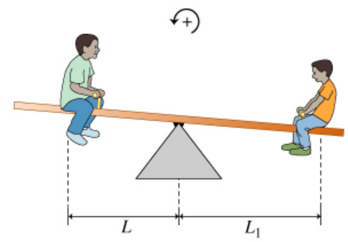
Physics 100A, Homework 12-Chapter 11 (part 2)

Torques on a Seesaw

A) Marcel is helping his two children, Jacques and Gilles, to balance on a seesaw so that they will be able to make it tilt back and forth without the heavier child, Jacques, simply sinking to the ground. Given that Jacques, whose weight is W, is sitting at distance L to the left of the pivot, at what distance L_1 should Marcel place Gilles, whose weight is w, to the right of the pivot to balance the seesaw?

B)Find the torque τ about the pivot due to the weight w of Gilles on the seesaw.
C)Determine the sum of the torques on the seesaw.



The torque produced by Gilles weight $\tau_G = -wL_1$

The torque produced by Jacques weight $\tau_J = WL$

The total torque about the pivot point must equal zero in equilibrium.

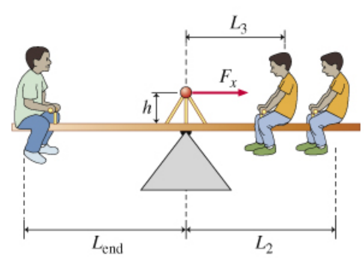
$$WL - wL_1 = 0 \qquad L_1 = WL / w$$

D) Gilles has an identical twin, Jean, also of weight w. The two twins now sit on the same side of the seesaw, with Gilles at distance L_2 from the pivot and Jean at distance L_3.

Where should Marcel position Jacques to balance the seesaw?

$$WL - wL_2 - wL_3 = 0 \qquad L = (w/W)(L_2 + L_3)$$

E) When Marcel finds the distance L from the previous part, it turns out to be greater than L_{end}, the distance from the pivot to the end of the seesaw. Hence, even with Jacques at the very end of the seesaw, the twins Gilles and Jean exert more torque than Jacques does. Marcel now elects to balance the seesaw by pushing sideways on an ornament (shown in red) that is at height h above the pivot.

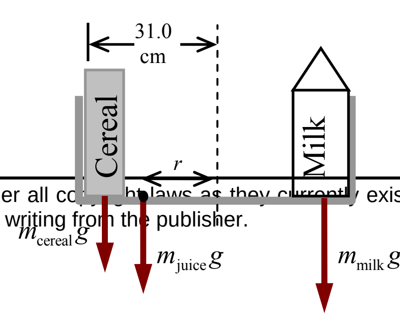


$$WL_{end} - hF_x - w(L_2 + L_3) = 0$$
$$F_x = (WL_{end} - w(L_2 + L_3))/h$$

11.41) A hand-held shopping basket 62.0 cm long has a 1.81 kg carton of milk at one end, and a 0.722 kg box of cereal at the other end.

Where should a 1.80 kg container of orange juice be placed so that the basket balances at its center?

Picture the Problem: The box of cereal is at the left end of the basket and the milk carton is at the right end.



Strategy: Place the origin at the center of the $L = 0.620$ m basket. Write Newton's Second Law for torque with the pivot axis at the center of the basket. Set the net torque equal to zero and solve for the distance *r* of the orange juice from the center of the basket. The orange juice will be placed on the cereal side of the basket because the cereal has less mass and exerts less torque than does the milk.

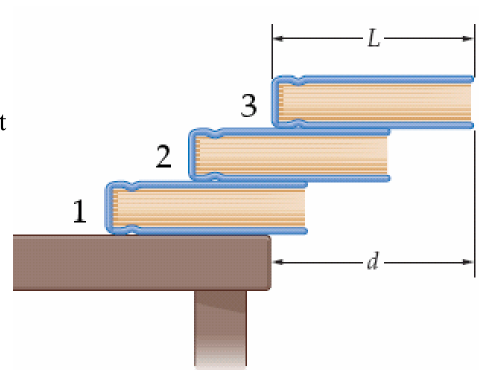
Solution: Set $\sum \tau = 0$ and solve for *r*:

$$\sum \tau = +\left(\tfrac{1}{2}L\right)m_{\text{cereal}}g + r\,m_{\text{juice}}g - \left(\tfrac{1}{2}L\right)m_{\text{milk}}g = 0$$

$$r = \frac{\tfrac{1}{2}L\left(m_{\text{cereal}} + m_{\text{milk}}\right)}{m_{\text{juice}}} = \frac{\tfrac{1}{2}(0.620\text{ m})(0.722 + 1.81\text{ kg})}{1.80\text{ kg}} = 0.187\text{ m} = \boxed{18.7\text{ cm}}$$

Insight: Another way to solve this question is ensure that the center of mass of the basket is at its geometric center, in a manner similar to problem 46 in Chapter 9. However, the balancing of the torques is actually a bit simpler in this case.

11.44) Maximum Overhang Three identical, uniform books of length *L* are stacked one on top the other .Find the maximum overhang distance d in the figure such that the books do not fall over.



Picture the Problem: The books are arranged in a stack as depicted at right, with book 1 on the bottom and book 3 at the top of the stack.

Strategy: It is helpful to approach this problem from the top down. The center of mass of each set of books must be above or to the left of the point of support. Find the positions of the centers of mass for successive stacks of books to determine *d*. Measure the positions of the books from the right edge of book 3 (right hand dashed line in the figure). If the center of mass of the books above an edge is to the right of that edge, there will be an unbalanced torque on the books and they'll topple over. Therefore we can solve the problem by forcing the center of mass to be above the point of support.

Solution: 1. The center of mass of book 3 needs to be above the right end of book 2:

$$d_3 = \frac{L}{2}$$

2. The result of step 1 means that the center of mass of book 2 is located at $d_2 = L/2 + L/2 = L$ from the right edge of book 3.

3. The center of mass of books 3 and 2 needs to be above the right end of book 1:

$$X_{\text{cm},32} = \frac{m(L/2) + m(L)}{2m} = \frac{3}{4}L$$

4. The result of step 3 means that the center of mass of book 1 is located at $d_1 = 3L/4 + L/2 = 5L/4$.

5. The center of mass of books 3, 2, and 1 needs to be above the right end of the table:

$$d = X_{\text{cm},321} = \frac{m(L/2) + m(L) + m(5L/4)}{3m} = \boxed{\frac{11}{12}L}$$

Insight: As we learned in problem 87 of Chapter 9, if you add a fourth book the maximum overhang is $(25/24)L$. If you examine the overhang of each book you find an interesting series: $d = \frac{L}{2} + \frac{L}{4} + \frac{L}{6} + \frac{L}{8} = \frac{25}{24}L$. The series gives you a hint about how to predict the overhang of even larger stacks of books.

11.49) You pull downward with a force of 35 N on a rope that passes over a disk-shaped pulley of mass 1.5 kg and radius 0.075m. The other end of the rope is attached to a 0.87 kg mass.

Picture the Problem: You pull straight downward on a rope that passes over a disk-shaped pulley and then supports a weight on the other side. The force of your pull rotates the pulley and accelerates the mass upward.

Strategy: Write Newton's Second Law for the hanging mass and Newton's Second Law for torque about the axis of the pulley, and solve the two expressions for the tension T_2 at the other end of the rope. We are given in the problem that

$T_1 = 35$ N. Let m be the mass of the pulley, r be the radius of the pulley, and M be the hanging mass. For the disk-shaped pulley the moment of inertia is $I = \frac{1}{2}mr^2$. .

Solution: 1. (a) The tension in the rope is not the same in both sides of the pulley. The tension in the rope on the other end of the rope accelerates the hanging mass, but the tension on your side both imparts angular acceleration to the pulley and accelerates the hanging mass. Therefore, the rope on $\boxed{\text{your side}}$ of the pulley has the greater tension.

2. (b) As stated in the problem, $T_1 = \boxed{35\text{ N}}$ for the rope on your side of the pulley.

3. Set $\sum \vec{\mathbf{F}} = m\vec{\mathbf{a}}$ for the hanging mass: $\sum F_y = T_2 - Mg = Ma$

4. Set $\sum \tau = I\alpha$ for the pulley: $\sum \tau = rT_1 - rT_2 = I\alpha = \left(\frac{1}{2}mr^2\right)\left(a/r\right) \Rightarrow a = 2\left(T_1 - T_2\right)/m$

5. Substitute the expression for a from step 4 into the one from step 3, and solve for T_2 (the tension on the other side of the pulley from you):

$$T_2 - Mg = M\left[2\left(T_1 - T_2\right)/m\right]$$
$$mT_2 - mMg = 2MT_1 - 2MT_2$$
$$T_2 = \frac{M\left(2T_1 + mg\right)}{2M + m}$$
$$= \frac{(0.87\text{ kg})\left[2(35\text{ N}) + (1.5\text{ kg})(9.81\text{ m/s}^2)\right]}{2(0.87\text{ kg}) + 1.5\text{ kg}} = \boxed{23\text{ N}}$$

Insight: The net force on the hanging mass is thus $T_2 - Mg = 23 - (0.87)(9.81) = 14.2$ N , enough to accelerate it upward at $a = 14.2/0.87 = 16.3$ m/s^2. The angular acceleration of the pulley is thus $a/r = \left(16.3\text{ m/s}^2\right)/\left(0.075\text{ m}\right) = 217\text{ rad/s}^2$.

11.50) You pull downward with a force of 35 N on a rope that passes over a disk-shaped pulley of mass 1.5 kg and radius 0.075 m. The other end of the rope is attached to a 0.87 kg mass.

This is the same problem as 11.49. The answer for the acceleration is above.

11.54) A 0.015 kg record with a radius of 15 cm rotates with an angular speed of $33\frac{1}{3}$ rpm.

Find the angular momentum of the record.

Picture the Problem: The disk-shaped record rotates about its axis with a constant angular speed.

Strategy: Use equation 11-11 and the moment of inertia of a uniform disk rotating about its axis, $I = \frac{1}{2}MR^2$, to find the angular momentum of the record.

Solution: Apply equation 11-11 directly:

$$L = I\omega$$
$$= \left(\tfrac{1}{2}MR^2\right)\omega = \tfrac{1}{2}(0.015\text{ kg})(0.15\text{ m})^2\left(33\tfrac{1}{3}\ \frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\text{ rad}}{\text{rev}}\right)\left(\frac{1}{60}\ \frac{\text{min}}{\text{s}}\right)$$
$$L = \boxed{5.9\times10^{-4}\ \text{kg}\cdot\text{m}^2/\text{s}}$$

Insight: The angular momentum of a compact disk rotating at 300 rev/min is about 7.5×10^{-4} kg·m^2/s. The compact disk (m = 13 g, r = 6.0 cm) is smaller than a record, but it spins faster, so the angular momenta are similar.

Spinning Situations.

Suppose you are standing on the center of a merry-go-round that is at rest. You are holding a spinning bicycle wheel over your head so that its rotation axis is pointing upward. The wheel is rotating counterclockwise when observed from above.

For this problem, neglect any air resistance or friction between the merry-go-round and its foundation.

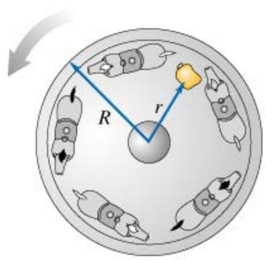
Suppose you now grab the edge of the wheel with your hand, stopping it from spinning. What happens?

Consider yourself, the merry-go-round, and the bicycle wheel to be a single **system**. When you stop the wheel from spinning, the angular momentum of the **system** about the vertical axis remains unchanged.

Then to conserve angular momentum the merry-go-round begins to rotate counterclockwise (as seen from above).

Change in Angular Velocity Ranking Task

A merry-go-round of radius R , shown in the figure, is rotating at constant angular speed. The friction in its bearings is so small that it can be ignored. A sandbag of mass m is dropped onto the merry-go-round, at a position designated by r. The sandbag does not slip or roll upon contact with the merry-go-round.



Rank the following different combinations of m and r on the basis of the angular speed of the merry-go-round after the sandbag "sticks" to the merry-go-round.

The guiding principle is that angular momentum is conserved.

$$L_i = I_{mgr}\omega_i$$
$$L_f = (I_{mgr} + mr^2)\omega_f$$
$$L_f = L_i$$
$$\omega_f = \frac{I_{mgr}\omega_i}{(I_{mgr} + mr^2)}$$ The value of ω_f depends of the value of the moment of inertia of the sandbag mr^2 .

case	m (kg)	r (R)	mr^2
1	40	0.25	2.5
2	10	0.50	2.5
3	20	0.25	1.25
4	10	1.0	10
5	15	0.75	8.4375
6	10	0.25	0.625

In decreasing order of omega (increasing order of the moment of inertia): 6, 3, (1,2), 5, 4

11.65) As an ice skater begins a spin, his angular speed is 3.17 rad/s. After pulling in his arms, his angular speed increases to 5.46 rad/s.

Find the ratio of the skater's final moment of inertia to his initial moment of inertia.

Picture the Problem: The skater pulls his arms in, decreasing his moment of inertia and increasing his angular speed.

Strategy: The angular momentum of the skater remains the same throughout the spin because there is assumed to be no torque of any kind acting on his body. Use the conservation of angular momentum (equation 11-15) together with equation 11-11, to find the ratio I_f/I_i .

Solution: Set $L_i = L_f$ and solve for I_f/I_i :

$$I_i\omega_i = I_f\omega_f \Rightarrow \frac{I_f}{I_i} = \frac{\omega_i}{\omega_f} = \frac{3.17 \text{ rad/s}}{5.46 \text{ rad/s}} = \boxed{0.581}$$

Insight: By rearranging his mass, especially by bringing his arms and legs in close to his axis of rotation, the skater has reduced his moment of inertia by an impressive 42% and increased his angular speed by 72%.

11.80) To prepare homemade ice cream, a crank must be turned with a torque of 3.95 N·m.

How much work is required for each complete turn of the crank?

. **Picture the Problem**: The torque acting through an angular displacement does work on the ice cream crank.

Strategy: Use equation 11-17 to find the work done by the torque acting through the given angular displacement. One complete turn corresponds to an angular displacement of 2π radians.

Solution: Apply equation 11-17 directly:

$$W = \tau\,\Delta\theta = (3.95 \text{ N}\cdot\text{m})(2\pi \text{ rad}) = \boxed{24.8 \text{ J}}$$

Insight: The work done on the ice cream crank is dissipated as heat via friction in the viscous ice cream mixture.

Introduction to Rotational Work and Power.

Consider a motor that exerts a constant torque of 25.0 N·m to a horizontal platform whose moment of inertia is 50.0 kg·m^2. Assume that the platform is initially at rest and the torque is applied for 12.0 rotations.

A) How much work does the motor do on the platform during this process?

$$W = \tau\Delta\theta = (25)(12\text{rev})(2\pi\text{rad/rev}) = 1{,}885 \text{ J}$$

B)What is the rotational kinetic energy of the platform $K_{rot,j}$ at the end of the process described above?

From the work energy theorem the total work is equal to the change in kinetic energy. So if the platform is initially at rest, the final kinetic energy is equal to the work or 1,885 J.

Now the slow approach:

$$\tau = I\alpha \qquad \alpha = \tau / I = 25/50 = 0.5 \text{ rad/s}$$

$$\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta$$

$$\omega_f = \sqrt{(0 + 2(0.5\text{rad/s})(12\text{rev})(2\pi\text{rad/rev}))} = 8.68 \text{ rad/s}$$

$$K_{rot,f} = \frac{1}{2}I\omega_f^2 = \frac{1}{2}(50)(868)^2 = 1{,}885 \text{ J}$$

C) What is the angular velocity ω_f of the platform at the end of this process?

As found above $\omega_f = 8.68 \text{ rad/s}$

D) How long does it take for the motor to do the work done on the platform calculated in Part A?

$\omega_f = \omega_i + \alpha t$

With $\omega_i = 0$ $t = \omega_f / \alpha = 8.68 / 0.5 = 17.4$ s

E) What is the average power delivered by the motor in the situation above?

$P_{avg} = W / t = 1{,}885 / 17.4 = 108$ Watts

F) Note that the instantaneous power P delivered by the motor is directly proportional to ω, so P increases as the platform spins faster and faster. How does the instantaneous power P_f being delivered by the motor at the time t_f compare to the average power P_{avg} calculated in Part E?

$P_f = \tau\omega_f = (25)(868) = 217$ Watts

$P_f = 2P_{avg}$

81. **Picture the Problem**: The drill spins the bit at a rapid rate while exerting a torque on the bit to keep it spinning.

Strategy: The power produced by the drill equals the torque it produces times its angular speed (equation 11-19).

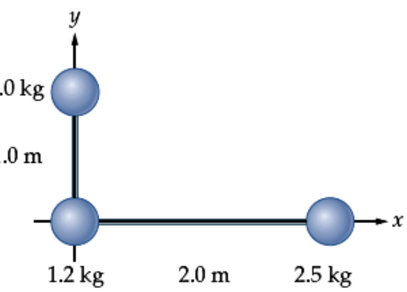
Solution: 1. Convert τ into units of $\text{N}\cdot\text{m}$:: $\tau = 3.68 \text{ oz}\cdot\text{in} \times \dfrac{1 \text{ lb}}{16 \text{ oz}} \times \dfrac{4.45 \text{ N}}{1 \text{ lb}} \times \dfrac{1 \text{ m}}{39.4 \text{ in}} = \underline{\underline{0.0260 \text{ N}\cdot\text{m}}}$

2. Convert ω into units of rad/s: $\omega = 42{,}500 \dfrac{\text{rev}}{\text{min}} \times \dfrac{2\pi \text{ rad}}{\text{rev}} \times \dfrac{1 \text{ min}}{60 \text{ s}} = \underline{\underline{4450 \text{ rad/s}}}$

3. Apply equation 11-19 directly: $P = \tau\omega = (0.0260 \text{ N}\cdot\text{m})(4450 \text{ rad/s}) = \boxed{116 \text{ W}}$

Insight: The same torque applied at 425 rev/min requires only 1.16 W of power.

82. **Picture the Problem**: The object gains rotational kinetic energy from an applied torque acting through an angular displacement.



Strategy: Find the kinetic energy that the L-shaped object has when it is rotated at 2.35 rad/s about the *x*, *y*, and *z* axes. The work that must be done on the object to accelerate it from rest equals its final kinetic energy (equations 11-18 and 10-17). From problem 15 we note that $I_x = 9.0 \text{ kg}\cdot\text{m}^2$, $I_y = 10 \text{ kg}\cdot\text{m}^2$, and $I_z = 19 \text{ kg}\cdot\text{m}^2$.

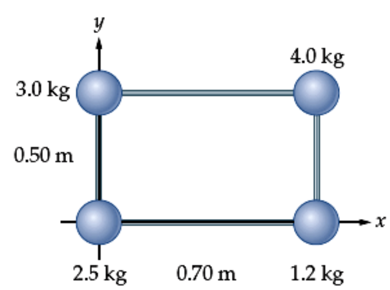
Solution: 1. (a) Find K_f for rotation about the *x* axis: $W = K_\text{f} = \tfrac{1}{2} I_x \omega_x^2 = \tfrac{1}{2}(9.0 \text{ kg}\cdot\text{m}^2)(2.35 \text{ rad/s})^2 = \boxed{25 \text{ J}}$

2. (b) Find K_f for rotation about the *y* axis: $W = K_\text{f} = \tfrac{1}{2} I_y \omega_y^2 = \tfrac{1}{2}(10 \text{ kg}\cdot\text{m}^2)(2.35 \text{ rad/s})^2 = \boxed{28 \text{ J}}$

3. (c) Find K_f for rotation about the *z* axis: $W = K_\text{f} = \tfrac{1}{2} I_z \omega_z^2 = \tfrac{1}{2}(19 \text{ kg}\cdot\text{m}^2)(2.35 \text{ rad/s})^2 = \boxed{52 \text{ J}}$

Insight: The larger the moment of inertia, the more work is required to obtain the same rotation rate.

83. **Picture the Problem**: The object gains rotational kinetic energy from an applied torque acting through an angular displacement.



Strategy: Find the kinetic energy that the rectangular object has when it is rotated at 2.5 rad/s about the *x*, *y*, and *z* axes. The work that must be done on the object to accelerate it from rest equals its final kinetic energy (equations 11-18 and 10-17). The power required to accomplish this in 6.4 s is the work divided by the time (equation 11-19). From problem 18 we note that $I_x = 1.8 \text{ kg}\cdot\text{m}^2$, $I_y = 2.5 \text{ kg}\cdot\text{m}^2$, and $I_z = 4.3 \text{ kg}\cdot\text{m}^2$.

Solution: 1. (a) Find *P* for rotation about the *x* axis: $P = \dfrac{W}{t} = \dfrac{\tfrac{1}{2} I_x \omega_x^2}{t} = \dfrac{\tfrac{1}{2}(1.8 \text{ kg}\cdot\text{m}^2)(2.5 \text{ rad/s})^2}{6.4 \text{ s}} = \boxed{0.88 \text{ W}}$

2. (b) Find *P* for rotation about the *y* axis: $P = \dfrac{W}{t} = \dfrac{\tfrac{1}{2} I_y \omega_y^2}{t} = \dfrac{\tfrac{1}{2}(2.5 \text{ kg}\cdot\text{m}^2)(2.5 \text{ rad/s})^2}{6.4 \text{ s}} = \boxed{1.2 \text{ W}}$

3. (c) Find *P* for rotation about the *z* axis: $P = \dfrac{W}{t} = \dfrac{\tfrac{1}{2} I_z \omega_z^2}{t} = \dfrac{\tfrac{1}{2}(4.3 \text{ kg}\cdot\text{m}^2)(2.5 \text{ rad/s})^2}{6.4 \text{ s}} = \boxed{2.1 \text{ W}}$

Insight: The larger the moment of inertia, the more work is required to obtain the same rotation rate.

84. **Picture the Problem**: The saw blade rotates on its axis and gains rotational kinetic energy due to the torque applied by the electric motor.

Strategy: The torque applied through an angular displacement gives the blade its rotational kinetic energy. Use equations 11-17 and 10-17 to relate the kinetic energy to the torque applied by the motor. Then use equation 11-17 again to find the kinetic energy and angular speed after the blade has completed half as many revolutions.

Solution: 1. (a) Find ω_f in units of rad/sec: $\omega_f = 3620\ \frac{\text{rev}}{\text{min}} \times \frac{2\pi\ \text{rad}}{\text{rev}} \times \frac{1\ \text{min}}{60\ \text{s}} = \underline{\underline{379\ \text{rad/s}}}$

2. Set $W = \Delta K$ and solve for τ : $W = \tau\,\Delta\theta = \frac{1}{2}I\omega^2$ and $I = \frac{1}{2}mr^2$

$$\tau = \frac{\frac{1}{2}mr^2\omega^2}{2\Delta\theta} = \frac{\frac{1}{2}(0.755\ \text{kg})(0.152\ \text{m})^2(379\ \text{rad/s})^2}{2(6.30\ \text{rev}\times 2\pi\ \text{rad/rev})} = \boxed{15.8\ \text{N}\cdot\text{m}}$$

3. (b) The time to rotate the first 3.15 revolutions is greater than the time to rotate the last 3.15 revolutions because the blade is speeding up. So more than half the time is spent in the first 3.15 revolutions. Therefore, the angular speed has increased to more than half of its final value. After 3.15 revolutions, the angular speed is $\boxed{\text{greater than}}$ 1810 rpm.

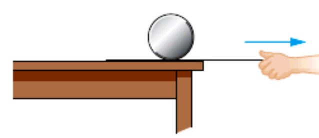
4. (d) Set $W = \Delta K$ and solve for ω : $\tau\,\Delta\theta = \frac{1}{2}I\omega^2 = \frac{1}{4}mr^2\omega^2$

$$\omega = \sqrt{\frac{4\tau\,\Delta\theta}{mr^2}} = \sqrt{\frac{4(15.8\ \text{N}\cdot\text{m})(3.15\ \text{rev}\times 2\pi\ \text{rad/rev})}{(0.755\ \text{kg})(0.152\ \text{m})^2}}$$

$$= (268\ \text{rad/s})\left(\frac{60\ \text{s}}{\text{min}}\right)\left(\frac{1\ \text{rev}}{2\pi\ \text{rad}}\right) = \boxed{2560\ \text{rev/min}}$$

Insight: The angular speed increases linearly upon time ($\omega = \omega_0 + \alpha t = \alpha t$) but depends upon the square root of the angular displacement: $\omega = \sqrt{\omega_0^2 + 2\alpha\,\Delta\theta} = \sqrt{2\alpha\,\Delta\theta}$.

85. **Picture the Problem**: A uniform disk stands upright on its edge, and rests on a sheet of paper placed on a tabletop. The paper is pulled horizontally to the right.



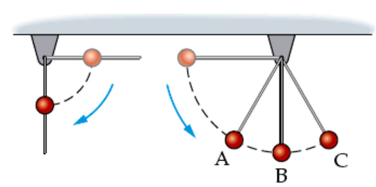
Strategy: Use Newton's Second Law for linear motion and for torques to predict the behavior of the disk.

Solution: 1. (a) There are three forces that act upon the cylinder, the force of friction from the paper, the force of gravity on the center of mass, and the normal force from the tabletop. The paper force is the only one that exerts a torque about the cylinder's center of mass, and it acts in the $\boxed{\text{counterclockwise}}$ direction to rotate the disk.

2. (b) The normal force and the force of gravity balance each other and do not produce any acceleration. The paper force is unbalanced and produces an acceleration that will cause the center of the disk to $\boxed{\text{move to the right}}$.

Insight: When the paper is removed the disk is translating toward the right but is rolling toward the left. What happens next depends upon the rotation and translation speeds as well as the magnitude of the friction force on the disk.

86. **Picture the Problem**: The two rotating systems shown at right each consists of a mass m attached to a rod of negligible mass pivoted at one end. On the left, the mass is attached at the midpoint of the rod; to the right, it is attached to the free end of the rod. The rods are released from rest in the horizontal position at the same time.

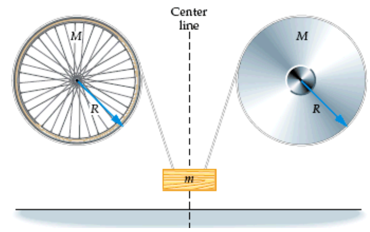


Strategy: Use Newton's Second Law for torques $\sum \vec{\boldsymbol{\tau}} = I\,\alpha$ to predict the behavior of the two rotating systems.

Solution: The angular acceleration of each system is given by $\alpha = \tau/I$. We can see that the right hand system experiences a larger torque due to its larger moment arm, but it also has a larger moment of inertia. Quantifying the two systems, we find that $\tau_{\text{left}} = \left(\frac{1}{2}L\right)(mg)$ and $I_{\text{left}} = m\left(\frac{1}{2}L\right)^2 = \frac{1}{4}mL^2$, so $\alpha_{\text{left}} = \left(\frac{1}{2}mg\,L\right)/\left(\frac{1}{4}mL^2\right) = 2g/L$, and $\tau_{\text{right}} = mg\,L$ and $I_{\text{right}} = mL^2$, so $\alpha_{\text{right}} = (mg\,L)/(mL^2) = g/L$. We can see that the left hand system has the larger angular acceleration, and we conclude that when the rod to the left reaches the vertical position, the rod to the right is $\boxed{\text{not yet vertical (location A)}}$.

Insight: The greater effect is the moment of inertia, because it depends on the square of the distance from the axis of rotation, whereas the torque depends only on the first power of the distance.

87. **Picture the Problem**: A disk and a bicycle wheel of equal radius and mass each have a string wrapped around their circumferences. Hanging from the strings, halfway between the disk and the hoop, is a block of mass m, as shown at right. The disk and the hoop are free to rotate about their centers.



Strategy: Use Newton's Second Law for torques $\sum \vec{\boldsymbol{\tau}} = I\,\alpha$ to predict the behavior of the two rotating systems.

Solution: 1. (a) Upon its release the mass exerts equal torques on the disk and the wheel. However, the disk has a smaller moment of inertia than the wheel and experiences the larger angular acceleration $\alpha = \tau/I$. The string on the disk will unravel faster than the string on the bicycle wheel, and we conclude that when the block is allowed to fall, it will $\boxed{\text{move toward the left}}$.

2. (b) The best explanation is $\boxed{\textbf{II.}\text{ The wheel has the greater moment of inertia and unwinds more slowly than the disk.}}$ Statement I is false, and statement III is true, but irrelevant.

Insight: Statement III is only true in terms of mass and radius. In terms of moment of inertia, the system is not symmetric, and that fact is what leads to the observed behavior.

88. **Picture the Problem**: A beetle sits at the rim of a turntable that is at rest but is free to rotate about a vertical axis.

Strategy: Use the conservation of angular momentum to answer the conceptual question.

Solution: 1. (a) As the beetle begins to walk, it exerts a force and a torque on the turntable. The turntable exerts an equal but opposite force and torque on the beetle. There are no torques on the beetle-turntable system, so there is no net change in its linear or angular momentum. If the turntable is much more massive than the beetle, it will barely rotate backward as the beetle moves $\boxed{\text{forward}}$. The beetle, then, will begin to circle around the perimeter of the turntable almost the same as if it were on solid ground.

2. (b) If the turntable is virtually massless, it will rotate backward with a linear speed at the rim that is almost equal to the forward linear speed of the beetle. The beetle will progress very slowly relative to the ground in this case—though as far as it is concerned, it is running with its usual speed. In the limit of a massless turntable, the beetle will $\boxed{\text{remain in the same location relative to the ground}}$.

Insight: In either case, massive turntable or nearly massless turntable, the angular momentum of the beetle in the laboratory frame of reference is balanced by the angular momentum of the turntable. The angular momentum of the beetle-turntable system must remain zero because there are no external torques on the system.

89. **Picture the Problem**: A beetle sits at the rim of a turntable that is at rest but is free to rotate about a vertical axis.

Strategy: Use the conservation of angular momentum to answer the conceptual question.

Solution: The angular momentum $L = I\,\omega$ of the system must remain constant because there are no external torques acting on it. Thus, as the beetle walks toward the axis of rotation, which reduces the moment of inertia of the system, the angular speed of the turntable will $\boxed{\text{increase}}$.

Insight: The beetle must do work against the "centrifugal force," or from another perspective the force of friction (that supplies the centripetal force to keep the beetle moving in a circle) does work on the beetle as it moves toward the center. The kinetic energy of the beetle therefore increases. A similar effect occurs when an ice skater does work to move her arms inward toward her body, and gains kinetic energy as she spins faster.

90. **Picture the Problem**: The Earth is imagined to magically expand, doubling its radius while keeping its mass the same.

Strategy: Use the conservation of angular momentum to answer the conceptual question.

Solution: The angular momentum $L = I\,\omega$ of the Earth must remain constant because there are no external torques acting on it. The moment of inertia $I = \frac{2}{5}M\,R^2$ would increase after the expansion, so the angular speed ω would decrease and the length of a day would $\boxed{\text{increase}}$.

Insight: The moment of inertia of the Earth in this case would increase by a factor of four, producing a day that is four times longer, or 96 hours!

91. **Picture the Problem**: The work the hamster does on the exercise wheel gives the wheel rotational kinetic energy.

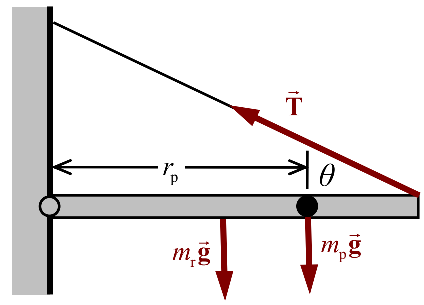
Strategy: Find the rotational kinetic energy of the wheel to determine the work done by the hamster (equation 11-18). Use Table 10-1 to find the moment of inertia of a hoop, $I = mr^2$. The hamster runs without slipping relative to the circumference of the exercise wheel, so that $\omega = v/r$ (equation 10-15) relates its linear speed with the angular speed of the wheel.

Solution: Set $W = \Delta K$ and substitute for I and ω:

$$W = \Delta K = \tfrac{1}{2}I\omega^2 = \tfrac{1}{2}\left(mr^2\right)\left(v/r\right)^2 = \tfrac{1}{2}mv^2 = \tfrac{1}{2}\left(0.0065\text{ kg}\right)\left(1.3\text{ m/s}\right)^2$$
$$= 5.5\times10^{-3}\text{ J} = \boxed{5.5\text{ mJ}}$$

Insight: Note that in this special case the rotational kinetic energy of the wheel in the laboratory frame of reference equals the linear kinetic energy the hamster has in the rotating frame of reference of the wheel.

92. **Picture the Problem**: The person's weight is supported by the hinge and the wire in the manner shown in the figure at right.



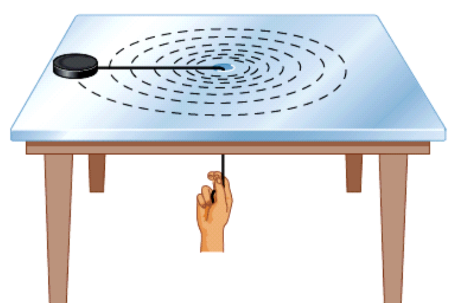
Strategy: Set the sum of the torques about the hinge equal to zero and solve for the moment arm of the person relative to the hinge. Let L = length of the rod, m_r = mass of the rod, m_p = mass of the person, and r_p = distance from the hinge to the person. Let $T = T_{max} = 1400\text{ N}$ and use equation 11-6 to solve for r_p.

Solution: Set $\sum\tau = 0$ and solve for r_p:

$$\sum\tau = L\left(T\sin\theta\right) - \left(\tfrac{1}{2}L\right)m_r g - \left(r_p\right)m_p g = 0$$
$$r_p = \frac{LT\sin\theta - \tfrac{1}{2}Lm_r g}{m_p g}$$
$$= \frac{\left(4.25\text{ m}\right)\left(1450\text{ N}\right)\sin\left(30.0°\right) - \tfrac{1}{2}\left(4.25\text{ m}\right)\left(47.0\text{ kg}\right)\left(9.81\text{ m/s}^2\right)}{\left(68.0\text{ kg}\right)\left(9.81\text{ m/s}^2\right)} = \boxed{3.15\text{ m}}$$

Insight: Note that when the person is 3.15 m from the hinge the tension in the cable (1450 N) is more than twice the weight of the person (667 N). This is because about half the tension is pulling horizontally toward the hinge and not supporting the downward weight of the person and the rod.

93. **Picture the Problem**: The puck travels in a circular path about the hole in the table, but the radius of the path can be adjusted by pulling on the string from underneath the table, as shown in the figure at right.



Strategy: Let the angular momentum of the puck remain constant, and use equation 11-12 to find the final speed of the puck.

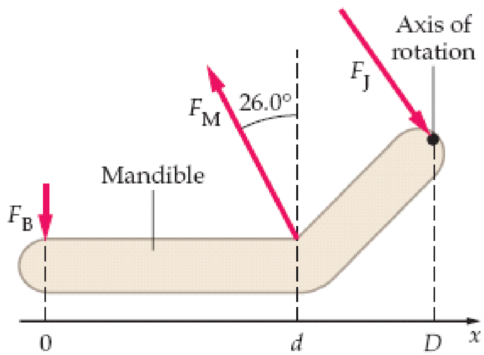
Solution: 1. (a) The angular momentum of the puck does not change because the string exerts no torque on the puck, but its moment of inertia decreases as the radius of its path decreases. Because $L = mvr$ we conclude the linear speed of the puck must $\boxed{\text{increase}}$ in order for L to remain the same while r decreases.

2. (b) Set $L_i = L_f$ and solve for v_f:

$$mvr = mv_f r_f$$
$$v_f = v\left(\frac{r}{r_f}\right) = v\left(\frac{r}{\tfrac{1}{2}r}\right) = \boxed{2v}$$

Insight: The puck gains kinetic energy in this process because pulling on the string exerts a force in the same direction as the radial displacement and therefore does work on the puck.

94. **Picture the Problem**: The masseter muscle and the biting force each produce a torque about the joint in a manner depicted by the figure at right.



Strategy: Find the torques produced by the two forces by finding the portion of each force that is perpendicular to the horizontal moment arms shown in the figure (equation 11-3). The torque from the biting force must be the same magnitude as the torque from the masseter muscle in order for the torques to be in equilibrium. Use the torque produced by the biting force together with the moment arm to find the magnitude of that force. Finally, apply Newton's Second Law in the horizontal and vertical directions to find the components of the force $\vec{\mathbf{F}}_\mathbf{J}$ that the mandible exerts on the joint.

Solution: 1. (a) The vertical component of $\vec{\mathbf{F}}_\mathrm{M}$ is the portion of the force that produces a torque about the moment arm $r_\mathrm{M} = D - d$.

$$\tau = r_\perp F = (D-d)(F_\mathrm{M}\cos\theta)$$
$$= (0.1085 - 0.0760\text{ m})\left[(455\text{ N})\cos 26.0°\right] = \boxed{13.3\text{ N}\cdot\text{m}}$$

2. (b) Use equation 11-3 again to find F_B:

$$\tau = r_\perp F = DF_\mathrm{B}$$
$$F_\mathrm{B} = \frac{\tau}{D} = \frac{13.3\text{ N}\cdot\text{m}}{0.1085\text{ m}} = \boxed{123\text{ N}}$$

3. (c) Set $\sum F_x = 0$ to find $F_{\mathrm{J},x}$:

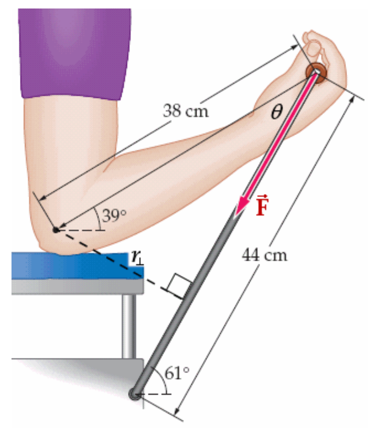
$$\sum F_\mathrm{x} = -F_{\mathrm{M},x} + F_{\mathrm{J},x} = 0$$
$$F_{\mathrm{J},x} = F_{\mathrm{M},x} = F_\mathrm{M}\sin\theta = (455\text{ N})\sin 26.0° = \boxed{199\text{ N}}$$

4. (d) Set $\sum F_y = 0$ to find $F_{\mathrm{J},y}$:

$$\sum F_y = -F_\mathrm{B} + F_{\mathrm{M},y} + F_{\mathrm{J},y} = 0$$
$$F_{\mathrm{J},y} = F_\mathrm{B} - F_{\mathrm{M},x}$$
$$= F_\mathrm{B} - F_\mathrm{M}\cos\theta = 123\text{ N} - (455\text{ N})\cos 26.0° = \boxed{-286\text{ N}}$$

Insight: While the biting force is large (123 N is equal to 27.6 lb) the 348-N total force on the joint is the same as 78.3 lb, and is an indicator of how strong the joints and muscles must be in order for the jaw to work correctly!

95. **Picture the Problem**: The force from the elastic cord produces a torque about the elbow joint in the manner indicated by the figure at right.



Strategy: Use the geometry in the figure to determine the component of the moment arm that is perpendicular to the force *F*, and then use equation 11-3 to determine the *F* that will produce the desired torque. Finally, use Hooke's Law (equation 6-4) to find the spring constant from the force and the stretch distance. Let *a* be the 38-cm length of the person's arm. The perpendicular component of the moment arm is $r_\perp = a\sin\theta$. A careful analysis of the geometry reveals that $\theta = 61° - 39° = 22°$. The stretch distance *x* is the difference between the 44-cm stretched length and the 31-cm unstretched length of the elastic cord.

Solution: 1. Solve equation 11-3 for *F*:

$$F = \frac{\tau}{r_\perp} = \frac{81\text{ N}\cdot\text{m}}{(0.38\text{ m})\sin 22°} = \underline{\underline{570\text{ N}}}$$

2. Solve equation 6-4 for *k*:

$$k = \frac{F}{x} = \frac{570\text{ N}}{0.44 - 0.31\text{ m}} = 4400\text{ N/m} = \boxed{4.4\text{ kN/m}}$$

Insight: The 570 N of force the elastic cord exerts on the hand is equivalent to 130 lb. A good workout!

96. **Picture the Problem**: This is a units conversion problem.

Strategy: The formula is a version of equation 11-19 but with non-metric units. The constant *C* simply converts the units from rev/min to rad/s and from ft·lb/s to horsepower. Use equation 11-19 to find the value of *C*, then use the given formula and the known value of *C* to find the engine torque in ft·lbs.

Solution: 1. (a) Use equation 11-19 to find *C*:

$$P(\text{hp}) = \tau(\text{ft}\cdot\text{lb})\,\omega(\text{rev/min})\times\frac{2\pi\ \text{rad}}{1\ \text{rev}}\times\frac{1\ \text{min}}{60\ \text{s}}\times\frac{1\ \text{hp}}{550\ \text{ft}\cdot\text{lb/s}}$$

$$\text{HP} = \frac{\text{Torque}\cdot\text{RPM}}{5252\ \text{ft}\cdot\text{lb}\cdot\text{rpm/hp}} = \frac{\text{Torque}\cdot\text{RPM}}{C}\ \text{hp}$$

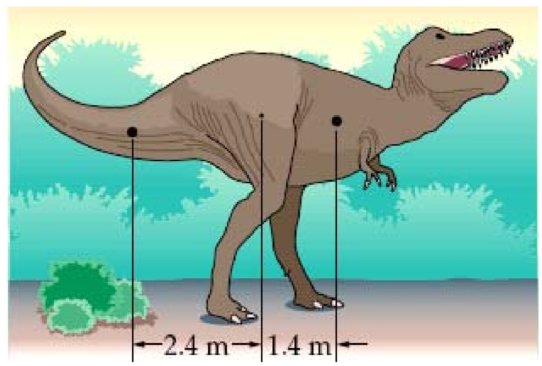
$$C = 5252\ \text{ft}\cdot\text{lb}\cdot\text{rev/min/hp} = \boxed{5250\ \text{ft}\cdot\text{lb}\cdot\text{rev/min/hp}}$$

2. (b) Use the given formula to find τ:

$$\text{Torque} = \frac{C\times\text{HP}}{\text{RPM}} = \frac{(5252\ \text{ft}\cdot\text{lb}\cdot\text{rpm/hp})(320\ \text{hp})}{(6500\ \text{rpm})} = \boxed{259\ \text{ft}\cdot\text{lb}}$$

Insight: The constant *C* can be considered "unitless" because it is basically power divided by power, but we retained the units to indicate how to accomplish the conversion. We bent the rules for significant figures for *C* a bit in step 2 to avoid rounding error.

97. **Picture the Problem**: The torque about the hip joint from the weight of the tail balances the torque from the weight of the upper torso of the dinosaur.



Strategy: Write Newton's Second Law for torque about the hip joint and solve for the mass of the tail. Let m_U be the mass of the upper torso, let m_T be the mass of the tail, and let $M = m_U + m_T$ be the total mass of the *T. rex*.

Solution: 1. Set $\sum\tau = 0$ and substitute for m_U:

$$r_T m_T g - r_U m_U g = 0$$

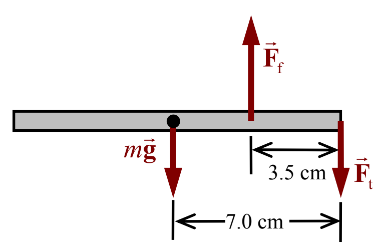
$$r_T m_T - r_U(M - m_T) = 0$$

2. Now solve for m_T:

$$m_T = \frac{r_U M}{r_T + r_U} = \frac{(1.4\ \text{m})(5400\ \text{kg})}{2.4 + 1.4\ \text{m}} = 2000\ \text{kg} = \boxed{2.0\times10^3\ \text{kg}}$$

Insight: Such a massive tail would not be necessary if the creature stood upright like humans do, placing its mass over the point of support of its feet. Other creatures like monkeys have large tails for better balance when doing acrobatics in the tree tops.

98. **Picture the Problem**: The weight of the pen, the thumb force, and the index finger force act on the pen in the manner indicated by the figure.



Strategy: Use Newton's Second Law for torque and Newton's Second Law for force in the vertical direction to determine the magnitudes of the forces. The forces and torques are each in equilibrium. The weight of the pen will act at the center of mass, 7.0 cm from the end of the pen.

Solution: 1. (a) $\boxed{\text{The force from the index finger}}$ will be greater in magnitude than the force from the thumb, because the finger force has to counteract both the thumb's force and the pen's weight.

2. (b) Set $\sum\tau = 0$ and solve for F_f:

$$\sum\tau = r_f F_f - r_{cm} mg = 0$$

$$F_f = \frac{r_{cm} mg}{r_f} = \frac{(7.0\ \text{cm})(0.028\ \text{kg})(9.81\ \text{m/s}^2)}{3.5\ \text{cm}} = \boxed{0.55\ \text{N}}$$

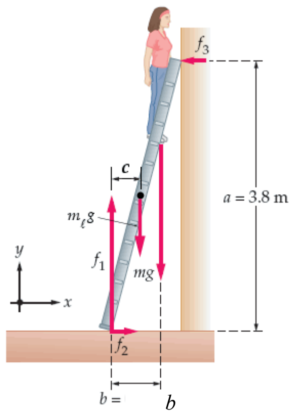
3. Set $\sum F_y = 0$ and solve for F_t:

$$\sum F_y = F_f - F_t - mg = 0$$

$$F_t = F_f - mg = (0.55\ \text{N}) - (0.028\ \text{kg})(9.81\ \text{m/s}^2) = \boxed{0.27\ \text{N}}$$

Insight: The largest force, 0.55 N, amounts to only 2.0 oz. The 28 g pen weighs about 1.0 oz.

99. **Picture the Problem**: The person stands on the 60.0-N ladder in the manner depicted by the figure at right.



Strategy: The problem can be solved by setting the vector sums of the forces and the torques equal to zero. The only differences between this problem and Active Example 11-3 are the addition of a vector $m_\ell \vec{\mathbf{g}}$ at the center of mass of the ladder, and the modification of the distance b. The horizontal distance between the base of the ladder and the vector $m_\ell \vec{\mathbf{g}}$ is $c = \sqrt{\left(\frac{1}{2}\times 4.0\text{ m}\right)^2 - \left(\frac{1}{2}\times 3.8\text{ m}\right)^2} = 0.62\text{ m}$.

Solution: 1. (a) Set $\sum \tau = 0$ and solve for f_3. Let $b = c$ because the person is halfway up the ladder:

$$\sum \tau = a\,f_3 - b\,mg - c\,m_\ell g = 0$$
$$f_3 = \frac{bmg + cm_\ell g}{a} = \frac{c\left(mg + m_\ell g\right)}{a}$$

2. Determine the numerical value of f_3:

$$f_3 = \frac{(0.62\text{ m})\left[(85\,\text{kg})(9.81\text{ m/s}^2) + 60.0\text{ N}\right]}{3.8\,\text{m}}$$
$$= 146\text{ N} = \boxed{0.15\text{ kN}}$$

3. Set $\sum F_x = 0$ and solve for f_2:

$$\sum F_x = f_2 - f_3 = 0$$
$$f_2 = f_3 = \boxed{0.15\text{ kN}}$$

4. Set $\sum F_y = 0$ and solve for f_1:

$$\sum F_y = f_1 - mg - m_\ell g = 0$$
$$f_1 = mg + m_\ell g = (85\text{ kg})(9.81\text{ m/s}^2) + 60.0\text{ N}$$
$$= 894\text{ N } = \boxed{0.89\text{ kN}}$$

5. (b) Set $\sum \tau = 0$ and solve for f_3. Let $b = \sqrt{\left(\frac{3}{4}\times 4.0\text{ m}\right)^2 - \left(\frac{3}{4}\times 3.8\text{ m}\right)^2} = 0.94\text{ m}$:

$$f_3 = \frac{bmg + cm_\ell g}{a}$$
$$= \frac{(0.94\text{ m})(85\text{ kg})(9.81\text{ m/s}^2) + (0.62\text{ m})(60.0\text{ N})}{3.8\text{ m}}$$
$$f_3 = 216\text{ N} = \boxed{0.22\text{ kN}}$$

6. Let $f_2 = f_3$ as in step 3:

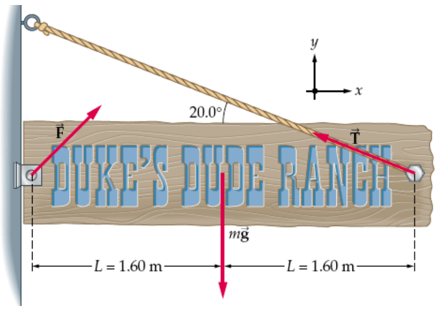
$$f_2 = f_3 = \boxed{0.22\text{ kN}}$$

7. The force f_1 is unchanged:

$$f_1 = 894\text{ N } = \boxed{0.89\text{ kN}}$$

Insight: As the person climbs higher on the ladder both f_3 and f_2 increase. The ladder leans with more force f_3 against the wall and relies more heavily on the static friction force f_2 to keep the base of the ladder from sliding out.

100. **Picture the Problem**: The sign is supported by a rope as indicated in the figure at right.



Strategy: Set the net torque about the bolt equal to zero and solve for the tension in the rope. The torque due to the rope is positive and the torque due to the weight is negative. Then write Newton's Second Law in the vertical and horizontal directions to find the vertical and horizontal components of the force $\vec{\mathbf{F}}$ exerted by the bolt on the sign.

Solution: 1. (a) Set $\sum \vec{\boldsymbol{\tau}} = 0$ and solve for T:

$$\sum \vec{\boldsymbol{\tau}} = (2L)(T\sin\theta) - (L)(mg) = 0$$

$$T = \frac{mg}{2\sin\theta} = \frac{(16.0\text{ kg})(9.81\text{ m/s}^2)}{2\sin 20°} = \boxed{229\text{ N}}$$

2. (b) Let horizontal forces sum to zero and solve for F_x:

$$F_x - T\cos\theta = 0$$

$$F_x = T\cos\theta = \left(\frac{mg}{2\sin\theta}\right)\cos\theta = \frac{mg}{2\tan\theta}$$

$$= \frac{(16.0\text{ kg})(9.81\text{ m/s}^2)}{2\tan 20.0°} = \boxed{216\text{ N}}$$

3. (c) Let vertical forces sum to zero and solve for F_y:

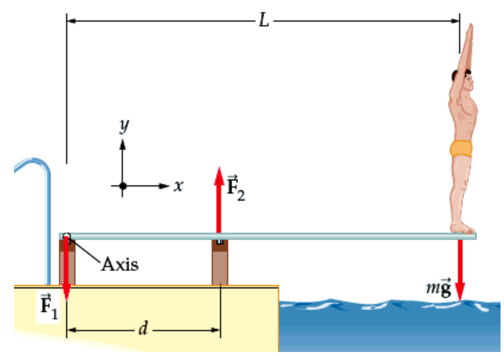
$$F_y - mg + T\sin\theta = 0$$

$$F_y = mg - T\sin\theta = mg - \left(\frac{mg}{2\sin\theta}\right)\sin\theta = \tfrac{1}{2}mg$$

$$= \tfrac{1}{2}(16.0\text{ kg})(9.81\text{ m/s}^2) = \boxed{78.5\text{ N}}$$

Insight: Note that the 229-N tension in the rope is almost 1.5 times larger than the 157-N weight of the sign because the rope is also pulling horizontally, and only the vertical portion is supporting the weight of the sign. It would take an infinite force to support the sign with a rope that is horizontal ($\theta = 0.0°$)!

101. **Picture the Problem**: The diver of mass m stands at the end of the diving board of negligible mass as shown at right. The pillars are
$d = 1.10$ m apart, the mass of the diver is 67.0 kg, and the magnitude of $F_1 = 828$ N.



Strategy: Write Newton's Second Law for rotation with the pivot point at the second pillar and solve for L. Then write Newton's Second Law in the vertical direction and solve for F_2.

Solution: 1. (a) Set $\sum \vec{\boldsymbol{\tau}} = 0$ about pillar 2 and solve for L:

$$\sum \vec{\boldsymbol{\tau}} = -d\,F_1 + (L-d)mg = 0$$

$$L = \frac{d(mg + F_1)}{mg} = \frac{(1.10\text{ m})\left[(67.0\text{ kg})(9.81\text{ m/s}^2) + 828\text{ N}\right]}{(67.0\text{ kg})(9.81\text{ m/s}^2)} = \boxed{2.49\text{ m}}$$

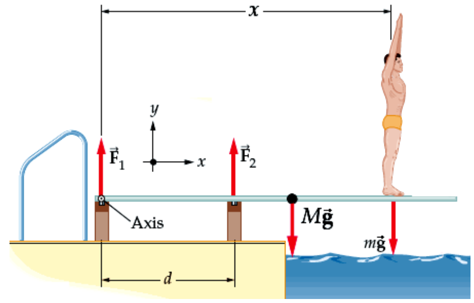
2. (b) Set $\sum F_y = 0$ and solve for F_2:

$$\sum F_y = -F_1 + F_2 - mg = 0$$

$$F_2 = mg + F_1 = (67.0\text{ kg})(9.81\text{ m/s}^2) + 828\text{ N} = 1490\text{ N} = \boxed{1.49\text{ kN}}$$

Insight: Pillar 1 must exert a downward force in order to balance the torque produced by the diver's weight. Pillar 2 must therefore exert a large force upward to balance the two downward forces $\vec{\mathbf{F}}_1$ and $m\vec{\mathbf{g}}$.

102. **Picture the Problem**: The diver of mass $m = 90.0$ kg stands at a distance x from the left end of the diving board of mass $M = 85$ kg and length $L = 5.00$ m as shown at right. The pillars are $d = 1.50$ m apart.



Strategy: Write Newton's Second Law in the vertical direction and Newton's Second Law for rotation with the pivot point at the left end of the board. The two equations can then be combined to find the two unknowns F_1 and F_2 as functions of x.

Solution: 1. Set $\sum F_y = 0$ and solve for F_1:

$$\sum F_y = F_1 + F_2 - m_{\text{diver}}g - W_{\text{board}} = 0$$
$$F_1 = m_{\text{diver}}g + W_{\text{board}} - F_2$$

2. Set $\sum \vec{\tau} = 0$ and solve for F_2:

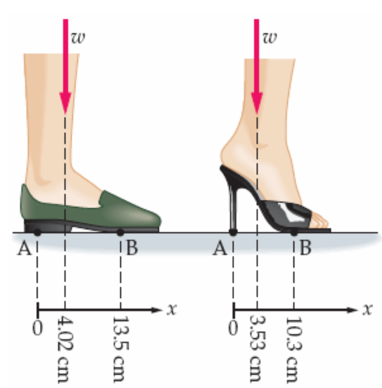
$$\sum \vec{\tau} = 0F_1 + (d)F_2 - (x)m_{\text{diver}}g - \left(\tfrac{1}{2}L\right)m_{\text{board}}g = 0$$
$$F_2 = \frac{1}{d}\left(m_{\text{diver}}gx + \tfrac{1}{2}m_{\text{board}}gL\right) = \frac{g}{d}\left(m_{\text{diver}}x + \tfrac{1}{2}m_{\text{board}}L\right)$$
$$= \frac{9.81\text{ m/s}^2}{1.50\text{ m}}\left[(90.0\text{ kg})x + \tfrac{1}{2}(85\text{ kg})(5.00\text{ m})\right]$$
$$F_2 = (589\text{ N/m})x + 1390\text{ N}$$
$$\vec{F}_2 = \boxed{\left[(0.589\text{ kN/m})x + 1.4\text{ kN}\right]\hat{\mathbf{y}}}$$

3. Use the value of F_2 in the equation from step 1 to find F_1:

$$F_1 = (90.0\text{ kg})(9.81\text{ m/s}^2) + (85\text{ kg})(9.81\text{ m/s}^2) - \left[(589\text{ N/m})x + 1390\text{ N}\right]$$
$$= -(589\text{ N/m})x + 330\text{ N}$$
$$\vec{F}_1 = \boxed{\left[-(0.589\text{ kN/m})x + 0.33\text{ kN}\right]\hat{\mathbf{y}}}$$

Insight: As the diver moves toward the end of the board, x increases, F_1 becomes larger in the negative (downward) direction, and F_2 becomes larger in the upward direction, with maximum values of $F_1 = -2.6$ kN and $F_2 = 4.3$ kN .

103. **Picture the Problem**: The weight of the person is distributed between the heel and the toe in different ways because of the shape of the shoe as shown in the figure at right.



Strategy: Write Newton's Second Law for torque about point A and solve for F_B. Then write Newton's Second Law in the vertical direction to find the force F_A. Note that the forces F_A and F_B are upward forces on the foot exerted by the floor.

Solution: 1. (a) Set $\sum \vec{\tau} = 0$ and solve for F_B:

$$r_w w - r_B F_B = 0$$
$$F_B = \frac{r_w}{r_B}w = \frac{4.02\text{ cm}}{13.5\text{ cm}}(279\text{ N})$$
$$F_B = \boxed{83.1\text{ N}}$$

2. Now set $\sum F_y = 0$ and solve for F_A:

$$F_A + F_B - w = 0$$
$$F_A = w - F_B = 279\text{ N} - 83.1\text{ N} = \boxed{196\text{ N}}$$

3. (b) Repeat step 1 for the high heel:

$$r_w w - r_B F_B = 0$$
$$F_B = \frac{r_w}{r_B}w = \frac{3.53\text{ cm}}{10.3\text{ cm}}(279\text{ N}) = \boxed{95.6\text{ N}}$$

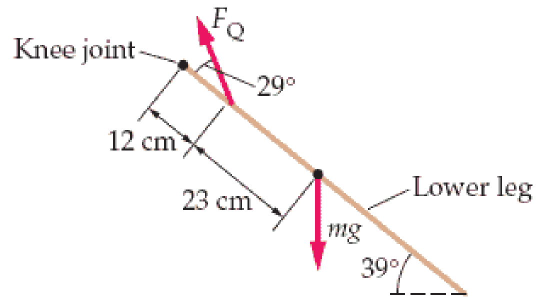
4. Repeat step 2 for the high heel:

$$F_A + F_B - w = 0$$
$$F_A = w - F_B = 279\text{ N} - 95.6\text{ N} = \boxed{183\text{ N}}$$

5. (c) The high heel has shifted more of the woman's weight to her toes.

Insight: Note that even a flat shoe exerts more force on the heel than the toes because $\vec{\mathbf{w}}$ is located closer to the heel .

104. **Picture the Problem**: The quadriceps muscle exerts a force just below the knee that supports the lower leg in the manner indicated in the figure.



Strategy: Write Newton's Second Law for torque about the knee joint and solve for F_Q. Note that the moment arm for the quadriceps force is: $r_{\perp,Q} = (12\text{ cm})\sin 29° = 5.8\text{ cm}$ and for the weight of the leg it is: $r_{\perp,W} = (35\text{ cm})\cos 39° = 27\text{ cm}$.

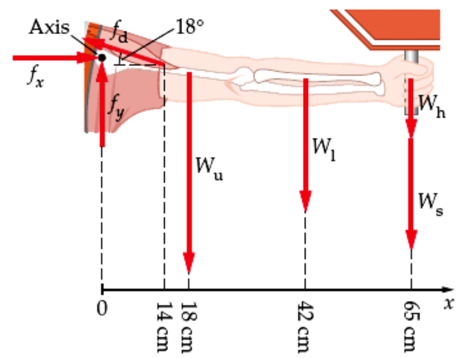
Solution: Set $\sum \vec{\tau} = 0$ and solve for F_Q:

$$r_{\perp,Q} F_Q - r_{\perp,W} mg = 0$$

$$F_Q = \frac{r_{\perp,W}}{r_{\perp,Q}} mg = \frac{27\text{ cm}}{5.8\text{ cm}}(3.4\text{ kg})(9.81\text{ m/s}^2) = 155\text{ N} = \boxed{0.16\text{ kN}}$$

Insight: Note that in order to produce the same torque as the leg's weight, but with a much smaller moment arm, the muscle must exert a force that is 4.7 times greater than the weight of the leg.

105. **Picture the Problem**: The deltoid muscle exerts a force just below the shoulder that supports the weight of the upper and lower arms, hand, and stop sign in the manner indicated by the diagram at right.



Strategy: Write Newton's Second Law for torque about the shoulder joint and solve for f_d. Note that the moment arm for the deltoid force is: $r_{\perp,d} = (14\text{ cm})\sin 18° = 4.3\text{ cm},$ and the moment arms for the weights are just those x components that are labeled in the diagram. Then write Newton's Second Law in the horizontal and vertical directions to find the forces f_x and f_y.

Solution: 1. (a) The magnitude of f_d is $\boxed{\text{greater than}}$ the magnitude of f_x because although f_x must equal the magnitude of the horizontal component of f_d (because they are the only two horizontal forces and the arm is in equilibrium), f_d also has a vertical component.

2. (b) Set $\sum \vec{\tau} = 0$ and solve for f_d:

$$r_{\perp,d} f_d - r_u W_u - r_l W_l - r_h (W_h + W_s) = 0$$

$$f_d = \frac{r_u W_u + r_l W_l + r_h (W_h + W_s)}{r_{\perp,d}}$$

$$= \frac{(18\text{ cm})(18\text{ N}) + (42\text{ cm})(11\text{ N}) + (65\text{ cm})(4.0 + 8.9\text{ cm})}{4.3\text{ cm}}$$

$$f_d = 380\text{ N} = \boxed{0.38\text{ kN}}$$

3. (c) Set $\sum F_x = 0$ and solve for f_x:

$$f_x - f_d \cos 18° = 0$$

$$f_x = f_d \cos 18° = (0.38\text{ kN})\cos 18° = \boxed{0.36\text{ kN}}$$

4. (d) Set $\sum F_y = 0$ and solve for f_y:

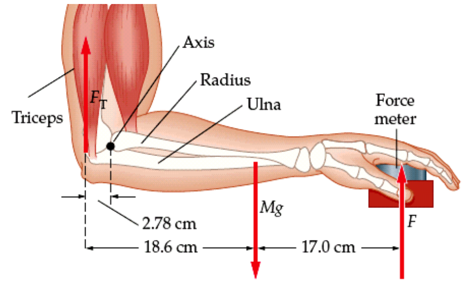
$$f_y + f_d \sin 18° - W_u - W_l - W_h - W_s = 0$$

$$f_y = W_u + W_l + W_h + W - f_d \sin 18°$$

$$= 18 + 11 + 4.0 + 8.9\text{ N} - (380\text{ N})\sin 18° = -80\text{ N} = \boxed{-0.08\text{ kN}}$$

Insight: The negative value of f_y indicates it actually acts in the downward direction on the shoulder joint, not upward as indicated in the figure. The rules of subtraction leave us with just one significant figure for the answer to part (d).

106. **Picture the Problem**: The triceps muscle exerts an upward force on the ulna at a point just behind the elbow joint as indicated in the figure at right.



Strategy: Write Newton's Second Law for torque about the elbow joint and solve for F_T.

Solution: 1. Set $\sum \vec{\tau} = 0$ and solve for F_T:

$$-r_T F_T - r_{cm} Mg + r_F F = 0$$

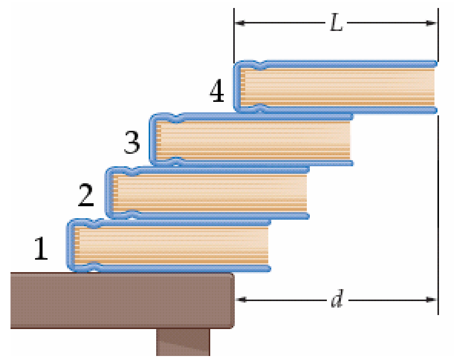
$$F_T = \frac{r_F F - r_{cm} Mg}{r_T}$$

2. Insert the numerical values:

$$F_T = \frac{(18.6 - 2.78 + 17.0 \text{ cm})(89.0 \text{ N}) - (18.6 - 2.78 \text{ cm})(15.6 \text{ N})}{2.78 \text{ cm}} = \boxed{962 \text{ N}}$$

Insight: The 962-N (216-lb!) force exerted by the triceps muscle is much greater than the 89.0-N (20.0-lb) force exerted by the hand because the moment arm of the triceps force is much smaller than that of the hand.

107. **Picture the Problem**: The books are arranged in a stack as depicted at right, with book 1 on the bottom and book 4 at the top of the stack.



Strategy: It is helpful to approach this problem from the top down. The center of mass of each set of books must be above or to the left of the point of support, otherwise there will be a net torque on the system and it will tip. Find the positions of the centers of mass for successive stacks of books to determine *d*. Measure the positions of the books from the right edge of book 1 (right hand dashed line in the figure).

Solution: 1. (a) The center of mass of book 4 needs to be above the right end of book 3.

$$d_3 = \frac{L}{2}$$

2. The result of step 1 means that the center of mass of book 3 is located at $L/2 + L/2 = L$ from the right edge of book 1.

3. The center of mass of books 4 and 3 needs to be above the right end of book 2:

$$d_2 = X_{cm,43} = \frac{m(L/2) + m(L)}{2m} = \frac{3}{4}L$$

4. The result of step 3 means that the center of mass of book 2 is located at $3L/4 + L/2 = 5L/4$.

5. The center of mass of books 4, 3, and 2 needs to be above the right end of book 1:

$$d_1 = X_{cm,432} = \frac{m(L/2) + m(L) + m(5L/4)}{3m} = \frac{11}{12}L$$

6. The result of step 3 means that the center of mass of book 1 is located at $11L/12 + L/2 = 17L/12$.

7. The center of mass of all four books needs to be above the right edge of the table:

$$d = X_{cm,4321} = \frac{m(L/2) + m(L) + m(5L/4) + m(17L/12)}{4m} = \boxed{\frac{25}{24}L}$$

8. (b) If the mass of each book is increased by the same amount, the answer to part (a) will $\boxed{\text{stay the same}}$ because it only depends upon the assumption that each book has the same mass, irregardless of the value of that mass.

Insight: If you examine the overhang of each book you find an interesting series: $d = \frac{L}{2} + \frac{L}{4} + \frac{L}{6} + \frac{L}{8} = \frac{25}{24}L$. The series gives you a hint about how to predict the overhang of even larger stacks of books!

108. **Picture the Problem**: The Earth spins on its axis with a nearly constant angular speed.

Strategy: Because the melting of the polar ice caps redistributes the Earth's mass a little bit but does not exert an external torque on the planet, the angular momentum of the Earth would remain constant. Combine equations 10-5, 11-11, and 11-15 to find the new rotation period for the Earth.

Solution: 1. (a) With conservation of angular momentum, an increase in the moment of inertia leads to a decrease in the speed of rotation. The length of a day would therefore $\boxed{\text{increase}}$.

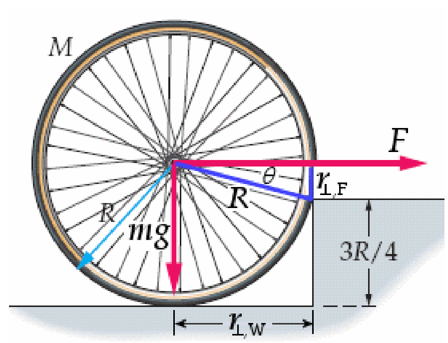
2. (b) Set $L_i = L_f$ and substitute $L = I\omega$: $\quad I_i\omega_i = I_f\omega_f$

3. Now let $\omega = 2\pi/T$ and solve for T_f: $\quad I_i\left(\frac{2\pi}{T_i}\right) = I_f\left(\frac{2\pi}{T_f}\right) \Rightarrow T_f = T_i\left(\frac{I_f}{I_i}\right) = T_i\left(\frac{I_f}{I_i}\right)$

4. Find $\Delta T = T_f - T_i$: $\quad \Delta T = T_f - T_i = \left(\frac{I_f}{I_i} - 1\right)T_i = \left(\frac{0.332 M_E R_E^2}{0.331 M_E R_E^2} - 1\right)(86,400\text{ s}) = \boxed{261\text{ s}}$

Insight: The longer day would be noticeable over time, as 261 s is equivalent to 4.35 min. The longer day would cause grief for time-sensitive astronomical observations and would mean that geosynchronous satellites would be in the wrong orbits and would drift slowly across the sky (see Active Example 12-1).

109. **Picture the Problem**: The force F is applied to the axis of the wheel in order to lift it over the step as shown in the figure at right.



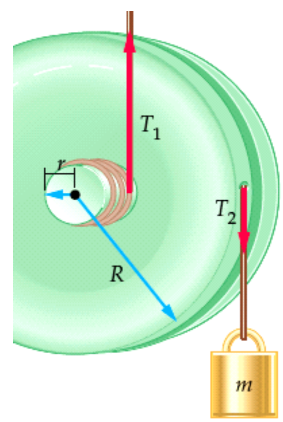
Strategy: In order to find the minimum force F that will lift the wheel over the step, we must balance the torques. The torque about the corner of the step that is produced by F must balance the torque produced by the downward force of gravity acting at the axle. The moment arm for the force F is $r_{\perp,F} = \frac{1}{4}R$ and the moment arm for the weight is $r_{\perp,W} = R\cos\theta$, where $\cos\theta = \frac{\sqrt{R^2 - \left(\frac{1}{4}R\right)^2}}{R} = \sqrt{\frac{15}{16}}$.

Solution: Set $\sum\vec{\tau} = 0$ and solve for F_{min}: $\quad \sum\tau = r_{\perp,W}Mg - r_{\perp,F}F_{min} = 0$

$$F_{min} = \frac{r_{\perp,W}Mg}{r_{\perp,F}} = \frac{\left(R\sqrt{15/16}\right)Mg}{R/4} = \boxed{\sqrt{15}\,Mg}$$

Insight: Less force is required if the step is smaller. For instance, a step height of $R/2$ would only require a force of $F_{min} = \sqrt{12}\,Mg$.

110. **Picture the Problem**: The yo-yo hangs in equilibrium under the influence of the two forces $\vec{\mathbf{T}}_1$ and $\vec{\mathbf{T}}_2$ as indicated in the diagram at right.



Strategy: Write Newton's Second Law for torque about the axis of the yo-yo, and then Newton's Second Law in the vertical direction for the yo-yo and for the hanging mass to obtain expressions for T_1, T_2, and m. The problem states that $R = 5.60r$.

Solution: 1. Set $\sum\vec{\tau} = 0$ and solve for T_1: $\quad r_{\perp 1}T_1 - r_{\perp 2}T_2 = 0$

$$T_1 = T_2\frac{r_{\perp 2}}{r_{\perp 1}} = T_2\frac{R}{r}$$

$$= T_2\frac{5.60r}{r}$$

$$\underline{\underline{T_1/5.60}} = T_2$$

2. Set $\sum F_y = 0$ for the yo-yo and solve for T_1:

$$T_1 - T_2 - Mg = 0$$
$$T_1 - (T_1/5.60) = Mg$$
$$T_1 = \frac{Mg}{1-1/5.60} = \frac{(0.101\text{ kg})(9.81\text{ m/s}^2)}{1-1/5.60} = \boxed{1.21\text{ N}}$$

3. Use the expression from step 1 to find T_2:

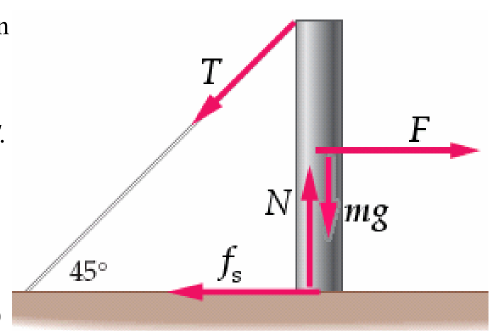
$$T_2 = \frac{T_1}{5.60} = \frac{Mg}{(1-1/5.60)5.60} = \frac{Mg}{4.60} = \frac{(0.101\text{ kg})(9.81\text{ m/s}^2)}{4.60} = \boxed{0.215\text{ N}}$$

4. Set $\sum F_y = 0$ for the hanging mass and solve for m:

$$T_2 - mg = 0$$
$$m = \frac{T_2}{g} = \frac{Mg}{4.60g} = \frac{M}{4.60} = \frac{0.101\text{ kg}}{4.60} = 0.0220\text{ kg} = \boxed{22.0\text{ g}}$$

Insight: If the hanging mass were not there, the weight of the yo-yo would create a torque with moment arm r relative to the point where $\vec{\mathbf{T}}_1$ contacts the axis, and the yo-yo would rotate counterclockwise and descend the string.

111. **Picture the Problem**: The various forces are applied to the rod, which is in equilibrium, as shown in the figure at right.



Strategy: Let L = the rod length and write Newton's Second Law for torque about the bottom of the rod in order to determine the wire tension T. Then write Newton's Second Law in the horizontal and vertical directions to determine the normal force N and the static friction force f_s. Then determine the maximum force F that can be applied to the rod without causing it to slip.

Solution: 1. (a) Set $\sum \tau = 0$ and solve for T:

$$\sum \tau = L(T\cos 45°) - \left(\tfrac{1}{2}L\right)F = 0$$
$$T = \frac{F}{2\cos 45°} = \underline{\underline{F/\sqrt{2}}}$$

2. Set $\sum F_y = 0$, substitute the expression for T from step 1, and solve for N:

$$\sum F_y = N - Mg - T\sin 45° = 0$$
$$N = Mg + T\sin 45° = Mg + \left(\frac{F}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right) = \underline{\underline{Mg + \tfrac{1}{2}F}}$$

3. Set $\sum F_x = 0$ and substitute for $f_s = \mu_s N = \mu_s\left(Mg + \tfrac{1}{2}F\right)$:

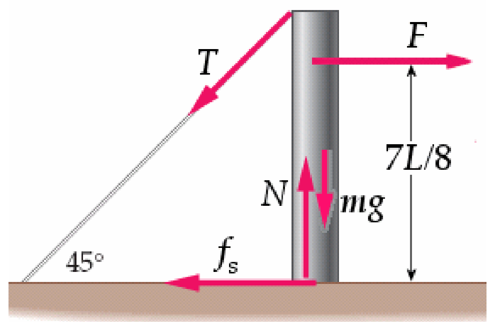
$$\sum F_x = F - f_s - T\cos 45° = 0$$
$$F = \mu_s N + T\cos 45°$$
$$= \mu_s\left(Mg + \tfrac{1}{2}F\right) + \left(\frac{F}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right) = \mu_s Mg + \tfrac{1}{2}F(\mu_s + 1)$$

4. Now solve for F:

$$F - \tfrac{1}{2}F(\mu_s + 1) = \mu_s Mg$$
$$F = \frac{\mu_s Mg}{1 - \frac{1}{2}(\mu_s + 1)} = \frac{\mu_s Mg}{\frac{1}{2}(1 - \mu_s)} = \boxed{\frac{2\mu_s Mg}{1 - \mu_s}}$$

Insight: The maximum force increases with μ_s until it becomes infinite when $\mu_s = 1$. If the coefficient of static friction is one or larger, it is impossible to pull the bottom of the rod out while applying the force at the midpoint; you would have to pull on a point below the midpoint.

112. **Picture the Problem**: The various forces are applied to the rod, which is in equilibrium, as shown in the figure at right.



Strategy: Let L = the rod length and write Newton's Second Law for torque about the bottom of the rod in order to determine the wire tension T. Then write Newton's Second Law in the horizontal and vertical directions to determine the normal force N and the static friction force f_s. Then show the maximum force F can be infinitely large and the rod will still not slip.

Solution: 1. (a) Use the expression from problem 96 to find the maximum F:

$$F = \frac{2\mu_s Mg}{1-\mu_s} = \frac{2\left(\frac{1}{7}\right)(2.3\text{ kg})(9.81\text{ m/s}^2)}{1-\frac{1}{7}} = \boxed{7.5\text{ N}}$$

2. (b) Set $\sum \tau = 0$ and solve for T:

$$\sum \tau = L(T\cos 45°) - \left(\tfrac{7}{8}L\right)F = 0$$

$$T = \frac{\frac{7}{8}F}{\cos 45°} = \frac{7\sqrt{2}}{8}F$$

3. Set $\sum F_y = 0$, substitute the expression for T from step 1, and solve for N:

$$\sum F_y = N - Mg - T\sin 45° = 0$$

$$N = Mg + T\sin 45° = Mg + \left(\frac{7\sqrt{2}F}{8}\right)\left(\frac{1}{\sqrt{2}}\right) = Mg + \tfrac{7}{8}F$$

4. Set $\sum F_x = 0$ and substitute for $f_s = \mu_s N = \mu_s\left(Mg + \frac{1}{2}F\right)$:

$$\sum F_x = F - f_s - T\cos 45° = 0$$

$$F = \mu_s N + T\cos 45°$$

$$= \mu_s\left(Mg + \tfrac{7}{8}F\right) + \left(\frac{7\sqrt{2}F}{8}\right)\left(\frac{1}{\sqrt{2}}\right) = \mu_s Mg + \tfrac{7}{8}F(\mu_s + 1)$$

5. Now solve for F:

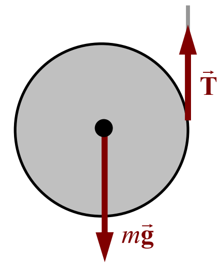
$$F - \tfrac{7}{8}F(\mu_s + 1) = \mu_s Mg$$

$$F = \frac{\mu_s Mg}{1 - \frac{7}{8}(\mu_s + 1)} = \frac{\mu_s Mg}{\frac{1}{8}(1 - 7\mu_s)} = \frac{8\mu_s Mg}{1 - 7\mu_s}$$

6. Now if we insert $\mu_s = 1/7$ into the above expression, the denominator becomes 1−1 = 0 and the force F becomes infinite. Thus the bottom of the rod will not slip under these conditions, no matter how hard you pull!

Insight: On the other hand, if the surface were frictionless ($\mu_s = 0$) the rod would slip with the smallest force applied anywhere along the length of the rod.

113. **Picture the Problem**: The cylinder rotates and falls downward along the length of the string.



Strategy: Write Newton's Second Law for torque about the center of the cylinder, then Newton's Second Law in the vertical direction for the cylinder in order to find its linear acceleration. From Table 10-1 the moment of inertia for a cylinder rotated about its axis is $I = \frac{1}{2}mr^2$. Let upward be the positive direction.

Solution: 1. Set $\sum \tau = I\alpha$ and solve for T:

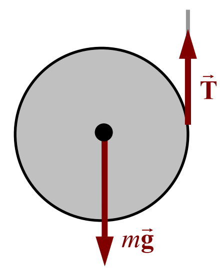
$$rT = I\alpha = \left(\tfrac{1}{2}mr^2\right)\left(a/r\right)$$
$$T = \tfrac{1}{2}ma$$

2. Let $\sum F_y = ma$ and solve for a:

$$T - mg = -ma$$
$$\left(\tfrac{1}{2}ma\right) - mg = -ma$$
$$\tfrac{1}{2}a + a = g \quad \Rightarrow \quad a = \boxed{\tfrac{2}{3}g}$$

Insight: Two ideas can help explain the slowing of the cylinder's acceleration: (1) the string exerts an upward force on the cylinder, reducing the net force that is accelerating it downward; and (2) the rotation of the cylinder stores some of the gravitational potential energy in the form of rotational as opposed to translational kinetic energy.

114. **Picture the Problem**: The sphere rotates and falls downward along the length of the string.



Strategy: Write Newton's Second Law for torque about the center of the sphere, then Newton's Second Law in the vertical direction for the sphere in order to find its linear acceleration. From Table 10-1 the moment of inertia for a sphere rotated about its axis is $I = \frac{2}{5}mr^2$. Let upward be the positive direction.

Solution: 1. Set $\sum \tau = I\alpha$ and solve for T:

$$rT = I\alpha = \left(\tfrac{2}{5}mr^2\right)\left(a/r\right)$$
$$T = \tfrac{2}{5}ma$$

2. Let $\sum F_y = ma$ and solve for a:

$$T - mg = -ma$$
$$\left(\tfrac{2}{5}ma\right) - mg = -ma$$
$$\tfrac{2}{5}a + a = g \quad \Rightarrow \quad a = \boxed{\tfrac{5}{7}g}$$

Insight: Two ideas can help explain the slowing of the sphere's acceleration: (1) the string exerts an upward force on the sphere, reducing the net force that is accelerating it downward; and (2) the rotation of the sphere stores some of the gravitational potential energy in the form of rotational as opposed to translational kinetic energy.

115. **Picture the Problem**: You pull straight downward on a rope that passes over a disk-shaped pulley and then supports a weight on the other side. The force of your pull rotates the pulley and accelerates the mass upward.

Strategy: Write Newton's Second Law for the hanging mass and Newton's Second Law for torque about the axis of the pulley. Let T_1 be the tension on the right side of the pulley and T_2 be the tension on the left side. Let *m* be the mass of the pulley, *r* be the radius of the pulley, and *M* be the hanging mass. The tension T_1 on the right side must equal the pulling force *F*. For the disk-shaped pulley the moment of inertia $I = \frac{1}{2}mr^2$ (Table 10-1).

Solution 1. (a) Set $\sum \vec{\mathbf{F}} = m\vec{\mathbf{a}}$ for the hanging mass:

$$\sum F_y = T_2 - Mg = Ma$$
$$T_2 = M(g+a)$$

2. Set $\sum \tau = I\alpha$ for the pulley:

$$\sum \tau = rT_1 - rT_2 = I\alpha = \left(\tfrac{1}{2}mr^2\right)(a/r)$$
$$a = 2(T_1 - T_2)/m$$

3. Substitute the expression for T_2 from step 1 into the one from step 2, and solve for *a*:

$$a = \frac{2(T_1 - T_2)}{m} = \frac{2[F - M(g+a)]}{m} = \frac{2(F - Mg)}{m} - \frac{2M}{m}a$$
$$a(1 + 2M/m) = 2(F - Mg)/m$$
$$a = \frac{2(F - Mg)}{m(1+2M/m)} = \frac{2(F-Mg)}{2M+m} = \boxed{\frac{F - Mg}{M + \frac{1}{2}m}}$$

4. (b) The tension on the right side of the pulley is $T_1 = \boxed{F}$ because there can only be one tension along the rope.

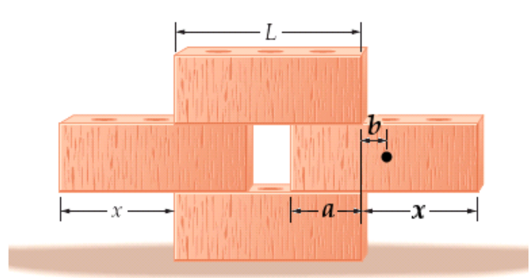
5. (c) Substitute for *a* in the expression from step 1:

$$T_2 = Mg + Ma = Mg + M\left(\frac{F - Mg}{M + \frac{1}{2}m}\right)$$
$$= \frac{Mg\left(M + \frac{1}{2}m\right)}{M + \frac{1}{2}m} + \frac{MF - M^2 g}{M + \frac{1}{2}m}$$
$$T_2 = \frac{M^2 g + \frac{1}{2}mMg + MF - M^2 g}{M + \frac{1}{2}m} = \boxed{\frac{2MF + mMg}{2M + m}}$$

6. (d) As $m \to 0$, $a \to F/M - g$ and $T_2 \to F$. These are the expected results for a massless, frictionless pulley. As $m \to \infty$, $a \to 0$ and $T_2 \to \dfrac{2FM}{2M+\infty} + \dfrac{\infty Mg}{2M+\infty} = 0 + Mg = Mg$. These are the expected results for a pulley that is too massive to rotate, so that the hanging mass is in equilibrium at rest.

Insight: The tension in the rope on the left side accelerates the hanging mass, but the tension on the right side both imparts angular acceleration to the pulley and accelerates the hanging mass. Therefore, the right hand rope has the greater tension T_1.

116. **Picture the Problem**: The bricks are stacked in the manner indicated by the figure at right.



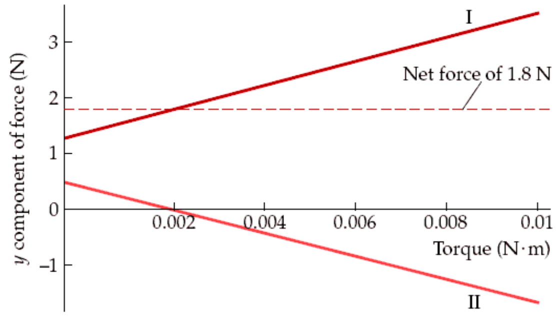
Strategy: Concentrate on the brick farthest to the right. The sum of the torques about the pivot point at the right edge of the bottom brick must be zero. There are two torques to consider, one caused by half the weight of the top brick acting on the upper-left corner, and one caused by the weight of the brick itself acting on the center of mass. An examination of the diagram reveals that $a = L - x$ and $b = x - \frac{1}{2}L$.

Solution: Set $\sum \tau = 0$ and solve for x:

$$a\left(\tfrac{1}{2}mg\right) - b(mg) = 0$$
$$(L-x)\left(\tfrac{1}{2}mg\right) = bmg = \left(x - \tfrac{1}{2}L\right)mg$$
$$L - x = 2x - L$$
$$2L = 3x \quad \Rightarrow \quad x = \boxed{\tfrac{2}{3}L}$$

Insight: The answer is independent of the mass of the bricks. It only assumes that the bricks all have the same mass and are placed symmetrically so that the weight of the top brick is evenly distributed between the two middle bricks.

117. **Picture the Problem**: A tooth is both moved and rotated by the application of two forces. The graph at right shows the values of the two forces necessary to produce a given torque, where the torque is measured about the center of the tooth.

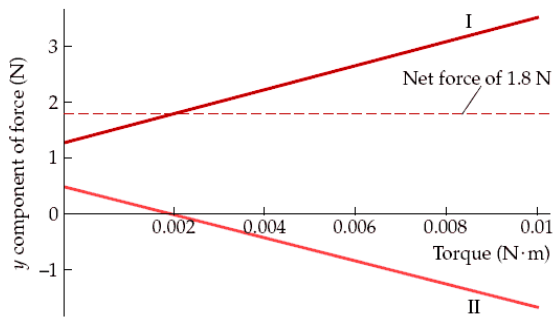


Strategy: A counterclockwise torque is desired to correct the clockwise rotation of the tooth. This means that the force F_2 must be larger than F_1.

Solution: Requiring that $F_2 > F_1$ means that $\boxed{\text{graph I corresponds to } F_2 \text{ and graph II corresponds to } F_1.}$

Insight: The forces as drawn do not have any x component, but if they did, the magnitudes of the two x components would need to cancel in order to avoid shifting the tooth in the x direction.

118. **Picture the Problem**: A tooth is both moved and rotated by the application of two forces. The graph at right shows the values of the two forces necessary to produce a given torque, where the torque is measured about the center of the tooth.

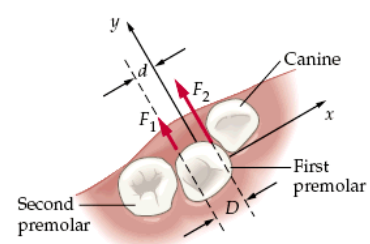


Strategy: Inspect the graph of line II to determine the value of the torque that corresponds to one of the forces being equal to zero.

Solution: Line II, corresponding to force F_1, crosses the zero force mark at a torque of $\boxed{0.0023 \text{ N·m.}}$

Insight: Although this arrangement puts less stress on the tooth, the torque is insufficient to rotate the tooth properly. We could also use equations to find the torque. Let $\sum F_y = F_1 + F_2 = 1.8 \text{ N} \Rightarrow F_2 = 1.8 \text{ N}$ if $F_1 = 0$. Then the torque on the tooth is $\sum \vec{\boldsymbol{\tau}} = 0 + (D-d)F_2 = (4.5 - 3.2 \text{ mm})(1.8 \text{ N}) = 0.0023 \text{ N}\cdot\text{m}$.

119. **Picture the Problem**: A tooth is both moved and rotated by the application of the two forces indicated in the figure at right.



Strategy: Set the torque about the center of the tooth equal to zero and the sum of the forces equal to 1.8 N in order to determine the magnitudes of the forces.

Solution: 1. Set $\sum \vec{\boldsymbol{\tau}} = 0$ and substitute $F_2 = F_{\text{total}} - F_1$:

$$\sum \vec{\boldsymbol{\tau}} = -d\,F_1 + (D-d)F_2 = 0$$
$$(D-d)\left(F_{\text{total}} - F_1\right) = d\,F_1$$

2. Now rearrange and solve for F_1:

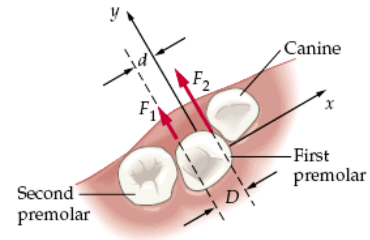
$$(D-d)F_{\text{total}} = \left[d + (D-d)\right]F_1$$
$$\frac{(D-d)}{D}F_{\text{total}} = F_1 = \frac{(4.5-3.2 \text{ mm})}{4.5 \text{ mm}}(1.8 \text{ N}) = \boxed{0.52 \text{ N}}$$

3. Solve for $F_2 = F_{\text{total}} - F_1$:

$$F_2 = 1.8 \text{ N} - 0.52 \text{ N} = \boxed{1.3 \text{ N}}$$

Insight: Although this arrangement puts the correct force on the tooth, there is no torque to rotate the tooth properly.

120. **Picture the Problem**: A tooth is both moved and rotated by the application of the two forces indicated in the figure at right.



Strategy: Set the torque about the center of the tooth equal to 0.0099 N·m and the sum of the forces equal to 1.8 N in order to determine the magnitudes of the forces.

Solution: 1. Set $\sum \vec{\boldsymbol{\tau}} = 0$ and substitute $F_2 = F_{\text{total}} - F_1$:

$$\sum \vec{\boldsymbol{\tau}} = -d\,F_1 + (D-d)F_2 = \tau_{\text{total}}$$
$$(D-d)(F_{\text{total}} - F_1) - \tau_{\text{total}} = d\,F_1$$

2. Now rearrange and solve for F_1:

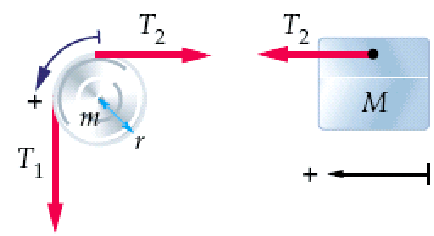
$$(D-d)F_{\text{total}} - \tau_{\text{total}} = \left[d + (D-d)\right]F_1$$
$$\frac{(D-d)F_{\text{total}} - \tau_{\text{total}}}{D} = F_1 = \frac{(0.0045 - 0.0032\text{ m})(1.8\text{ N}) - 0.0099\text{ N}\cdot\text{m}}{0.0045\text{ m}}$$
$$= \boxed{-1.7\text{ N}}$$

3. Solve for $F_2 = F_{\text{total}} - F_1$:

$$F_2 = 1.8\text{ N} - (-1.7\text{ N}) = \boxed{3.5\text{ N}}$$

Insight: Although this arrangement puts the correct force on the tooth, there is no torque to rotate the tooth properly.

121. **Picture the Problem**: The cart slides along a frictionless track because of a constant force exerted by a string that is passed over a pulley. As in Example 11-7, the cart has a mass of 0.31 kg, the pulling force is 1.1 N, and the pulley radius is 0.012 m. However, the pulley mass is doubled to 0.16 kg.



Strategy: Apply Newton's Second Law independently to the pulley and to the cart and solve for T_2. The pulley is a disk with moment of inertia $I = \frac{1}{2}mr^2$ (Table 10-1).

Solution: 1. (a) The value of T_2 will $\boxed{\text{decrease}}$ when the mass of the pulley is doubled because a larger net torque will be required to rotate the pulley, forcing T_2 to decrease if T_1 remains the same.

2. (b) Set $\sum F = ma$ for the cart and solve for a:

$$T_2 = Ma \;\Rightarrow\; a = \frac{T_2}{M}$$

3. Set $\sum \tau = I\alpha$ and solve for T_2:

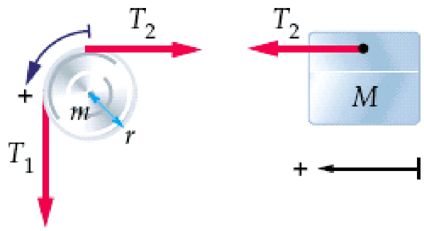
$$rT_1 - rT_2 = I\alpha = \left(\tfrac{1}{2}mr^2\right)(a/r)$$
$$T_2 = T_1 - \tfrac{1}{2}ma$$

4. Now substitute for a using the expression from step 2:

$$T_2 = T_1 - \tfrac{1}{2}m(T_2/M)$$
$$T_2\left(1 + \tfrac{1}{2}m/M\right) = T_1$$
$$T_2 = \frac{T_1}{1 + \frac{1}{2}m/M} = \frac{1.1\text{ N}}{1 + \frac{1}{2}(0.16\text{ kg})/(0.31\text{ kg})} = \boxed{0.87\text{ N}}$$

Insight: As predicted, the tension T_2 decreased from 0.97 N to 0.87 N when the mass of the pulley was doubled. If the mass of the pulley were infinitely large the tension T_2 would be zero and so would the acceleration of the system.

122. **Picture the Problem**: The cart slides along a frictionless track because of a constant force exerted by a string that is passed over a pulley. As in Example 11-7, the pulley has a mass of 0.080 kg, the pulling force is 1.1 N, and the pulley radius is 0.012 m. However, the cart mass is doubled to 0.62 kg.



Strategy: Apply Newton's Second Law independently to the pulley and to the cart and solve for T_2. The pulley is a disk with moment of inertia $I = \frac{1}{2}mr^2$ (Table 10-1).

Solution: 1. (a) The value of T_2 will $\boxed{\text{increase}}$ when the mass of the cart is doubled because a larger net force will be required to accelerate the cart, forcing T_2 to increase if T_1 remains the same.

2. (b) Set $\sum F = ma$ for the cart and solve for *a*: $T_2 = Ma \quad \Rightarrow \quad a = \dfrac{T_2}{M}$

3. Set $\sum \tau = I\alpha$ and solve for T_2:

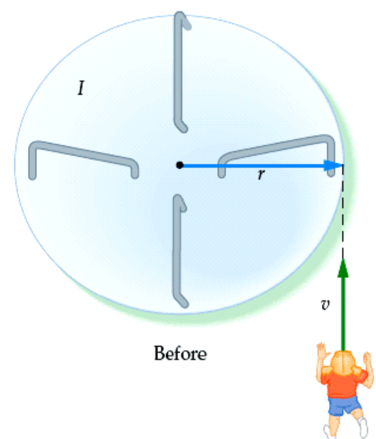
$$rT_1 - rT_2 = I\alpha = \left(\tfrac{1}{2}mr^2\right)(a/r)$$
$$T_2 = T_1 - \tfrac{1}{2}ma$$

4. Now substitute for *a* using the expression from step 2:

$$T_2 = T_1 - \tfrac{1}{2}m\left(T_2/M\right)$$
$$T_2\left(1+\tfrac{1}{2}m/M\right) = T_1$$
$$T_2 = \frac{T_1}{1+\frac{1}{2}m/M} = \frac{1.1\text{ N}}{1+\frac{1}{2}(0.080\text{ kg})/(0.62\text{ kg})} = \boxed{1.0\text{ N}}$$

Insight: As predicted, the tension T_2 increased from 0.97 N to 1.0 N when the mass of the cart was doubled. If the mass were infinitely large, the tension T_2 would be 1.1 N, and the acceleration would be zero because there would be no net torque on the pulley (and the cart is just too massive to accelerate).

123. **Picture the Problem**: The child runs tangentially to the merry-go-round and hops on. As in Active Example 11-5, the child has a mass of 34.0 kg, the merry-go-round has a moment of inertia of 512 kg·m^2 and a radius of 2.31 m, but the child's initial speed is different than 2.80 m/s.



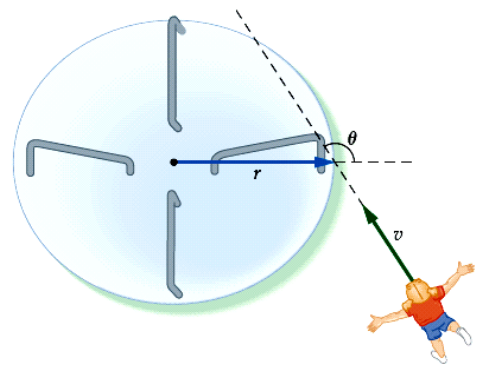
Strategy: Use equation 11-15 together with equation 11-11 to conserve angular momentum before and after the child jumps on the merry-go-round. Solve the resulting expression for the initial speed *v* of the child.

Solution: Set $L_i = L_f$ and solve for the initial speed *v*:

$$0 + rmv = I_f\omega_f = \left(I + mr^2\right)\omega$$
$$v = \frac{\left(I+mr^2\right)\omega}{rm} = \frac{\left[512+(34.0)(2.31)^2\text{ kg}\cdot\text{m}^2\right](0.425\text{ rad/s})}{(2.31\text{ m})(34.0\text{ kg})} = \boxed{3.75\text{ m/s}}$$

Insight: As we would expect, the child needs to run faster in order to get the merry-go-round spinning faster. The 34% increase in linear speed of the child results in a 34% increase in the angular speed of the merry-go-round because the initial and final angular momentum of the system depends linearly upon the speed of the child.

124. **Picture the Problem**: The child runs at an angle to the merry-go-round and hops on. As in Active Example 11-5 the child has a mass of 34.0 kg, the merry-go-round has a moment of inertia of 512 kg·m^2 and a radius of 2.31 m, and the child's initial speed is 2.80 m/s.



Strategy: Use equation 11-15 together with equation 11-11 to conserve angular momentum before and after the child jumps on the merry-go-round. The moment arm of the child's angular momentum is $r_\perp = r\sin\theta$. Solve the resulting expression for the approach angle θ of the child.

Solution: 1. Set $L_i = L_f$ and solve for $\sin\theta$:

$$0 + rmv\sin\theta = I_f\omega_f = \left(I + mr^2\right)\omega$$

$$\sin\theta = \frac{\left(I + mr^2\right)\omega}{rmv}$$

2. Solve for θ, keeping in mind that the calculator will return an angle equal to $180° - \theta$:

$$\theta = 180° - \sin^{-1}\left\{\frac{\left[512 + (34.0)(2.31)^2\ \text{kg}\cdot\text{m}^2\right](0.272\ \text{rad/s})}{(2.31\ \text{m})(34.0\ \text{kg})(2.80\ \text{m/s})}\right\}$$

$$= 180° - 59.1° = \boxed{121°}$$

Insight: If the child approaches at an angle θ that is greater than 90°, his initial angular momentum is smaller and the merry-go-round ends up spinning at a slower rate. If θ = 180°, the initial angular momentum would be zero and the merry-go-round would not rotate at all; in this case the child approaches the merry-go-round along the radial direction.