宙空間に漂っていた素粒子に質量を誕生させる粒子で重要な役割を果たした「神の粒子」と呼ばれている。標準理論で存在が指摘された他の素粒子は08年ま

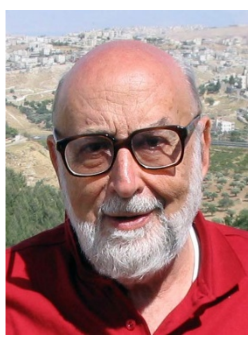
喜びあうヒッグス粒子の提唱者のピーター・ヒッグス博士(右)と研究代表者の一人ファビオラ・ジアノッティ博士＝ジュネーブで4日、ロイター

PLB 716 (2012) 1-29 ATLAS
PLB 716 (2012) 30-61 CMS



Higgs discovery paper (special edition)

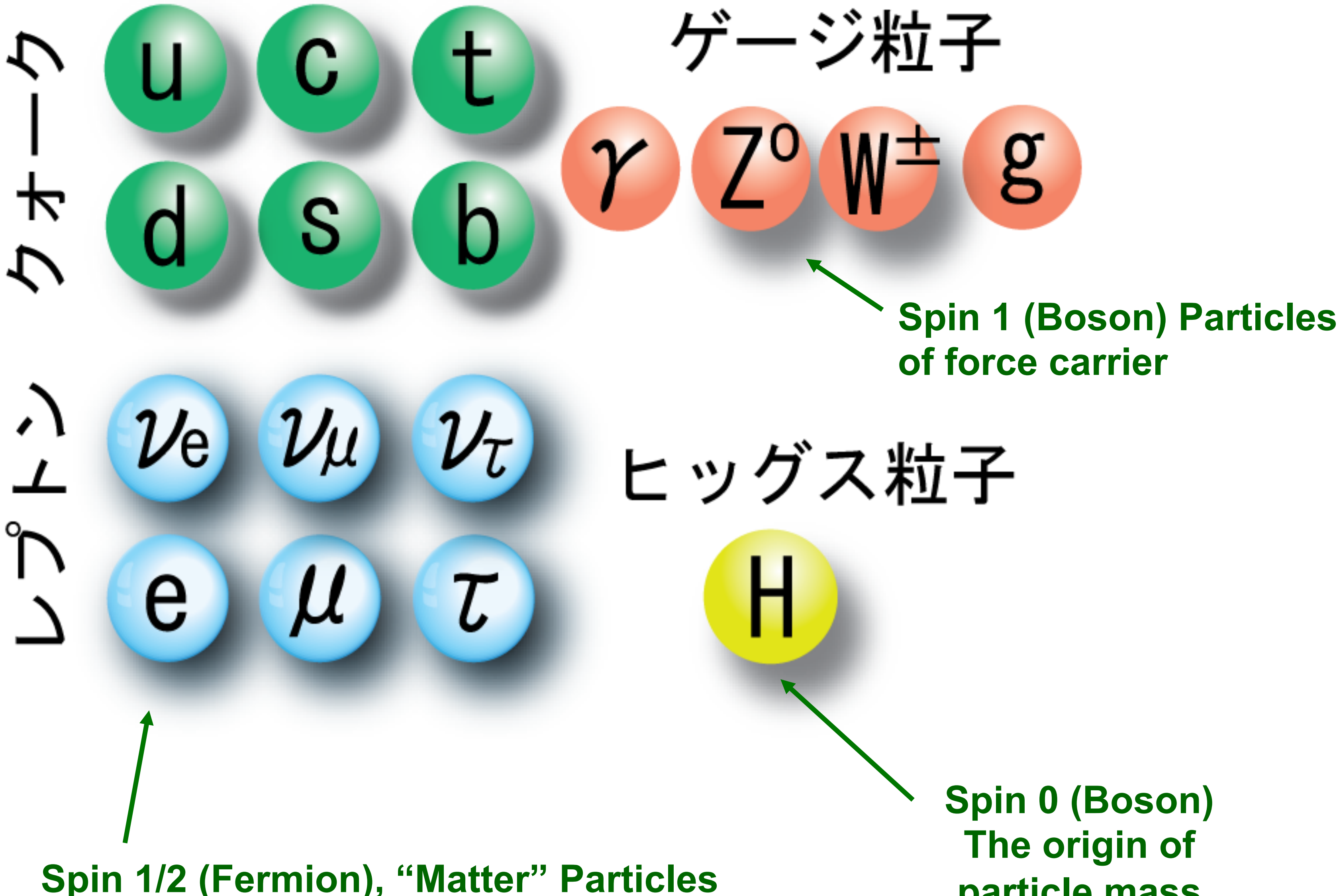
2013 NOBEL PRIZE IN PHYSICS
François Englert
Peter W. Higgs



The Standard Model Lagrangian

This stone monument is at CERN.
CERN = The European Organization
for Nuclear Research (Geneve Switzerland)

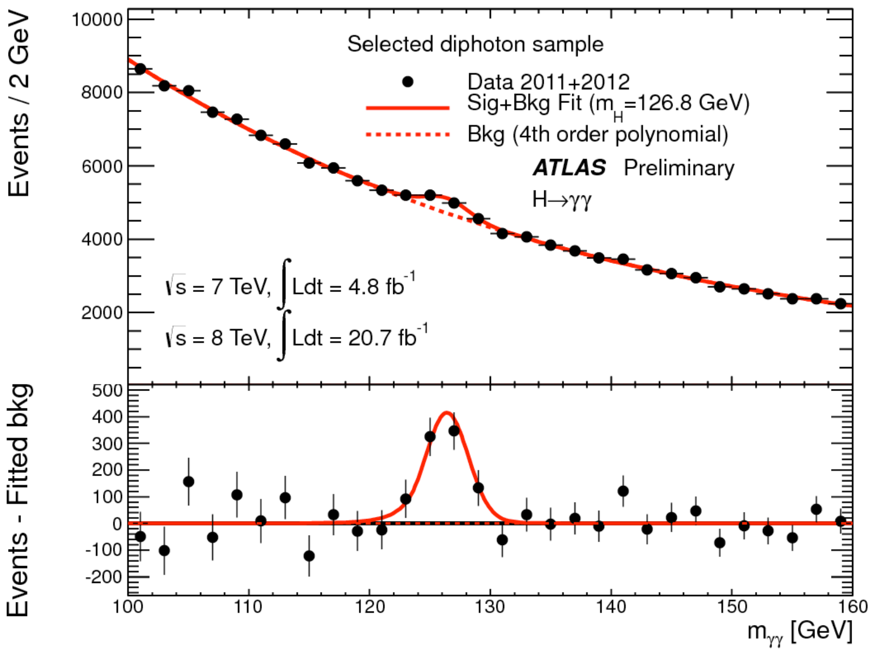
Characters



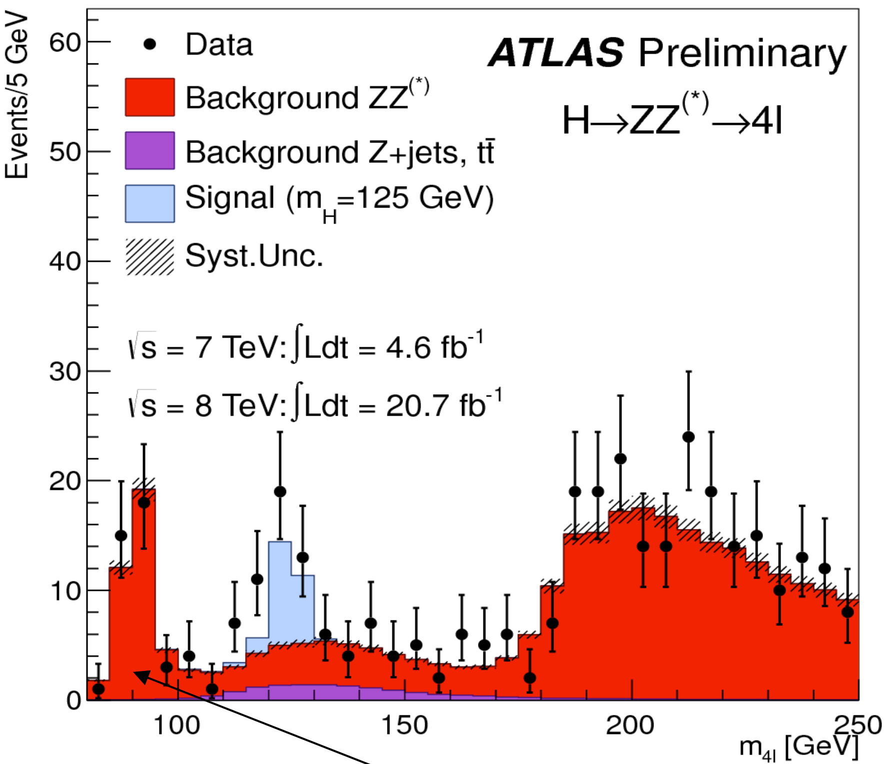
クォーク
u c t
d s b
レプトン
ν_e ν_μ ν_τ
e μ τ
ゲージ粒子
γ Z^0 $W^\pm$ g
ヒッグス粒子
H
Spin 1 (Boson) Particles of force carrier
Spin 0 (Boson) The origin of particle mass
Spin 1/2 (Fermion), “Matter” Particles

Higgs Discovery

H->γγ



H->ZZ*->llll



What is this peak?

ATLAS and CMS observed a Higgs particle at ~126GeV.
Experimental results will be discussed in the next lecture.

Gauge Theories

QED	QCD
U(1)	SU(3)
±e	±R, ±G, ±B
$\alpha = e^2/4\pi$	$\alpha_S = g^2/4\pi$
atoms	mesons, baryons, etc
molecules	nuclei

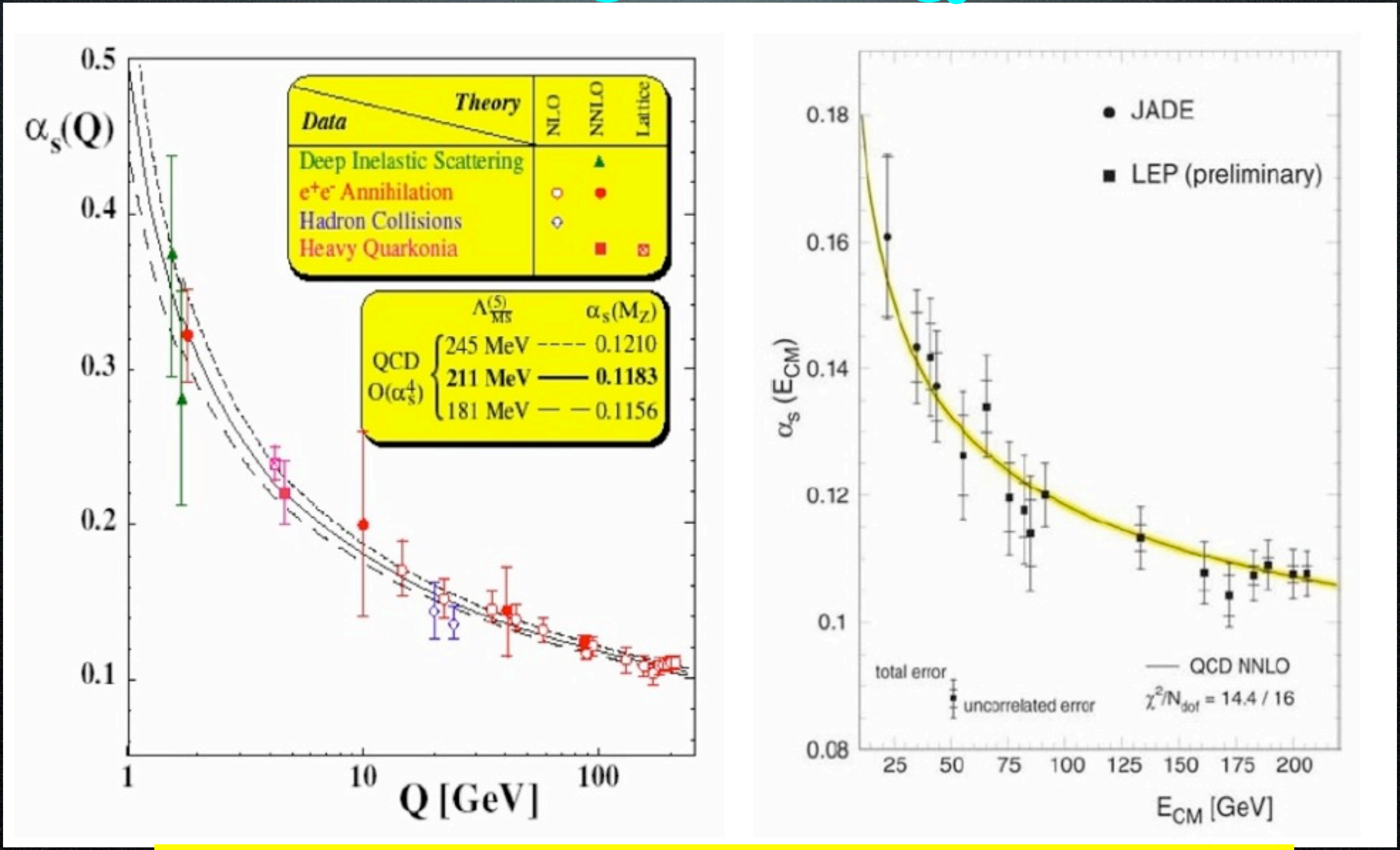
Simple theories give birth to rich phenomena!

	U(1)	$SU(2)_L$	SU(3)
Gauge bosons	B^0	W^0, W^+, W^- (3 bosons) (2x2-1=3)	Gluons (8) (3x3-1 = 8)
charge	Hyper Charge Y	Weak Isospin T	Color charge
fermions	Quark (L & R) Lepton (L & R)	Quark (**L only**) Lepton (**L only**)	Quark (L & R) No lepton
Coupling constant	α1	α2	α_s

Higgs Mechanism → MIXING

B^0, W^0 → photon, Z^0

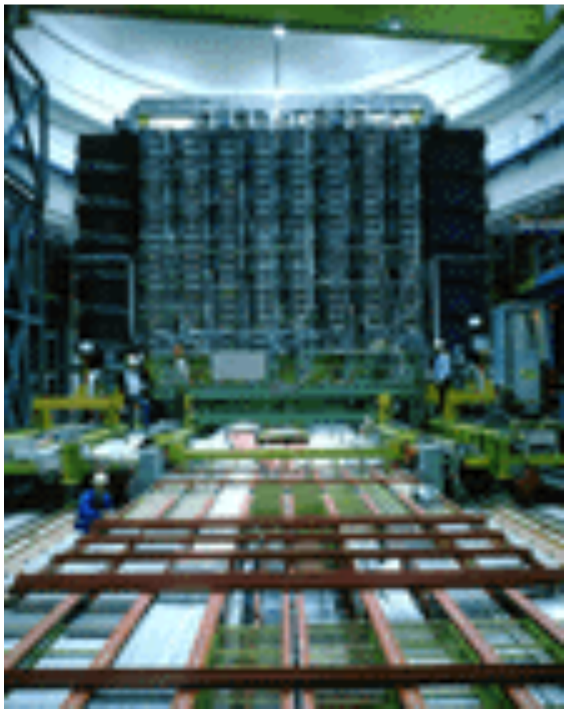
String interaction becomes weaker for higher energy

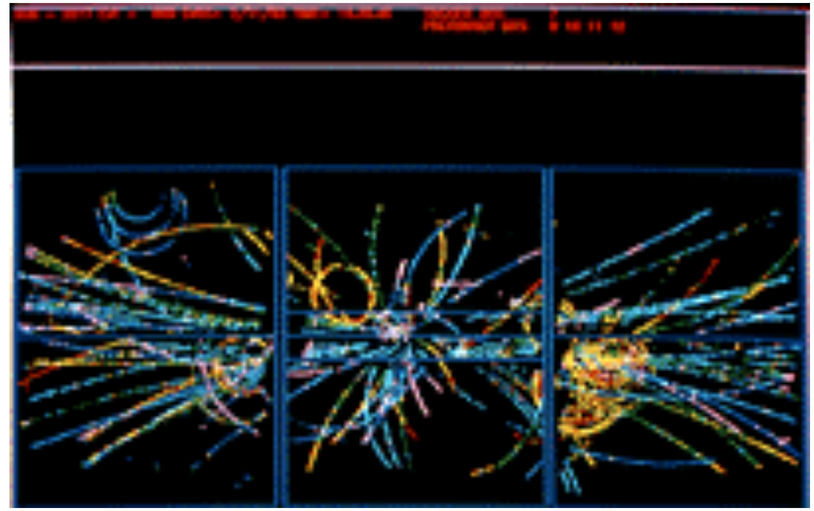


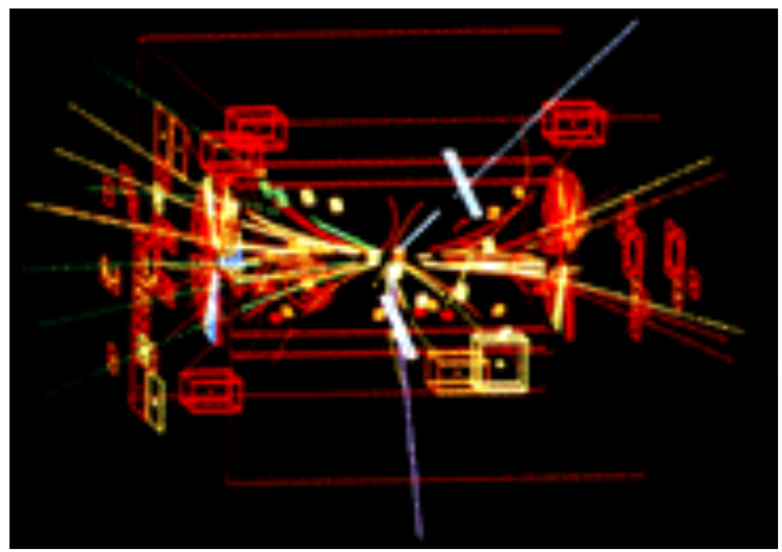
Asymptotic freedom of strong interaction

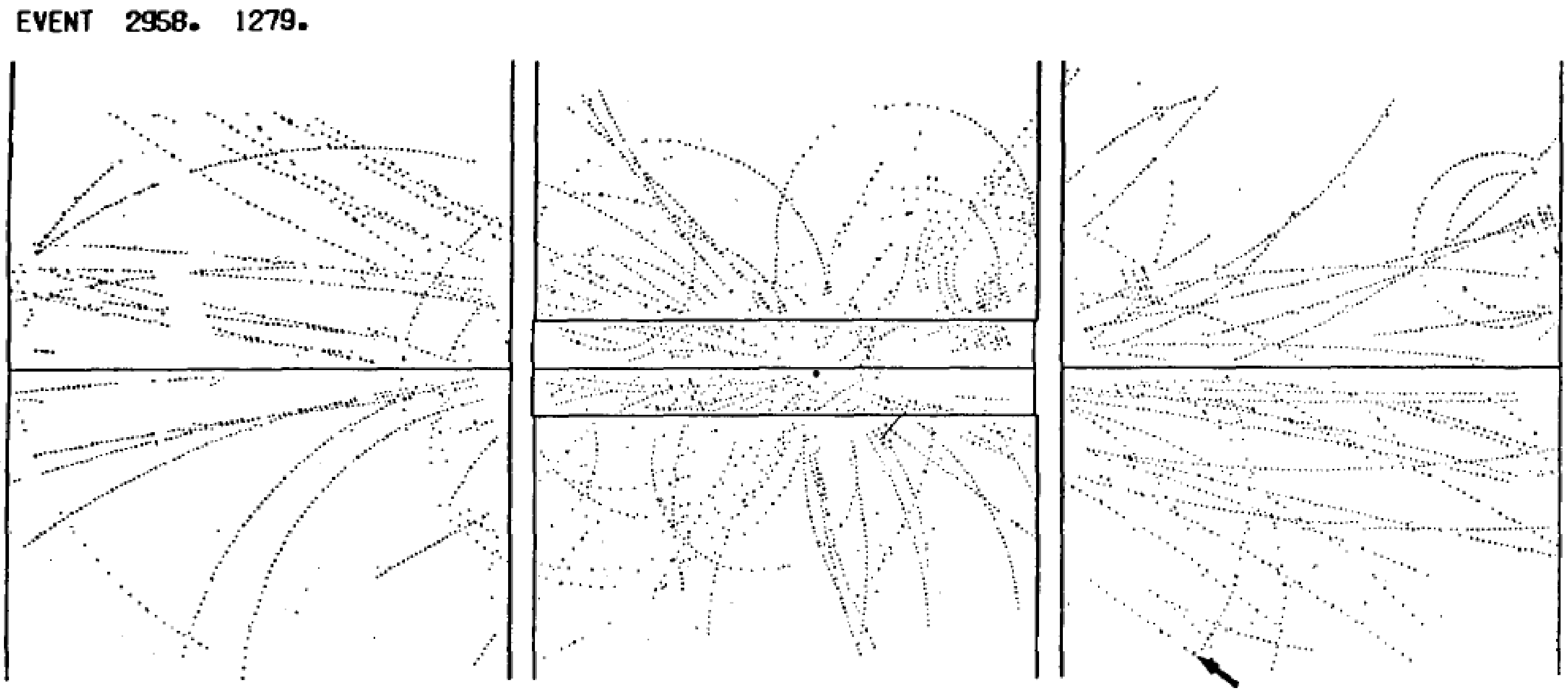
Discovery of W and Z bosons

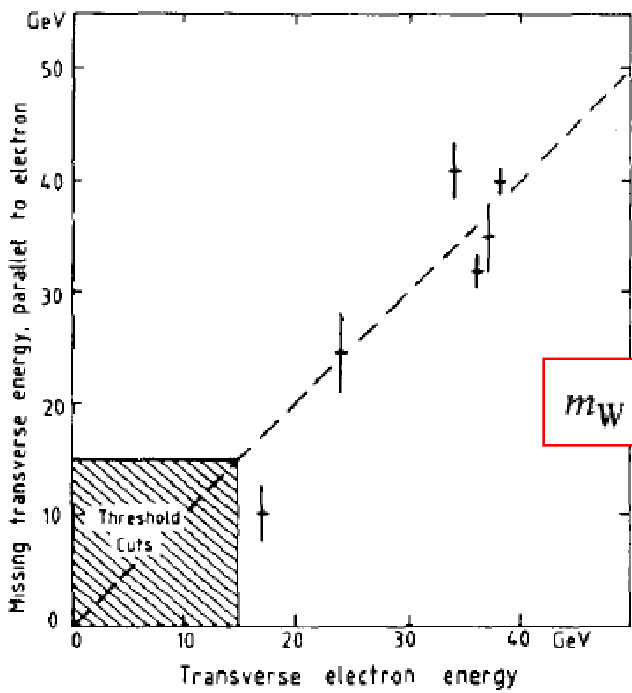
UA1 experiment detector











$m_W = (81 {}^{+5}_{-5})\ \mathrm{GeV}/c^2$

First W's from UA1

PL 122B(1983)103

and from UA2

PL 122B(1983)476

$M_W = (80^{+10}_{-6})\mathrm{GeV}/c^2$

First Z's from UA1

PL 126B(1983)398

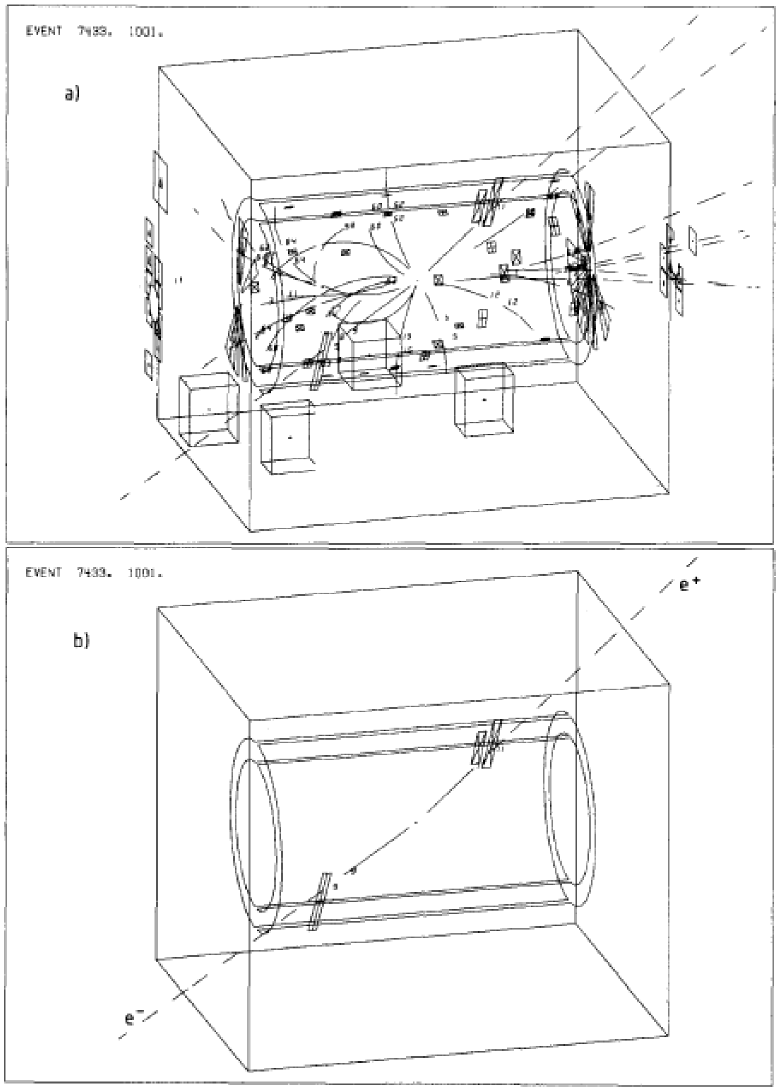
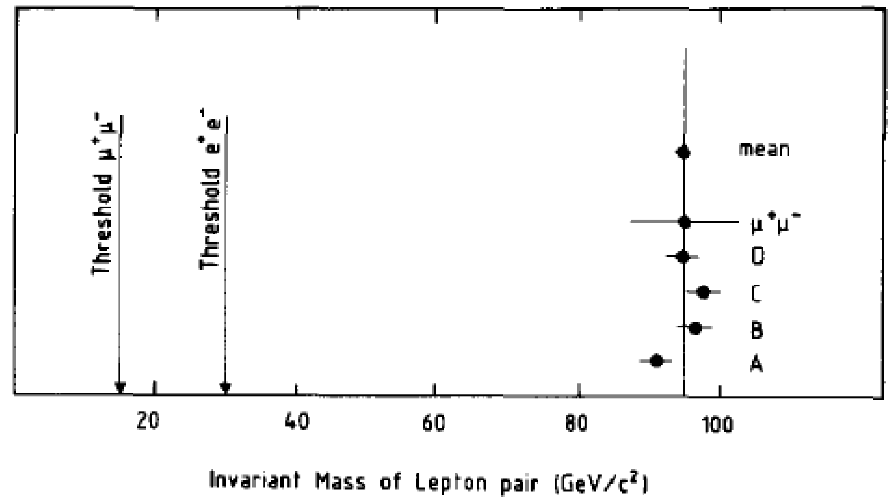


Fig. 2. (a) Event display. All reconstructed vertex associated tracks and all calorimeter hits are displayed. (b) The same, but thresholds are raised to $p_T > 2$ GeV/c for charged tracks and $E_T > 2$ GeV for calorimeter hits. We remark that only the electron pair survives these mild cuts.



$$m_{Z^0} = (95.2 \pm 2.5)\ \mathrm{GeV}/c^2$$

$$\sin^2\theta_w\,(m_W) = 0.226 \pm 0.011$$

and from UA2

PL 129B(1983)130

$$M_Z = 91.9 \pm 1.3 \pm 1.4\ \mathrm{GeV}/c^2$$

$$\sin^2\theta_W = 0.226 \pm 0.014$$

1st Key point to learn the framework of the Standard Model

- Left-handed quarks/leptons and Right-handed ones are originally different particles
  - u_L & $u_{R,}$ d_L & $d_{R,}$ e_L & $e_{R,}$,,,
  - e_L has weak-isospin and hyper-charge
  - e_R has only hyper-charge, no weak-isospin

Chirality, helicity and anti-particles

Q_R, e_R $\quad \frac{1+\gamma_5}{2}\psi$ Anti-particles ⟷ $\overline{Q}_R, \overline{e}_R$

Helicity ~ Left-hand

spin

E>>m

Momentum direction

helicity ~ Right-hand

E>>m

Q_L, e_L $\quad \frac{1-\gamma_5}{2}\psi$ Anti-particles ⟷ $\overline{Q}_L, \overline{e}_L$

helicity ~ Right-hand

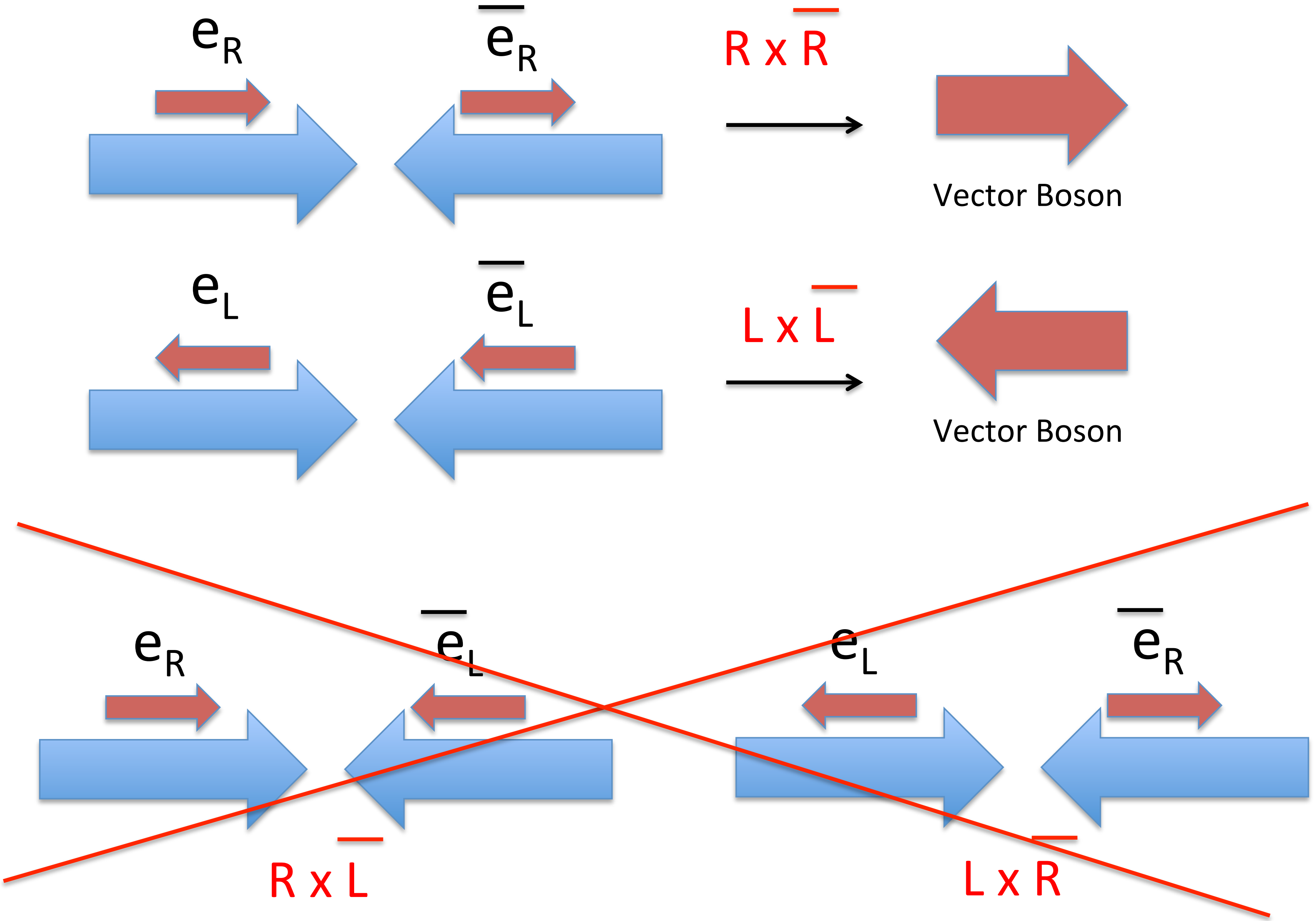
E>>m

Helicity ~ Left-hand

E>>m

e.g. neutrino

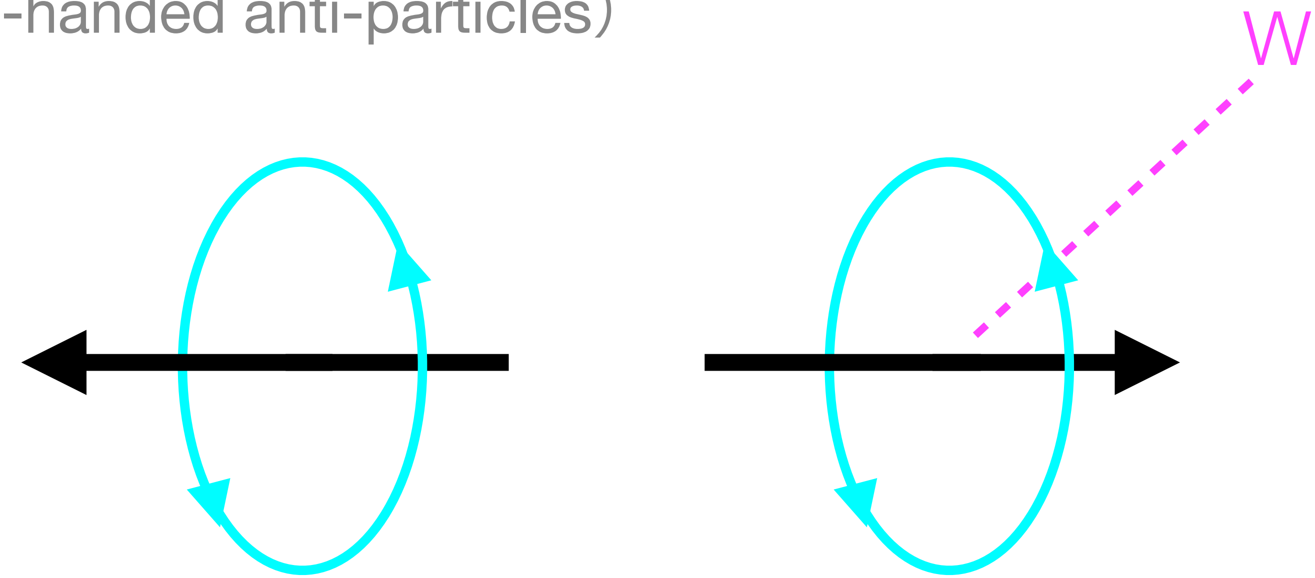
e.g. anti-neutrino



e_R
$\bar{e}_R$
$R \times \bar{R}$
Vector Boson
e_L
$\bar{e}_L$
$L \times \bar{L}$
Vector Boson
e_R
$\bar{e}_L$
e_L
$\bar{e}_R$
$R \times \bar{L}$
$L \times \bar{R}$

Weak Interaction is “left-handed”

- Operates on left-handed particles only
  (or right-handed anti-particles)



Iso-Vector $\mathbf{W}^{+} = (1,1)$, $\mathbf{W}^{0} = (1,0)$, $\mathbf{W}^{-} = (1.-1)$

Iso-Spin $u_L = (1/2, +1/2)$ $d_L = (1/2, -1/2)$

ν_{eL} e_L

....

Iso-Scalar $u_R = (0,0)$ $d_R = (0,0)$

$\nu_{eR} = (0,0)$ eR = (0,0)

....

gluons = (0,0), $B^0 = (0,0)$

Particles in Electroweak Theory

QCD color		weak isospin T	hypercharge Y
R/G/B	q_L =(u_L,d_L)	1/2	1/3
R/G/B	u_R	0	4/3
R/G/B	d_R	0	-2/3
0	l_L =(v_L,e_L)	1/2	-1
0	e_R	0	-2
0	ϕ	1/2	1

q_L, u_R and d_R were originally different particles!

Higgs ϕ=(0,v+H) does not interact with $Q = T_3 + Y/2$ = EM charge

The Glashow-Weinberg-Salam Model

The Glashow, Weinberg and Salam model treats EM and WEAK interactions as different manifestations of a single UNIFIED ELECTROWEAK force (Nobel Prize 1979)



Basic Idea:

Start with 4 massless bosons {W^+, W^0, W^-} and B^0. The neutral bosons MIX to give physical bosons (the particles we see), i.e. the W, Z^0 and γ.

$$\begin{pmatrix} W^+ \\ W^0 \\ W^- \end{pmatrix}, \; B^0 \quad \rightarrow \quad \begin{pmatrix} W^+ \\ Z^0 \\ W^- \end{pmatrix}, \; \gamma$$

Physical fields: W^+, Z^0, W^- and A (photon)

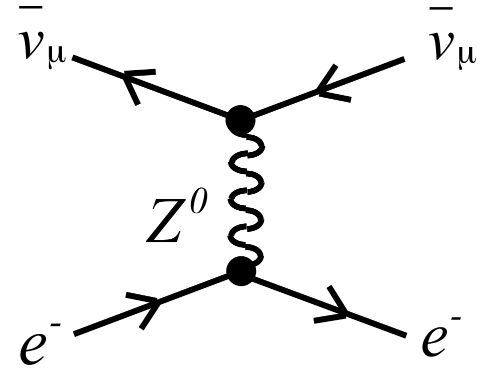
$$\begin{aligned} Z^0 &= W^0 \cos\vartheta_W - B^0 \sin\vartheta_W \\ A &= W^0 \sin\vartheta_W + B^0 \cos\vartheta_W \end{aligned}$$

ϑ_W WEAK MIXING ANGLE

W, Z^0 "acquire" mass via the HIGGS MECHANISM.

Evidence for GWS Model

- Discovery of Neutral Currents (1973)



The process $\bar{\nu}_\mu e^- \rightarrow \bar{\nu}_\mu e^-$ was observed.

ONLY possible Feynman diagram (no W diagram)

Indirect evidence for Z^0

- Direct Observation of $W^{\pm}$ and Z^0 (1983)

First DIRECT observation in $p\bar{p}$ collisions at $\sqrt{s} = 540\text{GeV}$ via decays into leptons.

$$p\bar{p} \rightarrow W^{\pm} + X \quad \hookrightarrow e^{\pm}\nu_e, \mu^{\pm}\nu_\mu$$

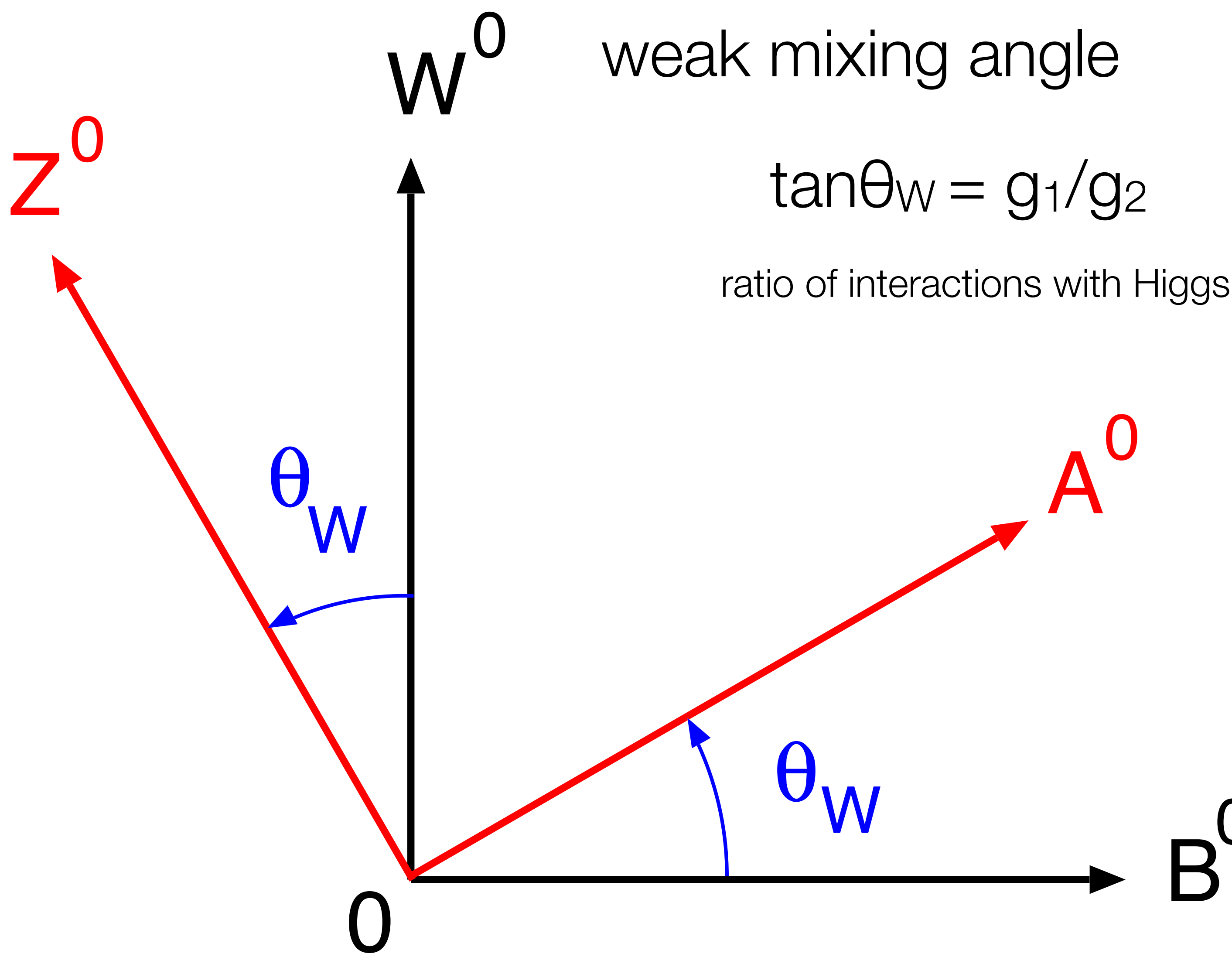
$$p\bar{p} \rightarrow Z^0 + X \quad \hookrightarrow e^+e^-, \mu^+\mu^-$$

- **Precision Measurements of the Standard Model (1989-2000)**

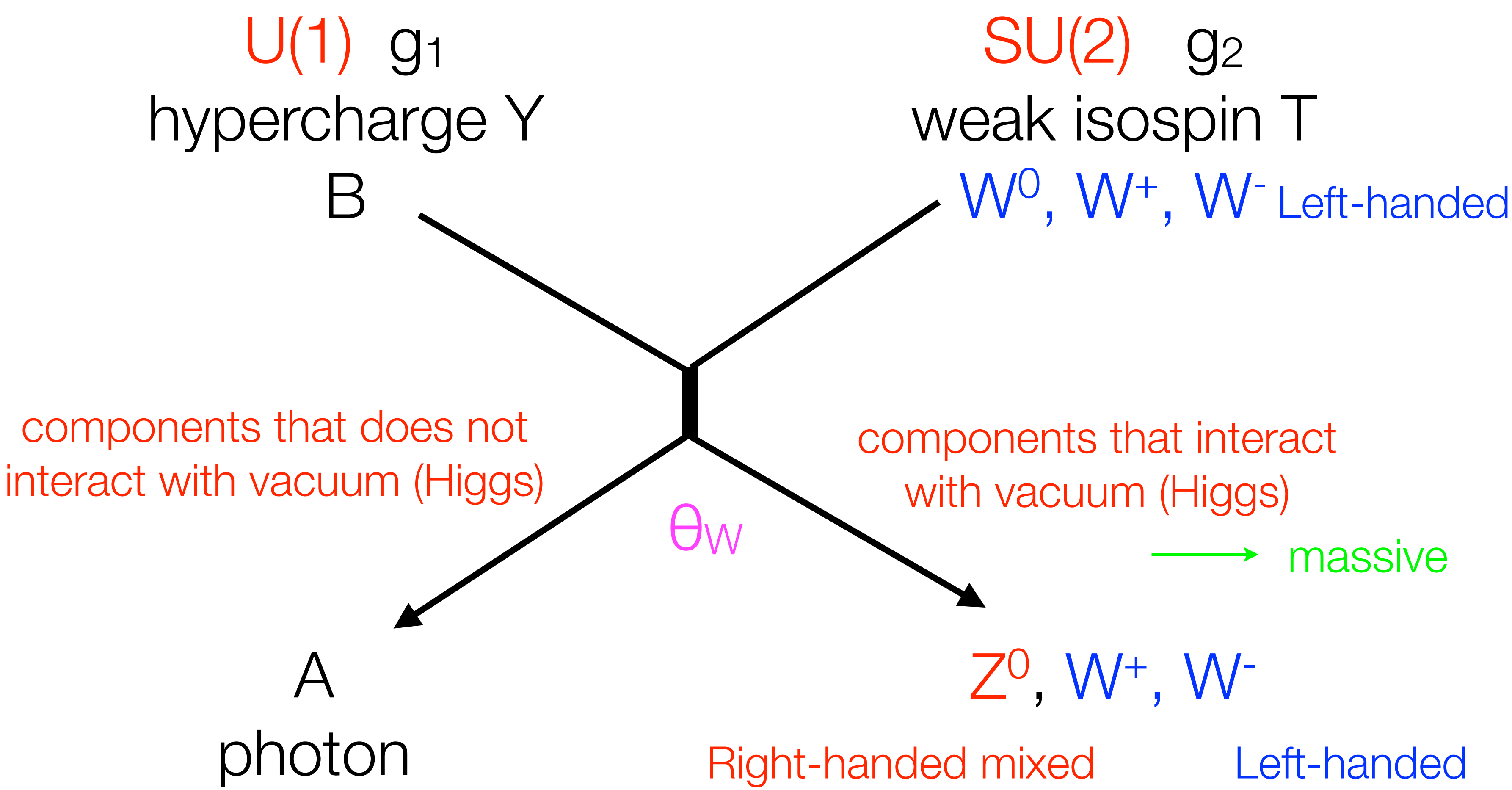
LEP e^+e^- collider (see later) provided many precision measurements of the Standard Model.

- Wide variety of different processes consistent with GWS model predictions and measure SAME VALUE of

$$\sin^2\vartheta_W = 0.231 \qquad \vartheta_W \approx 29^{\circ}$$


weak mixing angle
tanθW = g1/g2
ratio of interactions with Higgs
W0
Z0
A0
B0
θW
θW
0

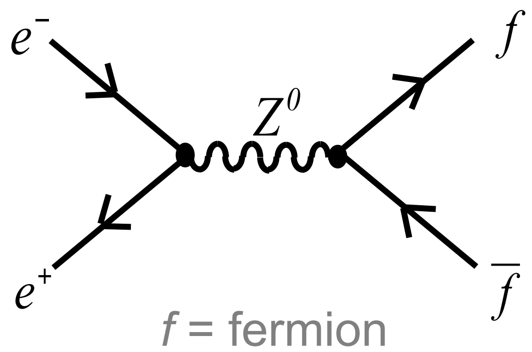
Photon & Z^0 in Electroweak Theory

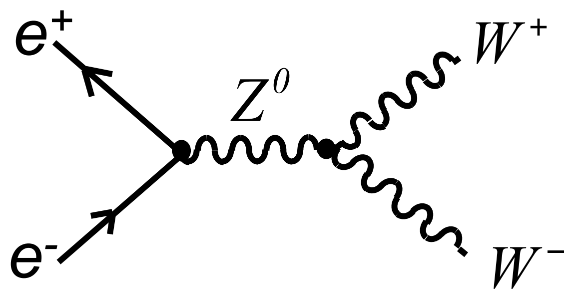


Experimental Tests of the Standard Model

The Large Electron Positron (LEP) collider at CERN provided precision measurements of the Standard Model (1989-2000).

Designed as a Z^0 and $W^{\pm}$ boson factory

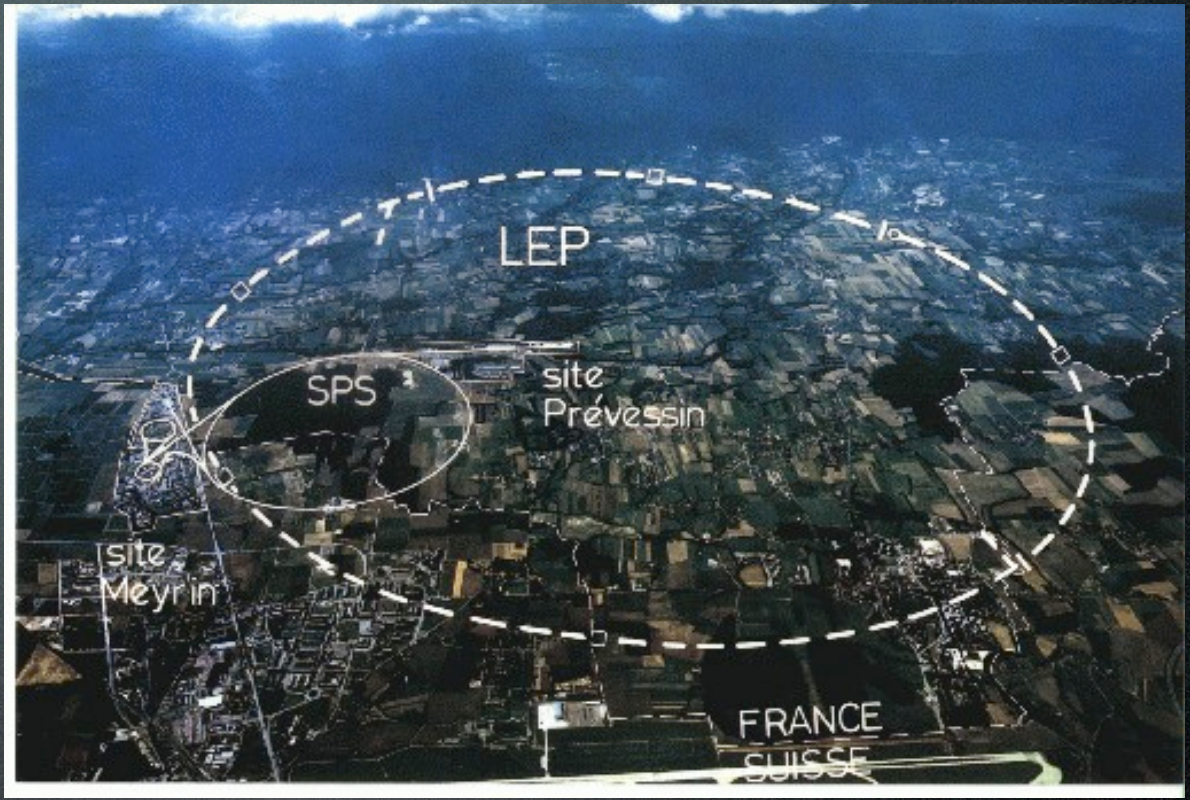




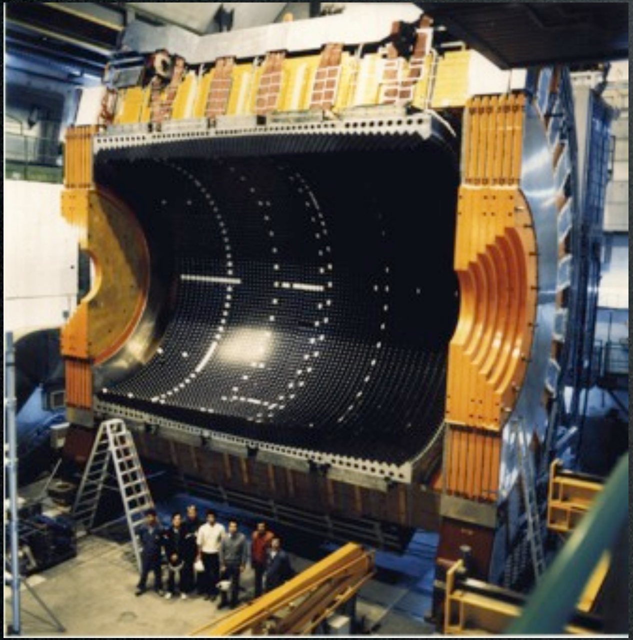
Precise measurements of the properties of Z^0 and $W^{\pm}$ bosons provide the most stringent test of our current understanding of particle physics.

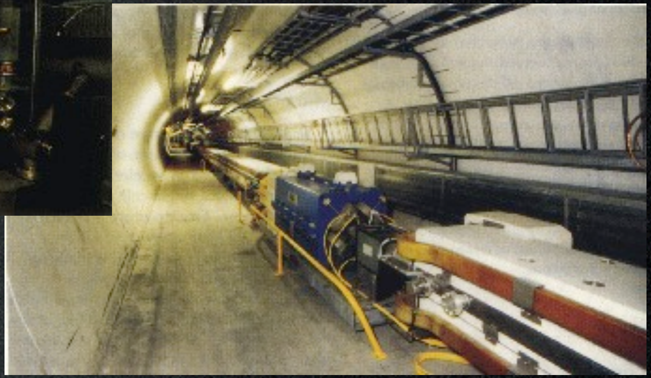
LEP

e^+e^- collider at CERN
operation: 1989 - 2000
circumference 27km
E_{CM} = ~89 - 209 GeV



OPAL detector (U.Tokyo)





Neutral current sectors

Fermion-boson interaction

$g' Y/2\ B^0 + g\ T_3\ W^0$

U(1) SU(2)$_L$

$B^0 = A \cos\theta_w + Z^0 \sin\theta_w$ U(1)

$W^0 = A \sin\theta_w - Z^0 \cos\theta_w$ SU(2)$_L$

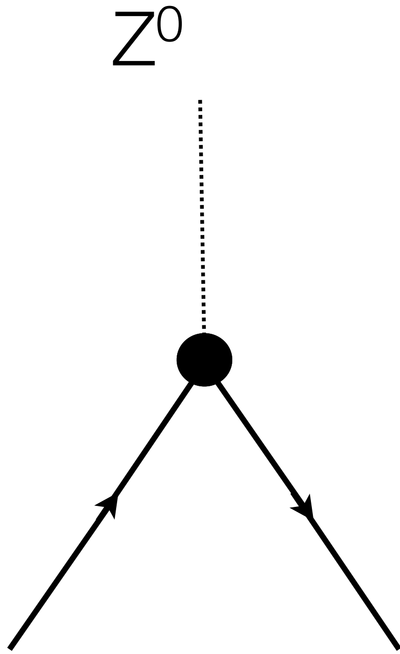
$Q = T_3 + Y/2$ (electric charge)

→ $g_{em}\ Q\ A + (g \cos\theta_w T_3 – g' \sin\theta_w Y/2)\ Z^0$

Photon exchange
= elecrtromagnetic

Z boson interaction

Interactions of Z^0



- left / right mixed
- V = L + R, A = L - R

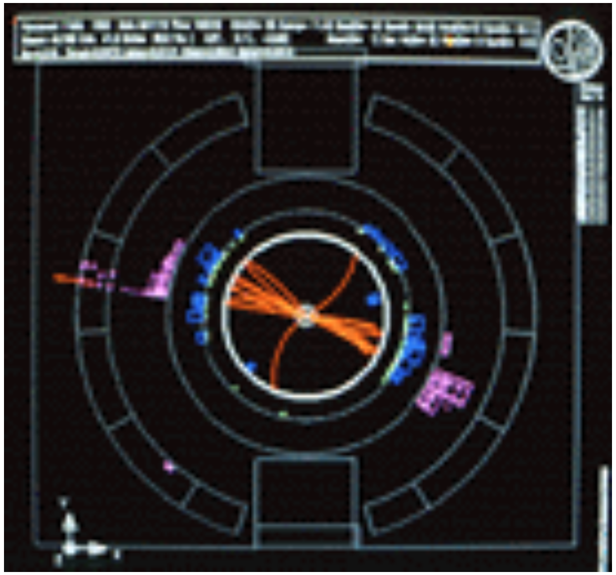
$$\bar{\psi}\gamma^{\mu}(g_V + g_A\gamma^5)\psi Z_\mu$$

$$g_A = \frac{1}{2}T_3$$

$$g_V = \frac{1}{2}T_3 - 2Q\sin^2\theta_W$$

1990

OPAL detector : decay Z0



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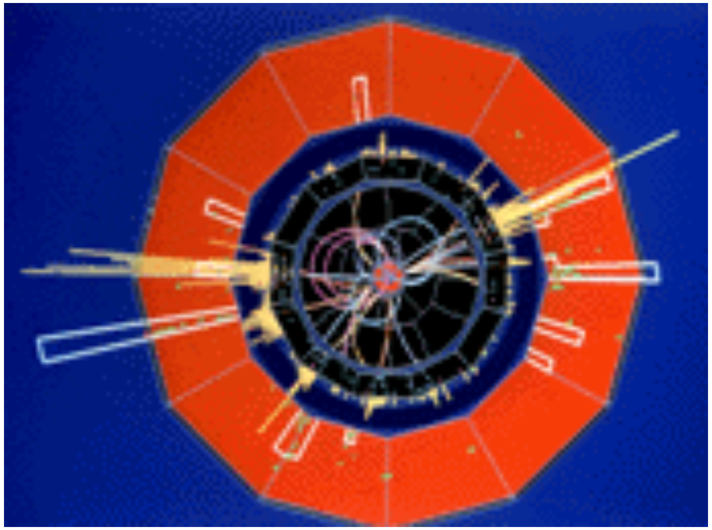
ALEPH : decay of a Z0



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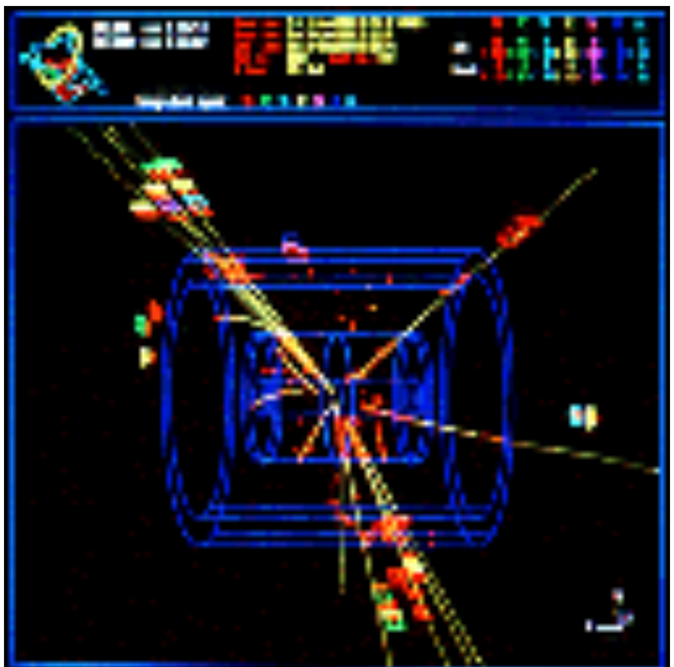
1990

DELPHI : decay of a Z0



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The first web server: this machine was used by Tim Berners-Lee in 1990 to develop and run the first WWW server, multi-media browser and web editor.

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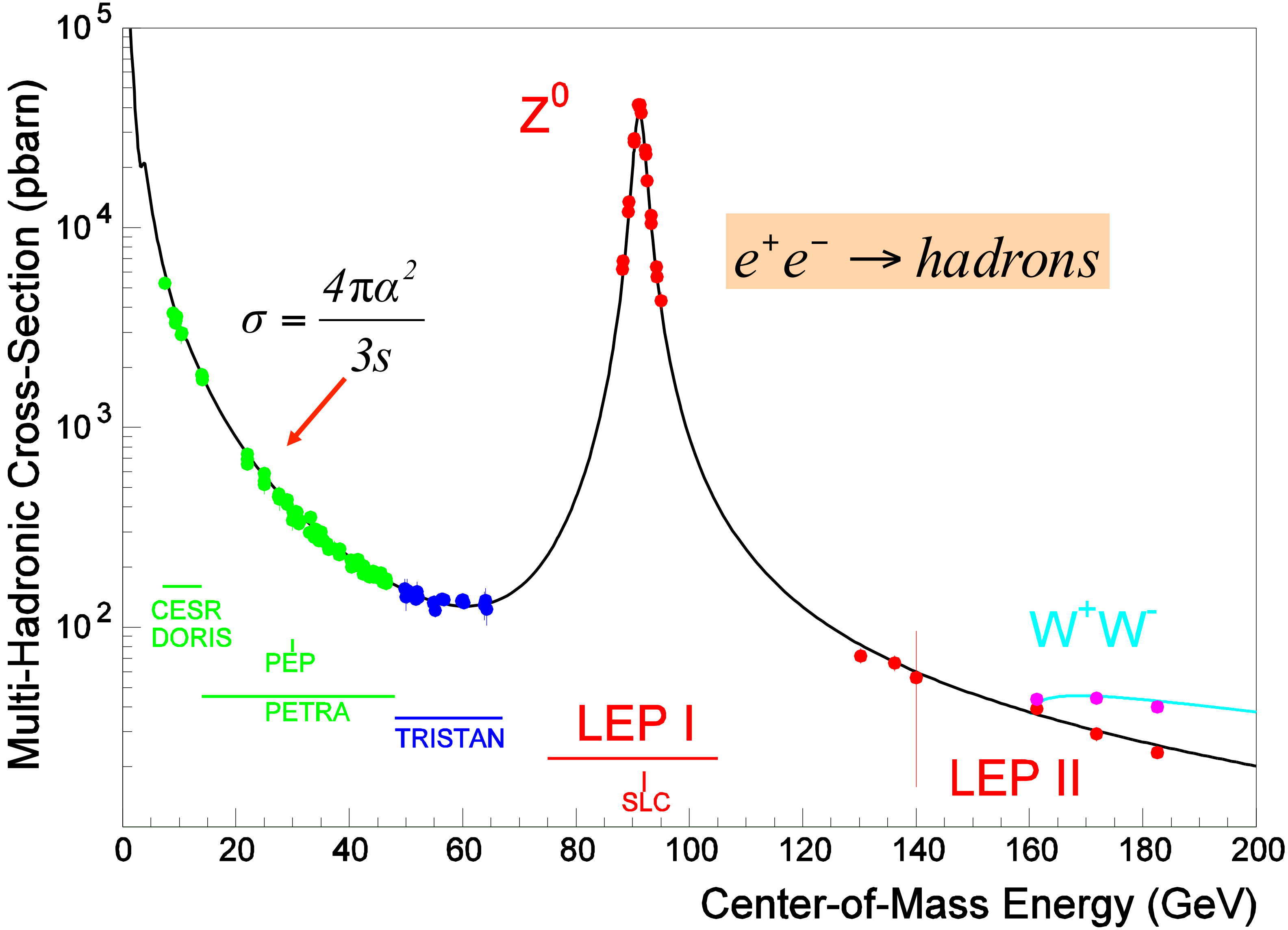
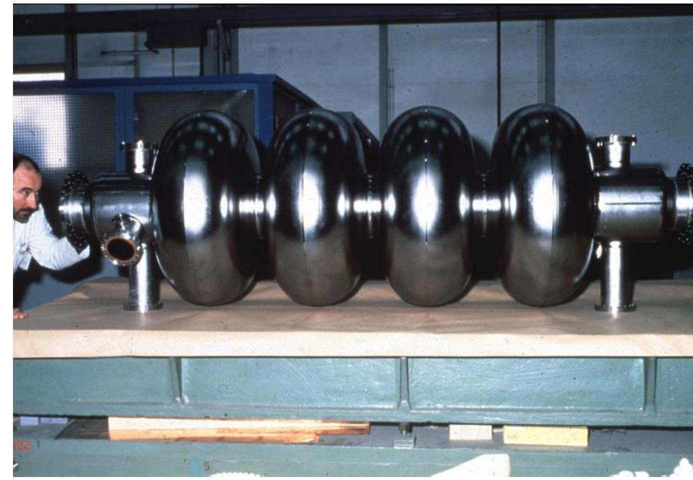
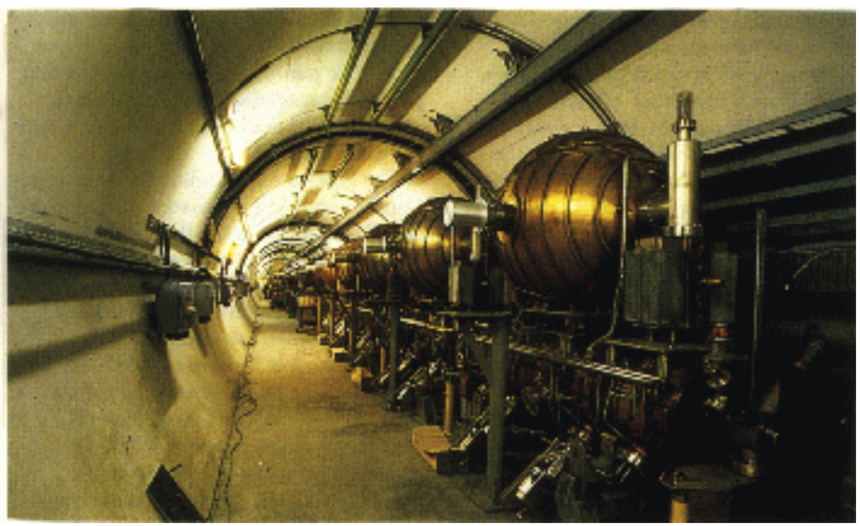
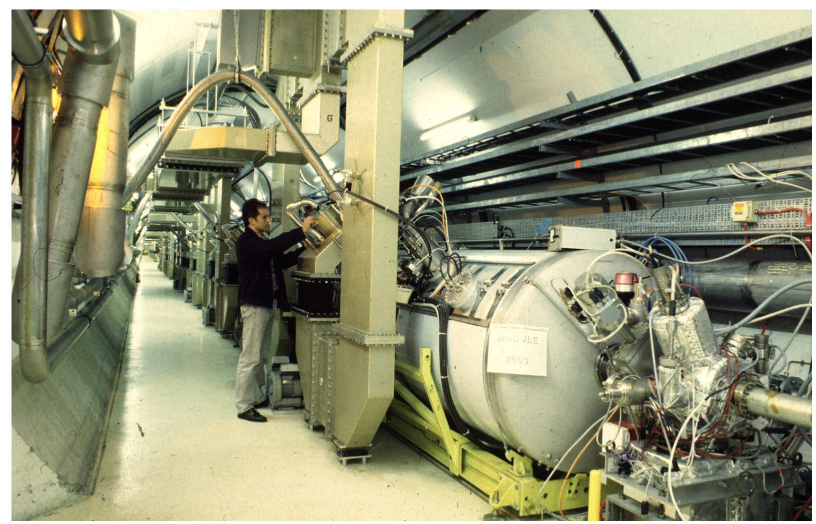

Multi-Hadronic Cross-Section (pbarn)
10^5
10^4
10^3
10^2
$\sigma = \frac{4\pi\alpha^2}{3s}$
Z^0
$e^+e^- \rightarrow hadrons$
W^+W^-
CESR
DORIS
PEP
PETRA
TRISTAN
LEP I
SLC
LEP II
0
20
40
60
80
100
120
140
160
180
200
Center-of-Mass Energy (GeV)

LEP



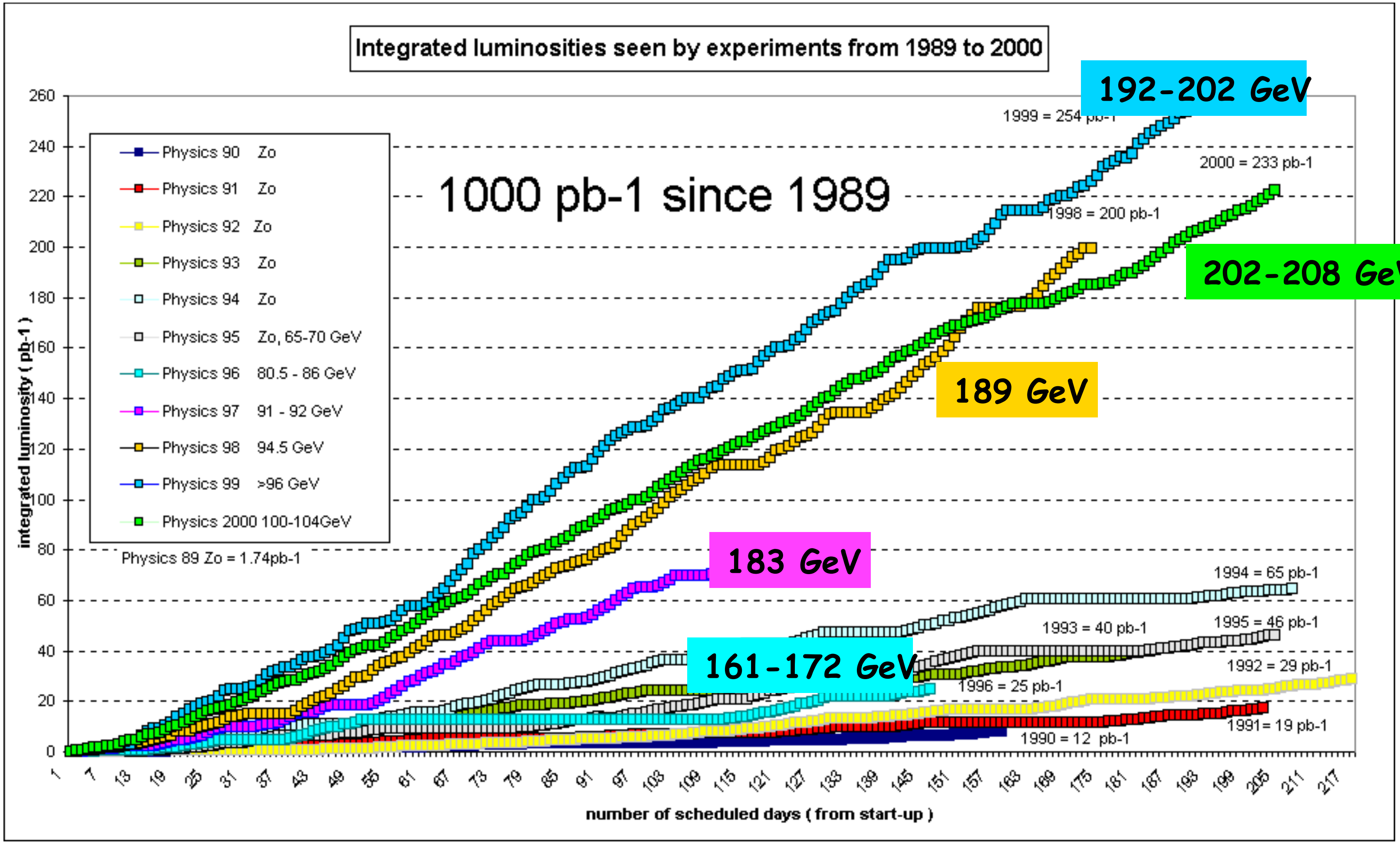




Accelerator cavities

Superconducting accelerator

LEP collider performance

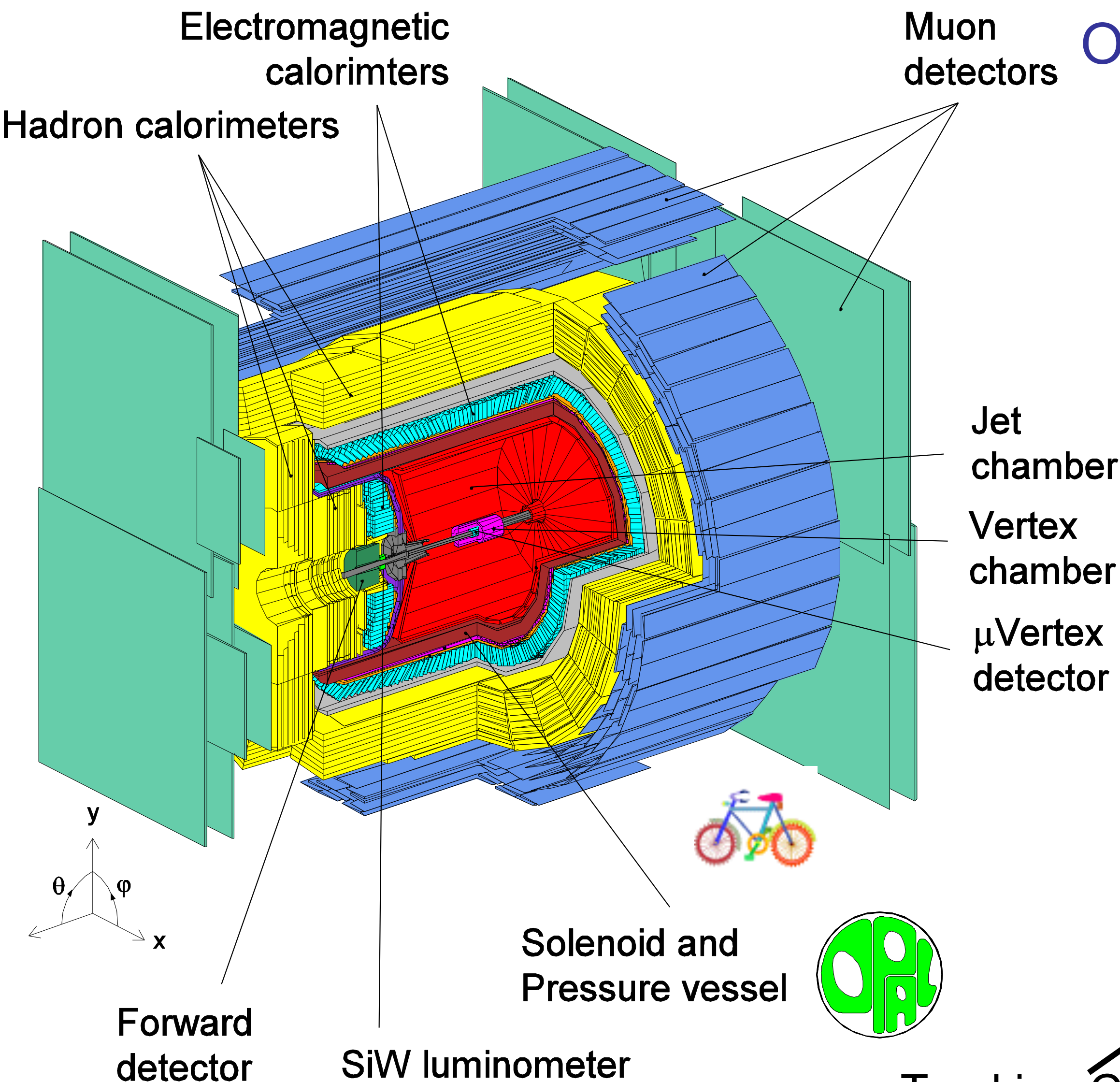


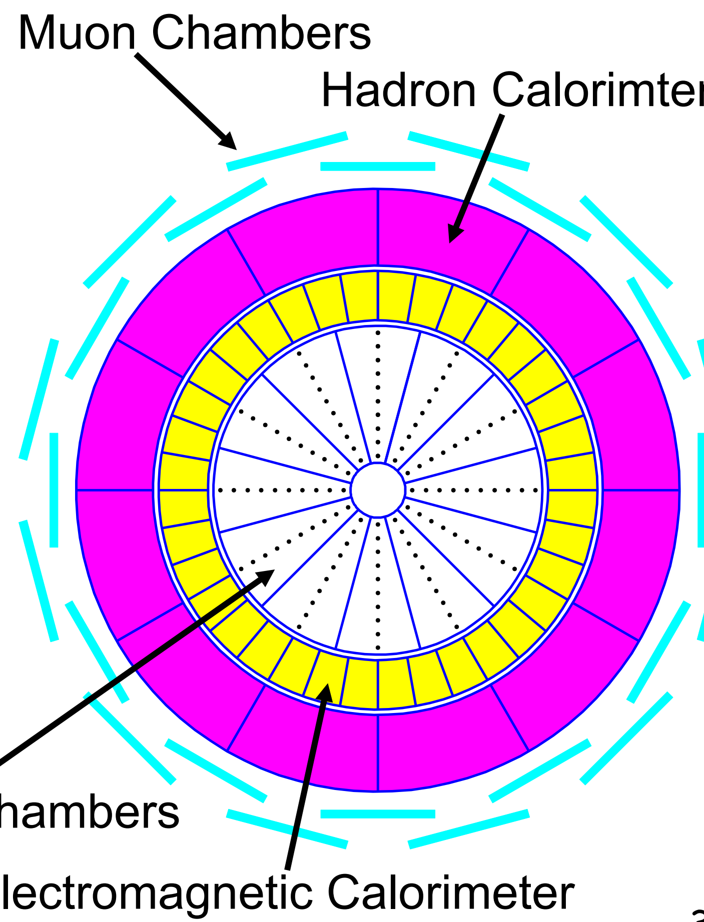
- LEP: e^+e^- collider at CERN
- LEP1: $E_{CM} \sim M_Z$ from 1989 to 1995
- LEP2: ~ 700 pb^{-1} /exp above WW production threshold from 1996 to 2000

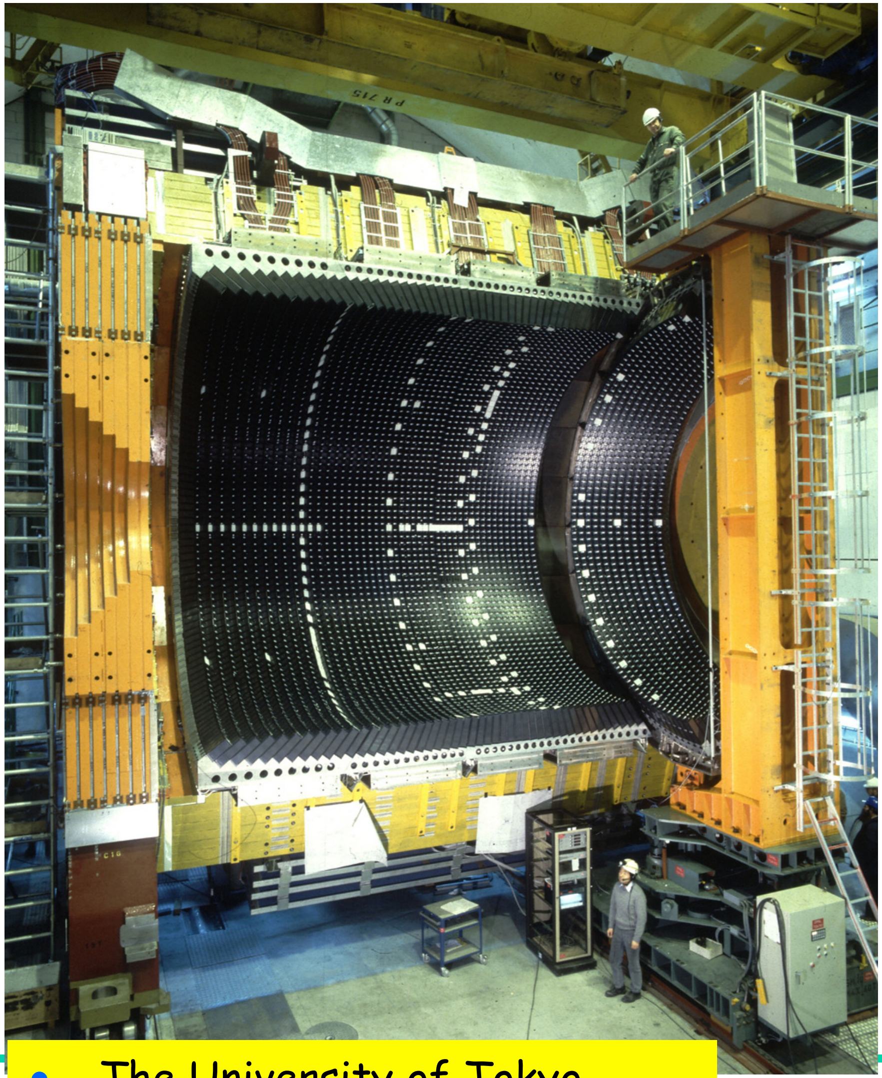
A LEP Detector: OPAL

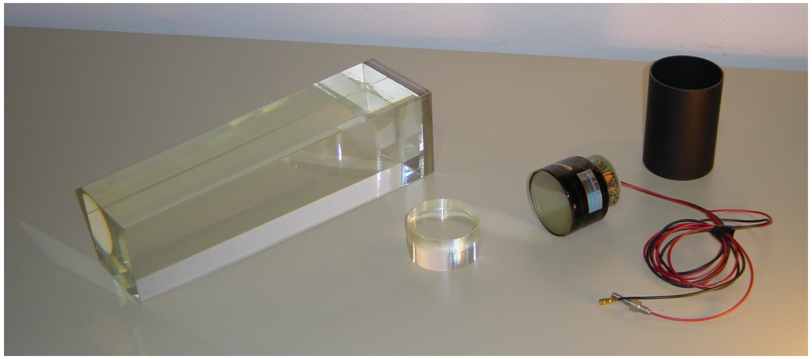
OPAL was one of the 4 experiments at LEP.

Size = 12 m x 12m x 15m







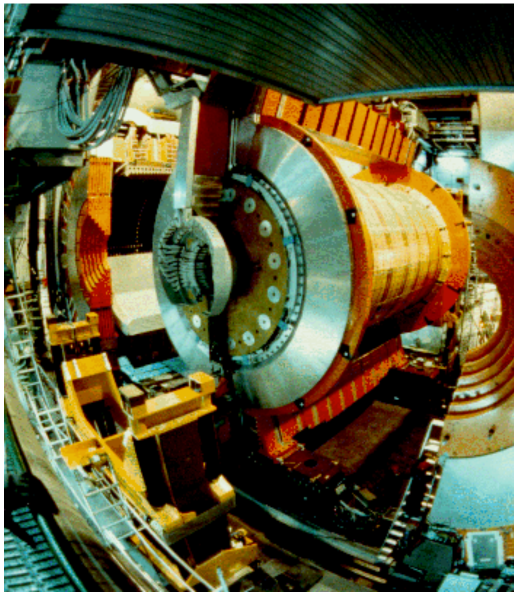


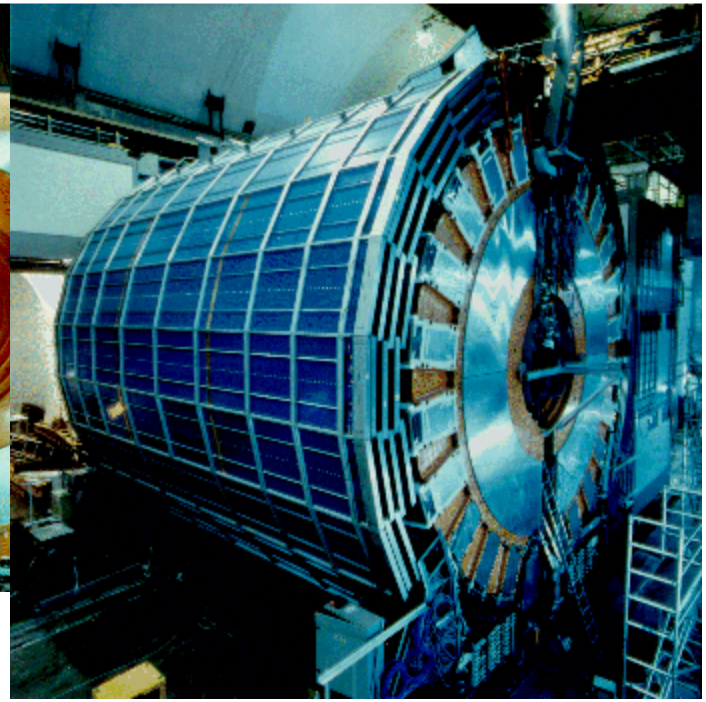
LEAD GLASS CALORIMETER

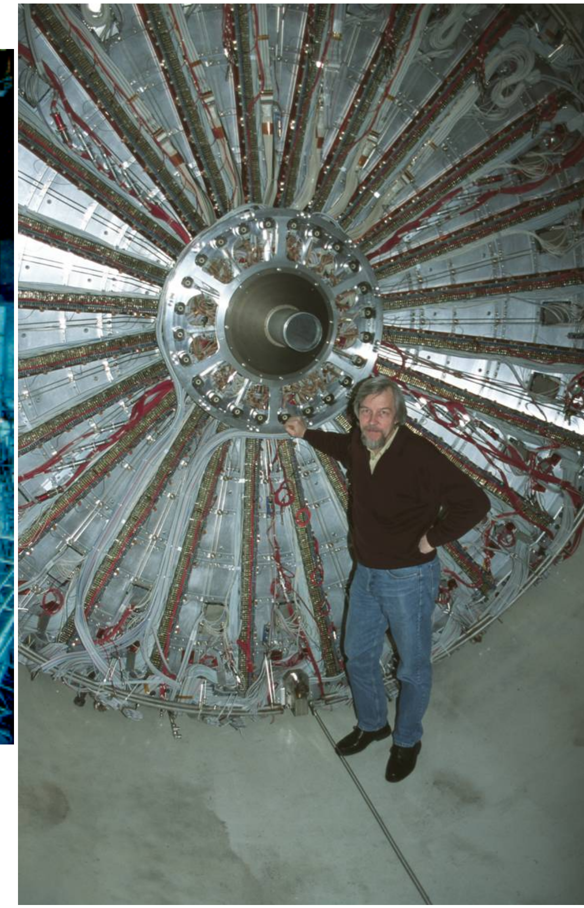
= Cherenkov type
Electromagnetic calorimeter

- The University of Tokyo

- Detector pictures

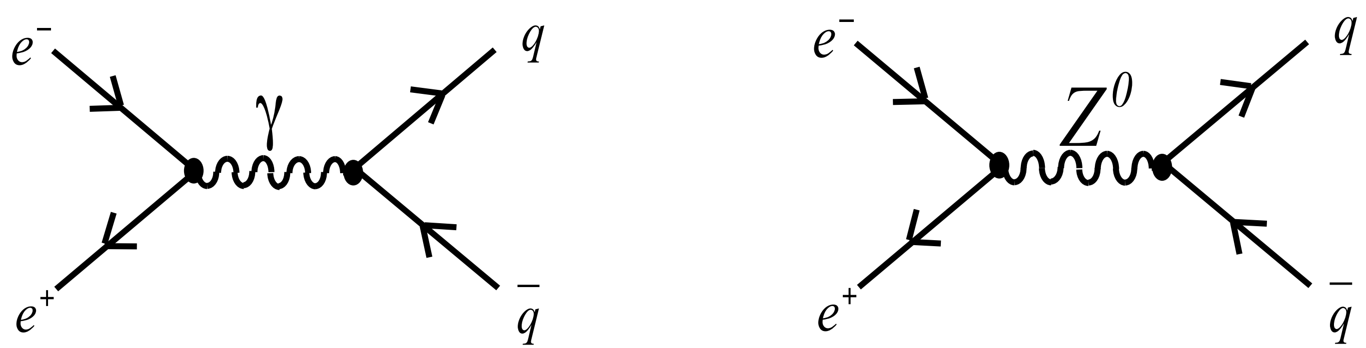






The Z^0 Resonance

Consider the process of $e^+e^- \rightarrow q\bar{q}$



$$\sigma(e^+e^- \rightarrow \gamma \rightarrow q\bar{q}) = \frac{4\pi\alpha^2}{3s}$$

- The Z^0 is a decaying intermediate massive states (lifetime ~ 10^{-25} s)

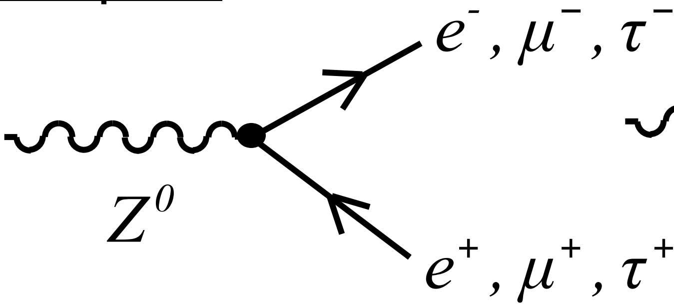
  BREIT-WIGNER RESONANCE

- At $\sqrt{s} \sim M_Z$ the Z^0 diagram dominates.

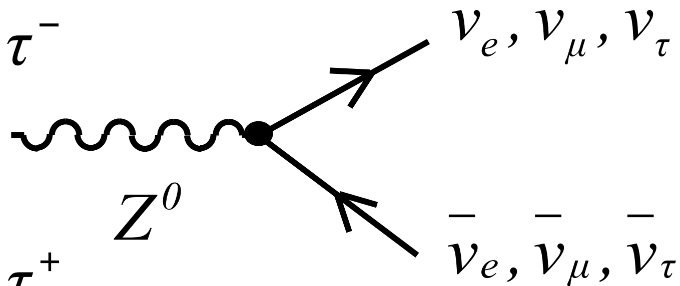
At Z resonance, we determined Standard Model fundamental parameters

- mass of Z (Mz → Weinberg angle)
- neutral current coupling constant (→Weinberg angle)
- Number of generations (N=3)
- strong coupling constant (QCD) (see lecture #3 for QCD)

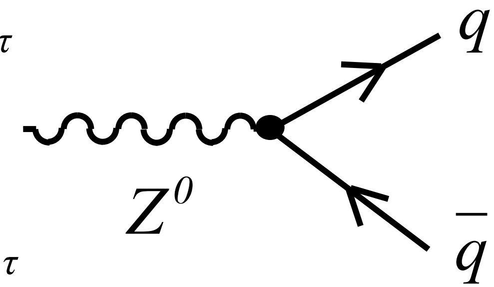
Examples:



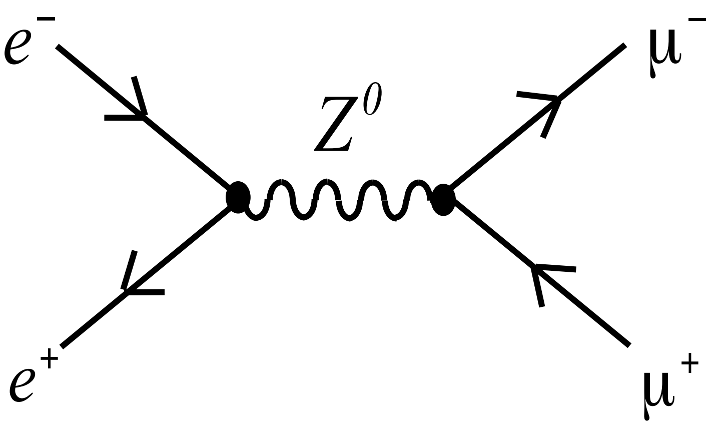
$$Z^0 \rightarrow e^+e^-, \mu^+\mu^-, \tau^+\tau^-$$



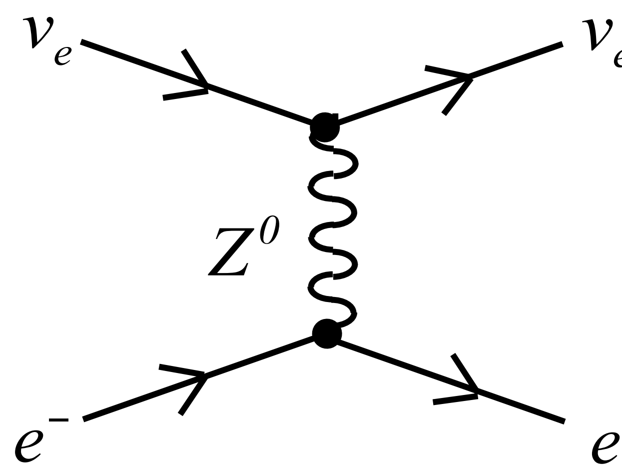
$$Z^0 \rightarrow \nu_e\bar{\nu}_e, \nu_\mu\bar{\nu}_\mu, \nu_\tau\bar{\nu}_\tau$$



$$Z^0 \rightarrow q\bar{q}$$



$$e^+e^- \rightarrow \mu^+\mu^-$$



$$\nu_e e^- \rightarrow \nu_e e^-$$

W Decay Width

$$\Gamma(W^+ \to e^+\nu_e) = \frac{G_F M_W^3}{6\sqrt{2}\pi} \approx 226.20 \pm 0.10 \text{ MeV}$$

$$\Gamma(W^+ \to u_i \bar{d}_j) = \frac{C G_F M_W^3}{6\sqrt{2}\pi} |V_{ij}|^2$$

C : color factor
V_{ij} : CKM matrix

W^+ DECAY MODES		Fraction (Γ_i/Γ)
$\ell^+\nu$	[b]	(10.80 ± 0.09) %
$e^+\nu$		(10.75 ± 0.13) %
$\mu^+\nu$		(10.57 ± 0.15) %
$\tau^+\nu$		(11.25 ± 0.20) %
hadrons		(67.60 ± 0.27) %

Z Decay Width

$$\Gamma(f\bar{f}) = \frac{C_f G_F M_Z^3}{6\sqrt{2}\pi}\left(g_V^{f2} + g_A^{f2}\right)$$

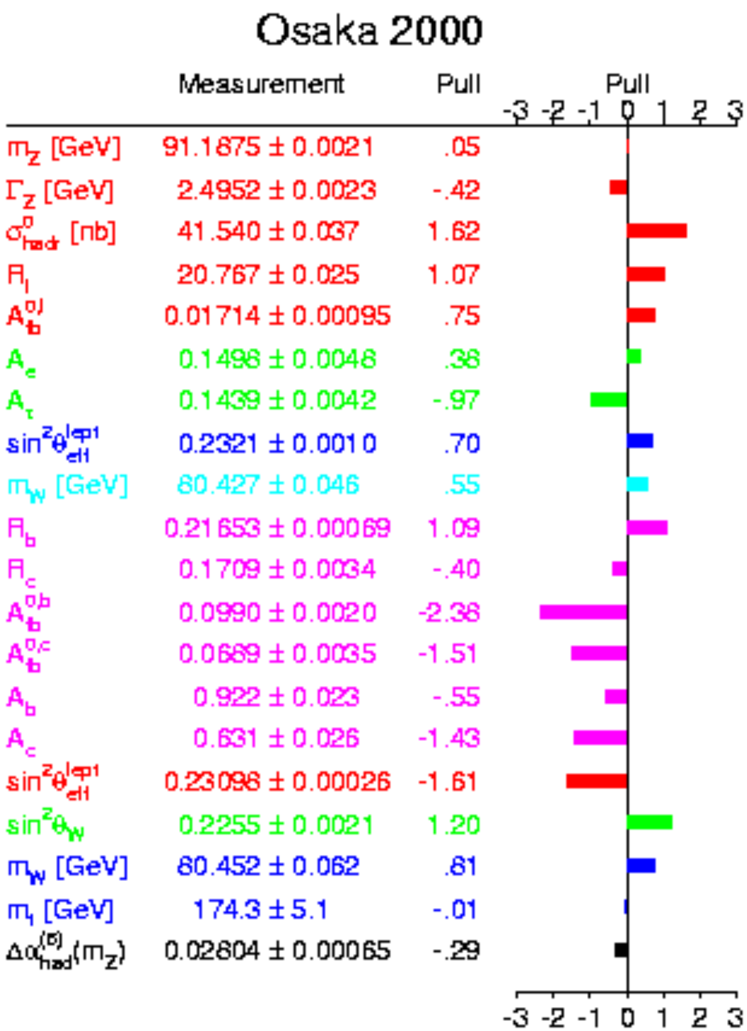
$$g_V^i \equiv t_{3L}(i) - 2q_i \sin^2\theta_W$$

$$g_A^i \equiv t_{3L}(i)$$

$C_f = 1$ (leptons) and 3 (quarks)

Z DECAY MODES		Fraction (Γ_i/Γ)
e^+e^-		(3.363 ±0.004) %
$\mu^+\mu^-$		(3.366 ±0.007) %
$\tau^+\tau^-$		(3.367 ±0.008) %
$\ell^+\ell^-$	[b]	(3.3658 ±0.0023) %
invisible		(20.00 ±0.06) %
hadrons		(69.91 ±0.06) %

Electroweak Precision Tests



The SM parameters are measured.

The SM relations are tested to unprecedented accuracy.

$U(1)\times SU(2)_L \times SU(3)$ structure and Mixing are fully verified.

precision data predicted top quark mass, Higgs mass, and indication of new physics at TeV scale and GUT scale (SUSY so on)

Now have 5 precise measurements of fundamental parameters of the Standard Model

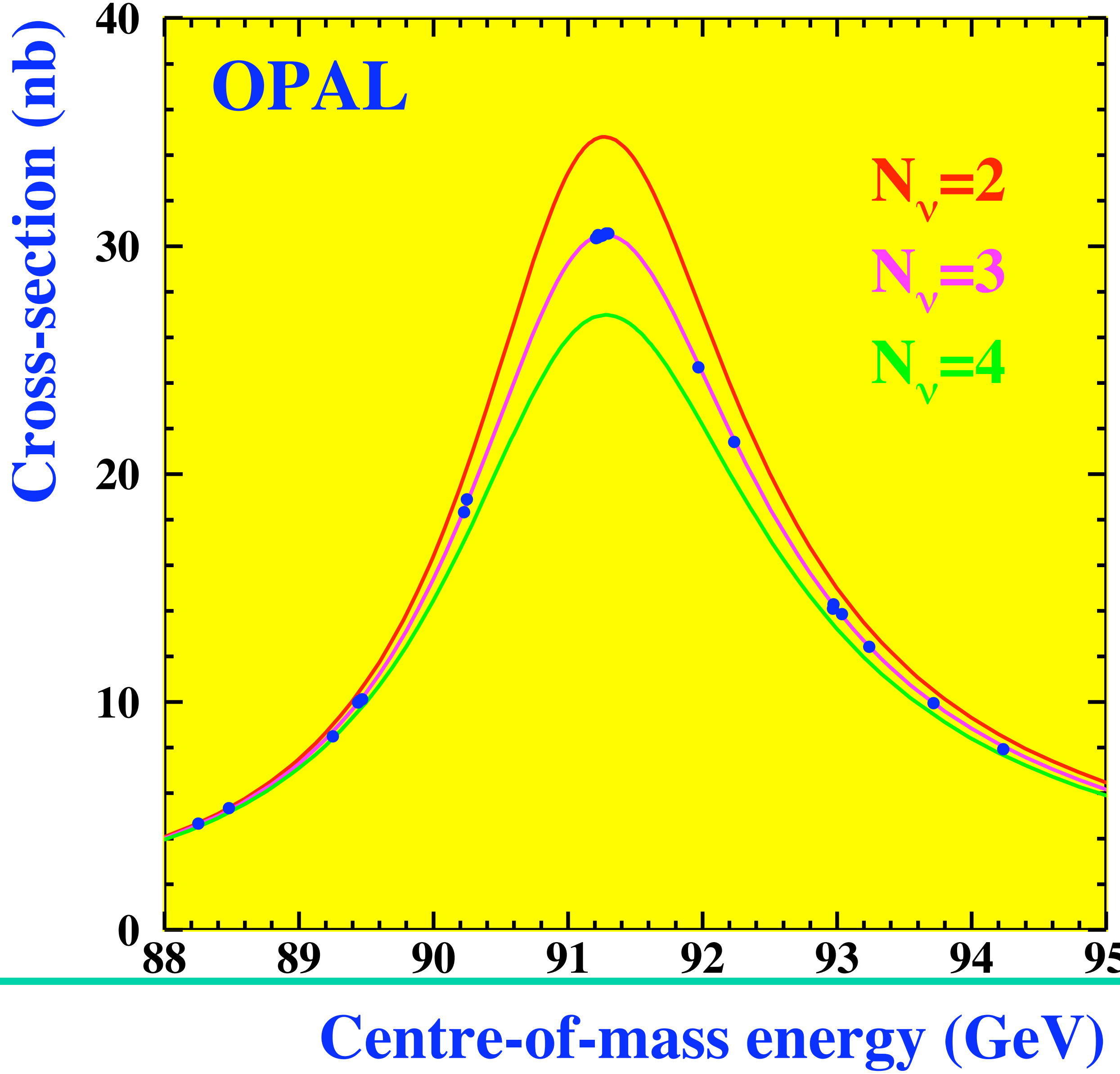
$$\begin{aligned} \alpha_{em} &= 1/\left(137.03599976 \pm 0.00000050\right) \\ G_F &= \left(1.16632 \pm 0.00002\right) \times 10^{-5}\ \text{GeV}^{-2} \\ M_W &= \left(80.423 \pm 0.038\right) \text{GeV} \\ M_Z &= \left(91.1875 \pm 0.0021\right) \text{GeV} \\ sin^2\vartheta_W &= 0.23143 \pm 0.00015 \end{aligned}$$

(at $q^2=0$)

In the Standard Model, ONLY 3 are independent.

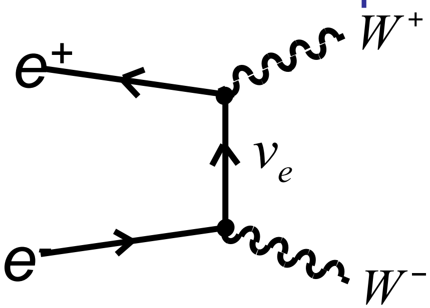
Their consistency is an incredibly powerful test of the Standard Model of Electroweak Interactions

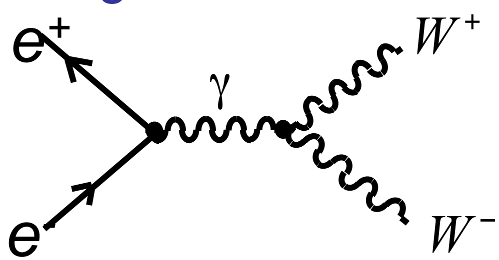
Number of Generations = 3

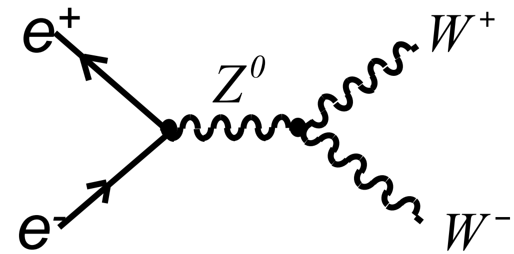


W^+W^- at LEP

- e^+e^- collisions W bosons are produced in pairs.
- Standard Model 3 possible diagrams:



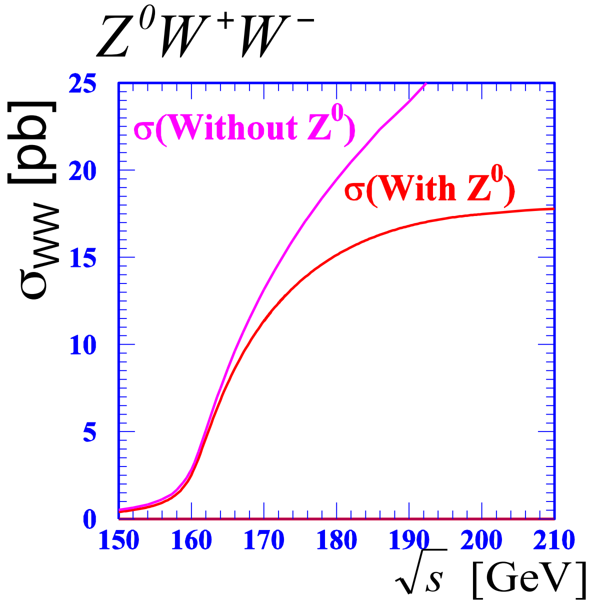


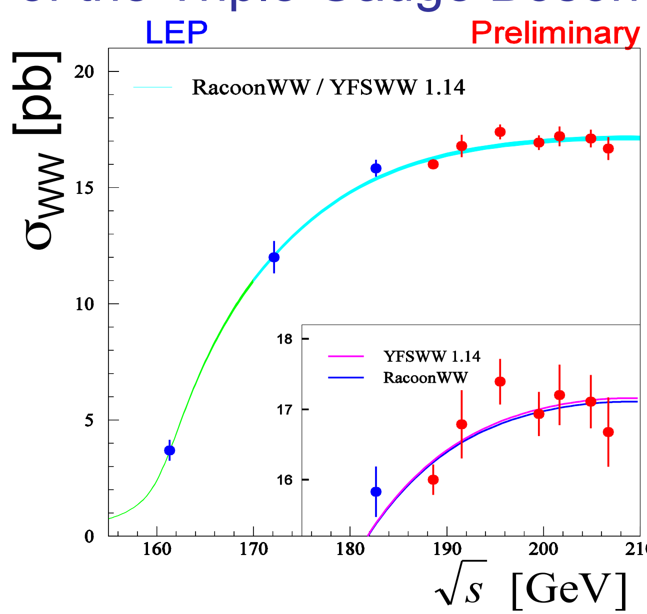


- LEP operated above the threshold for W^+W^- production (1996-2000)

$$\sqrt{s} > 2M_W$$

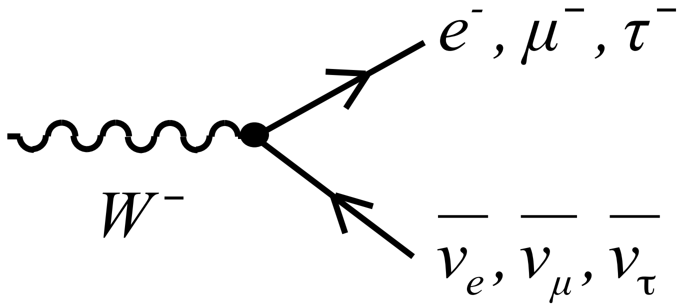
- Cross-section sensitive to the presence of the Triple Gauge Boson vertex

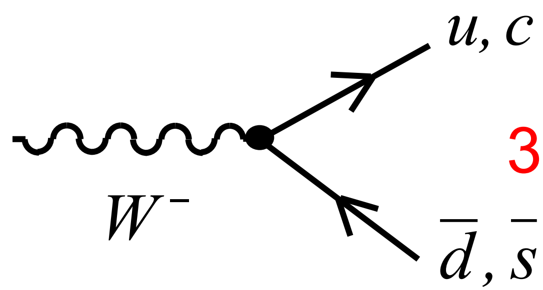




W^+W^- Decay at LEP

In the Standard Model $W^{\pm}\ell\nu$ and $W^{\pm}q\bar{q}$ couplings are ≈ equal.





$M_W < m_t$

3 FOR COLOUR

EXPECT (3 colours):

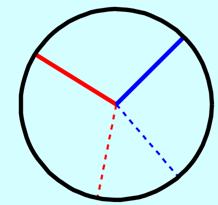
Branching fractions

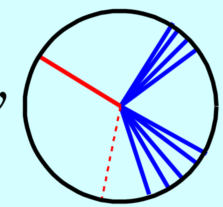
$$B\left(W^{\pm} \to q\bar{q}\right) = \frac{6}{9} = \frac{2}{3}$$

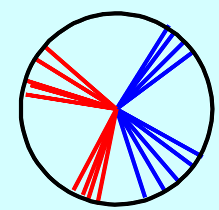
$$B\left(W^{\pm} \to \ell\nu\right) = \frac{3}{9} = \frac{1}{3}$$

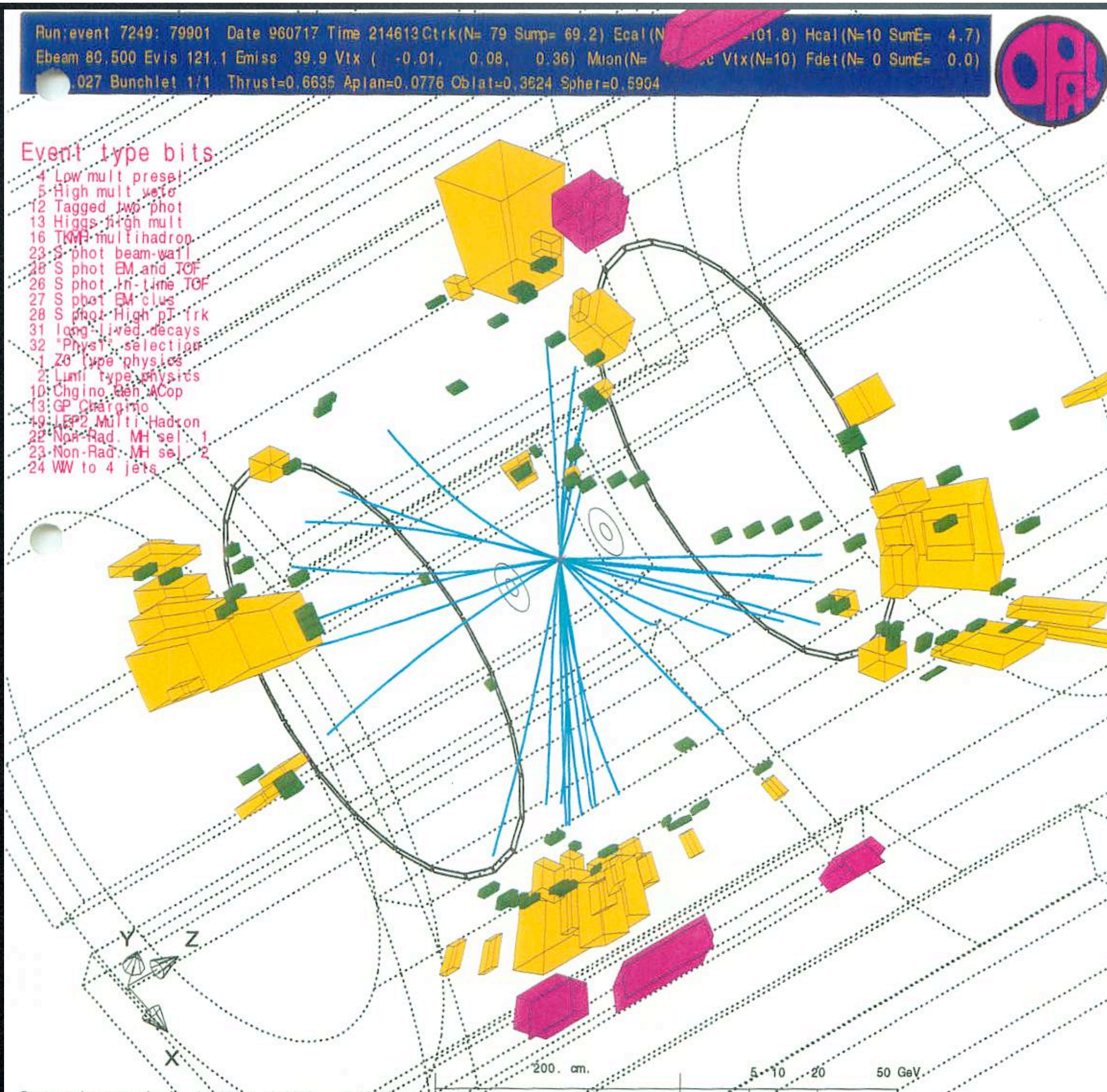
QCD corrections ~ $\left(1 + \alpha_s/\pi\right)$

$$\Rightarrow B\left(W^{\pm} \to q\bar{q}\right) = 0.675$$

$W^+W^- \to \ell\nu\ell\nu$  10.5%

$W^+W^- \to q\bar{q}\,\ell\nu$  43.9%

$W^+W^- \to q\bar{q}\,q\bar{q}$  45.6%

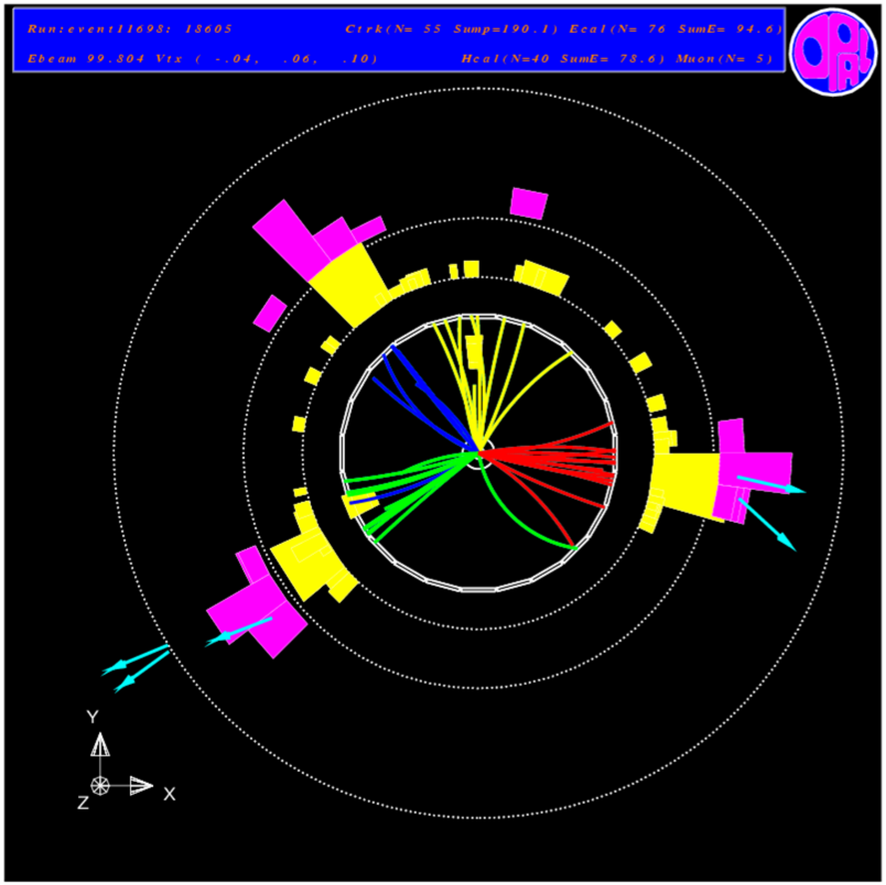


$e^+e^- \rightarrow W^+W^-$

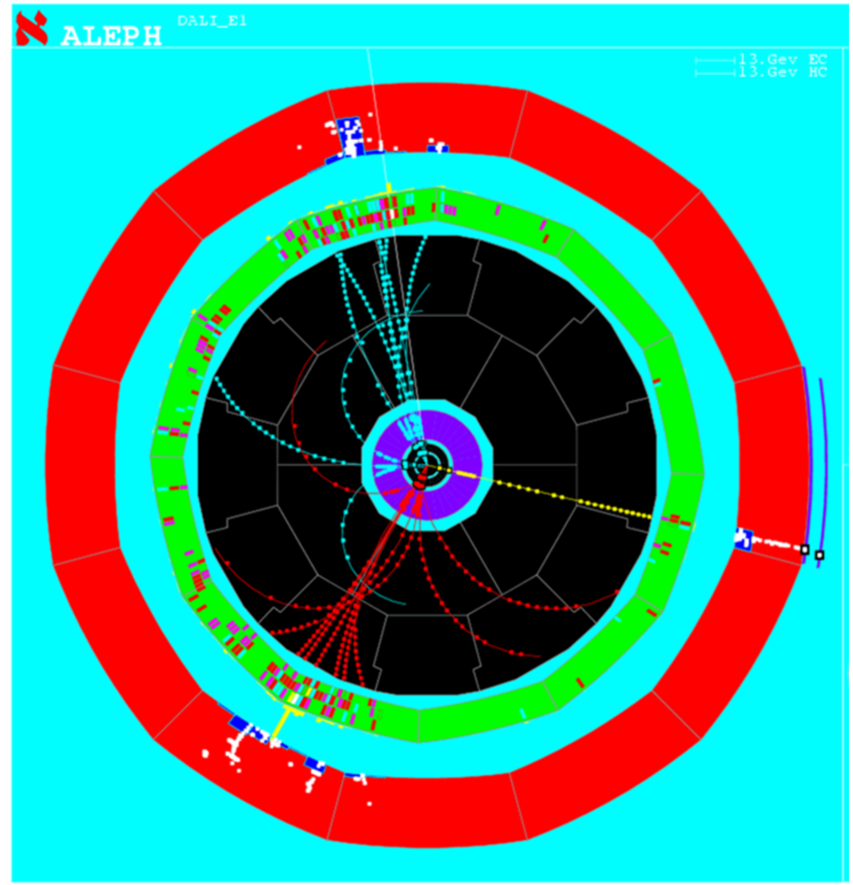
Both W's decayed into $q\bar{q}$, resulting in four jets

How can we reconstruct W's ?

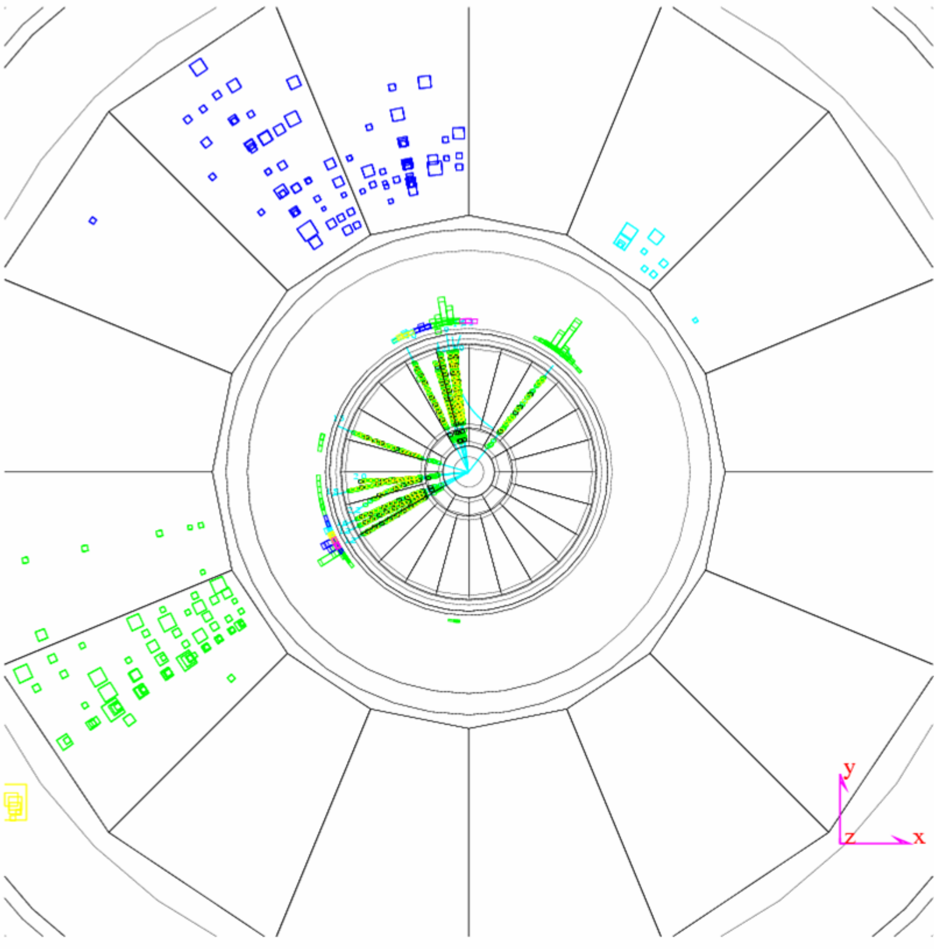
WW → qqqq (~45%)



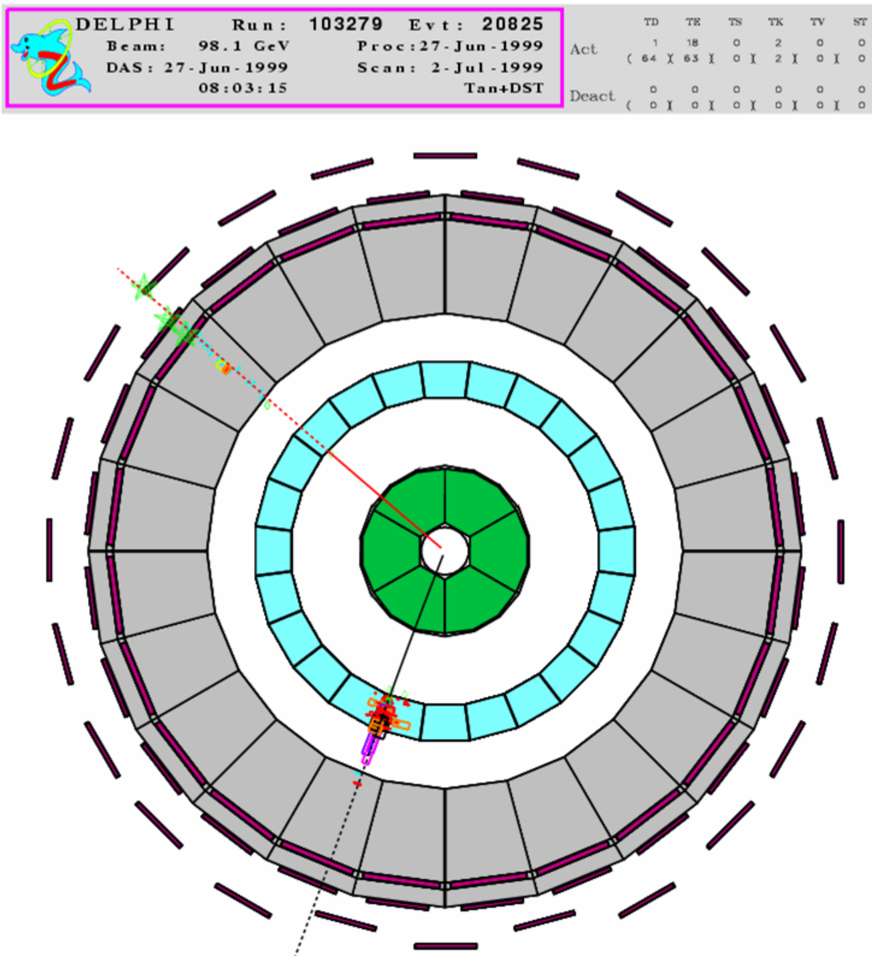
WW → μνqq (~15%)



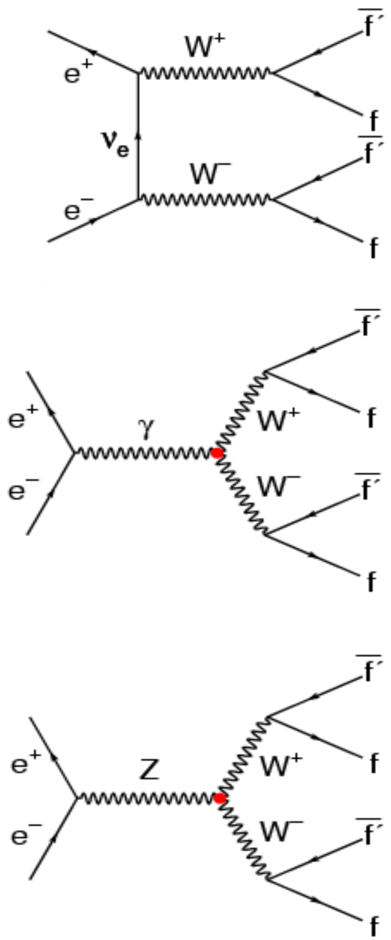
WW→ τνqq (~15%)

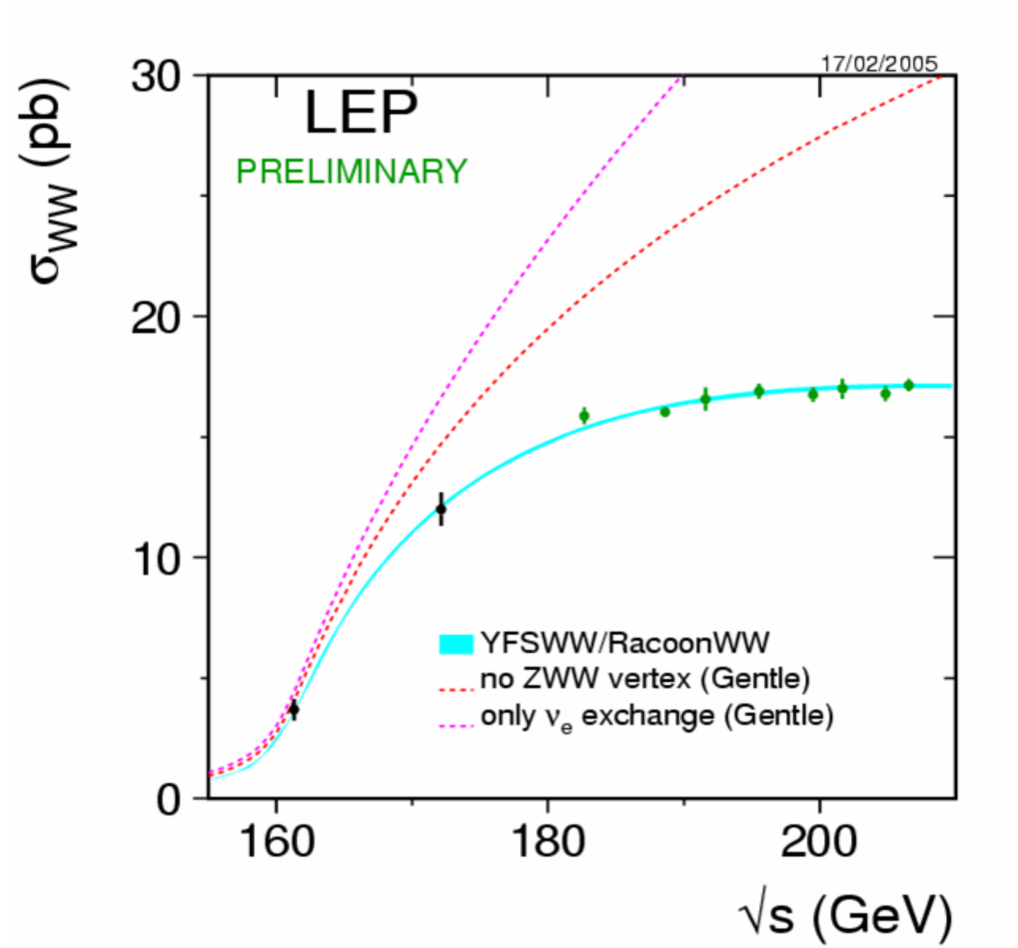


WW→ eνμν (~4%)



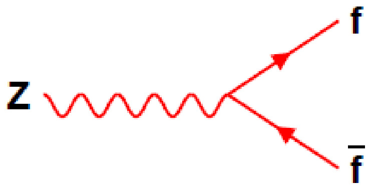
W-pair cross sections



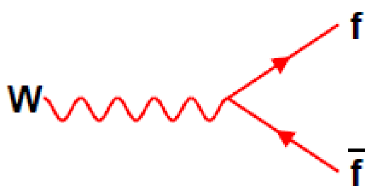


Clear proof of SU(2)xU(1) gauge couplings !

Higher Order Effects (Radiative Corrections)



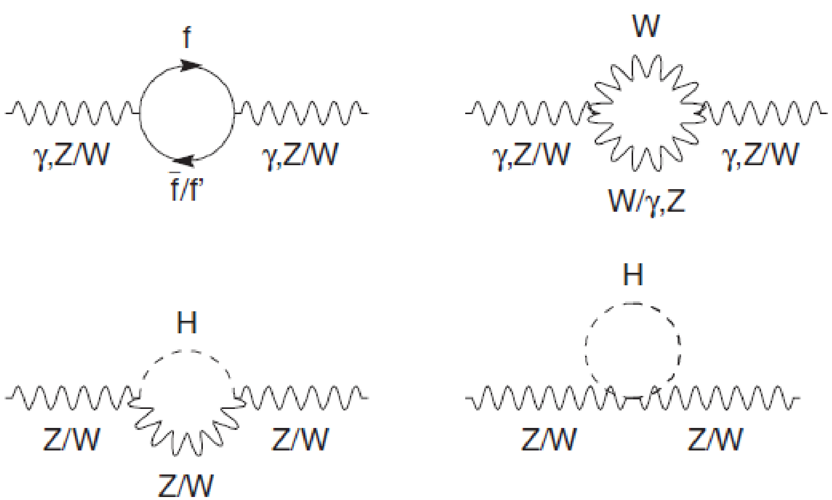
$$ie\gamma_\mu(g_v - g_a\gamma_5)\frac{1}{2\sin\theta_W\cos\theta_W}$$

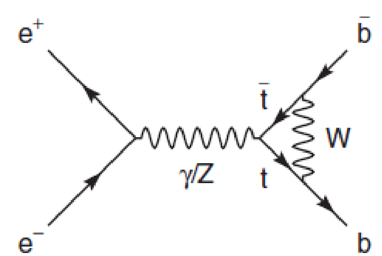


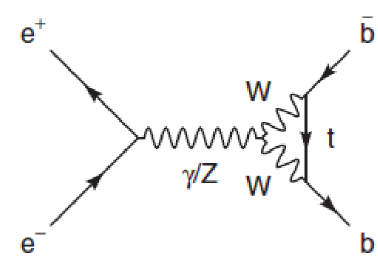
$$ie\gamma_\mu(1-\gamma_5)\frac{1}{2\sqrt{2}\sin\theta_W}$$

$$g_a = T^3 = \pm\frac{1}{2}$$

$$g_v = (T^3 - 2Q\sin^2\theta_W) = \pm\frac{1}{2}(1-4|Q|\sin^2\theta_W)$$







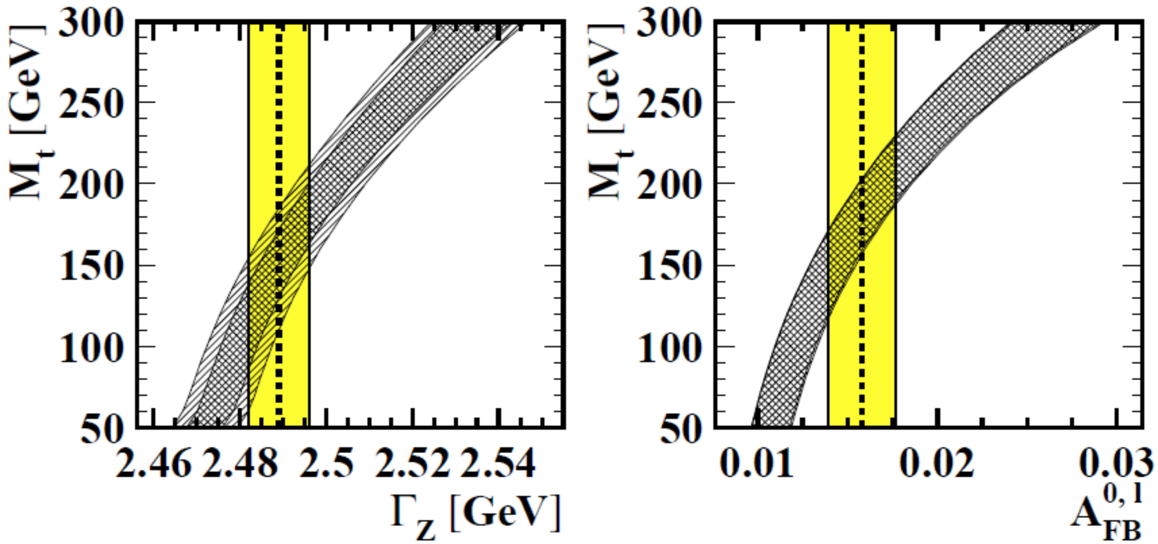
$$g_{Af} \equiv \sqrt{\rho_f}\,T_3^f$$

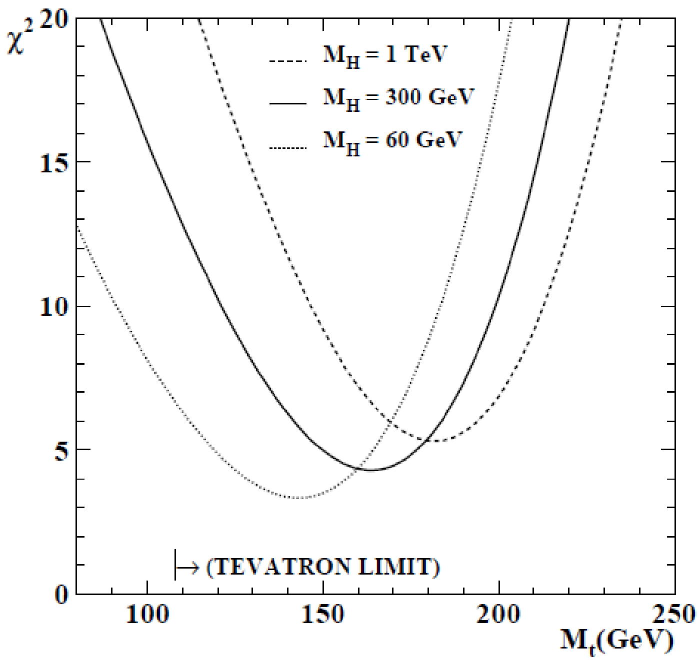
$$g_{Vf} \equiv \sqrt{\rho_f}\,(T_3^f - 2Q_f\sin^2\theta_{eff}^f)$$

$$\sin^2\theta_W = 1 - M_W^2/M_Z^2$$

$$\sin^2\theta_{eff}^f \equiv \kappa_f\sin^2\theta_W$$

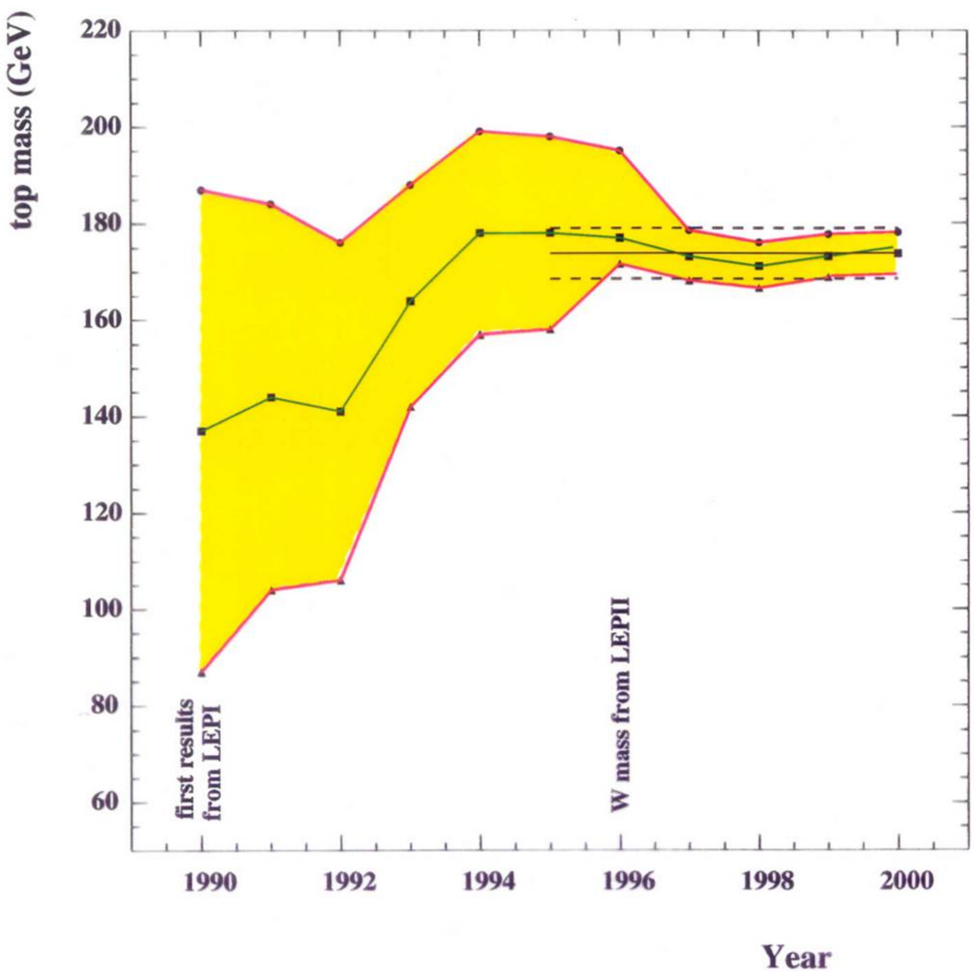
LEP Combined (Aug.1993)





	LEP	LEP + Collider and ν data
M_t (GeV)	$166^{+17}_{-19}\,^{+19}_{-22}$	$164^{+16}_{-17}\,^{+18}_{-21}$
$\alpha_s(M_Z^2)$	$0.120 \pm 0.006 \pm 0.002$	$0.120 \pm 0.006 \pm 0.002$
$\chi^2/(d.o.f.)$	3.5/8	4.4/11
$\sin^2\theta_{eff}^{lept}$	$0.2324 \pm 0.0005\,^{+0.0001}_{-0.0002}$	$0.2325 \pm 0.0005\,^{+0.0001}_{-0.0002}$
$1 - M_W^2/M_Z^2$	$0.2255 \pm 0.0019\,^{+0.0005}_{-0.0003}$	$0.2257 \pm 0.0017\,^{+0.0004}_{-0.0003}$
M_W (GeV)	$80.25 \pm 0.10\,^{+0.02}_{-0.03}$	$80.24 \pm 0.09\,^{+0.01}_{-0.02}$

Search for the Top Quark



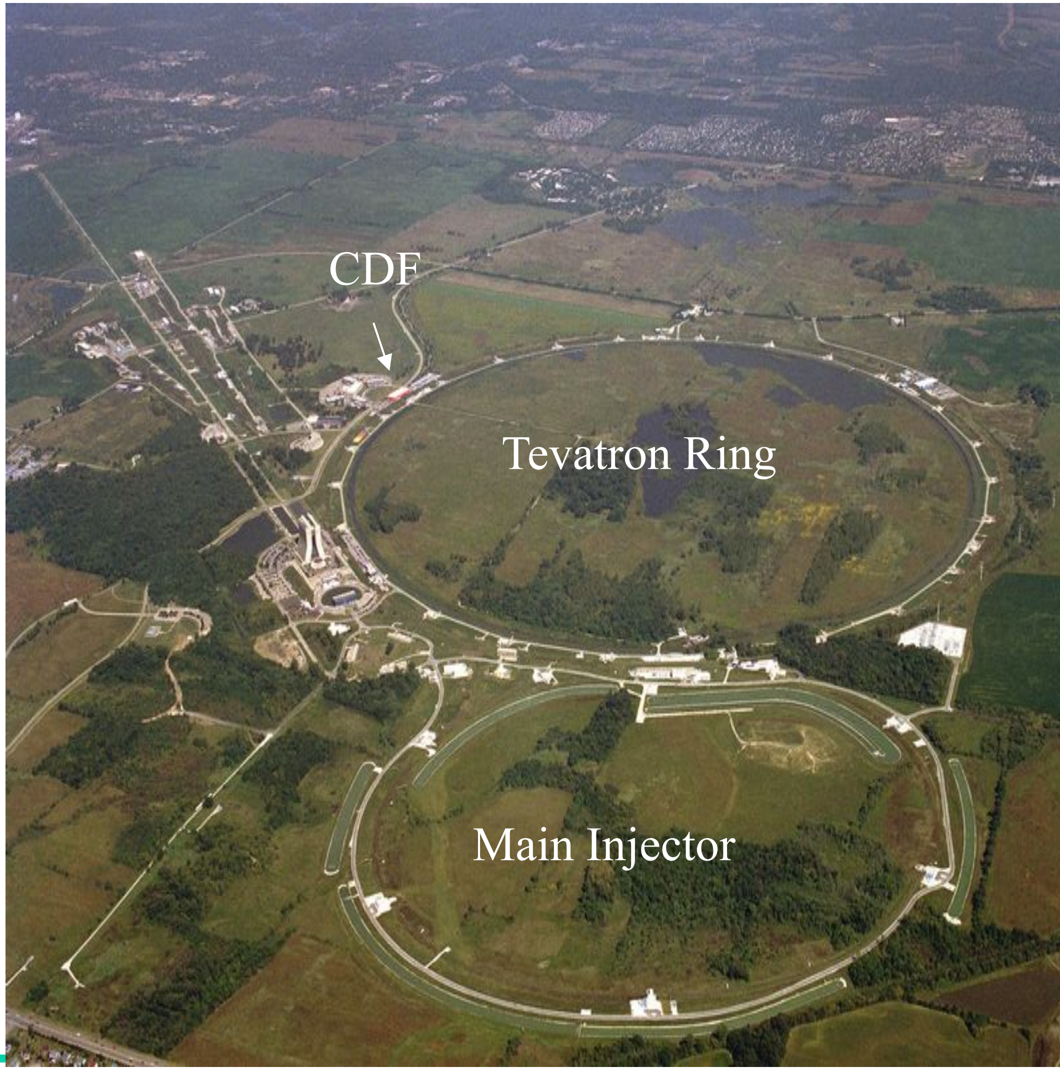
Indirect determination of the top mass

possible due to

- precision measurements
- known higher order electroweak corrections

$$\propto \left(\frac{M_t}{M_W}\right)^2, \ln\left(\frac{M_h}{M_W}\right)$$

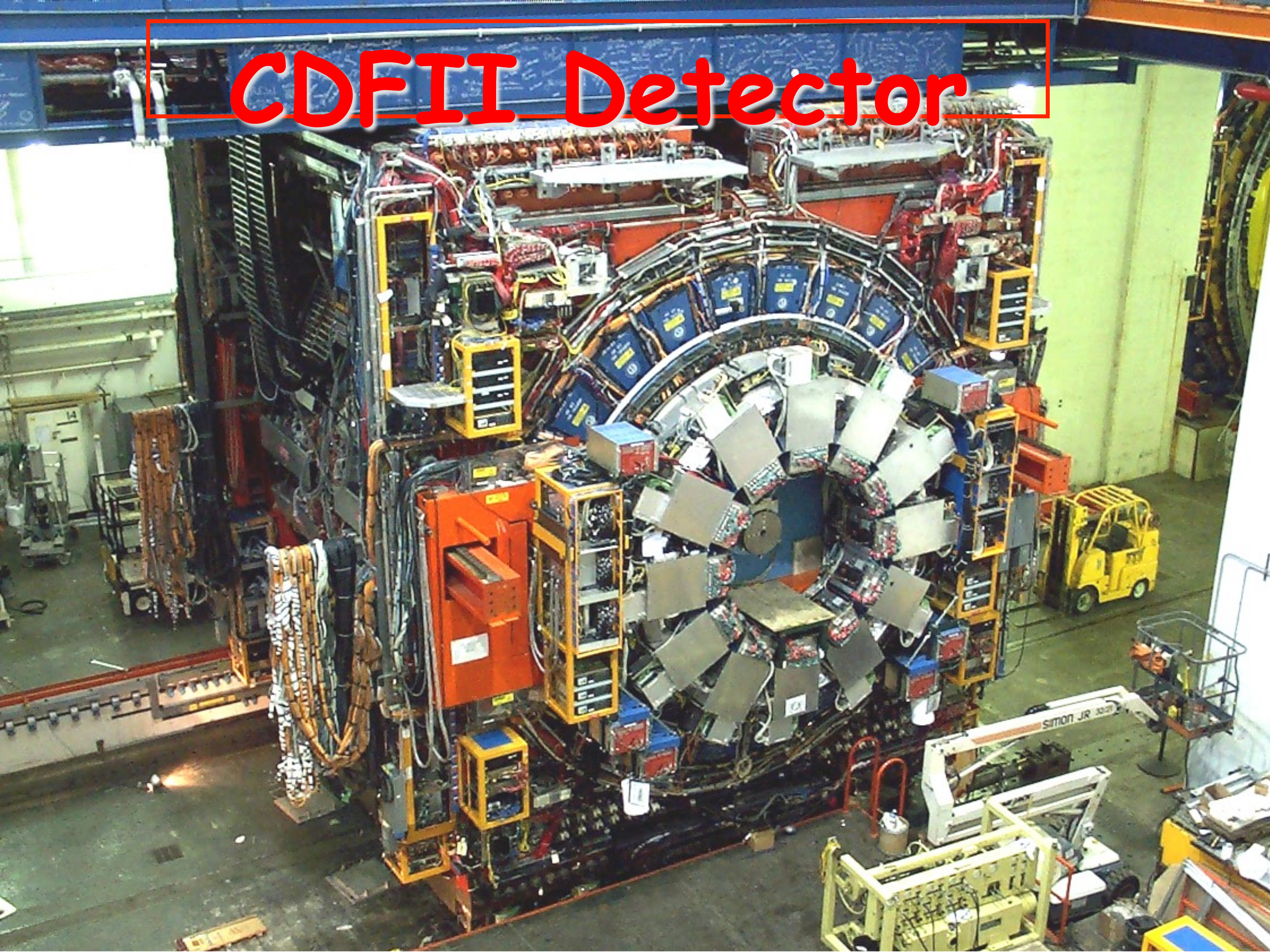
Tevatron



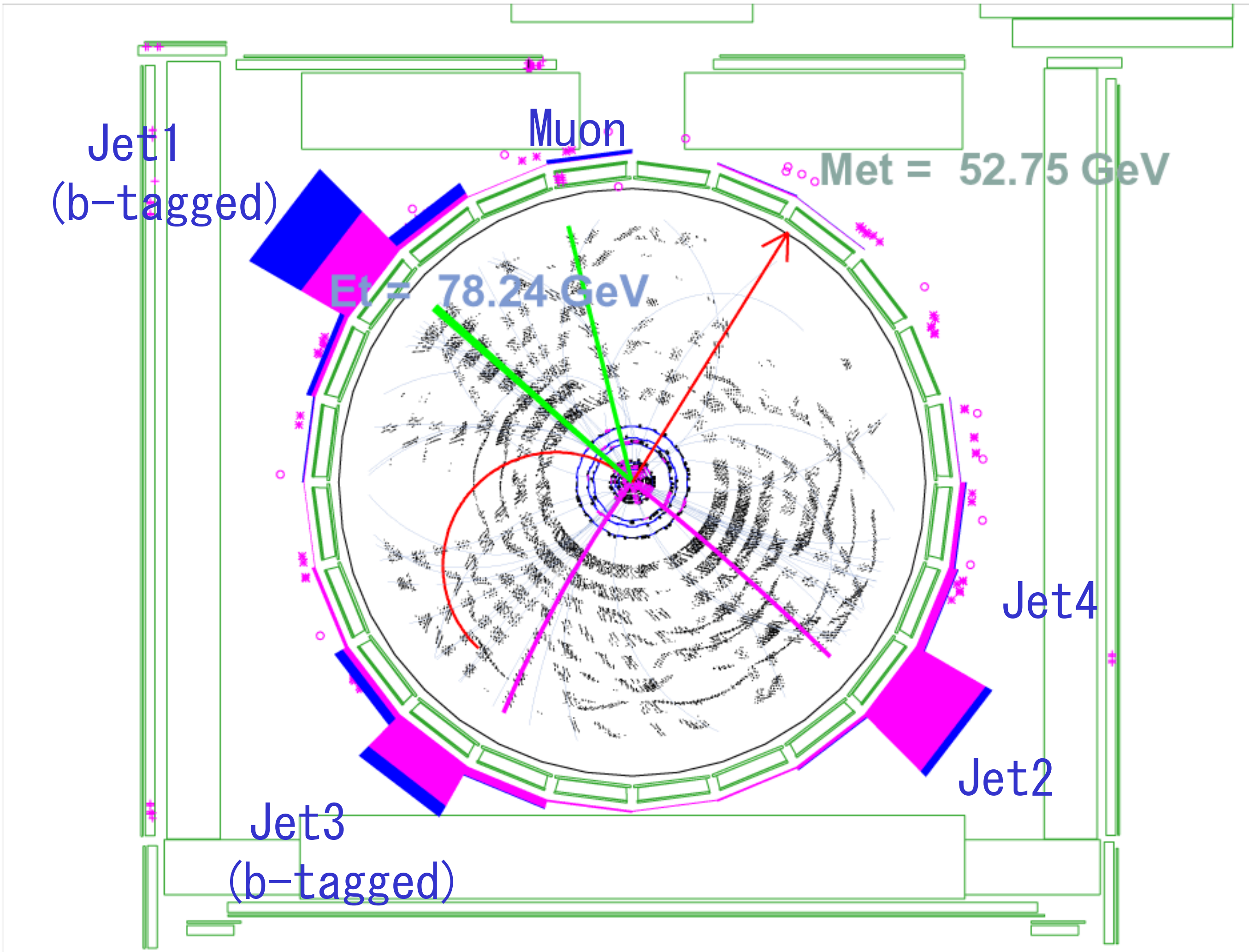
RunI (1992〜1996)
$\sqrt{s}$ = 1.8 TeV

RunII（2001〜）
$\sqrt{s}$ = 1.96 TeV
\+ Main Injector

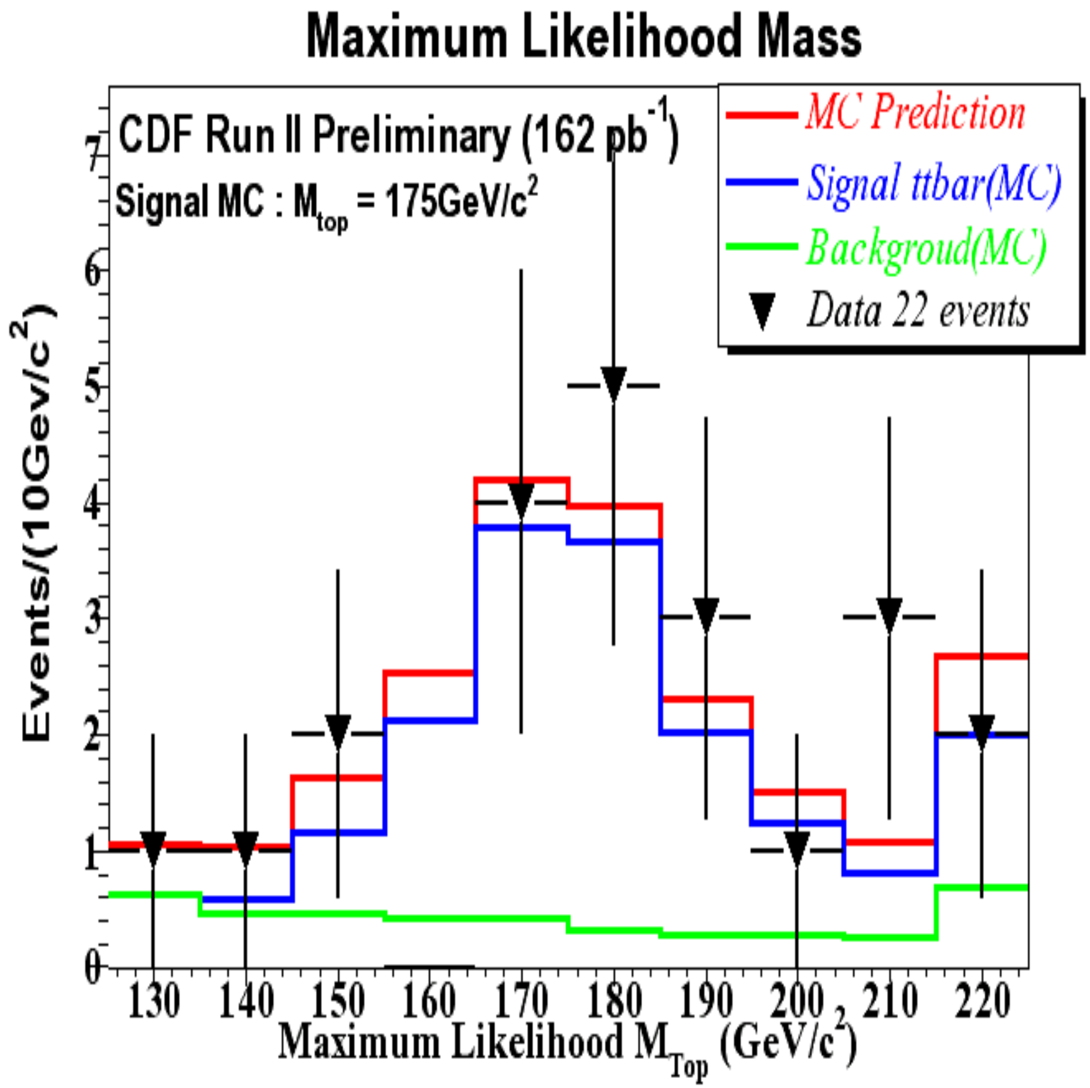
CDFII Detector



top quark production



Muon + Missing ET + 4Jets event(with 2b-tagged jets)



$$M_{top} = 177.8^{+4.5}_{-5.0}(\text{stat}) \pm 6.2(\text{syst})\ \text{GeV}/c^2$$

- Introduction of Higgs mechanism and Gauge Boson masses

Particles in Electroweak Theory

QCD color		weak isospin T	hypercharge Y
R/G/B	q_L $=(u_L,d_L)$	1/2	1/3
R/G/B	u_R	0	4/3
R/G/B	d_R	0	-2/3
0	l_L $=(\nu_L,e_L)$	1/2	-1
0	e_R	0	-2
0	ϕ	1/2	1

q_L, u_R and d_R were originally different particles!

Higgs $\phi=(0,v+H)$ does not interact with $Q = T_3 + Y/2$ = EM charge

The Glashow-Weinberg-Salam Model

The Glashow, Weinberg and Salam model treats EM and WEAK interactions as different manifestations of a single UNIFIED ELECTROWEAK force (Nobel Prize 1979)

Basic Idea:

Start with 4 massless bosons $\{W^+, W^0, W^-\}$ and B^0. The neutral bosons MIX to give physical bosons (the particles we see), i.e. the W, Z^0 and γ.

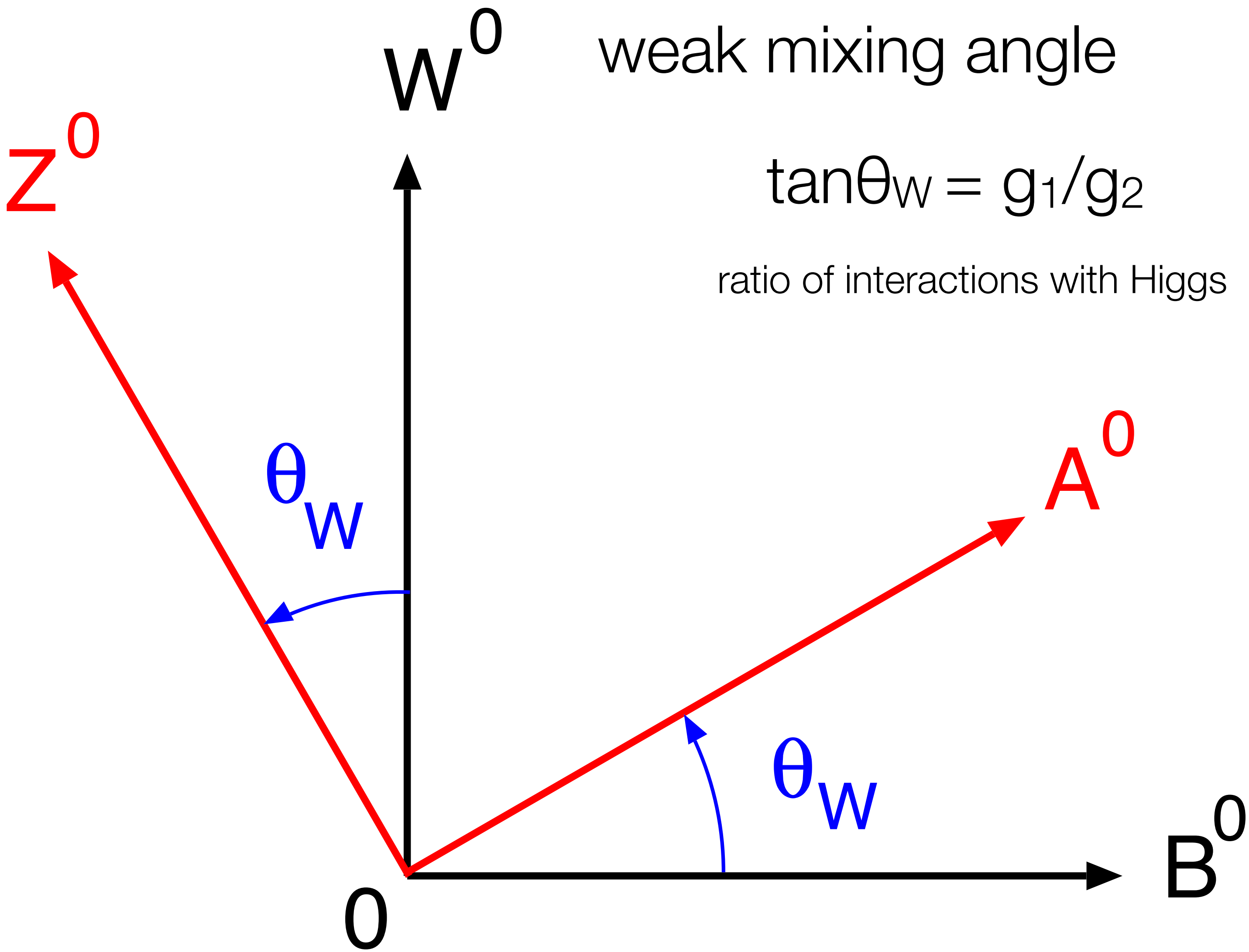
$$\begin{pmatrix} W^+ \\ W^0 \\ W^- \end{pmatrix}, \; B^0 \quad \rightarrow \quad \begin{pmatrix} W^+ \\ Z^0 \\ W^- \end{pmatrix}, \; \gamma$$

Physical fields: W^+, Z^0, W^- and A (photon)

$$Z^0 = W^0 \cos\vartheta_W - B^0 \sin\vartheta_W$$
$$A = W^0 \sin\vartheta_W + B^0 \cos\vartheta_W$$

ϑ_W WEAK MIXING ANGLE

W, Z^0 "acquire" mass via the HIGGS MECHANISM.


weak mixing angle
$\tan\theta_W = g_1/g_2$
ratio of interactions with Higgs
W^0
Z^0
A^0
B^0
θ_W
θ_W
0

SSB and Higgs Mechanism

- We need two steps to understand **SSB (Spontaneous Symmetry Breaking)** and Higgs mechanism. This is a way to get “massive” gauge bosons.

Step 1) Scalar particle in the global gauge symmetry

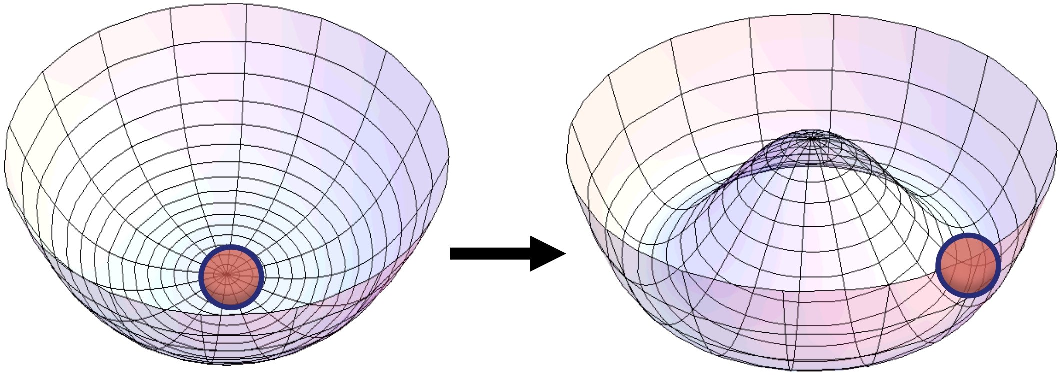
-> Produce a massive “Higgs candidate” and a massless particle (Nambu-Goldstone boson).

We call this procedure “Spontaneous Symmetry Breaking”.

2008 Novel prize

Step 2) Scalar particle in the local gauge symmetry

-> Drop the massless particle and produce a “massive gauge boson”.
We call this procedure “Higgs Mechanism”.



SSB: Scalar field U(1)

- Start from the easiest case;

$$L = \frac{1}{2}\left(\partial_\mu \phi\right)^* \left(\partial^\mu \phi\right) - V(|\phi|^2)$$

$$V = \mu^2 \phi^* \phi + \lambda\left(\phi^* \phi\right)^2$$

$$\phi = \frac{1}{\sqrt{2}}\left(\phi_1 + i\phi_2\right)$$ ϕ_1 and ϕ_2 … real

This Lagrangian has a global gauge invariance (symmetry).

$$L = \frac{1}{2}\left(\partial_\mu \phi_1\right)^2 + \frac{1}{2}\left(\partial_\mu \phi_2\right)^2 - \frac{1}{2}\mu^2\left(\phi_1^2 + \phi_2^2\right) - \frac{\lambda}{4}\left(\phi_1^2 + \phi_2^2\right)^2$$

$$x = \phi_1^2 + \phi_2^2$$

$$V = \frac{1}{2}\mu^2 x + \frac{\lambda}{4}x^2 = \frac{\lambda}{4}\left\{\left(x + \frac{\mu^2}{\lambda}\right)^2 - \frac{\mu^4}{\lambda^2}\right\}$$

$$V = \frac{1}{2}\mu^2 x + \frac{\lambda}{4}x^2 = \frac{\lambda}{4}\left\{\left(x + \frac{\mu^2}{\lambda}\right)^2 - \frac{\mu^4}{\lambda^2}\right\}$$

$$x = -\frac{\mu^2}{\lambda}(\equiv v^2) \rightarrow V = -\frac{\mu^4}{4\lambda}$$

$$L = \frac{1}{2}\left(\partial_\mu \phi\right)^*\left(\partial^\mu \phi\right) - V(|\phi|^2)$$

$$V = \mu^2 \phi^* \phi + \lambda\left(\phi^* \phi\right)^2$$

Klein-Goldon eq.

$$L = \frac{1}{2}\left(\partial_\mu \phi\right)\left(\partial^\mu \phi\right) - \frac{1}{2}m^2\phi^2$$

Comparing Klein-Goldon equation, the next condition is required to have no mass-term in the Lagrangian and to have the minimum in x≠0.

$$\lambda > 0, \mu^2 < 0$$

Then, we translate ϕ to a minimum energy. Without loosing generality, we can take as the point $\phi_1 = v, \phi_2 = 0$.

$$\phi \rightarrow \phi = \frac{1}{\sqrt{2}}\left(v + \eta + i\xi\right)$$

We may get the following expression;

$$L = \frac{1}{2}\left(\partial_\mu \xi\right)^2 + 0 + \frac{1}{2}\left(\partial_\mu \eta\right)^2 + \mu^2\eta^2 + \ldots$$

$$L = \frac{1}{2}(\partial_\mu \xi)^2 + 0 + \frac{1}{2}(\partial_\mu \eta)^2 + \mu^2\eta^2 + \ldots$$

Massless

This term has the form of a mass term!

Nambu-Goldstone boson

Mass $\sqrt{-2\mu^2}$

By breaking global gauge symmetry spontaneously, we have
(1) Massive scalar (maybe Higgs candidate!)
(2) Produce massless Nambu-Goldstone boson

SSB = Lagrangian is not changed at all but we change the ground state, which has a minimum energy.

Goldstone theorem

Massless scalar particles are produced whenever a continuous symmetry is spontaneously broken.

In the reality, the massless boson is not necessary.
We can drop it by applying SSB in the local gauge symmetry.

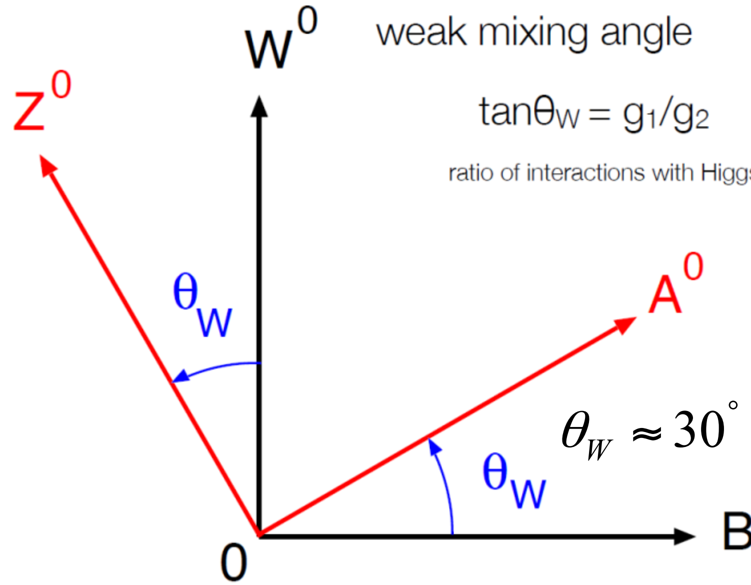
Keep in mind that this "massive" scalar is not a gauge boson.

1979 Nobel Prize

S.L.Glashow

A.Salam

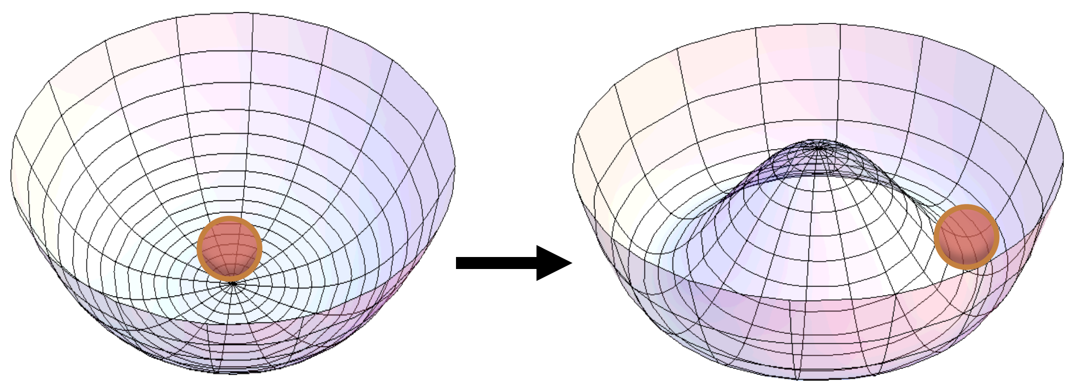
S.Weinberg



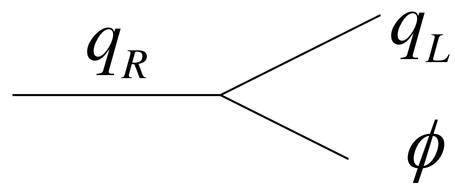
2013 Nobel Prize

F.Englert

P.W.Higgs



Higgs field



Higgs field has T and Y charges.
-> Chirality of fermions are flipped via the interaction with Higgs.

Particles in Electroweak Theory

QCD color		weak isospin T	hypercharge Y
R/G/B	q_L $=(u_L,d_L)$	1/2	1/3
R/G/B	u_R	0	4/3
R/G/B	d_R	0	-2/3
0	l_L $=(\nu_L,e_L)$	1/2	-1
0	e_R	0	-2
0	ϕ	1/2	1

q_L, u_R and d_R were originally different particles!

Higgs $\phi=(0,v+H)$ does not interact with $Q = T_3 + Y/2$ = EM charge

Construct Z and γ from W_3 and B.

$$\begin{pmatrix} W_{3\mu} \\ B_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} Z^0_\mu \\ A_\mu \end{pmatrix}$$

θ_W … Weinberg angle

$$ig'\frac{Y}{2}B_\mu + ig\frac{\tau_3}{2}W_{3\mu} = iA_\mu\left(g\sin\theta_W\frac{\tau_3}{2} + g'\cos\theta_W\frac{Y}{2}\right)$$

$$+iZ^0_\mu\left(g\cos\theta_W\frac{\tau_3}{2} - g'\sin\theta_W\frac{Y}{2}\right)$$

$$= ieA_\mu\left(\frac{\tau_3}{2} + \frac{Y}{2}\right) + iZ^0_\mu\left(\frac{eg}{g'}\frac{\tau_3}{2} - \frac{eg'}{g}\frac{Y}{2}\right)$$

$$g\sin\theta_W = g'\cos\theta_W \equiv e$$

$$= ieA_\mu\left(T_3 + \frac{Y}{2}\right) + iZ^0_\mu\left(\frac{eg}{g'}T_3 - \frac{eg'}{g}\frac{Y}{2}\right)$$

$T_3 = \frac{\tau_3}{2}$ Weak isospin

$$= ieA_\mu Q + iZ^0_\mu\left(\frac{eg}{g'}T_3 - \frac{eg'}{g}(Q - T_3)\right)$$

$T_3 + \frac{Y}{2} \equiv Q$ Charge operator

$$= ieA_\mu Q + i\frac{e}{\sin\theta_W\cos\theta_W}Z^0_\mu\left(T_3 - \sin^2\theta_W Q\right)$$

We can keep photon
as massless with this mixing.
(see later)

Higgs Mechanism to get W/Z mass

$$L_{scaler} = \left(D^{\mu}\phi\right)^{\dagger}\left(D_{\mu}\phi\right) - V(\phi^{\dagger}\phi)$$

$$V(\phi^{\dagger}\phi) = \mu^{2}\left(\phi^{\dagger}\phi\right) + \lambda\left(\phi^{\dagger}\phi\right)^{2}$$

$$\phi = \begin{pmatrix} \phi^{+} \\ \phi^{0} \end{pmatrix}, \begin{matrix} \phi^{+} = \left(\phi_1 + i\phi_2\right)/\sqrt{2} \\ \phi^{0} = \left(\phi_3 + i\phi_4\right)/\sqrt{2} \end{matrix}$$

$$\phi^{\dagger}\phi = \begin{pmatrix} \phi^{+*} & \phi^{0*} \end{pmatrix} \begin{pmatrix} \phi^{+} \\ \phi^{0} \end{pmatrix} = \phi^{+*}\phi^{+} + \phi^{0*}\phi^{0} \equiv \frac{1}{2}x$$

$x = -\frac{\mu^2}{\lambda} \equiv v^2$ -> minimum

$\phi = \begin{pmatrix} 0 \\ \left(v+h\right)/\sqrt{2} \end{pmatrix}, \phi_c = \begin{pmatrix} \left(v+h\right)/\sqrt{2} \\ 0 \end{pmatrix}$ then calculate $\left(D^{\mu}\phi\right)^{\dagger}\left(D_{\mu}\phi\right)$

$$D_\mu = \partial_\mu + ig'\frac{Y}{2}B_\mu + ig\frac{\vec{\tau}}{2}\vec{W}_\mu \qquad \vec{\tau}\vec{W}_\mu = \begin{pmatrix} W_{3\mu} & \sqrt{2}W_\mu^+ \\ \sqrt{2}W_\mu^- & -W_{3\mu} \end{pmatrix}$$

$$D_\mu \begin{pmatrix} 0 \\ (v+h)/\sqrt{2} \end{pmatrix} = \begin{pmatrix} i\frac{g}{2}(v+h)W_\mu^+ \\ \frac{1}{\sqrt{2}}\partial_\mu h + i\frac{v+h}{2\sqrt{2}}\left(g'YB_\mu - gW_{3\mu}\right) \end{pmatrix}$$

do the same thing for $(D_\mu\phi)^\dagger$. Then, $(D^\mu\phi)^\dagger(D_\mu\phi)$ becomes

$$\frac{1}{2}\left(\frac{g}{2}\right)^2 (v+h)^2\, 2W_\mu^+W^{-\mu} + \frac{(\partial_\mu h)^2}{2} + \frac{1}{2}\frac{(v+h)^2}{4}\left(g'YB_\mu - gW_{3\mu}\right)\left(g'YB^\mu - gW_3^\mu\right)$$

First, we can obtain W boson mass from the first term.

$$W_\mu^+W^{-\mu} = \frac{1}{2}\left(W_1^2 + W_2^2\right) = \frac{1}{2}\left(W_\mu^-W^{-\mu} + W_\mu^+W^{+\mu}\right)$$

$$m_W = \frac{gv}{2}$$

Next is Z mass：

$$g'YB_\mu - gW_{3\mu} = \sqrt{g^2+g'^2}\left(\frac{g'}{\sqrt{g^2+g'^2}}YB_\mu - \frac{g}{\sqrt{g^2+g'^2}}W_{3\mu}\right)$$

$$= \sqrt{g^2+g'^2}\left(\sin\theta_W YB_\mu - \cos\theta_W W_{3\mu}\right)$$

$$\cos\theta_W = \frac{g}{\sqrt{g^2+g'^2}}$$

$$\sin\theta_W = \frac{g'}{\sqrt{g^2+g'^2}}$$

$$\begin{pmatrix} Z^0_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix}\begin{pmatrix} W_{3\mu} \\ B_\mu \end{pmatrix}$$

Use Y=1
(Y=-1 cannot describe the universe.
This can be explained from the Higgs field has Q=0.)

$$\frac{1}{2}\frac{v^2}{4}\sqrt{g^2+g'^2}^{\,2} Z^0_\mu Z^{0\mu}$$

$$m_Z = \frac{\sqrt{g^2+g'^2}\,v}{2}$$

$$\frac{m_W}{m_Z} = \cos\theta_W$$

T_3, Y and Q

Show only the 1st generation but other generations have the same values.

Fermion	T_3	Y	Q	$T_3 - \sin\theta_W^2 Q$
ν_{eL}	+1/2	-1	0	+1/2
e_L	-1/2	-1	-1	-1/2+s2 ~ -1/4
ν_{eR}	0	0	0	0
e_R	0	-2	-1	s2 ~ 1/4
u_L	+1/2	+1/3	+2/3	1/2-2/3s2 ~ 1/3
d_L	-1/2	+1/3	-1/3	-1/2+1/3s2 ~ -5/12
u_R	0	+4/3	+2/3	-2/3s2 ~ -1/6
d_R	0	-2/3	-1/3	1/3s2 ~ 1/12

T: Weak isospion
Coupling to $W^\pm$

$$T_3 + \frac{Y}{2} \equiv Q$$

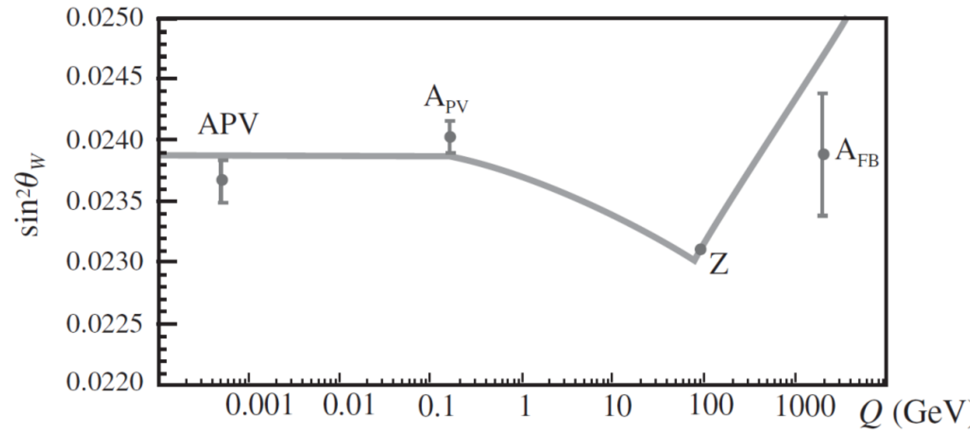
Coupling to Z

$$\sin^2\theta_W = 0.23 \sim 1/4$$
$$\theta_W \approx 30^\circ$$

Y: Hyper-charge

As you see in this table, the right-handed neutrino is a kind of ghost in the electroweak interaction.
We can never observe it via the EW interaction even if it exist.

W/Z Mass and vev values from measurements



- SU(2) coupling constant $g=e/\sin\theta_W$
- Weinberg angle $\sin^2\theta_W=0.23$
  - Several ways to determine this value. m_W and m_Z mass measurement (Z), electron-deuterium scattering (APV), Moller scattering (PV) etc
- Fine structure constant $\alpha=e^2/4\pi=1/137$
  - Measurement of a charge unit, that is, electron charge magnitude ("e")
- Fermi constant $G_F=1.166\times10^{-5}$ GeV^{-2}
  - Muon lifetime measurement

$$m_W = \frac{g}{\sqrt{4\sqrt{2}G_F}}$$

- W and Z mass relation $\dfrac{m_W}{m_Z} = \cos\theta_W$
- Vacuum expectation value (vev) v $v = \sqrt{\dfrac{1}{\sqrt{2}G_F}}$

(answers…)
$e=0.303$
$g=0.632$
$m_W=78$GeV
$m_Z=89$GeV
$v=246$GeV

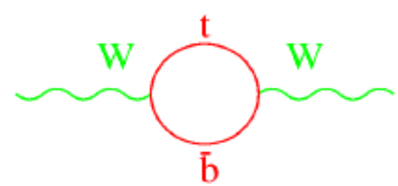
Electromagnetic constant
measured in atomic transitions,
e^+e^- machines, etc.

$$m_W = \left(\frac{\pi \alpha_{EM}}{\sqrt{2}\, G_F} \right)^{1/2} \frac{1}{\sin\theta_W \sqrt{1-\Delta r}}$$

Fermi constant
measured in muon
decay

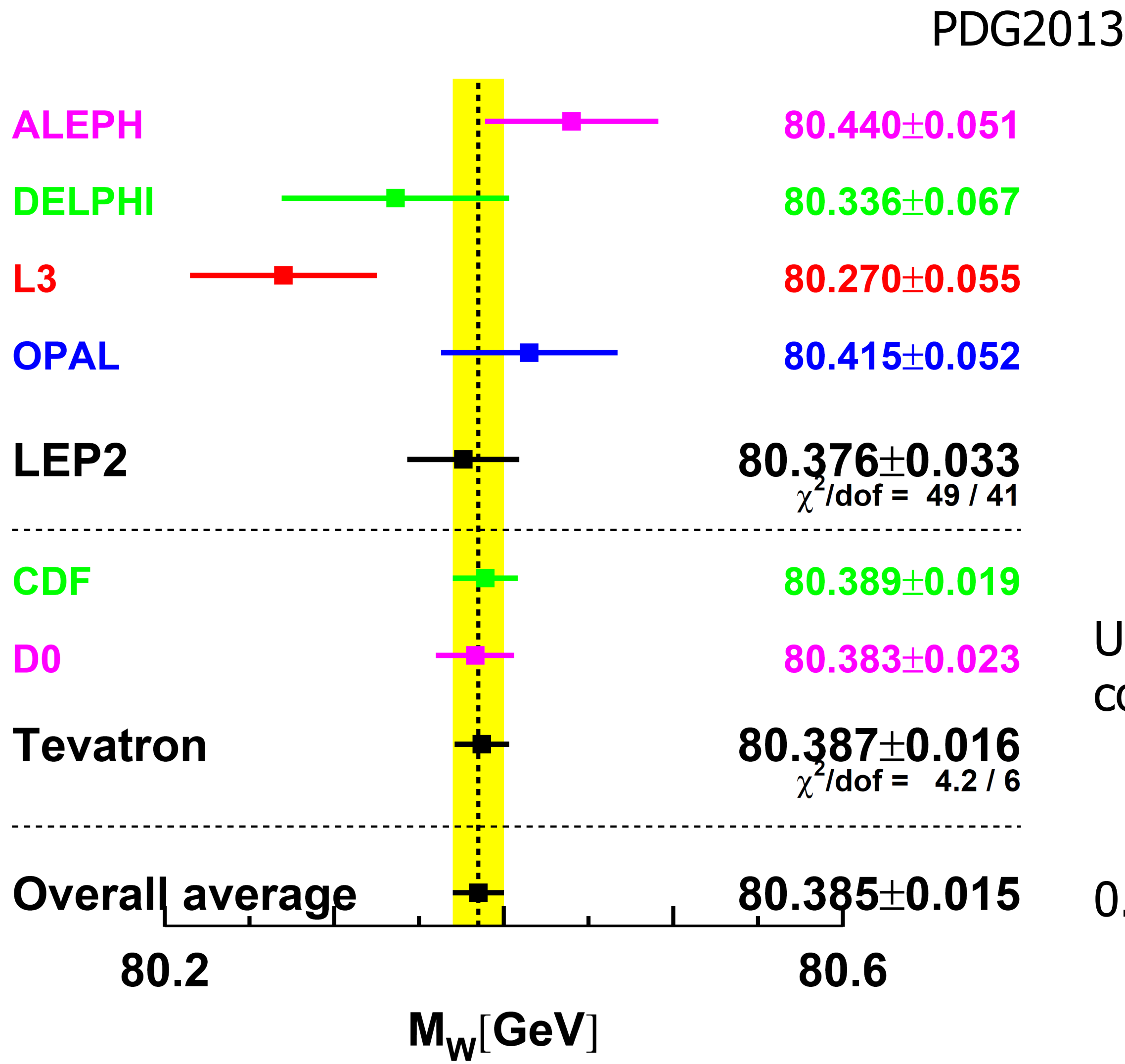
Weinberg angle
measured at
LEP/SLC

radiative corrections
$\Delta r \sim f\,(m_{top}^2, \log m_H)$
$\Delta r \approx 3\%$



Since the effect of m_H propagates with only "log",
even if the Higgs mass changes by 100GeV, the W mass is shifted by several 10MeV.
-> We need a few 10MeV precision on m_W to know Higgs mass with <100GeV level.
-> We already achieved this level! See later.

Precise measurement of W mass

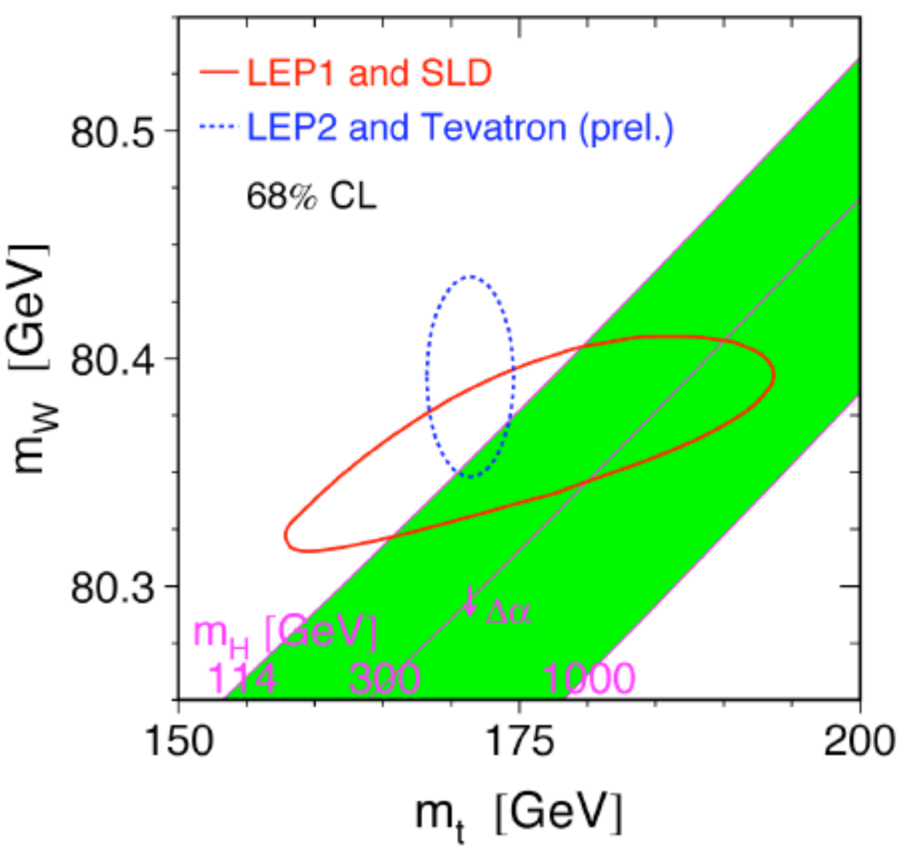


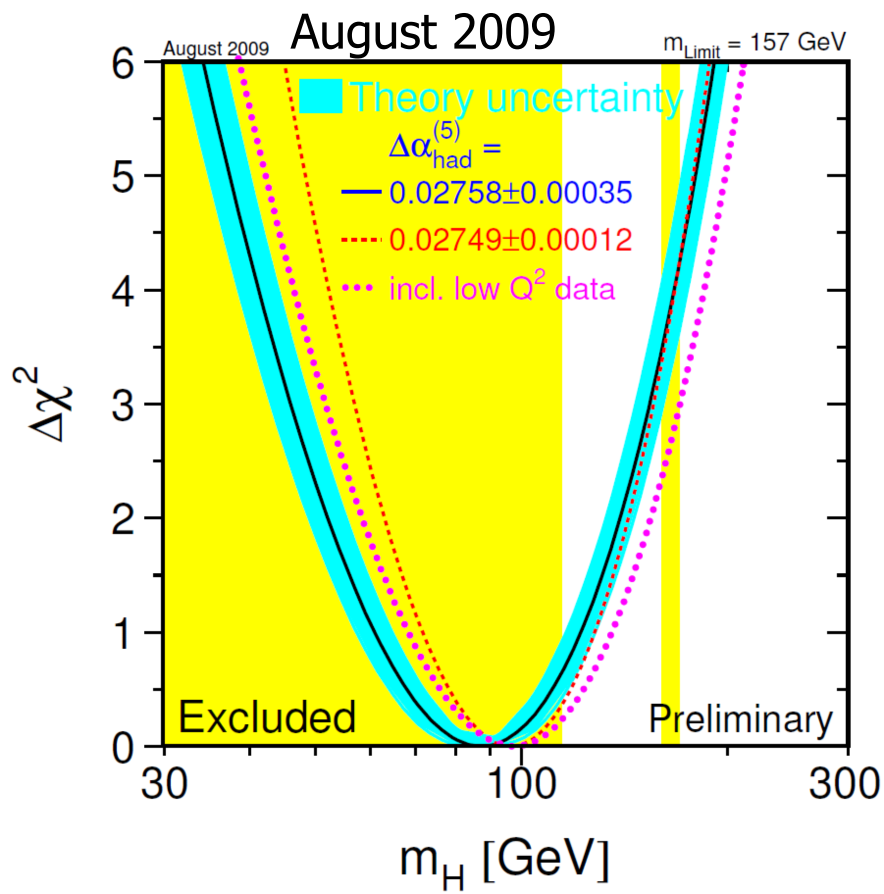
Updated in 2012 winter conference

0.02%!!!

One of the dominant systematic uncertainties of Tevatron is PDF (10MeV).

Higgs mass expectation from the radiative correction (precision measurements)





(These values from PDG2013 Table 10.4)

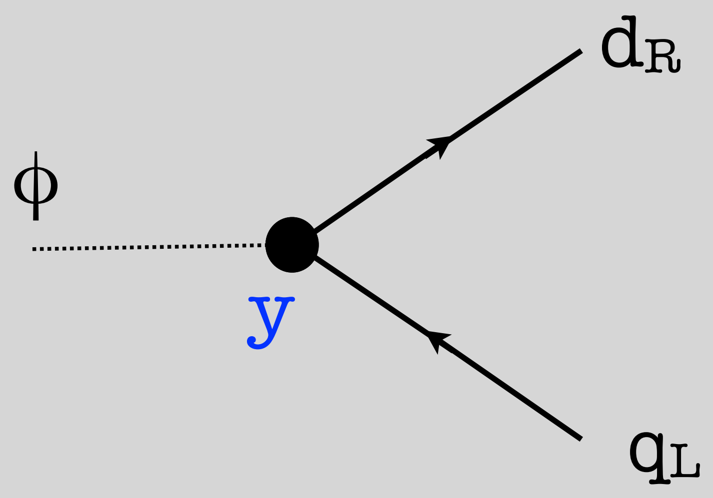
m_W(GeV) 80.420 ± 0.031 (Tevatron) 2009

80.376 ± 0.033 (LEP2) 2006

m_{Top}(GeV) 173.4 ± 1.0 2011
(Tevatron, ATLAS, CMS)

Fermion Masses

Yukawa interaction



new interaction!

$$y\left(\bar{q_L}\phi d_R + \bar{d_L}\bar{\phi}q_L\right)$$

$$\phi = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+H \end{pmatrix}$$

$$(yv)\bar{d}d$$ mass term

m_d

Yukawa coupling
$y = m_d/v$

Fermion Mass Term

$m\bar{\psi}\psi$ ← not gauge invariant

$$\mathcal{L}_{\text{f-s}} = -G_{\text{e}}\left[\overline{R}(\phi^{\dagger}L) + (\overline{L}\phi)R\right]$$

← $SU(2)_L \otimes U(1)_Y$ inv.

→ Spontaneous Sym. Breaking

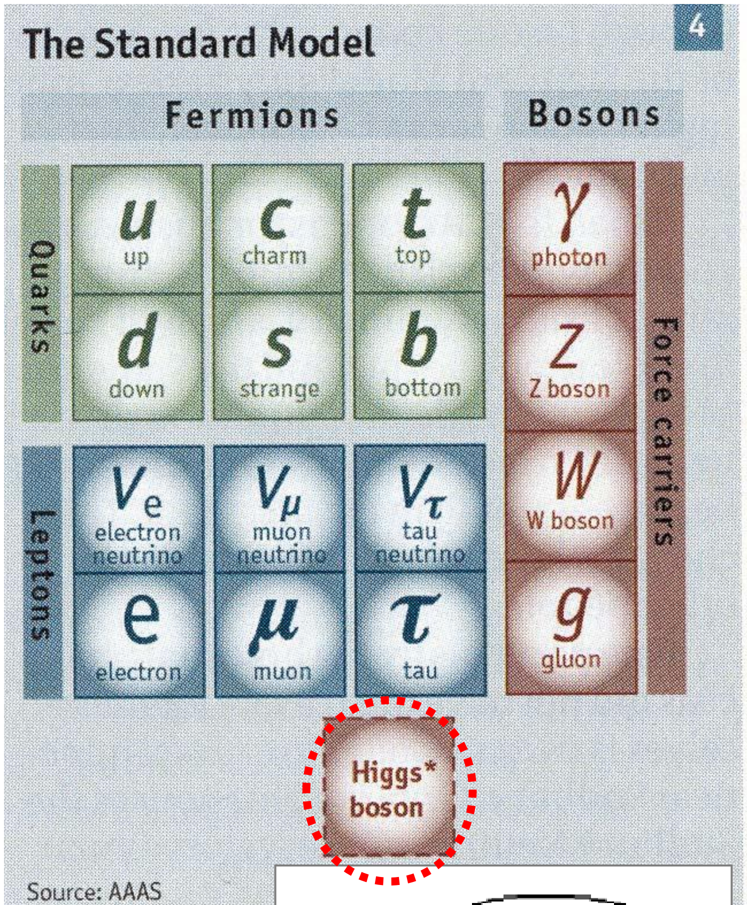
$$\mathscr{L}_{\text{Yukawa}} = -G_e\frac{(v+\eta)}{\sqrt{2}}(\bar{e}_{\text{R}}e_{\text{L}} + \bar{e}_{\text{L}}e_{\text{R}})$$

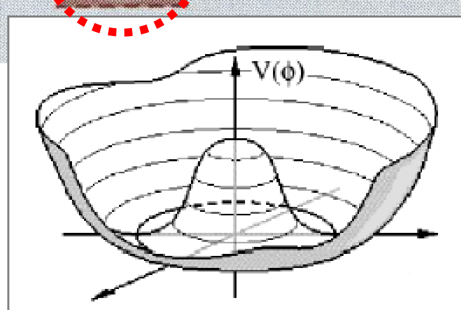
$$= \frac{-G_e v}{\sqrt{2}}\bar{e}e - \frac{G_e\eta}{\sqrt{2}}\bar{e}e$$

$$m_e = G_e v/\sqrt{2}$$

12.Jun.2013

Search for Higgs Boson



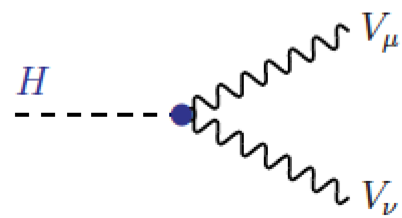


$$\mathcal{L}_{\text{scalar}} = (\mathcal{D}^\mu \phi)^\dagger (\mathcal{D}_\mu \phi) - V(\phi^\dagger \phi)$$

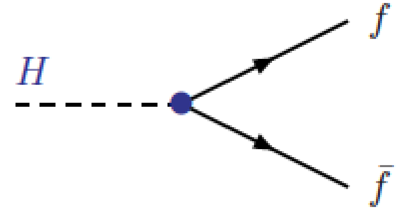
$$\mathcal{D}_\mu = \partial_\mu + i\frac{g'}{2}\mathcal{A}_\mu Y + i\frac{g}{2}\vec{\tau} \cdot \vec{b}_\mu$$

$$V(\phi^\dagger \phi) = \mu^2 (\phi^\dagger \phi) + |\lambda| (\phi^\dagger \phi)^2$$

$$\mathcal{L}_{\text{f-s}} = -G_{\text{e}} \left[\overline{R}(\phi^\dagger L) + (\overline{L}\phi) R \right]$$



$$g_{HVV} = 2M_V^2/v$$



$$g_{Hff} = m_f/v$$

Higgs Decay Width

"Higgs couples to mass"

$$\Gamma(H \to f\bar{f}) = C_f \frac{G_F m_f^2}{4\sqrt{2}\pi} \beta_f^3 M_H$$

$$C_f = 1 \text{ (leptons) or } 3 \text{ (quarks)}$$

$$\beta_f = (1 - 4m_f^2/M_H^2)^{1/2}$$

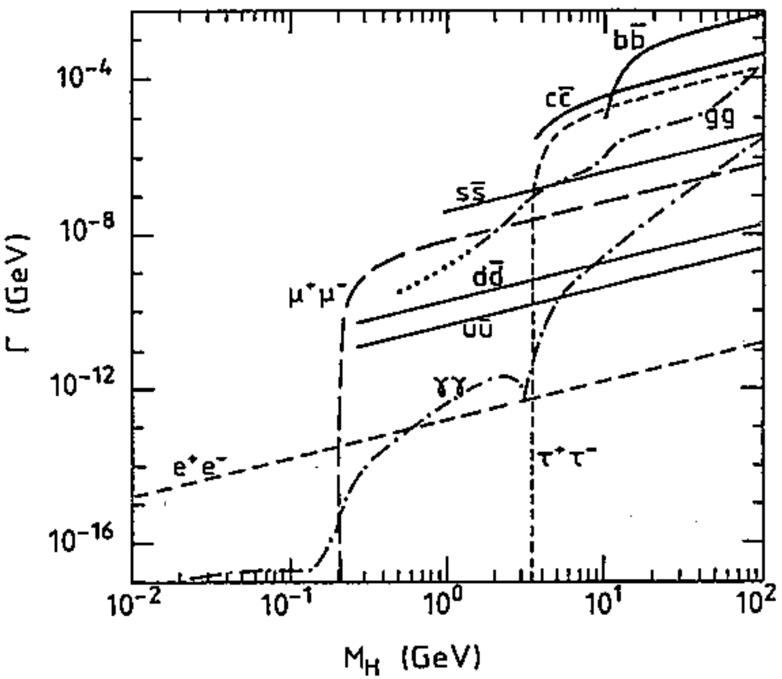
$$\Gamma(H \to VV^\dagger) = \delta_V \frac{G_F}{16\sqrt{2}\pi}(1 - x_V)^{1/2}\left(1 - x_V + \frac{3}{4}x_V^2\right) M_H^3$$

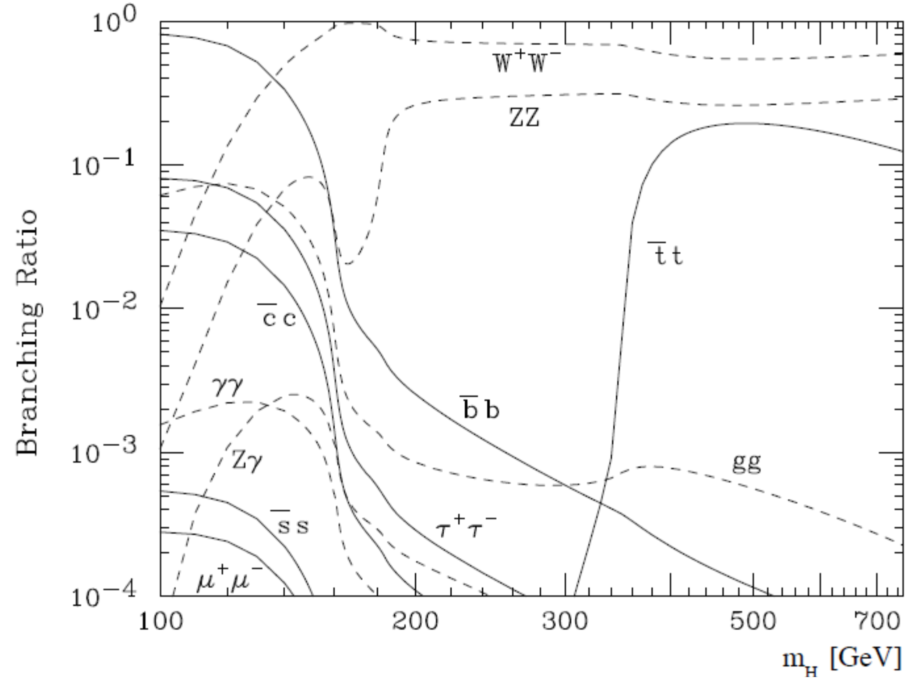
$$\delta_W = 2,\ \delta_Z = 1$$

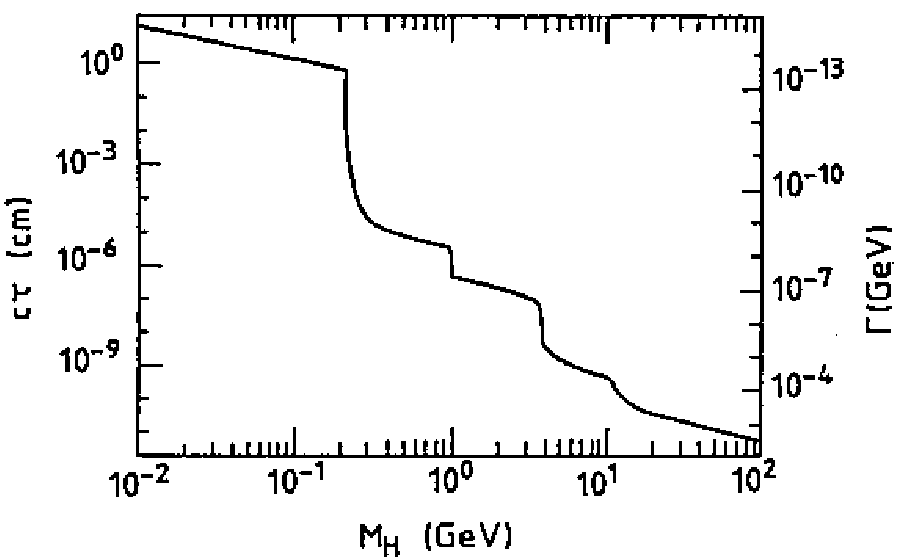
$$x_V = 4M_V^2/M_H^2$$

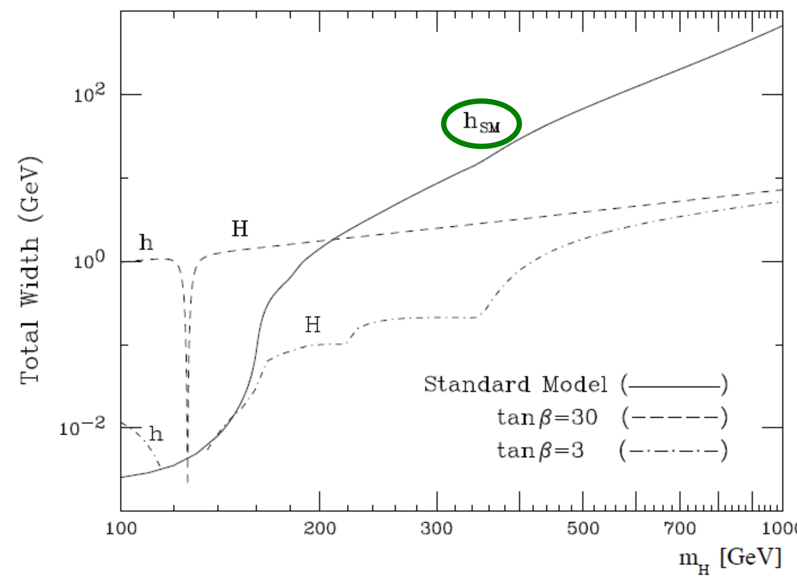
H* → *γγ **via *f*- and *W*-loops**
H* → *gg **via *q*-loops**

Higgs Decay Width





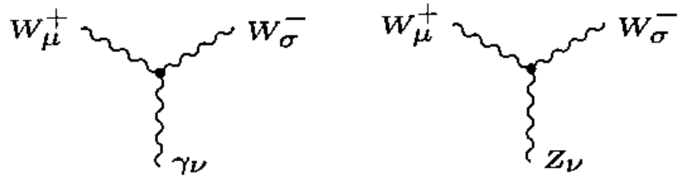


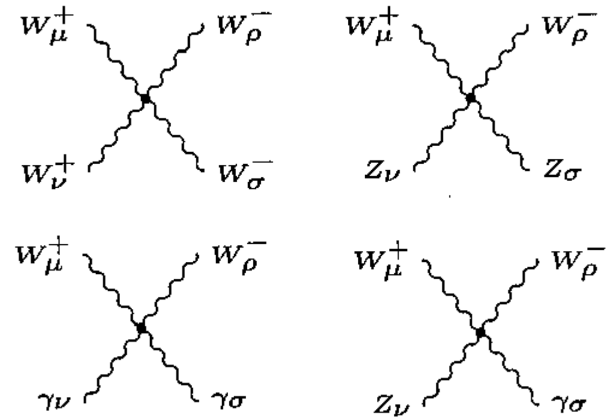


$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4} F^{\ell}_{\mu\nu} F^{\ell\mu\nu} - \frac{1}{4} f_{\mu\nu} f^{\mu\nu}$$

$$F^{\ell}_{\mu\nu} = \partial_\nu b^{\ell}_{\mu} - \partial_\mu b^{\ell}_{\nu} + g \varepsilon_{jk\ell} b^{j}_{\mu} b^{k}_{\nu}$$

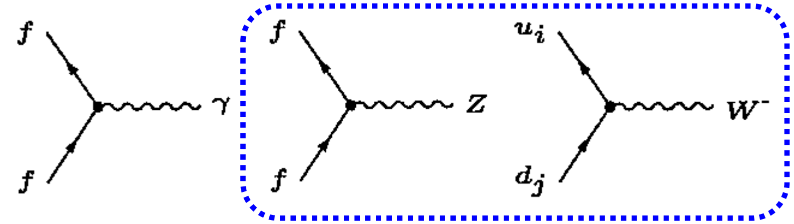
$$f_{\mu\nu} = \partial_\nu \mathcal{A}_\mu - \partial_\mu \mathcal{A}_\nu$$





$$\mathcal{L}_{\text{leptons}} = \overline{\mathsf{R}}\, i\gamma^\mu \left(\partial_\mu + i\frac{g'}{2}\mathcal{A}_\mu Y\right)\mathsf{R} + \overline{\mathsf{L}}\, i\gamma^\mu \left(\partial_\mu + i\frac{g'}{2}\mathcal{A}_\mu Y + i\frac{g}{2}\vec{\tau}\cdot\vec{b}_\mu\right)\mathsf{L}$$

→ $\mathcal{L}_{\text{quarks}}$ similar



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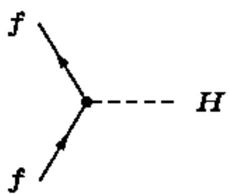
$$\mathcal{D}_\mu = \partial_\mu + i\frac{g'}{2}\mathcal{A}_\mu Y + i\frac{g}{2}\vec{\tau}\cdot\vec{b}_\mu$$

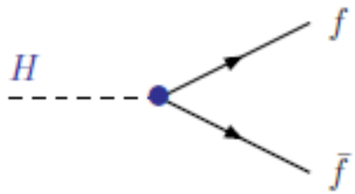
$$V(\phi^\dagger\phi) = \mu^2(\phi^\dagger\phi) + |\lambda|\,(\phi^\dagger\phi)^2$$

$$\mathcal{L}_{\text{f-s}} = -G_{\text{e}}\left[\overline{R}(\phi^\dagger L) + (\overline{L}\phi)R\right]$$

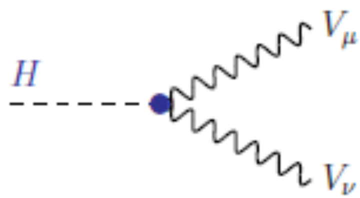


Summary of Higgs Couplings

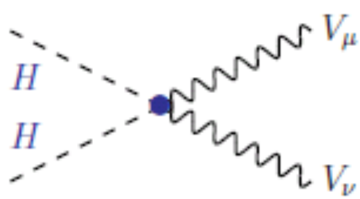
arXiv:hep-ph/0503172 (by A.Djouadi)
-> one of good textbooks for Higgs physics



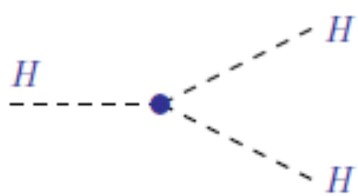
$$g_{Hff} = m_f/v = (\sqrt{2}G_\mu)^{1/2}\, m_f \qquad \times\, (i)$$



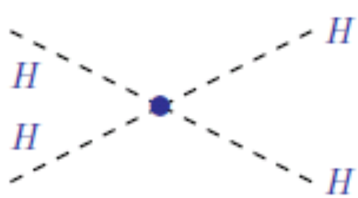
$$g_{HVV} = 2M_V^2/v = 2(\sqrt{2}G_\mu)^{1/2}\, M_V^2 \qquad \times\, (-ig_{\mu\nu})$$



$$g_{HHVV} = 2M_V^2/v^2 = 2\sqrt{2}G_\mu\, M_V^2 \qquad \times\, (-ig_{\mu\nu})$$



$$g_{HHH} = 3M_H^2/v = 3(\sqrt{2}G_\mu)^{1/2}\, M_H^2 \qquad \times\, (i)$$



$$g_{HHHH} = 3M_H^2/v^2 = 3\sqrt{2}G_\mu\, M_H^2 \qquad \times\, (i)$$

ATLAS+CMS seminar at CERN at the 4th July, 2012
(🡨 Japanese news paper)

ヒッグス粒子発見か

新素粒子検出 年内に結論

日米欧２チーム

あらゆる物質に質量を与えたと考えられる仮説上の素粒子「ヒッグス粒子」を探しているジュネーブの欧州合同原子核研究所（CERN＝セルン）は4日、大型粒子加速器「LHC」による実験で、ヒッグス粒子とみられる新たな素粒子を見つけたと発表した。ヒッグス粒子は、万物に含まれる素粒子の基本理論として1960年代に提唱された「標準理論」で存在が予言された18種の素粒子のうち、唯一未発見だ。　（3面にクローズアップ）

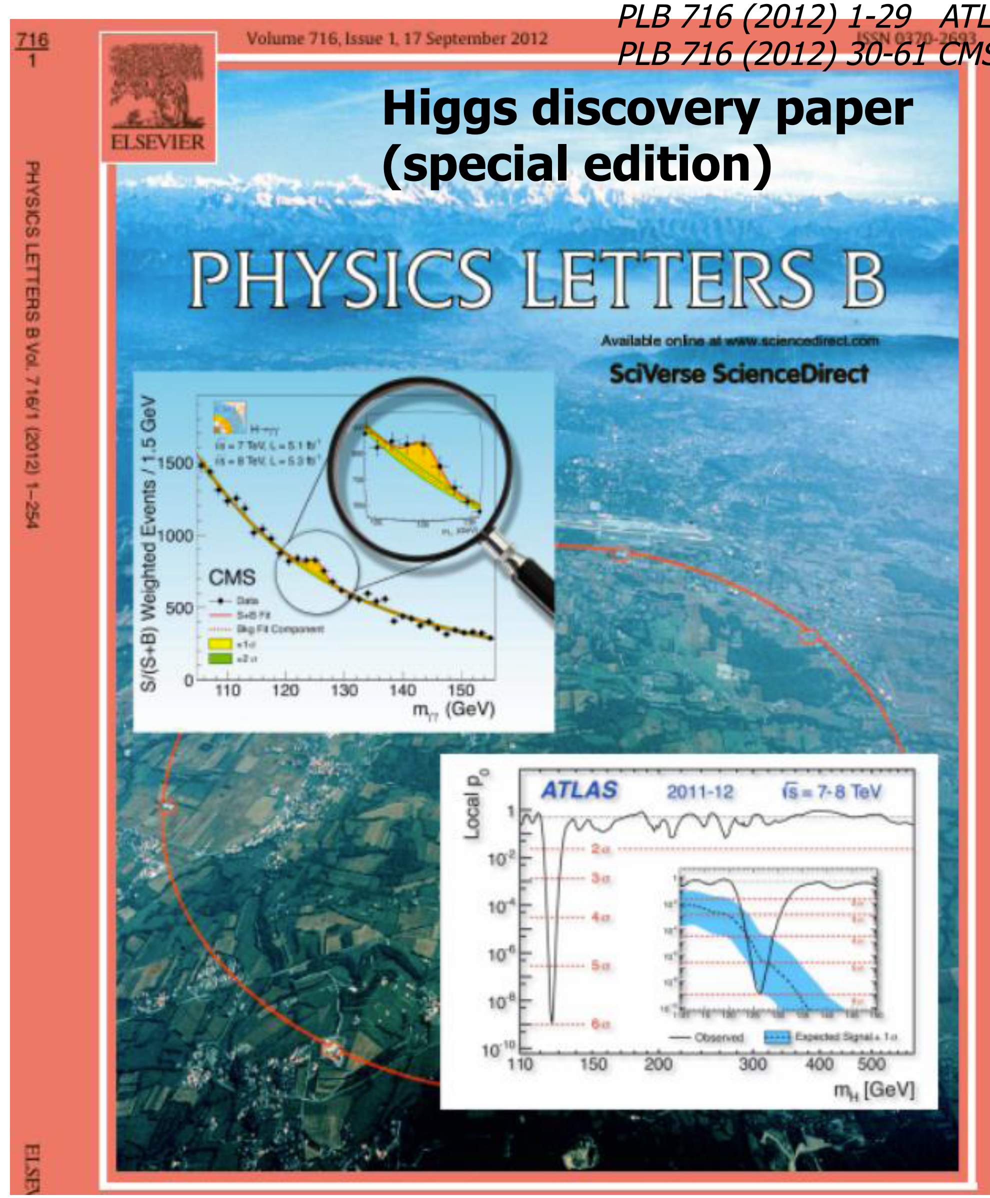
得られたデータからは発見された素粒子をヒッグス粒子と考えても矛盾がないが、ヒッグス粒子かを見極める方法は複数あり、さらなる実験や分析で確度を高める必要があるという。今年中には結論に至る見込み。

ヒッグス粒子は、英国の物理学者ピーター・ヒッグス博士（83）が提唱。宇宙誕生の大爆発（ビッグバン）直後、宇宙空間に漂っていた素粒子に質量を生じさせる粒子で重要な役割を果たした「神の粒子」と呼ばれている。標準理論で存在が指摘された18種の素粒子は88年ま

喜びあうヒッグス粒子の提唱者のピーター・ヒッグス博士（右）と研究代表者の一人ファビオラ・ジアノッティ博士＝ジュネーブで4日、ロイター

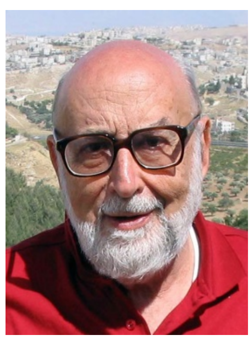
PLB 716 (2012) 1-29 ATLAS
PLB 716 (2012) 30-61 CMS

Higgs discovery paper (special edition)



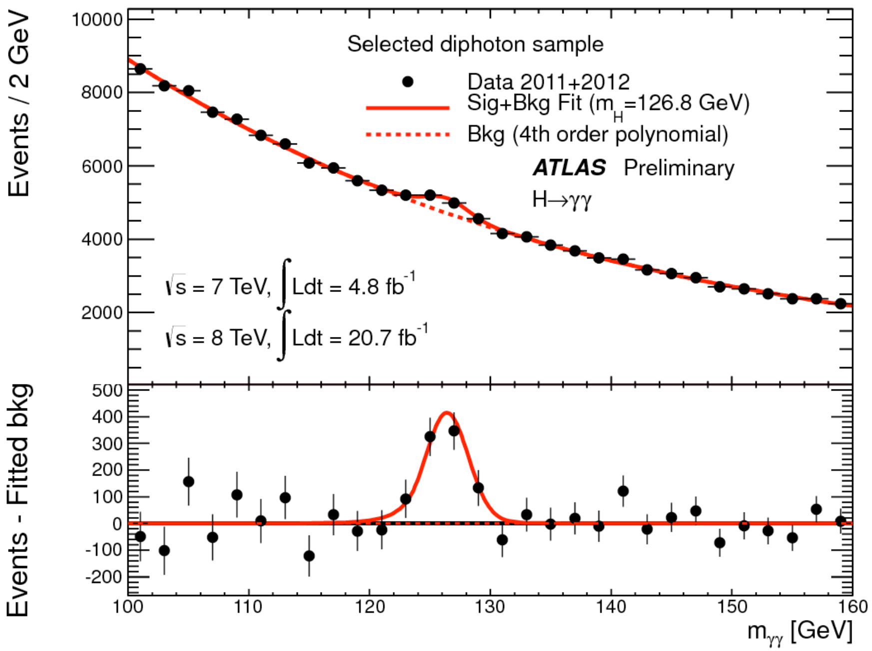
2013 NOBEL PRIZE IN PHYSICS

François Englert
Peter W. Higgs

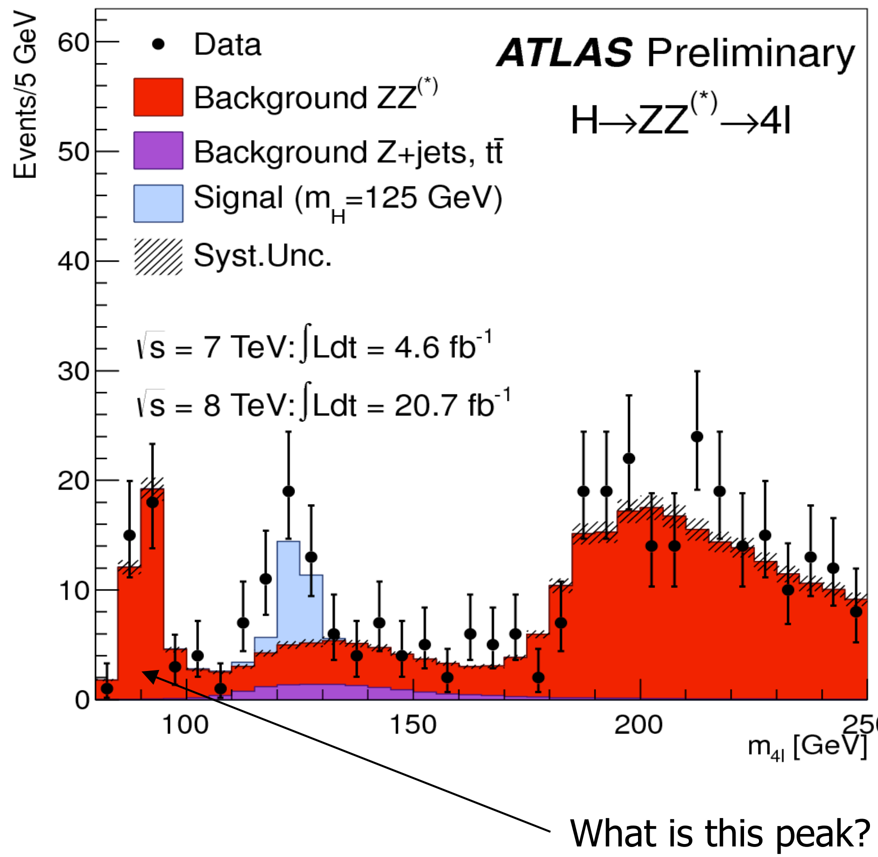


Higgs Discovery

H->γγ



H->ZZ*->llll



ATLAS and CMS observed a Higgs particle at ~126GeV.
Experimental results will be discussed in the next lecture.

NEXT WEEK

Search, Discovery, measurements of Higgs Boson(s)

Appendix

A note for Theory

To Understand the SM [SU(2)xU(1)]

Step A : still introduction…
- Overview of the SM Lagrangian

Step B : detail description of the SM Largragian
- Introduction of Lagrangian instead of motion of questions etc.
- Gauge symmetries
  - Gauge invariance
  - Gauge principal
- Global gauge symmetry
  - Spontaneous Symmetry Breaking with U(1)
- Local gauge symmetry
  - Higgs Mechanism with U(1)
  - Electroweak interaction SU(2)xU(1)

Strong interaction (SU(3)) was already studied in Yamashita-san's lecture.

Relativistic Wave Equations

- Klein-Goldon equation
  - Scalar particle, spin 0

$$\partial_\mu \partial^\mu \phi + m^2 \phi = 0$$

- Dirac equation
  - Fermion, spin 1/2

$$\left(i\gamma^\mu \partial_\mu - m\right)\Psi = 0$$

Ψ … spinor, 4 solutions -> spin up/down and particle and anti-particle

- Proca equation
  - Vector, spin 1

$$\partial_\nu F^{\mu\nu} + m^2 A^\mu = 0$$

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$$

Lagrangian

- Classical mechanics：Euler-Lagrange equation -> Newton's motion of equation

$$\frac{d}{dt}\left(\frac{dL}{d\dot{x}}\right)-\frac{dL}{dx}=0$$

$$L=T-V=\frac{1}{2}mv^2-V$$ Kinetic energy and potential energy

- The same idea can be applied to the quantum mechanics

$$L(x,\dot{x},t)\rightarrow L\left(\phi,\frac{\partial\phi}{\partial x^{\mu}},x_{\mu}\right)$$

Strictly speaking, *L* is Lagrangian density

$$\mathrm{L}=\int L d^3x$$

But for simplicity, we call *L* "Lagrangian".

$$\frac{\partial}{\partial x_{\mu}}\left(\frac{\partial L}{\partial\left(\frac{\partial\phi}{\partial x^{\mu}}\right)}\right)-\frac{\partial L}{\partial\phi}=0$$

$$\partial_{\mu}\left(\frac{\partial L}{\partial(\partial_{\mu}\phi)}\right)-\frac{\partial L}{\partial\phi}=0$$ （simplified）

Once we define a Lagrangian, we can calculate everything from it.

Examples of Lagrangian

- Scalar particle（spin 0, mass m）

$$L = \frac{1}{2}\left(\partial_\mu \phi\right)\left(\partial^\mu \phi\right) - \frac{1}{2} m^2 \phi^2$$

$$\rightarrow \frac{1}{2} 2\partial_\mu \partial^\mu \phi - \left(-\frac{1}{2} m^2 2\phi \right) = 0$$

$$\partial_\mu \partial^\mu \phi + m^2 \phi = 0$$ Klein-Goldon equation

- Dirac particle（spinor）（spin 1/2, mass m）

$$L = \overline{\Psi}\left(i\gamma^\mu \partial_\mu - m\right)\Psi$$

$\phi = \overline{\Psi}$ -> the 1st term of the Lagrange equation=0

$$\left(i\gamma^\mu \partial_\mu - m\right)\Psi = 0$$ Dirac equation

- Vector particle（spin 1, mass m） $L = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \frac{1}{2} m^2 A_\mu A^\mu$

Gauge Invariance

- We consider that "physics" should be invariant under a transformation. Such a transformation must exist to describe "physics". Here we consider "Gauge Symmetry".
  - "Symmetry and/or invariance" is an important concept in physics, for example,
    - Invariance in time -> Conservation of energy
    - Invariance under space transformation -> Conservation of momentum
    - Invariance under spatial rotation -> Conservation of angular momentum

- Since our observables are calculated from square of wave functions, these wave functions can have their phase.

$$\Psi \rightarrow \eta\Psi, \eta = e^{ig\theta}\left(|\eta| = 1\right)$$

  Now let's consider that "physics" is required to be invariant under this transformation.
  - For example, a free Dirac particle Lagrangian shown in the previous page is invariant under this transformation if θ does not depend on x.
    "No dependence on x" means that we apply this transformation at once in the universe. We call it "Global gauge invariance".

Gauge Invariance (cont'd)

- Then, now we have a question on "global"ization, that is, the universe adopt such a "global" idea. How do the universe change phases for all the spaces w/o any delay? It looks to impossible…?
  -> That's why we (people about 50 years ago) consider that "local" is a better idea than "global". Transformation should have dependence of coordination of "x".

$$\Psi \rightarrow e^{ig\theta(x)}\Psi$$

  We call it "local gauge invariance".

  Let's apply this invariance to the Dirac particle Lagrangian.

Local Gauge Invariance：U(1)

$$L = \overline{\Psi}\left(i\gamma^{\mu}\partial_{\mu} - m\right)\Psi \qquad \Psi \rightarrow e^{ig\theta(x)}\Psi$$

$$L \rightarrow L - g\overline{\Psi}\gamma^{\mu}\Psi\partial_{\mu}\theta(x)$$

Not invariance under the local gauge transformation.
To make it invariance, we introduce a field “A”, which is called a gauge field.
As a result, Lagrangian is rewritten with a covariant derivative.

$$\partial_{\mu} \rightarrow D_{\mu} = \partial_{\mu} + igA_{\mu}$$

D_{μ}… covariant derivative

When we perform the local gauge transformation, the field A is also changed.

$$A_{\mu} \rightarrow A_{\mu} - \partial_{\mu}\theta$$

The new Lagrangian is as follows;

$$L' = \overline{\Psi}\left(i\gamma^{\mu}D_{\mu} - m\right)\Psi$$

$$= \overline{\Psi}\left(i\gamma^{\mu}\partial_{\mu} - m\right)\Psi - gA_{\mu}\overline{\Psi}\gamma^{\mu}\Psi$$

Let's try the local gauge transformation.

$$g\left(A_\mu - \partial_\mu \theta\right)\overline{\Psi}\gamma^\mu\Psi = gA_\mu\overline{\Psi}\gamma^\mu\Psi - g\partial_\mu\theta\overline{\Psi}\gamma^\mu\Psi$$

then

$$L' \rightarrow L - g\overline{\Psi}\gamma^\mu\Psi\partial_\mu\theta - gA_\mu\overline{\Psi}\gamma^\mu\Psi + g\partial_\mu\theta\overline{\Psi}\gamma^\mu\Psi$$

$$= L - gA_\mu\overline{\Psi}\gamma^\mu\Psi = L'$$

New Lagrangian is invariant under the local gauge transformation.

$gA_\mu\overline{\Psi}\gamma^\mu\Psi$ describe the interaction between spinor field and gauge field.
-> We can write a Feynman diagram for this vertex!
(Keep in mind that all the terms of the Lagrangian can be written with Feynman diagrams.)

As you see in this example, to require the gauge invariance in a free field Lagrangian, we have to introduce a gauge field A.
Then, the interaction between spinor and gauge fields is defined.

Gauge Principle

- Gauge principle = The gauge invariance should be valid in our universe, that is, our universe should be described with Lagrangians, which have the gauge invariance.
  - Another important thing is that an interaction term is automatically obtained.

- So far, this principle looks work very well.
  - U(1) -> QED(Quantum electrodynamics) Electromagnetism interaction
  - SU(2) -> Weak interaction
  - SU(3) -> Strong interaction

  The SM describes these 3 interactions in the language of gauge theory.

- The L' explained in the previous pages is the Lagrangian to describe motion of Dirac particles and interaction between Dirac particles and photons. By adding a term for photon's motion, we can have QED Lagrangian.

$$L_{QED} = \overline{\Psi}\left(i\gamma^{\mu}\partial_{\mu} - m\right)\Psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - gA_{\mu}\overline{\Psi}\gamma^{\mu}\Psi$$

$$F_{\mu\nu} = \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu}$$

Mass term and Gauge invariance

If we have a mass term for photon (gauge field A),

$$L_{mass} = \frac{1}{2} m^2 A^\mu A_\mu$$

is added. This term apparently violate the gauge invariance.
If we want to keep the gauge principal, we cannot add this term to Lagrangian.
A photon mass must be ZERO under the gauge invariance.
So far there is no problem since a photon has zero-mass in the universe.

In general the mass of gauge fields must be zero because a mass term written with the form explained above violates the gauge invariance.

However we know that there are massive particles, that is, W and Z gauge bosons. (A little bit complicated for quarks and leptons. They also have mass in the reality but must be massless like gauge bosons…)

We have a choice to discard the gauge principal because the gauge principal seems not to describe the nature. However people, who have established the SM, didn't give up it. They/we believe(d) that "gauge principal" is one of best principals to build physics.

Our question is "Can we have massive particles w/o loosing the gauge principal?"

-> "Spontaneous Symmetry Breaking (SSB)"

SSB and Higgs Mechanism

- We need two steps to understand SSB (Spontaneous Symmetry Breaking) and Higgs mechanism. This is a way to get "massive" gauge bosons.

  Step 1) Scalar particle in the global gauge symmetry

  -> Produce a massive "Higgs candidate" and a massless particle (Nambu-Goldstone boson).

  We call this procedure "Spontaneous Symmetry Breaking".

  Step 2) Scalar particle in the local gauge symmetry

  -> Drop the massless particle and produce a "massive gauge boson".
  We call this procedure "Higgs Mechanism".

Higgs Mechanism at U(1) group

Let's consider the next Lagrangian;

$$L = \left(D_\mu \phi\right)^* \left(D^\mu \phi\right) - V(|\phi|^2) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

$$V = \mu^2 \phi^* \phi + \lambda \left(\phi^* \phi\right)^2 \qquad F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$$

$$\phi = \frac{1}{\sqrt{2}} \left(\phi_1 + i\phi_2\right)$$ ϕ_1 and ϕ_2 … real

$$D_\mu = \partial_\mu + igA_\mu$$

This Lagrangian is invariant under the local gauge transformation;

$$\phi \rightarrow e^{ig\theta(x)} \phi$$

$$A_\mu \rightarrow A_\mu - \partial_\mu \theta(x)$$

$$L = (\partial_\mu - igA_\mu)\phi^* (\partial^\mu + igA^\mu)\phi - \mu^2 \phi^* \phi - \lambda(\phi^* \phi)^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

$$\phi \rightarrow \phi = \frac{1}{\sqrt{2}}(v + \eta + i\xi) \qquad \lambda = -\frac{\mu^2}{v^2}, v = \sqrt{-\frac{\mu^2}{\lambda}}$$

ξ term $\quad \frac{1}{2}(\partial_\mu \xi)^2 + 0 + \ldots \quad m_\xi = 0$

η term $\quad \frac{1}{2}(\partial_\mu \eta)^2 + \mu^2 \eta^2 + \ldots \quad m_\eta = \sqrt{-2\mu^2} = \sqrt{2\lambda v^2}$

$A_\mu A^\mu$ term $\quad \frac{1}{2} g^2 v^2 A_\mu A^\mu - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \quad m_A = gv$

Scalar η has mass and also vector A has mass! This is what we want.
However, we still have a massless scalar ξ.

Anyway this procedure looks good to get "massive" gauge bosons (->W/Z).
But how do we treat the massless scalar ξ? What is it?

The 1st term of L becomes

$$\frac{1}{2}(\partial_\mu \eta)^2 + \frac{1}{2}(\partial_\mu \xi)^2 + \frac{1}{2} g^2 A_\mu A^\mu \left(v^2 + 2v\eta + \eta^2 + \xi^2\right) + gA^\mu \{(v+\eta)\partial_\mu \xi + \xi \partial_\mu \eta\}$$

and we can see a strange interaction in the last part.

The property of "**Local**" gauge transformation helps us!
We can drop this scalar ξ by using the local gauge symmetry.
-> We can adjust a phase (local gauge) to drop this scalar.

The present phase is

$$\phi \to \phi = \frac{1}{\sqrt{2}}(v + \eta + i\xi) \sim \frac{1}{\sqrt{2}}(v+\eta) e^{i\frac{\xi}{v}}$$

So, we can use the next gauge to drop the ξ term.

$$\phi \to e^{ig\theta(x)}\phi = e^{-i\frac{\xi(x)}{v}}\phi \qquad (\theta = -\frac{\xi}{gv})$$

$$A_\mu \to A_\mu - \partial_\mu \theta(x) = A_\mu + \frac{1}{gv}\partial_\mu \xi(x)$$

This transformation does not change the form of L. But since now ϕ is a real, we can substitute $\xi=0$ in L.

$$L = \frac{1}{2}\left(\partial_\mu h\right)^2 + \mu^2 h^2$$

Higgs kinetic energy and mass

$$-\frac{1}{2}\sqrt{-2\mu^2}^2 h^2$$

$$m_h = \sqrt{-2\mu^2}$$

$$-\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}g^2 v^2 A_\mu A^\mu$$

Gauge boson kinetic energy and mass

$$m_A = gv$$

$$+\frac{\mu^2}{v}h^3 + \frac{\mu^2}{4v^2}h^4$$

Higgs self coupling

$$+g^2 v A_\mu A^\mu h + \frac{1}{2}g^2 A_\mu A^\mu h^2$$

Higgs and gauge boson interaction

$$-\frac{1}{4}\mu^2 v^2$$

Constant (-> we can ignore it.)

$$L = \frac{1}{2}\left(\partial_\mu h\right)^2 + \mu^2 h^2$$ Higgs kinetic energy and mass $$-\frac{1}{2}\sqrt{-2\mu^2}^2 h^2$$ $$m_h = \sqrt{-2\mu^2}$$

$$-\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}g^2 v^2 A_\mu A^\mu$$ Gauge boson kinetic energy and mass $$m_A = gv$$

$$+\frac{\mu^2}{v}h^3 + \frac{\mu^2}{4v^2}h^4$$ Higgs self coupling

$$+g^2 v A_\mu A^\mu h + \frac{1}{2}g^2 A_\mu A^\mu h^2$$ Higgs and gauge boson interaction

$$-\frac{1}{4}\mu^2 v^2$$ Constant (-> we can ignore it.)

Mass term of Dirac fields

$m\overline{\Psi}\Psi$ violates the gauge invariance?

We can get the next expression by using Ψ_L, Ψ_R

$$m\overline{\Psi}\Psi = m\left(\overline{\Psi}_L \Psi_R + \overline{\Psi}_R \Psi_L\right)$$

This type of mass term does not conserve the chirality.
L and R is mixed.
Keep in mind that this is just about the chirality conservation not gauge invariance.

again

$m\overline{\Psi}\Psi$ violates the gauge invariance?

1. No if left-handed and right handed particles are in the same group.
2. Yes if they are in different groups.

In the SM Left-handed and right-handed fermions are in different groups.
So, we cannot have a mass term for Dirac fields in the SM Lagrangian.

SU(2)xU(1)

$$\Psi_1 = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \Psi_2 = u_R, \Psi_3 = d_R$$

Left-handed quarks … interacted with W

u_L … weak isospin up (T_3=1/2)

d_L … weak isospin down (T_3=-1/2)

Right-handed quarks … not interacted with W ($T=T_3=0$)

$$L_{fermion} = i\overline{\Psi_1}\gamma^{\mu}\partial_{\mu}\Psi_1 + i\overline{\Psi_2}\gamma^{\mu}\partial_{\mu}\Psi_2 + i\overline{\Psi_3}\gamma^{\mu}\partial_{\mu}\Psi_3$$

$$\partial_{\mu} \rightarrow D_{\mu} = \partial_{\mu} + ig'\frac{Y}{2}B_{\mu} + ig\frac{\vec{\tau}}{2}\vec{W}_{\mu}$$

$B, \vec{W}$ … 4 vector gauge fields

g′ … U(1) coupling constant

g … SU(2) coupling constant

$\vec{\tau}\vec{W}$ Only for Ψ_1

Y … Hyper charge

τ … Pauli Matrix

What's Hyper charge?

SU(2)xU(1)

We may need SU(2) for 3 weak gauge bosons and also due to the doublet of left-handed fermions. Then, how do we consider U(1)?
We need a photon-like gauge boson based on U(1) but this new U(1) should be common to weak isospin up/down, which have different charge (u=2/3 and d=-1/3). So we cannot use QED U(1) (electric charge) for this part.
That' why we introduce a new charge, which is common to the doublet.
We call it "Hyper charge".

$$L_{fermion} = i\overline{\Psi}_1\gamma^\mu\partial_\mu\Psi_1 + i\overline{\Psi}_2\gamma^\mu\partial_\mu\Psi_2 + i\overline{\Psi}_3\gamma^\mu\partial_\mu\Psi_3$$

$$\partial_\mu \to D_\mu = \partial_\mu + ig'\frac{Y}{2}B_\mu + ig\frac{\vec{\tau}}{2}\vec{W}_\mu$$

$B, \vec{W}$ … 4 vector gauge fields

g' … U(1) coupling constant
g … SU(2) coupling constant

$\vec{\tau}\vec{W}$ Only for Ψ_1

Y … Hyper charge
τ … Pauli matrices

$$L = i\overline{\Psi}_1\gamma^{\mu}\left(\partial_{\mu} + ig'\frac{Y}{2}B_{\mu} + ig\frac{\vec{\tau}}{2}\vec{W}_{\mu}\right)\Psi_1$$

$$+ i\overline{\Psi}_2\gamma^{\mu}\left(\partial_{\mu} + ig'\frac{Y}{2}B_{\mu}\right)\Psi_2 + i\overline{\Psi}_3\gamma^{\mu}\left(\partial_{\mu} + ig'\frac{Y}{2}B_{\mu}\right)\Psi_3$$

$$\tau^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \tau^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \tau^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\Psi_1 \longrightarrow e^{i\vec{\alpha}(x)\bullet\vec{\tau} + i\beta(x)Y}\Psi_1$$

$$\Psi_{2(3)} \longrightarrow e^{i\beta(x)Y}\Psi_{2(3)}$$

This Lagrangian is invariant under the local gauge transformation.
(B and W transformation are not written here but same as A.)

The interaction is defined from the local gauge invariance.

$$L = i\overline{\Psi}_1\gamma^{\mu}\left(\partial_{\mu} + ig'\frac{Y}{2}B_{\mu} + ig\frac{\vec{\tau}}{2}\vec{W}_{\mu}\right)\Psi_1$$

$$+ i\overline{\Psi}_2\gamma^{\mu}\left(\partial_{\mu} + ig'\frac{Y}{2}B_{\mu}\right)\Psi_2 + i\overline{\Psi}_3\gamma^{\mu}\left(\partial_{\mu} + ig'\frac{Y}{2}B_{\mu}\right)\Psi_3$$

$$\tau^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \tau^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \tau^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Where is $W^{\pm}$?

$$\vec{\tau}\vec{W}_{\mu} = \begin{pmatrix} W_{3\mu} & W_{1\mu} - iW_{2\mu} \\ W_{1\mu} + iW_{2\mu} & -W_{3\mu} \end{pmatrix}$$

$$W_{\mu}^{\pm} = \frac{1}{\sqrt{2}}\left(W_{1\mu} \mp iW_{2\mu}\right)$$

Interact with only Ψ_1(Left-handed fermions)

Where is Z and γ?

V-A coupling

$$\overline{\Psi}'_L \gamma^\mu \Psi_L = \overline{\Psi}' \frac{1+\gamma^5}{2} \gamma^\mu \frac{1-\gamma^5}{2} \Psi = \overline{\Psi}' \gamma^\mu \frac{1-\gamma^5}{2} \Psi$$

This type coupling is called "V-A coupling".
"-" is important. "V minus A"

This coupling means the interaction with left-handed fermions.

Let's rewrite the Lagrangian with Ψ(=u, d) instead of $\Psi_{L,R}$(=$u_{L,R}$, $d_{L,R}$).

$$L_{W-quark} = \frac{-g}{\sqrt{2}} \left(\bar{u} \gamma^\mu \frac{1-\gamma^5}{2} d W_\mu^+ + \bar{d} \gamma^\mu \frac{1-\gamma^5}{2} u W_\mu^- \right)$$

$$L_{Z-quark} = \frac{-g}{\cos\theta_W} \sum_{i=u,d} \bar{q}_i \gamma^\mu \left(L_i \frac{1-\gamma^5}{2} + R_i \frac{1+\gamma^5}{2} \right) q_i Z_\mu^0$$

$$L_i = T_3 - \sin^2\theta_W Q_i \qquad R_i = -\sin^2\theta_W Q_i$$

W boson … only V-A coupling
Z boson … can interact with both left- and right-handed fermions.
-> mixture of V-A and V+A.

SU(2)xU(1) Lagrangian

- So far we discussed only interaction between fermions and gauge bosons.
- Mass of fermions and gauge bosons are still zero.

$$L = \underline{L_{fermion}} + \boxed{L_{gauge} + L_{scalar} + L_{Yukawa}}$$

Let's see other terms.

① $$L_{fermion} = i\overline{\Psi}_1\gamma^{\mu}\left(\partial_{\mu} + ig'\frac{Y}{2}B_{\mu} + ig\frac{\vec{\tau}}{2}\vec{W}_{\mu}\right)\Psi_1 + i\overline{\Psi}_2\gamma^{\mu}\left(\partial_{\mu} + ig'\frac{Y}{2}B_{\mu}\right)\Psi_2 + i\overline{\Psi}_3\gamma^{\mu}\left(\partial_{\mu} + ig'\frac{Y}{2}B_{\mu}\right)\Psi_3$$

(this term was already discussed.)

② $$L_{gauge} = -\frac{1}{4} W^{i\mu\nu} W^{i}_{\mu\nu} - \frac{1}{4} B^{\mu\nu} B_{\mu\nu}$$

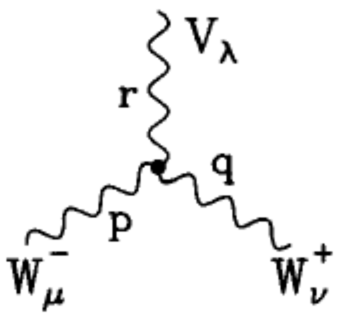
Gauge boson kinetic energy and interactions

$$W^{i}_{\mu\nu} = \partial_{\mu} W^{i}_{\nu} - \partial_{\nu} W^{i}_{\mu} - g \varepsilon^{ijk} W^{j}_{\mu} W^{k}_{\nu}$$

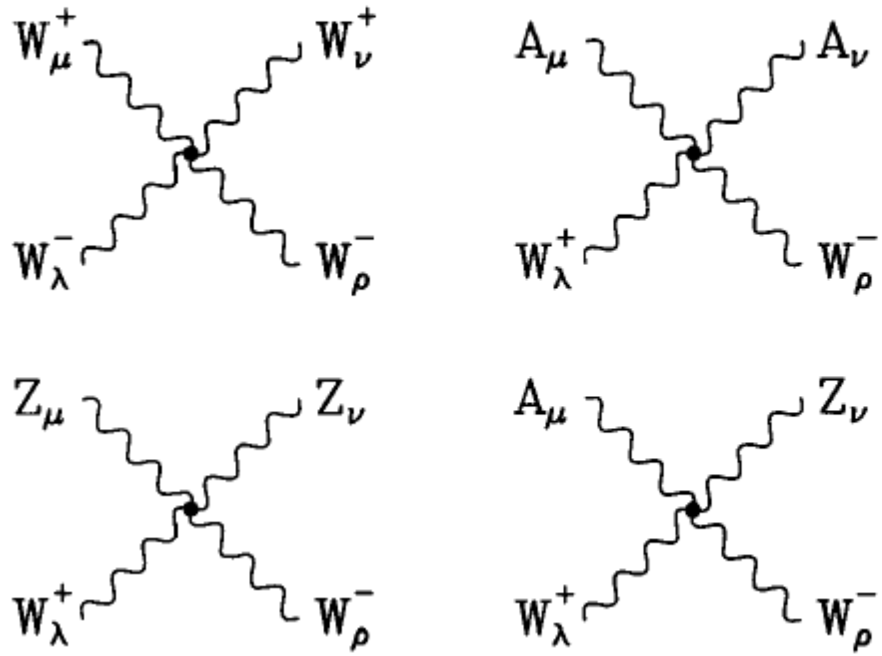
SU(2)->Non-Abelian

$$B_{\mu\nu} = \partial_{\mu} B_{\nu} - \partial_{\nu} B_{\mu}$$

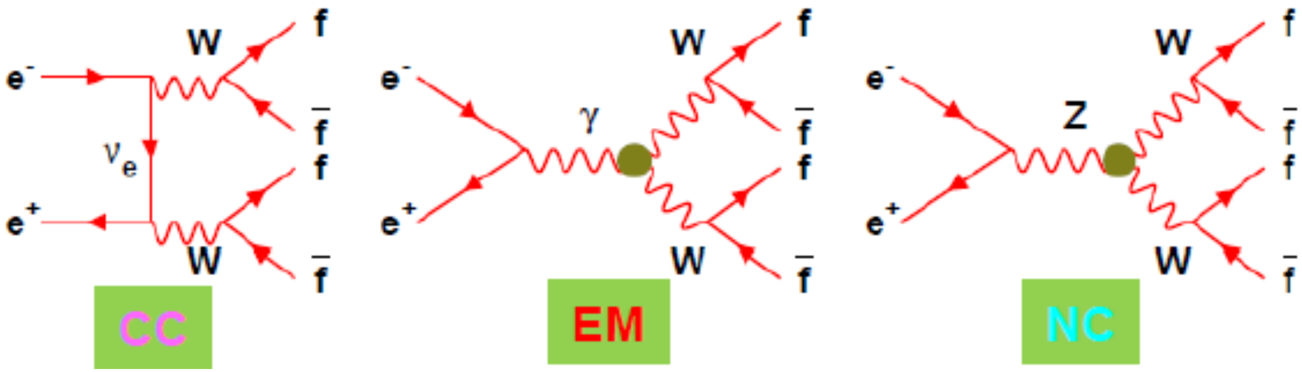
U(1)->Abelian

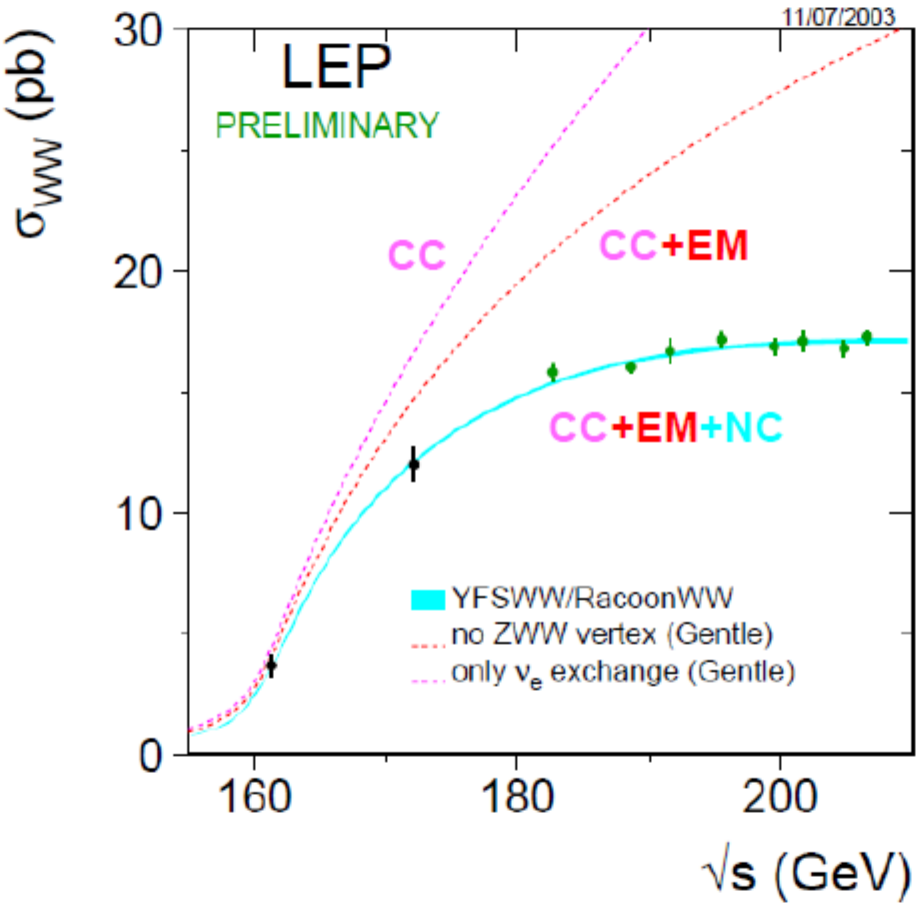


WWZ
WWγ



ZWW Coupling at LEP





③

$$L_{scalar} = \left(D^{\mu}\phi\right)^{\dagger}\left(D_{\mu}\phi\right) - V(\phi^{\dagger}\phi)$$

$$V(\phi^{\dagger}\phi) = \mu^{2}\left(\phi^{\dagger}\phi\right) + \lambda\left(\phi^{\dagger}\phi\right)^{2}$$

Higgs with gauge boson term
We can get gauge boson mass
from this term.

$$\phi = \begin{pmatrix} \phi^{+} \\ \phi^{0} \end{pmatrix}, \phi_{c} = \begin{pmatrix} \phi^{0*} \\ -\phi^{+*} \end{pmatrix}$$

One SU(2) doublet (="One Higgs doublet")
is the minimum case.

$$\phi_{c} = i\tau_{2}\phi$$

In case of SU(2), ϕ_c can be described with
components of ϕ.

④

$$L_{Yukawa} = -g_u\left(\overline{\Psi}_2\phi_c^\dagger\Psi_1 + \overline{\Psi}_1\phi_c\Psi_2\right) - g_d\left(\overline{\Psi}_3\phi^\dagger\Psi_1 + \overline{\Psi}_1\phi\Psi_3\right) - g_e\left(\overline{R}_e\phi^\dagger L_e + \overline{L}_e\phi R_e\right) - \ldots$$

(other generations)

Higgs with fermion term
We can get fermion mass from this term.

$$\Psi_1 = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \Psi_2 = u_R, \Psi_3 = d_R \qquad L_e = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}, R_e = e_R$$

Higgs Mechanism to get fermion mass

- We cannot use a typical mass term for fermions because of gauge invariance (based on chiral symmetry).

- We found/built other mass term, which should be invariant under SU(2)xU(1) and can produce fermion masses with Higgs mechanism.

$$
\begin{aligned}
L_{Yukawa} = & -g_u\left(\overline{\Psi_2}\phi_c^{\dagger}\Psi_1 + \overline{\Psi_1}\phi_c\Psi_2\right) \\
& - g_d\left(\overline{\Psi_3}\phi^{\dagger}\Psi_1 + \overline{\Psi_1}\phi\Psi_3\right) \\
& - g_e\left(\overline{R}_e\phi^{\dagger}L_e + \overline{L}_e\phi R_e\right) - \ldots
\end{aligned}
$$

$$\Psi_1 \rightarrow e^{i\vec{\alpha}(x)\bullet\vec{\tau}+i\beta(x)Y}\Psi_1 \qquad L_e \rightarrow e^{i\vec{\alpha}(x)\bullet\vec{\tau}+i\beta(x)Y}L_e$$

$$\Psi_{2(3)} \rightarrow e^{i\beta(x)Y}\Psi_{2(3)} \qquad R_e \rightarrow e^{i\beta(x)Y}R_e$$

$$\phi \rightarrow e^{i\vec{\alpha}(x)\bullet\vec{\tau}+i\beta(x)Y}\phi$$

Keep in mind that τ and Y are different for different particles.

$$\overline{R}_e \phi^\dagger L_e + \overline{L}_e \phi R_e =$$

$$\overline{e}_R \begin{pmatrix} \phi^{+*} & \phi^{0*} \end{pmatrix} \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} + \begin{pmatrix} \overline{\nu}_L & \overline{e}_L \end{pmatrix} \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} e_R =$$

$$\overline{e}_R \phi^{+*} \nu_L + \overline{e}_R \phi^{0*} e_L + \overline{\nu}_L \phi^+ e_R + \overline{e}_L \phi^0 e_R$$

Fermion	T_3	Y
ν_L	+1/2	-1
e_L	-1/2	-1
e_R	0	-2
ϕ^+	+1/2	1
ϕ^0	-1/2	1

Check the invariance

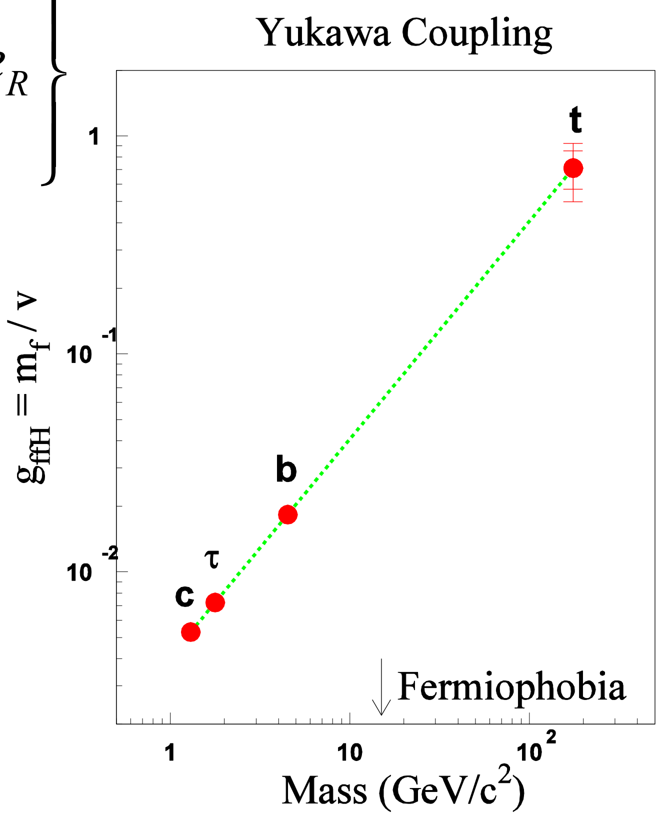
Higgs Mechanism to get fermion mass

$$g_e\left\{\bar{e}_R\begin{pmatrix}0 & \frac{v+h}{\sqrt{2}}\end{pmatrix}\begin{pmatrix}\nu_L \\ e_L\end{pmatrix}+\begin{pmatrix}\bar{\nu}_L & \bar{e}_L\end{pmatrix}\begin{pmatrix}0 \\ \frac{v+h}{\sqrt{2}}\end{pmatrix}e_R\right\}$$

$$= g_e\frac{v+h}{\sqrt{2}}\bar{e}_R e_L + g_e\frac{v+h}{\sqrt{2}}\bar{e}_L e_R$$

$$= \frac{g_e v}{\sqrt{2}}\left(\bar{e}_R e_L + \bar{e}_L e_R\right) + \frac{g_e}{\sqrt{2}}\left(\bar{e}_R e_L + \bar{e}_L e_R\right)h$$

$$= \frac{g_e v}{\sqrt{2}}\bar{e}e + \frac{g_e}{\sqrt{2}}\bar{e}eh$$



$$m_e = \frac{g_e v}{\sqrt{2}}$$

We call g Yukawa coupling.
From this expression, Yukawa coupling is proportional to mass.
This is parameter in the SM, that is, we cannot determine values for each fermion from the SM.