Simplification Of Switching Functions

Simplification Goals

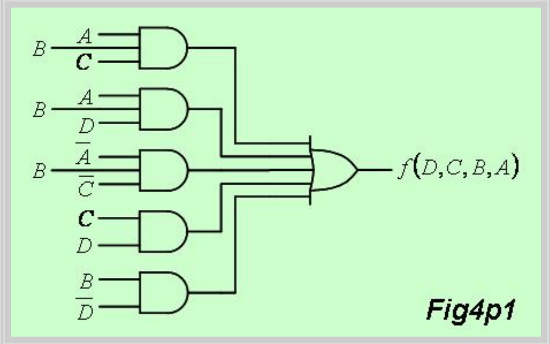
The goal of the simplification is to minimize the cost of realizing a function with physical circuit elements.

Save silicon area in VLSI circuits or reduce the number of SSI-circuits in implementation.

In general, it is desirable to minimize the number of circuit elements and to make each element as simple as possible.

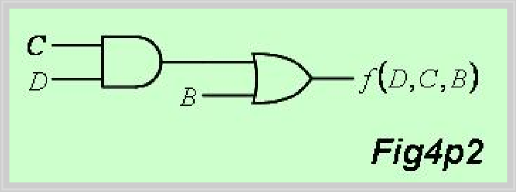
Reduce the number of product terms to reduce gate count.

Minimize the number of literals in each product terms to minimize the number of gate inputs.



Unsimplified two-level switching function has 4 variables, 13 literals and 5 product terms.

Two-level SOP function



Simplified switching function has 3 literals and 2 product terms (CD, B) and 3 variables.

SSI 1-10
MSI 10-100
LSI 100-10 000
VLSI >100 000

[2]

Simplification

Three "paper and pencil" methods

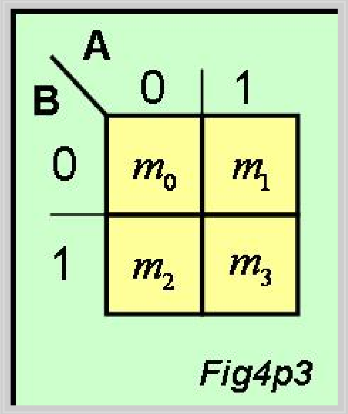
- **Karnaugh Map method up to six variables**
- **Quine-McCluskey Tabular Minimization method**
- **Switching algebra**

[2]

Simplification Of Switching Functions

Simplification

Karnaugh map for variables B and A



All possible minterms for variables B and A are : m_0 , m_1 , m_2 and m_3 .

$$m_0 = \overline{B} \cdot \overline{A}$$

$$m_1 = \overline{B} \cdot A$$

$$m_2 = B \cdot \overline{A}$$

$$m_3 = B \cdot A$$

All possible maxterms for variables B and A are : M_0 , M_1 , M_2 and M_3 .

$$M_0 = B + A$$

$$M_1 = B + \overline{A}$$

$$M_2 = \overline{B} + A$$

$$M_3 = \overline{B} + \overline{A}$$

Simplification

Karnaugh map for variables C, B and A

All possible minterms for variables C, B and A are : m_0 , m_1 , m_2 , ... and m_7 .

$$m_0 = \overline{C}\cdot\overline{B}\cdot\overline{A}$$
$$m_1 = \overline{C}\cdot\overline{B}\cdot A$$
$$m_2 = \overline{C}\cdot B\cdot\overline{A}$$
$$m_3 = \overline{C}\cdot B\cdot A$$
$$m_4 = C\cdot\overline{B}\cdot\overline{A}$$
$$m_5 = C\cdot\overline{B}\cdot A$$
$$m_6 = C\cdot B\cdot\overline{A}$$
$$m_7 = C\cdot B\cdot A$$

All possible maxterms for variables C, B and A are : M_0 , M_1 , M_2 , ... and M_7 .

$$M_0 = C + B + A$$
$$M_1 = C + B + \overline{A}$$
$$M_2 = C + \overline{B} + A$$
$$M_3 = C + \overline{B} + \overline{A}$$
$$M_4 = \overline{C} + B + A$$
$$M_5 = \overline{C} + B + \overline{A}$$
$$M_6 = \overline{C} + \overline{B} + A$$
$$M_7 = \overline{C} + \overline{B} + \overline{A}$$

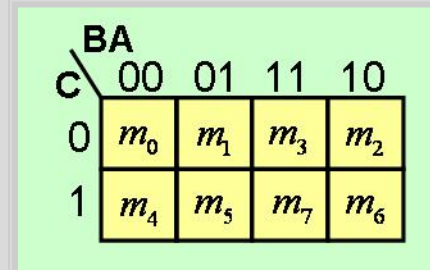


Fig4p4

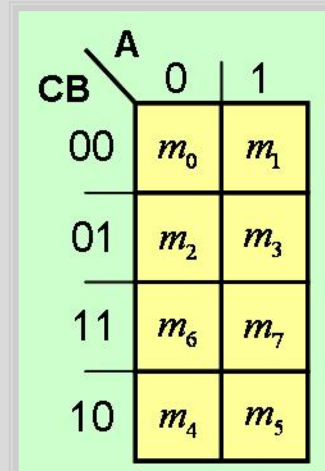


Fig4p5

Simplification

All possible minterms for variables D, C, B and A .

$$m_0 = \overline{D}\cdot\overline{C}\cdot\overline{B}\cdot\overline{A}$$
$$m_1 = \overline{D}\cdot\overline{C}\cdot\overline{B}\cdot A$$
$$m_2 = D\cdot\overline{C}\cdot B\cdot\overline{A}$$
$$m_3 = \overline{D}\cdot\overline{C}\cdot B\cdot A$$
$$m_4 = \overline{D}\cdot C\cdot\overline{B}\cdot\overline{A}$$
$$m_5 = \overline{D}\cdot C\cdot\overline{B}\cdot A$$
$$m_6 = \overline{D}\cdot C\cdot B\cdot\overline{A}$$
$$m_7 = \overline{D}\cdot C\cdot B\cdot A$$
$$m_8 = D\cdot\overline{C}\cdot\overline{B}\cdot\overline{A}$$
$$m_9 = D\cdot\overline{C}\cdot\overline{B}\cdot A$$
$$m_{10} = D\cdot\overline{C}\cdot B\cdot\overline{A}$$
$$m_{11} = D\cdot\overline{C}\cdot B\cdot A$$
$$m_{12} = D\cdot C\cdot\overline{B}\cdot\overline{A}$$
$$m_{13} = D\cdot C\cdot\overline{B}\cdot A$$
$$m_{14} = D\cdot C\cdot B\cdot\overline{A}$$
$$m_{15} = D\cdot C\cdot B\cdot A$$

All possible maxterms for variables D, C, B and A .

$$M_0 = D + C + B + A$$
$$M_1 = D + C + B + \overline{A}$$
$$M_2 = D + C + \overline{B} + A$$
$$M_3 = D + C + \overline{B} + \overline{A}$$
$$M_4 = D + \overline{C} + B + A$$
$$M_5 = D + \overline{C} + B + \overline{A}$$
$$M_6 = D + \overline{C} + \overline{B} + A$$
$$M_7 = D + \overline{C} + \overline{B} + \overline{A}$$
$$M_8 = \overline{D} + C + B + A$$
$$M_9 = \overline{D} + C + B + \overline{A}$$
$$M_{10} = \overline{D} + C + \overline{B} + A$$
$$M_{11} = \overline{D} + C + \overline{B} + \overline{A}$$
$$M_{12} = \overline{D} + \overline{C} + B + A$$
$$M_{13} = \overline{D} + \overline{C} + B + \overline{A}$$
$$M_{14} = \overline{D} + \overline{C} + \overline{B} + A$$
$$M_{15} = \overline{D} + \overline{C} + \overline{B} + \overline{A}$$

Karnaugh map for variables D, C, B and A

BA \ DC	00	01	11	10
00	m_0	m_4	m_{12}	m_8
01	m_1	m_5	m_{13}	m_9
11	m_3	m_7	m_{15}	m_{11}
10	m_2	m_6	m_{14}	m_{10}

Fig4p6a

DC \ BA	00	01	11	10
00	m_0	m_1	m_3	m_2
01	m_4	m_5	m_7	m_6
11	m_{12}	m_{13}	m_{15}	m_{14}
10	m_8	m_9	m_{11}	m_{10}

Fig4p6

[2]

Simplification Of Switching Functions

Simplification

Karnaugh map for variables E, D, C, B and A

K-map for variables E , D , C , B and A.

E=0

DC \ BA	00	01	11	10
00	m_0	m_1	m_3	m_2
01	m_4	m_5	m_7	m_6
11	m_{12}	m_{13}	m_{15}	m_{14}
10	m_8	m_9	m_{11}	m_{10}

E=1

DC \ BA	00	01	11	10
00	m_{16}	m_{17}	m_{19}	m_{18}
01	m_{20}	m_{21}	m_{23}	m_{22}
11	m_{28}	m_{29}	m_{31}	m_{30}
10	m_{24}	m_{25}	m_{27}	m_{26}

Fig4p7

Alternative K-map format for five variables

BA \ EDC	000	001	011	010
00	m_0	m_4	m_{12}	m_8
01	m_1	m_5	m_{13}	m_9
11	m_3	m_7	m_{15}	m_{11}
10	m_2	m_6	m_{14}	m_{10}

BA \ EDC	100	101	111	110
00	m_{16}	m_{20}	m_{28}	m_{24}
01	m_{17}	m_{21}	m_{29}	m_{25}
11	m_{19}	m_{23}	m_{31}	m_{27}
10	m_{18}	m_{22}	m_{30}	m_{26}

Fig4p9

[2]

Simplification Of Switching Functions

Simplification

Karnaugh map for variables E, D, C, B and A

All possible minterms for variables E, D, C, B and A are : m_0 , m_1 , m_2 , m_3 , ... and m_{31} .

$$m_0 = \overline{E}\cdot\overline{D}\cdot\overline{C}\cdot\overline{B}\cdot\overline{A} \quad m_4 = \overline{E}\cdot\overline{D}\cdot C\cdot\overline{B}\cdot\overline{A} \quad m_8 = \overline{E}\cdot D\cdot\overline{C}\cdot\overline{B}\cdot\overline{A} \quad m_{17} = E\cdot\overline{D}\cdot\overline{C}\cdot\overline{B}\cdot A$$

$$m_1 = \overline{E}\cdot\overline{D}\cdot\overline{C}\cdot\overline{B}\cdot A \quad m_5 = \overline{E}\cdot\overline{D}\cdot C\cdot\overline{B}\cdot A \quad m_9 = \overline{E}\cdot D\cdot\overline{C}\cdot\overline{B}\cdot A \quad m_{18} = E\cdot\overline{D}\cdot\overline{C}\cdot B\cdot\overline{A}$$

$$m_2 = \overline{E}\cdot D\cdot\overline{C}\cdot B\cdot\overline{A} \quad m_6 = \overline{E}\cdot\overline{D}\cdot C\cdot B\cdot\overline{A} \quad m_{10} = \overline{E}\cdot D\cdot\overline{C}\cdot B\cdot\overline{A} \quad \cdots$$

$$m_3 = \overline{E}\cdot\overline{D}\cdot\overline{C}\cdot B\cdot A \quad m_7 = \overline{E}\cdot\overline{D}\cdot C\cdot B\cdot A \quad \cdots \quad m_{31} = E\cdot D\cdot C\cdot B\cdot A$$

All possible maxterms for variables E, D, C, B and A are : M_0 , M_1 , M_2 , M_3 , ... and M_{31} .

$$M_0 = E+D+C+B+A \quad M_4 = E+D+\overline{C}+B+A \quad M_8 = E+\overline{D}+C+B+A \quad M_{17} = \overline{E}+D+C+B+\overline{A}$$

$$M_1 = E+D+C+B+\overline{A} \quad M_5 = E+D+\overline{C}+B+\overline{A} \quad M_9 = E+\overline{D}+C+B+\overline{A} \quad M_{18} = \overline{E}+D+C+\overline{B}+A$$

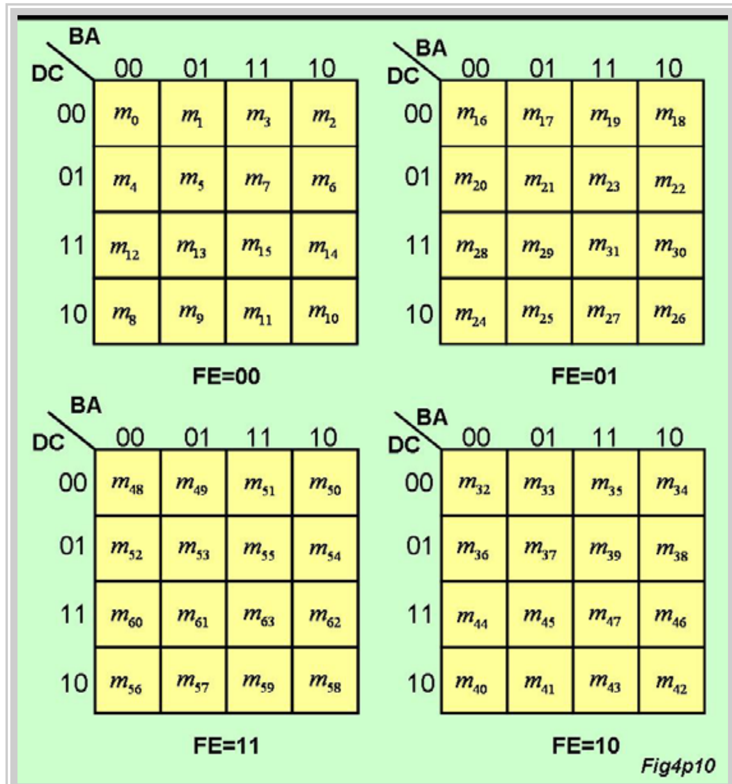
$$M_2 = E+D+C+\overline{B}+A \quad M_6 = E+D+\overline{C}+\overline{B}+A \quad M_{10} = E+\overline{D}+C+\overline{B}+A \quad \cdots$$

$$M_3 = E+D+C+\overline{B}+\overline{A} \quad M_7 = E+D+\overline{C}+\overline{B}+\overline{A} \quad \cdots \quad M_{31} = \overline{E}+\overline{D}+\overline{C}+\overline{B}+\overline{A}$$

[2]

Simplification

Karnaugh map for variables F, E, D, C, B and A



[2]

Simplification

Karnaugh map for variables F, E, D, C, B and A

All possible minterms for variables F, E, D, C, B and A are : m_0 , m_1 , m_2 , m_3 … and m_{63} .

$m_0 = \overline{F}\cdot\overline{E}\cdot\overline{D}\cdot\overline{C}\cdot\overline{B}\cdot\overline{A}$	$m_{16} = \overline{F}\cdot E\cdot\overline{D}\cdot\overline{C}\cdot\overline{B}\cdot\overline{A}$	$m_{32} = F\cdot\overline{E}\cdot\overline{D}\cdot\overline{C}\cdot\overline{B}\cdot\overline{A}$	$m_{48} = F\cdot E\cdot\overline{D}\cdot\overline{C}\cdot\overline{B}\cdot\overline{A}$
$m_1 = \overline{F}\cdot\overline{E}\cdot\overline{D}\cdot\overline{C}\cdot\overline{B}\cdot A$	$m_{17} = \overline{F}\cdot E\cdot\overline{D}\cdot\overline{C}\cdot\overline{B}\cdot A$	$m_{33} = F\cdot\overline{E}\cdot\overline{D}\cdot\overline{C}\cdot\overline{B}\cdot A$	$m_{49} = F\cdot E\cdot\overline{D}\cdot\overline{C}\cdot\overline{B}\cdot A$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
$m_{15} = \overline{F}\cdot\overline{E}\cdot D\cdot C\cdot B\cdot A$	$m_{31} = \overline{F}\cdot E\cdot D\cdot C\cdot B\cdot A$	$m_{47} = F\cdot\overline{E}\cdot D\cdot C\cdot B\cdot A$	$m_{63} = F\cdot E\cdot D\cdot C\cdot B\cdot A$

All possible maxterms for variables F, E, D, C, B and A are : M_0 , M_1 , M_2 , . . . and M_{63} .

$M_0 = F+E+D+C+B+A$	$M_{16} = F+\overline{E}+D+C+B+A$	$M_{32} = \overline{F}+E+D+C+B+A$	$M_{48} = \overline{F}+\overline{E}+D+C+B+A$
$M_1 = F+E+D+C+B+\overline{A}$	$M_{17} = F+\overline{E}+D+C+B+\overline{A}$	$M_{33} = \overline{F}+E+D+C+B+\overline{A}$	$M_{49} = \overline{F}+\overline{E}+D+C+B+\overline{A}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
$M_{15} = F+E+\overline{D}+\overline{C}+\overline{B}+\overline{A}$	$M_{31} = F+\overline{E}+\overline{D}+\overline{C}+\overline{B}+\overline{A}$	$M_{47} = \overline{F}+E+\overline{D}+\overline{C}+\overline{B}+\overline{A}$	$M_{63} = \overline{F}+\overline{E}+\overline{D}+\overline{C}+\overline{B}+\overline{A}$

[2]

Simplification

Plotting Functions in Canonical Form on the Karnaugh map

If switching function is expressed in canonical form, it may be readily plotted on a K-map. Each cell on the K-map corresponds to minterm/maxterm of the canonical form.

Suppose we have following Boolean function f_1 which is given in canonical SOP form :

$$f_1(B,A) = \overline{B}\cdot\overline{A} + B\cdot\overline{A} \quad f_1(B,A) = \sum m(0,2)$$

We have assumed that right side of variables (B, A) represent a "least significant bit" and left side (B) represent a "most significant bit".

$$\Rightarrow m_0 = \overline{B}\cdot\overline{A}$$
$$m_1 = \overline{B}\cdot A$$
$$m_2 = B\cdot\overline{A}$$
$$m_3 = B\cdot A$$

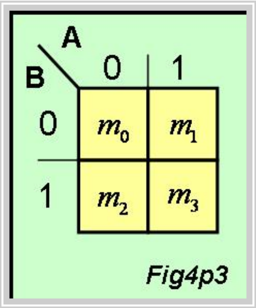


Fig4p3

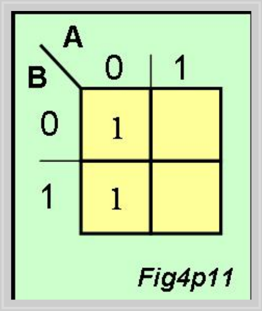


Fig4p11

If we plot minterms (1's) only on the K-map, then m_0 and m_2 are 1.

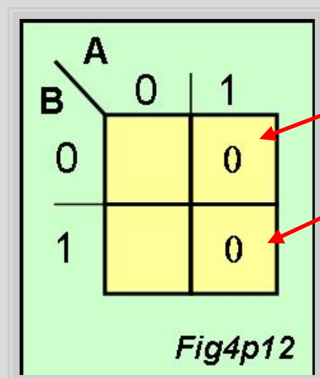
[2]

Simplification

Plotting Functions in Canonical Form on the Karnaugh map

$$f_1(B,A) = \sum m(0,2) = \prod M(1,3) = M_1 \cdot M_3 = \left(B + \overline{A}\right)\cdot\left(\overline{B} + \overline{A}\right)$$

If we plot maxterms (0's) only on the K-map, then M_1 and M_3 are 0.



$$M_1 = B + \overline{A}$$

$$M_3 = \overline{B} + \overline{A}$$

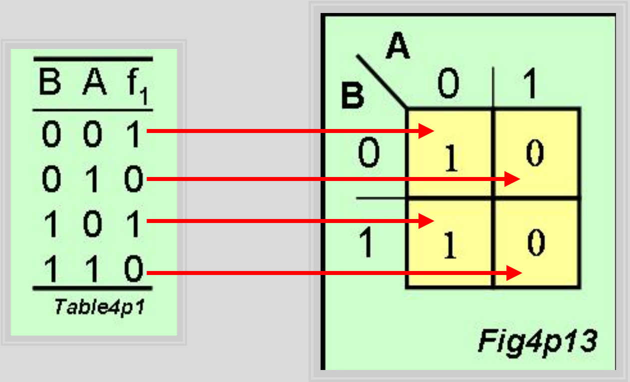
[2]

Simplification

Plotting Functions in Canonical Form on the Karnaugh map

If we plot all minterms (1) and maxterms (0) on the K-map, then the K-map corresponds directly to the truth table of the function f_1 .

The truth table for the function f_1 .



K-map for the function f_1 .

Simplification

Plotting Functions in Canonical Form on the Karnaugh map

$$f_2(C,B,A) = \sum m(0,3,5) = m_0 + m_3 + m_5$$
$$= \overline{C}\cdot\overline{B}\cdot\overline{A} + \overline{C}\cdot B\cdot A + C\cdot\overline{B}\cdot A$$
$$= \prod M(1,2,4,6,7) = M_1\cdot M_2\cdot M_4\cdot M_6\cdot M_7$$
$$= \left(C+B+\overline{A}\right)\cdot\left(C+\overline{B}+A\right)\cdot\left(\overline{C}+B+A\right)\cdot\left(\overline{C}+\overline{B}+A\right)\cdot\left(\overline{C}+\overline{B}+\overline{A}\right)$$

C \ BA	00	01	11	10
0	m_0	m_1	m_3	m_2
1	m_4	m_5	m_7	m_6

reference [2] Fig4p4

C	B	A	f_2
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	0

Table4p2

The truth table for the function f_2 .

C \ BA	00	01	11	10
0	1	0	1	0
1	0	1	0	0

Fig4p14

K-map for the function f_2 .

Simplification

Plotting Functions in Canonical Form on the Karnaugh map

SOP and **POS** form of switching function f_3 respectively.

$$f_3(D,C,B,A) = \sum m(0,3,5,7,10,11,12,13,14,15)$$

$$= \prod M(1,2,4,6,8,9)$$

DC \ BA	00	01	11	10
00	m_0	m_1	m_3	m_2
01	m_4	m_5	m_7	m_6
11	m_{12}	m_{13}	m_{15}	m_{14}
10	m_8	m_9	m_{11}	m_{10}

Fig4p6

DC \ BA	00	01	11	10
00	1	0	1	0
01	0	1	1	0
11	1	1	1	1
10	0	0	1	1

Fig4p15

Minterms $\mathbf{m_i}$ and maxterms $\mathbf{M_j}$

K-map for the function f_3 **.**

D	C	B	A	f_3
0	0	0	0	1
0	0	0	1	0
0	0	1	0	0
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	0
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

Table4p3

The truth table for the function f_3 **.**

Simplification

Plotting Functions in Canonical Form on the Karnaugh map

SOP and **POS** form of switching function f_3 respectively.

$$f_4(E,D,C,B,A) = \sum m(14,15,22,23,30,31)$$

$$= \prod M(0,1,\cdots,13,16,\cdots,21,24,\cdots,29)$$

E	D	C	B	A	f_4
0	0	0	0	0	0
0	0	0	0	1	0
0	0	0	1	0	0
..	..	..	..	..	..
0	1	1	1	0	1
0	1	1	1	1	1
1	0	0	0	0	0
..	..	..	..	..	..
1	0	1	1	0	1
1	0	1	1	1	1
1	1	0	0	0	0
..	..	..	..	..	..
1	1	1	1	0	1
1	1	1	1	1	1

Table4p4

The truth table for the function f_4.

E=0

DC \ BA	00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	0	0	1	1
10	0	0	0	0

E=1

DC \ BA	00	01	11	10
00	0	0	0	0
01	0	0	1	1
11	0	0	1	1
10	0	0	0	0

Fig4p16

K-map for the function f_4.

Simplification

Plotting Functions in Canonical Form on the Karnaugh map

$$f_5(F,E,D,C,B,A) = \sum m(9,11,25,27,41,43,57,59)$$
$$= \prod M(0,1,\cdots,8,10,12,13,\cdots,24,26,28,\cdots,40,42,44,\cdots,56,58,60,\cdots,63)$$

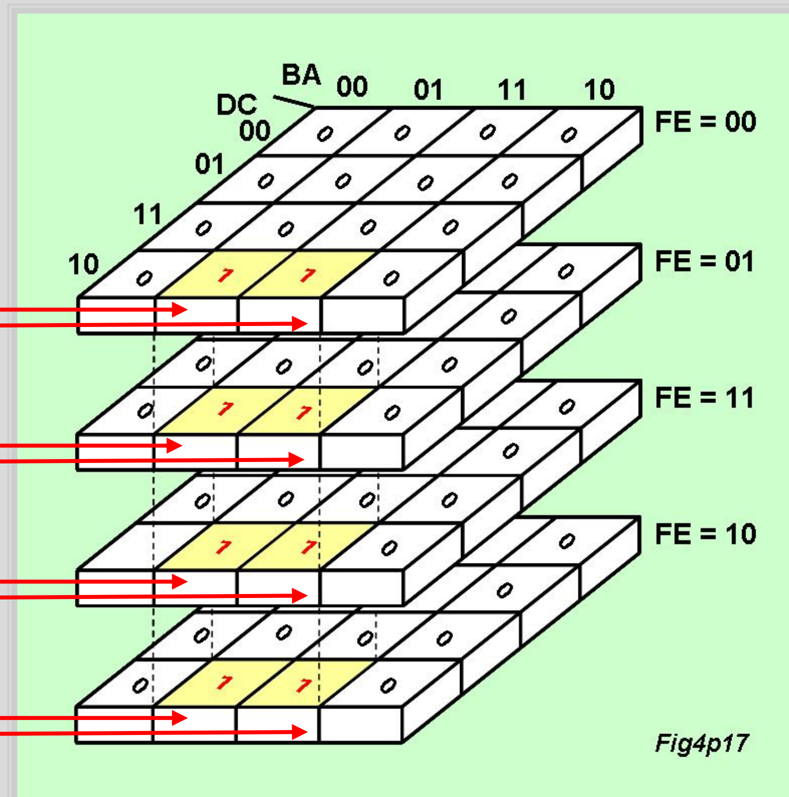
K-map for the function f_5.

m_9, m_{11}

m_{25}, m_{27}

m_{57}, m_{59}

m_{41}, m_{43}



Simplification

Plotting Functions in Standard SOP/POS Form on the K-map

In canonical form the each product or sum term must contain all the variables, either complemented or uncomplemented.

In standard form each product/sum term may have any number of literals.

Standard SOP form $f_6(C,B,A) = A + B \cdot \overline{A} + \overline{C} \cdot B$

Standard POS form $f_7(C,B,A) = (\overline{A} + B + \overline{C}) \cdot (\overline{B} + C) \cdot A$

[2]

Consider the following function $f_8(B,A)$, which is expressed as a **standard sum of products**.

Example

$$f_8 = \overline{A} + B \cdot A$$

The first term represents the portion of the map where A = 0 (minterms m_0 , m_2).

The second term represents the portion of the map where A = 1 and B = 1 (minterm m_3).

B is don't care

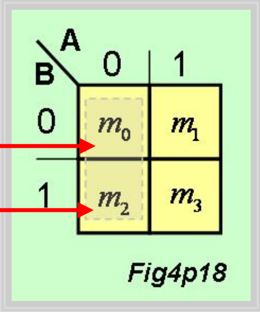


Fig4p18

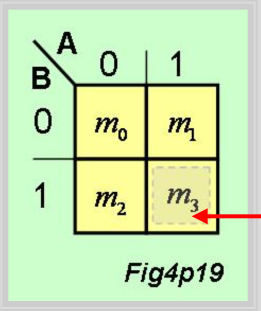


Fig4p19

Simplification

Plotting Functions in Standard SOP/POS Form on the K-map

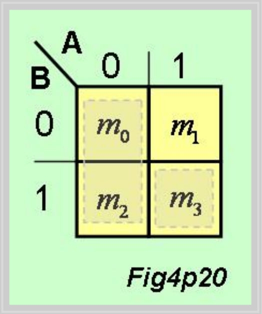


Fig4p20

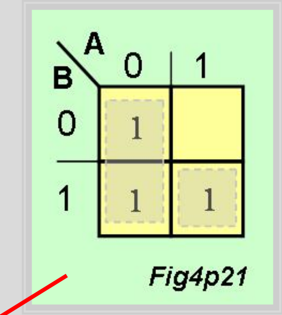


Fig4p21

Minterms m_0, m_2 and m_3 are **covered only once**.

Minterms (ones) plotted on the K-map.

The canonical sum of product form for function f_8 .

$$f_8 = \overline{B} \cdot \overline{A} + B \cdot \overline{A} + B \cdot A = \sum m(0,2,3)$$

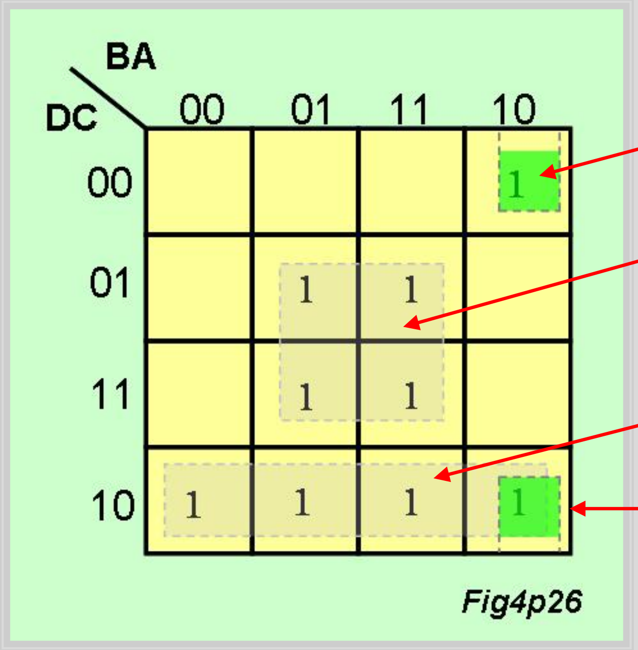
We can use K-map to make a conversion from standard SOP to canonical SOP form.

[2]

Simplification

Plotting Functions in Standard SOP/POS Form on the K-map

$$f_{10}(D,C,B,A) = D\overline{C} + CA + \overline{C}B\overline{A}$$ Standard SOP



D is don't care

D and B are don't care

A and B are don't care

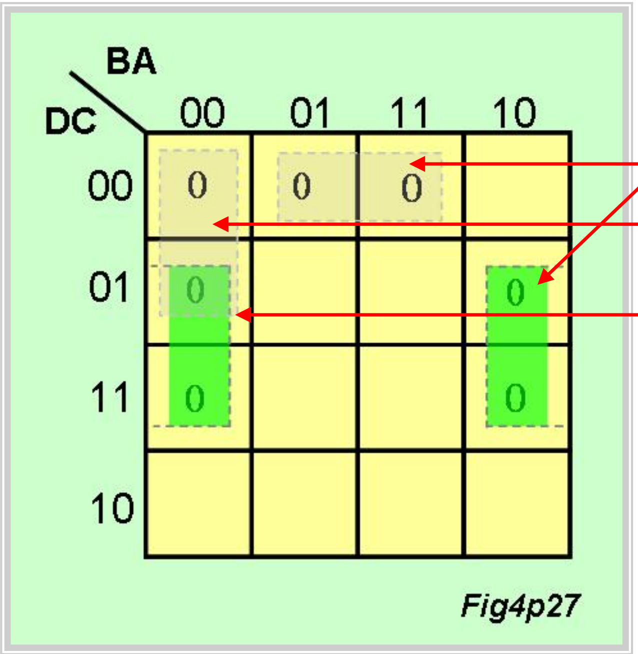
Minterm m_{10} is covered twice.

[2]

Simplification

Plotting Functions in Standard SOP/POS Form on the K-map

$$f_{10}(D,C,B,A)=\left(A+\overline{C}\right)\left(D+B+A\right)\left(D+C+\overline{A}\right)$$ **Standard POS**



Maxterm M_4 is covered twice.

The complement of the function f_{10}

Convert each 0 cell to a 1 cell and plot function in canonical or standard SOP form.

Simplification

Guidelines for Simplifying Functions Using K-maps

Five important points to keep in mind.

1. **Each minterm on a K-map of *n* variables has *n* logically adjacent minterms.**

Adjacent minterms : (m_0, m_1)

(m_1, m_3)

$(B\overline{A} \quad BA) \longleftrightarrow (m_2, m_3)$

$(\overline{B}\overline{A} \quad B\overline{A}) \longleftrightarrow (m_0, m_2)$

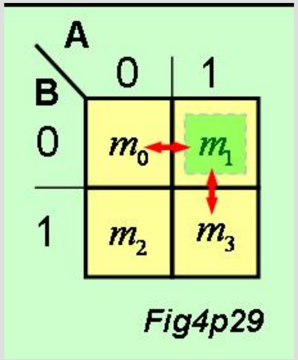


Fig4p29

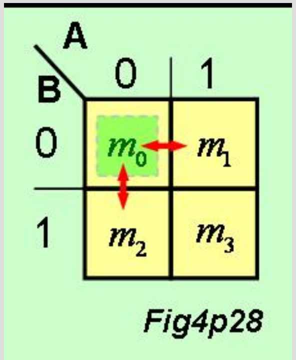


Fig4p28

Simplification

Guidelines for Simplifying Functions Using K-maps

Five important points to keep in mind.

Adjacent minterms :

Example :

$(\bar{C}BA \quad \bar{C}\bar{B}A) \longleftrightarrow (m_3, m_1)$

$(\bar{C}BA \quad \bar{C}B\bar{A}) \longleftrightarrow (m_3, m_2)$

$(\bar{C}BA \quad CBA) \longleftrightarrow (m_3, m_7)$

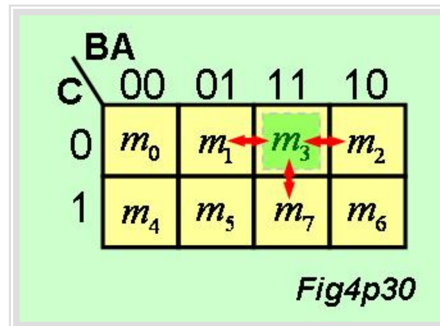


Fig4p30

Example :

(m_2, m_0)

(m_2, m_3)

(m_2, m_6)

(m_2, m_{10})

Or

(m_{13}, m_5)

(m_{13}, m_9)

(m_{13}, m_{12})

(m_{13}, m_{15})

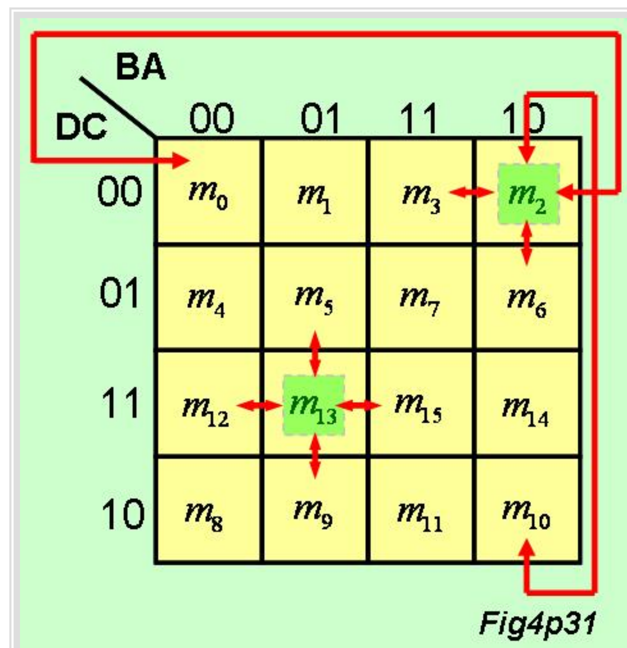


Fig4p31

[2]

Simplification

Guidelines for Simplifying Functions Using K-maps

Five important points to keep in mind.

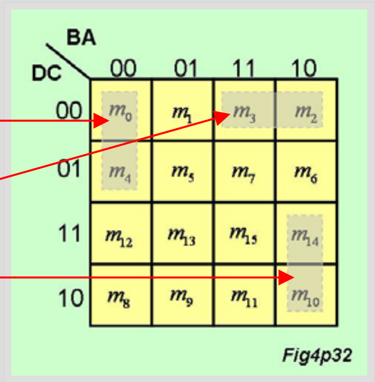
2. **Group squares in powers of 2 (2, 4, 8, 16, ..). Grouping 2^n squares, eliminates n variables.**

Eliminates one variable

C is eliminated

A is eliminated

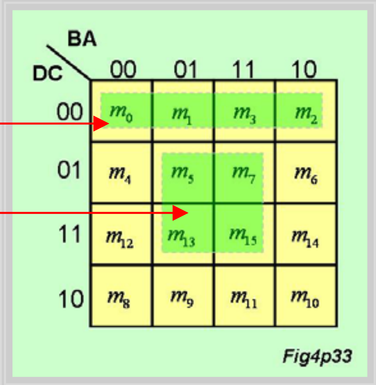
C is eliminated



Eliminates two variables

A and B are eliminated

D and B are eliminated



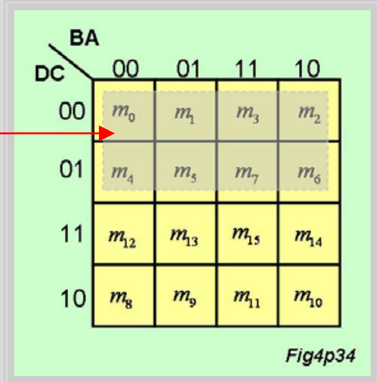
Simplification

Guidelines for Simplifying Functions Using K-maps

Five important points to keep in mind.

Eliminates three variables

A , B and C are eliminated



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Simplification

Guidelines for Simplifying Functions Using K-maps

Five important points to keep in mind.

3. Group as many squares together as possible. The larger the group is, the fewer the number of literals in the resulting product term.

4. Make as few groups as possible to cover all the minterms of function. The fewer the groups, the fewer the number of product terms in the minimized function. Stop, when all minterms are used at least once. The minterm is *covered* when it is used at least once in groups.

5. Start grouping with those minterms which has fewest number of logically adjacent minterms.

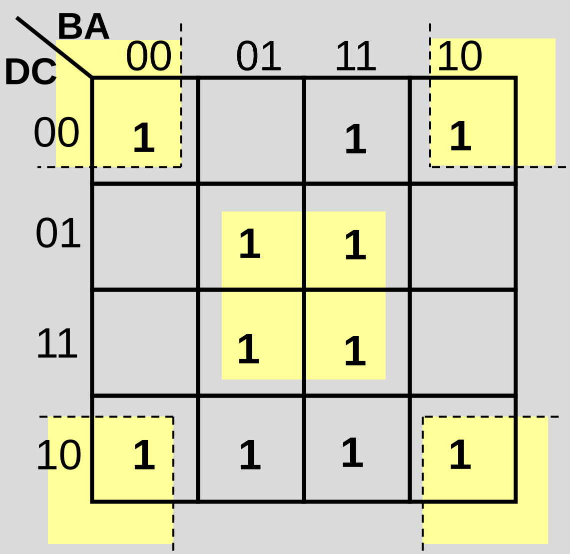
[2]

Simplification

We must ensure that all the minterms of the function are covered when we combine squares. There are no minterms already covered by other terms.

A *prime implicant* is a product term obtained by combining the maximum possible number of adjacent squares in the map.

If *minterm* in a square is *covered by only one prime implicant*, that prime implicant is said to be *essential*.



Essential Prime Implicants

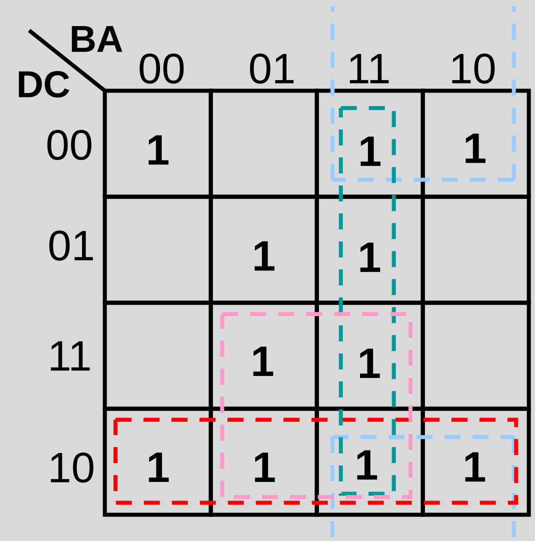
$C \cdot A$ $\quad \overline{C} \cdot \overline{A}$

Prime Implicants

$D \cdot \overline{C}$ $\quad D \cdot A$ $\quad B \cdot A$ $\quad \overline{C} \cdot B$

$$\begin{aligned} F &= C \cdot A + \overline{C} \cdot \overline{A} + D \cdot \overline{C} + \overline{C} \cdot B \\ &= C \cdot A + \overline{C} \cdot \overline{A} + D \cdot \overline{C} + B \cdot A \\ &= C \cdot A + \overline{C} \cdot \overline{A} + D \cdot A + \overline{C} \cdot B \\ &= C \cdot A + \overline{C} \cdot \overline{A} + D \cdot A + B \cdot A \end{aligned}$$

Four possible standard forms (SOP)



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Simplification

Examples with K-maps

Example with Five-Variable Map

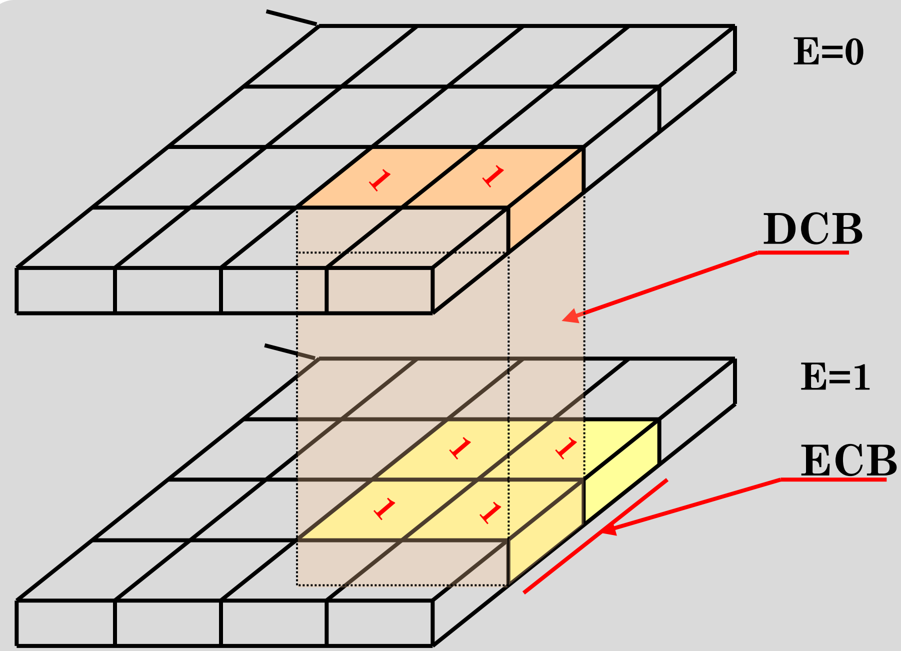
E=0

DC \ BA	00	01	11	10
00	m_0	m_1	m_3	m_2
01	m_4	m_5	m_7	m_6
11	m_{12}	m_{13}	m_{15} 1	m_{14} 1
10	m_8	m_9	m_{11}	m_{10}

E=1

DC \ BA	00	01	11	10
00	m_0	m_1	m_3	m_2
01	m_4	m_5	m_7 1	m_6 1
11	m_{12}	m_{13}	m_{15} 1	m_{14} 1
10	m_8	m_9	m_{11}	m_{10}

$F(E,D,C,B,A) = \sum(14,15,22,23,30,31)$ **Canonical form**



$F(E,D,C,B,A) = DCB + ECB$

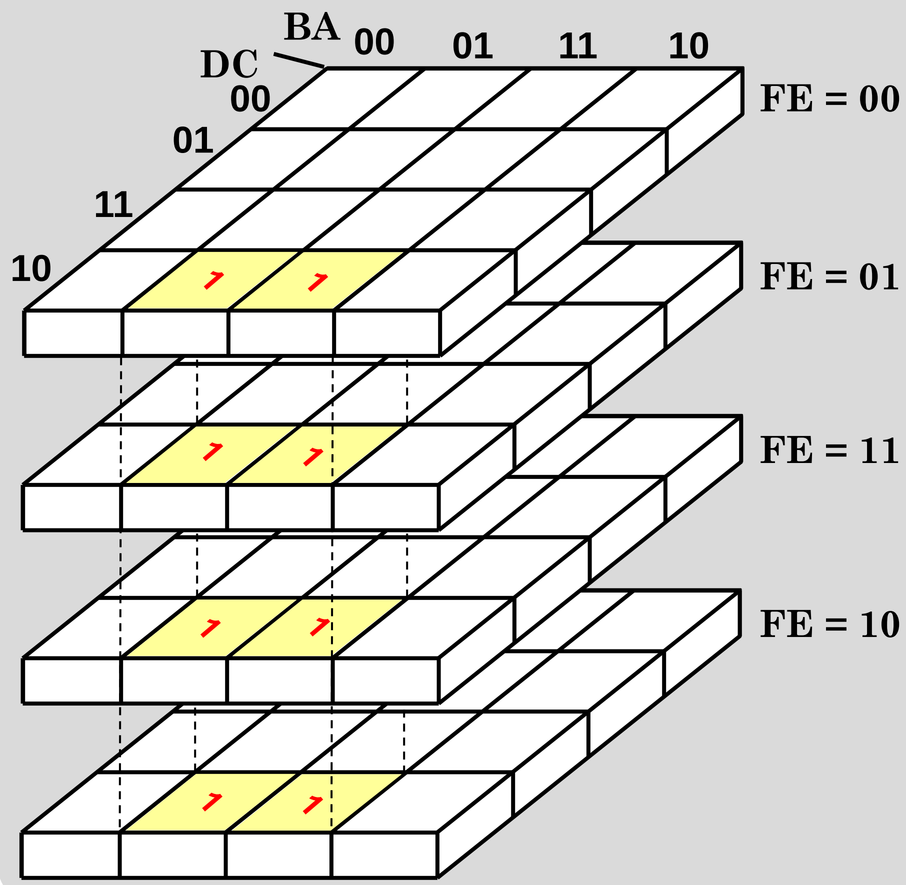
Minimized standard form (SOP)

Simplification

Examples with K-maps

Example with Six-Variable Map

$$f_5(F,E,D,C,B,A)=\sum m(9,11,25,27,41,43,57,59)$$



Simplified Boolean function f_5 :

$$F = D\overline{C}A$$

Minimized standard form (SOP)

Simplification

Examples with K-maps

$$F(D,C,B,A) = \sum m(0,1,2,5,8,9,10)$$
$$\prod M(3,4,6,7,11,12,13,14,15)$$

$\Rightarrow$

1's marked in the squares (minterms) : $m_0, m_1, m_2, m_5,$
m_8, m_9, m_{10}

0's marked in the squares (maxterms) : $M_3, M_4, M_6, M_7, M_{11},$
$M_{12}, M_{13}, M_{14}, M_{15}$

DC \ BA	00	01	11	10
00	m_0 1	m_1 1	m_3 0	m_2 1
01	m_4 0	m_5 1	m_7 0	m_6 0
11	m_{12} 0	m_{13} 0	m_{15} 0	m_{14} 0
10	m_8 1	m_9 1	m_{11} 0	m_{10} 1

If we take reduced maxterms, we obtain simplified function F.

If the squares marked with 0's are combined, (and take reduced minterms) we obtain the simplified *complemented* function: $\overline{F}$

$$\overline{F}(D,C,B,A) = B \cdot A + D \cdot C + C \cdot \overline{A}$$

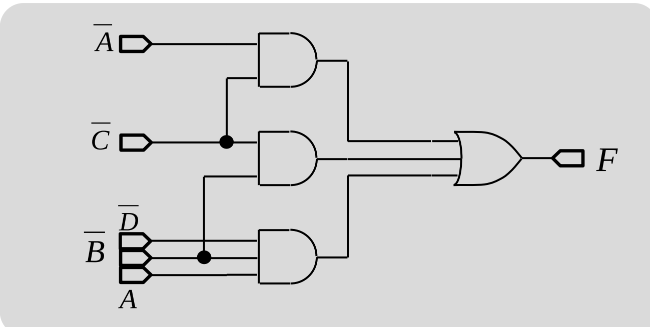
Applying DeMorgan's theorem we obtain simplified F in POS form.

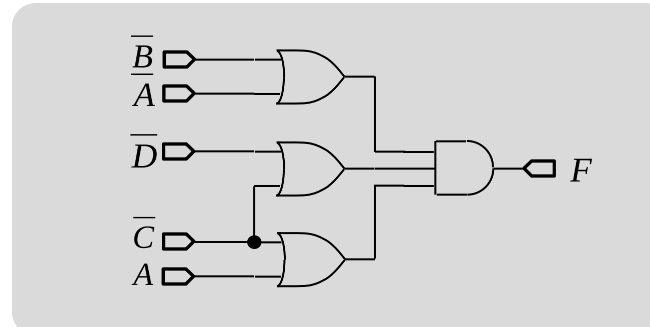
$$F = \overline{\overline{F}}(D,C,B,A) = \overline{B \cdot A + D \cdot C + C \cdot \overline{A}}$$
$$= \overline{(B \cdot A)} \cdot \overline{(D \cdot C)} \cdot \overline{(C \cdot \overline{A})}$$
$$= (\overline{B} + \overline{A}) \cdot (\overline{D} + \overline{C}) \cdot (\overline{C} + A)$$

Simplification

Examples with K-maps

Gate implementation of the function $F(D,C,B,A)=\sum(0,1,2,5,8,9,10)$





DC \ BA	00	01	11	10
00	m_0 1	m_1 1	m_3 0	m_2 1
01	m_4 0	m_5 1	m_7 0	m_6 0
11	m_{12} 0	m_{13} 0	m_{15} 0	m_{14} 0
10	m_8 1	m_9 1	m_{11} 0	m_{10} 1

$$F=\left(\overline{B}+\overline{A}\right)\cdot\left(\overline{D}+\overline{C}\right)\cdot\left(\overline{C}+A\right)$$

(standard POS form)

$$F=\overline{C}\cdot\overline{A}+\overline{C}\cdot\overline{B}+\overline{D}\cdot\overline{B}\cdot A$$

(standard SOP form)

DC \ BA	00	01	11	10
00	m_0 1	m_1 1	m_3 0	m_2 1
01	m_4 0	m_5 1	m_7 0	m_6 0
11	m_{12} 0	m_{13} 0	m_{15} 0	m_{14} 0
10	m_8 1	m_9 1	m_{11} 0	m_{10} 1

Simplification

Examples with K-maps

In practice, there are some applications where the function is not specified for certain combinations of the input variables.

For example, four-bit binary code for the decimal digits (0-9, P_0 - P_9).

Six unspecified outputs for some input (P_{10} - P_{15}) combinations.

These don't care conditions can be used on a map to provide further simplification of the Boolean function.

Mark the don't care minterm with X in the K-map.

$F = \overline{C}\cdot\overline{B} + \overline{D}\cdot\overline{B}\cdot A + D\cdot\overline{C}\cdot\overline{A}$ **(Simplified without X)**

(Simplified with X) $F = \overline{C}\cdot\overline{B} + \overline{D}\cdot\overline{B}\cdot A + \overline{C}\cdot\overline{A}$

DC \ BA	00	01	11	10
00	m_0: 1	m_1: 1	m_3: 0	m_2: X
01	m_4: 0	m_5: 1	m_7: 0	m_6: 0
11	m_{12}: 0	m_{13}: 0	m_{15}: 0	m_{14}: 0
10	m_8: 1	m_9: 1	m_{11}: 0	m_{10}: 1

The End