A Transformed Moody Chart that Is Read without Iterating

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Abstract:
Fluid flow problems are often solved using the Moody chart, a chart in which Darcy friction factor is plotted versus Reynolds number, with relative roughness as parameter. Moody's stated purpose in publishing the chart was to "*furnish the engineer with a simple means of estimating the friction factors to be used in computing the loss of head*"—ie to provide a simple way to estimate pressure drop (ΔP). For this purpose, the Moody chart is simple to use because it is read directly—ie without iterating.

The Moody chart is also used to estimate pipe diameter (D) and flow rate (W). For these purposes, the chart is *not* simple to use because it cannot be read directly. It must be read iteratively, or by trial-and-error.

In this article, a fluid flow chart is presented that is simple to use because it is read directly whether ΔP, W, or D is to be estimated. The new chart, Figure 1, is the result of transforming the Moody chart so that ΔP, W, and D are separated. The range of Figure 1 is Reynolds number from 10^4 to 10^8, and relative roughness from 0.00001 to 0.05. Over the range common to both charts, Figure 1 and the Moody chart are essentially *identical*—they differ only in form.

Introduction
The Moody [1] chart was presented at an ASME meeting in Pittsburgh in November, 1944. The paper was presented by Richard B. Willi on behalf of the author, Lewis W. Moody. Its title was "Friction Factors for Pipe Flow". The object of the paper is described in the first sentence:

> *The object of this paper is to furnish the engineer with a simple means of estimating the friction factors to be used in computing the loss of head* . . . .

For the stated purpose of estimating ΔP, the Moody chart yields precise results, and is simple to use. It allows ΔP to be estimated with a precision of 2 or 3%, and is read without iterating. (Moody [1] states that the chart allows ΔP to be estimated with an accuracy of ± 10%).

In addition to being used to estimate ΔP, the Moody chart is oftentimes used to estimate W or D. For these estimates, the Moody chart yields precise results, but it is not simple to use because it must be read by iterating, or by trial-and-error.

In this article, the Moody chart is transformed to a chart that is read without iterating, whether ΔP, D, or W is to be estimated.

Nomenclature

a	$\rho g \pi^2/8L$
b	$4/\pi\mu$
D	pipe diameter, ft.
f	Darcy friction factor
g	gravity constant, ft/sec^2
L	equivalent pipe length, ft
ΔP	pressure drop, psf
Re	Reynolds number
W	mass flow rate, pps
ε	absolute roughness, ft.
μ	viscosity, lbs/ftsec
ρ	density, lbs/ft^3

The reason the Moody chart must be read iteratively if W or D is to be estimated

The reason the Moody chart must be read iteratively if W or D is to be estimated can be seen by noting the following:

$$\text{y coordinate in Moody chart} = \text{friction factor } \alpha\ \Delta P D^5/W^2 \qquad (1)$$

$$\text{x coordinate in Moody chart} = \text{Reynolds number } \alpha\ W/D \qquad (2)$$

Expressions (1) and (2) indicate that x coordinates and y coordinates in the Moody chart depend on *both* W and D. Therefore, if *either* W or D is unknown and is to be estimated from the chart, neither the x nor the y coordinate can be calculated, in which case the chart must be read iteratively.

In short, the Moody chart must be read iteratively if W or D is to be estimated because both W and D appear in the x coordinate and in the y coordinate, and this makes it impossible to calculate the value of either coordinate if W or D is not known.

Separating the variables in the Moody chart

In order that a chart may be read directly to determine the variables of interest, the variables of interest must be separated—ie they must appear *singly* in the chart coordinates and in the chart parameter. For example, if x, y, and z are the chart variables of interest, the chart can be read directly for all variables of interest only if x, y, and z are separated—for example, if the chart is in the form y vs x with z as parameter. If two of the variables are combined in either coordinate or in the chart parameter, it is not possible to read the chart directly for all variables of interest.

In order that a fluid flow chart may be read directly whether ΔP, W, or D is to be estimated, it is necessary that ΔP, W, and D be separated. For example, ΔP, W, and D are separated if the y coordinate is dependent on W but is independent of ΔP and D, the x coordinate is dependent on D but is independent of ΔP and W, and the chart parameter is dependent on ΔP but is independent of W and D.

The Moody chart is readily transformed to a chart in which ΔP, W, and D are separated. The transformation is based on noting the following:

$$f = (\rho g \pi^2/8L)\,\Delta P D^5/W^2 = a\Delta P D^5/W^2 \qquad (3)$$

$$Re = (4/\pi\mu)W/D = bW/D \qquad (4)$$

$$f\,Re^2(\varepsilon/D)^3 = (a\Delta P D^5/W^2)(bW/D)^2(\varepsilon/D)^3 = ab^2\varepsilon^3\Delta P \qquad (5)$$

$$Re(D/\varepsilon) = (bW/D)(D/\varepsilon) = bW/\varepsilon \qquad (6)$$

Equation (5) indicates that $f\,Re^2(\varepsilon/D)^3$ is dependent on ΔP and independent of W and D. Equation (6) indicates that Re(D/ε) is dependent on W and independent of ΔP and D. Therefore Expressions (5) and (6) indicate that a chart in the form Re(D/ε) vs ε/D with parameter $fRe^2(\varepsilon/D)^3$ is in the form bW/ε vs ε/D with $ab^2\varepsilon^3\Delta P$ parameter. Since ΔP, W, and D are separated in a chart of this form, it is read directly whether ΔP, W, or D is to be estimated.

Figure 1, the result of transforming the Moody chart by separating the variables ΔP, W, and D

Figure 1 is the result of transforming the Moody chart by separating the variables ΔP, W, and D. The figure was generated by utilizing Expressions (5) and (6) to transform the Moody chart to a chart of log(Re(D/ε)) vs log(ε/D) with parameter $\log(0.5fRe^2(\varepsilon/D)^3)$—ie to a chart of log(4W/(πμε)) vs log(ε/D) with parameter $\log(\Delta P g\rho\varepsilon^3/(L\mu^2))$. The transformation was accomplished in the following steps:

1. Obtain f, Re, and ε/D coordinates for eleven curves in the Moody chart by reading the chart for relative roughness from .00001 to .05 at Reynolds numbers from 10^4 to 10^8.

2. Use the f, Re, and ε/D coordinates obtained in Step 1 to calculate coordinates of $0.5fRe^2(\varepsilon/D)^3$ and Re(D/ε).

3. Use the $0.5fRe^2(\varepsilon/D)^3$ and Re(D/ε) coordinates calculated in Step 2 to prepare Figure 1, a chart of log(Re(D/ε)) vs log(ε/D) with parameter $\log(0.5fRe^2(\varepsilon/D)^3)$. Note that Figure 1 is presented in terms of physical parameters rather than dimensionless parameters—it is labeled log(4W/(πμε)) vs log(ε/D) with parameter $\log(\Delta P g\rho\varepsilon^3/(L\mu^2))$.

(For illustration, the coordinates referred to in Steps 1 and 2 are listed in Table 1 for the curve in Figure 1 on which $\log(\Delta P g \rho \varepsilon^3/(L\mu^2)) = 2$.)

Note the following in Figure 1:

- The y coordinate is dependent on W, but is independent of ΔP and D.
- The x coordinate is dependent on D, but is independent of ΔP and W.
- The chart parameter is dependent on ΔP, but is independent of D and W.

Therefore, as desired, Figure 1 is read directly whether ΔP, D, or W is to be estimated.

It is important to note that, over the range of Reynolds number and relative roughness common to both charts, the Moody chart and Figure 1 are essentially *identical*—they differ only in form. The Moody chart is in the form of combined variables, and Figure 1 is in the form of separated variables.

The Moody chart vs Figure 1

It should be noted that the Moody chart describes both the laminar and turbulent ranges, whereas Figure 1 describes only the turbulent range. However, since laminar flow is described by a very simple equation, there is no need to describe it graphically.

A positive aspect of the Moody chart is that even a small chart can be read with good precision because the range of the y coordinate is quite small. The range is small because the y coordinate is a weak function of both the x coordinate and the chart parameter.

A negative aspect of the Moody chart is that W and D are combined in the coordinates, and this makes it necessary to read the chart iteratively if W or D is to be estimated.

A positive aspect of Figure 1 is that ΔP, W, and D are separated, and this makes it possible to read the chart without iterating, whether ΔP, W, or D is to be estimated.

A negative aspect of Figure 1 is that the y coordinate is a strong function of both the x coordinate and the chart parameter. Therefore the range of the y coordinate is large relative to the range of the y coordinate in the Moody chart. Because of the difference in range, a Moody chart of a given size can be read with better precision than a Figure 1 chart of the same size.

The end result is that:

- Figure 1 is better for estimating W and D because it is reasonably precise and can be read without iterating, whereas the Moody chart must be read by iterating.
- The Moody chart is better for estimating ΔP because it also can be read without iterating, and it is more precise than a Figure 1 chart of equal size.

The effect of imprecision in reading Figure 1 coordinates

It is estimated that, due to imprecision in reading Figure 1 coordinates, the results will usually be imprecise by one-half of a minor division or less. One-half of a minor division has the following impact:

Result	Imprecision
$\text{Log}(\varepsilon/D)$	.01
D	2.3%
$\text{Log}(4W/(\pi\mu\varepsilon))$	.025
W	6%
$\log(\Delta P g \rho \varepsilon^3/L\mu^2)$	.05
ΔP	12%

Example problem

A pipe line is to be laid to transport fluid from Plant A to Plant B. Estimate the pipe diameter that will result in a flow rate of 5 pps with a pressure drop of 4000 psf.

Given:

Pipe roughness ε = .0015 ft.
Equivalent length of pipe line = 200 ft
μ = .00050 lbs/ft sec
ρ = 60 lbs/ft^3

Analysis using the Moody chart in Appendix 1:

(To be completed by the reader):

Reader's answer obtained using the Moody chart in Appendix 1:

A pipe diameter of______ **ft.** will result in a flow rate of 5 pps with a pressure drop of 4000 psf.

Analysis using Figure 1:

- Use the given information to calculate the values of the y coordinate and the chart parameter:

$$\log(4W/(\pi\mu\varepsilon)) = \log(4x5/(\pi x.0005x.00015) = 7.93 \quad (7)$$

$$\log(\Delta Pg\rho\varepsilon^3/(L\mu^2)) = \log(4000x32.2x60x.00015^3/(200x.0005^2)) = -.28 \quad (8)$$

- Use the above values and Figure 1 to determine the value of the x coordinate. Then solve for the value of D.

$$\log(\varepsilon/D) = -2.85$$

$$\therefore (\varepsilon/D) = .00141$$

$$D = \varepsilon/(\varepsilon/D) = .00015/.00141 = .106 \text{ ft.}$$

Answer

A pipe diameter of **.106 ft.** will result in a flow rate of 5 pps with a pressure drop of 4000 psf.

(The Moody chart solution obtained by iterating is D = .104 ft., f = .0232, Re = $1.22x10^5$, and ε/D = .00144.)

Conclusions

- Figure 1 is better for estimating W and D because it is reasonably precise and can be read without iterating, whereas the Moody chart must be read by iterating.
- The Moody chart is better for estimating ΔP because it also can be read without iterating, and it is more precise than a Figure 1 chart of equal size. However, the precision of ΔP estimates obtained using Figure 1 will oftentimes be acceptable.

Acknowledgement

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References

[1] Moody, L.F. (1944), "Friction Factors for Pipe Flow", Trans. ASME, Nov., pp 671-684

Table 1 Background for the Figure 1 Curve on Which $Log(\Delta P g \rho \varepsilon^3/(L\mu^2)) = 2$

Read from Moody chart			Plotted in Figure 1		
ε/D	Re	f	$Log(\varepsilon/D)$	$Log(Re/(\varepsilon/D))$ $Log(4W/(\pi\mu\varepsilon))$	$Log(0.5fRe^2(\varepsilon/D)^3)$ $Log(\Delta P g \rho \varepsilon^3/(L\mu^2))$
0.03	1.11E+04	0.0600	-1.523	5.568	2.00
0.015	3.63E+04	0.045	-1.824	6.384	2.00
0.008	1.04E+05	0.0359	-2.097	7.114	2.00
0.004	3.29E+05	0.0288	-2.398	7.915	2.00
0.002	1.03E+06	0.0236	-2.699	8.712	2.00
0.001	3.18E+06	0.0197	-3.000	9.502	2.00
0.0006	7.30E+06	0.0175	-3.222	10.085	2.00
0.0002	4.27E+07	0.0137	-3.699	11.329	2.00

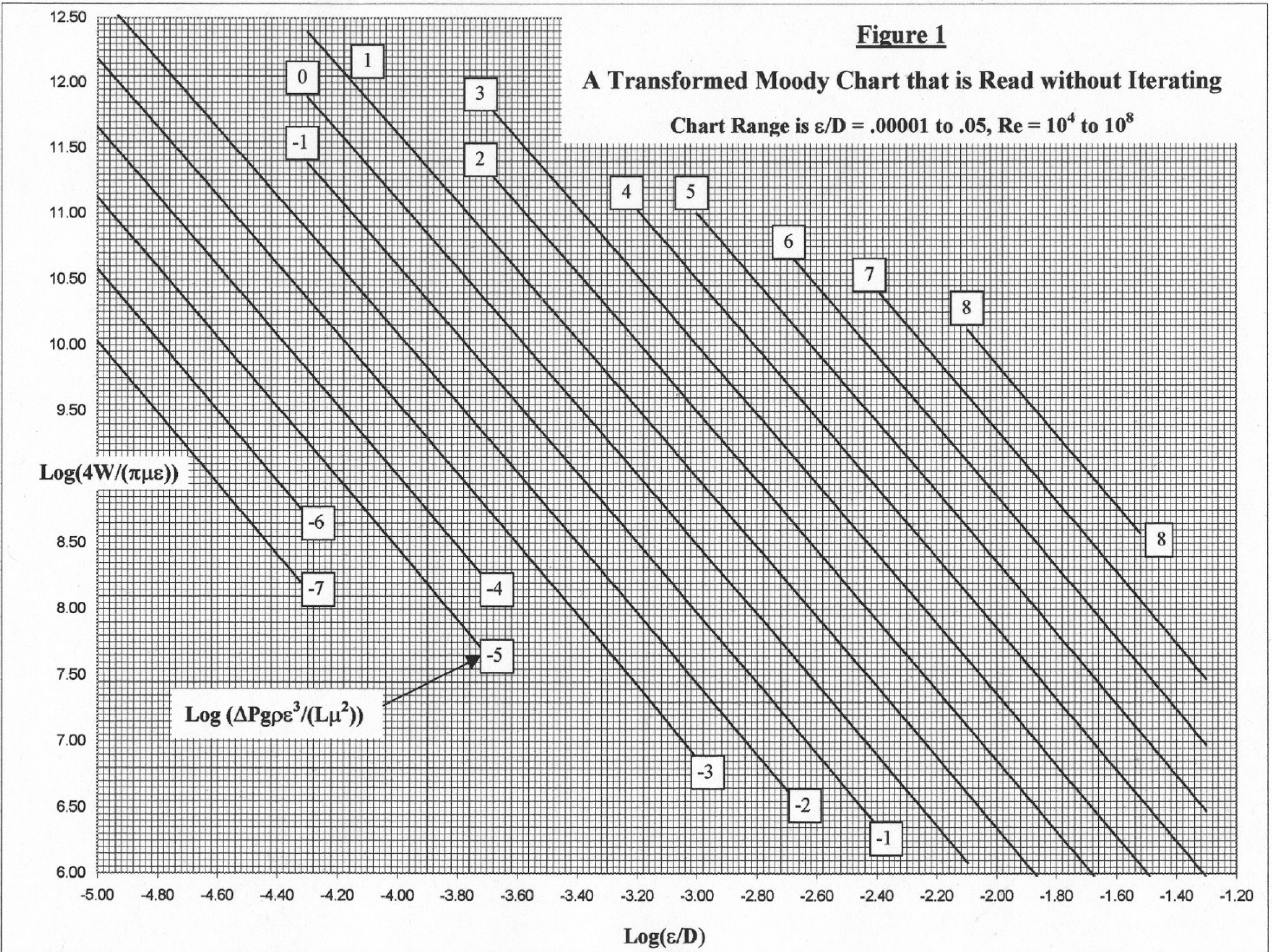
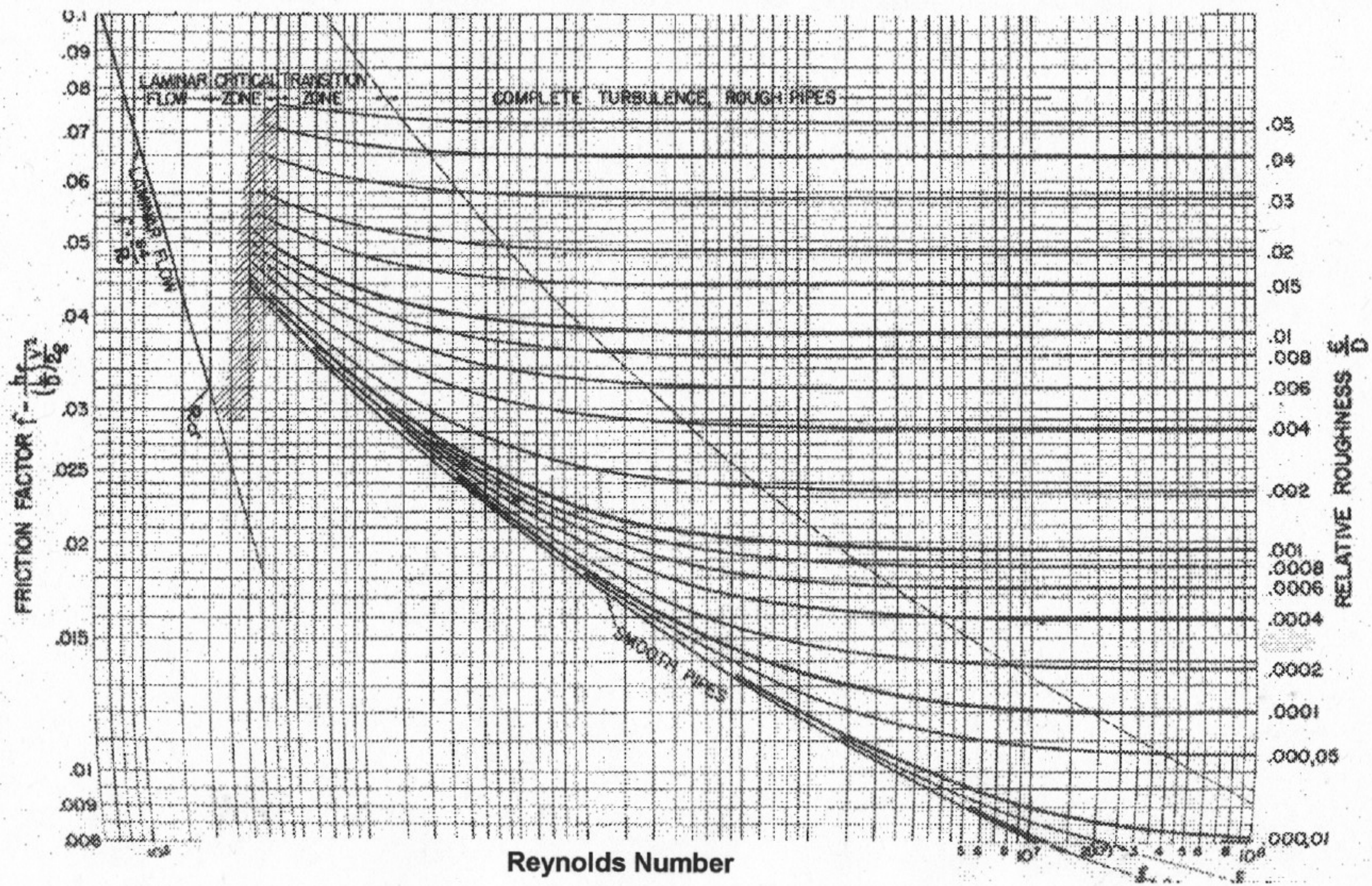


Figure 1
A Transformed Moody Chart that is Read without Iterating
Chart Range is ε/D = .00001 to .05, Re = 10^4 to 10^8
Log(4W/(πμε))
Log (ΔPgρε^3/(Lμ^2))
Log(ε/D)
12.50
12.00
11.50
11.00
10.50
10.00
9.50
8.50
8.00
7.50
7.00
6.50
6.00
-5.00
-4.80
-4.60
-4.40
-4.20
-4.00
-3.80
-3.60
-3.40
-3.20
-3.00
-2.80
-2.60
-2.40
-2.20
-2.00
-1.80
-1.60
-1.40
-1.20
0
1
3
-1
2
4
5
6
7
8
-6
-7
-4
-5
-3
-2
-1
8

<u>Appendix 1</u> Moody [1] chart



Reynolds Number